Lorentz-preserving fields in Lorentz-violating theories

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Abstract – We identify a fairly general class of field configurations (of spins 0, \(\frac{1}{2}\) and 1) which preserve Lorentz invariance in effective field theories of Lorentz violation characterized by a constant timelike vector. These fields concomitantly satisfy the equations of motion yielding cubic dispersion relations similar to those found earlier. They appear to have prospective applications in inflationary scenarios.

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Introduction. – Invariance under Lorentz transformation is known till date to be a global symmetry of the standard theory of elementary particles when gravitation is ignored. However, questions have been raised regarding the validity of this symmetry at small length scales owing to probable quantum gravity effects [1–37]. The natural mass scale of quantum gravity is the Planck mass \(M_{\text{Pl}}\).

Departures, suppressed by the Planck mass, from the standard special relativistic dispersion relation of free particles of mass \(m\) at large energies have been accepted as a signature of Lorentz invariance violation and has been the principal objet de l’attention of experimental and theoretical probes of Lorentz violation. These hypothesised ad hoc corrections due to Lorentz non-invariance must have their origin in new terms in the action of the system. Myers and Pospelov [38] have studied this issue within the framework of effective field theory involving fields of spins 0, \(\frac{1}{2}\) and 1, by incorporating into the action dimension five operators containing a constant timelike 4-vector \(n\) which ostensibly breaks Lorentz invariance. Choosing a Lorentz frame where \(n^\mu = (1, \vec{0})\), corrections of \(O(p^3)\) to the dispersion relation of each of the three fields have been obtained in [38] in the limit of relatively high energies \(E, M_{\text{Pl}} \gg m\).

For a complex scalar field this is given by

\[
\omega^2 \approx |\vec{p}|^2 + m^2 + \frac{\kappa}{M_{\text{Pl}}}|\vec{p}|^3. \tag{1}
\]

For the Maxwell field, the dispersion relation obtained takes the form (for circularly polarized photons) [22,38]

\[
\omega_{R,L}^2 \approx |\vec{p}|^2 \pm \frac{2\xi}{M_{\text{Pl}}}|\vec{p}|^3. \tag{2}
\]

In the case of a Dirac spinor one gets [38],

\[
\left[ \omega^2 - |\vec{p}|^2 - m^2 - \frac{2|\vec{p}|^3}{M_{\text{Pl}}}(\eta_1 + \eta_2\gamma_5) \right] \psi \approx 0,
\]

\[
\omega^2 - |\vec{p}|^2 - m^2 - \frac{2|\vec{p}|^3}{M_{\text{Pl}}}\eta_{R,L} \approx 0. \tag{3}
\]

In (3), the spinors have been chosen to be eigenstates of the chirality operator which is a valid assumption at high energies. \(\eta_{R,L} \equiv \eta_1 \pm \eta_2\).

Many experiments aimed at constraining the parameters \(\kappa, \xi, \eta_1, \eta_2\) quantifying Lorentz violation have been proposed in the past few years. Lorentz-violating effects scale with energy making astrophysical observations a perfect arena for detecting them. The simplest astrophysical observations that provide interesting constraints on lack of Lorentz symmetry at Planck scale measure the differences in arrival times of photons emitted simultaneously from distant sources of radiation like \(\gamma\)-ray bursts, active galactic nuclei and pulsars [4,8,9,12,20,27,29,33]. The authors of [23–25] found stringent limits on \(\xi\) by recording the timing of photons emitted during strong flares of the active galactic nucleus Markarian 501. The lowest-order corrections in the photon dispersion relation (2) also imply the birefringence of vacuum (different group velocities for different helicities of photons). In 2008, Maccione et al. [30] used polarimetric observations of hard X-ray from the Crab nebula to impose a bound on Lorentz
violation in quantum electrodynamics of $|\xi| < 9 \times 10^{-10}$ at 95% confidence level.

