Non-LTE Calculation for the Be Star Decretion Disk

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Abstract

The non-LTE state of the hydrogen gas in isothermal transonic decretion disks around a B1V star has been calculated by an iterative method in order to explore the basic physical process in the disk. This dynamical model is characterized by a density law in the equatorial plane of \( \rho(R) \propto R^{-3.5} \). The continuous radiation is calculated with the \( \Lambda \) iteration in the integral form, while we adopt a single-flight escape probability for lines. We describe the non-LTE state, the radiation flow and conversion in the disk. We conclude that the stellar Balmer continuum plays a key role in the non-LTE state of the disk. The examination of the local energy gain and loss suggests that the disk temperature has double minima along the equatorial plane in the optically thick case: the intermediate region caused by deficient ultraviolet radiation and the outer Lyman \( \alpha \) cooling region. We have also calculated some observable quantities, such as the spectral energy distribution, the \( UBV \) colors, the infrared excess and the Balmer line profiles. Our calculations with the mass loss rate less than \( 10^{-10} M_\odot \) yr\(^{-1} \) reproduce the observed continuum quantities. However, we could not get large H\( \alpha \) emission strength observed in Be stars. We suggest that the density gradient of the Be star disk is slower than that of the isothermal decretion disk.

Key words: stars: atmospheres — stars: circumstellar matter — stars: emission-line, Be — line: profiles — radiative transfer

1. Introduction

Be stars are classified as main sequence or giant B stars which have ever shown the emission in Balmer lines. Struve (1931), in his pioneering work on Be stars, proposed flattened equatorial disks around Be stars from the correlation between the emission line widths and the projected rotational velocities of underlying stars. Coyne & Kruszewski (1969) supported this view from the polarimetric observations. While, the rotation law in the disk had been unsettled for a long time. It seems that it has been recently established that the disk is rotating with a
Keplerian law, because the one-armed oscillation, which is excited only in the Keplerian or the near Keplerian disk, can explain the observational facts such as the V/R variation (Okazaki 1997) and the Balmer progression (Hirata & Okazaki 2000). The most confident evidence for the Keplerian disk is the interferometric detection of a size difference of the Hα emission region in the anti-direction measured from the central star, which correlates with the V/R variation (Vakili et al. 1998; Berio et al. 1999). Hummel & Vrancken (2000) obtained the same conclusion from the comparison between the observed and calculated emission profiles. The decretion disk, which was introduced first by Lee, Saio & Osaki (1991) and later developed by Porter (1999) and Okazaki (2001), gives the physical basis of the Keplerian disk, though the mechanism which supplies the mass and angular momentum is not yet known. The Be star disk model with magnetic field has been devised by Cassinelli et al. (2002), Maheswaran (2003) and ud Doula & Owocki (2002). However, Owocki & ud Doula (2003) concluded that the simulation with the stellar rotation does not form a magnetically torqued disk with enough density.

It is well known that the Be star disk is highly in non-LTE state, because the density is not so high that the collisional process is effective. It is necessary to solve non-LTE problem to derive physical quantities from observational materials. Then, it is highly desirable to calculate non-LTE state for the decretion disk model. Several non-LTE calculations have been published for the flattened disks with different velocity structures, namely, different density structures. Marlborough (1969) and Poeckert & Marlborough (1978) adopted the weighted mean of some extreme cases as the non-LTE solution. Millar & Marlborough (1998) and Millar & Marlborough (1999) expanded the model to the radiative equilibrium model. The latter took into account the diffuse radiation in an approximate way. The model has been developed further by Jones, Sigut & Marlborough (2004) and Sigut & Jones (2007), in which they examined the metal effect to the thermal structure of the disk. Sigut & Jones (2007) adopted a single-flight escape probability in the static plane-parallel geometry for lines and on-the-spot approximation for continuous radiation. They also took into account the diffuse continuous radiation in an approximate way. Non-LTE calculations were also performed by Krž (1979), Stee et al. (1995), and Stee & Bitter (2001), based on Sobolev approximation. This approximation is an adequate one when the velocity gradient is large (Hamann 1981; de Koter, Schmutz & Lamers 1993). However, the expansion velocity in the decretion disk is very small (Okazaki 1997). Hummel (1994) and Hummel & Hanuschik (1997) adopted the equivalent two level approximation and calculated Hα line profiles, using A iteration. Carciofi & Bjorkman (2006) performed the non-LTE calculation for the decretion disk under the radiative equilibrium by using Monte Carlo method. Carciofi et al. (2007) improved their model to take into account the vertical temperature structure. They showed that there is a temperature minimum in the several stellar radii on the equatorial plane.

In this paper, we present our results based on the full treatment of the diffuse continuous
radiation, as an extension of Iwamatsu & Hirata (2007). In section 2, we describe the central star and the disk structure for which we calculate the non-LTE state. In section 3, we describe our method of the non-LTE calculation. Section 4 gives the non-LTE state in our model, the local energy balance analysis, and also the total energy gain and loss in the whole disk. In section 5, we describe the continuum properties such as the spectral energy distributions, the $UBV$ colors, and the infrared excess. Section 6 treats the line properties such as the Balmer line profiles, the equivalent widths, and the Balmer decrement. The summary and the concluding remarks are given in section 7.

2. Central Star and Decretion Disk

The spectral type of the central star is assumed to be B1V and its basic parameters are chosen as the effective temperature $T_{\text{eff}} = 24000 \text{ K}$, the radius $R_\ast = 6.9 R_\odot$, and the mass $M_\ast = 11.0 M_\odot$, referred to $\pi$ Aqr (Bjorkman et al. 2000). The stellar flux is taken from ATLAS9 (Kurucz 1993) with the other parameters: the gravitational acceleration at the stellar surface, $\log g = 4.0$, the solar abundance, and the turbulent velocity of $2 \text{ km s}^{-1}$. For easier integration over the frequency, we adopt the stellar flux without line blanketing. Although the effective temperature for the stellar flux without line blanketing is actually 26400 K, we expect that the influence of line blanketing is small, because the flux difference near each shortward continuum limit is small. We assume the central star is spherical without limb darkening.

As a dynamical model, we adopt an isothermal transonic decretion disk in steady state with a viscosity parameter $\alpha = 0.1$ (Okazaki 2001). This one-dimensional model is characterized by almost Keplerian rotation and small expansion velocity. We assume that the disk is axisymmetric about the rotational axis and symmetric about the equatorial plane, and is in hydrostatic equilibrium in the vertical direction. It is convenient to adopt a cylindrical coordinate, whose origin and the polar axis are the center of the star and the rotation axis of the star, respectively. Let us denote the distance from the rotation axis by $\varpi$ and the vertical distance from the equatorial plane by $z$ in units of the stellar radius. We assume that the expansion and rotational velocities at any point are the same as those at $\varpi$ in the equatorial plane. The number density $N(\varpi,z)$ is related to the number density along the equatorial plane $N(\varpi,0)$ (Marlborough 1969) as

$$N(\varpi,z) = N(\varpi,0) \exp \left\{ -\frac{1}{Q} \left( \frac{1}{\varpi} - \frac{1}{\sqrt{\varpi^2 + z^2}} \right) \right\},$$

with

$$Q = \frac{kT_{\text{disk}}R_\ast}{Gm_H\mu M_\ast},$$

where $\mu$ is the mean molecular weight, which is fixed to 0.5 in our calculations. $M_\ast$ is the stellar mass and $T_{\text{disk}}$ is the temperature of the isothermal disk. The other notations have
usual meanings. The number density $N(\varpi, 0)$ is obtained from the column density of Okazaki (2001)’s model under the assumption that the disk is geometrically thin. This assumption is given by

$$-\frac{1}{Q} \left(\frac{1}{\varpi} - \frac{1}{\sqrt{\varpi^2 + z^2}}\right) \simeq -\left(\frac{z}{\sqrt{2Q\varpi}}\right)^2.$$  

(3)

This dynamical model is characterized by $N(\varpi, 0) \propto \varpi^{-3.5}$. The number density $N(\varpi, z)$ is derived from equation (1). Table 1 shows the mass loss rate $\dot{M}$, the total mass of the disk $M_{\text{disk}}$, the disk temperature $T_{\text{disk}}$, and the density $\rho(\varpi, 0)$ at $\varpi = 1$ and 1000 in our nine models. The number density $N(1, 0)$ is given by $N(1, 0) = \rho(1, 0)/m_H = 5.9754 \times 10^{23}\rho(1, 0)$.

