An overview of supersymmetry and its different applications is presented. We motivate supersymmetry in particle physics. We then explain how supersymmetry helps us analyze field theories exactly, and what dynamical lessons these solutions teach us. Finally, we describe how supersymmetry is used to derive exact results in string theory. These results have led to a revolution in our understanding of the theory.

1 Introduction

In this talk we will go on a guided tour through the superworld – the world of supersymmetric phenomena. We will explain what supersymmetry is and why many physicists expect to find it in the next generation of experiments. We will also show how powerful it is in leading to exact results in field theory and in string theory and how these results have revolutionized our understanding of string theory.

In section 2 we review the status of the standard model of particle physics, its underlying principles and its success. We also review some of the flaws in the standard model, which lead us to look for extensions of it. In section 3 we introduce supersymmetry as an extension of our ideas about the structure of space and time. In section 4 we explain why many physicist believe that supersymmetry is likely to exist at low energies (around TeV) and to be discovered soon.

In section 5 we turn to a different application of supersymmetry. It turns out that some aspects of supersymmetric field theories can be analyzed exactly. These are extremely complicated systems and the fact that they can be analyzed exactly is by itself surprising. More importantly, the exact solutions exhibit interesting phenomena with new lessons; among them is the crucial role played by electric-magnetic duality in the dynamics. The important applications of supersymmetry to mathematics will not be reviewed here.

In the final section we turn to string theory and show how using the magic of supersymmetry some nonperturbative information can be derived in string theory. This nonperturbative information has taught us many new facts, completely changing our perspective on the theory.
It is logically possible that string theory does not describe Nature and that supersymmetry will not be found in the TeV range. Then, only the applications in section 4 will survive. It is also possible that string theory describes Nature but supersymmetry is not present at low energies. Alternatively, it is also possible that string theory does not describe Nature but supersymmetry is found soon. My personal prejudice is that we should get the whole “package deal” including string theory and low energy supersymmetry. Clearly, the fact that supersymmetry naturally appears in one context makes it more likely that it also appears in another. It will be a shame if Nature does not use a beautiful and powerful idea like supersymmetry. However, as physicists, we should never forget that only experiments are the final judge about what constitutes a correct theory of Nature.

Many people have made crucial contributions to the developments of the subject. In order not to have a list of references longer than the text, we omit all references.

2 Review of the Standard Model

The standard model of particle physics is based on the following ingredients:

- The theory respects special relativity. In other words, space-time is invariant under the 3+1 dimensional Poincaré symmetry.

- The theory is based on the principles of quantum mechanics. It is generally believed that the only quantum theory which respects special relativity and is local (no action at a distance) and causal is local quantum field theory.

- The theory has local gauge symmetry. Unlike ordinary global symmetries (like isospin) gauge symmetry allows arbitrary symmetry transformation at different points in space-time. Therefore the symmetry group is really an infinite product of groups at different space-time points. Such a large symmetry group with arbitrary group element at different space-time points is familiar from general relativity and electrodynamics. The specific gauge group of the standard model is

\[ SU(3) \times SU(2) \times U(1). \]

The gauge symmetry leads to gauge interactions which are mediated by gauge particles. For example, the electro-magnetic interactions are mediated by photons. Similarly, in the standard model we also have gluons and W and Z gauge bosons.
• The matter particles in the standard model are in a representation of $SU(3) \times SU(2) \times U(1)$. These include the quarks, leptons and Higgs boson. Of these only the Higgs boson has not yet been experimentally found ($\nu_\tau$ has been “observed” only indirectly).

• The final ingredient of the standard model is its set of parameters. These include the masses of the various particles, the fine structure constant and a few others. Of these only one parameter, the Higgs mass, has not yet been measured.

The success of the standard model is unprecedented. It is a fully consistent theoretical theory. Furthermore, there are many experimental confirmations of the theory and there is no experiment which is manifestly in contradiction with it.

