Azimuthal asymmetries in unpolarized Drell-Yan events on nuclear targets

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We show that for Drell-Yan events by unpolarized hadronic projectiles and nuclear targets, azimuthal asymmetries can arise from the nuclear distortion of the hadronic projectile wave function, typically a spin-orbit effect occurring on the nuclear surface. The asymmetry depends on quantities that enter also the spin asymmetry in the corresponding Drell-Yan event on polarized free nucleonic targets. Hence, this study can be of help in exploring the spin structure of the nucleon, in particular the transverse spin distribution of partons inside the proton. All arguments can be extended also to antinucleon projectiles and, consequently, apply to possible future measurements involving nuclear targets at the foreseen HESR ring at GSI.

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I. INTRODUCTION

High-energy collisions of hadrons can be a very interesting source of information for testing QCD in the nonperturbative regime. In fact, in the past years several experiments have been performed that still await for a correct interpretation of the resulting data. Among others, hadronic collisions on transversely polarized proton targets [1, 2, 3, 4], where an azimuthally asymmetric distribution of final-state products (with respect to the normal of the production plane) is observed when flipping the transverse spin of the target or of the final products, the so-called transverse spin asymmetry. Perturbative QCD cannot accommodate for such asymmetries, sometimes as large as 40% also at high energy [1]. This problem has triggered a growing interest in this field of hadronic spin physics; new data and rapid developments are foreseen in the near future, as it emerged in a recently devoted workshop [5] (for a review covering also processes with lepton beams, see Refs. [6, 7]).

Here, we will concentrate on a series of measurements involving nuclear targets, namely high-energy collisions of pions and antiprotons on various unpolarized nuclei [8, 9, 10, 11] where the cross section for Drell-Yan events shows an unexpected largely asymmetric azimuthal distribution of the final lepton pair with respect to the production plane. It is usually convenient to study the problem in the so-called Collins-Soper frame [12] of Fig. 1. A lepton plane is defined by the direction of emission of the lepton pair and by the \textbf{z} axis which is approximately formed by the average of the directions of the colliding hadron momenta \textbf{P}_1 and \textbf{P}_2 (for a more rigorous definition, see Ref. [13]). Azimuthal angles are measured in a plane perpendicular to \textbf{z} and containing the axis \textbf{h} = \textbf{q}_\perp/|\textbf{q}_\perp|, where \textbf{q}_\perp is the transverse momentum of the final lepton pair detected in the solid angle \((\theta, \phi)\).

![FIG. 1: The Collins-Soper frame.](image_url)
At leading order in $\alpha_s$, the cross section can be parametrized as
\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right),
\]
with $\lambda \sim 1$ and $\mu \ll \nu \sim 30 \%$. Both Leading Order (LO) and Next-to-Leading Order (NLO) perturbative QCD calculations give $\lambda \sim 1$ and $\mu \sim \nu \sim 0$ [14], because they are based on the assumption of massless quarks and collinear annihilation. In fact, this result holds only if the annihilation direction is well defined and coincides with the $\hat{z}$ axis of Fig. 1 because the cross section would depend only on $\theta$. In general, the assumption of massless quarks leads to the so-called Lam-Tung sum rule $1 - \lambda = 2\nu$ [13], which is badly violated by data. More complicated mechanisms, like higher twists or factorization breaking terms at NLO, are not able to explain the size of $\nu$ in a consistent picture [16, 17, 18]. A promising interpretation relies on the observation that at leading twist the cross section for unpolarized Drell-Yan, when kept differential also in $d\alpha_s$, contains a term proportional to $\cos 2\phi (h_1^+ \otimes h_1^\perp)$, i.e. a specific convolution of the distribution function $h_1^\perp$ that describes the transverse momentum distribution of transversely polarized partons inside unpolarized hadrons [13], while $h_1^\perp$ describes the annihilating antiparton partner.

However, it is important to stress that the above results are obtained using nuclear target: before getting to the collision with a bound nucleon that generates the Drell-Yan event, the hadronic beam crosses the nuclear surface and travels inside the nuclear medium for some length depending on its energy. The aim of this paper is to try to understand whether effects due to the beam-nucleus interaction may assume a relevant role in contributing to the observed azimuthal $\cos 2\phi$ asymmetry. In other words, if an intrinsic transverse momentum distribution can affect the elementary annihilation of partons leading to an asymmetric azimuthal distribution of final lepton pairs through the function $h_1^\perp$, then a hadron deflected by nontrivial Initial-State Interactions (ISI) with the target nucleus can transfer its transverse momentum to the final lepton pair and contribute to the same azimuthal asymmetry. More, we will see that such ISI can originate azimuthal asymmetries also when no asymmetry is produced at the level of a free proton target.

