Hyperbolic Thermoelastic Analysis due to Pulsed Heat Input by Numerical Simulation

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Thermo-mechanical behavior in a rod subjected to a pulsed heat input was investigated by numerical simulation using the hyperbolic thermo-elasticity theory derived from the thermal dynamics in the present paper. Unlike the classical thermo-elastic theory with the parabolic energy equation and the hyperbolic motion equation, temperature response and thermal stress due to the temperature change exhibit significant wavy characteristics in the hyperbolic thermo-elasticity theory which is based on the non-Fourier heat conduction. The whole region of the rod is split into the heat disturbed region and the heat undisturbed region by the thermal wave front which is determined by the propagating velocity of the heat wave. The heat wave and elastic wave travel in the body at a finite velocity and reflect at the end of the rod. Thermal shock due to the discontinuous jump in thermal condition, and the reflection of thermal stress at the end terminate of the rod are significant during the heating process.

Key Words: Non-Fourier Heat Conduction, Hyperbolic Thermoelasticity, Finite Element Method (FEM), Newmark-β Method, Heat Wave, Finite Thermal Velocity

1. Introduction

The conventional thermo-elasticity involves the energy conservation equation based on the Fourier’s law of heat conduction and the equation of motion. Since the energy conservation equation takes a parabolic form and the equation of motion in terms of displacement within the infinitesimal deformation assumption is hyperbolic form, the conventional thermo-elasticity comprises a mixed hyperbolic-parabolic system\(^{(1),(2)}\). In contrast, the energy conservation equation considering the non-Fourier’s law together with the motion equation turn to be a hyperbolic-hyperbolic system, and this is also known as the hyperbolic thermo-elasticity. The non-Fourier law will be given a detailed introduction in the following section. Due to the energy conservation equation in the hyperbolic form upon the non-Fourier’s law can be reduced to the classical form with the parabolic equation when the relaxation time approaches to zero, this type of thermoelasticity is also called as the generalized thermo-elasticity or the extended thermo-elasticity\(^{(3),(4)}\).

Proposal of the hyperbolic thermo-elasticity based upon the non-Fourier heat conduction eliminates the paradox of infinite velocity of heat in a medium. According to the physical mechanism, heat transfer in a medium depends on the electron, photon's vibration at atomic level. However, the classical assumption of heat flow suggested by Fourier in 1822\(^{(5)}\) is diffusive with an infinite velocity. Recently, with the development in the transient heating technologies such as laser processes, especially ultrashort and high energy density laser’s applications on materials to enhance the properties of surface, investigations on the rapid heating combining mechanics behavior becomes an attractive topics\(^{(6)}–(8)\). It has been pointed out that the classical thermo-elasticity satisfy the necessary of general engineering applications, while, the modified thermo-elasticity based on the non-Fourier heat conduction is more suitable for the description of real transient heating\(^{(3)}\).

Tang and Araki have studied the dynamic thermal deformation in film considering the non-Fourier heat conduction by means of analytical method\(^{(9)}\). Although most of the present research works on the thermo-elasticity involving the non-Fourier effect are much concentrated on the analytical solution, it is also important to develop numerical methods available for a wide range of practical applications.
plications. In general, convenient numerical methods enable one to solve problems with much complicated geometry and non-linear problems on which material properties are dependent on temperature. Numerical methods further provide more detailed information about the transient displacement, strain and stress than analytical solutions.

The present paper proposes a numerical method to solve boundary-value problems of the hyperbolic thermo-elasticity employing the finite element method (FEM) combining with the Newmark-β method. We discuss both behavior of the heat wave and the elastic wave propagations in a medium under the non-Fourier heat conduction. Furthermore, comparisons of calculated results between the hyperbolic thermo-elasticity and the conventional thermo-elasticity are made.

