SO(10) SUSY GUT for Fermion Masses: Lepton Flavor and CP Violation

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Abstract

We discuss the results of a global $\chi^2$ analysis of a simple $SO(10)$ SUSY GUT with $D_3$ family symmetry and low energy R parity. The model describes fermion mass matrices with 14 parameters and gives excellent fits to 20 observable masses and mixing angles in both quark and lepton sectors, giving 6 predictions. Bi-large neutrino mixing is obtained with hierarchical quark and lepton Yukawa matrices; thus avoiding the possibility of large lepton flavor violation. The model naturally predicts small 1-3 neutrino mixing, with $\sin^2 \theta_{13} \simeq 0.05 - 0.06$. In this paper we evaluate the predictions for the lepton flavor violating processes, $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ and also the electric dipole moment of the electron, $d_e$, muon and tau, assuming universal squark and slepton masses, $m_{16}$, and a universal soft SUSY breaking $A$ parameter, $A_0$, at the GUT scale. We find $Br(\mu \rightarrow e\gamma)$ is naturally below present bounds, but may be observable by MEG. Similarly, $d_e$ is below present bounds; but is within the range of future experiments. We also give predictions for the light Higgs mass (using FeynHiggs). We find an upper bound given by $m_h \leq 127$ GeV, with an estimated $\pm 3$ GeV theoretical uncertainty. Finally we present predictions for SUSY particle masses in the favored region of parameter space.
1 Introduction

In this letter we present results for a global $\chi^2$ analysis of the $SO(10)$ SUSY GUT for fermion masses presented in Ref. [1]. The model also has a $D_3 \times [U(1) \times Z_2 \times Z_3]$ family symmetry. The three families of quarks and leptons are contained in three 16 dimensional representations of $SO(10)$ \{16_a, 16_3\} with $16_a, a = 1, 2$ a $D_3$ flavor doublet (see Ref. [2] for details on $D_3$). The third family, along with the pair of electroweak Higgs doublets, contained in the 10 dimensional representation of $SO(10)$, are $D_3$ singlets. Hence only the third generation has a renormalizable Yukawa coupling and, as a consequence, we have $\lambda_t = \lambda_b = \lambda_\tau = \lambda_\nu$, Yukawa unification at $M_{GUT}$. This forces us into the large $\tan \beta$ regime and several interesting predictions follow. We have derived the consequences of third generation Yukawa unification in several papers. In Ref. [5] we demonstrated that in order to fit the low energy values of the top, bottom and tau masses (with the typically large, of order 50%, radiative corrections to the bottom quark mass) the soft SUSY breaking parameters necessarily reside in a very narrow region of the possible parameter space. Hence we have definite predictions for SUSY spectra, see [5] and Section 4 for more details. In addition, in this region of parameter space the light Higgs mass necessarily has a central value of order 120 GeV. In Ref. [6] we demonstrated that this same minimal $SO(10)$ SUSY model [MSO$_{10}$SM] gives the correct abundance of dark matter, fitting the WMAP data, and gives observable values for the branching ratio $Br(B_s \rightarrow \mu^+ \mu^-)$ with a lower bound of order $10^{-8}$. The dark matter candidate in this model is the LSP, neutralino, which predominantly annihilates through a direct s-channel CP odd Higgs, $A$. In addition, it also dominates in the leptonic decay of $B_s$.

In the present model, all of the above results are retained (with small modifications), but in addition we fit the masses and mixing angles of all three families, including neutrino data. The model describes fermion mass matrices with 14 parameters and gives excellent fits to 20 observable masses and mixing angles in both quark and lepton sectors, giving 6 predictions. Both the charged fermion and neutrino mass matrices are hierarchical; thus suppressing large flavor violating interactions, even at large $\tan \beta$. The simple structure of the neutrino sector leads quite naturally to maximal atmospheric neutrino oscillations and large solar neutrino mixing [1]. We predict a very small value for $\sin \theta_{13} \simeq 0.05 \sim 0.06$. In addition, CP violation in the neutrino sector is fixed by the phases in the charged fermion mass matrices. At the same we can easily accommodate leptogenesis via non-thermal processes, see for example [7].

2 The Model

The full superpotential $W = W_{\text{ch.fermions}} + W_{\text{neutrino}}$ for fermion masses and mixing angles contains two terms. The first term, resulting in Dirac Yukawa matrices for charged fermions and neutrinos, is given by

$$W_{\text{ch.fermions}} = 16_3 10 16_3 + 16_a 10 \chi_a$$

$$+ \tilde{\chi}_a \left( M_\chi \chi_a + 45 \frac{\phi}{M} 16_3 + 45 \tilde{\phi}_a \frac{16_3}{16_a} + A 16_a \right).$$

1The charged fermion sector of this theory was considered in an earlier paper [2] and the neutrino sector of the theory was inspired by the previous analysis by one of us, R.D. [3].
The third family of quarks and leptons is contained in the superfield $16_3$ (transforming as a 16 of $SO(10)$); the first two families are contained in $16_a$, $a = 1, 2$ (with explicit $SO(10)$ transformations and transforming as a $D_3$ doublet) and the two Higgs doublets are contained in 10. The additional fields are an adjoint of $SO(10)$ (45) and several $SO(10)$ singlet flavon fields needed to break the full flavor symmetry $D_3 \times [U(1) \times Z_2 \times Z_3]$. Note, $M_\chi = M_0 (1 + \alpha X + \beta Y)$ includes $SO(10)$ breaking vevs in the X and Y directions, $\phi^a$, $\tilde{\phi}^a$ ($D_3$ doublets), $A$ ($1_B$ singlet) are $SO(10)$ singlet flavon fields, and $\tilde{M}$, $M_0$ are $SO(10)$ singlet masses. The fields 45, $A$, $\phi$, $\tilde{\phi}$ are assumed to obtain vevs $\langle 45 \rangle \sim (B - L) M_G$ (where $B$, $L$, $M_G$ is baryon and lepton number and the GUT scale, respectively), $A \ll M_0$ and

$$\langle \phi \rangle = \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right), \quad \langle \tilde{\phi} \rangle = \left( \begin{array}{c} 0 \\ \tilde{\phi}_2 \end{array} \right)$$

with $\phi_1 > \phi_2$. The second term gives large lepton number violating masses for “right-handed” neutrinos; necessary for the See-Saw mechanism. We have

$$W_{\text{neutrino}} = \overline{16} \left( \lambda_2 \ N_a \ 16_a + \lambda_3 \ N_3 \ 16_3 \right) + \frac{1}{2} \left( S_a \ N_a \ N_a + S_3 \ N_3 \ N_3 \right)$$

where the fields $N_3$; $N_a a = 1, 2$ are $SO(10)$ singlets and $\overline{16}$ is assumed to break $SO(10)$ to $SU(5)$ via a vev in the right-handed neutrino direction.

