The exact electron propagator for an electron in a constant uniform magnetic field as the sum over Landau levels is obtained by the direct derivation by standard methods of quantum field theory from exact solutions of the Dirac equation in the magnetic field. The result can be useful for further development of the calculation technique of quantum processes in an external active medium, particularly in the conditions of moderately large field strengths when it is insufficient to take into account only the ground Landau level contribution.

Keywords: Exact electron propagator; quantum processes calculation technique; external active medium

PACS numbers: 14.60.Cd, 12.20.Ds, 02.30.Gp

1. Introduction

Among astrophysical objects there exists a class of neutron stars which are called magnetars. Magnetic field values exceed there the critical value of $B_c = m_e^2/e \simeq 4.41 \times 10^{13}$ G, where $m_e$ is the electron mass. There is no longer possible to take into account the field influence by the perturbations theory under such conditions in the analysis of quantum processes. The detailed description of the calculation technique of processes in external fields one can find, for example, in reviews and books, see Refs. 1–6.

The magnetic field influence on the particle properties is determined by the specific charge, i.e. by the particle charge and mass ratio. Hence, the charged fermion which is the most sensitive to the external field influence is the electron. The calculations of specific physical phenomena in strong external field are based on the application of Feynman diagram technique generalization. It consists in the following procedure: in initial and final states the electron is described by the exact

---

*We use the Planck units: $\hbar = 1$, $c = 1$, and the Minkowski metric with the signature $(+, - , - , - )$. 
solution of the Dirac equation in the external field, and internal electron lines in quantum processes correspond to exact propagators that are constructed on the basis of these solutions.

The expression for the exact electron propagator in the constant uniform magnetic field was obtained by J. Schwinger in the Fock proper-time formalism. There are another propagator representations given in a number of works. Thus, in Ref. [9] the case was considered of superstrong field and the contribution of the ground Landau level to the electron propagator was obtained. In Ref. [10] see also Ref. [11], the propagator was transformed from the form of Ref. [7] into the sum over Landau levels. Also in Ref. [11] the electron propagator decomposition over the power series of the magnetic field strength was given.

In our opinion, it is quite important to know different representations of the electron propagator in the external magnetic field and the conditions of their applicability. There were some examples where misunderstanding of such conditions has led to erroneous papers. Thus, in Refs. [12, 13] the self-energy operator of neutrino in the magnetic field was calculated by the analysis of the one-loop diagram $\nu \rightarrow e^{-} W^{+} \rightarrow \nu$. The authors of the paper restricted themselves by consideration of the ground Landau level contribution to the electron propagator. As was shown in Ref. [14] the ground Landau level contribution is not dominant and the next levels give the contributions of the same order of magnitude.

As we know, there is no such methodologically important issue in the literature as a direct derivation by the standard quantum field theory methods of the exact electron propagator in the external magnetic field in the form of the sum over Landau levels from the exact solutions of the Dirac equation in a magnetic field. The present paper is intended to fill this gap. The exact solution of the Dirac equation for an electron in the external magnetic field on the $n$th Landau level is given in Sec. 2. On the basis of these solutions, the detailed derivation of the electron propagator is performed in Sec. 3. As a result, the propagator is written in $x$-representation as the sum over Landau levels. In Sec. 4 the identity is shown of the obtained expression for the propagator to the known result [10].

2. The Solution of the Dirac Equation for an Electron in the Magnetic Field

The Dirac equation for an electron with the mass $m$ and the charge $(-e)$, where $e > 0$ is the elementary charge, in magnetic field $B$ with the 4-potential $A^{\mu}(X)$, where $X$ denotes the 4-vector, $X^{\mu} = (t, x, y, z)$, takes the form:

$$\left(i (\partial \gamma) + e (A \gamma) - m\right) \Psi(X) = 0,$$

(1)

where $(\partial \gamma) = \partial_{\mu} \gamma^{\mu}$, $(A \gamma) = A_{\mu} \gamma^{\mu}$. For the case of the constant uniform magnetic field directed along the $z$-axis, choosing the 4-potential as $A^{\mu} = (0, 0, x B, 0)$, it is possible to write down the so-called solutions with positive and negative energy.
The solution with positive energy is:

