Cosmic equation of state from strong gravitational lensing systems

Marek Biesiada,* Aleksandra Piórkowska* and Beata Malec*

Department of Astrophysics and Cosmology, Institute of Physics, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

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ABSTRACT
The accelerating expansion of the Universe is a great challenge for both physics and cosmology. In light of lacking the convincing theoretical explanation, an effective description of this phenomenon in terms of a cosmic equation of state turns out useful.

The strength of modern cosmology lies in consistency across independent, often unrelated pieces of evidence. Therefore, every alternative method of restricting the cosmic equation of state is important. Strongly gravitationally lensed quasar–galaxy systems create such a new opportunity by combining stellar kinematics (central velocity dispersion measurements) with lensing geometry (Einstein radius determination from position of images).

In this paper we apply such a method to a combined data sets from Sloan Lens ACS and Lens Structure and Dynamics surveys of gravitational lenses. As a result we obtain the cosmic equation of state parameters, which generally agree with results already known in the literature. This demonstrates that the method can be further used on larger samples obtained in the future. Independently noticed systematic deviation between fits done on standard candles and standard rulers is revealed in our findings. We also identify an important selection effect crucial to our method associated with the geometric configuration of the lensing system along the line of sight, which may have consequences for sample construction from the future lensing surveys.

Key words: gravitational lensing: strong – cosmological parameters – dark energy.

1 INTRODUCTION
The present acceleration of the cosmic expansion is a fundamental challenge to standard models of both particle physics and cosmology. Discovery of this phenomenon on the Hubble diagrams obtained from the Type Ia supernova (SNIa) surveys (Riess et al. 1998, 2004; Perlmutter et al. 1999; Davis et al. 2007; Wood-Vasey et al. 2007; Kowalski et al. 2008) in combination with independent estimates of the amount of baryons and cold dark matter (Spergel et al. 2003; Eisenstein et al. 2005) led us to believe that most of the energy in the Universe exists in the form of mysterious dark energy.

The new physics of dark energy may lie in the nature of gravity, the quantum vacuum or extra dimensions. Concerning the first possibility there exists an increasing body of literature (e.g. Buchert 2001; Räsänen 2004; Ellis & Buchert 2005; Wiltshire 2007) pointing out that if one attempts to average out local sources of gravity (galaxies and clusters) in order to obtain the smoothed description of the Universe in the largest scales, such averaging procedure could manifest as an additional source term in the energy–momentum tensor. Within the second possibility our ideas about the quantum vacuum are expressed by either introducing cosmological constant $\Lambda$ or some time-evolving scalar field (quintessence). The last possibility is to contemplate modifications to the Friedman–Robertson–Walker models arising in brane-world scenarios. Irrespective of the theoretical approach chosen, a common point with the observations usually occurs at the level of the $w(z)$ coefficient in an effective equation of state $p = w(z)\rho$ for dark energy.

The potential of constraining dark energy models with SNIa data, even though ever increasing, would not be sufficient if taken alone in separation from the other approaches. Indeed, the power of modern cosmology lies in building up consistency rather than in single, precise, crucial experiments. Therefore, every alternative method of restricting cosmological parameters is desired. In this spirit a number of combined analyses involving lensing statistics (Silva & Bertolami 2003), cosmic microwave background radiation (CMBR) measurements (Spergel et al. 2003; Hinshaw et al. 2009), age–redshift relation (Alcaniz, Jain & Dev 2003), X-ray luminosities of galaxy clusters (Allen et al. 2008) or the large-scale structure considerations (Eisenstein et al. 2005) have been performed in the literature (references above being far from complete).

In this paper we use strongly gravitationally lensed systems for providing additional constraints on dark energy models. The idea of using such systems for measuring the cosmic equation of state was discussed in Biesiada (2006) and also in a more recent paper by Grillo, Lombardi & Bertin (2008). The first (to our knowledge) formulations of this approach can be traced back to Futamase &...
Yoshida (2001). The sections that follow outline the method, the sample used and cosmological scenarios tested. The last section presents the results and conclusions.

