Effects of residual stress and equivalent bending stiffness on the dimensional stability of the thin-walled parts

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Abstract
The monolithic thin-walled parts are widely used in the aeronautic and astronautic field because of its excellent mechanical performance and light weight, but the thin-walled parts are vulnerable to the machining deformation due to its low stiffness and high material removal rate. According to the relative basic theory, the stiffness and internal residual stress of the part are the critical factors affecting the dimensional stability. In this work, the influences of equivalent bending stiffness and residual stress on the dimensional stability of thin-walled parts are studied. Nine typical thin-walled parts in three groups with two materials (7075 aluminum alloy for A1~A3 and B1~B3, and Ti6Al4V titanium alloy for B4~B6) are machined and treated with different processes. Topology optimization technique is used to optimize the structure of parts to enhance the bending stiffness. Corresponding finite element method (FEM) simulations are carried out to further investigate the generation mechanism. The deformations in 312 h after machining are measured using coordinate measuring machine, and the deformation changes of the parts are obtained and analyzed. Finally, based on topological optimization and stress relief technology, a machining deformation control method for the monolithic thin-walled parts is proposed. Results show that the maximum and average deformations of thin-walled are evidently decreased using the proposed method.

Keywords Residual stress · Equivalent bending stiffness · Dimensional stability · Machining deformation · Topology optimization

1 Introduction
The monolithic thin-walled parts have been widely used in the aerospace products because of its high strength and low weight. However, due to the high material removal rate, low stiffness and complex structure, the machining accuracy is difficult to control. Therefore, the problem of machining deformation and dimensional stability of the monolithic thin-walled parts has always been the focus of scholars.

The machining is a highly nonlinear thermal-structural coupling process, which is affected by a lot of factors. Tang et al. [1] pointed out that the initial residual stress, machining-induced residual stress, clamping force, and cutting force, as well as cutting heat are the main influence factors of machining deformation. Fu et al. [2] mentioned that the release and rebalance of residual stress are the main factors of machining deformation, and they proposed a new method to calculate the initial residual. Huang et al. [3, 4] explored the impact of the initial residual stress of blank, the machining-induced residual stress, and the coupling of these two factors on the deformation. It was found that the initial residual stress of three-frame beam is the main factor of deformation, while for the aluminum alloy plate, the machining-induced residual stress is the major factor. On the basis of a large number of literature research, Li and Wang [5] systematically summarized the influence of residual stress on machining deformation as well as the control methods for aviation aluminum alloy.
which can offer a good guidance for researchers in related fields.

Scholars have conducted a series of research to study the machining process of the monolithic thin-walled parts, and then to predict and control the machining deformation. Ren et al. [6] regarded the multi pocket structure as a combination of Kirchhoff plates, combined the vibration model with the dynamic cutting model, and established the control equation which fully considers the dynamic characteristics of the thin-walled milling process. For the pre-bent plate, considering the coupling effect of bending stress and the machining-induced residual stress, Li et al. [7] analyzed the mechanism of deformation through theoretical analysis and numerical simulation, and corresponding experiments were carried out to validate the correctness of deformation mechanism.

In addition to the above theoretical research, finite element (FE) simulation plays an important role in machining deformation prediction. Considering the influence of deformation caused by the cutting force and multi-point contact structure dynamics in the contact area, Sun and Jiang [8] proposed a modeling method for dynamic milling system. Wang and Si [9] established a flexible iterative simulation model to predict deformation in the more complicated five-axis flank milling process. Similarly, Wang et al. [10] constructed an analytical model for frame parts that can predict deformation caused by residual stress and verified its accuracy through experiments.

In addition, plenty of achievements have been made in the control of machining deformation. Ma et al. [11] put forward a method to make the machining deformation uniform by planning the cutting amount. Kou et al. [12] used auxiliary support to balance the axial cutting force and control the deformation in milling process. To solve the process adjustment of the deformed workpiece in multiple processes, Hao et al. [13] proposed a geometry modeling method, which can be used to accurately reconstruct the deformation workpiece model, so as to conduct more accurate analysis in the next process.

