Dispersion theory of nucleon polarizabilities and outlook on chiral effective field theory

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Abstract

The polarizabilities of the nucleon are precisely studied and well understood due to recent experimental and theoretical work based on nonsubtracted dispersion relations. The recommended experimental values are $\alpha_p = 12.0 \pm 0.6$, $(12.0)$, $\beta_p = 1.9 \mp 0.6$, $(1.9)$, $\alpha_n = 12.5 \pm 1.7$, $(12.7 \pm 0.9)$, $\beta_n = 2.7 \mp 1.8$, $(2.5 \mp 0.9)$ in units of $10^{-4}\text{fm}^3$ and $\gamma^{(p)} = -36.4 \pm 1.5$, $(-36.6)$, $\gamma^{(n)} = 58.6 \pm 4.0$, $(58.3)$, $(\gamma^{(p)} = -0.58 \pm 0.20)$, $(\gamma^{(n)} = 0.38 \pm 0.22)$ in units of $10^{-4}\text{fm}^4$. The numbers given in parentheses are predicted values. Recently attempts to reanalyze low-energy Compton scattering data by making use of different versions of chiral effective field theory ($\chi$EFT) have led to sizable discrepancies between each other and with the recommended experimental data. An investigation is presented showing that these newly analyzed data should not be included in the recommendation.

1 Introduction

The description of Compton scattering by the nucleon via dispersion theory was developed at the beginning of the 1960s [2]. The theory made use of relations which may be denoted as $s$-channel dispersion relations and $t$-channel dispersion relations. The singularities entering into the $s$-channel dispersion relations may be taken from meson photoproduction experiments, whereas the singularities entering into the $t$-channel dispersion relations are related to $\pi\pi$ pairs created in two-photon reactions in case of the scalar $t$-channel, and to the $\pi^0$ meson in case of the pseudoscalar $t$-channel. The first application of the scalar $t$-channel to the polarizabilities of the nucleon came with the work of J. Bernabeu, T.E.O Ericson et al. (BEFT) in 1974 [3] where it was shown that the largest part of the electric polarizability and the total diamagnetic polarizability are due to the $t$-channel. The smaller part of the electric polarizability is due to the “pion cloud” showing up as a nonresonant meson photoproduction process. The paramagnetic polarizability is mainly due to the photoabsorption cross section provided by the $P_{33}(1232)$ resonance.

A convenient version of a dispersion theory applicable in a wide angular interval and at energies up to 1 GeV was developed by L’vov et al. [4]. This dispersion theory is of the fixed-$t$ variety where $s$-channel integrals are carried out along integration paths at constant $t$, and the $t$-channel contributions are taken care of in the form of “asymptotic” contributions, being an equivalent of the $t$-channel contributions. In principle this version

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of dispersion theory is completely equivalent to other versions as there are fixed-θ dispersions theories or hyperbolic dispersion theories. At a first sight there appears to be a disadvantage in case of fixed-θ dispersion theories because the integration paths are partly located in the unphysical range of the scattering plane. However, it has been shown by L’vov et al. [4] that this only leads to minor technical problems which can be solved without loss of precision. The validity of this latter statement has been clearly demonstrated by experiments, showing that fixed-θ dispersion theories lead to a precise representation of the experimental differential cross sections in the whole angular interval and at energies up to 1 GeV [4-6]. In this respect the unsubtracted dispersion theory differs from the subtracted dispersion theory [7] which loses validity already at the peak energy of the Δ resonance. For the spin-independent t-channel contribution the assumption was made that it is possible to represent it via a scalar σ-meson pole-term, in analogy to the well-known pseudoscalar π0 pole term as entering into spin-independent amplitudes. With this representation it was possible to arrive at agreement with experimental data in the angular and energy ranges described above, whereas without this representation of the pole term there remained a large discrepancy between prediction and the experimental data. This implies that at energies of the second resonance region of the nucleon and large scattering angles the σ-meson pole makes a dominant contribution to the Compton differential cross section. Furthermore, the σ-meson mass was determined to be \( m_\sigma = 600 \) MeV [5, 6] in agreement with other available data. These first investigations published in 2001 [5, 6] remained preliminary because the σ-meson pole was only an ansatz at that time and there was an uncertainty about its validity. This uncertainty has been removed in later investigations where it was shown that the σ-meson pole term has a very firm theoretical basis and that the quantitative predictions calculated from well known properties of the σ-meson are precise (see [1] for a summary). As a conclusion we may state that the dispersion theory of L’vov et al. [4] is precise and well tested. Therefore, there are good reasons to trust in the evaluations of electromagnetic polarizabilities from low-energy Compton-scattering data where use is made of this type of dispersion theory. This is the case for all experimental data which are summarized in [1, 8] leading to the recommended nucleon polarizabilities given in the abstract. For the neutron also electromagnetic scattering of slow neutrons has been taken into account. Furthermore, these recommended values are in excellent agreement with independent predictions obtained from high-precision CGLN amplitudes and well-investigated properties of the σ meson without making use of experimental Compton differential cross sections [1].

