PROBING THE SPATIAL DISTRIBUTION OF EXTRASOLAR PLANETS WITH GRAVITATIONAL MICROLENSING

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ABSTRACT

To search for extrasolar planets, current microlensing follow-up experiments are monitoring events caused by stellar-mass lenses, hoping to detect the planet’s signature of the short-duration perturbation in the smooth lensing light curve of the primary. According to this strategy, however, it is possible to detect only planets located within a narrow region of separations from central stars. As a result, even if a large sample of planets are detected and the separations from their central stars are determined, it will be difficult to draw meaningful result about the spatial distribution of planets. An additional channel of microlensing planet detection is provided if the monitoring frequency of survey experiments is dramatically increased. From high-frequency monitoring experiments, such as the recently proposed Galactic Exoplanet Survey Telescope mission, one can detect two additional populations of planets, which are free-floating planets and bound planets with wide orbits around central stars. In this paper, we investigate the lensing properties of events caused by wide-orbit planets and find that the light curves of a significant fraction of these events will exhibit signatures of central stars, enabling one to distinguish them from events caused by free-floating planets. Because of the large primary/planet mass ratio, the effect of the central star endures to considerable separations. We find that for a Jupiter-mass planet the signatures of the central star can be detected with fractional deviations of $\geq 80\%$ from the best-fitting single-lens light curves for $\geq 5\%$ from the best-fitting single-lens light curves for $\geq 0$ from the best-fitting single-lens light curves for $\geq 10$ AU, and the probability is still substantial for planets with separations up to $\sim$20 AU. Therefore, detecting a large sample of these events will provide useful information about the distribution of extrasolar planets around their central stars. Proper estimation of the probability of distinguishing wide-orbit and free-floating planets will also be important for the correct determination of the frequency of free-floating planets, whose microlensing sample will be contaminated by wide-orbit planets.

Subject headings: gravitational lensing — planetary systems — planets and satellites: general

1. INTRODUCTION

A Galactic microlensing event occurs when a compact massive object (lens) approaches very close to the observer’s line of sight toward a background star (source). Because of lensing, the source star image is split into two with different fluxes from that of the unlensed source. The locations and magnifications of the individual images are

$$\theta_{\pm} = \frac{1}{2} \left( u \pm \sqrt{u^2 + 4 \frac{u}{u}} \right) \theta_E$$

and

$$A_{\pm} = \frac{u^2 + 1}{2u\sqrt{u^2 + 4} + 2} \frac{1}{2},$$

where $u$ is the projected lens-source separation vector normalized by the Einstein ring radius $\theta_E$. The Einstein ring represents the effective lensing region around the lens within which the combined source star flux is magnified greater than $3/\sqrt{5} \sim 1.34$. For a typical Galactic bulge event with a lens and a source located at $D_{\text{ol}} = 6$ kpc and $D_{\text{os}} = 8$ kpc, respectively, the Einstein ring has a radius of

$$\theta_E \sim 0.32 \text{ mas} \left( \frac{m}{0.3 \, M_\odot} \right)^{1/2} ,$$

where $m$ is the lens mass. The angular size of $\theta_E$ corresponds to the physical distance at the lens location of

$$r_E = D_{\text{ol}} \theta_E \sim 2 \text{ AU} \left( \frac{m}{0.3 \, M_\odot} \right)^{1/2} .$$

For Galactic events, the separation between the two images is $|\theta_+ - \theta_-| = (u^2 + 4)^{1/2} \theta_E \sim 2\theta_E \sim 0.6$ mas, which is too small for the images to be resolved. However, the flux of the combined image varies with time because of the relative motion of the observer, lens, and source, and thus lensing events can be identified from the variation of source star fluxes. The light curve of a lensing event is represented by

$$A = A_+ + A_- = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}} .$$

Light variation caused by lensing can be distinguished from...
other types of variations because of the smooth, symmetric, and nonrepeating characteristics of the lensing light curves. The duration of a lensing event is characterized by the Einstein timescale $t_E$, which represents the time required for the source to transit $\theta_E$. For a typical lens-source proper motion of $\mu_{\text{rel}} \sim 25 \text{ km s}^{-1} \text{ kpc}^{-1}$, the Einstein timescale has a value of

$$t_E = \frac{\theta_E}{\mu_{\text{rel}}} \sim 20 \text{ days} \left(\frac{m}{0.3 \, M_\odot}\right)^{1/2}.$$  \hspace{1cm} (6)

If an event is caused by a lens having a planetary-mass companion and the position of the companion happens to be near the path of one of the two images created by the primary lens, the planet will perturb the light from the nearby image, causing deviation in the lensing light curve of the primary (Mao & Paczynski 1991; Gould & Loeb 1992). For a planet with a mass ratio $q$ to the central star, the deviation lasts for a duration of $t_{\text{E},p} \sim \sqrt{q}t_E$, which corresponds to $\sim 1$ day for a Jupiter-mass planet. Because of the short duration, it is difficult to detect the planet-induced perturbations from the current survey-type experiments, which have typical monitoring frequencies of 1–2 per night. To increase the monitoring frequency, current experiments are employing early warning systems (Alcock et al. 1996; Afonso et al. 2001; Bond et al. 2001; Udalski et al. 1994) to issue alerts of ongoing events detected in the early stage of lensing magnification and follow-up observation programs (Alcock et al. 1997; Rhie et al. 1999; Albrow et al. 1998) to intensively monitor the alerted events. Once the deviation is detected and analyzed, one can determine the mass ratio and the projected separation (normalized by $\theta_E$) between the planet and central star, $d$ (Gaudi & Gould 1997).