Increasing the stiffness of part can reduce its deformation effectively, and topology optimization technology is a very good simulation tool to obtain a structural with higher stiffness. Topology optimization refers to finding the optimal structure configuration for specific conditions in a given design domain. Due to the general demand for this purpose, topology optimization is widely used in various fields and has many approaches. Through comparison with other various topology optimization methods, both Sigmund et al. [14] and Deaton et al. [15] demonstrated that solid isotropic material with poverty model is the most popular and successful method due to its simplicity of the concept and numerical implementation. Topology optimization is often applied with 3D printing technology. Vanek et al. [16] proposed an optimization approach which can successfully complete the established support function with less support structure. Langelaar [17] constructed a system which can generate fully self-supporting structures in the additive manufacturing process using the topology optimization technology. Liu et al. [18] systematically analyzed the development trends of topology optimization for 3D printing technology, providing some feasible suggestions for relevant researchers.

Except for the field of 3D printing, Zhu et al. [19] and Wang et al. [20] comprehensively summarized the broad application prospects of topology optimization in the field of aerospace and medicine, respectively. Besides, Liu and Ma [21] investigated the manufacturing-oriented topology optimization methods, and pointed out some potential development directions of topology optimization in manufacturing, such as concurrent optimization of the related position path. From the above, it can be seen that topology optimization has been widely used in engineering practice successfully and can effectively solve the problem. In fact, some scholars have carried out similar research to enhance the stiffness through topology optimization. Based on the bionic topology optimization method, Li et al. [22] applied an improved growth concept to the structural design of machine tool, and its stiffness can be enhanced by optimizing the layout of inner stiffeners. Kim et al. [23] applied topology optimization technology to determine the mass distribution of each part, and solved the problem of system-level stiffness maximization of industrial robot. Therefore, it is feasible to improve the stiffness of thin-walled parts by topology optimization technology.

Reducing the internal residual stress of thin-walled parts is another approach to control the deformation. Dong et al. [24] believed that if there is still large residual stress in the part after machining, some degree of deformation will inevitably occur in the process of residual stress release. Therefore, some stress relief processes can be applied to homogenize the internal residual stress of parts, and the commonly used technologies include thermal stress relief (TSR), vibratory stress relief (VSR), thermal and vibratory stress relief (TVSR), ultrasonic vibration, mechanical deformation method, etc.

Gu et al. [25] established a unified ultrasonic vibration plasticity framework and revealed the mechanism of the ultrasonic stress relief, which has an important significance in ultrasonic stress homogenization. Gao et al. [26] investigated the effect of VSR on residual stress and fatigue life of Ti6Al4V part. Yan et al. [27] used creep model considering temperature effect to predict residual stress release during post weld heat treatment (PWHT) and carried out experiments for verification. The research results showed that there is a critical temperature (740 K), about which the residual stresses of Ti6Al4V start to decrease mathematically during PWHT, and the strain increment caused by creep
dominated the residual stress relief. Chen et al. [28] took 2219 aluminum alloy welding specimen as research object, and studied and compared the effects of TSR, VSR, and TVSR. Results showed that TSR reduced the peak residual stress more effectively, and the elimination effect of TVSR is more ideal under the resonance frequency and medium temperature. Wu et al. [29] applied the method of TVSR to SiCP/Al composite material and expanded the scope of its application.

In terms of mechanical deformation method, Koc et al. [30] compared the effects of two different cold working methods of compression and stretching on residual stress relief, and the results showed that both processes could reduce the residual stresses more than 90% while the former was more economical. Ran et al. [31] and Gong et al. [32] adopted the cold rolling and roll-bending process respectively to eliminate the residual stress of aluminum alloy generated during the quenching process.

Although the means to eliminate residual stress are various, their principles are similar, that is use certain methods to input energy into the aging piece, so that the stress state inside the part is more uniform. Compared with other stress homogenization methods, the TVSR process has the advantages of both TSR and VSR, with remarkable effect, relatively environmental protection, short time, and simple operation. Therefore, it has been widely concerned in recent years, and is the preferred method to reduce and homogenize the internal residual stress of thin-walled parts.

To sum up, it can be known that there are two effective ways to reduce the machining deformation of monolithic thin-walled structure: increasing its bending stiffness and reducing its residual stress. The residual stress is the driving force of deformation and the bending stiffness is the ability to resist deformation. Increasing the stiffness or decreasing the residual stress alone to reduce machining deformation has been studied by many scholars. However, the machining deformation control method by both increasing the bending stiffness and decreasing the residual stress is barely reported. The main innovation of this article is to explore the influence of their coupling effects on machining deformation, and proposed a control method for the machining deformation by combining the two aspects into one process flow.

