NEUTRON STARS WITH SMALL RADII—THE ROLE OF $\Delta$ RESONANCES

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ABSTRACT

Recent neutron star observations suggest that the masses and radii of neutron stars may be smaller than previously considered, which would disfavor a purely nucleonic equation of state (EoS). In our model, we use a flavor SU(3) sigma model that includes $\Delta$ resonances and hyperons in the EoS. We find that if the coupling of the $\Delta$ resonances to the vector mesons is slightly smaller than that of the nucleons, we can reproduce both the measured mass–radius relationship and the extrapolated EoS.

Key words: binaries: general – stars: neutron

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1. INTRODUCTION

The mass–radius relationship of neutron stars is of key importance if one wants to understand the high-density low-temperature region of the hadronic equation of state (EoS). Depending on this relationship, certain models for the hadronic EoS can either be confirmed or ruled out. A recent analysis of the masses and radii of neutron stars by Özel et al. (2008, 2010; Güver et al. 2010a, 2010b) seems to imply that they are smaller than previously assumed, with radii between 8 and 10 km and masses between 1.6 and 1.9 solar masses. This information is extracted from the thermonuclear bursts of X-ray emitting neutron stars in binary systems. Using the measured distance, the apparent surface area during the cooling phase of the bursts and the flux in certain types of bursts which exceed the Eddington limit, a constraint for the masses and radii of the measured neutron stars is deduced. From this, a backward-extrapolation of error bars for equations of state (Harada 2001; Lattimer & Prakash 2007; Lindblom 1992; Özel & Psaltis 2009) is performed. A comparison with various models of equations of state suggests that a purely nucleonic EoS may be too stiff, i.e., may produce too high masses at too large radii (Özel et al. 2010, and references therein). Note, however, that Chowdhury et al. (2010) are well able to reproduce a maximum mass of 1.92 $M_\odot$ for the static case using a density-dependent M3Y effective interaction for a nucleonic EoS. It is also possible that more exotic particles are realized within the interior of neutron stars (Weber 2001), which would soften the EoS.

Several attempts have already been made to understand the measured values. Fattoyev & Piekarewicz (2010) use relativistic mean field (RMF) models with a nucleonic EoS. While they can fit the EoS, they are not able to fully reach the small masses and radii reported by Özel et al. CiarciaIlli & Sandin (2010) explore the possibility of neutron stars having a dark matter core. Drago & Lavagno (2010) propose that this could be an experimental signal for the existence of two types of stars, ordinary neutron stars and “ultra-compact” neutron stars like quark stars. Steiner et al. (2010) performed a reanalysis of the experimental data with slightly different assumptions. They relax the assumption of the stellar radius at maximum temperature being equal to the photospheric radius, increasing the allowed radius interval. As a result, they reproduce mass–radius relationships which are in agreement with a purely nucleonic EoS.

In our approach, we want to explore the possibility of additional particle species being present, namely $\Delta$ resonances and hyperons, and investigate the effect on the EoS of neutron stars.

2. DESCRIPTION OF THE MODEL

A hadronic SU(3) sigma–omega model is used to describe the properties of neutron star matter. The included baryonic degrees of freedom are the baryon octet ($n,p$, and hyperons $\Lambda, \Sigma^{+,0,–}, \Xi^{0,–}$), the leptons ($e, \mu$), and the resonances from the spin 3/2 decuplet ($\Delta^{++,+0,–}, \Sigma^{++,0,–}, \Xi^{++,0,–}$, and $\Omega^{–}$). The mesons, that mediate the interactions between the baryons, are the vector–isoscalar $\omega$ and $\phi$, the vector–isovector $\rho$ and the scalar–isoscalar $\sigma$ and $\zeta$ (strange quark–antiquark state).

The Lagrangian density reads

$$L = L_{\text{Kin}} + L_{\text{Int}} + L_{\text{Self}} + L_{\text{SB}}.$$  

$L_{\text{Kin}}$ is the kinetic energy term for the hadrons and leptons. In addition there is an interaction term between the baryons and the scalar and vector mesons

$$L_{\text{Int}} = – \sum_i \bar{\psi}_i \left[ \gamma_0 (g_{i\omega}\omega + g_{i\phi}\phi + g_{i\rho}\rho) + M_i^* \right] \psi_i,$$

with the effective mass $M_i^*$ given by

$$M_i^* = g_{i\sigma}\sigma + g_{i\zeta}\zeta + \delta m_i,$$

with a small bare mass term $\delta m_i$. The coupling strengths of the baryons to the scalar fields are connected via SU(3) symmetry relations and the different SU(3) invariant coupling strengths are fitted to reproduce the baryon masses in vacuum (see Dexheimer & Schramm 2010 for the values of the couplings).

