Quantum phases of dimerized and frustrated Heisenberg spin chains with $s = 1/2$, 1 and $3/2$: an entanglement entropy and fidelity study

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Abstract
We study here different regions in phase diagrams of the spin-1/2, spin-1 and spin-3/2 one-dimensional antiferromagnetic Heisenberg systems with frustration (next-nearest-neighbor interaction $J_2$) and dimerization ($\delta$). In particular, we analyze the behaviors of the bipartite entanglement entropy and fidelity at the gapless to gapped phase transitions and across the lines separating different phases in the $J_2–\delta$ plane. All the calculations in this work are based on numerical exact diagonalizations of finite systems. (Some figures may appear in colour only in the online journal)

1. Introduction

Matter can appear in different quantum phases with exotic properties such as charge density waves, magnetism, superconductivity, and so on. Studies of these phases and the transitions from one phase to the other are important and interesting for both academic and technological reasons. Two major tools from quantum information theory have been used extensively in recent years for studying the quantum phases of a system: quantum entanglement and fidelity. The idea of quantum entanglement originated in the study of quantum correlations of many-body systems [1, 2]. It is expected that, even for moderately large system sizes, the entanglement entropy can identify the values of the parameters of the Hamiltonian where a quantum phase transition (QPT) occurs because a significant change of the quantum correlations of the systems occurs when one goes across such a transition [3]. In recent years, entanglement entropy has been used to study quantum critical regions in various systems [4–13]. Quantum fidelity is a measure of how little the ground state of a system changes as one changes the parameters of the Hamiltonian. A large change in the fidelity is anticipated close to a QPT even if the system size is not very large [14–28].

In this paper, we study different quantum phases and quantum critical regions with frustration $J_2$ (next-nearest-neighbor coupling) and dimerization $\delta$ (an alternation in the nearest-neighbor couplings) of the spin-1/2, 1 and 3/2 Heisenberg antiferromagnetic systems in one dimension by calculating the entanglement entropy and fidelity in the ground state of the system. In particular, we study the transition from a gapless to a gapped phase and the changes in the spin structure across different phase lines in the $J_2–\delta$ plane for the spin-1/2, 1 and 3/2 systems. For the spin-1/2 and spin-1 systems, our numerical study contains the locations of different critical points and lines separating different phases in the phase diagrams which had been found earlier by other methods, such as the density matrix renormalization group method [35–40]. We compare our results with those reported...
previously (using different techniques) whenever possible. For the spin-3/2 system, we find that the entanglement entropy and fidelity help us to estimate the locations of the various critical points and lines in the phase diagram.

There are some related works which we briefly mention here. The bipartite entanglement for the spin-1/2 J1-J2 model (without dimerization) has been studied in different contexts, like the transition from the Néel to the spiral phase and the gapless to gapped phase transition along the J2 axis (see, for example, [29, 30]). The gapless to gapped phase transition has also been studied using the fidelity (as a function of J2) of the first excited state with periodic boundary conditions [31] and the fidelity susceptibility of the ground state [32]. The role of entanglement between distant sites has been studied for this model with frustration and dimerization [33]. To the best of our knowledge, the phase diagram of the spin-1 and spin-3/2 J1-J2 model (with or without dimerization) has not yet been studied using entanglement entropy and fidelity. In this paper we report results on the entanglement entropy and fidelity of the J1-J2-δ model over the J2-δ plane.

Our paper is organized in the following way. In section 2, we discuss how to calculate the entanglement entropy and fidelity. We then introduce the Hamiltonian that we study in section 3. In section 4, we give a brief introduction to the numerical techniques we use in our work. We then present the entropy and fidelity results for the spin-1/2, spin-1 and spin-3/2 systems in section 5. We conclude our paper in section 6.

2. Entanglement entropy and fidelity

A pure state of a bipartite entangled system can be written as |ψ⟩ = ∑ |φi⟩ |ψi⟩′, where |φi⟩ and |ψi⟩′ are the basis states of the left and right blocks respectively. The reduced density matrix (RDM) of the left block, ρL = TrR(|ψ⟩⟨ψ|), is calculated by tracing out the degrees of freedom of the right block. The elements of the RDM ρL are given by

\[ ρ_{ij} = ∑_k C_{ik} C^*_{jk}. \]  

The von Neumann entropy of a block is given by

\[ S = -∑_i λ_i \log_2 λ_i, \]  

where the λi are the eigenvalues of ρ.