2 THE METHOD

Strong gravitational lensing occurs whenever the source, the lens and the observer are so well aligned that the observer–source direction lies inside the so-called Einstein ring of the lens. In a cosmological context the source is usually a quasar with a galaxy acting as the lens. Although strong lensing by clusters is known and the number of such cases increases, we will be concerned with galaxies acting as lenses. For detailed theory of gravitational lensing see e.g. Schneider, Ehlers & Falco (1992). Strong lensing reveals itself as multiple images of the source. The image separations in the system depend on angular diameter distances to the lens and to the source, which in turn are determined by background cosmology. This opens a possibility to constraining the cosmological model provided that we have good knowledge of the lens model.

Since the discovery of the first gravitational lens the number of strongly lensed systems increased to a hundred (in the CASTLES data base) and is steadily increasing following new surveys like the Sloan Lens ACS (SLACS) survey. It turns out that in the vast majority of cases the lens is a late-type E/S0 galaxy. This could be understood since ellipticals, being a latecomer in hierarchical structure formation, are created in mergers of low-mass spiral galaxies. Hence they are more massive than spirals and because the Einstein ring radius scales with mass, the probability of their acting as lenses is higher. Although they lack bright kinematic tracers at large radii (e.g. like H1 in disc galaxies) and thus their kinematics is more difficult to measure, there exists increasing evidence that their mass density profile can be well approximated by a singular isothermal sphere (SIS) model (or a variant thereof called singular isothermal ellipsoid (SIE)).

Now, the idea is that the formula for the Einstein radius in a SIS lens (or its SIE equivalent),

\[ \theta_E = 4\pi \sigma_0 D_h / c^2, \]

depends on the cosmological model through the ratio of (angular diameter) distances between lens and source and between observer and lens. The angular diameter distance in flat Friedmann–Robertson–Walker cosmology reads

\[ D(z; p) = \frac{c}{H_0} \int_0^z \frac{dz'}{h(z'; p)}, \]

where \( H_0 \) is the present value of the Hubble function and \( h(z; p) \) is a dimensionless expansion rate dependent on redshift \( z \) and cosmological model parameters \( p \). For example, in a flat \( \Lambda \) cold dark matter (LCDM) model \( h(z; p) = \sqrt{\Omega_m(1+z)^3 + \Omega_{\Lambda}; c = \Omega_m + 1 - \Omega_{\Lambda} \) in this case. Expansion rates in other cosmological scenarios are given in Section 4. From now on we will assume spatial flatness of the Universe since it is strongly supported by independent and precise experiments e.g. a combined 5-yr Wilkinson Microwave Anisotropy Probe (WMAP), baryon acoustic oscillations (BAO) and supernova data analysis gives \( \Omega_m = 1.005^{+0.005}_{-0.005} \) (Hinshaw et al. 2009). The sample upon which we work is small, and the addition of a (otherwise well constrained) curvature parameter would only distort the results.

Provided one has reliable knowledge about the lensing system, i.e. the Einstein radius \( \theta_E \) (from image astrometry) and stellar velocity dispersion \( \sigma_{\star} \) (form central velocity dispersion \( \sigma_c \) obtained from spectroscopy), one can use it to test the background cosmology. This method is independent of the Hubble constant value (which gets cancelled in the distance ratio) and is not affected by dust absorption or source evolutionary effects. It depends, however, on the reliability of lens modelling (e.g. SIS or SIE assumption) and measurements of \( \sigma_c \). Hopefully, starting with the Lens Structure and Dynamics (LSD) survey and the more recent SLACS survey, spectroscopic data for central parts of lens galaxies became available allowing to assess their central velocity dispersions. In practice central velocity dispersion \( \sigma_0 \) is estimated from the velocity dispersion within \( R_e / 8 \), where \( R_e \) is optical effective radius. Thorough discussion of these issues can be found in Treu et al. (2006a,b) and Grilli et al. (2008), where the arguments in favour of using \( \sigma_0 \) as a representative of \( \sigma_{\star} \) are presented. Moreover, there is a growing evidence for homologous structure of late-type galaxies (Koopmans et al. 2006, 2009; Treu et al. 2006a,b) supporting reliability of SIS/SIE assumption. In particular it was shown in Koopmans et al. (2009) that inside one effective radius massive elliptical galaxies are kinematically indistinguishable from an isothermal ellipsoid.

