Reliability Evaluation for Manufacturing System Based on Dynamic Adaptive Fuzzy Reasoning Petri Net

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ABSTRACT
Due to failure, partial failure, or maintenance, the capacity of each machine is multi-state. Therefore, the limited relationship between the capacity of each machine and the input raw materials has to be considered. Additionally, in order to utilize the machine more effectively, the capacity of the buffers cannot be ignored, too. In this paper, a dynamic adaptive fuzzy reasoning Petri net is proposed to evaluate reliability of a manufacturing system with multiple production lines. Firstly, the model of manufacturing system is conducted, and from the perspective of demand, the minimum capacity vector and loading vector of each machine are determined. Secondly, knowledge representation and rules are formulated to establish weighted fuzzy petri nets. And the weighted fuzzy Petri net is adaptive based on the real-time level of buffers, the minimum capacity vector and loading vector. Moreover, the efficiency of product production can be improved while ensuring system reliability by adjusting the buffer level. Finally, a numerical experiment is used to demonstrate the application of our method.

INDEX TERMS Reliability evaluation, manufacturing system, fuzzy reasoning petri net, multiple production lines.

I. INTRODUCTION
With the development of the manufacturing industry and the increasing customer’s demand, manufacturers are facing the challenge of improving reliability of products. The development of the industrial internet of things (IoT) has endowed modern manufacturing systems with the sensible ability and real-time production data which can be collected, and provides more opportunities for improving product reliability. For example, by opening up the data interface between the equipment and the enterprise resource planning system, real-time updates of production tasks and production schedules can be realized, and real-time monitoring of the production status and product production conditions of the manufacturing system can be achieved, thereby effectively improving the resource utilization of the manufacturing system. However, manufacturing system is a combination of people, machines and equipment, material flow and information flow [1], and many manufacturing systems are multiple production lines, such manufacturing systems produce products with higher efficiency and flexibility. Therefore, the manufacturing system is a complex dynamic process. Due to the non-ideal state of its constituent factors, which may lead to various errors inevitably in the manufacturing process. For example, due to failure, partial failure, or maintenance, the capacity of each machine is stochastic. The machine failure leads to defective product, which might be reworked or scrapped [2], [3]. This is because many defective products are still have substantial value, so it is meaningful to study the reliability of a multi-production line with reworking [4]. In some applications, the manufacturing system is divided into general manufacturing paths and reworking paths to meet production demands [4]–[6].

Many methodologies such as reliability block diagram, failure mode effect analysis (FMEA) and event tree analysis are used in the reliability analysis [7]–[9]. However, these traditional reliability analysis methods have many limitations. Their commonality is based on expert knowledge,
and a quantitative or qualitative reliability analysis can be obtained, but due to the lack of effective mathematical expressions, the automatic update of reliability evaluation cannot be completed. In the literature, fuzzy reasoning is an effective method to solve the above problem [10]. Consequently, fuzzy reasoning Petri net (FRPN) is introduced in this paper to address the above problem.

In this paper, the object we are focusing on is a manufacturing system of multiple production line with reworking path. In the manufacturing system, each machine has stochastic capacity levels with performance ranging from complete malfunction to a perfectly working level. Moreover, the input flow is defined as the input amount that each machine processes per cycle time, and the vector of all input flows is named as the machine’s loading. In addition, the minimum processing capacity of each machine to meet the loading is defined as the minimum capacity vector based on the minimal path theory [11], [12]. Further, due to the randomness of the machine’s capacity and the uncertainty of the loading size, the level of the capacity of machine and the level of loading are ambiguous. Consequently, system reliability can be inferred by the probability that the machine satisfies the loading. Moreover, the capacity of the buffer should be considered in order to make the system more effectively. Hence, a dynamic adaptive fuzzy reasoning Petri net (DAFRPN) is proposed to evaluate reliability of a manufacturing system with multiple production lines. The contributions of this paper can be summarized as follow.

(1) On the premise of comparing the machine’s capacity and the level of loading, the impact of the buffer capacity on the reliability of the manufacturing system is considered, and combining knowledge representation and rules, the DAFRPN is formulated.

(2) The DAFRPN infers reliability level of the manufacturing system at any time, and changes the buffer capacity dynamically, so that the resource allocation of the manufacturing system is more reasonable, and the efficiency and reliability of the manufacturing system are improved.

The outline of this paper is organized as follows. In section 2, the related works about reliability evaluation of manufacturing system and fuzzy Petri net are presented. Section 3 includes details of the proposed method. Section 4 contains details about a numerical experiments to illustrate and verify the proposed method. Finally, the conclusions and discussion are drawn in Section 5.