In addition to dispersion theory \( \chi \) EFT plays a prominent rôle in current investigations of nucleon Compton scattering and polarizabilities. The present investigation is motivated by the fact that recently \( \chi \) EFT has been used as a tool for analyses of experimental differential cross sections for Compton scattering by the proton, and results have been obtained which are considerable different from the well-founded standard values \( \alpha_p = 12.0 \pm 0.6 \) and \( \beta_p = 1.9 \pm 0.6 \). An example is the ChPT-investigation of Beane et al. [9] where the values \( \alpha_p = 12.1 \pm 1.1 \pm 0.5, \beta_p = 3.4 \pm 1.1 \pm 0.1 \) have been obtained. These values have been included in the data listing of the Particle Data Group [10] though the magnetic polarizability obtained deviates from the standard value by a factor 1.8, corresponding to 1.3 standard deviations. This new development implies that the correct values of the polarizabilities obtained in previous experiments may gradually be replaced by new values obtained by using barely understood theoretical procedures.
Summary of results of dispersion theory

In a recent article [1] a complete description of the present status of dispersion theory of nucleon Compton scattering and polarizabilities has been given. The recommended experimental polarizabilities introduced in the 2005 summary [8] have been confirmed in the 2013 summary [1] except for a slight correction applied to $\gamma(p,\pi)$. The reason for this slight correction was due to the fact that new high-precision analyses of CGLN amplitudes have become available [11] which made this revision necessary. These new high-precision analyses [11] were also of great importance in connection with the prediction of the $s$-channel components of the nucleon polarizabilities for all resonant and nonresonant excitation processes of the nucleon. Results of these analyses which are of interest here are summarized in Table 1. As documented in line 4 of Table 1 the $s$-channel predictions show a large deviation from the experimental data and these deviations are the same for the proton and the neutron. Furthermore, the differences obtained for the electric and the magnetic polarizabilities given in line 4 only differ by the signs. This means that these differences cancel in case of $\alpha + \beta$, i.e. $(\alpha + \beta)^t \equiv 0$, but make dominant contribution, viz. $(\alpha - \beta)^t$, in case of $\alpha - \beta$.

As has been pointed out in [1] the $t$-channel contribution differs from the $s$-channel contribution due to the fact that the former can be traced back to the mesonic structure of the constituent quarks. This means that the short-range contribution [12] introduced in some versions of $\chi$EFT may tentatively also be viewed in terms of properties of the constituent quarks. In spite of this interesting similarity there is an essential difference between $\chi$EFT and dispersion theory due to the fact that dispersion theory provides a method for a quantitative prediction of the $t$-channel contribution, whereas the $\chi$EFT prediction for the short-range contribution is treated as an adjustment, filling the gap between predictions and experimental data. This is the reason for naming them counterterms (c.t.) with $\delta\alpha$ corresponding to the electric part and $\delta\beta$ corresponding to the magnetic part.

2.1 Quantitative prediction of the $t$-channel component

The $t$-channel component of the electromagnetic polarizabilities of the nucleon has been first described by J. Bernabeo, T.E.O. Ericson et al (BEFT) [3] in the following way: If we restrict ourselves in the calculation of the $t$-channel absorptive part to intermediate states with two pions with angular momentum $J \leq 2$, the sum rule takes the convenient

