Simulation of leakage through mechanical sealing device

V P Tikhomorov, O A Gorlenko, M A Izmerov
Bryansk State Technical University, 50 years of October Boulevard, 7, Bryansk, 241035, Russia.
E-mail: dm-bgtu@yandex.ru

Abstract. The procedure of mathematical modeling of leakage through the mechanical seal taking into account waviness and roughness is considered. The percolation process is represented as the sum of leakages through a gap between wavy surfaces and percolation through gaps formed by fractal roughness, i.e. the total leakage is determined by the slot model and filtration leakage. Dependences of leaks on the contact pressure of corrugated and rough surfaces of the mechanical seal elements are presented.

1. Introduction
The task of sealing the joints of “metal-metal” type while observing the exact relative positioning of the mating parts is actual and is solved in many ways. When using sealing compounds, there are difficulties in disassembling the joints.

Particularly precise metal-to-metal joints are compacted by thin planar machining (grinding or scraping to Ra = 0.05 ... 0.5 μm). In addition, increased stiffness of the flanges and frequent arrangement of the tightening bolts are required. Other factors that significantly affect the tightness of the flat joint (tortuosity of percolation channels, porosity, etc.) are taken into account on the basis of experimental data. They enter unexplicitly into the Darcy equation for the filtration flow through a porous medium in the form of a permeability coefficient.

The calculation of leaks by none of the formulas does not give reliable results due to the presence in them of either coefficients determined experimentally for particular cases, or an approximation of the estimate of the height of the gap.

2. Mathematical model of leakage
A multilevel model of intercontact space, reflecting the undulation (prefractal) and roughness (fractal porous medium) is considered. The authors believe that in the presence of undulation and roughness, the leakage of liquid through the joint is determined by the expression:

$$Q = \alpha Q_r + (1 - \alpha) Q^*,$$

where, $\alpha = A_u / A_g$, $A_u$ - contour area determined by undulation; $A_g$ - nominal (geometric) surface area of the sealing ring; $Q^*$ - leakage through a gap formed by contact of corrugated surfaces, $Q_r$ – leakage through a porous layer, formed by a roughness on the contact contour spots.

3. Model of leakage through the gap
For an axisymmetric ring connection, the number of waves $n_u$ is determined by:
where \( r_m \) – average contact radius: \( r_m = \frac{r_1 + r_2}{2} \), \( r_1 \) and \( r_2 \) – internal and external radii of the axisymmetric seal; \( WSm \) – wavelength (the above-mentioned designation is in accordance with ISO4287: 1997 and ISO3274: 1996).

As a model of a wavy surface, let us take a set of radially arranged waves, which have a cylindrical shape in the upper part. This takes into account the random spread of the wave amplitudes, which is governed by the log-normal distribution law. The initial gap of the unloaded connection is assumed to be equal to \( H_0 = \text{distance between the average lines of the ripple profile} \).

Here \( W_p \) – the height of smoothing of waves 1 and 2 of a wavy surface (Fig. 1).

\[
Q^* = \frac{4 \pi h^3 \Delta p}{3 \eta \ln \left( r_2 / r_1 \right)}.
\]

Here \( \eta \) – dynamic viscosity; \( p \) – pressure; \( h \) – gap height; \( h = 0.5 \) \( (H_0 - \delta_{\text{max}}) \). \( \Delta p \) – gradient of pressure.

The magnitude of the approach of \( \delta_{\text{max}} \) surfaces under the action of compressive force \( F \) is determined using the simulation model, using the Hertz solution.

**The simulation model of contact interaction taking into account the waviness.** The mechanical seal surface is represented in the form of cylindrical waves. Let us assume that leakage of the condensed medium occurs between contour areas, the number of which is equal to \( n_w \). To estimate the elastic deformation of waves providing a given degree of leak tightness, let us represent the end surface in the form of a set of cylinders of the same \( r_w \) radius, located at different levels in height (Fig. 2).

