The linear optics holds high promise as a platform for implementing quantum algorithms. Here we present an all-optical experimental realization of the Fourier transformation-based phase estimation algorithm having a wealth of applications in quantum metrology. The employed setup is fully built on the linear optics devices such as beam splitters, mirrors and phase shifters, and the used technique opens route to realizing a rich diversity of quantum algorithms.

I. INTRODUCTION

Due to the rapid rise of quantum computation, increasing efforts are being invested into development of the necessary technologies. The list of promising solutions includes Josephson junctions [1], trapped ions [2], nuclear spins [3] and quantum dots [4]. Among other things, there have been great expectations regarding the optical implementation of the quantum algorithms [5–15]. To date, researchers have proposed a number of methods for realizing quantum logic by means of light. One example described in Ref. [13] is the cat-state scheme which encodes logical qubits in coherence states. Another approach is based on representing the qubit states by different polarization [14] or modes [5]. Beyond that, it has been shown that optics could in principle become a foundation for the quantum computing over continuous variables [15].

The aim of the present study is to practically demonstrate the quantum computing capacities of linear optics. We devise an experimental realization of the famous Fourier-based phase estimation algorithm, used namely in quantum metrology [16–18]. Our approach is predicated on the statement that any finite-dimensional unitary matrix can be realized by the means of 50:50 beam splitters, phase shifters and mirrors [5].

This paper is organized as follows. We will begin in Sec. II by giving a brief overview of the algorithm and our theoretical framework. In Sec. III, we will propose our experimental layout and obtain the corresponding formulae. Finally, we will discuss the results of the experiment in Sec. IV.

II. PRELIMINARIES

A. Algorithm description

We start with the description of the Fourier phase-estimation algorithm operating in the qudit regime. The algorithm starts from the initial qudit state which we take to be the superposition of all computational states:

$$|\Psi_\phi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{ik\phi} |k\rangle,$$

where \(\{|k\rangle\}_{k=0}^{d-1}\) is an orthonormal computational basis in the qudit’s Hilbert space. Additionally, we consider that \(\phi = \frac{m\pi}{d}\), \(m \in \{0, 1, \ldots, d - 1\}\). The purpose of the algorithm is to unambiguously determine the value of \(\phi\) via a single-shot measurement of the qudit state. This is performed by applying a base-\(d\) quantum Fourier transformation with a corresponding unitary operator \(\hat{F}\),

$$\hat{F} |n\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{-2\pi i nk/d} |k\rangle.$$

The action of \(\hat{F}\) on the initial state \(|\Psi_\phi\rangle\) yields one of the states from the computational set \(\{|k\rangle\}_{k=0}^{d-1}\) depending on \(\phi = \frac{m\pi}{d}\):

$$|\Psi_{\text{out}}\rangle = \hat{F} |\Psi_\phi\rangle = |m\rangle.$$ 

Accordingly, by measuring the output state \(|\Psi_{\text{out}}\rangle\) one can find the definitive value of \(\phi\).

The given algorithm can namely be used for the precision measurement of the magnetic field [17] [18]. Let us consider that each basis state \(|k\rangle\) corresponds to a particular angular momentum polarization; then the qudit...
residing in a state $|\Psi_\phi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle$ can be brought to the state $|1\rangle$ through the exposure to the magnetic field. Thus, assuming that the field accepts only $d$ values $H \in \{0, h, 2h, \ldots, (d - 1)h\}$, the task comes down to measuring the value of $\phi$.

**B. Optical scheme**

We proceed by introducing the optical framework of our study. In our setup the qudit is represented by $d$ coherent beams. Each element of its $d$-dimensional state vector is a complex amplitude of a corresponding beam. Accordingly, the state vector transforms when the light passes through the arrangement of beam splitters, phase shifters and mirrors. The task of constructing a particular unitary operator comes down to its decomposition into a sequence of the two-dimensional beam splitter transformations and individual phase shifts. In this section we devise base-3 (qutrit) scheme to carry out the Fourier transformation

$$\hat{F} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ e^{4\pi i/3} & e^{2\pi i/3} & e^{2\pi i/3} \end{pmatrix}. \quad (4)$$