The disk temperature in models 1–8 is fixed to two thirds of the effective temperature of the central star (Hummel & Vrancken 2000), i.e., 16000 K, while different mass loss rates are assigned from $1.0 \times 10^{-12}$ to $2.0 \times 10^{-10} M_\odot$ yr$^{-1}$ for models 1–8 in this order. Model 9 has the same mass loss rate as model 7, but with the disk temperature of 12000 K. This case is introduced for the examination of the energy balance. For models 1–8, we obtain the rotational velocities of $v_\phi(\varpi = 1) = 590$ km s$^{-1}$ and $v_\phi(\varpi = 1000) = 5.35$ km s$^{-1}$ and the expansion velocities of $v_\varpi(\varpi = 1) = 0.00472$ km s$^{-1}$ and $v_\varpi(\varpi = 1000) = 30.6$ km s$^{-1}$. For model 9, we obtain $v_\phi(\varpi = 1) = 591$ km s$^{-1}$, $v_\phi(\varpi = 1000) = 6.30$ km s$^{-1}$, $v_\varpi(\varpi = 1) = 0.00303$ km s$^{-1}$, and $v_\varpi(\varpi = 1000) = 22.4$ km s$^{-1}$. Hence, the rotational velocities at $\varpi = 1000$ are about one third of the Keplerian velocity, while the Mach numbers at $\varpi = 1000$ are about two. In figure 1, we show the iso-contour of the number density on the meridional plane for our model 7.

In conversion from the column density to the number density in Iwamatsu & Hirata (2007), there was an error that the column density is divided by $\mu = 0.5$. Therefore, the correct density is a half of the tabulated one, and all of the mass loss rate appeared in Iwamatsu & Hirata (2007) should be corrected by multiplying two.

Table 1. Physical quantities of our dynamical model sequence. We show the mass loss rate $\dot{M}$, the total mass of the disk $M_{\text{disk}}$, the disk temperature $T_{\text{disk}}$, and the density $\rho(\varpi, 0)$, at $\varpi = 1$ and 1000. $a(b)$ means $a \times 10^b$.  

| model | $\dot{M}$ | $M_{\text{disk}}$ | $T_{\text{disk}}$ | $\rho(\varpi, 0)$ [g/cm$^{-3}$] |
|-------|---------|-----------------|-----------------|-------------------|
|       | [$M_\odot$ yr$^{-1}$] | [$M_\odot$] | [K] | $\varpi = 1$ | $\varpi = 1000$ |
| 1     | 1.0(-12) | 1.02(-10) | 1.6(4) | 1.75(-12) | 8.60(-24) |
| 2     | 2.0(-12) | 2.04(-10) | 1.6(4) | 3.50(-12) | 1.72(-23) |
| 3     | 5.0(-12) | 5.11(-10) | 1.6(4) | 8.74(-12) | 4.30(-23) |
| 4     | 1.0(-11) | 1.02(-09) | 1.6(4) | 1.75(-11) | 8.60(-23) |
| 5     | 2.0(-11) | 2.04(-09) | 1.6(4) | 3.50(-11) | 1.72(-22) |
| 6     | 5.0(-11) | 5.11(-09) | 1.6(4) | 8.74(-11) | 4.30(-22) |
| 7     | 1.0(-10) | 1.02(-08) | 1.6(4) | 1.75(-10) | 8.60(-22) |
| 8     | 2.0(-10) | 2.04(-08) | 1.6(4) | 3.50(-10) | 1.72(-21) |
| 9     | 1.0(-10) | 1.68(-08) | 1.2(4) | 3.15(-10) | 1.35(-21) |
Following the notation of Kráž (1974), the statistical equilibrium equations are expressed separately as in Marlborough (1969). We assume complete redistribution for the line radiation.

3. Statistical Equilibrium Equation

3.1. Statistical Equilibrium Equation

We assume that the disk consists of only hydrogen atoms. When we designate the uppermost bound level of hydrogen atom by \( n_0 \), the charge conservation is expressed as

\[
N_e = N_+ = N(\varpi, z) - \sum_{n=1}^{n_0} N_n ,
\]

where \( N_e, N_+, \) and \( N_n \) are the electron number density, the proton number density, and the population of the \( n \)-th level at the point \((\varpi, z)\), respectively. The \( 2s \) and \( 2p \) sublevels are treated separately as in Marlborough (1969). We assume complete redistribution for the line radiation.

Following the notation of Kráž (1974), the statistical equilibrium equations are expressed as

\[
N_e \sum_{n' \neq n} N_{n'} C_{n'n} - N_n N_e \sum_{n' \neq n} C_{nn'} + N_e^2 N_+ C_{nn} - N_e N_n C_{nc} \\
+ \sum_{n' = n+1}^{n_0} \left( N_n A_{n'n} + N_{n'} B_{n'n} J_{nn'} - N_n B_{nn'} J_{nn'} \right) \\
- \sum_{n' = 1}^{n-1} \left( N_n A_{nn'} + N_{n'} B_{nn'} J_{nn'} - N_{n'} B_{n'n} J_{n'n} \right) + N_e N_+ \alpha_n \\
- 4\pi \int_{\nu_n}^{\infty} a_n(\nu) \frac{J_{vc}}{h\nu} \left\{ 1 - \frac{1}{b_n} \exp \left( - \frac{h\nu}{kT} \right) \right\} d\nu = 0 \quad (n = 1, 2, \ldots, n_0) ,
\]

where \( J_{vc}, J_{nn'} \) are the mean intensity of the continuum and line radiation, respectively. \( A_{n'n}, B_{nn'}, \) and \( B_{n'n} \) \((n' > n)\) are Einstein coefficients for the spontaneous emission, absorption, and stimulated emission, respectively. \( C_{nn'} \) is the collision coefficient, and \( a_n(\nu), \alpha_n \) are the ionization and recombination coefficients, respectively. Here, we neglect two-photon emission between \( 1s \) and \( 2s \) levels. Introducing a departure factor \( b_n = N_n/N_n^* \), where \( N_n^* \) is the LTE value, equations (5) are written as

\[
\sum_{n' = 1}^{n-1} b_n g_{n'} e^{x_{n'}} (N_e C_{n'n}) - b_n \left[ \sum_{n' = 1}^{n_0} g_{n'} e^{x_{n'}} (N_e C_{n'n}) \right] \\
+ g_n e^{x_n} \left\{ N_e \left( \sum_{n' = n+1}^{n_0} C_{nn'} + C_{nc} \right) + \sum_{n' = 1}^{n-1} A_{nn'} \{ n'n' \} + 4\pi \int_{\nu_n}^{\infty} a_n(\nu) \frac{J_{vc}}{h\nu} d\nu \right\} \\
+ \sum_{n' = n+1}^{n_0} b_{n'} \left\{ g_n e^{x_n} (N_e C_{nn'}) + g_{n'} e^{x_{n'}} A_{nn'} \{ nn' \} \right\} = -2 \left( \frac{2\pi m_e kT}{\hbar^2} \right)^{\frac{3}{2}} \alpha_n \\
- g_n e^{x_n} \left\{ N_e C_{nc} + 4\pi \int_{\nu_n}^{\infty} a_n(\nu) \frac{J_{vc}}{h\nu} \exp \left( - \frac{h\nu}{kT} \right) d\nu \right\} \quad (n = 1, 2, \ldots, n_0) ,
\]
where \( \{nn'\} \) is the net radiative bracket for \( H_{n,n'} \) line defined in Thomas & Athay (1961) and is given by

\[
\{nn'\} = 1 - \frac{(N_nB_{nn'} - N_{n'}B_{n'n})}{N_{n'}A_{n'n}} \quad (n < n').
\]  

(7)

3.2. Single-Flight Escape Probability

We approximate the net radiative bracket by a single-flight escape probability (e.g., Kastner 1994) for easier treatment of the line radiative process. This approximation is introduced as an extension of Sobolev approximation for the disk in which the velocity gradient is small. The single-flight escape probability \( \beta_{nn'} \) is defined as

\[
\beta_{nn'} = \frac{1}{4\pi} \int_{4\pi} d\Omega \int_{-\infty}^{\infty} d\nu \psi(\nu - \nu_{nn'}) \exp\{-\tau^{L}_{nn'}(\nu)\},
\]