II. RELATED WORKS

PNs have an inherent quality in representing logic in intuitive and visual way, and fuzzy PNs (FPNs) take all the advantages of PNs [13]. FPN are also used for fuzzy knowledge representation and reasoning, many results prove that FPN is suitable to represent and reason misty logic implication relations [14], [15]. There are some differences between PNs and FPNs since the features of fuzzy rule-based systems, which is shown as follow [16]–[18]:

(1) In FPNs, due to a token is associated with a truth value between 0 and 1, the number of tokens in the place cannot be greater than one.

(2) Owing to there is no “resource” concept in FPNs, FPNs are always conflict-free networks, and propositions may be shared by different rules simultaneously.

(3) After a transition is fired, the token will not be removed from the input position of the transition since the evaluation of the rule only means the true propagation of the proposition.

Although FPNs have been proved to be one of the most powerful knowledge representation techniques, however, traditional FPNs still have many shortcomings. First, the parameters in FPNs, such as weight, threshold, and certainty factor do not accurately represent increasingly complex knowledge-based expert systems and do not capture the dynamic nature of fuzzy knowledge [19]–[23]. Second, the fuzzy rules of most existing knowledge inference frameworks are static and cannot be adjusted dynamically according to variations of antecedent propositions [24]–[26]. In view of the complexity and the dynamic nature of knowledge-based systems, therefore, suitable models for knowledge-based systems should be adaptable [21]. Fuzzy reasoning Petri nets (FRPNs) have been used to represent fuzzy production rules (FPRs) and formulate fuzzy rule-based reasoning automatically [17]. The main features of a FRPN are its graphical representations and dynamic processing abilities to model knowledge-based systems [27].

In the past few decades, FPNs have been widely used to solve various engineering problems, and they are also very applicable in the field of manufacturing systems. Wu et al. [28] established a model combining fault tree analysis (FTA) with FRPNs to conduct reliability analysis of a solar array mechanical system, which can be used to find the most vital root causes and put forward propositions to improve reliability of the solar array. The proposed FRPN model uses the fuzzy reasoning algorithm, so it considered the uncertainty of relationship between the components in the system, which was suitable for the reliability analysis and apportionment of the mechanical system. Wu et al. [29] also proposed a reliability apportionment approach for spacecraft solar array using FRPNs and fuzzy comprehensive evaluation. Wu and Hsieh [30] explored a real-time FPN approach to diagnose progressive faults in discrete manufacturing systems. Hong et al. [31] proposed a reliability-based and cost-oriented product optimization method integrating FRPN, interval expert evaluation and cultural-based dynamic multi-objective particle swarm optimization (DMOPSO) using crowding distance sorting.

Lin and Chang [6] focused on reliability evaluation of a manufacturing system with multiple production lines based on the network-analysis perspective. Wu et al. [32] proposed a method for risk evaluation in assembly process based on the discrete-time SIRS epidemic model and information entropy. Both of the above papers considered the situation of reworking in a manufacturing system with multiple production line.
Moreover, they have established a graphical representation of the corresponding manufacturing system and assembly process. However, they ignored the fuzzy logic of the reliability of the manufacturing system due to the uncertain machining capabilities. In addition, the impact of buffer capacity on the reliability of the manufacturing system is ignored by them as well. Most importantly, the reliability of manufacturing system is dynamic due to the uncertainty of machine capabilities and the continuous changes in material flow and information flow. Therefore, this paper presents the DAFRPN to present and infer reliability of manufacturing system considering complicated coupling relationship of real-time machine capabilities, loadings and buffer levels.

III. PROPOSED METHOD

The research of Lin et al. showed that the reliability of the manufacturing system can be characterized by the strength of interference between the capabilities of the machine and the input material [6]. However, due to the rapid development of the IoT, real-time updates of the production capacity and material input of the manufacturing system have been realized. Therefore, it has become possible to realize the automatic evaluation of the reliability of the manufacturing system. In this paper, the proposed method will implemented in two parts. At first part, the manufacturing system is modeled and the capability vector and loading vector of the manufacturing system is obtained, all of which are detailed in Section 3.1 and 3.2. At second part, the capability and loading of each machine is fuzzed and the DAFRPN is defined and used to infer reliability of manufacturing system, all of which are detailed in Section 3.3 and 3.4.

Moreover, the proposed method require a number of iterations to achieve the optimal system reliability. We will get the system reliability for each calculation. When it is not optimal, we reset the buffer capacity allocation, the iteration is terminated until the system reliability reaches the optimal level. Moreover, the procedure of the proposed is shown in Figure 1 in APPENDIX.

A. SYSTEM DESCRIPTION AND ASSUMPTIONS

A flow-shop manufacturing system, that is a high-volume system with standardized machines and processes to produce the same or highly similar products is considered in this paper [33]. For such a manufacturing system, the input flow is defined as the input amount that each machine processes per cycle time. Obviously, the amount of input material for each machine must be less than the capacity of the machine. Moreover, assuming that the work-in-process (WIP) may have defects, the manufacturing system so as to be reworked. This makes the manufacturing system more complicated.

