Mode splitting transmission effect of surface wave excitation through a metal hole array

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The resonant frequencies of the excited surface waves on a metal hole array with respect to the incident angle were studied in the terahertz region. The experimental and theoretical results demonstrate that the resonant peak of surface wave excitation splits into two when transmitted through a metal hole array off-normally. The high-order mode with resonant frequency above the cutoff frequency $f_c$ (plasma frequency effect) has a shorter attenuation length than that of the low-order mode whose resonant frequency is below $f_c$. The reason is that the high-order mode is a coupled mode consisting of surface wave and hole modes, while the low-order mode is just an excited surface wave (which can be considered as the spoof surface plasmons). Our investigation may open a door to distinguish the spoof surface plasmons and the coupled modes of surface waves and hole modes.

Keywords: angular dependence; cutoff frequency; metal hole arrays; surface waves; terahertz

INTRODUCTION

In recent years, interest in the utilization of terahertz (THz) radiation has increased because of the various applications of THz technology. This trend has led to an increasing demand for THz components that can be used to manipulate THz beams. One of the most promising candidates for use as a component in THz devices is metal hole array (MHA). It has previously been demonstrated that a thin metal plate perforated with an array of periodically spaced holes exhibits bandpass characteristics when irradiated by electromagnetic waves of variable frequency. However, worldwide interest was not sparked until enhanced transmission in the visible region was first experimentally discovered on metal films featuring hole arrays with subwavelength periodicity. Further research theoretically revealed that the physical origin of this enhancement was due to surface plasmon polaritons. At THz frequencies, however, the dielectric properties of metals are those of perfect electric conductors, therefore, there are no surface plasmon polaritons with which to couple. Instead of surface plasmon polaritons, spoof surface plasmon polaritons can form on a metal surface with a designed periodic structure and contribute to an enhancement in transmission. However, the theory of spoof surface plasmon polaritons is limited to arrays with periodicities that are much smaller than the resonant wavelength.

Recently, researchers have found that the main mechanism for enhanced transmission is the excitation of surface waves, which are interfered and scattered by periodically arranged structures. Because MHAs are novel candidates for use as components in THz devices, such as filters, optical switches, sensors, etc., the features of MHA surface waves at normal incidence have been investigated in detail by changing parameters such as the number of holes, the hole diameter, the coating thickness of the dielectric film on the metal surface, etc. In general, this enhanced transmission characteristic is one type of plasmonic property that has been widely studied in THz plasmonic metamaterial systems, which is typically composed of periodically arranged unit cells. More importantly, the mode splitting effect observed in asymmetrical metamaterial systems results from the near-field coupling induced by the strong coupling between a bright eigenmode and a dark eigenmode. For example, a bright eigenmode is supported when two-gap split ring resonators are perfectly symmetric and interfere constructively at an identical frequency. When one of the gaps is off-centered, the resonance frequencies of the resonators differ slightly and mode splitting occurs due to destructive interference.

In this study, to better understand the characteristics of surface waves and mode splitting, the relationship between the incident angle and the transmission of surface waves through a MHA structure was investigated. In contrast with the mode splitting of near-field coupling coming from electromagnetically induced transparency in asymmetrical metamaterial systems, the mode splitting effect observed in this study depends on the in-plane component of the incidence vector. In addition, two splitting modes also have different properties: the high-order mode excited by a coupled mode of surface waves and the hole modes has a shorter attenuation length than that of the low-order mode, which is excited just by the spoof surface plasmons. A theoretical model is established to explain these characters. Our results harness the full potential of MHAs, which can find many applications in THz devices.
MATERIALS AND METHODS

To excite surface waves, we fabricated an aluminum MHA slab as shown in Figure 1a and 1b. Traditional micromachining was used to fabricate the arrays. Using a stepping motor control system, the spacing of the holes can be measured precisely. The MHA slab consisted of a triangular array of circular air holes on a \( t = 250 \) \( \mu \text{m} \) thick, 50 mm \( \times \) 50 mm rectangular aluminum plate. The holes had a diameter of \( d = 0.7 \) mm and were arranged on a hexagonal lattice with a pitch of \( s = 1.13 \) mm. The number of holes was large enough in our structure to avoid the finite size effect.

A transmitted pulse was measured using a THz time domain spectroscopy which was systematically similar to that used in Refs. 25 and 26. A collimated p-polarized THz wave, which was radiated from a 100 fs 800 nm laser pulse pumped photoconductive antenna emitter, was irradiated on the MHA. We rotated the MHA around its \( \Gamma K \) axis, as shown in Figure 1c, to vary the in-plane vector of the incident wave in the \( \Gamma M \) direction. By varying the time delay between the pumping and probe pulses, the waveform of the transmitted THz wave can be measured. An empty air signal was used as a reference for our measurements.

