First- and second-order gradient couplings to NV centers
engineered by the geometric symmetry

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Abstract

The magnetic fields with the first- and second-order gradient are engineered in several mechanically controlled hybrid systems. The current-carrying nanowires with different geometries can induce a tunable magnetic field gradient because of their geometric symmetries, and therefore develop various couplings to nitrogen-vacancy (NV) centers. For instance, a straight nanowire can guarantee the Jaynes-Cummings (JC) spin-phonon interaction and may indicate a potential route towards the application on quantum measurement. Especially, two parallel straight nanowires can develop the coherent down-conversion spin-phonon interaction through a second-order gradient of the magnetic field, and it can induce a bundle emission of the antibunched phonon pairs via an entirely different magnetic mechanism. Maybe, this investigation is further believed to support NV’s future applications in the area of quantum manipulation, quantum sensing, and precision measurement, etc.

Keywords: hybrid quantum system, nitrogen-vacancy center, gradient magnetic field

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I. INTRODUCTION

The precision measurement (PM) plays an important role during the development of physics, and the related investigations always maintain a considerable hot topic [1, 2]. Especially, the quantum precision measurement (QPM), namely carrying out some measurements utilizing the specific quantum methods, has promoted a great development in this area [3–8]. Because utilizing the existing quantum platforms or quantum methods, people can greatly improve the accuracy, resolution or sensitivity in comparison with the original measurements [1, 9–13]. To our knowledge, the optomechanical system [14–17], cold atoms system [8, 18], solid-state spins, et al [19–23], have made the important contributions to the QPM [24, 25]. Through the analysis and comparison of the above typical quantum systems, we note solid-state spins may be the more promising choice, owing to its unique superiorities, i.e., convenient preparation and applications but without complex trap, optical addressing and readout, easy manipulations even at room temperature, high sensitivity to electromagnetic fields or strain, and long coherence time, and so on [26–29]. Therefore the solid-state spins are expected to show a higher value of the study on QPM, and even to be developed into some practical quantum devices in the near future [26–29]. Among these solid-state spins’ family, the nitrogen-vacancy (NV) center may be considered as one of the most interesting spins to accomplish this QPM target very well. Its excellent spin properties have always attracted researchers to use it to carry out the quantum sensing [26, 27], quantum control and other aspects of investigations [30–33], and which correspond to the main reason that drive us to accomplish quantum manipulation or quantum sensing task using an NV-based hybrid system [34].

On the other hand, the investigations on hybrid quantum system and its applications have also been considered as a promising topic in recent years [35]. Especially, people can hybridize many different quantum units, such as the solid-state spins, cold atoms, superconductive device, mechanical resonator, and optical (or acoustic ) cavity, waveguide or lattice, and then utilize which to accomplish the target of quantum manipulations or simulations [35–44]. A more remarkable point to be mentioned is Li group’s fresh proposal for a hybrid quantum system with single NV center coupled to a direct-current (DC) nanotube, and the strong spin-phonon coupling at single-quantum level can be achieved in such theoretical design [45]. Undoubtedly, this NV-based hybrid system may not only provide a coherent
platform to sense the mesoscopic object only using a single spin [46–48], but also further broaden a wide area for manipulating such solid-state spins utilizing a novel mechanical approach [49–51].

Inspired by Li’s proposal and its basic physical mechanism, here we discuss a group of mechanics-controlled hybrid systems. The different magnetic interactions between an NV center and the current-carrying nanowire(s) with different geometric designs are studied in this work. Taking the circular and the straight-line nanowire(s) for example, we first discuss the magnetic fields and major gradients for different shapes or positions, and estimate the magnetic interactions between the NV spins and the gradient magnetic fields. We note that the first- and second-order gradient magnetic fields can be engineered in such different hybrid systems because of their different geometric symmetries. Therefore, a straight nanowire can guarantee the Jaynes-Cummings (JC) spin-phonon interaction and may indicate a potential route towards the application on quantum measurement. More interestingly, two parallel straight nanowires can develop the coherent down-conversion spin-phonon interaction through a second-order gradient of the magnetic field, and it can ensure a bundle emission of the antibunched phonon pairs via a fresh coupling mechanism. The numerical simulations evidently show that this investigation can not only provide an interesting prototype for a quantum sensor, but also further support the coherent manipulation of the spin and phonon(s). This theoretical study may be considered as an encouraging attempt, because it can also indicate a basic picture “peeking the classic world by a quantum observer”.

This work is mainly organized as follows: We first study several different designs for this proposal and study the major magnetic field gradients induced by nanowire(s) with different shapes in section II. We discuss the first- and second-order spin-phonon couplings and applications in section III and section IV, respectively. In section V, we also discuss another application on the mechanical manipulating the quantum phase transition (QPT) of the NV ensemble in this hybrid system. After that, we make a conclusion for this work in the last section VI.
FIG. 1. (Color online) The basic schematics for this proposal. (a) A ring type nanowire with the direct current (DC) is placed in the x-o-y plane, and an NV center attached at the end of the vibrating cantilever (with frequency $\omega$) is set around the center of this circular ring. This NV center can also be detected through the optical experimental method, namely the so-called optically detected magnetic resonance (ODMR). Similarly, an NV center is set near to (b) one straight line type nanowire (with distance $z_0$), and near to the center point of (c) two parallel and adjacent nanowires. (d) The basic energy-level structure and the microwave dressed-state design for this scheme, where we apply the microwave drive $\Omega_{1(2)}$ to ensure the available state transition process $|0\rangle \leftrightarrow |+1\rangle$ (or $|0\rangle \leftrightarrow |-1\rangle$).