$$
\Psi^{(+)}_{n, p_y, p_z, s}(X) = \frac{e^{-i(E_n t - p_y y - p_z z)}}{\sqrt{2 E_n (E_n + m)}} U^{(+)}_{n, p_y, p_z, s}(\xi^{(+)}) ,
$$

where $n$ indicates the Landau levels: $n = 0, 1, 2, \ldots$; $p_y, p_z$ are the electron “momentum components” along $y$ and $z$ axes (see below); $L_y, L_z$ are the normalization sizes along $y$ and $z$ axes; $s = \pm 1$ is the quantum number related to spin; $E_n$ is the electron energy on the $n$th Landau level:

$$
E_n = \sqrt{m^2 + p_z^2 + 2 \beta n} , \quad \beta \equiv eB .
$$

The bispinor $U^{(+)}$ has different forms for the cases $s = -1$ and $s = +1$:

$$
U^{(+)}_{n, p_y, p_z, s=-1}(\xi^{(+)}) = \begin{pmatrix}
0 \\
(E_n + m) V_n(\xi^{(+)}) \\
-1 \sqrt{2 \beta n} V_{n-1}(\xi^{(+)}) \\
-p_z V_n(\xi^{(+)})
\end{pmatrix} ,
$$

$$
U^{(+)}_{n, p_y, p_z, s=+1}(\xi^{(+)}) = \begin{pmatrix}
(E_n + m) V_{n-1}(\xi^{(+)}) \\
0 \\
p_z V_{n-1}(\xi^{(+)}) \\
1 \sqrt{2 \beta n} V_n(\xi^{(+)})
\end{pmatrix} ,
$$

and for the ground Landau level, $n = 0$, the solution exists just at $s = -1$. The variable $\xi^{(+)}$ is related with $x$ coordinate by the relation:

$$
\xi^{(+)} = \sqrt{\beta} \left(x + \frac{p_y}{\beta}\right) .
$$

$V_n(\xi)$ are the harmonic oscillator functions expressed in terms of the Hermite polynomials $H_n(\xi)$:

$$
V_n(\xi) = \frac{\beta^{1/4}}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\xi^2/2} H_n(\xi) , \quad H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n} e^{-\xi^2} ,
$$

$$
\int_{-\infty}^{+\infty} |V_n(\xi)|^2 dx = 1 .
$$

It should be noted that in the above expressions $p_z$ is the conserved momentum component of the electron along the $z$ axis, i.e. along the field direction, while $p_y$ is the generalized momentum, which determines the position of the center $x_0$ of the Gaussian packet along the $x$ axis, $x_0 = -p_y/\beta$, see [8].
The solution $\Psi^{(-)}$ with negative energy may be obtained from expressions (2), (4) – (6) by changing the sign of values $E_n, p_y, p_z$. The procedure of obtaining the solutions can be found, for example, in Ref. 6.

### 3. The Propagator Calculation on the Basis of the Dirac Equation Solutions

To calculate the electron propagator, the standard method is applied based on using the field operators which include the Dirac equation solutions in a magnetic field:

$$
\Psi(X) = \sum_{n, p_y, p_z, s} \left( a_{n, p_y, p_z, s} \psi^{(+)}_{n, p_y, p_z, s}(X) + b^{\dagger}_{n, p_y, p_z, s} \psi^{(-)}_{n, p_y, p_z, s}(X) \right) .
$$

Here $a$ is the destruction operator of the electron, $b^{\dagger}$ is the creation operator of the positron, $\psi^{(+)}$ and $\psi^{(-)}$ are the normalized solutions of the Dirac equation (1) in a magnetic field with positive and negative energy correspondingly.

The propagator is defined as the difference of time-ordered and normal-ordered productions of the field operators (8):

$$
S(X, X') = T(\Psi(X) \overline{\Psi}(X')) - N(\Psi(X) \overline{\Psi}(X')) .
$$

(9)

Using anticommutation relations for the creation and destruction operators, we obtain, that the propagator at $t > t'$ and at $t < t'$ is expressed in terms of the solutions with positive energy (2) and negative energy correspondingly:

$$
S(X, X') \bigg|_{t \gtrless t'} = \pm \sum_{n, p_y, p_z, s} \psi^{(\pm)}_{n, p_y, p_z, s}(X) \overline{\psi}^{(\pm)}_{n, p_y, p_z, s}(X') .
$$

