Abstract—We consider the full-duplex two-way relay channel with direct link between two users and propose two coding schemes: a partial decode-forward scheme, and a combined decode-forward and compute-forward scheme. Both schemes use rate-splitting and superposition coding at each node and generate codewords for each node independently. When applied to the Gaussian channel, partial decode-forward can strictly increase the rate region over decode-forward, which is opposite to the one-way relay channel. The combined scheme uses superposition coding of both Gaussian and lattice codes to allow the relay to decode the Gaussian parts and compute the lattice parts. This scheme can also achieve new rates and outperform both decode-forward and compute-forward separately. These schemes are steps towards understanding the optimal coding.

I. INTRODUCTION

The two-way channel in which two users wish to exchange message was first studied by Shannon [1]. A specific model is the two-way relay channel (TWRC) with a relay located between two users to help exchange messages. Two types of TWRC exist: one without a direct link between the two users, a model suitable for wired communication, and one with the direct link, more suitable for wireless communication. In this paper, we focus on the TWRC with direct link between the two users, also called the full TWRC.

A number of coding schemes have been proposed for the full TWRC. Different relay strategies, including amplify-and-forward, decode-forward based on block Markov coding, compress-forward and a combined decode-forward and compress-forward scheme, are studied in [2]. For the decode-forward strategy, the relay reliably decodes the transmitted messages from both users. It then re-encodes and forwards. For the compress-forward strategy, the relay compresses the noisy received signal and forwards. In [3], a decode-forward scheme based on random binning and no block Markovity was proposed, in which the relay broadcasts the bin index of the decoded message pair.

A new relaying strategy called compute-forward was recently proposed in [4], in which the relay decodes linear functions of transmitted messages. Nested lattice code [5] is used to implement compute-forward in Gaussian channels, since it ensures the sum of two codewords is still a codeword. Compute-forward has been shown to outperform DF in moderate SNR regimes but is worse at low or high SNR [4]. Compute-forward can be naturally applied in two-way relay channels as the relay now receives signal containing more than one message. In [6], nested lattice codes were proposed for the Gaussian separated TWRC with symmetric channel, i.e. all source and relay nodes have the same transmit powers and noise variances. For the more general separated AWGN TWRC case, compute-forward coding with nested lattice code can achieve rate region within 1/2 bit of the cut-set outer bound [7] [8]. For the full AWGN TWRC, a scheme based on compute-forward, list decoding and random binning technique is proposed in [9]. This scheme achieves rate region within 1/2 bit of the cut-set bound in some cases.

In this paper, we consider the ideas of decode-forward and compute-forward together and propose two new coding schemes for the full TWRC. The first scheme is a partial decode-forward scheme which extends the decode-forward scheme in [3]. Each user splits its message into two parts. The relay decodes one part of message from each user, re-encodes these two parts together and forwards. This scheme contains the original decode-forward scheme in [3] as a special case. Different from the one-way relay channel in which partial decode-forward brings no improvement on the achievable rate over decode-forward in Gaussian channels [10], somewhat surprisingly here for the full TWRC, partial decode-forward can achieve new rates and strictly increase the rate region over decode-forward.

The second scheme combines decode-forward scheme with compute-forward for the full Gaussian TWRC. Each user also splits its message into two parts, and encodes one part with a Gaussian codeword and the other with a lattice codeword. The relay chooses to decode-forward one part of the message from each user, while compute-forward the other part. This scheme can also achieve new rates and a better rate region than either decode-forward and compute-forward alone.

II. CHANNEL MODEL

A. Discrete memoryless TWRC model

The discrete memoryless two-way relay channel (DM-TWRC) is denoted by \((X_1 \times X_2 \times X_r, p(y_1, y_2, y_r|x_1, x_2, x_r), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_r)\), as in Figure 11. Here \(x_1\) and \(y_1\) are the input and output signals of user 1; \(x_2\) and \(y_2\) are the input and output signals of user 2; \(x_r\) and \(y_r\) are the input and output signals of the relay. We consider a full-duplex channel in which all nodes can transmit and receive at the same time.

