Does Dissipation in AGN Disks Couple to the Total Pressure?

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ABSTRACT

Recent work on the transport of angular momentum in accretion disks suggests that the Velikhov-Chandrasekhar instability, in which a large scale magnetic field generates small scale eddies in a shearing environment, may be ultimately responsible for this process. Although there is considerable controversy about the origin and maintenance of this field in accretion disks, it turns out that it is possible to argue, quite generally, using scaling arguments, that this process is sensitive to the total pressure in an AGN disk, rather than the pressure contributed by gas alone. We conclude that the resolution of the conceptual difficulties implied by the presence of strong thermal and viscous instabilities in radiation pressure and electron scattering dominated does not lie in models that couple the total dissipation rate to the gas pressure alone, or to some weighted mean of the gas and radiation pressures.

Subject headings: AGN
1. Introduction

The conventional model for AGN is one in which matter is absorbed by a supermassive black hole, typically with a mass of order $10^8 M_\odot$. The matter flows inward towards the black hole event horizon through a disk, which radiates away the heat generated by the dissipation of orbital energy. The dynamics of disk accretion are not well understood, but the dissipation of energy is caused by the outward transport of angular momentum. Following Shakura and Sunyaev (1973) this transport is modeled as the consequence of an effective viscosity $\nu \sim \alpha c_s H$, where $H$ is the disk thickness and $c_s$ is the sound speed. The mass accretion rate is taken to be close to the Eddington limit, i.e. some fraction of a solar mass per year. Given this basic model the disk temperatures are typically $10^5$ degrees or more. The opacity of the disk is dominated by electron scattering and its pressure is mostly due to radiation pressure.

Unfortunately, this model implies the presence of local instabilities so rapid and powerful that the basic assumption of a steady state disk is called into question (Pringle et al. 1973, Lightman & Eardley 1974, Shakura & Sunyaev 1976). Both thermal instabilities, in which a small perturbation to the local thermal energy density grows, and viscous instabilities, in which a small perturbation to the local surface density grows, are present on wavelengths almost as small as $H$. Typical growth rates are $\alpha \Omega$ for thermal instabilities and $\alpha (H/r)^2 \Omega$ for viscous instabilities.

Recently, Balbus and Hawley (Balbus & Hawley 1991, Hawley & Balbus 1991) have pointed out that the Velikhov-Chandrasekhar (VC) instability (which we have previously referred to as the Magnetic Shearing Instability) provides a natural way to explain the transport of angular momentum within an accretion disk. This instability (Velikhov 1959, Chandrasekhar 1961) appears when a magnetic field is present in a shearing flow. The
growth rate in a keplerian disk is roughly

$$\Gamma_{VC} = 3^{1/2}k_B V_A,$$

(1-1)

where $k_B$ is the component of the wavevector of the perturbation in the direction of the magnetic field and $V_A$ is the Alfvén speed. However, there is a minimum unstable wavelength $\sim V_A/\Omega$, and the growth rate reaches a maximum ($\sim \Omega$) at slightly larger scales. This instability has the virtue that it necessarily transports angular momentum in the right direction, i.e. outward. In the strongly shearing environment of an accretion disk we expect that the dominant role will be played by the azimuthal component of the magnetic field. We have previously argued on theoretical grounds ([Vishniac and Diamond 1992]) that in this case the turbulent transport is dominated by the small, rapidly growing eddys. This argument is apparently supported by preliminary numerical calculations ([Hawley & Balbus 1993]). We expect that the associated turbulent diffusivity is $\sim V_A^2/\Omega$, with an associated value of $\alpha = (V_A/c_s)^2$. If instead one considers axisymmetric modes, which necessarily couple only to $B_z$, one finds that the dominant eddys are large scale (or order $H$) with an associated turbulent diffusivity of $V_A H$, where the Alfvén speed here is $B_z/(4\pi \rho)^{1/2}$ rather than $B/(4\pi \rho)^{1/2}$ ([Hawley & Balbus 1992], [Zhang et al. 1993]), and an $\alpha$ of $V_A/c_s$. In real disks such modes should be suppressed by the more violent, smaller eddys associated with the azimuthal field, but we will see that our conclusions are insensitive to this point. A major controversy regarding the role of the VC instability is whether or not the motions induced by the turbulence are sufficient to drive the magnetic field to a large amplitude, as suggested by Balbus and Hawley (1991), or whether this instability plays a purely dissipative role which must be balanced by a dynamo effect arising from unrelated fluid motions (e.g. [Vishniac and Diamond 1992]). We will see that our conclusions are insensitive to the resolution of this controversy as well.