Complementary constraints have also been obtained from the threshold reactions of photon decay, fermion pair emission, synchrotron radiation, vacuum Cerenkov radiation and helicity changing decays. In [21], the authors analyzed synchrotron radiation from the Crab nebula to deduce $\eta > 7 \times 10^{-8}$. Observational details and their phenomenological consequences have been exhaustively discussed in [39–41].

It is clear that the deformed dispersion relation has been the object of extensive observational scrutiny of departure from Lorentz invariance. Does it unequivocally imply Lorentz violation? We first explore the possibility that Lorentz invariance violation? We first explore the possibility that Lorentz invariance violation near the Big Bang fluctuation of the inflaton field. We give an illustration of how Lorentz invariance violation near the Big Bang can be related to inhomogeneous Lorentz-preserving fields which in turn might give rise to structure formation. This scenario will be developed in the context of the Myers and Pospelov [38] model.

Lorentz invariance. –

Spin-0 fields. The action functional for a complex scalar field $\phi$ put forth in [38] is

$$ S = \int d^4x \mathcal{L}_{MP} \equiv \int d^4x \left[ |\partial \phi|^2 - m^2 |\phi|^2 \right] + \int d^4x \frac{i\kappa}{M_{Pl}} \phi^* \partial_{\mu} \phi \partial_{\mu} \phi. \quad (4) $$

with $\kappa$ being a real, dimensionless parameter and $\mathbf{n} \cdot \partial \equiv \partial_n$. $S_S$ and $S_{Vs}$ denote, respectively, the standard action for a complex scalar field and the new Lorentz-violating part. Under an infinitesimal Lorentz transformation, $\delta_{\alpha \beta} S_{Vs} = 0$ while

$$ \delta_{\alpha \beta} S_{Vs} = \int d^4x \phi^* n_\alpha \partial_{\beta} \partial_{\mu} \phi. \quad (5) $$

On the other hand, if the spacetime divergence of the Nöther current $\mathcal{J}$ corresponding to Lorentz transformations is computed, we get

$$ \partial_\mu \mathcal{J}_{\alpha \beta}^{\mu} = - \frac{\partial L_{MP}}{\partial (\partial_\nu \phi)} (\partial n_\nu)_{\alpha \beta} = n_\alpha \frac{\partial L_{MP}}{\partial (\partial_\nu \phi)} = \phi^* n_\alpha \partial_{\beta} \partial_{\mu} \phi. \quad (6) $$

If Lorentz transformations are symmetries of the system then we must simultaneously have $\delta_{\alpha \beta} S_{Vs} = 0$ and $\partial_\mu \mathcal{J}_{\alpha \beta}^{\mu} = 0 = n_\alpha \frac{\partial L_{MP}}{\partial (\partial_\nu \phi)}$. Requiring either of these yields the condition $n_\alpha \partial_{\beta} \partial_{\mu} \phi = 0$.

A possible non-trivial solution is, $\partial_\mu \phi = f(x \cdot n) = f(z)$ where $z \equiv x \cdot n$. It is convenient in flat spacetime to resolve the coordinate 4-vector along and orthogonal to $n$:

$$ x = x_0 + x_\perp, $$

where $n \cdot x_\perp = 0$ and $x_\perp = \frac{x \times n}{n \cdot n}$ $n = \frac{x}{n \cdot n}$. So, the derivative operator can be written as

$$ \partial = \partial_{\parallel} + \partial_{\perp}, $$

where for ease of notation $\partial_{\parallel} \equiv \partial x_0$, $\partial_{\perp} \equiv \partial x_\perp$. It is straightforward to show that $\partial_{\perp} \phi(x) = n^2 \partial_0 \phi(x_0, x_\perp)$ This implies in its turn

$$ \phi(x) = \phi_\parallel (x_0) + \phi_\perp (x_\perp) \quad (7) $$

where, $\phi_\parallel$ and $\phi_\perp$ are arbitrary functions of their arguments. Under a Lorentz transformation $x_0$ and $x_\perp$ will mix because they have not been defined in a Lorentz invariant fashion. Hence, the imprint of Lorentz violation introduced into the action is borne by the decomposition of the scalar field to ensure that the action retains Lorentz symmetry.