### Table 1: $s$-channel electromagnetic polarizabilities compared with experimental data.

|             | $\alpha_p$ | $\beta_p$ | $\alpha_n$ | $\beta_n$ |
|-------------|------------|------------|------------|------------|
| $s$-channel prediction | +4.48 | +9.44 | +5.12 | +10.07 |
| Experimental data | 12.0 ± 0.6 | 1.9 ± 0.6 | 12.5 ± 1.7 | 2.7 ± 1.8 |
| Difference line 3 - line 2 | +7.5 | -7.5 | 7.4 | -7.4 |
form for calculations \[3\]:

\[
(\alpha - \beta)^t = \frac{1}{16\pi^2} \int_{m_\pi^2}^{\infty} \frac{dt}{4m_\pi^2} \left( \frac{t - 4m_\pi^2}{4m_\pi^2} \right)^{1/2} \left[ f_+^0(t)F_0^{0*}(t) - \left( m^2 - \frac{t}{4} \right) \left( \frac{t}{4} - m_\pi^2 \right) f_+^2(t)F_0^{2*}(t) \right],
\]

where \( f_+^{(0,2)}(t) \) and \( F_0^{(0,2)}(t) \) are the partial-wave helicity amplitudes of the processes \( N\bar{N} \to \pi\pi \) and \( \pi\pi \to \gamma\gamma \) with angular momentum \( J = 0 \) and 2, respectively, and isospin \( I = 0 \). Though being the first who published the BEFT sum rule in its presently accepted form, Bernabeu and Tarrach \[3\] were not aware of the appropriate amplitudes to calculate the BEFT sum rule numerically. Later on evaluations of Eq. (1) also remained rather uncertain until recently, when Drechsel et al. \[7\] and Levchuk \[13\] carried out calculations with good precision. The results obtained in this way are listed in lines 2 and 3 of Table 2: Numerical evaluation of the BEFT sum rule and of the equivalent \( \sigma \)-meson pole.

| \((\alpha_p - \beta_p)^t\) | authors & methods                        |
|--------------------------|-----------------------------------------|
| +16.5                    | Drechsel, Pasquini, Vanderhaeghen, 2003 \[7\] |
| +14.0                    | Levchuk, 2004 \[13\]                    |
| +15.2                    | prediction based on the \( \sigma \)-meson pole \[1\] |

In the \( t \)-channel notation the two-photon process described by Eq. (1) may be written in the form

\[
\gamma\gamma \to \sigma \to \pi\pi \to \sigma \to N\bar{N},
\]

i.e. by a pion pair in the intermediate state, coupled to two photons on the one hand and to a \( N\bar{N} \) pair on the other, via correlations which may be understood as \( \sigma \) mesons. As has been justified in detail in \[1\] this composite intermediate state can be replaced by

\[
\gamma\gamma \to \sigma \to N\bar{N}
\]

which describes the \( t \)-channel pole contribution in complete analogy to the well known \( \pi^0 \) pole contribution

\[
\gamma\gamma \to \pi^0 \to N\bar{N}.
\]

This leads to the prediction derived in \[14, 15\]

\[
(\alpha - \beta)^t = \frac{g_{\sigma NN} M(\sigma \to \gamma\gamma)}{2\pi m_\sigma^2} + \frac{g_{f_0 NN} M(f_0 \to \gamma\gamma)}{2\pi m_{f_0}^2} + \frac{g_{a_0 NN} M(a_0 \to \gamma\gamma)}{2\pi m_{a_0}^2} r_3,
\]

where the \( \sigma \)-meson part is given by

\[
(\alpha - \beta)^t_\sigma = \frac{5\alpha_{em} g_{\sigma NN}}{6\pi^2 m_\sigma^2 f_\pi} = 15.2
\]

with \( \alpha_{em} = 1/137.04, g_{\sigma NN} \equiv g_{\pi NN} = 13.169 \pm 0.057, f_\pi = (92.42 \pm 0.26) \) MeV, \( m_\sigma = 666 \) MeV, as derived in \[16\] and given in line 4 of Table 2. Now with \((\alpha + \beta)^t = 0\) we arrive at

\[
\alpha^t = \frac{1}{2}(\alpha - \beta)^t, = +7.6
\]

\[
\beta^t = -\frac{1}{2}(\alpha - \beta)^t, = -7.6
\]
in excellent agreement with the findings in Table 1. The contributions from the $f_0(980)$ and $a_0(980)$ scalar meson entering into Eq. (5) are discussed in detail in [1]. These contributions are small and may be represented by

$$\alpha(f_0(980), a_0(980)) = \pm(0.3 - 0.4 \tau_3).$$

(9)

