Periodic and non-periodic brainwaves emerging via random synchronization of closed loops of firing neurons.

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Periodic and nonperiodic components of electrophysiological signals are modelled in terms of synchronized sequences of closed loops of firing neurons correlated in Markov chains. Single closed loops of firing neurons reproduce fundamental and harmonic components, appearing as lines in the power spectra at frequencies ranging about from 0.5 Hz to 100 Hz. Further interesting features of the brainwave signals emerge by considering multiple synchronized sequences of closed loops. In particular, we show that the fluctuations of the number of synchronized loops leads to the onset of broadband power spectral components. By effect of the fluctuations of the number of synchronized loops and the emergence of the related broadband component, highly distorted waveform and nonstationarity of the signal are observed, consistently with empirical EEG and MEG signals. The analytical relationships of the periodic and aperiodic components are evaluated by using typical firing neuron pulse amplitudes and durations.

1. INTRODUCTION

Periodic components of brain signals and their frequency bands (delta (1 – 3 Hz), theta (4 – 8 Hz), alpha (9 – 12 Hz), beta (12 – 30 Hz), gamma (> 30 Hz)) are central to neuroscience basic research and clinical protocols [1–3]. Aperiodic components, initially disregarded in comparison to periodic ones as considered just background noise, represent a significant part of signals. They manifest with power spectral densities varying approximately as $1/f^3$ and have been related to brain critical states [4, 5]. Recent studies have suggested that simultaneous changes of aperiodic and periodic brainwave components can underpin changes in functional and behavioural features, with the broadband components modulated by task performance and correlated with neuronal spiking activity. Synchronization between different neuronal groups may also manifest within arrhythmic brain activity with no apparent periodicity [6–13]. To keep pace with these findings, algorithms are being developed to the purpose of breaking complex electrophysiological signals down and transferring scientific findings into clinical practices [14, 15], an issue of is increasingly relevant to the development of brain–machine interfaces [16]. Despite remarkable advances in the interpreting and quantifying neurological signals, several problems mainly related to the dynamics of brain at various scales still remain unsolved [17].

In this work, a unified framework to quantify periodic and aperiodic power spectral components of electroencephalograms is developed based on a Markov chain description. The power spectral density is estimated in terms of time-sequences according to a statistical approach originally pioneered in the communication and information theory context [18–24]. Line spectral components are generated by Markov matrices corresponding to closed loop sequences of firing neurons characterized by a set of heterogeneous states. The oscillatory frequencies, observed in the EEG and MEG from 0.5 Hz to 100 Hz, can be reproduced by closed loops involving a few hundred neurons down to few neurons with firing intervals of the order of few milliseconds. The closed loop operates as an electric circuit where the neuron behaves as a rectifying diode, producing unidirectional currents, the synapses act as dissipative elements and the ionic currents as current generators. The formation of closed loops dissipate the excess energy accumulated in an active region of the brain by the local increase of circulating blood. A realistic description of the neurological signals require bunches of synchronized sequences of closed loops of firing neurons spontaneously formed in regions of high density, where neurons are connected to thousands other neurons nearby, that, if in a critical state, may be simultaneously fired.

The impulse generated by the bunch of synchronized sequence is represented by a Gaussian time function. The width of the Gaussian is related to the duration of the impulse emitted by the bunch and, in turn, to the characteristic cut-off filtering out the harmonic components of the power spectrum at high frequency. The amplitude of the Gaussian depends on the number of synchronized loops $N_i$. By taking into account the fluctuations of $N_i$ in the Markov chain model, a mixed power spectrum is obtained where lines and continuous spectral components co-exist. It is shown that the broadband component causes the line amplitude to change, while their frequency keeps on unchanged being only dependent on the reciprocal duration of the loops, and the distortion of the signal consistently with what is observed in the empirical EEG and MEG records. This distortion affects the lowest rather than the highest frequency components: thus delta waves with harmonics at frequencies lower than the cut-off are more distorted than gamma waves. In general the waves reported in electrophysiological graphs are at least distorted by second harmonic components, resulting in asymmetric triangular form of the wave.

The manuscript is organized as follows. In Sect.2 the general expression of the power spectral density of a se-
sequence of Markov correlated events is recalled. The conditions required for the onset of open or closed loops in the framework of the Markov chain description are also provided. Then, the approach is extended to an arbitrary number of synchronized closed loops, whose fluctuations cause the emergence of a broadband noise component. In Sect. 3, the proposed analytical framework is used as descriptive background to reproduce brainwaves signal features in relation to the mixed power spectrum. In Sect. 4, the proposed models and results are discussed, conclusions and suggestions for future work are also provided.