Fidelity measures how little a particular wavefunction (for instance, the ground state) changes with the parameters of a model Hamiltonian. This is quantified by the overlap of the wavefunction at two different parameter values. If p is a parameter then the fidelity is given by

\[ F = |⟨ψ(⟨p) |ψ(p + α)⟩|, \]  

where α is a small variation in p. In our case, both J2 and δ are parameters with respect to which we have calculated fidelity and we have taken the change in the parameter to be 10^−2 in the numerical calculations.

3. Description of the spin model

We study the Heisenberg Hamiltonian for the antiferromagnetic chain with both nearest-neighbor and next-nearest-neighbor couplings and dimerization [34, 36]. We will use this Hamiltonian for the spin-1/2, spin-1 and spin-3/2 systems; it is given by

\[ H = J_1 ∑_{i=1}^{2N-1} (1 - (-1)^i δ)  \vec{S}_i \cdot  \vec{S}_{i+1} + J_2 ∑_{i=1}^{2N-2} (1 + (-1)^i δ)  \vec{S}_i \cdot  \vec{S}_{i+2}, \]  

where J1 is the nearest-neighbor interaction (we take J1 = 1 for our study), δ (0 ≤ δ ≤ 1) is the dimerization and J2 (0 ≤ J2 ≤ 2) is the next-nearest-neighbor interaction. In our entropy contour plot we have taken the range of J2 from 0 to 2.

The phase diagrams of the spin-1/2 and spin-1 chains are different from each other in the J2-δ plane (see figure 1). The spin-1/2 system undergoes a QPT from a gapless phase to a gapped phase at J2c = 0.2411 ± 0.0001 without dimerization (δ = 0) [41], while the rest of the phase diagram is gapped. The line 2J2 + δ = 1 separates the Néel phase (region A) from the spiral phase (region B) [36].

![Figure 1. Phase diagrams of the (a) spin-1/2 and (b) spin-1 chains in the J2-δ plane.](image-url)
The spin-1 system has a number of distinct phases in the $J_2$–$\delta$ plane. Region I denotes the spin–Peierls gapped phase. Regions II and III are Haldane gapped and spiral regions respectively. In these two regions the ground state of an open chain is fourfold degenerate. Region IV is a spiral gapped phase with a non-degenerate ground state for an open chain. A gapless phase exists along the critical line ‘a’, that lies between (0, 0.25) and (0.22 ± 0.02, 0.20 ± 0.02) in the $J_2$–$\delta$ plane [38, 39, 42]. A line ‘c’ separating regions II and III extends from (0.73, 0) to (0.65, 0.05); on this line the gap appears to be zero (to numerical accuracy). Along the dotted lines ‘b’, which extends from (0.22 ± 0.02, 0.20 ± 0.02) to the point P = (0.432, 0.136), and ‘d’, which extends from P to (0.65, 0.05), the gap shows a minimum as a function of $\delta$ (see figure 12). This will be discussed in more detail in section 5.2.2. The line $2J_2 + \delta = 1$ starts at point P and extends up to (0, 1), separating regions I and IV. Another line (‘d’) starts at (0.39, 0) and ends at point P, separating regions II and III. The phase diagram of the spin-3/2 system has not been studied yet. We will show below that the entanglement entropy and fidelity of this system can give some insights into its phase diagram.

4. Numerical techniques

For our calculations, we use the $M_s$ basis (eigenstates of the $z$ component of total spin) [43]. These basis states are orthonormal and it is easy to obtain the RDM (which is used to calculate the entanglement entropy) when the states of the system are expressed in this basis. On the other hand, most of the results can be understood qualitatively using valence bond (VB) theory [44]. In this theory [45, 46], the basis states in the singlet space are expressed as products of pairwise singlets, which follow the Rumer–Pauling rules, to avoid overcompleteness of the VB states. Figure 2 shows some VB diagrams of a spin-1/2 chain with eight sites and total spin $S = 0$. The VB state (a) is a Kekulé state, which is a product of nearest-neighbor singlets.

