Viability of Reverse Pricing in Cellular Networks: A New Outlook on Resource Management

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Abstract—Reverse pricing has been recognized as an effective tool to handle the uncertainty of users’ demands in the travel industry (e.g., airlines and hotels). To investigate its viability in cellular networks, we study the practical limitations of (operator-driven) time-dependent pricing that has been recently introduced, taking into account demand uncertainty. Then, we endeavor to design the reverse pricing mechanism to resolve the weakness of the time-dependent pricing scheme. We show that the proposed pricing scheme can achieve “triple-win” solutions: an increase in the total revenue of the operator; higher resource utilization efficiency; and an increment in the total payoff of the users. Our findings provide a new outlook on resource management, and design guidelines for adopting the reverse pricing scheme.

I. INTRODUCTION

Since Kelly’s seminal work [1], [2], pricing has been recognized as an effective tool for the design and control of modern communication networks. In order to achieve various performance objectives (i.e., maximum social welfare from the resources or maximum revenue collected by the operators), there has been a substantial amount of work on the design of numerous network protocols and pricing schemes [3]–[7]. However, considerable mismatches between “theory” and “practice” have been observed in reality. The potential reason behind these gaps is that the high spatiotemporal user demand uncertainty is largely ignored in the existing literature. For example, the network demand in peak hours can be ten times more than that in off-peak hours [8]. In this work, we tackle this challenging issue, from a new pricing framework.

Traditional pricing (e.g., flat-rate or usage-based pricing) has been questioned by both academic and industry experts since it charges to users regardless of the actual network congestion [9]. This inefficiency, along with the ever-increasing demand in wireless networks, involves the proper design of new (congestion) pricing schemes for network management solutions. For instance, time-dependent pricing (TDP) has been recently introduced to reduce the peak-to-average ratios of network usage, taking into the actual network usage [10], [11]. In [10], the authors showed that 24% of traffic can be redistributed over a day, through incentivizing users to shift their usage to off-peak hours with lower prices. In [11], the comprehensive analytical study of usage-based and flat-rate schemes in TDP was presented. Nevertheless, we argue that resorting to (operator-driven) TDP still poses the problem of matching current network capacity with the network demand, because TDP still has to face or predict the demand uncertainty, rather than embracing it.

Motivated by the above discussions, in this paper, we propose reverse pricing to effectively handle unexpected demand fluctuations. Exemplified by Priceline.com [12]–[14], the essence of reverse pricing is that the operator (partially) delegates responsibility for pricing to the users. Unlike the (operator-driven) TDP [1] in which each user acts as a price taker and adjusts his amount of resources over time, the order of actions is reversed (as the term implies) in that each user can name the unit price that he is willing to pay for a given amount of resources, while the operator decides a resource recommendation rule and a hidden bid-acceptance threshold a priori. Intuitively, the superiority of reverse pricing is its natural ability to deal with the demand uncertainty over time. However, the challenge here is that such user participation in the pricing process may decrease the operator’s revenue, due to the negative cannibalization effect. Then, a natural question is: How to design reverse pricing in conjunction with forward pricing, with the goal of maximizing the operator’s revenue?

This paper endeavors to design such reverse pricing on top of forward pricing to answer the above question with stylized network models. To this end, we first model the interaction between a single operator and the users as a four-stage Stackelberg game, when employing reverse pricing along with forward pricing. To investigate its economic viability, we propose the proportional residual resource recommendation rule in which a user is allocated the available remaining resources in proportion to his payment via forward pricing, and examine the hidden bid-acceptance threshold conditions under which it is more profitable to adopt reverse pricing. We analyze the case of forward pricing only, as a baseline to evaluate the effects of utilizing the reverse pricing scheme. We show that our reverse pricing can achieve “triple-win” solutions: an increase in the total revenue of the operator; higher resource utilization efficiency; and an increment in the total payoff of the users.

The rest of this paper is organized as follows: Section II describes the system model. Section III introduces the reverse pricing mechanism and investigates its economic viability. Section IV examines the forward pricing scheme only, as a baseline to evaluate the impact of the proposed reverse pricing.

1For the rest of paper, we will use “(time-dependent) forward pricing” and “TDP” interchangeably to distinguish between TDP and reverse pricing.
scheme on the operator’s revenue, the users’ payoffs, and the resource utilization efficiency. Section V gives numerical results to validate the proposed studies, followed by concluding remarks in Section VI.

