Research on Adaptability of Multi-model Kalman Filter in Integrated Navigation System

Xiaolong Mi¹, Wangyang Zhao²*, Guangtuo Zhang², Xingling Wang² and Yuanming Wang²

¹Naval Equipment Department, Beijing 100841, China
²Tianjin Navigation Instrument research Institute, Tianjin 300131, China

*Corresponding author e-mail: zhaowangyang@163.com

Abstract: By applying multi-model adaptive control technology in data fusion of integrated navigation system, the multi-mode adaptive Kalman filter in the integrated navigation system is designed to overcome the problem of filtering accuracy reduction of inertial navigation system due to the changes in the external sensor status parameter with single mode Kalman filter model in the integrated navigation system. The simulation analysis is carried out on the system noise interference vector caused by the change of external environment and the change of the noise interference vector of measurement. The simulation results of a single model filter and the simulation results of multi-model adaptive filter are obtained. Based on the simulation results, it can be seen that compared with the single model Kalman filter, the multi-model adaptive filter is more accurate in the state error estimation of the system, and shows strong adaptability to the changes of the external environment.

1. Introduction

With the rapid development of navigation technology and control theory, the integrated navigation system has formed a multi-sensor navigation information system with many navigation sensor modules. The information fusion method is a strong instrument to solve the integrated processing of multi-sensor information. The optimal fusion of multi-source information can improve the adaptability of the system model to the external environment, and hence, the high-precision navigation information can be improved. There is no single model statistical feature in an integrated navigation system that accurately describes sensor errors under different operating conditions. Based on the actual situation for the engineering background, this paper proposes multi-model adaptive filter processing.

The conventional adaptive control has a poor control effect over the controlled object with jumping parameter. This is because of inaccurate parameter identification. This is particularly noticeable in multi-input-multi-output controlled object. The multi-input-multi-output controlled object requires identifying more parameters, and it is difficult to control the control effect once the parameter identification is inaccurate. In the inertial integrated navigation system, it is necessary to introduce a more optimized filtering algorithm to reduce or overcome the bias of state estimation caused by model error. In response to these circumstances, the application of multi model estimation theory in integrated navigation system is studied. The multi-model control is extended to discrete time system through analysis, and a multi-model is established for the navigation system with parameter jump, then a multi-model adaptive controller is constructed. It can ensure the input and output of the final navigation system to be stable and improve the transition process.
The multi-model adaptive control can greatly improve the transient response. However, the presence of many models resulting in a large amount of computation, and together with the “competition” of many models, the quality of control is reduced. Aiming at how to realize dynamic adjustment of model parameters without increasing the computational load of inertial navigation computer, this chapter studies the use of localization techniques to optimize model sets on the theoretical basis of multimode. It can not only ensure the control accuracy, but also reduce the calculation amount and improve the calculation speed. The localization technology, combined with the direct multi-model adaptive control, forms the multi-model adaptive controller of the system.

2. Limitations of Single Model Kalman Filter

The inertial navigation system works in a state of non-damping in this paper, and its main sources of error are zero position error of the accelerometer in the East and north directions and East, North and azimuth gyro drift. Refer reference [1] for its INS error model.

GPS error model

The main consideration in the GPS error model is the clock error \( \delta_t \) and the clock frequency error \( \delta_f \), and its positioning state equation can be expressed as:

\[
\begin{align*}
\delta \lambda_g &= -\beta_g \delta \lambda_g + \sigma_g \\
\delta \phi_g &= -\beta_g \delta \phi_g + \sigma_g \\
\delta V_{sg} &= -\beta_g \delta V_x + \sigma_g \\
\delta V_{yg} &= -\beta_g \delta V_y + \sigma_g \\
\end{align*}
\]

\( \beta_g \) is the reciprocal of the relevant time, \( \delta \lambda_g, \delta \phi_g, \delta V_{sg}, \delta V_{yg} \) are latitude and longitude error and velocity error. \( \sigma_g \) is white noise.

