Evolutionary population synthesis: models, analysis of the ingredients and application to high-$z$ galaxies

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ABSTRACT
Evolutionary population synthesis models for a wide range of metallicities, ages, star formation histories, initial mass functions, and Horizontal Branch morphologies, including blue morphologies at high metallicity, are computed. The model output comprises spectral energy distributions, colours, stellar M/L ratios, bolometric corrections, and near-infrared spectral line indices. The energetics of the post Main Sequence evolutionary phases are evaluated with the fuel consumption theorem. The impact on the models of the stellar evolutionary tracks (in particular with and without overshooting) is assessed. We find modest differences in synthetic broad-band colours as induced by the use of different tracks in our code (e.g., $\Delta(V-K) \sim 0.08$ mag; $\Delta(B-V) \sim 0.03$ mag). Noticeably, these differences are substantially smaller than the scatter among other models in the literature, even when the latter adopt the same evolutionary tracks. The models are calibrated with globular cluster data from the Milky Way for old ages, and the Magellanic Clouds plus the merger remnant galaxy NGC 7252, both for young ages of $\sim 0.1 - 2$ Gyr, in a large wavelength range from the $U$-band to the $K$-band. Particular emphasis is put on the contribution from the Thermally-Pulsing Asymptotic Giant Branch phase. We show that this evolutionary phase is crucial for the modelling of young stellar populations by the direct comparison with observed spectral energy distributions of Magellanic Clouds clusters, which are characterised by relatively high fluxes both blueward and redward the $V$-band. We find that the combination of the near-IR spectral indices $C_2$ and $H_2O$ can be used to determine the metallicity of $\sim 1$ Gyr stellar populations. As an illustrative application, we re-analyze the spectral energy distributions of some of the high-$z$ galaxies ($2.4 < z < 2.9$) observed with the Spitzer Space Telescope by Yan et al. (2004). Their high rest-frame near-IR fluxes are reproduced very well with the models including Thermally-Pulsing Asymptotic Giant Branch stars for ages in the range $\sim 0.6 - 1.5$ Gyr, suggesting formation redshifts for these objects around $z \sim 3 - 6$.

Key words: stars: evolution - stars: AGB and post-AGB galaxies: evolution - galaxies: stellar content - cosmology: early universe

1 INTRODUCTION
The evolutionary population synthesis (EPS) is the technique to model the spectrophotometric properties of stellar populations, that uses the knowledge of stellar evolution. This approach was pioneered by B. Tinsley in a series of fundamental papers, that provide the basic concepts still used in present-day computations. The models are used to determine ages, element abundances, stellar masses, stellar mass functions, etc., of those stellar populations that are not resolvable in single stars, like galaxies and extragalactic globular clusters. Due to the ubiquitous astrophysical applications of EPS models, a rich literature has been developed so far (Bruzual 1983; Renzini & Buzzoni 1986; Chiosi, Bertelli and Bressan 1988; Buzzoni 1989; Charlot & Bruzual 1991; Bruzual & Charlot 1993; Worthey 1994; Vazdekis et al. 1996; Tantalo et al. 1996; Fioc & Rocca-Volmerange 1997; Bressan, Granato & Silva 1998; Maraston 1998; Leitherer et al. 1999; Brocato et al. 2000; Thomas, Maraston & Bender 2003; Thomas, Maraston & Korn 2004).

In the simplest flavour of an EPS model, called Simple Stellar Population (hereafter SSP) by Renzini (1981), it is assumed that all stars are coeval and share the same chemical composition. The advantage of dealing with SSPPs is twofold. First, SSPs can be compared directly with globular cluster (GC) data, since these are the “simplest” stellar populations in nature. This offers the advantage of calibrating the SSPs with those GCs for which ages and element abundances are independently known, an approach introduced in the review by Renzini & Fusi Pecci (1988). This step is
crucial to fix the parameters that are used to describe that part of the model “input physics” - convection, mass loss, mixing - that cannot be derived from first principles. The calibrated models can be applied with more confidence to the study of extragalactic stellar populations. This step is taken in the models of Maraston (1998) and in their extension presented here. Second, complex stellar systems which are made up by various stellar generations are modelled by convolving SSPs with the adopted star formation history (e.g. Tinsley 1972; Arimoto & Yoshii 1986; Rocca-Volmerange & Guiderdoni 1987; Vazdekis et al. 1996; Kodama & Arimoto 1997; Barbaro & Poggianti 1997; Bruzual & Charlot 2003). Therefore the deep knowledge of the building blocks of complex models is very important.

Two techniques are adopted to compute SSP models, which differ according to the integration variable adopted in the post-Main Sequence: isochrone synthesis and ‘fuel consumption based’ algorithms. With the ‘isochrone synthesis’ (e.g. Chiosi, Bertelli & Bressan 1988; Charlot & Bruzual 1991) the properties of a stellar population are calculated by integrating the contributions to the flux in the various passbands of all mass-bins along one isochrone, after assuming an Initial Mass Function (IMF). Usually isochrones are computed up to the end of the Early Asymptotic Giant Branch phase. Later stellar phases like the Thermally-Pulsing Asymptotic Giant Branch are added following individual recipes or are neglected.

In the ‘fuel consumption’ approach (Renzini 1981; Renzini & Buzzoni 1986; Buzzoni 1989; Maraston 1998), the integration variable in Post Main Sequence is the so-called fuel, that is the amount of hydrogen and/or helium that is consumed via nuclear burning during a given post Main Sequence phase. The fuel at a given age is computed on the stellar evolutionary track of the turnoff mass (i.e., the mass completing the hydrogen-burning phase), thereby neglecting the dispersion of stellar masses in post main sequence. However since a mass difference of only few percent exists between the turnoff mass and the mass at any other post-Main Sequence phase, this assumption can be made safely, as also shown by Charlot & Bruzual (1991). The advantages of the fuel consumption approach are of two kinds. First, the fuel as integration variable is very stable since it is directly proportional to the contributions of the various phases to the total luminosity. This is very important in luminous, but short-lived evolutionary stages, e.g. the bright Red Giant Branch phase, where the evolutionary mass practically does not change. We note that the problem of the numerical instability on the RGB was early recognized by Tinsley & Gunn (1976). Second, and more important, there are several relevant stellar phases (e.g. blue Horizontal Branch, Thermally Pulsing Asymptotic Giant Branch, very hot old stars etc.) whose theoretical modeling is uncertain because of mass loss and for which complete stellar tracks are not available. The fuel consumption provides useful analytical relations that link the Main Sequence to the post Main Sequence evolution, by means of which one can include into the synthesis the energy contributions of these uncertain phases using, e.g. observations. The ‘isochrone synthesis’ technique is used in all models in the literature, with the exception of the models by Buzzoni (1989), Maraston (1998) and those presented here, that adopt the fuel consumption theorem.

Besides the method used to compute them, evolutionary population synthesis models keep the uncertainties inherent in the stellar evolutionary tracks and in the spectral transformations. Charlot, Worthey & Bressan (1996) investigated both classes of uncertainties by comparing their EPS models. The analysis is very illustrative for the three considered EPS, however does not yield information about the sole impact of the stellar tracks. Here we make a different exercise, because we can use the same EPS code and just vary the model ingredients. In this way we can isolate their impact on the final model. To quantify the model uncertainties is indeed very important since the cosmological inferences that are derived on the basis of galaxy ages and metallicities rely ultimately on the stellar population models.

Maraston (1998) presents a fuel-consumption-based code for evolutionary population synthesis, and SSP models for solar metallicity, and ages from 30 Myr to 15 Gyr. Distinct features of that work are:

(i) the extension of the fuel consumption theorem to compute models for young and intermediate-range ages;
(ii) the inclusion of a well-calibrated semi-empirical Thermally-pulsing Asymptotic Giant Branch phase, and the computation of realistic colors of intermediate-age ($t \lesssim 2$ Gyr) stellar populations
(iii) the modular structure of the code, that allows experiments with the EPS ingredients.

Subsequently the code has been updated to the computations of SSP models covering a wide range in metallicities and the model output have been extended to e.g. the spectral energy distributions, spectral indices, redshift evolution, with several applications being already published (Maraston & Thomas 2000; Saglia et al. 2000; Maraston et al. 2001; Maraston et al. 2003; Thomas, Maraston & Bender 2003; Thomas, Maraston & Korn 2004; Ferraro et al. 2004). This work is devoted to discuss comprehensively the overall EPS models and ingredients, in particular, the inclusion of the TP-AGB phase in the synthetic spectral energy distributions.

The paper is organized as follows. In Section 2 the properties of the evolutionary population synthesis code are recalled. The model ingredients, i.e. the fuel consumptions, the temperature distributions and the model atmospheres, for the various metallicities and ages, are described in detail in Section 3. In particular, the recipes for Horizontal Branch and Thermally Pulsing Asymptotic Giant Branch are presented in Section 3.4. The various model output are discussed in Section 4, where the comparisons with observational data and with models from the literatures are also presented. Section 5 deals with the model uncertainties, while Section 6 presents an high-redshift application of the model SEDs in which the TP-AGB has a primary importance. Finally a summary of the main results is given in Section 7.
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Each mass bins along a isochrone, from a lower mass limit $M_{\text{inf}}$ (usually 0.1 $M_{\odot}$) to the current turnoff mass $M_{\text{TO}}(t)$, having assumed an IMF $\psi(M)$

$$L^{\text{bol}}_{\text{MS}}(t; [Y, Z]) = \int_{M_{\text{inf}}}^{M_{\text{TO}}} L(M(t; [Y, Z]))\psi(M)dM$$

The PMS luminosity contributions are computed by means of the Fuel Consumption Theorem (RB86)

$$L^{\text{bol}}_{\text{PMS}}(t; [Y, Z]) = 9.75 \times 10^{10} b(t) \sum_j \text{Fuel}_j(M_{\text{TO}}(t; [Y, Z]))$$

where the evolutionary flux $b(t; [Y, Z])$,

$$b(t; [Y, Z]) = \psi(M_{\text{TO}}(t; [Y, Z]))|\dot{M}_{\text{TO}}|$$

provides the rate of stars evolving to any PMS phase $j$ at the age $t$ of the stellar population, and $\text{Fuel}_j(M_{\text{TO}}(t; [Y, Z]))$ is the amount of stellar mass to be converted in luminosity in each of these phases. The multiplicative factor in Eq. 3 stems from expressing the luminosity in solar units, the evolutionary flux $b(t)$ in years and the fuel in solar masses through the Einstein equation $E = \Delta M \cdot c^2$, with $\Delta = 6.55 \times 10^{-3}$. The latter is derived by considering that the transformation of 1 g of H into He releases $\sim 5.9 \times 10^{18}$ erg. This average value takes into account the dependence on whether the CNO cycle or the pp chain is at work and the different neutrino losses as a function of the temperature of the burning (Renzini 1981).

3 INGREDIENTS

Following M98, the ingredients of evolutionary synthesis models are:

(i) The energetics: mass-luminosity relations for the MS and fuel consumptions for PMS phases;

(ii) The surface parameters: the effective temperatures and surface gravities of the evolutionary phases;

(iii) The transformations to observables: spectra, or colours and bolometric corrections as functions of gravity and temperature, to convert the bolometric luminosity into a spectral energy distribution.

The key feature of the code is to have the three ingredients being allocated in three independent sets of matrices. This is very convenient as the code can be used to understand the impact of the various input selectively on the final result. We will use this structure to understand the discrepancies between EPS models that are based on different stellar evolutionary tracks. The adopted ingredients are described in the next subsections.

3.1 Energetics

The first matrix contains the energetics, i.e. the luminosities of MS stars and the fuel consumptions of PMS phases. In general, both are taken from stellar evolutionary models, except for those stellar phases that are poorly understood (Section 3.5), for which the energetics are estimated semi-empirically, by means of observations and with the aid of the fuel consumption theorem.

