Accuracy of the numerical computation of solar g modes

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From the recent work of the Evolution and Seismic Tools Activity (ESTA, Monteiro et al. 2006; Lebreton et al. 2008), whose Task 2 is devoted to compare pulsational frequencies computed using most of the pulsational codes available in the asteroseismic community, the dependence of the theoretical frequencies with non-physical choices is now quite well fixed. To ensure that the accuracy of the computed frequencies is of the same order of magnitude or better than the observational errors, some requirements in the equilibrium models and the numerical resolutions of the pulsational equations must be followed. In particular, we have verified the numerical accuracy obtained with the Saclay seismic model, which is used to study the solar g-mode region (60 to 140 µHz). We have compared the results coming from the Aarhus adiabatic pulsation code (ADIPLS), with the frequencies computed with the Granada Code (GraCo) taking into account several possible choices. We have concluded that the present equilibrium models and the use of the Richardson extrapolation ensure an accuracy of the order of 0.01 µHz in the determination of the frequencies, which is quite enough for our purposes.

1 Introduction

The interior of the Sun has been very well studied thanks to the information provided by pressure-driven modes (p modes). In the case of the dynamics of the solar interior, due to the very small number of non-radial p modes penetrating inside the core, neither the rotation profile (e.g., Chaplin et al. 1999; Thomson et al. 2003; García et al. 2001, 2004, 2008c) nor the dynamical processes (e.g., Mathis & Zahn 1999, see Moya et al. 2008) to the solar case and the calculation of the g-mode frequencies of the Sun. It is possible to fix the global uncertainties of the numerical schemes used (Moya et al. 2010a).

Gravity modes have been searched for a long time, almost since the beginning of helioseismology (e.g. Hill et al. 1991; Pallé 1991). But there is currently no undisputed detection of individual g modes for the Sun (Appourchaux et al. 2010). However, some peaks (e.g., Gabriel et al. 2002; Jiménez & García 2009) and groups of peaks (Turck-Chièze et al. 2004; García et al. 2008a) have been considered as reliable g-mode candidates as they are above 90% confidence level and they present several of their expected properties. Moreover, to increase the probability of detection, García et al. (2007, 2008b) searched for the global signature of such modes instead of looking for individual g modes. They have found the signature of the asymptotic-dipole g modes with more than 99.99% confidence level. The detailed study of this asymptotic periodicity revealed a higher

rotation rate in the core than in the rest of the radiative region and a better agreement with solar models (García et al. 2008b) computed with old-surface abundances (Grevesse, Noels, & Sauval 1993) compared to the new ones (Asplund, Grevesse & Sauval 2005). However, it was not possible to identify the sequence of individual peaks generating the detected signal because of the very small signal-to-noise ratio. Thus, to go further it is necessary to use theoretical g-mode predictions to guide our search (Broomhall et al. 2007; García 2010). For this purpose, we need to know the limits of the modeled physical processes and quantities as well as the internal numerical errors of the codes used to compute the predicted frequencies.

The accuracy of the present solar models has already been studied by Mathur et al. (2007) and Zaatri et al. (2007). They showed that models with different physical inputs and fixed surface abundances present differences in the frequencies of the g modes that are below 1 µHz in the range [60, 140] µHz. In the present work we study the numerical errors introduced by the approaches followed by the oscillation codes used to compute the g-mode frequencies of the Sun. It is a direct application of the study done in the ESTA group (whose Task 2 is devoted to the pulsational code’s comparison, see Moya et al. 2008) to the solar case and the calculation of the g-mode frequencies. This comparison makes it possible to fix the global uncertainties of the numerical schemes used (Moya et al. 2010a).
2 Modeling the Sun Interior: Computing \( p \) and \( g \) Modes

2.1 Solar Model

We have computed one solar model, which is based on the Seismic model developed by the Saclay team (Turck-Chièze et al. 2001; Couvidat et al. 2003). It is a 1-D model computed with the so called Code d’Evolution Stellaire Adaptatif et Modulaire (CESAM, Morel 1997). This solar model was tuned to better match helioseismic observations (e.g. the sound speed profile), especially in the radiative region. This model was also used to more accurately predict the neutrino fluxes. It is calibrated in terms of surface metallicity, luminosity, and radius at the age of 4.6 Gyrs with an accuracy of \( 10^{-5} \).

As one of the main aims of this study consists of calculating the frequencies of \( g \) modes, and these modes are mainly confined in the inner regions of the Sun, we have computed the model using the highest resolution in the core of the Sun (below 0.5 \( R_{\odot} \)).

