Paramagnons, weak disorder and positive giant magnetoresistance
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Abstract
At low temperature and for finite spin scattering in a weakly disordered metal, for a certain value, predicted from our theory, of the material-dependent paramagnon interaction, the total conductivity becomes highly sensitive to the orbital effects of a finite magnetic field. As a consequence, positive giant magnetoresistance and giant corrections to the Hall coefficient arise. We obtain very good agreement between this theory and recent positive giant magnetoresistance experiments, while making specific material-dependent predictions.

Recently, there has been a plethora of both experimental and theoretical investigations of giant magnetoresistance (GMR) in metallic systems [1]. In most cases experimentally observed so far, increasing the magnetic field \( H \) from zero causes the resistance to decrease to a fraction of its zero field value. This behavior persists for temperatures ranging from zero to well above room temperature. However, in the experiment of Tsui, Uher and Flynn [2] the observed GMR differs drastically from the usually observed GMR in four ways. 1) The effect only exists at low temperature \( T \), with the magnetoresistance correction reaching \( \sim 35\% \) of the zero field value for \( H \sim 6T \), but vanishing completely above 60°K for \( H \leq 6T \). 2) GMR is anisotropic with regards to the direction of the field \( H \), 3) it is not connected to the magnetization and does not saturate with increasing \( H \), for fields as big as at least 8T, and 4) it is positive, i.e. it increases with \( H \). Save for the giant magnitude of the effect, the four characteristics above can be explained in the frame of the metallic weakly disordered regime \( \epsilon_F \tau \gg 1 \), where \( \epsilon_F \) is the Fermi energy and \( \tau \) the elastic scattering time arising from disorder [3, 4]. In this regime, the conductivity corrections, due to disorder induced diffusion and to electron-electron interactions, are of order \( \sigma_o/(\epsilon_F \tau)^r \), where \( \sigma_o \) is the Drude term and \( r = 1, 2 \) for \( d = 2, 3 \) space dimensions.

We thereby propose a novel mechanism for giant corrections to the transport quantities, including GMR, due to the presence of paramagnons in a weakly disordered metal. At low temperature and for finite impurity spin scattering, for a certain value, predicted from our theory, of the material-dependent paramagnon interaction, the total conductivity becomes highly sensitive to the orbital effects [3] of a finite magnetic field. This is attributed to certain microscopic processes, otherwise negligibly small, which can be enhanced by a resonance factor, emanating from the spin-density channel. Thus an experimental signature like the one observed by Tsui et al. [2] is obtained. As we explain below, the samples used in ref. [2] contain the ingredients necessary for the appearance of GMR, in accordance with our theory.

We begin by considering a constant paramagnon interaction \( A_o \) acting only between particles of opposite spin and given by
\[
A_o = \Phi/N_F,
\]
where \( \Phi \) is dimensionless and positive, and \( N_F \) is the density of states at the Fermi level.

In the presence of weak disorder, which includes spin scattering, the ladder diagrams in the particle-hole channel give rise to a propagator \( A^j(q, \omega) \). \( j = -1, 0, 1 \) is the total spin difference between particle and hole of these spin-density propagators. \( A^j \) obey the coupled Bethe-Salpeter equations
\[
A^1 = A_o + A_o D^1 A^1 + A_o D^0 A^0,
\]
(2)
shown in fig. 1 - here we suppressed the variables $q, \omega$, which stand for the momentum and energy difference between particle and hole lines, respectively. Note the explicit spin indices in the figure. $\mathcal{D}^j$ are given by the components of the density and spin-density correlation functions $\mathcal{D}^{\pm 1} = \mathcal{D}^{1, \pm 1}$, $\mathcal{D}^{0} = [\mathcal{D}^{0, 0} - \mathcal{D}^{1, 0}] / 2$, $\mathcal{D}^{j, m}(q, \omega) = N_F \{ Dq^2 + j 4 \tau_S^{-1} / 3 - i \omega - i m \omega_H \}$, with $\tau_S^{-1}$ being the total spin scattering rate ($\hbar = c = 1$), $D$ the diffusion constant and $\omega_H$ the Zeeman energy. $A^{-1}$ obeys the same set of coupled equations with all spin indices reversed - equivalently $A^{-1}(\omega_H) = A^1(-\omega_H)$. The solution of these equations is written in the form

