When m-lindelof sets are mx-semi closed

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Abstract. This paper is devoted to introduce new concepts so called m-L(sc)-spaces. Several theorems related to these concepts are proved, further properties are studied as well as the relationships between these concepts with another types of m-L(sc)-spaces are investigated.

Key words. mx-open set, m-compact, m-lindelof, m-Lc-spaces, m-lindelof and mx -semi closed.

1. Introduction
It is known that there is no relation between m-Lindelof space and mx-closed sets, so this point stimulated some researchers to introduce a new concept namely m-Lc-spaces [1], these are the spaces (m-Lc-spaces) in which every m-Lindelof subset is mx-closed. In 2015 the author [2] introduced a new concept, namely, m-K2 (=A non-empty set X with an m-space is said to be m-K2 if mx-cl (A) is m-compact in X for a subset A of a m-space X, whenever X is m-compact). The basic definitions that are needed in this work are recalled. A space(X,mx) means a m-space where a sub family mx of the power set P(X), such that ∅ and X belong to mx [3]. Each member of mx is said to be mx-open set and the complement of an mx-open set is said to be mx-closed set. We denote the (X,mx) by m-space. For a subset A of a m-space X, the mx-interior of A and the mx-closure of A are defined as follows :
mx-cl(A) =∩{F: A ⊆ F, X-F is mx-open}
mx-int(A) = ∪{U: U ⊆ A, U ∈ mx}

Note that mx-cl(A)(mx-int(A)) is not necessarily mx-closed (mx-open). The m-space need not to be a topological space and the union and the intersection of any two mx-open sets are not necessarily to be mx-open [3], as we show in the following example:
Example: Let X = {a, b, c}, mx= {φ,X, {b, c}, {a, b}, {a}, {c}}.
Then (X, mx) is m-space but it is not topological space, since {b, c} ∩ {a, b} = {b} ∉ mx and {a}∪{c} = {a, c} ∉ mx. [2]. The authors [3] introduce the following definitions:
An m-space m_0 on a non-empty set X is said to have the property (γ) if the intersection of finite number of m_0-open sets is m_0-open. An m-space m_0 on a non-empty set X is said to have the property (β) if the union of any family of subsets of m_0 belong to m_0. A nonempty set X with m-space is said to be m-compact if every cover of X with m_0-open sets has a finite sub cover [4]. An empty set X with m-space m_0 is said to be m-lindelof if every cover of X with m_0-open sets has countable sub cover [5]. Every m-compact set is m-lindelof but the convers is not true [2]. For example:
The m-discrete space (X, τ_D), where X is infinite countable set, and τ_D = discrete m-Space, then (X, τ_D) is m-lindelof, which is not m-compact [2]. An m-space X which has (β) property is m-T1-
space if and only if every singleton set is \(m\)-closed [2]. A set \(A\) is an \(m\)-space \(X\) is called \(m\)-regular if for each \(m\)-closed set \(F\) in \(X\) and \(x \in X\) with \(x \notin F\), there exist disjoint \(m\)-open sets \(U, V\) in \(X\) with \(x \in U\) and \(F \subseteq V\) by [6]. Every \(m\)-compact subset of \(m\)-\(T_2\)-space has the properties (\(\beta\)) & (\(\gamma\)) is \(m\)-closed [7].

On \(m\)-lindelof sets are \(m\chi\) —semi closed

Definition (1): A space \(X\) is called \(m\)-\((sc)\)-space if every \(m\) -lindelof subset of \(X\) is \(m\)-semi closed.

Example (1): In the space \((X, \tau_D)\), such that \(X\) is a non-empty set, then every lindelof subset of \(X\) is closed means that \(X\) is \(L\)-c-space.

Remark (1): Every \(m\)-L c-space is \(m\)-\((sc)\)-space, but the convers may not be true.

