Symmetry-energy dependence of the dynamical dipole mode in the Boltzmann-Uehling-Uhlenbeck model

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Using an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model, we have studied the connection between the symmetry energy and features of the dynamical dipole mode in fusion reactions with charge-asymmetric entrance channel. The yield and angular distribution of the prompt photon emission are extracted by a bremsstrahlung approach. The experimental data of 36Ar+96Zr at 16 MeV/nucleon and 32S + 100Mo at 9.3 MeV/nucleon are compared with IBUG model calculations, and the soft symmetry energy is found to describe the data reasonably well.

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Heavy-ion collision using radioactive nuclei (especially $N > Z$) provides a unique possibility for exploring the structure of nuclei and the interaction between nucleons. One of related topic is the determination of the density dependence of symmetry energy ($E_{\text{sym}}(\rho)$), which is important in understanding the nuclear equation of state (EOS) of asymmetric nuclear matter as well as the properties of astrophysical objects [1, 2].

Giann dipole resonance (GDR), a well-established collective vibration of protons against neutrons in nuclei, can be thermally excited in dissipative heavy-ion reactions [3]. The existence of a different GDR-like excitation mode, called pre-equilibrium GDR or dynamical dipole mode, was also addressed experimentally [4–9] and theoretically [10–17]. During the early stage of charge-asymmetric heavy-ion collisions, a large amplitude collective dipole oscillation can be triggered along the symmetry axis of the strongly deformed composite system. This oscillation can emit prompt dipole photons, called dynamical dipole radiation, in addition to the photons originating from a thermal excited GDR of the hot compound nucleus (CN). The gamma production from such a pre-equilibrium evolution contains lots of information about the early stage of collisions when the CN is still in a highly deformed configuration. Especially, the excitation can reflect the density dependence of the $E_{\text{sym}}(\rho)$ below saturation in the fusion or deep-inelastic processes.

Several microscopic transport models, such as time-dependent Hartree-Fock (TDHF) [13], Boltzmann-Nordheim-Vlasov (BNV) [14], constrained molecular dynamics (CoMD) [15], and isospin-dependent quantum molecular dynamics (IQMD) [16, 17], have been successfully used to study the properties of dynamical dipole emission. A "bremsstrahlung" approach derived in Refs. [11, 12] was widely adopted to estimate the contribution of this pre-equilibrium collective dipole radiation emission. Here we use an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model [18] to calculate the gamma yield and the emission anisotropy of such prompt dipole radiation. The reactions of 36Ar + 96Zr at 16 MeV/nucleon and 32S + 100Mo at 9.3 MeV/nucleon, for which the experiments are well documented and the data are available in Refs. [7, 8], are considered in the present study.

The BUU model [19] was widely used for simulating heavy-ion collision dynamics and extracting important information of nuclear matter properties. The BUU equation reads

\[
\frac{df}{dt} + v \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = \frac{4}{(2\pi)^3} \int d^3p_2 d^3p_3 d\Omega \\
\begin{align*}
\frac{d\sigma_{NN}}{d\Omega} v_{12} \times [f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)] \\
\delta^3(p + p_2 - p_3 - p_4),
\end{align*}
\]

where $\frac{d\sigma_{NN}}{d\Omega}$ and $v_{12}$ are the in-medium nucleon-nucleon scattering cross section and the relative velocity for the colliding nucleons, respectively. The isospin dependence was incorporated into the model through the initializalation and the nuclear mean field potential $U$, which is parametrized as

\[
U(\rho, \tau_2) = a \left( \frac{\rho}{\rho_0} \right) + b \left( \frac{\rho}{\rho_0} \right)^{\sigma} + V_{\text{asy}}^{(p)}(\rho, \delta),
\]

where $\rho_0$ is the nuclear saturation density and $\rho = \rho_p + \rho_n$ with $\rho_p$, $\rho_n$, and $\rho$ being the nucleon, neutron, and proton densities, respectively. The coefficients $a$, $b$, and $\sigma$ are parameters for the isoscalar EOS. Here we choose the parameters corresponding to the soft EOS with the compressibility $K$ of 200 MeV ($a = -356$ MeV, $b = 303$ MeV, $\sigma = 7/6$). The single particle symmetry potential $V_{\text{asy}}$ has the form of

\[
V_{\text{asy}}^{(p)}(\rho, \delta) = \frac{\partial}{\partial \rho} \left[ \frac{C_{\text{sym}}}{2} \gamma \left( \frac{\rho}{\rho_0} \right)^\gamma \rho \delta^\gamma \right]
\]

\[
= C_{\text{sym}} \left( \gamma - 1 \right) \left( \frac{\rho}{\rho_0} \right)^\gamma \delta^2 \pm 2 \left( \frac{\rho}{\rho_0} \right)^\gamma \delta
\]

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where $C_{\text{sym}} = 35.19$ MeV is the symmetry energy coefficient. For simplicity, the $E_{\text{sym}}(\rho)$ with $\gamma = 0.5$ and $\gamma = 2$ in the present calculation is called as the soft and the stiff symmetry energy, respectively, with the former (latter) larger (smaller) at subsaturation densities but smaller (larger) at suprasaturation densities.