Generally, products are produced by multiple production lines, and this paper takes two production lines as an example to explain the method. The proposed method can be easily expanded and applied to the manufacturing system with more than two production lines. Hence, a serial discrete manufacturing system with two production lines is shown in Figure 2, and the assumptions are made as follow.

1. Machine \( M_i, i = 1, 2, \ldots, n \), has a variable processing capacity \( C_i \). In addition, each machine has a constant cycle time \( \tau_i \), and the cycle time of each machine in the manufacturing system is equal. That is, the system is synchronous lines.
2. Buffer \( B_i, i = 1, 2, \ldots, n - 1 \), has a variable buffer capacity \( F_i \), and \( \sum_{i=1}^{n-1} F_i \) is a finite value.
3. The dashed boxes represent the buffer while characterizing the inspection station in this paper. The inspection station checks whether the WIP can enter the next process or should be scrapped. If the defective WIP has been inspected by the inspection station, which means the WIP can be reworked from the inspection station to the previous inspection station. Moreover, the defective WIP is reworked at most one time by the same machine. And each inspection station is perfectly reliable.
4. At the beginning of a discrete time slot, \( M_i \) is starved if its upstream buffer \( B_{i-1} \) is empty and it is assumed that the first machine is never starved. Likewise, \( M_i \) is blocked if its downstream buffer \( B_i \) is full and the last machine is never blocked in this paper.

B. DETERMINATION OF MINIMUM CAPACITY VECTOR AND LOADING VECTOR

As shown in Figure 2, suppose each production line of the manufacturing system consists of 5 machines and 4 buffers,
and the repair path is from buffer 3 to buffer 1, thereby the manufacturing system can be decomposed, as in Figure 3. In the decomposition model, the manufacturing system consists of general line and reworking line, which represents manufacturing system of normal WIP and repaired WIP, respectively.

In order to meet the production requirements of the manufacturing system in each cycle time, each production line must input a certain amount of raw materials in advance. The demand for products produced by the manufacturing system is defined as $d$, and the product demand of each production line is $d_1$ and $d_2$ respectively, thereby $d = d_1 + d_2$. In the general line, the capacity of machine can be described as

$$O_j(G) = I_j p_j^n,$$  \hspace{1cm} (1)

where the success rate of each machine is $p$ and $n_j$ is the number of machines of production line $j$. In addition, the capacity of machine in reworking line is defined as

$$O_j(R) = I_j p_j^{n_j+k} q,$$  \hspace{1cm} (2)

where, $q = 1 - p$, $i = 1, 2, \ldots, n$ and $k$ is the number of inspection station through the reworking path. Hence, the amount of output products of line $j$ is written as

$$O_j = I_j p_j^{n_j} + I_j p_j^{n_j+k} q.$$  \hspace{1cm} (3)

Therefore, the maximum output for each line $j$ is

$$O_{\text{max}} = \min(M_j (p_j^{n_j+1} + \alpha_j p_j^{n_j+k+1})).$$  \hspace{1cm} (4)

where $r_{ij}$ is quantity of machines behind machine $M_{ij}$ in production line $j$. Meanwhile, $\alpha_{ij} = \begin{cases} 1, & M_{ij} \notin \text{reworking line} \\ 0, & M_{ij} \in \text{reworking line} \end{cases}$.

For a given product output demand $d$, the output demands of each production line are defined as $d_1$ and $d_2$, respectively, thereby $d = d_1 + d_2$. Moreover, for each demand pair $(d_1, d_2)$ has to meet $d_1 \leq O_{1, \text{max}}$ and $d_2 \leq O_{2, \text{max}}$. The input of each production line is

$$I_j = \frac{d_j}{p_j^n + p_j^{n_j+k} q}.$$  \hspace{1cm} (5)

And the input quantity of each machine in general line is

$$Q_{ij}(G) = I_j p_j^{n_j-1},$$  \hspace{1cm} (6)

where $s_{ij}$ is the sequence of the machine. The input quantity of each machine in reworking line is

$$Q_{ij}(R) = I_j p_j^{n_j+k-1} q,$$  \hspace{1cm} (7)

therefore, the total input loading is

$$l_{ij} = I_j p_j^{n_j-1} + \sigma_{ij} I_j p_j^{n_j+k-1} q,$$  \hspace{1cm} (8)

where, $\sigma_{ij} = \begin{cases} 1, & M_{ij} \in \text{reworking line} \\ 0, & M_{ij} \notin \text{reworking line} \end{cases}$. For each machine, the possible capacity is random and meets

$$C_{ij,c} \geq I_j c \geq C_{ij,c-1},$$  \hspace{1cm} (9)

where $c$ represents the sequences of capacity set of each machine. At this point, the smallest possible capacity of
each machine and the input materials of each machine are obtained, which are defined as the minimum capacity vector \( C \) and the loading vector \( L \), respectively.