(10)

Thus, the propagator is divided into the sum over Landau levels:

$$
S(X, X') = \sum_{n=0}^{\infty} S_n(X, X') .
$$

(11)

Further we will find the $n$th Landau level contribution into the propagator (10). It is convenient to come from the summation over the momenta $p_y$ and $p_z$ to the integration, by the substitution

$$
\frac{1}{L_y L_z} \sum_{p_y, p_z} \rightarrow \int \frac{dp_y dp_z}{(2\pi)^2} .
$$

(12)

For the $n$th level contribution we found:

$$
S_n(X, X') \bigg|_{t \gtrless t'} = \frac{1}{2 (\pm E_n)(\pm E_n + m)} \int \frac{dp_y dp_z}{(2\pi)^2} \times
$$

$$
\times \exp \{i [\mp E_n(t - t') \pm p_y(y - y') \pm p_z(z - z')]} \times
$$

$$
\times \sum_{s=\pm 1} \psi^{(\pm)}_{n, p_y, p_z, s}(\xi^{(\pm)}) \overline{\psi}^{(\pm)}_{n, p_y, p_z, s}(\xi^{(\pm)}) .
$$

(13)
The expression for energy (3) is taken into account: $E$ remains just in the sign at $p$.

One can see, that after changing the signs of integration variables $p$, where $p$ bispinors of the solution with negative energy, to: $\{ -1 \}$, which are constructed from the bispinors $\{ 1 \}$ and the corresponding bispinors of the solution with positive energy, to: $\{ 1 \}$, $\{ 1 \}$, $\{ 1 \}$, $\{ 1 \}$. As a result the propagator can be written at $(13)$, which taking into account $(14)$ and $(16)$ to the form of:

$$
\frac{1}{\sqrt{4\pi}} \exp \left[ -\frac{1}{2} (\xi^{(\pm)})^2 - \frac{1}{2} (\xi^{(\pm)})^2 \right] \left\{ (\pm E_n \gamma_0 \mp p_z \gamma^3 + m) \times \right.

\times \left[ \Pi - H_n(\xi^{(\pm)}) H_n(\xi^{(\pm)}) + \Pi + 2n H_{n-1}(\xi^{(\pm)}) H_{n-1}(\xi^{(\pm)}) \right] +

+ i 2n \sqrt{\beta} \gamma^1 \left[ \Pi - H_{n-1}(\xi^{(\pm)}) H_n(\xi^{(\pm)}) - \Pi + H_n(\xi^{(\pm)}) H_{n-1}(\xi^{(\pm)}) \right] \right\}, \tag{14}
$$

where the projection operators $\Pi_{\pm}$ are introduced:

$$
\Pi_{\pm} = \frac{1}{2} (I \pm i \gamma^1 \gamma^2), \quad \Pi_{\pm} \Pi_{\mp} = \Pi_{\pm}, \quad \Pi_{\pm} \Pi_{\mp} = 0. \tag{15}
$$

One can see, that after changing the signs of integration variables $p_y \rightarrow -p_y$ and $p_z \rightarrow -p_z$ in the expression $(13)$ at $t < t'$, the $\pm$ sign at $t > t'$ and $t < t'$ still remains just in the sign at $E_n$. It is appropriate to use the following relation, where the expression for energy $(3)$ is taken into account:

$$
\frac{f(\pm E_n)}{2 E_n} \left|_{t \leq t'} \right. \left. \int \frac{dp_y f(p_n) e^{-i p_0(t-t')}}{p^2 - m^2 - 2 \beta n + i \varepsilon}, \tag{16}
$$

where $p_y^2 = p_0^2 - p_z^2$. Hereafter we use the indices “$\parallel$” and “$\perp$” for denoting the 4-vector components, belonging to the pseudo-Euclidean subspace $(0, z)$ and the Euclidean plane $(x, y)$: $(ab)_\parallel = a_0 b_0 - a_z b_z, (ab)_\perp = a_x b_x + a_y b_y, (ab) = (ab)_\parallel - (ab)_\perp$.

