Relativistic magnetohydrodynamics with spin

Rajeev Singh,1,2, * Masoud Shokri,3, † and S. M. A. Tabatabae Mehr4, ‡

1Institute of Nuclear Physics, Polish Academy of Sciences, PL-31-342 Kraków, Poland
2Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York, 11794-3800, USA
3Institute for Theoretical Physics, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
4IPM, School of Particles and Accelerators, P.O. Box 19395-5531, Tehran, Iran

(Dated: November 9, 2022)

We extend the classical phase-space distribution function to include the spin and electromagnetic fields coupling and derive the modified constitutive relations for charge current, energy-momentum tensor, and spin tensor. Because of the coupling, the new tensors receive corrections to their perfect fluid counterparts and make the background and spin fluid equations of motion communicate with each other. We investigate special cases which are relevant in high-energy heavy-ion collisions, including baryon free matter and large mass limit. Using Bjorken symmetries, we find that spin polarization increases with increasing magnetic field for an initially positive baryon chemical potential. The corrections derived in this framework may help to explain the splitting observed in Lambda hyperons spin polarization measurements.

I. INTRODUCTION

Over the last few decades, relativistic heavy-ion collision experiments at RHIC and LHC have provided a unique opportunity for the study of the properties of hot and dense relativistic nuclear matter under extreme conditions [1]. The system produced by the colliding nuclei at high collision energies has been shown to quickly evolve from the initial nonequilibrium gluon plasma state to an equilibrated quark-gluon plasma (QGP) phase and eventually recombine into hadrons below the freeze-out temperature [2–5]. Using the theory of relativistic hydrodynamics it was shown that QGP matter produced this way forms the smallest fluid droplets ever observed which exhibit a nearly perfect fluid behavior [6–10]. The success of hydrodynamic description of relativistic heavy-ion collisions has opened new avenues in theoretical studies of relativistic matter, including the development of the theory of hydrodynamics and its off equilibrium applications [11].

Particularly interesting directions of study which are recently being followed include investigations of spin polarization phenomena in relativistic nuclear matter [12, 13] and its dynamics under the influence of electromagnetic fields (EM) [14–16]. Recent spin polarization measurements of emitted particles provided a new probe for studying QGP [17–24] and triggered many theoretical developments [25–34], see also Refs. [35–44]. Being oriented along the direction of the global angular momentum of the colliding system, microscopically, the spin polarization is believed to arise because of the spin-orbit coupling [45]. On the other hand, since at the macroscopic level, the QGP admits close-to-equilibrium dynamics, it is believed that the spin degrees of freedom undergo the thermalization as well. This may allow generation of spin polarization through the coupling between the fluid vorticity and spin of its constituents [46].

While the agreement between the global polarization measurements and “spin-thermal” models supports the hypothesis of polarization-vorticity coupling [47–52], the same theories do not quite explain the measurements related to differential observables [48, 51, 53]; though there are some recent advances in this direction [54–57].

Discrepancies between the theoretical predictions and the experimental data indicate that the current theoretical understanding of spin polarization dynamics in heavy-ion collisions is incomplete. If the spin degrees of freedom are thermalized, they should be incorporated into the hydrodynamic formalism on the same footing as the other macroscopic quantities, allowing for their non-trivial dynamics. To this end, several frameworks considering spin as a dynamical quantity were developed that used thermodynamic equilibrium [58], perfect fluid spin hydrodynamics [25, 26], effective action approach [38, 41, 59–61], method of entropy current analysis [35–37, 62–64], statistical operator formalism [65], nonlocal collisions [39, 42, 66–73], kinetic theory for massless fermions [66, 74–77], holographic method [40, 41, 78–81], anomalous hydrodynamics [82, 83], and Lagrangian method [84, 85]. Also see Refs. [86–91] for the studies related to helicity polarization. Relativistic hydrodynamics with spin proposed in Refs. [25, 26] was further developed [27–32] and also extended to dissipative systems in Refs. [33, 34]. In its current form, the spin hydrodynamics framework does not include interactions with the electromagnetic field which may possibly be present in the early-time evolution of QGP. Such a coupling may be crucial for the explanation of the splitting in Λ and Λ̄ spin polarization signal observed in the experiments [17, 92], which may arise because of opposite magnetic moments of these particles [47], for other approaches, see Refs. [88, 93].

In this work, we extend the spin hydrodynamics framework developed in Refs. [27–29] and include the coupling between spin and electromagnetic fields in the phase-space distribution function of the constituent particles. We obtain modified constitutive relations for the baryon charge current, energy-momentum tensor, and spin tensor arising because of the coupling. Then, we investigate special cases of these quantities for
the baryon-free system and the large mass limit, which are relevant to the physics of ultra-relativistic heavy-ion collisions. Finally, we study equations of motion in the case of Bjorken symmetry in the ideal MHD limit and obtain the evolution of the spin component. Although the qualitative features of spin polarization remains same as in Ref. [29] due to strong symmetries assumed herein, however, we find that magnetic field positively enhances the spin polarization for an initial positively enhanced baryon chemical potential. We think that in more realistic setup our framework will prove to be crucial in understanding the splitting between \( \Lambda \) and \( \bar{\Lambda} \) spin polarization.

II. CONVENTIONS

We use the mostly-minus Minkowski metric signature which reads \( g_{\mu\nu} = \text{diag} (+1, -1, -1, -1) \). As a result, the fluid four-velocity is normalized as \( U^\mu U_\mu = 1 \). The operator \( \Delta^\mu\nu = g^{\mu\nu} - U^\mu U^\nu \) projects tensors on the space transverse to \( U^\mu \). A tensor \( M_{\mu\nu} \) can be decomposed into a symmetric \( M_{[\mu\nu]} \equiv \frac{1}{2} (M_{\mu\nu} + M_{\nu\mu}) \) and an asymmetric \( M_{\{\mu\nu\}} \equiv \frac{1}{2} (M_{\mu\nu} - M_{\nu\mu}) \) part. We denote Levi-Civita symbol as \( \epsilon^{0123} \) which is totally antisymmetric and use the convention of \( \epsilon^{0123} = -\epsilon_{0123} = 1 \). Euclidean three-vectors are denoted with boldface, like \( \mathbf{B} \), as opposed to four-vectors. For the scalar and Frobenius product we use the notation \( a \cdot b \equiv a^\mu b_\mu \) and \( A : B \equiv A^{\mu\nu} B_{\mu\nu} \), respectively. Throughout the paper we assume natural units, i.e. \( c = \hbar = k_B = 1 \), unless stated otherwise.

III. ONE-PARTICLE DISTRIBUTION FUNCTION: COUPLING SPIN TO EM FIELDS

In this section we extend the classical phase-space spin distribution function [28, 33] by introducing a term coupling the spin to the external EM fields, in a way suggested in Ref. [12].