And \( C = \{ C_{11,c}, C_{21,c}, \ldots, C_{nl,c}, C_{12,c}, C_{22,c}, \ldots, C_{n2,c} \} \), \( L = \{ l_{11}, l_{21}, \ldots, l_{nl1}, l_{12}, l_{22}, \ldots, l_{nl2} \} \). Moreover, the capacity vector of buffers is defined as \( B = \{ b_{11}, b_{21}, \ldots, b_{n(b-1),1}, b_{12}, b_{22}, \ldots, b_{n(b-1),2} \} \), for the last machine on the production line is never blocked, thereby \( C_{nj,c} = b_{ij} \) is defined and \( C_{ij,c} - l_{ij} \leq b_{ij}(0 \leq i \leq n-1) \) has to be met.

C. KNOWLEDGE REPRESENTATION OF MANUFACTURING SYSTEM

Under the premise of meeting the production demand, to improve the production efficiency of the product production line, the buffer capacity must be optimized through quantitative allocation. However, due to failure, partial failure, or maintenance, the capacity of each machine is multi-state (i.e., stochastic). Moreover, the capacity of the buffer is uncertain in this paper, coupled with the capacity of machine, the complex relationship between the loading of machine and the capacity of buffer. It is difficult to decide a precise level of all machines, loads and buffers.

FPRs are a good tool for processing uncertain, imprecise, ambiguous real-world knowledge, and the uncertainty of the fulfillment of the conditions in rules [34]. Fuzzy knowledge representations for reliability of manufacturing system with multiple production lines based on weighted FPRs (WFPRs) is proposed in this paper, and the level of reliability of the manufacturing system is derived from the capabilities of machine and machines’ loading.

Let \( R_t \) be a set of WFPRs for reliability level reasoning. A specific WFPR is defined in this paper based on the definition of a general WFPR [35], and it is described as follows:

\[
R_t: \text{IF } c_{ij}(t) = FM_{ij} \text{ AND } l_{ij}(t) = FL_{ij} \text{ THEN } r_{ij}(t) = FR(\mu_{\text{FL}}(t) = \mu_x, w_{m_{ij}}, w_{l_{ij}}), \quad (10)
\]

where

1. \( c_{ij}(t) \) is the level of \( M_{ij} \) at time point \( t \).
2. \( l_{ij}(t) \) is the level of \( L_{ij} \) at time point \( t \).
3. \( r_{ij}(t) \) is the state of reliability, i.e., \( FR = \{ RL, RH \} \) at time point \( t \). RL means the state of reliability is low and RH represents the state of reliability is high.
4. \( FM_{ij} \) and \( FL_{ij} \) are the fuzzy sets of machine capacity levels and loading levels, respectively, which are described in 3 linguistic variables \{Low, Medium, High\} in this paper.
5. The parameter \( \mu_x \) are defined in the universe of discourse \([0,1]\), which are named as certainty factor (CF) reflecting the degree of certainty of event occurrence.
6. \( w_{m_{ij}} \) and \( w_{l_{ij}} \) are the set of weights, which are defined in the universe of discourse \([0,1]\). The weight reflects the relative importance of the proposition, and the sum of the proposition weights equals to 1.

In the antecedent propositions of the WFPRs, six abbreviations are written as \{ML, MM, MH, LL, LM, LH\} in this paper. For example, ML means the level of machine is low, LM means the level of load is medium. Each linguistic variable has a membership function. The triangle membership function is most commonly because of its computational efficiency and simplicity [36], [37]. Hence, the triangle membership function is adopt in this paper according to [27], [38], the membership function of machine capacity and load is shown in Figure 4.

At each decision time point, the real level of machine capacity and load is sampled and the degree of membership is obtained. The objective is that the reliability as large as possible, while making the buffer allocation as reasonable as possible. Moreover, three input fuzzy sets are associated with three sets of fuzzy quantities, thereby nine rules are developed and presented in Table 1.

Furthermore, the Mandani scheme [39], [40] is used to infer the state of reliability. Therefore, a certainty value of reliability is obtained based on the PN reasoning in the next section. In traditional WFPNs, the CF indicates the degree of certainty of a rule and is usually a constant determined by experts.

In this paper, the CF is dynamically change based on the limitation relationship of minimum capacity, the loading and the buffer level, which is defined as follow

\[
cf_{ij}(t) = \frac{C_{ij,c} - l_{ij}(t)}{b_{ij}(t)}, \quad (11)
\]
FIGURE 5. Membership function: (a) The limitation relationship of minimum capacity, load and buffer capacity of a machine; (b) Certainty value of low reliability decision; (c) Certainty value of high reliability decision.