2. MATHEMATICAL FRAMEWORK

In this section, the mathematical background for the calculation of the power spectral density of a sequence of events correlated in a Markov chain and conditions to yield a closed loop are recalled. The amplitude of the periodic and aperiodic components are derived for a single loop in subsection (A) and for an arbitrary number of synchronized closed loops in subsection (B).

Consider a neuron, in a state labelled α1, firing to a neuron, in a state labelled α2, and so on in a sequence α1, α2, ..., αN of N states. Let n1, n2, ..., nN indicate the numbers of neurons respectively in the states α1, α2, ..., αN. The firings of n1 synchronized neurons result in the subsequent firing of n2 neurons, yielding a total of n1 · n2 synchronized neurons, and so on. Hence, the number of synchronized neurons may reach a value of the order of several thousands in a relatively short time. For the sake of the example if each of the n1, n2, ... produces a pair of simultaneously firing neurons with firing characteristic time of 5 ms, a total number Nf of about 10^7 synchronized neurons would be produced in only 0.1 s.

The same process occurs on the neurons in the sequence α2, α3, ..., αN of the N connected states. Thus, a bunch of N groups of Nf synchronized neurons for each state α of the sequence could be expected.

The neuron firing pulses can be described in terms of functions Fa(t), where t is the time and αi is one of the N states which completely characterizes the firing. The superposition of individual firing Fα(t − t1), where t1 is the time origin arbitrarily chosen for each Fa(t) defines the relevant neurological signal I(t) = ∑a=−∞Fa(t − t1). The time interval between subsequent events Fa(t) and Fα+1(t) is indicated by the variable u, which depends on the states α through the distribution function qa,α,α+1(u), accounting for the correlation between the time intervals u.

The α states are represented by an homogeneous Markov chain characterised by the N × N matrix ||m|| with entries:

\[ m_{α,α+1} = P(α+1 | α) = α' | α = α \]  

(2.1)
defined as the conditional probability that if the event i is in a state α, the event i + 1 will be in the state α', where α and α' are a pair of N states. Then, the matrix ||M(α)|| can be built with entries:

\[ M_{α,α'}(u) = m_{α,α'}q_{α,α'}(u), \]  

(2.2)where α and α' are the states of a pair of successive events of the sequence and q_{α,α'}(u) the distribution function of the time interval between the pair. The Fourier transform of M_{α,α'}(u) is defined as:

\[ M_{α,α'}(ω) = \int_0^∞ q_{α,α'}(u) \exp(iωu)du, \]  

(2.4)where:

\[ Q_{α,α'}(ω) = \int_0^∞ q_{α,α'}(u) \exp(iωu)du, \]  

(2.3)

M_{α,α'}(ω) defines the entries of the correlation matrix ||M(ω)|| of the Markov process.

The power spectrum of random processes correlated in a Markov chain is given by:

\[ Φ(ω) = ν|Sα,α′(ω)|^2 + 2νRe \sum_{α,α′} Sα′,α(ω)Sα,α′(ω)pα||K(ω)||_{α,α′}, \]  

(2.5)where ν is the average number of events per unit time, Sα,α′(ω) is the Fourier transform of Fa(t), with the overline indicating the average over all the pulses in the sequence. Sα′,α(ω) and Sα,α′(ω) are the Fourier transforms of Fa(t) for a pair of states α and α′, pα is the fraction of states α in the sequence. Re means the real part. The matrix ||K(ω)|| is defined as:

\[ ||K(ω)|| = ||M(ω)|| · (||I|| − ||M(ω)||)^{-1} \]  

(2.6)with ||M(ω)|| is the correlation matrix of the Markov process with entries defined by Eq. (2.2) and ||I|| the identity matrix.

A. Line Power Spectral Density

In this subsection, we will show how to derive the power spectral density of a single closed loop of Markov chain correlated pulses. When the Markov matrix ||m|| in Eq. (2.1) describes random events organized in closed loop sequences, the correlation matrix ||M|| yields physically sound line spectra as those observed in brain waves.

The lines in the power spectrum correspond to singularities of Eq. (2.5) when det (||I|| − ||M(ω)||) = 0, i.e. for q_{α,α'}(u) = δ(u − u_{α,α'}) where u_{α,α'} is the time interval between consecutive events characterized by the states α, α', then:

\[ Q_{α,α'}(ω) = \exp(iωu_{α,α'}) \]  

(2.7)and Eq. (2.4) takes the form:

\[ M_{α,α'}(ω) = m_{α,α'} \exp(iωu_{α,α'}) \]  

