Federated Unbiased Learning to Rank

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ABSTRACT
Unbiased Learning to Rank (ULTR) studies the problem of learning a ranking function based on biased user interactions. In this framework, ULTR algorithms have to rely on a large amount of user data that are collected, stored, and aggregated by central servers.

In this paper, we consider an on-device search setting, where users search against their personal corpora on their local devices, and the goal is to learn a ranking function from biased user interactions. Due to privacy constraints, users’ queries, personal documents, results lists, and raw interaction data will not leave their devices, and ULTR has to be carried out via Federated Learning (FL).

Directly applying existing ULTR algorithms on users’ devices could suffer from insufficient training data due to the limited amount of local interactions. To address this problem, we propose the FedIPS algorithm, which learns from user interactions on-device under the coordination of a central server and uses click propensities to remove the position bias in user interactions. Our evaluation of FedIPS on the Yahoo and Istella datasets shows that FedIPS is robust over a range of position biases.

KEYWORDS
Federated learning; Unbiased learning to rank; Implicit feedback

1 INTRODUCTION
Learning to Rank (LTR) has been extensively studied in offline, online, and unbiased setups [3, 14, 16–18]. In this paper, we study LTR problems in the Federated Learning (FL) paradigm [20–22], where the goal is to learn a ranking function from user interactions without centralized data collection. In Federated LTR (FLTR), a ranker is trained locally (e.g. on mobile devices or PCs), under the federation of a central server. Local devices only send essential parameters (e.g. local gradients) to the server. On-device search is one of the most important applications that motivates this setup. In this application, users search against their personal corpora on their local devices, and the goal is to learn a ranker from biased user interactions. Due to privacy constraints, users’ queries, personal documents, results lists and raw interaction data will not leave their devices. Since raw data is not centrally collected, FL is not limited by data retention requirements, server-side storage, bandwidth and computational capacities, which enables learning with an unprecedented amount of data. More importantly, FL brings new potential for protecting user privacy. For example, with differential privacy, FL enjoys theoretical privacy guarantees [5, 10, 11, 22]. Instead of tackling all these important theoretical and practical aspects of FLTR, in this paper, we focus on an important and challenging problem: federated LTR under biased user interactions. To avoid wordiness we call this problem Federated Unbiased Learning to Rank (FULTR).

There are three main challenges in FULTR. Firstly, user interactions are almost always biased due to the nature of ranking [9, 14]. For example, position bias has a significant influence on user search behaviors: a document rendered at the top of a results list is more likely to be viewed and clicked than similar documents ranked at lower positions. Without addressing the position bias, a trained ranker is generally suboptimal. Secondly, in on-device search, the number of user interactions on each local device could be very limited compared with traditional server-side LTR, hence directly applying existing Unbiased Learning to Rank (ULTR) algorithms on each local device only leads to a suboptimal ranker. Finally, users only have access to their own corpora, and their behaviors are typically heterogeneous. Data distributions in each device could vary drastically [21, 22]. This violates the typical Independent Identically Distributed (i.i.d.) assumption in existing ULTR algorithms.

To tackle these challenges, we propose a simple FedIPS algorithm. FedIPS works in rounds. At the beginning of each round, a federator (central server) broadcasts an initialized ranker to participating devices. Then, local devices conduct an Inverse Propensity Score (IPS)-weighted Stochastic Gradient Descent (SGD) with their local data individually. The federator estimates the "pseudo global gradient" by combining all local gradients, and conducts a server-side SGD to update the ranker.

We showed that FedIPS generates an unbiased ranker. We also extensively evaluated the performance of FedIPS in simulated experiments on the Yahoo[7] and Istella [19] datasets. These simulations mimic a real-world scenario, where all user interact with their local devices independently and leave local interactive feedback. Our experimental results indicate that FedIPS is robust over a wide range of bias levels.