$$A^j(q, \omega) = \frac{K_{\Phi j} Dq^2 - i L_{\Phi j} \omega + M_{\Phi j}}{A_{\Phi} Dq^2 - i B_{\Phi} \omega + C_{\Phi}},$$

with the coefficients $A_{\Phi}, B_{\Phi}, C_{\Phi}, K_{\Phi j}, L_{\Phi j}, M_{\Phi j}$ depending on $\Phi$ and $H$. There are two limiting cases for $A_{\Phi}, B_{\Phi}, C_{\Phi}$ as a result of the finite spin scattering rate $\tau_S^{-1}$. For $Dq^2 > \omega$ ("static limit") we have

$$A^j = 4(1 - \Phi^2), \quad B^j = 12 - 22 \Phi + 10 \Phi^2 + O(\Omega_H^2), \quad C^j = \left[ (1 - \Phi)^2 + O(\Omega_H^2) \right] 4 \tau_S^{-1}/3,$$

while for $Dq^2 < \omega$ ("dynamic limit") we have

$$A^j = 12 - 20 \Phi + 15 \Phi^2/2 + O(\Omega_H^2), \quad B^j = 4 - 6 \Phi + 3 \Phi^2/2 + O(\Omega_H^2), \quad C^j = \left[ 1 - 2 \Phi + 3 \Phi^2/4 + O(\Omega_H^2) \right] 4 \tau_S^{-1}/3,$$

where $\Omega_H = \omega_H \tau_S$. It is explicitly assumed that $\Phi$ does not approach 1 closely, which would be the onset of the ferromagnetic transition.

The effect of giant magnetoresistance is attributed (see also below) to the combination of a negative $C_{\Phi}$, for $\Phi = 2/3 - 2$, and a vanishing $B_{\Phi}$, for $\Phi \approx \Phi_o \equiv 0.845$, in the dynamic limit. These two conditions can yield a resonance in the denominator of the propagator $A(q, \omega)$, which results in the enhancement of certain diagrammatic processes, otherwise negligibly small, which have a very high sensitivity to the presence of a magnetic field, thus causing the appearance of GMR. We will see below that there are materials satisfying $\Phi \approx \Phi_o$.

We observe that diagrams involving a factor $Z_m = \sum_{\omega} \int dq A^m(q, \omega) F(q, \omega)$ can yield a large contribution if $m = 2n$. This is the case because $Z_m = \sum_{\omega} z_m(\omega)$, with

$$z_{2n} \approx F \int_{x+x_o-a}^{x+a} \frac{dx}{(x-x_o-i\delta)^{2n}} \approx \frac{2 F}{(2n-1) \delta^{2n-1}}.$$

Here $\delta \equiv B_{\Phi} \omega / 2 \to 0$, $x \equiv A_{\Phi} Dq^2$, $x_o \equiv -C_{\Phi} > 0$, $a < x_o$ and $F \equiv F(x = x_o, \omega)$ (finally we will only make use of the case $n = 1$). After appropriate $\omega$ Matsubara summation, a term $B_{\Phi}^{2n-1}$ remains in the denominator yielding an overall enhancement factor.

The relevant dominant class of diagrams containing factors $A^2(q, \omega)$ are shown in fig. 2. These diagrams can be sandwiched between two pairs of $G^R(k, \epsilon)G^A(k, \epsilon)$ to yield conductivity contributions. Let us first take a close look at fig. 2a. The 2 Cooperons are inserted between the 2 $A(q, \omega)$’s for the following reasons. 1) They introduce a magnetic field dependence of the conductivity, which will be in accordance with the experiments of ref. [4] as to the magnitude and direction of the magnetic field etc. 2) If they were absent, the 2 $A(q, \omega)$’s would just collapse onto a single $A(q, \omega)$. 3) An even number of them is needed in order to properly conserve momentum in this diagram. 4) Since each Cooperon introduces a small
factor $1/(\epsilon_F \tau)$, we keep only diagrams with 2 Cooperons, and not 4,6,8,..., between any 2 $A$'s. This small prefactor is counterbalanced by the resonance of $A^{2n}$ we mentioned above.

