Radiative and non radiative muon capture on the proton in heavy baryon chiral perturbation theory

Harold W. Fearing, Randy Lewis, Nader Mobed, and Stefan Scherer

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

Department of Physics, University of Regina, Regina, Saskatchewan, Canada S4S 0A2

Institut für Kernphysik, Johannes Gutenberg-Universität, J. J. Becher-Weg 45, D-55099 Mainz, Germany

We have evaluated the amplitude for muon capture by a proton, $\mu + p \rightarrow n + \nu$, to $\mathcal{O}(p^3)$ within the context of heavy baryon chiral perturbation theory (HBChPT) using the new $\mathcal{O}(p^3)$ Lagrangian of Ecker and Mojžiš (E&M). We obtain expressions for the standard muon capture form factors and determine three of the coefficients of the E&M Lagrangian, namely, $b_7$, $b_{19}$, and $b_{23}$. We describe progress on the next step, a calculation of the radiative muon capture process, $\mu + p \rightarrow n + \nu + \gamma$.

1. INTRODUCTION

Chiral perturbation theory is an effective theory for QCD, formulated in terms of a series of effective Lagrangians of increasing orders in the momentum and quark mass expansion. It was originally formulated for mesons only, but can be extended as HBChPT to include heavy baryons. For a review see e.g. [1, 2].

The complete Lagrangian for a single nucleon coupling to pions and external fields up to third order in small momenta (denoted $\mathcal{L}^{EckM}_{\pi N}$) has only recently been constructed by Ecker and Mojžiš [3] (E&M), although calculations with earlier versions and for specific processes had been performed before. Here we study muon capture by a proton with the new Lagrangian, $\mathcal{L}^{EckM}_{\pi N}$. The form factors that appear in the muon capture amplitude have been considered previously within HBChPT, but not with the new Lagrangian $\mathcal{L}^{EckM}_{\pi N}$.

Our calculation gives explicit expressions for each of the muon capture form factors, in terms of parameters that appear in $\mathcal{L}^{EckM}_{\pi N}$. We use experimental data to determine the numerical values of the parameters, which are directly transferable to future calculations of other processes where $\mathcal{L}^{EckM}_{\pi N}$ is used. In particular, the parameters obtained for ordinary

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muon capture are a subset of those required for our calculation of radiative muon capture.

The external nucleon fields in our calculation are renormalized by defining a wave function renormalization factor, \( Z_N \), which is the residue of the full heavy baryon nucleon propagator at the pole. Even though \( Z_N \) is not measurable, it affects measurable quantities, and one must use the value consistent with the Lagrangian and the rest of the calculation. Our result differs from that of previous published work using different Lagrangians, but is the one appropriate for the new E&M Lagrangian, as can be checked for example by noting that our form is the one within our formalism that is necessary to ensure that the vector coupling is not renormalized.

There is an additional normalization factor relating the normalization of relativistic and heavy baryon wave functions, which we put in explicitly. An alternative approach recently suggested [5] absorbs this momentum dependent factor into the definition of \( Z_N \). This gives a different expression for \( Z_N \) than we obtain, but would lead to exactly the same physical results.

2. NON RADIATIVE MUON CAPTURE

The general amplitude for muon capture can be parameterized in terms of the form factors \( G_V(q^2), G_M(q^2), G_A(q^2) \) and \( G_P(q^2) \) as follows,

\[
\mathcal{M} = \frac{-iG_{\beta}}{\sqrt{2}} l_{\alpha} \bar{n}(p_n) \left[ G_V(q^2)\gamma^{\alpha} + \frac{iG_M(q^2)}{2m_N} \sigma^{\alpha\beta}q_{\beta} - G_A(q^2)\gamma^{\alpha}\gamma_{5} - \frac{G_P(q^2)}{m_{\mu}} q^{\alpha}\gamma_{5} \right] u(p_p),
\]

where \( l_{\alpha} = \bar{n}(p_n)\gamma_{\alpha}(1 - \gamma_{5})u(p_p) \) is the leptonic current, \( m_N \) is the physical nucleon mass, and \( G_\beta \) is the Fermi constant applicable to \( \beta \)-decay.