Despite this spectacular success, it cannot be over stressed that this is not “The End of Science.” In particular, all the ingredients of the standard model are problematic:

• We included special relativity but did not include general relativity or gravity. In Nature space-time is dynamical and can be curved. In the standard model, which does not include gravity, space-time is static and provides a passive arena for the interactions.

• Trying to add gravity to the standard model and in particular to combine general relativity with quantum mechanics leads to contradictions. Therefore, we must go beyond the framework of local quantum field theory.

• Regarding the other ingredients of the standard model, we would like to understand why the standard model is as it is. Why is this the gauge symmetry? Why is this the particle spectrum? Why are these the values of the parameters?

• All the experimental tests of the standard model have been performed at energies smaller than a few hundred GeVs. Therefore, the standard model should be viewed as an effective field theory valid up to that energy scale. At higher energies it can be extended to another theory. What is this theory?

• The last point allows for the possibility that there is no new physics in the TeV range and new degrees of freedom show up only at much higher energy, say $M_{\text{Planck}} \sim 10^{19}$ GeV, where gravitational effects cannot
be ignored and the theory must be modified. If this is the case, we face the hierarchy problem. This is essentially a problem of dimensional analysis. Why is the characteristic energy of the standard model, which is given by the mass of the W boson $M_W \sim 100\text{GeV}$ so much smaller than the next scale, $M_{\text{Planck}}$? It should be stressed that in quantum field theory this problem is not merely an aesthetic problem, but it is also a serious technical problem. Even if such a hierarchy is present in some approximation, radiative corrections tend to destroy it. More explicitly, loop diagrams like the loop of figure 1 restore dimensional analysis and move $M_W \rightarrow M_{\text{Planck}}$.

3 What is Supersymmetry?

Supersymmetry is a new kind of symmetry relating bosons and fermions. According to supersymmetry every fermion is accompanied by a bosonic superpartner. For example, the quarks which are fermions are accompanied by squarks which are bosons. Similarly, the gluons which are bosons are accompanied by gluinos which are fermions.

Another presentation of supersymmetry is based on the notion of superspace. We do not change the structure of space-time but we add structure to it. We start with four coordinates $X = t, x, y, z$ and add four odd directions $\theta_\alpha (\alpha = 1, \cdots , 4)$.

These odd directions are fermions

$$\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha;$$

i.e. they are quantum dimensions and have no classical analog. Therefore, it is difficult to visualize or to understand them intuitively. However, they can be treated formally.

The fact that the odd directions are anticommuting has important consequences. Consider a function of superspace

$$\Phi(X, \theta) = \phi(X) + \theta_\alpha \psi_\alpha(X) + \cdots + \theta^4 F(X).$$
Since $\theta_\alpha \theta_\alpha = 0$, and since there are only four different $\theta$s, the expansion in powers of $\theta$ terminates at the fourth order. Therefore, a function of superspace includes only a finite number of functions of $X$.

Hence, we can replace any function of superspace $\Phi(X, \theta)$ with the component functions $\phi(X), \psi(X) \cdots$. These include bosons $\phi(X), \cdots$ and fermions $\psi(X), \cdots$. This facts relates this presentation of supersymmetry, which is based on superspace, and the one at the beginning of this section, which is based on pairing between bosons and fermions.

A supersymmetric theory looks like an ordinary theory with degrees of freedom and interactions satisfying some symmetry requirements. A supersymmetric theory is a special case of more generic theory rather than being a totally different kind of theory.

The fact that bosons and fermions come in pairs in supersymmetric theories has important consequences. In some loop diagrams, like in figure 2, the bosons and the fermions cancel each other.

This boson-fermion cancellation is at the heart of most of the applications of supersymmetry. If supersymmetry is present in the TeV range, this cancellation solves the gauge hierarchy problem. Also, this cancellation is one of the underlying reasons for being able to analyze supersymmetric theories exactly.

4 Supersymmetry in the TeV Range?

There are several motivations for assuming that supersymmetry is realized in the TeV range. That means that the superpartners of all the particles of the standard model have masses of the order of a few TeV or less.

