Disagreement between capture probabilities extracted from capture and quasi-elastic backscattering excitation functions

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Abstract

Experimental quasi-elastic backscattering and capture (fusion) excitation functions are usually used to extract the s-wave capture probabilities for the heavy-ion reactions. We investigated the $^{16}\text{O}+^{120}\text{Sn}$, $^{144}\text{Sm}$, $^{208}\text{Pb}$ systems at energies near and below the corresponding Coulomb barriers and concluded that the probabilities extracted from quasi-elastic data are much larger than the ones extracted from fusion excitation functions at sub and deep-sub barrier energies. This seems to be a reasonable explanation for the known disagreement observed in literature for the nuclear potential diffuseness derived from both methods.

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Key words: capture cross section, quasi-elastic excitation function, capture probability
I. INTRODUCTION

In the investigation of reaction mechanisms between heavy ions, it is very important to know the diffuseness parameter of the nuclear potential between the colliding nuclei, since it affects the height and shape of the Coulomb barrier, and consequently the cross sections of the reaction mechanisms, particularly the capture or fusion process, for which this barrier has to be overcome. There are two very widely used approaches to derive the nuclear diffuseness parameter from experimental data. The first one is the use of fusion data at near barrier energies. The second approach is to extract this parameter from experimental elastic or quasi-elastic backscattering data. Both approaches should lead to the same value of this parameter. However, presently we find in literature large discrepancies in the nuclear potential diffuseness parameter extracted from the two mentioned analyzes as, for example, was found by Mukherjee et al. [1], Gasques et al. [2], and Evers et al. [3]. In this work we investigate the reasons for such discrepancies.

For the systems investigated in the present work ($^{16}$O + $^{120}$Sn, $^{144}$Sm, and $^{208}$Pb), the fusion process exhausts the capture cross section $\sigma_{cap}(E_{c.m.})$, and, thus, capture and fusion can be considered as similar or even identical processes. Furthermore, for those asymmetric and tightly bound systems, and at energies close or below the Coulomb barrier, the deep inelastic and breakup processes can be neglected, and consequently quasi-elastic process can be defined simply as the sum of elastic, inelastic and transfer processes. So, we have chosen to deal with very simple systems and conditions in the present work.

The present paper is organized as follows. In Sec. II, we present the methods used for the extraction of the s-wave capture probabilities from capture (fusion) and quasi-elastic backscattering excitation functions. The obtained results are given in Sec. III. We then summarize in Sec. IV.

II. EXTRACTION METHODS

A. Capture probabilities from experimental capture excitation function

We start with the capture cross section. The physical meaning of the first derivative of the function $E_{c.m.}\sigma_{cap}(E_{c.m.})$, in respect to the energy $E_{c.m.}$, can be elucidated by considering the penetration probabilities for different partial waves $J$. One can approximate the $J$
dependence of the transmission probability \( P_{\text{cap}}(E_{\text{c.m.}}, J) \), at a given \( E_{\text{c.m.}} \), by simply shifting the energy, as it was done recently by Sargsyan et al. \( \text{[4, 5]} \):

\[
P_{\text{cap}}(E_{\text{c.m.}}, J) \approx P_{\text{cap}}(E_{\text{c.m.}} - \frac{\hbar^2 \Lambda}{2\mu R_b^2} - \frac{\hbar^4 \Lambda^2}{2\mu^3 \omega^2 R_b^6}, J = 0),
\]

(1)

where \( \Lambda = J(J + 1) \), \( R_b = R_b(J = 0) \) is the position of the Coulomb barrier at \( J = 0 \), \( \mu = m_0 A_1 A_2 / (A_1 + A_2) \) is the reduced mass (\( m_0 \) is the nucleon mass), and \( \omega_b \) is the curvature of the s-wave potential barrier. Here, we use the same procedure of Sargsyan et al. \( \text{[4, 5]} \) for the expansion of the height \( V(R_b, J) = V_b(J) \) of the Coulomb barrier up to second order in \( \Lambda \):

\[
V_b(J) = V_b(J = 0) + \frac{\hbar^2 \Lambda}{2\mu R_b^2} + \frac{\hbar^4 \Lambda^2}{2\mu^3 \omega^2 R_b^6}.
\]

(2)

Now, if we use the formula for the capture cross section,

\[
\sigma_{\text{cap}}(E_{\text{c.m.}}) = \pi \lambda^2 \sum_{J=0}^{J_{\text{cr}}} (2J + 1) P_{\text{cap}}(E_{\text{c.m.}} - \frac{\hbar^2 \Lambda}{2\mu R_b^2} - \frac{\hbar^4 \Lambda^2}{2\mu^3 \omega^2 R_b^6}, J = 0),
\]

