Extended Bubble Cavitation Model to predict water hammer in viscoelastic pipelines

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Abstract. The equation of continuity describing the two-phase bubble flow was modified to take into account the influence of the viscoelasticity of the pipe walls. The modified equation of continuity together with the equation of motion was solved using the method of characteristics. Simulations carried out using in-house written program in Matlab indicate a high correspondence between simulated and experimental runs. The modified Shu model is characterized by simple construction, thanks to which it is easy to quickly implement this solution in commercial computer programs used to model transient states in pipes. It takes into account three basic phenomena accompanying these flows, namely: frequency-dependent hydraulic resistance, cavitation and viscoelastic effect off the pipe wall. Further work on its extension is underway.

1. Introduction

Today polymers are extensively used in pressurized pipe systems, due to their temperature, chemical and abrasion resistance, high pressure ratings, light weight, easy and fast installation, and low-price. The viscoelastic behaviour of polymers, which is characterized by a time-dependent strain, significantly influences the pressure response during transient events by attenuating the maximum or minimum pressure variations in the pipeline and by increasing the dispersion of the pressure wave [1]. Phenomena such as cavitation [2–4], frequency-dependent hydraulic resistance [5–8] or fluid structure interaction [9–11], commonly studied in metal conduits, are also found in plastic pipes.

In these pipes the equation of continuity is commonly used in the form presented by Rieutord and Blanchard [12,13]. The equation of motion has the same form as for metal pipes. Güney [14–16] was first who solved this set of hyperbolic partial differential equations. The classical method of characteristics was used to obtain ordinary differential equations, and next they were numerically solved by means of a finite difference technique. The creep compliance $J(t)$ function was obtained from dynamic tests carried out on a Rheovibron apparatus. From the frequency dependence of $E'(\omega)$ and $E''(\omega)$ determined experimentally the values of creep compliance coefficients ($J_i$) and retardation times ($T_i$) was found. This experimentally obtained creep function for the polyethylene pipe, many researchers are using today [17–20]. The experimental results of the water hammer obtained by Güney at that time are also used by the researcher to validate they own computer programs [17–21].

Covas presented other interesting papers [1,22–24]. She noticed that the nonlinear elastic behaviour of pipe walls is significant in plastic pipes and it cannot be ignored. The creep
compliance function was approximated as in earlier Güney works by a generalized Kelvin-Voigt model. Covas assumed that, the fluid velocity is negligible compared with the pressure wave speed, this allowed to neglect all convective derivatives. The final simplified solution was based on a rectangular method of characteristics computational grid.

Both Güney and Covas solutions were obtained using the same technique, namely: the method of characteristics. However, one can find papers in which general equations were solved by other methods. Hadj-Taïeb and Hadj-Taïeb [17] neglected convective terms, and following the technique described by Payret and Taylor [25] solved the set of partial differential equations with use of the two-step Lax-Wendroff finite difference scheme. Other authors [26–28] extended 2-D Pezzinga’s model [29] to simulate the nonlinear behaviour of pipe walls. Ferrante and Caponi [30,31] implemented the viscoelastic component in a frequency domain model which reduces the computational time.

Only a few authors studied the transient flow in plastic pipes with additional effects. Authors of [18,19,32] investigated the influence of fluid structure interaction in plastic pipes, and in the papers [17,33–35] the influence of cavitation on the flows in plastic pipes was investigated.

Hadj-Taïeb and Hadj-Taïeb [17] used the Henry’s law to calculate the instantaneous liquid volume fraction when pressure fall below the vapour pressure \( p_v \). The vapour phase was assumed to satisfy an ideal gas law. Consequently, the instantaneous mixture density was modelled.

Soares et al. [34,35] used popular discrete vapour cavity model (DVCM) based on the column separation hypothesis. The flow of liquid in the tube is instantaneously and completely separated by its vapour phase when the cavity is formed. Cavities were allowed to form at any of the computational sections if the pressure is computed to be below the vapour pressure. Pure liquid with a constant pressure wave speed was assumed to occupy the reach in between two computational sections. The absolute pressure in a cavity was set equal to the vapour pressure, then the upstream and downstream discharges at a cavity were computed ignoring mass transfer during cavitation. Having an instantaneous grid node discharges a cavitation volume were calculated with use of this model. Soares lumping the mass of free gas at computing sections also used the discrete gas cavity model (DGCM). In this model each isolated small volume of gas expands and contracts isothermally as the pressure varies, in accordance with the perfect gas law.