Example (2): Let \(R\) be the real line, \(N\) be a subset of \(R\) (where \(N\) is natural numbers) and \(\tau = \{U \subseteq R|U = R\) or \(U \cap N = \emptyset\}\). It is clear \((X, \tau)\) is an \(m\)-space. Then \((R, \tau)\) is \(m\)-\((sc)\)-space but it is not \(m\) -L-space, since \(R\) consists of rational numbers and irrational numbers, then if we take a \(m\)-lindelof subset \(K\) of irrational numbers so it is finite set (since the \(m\)-lindelof set in irrational numbers is just a finite set), \(K\) is not \(m\)-closed set since it is not intersects \(N\), but it is an \(m\)-semi closed, since there exists an \(m\)-closed subset \(K \cup N\) of irrational number \(Q\) such that \(K = m-\text{int}(K \cup N) \subseteq K \subseteq K \cup N\). Now if we take the rational number set \(Q\), it is a countable then it is an \(m\)-lindelof set, the set \(Q\) is an \(m\)-lindelof and \(m\)-semi closed but not \(m\)-closed, there exists an \(m\)-closed subset \((Q-N) \cup N\) of \(Q-N\) such that \(Q-N = m-\text{int}((Q-N) \cup N) \subseteq Q-N \subseteq (Q-N) \cup N\).

Remark (2): Every \(m\)-\((sc)\)-space is \(m\)-\((k)\)-space, but the convers may not be true.

Example (3): The \(m\)-space \((R, \tau_u)\) is a \(m\)-\(T_2\)-space, so it is a \(m\)-\((k)\)-space, but it is not \(m\) -\(L(sc)\)-space since if we take the natural \(N\) it is countable, then it is \(m\)-lindelof, but not \(m\)-semi closed because the only \(m\)-closed set which contain \(N\) is just \(R\) but \(R = m-\text{int}(R) \nsubseteq N \nsubseteq R\), so \((N, \tau_u)\) is not \(m\)-\(L(sc)\)-space.

Proposition (1): Every \(m\) -\(L(sc)\)-space is \(m\)-\(sT_1\)-space, whenever \(X\) has the property (\(s\beta\)).

Proof: If \(X\) is an \(m\)-\(L(sc)\)-space, then it is clear that the set\(\{x\}\) is an \(m\)-lindelof subset of \(X\) which is \(m\)-\(L(sc)\)-space, then \(\{x\}\) is \(m\)-semi closed subset of \(X\) for all \(x \in X\), hence, \(X\) is \(m\)-\(T_1\).

Definition (2): A subset \(F\) in an \(m\)-space \(X\) is called \(m\)-\(F_\theta\) —closed if it is the union of countably many \(m\)-closed sets. A set \(G\) is called \(m\)-\(G_\theta\)-open if it is the intersection of countably many \(m\)-open sets.

Remark (3): Every \(m\)-closed set is \(m\)-\(F_\theta\) —closed and \(m\)-open set is \(m\)-\(G_\theta\)-open, but the convers is not true for example (2).

Definition (3): A \(m\)-space \(X\) is called \(m\)-\(p\)-space if every \(m\)-\(G_\theta\)-open set in \(X\) is \(m\)-open.

Theorem (1): Every \(m\)-\(T_2\)-space and \(m\)-\(p\)-space is \(m\)-\(L(sc)\)-space, whenever \(X\) has the property (\(\beta\)).

Proof: Let \(L\) be an \(m\)-lindelof subset of \(X\) and \(x \notin L\), so for each \(\in L\), such that \(l \neq x\), but \(X\) is \(m\)-\(T_2\)-space means that there exists \(U, V \in m\chi\), such that \(\in U\), \(x \notin V\) and \(U \cap V = \emptyset\). Now, let \(\{U_{\alpha}\}_{\alpha \in A}\) be an \(m\) open cover to \(L\) which is \(m\)-lindelof, that is \(L = \bigcup_{\alpha \in A} U_{\alpha}\), thus, \(\bigcup_{\alpha = 1}^{n} U_{\alpha}\) is a cover to \(L\), put \(V' = \bigcup_{\alpha = 1}^{n} V_{\alpha}\), then \(V'\) is \(m\)-\(G_\theta\) —open set but \(X\) is \(m\)-\(p\)-space, so \(V'\) is \(m\)-open in \(X\), then \(x \in V' \subseteq L'\), that is \(x\) is an \(m\)-interior point to \(L'\) this implies \(L'\) is \(m\)-open set in \(X\), hence, \(L\) is \(m\)-closed in \(X\), but every \(m\)-closed is \(m\)-\(\theta\)-semi closed. Then \(X\) is \(m\)-\(L(sc)\)-space.

