Extra neutral gauge bosons arise in many extensions of the Standard Model. If one were to be discovered it would be necessary to measure its properties so that we could understand its origins. In this report we find that $Z'$ couplings can be measured at the ILC precisely enough to distinguish between models up to a $Z'$ mass of 2-3 TeV. An important ingredient in these measurements is polarization of the $e^-$ and to a lesser extent $e^+$ beams. $b$ and $c$-quark tagging would also give important additional information.

1. INTRODUCTION

Extra neutral gauge bosons ($Z'$) and other $s$-channel resonances are predicted by many models of new physics. Previous studies have examined $Z'$s predicted by string inspired models, LR symmetric models and numerous other extended gauge theories [1, 2, 3, 4, 5, 6]. Recently there has been a resurgence of interest in extra gauge bosons as they arise in many theories of current theoretical interest - in models with extra dimensions as Kaluza Klein excitations of the photon and $Z^0$ [7] and in the various manifestations of the Little Higgs Models [8, 9, 10, 11, 12]. In many scenarios it is possible that they are low enough in mass to be discovered by the LHC. However, this is but the first step to determine the underlying theory. To distinguish between the numerous possibilities it will be necessary to measure the properties of these new resonances.

The International Linear Collider is ideally suited to this task as it can make precision measurements of various observables starting with the basic process $e^+e^- \rightarrow f\bar{f}$ where $f$ could be leptons ($e, \mu, \tau$) or quarks ($u, d, c, s, b$) (and possibly the $t$-quark). This topic has been studied in a number of places, most notably in the work of S. Riemann as reported in Ref. [3]. In this contribution we expand on the work of Riemann by including a wider set of models of new physics and by exploring the sensitivity of the results to electron and positron polarization, to the $Z'$ mass, and to including additional observables in the fits. In particular, we examine what can be learned by including $c$ and $b$-quark tagging to remove the ambiguities when only leptonic observables are included. In this contribution we concentrate on our new results on $Z'$ properties. More generally other forms of $s$-channel resonances appear in theories of current interest. For example, theories of extra dimensions predict Kaluza Klein towers of massive gravitons. These are spin 2 objects so that they can be distinguished from $Z'$s by measuring the angular distribution of their decay products, either directly at the LHC or indirectly at the ILC.

We start with a short description of the observables with special emphasis on polarization. We then give our results for the various scenarios considered. A more detailed report will be given elsewhere [13].

2. OBSERVABLES IN $e^+e^-$ COLLISIONS

At $e^+e^-$ colliders, precision measurements see the effects of new $s$-channel resonances through deviations from standard model predictions due to interference between the $Z'$s and the photon and SM $Z^0$. The basic process is $e^+e^- \rightarrow f\bar{f}$ where $f$ could be leptons ($e, \mu, \tau$) or quarks ($u, d, c, s, b$). From the basic reactions a number of observables can be used to search for the effects of $Z'$s: The leptonic cross section, $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, the ratio of the hadronic to the QED point cross section, $R^{\text{had}} = \sigma^{\text{had}}/\sigma_0$, the leptonic forward-backward asymmetry, $A^{\ell}_{FB}$, the leptonic longitudinal asymmetry, $A^{\ell}_{LR}$, the hadronic longitudinal asymmetry, $A^{h}_{LR}$, the forward-backward asymmetry for specific quark or lepton flavours, $A^{f}_{FB}$, the $\tau$ polarization asymmetry, $A^{\tau}_{pol}$, and the polarized forward-backward asymmetry for specific fermion flavours, $A^{f}_{FB}(\text{pol})$. The indices indicate the final state fermions where $f = \ell$ and $q$.
with $\ell = (e, \mu, \tau), q = (c, b)$, and $\text{had} =$ ‘sum over all hadrons’. The expressions for these observables are given in ref. $[5, 14]$. A deviation for one observable is always possible as a statistical fluctuation so a more robust strategy is to combine many observables to obtain a $\chi^2$ figure of merit. We follow this approach by including various observables.