In HBChPT, the muon capture amplitude in terms of heavy baryon spinors is

\[
\mathcal{M} = \frac{g_{W}}{2\sqrt{2}m_{W}^{2}} l_{\alpha} \bar{n}(p_n) \left( \Gamma_{\mu W\pi}(q)^{(r)\alpha}(q) + \Gamma_{\pi N}(q)^{(r)\alpha}(q) \left[ \frac{i}{q^{2} - m_{\pi}^{2}} \right] \Gamma_{W\pi}(q) \right) n_{u}(p_p),
\]

with \( m_{\pi} \) the physical pion mass, and \( m_{W}, g_{W} \) the mass and weak coupling constant of the W boson.

The functions \( \Gamma_{\mu W\pi}(q), \Gamma_{\pi N}(q) \) and \( \Gamma_{W\pi}(q) \) are the fully renormalized vertex functions. To evaluate them we start with the Lagrangian, \( \mathcal{L}_{\pi N}^{EckM} = \hat{\mathcal{L}}_{\pi N}^{(1)} + \hat{\mathcal{L}}_{\pi N}^{(2)} + \hat{\mathcal{L}}_{\pi N}^{(3)} \), where the \( \mathcal{O}(p^{3}) \) part is the new part of the Lagrangian as derived by E&M. Using this Lagrangian the various contributing diagrams are calculated and completely renormalized, so that they can be expressed in terms of physical quantities. One then uses the relation between the heavy baryon spinors \( n(p) \) and the Dirac spinors \( u(p) \), i. e.,

\[
n_{u}(p) = \sqrt{\frac{2m_{N}}{m_{N} + v \cdot p_{p}}} \left( 1 + \frac{\not{p}}{2} \right) u(p) = \left[ 1 - \frac{k_{p}}{2m_{N}} + \frac{(m_{N} - m_{0N})}{2m_{N}} + \frac{k_{p}^{2}}{8m_{N}^{2}} + \mathcal{O}(\frac{1}{m_{N}^{3}}) \right] u(p),
\]

where \( p = m_{0N}v + k_{p} \), to express the amplitude in the original form and extract the form factors. The result is
an estimate of factor as measured in antineutrino-nucleon scattering (or pion electroproduction) gives and can be evaluated using the pion nucleon coupling constant as input.

where $G$ gives other known experimental quantities. $g$ thus giving the parameter magnetic moments, which leads to the standard value of $b$.

Our value of $b$ differs from the recently derived E&M $G$ which is in good agreement with the best value from non-radiative muon capture, $b_{19}$ is related to the so-called Goldberger-Treiman discrepancy, and can be evaluated using the pion nucleon coupling constant as input.

We thus find for the three constants of the E&M Lagrangian obtainable from this process $b_{17}(m_N) = -0.53 \pm 0.02$, $b_{23} = -3.1 \pm 0.3$, and $b_{19} = -0.7 \pm 0.4$. Recently the constant $b_{19}$ has also been determined by Mojžiš in the context of $\pi N$ scattering. His reported value of $b_{19} = -1.0 \pm 0.4$, corresponding to $g_{\pi N} = 13.0 \pm 0.1$, is consistent with our value of $b_{19}$ within errors, with the difference being due almost entirely to his choice of a different value of $F_{\pi}$.

We can now evaluate $G_P$ using these values and we obtain $G_P(-0.88m_{\mu}^2) = 8.21 \pm 0.09$ which is in good agreement with the best value from non-radiative muon capture, $G_P(-0.88m_{\mu}^2) = 8.7 \pm 1.9$.