The self-interaction terms for the scalar and vector mesons read...
\[ L_{\text{Self}} = -\frac{1}{2} \left( m_r^2 \omega^2 + m_r \rho^2 + m_\phi \phi^2 \right) 
+ g_4 \left( \omega^4 + \phi^4 + 3 \omega^2 \phi^2 + \frac{4 \omega^3 \phi}{\sqrt{2}} + \frac{2 \omega \phi^3}{\sqrt{2}} \right) 
+ k_0 (\sigma^2 + \xi^2) + k_1 (\sigma^2 + \xi^2)^2 
+ k_2 \left( \frac{\sigma^4}{2} + \xi^4 \right) + k_3 \sigma^2 \xi + k_4 \ln \frac{\sigma^2 \xi}{\sigma_0^2 \xi_0} \cdot (4) \]

and the explicit chiral symmetry breaking term is given by

\[ L_{SB} = m_n^2 f_\pi \sigma + \left( \sqrt{2} m_n^2 f_k - \frac{1}{\sqrt{2}} m_n^2 f_\pi \right) \xi. \cdot (5) \]

In the case of the baryonic octet we use an \( f \)-type coupling between the baryons and vector mesons, which yields coupling strengths as given by quark counting rules, i.e., \( g_{1o} = (n_i^i - n_j^j) g_0^0 \), \( g_{i0} = -(n_i^i - n_j^j) \sqrt{2} g_0^0 \), where \( g_{0} \) denotes the vector coupling of the baryon octet and \( n^i \) the number of constituent quarks of species \( i \) in a given hadron. A more detailed description of the model can be found in Papazoglou et al. (1999). The model has been tested extensively both for nuclear matter (Zschiesche et al. 2007) and neutron stars (Dexheimer & Schramm 2010; Dexheimer et al. 2008).

The parameters are fitted to reproduce the correct vacuum masses of the baryons and mesons, nuclear saturation properties at \( \rho_0 = 0.15 \) particles fm\(^{-3}\), the correct binding energy per nucleon, \( B/A = -16 \) MeV (from Myers & Swiatecki 1969 and Moller et al. 1988, slightly different from \( B = -15.26 \) MeV from Chowdhury & Basu 2006) and an incompressibility of \( K = 297.32 \) MeV and the asymmetry energy \( E_{\text{sym}} = 32.5 \) MeV. The incompressibility is a little bit high, but still acceptable.

As the values of the vector coupling of the decuplet are not constrained by the properties of saturated nuclear matter as in the case of the baryon octet, in the following we allow for moderate deviations from the overall vector coupling strength for the baryon decuplet compared to the octet. In order to study the effects of such a deviation in a most direct way we introduce a single parameter \( r_v \), which in the case of the \( \Delta \) resonances is defined as

\[ r_v = \frac{g_{\Delta o}}{g_{N o}} \cdot (6) \]

with the same rescaling of the other states of the decuplet.

Using this approach, we solve the equations of motion by differentiating the grand-canonical potential \( \Omega = -p = \epsilon - \mu Q_B \) of the model for \( T = 0 \) with respect to the fields (in mean-field approximation). Then we determine the EoS and use it to solve the Tolman–Oppenheimer–Volkoff equations for spherical static stars. A standard BPS crust EoS (Baym et al. 1971) is added to the outer layers of the star.

3. RESULTS

\( \Delta \) resonances have a repulsive vector potential which works to counteract gravity in a compact star. If the interaction strength of the \( \Delta \) resonances with respect to the nucleons is lowered, i.e., \( r_v < 1 \), then the EoS becomes softer and hence the maximum supported mass of the star decreases.

Figure 1 shows the impact of various coupling strengths \( r_v \) of the \( \Delta \) resonances on the overall mass–radius relation of the neutron star. The upper black curve represents \( r_v = 1 \), and in each subsequent curve the coupling strength is lowered by 0.05, arriving at 0.8 in the lowest curve. If the coupling strength of the

\[ \text{Figure 1. Mass–radius relationship of neutron stars for various couplings of the } \Delta \text{ resonances, starting from } r_v = 1 \text{ (upper line) to 0.8 (lowest line). Also included are the } 1 \sigma \text{ error bars for measured neutron stars from } \text{"Ozel et al. (2010).} \]

\[ \text{The black diamond on each curve represents the maximum stable configuration of the neutron star.} \]

\[ \text{(A color version of this figure is available in the online journal.)} \]

\( \Delta \) resonances is reduced, the maximum mass of the star likewise decreases. We found that for values of 0.7 or lower, the EoS has to be considered with care and may start to yield unphysical results such as the appearance of \( \Delta \) in the nuclear ground state. However, for values of \( r_v \) around 0.9, which corresponds to the physical case where the coupling strength of the \( \Delta \) resonances is only slightly lowered compared to normal nucleons, the mass–radius relation is in good agreement with the values given by \text{"Ozel et al.}\)