2.1. Polarization

An important ingredient in precision measurements at the ILC is the use of electron and positron polarization $[15, 16]$. These were included in the results given by Riemann in Ref. $[2]$. Although these details are given in Ref. $[15, 16]$ we reproduce the results here for the benefit of the interested reader. The cross section at an $e^+e^-$ collider with longitudinally-polarized beams can be written as:

$$
\sigma_{P_e^-P_e^+} = \frac{1}{4} \left\{ (1 + P_e^-)(1 + P_e^+)+ (1 - P_e^-)(1 - P_e^+)+ (1 + P_e^-)(1 - P_e^+)+ (1 - P_e^-)(1 + P_e^+) \right\}
$$

where $\sigma_{RL}$ stands for the cross section if the $e^-$ beam is completely right-handed polarized ($P_{e^-} = 1$) and the $e^+$ beam is completely left-handed polarized ($P_{e^+} = -1$) and analogously for $\sigma_{LR}, \sigma_{RR}, \text{and } \sigma_{LL}$. For $s$-channel production of vector bosons only $\sigma_{LR}$ and $\sigma_{RL}$ contribute. For this case the cross section for arbitrary polarizations can be written as

$$
\sigma_{P_e^-P_e^+} = (1 - P_{e^+}P_{e^-})\sigma_0[1 - P_{eff}A_{LR}]
$$

where $\sigma_0 = (\sigma_{RL} + \sigma_{LR})/4$ is the unpolarized cross section, $A_{LR} = (\sigma_{LR} - \sigma_{RL})/(\sigma_{LR} + \sigma_{RL})$ is the left-right asymmetry, and $P_{eff} = (P_{e^-} - P_{e^+})/(1 - P_{e^-} + P_{e^-})$ is the effective polarization. One sees that the collision cross sections can be enhanced if both beams are polarized and if $P_{e^-}$ and $P_{e^+}$ have different signs. This can be parametrized in an effective luminosity given by

$$
L_{eff} = \frac{1}{2}(1 - P_{e^-}P_{e^+})L
$$

Thus, one can obtain a $P_{eff}$ much higher than either of the two beam polarizations in addition to enhancements in $L_{eff}$. Another important result is that the uncertainty $\Delta P_{eff}/P_{eff}$ is less than the uncertainty of the individual polarizations $\Delta P_{e^-}/P_{e^-}$. The improvement in the measurements due to positron beam polarization can be substantial. One should see Ref. $[15, 16]$ for a more complete discussion.

3. RESULTS

We are interested in answering the question of how well we can distinguish between $Z'$s originating from different models. We take as our starting point the analysis of Riemann which used leptonic observables to demonstrate that one can extract $Z'$ couplings and discriminate between models $[2]$. In this brief report we explore the sensitivity of her results to variations in the assumptions used in obtaining those results. As mentioned in the introduction, numerous models exist. For the purposes of this study we consider $Z'$s coming from the $E_6$ $\chi$ model ($\chi$), LR-symmetric model (LR), Littlest Higgs model (LH) $[8, 11]$, Simplest Little Higgs model (SLH) $[4]$, and KK excitations originating in theories of extra dimensions (KK). We only present results for the Simplest Little Higgs model with a universal fermion sector. The KK case is problematic since, for this case, the couplings shown do not in fact correspond to the KK $Z'$ couplings because in this model there are both photon and $Z^0$ KK excitations roughly degenerate in mass.

The point is simply that the KK model cannot be distinguished from other models.

To obtain our plots, unless otherwise stated, we took $\sqrt{s} = 500$ GeV and $L_{int} = 1$ ab$^{-1}$ assuming electron and positron polarization of 80% and 60% respectively, $\Delta P_{e^-} = 0.5\%, \Delta P_{e^+} = 0.5\%$, and a systematic error of $\Delta_{sys} = 0.25\%$. The $Z'$ couplings shown in the figures are normalized such that the SM $Z^0$ couplings are $C_L^e = -\frac{1}{2} + \sin^2\theta_w$ and $C_R = \sin^2\theta_w$. A deviation for one observable is always possible as a statistical fluctuation so a more robust strategy is to combine many observables to obtain a $\chi^2$ figure of merit. We follow this approach by including various observables.
Figure 1: Resolving power (95% CL) for $M_{Z'} = 1, 2,$ and $3$ TeV and $\sqrt{s} = 500$ GeV, $L_{int} = 1ab^{-1}$. The smallest regions correspond to $M_{Z'} = 1$ TeV and the largest to $M_{Z'} = 3$ TeV. The left side is for leptonic couplings based on the leptonic observables $\sigma_{P_e^-P_e^+}^\mu, A_{LR}^\mu, A_{FB}^\mu$. The right side is for $b$ couplings based on the $b$ observables $\sigma_{P_b^-P_b^+}^b, A_{FB}^b, A_{FB}^b(pol)$ assuming that the leptonic couplings are known and a $b$-tagging efficiency of 70%.

