The strange transformation for point rotation coordinate frames and its experimental verification

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(Dated: March 31, 2022)

We consider the general form of the linear transformation for point rotation coordinate frames. The frames have the rotation axis at every point. In the transformation the frequency of one frame relative to another is not equivalent to the reverse frequency. Using symmetry of the direct and reverse transformation as well as symmetry of the frame coordinates we show that two different type of the transformation are possible. The first type is a generalization of the Lorentz transformation. This case can not be checked in optical measurements. In contrast to that the second unusual type allows us to observe consequences of the transformation in an optical experiment even though the characteristic constant inherent in the transformation is less than “nuclear time” of the order of $10^{-23}$ sec. We describe the experiment.

PACS numbers: 42.50.Xa, 03.65.Ta, 06.20.Jr, 06.30.Gv

I. INTRODUCTION

The concept of a “point rotation frame” arises in crystalloptics. Distinctive feature of the frame, in contrast to Cartesian one, is existence of the rotation axis at every point. In such a frame the axes are constructed on field amplitudes and only the axis direction is essential. Similar (non-rotating) frames have been used in quantum field theory for a long time. An example of the frame is the rotating optical indicatrix (index ellipsoid). One more difference in comparison with the Cartesian frame is the absence of centrifugal forces in the point rotation frame. The frame coordinates is an angle (phase) and time, the frequency of rotation is a parameter. In electro-optical crystals the rotation is stimulated by an applied rotating electric field. In crystals with the linear (Pockels) effect the frequency equals half (and even quarter) of that of the electric field. In the Kerr crystals this frequency is doubled\[2,3\]. The sense of rotation of the plane circularly polarized light wave moving through the electro-optical crystal with rotating optical indicatrix is reversed and the optical frequency is shifted if the amplitude of the applied electric field equals the half-wave value. The device for the shift by means of electro-optical crystals is the single-sideband modulator\[4,5,6\]. Note that optically the rotating phase plate is equivalent to the modulator but physically they are different as the plate has only one axis of rotation.

It is convenient to use for the description of circularly polarized plane light wave in the single-sideband modulator the transition to a frame with the resting optical indicatrix. Apparently, for the first time, convenience of that had been described in initial works on single-sideband modulation of light\[4\]. Such a transition results in change of the optical frequency. The change equals the frequency of the optical indicatrix. After the polarization reversal and returning back to the initial frame the frequency deviation is doubled. Emphasize that in the frame with the resting indicatrix the modulating electric field is also at rest in spite the fact that both the indicatrix and field rotate at different frequencies relative to the initial frame?! This is one further unusual and strange property of the point rotation frames.

The transition to the rotating frame always is connected with the question what is the frequency superposition law, is it linear or not. The nonlinear law always corresponds to an extra frequency shift. Emphasize that the consideration in the framework of the Maxwell equation can not give such an extra shift. The situation is analogous to a comparison between results obtained for rectilinear move with help of the Lorentz transformation and the Newton mechanics.

In Ref.\[7\] the question was considered in assumption that the combined frequency may be presented in terms of power series of two other frequencies. It was also assumed that the frequency of one frame relative to another equals to the reverse frequency. The only difference is the sign. The negative sign corresponds to the rotation in opposite direction. In this condition the extra shift in the first approximation is proportional to the product of the optical frequency squared and the modulation frequency. The characteristic constant in the extra shift has dimension of time. In Ref.\[7\] a optical experiment was also proposed for measurement of the term and it was shown that a lower limit for measurements of the characteristic constant with such a form of the shift is about $10^{-17}$ sec.

Shortly after it was shown that an analogy exists between the light propagation in medium with the rotating optical indicatrix and the motion particle in the rotating magnetic field and both the phenomena can be described in the framework of the Pauli equation\[8\]. In other words a plane circularly polarized light wave propagating along the optical axis of 3-fold electro-optical crystal under the action of an applied electric field possesses properties of two-component spinor. It means that measurements of the optical frequency shift in the single-sideband modulator is similar to measurements of the magnetic moment in the magnetic resonance and anomalous magnetic moment may be associated with the nonlinear frequency
shift. It was understood that the probable value of the characteristic constant is, as maximum, of the order of “nuclear time” \( \sim 10^{-23} \) sec. Such a small value excludes possibility to observe in optical experiments the term calculated in Ref. [5].

Meanwhile the immediate way to determine the frequency superposition law is the transformation for the point rotation frames. Emphasize that the Maxwell equation do not contain any information about the transformation. The transformation must be postulated.