II. SYSTEM MODEL

We consider a network with a total amount of available resource $Q$ (e.g., bandwidth, data rate, etc). A single operator allocates this resource to a set $I \equiv \{1, \cdots, I\}$ of users. We assume that the users have time-varying demand preferences for resources. These preferences are associated with their demand patterns over time. In this regard, we consider a time-slot system where the resource scheduling horizon is divided into a set $H \equiv \{1, \cdots, H\}$ of time slots. Note that this time slot can be based on time-varying demand patterns: peak network demand time slots, normal network demand time slots, and off-peak network demand time slots.

Each user $i \in I$ chooses a finite amount of resources based on his own time-varying demand preferences and the service prices over time. Specifically, user $i$ aims to maximize his own payoff, which is the difference between the utility of the allocated resource and the payment to the operator. Let $p \triangleq [p^h]_{h \in H}$ denote the (operator-driven) price vector per unit resource over the resource scheduling horizon $H$. We assume $p$ is known to the users before the resource scheduling horizon (i.e., ex-ante price), and remains constant at the end of the current scheduling horizon. Without loss of generality, the payoff function for user $i$ at time slot $h$ can be defined as

$$u_i(\theta_i^h, s_i^h, p^h) = \theta_i^h \ln(1 + s_i^h) - p^h s_i^h,$$

where $s_i^h$ is the allocated resource to user $i$, and $\theta_i^h$ is the time-varying willingness to pay of user $i$. As stated in [15], [16], the logarithmic utility function in (1) has been widely used to model the proportionally fair resource allocation in communication networks. We remark that $\theta_i^h$ is the maximum price per unit resource that user $i$ is willing to pay at time slot $h$, which indicates changing necessities of resource over the scheduling horizon (i.e., time-varying demand preferences).

We consider two types of pricing schemes:

1) Forward pricing only: The operator publishes the ex-ante unit price vector $p$ over the resource scheduling horizon $H$. The purpose of the operator is to maximize its expected revenue based on demand predictions. It implies that the operator has to encounter some level of demand uncertainty. This residual uncertainty stemming from users’ actual resource consumption may waste some available resources, which is a commonly observed phenomenon in reality [17].

2) Reverse pricing on top of forward pricing: In order to effectively react to unexpected demand fluctuations, the operator partially relinquishes its pricing control to the users. However, we assume that each user $i$ needs to report the amount of resources $s_i^h$ based on $p^h$, ahead of time slot $h$. From this contract, the operator calculates the residual resources $Q - \sum_{j \in I} s_j^h$, and suggests to allocate more resources to user $i$ in proportional to $s_i^h$, i.e., $x_i^h = s_i^h + \sum_{j \in I, j \neq i} s_j^h (Q - \sum_{j \in I} s_j^h)$. Under this proportional residual resource recommendation rule, the operator decides the hidden bid-acceptance threshold $\tau^h$ (price), and publicly specify the minimum participation unit price $p_{min}^h$ below which $x_i^h$ is not sold, before time slot $h$. Then, user $i$ can name his own unit price $b_i^h$ at will for $x_i^h$ and receives $x_i^h$ if $b_i^h$ exceeds $\tau^h$ at each time slot $h$ (If not, user $i$ receives $s_i^h$ with the unit price $p^h$).

Fig. 1 shows the timing of the game that characterizes the interaction between the operator and users. In Stage I, the operator announces the unit price vector to the users over the resource scheduling horizon, estimating the potential total demand from users. Due to the limited resource available, the operator should design the forward pricing scheme to guarantee that the aggregate user demand is no larger than what it can provide. In Stage II, each user acts as a price taker and chooses the amount of resources over time according to the published unit price vector. When the operator employs forward pricing only, the game ends. When the operator adopts reverse pricing on top of forward pricing, on the other hand, each user is required to report his resource usage via forward pricing, before the beginning of each time slot. In Stage III, the operator calculates the total demand from the users via forward pricing, and sets a resource recommendation rule and a hidden bid-acceptance threshold with a minimum participation unit price via reverse pricing. In Stage IV, each user names his own unit price for the given amount of resources, and the resource is allocated to him immediately and accordingly depending on