For the classical treatment of particles having spin one half and mass \( m \), phase-space distribution function with spin reads [28, 33]

\[
 f_0^\pm(x, p, s) = f_0^\pm(x, p) \exp \left[ \frac{1}{2} \omega_0(x) \cdot s(p) \right],
\]

where \( \omega_\varphi(p) \) is the spin polarization tensor and \( s_\varphi(p) \) represents the particle internal angular momentum expressed with spin four-vector \( s^\mu \) and four-momentum \( p^\mu \) as

\[
 s^\mu = \frac{1}{m} e^{\mu\nu\delta} p_\nu s_\delta.
\]

In Eq. (1), \( f_0^\pm(x, p) = \exp(-p \cdot \beta(x) + \xi(x)) \) is the Jüttner distribution, with \( \xi(x) \) being the ratio of the baryon chemical potential \( \mu(x) \) over temperature \( T(x) \), \( \xi = \mu(x) / T(x) \), and \( \beta_\mu(x) \) is the ratio of the fluid four-velocity \( U_\mu(x) \) to temperature, \( \beta_\mu = U_\mu / T \). Note that, the classical distribution function (1) is valid only for the case of local collisions between particles, however, it can be extended to include nonlocal effects through the gradients of \( f_0^\pm(x, p, s) \) [28].

We generalize the phase-space distribution function (1) to a case of interaction between the particle magnetic moment and the external EM field by introducing the modified distribution function in the form

\[
 f_\pm(x, p, s) = f_0^\pm(x, p, s) \exp \left[ \mp \alpha_M(x) F(x) \cdot s \right],
\]

where \( F_{\mu\nu} \) is the Faraday tensor expressed in terms of electric \( E_\mu \) and magnetic \( B_\mu \) four-vectors as

\[
 F_{\mu\nu} = E_\mu U_\nu - E_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha B^\beta,
\]

with

\[
 E^\mu \equiv F^{\mu\nu} U_\nu, \quad B^\mu \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} U_\beta.
\]

In Eq. (3), \( \alpha_M = \mu_M / T \) where \( \mu_M = g_M \mu_N \) is the magnetic moment of the quasiparticles and \( \mu_N \) being the nuclear magneton. In this work, for simplicity, we assume that the quasiparticles are \( \Lambda \) hyperons, with \( g_\Lambda = -0.6138 \pm 0.0047 \) [94]. However, it should be mentioned that, a more realistic setup needs multiple quark-like quasiparticles, with constitutive masses, see for example [31]. Keeping in mind the smallness of the amplitude of spin polarization in measurements [17], we assume the small polarization limit [29], \( \omega_\varphi \ll 1 \). We also assume weak EM fields \( eB \ll M^2 \), with \( M \) being a typical energy scale relevant to the system in consideration. If the quasiparticles are assumed to be electrically neutral Lambda hyperons, then \( M \sim m_\Lambda \). The aforementioned assumptions allow Eq. (3) to be approximated as

\[
 f_\pm(x, p, s) = f_0^\pm(x, p) \left( \frac{1}{2} \omega_0 : s \right) \left( 1 \mp \alpha_M F \cdot s \right),
\]

where \( f_\pm(f_-) \) represents the distribution function for particles (antiparticles). Similarly to the Faraday tensor, the spin polarization tensor \( \omega_\varphi \) in Eq. (6) can be decomposed with respect to the fluid four-velocity as [32]

\[
 \omega_{\mu\nu} = \kappa_{\mu}(U_{\nu} - \kappa_{\nu} U_{\mu} + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta),
\]

where the four-vectors \( \kappa_{\mu} \) and \( \omega_{\mu} \),

\[
 \kappa_{\mu} = \omega_{\mu} U^\mu, \quad \omega_{\mu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \omega^\nu U^\alpha,
\]

are orthogonal to \( U^\mu \) [32]. The above conditions leave \( \kappa_{\mu} \) and \( \omega_{\mu} \) with three degrees of freedom each, constituting together the same number of degrees of freedom as \( \omega_{\mu\nu} \). These four-vectors can be written in terms of three space-like orthonormal vectors \( X^\mu, Y^\mu, \) and \( Z^\mu \) which span the plane transverse to \( U^\mu \). These vectors, together with \( U^\mu \), form a basis satisfying [29, 32]

\[
 U \cdot U = 1, \quad X \cdot X = Y \cdot Y = Z \cdot Z = -1.
\]

Consequently,

\[
 \kappa^\mu = C_{\kappa X} X^\mu + C_{\kappa Y} Y^\mu + C_{\kappa Z} Z^\mu, \quad \omega^\mu = C_{\omega X} X^\mu + C_{\omega Y} Y^\mu + C_{\omega Z} Z^\mu.
\]

Here, \( C_{\kappa} = (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z}) \), and \( C_{\omega} = (C_{\omega X}, C_{\omega Y}, C_{\omega Z}) \) will be referred as the spin polarization components.
IV. CONSTITUTIVE RELATIONS IN THE PRESENCE OF EM FIELDS

In the following we derive the constitutive relations for the baryon charge current, energy-momentum tensor, and spin tensor from the distribution function (6) introduced in the previous section.

A. Charge current

The baryon charge current is the first moment of the modified distribution function (6) [27, 28]

\[ N^\lambda = \int dP \, dS \, p^\lambda \left[ f^+_s(x, p, s) - f^-_s(x, p, s) \right], \quad (12) \]

where the invariant integration measures for momentum \( dP \) and spin \( dS \) are defined, respectively, as [28]

\[ dP = \frac{d^3p}{(2\pi)^3} E_p, \quad \text{and} \quad dS = \frac{m}{\pi \delta} d^3s \, \delta(s \cdot s + \delta^2) \, \delta(p \cdot s). \quad (13) \]

Here \( E_p \) is the particle energy and \( \delta^2 \) is the length of the spin vector, which for spin half particles equals \( 3/4 \) [28].

Similarly to the current of a dissipative charged fluid [95], \( N^\lambda \) can be decomposed as

\[ N^\lambda = \mathcal{N} U^\lambda + N_s^\lambda = \left( \mathcal{N}_{\text{PF}} + \mathcal{N}_{\text{EM}} \right) U^\lambda + N_s^\lambda, \quad (14) \]

where \( \mathcal{N}_{\text{PF}} \) is the perfect fluid baryon charge density [32]

\[ \mathcal{N}_{\text{PF}} = 4 \, \sinh(\xi) \mathcal{N}_0. \quad (15) \]

The factor \( 4 \, \sinh(\xi) \) accounts for spin and particle-antiparticle degeneracies, and \( \mathcal{N}_0 \) is number density of the spinless neutral classical massive particles [1]

\[ \mathcal{N}_0 = g T^3 z^2 K_2(z), \quad (16) \]

with \( g = 1/(2\pi^2) \), \( z = m/T \) and \( K_n \) being \( n \)th modified Bessel function of second kind. In Eq. (14), \( \mathcal{N}_{\text{EM}} \) is the charge density modification and \( N_s^\lambda \) is the transverse current, both due to spin-EM coupling. They read

\[ \begin{align*}
\mathcal{N}_{\text{EM}} &= a_M \, \cosh(\xi) \mathcal{N}_0 \, e^{\gamma_T \mu \nu} \omega^\mu \nu U_\mu B_\nu, \\
N_s^\lambda &= a_M \, \sinh(\xi) \, A_3 \left( U^\lambda F^\mu \nu + 6U^\lambda U^\mu F^\nu \right) \\
&\quad - U^\lambda \delta^\mu F^\nu - g \delta^\mu \delta^\nu f^\lambda E^\nu) \theta_\mu, 
\end{align*} \quad (17) \]

where \( A_3 = \left( 2 \left( \epsilon_0 + P_0 \right) \right) / (T z^2) \) [25, 27, 28, 32] with \( P_0 \) and \( \epsilon_0 \) denoting the pressure, and energy density of the spinless and neutral classical massive particles, defined as [1]

\[ P_0 = \mathcal{N}_0 T, \quad \epsilon_0 = g z^3 T^4 K_1(z) + 3P_0. \quad (18) \]

respectively.

