Minimum optical depth multi-port interferometers for approximating any unitary transformation and any pure state

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Reconfigurable devices capable to implement any unitary operation with a given fidelity are crucial for photonic universal quantum computation, optical neural networks, and boson sampling. Here, we address the problems of approximating with a given infidelity any unitary operation and any pure state using multi-port interferometers, which are of current interest due to the recent availability of multi-core fiber integrated multi-port interferometers. We show that any pure state, in any dimension \(d\), can be prepared with infidelity \(\leq 10^{-15}\) with 3 layers of \(d\)-dimensional Fourier transforms and 3 layers of configurable phase shifters. In contrast, the schemes in [Phys. Rev. Lett. 73, 58 (1994) and Optica 3, 1460 (2016)], require optical depth \(2(d-1)\). We also present numerical evidence that \(d+1\) layers of \(d\)-dimensional Fourier transforms and \(d+2\) layers of configurable phase shifters can produce any unitary with infidelity \(\leq 10^{-14}\), while the scheme in [Phys. Rev. Lett. 124, 010501 (2020)] only achieves an infidelity in the order of \(10^{-7}\) for block-diagonal unitary transformations.

I. INTRODUCTION

A programmable universal multi-port array (PUMA) [1–3] is an interferometer composed of multiple beam splitters (BSs) and phase shifters (PSs), which can be reconfigured to implement any unitary operation in a \(d\)-dimensional complex Hilbert space. Together with single-photon sources and detectors, PUMAs allow for preparing any quantum state, implementing any quantum logic gate, and any arbitrary quantum measurement. Therefore, PUMAs are of fundamental interest for universal quantum computation with photons [4–7], optical neural networks for machine learning [8, 9], boson sampling [10], and the generation of higher-dimensional entanglement [11].

A well known example of a PUMA is the scheme of Reck et al. [1], consisting of a regular arrangement of \(d(d-1)\) 50 : 50 BSs and \(d^2\) tunable PSs aligned in \(2(2d-3)\) layers, as illustrated in Fig. 1(a). Integrated photonics [12] allowed implementing this scheme up to \(d = 6\) [13].

The number of layers in the interferometric array, or equivalently, the maximum number of BSs that a photon must pass through, is known as the optical depth \(N\). The performance of multi-port arrays is reduced by optical loss, which is directly proportional to \(N\). Therefore, a smaller \(N\) implies higher quality of the implemented unitary. Clements et al. [2] noticed that the BSs and PSs can be rearranged in a configuration with \(N = 2d\), shown in Fig. 1(b), which has lower depth than the scheme of Reck et al. and therefore, is more robust to noise. This scheme has been implemented up to \(d = 8\) [14].

Recent advances on multi-core optical fiber fabrication technology [15–19] have made it possible the realization of high-quality \(d \times d\) multi-core fiber multi-port BSs. For \(d = 4\) and \(d = 7\) fidelities of 0.995±0.003 and 0.992±0.008, respectively, have been achieved [17] with commercially available multi-core fibers. In addition, multi-core fibers and related technologies have been used for multidimensional entanglement generation [20], and demonstrating the computational advantage from the quantum superposition of multiple temporal orders for higher dimensions [21]. In the particular case of \(d = 4\) the generated multi-port BS corresponds to a Hadamard matrix. In parallel, the availability of multi-port BSs of \(d\) ports has stimulated the search for PUMAs with small optical depth based on sequences

\[
V = P_N T_d P_{N-1} T_d \cdots P_1 T_d P_0, \tag{1}
\]

where \(T_d\) is the transfer matrix of the multi-port beam splitter and \(P_j\) corresponds to a layer of PSs introducing
a different phase in each mode, that is,
\[
P_j = \text{diag}(e^{i\phi_{j1}}, e^{i\phi_{j2}}, e^{i\phi_{j3}}, \ldots, e^{i\phi_{jd}}),
\]
with \(\phi_{ji} = 0\) for \(j \leq N\). Tang et al. [22] numerically showed that, with a particular \(T_d\) introduced in [23] and \(N > d\), “[any] desired unitary matrix was generated with mean square error smaller than \(-20\) \text{dB for all tested cases}.” Zhou et al. [24] pointed out that when \(T_d\) is the Fourier transform (FT) matrix in dimension \(d\) (implemented by a “canonical multi-port beam splitter” [19]),
\[
F_d = \frac{1}{\sqrt{d}} \sum_{j,k=1}^{d} e^{2\pi i(j-1)(k-1)/d} |j\rangle \langle k|,
\]
then a sufficiently large \(N\) can approximate any arbitrary unitary in dimension \(d\). The question, in both cases, is which is the minimum \(N\) needed to obtain universality.

