Travelling salesman problem concerning to manipulator
Kawasaki FS03N trajectory formation

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Abstract: An idea of travelling salesman problem (TSP) application to an optimal manipulator path finding was implemented. A set of known poses robot should pass is given. The poses sequence may vary. A traveling time is a performance criteria to be minimized. This task was realized concerning to a special case of Kawasaki FS03N manipulator movement by dynamic programming method. A special Bellman function state notation was used that in some cases gives an ability to solve the task at industrial robot controller. The state notation rule is one-one mapping rule between TSP state and array index it is stored.

Introduction
In a number of cases a manipulator’s goal is some object motion with specific poses it should be placed in a case when the poses order is not important – some workpiece several sides examining task for example. Herewith a necessary aim is to reduce some additive quality factor [1]. In that case the problem is the same with the travelling salesman problem (TSP) [2-4]. The aim is to find a time-optimal path-tracing [5-7]. The set of poses is given. Initial and final pose is set, intermediate poses should be passed single time only and interleaving order may differ. Sometimes classical TSP is formalized as follows: a complete weighted loop-free graph G is presented with vertex set \( N = \{1, 2, \ldots, n\} \); any arc weight is nonnegative; a Hamilton circuit with minimal weight should be found [8].

This article contains a particular solution of such a task. A Kawasaki FS03N manipulator should place a workpiece mockup (just “workpiece” in further description) in fixed and known poses set (Fig. 1). Initial and final pose is the same and is labeled \(a\).

The intermediate poses are grouped into an ordered set \( M = \{b, c, d, e\} \) (Fig. 2). The workpiece should be examined by an instrument with fixed position in space. The fastest path should be found. The intermediate poses sequence is not fixed (may vary). The manipulator should stay in each pose from among \( M \) for examination purpose during a fixed time.

A time of detail examination – summarized time the manipulator should stay at each pose – we assume as a constant value, neglect in discussion and exclude from the travelling time.

All Kawasaki FS03N manipulator traveling time values \(T_{ij}\) (in seconds) from pose \(i\) to pose \(j\) \((i, j = 0..4)\) are presented in a Table 1 (for a definite motion speed). Nonzero \(i, j\) are the orders of the corresponding poses from the set \(M\). \(i, j = 0\) correspond to the pose \(a\).
Fig. 1. Kawasaki FS03N robot with the workpiece mockup in the initial pose a

Fig. 2. A schematic view of the workpiece mockup at the set of poses M={b, c, d, e} and the instrument.
Table 1. Manipulator traveling time values $T_{ij}$ (in seconds) from pose $i$ to pose $j$

| $a$, manipulator toward pose, (j) | $a$, 0 | $b$, 1 | $c$, 2 | $d$, 3 | $e$, 4 |
|-----------------------------------|-------|-------|-------|-------|-------|
| $a$, 0                            | 5.12  | 3.984 | 2.528 | 2.64  |
| $b$, 1                            | 5.112 | 6.032 | 5.52  | 5.408 |
| $c$, 2                            | 3.984 | 6.032 | 3.488 | 4.4   |
| $d$, 3                            | 2.528 | 5.52  | 3.488 | 2.704 |
| $e$, 4                            | 2.64  | 5.408 | 4.4   | 2.704 |

All possible path variants are shown in Fig. 3. On the right of Fig. 3, opposite each terminal vertex the total traveling time is given for each case. Each vertex corresponds to some state: current manipulator pose and the poses it have not passed yet (a letter before the brackets and the letters in the brackets mark that correspondingly).

The task was considered and solved as the TSP with time-optimal sequence of the robot-manipulator internal poses determination. The solution we have got was realized at Kawasaki FS03N robot-manipulator. A special state notation was used to implement ease of dynamic programming realization for the assigned TSP task at the robot-manipulator.

