Kaon photo-production on the nucleon and deuteron

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Abstract

Isobaric models for the photo-production of $K^+$ are discussed and their predictions are shown in the $K^0$ photo-production. The models are further used in spectator model calculations of the $K^0$ photo-production on deuteron. Considerable dependence of the inclusive cross section on the elementary amplitude was found.

1 Introduction

The photo-production of kaons on nucleons has been studied intensively last years [1-10]. Analysis of the process contributes to our understanding of dynamics in the strange sector. Special attention is payed to investigation of a resonance content of the amplitudes, particularly searching for “missing resonances”, the structure of hadrons (form factors), and to better fixing the effective couplings. This is possible especially after copious and good quality data were collected in JLab (CLAS) [11], ELSA (SAPHIR) [12], and SPring-8 (LEPS) [13]. The amplitude of the process is also an input information in calculations of excited spectra in the hypernucleus photo-production [14]. A good quality description of the elementary process can then minimise a theoretical uncertainty of the hypernuclear results.

There are several approaches to treat the elementary process. Among them the isobaric models based on the effective description utilizing only the hadronic degrees of freedom are suitable for their further use in more complex calculations. Other approaches are eligible either for higher energies ($E_\gamma > 4$ GeV), the Regge model [15], or to the threshold region, the Chiral Perturbation Theory [16]. Quark models [17] are too complicated for their further use in the hypernuclear calculations.

While there are many models which provide a satisfactory description of data in the $p(\gamma,K^+)\Lambda$ reaction, almost nothing is known about the $K^0$ photo-production, the only few data being in the $p(\gamma,K^0)\Sigma^+$ channel [18]. However, the first measurements of the $K^0$ photo-production from carbon and deuteron in the threshold region were performed at Tohoku University [19]. Utilizing these data the simple well known deuteron structure allows one to obtain information on the elementary process, $n(\gamma,K^0)\Lambda$, which is difficult to obtain otherwise.

2 Photo-Production on the Nucleon

In the effective hadronic Lagrangian approach various channels connected via the final state interaction have to be treated simultaneously to take unitarity properly into account [6, 7]. In Ref. [7] the coupled-channel approach has been used to include effects of $\pi N$ intermediate states in the $p(\gamma,K^+)\Lambda$ process. However, tremendous simplifications originate in neglecting the

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rescattering effects in the formalism assuming that they are included to some extent through effective values of the strong coupling constants fitted to data. This simplifying assumption was adopted in many of the isobaric models, e.g. Adelseck-Saghaï (AS1) [1], William-Ji-Cotanch (WJC) [2], Saclay-Lyon (SLA) [3], Kaon-MAID (K-MAID) [4], and Janssen et al. [5].

In these isobaric models the amplitude obtains contributions from the Born terms and exchanges of resonances. Due to absence of a dominant resonance in the process large number of possible combinations of the resonances with mass smaller than 2 GeV must be taken into consideration [1, 3]. This number of models is limited assuming constraints set by SU(3) symmetry [1, 3], crossing symmetry [2, 3], and duality hypothesis [2]. Adopting the SU(3) constraints to the two main coupling constants, however, makes the contribution of the Born terms nonphysically large [5]. To reduce this contribution either hyperon resonances [3] or hadronic form factors [4] must be added, or a combination of both [5]. The hadronic form factors which mimic a structure in the strong vertex are included in the K-MAID and Janssen models maintaining the gauge invariance of the amplitude [10]. In the analysis we use also our models M2 and H2 presented in Ref. [9]. In these models the SU(3) symmetry is assumed and the hadronic form factors were taken into account by the recipe of Ref. [10]. The models differ in the resonance content [9]. Free parameters were fitted to the ample set of CLAS data [11].

The strong coupling constants in the \(K^0\Lambda\) and \(K^+\Lambda\) channels are related via the SU(2) isospin symmetry: \(g_{K^+\Lambda p} = g_{K^0\Lambda n}\) and \(g_{K^+\Sigma^0 p} = -g_{K^0\Sigma^0 n}\). In the electromagnetic vertexes a ratio of the neutral to charged coupling constants have to be known. For the nucleon resonances it can be related to the known helicity amplitudes of the nucleons [25] whereas in the \(t\) channel it relates to the decay widths, which were measured only for the \(K^1\) meson [25]: \(r_{KK^1} = -\sqrt{\Gamma_{K^1\rightarrow K^0\gamma}/\Gamma_{K^+\rightarrow K^+\gamma}} = -1.53\) where the sign was set from the quark model prediction. Since the decay widths of the \(K_1\) meson are not known the appropriate ratio, \(r_{KK_1}\), have to be fixed in the models. It was fitted to the \(K^0\Sigma^+\) data in K-MAID [4], \(r_{KK_1} = -0.45\), but in the other models it is a free parameter. The \(u\) channel contributions remain unchanged in the \(K^0\) photo-production.