- The main motivation is a solution of the hierarchy problem. As we mentioned in the previous section in supersymmetric theories some loop diagrams vanish due to cancellations between bosons and fermions. In particular the loop diagram restoring dimensional analysis (figure 1) is cancelled as in figure 2. Therefore, in its simplest form supersymmetry solves the technical aspects of the hierarchy problem. More sophisticated ideas, known as dynamical supersymmetry breaking, also solve the
aesthetic problem.

• The second motivation for low energy supersymmetry comes from the idea of gauge unification. Recent experiments have yielded precise determinations of the strength of the $SU(3) \times SU(2) \times U(1)$ gauge interactions – the analog of the fine structure constant for these interactions. They are usually denoted by $\alpha_3$, $\alpha_2$ and $\alpha_1$ for the three factors in $SU(3) \times SU(2) \times U(1)$. In quantum field theory these values depend on the energy at which they are measured; i.e. these coupling constants run. The rates of change of these coupling constants depend on the particle content of the theory. Using the measured values of the coupling constants and the particle content of the standard model, we can extrapolate to higher energies and determine the coupling constants there. The result is that the three coupling constants do not meet at the same point. However, repeating this extrapolation with the particles of the standard model and their superpartners the three gauge coupling constants meet at a point (see figure 3). How much weight should we assign to this result? Two lines must meet at a point. Therefore, there are only two surprises here. First, the third line meets them at the same point – there is only one non-trivial number here. Second, which is more qualitative, the meeting point, the unification energy, is at a “reasonable value of the energy.” My personal view is that this is far from a proof of low energy supersymmetry but it is certainly encouraging circumstantial evidence.

• The next generation of experiments at Fermilab and CERN will explore the energy range where at least some of the superpartners are expected to be found. Therefore, in a few years we will know whether supersymmetry exists at low energies. If supersymmetry is discovered in the TeV range, the parameters of the superpartners like their masses and coupling constants will also be measured. These numbers will be extremely interesting as they will give us a window into the physics of higher energies.

• Finally we should point out that some of these superpartners might also be relevant for the dark matter of the Universe.

If supersymmetry is indeed discovered in the TeV range, this will amount to the discovery of the new odd dimensions. This will be a major change in our view of space and time, comparable to and perhaps bigger than the discovery of parity violation. It should be stressed that at the moment supersymmetry does not have a solid experimental motivation. If it is discovered, this will be one of the biggest successes of theoretical physics – predicting such a deep notion without any experimental input!
5 Exact Results in Quantum Field Theory

Quantum field theory is notoriously complicated. It is a non-linear system of an infinite number of coupled degrees of freedom. Therefore, (except in two dimensions) there are only a few exact results in quantum field theory. However, the special quantum field theories which are supersymmetric can be analyzed exactly!

The main point is that these systems are very constrained. The dependence of some observables on the parameters of the problem is so constrained that there is only one solution which satisfies all the consistency conditions. More technically, some observables vary holomorphically (complex analytically) with the coupling constants which are complex numbers. Due to Cauchy’s theorem, such analytic functions are determined in terms of very little data: the singularities and the asymptotic behavior. Therefore, if supersymmetry requires an observable to depend holomorphically on the parameters and we know the singularities and the asymptotic behavior, we can determine the exact answer. The boson-fermion cancellation, which we mentioned above in the context of the hierarchy problem, can also be understood as a consequence of a constraint following from holomorphy.

Another property of supersymmetric theories makes them tractable. They have a family of inequivalent vacua. To understand this fact recall first the situation in a magnet. It has a continuum of vacua, labeled by the orientation
of the spins. These vacua are all equivalent; i.e. the physical observables in one of these vacua are exactly the same as in any other. The reason is that these vacua are related by a symmetry and the phenomenon of many vacua leads to spontaneous symmetry breaking.

