Design of Attitude Loop Controller for Six-Rotor UAV Based on L1 Adaptive Method

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Abstract. As for the non-modeling dynamics of the six-rotor UAV during the modeling process, and the presence of uncertain external interferences such as crosswinds during flight, the L1 adaptive control method is used to design the attitude controller of the six-rotor UAV. The force analysis of the six-rotor UAV is performed first to establish a nonlinear dynamic model, and then the nonlinear factors and coupling terms of the model are considered as time-varying parameters and disturbances. The low-pass filter is introduced based on the traditional model reference adaptive algorithm, and the L1 adaptive attitude controller is designed to effectively suppress the high-frequency interference caused by mechanical vibrations of the aircraft. Finally, simulation results show that the algorithm has good dynamic performance and robustness while ensuring system stability.

1. Introduction
The six-rotor aircraft has the characteristics of vertical take-off and landing, hovering in the air, and has a simple control structure, low cost, and a certain load capacity, which can complete many inconvenient tasks for fixed-wing aircraft. It is precisely because of these characteristics of the Helicopter, it has been widely used in many industries and fields such as scientific research, civil commerce, military, forestry, agriculture, power, fire protection, etc. And the military’s attention has been greatly developed.

As a typical under-actuated nonlinear system, the position control process of the six-rotor aircraft is controlled by attitude control process, and all movements are based on the control of six motors. Therefore, the design of the six-wing aircraft attitude controller is the key to achieve its stable control. At present, there are many theoretical achievements in the research on attitude control of rotorcraft at home and abroad. For example, the paper [3] designed an attitude angle PID controller for a quadrotor. Based on PID method, the papers [4][5] proposed a fuzzy PID-based controller, and a control method based on neural network PID control for a quadrotor aircrafts proposed by the papers [6][7], which greatly enriched the PID control related content of the rotorcraft. The papers [8][9] proposed backstepping rotorcraft control method, its control effect is improved compared to the traditional PID control method, mainly in shorter adjustment time and higher stability. For rotorcraft with uncertain effects such as non-modeling dynamics and external interference, the references [10] [11] proposed a sliding mode control method, and the references [12][13] proposed a dynamic surface control (DSC) method base on sliding mode control. With the progressive application of these advanced control theories in rotorcraft control, it has shown great advantages in terms of fault-tolerant control and anti-interference. However, through
the above methods, the flight performance of the aircraft will be reduced, the mission load will be reduced, and the control accuracy will also be reduced when interference occurs, which will bring great harm to the rotorcraft system.

In this paper, a six-rotor unmanned aerial vehicle is taken as the research object, and an attitude inner loop controller based on L1 adaptive control method to eliminate uncertain factors such as external interference is designed. Through the introduction of adaptive law and state observer, real-time estimation of uncertainty disturbances and online observation of system states are achieved. By introducing a low-pass filter, the system’s adaptiveness and robustness are decoupled, which effectively suppresses the high-frequency noise signal caused by mechanical vibration of the aircraft breaks and break through the time-varying parameter frequency limitation inherent in the traditional model reference adaptive control algorithm. It can effectively track the target signal under adverse conditions such as external interference and unmodeled dynamics, and has good control performance, which improves the anti-interference ability and robustness of the system.

2. Dynamic model

The structure of the six-rotor aircraft is shown in Figure 1, which consists of a support, a platform and a power system. The main role of the support is to support the entire six-rotor aircraft on the ground, to provide a buffer for the six-rotor aircraft during landing, to prevent damage to the fuselage and electronic components caused by excessive force during landing; the main role of the platform is to provide space for the six-rotor aircraft control system, sensors and power supply; the main role of the power system is to provide flying power for the six-rotor aircraft.

![Structure diagram of the six-rotor aircraft.](image)

Based on the Newton-Euler equation to model the overall six-rotor UAV, let $\Omega = [p \ q \ r]^T$ and the velocity vector in the body coordinate system be $V = [u \ v \ w]^T$, then

\[
\begin{cases}
F = m\dot{V} + \Omega \times (mV) \\
M = I\dot{\Omega} + \Omega \times (I\Omega)
\end{cases}
\]  

(1)
In the above formulas, $F$ represents the combined external force that the six-rotor UAV receives in the body coordinate system, and $M$ represents the combined external torque that the six-rotor UAV receives in the body coordinate system. The external forces that the six-rotor UAV receives during flight are mainly gravity, the lift and resistance generated by the rotor, and the resistance between the body and the air. The external moments mainly include the aerodynamic moment generated by the rotor, the roll moment, the moment of inertia, air friction torque and rotor gyro effect to the body's torque. To simplify the modeling process, the following assumptions are made:

1. The body of the six-rotor aircraft is rigid, and the center of mass and the center of gravity of the structure coincide.
2. Except for the main rotor inertia, the rotor inertia of the six-rotor aircraft body is 0.
3. The resistance and torsional friction moments suffered by the rotor itself are ignored.

