Decomposition rate and interaction of fungi: A random perturbation differential equation approach

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Abstract. In order to describe the interaction between different fungi and their effects on lignocellulose decomposition, we established a fungal decomposition rate model, a fungal competition model and a dynamic disturbance model. Function fitting was used to represent the effects of different independent variables on the decomposition rate of fungus, and finally the expression and image of two important independent variables on the decomposition rate of Hyphal extension and moisture trade-off were obtained. Fungal interaction is mainly a competitive relationship, on the basis of which the differential relationship is used to establish a model to analyze the situation when the system reaches a stable state, and the stochastic dynamic disturbance equation is added to simulate the influence of environmental change on the system. The final analysis shows that there are four kinds of stable states: extinction, non-equilibrium persistence, weak equilibrium persistence and strong equilibrium persistence.

Keywords: Fungal decomposition rate, The differential equation, Random dynamic interference, Fungal interactions

1. Introduction
Ecological system is an open system, producers to collect material and energy in the system, disintegrator release them, the entire ecosystem balance and development is very important Fungus is rich in ecological system diversity, to maintain the stability of ecosystem diversity plays an important role Research on fungal decomposition rate of wood fiber and the interaction between different fungi on the maintenance of ecosystem stability has important significance Nutrients because space is limited, so the current on the interaction between different fungi is most as the rivalry between the population problem is analyzed However, the weather temperature in nature The environment is changing all the time, which will lead to deviation between practice and theory, especially for fungi sensitive to temperature and humidity. Therefore, the dynamic disturbance model can be established by adding stochastic dynamic disturbance equation to simulate environmental changes, which can effectively analyze the relative stable state of the ecosystem.
2. Model Establishment

2.1 Fungal Decomposition Rate Model (FDRM)

Through access to relevant data analysis found that the main factors influencing the rate of decomposition temperature humidity degree of overlap between community oxygen concentration light intensity time woody plant species site rotten year sampling mesh side extension rate, etc. Therefore, these factors as independent variables, with a certain amount of wood fiber to describe the decomposition rate and loss rate as the dependent variable fungal decomposition rate model is set up.

\[ y^* = a_0 + a_1x_1 + \cdots + a_kx_k \]  

(1)

Where, is the mass loss, is the value of the i-th influencing factor, namely the i-th independent variable, and is the assumed coefficient.

The least square multiplier is used as the fitting index to confirm the coefficient of the fitting equation in Equation (1) \[1\]. It can be seen from the following that the solution of the equation set (2) is the coefficient of the fitting equation.

\[
\begin{align*}
\varphi(a_0, a_1, \ldots, a_k) &= \sum_{n=1}^{N} (y_n - y_n^*)^2 = \sum_{n=1}^{N} (y_n - a_0 - a_1x_1n - \cdots - a_kx_kn)^2 \\
\frac{\partial \varphi}{\partial a_0} &= -2 \sum_{n=1}^{N} (y_n - a_0 - a_1x_1n - \cdots - a_kx_kn) = 0 \\
\frac{\partial \varphi}{\partial a_1} &= -2 \sum_{n=1}^{N} (y_n - a_0 - a_1x_1n - \cdots - a_kx_kn)x_1n = 0 \\
\frac{\partial \varphi}{\partial a_k} &= -2 \sum_{n=1}^{N} (y_n - a_0 - a_1x_1n - \cdots - a_kx_kn)x_kn = 0 \\
\end{align*}
\]

(2)

\[
\begin{align*}
a_0 + a_1l_{11} + a_2l_{12} + \cdots + a_kl_{1k} &= l_{1y} \\
a_1l_{21} + a_2l_{22} + \cdots + a_kl_{2k} &= l_{2y} \\
\vdots \\
a_1l_{ky} + a_2l_{k2} + \cdots + a_kl_{kk} &= l_{ky} \\
a_0 &= \bar{y} - a_1\bar{x}_1 - \cdots - a_k\bar{x}_k \\
\end{align*}
\]

(3)

(4)

Where,

\[
\begin{align*}
l_{ij} &= l_{ji} = \sum_{n=1}^{N} x_{in}x_{jn} - \frac{1}{N} (\sum_{n=1}^{N} x_{in})(\sum_{n=1}^{N} x_{jn}) \\
l_{iy} &= \sum_{n=i}^{N} y_nx_{in} - \frac{1}{N} (\sum_{n=1}^{N} x_{in})(\sum_{n=1}^{N} x_{jn}) \\
\end{align*}
\]

(5)

By analyzing the data provided by Nicky Lustenhouwer \[4\], we draw four violin maps to analyze the relationship between the Mass loss and Site, Years decayed, Mesh and Sampling side.