(8)

where \( \Omega \) is the solid angle, \( \nu_{nn'} \) is the central frequency of \( H_{nn'} \) line, and \( \tau^{L}_{nn'}(\nu) \) is the optical thickness from a point in the disk to the disk boundary for \( H_{nn'} \) line at the frequency \( \nu \) along the path, for which we must take into account the velocity field. The function \( \psi \) is an intrinsic broadening function and we assume thermal Doppler broadening. Then, \( \psi \) is given by

\[
\psi(\nu - \nu_{nn'}) = \frac{1}{\sqrt{\pi}\Delta\nu_D} \exp\left\{-\left(\frac{\nu - \nu_{nn'}}{\Delta\nu_D}\right)^2\right\},
\]

(9)

where \( \Delta\nu_D \) is the thermal Doppler width, defined as

\[
\Delta\nu_D = \frac{\nu_{nn'}}{c} \left(\frac{2kT_{\text{disk}}}{m_H}\right)^{1/2}.
\]

(10)

3.3. Continuous Radiation

The continuum mean intensity \( J_{\nu c} \) is the sum of the mean intensity of the direct and diffuse radiation:

\[
J_{\nu c} = J^*_\nu c + J^d_{\nu c}.
\]

(11)

The direct radiation from the star, \( J^*_\nu c \), is given by

\[
J^*_\nu c = \frac{1}{4\pi} \int_{\Omega_{\star}} I^*_\nu e^{-\tau^c_{\nu}} d\Omega,
\]

(12)

where \( I^*_\nu, \tau^c_{\nu}, \Omega_{\star} \) are the stellar radiation intensity at the stellar surface, the continuum optical depth at the frequency \( \nu \), and the solid angle subtended by the star, respectively. When \( \tau^c_{\nu} \approx 0 \), \( J^*_\nu c \) is approximated by

\[
J^*_\nu c \simeq WI^*_{\nu},
\]

(13)

where \( W \) is a geometrical dilution factor expressed by

\[
W = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{R^2}}\right).
\]

(14)
Here $R$ is the distance from the center of the star in units of $R_*$.

The diffuse radiation, $J^d_{vc}$, is expressed as

$$J^d_{vc} = \frac{1}{4\pi} \int_{4\pi} I^d_{vc}(s) d\Omega,$$

where $I^d_{vc}(s)$ is the diffuse radiation intensity towards the direction $s$, which is given by the formal integral of the radiative transfer equation as

$$I^d_{vc}(s) = \int S_\nu e^{-\tau^c_\nu} d\tau^c_\nu.$$

Here, $S_\nu$ is the source function and $\tau^c_\nu$ is the optical thickness from a point in the disk to the disk boundary. Although electron scattering yields the observed polarization, we do not include this effect in our calculations because it does not affect the energy conversion in the disk and because the scattering process makes the convergence in the iteration slower.

### 3.4. Physical Constants

$a_n(\nu)$ and $\alpha_n$ are adopted from Burgess (1964) for $n = 2s$ and $2p$. We adopt Mihalas (1978) and Johnson (1972) for the other $a_n(\nu)$ and $\alpha_n$, respectively. The gaunt factors for the bound-free and free-free processes are taken from Kurucz (1970). The free-free absorption coefficient $k_{ff}(\nu)$ and the continuous emission coefficient $j_{\nu}$ are adopted from Mihalas (1978) and Brussaard & van de Hulst (1962), respectively. $A_{n' n}$ for $n = 2s$ or $2p$ are from Karzas & Latter (1961), while the other $A_{n' n}$ values are from Wiese, Smith & Glennon (1966). $C_{nn'}$ and $C_{nc}$ are obtained from equations (36) and (39) in Johnson (1972) for $n, n' \neq 2s, 2p$. $C_{2s,2p}$ is computed from equations (3) and (55) in Seaton (1955). $C_{1,2s}$ is obtained through the interpolation of table X in Callaway (1985). Because $C_{2s,3}$ and $C_{2p,3}$ are the sum of $C_{2s,3s}$, $C_{2s,3p}$, $C_{2s,3d}$, and of $C_{2p,3s}$, $C_{2p,3p}$, $C_{2p,3d}$, respectively, these six coefficients are computed through the interpolation of table III in Callaway et al. (1987). The coefficients $C_{2s,n'}$ and $C_{2p,n'}$ for $n' \geq 4$ are estimated from the collision rate of Johnson (1972), in the same manner as in Krolik & McKee (1978). For $C_{2s,c}$ and $C_{2p,c}$, the assumption by Drake & Ulrich (1980) is adopted, i.e., $C_{nt,c} = C_{nc}$.

### 3.5. Computation

#### 3.5.1. Computational Scheme

By assuming trial values for $b_n$, the coefficients for $b_n$ in equations (6) are obtained through integrations for the escape probability defined by equation (8) and for the mean intensities of continuous radiation. Therefore, the calculations of equations (8), (12), and (15) and the solutions of equations (6) and (4) constitute an iteration loop in this order. We solve the transfer equation for the continuous radiation by this simple integral form, though it is known that the convergency is very slow when the disk becomes optically thick. In actual calculations, we assume the uppermost bound level, $n_0 = 10$, while we adopt $n_0 = 5$ when the energy balance
is discussed in section 4.2. For model 1, the initial values were set to $b_n = 1/W$. Then, the results of model 1 were adopted as the initial values for model 2, and so on.

3.5.2. Grid points and Interpolation

The vertical boundary of the disk is set to the surface with $N(\varpi, z) = 10^4 \text{cm}^{-3}$, while the disk radius, $\varpi_{\text{disk}}$, is cut off at 95% of the radius with $N(\varpi, 0) = 10^4 \text{cm}^{-3}$. The grid points for calculating the $b_n$ factors are set denser near the star and the equator. The fourteen grids are set exponentially for $\varpi$, while nine grids are set parabolically for $z$ ($z \geq 0$), including the boundary. Thus, the total number of the grids is 126. We set the grid points in the following form

$$\varpi_i = \varpi_{\text{disk}}^{i+1} (i = 1, 2, \cdots, 14), \quad z_{i,j} = z_{i,8} \left( \frac{j}{8} \right)^2 (j = 0, 1, \cdots, 8),$$

where $z_{i,8}$ is the vertical boundary with $N(\varpi_i, z) = 10^4 \text{cm}^{-3}$. Hereafter, we call these grid points as population grid points. Analytic setting of the grids is convenient for the interpolation in the $(i, j)$ space with an equal interval. Interpolation is made only for the $b_n$ factors, since all the relevant quantities are calculable through the $b_n$ factors in the isothermal disk. Actually, the $\log b_n$ values are interpolated. Easy and stable interpolation is required for precise integrations described below.

3.5.3. Integration

Equations (6) include many integrations in the coefficients of $b_n$. It is evident that the iteration converges to the false values if the integrations appeared in equations (6) are inaccurate. The 1% accuracy of the integration is aimed in our iterative calculations and also in calculations of the emergent flux and the line profile. We checked the accuracy for several test integrands in each integrations appeared in equations (6). Typical test integrands are based on the assumption of $b_n = 1/W$. The integration is made mostly by Simpson’s 3/8 formula.

The optical depth $\tau_{rin}^{L, \nu}$ in equation (8) is calculated with the meshpoints concentrated around the place where the relative velocity is zero along the path. The meshpoints for frequency integrals are set with an interval of $0.08\Delta \nu_D$ for the range of $-5\Delta \nu_D$ to $5\Delta \nu_D$. The angular integration in equation (8) is performed on a polar coordinate with a polar axis parallel to the rotational axis of the star. The meridional integration is performed with 25 meshpoints, which distribute denser near the equator, and 24 meshpoints are set for the azimuthal integration with an equal interval.

The optical depth $\tau_{r}^{c}$ in equation (12) is calculated with 120 meshpoints concentrated in the regions where the function $N^2(\varpi, z)/W$ is large. The angular integration is performed by introducing a polar coordinate whose polar axis passes the center of the star. We adopt 18 meshpoints from 0 to $\theta_*$ (the angular radius of the star) for the meridional integration, and 36 meshpoints from 0 to $2\pi$ for the azimuthal integration.