In the given paper we consider general linear transformation for the point rotation frames. We use symmetry of frame coordinates and assume that the reverse frequency is a function of the direct frequency with the same function in vice versa. We show that two different types of the transformation exist. The first type is a generalization of the Lorentz transformation. The type in an experimental sense corresponds to the case of Ref. [6]. The second type is principally different. The type give us a chance to measure the extra term. We describe an optical experiment for the measurement of the term. The experiment keeps the main features of that in Ref. [7].

II. GENERAL LINEAR TRANSFORMATION

The general form of the linear transformation for the transition from one frame to another can be written as follows

\[
\varphi' = q(\varphi - \nu t), \quad \tilde{t} = \frac{q^2 - 1}{q^2} \varphi - \frac{q}{\nu} t,
\]

(1)

where \( \varphi \) and \( t \) is an angle (phase) and time, tilde corresponds to the reverse transformation

\[
\varphi = \tilde{q}(\varphi - \tilde{\nu} \tilde{t}), \quad t = \frac{\tilde{q}^2 - 1}{\tilde{q}^2} \varphi - \frac{\tilde{q}}{\tilde{\nu}} \tilde{t},
\]

(2)

\( \nu \) is the frequency of second frame relative to first one. It is obvious that Eq. 1 turns out into Eq. 2 if variables with tilde change to variables without tilde and vice versa.

First of all we exclude from the consideration the Galilean transformation, i.e., the case \( q \equiv 1 \). This case with its infinite frequencies seems unbelievable from the viewpoint of contemporary physics.

Making normalization

\( \varphi \rightarrow \varphi \sqrt{\nu/\nu'}, \quad t \rightarrow t, \quad q \rightarrow q \nu/\nu', \quad \nu \rightarrow \sqrt{\nu/\nu'} \) or

\( \varphi \rightarrow \varphi \sqrt{\nu'}, \quad t \rightarrow t \sqrt{\nu'/\nu}, \quad q \rightarrow q \sqrt{\nu'/\nu}, \quad \nu \rightarrow \sqrt{\nu/\nu'}, \)

equa

except when the frames are at rest. In this case if the rotation axis coincides with some Cartesian axis we may associate \( \varphi \) and \( t \) with the Cartesian cylindrical angle and time. The above normalization would result, in particular, in the change of the speed of light. Therefore we must consider here the general case assuming that \( \nu \) is a function of \( \nu \). It means that rotations to the right and left are not equivalent in the approach.

For the point rotation frame we have not a general principle like the relativity principle for the Cartesian frames, however we use principle of symmetry instead. It means, in particular, that if \( \nu(\nu') = f(\nu) \) then \( \nu(\nu') = f(\nu') \).

The function \( \nu(\nu) \) and \( q(\nu) \) remain indeterminate except the condition at small \( \nu \), namely, \( \nu \rightarrow -\nu, \quad q \rightarrow 1 \) if \( \nu \rightarrow 0 \). If the characteristic constant \( \tau \) is of the order \( \sim 10^{-23} \) sec. then the normalized frequency \( \tau \nu \) even in microwave range is about \( 10^{-12} \). Therefore we assume that the function \( \nu(\nu) \) may be expanded in the powers series in \( \nu \)

\[
\nu(\nu) = -\nu + a_2 \nu^2 + a_3 \nu^3 + a_4 \nu^4 + ..., \quad (3)
\]

This expansion is compatible with the reverse expansion at certain conditions for the coefficients \( a_n \), namely, the expansion can not contain only odd powers of \( \nu \) and up term \( \nu^{2n} \) has only \( n \) independent coefficients \( a_n \). Obviously that any expansion \( 3 \) together with the reverse expansion can be written in the symmetric form with help of finite or infinite series

\[
\nu(\nu') = \sum_{n=1} b_n(\nu')^n \equiv F(\nu), \quad (4)
\]

where \( F \) is a function. Results below will be also valid for arbitrary \( F(\nu) \) with the only condition \( F(0) = 0 \).

The main problem in the given approach is the nonlinear frequency shift. However since it is defined by product \( \tilde{q} q \) we do not need to know the explicit form of function \( q(\nu) \). For finding the form of \( \tilde{q} q \) we use symmetry between \( \varphi \) and \( t \). Transformation 11 can be written as

\[
\tilde{t} = Q(t - \Lambda \varphi), \quad \tilde{\varphi} = \frac{Q \nu - 1}{Q \Lambda} t - Q \frac{\Lambda}{\Lambda} \varphi, \quad (5)
\]

where role of \( (\varphi, t, q, \nu) \) is played by \( (t, \varphi, Q, \Lambda) \) respectively and

\[
Q = -q \frac{\nu}{\nu'}, \quad \Lambda = (1 - \frac{1}{qq}) \frac{1}{nu} \nu'. \quad (6)
\]

It is naturally to assume that the equality in Eq. 3 would keep if \( \Lambda/\sigma, \tilde{\Lambda}/\sigma \) is substituted for \( \nu, \tilde{\nu} \). Here \( \sigma \) is a dimensional constant. Making use of the substitution and excluding \( (\nu + \nu) \) we obtain the equation for \( \tilde{q} q \)

\[
F \left( \frac{\Omega^2}{\nu^2} \right) - \frac{\Omega}{nu} F(\nu) = 0, \quad (7)
\]
where Θ = (1 − 1/qq)/σ. In the given case σ = ±r².