A \((n, 2^{R_1}, 2^{R_2}, P_e)\) code for a DM-TWRC consists of two message sets \(M_1 = [1 : 2^{R_1}]\) and \(M_2 = [1 : 2^{R_2}]\).
three encoding functions $f_{1,i}, f_{2,i}, f_{r,i}$, $i = 1, \ldots, n$ and two decoding function $g_1, g_2$.

\[
x_{1,i} = f_{1,i}(M_1, Y_{1,1}, \ldots, Y_{1,i-1}), \quad i = 1, \ldots, n
\]
\[
x_{2,i} = f_{2,i}(M_2, Y_{2,1}, \ldots, Y_{2,i-1}), \quad i = 1, \ldots, n
\]
\[
x_{r,i} = f_{r,i}(Y_{r,1}, \ldots, Y_{r,i-1}), \quad i = 1, \ldots, n
\]
\[
g_1 : \mathcal{Y}_{1}^{n} \times \mathcal{M}_1 \rightarrow \mathcal{M}_2, \quad g_2 : \mathcal{Y}_{2}^{n} \times \mathcal{M}_2 \rightarrow \mathcal{M}_1.
\]

The average error probability is $P_e = \Pr\{g_1(M_1, Y^n_1) \neq M_2$ or $g_2(M_2, Y^n_2) \neq M_1\}$. A rate pair is said to be achievable if there exists a $(n, 2^nR_1, 2^nR_2, P_e)$ code such that $P_e \rightarrow 0$ as $n \rightarrow \infty$. The closure of the set of all achievable rates $(R_1, R_2)$ is the capacity region of the two-way relay channel.

**B. Gaussian TWRC model**

The full additive white Gaussian noise (AWGN) two-way relay channel can be modeled as below.

\[
Y_1 = X_r + X_2 + Z_1
\]
\[
Y_2 = X_r + X_1 + Z_2
\]
\[
Y_r = X_1 + X_2 + Z_r
\]

where the noises are independent: $Z_1 \sim \mathcal{N}(0, N_1), Z_2 \sim \mathcal{N}(0, N_2), Z_r \sim \mathcal{N}(0, N_r)$. The average input power constraints for user 1, user 2 and the relay are $P_1, P_2, P_r$ respectively.

**III. A PARTIAL DECODE-FORWARD SCHEME**

In this section, we provide an achievable rate region for the TWRC with a partial decode-forward scheme. Each user splits its message into two parts and uses superposition coding to encode them. The relay only decodes one message part of each user and re-encodes the decoded message pair together and broadcast. It can either re-encode each message pair separately or divides these message pairs into lists and only encodes the list index, which is similar to the binning technique in [3]. Both strategies achieve the same rate region. The users then decode the message from each other by joint typicality decoding of both the current and previous blocks.

**A. Achievable rate for the DM-TWRC**

**Theorem 1.** The following rate region is achievable for the two-way relay channel:

\[
R_1 \leq \min\{I(U_1; Y_1|U_2, X_r) + I(X_1; Y_2|U_1, X_2, X_r), I(X_1, X_r; Y_2|X_2)\}
\]
\[
R_2 \leq \min\{I(U_2; Y_1|U_1, X_r) + I(X_2; Y_1|U_2, X_1, X_r), I(X_2, X_r; Y_1|X_1)\}
\]
\[
R_1 + R_2 \leq I(U_1, U_2; Y_1|X_r) + I(X_1; Y_2|U_1, X_2, X_r)
\]
\[
+ I(X_2; Y_1|U_2, X_1, X_r)
\]

(2)

for some joint distribution $p(u_1, x_1)p(u_2, x_2)p(x_r)$.

**Remark 1.** If $U_1 = X_1, U_2 = X_2$, this region reduces to the decode-forward lower bound in [3]. Therefore, the partial DF scheme contains the DF scheme in [3] as a special case.

**Proof:** We use a block coding scheme in which each user sends $B$ messages over $B$ blocks of $n$ symbols each.

1) **Codebook generation:** Fix $p(u_1, x_1)p(u_2, x_2)p(x_r)$. Split each message into two parts: $m_1 \in \{m_{10}, m_{11}\}$ with rate $(R_{10}, R_{11})$, and $m_2 \in \{m_{20}, m_{22}\}$ with rate $(R_{20}, R_{22})$.

- Generate $2^{nR_{10}}$ i.i.d. sequences $u^n_1(m_{10}) \sim \prod_{i=1}^{n} p(u_{i1})$, where $m_{10} \in \{1, 2^{nR_{10}}\}$. For each $u^n_1(m_{10})$, generate $2^{nR_{21}}$ i.i.d. sequences $x^n_1(m_{11}, m_{10}) \sim \prod_{i=1}^{n} p(x_{i1}|u_{i1})$, where $m_{11} \in \{1, 2^{nR_{21}}\}$.