Keramat et al. [33] also studied the cavitation flow in plastic pipes using a DVCM model of column separation similar to [34], but with different numerical implementation. The preliminary conclusion was that column separation (even for the fast closure of a valve) can hardly result in pressures higher than the Joukowsky pressure. The second conclusion was that the simplest model of column separation, DVCM, although sensitive to the numerical implementation, provided acceptable results for cavitating flow in viscoelastic pipes. In addition to simulation research, it is also worth distinguishing experimental research on the occurrence of cavitation in polyethylene and PVC pipelines made by Mitosek [36–38].

Retarded strain in all discussed papers is calculated in the same way, namely from the convolutional integral, which is the product of the local derivative from pressure and the weighting function, which is a derivative of the creep function. Today, most authors [30,39–47] use the Pezzinga-Scandura [48] and Covas [22–24] calibration method to determine coefficients describing the creep function to which experimental runs of water hammer in a conduit from a given type of polymer is needed. In this paper we will use creep function experimentally obtained by Güney.

Significant changes in the modelling approach can be noted in two recent works. Keramat et al. [21] developed a mathematical model taking the time dependency of Poisson’s ratio into account for linear viscoelastic pipes. Poisson’s ratio was written in terms of relaxation function and bulk modulus which is assumed to be constant. The relaxation function is obtained from creep function given as the viscoelastic property data of pipe material. This model it’s
recommended by its authors as more accurate than the conventional viscoelastic models but is much more complicated for numerical modelling. Ferrante and Caponi [30] presented the generalized Maxwell model which in the opinion of its authors performs slightly better than the well-known standard linear solid model, for both HDPE and PVC-O pipes.

In this work, which is the continuation of the previous two papers [20,49], the cavitational bubble flow model [50] will be extended so that the influence of retarded strain is properly taken into account in polymeric pipes.

2. Mathematical model

Starting from the equation of motion in the form of:

\[
\frac{\partial}{\partial t} \left( \rho_m v_m A \right) + \frac{\partial}{\partial x} \left( \rho_m v_m^2 A \right) + A \frac{\partial p}{\partial x} + \pi D \tau + g \rho_m A \sin \gamma = 0,
\]

where: \( t \) – time [s], \( x \) – distance along the pipe [m], \( A \) – pipe cross-sectional area \([m^2]\), \( v_m \) – mixture velocity \([m/s]\), \( p \) – pressure \([Pa]\), \( D \) – inner pipe diameter \([m]\), \( \tau \) – wall shear stress \([Pa]\), \( g \) – acceleration due to gravity \([m/s^2]\), \( \gamma \) – pipe slope angle \([\ell]\), \( \alpha \) – volumetric fraction of liquid phase \([-]\), \( \rho_v \) – density of vapour phase \([kg/m^3]\), \( \rho_l \) – density of liquid phase \([kg/m^3]\) and \( \rho_m \) – mixture density

\[
\rho_m = \alpha \rho_v + (1 - \alpha) \rho_l.
\]

After differentiation, one comes to the following form:

\[
\rho_m v_m \frac{dA}{dt} + \rho_m \frac{dv_m}{dt} + \rho_m v_m \frac{\partial v_m}{\partial x} + \rho_m \frac{d \rho_m}{dt} + \frac{\partial p}{\partial x} + \frac{4}{D} \tau + g \rho_m \sin \gamma = 0.
\]

In this paper, for simplicity, a horizontal pipeline \((\gamma = 0)\) was assumed, thus the last term on the left-hand side of the above equation will disappear. Then, using the continuity equations written separately for the gas phase:

\[
\frac{\partial}{\partial t} \left( \rho_v (1 - \alpha) A \right) + \frac{\partial}{\partial x} \left( \rho_v (1 - \alpha) A v_v \right) = 0,
\]

where \( v_v \) – average velocity of vapour flow \([m/s]\). And liquid phase:

\[
\frac{\partial}{\partial t} \left( \rho_l \alpha A \right) + \frac{\partial}{\partial x} \left( \rho_l \alpha A v_l \right) = 0,
\]

where \( v_l \) – average velocity of liquid flow \([m/s]\). By adding the equation (4) and (5), one will get:

\[
\frac{\partial}{\partial t} \left( \rho_v (1 - \alpha) A + \rho_l \alpha A \right) + \frac{\partial}{\partial x} \left( \rho_v (1 - \alpha) A v_v + \rho_l \alpha A v_l \right) = 0.
\]