The superpotential, (Eqn. 1) results in the following Yukawa matrices

$$Y_u = \left( \begin{array}{ccc} 0 & e' \rho & -\epsilon \xi \\ -e' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{array} \right) \lambda$$

$$Y_d = \left( \begin{array}{ccc} 0 & e' \xi & -\epsilon \xi \sigma \\ -e' \xi & \tilde{\epsilon} \xi & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{array} \right) \lambda$$

$$Y_e = \left( \begin{array}{ccc} 0 & -e' \omega & 3 \epsilon \xi \omega \\ e' \omega & 3 \tilde{\epsilon} \omega & 3 \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{array} \right) \lambda$$

$$Y_\nu = \left( \begin{array}{ccc} 0 & -e' \omega & \frac{3}{2} \epsilon \xi \omega \\ e' \omega & 3 \tilde{\epsilon} \omega & \frac{3}{2} \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{array} \right) \lambda$$

with

$$\xi = \frac{\phi_2}{\phi_1}; \quad \tilde{\epsilon} \propto \frac{\phi_2}{\tilde{M}}; \quad \epsilon \propto \frac{\phi_1}{\tilde{M}}; \quad \epsilon' \sim \frac{A}{M_0};$$

$$\sigma = \frac{1 + \alpha}{1 - 3\alpha}; \quad \rho \sim \beta \ll \alpha$$

$$\omega = 2 \sigma/(2 \sigma - 1).$$

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2In our notation, Yukawa matrices couple electroweak doublets on the left to singlets on the right. It has been shown in Ref. [8] that excellent fits to charged fermion masses and mixing angles are obtained with this Yukawa structure.
The Dirac mass matrices are then given by

\[
m_i \equiv Y_i \frac{v}{\sqrt{2}} \sin \beta \quad i = \nu, \ u
\]

\[
m_i \equiv Y_i \frac{v}{\sqrt{2}} \cos \beta \quad i = e, \ d
\]

Consider the neutrino masses. In the three 16s we have three electroweak doublet neutrinos (\(\nu_a, \nu_b\)) and three electroweak singlet anti-neutrinos (\(\bar{\nu}_a, \bar{\nu}_b\)). In addition, the anti-neutrinos get GUT scale masses by mixing with three \(SO(10)\) singlets \(\{N_a, a = 1, 2, 3\}\) transforming as a \(D_3\) doublet and singlet respectively. We assume \(\Phi_1\) obtains a vev, \(v_{16}\), in the right-handed neutrino direction, and \(\langle S_a \rangle = M_a\) for \(a = 1, 2\) (with \(M_2 > M_1\)) and \(\langle S_3 \rangle = M_3\)\(^3\). We thus obtain the effective neutrino mass terms given by

\[
W = \nu \ m_\nu \ \bar{\nu} + \bar{\nu} \ V \ N + \frac{1}{2} \ N \ M_N \ N
\]

with

\[
V = v_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad M_N = \text{diag}(M_1, M_2, M_3).
\]

The electroweak singlet neutrinos \(\{\nu, N\}\) have large masses \(V, M_N \sim M_G\). After integrating out these heavy neutrinos, we obtain the light neutrino mass matrix given by

\[
\mathcal{M} = m_\nu \ M_R^{-1} \ m_\nu^T,
\]

where the effective right-handed neutrino Majorana mass matrix is given by:

\[
M_R = V \ M_N^{-1} \ V^T \equiv \text{diag}(M_{R_1}, M_{R_2}, M_{R_3}),
\]

with

\[
M_{R_1} = (\lambda_2 \ v_{16})^2 / M_2, \quad M_{R_2} = (\lambda_2 \ v_{16})^2 / M_1, \quad M_{R_3} = (\lambda_3 \ v_{16})^2 / M_3.
\]

Defining \(U_e\) as the \(3 \times 3\) unitary matrix for left-handed leptons needed to diagonalize \(Y_e\) (Eqn. 4), i.e. \(Y_e^D = U_e^T \ Y_e \ U_e^T\) and also \(U_\nu\) such that \(U_\nu^T \ \mathcal{M} \ U_\nu = \mathcal{M}_D = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\), then the neutrino mixing matrix is given by \(U_{PMNS} = U_e^T \ U_\nu\) in terms of the flavor eigenstate (\(\nu_\alpha, \alpha = e, \mu, \tau\)) and mass eigenstate (\(\nu_i, i = 1, 2, 3\)) basis fields with

\[
\nu_\alpha = \sum_i (U_{PMNS})_{\alpha i} \ \nu_i.
\]

For \(U_{PMNS}\) we use the notation of Ref [9] with

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_3c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_3c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} \nu_1 \\ e^{i\alpha_2/2} \nu_2 \\ e^{i\alpha_3/2} \nu_3 \end{pmatrix}
\]

\(^3\)These are the most general set of vevs for \(\phi_a\) and \(S_a\). The zero vev for \(\phi_1\) can be enforced with a simple superpotential term such as \(S \ \phi_a \ \phi_a\).
Finally, we note that this theory is certainly not fundamental with many arbitrary symmetry breaking VEVs at the GUT scale. Nevertheless, it has two major features in its favor. As a result of the GUT and family symmetries, the Yukawa matrices, which are the only observables of the complicated GUT physics, have fewer parameters than low energy observables. Hence this theory is predictive. Secondly, the model has the advantage that it self-consistently fits the low energy data, and thus, at the very least, it is an excellent phenomenological ansatz for fermion masses. Thus it can be tested via additional low energy flavor violating processes.

3 Global $\chi^2$ Analysis

Yukawa matrices in this model are described by seven real parameters \{\(\lambda, \epsilon, \tilde{\epsilon}, \sigma, \rho, \epsilon', \xi\)\} and, in general, four phases \{\(\Phi_\sigma, \Phi_\xi, \Phi_\rho, \Phi_\xi\)\}. Therefore, in the charged fermion sector we have 11 parameters to explain 9 masses and three mixing angles and one CP violating phase in the CKM matrix, leaving us with 2 predictions. Note, these parameters also determine the neutrino Yukawa matrix. Finally, our minimal ansatz for the right-handed neutrino mass matrix is given in terms of three additional real parameters, i.e. the three right-handed neutrino masses. At this point the three light neutrino masses and the neutrino mixing matrix, \(U_{PMNS}\), (3+4 observables) are completely specified. Altogether, the model describes 20 observables in the quark and lepton sectors with 14 parameters, effectively having 6 predictions.

In addition to the parameters describing the fermion mass matrices, we have to input three parameters specifying the three gauge couplings: the GUT scale \(M_G\) defined as the scale at which \(\alpha_1\) and \(\alpha_2\) unify, the gauge coupling at the GUT scale \(\alpha_G\), and the correction \(\epsilon_3\) to \(\alpha_3(M_G)\) necessary to fit the low energy value of the strong coupling constant. Finally we have to input 7 supersymmetry parameters given by \(-\tilde{M}_1/2\), a universal gaugino mass; \(m_{10}\), a universal Higgs mass; \(m_{16}\), a universal squark and slepton mass; \(A_0\), a universal trilinear coupling; a small Higgs mass splitting parameter, \(\Delta m_H = 1/2(m^2_{H_u} - m^2_{H_d})/m^2_{10}\); the supersymmetric Higgs mass parameter \(\mu(M_Z)\) and the ratio of the two Higgs VEVs, \(\tan \beta\).