Since our Hamiltonian conserves total spin, all its eigenstates are also eigenstates of total spin. Therefore, the eigenstates that we obtain by diagonalizing the Hamiltonian in the constant $M_s$ basis will be linear combinations of the VB basis states. It may be worth mentioning here that appropriate linear combinations of constant $M_s$ basis states can give different VB basis states [47]. Now, a VB basis state contributes to the bipartite entanglement entropy of a state under study if the boundary between the two blocks of the system cuts a singlet line. If the boundary does not cut a singlet line, its contribution is assumed to be zero. For example, if the boundary goes through sites 4 and 5 of the system, the entropy contribution of diagram (a) will be zero while that of diagram (b) will be non-zero (figure 2). Depending upon the entropy contributions of the VB basis states and their relative weights in a state under study, we can qualitatively understand the entropy of the state [44]. In generating the contour plot of entropy, we have calculated the entropy over a grid of 201 $J_2$ values (0 $\leq$ $J_2$ $\leq$ 2) and 101 $\delta$ values (0 $\leq$ $\delta$ $\leq$ 1).

5. Results and discussion

In this section, we present numerical results for the entanglement entropy of finite size chains with two equal block sizes for spin 1/2, 1 and 3/2. We also present results for the ground state fidelity of both the systems in the $J_2$–$\delta$ plane.

5.1. The spin-1/2 system in the $J_2$–$\delta$ plane

5.1.1. Uniform chain ($\delta = 0$). The uniform spin-1/2 chain without dimerization ($\delta = 0$) goes through a gapless to gapped phase transition at $J_{2c} \approx 0.2411$ in the thermodynamic limit ($N \rightarrow \infty$) [41], and its Néel phase is separated from the spiral phase at $J_2 = 0.5$. To study the behavior of the system around these points, we calculate the ground state fidelity and entanglement entropy of the system.

We first consider the ground state entanglement entropy of finite size spin-1/2 chains with different $J_2$ values. The bipartite entropy for different chain lengths (and equal block size) can be seen in figure 3. At $J_2 = 0.5$, the entropy reaches a minimum; away from that point the entropy increases. For the systems with even block sizes the minimum of the entropy goes to zero, while for odd block sizes this minimum is one. This result can be explained by noting that, at $J_2 = 0.5$ (the Majumdar–Ghosh point [34]), the ground state has Kekulé state structure (as in figure 2(a)). Depending upon the block size being odd or even, entropy will be finite or zero respectively [44]. As $J_2$ moves away from this point, the presence of other VB basis states (as in figure 2(b)) in the ground state will become significant. Since these VB basis states will have a finite entropy contribution, the entropy of the ground state will increase as $J_2$ moves away from the point.

We do not observe any change in the behavior of the ground state entropy around $J_{2c}$. To investigate this further,
we plot the entropy versus \( \log_2 N \) for different \( J_2 \) values (see figure 4). The plots indicate that the present system sizes are too small to numerically verify the conformal field theory prediction of \( S = \frac{c}{6} \log_2 N \) (with \( c = 1 \)) at \( J_2 \) [10]. The first excited states in the singlet and triplet sectors cross as a function of \( J_2 \). We have calculated the entropy of these two states as a function of \( J_2 \). We find that the entropy of a state depends only on its spin and not on its energy. Therefore, as a function of energy level ordering, the entropy shows a jump at a value of \( J^*_2 \), which depends on the chain length. The jump in the value of the entropy can be seen from figure 5. In figures 5(a) and (b), we see that the ground state (lowest singlet) and lowest triplet state are non-degenerate for finite \( N \); they become degenerate in the thermodynamic limit [36, 41]. However, the first excited singlet and the first excited triplet states become degenerate near \( J^*_2 \) even for small values of \( N \). In figure 6 we plot the \( J^*_2(N) \) as a function of \( 1/N^2 \) and we see

Figure 3. Ground state entanglement entropy of the spin-1/2 system with different chain lengths at \( \delta = 0 \). The lower set of plots for \( 4n \) systems and the upper set of plots is for \( 4n + 2 \) systems.

Figure 4. For a spin-1/2 system, the entropy versus logarithm of the system size is shown for different \( J_2 \) values for a uniform chain.
that $J^*_2$ extrapolates to $J_2$ in the thermodynamic limit. The extrapolated value of $J^*_2$ (0.2414) is very close to the reported value of $J_2 = 0.2411$ [36, 41].

Upon calculating the ground state fidelity (taking $J_2$ to be the variable parameter), we do not observe any behavioral change at $J_2 = 0.5$ or $J_2c$. The fidelity of first excited states in both the singlet and the triplet sectors falls to zero near $J_2c$.