As usual, the test particle method is applied in the IBUU simulations, and the in-medium nucleon-nucleon cross section is also taken into account [20]. The calculations are performed at impact parameters of $b = 0, 2, 4$ fm where the fusion is an important reaction mechanism. In the study of dynamic dipole emission, the emitted nucleons with their local density below $\rho_0/8$ are excluded. Figure 1(a) and (b) show the time evolution of the emitted nucleons and the neutron-to-proton ratio ($R(n/p)$) from the soft and stiff symmetry energy, respectively, up to $t = 200$ fm/c when about 10% of the total nucleons become free. The stiff $E_{\text{sym}}(\rho)$ leads to enhanced emission of protons resulting in a smaller $R(n/p)$. The qualitative influence of $E_{\text{sym}}(\rho)$ on the $R(n/p)$ is similar to that predicted by the IQMD model [21] in fragmentation.

During pre-equilibrium dipole oscillation, a larger initial dipole moment will trigger a larger-amplitude isovector oscillation, which increases the chance of a clear experimental observation. The giant dipole moment in coordinate space is defined as [11]

$$D(t) = \frac{NZ}{A} X(t) = \frac{NZ}{A} (R_p - R_n), \quad (4)$$

where $A = N + Z$, $N = N_p + N_t$, and $Z = Z_p + Z_t$ are the total number of participating nucleons, neutrons, and protons from the projectile (p) and the target (t), respectively, while $X(t)$ is the distance between the centers of mass of protons and neutrons. The canonical momentum conjugate of $D(t)$ is expressed as

$$DK(t) = \frac{NZ}{A} (\frac{P_p}{Z} - \frac{P_n}{N}), \quad (5)$$

where $P_p$ and $P_n$ are the total momenta of center-of-mass system for the protons and neutrons. The initial ($t = 0$: touching configuration) giant dipole moment can be expressed as [11]

$$D(t = 0) = \frac{NZ}{A} X(t = 0) = \frac{R_p + R_t}{N + Z} Z_p Z_t \left( \frac{N_p}{Z_p} - \frac{N_t}{Z_t} \right), \quad (6)$$

where $R_p$ and $R_t$ are the radii of the projectile and target, respectively.

In Fig. 2 we plot the time evolution of the dipole moment $D(t)$ and the phase space correlation (spiraling) between $D(t)$ and $DK(t)$ for the $^{36}\text{Ar} + ^{96}\text{Zr}$ system at $16$ MeV/nucleon and $b = 4$ fm. It is clearly seen that the large amplitude of the first oscillation decays rapidly within several periods. The delayed dynamics for the stiff $E_{\text{sym}}(\rho)$ is related to the weaker isovector restoring force. Especially, the right panel of Fig. 2 shows that the collective oscillation initiates at about $20$ fm/c, corresponding to the touching configuration of the collision.

Finally, the gamma yield, as given by the bremsstrahlung approach, can be extracted as [11, 12]

$$\frac{dP}{dE_\gamma} = \frac{2e^2}{3\pi \hbar c E_\gamma} \left| D''(\omega) \right|^2, \quad (7)$$

where $E_\gamma = h\omega$ is the photon energy and $| D''(\omega) |^2$ is the Fourier transform of the dipole “acceleration” $D''(\omega) = \int_{t_0}^{t_{\text{max}}} D''(t) e^{i \omega t} dt$. For each event, $t_0$ represents the onset time of the collective dipole response (phase-space spiraling), and $t_{\text{max}}$ is the “damping time”. In this work, after investigating the $D(t)$ behavior, we define $t_{\text{max}} - t_0 = 300$ fm/c to make sure that the collective oscillation is basically over.

