CHARM AND BEAUTY IN PARTICLE PHYSICS

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ABSTRACT

The spectra of states containing charmed and beauty quarks, and their regularities, are reviewed.

I. INTRODUCTION

More than 20 years ago, two experimental groups announced the discovery of the first in a series of charm-anticharm bound states [1, 2]. During the first year in which the properties of these \( c\bar{c} \), or charmonium, states were mapped out, this system began to display experimental possibilities as rich as those in positronium. However, an important difference from positronium was predicted by theory and soon verified experimentally. Whereas the \( 2S \) and \( 1P \) positronium levels are nearly degenerate, the \( 1P \) charmonium level lies significantly below the \( 2S \) state. What does this say about the interquark force? M. A. Baqi Bég asked this question of André Martin during Martin’s visit to Rockefeller University in 1975. The result was the first [3] in a series of lovely theorems about the order of energy levels in nonrelativistic potentials [4, 5, 6], and a simple form of power-law potential [7] which has proved remarkably successful in predicting the masses of new states containing not only charm and beauty, but also strangeness.

My own involvement in similar questions began with the discovery of the upsilon \( (b - \bar{b}) \) levels [8], for which the \( 2S - 1S \) spacing appeared close to that in

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1Presented at CERN on September 28, 1994, at a symposium in honor of André Martin’s retirement. This article is dedicated to the memory of M. A. Baqi Bég.

2We label levels by \( n_r + 1 \), where the radial quantum number \( n_r \) is the number of nodes of the radial wave function between 0 and \( \infty \).
charmonium. Chris Quigg and I asked what kind of potential would give a level spacing independent of mass \[9\]. The result, a potential \( V(r) \sim \ln r \) whose properties had been investigated even before the discovery of the upsilons \[10\], was surprisingly simple, and led us to numerous related investigations of general properties of potential models \[11, 12\] and our own attempts at power-law fits \[13\]. It also stimulated work in the inverse scattering problem \[14\] as an outgrowth of attempts to construct the interquark potential directly from data.

These parallel efforts have been marked by a good deal of correspondence between the respective groups. We have greatly enjoyed hearing about each other’s results. It now appears that the first actual collaborative paper involving both our groups \[15\] will emerge as a result of this Symposium. For this, and for the opportunity to honor André, I am very grateful.

We begin in Section II by reviewing quarkonium spectra and their regularities. We next discuss the predictions of power-law potentials for level spacings in Sec. III and for dipole matrix elements in Sec. IV. Some inverse scattering results and the key role of information on the wave function at the origin are mentioned in Sec. V. We discuss hadrons with one charmed quark in Sec. VI, and relate their properties to those of hadrons containing a single \( b \) quark using heavy quark symmetry in Sec. VII. An overview of the properties of hadrons with beauty occupies Sec. VIII. These hadrons (particularly the mesons) are a prime laboratory for the study of the Cabibbo-Kobayashi-Maskawa (CKM) matrix (Sec. IX) and of CP violation (Sec. X). We note some issues for further study and conclude in Sec. XI.

**II. QUARKONIUM SPECTRA AND THEIR REGULARITIES**

Of all the known quarks, the charmed quark \( c \) and the beauty quark \( b \) offer the best opportunity for the study of bound states and for insights into the strong interactions using simple methods. Since the scale at which the interactions of quantum chromodynamics (QCD) become strong is several hundred MeV, the masses of the \( u, d, \) and \( s \) quarks are overwhelmed in bound states by QCD effects. The top quark is so heavy that it decays to \( W + b \) before forming bound states. Leptons, of course, being colorless, do not participate in this rich physics at all. In this Section we give a brief overview of levels containing only \( c \) and \( b \) quarks.

**A. Charmonium**

The charmonium spectrum is shown in Fig. 1. Masses of observed levels are based on the averages in Ref. \[16\]. The prediction of the \( \eta_c(2S) \) is based on Ref. \[17\]. Arrows are labeled by particles emitted in transitions. States above the horizontal dashed line can decay to pairs of charmed mesons (\( D \bar{D} \)) and are consequently broader than those below the line, which decay both electromagnetically and with appreciable branching ratios to non-charmed hadrons (not shown).

For many years, the major source of charmonium was the reaction \( e^+e^- \rightarrow \gamma^* \rightarrow (c\bar{c}) \), which can produce only states with spin \( J = 1 \), parity \( P = - \), and
charge-conjugation eigenvalue \( C = -1 \), namely the \(^3S_1\) and \(^3D_1\) levels. Other levels were reached by electric or magnetic dipole transitions from the \( J^{PC} = 1^{--} \) states, as indicated by the arrows labeled by \( \gamma \) in the figure. More recently, starting with an experiment in the CERN ISR \[18\] and continuing with studies in the Fermilab antiproton accumulator ring \[19\], it has been possible to perform \( \bar{p}p \) collisions with carefully controlled energy, forming charmonium states in the direct channel. The observation of the \( h_c(1P) \) level has been one benefit of these studies, which are expected to continue.

**B. Upsilonons**

We show \( \Upsilon (bb) \) levels in Fig. 2. The observed levels are as quoted in Ref. \[16\], while the \( J^{PC} = 0^{-+} \) levels are shown with masses predicted on the basis of Ref. \[17\]. The \( J^{PC} = 1^{+-} (“h_b”) \) levels are taken to have the spin-weighted average masses of the corresponding \( \chi_b \) levels. Since flavor threshold lies higher than for charmonium, there are *two* sets of narrow P-wave levels, and consequently a rich...
Figure 2: Spectrum of \( b \bar{b} \) states. Observed and predicted levels are denoted by solid and dashed horizontal lines, respectively. In addition to the transitions labeled by arrows, numerous electric dipole transitions and decays of states below \( B \bar{B} \) threshold to hadrons containing light quarks have been seen.

set of electric dipole transitions between the \( \Upsilon \) and \( \chi_b \) states, e.g., \( 3S \to 2P \to 2S \to 1P \to 1S \), \( 3S \to 1P \) (very weak), and \( 2P \to 1S \). The systematics of these transitions has been a subject of recent interest to André, our colleagues, and me [20, 21], which will be described in Sec. IV.