2 BACKGROUND AND RELATED WORK
We start with the notations used in this paper. Let \( Q \) be a set of queries and each \( q \in Q \) has a set of retrieved documents \( D_q \). Given query \( q \), each document \( d \in D_q \) has a binary relevance \( r_{q,d} \in \{0,1\} \). Let \( f_w \) be the ranking function parameterized by \( w \). We denote \( R_q,f_w \) as the ranked list generated by \( f_w \), \( R_{q,f_w}(k) \) as the \( k \)th document in \( R_q,f_w \), and \( R_{q,f_w}^{-1}(d) \) as the position of \( d \) in \( R_q,f_w \).

Federated learning. Federated Learning (FL) optimizes the following objective in a decentralized way [20, 21]:

\[
\min_w \ell_f(w) = \frac{1}{|U|} \sum_{u \in U} \frac{1}{|E_u|} \sum_{e \in E_u} \ell(e, f_w),
\]

(1)

where \( U \) is a subset of users, \( E_u \) is each user’s local dataset, \( \ell_f(w) \) is the global loss function, and \( \ell(e) \) is the local loss function. Each user interacts with local devices independently. FedAvg is the earliest FL algorithm [21]. Reddi et al. [24] proposed a more general framework FedOpt with adaptive optimizers.

Unbiased LTR. A common approach to address position bias is to assume that user click behavior follows Position Based Click...
Model (PBM) [8] and use Inverse Propensity Score (IPS) to compensate the difference between true relevance and clicks [1, 2, 14, 25]. Agarwal et al. [1] propose a general framework, which optimizes the IPS-weighted additive metric over clicked documents:

$$\ell(f_u|q, D_q) = \frac{g(\mathbb{R}^{-1}_{q,f_u}(d))}{p_{q,d}}.$$  \hspace{1cm} (2)

where $c_{q,d} \in \{0,1\}$ is the click indicator. $p_{q,d}$ is the examination probability also known as propensity. $g(\cdot)$ is a position-based weighting function capturing different ranking metrics: for example DCG, Precision@k, etc. [1, 13]. For simplicity, in this paper we choose $g(\mathbb{R}^{-1}_{q,f_u}(d)) = \mathbb{R}^{-1}_{q,f_u}(d)$, and our results hold for any other additive ranking metrics.

The additive metric in Eq. (2) is not differentiable w.r.t. $w$. In practice, we consider a surrogate loss [1], which upper bounds $\mathbb{R}^{-1}_{q,f_u}(d)$ in Eq. (2) as follows:

$$\mathbb{R}^{-1}_{q,f_u}(d) \leq 1 + \sum_{d' \in D_q} \max(0, 1 - (f_u(q,d) - f_u(q,d'))).$$  \hspace{1cm} (3)

Combining Eqs. (2) and (3), we reach to the following surrogate loss function, which is used in our FULTR setup:

$$\ell(w) = \frac{1}{|Q|} \sum_{q \in Q} \sum_{d \in D_q} \frac{h_{q,d}(w)}{p_{q,d}},$$  \hspace{1cm} (4)

where $h_{q,d}(w) = \sum_{d' \in D_q} \max(0, 1 - (f_u(q,d) - f_u(q,d')))$. In IPS-based methods, the propensity score is usually assumed to be known. We make a similar assumption in this paper. In practice, the propensity is either estimated by controlled experiments [2, 14] or inferred from interaction data [3, 25].

**Federated LTR.** FL has recently drawn attentions from the LTR community due to growing body of research addressing privacy preservation. Kharitonov [15] studies the federated online LTR and proposed FOLTR. Hartmann et al. [12] uses FL to solve the URL suggestion task for web browsing experience. Anelli et al. [4] propose FedeRank, which is a FL version of matrix factorization. Different from these methods, our focus in this paper is to deal with the position bias in logged data.

3 FEDERATED UNBIASED LTR

In our FULTR setup, every client shares the same production ranking policy $f_0$, which can be synchronized during software updates. Each user interacts with the ranking policy independently and leaves implicit feedback. The issued queries, displayed documents, and interactive feedback are stored in local devices. We do not make any assumption on the local data $Q_u$, meaning that their distributions can be heterogeneous. Learning in this setup has at least three challenges: (1) The feedback is influenced by the position bias. (2) The amount of local data $Q_u$ can be small. (3) The i.i.d. assumption does not hold, hence existing ULTR algorithms may not be directly applied.

