Effect of the Spin 3/2 Nucleon Resonances in Kaon Photoproduction

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Abstract. We have studied two different formulations of spin 3/2 nucleon resonance by means of kaon photoproduction on the proton, \( \gamma p \rightarrow K^+ \Lambda \). The formulations of spin 3/2 nucleon resonances proposed by Adelseck (model A) and Pascalutsa (model B) have been used in deriving the scattering amplitudes. The amplitudes are calculated by means of the relevant Feynman diagrams for the process. All nucleon resonances with spin up to 3/2 listed by the Particle Data Group are included in the model. Both formulations are then compared with the experimental data, which include differential cross section and polarization observables, through \( \chi^2 \) minimization. It is found that the Pascalutsa’s formulation of the spin 3/2 leads to a better agreement with the experimental data.

1. Introduction
It has been realized that kaon photoproduction provides an essential tool for the studies of strange hadron, because as a probe photon is well understood and well under-controlled. Most analyses so far have focused on the process of \( p(\gamma, K^+)\Lambda \). This is due to the abundant available data on this channel. By studying this process, one can investigate the reaction mechanism, coupling constants, and hadronic form factors. Furthermore, one is also able to extend this study to hypernuclear sector, where in this case a conventional nucleus is used as a target.

Although kaon photoproduction has been investigated since the late of 50s, a comprehensive description of kaon photoproduction is still far from satisfactory. This happens because the strange particles can create an additional degree of freedom. Moreover, the high threshold energy of the \( p(\gamma, K^+)\Lambda \) process allows for a number of nucleon resonances to contribute in the reaction mechanism. Finally, the lack of knowledge on the corresponding coupling constants increase the complexity of the theoretical investigation.

The construction of new generation accelerators combined with the precise detectors has contributed to a great advancement in the experimental side of kaon photoproduction. In the last two decades, abundant data on kaon photoproduction have been published by a number of experimental collaborations, which deserved accurate analyses by means of theoretical of phenomenological models. In the present analysis, we have included almost 7500 data points in our fitting database, which is twice larger than those in our previous study [1].

In spite of the problems mentioned above, theoretical models of kaon photoproduction have been also continuously improved. Isobar model used in our calculation has also been progressively improved. The formulation of intermediate states such as that of meson with spin 0 and 1, nucleon and hyperon with spin 1/2, along with their resonances have been well
Nevertheless, the formulation of higher spin resonances is still plagued with the problem of consistency.

In the present work, we aim to study the effect of inclusion of spin 3/2 nucleon resonances with two different formulations. The new formulation proposed by Pascalutsa [3] will be compared with the previous one put forwarded by Adelseck [4]. A number of previous studies found that Pascalutsa’s formulation is more consistent. A more detailed explanation is given in Ref. [5].

This paper is organized as follows. In the next section, the ingredient of our isobar model and the formulation of propagator and vertex factors of spin 3/2 nucleon resonances used in our calculation will be presented. After that we will present the numerical result as well as the comparison between our calculation and the experimental data. Summary and conclusion will be presented in the last section.

2. Formalism

2.1. The isobar model

We investigate kaon photoproduction

$$\gamma(k) + p(p) \rightarrow K^+(q) + \Lambda(p_\Lambda),$$

by utilizing an isobar model, where the corresponding amplitude is obtained from Feynman diagrams shown in Fig. 1.

From the Feynman diagrams, we can obtain the scattering amplitude $\mathcal{M}$. In our calculation, we decompose the amplitude into six gauge and Lorentz invariant matrices $M_i$,

$$\mathcal{M} = \mathcal{M}_{\text{back.}} + \mathcal{M}_{\text{res}} = \bar{u}_\Lambda \left( \sum_{i=1}^{6} A_i M_i \right) u_p,$$

where $\bar{u}_\Lambda$ and $u_p$ are the spinors of $\Lambda$ and proton. The functions $A_i$ are Lorentz invariant scalar and merely functions of the Mandelstam variables $s$, $t$, and $u$. For photoproduction, Lorentz gauge implies that $k \cdot \epsilon = k^2 = 0$, and as a consequence only the matrices $M_1$ to $M_4$ contribute. The cross section and other polarization observables can be directly calculated from the amplitudes $A_i$.