The highly energetic pions and antiprotons used in measurements reported in Refs. [8, 9, 10, 11] demand a description of their internal structure in terms of elementary degrees of freedom (quarks and gluons) in order to correctly describe their propagation and to study effects that can modify high-energy azimuthal asymmetries at RHIC or LHC (see, for example, Refs. [13, 20]). Here, we will consider Drell-Yan processes at lower energies of interest for the foreseen HESR ring at GSI, where collisions of (polarized) protons and antiprotons with energy in the range 10 ÷ 30 GeV will be studied [21, 22, 23, 24, 25, 26, 27]. Hence, we will adopt a description based on "traditional" degrees of freedom as nucleons and pions; this justification is followed by the following Sec. III.

The paper is organized as follows. In Sec. II the nuclear damping of the projectile wave function, based on the eikonal approximation, is reviewed and the induced modifications to the scattering amplitude are discussed. In Sec. III the spin-orbit effect on the nuclear surface is discussed and the formulae are implemented with an additional damping that depends on the polarization component of the incoming hadron beam. In Sec. IV it is shown how all this can produce azimuthal asymmetries in Drell-Yan events from unpolarized hadron-nucleus collisions even when no asymmetry is generated using a free proton target. Finally, in Sec. V the discussion is presented about the issues of the size of this nuclear effect and of the possibility of disentangling it from genuine asymmetries related to the Drell-Yan hard event. Results are summarized in Sec. VI.

II. SPIN-INDEPENDENT NUCLEAR DAMPING

As already stressed in Sec. II we will consider Drell-Yan processes involving nuclear targets at energies of interest for the foreseen HESR ring at GSI. At present, the discussion is still open about the two options of an antiproton beam of energy 15 GeV hitting a fixed proton target or colliding on a proton beam of energy around 3 GeV, such as the center-of-mass energy ranges from 5 to approximately 14 GeV [21, 22, 23, 24, 25, 26, 27]. At this scale, the use of traditional "nuclear" degrees of freedom (nucleons and pions) is still appropriate for describing the propagation of a hadron inside the nucleus, also because spin-orbit effects are still significant. Moreover, a distance in space (the so-called formation length) must separate the Drell-Yan hard event from the ISI rescatterings in such a way that the two processes do not influence each other; the formation length increases with energy but it is already significant at 15 GeV beams. In fact, the same problem occurs in the semi-inclusive quasi-elastic electroproduction of protons on nuclei about the Final-State Interactions (FSI) of the proton travelling inside the residual nuclear medium after the hard event of virtual photon absorption. The NE18 experiment about the $^{12}$C($e, e'p$)X reaction has shown that, at 10 GeV energies, the ejectiles interact with the nuclear medium exactly as "normal" protons, before emerging and being actually detected as protons.

At the same time, a projectile with energy 15 GeV impinging on a target nucleus is energetic enough such that the collision can be safely considered in the Born Approximation. The main effect of the nuclear medium is, therefore,
distort the projectile wave function. The most popular and widely adopted approach to such a situation is the Glauber method \[29\], which has a long well established tradition of successfull results in the field of high-energy proton-nucleus elastic scattering (for a review, see also Ref. \[31\]). Again, a parallel with quasi-elastic \(A(e,e'p)X\) reactions can be established, since above the inelastic production threshold, i.e. for proton momenta \(p \gtrsim 1\,\text{GeV/c}\), it has been shown \[31, 32, 33\] that this method gives the same FSI effects as the usual approach based on the Distorted Wave Impulse Approximation (DWIA), where the projectile wave function is expanded in partial waves and a Schrödinger equation is solved for each partial wave \(L\) with a complex optical potential up to a \(L_{\text{max}}\) depending on the projectile energy and satisfying a stringent convergency criterion (for a review, see Ref. \[34\]). Actually, it was directly checked that the equivalence of these methods holds when the Glauber scattering wave is replaced by an eikonal wave function \[35\]. Because of the similarity between the FSI problem in \(A(e,e'p)\) reactions and the ISI problem in \(p-A\) collisions, we will adopt here the same eikonal approximation for the projectile wave function.

Since for a fastly moving object the nuclear density can be considered roughly constant but for a small portion corresponding to the nuclear surface, the eikonal wave function of the projectile with momentum \(p\) can be represented by the damped plane wave

\[
\psi(\mathbf{r}) \approx e^{-p_r \mathbf{r}} e^{ip \mathbf{P} \cdot \mathbf{r}} e^{-p_z \mathbf{r}},
\]

which corresponds to an approximate solution of the Schrödinger equation inside homogeneous nuclear matter with \(|p_f| = p_f\) proportional to the ratio between the absorptive (imaginary) potential and \(|p| = p\); it is equivalent to neglect the solution of the Schrödinger equation that propagates backward in space \[29\]. For a non-homogeneous nucleus, the same approximation leads to the Glauber wave function. The factor \(\exp(-p_r \mathbf{R})\) is due to the incoming proton wave being properly normalized to 1, where \(\mathbf{R}\) is a constant vector whose modulus gives the nuclear radius. A reasonable value for \(p_f\) can be estimated by reproducing, among other things, the NE18 data \[28\] for the \(^{12}\text{C}(e,e'p)\) reaction where the flux of the outgoing proton with \(p = 1.4\,\text{GeV/c}\) is overall reduced by 40%, including the \(Q^2\) behaviour of the related transparency coefficient \[30\]. It turns out that \(p_f \ll p\) for the above NE18 kinematics typically \(p_f \sim 50\,\text{MeV/c}\). Nevertheless, this damping is able to generate “geometric” FSI responsible for the asymmetric proton knockout with respect to the scattering plane in exclusive \(^{12}\text{C}(e,e'p)\) reactions with a polarized lepton beam (the so-called fifth structure function problem) \[36\].