We have also imposed some physically motivated constraints on the $\chi^2$ analysis. We demand a lower bound on the lightest stop mass given by \(m_\tilde{t} = 500\) GeV. Lower values of \(m_\tilde{t}\) actually give even better fits. On the other hand, a chargino-stop loop gives the dominant SUSY contribution to the process \(b \to s\gamma\) and lighter stop values make it difficult to fit this process. In addition we fix the CP odd Higgs mass, \(m_A = 700\) GeV. Lower values would result in a branching ratio \(Br(B_s \to \mu^+\mu^-)\) which approaches the experimental lower bound, see Ref. \([6]\). Our results however are not sensitive to this latter constraint. Further discussion of the former constraint is given in Sect. \([4]\).

\(^4\)Of course, in any supersymmetric theory there is one additional parameter in the fermion mass matrices, i.e. \(\tan \beta\). Including this parameter, there is one less prediction for fermion masses, but then (once SUSY is discovered) we have one more prediction. This is why we have not included it explicitly in the preceding discussion.

\(^5\)In principle, these parameters can be complex. We will nevertheless assume that they are real; hence there are no additional CP violating phases in the neutrino sector.

\(^6\)Note, the two Majorana phases are in principle observable, for example, in neutrinoless double-beta decay \([10]\), however, the measurement would be very difficult (perhaps too difficult \([11]\)). If observable, this would increase the number of predictions to 8.
All the parameters (except for $\mu(M_Z)$) are run from the GUT scale to the weak scale ($M_Z$) using two (one) loop RGEs for dimensionless (dimensionful) parameters. At the weak scale, the SUSY partners are integrated out leaving the two Higgs doublet model as an effective theory. We require proper electroweak symmetry breaking. Moreover, the full set of one loop, electroweak and SUSY, threshold corrections to fermion mass matrices, as well as to the three precision electroweak observables ($G_{\mu}$, $\alpha_{EM}^{-1}$ and $\alpha_s(M_Z)$) are calculated at $M_Z$.

Below $M_Z$ we use 3 loop QCD and 1 loop QED RG equations to calculate light fermion masses and $\alpha_{EM}$. More details about the analysis can be found in [5] or [6].

In addition, we self-consistently include the contributions of the right-handed neutrinos to the RG running between the GUT scale and the mass of the heaviest right-handed neutrino [12].

The $\chi^2$ function is constructed from observables given in Table 2. We have used the top quark mass from the PDG reviews of particle properties 2005 update by T. M. Liss and A. Quadt. Note that we have included several redundant observables in the quark sector. We do this because quark masses are not known with high accuracy and different combinations of quark masses usually have independent experimental and theoretical uncertainties. Thus we include three observables for the charm and bottom quark masses: the $\overline{MS}$ running masses ($m_c(m_c)$, $m_b(m_b)$) and the difference in pole masses $M_b - M_c$ obtained from heavy quark effective theory. The same is true for observables in the CKM matrix. For example, we include $V_{td}$ and the two CP violating observables $\epsilon_K$ and the value for $\sin(2\beta)$ given by the world average measured via the process $B \to J/\psi K_s$ [13]. However we have doubled the error to take into account the significant difference between the BaBar and Belle central values. We thus have 16 observables in the quark and charged lepton sectors. We use the central experimental values and one sigma error bars from the particle data group [9]. However, in the case that the experimental error is less than 0.1% we use $\sigma = 0.1\%$ due to the numerical precision of our calculation.

At present only four observables in the neutrino sector are measured. These are the two neutrino mass squared differences, $\Delta m_{31}^2$ and $\Delta m_{21}^2$, and two mixing angles, $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$. For these observables we use the central values and 2$\sigma$ errors from Ref. [14]. The other observables: neutrino masses, 1-3 mixing angle and the phase of the lepton mixing matrix are predictions of the model. In addition, the new feature of this paper is the predictions for several lepton flavor violating processes and lepton electric dipole moments.

### 4 Results

Let us now discuss our results. We performed the global $\chi^2$ analysis for values of the soft SUSY breaking scalar mass at $M_G$ given by $m_{16} = 3, 4$ and 5 TeV. However, we only present the results for the latter two cases. Good fits prefer the region of SUSY

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7In a top down analysis, the $\overline{DR}$ value of $sin^2\theta_W(M_Z)$ is obtained directly via RG running $\alpha_{(1,2)}$ from the GUT scale. Then with the calculated value of $M_Z$ and all one loop threshold corrections included in $\Delta r$, we obtain the observed value of $G_{\mu}$ [4].

8The only difference is that in the present analysis we include all three families of fermions.

9This value of the top quark mass (172.7 ± 2.9 GeV/c$^2$) is not so different from the most recent value (172.5 ± 1.3$_{stat}$ ± 1.9$_{syst}$ GeV/c$^2$) from CDF II and DZero at the Tevatron [15].
parameter space characterized by

\[ \mu, M_{1/2} \ll m_{16} \]
\[-A_0 \simeq \sqrt{2} m_{10} \simeq 2m_{16}. \]  

(16)

This is required in order to fit the top, bottom and tau masses when the third generation Yukawa couplings unify [5]. Note, the three input parameters \((\mu, M_{1/2}, m_{16})\) are not varied when minimizing \(\chi^2\). As a consequence of the relations, (Eqn. 16), we expect heavy first and second generation squarks and sleptons, while the third generation scalars are significantly lighter (with the stop generically the lightest). In addition, charginos and neutralinos are typically the lightest superpartners. We predict values of \(\tan \beta \sim 50\) and a light Higgs with mass, \(m_h \leq 127\) GeV (with a theoretical uncertainty \(\pm 3\) GeV). The specific relations between the SUSY breaking parameters also lead to an interesting prediction for the process \(B_s \rightarrow \mu^+\mu^-\) with branching ratio in the region currently being explored at the Tevatron [1]. Furthermore, the neutralino relic density obtained for our best fit parameters is consistent with WMAP data [6] and direct neutralino detection is possible in near future experiments. Finally, this region maximally suppresses the dimension five contribution to proton decay [16] and suppresses SUSY flavor and CP violation in general. For more information on the SUSY and Higgs spectra and related phenomenology in this region of SUSY breaking parameter space, see Refs. [5] and [6].

In Figs. 1 and 2 we present contours of constant \(\chi^2\) for \(m_{16} = 4\) and 5 TeV with \(m_A = 700\) GeV, as a function of \(\mu, M_{1/2}\). The best fits are obtained for small values of \(M_{1/2} \leq 300\) GeV (where the lower bound on \(M_{1/2}\) is determined by the experimental bound on the chargino mass, \(m_{\chi^+} > 104\) GeV). We find that the value of \(\chi^2\) decreases as \(m_{16}\) increases. This is solely due to the lower bound of 500 GeV on the lightest stop mass and the resultant difficulty in fitting the bottom quark mass with heavier stop.

