Numerical study of detonation flows in a supersonic annular chamber

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Abstract. The presented paper is devoted to the study of detonation structures in a flow-type supersonic chamber of a new design, previously proposed by us. The detonation chamber is an annular cylindrical channel located between the inner and outer cylindrical walls. The oblique detonation wave is formed by a compression body inside the annular channel, shaped as a continuous monofilar helix with a constant pitch angle. Numerical simulations are performed for a supersonic flow of a stoichiometric hydrogen-air mixture with inflow Mach number $M_0=3$ and the pitch angle $\alpha<\alpha_{CJ}$. A mathematical model of the reacting flow in the combustor is developed in a two-dimensional unsteady formulation. A two-stage model of the detonation kinetics is used to describe the chemical process in reacting gas. The flow evolution for the different conditions for starting the detonation chamber operation is numerically studied. The final structures of the steady flow in the combustor are obtained. The bifurcation effect of a stationary solution according to the initial conditions of the problem is discovered for some combinations of the geometric parameters of the detonation combustor.

1. Introduction

A flow-type detonation chamber (DC) with a new design was proposed in our previous publications [1,2]. The DC is an annular channel located between the inner and outer cylindrical walls. A supersonic flow of a uniformly mixed stoichiometric hydrogen-air mixture with Mach number $M_0$ higher than the Mach number $M_{CJ}$ of the detonation wave (DW) in the Chapman-Jouguet (CJ) regime enters the combustor. The reacting mixture is burned in a steady oblique detonation wave (ODW) or system of oblique and normal DWs. The ODW is formed above a compression surface, which we proposed in the form of an infinitely thin solid monofilar helix. This helix overlaps the entire cross-section of the annular channel from the inner to the outer wall of the combustor (see figure 1 in [1]). The helix has a constant angle $\alpha$ with respect to the combustor axis.

The width $\Delta r$ of the annular channel is assumed to be much smaller than the average channel radius $r_c$, therefore, the gas flow over the combustor radius is assumed to be uniform. As a result, the flow in the combustor can be modeled in a two-dimensional formulation, as it is usually done when studying continuously rotating DWs in annular combustors. The two-dimensional solution domain shown in figure 1 is an annular channel, described above, cut and unfolded onto a plane. The channel cut is made as follows: for $0<x<L_1$ it is parallel to the axis of the channel, then up to $x=L$ goes along the surface of the helix. Thus, for $x>L_1$, the lower and upper boundaries of the solution domain (figures 1-5) represent the same rigid wall — the compression body in the form of a helix. The lower boundary is the windward side of the helix; the upper boundary is the leeward side of the helix. Here $L$ is the total...
DC length, \( x=L_1 \) is the location of the beginning of the helix, \( H=2\pi r_c \) is the height of two-dimensional solution domain.

The steady detonation flow structures in this type of DC were studied in [1] for the Mach number \( M_0=5 \) (hypersonic inflow) and \( M_0=3 \) [2] (supersonic inflow), and varied geometric parameters of the combustor (\( L, H, \) and helix angle \( \alpha \)). But all these studies were carried out at relatively large helix angles, larger than the Chapman-Jouguet angle, \( \alpha > \alpha_{CJ} \). The main intriguing result detected was the effect of bifurcation of the steady structure of the detonation flow in the combustor over the initial \((r=0)\) vector of the solution at certain combinations of the geometric parameters of the combustor.

The present paper describes further investigations of the flow structure and steady flow regimes with a steady ODW in the annular DC based on the new concept. Two-dimensional numerical simulations are performed at the inflow Mach number \( M_0=3 \) for \( \alpha=11^{\circ} < \alpha_{CJ} \approx 13^{\circ} \) and different combustor geometries.

2. Governing equations and chemical kinetics model

The dynamics of the compressible chemically reactive medium is described by the two-dimensional Euler equations.

The chemical reaction in the DW is described by the two-stage model of the detonation kinetics (stage of the induction period, and main heat release stage). The duration of the induction period is determined by known empirical Arrhenius-type formula for hydrogen mixtures. The main heat release takes place after the induction period. The value of heat release and all thermodynamic parameters of the reacting gas is described by equations of the generalized kinetic model. This two-stage detonation kinetics is described in more detail in our works on the numerical simulation of the multifront (cellular) structure of detonation waves in hydrogen mixtures [3], hydrocarbon mixtures [4,5], and two-fuel mixtures [6].