3.1.1 Stellar models

The bulk of input stellar models (tracks and isochrones) is from Cassisi et al. (1997a,b; 2000; see also Bono et al. 1997). Their main features are summarized in the following. These are canonical stellar evolutionary tracks, i.e. the efficiency of the overshooting parameter is assumed to be zero. The actual size of the overshooting is a matter of debate since several years and work is in progress to calibrate the overshooting parameter with observational data (Bertelli et al. 2003; Woo et al. 2003). These articles favour moderate amounts of overshooting, but the results on different stellar evolutionary tracks are discrepant. The Cassisi tracks are used to compute models we will refer to as standard SSP models. The choice of these tracks as basis is due to the following reasons: i) extensive calibrations with galactic globular clusters (GCs) have been performed (Cassisi & Salaris 1997; Cassisi, Degr`I‘nocenti, Salaris 1997; De Santis & Cassisi 1999); ii) tracks (isochrones) are provided with very fine time (mass) spacing (e.g., a typical Red Giant Branch track contains $\sim 5000$ models) which is essential to perform good numerical integrations; iii) these tracks are the closest to the ones (from Castellani, Chieffi & Straniero 1992) that were adopted for the solar metallicity models presented in M98. For sake of homogeneity, the M98 models have been re-computed with the solar metallicity tracks of the Cassisi’s database. Minimal differences have been found, that are due to the temperature/colour transformations rather than to the stellar tracks. The metallicity of the Cassisi tracks range from $Z_{\odot}/200$, typical of the Milky Way halo to $2Z_{\odot}$, the helium enrichment law being $\Delta Y/\Delta Z \sim 2.5$. In order to extend the metallicity range, we implement a set of tracks with 3.5 solar metallicity, and the same $\Delta Y/\Delta Z$, from the Padova database (see below). The exact values of helium and metals for the SSP grid are tabulated in the section presenting the results.

Most SSP models in the literature are based on the tracks by the Padova group (Fagotto et al. 1994; Girardi et al. 2000; Salasnich et al. 2000). Therefore it is interesting to explore the effects of other stellar evolutionary tracks, on the ages and metallicities inferred for real stellar populations. To this aim several SSPs have been computed by means of the Padova stellar models. The various comparisons will be shown in Section 3. The issue is a very important one as at metallicities above solar, i.e. in the range more relevant to massive galaxies, the calibration of the tracks is hampered by the lack of GCs with ages and chemical compositions known independently.

Finally the solar metallicity isochrones/tracks of the Geneva database (Schaller et al. 1992; Meynet et al. 1994) are adopted in order to compute very young SSPs ($10^{-3} \leq t/\text{Myr} < 30$).

3.1.2 Main Sequence: Mass Luminosity relations

Isochrones are adopted up to the turnoff, and the MS luminosity contributions are evaluated by means of Equation 2. Therefore the results depend on the mass-luminosity relations of the isochrones. Figure 4 compares the mass-luminosity relations of the isochrones from Cassisi and Padova (solid and dotted lines, respectively) for various ages and metallicities. In general, a fairly good agreement is found for masses $\gtrsim 0.5 M_{\odot}$, independent of the metallicity, while the low MS of the Padova tracks with high metallicity (solar and above) is brighter than that of the Cassisi tracks, by nearly a factor 2. However this effect is not important, because the contribution of the low MS to the total light is very small, unless the stellar population has a very steep IMF (e.g. $\gtrsim 3.5$ in the notation in which the Salpeter exponent is 2.35, M98), so that its light is dwarf dominated. Noticeable is instead the effect of overshooting, because of which the Padova MS has a turnover mass at given age that is larger than that of canonical tracks (e.g. at 3 Gyr and solar metallicity the turnover masses are $1.45 M_{\odot}$ and 1.37 $M_{\odot}$, respectively). Stel-
lar models with overshooting have more massive convective cores, therefore they run to higher luminosities and live longer than classical models. This effect stems from the higher fuel for given mass when overshooting is considered, which prolongs the MS lifetime. The effect lasts until the mass has a convective core on the MS, i.e. it disappears for \( M \lesssim 1 \, M_\odot \).

As to metallicity effects, at given mass a higher metal content makes the star fainter, because of the combined effects of the less amount of hydrogen and the higher opacity. For example, a \( 0.8 \, M_\odot \) star with \( 2 \, Z_\odot \) metallicity is a factor 3 fainter on the MS than one with the same mass but metallicity \( Z_\odot/200 \). Instead, there is no difference between the solar and twice-solar metallicity MS relations. This comes from the fact that helium increases along with metallicity in both tracks, according to the helium enrichment law \( \Delta Y/\Delta Z \sim 2.5 \). The higher helium at larger metallicity counter-balances the metallicity effects and keeps the star at roughly the same brightness.

In order to link the PMS evolution of the turnoff mass to its MS, one needs to know the rate of evolution of turnoff-like stars off the MS through the later evolutionary stages, as a function of the SSP parameters (age, metallicity, IMF). According to RB86 this quantity is expressed analytically by the evolutionary flux \( b(t) \) (Eq. 4), that is proportional to the time derivative of the relation turnoff mass/age and the adopted IMF. The dependence of the function \( b(t) \) on the SSP parameters is shown in Figure 2. The main panel focuses on age and chemical composition effects, the small one on the IMF. Obviously \( b(t) \) depends mainly on the age of the SSP, as the derivative of the turnoff mass/age and the adopted IMF. The dependence of the function \( b(t) \) on the IMF scale factor is steeper than that referred to the Salpeter IMF. The IMF dependence is highlighted in the small internal box, where the \( b(t) \) vs age for solar metallicity are shown for the Kroupa and the Salpeter IMF. The Kroupa SSP has a \( \sim 30 \) per cent higher evolutionary rate with respect to the Salpeter SSP.

### 3.1.3 Post Main Sequence: Fuel consumptions

The luminosity contributions of the PMS phases are computed by means of the fuel consumption theorem, according to Eq. 1. The nomenclature of the main PMS evolutionary phases, in order after the MS, are: Sub Giant Branch (SGB); Red Giant Branch (RGB); Helium Burning or Horizontal Branch (HB); Early Asymptotic Giant Branch (E-AGB); Thermonally Pulsing Asymptotic Giant Branch (TP-AGB). The stellar evolutionary phases are highlighted on the observed colour magnitude diagram of NGC 1851, a metal-poor GC of the Milky Way (data from Piotto et al. 2002).

In order to compute the fuels, evolutionary tracks are adopted up to the completion of the E-AGB. Complete tracks for the TP-AGB phase are not available. In fact it is difficult to follow the physics of the stellar interiors during the thermal pulses and stellar tracks are actually restricted to envelope models (e.g. Renzini & Voli 1981; Iben & Renzini 1983; Lattanzio 1986; Boothroyd & Sackmann 1988; Blocker & Schonberner 1991; Marigo, Bressan & Chiosi 1996; Wagenhuber & Gronewegen 1998; Marigo 2001; Mouchine & Lancón 2002), and even the latter are uncertain due to the occurrence of strong mass-loss that aborts the phase (the superwind phase, Iben & Renzini 1983). In the present models the energetics of the TP-AGB comes from the semi-empirical calibration of M98 (see Section 3.4.3).

The fuel for the evolutionary phase \( j \) (until the E-AGB) is computed by integrating the product of the evolutionary time and the emergent luminosity along the track appropriate to that phase. The tracks for the given turnoff masses are obtained from interpolation in log mass. The evolutionary mass for the helium burning phase \( M_{11} \) is obtained from \( M_{11} \) after evaluation of the mass loss during the RGB. This allows to play with various HB morphologies (see Section 3.4). Finally, the separation between HB and E-AGB...
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Figure 4. The fuel consumptions (in $M_\odot$) in the various PMS phases, as functions of the turnoff mass, for various metallicities.

is set when the mass of the CO core along the track is different from zero. Note that each phase is divided suitably into a certain number of subphases, in order to map appropriately the spectral type changes (M98). The criterion for the subdivision into suphases depends on the temperature, therefore is described in Section 3.2.

Figure 4 shows the fuels (in $M_\odot$) for the PMS evolutionary phases, as functions of the turnoff mass, for various metallicities. As already pointed out by RB86, the most relevant PMS phase changes with the evolutionary mass, i.e. with the age of the stellar population, and we find here that the trend does not depend on the chemical composition.

In massive stars ($M > 3 M_\odot$), i.e. those dominating young - $t \lesssim 0.2$ Gyr - stellar populations, the dominant PMS phase is the HB, its fuel decreases with the decreasing stellar mass. In low-mass stars ($M < 2 M_\odot$), i.e. those dominating old - $t \gtrsim 2$ Gyr - stellar populations, the RGB phase is the most important PMS phase, when a He-degenerate core is developed (RB86; Sweigart, Greggio & Renzini 1989). Stars with masses in the narrow mass range between 3 and 2 $M_\odot$, i.e. those dominating $0.2 \lesssim t / \text{Gyr} \lesssim 2$ old stellar populations, spend a conspicuous amount of fuel on the TP-AGB phase. The onset of the development of the TP-AGB and RGB phases has been called by RB86 “phase transitions”.

The HB fuel (left-hand upper panel) of massive stars is affected by metallicity in the sense of a higher fuel at a lower metal content, owing to the higher relative abundance of hydrogen, an effect similar to what pointed out for the MS luminosity (Section 3.1.2). For example, at masses $\gtrsim 4 M_\odot$, the $Z_\odot / 20$ metallicity has nearly a factor 2 more fuel than the solar one. In the small mass regime ($M \lesssim 1.5 M_\odot$, $\tau_{\text{SSP}} \gtrsim 3$ Gyr) metallicity effects are negligible.

The RGB phase (right-hand upper panel) starts to develop at masses around $2 M_\odot$ almost independent of metallicity in these classical (no overshooting) tracks, in excellent agreement with the early findings by Sweigart, Greggio & Renzini (1990). The RGB fuel is rather insensitive to metallicity until $Z_\odot$. For reference, a 10 Gyr, $Z_\odot / 200$ stellar population has 0.24 $M_\odot$ of RGB fuel and a coeval one with solar metallicity 0.23 $M_\odot$. However, at higher metallicities the RGB fuel starts decreasing with increasing metal content, and for example a stellar population with 3.5 $Z_\odot$ (see Figure 5) has 0.15 $M_\odot$ of RGB fuel, nearly 35 per cent less than the solar chemical composition. This is the effect of the very high helium abundance associated to the high metal content because of the helium enrichment law $\Delta Y / \Delta Z \sim 2.5$. However, at high metallicities the RGB fuel starts decreasing with increasing metal content, and for example a stellar population with 3.5 $Z_\odot$ has 0.15 $M_\odot$ of RGB fuel, nearly 35 per cent less than the solar chemical composition. This is the effect of the very high helium abundance associated to the high metal content because of the helium enrichment law $\Delta Y / \Delta Z \sim 2.5$.

The effect of the chemical composition that we discussed previously has little effect on the TP-AGB fuel. The TP-AGB fuel (middle upper panel) is one of the most conspicuous, together with that in HB and RGB, and is a strong function of the stellar mass, therefore of the age of the stellar population. It reaches a maximum for masses between 3 and $2 M_\odot$ and is negligible for masses outside this narrow intermediate-mass range. Furthermore, the TP-AGB fuel does not depend appreciably on metallicity (see Section 3.4.3).

Finally the SGB and E-AGB (lower panels) are the least important phases, providing at most $\sim 20$ per cent and $\sim 10$ per cent, respectively of the total PMS luminosity (see also Section 4.1).

Figure 5 shows the total fuel consumption in the whole PMS, as a function of age, for the various metallicities. In interpreting this figure in a stellar population perspective it is useful to remind that the fuel scales directly with the PMS luminosity of a stellar population.

The effect of the chemical composition that we discussed pre-
previously is evident on the total fuel of stellar populations with ages smaller than $\sim 1$ Gyr, that is larger the lower the metal content is. A young metal-poor stellar population with $Z = Z_\odot/20$ has $\sim 20$ per cent more fuel to be burned, e.g. is 20 per cent brighter than a coeval one in which the metallicity is twice solar. In older stellar populations metallicity effects become less important, as a consequence of the small metallicity dependence of the RGB fuel, that is the largest source of energy at high ages. An exception is however the very metal-rich stellar population ($Z = 3.5 Z_\odot$, dashed-dotted line), that at old ages has $\sim 25$ per cent less fuel than all other chemical compositions, for which the fuel scatter around a value $\sim 0.4 M_\odot$. This is due to the very high abundance of helium in these tracks, caused by the assumed helium enrichment law ($\Delta Y/\Delta Z \sim 2.5$), that implies the abundance of hydrogen to be only 0.45. This explains the sharp decrease in fuel since hydrogen is its most important source.