Fig. 1(a) shows the resolving power of the lepton couplings assuming lepton universality and using the three observables: $\sigma_{P_e^-P_e^+}^\mu, A_{FB}^\mu, A_{LR}^\mu$ for $M_{Z'} = 1, 2$ and $3$ TeV. As noted by Riemann there is a two-fold ambiguity in the signs of the lepton couplings since all lepton observables are bilinear products of the couplings. The hadronic observables can be used to resolve this ambiguity since for this case the quark and lepton couplings enter the interference terms linearly. Fig. 1(b) shows the resolving power for $b$-quark couplings based on the $b$-quark observables $\sigma_{P_b^-P_b^+}^b, A_{FB}^b, A_{FB}^b(pol)$ assuming that the leptonic couplings are accurately known from other measurements and a $b$-tagging efficiency of 70%. One could gain additional information by studying other observables with hadron final states such as $R_{had}, A_{LR}^{had}$, and observables involving the $c$-quark.

We next consider the importance of polarization. In Fig. 2 we show results for the cases of no polarization, only the electron is polarized, and both the electron and positron are polarized. The results are shown for $M_{Z'} = 2$ TeV, $\sqrt{s} = 500$ GeV and $L_{int} = 1ab^{-1}$ using the three observables $\sigma_{P_e^-P_e^+}^\mu, A_{LR}^\mu, A_{FB}^\mu$. Note that the appropriate values of $P_e^-$ and $P_e^+$ are used in eqn. 1 and for the unpolarized case $A_{LR}$ does not contribute. Clearly polarization will be important for measuring couplings and disentangling models if a $Z'$ were discovered although positron polarization does not appear to be an important factor for these measurements.

In Fig. 2 we assumed a $Z'$ mass of 2 TeV. But the LHC has the potential of discovering a heavy neutral gauge boson up to 5 TeV or higher. Supposing that this is the case, can the ILC still give us useful information? In Fig. 3 we show the resolving power for $Z'$s with $M_{Z'} = 1, 2, 3,$ and $4$ TeV, again using only the three $\mu$ observables assuming the $e^-$ and $e^+$ polarizations given above. Reasonably good measurements can be made for the $M_{Z'} = 2$ TeV case. For $M_{Z'} = 3$ TeV the resolving power deteriorates but the measurements can still distinguish between many of the currently popular models. At $M_{Z'} = 4$ TeV it becomes quite difficult to distinguish among the models although some models could still be ruled out.

In Fig. 4 we examine possible improvement in the resolving power by including more observables. In the previous figures we only included three observables with final state muons. If $\tau$ leptons could be observed with reasonable efficiency an additional five observables ($\sigma_{P_\tau^-P_\tau^+}^\mu, A_{LR}^\mu, A_{FB}^\mu, P_\tau$ the $\tau$ polarization, and $A_{FB}^\mu(Pol)$) can be included.

ALCPG0108
Figure 2: The effect of polarization on coupling measurements. Resolving power (95% CL) for $M_{Z'} = 2$ TeV and $\sqrt{s} = 500$ GeV, $L_{int} = 1 \text{ab}^{-1}$ for leptonic couplings based on the leptonic observables $\sigma_{P_{-}P_{+}}, A^{L}_{LR}$, and $A^{F}_{FB}$. The largest region corresponds to the unpolarized case while the smallest region corresponds to electron and positron polarization of 80% and 60% respectively with the middle region corresponding to only electron polarization.

Figure 3: Resolving power (95% CL) for $M_{Z'} = 1, 2, 3,$ and 4 TeV, and $\sqrt{s} = 500$ GeV, $L_{int} = 1 \text{ab}^{-1}$ for leptonic couplings based on the leptonic observables $\sigma_{P_{-}P_{+}}, A^{L}_{LR}$, and $A^{F}_{FB}$.

in the $\chi^2$. Fig. 4 shows the improvement one gains by including the $\tau$ observables for $M_{Z'} = 2$ TeV (left figure) $M_{Z'} = 4$ TeV (right figure). For lack of a better estimate we simply take the $\tau$ efficiency equal to one which is clearly overly optimistic. For the $M_{Z'} = 2$ TeV case the improvement is not so impressive but for the $M_{Z'} = 4$ TeV case the extra observables could be important for disentangling the models.

