Reasoning about Object-Z formal specification with LTLC

Haijun Huang1 and Zhicheng Wen1,*
1Jiangxi University of Engineering, Xinyu, Jiangxi Province, 338000, China
*Corresponding author e-mail: zcwen@mail.shu.edu.cn

Abstract: It presents a method that linear temporal logic with clock (LTLC) is added to formal specification language Object-Z. LTLC Extending Object-Z is a modular formal specification language, which is the smallest extension for Object-Z. The complex real-time software formal specification can be described and verified easily by the extended Object-Z. Finally, an example is given to show that the formal specification of Object-Z can be reasoned and the correctness of the method is verified.

1. Introduction
Object-oriented formal specification is very suitable described by Object-Z[1]. Through the application of object-oriented technology[2-4], it can describe complex data and algorithms. There is only the abstract concept of termination and no concept of duration associated with operations. Therefore, a real-time reaction system is very difficult to be described by Object-Z. In order to describe the real-time specifications[5] about continuous time relationships or properties, this paper introduces a method that Object-Z is extended with Linear Temporal Logic with Clock(LTLC). The extended Object-Z using LTLC is the smallest extension of Object-Z syntax and semantics. For object-oriented real-time system modeling, it can provide an elegant symbol. In addition, the verification becomes convenient. Finally, this article uses an example to indicate that it can infer the Object-Z formal specification to verify that formal specification developed is correct.

2. Adding LTLC to Object-Z
2.1 Syntax for extended Object-Z
The syntax for extending Object-Z, we will use LTLC to introduce in this section. In general, symbolic logic formulas in the standard Object-Z are used to describe the invariants of classes and the predicates of operations. You can use LTLC formula to describe the invariants of the class and an operation predicates in the extended Object-Z.

2.1.1 Structure for Object-Z class. An optional operation MAIN is introduced in the extended Object-Z class. There are several nondeterministic operations[6,7] in this class structure. In Figure 1, the structure is shown where the clock variable $t_{now}$ can be used to initialize the class system. In an operation, it is only available for the other clock variables defined in a corresponding operation (explained in the next section).


2.1.2 Structure for Object-Z operation. The operation predicate part of an extended Object-Z mode consists of some optional parts \(^8\). The operation mode structure is shown in Figure 2.

**Definition:** The two corresponding clock variables are \(t_v\) and \(t'_v\) for any variable \(v\).

2.2 Example

For example, there are three alternate signals of road traffic lights, red, yellow and green. Four indicator lights (green, yellow, red) and four states (green, green, yellow, red, yellow, red, yellow) are a simple example for the intersection indicator system. First, the green light lasts 60 seconds (green state). There is a central (green, yellow) transition state after the green light is switched to the red light. This will last 3 seconds. Then, red light for 50 seconds is held (red state). After that, the red light will switch to green light, but also shifted to the middle (red, yellow) which lasts for 4 seconds. These four signal states may change in sequence.

If the state variable is \(p\) in the state schema, it can be seen the state conversion defined, and it is clear that the corresponding clock variable is \(t_p\) and \(t'_p\). This state variable \(p\) is a delay amount, and therefore corresponds to a delay conversion. It is shown for extended Object-Z style operation light switch in Fig. 3.

\[
\begin{align*}
\Delta (p) \\
\text{t}_p, \text{t}'_p: \text{Time} \\
\text{t}_p=0 \land \\
((p=1 \land t_p \leq 60) \land (p'-2 \land t'p > 60 \land t'p = 0) \\
\lor ([p=2 \land t_p \leq 3] \land (p'=3 \land t'p > 3 \land t'p = 0) \\
\lor ([p=3 \land t_p \leq 50] \land (p'=4 \land t_p > 50 \land t'p = 0) \\
\lor ([p=4 \land t_p \leq 4] \land (p'=1 \land t_p > 4 \land t'p = 0))
\end{align*}
\]

Fig 3 Operation lightswitch

It indicates for a predicate part that the clock variable \(t_p\) starts first and it may convert from current state to the next state which will continue for a while. The integers 1, 2, 3 and 4 simply present the corresponding four states. For example, the notation of \([p=2 \times t_p \leq 3] \land (p'=3 \times t_p > 3 \times t'_p = 0)\) means that the current state 2 will last for 3 seconds, but the clock variable is reset last (\(t'_p = 0\)).
To better illustrate the use of extended Object-Z to describe formal specifications, here is an example. The extension Object-Z specification is considered as follows: describing a formal system that includes two classes of console and buttons.

