Lamb Mossbauer factor using non-extensive Statistics

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Abstract:
Lamb Mossbauer Factor is derived using Tsallis Statistics. Recoil free factor so obtained, has weak temperature dependence which is similar to the temperature dependence observed for Lamb Mossbauer factor of a fractal.

Introduction:
Anomalous behavior observed in properties of various physical system have been explained by using non-extensive Tsallis statistics. These physical systems /phenomena include turbulence in plasma [1], Cosmic ray background radiation [2], self gravitating systems [3], econo-physics[4], electron - positron annihilation [5], classical and quantum chaos [6], linear response theory [7], Levy type anomalous super diffusion [8], thermalization of electron - phonon systems [9], low dimensional dissipative systems [10], etc.

It has been shown that non-extensive features get manifested in those systems which have long range forces, long memory effects or in those systems which evolve in (non Euclidean like space-time) fractal space time [11 and reference therein]. Non - extensive statistics is based on two postulates [11 and reference therein]. First is the definition of non- extensive entropy

\[ S_q = \frac{1 - \sum P_i^q}{q - 1} \]  

where q is the non-extensive entropic index, k is a positive Constant and \( P_i \) are the probabilities of the microscopic states with \( \sum P_i = 1 \).
The second postulate is the definition of energy $U_q = \sum P_i^q E_i$, where $E_i$ is the energy spectrum. As $q \to 1$, $S_q = -\sum p_i \ln p_i$, which is the Boltzmann Gibbs Shannon entropy.

Apart from other applications it has been suggested [11] that non-extensive statistics can be applied to complex systems like glassy materials and fractal / multi-fractal or unconventional structures also. Anomalous Lamb Mossbauer factor behavior observed in glasses and biological system have motivated us to use non-extensive statistics. We will also compare Lamb Mossbauer factor using Tsallis statistics with Lamb Mossbauer factor of a fractal.

Anomalous $f$ - factor:

Lamb Mossbauer factor or recoil free factor $f$ is given by

$$\log f = -\frac{4\pi^2}{\lambda^2} \frac{x_m^2}{\bar{\chi}}$$

(2)

Where $x_m^2$ is mean square displacement of Mossbauer atom of mass $m$, averaged over the nuclear life time. For Debye model of lattice vibrations, the Lamb Mossbauer factor can be written as [12]

$$f = \exp\left[-\frac{6R}{k\theta_D} \left(\frac{1}{4} + \frac{T}{\theta_D} \int_{0}^{\theta_D} \frac{xdx}{\exp(x) - 1}\right)\right]$$

(3)

Where $x = \frac{\hbar \omega}{kT}$, $R$ is recoil energy and $\theta_D$ is the Debye temperature defined by $\hbar \omega_{max} = k \theta_D$.

Equation (3) has been obtained using Boltzman Gibbs statistical mechanics. $f$-factor is a very important parameter to study temperature dependence of dynamics of Mossbauer atom. In most of the cases equation (3) explains the $f$-factor experimental data very effectively. However, there are various system like superconductors, glasses, biological system etc. where $f$ - factor experimental data can not accounted by equation (3). Anomalous behavior of experimental $f$ - factor data in various types of superconductors have been explained by incorporating anharmonicity in the dynamics of Mossbauer atom [13,14]. For many biological systems e.g. in deoxy -myoglobin above 265 K, $f$ - factor experimental data [15,16]can not explained by equation (3). Again in various types of glassy
Theory:

To derive non-extensive form of Lamb Mossbauer factor we have to use non-extensive quantum distribution function of bosons. It is very difficult to derive exact analytical expression for non-extensive distribution function. However, there are many studies which provide the approximate form of non-extensive distribution functions [2,18]. In the present paper we use dilute gas approximation (DGA) for boson distribution function. For DGA case, the average occupation number is given as [18]

\[
\langle n_q \rangle = \frac{1}{(1 + (q-1)\beta(E_i - \mu)^{1/(q-1)} - 1)} \tag{4}
\]

When dealing with system in contact with the heat bath at the temperature \(\beta\), we have

\[
\langle n_q \rangle = \frac{\hbar \omega_i}{(1 + (q-1)\frac{\hbar \omega_i}{kT})^{1/(q-1)} - 1} \tag{5}
\]

