Robust adaptive control for continuous wheel slip rate tracking of vehicle with state observer

Jiaxu Zhang¹,², Zhengtang Shi³, Xiong Yang³ and Jian Zhao¹

Abstract
This article proposes a novel robust adaptive wheel slip rate tracking control method with state observer. First, a modified tracking differentiator is proposed based on a combination of tangent sigmoid function with terminal attraction factor and linear function to improve convergence speed and avoid chattering phenomenon, and then, the modified tracking differentiator is used as state observer to smooth and estimate the states of the system. Second, a robust adaptive wheel slip rate tracking control law with fuzzy uncertainty observer and modified adaptive laws is derived based on Lyapunov-based method. The fuzzy uncertainty observer is used for estimating and compensating the additive uncertainty, and the modified adaptive laws are used for estimating the unknown optimal weight vector of the fuzzy uncertainty observer and the multiplicative uncertainty. Finally, the performance of the robust adaptive wheel slip rate tracking control method is verified based on the model-in-the-loop simulation system.

Keywords
Wheel slip rate tracking control, state observer, robust control, adaptive control, fuzzy uncertainty observer

Introduction
Nowadays, self-driving electric vehicle has become one of the main goals of automobile industry among China, Europe, the United States, and Japan. However, continuous, fast, and stable wheel slip rate tracking control is an extremely important foundation of achieving braking energy regeneration, fully automatic parking control, adaptive cruise control, and autonomous emergency braking control for self-driving electric vehicle. Therefore, continuous, fast, and stable wheel slip rate tracking control has been widely concerned by many scholars and automobile manufacturers.

In the early days, the wheel slip rate tracking control is mostly used in the antilock braking system (ABS), which can effectively both improve the braking stability and minimize the braking distance by making the wheel slip rate converge to the desired value that corresponds to the maximum value of tire-road friction coefficient. Since the vehicle exhibits the characteristics of complex nonlinearity, time variation, strong coupling, and uncertainty under braking condition, early research results adopt mostly logic switching rule to make the wheel slip rate tracking control system have strong robustness against lumped uncertainty. Fu et al. gave the periodic solution of the single-wheel antilock braking system and adopted Poincare map to analyze the stability of the periodic solution. Jing et al. proposed a switched wheel slip rate tracking control strategy with few tuning parameters for antilock braking system, and the constraints of the tuning parameters were given by Filippov-based stable theory framework to make the wheel slip rate converge to the desired value that corresponded to the peak point of tire-road friction coefficient. Kiencke and Nielsen utilized wheel deceleration information to design logic switching rule to keep the wheel slip rate in the neighborhood of the optimal value. Kuo and Yeh presented a four-state control strategy for antilock braking system based on piecewise linear tire model, and the proposed scheme was proved to accommodate all road conditions via Poincare maps. Pasillas-Lepine adopted wheel deceleration logic-based switching to design five-phase antilock brake

¹State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun, China
²Intelligent Network R&D Institute, China FAW Group Co., Ltd., Changchun, China
³Intelligent Vehicle Control System Research Institute, Zhejiang Asia-Pacific Mechanical and Electronic Co., Ltd., Hangzhou, China

Corresponding author:
Jian Zhao, State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun 130022, China.
Email: zhaojian@jlu.edu.cn
algorithm, and the robustness of the designed algorithm with respect to friction coefficient changes was proved based on Poincaré map. Tanelli et al. proposed a second-order sliding mode wheel slip rate tracking controller with a sliding mode observer, which was used for estimating the tire–road friction coefficient. The proposed controller generated continuous control actions to eliminate the chattering phenomenon of conventional sliding mode controller. Harifi et al. designed a novel sliding mode controller with integral switching surface for wheel slip rate tracking control based on two-axle vehicle dynamic model, and the integral switching surface can effectively reduce the chattering caused by the discontinuous term of the sliding mode controller. Johansen et al. established a family of nominal models to describe the wheel slip rate dynamics via linearizing locally the wheel slip rate dynamic model and adopted linear quadratic regulator (LQR) approach to design a gain-scheduled wheel slip rate tracking controller. Lin and Hsu proposed a hybrid wheel slip rate tracking control system with a system uncertainty observer by combining a nominal controller with a compensation controller. The nominal controller was used for making the wheel slip rate track the desired value in the presence of the system uncertainty, and the compensation controller was used for improving the wheel slip rate tracking precision by compensating the estimation error of the system uncertainty observer. He et al. presented a novel wheel slip rate output feedback tracking controller by the time-varying asymmetric barrier Lyapunov function–based control method, and then, the proposed controller was proved to have strong robustness against external disturbance and parameter deviation. Mirzaei et al. presented an optimized fuzzy wheel slip rate tracking controller, and all the components of the fuzzy system were estimated based on hybrid optimization algorithm composing of genetic algorithm and error-based global optimization approach. Mirzaei and Mirzaeinejad presented a nonlinear optimal wheel slip rate tracking controller with a free weight ratio based on wheel dynamic model, and the free weight ratio was used to compromise control accuracy and control energy. Mirzaeinejad presented a nonlinear robust control law with radial basis function (RBF) neural network–based observer for wheel slip rate tracking control, and the RBF neural network–based observer was used for estimating and compensating the unknown uncertainty caused by changing vehicle parameters and unmodel dynamics. Pasillas-Lepine et al. proposed a nonlinear wheel slip rate tracking controller with the wheel slip rate and the difference between the wheel and vehicle accelerations for the feedback variables, and the robustness of the proposed wheel slip rate tracking control algorithm with respect to vertical load variations and uncertainties in tire–road friction was proved via Lyapunov-based method. Qiu et al. proposed a nonlinear backstepping control law to track the desire wheel slip rate and avoid the wheel slip rate in the unstable region via asymmetric barrier Lyapunov function–based dynamic surface control approach, which can both relax the required initial conditions of the closed-loop system and eliminate repeated differentiation. Tanelli et al. presented a nonlinear output feedback wheel slip rate tracking controller based on wheel dynamic model with input constraints, and the proposed controller could make a distinction between the unstable region and the stable region of the friction curve by Poincaré–Bendixson theorem. Yu et al. established four nominal models corresponding to different tire-road friction conditions based on quarter-vehicle model with LuGre tire model and designed four wheel slip rate tracking controllers for the four nominal models. Moreover, the index function was used for achieving the smooth transformation of the four wheel slip rate tracking controllers. He et al. established a quarter-vehicle model with Burckhardt tire model as control model and then proposed a novel nonlinear robust wheel slip rate tracking controller based on barrier Lyapunov function. Zhang and Li presented an adaptive sliding mode wheel slip rate tracking controller with uncertainty observer based on backstepping design framework and adopted Lyapunov-based method to prove that the wheel slip rate tracking error was uniformly ultimately bounded in the presence of uncertainty. The above wheel slip rate tracking control methods based on dynamic model usually depend on simplified dynamic models to design simple and effective controllers. However, the simplified dynamic models cannot accurately characterize complex nonlinearity, time variation, strong coupling, and uncertainty under braking condition. Therefore, the research on wheel slip rate tracking control method with strong robustness and high precision has profound theoretical and practical value.