| Stage 1: Operator | Stage 2: Each user $i$ | Stage 3: Operator | Stage 4: Each user $i$ |
|-------------------|------------------------|-------------------|-----------------------|
| $\forall p$ is set and announced based on demand predictions over $H$ | Use $s_i^h$ based on $p^h$ at each time slot $h$ | Set a resource recommendation rule $x_i^h$ and a hidden bid-acceptance threshold $\tau^h$ based on $\sum_{i \in I} s_i^h$ ahead of each time slot $h$ | Name own price $b_i^h$ at each time slot $h$ |
| | Report $s_i^h$ based on $p^h$ ahead of each time slot $h$ | Specify a minimum participation unit price $p_{min}^h$ of each time slot $h$ | If $b_i^h \geq \tau^h$, allocated resource $x_i^h$ payment $b_i^h \cdot x_i^h$ |
| | | | If $b_i^h < \tau^h$, allocated resource $s_i^h$ payment $p^h \cdot s_i^h$ |

Fig. 1. Timing of the game.
III. REVERSE PRICING ON TOP OF FORWARD PRICING

We first consider that the operator adopts the reverse pricing scheme on top of the forward pricing scheme. Given the (operator-driven) unit price vector and the corresponding total demand of the users over the resource scheduling horizon, we investigate the users’ participation decisions and corresponding bidding strategies, as well as the operator’s resource recommendation rule and hidden bid-acceptance threshold with the minimum participation unit price. We start with the users’ cases, with the hope to shed light on the feasibility of utilizing the reverse pricing scheme.

A. Users’ Participation Decisions and Subsequent Bidding Strategies in Stage IV

Without loss of generality, we assume that \( x_i \triangleq \{x_i^h\}_{h \in \mathcal{H}, i \in \mathcal{I}} \) denote the user \( i \)'s resource vector declared by the operator in Stage III. In this case, the operator can publicly specify any minimum participation unit price \( 0 \leq p_{min}^h < p^h \) below which \( x_i^h \) is not sold. Given \( x_i^h \) and \( p_{min}^h \) at time slot \( h \), user \( i \) decides whether to participate in the pricing process, i.e.,

\[
\theta_i^h \ln (1 + x_i^h) - p_{min}^h x_i^h \geq \theta_i^h \ln (1 + s_i^h) - p^h s_i^h, \forall i \in \mathcal{I}, \forall h \in \mathcal{H}. \tag{2}
\]

where \( s_i^h \) is the user \( i \)'s reported resource usage in Stage II, based on the announced price per unit resource \( p^h \) in Stage I.

For inducing user \( i \) to take part in the pricing process, \( p_{min}^h \) must satisfy the constraint (2), i.e.,

\[
p_{min}^h \leq \frac{\theta_i^h \ln \left( \frac{(1 + x_i^h)}{(1 + s_i^h)} \right) + p^h s_i^h}{x_i^h}. \tag{3}
\]

Otherwise, user \( i \) has no incentive to name his own unit price \( b_i^h \) (i.e., \( b_i^h = 0 \)). Based on it, we define \( \mathcal{I}^+ (p_{min}^h) = \left\{ i \in \mathcal{I} : p_{min}^h \leq \theta_i^h \ln (1 + x_i^h)/(1 + s_i^h) + p^h s_i^h \right\} \), i.e., the set of users that would name strictly positive prices at time slot \( h \) in Stage IV.

Since each user \( i \in \mathcal{I}^+ (p_{min}^h) \) is uncertain to the hidden bid-acceptance threshold \( \tau_h \), for analytical tractability, we assume that user \( i \) updates his belief that \( \tau_h \) follows a uniform distribution in \( [p_{min}^h, p^h] \) at any time slot \( t \). Then, the expected payoff maximization problem for user \( i \) can be formulated as

\[
\textbf{P1} : \max_{p_{min}^h \leq b_i^h \leq u_i} \left\{ \theta_i^h \ln \left( \frac{1 + x_i^h}{1 + s_i^h} \right) + s_i^h p^h + x_i^h p_{min}^h \right\} + u_i(b_i^h < \tau_h), \forall i \in \mathcal{I}^+ (p_{min}^h), \forall h \in \mathcal{H}. \tag{4}
\]

where \( b_i^h \) is the named unit price to the operator that user \( i \) is willing to pay for the given amount of resources \( x_i^h \).