The same thing has already been observed for the system with periodic boundary conditions [31].

5.1.2. Dimerized chain ($0 < \delta \leq 1$). In the phase diagram of the spin-1/2 system with dimerization, the Néel phase is separated from the spiral phase by the line $2J_2 + \delta = 1$. We study the ground state entanglement entropy of finite size chains in these phases with different $\delta$ values (see figure 7). We observe that the entropy of the system is minimum for values of $J_2$ and $\delta$ which fall on this line. For systems with even sized blocks this minimum value is zero, while for odd sized blocks the minimum is one. The reason for this is similar to the case $\delta = 0$ as given earlier.

We have calculated the ground state fidelity (with $J_2$ as the variable parameter) of finite size systems in the $J_2-\delta$ plane; we do not observe any sudden change in fidelity along the $2J_2 + \delta = 1$ line. This can be explained by the fact that the phases of the system on both sides of the line are gapped, which implies that the ground state does not cross any excited state (i.e., the ground state does not change its character of being a singlet or a triplet) when we cross this line in the parameter space.

5.1.3. Spin-1/2 entropy phase diagram (contour plot). We study the gapless to gapped phase transition and change in the

Figure 6. Convergence of the crossing points of the excited states for a spin-1/2 chain.

Figure 7. Ground state entanglement entropy of a spin-1/2 chain with lengths of 6, 8, 10, 12, 14, 16, 18 and 20, for different values of $\delta$ values; the line types for different lengths are the same as in figure 3. The upper set of curves is for chain length $4n + 2$ while the lower one is for chain length $4n$ ($n$ being a positive integer). Near the minimum, the finite size dependence is very weak.
spin structure (order–disorder change) along the $2J_2 + \delta = 1$ line using the entropy contour plot for a spin-1/2 chain with 20 sites (see figure 8). The entropy is zero along this line. The density of the entropy contour lines shows whether the system is in a gapless or gapped phase. From this figure we see that the gapless region between $J_2 = 0$ and $J_2_c$ has a higher density of entropy contour lines. In the rest of the figure the density of the contour lines is lower, which shows that the rest of the phase diagram is gapped. At higher values of $J_2$, the spin-1/2 chain behaves like two decoupled chains with a weak coupling ($J_1$) between them. The stronger interaction ($J_2$) is responsible for the higher entropy. For higher values of both $J_2$ and $\delta$ values, the spin-1/2 chain behaves like a spin ladder with a gapped phase.

5.2. The spin-1 system in the $J_2–\delta$ plane

As mentioned in section 3, there are many phases in the $J_2–\delta$ phase diagram of the spin-1 system. To study these phases, we first calculate the bipartite entanglement entropy in the ground state for finite systems. We calculate the ground state entropy for different values of $\delta$ (with large $\delta$) and for different chain lengths (see figure 9). The entropy is zero along the line $2J_2 + \delta = 1$ for the systems with even sized blocks and non-zero for systems with odd sized blocks just as in the spin-1/2 case. As in the spin-1/2 case, the finite size effects are very weak near the minima of the entropy (figure 9).

5.2.1. Spin-1 entropy phase diagram (contour plot). In figure 1(b), we know that the ground state is fourfold degenerate in regions II and III for an open chain. Hence we used the spin parity symmetry to break the degeneracy.
between the states corresponding to the total spin \( S = 0 \) and 1. Then we calculate the entanglement entropy for the lowest state in the even parity subspace (which is a singlet). We study the quantum phases and QPTs of the spin-1 chain with 16 sites using the contour plot of the ground state entropy in the \( J_2–\delta \) plane, where \( J_2 \) goes from 0 to 2 and \( \delta \) goes from 0 to 1 (see figure 10). We see in the figure that the line \( 2J_2 + \delta = 1 \) starts approximately at point \( P = (0.432, 0.136) \) and extends up to \( (0, 1) \). Along this line the entropy is zero. Along the gapless lines ‘a’ and ‘c’ (in figure 1(b)) the density of the entropy contour lines is higher, while the density is lower in the rest of the phase diagram. Concerning line ‘d’ (in figure 1(b)), we observe that the density of contour lines is much lower compared to the regions nearby.