The conservation of charge current, \( \partial_\mu N^\mu = 0 \), leads to the first equation of motion

\[ \begin{align*}
\mathcal{N}_{\text{PF}} + \mathcal{N}_{\text{EM}} + (\mathcal{N}_{\text{PF}} + \mathcal{N}_{\text{EM}}) \theta_U &= - \partial_\mu N^\mu, 
\end{align*} \quad (19) \]

wherein \( \cdot \cdot \cdot \equiv U_\mu \partial_\mu \cdot \cdot \cdot \) denotes the co-moving temporal derivative [96] and \( \theta_U \equiv \partial_\mu U^\mu \) is the expansion scalar. If the quasiparticles carry a electric charge \( q \), the electric current is

\[ J^\mu = q \, N^\mu, \quad (20) \]

resulting in a “back-reaction” of spin-EM coupling with EM fields via Maxwell equations. Since we are taking Lambda hyperons, which are electrically neutral, as the quasiparticles of the fluid the electric current vanishes [97].

B. Energy-momentum tensor

The energy-momentum tensor is the second moment of the distribution function (6), namely [27, 28]

\[ T^{\mu \nu} = \int dP \, dS \, p^\mu p^\nu \left[ f^+_s(x, p, s) + f^-_s(x, p, s) \right]. \quad (21) \]

Performing the spin integration followed by the momentum integration one obtains

\[ T^{\mu \nu} = \mathcal{E} U^\mu U^\nu - \mathcal{P} \Delta^{\mu \nu} + \mathcal{Q} \mu U^\nu + \mathcal{Q}^\nu U^\mu + \tau^{\mu \nu}. \quad (22) \]

Here \( \mathcal{E} \) is the modified energy density

\[ \mathcal{E} \equiv U_\mu U^\mu T^{\mu \nu} = \mathcal{E}_{\text{PF}} + \mathcal{E}_{\text{EM}}, \quad (23) \]

with

\[ \begin{align*}
\mathcal{E}_{\text{PF}} &= 4 \, \cosh(\xi) \mathcal{E}_0, \\
\mathcal{E}_{\text{EM}} &= a_M \, \sinh(\xi) \left\{ \mathcal{E}_0 \omega : F \\
&+ 2 \left\{ \left( I_{40}^{(1)} + I_{41}^{(0)} \right) \omega : E - 2I_{41}^{(0)} \omega : B \right\} \right\}, 
\end{align*} \quad (24) \]

and \( \mathcal{P} \) is the modified pressure

\[ \mathcal{P} \equiv -\frac{1}{3} \Delta : T = \mathcal{P}_{\text{PF}} + \mathcal{P}_{\text{EM}}, \quad (25) \]

with

\[ \begin{align*}
\mathcal{P}_{\text{PF}} &= 4 \, \cosh(\xi) \, P_0, \\
\mathcal{P}_{\text{EM}} &= a_M \, \sinh(\xi) \left\{ P_0 \omega : F \\
&- 2 \left\{ I_{41}^{(0)} + \frac{5}{3} I_{42}^{(0)} \right\} \omega : E - \frac{10}{3} I_{42}^{(0)} \omega : B \right\}. 
\end{align*} \quad (26) \]

In Eq. (22), \( Q^{\mu \nu} \) is a transverse vector current that resembles the heat current, and \( \tau^{\mu \nu} \) is a transverse traceless tensor similar to the stress tensor of a dissipative fluid [95] given by

\[ \begin{align*}
\mathcal{Q}^{\mu \nu} &\equiv \Delta^{\mu \nu} U_\mu T^\beta \beta \\
&= 2 a_M \, \sinh(\xi) I_{41}^{(0)} \epsilon^{\mu \nu \alpha \beta} U_\nu \left( E_\alpha \omega_\beta - B_\alpha \kappa_\beta \right), \quad (27) \\
\tau^{\mu \nu} &\equiv \Delta^{\mu \nu} T^\alpha \beta \\
&= 4 a_M \, \sinh(\xi) I_{42}^{(0)} \left( E^{(\mu} \kappa^{\nu)} + B^{(\mu} \omega^{\nu)} \\
&- \frac{1}{3} \Delta^{\mu \nu} \left( \kappa \cdot E + \omega \cdot B \right) \right). \quad (28) 
\end{align*} \]
where $\Delta_{\alpha\beta}^{\mu\nu} \equiv (1/2) \left[ \Delta^\mu_{\alpha} \Delta^\nu_{\beta} + \Delta^\nu_{\alpha} \Delta^\mu_{\beta} - (2/3) \Delta^{\mu\nu} \delta_{\alpha\beta} \right]$. The thermodynamic integrals $I_{\alpha\beta}^{\mu\nu}$ appearing in the equations above are defined in appendix A.

As usual, the conservation of energy and momentum, $\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu$, can be decomposed into longitudinal and transverse parts with respect to $U$. The longitudinal one, the so-called energy equation, reads

$$\dot{\mathcal{E}} + (\mathcal{E} + P) \theta_U = -qE \cdot N_\perp + \mathcal{U} \cdot Q - \nabla \cdot Q + \frac{1}{2} \mathcal{T} : \sigma,$$

where $\nabla \mu \equiv \partial_\mu - U_\mu U^\nu \partial_\nu$ and $\sigma^{\mu\nu} \equiv \Delta_{\alpha\beta}^{\mu\nu} \nabla^\alpha U^\beta$. In the direction transverse to $U$ we have

$$(\mathcal{E} + P) \mathcal{U}^\mu = \nabla^\mu P + \left( \mathcal{N} E^\mu + e^{\mu\alpha\beta} B_\alpha N_\perp U_\beta \right) \left( \mathcal{U} \cdot Q + \frac{1}{2} \mathcal{T} : \sigma \right) \mathcal{U}^\mu - \partial_\sigma T^{\mu\sigma} - \dot{\mathcal{Q}}^\mu - \partial_\sigma (U^\sigma Q^\mu).$$

C. Spin tensor

The spin tensor is defined as [28]

$$S^{\lambda,\mu\nu} = \int d^3 p \, \rho^S \, \jmath^{\mu\nu}[f^S_+(x,p,s) + f^S_-(x,p,s)].$$