The eikonal approximation of Eq. \(2\) is, by construction, a high-energy approximation and its reliability increases with the projectile energy, ideally corresponding to the limit of an infinite number of partial waves for the solution in the DWIA. In these conditions, as a first step we neglect spin contributions to Eq. \(2\) (but we will reconsider the problem later). Moreover, if the projectile propagates along the \(\hat{z}\) direction, we will assume that also the damping is uniformly concentrated along the same direction, i.e. \(p_f \parallel p = p \hat{z}\); consequently,

\[
\psi(\mathbf{r}) \approx e^{-p_z \mathbf{r}} e^{ip \mathbf{P} \cdot \mathbf{r}} \equiv \psi(z).
\]

This introduces a (small) error in the asymptotic angular distribution of scattered projectiles, but to our purpose of considering only hard events inside the target nucleus it is irrelevant.

The function \(\psi(\mathbf{r})\) can be considered as a plane wave with the complex momentum \(\mathbf{p} + ip_f = (p + ip_f) \hat{z} = P \hat{z} = \mathbf{P}\). Therefore, as a simple plane wave gets Fourier transformed to a \(\delta\) distribution, similarly the "complex" plane wave exp \((ip_z)\) can be Fourier transformed by extending the definition of the \(\delta\) distribution to the complex plane. Following Ref. \[37\], since the \(\delta\) distribution of a real argument can be defined as

\[
\delta(x - \bar{x}) = \frac{1}{\pi} \lim_{\epsilon \to 0} \frac{\epsilon}{(x - \bar{x})^2 + \epsilon^2} = \lim_{\epsilon \to 0} \frac{1}{2\pi i} \left( \frac{1}{x - \bar{x} - i\epsilon} - \frac{1}{x - \bar{x} + i\epsilon} \right) ,
\]

the analytic continuation to a complex argument becomes

\[
\delta(z - \bar{z}) = \delta(z - (\bar{x} + i\bar{y})) = \frac{1}{2\pi i} \left( \frac{1}{z - \bar{x} - i\bar{y}} - \frac{1}{z - \bar{x} + i\bar{y}} \right) = \frac{1}{2\pi i} \left( \frac{1}{z - \bar{z}} - \frac{1}{z - \bar{z}^*} \right).
\]

In other words, when a function \(f(x)\) of the real argument \(x\) is convoluted with \(\delta(x - \bar{x})\), the integrand is pinched between the two singularities \(x = \bar{x} \pm i\epsilon\), which eventually collapse to the same value in the limit \(\epsilon \to 0\). For a function \(f(z)\) of a complex argument \(z\), the effect is similar if \(f(z)\) is analytic and \(f(z) \to 0\) for \(|z| \to \infty\). In fact, in this case we have

\[
\int_C dz \delta(z - \bar{z}) f(z) = f(\bar{z}) ,
\]

where \(C\) is an integration contour in the complex plane extending to Re\((z) \to \pm \infty\) on the real axis and closing in the upper plane for Im\((\bar{z}) = \bar{y} > 0\) (and viceversa).
How to interpret the \( \delta \) distribution with a complex argument? With no damping, the eikonal wave in momentum space is represented by \( \delta(k - p) \), a singular distribution in \( k \) peaked on the momentum \( p \) of the projectile. With a nuclear damping \( (p_T \neq 0) \), the momentum distribution is still peaked around \( p \) but is no longer singular, since the potential inside the nuclear volume distorts the wave function and introduces a width \( \Delta p \sim 1/R \) around \( p \). For energies above the inelastic proton-nucleon threshold, the nuclear potential is mainly imaginary and generates an absorption of the projectile flux (here represented by the damping \( p_T \)); hence, \( \Delta p \sim 1/z \), where \( z \) is the absorption length if the projectile is propagating along the \( z \) axis. If \( z < R \), then \( \Delta p \) is mostly given by the damping, i.e. by \( p_T \). In conclusion, the distribution \( \delta(k - p - p_T) \) is peaked around \( p \) but has a finite width \( \Delta p \), or equivalently it describes a projectile with a finite absorption length \( z \sim 1/\Delta p \), that is mainly given by the damping \( p_T \).

