Quasioptimal control in diversified signal transmission

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Abstract. To increase the noise immunity of signal transmission, diversity methods are now widely used, consisting in obtaining and combining several copies of the transmitted signal. In this case, it is possible to perform a combination either before the detection procedure or after it. If you do not take into account the possible use of non-linear types of modulation, then the pre-detector combination always has advantages over the post-detector combination. However, taking into account the nonlinear properties of the transmitted signals, new possibilities appear for increasing the noise immunity in combination and simplifying the processing. In the case of using analog signals, in particular frequency modulation, at certain points in time, the pre-detection combination can lose to the post-detection combination. At the same time, by combining pre-detector and post-detector combining circuits, it is possible to lower the threshold level during demodulation and increase noise immunity. In the case of using digital modes of modulation, it is possible to process only the signals after demodulation without reducing the noise immunity and to eliminate the need for preliminary phasing of the diversity signals before detection.

1. Formulation of the problem
Diversity methods have been known for a long time, but even now they are quite widely used in signal transmission ([1-11]). Space diversity, despite the fact that it requires the use of several antennas, can significantly improve the noise immunity of signal transmission when transmitting over non-stationary channels. With the correct organization of the communication system, fading in various channels occurs independently, which makes it possible to reduce the likelihood of communication disruptions.

Diversity techniques are based on the transmission and reception of multiple identical copies of a signal and their sharing. The received copies must be combined in order to obtain the maximum gain in noise immunity. In this case, a number of peculiarities arise, in particular, the combination can be performed both before and after detection. However, when using the features of the structure of the transmitted signals, it becomes possible to obtain an additional gain in transmission noise immunity.

It is a generally accepted fact that the pre-detection combination is superior to the post-detection one ([2-4]). Provided that the useful components are preliminarily phased, they add up "in amplitude", while noises with random mutual phase shifts add up "in power". This increases the signal-to-noise ratio before demodulation. However, the non-linear properties of the transmitted signals can make their own "corrections". The same is true for both analog and digital types of modulation.

2. Implementation of nonlinear features of the signals used
Among the nonlinear types of modulation, features can be observed that indicate the temporal advantages of post-detector addition over pre-detector addition ([3-5]).
of signal voltage and noise before detection; \( \gamma_{\text{out}} \) - the same ratio after detection. Let us denote through \( L[\bullet] \) relationship between \( \gamma_{\text{in}} \) and \( \gamma_{\text{out}} \) for this type of modulation, \( \gamma_{\text{out}} = L[\gamma_{\text{in}}] \). We also denote \( Q[\bullet] \) by denote the operation of determining the signal-to-noise ratio after combining the diversity signals, in which the signal-to-noise ratio values are equal \( \gamma_1 = \gamma_N \), i.e. \( \gamma_{\text{out}} = Q[\gamma_1, \ldots, \gamma_N] \). Thus, as a result of pre-detector addition, the resulting signal-to-noise ratio will be equal, and as a result of post-detector addition \( \gamma_D = L[Q[\gamma_1, \ldots, \gamma_N]] \), the result will be \( \gamma_p = Q[L[\gamma_1], L[\gamma_2], \ldots, L[\gamma_N]] \). It can be shown that instantaneous values \( \gamma_D \) are not always greater than instantaneous values \( \gamma_p \).

As an example, when using analog signals, consider frequency modulation. The result of summing the received signal \( U(t) \) and \( n(t) \) noise can be represented in the form \( x(t) = U(t) + n_1(t) + n_2(t) \), where \( n_1(t) \)-noise component in phase with the useful signal, \( n_2(t) \)-noise component orthogonal to the useful signal. At FM, the value of the amplitude \( U_c \) of the useful signal during an interval of time less than the quasi-period of fading can be considered constant ([2-6]). Orthogonal noise components:

\[
n_c = n_u \cos \varphi_n; \quad n_s = n_u \sin \varphi_n
\]

mutually independent. The instantaneous phase changes \( \varphi_n \) are the sum of the independent instantaneous phase changes of the original noise and the phase change of the useful signal, i.e. distribution \( \varphi_n \) can be considered uniform within \( 0 \div 2\pi \). Thus, the error of the instantaneous phase of the sum of the useful signal and noise with respect to the phase of the useful signal will be equal to

\[
\varphi_1 = \arctg \frac{n_u \sin \gamma_n}{U_c + n_u \cos \gamma_n} = \arctg \frac{a \sin \gamma_n}{1 + a \cos \gamma_n},
\]

where \( a = \frac{n_u}{U_c} \).