C. Quarkonium and QCD

As anticipated [22], quarkonium has proved a remarkable laboratory for the study of quantum chromodynamics.

1. Forces between a quark and an antiquark are best visualized with the help of Gauss’ Law. At short distances, the interquark potential is described by an effective potential \( V(r) = -(4/3)\alpha_s(r)/r \), where the 4/3 is a color factor and the strong fine structure constant \( \alpha_s \) decreases as \( 1/\ln r \) at short distances as a result of the asymptotic freedom of the strong interactions [23]. Lines of force behave
approximately as they do for a Coulomb potential. They spread out in a typical dipole pattern; one cannot tell the scale of the interaction by looking at them. At long distances, on the other hand, the chromoelectric lines of force bunch up into a flux tube of approximately constant area, much as magnetic flux in a type-II superconductor forms tubes. The force between a quark and antiquark at long distances is then independent of distance \[24\], so the potential \(V = kr\) rises linearly with distance. Experimentally \(k\) is about 0.18 GeV\(^2\).

2. Decays of quarkonium states are a source of information about the strength of the strong coupling constant. For example, the ratio of the three-gluon and \(\mu^+\mu^-\) decay rates of the \(\Upsilon\) is proportional to \(\alpha_s^3/\alpha^2\), where \(\alpha\) is the electromagnetic fine-structure constant, and leads \[25\] to a value of \(\alpha_s(M_Z) = 0.108 \pm 0.010\) consistent with many other determinations. (It has become conventional to quote \(\alpha_s\) at \(M_Z\) even though the decay of the \(\Upsilon\) probes \(\alpha_s\) at \(m_b \approx 5\) GeV.)

3. Lattice QCD calculations \[26\] deduce the value of \(\alpha_s\) from the observed \(1P - 1S\) level spacing in the \(\Upsilon\) system (Fig. 2), leading to \(\alpha_s(M_Z) = 0.110 \pm 0.006\). Both this value and that determined from \(\Upsilon\) decays are consistent with the world average \[27\] \(\alpha_s(M_Z) = 0.117 \pm 0.005\).

III. LEVEL SPACINGS IN POWER-LAW POTENTIALS

The spectra of the Coulomb \((V \sim -r^{-1})\) and three-dimensional oscillator \((V \sim r^2)\) potentials are familiar to students of quantum mechanics, some of whom even are aware (as was Newton \[28, 29\]) that the two problems are related to one another. These spectra are illustrated in Figs. 3(a) and 3(b). In the Coulomb potential, the energy levels are proportional to \(-(n_r + L + 1)^{-2} = -n^{-2}\), where \(n\) is the principal quantum number, and thus are highly degenerate. A different type of degeneracy is present in the harmonic oscillator, for which the energies are proportional to \(2n + L + 3\). An intermediate case, \(V \sim \ln r\) (equivalent to the limit of \(V = (r^\nu - 1)/\nu\) as \(\nu \to 0\)) is shown in Fig. 3(c). (Further examples may be found in Ref. \[15\]). The quarkonium spectrum is rather similar to this. Indeed, a potential \(V(r) = -(4/3)\alpha_s(r)/r + kr\) can be approximated by some power intermediate between \(-1\) and \(1\) for a limited range of distance \[30\]. It so happens that for \(c\bar{c}\) and \(b\bar{b}\) states, which are sensitive to the range between 0.1 and 1 fm \[12\], this power turns out to be close to zero.

A. Predictions of the Martin potential

The \(2S - 1S\) level spacing in the \(\Upsilon\) family is slightly smaller than that in charmonium. Since level spacings in a potential \(V \sim r^\nu\) behave with reduced mass \(\mu\) as \(\Delta E \sim \mu^{-\nu/(\nu+2)}\) \[3, 14\], a small positive power will be able to reproduce this feature. What is remarkable is how much else is fit by such a simple ansatz. A potential \(V(r) \sim r^{0.1}\) \[1\] (we refer the reader to the original articles for precise constants and quark masses) not only fits charmonium and upsilon spectra remarkably well,
Figure 3: Patterns of lowest-lying energy levels in various potentials $V(r) = (r^n - 1)/n$. (a) Coulomb potential ($n = -1$) (the dashed line indicates the onset of continuum levels); (b) three-dimensional oscillator ($n = 2$); (c) $V(r) \sim \ln r$, corresponding to the limit $n \to 0$. 

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as shown in Fig. 4, but also has been successful in fitting and anticipating masses of states containing strange quarks, using the mass of the $\phi(1020)$ as the input for the $1^3S_1 ss$ level. We compare these predictions with observations \[16, 31, 32, 33\] in Table 1. Standard assumptions regarding spin-spin interactions have been made in order to estimate hyperfine splittings between $^1S$ and $^3S$ levels.

**B. Remarks on levels**

The agreement between predictions and experiment in Fig. 4 and Table 1 is so good that many predictions are hard to distinguish from the observations. Even the discrepancies are interesting.

