Obstacles avoidance for autonomous marine vehicles based on the model predictive control

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Abstract. In this paper, a new obstacle collision avoidance algorithm based on the model predictive control is proposed. The algorithm has the following advantages. First, the vehicle nonlinear dynamics as well as restrictions on the phase variables and the controls defining forbidden areas for the vehicle are taken into account. Second, an adaptation of the algorithm to the time-varying number of obstacles according to information coming in real-time from sensors is implemented. Third, the return to the prescribed trajectory after obstacles avoidance is provided. The ship collision avoidance on the sea surface is considered as an example.

1. Introduction
Obstacles avoidance is an important problem for autonomous vehicles moving in unknown dynamic environments. Quite a number of methods have been offered to solve it. Two of the earliest and computationally simple approaches are based on potential fields [1, 2] and the dynamic window approach [3]. In the framework of the first one, the vehicle is considered as a particle that moves in a potential field generated by the goal and obstacles in the environment. Obstacles are either a priori known or on-line detected by the on-board sensors. A shortcoming of this approach is that the vehicle can get stuck in a local minimum of the potential. Another problem with potential fields is that they specify a direction that may not respect nonholonomic constraints. The second one restricts the search space to the velocities reachable within a short time and free interval from collisions.

At present, the model predictive control (MPC) is one of the most common approaches for constructing controls of autonomous vehicles in real time, see, e.g., [4] and references therein. This is due to the fact that MPC allows one to take into account the nonlinear vehicles dynamics, environmental information and different restrictions on controls and phase variables. The MPC formulation requires the online solution of the optimization problem on a prediction horizon. For the nonlinear vehicle, obstacles and constraints models, this is a computationally intensive nonlinear optimization problem. It follows that real-time implementation of nonlinear MPC on autonomous vehicles is a difficult task and this is the main obstacle to using MPC in plants with fast dynamics such as vehicles.

This paper presents a new obstacles avoidance algorithm for autonomous marine vehicles using MPS. The proposed algorithm takes into account the nonlinear vehicle dynamics as well as restrictions on the phase variables and the control defining forbidden circle areas to the vehicle. Besides, an adaptation of the algorithm to time-varying number of obstacles according to information coming in real-time from sensors and the return to the prescribed trajectory after obstacles avoidance are
implemented. As a possible application of the proposed algorithm, the ship collision avoidance on the sea surface is considered.

2. Problem statement

We consider the following scenario. The ship is moving along a specified route fixing possible obstacles that can be both static and dynamic. The ship model is described by a nonlinear discrete system of the form:

\[ q_{i+1} = f^*(q_i, u_i), \quad \eta_i = (x_i, y_i, \varphi_i)^T, \quad q_0 = \bar{q}, \quad t = 0, 1, ..., \]

where \( q_i \in \mathbb{R}^n \) is the state vector; \( \eta_i \in \mathbb{R}^3 \) is the output vector; \( u_i \in \mathbb{R}^m \) is the control; \( x_i, y_i, \varphi_i \) are ship coordinates and the ship heading angle in the earth coordinate system, respectively; \( f^* (\cdot, \cdot) \) is the known vector function; \( T \) is the transpose operation; \( q_i \) includes \( x_i, y_i, \varphi_i \).

We assume that the estimation task of the systems state (1) has been solved and \( q_i \) are known vectors as well as the obstacles coordinates \( x_{0i}^o, y_{0i}^o \) for all \( t = 0, 1, ..., i = 1, 2, ..., N \) (for example, see [5]).

Let us introduce the forbidden areas including the obstacles for the ship as a set of \( x_i, y_i \) values satisfying the condition:

\[ D_i^j (x_i, y_i) = \left[ (x_i - x_{0i}^o)^2 + (y_i - y_{0i}^o)^2 \right]^{1/2} < \overline{D}, \quad t = 0, 1, ..., i = 1, 2, ..., N, \]

where \( \overline{D} \) is the radius of the prohibited circles areas. It is required to choose the control to ensure the ship movement approximately along the desired route avoiding the forbidden area.