3.1 FedIPS

We propose FedIPS to address these challenges. FedIPS employs FedOpt, a general FL algorithm, to deal with the non-i.i.d. challenge in FL. Each client $u$ conducts the IPS-weighted SGD and minimizes the following objective function:

$$\ell_{u,IPS}(w_u) = \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} \frac{g(\mathbb{R}^{-1}_{q,f_u}(d))}{p_{q,d}} c_{q,d}.$$  \hspace{1cm} (5)

The goal of the global optimizer is to solve the following problem:

$$w^* = \arg \min_w \ell_{u,IPS}(w) = \arg \min_w \frac{1}{|U|} \sum_{u \in U} \ell_{u,IPS}(w),$$  \hspace{1cm} (6)

where $\ell_{u,IPS}(w)$ is the global objective.

Details of FedIPS are summarized in Algorithm 1. Inputs of the algorithm are: the initialized model $w_0$, a global learning rate $\eta_g$, and a local learning rate $\eta_l$. At the beginning of each round $t$, a subset $U$ of users is randomly drawn and the model $w_t$ is broadcast (Line 2 - 3). Each client $u \in U$ conducts SGD according to its local data $Q_u$ (Line 5 - 8). As the objective function in Eq. (5) is not differential, the additive metric is replaced by a surrogate metric, e.g. Eq. (3). Thus, each client $u$ minimizes the following objective function:

$$\ell_{u,S}(w_{t,u}) = \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} \frac{c_{q,d} h_{q,d}(w_{t,u})}{p_{q,d}},$$  \hspace{1cm} (7)

where the surrogate loss $h_{q,d}(w_{t,u})$ is defined in Eq. (4). After the local optimization, local updates $\Delta_{t,u} = w_{t,u}^* - w_{t-1,u}$, are sent to the federator (Line 9). On the server side, the federator estimates the "pseudo gradient" by taking the average of all local updates, and use the server-side SGD to minimize the global loss (Line 11). The algorithm runs for $T$ rounds and outputs the optimized ranker $w^*$.

3.2 Unbiased Estimator of FULTR

In a full-information LTR setup, we have relevance labels, and optimize the following additive metric:

$$\Psi(f_u|q, D_q) = \sum_{d \in D_q} g(\mathbb{R}^{-1}_{q,f_u}(d)) r_{q,d}.$$  \hspace{1cm} (8)

In FULTR, each client solves the problem in Eq. (5) using the feedback as "pseudo label" instead of the relevance ground truth. In this section, we show that the metric in Eq. (5) is an unbiased estimator of the full-information metric in Eq. (8). The proof methodology is inspired by [1]. First, we denote $c_{q,d} \in \{0,1\}$ as the examination indicator of a document $d$. $c_{q,d} = 1$ means $d$ is examined by the user, otherwise it is 0.
Algorithm 1 FedIPS

Input: $w_0$, $\eta_g$ and $\eta_l$

Output: $w^*$

1: for $t = 0, 1, \ldots, T - 1$
2: Sample a client subset $\mathcal{U}$
3: $w_{t,u} \leftarrow w_t, \forall u \in \mathcal{U}$ // Broadcast $w_t$
4: for $u \in \mathcal{U}$ do // In parallel
5: for $d_i \in D_u$ and $q \in Q_u$ do
6: $g_u \leftarrow \frac{1}{p_u} \nabla h_{q,d_i}(w_{t,u})$ // Client gradient
7: $w_{t,u} \leftarrow w_{t,u} - \eta_g g_u$ // Client update
8: end for
9: $\Delta_{t,u} \leftarrow w_{t,u} - w_t$
10: end for
11: $w_{t+1} \leftarrow w_t + \eta_g \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \Delta_{t,u}$ // Sever update
12: end for
13: $w^* \leftarrow w_T$