State notation method for using dynamic programming for TSP

A calculated Bellman function value for almost any state should be stored at some memory address for further usage at dynamic programming method [9-13]. In this work the state notation is organized to compute the state memory adress and do a reverse computation – define the state by the corresponding address. We use very simple rule for that:

- the state is represented by a binary number-word (a "0110" as an example)
- a bit order is a manipulator pose number in the ordered set $M$;
- the passed poses are units, the poses are not yet passed – zeros;
- current pose is a bold unit;

For the 0110 state $c$, $d$ poses are passed, $d$ pose is current, $b$, $e$ poses aren’t passed yet (the initial and final pose $a$ is not used in state recording) (Fig. 3).

This state notation in Bellman function arguments gives visually quite understandable expression as follows:

$$B(0000) = \min\{T_{01} + B(1000), T_{02} + B(0100)\},$$

$$T_{03} + B(0010), T_{04} + B(0001)\},$$

$$B(1000) = \min\{T_{12} + B(1100), T_{13} + B(1010), T_{14} + B(1001)\},$$

$$B(0100) = \min\{T_{21} + B(1100), T_{23} + B(0110), T_{24} + B(0101)\},$$

... etc.
Fig. 3. Possible variants of the workpiece poses sequence during its handling time.
and also (see the Table 1):

\[
B(1100) = \min\{T_{23} + T_{34} + T_{40}, T_{24} + T_{43} + T_{30}\},
\]

\[
B(1100) = \min\{T_{13} + T_{34} + T_{40}, T_{14} + T_{43} + T_{30}\},
\]

… etc.

Here \( T_{ij} \) is a manipulator traveling time from pose \( i \) to pose \( j \) according to the Table 1.

The main features of the state notation rule for memory adress computing and ability of programming usage are the follows:

- A state is described by the couple of numbers \((p, n)\), where: \( p \) is a passed pose index (decimal representation of the binary number-word); \( n \) is a bit order of the current pose (unit). For example:

\[
B(0000) = B(0, 0), \quad B(1000) = B(8, 1),
\]

\[
B(0100) = B(4, 2), \quad B(1010) = B(10, 3).
\]

etc.

- Let enter a \( Bc \) array (an array element is a traveling time from corresponding current state to final pose – i.e. a Bellman function value) and a \( Bs \) array (an array element is a next optimal pose from corresponding current state). A matrix row corresponds to passed poses, a matrix column corresponds to current pose index (a pose serial number in the set of poses \( M \)). Then let Bellman function argument \((p, n)\) be cell adress of the arrays: \( p \) – row numer, \( n \) – column number.

- With such addressing mode approximately half of all \( Bc \) and \( Bs \) array cells will be empty. For example, for the 10-th array row only the first element (state “1010”) and the third element (state “1010”) will be filled. For reduction of unused address space it is rational to use a symmetry of binary notation of serial natural numbers, demonstrated in a Table 2. Asymmetry law mathematically is shown below:

\[
bin(i) = \overline{bin(2^N-i-1)}
\]

where \( i \) is a Table 2 row number in decimal representation, \( N \) is a binary digit number we use.

If only «unit cells» is filled with some information then using the described at the Table 2 symmetry we can write information to a Table 3 that don’t have empty/zero cells.

The row number \( k \) of the \( Bc \) and \( Bs \) arrays will be defined as

\[
k = \begin{cases} i & \text{at } i < 2^{N-1} \\ 2^N - i - 1 & \text{at } i \geq 2^{N-1} \end{cases}
\]
Table 2. Binary notation symmetry of row serial number

| № = i | 0  | 0  | 0  | 0  |
|-------|----|----|----|----|
| 0     |    | 0  | 0  | 0  |
| 1     |    | 0  | 0  | 1  |
| 2     |    | 0  | 0  | 1  |
| 3     |    | 0  | 1  | 0  |
| 4     |    | 0  | 0  | 1  |
| 5     |    | 0  | 1  | 0  |
| 6     |    | 0  | 1  | 1  |
| 7     |    | 0  | 1  | 1  |
| 8     |    | 1  | 0  | 0  |
| 9     |    | 1  | 0  | 0  |
| 10    |    | 1  | 0  | 1  |
| 11    |    | 1  | 0  | 1  |
| 12    |    | 1  | 1  | 0  |
| 13    |    | 1  | 1  | 0  |
| 14    |    | 1  | 1  | 1  |
| 15    |    | 1  | 1  | 1  |

