Heliocentric radial velocity (km s\(^{-1}\))
| Date (UT) | MJD     | Velocity\(^{\text{km s}^{-1}}\) | Phase \((O-C)\) km \text{s}^{-1} | \(v \sin i\) km \text{s}^{-1} |
|----------|---------|---------------------------------|---------------------------------|--------------------------|
| 2003 Mar. 10.279* | 52708.279 | +44.4 | -6.887 | -2.3 |
| Apr. 30.159* | 759.159 | +44.7 | 0.915 | +0.8 |
| 2004 Sept. 26.505* | 53274.505 | +34.7 | 0.974 | +0.3 |
| 2005 Nov. 14.178 | 53688.178 | +31.2 | 0.986 | -0.7 |
| 19.176 | 693.176 | +17.5 | 1.578 | +1.6 |
| 25.182 | 699.182 | -18.4 | 2.290 | -0.9 |
| 30.183 | 704.183 | +45.6 | 0.882 | -1.5 |
| Dec. 15.131 | 719.131 | +26.9 | 4.653 | -2.4 |
| 17.172 | 721.172 | +47.9 | 0.895 | +1.9 |
| 18.119 | 722.119 | +29.7 | 5.007 | +2.5 |
| 2006 Jan. 26.016 | 53761.016 | +22.4 | 9.616 | -0.3 |
| Feb. 15.991 | 781.991 | +3.7 | 12.101 | -1.5 |
| 16.905 | 782.905 | -12.6 | 0.209 | +0.6 |
| 17.998 | 783.998 | -15.9 | 0.339 | +0.4 |
| 18.997 | 784.997 | -7.7 | 0.422 | -1.2 |
| 20.887 | 786.887 | +35.4 | 0.681 | +1.4 |
| Mar. 1.920 | 795.920 | +41.1 | 13.751 | -2.5 |
| 4.927 | 798.927 | +3.2 | 14.108 | -0.5 |
| 6.006 | 800.006 | -18.6 | 0.236 | -3.1 |
| 6.909 | 800.909 | -15.6 | 0.343 | +0.5 |
| 24.918 | 818.918 | -4.5 | 16.476 | -3.0 |
| Apr. 3.994 | 828.994 | +33.5 | 17.670 | +1.4 |
| 4.938 | 829.938 | +44.0 | 0.782 | -2.4 |
| 6.057 | 831.057 | +41.5 | 0.915 | -2.5 |
| 8.962 | 833.962 | -16.3 | 18.259 | +0.5 |
| 10.915 | 835.915 | +3.5 | 0.490 | +2.9 |
| 12.022 | 837.022 | +24.4 | 0.621 | -2.3 |
| 14.872 | 839.872 | +37.9 | 0.959 | +0.7 |
| 25.915 | 850.915 | -17.6 | 20.267 | -0.5 |
| 27.898 | 852.898 | +1.1 | 0.502 | -1.5 |
| 29.854 | 854.854 | +42.7 | 0.734 | +1.2 |
| May 1.947 | 856.947 | +31.5 | 0.982 | -1.2 |
| 2.974 | 857.974 | +4.7 | 21.104 | +0.1 |
| 5.970 | 860.970 | -6.1 | 0.459 | -2.0 |
| 9.866 | 864.866 | +42.8 | 0.920 | -0.5 |
| 10.892 | 865.892 | +19.4 | 22.042 | +0.3 |
| 11.901 | 866.901 | -5.0 | 0.161 | +1.6 |
| 15.928 | 870.928 | +27.5 | 0.638 | +0.8 |
| 16.868 | 871.868 | +43.6 | 0.750 | +0.2 |
| 18.876 | 873.876 | +31.3 | 0.988 | -0.2 |
| 21.885 | 876.885 | -14.0 | 23.344 | +2.0 |
| 23.886 | 878.886 | +19.3 | 0.581 | +2.9 |
| 29.893 | 884.893 | -16.1 | 24.293 | +1.4 |
| 30.896 | 885.896 | -8.9 | 0.412 | +1.3 |
| 31.896 | 886.896 | +6.8 | 0.530 | -0.6 |
| June 2.893 | 888.893 | +47.6 | 0.767 | +2.5 |
| 3.893 | 889.893 | +50.8 | 0.885 | +4.0 |

*Observation published by Henry et al.\(^9\).

All others observed with the Cambridge Coravel.
### Table III

Radial-velocity observations of HD 80731

| Date (UT) | MJD   | Velocity (km s⁻¹) | Phase | (O - C) (km s⁻¹) | \(v\) sin \(i\) (km s⁻¹) |
|-----------|-------|-------------------|-------|------------------|------------------------|
| 2003 Mar. | 8.307 | 52706.307         | -8.7  | 95.987           | -1.1                   |
|           | 9.283 | 707.283           | +17.2 | 57.079           | -0.8                   |
|           | 10.314| 705.314           | +11.6 | 17.157           | -9.0                   |
| May       | 1.258 | 760.258           | +12.4 | 87.041           | +2.0                   |
| 2004 Apr. | 24.202| 53119.202         | -9.2  | 54.668           | +2.3                   |
|           | 25.192| 120.192           | -18.7 | 7.761            | -0.4                   |
|           | 28.215| 123.215           | +11.0 | 53.044           | -0.1                   |
| Nov.      | 19.213| 53693.213         | +0.7  | 0.443            | -3.4                   |
|           | 25.191| 699.191           | -2.6  | 1.003            | -0.4                   |
|           | 30.193| 704.193           | +0.9  | 0.471            | -1.3                   |
| Dec.      | 15.148| 719.148           | -24.8 | 2.872            | -0.6                   |
|           | 17.177| 721.177           | +15.9 | 3.042            | +0.6                   |
|           | 18.131| 722.131           | +17.7 | 0.152            | -3.6                   |
|           | 26.946| 730.946           | -11.8 | 0.978            | -1.0                   |
|           | 28.022| 732.022           | +19.3 | 4.078            | +1.4                   |
| 2006 Jan. | 26.007| 53761.007         | -20.3 | 6.794            | +0.2                   |
|           | 27.242| 762.242           | -22.5 | 0.909            | +0.9                   |
| Feb.      | 15.997| 781.997           | -16.4 | 8.760            | +1.8                   |
|           | 16.919| 782.919           | -23.9 | 0.847            | -0.4                   |
|           | 18.009| 784.009           | -16.8 | 0.941            | +1.6                   |
|           | 18.876| 784.876           | +5.7  | 9.030            | -1.4                   |
|           | 20.896| 786.896           | +17.4 | 0.219            | -1.2                   |
| Mar.      | 1.035 | 795.035           | -8.4  | 0.982            | +1.1                   |
|           | 1.931 | 795.931           | +15.8 | 10.066           | -0.1                   |
|           | 3.042 | 797.042           | +21.1 | 0.170            | +0.3                   |
|           | 3.865 | 797.865           | +17.1 | 0.247            | +0.1                   |
|           | 4.936 | 798.936           | +13.0 | 0.347            | +2.5                   |
|           | 6.016 | 800.016           | +5.4  | 0.448            | +1.6                   |
|           | 6.917 | 800.917           | +0.5  | 0.533            | +2.5                   |
|           | 22.921| 816.921           | +10.1 | 12.032           | +2.4                   |
| Apr.      | 4.002 | 829.002           | +22.2 | 13.164           | +1.2                   |
|           | 5.053 | 830.053           | +17.6 | 0.262            | +1.5                   |
|           | 6.068 | 831.068           | +9.7  | 0.357            | -0.2                   |
|           | 8.957 | 833.957           | -9.6  | 0.628            | -1.0                   |
|           | 10.933| 835.933           | -23.8 | 0.813            | -2.0                   |
|           | 12.029| 837.029           | -23.5 | 0.916            | -0.5                   |
|           | 14.880| 839.880           | +21.7 | 14.183           | +1.3                   |
|           | 25.925| 850.925           | +19.7 | 15.217           | +1.0                   |
|           | 27.912| 852.912           | +6.5  | 0.404            | -0.3                   |
| May       | 1.961 | 856.961           | -19.9 | 0.783            | -0.1                   |
|           | 2.964 | 857.964           | -22.4 | 0.577            | -1.9                   |
|           | 5.996 | 860.996           | +21.9 | 16.161           | +0.8                   |
|           | 9.901 | 864.901           | -2.9  | 0.527            | -1.3                   |
|           | 10.915| 865.915           | -6.0  | 0.622            | +2.2                   |
|           | 11.915| 866.915           | -13.2 | 0.715            | +1.8                   |
|           | 15.941| 870.941           | +20.1 | 17.093           | +0.6                   |
|           | 16.969| 871.969           | +20.0 | 0.189            | -0.1                   |
|           | 18.915| 873.915           | +7.0  | 0.371            | -1.9                   |
|           | 21.913| 876.913           | -13.2 | 0.652            | -2.9                   |
|           | 26.943| 881.943           | +21.8 | 18.123           | +0.6                   |
|           | 29.921| 884.921           | +8.9  | 0.402            | +2.0                   |
|           | 30.929| 885.929           | +0.3  | 0.497            | -0.2                   |
|           | 31.908| 886.908           | -6.1  | 0.588            | -0.3                   |
| June      | 2.910 | 888.910           | -19.4 | 0.776            | -0.1                   |
|           | 3.917 | 889.917           | -27.1 | 0.870            | -2.9                   |
|           | 5.914 | 891.914           | +10.2 | 19.057           | -4.1                   |
|           | 8.925 | 894.925           | +9.5  | 0.340            | -1.6                   |
|           | 9.923 | 895.923           | +2.9  | 0.433            | -1.9                   |
|           | 10.926| 896.926           | -1.8  | 0.527            | -0.2                   |
|           | 11.917| 897.917           | -7.4  | 0.620            | +0.6                   |
|           | 12.920| 898.920           | -14.3 | 0.714            | +0.5                   |