Two type solutions of Eq. (7) exist. First type is exact solution Θ = δν or
\[ 1 - \frac{1}{qq} = \sigma \delta \nu. \]  
(8)

Transformation (1) for this case is a generalization of the Lorentz transformation. Without loss generality we use here the term the Lorentz transformation in spate the fact that σ may be as positive as negative. From the viewpoint of the experimental checking this case is equivalent to results of Ref. [7].

The second type of solutions may be presented as series
\[ 1 - \frac{1}{qq} = \sigma (r \delta \nu)^{\frac{1}{2}} + \sigma \sum_{n=2} (r_n \delta \nu)^{\frac{1}{2^n}}, \]  
(9)
where r is a negative root of equation \( F(r) = 0 \). The number of such solutions equals the number of zeros of \( F(r) \). A necessary condition for existence of the solutions is \( n \geq 2 \) in expansion (1). First term in the right part of Eq. (4) determines the frequency superposition law in the first approximation. The characteristic constant in the given case is
\[ \tau = \sigma \sqrt{-r}. \]  
(10)
For simplicity we use the same letter for the characteristic constant in both the types of solutions. Note that expansion (9) is valid for small ν. However at zeros of \( F(\delta \nu) \) exact equality \( \delta \nu = -\nu \) holds, i.e., values \( ν = \pm \sqrt{|r|} \) are some distinctive points.

The second type of solutions adds to the list one further strange property of the point rotation frames. Normalization
\[ \nu^* = \sqrt{-\nu}, \quad \varphi^* = \varphi \nu \nu, \quad t^* = t, \quad q^* = q \nu \nu \]  
(11)
imparts the Lorentz shape to Eq. (1).

\[ \varphi^* = q^* (\varphi^* - \nu^*) t, \quad \tilde{t}^* = q^* (-\nu^* \varphi^* + t). \]  
(12)
If \( \nu^* \to 0 \) then \( \nu^* \to \infty \). On the other hand the non-normalized transformation at \( \nu \to 0 \) tends to the Galilean form
\[ \varphi = \varphi, \quad \tilde{t} = \tau \varphi + t, \]  
(13)
where \( \varphi \) and \( t \) switch places. Term \( \tau \varphi \) is very small because of the small value of \( r \).

In accordance with Eq. (10) consider the time \( \Delta \tilde{t} = \tau \Delta \varphi + \Delta t \) and angle \( \Delta \tilde{\varphi} = \Delta \varphi \) intervals. The time interval measured at the same value of angle \( \Delta \varphi = 0 \) is quite determined \( \Delta \tilde{t} = \Delta t \) whereas at the same time \( \Delta (\tilde{t} = 0) \) a time leap \( \Delta \tilde{t} = \tau \Delta \varphi \) exists. The leap is the time of the rotation through angle \( \varphi \) at frequency \( 1/\tau \).

Since Eq. (10) is the frame transformation into itself the result may be interpreted as an uncertainty of the time determination. The maximal value of the leap is \( 2\pi \tau \) as at \( \varphi = 2\pi \) the frame also coincides with itself.

If the second type truly corresponds to physical reality then a lower limit for measurements of the characteristic constant may be drastically decreased.

### III. Frequency Superposition

Consider a plane circularly polarized light wave moving through an electro-optical crystal with the rotating optical indicatrix. The light and the indicatrix, for definiteness, are assumed to rotate in the same direction with frequencies \( \omega \) and \( \nu \). In correspondence with Eq. (1) the optical frequency in the frame with the resting optical indicatrix is
\[ \omega' = \frac{\omega - \nu}{\sigma \Theta \omega/\nu - \nu/\nu}. \]  
(14)

It is obvious that the reverse transition result to exact equality
\[ \frac{\omega' - \nu}{\sigma \Theta \omega'/\nu - \nu/\nu} \equiv \omega. \]  
(15)

However if the reversal rotation occurs then instead of \( \omega' \) we must substitute \( -\omega' \) in Eq. (15). After simplification we obtain for the output frequency
\[ \omega_{\text{out}} = \frac{-\omega + 2\nu - \sigma \Theta \omega}{2\sigma \Theta \omega/\nu + \sigma \Theta + 1}. \]  
(16)

For the solution of the first type \( 1 - 1/qq = \sigma \nu \). Taking into account that \( \nu < \omega \) and \( \tilde{\nu} = -\nu \) for small \( \nu \) we obtain from Eq. (10) in the first approximation
\[ \omega_{\text{out}} \approx -\omega + 2\nu + 2\tau^2 \nu^2. \]  
(17)

The extra frequency shift \( 2\tau^2 \nu^2 \) is an equivalent of that in Ref. [7]. The shift cannot be measured optically because the very small characteristic constant \( \tau \). Even if the modulating frequency is a powerful optical wave \( \nu \approx 10^{14} \) Hz then at \( \tau \approx 10^{-23} \) sec. the extra shift \( 2\sigma \nu^2 \) can not be picked out in laser or photodetector noises.