In order to understand the modifications inside the scattering amplitude, it is useful to consider first the case without damping; for the projectile-nucleus collision we have

\[
a(p) = \int dq d\mathbf{k} \psi_p^*(q) A(q, \mathbf{k}) \approx \int dq d\mathbf{k} \delta(q - p\hat{z}) A(q, \mathbf{k}) = \int d\mathbf{k} A(p\hat{z}, \mathbf{k}) ,
\]

where \( \psi_p \) represents the incoming wave function in momentum space centered around \( p \), the function \( A \) contains all the dynamics of the hard collision, and the integration upon \( d\mathbf{k} \) covers the portion of phase space complementary to the one pertaining the incoming projectile, i.e. it represents the integration upon momenta of nucleons participating to the collision and of final lepton products. Then, if the function \( A \) is well behaved and vanishes for asymptotically large momenta, Eq. (7) can be generalized to

\[
a(p + ip_T) = \int d\mathbf{Q} d\mathbf{k} \delta(\mathbf{Q} - \mathbf{P}) A(\mathbf{Q}, \mathbf{k}) = \int d\mathbf{k} A((p + ip_T)\hat{z}, \mathbf{k}) .
\]

FIG. 2: The leading-twist contribution to the Drell-Yan process.

If the invariant mass \( M \) of the Drell-Yan lepton pair is kinematically constrained in a range where the annihilation proceeds through a virtual photon, the dominant contribution at leading twist is represented by the handbag diagram of Fig. 2, where the blobs represent all the possible residual states of the two hadrons after the hard event that are not observed. Hence, a sum over all possible configurations is understood in the two cuts. As such, the process is by construction semi-inclusive and the cross section will be proportional to an incoherent sum of several transition probabilities, each one leading to the same final lepton pair. Neglecting the irrelevant phase space coefficient, the cross section with no nuclear damping will be related to

\[
W^o(p) = \sum_{n} |a_n(p)|^2 ,
\]

where the number of terms depends on the model used to describe the blobs in Fig. 2.

From Eq. (8), it is evident that the damping of the projectile wave function implies a shift in the argument of the elementary scattering amplitude, namely \( a_n(p) \rightarrow a_n(p + ip_T) \). Since \( p_T \ll p \), we have

\[
a_n(P = p + ip_T) = a_n(p) + ip_T \left. \frac{\partial a_n}{\partial P} \right|_{p_T=0} + o(p_T) ,
\]

so that Eq. (8) gets modified into

\[
W(p, p_T) = W^o(p) - 2p_T \text{Im}[G(p)] + o(p_T) ,
\]
with

$$G(p) = \sum_n \left( a_n^* \frac{\partial a_n}{\partial P} \right) \bigg|_{p_I=0}.$$  (12)

Equation (11) immediately suggests that, when building an azimuthal asymmetry of the type described in Sec. II, if for some reason the leading term $W^0(p)$ gets cancelled, an asymmetry could arise from the projectile-nucleus interaction through the interference term $G(p)$.

Therefore, small contributions ($p_I \ll p$) can become important in an asymmetry. Consequently, we have to go back to the problem of the spin dependence of the projectile-nucleus interaction, which was initially neglected.

### III. SPIN-ORBIT EFFECTS

The eikonal approximation is based on the assumption that a fast moving object experiences a roughly constant nuclear density but on the nuclear surface, where modifications of the projectile wave function can happen due to soft spin-orbit interactions. However, it is well known that the spin-orbit potential modifies the nuclear density but on the nuclear surface, where modifications of the projectile wave function can happen due to soft spin-orbit interactions. For a projectile travelling along the usual $\hat{z}$ axis, the spin-orbit contribution $\psi_{so}$ to its wave function in the eikonal approximation reads

$$\psi_{so}(z) \approx \int_{-\infty}^{z} dz' V_{so}(z') \propto \int_{-\infty}^{z} dz' \frac{dp}{dz} = \rho(z).$$  (13)

In other words, the modifications to the wave function induced by the spin-orbit potential do not depend on the details of the surface of the nucleus but, rather, on its bulk density.

Since the projectile momentum is $p = p_\hat{z}$, it is natural to quantize the spin along the $\hat{y}$ direction such that the operator form of the spin-orbit potential is

$$V_{so}(r) \sigma \cdot r \times p \propto -\frac{d\rho}{dz} p_x \sigma_y,$$  (14)

where $\sigma$ is the vector of 2x2 Pauli matrices. Consequently, the two spin components $\psi_{\pm}$ of the incoming wave function (if the projectile has spin $\frac{1}{2}$) feel a spin-orbit potential of opposite sign: at large energies, $V_{so}$ is also purely absorptive so that the net effect is that different polarization components of the incoming hadron beam feel different dampings.

In the given conditions, the asymmetry should be reasonably independent from $\hat{y}$, because it is created when the projectile enters the nuclear surface, and it is along the $\hat{z}$ direction such that the nucleus is divided in two hemispheres (e.g., $x > 0$ and $x < 0$ in the frame where the target is at rest and centered in the origin), each one being crossed by a flux of projectiles with opposite polarization.

