Josephson Current in the Presence of a Precessing Spin

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The Josephson current in the presence of a precessing spin between various types of superconductors is studied. It is shown that the Josephson current flowing between two spin-singlet pairing superconductors is not modulated by the precession of the spin. When both superconductors have equal-spin-triplet pairing state, the flowing Josephson current is modulated with twice of the Larmor frequency by the precessing spin. It was also found that up to the second tunneling matrix elements, no Josephson current can occur with only a direct exchange interaction between the localized spin and the conduction electrons, if the two superconductors have different spin-parity pairing states.

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There is a growing interest in a number of techniques that allow one to detect and manipulate a single spin in the solid state. Partial list includes optical detection of ESR in a single molecule, tunneling through a quantum dot, and, more recently, electron-spin-resonance-scanning tunneling microscopy (ESR-STM) technique. There is a growing recognition that the ESR-STM technique is capable of detecting the precession of a single spin through the modulation of the tunnel current. Interest in ESR-STM lies in the possibility to detect and manipulate a single spin, which is crucial in spintronics and quantum information processing.

Several proposals have been made for the mechanism of the spin detection with the ESR-STM. One is the effective spin-orbit interaction of the conduction electrons in the two-dimensional surface coupling the injected unpolarized current to the precessing spin. Another one is the interference between two resonant tunneling components through the magnetic field split Zeeman levels. Both of these mechanisms rely on a spin-orbit coupling to couple the local spin to the conduction electrons and have assumed no spin polarization of tunneling electrons. On the other hand, one can perform ESR-STM measurements on samples with much smaller spin-orbit coupling. Theoretically it is also important to investigate the role of direct exchange in ESR-STM measurements, without any spin-orbit coupling. Exchange interaction has a tremendous effects on the physics of conducting substances when magnetic impurities are present.

The above mentioned experimental and theoretical studies are concentrated on the tunneling between two normal metals. A natural extension is a question of the role of a precessing spin localized inside a tunneling barrier on the Josephson current between two weakly coupled superconductors. This is the problem we address in this Letter.

Previously, the Josephson effect between superconductors with nontrivial pairing symmetry has been extensively studied, see e.g., [1]. There are two main aspects of current study that differ from the previous work: 1) we will consider the effect of the precessing localized spin in the junction on the Josephson current. This effect, to our knowledge, has not been addressed before; 2) we will assume no spin-orbit coupling between the superconductors. The role of the spin-orbit coupling will be addressed elsewhere.

The model system under consideration is illustrated in Fig. 1. It consists of two ideal superconducting leads coupled to each other by a single magnetic spin. In the presence of a magnetic field, the spin precesses around the field direction. We neglect the interaction of the spin with two superconducting leads. The Hamiltonian for the Josephson junction can then be generally written as [2]:

$$H = H_L + H_R + H_T.$$  (1)

The first two terms are respectively the Hamiltonian for electrons in the left and right superconducting leads of the tunneling junction:

$$H_{L(R)} = \sum_{k(p),\sigma} \epsilon_{k(p),\sigma} c_{k(p),\sigma}^\dagger c_{k(p),\sigma} + \frac{i}{2} \sum_{k(p),\sigma,\sigma'} [\Delta_{\sigma\sigma'}(k(p)) c_{k(p),\sigma}^\dagger c_{-k(-p),\sigma'}^\dagger + \text{H.c.}],$$  (2)