In this paper, nine typical parts in three groups with two materials (7075 aluminum alloy for A1~A3 and B1~B3, and Ti6Al4V titanium alloy for B4~B6) are machined and treated with different processes. Topology optimization technique is used to optimize the structure of parts to enhance the bending stiffness. Corresponding FEM simulations are carried out to further investigate the generation mechanism. The deformations in hundreds of hours after machining are measured using coordinate measuring machine, and the deformation changes of the parts are obtained and analyzed. Finally, a machining deformation control method for thin-walled parts based on topological optimization and stress relief process is summarized and proposed.

2 Theory

2.1 Equivalent bending stiffness

The expression of plate’s bending stiffness is shown as follows [33],

$$D = \frac{Eh^3}{12(1-\mu^2)}$$  (1)

where $D$ is the bending stiffness, $h$ is the thickness of the plate, and $E$ and $\mu$ are the elastic modulus and Poisson’s ratio of the material respectively.

The equivalent thickness and equivalent bending stiffness of frame parts can be calculated by static analysis using FEM. Thus, the analytical model of plate part can be extended to the general thin-walled parts, and this process is shown in Fig.S1. The maximum deformation in $X$ and $Y$ directions ($dis_x$ and $dis_y$) are obtained respectively under uniform load $q_x$ and $q_y$. The thin-walled part is equivalent to a cantilever beam and its maximum deformation can be expressed as Eq. (2) if a vertical force $F$ is applied to the cantilever end,

$$wl = \frac{4F\, len^2}{E\, bl\, hl^3}$$  (2)

where $wl$ is the maximum deformation, and $len$, $bl$, and $hl$ are the length, width, and thickness of the cantilever beam respectively.

Therefore, the equivalent thickness and equivalent bending stiffness of frame parts in $X$ and $Y$ directions could be expressed as follows,

$$h_{eqx} = \left(\frac{4F_xL^3}{dis_x\, E\, W}\right)^{\frac{1}{3}}$$  (3)

$$h_{eqy} = \left(\frac{4F_yW^3}{dis_y\, E\, L}\right)^{\frac{1}{3}}$$  (4)

$$D_{eqx} = \frac{E\, h_{eqx}^3}{12(1-\mu^2)}$$  (5)

$$D_{eqy} = \frac{E\, h_{eqy}^3}{12(1-\mu^2)}$$  (6)
where \( L \) and \( W \) are the length and width of the workpiece respectively, \( F_x = q_x W \), \( F_y = q_y L \).

2.2 Machining deformation calculation

As shown in Fig. 1, the initial and machining-induced residual stress will lead to the inevitable deformation. Therefore, when calculating the deformation of the plate workpiece with bi-directional initial residual stress, it can be equivalent to apply uniform bending moment on both opposite sides [34]. Through theoretical derivation, the bottom deflection function of thin-walled frame part is obtained,

\[
 w_i(x, y) = \frac{1}{2} \left( \frac{1}{R_{x,i}} x^2 + \frac{1}{R_{y,i}} y^2 \right), \quad (-\frac{L}{2} \leq x \leq \frac{L}{2}, -\frac{W}{2} \leq y \leq \frac{W}{2}) \tag{7}
\]

where \( R_{x,i} \) and \( R_{y,i} \) are the curvature radius of workpiece in \( X \) and \( Y \) directions after the layer \( i \) is stripped, and their expressions are as follows:

\[
 \frac{1}{R_{x,i}} - \frac{1}{R_{x,i-1}} = \frac{6f_i \left( \sigma_{x,i-1} t h_i - \mu \sigma_{x,i-1} t h_i + \sigma_{x,i} P_{x,i} t - \mu \sigma_{x,i} P_{x,i} t \right)}{E h_{eqx,i}} \tag{8}
\]

\[
 \frac{1}{R_{y,i}} - \frac{1}{R_{y,i-1}} = \frac{6f_i \left( \sigma_{y,i-1} t h_i - \mu \sigma_{y,i-1} t h_i + \sigma_{y,i} P_{y,i} t - \mu \sigma_{y,i} P_{y,i} t \right)}{E h_{eqy,i}} \tag{9}
\]

Among them, \( h_{eqx,i} \) and \( h_{eqy,i} \) are the equivalent thickness of the remaining workpiece in \( X \) and \( Y \) directions after the layer \( i \) is removed, \( f_i \) is the volume fraction of removed material in layer \( i \), \( h_i \) is the total thickness of the workpiece before stripping layer \( i \), \( t \) is the stripped thickness of each layer, and \( E \) and \( \mu \) are the elastic model and Poisson’s ratio respectively.