II. THE FIRST- OR SECOND-ORDER GRADIENT OF THE MAGNETIC FIELDS FOR DIFFERENT DESIGNS

In this section, we mainly discuss three different designs in our proposal, and plot the basic schematics in Fig. 1 (a-c). For example, Fig. 1 (a) means the circular nanowire with the DC, (b) is the single straight line with the DC, and (c) demonstrates a pair of parallel nanowires with the equal and opposite DCs.
A. The circular nanowire case

As shown in Fig. 1 (a), this hybrid quantum system is mainly composed of a ring type current-carrying nanowire (with the radius $r$ and the DC $I$) and an NV spin attached at the end of a horizontal cantilever (with the dimension ($l$, $w$, $t$) and the fundamental frequency $\omega$). The NV spin is just placed at (or around) the center point of this ring nanowire in the x-o-y plane, and this vibrating resonator can induce a time-dependent oscillating displacement around the NV spin’s equilibrium position. This design can result in a gradient magnetic field to this NV spin, which are mainly induced by the cooperation of the vibrating resonator and the nanowire. To be specific, this current-carry nanowire can produce the static magnetic field according to the Biot-Saval law. For example, the magnetic field at the center point is $B_o = \mu_0 I/2r$ with $z = 0$; while for the general case, the magnetic field along the $z$ axis can also be expressed as $B_{zo} = \mu_0 I r^2/[2(r^2 + z_0^2)^{3/2}]$, with the new equilibrium coordinate $z = z_0$.

We assume this vibrating spin around its equilibrium point will induce a tiny displacement along the $z$ direction, and this displacement can be defined as $d\hat{z}$. Here we will discuss two different cases: For case (i), if the NV spin’s equilibrium position is $z = 0$, owing to its inherent geometric symmetry, we can mainly get the two-order gradient field along $z$ direction, with expression

$$B_o(z) = \frac{\mu_0 I}{2r}[1 + (d\hat{z}/r)^2]^{3/2} \approx \frac{\mu_0 I}{2r}[1 + \frac{3}{2}\left(\frac{d\hat{z}}{r}\right)^2] = B_o + B^{(2)}_o(z).$$

Here the static magnetic field is $B_o = \mu_0 I/2r$ and its second-order gradient is expressed as $B^{(2)}_o(z) = \frac{3\mu_0 I}{2r}\times (d\hat{z})^2$. While for case (ii), and we can break its inherent symmetry through modifying the NV spin’s equilibrium position $z = z_0 \neq 0$, then we can obtain the total magnetic field

$$B_{zo}(z) = \frac{\mu_0 I r^2}{2[r^2 + (z_0 + d\hat{z})^2]^{3/2}} \approx \frac{\mu_0 I}{2r}\left\{\frac{1}{1 + \left(\frac{z_0}{r}\right)^2} - \frac{3}{2} \times \frac{z_0}{[1 + \left(\frac{z_0}{r}\right)^2]^{5/2}} \times \frac{d\hat{z}}{r}\right\} = B_{zo} - B^{(1)}_{zo}(z).$$

So its static and first-order magnetic fields are $B_{zo}$ and $B^{(1)}_{zo}(z) = \frac{3\mu_0 I z_0}{4r^2}\left(\frac{1}{r^2 + k^2}\right)^{5/2} \times d\hat{z}$, respectively, with a dimensionless constant $k \equiv \frac{z_0}{r}$. Although the static and the gradient
magnetic fields are all different for both different cases, the NV spin can experience both the static parts and the different gradient parts, and the interaction acting on an NV spin can be generally described as $\hat{H} = g_e \mu_B \hat{\mathbf{B}} \cdot \hat{\mathbf{S}}$. In which, the the NV’s Landé factor is $g_e \simeq 2$, Bohr magneton is $\mu_B = 14$ GHz/T, $\hat{\mathbf{B}}$ means the total magnetic field including the static parts $B_o$ or $B_{z0}$ and the gradient parts $B_o^{(2)}(z)$ or $B_{z0}^{(1)}(z)$, and $\hat{\mathbf{S}} \equiv (\hat{S}_x, \hat{S}_y, \hat{S}_z)$ stands for the spin’s operator with $m_s = 0, \pm 1$. In addition, we can also apply another homogeneous static magnetic field $B_{za}$ to this NV spin, which can ensure the easy and feasible modifications of the total static field acting on this spin.

Then we assume that this cantilever is cooled down to its ground state or low excited state, and this time-dependent displacement $d\hat{z}$, namely the quantum fluctuation, can also be quantized through the standard process, with $d\hat{z} = \delta_{zf} (\hat{b} + \hat{b}^\dagger)$ and the zero field quantum fluctuation $\delta_{zf} = \sqrt{\hbar/2m\omega}$. So this gradient magnetic field experienced by the NV can be quantized as: for case (i), the second-order gradient