If $n$ is timelike, we can choose coordinates such that $x^0$ lies along $n$. Then our condition (7) implies that when the full scalar field is a linear combination of a time-dependent, spatially homogeneous piece and a static spatially inhomogeneous piece, the theory will possess Lorentz symmetry. As a matter of fact, these fields indeed provide a representation of Lorentz algebra as we hope to show in our future publication.

Maxwell (spin-1) field. In this case, the usual kinetic term of the free Maxwell field and a dimension five, $n_\text{d}$ dependent operator constitute the modified Lagrangian density proposed in [38],

$$ S = \int d^4x \left[ - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{\xi}{M_{Pl}} n^\mu F_{\mu \nu} n^\alpha \partial_\alpha n_\rho \bar{F}^{\rho \nu} \right], \quad (8) $$

where $\xi$ is a dimensionless parameter constraining Lorentz violation. For convenience, we define $n^\mu F_{\mu \nu} \equiv F_{\nu \mu}, n_\rho \bar{F}^{\rho \nu} = \bar{F}_{\nu \mu}$. Directly studying the variation of
the action or the divergence of the Nöther current of Lorentz transformation in parallel with the argument given in case of the scalar field, the condition for the theory to be Lorentz invariant is

\[ n_\alpha F_{\beta j} \partial_n F^{\alpha j} + F_{\nu n} n_\alpha \partial_\beta \tilde{F}^{\nu j} + F^{\nu j} \partial_n n_\alpha \tilde{F}_{\beta j} = 0. \]  

(9)

The last term is always zero because \( F^{\nu j} \partial_n n_\alpha \tilde{F}_{\beta j} = F^{\nu j} \partial_n n_\alpha \epsilon_{\beta j \lambda \alpha} \tilde{F}^{\lambda \alpha} \) contains a fifth rank completely antisymmetric tensor \((\epsilon^{\beta j \lambda \alpha})\) in four spacetime dimensions.

We transform to the Lorentz frame defined by \( n = (1, \vec{0}) \), to get a better physical picture of the problem in terms of electric and magnetic field 3-vectors identified as \( E_i = F_{0i} = \frac{1}{2} \varepsilon_{ijk} F_{jk} \), \( B_i = \tilde{F}_{0i} = \frac{1}{2} \varepsilon_{ijk} F_{jk} \), where we have used \( \varepsilon_{ijk} = \varepsilon_{ij} \). The condition for Lorentz invariance becomes

\[ \varepsilon_{ijm} \partial_0 B_j B_m - E_j \partial_0 B_j = 0, \]  

(10)

\[ \vec{B} \times \vec{B} - \nabla \vec{B} \cdot \vec{E} = 0. \]  

(11)

It is easy to see that if the fields are harmonic functions of spacetime as

\[ \vec{E} = \text{Re}( \vec{E}_0 \exp(-iwt + i\vec{k} \cdot \vec{x})) , \]  

\[ \vec{B} = \text{Re}( \vec{B}_0 \exp(-iwt + i\vec{k} \cdot \vec{x})) , \]  

they satisfy (11) when the relation \( \vec{E} \cdot \vec{B} = 0 \) (deduced from Bianchi identity) is incorporated. This ensures that it is possible to have Lorentz symmetric electromagnetic fields in the modified electrodynamics of [38].

**Spinor field.** In [38], the action describing a Dirac spinor has been modified to

\[ S = \int d^4 x \bar{\psi} \left[(i \gamma \cdot \partial - m) \psi + \frac{\gamma \cdot n (\eta_1 + \eta_2 \gamma_5) }{M_{Pl} } \partial_\alpha \psi \right] \]

\[ \equiv S_D + S_{V_\parallel} , \]  

(14)

where \( S_D \) is the standard Dirac action of a spinor field \( \psi \) and \( S_{V_\parallel} \) accounts for Lorentz violation. The dimensionless parameters \( \eta_1, \eta_2 \) give the measure of Lorentz violation.