Since the continuous emission produced in the denser part near the star is effective for
the diffuse radiation, $J^d_{\nu c}$, in equation (15), we introduce a polar coordinate whose polar axis
passes the footpoint, ($\varpi, z) = (1,0)$ for the angular integration. The angular meshpoints are
set denser near the footpoint and towards the equatorial plane, depending on the population
grid point. The meridional integration is performed first, and then the azimuthal one. In the
case of the population grid point distant from the star, about 100 meshpoints are required for
the meridional integrals, while 12 meshpoints is enough to satisfy the 1% precision when the
population grid point is located in the dense part, due to its isotopic nature. For the azimuthal
integration, 26 meshpoints are set, which are distributed denser near the equator.

The path integral in equation (16) is made by using Simpson’s 3/8 rule for the calculation
of the optical depth, while the trapezoidal rule is applied for the integration of the source
function. The segments for the calculation of the optical depth are divided repeatedly until they
have satisfied the constraint that the absorption coefficient ratio for both edges of the segment
is between 1/1.2 and 1.2 for all frequency meshpoints. Moreover, the additional constraint is
added for the optical depth of the segment, $\Delta \tau_\nu$, at frequency $\nu$: the segment is divided until
satisfying $\Delta \tau_\nu < 0.2$ for $0 < \tau_\nu < 2$, $\Delta \tau_\nu < 2$ for $2 < \tau_\nu < 5$, $\Delta \tau_\nu < 5$ for $5 < \tau_\nu < 10$, and we
neglect the contribution from the more distant region with $\tau_\nu > 10$. As a result, the number of
the meshpoints along the path which passes only the optically thin region is several tens, while
it reaches several thousands when the path includes the optically thick region.

Totally sixty four frequency meshpoints are set for the integration over the frequency
to calculate the ionization rate in equation (6). They consist of 13 meshpoints for the Lyman
continuum, 10 meshpoints for the Balmer continuum, 7 meshpoints for the Pashen, Brackett,
and Pfund continua, and 4 points for the other continua with $n > 5$.

3.5.4. Convergency

Our simple iteration scheme requires many iterations in the case of optically thick disk.
In such a case, the convergence is slowest for $b_1$ at the place slightly distant from the star
around the equatorial plane where the Lyman optical thickness towards the disk boundary is
very large. When the ratio of the new $b_1$ value to its previous one keeps monotonous variation
during five consecutive iterations, the five power of the ratio is multiplied to the new $b_1$ factor
in order to accelerate the convergence, while the root of previous and new $b_n$ factors is adopted
as a next trial value in order to relax the solution.

By examining the convergence behavior of $b_n$, we found that all the $b_n$ factors except
$b_1$ in the dense region converge when all the ratios of newly calculated departure factors $b_1$
to the previous factors are within $1 \pm 0.01$ at all the population grid points. Such a behavior
in iteration is understandable since our models are not optically thick towards the vertical
direction in the Balmer and the other continua with longer wavelength, and since the fully
ionized state is assured from the assumed high disk temperature. As will be shown in section
4.3, Lyman continuum does not play so important role in the radiation field in the disk. Then,
we decided to stop the iteration when the above 1% criterion is fulfilled for all $b_n$ values. When we compare such a result with the results from further iteration and from the iteration with the initial $b_1$ values in opposite sense, a few tens percents deviation at most from the guessed convergent values was noticed for $b_1$ in the dense region of model 7.

### 3.5.5. Computational Time

All calculations in this paper were made with the computer with the CPU of the AMD Opteron 250 (2.4 GHz) at the Astronomical Data Analysis Center, ADAC, of the National Astronomical Observatory of Japan. The calculation times for one iteration are about twelve hours for the optically thin disk, about eighteen hours for the optically thick disk. The 1% criterion was achieved after ten or less iterations for the optically thin case, and after about forty iterations for the optically thick case. The total time is approximately several days for the optically thin case and two weeks for the optically thick case.

### 4. Global Features

#### 4.1. Non-LTE State

In order to show the optical property of our model sequence, we list, in table 2, the respective continuum optical thicknesses, $\tau_r, \tau_z$, and $\tau_\phi$ at each continuum limit from the foot-point ($\varpi = 1, z = 0$) to the disk boundary towards the radial, vertical, and azimuthal directions for our models 1, 4, and 7. Reminding an observational fact that the second Balmer jump due to the strong Balmer shell lines appears in strong shell stars (Divan 1979), we cover the range of the mass loss rate for which the radial optical thickness at the Balmer shortward edge is larger than unity. Model 1 is optically thin, while model 7 is optically thick at all shortward edges. In the wavelength where the optical depth for the continuum is large, the role of diffuse radiation becomes important in the non-LTE state of the disk. In model 1, the contributions of the direct and diffuse radiation to the ionization from the ground level are comparable in the vicinity of the star, while the direct stellar radiation overwhelms in the longer wavelength region. In model 7, the diffuse radiation overwhelms the direct radiation in the dense part of the disk. Taking into consideration that our B1V star emits the radiation in the Lyman and Balmer continua by 0.2% and 93.3% of the total luminosity, respectively, we classify the optical property of the disk by the optical thickness for the Balmer continuum. We regard that models 1 and 7 are the respective typical cases for the optically thin and thick disks. Notice that the optical thickness at the shortward edges of Balmer and Paschen series has the smallest values among those at all the shortward edges. These facts imply that the stellar Balmer continuum principally governs the radiation field in Be stars. We also note that an inequality $\tau_z < \tau_r < \tau_\phi$ holds not only at the footpoint but also in the main part along the equator.

Figure 2 shows the variation of the $b_n$ factors along the equatorial plane as a function
Table 2. Continuum optical thickness towards the radial direction ($\tau_r$), the vertical direction ($\tau_z$) and the azimuthal direction ($\tau_\phi$) from the footpoint to the boundary in our three models. All the bound-free processes and the free-free process are included. The first row corresponds to the threshold wavelength: the integer means the bound level related to the bound-free transition, while the ‘−’ and ‘+’ signs indicate the shortward and the longward of the threshold, respectively. For example, ‘1c−’ means the shortward edge of the Lyman limit. a(b) means $a \times 10^b$.

|       | model 1 |          |          |          |          |          |          |          |          |
|-------|---------|----------|----------|----------|----------|----------|----------|----------|----------|
|       | $\tau_r$ | $\tau_z$ | $\tau_\phi$ | $\tau_r$ | $\tau_z$ | $\tau_\phi$ | $\tau_r$ | $\tau_z$ | $\tau_\phi$ |
| 1c−   | 5.75(0)  | 3.52(−1)| 1.26(+1) | 4.65(+2) | 2.45(+1) | 9.25(+2)  | 1.51(+4) | 2.41(+3) | 4.85(+4)  |
| 1c+   | 4.68(−4)| 3.01(−5)| 1.05(−3)| 2.59(−2)| 2.24(−3)| 6.50(−2)| 1.88(0)  | 2.40(−1)| 5.68(0)  |
| 2c−   | 2.53(−2)| 1.56(−3)| 5.61(−2)| 1.36(0) | 1.13(−1)| 3.35(+0) | 9.64(+1) | 1.21(+1)| 2.89(0)  |
| 2c+   | 2.29(−3)| 2.49(−4)| 6.56(−3)| 1.85(−1)| 2.38(−2)| 5.52(−1)| 1.61(+1)| 2.38(+0)| 5.16(0)  |
| 3c−   | 1.98(−2)| 1.98(−3)| 5.52(−2)| 1.51(0) | 1.87(−1)| 4.43(+0) | 1.26(+2)| 1.87(+1)| 4.05(0)  |
| 3c+   | 7.54(−3)| 1.02(−3)| 2.37(−2)| 7.12(−1)| 1.02(−1)| 2.25(+0) | 6.87(+1)| 1.02(+1)| 2.21(+2)|
| 4c−   | 3.04(−2)| 3.99(−3)| 9.46(−2)| 2.82(+0)| 4.00(−1)| 8.84(+1)| 2.68(+2)| 3.99(+1)| 8.65(+2)|
| 4c+   | 2.11(−2)| 3.06(−3)| 6.80(−2)| 2.10(+0)| 3.09(−1)| 6.72(+0)| 2.07(+2)| 3.08(+1)| 6.68(+2)|
| 5c−   | 5.88(−2)| 8.44(−3)| 1.89(−1)| 5.82(+0)| 8.53(−1)| 1.86(+1)| 5.73(+2)| 8.52(+1)| 1.85(+3)|
| 5c+   | 5.05(−2)| 7.45(−3)| 1.63(−1)| 5.06(+0)| 7.49(−1)| 1.63(+1)| 5.03(+2)| 7.49(+1)| 1.62(+3)|