The double, $\pm$, sign on r.h.s. of Eq. (9) indicates that we leave the sign of $\alpha(f_0(980), a_0(980))$ undetermined and, as a consequence, include this quantity into the error when calculating the neutron electromagnetic polarizability $\alpha_n$ from the experimental proton electric polarizability $\alpha_p$ and the predicted difference $(\alpha_n - \alpha_p)$, leading to [1]

$$\alpha_n = 12.7 \pm 0.9, \quad \beta_n = 2.5 \mp 0.9.$$  

(10)

This result for the neutron polarizabilities is extremely important because it rests on very firm arguments, on very precise experimental data for the CGLN amplitudes and probably is more precise than any value to be obtained from future experiment.

The conclusion we may draw from this result is that dispersion theory provides us with a quantitative prediction of the three components of the electromagnetic polarizabilities, as there are the $s$-channel nonresonant and $s$-channel resonant excitations of the nucleon, and the $t$-channel part which may be understood as scattering by the $\sigma$ meson while being part of the structure of the constituent quark. This latter process is expected to take place because the $\sigma$ meson mediates the generation of mass of the constituent quark via chiral symmetry breaking and, therefore, has to be a part of the constituent-quark structure. This very consistent result obtained from dispersion theory contrasts with the unspecified short-range contribution discussed in case of $\chi$EFT.

2.2 Dependence of the polarizabilities on the photon energy

Compton scattering experiments aimed to determine the polarizabilities of the nucleon are carried out typically at energies above 50 MeV up to energies well below the $\Delta$ peak. In the upper part of this energy interval fits to the experimental differential cross sections require a general knowledge of the photon-energy, $\omega$, dependences of the polarizabilities. Dispersion relations evaluated in the backward direction at $\theta = \pi$ are the appropriate tool to determine these $\omega$ dependences. Figure 1 shows the result of the calculation when including the empirical $E_{0+}$ CGLN amplitude, the $P_{33}(1232)$ resonance and the $\sigma$-meson pole contribution. The latter contribution has been calculated for the mass $m_\sigma = 600$ MeV. This mass is the appropriate value because it fits the experimental differential cross sections in the second resonance region of the proton. Details may be found in [1].

It may be of interest to disentangle the curves shown in Figure 1 into an electric part and a magnetic part. The results obtained are shown in Figure 2. The $\omega$-dependent electric polarizability as shown in the left panel of Figure 2 is a superposition of the $E_{0+}$ contribution as given by the nonresonant photoabsorption cross section and a positive part of the $\sigma$-meson pole contribution. In a similar way the magnetic polarizability as shown in the right panel of Figure 2 can be traced back to the $P_{33}(1232)$ resonance as the main component of the paramagnetic polarizability, and a negative part of the $\sigma$-meson pole contribution which represents the diamagnetic polarizability.
Figure 1: Real part of the three main components of \((\alpha - \beta)(\omega)\) for the proton. 1) Meson-cloud component as given by the \(E_{0+}\) CGLN amplitude. 2) Resonant component due to the \(P_{33}(1232)\) nucleon resonance. 3) \(t\)-channel component as given by the \(\sigma\)-meson pole. The curve is calculated for the \(\sigma\)-meson mass \(m_\sigma = 600\).

For sake of completeness we also investigate the \(\omega\) dependencies of the spinpolarizabilities. In the left panel of Figure 3 the components of the backward spinpolarizability are shown. There is a constructive interference of the \(E_{0+}\) component (curve 1) and the \(P_{33}(1232)\) component (curve 2). The main components are due to the \(\pi^0\) \(t\)-channel which lead to a destructive interference in case of the proton and to a constructive interference in case of the neutron. In case of the forward spinpolarizability as shown in the right panel of Figure 3 the \(E_{0+}\) component (curve 1) and the \(P_{33}(1232)\) component (curve 2) interfere destructively. This leads to very small values for the forward spinpolarizabilities for the proton as well as for the neutron. Furthermore, since the \(E_{0+}\) component is larger for the neutron than for the proton, whereas the \(P_{33}(1232)\) components are the same, it is not a surprise that the forward spinpolarizabilities of the proton and neutron have different signs (see \[1\] and references therein).