The background terms consist of the Born terms along with the $K^{*+}(892)$ and $K1(1270)$ intermediate states, as well as two hyperon resonances. Meanwhile, the resonance terms consist of the nucleon resonances adopted from the PDG [6], as shown in Table 1, with the status of at least two-star rating and have spin up to 3/2. All intermediate states involved in the process are depicted in Fig. 1. Furthermore, the amplitudes of the background terms and spin 1/2 resonances are not presented in following section because they are not our main concern here and their formulations have been rather well established [2].
where we have defined

\[ \text{Adelseck (Model A). The propagator reads} \]

In this paper, we only present the Pascalutsa [3] formulation (Model B) of spin 3/2 propagator

2.2. Spin 3/2 nucleon resonances

In this paper, we only present the Pascalutsa [3] formulation (Model B) of spin 3/2 propagator and vertex factors. We refer the reader to Ref. [4] for a detailed description of the formulation by Adelseck (Model A). The propagator reads

\[
S_{\mu\nu}^{3/2} = \frac{\not{p} + \not{k} + m_{N^*}}{3(s - m_{N^*}^2 + im_{N^*} \Gamma_{N^*})} \times (3P_{\mu\nu} + \gamma^\rho \gamma^\sigma P_{\rho\sigma} P_{\mu\nu}),
\]

where \( P_{\mu\nu} = -g_{\mu\nu} + \frac{1}{s}(p + k)(\mu(p + k)_\nu) \). Here, \( m_{N^*} \) and \( \Gamma_{N^*} \) are the mass and width of the resonance, respectively. The electromagnetic vertex factor \( N^* p_\gamma \) for the nucleon resonance with parity \( \pm 1 \) is written as

\[
\Gamma_{N^* p_\gamma}^{\nu(\pm)} = -\frac{i}{m_{N^*}} \left[ g^{(1)} (\epsilon^{\nu} \not{k} - k^{\nu} \not{p}) \not{p} + g^{(2)} (k^{\nu} \not{p} \cdot \epsilon - \epsilon^{\nu} \not{p} \cdot k) + g^{(3)} \gamma^\nu (\not{k} - \not{p}) \right] \Gamma_{\pm},
\]

where we have defined

\[
g^{(1)} = -2i g^{a}_{N^* p_\gamma} + 3i g^{c}_{N^* p_\gamma} - g^{d}_{N^* p_\gamma},
\]

\[
g^{(2)} = -2i g^{a}_{N^* p_\gamma} + g^{b}_{N^* p_\gamma} + 2i g^{c}_{N^* p_\gamma} - 2g^{d}_{N^* p_\gamma},
\]

\[
g^{(3)} = -i g^{a}_{N^* p_\gamma} + i g^{b}_{N^* p_\gamma},
\]

\[
g^{(4)} = -i g^{a}_{N^* p_\gamma} + i g^{c}_{N^* p_\gamma},
\]

\[
g^{(5)} = -2i g^{a}_{N^* p_\gamma} + i g^{c}_{N^* p_\gamma} - g^{d}_{N^* p_\gamma},
\]

and \( \Gamma_+ = i\gamma_5 \) and \( \Gamma_- = 1 \) for parity. The hadronic \( KY N^* \) vertex factor reads

\[
\Gamma_{KY N^*}^{\mu(\pm)} = \frac{g_{KY N^*}}{m_{N^*}^2} \Gamma_{\pm} \left[ (p_{\lambda} \cdot q - \not{p}_{\lambda} \not{q}) \gamma^\mu + \not{p}_{\lambda} \gamma^\mu - \not{q} \gamma^\mu \right].
\]

