The Hubble tension in the non-flat Super-ΛCDM model
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Abstract
We investigate the Hubble tension in the non-flat Super-ΛCDM model. The non-flat Super-ΛCDM model extends the Super-ΛCDM model by including the spatial curvature as a free parameter. The Super-ΛCDM model extends the standard ΛCDM model of cosmology through additional parameters accounting for the possible effect of a trispectrum in the primordial fluctuations. In the cosmic microwave background data, this effect can be parameterized using parameters that change the observed angular power spectrum from the theoretical power spectrum due to a trispectrum that couples long and short wavelength modes. In this work, we perform Markov Chain Monte Carlo (MCMC) data analysis on the recent Planck 2018 temperature and polarization fluctuations data and the local Hubble constant measurements from supernovae data assuming a non-flat Super-ΛCDM model. We find that there is a preference for non-zero values of the spatial curvature parameter Ω_k and the Super-ΛCDM parameter A_0 at a level of Δχ^2 improvement of approximately 23.

1. Introduction
The Planck 2018 temperature and polarization fluctuations data currently provide very tight constraints on the standard ΛCDM model of cosmology [1]. One of the important cosmological parameters that can be inferred from the Planck cosmic microwave background (CMB) fluctuations data is the Hubble constant [2], H_0, the current expansion rate of the universe. The Planck 2018 temperature and polarization data, assuming the standard ΛCDM model, provide constraints (H_0 = 67.27 ± 0.60 km/s/Mpc) that are tighter than the direct measurements of the Hubble constant using Type Ia supernovae from the SH0ES collaboration (H_0 = 73.2 ± 1.3 km/s/Mpc) [3]. These measurements of the Hubble constant, H_0, from the early and late universe disagree at more than 4σ [4]; see also [5], however. A more recent work from the SH0ES team [6] presents a tighter measurement of the Hubble constant at H_0 = 73.04 ± 1.04 km/s/Mpc which brings the disagreement with the Planck CMB prediction to 5σ. This disagreement between the measurement of Hubble constant from Type Ia supernovae and that derived from CMB is a very active area of investigation in cosmology. It is possible that the resolution of the discrepancy can be due to currently unaccounted for systematic effects in one or more of the data sets that are used in the analyses. It is also possible that the resolution of the discrepancy comes through an extension to the cosmological model. See recent review articles [6, 7, 8, 9] for discussions of several proposed solutions to the Hubble tension problem.

One potential solution to the Hubble tension was presented in [10]: it extends the ΛCDM model by allowing the primordial fluctuations to deviate from the assumption of Gaussianity. The extended model is called the Super-ΛCDM model and it requires the primordial fluctuations that seed the CMB fluctuations have a deviation from Gaussianity. In [10], using Planck 2015 temperature fluctuations data, it was shown that the Super-ΛCDM model can alleviate the Hubble tension; the model reduced the tension (2.5σ), but did not completely resolve it. Most of the potential solutions to the Hubble tension problem presented so far similarly fit this description: they are able to reduce the level of tension but unable to solve the tension completely.

It is therefore interesting to consider if we can add another parameter to the ΛCDM model on top of the additional parameter(s) of the Super-ΛCDM model, and whether such an analysis can further alleviate the Hubble tension. We
do such an analysis in this work by adding the spatial curvature as a free parameter in the Super–ΛCDM model. We investigate parameter constraints on the spatial curvature Ω_k and the Super–ΛCDM parameters A_0, ϵ using the final Planck satellite CMB temperature+polarization data in combination with the Riess et. al 2020 Hubble constant measurement [3]. In the non-flat ΛCDM model, the expansion history of the universe depends on the curvature density parameter today (Ω_k) in addition to matter (Ω_m), radiation (Ω_r) and dark energy (Ω_Λ) densities:

$$\left( \frac{H}{H_0} \right)^2 = \Omega_k (1+z)^2 + \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda$$

(1)

The Super–ΛCDM model considers the effect of a primordial trispectrum on the observed angular power spectrum. In the presence of a trispectrum, it is possible that long-wavelength fluctuations modulate the small scale modes and therefore the observed power spectrum. The effect can be modeled by modifying the theoretical angular power spectrum (C_ℓ) in the following manner [10]:

$$C_\ell \rightarrow C_\ell + A_0 C_\ell(n_s + \epsilon),$$

(2)

where A_0 and ϵ are additional parameters of the Super–ΛCDM model, and C_\ell(n_s + ϵ) are the angular power spectra evaluated by changing the spectral index from n_s to n_s + ϵ with the other cosmological parameters fixed. It can be shown [10] that the effect of an additional modulation term such as A_0 C_\ell(n_s + ϵ) on a pseudo-C_ℓ power spectrum estimator is equivalent to a non-Gaussian term in the covariance matrix due to a primordial trispectrum of the form [11]:

$$T(k_1, k_2, k_3, k_4) = 4\tau_{NL} \left( \frac{K}{\sqrt{k_1 k_2}} \right)^{-2\epsilon} P_\phi(k_1) P_\phi(k_2) P_\phi(K),$$