$$B_o^{(2)}(z) = G^{(2)} \delta_{zf}^2 (\hat{b} + \hat{b}^\dagger)^2;$$

and for case (ii), the first-order gradient

$$B_{z0}^{(1)}(z) = G^{(1)} \delta_{zf} (\hat{b} + \hat{b}^\dagger).$$

We can obtain the $G^{(2)} = \frac{3\mu_0 I}{4r^3}$ and $G^{(1)} = \frac{3\mu_0 l z_0}{4r^2} \left( \frac{1}{1+\alpha^2} \right)^{5/2}$ from Eq. (1) and Eq. (2), respectively. We state the basic spin-phonon interaction in this hybrid system can be described as $\hat{H}^{(1,2)} \approx g_e \mu_B \hat{\mathbf{B}}_o^{(1,2)}(z) \hat{S}_z \approx g^{(1,2)} (\hat{b} + \hat{b}^\dagger)^{1,2} \hat{S}_z$, with the first- or second-order gradient coupling $g^{(1,2)} = g_e \mu_B G^{(1,2)} \delta_{zf}^{1,2}$. In general case, we may consider the silicon-based cantilever with dimension ($l = 11$, $w = 0.05$, $t = 0.04$) $\mu$m. So its fundamental frequency and the zero-field fluctuation can also be expressed as $\omega = 3.516 \times (t/l^2) \sqrt{E/12\varrho} \approx 2\pi \times 2.5$ MHz and $\delta_{zf} = \sqrt{\hbar/2m\omega} \approx 10^{-12}$ m, respectively. For this mechanical mode, the quality factor is estimated as about $Q_m \in [10^3, 10^6]$, with the Young’s modulus $E \approx 10^{11}$ Pa, the mass density $\varrho \approx 2.33 \times 10^3$ kg/m$^3$, and its effective mass of this NV attached cantilever resonator is estimated as $m \approx 10^{-17}$ kg. This circular nanowire is assumed $r \approx 10^{-6}$ m and the DC $I \in [10, 1000]\mu$A, the NV’s equilibrium position is $z_0 \approx 0.5r$, and the permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}$ H/m. However, we estimate this design can not ensure a strong coupling to single NV center by taking a group of the general experimental parameters above for example.
FIG. 2. (Color online) The spin-phonon coupling strength $g^{(1)}$ varying with the current intensity $I$, which is induced by this first-order gradient of the nanowire. The parameters are set as: the permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\delta_{zf} = 10^{-12}$m, and $z_0 = 10^{-7}$m.

B. The case of single straight nanowire with the DC

To deal with this problem of the weak coupling strength, we modulate our proposal according to Li’s investigation of NV centers coupled to a nanotube [45]. As shown in Fig. 1 (b), a straight-line type nanowire with the DC $I$ is set near to an NV center (the distance is $z_0$), and this vibrating cantilever will also develop a time-dependent oscillating displacement of the NV center. Firstly, the static nanowire with current $I$ can develop the static magnetic field $B_x = \mu_0 I/(2\pi z_0)$, but the NV center can experience a timely oscillating magnetic field with expression $B_{z_0}(t) = \mu_0 I/[2\pi(z_0 + d\tilde{z})] \approx B_0 - B^{(1)}_{z_0}(t)$. In which, $B_0 = B_x$ is the static magnetic field, and $B^{(1)}_{z_0}(t) \approx \frac{\mu_0 I}{2\pi z_0} \times d\tilde{z}$ is the first-order gradient of the magnetic field. Through the similar standard quantization process with $d\tilde{z} = \delta_{zf}(\hat{b} + \hat{b}^\dagger)$, we can get $B^{(1)}_{z_0}(t) = \frac{\mu_0 I \delta_{zf}}{2\pi z_0}(\hat{b} + \hat{b}^\dagger)$. Here we assume that the spin’s direction $\hat{S}_z$ is also along the direction of this gradient for simplicity. Then we can obtain this type of magnetic interaction in the Schrödinger picture (SP) with expression

$$\hat{H}_{sp}^{(1)} = g^{(1)}(\hat{b} + \hat{b}^\dagger)\hat{S}_z,$$

(5)
TABLE I. The list of coupling strength $g^{(1)}$ with different DC $I$.

| The different $I$ (mA) | Coupling strength $g^{(1)}$ (kHz) |
|------------------------|----------------------------------|
| 0.1                    | 5.6                              |
| 0.5                    | 28                               |
| 1.0                    | 56                               |
| 2.0                    | 112                              |
| 3.0                    | 168                              |
| 4.0                    | 224                              |
| 5.0                    | 280                              |

with its coupling strength $g^{(1)} = g_e \mu_B \frac{\mu_0 I \delta_f}{2 \pi z_0^2}$. We also estimate this coupling strength and plot the results in Fig. 2. From which, we note that we can obtain the strong spin-phonon coupling strength for an NV center in this hybrid system, with the coupling strength in regime $g^{(1)} \sim [10^3, 10^5]$Hz as $I \sim [0.1, 10]$mA. We also make a brief list as shown in Table I.

C. The case of a pair of parallel nanowires with the equal and opposite DCs

We can also only engineer the second-order gradient field via another symmetric design [52, 53]. According to Fig. 1 (c), we apply a pair of parallel nanowires carried with the equal but opposite DCs to this scheme. Similarly, we can also obtain the whole magnetic field effectively, which also means the NV’s being experienced. So this whole magnetic field is

$$B_{z_0}(t) = \frac{\mu_0 I}{2 \pi \left[ (z_0 + d\hat{z}) + (z_0 - d\hat{z}) \right]} \approx B_0 + B^{(2)}_{z_0}(t), \quad (6)$$

where, $B_0 = \mu_0 I / \pi z_0$ is the static magnetic field, and $B^{(2)}_{z_0}(t) \approx (d\hat{z})^3 \times \mu_0 I / \pi z_0^3$ means the second-order gradient field. Here we also state, the first-order gradient is counteracted to zero because of its inherent geometric symmetry. As a result, we can also obtain this spin-phonon interaction with the Hamiltonian expression

$$\hat{H}_s^{(2)} = g^{(2)} (\hat{b} + \hat{b}^\dagger)^2 \hat{S}_z. \quad (7)$$

Here its second-order-gradient coupling strength $g^{(2)} = g_e \mu_B \frac{\mu_0 I z_0^2}{2 \pi \pi z_0}$, and utilizing the same parameters as in Fig. 2, we also estimate this coupling strength and plot the results in Fig. 3.
FIG. 3. (Color online) The spin-phonon coupling strength $g^{(2)}$ varying with the current intensity $I$, which is induced by this second-order gradient of this pair of nanowires. The parameters are set as same as the Fig. 2.