(3)

convert the sum over the partial waves \( J \) into an integral, and express \( J \) by the variable \( E = E_{\text{c.m.}} - \frac{\hbar^2 \Lambda}{2\mu R_b^2} \), we obtain the following simple expression \( \text{[4, 5]} \):

\[
\sigma_{\text{cap}}(E_{\text{c.m.}}) = \frac{\pi R_b^2}{E_{\text{c.m.}}} \int_{E_{\text{c.m.}} - \frac{\hbar^2 \lambda}{2\mu R_b^2}}^{E_{\text{c.m.}}} dE P_{\text{cap}}(E, J = 0)[1 - \frac{4(E_{\text{c.m.}} - E)}{\mu \omega^2 R_b^2}],
\]

(4)

where \( \lambda^2 = \hbar^2 / (2\mu E_{\text{c.m.}}) \) is the reduced de Broglie wavelength and \( \Lambda_{\text{cr}} = J_{\text{cr}}(J_{\text{cr}} + 1) \). For values \( J \) larger than the critical angular momentum \( J_{\text{cr}} \), the potential pocket in the nucleus-nucleus interaction potential vanishes and the capture does not occur. To calculate \( J_{\text{cr}} \) and \( R_b \), we use the nucleus-nucleus interaction potential \( V(R, J) \) of Ref. \( \text{[6, 7]} \). For the nuclear part of the nucleus-nucleus potential, the double-folding formalism with the Skyrme-type density-dependent effective nucleon-nucleon interaction is used. For the systems that we investigate in the present work, with \( Z_1 \times Z_2 < 2000 \), where \( Z_{1,2} \) are the atomic numbers of interacting nuclei, the critical angular momentum \( J_{\text{cr}} \) is large (from 54 to 62), \( P_{\text{cap}}(E_{\text{c.m.}}, J = 0) \gg P_{\text{cap}}(E_{\text{c.m.}} - \frac{\hbar^2 \Lambda}{2\mu R_b^2}, J = 0) \) for energies around and below the barrier, and the factor \( 1 - \frac{4(E_{\text{c.m.}} - E)}{\mu \omega^2 R_b^2} \) in Eq. (4) very weakly influences the results of the calculations at this energy range \( \text{[4]} \). Therefore, Eq. (4) can be approximated as

\[
\sigma_{\text{cap}}(E_{\text{c.m.}}) = \frac{\pi R_b^2}{E_{\text{c.m.}}} \int_0^{E_{\text{c.m.}}} dE P_{\text{cap}}(E, J = 0).
\]

(5)
Multiplying this equation by $E_{\text{c.m.}}/(\pi R_b^2)$ and differentiating over $E_{\text{c.m.}}$, one obtains

$$\frac{d}{dE_{\text{c.m.}}} [E_{\text{c.m.}} \sigma_{\text{cap}}(E_{\text{c.m.}})] = \frac{1}{\pi R_b^2} P_{\text{cap}}(E_{\text{c.m.}}, J = 0) = 1$$

(6)

From Eq. (6) one can observe that $\frac{d}{dE_{\text{c.m.}}} [E_{\text{c.m.}} \sigma_{\text{cap}}(E_{\text{c.m.}})]$ has a physical interpretation in terms of the $s$-wave transmission in the entrance channel, and therefore the $s$-wave transmission probability can be extracted with a good accuracy from the experimental capture cross sections $\sigma_{\text{cap}}(E_{\text{c.m.}})$ at energies near and below the Coulomb barrier. There are other methods to derive Eq. (6), as it was previously done by Balantekin et al. [8].

**B. Capture probabilities from experimental quasi-elastic backscattering data**

Now lets consider the quasi-elastic scattering at backward angles. For reactions involving only tightly bound nuclei at low energies, one can write the direct relationship between capture and backward quasi-elastic scattering probabilities as

$$P_{\text{qe}}(E_{\text{c.m.}}, J) + P_{\text{cap}}(E_{\text{c.m.}}, J) = 1$$

(7)

This relation is due to the conservation of the reaction flux, since any loss from the backward quasi-elastic scattering channel contributes directly to the capture and vice-versa [4, 5, 9]. For experimentalists, usually it is much easier and simpler to measure quasi-elastic scattering than capture (fusion). By this reason, using Eq. (7) one finds the relation

$$P_{\text{cap}}(E_{\text{c.m.}}, J = 0) = 1 - \frac{d\sigma_{\text{qe}}(E_{\text{c.m.}})}{d\sigma_{\text{Ru}}(E_{\text{c.m.}})}$$

(8)

which is well suited for the extraction of the $s$-wave capture probability from the experimental quasi-elastic backscattering probability $d\sigma_{\text{qe}}/d\sigma_{\text{Ru}}$. In Eq. (8) the $P_{\text{qe}}(E_{\text{c.m.}}, J = 0) = d\sigma_{\text{qe}}/d\sigma_{\text{Ru}}$ was assumed to be the ratio of the quasi-elastic scattering differential cross section and Rutherford differential cross section at 180 degrees [10]. However, experimentally it is not possible to take quasi-elastic data at 180 degrees, but rather at backward angles in the range from 150 to 170 degrees. So, the corresponding center of mass energies have to be corrected by the centrifugal potential at the experimental angle [10].