Figure 3 shows the comparison of calculated photon yields with the experimental data of $^{36}\text{Ar} + ^{96}\text{Zr}$ at $16$ MeV/nucleon [7] and $^{32}\text{S} + ^{100}\text{Mo}$ at $9.3$ MeV/nucleon [9], respectively. It is seen that the soft $E_{\text{sym}}(\rho)$ gives a stronger restoring force for the dynamical dipole in dilute neck region, and leads to a faster and stronger dynamical dipole oscillation, resulting in a larger centroid and stronger dynamical dipole emission. Another feature of the dynamical dipole emission is that the gamma-ray yields in both cases show a decreasing tendency towards larger impact parameter inducing a larger dipole moment in the oscillation process.
of the centroid energy $E_{\text{calc}}$ calculation of the soft energy factor ($E_{\text{sym}}$) is expected as averaging over all angles and orthogonalizing them into the beam axis, see e.g., statistical compound nucleus GDR radiation in Ref. [22]. The angular distribution of the prompt gamma emission was extracted by using the formalism from Ref. [14].

First, we investigate in a very small time interval $\Delta t$. We denote $\phi_i$ and $\phi_f$ as the initial and final angles of the oscillation axis with respect to the beam axis. So $\Delta \phi = \phi_f - \phi_i$ is the rotation angle during the small time interval $\Delta t$. From preceding text, the angular distribution in the time interval $\Delta t$ can be averaged over the angle $\Delta \phi$ as $M(\theta) \sim 1 - (\frac{1}{4} + \frac{3}{4}x)P_2(\cos \theta)$, where $x = \cos(\phi_f + \phi_i)\sin(\phi_f - \phi_i)$ [14]. Then, for the whole time of the dynamical dipole, we can extract the angular distribution by using a weighted form for every time step and summing over all of them, i.e., $M(\theta) = \sum_{i=0}^{\Delta t} \beta_i M(\theta, \Phi_i)$, where $\Phi_i$ is the rotation angle and $\beta_i$ is the radiation emission probability. From Eq. (7) we get $\beta_i = P(t_i) - P(t_{i-1})$ where $P(t) = \int_{t}^{t_{\text{max}}} |D''(t)|^2 dt/P_{\text{tot}}$ with $P_{\text{tot}}$ given by $P(t_{\text{max}})$, which is the total emission probability at the final dynamical dipole damped time. Finally we have:

$$M(\theta) = M_0 \left[ 1 - \sum_{i=0}^{\Delta t} \beta_i \left( \frac{1}{4} + \frac{3}{4}x \right) P_2(\cos \theta) \right], \quad (8)$$

where $M_0$ can be obtained from the experimental data when $P_2(\cos \theta) = 0$. For the $^{36}\text{Ar} + ^{96}\text{Zr}$ system at 16 MeV/nucleon, $M_0 \approx 0.6 \times 10^{-4}$ from Ref. [7]. To study the effect of the symmetry energy on the radiation emission probability, we plot the time evolution of radiation emission probability from different symmetry energies with different impact parameters in Fig. 5. It is found that the photon emission occurs before 200 fm/c, after which $P(t)$ levels off, and the soft symmetry energy leads to a shorter emission period. The line is smoother with a larger impact parameter, because the rotation of the neck region, i.e., the oscillation axis, makes possible the radiation emission to different angles in peripheral collision.

The angular distribution of the dynamical dipole is shown in Fig. 6 together with the experimental data from Ref. [7]. From different symmetry energies with small impact parameters, little rotation of the oscillation axis leads to almost the same angular distribution with the coefficient $a_2 = -1$, suggesting the $\sin^2(\theta)$ form of the emission from the dipole oscillation along the beam axis. The experimental data from Refs. [7, 8] sustains our calculation. In the non-central collisions a little more photons are emitted at forward/backward angles for the stiff symmetry energy, indicating that the angular distribution of the dynamical dipole seems weakly dependent on the symmetry energy at subsaturation densities.

To further check the systematic effect of symmetry energy factor ($\gamma$) on the centroid, we plot the $\gamma$ dependence of the centroid energy $E_c$ in Fig. 4(a). A monotonic decreasing behavior is exhibited in the correlation between $E_c$ and $\gamma$. A constant symmetry energy ($\gamma=0$) gives the largest value of $E_c$. Because the GDR takes place in the subsaturation density region, the overall strength of the symmetry energy decreases with increasing $\gamma$. Figure 4(b) indicates that the soft symmetry energy ($\gamma=0.5$) is in agreement with the experimental data from Ref. [7], where a mean impact parameter $b_{\text{mean}} = 2.3-2.5$ fm was deduced. However, we should keep in mind that if one considers momentum-dependent effective interaction which is absent in the current IBUU model, the centroid and width of dynamical dipole emission could be shifted [17] and the above conclusion might be modified.