3 Outlook on chiral perturbation theory

3.1 Chiral perturbation theory and the \(E_{0+}\) CGLN amplitude

In 1993 A. I. L’vov published a paper \[17\] with the title “A dispersion look at the chiral perturbation theory. Nucleon electromagnetic polarizabilities”. In this paper it is shown that chiral perturbation theory as discussed at that time \[18,19\] corresponds to dispersion theory applied to the \(E_{0+}\) CGLN amplitude in different versions. The relativistic version of chiral perturbation theory (ChPT) corresponds to the \(E_{0+}\) CGLN amplitude in the
Figure 2: Left panel: Energy dependent electric polarizability \( \alpha(\omega) \) due to the pion cloud and the \( \sigma \)-meson pole (see Fig. 1). Right panel: Energy dependent magnetic polarizability \( \beta(\omega) \) due to the \( P_{33}(1232) \) resonance and the \( \sigma \)-meson pole (see Fig. 1).

Figure 3: Photon-energy dependent spinnpolarizabilities. Left panel: Components of backward spinnpolarizabilities, 1) \( E_{0+} \) component, 2) \( P_{33}(1232) \) component, 3) \( \pi_0 \)-pole component for the proton, 4) \( \pi_0 \)-pole component for the neutron. Right panel: Components of forward spinnpolarizabilities, 1) \( E_{0+} \) component, 2) \( P_{33}(1232) \) component.
Born approximation, whereas the heavy baryon version (HBChPT) of chiral perturbation theory also corresponds to the Born approximation but with the further modification that terms of the order $m_\pi/m_p$ are shifted to zero. This shift may be viewed in terms of a shift $m_\pi \to 0$ corresponding to a shift to the chiral limit, or in terms of a shift $m_p \to \infty$ which justifies the denotation HBChPT.

The results obtained are given in Table 3 together with the corresponding results obtained on the basis of the empirical $E_{0+}$ CGLN amplitude (see [1,20]) and the recommended experimental values of the polarizabilities. In Table 3 it can be seen that the modifications of the $E_{0+}$ CGLN amplitude inherent in the two versions of chiral perturbation theory lead to large shifts of the predicted electromagnetic polarizabilities as compared to the empirical values given in line 4. On the other hand these shifts contained in lines 2 and 3 lead to a much better agreement with the experimental electromagnetic polarizabilities as given in line 5. This is especially true for the HBChPT version.

Table 3: Predicted electromagnetic polarizabilities of the nucleon compared with experimental data. The predictions given in lines 2 and 3 correspond to different modified versions of the $E_{0+}$ CGLN amplitude, the prediction in line 4 to the empirical $E_{0+}$ CGLN amplitude.

|       | $\alpha_p$ | $\beta_p$ | $\alpha_n$ | $\beta_n$ |
|-------|------------|------------|------------|------------|
| ChPT  | 7.4        | -2.0       | 10.1       | -1.2       |
| HBChPT| 12.6       | 1.3        | 12.6       | 1.3        |
| Empirical $E_{0+}$ | 3.2 | -0.3 | 4.1 | -0.4 |
| Experiment | $12.0 \pm 0.6$ | $1.9 \mp 0.6$ | $12.5 \pm 1.7$ | $2.7 \mp 1.8$ |

The remarkable agreement of the predictions based on the two versions of chiral perturbations theory with the experimental data may lead to the (erroneous) conclusion that these predictions are essentially complete and only need some minor corrections in order to represent the true theoretical prediction of the electromagnetic polarizabilities. This conclusion makes use of the supposition that chiral perturbation theory contains the appropriate description of the low-energy properties of the nucleon-pion system and, therefore, necessarily should lead to the correct description of the electromagnetic polarizabilities of the nucleon. Other degrees of freedom of the nucleon as there are the resonant excitation of the nucleon and the $t$-channel are “high-energy” phenomena and, therefore, do not have to be taken into consideration.