The observation equation of INS/GPS integrated system is:

\[
Z(t) = \begin{bmatrix}
\lambda_t - \lambda_g \\
\phi_t - \phi_g \\
V_{x,t} - V_{x,g} \\
V_{y,t} - V_{y,g}
\end{bmatrix} = \begin{bmatrix}
\delta \lambda + \delta m_{gx} \\
\delta \phi + m_{gy} \\
\delta V_x + m_{sx} \\
\delta V_y + m_{sy}
\end{bmatrix} = H_c(t)X_c(t) + V_c(t)
\]

Doppler error model

The Markov process can be used to describe the speed error \( \delta V_d \) and the deflection angle measurement error \( \Delta \) of the Doppler meter, and the error of ground speed scale coefficient \( \Delta k_d \) is a random constant, we have:

\[
\begin{align*}
\delta V_d &= -\beta_d V_d + w_{dV} \\
\Delta &= -\beta_d V_d + w_{d\Delta} \\
\Delta k_d &= 0
\end{align*}
\]

Observation equation of INS / Doppler integrated navigation system:

\[
Z(t) = \begin{bmatrix}
V_{cx} - V_{dx} \\
V_{cy} - V_{dy}
\end{bmatrix} = \begin{bmatrix}
\delta V_x - V_y + V_x \Delta - V_x \Delta k_d - \delta V_y \sin K + m_{vx} \\
\delta V_y + V_x \Delta - V_y \Delta k_d - \delta V_x \cos K + m_{vy}
\end{bmatrix}
\]

When GPS, Doppler and INS are combined, if single model Kalman filter is used, then the observation equation of its integrated navigation system is:

\[
Z(t) = H_c(t)X_c(t) + V_c(t)
\]
Where $\delta V_x$, $\delta V_y$ are the velocity errors of INS, $\delta \lambda$, $\delta \phi$ are the latitude and longitude errors of INS, $\alpha$, $\beta$, $\gamma$ are error angles, $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$ are gyro drifts, $\delta \lambda_g$ and $\delta \phi_g$ are the latitude and longitude errors of GPS, $\delta V_d$, $\Delta p$ and $k_d$ are Doppler velocity measurement error, drift angle error and ground velocity scale coefficient error.

$$
H_c(t) = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -V_y & 0 & 0 & 0 & 0 & 0 & -\sin k_v & -V_y & -V_x \\
0 & 1 & 0 & 0 & 0 & 0 & V_x & 0 & 0 & 0 & 0 & -\cos k_v & V_x & -V_y
\end{bmatrix}
$$

(5)

$V_c(t) = [m_{gx} \quad m_{gy} \quad m_{vx} \quad m_{vy}]^T$ is white observation noise with zero mean value.

So, the observation equation of INS / GPS / Doppler integrated navigation system is as follows:

$$
Z(t) = \begin{bmatrix}
\lambda_i - \lambda_g \\
\varphi_i - \varphi_g \\
V_x - V_{ds} \\
V_y - V_{dg}
\end{bmatrix} = \begin{bmatrix}
\delta \lambda_i - \delta \lambda_g + \delta m_{gx} \\
\delta \varphi_i - \delta \varphi_g + \delta m_{gy} \\
\delta V_x - V_y \times \gamma - V_y \times \Delta p - V_x \times k_d - \delta V_d \times \sin k_v + m_{vx} \\
\delta V_y - V_y \times \gamma - V_x \times \Delta p - V_y \times k_d - \delta V_d \times \cos k_v + m_{vy}
\end{bmatrix} = H_c(t)X_c(t) + V_c(t)
$$

(6)

$$
H_c(t) = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -V_y & 0 & 0 & 0 & 0 & -\sin k_v & -V_y & -V_x \\
0 & 1 & 0 & 0 & 0 & 0 & V_x & 0 & 0 & 0 & 0 & -\cos k_v & V_x & -V_y
\end{bmatrix}
$$

(7)

$V_c(t) = [m_{gx} \quad m_{gy} \quad m_{vx} \quad m_{vy}]^T$ is the white observation noise with zero mean value.

