On the nature of the depinning transition in type-II superconductors

N. Lütke-Entrup, B. Plaçais, P. Mathieu and Y. Simon
Laboratoire de Physique de la Matière Condensée de l’Ecole Normale Supérieure
CNRS URA 1437, 24 rue Lhomond, F-75231 Paris Cedex5
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The surface impedance $Z(f)$ of conventional isotropic materials has been carefully measured for frequencies $f$ ranging from 1 kHz to 3 MHz, allowing a detailed investigation of the depinning transition. Our results exhibit the irrelevance of classical ideas on the dynamics of vortex pinning. We propose a new picture, where the linear ac response is entirely governed by disordered boundary conditions of a rough surface, whereas in the bulk vortices respond freely. The universal law for $Z(f)$ thus predicted is in remarkable agreement with experiment, and tentatively applies to microwave data in YBaCuO films.

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A perfect sample of a type-II superconductor in the vortex (or mixed) state would be transparent to an electromagnetic wave at very low frequencies. But defects are always present and strongly alter the quasi-static and low-frequency response; low frequencies here means $\Omega = 2\pi f \ll \Omega_d$, a so-called “depinning frequency” depending on the material and vortex density. It is important for applications to know what kind of defects can pin vortices, how they hinder small vortex oscillations and thereby restrain the penetration of an ac ripple. In this respect, a study at low levels of excitation can pin vortices, how they hinder small vortex oscillations and thereby restrain the penetration of an ac ripple. It is generally accepted that bulk pinning centers provide much information about the dynamics of pinning. It is generally accepted that bulk pinning centers provide much information about the dynamics of pinning. However, it has been able to account for variations of $Z$ at higher frequencies. In particular, as the first increasing of $R(f)$ is stronger than expected, the understanding of dissipation remains a puzzling problem, including in high $T_c$ materials.

Experiments are performed on a series of slabs of cold-rolled polycrystalline PbIn and pure single-crystalline Nb. The slabs $(xy)$ are immersed in a normal magnetic field $B$; unless specified their thickness $2d$ is much larger than the flux-flow penetration depth $\delta_f$ (see below). At equilibrium, up to the upper critical field $B_{c2}$, a regular lattice of vortex lines parallel to $z$ is formed, with the density $n = B/\varphi_0$, where $\varphi_0$ is the flux quantum. Both faces of the slab, $z = \pm d$, are then subjected to an ac magnetic field $b_0 e^{-i\Omega t}$ parallel to the length ($x$-direction) of the sample. Under such conditions, induced currents and electric fields, $J(z)$ and $e(z) e^{-i\Omega t}$, are along the $y$-direction, while vortices oscillate in the $x$-$z$-planes. For low exciting fields ($b_0 \sim 1 \mu T$), vortex displacements $u(z) \sim 1$ Å are very small compared with the vortex spacing $a \simeq n^{-\frac{1}{2}}$ ($\sim 1000$ Å, for $B \sim 0.1$ T) (Fig. 1a). The electric field $e_0$ at the surface $z = d$, $e_0 = e(d) = -e(-d)$, is measured by means of a pick-up wound coil. The main experimental difficulty in such measurements is to ensure a precise calibration of the phase $\varphi$ of $e_0$ (within better than 0.5° at 100 kHz). Thus we get the surface impedance of the slab, defined as the ratio $\mu_0 e_0/b_0$. Putting

$$\frac{iZ}{\mu_0 \Omega} = \frac{ie_0}{\Omega b_0} = \lambda^* = \lambda' + i\lambda'' = \Lambda e^{i\varphi},$$

the ac response will be conveniently expressed in terms of the complex penetration length $\lambda^*$. As easily seen, $2b_0 \Lambda$ (a factor 2 for two faces) represents the amplitude of the ac magnetic flux penetrating the slab per unit length along $y$. The length $\lambda'$ measures the dissipation, as $\lambda'' = \Lambda/\lambda = R/|Z|$ is the sine of the loss angle $\varphi$.