We now study a situation with inequivalent vacua. Consider the case of degrees of freedom with the potential $V(x, y)$ in figure 4. The vacua of the system correspond to the different points along the valley of the potential, $y = 0$ with arbitrary $x$. However, as we tried to make clear in figure 4, these points are inequivalent – there is no symmetry which relates them. More explicitly, the potential is shallow around the origin but becomes steep for large $x$. Such “accidental degeneracy” is usually lifted by quantum effects. For example, if the system corresponding to the potential in the figure had no fermions, the zero point fluctuations around the different vacua would have been different. They would have led to a potential along the valley pushing the minimum to the origin.

However, in a supersymmetric theory the zero point energy of the fermions exactly cancels that of the bosons and the degeneracy is not lifted. The valleys persist in the full quantum theory. Again, we see the power of the boson-fermion cancellation.

We see that a supersymmetric system typically has a continuous family of vacua. It is referred to as *moduli space of vacua*, and the modes of the system
corresponding to motion along the valleys are called moduli.

The analysis of the systems is usually simplified by the presence of these manifolds of vacua. Asymptotically, far along these flat directions of the potential the analysis of the system is simple and various approximate techniques are applicable. Then by using the asymptotic behavior along several such asymptotics as well as the constraints from holomorphy the solution is unique.

This is a rather unusual situation in physics. We perform approximate calculations which are valid only in some regime and this gives us the exact answer. This is a theorist’s heaven – exact results with approximate methods!

Once we know how to solve these system, we can analyze many examples. The main lesson which was learned is the fundamental role played by electric-magnetic duality. It turns out to be the underlying principle controlling the dynamics of these systems.

When faced with a complicated system with many coupled degrees of freedom it is common in physics to look for weakly coupled variables which capture most of the phenomena. For example, in condensed matter physics we formulate the problem at short distance in terms of interacting electrons and nuclei. The solution is the macroscopic behavior of the matter and its possible phases. It is described by weakly coupled effective degrees of freedom. Usually they are related in a complicated, and in most cases unknown, way to the microscopic variables. Another example is hydrodynamics, where the microscopic degrees of freedom are molecules and the long distance variables are properties of a fluid which are described by partial differential equations.

In one class of supersymmetric field theories the situation is similar to that. The long distance behavior is described by a set of weakly coupled degrees of freedom. As the characteristic length scale becomes longer, the interactions between these effective degrees of freedom become weaker, and the description in terms of them becomes more accurate.

In another class of examples there are no variables in terms of which the long distance theory is simple. The theory remains interacting (it is in a non-trivial fixed point of the renormalization point). In these situations there are two (in some cases more than two) descriptions of the physics leading to identical results for the long distance interacting behavior.

In both classes of examples an explicit relation between the two sets of variables is not known. However, there are several reasons to consider these pairs of descriptions as being electric-magnetic duals of one another. The original variables at short distance are referred to as the electric degrees of freedom. The other set of variables are magnetic.

These two dual descriptions of the same theory give us a way to address strong coupling problems. When the electric variables are strongly coupled,
they fluctuate rapidly and their dynamics is complicated. However, the magnetic degrees of freedom are weakly coupled. They do not fluctuate rapidly and their dynamics is simple. In the first class of examples the magnetic degrees of freedom are the macroscopic ones. They are massless bound states of the elementary particles. In the second class of examples there are two valid descriptions of the long distance theory: the electric and the magnetic ones. As one of them becomes more strongly coupled, the other becomes more weakly coupled.

Finally, using this electric-magnetic duality we can find a simple description of complicated phenomena associated with the phase diagram of the theories. For example, as the electric degrees of freedom become strongly coupled, they can lead to confinement. In the magnetic variables, this is simply the Higgs phenomenon (superconductivity) which is easily understood in weak coupling.

We summarize the electric-magnetic relations in the following table:

|                | electric | magnetic |
|----------------|----------|----------|
| coupling       | strong   | weak     |
| fluctuations   | large    | small    |
| phase          | confinement | Higgs    |

Apart from the “practical” application to solving quantum field theories, the fact that a theory can be described either in terms of electric or magnetic variables has deep consequences:

- In theories of the first class of examples it is natural to describe the magnetic degrees of freedom as composites of the elementary electric ones. The magnetic particles typically include massless gauge particles reflecting a new magnetic gauge symmetry. These massless composite gauge particles are associated with a gauge symmetry which is not present in the fundamental electric theory. This is rather surprising because most people believed that such a phenomenon cannot take place in four dimensions. The lesson from these examples is that gauge invariance cannot be fundamental.