Recently, it has been proven that universality can be achieved with \(6d + 1\) phase layers and \(6d\) Fouriers [3]. See Fig. 1(c). However, this design has larger depth than previous configurations \(N = 6d\), and then is less robust against imperfections.

Also recently, a very compact scheme has been proposed by Saygin et al. [25] consisting on a sequence of \(d+1\) layers of PSs and \(d\) layers of a randomly chosen unitary transformation \(T_d\) (i.e., with \(N = d\)). See Fig. 1(d). This scheme has two important properties: its number of PSs agrees with the number of parameters of a general unitary transformation, and it is universal and error insensitive [25]. This last property was demonstrated using numerical simulations in \(d \geq 10\). In contrast to previous PUMAs [1–3], which can be programmed following an algorithm, the scheme in [25] requires to solve a global optimization problem in order to derive the best settings for the PSs.

Non-universal multiport arrays also can find applications in particular tasks. In these cases, the multiports have lower depth, are more robust against noise [26], and require less tunable phases. An example is the preparation of arbitrary pure states in a given dimension. For that aim it is enough to use a subset of the scheme of Reck et al., as shown in Fig. 1(f), which has optical depth of \(2(d-1)\). The same subset of operations is also contained in the design by Clements et al.

In this article, we address the problems of approximating with a given fidelity any unitary transformation and any pure state using devices composed of several FT and PS layers. We call a multiport array with these characteristics as pseudo-universal. Our main result is that the scheme with \(d+1\) layers of \(d\)-dimensional Fourier transforms and \(d+2\) layers of configurable phase shifts shown in Fig. 1(e) can produce any unitary transformation with infidelity \(\leq 10^{-14}\). This result improves the scheme based on \(d+1\) layers of PSs and \(d\) layers of a randomly chosen unitary transformation \(T_d\) [25], which according to our numerical experiments does not exhibit pseudo-universality in dimension \(d \leq 10\). In particular, this scheme generates block-diagonal unitary transformations with infidelity in the order of \(10^{-7}\). Then, we show that an array with just 3 layers of \(d\)-dimensional FTs and 3 layers of configurable PS, as shown in Fig. 1(h), can prepare any pure state with infidelity in the order of \(10^{-15}\) in any dimension from \(d \leq 10\). A similar scheme with just 2 layers of FTs and PSs only achieves infidelity in the order of \(10^{-7}\).

II. GENERAL CONSIDERATIONS

We define the programmable pseudo-universal multiport arrays (PPUMAs) as programmable multi-port arrays that allow implementing any pure state or any unitary transformation with infidelity of at least order \(O(\epsilon)\). The infidelity between two pure states is given by
\[
I(|\psi\rangle, |\phi\rangle) = 1 - |\langle \psi|\phi \rangle|^2.
\]
Analogously, the infidelity between two unitary transformations is given by
\[
I(U, V) = 1 - \frac{1}{d^2} |\text{tr}(U^\dagger V)|^2.
\]

The definition of pseudo-universality establish a hierarchy in PPUMAs, since PPUMAs with lower \(\epsilon\) are better candidates for true universality. We focus on the lower dimensional regime \((d = 3, \ldots, 10)\), since it is still possible to carry out extensive numerical simulations covering a substantial part of the pure state and unitary transformation spaces.

In order to study the pseudo-universality of multiport arrays, we randomly generate, according to a Haar-uniform distribution, thousands of elements from the space of pure states and from the space of unitary transformations. Afterward, we minimize the relevant infidelity to find the angles of the PSs in Eqs. (1) and (2) that best approximate each element generated.

Since the infidelities \(I(|\psi\rangle, |\phi\rangle)\) and \(I(U, V(\phi))\) involve trigonometric functions, they have many local minima, so a global optimization method has to be used. We adopt a multi-starting strategy where optimization for a given element (pure state or unitary transformation) is executed for a large set of different initial conditions in order to find multiple local minimal. The best local minimum is chosen as the optimal solution. The optimization problem is solved via Julia Optim [27].