| The different $I$ (mA) | Coupling strength $g^{(2)}$ (kHz) |
|------------------------|-----------------------------------|
| 5                      | 0.056                             |
| 10                     | 0.112                             |
| 50                     | 0.56                              |
| 100                    | 1.12                              |

We also note that this second-order coupling is much less than the first-order one, and it also satisfies $g^{(2)} \approx 10^3$Hz even if the current intensity $I$ is increased to about 100mA. The relevant part of the results are also list in Table II.

III. THE FIRST-ORDER COUPLING MECHANISM AND APPLICATION

For this hybrid system, the spin-phonon interaction caused by the first-order gradient field can reach to the strong coupling regime, but the fundamental frequency of this mechanical
mode $\omega$ is far less than the NV’s transition frequency $D \pm \delta$. Here we state $\omega \sim 2$MHz and $D \pm \delta \sim 2 - 4$GHz. So we have to introduce the so-called dressed-state design to this scheme, and the major schematic is plotted in Fig. 1 (d). For example, we can apply the two-tune classical microwave fields ($\omega_{1,2}$, $\Omega_{1,2}$) to drive this NV center, and which can both develop the different near-resonance transitions $|0\rangle \leftrightarrow |\pm 1\rangle$, respectively. We can obtain the strong coherent spin-phonon coupling via the standard dressed-state method (Appendix A).

In the interaction picture (IP), we can also obtain the standard Jaynes-Cummings (JC) Hamiltonian for describing this coupled NV center and nanowire via the rotating-wave approximation,

$$\hat{H}_{JC}^{IP} = \lambda (\hat{\sigma}_+ \hat{b} + \hat{\sigma}_- \hat{b}^\dagger).$$  \hspace{1cm} (8)

Here we have made the assumption of the resonance condition $\omega = \omega_-$ for simplicity. Furthermore, if we assume $\omega = -\omega_-$, we can also get the anti-Jaynes-Cummings (AJC) Hamiltonian with expression

$$\hat{H}_{AJC}^{IP} = \lambda (\hat{\sigma}_+ \hat{b}^\dagger + \hat{\sigma}_- \hat{b}).$$  \hspace{1cm} (9)

Here, its coupling strength of Eq. (8) and Eq. (9) satisfies $\lambda = -g^{(1)} \sin \theta \in [10^3, 10^5]$Hz with $I \leq 10$mA.
In view of this type of JC or ATC Hamiltonian, this design may provide a feasible platform to perform the coherent manipulation of a single (or a group of) NV center(s), and this hybrid system can also mimic the standard model, with respect to a single atom coupled to a single cavity mode in cavity-QED system. We think this hybrid system may surely accomplish many applications in the area of quantum manipulation, such as the universal logic operations, entanglement state, and so on [54, 55].

And here, we also hope this hybrid system may provide another interesting application, the quantum measurement or quantum sensor. For example, as we modify the intensity of this DC $I$, and this change will induce the direct variation of the coherent coupling strength $\lambda \approx g^{(1)}$. Then we can either catch this variation through the basic output spectrum theoretically, or observe it experimentally via the “optically detected magnetic resonance (ODMR)” platform. To illustrate its feasibility for this potential application, The basic output spectrum is defined as $S(\omega) = \int_{-\infty}^{\infty} \lim_{t \to \infty} \langle \hat{A}_+(t + \tau) \hat{A}_-(t) \rangle e^{-i\omega \tau} d\tau$ with $\hat{A} = \hat{\sigma}_\pm$ or $\hat{b}$ [12]. Here we choose $\langle \hat{\sigma}_+(t+\tau)\hat{\sigma}_-(t) \rangle$ for example, and the results for this JC type interaction and the AJC interaction are plotted in Fig. 4 and in Fig. 5, respectively. From which, we note that as the DC $I$ has a bit variation $\Delta I = \pm 0.1\text{mA}$, the output spectrum will cause a
FIG. 6. (Color online) The dynamical population of $\hat{b}^\dagger \hat{b}$ and $\hat{\sigma}_+ \hat{\sigma}_-$. In which, $\langle \hat{b}^\dagger \hat{b} \rangle$ correspond to $\gamma = \kappa = 0$ (the blue solid line) and $\gamma = \kappa \approx 0.1 \chi'$ (the wine dash dot line), and $\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle$ mean $\gamma = \kappa = 0$ (the red dash line) and $\gamma = \kappa \approx 0.1 \chi'$ (the pink dash dot dot line).

significant frequency shift $\Delta \omega \approx \pm 10$kHz (for example, the blue solid line and the black solid line), and this result can also be demonstrated by the JC and the AJC models, equivalently. Additionally, we can also apply the optical method to further improve its resolution in this hybrid system, for example the so-called “ODMR” platform. To detect this weak-current signal in optical frequency regime (with frequency $\sim 700$THz), we consider its resolution can be surely further enhanced utilizing this platform.

