A new deterministic model of strange stars

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Abstract: The observed evidence for the existence of strange stars and the concomitant observed masses and radii are used to derive an interpolation formula for the mass as a function of the radial coordinate. The resulting general mass function becomes an effective model for a strange star. The analysis is based on the MIT bag model and yields the energy density, as well as the radial and transverse pressures. Using the interpolation function for the mass, it is shown that a mass-radius relation due to Buchdahl is satisfied in our model. We find the surface redshift ($Z$) corresponding to the compactness of the stars. Finally, from our results, we predict some characteristics of a strange star of radius 9.9 km.

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I. INTRODUCTION

The constitution of the interior of neutron stars is still considered an open question by the scientific community. Not only are neutron stars highly dense compact astrophysical objects, the extreme conditions near the center have led to the hypothesis that the core consists entirely of quark matter, in which the constituent quarks comprising the neutrons become deconfined. Moreover, based on the MIT bag model, it was first argued explicitly by Witten [1] that the most stable state of matter is strange quark matter, which is a mixture containing roughly the same number of up, down, and strange quarks.

Quark matter is of interest for a number of reasons. For example, that compact stars could result in a topology change, that is, in the formation of wormholes, had already been suggested in Ref. [2]. It is shown in Ref. [3] that the topology change inside a neutron star requires a quark-matter core of a certain minimal radius. The survival of quark nuggets from the Early Universe is also a possibility. Using the MIT bag model, it is shown in Ref. [4] that quark matter may be a suitable candidate for dark matter. In fact, a strange star may be regarded as a huge strangelet.

In QCD, the interaction between quarks becomes weak for a large exchange of momentum. So, for a sufficiently large temperature or density, or both, interaction between constituent quarks becomes very weak and, consequently, they become deconfined. In heavy ion collider experiments deconfinement of quarks may be brought about at high temperatures (∼ 180 MeV or above). But neutron stars are cold (∼ few KeV). Extremely large chemical potential at the core of the neutron star plays the central role for deconfinement of the quarks. At densities of nearly twice nuclear density, hyperons appear in neutron star matter and this state is known as the hadronic phase (HP), whereas a deconfined quark matter phase is obtained as a phase transition from the hadronic phase at densities much higher than the nuclear density.

If the conversion of neutron-star matter to quark matter is not confined to the core, then the result is a quark star. Under certain conditions, it is theoretically possible for some up and down quarks to be transformed into strange quarks. Since the strange matter is the true ground state of matter, nothing can stop the conversion of the entire quark star into strange matter once the core gets converted into strange matter. Thus a neutron star gets converted into a strange star. In this paper, we propose a new deterministic model for strange stars based on the MIT bag model.

We organize our paper as follows:

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TABLE I: The values of the mass and radius for various strange stars [3, 8]

| Strange Stars | Radius (in km) | Mass \((M_\odot)\) | The mass in km \((1 M_\odot = 1.475 \text{ km})\) |
|---------------|----------------|---------------------|-------------------------------------------------|
| PSR J1614-2230 | 10.3           | 1.97                | 2.9057                                          |
| Vela X - 12   | 9.99           | 1.77                | 2.6107                                          |
| PSR J1903+327 | 9.82           | 1.667               | 2.4588                                          |
| Cen X - 3     | 9.51           | 1.49                | 2.1977                                          |
| SMC X - 1     | 9.13           | 1.29                | 1.9027                                          |

In Sec II, we have provided the basic equations. In Sec. III, we have obtained the solutions of physical parameters. In section IV, we have studied mass-radius relation & surface redshift of the stranger stars. The article is concluded with a short discussion.

II. BASIC EQUATIONS

To describe the spacetime of the interior of the strange star, we assume the metric to be

\[
ds^2 = -e^{\nu(r)}dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \tag{1}
\]

and then recall that the most general energy momentum tensor compatible with spherically symmetry is

\[
T^\mu_\nu = (\rho + p_r)u^\mu u_\nu - p_r g^\mu_\nu + (p_t - p_r)\eta^\mu_\eta_\nu \tag{2}
\]

with

\[
u^\mu u_\mu = -\eta^\mu_\eta_\mu = 1.
\]

The Einstein field equations are listed next.

\[
e^{-\lambda} \left[ \frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} = 8\pi \rho, \tag{3}
\]

\[
e^{-\lambda} \left[ \frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = 8\pi p_r, \tag{4}
\]

\[
\frac{1}{2} e^{-\lambda} \left[ \frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{1}{r}(\nu' - \lambda') \right] = 8\pi p_t. \tag{5}
\]

Our analysis begins with a set of astrophysical objects considered to be candidates for strange stars. The masses and radii of these compact objects are listed in Table 1. The interpolation technique has been used to estimate the cubic polynomial that yield the following expression for the mass as a function of the radial coordinate \(r\):

\[
m(r) = ar^3 - br^2 + cr - d, \tag{6}
\]

where \(a = 0.01492\), \(b = 0.3296\), \(c = 3.03\), and \(d = 9.6453\). The graph of \(m(r)\) is shown in Fig. 1. Observe that the best fit is obtained in the range around 7 km to 12 km. We will see later that the accepted range is \(r = 6.2\) km to \(r = 12.2\) km.