Figure 1. Violin maps between Mass loss and other variables
Using the least square method, we can get the function expression of Mass loss regarding Extension rate under different Years decayed conditions as follows:

\[
\begin{align*}
y &= 1.54x + 17.31, \text{when } c = 3; (R^2 = 0.34) \\
y &= 1.237x + 57.87, \text{when } c = 5; (R^2 = 0.20)
\end{align*}
\]  

(6)

Where, \( y \) is Mass loss, \( x \) is Extension rate, \( c \) is Years decayed, and \( R^2 \) is the coefficient of determination.

By observing and analyzing the two images obtained from the research results of Nicky Lustenhouwer et al.\[4\], and using the data in the two papers related to the attachment to reproduce the images, it can be found that the hyphal extension rate is approximately in logarithmic relation with the decomposition rate, and the moisture trade-off is approximately in a linear relationship with the logarithm of the decomposition rate\[2\].

Therefore, assuming the decomposition rate as a constant value \( Y \), the approximate relationship function between the hyphal extension rate and the decomposition rate and the approximate relationship function between the moisture trade-off and the decomposition rate can be selected as follows:

\[
\begin{align*}
Y &= a_1 h + b_1, \ln Y = a_2 m + b_2
\end{align*}
\]

(7)

Where, \( h \) is the hyphal extension rate, \( m \) is moisture trade-off, \( a_1, b_1, a_2, b_2 \) are assumed coefficients.

Then, the influences of the hyphal extension rate and moisture trade-off on decomposition rate are combined in a linear relationship, and the results were simplified.

\[
y = r_1 \ln(r_0 h + r_d) + r_2 h + r_3
\]

(8)
Where \( r_0, r_0', r_1, r_2, r_3 \) are coefficients.

Using the above model, we can fit the relationship between decomposition rate and hyphal extension rate at different temperatures:

\[
\begin{cases}
y = 2.73x + 1.855, & \text{when temperature} = 10; \quad (R^2 = 0.50) \\
y = 2.247x + 4.799, & \text{when temperature} = 16; \quad (R^2 = 0.41) \\
y = 2.509x + 11.48, & \text{when temperature} = 22; \quad (R^2 = 0.25)
\end{cases}
\]

And the relationship between the decomposition rate and moisture trade-off is as follows:

\[
y = x + 1.888; \quad (R^2 = 0.40)
\]

Then, the effects of the hyphal extension rate and moisture trade-off on decomposition rate are combined with a linear relationship, and the relationship expression among the three can be obtained:

\[
y = 0.37x_1 + 0.37x_2 + 2.07; \quad (R^2 = 0.61)
\]

Where, \( x_1 \) is the log(Hyphal extension rate) , \( x_2 \) is the moisture trade-off, \( y \) is the log(decomposition rate)

**Figure 4.** Combine hyphal extension rate and moisture trade-off on decomposition rate

### 2.2 Lotka-Volterra Model (LVM)

When fungus 1 and fungus 2 are present, an increase in the number of one inhibits the growth of the other, given limited space and resources. Several parameters are used to represent the rules that the two fungi obey in the process of competition as follows[4]:

\[
\begin{align*}
\frac{dN_1}{dt} &= r_1N_1(1 - \frac{N_1}{k_1} - \alpha\frac{N_2}{k_1}) \\
\frac{dN_2}{dt} &= r_2N_2(1 - \frac{N_2}{k_2} - \beta\frac{N_1}{k_2})
\end{align*}
\]

Where, \( N_1 \) and \( N_2 \) are the size (expressed in number) of the two fungi, \( k_1 \) and \( k_2 \) are the maximum environmental tolerance of the two fungi, \( r_1 \) and \( r_2 \) are the growth rate of the two fungi, \( \alpha \) is the coefficient of competition of 1 to 2, \( \beta \) is the coefficient of competition of 2 to 1.