1. The $\eta'_c$, when predicted, disagreed with a claimed state \[34\] which has not been confirmed in a new proton-antiproton experiment \[19\].
Table 1: Masses of states containing strange quarks predicted in a potential $V \sim r^{\nu}$ and observed experimentally.

| Level | Predicted Mass (GeV) | Observed Mass (GeV) | Level | Predicted Mass (GeV) | Observed Mass (GeV) |
|-------|----------------------|---------------------|-------|----------------------|---------------------|
| $(s\bar{s})_{2S}$ | 1.634 | 1.650$^a$ | $b\bar{s}$ | 5.364 | 5.368$^d$ |
| $c\bar{s}$ | 1.99 | 1.97 | $(b\bar{s})^*$ | 5.409 | 5.422$^c$ |
| $(c\bar{s})^* 3P$ | 2.11 | 2.11 | $b\bar{c}$ | 6.25 | 6.32 |
| $(c\bar{s}) 3P$ | 2.54 | 2.54$^b$ | |

a) Ref. [16]; b) $3P_1$ level [16]; c) $3P_2$ level [31];
d) Ref. [32]; e) Ref. [33]; see discussion in text.

2. The observed $\psi(1D)$ level, the $\psi(3770)$, is a $1^3D_1$ state, whereas the prediction has been shown for the spin-averaged 1D mass. The other 1D levels (the $1^3D_2$ and $3^3D_3$) probably lie higher, and are accessible in $\bar{p}p$ interactions. The $\psi(3770)$ is a good source of $D\bar{D}$ pairs, soon to be exploited by the Beijing electron-positron collider. The $1^3D_2$ levels cannot decay to $D\bar{D}$ and probably lie below $D\bar{D}^*$ threshold, so they are expected to be narrow.

3. The observed $\psi(4160)$ level is not really understood on the basis of any simple potential models, Martin’s or otherwise. Is it the $2^3D_1$ level, mixed with S-waves so as to have an appreciable coupling to $e^+e^-$? Its mass and couplings are undoubtedly strongly affected by coupled channels. A similar distortion is visible near $BB$ threshold in the $\Upsilon$ family [35].

4. The $\chi_b(9900)$ levels lie higher than Martin’s prediction, exposing the limitations of a universal power-law potential. Their position relative to the $1S$ and $2S$ levels, when compared to that of the $\chi_c$ levels in charmonium, is weak evidence that the interquark potential is becoming more singular at short distances, as predicted by QCD [36].

5. The $1D$ and $2D bb$ levels can be searched for in the direct $e^+e^- \rightarrow \gamma^* \rightarrow 3D_1$ reaction, in cascade reactions involving electric dipole transitions to and from $P$-wave levels, and possibly in transitions to $\Upsilon(1S)\pi\pi$ [37].

6. The $D^*_s - D_s$ splitting is about the same as the $D^* - D$ splitting. Since the hyperfine splitting is proportional to $|\Psi(0)|^2/m_c m_q$, where $\Psi(0)$ is the nonrelativistic wave function of the charmed quark and the light quark $q = d$, $s$ at zero separation, one expects $|\Psi(0)|^2_{cs} \approx |\Psi(0)|^2_{cd}(m_s/m_d)$, a relation useful in determining the ratio of the $D_s^+$ and $D^+$ decay constants [38].

7. The $B^*_s - B_s$ splitting in Martin’s approach, as well as in an expansion in
inverse powers of heavy quark masses performed much later \[39\], is predicted to be the same as the $B^0 - \bar{B}^0$ splitting. A tentative observation by the CUSB group \[33\] is consistent with this expectation.

8. The ratio of level spacings $(3S - 2P)/(2P - 1D)$ is an interesting quantity. In a power-law potential $V \sim r^{\nu}$, for a wide range of values of $\nu$, this quantity is expected to be very close to unity \[13\]. This circumstance can be useful to anticipate the position of the $b\bar{b}$ 1D levels, discussed above, and the $c\bar{c}$ 2P levels, which may play a role \[40\] in the hadronic production of the $\psi(2S)$ \[41\]. This ratio for $b\bar{b}$ states is very far from unity in Ref. \[5\], where Martin quoted a prediction for the 1D levels from another source \[12\].

9. The $1P_1$ levels of quarkonium were predicted by Stubbe and Martin \[43\] to lie no lower than the spin-weighted average of the corresponding $\chi^{(3P_{0,1,2})}$ levels. A candidate for the $1P_1 b\bar{b}$ level proposed by the CLEO Collaboration several years ago \[44\] violated this bound; it was subsequently not confirmed. The corresponding $c\bar{c}$ level has been discovered just at the lower limit of the Stubbe-Martin bound \[45\]; its mass is $3526.14 \pm 0.24$ MeV, close to the spin-weighted average of the $\chi_c$ levels, $3525.3 \pm 0.1$ MeV.

C. Mesons with charm and beauty

An interesting system in which the quarks are heavy but unequal in mass is the set of $b\bar{c}$ levels, recently discussed in detail by Eichten and Quigg \[46\]. The positions of their predicted $1S$ levels are very close to those anticipated by Martin (see Table 1). If the fine structure of the $1P$ levels (predicted to lie around 6.7 GeV) can be observed, it may provide new information about spin-dependent forces not accessible in equal-mass systems. The $2S - 1S$ spacing is predicted to be somewhat below 0.6 GeV. A narrow set of 1D levels is predicted at 7.0 GeV. The $2P$ levels are expected to lie very near the $\bar{B}D$ threshold at 7.14 GeV.

IV. DIPOLE TRANSITIONS IN POWER-LAW POTENTIALS

The pattern of electric dipole matrix elements in atomic transitions can be understood on very intuitive grounds, in terms of overlaps of wave functions and semiclassical arguments \[47\]. The $\Upsilon$ system is rich enough to display some aspects of this pattern, as shown in Fig. 5 \[21\].