Plugging the distribution function (6) into the above equation and integrating over the spin and momenta gives

$$S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}_P - 2 a_M \tanh(\xi) S^{\lambda,\mu\nu}_EM,$$

where [27–29, 32]

$$S^{\lambda,\mu\nu}_P = \cos(\xi) \left( A_1 U^\alpha_{\mu} \omega^\beta T^\gamma_{\nu} + A_2 U^\alpha_{\mu} U^{(\beta} \omega^{\gamma)T_{\nu}} + A_3 \left( U^{(\beta} \omega^{\gamma)T_{\nu}} + g^{\alpha\beta} k^\gamma \right) \right),$$

$$S^{\lambda,\mu\nu}_EM = \cos(\xi) \left( [U^\alpha T^\mu A_{\beta\gamma} + A_3 U^{(\beta} \omega^{\gamma)T_{\nu}} + A_3 \left( U^{(\beta} \omega^{\gamma)T_{\nu}} + g^{\alpha\beta} k^\gamma \right) \right),$$

with the thermodynamic coefficients defined as

$$A_1 = N_{(0)} - A_3, \quad A_2 = 2 \left[ A_1 - 2 A_3 \right].$$

As the phase-space distribution function (6) is valid only for the local collisions of the particles, it suggests that the orbital contribution ($J^{\lambda,\mu\nu}_{\alpha\beta}$), in the total angular momentum ($J^{\lambda,\mu\nu}_{\alpha\beta}$), can be eliminated and spin ($S^{\lambda,\mu\nu}_{\alpha\beta}$) can be conserved independently [28]. Here, the energy-momentum tensor (21) is, by definition, symmetric. However, it may have antisymmetric contribution if we include nonlocal collisional effects [72, 73], which are neglected in the current study for simplicity.

Thus, neglecting nonlocal collisions we observe that the spin tensor is a conserved current

$$\partial_\alpha S^{\lambda,\mu\nu}_{\alpha\beta} = 0.$$  

From above we get six equations of motion for the six spin polarization components ($C_C$ and $C_{\omega}$) which, in general, are coupled to each other.

V. SPECIAL CASES

In this section we investigate our formalism in some special situations.

A. Baryon-free system

In ultra-relativistic heavy-ion collisions, the QGP can be considered approximately as baryon-free matter [3]. In the absence of the baryon chemical potential, i.e., $\mu = 0$, the spin-EM coupling can only affect the charge current sector. The spin-EM coupling results in a non-vanishing EM component of the number density $N^\mu_{EM}$ and transverse current $N^\mu_\perp$ in Eq. (14). Furthermore, the heat current $Q^\mu$ (27) and stress tensor $T^{\mu\nu}$ (28) vanish, and the energy-momentum tensor is reduced to its perfect fluid form

$$T^{\mu\nu} = E_{PF} U^\mu U^\nu - P_{PF} \Delta^{\mu\nu}. \tag{37}$$

Similarly, the spin tensor (32) reduces to

$$S^{\alpha,\beta\gamma}_{(\mu=0)} = U^\alpha [A_1 \omega^{\beta,\gamma} + A_2 U^{(\beta} \omega^{\gamma)T_{\nu}}] + A_3 \left( U^{(\beta} \omega^{\gamma)T_{\nu}} + g^{\alpha\beta} k^\gamma \right),$$

with no spin-EM coupling terms.

B. Large mass limit

Another special case is when the constituents of the fluid, say hyperons, have masses much larger than the temperature, i.e., $z = m/T \gg 1$. Then, we can neglect $A_3$ in Eq. (33), and the electric field no longer appears in the spin tensor (32), giving

$$S^{\alpha,\beta\gamma}_{z \gg 1} = \cos(\xi) N_{(0)} U^\alpha e^{\beta\gamma T_{\nu}} U_{\nu} \left( \omega_{\nu} - 2 a_M \tanh(\xi) B_{\nu} \right) \tag{39}$$

VI. 0+1D: BJORKEN FLOW

In this section, we examine our formulation with the Bjorken setup as a special situation relevant to the ultra-relativistic heavy-ion collisions. Bjorken symmetries simplify equations of motion significantly; the hydrodynamic variables become functions of proper time $t = \sqrt{t^2 - z^2}$ alone, and energy-momentum conservation in the transverse direction (30) is trivially satisfied. Furthermore, the heat current $Q^\mu$ (27) and the stress tensor $T^{\mu\nu}$ (28) disappear. Yet, even in this situation, the equations of motion of background and spin are highly coupled due to spin-EM contribution. The fluid flow is affected by the spin

\footnote{In the case of Bjorken expansion the basic vectors are $U^a = (\cosh(\eta), 0, 0, \sinh(\eta))$, $X^a = (0, 1, 0, 0)$, $Y^a = (0, 0, 1, 0)$, and $Z^a = (\sinh(\eta), 0, 0, \cosh(\eta))$, where $\eta$ is the space-time rapidity [29].}
dynamics, which is different from the previous studies [29–31].

We start with the baryon charge conservation (19). The transverse current trivially satisfies $\partial_{\tau} N^{\mu}_Y = 0$ due to Bjorken symmetries. Therefore the charge conservation reduces to an equation formally similar to that of Bjorken perfect fluid [29],

$$\frac{dN}{d\tau} + N^\tau = 0.$$  \hspace{1cm} (40)

For simplicity, we assume uncharged particles and the ideal MHD limit. Therefore, the profile of the magnetic field is [98]

$$B^\mu = aB_0 \frac{\tau_0}{\tau} Y^\mu,$$  \hspace{1cm} (41)

with $B_0 = (1/e) m_\pi^2$, with $m_\pi$ being the Pion’s mass, and $a$ being a positive dimensionless number. At the initial proper time $\tau_0$ the initial magnetic field is $B_0 = aB_0$. Setting $a = 0$ switches off the magnetic field. Here, $Y^\mu = (0, 0, 1, 0)$. Due to the magnetic field profile adopted herein, Eq. (41), we suspect that the only spin component $C_{\omega Y}$ will be coupled to the EM fields, which is indeed the case shown below.

The spin-EM coupling occurs through the term $F : \omega = -2B_0 C_{\omega Y} \tau_0/\tau$, which gives rise to

$$N' = N_{PF} + 2gT^3 a_M B_0 C_{\omega Y} \frac{\tau_0}{\tau} \cosh(\xi) \left[ z^2 K_2(z) + 2z K_1(z) + 2(K_2(z) + 3K_3(z)) \right],$$  \hspace{1cm} (42)

where $N_{PF}$ is given by Eq. (15). Here $T$, $\xi$, and $C_{\omega Y}$ are unknown functions of $\tau$.