In a later versions of chiral perturbation theory [21,22] which are considered in the next subsections in more detail it was recognized that the resonant excitation of the nucleon plays a dominant rôle and provides a large paramagnetic component. Furthermore, quantities $\delta \alpha$ and $\delta \beta$ denoting the electric and the magnetic counterterms (c.t.), respectively, have been introduced.
3.2 Chiral dynamics in low-energy Compton scattering off the nucleon versus dispersion theory

For a comparison of dispersion theory (DR) as outlined in the foregoing and in [1] we start with the work of Hildebrandt et al. [21] from the following reasons. This work takes into account a complete list components and provides numerical results for them. These components are

(i) the nonresonant Nπ component which is known to have the $E_{0+}$ CGLN amplitudes as the main part,

(ii) the short-range or counterterm (c.t.) component which may tentatively be compared with the $t$-channel component of DR,

(iii) the $\Delta$-pole component calculated by the small scale expansion (SSE) method, which may be compared with the resonant $s$-channel component of DR, and

(iv) the $\Delta\pi$ component which may be compared with the $\gamma \rightarrow \pi\Delta \rightarrow N\pi\pi$ component of the photoabsorption cross section.

Table 4: Components contributing to the electromagnetic polarizabilities of the proton obtained in the SSE-version of HBChPT compared with dispersion theory (DR) as outlined in the foregoing sections and in [1]. The $t$-channel components of DR are tentatively compared with the electric, $\delta\alpha$, and magnetic, $\delta\beta$, counterterm (c.t.).

| Component  | HBChPT | DR  | Component  | HBChPT | DR  |
|------------|--------|-----|------------|--------|-----|
| Nπ         | +11.87 | +3.09 | Nπ         | +1.25  | +0.48 |
| c.t.       | -5.92  | +7.6 | c.t.       | -10.68 | -7.6 |
| $\Delta$-pole | 0.0     | -0.01 | $\Delta$-pole | +11.33 | +8.56 |
| $\Delta\pi$ | +5.09   | +1.4 | $\Delta\pi$ | +0.86  | +0.4 |
|            | 11.04  | 12.08 |            | 2.76   | 1.84 |

For the Nπ component of $\alpha_p$, given in line 2 of Table 4 we find a remarkable discrepancy between the results obtained by the HBChPT-SSE calculation and the results obtained by dispersion theory (DR), amounting to a factor 3.8. This factor is due to the fact that HBChPT replaces the empirical $E_{0+}$ CGLN amplitude by a modified version of the Born approximation which leads to this large factor as discussed in the foregoing subsection.

The counterterms (c.t.) $\delta\alpha$ and $\delta\beta$ given in line 3 of Table 4 are not obtained by a parameter-free prediction, but by adjustments to experimental data. A disadvantage of the result obtained is that the sum $\delta\alpha + \delta\beta$ is unequal to zero and negative. Then, according to Baldin’s sum rule there should be a negative component in the total photoabsorption cross section of the nucleon which is related to this quantity $\delta\alpha + \delta\beta$. Such a component does not exist. On the other hand the predictions in line 3 of Table 4 obtained from the $\sigma$-meson pole term of the $t$-channel (columns DR) obey the condition $\alpha_t + \beta_t = 0$.

The paramagnetic polarizability given in line 4 of Table 4 shows that the SSE-method leads to the right order of magnitude for this quantity. The deviation from the prediction obtained via dispersion theory amounts to 32%.
This is different for the $\Delta \pi$ component which is too big by a factor of 3.6 in case of the electric polarizability and by a factor of 2 in case of the magnetic polarizability. In this case similar effects may play a rôle as in case of the $N \pi$ component.

Summarizing it may be stated that the HBChPT-SSE version of chiral perturbation theory as discussed in this sections has the advantage of containing all the degrees of freedom which also are expected from the point of view of dispersion theory. There is the nonresonant $N \pi$ component, a formal analog (c.t.) of the $t$-channel component, a component from resonant excitation of the nucleon and a $\Delta \pi$ component. However, the numbers obtained are not even approximations and partly do not have the right properties as it is the case for the c.t. component.

### 3.3 Different variants of chiral EFT

A deeper insights into the methodology of ChPT is given in a recent publication of Lensky et al. [22]. One interesting piece of information is given in terms of counterterms $\delta \alpha^{(p)}_E$ and $\delta \beta^{(p)}_{\pi \Delta}$ entering into the different variants of chiral EFT. These are given in the following Table 5 together with other contributions. The variants are as follows:

Table 5: Values for the nonresonant $N \pi$ contribution, the counterterm (c.t.), the $\Delta$ pole and the $\Delta \pi$ contribution to the electric $\alpha$ and magnetic $\beta$ polarizabilities in different variants of $\chi$EFT considered in [22]

|     | I $\alpha$ | I $\beta$ | II $\alpha$ | II $\beta$ | III $\alpha$ | III $\beta$ | IV $\alpha$ | IV $\beta$ | V $\alpha$ | V $\beta$ | VI $\alpha$ | VI $\beta$ |
|-----|------------|-----------|-------------|-------------|--------------|-------------|-------------|-------------|-------------|------------|-------------|-------------|
| $N \pi$ | 0.0        | 0.0       | 0.0         | 0.0         | 12.6         | 1.3          | 6.9         | -1.8        | 12.6        | 1.3        | 6.9         | -1.8        |
| c.t.      | 10.5       | 27.0      | 10.6        | -4.4        | -2.1         | 1.4          | 3.6         | 4.5         | -9.8        | -7.1       | -0.8        | -1.2        |
| $\Delta$ pole | 0.0        | 0.0       | -0.1        | 7.1         | 0.0          | 0.0          | 0.0         | 0.0         | -0.1        | 7.1        | -0.1        | 7.1         |
| $\Delta \pi$ | 0.0        | 0.0       | 0.0         | 0.0         | 0.0          | 0.0          | 0.0         | 0.0         | 7.8         | 1.4        | 4.5         | -1.4        |

I: The results for Compton scattering with nucleon and $\pi^0$ Born graphs (Tree graphs), plus polarizabilities as given by the counterterm (c.t.).
II: Tree graphs plus the effects of the (dressed) $\Delta$ s- and u-channel pole graphs.
III: Tree graphs plus $\pi N$ loops: the $O(e^2 \delta^2)$ calculation in heavy-baryon $\chi$EFT without an explicit $\Delta$ degree of freedom.
IV: as (III) but with relativistic nucleon propagator in the $\pi N$ and $\pi \Delta$ loops.
V: the $O(e^2 \delta^3)$ calculation in heavy-baryon $\chi$EFT with explicit $\Delta$, including tree graphs, $\Delta$ poles and HB $\pi N$ and $\pi \Delta$ loops.
VI: as (V), but with relativistic nucleon propagators in the $\pi N$ and $\pi \Delta$ loops.

According to the authors [22] all of these variants of $\chi$EFT are based on the same low-energy symmetries of QCD and in this sense are equivalent. Clearly only the last two versions V and VI in Table 5 are realistic calculations which can be compared with experimental Compton differential cross sections in the resonance region and some way below. A comparison with experimental data at angles from $\theta_{\text{lab}} = 60^\circ$ to $135^\circ$ shows that these two variants of $\chi$EFT which both include the leading $\pi N$ loop effects and an
explicit $\Delta$ are quite similar, provided the counterterms (c.t.) are included and adjusted to yield identical values for the scalar dipole polarizabilities. However, the values found for the two sets of counterterms in the $\chi$EFT variants considered here are rather different. They are particularly large in the $\pi^\text{HB}\Delta$ calculation (column V), especially as compared to the relativistic $\pi\Delta$ one (column VI).

The statements of the authors cited in the foregoing section have to be supplemented in the following way. All the numbers contained in Table 5 are far away from the true values provided by dispersion theory (see Table 4), except perhaps for the $\Delta$ pole contribution. In Table 4 the $\chi$EFT prediction for this contribution is 32% larger than the true value, whereas in Table 5 it is 20% smaller. On the other hand, after decades of experimental work the true value of the $\Delta$ pole contribution has arrived at a precision of close to 1% and it certainly is not acceptable to throw away this precision in an investigation which is aimed to analyze experimental data. Also the use of a counterterm which has no explanation in terms of degrees of freedom of the nucleon is completely unacceptable.

4 Discussion and conclusions

(i) The analysis of Compton scattering and polarizabilities in terms of unsubtracted dispersion relations is precise and well tested experimentally in a large angular range and at energies up to 1 GeV. In this respect the unsubtracted dispersion theory differs from the subtracted dispersion theory \[7\] where the former loses validity already at the peak energy of the $\Delta$ resonance.

(ii) The polarizabilities of the nucleon are determined from experimental data in two ways, firstly by adjusting the predictions of the unsubtracted dispersion theory to the experimental differential cross sections for Compton scattering in the low-energy domain below the $P_{33}(1232)$ peak and secondly by calculating them from the $s$-channel and $t$-channel singularities as provided by high-precision analyses of CGLN amplitudes and well known properties of the scalar $t$-channel. In the latter case no use is made of Compton scattering differential cross sections. The results obtained by these two methods are in excellent agreement with each other.