In passing, let us note that Finkelstein, Castellani, Di Castro, Lee and Ma, and Chang and Abrahams, have shown that the diffusive correlators (diffusion and Cooperon) retain their form in the presence of interactions - modulo a renormalization of the diffusion coefficients, inelastic scattering rate etc. As a result we do not need to consider explicit interaction contributions in the diffusion and the Cooperon.

In the following, we restrict ourselves to the low temperature limit $T \to 0$.

Now we sum the diagrams shown in figs. 2 a,b,c, which give the dominant contribution to the paramagnon conductivity here (to lowest order in the parameter $b_H$ in eq. (3) below). We also take into account the same diagrams but with the propagator $DAD_i$, i.e. a propagator $A(q, \omega)$ sandwiched between two diffusons $D(q, \omega)$. This latter combination has appeared in the works of Altshuler, Aronov, Larkin and Khmelnitskii, Lee and Ramakrishnan, and Millis and Lee, where the usual small magnetoresistance due to weak disorder and interactions was investigated. We find that the total contribution of these diagrammatic blocks is given by the block $\Gamma_H$

$$
\Gamma_H(k, \epsilon) = \Gamma_{1H} - \Gamma_{2H} \frac{G^R(k, \epsilon)G^A(k, \epsilon)}{2\pi \tau \epsilon_F}, \quad \Gamma_{1H} = \alpha \tau^2 b_H, \quad \Gamma_{2H} = \frac{b_H}{2\pi \tau \epsilon_F}.
$$

(8)

Here $G^{R,A}(k, \epsilon) = 1/\{\epsilon - \epsilon_k \pm i/2\tau\}$, $c_H = \sum_q C^2(q, \omega = 0)$ (see eqs. (14),(15) below) and $s = M_{1\Phi}^2 [-C\Phi/(A\Phi D)]/d^{1/2}/(2A\Phi D)$,

$$
b_H = \frac{s^2 t c_H}{B_o^2},
$$

(9)

$t = 5\pi N_F (2\tau)^7/16$, $\alpha = 1 - (2\tau)^2 \left(\frac{1}{3\tau S} + \frac{1}{3\tau S_1}\right)$ and $\tau^{-1} = \tau^{-1} - \tau_{S}^{-1}$ is the scattering rate due to non-magnetic impurities.

Then we take the ladder sum of $\Gamma_H$ as follows

$$
B(k, \epsilon) = \Gamma_H(k, \epsilon) + \Gamma_H(k, \epsilon)G^R(k, \epsilon)G^A(k, \epsilon)B(k, \epsilon) = \frac{\Gamma_H(k, \epsilon)}{1 - \Gamma_H(k, \epsilon)G^R(k, \epsilon)G^A(k, \epsilon)}.
$$

(10)

The total conductivity due to paramagnons is given by

$$
\sigma_P = \frac{2e^2}{m^2} \int d\vec{k} \frac{k^2}{2} \{G^R(k, \epsilon_F)G^A(k, \epsilon_F)\}^2 B(k, \epsilon_F) = -\sigma_o + \sigma_c.
$$

(11)

$\sigma_o$ is the well known Drude term. Finally the total conductivity is given by

$$
\sigma(H) = \sigma_o + \sigma_P = \sigma_c.
$$

(12)

$$
\sigma_c = \frac{4N_F e^2 \epsilon_F}{3m S_H} \left\{ \frac{\Gamma_{1H} - \Gamma_{2H}}{y_+} \arctan \left( \frac{\epsilon_F}{\sqrt{\sigma^2 - y_+}} \right) - \frac{\Gamma_{1H} - \Gamma_{2H}}{y_-} \arctan \left( \frac{\epsilon_F}{\sqrt{\sigma^2 - y_-}} \right) \right\}.
$$

(13)