Since the spin-orbit damping adds to the nuclear damping described in the previous section, we can formally write the whole wave function as

$$\psi_{\pm}(r) = \psi(z)(1 \pm \alpha(p) p_x),$$  (15)

where $\psi(z)$ is defined in Eq. (8). The coefficient $\alpha(p)$ is largely energy dependent. For sake of simplicity (and noting that the overall spin-orbit effect is not an increasing function of $p$ at large energies), we rewrite the asymmetric factor as $\alpha(p) \equiv \eta(p)$. Since $\eta R \ll 1$ at large energies, we may approximate $1 \pm \eta x \approx \exp(\pm \eta x)$ and write the components of the projectile wave function as

$$\psi_{\pm}(r) \approx e^{ipz - p_I z \pm \eta x},$$  (16)

which correspond to two plane waves with complex momenta $P_{\pm} = P \hat{z} \mp P_x \hat{x} = (p + ip_I) \hat{z} \mp \eta \hat{x}$, respectively.

We can now repeat the previous steps leading to Eq. (8) and write the scattering amplitude as

$$a_{\pm}(p + ip_I, \eta) = \int dk A((p + ip_I) \hat{z} \mp \eta \hat{x}, k).$$  (17)

Again, following Eqs. (11-12), we can perform a Taylor expansion including only terms linear in $p_I$ and $\eta$, namely

$$a_{\pm n}(P = p + ip_I, P_x = \eta) \approx a_{\pm n}(p) + ip_I \frac{\partial a_{\pm n}}{\partial P} \bigg|_{p_I=0} \mp \eta \frac{\partial a_{\pm n}}{\partial P_x} \bigg|_{p_I=0},$$  (18)
which leads to the response
\[ W_\pm (p, p_I, \eta) = W_\pm^o (p) - 2p_I \Im [G_L^\pm (p)] \pm 2\eta \Im [G_T^\pm (p)] , \]  
where
\[ G_L^\pm (p) = \sum_n \left( a_n^\pm \frac{\partial a_n^\pm}{\partial p} \right) \bigg|_{p_I = \eta = 0} , \quad G_T^\pm (p) = \sum_n \left( a_n^\pm \frac{\partial a_n^\pm}{\partial p} \right) \bigg|_{p_I = \eta = 0} . \]  

IV. AZIMUTHAL ASYMMETRY IN UNPOLARIZED DRELL-YAN

When building an azimuthal asymmetry from a hard event, the response \( W_\pm \) must depend also on a variable, say \( \xi \), which is characteristic of the asymmetric distribution of the final products. In the example of Fig. 11 \( \xi \) can be the angle \( \phi \) which identifies the azimuthal direction of the lepton pair production in the Collins-Soper frame. By making this dependence explicit in \( W_\pm \), a typical feature of these asymmetric distributions for a free proton target (or when no distortion is taken into account, i.e. for \( p_I = \eta = 0 \), hence the superscript \( o \) in the following formula) is that for each \( \xi \) there exists a correlated \( \xi' \) such that
\[ W_\pm^o (\xi, p) = W_\mp (\xi', p) . \]  

In the previous example, the correspondence \( \xi \rightarrow \xi' \) could be \( \phi \rightarrow -\phi \) or \( \phi \rightarrow \phi + \pi \). In other words, there are always two alternative ways for defining and/or calculating an azimuthal spin asymmetry when the effect of nuclear medium is not included: fix \( \xi \) and reverse the spin of the projectile, or fix its spin and replace \( \xi \) by \( \xi' \).

Then, it is natural to define the spin asymmetry for a polarized Drell-Yan event on a free proton target as
\[ A_{TT} = \frac{W_+^o (\xi, p) - W_-^o (\xi, p)}{W_+^o (\xi, p) + W_-^o (\xi, p)} = \frac{W_+^o (\xi, p) - W_+^o (\xi', p)}{W_+^o (\xi, p) + W_+^o (\xi', p)} . \]  

In addition, we also define the ”polarization averaged” asymmetry
\[ A^o = \frac{1}{2} \left( W_+^o (\xi, p) + W_-^o (\xi, p) - W_+^o (\xi', p) - W_-^o (\xi', p) \right) . \]  

Because of Eq. 21, the latter \( A^o \) turns out to vanish. It means that those scattering amplitudes that contribute to \( A_{TT} \) simply average to zero. Effects of a different origin may create a nonvanishing \( A^o \). In fact, because of the nuclear damping summarized in Eq. 19, the analogue of Eq. 21 for a real nuclear target becomes
\[ W_\pm (\xi, p, p_I, \eta) - W_\mp (\xi', p, p_I, \eta) = 2p_I \Im [G_L^\pm (\xi, p) - G_L^\pm (\xi, p)] + 2\eta \Im [G_T^\pm (\xi, p) + G_T^\pm (\xi', p)] , \]  