TABLE 1. The parameters of fuzzy rules for reasoning.

| No. | $c_i(t)$ | $w_{a_i}$ | $l_i(t)$ | $w_{a_i}$ | $r_i(t)$ | $cf_i(t)$ |
|-----|----------|-----------|----------|-----------|----------|-----------|
| 1   | Low      | 0.8       | Low      | 0.2       | RL       | $\mu_l$   |
| 2   | Low      | 0.6       | Medium   | 0.4       | RL       | $\mu_l$   |
| 3   | Low      | 0.5       | High     | 0.5       | RL       | $\mu_l$   |
| 4   | Medium   | 0.4       | Low      | 0.6       | RH       | $\mu_h$   |
| 5   | Medium   | 0.5       | Medium   | 0.5       | RH       | $\mu_h$   |
| 6   | Medium   | 0.3       | High     | 0.7       | RL       | $\mu_l$   |
| 7   | High     | 0.5       | Low      | 0.5       | RH       | $\mu_h$   |
| 8   | High     | 0.6       | Medium   | 0.4       | RH       | $\mu_h$   |
| 9   | High     | 0.2       | High     | 0.8       | RH       | $\mu_h$   |

Moreover, the rules are defined to infer the CFs of WFPRs as follows:

$$R_y : \text{IF } cf_i(t) \text{ is } CR_{ij} \text{ THEN } \mu_l \text{ is } CVL \text{ AND } \mu_h \text{ is } CVH,$$

where, $CR_{ij}$ is a fuzzy set, $CVL$ and $CVH$ are the certain values of the level of low and high reliability decision respectively. Moreover, the $CR_{ij}$ is represented with a fuzzy term set \{Low, Medium, High\}, and the symmetrical triangle membership function \{0, 0.5, 1\} is used for $CR_{ij}$ shown in Figure 5a. In addition, $CVL$ and $CVH$ have a fuzzy term set \{Small, Middle, Big\} with the symmetrical triangle membership function \{0, 0.5, 1\}, which is shown in Figure 5b,c. If the limitation relationship of minimum capacity, load and buffer capacity of a machine is higher in a decision cycle, a bigger CF for a high reliability state and a smaller CF for a low reliability state will be assigned to the WFPRs. Hence, the knowledge of rules is represented in Table 2.

TABLE 2. The fuzzy rules for certainty factor reasoning.

| No. | $c_i(t)$ | $\mu_l$ | $\mu_h$ |
|-----|----------|---------|---------|
| 1   | Low      | Big     | Small   |
| 2   | Medium   | Middle  | Middle  |
| 3   | High     | Small   | Big     |

D. DYNAMIC ADAPTIVE FUZZY REASONING PETRI NET FOR RELIABILITY EVALUATION

WFPRNs are an important modeling tool for knowledge representation and reasoning, which have been extensively used in a lot of fields. In this paper, the DAFRPN is used to evaluate the reliability of manufacturing system. And it’s defined as an eight-tuple:

$$\text{DAFRPN} = \{P, T, I, O, E, W, \lambda, \alpha, \beta\},$$

where

1. $P = \{p_1, p_2, \ldots, p_m\}$ indicates a finite nonempty set of places, $m$ is the number of propositions. Further, the places include a set of starting places $PS$ and a set of terminating places $PT$.
2. $T = \{t_1, t_2, \ldots, t_h\}$ is a finite nonempty set of transitions, which represent a set of fuzzy inference rules, and $h$ is the number of transitions.
3. $I : PS \rightarrow T$, represents an input function and means the directed arcs from places to transitions.
4. $O : T \rightarrow PT$, represents an output function and means the directed arcs from transitions to places.
5. $E = \{e_1, e_2, \ldots, e_m\}$ is a finite set of proposition, $P \cap T \cap E = \emptyset, |P| = |E|$, defining there is a one-to-one correspondence between the place and the proposition.
6. $W : PS \times T \rightarrow [0, 1]$, represents an input function whose elements are weights of $PS$, and the sum of $PS$ weights of a transition is equal to 1.
7. $\lambda : T \times PT \rightarrow [0, 1]$, represents an output function whose elements are CFs of transitions.
8. $\alpha : PS \rightarrow [0, 1]$, represents a mapping from a set of places to a collection of real values in $[0,1]$. $\alpha(P_i)$ is the fuzzy value of input place, defining the truth degree of places.
9. $\beta : P \rightarrow E$, represents a bidirectional mapping between a set of places to a set of propositions.

Further, according to the above knowledge representation and reasoning, a DAFRPN for reliability evaluation can be established, which is shown in Figure 6 in APPENDIX.