The analysis of the ideal response, though it is not observed (unless $\Omega \gg \Omega_d$), is an important step in our argument. A perfect slab would behave like a linear continuous medium, of resistivity $\rho_f$ and permeability $\mu = \mu_r \mu_0$ ($0 < \mu_r < 1$). Here $\rho_f \simeq \rho_0 B/B_{c2}$ is the flux-flow resistivity, and $\mu$ is the “diamagnetic permeability” of the mixed state $\mu_r$. $\mu_r$ increases steadily with the vortex density and rapidly approaches unity (typically $\mu_r > 0.9$ for $B \gtrsim 0.2 B_{c2}$). In the absence of pinning, an electromagnetic wave, $b \propto e^{\pm ik z} e^{-i\Omega t}$, $e = \mp \Omega b/\kappa_1$, can propagate in the bulk, according to the simple equation of dispersion $k_0^2 = \mu_0 \Omega/\rho_f = 2i/\delta_f^2$. The wave field $b(z)$ would be accordingly: $b_{10} \cosh(ik_1 z)/\cosh(ik_1 d)$, where $b_{10} = \mu_r b_0$ if one makes allowance for a surface screening by diamagnetic currents. This leads to $\lambda'_{\text{ideal}} = \mu_r \lambda_f \tanh(d/\lambda_f)$, where $\lambda_f = (1 + i)\delta_f/2$. Assuming $\mu_r \simeq 1$, this (undisputed) result involves all features of a normal skin effect. In the so-defined thick limit (thick slabs and/or high frequencies), say $d \gtrsim 2\delta_f$, $\lambda'_{\text{ideal}} = \mu_r \lambda_f \simeq \lambda_f$, so that $\lambda' = \lambda'_{\text{ideal}} = \delta_f/2$. In the thin limit, say $d \lesssim \delta_f$, $\lambda'_{\text{ideal}} = \mu_r d \simeq d$, which means perfect transparency.

As shown in Fig. 2 the actual response is quite different: after a low-frequency plateau, $\lambda' \simeq \lambda'(0)$ ($\lambda'' \simeq 0, \varphi \simeq 0$), the loss-angle increases with frequency, so that the ideal skin effect ($\lambda' \simeq \lambda_f, \varphi = \pi/4$) is recovered beyond some depinning frequency $\Omega_d$; this can be pre-
ciscely defined as the mid-frequency for which $\varphi = \pi/8$. Note, in passing, that the observation of a linear response is not consistent with predictions of a naive critical state model: A critical-current density as small as $J_c \sim 10 \text{A/cm}^2$ should restrict the penetration of fields $b_0 \sim 1 \mu T$ to depths $\Lambda = b_0/\mu_0 J_c \lesssim 1 \mu m$, which is much smaller than observed, and seeing that $\Lambda \propto b_0$, no linear regime could exist at all. The linear skin effect over depths $\Lambda \sim 100 \mu m$ was first reported by P. Alais and Y. Simon (MS) continuum theory of the mixed state \cite{9,10}, and then misinterpreted by considering the vortex curvature \cite{11} is lowered, and a large normal-like skin effect is observed. For a real rough surface, a vortex-slipage effect at the surface induces a relatively strong bending of vortices over the depth $\lambda_V \sim a$, so that non-dissipative currents associated with this vortex curvature \cite{9,10} are greatly screening the exciting field. 

b) An isolated superfluid vortex in a rotating box of helium II terminates at a wall asperity. If it is acted on in the bulk, the vortex line bends near the wall so as to keep on ending normal to the surface, making thus an angle with the mean smooth surface.

The model of the critical state based on the Mathieu-Simon (MS) continuum theory of the mixed state \cite{9,10}, has prompted us to an alternative interpretation. We briefly recall the points of importance in the MS theory: i) each vortex line (unit vector $\mathbf{u}$) must terminate normal to the surface ($\mathbf{u} \times \mathbf{n} = 0$); whence the leading part of the boundary conditions (rough or smooth surface) in any problem of equilibrium or motion of vortices. ii) Vortex lines are not always field lines, so that the vortex field $\mathbf{w} = \nu_0 \mathbf{v}$ and the mesoscopic field $\mathbf{B}$ must be regarded as two locally independent variables. The conjugate variable of $\omega$, $\varepsilon = \varepsilon(\omega, T, \mathbf{v})$, appears as a local line tension $\varepsilon(\mathbf{v})$ in the MS equation for vortex equilibrium or non-dissipative motion $\mathbf{J}_s + \text{curl} \varepsilon = 0$. iii) The classical picture of a local diamagnetism is misleading. A diamagnetic current, just like a subcritical transport current, is a true non-dissipative supercurrent $\mathbf{J}_s = -\text{curl} \varphi$ flowing near the surface over a small vortex-state penetration depth $\lambda_V$ ($\lesssim a_0$, the zero-field London depth). Any deviation $\omega - B$ also heals beyond the depth $\lambda_V$, so that $\omega \equiv B$ in the bulk sample. Although the mean magnetic-moment density of a perfect body turns out to be $-\varepsilon$, the quantity $-\varepsilon$ has not the primary physical meaning of a local magnetization, nor $\mu_r$, conveniently defined as the ratio $\omega/(\omega + \mu_0 \varepsilon)$, that of a true local permeability \cite{11}.