Now, assuming that the gas phase has the same velocity as the liquid phase (non-slip flow) \( v_v = v_l = v_m \):

\[
\frac{\partial}{\partial t} (A \rho_m) + A \frac{d \rho_m}{dt} + A \frac{\partial v_m}{\partial x} = 0.
\]

After differentiation, one will get:

\[
A \frac{d \rho_m}{dt} + \rho_m \frac{dA}{dt} + A \rho_m \frac{\partial v_m}{\partial x} = 0.
\]

Dividing equation (8) by \( A \rho_m \):

\[
\frac{1}{\rho_m} \frac{d \rho_m}{dt} + \frac{1}{A} \frac{dA}{dt} + \frac{\partial v_m}{\partial x} = 0.
\]
However, multiplying the equation (8) with $v_m$ and dividing by $A$, one gets another useful form:

$$v_m \frac{d \rho_m}{dt} + \frac{\rho_m v_m}{A} \frac{dA}{dt} + v_m \rho_m \frac{\partial v_m}{\partial x} = 0,$$

what makes three terms disappear in the equation (3), and the resulting final form of the equation of motion is as follows:

$$\rho_m \frac{dv_m}{dt} + \frac{\partial p}{\partial x} + \frac{2}{R} = 0.$$

From [23,51] it is known that in plastic pipes:

$$\frac{1}{\rho_l} \frac{dp_l}{dt} = \frac{1}{\varepsilon_l} \frac{dp_v}{dt} \quad \text{and} \quad \frac{1}{\rho_v} \frac{dp_v}{dt} = \frac{1}{\varepsilon_v} \frac{dp_v}{dt},$$

The total derivative of $\rho_m$ is calculated from the following equation:

$$\frac{d \rho_m}{dt} = \rho_m \frac{\partial p}{\partial x} + v_m \frac{\partial \rho_m}{\partial x}.$$

Substituting $\rho_m$ from (2) and after differentiation:

$$\frac{d \rho_m}{dt} = \alpha \frac{d \rho_l}{dt} + (\rho_l - \rho_v) \frac{d \alpha}{dt} \quad \text{and} \quad \frac{d \rho_v}{dt}.$$

Introducing solution (14) into (9) and using the last relationship presented in (12), one will get:

$$\left[ \rho_m \frac{D}{E} + \alpha \rho_l \frac{1}{K_l} + \frac{(1 - \alpha) \rho_v}{K_v} \right] \frac{dp}{dt} + (\rho_l - \rho_v) \frac{d \alpha}{dt} + 2 \rho_m \frac{d \varepsilon_r}{dt} + \rho_m \frac{\partial v_m}{\partial x} = 0.$$

As in the method of characteristics the pressure wave speed $c$ must be constant, the assumption in this work (also J.J. Shu assumed it) is that it value will be calculated as pre-transient. At that moment, only liquid flow occurs so for starting time $\alpha = 1$, and the expression in bracket of above equation reduces to:

$$c^{-2} = \left[ \rho_l \left( \frac{D}{E} + \frac{1}{K_l} \right) \right].$$

Then, finally the equation of continuity has such form:

$$\frac{1}{c^2} \frac{dp}{dt} + (\rho_l - \rho_v) \frac{d \alpha}{dt} + 2 \rho_m \frac{d \varepsilon_r}{dt} + \rho_m \frac{\partial v_m}{\partial x} = 0.$$

In non-slip flows, the share of the dispersed phase of a statistical nature, i.e. the volume $\alpha$ and mass $c_m$ concentration, are equal to the relevant proportions of dynamic nature, i.e. transport concentration and the degree of dryness [52]. The following relationship applies then: $v_m = \frac{v}{\alpha}$, where $v$ is superficial velocity of liquid phase.