Given a user $u$, we have:

$$
\mathbb{E}_c \left[ \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)c_{q,d} \right] = \mathbb{E}_c \left[ \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)e_{q,d} \right] = \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)e_{q,d} P_{q,d}
$$

$$
\mathbb{E}_c \left[ \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)c_{q,d} \right] = \mathbb{E}_c \left[ \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)e_{q,d} \right] = \frac{1}{|Q_u|} \sum_{q \in Q_u} \mathbb{E}_c \left[ \frac{1}{p_{q,d}} g^{-1}(d) \right]
$$

where $g(d)$ is a shorthand for $g(R^{-1}_q(d))$. Equation (a) comes from the PBM [8]: where $c_{q,d} = e_q \cdot e_{q,d}$. Equation (b) holds because $p(e_d = 1) = p_{q,d}$. This shows that each client conducts an unbiased learning.

Similarly we show that on the server side the global objective function $f_{q,IPS}(w)$ in Eq. (6) is also unbiased:

$$
\mathbb{E}_c \left[ f_{q,IPS}(w) \right] = \mathbb{E}_c \left[ \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} g(d)c_{q,d} P_{q,d} \right]
$$

$$
\mathbb{E}_c \left[ f_{q,IPS}(w) \right] = \mathbb{E}_c \left[ \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \frac{1}{|Q_u|} \sum_{q \in Q_u} \sum_{d \in D_q} \mathbb{E}_c \left[ \frac{1}{p_{q,d}} g^{-1}(d) \right] \right]
$$

$$
\mathbb{E}_c \left[ f_{q,IPS}(w) \right] = \mathbb{E}_c \left[ \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \frac{1}{|Q_u|} \sum_{q \in Q_u} \mathbb{E}_c \left[ \Psi(f_w(q), D_q) \right] \right]
$$

4 EXPERIMENTAL SETUP

Dataset. Using offline collected datasets and simulated experiments to evaluate FL and ULTR algorithms is a widely adopted practice [13, 14, 21, 24]. In our experiments, we choose two public available LTR datasets: Yahoo [7] and Istella [19]. These datasets contain large collections of queries $Q$ from real-world search engines. Each query $q$ is attached with a set of candidate documents $D_q$. Each query-document pair is represented by a feature vector $x_{q,d}$. For pre-processing, we filter out queries where all documents have the same score. After pre-processing, the Yahoo dataset contains 14,377 queries with 32.51 documents per query on average. The Istella dataset contains 32,625 queries with 103.73 documents per query on average. We conduct the query-level normalization, and scale features to the range of $[0, 1]$.

Logging policy. The logging policy $f_0$ mimics the production ranker in a real-world search engine. We follow a commonly used setup [13, 14] and sample 1% queries to train a linear ranker in a full-information manner.

Click simulator. We follow the PBM designed by [13] and simulate clicks as follows:

$$
p(c_{q,d} = 1) = \begin{cases} p(e_{q,d,u}), & \text{if } r_{q,d} \in \{3, 4\} \\
0.1p(e_{q,d,u}), & \text{otherwise} \end{cases}
$$

where $p(e_{q,d,u})$ is the user-dependent examination probability. This is to mimic the real-world scenario where every user’s position bias is different. Give user $u$, we define $p(e_{q,d,u}) = (\frac{1}{\eta_1} - 1)^{y_1}$, where $y_1 \geq 0$ is a user-dependent position bias factor. In our experiments, we sample $y_1$ from a left-truncated Gaussian distribution $N(y, 0.1)$.

Federated learning setup. We simulate $|\mathcal{U}|$ clients, each of which issues 5 queries per round. On all devices, $K$ documents are displayed for each query. During each global round, a user clicks $m$ documents. To study the impact of position bias, we choose $|\mathcal{U}| = 2,000$, $K = 5$ and $m = 10$.

Baselines. To our best knowledge, FedIPS is the first FULTR algorithm. So we design two non-FULTR baselines for comparison. The first one is the vanilla FedAvg algorithm [21]. The second baseline is the linear LambdaRank [6] trained in a full-informational manner. We name it $\lambda$Linear. We follow the setup in [13, 14] and use the linear ranker:

$$
f_w(q,d) = w^T x_{q,d}
$$

We believe that the proposed FedIPS is general enough to accommodate more sophisticated models, such as neural networks [23].