Hence, the amplitude of spin 3/2 nucleon resonances with parity \( \pm 1 \) becomes

\[
M_{res}^{3/2} = \bar{u}_{\lambda} \left( \Gamma_{KY N^*}^{\mu(\pm)} \times P_{\mu\nu} \times \Gamma_{N^* p_\gamma}^{\nu(\pm)} \right) u_p,
\]

\[\text{Table 1. Status, mass and width of nucleon resonances used in our calculation [6].}\]

| Resonances | Status | Mass (MeV) | Width (MeV) |
|------------|--------|------------|-------------|
| N(1440)P_{11} | **** | 1430 ± 20 | 350 ± 100 |
| N(1520)D_{13} | **** | 1515 ± 5 | 115^+10_{-15} |
| N(1535)S_{11} | **** | 1535^+20_{-10} | 150 ± 25 |
| N(1650)S_{11} | **** | 1655^+10_{-12} | 140 ± 30 |
| N(1700)D_{13} | *** | 1700 ± 50 | 150^+50_{-100} |
| N(1710)P_{11} | *** | 1710 ± 30 | 100^+150_{-50} |
| N(1720)P_{13} | **** | 1720^+30_{-20} | 250^+150_{-100} |
| N(1875)D_{13} | *** | 1875^+55_{-20} | 200 ± 25 |
| N(1880)P_{11} | ** | 1870 ± 35 | 235 ± 65 |
| N(1895)S_{11} | ** | 1895 ± 15 | 90^+30_{-15} |
| N(1900)P_{13} | *** | 1900 | 250 |
| N(2120)D_{13} | ** | 2140 | 330 ± 45 |
By decomposing Eq. (11) into the gauge and Lorentz invariant matrices $M_i$, we obtain the following amplitudes,

$$A_1 = m_p \left\{ \frac{1}{2} (m_p + m_A) (c_A - m_A m_p - 3 s c_s) + m_A (2 s c_s - c_k) \right\} \pm m_N \left\{ \frac{1}{2} (m_p + m_A) \times (m_A - 3 m_p c_s - \frac{1}{2} s c_A m_p) + 2 c_A - 2 m_A^2 - 3 c_1 - \frac{1}{2} s c_A c_k \right\} \right\} G^1$$

$$+ \left\{ \frac{1}{2} (m_p + m_A) (m_p c_A + m_A (k^2 - s) + b_p c_A) \right\} \pm m_N \left\{ m_A b_p + \frac{1}{2} (m_p + m_A) \times (3 (c_1 - b_p c_s) + \frac{1}{2} s c_A (k^2 - s) + m_A m_p) \right\} \right\} G^2$$

$$+ 2 \left\{ b_p c_A - m c_A m_p - 3 s c_1 \right\} \pm m_N m_A \left\{ c_k c_s - 3 c_1 - k^2 \right\} G^3$$

$$A_2 = \frac{m_p}{t - m_k^2} \left\{ m_A k^2 \pm m_N \right\} G^1$$

$$+ \left\{ 3 (c_1 - b_p c_s) - m_A m_p k^2 \right\} G^2$$

$$A_3 = \frac{1}{t} m_p \left\{ 3 s - m_A m_p \right\} \pm m_N \left\{ 3 (m_p - m_A) \mp 2 s c_A m_p \right\} G^1$$

$$+ \left\{ m_p c_A + m_A (k^2 - s) \right\} \pm m_N \left\{ m_A m_p + \frac{1}{2} s c_A (k^2 - s) + 3 (c_1 + b_p (1 + \frac{1}{2} s c_A)) \right\} G^2$$

$$- 2 \left\{ m_A c_k \pm m_N \left\{ 3 c_1 + \frac{1}{2} s c_A c_k \right\} \right\} G^3$$

$$A_4 = -\frac{1}{t} m_p \left\{ 3 (c_A + s c_s) + m_A m_p \right\} \pm m_N \left\{ 3 (m_p + m_A) \mp 2 m_p c_A \right\} G^1$$

$$+ \left\{ m_p c_A + m_A (k^2 - s) \right\} \pm m_N \left\{ m_A m_p + \frac{1}{2} s c_A (k^2 - s) + 3 (c_1 - b_p c_s) \right\} G^2$$

$$- 2 \left\{ m_A c_k \pm m_N \left\{ 3 c_1 + \frac{1}{2} s c_A c_k \right\} \right\} G^3$$