3. The MPC algorithm

We can consider the obstacles avoidance problem within the MPC framework. Let us assume that the system:

\[ q_{i+1} = f^*(q_i, u_i^i), \quad \varphi_i^i = (x_i^i, y_i^i, \varphi_i^i)^T, \quad q_0^i = \bar{q}^i, \quad t = 0, 1, ... \]

describes the prescribed ship movement. We solve the following constrained optimization problem:

\[ \min J = \min \sum_{j=1}^{h} \left[ (\eta_{i+j} - \eta_{i+j}^i)^T \Lambda (\eta_{i+j} - \eta_{i+j}^i) + \lambda \| u_{i+j-1} - u_{i+j-1}^i \|^2 \right] \]

subject to \( D_i^j (x_{i+j}, y_{i+j}) > \overline{D} \), where \( h \) is the horizon length; \( \Lambda \in \mathbb{R}^{3h} \), \( \lambda \) are some positive-definite matrix and positive constant, \( \overline{D} \geq \overline{D} \) is such user-defined function depending on the maneuvering properties of the vehicle, obstacles and the size of the forbidden area that the condition \( D_i^j (x_i, y_i) > \overline{D}, \quad t = 0, 1, ..., i = 1, 2, ..., N \) are held. The optimal control must return the ship to the desired trajectory (the stability condition of the closed optimal system): there is such \( \delta > 0 \) that if \( \| \bar{q}_m - \bar{q}_m^i \| < \delta \), then:

\[ \lim_{t \to \infty} \| \bar{q}_{m+t} - \bar{q}_{m+t}^i \| = 0, \]

where \( m \) is the time required for obstacles avoidance.

Now we find a linear approximation to the formulated problem. Linearizing the right parts of the system (1) \( f^* (q_i, u_i) \) gives:

\[ f^* (q_i, u_i) = f^* (q_i^*, u_i^*) + \frac{\partial f^* (x, u)}{\partial x} \|_{x=q_i^*, u=u_i^*} (q_i - q_i^*) + \frac{\partial f^* (x, u)}{\partial u} \|_{x=q_i^*, u=u_i^*} (u_i - u_i^*) =
\]

\[ f^* (q_i, u_i) + A_i (q_i - q_i^*) + B_i (u_i - u_i^*), \]

where \( A_i = A (q_i^*, u_i^*) \), \( B_i = B (q_i^*, u_i^*) \). Introducing the notations \( \bar{q}_i = q_i - q_i^* \), \( \bar{u}_i = u_i - u_i^* \) gives:

\[ \bar{q}_{i+1} = A_i \bar{q}_i + B_i \bar{u}_i, \quad \bar{\eta}_i = C_i \bar{q}_i. \]
Linearizing (2) gives:

\[ D^{i}_{t+h} = \left[ (x_{t+h} - x_{t+h}^{0,i})^2 + (y_{t+h} - y_{t+h}^{0,i})^2 \right]^{1/2} = \]

\[ \tilde{D}^{i}_{t+h} + \frac{\partial D_{t+h}(x, y)}{\partial x} \tilde{x}_{t+h} + \frac{\partial D_{t+h}(x, y)}{\partial y} \tilde{y}_{t+h} = \]

\[ \tilde{D}^{i}_{t+h} + C^{x,i}_{t+h} \tilde{x}_{t+h} + C^{y,i}_{t+h} \tilde{y}_{t+h} > D_{i}, \quad i = 1, 2, \ldots, N. \tag{7} \]

In new variables, the quality criterion will take the form:

\[ J = \sum_{j=1}^{h} \left( \tilde{n}_{t+j}^T \Lambda \tilde{n}_{t+j} + \lambda \tilde{u}_{t+j}^2 \right). \tag{8} \]

Using (6), we obtain the relations:

\[ \tilde{n}_{t+1} = CA_{q} \tilde{u}_{t} + CB_{i} \tilde{u}_{t}, \]
\[ \tilde{n}_{t+2} = CA_{q+1} \tilde{u}_{t} + CA_{q+1}B_{i} \tilde{u}_{t} + CB_{i+1} \tilde{u}_{t+1}, \]
\[ \tilde{n}_{t+h} = CA_{q+h-1} \tilde{u}_{t+h-2} \cdots A_{q} \tilde{u}_{t} + CA_{q+h-1} \tilde{u}_{t+h-2} \cdots A_{q+1}B_{i} \tilde{u}_{t} + \ldots + CB_{i+h-1} \tilde{u}_{t+h-1}. \]