Let us denote the orbital angular momentum by $L$, the radial quantum number by $n_r$, and the principal quantum number by $n = n_r + L + 1$. (We have been labeling our levels by $n_r + 1$.) As in atoms, transitions in which $n$ and $L$ change in opposite directions are highly disfavored. For example, in the transition $3S \rightarrow 1P$, $n$ decreases from 3 to 2 while $L$ increases from 0 to 1. Such transitions are just barely visible in the $\Upsilon$ system \[45\]. The ratio $r_1 \equiv \langle 1P|r|3S \rangle / \langle 2P|r|3S \rangle$ is highly suppressed in power-law potentials for a large range of interesting powers, as seen in Fig. 6.
Figure 5: Observed electric dipole transitions in the $\Upsilon$ system. Arrows denote favored transitions. The very weak $3S \rightarrow 1P$ transition is denoted by a dotted line. The $2P \rightarrow 1S$ transition, denoted by a dashed line, is also somewhat suppressed.

Figure 6: Ratios $r_1 \equiv \langle 1P|r|3S \rangle / \langle 2P|r|3S \rangle$ (dot-dashed) and $r_2 \equiv \langle 1S|r|2P \rangle / \langle 2S|r|2P \rangle$ (dashed) as a function of $\nu$ in power-law potentials $V(r) \sim r^\nu$. 
There is also a tendency for transitions to favor levels whose wave functions are as similar to one another as possible. Thus, the transition \(2P \rightarrow 1S\) (involving a change of two units of \(n\)) is suppressed in comparison with \(2P \rightarrow 2S\), where \(n\) changes by only one unit. Fig. 6 shows that the ratio \(r_2 \equiv \langle 1S|r|2P\rangle/\langle 2S|r|2P\rangle\) is moderately suppressed in power-law potentials. Both \(r_1\) and \(r_2\) would vanish in a harmonic oscillator potential, as can be seen by expressing the dipole operator as a sum of creation and annihilation operators.

While working on dipole transitions [21], we had enjoyable correspondence with André, who shared with us a number of interesting rigorous results [20] on the signs of dipole matrix elements in various potentials. A number of years ago, André had already shown that the \(2P \rightarrow 1S\) matrix element could not vanish and had the same sign as the product of the two radial wave functions at infinity [19].

\section*{V. INVERSE SCATTERING RESULTS}

One can construct an interquark potential directly from the masses and leptonic widths of S-wave quarkonium levels [14]. A potential constructed from \(b\bar{b}\) levels agrees remarkably well with that constructed using charmonium data, except at the shortest distances, where the heavier \(b\bar{b}\) system provides the more reliable information. (We refer the reader to Refs. [14] for illustrations.) Consistency between the two constructions leads to a rather tight constraint on the difference between charmed and \(b\) quark masses, \(m_b - m_c \simeq 3.4\) GeV.

Supersymmetric quantum mechanics [50] has proved very helpful in the construction of potentials [51]. A Hamiltonian with a given spectrum can be factorized into the product of two operators, \(H_+ = A^\dagger A\). A Hamiltonian \(H_- = AA^\dagger\) (related by supersymmetry to \(H_+\)) has the same spectrum aside from any state \(|0\rangle\) annihilated by the operator \(A\), in which case \(|0\rangle\) is the (zero-energy) ground state of \(H_+\), but does not belong to the spectrum of \(H_-\). Starting from a potential \(V_- = \kappa^2\) in \(H_-\) which has no bound states, we then find a potential \(V_+ = \kappa^2[1 - 2\text{sech}^2\kappa(x - x_0)]\) in \(H_+\) with a single zero-energy bound state. The integration constant \(x_0\) may be chosen to give a symmetric potential whose odd-parity levels are suitable S-wave wave functions for the radial equation of a three-dimensional problem. By appropriate shifts of the energy after each supersymmetry transformation, one can construct potentials with an arbitrary spectrum. This construction bears an interesting relation to the vertex operator in string theory [52].

The key role of leptonic widths in solving the inverse scattering problem arises from the information they provide on the squares of wave functions at zero interquark separation. These quantities obey beautiful regularities and inequalities in power-law potentials [11, 53].
VI. CHARMED HADRONS

The ground states of levels containing a single charmed quark are shown in Fig. 7, adapted from Ref. [36] using data quoted in Ref. [16]. All the levels except baryons with spin 3/2 (dashed lines) have been seen, including a recently reported excited state of the $\Xi_c$ found in a CERN experiment [54]. What follows is a small sample of some interesting questions in charmed-hadron physics.

A. $D$ meson semileptonic decays

A free-quark model of $D$ meson semileptonic decays in which the charmed quark undergoes the transition $c \rightarrow s\ell^+\nu_\ell$ would predict, in the limit of zero recoil of the strange quark, the ratio of 1:3:0 for $\bar{K}:\bar{K}^*:\bar{K}^{**}$, where $\bar{K}^{**}$ stands for any excited state of the strange quark and nonstrange spectator antiquark. The observed ratio is more like 7:4:(0 to 4) [10, 55, 56]; it is still not certain how much
of the $D$ semileptonic branching ratio is associated with states other than $K$ and $K^*$. ($B$ meson semileptonic decays lead to final states other than $D$ and $D^*$ \cite{57}, so one should expect similar behavior for lighter-quark systems.)

Jim Amundson and I have looked at this process \cite{56} from the standpoint of heavy quark effective theory, treating the strange quark as heavy in a manner reminiscent of André’s bold assumption for quarkonium spectra, mentioned in Sec. III. We can identify several sources of the discrepancy with the heavy-quark limit, including an overall QCD suppression of $K$ and $K^*$ production, a phase-space suppression of $K^*$ relative to $K$, and a spin-dependent (hyperfine) interaction between the strange quark and the spectator antiquark which increases the $K$ rate and decreases the $K^*$ rate.