Definition (4): An \(m\)-space \(X\) is said to be \(m\)-\(L(\theta_\beta)\)-space if every \(m\)-lindelof subset of \(X\) is \(m\)-\(\theta\)-closed.
Example (4): \((X, \tau_D)\), where \(X\) is countable set, satisfy definition (4).

Definition (5): A space \(X\) is called \(\theta-p\)-space if every \(\theta-G_\delta\)-open set in \(X\) is \(\theta\)-open.

Definition (6): An \(m\)-space \(X\) is called \(m-\theta-p\)-space if every \(m,\theta\)-open set in \(X\) is \(m,\theta\)-open.

Corollary (1): Every \(m-\theta-T_2\) and \(m-\theta-p\)-space are \(m-L(\theta)c\)-space.

Proof: Let \(M\) be an \(m\)-lindelöf subset and \(x \notin M\), for each \(m \in M\) such that \(m \neq x\), but \(X\) is \(m -\theta T_2\) (means that) there exists \(U, V \in \tau_\theta\) such that \(m \in U, x \in V\) and \(U \cap V = \emptyset\). Now, let \(\{U_\alpha\}_{\alpha \in \Omega}\) be an \(m,\theta\)-open cover of \(M\), which is a lindelöf; that is \(M = \bigcap_{\alpha = 1}^\infty U_\alpha\), so \(\bigcup_{\alpha = 1}^\infty U_\alpha\) is a cover to \(M\). Put \(\forall \alpha \in \Omega\), then \(V_\alpha = m,\theta -\emptyset\) but \(X\) is \(m,\theta-p\)-space, so \(V_\alpha\) is \(m,\theta\)-open in \(X\). Then \(x \in V_\alpha \subseteq M^c\) that is \(x\) is an \(m,\theta\)-interior points to \(M^c\). This implies, \(M^c\) is \(m,\theta\)-open set in \(X\), so \(M\) is \(m,\theta\)-closed in \(X\). Then \(X\) is \(m\) - \(L(\theta)c\)-space.

Definition (7): Let \(X\) be an \(m\)-space and \(Y\) be a subspace of \(it\), a subset \(U_Y\) is said to be \(m,\theta\)-open in \(Y\) if there exists an \(m,\theta\)-open set \(U_X\) in \(X\) such that \(U_Y = U_X \cap Y\).

Definition (8): let \(X\) be an \(m\)-space and \(Y\) be a subspace of \(it\), a subset \(U_Y\) is said to be \(m,\theta\)-closed in \(Y\) if there exists an \(m,\theta\)-closed set \(U_X\) in \(X\) such that \(U_Y = U_X \cap Y\).

Example (5): \(X = \{a, b, c, d\}, Y = \{a, c\}\), \(\tau_X = \{\emptyset, X\}, \tau_Y = \{\emptyset, Y\}\) satisfies definition (8).

Definition (9): Let \((X, \tau_X)\) be an \(m\)-space and \(Y \subseteq X\), then the subspace topology of \(Y\) in \(X\) is \(\tau_Y = \{Y \cap U | U \in \tau_X\}\).

Proposition (2): Let \(X\) be an \(m\)-space, a subset \(U\) of \(X\) is \(m,\theta\)-open if and only if, \(\forall x \in U, x \in m,\theta -\text{int} (U)\), wherever \(X\) has the property(\(\beta\)).

Proof: Let \(U\) be \(m,\theta\)-open in \(X\), and \(x \in U\) by taking \(U = G\), then \(x \in U = G \subseteq U\). When \(x\) is an \(m,\theta\)-interior point of \(U\) but \(x\) is an arbitrary, hence, \(\forall x \in U, x\) is an \(m,\theta\)-interior point to \(U\). Consequently, \(\forall x \in U\), there exists an \(m,\theta\)-open set \(G\) containing \(x\) and contained in \(U\), so \(U = \bigcup_{\alpha \in \Lambda} G_\alpha\), but \(X\) has a property(\(\beta\)), thus, \(\bigcup_{\alpha \in \Lambda} G_\alpha\) is \(m,\theta\)-open, that is \(U\) is \(m,\theta\)-open sets.