The sizable increase of PMS fuel at ages around 0.3 Gyr ($log t \sim 8.47$) marks the onset of the AGB phase transition, that is dominated by the TP-AGB (M98, Figure 4). At later ages the RGB phase transition occurs, which is barely visible in Figure 5 as a small bump around 1 Gyr, due to the simulatenous decrease of TP-AGB fuel. Finally, the bumps in the fuels at late ages ($t \gtrsim 6$ Gyr) reflect the trend of the SGB fuel (Figure 6). After the development of the TP-AGB, $\sim 70$ per cent of the total energy of a stellar population comes from PMS stars (Figure 6).

The dependence of MS and PMS energetics on stellar evolutionary tracks is illustrated in Figure 7. The transition from a not degenerate to a degenerate helium core, that marks a well developed RGB phase and a drop in the HB fuel, occurs at later epochs if overshooting is taken into account in the stellar models. This is shown in the top panels of Figure 7 where the fuels obtained with classical (solid) and overshooting (dotted) tracks as functions of the age of the stellar populations are compared. The RGB phase transition occurs at 0.7 Gyr in classical models and at 1 Gyr in models with overshooting.

The age marking the onset of the RGB phase transition was early recommended by Barbaro & Pigatto (1984) as a suitable observational check of the overshooting hypothesis. This test has been recently performed by Ferraro et al. (2004), where deep, VLT-
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**Figure 7.** Influence of the stellar tracks. Comparison between the EPS ingredients obtained with classical (solid line, from Cassisi et al.) and overshooting tracks (dotted line, from Girardi et al. 2000 and Salasnich et al. 2000). From top left to bottom right the various panels show: the total fuel consumptions in $M_\odot$ for the various PMS phases, as a function of the age of the SSP; the evolutionary flux; the bolometric luminosity of MS and post-MS. For both tracks the metallicity is half-solar ($Z = 0.008$), and only ages larger than 0.3 Gyr are plotted. The TP-AGB fuel is not considered for this comparison since it does not depend on the adopted tracks, as explained in the text.

**Figure 8.** Relation between fuel consumption and temperature subphases. The left-hand panel shows the subphases along the RGB of 12 Gyr SSPs with solar and $Z_\odot/200$ metallicities (circles and triangles, respectively). The right-hand panel shows the subphases along the HB for the same metallicities, filled and open symbols refer to blue and red horizontal branch morphologies, respectively. In both panels the symbol sizes scale with the fuel consumptions. The big circles in the left-hand panel correspond to the position of the so-called RGB bump.

3.2 Temperatures

The second matrix contains the distribution of effective temperatures and surface gravities of the evolutionary phases $j$ for the various ages and metallicities. As already mentioned, every evolutionary phase is split into a certain number of so-called photometric sub-phases, inside which the spread in effective temperatures $T_{\text{eff}}$ is $\lesssim 100$ K. This was found to be appropriate for a good tracing of the varying spectral type along a phase. What is relevant to evolutionary synthesis models is that the fuel consumption is evaluated specifically in each subphase. In general the consumption of energy is not homogeneous with temperature. This is visualized in Figure 8 where the temperature/gravity subphases for old (12 Gyr) RGBs and HBs are displayed for two metallicities (circles for solar; triangles for $[Z/H] = -2.25$), with the symbol size being proportional to the fuel. The fuel consumption along the RGB of a metal-rich stellar population is enhanced at the so-called RGB bump (big circles in the left-hand panel of Figure 8), when the H-burning shell reaches the internal layer that was previously mixed through the first convective dredge-up, gets fresh fuel and therefore spends a longer time in this location (Sweigart, Greggio & Renzini 1989).

The RGB bump is rather close to the He clump. If the total RGB fuel would be assigned homogeneously along the RGB track, the weight of the bump would be unappropriately distributed along the whole track, in particular would be given to the tip, with the effect of overestimating the near-IR flux of the SSP. The fuel consumption approach implemented here where the evolutionary timescale is considered is an efficient way of taking the bump into account. The bump is predicted to almost disappear at decreasing metallicity (Sweigart, Greggio, Renzini 1989), and indeed the RGB fuel in the metal-poor stellar population results to be rather homogeneously distributed (triangles in Figure 8). Note that the significant contribution of the RGB tip to the fuel comes from the high luminosity of that subphase.

The right-hand panel of Figure 8 illustrates the effect of the Horizontal Branch morphology on the temperature of the subphases. For the same ages and metallicities two options for the HB morphology are shown, red (open symbols) and blue (filled symbols). We remind that blue/red HBs mean that the whole HB life-

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1 It should be also noted that model atmospheres are provided with $\Delta T_{\text{eff}} \sim 200$ K.
time is spent on the blue/red side of the RR-Lyrae strip, while intermediate HB is used to refer to the mixed cases. In order to trace properly the HB evolution, we use the evolutionary track for the helium burning phase of the mass that is obtained after mass-loss is applied to the RGB track (see Section 3.4.3). In the HB phase, most fuel consumption occurs on the so-called Zero Age Helium Burning (ZAHB) that usually corresponds to a very narrow temperature range, the evolution from the tip-RGB to the ZAHB and to the ZAHB to the E-AGB happening on very short timescales (∼ 1 Myr). However in presence of strong mass loss like in the case of a BHB at high Z (filled circles) the evolutionary timescale can be significant at various temperature locations.

Stellar effective temperatures depend crucially on the efficiency of convective energy transfer, parametrized by the mixing length parameter $\alpha$. The latter cannot be derived from first principles, and as far as we know it could be connected to several stellar parameters, such as the stellar mass, the evolutionary status or the metallicity, and its calibration with observational data is certainly required. The use of uncalibrated theoretical effective temperatures is the synchro-

The calibration of the mixing-length for the Cassisi’s tracks is described in Salaris & Cassisi (1996). The tracks with solar metallicity are computed for $\alpha = 2.25$, a value that matches the Sun. This same value is kept in the tracks with supersolar metallicities (Bono et al. 1997). At sub-solar metallicities instead the mixing-length parameter that is calibrated with the Sun is assumed at all metallicities.

The resulting differences in the RGB temperatures are shown in Figure 9, where the RGBs of old (10 Gyr) isochrones from Cassisi et al. (solid lines) and from Padova (Girardi et al. 2000, dotted lines) are shown for three metallicities ($Z_{\odot}/200$, $Z_{\odot}$, $2Z_{\odot}$, from left to right). At high metallicities, the Padova RGBs are cooler than those of the Cassisi’s tracks, in such a way that the 2 $Z_{\odot}$ RGB of Cassisi coincides with the solar one by Girardi et al. Since the cooler temperatures are proper to the whole RGB, and not only to the tip, for what said before (see also Figure 8) it has to be expected that the optical/IR flux ratio of metal-rich SSP models will depend on the choice of the tracks. This effect will be quantified in Section 5. Unfortunately, as already mentioned, at these high metallicities, that are the most relevant to massive galaxies, the calibration of the models is hampered by the lack of metal-rich GCs with independently known ages and metallicities. The only two objects useful to this purpose are the two metal rich GCs of the Baade window (NGC 6553 and NGC 6528, Ortolani et al. 1995), whose total metallicity is around solar (Thomas, Maraston & Bender 2003a). The complication here is that these two objects have enhanced $\alpha/Fe$ ratios (Barbuy et al. 1999; Cohen et al. 1999), therefore the proper calibration requires the use of stellar tracks accounting for this effect. Element ratio effects are being incorporated in stellar models (e.g. Bergbusch & Vandenberg 1992, 2001; Salasnich et al. 2000; Kim et al. 2002), but for the stellar tracks considered here, their $\alpha/Fe$-enhanced version was either not yet available at the time these models were computed (Cassisi’s tracks) or does not appear to be convincing (see Thomas & Maraston 2003 for the Salasnich et al. 2000 tracks).

At lower metallicities the opposite effect is found, i.e. the Cassisi tracks appear to have cooler RGBs than the Padova ones. At least part of the effect must originate from the treatment of the mixing-length. However the methodology by Salaris & Cassisi (1996) depends necessarily on temperature/colour transformations, as the RGB theoretical temperature luminosity relation must be converted into the observed colour-magnitude diagram. Therefore it is hard to push any strong conclusion on which parameterization of the mixing-length is better. Generally, any calibration of stellar tracks depends upon the adopted temperature/colour transformations, and in particular for the coolest part of the RGB these are notoriously uncertain.

The comparison with the Padova tracks is the most relevant to evolutionary population synthesis issues, since these tracks are adopted by all existing EPS codes except the one presented here. However several other isochrones exist in the literature. In Figure 9, the latest Yale models ($Y^2$, Yi et al. 2003) and the models of Victoria (D.A. Vandenberg, private communication for solar metallicity; Bergbusch & Vandenberg 2000) for the subsolar metallicity are shown. Both sets of tracks include overshooting. The $Y^2$ isochrones (dot-dashed) behave very similarly to the Padova ones.
at all metallicities. At solar metallicity the Victoria models (dotted line) agree quite well with the Cassisi’s one till the early portion of the RGB, after which the track departs toward cooler temperatures reaching values around the tip that are more similar to those of the Padova or Yale isochrones. Interestingly the metal-poor isochrone of Victoria has a RGB rather similar to that of Cassisi, which suggests that the calibration of the mixing-length is not the whole story. At supersolar metallicities the Victoria isochrones are not available. In Section 5 we will show the impact of the various RGBs’ on the integrated colours of SSPs.

3.3 Transformations to observables

The third matrix contains the transformations to the observables, used to convert effective temperatures and surface gravities into a spectral energy distribution (SED). The transformations can be either theoretical or empirical. The models of M98 used a mix bag of ingredients. They rely on the classical Kurucz (1979 and revisions) model atmospheres for 3500 \( \leq T_{\text{eff}}(K) \leq 35000 \), complemented with models for M giants by Bessell et al. (1989) for cooler temperatures, and with empirical colours for TP-AGB stars (see M98 for references). The models presented here are partially revised in this respect because the ingredients of the mixed matrix constructed by M98 have been in the meantime compiled into one library (see Section B.3.1), in which border effects between the merged libraries are considered in far more detail than it was done in M98. Also the empirical ingredients for TP-AGB stars have been revised, because of the availability of complete SEDs (Section B.3.2).

3.3.1 Model atmospheres

The synthetic stellar spectra are taken from the spectral library compiled by Lejeune, Cuisinier and Buser (1998, in its latest version as available on the web, hereafter the BaSel library). This library has become widely used in population synthesis studies and was obtained by merging the Kurucz library of model atmospheres \( T_{\text{eff}} \geq 3500 \, \text{K} \) with model atmospheres for cooler stars (for references and full details see Lejeune et al.). As in M98, a quadratic interpolation in \( (T_{\text{eff}}, \log g) \) is performed on this library to compute the stellar spectra appropriate to each subphase. The BaSel library is provided for various iron abundances. We obtain the appropriate one to each set of tracks by interpolating linearly in [Fe/H]. Finally, a blackbody spectrum is assigned \( T_{\text{eff}} \geq 50,000 \, \text{K} \). The use of the BaSel library allows the computation of model SEDs with low spectral resolution, i.e. 5-10 Å to the visual region, 20 to 100 Å in the near-IR. Bruzual & Charlot (2003) adopt STELIB (Le Borgne et al. 2003, see also Le Borgne et al. 2004), an empirical spectral library of stars with metallicity around solar and a much higher spectral resolution (3 Å), to compute a set of SSPs with solar metallicity. They compare the integrated colours \( B-V \) and \( V-K \) obtained with both BaSel and STELIB. The colour differences as induced by the spectral transformations result to be at most a few hundreds of magnitude in a wide range of ages.

3.3.2 Empirical spectra for TP-AGB stars

As is well known, current synthetic spectral libraries do not include spectra for Carbon-rich and Oxygen-rich stars populating the TP-AGB phase, although some theoretical computations begin to be available (Lloyd et al. 2001). For the spectra of this type of stars we use the empirical library by Lançon & Mouchine (2002). The latter is based on the library of individual stellar spectra by Lançon & Wood (2000), that collects observations of C-, O-type stars in the Milky Way and Magellanic Clouds. Since individual stellar spectra of such cool, variable stars are subjected to strong star-to-star or observation-to-observation variations, Lançon & Mouchine (2002) have constructed mean templates of the Lançon & Wood (2000) library, obtained by averaging observations of individual stars. In this work we will use the average templates.