† uncertain.

of the distance for our models 1, 4, and 7. The geometric dilution factor $W$ is also shown with the dashed lines. The $b_n$ factors once increase outwards from $b_n \sim 1$ in all our models, because the available stellar radiation decreases outwards. The overpopulation is less remarkable in the higher levels, since the collisional process operates more effectively in the higher levels. The 1s and 2s departure factors increase towards the outer boundary, because they are the ground level or the metastable level. It is noteworthy that the second level population approaches to $1/W$, but does not exceed $1/W$, reflecting a fact that the stellar Balmer continuum is a principal source of ionization. While, the $b_n$ factors of the excited levels ($n = 2p, 3, 4...$) turn to decrease at some distance, and finally converge to the nebular case A. Similar behavior was concluded by Poeckert & Marlborough (1978) (see figure 2 of Hirata & Kogure 1984). We confirmed that the turning point with maximum $b_n$ reflects the change from optically thick nature to thin nature in H$_{n-1,n}$ line, i.e., the leading member of the series, thus degrading the $n$-th level population. The turning point shifts outwards in the denser disk with larger mass loss rate, caused by optically thicker nature. In the outer region, the disk becomes optically thin for the lines, first, from the higher series members, then, to the lower series members. In the optically thick disk, the LTE region is added in the innermost part where it is optically thick in all wavelength regions. Figure 3 shows the iso-contour map of the departure factors $b_n$ on the meridional plane for our standard model 7 with $n_0 = 5$. The $b_n$ factors become smaller as leaving from the equator, reflecting the smaller optical thickness and subsequent more important role of the direct stellar
Fig. 2. $b_n$ factor variation on the equatorial plane for the models 1, 4, and 7. The dashed curves indicate $1/W$.

Fig. 3. Contour map of the departure factor $b_n$, where $n = 1, 2, 2p, 3, 4, 5$ on the meridional plane for model 7. The × signs indicate the position of the population grid points in our computation. The iso-contours of the dilution factor $W$ are also shown in each panel.

radiation from the polar region. For comparison, we also show the $W^{-1}$ contours for some typical values.

4.2. Energy Balance

Next, we calculate the radiation energy gain and loss in our isothermal disk and examine the deviation from the radiative equilibrium. The calculations were made for $n_0 = 5$. The energy loss at any point in the disk is given by

$$E_{\text{loss}} = 4\pi \int j_\nu d\nu + \sum_{n=2}^{n_0} \sum_{n=1}^{n'-1} h\nu_{nn'} N_{nn'} A_{nn'} \beta_{nn'},$$

(18)

where the first and second terms correspond to the loss through the continuous process and the line process, respectively. The energy gain is expressed as

$$E_{\text{gain}} = 4\pi \int k_\nu J_{\nu c} d\nu.$$  

(19)

If the energy loss is larger than the energy gain, the disk temperature should become lower than the assumed one when the radiative equilibrium holds. We define the ratio of $E_{\text{loss}}$ to $E_{\text{gain}}$ by $R_{lg}$. Figure 4 shows the ratio, $R_{lg}$, along the equatorial plane for models 1, 7, and 9. The model 9 ($T_{\text{disk}} = 12000$ K) is shown for illustrating the case of the lower disk temperature. The label T represents $R_{lg}$, while the labels C and L indicate the continuum and line contributions, respectively.

In the case of optically thin disk (model 1), our isothermal assumption with $T_{\text{disk}} = 2/3 T_{\text{eff}}$ fills almost the radiative equilibrium on the equatorial plane. In model 7, however, the energy loss is larger than the energy gain in almost all part of the equator. There exist two regions where the value of $R_{lg}$ reaches maximum: the inner part around $4R_*$ and the outer part around $60R_*$. The formation of the inner cool region is caused by the ultraviolet radiation deficiency due to the optically thick nature. The outer cool region is formed because Lyman $\alpha$ becomes optically thin, thus the $L\alpha$ cooling becomes effective. Outside of the $L\alpha$ cooling region, the $b_{2p}$ factor drops drastically, so the $L\alpha$ cooling becomes ineffective. The energy loss by the line process is negligibly small in the inner part, while it reaches a half of the total loss in the outer $L\alpha$ cooling region. This $L\alpha$ cooling region will not be observed in the optical spectral region, because no contribution is expected due to its low density. In model 9 ($T_{\text{disk}} = 12000$ K), the global feature resembles to that in model 7. Note that the disk temperature in the innermost LTE region should be higher than the assumed one, in the sense that it approaches to the
4.3. Energy Flow and Conversion

Let us discuss the radiation flow and conversion in the disk. Since the continuum optical thickness is smallest towards the vertical direction (see table 2), the continuous radiation emitted in the disk tends to escape from the upper and lower surfaces of the disk. The inclination-angle dependency of the emergent flux will be given in the next section.

Although the L\(\alpha\) cooling is effective in the outer part of the disk, it is found that the total amount of energy emitted in the lines from the whole disk is negligible, compared with that in the continuum. This is because the amount of the energy emitted in the region far from the star is much smaller than that near the star. Hence, we consider only the continuous radiation in the following. We define the continuum luminosity in Lyman, Balmer, … continua \((L_1, L_2, \ldots; n = 1 – 6)\) for \(n_0 = 5\) by

\[
L_n = 4\pi \int_0^\pi \sin i \, di \int_{\nu_n}^{\nu_{n-1}} F_\nu(i) \, d\nu,
\]

where \(F_\nu(i)\) is the flux integrated over the disk and the star towards the inclination angle \(i\) at the frequency \(\nu\), \(\nu_n\) being the threshold frequency of each continuum band with the exceptions of \(\nu_0 = \infty\) and \(\nu_6 = 0\). We also define the similar quantity \(L_n\) for the stellar radiation.

In figure 6, we show, for models 1, 7, and 9, the difference \(L_n - L_n^*\) at each band, as well as the difference of the total luminosity, \(L - L_*\) (designated as ”all”), all being normalized by the stellar bolometric luminosity \(L_*\). The total luminosity of model 1 is slightly less than the stellar luminosity, indicating the slightly higher isothermal equilibrium temperature. Total luminosity of model 7 is larger than the stellar luminosity, while that of model 9 is less than the stellar luminosity. This indicates that the isothermal equilibrium temperature is in between 12000 K and 16000 K for model 7. This conclusion is consistent with the results in the radiative equilibrium models calculated by Millar & Marlborough (1999), Carciofi & Bjorkman (2006), and Sigut & Jones (2007), since they all gave \(T_{\text{disk}}\) around \(0.6T_{\text{eff}}\) as an equivalent isothermal disk temperature.
Fig. 6. Disk luminosity in each band relative to the stellar bolometric luminosity for models 1, 7, and 9. The abscissa expresses each band, e.g., 1c=Lyman continuum and 'all' indicates the total luminosity. The scale of the ordinate in the upper and lower two panels are different.

Fig. 7. Spectral energy distribution for three models.

Next, we discuss the radiation energy conversion in the disk. It is well known that stellar ultraviolet radiation is converted into radiation with longer wavelength and line radiation in the Be star disk. Figure 6 clearly shows that Balmer continuum photons are converted into photons with longer wavelength. Similar conclusion was reported by Millar & Marlborough (1999). Stellar Lyman continuum plays little role in the disk except the ground level regulation in the outer envelope of the disk where the optical depth for Lyman continuum towards the central star is small. In the optically thick disk, only a small fraction of the Lyman continuum is converted into the optical and infrared radiation (models 7 and 9). While, in the optically thin disk (model 1), Lyman continuum photons are even produced in the disk from the Balmer continuum photons. This result comes from a fact that a B1V star emits almost all the radiation not in the Lyman continuum but in the Balmer continuum. This is a general conclusion for Be stars, because the stellar Lyman continuum is more deficient in Be stars with later spectral types.

5. Continuum Property

5.1. Spectral Energy Distribution

We describe several observable quantities for our models with $T_{\text{disk}} = 16000$ K in present and next sections. Figure 7 shows the spectral energy distribution (SED) for models 1, 4, and 7. The SEDs for different inclination angles are shown, together with the stellar SED. The infrared excess develops as the mass loss rate increases and as the inclination angle decreases. The Lyman continuum shows the deficiency or excess, depending on the inclination angle in the case of optically thick disk. In model 7, the disk continuum contributes to the SEDs even in the Balmer and Paschen continua.