Proposition (3): Let \(X\) and \(Y\) be two \(m\)-spaces, a function \(f: X \rightarrow Y\) is \(m\)-continuous iff the inverse image under \(f\) of every \(m,\theta\)-open (\(m,\theta\)-closed) set in \(Y\) is \(m,\theta\)-open (\(m,\theta\)-closed) in \(X\) with property(\(\beta\)).

Proof: Let \(f\) be \(m\)-continuous function and let \(H\) be an \(m,\theta\)-open in \(Y\) (to prove that \(f^{-1}(H)\) be \(m,\theta\)-open in \(X\)). If \(f^{-1}(H) = \emptyset\), then there is nothing to prove. Thus, let \(f^{-1}(H) \neq \emptyset\) and let \(x \in f^{-1}(H)\) that is \(f(x) \in H\). Since \(f\) is \(m\)-continuous there exists an \(m,\theta\)-open set \(G\) in \(X\) such that \(x \in G\) and \(f(G) \subseteq H\) (means that) \(x \in G \subseteq f^{-1}(H)\), so \(x\) is an interior point, \(f^{-1}(H)\) is \(m,\theta\)-open in \(X\). Conversely, assume that \(f^{-1}(H)\) is \(m,\theta\)-open in \(X\), \(y \in m,\theta\)-open set \(H\) in \(Y\) (to prove that \(f\) is \(m\)-continuous at \(x \in X\)). Let \(H\) be any \(m,\theta\)-open set in \(Y\) such that \(f(x) \in H \rightarrow x \in f^{-1}(H)\), by hypothesis \(f^{-1}(H)\) is \(m,\theta\)-open in \(X\). If \(f^{-1}(H) = \emptyset\), then \(G\) is an \(m,\theta\)-open set in \(X\) containing \(x\) such that \(f(G) = f(f^{-1}(H)) \subseteq H\), hence, \(f\) is \(m\)-continuous function.

The proof of closeness is the same.

Theorem (2): Let \((X, m_X)\) and \((Y, m_Y)\) be \(m\)-spaces and \(B\) be a base for \(m_Y\) on \(Y\), a function \(f: (X, m_X) \rightarrow (Y, m_Y)\) is \(m\)-continuous if and only if \(\forall B \in B, f^{-1}(B)\) is \(m,\theta\)-open in \(X\), whenever \(X\) and \(Y\) have the property(\(\beta\)).

Proof: Let \(f\) be an \(m\)-continuous and \(B \in B\) since \(B\) be \(m,\theta\)-open in \(Y\), then \(f^{-1}(B)\) is \(m,\theta\)-open in \(X\) (by proposition (3)).

Conversely, (Given \(f^{-1}(B)\) is \(m,\theta\)-open in \(X\)) \(\forall B \in B\) and let \(H\) be an \(m,\theta\)-open in \(Y\), then \(H\) is a union of \(B\) \(m,\theta\)-open members of \(B\) (means that) \(H = \bigcup B_\alpha\) for some \(\alpha \in \Lambda\). Therefore, \(f^{-1}(H) = f^{-1}(\bigcup B_\alpha)\) for some \(\alpha \in \Lambda\)=\(f^{-1}(B_\alpha)\) which is \(m,\theta\)-open in \(X\). Since each \(f^{-1}(B_\alpha)\) is \(m,\theta\)-open in \(X\) and \(X\) has the property(\(\beta\)), hence \(f\) is \(m\)-continuous.
Theorem (3): Let $X$ and $Y$ be an m-spaces a function $f$ from a space $X$ to a space $Y$ is m-continuous if $f(mx-cl(A)) \subseteq mx-cl(f(A))$, for every $A \subseteq X$, whenever $X$ and $Y$ have the property $(\beta)$.