- Generate $2^{nR_{20}}$ i.i.d. sequences $u^n_2(m_{20}) \sim \prod_{i=1}^{n} p(u_{i2})$, where $m_{20} \in \{1, 2^{nR_{20}}\}$. For each $u^n_2(m_{20})$, generate $2^{nR_{22}}$ i.i.d. sequences $x^n_2(m_{22}, m_{20}) \sim \prod_{i=1}^{n} p(x_{i2}|u_{i2})$, where $m_{22} \in \{1, 2^{nR_{22}}\}$.

- Uniformly throw each message pair $(m_{10}, m_{20})$ into $2^{nR_{2}}$ bins. Let $K(m_{10}, m_{20})$ denote the index of bin.

- Generate $2^{nR_{R}}$ i.i.d. sequences $x^n_r(K) \sim \prod_{i=1}^{n} p(x_{ri})$, where $K \in \{1, 2^{nR_{R}}\}$. If $R_r = R_{10} + R_{20}$, there is no need for binning.

The codebook is revealed to all parties.

2) **Encoding:** In each block $b \in \{1, B - 1\}$, user 1 and user 2 transmit $x^n_1(b_{10}), x^n_1(b_{11})$ and $x^n_2(b_{20}), x^n_2(b_{22})$ respectively. In block $B$, user 1 and user 2 transmit $x^n_1(1,1)$ and $x^n_2(1,1)$, respectively.

At the end of block $b$, the relay has an estimate $(\hat{m}_{10}(b), \hat{m}_{20}(b))$ from the decoding procedure. It transmits $x^n_r(K(\hat{m}_{10}(b), \hat{m}_{20}(b)))$ in block $b+1$.

3) **Decoding:** We explain the decoding strategy at the end of block $b$.

**Decoding at the relay:** Upon receiving $y^n_r(b)$, the relay searches for the unique pair $(\hat{m}_{10}(b), \hat{m}_{20}(b))$ such that

\[
\{(u^n_1(\hat{m}_{10}(b)), u^n_2(\hat{m}_{20}(b))), g^n_r(b, x^n_r(\hat{m}_{10}(b), \hat{m}_{20}(b)))\} \subseteq A^n_r.
\]

Following the analysis in multiple access channel, the error probability will go to zero as $n \rightarrow \infty$

\[
R_{10} \leq I(U_1; Y_r|U_2, X_r)
\]
\[
R_{20} \leq I(U_2; Y_r|U_1, X_r)
\]
\[
R_{10} + R_{20} \leq I(U_1, U_2; Y_r|X_r).
\]

(4)

**Decoding at each user:** By block $b$, user 2 has decoded $m_{21}(b-2)$. At the end of block $b$, it searches for a unique message pair $(\hat{m}_{10}(b-1), \hat{m}_{11}(b-1))$ such that

\[
\{(x^n_r(K(\hat{m}_{10}(b-1), \hat{m}_{11}(b-1))), y^n_2(b, x^n_2(\hat{m}_{10}(b-1), \hat{m}_{11}(b-1))), g^n_2(b-1, x^n_2(K(m_{10}(b-2), m_{20}(b-2))), x^n_2(b-1))\} \subseteq A^n_r.
\]

Following joint decoding analysis, the error probability will go to zero as $n \rightarrow \infty$

\[
R_{11} \leq I(X_1; Y_2|U_1, X_2, X_r)
\]
\[
R_{10} + R_{11} \leq I(X_r; Y_2|X_2) + I(U_1, X_1; Y_2|X_2, X_r)
\]
\[
= I(X_1, X_r; Y_2|X_2).
\]

(5)
Similarly, user 1 can decode \((m_{20}(b-1), m_{22}(b-1))\) with error probability goes to zero as \(n \to \infty\) if
\[
R_{22} \leq I(X_2; Y_1 | U_2, X_1, X_r)
\]
\[
R_{20} + R_{22} \leq I(X_2, X_r; Y_1 | X_1).
\]

By applying Fourier-Motzkin Elimination to the inequalities in (4)-(6), the achievable rates in terms of \(R_1 = R_{10} + R_{11}\) and \(R_2 = R_{20} + R_{22}\) are as given in Theorem 1.

**B. Rate region for the Gaussian TWRC**

Now we apply the proposed partial decode-forward scheme to the AWGN TWRC in [1]. Using jointly Gaussian codewords, we can derive an achievable rate region as follows.