3.4 Recipes for critical stellar phases: TP-AGB and HB

3.4.1 Mass loss in red giants

The stellar temperatures and luminosities during those evolutionary phases that follow episodes of, or suffer themselves of, stellar mass loss, cannot be predicted by stellar tracks. This comes from the fact that a theory relating mass loss rates to the basic stellar parameters does not exist. Therefore mass loss has to be parametrized and its efficiency be calibrated with data. Due to such complication we call these stellar phases ‘critical’. The amount of mass loss is usually parametrized by means of the Reimers (1977) empirical formula

\[
\frac{dM}{dt} = -\eta \frac{(L/\log g - R)}{dt},
\]

where \( M, L, \log g, R \) stand for mass, emergent luminosity, surface gravity and radius, respectively, of the stellar configuration in its lifetime \( dt \). The parameter \( \eta \) introduced by Fusi Pecci & Renzini (1976) takes into account the efficiency of mass-loss. In stars of intermediate mass \( (2 \leq M/M_\odot \leq 8) \), mass loss influences significantly their post Main Sequence evolution, i.e. the energetics and the temperature, along the TP-AGB (Section 3.1). In low-mass stars \( (M \lesssim 2 M_\odot) \) mass-loss occurs also during the RGB, particularly towards the tip, affecting the subsequent HB evolution and hence the HB morphology (Section 3.4.2).

It is important to notice that the efficiency \( \eta \) cannot depend too much on metallicity (Renzini 1981), because the observed HB morphology of GCs is almost always red at high-metallicity, implying that mass-loss does not increase significantly with the metal abundance. Renzini (1981) evaluates \( \eta \propto Z^5 \cdot x \leq 0.2 \).

In the following sections we describe the modelling of HB and TP-AGB.

3.4.2 The Horizontal Branch morphology

The amount of mass loss during the RGB phase is computed by integrating the Reimers formula along the RGB track. The efficiency \( \eta \) was calibrated by Fusi Pecci & Renzini (1976) by comparing the mass of RR-Lyrae in the instability strip of the metal-poor MW GC M13 \( ([Z/H] \sim -1.5) \), with its turnoff mass, and found to be \( \sim 0.33 \). This value of \( \eta \) is by definition appropriate only at this metallicity and for the RGB tracks used by Fusi Pecci & Renzini (1976) and it has to be re-obtained for other chemical compositions and when other stellar models are used.

The approach followed by Maraston & Thomas (2000) was to compute the integrated H\( \beta \) line, that is very sensitive to the HB morphology (see also de Freitas Pacheco & Barbuy 1995; Poggianti & Barbaro 1997; Lee et al. 2000) and to compare it with galactic GCs of known ages, metallicities and HB morphologies. The value of \( \eta \) appropriate to \( [Z/H] \sim -2.25 \) was found to be 0.2.

The procedure of using one value of \( \eta \) per metallicity (and age) aims at recovering the average trend of a bluer HB morphology with decreasing metallicity, the latter being the 1-st parameter ruling the HB morphology. The trend is easy to understand in terms of stellar evolution. At low metallicity the evolutionary mass is smaller at given MS lifetime (because the stars are more compact...
and hotter and the nuclear burning more efficient), therefore the production of hotter effective temperatures by envelope removal is easier (helped also by the lower metal content). However, as well known a large scatter is found in the HB morphology of GCs with the same nominal metallicity, a still unexplained fact that is recalled as the 2-nd parameter effect. The account of all possible HB morphologies, that are able to be red in almost all cases. However, we know at least two examples of metal-rich GCs that have extended HB morphologies. These are the Bulge GCs NGC 6441 and GC 6388 (Rich et al. 1997) with twice solar metallicity, introducing an analytical recipe that contains the observed value.

The finding of Rich et al. (1997) calls for the need of models with BHBs also at high-metallicities. For this purpose we seek the \( \eta \) value at which the HB fuel is spent (almost) entirely blueward the RR-Lyrae strip. For the Cassisi tracks we find \( \eta \approx 0.85, 0.45; 1.0, 0.7; 0.94, 0.66 \) for 10 and 15 Gyr and metallicities half-solar, solar and twice-solar, respectively. The masses at the HB phase are then \( \sim 0.5/0.55 \), whose temperatures reach \( \sim 9000 \) K. This amount of mass-loss means to remove nearly half of the initial stellar mass during the RGB. Note finally that by choosing a larger \( \eta \), one gets an even bluer HB morphology. The SSPs with high-Z and BHB are computed for the ages of 10 and 15 Gyr.

As a last remark, the variation of the HB fuel between models with and without mass-loss is of having less fuel in the models in which mass loss is applied, that stems from the lower evolutionary mass at high-metallicities half-solar, solar and twice-solar, respectively. The masses at the HB phase are then \( \sim 0.5/0.55 \), whose temperatures reach \( \sim 9000 \) K. This amount of mass-loss means to remove nearly half of the initial stellar mass during the RGB. Note finally that by choosing a larger \( \eta \), one gets an even bluer HB morphology. The SSPs with high-Z and BHB are computed for the ages of 10 and 15 Gyr.

### 3.4.3 The Thermally-Pulsing AGB: inclusion in the integrated spectra

M98 present SSPs in which the TP-AGB phase was included semi-empirically in the models, using a table of theoretical fuel consumptions (from Renzini 1992) and calibrating them with measurements of the bolometric contribution of the TP-AGB phase to the total light in intermediate age LMC GCs (from Frogel, Mould & Blanco 1990, see Figure 3 in M98). Basically, the observed contributions fix the left-hand side member of Eq. (4) written for \( j = \)TP AGB, which allows the evaluation of the fuel, using the evolutionary flux appropriate for the given age. This calibration of the fuel equals to determine empirically the mass-loss efficiency along the TP-AGB phase, which is found to be \( \eta \sim 1/3 \div 2/3 \).

Maraston et al. (2001) extend the SSP models to half and twice solar metallicity, introducing an analytical recipe that connects the amount of TP-AGB fuel to the envelope mass at the first thermal pulse (beginning of the TP-AGB phase). Briefly recalling,
to metallicity, the total TP-AGB fuel does not depend too much on metallicity. As a result of this assumption the mass-loss efficiency (which is derived from the Magellanic Clouds GCs) does not depend on metallicity. The TP-AGB fuel decreases very little with increasing mass-loss efficiency. Renzini & Voli (1981) show that by doubling the mass-loss from 0.33 to 0.66 the fuel in a 2M⊙ star with solar metallicity decreases by only 20 per cent.

Instead the metallicity influences the partition of the total fuel between C and M stars, according to the rationale of Renzini & Voli (1981, adopted in both M98 and Maraston et al. 2001). Briefly summarizing, in metal-poor chemical compositions the abundance of oxygen in the envelope is lower and a lower amount of carbon has to be dredged-up in order to reach C/O > 1, hence binding the whole oxygen into CO molecules (those with the highest binding energy). The residual carbon is then available to produce CH, CN, C2 molecules and carbon stars are made. Therefore a metal-poor stellar population is expected to have more carbon stars than a metal-rich one. Quantitatively, the fuel in C-stars doubles by halving the metallicity. The adopted recipe is given in Figure 12 where the percentage of the TP-AGB fuel that is spent by Carbon-type stars as a function of metallicity is shown. The reference metallicity to which the scaling of Renzini & Voli (1981) is applied, is the 0.5 Z⊙ metallicity (dotted line), because this is the chemical composition of the Magellanic Clouds GCs that are used as calibrators³. Solar and twice solar metallicities (solid and short-dashed line, respectively) are assigned halves and one quarter of the fuel in C-star of the 0.5 Z⊙ metallicity, respectively.⁴ At [Z/H] ∼ −1.35 (long-dashed line) the fuel in C-star is a factor 10 larger than at 0.5 Z⊙, therefore the TP-AGB fuel is almost always spent by C-stars. Finally, at [Z/H] ∼ −2.25 the whole TP-AGB fuel is assigned to C-stars. These numbers can be compared to data of resolved C-stars in galaxies and help constraining the age distribution of the stellar populations. For example, Davidge (2003) finds that the C-star component is 10% of the whole AGB in the dwarf galaxy NGC 205. According to Figure 12 this implies that either a burst has occurred less than ∼ 200 Myr ago (as favoured by Davidge 2003) or the stellar population is globally old. Also interesting is the finding of a conspicuous population of C stars in the arms of M33 (Block et al. 2004), which could help constraining the star formation history of this galaxy.

Note that the C/M ratio as a function of metallicity by Renzini & Voli (1981) is confirmed by the recent TP-AGB models of Marigo et al. (1999), as inferred from their figures since quantities are not tabulated.

Our previous models were restricted to broad-band colours due to the unavailability at the time of spectra, either theoretical or empirical, appropriate to TP-AGB stars. However, in order to be useful for high redshift studies, the TP-AGB phase has to be included in the synthetic spectral energy distribution. This improvement of the models is performed here. The fuel is distributed among the empirical spectra of the individual C-,O-type stars by Lancôme & Mouchine (2002). The spectral type that starts the TP-AGB phase is chosen to be close to that of the subphase terminating the E-AGB phase. Spectral types of both O- and C-stars are then included, as described above.

4 RESULTS

The model output are the integrated spectrophotometric properties of SSPs, namely spectral energy distributions (SEDs), broad-band colours (Johnson-Cousins, SLOAN, HST filter systems), spectral line indices, mass-to-light ratios, bolometric corrections, etc. These are functions of the main parameters of the SSP model: the age t and the chemical composition X, Y, Z (referred to as [Z/H]⁵). Ages and metallicities of the SSP grid are listed in Table 1. Models are generally given in time steps of 1 Gyr, for ages larger than 1 Gyr, and of 0.1 Gyr for smaller ages. The Table indicates also the position of the Magellanic Clouds GCs that are used as calibrators. In any case the reader is referred to the present Figure 12 and to the tables provided at the model WEB page.

³ Frogel, Mould & Blanco (1990) measure also the luminosity contributions of C and M-type stars as functions of the GC age.

⁴ L. Greggio and M. Mouchine pointed out that the percentage of fuel spent as C and M stars in Table 4 and Figure 2 of Maraston (1998) is reversed. This mistake concerns only the graphics and not the models. In any case the reader is referred to the present Figure 12 and to the tables provided at the model WEB page.

⁵ The notation [Z/H] is used to indicate total metallicities, i.e. the total abundance of heavy elements with respect to hydrogen normalized to the solar values, i.e. [Z/H] = log(Z/Z⊙) − log(H/H⊙). By [Fe/H] we mean the abundance of iron with respect to hydrogen normalized to the solar values, i.e. [Fe/H] = log(Fe/Fe⊙) − log(H/H⊙). If elements have solar proportions then [Fe/H] = [Z/H]. In case of α-element enhancement, the relation between [Fe/H] and [Z/H] is: [Fe/H] = [Z/H] − 0.93 × ([α/Fe]) (Thomas, Maraston & Bender 2003).

Figure 12. The percentage of the TP-AGB fuel spent in Carbon star mode for the various metallicities, as a function of age.
concern the initial mass function (IMF) and the morphology of the Horizontal Branch. The models are computed for two choices of the IMF, namely Salpeter (1955) and Kroupa (2001), and are provided for two different assumptions regarding the mass loss along the RGB (Section 3.4.2), which result into various HB morphologies. These are indicated in Table 1. In the following subsections we discuss the various model output and their comparisons with observational data and with similar models from the literature.

4.1 Luminosity contributions by stellar phases

Figure 13 shows the contributions from the various evolutionary phases to the total bolometric, V and K luminosities (from top to bottom) of metal-poor, solar metallicity, and metal-rich SSPs, respectively (from left to right).