As a result of frequency detection, this instantaneous phase error will lead to an instantaneous frequency error that determines the noise after detection, which is determined by the formula:

\[
\varepsilon_1 = \frac{d \varphi_1}{dt} = a \frac{a + \cos \varphi_n}{1 + 2a \cos \varphi_n + a^2} \frac{d \varphi_n}{dt}
\]

The quantities \( \varphi_n \) and \( \frac{d \varphi_n}{dt} \) are mutually independent, just like the quantities \( a \) and \( \varphi_n \). Let's single out the coefficient

\[
b_1 = a \frac{a + \cos \varphi_n}{1 + 2a \cos \varphi_n + a^2}
\]

This factor will determine the degree of transformation of the instantaneous frequency of the noise before detection to the instantaneous frequency of the noise after the detection. Let's average it over the phase taking into account its uniform distribution in the interval \( 0 \div 2\pi \). We will also perform averaging in the root-mean-square sense because of the alternation of signs \( \beta_1 \).

\[
\beta_1 = a \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left( \frac{a + \cos \varphi_n}{1 + 2a \cos \varphi_n + a^2} \right)^2 d \varphi_n}
\]

As a result of mathematical transformations, we obtain
The obtained expressions correspond to the situation of ideal frequency detection, when the amplitude fluctuations at the BH input are completely removed from its result, i.e. there is a deep signal limitation.

After calculating the RMS value by phase averaging, i.e.

\[ \beta_2 = \sqrt{\frac{1}{2\pi} \int_{0}^{2\pi} b_2^2 d\varphi_n} \]

as a result we have

\[ \beta_2 = \begin{cases} \frac{a}{\sqrt{2\sqrt{1 - a^2}}}; & |a| < 1 \\ \sqrt{\frac{2a^2 - 1}{2(a^2 - 1)}}; & |a| > 1 \end{cases} \]

The clipping method (which is usually always done before frequency detection) and the clipping level can be different. Therefore, the dependence of the corresponding averaging result for a real circuit will lie between the graphs \( \beta_1(a) \) and \( \beta_2(a) \). Similar graphs are shown in figure 1. The graphs show the dependences: 1 - for \( \beta_1(a) \), 2 – for \( \beta_2(a) \). When the signal level falls below the limiting threshold, it is necessary to switch from the shape of the graph 1 to the shape of the graph 2 with a certain coupling coefficient before \( \beta_2(a) \), so that during this transition there are no gaps in the instantaneous values of the signals.

Figure 1 shows that the dependence \( \gamma_{out} = L(\gamma_{in}) \) at FM is by no means monotonic. It has an area near the value \( a = 1 \) where the noise level rises sharply. As is known [3, 6], this manifests itself in the
appearance of the so-called “anomalous” noise in the form of impulse surges. As the input signal-to-
noise ratio deteriorates, the emission rate increases rapidly, so the output signal-to-noise ratio falls faster
than the input signal-to-noise ratio. This manifests itself as a known threshold in FM demodulation. This
phenomenon is usually explained as follows (figures 2, 3 and 4). The explanation is that for close values
of the vectors of the signal U and noise n, their total vector y = U + n rotates around the center of
coordinates very quickly, even if the rate of rotation of the vector n is small. This takes place when the
end of the vector y falls into some region Ω near the origin (for n ≈ –U).

![Figure 2. Vector sum of signal and noise.](image)

Such a rapid turn turns into an overshoot as a result of frequency detection. When the level of the
useful signal decreases (for example, as a result of fading), the probability that the condition | n | ≈ | U |
increases, so the frequency of occurrence of emissions also increases.

In the case of diversity reception, the absence of monotonicity of the function $L\{\bullet\}$ provides
additional opportunities for improving noise immunity as a result of lowering the threshold level. This
is due to the fact that in this case the condition is $L\{Q_{y_1},...,Q_{y_N}\} > Q\{L_{y_1},...,L_{y_N}\}$ not always
met. This is illustrated in figure 3 for a diversity factor of N = 2. (The addition is assumed to be linear.)