The only source of Lorentz violation is, by assumption, the appearance of the constant 4-vector \( n \) in \( S_{V_\parallel} \). Thus, there are no constant vectors in the theory independent of \( n \). It is straightforward to show that, under an infinitesimal Lorentz transformation, the action changes by

\[ \delta_{\alpha \beta} S_{V_\parallel} = \frac{1}{M_{Pl} } \int d^4 x \bar{\psi} \left[ n_\alpha \gamma_\beta (\eta_1 + \eta_2 \gamma_5) \partial_\alpha \psi + \gamma \cdot n (\eta_1 + \eta_2 \gamma_5) n_\beta \partial_\alpha \psi \right] \]  

(15)

If we set \( \partial_n \psi = \chi (z) \), where \( z = x - n \), then the second term in (15) vanishes. After a partial integration (dropping the surface term), the first term reduces to

\[ \delta_{\alpha \beta} S_{V_\parallel} = - \frac{1}{M_{Pl} } \int d^4 x \left[ n_\eta n_\alpha \gamma_\beta \chi + n_\eta n_\alpha \bar{\chi} \gamma_5 \gamma_5 \chi \right] \]

\[ = - \frac{1}{M_{Pl} } \int d^4 x \left[ n_\eta n_\alpha J_\alpha (z) + n_\eta n_\alpha J^5_\alpha (z) \right] , \]  

(16)

where \( J_\alpha (z) \equiv \bar{\chi} \gamma_\alpha \chi, J^5_\alpha (z) \equiv \bar{\chi} \gamma_5 \gamma_\alpha \chi, \) etc.

Now, one can decompose the currents \( J \) and \( J^5 \) as \( J = (\frac{n \cdot \gamma}{M_{Pl} } ) n + J_\perp \) and \( J^5 = (\frac{n \cdot \gamma}{M_{Pl} } ) n + J^5_\perp \), where \( n \cdot J_\perp = 0, n \cdot J^5_\perp = 0 \). Inserting this decomposition into (16), it is clear that

\[ \delta_{\alpha \beta} S_{V_\parallel} = - \int d^4 x \left[ n_\eta n_\alpha J_{\perp \beta} (z) + n_\eta n_\alpha J^5_{\perp \beta} (z) \right] , \]  

(17)

so that Lorentz violation now depends on the current 4-vectors \( J_\perp \) and \( J^5_\perp \).

It should be noted, however, that these current 4-vectors are orthogonal to \( n \) and are constants in the direction they point! If, for example, \( n \) is timelike, the currents \( J_\perp \) and \( J^5_\perp \) must be spacelike and yet must be spatially homogeneous, being functions of \( z \). This makes them constant 4-vectors independent of \( n \). Since, by assumption there are no constant 4-vectors in the problem apart from \( n \), these currents must vanish.

As illustrated for the scalar field, requirement of Lorentz invariance of the action implies that

\[ \psi(x) = \bar{\psi}_\parallel (\chi_{\parallel}) + \psi_\perp (\chi_{\perp}) . \]  

(18)

In the preferred frame \( n = (1, \vec{0}) \), \( \psi_\parallel (\chi_{\parallel}) \) is a spatially homogeneous spinor, whereas the spinor \( \psi_\perp (\chi_{\perp}) \) is time independent.

Hence, the invariance of the action under infinitesimal Lorentz transformations in our case does indeed lead to non-trivial restrictions on the functional form of fields in that the fields decouple into two parts one of which will be function only of certain projections of the coordinate vector along the fixed vector \( n \), the other part being a function of projections of the coordinate vector orthogonal to \( n \). These constraints have not been imposed by hand: they are the most general solutions of the equations that result on requiring invariance under infinitesimal Lorentz transformations.