*Observations published by Harvard et al.
### TABLE V

**Radial-velocity observations of HD 17310**

The sources of the observations are as follows:

- 2002–2004 — published by Henry et al.\(^9\); 2005/2006 — Cambridge Coravel

| Date (UT) | MJD     | Velocity \(\text{km s}^{-1}\) | Phase \((O - C)\) \(\text{km s}^{-1}\) | \(v\sin i\) \(\text{km s}^{-1}\) |
|-----------|---------|-------------------------------|---------------------------------|--------------------------------|
| 2002 Sept. 25.411 | 52542.411 | -3.7                          | 39.104                         | -2.9                           |
| 2003 Sept. 20.408 | 52902.408 | -1.9                          | 26.044                         | -4.5                           |
|          | 21.425  | 903.425                        | +1.1                           | .081                           | +1.4                           |
|          | 22.402  | 904.402                        | -3.7                           | .116                           | -2.9                           |
| Oct. 28.379 | 940.379  | +28.6                         | 25.409                         | +0.2                           |
|          | 29.369  | 941.369                        | +28.6                          | .445                           | -3.4                           |
| 2004 Sept. 25.427 | 53273.427 | +29.7                         | 13.381                         | +4.4                           |
|          | 26.385  | 274.385                        | +23.7                          | .416                           | -5.4                           |
|          | 27.371  | 275.371                        | +35.7                          | .451                           | +3.1                           |
|          | 28.344  | 276.344                        | +31.1                          | .486                           | -4.8                           |
|          | 29.370  | 277.370                        | +40.4                          | .523                           | +1.4                           |
| 2005 Sept. 29.092 | 53642.092 | +45.4                         | 0.633                          | -0.3                           | 9                              |
| Nov. 29.949 | 703.949  | +37.0                         | 2.857                          | -0.2                           | 4.5                            |
| Dec. 8.937 | 712.937  | +2.9                          | 3.180                          | +0.2                           | 11                             |
|          | 10.921  | 714.921                        | +9.5                           | .251                           | -0.6                           | 11                             |
|          | 16.940  | 720.940                        | +32.4                          | .468                           | -1.8                           | 11.5                           |
|          | 17.917  | 721.917                        | +40.4                          | .503                           | +3.0                           | 6.5                            |
|          | 19.858  | 723.858                        | +44.3                          | .572                           | +1.7                           | 9                              |
|          | 26.893  | 730.893                        | +40.1                          | .825                           | -1.1                           | 12.5                           |
| 2006 Jan. 4.795 | 53739.795 | +3.7                          | 4.145                          | +3.4                           | 9.5                            |
|          | 25.836  | 760.836                        | +30.3                          | .902                           | +0.8                           | 8.5                            |
|          | 28.814  | 763.814                        | +9.3                           | 5.009                          | +1.6                           | 13                             |
SPECTROSCOPIC BINARY ORBITS
FROM PHOTOELECTRIC RADIAL VELOCITIES
PAPER 191: HD 17310, HD 70645 AND HD 80731

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The three objects have been identified as members of the recently recognized class of γ Doradûs stars, which exhibit multi-periodic photometric variations that are thought to arise from non-radial pulsation. The particular objects treated here also prove to be spectroscopic binaries, for which we provide reliable orbits. The radial velocities exhibit unusually large residuals, in which some of the photometric periodicities can be traced. Some of the same periodicities are also demonstrated by the observed variations in the line profiles, which are quantified here simply in terms of the line-widths.

Introduction

The characters of a few stars that showed small photometric variations with multiple periodicities of the order of one day — longer than typical δ Scuti variations — gradually became apparent in the late years of the last century. As recently as 1999 Kaye et al.1 defined a new class of variables, having γ Doradûs as the type star, to accommodate such objects, whose photometric instability has been attributed to high-order gravity-mode pulsations, in which the motions are mainly tangential (rather than radial, as in the case of δ Scuti pulsations, which have typical periods of the order of 0.1 day). The periods are in the range 0.3–3 days; the pulsations are likely to affect the line profiles, more particularly in the wings of the lines, but since they are largely non-radial and slower than in the δ Scuti case their effects on stellar radial velocities are likely to be more muted.

In the same year as the new class was recognized, Handler2 presented a list of membership candidates, identified from a comprehensive search of the Hipparcos ‘epoch photometry'; the list contained 70 entries, of which 46 were considered to be ‘prime candidates’. Many, but not all, of those that have been investigated have proved to be spectroscopic binaries. Paper 1873 in this series gave orbits for two of them; in an introductory section it referred to the observational history (salienc parts of which were published in this Magazine), of γ Dor itself, and to the recognition4,5 in 9 Aur, a non-binary member of the class, of a sub-set of the photometric periods in the star’s radial velocities and line-profile variations. One of the present authors was also responsible for the radial-velocity measurements that led to a double-lined orbit6 for HD 221866 and the tentative identification of the secondary component in that system as a γ Dor star.

The observational histories of the three stars that form the subject of this paper, all of which are of HD type F0, run extremely parallel with one another, because (apart from some uvbyHβ photometry, not referenced here) it is only since their appearance in Handler’s list2 of ‘prime γ Dor candidates’ that any interest has been taken in them. Even since then, there have been only three papers — by Martin, Bossi & Zerbi7 in 2003, Mathias et al.8 in 2004, and Henry, Fekel & Henry9 in 2005 — that have shed much light on them; all three stars feature in the last one9, but the other two do not include HD 17310.
The latter star, however, does feature in a few papers that are not concerned with the others. It was among a large number of stars observed for radial velocity by Grenier et al., who found a mean of $-21.4 \pm 3.6$ km s$^{-1}$ from three 80-Å mm$^{-1}$ spectrograms, the uncertainties of the results being such that the plate-to-plate discrepancies did not suggest real variability; they classified the spectrum as F2 IV–V. Then Koen & Eyer identified periods in the Hipparcos photometry; and Nordström et al. gave basic data about the star in their survey of F and G dwarfs, but they did not measure any radial velocities for it. HD 70645 is mentioned in one ‘extra’ paper, in which Topka et al. remarked that it is within a field surveyed by the Einstein X-ray satellite, but no X-rays from it were detected. HD 80731 was observed twice by Moore & Paddock long ago with the Lick 36-inch refractor and a prismatic spectrograph giving 75 Å mm$^{-1}$ at H$_7$; they listed a spectral type of F0 V and a mean radial velocity of +4 km s$^{-1}$ with a ‘probable error’ of 1.7 km s$^{-1}$, so the binary nature of the object was not discovered.