It can be seen that equivalent thickness is the pivotal parameter to deduce the deformation from the Eq. (7) to Eq. (9). Moreover, Eqs. (5–6) show that the equivalent stiffness is proportional to the cubic of equivalent thickness when the material is fixed. Therefore, increasing the thin-walled part’s equivalent stiffness can effectively reduce its machining deformation.

The theoretical model above presents the relationship among the machining deformation, residual stress, and bending stiffness. It provides theoretical support for the later application of simulation, optimization, and the proposed of the deformation control method.
3 Specimen design

The release and redistribution of residual stress will lead to the deformation. With the same stress distribution, increasing the equivalent stiffness of the workpiece can effectively reduce its deformation. In this study, some stiffening ribs are designed in specific positions through topology optimization, which can significantly improve the equivalent stiffness of the workpiece with the similar material removal rate.

3.1 Topology optimization

The size and shape of original thin-walled parts are shown in Fig. 2. Then, the simulation of topological optimization is carried out using the OptiStruct module of Hypermesh software to improve the bending stiffness.

As is presented in Section 2.2, the initial and machining-induced residual stress are the primary factors on the machining deformation of thin-walled parts. The load formed by residual stress release could be equivalent to uniform bending moment of two opposite sides, so the static analysis is selected to simulate the release of residual stress. Uniform loads are applied in length and width direction respectively, and the fixed constraints are applied in the center line (Figs. S2 and S3). The obtained displacements are taken as the constraint variables of topology optimization, and the volume fraction of the optimization region is taken as the optimization objective. Thus, the minimum volume fraction of the optimization region that can also satisfy the displacement constraint is obtained, and the morphology of the stiffening ribs are outputted as the optimization results.

3.2 Determination of specimen

The optimal A-type and B-type parts are calculated after 49 and 123 iterations in Hyperworks software, and their topology optimization results are shown in Figs. 3 and 4. It shows that the topological morphology has initially appeared when the number of iterations reaches about 50%. The subsequent iteration calculations further modify and refine the basic morphology. Results show that a thick stiffener is generated parallel to the central axis of the long side direction, while some relatively short and thin stiffeners are generated at the local position to strengthen its stiffness.

Based on the topological optimization results and considering the machinability at the same time, some stiffening ribs are added to improve the bending stiffness during machining process (Fig. 5). Then, the original model and optimized model are imported into ANSYS to calculate their equivalent bending stiffness, respectively. The comparison results are shown in Table 1. It shows that the equivalent bending stiffness after topology optimization is significantly improved. Meanwhile, due to the addition of stiffeners, the volume of retained materials increases, resulting in a slight reduction of material removal rate. Results show that the $D_{eqxx}$ (equivalent bending stiffness in X direction) of types...
A and B increases by 63.23% and 89.34% respectively; $D_{eqy}$ (equivalent bending stiffness in $Y$ direction) increases by 427.64% and 174.59% respectively, and the material removal ratio reduces by 8.54% and 7.71% respectively.

4 Simulations and experiments

4.1 Initial residual stress measurement tests

The initial stress of specimen blank should be tested before machining simulations and experiments. One 7075 aluminum alloy (Fig. 6b) and one Ti6Al4V specimen (Fig. 6e), whose sizes are both $300 \times 150 \times 30$ mm, are used for residual stress measurement tests. The blind-hole and ring-core methods are combined to measure the residual stress along the depth direction. RS-200 micro-measurement device produced by Vishay Co., Ltd. (USA), is used for blind-hole tests, which can measure the residual stress within 0–2-mm depth under the surface (Fig. 6a and d). And RKV-18 device produced by Laubinger + Rickmann GmbH & Co. KG (Germany) is used for the ring-core tests, which can measure the residual stress within 2–4-mm depth under the surface (Fig. 6c and f).

Thus, the average values within 0–2- and 2–4-mm depth under the surface are measured by the blind-hole and ring-core method, respectively. Subsequently, the material within 4 mm under the surface is removed by a YCM-CNC-116B machining center (produced by Yongjin machinery Co., Ltd., China) with q125-mm milling tools, and the combination of the two methods is utilized again to obtain the residual stress under new surface using layer-removed method. As a result, residual stresses within 0–16-mm depth can be measured, and the measured value is corrected by the modified formula of layer-removed method.