![Figure 3. Vector sum for diversity factor N=2.](image)

Figure 3 shows vector diagrams of signals in two diversity branches. The total vectors $y_1 = U_1 + n_1$
and $y_2 = U_2 + n_2$ in both cases separately do not fall into the region Ω corresponding to the appearance
of an outlier, i.e. after frequency detection, no spike occurs in any of the branches. Figure 4 shows vector
diagrams of signals after pre-detector addition. In this case, the total vector $U_1 + U_2 + n_1 + n_2$ falls into
the region Ω, as a result of which, after detection, an overshoot will take place.

Thus, with such a combination of signal and noise vectors, the pre-detector addition of signals
generates an overshoot, while the post-detector addition does not. (Although, in general, the total number
of emissions during the post-vector addition is naturally greater than during the pre-detector addition,
however, their appearance basically does not coincide in both types).

![Figure 4. Situation of overshoot.](image)

The considered phenomenon can be used to eliminate part of the surges during pre-detector addition,
if at the same time the post-detector addition signal is used for this time. To do this, consider the circuit
shown in figure 5.
At the moments of time corresponding to the presence of an anomalous noise emission in the pre-detector addition signal and its absence in the post-detector addition signal, the post-detector addition signal is switched to the output, and the rest of the time - to the pre-detector addition signal. In the adder (S2), the pre-detector diversity combining is carried out (it is assumed that they are phased before). In frequency detectors, each diversity signal is detected separately (FD1 and FD2) and their sum (FD3). In the adder (S1), post-detector addition is performed. All frequency detectors are assumed to be sufficiently identical, i.e. in the absence of noise, their output useful signals are the same.

In subtraction blocks (−), the signals of each branch are subtracted from the total signal. The received voltages are fed to the input of threshold devices (PU1 and PU2), which generate a logic signal "1" if there is an overshoot at the input, and a logic signal "0" if there is no overshoot. These threshold devices determine the voltage level that can be considered an overshoot. The logical signals of the threshold devices are combined by the logical operation "AND", the result of which controls the operation of the switch (K). Delay elements (LZ) are used to compensate for the time delay in switching the switch due to the finite response time of the threshold devices and ensure the simultaneous arrival of surges and the control signal to the switch.

Let us denote the probabilities of the appearance of outliers in the first channel, in the second channel, and in the case of pre-detector addition through, respectively \( P\{y_1 \in \Omega\} \), \( P\{y_2 \in \Omega\} \) and \( P\{y_1 + y_2 \in \Omega\} \), as the probabilities of getting the ends of the vectors, \( y_1 \), \( y_2 \) and \( y_1 + y_2 \) into the region \( \Omega \). Hence, we obtain the probability of an outlier occurring during post-detector addition:

\[
P_{AD} = P\{y_1 \in \Omega\} + P\{y_2 \in \Omega\} - P\{y_1 \in \Omega\}P\{y_2 \in \Omega\}
\]

The probability of an outlier in the circuit in figure 5.

\[
P_C = P\{y_1 \in \Omega\}P\{y_2 \in \Omega\}
\]

Disregarding emission levels, inequalities can guide the relative effectiveness of schemes:

\[
P\{y_1 \in \Omega\} + P\{y_2 \in \Omega\} - P\{y_1 \in \Omega\}P\{y_2 \in \Omega\} \rightarrow P\{y_1 + y_2 \in \Omega\} \gg P\{y_1 \in \Omega\}P\{y_2 \in \Omega\}
\]

Thus, the circuit in figure 5 allows to eliminate a large number of surges at the FD output during pre-detector addition, thereby significantly reducing the detection threshold level. The magnitude of the threshold level reduction is determined by many factors, such as the level of limitation before detection, the frequency modulation index, the shape of the signal spectrum, etc. Computer simulation has shown the possibility of lowering the threshold level in different conditions by 3.7 dB. Efficiency in practice will be determined largely also by technical characteristics, mainly by the speed of the switch at the output of the circuit.

![Figure 5. Circuit for diversity factor N=2.](image-url)
A similar approach can be applied for the diversity factor $N > 2$. In this case, in a simple version, the switch connects to the output either the signal obtained after the FD general pre-detector addition, or the post-detector addition signal of all separately detected diversity signals. In a more sophisticated version, a multi-input switch can be used. It selects the best signal from several combinations of input signals obtained by summing with different sets of weights.