All three stars feature in the Hipparcos survey. For HD 17310, the Hipparcos catalogue preferred to give ground-based values of $V$ and $(B - V)$, $7^m.76$ and $0^m.378$, respectively, to its own measurements, but we have not been able to discover whence it obtained them. The $V$ magnitude was flagged as variable, but no period was found; Handler and Koen & Eyer, however, thought that they saw a period of 2.0296 days in the Hipparcos photometry. The parallax, $0^\prime.00914 \pm 0^\prime.00114$, leads to a distance modulus of $5^m.2 \pm 0^m.3$ and thus to an absolute magnitude of about $+2^m.6$. HD 70645 is attributed a Hipparcos-based mean $V$ magnitude of 8.12, appearing to vary with a period (also noted by Handler) of 0.82488 days, and a ground-based $(B - V)$ of $0^m.344$. The parallax of $0^\prime.00755 \pm 0^\prime.00087$ yields a distance modulus of $5^m.6 \pm 0^m.3$, implying $M_V \sim 2^m.5$. HD 80731 is listed with ground-based magnitudes of $V = 8^m.46$, $(B - V) = 0^m.345$, the former variable in a period of 1.11556 days (again confirmed by Handler), and with a parallax of $0^\prime.00677 \pm 0^\prime.00103$, leading to $m - M = 5^m.85 \pm 0^m.3$ and thus to $M_V \sim 2^m.6$. The three stars are all seen to have just the colour indices and absolute magnitudes that would be expected for early-F dwarfs.

We next give a brief synopsis of what the three post-Handler papers have discovered about the stars. Martin et al. obtained fresh photometric data on eight stars, including HD 70645 and HD 80731, on the five useable nights during a single 14-night observing run on the 90-cm telescope in the Sierra Nevada; the number of observations per star was only about 50. They also re-investigated the Hipparcos photometry with a computer program that they considered to be superior to other people’s. For HD 70645, Martin et al. did not confirm the 0.825-day period that both Hipparcos and Handler thought to exist in the Hipparcos photometry, but instead found periods of 0.792 and 1.297 days; their own data indicated 1.14 and 0.690 days. For HD 80731 they confirmed the Hipparcos/Handler periodicity of 1.1156 days in the Hipparcos photometry, and found an additional one of 0.745 days. Their own measures were considered to indicate possible periods of 7.00, 2.23, and 1.401 days, but it is difficult to believe that periods of such lengths could be reliably identified on the basis of only five nights’ data.

Mathias et al. obtained repeated spectra of a number of γ Dor candidates, including HD 70645 and HD 80731, with the Aurelie spectrograph on the Haute-Provence 1.52-m telescope, mainly to look for line-profile variations that would corroborate the γ Dor natures of the stars concerned. They concentrated attention on just two ionic lines in the blue part of the spectrum. Over a total interval of a little more than a year they obtained 10 spectra of HD 70645 and 11 of HD 80731, finding line-profile variations to be “evident” in both cases and discovering the binary natures of both stars. They derived orbits for them both;
that of HD 70645 is in principle correct, but that of HD 80731 is completely mistaken, having a period of 13.572 ± 0.011 days, whereas we shall show below that the true value is 10.674 days. Unfortunately they did not publish their radial velocities, so we cannot take them into account in our own orbital solutions. We note that we have already found another mistaken period, that of HD 100215; without the data in front of us we cannot see exactly how the errors arose, but in all cases the number of velocities was very small upon which to base orbits. Mean $v \sin i$ values of 11 and 13 km s$^{-1}$ were listed for HD 70645 and HD 80731, respectively.

Henry, Fekel & Henry observed, in the course of a year’s campaign with an automated 0.4-m telescope at the Fairborn Observatory, all three of the stars discussed in the present paper. They obtained much more satisfactory photometric coverage than either Hipparcos (very bad temporal distribution of the data points, but of course the photometry was only a by-product of the principal objective) or Martin et al. (very small data set). For HD 70645 and HD 80731 they had a total of more than 400 measurements in both $B$ and $V$, obtained on about 250 nights in one season, so the derived periodicities ought to be entirely reliable. Martin et al.’s rejection of the Hipparcos- and Handler-derived period of 0.825 days for HD 70645 from the Hipparcos photometry in favour of 0.792 days was corroborated, and the Martin primary period of 1.14 days was confirmed. The Henry et al. periods for HD 70645, in descending order of amplitude, were 1.1032, 0.7929, 0.8593, 1.2405, and 1.1461 days, all of them found in both the $V$ and the $B$ data and all having uncertainties typically of 2 in the fourth decimal place. In the case of HD 80731, Henry et al. confirmed the 1.1156-day period that all of the previous investigations had found in the Hipparcos photometry, but not the periods proposed by Martin et al. from their own data. The Henry et al. periods, in order, were 1.1159, 1.2783, 1.5154, and 0.7623 days.

Henry et al. also obtained spectroscopy in the red ($\lambda \sim 6400$ Å) with the Kitt Peak coudé-feed system. They classified the stars, finding both HD 70645 and HD 80731 to be of type F1 (and they knew them to be main-sequence objects from the Hipparcos parallaxes), and they gave $v \sin i$ values of 11 and 14 km s$^{-1}$, respectively. They measured (and tabulated, so we are able to utilize them) three radial velocities for HD 70645 and seven for HD 80731. They commented that two of their velocities of HD 70645 are consonant with Mathias et al.’s orbit, but that the third showed a residual of about 10 km s$^{-1}$, so the orbit must require some revision; and they saw that their velocities of HD 80731 were not consistent with the Mathias et al. orbit.

We have left till last our reference to the work of Henry et al. on the third of the stars discussed in the present paper, HD 17310, simply because their paper is the only one that deals with it. The paper lists as many as eleven radial velocities, with a range of over 40 km s$^{-1}$, and gives a spectral type of F2 and a projected rotational velocity of 10 km s$^{-1}$. Three photometric periods were established from more than 200 measurements in both $V$ and $B$; they were 2.138, 1.825, and 2.452 days, with uncertainties near 0.001. The 2.0296-day period derived from the Hipparcos photometry by both Handler and Koen & Eyer was not confirmed.

Henry et al. performed a period search on their radial velocities of HD 17310 in an effort to identify the orbital period. The observational ‘window function’ was far from ideal, because the measurements were made during just four observing runs, in which the star was observed on one night and on three, two, and five consecutive nights, respectively. The best period formally was 0.9653 days, but the authors did not trust it. “Instead,” (they said) “we prefer periods in the 20–30 day range, the best of which is 27.793 days.” We are able to commend both their instinct and their conclusion, as we shall show below that that period is correct. Their preferred period is a 1-day$^{-1}$ alias of the short one; expressed as frequencies, they are 0.03598 and 1.03595 day$^{-1}$, respectively.
The paper\textsuperscript{9} by Henry, Fekel & Henry, which is entitled \textit{11 New $\gamma$ Doradus Stars}, was published in mid-2005 and caught the attention of one of the present authors, who found particular interest in a column in Table 1 where the $v\sin i$ values were given for the 11 stars. Three of the values — those assigned to the stars that we are discussing now — were from 10 to 14 km s$^{-1}$, whereas the others ranged from 38 to 150 km s$^{-1}$. Stars that rotate rapidly are difficult or impossible to measure for radial velocity with the Coravel at the Cambridge 36-inch telescope, but when a short investigation of the literature had revealed that the three $\gamma$ Dor stars that rotated slowly lacked reliable orbits those objects were placed on the Coravel observing programme.