However, a small number of waves do not allow obtaining statistically significant results of the contact interaction. Simulation modeling in this case is a suitable tool for obtaining acceptable results. **Procedure for conducting statistical tests.**

1. The authors set load \( F \), which falls on the \( n_w \) waves, and the radius of upper part curvature \( r_w \).
Having adopted the logarithmically normal law of distribution of the vertices of waves, the authors simulate a wave consisting of random variables (RV). Let us define the initial approach of $\delta_{\text{max}}$ waves [2], assuming that there is only one wave, according to the formula:

$$\delta = \frac{F}{L} \left( \lambda_1 + \lambda_2 \right) \ln \left( \frac{L}{4(\lambda_1 + \lambda_2)F \cdot r_w} \right) + 2.38629.$$ 

Here $L$ – contact line length ($L = r_2 - r_1$); $\lambda_i = \left(1 - \mu_i^2\right)/(\pi E_i)$, where $\mu$ – Poisson's ratio, $E$ – modulus of elasticity; $r_w$ – wave radius, $F$ – load incident on the wave, which is deformed before approaching $\delta$.

At the previously calculated value of approach $\delta_{\text{max}}$, the deformation of the $i$-th wave, according to Fig. 2 (taking into account the height of each projection $h_{wi}$), turns out to be equal to:

$$\delta_i = h_{wi} - (W_p - \delta_{\text{max}}).$$

2. Let us find the reaction of the $i$-th wave $F_i$ corresponding to deformation $\delta_i$. Let us compare the sum of the $\Sigma F_i$ reactions per $n_w$ wave with external load $F$. If $\Sigma F_i$ differs from $F$ by more than the value of given accuracy $[\varepsilon]$ (for example, $\varepsilon = 0.01$), then by half-dividing the method or by decreasing the approach value, or increasing it, the authors will pick up such value that corresponds to the given external load.

3. Let us perform $N$ runs of wave modeling (first, let us take $N = 20$) and determine in each case the approach $\delta_i$, $i = 1, \ldots, N$.

4. Based on the results of $N$ runs, let us calculate the average arithmetic deviation $\overline{\delta}(N)$ and half the confidence interval $d(N, \alpha)$, where $\alpha$ is the significance level. With the help of the Student's criterion, it is possible to determine whether the initially chosen number of $N$ experiments is sufficient, or not. If the number of experiments is not enough to obtain a reliable result, then the number of runs should be increased.

5. Changing the load on the waves, the authors find, in accordance with the proposed procedure, ratios $F_2 \sim \overline{\delta}(N), F_3 \sim \overline{\delta}(N), \ldots, F_n \sim \overline{\delta}(N)$.

Filtration leakage for a steel-steel pair occurs when $p_a = 80 ... 100$ MPa; and before that, the flow of liquid through the gap is observed. Calculations for the proposed algorithm are presented in the form of the graphs in Fig. 3. Initial data: $W_p = 10 \mu$m; $\Delta p = 0.9$ MPa; $r_2 - r_1 = 5$ mm; dynamic viscosity $\eta = 0.001 \text{ Pa} \cdot \text{s}$ (water). To assess the effect of macrogeometry on sealing tightness, each graph is constructed in three versions for different wave numbers.

![Figure 3](image-url)  
**Figure 3.** Tightness of the connection: a - dependence of leakage $Q, 1/h$, on load $F$, N; b - dependence of leakage $Q, 1/h$, on the wave curvature radius $r_w$, $\mu$m

The suggested approach allows one to develop a technique of calculation of a joint tightening to
maintain a required degree of tightness and quality of real mechanical seals.

4. Filtration leakage model
This leakage model uses a representation of the intercontact gap on the contour areas in the form of a fractal porous medium. Let us assume that the contour area is equal to the nominal (typical for miniature sealing devices).

The microroughness of the surface is described quite accurately by random fractal functions. As such function, let us choose the cosine fractal Weierstrass-Mandelbrot function [3, 4]:

\[
h_n(x) = \sum_{n=-\infty}^{\infty} \frac{1 - \cos(\gamma^n x)}{\gamma^{2-D_f} n^n}.
\]

Here \( h_n(x) \) – the ordinate of the profile height at point \( x \); \( n \) – frequency index belonging to the interval \([-\infty; \infty]\); the value of \( \gamma \) is chosen in the range of values \( \gamma = 0.8 \ldots 1.8 \) (an acceptable value, according to A. Majumdar [3], is \( \gamma = 1.5 \)). The \( D_f \) parameter determines the fractal dimension of the curve.