Based on the above assumptions, the dynamic model of the system can be obtained:

$$
\begin{align*}
\dot{x} &= (A_m + \Delta A)x(t) + B_m \omega u(t) + f(t, x(t)), \quad x(0) = x_0 \\
y(t) &= Cx(t)
\end{align*}
$$

where $x(t) \in R^n$ is the observable 6-dimensional system state, $u(t) \in R^m$ is the system 6-dimensional control input $(m \leq n)$, $y(t) \in R^p$ is the 3-dimensional system control output, and $A_m = A - BK\Sigma$ is the known $n \times n$ Herwoods matrix (the real parts of all eigenvalues are negative). $\Delta A \in R^{n \times n}$ is the uncertain parameters of the system generated during the modeling process, $K\Sigma$ is state feedback gain. $B_m \in R^{m \times n}$ is the known full-rank constant matrices, and $(A_m, B_m)$ is controllable. $C \in R^{p \times n}$ is the known constant full-rank matrices, and $(A_m, C)$ is observable. $\omega \in R^{m \times n}$ is uncertain system input gain matrix, and $f : R \times R^n \times R^m \to R^n$ is the unknown nonlinear part of the system.

3. Design of the controller

3.1 Description of attitude loop control problem.

The dynamic model of the attitude loop of a six-rotor aircraft with system uncertainty can be written as follows:[14]:

$$
\begin{align*}
\dot{x} &= (A_m + \Delta A)x(t) + B_m \omega u(t) + f(t, x(t)), \quad x(0) = x_0 \\
y(t) &= Cx(t)
\end{align*}
$$

where $x(t) \in R^n$ is the observable 6-dimensional system state, $u(t) \in R^m$ is the system 6-dimensional control input $(m \leq n)$, $y(t) \in R^p$ is the 3-dimensional system control output, and $A_m = A - BK\Sigma$ is the known $n \times n$ Herwoods matrix (the real parts of all eigenvalues are negative). $\Delta A \in R^{n \times n}$ is the uncertain parameters of the system generated during the modeling process, $K\Sigma$ is state feedback gain. $B_m \in R^{m \times n}$ is the known full-rank constant matrices, and $(A_m, B_m)$ is controllable. $C \in R^{p \times n}$ is the known constant full-rank matrices, and $(A_m, C)$ is observable. $\omega \in R^{m \times n}$ is uncertain system input gain matrix, and $f : R \times R^n \times R^m \to R^n$ is the unknown nonlinear part of the system.

3.2 Design of the control law

As shown by the dotted line in Figure 2, the structure of the L1 adaptive controller is composed of a state estimator, a parameter adaptive law, and a control law with a low-pass filter. The role of the state estimator is to estimate the real-time state of the system and its changing laws. The role of the adaptive law is to adjust the parameters estimated by the state estimator. The controller can be divided into a
preliminary control law and a low-pass filter $C(x)$ that suppresses high-frequency signals, and the preliminary control law is designed by the tracking signal of the state estimator and the parameters adjusted by the adaptive law. The high-frequency signal in the control signal is filtered by a low-pass filter, and the low-frequency signal is input into the system, to achieve the desired performance[15].

![L1 adaptive controller](image)

Figure 2. Structure diagram of the L1 adaptive controller.

(1) State estimator
As for the system (3), the state estimator is designed as follow:

\[
\dot{x} = A_m \dot{x}(t) + B_m (\hat{\omega}(t)u(t) + \hat{\theta}_1(t)\parallel x\parallel_{\infty} + \hat{\sigma}_1(t)) + B_{sm} (\hat{\theta}_2(t)\parallel x\parallel_{\infty} + \hat{\sigma}_2(t))
\]

\[
\hat{y}(t) = C\dot{x}(t)
\]

In above formula, $\hat{\omega}(t) \in R^{m \times n}$, $\hat{\theta}_1(t) \in R^m$, $\hat{\theta}_2(t) \in R^{n-m}$, $\hat{\sigma}_1(t) \in R^n$, $\hat{\sigma}_2(t) \in R^{m-m}$ both are adaptive estimates of parameters.