**Evaluation of dispersion relation.** Now that we have found quite general and non-trivial field configurations that make the modified scalar, vector and spinor actions of [38] Lorentz invariant, the next step will entail calculating the dispersion relations obeyed by these special fields.

**Scalar field.** The scalar field \( \phi(x) \) assumed to be given by (7) leads to the equation of motion \( (\Box + m^2) \phi = \frac{i}{M_{Pl} } \partial_\alpha \partial^\alpha \phi \) to be written as

\[ (\Box + m^2) \phi_\parallel = -(\Box^5 + m^2) \phi_\parallel + \frac{i \kappa}{M_{Pl} } \partial_\alpha \partial^\alpha \phi_\perp . \]  

(19)

Here, we have used the decomposition,

\[ \Box \phi = \Box^5_\parallel \phi_\parallel + \Box^5_\perp \phi_\perp \]

when \( n \) is a unit vector. It is obvious that to make sense of (19) we must set both sides to a constant which we choose to vanish for convenience. By taking the simple ansatz \( \phi_\parallel \sim \exp(-i \vec{k}_\parallel \cdot \vec{x}_\parallel) \) and \( \phi_\parallel \sim \exp(-i \vec{k}_\parallel \cdot \vec{x}_\parallel) \), it is easy to
see that the dispersion relations for the fields $\phi_\parallel$ and $\phi_\perp$ are, respectively,

$$E_\parallel^2 = |\vec{k}_\parallel|^2 + m^2 + \frac{\kappa}{M_{Pl}} (\vec{n} \cdot \vec{k}_\parallel)^3, \quad (20)$$

$$E_\perp^2 = |\vec{k}_\perp|^2 - m^2. \quad (21)$$

If we go to the inertial frame where $\vec{n} = (1,0)$ the dispersion relations take the simplified forms: $E_\parallel^2 = m^2 + \frac{\kappa}{M_{Pl}} |\vec{k}_\parallel|^3$ provided the four-momentum $\vec{k} = (E, \vec{k})$. One can now eliminate $m^2$ from these equations to get the dispersion relation of the complete scalar field $\phi(\vec{x})$ in the high-energy regime $\mathcal{E} \simeq |\vec{k}| \gg m$,

$$E^2 \simeq |\vec{k}|^2 + \frac{\kappa}{M_{Pl}} |\vec{k}|^3$$

(22)

which is same as the dispersion relation (1) computed in [38].

**Vector field.** The equation of motion obtained by the variation of the action (8) is (derived in [43]),

$$\partial_\mu F^{\mu\nu} + \frac{\xi}{M_{Pl}} (\eta_{\nu \sigma} \partial_\mu F_{\nu \sigma} - \partial_\mu F^{\nu \sigma} \eta_{\sigma \nu}) = 0.$$ \hspace{1cm} (23)

The above equation and the Bianchi identity $\partial_\mu F^{\mu \rho} = 0$ in the chosen reference frame are equivalent to the following equations:

$$\vec{\nabla} \cdot \vec{E} = 0 = \vec{\nabla} \cdot \vec{B}, \quad (24)$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}, \quad (25)$$

$$\vec{\nabla} \times \vec{B} = \frac{\xi}{M_{Pl}} \left( \vec{B} - \vec{\nabla} \times \vec{E} \right) = 0.$$ \hspace{1cm} (26)

These are the modified free Maxwell equations. If we take the curl of both sides of (26), simplify using the Bianchi identity (24), substitute the Li solution for the magnetic field and assume $\vec{k} = (\omega, 0, 0, k^3)$, the dispersion relations at high energy $\omega \simeq |\vec{k}|$ are

$$\omega_{R,L}^2 - |\vec{k}|^2 \simeq \pm \frac{2\xi}{M_{Pl}} |\vec{k}|^3.$$ \hspace{1cm} (27)

The plus and minus signs appear for right and left circularly polarised electromagnetic waves, respectively.