HD 17310 is very unfavourably placed, at a declination of nearly $-7^\circ$ in Eridanus, on the border by Cetus, about $3^\circ$ north-preceding the fourth-magnitude star $\eta$ Eri. Strictly speaking it ought not to be observed with the Cambridge telescope, whose coudé beam is increasingly vignetted by the telescope structure below $-5^\circ$ declination, but the observer persuaded himself that the vignetting at $-7^\circ$ was not so great as to be likely to produce errors as bad as those that could be expected from other sources. Observations were necessarily confined to the vicinity of the meridian, so the observing season was short and yielded only 11 measurements.

HD 70645 and HD 80731, in contrast, are at high declinations ($68^\circ$ and $62^\circ$ respectively), quite close together in the north-preceding corner of Ursa Major. Preliminary orbits were established very quickly, and for a time thereafter the radial velocities were measured only where they would serve to fill gaps in the phase distribution. Later, when it became apparent that the residuals from the orbits might yield (or at least exhibit) some of the pulsational periodicities, a measurement was made on each fine night regardless of orbital phasing. The two stars, though easily circumpolar as seen from Cambridge, cannot be observed more than about 6 hours from upper culmination because structures associated with the northward-going coudé focus (and indeed the Coravel instrument itself) occupy the northern part of the dome of the 36-inch reflector. The total numbers of new radial velocities are 44 for HD 70645 and 54 for HD 80731.

Although the data are (unusually, for this series of papers) confined to a single observing season, by reason of their continuity and compact distribution in time they lend themselves tolerably well to the investigation of short periods superimposed upon the orbital variation. The likelihood that pulsational instabilities would be traceable in the radial velocities is indicated by substantial variations in the profiles of the cross-correlation dips from which the velocities are determined. That is illustrated by Fig. 1, which compares the dips given by HD 80731 on different occasions. The S/N ratios achievable for radial-velocity traces of the stars concerned are not usually adequate to delineate with confidence any real asymmetries that may be present, but the overall widths of the line profiles certainly change from one occasion to another. The widths are characterized numerically here as if they were projected rotational velocities, $v\sin i$, but our use of that expression is to be regarded merely as a name for the line-width parameter that is routinely calculated for each radial-velocity trace by the Coravel reduction software as if the broadening of the spectral lines, beyond the minimum width given by other stars, were due simply to rotation of the stars as solid bodies. The values are quantized in $\frac{1}{2}$-km s$^{-1}$ steps, owing to the manner in which they are calculated\textsuperscript{16}.

Straightforward orbital solutions of the radial-velocity data give orbits that are quite satisfactory but are characterized by unusually large residuals, of the order of 1.5 km s$^{-1}$ or so — two if not three times as large as might be expected from the character of the data. We show below that pulsational periods can be traced in the residuals. We can try to model the residuals by sine waves, by regarding the residuals as radial velocities in their own right and solving them with a program that derives circular orbits from such data. If there were only one short period, it would perhaps be appropriate to treat the raw radial-velocity measurements with
the orbit program that solves simultaneously the outer and inner orbits of single-lined triple systems. That, however, manages to improve slightly the fit to the short-period, low-amplitude ‘inner orbit’ by making slight changes to the ‘outer’ — in this case, the only true — orbit. Not only would such changes not be likely to suit more than one periodicity among the pulsational ‘orbits’ but, as the number of data points increases, the scope for adjusting the true orbit to accommodate residuals arising from pulsation decreases until in the limit of an indefinitely large number of data it would vanish altogether. We conclude that the proper procedure is first to derive the actual orbit by a straightforward application of the single-line orbit-solving program, and then to use the resulting set of residuals as the dataset to be investigated for evidence of pulsational periodicities. In the sections below, we discuss the stars out of conventional right-ascension sequence in order to treat the two comparatively well-observed ones first.

It was only as our observations accumulated, and we realized that the velocity residuals were not random but exhibited one or more periods associated with the $\gamma$ Dor pulsations, that concern arose as to whether the quality of the data would be adequate to support an analysis of pulsations whose amplitudes would be very much smaller than those of the orbits that we initially set out to determine. Most of the later observations, therefore, were integrated to more generous levels, usually $>10,000$ counts per bin, than most of the earlier ones, which were nearer 5000. In analyzing the final datasets, we experimented with flagging them (a) by temporal halves, and alternatively (b) by the count levels. There proved not to be significant differences between any of the divisions thus made: the conclusion to be drawn from the exercise seems to be that observational error is either not the principal contributor to the velocity residuals (the analysis of which is therefore valid), or is not significantly reduced by approximately doubling the integrations (which is difficult to believe). It also appears, therefore, that our concern over the data quality was misplaced, and that the extra time spent on many of the later integrations may largely have been wasted!

**HD 70645**

The Cambridge radial-velocity measurements began in 2005 November, soon after the star was first observable on the dawn meridian, and continued until it was beyond reach in the north-west at dusk at the beginning of the following June. Forty-four observations were made of it; they are listed in Table I, together with the three velocities published by Henry et al. and the phases and residuals obtained from a solution performed as for a normal single-lined binary star. All the velocities were given equal weight. Initially the Cambridge measurements were solved alone, and gave the period with a standard error of 0.0022 days. Then the published data were brought in to the solution, producing negligible changes to the elements but (by increasing the time base from 200 to 1200 days) reducing the standard error of the period to 0.0008 days, a worthwhile improvement. The solution is illustrated in Fig. 2 and the orbital elements are set out in the informal table below.

\[
\begin{align*}
  P &= 8.4402 \pm 0.0008 \text{ days} \\
  \gamma &= +14.53 \pm 0.28 \text{ km s}^{-1} \\
  K &= 33.1 \pm 0.4 \text{ km s}^{-1} \\
  e &= 0.077 \pm 0.012 \\
  \omega &= 66 \pm 9 \text{ degrees} \\
  (T)_{10} &= \text{MJD 53764.26} \pm 0.21 \\
  a_1 \sin i &= 3.82 \pm 0.05 \text{ Gm} \\
  f(m) &= 0.0314 \pm 0.0011 \text{ M}_\odot \\
  \text{R.m.s. residual} &= 1.72 \text{ km s}^{-1}
\end{align*}
\]

Unfortunately the potential pulsational periods cannot be determined accurately enough for the cycle count back to the published observations to be secure, so from this point onwards the investigation is limited to
the Cambridge measures. A column has been added to Table I to give the apparent \( v \sin i \) value determined individually from each observation.

We have treated the velocity residuals, with their corresponding observational epochs, as an autonomous dataset, to be examined for pulsational periodicities as explained at the end of the section above. Equally, we regarded the line-widths as constituting a parallel dataset meriting an analogous examination. Rather than choosing whether to test for the presence of periods already proposed by others, or instead to make an independent search for periodicities, we decided to adopt first the one strategy and then the other. It could be argued that the photometric periods found by Henry \textit{et al.} come from such a rich database that their validity in the magnitude data is practically guaranteed, so all we need to do is to test for their presence in the radial velocities and the line-widths; but it would be a pity to overlook other periods, that might be more conspicuous in radial velocities or in line-widths than in brightness, simply by neglect of an unprejudiced search of our own data. At the same time we need to be careful not to fall into the error, of which we have sometimes\textsuperscript{18,19} suspected others, of placing too much reliance on short periods that may be mathematically present in sparse data strings.