The next equation is the energy equation which reads

$$\frac{dE}{d\tau} + E + P = 0,$$  \hspace{1cm} (43)

where

$$E = gT^4 z^3 \left[ 2K_1(z) + 3K_3(z) \right] \times \left[ 2 \cosh(\xi) + a_M B_0 C_{\omega Y} \frac{\tau_0}{\tau} \sinh(\xi) \right],$$  \hspace{1cm} (44)

$$P = N_{PF} \left( \coth(\xi) + a_M B_0 C_{\omega Y} \frac{\tau_0}{\tau} \right).$$  \hspace{1cm} (45)

We solve the coupled set of equations (40), (43), and (36) to obtain the evolution of temperature $T$, baryon chemical potential $\mu$, and spin polarization components. In this setup, each spin polarization component evolve independently of each other [29]. Due to our magnetic field profile (41), the evolution of the spin component $C_{\omega Y}$ is modified by the spin-EM coupling, while other components are not affected, thus we will only study the evolution of $C_{\omega Y}$. As we have assumed small polarization limit, $\omega_{\omega Y} \ll 1$, the spin component must be initialized accordingly, i.e., it can take any positive value less than 1. Therefore, we choose initial value of the spin component as $C_{\omega Y_0} = 0.3$ at $\tau_0 = 0.5$ fm/c. Let us consider four different cases of initial temperature $T_0$ and baryon chemical potential $\mu_0$ corresponding to four different collision energies

$$\begin{align*}
T_0 &= 0.2 \text{ GeV}, & \mu_0 &= 0.4 \text{ GeV} & \text{for } \sqrt{s_{NN}} &= 7.7 \text{ GeV}, \\
T_0 &= 0.3 \text{ GeV}, & \mu_0 &= 0.3 \text{ GeV} & \text{for } \sqrt{s_{NN}} &= 27 \text{ GeV}, \\
T_0 &= 0.3 \text{ GeV}, & \mu_0 &= 0.15 \text{ GeV} & \text{for } \sqrt{s_{NN}} &= 62.4 \text{ GeV}, \\
T_0 &= 0.6 \text{ GeV}, & \mu_0 &= 0.05 \text{ GeV} & \text{for } \sqrt{s_{NN}} &= 200 \text{ GeV},
\end{align*}$$

such as [99]

$T_0 = 0.2 \text{ GeV}, \quad \mu_0 = 0.4 \text{ GeV}$ for $\sqrt{s_{NN}} = 7.7 \text{ GeV}$, $T_0 = 0.3 \text{ GeV}, \quad \mu_0 = 0.3 \text{ GeV}$ for $\sqrt{s_{NN}} = 27 \text{ GeV}$, $T_0 = 0.3 \text{ GeV}, \quad \mu_0 = 0.15 \text{ GeV}$ for $\sqrt{s_{NN}} = 62.4 \text{ GeV}$, $T_0 = 0.6 \text{ GeV}, \quad \mu_0 = 0.05 \text{ GeV}$ for $\sqrt{s_{NN}} = 200 \text{ GeV}$,

(46)

to observe the dependence of spin-EM coupling on different initial values of $T$ and $\mu$.

The spin component $C_{\omega Y}$ also has physical relevance with respect to ultra-relativistic heavy-ion collisions, as it is expected that the total angular momentum is initially purely orbital, negative and directed orthogonal to the reaction plane (i.e. along $-y$ axis) [17–19, 21, 22]. After the collision, this initial orbital angular momentum can be converted to spin angular momentum in the same direction. Therefore, to address such a physical system it is enough to assume a non-zero and small initial $C_{\omega Y}$ and set all other spin components to zero, see Ref. [29] for more details.

Figure 1 shows the evolution of $C_{\omega Y}$ for two different sets of $T_0$ and $\mu_0$ for $\sqrt{s_{NN}} = 7.7 \text{ GeV}$ (upper panel) and $\sqrt{s_{NN}} = 27$
for which, when transiting to the so-called high collision energies, an increase in the collision energy, gives rise to a decrease in the amplitude of the spin component even for very small values of $\alpha$.

The evolution of the spin component $(C_{\omega Y})$ is directly connected to the spin polarization of the $\Lambda(\bar{\Lambda})$ hyperon emitted at the freeze-out hypersurface, $\Delta \Sigma_j = \tau U_j \, dx \, dy \, d\eta$, through the following expression of average spin polarization per particle $\langle \pi_\mu \rangle_\rho$ (suffix $p$ signifies it is a momentum dependent quantity) [32, 100]

$$
\langle \pi_\mu \rangle_\rho = \frac{E_\rho \, d\Pi_\rho^\mu(p)}{E_\rho \, d^3p},
$$

where the numerator is the momentum distribution of the total Pauli-Lubański (PL) four-vector and denominator is the momentum density of particles and antiparticles given, respectively, as [32]

$$
E_\rho \frac{d\Pi_\rho^\mu(p)}{d^3p} = - \frac{1}{(2\pi)^3 m} \left[ \int \cosh(\xi) \, \Delta \Sigma_\rho \rho^\mu \, e^{-\beta \rho} 
\times \left( (\tilde{\omega}_{\rho\beta} + \tilde{F}_{\rho\beta}) \rho^\beta \right) \right].
$$

Integrating over momentum coordinates we obtain momentum independent spin polarization using Eq. (47) as [32]

$$
\langle \pi_\mu \rangle = \frac{\int d\rho \, \langle \pi_\rho \rangle_\rho \, E_\rho \, d\Pi_\rho^\mu(p)}{\int d\rho \, E_\rho \, d\Pi_\rho^\mu(p)} = \frac{\int d^3p \, \frac{d\Pi_\rho^\mu(p)}{d^3p}}{\int d^3p \, \frac{d\mathcal{N}^\rho(p)}{d^3p}}.
$$

Since Bjorken symmetries force the hydrodynamic variables to be the functions of $\tau$ only, the factor $\cosh(\xi)$, in Eq. (47), comes out of the integration and cancels out [29]. This makes Eq. (47) independent of the baryon chemical potential and therefore, within the Bjorken setup, we cannot observe the explicit $\mu$ dependency of $\Lambda - \bar{\Lambda}$ spin polarization. However, in more realistic setups [32, 101] this might not be the case. Since we assume that magnetic fields do not break Bjorken symmetries, the spin polarization evolution remains independent of the chemical potential with $\alpha \neq 0$. Thus, we refrain to show them here as they are qualitatively similar to the spin polarization results for Bjorken background without electromagnetic fields [29].

VII. SUMMARY

In this work, we introduced a spin-EM coupling term into the classical phase-space distribution function and derived the charge current, energy-momentum tensor, and spin tensor. In contrast to previous studies [29–31], the background and spin equations of motion are highly coupled due to the spin-EM coupling. We also mentioned certain special cases relevant to ultra-relativistic heavy-ion collisions. Lastly, we examined our formalism with Bjorken flow for four different sets of initial
temperature and baryon chemical potential and found that with increasing collisional energy, the amplitude of spin component decreases in the low collision energy regime, while it increases otherwise. With some further considerations, the presented formalism may be relevant in explaining the observed $\Lambda - \bar{\Lambda}$ spin polarization splitting. In particular, it is required to consider more realistic setups, which is the focus of our future investigation.

ACKNOWLEDGMENTS

We thank R. Ryblewski for constructive feedback and critical reading of the manuscript and also thank V. E. Amburs, S. Bhadury, A. Dash, W. Florkowski, A. Jaiswal, D. Rischke, N. Sadooghi, and G. Sophys for fruitful discussions. M.S. acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC-TR 211 ‘Strong-interaction matter under extreme conditions’– project number 315477589 – TRR 211. R.S. acknowledges the support of Polish NAWA Bekker program no.: BPN/BEK/2021/1/00342 in the completion of this work. This research was also supported in part by the Polish National Science Centre Grants No. 2016/23/B/ST2/00717 and No. 2018/30/E/ST2/00432.