From paper [20] it follows that:

$$\frac{\partial \varepsilon_r(t)}{\partial t} = \frac{\theta D}{2e} \int_0^t \frac{\partial}{\partial t} \left( p(t - u) - p(0) \right) \left( \sum_{i=1}^k \frac{J_i}{T_i e^{-\frac{u}{T_i}}} \right) du,$$
\( T_i \) – retardation time of the dashpot of \( i \)-th element \([s]\). Recognizing the analogy of the above equation with the convolution integral representing the wall shear stress, let us write:

\[
\sum_{i=1}^{k} \frac{J_i}{T_i} e^{-\frac{t}{T_i}} = w_i(u).
\]

Equations (17) and (11), after taking into account the formulas for the wall shear stress and the derivative of the retarded strain, form the following system:

\[
\begin{align*}
\frac{1}{\text{c}e} \frac{\partial p}{\partial t} + (\rho_l - \rho_v) \frac{\partial \alpha}{\partial t} + \rho_m \frac{\partial}{\partial t} \left( \frac{\alpha}{\text{c}_x} \right) + \rho_n \Xi \int_0^t \frac{\partial p}{\partial t} (u) \cdot w_J(t-u) \, du &= 0, \\
\rho_m \frac{\partial}{\partial t} \left( \frac{\alpha}{\text{c}_x} \right) + \frac{\partial \rho_m}{\partial t} \left( \frac{\alpha}{\text{c}_x} \right) + \frac{2 \rho_m}{\text{c}_x} \int_0^t \frac{\partial p}{\partial t} (u) \cdot w_{U_F}(t-u) \right) &= 0,
\end{align*}
\]

(20)

where: \( \Xi = \frac{2B\theta}{\text{c}} \) – enhanced \( \theta \) parameter \([-\]), \( w_{J} \) – creep weighting function \([Pa^{-1} \cdot s^{-1}]\), \( w_{U_F} \) – unsteady friction weighting function \([-\]), \( \mu_m \) – Dukler’s \([53,54]\) two-phase mixture dynamic viscosity \([Pa \cdot s]\). Using the numerical solution of convolution integral presented by Schohl \([55]\) to solve equation (18), we obtain:

\[
\frac{\partial \varepsilon}{\partial t} (t + \Delta t) \approx \frac{D\theta}{2\text{c}} \sum_{i=1}^{k} \left( x_i(t) \cdot e^{-\frac{\Delta t}{T_i}} + \frac{J_i}{\Delta t} \left[ 1 - e^{-\frac{\Delta t}{T_i}} \right] \left( p_{t+\Delta t} - p(t) \right) \right),
\]

(21)

where \( \Delta t \) – a constant time step in method of characteristics \([s]\). The above equation can be written in the following simple form:

\[
\frac{\partial \varepsilon}{\partial t} (t + \Delta t) = p(t + \Delta t) F - G(t),
\]

(22)

\[
F = \frac{D\theta}{2\text{c}} \sum_{i=1}^{k} M_i, \quad G(t) = \frac{D\theta}{2\text{c}} \sum_{i=1}^{k} \left( M_i \cdot p(t) - x_i(t) \cdot N_i \right), \quad M_i = \frac{J_i}{\Delta t} \left[ 1 - e^{-\frac{\Delta t}{T_i}} \right], \quad N_i = e^{-\frac{\Delta t}{T_i}}.
\]

The solution of the convolutional integral describing the wall shear stress is as follows \([5,56]\):

\[
\tau(t + \Delta t) \approx \frac{\rho_m f(\alpha(t)) |v(\alpha(t))|}{\alpha(t) \text{c}_x} + \frac{2 \rho_m}{\text{c}_x} \sum_{i=1}^{3} \left[ A_i y_i(t) + \eta B_i \left[ \frac{v(t)}{\alpha(t)} - \frac{v(t-\Delta t)}{\alpha(t-\Delta t)} \right] + \left[ 1 - \eta \right] C_i \left[ \frac{v(t-\Delta t)}{\alpha(t-\Delta t)} - \frac{v(t-2\Delta t)}{\alpha(t-2\Delta t)} \right] \right]
\]