At this point a brief aside is necessary. In our analysis, we have not evaluated several significant pieces of data. These include the branching ratios \(Br(b \rightarrow s\gamma)\), \(Br(B \rightarrow X_s l^+ l^-)\), and \(B_s - \bar{B}_s\) mixing. These also provide significant constraints on the theory. However, we have used the code of T. Blažek [17] [8] to check our analysis and also evaluate the branching ratio \(Br(b \rightarrow s\gamma)\). This process is enhanced at large \(\tan \beta\). The dominant SUSY contribution comes from the chargino-stop loop. We find that the most significant constraint is a lower bound on the lightest stop mass of order 500 GeV. We have thus imposed this bound on the stop mass by introducing a penalty to \(\chi^2\). The best fit for the branching ratio \(Br(b \rightarrow s\gamma)\) is then fit with the minimal stop mass. Note, we find that \(\chi^2\) increases as the lower bound on the stop mass increases. This is due to the fact that a good fit for \(m_b\) prefers a light stop [5, 6]. In addition, as the lower bound on the stop mass increases, we find it necessary to increase \(m_{16}\). For example, with the light stop mass, \(m_{\tilde{t}} = 300\) GeV, we find good fits with \(m_{16} = 3\) TeV [5]. Now with \(m_{\tilde{t}} = 500\) GeV, good fits, with \(\chi^2 \leq 8\), are only obtained with  

\[ m_{16} \gtrsim 3\]  

In addition, the best fit requires a non-universal Higgs masses at the GUT scale with \(\Delta m_H = 1/2(m_H^2 - m_{H_d}^2)/m_{10}^2 \sim 07\). Note this is significantly smaller than needed in the past [5]. That is because the RGE running of neutrino Yukawas from \(M_G\) to the heaviest right handed neutrino has been included self-consistently. As noted in [5], such running was a possible source for Higgs splitting. Evidently it can not be the only source.

This process is sensitive to the CP odd Higgs mass, \(m_A\), which can be adjusted in theories with non-universal Higgs masses.
$m_{16} \geq 4 \text{ TeV}$. As a final note, for a light stop ($m_{\tilde{t}} = 500 \text{ GeV}$) the Wilson coefficient of the dominant operator for the process $b \to s\gamma$, $O_7$, has the opposite sign in the MSSM than for the standard model, i.e. $C_7(MSSM) \sim -C_7(SM)$ \cite{18}. Recent measurements of the branching ratio $Br(B \to X_s l^+ l^-)$ \cite{19} suggest that the same sign is preferred, $C_7(MSSM) \sim C_7(SM)$; while the latest data on the forward-backward asymmetry does not seem to distinguish these two possibilities \cite{20}. Clearly a more detailed $\chi^2$ analysis including all these processes would be necessary to better test the theory. This is however not the focus of the present paper.  

In Figs. \[3\] and \[4\] we present contours of constant light Higgs mass for the case $m_{16} = 4$ and $5 \text{ TeV}$. There is not much difference in the range of light Higgs mass in the two cases. We find an upper bound on the Higgs mass given by $m_h \leq 127 \text{ GeV}$. In our analysis, we use the output of our RG running as input to FeynHiggs \cite{21} to obtain the Higgs pole mass at two loops. Note, in the region of parameter space with $|A_0| > m_{16}$ the radiative corrections to the Higgs mass are significant, i.e. the two loop correction, using FeynHiggs, is of order $30 \text{ GeV}$. One might then worry that the theoretical uncertainty in the light Higgs mass is just as big. However, Heinemeyer \cite{22} (see sec 2.5), estimates the uncertainties in the light Higgs mass from yet-to-be-calculated two-loop corrections and higher to be at most $3 \text{ GeV}$. We have thus taken

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\textsuperscript{12}We have also not calculated $(g-2)_{\mu}$ in this paper. However in a previous analysis \cite{6} we found typical values of $(g-2)_{\mu} < 3 \times 10^{-10}$.  

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Figure 1: Contours of constant $\chi^2$ for $m_{16} = 4 \text{ TeV}$ and $m_A = 700 \text{ GeV}$. The yellow (very light) shaded region at the bottom and top (and the region bounded by the extended solid boundary line) has $\chi^2 \geq 8$. The blue (light shaded) region on the left (and below the extended dotted line) is excluded by $m_{\chi^+} < 104 \text{ GeV}$ and the green (darker shaded) region is excluded by the Higgs mass bound $m_h < 111 \text{ GeV}$. 

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Figure 2: Contours of constant $\chi^2$ for $m_{16} = 5$ TeV and $m_A = 700$ GeV. The yellow (very light) shaded region at the bottom and top (and the region bounded by the extended boundary line) has $\chi^2 \geq 6$. The blue (light shaded) region on the left (and below the extended dotted line) is excluded by $m_{\chi^+} < 104$ GeV and the green (darker shaded) region is excluded by the Higgs mass bound $m_h < 111$ GeV.

±3 GeV as the estimated total theoretical uncertainty in the light Higgs mass.

4.1 Lepton flavor violation and electric dipole moments

Let us now focus on our results for lepton flavor violating (LFV) processes $l_j \rightarrow l_i \gamma$ and charged lepton electric dipole moments (EDMs) in this theory. We start with universal squark and slepton masses and a universal A parameter at the GUT scale. This is thus a GUT version of minimal flavor violating boundary conditions, giving minimal flavor violation at low energies. Thus the dominant contribution to lepton flavor violation results from the RG running of slepton masses and the effect of neutrino Yukawa couplings on this running from the GUT scale to the heaviest right-handed neutrino Majorana mass of order $10^{14}$ GeV. See for example the seminal paper on this subject [24]. There is also ample literature regarding LFV and EDMs, see for instance [23].

Our results for the light Higgs mass are about 5 GeV higher than in our previous analyses [5, 6]. This is due to the fact that we now use FeynHiggs to calculate the Higgs mass, where as in previous papers we used an effective potential analysis. The disagreement of the effective potential method with FeynHiggs (a perturbative analysis) is well known, see for example [23]. In addition one sees that the light Higgs mass decreases as $M_{1/2}$ increases. This fact is completely due to the global $\chi^2$ analysis and the need to fit the bottom quark mass starting with 3rd family Yukawa unification. For a detailed discussion of this effect, see the second reference in [6].
Figure 3: Contours of constant light Higgs mass $m_h$ for $m_{16} = 4$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 1.

\[ \text{[25, 26] for LFV and [27, 28] for EDMs. Therefore we simply quote the results of [25] and [27] here and refer the reader to those references for more detail.}

Following the notation of [25], the effective Lagrangian $\mathcal{L}$ relevant for the decay $l_j \rightarrow l_i \gamma$ is

\[
\mathcal{L} = -\frac{e}{2} m_{l_j} \bar{u}_i \sigma_{\alpha\beta} (A_{2}^L P_L + A_{2}^R P_R) u_j F^{\alpha\beta},
\]

where $e$ is the electric charge, $m_{l_j}$ is the mass of the decaying lepton, $P_{R/L}$ is the chirality projection operator, $u_i$ and $u_j$ are Dirac spinors describing $l_i$ and $l_j$, respectively. $A_{2}^{L/R}$ is obtained by calculating Feynman diagrams depicted in Figure 5 at one loop, and found in [25].