where the ”undistorted” contributions \( W_\pm^o \) cancel just because of Eq. 21. If \( p_I \) and/or \( \eta \) are not vanishing, then also Eq. 24 is not vanishing. Consequently, the generalization of \( A^o \) to the case of a real nuclear target reads
\[ A = \frac{1}{2} \left( W_+ (\xi, p, p_I, \eta) + W_- (\xi, p, p_I, \eta) - W_+ (\xi', p, p_I, \eta) - W_- (\xi', p, p_I, \eta) \right) \]
\[ = \frac{1}{2} \left\{ 2p_I \Im [G_L^\pm (\xi, p) + G_L^\pm (\xi', p) - G_L^\pm (\xi, p) - G_L^\pm (\xi', p)] \right. \]
\[ + 2\eta \Im [G_T^\pm (\xi, p) - G_T^\pm (\xi, p) - G_T^\pm (\xi', p) + G_T^\pm (\xi', p)] \right\} . \]  

When nuclear distortion is neglected or a free proton target is considered, i.e. for \( p_I = \eta = 0 \), then Eq. 21 holds and \( A = A^o = 0 \). It means that for a nuclear target the projectile can equivalently follow different paths (\( \xi \) and \( \xi' \)) inside the nucleus. When including the nuclear distortion, from Eq. 25 we get in general that \( A \neq 0 \): because of the nuclear damping, the two paths \( \xi \) and \( \xi' \) are not equivalent and the nucleus acts as a polarizer for the projectile.

We can now put this picture in the physical context described in Sec. I. If the Lam-Tung sum rule were correct, the cross section would not depend on the azimuthal \( \phi \) distribution of the final lepton pair in an unpolarized Drell-Yan event in nuclear targets. This is somewhat the same content of Eq. 21 and of the condition \( A^o = 0 \) for \( \xi = \phi \).
However, as already stressed in Sec. I the Lam-Tung sum rule is valid only for the direction of annihilation along the \( \hat{z} \) axis (see Fig. 1), i.e. for collinear partons in the hard event. Violation of the sum rule emphasizes the role of transverse dynamics of partons inside hadrons. A similar azimuthal asymmetry can be obtained by considering that the transverse momentum of the Drell-Yan pair originates from the transverse momentum of the colliding hadron when it is deflected by nuclear damping (particularly by the “transversely asymmetric” spin-orbit effect). This sort of duality that is intriguingly established between the partonic and hadronic interpretations of azimuthal asymmetries in terms of transverse (spin) degrees of freedom, might suggest that spin-orbit effects play a role also at the elementary level in characterizing the transverse dynamics of partons inside hadrons.

In conclusion, when a Drell-Yan hard event is considered in a collision of a hadronic unpolarized projectile with an unpolarized nuclear target, the nuclear damping and the soft spin-orbit interaction happening on the nuclear surface can distort the projectile wave function and produce an azimuthal asymmetric distribution of the final lepton pair (\( \vec{A} \neq 0 \)) even if there is no asymmetry in the corresponding collision on a free proton target (\( \vec{A}^c = 0 \)). The necessary condition for this to happen is that for the corresponding Drell-Yan with a polarized projectile and/or proton target the transverse spin asymmetry is not vanishing (\( \mathcal{A}_{TT} \neq 0 \)), because both observables are expressed in terms of the same scattering amplitudes \( a_n \).

V. DISCUSSION AND PERSPECTIVES

Given the results summarized in the last paragraph of previous section, the question arises about the interpretation of the experimental data of Refs. [8, 9, 10, 11] and of possible future Drell-Yan experiments with (anti)proton beams at the foreseen HESR at GSI: when a nuclear target is employed, how big is the distortion of the beam wave function at a given kinematics? And how can this effect be disentangled from the genuine azimuthal asymmetry due to the hard event?

Concerning the first question, the main problem is the lack of knowledge about the microscopic mechanisms that build up the elementary scattering amplitudes \( a_n \) in Eq. (20). We can attempt some educated guess based on simplifying hypothesis and on the general properties of known parton distributions and of hadron-hadron phenomenology.

For sake of simplicity, let \( W^\alpha \) in Eq. (9) be dominated by a single amplitude \( a(P, P_x) = a_n + ia_i \) with \( \partial a_i / \partial P(x) \approx 0 \) and \( a_n \approx 0 \) at \( p_f = \eta = 0 \). Then,

\[
W^\alpha(p) \approx a^2_i, \quad G^i(p) \approx \left( a_i \frac{\partial a_n}{\partial P_t} \right)_{p_f=\eta=0},
\]

where \( i = L, T \) correspond to longitudinal and transverse components with \( P_L = P + ip_I \) and \( P_T = P_x = i\eta \).

We define the ratios

\[
r_I = \frac{2p_I G^L(p)}{W^\alpha(p)}, \quad r_{so} = \frac{2\eta G^T(p)}{W^\alpha(p)},
\]

that should tell us about the relative size between the asymmetry of final lepton pairs generated by the collision of an unpolarized hadron beam on a nucleus, and by the collision of the same beam, but polarized, on a free proton target. In other words, \( r_I \) and \( r_{so} \) should tell us how big is the asymmetry produced by nuclear damping and spin-orbit, respectively, with respect to the asymmetry produced by the genuine polarized Drell-Yan hard event.