In curve (i) of figure 11, we plot \( \delta_{\text{cal}} \) versus \( J_2 \) for the points corresponding to the minimum value of the entropy in the \( J_2–\delta \) plane; this curve goes from point \( P \) to the point \( (0, 1) \). This curve is seen to follow the line \( 2J_2 + \delta = 1 \) as expected; we see a step staircase instead of a straight line because we have calculated the entropy for discrete values of \( J_2 \) and \( \delta \). To detect the gapless phase along lines ‘a’ and ‘c’, we calculate the first order derivative of the entropy (using the three-point differentiation formula) along the \( \delta \)-axis for different values of \( J_2 \) in this plane. In curve (ii) of figure 11, we plot \( \delta_{\text{cal}} \) versus \( J_2 \) corresponding to the maximum of the absolute value of the derivative. This curve follows lines ‘a’ and ‘c’ closely.

### 5.2.2. Spin-1 chain fidelity and gap

We calculate the ground state fidelity of spin-1 chains with finite sizes in the \( J_2–\delta \) plane. For small values of \( \delta \), the ground states of finite size systems have multiple energy level crossings with the excited states. Because of these finite size effects, the fidelity of the ground states is not a reliable tool for studying phases in regions II and III. However, in regions I and IV, we find no energy level crossings in the ground states of finite systems. We calculate the ground state fidelity for systems with 6, 8, 10, 12 and 16 sites for different values of \( \delta \) along the \( J_2 \) axis in this plane. We find no sudden changes in the fidelity. We show the plot of \( J_2 \) versus \( \delta_{\text{cal}} \) corresponding to the minimum fidelity for a chain of 16 sites (see the inset of figure 11). This curve follows lines ‘a’, ‘b’, ‘b’ and ‘c’, qualitatively separating regions II and III from regions I and IV.

To understand the quantum phase transition on line ‘b’ in figure 1, we calculate the spin gap of the system near this line (see figure 12). Since the ground state of an open chain with spin 1 is fourfold degenerate in regions II and III, we calculate the excitation energy gap as the difference between the lowest energy state in the \( M_s = 0 \) sector and the first excited state in the \( M_s = 1 \) sector. Our current calculations based on a finite size analysis shows that the gap could vanish in the thermodynamic limit. This improves our earlier report, which had convergence difficulties [38, 39]. We also studied the behavior of the gap across lines ‘b’ and ‘b’. For example, for the 16 site chain, the gap is minimum at \( \delta \simeq 0.13 \) at \( J_2 = 0.4 \) and \( \delta \simeq 0.09 \) at \( J_2 = 0.5 \) (see the insets of figure 12). Moreover, there is a change in the behavior of the gap versus \( 1/N \) as \( \delta \) varies; for \( \delta \) lying below the phase transition line the gap saturates to a finite value, while for \( \delta \) lying above the line the gap continues to decrease steadily as \( N \) increases.

Line ‘b’ appears to be a phase transition line which separates the Haldane and spin–Peierls phases, which are both gapped. These two phases differ in several ways. For an open chain, the ground state has a fourfold degeneracy (a spin singlet and a spin triplet which are degenerate) with spin-1/2 states at the ends in the Haldane phase, but is non-degenerate (spin singlet) in the spin–Peierls phase. Further, the Haldane phase has a non-local string order parameter [48].

### 5.3. The spin-3/2 system in the \( J_2–\delta \) plane

In this section we study the phase diagram of the spin-3/2 system in the \( J_2–\delta \) plane. To the best of our knowledge, the
quantum phases of the Heisenberg spin-3/2 antiferromagnetic chain in the \( J_2-\delta \) plane have not been studied earlier. However, from the field theory analysis of the spin chain, half-odd integer systems are gapless at \( \delta = 0 \) and for small values of \( J_2 \). With dimerization \( (\delta \neq 0) \), it is predicted that the spin-3/2 system should be gapless at \( \delta = 2/3 \) for \( J_2 = 0 \) [39, 49].