The system of governing equations was closed by the well-known thermal equation of state for an ideal gas.

At the inlet boundary \( x=0 \) of the solution domain, incoming unperturbed supersonic flow parameters are specified. In the external virtual peripheral grid cells behind the outlet boundary \( x=L \), low counterpressure is specified for the guaranteed critical sonic gas outflow from the DC outlet into the surrounding virtual space. At the lower and upper boundaries, periodic boundary conditions are defined at \( x\leq L_1 \) for \( y=0 \) and \( y=H \) as well as the conditions of an impervious solid wall at \( x>L_1 \).

3. Numerical method

The resultant systems of equations are solved numerically, using the code based on the Godunov-type finite-volume scheme with the fourth-order MUSCL TVD reconstruction and the advanced HLLC algorithm for an approximate solution of the Riemann problem. In the implementation of HLLC algorithm for the case of a chemically reacting mixture, the “energy relaxation method” is used. This method eliminates the problem of numerical solution of the Riemann problem for a medium with a complicated nonlinear equation of state (including that with a variable ratio of specific heats). Integration in time is performed with second-order accuracy by using additive semi-implicit Runge-Kutta methods. All details about these numerical algorithms can be found in [1–6].

A uniform fixed grid is used in both directions. In these calculations, the characteristic numbers of numerical cells are \( N_x=2160 \) and \( N_y=1280 \).

The code is parallelized with MPI library using the domain decomposition technique. The numbers of CPU cores along \( x \) and \( y \) axes are \( Npr_x=18 \) and \( Npr_y=16 \).

4. Results and Discussion

The present numerical simulations are performed for a stoichiometric hydrogen-air mixture. A uniform flow of the mixture with a static pressure \( p_0=0.166 \) atm and static temperature \( T_0=850 \) K is set
at the DC inflow ($x=0$). For these initial parameters of the mixture, the Mach number of a self-sustained DW is $M_{CJ}=2.73$. The helix angle is $\alpha=11^\circ$, and $L, H$ are varied.

With a decrease in the angle $\alpha$, the characteristic dimensions of the DC were expectedly increased, compared with [1,2], from about ten centimeters to about one meter. Figure 1 shows the flow field in a DC at $L = 120 \text{ cm}$, $H = 90 \text{ cm}$, and figure 2 at $L = 120 \text{ cm}$, $H = 100 \text{ cm}$. Here, the flow field of numerical Schlieren visualization, the temperature field in Kelvin degrees, and the density field normalized to the initial density in the inflow are shown. The solid red line in these and subsequent figures shows the boundary of the end of the induction zone and the beginning of the heat-release zone. The final stationary field flow is shown.

Figure 1. Stationary structure of flow field in a detonation chamber for $H=90$ cm, $L=120$cm:
(a) numerical Schlieren; (b) temperature, K.

Figure 2. Stationary structure of flow field in a detonation chamber for $H=100$ cm, $L=120$cm:
(a) numerical Schlieren; (b) normalised density.
Figure 3. The evolution of the flow structure at the SW start. Numerical Schlieren for the time moments: (a) 0.5 ms; (b) 8 ms; (c) 16 ms; (d) 48 ms.

We have a classical $\lambda$-structure of the flow, i.e., the process of acceleration of heat release behind the front of the leading inclined shock wave (SW) arising on the windward side of the helix and the further formation from this unreacted shock the oblique DW at $x \approx 75$ cm. In the upper part of the flow, on the leeward side of the helix, a fan of a classical centered rarefaction wave (RW) is formed. For a given channel heights $H$, the rarefaction wave interacts with the ODW. In Figure 1, we have the Mach reflection of the ODW from the leeward side of the helix. With increasing parameter $H$ (figure 2), the reflection condition disappears and we have an almost straight ODW. In the upper part of the front of this ODW, we observed the development of two-dimensional instabilities and the beginning of the formation of a multi-front (cellular) DW structure.

Our studies have shown that at $H = 80, 90, 100,$ and $120$ cm for $L = 120$ cm, the $\lambda$-configuration does not change its structure and the coordinates of the triple point remain unchanged. At $H = 80$ and 90 cm, there is the Mach reflection of the ODW from the upper boundary, which also does not affect the coordinates of the triple point of the $\lambda$ – configuration.