In the DAFRPN, each input place has a token, and the enabling criterion is and the firing criterion are defined as follow:

$$\forall t_h \in T, t_h \text{ is enabled for } p_i \in I(t_h), \quad \alpha(P_i) > 0 \quad (14)$$
When \( t_0 \) is fired, the tokens in input places are copied, and a token with certain degree of truth is deposited into each of the output places.

If \(|*P|=1\), which represents the output place has only one input transition. Thereby the fuzzy value of output place is described as:

\[
\alpha(P_k) = \lambda_j \times \sum [\alpha(P_i) \times w_j], \quad k = 7, 8.
\]

where \( j = 1, 2, \ldots, 9 \), and represents the sequence of transitions. If \(|*P|>1\), which represents the output place has more than one input transition, and the fuzzy value of output place is defined as:

\[
\alpha(P_k) = \max(\lambda_j \times \sum [\alpha(P_i) \times w_j]), \quad k = 7, 8.
\]

**IV. NUMERICAL EXPERIMENTS**

**A. CASE DESCRIPTION**

An oil pump manufacturing system is utilized to demonstrate the proposed algorithms. In this case, two identical production lines are installed, and each production line has one opportunity to rework. Moreover, each production line with 5 machines and 4 buffers, and the reworking paths from buffer 4 to buffer 2 and buffer 9 to buffer 7, respectively.

Therefore, the oil pump manufacturing system is shown in Figure 7.

**B. CALCULATION OF MINIMUM CAPACITY VECTOR AND LOAD VECTOR**

It is assumed that the success rate of each machine \( p=0.95 \) and the machine data is given in Table 3. According to the production plan, 360 pieces of oil pump shall be produced, and every 30 pieces of pump shall be packaged for transportation.

Moreover, for reworking path 1 and 2, the sequence of machine is shown as in Table 4.

- Step 1: Find the maximum output for each path. Through calculation, the maximum output of each line is:
  
  \[
  O_{1,\text{max}} = \min \{400 \times (0.95^{4+1} + 0.95^{4+1} \times 0.05), \ 600 \times (0.95^{3+1} + 0.95^{3+1} \times 0.05), \ 300 \times (0.95^{2+1} + 0), \ 300 \times (0.95^{1+1} + 0), \ 300 \times (0.95^{0+1} + 0) \}\n  \]
  
  \[
  = \min \{324.214, \ 511.917, \ 257.213, \ 315.875, \ 285.0 \} = 257.213
  \]

  \[
  O_{2,\text{max}} = \min \{226.950, \ 255.959, \ 231.492, \ 270.750, \ 342.0 \} = 226.950
  \]

- Step 2: Find the demand assignment \((d_1, d_2)\) satisfying \(d_1 + d_2 = 360, d_1 \leq 257.213, d_2 \leq 226.950\). Hence, there are three feasible combinations, which are \(D_1 = (150, 210), D_2 = (180, 180)\) and \(D_3 = (210, 150)\), respectively.

- Step 3: For demand pair \((d_1, d_2)\), do the following steps. For demand pair \(D_1 = (150, 210)\).

  - 3.1a. The input materials of each production line is
    
    \[
    I_1 = 150 \big/ (0.95^5 + 0.95^0 \times 0.05) = 185.063 \text{ and } I_2 = 210 \big/ (0.95^5 + 0.95^0 \times 0.05) = 259.088.
    \]

  - 3.2a. For demand pair \((d_1, d_2)\), the input flow of each machine is shown in Table 5.

  - 3.3a. Transform input flows from both general path and reworking path into each machine’s loading vector
    
    \[
    L_1 = \{185.063, 175.810, 174.953, 166.205, 157.895, 259.088, 246.134, 244.933, 232.687, 221.035\}.
    \]
For each machine, find the smallest possible capacity $C_{ij,c}$ satisfying $C_{ij,c} \geq l_{ij} \geq C_{ij,c-1}$. And the minimal capacity vector of demand pair $(d_1, d_2)$ is $C_1 = [200, 200, 200, 200, 200, 200, 200, 200, 200, 200]$. Likewise, for demand pair $D_2 = (180, 180)$, the input materials of production line 1 is $I_1 = 222.076$ and for production line 2 is $I_2 = 222.076$; for demand pair $D_3 = (210, 150)$, the input materials of production line 1 is $I_1 = 259.088$ and for production line 2 is $I_2 = 185.063$; Additionally, the input flow of each machine for demand pair $D_2$ and $D_3$ are shown in Table 6 and 7, respectively.

According to Table 3, the minimum capacity vectors for three demand pairs are shown in Table 8.