The decay amplitude is given by

\[
T = i e \epsilon^* m_{l_j} \bar{u}_i (p - q) \sigma_{\alpha\beta} q^\beta (A_{2}^L P_L + A_{2}^R P_R) u_j (p),
\]

where $\epsilon^*$ is the photon polarization vector. Then, the decay rate is

\[
\Gamma(l_j \rightarrow l_i \gamma) = \frac{e^2}{16\pi} m_{l_j}^5 (|A_{2}^L|^2 + |A_{2}^R|^2).
\]

On the other hand, the lepton electric dipole moment $d_{l_i}$ is defined as the coefficient of the effective Lagrangian $\mathcal{L}$ of the form

\[
\mathcal{L} = -\frac{i}{2} d_{l_i} \bar{u}_i \gamma_5 \sigma_{\alpha\beta} u_i F^{\alpha\beta}.
\]
Figure 4: Contours of constant light Higgs mass $m_h$ for $m_{16} = 5$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 2.

Figure 5: One-loop Feynman diagrams relevant for the decay of $l_j \rightarrow l_i \gamma$. (a) involves charginos $\tilde{\chi}_A^c$ and sneutrinos $\tilde{\nu}_a$, and (b) involves neutralinos $\tilde{\chi}_A^0$ and sleptons $\tilde{l}_\alpha$ in the loop with $A = 1, 2$ for charginos, $A = 1, 2, 3, 4$ for neutralinos, $a = 1, 2, 3$ and $\alpha = 1, \ldots, 6$.

Let us write

$$d_{l_i} \equiv d_{l_i}^{ch} + d_{l_i}^{nt},$$

where $d_{l_i}^{ch}$ and $d_{l_i}^{nt}$ are contributions to the EDM from loops in Figure 5 (a) and (b).
with replacing $l_j$ by $l_i$, respectively. Then we find

$$d_{l_i}^{ph} = -\frac{e}{16\pi^2} \sum_{A=1}^{2} \sum_{a=1}^{3} \text{Im}(C_{iAa}^L C_{iAa}^R) \frac{m_{\tilde{\chi}_A}^+}{m_{\tilde{\nu}_A}^2} \frac{3 - 4x_{Aa} + x_{Aa}^2 + 2\ln x_{Aa}}{2(1 - x_{Aa})^3},$$

$$d_{l_i}^{mt} = -\frac{e}{16\pi^2} \sum_{A=1}^{4} \sum_{a=1}^{6} \text{Im}(N_{iAa}^L N_{iAa}^R) \frac{m_{\tilde{\chi}_A}^0}{m_{\tilde{\nu}_A}^2} \frac{1 - x_{Aa}^2 + 2x_{Aa} \ln x_{Aa}}{2(1 - x_{Aa})^3}.$$

Here $C_{iAa}^L$, $C_{iAa}^R$, $N_{iAa}^L$ and $N_{iAa}^R$ are read from vertices shown in Figure 6 and the expression for them is given in [25]. $m_{\tilde{\chi}_A}$ is chargino mass, $m_{\tilde{\nu}_A}$ is sneutrino mass, $m_{\tilde{\chi}_A}^0$ is neutralino mass, $m_{\tilde{\nu}_A}$ is selectron mass, $x_{Aa} \equiv m_{\tilde{\chi}_A}^2/m_{\tilde{\nu}_A}^2$ and $x_{Aa} \equiv m_{\tilde{\nu}_A}^2/m_{\tilde{\nu}_A}^2$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Vertices (a) define $C_{iAa}^L$ and $C_{iAa}^R$ and vertices (b) $N_{iAa}^L$ and $N_{iAa}^R$ with $A = 1, 2$ for charginos, $A = 1, 2, 3, 4$ for neutralinos, and $\alpha = 1, \ldots, 6$.}
\end{figure}

Table 1 shows the current limits on various lepton flavor violating processes and EDMs with an estimated sensitivity for future experiments. See for example [32] for a summary of the current and future experimental status on searches for LFV and the muon EDM.

| Process | Current limit | Expected accuracy |
|---------|--------------|------------------|
| $Br(\mu \to e\gamma)$ | $< 1.2 \times 10^{-11}$ (29) | $5 \times 10^{-14}$ (30) |
| $Br(\tau \to e\gamma)$ | $< 1.1 \times 10^{-7}$ (31) | – |
| $Br(\tau \to \mu\gamma)$ | $< 6.8 \times 10^{-8}$ (32) | – |
| $Br(\mu \to 3e)$ | $< 1.0 \times 10^{-12}$ (33) | – |
| $Br(\tau \to 3\ell)$ | $< 1 - 3 \times 10^{-7}$ (34) | – |
| $Br(\mu Ti \to e Ti)$ | $< 1.7 \times 10^{-12}$ (35) | $10^{-18}$ (36) |
| $d_e$ [e·cm] | $< 1.6 \times 10^{-27}$ (37) | $10^{-34}$ (38) |
| $d_\mu$ [e·cm] | $< 10^{-18}$ (39) | $10^{-24} - 10^{-25}$ (40) |
| $d_\tau$ [e·cm] | $< 10^{-16}$ (41) | – |

Table 1: Current upper limit on various LFV processes and lepton EDMs. The limits on the LFV processes are all at 90 % CL. Estimated accuracy for the future experiment is given for $\mu \to e\gamma$, $\mu e$ conversion, electron EDM and muon EDM.

We note that the MEG collaboration [30] (searching for $\mu \to e\gamma$ with a sensitivity of order $\geq 5 \times 10^{-14}$) should start taking data by September 2006 and may have significant results by 2008. It will be an excellent test for any new physics beyond the standard
model; and in particular, the DR model [1]. Let us also note here that we can calculate \( Br(l_i \rightarrow 3l_j) \) to a good approximation by using

\[
\frac{Br(l_i \rightarrow 3l_j)}{Br(l_i \rightarrow l_j \gamma)} \approx \frac{\alpha}{3\pi} \left( \log \frac{m_{l_i}^2}{m_{l_j}^2} - \frac{11}{4} \right),
\]

which has been verified in Ref. [43] for all values of \( \tan \beta \). In particular

\[
\frac{Br(\tau \rightarrow 3\mu)}{Br(\tau \rightarrow \mu \gamma)} \approx \frac{1}{440}, \quad \frac{Br(\tau \rightarrow 3e)}{Br(\tau \rightarrow e \gamma)} \approx \frac{1}{94}, \quad \frac{Br(\mu \rightarrow 3e)}{Br(\mu \rightarrow e \gamma)} \approx \frac{1}{162}.
\]

This means that if we satisfy the constraint from \( Br(l_i \rightarrow l_j \gamma) \), we automatically also satisfy the constraint from \( Br(l_i \rightarrow 3l_j) \).