On the other hand, the different versions of $\chi$EFT make use of uncertain theoretical tools and of predictions which deviate from the true values by large factors. The only possibility to make use of $\chi$EFT in case of Compton scattering and polarizabilities is to introduce counterterms which swallow the large differences between predicted and experimental polarizabilities. These counterterms do not have any interpretation in terms of degrees of freedom of the nucleon and are quite different in the different versions of $\chi$EFT. It is impossible to see how improvements for the precision of the polarizabilities can be obtained when applying these uncertain procedures to the analysis of experimental Compton differential cross sections, as claimed in recent investigations (see \[9\] 23–25 and references therein).

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References

[1] M. Schumacher, Fortschr. Phys. 61, 703 (2013), arXiv:1301.1567 [hep-ph].

[2] A.C. Hearn, E. Leader, Phys. Rev. 126, 789 (1962); R. Köberle, Phys. Rev. 166, 1588 (1968).

[3] J. Bernabeu, T.E.O Ericson, C. Ferro Fontan, Phys. Lett. 49, B 381 (1974); J. Bernabeu, B. Tarrach, Phys. Lett. 69 B, 484 (1977).

[4] A.I. L’vov, V.A. Petrun’kin, M. Schumacher, Phys. Rev. C 55, 359 (1997).

[5] G. Galler et al., Phys. Lett. B 503, 245 (2001).

[6] S. Wolf et al., Eur. Phys. J. A 12, 231 (2001).

[7] D. Drechsel, B. Pasquini, M. Vanderhaeghen, Phys. Rep. 378, 99 (2003), arXiv:hep-ph/0212124.

[8] M. Schumacher, Progress in Particle and Nuclear Physics 55, 567 (2005), hep-ph/0501167.

[9] S. R. Beane, M. Malheiro, J.A. McGovern, D.R. Phillips, U. van Kolck, Phys. Lett. B 567, 200 (2003); Erratum: Phys. Lett. B 607, 320 (2005), arXiv:nucl-th/0209002.

[10] J. Beringer, et al. (Particle Data Group) Phys. Rev. D 86, 010001 (2012).

[11] D. Drechsel, S.S. Kamalov, L. Tiator, Eur. Phys. J. 34, 69 (2007), arXiv:0710.0306 [nucl-th].

[12] A.I. L’vov, S. Scherer, B. Pasquini, C. Unkmeir, D. Drechsel, Phys. Rev. C 64, 015203 (2001), arXiv:hep-ph/0103172.

[13] M.I. Levchuk, private communication (2004).

[14] M. Schumacher, Eur. Phys. J. A 34, 293 (2007), arXiv:0712.1417 [hep-ph].

[15] M. Schumacher, Nucl. Phys. A 826, 131 (2009), arXiv:0905.4363 [hep-ph].

[16] M. Schumacher, Eur. Phys. J. A 30, 413 (2006), Erratum 32, 121 (2007) hep-ph/0609040; M.I. Levchuk, A.I. L’vov, A.I. Milstein, M. Schumacher, Proceedings of the Workshop on the Physics of Excited Nucleons, World Scientific, NSTAR 2005, 389 (2005) hep-ph/0511193.

[17] A.I. L’vov, Phys. Lett. B 304, 29 (1993).

[18] V. Bernard et al., Phys. Rev. Lett. 67, 1515 (1991).

[19] V. Bernard et al., Nucl. Phys. B 373, 346 (1992).

[20] M. Schumacher, Eur. Phys. J. A 31 327 (2007), arXiv:0704.0200 [hep-ph].

[21] Hildebrandt, et al., Eur. Phys. J. A 20, 293 (2004), [nucl.-th/0307070]
[22] V. Lensky, J. M. McGovern, D.R. Philips, V. Pascalutsa, Phys. Rev. C 86, 048201 (2012), arXiv:1208.4559 [nucl-th].

[23] H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman, Prog. Part. Nucl. Phys. 67, 841 (2012), arXiv:1203.6834 [nucl-th].

[24] N. Krupina, V. Pascalutsa, arXiv:1304.7404 [nucl-th].

[25] H.W. Griesshammer, D.R. Philips, J. McGovern, arXiv:1306.2200 [nucl-th].