If \(x^2_m\) is the mean square displacement of Mossbauer atom of mass m, and for the case of harmonic motion, we have

\[
3N mx^2_m \omega^2 = (n_q + \frac{1}{2})\hbar \omega_i \tag{6}
\]

The harmonic approximation in (6) means that each of 3N as oscillators modes have \(\delta\) function distribution. Let \(f(\omega_i)\) be the spectral distribution of the lattice vibration frequencies of the lattice. Let us find average of \(4\pi^2 x^2_m\) over the entire vibration spectrum.

\[
-\frac{4\pi^2}{\lambda^2} < x^2_m > = \int_0^{\omega_{max}} \frac{\hbar \omega_i}{m \omega_i^2} \left( \frac{1}{2} + n_q \right) \frac{4\pi^2 f(\omega_i) d\omega_i}{3N \lambda^2} \tag{7}
\]

For the case of Debye approximation

\[
f(\omega_i) = 9N \frac{\omega_i^2}{\theta_D^3} \frac{\hbar^3}{k^3} \tag{8}
\]

where \(\hbar \omega_{max} = k \theta_D\).

Using equations (3) and (7) and (8), we have

materials f-factor experimental data [17] shows temperature dependence similar to biological systems.
Equation (9) represents Lamb Mossbauer factor using Tsallis statistics where \( n_q \) is given by equation (5).

Results and Discussion:

We have numerically solved equation (9) for \( q = 2 \) shown as (+) corresponding to curve 'b' of figure 1. The curve 'c' of figure 1 corresponds to Lamb Mossbauer factor of a fractal. It has been shown that f factor for a fractal is given by [19,20]

\[
\log f \propto T^{(0.74+\frac{4}{3})} \exp(-cT^{0.74}) \tag{10}
\]

We have plotted proportional temperature dependence of equation (10) (Lamb Mossbaur factor of a fractal) in curve 'c'. curve 'a' in this figure has been obtained using equation (3), corresponding to Lamb Mossbauer factor using Boltzman Gibbs statistical mechanics. It is clear from figure 1 that Curve 'b' and 'c' have similar or overlapping behavior. Comparison of Curve 'a' on one hand, to Curves 'b' and 'c' on the other hand makes it clear that the Lamb Mossbauer fraction in non-Euclidean case is smaller than in the case of Euclidean.
Figure 2: Lamb Mossbauer factor defined in equation 10 for (a) $q=3.0$, (b)$q=1.2$ and (c) $q=0.6$. Experimental data has been taken from reference 17.

It is also clear that both fractal and non-Euclidean Lamb Mossbauer fraction is less temperature dependent than fraction for Euclidean solids. The similarity between Curve 'b' and 'c' is expected because non-extensive statistics corresponds to non-Euclidean space time which is fractal in nature.

In Fig. 2 we are plotting experimental data corresponding to Fecl$_2$ dissolved in glycerol-water mixture in super cooled liquid state. The experimental data has been taken from reference [17]. The three curves corresponding to non-extensive Lamb Mossbauer factor for different values of parameter $q$. It is clear that temperature dependence of glassy materials above some characteristic temperature is very strong whereas non-extensive Lamb Mossbauer factor has continuous weak temperature dependence. Thus non-extensive f-factor cannot account for anomalous f-factor data of glasses and bio systems. It is interesting to mention here for the sake of completeness that Lamb Mossbauer factor using generalized algebra (of deformation concepts) has been derived earlier and deformation parameter $q$ has been identified as anharmonic coefficient when applied to experimental Mossbauer data [13,14]. Now with the advent of generalized statistics, Lamb Mossbauer factor seems to behave more like an f-factor of a fractal. It will be interesting exercise to use both generalized algebra and generalized statistics to derive Lamb Mossbauer factor. Combination of gen-
eralized statistics and generalized algebra has been successfully applied to the study of COBE [21] results.

**Conclusion:**

In this paper we have derived Lamb Mossbauer factor for generalized statistics and found it very similar to f-factor of a fractal.

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