Thus we have obtained the form factors of muon capture by a proton in the framework of the recently derived E&M $O(p^3)$ heavy baryon chiral Lagrangian, and used experimental data to extract numerical values for some of the Lagrangian’s parameters, which will be needed for the calculation of radiative muon capture. Further details are given in [8].
3. RADIATIVE MUON CAPTURE

The radiative muon capture process, $\mu + p \rightarrow n + \nu + \gamma$, is particularly interesting because it is especially sensitive to the value of the induced pseudoscalar form factor, $G_P(q^2)$. A recent TRIUMF experiment [9] measured the rate for the radiative process and found a value of $G_P$ approximately 1.5 times the value expected from the Goldberger-Treiman relation. While consistent with radiative capture measurements in nuclei, this value is at variance with results from the nonradiative capture both in nuclei and on the proton, which agree with the Goldberger-Treiman relation.

The standard theoretical calculation for radiative capture [10] involves four tree level Feynman graphs, consisting of radiation from the external particles and from the intermediate pion generating $G_P$, together with a gauge term generated by minimal substitution. Corrections due to $\Delta$ intermediate states have been calculated [11] but are small.

In view of the discrepancy with experiment it is of interest to apply the same heavy baryon chiral perturbation theory approach used for ordinary capture to the radiative capture process. This allows a microscopic calculation of the gauge term and one would expect a number of new contributions for which there is no counterpart in the standard Feynman graph approach.

We have approached this process using the same techniques used for ordinary muon capture. In particular we use the same $\mathcal{O}(p^3)$ Lagrangian of Ecker and Mojžiš [4] and our same expression for the wave function renormalization factor [8]. The strategy is to evaluate individually the complete renormalized irreducible vertex functions for the separate interactions, i.e. for the weak-NN, $\pi$NN, $\gamma$NN, $\gamma\pi\pi$ and weak-$\pi$ vertices. We then put the pieces together to get the radiative muon capture amplitude.

We expect that the results corresponding to the first four graphs of the standard approach will be similar to the standard calculation. These graphs, corresponding to radiation from external legs and from an intermediate pion, involve situations where the electromagnetic and weak vertices are separated. Thus any approach which uses the nonradiative muon capture and electromagnetic vertices to fix the parameters of the interaction, as we have done, should pretty much reproduce the standard calculation of these graphs. There may be possible off shell effects and the main terms and their leading relativistic corrections may arise from various pieces of the ChPT Lagrangian, but the basic physics is the same as in the standard approach.

This is not the case for the gauge term however, which is included in the standard calculation only by way of a minimal substitution. In contrast, the ChPT approach makes an explicit prediction for contributions to this diagram and there are many such contributions. Some contain loops, with both weak and electromagnetic vertices attaching to the loop, and some arise from contact terms from the $\mathcal{O}(p^3)$ Lagrangian. Most of these contributions do not appear in the standard minimal coupling diagram and, except that they are $\mathcal{O}(p^3)$ terms, one does not know a priori how large they will be.

We have explicitly evaluated one such contribution arising from the Wess-Zumino-Witten part of the Lagrangian. This contribution leads to a diagram with intermediate
pion coupling at a point to weak and electromagnetic currents. It contains the pion propagator and so contributes to $G_P$. An analogous contribution was important for the non soft photon corrections to the spin dependent part of the virtual Compton scattering amplitude [12]. Like radiative muon capture, that process involves the coupling to two external currents. Furthermore this term is gauge invariant by itself, and so can be evaluated individually. Unfortunately it turns out to be negligibly small, apparently because for radiative muon capture, unlike Compton scattering, there are leading contributions from lower order parts of the Lagrangian.

There are however many other diagrams involving both loops and contact terms which may contribute. These can be combined into the renormalized, irreducible, weak-$\gamma$NN and $\pi\gamma$NN vertices which are now being calculated. When combined appropriately with the other vertices these will allow an explicit comparison with the radiative muon capture data. [13]

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