(2.8)
A closed loop involving $N$ states $\alpha$ can be expressed by a $N \times N$ matrix $[M(\omega)]$ where all the rows are made up of zeroes except one entry equal to 1. with $m_{\alpha,\alpha'} = 0$ except if $\alpha_{i+1} = \alpha'$ and $\alpha_i = \alpha$ then $m_{\alpha,\alpha'} = 1$. The condition $m_{N-1,N} = m_{N,1} = 1$ ensures closeness of the loops. With the above choices, the relationship det $([I] - [M(\omega)]) = 0$ becomes:

$$1 - \exp[i(\omega u_{1,2} + u_{2,3} + \ldots + u_{N,1})] = 0 \quad (2.9)$$

with solutions:

$$\omega = \omega_n = n\omega_o = \frac{2\pi}{\sum_{i=1}^{N} u_{i,i+1}} \quad (2.10)$$

Eq. (2.10) yields the fundamental frequency $\omega_o$ and the harmonics of the periodic component. $n$ is an integer. $\omega_o$ decreases as $N$ and $u_{i,i+1}$ increase. The amplitude of the spectral lines at $\omega = \omega_n$ is given by:

$$A_n = \frac{2\nu^2}{N^2} \text{Re} \sum_{\alpha\alpha'} S_{\alpha}^* (\omega_n) S_{\alpha'} (\omega_n) C_{\alpha \alpha'} (\omega_n) \quad (2.11)$$

where $\nu$ and $S_{\alpha}^* (\omega) S_{\alpha'} (\omega)$ have been defined after Eq. (2.5) and $C_{\alpha \alpha'}$ are the entries of the adjoint matrix: $||C(\omega_n)|| = \text{adj} ([I] - [M(\omega_n)])$. The amplitude given by Eq. (2.11) holds only at the singular frequency values $\omega = \omega_n$, when the determinant is zero. At frequencies $\nu$ different from the singularities $\omega_n$, the amplitude is equal to zero as expected for strictly periodic functions. Thus, the power spectral density of the periodic components writes:

$$\Phi_\nu(\omega) = \sum_{n=-\infty}^{\infty} A_n \delta(\omega - \omega_n) \quad (2.12)$$

The amplitude $A_n$ has been estimated for different values of the parameters $\nu$, $N$ and Fourier transform of the single pulse $S_\nu$ in [23].

The power spectral density of an arbitrary Markov correlated pulse sequence Eq. (2.5) has been worked out by averaging over time from $-\infty$ to $\infty$, by assuming stationarity. In the case of a single closed loop sequence, the power spectrum of a perfectly periodic signal is obtained, i.e. a line power spectrum taking discrete positive values at $n\omega_o$ and zero at any other frequency and no broadband noise is generated. The expression within square brackets in Eq. (2.5) corresponds to the real part of the sum of $N$ elements of the principal diagonal of the matrix $||K(\omega)||$ multiplied by $S_\nu (\omega) S_\nu (\omega^*)$ and by $p_\alpha$, while the off-diagonal elements do not contribute. The real parts of the diagonal elements are equal to $-1/2$, multiplied by $p_\alpha = 1/N$ and by 2, give $-S(\omega)^2$, that summed to the first term $|S(\omega)|^2$ of the same equation, cancel each other. The term $-|S(\omega)|^2$ is the average over all the $N$ states $\alpha$ of the square modulus of products relative to the two complex conjugate transforms of a couple of identical impulses symmetric with respect to zero within the two half of the sequence extending from $-\infty$ to $\infty$, while similar products within square brackets in the same equation refers to a couple of different impulses relative to the same state $\alpha$ along the sequence. When each state $\alpha$ remain identical along the sequence from $-\infty$ to $\infty$ and only their position along the sequence change according to the conditional probability expressed by the Markov matrix $[m]$, averaging over the statistical ensemble does not change the above conclusions. The line spectrum, generated by a periodic function, remain unchanged by averaging along the sequence.

### B. Broadband Power Spectral Density

In this subsection, the Markov matrix approach is extended to an arbitrary number $N_i$ of synchronized closed loops of firing neurons. Under the assumption of fluctuations of $N_i$ the expression of the power spectrum containing the broadband components is derived. The synchronized firing of neurons belonging to the same state $\alpha$ will be described by the superposition of individual firing functions in terms of the $F_\alpha (t) = \sum_{i=1}^{N_i} F_{i,\alpha} (t)$, where the angle brackets $\langle \rangle$ refers to the average over the ensemble of the $N_i$ loops. Owing to the large number $N_i$ of loops and the imperfect synchronization of the elementary impulses belonging to the same state $\alpha$, the firing events are described as Gaussian functions, hence $\langle F_{i,\alpha}(t) \rangle$ can be written as:

$$\langle F_{i,\alpha}(t) \rangle = \frac{\langle A_\alpha \rangle}{\sigma_\alpha \sqrt{2\pi}} \exp \left( -\frac{t^2}{2\sigma_\alpha^2} \right) \quad , (2.13)$$