In the method used in this paper, the cosmological model enters not through a distance measure directly, but rather through a distance ratio

\[ \frac{D^\text{obs}(z_l, z_s; p)}{D^\text{obs}(z_l, z_s; p)} = \frac{D_\theta(z_l; p)}{D_\theta(z_l; p)} \]

and respective observable counterpart reads

\[ \frac{D^\text{obs}(z_l, z_s; p)}{D^\text{obs}(z_l, z_s; p)} = \frac{4\pi \sigma_0^2 c / 3\theta_E^2}{3\theta_E^2} \]

This has certain consequences both advantageous and disadvantageous. The positive side is that the Hubble constant \( H_0 \) gets cancelled, hence it does not introduce any uncertainty to the results. On the other hand, we have a disadvantage that the power of estimating \( \Omega_m \) is poor (which could be seen by inspection into specific formulae for \( h(z; p) \) – see Table 1). Therefore we only attempted to fit through a combination of 5-yr WMAP, baryon acoustic oscillations (BAO) and supernova data analysis gives \( \Omega_m = 1.005^{+0.005}_{-0.005} \) (Hinshaw et al. 2009). The sample upon which we work is small, and the addition of a (otherwise well constrained) curvature parameter would only distort the results.

1 http://www.cfa.harvard.edu/castles/

2 http://www.slacs.org/
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\( \Omega_m \) in the case of a \( \Lambda \)CDM model (where it is the only free parameter in flat cosmology) and it was successful only for the restricted sample (see below). In other cases we assumed fixed values for \( \Omega_m \). Cosmological model parameters (coefficients in the equation of state) have been estimated by minimizing the chi-square:

\[
\chi^2(p) = \sum_i \frac{(D_{\text{obs}}^i - D_{\text{pl}}^i(p))^2}{\sigma_D^2},
\]

(4)

where the sum is over the sample and \( \sigma_D^2 \) denotes the variance of \( D \) (contextual use of the same symbol for variances and velocity dispersions should not lead to confusion). In calculating \( \sigma_D \) we assumed that only velocity dispersion errors contribute and the Einstein radii are determined accurately.

3 SAMPLES USED

We used a combined sample of \( n = 20 \) strong lensing systems with good spectroscopic measurements of central dispersions from the SLACS and LSD surveys (essentially the same sample as used by Grilli et al. 2008). Original data concerning the SLACS sample came from Treu et al. (2006a) (see also an erratum in Treu et al. 2006b – very important one). Data concerning LSD lenses are taken after Treu & Koopmans (2004) and Koopmans & Treu (2002, 2003).

As already noticed in Treu et al. (2006a) the SLACS sample has an average \( D_h/D_t \) ratio equal to 0.58 with an rms scatter 0.15. Whereas for their purpose it was advantageous, in our context it weakens the performance of the method. Therefore we selected a subsample of \( n = 7 \) lenses with the \( D \) ratio deviating from the mean more than rms in either direction. It is summarized in Table 1 where the names of lenses in the restricted sample are given in bold.

For comparison of our results with the data which triggered the dark energy problem, we also performed fits to the SNIa data (\( n = 307 \) supernovae) using the Union08 compilation of Kowalski et al. (2008). The \( \Omega_m = 0.27 \) prior was used throughout, except in the \( \Lambda \)CDM model where the fit was attempted.

4 COSMOLOGICAL MODELS TESTED

Several scenarios have been put forward as an explanation of the presently accelerating expansion of the Universe. The most obvious candidate is the cosmological constant \( \Lambda \) representing the energy of the vacuum. The corresponding cosmological model, which turned out to be in agreement with all existing (independent and alternative) observations, is the \( \Lambda \)CDM model. It is equivalent to \( w = -1 \) in the cosmic equation of state \( p = w \rho \) and the only free parameter here is \( \Omega_m \) representing the density of baryonic plus CDM as a fraction of critical density (as already said, spatial flatness is assumed). On one hand it is therefore the most parsimonious one, but well known fine-tuning problems led many people to seek beyond the \( \Lambda \) framework and develop the concept of quintessence. Usually the quintessence is described in a phenomenological manner, as a scalar field with an appropriate potential. In first approximation it could be tested observationally by promoting \( w \) to the role of a free parameter to be fitted from the data. However there is no a priori reason to expect that \( w \) should then be a constant. The parametrization of \( w(z) = w_0 + w_z(z)/(1 + z) \) developed by Chevalier & Polarski (2001) and Linder (2003) turned out to be well suited and robust for such a case. In the past, the alternative parametrization \( w(z) = w_0 + w_zz \) was used (which is a truncated Taylor series representation of \( w(z) \)). The Chevalier–Polarski–Linder parametrization instead uses an expansion with respect to the physical degree of freedom, i.e. the scalefactor (expanded around its present value). Dimensionless (i.e. with \( H_0 \) factored out) expansion rates for respective models are given in Table 2.