In this article, we propose a novel robust adaptive wheel slip rate tracking control method with state observer based on wheel slip rate dynamic model with additive and multiplicative uncertainties. First, a modified tracking differentiator is proposed based on a combination of tangent sigmoid function with terminal attraction factor and linear function. The tangent
sigmoid function with terminal attraction factor can effectively cut short the convergence time if the state trajectory of the system is away from the equilibrium point, and the linear function can avoid chattering phenomenon if the state trajectory of the system approaches the equilibrium point. Then, the modified tracking differentiator is used as state observer to smooth and estimate the states of the system to solve the excessive noise problem caused by the noise amplification effect of the traditional first-order inertial link with small inertia time constant when being used for calculating the first-order derivative of the signal. Second, a simplified wheel slip rate dynamic model with additive and multiplicative uncertainties caused by the neglect of suspension dynamics, tire rolling resistance, and tire sideslip characteristics and the estimation error of the state observer is derived for control strategy design, and a robust adaptive wheel slip rate tracking control law with fuzzy uncertainty observer is derived based on the simplified wheel slip rate dynamic model with additive and multiplicative uncertainties and Lyapunov-based method. The fuzzy uncertainty observer is used for estimating and compensating the additive uncertainty of the system, and the unknown weight vector of the fuzzy uncertainty observer and the multiplicative uncertainty of the system are estimated by modified adaptive laws. Therefore, the fuzzy uncertainty observer with the modified adaptive laws not only improves the robust and adaptive ability of the system but also solves the problem of high computational complexity and unreasonable sensitivity to system uncertainties resulting from overly complicated model-based control strategy. Finally, the effectiveness and feasibility of the robust adaptive wheel slip rate tracking control method are verified based on the model-in-the-loop simulation system.

This article is organized as follows: in section “The dynamic model,” the wheel slip rate dynamic model with the uncertainties is derived; in section “The proposed method,” the robust adaptive wheel slip rate tracking control method with state observer is developed and analyzed based on Lyapunov-based method; in section “Simulation results,” various maneuvers are carried out to compare the proposed robust adaptive wheel slip rate tracking control method with the wheel slip rate sliding mode tracking control method; section “Conclusion” gives the conclusion of our work.

The dynamic model

Basically, the wheel slip rate dynamic model is the basis of control strategy design, but overly complicated model will easily lead to control strategy with high computational complexity and unreasonable sensitivity to system uncertainties, and the simplified model cannot accurately characterize complex nonlinearity, time variation, strong coupling, and uncertainty under braking condition. Hence, the wheel slip rate dynamic model with the additive uncertainty caused by ignoring suspension dynamics, tire rolling resistance, and tire sideslip characteristics is derived for control strategy design. First, the quarter-vehicle model with wheel angular speed and vehicle longitudinal speed for the states is established to describe the wheel slip rate dynamics by ignoring suspension dynamics, tire rolling resistance, and tire sideslip characteristics

\[
\begin{align*}
J_\omega &= r F_x - T_b \\
\dot{m} &= -F_x
\end{align*}
\]

where \( m, r, \) and \( J \) denote the quarter-vehicle mass, the effective rolling radius, and inertia of the wheel, respectively; \( \omega, T_b, \) and \( v \) denote the wheel angular speed, the brake torque, and the vehicle speed, respectively; \( F_x \) denotes the longitudinal force that can be represented as the function of the vertical force \( F_z \) and the tire–road friction coefficient curve \( \mu(\lambda) \)

\[
F_x = F_z \mu(\lambda)
\]

According to Burckhardt tire model,\(^{26}\) \( \mu(\lambda) \) can be represented as the function of the wheel slip rate \( \lambda \)

\[
\mu(\lambda) = \vartheta_1(1 - e^{-\lambda \vartheta_2}) - \lambda \vartheta_3
\]

where \( \vartheta_1, \vartheta_2, \) and \( \vartheta_3 \) denote the maximum value, shape, and difference in the tire–road friction coefficient curve, respectively.