Further analysis is going to be based on the MIT bag model. In this model, the strange matter is assumed to have the following equation of state [7]:

\[
p_r = \frac{1}{3}(\rho - 4B), \tag{7}
\]

where \(B\), the bag constant, is in units of MeV/(fm)\(^3\). To obtain a value suitable for our analysis, it is important to note that for the strange-star candidates, \(B\) has been found to lie in the range 60-80 MeV/(fm)\(^3\) for a \(\beta\) - equilibrium stable strange-matter configuration [8, 9]. A convenient value for plotting purposes is \(B = 0.0001\). Since this choice corresponds to 83 MeV/(fm)\(^3\), it is close to the accepted values. Indeed,

\[
83 \frac{\text{MeV}}{\text{(fm)}^3} \times (10 \text{ m})^6 = 0.0001(\text{km})^{-2}.
\]

For other possible values of \(B\), see Ref. [10].
FIG. 1: Interpolation curve from the observed values of mass and radius of the strange-star candidates.

III. SOLUTIONS

From the metric potential $e^\lambda$ in Eq. (3), we get

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} = 1 - 2ar^2 + 2br - 2c + \frac{2d}{r}.$$  (8)

Next, making use of Eq. (7), we obtain from the Eqs. (3), (4), and (5),

$$\rho = \frac{1}{8\pi} \left( 6a - \frac{4b}{r} + \frac{2c}{r^2} \right),$$  (9)

$$p_r = \frac{1}{24\pi} \left( 6a - \frac{4b}{r} + \frac{2c}{r^2} \right) - \frac{4B}{3},$$  (10)

and

$$p_t = \frac{1}{16\pi} \left( 1 - 2ar^2 + 2br - 2c + \frac{2d}{r} \right) \left( I_1 - I_2 + I_3 + I_4 - I_5 \right),$$  (11)

where

$$I_1 = \frac{12ar^3 - 10br^2 - 4cr - \frac{8r^3}{2\pi}}{2(3r^2 - 6ar^4 + 6br^3 - 6cr^2 + 6dr)},$$

$$I_2 = \frac{(4ar - 2b + \frac{2d}{r})(12ar^3 - 10br^2 - 4cr - \frac{8r^3}{2\pi} - 6d)}{2(1 - 2ar^2 + 2br - 2c + \frac{2d}{r})(3r^2 - 6ar^4 + 6br^3 - 6cr^2 + 6dr)},$$

Cubic polynomial

$$y = 0.014929x^3 - 0.32964x^2 + 3.03x - 9.6453$$

Experimental/Observed data

Spline interpolation

Cubic polynomial fitting
FIG. 2: (Left) The plot of $\rho$ as a function of $r$ (in km) (Middle) Plot of $p_r$ vs. $r$ (in km) with $B = 10^{-4}$. (Right) Plot of $p_t$ vs. $r$ (in km) with $B = 10^{-4}$.

TABLE II: Verification of Buchdahl’s condition for various radii

| Radius (km) | $\frac{2M}{R}$ | Comments |
|------------|----------------|----------|
| 6.1        | -0.013         | negative |
| 6.2        | 0.0086         | $\leq 0.89$ |
| 7          | 0.15196        | $\leq 0.89$ |
| 8          | 0.28483        | $\leq 0.89$ |
| 9          | 0.4008         | $\leq 0.89$ |
| 10         | 0.5229         | $\leq 0.89$ |
| 11         | 0.6657         | $\leq 0.89$ |
| 12.2       | 0.8779         | $\leq 0.89$ |
| 12.3       | 0.8999         | $\geq 0.89$ |

\[ I_3 = \frac{12ar^3 - 10br^2 - 4cr - \frac{3Br^3}{2\pi} - 6d}{3r^2 - 6ar^4 + 6br^3 - 6cr^2 + 6dr} \times \frac{4ar - 2b + \frac{4d}{r}}{1 - 2ar^2 + 2br - 2cr + \frac{6d}{r^2}}, \]

\[ I_4 = \frac{36ar^2 - 20br - 4c - \frac{3Br^2}{2\pi}}{3r^2 - 6ar^4 + 6br^3 - 6cr^2 + 6dr}, \]

\[ I_5 = \frac{(12ar^3 - 10br^2 - 4cr - \frac{3Br^3}{2\pi} - 6d)(6r - 24ar^3 + 18br^2 - 12cr + 6d)}{(3r^2 - 6ar^4 + 6br^3 - 6cr^2 + 6dr)^2}. \]

The plots of Eqs. (9) - (11) are shown in Figs. 2.