Our procedure for assessing the significance of possible periods, whether taken from the literature or found by ourselves, was as follows. We set up the datasets as if for single-lined solutions of circular orbits, with the period to be tested, a nominal amplitude of 1 km \( s^{-1} \), and with the epoch set at that of the largest positive (residual) velocity or the largest line-width. If the solution then ran and converged, that result was taken as qualitative evidence in favour of the existence of the relevant period in the data. If it did not, we ran a ‘plot-only solution’, in which we did not ask the computer to improve the elements that we had supplied but simply to plot the solution as it stood; that would enable us to see from the plot whether there appeared to be a significant variation with the relevant period but not with the phasing implied by our inevitably crude guess. Any apparent variation could be followed up by re-running an optimized solution with an appropriately adjusted initial epoch. Where no evidence of a systematic phase-related variation could be seen in a ‘plot-only solution’ and the computer could not be persuaded to pull in to any solution and improve on it, we concluded that no significant periodicity existed. We feel quite secure in doing that, since (as we proceed to show) several of the results where there \textit{did} appear to be \textit{some} evidence of phase dependence, and where the computer \textit{did} grasp the solution and improve on it, have turned out to be without statistical significance.

The method that we have selected for quantifying significance is to compare, in the light of the \( F \) test, the sums of the squares of the residuals from the solutions obtained with and without the prospective period. We consider first the radial-velocity case. The ‘without’ sum is always the same, being just the sum of the squares of the residuals that are given in Table I for the 44 Cambridge observations and that form the data set that we are testing for periodicities. That sum is 132.24 (km \( s^{-1} \))^2. In solving those data for a circular ‘orbit’ we attach optimal values to four independent variables, \textit{viz.}, period, epoch, amplitude, and \( \gamma \)-velocity. (Although, from the manner in which they were obtained, the velocities that constitute the dataset must have a weighted mean of zero, it does not follow that the \( \gamma \)-velocity of an optimized pulsational ‘orbit’ derived from them will be exactly zero.) Especially if the imposed or resulting period is a significant one, the sum of squares of the new set of residuals will be reduced. The amount of the reduction will be assignable to the four degrees of freedom represented by the four fitted variables, while the remaining sum is to be laid at the door of the other 40 degrees of freedom; the significance of the reduction is found by taking the ratio of the mean-square per degree of freedom between the four and the 40 and comparing it with tabular values of \( F_{4,40} \) for various levels of significance.
We clarify the procedure by an initial 'worked example' based on the most significant pulsational period that we have found for HD 70645, viz., the 0.792-day one found first by Martin et al. and corroborated by Henry et al. The computer pulls into a solution with a period of 0.7919 ± 0.0003 days and an amplitude of 1.62 ± 0.31 km s\(^{-1}\) (which at >5σ looks promising) and yields a sum of squares of 78.81 (km s\(^{-1}\))^2 for the 44 residuals, which represent the 40 degrees of freedom left after we used four in fitting the four variables. Thus the 40 degrees cost 78.81/40 = 1.97 (km s\(^{-1}\))^2 each. The four degrees represented by the variables cost (132.24 − 78.81), or 53.43 (km s\(^{-1}\))^2, i.e. 13.36 per degree, so we obtain \(F_{4,40} = 13.36/1.97 = 6.80\). The tabular values of \(F_{4,40}\) for various degrees of significance are as follows: 10% 2.09, 5% 2.61, 2\(1\)/2% 3.13, 1% 3.83, 0.5% 4.37, 0.1% 5.70. Our value is therefore comfortably beyond even the 0.1% value of \(F\), and it follows that the period is to all intents and purposes certainly present in the data. The plot of the fitted sine-wave and the velocities to which it is fitted (the residuals from the orbit derived above and plotted in Fig. 2) is shown in Fig. 3.

We have been through all of the periods found by Henry et al. in the same way, and present the results very succinctly in Table II. The successive lines of the Table give the successive quantities specified in our illustration of the procedure above, as follows:

(a) Period (days) given by Henry et al.;
(b) Period (days) found in our velocities, with its standard error in units of the last decimal place in brackets;
(c) Amplitude (km s\(^{-1}\)) found in our velocities, with its standard error similarly;
(d) Sum of squares of the velocities ((km s\(^{-1}\))^2) before the period is fitted;
(e) Sum of squares after the period is fitted, followed by that quantity divided by 40 (the number of degrees of freedom that it represents), so the second number is the mean square per degree of freedom;
(f) (d) minus (e), the remaining portion of the sum of squares, attributable to the four degrees of freedom represented by the fitted period, and the same quantity divided by four to give the mean square per degree;
(g) the ratio of the mean squares in (e) and (f) immediately above, \(F_{4,40}\); and finally
(h) the significance of that \(F\) ratio (n. s. = 'not significant').

The ensuing lines (b1) to (h1) will be explained shortly.

**Table II**

*Significances of periods in the radial- and rotational-velocity data on HD 70645*

|   | a |   | b |   | c |   | d |   | e |   | f |   | g |   | h |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|   | 0.7929 | 0.8593 | 1.1032 | 1.2405 |
| (b) | 0.7919 (3) | 0.8606 (5) | 1.1049 (8) | 1.2428 (16) |
| (c) | 1.62 (31) | 1.35 (34) | 1.17 (37) | 0.93 (37) |
| (d) | 132.24 | 132.24 | 132.24 | 132.24 |
| (e) | 78.81 | 1.97 | 95.03 | 2.38 | 105.99 | 2.65 | 113.52 | 2.84 |
| (f) | 53.43 | 13.36 | 37.21 | 9.30 | 26.25 | 6.56 | 18.72 | 4.68 |
| (g) | 6.80 | 3.91 | 2.48 | 1.64 |
| (h) | 0.1% | 1% | 10% | n. s. |
| (b1) | 0.7927 (5) | 0.8594 (9) | 1.1018 (10) | 1.2475 (16) |
| (c1) | 1.26 (35) | 0.84 (38) | 0.99 (38) | 0.9 (4) |
| (d1) | 134.94 | 134.94 | 134.94 | 134.94 |
| (e1) | 100.90 | 2.52 | 120.48 | 3.01 | 114.37 | 2.86 | 120.26 | 3.01 |
| (f1) | 34.04 | 8.51 | 14.46 | 3.61 | 20.57 | 5.14 | 14.68 | 3.67 |
| (g1) | 3.38 | 1.20 | 1.79 | 1.22 |
| (h1) | 2\(1\)/2% | n. s. | n. s. | n. s. |
It is to be noticed that the order of the periods listed in Table II with progressively decreasing significance is not the same as the order of the photometric amplitudes found by Henry et al., which is 2, 3, 1, 4 for the successive columns in our Table; the uncertainties in the photometric amplitudes, however, are large enough in relation to the amplitudes themselves to mean that the ordering of the photometric periods is not really determinate. Henry et al.’s fifth period is omitted from our Table because it did not produce a plot that looked at all significant, and the sum of squares fell only to about 125, so there is no evidence for its presence at all.

An analysis can be made of the significance of the various periods in the $v\sin i$ data in exactly the same fashion as for the radial velocities. The mean $v\sin i$ is 12.16 km s$^{-1}$, and the r.m.s. spread of the individual values is 1.75 km s$^{-1}$, the mean square being therefore $(1.75)^2$ or 3.07, and the sum of squares 44 times that, 134.94 (km s$^{-1}$)$^2$ (coincidentally very close to that of the radial velocities); that total is apportioned between the four degrees of freedom represented by the fitted sine-wave and the remaining 40 degrees, exactly as in the case of the radial velocities, so we have simply added to Table II another set of lines corresponding precisely with the first set, labelled (b1) to (h1) and pertaining to the $v\sin i$ data.