The matrix $\hat{A}_{jk}(\alpha, \theta)$ of an arbitrary lossless beam splitter with the $j$th and $k$th input beams may be expressed in the form

$$\hat{A}_{01}^\chi(\alpha, \theta) = \begin{pmatrix} \cos \chi e^{i\theta} & \sin \chi e^{i(\theta+\alpha)} & 0 \\ -\sin \chi e^{i(-\theta-\alpha)} & \cos \chi e^{i\theta} & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad (5)$$

$$\hat{A}_{12}^\chi(\alpha, \theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \chi e^{i\theta} & \sin \chi e^{i(\theta+\alpha)} \\ 0 & -\sin \chi e^{i(-\theta-\alpha)} & \cos \chi e^{i\theta} \end{pmatrix}, \quad (6)$$

where $\chi$ determines the split ratio ($\sqrt{T} = \cos \chi, \sqrt{R} = \sin \chi$); $\alpha$ and $\theta$ are certain phases. The matrix $\hat{P}_{0,1,2}^\beta$ corresponding to the phase change by $\beta$ of the $j$-th beam can be defined as

$$\hat{P}_{0,1,2}^\beta = \left\{ \begin{pmatrix} e^{i\beta} & 0 & 0 \\ 0 & e^{i\theta} & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} : \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} : \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \right\}. \quad (7)$$

Naturally, being limited in instruments, one may need to prepare a beam splitter matrix with a desired ratio of reflection to transmission. This can be done by assembling a Mach–Zehnder interferometer [19] using two symmetric 50:50 beam splitters (for convenience, henceforth we will omit the notation for dependence on $\alpha$.

FIG. 2. Experimental scheme for the qutrit case of the metrological algorithm.
and θ if \((\alpha, \theta) = (\pi/2, 0)):
\[
\hat{A}_{01} = \hat{A}_0^x (\pi/2, 0) = \hat{p}_0^{\pi/4} \hat{p}_1^{\pi/4} \hat{A}_{01}^x \hat{p}_0^{\pi/4} \hat{p}_1^{\pi/4} \hat{p}_0^{\pi/4} \hat{p}_1^{\pi/4}, \tag{8}
\]
with \(\hat{A}_{01}^x \) corresponding to the ideal symmetric beam splitter.

As shown in Ref. [10], the Fourier transformation \(\hat{F}\) may be factorized as follows:
\[
\hat{F} = \hat{p}_1^{\pi/4} \hat{A}_{12}^x \hat{p}_0 \hat{A}_{01}^x \hat{p}_1^{\pi/4} \hat{p}_2^{\pi/4} \hat{p}_1^{\pi/4} \hat{A}_{01}^x \hat{p}_0^{\pi/4} \hat{p}_1^{\pi/4} \hat{p}_2^{\pi/4}, \tag{9}
\]
where \(\chi = \tan^{-1}(\sqrt{2})\). It is easily seen from this expression that the experimental realization of \(\hat{F}\) requires no more than 4 symmetric 50:50 beam splitters. The optical circuit for \(\hat{F}\) is depicted in Fig. 1.

III. EXPERIMENTAL SETUP

The experimental layout, which can be divided into two modules, is depicted in Fig. 2.

In the state preparation module, the incident laser beam is converted into the qutrit initial state given by Eq. (1). The beam splitters BS\(_a\) and BS\(_b\) generate three beams each representing a particular basis state \(|j\rangle\) (\(j = \{0, 1, 2\}\)). The |1⟩ and |2⟩ beams then pass through respectively one (PS\(_\phi\)) and two (PS\(_{2\phi}\)) phase shifters attached to a swivel platform which sets the relative phases 0, \(\phi\), and \(2\phi\). The value of \(\phi\) depends on the position of the platform: by rotating the platform one alters the length of the optical paths through the phase shifters and, therefore, changes \(\phi\) without affecting the ratio between the relative phases.

The primary module corresponds to Eq. (9) and Fig. 1. However, although Eq. (9) directly translates the Fourier transformation into the optical setting, it fails to take account of certain limitations intrinsic to the real equipment. Let us provide the list of equipment and make further corrections of Eq. (9).

**Phase shifters.**- The phase shifters mainly serve to adjust the relative phases of the beams. In our setup, we use pieces of thick glass; the transmission through these elements is associated with a near 12.5% intensity loss. In order to take such losses into account we should employ the corresponding operators \(\hat{L}_{0,1,2}\):
\[
\hat{L}_{0,1,2}^t = \begin{cases}
\begin{pmatrix}
1 & 0 & 0 \\
0 & t & 0 \\
0 & 0 & t
\end{pmatrix} & \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\end{cases},
\]
where \(t\) is the absolute value of the transmission coefficient of and individual phase shifter.