(3)

where P_\phi(k) = (2\pi^2 A_0/k^3)(k/k_0)^{n_\phi - 1} is the power spectrum of potential fluctuations, k_0's are the four wavenumbers of the trispectrum in momentum space such that k_1 + k_2 + k_3 + k_4 = 0 and K = |k_1 - k_2| = |k_3 - k_4|. A negative A_0 parameter (as is preferred by the data analysis presented later) means that the actual value of the amplitude of fluctuations is larger than what is inferred from CMB power spectra assuming ΛCDM. However, note that in the Super–ΛCDM model the exact value of A_0 cannot be calculated given a trispectrum. Only the variance of A_0 i.e. ⟨A_0^2⟩ can be calculated. As such, it is not easy to directly translate the constraints obtained on A_0 and ϵ parameters in this work to the trispectrum amplitude T_{NL} of the primordial trispectrum. The ϵ parameter in the primordial trispectrum Eq. [3] is related to the mass of the additional scalar field in the quasi-single field model that generates the trispectrum [12-13]. From Eq. [2], we can see that ϵ has degeneracy with the spectral index n_s.

The modified angular power spectra in Eq. [2] are used to fit with the experimental angular power spectra. In the MCMC sampling, the cosmological parameters, the calibration parameters and the Super–ΛCDM parameters (A_0, ϵ) are all sampled together. To calculate the theoretical power spectra, C_ℓ, we make use of camb [14][15]. To sample the posterior parameter distribution, we use the cobaya [16] package and its implementation of the Metropolis sampler [17, 18, 19]. With the assumption that the long-wavelength modes that are responsible for the exact realization of both the temperature and polarization fluctuations of the Planck satellite data are mostly the same, we use a single A_0 parameter for all TT, TE, EE power spectra. The amount of overlap of these long-wavelength modes with the CMB lensing power spectra φφ needs to be studied carefully, and as such, we will omit using the CMB lensing power spectra in this work.

There have been a few analyses so far that have added the spatial curvature as an additional parameter to the ΛCDM cosmological model and its extensions in the context of current discussions of cosmic parameter tensions [20, 21, 22-25, 26, 27], but in most of the cosmological analyses the universe is assumed to be flat. It is known that the Planck-only data favors a negative value of Ω_k at roughly 3σ [27]; this preference is not robust to addition of CMB lensing and BAO data [1].

2. Data Used

The main (most constraining) data we use are the Planck 2018 temperature and polarization fluctuations data. There are three separate likelihood calculations for these data, which are listed below. The descriptions of the Planck
Table 1: 1D Marginalized constraint on some parameters of interest in the non-flat Super–ΛCDM model fits performed using the Planck + $H_0$ + Pantheon data. The second column shows mean values and 1σ constraint whereas the third column shows the 3σ range of the parameter posteriors.

| Parameter   | Best-fit | Constraint (68%) | 99.7% (3σ) Range |
|-------------|----------|------------------|-------------------|
| $A_0$       | -0.168   | $-0.169 \pm 0.045$ | [-0.299, -0.040] |
| $\Omega_k$  | 0.00666  | $0.0075^{+0.0025}_{-0.0023}$ | [-0.0003, 0.0140] |
| $H_0$       | 72.08    | $72.30 \pm 1.21$  | [68.72, 75.85]    |
| $\Omega_m$  | 0.2718   | $0.2713 \pm 0.0095$ | [0.2454, 0.3009]  |
| $100\theta_{MC}$ | 1.04116  | $1.04109 \pm 0.00033$ | [1.04008, 1.04206] |
| $\Omega_b h^2$ | 0.02266  | $0.02260 \pm 0.00018$ | [0.02210, 0.02310] |
| $\log(10^{10} A_s)$ | 3.220    | $3.221 \pm 0.051$  | [3.078, 3.375]    |
| $n_s$       | 0.9487   | $0.946^{+0.017}_{-0.014}$ | [0.9043, 0.9787]  |
| $\epsilon$ | -0.117   | $-0.120^{+0.088}_{-0.069}$ | [-0.334, 0]       |