Figure 2 shows the EoS for various cases considered. For comparison reasons, we use the same units as \text{"Ozel et al. (2010)\) and include their error bars (black perpendicular lines). The stiffer line represents a purely nucleonic e, n, p, and e being present. In all subsequent cases, \( \Delta \) resonances are included. We look at two cases, \( r_v = 1 \) and \( r_v = 0.9 \) (both dashed). The dotted lines below each of the curves represent the effects of muons and hyperons being added. The inclusion of additional particle species generally softens the EoS. The major effect comes from the delta resonances, while the hyperons and muons only make minor changes.

Figure 3 shows the particle abundances for the best-fit case of \( r_v = 0.9 \), with nucleons, leptons, \( \Delta \) resonances and hyperons present. At densities between 0.2 and 0.5 particles fm\(^{-3}\), where \( Q_0 = 0.15 \) fm\(^{-3}\) represents normal nuclear matter density, the \( \Delta \) resonances start to appear, first the \( \Delta^+ \) then the others. At very high densities beyond six times \( Q_0 \), the hyperons start to appear. In principle, all the listed particles are present in the model and can appear, including the excited spin 3/2 states. However, as one can see in Figure 3, only the \( \Lambda \) starts to appear in a small fraction in the relevant regime.

It is noteworthy that negatively charged particles like the muon and the \( \Lambda^- \) at a certain point take up the role of the electrons in charge conservation and thus reduce the electron...
Considered equations of state (from stiffest to softest): purely nucleonic (solid), including \( \Delta \) resonances at \( r_v = 1 \) (dashed), \( \Delta \) resonances and hyperons at \( r_v = 0.9 \) (dotted). The additional inclusion of hyperons only creates a minor effect.

(A color version of this figure is available in the online journal.)

Particle abundancy as a function of the baryonic density in \( 1 \text{ fm}^{-3} \) for the case with hyperons and Delta resonances present, \( r_v = 0.9 \).

(A color version of this figure is available in the online journal.)

Effective mass of the particles as a function of the baryonic density.

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Figure 4. Effective mass of the particles as a function of the baryonic density.

(A color version of this figure is available in the online journal.)

4. DISCUSSION

Recent experimental data from Özel et al. (2010) seem to favor neutron star masses and radii that are smaller than previously considered. The question is whether a purely nucleonic EoS, which naturally is very stiff, can describe such small masses and radii. Chowdhury et al. (2010) reach a maximum mass for the static case of \( 1.92 M_{\odot} \), which is in agreement with the upper limit of Özel et al. even for a nucleonic EoS.

In a different analysis of the experimental data, Steiner et al. (2010) relax the assumption of the stellar radius at maximum temperature being equal to the photospheric radius, increasing the allowed radius interval. As a result, they reproduce mass–radius relationships which are in agreement with a purely nucleonic EoS. If the analysis of Özel et al. is correct, and neutron stars have smaller masses and radii than previously considered, then this may hint at more exotic particle species being present in the EoS. We have shown that an EoS with \( \Delta \) resonances in such a case would be in accordance with the experimental data. While our approach includes hyperonic degrees of freedom in a natural way, the results are dominated by the impact of the \( \Delta \) resonances, whereas hyperons only have a minor effect on the result.

If, on the other hand, Steiner et al. should be correct with their analysis, this would point to a more conservative mass–radius relation and EoS, which can be described by the presence of nucleons and electrons alone. A greater amount of analyzed experimental data may be necessary to resolve this issue.

5. CONCLUSIONS

We have applied a nonlinear realization of an extended sigma–omega model, including \( \Delta \) resonances and hyperons in the EoS of a neutron star calculation. A parameter scan for the coupling strength of the \( \Delta \) resonances has been performed. By slightly reducing the strength of the vector coupling by about 10% compared to the values for the baryonic octet we obtain good agreement with the results from Özel et al. (2010), both

abundance. At a certain point, the \( \Delta \) particles, due to their reduced repulsive potential, become the dominant particle species.

Figure 4 shows the effective mass of the nucleons, \( \Delta \) and \( \Lambda \) particles within the chiral model as a function of the density.
in the mass–radius relation and EoS. For the best-fit case with $r_v = 0.9$, we obtain a maximum mass of $1.65 \, M_\odot$ and a radius of 10.4 km. It is noteworthy that this agreement can be achieved by the not too exotic inclusion of Δ resonances in the EoS without considering quark cores or quark stars. Future work that analyzes the effect of Δ resonances on rotating neutron stars is underway.

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