**Nomenclature**

\( (\cdot) \) : Time derivative
\( (\cdot)_i \) : Partial differentiation with respect to coordinate
\( x_i \) \((i = 1, 2, 3)\)
\( c \) : Specific heat
\( C_h \) : Velocity of heat propagation
\( E_{ijkl} \) : Stiffness tensor
\( F \) : Helmholtz free energy density
\( f_i \) : Body force
\( h_t \) : Heat flux
\( L \) : Propagating distance of heat wave front
\( r \) : Heat resource
\( S \) : Entropy density
\( T \) : Temperature
\( t \) : Time
\( V \) : Volume of a continuum
\( x, y \) : Coordinates
\( \sigma \) : Thermal expansion coefficient
\( \beta \) : Parameter in Newmark-β method
\( \Gamma \) : Surface of continuum body
\( \beta_2 \) : Thermal expansion parameter
\( \xi \) : Parameter in Newmark-β method
\( \delta_{ij} \) : Kronecker’s delta
\( e \) : Strain tensor
\( \theta \) : Dimensionless temperature
\( \kappa \) : Thermal conductivity
\( \lambda \) : Lamé constant
\( \mu \) : Lamé constant
\( \nu \) : Poisson’s ratio
\( \xi, \eta \) : Dimensionless coordinate
\( \rho \) : Mass density
\( \sigma \) : Stress tensor
\( \tau \) : Dimensionless time
\( \tau_0 \) : Relaxation time
\( u_i \) : Velocity of particle on direction \( x_i \)
\( u \) : Displacement of particle on direction \( x_i \)

2. **Formulations**

The governing equations for mechanics and heat conduction are briefly reviewed within the framework of infinitesimal deformation theory making use of the thermodynamics. The thermodynamics, involving the statement of energy conservation and the statement of momentum conservation, has been derived by a number of authors. Here we apply these equations to solid.

2.1 **Fundamental equations**

Consider a moving continuum body which occupies a volume \( V \) bounded by a closed surface \( \Gamma \). When the body undergoes certain mechanical loads and/or heat loads, mass, linear momentum, angular momentum and energy conservation hold within the continuum. Then the mass conservation is reduced to so-called the continuity equation, for the linear momentum is reduced to the local form of so-called the Eulerian equation of motion as

\[ \rho \ddot{u}_i = \sigma_{ij, \cdot j} + f_i, \]  

in which \( \rho \) denotes the mass density, \( u_i \) the components of displacement vector, \( \sigma_{ij} \) the components of stress tensor and \( f_i \) stands for the components of body force vector. Equation (1), obtained from Newton’s laws of motion through the material derivative, states that in an inertial frame of reference the material rate of change of the linear momentum of a body is equal to the resultant of applied external forces. Essentially, \( \ddot{u}_i \) on the left-hand side of Eq. (1) is the acceleration \( D\dot{u}_i / Dt \). The equation of motion (1) is a typical hyperbolic partial differential equation in terms of displacement. Here, a superposed dot denotes partial differentiation with respect to time. A comma denotes partial differentiation with respect to a variable or spatial coordination in the following context.

We write down all the equations in terms of the Cartesian coordinate system. Hence, the other conservation for the angular momentum states that the stress is symmetric, i.e. \( \sigma_{ij} = \sigma_{ji} \).

Generally, the constitutive equation is defined for the stress with Hooke’s law as

\[ \sigma_{ij} = D_{ijkl} \epsilon_{kl}, \]  

The stress is related to the kinematic measure of strain. We restrict that the field variables are infinitely small so that nonlinear terms in these variables are negligible. Thus, in such a linearized theory, the infinitesimal strain is defined by

\[ 2\epsilon_{ij} = u_{i,j} + u_{j,i}. \]  

Then the governing equation (1) further takes a hyperbolic form in terms of displacement, i.e. Navier’s equation

\[ \rho \ddot{u}_i + \mu \dot{u}_{i,jj} + (\lambda + \mu) u_{j,k,k} = -f_i. \]  

Under the certain circumstances of both the prescribed displacement on a part of the domain and the sur-
face force on a complimentary domain, the above equations comprise boundary-value problems with a proper choice of initial values.

It is based on the energy conservation law to derive the heat conduction equation. We are only concerned with the work done by the external force and the thermal exchange with environment in an enclosed body. Hence when we consider the conservation of energy, an additional conservation law, i.e. the first law of thermodynamics, must be prescribed as

$$\dot{e} = \dot{h} + \dot{r} + \sigma_{ij} \dot{u}_{ij},$$

in which $e$ stands for the internal energy density, $h$ the heat inflow and $r$ the heat source inside the domain. The mechanical power contributes to the change in internal energy density. Relating the internal energy with the temperature $T$ and specific heat $c$, further we may find the basic equation for the heat conduction regarding of the constitutive equation for heat flux.