Let us return to the ac response of the slab in the standard geometry of Fig. \ref{fig:fig1} and suppose that bulk motions are unrestricted (no bulk pinning). As pointed out by Sonin et al. \cite{11}, the distinction between $\omega$ and $\mathbf{B}$ implies additional degrees of freedom, and a second $k_2$-mode appears besides the classical $k_1$-mode ($k_1 = \pm \pi/\lambda_f$); this is a London-like non-dispersive evanescent mode, which dies off over the depth $\lambda_V$ ($k_2 = \pm \pi/\lambda_V$). \cite{11}. Note that, in practice, $\lambda_V \ll \delta_f$, $\Lambda$ and $d$. In a pure $k_2$-mode vortex and field lines bend in opposite directions, whereas they coincide in a pure $k_1$-mode. More explicitly \cite{11},

$$\nu_{1x} = \frac{b_1}{B} = \frac{u_1}{\lambda_f}, \quad \nu_{2x} = -\frac{b_2}{B} = \frac{\mu_r}{\lambda_V}, \quad \nu_{2z} = \frac{u_2}{\lambda_V}, \quad (3)$$

where $\nu_{xz} = \partial u_1/\partial z$, $\nu_{2z} = i k_1 u_{1z}$, $\nu_{2z} = i k_2 u_{2z}$ and $b/B$ measure the slopes of vortex and field lines respectively. Then the response $b(z)$ will be that combination of the modes, $b_1 + b_2$, which satisfies the field continuity, $b_0 = b_{10} + b_{20}$, as well as the correct boundary condition for vortex lines. The latter condition will determine the relative weights $\beta_1 = b_{10}/b_0$ and $\beta_2 = b_{20}/b_0$ of the modes ($\beta_1 + \beta_2 = 1$), and, therefore, the effective penetration

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{a) Vortex lines $u(z)$ (full lines), and field profiles $b(z)$ (dashed lines) near one face of a thick slab are sketched with arbitrary units; for clarity, the actual length scaling, $u \ll a \ll \delta_f$, is not preserved. For an ideal surface, vortices normal to the plane boundary, the weight of the $k_2$-mode \cite{11} is lowered, and a large normal-like skin effect is observed. For a real rough surface, a vortex-slipage effect at the surface induces a relatively strong bending of vortices over the depth $\lambda_V \sim a$, so that non-dissipative currents associated with this vortex curvature \cite{9,10} are greatly screening the exciting field. b) An isolated superfluid vortex in a rotating box of helium II terminates at a wall asperity. If it is acted on in the bulk, the vortex line bends near the wall so as to keep on ending normal to the surface, making thus an angle with the mean smooth surface.}
\end{figure}
depth $\lambda^* = \beta_1 \lambda_f + \beta_2 \lambda_V \simeq \beta_1 \lambda_f$ (since $\lambda_V \ll \delta_f$ and $\Lambda$).

For an ideal surface, the vortex boundary condition is clearly $\nu_x = 0$ (point 1 above); then using Eq. (3), we just recover the simple classical-diamagnetism result, that is $\beta_1 = \mu_x$. Now, the point is that the surface roughness may considerably change this boundary condition, so as to enhance the weight of the second mode (Fig. 1b). Thus we argue that small effective skin depths at low frequencies should not result from restricting the penetration of the $k_1$-mode, but from its amplitude being reduced due to the screening effect of the second mode.

According to the MS model [1][2][3], if the surface has irregularities on the scale of $a$, vortex lines can bend over a depth $\lambda_V$, making thus an angle $\alpha$ with the mean smoothed surface $z=d$. On the average, and in any direction $x$, $\alpha$ should not exceed a critical value $\alpha_c \sim 1-10^\circ$ : $\langle \nu_x \rangle_{z=d} \leq \nu_{xc} = \sin \alpha_c \sim 10^{-2}-10^{-1}$ [4][5].

As stated above, superficial non-dissipative supercurrents ($J_s = -\text{curl}$) can result from such distortions of the vortex array; integrating $-\text{curl}$, the net current density in the $y$-direction is $\tilde{J}_y(\Lambda/m) = \langle \nu_z \rangle \simeq \varepsilon \langle \nu_x \rangle$, where $\varepsilon \simeq B(1-\mu_x)/\mu_x$. A dc subcritical current can be regarded as a frozen pure $k_2$-mode in the limit $\Omega \to 0$. To the maximum, $\tilde{J}_y = \tilde{J}_x = -\nu_{xc} B(1-\mu_x)/\mu_x$ is the surface critical-current density. Starting from an equilibrium, where $\langle \nu_x \rangle = 0$, a shift of the bulk vortex array is expected to induce a vortex curvature in the opposite direction, $\nu_x = f(u_{\text{bulk}})$ with $\nu_{xc} = f(u \sim a)$. Perhaps such a vortex slippage ($\nu_x < 0$ if $u > 0$) is more intuitive when dealing with superfluid vortices in a rotating box (see Fig. 1b). In helium, bulk pinning does not exist, and only asperities of the walls can pin vortices [14]. We are just extending this idea to collective motions of a vortex array along a rough surface of superconductors.