Using the relation $(16)$ we add to the expression $(13)$ an integration over the zero momentum component. As a result the propagator can be written at $t > t'$ and at $t < t'$ identically. Renaming the variables $\xi^{(\pm)} = \xi, \xi^{(\pm)} = \xi'$, we reduce $(13)$ with taking into account $(14)$ and $(16)$ to the form of:

$$
S_n(X, X') = \left. \frac{1}{\sqrt{4\pi}} \exp \left( -\beta \frac{x^2 + x'^2}{2} \right) \int \frac{dp_0 dp_y dp_z}{(2\pi)^3} \times \right.

\times \left. \exp \left\{ -\frac{p_y^2}{\beta} - p_y [x + x' - i (y - y')] \right\} \times \right.

\times \left\{ \left[ (p_y)_\parallel + m \right] \left[ \Pi - H_n(\xi) H_n(\xi') + \Pi + 2n H_{n-1}(\xi) H_{n-1}(\xi') \right] +

+ i 2n \sqrt{\beta} \gamma^1 \left[ \Pi - H_{n-1}(\xi) H_n(\xi') - \Pi + H_n(\xi) H_{n-1}(\xi') \right] \right\}. \tag{17}
$$
It is worthwhile to note that the expression (11) with (17) for the electron propagator in a constant uniform magnetic field as the sum over Landau levels in the \( x \)-space has its own significance. In some cases, this form of the propagator can be more convenient than other representations.

One can make an integration over \( p_y \) in the propagator (17) by introducing a new variable

\[
u = \frac{p_y}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{2} \left[ x + x' - i(y - y') \right],
\]

and using the well-known integrals[15]

\[
\int_{-\infty}^{\infty} e^{-u^2} H_n(u + a) H_n(u + b) \, du = 2^n n! \sqrt{\pi} L_n(-2a b),
\]

\[
\int_{-\infty}^{\infty} e^{-u^2} H_n(u + a) H_{n-1}(u + b) \, du = 2^{n-1} n! \sqrt{\pi} \frac{1}{b} \left[ L_n(-2a b) - L_{n-1}(-2a b) \right],
\]

(18)

where \( L_n(x) \) are the Laguerre polynomials:

\[
L_n(x) = \frac{1}{n!} e^x \frac{d^n}{dx^n} (x^n e^{-x}).
\]

(19)

As a result, the \( n \)th Landau level contribution into the electron propagator in a magnetic field can be presented in the form:

\[
S_n(X, X') = e^{i\Phi(X, X')} \hat{S}_n(X - X'),
\]

(20)

where \( \Phi(X, X') \) is the translational and gauge non-invariant phase, which is equal for all Landau levels:

\[
\Phi(X, X') = -\frac{\beta}{2} (x + x')(y - y').
\]

For more details about properties of the phase, see, e.g., Ref. [6]. \( \hat{S}_n(Z) \) is the gauge and translational invariant part of the propagator \( (Z = X - X') \), represented in the form of the double integral over \( p_\parallel \):

\[
\hat{S}_n(Z) = \frac{i}{2\pi} \exp \left( -\frac{\beta}{4} Z_\perp^2 \right) \int \frac{d^2 p_\parallel}{(2\pi)^2} \frac{e^{-i(p\gamma)_\parallel}}{p_\parallel^2 - m^2 - 2\beta n + 1\varepsilon} \times \]

\[
\times \left\{ \left[ (p\gamma)_\parallel + m \right] \left[ \Pi_- L_n \left( \frac{\beta}{2} Z_\perp^2 \right) + \Pi_+ L_{n-1} \left( \frac{\beta}{2} Z_\perp^2 \right) \right] + \right.
\]

\[
\left. + 2 i n \frac{(Z\gamma)_\perp}{Z_\perp^2} \left[ L_n \left( \frac{\beta}{2} Z_\perp^2 \right) - L_{n-1} \left( \frac{\beta}{2} Z_\perp^2 \right) \right] \right\},
\]

(21)
4. The Electron Propagator in a Magnetic Field as the Sum over Landau Levels

Let us compare the obtained expression (21) with the available expansion of the propagator over Landau levels, given in Ref. [10]. In that paper the propagator had the form analogous to (20), but its gauge and translational invariant part was presented in the form of integral over 4-momentum (see note added in proof):

\[
\hat{S}(Z) = \int \frac{d^4p}{(2\pi)^4} S(p) e^{-i(pZ)} , \quad S(p) = \sum_{n=0}^{\infty} S_n(p) ,
\]

where

\[
S_n(p) = \frac{i}{p_\parallel^2 - m^2 - 2\beta n + i\varepsilon} \left[ (p\gamma)_\parallel + m \right] \left[ d_n(v) - \frac{i}{2} (\gamma \varphi \gamma) d_n'(v) \right] - 2n \frac{d_n(v)}{v} (p\gamma)_\perp , \quad v = \frac{p_\parallel^2}{\beta} .
\]