3 Photo-Production on the Deuteron

Our aim is to demonstrate a dependence of the inclusive cross section in \(d(\gamma, K^0)\Lambda p\) process on the input elementary amplitudes. We show it in a simple model based on the impulse approximation in which the proton acts as a spectator. Since a part of the \(\Lambda\Lambda\) interaction in the final state (FSI) is absorbed in the coupling constants of the elementary amplitude and the KN interaction is weak on the hadronic scale the main lack of precision comes from ignoring the \(\Lambda\Lambda\) FSI. However, it has been shown in Ref. [22] that the \(\Lambda\Lambda\) interaction is important for the exclusive process \(d(\gamma, K^+\Lambda)n\) near the threshold but is not so significant for the inclusive \(d(\gamma, K^+)\Lambda n\) one. Assuming that the nature of the FSI in the \(K^+\) and \(K^0\) production is not too much different we suppose our simple model as a good approach here.

In the spectator approximation we can write the cross section as

\[
d^0\sigma = \frac{m_{\Lambda} m_N'}{64\pi P_{\gamma} \cdot P_d E_{\Lambda} E_{N'}} \int d^4 P_N \frac{\delta^4(P_f^e - P_e^e) (s - m^2_N)^2}{m_{\Lambda} m_N} \times \\
\times \frac{d\sigma^e}{dt} \delta^4(P_d - P_N - P_{N'}) \left( \frac{1}{2} \sum |M_{fi}|^2 \right) \frac{1}{2} \sum |M_{ef}|^2 d^3 p_K d^3 p_\Lambda d^3 p_{N'} ,
\]

where we have added the integration over the four-momentum of the target nucleon \(P_N\), splitting the \(\delta\)-function into two parts, and have introduced the invariant cross section for the amplitudes.
two-body process

\[ \frac{d\sigma^e}{dt}(s, t) = \frac{1}{4\pi} \frac{m_N m_\Lambda}{(s - m_K^2)^2} \frac{1}{4} \sum |M_{fi}|^2, \]  

(2)

with \( P_i^e = P_\gamma + P_N, P_f^e = P_K + P_\Lambda, s = (P_\gamma + P_N)^2, \) and \( t = (P_\gamma - P_K)^2. \) The spin averaged matrix element \( \frac{1}{6} \sum |M_{fi}|^2 \) in (1) can be expressed via the elementary one in the deuteron laboratory frame

\[ \frac{1}{6} \sum |M_{fi}|^2 = (2\pi)^3 \frac{2m_d E_{N'}}{E_N} \frac{1}{4} \sum |M_{fi}^e|^2 u_d(p_{N'})^2, \]  

(3)

where we have followed the formalism of Ref. [23]. We have assumed the isospin formalism and have added the isospin factor \( \sqrt{2} \) to the elementary amplitude to take account of the antisymmetrization of two nucleons in the intermediate state. In Eq. (3) the energy of the target nucleon is given by \( E_N = E_d - E_{N'} = E_K + E_\Lambda - E_\gamma, \) for the off-shell approximation and by \( E_N = \sqrt{m_N^2 + \vec{p}_N^2} \) for the on-shell one. In the on-shell approximation the energy conservation law is obviously violated in the two-body vertex, \( E_\gamma + E_N > E_K + E_\Lambda, \) as well as in the deuteron one, \( E_N + E_{N'} > E_d. \)

The final expression for the inclusive cross section in the lab frame is

\[ \frac{d^3\sigma}{d|\vec{p}_K|d\Omega_K} = \int \frac{m_N(s - m_N^2)^2 \vec{p}_K^2}{4m_N E_\gamma E_K E_N |\vec{p}_\gamma - \vec{p}_K|} \frac{u_d(p_{N'})^2}{\pi} \frac{d\sigma^e}{dt} d|\vec{p}_{N'}|d\Phi_{N'}, \]  

(4)

where all energies and momenta are in the lab frame, the target nucleon mass is taken to be consistent with kinematics, \( m_N = \sqrt{P_N^2}, \) \( \vec{p}_N = -\vec{p}_{N'}. \) In our calculations we used the non relativistic Bonn wave functions[24].