2.2 Pulsation Codes Brief Description: Aarhus and GraCo

To calculate the \( p \)- and \( g \)-mode frequencies we have used two oscillation codes: the Granada Code (GraCo) and the Aarhus adiabatic pulsation code (ADIPLS), both being part of the ESTA group.

GraCo (Moya et al. 2004; Moya & Garrido 2008) is a non-radial non-adiabatic linear pulsational code using a second-order integration scheme to solve the set of differential equations. The code also provides frequencies in the adiabatic approximation. This code has been used as a reference code for the ESTA study. Thus all of the possible possibilities explained in the text except the case not using of the Richardson extrapolation are shown (see text for details).

The ADIPLS code (Christensen-Dalsgaard 2008) is one of the first and most used adiabatic pulsation codes in the world. It is also based on a second-order integration scheme, and it uses the Eulerian variation of the pressure as eigenfunction. We have used the relaxation method where the equations are solved together with a normalization condition. The frequencies are found by iterating on the outer boundary condition. The calculation does not use the Cowling approximation. Finally, to compute these frequencies, we have remeshed the model onto 2400 points and extrapolated the parameters below 0.05 \( R_{\odot} \).

3 Results

Using these two codes, we have obtained the adiabatic frequency spectrum of the Saclay equilibrium model. A complete comparison following the work done by the ESTA group has been carried out. For this purpose, the global characteristics of the numerical resolution for the ADIPLS code’s results have been fixed, that is: the use of the Richardson extrapolation, \( P' \) as eigenfunction and, \( \ln r \) as integration variable. The frequencies obtained with ADIPLS are our reference frequencies.

On the other hand, the frequencies from GraCo have been obtained using all of the possible options for the numerical resolution (see Moya et al. 2008).

In Figure 1, we first show, an overview of the complete frequency spectrum. The ADIPLS frequencies are the reference. The differences obtained with all of the possibilities explained in the text except the case not using of the Richardson extrapolation are shown (see text for details).

Fig. 1 Overview of the differences found throughout the complete frequency spectrum. The ADIPLS frequencies are the reference. The differences obtained with all of the possibilities explained in the text except the case not using of the Richardson extrapolation are shown (see text for details).
iii) When the Richardson extrapolation is not used, the differences found are in the range $[0.01, 0.08]$ $\mu$Hz, which can be up to four times bigger than the other comparisons.

iv) The rest of the possible choices for the numerical resolution provide differences in the range $[-0.02, 0.02]$ $\mu$Hz. This is similar to the range obtained when the same configuration is used in both codes.

### 4 Conclusions

The search for $g$ modes has been a long quest as they would be the best probes of the solar core, thus representing a huge potential to better constrain its structure and dynamics. Up to now, a few candidates have been detected and recently the global properties of dipole $g$ modes have been detected with more than 99% confidence level. The next step in the search for individual $g$ modes would consist in being guided by the theoretical predictions of their frequencies obtained with an oscillation code for a given solar model.

This is the reason why the accuracy of the frequencies calculated with numerical algorithms is important. In this paper we have taken the advantage of the previous studies of the ESTA group and we have tested the accuracy of the equilibrium model used for the search of $g$ modes in the Sun under changes of methodology in the numerical integration of the pulsational equations. A model based on the Saclay-Seismic model has been used as an input to the pulsational codes ADIPLS and GraCo. Two comparisons have been studied: i) an overview of the differences obtained along the complete frequency spectrum and, ii) a special analysis of the $g$-mode region $[60, 140]$ $\mu$Hz.

This first comparison has shown that among the different methodologies in several zones of the spectrum, the present equilibrium model provides differences of the order of $0.1$ $\mu$Hz. This also happens when the same choices are used for both codes. This means that we need a larger number of mesh points if we want to accurately fit the observed frequencies in these regions.

On the other hand, the $g$-mode region presents an accuracy of the order of $\pm 0.02$ $\mu$Hz for any methodology choice when the Richardson extrapolation is used which is much better than the uncertainties given by the physical prescriptions used in the models. This study has shown that, if we want to pursue a search related to observed $g$ modes in this region and based on theoretical models, the numerical accuracy of the Saclay-Seismic model and the use of either ADIPLS or GraCo codes, in terms of number of mesh points and numerical accuracy of sensitive quantities, are enough. Thus, such a guided search will be mainly sensitive to uncertainties coming from the physical inputs of the models.

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