IV. MASS-RADIUS RELATION AND SURFACE REDSHIFT

Our final task is to study the maximum allowed mass-to-radius ratio in our model. Buchdahl [11] showed that for a perfect-fluid sphere, twice the ratio of the maximum allowed mass to the radius is 8/9, i.e., $2M/R \leq 8/9$. Moreover, if the trace of the energy-momentum tensor is postulated to be nonnegative, then the ratio of the total mass to the coordinate radius is $\leq 5/18$. In other words, $M/R$ is strictly less than $4/9$. Now, while we would expect the quantity $1 - 2M/R$ to be nonnegative, Buchdahl’s condition actually does not allow the value to be less than $1/9$.

To see how these conditions apply to our model, we return to Eq. (6) and recall that the best fit occurs between the radii 6.2 km and 12.2 km. In this range, the condition $1 - 2M/R > 1/9$ is met, as required by Buchdahl’s condition. As can be seen from the Table II that for different radii this condition is obeyed in our model of strange stars.
TABLE III: Redshift for various radii

| Radius (km) | Z       |
|------------|---------|
| 7          | 0.0859  |
| 8          | 0.1824  |
| 9          | 0.29189 |
| 10         | 0.4478  |
| 11         | 0.7296  |
| 11.5       | 0.9924  |

FIG. 3: (Left) The variation of \( m(r) \) with respect to \( r \) (in km). (Right) The variation of the redshift function with respect to \( r \) (in km).

The compactness of the star is obtained as

\[
u = \frac{m(r)}{r} = ar^2 - br + c - \frac{d}{r}.
\]  

(12)

The nature of the compactness of the star is shown in Fig. 3(left). The surface redshift (\( Z \)) corresponding to the above compactness (\( u \)) is given by

\[
1 + Z = [1 - (2u)]^{-\frac{1}{2}},
\]

(13)

where

\[
Z = \frac{1}{\sqrt{2ar^2 - 2br + 2c - \frac{2d}{r}}} - 1
\]

(14)

The redshift \( Z \) can be measured from the X-ray spectrum. This \( Z \) actually gives the compactness of the star. High observed redshifts (0.35-0.45) are consistent with strange stars which have mass-radius ratios higher than neutron stars. Thus, it is easy to find the maximum surface redshift for the anisotropic strange stars of different radii from equation (14). We calculate the maximum surface redshift for different strange stars with different radii, which is shown in Table III. The nature of the surface redshift of the star is shown in Fig. 3 (right).

V. CONCLUSION

The observed evidence for the existence of strange stars has also led to observed masses and radii. These observations are used in this paper to obtain an interpolation function \( m(r) \), which proved to be an effective model for strange stars. The subsequent analysis is based on the MIT bag model and yields both the radial pressure \( p_r \) and the transverse pressure \( p_t \), as well as the energy density of the strange star. Subsequently, \( m(r) \) was used to show that a mass-radius relation due to Buchdahl is satisfied in our model of a strange star. From our results we can predict some characteristics of a strange star of radius, say 9.9 km. One can see that we have set \( c = G = 1 \) in our calculations. Now, if one substitutes \( G \) and \( c \) into relevant equations, the value of the surface density of the predicted strange star...
of radius 9.9 km turns out to be \( \rho_s = 0.23 \times 10^{15} \text{ gm cm}^{-3} \). Note that our model cannot predict the central density since we cannot get the values of the physical parameters less than 6.2 km radius. Also, we can predict the mass of strange star of radius 9.9 km as \( 1.71M_\odot \). The compactness of the star will be 0.2548 and corresponding redshift is \( Z = 0.4285 \). This high redshift is convenient for explaining strange stars. We can definitely state that our predicted strange star is more compact than neutron stars. In 2008, Cackett et al. \[12\] reported that redshift of a strange star in the low-mass X-ray binary 4U 1820-30 is \( Z = 0.43 \). This supports our prediction on strange stars in the low-mass X-ray binary 4U 1820-30. Recently, X-ray binaries XTE J1739-285 were suggested as strange stars \[13\]. We hope our method can be used to determine different characteristics of these strange stars.

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