**Corollary 1.** The rate region in (3) is achievable for the AWGN two-way relay channel.

Achievability follows from Theorem 1 by setting \(X_1 = U_1 + V_1\), where \(U_1 \sim \mathcal{N}(0, \alpha P_1)\) and \(V_1 \sim \mathcal{N}(0, \alpha P_1)\) are independent, and by setting \(X_2 = U_2 + V_2\), where \(U_2 \sim \mathcal{N}(0, \beta P_2)\) and \(V_2 \sim \mathcal{N}(0, \beta P_2)\) are independent.

**Corollary 2.** Partial decode-forward achieves strictly better region region than the decode-forward scheme in [3] when the following condition holds:
\[
N_r > \min\{N_1, N_2\}
\]
or
\[
C(P_1/N_2) + C(P_2/N_1) > C((P_1 + P_2)/N_r).
\]

The larger rate region of partial decode-forward can come from time sharing of decode-forward and direct transmission (without using the relay). But for asymmetric channels, new rates outside this time-shared region are also achievable as shown in the numerical results section.

**IV. A COMBINED DECODE-FORWARD AND COMPUTE-FORWARD SCHEME FOR THE GAUSSIAN TWRC**

In this section, we propose a combined decode-forward and compute-forward scheme and analyze its rate regions for the Gaussian TWRC. Each user splits its message into two parts. One part is encoded by a random Gaussian code, while another part is encoded by a lattice code. The user transmits a superposition codeword of the two parts. The relay decodes the Gaussian codewords of both users and a function (the sum) of the two lattice codewords. It then jointly encodes all 3 decoded parts and forwards. Again the relay can assign a separate codeword to each set of the 3 decoded parts or it can encode only the list index as in [3] without affecting the achievable rate. The users apply both joint typicality decoding and list lattice decoding [9] to decode the message from each other. The combined scheme achieves genuinely new rate. An example will be given in the numerical result section.

**Theorem 2.** The following rate region is achievable for the AWGN two-way relay channel:

\[
R_{10} \leq C\left(\frac{\alpha P_1}{\alpha P_1 + \beta P_2 + N_r}\right) = I_1
\]
\[
R_{20} \leq C\left(\frac{\beta P_2}{\alpha P_1 + \beta P_2 + N_r}\right) = I_2
\]
\[
R_{10} + R_{20} \leq C\left(\frac{\alpha P_1 + \beta P_2}{\alpha P_1 + \beta P_2 + N_r}\right) = I_3
\]
\[
R_{11} < \frac{1}{2} \log \left(\frac{\alpha P_1 + \beta P_2 + \bar{\alpha} P_1}{N_r}\right) = I_4
\]
\[
R_{22} < \frac{1}{2} \log \left(\frac{\beta P_2 + \bar{\beta} P_2}{N_r}\right) = I_5
\]
\[
R_{10} \leq C\left(\frac{\alpha P_1 + \gamma P_r}{\alpha P_1 + \gamma P_r + N_2}\right) = I_6
\]
\[
R_{20} \leq C\left(\frac{\beta P_2 + \gamma P_r}{\beta P_2 + \gamma P_r + N_1}\right) = I_7
\]
\[
R_{11} \leq C\left(\frac{\gamma P_r}{P_1 + N_2}\right) + C\left(\frac{\bar{\alpha} P_1}{N_2}\right) = I_8
\]
\[
R_{22} \leq C\left(\frac{\gamma P_r}{P_2 + N_1}\right) + C\left(\frac{\bar{\beta} P_2}{N_1}\right) = I_9
\]

where \(0 \leq \alpha, \beta, \gamma \leq 1\) and \(|x|^+ \triangleq \max\{x, 0\}\). By applying Fourier-Motzkin Elimination to the above inequalities, the achievable rates in terms of \(R_1 = R_{10} + R_{11}\) and \(R_2 = R_{20} + R_{22}\) can be expressed as

\[
R_1 \leq \min(I_1, I_6) + \min(I_4, I_8)
\]
\[
R_2 \leq \min(I_2, I_7) + \min(I_5, I_9)
\]
\[
R_1 + R_2 \leq I_3 + \min(I_4, I_8) + \min(I_5, I_9).
\]

**Proof:** We use block coding scheme in which each user sends \(B-1\) messages over \(B\) blocks of \(n\) symbols.

1) **Codebook generation:** Let \(P_1 = \alpha P_1 + \bar{\alpha} P_1\) and \(P_2 = \beta P_2 + \beta P_2\). Without loss of generality, assume \(\bar{\alpha} P_1 \geq \beta P_2\), construct a chain of nested lattices \(\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda_{c_1} \subseteq \Lambda_{c_2}\), where \(\sigma^2(\Lambda_1) = \bar{\alpha} P_1\) and \(\sigma^2(\Lambda_2) = \beta P_2\). \(\Lambda_1\) and \(\Lambda_2\) are Rogers-good and Poltyrev-good, while \(\Lambda_{c_1}\) and \(\Lambda_{c_2}\) are Poltyrev-good [5, 11].