Here $y_\pm = (\Gamma_{1H} \pm S_H)/2$, $S_H = \sqrt{\Gamma_{1H}^2 - 4\Gamma_{2H}^2}$, and $\sigma \equiv 1/2\tau$. Note that as the temperature $T$ is increased, $\Gamma_H$ decays due to the increasing dephasing rate $\tau_{\phi}(T)^{-1}$ in the denominator of the Cooperons. As a result, the overall magnitude of $\sigma_P$ - and of the magnetoconductivity -
decays, and we recover the usual Drude conductivity. In $d = 2$ with $H$ perpendicular to the system we have

$$c_H = \sum_q C^2(q, \omega = 0) = \frac{y}{H} \sum_{n=0}^{\infty} \left( \frac{1}{aH} + n + \frac{1}{2} \right)^{-2},$$

with $a = 4De\tau_\phi(T)$ and $y = 1/(2(4\pi)^3N_F^2\tau^4e^2D)$. Moreover in $d = 3$ - where we restrict ourselves henceforth -

$$c_H = \sum_q C^2(q, \omega = 0) = \frac{y}{\sqrt{H}} \sum_{n=0}^{\infty} \left( \frac{1}{a_iH} + n + \frac{1}{2} \right)^{-3/2},$$

is the contribution of the pair of Cooperons for a field $H$ along the $i$ space direction, $a_i = 4D_{i\perp}\tau_\phi(T)$, $y_i = 1/(4\pi^3N_F^2\tau^4\sqrt{eD_{i\perp}^3D_i})$, and $i \perp \perp$ stands for the plane perpendicular to the axis $i$. Here we assume that $\tau_\phi^{-1}(T) \gg \tau_S^{-1}$. For materials which are isotropic in the plane ($\parallel$) but anisotropic in the direction perpendicular to the plane ($\perp$), the following relation holds (c.f. also [10] for a similar result)

$$\frac{a_\perp}{a_\parallel} = \left( \frac{y_\perp}{y_\parallel} \right)^{-2} = \frac{D_\perp}{D_\parallel}.\quad (16)$$

Eqs. (13) and (16) yield a very good fit to the GMR data, with $a_\perp = 0.190 T^{-1}$, $a_\parallel = 0.085 T^{-1}$, $\Gamma_1H(H_\perp = 4.58 T) = 0.4524 \sigma^2$ and $r = 4a^3/(\pi\epsilon_Fa) = 0.03 \sigma^2$. We have assumed that $\epsilon_F \gg \sqrt{\sigma^2 - y_\perp}$, thus using a total of 2 parameters per curve.

The disorder in the Dy/Sc superlattices arises probably mostly from the interfaces, with Dy ($A_{Dy} = 66$) providing spin-orbit scattering. Interface disorder (roughness) in superlattices amounts to effective “bulk” disorder and anisotropic diffusion coefficients [12].

A big effective mass $m^*$ is required in order to fit this theory with the data of ref. [2].

This condition follows from the decay of the Cooperon for $H > H_\phi$, where the dephasing field $H_\phi \sim m^*/\epsilon_F\tau_\phi[3]$. Actually $m^* > 40m$, $m$ being the free electron mass, for hcp Sc near the Fermi surface and close to the points H and L [13, 14]. However, GMR due to paramagnons can in principle appear for any $m^*$.

The sine qua non condition for paramagnon-induced GMR is the closeness of the constant $\Phi$ of the material in question to $\Phi_o$. As in the limit of high electronic density the RPA interaction is appropriate we ought to compare the corresponding renormalized values $\Phi_{eff}$ with $\Phi_o$, not the bare $\Phi$ itself. In the $q \to 0$ static limit this procedure leads to the substitution

$$\Phi \to \Phi_{eff} = \frac{\Phi}{1 - \Phi^2}.\quad (17)$$

The bulk constant $\Phi$ has already been calculated within a band structure scheme for a number of elements by Sigalas and Papaconstantopoulos [4]. Inserting $\Phi$ for hcp Sc in $\Phi_{eff}(\Phi)$ yields a difference of 4.6% from $\Phi_o = 0.845$. Hence it is consistent to anticipate gigantic conductivity corrections for Sc and Sc based materials.