In Fig. 7 we plot contours of constant branching ratio \( Br(\mu \rightarrow e \gamma) \times 10^{14} \) for \( m_{16} = 4 \) TeV. The prediction is significantly below the present experimental bounds. Moreover, comparing this result with the future sensitivity of the MEG experiment [30] (\( Br(\mu \rightarrow e \gamma) > 5 \times 10^{-14} \)) we find that our prediction is below the MEG sensitivity in most of the parameter space. Note, however, the narrow region in the upper right hand corner with \( \chi^2 \leq 8 \) which is within the sensitivity of the MEG experiment. In Fig. 8 we present results for \( m_{16} = 5 \) TeV. Of course, larger scalar masses suppress the branching ratio, so that now the entire allowed region is below the projected MEG sensitivity. The results for the decays \( \tau \rightarrow e \gamma \) and \( \tau \rightarrow \mu \gamma \) are given in Fig. 9. Unfortunately the results are significantly below the present bounds and we are not aware of any experiments to significantly improve these bounds.

Figure 7: Contours of constant branching ratio \( Br(\mu \rightarrow e \gamma) \times 10^{14} \) for \( m_{16} = 4 \) TeV and \( m_A = 700 \) GeV. The shaded regions are the same as in Fig. 1. Note the narrow region in the upper right hand corner with \( \chi^2 \leq 8 \) which is within the sensitivity of the MEG experiment.
Figure 8: Contours of constant branching ratio $\text{Br}(\mu \to e, \gamma) \times 10^{15}$ for $m_{16} = 5 \text{ TeV}$ and $m_A = 700 \text{ GeV}$. The shaded regions are the same as in Fig. 2. Note all points are below the sensitivity of the MEG experiment.

We have also evaluated the predictions for the electric dipole moment of the electron, muon and tau. In Figs. 10 and 11 we present the results for the electric dipole moment of the electron. Note, in both cases, the entire region is below the present bounds, and also within the projected sensitivity of future experiments [38]. In Fig. 12 we present the results for the electric dipole moments of the muon and tau. In all cases the results are below the present bounds and for $d_\mu$ the result is below the projected sensitivity of future experiments [40].

In Figs. 13 and 14 we evaluate the neutrino mixing angle $\sin^2 \theta_{13}$. Recall that measuring this mixing angle is the goal of several future reactor and long baseline neutrino experiments. Moreover, a sufficiently large value for $\sin^2 \theta_{13}$ is needed in order to have the possibility of observing CP violation in neutrino oscillations. The value of $\sin^2 \theta_{13}$ is somewhat sensitive to the value of $m_{16}$; with a central value changing from $\sin^2 \theta_{13} \sim 0.0030 \pm 0.0007$ for $m_{16} = 4 \text{ TeV}$ to $\sin^2 \theta_{13} \sim 0.0024 \pm 0.0004$ for $m_{16} = 5 \text{ TeV}$ (where the uncertainty corresponds to varying over the range for $\mu$, $M_{1/2}$ with $\chi^2 \leq 8$ or $\leq 6$ in the two cases). Note, while $\sin^2 \theta_{13}$ is relatively insensitive to varying $\mu$, $M_{1/2}$; the CP violating parameter $\sin \delta$, on the other hand, is quite sensitive. We find that $\sin \delta$ can vary between 0.1 and 1.0 for different values of $\mu$, $M_{1/2}$, and as a result the CP violating Jarlskog parameter $J$ ranges between 0.0013 and 0.013. CP violation in the latter case may be observable at long baseline experiments. For example, the JPARC-SK experiment has a potential sensitivity to $\sin^2 2\theta_{13} < 1.5 \times 10^{-3}$ and $\delta \sim \pm 20^\circ$ and a comparable sensitivity is expected from the “Off-axis NUMI” proposal [44].

In Tables 2 and 3 we present the $\chi^2$ fit for a particular point in SUSY parameter space for $m_{16} = 4$ and 5 TeV, respectively. The points give a value of $\chi^2 = 7.65$ and 4.99. The former is acceptable while the latter is quite good. In the table caption we
Figure 9: Contours of constant branching ratio $Br(\tau \to e\gamma) \times 10^{12}$ (upper) and $Br(\tau \to \mu\gamma) \times 10^{11}$ (lower) for $m_{16} = 4$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 1.

present the input data at the GUT scale. We also show the heavy Majorana neutrino masses (roughly $10^{10}, 10^{12}, 10^{14}$ GeV) responsible for the See-Saw mechanism and the light neutrino masses.

Note, that the pull from $m_b$ and $M_b - M_c$ is significantly lower for $m_{16} = 5$ TeV than for $m_{16} = 4$ TeV. This accentuates the “tug of war” between the gluino and chargino loop contributions to the bottom quark mass at large $\tan \beta$. The light quark mass ratio $m_d/m_s$ is difficult to fit and contributes significantly to the pull in both cases. This mass ratio is particularly sensitive to the Georgi-Jarlskog ansatz relating first and second generation quark and lepton masses. The conflict here is with the very low value of the strange quark mass, of order 105 MeV, preferred by lattice gauge theory.
Figure 10: Contours of constant electric dipole moment of the electron, $d_e [e \cdot cm] \times 10^{29}$, for $m_{16} = 4$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 1. Note the entire region is within the sensitivity of future experiments.

calculations. Finally, we note that in a previous analysis [1] $\sin 2\beta$ contributed a value of 1.5 to the pull. However the recent Belle data gives a significantly smaller central value for $\sin 2\beta$ and now the fit is significantly improved.

We present the additional predictions for squark, slepton and Higgs masses, at these two points in SUSY parameter space, in Table 4 and for neutrino masses and mixing parameters, in Table 5. We have given the value for the effective mass parameter observable in neutrinoless double beta decay

$$\langle m_{\beta\beta} \rangle = \left| \sum_i U_{\ell i}^2 m_{\nu_i} \right| = \left| \sum_i |U_{\ell i}|^2 m_{\nu_i} e^{i\alpha'_{i}} \right|$$

(17)

(where $\alpha'_{i} = \alpha_i + 2\delta, \ i = 1, 2$ [45]). It is predicted to be of order $2 \times 10^{-4}$ eV which is too low to see in near-future experiments [16, 44]. We also give the effective electron-neutrino mass observable, relevant for the analysis of the low energy beta decay of tritium. This mass parameter is unaffected by Majorana phases and is predicted to be an order of magnitude larger. The observable,

$$m_{\nu_e}^{\text{eff}} = \left( \sum_i |U_{\ell i}|^2 m_{\nu_i}^2 \right)^{1/2}$$

(18)

is predicted to be of order $6 \times 10^{-3}$ eV. The current experimental limit is $m_{\nu_e}^{\text{eff}} \leq 2.5$ eV with the possibility of future experiments, such as KATRIN, reaching bounds on the order of 0.35 eV [44]. Unfortunately, both mass parameters may be unobservable by presently proposed experiments. Finally, in Table 6 we present the predictions for
Figure 11: Contours of constant electric dipole moment of the electron, $d_e [e \cdot cm] \times 10^{29}$, for $m_{16} = 5$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 2. Note the entire region is within the sensitivity of future experiments.

lepton flavor violation and the electric dipole moments at the same points in SUSY parameter space.