(a) The coefficient matrix characteristic equation of the linear equations is \((\lambda - r_1)(\lambda - r_2) = 0\), so \(E_1(0,0)\) is the unstable point.

(b) The characteristic equation of the corresponding coefficient matrix is \((\lambda + r_1)(\lambda - r_2) = 0\), so \(E_2(k_1, 0)\) is the saddle point.

(c) Similarly, the coefficient matrix characteristic equation of the linear system corresponding to \(E_3(0, k_2)\) can be obtained as \([\lambda - (r_1 - \frac{r_1\alpha k_2}{k_1})](\lambda + r_2) = 0\). So when \((k_1 - \alpha k_2) > 0\), \(E_3(0, k_2)\) is the saddle point; when \(k_1 - \alpha k_2 < 0\), \(E_3(0, k_2)\) is the stable node.

(d) Transform the singularity \(E_4(k_1 - \alpha k_2, k_2 - \beta k_1)\) as follows:
\[
\frac{dN_1}{dt} = \frac{r_1}{k_1} (N_1 \cdot (k_1 - \alpha k_2)) (N_2 - N_2)
\]
\[
\frac{dN_2}{dt} = \frac{r_2}{k_2} (N_2 \cdot (k_2 - \beta k_1)) (N_1 - N_1)
\]
\[
N_1 = N_1 - (k_1 - \alpha k_2), N_2 = N_2 - (k_2 - \beta k_1)
\]

The corresponding characteristic equation is \[ \lambda + \frac{r_1(k_1 - \alpha k_2)}{k_1} [\lambda + \frac{r_2(k_2 - \beta k_1)}{k_2}] = 0, \]
so \(E_4(k_1 - \alpha k_2, k_2 - \beta k_1)\) is the stable point. According to the above stability analysis, four results as shown in the following table can be seen under different inhibition relationships.

| Different inhibition relationships | Fungus 1 inhibits 2 \((k_1 > \frac{k_2}{\beta})\) | Fungus 1 disinhibits 2 \((k_1 < \frac{k_2}{\beta})\) |
|----------------------------------|---------------------------------|---------------------------------|
| Fungus 2 inhibits 1 \((k_2 > \frac{k_1}{\alpha})\) | Either Fungus 1 or 2 survives(I) | Fungus 2 survives(II) |
| Fungus 2 disinhibits 1 \((k_2 < \frac{k_1}{\alpha})\) | Fungus 1 survives(III) | A balance of Fungus 1 and 2(IV) |

By using the curve representation, we can more intuitively observe the interaction between the two fungi and the different evolution of the system under different inhibition relations.

**Figure 5.** Four results represented by curves

### 2.3 Dynamic Interference Model (DIM)

On the basis of LVM, another expression of Lotka-Volterra model\(^5\) is used as follows:

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= x_1(t) [r_{10} - a_{11} x_1(t) - a_{12} x_2(t)] \\
\frac{dx_2(t)}{dt} &= x_2(t) [r_{20} - a_{21} x_1(t) - a_{22} x_2(t)] \\
x_1(t) > 0, r_{10} > 0, a_{ij} > 0, (i = 1,2; j = 1,2).
\end{align*}
\]

(13)

Where, \(x_1(t)\) and \(x_2(t)\) are the scale of fungi 1 and 2 at time \(t\) (described by number), \(r_{10}(0)\) is the growth rate within the population, \(a_{ij}\) is the competition coefficient within the population (when \(i = j\)) or the competition coefficient between the populations (when \(i \neq j\)).