The wheel slip rate \( \lambda \) is defined as

\[
\lambda = \frac{v - \omega r}{v}
\]

Differentiating equation (4) yields

\[
\dot{\lambda} = \frac{1}{v} ((1 - \lambda) \dot{v} - \omega r)
\]

Differentiating equation (5) yields

\[
\ddot{\lambda} = \frac{1}{v} (-2 \dot{\lambda} + \dot{v}(1 - \lambda) - \ddot{\omega} r)
\]

Differentiating equation (1) yields

\[
\begin{align*}
J_\ddot{\omega} &= r F_z \mu(\lambda) - \dot{T}_b \\
\dot{m} &= -F_z \mu(\lambda)
\end{align*}
\]

Substituting equation (7) into equation (6), we can obtain

\[
\ddot{\lambda} = -\frac{1}{v} \left( -\frac{2 F_z \mu(\lambda)}{m} \dot{\lambda} + \frac{1 - \lambda}{m} + \frac{r^2}{J} F_z \mu(\lambda) \right) + \frac{r}{JF_b} \dot{T}_b
\]

Supposing that \( \dot{\lambda}_d \) denotes the desired wheel slip rate and \( d \) denotes the additive uncertainty caused by ignoring suspension dynamics, tire rolling resistance and tire sideslip characteristics. Meanwhile, the state vector and the control input are defined as \( x = [x_1, x_2]^T = [\dot{x}_1, \lambda - \dot{x}_2]^T \) and \( u = \dot{T}_b \), respectively, and we can obtain the following wheel slip rate
dynamic model with the additive uncertainty according to equation (8)

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{\nu} f(x) + Gu + d_i \\
&\quad - \left( \frac{m}{1 + \frac{m}{2}} \right) \mu \left( k_d - x_2 \right)
\end{aligned}
\]

where \( f(x) = 1/\nu \cdot G \cdot r \) and \( G = r/(Jv) \).

### The proposed method

The overall structure of the robust adaptive wheel slip rate tracking control method with state observer is shown in Figure 1. The outputs of the state observer with the wheel slip rate tracking error for the input are the smoothed wheel slip rate tracking error and the first-order derivative of the smoothed wheel slip rate tracking error, which are used as the inputs of the robust adaptive wheel slip rate tracking control law, fuzzy uncertainty observer, the modified adaptive law for optimal weight vector, and the modified adaptive law for multiplicative uncertainty. The fuzzy uncertainty observer is used for estimating and compensating the additive uncertainty of the system, and the unknown optimal weight vector of the fuzzy uncertainty observer and the multiplicative uncertainty of the system are estimated by modified adaptive laws. Taking into accounting the online estimation and compensation of the additive and multiplicative uncertainties by fuzzy uncertainty observer and modified adaptive laws, the output of the robust adaptive wheel slip rate tracking control law is the derivative of the brake torque, the integral operation of which is used as the input of the controlled plant.

### State observer design

Traditionally, the first-order inertial link with small inertia time constant is used for calculating the first-order derivative of the signal, but the noise amplification effect of the first-order inertial link will lead to excessive noise in the derivative of the signal. Therefore, a modified tracking differentiator described by Theorem 1 is proposed based on a combination of tangent sigmoid function with terminal attraction factor and linear function, and then, the modified tracking differentiator is used as state observer to smooth and estimate the states of the system.

**Theorem 1.** Let the origin \((x_1, x_2) = (0, 0)\) be an equilibrium point for the following system

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\kappa_0 x_1 - \kappa_1 |x_1|^{\alpha} \text{sig}(\eta_0 x_1) - \kappa_2 x_2 \\
&\quad - \kappa_3 |x_2|^{\beta} \text{sig}(\eta_1 x_2)
\end{aligned}
\]

where \(x_1\) and \(x_2\) are the system state variables; \(\kappa_0, \kappa_1, \kappa_2, \kappa_3, \eta_0, \eta_1, \alpha, \text{ and } \beta\) are the positive design parameters; \(\text{sig}()\) denotes tangent sigmoid function, which can be described as

\[
\text{sig}(x) = \frac{2}{1 + e^{-2x}} - 1
\]

The equilibrium point of the system described by equation (10) is asymptotically stable.

**Proof.** Defining the Lyapunov candidate function as follows

\[
V = \int_0^{\chi_1} k_1 |\tau|^\alpha \text{sig}(\eta_1 \tau)d\tau + \frac{1}{2} k_0 x_1^2 + \frac{1}{2} x_2^2
\]

Since the integrable function \(k_1 |\tau|^\alpha \text{sig}(\eta_1 \tau)\) and the corresponding upper limit \(\chi_1\) of integral term in the Lyapunov candidate function described by equation (12) have the same sign, we can obtain

\[
\int_0^{\chi_1} k_1 |\tau|^\alpha \text{sig}(\eta_1 \tau)d\tau > 0
\]

According to inequality (13), we can obtain

\[
V > 0
\]

Differentiating equation (12) with respect to time, we can obtain

\[
\dot{V} = -\kappa_1 x_1 |x_1|^{\alpha} \text{sig}(\eta_0 x_1) + k_0 x_1 x_2 + \kappa_2 x_2 \text{sig}(\eta_0 x_1) - \kappa_3 x_2^{\beta} \text{sig}(\eta_1 x_2)
\]

\[
= -\kappa_2 x_2^2 - \kappa_3 |x_2|^{\beta} x_2 \text{sig}(\eta_1 x_2)
\]

According to equation (15), we can obtain that \(\dot{V} = 0\) if and only if \((x_1, x_2) = (0, 0)\). Therefore, on the basis of LaSalle invariance principle, the equilibrium point of the system described by equation (10) is asymptotically stable.