The residual stress profiles of 7075 aluminum alloy and Ti6Al4V specimens are presented in Fig. 7. The stress values of 16–32 mm and 0–16 mm are symmetrical.

4.2 Experiments

Based on the model before and after topology optimization, different machining and stress relief processes are applied to 7075 aluminum alloy and Ti6Al4V titanium alloy, respectively (Fig. 8). There are two kinds of parts with different shapes, named A1 ~ A3 and B1 ~ B6, respectively. Type A parts are divided into one group, whose material is 7075 aluminum alloy, while type B parts are divided into two groups, whose materials are 7075 aluminum alloy (B1 ~ B3) and Ti6Al4V titanium alloy (B4 ~ B6), respectively. Each group contains three parts, corresponding to three different processes:

- Process 1 (A1, B1, B4): the blank is machined to the final shape directly.
- Process 2 (A2, B2, B5): the stiffening ribs are reserved in the 1st machining, and after 96-h natural aging treatment (waiting for 96 h in the unsteady room temperature), the reserved stiffeners are removed in the 2nd machining.
- Process 3 (A3, B3, B6): the stiffening ribs are reserved in the 1st machining, and TVSR is conducted to further reduce the residual stress. Finally, the reserved stiffeners are removed in the 2nd machining.

Among them, the TVSR mentioned in process 3 is a new residual stress control method. The traditional TSR has a significant effect on global stress relief yet requires high energy consumption. VSR is green and environmentally friendly, but it is mainly used to eliminate the stress value of key parts. In some cases, the overall stress relief effect is not so significant. The method of TVSR integrates the advantages of TSR and VSR. At the same time of TSR, it provides vibration excitation to the working platform, which can significantly reduce the internal residual stress of workpiece and achieve a good stress homogenization effect. The mechanism of TVSR is as follows: the yield strength of material will be reduced at high temperature. Under the action of vibration excitation, the residual stress of workpiece will be superimposed with the vibration dynamic stress, and plastic
strain occurs in parts exceeding the yield strength of material, which will release the residual stress.

The experimental equipment of TVSR is developed and built by the author’s team. As shown in Fig. 9, the equipment is composed by heating system and vibration system. When working, the workpiece is clamped on the vibration platform, and the vibration exciter stimulates the platform to drive the overall vibration of workpiece. At the same time,

Table 1  Comparison of equivalent bending stiffness before and after topology optimization (material removal rate equals material removal volume/blank volume)

| Performance comparison | Type A | | | Type B | | |
|------------------------|--------|--------|--------|--------|--------|
|                        | Before optimization | After optimization | Percentage | Before optimization | After optimization | Percentage |
| $D_{eqx}(N\cdot mm)$   | $2.01 \times 10^7$  | $3.28 \times 10^7$  | $+63.23\%$ | $4.23 \times 10^7$  | $8.01 \times 10^7$  | $+89.34\%$ |
| $D_{eqy}(N\cdot mm)$   | $9.27 \times 10^6$  | $4.89 \times 10^7$  | $+427.64\%$| $5.27 \times 10^6$  | $1.45 \times 10^7$  | $+174.59\%$|
| Mass(kg)               | 75.17\%          | 66.63\%          | $-8.54\%$  | 82.47\%          | 74.75\%          | $-7.71\%$  |

Fig. 5  Design results of a type A, b type B
the thermal aging furnace covers the vibration platform, and the aging temperature of heating furnace is adjusted by the temperature controller. The workpiece releases its internal residual stress under the coupling effect of vibration and heat.

Taking A3 and B3 for example to introduce the process flow in Fig. 8, their bottom surface deformations are measured at 24 h after machining, and the TVSR is carried out. The holding temperatures of titanium alloy and aluminum alloy of TVSR are 280 °C and 175 °C, respectively. According to the measured resonance frequency, the vibration frequency of A3 and B3 is set as 60.5 Hz and 60.3 Hz respectively, and the corresponding rotational speed of eccentric motor is 3630 r/min and 3618 r/min respectively. The process flow of TVSR for aluminum alloy workpiece (A3, B3) is as follows:

1) 0–30 min: heat the workpiece from 25 to 175 °C evenly
2) 30–60 min: keep the temperature within 175 °C ± 2 °C for 0.5 h
3) 60–65 min: vibrate with resonance frequency for 5 min at 175 °C
4) 65–95 min: keep the temperature within 175 °C ± 2 °C for 0.5 h
5) 95–100 min: vibrate with resonance frequency for 5 min at 175 °C
6) 100–340 min: stop heating, cool with the furnace for 4 h, and air cool to room temperature after removing the fixture.