One important drawback of the current microlensing planet search strategy is that one can detect only planets located within a narrow region of separations from central stars. This is because the images produced by the central star are located close to the Einstein ring during the event, and thus, only planets located near the ring can effectively perturb the images (see more details in § 2). As a result, even if a large sample of planets are detected and their separations are determined, no meaningful result can be drawn about the spatial distribution of planets around central stars.

There are two possible channels for detecting wide-orbit planets. One is through “repeating” events in which the source trajectory passes close to both the planet and the primary star, and the other is through “isolated” events in which the trajectory passes close to the planet only. Planets via the channel of repeating events can be detected by extending the time of follow-up monitoring of events (see the extensive works of Di Stefano & Scalzo 1999a, 1999b on this channel of planet detections). Detecting planets via the channel of isolated events, on the other hand, is difficult with the current experiments because these events occur without any warning and last for very short period of time. However, if the monitoring frequency is dramatically increased, it will be possible to detect a large sample of wide-orbit planets through this channel. Recently, such a high-frequency survey experiment, *Galactic Exoplanet Survey Telescope (GEST)*, was proposed to NASA by Bennett & Rhie (2002). The GEST mission is designed to continuously monitor $\sim 10^6$ Galactic bulge main-sequence stars with a frequency of several times per hour by using a 1–2 m aperture space telescope. Another population of planets that can also be detected by the high-frequency lensing survey is free-floating planets.

In this paper, we investigate the lensing properties of events caused by wide-orbit planets that will be detected by future high-frequency lensing experiments. Unlike the negligible effect of the wide-orbit planet on the lensing behavior of the primary, the effect of the primary on the lensing behavior of the planet may be important even at very large separations because of the large primary/planet mass ratio. If this is so, the signature of the central stars might be noticed for a significant fraction of events produced by wide-orbit planets, enabling one to distinguish them from events caused by free-floating planets.

The paper is organized as follows. In § 2, we discuss the basics of planetary microlensing that are required to describe the lensing behavior of events caused by wide-orbit planets. In § 3, we investigate the properties of wide-orbit planetary lensing in detail. In § 4, we estimate the probability of distinguishing events produced by wide-orbit and free-floating planets. In § 5, we summarize the results and discuss the implications of the results.

### 2. Basics of Planetary Microlensing

The lensing behavior of a system having a planetary-mass companion is described by the formalism of binary lensing with a very low mass ratio companion. If a source star located at $\zeta = \xi + i\eta$ in complex notation is lensed by two point-mass lenses with the individual locations of $z_{L,1} = x_{L,1} + iy_{L,1}$ and $z_{L,2} = x_{L,2} + iy_{L,2}$ and the mass fractions of $m_1$ and $m_2$, respectively, the locations of the resulting images $z = x + iy$ are obtained by solving the lens equation, which is represented by

$$\zeta = z + \frac{m_1}{z_{L,1} - z} + \frac{m_2}{z_{L,2} - z},$$  \hspace{1cm} (7)

where $\bar{z}$ denotes the complex conjugate of $z$ and all lengths are normalized by the Einstein ring radius corresponding to the total mass of the binary (combined Einstein ring radius). Since the lens equation describes a mapping from the lens plane to the source plane, finding image positions $(x, y)$ for a given source position $(\xi, \eta)$ requires inverting the lens equation. Although the lens equation for a binary lens system cannot be algebraically inverted because of its nonlinearity, it can be expressed as a fifth-order polynomial in $z$ and the image positions can be obtained by numerically solving the polynomial (Witt 1990). Since the lensing process conserves the source star surface brightness, the magnification of each image equals to the area ratio between the image and the unlensed source and mathematically it is obtained by computing the Jacobian of the mapping equation evaluated at the image position,

$$A_i = \left| 1 - \frac{\partial \zeta}{\partial z} \right|^{-1}.$$  \hspace{1cm} (8)

Then, the total magnification is given by the sum of the magnifications of the individual images, i.e., $A = \sum A_i$.