where we have denoted the electron creation (annihilation) operators in the left (L) lead by $c_{k\sigma}$ ($c_{k\sigma}^\dagger$) while those in the right (R) lead by $c_{p\sigma}$ ($c_{p\sigma}^\dagger$). The quantities $k(p)$ are momenta and $\sigma$ is the spin index. The quantities $\epsilon_{k(p),\sigma}$, $\Delta_{\sigma\sigma'}(k(p))$ are, respectively, the single particle energies of conduction electrons, and the pair potential (also called gap function) in the leads. For the purpose
The exchange term in the exponent of the tunneling matrix element $T_{\sigma\sigma'}(k, p)$ transfer electrons through an insulating barrier. When a local spin is embedded into the tunneling barrier, the tunneling matrix can be written in the spin space as \[ T = T_0 \exp \left[ -\frac{\Phi}{2} \right] \left[ \begin{array}{c} \cos \left( \frac{JS \cdot \sigma}{2} \right) \Phi_0 \right] + n(t) \cdot \dot{\sigma} \sinh \left( \frac{JS \cdot \sigma}{2} \right) \Phi_0 \right] . \] where $\Phi$ is the spin-independent potential barrier, and $\Phi_0 = \hbar^2/2m_e d^2$ is the characteristic energy scale for the barrier width $d$, $J$ is the exchange interaction between the local spin $S$ and the tunneling electrons denoted by the Pauli matrix $\sigma$. In an external magnetic field $B$, a torque will act on the magnetic moment $\mu$ of amount $\mu \times B$, where $\mu = \gamma S$ with $\gamma$ the gyromagnetic ratio. The equation of motion of the local spin is given by $d\Phi/dt = \mu \times (\gamma B)$. For a static magnetic field applied along the $z$ direction, we shall see that the local spin would precess about the field at the Larmor frequency $\omega_L = \gamma B$, i.e., $S = n(t)S_z$, where $S$ is the magnitude of the local spin and $n(t) = (n_x, n_y, n_z) = (n_L \cos(\omega_L t), -n_L \sin(\omega_L t), n_0)$ the unit vector for the ‘instantaneous’ spin orientation. Here $n_L$ and $n_\perp$ are the magnitude of the longitudinal and transverse components of $S$ to the field direction. They obey the sum rule $n^2 + n^2 = 1$. We note that the expression for $n(t)$ shows the constant left-handed precession, and the $z$ component of $S$ is time-independent. The precession of the spin can also be obtained quantum mechanically by replacing the local spin operator with its average value. The exchange term in the exponent of the tunneling matrix element is very small as compared with the barrier height $\Phi$. We then perform the Taylor expansion in $JS$ and arrive at:

$$
\tilde{T} = T_0 \exp \left( -\frac{\Phi}{2\Phi_0} \right) \left[ \cos \left( \frac{JS \cdot \sigma}{2\Phi_0} \right) \Phi_0 \right] + n(t) \cdot \dot{\sigma} \sinh \left( \frac{JS \cdot \sigma}{2\Phi_0} \right) \Phi_0 \right] .
$$

Since the energy associated with the spin precession, $\hbar \omega_L \sim 10^{-6}$ eV is much smaller than the typical electronic energy on the order of 1 eV, the spin precession is very slow as compared to the time scale of all conduction electron process. This fact allows us to treat the electronic problem adiabatically as if the local spin is static for every instantaneous spin orientation $\tilde{T}$. Our remaining task is to calculate the Josephson current in the presence of the spin. The current operator is given by

$$
\tilde{I} = i e \sum_{k, p, \sigma, \sigma'} [T_{\sigma\sigma'}(k, p; t)c_{k\sigma}^\dagger c_{p\sigma'} + H.c.] .
$$

When a voltage bias $eV = \mu_L - \mu_R$ is applied across the junction, following the standard procedure \[14\], we can write the phase dependent contribution, i.e., the Josephson current as:

$$
I_J(t) = e \int_{-\infty}^t dt' e^{i eV(t'+t)} \left\{ [A(t), A(t')] - e^{-i eV(t'+t)} [A^\dagger(t), A^\dagger(t')] \right\} ,
$$

where the operator $A(t) = \sum_{k, p; \sigma, \sigma'} T_{\sigma\sigma'}(k, p; t)c_{k\sigma}^\dagger c_{p\sigma'}(t)$. Here the operators $c_{k\sigma}(t) = e^{iK_{L(R)}t}c_{k\sigma}e^{-iK_{L(R)}t}$ with $K_{L(R)} = H_{L(R)} - \mu_L N_{L(R)}$ and $N_{L(R)} = \sum_{k, p, \sigma} c_{k\sigma}^\dagger c_{k\sigma}$. For either spin-singlet or spin-triplet superconductors, we can perform the Bogoliubov transformation to express the electron operators in terms of quasiparticle operators:

$$
c_{k\sigma} = \sum_{\sigma'} (u_{k\sigma\sigma'} \gamma_{k\sigma\sigma'} - \sigma v_{k\sigma\sigma'} \gamma_{k\sigma\sigma'}^\dagger)
$$
to diagonalize the unperturbed Hamiltonian, where $(u_{k\sigma\sigma'}, v_{k\sigma\sigma'})^T$ is the Bogoliubov quasiparticle wavefunction. For a spin singlet superconductor, the order parameter matrix can be written as: $\Delta(k) = (i\tilde{\sigma}_y)\psi(k)$, where $\psi(k)$ is an even function of $k$. The quasiparticle wavefunction is then given by:

$$
\begin{pmatrix}
u_{k\sigma\sigma'} \\
\nu_{k\sigma\sigma'}
\end{pmatrix} =
\begin{pmatrix}
u_{k\sigma\sigma'} e^{i(\varphi_k + \varphi)} \\
\nu_{k\sigma\sigma'} e^{i\varphi_k} \delta_{\sigma\sigma'}
\end{pmatrix} ,
$$

with

$$
\begin{pmatrix}
u_{k} \\
\nu_{k}
\end{pmatrix} =
\begin{pmatrix}
u_{k} \\
\sqrt{\frac{1}{2}} \left( 1 + \frac{\xi_k}{E_k} \right)
\end{pmatrix} ,
$$

where we have introduced $\psi(k) = |\psi(k)|e^{i(\varphi_k + \varphi)}$ with $\varphi_k$ and $\varphi$ being the internal and global phase, and $\xi_k = \epsilon_k - \mu$, and $E_k = \sqrt{\xi_k^2 + |\psi(k)|^2}$. For the spin-triplet pairing state, the order parameter can be written as: $\Delta(k) = i(d(k) \cdot \tilde{\sigma})\delta_y$, where $d = (d_u, d_v, d_w)$ is an odd vectorial function of $k$ defined in a three-dimensional spin space spanned by $(u, v, w)$. We shall be typically concerned with two types of triplet pairing states—non-equal-spin pairing, where the Cooper pairs are formed by electrons with anti-parallel spins, and equal-spin pairing, where the Cooper pairs are formed by electrons with parallel spins. The non-equal spin pairing state has the form:

$$
\Delta(k) =
\begin{pmatrix}
0 \\
d_I(k)
\end{pmatrix} .
$$
corresponding to \((d_{i\uparrow}, d_{i\downarrow}, d_{i\sigma}) = (0, 0, d_I(k))\). This type of pairing state may be realized in the recently discovered superconducting \(\text{Sr}_2\text{RuO}_4\) \([13, 14]\). The equal spin pairing state has the form:

\[
\hat{\Delta}(k) = \begin{pmatrix} 2d_{I\uparrow}(k) & 0 \\ 0 & 2d_{I\downarrow}(k) \end{pmatrix},
\]

(12)
corresponding to \((d_{i\uparrow}, d_{i\downarrow}, d_{i\sigma}) = (0, -i2d_I(k), 0)\). This state may be relevant to the A-phase of superfluid \(^3\text{He}\) \([17]\) and of heavy fermion UPT3 \([18]\). A little algebra yields the quasiparticle wavefunction:

\[
\begin{bmatrix} u_{k\sigma'} \\ v_{k\sigma'} \end{bmatrix} = \begin{pmatrix} \sigma u_{I\uparrow,k}e^{i(\varphi_k + \varphi_0)\delta_{\sigma\sigma'}} \\ v_{I\downarrow,k}\delta_{\sigma,-\sigma'} \end{pmatrix},
\]