5.2. UBV Bands

The $UBV$ magnitudes were calculated by using the response functions for the photometric filters given in Buser & Kurucz (1978). Figure 8 shows the $V$ magnitude variation as a function of the mass loss rate. The brightness in the $V$ band increases as the mass loss rate increases and the inclination angle decreases. The brightness decrease in the $V$ band is not resulted and only slight brightening in the $V$ band is obtained in the equator-on case in our present model sequence ($T_{\text{disk}} = 16000$ K). We note, however, that the $V$ band brightness
Fig. 8. \( V \)-band magnitude variation as a function of the mass loss rate for the model sequence with \( T_{\text{disk}} = 16000 \) K. We also show the disk mass in the upper part.

Fig. 9. Flux variation as a function of \( \cos i \) in the \( UBV \) bands for model 7.

decrease by 0.2 mag is obtained for \( i = 90^{\circ} \) in model 9 (\( T_{\text{disk}} = 12000 \) K). The observed range of \( \Delta V \approx 0.5 \) mag is well covered by the mass loss rate less than \( 10^{-10} M_{\odot} \) yr\(^{-1} \).

Next, we examine the inclination angle dependency of the emergent flux in each band. Figure 9 shows the fluxes in the \( UBV \) bands for model 7 as a function of \( \cos i \), to which the projected area is proportional. It is seen that the emergent fluxes are, roughly speaking, proportional to \( \cos i \), though they are slightly concentrated towards the lower inclination angle.

Figure 10 shows the variations on the color-magnitude (CM) and color-color (CC) diagrams. The \( B-V \) color becomes redder and the brightness in the \( V \) band becomes larger as the mass loss rate increases. The amount of variation is larger for smaller inclination angle. It is interesting to note that the variation on the CM diagram almost lies on the same locus, regardless the mass loss rate and the inclination angle. The color-color plot in figure 10b indicates that the \( U-B \) color becomes bluer for the low inclination angle and redder for the high inclination angle, as the disk develops. We confirmed that no color change in \( U-B \) occurs at \( i \approx 70^{\circ} \). We note that our present results are consistent with the observed variations on the CM and CC diagrams for early Be stars (e.g., Hirata & Hubert 1981).

5.3. Far-Infrared Region

The free-free transition is a dominant process in this wavelength region. Therefore, the emergent flux is rather free from the non-LTE state and reflects the density and temperature structure of the disk more directly. Figure 11 shows the magnitude variation at four IRAS wavelengths as a function of the mass loss rate. The monochromatic magnitudes were calculated at 12, 25, 60, and 100\( \mu \)m, since the SED in this wavelength region is not complex. The brightening in magnitude scale increases linearly with the increase of the mass loss rate in the pole-on case.

The difference between \( i = 0^{\circ} \) and \( 90^{\circ} \) reaches 1.5–1.8 mag in the case of the larger mass loss rate, resulting one order of magnitude difference in the mass loss rate in our models. We suggest that the pole-on approximation in Waters (1986) is valid for \( i \leq 30^{\circ} \) and the systematic deviation is expected for the higher inclination angle. Similar result has been concluded by Carciofi et al. (2007). We found such a correlation between the mass loss rate given by Waters, Coté & Lamers (1987) and the projected rotational velocity \( v \sin i \) given by Coté & Waters (1987), as is shown in figure 12. There is a tendency that the mass loss rate becomes larger for

Fig. 10. (a) Variation on the Color-Magnitude diagram. (b) Variation on the Color-Color diagram.
Fig. 11. Monochromatic magnitude change at four IRAS wavelengths (12, 25, 60, and 100 µm).

Fig. 12. Mass loss rate given in Waters, Coté & Lamers (1987) against $v \sin i$. Different spectral types are distinguished by different symbols.

larger stellar projected rotational velocity $v \sin i$. We also note that, in figure 12, the stars with large $v \sin i$ ($\sim 400$ km s$^{-1}$) have the mass loss rates one order of magnitude larger than that with the smaller $v \sin i$ ($\sim 100$ km s$^{-1}$) for all the spectral type groups. Our rough estimate suggests that the mass loss rate based on our model is $10^{-12} - 10^{-10}$ $M_\odot$ yr$^{-1}$, three orders of magnitude smaller than that of Waters, Coté & Lamers (1987), reflecting the difference in the initial velocity $v_\infty(\varpi = 1)$.

Figure 13 shows the flux variation at four IRAS wavelengths as a function of cos$i$ for model 7. Note that the disk contribution overwhelms the stellar flux in the IRAS wavelength region. Again, the flux increases almost linearly with cos$i$. However, due to the opaque nature in this wavelength region, the disk has a finite opaque area, thus resulting the infrared excess even in the equator-on view. Note that the cos$i$ dependence of the infrared flux is convex, in contrast to that in the optical region (see figure 9).

6. Line Property

6.1. Balmer Line Profiles

Balmer line profiles were calculated under the assumption that the intrinsic broadening function is given by the thermal motion to see the effect of velocity field, although Stark broadening becomes important in the denser part of the optically thick disk. Our models cover the large range of the continuous optical thickness at H$\alpha$ from $\tau_r = 0.01$ (model 1) to $\tau_r = 375$ (model 8) (see also table 2). In the optically thick models, the contribution from the disk continuum is noticeable even in the visual region, as is shown in the previous section. This means that the resulting Balmer line profiles are complex, disturbed not only by the shell absorption, that is, the line absorption of the photospheric radiation, but also by the line absorption of the disk continuum. The latter is attributable to the so-called pseudo-photosphere (Harmanec 1994, 2000).

In figure 14 we show the rectified H$\alpha$, H$\beta$, and H$\gamma$ line profiles in three models 1, 4 and 7 for various inclination angles. The photospheric component is not included in this figure. In the case of $i \sim 0^\circ$, the narrow double-peak profile appears. This special case will be discussed in section 6.2. The wine-bottle type profile is traced for $i = 15^\circ$ and $30^\circ$ in model 7. The broad double-peak profile develops for the larger inclination angle. The peak separation is larger

Fig. 13. Flux variation as a function of cos$i$ in the IRAS bands for model 7.
Fig. 14. Hα, Hβ, and Hγ line profiles for our three models. The photospheric line is not considered in these profiles. The dotted, dashed and solid lines are the line profiles for models 1, 4 and 7, respectively. Note that the top panels have the vertical scales different from the other lower panels.

Fig. 15. Hα line profiles with the inclusion of the photospheric component in model 4. The contribution of the photospheric line, the shell absorption and the disk emission component are also shown. The ordinate is in the flux form.

for the smaller mass loss rate and for the larger inclination angle. The formation of the peak separation will be also discussed in section 6.2. The strong shell component appears for $i = 90^\circ$ in models 4 and 7. These behaviors, including the narrow double-peak emission line profile in the pole-on case, are similar to those obtained by Hummel (1994). We also note that the central quasi-emission (CQE) is noticed for $i = 90^\circ$ of model 1 (for CQE, see Hanuschik 1995).

The Hα line profiles, in the flux form, with the inclusion of the photospheric component are illustrated in figure 15 for model 4, where we also show the contribution from various components: the emission component, the shell absorption component, and the photospheric component. The emission component includes the line and continuum originated in the disk. The intrinsic photospheric profile was taken from ATLAS9 (Kurucz 1993) with the same parameters as for the stellar SED. This profile is put on the rotating stellar surface with the critical equatorial rotational velocity of 591 km s$^{-1}$ without limb darkening. One can see the veiling effect by the disk continuum, which is more noticeable in the smaller inclination angle.

For illustrating a case contaminated heavily by the disk continuum, we show, in figure 16, the Hα profiles in model 7. In addition to the veiling effect, the emission component has absorption features below the disk continuum in the wing part for $i = 30^\circ - 75^\circ$ and in the central part for $i = 90^\circ$. These absorption features are formed by the line absorption of the disk continuum, i.e., in the pseudo-photosphere. This suggests that 1) for the intermediate inclination angle, it is difficult to segregate the photospheric component from the observed profile, caused by the broad shell absorption and the pseudo-photospheric absorption, and 2) for $i \sim 90^\circ$, the observed central intensity could be almost zero even if the disk continuum is present. We note that the emission equivalent width rectified by the disk continuum and integrated over the line feature may become even negative, caused by the dominant pseudo-photospheric absorption feature. This is notable in the higher Balmer lines in our models. We also note that a weak central shell absorption feature appears for $i \geq 75^\circ$ in model 7. Our major results on Hα line are similar to those of Hummel (1994) except the veiling effect and the pseudo-photospheric absorption, which are newly introduced in this paper.