As in the solar metallicity SSP already discussed in M98, most bolometric energy is shared by the three main phases MS, TP-AGB and RGB independent of metallicity (cf. Figure 4). The energetics of young SSPs ($t \lesssim 0.3$ Gyr) is dominated by MS stars, that of SSPs with $t \gtrsim 2$ Gyr by RGB stars. SSPs with ages in the range

![Figure 13. Luminosity contributions in bolometric, V and K (from top to bottom) of evolutionary phases and their dependences on age and metallicity. From left to right metal-poor, solar metallicity and metal-rich SSPs (with a Salpeter IMF) are shown, respectively. Note that the y-scale of the bottom panel is not the same as in the other two panels.](image)

### Table 1. Ages, metallicities and input tracks of the SSP grid

| $t$ (Gyr) | $(Y, Z)$ | $[Z/H]$ | Stellar tracks       |
|-----------|---------|---------|----------------------|
| 1–15      | 0.230, 10$^{-4}$ | -2.25   | Cassisi              |
| 3 · 10$^{-6}$–15 | 0.230, 0.001 | -1.35   | Cassisi + Geneva     |
| 3 · 10$^{-6}$–15 | 0.255, 0.01 | -0.33   | Cassisi + Geneva     |
| 3 · 10$^{-6}$–15 | 0.289, 0.02 | +0.00   | Cassisi + Geneva     |
| 3 · 10$^{-6}$–15 | 0.340, 0.04 | +0.35   | Cassisi + Geneva     |
| 1–15      | 0.390, 0.07 | +0.67   | Padova               |

...
0.3 \lesssim t \lesssim 2 \text{ Gyr} are dominated by TP-AGB stars except in the metal-poor stellar population, where AGB, MS and HB have similar contributions. For ages older than \sim 6 \text{ Gyr}, the MS contributions tend to rise. This effect is caused by the MS integrated luminosity decreasing slower than \ln(t). As the total fuel keeps nearly constant, the net result is a lower total luminosity as mainly due to a lower PMS luminosity.

The main contribution to the V-luminosity comes always from MS stars, except for the metal-poor SSP, where at young ages the HB phase has a relatively larger contribution, and at very old ages the RGB is the dominant phase for the V-luminosity, which is due to the warm RGB temperatures at such low metallicity.

Finally, the light in the K-band (and in the near-infrared in general) in the age range 0.3 \lesssim t \lesssim 2 \text{ Gyr} is dominated by TP-AGB stars at every metallicity, a rôle taken over by the RGB at old ages. These results demonstrate the importance of including the TP-AGB phase for a correct interpretation of rest-frame near-infrared colours and luminosities of 1 Gyr stellar populations

### 4.2 Spectral energy distributions with TP-AGB

The synthetic SEDs of old stellar populations, in particular the Horizontal Branch morphology, have been compared with IUE data of GCs in Maraston & Thomas (2000) up to the metallicity of 47 Tucanae. In this section we focus on the most relevant features of the new model SEDs, namely the inclusion of the spectra of C- and O-type TP-AGB stars. We will compare the SEDs with a sample of Magellanic Clouds GCs for which data in the whole spectral range from \text{V} to \text{K} are available, and with models from the literature. Further comparisons with both observational data and models using broad-band colours and spectral indices will find place in following sections.

#### 4.2.1 Fingerprints of \sim 1 \text{ Gyr populations}

The inclusion of the TP-AGB phase in a model SED is substantial at ages in the range 0.3 \lesssim t/\text{Gyr} \lesssim 2 where the fuel consumption in this phase is maximum (cf. Section 4.4.3). This is shown in Figure 14 where two model SEDs of 0.8 Gyr, 0.5 \Zsol with and without the TP-AGB phase are compared. While the optical part of the spectra is insensitive to the presence of the cool TP-AGB stars, the near-IR one changes dramatically. Not only the absolute flux in the near-IR region increases by nearly a factor 3, also peculiar absorption features appear (e.g. CN, C2 in C-stars and H2O and CO in O-stars, see e.g. Lançon & Wood 2000 for more details). These absorption features besides the integrated colors can be used as indicators of \sim 1 \text{ Gyr} stellar populations in the integrated spectra of stellar systems. For example, Mouchine et al. (2002) detected Carbon molecules absorptions in the near-IR spectrum of W3, a massive GC of the merger-remnant galaxy NGC 7252 that we previously suggested to be in the AGB phase transition based on its \text{B}, \text{V}, \text{K} colours (Maraston et al. 2001).

As discussed in section 4.4.3 the relative proportions of C- and O-stars depend on metallicity, for the model of Figure 14 the ratio being 9:1. The effect of considering other metallicities is displayed in Figure 15 where 0.4, 1 and 2 Gyr SSPs with chemical compositions \Zsol/20, \Zsol and 2 \Zsol, are shown. The larger total metallicity favours the production of O-rich stars over that of C-rich stars. This is evident in the integrated SEDs as the disappearing of the CN and C2 bandheads at 1 \lesssim \lambda/\mu m \lesssim 1.5 typical of C-stars in favour of the H2O molecules around 1.6 \mu m. Several line indices are defined that trace these spectral features (e.g. Aaronson et al. 1978; Frogel et al. 1978; Alvarez et al. 2000) and can be used in extragalactic studies. With the new spectra we are in the position of computing the integrated indices for the SSP models (see also Lançon et al. 1999).

Figure 14 shows three of these indices. The C2 index (Alvarez et al. 2000) measures the strength of the bandhead at 1.77 \mu m (Balkil & Ramsay 1963) and is defined as the ratio between the fluxes in the regions 1.768 – 1.782 \mu m and 1.752 – 1.762 \mu m. The water
Figure 15. Effect of the metallicity of the TP-AGB stars on the SEDs of 0.4, 1 and 2 Gyr stellar populations. From left to right, the metallicity increases from $Z_\odot/20$ to $2 Z_\odot$, and with it the relative importance of Oxygen-rich stars over Carbon-rich stars.

Vapour absorption band index $H_2O$ measures the ratio of the flux densities at 2.04 µm and 2.22 µm, based on the HST/NICMOS filters F204M and F222M, or at 1.71 µm and 1.80 µm, based on the HST/NICMOS filters F171M and F180M, and the CO index the flux densities at 2.37 µm and 2.22 µm, based on HST/NICMOS filters F237M and F222M. The indices are defined in magnitudes and normalized to Vega$^7$.

The $C_2$ index is a strong function not only of the age, peaking at the time of the AGB phase transition, but especially of the chemical composition, its value decreasing with increasing metallicity, following the decrease of the fuel in C-stars. The water vapour is rather insensitive to the chemical composition, hence is a good age indicator for ages between 0.4 and 2 Gyr. The CO index behaves similarly to the $H_2O$.

The combination of indices that works best to unveil the presence of $\sim 1$ Gyr stellar populations and their chemical composi-

$^7$ Using a spectrum of Vega taken with HST and kindly provided by R. Bender, the zeropoints are: $C_2 = 0.038$; $H_2O_{1.71 \mu m} = 0.160$; $H_2O_{2.04 \mu m} = -0.360$; $CO = 0.263$, that have to be subtracted to the indices.
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4.2.2 Comparison with literature and data

Figure 18 shows our SED of a 1 Gyr, $Z = Z_\odot$ SSP (thick line) and similar models from Bruzual & Charlot (2003, hereafter BC03), from the Pegase code (Fioc & Rocca-Volmerange 1997, version 2 as available at http://www.iap.fr/pegase, hereafter PEGASE.2 models) and from the latest version of Starburst99 (Vázques & Leitherer 2005, long-dashed-short-dashed line). The BC03, PEGASE.2 and Starburst99 are very similar one to each other which is due to their use of the Padova tracks, and are more consistent with the version of our models not including the TP-AGB phase than with that in which the TP-AGB is accounted for. This result is not easy to understand. In the BC03 models the inclusion of C-stars is accounted for by means of unpublished theoretical spectra and an unspecified temperature/luminosity relation, therefore it is hard to make a
meaningful comparison with our models. In the PEGASE.2 models and in the latest version of the Starburst99 models, the TP-AGB phase is included by adopting theoretical prescriptions for luminosities and lifetimes (Gronewegen & de Jong 1993 and Vassiliadis & Wood 1993, respectively), but the key information concerning the TP-AGB fuel consumption as a function of mass, its comparison with the Frogel, Mould & Blanco data, the inclusion of Carbon stars and which spectra are assigned to TP-AGB stars are not specified.

We now compare the model SEDs with Magellanic Clouds GCs, that are the ideal templates in the age range relevant to the TP-AGB. In M98 we used the data available at the time, namely $U, B, V, K$. Here we complete the wavelength coverage by adding data in the R, I-Cousins bands from new observations of a sample of MC GCs in the relevant age range (Goudfrooij et al. 2005, in preparation). For this sample we can perform the SED fittings. These are shown in Figure 19 in which nine out of the 20 objects of the Goudfrooij et al. (2005) sample are considered, according to the availability of all bands and also to reasons of space, since some objects display basically the same spectral energy distributions. The selected GCs span the interesting range in SWB (Searle, Wilkinson & Bagnuolo 1980) types from 3 to 5.5. The SWB scheme is a rank of C- and O- stars. The type 7, for example, corresponds to Milky Way-like objects, with ages $\sim 13$ Gyr and $[Z/H] \sim -2$. As discussed by Frogel, Mould & Blanco (1990) the exact ranking of some individual clusters might be not so meaningful, but the range from 4 to 5.5 corresponds to objects with the largest numbers of AGB stars, that are the only ones in which Carbon stars are detected (see Fig. 3 in M98). Therefore the GCs in this SWB range span average ages between $\sim 0.3$ and $\sim 2$ Gyr (cf. Table 3 in Frogel, Mould & Blanco 1990). Indeed, ages determined by various methods agree generally well with the SWB ranking. For example, in order to fit NGC 1987 we use an SSP with the age and the metallicity as determined in the literature (from Girardi et al. 1995 and Ferraro et al. 1995), and the result is very good. However we emphasise that with the exercise of Figure 19 we do not aim at deriving precise ages for the objects, but rather at illustrating the effect of the TP-AGB phase. Figure 19 shows that their SEDs can be fit only with a proper inclusion of the TP-AGB energy contributions and the spectra of C- and O- stars.

The SSPs have metallicities either half-solar or $1/20 Z_\odot$ (i.e. $[Z/H] = -0.33$ and $[Z/H] = -1.35$), with thick lines showing our models, and dotted and dashed thin lines those by BC03 and PEGASE.2, respectively. For the latter the lower metallicity is $[Z/H] \sim -0.7$. For our models we show also the broad-band fluxes (empty circles). Each panel presents an individual GC SED (filled points). Starting from the top left, object NGC 265 with SWB = 3 is a pre-AGB GC whose SED does not yet display evidences of TP-AGB stars redwards the $R$-band. The situation changes completely when later SWB types are considered and the typical features of the cool TP-AGB stars appear in the near-IR spectrum. Their complete spectral energy distributions are well fit by our models.

The models by BC03 and PEGASE.2 do not reproduce the observed SEDs with the same SSP parameters, displaying substantially less flux redward the $R$-band. The same comparison holds for the Starburst99 models (not shown, see Figure 18). The obvious conclusion is that the recipes for the TP-AGB adopted in those models are not adequate at describing real stellar populations with TP-AGB stars. Further discussions on these models is found in the next section. There is no combination of age and metallicity that allows to fit the high fluxes both blueward and redward the $V$-band unless the TP-AGB phase is included as shown by the thick solid lines. As we will see in Section 5, in case of galaxies a composite stellar population in which a late burst is superimposed to an old component can fulfill the need for high blue and near-IR fluxes (relative to $V$). This option is clearly not viable in case of GCs.

In the next section we will show the comparison with more objects by means of broadband colours.

### 4.3 Broadband colours

Broadband colors are computed for several filter systems (e.g. Johnson-Cousins, SLOAN, HST). In the following subsections we will compare them with Magellanic Clouds GCs and Milky Way Halo and Bulge GCs, which allows the check of young and intermediate age models with slightly subsolar metallicities, and old ages for a wide range of metallicities, respectively. Comparisons will be also made with models from the literature.

#### 4.3.1 TP-AGB-dominated $\sim 1$ Gyr models

In Figure 20 we perform a comparison similar to that shown in Figure 19 by means of broadband colours which allows us to use a larger data sample. We also include the data of the GCs in the merger remnant galaxy NGC 7252 (Maraston et al. 2001, open triangles). The models are selected to have ages smaller than 2 Gyr and half-solar metallicity.