## 5 Summary and Conclusions

In this paper we have performed a global $\chi^2$ analysis on a well-motivated, phenomenologically acceptable minimal SO(10) SUSY GUT with a $D_3$ family symmetry. The most stringent constraint comes from assuming Yukawa coupling unification for the third family of quarks and leptons. The $\chi^2$ contours as functions of $\mu$ and $M_{1/2}$ for $m_{16} = 4$ and 5 TeV are given in Figs. 1 and 2. We find acceptable solutions with $\chi^2 < 8$ (6) for $m_{16} = 4$ (5) TeV, respectively. We find the light Higgs mass, found using FeynHiggs, has an upper bound given by $m_h \leq 127$ GeV (see Figs. 3 and 4). The additional predictions for the SUSY spectrum, and neutrino masses and mixing angles at two particular points in SUSY parameter space are given in Tables 4 and 5.

In addition to the global $\chi^2$ analysis, we focused on obtaining the rates for several lepton flavor violating processes and also for the charged lepton electric dipole moments. We calculated the branching ratios for the lepton flavor violating processes $Br(\mu \rightarrow e\gamma)$ (Figs. 7 and 8) and $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ (Fig. 9). There is only a narrow region in the upper right hand corner (Fig. 7) with $\chi^2 \leq 8$ which is within the sensitivity of the MEG experiment. We have also evaluated the electric dipole moments of the electron (Figs. 10 and 11) and muon and tau (Fig. 12). In all cases the results are below the present experimental bounds, and for $d_\mu$ the result is below the projected sensitivity of future experiments [40]. However, in both cases, the entire region is within the
Table 2: The fit for fermion masses and mixing angles at one particular point in SUSY parameter space defined by \( m_{16} = 4 \) TeV, \( \mu = 300 \) GeV and \( M_{1/2} = 200 \) GeV.

Initial parameters: \( (1/\alpha_G, M_G, \varepsilon_3) = (24.84, 3.30 \times 10^{16} \text{ GeV}, -3.57 \%) \),
\( (\lambda, \lambda \varepsilon, \sigma, \lambda \varepsilon', \rho, \lambda \varepsilon', \lambda \varepsilon \xi) = (0.62, 0.030, 0.87, 0.0063, -0.059, -0.0021, 0.0040) \),
\( (\Phi_\sigma, \Phi_\xi, \Phi_\rho, \Phi_\xi) = (0.637, 0.453, 0.709, 3.609) \) rad,
\( (m_{16}, M_{1/2}, A_0, \mu(M_Z)) = (4000, 200, -7809.1, 300) \) GeV,
\( ((m_{H_u}/m_{16})^2, (m_{H_u}/m_{16})^2, \tan \beta) = (1.91, 1.61, 50.34) \)
\( (M_{R_3}, M_{R_2}, M_{R_1}) = (4.6 \times 10^{13} \text{ GeV}, -8.1 \times 10^{11} \text{ GeV}, 1.1 \times 10^{10} \text{ GeV}) \)

| Observable (masses in GeV) | Data (\sigma) | Theory | Pull |
|-----------------------------|---------------|--------|------|
| \( G_\mu \times 10^9 \)     | 1.16637 (0.1\%) | 1.16638 | < 0.01 |
| \( \alpha_{EM}^{-1} \)      | 137.036 (0.1\%) | 137.035 | < 0.01 |
| \( \alpha_s(M_Z) \)         | 0.1187 (0.002) | 0.1174 | 0.37 |
| \( M_t \)                   | 172.7 (2.9)    | 173.11 | 0.02 |
| \( m_b(M_b) \)              | 4.25 (0.25)    | 4.49   | 0.94 |
| \( M_b - M_c \)             | 3.4 (0.2)      | 3.61   | 1.16 |
| \( m_c(m_c) \)              | 1.2 (0.2)      | 1.16   | 0.03 |
| \( m_s \)                   | 0.105 (0.025)  | 0.107  | 0.01 |
| \( m_d/m_s \)               | 0.0521 (0.0067) | 0.0638 | 3.09 |
| \( Q^{-2} \times 10^3 \)    | 1.934 (0.334)  | 1.815  | 0.12 |
| \( M_t \)                   | 1.777 (0.1\%)  | 1.777  | < 0.01 |
| \( M_\mu \)                 | 0.10566 (0.1\%) | 0.10566 | < 0.01 |
| \( M_c \times 10^3 \)       | 0.511 (0.1\%)  | 0.511  | < 0.01 |
| \( V_{us} \)                | 0.22 (0.0026)  | 0.2193 | 0.06 |
| \( V_{cb} \)                | 0.0413 (0.0015) | 0.0410 | 0.03 |
| \( V_{ub} \)                | 0.00367 (0.00047) | 0.00316 | 1.15 |
| \( V_{td} \)                | 0.0082 (0.00082) | 0.00824 | < 0.01 |
| \( \epsilon_K \)           | 0.00228 (0.000228) | 0.00234 | 0.08 |
| \( \sin(2\beta) \)         | 0.687 (0.064)  | 0.6435 | 0.46 |
| \( \Delta m^2_{31} \times 10^3 \) | 2.3 (0.6) | 2.382 | 0.01 |
| \( \Delta m^2_{21} \times 10^5 \) | 7.9 (0.6) | 7.880 | < 0.01 |
| \( \sin^2 \theta_{12} \)   | 0.295 (0.045)  | 0.289  | 0.01 |
| \( \sin^2 \theta_{23} \)   | 0.51 (0.13)    | 0.532  | 0.03 |
| **TOTAL \( \chi^2 \)**     |                | 7.65   |      |
Table 3: The fit for fermion masses and mixing angles at one particular point in SUSY parameter space defined by $m_{16} = 5$ TeV, $\mu = 300$ GeV and $M_{1/2} = 200$ GeV.

Initial parameters: $(1/\alpha_G, M_G, \epsilon_3) = (24.90, 3.29 \times 10^{16}$ GeV, $-3.45 \%)$,
$(\lambda, \lambda\epsilon, \sigma, \lambda\epsilon, \rho, \lambda\epsilon', \lambda\epsilon\epsilon) = (0.63, 0.030, 0.77, 0.0070, -0.054, -0.0022, 0.0035)$,
$(\Phi_\sigma, \Phi_\epsilon, \Phi_\rho, \Phi_\epsilon') = (0.643, 0.410, 0.692, 3.618)$ rad,
$(m_{16}, M_{1/2}, \Delta, \mu(M_Z)) = (5000, 200, -9918.9, 300)$ GeV,
$((m_{H_u}/m_{16})^2, (m_{H_d}/m_{16})^2, \tan \beta) = (1.91, 1.62, 50.53)$
$(M_{R_3}, M_{R_2}, M_{R_1}) = (6.1 \times 10^{13}$ GeV, $-8.6 \times 10^{11}$ GeV, $9.6 \times 10^9$ GeV)