In this note we will argue that the VC instability in an AGN disk is strongly coupled
to both matter and radiation, so that its dissipative effects should scale with the total (mostly radiation) pressure. We also point out, contrary to previous arguments, that magnetic buoyancy does not quickly eliminate any field whose saturation Alfvén speed is a small fraction of the local sound speed. Both of these results are insensitive to the current controversy concerning the maintenance of magnetic fields in disks.

2. The Velikhov-Chandrasekhar Instability in an AGN Disk

How can we tell if the VC instability couples to the radiation pressure? Clearly if the motions of the magnetic field and the charged particles tied to it have no significant dynamical interactions with the photons on the relevant length and time scales then we can ignore the photons in applying the instability to angular momentum transport. The relevant equipartition energy density for the magnetic field would then be the thermal energy density of the gas. This suggests a series of questions. First, can photons free stream through the eddys created by the VC instability? Second, can photons damp the smallest VC eddys, thereby shifting the dynamics to a larger scale, or blocking the Balbus-Hawley mechanism altogether? Third, can a photon contained within an eddy escape in one eddy turn-over time or less? We will see that the answers to these questions are ”No”, “No”, and “Yes, but it doesn’t much matter”.

First, we consider whether or not photons can free stream through the eddys created by the VC instability. Let’s consider the ratio of the photon mean free path, $\lambda_{mfp}$, to the typical eddy size, $V_A/\Omega$, where $V_A$ is the Alfvén speed and $\Omega$ is the local rotational velocity. As mentioned above, it has been claimed that the important eddys are much larger, of order the disk thickness $H$, but clearly if $\lambda_{mfp} < V_A/\Omega$ then the same conclusion must also
follow for larger eddys. We have

$$\frac{\Omega \lambda_{mfp}}{V_A} \sim \frac{\Omega}{V_A \sigma_T n_e}, \quad (2-1)$$

where $n_e$ is the electron density. However, for a disk supported by radiation pressure and dominated by electron scattering the vertical height can be obtained by balancing the average gravity $\sim H \Omega^2$, with the acceleration per particle due to the radiative flux $\dot{M} \Omega^2$. We find that

$$H \sim \frac{\dot{M} \sigma_T}{\mu c}, \quad (2-2)$$

where $\mu$ is the mean mass per electron. It follows that

$$\frac{\Omega \lambda_{mfp}}{V_A} \sim \frac{\Omega \dot{M}}{\rho c H V_A}. \quad (2-3)$$

Now since

$$\dot{M} \sim \frac{\alpha \Sigma c_s^2}{\Omega}, \quad (2-4)$$

we have

$$\frac{\Omega \lambda_{mfp}}{V_A} \sim \left( \frac{c_s^2}{c V_A} \right) \alpha \quad (2-5)$$

Assuming that small scale eddys dominate the angular momentum transport $\alpha \sim (V_A/c_s)^2$ and this ratio is $\sim V_A/c$, i.e. small. If instead we had assumed that the angular momentum transport was dominated by eddys on a scale $\sim H$, then we would have $\alpha \sim V_A/c_s$, but the relevant length scale for the eddys would have been $H$ instead of $V_A/\Omega$ so the final estimate of the ratio would have been unchanged. If we had taken a smaller estimate of $\alpha$, consistent with the notion that radiation pressure doesn’t contribute to the rate of angular momentum transport, our conclusion would only have been strengthened. Photons do not free stream through VC induced eddys in an accretion disk.

Second, we can ask if photons damp the smallest VC eddys, thereby shifting the dynamics to a larger scale, or blocking the Balbus-Hawley mechanism altogether? Now
the appropriate ratio is the radiative damping rate to the eddy turnover time. Assuming the relevant eddys are small scale twists in the large scale azimuthal field then the eddy turnover time is just \( \sim \Omega^{-1} \) and

\[
\frac{\tau_{eddys}}{\tau_{damping}} \sim \left( \frac{\Omega}{V_A} \right)^2 c \lambda_{mfp} \frac{\rho \gamma}{\rho} \Omega^{-1}.
\]  

This can be rewritten as

\[
\frac{\tau_{eddys}}{\tau_{damping}} \sim \left( \frac{c_s}{V_A} \right)^2 \left( \frac{P}{P} \right) \frac{\Omega M}{\rho c^2 H},
\]  

or using the usual relationship between \( H \) and \( \dot{M} \)

\[
\frac{\tau_{eddys}}{\tau_{damping}} \sim \left( \frac{c_s}{V_A} \right)^2 \left( \frac{P}{P} \right) \left( \frac{c_s}{c} \right)^2 \alpha.
\]  

Since \( \alpha \sim (V_A/c_s)^2 \) and \( P_{\gamma} \sim P \) this is just \( (c_s/c)^2 \). Again, a smaller value of \( \alpha \) would only strengthen our conclusions. If we had considered large scale, axisymmetric eddys, then the eddy turnover time would have been longer, i.e. \( H/V_A \), the eddy length scale would have been longer, i.e. \( H \), and the value of \( \alpha \) would have been larger, i.e. \( V_A/c_s \). Our conclusion would be unchanged. The shear viscosity due to photons has a negligible effect on the VC instability in AGN disks.