For the purpose of convenient deducing the constitutive equations governing the mechanics behavior, an additional energy function is to be introduced, e.g. Helmholtz free energy. When the continuous body is just considered the heat energy and deformation in the thermo-elasticity theory, Helmholtz free energy density has the definition as

$$F(\varepsilon, T) = e(\varepsilon, T) - TS.$$  

(6)

Obviously, the Helmholtz free energy density depends on the strain and temperature. $S$ is the entropy density, which is derived as

$$S = -\frac{\partial F}{\partial T}.$$  

(7)

In addition, the entropy density and the heat energy density has following relation:

$$\dot{S} = \frac{\dot{q}}{T},$$  

(8)

$\dot{q}$ is the heat input from outer surface. Together with Eqs. (5) – (7), then we have an expression for $\dot{S}$ in terms of the Helmholtz free energy density

$$\dot{S} = -\left( \frac{\partial^2 F}{\partial \varepsilon_{ij} \partial T} \dot{\varepsilon}_{ij} - \frac{\partial^2 F}{\partial \varepsilon_{ij} \partial T} \right).$$  

(9)

Expanding the Helmholtz free energy density in Taylor’s series at the reference state under $\varepsilon_{ij} = 0$ at temperature $T_0 = 0$, since these higher order terms can be neglected, we have the following definitions:

$$\frac{\partial^2 F}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} \equiv E_{ijkl}, \quad \frac{\partial^2 F}{\partial \varepsilon_{ij} \partial T} \equiv -\beta_{ij}. \tag{10}$$

Thus, the relation of the thermal energy density rate with the strain rate and temperature rate deduced from the law of energy conservation (5) can be written as:

$$\dot{q} = T(-\beta_{ij} \dot{\varepsilon}_{ij} + \frac{\rho c}{T} \dot{T}),$$  

(11)

or

$$\dot{h} + \dot{r} = T(-\beta_{ij} \dot{\varepsilon}_{ij} + \frac{\rho c}{T} \dot{T}).$$  

(11')

### 2.2 Constitutive equations

Constitutive equations for the extended thermo-elasticity are formulated based upon the non-Fourier heat conduction law. Before that, the conventional linear thermo-elasticity model is summarized and we discuss the difference of the two types of model. Here we consider the stress-strain relation in a general thermal environment. The strain energy density is assumed to be expressed in terms of strain and temperature\(^{(11)}\). Therefore, the linear thermo-elasticity is given in the form of the well-known Duhamel-Neumann’s relation as

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl} - \beta_{ij} T,$$  

(12)

in which forth-rank tensor $E_{ijkl}$ stands for the stiffness tensor and second-rank tensor $\beta_{ij}$ the thermo-elasticity tensor. We tacitly assume the reference temperature to be zero. According to the definition of the Helmholtz free energy density given in Eq. (6), these coefficients for isotropic materials are further expressed as

$$E_{ijkl} = \mu (\delta_{ia} \delta_{bj} + \delta_{ib} \delta_{aj}) + \lambda \delta_{ij} \delta_{kl},$$  

(13)

$$\beta_{ij} = \alpha (3 \lambda + 2 \mu) \delta_{ij},$$  

(14)

where $\mu$ and $\lambda$ are the Lamé constants and $\alpha$ is the thermal expansion coefficient.

#### 2.2.1 Conventional heat conduction based on the Fourier law

The Fourier law states that the heat flux vector is proportional to the temperature gradient. This implies the natural interpretation of the heat flow in such a way that the heat always flows to lower temperature state. The heat flux vector $\mathbf{h}$ takes form as

$$h_i = -k T_i,$$  

(15)

in which $k$ is the thermal conductivity. Combining these effects to Eq. (11), the conventional heat conduction equation is expressed as

$$\rho c \dot{T} = k T_i + \dot{r} - \beta_{ij} \dot{\varepsilon}_{ij} T,$$  

(16)

where the last term means the heat generation due to the deformation of the body. The heat conduction equation (16) is a parabolic type equation. It is pointed out that, under some circumstances of both prescribed temperature and heat flow, the Eq. (16) forms a boundary-value problem for temperature.