The tangent sigmoid function described by equation (11) is shown in Figure 2, it has good linear characteristic when its independent variable is small, and it has...
the characteristic of global smoothness and saturation when its independent variable is large. Therefore, the tangent sigmoid function is especially suited as the activation function of the terminal attraction factor to improve the convergence rate of the system and avoid the high-frequency chatter of the system.

Moreover, the tangent sigmoid function described by equation (11) can be transformed into Taylor series as follows

\[
sig(x) = x - 2x^3 + O(x^3)
\]

where \(O(x^3)\) denotes the higher order term of \(x^3\).

It can be seen from equation (16) that the tangent sigmoid functions with terminal attraction factors play a dominant role in making the state trajectory of the system converge to the equilibrium point when

\[
\sqrt{(\eta_0\chi_1)^2 + (\eta_1\chi_2)^2} > 1,
\]

which means the state trajectory of the system is far from the equilibrium point. Therefore, the system described by equation (10) can be simplified as follows

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\kappa_1\chi_1^\alpha \sig(\eta_0\chi_1) - \kappa_3\chi_2^\beta \sig(\eta_1\chi_2)
\end{aligned}
\]

According to equation (17), the tangent sigmoid functions with terminal attraction factors can effectively improve the convergence speed when the state trajectory of the system is far from the equilibrium point.

It can be seen from equation (16) that the linear functions play a dominant role in making the state trajectory of the system converge to the equilibrium point when

\[
\sqrt{(\eta_0\chi_1)^2 + (\eta_1\chi_2)^2} \leq 1,
\]

which means the state trajectory of the system is in the neighborhood of the equilibrium point. Therefore, the system described by equation (10) can be simplified as follows

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\kappa_0\chi_1 - \kappa_2\chi_2
\end{aligned}
\]

According to equation (18), the linear functions can effectively avoid chattering phenomenon when the state trajectory of the system approaches the equilibrium point.

According to the Lemma given by Han and Wang,\(^\text{27}\) the following state observer is designed based on Theorem 1

\[
\begin{aligned}
\dot{x}_1 &= \dot{x}_2 \\
\dot{x}_2 &= -R^2\left(\kappa_0(x_1 - x_1) + \kappa_1|x_1 - x_1|^\alpha \sig(\eta_0(x_1 - x_1)) + \kappa_2\frac{x_2}{R} + \kappa_3\frac{x_3}{R} \sig\left(\eta_1\frac{x_2}{R}\right)\right)
\end{aligned}
\]

where \(R > 0\) is the design parameter, and when \(R \to \infty\), the estimate denoted by \(\dot{x}_1\) converges weakly to the wheel slip rate tracking error and the estimate denoted by \(\dot{x}_2\) is approximately equal to the derivative of the wheel slip rate tracking error. However, the noise-rejection ability of the state observer deteriorates with the increasing of the design parameter \(R\). Therefore, the design parameter \(R\) selection must consider the estimation error and the noise-rejection ability of the state observer at the same time.

### Control law design

According to the proposed state observer, the wheel slip rate dynamic system described by equation (9) can be modified as follows

\[
\begin{aligned}
\dot{x}_1 &= \dot{x}_2 \\
\dot{x}_2 &= f(\dot{x})\theta + \Delta f(\dot{x}) + Gu + d_i
\end{aligned}
\]

where \(\dot{x} = [\dot{x}_1, \dot{x}_2]^T\) denotes the estimate of the system state vector; \(\theta\) and \(\Delta f(\dot{x})\) denote the multiplicative and additive uncertainties caused by the estimation error of the system state vector, respectively

\[
\begin{aligned}
\dot{x}_1 &= \dot{x}_2 \\
\dot{x}_2 &= f(\dot{x})\theta + Gu + d_a
\end{aligned}
\]

where \(d_a = \Delta f(\dot{x}) + d_i\) denotes the sum of the additive uncertainties of the system.

Since any nonlinear function can be approximated with arbitrary accuracy by the fuzzy logic system,\(^\text{28}\) we adopt the fuzzy logic system as the fuzzy uncertainty observer to realize the estimation and compensation of the sum of the additive uncertainties of the system. According to weighted average method, the sum of the additive uncertainties of the system can be expressed as

\[
d_a(\xi, w^*) = (w^*)^T\phi(\xi) + \varepsilon
\]

where \(\varepsilon\) denotes the approximation error satisfying \(|\varepsilon| \leq \delta\). \(\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T\) denotes the input vector, and \(w^* = [(w_1)^*, (w_2)^*, \ldots, (w_n)^*]^T\) and \(\phi(\xi) = [\phi^1(\xi), \phi^2(\xi), \ldots, \phi^n(\xi)]\).
Considering the system described by equation (36), we can obtain