4 Results and Discussion

Figure 1: Calculated differential cross section in the photo-production of \( K^+ \) on proton are compared with data from Refs. [11](circle), [12](square), [20](triangle), and [21](dot).

The isobaric models being fitted to the data on the \( K^+ \) photo-production on proton are in satisfactory agreement with the data, except for the forward kaon angles, as it is demonstrated in Fig. 1 for the SLA, K-MAID, M2, and H2 models. The SLA and K-MAID models were
fitted to the older data (triangles ans dots) whereas the M2 and H2 ones were adjusted only to the latest CLAS data [11]. The obvious and most serious, in the view of the hypernuclear calculations, discrepancy of the results is at $\theta_{K}^{\text{lab}} < 40$ deg. In this region, however, one observes an inconsistency of the experimental data too, further stressed by the new CLAS and SAPHIR data. The systematic discrepancy of results for a very small kaon angle is shown in Fig. 2. Models with hadronic form factors, K-MAID, M2, and H2, provide much smaller cross sections at $E_{\gamma}^{\text{lab}} > 1.5$ GeV than the others.

![Figure 2](image2.png)

Figure 2: Calculated differential cross sections for the forward angle are shown as a function of energy in the laboratory frame.

![Figure 3](image3.png)

Figure 3: Predictions of the models for the differential cross section in the photo-production of $K^0$ on neutron are shown for the photon energy of 1.1 GeV.

In Figure 3 predictions of the models for the $K^0$ photo-production on neutron are shown. As mentioned already in Sect. 2, the only free parameter in this channel is the ratio $r_{KK1}$. We have used the value of -0.45, as it was fixed in the K-MAID [4], in the models M2 and H2 too. In the case of the SLA model we have found two values of the ratio, -1.6 (SLA1) and -3.4 (SLA2), which provide results very near to those of the K-MAID at forward angles, see Fig. 3. The models reveal more different results than in the $K^+$ production. The bump structure is produced by the $N^*(1720)$ exchange.

In Figure 4 we demonstrate that the K-MAID and SLA are much more sensitive to a
Figure 4: Contribution of the $K_1$ exchange to the differential cross section in the photo-production of $K^0$ on neutron is shown as a function of energy at kaon angle of 10 deg.

The contribution of the $K_1$ exchange than the M2 and H2 models. This phenomenon is valid for energies up to 1.5 GeV and kaon lab angles up to 50 deg. That makes a choice of values for $r_{K K 1}$ less important in the M2 and H2 than in the SLA and K-MAID models.

A sensitivity of the cross section $S_2$ to the $r_{K K 1}$ parameter is shown again for the $d(\gamma, K^0) \Lambda p$ reaction in Fig. 5. The figure shows that the value of $r_{K K 1}$ is not too important for the M2 and H2, so that they provide really predictions in the $K^0$ channel at the kinematical region assumed here. On the contrary, results of the SLA model vary very strongly with values of $r_{K K 1}$ (notice the scale of the appropriate figure). Predictions of the models for the inclusive cross section $S_2$ are shown for the $K^0$ photo-production on the deuteron for small kaon lab angles and photon energies in Fig. 6. The results differ significantly in some cases which enable the data from Tohoku experiment [19] to discriminate between the elementary models.

To summarise, we showed that the isobaric models still provide different predictions for the cross section of the $K^+$ photo-production at forward angles which then causes large input uncertainty of the hypernuclear calculations. Predictions for the inclusive cross sections of the simple model for the $d(\gamma, K^0) \Lambda p$ reaction display considerable sensitivity to elementary amplitudes and therefore we expect that the experimental data will allow to discriminate between various elementary amplitudes which otherwise fit the $p(\gamma, K^+ ) \Lambda$ data equally well.

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Figure 5: Double differential cross sections as a function of the kaon lab momentum for the photo-production of $K^0$ on deuteron are plotted in dependence of the ratio of electromagnetic couplings for $K_1$, $r_{KK1}$, for the models M2, H2, K-MAID, and SLA. The fat lines indicate results for the parameters (see also the text) used in Fig 6. Calculations were performed at the photon lab energy of 1 GeV and kaon lab angle of 10 deg.

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Figure 6: Calculated double differential cross sections in the photo-production of $K^0$ on deuteron are shown for two energies and angles.

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