- For theories of the second kind the notion of elementary particle breaks down. There is no invariant meaning to which degrees of freedom are elementary and which are composite. The magnetic degrees of freedom are composites of electric ones and vice versa. Again, such behavior is very surprising in four dimensions.
6 The String Revolution

We do not know how to formulate string theory nor do we know its underlying principles. Surprisingly, this fact does not stop us from making progress. In particular, as in field theory, the magic of supersymmetry allows us to obtain some exact results and to control the theory in extreme situations. These results have completely changed our perspective on the theory. In the remainder of this talk we will briefly mention some of the main lessons:

- Just as in supersymmetric field theories, string theory has many inequivalent vacua – a moduli space of vacua. It turns out that the supersymmetric compactifications of all five string theories are connected. A “map” of these vacua is given in figure 5. At different boundaries of the map we find the five known string theories as well as the mysterious eleven-dimensional theory whose low energy limit is eleven-dimensional supergravity. Without the magic of supersymmetry only the vicinity of each boundary could be explored in perturbation theory and there was no way to extrapolate from one boundary to another. Now, with these extrapolations, it is clear that all the vacua are connected. We conclude that instead of five string theories there is only one theory with many solutions. The theory is unique!

- As we extrapolate from one boundary to another a phenomenon, which we have already discussed in the previous section, takes place. The “elementary” degrees of freedom at one boundary appear composite else-
There is no universal object which appears elementary everywhere and should therefore be viewed as preferred. Furthermore, in the boundary where the theory becomes eleven-dimensional there are no strings at all. We conclude that the theory is not a theory of strings. Therefore it is appropriate to change its name and it is often being referred to as M-theory (M stands for magic, mysterious, membrane, mother, ...).

• At various boundaries of the map in figure 5 there is a preferred notion of space-time. However, as we extrapolate from one boundary to another, the underlying space-time becomes ambiguous. The theory can be described either as one kind of strings propagating on one background or as another kind of strings propagating on another background. This ambiguity is known as string duality. It has led to a proposal to formulate the full theory in terms of the dynamics of large matrices – the coordinates of space-time are non-commuting matrices in this approach.

• The map in figure 5 includes the value of a parameter which can loosely be called $\hbar$. As we approach various boundaries we seem to take it to zero or infinity. However, a more careful examination of the theory shows that even as we set $\hbar \to 0$ the theory still includes sectors, which remain quantum mechanical. Furthermore, in the eleven-dimensional vacuum there is no parameter like $\hbar$. We see that there is no classical theory whose quantization leads to string theory. Instead, the theory is inherently quantum mechanical!

• Certain black-hole solutions of string theory where examined. Using the magic of supersymmetry an extrapolation from weak coupling to strong coupling can be performed and one can exactly enumerate the black-hole states. It turns out to coincide with the number predicted by the Bekenstein-Hawking entropy formula. Therefore, the black-hole entropy reflects the existence of many microscopic states. This is a crucial step toward resolving the black-hole information paradox. It points in the direction that the full theory is unitary and no information is being lost in Hawking radiation.

Unfortunately, these exciting developments have not yet led to direct comparison with experiment. The situations where exact answers are possible are very idealized and have a lot of supersymmetry – even more than the amount of supersymmetry we expect to find in the TeV range. Even worse, before these developments one could have hoped that the ten or eleven-dimensional vacua are somehow inconsistent. Now, they appear perfectly consistent and
are unified into a beautiful picture. Therefore, the question “why don’t we live in ten or eleven dimensions?” becomes sharper.

However, these developments are an enormous step toward uncovering the underlying dynamical principles of string theory.

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