Recently, a clear anisotropy for the angular distribution of the total gamma spectrum was observed [7, 8]. For the dipole oscillation just along the beam axis, the angular distribution is expected to be given by the Legendre polynomial expansion $M(\theta) \sim \sin^2 \theta \sim 1 + a_2 P_2(\cos \theta)$ with $a_2 = -1$, where $\theta$ is the polar angle between the emitted photon direction and the oscillation axis. For the dynamical dipole mode, the prompt dipole axis will rotate during the radiative emission. If the oscillation is a stable and uniform rotation, an anisotropy parameter $a_2 = -1/4$ is expected as averaging over all angles and orthogonalizing them into the beam axis, see e.g., statistical compound nucleus GDR radiation in Ref. [22]. The angular distribution of the prompt gamma emission was extracted by using the formalism from Ref. [14].
as well as to investigate the early entrance channel densities. Similar results were also found in Ref. \[14\], which provides a suitable probe to test the symmetry energy as well as to investigate the early entrance channel dynamics in dissipative reactions with asymmetric nuclear collisions. The present quantitative comparisons with the dynamical dipole gamma-ray spectra data seem to favor a softer symmetry energy. However, a caution is needed for the above conclusion of the symmetry energy if the momentum dependent effective interaction is taken into account. On the other hand, we hope new experiments could provide more accurate angular distribution data for the prompt dipole radiation as similarly expected in Ref. \[14\] for \(_{132}^{82}\text{Sn}\) beams, which may give another constraint on the symmetry energy.

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[1] A.W. Steiner \textit{et al.}, Phys. Rep. \textbf{411}, 325 (2005).
[2] V. Baran, M. Colonna, V. Greco, M. Di Toro, Phys. Rep. \textbf{410}, 335 (2005).
[3] K. A. Snover, Annu. Rev. Nucl. Part. Sci. \textbf{36}, 545 (1986).
[4] S. Filibotte \textit{et al.}, Phys. Rev. Lett. \textbf{77}, 1448 (1996).
[5] D. Pierroutsakou \textit{et al.}, Eur. Phys. J. A \textbf{16}, 423 (2003).
[6] D. Pierroutsakou \textit{et al.}, Phys. Rev. C \textbf{71}, 054605 (2005).
[7] D. Pierroutsakou \textit{et al.}, Phys. Rev. C \textbf{80}, 024612 (2009).
[8] B. Martin \textit{et al.}, Phys. Lett. B \textbf{664}, 47 (2008).
[9] D. Pierroutsakou \textit{et al.}, Eur. Phys. J. A \textbf{17}, 71 (2003).
[10] Ph. Chomaz, M. Di Toro, and A. Smerzi, Nucl. Phys. A \textbf{563}, 509 (1993).
[11] V. Baran \textit{et al.}, Nucl. Phys. A \textbf{679}, 373 (2001).
[12] V. Baran, D. M. Brink, M. Colonna, and M. Di Toro, Phys. Rev. Lett. \textbf{87}, 182501 (2001).
[13] C. Simenel, Ph. Chomaz, G. de France, Phys. Rev. Lett. \textbf{86}, 2971 (2001); Phys. Rev. C \textbf{76}, 024609 (2007).
[14] V. Baran, C. Rizzo, M. Colonna, M. Di Toro, and D. Pierroutsakou, Phys. Rev. C \textbf{79}, 021603(R) (2009).
[15] M. Papa \textit{et al.}, Phys. Rev. C \textbf{72}, 064608 (2005).
[16] H. L. Wu \textit{et al.}, Phys. Rev. C \textbf{81}, 047602 (2010); J. Wang \textit{et al.}, Nucl. Sci. Tech. \textbf{24}, 030501 (2013).
[17] C. Tao, Y. G. Ma, G. Q. Zhang \textit{et al.}, Phys. Rev. C \textbf{87}, 014621 (2013); Nucl. Sci. Tech. \textbf{24}, 030502 (2013).
[18] Y. G. Ma \textit{et al.}, Phys. Rev. C \textbf{85}, 024618 (2012).
[19] W. Bauer, G. F. Bertsch, W. Cassing, U. Mosel, Phys. Rev. C \textbf{34}, 2127 (1986).
[20] G. Q. Li, R. Machleidt, Phys. Rev. C \textbf{48}, 1702 (1993); Phys. Rev. C \textbf{49}, 566 (1994).
[21] S. Kumar, Y. G. Ma, Phys. Rev. C \textbf{86}, 051601(R) (2012); Phys. Rev. C \textbf{84}, 044620 (2011).
[22] M.N. Harakeh, A. van der Woude, \textit{Giant Resonances}, Oxford Univ. Press 2001.