**Beam splitters.**- We employ beam splitters with dielectric coating optimized for the 400 – 700 nm range. The nominal split ratio is 50:50. In practice however, this holds only if the incident laser beam is unpolarized. For the case of the linearly polarized beam used in our experiment, the split ratio is close to 55:45.

**Mirrors.**- Dielectric mirrors optimized for the 400 – 700 nm range.

**Laser.**- Diode pumped solid state laser, 532 nm, 150 mW. Let us write the explicit equation for the operation realized in the primary module:
\[
\hat{U} = [\hat{A}_{12}^x (\alpha_4, \theta_4) \hat{p}_2^{\psi_4} \hat{L}_1^{\psi_4} \hat{p}_1^{\psi_4} |4 \rangle [\hat{A}_{01}^x (\alpha_3, \theta_3) \hat{p}_1^{\psi_3} \hat{p}_0^{\psi_3} \hat{L}_1^{\psi_3} \hat{p}_0^{\psi_3} |3 \rangle \\
\times [\hat{A}_{01}^x (\alpha_2, \theta_2) \hat{p}_1^{\psi_2} \hat{L}_1^{\psi_2} \hat{p}_0^{\psi_2} |2 \rangle [\hat{A}_{12}^x (\alpha_1, \theta_1) \hat{L}_2^{\psi_2} \hat{p}_2^{\psi_1} |1 \rangle, \tag{10}
\]
where \( t_{ps} \) is the modulus of the transmission coefficient of PS\(_1\), \ldots, PS\(_4\); \( \chi_0 \) defines the beam splitters’ split ratio \( \sqrt{T} = \cos \chi_0, \sqrt{R} = \sin \chi_0 \); \( \alpha_i \) and \( \theta_i \) correspond to BS\(_i\) (see Eq. (5)); \( \psi_i \) is the phase change due to reflection of \( M_i \); \( x_i \) is the phase change on PS\(_i\). In our experiment \( t_{ps} = 0.935, T = 0.445 \) and \( R = 0.555 \). The notation \( [\ldots] \) will be needed later in the text. For simplicity, the above formula does not explicitly include discrepancies in the optical distances. In this respect, we can suppose that \( x_i \) is a relative phase in which such terms along with the phase shift on PS\(_i\) are taken into account. The output state vector can be written as

\[
|\Psi_{out}(x_1, x_2, x_3, x_4, \phi)\rangle = \hat{U} \{ \hat{P}_0^{\psi_a} \hat{L}_1^{t_{ps}} \hat{P}_1^{\lambda} \hat{L}_2^{x_t} \hat{P}_2^{x_{2\phi}} \hat{A}_{01}^{\lambda_0} (\alpha_a, \theta_a) \hat{A}_{02}^{\lambda_0} (\alpha_b, \theta_b) |2\} \}_{sp},
\]

where the brackets \( \{ \ldots \} \) denote the state prepared in the first module of the scheme; \( t_\phi \) and \( t_{2\phi} \) are the absolute values of the transmission coefficient of PS\(_\phi\) and PS\(_{2\phi}\) respectively (in our experiment \( t_\phi = 0.922, t_{2\phi} = 0.894 \)); \( (\alpha_a, \theta_a), (\alpha_b, \theta_b) \) and \( \psi_a \) correspond respectively to BS\(_a\), BS\(_b\) and \( M_0 \).

For certain values of \( x_i \) (which we denote by \( x_i^* \); see Appendix A and Eq. (11)) the transformation implemented in the scheme is similar to Eq. (9):

\[
\hat{F}_{exp} = [\hat{A}_{10}^{x_0} \hat{P}_0^{x_{2\phi}} \hat{L}_1^{x_t} |4] \times [\hat{A}_{01}^{x_0} \hat{P}_0^{x_{2\phi}} |3] \\
\times [\hat{A}_{01}^{x_0} \hat{L}_1^{x_t} |2] \\
\times [\hat{A}_{12}^{x_0} \hat{L}_2^{x_{2\phi}} \hat{F}_2^{x_{3\phi}/2} |1].
\]

Here we ignored the phases of the resulting beams incident on the detectors. Further clarification of this expression along with the description of the adjustment procedure is given in Appendix A.