with variance $\sigma_\alpha$, amplitude $\langle A_\alpha \rangle = \int_{-\infty}^{\infty} \langle F_{i,\alpha}(t) \rangle \, dt$ and the time origin of the impulses taken at the maximum of the Gaussian function relative to every state $\alpha$. The Fourier transform of the function $F_{i,\alpha}(t)$ can be written as $S_{\alpha} (\omega) = N_i \langle S_{i,\alpha}(\omega) \rangle$ where:

$$\langle S_{i,\alpha}(\omega) \rangle = \langle A_\alpha \rangle \exp \left( -\frac{\omega^2 \sigma_\alpha^2}{2} \right) \quad . (2.14)$$

When the fluctuations of the number $N_i$ of synchronized loops are taken into account in the general expression of the power spectral density, the quantity $S_{\alpha} (\omega) S_{\alpha} (\omega^*)$ writes $\langle N_i \rangle^2 \langle |S_{i,\alpha}(\omega)|^2 \rangle$. The first term in the same equation is $\langle N_i^2 \rangle$ $\langle |S_{i,\alpha}(\omega)|^2 \rangle$. Hence, the power spectrum writes:

$$\Phi(\omega) = \nu \left( \frac{N_i^2 - N_i^2}{N_i^2} \right) \langle |S_{i,\alpha}(\omega)|^2 \rangle \quad (2.15)$$

The quantity $N_i^2 - N_i^2$, yielding the fluctuations of $N_i$ around its average value $N_i$, can be estimated by assuming that $N_i$ is a stationary random variable described by a normalized Gaussian probability function $P(N_i)$:

$$P(N_i) = \frac{1}{\sigma N_i \sqrt{2\pi}} \exp \left( -\frac{(N_i - \overline{N_i})^2}{2\sigma^2 N_i^2} \right) \quad , (2.16)$$
with \((N_i)^2 = \left(\int_{-\infty}^{\infty} (N_i \cdot P(N_i)) dN_i\right)^2\) and \((N_i^2) = \int_{-\infty}^{\infty} (N_i^2 \cdot P(N_i)) dN_i = (N_i)^2 + \sigma^2_{N_i}\) and the variance is \(\sigma_{N_i}\). Hence Eq. (2.15) writes:

\[
\Phi(\omega) = \nu \sigma^2_{N_i} |\mathbb{E}_{\nu,\alpha}(\omega)|^2 , \tag{2.17}
\]

which yields the broadband component of the noise power spectrum, whose intensity has been estimated for normally distributed fluctuations of the number of synchronized loops. The amplitude of the broadband component depends on the fluctuations of \(N_i\) through the variance \(\sigma_{N_i}\).

The width of the Gaussian \(\sigma_\alpha\) for each state \(\alpha\) in Eq. (2.13) is related to the cut-off frequency of the power spectrum and thus, to the cut-off of the highest harmonic frequencies of the periodic components. If the pulses are Gaussian with Fourier transform given by Eq. (2.13), the cut-off angular frequency may be assumed to be \(\omega_c = 1/\tau_\alpha\), giving a reduction factor of \(1/e \approx 0.36\). For instance, \(\tau_\alpha = 2 \cdot 10^{-3} s\) yields a cut off frequency \(\omega_c = 500 \text{ rad s}^{-1}\) and \(f_c = 79.5 \text{ Hz}\), that for an \(\alpha\) wave of \(8 \text{ Hz}\) would allow about 10 harmonics to stand in the power spectrum up to \(80 \text{ Hz}\) with only a slight reduction, while for a wave of \(20 \text{ Hz}\) a reduction of the amplitude would occur after the \(4\)th harmonic. At frequencies lower than the cut-off \(\omega_c\), number and amplitude of the harmonics merely depend on the states characterizing the neurons forming the loops.

The main effect of the fluctuations of \(N_i\) is to give rise to a mixed power spectrum. The effect of this noise adds to the signal, which now includes spectral lines at the angular frequencies \(n \cdot \omega_0\) and a continuous broadband noise. Fluctuations of the parameters intrinsic to a single loop give rise to the changes in the synchronization with the other loops by changing the number \(N_i\). As observed in empirical EEG records, amplitude and waveform vary almost at every period of the detected signal. According to our model, this change may be due to fluctuations of the number of synchronized loops \(N_i\) and onset of synchronization with slightly different characteristics.