Fig. 16. Same as figure 15 but in model 7.
Fig. 17. Vertical Hα source function variation at $\zeta = 1.06$ in model 7. Several optical depths at the line center are indicated.

6.2. Peak Separation

As stated in section 6.1, the narrow double-peak emission profile appears in the case of $i \sim 0^\circ$. The same characteristics was concluded and interpreted by Hummel (1994) in terms of the non-coherent scattering (NCS). Figure 17 illustrates the vertical change of the Hα source function at the distance of $\zeta = 1.06$ in model 7. The optical depth at the line center is also shown. One can see that the source function increases as the optical depth increases in the effective line formation region. Similar tendency is kept in the inner part of the disk, where the source function is large. Recalling that no velocity gradient exists in the vertical direction in our model, the central reversal is naturally formed.

In figure 18a, we show the resulting peak separation as a function of $\sin i$. As $\sin i$ increases, the profile characteristic changes from double-peak profile caused by NCS, passing the single-peak profile, to the double-peak profile of rotation origin. The peak separation caused by NCS is within two or three Doppler widths in our calculations and increases as the mass loss rate increases, caused by the stronger central reversal. The domain of this type of double-peak profiles expands further to the larger inclination angle as the mass loss rate increases. The same type profiles are also formed in Hβ and Hγ lines with the peak separations narrower than those in Hα line (see figure 14), reflecting their smaller optical thickness.

The peak separation originated in the rotational velocity field may be broadened by the shell and the other absorption components. In figure 18a, we also show the peak separations derived from the emission component only, the profile including the shell absorption component, and the profile including further the photospheric component for models 4 and 7. While, we show only the peak separation derived from the emission component for model 1, because the the emission component is very weak, so the determination of peak position is difficult when the shell absorption and the photospheric component are taken into account. One can see that the effect of making peak separation larger by such absorption features is small even in the weak emission profiles for $i \sim 90^\circ$ in model 4. The peak separations in model 7 is little influenced by the background absorption features, since the pure emission profiles have sharper peaks. Note that the large deviation of the peak separation at $i = 90^\circ$ from those at the other inclination angles is originated in the pure emission component itself.

Now, let us examine the peak separation of the double-peak profile in the higher inclination angle. The peak separation of the double-peak emission line profile in Be stars has been often used for estimating the outer radius of the disk. Observationally, it is well known that the peak separation is smallest in Hα and becomes larger for the higher Balmer lines, reflecting the emitting sizes in each lines. The outer radius of the disk, $R_{\text{disk}}$, in the Keplerian rotation law is estimated as

18
\[ \frac{R_{\text{disk}}}{R_*} = \left( \frac{2v_\phi(\pi - 1)\sin i}{\Delta V_{\text{peak}}} \right)^2, \]  

(21)

where \( \Delta V_{\text{peak}} \) is the peak separation of the double peak profile. This formula is derived from the profile calculation for the rotating flat disk with a finite outer radius, under the assumption that the line surface brightness distribution is given (e.g., Huang 1972, 1973, Hirata & Kogure 1984). The velocity at the maximum intensity corresponds to that at which the iso-line-of-sight velocity curve contacts with the outer radius, unless the surface brightness distribution is a steeply decreasing function of the distance. If the surface brightness is locally proportional to \( \varpi^{-m} \), it is easy to show that the region with \( m > 2.5 \) does little contribute to the emission profile in the Keplerian disk (see equations (4)–(7) in Hirata & Kogure 1984). Then, the peak separation in our model which extends infinitely towards the radial direction is thought to reflect the maximum radius with \( m < 2.5 \).

The peak separation given by equation (21) is proportional to \( \sin i \) when the outer radius is fixed. While, in figure 18a, we notice a quasi-linearity except for \( i \sim 0^\circ \) and \( i \sim 90^\circ \). Hence, the calculated peak separation reflects somehow the effective disk radius, although the quasi-linear part does not pass the zero point, in contrast to the implication from equation (21). It is possible to derive the effective outer radius by fitting the slope in the quasi-linear part of figure 18a to that in equation (21). Then, we obtain \( R_{\text{disk}} = 1.7, 4.9 \) and \( 12.3R_* \) for models 1, 4, and 7, respectively.

Although we did not calculate the line surface brightness for our models, it is possible to examine how the peak position is formed in our models. Figure 19 shows the variation of the escape probability \( \beta \), the source function \( S \), and \( p = N_{3} \sum_{n=2s,2p}A(3,n)\beta(n,3) \) for \( \text{H}\alpha \) line in model 7 along the equatorial plane. The last quantity is proportional to the \( \text{H}\alpha \) photon energy which escapes from the equatorial plane to the outside of the disk and may be regarded as the angle-averaged surface brightness distribution on the equatorial plane. With a slope of \( m = 2.5 \) for \( p \), we obtain \( R_{\text{disk}} = 1.3, 3.5, \text{and } 7.8R_* \) for models 1, 4, and 7, respectively. These values are, roughly speaking, consistent with those estimated from the slopes in figure 18a, mentioned before. The source function illustrated in figure 19 essentially behaves in the same manner as \( p \) in the main part of the disk, having a slower decline in the inner part and a steeper decline in the outer part. Hence, we can guess the formation region of any lines from their source function behavior along the equatorial plane. The source function for \( \text{H}_{mn} \) line is approximately proportional to \( b_{n'}/b_{n} \). From figure 2, it is evident that the higher Balmer line is formed in the region nearer to the star than the region where the lower Balmer line is formed, thus has a larger peak separation.

In figure 18b, we show the \( \text{H}\alpha \) outer radii estimated from equation (21), together with the radii estimated from \( p \) and \( \beta(2p,3) = 0.5 \) at the equatorial plane. We choose \( \beta(2p,3) = 0.5 \) as the effective boundary between the optically thick and thin regions at \( \text{H}\alpha \). Roughly speaking, the radii estimated by equation (21) fall in between our two rough estimates. However, the
Fig. 18. (a) Hα peak separation against sin i for three models. The peak separation obtained from the emission component, the emission+shell component, and the emission+shell+photospheric components are designated by "+", ◦, and ×, respectively. (b) Disk radius calculated from equation (21). Symbols are the same as in (a). We also show two rough theoretical estimations: the distance where the escaping Hα photons begins to decrease rapidly from the equatorial plane (thin dotted line with a designation of m = 2.5) and the distance where the escape probability of the Hα photon becomes 0.5 at the equatorial plane (thick dashed line with a designation of β = 0.5). See the text for more details.

Fig. 19. Escape probability, source function, and $p = N_3 \sum_{n=2s,2p} A(3,n) \beta(n,3)$ at the equator for Hα line in model 7. See the text for the details.

direct application of equation (21) to the observation gives only a rough estimation of the disk radius. Hummel (1994) obtained the same conclusion and discussed the peak separation formation, based on the emissivity behavior for his $N(\varpi,0) \propto \varpi^{-3}$ models.

6.3. Equivalent Width and Balmer Decrement

Physically meaningful line emission strength would be the total flux calculated from the emission component above the disk continuum, illustrated in figures 15 and 16. We call it a line emission power, hereafter. However, the synthesized emission profiles are more complex one, disturbed by the disk continuum, by the shell component in the case of large inclination angle, and by the pseudo-photospheric absorption in the optically thick disk. Here, we simply define the emission equivalent width, $W_\alpha$, as the area above the apparent continuum level in figure 14. This corresponds approximately to the equivalent width above the photospheric absorption profile in actual observation.