Split each message into two parts: \(m_1 = (m_{10}, m_{11})\) with rate \((R_{10}, R_{11})\), and \(m_2 = (m_{20}, m_{22})\) with rate \((R_{20}, R_{22})\).

- Generate \(2^{nR_{10}}\) random Gaussian codewords \(u_1^{n_{10}}(m_{10})\) with power constraint \(\alpha P_1\). Associate each message
the codebook is revealed to all nodes.

In block $b$, it can then decode $r_20$, $m_{20}$ lattice codewords $u_2^* (m_{20})$. Let $x_2^* (m_2) = u_2^* (m_{20}) + v_2^* (m_{22})$.

- Uniformly throw each pair $(m_{10}, m_{20})$ into $2^{nR_{c1}}$ bins.
- Form the computed codewords $T^* = (t_2^* (m_{11})), Q_2 (t_2^* (m_{22}) + u_2^* (m_{22})))$ mod $A_1$, where $Q_2 (t_2)$ is the lattice quantizer mapping $t_2$ to the nearest lattice point. Uniformly throw $T^*$ into $2^{nR_{c1}}$ bins. Let $S(T^*)$ denote the bin index.
- Generate $2^{nR_{c1}}$ Gaussian codewords $u_r^* (K)$ with power constraint $\gamma P_2$ and $2^{nR_{c1}}$ Gaussian codewords $v_r^* (S)$ with power constraint $\gamma P_r$. Let $x_r^* = u_r^* (K) + v_r^* (S)$.

The codebook is revealed to all nodes.

2) Encoding: In block $b$, user 1 sends $x_1^* (m_1 (b))$ and user 2 sends $x_2^* (m_2 (b))$. Assume the relay has decoded $(m_{10}(b-1), m_{20}(b-1))$ and $T^* (b-1)$ in block $b-1$. It then sends $x_r^* (b) = u_r^* (K(m_{10}(b-1), m_{20}(b-1))) + v_r^* (S(T^*(b-1)))$ in block $b$.

3) Decoding: We explain the decoding strategy at the end of block $b$.

Encoding at the relay: The relay first decodes $m_{10}(b)$ and $m_{20}(b)$ using joint typicality decoding. Similar to the analysis in multiple access channel, $P_r \rightarrow 0$ as $n \rightarrow \infty$ if

$$R_{10} \leq I(U_1; Y_r | U_2, X_r)$$

$$R_{20} \leq I(U_2; Y_r | U_1, X_r)$$

$$R_{10} + R_{20} \leq I(U_1, U_2; Y_r | X_r).$$

(10)

The relay then subtracts $u_r^* (m_{10}(b))$ and $u_r^* (m_{20}(b))$ from its received signal. Following arguments similar to those in [8], it can then decode $T^*(b)$ with vanishing error as long as

$$R_{11} \leq \frac{1}{2} \log \left( \frac{\alpha P_1}{\alpha P_1 + \beta P_2 + \frac{\beta P_2}{N_r}} \right).$$

$$R_{22} \leq \frac{1}{2} \log \left( \frac{\beta P_2}{\alpha P_1 + \beta P_2 + \frac{\beta P_2}{N_r}} \right).$$

(11)

Decoding at each user: At the end of block $b$, user 2 first decodes the unique $m_{10}(b-1)$ such that

$(u_2^* (K(m_{10}(b-1), m_{20}(b-2))), y_2^* (b), x_2^* (b)) \in A_r^c$

and

$(u_1^* (m_{10}(b-1)), u_1^* (K(m_{10}(b-2), m_{20}(b-2))), x_2^* (b-1), y_2^* (b-1)) \in A_r^c$

This decoding has vanishing error probability if

$$R_{10} \leq I(U_r; Y_2 | X_2) + I(U_1; Y_2 | U_r, X_2)$$

$$= I(U_1, U_r; Y_2 | X_2).$$

(12)