In the equation, $\delta \lambda_g$ and $\delta \varphi_g$ are the latitude and longitude errors of GPS, $\delta V_d$, $\Delta p$ and $k_d$ are Doppler velocity measurement error, drift angle error and ground velocity scale coefficient error. The single model Kalman filter is used, so in INS / GPS / Doppler integrated navigation system, GPS information is used for position information, and Doppler information is used for velocity information.

3. Multi-model Kalman Filter for Integrated Navigation System

Any actual system has varying degrees of uncertainty. These uncertainties are sometimes expressed within the system or outside the system. When expressed within the system, the designer may not know the structure and parameters of the mathematical model of the controlled object in advance. In terms of the impact of external environment on the system, it can be expressed in terms of many disturbances. These disturbances are often unpredictable, and they may be deterministic or random. Furthermore, there are some measurement noises from different measurement feedback loops into the system, and the statistical characteristics of these random disturbance noises are often unknown. Refer Figure 1 for the schematic of the multi-model estimation method.

![Figure 1 Schematic diagram of multi model estimation method](image-url)
A set of parallel filters is used for different operation modes for stochastic hybrid system. The input of each filter is the control input and measurement information of the system, and the output of each filter is the output residual and state estimation based on a single model. The model probability of the model corresponding to each filter is designed based on the residual information and some hypothesis testing principles. The overall state estimation of the system is the weighted average of the state estimation of each filter. In the multi model estimation method, the design of the model set, the selection of the filter, the estimation fusion and the input reset of the filter are all important links and key factors affecting the performance of the navigation system.

3.1 Multi-model adaptive estimation

The multi-model adaptive estimator consists of a parallel Kalman filter library and a hypothesis test algorithm. Each filter in the library has a specific system model, and its inner Kalman filter model is described by independent vector parameters \( a_i; i = 1, 2 \cdots N \). Each Kalman filter unit independently forms an estimate of the current system state \( \hat{X}_i \) based on its own model and input vector \( u \). Then, this estimate is used to form a prediction value for the measurement vector, and the residual difference \( r_i \) between the value and the actual measurement vector \( Z \) is used as an indication of how similar each filter model is to the actual system model. The smaller residual indicates that more matching the filter model with the actual system model. The hypothesis is made that the test algorithm uses residuals to calculate the conditional probability \( p \) of each Kalman filter model under the conditions of measurement and actual vector parameter \( a \). These conditional probabilities are used to measure the validity of the state estimates of each Kalman filter. Take the probability weighted average of the estimation of each state, so the mixed state estimation of the actual system \( \hat{X}_{MMAE} \) is formed. Refer Figure 2 for the multi-model adaptive estimator.

\[
\begin{align*}
X_{i}(t_k) &= \Phi_{i} X_i(t_{k-1}) + B_i u(t_{k-1}) + \Gamma_{i} w_i(t_{k-1}) \\
Z_i(t_k) &= H_i X_i(t_k) + V_i(t_k)
\end{align*}
\]  

(8)

Where \( X_i \) is the state vector of the \( i \) the Kalman filter model, \( i = 1, 2 \cdots N \); \( \Phi^i \) is the state transition matrix of the \( i \) the Kalman filter model; \( B^i \) is the control input matrix of the \( i \) the Kalman filter model; \( u \) is the system control input vector; \( \Gamma^i \) is the noise input matrix of the \( i \) the Kalman filter model;

![Figure 2 Structure diagram of multi-model adaptive estimator](image-url)
$W_i$ is the discrete dynamic interference white noise input of the $i$ the Kalman filter model. It has zero mean and its autocorrelation is:

$$E\left(w_i(t_k)w_i^T(t_j)\right) = \begin{cases} Q_i, & t_k = t_j \\ 0, & t_k \neq t_j \end{cases}$$