Finally, can a photon contained within an eddy escape in an eddy turn-over time or less? This is not equivalent to the preceding question because such photons carry away a share of the eddy momentum which is proportional to \( \rho \gamma/\rho \sim c_s^2/c^2 \), which is small. In fact, going through the argument in the preceding paragraph, but omitting the factor of \( \rho \gamma/\rho \), we get that a typical photon can just diffuse out of an eddy in one eddy turn-over time. This means that the bulk viscosity of the fluid is not completely negligible and the thermal conductivity of the fluid is high enough that fluid in an eddy remains roughly at a constant temperature. Nevertheless, the VC instability is a low frequency, incompressible mode whose behavior is not sensitive to the bulk viscosity or the thermal conductivity. In particular we note that the fraction of the mode energy which is contained in pressure
fluctuations is of order \((V_A/c_s)^2\), the remainder residing in kinetic energy and magnetic field fluctuations. Since only this fraction of the mode energy can be dissipated by the bulk viscosity in one eddy turn over time (given our result that photon’s diffuse from an eddy on this time scale), it follows that this damping mechanism can have an effect of order unity only when the \(\alpha \sim 1\) and \(V_A \sim c_s\).

3. Magnetic Buoyancy and the Saturation of the Magnetic Field

One final loophole is that the saturation mechanism for the magnetic field strength might depend on some process which does discriminate between gas pressure and radiation pressure. The proposal by Balbus and Hawley (1991) that the VC instability leads to a runaway growth of the magnetic field, at a rate \(\sim \Omega\), until \(V_A \rightarrow c_s\) does not seem to allow for this possibility. That is, if the local dynamics of the VC instability are responsible for the growth and maintenance of the magnetic field then the fact that this instability is insensitive to the distinction between gas pressure and radiation pressure implies that the dissipation of orbital energy in the disk is also insensitive to this distinction.

On the other hand, the theory that the magnetic field is driven by a separate dynamo mechanism might allow for this possibility. It becomes a question of whether or not the buoyancy of the field depends in some critical way on whether the surrounding pressure is due to radiation or gas. For example, Sakimoto and Coroniti (1989) have claimed that the high thermal conductivity of an AGN disk implies that flux tubes within such a disk would be so buoyant, assuming that dissipation couples to the total pressure, that the loss of magnetic flux due to buoyancy would remove the disk’s magnetic field faster than it could conceivably be regenerated. We will now reëxamine this argument, bearing in mind the somewhat different magnetic field dynamics implied by the VC instability.
We begin by reviewing the critical elements of their argument. They modeled the buoyancy of small isolated flux tubes within a disk and found that if the strength of the field was allowed to scale with the local radiation pressure, then the flux tubes were transparent and buoyancy losses were rapid. The flux tubes were assumed to be almost axisymmetric (that is, $B_r$ much less than $B_\theta$ and both constant within a flux tube) and to rise at a speed limited only by their ability to adjust their internal temperature to match their environment, and by the drag on the flux tube. The latter quantity was set equal to

$$-\left(\frac{C_d}{2\pi}\right)(\pi aL)\rho_e v^2 \frac{v}{|v|},$$

which is appropriate for the drag on a solid tube of radius $a$ and length $L$ moving with a speed $v$ through an approximately inviscid medium with a density $\rho_e$. The constant $C_d$ was set equal to 0.1 in their simulations. Assuming that the thermal conductivity of the fluid is large we can estimate the rise velocity by balancing the drag with the buoyant force

$$\left(\frac{V_A}{c_s}\right)^2 z \Omega^2 \sim \frac{V_{fluxtube}^2}{a}.$$  \hspace{1cm} (3-2)

In other words, magnetic flux will be ejected from the disk at a rate of approximately

$$\frac{V_{fluxtube}}{H} \sim \frac{V_A}{c_s} \Omega \left(\frac{a}{H}\right)^{1/2}.$$ \hspace{1cm} (3-3)