**B. Strange $D$ meson decay constants**

Recent observations of the decay $D_s \to \mu \nu$ \cite{58} have led to a measurement of the quantity $f_{D_s} \simeq 300$ MeV (in units where the pion decay constant is 132 MeV). This value agrees with one obtained earlier \cite{38, 59} from the decay $\bar{B} \to D^-s D$ under the assumption that the weak current in the decay of a $b$ quark to a charmed quark creates a $D^-_s$ meson. Through the expression $f_{D_s}^2 = 12|\Psi(0)|^2/M_{D_s}$, where $\Psi(0)$ is the wave function of the charmed quark and strange antiquark at zero separation, and the use of heavy quark symmetry, one can extrapolate this observation to predict other heavy meson decay constants, such as $f_D$, $f_B$, and $f_{B_s}$. A measurement of $f_D$ may be available in the near future at the Beijing Electron-Positron Collider (see Sec. III B 2). The last two decay constants are of particular interest in the study of CP violation in $B$ meson decays, as we shall see.

**C. Charmed baryons**

1. *Excited strange baryons* ought to be visible in semileptonic decays of the $\Lambda_c$. The nonstrange quarks in a $\Lambda_c$ are in a state of spin and isospin zero. In a spectator model, they should remain so. If the strange quark is given a sufficient “kick,” the nonstrange quarks should be able to form not only a $\Lambda$, but also the lowest-lying excitations in which the nonstrange quarks have zero spin and isospin, the states $\Lambda(1405)$, with $J^P = 1/2^-$, and the $\Lambda(1520)$, with $J^P = 3/2^-$. No such states have yet been seen \cite{60}; why not?

Many decays of $\Lambda(1405)$ and $\Lambda(1520)$ are elusive, consisting of charged $\Sigma \pi$ modes, and $K^0n$ for the $\Lambda(1520)$. However, the decays $\Lambda(1405) \to \Sigma^0 \pi^0 \to \Lambda \gamma \pi^0$ and $\Lambda(1520) \to K^-p$ are visible in CLEO. The importance of such final states in semileptonic $\Lambda_c$ decays consists not only in the degree to which semileptonic decays of heavy-quark hadrons populate excited states, but in the normalization of numerous branching ratios of the $\Lambda_c$ \cite{61}.

2. *Excited charmed baryons* have recently been identified \cite{62}, consisting of states lying 308 and 342 MeV above the $\Lambda_c$. Since the light-quark system in a $\Lambda_c$
baryon consists of a $u$ and $d$ quark bound to a state $[ud]$ of zero spin, zero isospin, and color antitriplet, the $\Lambda_c$ is a simple object in heavy-quark symmetry, easily compared with the $\Lambda_b = b[ud]$ and even with the $\Lambda = s[ud]$.

The $[ud]$ diquark in the $\Lambda$ can be orbitally excited with respect to the strange quark. The $L = 1$ excitations consist of a fine-structure doublet, the $\Lambda(1405)$ with spin-parity $J^P = 1/2^−$ and the $\Lambda(1520)$ with $J^P = 3/2^−$ mentioned above. The spin-weighted average of this doublet is 366 MeV above the $\Lambda$. These states are illustrated on the left-hand side of Fig. 8.

The candidates for the charmed counterparts of the $\Lambda(1405)$ and $\Lambda(1520)$ are shown on the right-hand side of Fig. 8. The spin-weighted average of the excited $\Lambda_c$ states is 331 MeV above the $\Lambda_c$, a slightly smaller excitation energy than that in the $\Lambda$ system. The difference is easily understood in terms of reduced-mass effects. The $L \cdot S$ splittings appear to scale with the inverse of the heavy quark ($s$ or $c$) mass. The corresponding excited $\Lambda_b$ states probably lie 300 to 330 MeV above the $\Lambda_b(5630)$, with an $L \cdot S$ splitting of about 10 MeV.

D. Excited charmed mesons

A good deal of progress has been made recently in the study of the P-wave resonances of a $c$ quark and a $\bar{u}$ or $\bar{d}$, generically known as $D^{**}$ states. Present data [16, 31] and predictions [63] are summarized in Fig. 9.

The observed states consist of the $1S$ (singlet and triplet) charmed mesons and all six (nonstrange and strange) $1P$ states in which the light quarks’ spins
combine with the orbital angular momentum to form a total light-quark angular momentum \( j = 3/2 \). These states have \( J = 1 \) and \( J = 2 \). They are expected to be narrow in the limit of heavy quark symmetry. The strange \( 1P \) states are about 110 MeV heavier than the nonstrange ones. In addition, there are expected to be much broader (and probably lower) \( j = 1/2 \) \( D^{\ast \ast} \) resonances with \( J = 0 \) and \( J = 1 \).

For the corresponding \( B^{\ast \ast} \) states, one should add about 3.32 GeV (the difference between \( b \) and \( c \) quark masses minus a small correction for binding). One then predicts [63] nonstrange \( B^{\ast \ast} \) states with \( J = (1, 2) \) at (5755, 5767) MeV, to which we shall return in Sec. X A.

**E. Lifetime differences**

Charmed particle lifetimes range over a factor of ten, with

\[
\tau(\Xi_c^0) < \tau(\Lambda_c) < \tau(\Xi_c) \simeq \tau(D^0) \simeq \tau(D_s) < \tau(D^+) \quad .
\]

(1)

Effects which contribute to these differences [64] include (a) an overall nonleptonic enhancement from QCD [65], (b) interference when at least two quarks in the final state are the same [66], (c) exchange and annihilation graphs, e.g. in \( \Lambda_c \) and \( \Xi^0_c \) decays [67], and (d) final-state interactions [68].

In the case of \( B \) hadrons, theorists estimate that all these effects shrink in importance to less than ten percent [69]. However, since the measured semileptonic branching ratio for \( B \) decays of about 10 or 11% differs from theoretical calculations
of 13% by some 20%, one could easily expect such differences among different $b$-flavored hadrons. These could arise, for example, from final-state interaction effects. There are many tests for such effects possible in the study of decays of $B$ mesons to pairs of pseudoscalars [70].