The classical thermo-elasticity theory which involves Eqs. (1) – (16) based on the Fourier law was applied successfully to all aspects of engineering and science calculation. So these are called as the conventional thermo-elasticity theory (CTE)\(^{(9)}\), which implies the parabolic nature of the equation of heat conduction and the hyperbolic nature of the equation of motion are more transparent in isotropic body.

#### 2.2.2 Heat conduction based on the non-Fourier law

As is readily understood in Eq. (15), the Fourier law predicts the heat conduction with infinite speed. This means that an incidental signal of temperature change at
where \( \tau_0 \) is the relaxation time in heat conduction. Together with Eqs. (11) and (17), then we have the following equation in terms of the Helmholtz free energy density

\[
-kT_{\theta} = r + \tau_0 \dot{T} - T \left[ \frac{\partial F}{\partial T} (\dot{T} + \tau_0 \dot{T}) + \frac{\partial^2 F}{\partial \varepsilon_i \partial T} (\dot{\varepsilon}_i + \tau_0 \dot{\varepsilon}_i) \right] + \tau_0 \left[ \frac{\partial^2 F}{\partial T^2} + T \frac{\partial^2 F}{\partial T^2} \right] + \varepsilon_i (2T \frac{\partial^2 F}{\partial \varepsilon_i \partial T^2} + \dot{\varepsilon}_i T \frac{\partial^3 F}{\partial^2 \varepsilon_i \partial T}) \tag{18}
\]

The last bracketed set of terms on the right-hand side can be neglected within the framework of usual assumptions of the linear theory.

Thus, the energy equation for heat conduction becomes

\[
-kT_{\theta} = r + \tau_0 \dot{T} - T \left[ \frac{\partial F}{\partial T} (\dot{T} + \tau_0 \dot{T}) + \frac{\partial^2 F}{\partial \varepsilon_i \partial T} (\dot{\varepsilon}_i + \tau_0 \dot{\varepsilon}_i) \right]. \tag{19}
\]

Furthermore, there exist additional relation expressions according the definition of the Helmholtz free energy function as

\[
\frac{\partial^2 F}{\partial T^2} = -\frac{\rho c}{T}, \quad \frac{\partial^2 F}{\partial \varepsilon_i \partial T} = -\beta_{ij}, \tag{20}
\]

Hence, we finally obtain the following heat conduction equation

\[
kT_{\theta} = r + \tau_0 \dot{T} + [pc(T + \tau_0 \dot{T}) - \beta_{ij} T (\dot{\varepsilon}_i + \tau_0 \dot{\varepsilon}_i)]. \tag{21}
\]

When the magnitude of the relaxation time is negligible, this equation is reduced to Eq. (16). Further if we neglect the relaxation effect both in heat source and deformation-induced term, a simplified heat conduction equation can be expressed in

\[
kT_{\theta} = [pc(T + \tau_0 \dot{T}) - (3\lambda + 2\mu)\tau \dot{\varepsilon}_i T]. \tag{22}
\]

Equation (22) is also a hyperbolic differential equation. Together with Eq. (1) in terms of displacement, then the thermo-elasticity theory is constructed. Since both governing equations for temperature and displacement are hyperbolic differential forms, the thermo-elasticity theory is called as the hyperbolic thermo-elasticity theory in contrast to the mixed hyperbolic-parabolic thermo-elasticity. In addition, because Eq. (22) can be reduced into Eq. (16) once the relaxation time is assumed to be zero, the hyperbolic thermo-elasticity is also referred as the extended thermo-elasticity or generalized thermo-elasticity\(^{(14)}\).

The relaxation time, which characterizes the scattering processes in phonon from the point of view of physics, is best regarded as the constant parameter extracted from experimental data, although it may depend on temperature. Calculation of the relaxation time requires detailed information on the harmonic phonon spectrum and the physical condition of solid at the microscopic level. This kind of information is not easily obtained. In general, the order of magnitude of the relaxation time for homogeneous solid is \(10^{-6} \sim 10^{-10}\) second. Therefore, in some cases of normal prediction of temperature, influence from the relaxation time is negligible. In extreme conditions, however, such as rapid heating in nanometer scale, the effect of the relaxation time becomes dominant.