Let us write the well-known and widely used integral distribution function of the number of pores of certain sizes as (Korczak's law [5]):

\[
N(L > r) = \left( \frac{r_{\max}}{r} \right)^{D_f}.
\]

Here \( r_{\max} \) – maximum pore radius; \( D_f \) – fractal dimension of pores \((0 < D_f < 2)\).

Differentiating the Korczak equation with respect to the pore radius, one obtains the number of pores whose radius is in the range from \( r \) to \( r+dr \):

\[
-dN = D_f r_{\max}^{D_f} (r_{\max}^{D_f})' dr.
\]

It follows from the equation that the number decreases with increasing a pore radius (-dN > 0).

5. Evaluation of the fractal tortuosity of the leakage channels
The authors believe that the contact of the fractal surfaces of the sealing device can be represented in the form of a fractal porous medium. The length of a chain of connected pores in the plane of the connector through which a leak occurs can be determined (according to the Mandelbrot [4]) by the following relationship:

\[
L = \lambda \cdot \delta^{1-D_f},
\]

where \( L \) – length of the curve (leakage channel), \( \lambda \) – experimentally determined parameter, \( \delta \) – the measurement scale (a segment or standard, by which the length of the curve is measured); \( D_f \) – fractal dimension of the curve (chain of pores).

The length of leakage channel \( L \) is determined by expression \( L = N(\delta) \cdot \delta \). With decreasing \( \delta \) size, the length increases nonlinearly. By logarithmizing the equation relating the length to the fractal dimension, one obtains:

\[
\lambda = \frac{L_1}{\delta_1^{1-D_f}}.
\]

Dependence \( \ln L = f(\ln \delta) \) is presented in the form of a graph. Then the slope of the straight line in sector \( \delta_1 \ldots \delta_2 \) allows us to find the fractal dimension:

\[
D_f = 1 - tg \alpha = 1 - \frac{lg L_1 - lg L_2}{lg \delta_1 - lg \delta_2}, \quad 1 < D_f < 2.
\]

Then the length of curve \( L_f \) (of the main channel of leakage) will be equal to:

\[
L_f = \lambda \left( \frac{A_{1/2}}{L_0} \right)^{1-D_f}.
\]
Let us imagine the main leakage channel as shown in Fig. 4, assuming that the side walls separated by bold lines are isolated, liquid injection occurs from the bottom up. In this case, the coefficient of tortuosity is defined as:

$$K_t = \frac{L_T}{L_0} = L_0^{D_t-1} (2r)^{1-D_t}.$$  

For $D_t = 1$, there is $K_t = 1$. Taking into account the self-similarity ($K_t = K_t^0$), let us write:

$$L_0^{D_t-1} (2r)^{1-D_t} = \lambda \cdot \frac{\delta^{1-D_t}}{A_c^{\frac{1}{2}}} \cdot (2r)^{1-D_t} = \lambda \cdot \frac{\delta^{1-D_t}}{A_c^{\frac{1}{2}}} \cdot L_0^{D_t-1} r^{1-D_t} = \lambda \cdot \frac{\delta^{1-D_t}}{A_c^{\frac{1}{2}}} r^{1-D_t}.$$  

From where the average diameter of the leakage channel and the average hydraulic radius of the channel:

$$d_K = 2r = \frac{\lambda^{1-D_t}}{A_c^{\frac{1}{2}}} \cdot \delta; \quad r = \frac{\lambda^{1-D_t}}{2 \cdot A_c^{\frac{1}{2}}} \cdot \frac{1}{(2r)^{1-D_t}}.$$  

The flow rate of a liquid through a rectilinear cylindrical channel according to the Poiseuille [1] is determined by the equation:

$$q^*(r) = -\frac{\pi r^4}{8\eta} \frac{dp}{dL_T}.$$  

Then the expression for leakage through a porous medium has the form:

$$Q_F = -\int_{r_{min}}^{r_{max}} q^*(r) dN(r),$$  

where $N(r)$ – number of pores, whose radius is in the range from $r$ to $r+dr$.