### 3. DISCUSSION

In this section, the power spectrum generated by the synchronized closed loop sequences of Markov correlated firing neurons will be confronted with typical features observed in EEG and MEG measurements. As discussed in Sect. 2, the sequences of electric and magnetic impulses received by the sensors on the scalp at every cycle correspond to \(N\) groups of \(N_i\) synchronized impulses generated by the firing of a single neuron in each loop. The average time intervals \(u\) between subsequent neuron firings along the loop multiplied by \(N\) yields the loop duration and its reciprocal the lowest frequency component of the mixed power spectrum. In principle \(N_i\) could be estimated by considering the intensity of either the elementary electric or the magnetic impulses, which are almost simultaneously produced by a single firing neuron in each sequence. Better estimates are obtained by using the magnetic component, not attenuated by the cerebral matter and the scalp, contrarily to the electrical components of the signal [22 27]. The magnetic and electric field components \(\vec{B}\) and \(\vec{E}\) generated by the firing of a single neuron obey the Maxwell equation \(\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \delta \vec{E}/\delta t\), with \(\vec{J}\) and \(\epsilon_0 \mu_0 \delta \vec{E}/\delta t\) the conduction and displacement current density. The conduction charges in the axon move at speed ranging between 0.5 \(\text{ms}^{-1}\) and 5.0 \(\text{ms}^{-1}\), thus much faster than the charges moving in the outer regions. Therefore, the electric field in the axon is partly screened by the conductive cerebral matter.

The magnetic field generated by the conduction charge inside the axon can be estimated by using the relationship valid for metallic conductors:

\[
\vec{B}(t) = \frac{\mu_0}{4\pi} I(t) \Delta t \frac{\vec{u}_A \times \vec{u}_D}{r_s} ,
\]

with \(I(t)\) the current intensity, \(r_s\) the distance between the midpoint of the axon and the point where \(\vec{B}(t)\) is measured, \(\Delta t\) the length of the axon, which, for neurons connected in the same area of the brain, ranges between 50 \(\mu\text{m}\) and 200 \(\mu\text{m}\). The unit vector \(\vec{u}_A\) and \(\vec{u}_D\) indicate respectively the directions of \(\Delta \vec{t}\) and \(r_s\). The amplitude of the magnetic field \(\vec{B}(t)\) writes:

\[
B(t) = \frac{\mu_0}{4\pi} I(t) \Delta t \frac{\sin \theta}{r_s} , \tag{3.1}
\]

where \(\theta\) is the angle between \(\vec{u}_A\) and \(\vec{u}_D\). \(I(t)\) can be estimated by considering the charge transferred by the firing process to nearby neurons. When the neuron receives positive inputs from its dendrites over a short time interval, its resting membrane potential increases from about \(-75\text{mV}\) to a critical value of about \(-45\text{mV}\). At this point, a positive charge \(Q\) enters the soma from the ionic channels making the membrane potential slightly positive a process lasting about 1 \(\text{ms}\). During a subsequent time interval with approximately the same duration (1 \(\text{ms}\)) the excess positive charge \(Q\) is ejected through its axon, restoring the membrane potential to its resting value \(-75\text{mV}\) after a small over-shut. As a good approximation, the ejection of this charge corresponds to a variation of the membrane potential of about 100 \(\text{mV}\). By approximating the soma as a sphere of radius \(R\) with uniform inner charge density \(\rho\), the electric field can be written as \(E(r) = \rho/4\pi r^3/4\pi \epsilon_0 r^2 = \rho r/3\epsilon_0\) at \(r \leq R\) with \(r\) the distance from the center of the sphere. The membrane potential \(V_m\) writes:

\[
V_m = \int_0^R E(r)dr = \frac{\rho}{3\epsilon_0} \int_0^R rdr = \frac{\rho}{6\epsilon_0} R^2 = \frac{Q}{8\pi\epsilon_0 R} \tag{3.2}
\]

where \(Q = \rho R^3/4\pi\) is the total charge within the sphere. A membrane potential of about \(V_m = 100\text{mV}\) and a soma radius of about \(R = 25.0 \mu\text{m}\) result in an ejected charge
\[ Q = 8\pi\varepsilon_0 RV_m = 5.55 \cdot 10^{-16} \text{ C}. \] If the ejected charge crosses the axon in about 1\text{ms}, the current impulse \( I(t) \) can be described by a Gaussian function of time with variance \( \sigma = 1\text{ms} \):

\[ I(t) = \frac{Q}{\sigma\sqrt{2\pi}} \exp\left(\frac{t^2}{2\sigma^2}\right) \quad (3.3) \]

with the condition \( \int_{-\infty}^{\infty} I(t)dt = Q \). The current \( I(t) \) can be introduced in the magnetic field (Eq. (3.1)), i.e.:

\[ B(t) = \frac{\mu_0}{4\pi} \frac{Q}{\sigma\sqrt{2\pi}} \exp\left(\frac{t^2}{2\sigma^2}\right) \Delta \ell \sin(\theta) \frac{r_\ell}{r^5}. \quad (3.4) \]