The Pantheon sample consists of 1048 type Ia supernovae distance measurements in the redshift range $0.01 < z < 2.3$ [29]. We can add the Pantheon supernova data to our analysis without making any change to the available Pantheon likelihood. This is because the model prediction of distance measurements only depend on background parameters: $\Omega_m$, $\Omega_A$, $\Omega_k$, and $H_0$, and not on the additional Super–ΛCDM parameters $A_0$ and $\epsilon$. We omit some cosmological datasets such as the Baryon Acoustic Oscillation (BAO) and CMB lensing for which detailed understanding of the theoretical prediction of the Super–ΛCDM model is lacking. It will be interesting to check whether the result we obtain in this work holds when BAO and CMB lensing predictions for Super–ΛCDM model are worked out and the relevant data are included.

In addition to the comparison of ΛCDM and non-flat Super–ΛCDM, we will also briefly discuss MCMC results for the non-flat ΛCDM model, in which $\Omega_k$ is allowed to be a free parameter in addition to the six ΛCDM cosmological parameters.

3. Results

In Table 1, we list 1D marginalized posterior constraints for several parameters of the Super–ΛCDM + $\Omega_k$ model. The second column gives the best-fit parameter values whereas the third column gives the posterior mean values with the corresponding one standard deviation constraint on a parameter. The table, in the third column, lists the three standard deviation range of posterior values for each of the parameters.
3.1. Constraint on $\Lambda$CDM parameters

In Figure 1, we show 1D marginalized posterior distribution for six $\Lambda$CDM parameters for our MCMC results from both $\Lambda$CDM (red dashed) and Super--$\Lambda$CDM + $\Omega_k$ (black solid) models. Large parameter shifts occur for the Hubble constant ($H_0$), which is desired as our study of the non-flat Super--$\Lambda$CDM model is motivated to solve the Hubble tension. When the spatial curvature is made a free parameter, the Planck and Pantheon data do not simultaneously constrain $\Omega_m$, $\Omega_k$ and $H_0$ parameters; the situation improves when the SH0ES Hubble constant measurement is included and the combined data set prefers a higher value of the Hubble constant in the non-flat Super--$\Lambda$CDM model: $H_0 = 72.30 \pm 1.21$ (68%).

With a shift in the expansion rate to a higher value, there is a corresponding shift in the matter density to a lower value in the non-flat Super--$\Lambda$CDM model: $\Omega_m = 0.2713 \pm 0.0095$ (68 %). The strong degeneracy between the Hubble constant $H_0$ and the matter density $\Omega_m$ can be seen in the 2D posterior probability density plot of Figure 2.

There is also a preference in the non-flat Super--$\Lambda$CDM model for the amplitude of fluctuations $A_\text{S}$ and therefore log$(10^{10}A_\text{S})$ to a higher value than in the $\Lambda$CDM model; see Figure 1. This behavior is consistent with the previous work in [10] and is due to the preference for a negative value of the Super--$\Lambda$CDM parameter $A_0$; the strong degeneracy between $A_0$ and log$(10^{10}A_\text{S})$ can be seen in Figure 2.

3.2. 1D Marginalized constraint on $A_0$, $\epsilon$ and $\Omega_k$

The constraint on $A_0$, marginalized over all other parameters, is $A_0 = -0.169 \pm 0.045$ (68%). Compared to the constraint in [10] where the $\Omega_k$ parameter was fixed to zero, the marginalized constraint on $A_0$ in this work is tighter — preferring non-zero value of $A_0$ at approximately 3σ even when marginalized over all other parameters. The 1D marginalized posterior distribution of the three new parameters in the extended model ($A_0$, $\epsilon$ and $\Omega_k$) is shown in Figure 1 (bottom panel). Note that the data we used do not constrain the scale parameter $\epsilon$ of the Super--$\Lambda$CDM model very well. The 1D marginalized constraint on $\Omega_k$ is $\Omega_k = 0.0075^{+0.0025}_{-0.0023}$, with a preference for a non-zero value of spatial curvature at slightly less than 3σ once we account for the non-Gaussian nature of the 1D marginalized distribution. The posteriors for $\Omega_k$ and $A_0$ are not significantly correlated. Next, we discuss how their joint posterior probability distribution significantly excludes the $\Lambda$CDM values of ($A_0 = 0, \Omega_k = 0$).