Rewriting them in a more compact form gives:

\[ \tilde{Q}_{t} = F \tilde{q}_{t} + H_{t} \tilde{u}_{t}, \]

where \( \tilde{U}_{t} = (\tilde{u}_{t}, \tilde{u}_{t+1}, \ldots, \tilde{u}_{t+h-1})^T \), \( \tilde{Q}_{t} = (\tilde{n}_{t+1}, \tilde{n}_{t+2}, \ldots, \tilde{n}_{t+h-1})^T \),

\[ F_{t} = F_{t}\left[q_{t}, u_{t}, q_{t+1}, u_{t+1}, \ldots, q_{t+h-1}, u_{t+h-1}\right], \quad H_{t} = H_{t}\left[q_{t}, u_{t}, q_{t+1}, u_{t+1}, \ldots, q_{t+h-1}, u_{t+h-1}\right]. \]

Substitution (9) in (7), (8) gives:

\[ J = \tilde{q}_{t}^T F_{t}^T \Lambda F_{t} \tilde{q}_{t} + 2\tilde{q}_{t}^T F_{t}^T \Lambda H_{t} \tilde{u}_{t} + \tilde{u}_{t}^T \left(H_{t}^T H_{t} + \lambda I\right) \tilde{u}_{t}, \tag{10} \]

\[ D^{i}_{t+h} = \tilde{D}^{i}_{t+h} + \left(C^{x,i}_{t+h} F_{t}^x + C^{y,i}_{t+h} F_{t}^y \right) \tilde{q}_{t} + \left(C^{x,i}_{t+h} H_{t}^x + C^{y,i}_{t+h} H_{t}^y \right) \tilde{u}_{t} > D_{i}, \tag{11} \]

where \( F_{t} = (0, 0, 1, 0, 0)F_{t} \), \( F_{t} = (0, 0, 0, 0, 0)F_{t} \), \( H_{t} = (0, 0, 1, 0, 0)H_{t} \), \( H_{t} = (0, 0, 0, 0, 0)H_{t} \).

Taking into account the expressions (10), (11) the problem of the optimal predictive control by the system (6) can be presented in the form:

\[ u_{t} = (1, 0, \ldots, 0) \tilde{U}_{t} = \varphi_{t}\left(\beta_{t}, \tilde{q}_{t}\right) = \varphi_{t}\left(\beta_{t}, q_{t} - q_{t}^0\right), \tag{12} \]

where \( \tilde{U}_{t} \) is for each \( t = 0, 1, \ldots \) the solution of the optimization problem:

\[ \min\left\{ \tilde{U}_{t}^T W_{t} \tilde{U}_{t} + M_{t}^T \tilde{U}_{t} \right\}, \quad \tilde{U}_{t} = (\tilde{U}_{t}^1, \tilde{U}_{t}^2, \ldots, \tilde{U}_{t}^{h})^T \in M_{t}, \quad M_{t} = \left\{ \tilde{U}_{t} : L_{t} \tilde{U}_{t} \leq b_{t}\right\}, \]

\[ W_{t} = H_{t}^x H_{t} + \lambda I, \quad M_{t}^T = 2\tilde{q}_{t}^T F_{t}^T \Lambda H_{t}, \quad L_{t} = C_{t+h}^x H_{t}^x + C_{t+h}^y H_{t}^y, \quad b_{t} = \tilde{D}_{t+h} + C_{t+h}^x F_{t}^x + C_{t+h}^y F_{t}^y, \]

\[ \beta_{t} = \left(\tilde{q}_{t}, u_{t}, \tilde{q}_{t+1}, u_{t+1}, \ldots, \tilde{q}_{t+h-1}, u_{t+h-1}\right)^T. \]

At the same time, (12) can be used as a predictive linearized control of the original nonlinear system (1) in the vicinity of the prescribed motion. More much attractive use is that of the linearization in the vicinity of the predictive motion. Let the predictive trajectory of the ship be described by the system:

\[ \tilde{q}_{t+1} = f^{x}(\tilde{q}_{t}, \tilde{U}_{t}) \quad i = t, t + 1, \ldots, t + h - 1. \tag{13} \]

Then replacing \( \beta_{t} \) with the realltion \( \tilde{\beta}_{t} = [\tilde{n}_{t+1}, \tilde{U}_{t}^1, \tilde{n}_{t+2}, \ldots, \tilde{n}_{t+h-1}, \tilde{U}_{t}^{h-1}] \), we obtain:

\[ \tilde{u}_{t} = \varphi_{t}\left(\tilde{\beta}_{t}, q_{t} - q_{t}^0\right). \tag{14} \]
4. Simulation