The maximum value of the magnetic field amplitude is achieved at \( t = 0 \) with \( \sin \theta = 1 \). This situation, where \( \sin(\theta) = 1 \), occurs when the axon of the firing neuron is tangent to the surface of the skull in the point where the magnetic sensor is placed. In the cortical area of the brain this may happen when the firing neuron is in one of the numerous regions called gyri \([28, 29]\). Then by using an average value of \( \Delta \ell = 100 \mu m \) and of the distance \( r_s = 2 \cdot 10^{-2} \text{ m} \) of the magnetic sensor detecting the impulse, an average peak intensity \( B(0) \approx 0.5 \cdot 10^{-20} T \) is obtained. To generate magnetic impulses of the order of \( 10^{-13} T \), value generally found in MEG and EEG, about \( N_i = 10^7 \) synchronized firing neurons would be needed \([?]\).

While it is generally accepted that the magnetic signal generated by firing neurons (MEG) is due to the ejected excess charge within the axon, the origin of the electric signal (EEG) is more controversial. A common assumption is that the signal is generated in correspondence of the chemical synapses connecting the firing neuron to the dendrites of numerous postsynaptic ones. The charge emitted from the axon of the firing neuron is split in hundreds or thousands fractions which create ionic currents external to the synapses, then detected by the sensors of the EEG setup. The synchronized firing of a large population of neurons and the role played by the excitatory and inhibitory synapses yield a complex situation of small current impulses, where spontaneous oscillations can be generated under suitable assumptions \([30, 31]\). An alternative interpretation relates the electric signals to the transient potential impulses radially emitted externally to the soma of the firing neurons, due to the rapid variations of the external electric field generated by the charge variations inside the neuron. Within this description, the potential impulse peak value, detected by an electrode internal to the soma during firing, is assumed of the order of 100mV, the electrode external to the soma peak value of about 0.1mV, the distance of the second electrode from the membrane of the neuron of the order of 200\mu m. In order to evaluate the intensity of the electric potential impulse, we consider again the neuron with the sferical soma of radius \( R = 25\mu m \) and the electric field \( E(r) \) at \( r < R \) used above for the calculation of the magnetic impulse. If the conductive medium within the cranial bone is neglected, the electric field \( E(r) \) at \( r > R \) is given by \( E(r) = Q/4\pi\varepsilon_0r^2 \) where \( Q \) is the whole charge within the soma. The electric potential, at distance \( r > R \) from the center of the sphere, is given by \( V = Q/4\pi\varepsilon_0r \), with \( V = 0 \) as \( r = \infty \). The field \( E(r) \) in the presence of conductive liquids, as it is the case for the brain, in stationary condition would be zero. A charge layer equal to the internal charge but with opposite sign, would be attracted and surround the membrane of the soma. This charge with spherical symmetry cancels out the field for \( r > R \). When a fast transient process occurs to the charge within the soma, as it is the case during the firing of the neuron, and the conductive liquid within the cranial bone contains positive and negative ions, the screening of the charge inside the soma is expected to occur only in part, which justifies the impulse \( \Delta \ell \). The amplitude of this impulse, compared to the amplitude expected from Eq. (3.2), allows to evaluate the effective charge \( Q_{eff} \) at the center of the sphere, and thus the amplitude of the impulse at an arbitrary distance \( r > R \). By assuming a distance \( r \) from the center of the sphere of the order of 200\mu m and \( Q = 5.55 \cdot 10^{-16} \text{ C}, Eq. (3.2) \) gives \( V = 22.28 mV \), exceeding the typical measured value 0.1mV. This result could imply that the charge internal to the soma is screened by an external opposite charge which reduces the \( Q \) value to \( Q_{eff} \). A value of \( Q_{eff} \) of the order of \( 10^{-18} \text{C} \), i.e. two orders of magnitude lower than \( Q \) would yield a potential value of the order of 0.1mV at a distance of about 200\mu m and a potential value of the order of 10^{-8}V at a distance of about 10^{-2}m. It should be also considered that the electrometer input impedance is close to \( \infty \). The impulse amplitude is expected to be reduced by the cranial bone and by the input impedance of the operational amplifiers connected to the electric sensors of the EEG set-up, with a gain inversely proportional to the input resistance, that must be kept low but large enough to drive the signal recorder. By keeping into account all these effect a smaller value of the potential could be obtained.

