Flow dependence of high $p_T$ parton energy loss in heavy-ion collisions

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The measured transverse momentum spectra and HBT correlations of bulk (i.e., low $p_T$) matter has long been recognized as a promising tool to study the initial high-density phases of ultrarelativistic heavy-ion collisions (URHIC)\cite{1,2,3,4,5,6,7,8,9,10}. In the present paper we focus on the energy loss of the leading jet particle and leave predictions for the distortion of the jet cone to a subsequent publication.

I. INTRODUCTION

Energy loss of a high $p_T$ ‘hard’ parton travelling through low $p_T$ ‘soft’ matter has long been recognized as a promising tool to study the initial high-density phases of ultrarelativistic heavy-ion collisions (URHIC)\cite{1,2,3,4,5,6,7,8,9,10}. In the present paper we focus on the energy loss of the leading jet particle and leave predictions for the distortion of the jet cone to a subsequent publication.

II. THE FORMALISM

The parton’s energy loss depends on the position of its production point $\vec{r}_0$ and the angular orientation of its trajectory $\vec{r}$. In order to determine the probability for a hard parton $P(\Delta E)$ to lose the energy $\Delta E$ while traversing the medium on its trajectory, we make use of a scaling law\cite{14} which allows to relate the dynamical scenario to a static equivalent one whose density has been rescaled. This probability distribution of the energy loss $\Delta E$ is given as a functional of the distribution $\omega \frac{dI}{d\omega}$ of gluons emitted into the jet cone\cite{15}.

\begin{equation}
P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int_{I} d\omega_i \frac{dI(\omega_i)}{d\omega} \right] \delta \left( \Delta E - \sum_{i=1}^{n} \omega_i \right) \exp \left[ -\int d\omega \frac{dI}{d\omega} \right]. \tag{1}
\end{equation}

The explicit expressions for this quantity are different if one calculates $\omega \frac{dI}{d\omega}$ under the assumption that multiple soft scattering processes or a few hard scattering processes are responsible for the energy loss. For both cases they can be found in\cite{15}. Using these results we obtain $P(\Delta E)$ from the key quantity of jet energy loss, namely the local transport coefficient $q(\eta_s, r, \tau)$. The transport coefficient characterizes the squared average momentum transfer from the medium to the hard parton per unit pathlength. Since we consider a time-dependent inhomogeneous medium, this quantity depends on spacetime rapidity rapidity $\eta_s = \frac{1}{2} \ln(t + z)/(t - z)$, radius $r$ and proper time $\tau = \sqrt{t^2 - z^2}$. For simplicity we focus on central collisions and assume azimuthal symmetry. The
transport coefficient is related to the energy density of the medium as \( \hat{q} = c^3 \rho / 4 \). In the case of an ideal quark-gluon plasma (QGP), one would expect \( c \approx 2 \). It depends also on the jet trajectory \( \hat{r} = r_0 + \xi \vec{n} \) (production point \( r_0 \) and orientation \( \vec{n} \)). In order to calculate the transport coefficient in the presence of flow, we follow the prescription suggested in [10] and replace

\[
\hat{q} = c^3 \rho (p) \rightarrow c^3 \rho (T^{n \perp n \perp})
\]

(2)

with

\[
T^{n \perp n \perp} = T^\rho (\epsilon) + \left[ \epsilon + p(\epsilon) \right] \beta_n^2 /
\]

(3)

where \( \beta_n \) is the spatial component of the flow field orthogonal to the parton trajectory. \( T^{n \perp n \perp} \) indicates the component of the energy-momentum tensor, where \( n \perp = \perp \) is orthogonal to the jet’s trajectory. In the absence of any flow effects the original result \( \hat{q} = c^3 \rho (\rho) \) is recovered. The transport coefficient enters the calculation of \( P(\Delta E) \) via \( \omega_n \) and \( \langle \hat{q}L \rangle \), which are the linearly line-averaged characteristic gluon energy:

\[
\omega_n (r_0, \phi) = \int_0^\infty d\xi \xi \hat{q} (\xi)
\]

(4)

and the time-averaged total transverse momentum squared:

\[
\langle \hat{q}L \rangle (r_0, \phi) = \int_0^\infty d\xi \hat{q} (\xi),
\]