where

$$G^{(i)} = \frac{g^{(i)} g_{KAN^*}}{3m_N^2 (s - m_N^2 + im_N^2 \Gamma_N^*)}.$$
As shown in Table 2, Model B gives a smaller value of $\chi^2/N$ compared to Model A, which indicates that Model B leads to a better agreement with the experimental data. The value of $\chi^2/N$ are as expected because Model B has more free parameters, which consist of coupling constants, masses and widths of the resonances. The two main coupling constants $g_{K\Lambda N}$ and $g_{K\Sigma N}$ tend to reach their limits. We provide some figures to investigate the models in details.

Table 2. Extracted parameters from fit to the experimental data for both models.

| Parameter          | Model A | Model B |
|--------------------|---------|---------|
| $g_{K\Lambda N}/\sqrt{4\pi}$ | -3.00   | -4.40   |
| $g_{K\Sigma N}/\sqrt{4\pi}$   | 0.90     | 0.90    |
| $\chi^2$            | 25548   | 20719   |
| $N_{\text{par}}$    | 54      | 66      |
| $\chi^2/N$          | 3.46    | 2.81    |

The total cross section obtained by using Model A and B are compared with the prediction of Kaon-Maid [14] and the experimental data from CLAS 2006 [8], as shown in Fig. 2. Both models seem to underpredict the data. This happens because the new data from CLAS 2010 [10] are tend to be smaller than those of CLAS 2006 in small fraction of the differential cross section. Note that the experimental data shown in this figure are not included in the fitting database.

The predicted differential cross section of the two models have a little discrepancy with the experimental data. However, both models show different strategies to fit the data. Model A tends to oscillate to fit the data due to the large error bars or uncertainties in the Crystal Ball data [7]. Meanwhile, Model B takes an average value of the cross section to arrive at the best fit. Nevertheless, Model B gives a better result to reproduce the data. As we can see, the comparison of differential cross sections is insufficient to determine the best model. Therefore, comparison in polarization observables becomes very decisive in our investigation, since polarization observables are very sensitive to the different ingredients in the scattering amplitudes. As a consequence, polarization observables provide a robust constraint for phenomenological approaches.

Comparison of single polarization observables which consists of recoil polarization $P$, target asymmetry $T$, and photon asymmetry $\Sigma$, are depicted in Fig. 3. As predicted, both models
deviate significantly from the target and photon asymmetries data at the forward angle. Although the experimental data from GRAAL collaboration [13] are only available up to 1.9 GeV, it is still possible to pin down the best model in this case. As shown in Fig. 3, Model B displays the best agreement with data rather than Model A. Meanwhile, both models do not have a substantial problem to fit the recoil polarization $P$. Therefore, in spite of the abundant experimental database available for recoil polarization $P$, here it is difficult to determine the
best model, since both models predict the observables perfectly.

For completeness, we also present the beam-recoil double polarizations $O_x$ and $O_z$ at forward angle for comparing the models, as depicted in Fig. 3. Interestingly, Model B can also nicely reproduce the experimental data and has different characteristic compared to model A. Although other double polarization observables are available in kaon photoproduction, we will not present them in this paper, for the sake of brevity.

We also note that both models satisfy the SU(3)-symmetry constraints for the two leading coupling constants. Hence, Model B seems to meet all requirements for a good phenomenological model of $K^+\Lambda$ photoproduction. Nonetheless, the inclusion of higher spin nucleon resonances could also lead to better agreement to the experimental data.

4. Summary and conclusion

We have analyzed kaon photoproduction process $\gamma + p \rightarrow K + \Lambda$ by using an isobar model that contains different formulations of spin 3/2 propagators and vertices. For this purpose we have derived the corresponding amplitudes and decomposed them into the standard gauge and Lorentz invariant matrices. All available nucleon resonances with spins up to 3/2 listed by PDG and all latest $K^+\Lambda$ photoproduction data consisting of around 7500 data points are taken into account in the model. It is found that Model B, which utilizes the Pascalutsa’s prescription, yields the best agreement with the experimental data.

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