To illustrate the capability of the proposed algorithm, we use the ship model [6]:

\[ x_{t+1} = x_t + v \cos(\varphi_t) \Delta t, \quad y_{t+1} = y_t + v \sin(\varphi_t) \Delta t, \]

\[ \varphi_{t+1} = \varphi_t + \frac{r_t \Delta t}{r(s)} / (K(T_s + 1)/(T_s s + 1)) \]

and the obstacle model \[ x^o_{t+1} = x^o_t + v^o \cos(\psi_t) \Delta t, \quad y^o_{t+1} = y^o_t + v^o \sin(\psi_t) \], where \( \delta(s) \) is transformed by Laplace; steering angle - \( u_t = \delta_t \); \( \Delta \) is the sampling period. The prescribed ship motion is described by the system \( x_{t+1} = x_t + v \cos(\varphi_t) \Delta t, \quad y_{t+1} = y_t + v \sin(\varphi_t) \).

Input data for the simulation are listed in Table 1. We consider two different scenarios. In the first one, the obstacle is stationary. In the second one, the obstacle is dynamic and moves toward the ship. As it is seen from Figures 1 and 2, the control system provides the solution to the problem (the obstacles avoidance and the ship return to the prescribed route). Let us note that in both scenarios the control parameters were the same.

**Table 1. Input data for simulation**

| Components of the system | Parameters |
|--------------------------|------------|
| Ship                     | \( x_0 = 10m, y_0 = 10m, v = 5m/s, \varphi = 60^\circ, \Delta = 0.3s, \Delta = 0.3s, K = 0.36, T_1 = 1.19, T_2 = 1.44, T_3 = 1.57 \) |
| Obstacle                 | \( x^o_0 = 574m, y^o_0 = 995m, v^o = 0m/s \) for the stationary obstacle, \( v^o = 10m/s \) for the dynamic obstacle, \( \varphi^o = 240^\circ \) |
| Quality criterion        | \( \Lambda = 5 \times 10^{-5} I_{10}, \lambda = 0.5, h = 10 \) |

Restrictions and control \( \overline{D} = 45m, \overline{D}_t = 400m, u_t = \begin{cases} 6^\circ & | \hat{u}_t | \leq 6^\circ \\ \hat{u}_t & \text{otherwise} \end{cases} \)

**Figure 1.** The case of the static obstacle: a – the trajectories of the ship and obstacle; b, c, d – dependencies of the control, the ship heading angle and the distance to the obstacle on time, respectively.
5. Conclusion
The main contributions of this paper are as follows. The new obstacle avoidance algorithm based on the model predictive control for autonomous marine vehicles was proposed. It takes into account the nonlinear vehicle dynamics and restrictions on the phase variables, and control defining forbidden circle areas to the vehicle. The MPC linearization is performed in the vicinity of the predicted trajectory of the vessel. An adaptation of the algorithm to time-varying number of obstacles according to information coming in real-time from sensors and the return to the prescribed trajectory after obstacles avoidance is provided. The results of simulations show the effectiveness of the proposed algorithm.

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References
[1] Rimon E Koditschek D 1992 Exact robot navigation using artificial potential functions IEEE Trans. Robot. Autom. 8(5) 501–518
[2] Khatib O 1995 Real-time obstacle avoidance for manipulators and mobile robots International J. of Robotics Research 5(1) 90–98
[3] Fox D Burgard W, and S Thrun S 1997 The Dynamic Window Approach to Collision Avoidance IEEE Robotics & Automation Magazine 4(1) 23–33
[4] Yoon Y Shin J Kim J, Park Y and Sastry S. 2009 Model predictive active steering and obstacle avoidance for autonomous ground vehicles Control Engineering Practice 17(7), 741–750
[5] Skorohod B 2018 Receding Horizon Unbiased FIR Filters and their Application to Sea Target Tracking J. of Control Science and Engineering 5 1–14
[6] Moreno-Salinas D Chaos, Manuel de la Cruz and Aranda J 2013 Identification of a Surface Marine Vessel Using LS-SVM J. of Applied Mathematics Article ID 803548 http://dx.doi.org/10.1155/2013/803548