(5)

respectively [15]. The calculation of the transport coefficient and subsequent \( \omega_n (r_0, \phi) \), \( \langle \hat{q}L \rangle (r_0, \phi) \) and finally \( P(\Delta E) \) requires a model of the fireball evolution which determines the local energy density and the flow profile. We discuss details of that model in the next section. In our calculation we average over all possible angles and production vertices, weighting the distribution of the jets with the nuclear overlap factor \( T_{AA} (b) = \int d z \rho^2 (b, z) \) (for central collisions) with \( \rho \) the nuclear density as a function of impact parameter \( b \) and longitudinal coordinate \( z \). This formalism sets the stage for the calculation of the nuclear modification factor. In the case of central collisions it is given by

\[
R_{AA}(pt, y) = \frac{d^2 N_{AA} / dp tr dy}{T_{AA}(0) d^2 N_{NN} / dp tr dy}.
\]

(6)

To obtain \( R_{AA} \) we use the formalism and averaging procedure as described above and calculate the inclusive charged hadrons production in LO pQCD. This amounts to folding the average energy loss probability into the factorized expression for hadron production (for brevity we give schematical expressions, an explicit representation can be found in [17, 18]):

\[
d\sigma_\text{med}^{A \rightarrow h + X} = \sum_f d\sigma_\text{vac}^{A \rightarrow f + X} \otimes P_f (\Delta E) \otimes D_f (z, \mu_F^2)
\]

(7)

where

\[
d\sigma_\text{vac}^{A \rightarrow f + X} = \sum_{ijk} f_{ij} / A (x_1, Q^2) \otimes f_{jk} / A (x_2, Q^2) \otimes \hat{\sigma}_{ij \rightarrow f + k}.
\]

(8)

Here, \( f_{ij} / A (x, Q^2) \) denotes the distribution of the parton \( i \) inside the nucleus as a function of the parton momentum fraction \( x \) and the hard scale \( Q^2 \) of the scattering. Likewise, \( D_f (z, \mu_F^2) \) denotes the fragmentation function of a parton \( f \) into a hadron with the hadron taking the fraction \( z \) of the parton momentum and a fragmentation scale \( \mu_F^2 \) which should be a typical hadronic scale \( O(1 \text{ GeV}) \). The expressions for the hard pQCD cross sections \( \hat{\sigma}_{ij \rightarrow f + k} \) can e.g. be found in [13]. We use the CTEQ6 parton distribution functions [21] for the pp reference, the NPDF set [22] for production in nuclear collisions and the KKP fragmentation functions [23]. Unless stated otherwise, we assume for the strong coupling \( \alpha_s = 0.3 \) in the calculation of \( P(\Delta E) \) in following.

III. THE FIREBALL EVOLUTION MODEL

The fireball evolution enters this framework in the shape of \( c(\eta_s, r, \tau) \) and the flow profile \( u^\mu (\eta_s, r, \tau) \). For the description of the evolution, we base our investigation on the formalism outlined in [13].

The main assumption for the model is that an equilibrated system is formed a short time \( \tau_0 \) after the onset of the collision. Furthermore, we assume that this thermal fireball subsequently expands isentropically until the mean free path of particles exceeds (at a time \( \tau_f \)) the dimensions of the system and particles move without significant interaction to the detector.

For the entropy density at a given proper time we make the ansatz

\[
s(\tau, \eta_s, r) = NR(r, \tau) \cdot H(\eta_s, \tau)
\]

(9)

with \( \tau \) the proper time as measured in a frame co-moving with a given volume element and \( R(r, \tau), H(\eta_s, \tau) \) two functions describing the shape of the distribution and \( N \) a normalization factor. We use Woods-Saxon distributions

\[
R(r, \tau) = 1 / \left( 1 + \exp \left[ \frac{r - R_e(\tau)}{d_{ws}} \right] \right)
\]