\[
\dot{\hat{\theta}} = z_2 f(\hat{x}) - \sigma_1 (\hat{\theta} - \hat{\theta}_0) \\
\dot{\hat{w}} = z_2 f(\hat{x}) - \sigma_2 (\hat{w} - \hat{w}_0)
\]  

where \(\sigma_1 > 0\) and \(\sigma_2 > 0\) denote the design parameters and \(\hat{\theta}_0\) and \(\hat{w}_0\) denote the initial estimates of the multiplicative uncertainty and the optimal weight vector, respectively. All signals of the wheel slip rate tracking control closed-loop system are uniformly ultimately bounded, and the tracking error satisfies

\[
\limsup_{t \to \infty} |x_1| \leq \frac{1}{\eta} \left( \frac{1}{2} \sigma_1^2 + \frac{\sigma_1}{2} \hat{\theta}^2 + \frac{\sigma_2}{2} \hat{w}^T \hat{w} \right) \tag{29}
\]

where \(\hat{\theta}_0 = \theta - \hat{\theta}_0\) and \(\hat{w}_0 = w^* - \hat{w}_0\) denote the initial estimation errors of the multiplicative uncertainty and the optimal weight vector, respectively; \(\eta = \min (\kappa_4, \kappa_5, (1/2), (\sigma_1/2), (\sigma_2/2))\).

**Proof.** Defining the Lyapunov candidate function as follows

\[
V_1 = \frac{1}{2} z_1^2 \tag{30}
\]

differentiating equation (30) with respect to time, we can obtain

\[
\dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 + \nu) \tag{31}
\]

choosing the virtual control input as follows

\[
\nu = -\kappa_4 z_1 \tag{32}
\]

substituting equation (32) into equation (31), we can obtain

\[
\dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 - \kappa_4 z_1) \tag{33}
\]

augmenting the Lyapunov candidate function defined by equation (30), we can obtain

\[
V_2 = V_1 + \frac{1}{2} \dot{z}_2^2 + \frac{1}{2} \hat{\theta}^2 + \frac{1}{2} \hat{w}^T \hat{w} = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2
+ \frac{1}{2} \dot{\theta}^2 + \frac{1}{2} \hat{w}^T \hat{w} \tag{34}
\]

where \(\hat{\theta} = \theta - \hat{\theta}\) and \(\hat{w} = w^* - \hat{w}\) denote the estimation errors of the multiplicative uncertainty and the optimal weight vector, respectively.

Differentiating equation (34) with respect to time, we can obtain

\[
\dot{V}_2 = z_1 (z_2 - \kappa_4 z_1) + z_2 (\dot{z}_2 - \nu) - \dot{\theta} \hat{\theta} - \hat{w}^T \hat{w}
\]

where \(\kappa_4 > 0\) and \(\kappa_5 > 1/2\) denote the design parameters; the estimate \(\hat{\theta}\) of the multiplicative uncertainty of the system and the estimate of the optimal weight vector are updated based on the following modified adaptive laws

\[
\dot{\hat{\theta}} = z_2 f(\hat{x}) - \sigma_1 (\hat{\theta} - \hat{\theta}_0) \tag{27}
\]

\[
\dot{\hat{w}} = z_2 f(\hat{x}) - \sigma_2 (\hat{w} - \hat{w}_0) \tag{28}
\]

where \(\sigma_1 > 0\) and \(\sigma_2 > 0\) denote the design parameters and \(\hat{\theta}_0\) and \(\hat{w}_0\) denote the initial estimates of the multiplicative uncertainty and the optimal weight vector,

\[
\phi^j(\xi), \ldots, \phi^j(\xi) \tag{23}
\]

respectively. All signals of the wheel slip rate tracking control closed-loop system are uniformly ultimately bounded, and the tracking error satisfies

\[
\limsup_{t \to \infty} |x_1| \leq \frac{1}{\eta} \left( \frac{1}{2} \sigma_1^2 + \frac{\sigma_1}{2} \hat{\theta}^2 + \frac{\sigma_2}{2} \hat{w}^T \hat{w} \right) \tag{29}
\]

where \(\hat{\theta}_0 = \theta - \hat{\theta}_0\) and \(\hat{w}_0 = w^* - \hat{w}_0\) denote the initial estimation errors of the multiplicative uncertainty and the optimal weight vector, respectively; \(\eta = \min (\kappa_4, \kappa_5, (1/2), (\sigma_1/2), (\sigma_2/2))\).
\[ V_1 = -\kappa_4 \xi^2 - \kappa_5 \xi^2 z + z_2 \xi + z_3 f(\xi) \hat{\theta} - \ddot{\theta} \]
\[ + z_4 \frac{\dot{t}}{\xi} \phi(\xi) - \bar{\omega}^T \bar{\omega} \]  
(37)

Substituting the modified adaptive laws described by equation (27) and equation (28) into equation (37), we can obtain
\[ V_1 = -\kappa_4 \xi^2 - \kappa_5 \xi^2 z + z_2 \xi + \sigma_1 \dot{\theta}(\theta - \dot{\theta}_0) \]
\[ + \sigma_2 \dot{\theta}^T (\bar{w} - \bar{w}_0) \]  
(38)