Table 3. Information record rule illustration for the case of data filling only at nonzero array cells in the Table 2

| № = k | 1  | 1  | 1  | 1  |
|-------|----|----|----|----|
| 0     | 15 | 1  | 1  | 1  |
| 1     | 14 | 1  | 1  | 1  |
| 2     | 13 | 1  | 1  | 1  |
| 3     | 12 | 1  | 1  | 1  |
| 4     | 11 | 1  | 1  | 1  |
| 5     | 10 | 1  | 1  | 1  |
| 6     | 9  | 1  | 1  | 1  |
| 7     | 8  | 1  | 1  | 1  |

The $Bc$ and $Bs$ arrays are listed below at the Tables 4 and 5 correspondingly.
Table 4. $B_c$ array – optimal traveling time from a current state to the final pose

| № $i$ | $bin(i)$ | № $i$ | $bin(i)$ | $B$  | $C$  | $D$  | $E$  |
|------|---------|------|---------|------|------|------|------|
| 1    | 0001    | 14   | 1110    | –    | –    | –    | 17.34|
| 2    | 0010    | 13   | 1101    | –    | –    | 17.57| –    |
| 3    | 0011    | 12   | 1100    | 10.64| 8.83 | 14.63| 15.42|
| 4    | 0100    | 11   | 1011    | –    | 16.67| –    | –    |
| 5    | 0101    | 10   | 1010    | 13.07| 14.08| 10.53| 13.34|
| 6    | 0110    | 9    | 1001    | 12.05| 14.08| 13.22| 10.18|
| 7    | 0111    | 8    | 1000    | 14.86| –    | –    | –    |

Table 5. $B_s$ array – optimal further pose from a current state

| № $i$ | № $i$ | $B$ | $C$ | $D$ | $E$ |
|-------|-------|-----|-----|-----|-----|
| 1     | 14    | –   | –   | –   | 3   |
| 2     | 13    | –   | –   | 2   | –   |
| 3     | 12    | 4   | 3   | 2   | 1   |
| 4     | 11    | –   | 1   | –   | –   |
| 5     | 10    | 2   | 1   | 2   | 3   |
| 6     | 9     | 2   | 1   | 4   | 3   |
| 7     | 8     | 2   | –   | –   | –   |

The algorithm program realization

The described above memory addressing method was tested in the algorithm for optimal sequence choice of the manipulator - workpiece poses.

During dynamic programming realization at first step an algorithm consider all the states that correspondes with any two havn’t passed poses and the appropriate $B_c$ and $B_s$ cells are filling according the rule (1). Then at the second step all the states with three not passed poses are considered, etc.

The algorithm was realized in Matlab. Some of used built-in functions are:
- `dec2bin`, `bin2dec`, `str2num`, `num2str`, `strfind` – for translation between binary and decimal notations;
- `nchoosek` – all possible combinations of not passed poses at each step.

The algorithm work result is in the Tables 4 and 5. The cells, corresponding to $i = 7, 11, 13, 14$ of the Tables are empty, since the corresponding states don’t have variants of further trajectory.

Analysis of results and discussion

For better illustration of presented work the case with limit to four intermediate manipulator - workpiece poses is explored. Maximum time saving is defined by the variants with maximum and minimum traveling time. In this case it is 8.5%.

In general case many conditions affect on manipulator traveling time from one pose to another in the general case [14-17]. Some of them:
- the manipulator speed parameters;
- the instrument and base coordinate systems relative position choice [18];
– required movement accuracy;
– some type of movement parameters, etc.

In spite of above listed factors we got result that could say of:
– a manipulator path choice influence on time traveling by the given example;
– principal TSP applicability to manipulator path selection at the given conditions.

The dynamic programming state memory adressing was considered that gives some potential benefits in industrial computer realization of the TSP.

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