It has been found that at the above values $L$ and $H$, the final steady-state flow structure is not affected by the conditions for starting the DC operation. The following is meant.
Figure 4. The evolution of the flow structure at the RW start. Numerical Schlieren for the time moments: (a) 0.5 ms; (b) 1 ms; (c) 4 ms; (d) 8 ms.

It has been shown in [1,2] that there are two extremely different start conditions. Starting DC operation \((t = 0)\) from motionless mixture \(u = (0,0)\) inside the solution region leads to the formation of a normal shock wave at the inflow DC boundary \(x = 0\), which then propagates along the channel until its outlet at \(x = L\). This process is shown in figure 4a. Let us denote this as a SW start.

The start \((t = 0)\) from the homogeneous mixture flow \(u = (M_0 c_0,0)\) in computational domain is equivalent to the instantaneous input the DC channel in a homogeneous supersonic flow with the subsequent formation of an inclined SW and RW at the lower and upper boundaries of the spiral, respectively. This process is schematically shown in figure 4. Let's designate it as RW start.

Here \(u\) is the two-dimensional vector of the mass velocity of the mixture, and \(c_0\) is the initial speed of sound in the inflow.

With a decrease in the height of the solution domain \(H\), the evolution of the flow in the DC after the start leads to various stationary structures. Figure 3 shows the dynamics of the final flow structure formation for SW start, while in figure 4 – for RW start, at \(H = 60\) cm.

Figure 3a shows the DW at \(x_0 \approx 95\) cm previously formed from the shock wave at the DC inlet at the start and then going through the motionless mixture to the chamber outlet. Another DW
is shown there at $x_p \approx 5 \text{ cm}$, which is directed against the inflow. Figure 3b shows that the first DW has already left the solution domain, and the second DW is gradually retreating downstream. Further, this wave is gradually carried away by the flow and after a brief oscillatory process (Figure 3c and 3d) occupies its stationary position shown in figure 3d.

Figure 4 shows a completely different process. Here we see the formation of the leading oblique shock wave and the induction zone behind it, the subsequent formation of the oblique DW, its reflection from the leeward surface of the helix, the appearance of the Mach stem and its sequential forward movement, against the flow. A centered rarefaction wave forms in the upper part of the flow. The final stationary position of the DW is shown in Figure 4d.

So, figures 3d and 5a show the final stationary flow structure for SW start, and figures 4d and 5b – for RW start. Figure 5 shows the fields of static temperature (K), and the solid black line shows the boundary $M = 1.0$ of the subsonic region.

In the first case, we have a DW normal to the helix surface, standing stationary at the very beginning of the heat-release zone (see figures 1 and 2) behind an inclined SW. In the middle part of the flow we see an oblique DW, and then its Mach reflection from the upper wall. In the second case, we have a fully formed induction zone on the leading inclined SW and the almost normal to the upper boundary DW, standing stationary in the vicinity of the triple point of the $\lambda$-configuration (see figures 1 and 2).

It remains unclear why exactly these two points, namely the beginning of the heat-release zone at the very surface of the windward side of the helix and the triple point in the $\lambda$-configuration, are a stabilizing factor that determines the final DW position.

So, for $H = 60 \text{ cm}$, we have a bifurcation of the stationary solution depending on the initial state of the flow.

**Conclusions**

The dynamics of flow evolution for two different operation start types of the detonation chamber (DC) has been studied numerically. Different steady solutions have been obtained for a supersonic flow of stoichiometric hydrogen-air mixture at inflow Mach number $M_0 = 3$ for helix angle $\alpha = 11^\circ$. The DC geometry is varied.

It has been shown that within a wide range of the DC length $L$ and the height of the solution domain $H = 2\pi r_c$, a classical $\lambda$-structure with an oblique DW is formed. Within this range of $H$ and $L$
parameters, neither the type of this structure, nor the coordinates of the triple point depend on the initial conditions of the DC operation, as well as on the values of $H$ and $L$.

As $H$ decreased below the critical value, the bifurcation effect of the stationary solution was discovered. Different operation start types of DC led to significantly different types of stationary solution structure. This phenomenon was observed in our previous study [1,2] of this DC with the new design for $M_0=5$ and $M_0=3$ at $\alpha > \alpha_{CJ}$.

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