User 2 then subtracts $u_1^* (m_{10}(b-1))$ from $y_2^* (b-1) and uses a lattice list decoder [9] to decode a list of possible $m_{11}(b-1)$ of size $2^{n(R_{c1} - C(\alpha P_1/N_r))}$, denoted as $L(m_{11}(b-1))$. To decode which message in this list was sent, it uses the received signal in block $b$. That is to say, it decodes the unique $m_{11}(b-1)$ such that

$$(y_2^* (b), x_2^* (b), u_1^* (K(m_{10}(b-1), m_{20}(b-1))),$$

$$x_1^* (K(m_{10}(b-1), m_{20}(b-1)), S(T^*(b-1)))) \in A_r^c$$

and

$$m_{11}(b-1) \in L(m_{11}(b-1)).$$

This decoding has vanishing error probability if

$$R_{11} \leq I(X_r; Y_2 | X_1, U_r) + C(\alpha P_1/N_r).$$

(13)

Similarly, user 1 can decode $m_{20}(b-1), m_{22}(b-1)$ with vanishing error as long as

$$R_{20} \leq I(U_2; U_r | Y_1)$$

$$R_{22} \leq I(X_r; Y_1 | U_1, U_r) + C(\beta P_2/N_1).$$

(14)

Finally, by setting

$$X_1 = U_1 + V_1; \quad U_1 \sim \mathcal{N}(0, \alpha P_1), \quad V_1 \sim \mathcal{N}(0, \alpha P_1)$$

$$X_2 = U_2 + V_2; \quad U_2 \sim \mathcal{N}(0, \beta P_2), \quad V_2 \sim \mathcal{N}(0, \beta P_2)$$

$$X_r = U_r + V_r; \quad U_r \sim \mathcal{N}(0, \gamma P_r), \quad V_r \sim \mathcal{N}(0, \gamma P_r)$$

the achievable rate in Theorem 2 can be derived from inequalities (10)-(13). [11]

V. NUMERICAL RESULTS

In this section, we compare the achievable rate regions of the two proposed schemes with pure decode-forward (DF) [3] and pure compute-forward [2].

Figures 2 and 3 show the achievable rate regions of pure DF [3], of direct transmission (without using the relay) and of the proposed partial DF for 2 different channel configurations. Figure 2 shows that partial DF can achieve new rates outside the time sharing region of pure DF and direct transmission. For example, by setting $\alpha = 1, \beta = 0.5$ in [3], partial DF can achieve the rate $(R_1, R_2) = (0.58, 1.47)$ which is outside the convex hull of direct transmission and pure DF. This is notably different from the one-way relay Gaussian channel in which partial DF brings no improvement. In Figure 2, the channel from the users to the relay is stronger than the channel between two users, thus the relay chooses partial DF to obtain a better rate region than DF. Figure 3 shows performance for another channel configuration which is symmetric. In this case, the channels from two users to the relay are significantly stronger than the direct channel, and the relay will fully decode the messages.

Figures 4 and 5 present the achievable rate regions for DF, compute-forward and the combined scheme. The cut-set outer bound is obtained by assuming correlated channel inputs and independent channel noise, which is different from the cut-set bound in [9] for physically degraded channels. Both Figures 4 and 5 show that the combined scheme can achieve a better rate region than either DF and compute-forward alone. Figure 4 for an asymmetric channel shows new rates outside the time-shared region of DF and compute-forward. For example, by setting $\alpha = 0.5, \beta = 0$ in [3], the combined scheme can achieve the rate $(R_1, R_2) = (0.678, 0.859)$ which is outside the convex hull of pure DF and pure compute-forward. In
also achieve a boundary point of Figure 5 for a symmetric channel, the combined scheme can separate scheme. These results suggest more comprehensive both Gaussian and lattice codes can strictly outperform each with compute-forward by rate splitting and superposition of Gaussian relay channel. In addition, combining decode-forward decode-forward. This result is opposite to the one-way Gaussian relay channel: a partial decode-forward scheme and a combined decode-forward and compute-forward scheme. Analysis for asymmetric channels. But because of space limitation, analyses of these new rates are left for future work.

VI. CONCLUSION

We have proposed two new coding schemes for the two-way relay channel: a partial decode-forward scheme and a combined decode-forward and compute-forward scheme. Analysis for the Gaussian channel shows that partial decode-forward can strictly increase the rate region of the TWRC over pure decode-forward. This result is opposite to the one-way Gaussian relay channel. In addition, combining decode-forward with compute-forward by rate splitting and superposition of both Gaussian and lattice codes can strictly outperform each separate scheme. These results suggest more comprehensive coding schemes possible for the TWRC.

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