This same sort of GMR has been seen in Sc films with disorder and also Er/Sc and Dy/Sc/Y superlattices [15], i.e. only materials containing Sc. One may hypothesize that the very high density of states of Sc around the Fermi level [3] make Sc the dominant element and it is appropriate to use the value of $\Phi$ calculated for bulk Sc, as it would not change much for a Sc based superlattice. GMR is not seen in Y films, or Y based superlattices [12], although Y, lying in the same column of the periodic table as Sc just one row below it, has a very similar
band structure to Sc. Nevertheless here the difference of $\Phi_{\text{eff}}$ from $\Phi_o$ is -14%, and large transport corrections cannot arise.

With regards to further predictions (or retrodictions) now, $\Phi_{\text{eff}}$ of (fcc) Pt differs from $\Phi_o$ by 6.9% and $\Phi_{\text{eff}}$ of (bcc) Rh differs by 0.4% (they both have high $m^*$’s and $N_F$ as well). In an experiment done with polycrystalline samples of Pt and Rh by Schulze [14] in 1941, positive GMR was seen for both materials. It is likely that this was the first signature of the paramagnon GMR seen experimentally. Further, $\Phi_{\text{eff}}$ for (fcc) Cr differs by 4.5% from $\Phi_o$. More, the high-$T_c$ superconductor Tl$_2$Ba$_2$CuO$_{6+\delta}$ (Tl-2201) exhibits this sort of positive GMR [17], and in all appearances the quasi 2-d metal $\alpha$-(BEDT-TTF)$_2$KHg(SCN)$_4$ [18]. Apparently the value of $\Phi$ for these materials is not known at present.

Our discussion so far presupposes that the microscopically calculated bare value of $\Phi$ [14] is not really renormalized by paramagnons, disorder etc. - except $\Phi_{\text{eff}}$ of course. Hertz et al. [19] have shown that there is no Migdal’s theorem for paramagnons. In that case, the first and second order vertex corrections are of the same order of magnitude as the bare paramagnon vertex, and presumably this is so for higher vertex corrections. However already the 2nd order correction comes with a minus sign, and it is possible that a converging power series is thus formed for the total paramagnon vertex, yielding a result close to the bare value. Also, it is not unlikely that the self-energy and vertex corrections cancel each-other, as far as the $\Phi$ renormalization is concerned, with the proviso above for $\Phi_{\text{eff}}$, in a manner analogous to ref. [20].

A few more comments about the ramifications of this theory are in order. The correction to the density of states is small. The correction to the Hall coefficient $R_H = \rho_{xy}/H$, with $\rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$, is usually given with the assumption that $\Delta R/R$ is small, as e.g. in the work of Houghton et al. [21]. Here this is not the case and, assuming the cyclotron energy $\omega_c \ll \tau^{-1}$, we obtain

$$\frac{\delta R_H}{R_H} = \frac{1 + \delta_{xy} - (1 + \delta_{xx})^2}{(1 + \delta_{xx})^2}, \delta_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}}$$

where $\sigma_{ij}$ is the usual Drude term and $\sigma_{ij}$ the paramagnon contribution discussed above. Probably this explains the gigantic correction to the Hall coefficient at low temperature seen in ref. [22].

In summary, we have shown that paramagnons in the weakly disordered regime can yield positive giant magnetoresistance at low temperatures. The theory not only agrees with experiment so far, but makes specific material-dependent predictions for future experiments as well.

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Figure Captions

Fig. 1. The coupled Bethe-Salpeter equations obeyed by $A^i$. Note the explicit spin indices corresponding to the various parts of the diagrams.

Fig. 2. The basic diagrammatic blocks. The dashed lines in figs. b and c denote impurity scattering.

Fig. 3. Plot of the giant magnetoresistance $\Delta R/R(0)$, $\Delta R = R(H) - R(0)$. The points are the experimental data of ref. [2] for a typical Dy/Sc superlattice at $T = 10^\circ$K and the lines the theoretical fits, from eq. (13), with the constraint (16). The upper and lower lines correspond to the field $H$ being parallel and perpendicular to the superlattice growth axis.
\[ A' = A_0 + A_1 + A_2 + A_3 \]

\[ A^0 = A_0 + A_1 + A_2 + A_3 \]

Fig. 1