According to equation (38), we can obtain
\[ V_1 = -\kappa_4 \xi^2 - \kappa_5 \xi^2 z + z_2 \xi + \sigma_1 \dot{\theta}(\theta - \dot{\theta}_0) \]
\[ + \sigma_2 \dot{\theta}^T (\bar{w} - \dot{w}^* + \dot{w}^* - \bar{w}_0) \]
\[ = -\kappa_4 \xi^2 - \kappa_5 \xi^2 z + z_2 \xi + \sigma_1 \dot{\theta}^2 + \sigma_1 \dot{\theta} \dot{\theta}_0 - \sigma_2 \dot{\theta}^T \bar{w} \]
\[ + \sigma_2 \dot{\theta}^T \bar{w}_0 \]  
(39)

According to Young inequality,\(^2^9\) we can obtain
\[ \ddot{\theta}_0 \leq \frac{1}{2} \dot{\theta}^2 + \frac{1}{2} \dot{\theta}_0^2 \]  
(40)
\[ \bar{w}^T \bar{w}_0 \leq \frac{1}{2} \dot{w}^T \dot{w} + \frac{1}{2} \dot{w}_0^T \dot{w}_0 \]  
(41)
\[ z_2 \xi \leq \frac{1}{2} \xi^2 + \frac{1}{2} \dot{z}_2^2 \leq rac{1}{2} z_2^2 + \frac{1}{2} \dot{z}_2^2 \]  
(42)

Substituting inequality (40) to inequality (42) into equation (39), we can obtain
\[ V_1 \leq -\kappa_4 \xi^2 - \left( \kappa_5 - \frac{1}{2} \right) \xi^2 + \frac{1}{2} \dot{z}_2^2 - \sigma_1 \dot{\theta}^2 - \sigma_2 \dot{\theta}^T \bar{w} \]
\[ + \frac{\sigma_1}{2} \dot{\theta}_0^2 + \frac{\sigma_2}{2} \dot{\theta}^T \bar{w}_0 \]  
(43)

Substituting \( \eta = \min(\kappa_4, \kappa_5, (1/2), (\sigma_1/2), (\sigma_2/2)) \) into equation (43), we can obtain
\[ V_1 \leq -2\eta V_2 + \frac{1}{2} \dot{v}^2 + \sigma_1 \dot{\theta}_0^2 + \sigma_2 \dot{\theta}^T \bar{w}_0 \]  
(44)

According to comparison principle,\(^3^0\) we can obtain
\[ V_2(t) \leq e^{-2\eta t} V_2(0) + \frac{1}{2\eta} \left( \frac{1}{2} \dot{v}^2 + \sigma_1 \dot{\theta}_0^2 + \sigma_2 \dot{\theta}^T \bar{w}_0 \right) \]  
(45)

where \( V_2(0) \) denotes the initial value of the Lyapunov candidate function defined by equation (34).

According to inequality (45), we can deduce that all signals of the wheel slip rate tracking control closed-loop system is uniformly ultimately bounded, and the estimate of the tracking error satisfies
\[ \limsup_{t \to \infty} |\hat{x}_1(t)| \leq \frac{1}{\eta} \left( \frac{1}{2} \dot{v}^2 + \sigma_1 \dot{\theta}_0^2 + \sigma_2 \dot{\theta}^T \bar{w}_0 \right) \]  
(46)

Since the estimate \( \hat{x}_1 \) of the proposed state observer described by equation (19) can asymptotically converge to the state \( x_1 \) of the system, the wheel slip rate tracking error satisfies
\[ \limsup_{t \to \infty} |x_1(t)| \leq \frac{1}{\eta} \left( \frac{1}{2} \dot{v}^2 + \sigma_1 \dot{\theta}_0^2 + \sigma_2 \dot{\theta}^T \bar{w}_0 \right) \]  
(47)

It can be seen from equation (47) that the wheel slip rate tracking error is uniformly ultimately bounded. Meanwhile, reducing the initial estimation error of the multiplicative uncertainty of the system and the initial estimation error of the optimal weight vector can reduce the upper bound of the wheel slip rate tracking error.

### Simulation results

In this section, the full-vehicle dynamics simulation software (MSC CarSim\(^8\)), which can accurately reflect the whole vehicle dynamics, is used to construct the model-in-the-loop simulation system to validate the effectiveness and feasibility of the proposed robust adaptive wheel slip rate tracking control method. Three maneuvers with desired step signal, desired sinusoidal signal, and desired ramp signal are carried out to compare the proposed robust adaptive wheel slip rate tracking control method with the sliding mode control (SMC) method in terms of robustness, smoothness, and tracking accuracy. In the simulation process, the dimension of the basis function vector of the uncertainty observer is set to 6, the centers of the basis function vector are equally spaced in \([0, 0.15; -10, 10]\), and the widths of the basis function vector are set to 1025. In addition, the vehicle parameters and other parameters of the proposed method are shown in Table 1.

| Meaning | Symbol | Value |
|---------|--------|-------|
| The quarter-vehicle mass | \( m \) | 354 kg |
| The wheel inertia | \( J \) | 0.9 kg m\(^2\) |
| The effective rolling radius of wheel | \( r \) | 0.31 m |
| The design parameter | \( \kappa_0 \) | 10 |
| The design parameter | \( \kappa_1 \) | 5 |
| The design parameter | \( \kappa_2 \) | 10 |
| The design parameter | \( \kappa_3 \) | 5 |
| The design parameter | \( \alpha \) | 3 |
| The design parameter | \( \beta \) | 3 |
| The design parameter | \( \eta_0 \) | 2 |
| The design parameter | \( \eta_1 \) | 2 |
| The design parameter | \( R \) | 80 |
| The design parameter | \( \kappa_4 \) | 350 |
| The design parameter | \( \kappa_5 \) | 200 |
| The design parameter | \( \sigma_1 \) | 0.01 |
| The design parameter | \( \sigma_2 \) | 0.01 |