The AGB phase transition among the clusters appears as an enhancement of the IR luminosity with respect to the optical one, that increases with increasing wavelength, with e.g., the $V - K$ colour reaching values larger than 3. All bands redwards $R$ are affected. The models of this paper provide a good description of TP-AGB-dominated stellar populations in a large wavelength range. As
already shown in Maraston et al. (2001), the models reproduce very well also the integrated colours of the young GCs of the merger remnant galaxy NGC 7252.

The other models considered here, namely those by BC03 and PEGASE.2\(^8\) do not exhibit the “jump” in the near-IR colours displayed by the MC GCs, remaining systematically bluer than the data and evolving slowly towards the red colours produced by the rise of the RGB phase at \(t \gtrsim 1\) Gyr. This pattern is equivalent to the SED comparison discussed in the previous section, again sug-

\(^8\) For this comparison we focus on those models for which the TP-AGB is stated to be included. The Worthey (1994), the Vazdekis et al. and the Starburst99 (Leitherer et al. 1999) models do not treat the TP-AGB phase. Mouchine & Lançon (2002) include the TP-AGB phase in such a way as to reproduce the \(B - V\) vs. \(V - K\) synthetic diagram published by Maraston et al. (2001). However the TP-AGB bolometric contribution in their models is up to 20 per cent (Mouchine et al. 2002), smaller than the observed \(\sim 40\) per cent one (Frogel, Mould & Blanco 1990) reproduced by our models (Figure 13).
gesting that the TP-AGB phase is not fully accounted for in these models.

BC03 argue that stochastical fluctuations in the number of TP-AGB stars suffice to explain the full range of observed $V-K$ colours of MC GCs. These effects are mimicked by means of a stochastic IMF in a $\sim 2 \cdot 10^4 M_\odot$ model star cluster. From their Figure 8 one sees that the simulations match very well the colours of the youngest MC GCs, whose near-IR light is dominated by red supergiants with lifetimes $\sim 10^6$ yr. The TP-AGB phase instead lasts ten times longer and is therefore less affected by stochastical fluctuations. As a consequence the simulations are less successful in covering the observed colours of the TP-AGB-dominated MC star clusters. Moreover, the probability distributions of the points in their simulations is squewed towards bluer colors. This implies that it would be highly unlikely to observe a GC on the red side of the models if stochastical fluctuations were dominating the distribution of the data. Instead the data scatter exactly to the red side of the models.

A further important point is that the importance of stochastical fluctuations depends dramatically on the mass of the globular cluster. While the effect is relatively large for masses of the order $10^5 M_\odot$ (considered in the BC03 simulations), it is significantly smaller at $10^6 M_\odot$ and completely negligible around $10^7 M_\odot$. Hence, the stochastical fluctuations, based on a $10^3 M_\odot$ cluster, are not appropriate to describe the colours of the star clusters of the merger-remnant galaxy NGC 7252 shown as open triangles in Figure 20. Their luminous masses are estimated to be at least $10^6 M_\odot$ (Schweizer & Seitzer 1998) ranging up to even $10^8 M_\odot$ as confirmed dynamically for the most luminous object (Maraston et al. 2004). The expected stochastical fluctuations for such very massive objects are of the order of a few percent (Maraston et al. 2001) and their colours are not explained by the BC03 models without fluctuations (Figure 20). Instead, their colours are nicely explained by a TP-AGB phase like in the MC clusters (Maraston et al. 2001 and Figure 20), a conclusion that is supported by the direct observation of Carbon stars in their spectra (Mouhcine et al. 2002 with SOFI observations).

To conclude, while we fully agree that stochastical fluctuations in the number of stars along short evolutionary phases scatter the near-IR colours of small-mass star clusters, the account of these effects should be considered on top of the stellar evolutionary phases. For example it seems more likely to us that the lack of the AGB phase transition in the BC models comes from the fact that the TP-AGB bolometric contribution never exceeds $\sim 10$ per cent (Figure 3 in Charlot & Bruzual 1991). This is at variance with the measured bolometric contribution of the TP-AGB phase ($\sim 40$ per cent, Figure 3 in M98) which was evaluated by taking stochastical fluctuations into account.

In the PEGASE.2 models the details of the inclusion of the TP-AGB phase are not specified. However from Figure 2 in Fioc & Rocca-Volmerange (1997) one sees that a jump in $V-K$ of $\sim 0.6$
mag (up to \(V - K \sim 2.2\)) occurs at \(t \sim 0.1\) Gyr, after which the evolution of this colour is nearly constant. The AGB phase-transition as exhibited by the MC star clusters does not take place in their models.

### 4.3.2 Age and metallicity relations of old models

Milky Way GCs spanning a wide range of metallicities at nearly the same age are ideal to calibrate the synthetic colours/metallicity relations. Figure 22 shows a comprehensive comparison of old (13 Gyr) models with all available photometry of Milky Way GCs. Data are plotted towards the \([\text{Fe}/\text{H}]\) parameter of the Zinn & West (1984) scale, that we show to reflect total metallicities (Thomas, Maraston & Bender 2003). Data for the two metal-rich GCs of the galactic Bulge NGC 6528 and NGC 6553 are the open triangles. Models are shown for both the Kroupa and the Salpeter IMF (solid and dashed lines, respectively) only for completeness since we found that the optical colors of SSPs are virtually unaffected by plausible IMF variations (M98). The dotted line shows the BC03 models for the same age and the Salpeter IMF.

The match between models and data is very good, except for the \(B - V\) colour, that appears to be systematically redder in the models (of 0.05 mag). We have checked that this feature is common to all SSP models considered in the paper, namely, the BC03, PEGASE.2, Worthey (1994) and Vazdekis et al. (1996), and was discussed by Worthey (1994) and Charlot, Worthey & Bressan (1996). We now investigate the origin of the offset.

The offset cannot be attributed to HB morphologies. Moreover, the blue HB morphology\(^{10}\) of most GCs with metallicities below \(-1.3\) is already accounted for in our models at 13 Gyr and \([\text{Z}/\text{H}] = -2.25\) (cf. Figure 17). At 13 Gyr and \([\text{Z}/\text{H}] = -1.35\) the HB morphology is intermediate (cf. Figure 17). A purely blue HB, with \(T_{\text{eff}}\) up to \(10^4\) K would make the \(B - V\) bluer by 0.03 mag. But the GCs with metallicities \(\geq -1\). have a red HB and still their \(B - V\) is bluer than the models. The latter data can be better match with a younger age, e.g. 8 Gyr, which is perhaps not excluded (e.g. Rosenberg et al. 1999). However this younger age worsens the fit to \(U - V\) and \(U - B\).

A more likely explanation is a defect in the colour-temperature transformations. This issue is extensively addressed by VandenBerg & Clem (2003), who provide a set of transformations to cure the \(B - V\) problem, which is attributed to deficiencies of the model atmospheres. We have found that the \(B - V\) colors in their transformations are very close to those of the BaSel library for the turnoff temperatures of the 13 Gyr models (5800 \(\lesssim T_{\text{eff}}/K \lesssim 6700\), \(\log g \sim 4.5\)), but are bluer for the RGB base (4900 \(\lesssim T_{\text{eff}}/K \lesssim 5300\), \(\log g \sim 3\)), by 0.1 mag at \([\text{Z}/\text{H}] \sim -2.25; -1.35\). Since the RGB contributes \(\sim 40\) per cent to the V-luminosity at \([\text{Z}/\text{H}] = -1.35\) and below (cf. Figure 17) the use of the VandenBerg & Clem (2003) transformations would be able to reduce significantly the discrepancy between data and models. It is intriguing however that the Vazdekis et al. models that adopt an empirical temperature-colour relation instead of the theoretical one suffer from the same problem. Also, Bruzual & Charlot (2003) compare the integrated \(B - V\) of solar metallicity SSPs based on the BaSel and the Pickles empirical library, and found negligible differences. Though their exercise refers to solar metallicity, while our comparison focuses on sub-solar chemical compositions, it has to be expected that if a fundamental problem in the treatment of the opacity would be responsible for the colour offset, this would be even more serious at higher metallicities.

We are left with one alternative explanation, that is an offset in the photometric bands, due to the broadband filters. Indeed, two different B filters are used, the so-called B2 and B3 (Buser), in order to compute \(U - B\) and \(B - V\), respectively. These are commonly defined as “standard Johnson B”. The colour \(U - V\) is then evaluated by adding \(U - B\) and \(B - V\).\(^{11}\) It is hard to trace back which exact filter was used for the data, therefore we vary the B-filter in the models. The result is that if we use the B2 filter to compute the \(B - V\) colour, the latter gets bluer by 0.03 mag., again in the direction of improving the comparison, while leaving the \(U - V\) colour almost unchanged.

To conclude, either a defect of the model atmospheres or a filter mismatch or a combination of both are the most plausible sources of the discrepancy between data and models in \(B - V\). However since at redshift > 0 one needs the whole spectral energy distribution, the use of spectral libraries like the BaSel one is unavoidable. Instead, in comparing the models with observed colours, it is safer to take the offset of 0.05 mag. into account.

Back to Figure 22 we see that the BC03 models behave very similarly to ours, with the exception of the \(V - R\) and \(V - I\) colours, that in the BC03 models appear to be too red when compared to our models and to the data. Reducing the age does not improve the comparison. We cannot explain the reasons for this offset.

### 4.4 Mass-to-light ratios

The stellar mass-to-light ratios \(M^{\ast}/L\) are computed as in M98, by taking the stellar mass losses into account. The relation between living stars and remnants is from Renzini & Ciotti (1993), where the remnant mass as a function of the initial mass is given for three types of stellar remnants, i.e. white dwarfs, neutron stars and black holes. These relations are used for every metallicity, the dependence on the chemical composition entering through the turnoff mass. Figure 24 shows the stellar mass of an evolving stellar population with initial mass of 1 \(M_\odot\), solar metallicity and Salpeter IMF (solid and dotted line, respectively). In case of a Salpeter IMF the stellar mass decreases by \(\sim 30\) per cent in 15 Gyr, \(\sim 23\) per cent being lost in the first Gyr. The Kroupa IMF with less power in low-mass stars has \(\sim 16\) per cent less mass, and correspondingly lower \(M^{\ast}/L^{\ast}\). The stellar mass of the BC03 models is smaller than that computed in our models.

The basic dependences of the \(M^{\ast}/L^{\ast}\) on the SSP parameters, ages, metallicities and the IMF are shown in Figure 26. The \(M^{\ast}/L^{\ast}\) increases with age because the luminosities decrease, an effect independent of the photometric band. An important exception is the occurrence of the AGB phase transition during which the near-IR luminosity increases with age and therefore the \(M^{\ast}/L_{\text{near-IR}}\) decreases, by a factor 3-5 with respect to models not including the TP-AGB phase.

Metallicity effects are strongly dependent on \(\lambda\). The \(M^{\ast}/L^{\ast}\) generally increase with increasing metallicity because the luminosities decrease (which is explained by lower MS and PMS luminosities with increasing metal content, Section 5.1). However in the near-IR, a transition occurs at \(\lambda \gtrsim 1\) band and the \(M^{\ast}/L^{\ast}\) become independent of metallicity at ages \(\gtrsim 2\) Gyr. The reason for this is that a high Z SSP produces less light than one with a lower amount.

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\(^{10}\) as derived from the HBR parameter listed in the Harris (1996)

\(^{11}\) We acknowledge a discussion with R. Buser who confirmed this fact to us.
of metals, but most of the energy is emitted preferentially at longer wavelengths because of the cooler stellar temperatures. Note however that the effect holds until the metallicity is not too large. The very metal-rich SSP of our set \( (Z = 3.5 \, Z_\odot) \) has \( M^*/L \) smaller than the others SSPs.

The effect of the IMF was amply discussed in M98. The main result was that the \( M^*/L \) of SSPs is minimal for Salpeter and increases for both a flattening or a steepening of the IMF. The reason is that a larger amount of mass in massive remnants or in living stars is present. This interesting point stems from considering the evolution of the stellar mass. The models in Figure 21 refer to a Kroupa IMF, the effect of having a Salpeter IMF is shown only in the bolometric (solid thin line), with the \( M^*/L \) being larger by a factor \( \sim 1.5 \). A similar scaling factor is found at the various \( \lambda \)'s, almost independent of metallicity.