Consider the second type of solutions with \( 1 - 1/qq \approx \tau \sqrt{-\nu} \nu \). In the first approximation
\[ \omega_{\text{out}} \approx -\omega + 2\nu - 2\tau \omega^2. \]  
(18)

The extra shift \( 2\tau \omega^2 \) do not depend on \( \nu \), i.e., such a shift must be produced by the usual half wave plate! Emphasize that from the viewpoint the Maxwell equation the frequency shift in the single-sideband modulator is a consequence of the phase difference between two component of the electric field of light wave whereas from the viewpoint of photons it is something different.

The extra shift may be interpreted as an energy of the polarization reversal. The sign difference of the energy corresponds to the assumption on inequivalence of the right and left rotation. The shift of the second type may far exceed the shift of the first type. The relative value of the extra shift for \( \tau \approx 10^{-23} \) sec. is \( 2\tau \omega \approx 10^{-8} \) in visible range.
FIG. 1: Schematic of experiment. P is the polarizer

IV. MEASUREMENT OF THE EXTRA SHIFT

Schematic of the experiment for measuring the extra shift of the second type is shown in Fig. 1. Linearly polarized light from laser passes through the single-sideband modulator (for example Lithium Niobate modulator [5]). A electric field rotating at frequency $\Omega = 2\nu$ is applied to the modulator. Evolution of the laser spectrum under change of the amplitude and frequency of the electric field may be observed by means of the scanning interferometer. The linearly polarized light is a sum of two circularly polarized waves of frequencies $\omega$ and $-\omega$. The modulator changes the frequencies to $\omega + \Omega - 2\tau\omega^2$ and $-\omega + \Omega - 2\tau\omega^2$ respectively. After the paralyzer light is modulated in intensity at frequency $2\Omega - 4\tau\omega^2$. The extra shift $4\tau\omega^2$ could be extracted by heterodyning as it is in Ref. [7]. However in the given case the schematic is simplified (heterodyne and doubler are crossed out in Fig. 1) since resonance $\Omega = 2\tau\omega^2$ can be used with matching the sign of $\tau$ by the reversal of the applied electric field. The shift can be measured with confidence if the characteristic constant is about $10^{-23}$ sec. Note that if the extra shift really exists then similar schematic may be effectively used for precision measurements in spectroscopy.

V. CONCLUSION

The idea of inequivalence of the direct and reverse frequencies and the principle symmetry leads to two types of the transformation for the point rotation frames. The first type is a generalization of the Lorentz transformation. The type seems more applicable to the Cartesian frames. Measurements of the extra frequency shift in this case lies beyond possibilities of optics. The second type is applicable only to the point rotation frames. An unusual and strange property of this type is uncertainty of the time determination. The extra shift in this case may be verified in optical measurements if the characteristic constant in the transformation is about (and even less) $10^{-23}$ sec.

Another question arises in the above construction: is parity violation connected with the inequivalence of the direct and reverse frequency?

[1] B. V. Gisin, Kristallografia, 37, 218 (1992) [Sov. Phys. Crystallogr. 37(1), (1992)].
[2] L. P. Kaminov, An introduction to electrooptical devices (Academic press, New York, 1974).
[3] J. F. Nye, Physical Properties of Crystals (University Press, London, 1964).
[4] D. H. Baird and C. F. Buhrer, Single-Sideband Light Modulator (U.S. Patent 3204104, 1965); C. F. Buhrer, D. H. Baird, and E. M. Conwell, Appl. Phys. Lett. 1, 46 (1962).
[5] J. P. Campbell, and W. H. Steier, IEEE J. Quantum Electron. QE-7, 450 (1971).
[6] B. V. Gisin, Electro-optical single-sideband modulators: problems and applications, Selected Papers from Photonics-98, SPIE Proceedings, 3666, 132 (1999).
[7] B. V. Gisin, Phys. Rev. A 50, 2003 (1994).
[8] B. V. Gisin, Phys. Lett. A 209, 285 (1995).
[9] B. V. Gisin, Phys. Rev. A 61, 53808 (2000).