Proof: Let $f$ be an m-continuous, since $m_x$-cl($f(A)$) is $m_x$-closed in $Y$, then $f^{-1}(m_x-cl(U))$ is $m_x$-closed in $X$ and therefore
\[
\forall A \subseteq f^{-1}(m_x-cl(f(A))) \Rightarrow A \subseteq f^{-1}(m_x-cl(f(A)))
\]
\[
\Rightarrow m_x$-cl($A) \subseteq m_x$-cl($f^{-1}(m_x-cl(f(A))) = f^{-1}(m_x-cl(f(A)))$ by (1)
\]
\[
\Rightarrow f(m_x-cl(A)) \subseteq m_x$-cl($f(A))$
\]

Lemma (1): Let $f:X \rightarrow Y$ be an injective and m-continuous function from an m-space $X$ in to a space $Y$ if $F$ is a $m_x$-semi closed set in $X$, then $f(F)$ is also $m_x$-semi closed in $Y$ with property $(\beta)$.

Proof: let $F \subseteq X$, for some $m_x$-semi closed $F$ in $X$, since $f$ is (one to one), so $F=f^{-1}(F(F))$, so $F$ is $m_x$-open in $X$, that is there is $U$ is $m_x$-open subset of $X$ such that $U \subseteq F \subseteq m_x-cl(U), f(U) \subseteq f(F) \subseteq f(m_x-cl(U))$, and since $f$ is $m$ - continuous, then $f(m_x-cl(U)) \subseteq m_x-cl(f(U))$. This yields,
\[
f(U) \subseteq f(F) \subseteq m_x-cl(f(U))$, then $f(F)$ is $m_x$-semi open set in $Y$. Hence, $f(F)$ is $m_x$-semi closed in $Y$.

Lemma (2): Let $(X, m_x)$ and $(Y, m_y)$ be two m-spaces and let $f:X \rightarrow Y$ be an m-homeomorphism function from a space $X$ to a space $Y$. If $F$ is $m_x$-semi closed in $X$, then $f^{-1}(F)$ is also $m_x$-semi closed in $X$ with property $(\beta)$.

Proof: let $f^{-1}(F) \subseteq X$, for some $m_x$-semi closed $F$ in $X$ and $f(f^{-1}(F))$ (fis onto), then there exists $G$ be an $m_x$-closed set in $Y$, such that $m_x$-int($G) \subseteq F \subseteq G$, $f^{-1}(m_x$-int($G)) \subseteq f^{-1}(F) \subseteq f^{-1}(G)$, and since $f$ is m-homeomorphism, then $m_x$-int($f^{-1}(G)) \subseteq f^{-1}(m_x$-int($G))$. Thus, $m_x$-int($f^{-1}(G)) \subseteq f^{-1}(F)$ is $m_x$-semi closed in $Y$.

Lemma (3): If $F$ is $m_x$-semi closed in $X$, and $Y$ is a subspace of $X$, then $F \cap Y$ is $m_x$-semi closed in $Y$.

Proof: Since $F$ is $m_x$-semi closed in $X$, then there exists $m_x$-closed set $G$ of $X$ such that $G \subseteq F \subseteq G_{int}$, $m_x = $ int($G_{int}$) $\subseteq F \subseteq G_{int}$, $m_x = $ int($G_{int}$) $\subseteq F \subseteq G_{int}$, therefore, $F \cap Y$ is $m_x$-semi closed in $Y$.

Proposition (4): The property of space being $m$-$L(sc)$-space is a topological property, whenever $X$ has the property $(\beta)$.

Proof: let $f:X \rightarrow Y$ be an m-homeomorphism function from an m-$L(sc)$-space $X$ into a space $Y$, we aim to show that $Y$ is also m-$L(sc)$-space. Let $M$ be a m-lindelof subset of $Y$, but $f^{-1}$ is m-continuous $m_x$-semi closed in $X$. Since $f$ is onto, then $M=f(f^{-1}(M))$ and by lemma (2), then $M$ is $m_x$-semi closed of $Y$, therefore, $Y$ is an m-$L(sc)$-space.

Definition (10): An m-space $X$ is called m-$sLc$-space if every $m_x$-semi lindelof is $m_x$-closed. For example, $(\mathbb{R}, \tau_{cof})$, where $\tau_{cof}$ be a co-finite m-space.

Definition (11): An m-space $X$ is called m-$sL(sc)$-space if every $m_x$-semi lindelof is $m_x$-semi closed.