In order to estimate \( r_I \) and \( r_{so} \), we observe that the parton densities significantly change over the scale of their parent hadron size \( R_h \sim 0.2 \div 0.5 \) fm (which corresponds to a momentum scale of \( p_h \sim 1 \div 0.4 \) GeV/c, respectively). We assume that also the amplitude \( a \) changes significantly only over the same scale; neglecting fine details and looking only to the order of magnitudes, we can roughly conclude that

\[
\frac{G^i(p)}{W^\alpha(p)} \approx \left( a_i \frac{\partial a_n}{\partial P_t} \right)_{p_f=\eta=0} \sim \frac{1}{p_h} \sim R_h.
\]

In other words, we safely assume that the relevant mechanisms involve correlations at most over the scale \( R_h \) excluding long-range interactions that would imply large derivatives of \( a \) at \( p_f = \eta = 0 \) and, consequently, large values of \( G^i \). In this hypothesis, the ratios of Eq. (27) become \( r_I \sim 2p_I/p_h \) and \( r_{so} \sim 2\eta/p_h \), respectively. As already mentioned in Sec. IV experimental data about nuclear transparency on \(^{12}\text{C}\) seem to suggest \( p_f \sim 50 \) MeV/c. This value can be interpreted as the mean free path of a hadron travelling in the nuclear medium, and it should not depend on the mass number but for very light nuclei. The corresponding ratio \( r_I \), then, could be in the range \( 5 \div 10 \% \).

In order to estimate \( \eta \), it is useful to consider the nuclear analyzing power \( A_N \), i.e. the asymmetry between cross sections at a given transferred momentum for spin up and spin down projectiles colliding on an unpolarized nucleus.
If the typical range associated with spin-orbit effects is given by the surface thickness, we can approximate this scale in a nucleus again with $R_h$. Hence, we can assume that $\langle A_N \rangle \approx \eta R_h \sim \eta/p_h$, where $\langle A_N \rangle$ is the analyzing power averaged over the transferred momenta relevant to the ISI occurring during the propagation of the projectile inside the nuclear target, typically in the range $0 \div 0.5 \text{ GeV}/c$. The problem of estimating $r_{so}$ is, therefore, translated into the determination of the typical size of $\langle A_N \rangle$. In the literature, recent measurements and attempts of theoretical interpretations have been published for elastic scatterings at energies beyond 10 GeV with proton-proton $^{32, 40}$, proton--$^{12}$C $^{41, 42, 43}$, and antiproton-proton $^{38}$ systems (for a collection of older experimental data, see Ref. $^{40}$ and references therein). Results for $A_N$ with proton and $^{12}$C targets are similar in magnitude and shape and they show a steep decrease from 10% at 10 GeV down to 2% at 20 GeV, with somewhat higher values at larger energies around 30 GeV but with large error bars. At large energies, the antiproton-proton results have also a similar magnitude to the previous ones $^{39}$. However, extrapolation of these findings to other nuclear targets (maybe, isoscalar nuclei like $^{12}$C) should be taken with great care, since the reproduction of "nuclear" $A_N$ in terms of proton-proton and antiproton-proton amplitudes is not theoretically well established $^{42, 43}$. For transferred momenta in the range of interest $0 \div 0.5 \text{ GeV}/c$, $A_N$ already gets its maximum value for almost all beam energies and the other values are not much smaller than that. At larger angles, $A_N$ can reach large values at all energies, but this situation is not relevant here and it won’t be considered. In conclusion, with the above caveat we can assume $\langle A_N \rangle \approx A_N$ and finally deduce that the approximate size of the spin-orbit nuclear damping is in the range 4% to 10%, depending on the projectile energy.

The previous estimates about $r_I$ and $r_{so}$ are rather conservative, since they are based on hard interactions occurring at most within the short range $R_h$. However, we cannot exclude the case of a scattering amplitude $a$ having a much steeper dependence on small values of $P(P_x)$ because of correlations at a longer range. For example, if $a$ has a resonance-like behaviour with a pole as

$$a(k) = \frac{\alpha}{k - i \gamma},$$  \hspace{1cm} (29)

where $k = P$ or $P_x$, then from Eq. (20) we get

$$r_I = \frac{2pt G^L(p)}{W^o(p)} = \frac{2pt}{\gamma}, \hspace{1cm} r_{so} = \frac{2\eta G^T(p)}{W^o(p)} = \frac{2\eta}{\gamma}.$$  \hspace{1cm} (30)

In the "short-range" hypothesis, $\gamma \approx 1/R_h$. But if $a$ represents an intermediate state with a narrow width, then the nuclear damping is largely enhanced. This is not surprising, since a narrow-width resonance would propagate inside the nucleus with a mean free path $\sim 1/\gamma$ increasing the importance of nuclear effects. A possible example of this phenomenon is represented by the Drell-Yan process at the $J/\psi$ mass $^{26}$, but involving a nuclear target. Since this resonance is very narrow and it is present in all possible amplitudes, if the process is studied via a hadron-nucleus collision we would have the standard conditions of nuclear shadowing, i.e. the propagation of a quasi-real particle through large distances inside the nucleus.