(13)
for the non-equal-spin-triplet pairing state; while

\[
\begin{bmatrix} u_{k\sigma'} \\ v_{k\sigma'} \end{bmatrix} = \begin{pmatrix} \sigma u_{I\uparrow,k}e^{i(\varphi_k + \varphi_0 + \pi)\delta_{\sigma\sigma'}} \\ v_{I\downarrow,k}\delta_{\sigma,-\sigma'} \end{pmatrix},
\]

(14)
for the equal-spin-triplet pairing state. Here \(u_{I\uparrow(k),k}, v_{I\downarrow(k),k}\) has the same form as that given by Eq. (10) except \(\sigma \equiv \text{spin-triplet, and equal-spin-triplet pairing state:} \) above symmetry properties, one can find the expectation value of two annihilation operators in one superconductor is combined with the expectation value of two creation operators in the other superconductor, that is, \(\langle \hat{c}_{k\sigma'}^\dagger(t)\hat{c}_{k\sigma}(t')\rangle\langle \hat{c}_{p\sigma'}^\dagger(t')\hat{c}_{-p\sigma}(t')\rangle\). Using the above symmetry properties, one can find the expectation values for superconductors with a spin-singlet, non-equal-spin-triplet, and equal-spin-triplet pairing state:

\[
\langle \hat{c}_{k\sigma'}^\dagger(t)\hat{c}_{-k\sigma}(t')\rangle = \begin{pmatrix} \sigma \delta_{\sigma,-\sigma'}u_{k\sigma'}v_{I\downarrow,k} \\ \delta_{\sigma,-\sigma'}u_{I\uparrow,k}v_{k\sigma} \\ -\delta_{\sigma,\sigma'}u_{I\downarrow,k}v_{I\downarrow,k} \end{pmatrix} \times \langle e^{iE_k(t-t')}f(E_k) - e^{-iE_k(t-t')}f(-E_k) \rangle,
\]

(15a)
and

\[
\langle \hat{c}_{p\sigma}(t)\hat{c}_{-p\sigma'}(t')\rangle = \begin{pmatrix} \sigma \delta_{\sigma,-\sigma'}u_{p\sigma'}v_{I\downarrow,p} \\ \delta_{\sigma,-\sigma'}u_{I\uparrow,p}v_{p\sigma} \\ -\delta_{\sigma,\sigma'}u_{I\downarrow,p}v_{I\downarrow,p} \end{pmatrix} \times \langle e^{-iE_p(t-t')}f(-E_p) - e^{iE_p(t-t')}f(E_p) \rangle,
\]

(15b)
where the Fermi distribution function \(f(E) = 1/\exp(E/T) + 1\). We evaluate the Josephson current in various types of superconducting junctions. First let us consider that both the left and right superconductors are of spin-singlet pairing symmetry, one can arrive at the Josephson current as:

\[
I_J = \frac{e}{\hbar} \sum_{k,p} \sum_{\sigma\sigma'} \langle \sigma\sigma' | \text{Im}[T_{\sigma\sigma'}(t)T_{-\sigma,-\sigma'}(t)e^{i(2eVt + \delta\varphi)}] \rangle \times \frac{|\psi_k|^2|\psi_p|^2\Omega_{k,p}(eV)}{2E_kE_p},
\]

(16)
where the phase difference \(\delta\varphi = (\varphi_R - \varphi_L) + (\varphi_p - \varphi_k)\), and

\[
\Omega_{k,p}(eV) = \left[\frac{1}{eV + E_k - E_p} - \frac{1}{eV - E_k + E_p}\right] \times |f(E_k) - f(E_p)| + \left[\frac{1}{eV + E_k + E_p} - \frac{1}{eV - E_k - E_p}\right] \times |1 - f(E_k) - f(E_p)|.
\]

(17)