### 3. Numerical Solution

For the primary purpose of the present paper, we attempt to perform the investigation of propagation behavior of the heat wave as well as the yielded thermal stress by temperature variation in the non-Fourier heat conduction. The coupled effect in the thermo-elasticity problem is supposed to be comparatively small in the infinitesimal deformation range so that it is not necessary to calculate the influence from heat generated by the elasticity deformation\(^{(15)}\). So the coupling term in the equation is omitted here.

For convenient analysis on the thermo-elastic problems, we use such dimensionless variables given as \(\theta = \frac{T(x,y,t) - T_0}{\tau_0}, \tau = \frac{t}{2\tau_0}, \xi = \frac{\rho c}{\tau_0 k}, \eta = \frac{\rho c}{\tau_0 k} \sqrt{2} \) to transform the governing equation of the non-Fourier heat conduction into the dimensionless form as follows:

\[
\dot{\theta} + 2\theta = \theta', \tag{23}
\]

Thus, discretization of the governing equations for energy and motion in the finite element method are given as:

\[
[M_\theta] [\ddot{\theta}] + [C_\theta][\dot{\theta}] - [K_\theta][\theta] = [q], \tag{24}
\]

\[
[M_u][\ddot{u}] - [K_u][u] + [C_u][\theta'] = [f], \tag{25}
\]

where

\[
[M_\theta] = \sum_{i,e} \int_{r_i} [N_i]^T [N_i] dV, \tag{26}
\]

\[
[C_\theta] = \sum_{i,e} \int_{r_i} 2[N_i]^T \{\nabla N_i\} dV, \tag{27}
\]

\[
[K_\theta] = \sum_{i,e} \int_{r_i} [\nabla N_i]^T [\nabla N_i] dV, \tag{28}
\]

\[
[q] = \sum_{i,e} \int_{r_i} [N_i]^T [q] dV, \tag{29}
\]

\[
[M_u] = \sum_{i,e} \int_{r_i} \rho [N_i]^T [N_i] dV, \tag{30}
\]

\[
[K_u] = \sum_{i,e} \int_{r_i} [B]^T [D][B] dV, \tag{31}
\]

\[
[C_u] = \sum_{i,e} \int_{r_i} \alpha [B]^T [D] dV, \tag{32}
\]

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\( f = \sum \int_{\Gamma} [N]^T[f]d\Gamma. \)  

(33)

Boundary-value problems involving both the hyperbolic equations given in previous section under transient conditions are rather difficult to solve using analytical solutions. However, the finite element method combining with the Newmark-\( \beta \) method presented here can solve the thermo-elasticity problems with multi-dimensions effectively. Being a time discretization approach, the Newmark-\( \beta \) method, which is essentially an extension of the acceleration method in the integration of time differential equations, provides an unconditionally stable scheme, when it satisfies such conditions(16):

\[
\beta \geq 0.25(0.5 + \zeta)^2, \quad (\zeta \geq 0.5),
\]

(34)

where \( \zeta \) and \( \beta \) are the numerical parameters for the sufficient accuracy and stability. Here we take \( \zeta = 0.5 \) and \( \beta = 0.25 \) which satisfy the requirements at same time at least.

Combining the time integral approach be means of the Newmark-\( \beta \) method, we have

\[
\begin{align*}
\left( \frac{4}{\Delta \tau^2} [M_u] + \frac{2}{\Delta \tau} [C_u] + [K_u] \right) \{\theta\}_{\tau+\Delta\tau} &= \{q\} + \{M_u\} \left( \{\ddot{u}\}_{\tau} + \frac{4}{\Delta \tau} \{\dot{u}\}_{\tau} + \frac{4}{\Delta \tau^2} \{u\}_{\tau} \right) \\
+ [C_u] \left( \{\ddot{u}\}_{\tau} + \frac{2}{\Delta \tau} \{\dot{u}\}_{\tau} \right). & \quad \text{(35)}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{4}{\Delta \tau^2} [M_u] + [K_u] \right) \{a\}_{\tau+\Delta\tau} &= -[C_u] \{\theta\} + [M_u] \left( \{\ddot{u}\}_{\tau} + \frac{4}{\Delta \tau} \{\dot{u}\}_{\tau} + \frac{4}{\Delta \tau^2} \{u\}_{\tau} \right). \\
& \quad \text{(36)}
\end{align*}
\]