(9)

$z_i$ is the measurement vector of the $i$ the Kalman filter model; $H_i$ is the output matrix of the $i$ the Kalman filter model; $v_i$ is the discrete dynamic measurement white noise of the $i$ the Kalman filter model. $v_i$ and $w_i$ are uncorrelated and have zero mean, and their autocorrelation is:

$$E\left(v_i(t_k)v_i^T(t_j)\right) = \begin{cases} R_i, & t_k = t_j \\ 0, & t_k \neq t_j \end{cases}$$

(10)

The Kalman filter model and the actual system model are both linear models. However, these two models may differ in dimensions. In many cases, the Kalman filter model is a simplification of the actual system model (in general, the state of the Kalman filter is a subset of the state of the actual model).

With the above model, Kalman filter algorithm determines optimal prediction and measurement of time update for Kalman filter state estimation, update optimal estimate equation and covariance matrix of state estimation error. Based on Kalman filter model, the time update equation of Kalman filter state estimation is:

$$\begin{align*}
\dot{\hat{X}}_i(t_k^-) & = \Phi_i \hat{X}_i(t_{k-1}^-) + B_i u(t_{k-1}) \\
Z_i(t_k^-) & = H_i X_i(t_k^-)
\end{align*}$$

(11)

Where $\hat{X}_i$ is the $i$ the Kalman filter state estimation vector; $\dot{Z}_i(t_k^-)$ is the prediction of the measurement vector by the $i$ the Kalman filter before $t_k$ time; $t_k$ is the time before the $k$ -the measurement update; $t_{k-1}$ is the time after the $k-1$ measurement update; The time update equation of state estimation error covariance matrix is:

$$P_i(t_k^-) = \Phi_i P_i(t_{k-1}^-) \Phi_i^T + \Gamma_i Q_i \Gamma_i^T$$

(12)

The state estimation of Kalman filter can realize the measurement update by the following formula:

$$\hat{X}_i(t_k^-) = \hat{X}_i(t_k^-) + K_i(t_k) r_i(t_k)$$

(13)

Here, the gain of Kalman filter is:

$$K_i(t_k) = P_i(t_k^-) H_i^T A_i(t_k^-)^{-1}$$

(14)

The residual variance matrix calculated by Kalman filter is:

$$A_i(t_k) = H_i P_i(t_k^-) H_i^T + R_i$$

(15)

The residual vector of Kalman filter is the difference between the measured value $Z$ and the estimate $H_i \dot{X}_i(t_k^-)$ based on the previous measured value of Kalman filter, that is:

$$r_i(t_k) = Z(t_k) - H_i \dot{X}_i(t_k^-)$$

(16)

The update equation of the variance matrix of state estimation is:

$$P_i(t_k^-) = P_i(t_k^-) - K_i(t_k) H_i P_i(t_k^-)$$

(17)

The covariance matrix of state estimation error can be pre calculated by (8), (9), (10) and (11). The time update of the state vector by Kalman filter is determined by equation (7), and measurement
3.3 The establishment of model set

Due to reasons such as environmental changes, carrier mobility etc., the random characteristics of the inertial components in the integrated inertial navigation system change. From the perspective of system state and measurement equation, the statistical characteristics of random noise and measurement noise of the system change with mobility; that is, the variance \(Q(k)\) and \(R(k)\) of \(W(k)\) and \(V(k)\) is getting bigger. The sensor error depends on the dynamic environment in which the sensor is operating. With the different path of carrier operation, the sensor error model will change, which will have a great impact on the accuracy of the system Kalman filter. The emergence of maneuvering is mainly reflected in the random noise changes of the sensor, that is, the variance \(Q(k)\) and \(R(k)\) of \(W(k)\) and \(V(k)\) is getting bigger. Therefore, we have introduced a set of \(Q(k)\) and \(R(k)\) (that is, \(Q^{j_1}(k), R^{j_2}(k)\)) when describing the above integrated navigation system in a multi-model approach, and a model library composed of \(r\) models \((r \leq j_1 \cdot j_2)\) is established based on this set of model, and the system is described with multiple models as follows:

\[
\begin{align*}
X(k + 1) &= \Phi(k + 1, k)X(k + 1) + \Gamma(k + 1, k)W_i(k) \\
Z(k + 1) &= H(k + 1)X(k + 1) + V_i(k + 1)
\end{align*}
\]

(18)

Where the number of \(W\) is \(j_1\), and the number of \(V\) is \(j_2\).