The functions \(d_n(v)\) have the form:

\[
d_n(v) = (-1)^n e^{-v} \left[ L_n(2v) - L_{n-1}(2v) \right] ,
\]

where \(L_n(x)\) are the Laguerre polynomials [19] with an additional definition \(L_{-1}(x) \equiv 0\). The expression \((\gamma \varphi \gamma) = \gamma_\alpha \varphi^{\alpha\beta} \gamma_\beta\) contains the dimensionless tensor of the external magnetic field \(\varphi^{\alpha\beta} = F^{\alpha\beta}/B\). In the frame, where the field is directed along the \(z\) axis, one has \((\gamma \varphi \gamma) = -2\gamma^1 \gamma^2\).

To ensure that our expression for the propagator (21) is consistent with Eqs. (22), (23), it is enough to perform in Eq. (22) the integration over the momentum components \(p_x, p_y\), which are transverse to the field. Thus, the \(n\)th Landau level contribution to the propagator is expressed via three different integrals \(I_{1,2,3}(Z_\perp)\) in the Euclidean plane \((p_x, p_y)\):

\[
\hat{S}_n(Z) = \int \frac{d^2p_\parallel}{(2\pi)^2} \frac{i e^{-i(pZ)_\parallel}}{p_\parallel^2 - m^2 - 2\beta n + i\varepsilon} \times
\]

\[
\times \left\{ \left[ (p\gamma)_\parallel + m \right] \left[ I_1(Z_\perp) - \frac{i}{2} (\gamma \varphi \gamma) I_2(Z_\perp) \right] - 2n I_3(Z_\perp) \right\} .
\]

An integration over the polar angle leads to the Bessel integral:

\[
\int_0^{2\pi} e^{i(\xi \cos \varphi - n \varphi)} d\varphi = 2\pi i^n J_n(\xi) ,
\]

where \(J_n(\xi)\) is the Bessel function. As a result, the integrals \(I_{1,2,3}(Z_\perp)\) take the
form:

\[ I_1(Z_\perp) = \int \frac{d^2 p_\perp}{(2\pi)^2} d_n(v) e^{i(p Z_\perp)} = \frac{\beta}{4\pi} \int_0^\infty dv J_0(\sqrt{\beta} Z_\perp \sqrt{v}) d_n(v), \]

\[ I_2(Z_\perp) = \int \frac{d^2 p_\perp}{(2\pi)^2} d'_n(v) e^{i(p Z_\perp)} = \frac{\beta}{4\pi} \int_0^\infty dv J_0(\sqrt{\beta} Z_\perp \sqrt{v}) d'_n(v), \]

\[ I_3(Z_\perp) = \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{d_n(v)}{v} e^{i(p Z_\perp)} (p \gamma) = \]

\[ = i \frac{\beta^{3/2}}{4\pi} \frac{(Z \gamma)_\perp}{Z_\perp} \int_0^\infty dv J_1(\sqrt{\beta} Z_\perp \sqrt{v}) \frac{d_n(v)}{\sqrt{v}}, \]

where \[ Z_\perp = \sqrt{Z_\perp^2} = \sqrt{(x-x')^2 + (y-y')^2}. \]

Calculating the integrals:

\[ I_1(Z_\perp) = \frac{\beta}{4\pi} \exp\left(-\frac{\beta}{4} Z_\perp^2\right) \left[ L_n\left(\frac{\beta}{2} Z_\perp^2\right) + L_{n-1}\left(\frac{\beta}{2} Z_\perp^2\right)\right], \]

\[ I_2(Z_\perp) = -\frac{\beta}{4\pi} \exp\left(-\frac{\beta}{4} Z_\perp^2\right) \left[ L_n\left(\frac{\beta}{2} Z_\perp^2\right) - L_{n-1}\left(\frac{\beta}{2} Z_\perp^2\right)\right], \]

\[ I_3(Z_\perp) = -i \frac{\beta^{3/2}}{2\pi} \frac{(Z \gamma)_\perp}{Z_\perp} \exp\left(-\frac{\beta}{4} Z_\perp^2\right) \left[ L_n\left(\frac{\beta}{2} Z_\perp^2\right) - L_{n-1}\left(\frac{\beta}{2} Z_\perp^2\right)\right], \]

and substituting them into (25), one finally obtains the expression, which coincides with (21).