IV. THE SECOND-ORDER COUPLING MECHANISM AND APPLICATION

By introducing the second-order coupling $\hat{H}_{sp}^{(2)}$ to the Appendix A, we can equivalently get the total Hamiltonian in the same dressed-state basis, with expression

$$\hat{H}_{so} = \hat{H}_0 + (\chi' |g\rangle \langle d| + \chi |d\rangle \langle e| + \text{H.c.}) \times (\hat{b}^\dagger + \hat{b})^2.$$  \hspace{1cm} (10)

Here $\hat{H}_0 = \omega \hat{b}^\dagger \hat{b} + \omega_{eg} |e\rangle \langle e| + \omega_{dg} |d\rangle \langle d|$, and $\chi' = -g^{(2)} \sin \theta$ and $\chi = g^{(2)} \cos \theta$. In the interaction picture (IP), we can get the Hamiltonian with expression by discarding the
energy shift items $\sim |g(d)\rangle\langle d(g)| \times \hat{b}^{\dagger}\hat{b}$ and $\sim |e(d)\rangle\langle d(e)| \times \hat{b}^{\dagger}\hat{b}$,

$$
\hat{H}_{so}^{IP} \approx (\chi|g\rangle\langle d| e^{i\omega_{+}t} + \chi|d\rangle\langle e| e^{-i\omega_{+}t} + \text{H.c.}) \times (\hat{b}^{\dagger}e^{-2i\omega t} + \hat{b}^{\dagger}e^{2i\omega t})
$$

(11)

Under the resonance condition with $\omega_{+} = 2\omega$, we can discard the off-resonance items safely, and get the generalized JC model with the two-phonon transition,

$$
\hat{H}_{so}^{eff} \approx \chi(|d\rangle\langle e| \hat{b}^{\dagger} + |e\rangle\langle d| \hat{b}^{\dagger}) \equiv \chi(\hat{\sigma}_{+}^{\dagger}\hat{b}^{\dagger} + \hat{\sigma}_{-}^{\dagger}\hat{b}^{\dagger}),
$$

(12)

which shows a perfect down-conversion for a single input photon [52, 53, 56–58], i.e., it can mediate the conversion of a photon in the qubit to two phonons in the cantilever. The dynamical population of $\hat{b}^{\dagger}\hat{b}$ and $\hat{\sigma}_{+}\hat{\sigma}_{-}$ are numerically simulated via the master equation

$$
\frac{d\rho}{dt} = -i[\hat{H}_{so}^{eff}, \rho] + \frac{\kappa}{2}D[\hat{b}]\rho/2 + \gamma D[\hat{\sigma}_{-}]\rho/2,
$$

and the curves with different conditions are plotted in Fig. 6. Here, $D[\hat{o}]\rho = 2\hat{o}\rho\hat{o}^{\dagger} - \hat{o}^{\dagger}\hat{o}\rho - \hat{o}\rho\hat{o}^{\dagger}$ is the standard Lindblad operator for a given operator $\hat{o}$ ($\hat{o} = \hat{b}, \hat{\sigma}_{\pm}$), and $\kappa$ ($\gamma$) represents the decay rate of the resonator (qubit).

The results in Fig. 6 clearly indicate that this type of generalized two-phonon JC model can dominate a periodic oscillating behavior between the phonon pair and single spin.

In the presence of system dissipation, if a weak continuous driving is applied to the qubit, such a system can also act as a nonclassical light source for antibunched phonon-pair emissions. In this case, when the system-environment coupling is treated in the Born-Markov approximation, the dynamics of the system is governed by the following master equation

$$
\frac{d\rho}{dt} = -i[\hat{H}_{so}^{eff} + \hat{H}_{d}, \rho] + \frac{\kappa}{2}D[\hat{b}]\rho + \frac{\gamma}{2}D[\hat{\sigma}_{-}]\rho, \quad (13)
$$

where $\hat{H}_{d} = \Omega(\hat{\sigma}_{+}^{\dagger} + \hat{\sigma}_{-}^{\dagger})$ describes the weak driving applied to the qubit with amplitude $\Omega$.

It can be readily seen from the master equation that when the qubit is driven to the excited state, two phonons will be created in the resonator due to the down-conversion interaction $\hat{H}_{so}^{eff}$. For large resonator dissipation, each phonon in the resonator has a very large probability to leave the resonator. It is a probabilistic event determined by the coupling strength $\chi$ and the resonator decay rate $\kappa$. Once one of the two phonons is transferred outside the resonator, the resonance condition is no longer satisfied and the energy exchange between the qubit and the resonator stops. The remaining phonon has no choice but to leak out of the resonator within the resonator lifetime, resulting in a phonon-pair emission. Since the two phonons are emitted in a very small time window, they are strongly correlated. However, the weak continuous driving greatly suppresses the resonator transitions to the higher Fock
FIG. 7. (Color online) The standard second-order correlation function $g^{(2)}(\tau)$ and the generalized second-order correlation function $g^{(2)}_2(\tau)$ versus $\chi\tau$. The parameters are chosen as $\kappa = 3\chi$, $\gamma = 0.01\chi$, and $\Omega = 0.1\chi$.

states and guarantees that the emitted phonon pairs are well separated, leading to the release of the resonator energy in the form of antibunched phonon pairs [57, 59–66].