**Spinor field.** If we take the spacetime dependence of the fields in (18) to be $\psi_\parallel \sim \exp(-i\vec{k}_\parallel \cdot \vec{x}_j)$, $\psi_\perp \sim \exp(-i\vec{k}_\perp \cdot \vec{x}_j)$ and proceed as for the scalar field then the dispersion relations for the fields $\psi_\parallel$ and $\psi_\perp$ turn out, respectively, to be

$$\left[ E_\parallel^2 - |\vec{k}_\parallel|^2 - m^2 - \frac{2(\vec{n} \cdot \vec{k}_\parallel)^3}{M_{Pl}} (\eta_1 + \eta_2 \gamma_5) \right] \psi_\parallel = 0, \quad (28)$$

$$\left[ E_\perp^2 - |\vec{k}_\perp|^2 + m^2 \right] \psi_\perp = 0. \quad (29)$$

We are interested in high-energy phenomena and at sufficiently high energies the massive spinors can be treated as chirality operator eigenstates. Redefining $\eta_{R,L} \equiv \eta_1 \pm \eta_2$, the dispersion relations in the special Lorentz frame we have selected in earlier instances simplify to $E^2 - m^2 - \frac{2(\vec{n} \cdot \vec{k})^3}{M_{Pl}} |\vec{k}|^2 = 0$. Here, the four-momentum of field $\psi(\vec{x})$ is $\vec{k} = (E, \vec{k})$. We can combine these two equations and get the dispersion relation of $\psi(\vec{x})$:

$$E^2 - |\vec{k}|^2 - \frac{2\gamma_5}{M_{Pl}} \eta_{R,L} \simeq 0 \quad (30)$$

in the limit of high energy $\mathcal{E} \simeq |\vec{k}| \gg m$.

We have demonstrated invariance for transformations only close to the identity in the parameter space of the Lorentz group. However, in dealing with Lorentz transformations in field theory, one is invariably dealing with the simply-connected universal cover $SL(2,\mathbb{C})$ of the Lorentz group and so it is quite adequate to consider only the Lorentz Lie algebra and its action on the fields. Indeed, the Lorentz Lie algebra is realized on our special field configurations because the Noether current appropriate to the transformations is conserved and also the full action is invariant under infinitesimal transformations. This should guarantee invariance under the connected part of the Lorentz group which is of concern here provided we adhere to the special field configurations. In fact, this is sufficient to obtain the non-standard dispersion relations as well, as has been illustrated.

The subject of our next project is to explicitly exhibit the closure property of Lorentz algebra as realised on the Lorentz-preserving fields (manuscript under preparation). The first step towards this is to appropriately choose the generalised coordinates and construct the canonically conjugate momenta for the higher-derivative Myers-Pospelov model. It appears that one can set up a well-posed initial-value formulation of this theory with respect to our field solutions, thus eliminating the possibility of appearance of ghost states (auxiliary degrees of freedom that do not contribute towards the dynamics).

### Possible application in cosmology.

- The field configurations that we have obtained have aspects of intrinsic interest when one considers prospective application to cosmology as in inflationary scenarios. A peculiarity of almost all models of inflation [44,45] is that the phase of accelerated expansion lasted long enough for present scales of cosmological interest to be redshifted from trans-Planckian length scales at the onset of inflation. Hence, Lorentz violation must have been an intrinsic feature of the nascent Universe. Here again, one of the phenomenological approaches towards studying this era consists of modifying the standard dispersion relation of the scalar inflaton field together with the introduction of a standard time-like unit vector in the effective Lagrangian to define the preferred frame. However, the altered dispersion relation must obviously reduce to the standard linear behaviour at energies much smaller than $M_{Pl}$. The Myers and Pospelov theory shares these criteria which make it suitable for studying the physics of the Universe at the beginning of
inflation. The fact that there is a natural decomposition in Lorentz-preserving (scalar) fields between spatially homogeneous and inhomogeneous parts implies that while the former, in a Friedmann-Robertson-Walker (FRW) background spacetime, can play the role of the inflaton field, the latter, acting as a perturbation on the former, may provide natural seeds for the growth of inhomogeneities in the Universe. Moreover, in Friedmann-Robertson-Walker spacetime, a chosen frame exists by construction. Hence, the constant vector $n$ can be taken to be orthogonal to the homogeneous isotropic spatial sections in FRW spacetime such that $n = (1, 0)$ in the comoving frame.