Appendix A: Thermodynamic identities

In this appendix, we list the thermodynamic identities we used in Sec. IV

\[
I^{\mu_4 \nu_4}_{\rho_4} = I^{\mu_4}_{\nu_4} U^{\rho_4} U^{\nu_4} U^{\rho_4} + I^{\mu_4}_{\nu_4} \left( \Delta^{\mu_4 \nu_4} U^{\rho_4} + \Delta^{\rho_4 \nu_4} U^{\rho_4} + \Delta^{\rho_4 \nu_4} U^{\rho_4} + \Delta^{\rho_4 \nu_4} U^{\rho_4} \right) \]

with

\[
I^{(0)}_{\nu_4} = \frac{T^6}{64m^2} \left[ K_6(z) + 2K_4(z) - 2K_2(z) - 2K_0(z) \right], \quad (A2)
\]

\[
I^{(1)}_{\nu_4} = -\frac{T^6}{192m^2} \left[ K_6(z) - 2K_4(z) - 2K_2(z) + 2K_0(z) \right], \quad (A3)
\]

\[
I^{(2)}_{\nu_4} = \frac{T^6}{960m^2} \left[ K_6(z) - 6K_4(z) + 15K_2(z) - 10K_0(z) \right].
\]

[1] W. Florkowski, Phenomenology of ultra-relativistic heavy-ion collisions. World Scientific, Singapore, 2010. [https://cds.cern.ch/record/1321594]

[2] D. Kharzeev and M. Nardi, “Hadron production in nuclear collisions at RHIC and high density QCD,” Phys. Lett. B 507 (2001) 121–128, arXiv:nucl-th/0012025.

[3] U. W. Heinz and P. F. Kolb, “Early thermalization at RHIC,” Nucl. Phys. A 702 (2002) 269–280, arXiv:hep-ph/0111075.

[4] M. Gyulassy and L. McLerran, “New forms of QCD matter discovered at RHIC,” Nucl. Phys. A 750 (2005) 30–63, arXiv:nucl-th/0405013.

[5] E. V. Shuryak, “What RHIC experiments and theory tell us about properties of quark-gluon plasma?,” Nucl. Phys. A 750 (2005) 64–83, arXiv:hep-ph/0405066.

[6] P. Romatschke and U. Romatschke, “Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?,” Phys. Rev. Lett. 99 (2007) 172301, arXiv:0706.1522 [nucl-th].

[7] U. Heinz and R. Snellings, “Collective flow and viscosity in relativistic heavy-ion collisions,” Ann. Rev. Nucl. Part. Sci. 63 (2013) 123–151, arXiv:1301.2826 [nucl-th].

[8] D. T. Son and A. O. Starinets, “Hydrodynamics of r-charged black holes,” JHEP 03 (2006) 052, arXiv:hep-th/0601157.

[9] T. Schäfer and D. Teaney, “Nearly Perfect Fluidity: From Cold Atomic Gases to Hot Quark Gluon Plasmas,” Rept. Prog. Phys. 72 (2009) 126001, arXiv:0904.3107 [hep-ph].

[10] B. Schenke, “The smallest fluid on Earth,” Rept. Prog. Phys. 84 no. 8, (2021) 082301, arXiv:2102.11189 [nucl-th].
[20] ALICE Collaboration, S. Acharya et al., “Evidence of Spin-Orbital Angular Momentum Interactions in Relativistic Heavy-Ion Collisions,” Phys. Rev. Lett. 125 no. 1, (2020) 012301, arXiv:1910.14408 [nucl-ex].

[21] ALICE Collaboration, S. Acharya et al., “Global polarization of ΛΛ hyperons in Pb-Pb collisions at √sNN = 2.76 and 5.02 TeV,” Phys. Rev. C 101 no. 4, (2020) 044611, arXiv:1909.01281 [nucl-ex].

[22] F. J. Kornas, “A polarization in au + au collisions at √sNN= 2.4 gev measured with hades,” in The XVIII International Conference on Strangeness in Quark Matter (SQM 2019), D. Elia, G. E. Bruno, P. Colangelo, and L. Cosmai, eds., pp. 435–439. Springer International Publishing, Cham, 2020.

[23] STAR Collaboration, M. S. Abdallah et al., “Global Λ-hyperon polarization in Au+Au collisions at √sNN=3 GeV,” Phys. Rev. C 104 no. 6, (2021) L061901, arXiv:2108.00044 [nucl-ex].

[24] ALICE Collaboration, S. Acharya et al., “Polarization of Λ and ̅Λ hyperons along the beam direction in Pb-Pb collisions at √sNN = 5.02 TeV,” arXiv:2107.11183 [nucl-ex].

[25] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, “Relativistic fluid dynamics with spin,” Phys. Rev. C 97 no. 4, (2018) 041901, arXiv:1705.00587 [nucl-th].

[26] W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, “Spin-dependent distribution functions for relativistic hydrodynamics of spin-1/2 particles,” Phys. Rev. D97 no. 11, (2018) 116017, arXiv:1712.07676 [nucl-th].

[27] W. Florkowski, A. Kumar, and R. Ryblewski, “Thermodynamic versus kinetic approach to polarization-vorticity coupling,” Phys. Rev. C98 no. 4, (2018) 044906, arXiv:1806.02616 [hep-ph].

[28] W. Florkowski, R. Ryblewski, and A. Kumar, “Relativistic hydrodynamics for spin-polarized fluids,” Prog. Part. Nucl. Phys. 108 (2019) 103709, arXiv:1811.04409 [nucl-th].

[29] W. Florkowski, A. Kumar, R. Ryblewski, and R. Singh, “Spin polarization evolution in a boost invariant hydrodynamical background,” Phys. Rev. C 99 no. 4, (2019) 044910, arXiv:1901.09655 [hep-ph].

[30] R. Singh, G. Sophys, and R. Ryblewski, “Spin polarization dynamics in the Gubser-expanding background,” Phys. Rev. D 103 no. 7, (2021) 074024, arXiv:2011.14907 [hep-th].

[31] R. Singh, M. Shokri, and R. Ryblewski, “Spin polarization dynamics in the Bjorken-expanding resistive MHD background,” Phys. Rev. D 103 no. 9, (2021) 094034, arXiv:2103.02692 [hep-ph].

[32] W. Florkowski, R. Ryblewski, R. Singh, and G. Sophys, “Spin polarization dynamics in the non-boost-invariant background,” arXiv:2112.01856 [hep-ph].

[33] S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, “Relativistic dissipative spin dynamics in the relaxation time approximation,” Phys. Lett. B 814 (2021) 136096, arXiv:2002.03937 [hep-ph].

[34] S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, “Dissipative Spin Dynamics in Relativistic Matter,” Phys. Rev. D 103 no. 1, (2021) 014030, arXiv:2008.10976 [nucl-th].

[35] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, “Fate of spin polarization in a relativistic fluid: An entropy-current analysis,” Phys. Lett. B 795 (2019) 100–106, arXiv:1901.06615 [hep-th].

[36] K. Fukushima and S. Pu, “Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –,” Phys. Lett. B 817 (2021) 136346, arXiv:2010.01608 [hep-th].