We point out that potential periodicities may lack statistical significance but nevertheless be present in the data. That may be suggested in some cases by the simple fact that the attempt to compute a solution with a given period does actually produce convergence, and does so at a period that is close to the suggested one. If the process is initiated with an arbitrary period, it tends either to diverge or to pull into a period that is not plausibly similar to the one under trial.

The final results of our analysis, seen in lines (h) and (h1), are that three of the Henry et al. photometric periods are traceable in the radial velocities, with diminishing degrees of significance, but only one is significant in the rotational velocities. It is the same period that is much the most significant one in the radial velocities that is also traceable in the rotational ones. As a general comment on the results of Table II and the analogous tables to follow for the other two stars, we remark that the significances that are found from the $F$ test are smaller than might be anticipated from a comparison of the amplitudes in lines (c) and (c1) with their respective standard errors. Although we cannot offer any mathematical reason for the apparent discrepancy in significances, we understand that it is well known to period-search experts that ratios less than 4 for $K$ to $\sigma(K)$ are not usually significant, notwithstanding that in a ‘normal distribution’ a significance of 1% is reached at a ratio of 2.58.

**HD 80731**

Just as in the case of HD 70645, radial-velocity measurements with the Cambridge *Coravel* began in 2005 November and continued until the observing season closed in the following June, and (again like HD 70645) it was observed with increased assiduity towards the end of the season in order to improve the chances of documenting pulsational periods. The Cambridge measurements number 54; they are set out in Table III, along with the seven velocities published by Henry et al. as well as with the phases and residuals that stem from a normal single-lined orbital solution, and also with the ‘$v\sin i$’ values for each of the observations. A solution based on the Cambridge observations alone gave a period of 10.678 ± 0.004 days; the inclusion of six out of the seven Henry et al. measures did not change the elements significantly but reduced the standard error of the period to 0.0004 days. Among the published measurements, the third one has such an extreme residual (9 km s$^{-1}$, more than twice as great as any other) that it seems to be beyond any combination of
pulsational velocities plus normal accidental error, so it may be suspected of some sort of qualitative error and has therefore been omitted from the solution. The orbit has the elements given below and is plotted in Fig. 4.

\[
P = 10.6744 \pm 0.0004 \text{ days} \quad (T)_4 = \text{MJD} 53731.19 \pm 0.05
\]
\[
\gamma = -0.78 \pm 0.22 \text{ km s}^{-1} \quad a_1 \sin i = 3.08 \pm 0.05 \text{ Gm}
\]
\[
K = 22.82 \pm 0.33 \text{ km s}^{-1} \quad f(m) = 0.0103 \pm 0.0005 \, M_\odot
\]
\[
e = 0.392 \pm 0.012
\]
\[
\omega = 265.7 \pm 2.3 \text{ degrees} \quad \text{R.m.s. residual} = 1.57 \text{ km s}^{-1}
\]

To investigate the presence of pulsational periods we have followed exactly the same procedure for HD 80731 as for HD 70645, so we can proceed immediately to present the results, which are shown in Table IV. The first three periods given in line (a) are those of Henry et al.\textsuperscript{9}, and the fourth is one of those proposed by Martin et al.\textsuperscript{7}. As in Table II, the first section (as far as line (h)) refers to pulsations seen in the radial-velocity residuals, while the second section (lines (b1) – (h1)) refers to those seen in the rotational velocities. The third section (lines (b2) to (h2)) gives information about two additional periods identified by ourselves in the rotational velocities (clearly line (a) does not apply there). Opportunity is taken to use the spare space in that section to include brief reminders of the significance (described in full immediately before Table II) of the successive lines.

**Table IV**

*Significances of periods in the radial- and rotational-velocity data on HD 80731*

| (a)  | 1.1159 | 1.2783 | 1.5154 | 2.23 |
| (b) | 1.1159 (9) | 1.5157 (22) | 2.236 (4) |
| (c) | 1.00 (32) | 0.91 (28) | 1.05 (28) |
| (d) | 137.1 | 137.1 | 137.1 |
| (e) | 113.9 | 2.28 | 112.5 | 2.25 | 106.0 | 2.12 |
| (f) | 23.2 | 5.80 | 24.6 | 6.15 | 31.1 | 7.77 |
| (g) | 2.55 | 2.79 | 3.67 |
| (h) | ~5% | 5% | 21/2% |
| (b1) | 1.1147 (7) | 1.2774 (14) | 1.5137 (25) |
| (c1) | 2.3 (5) | 1.6 (5) | 1.4 (6) |
| (d1) | 432.1 | 432.1 | 432.1 |
| (e1) | 282.1 | 5.64 | 357.9 | 7.16 | 385.6 | 7.71 |
| (f1) | 150.0 | 37.05 | 74.2 | 18.55 | 46.5 | 11.62 |
| (g1) | 6.71 | 2.59 | 1.51 |
| (h1) | 0.1% | 5% | n.s. |
| (b2) | 0.8210 (5) | 1.1617 (7) | Period and (in brackets) its standard error |
| (c2) | 1.9 (4) | 2.2 (5) | Pulsational amplitude and its standard error |
| (d2) | 432.1 | 432.1 | Sum of squares, apportioned between: |
| (e2) | 314.3 | 6.29 | 328.5 | 6.57 | from 50 degrees of freedom, and per degree |
| (f2) | 117.8 | 29.45 | 103.6 | 25.90 | from 4 degrees (pulsation), and per degree |
| (g2) | 4.69 | 3.95 | 3.95 | $F_{4,50}$ (quotient of the above 2 lines) |
| (h2) | 0.5% | 1% | Significance of the $F$ ratio above |

**HD 17310**

Although the first Cambridge radial-velocity measurement was made in 2005 September, it was not till late November that routine measurements began, and after the observer was absent for much of 2006 January the observing season was practically at its close. There are only 11 Cambridge measurements, listed in Table V, to add to the same number published by Henry et al.\textsuperscript{9}. The Cambridge data are, however, better...
distributed in time, and when solved by themselves for the orbital elements they yield an unambiguous period of $27.67 \pm 0.22$ days, very similar to the best (but not uniquely determined) value of 27.793 days favoured by Henry et al. The precision of the Cambridge period is plenty good enough to extrapolate back to the Henry et al. epochs without any possible error in the cycle count, so the two sets of velocities can be solved together.

The joint solution does, however, throw up problems that were mercifully lacking in the cases of the other two stars. For them the data from the two sources seemed to agree well both in zero-point and in the sizes of the residuals from the orbits, so neither a zero-point shift nor unequal weighting was called for. In the present case, a straightforward solution with equal weights shows fairly serious disparities both in zero-point and in residuals. The means of the residuals from the two sources differ by $1.62 \pm 1.16$ km s$^{-1}$, Cambridge being more positive, while the mean squares (the variances) are 3.81 and 10.82 (km s$^{-1}$)$^2$ for Cambridge and Henry et al., respectively, a difference of a factor of 2.85. With six elements fitted to 22 equally weighted observations, we could consider that each source has 8 degrees of freedom; the factor that we have found exceeds the 10% point of $F_{8,8}$, which is 2.54, so we have seen fit to attribute half-weight to the published velocities. The weighted variances then turn out to be 2.93 and 6.05 (km s$^{-1}$)$^2$, a ratio still as high as 2.06, but no longer very significant; we would have to go a lot further to equalize them, but we do not care to do that, particularly in the light of the apparent quasi-equality in the cases of the other stars. We could also worry about the zero-points, which in the revised (weighted) solution differ by $1.84 \pm 1.15$ km s$^{-1}$, or 1.60σ, for which the probability according to the ‘normal distribution’ is about 11%. Not wishing to tamper too much with an already minimal dataset, we decided not to make any adjustment to the relative zero-points. On the basis, then, of no interference with the observed velocities apart from half-weighting the published ones, we obtain the orbit that is plotted in Fig. 5 and whose elements are:

\[
\begin{align*}
P & = 27.819 \pm 0.022 \text{ days} \\
\gamma & = +25.8 \pm 0.7 \text{ km s}^{-1} \\
K & = 24.0 \pm 1.0 \text{ km s}^{-1} \\
e & = 0.19 \pm 0.04 \\
\omega & = 125 \pm 15 \text{ degrees} \\
(T)_{-5} & = \text{MJD 53485.4} \pm 1.2 \\
a_1 \sin i & = 9.0 \pm 0.4 \text{ Gm} \\
f(m) & = 0.038 \pm 0.005 M_{\odot} \\
\end{align*}
\]