Proposition 1: For each user \( i \in \mathcal{I} \), the optimal solution for Problem P1 at each time slot \( h \in \mathcal{H} \) is given by

\[
b_i^h = \begin{cases} 
\frac{1}{2x_i^h} \left[ \theta_i^h \ln \left( \frac{1 + x_i^h}{1 + s_i^h} \right) + s_i^h p^h + x_i^h p_{min}^h \right], & \forall i \in \mathcal{I}^+ (p_{min}^h), \\
0, & \forall i \in \mathcal{I} \setminus \mathcal{I}^+ (p_{min}^h). 
\end{cases} \tag{5}
\]

Proof: To verify that Problem P1 is a convex optimization with respect to \( b_i^h \), let us rewrite this problem as follows:

\[
\max_{p_{min}^h \leq b_i^h \leq u_i} \left( \theta_i^h \ln (1 + x_i^h) - b_i^h x_i^h \right) + p_{min}^h + \left( \theta_i^h \ln (1 + x_i^h) - p^h x_i^h \right) + b_i^h x_i^h , \forall_i \in \mathcal{I}, \forall h \in \mathcal{H}. \tag{6}
\]

Apparently, the constraint is a convex set. Thus, it suffices to examine that the second order derivative is less than 0. Let \( f_i(b_i^h) \) denote the user \( i \)'s objective function in (4). Then we have

\[
\frac{\partial^2 f_i}{\partial b_i^h^2} = \frac{2x_i^h}{p^h - p_{min}^h} < 0, \tag{7}
\]

which proves that Problem P1 is a convex optimization with respect to \( b_i^h \). Exploiting the first order necessary condition yields

\[
b_i^h = \frac{1}{2x_i^h} \left[ \theta_i^h \ln \left( \frac{1 + x_i^h}{1 + s_i^h} \right) + s_i^h p^h + x_i^h p_{min}^h \right], \tag{8}
\]

which completes the proof.

Proposition 1 characterizes the optimal bidding strategies of the users in Stage IV. Since the users maximize their expected payoff against the probability of being accepted via reverse pricing, they tend to name their own unit prices below the unit price that the operator sets (i.e., \( b_i^h = (p_{min}^h + p^h)/2 < p^h \) when \( x_i^h = s_i^h \)). It hints on how to design the reverse pricing mechanism to extract more revenue from the users.

B. Operator’s Resource Recommendation Rule and Hidden Bid-Acceptance Threshold in Stage III

We now consider how to utilize the reverse pricing scheme along with the forward pricing scheme. The purpose of the operator is to maximize its expected revenue, considering the total demand from the users in Stage II. To this end, the operator needs to examine the following issues:

1) Resource recommendation rule: How to design a resource recommendation rule based on the total demand from the users in Stage II, subject to the total limited resource?

2) Hidden bid-acceptance threshold: How to set a hidden bid-acceptance threshold for revenue maximization?

To address these issues one by one, we first investigate the resource recommendation rule in the reverse pricing scheme.

Let \( Q^h \triangleq \left( \sum_{i \in \mathcal{I}} x_i^h \right)^+ \) denote the residual resource available over the resource scheduling horizon \( \mathcal{H} \), where \( (\cdot)^+ \) denotes \( \max(\cdot, 0) \). In the special case where no resource is left to the operator at time slot \( h \) (i.e., \( Q^h = 0 \)), the operator has no incentive to involve the users in pricing decisions as described in the previous subsection. In this case, the operator only adopts the forward pricing scheme, or allocates the same amount of resources submitted by the users in Stage II (i.e., \( x_i^h = s_i^h \)) and sets the threshold \( \tau_h = p^h \) to force them to buy it via forward pricing.

When \( Q^h > 0 \), on the other hand, the operator decides the resource recommendation rule to satisfy the following conditions:

\[
Q \geq \sum_{i \in \mathcal{I}} x_i^h \geq \sum_{i \in \mathcal{I}} s_i^h, \quad \forall h \in \mathcal{H} \tag{9}
\]
where the first inequality in (9) corresponds to the maximum available resource constraint that the operator can provide via reverse pricing, and the second inequality in (9) comes from the operator’s revenue maximization standpoint through the disposal of residual available resources, as noted in the previous subsection. In this case, intuitively, the operator seeks to clear the market, i.e., \(Q = \sum_{i \in \mathcal{I}} x^h_i\) for revenue maximization.