For FedIPS and FedAvg, we use a grid search to choose parameters from ranges of: $\eta_l \in \{0.00001, 0.0001, 0.001\}$ and $\eta_g \in \{0.05, 0.3, 1, 2\}$. For $\lambda$Linear, we choose the learning rate from \{0.001, 0.005, 0.01, 0.03, ..., 0.09, 0.1, 0.3, ..., 0.9\}.

NDCG@5 is used as the evaluation metric.

5 EXPERIMENTAL RESULTS

We first compare FedIPS with non-FULTR baselines. As demonstrated in the left column of Fig. 1 where $y = 1$ is picked as an example, on both Istella and Yahoo datasets, FedIPS significantly outperforms FedAvg. In Yahoo dataset, FedAvg starts to taper off after only a few iterations, while FedIPS’s NDCG keep growing even after 500 iterations, almost approaching the full-informational $\lambda$Linear.

We then study the impact of varying the level of position bias $y$ and results are shown in Fig. 1. We run FedIPS and FedAvg with $y \in \{0.5, 1.0, 1.5, 2.0\}$. We observe that FedIPS is fairly robust to the...
different degrees of position biases. Although $y = 0.5$ gives the best NDCG, their differences are small. On the other hand, we notice a huge drop in the performance of FedAvg when $y$ becomes large. This indicates that our FedIPS handles position bias pretty well.

We also study the impact of different numbers of clients (local devices). We expect better performance of FedIPS with more clients registered for FL. We simulated $|U| \in \{100, 500, 2500, 12500, 62500\}$. Results are reported in the Fig. 2 (Left). 2 We observe that when $|U|$ reaches a certain level, e.g., $|U| = 2500$, increasing $|U|$ only marginally improves the metrics of FedIPS. Specifically, FedIPS with $|U| = 100$ falls behind others and the learning curves of $|U| \in \{100, 500\}$ have large fluctuations since the estimated global gradient is noisy with small $|U|$. When $|U| \geq 2500$, FedIPS exhibits similar convergence trends. Our speculation is that when a sufficient amount of clients are involved, FedIPS estimates a rather accurate global gradient, and the variance in the estimator is no longer a bottleneck.

Finally, we experiment with a simple way to estimate IPS in FULTR without intervention. We extend the regression-based EM approach by Wang et al. [25] to FULTR. Briefly, a function $F$ is used to estimate the hidden relevance $r_{q,d}$, which is then used in the M-step to infer the propensity score [25]. In our experiments, we choose $F$ as a linear function and use FedOpt to optimize it. The updated $F$ is used to estimated personalized IPS, i.e., running a similar version of Algorithm 1 in [25] on local client.

We conduct experiments with different numbers of clicks $m$, and report the results in Fig. 2 (Right). We observe that for all $m$ values, FedIPS with the estimated IPS eventually outperforms FedAvg. In the most sparse setting where users only provide $m = 5$ clicks, FedIPS outperforms FedAvg only after about 100 rounds. We also observe performance gaps between the estimated IPS and real IPS (in contrast to Fig. 1). This indicates that although our estimated IPS is helpful in addressing the position bias, there are still a lot of room for improvement.

6 CONCLUSION

In this paper, we study the Federated Unbiased Learning to Rank (FULTR) problem, an important challenge in federated LTR. We propose the FedIPS algorithm as a solution. We conduct extensive experiments on public datasets to evaluate FedIPS, and show that FedIPS outperforms the biased FedAvg, and approaches the performance of the full-information $\lambda$Linear.

We point out a few directions for future research. First, to enjoy theoretical guarantees on privacy preservation, it is a common practice to apply randomized mechanisms to FL algorithms, as studied by the differential privacy [10, 22] community. Another challenging problem is to better estimate propensities in the FL setup. This will help bridge the gap that we observed in our last set of experiments. Evaluating FedIPS with sophisticated models such as nonlinear neural networks or boosting trees will also be very interesting.

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Due to limited space, in the following experiments, we only report results on the Yahoo datasets, but similar results are observed on the Istella dataset.
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