Figure 20 shows the variation of $W_\alpha$ as a function of the mass loss rate. The computed Hα equivalent widths are rather small and are similar to those reported by Hummel (1994) and Hummel & Hanuschik (1997) for their models with $N(\varpi,0) \propto \varpi^{-3}$. They are 6 Å at most and do not reproduce the observed ones (e.g., Briot 1971; Dachs, Hanuschik & Kaiser 1986; Dachs, Rohe & Loose 1990, 1992). Sigut & Jones (2007) have also concluded that the Hα luminosity for their models with $\rho \propto \varpi^{-3.5}$ is too faint when compared with the observations. This problem will be discussed in the last section. At the high end of the mass loss rate larger than $5 \times 10^{-11} M_\odot$ yr$^{-1}$, the equivalent width gradually decreases due to the veiling effect for $i \leq 60^\circ$. In figure 20, we also show the equivalent width integrating over the profile, including the absorption feature in the case of $i = 90^\circ$ (dotted line) in order to demonstrate the effect of the absorption feature: the broad shell absorption component overwhelms the emission component when the mass loss rate is low (see figures 14 and 15). This implies that the development of a broad absorption feature is the first spectroscopic evidence of a new mass loss event in the case of $i \sim 90^\circ$.

Figure 21 shows (a) the line emission power and (b) the equivalent width of Hα against cos i. The Hα emission power is free from the veiling effect and from the shell absorption by its
Fig. 20. Variation of the Hα emission equivalent width as a function of the mass loss rate. The disk mass is also shown in the upper part. The dotted line corresponds to the equivalent width with the inclusion of the absorption feature below the apparent continuum level.

Fig. 21. (a) Emission power, (b) emission equivalent width of Hα as a function of cos i.

definition. Declines at \( i = 90^\circ \) in models 4 and 7 are caused by the central pseudo-photospheric absorption in figure 21a and by the central shell and pseudo-photospheric absorption in figure 21b. The Hα emission equivalent width in figure 21b suffers strong veiling effect for \( i \leq 60^\circ \) in model 7. From figure 21a, we conclude that Hα photons also tend to escape towards the polar direction, in spite of the existence of velocity gradient. However, such a tendency is less remarkable, compared with the continuous radiation (see figures 9, 13, and 21a). We confirmed that Hβ and Hγ lines show the same tendency.

The Balmer decrement may be defined as the ratio of the fluxes above the apparent continuum level in figures 15 and 16, corresponding to our definition of the emission equivalent width. Here, we call it observable Balmer decrement. We also define theoretical Balmer decrement derived from the line emission powers. Figure 22 shows (a) theoretical and (b) observable Balmer decrements in the form of the ratio of Hα to Hβ \( (D_{34}) \) versus the ratio of Hγ to Hβ \( (D_{54}) \). For the observable Balmer decrement, we do not adopt the cases in which \( W_\alpha \) is less than 0.1 Å, because the results are less accurate and are also meaningless from the observational point of view. Several cases for high inclination angles are lost in figure 22b due to this cut-off in Hγ and Hβ lines. For comparison, we also show a nebular case B solution for \( T_e = 10000 \) K (Brocklehurst 1971) by a large plus marked ”Case B”. The theoretical Balmer decrements are located in the left of case B, while the observable Balmer decrements extend towards right and lower direction, caused by the shell absorption effect for high inclination angle (see the cases for \( i = 75^\circ \)). Our results in figure 22b fall in the observed area given in Dachs, Rohe & Loose (1990). In both diagrams, the Balmer decrement is steeper for higher inclination angle. This is consistent with the observational implication of steeper decrement for shell stars suggested by Dachs, Rohe & Loose (1990).

7. Summary and Concluding Remarks

The non-LTE state of hydrogen atoms in the isothermal steady-state decretion disk around a B1V star has been calculated, based on the \( \Lambda \) iteration for the continuous radiation and the single-flight escape probability for the line radiation. Thus, we obtained the continuous
radiation field without approximation. The basic physical process in our models was presented. We described the non-LTE state in terms of the $b_n$ factors. We examined the radiation flow and radiation conversion in the disk. The local and global deviations from the radiative equilibrium in our iso-thermal models were examined through counting the energy gain and loss. We also calculated several photometric and spectroscopic quantities and examined their dependency on the mass loss rate and the inclination angle. Some implications to the observable quantities were also presented.

Our main conclusions are summarized as follows:

1. The $b_n$ factors increase outwards from $b_n \sim 1$. The $b_n$ factors of $2p$ and higher levels then turn to decrease at the distance where the $H_{n,n-1}$ line becomes optically thin and finally tend to a nebular case A solution. In the optically thick disk, the LTE region is added in the innermost region of the disk.

2. Stellar Balmer radiation plays a definitive role in the radiation field of the disk. The conversion of the stellar Balmer radiation into the longer wavelength continuous radiation and the line emission is the main process in radiation field of the Be star disk. Stellar Lyman continuum contributes only to the ionization in the outer envelope of the disk where the disk is optically thin towards the central star.

3. The radiative equilibrium almost holds in the disk with $T_{\text{disk}} = 2/3T_{\text{eff}}$ in the optically thin case for the Balmer continuum. Similar conclusion is obtained by Carciofi & Bjorkman (2006). When the disk becomes optically thick, there appear two cool regions: the inner region caused by the deficient Balmer continuous radiation and the outer region caused by the Lyman $\alpha$ cooling.

4. The optical thickness of the disk is smallest towards the vertical direction for the continuum. Hence, the continuous radiation tends to escape from the disk vertically. Our models with the mass loss rate less than $10^{-10} M_\odot \text{yr}^{-1}$ cover well the observed variations in the $UBV$ and IRAS bands. The infrared excess in the IRAS band for $90^\circ$ is smaller than that for $i = 0^\circ$ by $1.5-1.8$ mag. We suggest that the pole-on approximation in Waters (1986) is valid only for $i \leq 30^\circ$. The infrared mass loss rate based on our model is $10^{-12} - 10^{-10} M_\odot \text{yr}^{-1}$, three order of magnitude smaller than that of Waters, Coté & Lamers (1987).

5. General characteristics of the observed profiles of the classical Be stars like the wine-bottle, double peak, and shell profiles are reproduced in our non-LTE model, just as was shown by Hummel (1994). However, the isothermal steady-state decretion disk does not give the observed large H$\alpha$ emission intensity.

6. The pseudo-photospheric absorption feature through the line absorption of the disk continuum appears in the case of the large mass loss rate.

7. The peak separation of the double-peak profile is almost determined from the emission
profile, that is, the shell absorption and the photospheric absorption affect the peak position only a little bit. Though the peak separation gives a rough measure of the outer radius of the disk, the resultant radius still depends on the inclination angle. This is just the same conclusion given by Hummel (1994).

Most serious result in our calculation based on the $N(\varpi,0) \propto \varpi^{-3.5}$ law is that we did not get the large H$\alpha$ emission equivalent width observed in early Be stars. As the mass loss rate increases, our isothermal model sequence with $T_{\text{disk}} = 2/3 T_{\text{eff}}$ deviates more from the radiative equilibrium in the sense that the radiation loss from the disk is larger than the energy input from the star. This means that the radiation energy lost from the disk comes partially from the compensation of the internal energy in the disk with the temperature higher than the equilibrium temperature. Hence, the H$\alpha$ intensity will not increase even if we introduce the radiative equilibrium model with the same density structure.

It is highly probable that we should introduce some factors which enhance the line emission, compared with the continuous emission. The Be star disk may extend more towards the vertical direction. We introduced the turbulence velocity which is equal to the sound velocity. However, we obtained only a small enhancement in the H$\alpha$ equivalent width. Introduction of high velocity gradient may improve the situation. We calculated the non-LTE state for the model proposed by Stee & Araújo (1994), keeping the disk mass of our standard disk model. However, the resultant H$\alpha$ equivalent width does decrease drastically, since the high density region becomes compact. Sigut & Jones (2007) adopted the $N(\varpi,0) \propto \varpi^{-2.5}$ law and obtained the H$\alpha$ luminosity comparable to the observed values. We also made the preliminary calculation for the $N(\varpi,0) \propto \varpi^{-2.5}$ law with the same mass loss rate as in model 7 and obtained the H$\alpha$ emission equivalent width of 80 $\text{Å}$ for $i = 90^\circ$, reflecting the large projected area of the optically thick region at H$\alpha$. This result may suggest that the density in actual Be stars decreases more slowly than our model with $N(\varpi,0) \propto \varpi^{-3.5}$, as suggested by Sigut & Jones (2007). This is possible if the disk temperature decreases outwards, caused by deficient Balmer continuous radiation in the optically thick region.

Our adoption of the single-flight escape probability for the lines may not be valid in the solution of the line radiative transfer. It is evident that this approximation underestimates the net radiative bracket, because the local escape of photons is replaced by the global escape from the disk. It is also important to get the exact solution numerically for the line transfer problem.

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