Lemma (4): If a set $A$ is $m_x$-semi closed in $X$ and $F$ is a $m_x$-closed set in a space $X$, then $A \cup F$ is $m_x$-semi closed in $F$ and also in $X$. 


Proposition (5): Suppose that $X = X_1 \cup X_2$. Then, $X_1$ is $m_x$-closed L(sc)-subspace of $X$ and $X_2$ is $m_x$-closed Lc-subspace of $X$, then $X$ is m-L(sc)-space.

Proof: Let $M$ be an m-lindelof subset of $X$, then $M \cap X_1$ is an $m_x$-closed in $M$, which is m-lindelof in $X$ and so $M \cap X_1$ is $m_x$-closed in $X$, but $M \cap X_1 \subseteq X_1$, which is m-L(sc)-space, then $M \cap X_1$ is $m_x$-semi closed in $X_1$, and $M \cap X_2$ is $m_x$-closed in $M$ which is m-lindelof. Then, $M \cap X_2$ is m-lindelof of $X$, but $M \cap X_2 \subseteq X_2$, which is m-Lc-space, then $M \cap X_2$ is $m_x$-closed in $X_2$, but $M = (M \cap X_1) \cup (M \cap X_2)$, and so $M$ is $m_x$-semi closed in $X$ (by lemma (4)).

Proposition (6): Let $(Y, \tau_Y)$ be an m-subspace of $(X, \tau_X)$, then for every $A \subseteq Y$ we have $m_X - cl(A) \cap Y = m_X - cl(A) \cap X \cap Y$.

Proof: $m_X - cl(A) \cap Y = \cap \{k: k \text{ is } m_X - \text{closed in } X \text{ and } A \subseteq k\} = \cap \{F \cap Y: F \text{ is } m_X - \text{closed in } X \text{ and } A \subseteq F \cap Y\}$.

Lemma (5): If $W$ is m-semi closed in $X$ and $Y \subseteq X$, then $M \cap Y$ is $m_x$-semi closed in $Y$.

Proof: Since $W$ is m-semi closed in $X$, then there exists $m_x$-closed set $F$ of $X$ such that $m_X - int(F) \cap X \subseteq W \subseteq F \cap X$, so $m_X - int(F) \cap X \cap Y \subseteq W \cap Y \subseteq F \cap Y$. Then $W \cap Y$ is $m_x$-semi closed in $Y$.

Proposition (7): Every m-locally compact L(sc)-space is an m-sT2-space.

Proof: Let $X$ be an m-locally compact L(sc)-space, then it is m-locally compact K(sc)-space, hence $X$ is an sT2-space. Let $X$ be m-locally compact and m-K(sc)-space, then $X$ is m-T2-space.

Proposition (8): The property of space being m-L(sc)-space is a hereditary property.

Proof: Let $(X, \tau)$ be an m-L(sc)-space and let $(Y, \tau_Y)$ be a subspace of $(X, \tau)$ (to prove that $Y$ is m-L(sc)-space). Let $M$ be an m-lindelof in $Y$. Since $M \subseteq Y \subseteq X$, then $M$ is m-lindelof in $X$ and $M \cap Y$ is m-semi lindelof closed in $Y$.

Definition (12): Let $X$ be an m-lindelof space, and let $A \subseteq X$. A subset $A$ is said to be m-semi lindelof if $A$ is m-semi lindelof subspace of $X$.

Remark (4): Every m-semi lindelof space is m-lindelof space, but the converse may be not true.

Example (6): Let $(X, \tau)$ be an m-space such that, $X = \mathbb{R}$ (the real line), and $\tau = \{r_a: a \in \mathbb{R}\} \cup \{\mathbb{R}\}$ be an m-space defined on $X$ such that $r_a = \{x: x \in \mathbb{R} \text{ and } x \geq a\}$. Then, $(X, \tau)$ is an m-lindelof but not m-semi lindelof space, since if we take a family $B=\{r_1\} \cup \{a: a < 1\}$ m-semi open cover to $X$, $\{r_1\} \cup \{a\}$ is m-semi open set for each $a \in \mathbb{R}$, and since $\{r_1\} \subseteq \{r_1\} \cup \{a\} \subseteq m_X - cl(\{r_1\}) = R$, such that this family of m-semi open sets cannot be reducible to a countable subcover since $\mathbb{R} - r_a$ uncountable set.