(24)

where:

\[
\eta = \frac{\int_0^{\Delta t} w_{\text{class}}(u) \, du}{\int_0^{\Delta t} w_{\text{eff}}(u) \, du}, \quad A_i = e^{-\alpha_i \Delta t}, \quad B_i = \frac{m_i}{\Delta t n_i} \left[ 1 - A_i \right], \quad C_i = A_i B_i.
\]

(25)

The system of partial differential equations (20) was solved in this work using the method of characteristics and finite differences method:

\[
\begin{align*}
\frac{p_D}{\alpha_D} &+ \frac{s_D}{\alpha_D} + \frac{p_D}{\alpha_D} + \frac{\Xi}{2} \ln \frac{\rho_m D}{\rho_D} + p_D c \Xi F \Delta t = C_A, \\
\frac{p_D}{\alpha_D} &+ \frac{s_D}{\alpha_D} + \frac{p_D}{\alpha_D} - \frac{\Xi}{2} \ln \frac{\rho_m D}{\rho_D} - p_D c \Xi F \Delta t = C_B,
\end{align*}
\]

(26)
where:
\[
\begin{align*}
    C_A &= \frac{v_A}{\alpha_A} + \frac{p_A - p_{pv}}{c_{\rho v}} - \frac{f_A \Delta v A |v_A|}{4 \rho_0 A} + \frac{2}{\kappa} \ln \frac{\rho m E \rho m_E}{\rho L \rho m L} + G_E (t) \Xi c \Delta t, \\
    C_B &= \frac{v_B}{\alpha_B} - \frac{p_B - p_{pv}}{c_{\rho v}} - \frac{f_B \Delta v B |v_B|}{4 \rho_0 B} - \frac{2}{\kappa} \ln \frac{\rho m E \rho m_E}{\rho L \rho m L} - G_E (t) \Xi c \Delta t
\end{align*}
\]
and \( \kappa = 1 + c^2 \rho_l \Xi F \Delta t \). From the above equations, solutions for the internal node of the characteristic grid were obtained (Fig. 1) for averaged cross-section velocity:
\[
v_D = \frac{\alpha_D (C_A + C_B)}{2},
\]
and for the instantaneous pressure value:
\[
p_D = \frac{(C_A - C_B) c_{\rho l}}{2 \kappa} + p_v - \frac{c^2 \rho_l}{2 \kappa} \ln \frac{\rho_m D}{\rho_l}.
\]
From the analysis of the above formulas it follows that in order for \( p_D > p_v \) (then \( \rho_m D = \rho_l \)), the condition that \( C_A \geq C_B \) must be satisfied. When \( C_A \geq C_B \), then \( \alpha_D = 1 \) and \( p_D = \frac{(C_A - C_B) c_{\rho l}}{2 \kappa} + p_v \). And if \( C_A < C_B \) cavitation occurs, then \( p_D = p_v \) and mixture density is calculated \( \rho_m D = \rho_l e^{\frac{(C_A - C_B)}{c_{\rho l}}} \). Having the instantaneous value of the density of the mixture \( \rho_m D \), the vapor density \( \rho_v \) and the density of the liquid \( \rho_l \), one should determine from the equation (2) the instantaneous value of the liquid phase concentration \( \alpha_D \):
\[
\alpha_D = \frac{\rho_l e^{\frac{(C_A - C_B)}{c_{\rho l}}} - \rho_v}{\rho_l - \rho_v}.
\]