For comparison we also performed fits of the models considered above to the SNIa data with the same prior assumptions (spatial flatness of the Universe and \( \Omega_m \)). We have taken the Union08 compilation (Kowalski et al. 2008) and instead of straightforward \( \chi^2 \) fitting \( m(z) \) versus \( D_L(z) \) (where \( m \) is visible magnitude, \( D_L \) denotes luminosity distance) we used a well-known modified approach equivalent to marginalizing over the intercept (Nesseris & Perivolaropoulos 2005).

5 RESULTS AND CONCLUSIONS

Performing fits of different cosmological scenarios (shown in Table 2) on the full SLACS + LSD sample of \( n = 20 \) strong lensing systems we obtained the equation of state parameters displayed in Table 3. In \( \Lambda \)CDM model, where \( \Omega_m \) was the only free parameter we were not able to make a reliable fit on the sample considered. As already mentioned the reason for this is twofold. First, our theoretical quantity \( D^\text{th} \) was the ratio of two integrals differing only by the limits of integration. Second, in the full sample the \( D_h/D_t \) ratio is concentrated around a central value of 0.54. Therefore, in case of ‘simple’ dependence (just a factor) of \( h(z) \) on a parameter (as is the case for \( \Omega_m \)) the bulk of the sample only introduces an undesired scatter. More ‘sophisticated’ dependence (exponent of the integration variable) on the equation of state parameters makes it possible to obtain fits on this sample.

For comparison we also report (Table 5) values of these parameters best fitted to the Union08 SNIa compilation (Kowalski et al. 2008). One can see that the \( w \) coefficient obtained from the full strong lensing sample agrees with the respective value derived from supernovae data (almost the whole 2\( \sigma \) confidence interval for \( w \) from the Union08 data set lies within the 1\( \sigma \) CI from lenses). The value inferred is also in agreement with the WMAP5 results presented in Hinshaw et al. (2009) including also combined WMAP5, BAO and SNIa analysis. Note that this is also consistent with the \( \Lambda \)CDM model. Concerning the evolving equation of state in the Chevalier–Polarski–Linder parametrization, confidence regions in the \((w_0, w_z)\) plane are shown in Fig. 1. It can be seen that the concordance model (\( \Lambda \)CDM) while consistent with SNIa data (at 2\( \sigma \) level) is inconsistent with the strong lensing systems method applied here. SNIa results and strong lensing results are marginally

| Model | EOS \( p = w \rho \) | Cosmological expansion rate \( h(z) \) |
|-------|-----------------|-------------------------------|
| \( \Lambda \)CDM | \( w = -1 \) | \( h(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} \) |
| Quintessence | \( w = \text{const.} \) | \( h(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_Q (1 + z)^{3(w+1)/2}} \) |
| Chevalier–Polarski–Linder | \( w(z) = w_0 + w_z (1+z)^{-1} \) | \( h(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_Q (1 + z)^{3(w_0+w_z+2z)/2}} \exp(-3w_0w_z) \) |
consistent with each other. Most probably this is due to the small sample of strong lenses combined with the systematics discussed above.

Working on the restricted sample (containing lenses with a $D_{ls}/D_s$ ratio greater than the rms spread around the mean value), although a dramatically decreased sample size (down to $n = 7$), allowed to obtain fits on $\Omega_m$ in $\Lambda$CDM (Table 4) which turned out to agree with SNIa fits (Table 5) and WMAP5 data (Hinshaw et al. 2009). Although the best fit for the $w$ parameter quintessence scenario is higher than inferred from SNIa or WMAP5, yet the 2σ interval for the Union08 data falls within the 2σ interval from the lenses. Hence the agreement is quite good. Similarly, fits for $w_0$ and $w_a$ are improved (even though confidence regions get larger – see Fig. 2).