Definition (13): An m-space $X$ is called hereditary m-semi lindelof if every subspace of $X$ is m-semi lindelof.

Proposition (9): For a hereditary m-semi lindelof space $X$, the following statements are equivalent: $X$ is an $m$sLc-space. $X$ is a countable discrete space.
Proof: To prove \(X\) is discrete space, let \(A\) be any subset of \(X\), then it is an \(m\)-semi lindelof, which is an \(m\)-sLc-space, then \(A\) is \(m\)-closed. Also, \(X - A\) is \(m\)-semi lindelof subset of \(X\), then it is an \(m\)-closed in \(X\), hence, \(A\) is \(m\)-open, which implies \(X\) is discrete and \(X\) is countable, since every \(m\)-semi lindelof is \(m\)-lindelof and as follows the fact \(m\)-lindelof is discrete space if it is countable.

Conversely, let \(G\) is an \(m\)-semi lindelof so if it is an \(m\)-lindelof in \(X\), which is a discrete space, so \(G\) is an \(m\)-closed subset of \(X\). Therefore, \(X\) is an \(m\)-sLc-space.

Lemma (6): Every \(m\)-semi closed subset of \(m\)-semi lindelof space is \(m\)-semi lindelof.

Proof: let \(W\) be an \(m\)-semi closed subset of \(X\), where \(X\) is \(m\)-semi lindelof space and suppose \(\{G_\alpha\}_\alpha\subseteq\) be an \(m\)-semi open cover of \(W\) (means that) \(W = \bigcup_{\alpha\in\Omega} G_\alpha\), so \(X = \bigcup_{\alpha\in\Omega} G_\alpha \cup W^c\), but \(X\) is \(m\)-semi lindelof space, then \(X = \bigcup_{\alpha=1}^\infty G_\alpha \cup W^c\), \(W \subseteq \bigcup_{\alpha=1}^\infty G_\alpha\), therefore \(W\) is \(m\)-semi lindelof.

Proposition (10): Suppose that \(X= X_1 \cup X_2\), where \(X_1\) and \(X_2\) are \(m\)-closed sLc-subspaces in \(X\), then \(X\) is also an \(m\)-sLc-space.

Proof: let \(W\) be \(m\)-semi lindelof of \(X\), then \(W\) is \(m\)-lindelof space, \(W \cap X_1\) and \(W \cap X_2\) are \(m\)-semi closed in \(W\), then it is \(m\)-semi closed in \(W\), which is \(m\)-semi lindelof. Thus, \(W \cap X_1\) and \(W \cap X_2\) are \(m\)-semi lindelof subsets of \(X\) (by lemma(6)). Since \(W \cap X_1\) is subset of \(X_i, i=1,2\) which is \(m\)-sLc-space, thus \(W\) is \(m\)-closed in \(X_i\). Since \(X_i\) is \(m\)-closed in \(X\), then \(W \cap X_i\) is \(m\)-closed in \(X\), but \(W = (W \cap X_1) \cup (W \cap X_2)\), and so \(W\) is \(m\)-closed in \(X\), therefore, \(X\) is \(m\)-sLc-space.

Definition (14): An \(m\)-space \((X, mX)\) is said to be an \(m\)-locally \(L(sc)\)-space if each point has \(m\)-x-open neighborhood, which is an \(m\)-L(sc)-subspace. Clearly, every \(m\)-(lc)-space is \(m\)-locally \(L(sc)\)-space, but the converse may be not true.

Example (7): let \((R, \tau_D)\) be a discrete \(m\)-space, then \(R\) be an \(m\)-locally \(L(sc)\)-space, where it is an \(m\)-L(sc)-space.

Lemma (7): let \(A \subseteq Y \subseteq X\), and \(A\) is \(m\)-semi open set in \(Y\). A set \(A\) be an \(m\)-semi open set in \(X\) if \(Y\) is an \(m\)-open subspace of \(X\).