Whether this is fast or not depends somewhat on the size of the flux tube, but unless $a \ll H$ a typical flux tube will be ejected from the disk at the Alfvén speed. In practice the ejection rate can be somewhat higher if the flux tube has a nonzero $B_r$ (as is necessary to obtain an angular momentum flux from the global stress on the field lines). In that case the magnetic field pressure in the flux tube rises quickly in the course of the ejection. On the other hand, for a flux tube of fixed size whose magnetic pressure scales with the gas pressure alone, the thermal conductivity is sharply reduced and the magnetic flux loss rate will be much smaller. Sakimoto and Coroniti used this difference to argue that it isn’t possible to construct self-consistent disk models whose angular momentum transport is due
to the shearing of the magnetic field lines and whose magnetic field pressure is proportional to the total pressure.

The presence of the VC instability affects this argument in a number of ways. First, no global value of $B_r$ is necessarily implied in a model in which angular momentum transport is due to magnetic field stresses. Instead the angular momentum transport is due to transient ripples in the field lines on scales of $V_A/\Omega$. The globally averaged value of $B_r$ depends on the nature of the dynamo mechanism supporting the field, but $B_rB_\theta$ is not a significant source of angular momentum transport. Second, the typical size of magnetic field structures, including flux tubes, is certainly no larger than the scale of the VC induced turbulent eddies, $\sim V_A/\Omega$. For a weak magnetic field this scale will be proportionately smaller. As we have shown in the previous section, this implies that whether the magnetic field pressure scales with the total pressure or just the gas pressure typical structures in the field are always fairly conductive thermally. In neither case is buoyancy greatly hindered by the ability of the flux tubes to drop to temperatures much below the surrounding fluid. Together these two points imply that the distinction suggested by Sakimoto and Coroniti between models with a coupling between dissipation and radiation pressure, and those without, is not a meaningful one. Magnetic buoyancy is a problem for both, or for neither.

In fact, it appears to be a problem for neither, a point which we have addressed in previous papers (Vishniac and Diamond 1992, Vishniac & Diamond 1993). It turns out that the ejection rate given in Eq. (3-3) is a significant overestimate. Briefly put, the fact that the field lines form ripples of size $V_A/\Omega$ at a rate of $\Omega$ implies that a flux tube of size $V_A/\Omega$ will entrain a shifting volume of surrounding material of roughly equal size on a time scale of roughly $\Omega^{-1}$. This implies a drag of approximately $V_{\text{fluxtube}}\Omega$. Equating this to the buoyancy force implies

$$\left(\frac{V_A}{c_s}\right)^2 z\Omega^2 \approx V_{\text{fluxtube}}\Omega,$$  

(3-4)
or a typical upward speed of $\sim V_2 A / c_s$. This in turn implies a magnetic flux loss rate of $\sim (V_A / c_s)^2 \Omega$. For a strong field this is very rapid, and for a weak field this is arbitrarily slow. In any disk dynamo model balancing this rate with the dynamo growth rate will give a saturation level for the magnetic field (and consequently for $\alpha$). Of course, we could have assumed still smaller flux tubes, but then the turbulent entrainment would have been even more effective. Larger flux tubes will self-destruct due to the VC instability.

The bottom line is that the ejection of magnetic flux from a disk is not particularly efficient unless the Alfvén speed is comparable to the local sound speed, or equivalently unless the magnetic field energy density is a large fraction of the total pressure. There is no mechanism that would tend to pin the magnetic field pressure to the gas pressure. This in turn implies that the dissipation of orbital energy in an AGN disk couples to the total pressure, rather than just the gas pressure.

4. Discussion

We have shown that if the Balbus-Hawley mechanism, which relies on the VC instability, is responsible for angular momentum transport in accretion disks, then the associated dissipation scales with the total disk pressure, rather than just the gas component. In itself this is a somewhat negative result. Its implication is that the strong thermal and viscous instabilities found in AGN disks, cannot be due to coupling to an inappropriate pressure. Evidently, the pathological behavior of such disks needs another cure. Exactly what that cure might be is beyond the scope of this paper, but we will mention two possibilities. First, the pathology might be real, and might drive the disk to some extremely inhomogeneous and/or optically thin state. This outcome is implied by the view that the VC instability is a self-sustaining process. Second, the angular momentum transport might not be described
by purely local processes. In this context this would mean that while the VC instability is a
local process responsible for energy dissipation and angular momentum transport, that the
magnetic field dynamo is dependent on nonlocal effects. The wave driven dynamo (Vishniac
et al. 1990, Vishniac and Diamond 1992) is an example of such a process. Its implications
for thermal and viscous instabilities in AGN disks are discussed in Vishniac & Diamond
(1993).

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