[37] S. Li, M. A. Stephanov, and H.-U. Yee, “Nondissipative Second-Order Transport, Spin, and Pseudogauge Transformations in Hadronic Dynamics,” Phys. Rev. Lett. 127 no. 8, (2021) 082302, arXiv:2011.12318 [hep-th].

[38] D. Montenegro and G. Torrieri, “Linear response theory and effective action of relativistic hydrodynamics with spin,” Phys. Rev. D 102 no. 3, (2020) 036007, arXiv:2004.10195 [hep-th].

[39] N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, “Generating Spin Polarization from Vorticity through Nonlocal Collisions,” Phys. Rev. Lett. 127 no. 5, (2021) 052301, arXiv:2005.01506 [hep-ph].

[40] M. Garbiso and M. Kaminski, “Hydrodynamics of simply spinning black holes & hydrodynamics for spinning quantum fluids,” JHEP 12 (2020) 112, arXiv:2007.04345 [hep-th].

[41] A. D. Gallegos, U. Gürsoy, and A. Yarom, “Hydrodynamics of spin currents,” SciPost Phys. 11 (2021) 041, arXiv:2101.04759 [hep-th].

[42] X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, and Q. Wang, “From Kadanoff-Baym to Boltzmann equations for massive spin-1/2 fermions,” Phys. Rev. D 104 no. 1, (2021) 016029, arXiv:2103.10636 [nucl-th].

[43] E. Speranza and N. Weickgenannt, “Spin tensor and pseudo-gauges: from nuclear collisions to gravitational physics,” Eur. Phys. J. A 57 no. 5, (2021) 155, arXiv:2007.00138 [nucl-th].

[44] S. Bhadury, J. Bhatt, A. Jaiswal, and A. Kumar, “New developments in relativistic fluid dynamics with spin,” Eur. Phys. J. ST 230 no. 3, (2021) 655–672, arXiv:2101.11964 [hep-ph].

[45] Z.-T. Liang and X.-N. Wang, “Globally polarized quark-gluon plasma in non-central A+A collisions,” Phys. Rev. Lett. 94 (2005) 102301, arXiv:nucl-th/0410079 [nucl-th]. [Erratum: Phys. Rev. Lett.96,039901(2006)].

[46] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, “Relativistic distribution function for particles with spin at local thermodynamical equilibrium,” Annals Phys. 338 (2013) 32–49, arXiv:1303.3431 [nucl-th].

[47] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, “Global hyperon polarization at local thermodynamic equilibrium with vorticity, magnetic field and feed-down,” Phys. Rev. C95 no. 5, (2017) 054902, arXiv:1610.02506 [nucl-th].

[48] I. Karpenko and F. Becattini, “Study of Λ polarization in relativistic nuclear collisions at √sNN = 7.7 –200 GeV,” Eur. Phys. J. C77 no. 4, (2017) 213, arXiv:1610.04717 [nucl-th].

[49] L.-G. Pang, H. Petersen, Q. Wang, and X.-N. Wang, “Vortical Fluid and Λ Spin Correlations in High-Energy Heavy-Ion Collisions,” Phys. Rev. Lett. 117 no. 19, (2016) 192301, arXiv:1605.04024 [hep-ph].

[50] Y. Xie, D. Wang, and L. P. Csernai, “Global Λ polarization in high energy collisions,” Phys. Rev. C 95 no. 3, (2017) 031901, arXiv:1703.03770 [nucl-th].