Capturing the idea that more resources reported from the users in Stage II implicitly mean higher willingness to pay of them, in this work, we propose the proportional residual resource recommendation rule. To be more specific, the operator simply suggests to allocate the remaining amount of resources to each user \(i\) in proportion to his reported amount of resources \(s^h_i\) in Stage II (i.e., \(x^h_i = \frac{s^h_i}{\sum_{j \in \mathcal{I}} s^h_j} Q^h\)). This recommendation rule yields to obtain “triple-win solutions”, as will be discussed in Sec. V.

Next we explain how the operator determines a hidden bid-acceptance threshold \(\tau^h\) given the proportional residual resource recommendation rule. To this end, the operator should examine the conditions under which it is more profitable to employ the reverse pricing scheme. The answer to these conditions is intertwined with the minimum participation unit price \(p_{\min}\).

Given that user \(i\) reports his resource consumption \(s^h_i\) via forward pricing in Stage II, the operator tries to set \(p_{\min}\), i.e.,

\[
p_{\min} x^h_i \geq p^h s^h_i, \quad \forall i \in \mathcal{I}, \forall h \in \mathcal{H},
\]

where the constraint (10) indicates that the operator avoids the revenue loss via reverse pricing, by setting \(p_{\min}\) to earn at least the revenue via forward pricing. We then have the following results as shown in Lemma 1.

**Lemma 1**: Assume that an operator adopts a proportional residual resource recommendation rule based on the reported aggregate user demand in Stage II. The minimum participation unit price that the operator sets should satisfy the following condition

\[
p^h > p_{\min} \geq \frac{p^h \sum_{i \in \mathcal{I}} s^h_i}{Q^h}, \quad \forall h \in \mathcal{H}.
\]

**Proof**: The proportional residual resource recommendation rule leads to \(Q = \sum_{i \in \mathcal{I}} x^h_i\). Summing over all \(i\) under the constraint (10) completes the proof.

Perhaps surprisingly, the result implies the viability of utilizing the reverse pricing scheme on top of the forward pricing scheme. By exploiting the revealed total user demand in Stage II and partially outsourcing pricing to the users, the operator can boost its revenue.

For revenue maximization, however, the operator should further investigate its expected revenue under different values of \(p_{\min}\). When \(p_{\min}\) is small, in general, the users tend to bid with lower prices (see the last term in Eq. [3]). This implies that more potentially profitable trades are expected to occur, at the cost of the higher increase in its revenue per user via reverse pricing. When \(p_{\min}\) is large, on the other hand, the reverse is generally true. Thus, the operator should set \(p_{\min}\) as a strategic variable, taking this trade-off into account. An example of this will be discussed in Sec. V.

Given any minimum participation unit price \(p_{\min}\), subject to the constraint (11), the operator can determine the hidden bid-acceptance threshold \(\tau^h\) at each time slot \(h \in \mathcal{H}^+\).

**Proposition 2**: For any minimum participation unit price \(p_{\min}\) subject to the constraint (11), the hidden bid-acceptance threshold is given by

\[
\tau^h = p_{\min}, \quad \forall h \in \mathcal{H}.
\]

**Proof**: Suppose that \(\tau^h > p_{\min}\), where \(p_{\min}\) is any minimum participation unit price that satisfies the constraint (10). Assume that user \(i \in \mathcal{I}^+(p_{\min})\) names his own price \(b^h_i\), i.e., \(p_{\min} \leq b^h_i < \tau^h\). Since \(b^h_i < \tau^h\), the payment of user \(i\) is \(s^h_i p^h\). However, if \(\tau^h = p_{\min}\), the resultant payment of user \(i\) is

\[
x^h_i p_{\min} \geq \left( s^h_i + \frac{s^h_i Q^h}{\sum_{j \in \mathcal{I}} s^h_j} \right) \frac{p^h \sum_{i \in \mathcal{I}} s^h_i}{Q^h} \geq p^h s^h_i,
\]

which completes the proof.

### IV. BENCHMARK SCENARIO: FORWARD PRICING ONLY

In the previous section, we have investigated the viability of introducing the reverse pricing scheme on top of the forward pricing scheme. In this section, we consider the case of the forward pricing scheme only. This study serves as a baseline to evaluate the impact and significance of the reverse pricing scheme on the operator’s revenue, the users’ payoffs, and the resource utilization efficiency. By backward induction again, we start from Stage II where each user acts as a price taker and adjusts his amount of resource over time.