Proof: Since \(A\) is \(m\)-semi open in \(Y\), then there exists an \(m\)-open set \(G\) of \(Y\) such that \(G \subseteq A \subseteq mX - cl(G)_{in,Y}\), but \(mX - cl(G)_{in,Y} = mX - cl(G)_{in,X} \cap Y\), then \(G \subseteq A \subseteq mX - cl(G)_{in,Y} \subseteq mX - cl(G)_{in,X}\). Then, \(A\) is \(m\)-semi open in \(X\).

Theorem (4): An \(m\)-space \(X\) is an \(m\)-L(sc)-space if and only if each point has \(m\)-cl open neighborhood that is \(m\)-L(Sc)-space.

Proof: If \(X\) is an \(m\)-L(sc)-space, then for each \(x \in X\), \(X\) itself is \(m\)-cl open neighborhood, which is an \(m\)-L(sc)-space. Conversely, let \(L \subseteq X\) be an \(m\)-lindelof subset \(X\) and let \(x \notin L\), choose an \(m\)-cl open neighborhood \(W_x\) of \(x\) in \(X\) such that \(W_x\) is an \(m\)-L(sc)-subspace \(W_x \cap L\) is \(m\)-lindelof in the sub space \(W_x\), which is an \(m\)-L(sc)-space. Therefore, it is \(m\)-semi closed in \(W_x\) but \(W_x - (W_x \cap L) = W_x - L\) is \(m\)-semi open in \(W_x\) which is \(m\)-open in \(X\). Thus, \(W_x - L\) is \(m\)-semi open in \(X\) (by lemma (7)), that is \(L\) is \(m\)-semi closed in \(X\), then \(X\) is an \(m\)-L(sc)-space.

Proposition (11): Every \(m\)-locally \(L(sc)\)-space is a \(m\)-sT1-space with property (B).

Proof: If \(X\) is not \(sT_1\)-space (means that) there exist \(x, y \in X, x \neq y\) such that for all \(m\)-semi open set contains \(y\), also contains \(x\). Since \(X\) is \(m\)-locally \(L(sc)\)-space, let \(U\) be an \(m\)-open neighborhood of \(x\) such that \((U, \tau_U)\) is \(m\)-L(sc)-space, so \((U, \tau_U)\) is \(m\)-sT1-space, thus \(\{x\}\) is \(m\)-semi open in \(U\), then \(U - \{x\}\) is \(m\)-semi open in \(U\) and since \(U\) is \(m\)-open in \(X\), then \(U - \{x\}\) is \(m\)-open in \(X\) (by lemma (7)), but \(y \in U - \{x\}\) and \(U - \{x\}\) does not contain \(x\), but this is a contradiction, then \(X\) is \(m\)-sT1-space.
Proposition (12): The property of a m-space being m-locally L(sc)-space is H-property.

Proof: Let \((X, m_X)\) be an m-locally L(sc)-space, and let \(A \subseteq X\). By assumption for \(x \in A\), there exists \(U \in m_X\) such that \(U\) is an \(m\)-L(sc)-subspace of \(X\) not that \(V = U \cap A\) is an \(m\)-open neighborhood of \(x\) in \(A\) and \(V\) is an \(m\)-L(sc)-subspace of \(A\) (since \(m\)-L(sc)-space is hereditary property), then \((A, m_{X_A})\) is an m-locally L(sc)-space.

Proposition (13): The property of a m-space being locally L(sc)-space is a topological property.

Proof: Let \(f : (X, m_X) \rightarrow (Y, m_Y)\) be an m-homeomorphism function and let \((X, m_X)\) be an m-locally L(sc)-space. Let \(y \in Y\), choose \(x \in X\) such that \(f(x) = y\). Since \((X, m_X)\) is m-locally L(sc)-space, then there exists \(U \in m_X\) such that \(x \in U\) and \(U\) is an \(m\)-L(sc)-space. Since \(f\) is \(m\)-X-open, then \(f(U)\) is an \(m\)-X-open neighborhood of \(y\) in \((Y, m_Y)\), and since \(f\) is m-homeomorphism function, then \(f(U)\) is an \(m\)-L(sc)-subspace of \((Y, m_Y)\), therefore, \((Y, m_Y)\) is an m-locally L(sc)-space ●

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