RESULTS AND DISCUSSION

First, for comparison, we studied the MHA at normal incidence (\( \theta = 0^\circ \)). Figure 2a shows the measured transmitted pulse of the reference (air) and MHA at the normal angle. The reference signal (air) was a single-cycle pulse. After inserting the MHA, the signal showed an additional off-plane oscillating attenuation tail, which probably originated from the interaction between the incident waves and the surface waves on the metal surface. To characterize this oscillation, we performed a Fourier transformation to obtain spectra of the frequency domain, as shown in Figure 2b. The resonant peak of the normal incidence is located at 0.265 THz, which agrees well with the results reported in Ref. 10. It is worth noting that the cutoff frequency is 0.251 THz in our structure based on Equation (5) in Ref. 27. According to Pendry’s theory, the cutoff frequency represents the marginal value of the spoof surface plasma frequency. The experimental results show that the resonant frequency is larger than the cutoff frequency. Therefore, spoof surface plasmon polariton theory cannot be applied directly to our MHA at normal incidence.
We then conducted an experimental study on the angular-dependent transmission features of the MHA, the results of which are shown in Figure 3a. We found that by changing the incident angle $\theta$, the resonant peak of the surface waves splits into two peaks, one with a lower frequency and the other with a higher frequency relative to the frequency of the non-split peak at an incident angle of 0°. Moreover, the low-frequency peak moved toward lower frequencies as the incident angle increased. We also performed numerical calculations on the transmission spectra using CST software, inputting the same parameters used in the experiment; the results are shown in Figure 3b. The calculation results show similar behavior to those observed in the experiments. We also measured the transmission spectra by rotating the MHA around the ΓM axis to obtain a variation of the in-plane component in the ΓK direction and obtained similar results.

We attribute this phenomenon to the excitation of surface waves. The surface wave vector will couple with the in-plane vector of the incident wave on the MHA slab at a finite incident angle. To quantitatively describe the transmission spectra that depend on the incident wave vector direction, a surface wave dispersion relationship is used. This dispersion relationship is appropriate for smooth metal/dielectric surfaces, therefore, the influence of the hole arrays was neglected for convenience in performing the calculations. If an incident wave is irradiated on a grating at an angle of $\theta$, the energy and momentum conservation of the coupling effect between the surface wave and the incident wave vector is given by

$$k_{sw} = k_{in} + mG_x + nG_y$$

where $k_{sw}$ is the surface wave vector; $k_{in} = (2\pi \sin \theta/c) \times (1,0,0)$ is the in-plane vector of the incident wave; $m$ and $n$ are integers; $G_x$ and $G_y$ represent the grating momentum, where, for a hexagonal lattice, $G_x = (2\pi/s) \times (2/(3)^{1/2},0,0)$ and $G_y = (2\pi/s) \times (1/(3)^{1/2},1,0)$; $c$ is the velocity of light in vacuum. The surface wave vector amplitude is equal to $|k_{sw}| = n_{sw} \times 2\pi f/c$, where the effective index $n_{sw} = 1$ due to the large permittivity of metals. Thus, we can obtain an equation that describes the conditions under which surface waves are excited:

$$f_{sw} = \frac{2(m^2 + mn + n^2)}{\sqrt{(2m+n)^2 + 3n^2 \cos^2 \theta - (2m+n) \sin \theta}} \times f_0$$

where $f_{sw}$ is the resonant frequency of surface waves and $f_0$ is the lowest surface wave resonance frequency at normal incidence, which is equal to $2\omega/(3)^{1/2}$. This expression generalizes the relationship between $f_{sw}$ and the incident angle $\theta$, where the resonant formula discussed in Refs. 11–13 is a special case ($\theta = 0°$) of this equation. From Equation (2), we can see that the resonant frequency $f_{sw}$ is a function of the incident angle $\theta$. At normal incidence, the resonant frequencies can be calculated as follows:

$$f_{sw} = f_0 \sqrt{m^2 + mn + n^2}$$

where $s = 1.13$ mm and $f_0 = 0.307$ THz at the lowest mode, slightly higher than the peak frequency observed in Figure 2b. This is similar to the results described in Ref. 11.

When the incident angle $\theta$ becomes finite, the lowest mode splits into two branches. These newly emerged branches are excited as a result of the non-zero parallel vector of the incident wave. For an accurate estimate of the shift in surface wave resonances, we analyzed the position of the strongest transmission peak modes, $(-1,0)$ and $(0,-1)/(-1,1)$. Here, the $(0,-1)$ and $(-1,1)$ modes are degenerate, and both have the same dispersion relationship according to Equation (2). The dispersion curves for the surface wave modes $(-1,0)$ and $(0,-1)/(-1,1)$ are displayed in Figure 4. The numerical and experimental results extracted from Figure 3 are also presented.