(10)

\[
H(\eta_s, \tau) = 1 / \left( 1 + \exp \left[ \frac{\eta_s - H_e(\tau)}{\eta_{ws}} \right] \right).
\]

to describe the shapes for a given \( \tau \). Thus, the ingredients of the model are the skin thickness parameters \( d_{ws} \) and \( \eta_{ws} \) and the parametrizations of the expansion of the spatial extensions \( R_e(\tau), H_e(\tau) \) as a function of proper time. For a radially non-relativistic expansion and constant acceleration we find \( R_e(\tau) = R_0 + \frac{2 \eta \tau^2}{\mu^2} \). \( H_e(\tau) \) is obtained by integrating forward in \( \tau \) a trajectory originating from the collision center which is characterized by
investigation we also test a flow profile \( \rho_{r/R} \). We allow for the possibility of accelerated longitudinal velocity \( \rho_{c} \), which is a free parameter and we choose to use the transverse velocity \( \rho_{\perp} = a_{\perp} \tau_{f} \) and rapidity at decoupling proper time \( \eta' = \eta_{0} + a_{\tau} \tau_{f} \) as parameters. Thus, specifying \( \eta_{0}, \eta_{f}, \rho_{\perp}, \rho_{c} \) and \( \tau_{f} \) sets the essential scales of the spacetime evolution and \( d_{m} \) and \( \eta_{s} \) specify the detailed distribution of entropy density.

For transverse flow we assume a linear relation between radius \( r \) and transverse rapidity \( \rho = \alpha = \atanh(\eta_{0} + \alpha_{\tau} \tau_{f}) \) with \( \rho_{c}(\tau) = \atanh(\rho_{c}) \). In the following investigation we also test a flow profile \( \rho \sim r^{2} \).

We allow for the possibility of accelerated longitudinal expansion which in general implies \( \eta \neq \eta_{s} \). Here, \( \eta = \frac{1}{2} \ln \frac{m_{0}^{2} - p_{\perp}^{2}}{m_{0}^{2} - p_{\tau}^{2}} \) denotes the longitudinal momentum rapidity of a given volume element. We can parametrize this mismatch between spacetime and momentum rapidity as a local \( \Delta \eta = \eta - \eta_{s} \) which is a function of \( \tau \) and \( \eta_{s} \).

IV. RESULTS

In the following we calculate \( \omega d\Omega \) in the multiple soft scattering approximation. The largest remaining uncertainty is the detailed choice of the parameter \( c \) in the relation \( \hat{q} = c(T^{m,n}) \) connecting transport coefficient and local energy density. As the highest temperature reached in the model evolution is only approximately two times the phase transition temperature \( T_{C} \), there is no a priori reason to assume that the ideal QGP value \( c \approx 2 \) would be realized. Hence, unless stated otherwise, we adjust \( c \) such that the scenario \( \text{without flow} \) reproduces the large \( p_{T} \) region (we do not include any intermediate \( p_{T} \) physics such as the Cronin enhancement in this study) and discuss changes induced by flow relative to this baseline.

As in [17], we find that \( R_{AA} \) stays rather flat as a function of \( p_{T} \) out to more than 50 GeV. If the flattening which may be seen in the data can be trusted, a value of \( c = 4-6 \) would lead to agreement with the data by the STAR collaboration for 5% central 200 AGeV Au-Au collisions [24] (see Fig. 1). Since taking into account the effect of flow transverse to the jet axis is expected to increase energy loss, we choose the value \( c = 4 \) as a baseline for the following investigation.

Taking the transverse flow field into account we indeed observe increased suppression, i.e. a reduction of \( R_{AA} \). Its magnitude shows some dependence on how the flow field depends on the radial position, we investigate \( \rho \sim r \) and \( \rho \sim r^{2} \). In both cases the maximum transverse velocity is adjusted such that the measured \( \pi^{-}, K^{-} \) and \( p \) transverse mass spectra are reproduced. The emerging trend is that shuffling more flow towards the fireball edge leads to a greater reduction of \( R_{AA} \). The reasons are rather involved: In order to experience transverse flow at all, the jet production vertex has to be away from the center of the transverse plane and the hard parton needs to propagate transverse to the local radial direction \( \hat{e}_{r} \). On the other hand, hard partons produced close to the surface tend to escape before a significant amount of transverse flow could be generated. In the end, hard partons still in the medium at relatively late times will be close to the surface and experience additional energy loss if the flow profile leads to larger flow for the periphery.