\section*{IV. RESULTS AND DISCUSSION}

Fig. 4 shows the measured intensities as functions of \( \phi \). The data on D\(_i\) is fit by the square of \( i\)th element of the output vector function given by Eq. (11):

\[
p_i = a_i \ |i| |\Psi_{out}(x_1, x_2, x_3, x_4, \lambda \cdot \phi + \mu)\rangle|^2 + b_i, \tag{13}
\]

where \( a_i \) is the intensity scaling parameter; \( b_i \) is the intensity bias simulating the interference visibility loss; \( \lambda \) and \( \mu \) are respectively phase scaling parameter and phase shift independent of \( i \). The phases \( x_i \) determined from the fit are given by

\[
(x_1, x_2, x_3, x_4) = (x_1^F, x_2^F, x_3^F, x_4^F) \\
+ (-0.25, 0.16, -0.20, -0.28). \tag{14}
\]

Despite the discrepancies described by the second term, we believe our data compare well with the theoretical plots depicted in Fig. 3. The results show that the interference can to a large extent be controlled notwithstanding the complexity of the optical scheme. It can thus be reasonably considered that the optical framework provides the capacity of a small scale quantum computer. Nevertheless, such approach has its own limitations.

One drawback of our experiment was the divergence of the interfering beams. The estimated angles between the beams are \( \sim 10^{-3} \) rad. Such divergence results in the complex interference pictures which can no longer be considered one-dimensional; the emerging effects are not taken into account by our model. Another shortcoming was the intensity fluctuations caused by the mechanical oscillations in the system. The latter, together with the intrinsic instability of the optical mounts, appear to cause the drift of \( x_i \).

In the future, we plan to practically realize the base-4 (quartet) version of the algorithm. To further our research we intend to employ a single-photon source. Apart from that, future work will also entail refining our theoretical model along with the further upgrade of the equipment. Namely, we aim to implement the machine learning algorithms to compensate the imprecision in adjustment of the optical elements.

\[\text{FIG. 4. The measured intensities as functions of } \phi. \text{ The solid line shows the theoretical fit to the data.}\]
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Appendix A: Adjustment

In this Appendix we describe the adjustment procedure for the qutrit quantum Fourier transformation setup illustrated in Fig. 2. Our step-by-step approach lays in tuning the signal at the intermediate points of the scheme (see Fig. IV). At each consecutive step, the interference intensity at the given point is matched with the theoretical value obtained through the breakdown of Eq. (12). At the first two stages, we receive the signal reflected from the phase shifters PS2 and PS3 using the detectors AD1 and AD2, respectively. In turn, the last two stages involve the signals from the detectors AD3 and AD4. The adjustment is performed via rotating the phase shifters (i.e., altering the optical path length) preceding the given point. By doing so, one changes the phases $x_i$ which in the end should be equal to $x_i^F$:

$$x_1^F = -a_1 + a_3 - a_b + \theta_1 + \psi_2 + \pi;$$
$$x_2^F = -a_2 + a_3 - \theta_1 + \psi_1 - \psi_2 - \pi/2;$$
$$x_3^F = -a_2 + a_3 - \psi_3 + \psi_4 + \pi - 2\tilde{\chi};$$
$$x_4^F = -a_1 + a_2 + a_4 - a_b + \theta_1 - \theta_2 - \theta_3 - \psi_4 - \psi_5 - \psi_6 - \psi_a - \pi + \tilde{\chi}. \quad (A1)$$

Let us now examine each step of the procedure in details. 

**Step 1.** Since $x_1$, $x_2$, $x_3$, and $x_4$ essentially determine the initial relative phases between the $|0\rangle$, $|1\rangle$ and $|2\rangle$ beams, we have a freedom in choosing $\phi$. Here and throughout the whole procedure we put $\phi = \pi/3$.