The experimental results agree well with the numerical results and the results of the calculations performed using Equation (2). From Equation (1), we can also see that $k_{in}$ increases or decreases with the reciprocal vectors, which depend on the incident angle. The in-plane wave vector $k_{in}$ increases with the incident angle $\theta$, leading to a shift in the photonic bands to lower frequencies. The physical origins of this phenomenon lie in the folding of the in-plane Brillouin zone and have been discussed in detail by Ghaemi et al. with respect to the visible

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**Figure 2** (a) The incident and transmitted THz waveform of the MHA at normal incidence. (b) Corresponding Fourier-transformed spectra in the frequency domain. MHA, metal hole array; THz, terahertz.

**Figure 3** Transmission spectra of the MHA at incident angles ranging from 0° to 40° with a step of 10°. For clarity, each curve was arbitrarily shifted vertically by 0.15. (a) Experimental; (b) numerical results. MHA, metal hole array; THz, terahertz.
conditions and the perfect electrical conductor approximation were applied at the resonant frequency with the excitation of surface waves. An

...presented the opposite symmetry. The intensity of the electric field along the z-direction is strongly localized near the circular hole edge and decays gradually along the x-direction away from the metal surface. This is because the zero-order diffraction mode becomes apparent at the resonant frequency with the excitation of surface waves. An attenuation length on the order of the incident wavelength can be achieved even though a perfect electric conductor is used.\textsuperscript{32} Figure 5b and 5c show the electric field distribution $E_x$ of two resonant frequencies, 0.222 THz ((−1,0) mode) and 0.273 THz ((0,−1)/(−1,1) mode), respectively, at an incident angle of 20°. The simulation results also show a clear intensity modulation of $E_x$ along the x-direction within individual holes at both resonant frequencies. However, compared with the $E_x$ at normal incidence, the distributions of $E_x$ on the two surfaces of the hole array interface are asymmetric for the $f=0.222$ THz map, and this asymmetry is slightly stronger for the $f=0.273$ THz map. In addition, the spatial attenuation length of the surface wave at a resonant frequency of $f=0.222$ THz is longer than that of $f=0.273$ THz. This localized effect is clearly illustrated in the (0,−1)/(−1,1) mode map. The differences between Figure 5b and 5c can be explained by the fact that the MHA has its own cutoff frequency $f_c$ for incident wave propagation. When the (−1,0) order wave with resonant frequency $f_{sw}<f_c$ transmits through the MHA, the air holes can be regarded as barriers. Consequently, the incident wave is converted to an evanescent wave and tunnels through the air holes resonantly. On the contrary, for the (0,−1)/(−1,1) mode whose resonant frequency, $f_{sw}$ is $>f_c$, the strong coupling between the surface waves and the hole modes should be the dominant mechanism for the transmitted wave.\textsuperscript{3} Eventually, we could distinguish spoof surface plasmons from coupled modes by modes splitting effect.

CONCLUSIONS

In summary, we succeeded in investigating the resonant properties of surface waves by using a thick metal material. The resonant mode splitting of the surface waves due to the non-zero in-plane vector of the incident wave was observed and theoretically analyzed. The features of two splitting resonances were observed and distinguished by investigating the electric field distribution of these two modes around the hole edge at the peak transmission frequencies. The $E_x$ distribution that was obtained by CST simulation for the peak transmission clearly confirms that the low-order mode, with $f_{sw}<f_c$ shows an attenuation length that is longer than that of the high-order mode, with $f_{sw}>f_c$. The mechanism of this effect is due to the coupling between the surface waves and the hole modes. These results demonstrate that the spoof surface plasmons and the coupled mode consisting of excited surface mode and the hole mode can be easily distinguished, although both of them are bounded in the surface of the MHA. Furthermore, our research reveals the fact that the physical mechanism of modes splitting in MHAs is different from that of metamaterial systems, even though both of these effects are due to the break of symmetry in broad sense.

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Figure 4 Dispersion relationship of modes (−1,0) and (0,−1)/(−1,1). The numerical results were obtained from Figure 3a. The theoretical results (solid line) were calculated using Equation (2). $f_c$ is the cutoff frequency of the MHAs. MHA, metal hole array; THz, terahertz.

Figure 5 Simulated electric field amplitude $E_x$ of transmission maxima located at (a) a resonant frequency of 0.265 THz at normal incidence, (b) the (−1,0) mode at a resonant frequency of 0.222 THz at 20° incidence and (c) the (0,−1)/(−1,1) mode at a resonant frequency of 0.273 THz at 20° incidence. In all three maps, the electric field antinodes of the surface waves are located around the hole corners for the input and output sides of the hole array. This surface wave distribution allows for the transmission of incident light through the hole array. THz, terahertz.
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