It is well known that the quantum optical coherence for isolated phonons emitted by conventional emitters can be well described by the standard second-order correlation function

$$g^{(2)}(\tau) = \frac{\langle b_\dagger(0)b_\dagger(\tau)b(\tau)b(0) \rangle}{\langle b_\dagger(0)b(0) \rangle\langle b_\dagger(\tau)b(\tau) \rangle}. \quad (14)$$

However, for phonon-pair emissions, the unit of emission is replaced by a bundle of two phonons. The phonons in the same pair are strongly correlated, while those in different pairs are antibunched. Because the standard correlation functions are just valid for single phonons, they can not capture the correlation between separated phonon pairs [57, 59–62, 64, 65]. To reveal the quantum statistics of the emitted phonon pairs, we introduce the generalized second-order correlation function

$$g^{(2)}_2(\tau) = \frac{\langle b_\dagger^N(0)b_\dagger^N(\tau)b^N(\tau)b^N(0) \rangle}{\langle b_\dagger^N(0)b^N(0) \rangle\langle b_\dagger^N(\tau)b^N(\tau) \rangle}. \quad (15)$$
FIG. 8. (Color online) The dressed-state energy level for single NV center. Here we have assumed $\Delta_0 > 0$ and $\Omega_0 \approx 2\Delta_0$, then we can get $\omega_{ed} \approx 2\omega_{dg}$ with $\omega_{dg} = (-\Delta_0 + \sqrt{\Delta_0^2 + 2\Omega_0^2})/2$ and $\omega_{ed} = (\Delta_0 + \sqrt{\Delta_0^2 + 2\Omega_0^2})/2$. As a result, this design can ensure the first- and second-order couplings to NV spin, with the single-phonon and two-phonon resonant transitions, respectively.

where the phonon pairs are considered as single entities (here $N = 2$ for this work) [59]. In Fig. 7, the standard and generalized second-order correlation functions are illustrated as functions of dimensionless parameter $\chi\tau$ by numerically solving the master equation (13). The chosen parameters are shown in the caption of the figure. It can be easily found that the emitted phonon pairs are indeed antibunched (the red dash-dotted curve) while the single phonons are strongly correlated (blue solid curve).

V. MECHANICAL MANIPULATING QPT OF NV ENSEMBLE IN THIS PROPOSAL

In addition, we also note this design can also provide us another potential application on quantum manipulation. As shown in Fig. 1 (c), if we assume that the DC current of one nanowire is an invariant constant $I_0$, and the other one is the tunable DC, with the definition $i(t) = I_0 \times f(t)$ and $f(t)$ is a controllable function. Then we can obtain its interaction Hamiltonian mainly with expression

$$\hat{H}_{sp}^{(2)} = [G^{(1)}(i)\hat{X} + G^{(2)}(i)\hat{X}^2]\hat{S}_z.$$  

(16)
Here, $G^{(1,2)}(i)$ mean the effective first- and second-order gradient couplings to spins, respectively, and the generalized coordinate operator is $\hat{X} = (\hat{b} + \hat{b}^\dagger)$. For single NV center, we can also apply the MW dressed-state operation (see Appendix A), and the dressed-state energy level is illustrated in Fig. 8. As long as $\Delta_0 > 0$ and $\Omega_0 \approx 2\Delta_0$, we can get $\omega_{ed} \approx 2\omega_{dg}$, and its total interaction Hamiltonian is expressed as

$$\hat{H}^{(1,2)}_{TP} = \lambda(i)(\hat{\sigma}_+ \hat{b} + \hat{\sigma}_- \hat{b}^\dagger) + \chi(i)(\hat{\sigma}_+^\dagger \hat{b}^2 + \hat{\sigma}_-^\dagger \hat{b}^{2\dagger}).$$  \hspace{1cm} (17)

In which, $\lambda(i)$ and $\chi(i)$ mean the single-phonon and two-phonon interaction of single NV center, respectively, and we also stress the above two parameters are both tunable in this design. To indicate this interesting interaction system for different coupling mechanism, we can introduce the homogenous collective NV spins to this proposal, and then discuss the potential application on the manipulation of the quantum phase transitions (QPTs) process as we tune $\lambda(i)$ and $\chi(i)$ continuously. Therefore, this interaction system can be described as

$$\hat{H}_{Total} = \lambda(i)(\hat{J}_+ \hat{b} + \hat{J}_- \hat{b}^\dagger) + \chi(i)(\hat{J}_+^\dagger \hat{b}^2 + \hat{J}_-^\dagger \hat{b}^{2\dagger}),$$  \hspace{1cm} (18)

where the collective spin operators are $\hat{J}_\pm^{(l)} = \sum_{j=1}^N \hat{\sigma}_\pm^{(lj)}$ and $\hat{J}_l^{(l)} = \sum_{j=1}^N \hat{\sigma}_l^{(lj)} / 2 (\alpha = x, y, z)$, with the $j$th two-level qubit $\hat{\sigma}_\pm^{(lj)}$ in basis space $\{|d\rangle, |g\rangle\} \{\{|e\rangle, |d\rangle\}$). This dynamical process can be described as the master equation effectively

$$\frac{d\rho}{dt} = -i[\hat{H}_{Total}, \rho] + \kappa_{eff} D[\hat{b}]\rho + \gamma D[\hat{J}_z]\rho + \gamma' D[\hat{J}_z^\dagger]\rho,$$  \hspace{1cm} (19)

where, $\kappa_{eff}$ means the effective mechanical dissipation rate, $\gamma^{(l)}$ mean the collective dephasing rates for two different bases. We also note that the dephasing rate will be suppressed at a certain degree in a group of dressed bases with $\kappa_{eff} \gg \gamma, \gamma'$, and we assume $\gamma = \gamma'$ for simplicity.