Particularly, compared with the aluminum alloy specimen, the titanium alloy one has different vibration frequencies as well as the holding temperature and time. The titanium alloy keeps the holding temperature within \(280^\circ C \pm 2^\circ C\) for 60 min and the total time is 370 min. The NC machining and TVSR process of A-type and B-type parts are shown in Fig. 10.

In order to explore the influences of topological structure and TVSR on the dimensional accuracy and stability, deformation measurements are carried out for many times.
in hundreds of hours. The numbers and time of deformation measurement are shown in Table 2. M1 and M2 in the table represent the 1st and 2nd milling, respectively. Figure S4 is the location and distribution of the measuring points of each workpiece, and Fig. 11 presents the measurement device.

### 4.3 Machining simulations

To further study the influences of equivalent bending stiffness and residual stress on the machining deformation and the dimensional stability, FE models of machining process are established by the combination of AdvantEdge and ANSYS software. The nonlinear thermal-structure coupling model of milling process is built using AdvantEdge to calculate the machining-induced residual stress, and the milling force as well as the temperature at local positions. The milling parameters of 7075 aluminum alloy and Ti6Al4V are the same as the experiments.

The milling process model of the whole workpiece is built in ANSYS software. The solid 186 element with 20 nodes is selected, and the geometry model with machining and non-machining area are imported into ANSYS. Then, the geometric model is discretized into finite element model through meshing (Fig. 12). The initial and the machining-induced residual stress, as well as other results calculated by AdvantEdge, are assigned to the layer of FE model, and the material-removed process is simulated with birth and death element method. Thus, the deformation and stress during milling process can be calculated after the material is removed from the blank layer by layer.

### 5 Results and discussions

#### 5.1 Comparison of FEM and experiment results

Figure S5 presents the machining-induced residual stresses calculated by AdvantEdge software. Stress results on three quartering points of the machined surface are extracted and summarized, and the average values of the three points are taken as the simulation results. It shows that a compressive stress layer is formed on the machined surface, and some tensile stress exists in the subsurface layer. For 7075 aluminum alloy, the maximum compressive stress, average stress, and stress layer depth in the X direction are $-108$ MPa, $-51$ MPa, and 0.14 mm, respectively, and those in Y direction are $-141$ MPa, $-94$ MPa, and 0.04 mm, respectively. For Ti6Al4V, those in X direction are $-238$ MPa, $-120$ MPa, and 0.12 mm, respectively, and...
those in $Y$ direction are $-258$ MPa, $-121$ MPa, 0.09 mm, respectively.

The deformations after the 1st milling caused by the blank initial and machining-induced residual stresses of the specimens are demonstrated in Fig. 13. Results show that the bottom surface of the parts is approximately an elliptical surface, and the peak deformation occurs at four corners, which is in accordance with the theory in Section 2.

Figure 14 presents the FEM and experimental deformation results of the measuring points on the bottom surface. It can be seen the deformation distribution of FEM agrees well with experiment. Because A2 and A3, B2 and B3, and B5 and B6 have the same shape after 1st milling, the calculated results between two parts are the same, while the measured results are different. Therefore, the calculated results are compared to the average measured value of two parts (A2/ A3, B2/B3, and B5/B6).

The calculated peak deformation of A1 and A2/A3 are 0.297 and 0.187 mm, and the measured results are 0.315 and 0.271 mm; the calculated average values of 28 points (Fig.S4a) are 0.162 and 0.089, and the measured results are 0.143 and 0.119 mm. For group A, the maximum relative errors of calculated peak and average deformation are $-31.04\%$ and $-26.04\%$.

The calculated peak deformation of B1 and B2/B3 are 0.213 and 0.160 mm, and the measured results are 0.216 and 0.173 mm; the calculated average values of 33 points (Fig. S4b) are 0.087 and 0.059 mm, and the measured results are 0.157 and 0.103 mm. The calculated peak deformation of B4 and B5/B6 are 0.346 and 0.314 mm, and the measured results are 0.393 and 0.311 mm; the calculated average value are 0.111 and 0.116 mm, and the measured results are 0.186
and 0.153 mm. For group B, the maximum relative errors of calculated peak and average deformation are $-44.62\%$ and $-12.04\%$.