With the use of digital signals, diversity as a technique has retained its relevance. At the same time, it became possible to implement the combination much easier. To clarify this, it is necessary to consider the transmission of BPSK signals or any other binary signals that, upon detection, take two logical values. It is assumed that the analog value of the signal is compared with a certain level, as a result of which one of two possible logical values is selected.

When detection is performed by two separate detectors, their output signals may or may not coincide. If the output signals of the detectors match, it means that either there is no error in both signals, or both signals are affected by an error. Naturally, the overall result of the analysis will also be erroneous.

Let's analyze 2-fold diversity. Let $s_1 = u_1 + n_1$ and $s_2 = u_2 + n_2$ denote the signals arriving at the detectors of the first and second branches, where: $u_1$, $u_2$ are useful components, $n_1$, $n_2$ are the noise components of the first and second diversity signals. When using addition before the detector, the signal in front of the detector will be $s_C = (u_1 + u_2) + (n_1 + n_2) = u_C + n_C$. In practice, the power of both binary signals is the same, and the appearance of both variants is equally probable. Without taking into account noise, signal "1" corresponds to level «$a$» at the detector input, signal "0" corresponds to level «$-a$».

The decision is made by comparing the obtained level with zero. Let the character "0" be transmitted. (If a symbol equal to a logical one is transmitted, all reasoning is similar). An error occurs on the first diversity channel if $n_1 > a$. In the second diversity channel, it will occur if $n_2 > a$. At the same time, an error during addition to the detector will occur if $n_1 + n_2 > 2a$.

To implement the described quasi-optimal control, the scheme shown in figure 1 is proposed.

![Figure 6. Circuit for diversity factor N>2.](image-url)
When signals $x_1$ and $x_2$ are the same, any of these signals (let signal $x_1$) go to the output. When signals $x_1$ and $x_2$ are different, signal $x_3$ is output. As a result, the following result is observed. Let a signal equal to $-a$ be transmitted at some moment in time, which corresponds to the transmission of a logical zero. An error can be when, or both PU generate the same signal "1", or both PU generate different signals, but the wrong one will be selected.

Let's analyze the last situation in more detail. Let the signal values be equal: $x_1 = 1$ and $x_2 = 0$, (or $x_1 = 0$ and $x_2 = 1$). In this case, the decision will be made based on the value of the variable $x_3$. An error will occur if a signal $x_3 = 1$ is generated and a decision is made that a logical unit was transmitted. This situation corresponds to the fulfillment of the inequality: $s_1 + s_2 > 0$, which means that either there are inequalities $n_1 - a + n_2 - a > 0$, or $n_1 + n_2 > 2a$. In this case, the "boundary" between correct and erroneous solutions is determined by the equation $n_1 + n_2 = 2a$. When one is passed, the result is the same. It follows from this that the considered fragment of the circuit has the same properties and operates in the same way as the aforementioned known pre-detector linear addition circuit. The advantage is that it does not require the organization of a preliminary operation of phasing the added signals.

The part of the circuit outside the dotted line is a union of linear addition and auto selection. As shown in ([5]), it implements quasi-optimal processing, which is close in properties to optimal addition, but does not require complex-to-implement phasing and signal level control units. The circuit operates as follows. Using the amplitude detector (AD) and averager (AV) is determined by the current level of the signals in diversity branches. In the adder ($\Sigma$), their sum is formed with a weighting coefficient, the value of which must be equal to the value ([4]). All three received signals are compared, and the largest of them is selected (using the MAX block). The control signals generated by this unit, with the help of keys (K), are connected to the output through the OR circuit, either the output signals of one of the threshold devices (if there is an advantage in auto selection), or signal $y$, which corresponds to linear addition in properties.

Application of the channels described quasioptimal diversity signal transmission control algorithms can significantly simplify practical implementation of control.

### 3. Output

Pre-detector diversity combining always has advantages in noise immunity over post-detector diversity combining when considering only the linear properties of the signals. In the case of taking into account the nonlinear properties of analog signals, there are times when this advantage is lost. The use of nonlinear features of the combined diversity signals allows to increase the noise immunity of transmission and, with digital signals, to simplify the implementation of combination schemes.

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