(2) Parameter adaptive law
The adaptive laws of $\hat{\omega}(t), \hat{\theta}_1(t), \hat{\sigma}_1(t), \hat{\theta}_2(t), \hat{\sigma}_2(t)$ are defined as follows:

\[
\dot{\hat{\omega}}(t) = \Gamma \text{Pr} \text{oj}(\hat{\omega}(t), -(\hat{x}^T(t)PB_m)^T u^T(t))
\]

\[
\dot{\hat{\theta}}_1(t) = \Gamma \text{Pr} \text{oj}(\hat{\theta}_1(t), -\hat{x}_1(t)PB_m)^T x\parallel_{\infty})
\]

\[
\dot{\hat{\sigma}}_1(t) = \Gamma \text{Pr} \text{oj}(\hat{\sigma}_1(t), -\hat{x}_1(t)PB_m)^T)
\]

\[
\dot{\hat{\theta}}_2(t) = \Gamma \text{Pr} \text{oj}(\hat{\theta}_2(t), (\hat{x}_1(t)PB_m)^T_\infty)
\]

\[
\dot{\hat{\sigma}}_2(t) = \Gamma \text{Pr} \text{oj}(\hat{\sigma}_2(t), (\hat{x}_1(t)PB_m)^T)
\]

Where $\hat{x}^T = \hat{x}(t) - x(t)$, $\Gamma \in R^+$ is the adaptive gain. For arbitrary $P = P^T > 0$, it satisfies Lyapunov’s equation, $A_m^T P + P A_m = -Q$. $\text{Pr} \text{oj}(\cdot)$ represents the projection operator.

(3) Control law
The control law of the system can be designed as:

\[
u(s) = -KD(s)\dot{\eta}(s)
\]
in which $K \in \mathbb{R}_{>0}^m$ is the bandwidth of the filter, $D(s)$ must be a strictly real transfer function to ensure that the filter is strictly stable. In order to ensure the low-pass gain of the filter be 1, $D(s)$ must include an integral link $\frac{1}{s} I_n$, $\tilde{\eta}(s)$ is the Laplace transform of $\tilde{\eta}(t)$.

$$\tilde{\eta}(t) = \tilde{\omega}(t)u(t) + \tilde{\eta}_1(t) + \tilde{\eta}_{2m}(t) - r_g(t)$$

Where

$$r_g(s) = K I(s)$$

$$\tilde{\eta}_{2m}(s) = (C(sI_n - A_m)^{-1} B_m)^{-1} (C(sI_n - A_m)^{-1} B_{uu}) \tilde{\eta}_2(s)$$

And $\tilde{\eta}_1(t)$ and $\tilde{\eta}_2(t)$ can be defined as

$$\tilde{\eta}(t) = \tilde{\theta}(t)\|x\|_u + \tilde{\sigma}(t), i = 1, 2$$

3.3 Stability analysis.
Define

$$\theta_m(\rho) = 4(\max_{oct \in \Omega} \mathrm{tr}(\omega^T \omega) + (\sigma^2_1 + \sigma^2_2)m + (\sigma^2_1 + \sigma^2_2)(n - m)) +$$

$$4 \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)} ((\theta_1 d_{\theta_1} + \sigma_{\theta_1} d_{\sigma_1})m + (\theta_2 d_{\theta_2} + \sigma_{\theta_2} d_{\sigma_2})(n - m))$$

Select Lyapunov function

$$V(\tilde{x}(t), \tilde{\omega}(t), \tilde{\theta}(t), \tilde{\sigma}(t))$$

$$= \tilde{x}^T(t)P(t)\tilde{x}(t) + \frac{1}{\Gamma} (\mathrm{tr}(\tilde{\omega}^T(t)\tilde{\omega}(t)) + \sum_{i=1}^{2} (\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \tilde{\sigma}_i^T(t)\tilde{\sigma}_i(t)))$$

Differentiate the Lyapunov function, it can be obtained that for any $\tau \geq 0$, then for any $t \in [0, \tau]$, there are the following boundary conditions

$$\|\tilde{\theta}(t)\|_x < \theta_{\theta_1} = \theta_{\theta_2}(\rho)$$

$$\|\tilde{\theta}(t)\|_x < d_{\theta_1} = d_{\theta_2}(\rho)$$

$$\|\tilde{\sigma}(t)\|_x < \sigma_{\theta_1} = \sigma_{\theta_2}(\rho)$$

$$\|\tilde{\sigma}(t)\|_x < d_{\sigma_1} = d_{\sigma_2}(\rho)$$

Let $\tau_i \in (0, \tau]$ be the first discontinuity point of $\tilde{\theta}(t)$ or $\tilde{\sigma}(t)$. When using the equation (5) and the parameter adaptive law with the projection operator, for all $t \in [0, \tau_1]$, the equation (11) has an upper bound

$$\dot{V}(t) \leq -\tilde{x}^T(t)Q\tilde{x}(t) + \frac{2}{\Gamma} \sum_{i=1}^{2} (\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \tilde{\sigma}_i^T(t)\tilde{\sigma}_i(t))$$

Since $\tilde{\theta}(t)$ and $\tilde{\sigma}(t)$ are both continuous at any time $t \in [0, \tau_1]$ and according to the boundary conditions, it can be obtained

$$\dot{V}(t) \leq -\tilde{x}^T(t)Q\tilde{x}(t) + \frac{4}{\Gamma} (\theta_1 d_{\theta_1} m + \sigma_{\theta_1} d_{\sigma_1} m + \theta_2 d_{\theta_2} (n - m) + \sigma_{\theta_2} d_{\sigma_2} (n - m))$$