The feature of a button object can be described by the class button (shown in Fig. 4). The button contains the state variable recording whether the button is in the "0" or "1" position. First, the button object is in state 0. You just want to think the specific time $m_{t1}$ you can switch between cities of buttons. As a result, the state of "0 / 1" is changed. This operation is designated by a toggle operation. Therefore, button $on$ is a toggle delay variable corresponding to two delay transformations.

The corresponding clock variable: $t$ and $t'$ are defined. The first line of the button class specification indicates that the interface between the button class and the environment object is limited to the execution and manipulation toggle of the initialization schema. Since toggle operations include only delay variables, it does not lead to time-out and is not required to be combined with restoration. The current time has two constraints every time ($[\mathbf{t}_\text{now}>\text{sometime}$, it must be time invariant) and the delay time $m_{t1}$ is smaller than the maximum delay class (updelay). The clock variable $t_{\text{now}}$ is defined from any class.

4. Reasoning formal specification
Formal verification includes model checking and theorem proving. In this section, we prove the extended Object-Z by theorem proving. You can infer the characteristics and behavior of the specification. The inference rules and methods of standard object Z reasoning are shown in [12]. The properties of this class are expressed as $A:: d$ $\vdash \Psi$ $\Phi$. When the statements d and predicate are given, when the order of at least one predicate is true, the predicate is valid. That is to say, the predicate attribute is true. Variables and constants only define the predicate "$\Phi". It is accessible within the class, or is declared $d$.
Console

↑ (Init, ToggleOneButton, ShutDown)
| mt2, mt3, mt4, mt5: Time
| max: N

busyv, lazyv: Button
busy_c, lazy_c: N

[[busylazy]](t_{now}>0)

Init

busy_c=0 ∧ lazy_c=0
busyv.Init ∧ lazyv.Init[optional]

Operation: SelectAndIncrement\(_0\) (as shown in Fig 6.)
Operation: ShutDown\(_0\) (as shown in Fig 7.)

SelectAndIncrement\(_0\) \(_\Delta\) \{t\} Restore
ShutDown\(_0\) \(_\Delta\) \{t\} Restore
ToggleOneButton[SelectAndIncrement \(b?\).Toggle
MAIN=(ToggleOneButton[ShutDown] ; MAIN

Fig 5. Class Console

Fig 6. Class SelectAndIncrement\(_0\)

Fig 7. Class Console

4.1 Reasoning the state variables

We may consider invariants, pre- and postconditions developed by Object-Z for LTLC. The symbol invariants(x_var) in the state pattern represent invariants. In the state mode of class Class, variable X represents the state variables. It has:

Precondition\(_{op}\) = \(\forall x_{var}:X; output\_var: OUTPUT\)
OP(x_var, x_var’, input_var, output_var) ∧ Invariant(x_var) ∧ Invariant(x_var’)
Postcondition\(_{op}\) = Invariant(x_var’) ∧ OP(x_var’, output_var)

The postcondition of the operation keeps the predicate expression, containing poststate or output
variables. Precondition and postcondition state variables can be expressed synchronously in predicates.

Similarly, in real-time object-Z, efficient classes have invariants, and efficient operations have their pre- and postconditions.