**III. RESULTS OF CALCULATIONS**

From Eqs. (6) and (8) one observes that the $P_{\text{cap}}(E_{\text{c.m.}}, J = 0)$ could be extracted either from the experimental capture or fusion [Eq. (6)] or from quasi-elastic backscattering [Eq.
excitation functions. One could also say that the proposed methods of extracting the s-wave transmission probabilities are almost model-independent. In the following we show the results obtained by both methods, for the three systems under investigation.

In Fig. 1 we show the results for the $^{16}\text{O} + ^{120}\text{Sn}$ system. One can see a good agreement between the extracted probabilities for the $^{16}\text{O} + ^{120}\text{Sn}$ reaction by both methods [Eqs. (6) and (8)] at energies near the Coulomb barrier, but there is disagreement at deep sub-barrier energies. In Fig. 2 we show the results for the $^{16}\text{O} + ^{208}\text{Pb}$ system. Now, the deviations are dramatic, even at near barrier energies, and they increase with decreasing energy under the barrier. Figure 3 shows the results for the $^{16}\text{O} + ^{144}\text{Sm}$ system. Again, one can observe disagreement even at energies not too much below the barrier. The capture probabilities (closed squares) extracted from the experimental capture data [16] were shifted by 1 MeV to the lower energies, in order to try to understand the mismatching between the probabilities extracted from the experimental quasi-elastic backscattering and capture (fusion) data. Indeed, one finds an improvement at near barrier energies, but the disagreement is still important at deep sub-barrier energies.

So, clearly there is a mismatch between quasi-elastic backscattering and fusion (capture) experimental data. The explanation of this disagreement is, therefore, required, either experimentally or theoretically. One consequence of the overestimation of the capture probability at deep sub-barrier energies when one uses quasi-elastic backscattering data, in comparison with those when one uses fusion cross section data, is that the nuclear potential diffuseness parameter extracted from quasi-elastic scattering data should be larger than that extracted from the fusion excitation function data. This is, indeed, what has been reported in literature [2, 3]. Since the theoretical predictions agree with the experimental capture (fusion) cross sections for these systems (for example, see Ref. [7]), we might suspect that a possible reason for the overestimation of the capture probability from the experimental quasi-elastic data at deep sub-barrier energies is the underestimation of the total reaction differential cross section, that is taken as the Rutherford differential cross section at this energy regime.

IV. CONCLUSIONS

We have found an overestimation of the s-wave capture probability, at very low energies, extracted from the experimental quasi-elastic backscattering data with respect to that ex-
tracted from the experimental capture (fusion) excitation function. Then, it is not surprising that there are reported large discrepancies in the nuclear potential diffuseness parameters extracted from the analyzes of the experimental quasi-elastic (or elastic) backscattering and capture (fusion) data. We suggest that it is desirable to have experimental efforts to measure precisely quasi-elastic backscattering excitation functions, as well as further theoretical investigation on this subject. Our study may be useful for current experimental activities in the field, as it puts together different processes.

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FIG. 1: (Color online) The extracted s-wave capture probabilities for the $^{16}\mathrm{O} + ^{120}\mathrm{Sn}$ reaction by employing Eqs. (6) [squares] and (8) [circles]. The used experimental quasi-elastic backscattering and capture (fusion) excitation functions are from Ref. [11].
FIG. 2: (Color online) The extracted s-wave capture probabilities for the $^{16}$O + $^{208}$Pb reaction by employing Eqs. (6) [squares] and (8) [circles]. The used experimental quasi-elastic backscattering data are from Refs. [12]. The used experimental capture (fusion) excitation functions are from Refs. [13] (open squares) and [14] (closed squares).

FIG. 3: (Color online) The extracted s-wave capture probabilities for the $^{16}$O + $^{144}$Sm reaction by employing Eqs. (6) [squares] and (8) [circles]. The experimental quasi-elastic backscattering and capture (fusion) excitation functions from Refs. [10] and [15, 16], respectively. The capture probabilities (closed squares) extracted from the experimental capture (fusion) excitation function [16] are shifted by 1 MeV to the lower energies (see text).