Using AD$_1$ we measure the intensity of the $|1\rangle$ beam after it passes through BS$_{12}$. This intensity may be regarded as the probability $p_1$ of finding the qutrit in the state $|1\rangle$ after the action of the first block of operators (denoted by $[\ldots]$) in Eq. (10) and may be written...
\[ p_1 = \sin^2(\chi_0) \cos^2(\chi_0) \left( t_{ps} t_{2\phi} (t_{ps} t_{2\phi} + 2 t_{\phi} \sin(\chi_0) \cos(\Delta x_1 + \phi)) + t_{\phi}^2 \sin^2(\chi_0) \right), \quad (A2) \]

where we denote \( \Delta x_i = x_i - x_i^F \).

Our object is to set the value of \( \Delta x_1 \) to zero so that the measured signal would comply with the action of the first block in Eq. (12). Experimentally we achieve this by rotating PS_1 and controlling the intensity on AD_1. According to Eq. (A2), the target intensity can be expressed in terms of the experimentally measurable values as \( p_1 = \min_{x_1} p_1 + 0.75 (\max_{x_1} p_1 - \min_{x_1} p_1) \). Fig. 6(a) shows the theoretical plot of the signal as function of \( \Delta x_1 \), where the dot marks the point to which we adjust PS_1.

Step 2.– Using AD_2 we measure the intensity of the \(|0\rangle\) beam after it passes through BS_{01}. Bearing in mind the second block of operators \([\ldots]_2\), we write the corresponding probability \( p_2 \):

\[ p_2 = \sin^2(\chi_0) \cos^2(\chi_0) \left( \cos^2(\chi_0) + \frac{1}{2} t_{ps}^2 \sin^2(\chi_0) t_{\phi}^2 + 2 t_{ps}^2 t_{2\phi}^2 - t_{\phi}^2 \cos(2\chi_0) + 4 t_{ps} t_{\phi} t_{2\phi} \sin(\chi_0) \cos(\phi) \right) \]
\[ - 2 t_{ps} \cos(\chi_0) \sin(\chi_0) (t_{ps} t_{2\phi} \sin(\Delta x_2 + 2\phi) + t_{\phi} \sin(\Delta x_2 + \phi) \sin(\chi_0)) \quad (A3) \]

The condition \( \Delta x_2 = 0 \) corresponds to a minimum of \( p_2 \) (see Fig. 6(b)).

Step 3.– Using AD_3 we measure the intensity of the \(|0\rangle\) beam after it passes through BS_{01}. The corresponding probability \( p_3 \) after the action of the third block operators \([\ldots]_3\) is given by
\[ p_3 = \sin^2(\chi) \cos^2(\chi)(t_{ps}^2 \cos^4(\chi) + t_{ps}^2 \sin^2(\chi) \cos^2(\chi)(-2t_{ps}t_{ps} t_{2\phi} \sin(2\phi) + t_{\phi} \sin(\chi)) \sin(\phi) \\
+ t_{ps} \sin(\Delta x_3 - 2(\phi + \tan^{-1}(\sqrt{2}))) - t_{phi} \cos(-\chi_0 + \Delta x_3 - \phi - 2\tan^{-1}(\sqrt{2})) \\
+ t_{ps} \cos(\chi_0 + \Delta x_3 - \phi - 2\tan^{-1}(\sqrt{2})) + t_{ps} \sin^2(\chi_0) \cos(\chi_0)(-2t_{ps}^2 t_{2\phi} \sin(\Delta x_3 + 2\phi - 2\tan^{-1}(\sqrt{2}) \\
+ 2t_{ps} t_{2\phi} \sin(2\phi) - t_{ps} t_{\phi} \cos(\chi + \Delta x_3 + \phi - 2\tan^{-1}(\sqrt{2})) + t_{ps} t_{\phi} \cos(\chi + \Delta x_3 + \phi - 2\tan^{-1}(\sqrt{2}) \\
+ 2t_{\phi} \sin(\chi_0) \sin(\phi)) + \frac{1}{2} t_{ps} \sin^2(\chi_0) \cos^2(\chi_0)(-t_{ps}^2 t_{2\phi} + t_{\phi}^2 \cos(2\phi) \\
- 2(2t_{ps}^3 t_{2\phi}^2 + t_{ps}^2 t_{\phi}^2 - 2) \cos(\Delta x_3 - 2\tan^{-1}(\sqrt{2})) + t_{ps}^2 t_{2\phi}^2 + 4t_{ps}^3 \sin(\chi_0) \cos(\phi) \\
+ t_{ps}^2 t_{\phi}^2 + 2t_{ps}^2 t_{2\phi}^2 + \frac{8}{3} t_{ps} t_{\phi} t_{2\phi} \sin(\chi_0) \cos(\Delta x_3) \cos(\phi) - \frac{16}{3} \sqrt{2} t_{ps} t_{\phi} t_{2\phi} \sin(\chi_0) \sin(\Delta x_3) \cos(\phi) \\
+ 4t_{ps} t_{\phi} t_{2\phi} \sin(\chi_0) \cos(\phi) + t_{ps}^2 t_{\phi}^2 \cos(2\chi_0 + \Delta x_3 - 2\tan^{-1}(\sqrt{2})) + t_{ps} t_{\phi}^2 \sin(\Delta x_3 - 2(\chi_0 + \tan^{-1}(\sqrt{2})) + t_{\phi}^2)) + \sin^4(\chi_0) \) \]