3 Simulation Study

In the case of change of system noise disturbance vector and measurement noise disturbance vector caused by the change of external environment, simulation results of single model filter and multi-model adaptive filter are presented. For the case that the measurement noise vector \(V\) changes, this paper introduces a set of \(R(k)\) to select three typical values (that is: \(R^{j}(k)\)) to build three models. In model 1, the measured noise vector variance matrix \(R\) is \(R_0\) (\(R_0\) is the measurement noise variance matrix of a single model); in model 2, \(R\) is \(100R_0\); in model 3, \(R\) is \(400R_0\). Based on the selection of the model set above, multi-model adaptive Kalman filter and single model Kalman filter with integrated inertial navigation system are selected in this paper.

(1) Simulation when the system noise vector changes

During the 72-hour simulation of the system, hypothesis is made that changes in the external environment cause the variance of the system noise vector to change as follows: \(50Q_0\) in 0–12 hours; \(Q_0\) in 12–36 hours; \(100Q_0\) in 36–48 hours; \(Q_0\) in 48–60 hours; \(25Q_0\) in 60–66 hours and \(Q_0\) in 60–72 hours. That is:

\[
Q(t_k) = \begin{cases} 
50Q_0; t_k = (0, 12)h & \\
Q_0; t_k = (12, 36)h & \\
100Q_0; t_k = (36, 48)h & \\
Q_0; t_k = (48, 60)h & \\
25Q_0; t_k = (60, 66)h & \\
Q_0; t_k = (66, 72)h & 
\end{cases}
\]

(19)

Single model Kalman filter is used to obtain Figure 3; performing multi-model adaptive filtering. Finally, assumption is made that the accurate model parameters are known in advance, and single model Kalman filter with accurate model is simulated to obtain Figure 4.
Based on the simulation results, it can be seen that single model Kalman filter is fixed, causing poor filtering effect, even lead to the system divergence. Compared with single model Kalman filtering, multi-model adaptive filtering and interactive multi-model filtering are more accurate in state error estimation. The simulation results are optimal only by assuming that the system model parameters are the same as the real model parameters (i.e., the simulation model is accurate). MMAE is closer to the accurate filtering accuracy of simulation model, so MMAE has better filtering effect than that of IMM. According to the probability map of MMAE and IMM, the probability of model 1 is close to 1 in 0-24 hours, 36-48 hours and 60-72 hours; the probability of model 2 is close to 1 in 24-36 hours; and the probability of model 3 is close to 1 in 48-60 hours. These are consistent with the assumption of the system model. It indicates that MMAE and IMM filters present with strong adaptability to the changes in the external environment.

4. Simulation Comparison

Carry out 72-hour simulation of the system. Refer Reference [5] for the single model Kalman filter model. For both single model Kalman filter and multi-model Kalman filter, predict once per second, filter every 15 seconds. The correlation time of inertial navigation system gyro is 5 hours, and that of Doppler system is 15 minutes.

5. Conclusion

In the multi-sensor data fusion processing method, multi-model adaptive control is applied to the integrated navigation system. Multi-model control has the following advantages over single model
algorithm: it can refine the modeling by appropriately expanding the model, which can effectively improve the transient effect of the system. Under the condition of satisfying the prior hypothesis, the estimate is the optimal estimate under the meaning of the mean square error. The algorithm parallel structure is conducive to the realization of parallel operation.

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