The effectiveness and accuracy of simulation model can be verified via the comparison between FEM and experiment results. In addition, as the main driving force, the effects of residual stress can be demonstrated via FEM simulation.

5.2 Analysis of the deformation with time

In most cases, FE models can only calculate the machining deformation after 1st milling due to the uncertain evolution of residual stress during 96-h natural aging; therefore, the variation of deformation with time can only be obtained by measurement tests rather than FEM simulations. Results show that (Figs. 15 and 16), compared with A1

![FEM deformation contours after 1st milling](image-url)
Fig. 14  FEM and experiment results after 1st milling
(0.315 mm), the peak deformations of A2 (0.264 mm) and A3 (0.264 mm) are reduced by 16.33% and 11.85%, and the average deformation of A2 (0.110 mm) and A3 (0.127 mm) is reduced by 23.10% and 10.86%, compared with A1 (0.143 mm) at 24 h after 1st milling.

Similarly, compared with B1 (0.216 mm), the peak deformations of B2 (0.172 mm) and B3 (0.174 mm) are reduced by 20.33% and 19.69%. Compared with B1 (0.143 mm), the average deformations of B2 (0.110 mm) and B3 (0.127 mm) are reduced by 25.02% and 43.74% at 24 h after 1st milling. Compared with B4 (0.393 mm), the peak deformations of B5 (0.320 mm) and B6 (0.324 mm) are reduced by 18.66% and 17.76%. Compared with B4 (0.186 mm), the average deformations of B5 (0.149 mm) and B6 (0.168 mm) are reduced by 19.76% and 9.64% at 24 h after 1st milling.

Besides, Fig. 15a, d, and g also indicate that the overall dimension continuously changes with the time during 24–312 h after machining. The change is significant within 24–72 h after machining; it preliminarily reaches a stable state after 72 or 96 h, and there is no significant change during 96–312 h.

Other specimens, such as A2, A3, B2, B3, B5, and B6 (Fig. 15), also have the same changing trend, namely reach a stable state after 72 or 96 h, which means the strain energy accumulated by residual stress has been released in 72 or 96 h. In this situation, the 2nd milling will not cause obvious deformation.

According to the measurement results, the changes of the maximum deformation of A2, A3, B2, B3, B5, and B6 caused by the 2nd milling are only 0.027, 0.030, 0.027, 0.026, 0.038, and 0.022 mm because of the energy release

![Fig. 15 Measured variation of deformation with time of a A1, b A2, c A3, d B1, e B2, f B3, g B4, h B5, and i B6. (Except A1, B1 and B4, time after 100 h represents the time after second milling + 100 h)](image-url)
after 1st milling, and the dimensions do not fluctuate evidently after the 2nd milling. In this study, the measured values at 96 h after the final (2nd) machining are taken as the final deformations. Thus, compared with A1, B1, and B4, the final maximum deformations of A2, B2, and B5 are reduced by 20.39%, 14.92%, and 3.80%; the average deformations are reduced by 1.60%, 15.39%, and 6.70%; and the dimension fluctuation in 96 h after final milling are reduced by 74.69%, 68.56%, and 63.10%, respectively.

The deformation is determined by the total elastic strain energy $\Pi_P$ of the part, which can be expressed as in Eq. (10) (Wang et al. [37] and Liu et al. [38]),

$$\Pi_P = U + W = \int \left( \frac{1}{2} \{ \sigma_i \varepsilon_i + \sigma_{ij} \varepsilon_{ij} + \sigma_{ij} \varepsilon_{ij} + \sigma_{ij} \varepsilon_{ij} + \sigma_{ij} \varepsilon_{ij} + \sigma_{ij} \varepsilon_{ij} \right) dV + W$$

where $U$ is the strain energy density function, $W$ is the work by external loads, $\sigma_i$ and $\tau_{ij}$ are normal and shear stresses, $\varepsilon_i$ and $\gamma_{ij}$ are normal and shear strains, and $V$ is the volume of the body. The external work $W$ is 0 in a free state.