III. GENERATION OF ARBITRARY UNITARY TRANSFORMATIONS

Here, we address the problem of generating arbitrary unitary transformations. For that, we compare the performance of three schemes which are candidates to be PPUMA: (i) \(d+1\) layers of PSs and \(d\) layers of FTs [25], (ii) \(d+2\) layers of PSs and \(d+1\) layers of FTs, and (iii) \(d+3\) layers of PSs and \(d+2\) layers of FTs.
We explore all dimensions from \( d = 3 \) to \( d = 10 \). For each arrangement in each dimension, we try \( 10^3 \) unitary matrices from a Haar-uniform distribution. In all dimensions, 30 randomly generate initial conditions are employed. The results of the optimization are shown in Fig. 2, where histograms for the best-achieved infidelity (in logarithmic scale) are exhibited. We can see that, in \( d = 3 \) and 4, the use of \( d + 1 \) layers of PSs and \( d \) layers of FTs leads to approximately a \( 5\% \) of matrices which cannot be constructed with high fidelity. These are constructed with an infidelity of order \( O(10^{-2}) \). This can be due to two reasons: either the configuration is a PPUMA with error \( O(10^{-2}) \), or the optimization algorithm was unable to find the global minimum. To rule out this last possibility, this \( 5\% \) of matrices is optimized again with a larger set of initial conditions. However, no significative reduction in infidelity is achieved. Besides, numerical simulations are also implemented using Matlab GlobalSearch and Python BasinHopping. These maintain the main features exhibited in Fig. 2 but an overall increase in the infidelity for the three tests. This indicates the existence of unitary transformations that cannot be accurately implemented by means of \( d + 1 \) layers of PSs and \( d \) layers of FTs, and moreover, that this configurations is a PPUMA \( O(10^{-2}) \) and not completely universal for \( d = 3 \) and \( d = 4 \). Interestingly, it can be seen that for \( d + 1 \) and \( d + 2 \) layers of FTs, the implementation of the same matrices is always done with better infidelity than the one achieved by means of \( d \) layers, and that all schemes are PPUMA \( O(10^{-13}) \) in high dimensions.

Randomly chosen unitary matrices according to the Haar distribution uniformly represent the space of unitaries. However, there are matrices that will never appear in the sample since they span a subspace of null measure. To check the universality of the scheme, we look for this type of subsets whose elements cannot be constructed with high fidelity. Specifically, we consider matrices of the form

\[
U = U_{d_1} \oplus U_{d_2} = \begin{pmatrix} U_{d_1} & 0 \\ 0 & U_{d_2} \end{pmatrix},
\]

(6)

where \( U_{d_i} \) is a unitary matrix of size \( d_i \). We choose \( d_j = d/2 \) for \( d \) even and \( d_1 = (d-1)/2 \) and \( d_2 = (d+1)/2 \) for \( d \) odd. Fig. 3 presents histograms of the infidelity (in logarithmic scale) that result from optimizing \( 10^3 \) randomly generated block-diagonal unitary transformations of the form (6) from a Haar-uniform distribution with \( d = 3, \ldots, 10 \). The optimization is done as explained before. In the case of \( d \) layers of FTs, the histograms are spread along two to four orders of magnitude of the infi-
delity and achieve values that are at least two orders of magnitude worse than the case of $d+1$ layers of FTs. Particularly, for $d = 3, 4$ the infidelities are order $O(10^{-2})$, while for $d = 5$ to $d = 10$ they are at most $O(10^{-5})$. These last results do not coincide with those obtained on unitary matrices in Haar distribution, where all were reconstructed with infidelities of order $O(10^{-13})$. On the other hand, infidelities obtained with $d + 1$ FTs are concentrated in a narrow interval around infidelities of order $O(10^{-13})$. Furthermore, the histograms obtained with $d + 1$ layers in Figs. 2 and 3 are very similar.

All of this indicates that the configuration based on $d + 1$ layers of PSs and $d$ layers of FTs is a PPUMA $O(10^{-5})$, and that supplementing it with an additional layer allows us to implement a PPUMA $O(10^{-14})$. Clearly, the addition of an extra layer of FT and PS leads to a large increase in accuracy of the implemented unitary transformation for block-diagonal unitary transformations. This leads to the question of whether adding more layers could lead to even better accuracy. To examine this, we carry out a third test adding an extra layer of FTs and PSs. Simulations with this later configuration do not exhibit a significant increase in accuracy with respect to the case of $d + 1$ layers of FTs. Therefore, we have that the scheme with $d + 1$ layers of FT and $d + 2$ layers of PS is the PPUMA with lower depth.

Since the scheme with $d + 1$ layer of FTs has $d − 1$ more phases than is needed to characterize a unitary transformation in $U(d)$, we try three strategies to reduce the number of phases: (i) deleting a randomly chosen phase, (ii) fixing $d − 1$ phases at the inner layers, and (iii) fixing $d − 1$ phases of the last layer. In all cases, simulations show that each intervention conveys the decrease of the pseudo-universality.

IV. GENERATION OF ARBITRARY PURE STATES

Pure states in a $d$-dimensional Hilbert space are defined by $2d − 2$ independent real parameters. This indicates that a configuration with lower numbers of layers than a PPUMA for unitary matrices could be sufficient to generate any pure state in any dimension, being the configuration with 2 layers of FTs, which has exactly $2d − 2$ tunable phases, the smallest possible candidate, as is shown in Fig. 1(g). To test this hypothesis, we have consider three different candidates to PPUMA, with 2, 3 and 4 layers of FTs and PSs. A given target state $|\phi\rangle$ is generated applying the unitary transformation $U$ onto a fixed state $|1\rangle$, where $U$ corresponds to the action of the different layers of FTs and PSs. The infidelity $I(U|\phi\rangle)$ is minimized with respect to the phases introduced by the layers of PSs. For each dimension, a set of $5 \times 10^3$ pure states Haar-uniform distributed was generated and each of them was reconstructed 20 times to avoid local minimums.