3.3. 2D Posterior distribution on ($A_0$, $\Omega_k$)

In Figure 3, we show the 2D posterior distribution for parameters ($A_0$, $\Omega_k$) produced in the non-flat Super--$\Lambda$CDM fit using the Planck + SH0ES + Pantheon data. In the figure, the dashed lines show the value of these parameters in the standard $\Lambda$CDM model i.e. $A_0 = 0$ and $\Omega_k = 0$. The $\Lambda$CDM value in the 2D plane is shown as a red cross at ($A_0 = 0, \Omega_k = 0$). As can be seen in the figure, the $\Lambda$CDM value is outside the 3σ (99.7%) confidence contour. If we add a 99.95% confidence contour in Figure 3, which corresponds to about 3.5σ, the $\Lambda$CDM value (red cross) still lies comfortably outside the contour. However, we start to observe that the contour has a noisier shape indicating that we need a larger sample of MCMC points to make robust inference at 99.95% confidence level; therefore, we do not plot beyond the 99.7% confidence contour.

4. Discussion

We now discuss the improvement in fits in the non-flat Super--$\Lambda$CDM model compared to the flat $\Lambda$CDM model. The model fit improvement of the non-flat Super--$\Lambda$CDM model over $\Lambda$CDM model for our data combination is substantial: $\Delta \chi^2 = -23.1$. But it is important to check whether the improvement in overall fit of the data occurs at the expense of significantly reducing the quality of fit to a subset of the data.

In Table 2, we list and compare the $\chi^2$ of fits for different models and data. In the table we can see if any improvement in fits is largely driven by one of the data at the expense of significantly degrading the quality of fit to other data. Comparing the $\chi^2$ of fits for the non-flat Super--$\Lambda$CDM (second column in Table 2) with the $\chi^2$ of fits for the flat $\Lambda$CDM (fourth column in Table 2), it is clear that the non-flat Super--$\Lambda$CDM model’s large improvement in fit of the Planck + $H_0$ + Pantheon data does not deteriorate individual Planck temperature and polarization likelihood fits. The large improvement in $\Delta \chi^2$ is also consistent with the the 2D posterior plot shown in Figure 3, in which we can observe that the combination of data Planck + SH0ES + Pantheon prefers a non-zero value of $\Omega_k$ at a statistical significance exceeding 3σ. We note, however, that the preference for non-zero $\Omega_k$ is for the sign opposite to what is
Figure 1: Top and middle panels: 1D marginalized constraint on six $\Lambda$CDM parameters in two models: standard $\Lambda$CDM model (red dashed) and the extended Super-$\Lambda$CDM model (solid black). Relatively large shifts in parameter constraint values are between the two models are found for the matter density, the Hubble constant and the amplitude of fluctuations. The values of $H_0$ posterior in the Super-$\Lambda$CDM + $\Omega_k$ model (solid black) are larger compared to the $\Lambda$CDM derived value of $H_0$ (red dashed), but are consistent with the SHOES $H_0$ measurement shown as gray bands (1 and 2 $\sigma$) in the middle panel. Bottom panel: 1D marginalized constraint on three new parameters in the non-flat Super-$\Lambda$CDM model. In all cases, the data set used in our main data combination: Planck + SHOES + Pantheon. The constraint on $\epsilon$ is weak as it has a strong degeneracy with $n_s$. On the other hand, the data combination generally prefers values of $\Omega_k$ and $A_0$ away from the $\Lambda$CDM null value of zero. See also Table 1.
Figure 2: 2D posterior probability distributions for the parameters \((A_0, \epsilon, \Omega_k)\) sampled in our non-flat Super-\(\Lambda\)CDM model fits, with the Hubble constant parameter, \(H_0\). The data sets used are the Planck 2018 temperature and polarization CMB data, the Pantheon supernovae data and the Riess et. al Hubble constant measurement of [30]. We also show the 2D posterior probability distributions of these parameters with the parameter for amplitude of fluctuations \(\log(10^{10}A_s)\) in the top panel.

| Likelihood                  | non-flat Super-\(\Lambda\)CDM | non-flat \(\Lambda\)CDM | flat \(\Lambda\)CDM (Planck only) | flat \(\Lambda\)CDM (Planck + SH0ES) |
|-----------------------------|-------------------------------|--------------------------|-----------------------------------|-----------------------------------|
| planck_2018_lowl.TT         | 21.43                         | 23.67                    | 22.10                             | 23.26                             |
| planck_2018_lowl.EE         | 395.75                        | 396.52                   | 396.02                            | 396.05                            |
| planck_2018_highl_plik.TTTEEE | 2338.82                     | 2350.32                  | 2349.14                           | 2344.6                            |
| H0.riess2020                | 0.74                          | 1.75                     | 13.93                             | -                                 |
| SN.pantheon                 | 1036.08                       | 1035.33                  | 1034.76                           | -                                 |
| Total \(\chi^2\)           | 3792.82                       | 3807.59                  | 3815.95                           | -                                 |
| \(\Delta \chi^2\) (relative to flat \(\Lambda\)CDM) | -23.1                        | -8.4                    | 0                                 | -                                 |