Linearizing $\nu_x = f(u_{\text{bulk}})$ for small displacements we can write $\nu_x = -u_{\text{bulk}}/l$, where $l$ is a real length characterizing the surface roughness: $l = \infty$ would correspond to an ideal surface; in practice we expect that $l \sim a/\nu_x \sim 0.1-10 \mu m$. As far as $\lambda_V \ll \delta_f$, this condition applies quasistatically in the ac response by taking $u_{\text{bulk}} = u_{10}$, so that the vortex-slippage condition reads

$$\nu_x = -\frac{u_{10}}{l} \quad .$$

Now, substituting Eq. (4) for $\nu_x = 0$ in the above calculation of $\beta_1$ and $\beta_2$, and considering the thick limit, we obtain:

$$\frac{1}{\lambda^*} = \frac{1}{\mu_x \lambda_f} + \frac{1}{\lambda_S} \quad ,$$

where $\lambda_S = l_{\mu_x}/(1-\mu_x) \sim B_0/\mu_0 l_e$ is the real limit of $\lambda^* \simeq \beta_1 \lambda_f$ as $\Omega \to 0$ ($\lambda_f \simeq (1+i)\delta_f/2$). Note that setting $\mu_x = 1$ from the beginning would lead wrongly to $\lambda^* = \lambda_f$, that is the ideal response. While giving the same low and high-frequency limits as Eq. (2), $\lambda^*(0) = \lambda_S$ (real) and $\lambda^*(\infty) = \mu_x \lambda_f$ (depinning), Eq. (3)
be reexamined, especially because of the anisotropy and high-frequency correcting terms in the dispersion equation for the two-modes \(11\). Nevertheless, it is worth noting that Eq. \((5)\) may account for the observed \(f\)-dependence of \(R\) in YBaCuO from 1 to 20 GHz (Fig. \(4\)) \(9\). These results support the argument, developed in previous works, \(9\), that bulk pinning is absolutely ineffective in a large class of “soft” materials (void of strong bulk inhomogeneities). Contrary to the common idea that any crystal defect may be a pinning center, we are led to the conclusion that a normally homogeneous sample in the mixed state rather imitates the behaviour of a superfluid vortex array enclosed in a rough box.

From data taken below the thick limit (not shown in the figures), we retain an important result. When \(2d\) is decreased under conditions where \(\lambda_C\) or \(\lambda_S \ll \delta_f\) (\(\Omega \ll \Omega_d\)), we observe that the ac response becomes thickness-dependent as soon as \(d \lesssim 2d_f\), as predicted: just substitute \(\lambda_f \tanh(d/\lambda_f)\) for \(\lambda_f\) in Eq. \((2)\). In particular, the loss angle is significantly smaller than stated by Eq. \((2)\). According to the one-mode Campbell model, size effects should arise for much thinner slabs such as \(d \lesssim \lambda_C\). Note in this respect that our “pinning length” \(\lambda_S\), contrary to \(\lambda_C\), does not represent an actual penetration depth. The mere observation that size effects arise for \(d \sim \delta_f\), not \(d \sim \lambda_C\), attests that the bulk \(k_1\)-mode propagates freely, and reveals the need for a two-mode electrodynamics.

In conclusion, the MS model of the critical state completed by the vortex-slippage condition \((4)\) accounts quantitatively for the surface impedance of a variety of conventional samples, which all have standard critical-current densities (\(\kappa_c \sim 10\) A/cm): polycrystalline lead-indium slabs and single-crystal slabs of pure niobium (Figs. \(2\) and \(3\)). For the application to the case of YBaCuO at \(f \sim 10\) GHz, our derivation of Eq. \((7)\) has to

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**FIG. 4.** The microwave surface resistance \(R\) of an YBaCuO film (sample No. 2 of Ref. \(3\)) vs frequency plotted in a log-log scale. (●) are experimental data taken from Fig. 6 of Ref. \(3\). The full line is a fit using our Eq. \((5)\), and taking \(\lambda_S\) and \(\rho_f\) (or \(\lambda_f\)) as two adjustable parameters (we find \(\lambda_S = 0.07\mu m\), \(\rho_f = 0.4\mu \Omega cm\), \(\Omega f/2\pi = 100\) GHz). This fit is very close to the empirical power-law \(R \sim f^{0.16}\) proposed by the authors. The dashed line shows the best fit obtained in Ref. \(3\) from the Coffey-Clem flux creep-model \((4)\).

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