4. Problem Statements

The present sample evaluated is concerned with a finite medium in order to describe the behavior of the hyperbolic thermo-elasticity theory. For simplicity, it is desirable to assume a rod whose one side is irradiated to a pulsed heat representing a laser beam, while another end side is assumed to be an adiabatic boundary as shown in Fig. 1. In order to describe the mechanical behavior in the same domain, the adiabatic end of the rod is assumed as constrained boundary. A little more practical example, which was only concern on the heat conduction problem, has been investigated in the previous report in a two-dimensional domain under the non-Fourier(18). The same objective domain is condensed into the rod which is long enough compared with its width. Thus, from this viewpoint, the sample under investigation in the current paper may be regarded as one-dimensional domain. The rod is discretized on the transverse section along its axis for finite element analysis. All physical properties are independent of temperature.

As schematically illustrated in Fig. 2, the heat flux is acted on the right boundary (\( \eta = 1 \)) of the rod in the following dimensionless forms

\[
q = \begin{cases} 
q_0(1 - \cos(\omega \tau)) / 2 & \text{for } 0 \leq \tau \leq \tau_p \\
0 & \text{for } \tau_p \leq \tau \leq \infty 
\end{cases} \quad \text{(37)}
\]

with \( \omega = 2\pi / \tau_p \), where \( \tau_p = 0.6 \) is the period of the pulsed heat flux.

Material parameters, referring to the data for SUS 304 stainless steel, are adopted. The Young’s modulus takes as \( 2.0 \times 10^{11} \) Pa, the thermal expansion coefficient \( 1.7 \times 10^{-6} / \text{K} \), the mass density \( 7920 \text{kg/m}^3 \), the specific heat \( 0.502 \text{kJ/kg-K} \), the thermal conductivity \( 15.9 \text{W/m-K} \), and the relaxation time for steel \( 10^{-10} \text{second} \). In order to prevent unexpected oscillation in numerical calculation of FEM, the time step \( \Delta \tau \) and the representative mesh size \( \Delta x = \Delta y \) are take \( 2.0 \times 10^{-10} \) second and \( 4.5 \times 10^{-8} \) meter, respectively.

5. Results and Discussion

Figure 3 shows the relationship between the axial stress \( \sigma_y \) distributions and the dimensionless temperature responses under the non-Fourier heat conduction at dimensionless time 0.45, 0.85, 1.65 respectively. Dotted lines represent the dimensionless temperature distributions on the axis \( \eta \) of the rod, and solid lines denote the axial stresses \( \sigma_y \). In the calculated results of dimension-
less temperature distributions, clearly, sharp heat waves exist as indicated in Fig. 3. For a short time after the initial transient when the heat flux penetrated one side of the rod, appearance of the step jump discontinuities in the temperature distributions is due to the finite propagating speed of the thermal wave as mentioned in Ref. (9). Therefore, the whole region of the rod will be split into two segments: heat disturbed region and heat undisturbed region. The interface of the disturbed region and the undisturbed region is determined by the thermal wave front. In the disturbed region, temperature raises due to influence of the heat wave, whereas, in the undisturbed region, temperature still remains initial value of zero \( \theta = \frac{T(x,y,t) - T_0}{T_0} \). The arrows in Fig. 3 represent the propagation directions of the heat waves.

It is also illustrated in the same figure that, in the case of the non-Fourier heat conduction, the thermal expansion due to the temperature ascent generates compressive stress, which is denoted as negative value in the disturbed region where the temperature is changed as shown in Fig. 3 (a) and (b). On the other hand, ahead of the heat wave front, the axial thermal stress still is zero without any thermal deformation occurrence. Further more, due to the wavy nature of heat travel in solid, the thermal stress caused by the variation of temperature also exhibit remarkable wavy characteristics. Therefore it can be believed that such yielded significant thermal stress would result in severe damage on the domain when subjected to a rapid thermal disturbance under the non-Fourier heat conduction.