The main reason that the influence of transverse flow is much less pronounced than in the findings of [10] is that in a realistic evolution scenario the transverse flow field does not build up instantaneously but rather develops gradually over time — as jet energy loss is a phenomenon predominantly sensitive to earlier times when the medium energy density is large, the spacetime region of the evolution where transverse flow is small is probed with \( R_{AA} \). We demonstrate this in Fig. 2 where we vary the amount of pre-equilibrium transverse flow \( v_{\perp} \) at the fireball edge. (We take care that the final transverse flow agrees with the measured \( m_{T} \) spectra by reducing the flow gained during the thermalized evolution accordingly). The amount of primordial transverse flow is expected to be small, but our results demonstrate clearly that we recover the comparatively large sensitivity seen in [10] when we assume that a strong flow field is present \( \text{ab initio} \).

However, just increasing the amount of primordial transverse expansion is hardly an appropriate comparison as in addition to the increased role of flow in the energy loss there is a rather trivial geometrical effect: The fast initial expansion of the fireball radius in the presence of primordial flow increases the average pathlength of hard partons in matter. In order to separate the two effects,
we show a calculation in which only the geometrical effect was taken into account but where we neglected the flow effect on $\hat{q}$. The results are shown in Fig. 3.

In essence, for not too large primordial expansion velocities, geometry accounts for about half of the observed effect. Note that a small primordial flow field cannot be ruled out by current experimental evidence: One of the observables most sensitive to the initial state is the emission of thermal photons, but using the framework outlined in \[17\] we observe that the increased radial flow tends to compensate the faster cooling by the increased expansion so that it is very difficult to distinguish the scenarios.

In principle, in a non-boost invariant accelerated longitudinal expansion there can be a component of longitudinal flow transverse to the jet axis (this is not so in a Bjorken expansion where the hard parton is always comoving with the surrounding matter in longitudinal direction). The magnitude of this component depends on the amount of longitudinal acceleration the system has undergone and the distance of the jet from midrapidity. In the present framework, it turns out that longitudinal flow leads to an additional 2% suppression at $y = 0.5$ as compared to a situation in which only transverse flow is taken into account. This is small compared to other uncertainties, hence we do not discuss the effect of longitudinal flow with regard to additional energy loss any further.

V. THE RELATION BETWEEN ENERGY DENSITY AND TRANSPORT COEFFICIENT

In \[17\], an estimate within a Bjorken expansion model was made for the value of the parameter $\alpha$ relating the transport coefficient and energy density. The analysis found $\alpha > 8...19$ far from the perturbative value $\alpha \approx 2$ valid for the ideal QGP. This is in fact consistently larger than our findings neglecting flow. The reason is that we do not use a Bjorken expansion. This leads to stronger initial compression of matter (and thus higher densities) as well as slower subsequent cooling in our model. If we assume a Bjorken expansion and neglect the effect of transverse flow on energy loss we find good agreement with the data for $\alpha = 10$, quite consistent with the estimate in \[17\].

However, if we take transverse flow into account in a moderately optimistic scenario, i.e. assuming a quadratic flow profile, a small value of primordial flow with $v_T^i = 0.1$ and furthermore increase $\alpha_s$ to 0.45 instead of 0.3 then we find that $\alpha = 2$ in fact gives a fair description of the data. Thus, there may not be a huge quantitative discrepancy between pQCD predictions and the measured energy loss. This is shown in Fig. 4.
VI. SUMMARY

We have investigated the effect of flow transverse to the jet direction on the energy loss of the leading parton. The net effect arises from an intricate interplay of several different effects, among them the quadratic pathlength dependence of energy loss which tends to emphasize the role of late times, the expansion and cooling matter which dilutes the system and emphasizes early times and gradual buildup of the transverse flow field over time. Jets need to be produced away from the center of the transverse distribution in order to see a component transverse to the jet direction, but if they are produced too close to the edge they are likely to escape before flow could build up. As a result, we find great sensitivity to the presence of a primordial flow field.

We have also studied the role of a longitudinal flow component orthogonal to the outgoing jet. We observe that away from midrapidity additional energy loss is induced if the jet is not longitudinally co-moving with the thermalized matter, however around midrapidity this effect turns out to be small compared to the influence of transverse flow.

If we include the additional energy loss induced by transverse flow in a favourable (but still realistic) scenario, we find that the ideal QGP relation $\hat{q} \approx 2e^{\frac{3}{4}}$ gives a fair description of the data. Thus, deviations from the ideal QGP energy loss may not be large even in the temperature range $<2T_{C}$ probed at RHIC energies.

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