In summary, nuclear dumping effects in hadron-nucleus collisions will affect azimuthal asymmetries from Drell-Yan hard events most likely by few percents, unless quasi-real intermediate states are involved in the propagation of the hadron projectile inside the target nucleus before the hard event. This condition is not so unlikely, if higher-twist contributions are relevant in Drell-Yan processes. Indeed, if $M_p/M$ is related to the ratio between subleading and leading twists (with $M_p$ the proton mass), a simulation in the typical kinematics foreseen at HESR at GSI $^{44}$ reveals that the cross section has a steep $M$ dependence with most events concentrated at the lowest allowed $M = 4$ GeV, where $M_p/M \approx 25\%$. However, new measurements of Drell-Yan events are of course needed to confirm and substantiate previous arguments, since the available data pertain a completely different regime $^{38, 39, 41, 41}$, where, incidentally, large error bars in the azimuthal asymmetries do not exclude nuclear effects when changing from the deuteron to the tungsten targets (see Fig.8 of Ref. $^{4}$).

Going back to the beginning of the section, we consider the second question about the possibility of disentangling the effect of nuclear damping from the genuine azimuthal asymmetry produced by a Drell-Yan hard event. Since spin-orbit effects are mostly associated with a single quasi-elastic projectile-nucleon scattering, detection of the Drell-Yan lepton pair in coincidence with a recoil proton with small longitudinal momentum (< 300 MeV/c) and large transverse momentum (400 $\div$ 800 MeV/c) would allow for a quantitative check of the importance of ISI (responsible for this large-angle emission) and for an indirect determination of the polarization of the projectile before the hard event, because this would determine the direction of the recoil proton momentum. With this tagging technique, it should be possible, in principle, to study a polarized Drell-Yan event from unpolarized beams and targets.
VI. CONCLUSIONS

Since several years, a series of measurements of Drell-Yan events in unpolarized high-energy hadronic collisions on nuclei have been published in the literature \[8, 9, 10, 11\] that still await for a coherent quantitative explanation. In fact, the differential cross section shows an unexpected largely asymmetric azimuthal distribution of the final lepton pair with respect to the production plane (see Fig. 1). At present, the most promising interpretation identifies the source of the observed $\cos 2\phi$ asymmetry in a leading-twist contribution to the cross section containing a peculiar distribution function, that describes the intrinsic transverse momentum distribution of transversely polarized partons inside unpolarized hadrons \[13\].

In this paper, we have explored the consequences of the fact that the above experiments involve target nuclei, namely that the hadronic beam (pions or antiprotons) has to cross the nuclear surface and travel inside the nuclear matter for some length (related to its energy) before colliding on a bound nucleon to produce the Drell-Yan hard event. The wave function of the projectile is distorted by the nuclear damping and by spin-orbit interactions happening on the nuclear surface, producing also an azimuthal asymmetry in the distribution of the Drell-Yan lepton pair; this effect should be added to the one originating from the elementary hard event. The necessary condition for this to happen is that for the corresponding Drell-Yan with polarized projectile and/or free proton target the asymmetry produced by flipping one of the spins is not vanishing, since both asymmetries are expressed in terms of the same elementary scattering amplitudes.

Using some educated guess based on simplifying hypothesis and on the general properties of known parton distributions and of hadron-hadron phenomenology, we estimated that the nuclear damping should affect the azimuthal asymmetry from Drell-Yan hard events most likely by few percents, unless quasi-real intermediate states are involved in the propagation of the hadron projectile inside the target nucleus before the hard event. This condition is not so unlikely if nuclear targets will be possibly adopted at the HESR ring at GSI, where in the considered kinematics either higher-twist contributions are relevant or the invariant mass of hadronic resonances with narrow width is reached in the annihilation (see Refs. \[21, 22, 23, 24, 25, 26, 27\] for further details).

It should be possible to isolate "nuclear" asymmetries from "hard" asymmetries by considering a Drell-Yan process where the final lepton pair is detected in coincidence with a recoil proton with small longitudinal momentum and large transverse momentum. In fact, only the rescatterings of the hadron beam propagating inside the nuclear medium would be responsible of such large-angle emission. It is interesting to note that, as the $\cos 2\phi$ asymmetry could be produced by the transverse dynamics of partons inside hadrons leading to a violation of the Lam-Tung sum rule \[15\], similarly the same asymmetry can be obtained by considering the transverse momentum of the colliding hadron when it is deflected by nuclear damping (particularly by the "transversely asymmetric" spin-orbit effect). This parallelism might suggest that spin-orbit effects play a role also at the elementary level in characterizing the transverse dynamics of partons inside hadrons.

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