By checking the positions of the heat wave front at different times, it is interesting to observe that the passed distance by the heat wave front equal to the moment, i.e. \( L = \tau \), where \( L \) is the propagated distance of the heat wave front at time \( \tau \) in dimensionless form in the medium, thus the coordinate position in dimensionless form is now denoted by \( \eta = 1 - \tau \). For instance, at dimensionless time 0.45, the heat wave front arrives at position \( \eta = 0.55 \), which indicates the propagated distance \( L = 0.45 \). This is determined by the relation that all variables of heat in dimensionless forms are given as Eq. (23), where the dimensionless velocity of heat wave traveling in medium is unit. Actually, the propagated distance of the heat wave front determined by \( L = C_h \cdot t \), where the velocity is defined as \( C_h = \sqrt{\frac{k}{\rho \cdot c \cdot \tau_0}} \), depending upon the thermal conductivity and the relaxation time for materials. In the non-Fourier heat conduction, the propagations of axial stresses \( \sigma_y \) are in accordance with the heat waves as shown in Fig. 3, e.g. the thermal stress \( \sigma_y \) also reaches the position of \( \eta = 0.55 \) at dimensionless time \( \tau = 0.45 \) when the heat wave arrives at the position of \( \eta = 0.55 \). Moreover, when the heat wave arrives at the adiabatic end of the rod at dimensionless time of \( \tau = 1.0 \), the heat wave inverses its propagating direction and then reflects, consequently, the stress reflection also occurs at the constrained end of the rod. Generally, when an elastic wave meets a boundary surface, the same wave is also reflected as a result of conservation characteristic. In Fig. 3 (c), the reflected waves of the heat and the thermal stress at dimensionless time 1.65, indicate that the positions of the heat wave front and the thermal stress front are \( \eta = 0.65 \). Each of these ther-
mal wave peaks simultaneously dissipates thermal energy along the propagating path, hence the thermal stress becomes weaker with the decreasing temperature in the non-Fourier heat conduction.

On the contrary, for the Fourier heat conduction, analysis reveals, the diffusive features in temperature variation, and the thermal stress distributions caused by temperature changes are rather flat and smooth at corresponding times. Figure 4 represents the thermal stress distributions and the dimensionless temperature responses for the Fourier heat conduction at dimensionless time of 0.45, 0.85 and 1.65 respectively. Compared with Fig. 3 (a), obviously, appearances of temperature rising and corresponding thermal stress are over the whole region of domain at dimensionless time 0.45 for the Fourier heat conduction. At dimensionless time $\tau = 1.65$ in Fig. 4 (c), the thermal stress further becomes almost flat and constant in the whole region.

Variations of the axial stress at the end side of the rod in the non-Fourier heat conduction and the Fourier heat conduction are plotted in Fig. 5. It can be observed that due to the reflection of the heat wave in the non-Fourier at the end of the rod, the yielded thermal stress peak is rather higher than that in the Fourier case. Such significant thermal shock caused by the reflection of the heat wave at the end was not focused in the present because of negative of the heat wavy nature in the conventional Fourier law. We can only mention that Brorson et al. have observed the existence of similar behavior at the rear side of gold film experimentally(19).

6. Concluding Remarks

It presents the results of the extended thermoelasticity under the non-Fourier heat conduction using the numerical simulation involving the finite element method and the Newmark-\(\beta\) integral approach. In order to clarify the effect of unconventional heat conduction and mechanics, the objective domain is imaged as a simple rod which may be regarded as one dimensional problem approximately. Overall results are as follows:

1. It is found that in the non-Fourier heat conduction the whole region of the rod will be split into two segments: the disturbed region by heat wave and the undisturbed region. The interface of the disturbed region and the undisturbed region is determined by the heat wave front which it travels in the medium at a finite velocity. In the disturbed region, the temperature rises due to the influence of the heat wave, whereas, in the undisturbed region, it still remains at the initial values.

2. Such a distinct temperature difference between
the disturbed region and the undisturbed region leads to the appearance of sharp thermal stress distribution. The thermal stress also shows wavy features just as the heat wave, and the axial distribution will be also divided into two parts by the heat wave front. In the disturbed region, compressive stress appeared and no thermal stress took place in the undisturbed region during the propagating procedure.

(3) The analytical results show that the thermal stress becomes significant at the end of the rod because of the reflection of the heat wave. It is believed that such obvious thermal stress, or thermal shock, will make more serious damages on the material in the non-Fourier heat conduction. In the case of the Fourier heat conduction, however, the thermal stress distributions are always taken place smooth and show flat distributions in the objective domain subjected to a thermal disturbance suddenly.

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