Because the projection operator ensures that for any $t \in [0, \tau_1]$, there is
If there are for all $\tau' \in \{0, \tau_1\}$, then $V(\tau') > \frac{\Theta_m(\rho_c)}{\Gamma}$, so that combining the definitions of Lyapunov functions (9) and the definition of $\Theta_m(\rho_c)$ in equation (8), we can know that

$$\ddot{\mathbf{x}}^T (\tau') \mathbf{P} \dot{\mathbf{x}}(\tau') > \frac{4 \lambda_{\max}(P)}{\lambda_{\min}(Q)} \left( \Theta_{\hat{\omega}} d_{\hat{\omega}} m + \Theta_{\hat{\theta}_m} d_{\hat{\theta}_m} (n-m) + \Theta_{\hat{\sigma}} d_{\hat{\sigma}} (n-m) \right)$$

then

$$\ddot{\mathbf{x}}^T (\tau') Q \dot{\mathbf{x}}(\tau') \geq \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} \mathbf{P} \ddot{\mathbf{x}}^T (\tau') \mathbf{P} \dot{\mathbf{x}}(\tau')$$

$$> \frac{4 \Gamma}{\lambda_{\max}(P)} (\Theta_{\hat{\omega}} d_{\hat{\omega}} m + \Theta_{\hat{\theta}_m} d_{\hat{\theta}_m} (n-m))$$

Finally, according to the equation (12) and equation (15), it can be obtained

$$V(\tau') < 0$$

Therefore, the stability of the system based on L1 adaptive control law is proven.

4. Simulation and analysis

4.1 Parameters of the simulation system.

The parameter matrix of the inner loop model of the six-rotor aircraft system is as follows:

$$A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}_{6 \times 6}$$

$$B = \begin{bmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \end{bmatrix}_{6 \times 3} = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}_{6 \times 3}$$

$$C = [I_{3 \times 3} \ 0_{3 \times 3}]_{3 \times 6}$$

$$\omega = \begin{bmatrix} 0.6 & -0.2 & 0.1 \\ 0.2 & 1.2 & 0.2 \\ -0.2 & 0.2 & 1 \end{bmatrix}_{3 \times 3}$$
The outer ring of the six-rotor aircraft will be controlled by PID control method. The specific parameters of the PID controller are shown in Table 1.

Table 1. PID controller parameters of the outer loop.

|   | P | I | D |
|---|---|---|---|
| x | 1 | 0 | 2 |
| y | 2 | 0 | 2 |
| z | 3 | 0 | 3 |

4.2 Experimental simulation and analysis.

Initial state: \((\phi \ \theta \ \psi)^T = (0 \ 0 \ 0)^T\)

Target state: \((\phi \ \theta \ \psi)^T = (2 \ 5 \ 10)^T\)

Note: The meaning of the axis symbols in all the figures below is as follows:
Attitude angle (unit: rad): yaw angle \(\psi\), roll angle \(\phi\), pitch angle \(\theta\).

It can be seen from Figure 3 that the each state of the simulation can reach a stable state at about 8 seconds, indicating that the inner attitude loop of the six-rotor aircraft can be controlled using the L1 adaptive control method, but the pitch angle tracking has a certain steady-state error that will cause the
direction x displacement and direction y displacement to reach the desired position slightly longer, which will lead to a longer time for the outer position ring to stabilize.

Under the influence of crosswind interference so that the efficiency of rotor 1 dropped to 50%, the simulation comparison between the L1 adaptive method and the PID method was performed using the roll angle as an example.

![Comparison of L1 method and PID method on interference state](image)

Figure 4. Comparison of L1 method and PID method on interference state.

It can be seen from Figure 4 that after the rotor 1's efficiency is reduced due to external interference, both the L1 adaptive control method and the PID method can be effectively adjusted to make the roll angle return to the target state. However, the L1 adaptive control has faster convergence speed, smaller overshoot, and better dynamic performance and robustness.

5. Conclusion
In this paper, the attitude control of a six-rotor aircraft is taken as the research object. Based on the L1 adaptive control, an attitude inner-loop controller that can eliminate uncertain effects such as unmodeled dynamics and external interferences is designed. Based on the traditional model reference adaptive algorithm, a low-pass filter is introduced, and the L1 adaptive attitude controller is designed, which can effectively suppress the high-frequency interference caused by mechanical vibrations of the aircraft and break through inherent frequency limitation of time-varying parameters of the traditional model reference adaption control algorithm, and has strong anti-interference ability and good robustness. Simulation results show that the designed L1 adaptive attitude inner loop controller can guarantee the stable control with good performance of the UAV, and has certain practical value.

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