5.3.1. Spin-3/2 entropy phase diagram (contour plot). We study different quantum phases of spin-3/2 chain. We use spin parity symmetry to break the degeneracy (within numerical accuracy) of the ground state of this system. For a 12 site chain, a contour plot of the entropy is shown in figure 13. As in the spin-1/2 and spin-1 cases, we observe an order–disorder transition along the \( 2J_2 + \delta = 1 \) line. The line starts approximately from point \( Q = (J_2 = 0.38, \delta = 0.24) \) and extends up to \( (J_2 = 0, \delta = 1) \). For small \( J_2 \) values near \( \delta = 0 \) in the contour plot, the line pattern is similar to that for the spin-1/2 case and quite different from the spin-1 case. This suggests that there can be a gapless phase at \( \delta = 0 \) as predicted. For \( J_2 = 0 \), the line density is very high between \( \delta = 0.4 \) and 0.5; this suggests another gapless phase in this region as predicted by field theory. As in the spin-1 case, the density of lines is high at larger \( J_2 \) values (about \( J_2 = 1 \)). This suggests a numerically gapless phase in this region. We also confirm these gapless phases by comparing numerical energy gaps in those regions with that of a phase at large values of \( J_2 \) and \( \delta \) where the density of lines is very low. For better understanding of these quantum phases we plot \( \delta_{\text{cal}} \) versus \( J_2 \) values corresponding to the minimum entropy (curve (i) from figure 14) above the ‘Q’ point. This curve follows the \( 2J_2 + \delta = 1 \) line, similar to the spin-1/2 and spin-1 cases. We also plot the points corresponding to the maximum absolute values of the first order derivative of entropy with respect to \( \delta \); this is shown as curve (ii) in figure 14. This is similar to the curve representing a numerically gapless phase for a spin-1 system (curve (ii) in figure 11). This suggests that there can also be a gapless region along curve (ii) for the spin-3/2 system. The gapless point (curve (ii) of figure 14) at \( \delta = 0.45 \) at \( J_2 = 0 \) is consistent with the value \( \delta = 0.431 \) reported [50]. Note that this value is different from the field theory prediction of 2/3 [39, 49].

We calculate the ground state fidelity of the 12 site spin-3/2 chain along different \( J_2 \) values with \( \delta \) as the variable parameter. We plot \( J_2 \) versus \( \delta \) correspond to minimum fidelity in this plane (see inset of figure 14). The curve approximately follows curve (ii) in figure 14.

To further investigate the gapless points which are expected to occur at certain non-zero values of \( \delta \) at
Figure 14. Curve (i) shows the \((J_2, \delta_{\text{cal}})\) values (squares) corresponding to the minimum entropy. Curve (ii) shows the \((J_2, \delta_{\text{cal}})\) values corresponding to the maximum absolute values of the first order derivative of the entropy w.r.t. \(\delta\). The inset shows the points corresponding to minimum fidelity (taking \(\delta\) as the variable parameter). All the results are obtained for the 12 site spin-3/2 chain.

\[ J_2 = 0 \] for the spin-1 and 3/2 systems, we have shown the entropy versus the logarithm of the system size for different \(\delta\) values in figure 15. The present system size appears to be too small to numerically verify the conformal field theory prediction \([10]\) of \(S = \frac{c}{4}\log_2 N\) (with \(c = 1\)) at the critical points which occur at certain values of \(\delta\) (numerically estimated to be 0.24 for spin-1 and 0.43 for spin-3/2).

6. Conclusion

We have used entanglement entropy and fidelity as tools to study the different quantum phases and quantum critical regions of the spin-1/2, 1 and 3/2 chains in the \(J_2-\delta\) plane. For this study, we have employed extensive exact diagonalization of spin chains with up to 20 sites depending on the site spin. We have considered 201 values of \(J_2\) in the range 0–2 and 101 values of \(\delta\) in the range 0–1 corresponding to over 20 000 grid points.

We have studied the complete phase diagrams of these three systems using entropy contour plots and fidelity in the \(J_2-\delta\) plane. We have been able to identify the quantum phase transitions from gapless to gapped phases using the density of the contour lines of the entropy and the minimum fidelity. Though the full phase diagram of the spin-3/2 system has not been investigated before, we have conjectured the existence of some gapless regions and an order–disorder line by studying its phase diagram and comparing it with the phase diagrams of the spin-1/2 and spin-1 systems. Our main results are that we find indications of a gapless region near \(\delta = 0\) and small values of \(J_2\) in the spin-1/2 system, a gapless region at finite \(\delta\) in the spin-1 system (lines ‘a’ and ‘b’ in figure 1(b)), and two gapless regions near \(\delta = 0\) and around \(\delta \sim 0.4–0.5\) for \(J_2 = 0\) in the spin-3/2 system.

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Figure 15. For spin-1 and 3/2 systems, the entropy versus logarithm of the system size is shown for different \(\delta\) values with \(J_2 = 0\).
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