Finally our models are compared with others in the literature in Figure 24. The models by BC03 have a bolometric \( M/L \) that...
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Figure 22. The total stellar mass (normalized to 1 $M_\odot$) of an evolving stellar populations in which the stellar mass losses are taken into account.

Figure 23. The dependence of the $M^*/L$ (in solar units) on age and metallicity. Models refer to the Kroupa IMF, for the bolometric the solar-metallicity models for a Salpeter IMF are shown as thin solid line. Note that the 3.5 $Z_\odot$ SSP is computed with the Padova tracks and does not contain the TP-AGB phase.

Figure 24. Comparison between model $M/L$ in various bands from different authors (see Footnote 12). All SSPs have solar metallicity and Salpeter IMF.

The total stellar mass is larger than ours around ages of a few Gyr, because the luminosity of the Padova tracks is smaller due to the delayed development of the RGB phase (Figure 7). The effect is partly counterbalanced by a slightly smaller stellar mass (Figure 22). The largest differences are found in the near-IR. The $M/L_K$ of the BC03 models is higher than ours around 0.8-1 Gyr due to the lower contribution of the TP-AGB in their models. At old ages it is instead smaller, due to the cooler RGBs of the Padova tracks (cf. Figure 9). The models by Worthey (1994) have smaller $M/L_{bol}$ and $M/L_K$, due to larger bolometric and near-IR luminosities. This is possibly due to a larger number of upper RGB stars with respect to that predicted by the Padova isochrones (Charlot, Worthey & Bressan 1996). Note also that the Worthey (1994) $M/L$'s do not take the stellar mass losses into account. Finally, the models by Vazdekis et al. (1996) behave in general similarly to the BC03 ones, which is due to the use of the Padova tracks in both models. Some differences appear in the $K$-band, however. At $t \sim 10^9$ yr a dip in the Vazdekis models is present that is not found in the BC03 models, therefore it can hardly be connected to the Padova tracks, unless the two models adopt a different release.

4.5 Lick indices

The Lick indices of SSP models computed by means of the Lick fitting functions (Worthey et al. 1994) have solar-scaled element abundance ratios at high-metallicities $[Z/H] \gtrsim 0.5 Z_\odot$ (Worthey et al. 1994; Maraston et al. 2003). Therefore they are not adequate to model stellar systems with high-metallicities and enhanced $\alpha$/Fe abundance ratios, like Bulge globular clusters, elliptical galaxies and bulges of spirals (Maraston et al. 2003). At low metallicities, these models trace element-abundance ratios that are proper to the stars used to compute the fitting functions, which is a mix of solar-scaled and enhanced ratios (Maraston et al. 2003; Thomas, Maraston & Bender 2003). Finally, the models do not contain the explicit dependence on the relative elemental proportions, which is instead a powerful tool to constrain the star formation history of stellar systems (Matteucci 1994; Thomas, Greggio & Bender 1999).

To circumvent the limitations quoted above, we have computed new-generation stellar population models of Lick indices that include the dependence of the element ratios. The models allow several non standard elemental mixtures, and are checked to reproduce the Lick indices of Milky Way Halo and Bulge globular clusters for their proper element ratios. The models of the classical Lick
The equivalent width of the triplet absorption line of calcium at $4.7\,\text{Å}$, the Ca II triplet index, is investigated in detail in a forthcoming paper.

Typically used as pure age indicators, the D-4000 is very sensitive to metallicity (Figure 25, left-hand panel), and at low metallicity it is actually completely insensitive to age. Interestingly the D-4000, unlike the Balmer lines (Maraston & Thomas 2000) is insensitive to the morphology of the Horizontal Branch at low metallicity. This suggests its use as metallicity indicator for low-metallicity extragalactic GCs, since at low metallicity the break is also completely insensitive to age. In contrast we find the break to be very sensitive to blue HB at high metallicity. This is also shown in Figure 25, left-hand panel, where the arrow indicates the value of the break of a 10 Gyr, solar metallicity model with a BHB (see Section 4.3). The D-4000 break is dimished by $\sim 0.6\,\text{Å}$. The break is completely insensitive to the IMF, when modified from Salpeter and Kroupa (Figure 25, central panel), which is obvious because the blue spectral region is not sensitive to the percentage of stars with masses lower than $\sim 0.5\,M_\odot$.

The presence of several metallic lines in the wavelength windows defining the D-4000 break makes this index possibly dependent on the details of the chemical abundance pattern. Such an effect is likely to occur at metallicities larger than solar, since the solar-scaled models and the $[\alpha/\text{Fe}]$-enhanced GCs data agree up to solar metallicity (Figure 25, right-hand panel). This issue is investigated in detail in a forthcoming paper.

The dependence of the indices on the IMF is discussed in Section 4.8. The D-4000 and the CaII index change consistently and would yield an age of approximately 2 Gyr. The effect on index-index diagrams is shown in Thomas et al. (2004).

In order to break the age-BHB degeneracy the best strategy appears to be a combination of Balmer lines (or blue colours) and TP-AGB sensitive colours like $H - K$. Indeed as one sees from Table 3 the latter is insensitive to the presence of a BHB while it increases substantially in presence of a real 2 Gyr old population containing TP-AGB stars (Figure 20).

The values provided in Table 3 allow the evaluation of BHB effects in arbitrary proportions on a stellar population, also when

In order to evaluate the effect on several other SSP output the readers can compare the models with BHB and RHB that are available at the model web page.
Figure 25. The D-4000 break all at once. **Left-hand panel.** The dependence on age, metallicity and Horizontal Branch. The two lowest metallicity SSPs are shown with both blue and red HBs (thick and thin lines, respectively), and a negligible impact is found. The arrow points to the value of the break when a blue HB is considered in the 10 Gyr, solar metallicity SSP. A sizable decrease of $\sim 0.6 \AA$ occurs as a consequence of a blue instead of a red HB. **Central panel.** The dependence of the D-4000 break on the input stellar tracks, shown using the BC03 models and solar metallicity SSPs, is negligible. The dependence on the star formation history is highlighted by means of Composite Stellar Population (CSP) models with various exponentially-decreasing star formations (dotted lines, the e-folding times are labelled beside each curve). **Right-hand panel** Comparison of the synthetic D-4000 break of SSPs with various metallicities and the constant age of 13 Gyr with GC data.

Figure 26. The dependence of the indices $\text{CaT}^*$, $\text{CaT}$ and $\text{PaT}$ on age at fixed solar metallicity (upper row), and on metallicity at the fixed age of 12 Gyr (lower row). Solid and dotted lines refer to the models of this paper and of Vazdekis et al. (2003), respectively, both for a Salpeter IMF. In the lower central panel the models are compared with Milky Way GC data (from Armandroff & Zinn 1988).
other models from the literature are adopted. In particular, the values in the Table can be applied to the models with variable element abundance ratios by Thomas, Maraston & Bender (2003, for the Lick indices) and by Thomas, Maraston & Korn (2004, for the higher-order Balmer lines).

5 UNCERTAINTIES ON SSPS DUE TO STELLAR TRACKS OR EPS CODES

Two types of uncertainties affect the interpretations of the age and the metallicity of an unresolved stellar population by means of SSPs models. The first group collects what we can call *intrinsic uncertainties*, in the sense that they belong to the physiology of the model. These are led by the "age-metallicity degeneracy" (e.g. Faber 1972; O'Connell 1980; Worthey 1994; Maraston & Thomas 2000), the effect by which a larger metal content produces similar spectral changes as an older age, and vice versa. To this group belongs the HB morphology and the chemical abundance pattern. These uncertainties cannot be removed, however they can be taken into account by a proper use of the models.

The other class should be called *transient uncertainties*, in the sense that a solution can be found at some point. To this group belong the discrepancies in the model output that are generated by: 1) stellar evolutionary tracks; 2) transformations to the observables; 3) EPS codes. Charlot, Worthey & Bressan (1996) studied the origin of the discrepancies in their model $B-V$, $V-K$ and $M/L_V$. Their main conclusions are: a) the 0.05 mag discrepancy in $B-V$ colour originates from a known limitation of the theoretical stellar evolutionary input, i.e. the - energetics and temperatures - matrices while keeping constant the temperature/color/spectra-transformation matrix. A similar exercise can be found in Fioc & Rocca-Volmerange (1997), where the impact of varying the stellar tracks from the Padova to the Geneve set is tested. Major differences were found at very young ages (tenths of Myr), that are connected with the recipes of mass-loss in massive stars. However both sets of tracks adopt overshooting, while in the exercise presented here the Frascati tracks do not include this parameter, thereby allowing to explore its impact on the spectral evolution. In the following we will focus on colours and spectra, since the influence of the stellar tracks on the Lick absorption indices has been discussed by Maraston et al. (2001b; 2003). We also leave out the investigation of the uncertainties in the spectral libraries since it is made in BC03.

In Figure 27(left-hand panel) we compare selected broadband colours with solar metallicity and various ages as computed with our code and the Frascati and the Padova stellar tracks (solid thick and thin lines, respectively).

The $V-K$ is the most affected by the stellar models. At $t \gtrsim 2$ the value obtained with the Padova tracks is redder by 0.08 mag than that obtained with the Frascati tracks. This was expected since the largest difference among the two sets of tracks is the RGB temperature (Figure 3). The difference is not dramatic since the tracks deviate especially towards the tip, where the evolutionary timescale is faster, therefore the fuel consumption smaller. Similar values are found at twice solar metallicity.

The $V-I$ is rather insensitive to the choice of the tracks, while the $B-V$ relative to the Padova tracks is bluer at the ages at which the overshooting is important, in agreement with the higher MS luminosity of the Padova tracks (Figure 7). At old ages the two model $B-V$ are in perfect agreement. As for the metallicity scale (right-hand panels), the largest discrepancies are found when the colors involving near-IR bands are considered, which again stems from the different RGB temperatures. For example at low metallicities the Padova tracks are hotter than the Frascati ones, and the relative colours bluer. Therefore an observed colour (at given age) is interpreted as connected to a higher metallicity (e.g. a $V-K$ of 2 requires $[Z/H] \sim -1.7$ for SSPs based on the Padova tracks, but just $[Z/H] \sim -2.2$ for those based on the Frascati tracks). The differences are much smaller in the optical bands.

Also plotted are models by other EPS codes (Worthey 1994, Vazdekis et al. 1996, PEGASE.2, BC03 and Vázquez & Leitherer 2005). All these models except those by Worthey (1994) adopt the stellar evolutionary tracks from the Padova database and are computed by means of the isochrone synthesis technique. In spite of such harmony of model inputs, sizable discrepancies exist between these models. Interestingly, these type of discrepancies are larger than those induced by the use of different stellar tracks in our code. For example, the $V-K$ of the PEGASE.2 models with solar metallicity is nearly 0.2 mag. redder than that of the Vazdekis models, while BC03 agree with the latter in $V-I$ and with PEGASE.2 in $V-K$.

Note that in none of these other models the AGB phase transition is appreciable.

Also the metallicity scale shows quite a discrepancy, especially approaching large chemical abundances.

The scatter in the inferred ages and metallicities can be estimated directly from Figure 28. We did not attempt a more quantitative evaluation since after all metallicities and ages should not be determined by means of one indicator, rather through a "grid-like" approach.

As a final remark, the integrated colours of the Worthey (1994) models are typically redder than those of the other models. Charlot, Worthey & Bressan (1996) attribute the effect to a factor 2

Note that our SSPs based on the Padova tracks do not include the TP-AGB phase therefore the differences at $t \lesssim 2$ Gyr are not relevant in this context.
more stars on the upper RGBs of G. Worthey isochrones than on the Padova isochrones used in the other two EPS. The fuel consumption approach helps to avoid such uncertainties.

6 A JUMP TO HIGH-z: TP-AGB STARS IN SPITZER GALAXIES?

As discussed in Section 3.1, the onset and development of the TP-AGB phase occupies a narrow age range in the evolutionary path of stellar populations, being confined to ages $0.3 \lesssim t/\text{Gyr} \lesssim 2$. During this short epoch the TP-AGB phase is the dominant one in a stellar population providing $\sim 40$ per cent of the bolometric contribution, and up to $\sim 80$ per cent of that in the $K$-band (Figure 13). Therefore the inclusion of the TP-AGB phase in a stellar population model is essential for a correct interpretation of galaxies with stellar populations in this age range as discussed in Section 4.2.2.