4. CONCLUDING REMARKS

A mathematical framework, leading to the unified description of the discrete and continuous power spectral components of the EEG and MEG signals, has been proposed. The model is based on the assumption that the complex network of interacting neurons spontaneously form closed loops chains of \( N \) firing neurons giving rise to repetitive emission of electric and magnetic impulses. By describing the electric and magnetic impulses generated by the firing of a neuron as Gaussians of given amplitude and width, the line power spectrum of this periodic function, fundamental and harmonic components, are shown to depend on the distributions of the amplitude and variance of the emitted impulses and of the time intervals between their emission, which are expression of the \( N \) states \( \alpha \) of the sequence. A general matrix equation gives the line power spectrum.

Furthermore, by considering the fluctuations of the
number $N_i$ of synchronized firing neurons belonging to different closed loop chains, a broadband component emerges in the power spectrum. The synchronized pulses are represented as Gaussian time functions whose amplitude and variance depend on the degree of synchronization and simultaneousness of the firing of the neurons belonging to different single loops but in the same state $\alpha$. T

As discussed in Sect.3, the fluctuations of the number of synchronized loops $N_i$ may change the amplitude of the signal and its waveform which is determined by the harmonic components generated by the loops. It is worthy of note that, even if the fluctuations change the waveform of the signal, the frequency of the wave remains unchanged, as the frequency of the wave depends on the duration of the loop $\sum_i u_i$, which is unchanged by the fluctuation of $N_i$. An additional effect of the presence of the broadband component concerns the onset of a cut-off in the line power spectrum, which is related to the width of the electric impulses represented by the variance of the Gaussian time functions. Larger variance implies lower cut-off frequency and smaller number of harmonic components, filtering out a signal closer and closer to the pure sinusoid.

The main features explained by the proposed model are summarized here below. The broad range of discrete frequencies observed in the EEG and MEG signals is obtained by considering different duration of closed loops sequences of firing neurons. By assuming an average firing interval between successive neurons of $5 \cdot 10^{-3}$ s, sequences ranging from few hundred neurons to only few neurons cover the range from 0.5 Hz (lowest frequency of delta waves) to 50 Hz (highest frequency of gamma waves). The presence of several harmonic components of the fundamental sinusoidal wave in the graph of EEG is also accounted for, particularly at low frequency, as the delta waves, where a pure sinusoid is never observed. High frequency waves are less distorted since harmonics stand at frequencies multiple of the fundamental one, and the cut-off frequency of the detected power spectrum cuts the harmonics exceeding that frequency. In a few particular cases all harmonics are cut-off, or strongly reduced, giving a sinusoidal, or nearly sinusoidal, wave. Part of the distortion can be due to the broadband noise created by the fluctuation of the number of $N_i$. One of the effects discussed above, and observed in almost all EEG graphs, is the continuous change of the waveform, practically at every period, of the received signal produced by the loops. This change, which is associated to a heavy distortion of the fundamental wave, is due to the presence of several harmonic waves within the periodic signal, and it is enough a change of the amplitude or the phase of one or few of these harmonics during the fluctuation of $N_i$, to change the waveform over the period. A signal constituted of a fundamental wave and many harmonics is just what is expected from a sequence of heterogeneous electric impulses characterized by different states $\alpha$, as considered in the present paper.