Table 2: \(\chi^2\) values for best-fit parameters for individual data likelihoods for several models of interest. The data combination used is Planck + SH0ES + Pantheon except when indicated in the column heading.
found with the Planck-only data in the $\Lambda$CDM + $\Omega_k$ model in [27], which finds a preference for $\Omega_k < 0$ whereas our analysis with a different data combination finds a preference for $\Omega_k > 0$.

The mild preference of the Planck CMB data for negative $\Omega_k$ in the $\Lambda$CDM + $\Omega_k$ model seems to come from its ability to better fit the apparent larger lensing effect in the temperature data (sometimes parameterized by the phenomenological $\alpha_{\text{lens}}$ parameter) in addition to the better fit to the low-$\ell$ temperature data [1]. However, those fits were done without adding the SH0ES data. In the fits with Planck+SH0ES+Pantheon data, we find an improvement in fit in the $\Lambda$CDM + $\Omega_k$ model over the $\Lambda$CDM model by $\Delta \chi^2 = -8.4$, but with a preference for $\Omega_k > 0$. As can be seen in Table 1, most of the improvement in fit is due to better fitting the SH0ES data, while the fit to the Planck CMB data worsens by $\Delta \chi^2 = 6.6$ (compared to flat $\Lambda$CDM using Planck-only data). This is not desirable and points to the fact that the two data sets are discrepant in the $\Lambda$CDM + $\Omega_k$ model. On the contrary, the non-flat Super-$\Lambda$CDM model improves fitting of the Planck CMB data compared to the flat $\Lambda$CDM model (with Planck-only data) by $\Delta \chi^2 = -7.9$.

In Figure 4, we plot the residuals of Planck power spectrum data. The residuals of $D_\ell = \ell(\ell+1)C_\ell/(2\pi)$ for $TT$, $EE$, and $TE$ spectra are calculated with respect to the corresponding best-fit $\Lambda$CDM Planck 2018 Planck spectra provided by the Planck Collaboration[1] and plotted after normalizing by the cosmic variance for each multipole. In the figure we also plot the corresponding residuals of the best-fit non-flat Super-$\Lambda$CDM power spectra; the best-fit model parameters used can be found in Table 1. For the $TT$ spectra, we can observe that the non-flat Super-$\Lambda$CDM model captures some of the oscillatory features of the data residuals ($800 \lesssim \ell \lesssim 1800$) leading to an improvement in CMB $\Delta \chi^2$.

5. Summary

In this work, we present a concrete example of how the Hubble tension could be a hint of two different extensions to the standard $\Lambda$CDM model. We find that a non-flat Super-$\Lambda$CDM model can significantly alleviate the Hubble tension. We find that the model greatly improves the fit to the combination of Planck temperature plus polarization, the local measurement of the Hubble constant, and the Pantheon supernovae distance measurements. With three extra model parameters, the fit improvement of the Super-$\Lambda$CDM model with respect to the standard $\Lambda$CDM for the data combination Planck + SH0ES + Pantheon is found to be $\Delta \chi^2 = -23.1$. The Hubble constant value preferred is larger compared to the Planck-only $\Lambda$CDM derived Hubble constant. We find that the better fit is obtained without degrading the fit to the Planck likelihood.

\[\text{COM\_PowerSpect\_CMB-base-plikHM_TTTEEE-lowl-lowE-lensing-minimum-theory}\_R3.01.txt\]
Figure 4: Residuals of Planck $TT$, $EE$, and $TE$ angular power spectra with respect to the corresponding best-fit $ΛCDM$ theory fit using only Planck data. The (binned) residuals and error bars plotted above are normalized by cosmic variance. The corresponding best-fit non-flat Super-$ΛCDM$ theory residual (blue line) with respect to the best-fit Planck $ΛCDM$ theory is also plotted. It can be observed that the non-flat Super-$ΛCDM$ model follows the oscillatory features in the temperature power spectrum residuals between multipoles $ℓ \approx 800$ and $ℓ \approx 1800$. This likely accounts for some of the improvement in CMB $Δ\chi^2$ in the non-flat Super-$ΛCDM$ model.
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