One should note, however, that a systematic shift downwards in the $(w_0, w_a)$ plane persists. Such a shift in best-fitting parameters inferred from supernovae (standard candles, sensitive to luminosity distance) and BAO or acoustic peaks (standard rulers, sensitive to angular diameter distance) has already been noticed and discussed by Lazkoz, Nesseris & Perivolaropoulos (2008) and by Linder & Roberts (2008). Bearing in mind the similar mutual inconsistency in the Hubble constant values inferred from lensing time delays $H_0 = 52 \pm 6\, \text{km}\, \text{s}^{-1}\, \text{Mpc}^{-1}$ (Kochanek & Schechter 2004) and from the Hubble Space Telescope (HST) Key Project

$H_0 = 72 \pm 8\, \text{km}\, \text{s}^{-1}\, \text{Mpc}^{-1}$ (Freedman et al. 2001), our result suggests the need for taking a closer look at the compatibility of results derived by using angular diameter distances and luminosity distances, respectively. It is also worth noticing that the ideas of testing the Etherington reciprocity relation between these two distance measures have been discussed by Bassett & Kunz (2004) and by Uzan, Aghanim & Mellier (2004).

In conclusion our results demonstrated that the method discussed in Biesiada (2006) and extensively investigated by Grillo et al. (2008) on simulated data can be used in practice to constrain cosmological models. It turned out to give reasonable results on already accessible samples of strongly lensed systems. Besides the
uncertainties related to velocity dispersion measurements and their conversion to relevant lens model parameters (as well as the impact of the SIS assumption) the issue of systematics associated with $D_s/D_e$ ratio in the sample turned out to be important. In particular it implies that strong lensing survey strategies like the one adopted in the SLACS survey are better from this point of view. Lensing systems are gathered around something like 0.58 in distance ratio because it is roughly the configuration for which the lensing probability (for a given lens mass) is the highest. Earlier searches were focused on source population (quasars), seeking close pairs or multiples and checking if they are multiple images of a single source lensed by an intervening galaxy. Therefore a high lensing probability was an important selection factor there. On the other hand, the SLACS survey is focused on possible lens population (massive ellipticals) with good spectroscopic data. Using Sloan Digital Sky Survey (SDSS) templates, spectra are carefully checked for residual emission (at least three distinct common atomic transitions) coming from higher redshifts. Such candidates undergo image processing by subtracting the parametrized brightness distribution typical for early-type galaxies in order to reveal multiple images of the quasar. Details can be found in Bolton et al. (2006). Therefore, besides the obvious bonus of having the central velocity dispersion measured, such a strategy is better suited for discovering systems with larger $D_s/D_e$ ratios which in turn can be used for testing cosmological models.

Finally, one important effect – neglected here – should be mentioned, which is the influence of line-of-sight mass contamination. The debate on this issue started with Bar-Kana (1996) and Keeton, Kochanek & Seljak (1997) who were among the first to convincingly demonstrate that the effect of large-scale structure on strong lensing could be significant. More recent results on this issue can be found in Dalal, Hennawi & Bode (2005) (in the context of cluster lensing) or Momcheva et al. (2006). This raises the issue of what impact this effect might have on our results, since the sample was small. A straightforward naive first guess (based on Poissonian statistics) might suggest that a sample size of order of a few hundred lenses might reduce the line-of-sight ‘noise’ contamination down to a few per cent. This is however not that simple since the line-of-sight contamination is in fact a systematic effect. Namely, massive early-type galaxies (i.e. typical lenses) prefer overdense environments, so one consistent approach would be to follow light rays (ray-shooting simulation) through many lens planes (obtained from cosmological N-body simulation) up to high source redshift. This was done by Wambsganss, Bode & Ostriker (2005) with the result that up to $z_s = 1$ most (i.e. 95 per cent) of lenses involved only a single mass concentration, whereas for sources at $z_s = 3.8$ the important contribution of intervening mass could be significant in 38 per cent of strong lensing systems. This result suggests that the line-of-sight contamination should be addressed separately for each particular survey. For the SLACS survey (where the bulk of our sample comes from) this was assessed in Treu et al. (2009) where the authors found that SLACS lens galaxies are an unbiased population (i.e. with environmental effects typical to the overall population of early-type galaxies) and the typical contribution from external mass distribution is small – no more than a few per cent. Fortunately, the SLACS survey is ongoing, i.e. the sample of spectroscopically investigated strong lenses is growing. However this survey is relatively shallow, so for cosmological applications one is forced to combine it with deeper surveys (with different designs – hence different systematics) and the problem of line-of-sight contamination remains challenging.

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