### C. RELIABILITY EVALUATION OF THE OIL PUMP MANUFACTURING SYSTEM

Suppose that in the two production lines, the total limited capacity of buffers 1 to 4 and buffers 5 to 9 is 240, and the buffer capacity of each machine in each production line is random. Moreover, the initial value of the buffer capacity of each machine in different demand pairs is as shown in Table 9.
TABLE 4. The sequence of machine.

| Line (j) | Machine       | Following machines | \( r_j \) | \( \sigma_0 \) | \( \sigma_y \) |
|----------|---------------|--------------------|----------|------------|------------|
| 1        | \( M_1 \)     | \( M_2, M_3, M_4, M_5 \) | 4        | 1          | 1          |
| 2        | \( M_2 \)     | \( M_3, M_4, M_5 \)   | 3        | 2          | 1          |
|          | \( M_3 \)     | \( M_4, M_5 \)        | 2        | 3          | 0          |
|          | \( M_4 \)     | \( M_3 \)             | 1        | 4          | 0          |
|          | \( M_5 \)     | \( - \)               | 0        | 5          | 0          |

TABLE 5. The input flow of each machine for \( D_1 \).

| \( O_i(G) \) | \( O_i(R) \) | \( l_i \) | \( Q_i(G) \) | \( Q_i(R) \) | \( l_i \) |
|--------------|--------------|----------|--------------|--------------|----------|
| 185.063      | 0            | 185.063  | 259.088      | 0            | 259.088  |
| 175.810      | 0            | 175.810  | 246.134      | 0            | 246.134  |
| 167.019      | 7.933        | 174.953  | 233.827      | 11.106       | 244.933  |
| 158.668      | 7.537        | 166.205  | 222.135      | 10.552       | 232.687  |
| 150.735      | 7.160        | 157.895  | 211.029      | 10.024       | 221.035  |

TABLE 6. The input flow of each machine for \( D_2 \).

| \( O_i(G) \) | \( O_i(R) \) | \( l_i \) | \( Q_i(G) \) | \( Q_i(R) \) | \( l_i \) |
|--------------|--------------|----------|--------------|--------------|----------|
| 222.076      | 0            | 222.076  | 222.076      | 0            | 222.076  |
| 210.972      | 0            | 210.972  | 209.944      | 0            | 209.944  |
| 200.424      | 9.520        | 209.447  | 190.403      | 9.044        | 190.403  |
| 190.883      | 8.592        | 189.475  | 189.475      | 10.024       | 189.475  |

TABLE 7. The input flow of each machine for \( D_3 \).

| \( O_i(G) \) | \( O_i(R) \) | \( l_i \) | \( Q_i(G) \) | \( Q_i(R) \) | \( l_i \) |
|--------------|--------------|----------|--------------|--------------|----------|
| 259.088      | 0            | 259.088  | 259.088      | 0            | 259.088  |
| 246.134      | 0            | 246.134  | 244.933      | 0            | 244.933  |
| 233.827      | 11.106       | 232.687  | 222.135      | 10.552       | 222.135  |
| 211.029      | 10.024       | 221.035  | 221.035      | 10.024       | 221.035  |

Without considering the changes in the production requirements of the 360 pieces of oil pump, normalization is used to process the data in Table 10, and the results are shown in Table 11.

Based on Table 11, the comparison of the reliability states of each machines with three different demands is shown in Figure 8.

In addition, the final reliability of each machine is the value of the state of RH. Further, the reliability of manufacturing system is determined by the smallest value of the state of RH for each machine in the demand pair, which is defined as follows:

\[ R = \min\{\alpha(P_{ij})\}, \quad j = 1, 2, \ldots, 30, \]  \hspace{1cm} (17)

therefore, the reliability of the manufacturing system is 0.007. Moreover, according to Table 11 and Figure 7, in the demand \( D_1 \), the buffer capacity of machines 6 and 8 needs to be adjusted firstly, the same as machines 4, 8 and 9 in the demand \( D_2 \). This is because the value of reliability state of RL of these machines is more higher than that of RH. And the reliability of each machine after adjustment is shown in Figure 9.

Hence, the reliability of the manufacturing system is changed as 0.512. Likewise, many of these machines have similar values for reliability state of RL and RH, such as machine 7 in demand \( D_1 \). Moreover, there are also some machines whose values of reliability state of RH value is not optimal, and there is room for further optimization. Eventually, the reliability of the manufacturing system becomes higher and higher after continuous adjustment. Moreover, the reliability of the manufacturing system after each adjustment is shown in Figure 10. According to Figure 10, the reliability of the oil pump manufacturing system has reached its optimum after 24 evolutions, which is 1. Since the capacity of buffers is limited, when the reliability of all demands is optimal, that is, when the reliability of all machines is 1, then the solution with the smallest buffer capacity is better for production efficiency and should be adopted. In this paper,
table 10. the initial reliability state of each machine.