The summation over spin indices involves the term, \(T_{\uparrow\uparrow}T_{\downarrow\downarrow}, \) and \(T_{\uparrow\downarrow}T_{\downarrow\uparrow}\). It then follows from the structure of the tunneling matrix as given by Eq. \([10]\), which has the property \(T_{\uparrow\downarrow} = T_{\downarrow\uparrow}^\dagger\), that the flowing Josephson current is not modulated with time by the precessing spin. Similarly, one can find that this conclusion is also true for the Josephson current between two superconductors of dissimilar spin parity. However, when each side of the junction is a superconductor having equal spin-triplet pairing symmetry, the Josephson current becomes:

\[
I_J = -e \sum_{k,p} \sum_{\sigma\sigma'} \text{Im}[T_{\sigma\sigma'}(t)T_{-\sigma,-\sigma'}(t)e^{i(2eVt + \delta\varphi)}] \times \frac{|d_{I\uparrow,k}||d_{I\downarrow,p}|\Omega_{k,p}(eV)}{2E_kE_p},
\]

(18)
which will be time dependent even in the absence of the voltage bias when the spin is precessing at \(\omega_L\). In some detail, because \(T_{\uparrow\downarrow} = T_{\downarrow\uparrow} = T_{\uparrow\downarrow}^\dagger\), the term with a pre-factor \(e^{i\omega_Lt}\), \(I_J\) contains a term with a pre-factor \(\cos(2\omega_Lt)\). This implies that the Josephson current flowing between two equal spin-triplet pairing superconductors is modulated in time at a frequency of \(2\omega_L\), i.e., twice of the Larmor frequency. The relative ratio between the Larmor modulation part \(\delta I_J\) and the constant part \(I_{J0}\) is:

\[
\frac{\delta I_J}{I_{J0}} = \frac{J^2S^2}{2\Phi_0} \sim 10^{-2}.10^{-3},
\]

(19)
for \(\Phi = 1 \text{ eV}, \Phi_0 = 0.05 \text{ eV}, JS = 0.1 \text{ eV}, \) which is experimentally detectable. The modulation of a Josephson current by a precessing spin could be used for a single spin detection.

If we suppose that the left superconductor is a spin-singlet superconductor which is weakly coupled to the
right superconductor having non-equal-spin-triplet pairing symmetry, the Josephson current is found to be:

\[ I_J = \frac{e}{2E_kE_p} \sum_{k,p} |\langle \Psi_R' | d_{i,p} | \Psi_L,\sigma \rangle|^2 \delta(\mathcal{E}_L - \mathcal{E}_R) \]  

\[ \times |\langle \Psi_R' | d_{i,p} | \Psi_L,\sigma \rangle|^2 \delta(\mathcal{E}_L - \mathcal{E}_R) \]  

\[ \times \sum_{\sigma' \sigma} \sigma T_{\sigma \sigma'}(t) T_{-\sigma',-\sigma} \frac{\sin(2eVt + \delta \phi)}{2E_kE_p}. \]  

(20)

Notice that the summation \[ \sum_{\sigma' \sigma} \sigma T_{\sigma \sigma'}(t) T_{-\sigma',-\sigma} \] is zero. Also the Josephson current cannot occur when a spin-singlet superconductor is weakly coupled to a superconductor of equal-symmetry. We define \( \Psi \) to establish a simple phenomenological theory for the Josephson tunnel barrier.

In summary, we have studied the Josephson current in the vicinity of an atomic spin on the superconducting surface. The spin-relaxation time in superconductor is governed by the coupling of spin with its environment and is strongly suppressed due to the gapped nature of quasi-particles, we can expect a very long spin-relaxation time when the local spin is embedded in a superconducting junction. Therefore, it would be very interesting to extend the JSTM technology by using a superconducting tip to study the Josephson current in the vicinity of an atomic spin on the superconducting surface.

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namics, a so-called back action effect, have been considered in [6, 7, 8]. It has been shown that the back-action effects are small and precession is not drastically affected by a tunneling current, provided the coupling is small enough. This allows us to treat the problem with a long decay time $\tau$ of precessing state $\omega_L \tau \gg 1$.

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