Let \(C_0(t)\) represent the amount of external factors influencing individuals at time \(t\), and \(C_c(t)\) represent the amount of external factors influencing the environment at time \(t\)\(^6\). Assuming that the fungal population does not migrate and is homogeneous and introduces random disturbance\(^7\), then:

\[
r_{10} \rightarrow r_{10} + \sigma_i^2 dB_i(t)
\]

(14)
Where, $dB_i(t)$ is the interference of external variables, $\sigma_i^2$ is Brown’s exercise intensity.

\[
\begin{align*}
\frac{dC_0(t)}{dt} &= kC_c(t) - (g + m)C_0(t) \\
\frac{dC_c(t)}{dt} &= -hC_c(t) + u(t)
\end{align*}
\] (15)

The initial values of the two influences are $C_0(0) = C_c(0) = 0$

Where, $r_{i1}$ is the influence intensity of external factors on the dynamical system, $k, g, m, h$ are all normal numbers, $kC_c(t) - gC_0(t)$ is the amount of external influence absorbed by the population from the environment at time $t$, $mC_0(t)$ is the self-digesting amount of the population to the external environment at time $t$, $-hC_c(t)$ is the self-digesting amount of the primary environment to external factors at time $t$, $u(t)$ is the input rate of external disturbance to the environment.

Then we can get the following conclusion, When $\lim_{t \to +\infty} x(t) = 0$ a.s fungal population $x(t)$ will extinct. When $\lim_{t \to +\infty} [x(t)] = 0$ a.s. fungal population $x(t)$ is not average persistent. When $[x(t)] > 0$ a.s. fungal population $x(t)$ is weakly average persistent. When $[x(t)]^* \geq 0$ a.s. fungal population $x(t)$ is strongly average persistent.

The above four results correspond to the following four dynamic interference situations:

a) If $[C_0(t)]^* > \varphi(\psi)$, then $\lim_{t \to +\infty} x(t) = 0$, the system is in a extinction state.

\[
\varphi = \begin{cases} 
\frac{r_{10} - \sigma_1^2}{2}, & \delta \leq 0 \\
\frac{r_{11}}{\delta}, & \delta > 0
\end{cases}, \quad \psi = \begin{cases} 
\frac{r_{20} - \sigma_2^2}{2}, & \delta > 0 \\
\frac{r_{21}}{\delta}, & \delta \leq 0
\end{cases}
\] (16)

b) If $[C_0(t)]^* = \varphi([C_0(t)]^* = \psi)$, then $\lim_{t \to +\infty} [x(t)] = 0$, the system is in a non-equilibrium persistence state.

c) If $[C_0(t)]^* < \varphi(\psi)$, then $[x(t)]^* > 0$, the system is in a weak equilibrium persistence state.

d) If $\lim_{t \to +\infty} [C_0(t)]$ is bounded and satisfies the following conditions, the system is in a strong equilibrium state.

\[
\begin{align*}
\Delta - \Phi_1 \lim_{t \to +\infty} [C_0(t)] &\geq 0 \\
\Delta - \Phi_2 \lim_{t \to +\infty} [C_0(t)] &\geq 0
\end{align*}
\] (17)

\[
\lim_{t \to +\infty} [X_1(t)] = \frac{\Delta - \Phi_1 \lim_{t \to +\infty} [C_0(t)]}{\Delta} \geq 0, \quad \lim_{t \to +\infty} [X_2(t)] = \frac{\Delta - \Phi_2 \lim_{t \to +\infty} [C_0(t)]}{\Delta} \geq 0
\] (18)

The comparative advantages and disadvantages of different types of fungi and different environments can be predicted through the above model analysis.

Figure 6. Four dynamics of fungi 1 and 2
3. Conclusion

The dynamic disturbance model takes into account the effects of environmental changes in nature on fungi and can be combined with the fungal decomposition rate model to predict the future change of fungal decomposition rate on lignofiber. From the above demonstration, fungi with the ability to inhibit other fungi always have a comparative advantage, regardless of whether the fungal population is in an average persistent state or not. If none of the fungal populations can maintain a strong average persistence, then at least one of them will become extinct. Fungi that are inhibited by other fungi, the less competitive fungi are at a relative disadvantage. In arid and semi-arid climate areas, the temperature difference between day and night is large, and the humidity is low, so the fungi that grow slowly have a comparative advantage. In temperate climates, where temperatures change with the seasons and ecosystems vary from season to season, slow-growing fungi can periodically gain an advantage. Rainforests are hot, rainy and relatively humid, so fungi that grow slowly and can tolerate heat and humidity are at an advantage.

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