|        | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | M9 | M10 |
|--------|----|----|----|----|----|----|----|----|----|-----|
| D1     | RL | 0.303 | 0.479 | 0.223 | 0.123 | 0 | 0.592 | 0.474 | 0.474 | 0.488 | 0 |
|        | RH | 0.782 | 0.677 | 0.503 | 0.638 | 0.769 | 0.062 | 0.588 | 0.317 | 0.575 | 0.597 |
| D2     | RL | 0.161 | 0.208 | 0.151 | 0.351 | 0 | 0.239 | 0.267 | 0.605 | 0.442 | 0 |
|        | RH | 0.451 | 0.497 | 0.385 | 0.091 | 0.789 | 0.583 | 0.339 | 0.004 | 0.070 | 0.702 |
| D3     | RL | 0.327 | 0.258 | 0.536 | 0.458 | 0 | 0.201 | 0.128 | 0.457 | 0.215 | 0 |
|        | RH | 0.343 | 0.672 | 0.631 | 0.496 | 0.748 | 0.316 | 0.458 | 0.595 | 0.481 | 0.974 |

Table 11. the data in Table 10 after processing.

|        | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | M9 | M10 |
|--------|----|----|----|----|----|----|----|----|----|-----|
| D1     | RL | 0.279 | 0.414 | 0.303 | 0.162 | 0 | 0.905 | 0.446 | 0.599 | 0.459 | 0 |
|        | RH | 0.721 | 0.586 | 0.697 | 0.838 | 1 | 0.095 | 0.554 | 0.401 | 0.541 | 1 |
| D2     | RL | 0.263 | 0.295 | 0.282 | 0.794 | 0 | 0.291 | 0.441 | 0.993 | 0.863 | 0 |
|        | RH | 0.737 | 0.705 | 0.718 | 0.206 | 1 | 0.709 | 0.559 | 0.007 | 0.137 | 1 |
| D3     | RL | 0.488 | 0.277 | 0.459 | 0.480 | 0 | 0.389 | 0.218 | 0.434 | 0.309 | 0 |
|        | RH | 0.512 | 0.723 | 0.541 | 0.520 | 1 | 0.611 | 0.782 | 0.566 | 0.691 | 1 |

Figure 8. The comparison of the reliability states of each machines.

Figure 9. The comparison of the reliability of each machines.

Figure 10. The reliability of the oil pump manufacturing system after each adjustment.

The total amount of buffers capacity of demand pair D1, D2 and D3 are 176.2, 236.2 and 196.2 respectively. Obviously, the demand pair D1 is more efficient of the manufacturing system, because it has more buffer capacity surplus that can be used to transport more materials and WIP.

V. CONCLUSION AND DISCUSSION

Due to failure, partial failure, or maintenance, the capacity of each machine is stochastic, thereby defective products may come out. However, in many cases, defective products still have substantial value, and thus leads to rework. In this paper, a manufacturing system of multiple production lines with reworking path is considered to evaluate its reliability. Moreover, from the perspective of demand, the minimum capacity vector and loading vector of the machine on each production line can be determined. Further, due to the randomness of the machine’s capacity and the uncertainty of the loading size, the level of the capacity and loading of
machine are ambiguous. Thus, a dynamic adaptive fuzzy reasoning Petri net is proposed to infer reliability level of manufacturing system. Weighted fuzzy production rules with certain values are used to describe fuzzy knowledge of reliability state, and the capacity and load of machine, together with limitation relationship of minimum capacity, the loading and the buffer level are formatted as linguistic fuzzy sets to represent the imprecise knowledge of reliability level. The dynamic adaptive fuzzy reasoning Petri net is implemented to continuously change the capacity of the buffer and make the reliability of the system more and more higher. And an oil pump manufacturing system is utilized to conduct simulation experiments, the result shows the effectiveness and flexibility of our method for evaluating reliability of manufacturing system and automatically optimize the production efficiency of the manufacturing system. In general, the method in this paper is suitable for solving the reliability evaluation and automatic optimization of manufacturing systems of multiple line with reworking. However, it still has many limitations.

In this paper, the demand for production is assumed to be a fixed value, which may not be consistent with the actual production situation. Therefore, as for future work, we can set the demand to a random value and expect to find an optimal production plan. In other words, the next step of the research is to simultaneously optimize the buffer capabilities, input materials, and machines’ capabilities at the same time. This will make the research in this paper more meaningful. In this regard, many scholars [41]–[43] in the manufacturing field have done similar work from different angles. Moreover, the reliability evaluation based on dynamic adaptive fuzzy reasoning Petri nets can be extended and applied to manufacturing systems with parallel, assembly or disassembly structures.

APPENDIX

See Figs. 1 and 6.

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