4. CONCLUSIONS

In this contribution we examined the potential of the ILC to distinguish between different models that predict $Z'$ bosons. What we found is that it is an extremely powerful tool and would be crucial for disentangling this sort of physics if a discovery were made at the LHC. In previous work that concentrated on leptonic couplings there were ambiguities. If the ILC detectors have reasonable $b$ and $c$-quark tagging efficiencies additional useful information could be obtained. We also demonstrated the importance of polarization. In this report we touched upon the couplings of variations of the Little Higgs models. A more detailed account of this aspect of our work will be given elsewhere.

Acknowledgments

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

ALCPG0108
Figure 4: Resolving power (95% CL) of leptonic couplings for $M_{Z'} = 2$ TeV (left side) and $M_{Z'} = 4$ TeV (right side) and $\sqrt{s} = 500$ GeV, $L_{int} = 1 ab^{-1}$. The outer region only includes the three muon observables $\sigma_{\mu e-\mu e+}$, $A_{LR}^{LR}$, and $A_{FB}^{FB}$ while the smaller region includes, in addition, the five tau observables ($\sigma_{\tau e-\mu e+}$, $A_{LR}^{\tau}$, $A_{FB}^{\tau}$, $A_{FB}^{\tau}(pol)$, and $P_{\tau}$).

References

[1] A. Leike, Phys. Rept. 317, 143 (1999) [arXiv:hep-ph/9805494].
[2] M. Cvetic and S. Godfrey, arXiv:hep-ph/9504216.
[3] J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group], hep-ph/0106315.
[4] G. Weiglein et al. [LHC/LC Study Group], hep-ph/0410364.
[5] S. Godfrey, Phys. Rev. D 51, 1402 (1995) [arXiv:hep-ph/9411237].
[6] S. Capstick and S. Godfrey, Phys. Rev. D 37, 2466 (1988).
[7] Recent pedagogical introductions to extra dimensions are given by T. G. Rizzo, eConf C040802, L013 (2004) hep-ph/0409309, and K. Cheung, hep-ph/0409028.
[8] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002) arXiv:hep-ph/0206021.
[9] M. Schmaltz, JHEP 0408, 056 (2004) arXiv:hep-ph/0407143.
[10] M. Schmaltz and D. Tucker-Smith, arXiv:hep-ph/0502182.
[11] T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Rev. D 67, 095004 (2003) arXiv:hep-ph/0301040.
[12] T. Han, H. E. Logan and L. T. Wang, arXiv:hep-ph/0506313.
[13] Preliminary results were given in S. Godfrey, contribution to Physics interplay of the LHC and the ILC (unpublished), see [4]; S. Godfrey, P. Kalyniak, A. Tomkins, in preparation.
[14] For some early references see: F. Boudjema, B.W. Lynn, F.M. Renard, C. Verzegnassi, Z. Phys. C48, 595 (1990); A. Blondel, F.M. Renard, P. Taxil, and C. Verzegnassi, Nucl. Phys. B331, 293 (1990); G. Belanger and S. Godfrey, Phys. Rev. D 34, 1309 (1986); Phys. Rev. D 35, 378 (1987). P.J. Franzini and F.J. Gilman, Phys. Rev. D35, 855 (1987); M. Cvetic and B. Lynn, Phys. Rev. D35, 1 (1987); B.W. Lynn and C. Verzegnassi, Phys. Rev. D35, 3326 (1987); T.G. Rizzo ibid, 36, 713(1987); A. Bagned, T.K. Kuo, and G.T. Park ibid, 44, 2188 (1991); A. Djouadi et al, Z. Phys. C56 289 (1992); A. Leike, Z. Phys. C62, 265 (1994);
[15] G. Moortgat-Pick et al., arXiv:hep-ph/0507011.
[16] K. Fujii and T. Omori, KEK-PREPRINT-95-127

ALCPG0108