The condition of thermodynamic limit is $N \to \infty$, $V \to \infty$ and the finite particle-number density $n = N/V$. We set $\langle \hat{J}_k \hat{J}_l \rangle \to \langle \hat{J}_k \rangle \langle \hat{J}_l \rangle$, $k, l \in \{x, y, z\}$ by safely discarding their fluctuations. The differential equations of the collective spins’ expected values are derived from $\langle \hat{J}_k \rangle = Tr(\rho \hat{J}_k)$, and $d\langle \hat{J}_k \rangle / dt = Tr(\dot{\rho} \hat{J}_k)$. Then we can also introduce the normalization operations with respect to $\langle \hat{J}_k^{(l)} \rangle = X^{(l)}$, $\langle \hat{J}_y^{(l)} \rangle = Y^{(l)}$, $\langle \hat{J}_z^{(l)} \rangle = Z^{(l)}$, and $\langle \hat{b} \rangle = \beta$, where $\beta = \beta_{Re} + \beta_{Im}$ [30, 67–74].

We can acquire the analytical solutions of the relevant order parameters in this parameter space $\{\lambda, \chi, \kappa_{eff}, \gamma\}$, and then exhibit these analytical results in Appendix (B). Without
FIG. 9. (Color online) The steady-state order parameters $|\beta|$, $|Z|$, and $|Z'|$ varying with the nonlinear coupling strength $\lambda$ and $\chi$. In which, the navy solid line with sphere, the red dash line with diamond, and the blue dash line denote $|\beta|$, $|Z|$, and $|Z'|$, respectively. In which, we choose the parameters conditions for example: (a) $\chi = 0.1\kappa_{\text{eff}}$, (b) $\lambda = 0.1\kappa_{\text{eff}}$, and the other parameters are $\gamma = 0.1\kappa_{\text{eff}}$, $N = 10$.

loss of generality, we take one of steady-state solutions $\beta = \beta_1$ into our considerations. According to the achieved steady-state solution of the order parameter $|\beta|$, $|Z|$, and $|Z'|$ in Appendix (B), we plot these steady-state absolute values $|\beta|$, $|Z|$, and $|Z'|$ varying with the first-order coupling strength $\lambda$ in Fig. 9 (a), and with the second-order coupling strength $\chi$ in Fig. 9 (b). The results in Fig. 9 show that the critical points of this NV ensemble can be demonstrated around the points of $\lambda \sim 0.06\kappa_{\text{eff}}$ and $\chi \sim 0.26\kappa_{\text{eff}}$ when we modulate the first- and second-order couplings, respectively. And we believe this fresh attempt may attract many attentions at a certain degree in the field of QPTs or quantum manipulations.

VI. CONCLUSION

In conclusion, we propose a hybrid system of an NV center coupled to the different shape of nanowires carried with the DC. Assisted by a cantilever design, the magnetic fields with the first- and second-order gradient are engineered. In detail, several nanowires with different geometries can induce a tunable magnetic field gradient because of their symmetries, which can therefore result the different couplings to NV centers. Especially, the straight-line nanowire(s) can develop the coherent coupling at single-quantum level, through the first- or second-order gradient of the magnetic field. This theoretical attempt may not only guarantee an interesting coherent platform to perform the correlated two-phonon emission,
but also provide another potential route towards the detection of extremely weak signal. Maybe, this investigation is believed to further support the NV’s future applications in the area of quantum manipulation, and quantum sensing, etc.

VII. APPENDIX

A. The explanation in detail for dressed states process

For describing an NV center driven by the dichromatic classical MW fields, we can get

\[
\hat{H}_{NV} = D \hat{S}_z^2 + \delta \hat{S}_z/2 + \mu_B g_e [(B_x^0(t) + B_x^2(t))] \hat{S}_x, \tag{20}
\]

and the classical driving fields are \( B_x^{(1,2)}(t) = B_0^{(1,2)}(t) \cos(\omega_{1,2}t + \phi_{1,2}) \), respectively. The NV’s Hamiltonian in the rotating frame with \( \omega_{1,2} \) is

\[
\hat{H}_{NV} = \sum_{j=1,2} -\Delta_j |j\rangle \langle j| + \frac{\Omega_j}{2} (|0\rangle \langle j| + |j\rangle \langle 0|), \tag{21}
\]

where \( \Delta_{1,2} \equiv |D - \omega_{1,2} \pm \delta/2| \) and \( \Omega_{1,2} = \mu_B g_e B_0^{(1,2)}/\sqrt{2} \). Here we set \( h = 1 \), \( \Delta_{1,2} = \Delta_0 \) and \( \Omega_{1,2} = \Omega_0 \) for simplicity, and notice that the state \( |0\rangle \) is coupled to a “bright” state \( |b\rangle = (|+1\rangle + |-1\rangle)/\sqrt{2} \), but the “dark” state \( |d\rangle = (|+1\rangle - |-1\rangle)/\sqrt{2} \) is decoupled from which.