The result of the optimization procedure for 2, 3, and
4 layers is summarized in Fig. 4, which correspond to an infidelity histogram. As is apparent from this figure, the configuration with 2 layers leads to two well defined populations: a set of states that, in the best case, are generated with an infidelity in the order $O(10^{-5})$ and set of states generated with very low infidelities in the order of $O(10^{-15})$. These two sets are present across all inspected dimensions. The size of set of high infidelity states slowly decreases with the dimension. To rule out the possibility of failure of the optimization algorithm on the states of the later set, a second optimization attempt was carried out. This time, different initial guesses were employed and other two optimization algorithms, Matlab GlobalSearch and Python Basin-Hopping, were used. In spite of this, our main findings remain unchanged. This strongly suggests the existence of pure states that cannot be accurately generated with a configuration based on two layers of FTs and two layers of PSs. This result can be greatly improved, as Fig. 4 shows, by increasing the number of layers from 2 to 3. In this case, all states were generated with an infidelity in the order of $O(10^{-15})$, which suggests that 3 layers form a PPUMA $O(10^{-15})$ for the generation of states. This result raises the question whether a further increase in the number of layers can lead to a reduction in infidelity. To study this, we performed numerical simulations with 4 layers of FTs and PSs. These, however, also lead to infidelities in the order of $O(10^{-15})$ for all simulated states. This shows that the PPUMA based on 3 layers could provide the configuration with the smallest optical depth to generate pure states and, consequently, with the highest robustness to errors.

The optical depth of our proposal compares favourably with other schemes. In particular, the design by Reck et al. generates any pure state employing a Mach-Zehnder interferometer between neighboring paths and a final layer of PSs, which corresponds to the first diagonal in Fig. 1(f). The same array of BSs and PSs generates any arbitrary state in Clemens et al. configuration, which corresponds to the main anti-diagonal in Fig. 1(b). In both designs, a pure state is given as a lineal combination of the bases states where the probability amplitudes are given by complex phases multiplied by hyperspherical coordinates. This leads to an optical depth of $2(d - 1)$, which is greater than the optical depth of 3 achieved by our proposal.

V. CONCLUSIONS

For photonic universal quantum computation and many applications, it is of fundamental importance to identify the PUMA with the smallest optical depth. In this article, we have advanced in the solution of this problem by considering multiport arrays based on Fourier transform through several Monte Carlo numerical simulations for $d = 3, \ldots, 10$. We show that an array composed of $d + 1$ layers of FTs and $d + 2$ layers of PSs is a PPUMA that generates arbitrary unitary transformations with an infidelity of at least order $O(10^{-14})$. This
configuration improves the infidelity achieved using $d$ layers of FTs and $d + 1$ layers of PSs, which achieves in $d = 3, 4$ an infidelity in the order of $O(10^{-2})$ and for block diagonal unitary transformations an infidelity of the order of $O(10^{-7})$. Furthermore, we show that increasing the number of FT layers to $d + 2$ does not result in any further improvement in infidelity. All of this points out that the best candidate to be the PPUMA with minimal optical depth may be the one with $d + 1$ layers of FTs and $d + 2$ layers of PSs shown in Fig. 1(e). However, the problem of the PUMA with the minimum depth remains open. While, for some purposes, the scheme in [25] may be good enough, the configuration with $d + 1$ layers delivers much better accuracy for general purposes.

We have also studied the problem of generating pure states through multiport arrays based on layers of FTs and PSs. Since pure states have $2d - 2$ independent real parameters, a first guess is a configuration based on 2 layers of FTs and 2 layers of PSs. However, we show that this configuration generates a set of states, in the best case, with an infidelity in the order of $O(10^{-5})$. Another set of states is generated with very low infidelities in the order of $O(10^{-15})$. These two sets are present across all inspected dimensions. Increasing the number of layers to 3 we obtain a configuration that consistently generates all states with an infidelity in the order of $O(10^{-15})$. As in the case of unitary transformations, a further increase to 4 layers does not lead to a decrease in the infidelity. This shows that the PPUMA based on 3 layers could provide the configuration with the smallest optical depth and, consequently, with the highest robustness to errors.

As long as no universal PUMA from an analytical argument is found, it is important to continuously study numerically the problem, in order to find new sets of unitary matrices that are implemented with low infidelity with the tested configurations, and thus fine-tune the best candidate to be universal.

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