When $\partial \Pi_P$ is close to 0, there is no sufficient energy for the part to deform. In this situation, the change rate of internal stress and strain is also 0; as a result, a relative stable state is achieved. This explains the mechanism of deformation caused by stress release to a certain extent.

If 96-h natural aging is replaced by TVSR, the final deformation can also be deceased. Compared with A1, B1, and B4, the final maximum deformations of A3, B3, and B6 are reduced by 23.51%, 24.16%, and 0.75%; the average deformations are reduced by 23.74%, 44.70%, and 21.30%; and the dimension fluctuation in 96 h after final milling are reduced by 72.99%, 82.75%, and 78.94%, respectively. Results indicate that the TVSR process could significantly reduce strain energy and residual stress in the thin-walled part, and the deformation control effect is even better than 96-h natural aging.

![Fig. 16 Deformation comparison](image-url)
5.3 Summary of deformation control method

In summary, the influences of equivalent bending stiffness and residual stress on the dimensional accuracy and stability in hundreds of hours after machining are investigated. It is found that the dimensions of the thin-walled parts continuously change with time after machining, and it preliminarily reaches a stable state after 72 or 96 h with the release and rebalance of residual stress. This process can be accelerated using TVSR process for 5.7 h. Compared with the original part (process 1), the final deformation and dimension fluctuation are evidently decreased using processes 2 and 3 in this study.

At present, materials are usually reserved as the machining allowance in the position that is easy to be out of tolerance. However, according to the theory model of machining deformation, bending stiffness of the workpiece directly determines the magnitude of deformation. The machining deformation and dimension fluctuation will be further controlled if the allowance can enhance the bending stiffness.

According to the experiment and simulation results of this study, a machining deformation control method based on topological optimization and stress relief process can be proposed, which has been proved to be effective in the machining of 7075 aluminum alloy and Ti6Al4V titanium alloy. The steps can be summarized as follows and Fig. 17 shows the flow chart of this deformation control method:

Step 1: Designing the stiffening ribs as the machining allowance using FEM topological optimization.
Step 2: Determining the intermediate workpiece according to topological optimization results.
Step 3: Machining the intermediate workpiece in rough machining process.
Step 4: Removing the fixture, and placing the workpiece statically in a free state for 96 h or conducting TVSR process for 5.7 h.
Step 5: Machining the reserved stiffening ribs in the finish machining.

The proposed method could effectively decrease machining deformation and improve machining accuracy of monolithic thin-walled parts without changing process device and parameters. The experiment and simulation data as well as the proposed method can offer a theoretical foundation and data support for technicians to control deformation of the thin-walled parts.

6 Conclusions

In order to study the influence of equivalent bending stiffness and stress relief process on dimensional stability of monolithic thin-walled parts, nine typical parts in three groups with two materials (7075 aluminum alloy for A1 ~ A3 and B1 ~ B3, and Ti6Al4V titanium alloy for B4 ~ B6) are machined and treated with different processes. Through comparing the machining deformation value under different processes, a machining deformation control method is summarized. Analyze the experimental results and get the following conclusions:

(1) The reserved stiffening ribs obtained by topological optimization can evidently enhance the bending stiffness, and the dimension of machined parts reaches a basically stable state after 96 h. When the 2nd milling for removing the reserved stiffening ribs is performed after 96 h natural aging, no obvious deformation occurs because of the release and rebalance of residual stress.

(2) The final maximum deformations of A2, B2, and B5 are reduced by 20.39%, 14.92%, and 3.80% with the process flow “1st milling reserving the stiffening ribs- > waiting 96 h - > 2nd milling removing the ribs,” respectively. And the final maximum deformations of A3, B3, and B6 are reduced by 23.51%, 24.16%, and 0.75%, when “waiting for 96 h” is replaced by TVSR process.
(3) A machining deformation control method is proposed based on the combined topological optimization and stress relief process. The process flow can be simply described as “1st milling reserving the stiffening ribs -> waiting 96 h or conducting TVSR process -> 2nd milling removing the ribs.” This method could effectively decrease machining deformation and improve machining accuracy of thin-walled parts.

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Author contribution The authors declare that they are all participants in the work and none of them performed only administrative functions.

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Data availability The data and material supporting the results of this study are included within the article, and the original data of experiment and simulation are available from the corresponding author on reasonable request for academic communication.

Code availability Not applicable.

Declarations Competing interests The authors declare no competing interests.

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