| Observable (masses in GeV) | Data ($\sigma$) | Theory | Pull |
|---------------------------|----------------|--------|------|
| $G_{\mu} \times 10^9$    | 1.16637 (0.1%) | 1.16638 | < 0.01 |
| $\alpha_{EM}^{-1}$       | 137.036 (0.1%) | 137.036 | < 0.01 |
| $\alpha_s(M_Z)$          | 0.1187 (0.002) | 0.1178  | 0.18  |
| $m_t$                     | 172.7 (2.9)    | 173.64  | 0.10  |
| $m_b(M_b)$                | 4.25 (0.25)    | 4.29    | 0.03  |
| $M_b - M_c$               | 3.4 (0.2)      | 3.49    | 0.23  |
| $m_c(m_c)$                | 1.2 (0.2)      | 1.08    | 0.34  |
| $m_s$                     | 0.105 (0.025)  | 0.113   | 0.12  |
| $m_d/m_s$                 | 0.0521 (0.0067)| 0.0629  | 2.60  |
| $Q^{-2} \times 10^3$     | 1.934 (0.334)  | 1.824   | 0.10  |
| $M_t$                     | 1.777 (0.1%)   | 1.777   | < 0.01|
| $M_\mu$                   | 0.10566 (0.1%) | 0.10566 | < 0.01|
| $M_\tau \times 10^3$     | 0.511 (0.1%)   | 0.511   | < 0.01|
| $V_{us}$                  | 0.22 (0.0026)  | 0.2195  | 0.03  |
| $V_{cb}$                  | 0.0413 (0.0015)| 0.0413  | < 0.01|
| $V_{ub}$                  | 0.00367 (0.00047)| 0.00325 | 0.77  |
| $V_{td}$                  | 0.0082 (0.00082)| 0.00812 | < 0.01|
| $\epsilon_K$             | 0.00228 (0.000228)| 0.00231 | < 0.01|
| $\sin(2\beta)$           | 0.687 (0.064)  | 0.6511  | 0.31  |
| $\Delta m^2_{31} \times 10^9$ | 2.3 (0.6) | 2.408  | 0.03  |
| $\Delta m^2_{21} \times 10^5$ | 7.9 (0.6) | 7.880  | < 0.01|
| $\sin^2 \theta_{12}$     | 0.295 (0.045)  | 0.288   | 0.01  |
| $\sin^2 \theta_{23}$     | 0.51 (0.13)    | 0.537   | 0.04  |
| **TOTAL $\chi^2$**       |                | 4.99    |      |
Table 4: Predictions for SUSY and Higgs spectra for the fit given in Tables 2 and 3 in units of GeV.

| Particle | $m_{16} = 5$ TeV | $m_{16} = 4$ TeV |
|----------|-----------------|-----------------|
| $h$      | 120             | 124             |
| $H$      | 699             | 699             |
| $A^0$    | 700             | 699             |
| $H^+$    | 700             | 701             |
| $\chi^0_1$ | 81             | 80             |
| $\chi^0_2$ | 151            | 150            |
| $\chi^+_1$ | 151            | 150            |
| $i \tilde{g}$  | 605            | 597            |
| $i \tilde{t}_1$ | 498            | 496            |
| $i \tilde{b}_1$ | 902            | 846            |
| $i \tilde{\tau}_1$ | 1686        | 1421           |

Table 5: Predictions for neutrino masses, $\sin^2 \theta_{13}$ and CP violation in the lepton sector for the fit given in Tables 2 and 3.

|          | $m_{\nu_3}$ (eV) | $m_{\nu_2}$ (eV) | $m_{\nu_1}$ (eV) | $\sin^2 \theta_{13}$ | $J$   | $\sin \delta$ | $\alpha_1$ (rad) | $\alpha_2$ (rad) | $\langle m_{\beta\beta} \rangle$ (eV) | $m_{\text{eff}}$ (eV) | $\epsilon_1$          |
|----------|-----------------|-----------------|-----------------|-------------------|------|---------------|-----------------|-----------------|----------------------------|---------------------|---------------------|
| $m_{16}$ = 5 TeV | 0.0492          | 0.0097          | 0.0041          | 0.0025            | 0.0013 | 0.119        | -2.974          | 0.136            | 0.00010                      | 0.0067              | 0.92 x 10^{-7}        |
| $m_{16}$ = 4 TeV | 0.0489          | 0.0096          | 0.0036          | 0.0037            | 0.013  | 0.996        | -1.771          | 1.315            | 0.00038                      | 0.0067              | 1.61 x 10^{-7}        |

Table 6: Predictions for branching ratios for lepton flavor violating processes and the electric dipole moment of leptons for the fit given in Tables 2 and 3.

|          | $m_{16} = 5$ TeV | $m_{16} = 4$ TeV |
|----------|-----------------|-----------------|
| $Br(\mu \to e\gamma)$ | $4.69 \times 10^{-15}$ | $1.40 \times 10^{-14}$ |
| $Br(\tau \to e\gamma)$ | $1.26 \times 10^{-12}$ | $2.40 \times 10^{-12}$ |
| $Br(\tau \to \mu\gamma)$ | $6.13 \times 10^{-11}$ | $1.22 \times 10^{-10}$ |
| $d_e$ [e·cm] | $9.15 \times 10^{-30}$ | $2.43 \times 10^{-29}$ |
| $d_\mu$ [e·cm] | $-3.58 \times 10^{-28}$ | $-7.93 \times 10^{-28}$ |
| $d_\tau$ [e·cm] | $-1.54 \times 10^{-27}$ | $-2.64 \times 10^{-27}$ |
Figure 12: Contours of constant electric dipole moment of the muon, $d_\mu [e \cdot cm] \times 10^{28}$, (upper) and the tau, $d_\tau [e \cdot cm] \times 10^{27}$, (lower) for $m_{16} = 4$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 1. Note, the entire region is below present bounds and for $d_\mu$ the result is below the projected sensitivity of future experiments.

projected sensitivity of future experiments for $d_e$. The results for the lepton flavor violating processes and electric dipole moments at the same two particular points in SUSY parameter space are given in Table 6.

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Figure 13: Contours of constant neutrino mixing angle, $\sin^2 \theta_{13}$, for $m_{16} = 4$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 1.

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Figure 14: Contours of constant neutrino mixing angle, $\sin^2 \theta_{13}$, for $m_{16} = 5$ TeV and $m_A = 700$ GeV. The shaded regions are the same as in Fig. 2.

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\( d_\mu \, [\text{e cm}] \times 10^{28} \)

\( \chi^2 = 6 \)

\( m_{\nu} = 104 \)

\( m_h = 114.4 \)

\( m_h = 111 \)

\( \chi^2 = 6 \)
\[ d \tau \text{ [e cm]} \times 10^{27} \]

\[ \chi^2 = 6 \]

\[ m_{\nu} = 114.4 \]

\[ m_{\chi} = 111 \]

\[ m_{\chi} = 104 \]
$\text{Br}(\tau \to e, \gamma) \times 10^{13}$

$\chi^2 = 6$

$m_{h} = 114.4$

$m_{h} = 111$
\[ \text{Br}(\tau \to \mu, \gamma) \times 10^{11} \]

\[ \chi^2 = 6 \]

\[ m_{\chi} = 104 \]

\[ m_{\chi} = 114.4 \]

\[ m_{\chi} = 111 \]
$$\sin^2 \theta_{13}$$

$\chi^2 = 6$

$m_{\beta} = 111$

$m_{\beta} = 114.4$

$m_{\beta} = 104$

$\mu [\text{GeV}]$

$M_{1/2} [\text{GeV}]$