[1] L. J. Hirsch, and R.P. Brenner, “Atlas of EEG in Critical Care”, Wiley, (2010).
[2] da Silva, F.L., 2013. EEG and MEG: relevance to neuroscience. Neuron, 80(5), pp.1112-1128.
[3] Baillet, S. (2017). Magnetoencephalography for brain electrophysiology and imaging. Nature neuroscience, 20(3), 327-339.
[4] de Arcangelis, L., & Herrmann, H. J. (2010). Learning as a phenomenon occurring in a critical state. Proceedings of the National Academy of Sciences, 107(9), 3977-3981.
[5] Tagliazucchi, E., Balenzuela, P., Fraiman, D., & Chialvo, D. R. (2012). Criticality in large-scale brain fMRI dynamics unveiled by a novel point process analysis. Frontiers in physiology, 3, 15.
[6] Buzsaki, G., & Draguhn, A. (2004). Neuronal oscillations in cortical networks. science, 304(5679), 1926-1929.
[7] Canolty, R. T., Edwards, E., Dalal, S. S., Soltani, M., Nagarajan, S. S., Kirsch, H. E., ..., & Knight, R. T. (2006). High gamma power is phase-locked to theta oscillations in human neocortex. science, 313(5793), 1626-1628.
[8] He, B. J., Zempel, J. M., Snyder, A. Z., & Raichle, M. E. (2010). “The temporal structures and functional significance of scale-free brain activity”. Neuron, 66(3), 353-369.
[9] Buzsáki, G. and Wang, X.J., 2012. Mechanisms of gamma oscillations. Annual review of neuroscience, 35, p.203.
[10] Becker, R., Van De Ville, D., & Kleinschmidt, A. (2018). Alpha oscillations reduce temporal long-range dependence in spontaneous human brain activity. Journal of Neuroscience, 38(3), 755-764.
[11] Wairagkar, M., Hayashi, Y., & Nasuto, S. J. (2021). Dynamics of long-range temporal correlations in broadband EEG during different motor execution and imagery tasks. Frontiers in neuroscience, 15, 413.
[12] Cabral, Joana, et al. "Metastable oscillatory modes emerge from interactions in the brain spacetime connectome.". Commun Phys 5, 184 (2022)
[13] Brady, B., & Bardouille, T. (2022). “Periodic/Aperiodic parameterization of transient oscillations (PAPTO)–Implications for healthy ageing". Neuroimage, 251, 118974.
[14] Donoghue, T., Haller, M., Peterson, E. J., Varma, P., Sebastian, P., Gao, R., ..., & Voytek, B. (2020). “Parameterizing neural power spectra into periodic and aperiodic components”. Nature Neuroscience, 23(12), 1655-1665.
[15] Gerster, M., Waterstraat, G., Litvak, V., Lehertz, K., Schnitzler, A., Florin, E., ..., & Nikulin, V. (2022). “Separating neural oscillations from aperiodic 1/f activity: challenges and recommendations”. Neuroinformatics, 1-22.
[16] Zhang, M., Tang, Z., Liu, X., & Van der Spiegel, J. (2020). Electronic neural interfaces. Nature Electronics, 3(4), 191-200.
[17] Goodfellow, M., Andrzejak, R. G., Masoller, C., & Lehertz, K. (2022). What Models and Tools Can Contribute
to a Better Understanding of Brain Activity. Front. Netw. Physiol. 2: 907995. doi: 10.3389/fnetp.

[18] W. H. Huggins, "Signal-Flow Graphs and Random Signals," in Proceedings of the IRE, vol. 45, no. 1, pp. 74-86, (1957).

[19] R. D. Barnard, "On the discrete spectral densities of Markov pulse trains," in The Bell System Technical Journal, vol. 43, no. 1, pp. 233-259, (1964).

[20] Mazzetti, P., and C. Oldano. "Spectral properties of physical processes of Markov correlated events. I. Theory." Journal of Applied Physics 49, no. 11 (1978): 5351-5356.

[21] P. Galko and S. Pasupathy, "The mean power spectral density of Markov chain driven signals," in IEEE Transactions on Information Theory, vol. 27, no. 6, pp. 746-754, (1981).

[22] Bilardi, G., Padovani, R., & Pierobon, G. (1983). "Spectral analysis of functions of Markov chains with applications." IEEE Transactions on Communications, 31(7), 853-861.

[23] Mazzetti, P., and Carbone, A. "Harmonic spectral components in time sequences of Markov correlated events." AIP Advances, 7(7), (2017): 075216.

[24] Centers, J., Tan, X., Hareedy, A., & Calderbank, R. (2021). Power spectra of constrained codes with level-based signaling: Overcoming finite-length challenges. IEEE Transactions on Communications, 69(8), 4971-4986.

[25] Izhikevich, E.M., 2007. Dynamical systems in neuroscience. MIT press.

[26] Delorme, A., Palmer, J., Onton, J., Oostenveld, R. and Makeig, S., 2012. Independent EEG sources are dipolar. PLoS one, 7(2), p.e30135.

[27] Vorwerk, J., Cho, J.H., Rampp, S., Hamer, H., Knösche, T.R. and Wotlers, C.H., 2014. A guideline for head volume conductor modeling in EEG and MEG. NeuroImage, 100, pp.590-607.

[28] Jiang, X., Zhang, T., Zhang, S., Kendrick, K. M., & Liu, T. (2021). “Fundamental functional differences between gyri and sulci: implications for brain function, cognition, and behavior. Psychoradiology”, 1(1), 23-41.

[29] Qiyu Wang, Shijie Zhao, Zhibin He, Shu Zhang, Xi Jiang, Tuo Zhang, Tianming Liu, Cirong Liu, Junwei Han, “Modeling functional difference between gyri and sulci within intrinsic connectivity networks”. Cerebral Cortex, 2022;, bhac111, https://doi.org/10.1093/cercor/bhac111

[30] A. Hutt, Oscillatory activity in excitable neural systems.” Contemporary Physics 51, no. 1 (2010): 3-16.

[31] M. Hashemi A. Hutt, J. Sleigh, “How the corticothalamic feedback affects the EEG power spectrum over frontal and occipital regions during propofol-induced sedation”, J Comput Neurosci (2015) 39:155–179 DOI 10.1007/s10827-015-0569-1