### Step signal maneuver

The step signal is usually treated as typical signal to test the steady and dynamic characteristics of the wheel slip rate tracking closed-loop system. Therefore, the step
signal maneuver with zero steering wheel angle is implemented on a flat dry asphalt road. In the step signal maneuver, the final value of the desired step signal and the initial vehicle speed are set to 0.09 and 27.78 m/s, respectively. The front left wheel simulation results of the step signal maneuver are represented in Figure 3. As shown in Figure 3(a) and (b), the states $x_1$ and $x_2$ can be smoothly estimated by the proposed state observer. As shown in Figure 3(c) and (e), both the proposed method and the SMC method have strong robustness against the additive uncertainty, but the proposed method has better performance than the SMC method in terms of tracking accuracy and smoothness. As shown in Figure 3(f), the brake torque of the SMC method exists chattering phenomenon at the steady stage of the desired wheel slip rate, but the brake torque of the proposed method has more smoothness. As shown in Figure 3(g), compared with the tire–road friction coefficient of the SMC method, the tire–road friction coefficient of the proposed method is less fluctuant at the steady stage of the desired wheel slip rate. Therefore, the proposed method can effectively both cut short the convergence time of the braking system and improve the smoothness of the braking system.

**Sinusoidal signal maneuver**

The sinusoidal signal is usually treated as typical signal to test the delay characteristic of the wheel slip rate tracking closed-loop system. Therefore, the sinusoidal signal maneuver with zero steering wheel angle is implemented on a flat wet asphalt road. In the sinusoidal signal maneuver, the frequency, the amplitude, and the bias of the sinusoidal signal are set to 6.28 rad/s, 0.045, and 0.046, respectively, and the initial vehicle speed is set to 30.56 m/s. The front left wheel simulation results of the sinusoidal signal maneuver are represented in Figure 4. As shown in Figure 4(a) and (b), the
states $x_1$ and $x_2$ can be smoothly estimated by the proposed state observer. As shown in Figure 4(c)–(e), both the proposed method and the SMC method have robustness against the additive uncertainty, and the wheel slip rate of the proposed method has less time delay and more smoothness than that of the SMC method. As shown in Figure 4(f), the chattering phenomenon becomes more obvious around the peaks of the brake torque of the SMC method, but the brake torque of the proposed method has more smoothness. As shown in Figure 4(g), the tire–road friction coefficients of both the SMC method and the proposed method change with the wheel slip rates, and compared with the SMC method, the tire–road friction coefficient of the proposed method exerts more smoothness because of the smoothness of the wheel slip rate of the proposed method. Therefore, the proposed method can effectively both cut short the convergence time of the braking system and improve the smoothness of the braking system.

**Ramp signal maneuver**

The ramp signal is usually treated as typical signal to test the tracking characteristic of the wheel slip rate tracking closed-loop system. Therefore, the ramp signal maneuver with zero steering wheel angle is implemented on a flat ice-snow road. In the ramp signal maneuver, the maximum value and the ascending and descending slopes of the desired ramp signal are set to 0.9, 0.5, and 0.5, respectively, and the initial vehicle speed is set to 33.33 m/s. The front left wheel simulation results of the ramp signal maneuver are represented in Figure 5. Figure 5(a) and (b) shows that the proposed state observer can estimate the states $x_1$ and $x_2$ without excessive noise. Figure 5(c)–(e) show that both the wheel slip rates of the proposed method and the SMC method can track the desired ramp signal accurately, but the wheel slip rate of the proposed method has more smoothness than that of the SMC method. As shown in Figure 5(f), the chattering phenomenon becomes more obvious...
around the peaks of the brake torque of the SMC method, but the brake torque of the proposed method has more smoothness. As shown in Figure 5(g), the tire–road friction coefficients of both the SMC method and the proposed method change with the wheel slip rates, and compared with the SMC method, the tire–road friction coefficient of the proposed method exerts more smoothness since the wheel slip rate of the proposed method has more smoothness. Therefore, the proposed method can effectively both cut short the convergence time of the braking system and improve the smoothness of the braking system.

**Conclusion**

This article has proposed a novel robust adaptive wheel slip rate tracking control method with state observer. First, a state observer is presented based on the modified tracking differentiator to lay the foundation for the full-state feedback control law design, and the modified tracking differentiator can effectively improve convergence speed and avoid chattering phenomenon by combining tangent sigmoid function with linear function. Second, a robust adaptive wheel slip rate tracking control law with fuzzy uncertainty observer is derived based on Lyapunov-based method. The fuzzy uncertainty observer is used for estimating and compensating the additive uncertainty, and the unknown optimal weight vector of the fuzzy uncertainty observer and the multiplicative uncertainty are estimated by modified adaptive laws. Finally, three maneuvers with typical signals are implemented based on the model-in-the-loop simulation system to validate the effectiveness and feasibility of the proposed method, and the
simulation results show that the proposed method has the advantages over the SMC method in smoothness and tracking accuracy.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Nature Science Foundation of China under Grant No. 51575225.