[51] F. Becattini and I. Karpenko, “Collective Longitudinal Polarization in Relativistic Heavy-Ion Collisions at Very High Energy,” Phys. Rev. Lett. 120 no. 1, (2018) 012302, arXiv:1707.07984 [nucl-th].
[52] B. Fu, K. Xu, X.-G. Huang, and H. Song, “A hydrodynamic study of hyperon spin polarization in relativistic heavy ion collisions,” arXiv:2011.03740 [nucl-th].
[53] W. Florkowski, A. Kumar, R. Ryblewski, and A. Mazeliauskas, “Longitudinal spin polarization in a thermal model,” Phys. Rev. C 100 no. 5, (2019) 054907, arXiv:1904.00002 [nucl-th].
[54] F. Becattini, M. Buzzegoli, and A. Palermo, “Spin-thermal shear coupling in a relativistic fluid,” Phys. Lett. B 820 (2021) 136519, arXiv:2103.10917 [nucl-th].
[55] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, “Local Polarization and Isothermal Local Equilibrium in Relativistic Heavy Ion Collisions,” Phys. Rev. Lett. 127 no. 27, (2021) 272302, arXiv:2103.14621 [nucl-th].
[56] B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, “Shear-Induced Spin Polarization in Heavy-Ion Collisions,” Phys. Rev. Lett. 127 no. 14, (2021) 142301, arXiv:2103.10403 [hep-ph].
[57] W. Florkowski, A. Kumar, A. Mazeliauskas, and R. Ryblewski, “Effect of thermal shear on longitudinal spin polarization in a thermal model,” arXiv:2112.02799 [hep-ph].
[58] F. Becattini and L. Tinti, “The Ideal relativistic rotating gas as a perfect fluid with spin,” Annals Phys. 325 (2010) 1566–1594, arXiv:0911.0864 [gr-qc].
[59] D. Montenegro and G. Torrieri, “Causality and dissipation in relativistic polarizable fluids,” arXiv:1807.02796 [hep-th].
[60] W. M. Serenone, J. a. G. P. Barbon, D. D. Chinellato, M. A. Lisa, C. Shen, J. Takahashi, and G. Torrieri, “A polarization from thermalized jet energy,” arXiv:2102.11919 [hep-ph].
[61] G. Torrieri and D. Montenegro, “Linear response hydrodynamics of a relativistic dissipative fluid with spin,” arXiv:2207.05037 [hep-th].
[62] D. She, A. Huang, D. Hou, and J. Liao, “Relativistic Viscous Hydrodynamics with Angular Momentum,” arXiv:2105.04060 [nucl-th].
[63] A. Daher, A. Das, W. Florkowski, and R. Ryblewski, “Equivalence of canonical and phenomenological formulations of spin hydrodynamics,” arXiv:2202.12609 [nucl-th].
[64] Z. Cao, K. Hattori, M. Hongo, X.-G. Huang, and H. Taya, “Gyrohydrodynamics: Relativistic spinful fluid with strong vorticity,” JPET 2022 no. 7, (2022) 071D01, arXiv:2205.08051 [hep-th].
[65] J. Hu, “Kubo formulae for first-order spin hydrodynamics,” Phys. Rev. D 103 no. 11, (2021) 116015, arXiv:2101.08440 [hep-ph].
[66] Y. Hidaka and D.-L. Yang, “Nonequilibrium chiral magnetic/vortical effects in viscous fluids,” Phys. Rev. D 98 no. 1, (2018) 016012, arXiv:1801.08253 [hep-th].
[67] D.-L. Yang, K. Hattori, and Y. Hidaka, “Effective quantum kinetic theory for spin transport of fermions with collisional effects,” JHEP 20 (2020) 070, arXiv:2002.02612 [hep-ph].
[68] Z. Wang, X. Guo, and P. Zhuang, “Local Equilibrium Spin Distribution From Detailed Balance,” arXiv:2009.10930 [hep-th].
[69] N. Weickgenannt, E. Speranze, X.-l. Sheng, Q. Wang, and D. H. Rischke, “Derivation of the nonlocal collision term in the relativistic Boltzmann equation for massive spin-1/2 particles from quantum field theory,” Phys. Rev. D 104 no. 1, (2021) 016022, arXiv:2103.04896 [nucl-th].
[70] J. Hu, “Relativistic first-order spin hydrodynamics via the Chapman-Enskog expansion,” arXiv:2111.03571 [hep-ph].
[71] J. Hu, “The linear mode analysis from spin transport equation,” arXiv:2202.07373 [hep-ph].
[72] A. Das, W. Florkowski, A. Kumar, R. Ryblewski, and R. Singh, “Semiclassical kinetic theory for massive spin-half fermions with leading-order spin effects,” arXiv:2203.15562 [hep-th].
[73] N. Weickgenannt, D. Wagner, E. Speranza, and D. Rischke, “Relativistic second-order dissipative spin hydrodynamics from the method of moments,” arXiv:2203.04766 [nucl-th].
[74] M. A. Stephanov and Y. Yin, “Chiral Kinetic Theory,” Phys. Rev. Lett. 109 (2012) 162001, arXiv:1207.0747 [hep-th].
[75] J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, and Y. Yin, “Lorentz Invariance in Chiral Kinetic Theory,” Phys. Rev. Lett. 113 no. 18, (2014) 182302, arXiv:1404.5963 [hep-th].
[76] E. V. Gorbar, D. O. Rybalka, and I. A. Shovkovy, “Second-order dissipative hydrodynamics for plasma with chiral asymmetry and vorticity,” Phys. Rev. D95 no. 9, (2017) 096010, arXiv:1702.07791 [hep-th].
[77] S. Shi, C. Gale, and S. Jeon, “Relativistic Viscous Spin Hydrodynamics from Chiral Kinetic Theory,” arXiv:2008.08618 [nucl-th].
[78] M. P. Heller, A. Serantes, M. Spaliński, V. Svensson, and B. Withers, “Convergence of hydrodynamic modes: insights from kinetic theory and holography,” SciPost Phys. 10 no. 6, (2021) 123, arXiv:2012.15393 [hep-th].
[79] A. D. Gallegos and U. Gürsoy, “Holographic spin fluids and Lovelock Chern-Simons gravity,” JHEP 11 (2020) 151, arXiv:2004.05148 [hep-th].
[80] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, “Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation,” arXiv:2107.14231 [hep-th].
[81] A. D. Gallegos, U. Gürsoy, and A. Yarom, “Hydrodynamics, spin currents and torsion,” arXiv:2203.05044 [hep-th].
[82] D. T. Son and P. Surowka, “Hydrodynamics with Triangle Anomalies,” Phys. Rev. Lett. 103 (2009) 191601, arXiv:0906.0504 [hep-th].
[83] D. E. Kharzeev and D. T. Son, “Chiral magnetic and vortical effects in heavy ion collisions,” JHEP 106 (2011) 062301, arXiv:1010.0038 [hep-ph].
[84] D. Montenegro, L. Tinti, and G. Torrieri, “The ideal relativistic fluid limit for a medium with polarization,” Phys. Rev. D96 no. 5, (2017) 056012, arXiv:1701.08263 [hep-th].
[85] D. Montenegro, L. Tinti, and G. Torrieri, “The ideal relativistic fluid limit for a medium with polarization,” Phys. Rev. D96 no. 5, (2017) 056012, arXiv:1701.08263 [hep-th].
[86] V. E. Ambrus and M. N. Chernodub, “Vortical effects in Dirac fluids with vector, chiral and helical charges,” arXiv:1912.11034 [hep-th].
[87] V. E. Ambrus, “Helical massive fermions under rotation,” JHEP 08 (2020) 016, arXiv:1912.09977 [nucl-th].
[88] V. E. Ambrus and M. N. Chernodub, “Hyperon–anti-hyperon polarization asymmetry in relativistic heavy-ion collisions as an interplay between chiral and helical vortical effects,” Eur. Phys. J. C 82 no. 1, (2022) 61, arXiv:2010.05831 [hep-ph].
[89] F. Becattini, M. Buzzegoli, A. Palermo, and G. Prokhorov, “Polarization as a signature of local parity violation in hot QCD matter,” arXiv:2009.13449 [hep-ph].

[90] J.-H. Gao, “Helicity polarization in relativistic heavy ion collisions,” arXiv:2105.08293 [hep-ph].

[91] C. Yi, S. Pu, J.-H. Gao, and D.-L. Yang, “Hydrodynamic helicity polarization in relativistic heavy ion collisions,” Phys. Rev. C 105 no. 4, (2022) 044911.

[92] H. Li, X.-L. Xia, X.-G. Huang, and H. Z. Huang, “Global spin polarization of multistrange hyperons and feed-down effect in heavy-ion collisions,” Phys. Lett. B 827 (2022) 136971, arXiv:2106.09443 [nucl-th].

[93] O. Vitiuk, L. V. Bravina, and E. E. Zabrodin, “Is different Λ and ̄Λ polarization caused by different spatio-temporal freeze-out picture?,” Phys. Lett. B 803 (2020) 135298, arXiv:1910.06292 [hep-ph].

[94] Particle Data Group Collaboration, P. A. Zyla et al., “Review of Particle Physics,” PTEP 2020 no. 8, (2020) 083C01.

[95] P. Kovtun, “Lectures on hydrodynamic fluctuations in relativistic theories,” J. Phys. A 45 (2012) 473001, arXiv:1205.5040 [hep-th].

[96] P. Romatschke and U. Romatschke, Relativistic Fluid Dynamics In and Out of Equilibrium. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5, 2019. arXiv:1712.05815 [nucl-th].

[97] G. S. Denicol, E. Molnár, H. Niemi, and D. H. Rischke, “Resistive dissipative magnetohydrodynamics from the Boltzmann-Vlasov equation,” Phys. Rev. D 99 no. 5, (2019) 056017, arXiv:1902.01699 [nucl-th].

[98] S. Tabatabaee and N. Sadooghi, “Wigner function formalism and the evolution of thermodynamic quantities in an expanding magnetized plasma,” Phys. Rev. D 101 no. 7, (2020) 076022, arXiv:2003.01686 [hep-ph].

[99] I. A. Karpenko, P. Huovinen, H. Petersen, and M. Bleicher, “Estimation of the shear viscosity at finite net-baryon density from A + A collision data at √s_{NN} = 7.7 – 200 GeV,” Phys. Rev. C 91 no. 6, (2015) 064901, arXiv:1502.01978 [nucl-th].

[100] R. Singh, “Theoretical Aspects of Relativistic Perfect-fluid Spin Hydrodynamic Framework,” Acta Phys. Polon. Supp. 15 no. 3, (2022) 35.

[101] S. Shi, S. Jeon, and C. Gale, “Family of new exact solutions for longitudinally expanding ideal fluids,” Phys. Rev. C 105 no. 2, (2022) L021902, arXiv:2201.06670 [hep-ph].