3. Exemplary simulation

In this section, exemplary simulation studies will be carried out. Received results will be compared with the experimental results presented by Güney [14–16]. The experimental setup build in INSA Lyon (France) was composed of a horizontal low-density polyethylene test pipe (LDPE, Poisson’s ratio \( \nu_F = 0.38 [-] \)), a constant pressure reservoir and a quick-closing piston valve at the downstream end of the pipe (Fig. 2). The pipe length was \( L = 43.1 \) [m], inner diameter \( D = 0.0416 \) [m], wall thickness \( e = 0.0042 \) [m], and characteristic polymer pipe roughness size \( k_s = 1.5 \cdot 10^{-6} \) [m] were assumed. The fluid was water with temperature 13.8 \(^{\circ}C \), density 999.3 [kg/m\(^3\)] and kinematic viscosity \( \nu = 1.17 \cdot 10^{-6} \) [m\(^2\)/s]. Initial steady-state
flow \((v_0 = 1.28 \text{ [m/s]})\) was determined using a simple method involving measuring volumes of water collected in a fixed time. It is not clear, as the author of the experimental setup do not mention about it, but according to the schematic diagram (Fig. 2) it looks like that in all cases the liquid flow into the atmosphere. The absolute reservoir pressure can be then derived from following formula:
\[
p_R = p_{atm} + hg\rho,
\]
where \(g\) – acceleration due to gravity \([\text{m/s}^2]\) and according to table from Güney’s PhD \([14]\) \(h=2.88 \text{ [m]}\). The experimental uncertainty was estimated within \(\pm 3 \text{ [kPa]}\) for pressure and \(\pm 1\%\) for the initial flow velocity. In analyzed case the closure time of the valve was \(T_c = 0.012 \text{ [s]}\). The creep function is an filtered experimental function characterized by following creep and retardation time coefficients: \((J_1 = 0.637 \cdot 10^{-9}, J_2 = 0.871 \cdot 10^{-9}) \text{ [Pa}^{-1}\text{]}\) and \((T_1 = 0.0166, T_2 = 1.747) \text{ [s]}\).

![Figure 2. Schematic diagram of Guney’s experimental test stand](image)

Due to the fact that the Reynolds number in the stationary flow preceding the unsteady state was \(Re_0 = 45511 \text{ [-]}\), in this study examined will be the effect of calculated resistance. Four cases were examined:

a) weighting function calculated for laminar flow (the shape of the filtered simplified function similar to the classic weighting function according to Zielke \([57]\), the coefficients were calculated using the analitycal method described in \([7]\)),

b) weighting function calculated for turbulent flow (for initial condition, coefficients were scaled using the procedure discussed in the paper \([58]\)),

c) variable values of weighting function coefficients (constant values during the single dimensionless time step, followed by a step change to the new values calculated on the basis of the instantaneous Reynolds number),

d) resistances calculated from quasi-steady formula (convolutional integral not taken into account).

In all numerical simulations the pipeline was divided for 64 equally long elements \((N = 64 \text{ [-]}, \Delta x = 0.6734 \text{ [m]}\)). With that assumption and knowing the value of calculated pressure wave speed \(c = 300 \text{ m/s}\), a constant time step were calculated \(\Delta t = 2.245 \cdot 10^{-3} \text{ [s]}\) \((\Delta \hat{t} = 6.07 \cdot 10^{-6} \text{ [-]}\)). The graphical representations of simulated transient states are presented on Fig. 3 and Fig. 4.

From the results shown (Fig. 3 and Fig. 4) it can be seen that:

• transient cavitation runs are simulated with a modified bubble cavitation model with sufficient accuracy,

• in the runs taking into account the frequency-dependent hydraulic resistance, the selection of the weighting function necessary to calculate the values of frequency-dependent resistances were not crucial (Fig. 4),
Figure 3. Comparison of simulated and experimental results ($Re_0 = 45511$)

Figure 4. Enlarge of selected time range

- the simplest weighting function, which is nominally dedicated to laminar flow, gave the best results,
- the use of a quasi-steady friction model resulted in results that were clearly different from those simulated (Fig. 4)

Simulation results obtained for other initial conditions will be presented during the conference.

4. Conclusions

The presented modified bubble cavitation flow model, which takes into account the retarded deformation of the plastic pipe walls, very accurately models the analyzed transient states. The numerical simulation carried out showed that the inclusion of frequency-dependent hydraulic resistance approaches the simulation results to the experimental one. However, the significance of frequency-dependent friction is much smaller than the creep function. Its derivative determined in this work as a weighting function $w_J$ has a fundamental influence on the modeled pressure changes runs.

New modified model is characterized by simple construction, thanks to which it is easy to quickly implement this solution in commercial computer programs used to model transient states in pipes. It takes into account three basic phenomena accompanying these flows, namely: frequency-dependent hydraulic resistance, cavitation and viscoelastic effect of the pipe wall material.

Due to the fact that two other models of cavitation flow in plastic pipes are known from the literature, namely the discrete vapor cavity model (DVCM) and the discrete gas cavity model (DGCM), a comparative analysis of all these models is necessary.

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