**ORCID iD**

Jiaxu Zhang https://orcid.org/0000-0001-6159-1965

**References**

1. Lv C, Zhang JZ, Li YT, et al. Novel control algorithm of braking energy regeneration system for an electric vehicle during safety-critical driving maneuvers. *Energy Convers Manage* 2015; 106: 520–529.

2. Xu Y, Lu ZF, Shan X, et al. Study on automatic parking method based on the sliding mode variable structure and fuzzy logical control. *Symmetry* 2018; 10(10): 523–535.

3. Kim W, Kang C, Son Y, et al. Vehicle path prediction using yaw acceleration for adaptive cruise control. *IEEE Trans Intell Transp Syst* 2018; 19(12): 3818–3829.

4. Segata M and Cigno RL. Automatic emergency braking: realistic analysis of car dynamics and network performance. *IEEE Trans Veh Technol* 2013; 62(9): 4150–4161.

5. Fu WP, Fang ZD and Zhao ZG. Periodic solutions and harmonic analysis of an anti-lock brake system with piecewise-non-linearity. *J Sound Vib* 2001; 246(3): 543–550.

6. Jing HH, Liu ZY and Chen H. A switched control strategy for antilock braking system with on/off valves. *IEEE Trans Veh Technol* 2011; 60(4): 1470–1484.

7. Kiencke U and Nielsen L. *Automotive control systems*. Berlin: Springer, 2003.

8. Kuo CY and Yeh EC. A four-phase control scheme of an anti-skid brake system for all road conditions. *Proc IMechE, Part D: J Automobile Engineering* 1992; 206(44): 275–283.

9. Pasillas-Lepine W. Hybrid modelling and limit cycle analysis for a class of five-phase ABS algorithms. *Veh Syst Dyn* 2006; 44(2): 173–188.

10. Tanelli M, Osorio G, Bernardo MD, et al. Existence, stability and robustness analysis of limit cycles in hybrid anti-lock braking systems. *Int J Contr* 2009; 82(4): 659–678.

11. Yeh EC and Day GC. Parametric study of anti-skid brake systems using Poincare map concept. *Int J Veh Des* 1992; 13(3): 210–232.

12. Amodeo M, Ferrara A, Terzaghi R, et al. Wheel slip control via second-order sliding-mode generation. *IEEE Trans Intell Transp Syst* 2010; 11(1): 122–131.

13. Harifi A, Aghagolzadeh A, Alizadeh G, et al. Designing a sliding mode controller for slip control of antilock brake systems. *Trans Res Part C* 2008; 16(6): 731–741.

14. Johansen TA, Petersen I, Kalikukh J, et al. Gain-scheduled wheel slip control in automotive brake systems. *IEEE Trans Contr Syst Technol* 2003; 11(6): 799–811.

15. Lin CM and Hsu CF. Neural-network hybrid control for antilock braking systems. *IEEE Trans Neural Netw* 2003; 14(2): 351–359.

16. He YG, Lu CD, Shen J, et al. Design and analysis of output feedback constraint control for antilock braking system with time-varying slip ratio. *Math Prob Eng* 2019; 2019: 1–11.

17. Mirzaei A, Moallem M, Dehkordi BM, et al. Design of an optimal fuzzy controller for antilock braking systems. *IEEE Trans Veh Technol* 2006; 55(6): 1725–1730.

18. Mirzaei M and Mirzaeinejad H. Optimal design of a nonlinear controller for anti-lock braking system. *Trans Res Part C* 2012; 24: 19–35.

19. Mirzaeinejad H. Robust predictive control of wheel slip in antilock braking systems based on radial basis function neural network. *Appl Soft Comput* 2018; 70: 318–329.

20. Pasillas-Lepine W, Loria A and Gerard M. Design and experimental validation of a nonlinear wheel slip control algorithm. *Automatica* 2012; 48: 1852–1859.

21. Qiu YN, Liang XG and Dai ZY. Backstepping dynamic surface control for an anti-skid braking system. *Contr Eng Pract* 2015; 42: 140–152.

22. Tanelli M, Astolfi A and Savaresi SM. Robust nonlinear output feedback control for brake by wire control systems. *Automatica* 2008; 44: 1078–1083.

23. Yu HX, Qi QZ, Duan JM, et al. Multiple model adaptive backstepping control for antilock braking system based on LuGre dynamic tyre model. *Int J Veh Des* 2015; 69(1–4): 168–184.

24. He YG, Lu CD, Shen J, et al. Design and analysis of output feedback constraint control for antilock braking system based on Burckhardt’s model. *Assem Autom* 2019; 39(4): 497–513.

25. Zhang JX and Li J. Adaptive backstepping sliding mode control for wheel slip tracking of vehicle with uncertainty observer. *Measur Contr* 2018; 51(9–10): 396–405.

26. Savaresi SM and Tanelli M. *Active braking control systems: design for vehicles*. Berlin: Springer, 2010.

27. Han JQ and Wang W. Nonlinear tracking differentiator. *J Syst Sci Math Sci* 1994; 14(2): 177–183.

28. Dash PK, Mishra S and Panda G. A radial basis function neural network controller for UPEC. *IEEE Trans Power Syst* 2000; 15(4): 1293–1299.

29. Alzer H, Fonseca CMD and Kovacec A. Young-type inequalities and their matrix analogues. *Lin Multilin Algebra* 2015; 63(3): 622–635.

30. Khalil HK. *Nonlinear systems*. 3rd ed. New York: Prentice Hall, 2001.