5. Conclusion

We have constructed the \( n \)th Landau level contribution to the exact electron propagator in an external magnetic field, based on the Dirac equation exact solutions. It is performed in the \( x \)-representation. The expression (11) with (17) for the propagator can be more convenient than other representations in some cases. This result could have a methodological significance for further developments of the calculation technique for the analysis of quantum processes in an external active medium such as hot dense plasma and strong electromagnetic fields. In particular, it could be useful in the situation of the moderately large field strengths, when it is insufficient to take into account only the ground Landau level contribution.

Note added in proof

There was an error in expression for the propagator in Ref. [10], namely, the term in the second line of Eq. (4.33) should contain the factor \((-i)\). This error was corrected.
in Ref. [11] Eqs. (39) and (40), and also in Ref. [16] Eqs. (13) and (14), but without any comments. We thank M. I. Vysotsky for a discussion clarifying this point.

Acknowledgments

We are grateful to N. V. Mikheev, M. V. Chistyakov and D. A. Rumyantsev for useful remarks.

This work was performed in the framework of realization of the Federal Target Program “Scientific and Pedagogic Personnel of the Innovation Russia” for 2009–2013 (State contract no. P2323) and was supported in part by the Ministry of Education and Science of the Russian Federation under the Program “Development of the Scientific Potential of the Higher Education” (project no. 2.1.1/13011), and by the Russian Foundation for Basic Research (project no. 11-02-00394-a).

References

1. V. I. Ritus, Quantum effects of the interaction of elementary particles with an intense electromagnetic field (in Russian), in Quantum Electrodynamics of Phenomena in an Intense Field, Proc. P. N. Lebedev Physical Institute, vol. 111 (Nauka, Moscow, 1979), pp. 5–151.

2. C. Itzykson and J.-B. Zuber, Quantum Field Theory (McGraw-Hill, New York, 1985).

3. V. O. Papanian and V. I. Ritus, Three-photon interaction in an intense field, in Issues in Intense-Field Quantum Electrodynamics, ed. by V. L. Ginzburg (Nova Science Publishers, New York, 1989), pp. 153–179.

4. A. E. Shabad, in Polarization of the Vacuum and a Quantum Relativistic Gas in an External Field, ed. by V. L. Ginzburg (Nova Science Publishers, New York, 1992).

5. I. M. Ternov, V. Ch. Zhukovskii and A. V. Borisov, Quantum Processes in Strong External Field, in Russian (Moscow State Univ., Moscow, 1989).

6. A. V. Kuznetsov and N. V. Mikheev, Electroweak Processes in External Electromagnetic Fields (Springer-Verlag, New York, 2003).

7. J. Schwinger, Phys. Rev. 82, 664 (1951).

8. V. A. Fock, Physik. Z. Sowjetunion 12, 404 (1937).

9. Yu. M. Loskutov and V. V. Skobelev, Phys. Lett. A 56, 151 (1976).

10. A. Chodos, K. Ecerding and D. A. Owen, Phys. Rev. D 42, 2881 (1990).

11. T.-K. Chyi, C.-W. Hwang and W. F. Kao et al., Phys. Rev. D 62, 105014 (2000).

12. E. Elizalde, E. J. Ferrer and V. de la Incera, Ann. of Phys. 295, 33 (2002).

13. E. Elizalde, E. J. Ferrer and V. de la Incera, Phys. Rev. D 70, 043012 (2004).

14. A. V. Kuznetsov, N. V. Mikheev, G. G. Raffelt and L. A. Vassilevskaya, Phys. Rev. D 73, 023001 (2006).

15. A. P. Prudnikov, Yu. A. Brychkov and O. I. Marichev, Integrals and Series. Special Functions (Gordon-Breach, New York, 1990).

16. V. P. Gusynin and A. V. Smilga, Phys. Lett. B 450, 267 (1999).