R.m.s. residual = 2.1 km s$^{-1}$

For the purposes of searching for pulsational effects, potentially of multiple periods that are all of short periods and small amplitudes, our data are woefully few. The three periodicities tabulated by Henry et al. and the one identified in the Hipparcos ‘epoch photometry’ by Koen & Eyer, however, are all near 2 days (not 1 day, as in the cases of HD 70645 and HD 80731), and that circumstance makes it more plausible to search the whole run of 22 velocities for the relevant periods. There are gaps of about one year each, or 180 cycles, in the data set, and the Henry et al. periods being tested have uncertainties between one and two thousandths of a day, so it looks as if the phasing error after a year is only of the order of 0.3 days and there should be little chance of getting out of phase by whole cycles.

We did first test the various periods against the Cambridge data alone, but since we used up six degrees of freedom by determining the basic orbit from the 11 observations, by the time we had fitted another ‘orbit’ (circular, so using 4 degrees of freedom) to the observations there could be said to be almost nothing left — with one more variable one could in principle fit all the velocities exactly! To do so, however, one
would need to solve the two orbits simultaneously rather than *seriatim* as in our tests. Having determined
the orbit of the spectroscopic binary, we could possibly regard the residuals as constituting a fresh data
source, notwithstanding that those residuals will have been reduced (*i.e.* the information in which we are
then interested will have been diluted) by some accommodation of the pulsational velocities by the orbital
solution. The upshot of the investigation, in any case, was that we found nothing significant at any of Henry
et al.’s three periods, but there was a remarkably large signal at the Koen & Eyer period of 2.0296 days.
Indeed, a periodicity very close to 2 days is conspicuous in the data of Table V just upon inspection — the
violent reversal of the signs of residuals between alternate days gives it away — and a period of 1.966 days,
that gave an even more dramatic signal than the Koen & Eyer period, was noticed before any period-search
program was brought to bear. The search program identified a still more potent period at 0.6618 days.
The actual statistical significances of the various periods are very doubtful owing to the scarcity of data, and
comment is withheld until we have presented an analysis of the complete dataset including the Henry et al.
velocities.

Before going on to do that, however, we may refer to the ‘*v sin i*’ data for HD 17310, of which there are
just the 11 Cambridge values. Their mean is $9.6 \pm 0.8 \text{ km s}^{-1}$, and the sum of the squares of the deviations
from that value is about 65 (km s$^{-1}$)$^2$. We can state the results of our trials quite briefly. The three Henry
et al. periods did not reduce the sum of squares to any great extent, but the periods of 2.0296, 1.966, and
0.6618 days all produced sums of squares reduced to near 20 (km s$^{-1}$)$^2$. Since the rotational-velocity numbers
were not utilized in the determination of the binary-star orbit, the 11 values could reasonably be regarded
as independent data and therefore as possessing jointly 11 degrees of freedom, of which four are used up in
fitting any pulsational ‘orbit’. So, in exact analogy with the treatment described before the presentation of
Table II for HD 70645, we may say that the sum of squares remaining after the derivation of a pulsational
periodicity is associated with seven degrees of freedom, while the reduction from the original sum represents
the cost of the four degrees used in the fit. In that case, the three periods that caused reductions of about
45 and left sums of about 20 (km s$^{-1}$)$^2$ gave values of $F_{4,7}$ of about $(45/4)/(20/7)$, ~ 4, which is nearly the
5% point (4.12). Thus, in view of the fact that we were not trying to find fresh periods but were merely
making individual tests of ones that had been proposed on the basis of quite independent data, there are
grounds for cautious optimism in thinking that those periods may be discernible in the rotational velocities.
Conversely, the likelihood of the existence of the periods in the rotational velocities provides some support
for their presence in the radial ones.

We next extend the investigation of periodicities in the orbital radial-velocity residuals to the complete
dataset, including the Henry et al. velocities. The results are presented in exactly the same way as for the
other two stars, in Table VI below. In the first part of the table we test the four periods (three from
Henry et al. and one from Koen & Eyer) that have been identified photometrically, and then give the results
from the other two periods that seemed so significant in the Cambridge radial-velocity data and gained some
support from the rotational velocities. The figure for the sum of squares of the ‘raw’ orbital-velocity residuals,
appearing throughout in lines (d) and (d1), is seen to be 98.51 (km s$^{-1}$)$^2$. The 1% point of $F_{4,18}$ that is very
nearly reached by two of the periods in the first section is 4.58. The 0.1% point is 7.46, so both the values in
the second section of the table are far beyond that. Even the highest significance that we have seen tabulated
for $F_{4,18}$, 0.05%, is ‘only’ 8.47. The periods quoted in line (a1) are those found first, when the Cambridge
velocities were considered alone, and are otherwise unsupported. The 1.968-day period is illustrated in Fig. 6.
Table VI

Significances of periods in the radial-velocity data on HD 17310

(a) 2.137 1.825 2.451 2.0296
(b) 2.1360 (7) 1.8220 (4) 2.4510 (10) 2.0292 (4)
(c) 1.8 (7) 2.4 (6) 1.9 (7) 2.6 (6)
(d) 98.51 98.51 98.51 98.51
(e) 71.57 3.97 49.40 2.74 68.93 3.83 49.77 2.76
(f) 26.94 6.73 49.11 12.28 29.58 7.39 48.74 12.18
(g) 1.70 4.49 1.93 4.41
(h) n. s. almost 1% n. s. almost 1%

(a1) 1.9650 (26) 0.6617 (2) Period being tested and its standard error
(b1) 1.9683 (3) 0.66173 (3) Period found and its standard error
(c1) 2.9 (4) 3.3 (4) Pulsational amplitude and its standard error
(d1) 98.51 98.51 Sum of squares, apportioned between:
(e1) 24.65 1.36 21.27 1.18 from 18 degrees of freedom, and per degree
(f1) 73.86 18.46 77.24 19.31 from 4 degrees (pulsation), and per degree
(g1) 13.6 16.3
(h1) 0.1% 0.1% Significance of the $F_4,18$ (quotient of the above 2 lines)

Discussion

We have gone some way towards demonstrating, by a formal (if elementary) statistical analysis, that certain periods, mostly already recognized in photometric datasets that are much richer than our kinematic ones, are present in the radial and quasi-rotational velocities that we have measured for the three $\gamma$ Dor stars. There remain questions, however, as to how far the statistical results should be trusted.

We have already indicated an inclination towards trusting them where we are simply testing already-defined periods for their presence in our data. In such cases we are not searching a sparse data string for a short period, a procedure that we know can lead to ‘false positives’. If the test calculation immediately converges and gives a period that is, within the joint uncertainties of itself and of the trial period, the same as the one being tested, there are grounds for thinking that the result is secure. Misgivings start to creep in, however, when we consider the results of multiple periodicities identified in the same dataset. If we look at row (f) of Table II, for example, we see that the sum of contributions listed as being made by the four tested periods to the total sum of squares is more than that whole total! A greater excess of the individual contributions over the whole total is seen in row (f) of Table VI. It could, however, be argued that we should not include contributions from periods that have turned out not to be significant. If we pretend for a moment to be really na"ıve operators, we could imagine ourselves trying any number of periods at random, and most of them would yield a ‘solution’ that was better than nothing, in the sense that it would produce some reduction in the sum of squares; but it would be nonsensical to add up all the reductions and say that we had thereby accounted for all and more of the apparent raggedness of our velocities. Clearly some consideration ought to be given to the total number of degrees of freedom used in fitting multiple periods to the same data, but we are unable to suggest how to do that in a constructive fashion.