Having adopted the pressure gradient (pressure drop) $dP/dL_0 = \Delta P / L_0$ and substituting the above-mentioned relations in the previous expression, let us find:

$$Q_F = -\frac{\pi}{8\eta(4-D_f)} \left( D_f - 1 \right) \lambda \cdot A_c^{\frac{3}{2}(1-D_t)} \delta^{1-D_t} \cdot L_0^{D_f-1} \left( r_{max}^4 - r_{min}^4 \right) = -A \cdot r_{max}^4 \left( 1 - \left( \frac{r_{min}}{r_{max}} \right)^{4-D_f} \right).$$  

In the calculations, the authors take $r_{min}^2 = 10^{-2}$. The cross-sectional area of the porous medium is determined through the porosity, and finally one can write:

$$Q_F = -\frac{\pi}{8\eta(4-D_f)} \left( D_f - 1 \right) \lambda \cdot A_c^{\frac{3}{2}(1-D_t)} \delta^{1-D_t} \cdot L_0^{D_f-1}.$$
Fig. 5 shows the dependence of leakage $Q$, mm$^3$/s, transformer oil (dashed line) and oil (solid line), on: a) maximum pore radius $r_{\text{max}}$, μm; b) the fractal dimension of porosity $D_f$; c) the fractal dimension of tortuosity $D_T$.

![Figure 5. Dependence of leakage $Q$, mm$^3$/s, transformer oil (dashed line) and oil (solid line)](image)

Initial data: axisymmetric compaction with parameters $L_0 = 0.8$ mm, $A_c = 10$ mm$^2$, $\delta = 2$ μm, $D_f = 1.5$, $D_T = 1.5$ and sealing medium transformer oil (dynamic viscosity $\eta = 0.0316$ Pa·s) and oil ($\eta = 0.2$ Pa·s); pressure drop $\Delta p = 10^6$ Pa. The data obtained are in good agreement with the data from literature sources [6].

6. Conclusion

Thus, it is possible to simulate the contact of corrugated and rough surfaces with the determination of the tightness of the connection of metal surfaces depending on the load and approach, taking into account the adopted law of distribution of random variables. The proposed approach makes it possible to develop a procedure for calculating the tightening of the joint to ensure the required degree of tightness and the quality of real end sealing devices.

References

[1] Mayer E 1978 Mechanical seals (Mechanical engineering) 288 p.
[2] Norden B N 1973 On the compression of a cylinder in contact with a plane surfaces (Washington: Institute for Basic Standards National Bureau of Standards)
[3] Majumdar A Tien C L 1991 Fractal characterization and simulation of rough surfaces 136 313 - 327
[4] Mandelbrot B 2002 Fractal Geometry of Nature (Institute of Computer Science)
[5] Bogomolov D Yu Poroshin V.V. Radygin V. Yu. Syromyatnikova A.A. Sheipak A.A. 2010 Mathematical modeling of fluid flow in the slot channels taking into account the real microtopography of the surface of their walls (MGIU)
[6] Kou J Y Liu F Wu J Fan H Lu Y Xu 2009 Fractal analysis of effective thermal conductivity for three-phase (unsaturated) porous media Journal of applied physics 106
[7] Yu B M Le J H 2001 Fractals 365 p
[8] Grimmet G 1989 Percolation (New York NY Springer)
[9] DeHon A 2005 Nanowire-based programmable architectures ACM Journal on Emerging Technologies in Computing Systems 1(2) 109 – 162
[10] Huang J Tahoori M Lombardi F 2004 On the defect tolerance of nano-scale two-dimensional crossbars Proc. IEEE Int'l Symp. on Defect and Fault Tolerance of VLSI systems p 96 - 104