1) Invariant rule:
   \[ \text{InvariantClass}(x_{\text{var}}, t_{\text{now}}) = \text{Invariant}_{\text{filter}}(t_{\text{now}}) \land \text{InvariantClass}(x_{\text{var}}) \]

For the Console operation, we have:

\[ \text{InvariantConsole}(x_{\text{var}}, t_{\text{now}}) = (t_{\text{now}} > 0) \land (\text{busy} \land \text{lazy}) \]

2) Precondition rule:
\[ \text{PreOP} = \text{PreOP}(x_{\text{var}}, x_{\text{var}'}, T_{\text{variable}}, T_{\text{variable}'}) \]

3) Postcondition rule:
\[ \text{PostOP} = \text{Invariant} \land \text{PostOP}(x_{\text{var}}, x_{\text{var}'}, T_{\text{variable}}, T_{\text{variable}'}) \]

Where, \( T_{\text{variable}} \) represents the time variable; \( \text{PreOP}(x_{\text{var}}, x_{\text{var}'}, T_{\text{variable}}, T_{\text{variable}'}) \) represents the preconditions, and can be calculated using this method. In normalization of object Z, \( \text{InvariantClass}(t_{\text{now}}), \text{PostOP}(x_{\text{var}}, x_{\text{var}'}, t, t') \) and \( \text{InvariantClass}(x_{\text{var}}, t) \) have similar meanings.

4.2 Reasoning the properties

For the class \( \text{Button} \) of the example above, we have:

\[ \text{Button:: Toggle} \vdash ((\text{on}'=1 \land t_{\text{on}}>m_{t}) \land (t_{\text{on}}'=0)) \lor ((\text{on}'=0 \land t_{\text{on}}=m_{t}) \land (t_{\text{on}}>0) \land \text{busy} \land \text{lazy}) \]

and

\[ \text{Button:: Toggle} \vdash ((\text{on}'=1 \land t_{\text{on}}>m_{t}) \land (t_{\text{on}}'=0)) \lor ((\text{on}'=0 \land t_{\text{on}}=m_{t}) \land (t_{\text{on}}>0) \land \text{busy} \land \text{lazy}) \]

Then, we can use the theorem, it has:

\[ \text{Button:: Toggle} \vdash ((\text{on}'=1 \land t_{\text{on}}>m_{t}) \land (t_{\text{on}}'=0)) \lor ((\text{on}'=0 \land t_{\text{on}}=m_{t}) \land (t_{\text{on}}>0) \land \text{busy} \land \text{lazy}) \]

and

\[ \text{Button:: Toggle} \vdash ((\text{on}'=1 \land t_{\text{on}}>m_{t}) \land (t_{\text{on}}'=0)) \lor ((\text{on}'=0 \land t_{\text{on}}=m_{t}) \land (t_{\text{on}}>0) \land \text{busy} \land \text{lazy}) \]

That is, in the Toggle operation, \( \text{on}=0 \land t_{\text{on}}=m_{t} \) remains until \( \text{on}'=1 \land t_{\text{on}}>m_{t} \land t_{\text{on}}'=0 \lor \text{on}=1 \land t_{\text{on}}\leq m_{t} \)

and \( \text{on}=0 \land t_{\text{on}}\leq m_{t} \) remains until \( \text{on}'=0 \land t_{\text{on}}>m_{t} \land t_{\text{on}}'=0 \). Symbol \( t_{\text{on}}'=0 \) indicates a reset of clock variables. It indicates that the switching state changes within 0/1 and the corresponding delay time is \( m_{t} \), which is consistent with the design requirements.

5. Conclusion

In this paper, we propose a method for extending LTLC to Object-Z. The extension with multithreaded module style language LTLC is the minimum extension of Object-Z, and it is useful to describe and verify formal specifications of complex system. Finally, use an example to instruct the Object-Z format specification to make sure it is correct.

Acknowledgements

Haijun Huang (1980.11 -), male, Han, NanHang city, Jiangxi Province, master’s degree, associate professor, majoring in: communication engineering and electronic science. Corresponding author: Zhicheng Wen (1972.11 -), male, Han, Dong’an county, Hunan province, Ph.D., professor, majoring in: network security, trusted software.

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