F. Anomalous electroweak couplings of charm?

A curious item was reported [71] at the DPF 94 conference in August in Albuquerque. The forward-backward asymmetries in heavy-quark production, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$, have been measured both on the $Z$ peak and 2 GeV above and below it. All quantities are in accord with standard model expectations except for $A_{FB}^{0,c}$ at $M_Z - 2$ GeV, which is considerably more negative than expected. It would be interesting to see if this effect is confirmed by other groups.

VII. HEAVY QUARK SYMMETRY

In a hadron containing a single heavy quark, that quark ($Q = c$ or $b$) plays the role of an atomic nucleus, with the light degrees of freedom (quarks, antiquarks, gluons) analogous to the electron cloud. The properties of hadrons containing $b$ quarks then can calculated from the corresponding properties of charmed particles by taking account [72] of a few simple “isotope effects.” For example, if $q$ denotes a light antiquark, the mass of a $Qar{q}$ meson can be expressed as

$$M(Qar{q}) = m_Q + \text{const.}[n, L] + \frac{\langle p^2 \rangle}{2m_Q} + a \frac{\langle \sigma_q \cdot \sigma_Q \rangle}{m_q m_Q} + O(m_Q^{-2}) . \quad (2)$$

Here the constant depends only on the radial and orbital quantum numbers $n$ and $L$. The $\langle p^2 \rangle/2m_Q$ term expresses the dependence of the heavy quark’s kinetic energy on $m_Q$, while the last term is a hyperfine interaction. The expectation value of $\langle \sigma_q \cdot \sigma_Q \rangle$ is (+1, −3) for $J^P = (1^-, 0^-)$ mesons. If we define $\overline{M} \equiv [3M(1^-) + M(0^-)]/4$, we find

$$m_b - m_c + \frac{\langle p^2 \rangle}{2m_b} - \frac{\langle p^2 \rangle}{2m_c} = \overline{M}(Bar{q}) - \overline{M}(car{q}) \simeq 3.34 \text{ GeV} . \quad (3)$$

so $m_b - m_c > 3.34 \text{ GeV}$, since $\langle p^2 \rangle > 0$. Details of interest include (1) the effects of replacing nonstrange quarks with strange ones, (2) the energies associated with orbital excitations, (3) the size of the $\langle p^2 \rangle$ term, and (4) the magnitude of hyperfine effects. In all cases there exist ways of using information about charmed hadrons to predict the properties of the corresponding $B$ hadrons. In search of methods without theoretical bias, we have even resorted [73] on occasion to numerical interpolation!
VIII. OVERVIEW OF HADRONS WITH BEAUTY

The use of heavy quark symmetry allows us to extrapolate from the spectrum shown in Fig. 7 of hadrons containing a single charmed quark to that of hadrons containing a single \( b \) quark. Taking account of the effects mentioned in the previous section, we obtain the spectrum shown in Fig. 10, updated and adapted from Ref. \[36\]. Some similarities and differences with respect to the charmed-hadron spectrum can be seen.

The \( B^* - B \) hyperfine splitting scales as the inverse of the heavy-quark mass: \( B^* - B = (m_c/m_b)(D^* - D) \). Consequently, while \( D^{**} \rightarrow D^0\pi^+ \) and \( D^{**} \rightarrow D^+\pi^0 \) are both allowed, leading to a useful method \([4]\) for identifying charmed mesons via the soft pions often accompanying them, the only allowed decay of a \( B^* \) is to \( B\gamma \). No soft pions are expected to accompany \( B \) mesons.

The \( B^*_s - B_s \) hyperfine splitting is expected to be the same as that between

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Figure 10: Lowest-lying S-wave levels of hadrons containing a single \( b \) quark. The lowest level in each group decays weakly. Dashed lines indicate levels not yet observed.
$B^{*0}$ and $B^0$ [32], as mentioned earlier. The observation by the CUSB group [33] consistent with this expectation needs confirmation.

In the $\Lambda_b$, the $u$ and $d$ quarks are in a state of zero spin and isospin, so the $b$ quark carries the spin of the $\Lambda_b$. This fact may be useful in probing the weak interactions of the $b$ quark [75]. Although the $\Lambda_b$ is listed as established by the Particle Data Group [16] (see the experiments in Ref. [76], yielding an average mass of $5641 \pm 50$ MeV), its confirmation in Fermilab [77] and LEP experiments has remained elusive up to now. Bounds on its mass were derived some time ago by Martin [78] and refined by Martin and Richard [79].

Many other states are expected to be rather similar to those in the charm system, once the added mass of the $b$ quark has been taken into account. The precise value of the splitting between the $\Sigma_b^*$ and $\Sigma_b$ is important [80] in estimating the amount of depolarization undergone by a $b$ quark as it fragments into a $\Lambda_b$.

IX. THE CKM MATRIX

Our present understanding of CP violation links the observed effect in the neutral kaon system to a phase in the unitary Cabibbo-Kobayashi-Maskawa [81] (CKM) matrix describing weak charge-changing transitions among quarks. A sound understanding of the way in which heavy quarks are incorporated into hadrons is essential to specify the CKM parameters precisely as possible in order to test the theory.

A. Measuring CKM elements

We write the matrix in the form [82]:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$ (4)

The upper left $2 \times 2$ submatrix involves only one real parameter $\lambda = \sin \theta_c$, where $\theta_c$ is the Cabibbo angle. The couplings involving the third family of quarks ($b, t$) require three additional parameters $A$, $\rho$, and $\eta$. We outline the means [83] by which these quantities are measured.

1. The parameter $\lambda$ is specified by comparing strange particle decays with muon decay and nuclear beta decay, leading to $\lambda \approx \sin \theta \approx 0.22$.