Just as an illustration, we demonstrate how inhomogeneities appear in the energy momentum tensor in a Minkowski spacetime where $T_{\mu\nu} = \frac{\partial L_{\text{MFP}}}{\partial (\partial_{\mu} \phi)} \partial_{\nu} \phi - \eta_{\mu\nu} L_{\text{MFP}}$. For fields of the form (7) described by the Lagrangian density (4), the energy density $\rho$ and pressure $p$ in the massless limit as measured in the chosen frame are

$$\rho = T_{00} = \phi_{\parallel}^2 - \frac{ik}{M_{\text{Pl}}} \phi_{\parallel} \phi_{\parallel}^* - \frac{1}{3} (\nabla \phi_{\perp})^2, \quad (31)$$

$$p = -\frac{1}{3} T_{ii} = \phi_{\parallel}^2 + \frac{ik}{M_{\text{Pl}}} \phi_{\parallel} \phi_{\parallel}^* + \frac{ik}{M_{\text{Pl}}} \phi_{\parallel} \phi_{\parallel}^* - \frac{1}{3} (\nabla \phi_{\perp})^2. \quad (32)$$

The additional second and third terms in the energy density $\rho$ which follow from the new term in the action, may serve as sources of perturbations on the flat Minkowski metric when gravity is considered. This indicates that presence of Lorentz violation in an originally flat spacetime will automatically introduce non-trivial curvature. The inhomogeneous field $\phi_{\perp} (x_{\perp})$ appears only in the last two terms of the expression for $\rho$ which can be understood as perturbations over the homogeneous energy density of the field $\phi_{\parallel} (x_{\parallel})$. The Lorentz-preserving perturbations due to the field $\phi_{\perp} (x_{\perp})$ in 31, 32 might lead to growth of Lorentz invariant inhomogeneities in spacetime.

Reference [46] provides an interesting exposition on the impact of Lorentz violation on the inflationary scenario in the scalar-vector-tensor model of gravity, where the Lorentz-violating vector $u^\mu$ (as per the notation of [46]) is constrained to be unit and timelike. The authors here have deduced that the preferred frame, determined by $u^\mu$, practically aligns with the Cosmic Microwave Background rest frame and parameterised the metric for a homogeneous isotropic spacetime as

$$ds^2 = N^2(t)dt^2 - e^{2\alpha(t)} \delta_{ij} dx^i dx^j.$$ 

The lapse function $N(t)$ also appears in the components of $u^\mu$: $u^\mu = (\frac{1}{N}, 0)$. Obviously, this choice of the metric and Lorentz-violating vector is more general and far more appropriate for understanding cosmological aspects than Minkowski spacetime. Our cosmological example above is a special case where $N = 1, \alpha = 0$. References [46–49] illustrate how Lorentz-violating inflationary solutions for a family of models can be found even in the absence of any inflationary potential.

True significance of our Lorentz-preserving fields in the study of inflation, particulary trans-Planckian modes, will be realised only when we suitably modify the Myers-Pospelov effective action to hold in a spacetime with non-trivial curvature. We are interested in exploring how the inhomogeneous term appearing in the energy density may lead to the Jeans instability or some other aspect of structure formation in an FRW background. We hope to report on this in future.

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