Therefore, \( \hat{H}_{NV} \) results in a group of basis including the state \( |d\rangle \), and the two dressed states \( |g\rangle = \cos \theta |0\rangle - \sin \theta |b\rangle \) and \( |e\rangle = \cos \theta |b\rangle + \sin \theta |0\rangle \). Where the phase \( \theta \) is determined by \( \tan 2\theta = -\sqrt{2} \Omega_0/\Delta_0 \), and the eigenfrequencies are \( \omega_d = -\Delta_0 \), and \( \omega_{e/g} = (-\Delta_0 \pm \sqrt{\Delta_0^2 + 2\Omega_0^2})/2 \), respectively. In this dressed-state basis, the parameters \( \Omega_0 \) and \( \Delta_0 \) are both tunable, and we can get the suitable energy level which can just match to the mechanical frequency \( \omega \). Then we can rewrite the Hamiltonian via this new basis,

\[
\hat{H}_{fo} = \omega \hat{b}^\dagger \hat{b} + \omega_{eg} |e\rangle \langle e| + \omega_{dg} |d\rangle \langle d| + (\lambda |g\rangle \langle d| + \lambda' |d\rangle \langle e| + H.c.) \times (\hat{b}^\dagger + \hat{b}). \tag{22}
\]

In above equation, the coefficients are defined as \( \omega_{eg} = \omega_e - \omega_g = \sqrt{\Delta_0^2 + 2\Omega_0^2} \), \( \omega_{dg} = \omega_d - \omega_g = (-\Delta_0 + \sqrt{\Delta_0^2 + 2\Omega_0^2})/2 \), \( \lambda = -g^{(1)} \sin \theta \), and \( \lambda' = g^{(1)} \cos \theta \). As long as \( \omega_{ed} = \omega_{eg} - \omega_{dg} \gg \omega_{dg} \approx \omega \), we can obtain the first-order coupling Hamiltonian effectively in this two-level subspace \( \{|d\rangle, |g\rangle\} \),

\[
\hat{H}'_{fo} = \omega \hat{b}^\dagger \hat{b} + \frac{\omega}{2} \hat{\sigma}_z + \lambda (\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{b}^\dagger + \hat{b}). \tag{23}
\]

Here we define \( \omega_- = \omega_{dg}, \omega_+ = \omega_{eg} - \omega_{dg}, \hat{\sigma}_z \equiv |d\rangle \langle d| - |g\rangle \langle g|, \hat{\sigma}_+ \equiv |d\rangle \langle g|, \text{and } \hat{\sigma}_- \equiv |g\rangle \langle d| \).
B. The necessary physical description on QPTs process with the average-field method

A group of steady-state average-field equations for describing this dynamical QPTs process is written as [30, 67, 69–74]

\[ \begin{align*}
0 &= i\lambda(\beta - \beta^*)Z - \gamma X, \\
0 &= -\lambda(\beta + \beta^*)Z - \gamma Y, \\
0 &= \lambda(\beta + \beta^*)Y - i\lambda(\beta - \beta^*)X, \\
0 &= -2\kappa_{eff}\beta - i\lambda(X - iY) + i2\chi\beta^*(X' - iY'), \\
0 &= i\chi(\beta^2 - \beta'^2)Z' - \gamma X', \\
0 &= -\chi(\beta^2 + \beta'^2)Z' - \gamma Y', \\
0 &= \chi(\beta^2 + \beta'^2)Y' - i\chi(\beta^2 - \beta'^2)X'.
\end{align*} \tag{24} \]

In addition to the total spins' conservation \(X^2 + Y^2 + Z^2 = N^2/4\) and \(X'^2 + Y'^2 + Z'^2 = N^2/4\), we can get a group of effective equations

\[ \begin{align*}
0 &= i\lambda(\beta - \beta^*)Z - \gamma X, \\
0 &= -\lambda(\beta + \beta^*)Z - \gamma Y, \\
0 &= -2\kappa_{eff}\beta - i\lambda(X - iY) + i2\chi\beta^*(X' - iY'), \\
0 &= i\chi(\beta^2 - \beta'^2)Z' - \gamma X', \\
0 &= -\chi(\beta^2 + \beta'^2)Z' - \gamma Y'.
\end{align*} \tag{25} \]

We can acquire the analytical solutions of the relevant order parameters in this parameter space \(\{\lambda, \chi, \kappa_{eff}, \gamma\}\)

\[ \begin{align*}
\beta_1^2 &= \frac{3}{2} - \frac{q}{2} + \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} + 3 \sqrt{\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} - \frac{b}{3a}}, \\
\beta_2^2 &= \omega^{\frac{3}{2}} \left[ -\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} + \omega^{\frac{3}{2}} \left[ -\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} - \frac{b}{3a} \right] \right], \\
\beta_3^2 &= \omega^{\frac{3}{2}} \left[ -\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} + \omega^{\frac{3}{2}} \left[ -\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + \left(\frac{p}{3}\right)^3} - \frac{b}{3a} \right] \right], \\
Z &= \frac{N}{2} \times \frac{\gamma}{\sqrt{4\lambda^2\beta^2 + \gamma^2}}, \\
Z' &= \frac{N}{2} \times \frac{\gamma}{\sqrt{4\chi^2\beta'^2 + \gamma^2}}.
\end{align*} \tag{26} \]
In Eq. (26), the coefficients are
\[ \omega = \frac{-1 + \sqrt{3}i}{2}, \]
\[ p = -\frac{b^2}{3a^2}, \]
\[ q = \frac{27a^2d + 2b^3}{27a^3}, \]
\[ a = 16\lambda^2\chi^4, \]
\[ b = 4\chi^2(\chi^2\gamma^2 - \lambda^4), \]
\[ d = -\lambda^4\gamma^2. \]  

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