A more extreme situation than those already mentioned is the one referred to in the paragraph next but one before Table VI above, where each of three periods is apparently found to be responsible for more than two-thirds of the total sum of squares! Two of those periods, however, have found no support from photometry and might be dismissed as mere idiosyncrasies in the very small Cambridge dataset, especially as they result from doing just what we have warned against, viz., searching a data string for periods short in comparison with the mean interval between observations. But in that case why would they be overwhelmingly reinforced when the dataset in which they were first noticed was expanded to include the
published velocities? Row (f1) in Table VI appears to show that each of the two ‘new’ periods accounts for three-quarters of the total variance!

We can see that at least part of the answer is that there is really only one period: the two new ones are 1-day$^{-1}$ aliases of one another (although not within their joint formal uncertainties). Labelling 1.9683 days as $P_1$ and 0.66173 days as $P_2$, we find the corresponding frequencies to be $\nu_1 = 0.5081$ and $\nu_2 = 1.5112$ day$^{-1}$, respectively. Moreover, the Koen & Eyer period of 2.0296 days ($P_3$) inverts to — in fact it was actually given by those authors as — a frequency of 0.4927 day$^{-1}$ ($\nu_3$), which is seen to be very closely the 1-day$^{-1}$ complement of $\nu_1$. The close numerical relationships between all three of the periods that seem to be so powerfully present in the small dataset of HD 17310 radial velocities warns us of the likelihood that at most one of the three periods can be real, the others being mere mathematical artefacts. When plotted modulo the three periods in turn, the \textit{Hipparcos} ‘epoch photometry’ seems unrelated to $P_2$, but at least to a subjective view its phase-dependence on $P_1$ is scarcely less convincing than that on $P_3$, which itself inspires little confidence; the Henry \textit{et al.} periods are even less visible in the \textit{Hipparcos} photometry.

Clearly we cannot claim to have the last word, let alone the greatest wisdom, on these matters, which would become clearer, even in the absence of fresh insight or inspiration, if we could bring to bear a much greater quantity of data. What we \textit{can} claim in this paper is to have established the spectroscopic orbits of the three stars; in descending order of certainty we believe also that we can trust the demonstration of two of the already-known photometric periods in the radial velocities of HD 70645, and probably also in HD 80731, and we think that we have traced the dominant radial-velocity period in each of those stars in the line-width parameter too. It furthermore seems likely, from our very parsimonious data, that at any rate two of the four photometric periods that have been identified in HD 17310 are present in the radial velocities; one of them appears also to be present in the line-widths. Those widths, as well as the radial velocities, are also represented extraordinarily well by either of two periods that are aliases of one another and of one of the photometric ones, but we are not able to adjudicate on the reality of those periods.

As far as our observations (or those of Mathias \textit{et al.}) are concerned, all three of the systems with which we are concerned are single-lined. We have not noticed secondary dips in any of the radial-velocity traces, although we regret having omitted to make a specific search for them by taking long integrations at the appropriate velocity ranges near the nodes of any of the orbits. We can say only that the secondaries are probably at least two magnitudes fainter than the primaries. None of the mass functions is particularly large: those of HD 70645 and HD 17310 are both between 0.03 and 0.04 $M_{\odot}$, while that of HD 80731 is only 0.01. For an early-F star whose own mass may be estimated at 1.6 $M_{\odot}$, a mass function of 0.04 $M_{\odot}$ requires the secondary to have a minimum mass of about 0.55 $M_{\odot}$, corresponding to a spectral type not much earlier than M0 V and an absolute magnitude about six magnitudes fainter than that of the primary. If the orbital inclination is far from 90°, however, the secondary may be more massive than the minimum value, to any extent, though statistically inclinations are high.

In their study of 59 potential $\gamma$ Dor stars whose candidatures were mostly based on the \textit{Hipparcos} photometry, Mathias \textit{et al.} were able, in a very few cases, to identify the main \textit{Hipparcos} frequency in their radial-velocity curves. For those cases, they deduced ratios between the radial-velocity and photometric amplitudes in the range 35 to 96 km s$^{-1}$ per magnitude. We can perform the same exercise for the three stars whose radial velocities we have analyzed, all of which have periods in common with the photometric ones found by Henry \textit{et al.}. For HD 70645, we find ratios of radial velocities to $B$- and $V$-band signals between 56 and 81, and 74 and 110 km s$^{-1}$ per magnitude, respectively. The corresponding numbers are 33
to 95 and 42 to 107, respectively, for HD 80731, while for HD 17310, which has particularly low photometric amplitudes and suspiciously high radial-velocity ones, the ratios are all very large, between 170 and 430. We suppose that the observed ratios will be of significance for modellers of the γ Dor phenomenon.

Stars in binary systems with short orbital periods often have synchronized rotations. (They often have circular orbits, too, but since the time-scale for circularization is much longer than that for capture of the rotation\textsuperscript{20,21}, the non-zero orbital eccentricities of the three stars with which we are concerned here does not necessarily imply that the rotations are not synchronized.) We have seen in the introductory section of this paper that all three stars have colours and luminosities appropriate to early-F dwarfs, so they must also have normal radii of about 0.9 Gm. The pseudo-synchronous\textsuperscript{22} rotation periods appropriate to the periods and orbital eccentricities of the three stars are about 23, 8.2, and 5.2 days, corresponding to equatorial rotational speeds of about 3, 8, and 13 km s\textsuperscript{-1} for HD 17310, HD 70645, and HD 80631, respectively. The variability of the observed line-widths of all three stars demonstrates that those widths are not to be interpreted purely in terms of rotation; we do not know whether the minimum width observed for each is or is not still largely increased from the value set mainly by rotation, but it must represent an upper limit to the rotational velocity.

The minimum observed value is very likely to have been minimized partly by accidental observational error; allowing subjectively for such an effect, we might say that the minimal values for the three stars are about 6, 9, and 10 km s\textsuperscript{-1}, respectively. Comparison with the values calculated on the basis of pseudo-synchronism leads to the conclusions that HD 17310 and HD 70645 are either rotating faster than synchronism or else their minimum line-widths still owe something to pulsation, whereas HD 80731 is either rotating more slowly than synchronism or, if synchronized, has an orbital inclination no greater than about 50\degree.

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Radial-velocity traces of HD 80731, obtained with the Cambridge Coravel on 2006 May 9 (left) and June 3, illustrating the variability of the ‘dip’ profile.

The observed radial velocities of HD 70645 plotted as a function of phase, with the velocity curve corresponding to the adopted orbital elements drawn through them. Cambridge observations are represented by the filled squares; those published by Henry et al. are plotted as open circles. All were give equal weight in the solution of the orbit.

Illustrating the most convincing pulsational period detected in the orbital radial-velocity residuals of HD 70645. The data points are the times of observation and the velocity residuals tabulated for the Cambridge observations in the fifth column of Table I. The Henry et al. velocities cannot usefully be plotted, because the pulsational period is not determined well enough to maintain phases back to previous seasons.

As Fig. 2, but for HD 80731. The errant open-circle point below the maximum of the velocity curve was omitted from the solution of the orbit.

As Fig. 2, but for HD 17310.

Illustrating one of the apparent periods noticed in the radial-velocity residuals from the orbit of HD 17310 but (it seems) not in the photometry. The period was first noticed in the Cambridge observations alone (the filled squares); if it does not manifest any underlying reality it is amazing that it should have been reinforced as cogently as it obviously is by the inclusion of the altogether independent Henry et al. velocities (the open circles).