For \( \Delta x_3 = 0 \) we have \( p_3 = \min_{x_3} p_3 + 0.60(\max_{x_3} p_3 - \min_{x_3} p_4) \) (see Fig. 4(c)).

**Step 4.** Using AD3 we measure the intensity of the |1\> beam after it passes through BS\(_{4a}\). The corresponding probability \( p_4 \) after the action of the fourth block operators ([..]_4) is given by

\[ p_4 = \frac{1}{12} \sin^2(\chi) \cos^2(\chi)(2t_{ps}^4 t_{2\phi}^2 \cos^2(2\phi)(t_{ps}^2 6t_{ps} \sin^4(\chi_0) - \sin^2(2\chi_0)) + 6 \cos^4(\chi_0) \\
- 8t_{ps}^4 t_{2\phi} \cos(2\phi) \cos(\phi)(2t_{ps} t_{2\phi} \sin^3(\chi_0) \cos^2(\chi_0) - 3t_{\phi} \sin(\chi_0) \cos(\chi_0)) + \sqrt{2} \sin(2\chi_0) \\
+ 12t_{ps}^4 \cos^4(\chi_0)(t_{ps}^2 t_{2\phi}^2 \cos^2(2\phi) + t_{\phi} \sin(\chi_0)(4t_{ps} t_{2\phi} \sin^2(\phi) \cos(\phi) + t_{\phi} \sin(\chi_0))) \\
- 8t_{ps}^2 \sin^4(\chi_0) \cos(\chi_0)(2\cos(\phi)(t_{ps}(3t_{ps} + 1)t_{2\phi} \sin(\phi) + \sqrt{2} t_{\phi} \sin(\chi_0) + (3t_{ps} + 1)t_{\phi} \sin(\chi_0) \sin(\phi)) \\
+ 12 \sin^2(\chi_0) \cos^2(\chi_0)(2t_{ps}^2 t_{2\phi}^2 \sin^2(2\phi) \cos(2\tan^{-1}(\sqrt{2})) - t_{ps}^2 t_{\phi} \cos(2\chi_0) \sin(2\tan^{-1}(\sqrt{2}) + 1) \\
+ t_{ps}^2(24t_{ps}^4 t_{2\phi} t_{2\phi} \sin(\phi) + 6t_{ps}^3 \sin(\chi_0) \cos(\phi) - 4t_{ps} t_{\phi} t_{2\phi} \sin^2(2\chi_0) \sin(\phi) \sin(2\phi) \\
+ (-t_{ps}^2 t_{\phi}^2 + 3t_{ps} + 2) \sin^2(2\chi_0) - 3t_{ps}^4 \sin^4(\chi_0)(2t_{ps}^2 t_{2\phi} \cos(4\phi) - 2t_{ps}^2 t_{2\phi} + t_{\phi}^2 \cos(2\chi_0) - t_{\phi}^2)) \\
+ 8t_{ps} \sin(\chi_0) \cos^3(\chi_0)(3t_{\phi} \sin(\chi_0)(\sin(\phi) - t_{ps} \sin(\phi + 2\tan^{-1}(\sqrt{2}))) + t_{ps}(t_{ps} + 3)t_{2\phi} \sin(2\phi)))) \] \]

For \( \Delta x_4 = 0 \) this becomes

\[ p_4 = \min_{x_4} p_4 + 0.64(\max_{x_4} p_4 - \min_{x_4} p_4) \] (see Fig. 4(d)).

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