2. The element $V_{cb} = A\lambda^2$ is responsible for the dominant decays of $b$-flavored hadrons. The lifetimes of these hadrons and their semileptonic branching ratios then lead to an estimate $V_{cb} = 0.038 \pm 0.003$, or $A = 0.79 \pm 0.06$. One must relate processes at the quark level to those at the hadron level either using a QCD-corrected free quark estimate or specific models for final states. The constraints on $m_b - m_c$ arising in charmonium and upsilon spectroscopy, whereby this difference lies between 3.34 and 3.4 GeV, are proving useful in this regard.
3. The magnitude of the element $V_{ub}$ governs the rate of decays of $b$-flavored hadrons to charmless final states. One infers $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ or $\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09$ from leptons emitted in semileptonic decays $b \to u\ell\nu$ with energies beyond the endpoint for $b \to c\ell\nu$. The error reflects the uncertainty associated with models relating this small part of the spectrum to the whole rate.

4. The phase of $V_{ub}$, $\text{Arg}(V_{ub}^*) = \arctan(\eta/\rho)$, is the least certain quantity. Information on it may be obtained by studying its effect on contributions of higher-order diagrams involving the top quark, such as those governing $B^0 - \bar{B}^0$ mixing and CP-violating $K^0 - \bar{K}^0$ mixing, with $m_t = 174 \pm 17$ GeV.

The most recent estimate for the $B^0 - \bar{B}^0$ mixing amplitude, incorporating recent observations of time-dependent oscillations [32], is $\Delta m / \Gamma = 0.71 \pm 0.07$. The dominant contribution to the mixing is provided by one-loop diagrams ("box graphs") involving internal $W$ and top quark lines, leading to $\Delta m \sim f_B m_t^2 |V_{td}|^2$ (times a slowly varying function of $m_t/M_W$). Here the "$B$ decay constant," $f_B$, describes the amplitude for finding a $b$ antiquark and a light quark at the same point in a $B$ meson. Since $|V_{td}| \sim |1 - \rho - i\eta|$, the $B^0 - \bar{B}^0$ mixing amplitude leads to a constraint in the $(\rho, \eta)$ plane consisting of a circular band with center (1,0). The main contribution to the width of this band is uncertainty in $f_B$.

A similar set of box diagrams contributes to the parameter $\epsilon$ describing CP-violating $K^0 - \bar{K}^0$ mixing. The imaginary part of the mass matrix is proportional to $f_K m_t^2 \text{Im}(V_{td}^2)$ times a slowly varying function of $m_t$, with a small correction for the charmed quark contribution and an overall factor $B_K$ describing the degree to which the box graphs account for the effect. Since $\text{Im}(V_{td}^2) \sim \eta(1 - \rho)$, the constraint imposed by CP-violating $K^0 - \bar{K}^0$ mixing consists of a hyperbolic band in the $(\rho, \eta)$ plane with focus at (1,0), whose width is dominated by uncertainty in the magnitude of $V_{cb}$.

B. Constraints on parameters

The allowed region in the $(\rho, \eta)$ plane is bounded by circular bands associated with the $|V_{ub}/V_{cb}|$ and $B^0 - \bar{B}^0$ mixing constraints, and a hyperbolic band associated with the CP-violating $K^0 - \bar{K}^0$ mixing constraint. In a recent determination [32] we used parameters, in addition to those mentioned above, including $B_K = 0.8 \pm 0.2$, $f_B = 180 \pm 30$ MeV (in units where $f_\pi = 132$ MeV), $\eta_{QCD} = 0.6 \pm 0.1$ (a correction to the $B - \bar{B}$ mixing diagrams), and $B_B = 1$ for the factor analogous to $B_K$, and found $-0.3 \leq \rho \leq 0.3$, $0.2 \leq \eta \leq 0.4$. The main uncertainty in $\rho$ stems from that in $f_B$, while model-dependent sources of error in $V_{cb}$ and $V_{ub}$ are the main sources of uncertainty on $\eta$. Thus, improved knowledge about hadron physics can have a major impact on our present understanding of weak interactions.

C. $B_s - \bar{B}_s$ mixing

In contrast to $B^0 - \bar{B}^0$ mixing, which involves the uncertain CKM element $V_{td}$, the $B_s - \bar{B}_s$ mixing amplitude involves the elements $V_{ts} \approx -V_{cb} = -0.038 \pm 0.003$.
and \(V_{tb} \approx 1\), so that the main source of uncertainty in \(x_s \equiv (\Delta m/\Gamma)_{B_s}\) is the decay constant \(f_{B_s}\). For \(f_{B_s} = 200 \pm 50\) MeV and \(m_t = 174 \pm 17\) GeV we find \(x_s = 16 \times 2^{\pm 1}\). If this mixing rate can be measured and the uncertainties on \(V_{ts}\) and \(m_t\) reduced, a useful value for \(f_{B_s}\) (and hence, via SU(3), for \(f_B\)) can be obtained. Estimates for \(f_B/f_{B_s}\) range from about 0.8 to 0.9 [86].

X. CP VIOLATION IN \(B\) DECAYS

If the phase in the CKM matrix is responsible for CP violation in the neutral kaon system, dramatic CP-violating effects are expected in decays of \(B\) mesons. In order to exploit and interpret these effects, many aspects of hadron spectroscopy are important. I would like to mention just two areas of recent progress.

A. Decays to CP eigenstates

1. \(\pi - B\) correlations are useful in identifying the flavor of neutral \(B\) mesons at the time of production. Once produced, these mesons can undergo \(B^0 - \bar{B}^0\) mixing, leading to time-dependent asymmetries in decays to CP eigenstates like \(J/\psi K_S\). Time-integrated decays also can display rate asymmetries, whose interpretation is often independent of final-state effects. For example, the asymmetry in decays of \(B^0\) or \(\bar{B}^0\) to \(J/\psi K_S\) is equal to \(-[x_d/(1 + x_d^2)]\sin[\text{Arg}(V_{td}^*)/2]\), where \(x_d = (\Delta m/\Gamma)|_d = 0.70 \pm 0.07\) is the mixing parameter mentioned earlier. One has to know the flavor of the neutral \(B\) at time of production. One proposed means for “tagging” the \(B\) involves its correlation with charged pions produced nearby in phase space [87]. The existence of such a correlation is predicted both by fragmentation and resonance decay pictures.