**A. Users’ Desired Resources in Stage II**

As described in Sec. II, if the operator announces a unit price \(p^h\) at each time slot \(h \in \mathcal{H}\) in Stage I, each user \(i \in \mathcal{I}\) solves the following payoff maximization problem:

\[
P2 : \max_{s^h_i} u_i(\theta^h_i, s^h_i, p^h) = \frac{\theta^h_i}{p^h} (1 + \frac{s^h_i}{p^h}) - p^h s^h_i,
\]

which leads to

\[
s^h_i^* = \left( \frac{\theta^h_i}{p^h} - 1 \right) ^+.
\]

In the following analysis, throughout the paper, we only focus on a special case of Problem P2 by assuming that all the users are always admitted (i.e., \(s^h_i^* > 0, \forall i \in \mathcal{I}, \forall h \in \mathcal{H}\)).

**B. Operator’s Forward Pricing in Stage I**

In Stage I, the operator tries to maximize its expected revenue over the resource scheduling horizon based on demand predictions. For the operator experiencing a variety of time-varying demand patterns, however, the assumption of the complete information of the network demand a priori is not valid in general. To limit the dimension of this issue, we assume that the operator knows the payoff function of each user \(i\) in (4), but \(\theta^h_i\) is a random variable with a finite mean with a bounded magnitude at each time slot \(h\). For the ease of exposition, the time-varying willingness to pay for user \(i\) can

\footnote{This assumption does not change the main insights obtained in this paper.}
be expressed as $\theta^h_i = \hat{\theta}^h_i + \delta^h_i$, where $\hat{\theta}^h_i$ is a finite mean and $\delta^h_i$ is a random variable that indicates a time variance in user $i$’s willingness to pay. We further assume that $\delta^h_i$ is a zero-mean random variable with a bounded magnitude known to the operator. Mathematically, $E(\delta^h_i) = 0$ and $|\delta^h_i| \leq \epsilon^h_i$, with $\epsilon^h_i > 0$ denoting the maximum magnitude of user $i$’s demand uncertainty, $\forall i \in I$, $\forall h \in H$. Without loss of generality, we assume that $\hat{\theta}^h_1 - \epsilon^h_1 > \hat{\theta}^h_2 - \epsilon^h_2 > \cdots > \hat{\theta}^h_h - \epsilon^h_h$ at each time slot $h$.

Under the above assumption, the operator seeks to announce the optimal price vector $p$ to maximize its expected revenue over the resource scheduling horizon $H$, subject to the limited total resource $Q$. This is obtained by solving the following optimization problem:

$$P_3 : \max_{p \succeq 0} \sum_{h \in H} \sum_{i \in I} E_{\phi_h} (\theta^h_i - p^h),$$

$$\text{s.t.} \sum_{i \in I} \left( \frac{\theta^h_i}{p^h} - 1 \right) \leq Q, \forall h \in H.$$  \hfill (16)

Note that we do not take the expectation of the resource constraint in (17) with respect to $\theta^h_i$, to always guarantee that the aggregate user demand in Stage II is no larger than what the operator can provide.

For satisfying the resource constraint in (17), the operator should set the ex-ante price $p^h$, i.e.,

$$\sum_{i \in I} \left( \frac{\theta^h_i}{p^h} - 1 \right) \leq \sum_{i \in I} \left( \frac{\hat{\theta}^h_i + \epsilon^h_i}{p^h} - 1 \right) \leq Q, \forall h \in H,$$  \hfill (18)

where (a) follows from the maximum value of $\theta^h_i$. Since the objective function in (16) is a decreasing function of $p^h$ at each time slot $h$, we can reformulate Problem $P_3$ as follows:

$$P_4 : \min_{p \geq 0} \sum_{h \in H} \sum_{i \in I} p^h,$$

$$\text{s.t.} \sum_{i \in I} \left( \frac{\hat{\theta}^h_i + \epsilon^h_i}{p^h} - 1 \right) \leq Q, \forall h \in H.$$  \hfill (19)

Proposition 3: Given $Q > \sum_{i \in I} \frac{\theta^h_i + \epsilon^h_i}{\epsilon^h_i} - I$, the optimal solution to Problem $P_4$ is given by

$$p^{h*} = \frac{\sum_{i \in I} \hat{\theta}^h_i + \epsilon^h_i}{Q + I}, \forall h \in H.$$  \hfill (20)

Proof: The objective function in (19) is an increasing function of $p^h$ at time slot $h$. On the other hand, the left hand side of the resource constraint in (20) is decreasing in $p^h$. Thus the optimal solution can be obtained when the equality in (20) holds. Since all the users are always admitted, the optimal solution (21) should be less than $\theta^h_1 - \epsilon^h_1$, which completes the proof.