In particular, the TP-AGB phase can be used as an age indicator of intermediate-age stellar populations and its power is to be relatively robust against the age/metallicity degeneracy. In this direction goes the work by Silva & Bothun (1998), in which the two AGB-sensitive colours $J-H$ and $H-K$ were used to constrain the
amount of intermediate-age stars in local, disturbed, field ellipticals. The authors rely on the comparison with colours of real TP-AGB stars after noticing that the population models they explored (Worthey 1994 and Bruzual & Charlot 1993) neither reproduced the near-IR colours of local ellipticals nor were in agreement one with each other (the latter is confirmed by our comparison, see Figure 27). Here we add that those models would not be suitable to the aim since they do not include the calibrated contribution by the TP-AGB phase (see 4.2.2). The possible use of the TP-AGB as age indicator for high-z galaxies was firstly suggested by Renzini (1992).

At high redshifts when galaxies are dominated by \( \sim 1 \) Gyr old populations as inferred from the ages and element ratios of local early-type galaxies (Thomas et al. 2004), the TP-AGB signature in the rest-frame near-IR must show up, with e.g. the rest-frame \( V - K \) colour mapping into the observed \( K - 10 \mu m \) at \( z = 3 \). This portion of the spectrum became recently available thanks to the advent of the Spitzer Space Telescope (SST). Therefore a straightforward application of the model SEDs presented in this paper is the interpretation of the high-z Spitzer galaxies as also discussed in Maraston (2004).

Recently Yan et al. (2004) published SST-IRAC data of galaxies in the Hubble Ultra Deep Field, with photometric redshifts ranging from 1.9 to 2.9. In order to explain the observed SEDs in the whole spectral range from the rest-frame \( B \) to \( K \), in particular the high fluxes in both the blue and the near-IR, the authors must combine a dominant stellar population being at least \( 2.5 \) Gyr old, with traces (less than 1 per cent in mass) of a 0.1 Gyr old one. The relatively high age of the (dominant!) old component generally implies very high formation redshifts (\( z_f \gg 10 \)), and even exceeds the age of the universe for the highest redshift objects.

This contrived solution is imposed by the evidence of very high observed fluxes in both the optical and near-IR rest frames. As we saw in Section and Figure 19, high near-IR fluxes in young stellar populations are the fingerprints of TP-AGB stars. It is therefore interesting to explore which ages are inferred from the models of this paper.

Figure 28 shows two-colour diagrams in the IRAC+K observed frame, for the three highest-z objects of the Yan et al. (2004) sample. The \( y \)-axis in the right-hand panel corresponds to the rest-frame \( V - K \), that is very sensitive to TP-AGB stars (cf. Figure 20). At the nominal redshift of the objects (\( z \sim 2.9 \)) the universe is at most 2.3 Gyr old in current cosmologies. The BC03 models (dotted lines) used by Yan et al. (2004) do not match the intrinsic \( V - K \) of the reddest object (#3, right-hand panel) unless the age becomes 4 Gyr, that exceeds the age of the universe. Our model (solid line) reaches the observed colours using ages around \( \sim 1.5 - 2 \) Gyr, instead. The inferred stellar mass for object #3 is \( \sim 2.5 \times 10^{10} \) \( M_\odot \). The other two objects in Figure 28 look younger in this simple diagrams and seem to be in the pre-TP-AGB phase. However, object #8 has a rest-frame \( V - K \) that is too red (or a rest-frame \( I - J \) that is too blue, x-axis) also for our standard models. A possible explanation could be reddening by dust, that would affect mostly the rest-frame \( V \). Dust reddening was not found to be a promising solution by Yan et al. (2004), however this might be also connected with their modelling lacking intrinsically red stars in the stellar population. A more detailed investigation is clearly beyond the scope of this paper and will be addressed in a subsequent work.

As a further example Figures 29 and 30 show the fits to the whole SEDs for other two objects of the Yan et al. (2004) sample. In the first case the best “by-eye” solution was found to be a composite stellar population (with solar metallicity) in which stars started to form 0.8 Gyr ago, with an exponentially-declining star formation mode with e-folding time of 0.1 Gyr. In the second case, a single burst (i.e. an SSP) of 0.6 Gyr with low-metallicity (\( Z_\odot / 20 \)) was instead used. For both objects the whole spectral energy distributions are fitted reasonably well with lower ages, corresponding to formation redshifts of the order \( z \sim 3 \). The derived stellar masses are \( 1.56 \times 10^{10} \) \( M_\odot \) and \( 1.54 \times 10^{9} \) \( M_\odot \), respectively. Interestingly, in the latter case the mass and the metallicity fits very well the corresponding relation obtained in the local universe for dwarf galaxies.

Figure 28. Two-colour diagrams in the IRAC colours (left-hand panel) and IRAC and \( K \) (right-hand panel), of the three highest-z objects (# 3, # 8, # 9) of the Yan et al. (2004) sample. Magnitudes are in the AB system, effective wavelengths of the IRAC filters are indicated. The \( y \)-axis of the right-hand panel corresponds to the rest-frame \( V - K \). Dotted and solid lines refer to the models by BC03 (solar metallicity) and ours for 0.5 \( Z_\odot \) plotted until the age of 2 Gyr, in order not to exceed the local age of the Universe (adopted cosmology \( \Omega_M = 0.3, \Omega_\Lambda = 0.7, H_0 = \) 73 km s\(^{-1}\) Mpc\(^{-1}\)).
Evolutionary population synthesis: models, analysis of the ingredients and application to high-z galaxies

7 SUMMARY

Evolutionary population synthesis models with various ages, metallicities, star formation histories, Horizontal Branch morphologies, are presented. The evolutionary code is based on the fuel consumption theorem (Renzini & Buzzoni 1986) for the evaluation of the energetics of the post Main Sequence evolutionary phases. The code was introduced in our previous work (Maraston 1998) in which stellar population models complete in all main stellar evolutionary phases, but only for solar metallicity were computed. The present paper provides the extension of the modelling to a wider range of stellar population parameters. The most important features of the models are: i) the inclusion of the energetics and the spectra of TP-AGB stars and their calibrations with observations; ii) the allowance for various Horizontal Branch morphologies, in particular blue morphologies at high metallicities. We will come back to both points below.

In the first part of the paper we perform a comprehensive analysis of the model ingredients, focusing in particular on: i) the metallicity effects on energetics and temperatures; ii) the impact of different stellar evolutionary tracks. The most relevant results can be summarized as follows.

The energetics of stellar populations are higher at lower metallicity because of the greater hydrogen abundance. The most affected stellar phases are the main sequence and the helium burning phase. The differences become smaller at ages later than the RGB phase-transition (\(t \gtrsim 1\) Gyr) when the RGB phase becomes the most important contributor to the total bolometric. This is due to the RGB fuel consumption being not affected dramatically by the chemical abundance, a result consistent with previous findings (Sweigart, Greggio & Renzini 1989).

The impact of stellar evolutionary tracks is evaluated by comparing energetics and temperatures of two different stellar models, namely the set of Frascati tracks by Cassisi and collaborators, that are classical tracks without overshooting, with those by the Padova school, that include the effects of overshooting. We find that the mass- luminosity relations for MS stars are rather similar among the explored tracks, which is explained by the modest amount of overshooting included in recent computations. Instead, major differences are found in post-MS phases, which involve both the energetics and the temperatures of the Red Giant Branch. In the Padova tracks the onset and development of the RGB is delayed with respect to the Frascati tracks, an effect of overshooting. For example, a \(~0.8\) Gyr stellar population model based on the Padova tracks has \(~30\) per cent less light than one based on the Frascati tracks, a result influencing the integrated output, especially the \(M/L\). It should be noticed that a recent comparison of the onset age of the RGB with what is observed in MC GCs favours the earlier RGB development of the Frascati tracks (Ferraro et al. 2004).

The temperatures of the RGB, especially towards the tip, are cooler in the Padova tracks, at solar metallicity and above, a feature also found in the Yale tracks of Yi et al. (2003). The RGB temperature is the ingredient having the largest influence on the SSP output. For example, the integrated \(V-K\) colours of old, metal-rich, stellar populations change by \(\Delta V-K \sim 0.08\) mag. The RGB temperatures are connected to the mixing-length and the underlying stellar models. The calibration of these effects is difficult since the comparisons with data require the adoption of temperature-colors

Figure 29. Comparison between the observed narrow-band fluxes of object #11 from Yan et al. (2004) and those of a 0.8 Gyr old stellar population model with solar metallicity in which the stars are forming with an exponentially-declining star formation rate with e-folding time of 0.1 Gyr.

Figure 30. Comparison between the observed narrow-band fluxes of object #15 from Yan et al. (2004) and those of a 0.6 Gyr old metal-poor simple stellar population.
transformations, that are uncertain as well. In spite of this, it would be highly valuable to understand the origin of such discrepancies at least theoretically.

Relevant to evolutionary population synthesis models is the following result. The differences induced by the use of various stellar tracks in our code are substantially smaller than the scatter among stellar population models in the literature that adopt the same stellar tracks.

In the second part of the paper we describe in detail the recipes for the stellar phases that are affected by mass-loss and therefore not described sufficiently by stellar evolutionary tracks. These are the Horizontal Branch (HB) in old stellar populations and the Thermally-Pulsing Asymptotic Giant Branch (TP-AGB) in stellar populations with ages between 0.2 and 2 Gyr. For the HB we apply mass-loss to the RGB tracks such that we obtain a good match to stellar population spectrophotometric indicators that are sensitive to the HB morphology (e.g. the Balmer lines). We therefore calibrate the amount of mass-loss with MW GCs of known HB morphology. In order to account for the observed scatter in the HB properties at a fixed metallicity, we provide the models with a couple of choices for the HB morphology. In particular, we also compute blue HB at high Z and show how much blue indicators like e.g. the Balmer lines and the U, B, V magnitudes are affected. The effects of BHBs at high Z are quantified in a form that can be easily used also when other evolutionary population synthesis are adopted.

A result interesting for extra-galactic GC studies is that the D-4000 A break, unlike the Balmer lines is insensitive not only to age but also to the Horizontal Branch morphology at low metallicity ([Z/H] \lesssim 0.1). Therefore this colour can be used as metallicity indicator in the metal-poor regime.

The energetic of the TP-AGB phase was calibrated with Magellanic Clouds GCs data in Maraston (1998). It was shown that this phase is the dominant one in stellar populations with ages between 0.3 and 2 Gyr, providing 40 per cent of the bolometric and up to 80 per cent of the luminosity in the near-IR. Here we make a step forward by including the spectra of TP-AGB stars in the synthetic spectral energy distributions (SEDs) of the stellar population models, using available empirical spectra of TP-AGB C-rich and O-rich stars. We compare the resulting model SEDs with the observed SEDs of a sample of MC GCs with data from U to K. The models provide a very good fit to the data in the whole age range relevant to the TP-AGB, a success that is entirely due to the proper inclusion of the phase in the models. We further compute the integrated spectral indices C2, H2O, CO in the near-IR (1.7 \lesssim \lambda/\mu m \lesssim 2.4), that are effective tracers of C and O stars. We find that the combination of C2 and H2O can be used to determine the metallicity of \sim 1 Gyr stellar populations, independently of the star formation history.

The model SEDs presented here can be applied to the analysis of high-z galaxies extending the concept of TP-AGB as age indicator for \sim 1 Gyr stellar populations to primeval objects in the early universe. As an illustrative application we re-analyze some high-z (2.4 \lesssim z \lesssim 2.9) galaxies recently observed with the Spitzer Space Telescope (Yan et al. 2004). The distinctive features of these objects is to have strong fluxes in both the rest-frames blue and near-IR relative to the flux in the V-band. This is what characterizes TP-AGB dominated stellar populations. We show that the data can be explained by stellar populations with ages in the range 0.6 to 1.5 Gyr. These ages are comfortable given the age of the universe at those redshifts (~ 2 Gyr) and imply formation redshifts between 3 and 6.

Models are available at www-astro.physics.ox.ac.uk/~maraston.

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