2. \(B^{**}\) resonances can serve as explicit sources of \(\pi - B\) correlations. One expects resonances in the \(\pi^+B^0\) and \(\pi^-\bar{B}^0\) channels but not in the \(\pi^-B^0\) and \(\pi^+\bar{B}^0\) channels. If these resonances are narrow, they can help in suppressing backgrounds.

The expected spectrum of \(B^{**}\) resonances can be roughly anticipated by adding about 3.32 GeV to the masses of excited charmed states shown in Fig. 9. One expects narrow P-wave levels of spins 1 and 2 around 5.76 GeV, and broader levels of spins 0 and 1 somewhat lower in mass. Recently two groups at LEP [88] have presented evidence for \(\pi - B\) correlations which appear to show at least some resonant activity in the “right-sign” combinations.

B. Decays to CP non-eigenstates

A difference between the rates for a process and its charge-conjugate, such as \(B^+ \to \pi^+K^0\) and \(B^- \to \pi^-K^0\), signifies CP violation. Under charge conjugation, weak phases change sign, but strong phases do not. In order for a rate difference to appear, there must be both a weak phase difference and a strong phase difference in the channels with isospins \(I = 1/2\) and \(3/2\). Recently it has been shown that one may be able to measure weak phases via the rates for \(B\) decays to pairs of
light pseudoscalar mesons without having any strong phase differences[70]. The presence of electroweak penguins[89] is one possible obstacle to this program, which is under further investigation.

XI. FOR THE FUTURE

A. Charmonium

The study of charmonium levels not limited to those with \( J^{PC} = 1^{--} \) will benefit from further experiments with stored antiprotons[19]. One can look forward to discovery of the \( \eta'_c \), the narrow \( 1^{1/2}D_2 \) levels, and perhaps one or more narrow \( 2P \) levels. The Beijing Electron-Positron Collider will turn its attention to the \( \psi(3770) \), a copious source of \( D\bar{D} \) pairs, leading to an eventual measurement of the valuable \( D \) meson decay constant when the process \( D \rightarrow \mu\nu \) is seen.

B. Upsilon

A number of interesting states remain to be found. These include the \( \eta_b \) (probably reachable from the \( \Upsilon(2S) \)), the \( \eta'_b \), the lowest \( 1P_1 \) level (around 9.9 GeV), and various \( \Upsilon(1D) \) and \( \Upsilon(2D) \) states. A careful scan in \( e^+e^- \) center-of-mass energy around 10.16 and 10.44 GeV may be able to turn up the predicted \( 3D_1 \) levels.

C. Charmed hadrons

We can look forward to more precise measurements of the \( D_s \) decay constant and to the first observations of \( D \rightarrow \mu\nu \). The \( \tau\nu \) final state may be the largest single decay mode of the \( D_s \), with a branching ratio approaching 9%!

The \( 2S \) charmed hadrons are expected to have masses of around 2.7 GeV, and thus to be able to decay to \( D_s^{(*)}K \). The discovery of such modes would encourage us to look for similar correlations in \( B_sK \) systems, which would be useful in identifying the flavor of strange \( B \) mesons at time of production[10].

Great progress has already been made, and more is expected, in the study of charmed baryons (both \( S \)-wave and \( P \)-wave) and of \( P \)-wave charmed mesons. We can look forward to the eventual discovery of charmed baryons with spins of \( 3/2 \), the partners of the familiar \( \Delta \) and \( \Omega^- \). The interest in the masses and decays of these states transcends the charm sector alone, and is important for anticipating properties of baryons containing a single \( b \) quark.

The differences in charmed particle lifetimes have provided a wealth of information about how strong interactions affect weak decays. These differences are expected to be much less marked for hadrons with beauty. One baryon whose lifetime is expected to be very short[34] is the \( \Omega_c \); we look forward to a determination (or at least an upper limit) in the near future.

Hadrons with more than one charmed quark (such as the \( ccu \) baryon) provide an interesting testing ground for theorems concerning the masses of multi-quark
systems \cite{[21]}. Perhaps such hadrons can be produced in $e^+e^-$ interactions, where one does not have to pay a heavy penalty for production of the first charmed quark.

**D. Hadrons with beauty**

In a few years, we will have confirmed the existence of the $B_s^*$, the $\Lambda_b$, the narrow $1P$ mesons, and perhaps some $2S$ states as well. The $1P$ mesons in particular may prove a valuable adjunct to CP-violation studies in the $B$ meson system.

A great deal remains to be learned about the weak decays of hadrons with beauty, especially to charmless final states. Here experimental work has outstripped theory in most cases, requiring us to come up with more reliable models for the way in which quarks are incorporated into hadrons. One area of future experimental progress may be in the determination of the full kinematics of semileptonic decay processes (including the momentum of the missing neutrino), which will reduce dependence on models.

With luck and ingenuity, we may yet learn the amplitude for $B_s - \bar{B}_s$ mixing, which will help fix the decay constant $f_{B_s}$ and, via SU(3), the constant $f_B$ which is of great importance in anticipating CP-violating effects in the $B$ meson system.

Finally, we can look forward to many years of fine data from CESR, Fermilab, LEP, and future facilities, culminating in the discovery of CP violation in $B$ decays. This would represent a triumph of Standard Model physics based on our present picture of the CKM matrix. We would then have to figure out where that curious phase in the CKM matrix actually comes from!

**E. Conclusion**

In conclusion, let me express thanks on behalf of all of us at this symposium to André Martin for showing us physics with charm and beauty!

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