Proposition 3 reveals the practical limitation, from the operator’s revenue maximization perspective. As the level of demand uncertainty $\epsilon^h_i$ is higher, the operator has to charge a higher price $p^{h*}$ to the users to satisfy the resource constraint in (20). It may lead to large demand fluctuations at the operator in accordance with users’ actual resource consumption. In other words, capturing user demand precisely is of great importance to extract more revenue from the users. This inspires us to introduce the reverse pricing scheme that reveals their demand, at the expense of allowing them to quote their own unit prices.

V. NUMERICAL RESULTS

In this section, we provide numerical examples to study several key properties of reverse pricing on top of forward pricing. For illustration convenience, we consider a 10-user in the network where the total amount of available resource is chosen as $Q = 100$. The resource scheduling horizon is set as $H = \{1, 2, \cdots, 12\}$. The mean willingness to pay for each user follows the uniform distribution on $[10, 20]$ at each time slot. The time variance of each user’s willingness to pay follows the uniform distribution on $[-\epsilon^h, \epsilon^h]$, where $\epsilon^h$ is randomly selected from $[1, 5]$ for each user at each time slot. At each time slot, the minimum participation unit price is fixed as $p_{\text{min}} = \frac{Q^h}{Q}$ unless specified otherwise.

We first investigate the economic viability of utilizing reverse pricing on top of forward pricing. Fig. 2(a) shows how the aggregate user demand changes over the resource scheduling horizon with forward pricing only or both reverse and forward pricing. With forward pricing only, there are still fluctuating residual resources due to the intrinsic demand uncertainty. With reverse pricing on top of forward pricing, on the other hand, because of no demand uncertainty, no resource is left over the resource scheduling horizon, improving resource utilization efficiency. From this, it is observed from Fig. 2(b) that our proposed pricing scheme always outperforms the forward pricing scheme only, in terms of the total payoff of the users. Even if each user reports his actual resource usage via forward pricing, the flexible incentives to name his own
price brings benefits to him. Surprisingly, Fig. 2(c) shows that the operator can extract more revenue by partially outsourcing pricing to the users. This result implies that the operator should re-examine the idea of involving the users in pricing decisions not only to deal with the demand uncertainty, but also to boost its revenue.

We then examine the effects of the minimum participation unit price \( p_{\text{min}}^1 \) at a particular time slot (i.e., time slot 1). Figs. 3(a) shows that the total user demand is non-increasing over \( p_{\text{min}}^1 \). This is due to the fact that the users are induced to bid with higher prices as \( p_{\text{min}}^1 \) increases. In this example, when \( 1.515 < p_{\text{min}}^1 < 1.519 \), only some users name their own prices, decreasing the resource utilization efficiency. When \( 1.519 \leq p_{\text{min}}^1 \leq p^1 = 1.5616 \), no users name their own prices, degenerating the forward pricing only case. Intuitively, the total user payoff is monotonically decreasing over \( p_{\text{min}}^1 \) as shown in Fig. 3(b). Fig. 3(c) shows how the total revenue of the operator changes over \( p_{\text{min}}^1 \). With the goal of maximizing its revenue, \( p_{\text{min}}^1 \) is chosen as 1.515, achieving a large revenue gain (e.g., 28%) compared to forward pricing only.

VI. CONCLUDING REMARKS

This paper studies the economic viability for a cellular operator to employ reverse pricing on top of (operator-driven) time-dependent pricing. By exploring the practical limitations of time-dependent pricing, we design the reverse pricing mechanism to resolve them. We show that the proposed pricing scheme can increase the operator’s revenue, and improve the resource utilization efficiency and the total user payoff in the meantime. These findings provide a new outlook on resource management, and design guidelines for adopting reverse pricing.

A weakness of this study is the very simple user behavior model via reverse pricing, i.e., using the uniform distribution for the users’ belief in the hidden bid-acceptance threshold. This should be verified by the realistic evaluation with real users. Further work should therefore build a system prototype, and conduct an experiment with the real users, where our current research is heading.

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