Because of the very small mass ratio of the planet to the primary, the planetary lensing behavior is well described by that of a single lens event for most of the event duration. However, noticeable deviations can occur if the planet is
located close to one of the images produced by the primary lens. The region around the image-perturbing planet’s location in the lens plane corresponds to the region around caustics in the source plane. Therefore, noticeable deviations occur when the source approaches the region around caustics. The caustics are the main new features of binary lensing and refer to the set of source positions at which the magnification of a point source becomes infinity. For the case of a wide-orbit planet, the caustic is located along the primary planet axis, and its location on the axis is approximated by

$$x_c \approx x_p - \frac{1}{x_p}$$  \hspace{1cm} (9)  

where $x_p$ is the position of the planet (in units of $\theta_E$) with respect to the primary, which is located at the origin (Griest & Safizadeh 1998). Then, caustics are located within the Einstein ring when the planetary separation is in the range of $0.6 \leq x_p \leq 1.6$, which is called the “lensing zone” (Gould & Loeb 1992; Wambsganss 1997). In addition, the size of the caustic, and thus the probability of detecting planet-induced deviations, is maximized when the planet is in the lensing zone. Under the current planet search strategy of monitoring events caused by stellar-mass lenses, therefore, only planets within the lensing zone can be effectively detected.

**Fig. 1.**—Contour maps of fractional magnification excesses, $\epsilon$, of planets with wide separations from central stars. The left-side panels show the maps for planets with projected separation from central stars of $l = 10, 14,$ and $18$ AU, respectively, and the right-side panels show the enlargements of the maps in the central region of the corresponding left-side maps. For all three cases, planets have a common mass ratio to the central star of $q = 0.003$. The locations are set so that the effective position of the planet (plus sign) is at the origin and all lengths are normalized by the Einstein ring radius of the planet, $\theta_{E_P}$. The dotted circle on each of the left-side panel represents the Einstein ring of the planet. The closed figure on each of the right panels (thick solid line) is the caustics. Both the planet and the central star are located on the $\xi$ axis and the central star is on the right. The position of the true planet position is marked by a filled dot. The physical separation between the planet and the central star, $l$, is set by assuming $r_E = 2$ AU. The source star is assumed to have a radius of $R_\star = 1 R_\odot$. Both gray scales and contours are used to represent the regions of significant deviations with $\epsilon \geq 3\%, 5\%$, and $10\%$, respectively. The straight lines are the source trajectories of events whose resulting light curves are presented in Fig. 2.
3. LENSING BY WIDE-ORBIT PLANET

As the separation between the planet and central star increases, the caustic shrinks rapidly. If the separation is significantly larger than the Einstein ring radius, both lens components behave as if they are two independent single lenses. Because of the deflection of light produced by the presence of the companion, however, the position of each lens is effectively shifted toward the companion (Di Stefano & Mao 1996). The effective positions of the individual lenses are given by

\[ \tilde{x}_{L,j} = x_{L,j} - \frac{m_j}{m_1/m_j} \frac{1}{\theta_{E,j}} \text{sign}(x_{L,j} - x_{L,j}), \]

where \( \theta_{E,j} \) represents the Einstein ring radius of each lens, the subscripts \( i \) and \( j \) are used to denote one lens component and its companion, respectively, and the term \(-\text{sign}(x_{L,j} - x_{L,j})\) implies that the shift is toward the direction of the companion. Then, the amount of the positional shift of the central star due to the planet is \( \delta_x = [\tilde{x}_{E} - x_s] \sim q/d, \) which is negligible because of the combination of small \( q \) and large \( d. \) On the other hand, the amount of the shift of the planet’s position due to its central star, \( \delta_p = [\tilde{x}_p - x_p] \sim 1/d, \) is not negligible because it does not depend on \( q. \) Note that the effective position of the wide-orbit planet is

\[ \tilde{x}_p \sim x_p - \frac{1}{d} = x_p - \frac{1}{x_p}, \]

implying that it corresponds to the position of the caustic (cf. eq. [9]). Since significant deviations can occur only if the source trajectory passes close to caustics, although most of the light curve of the event caused by a wide-orbit planet is approximated by that of a single-lens event produced by the planet at its effective position, its central part can be distorted because of the effect of the central star.

Another effect to be taken into consideration in describing the lensing behavior of events caused by wide-orbit planets is the finite size of the source star. The finite source effect for these events is important, because the source size compared to the Einstein ring radius of the planet, \( \theta_{E,p} \equiv \sqrt{\theta_{E}^2 + \theta_s^2} \), is no longer negligible (Bennett & Rhie 1996). For events caused by a Jupiter-mass planet, for example, the radii of source stars correspond to \( \theta_s \sim 3, 5, 7 \) \( \text{mas}, \) and the most common stars to be monitored by the GEST mission will be main-sequence stars, the maps are constructed assuming that the source star has a radius of \( R_s = 1 R_\odot. \) Each map is centered at the effective position of the planet (plus sign) and the true position of the planet is marked by a filled circle. All lengths are normalized by \( \theta_{E,p}. \)

From the maps, one finds that, as expected, the effective planet position matches very well the center of the caustic. One also finds that the size of significant deviation regions (compared to the size of the Einstein ring of the planet) is not negligible even for planets located at large separations.

In Figure 2, we present several example light curves of events resulting from the source trajectories marked in Figure 1.

![Light curves of events caused by wide-orbit planets with various separations from central stars](image-url)

**Fig. 2.** Light curves of events caused by wide-orbit planets with various separations from central stars. The source trajectories responsible for the events are marked in the corresponding maps in Fig. 1. The time is normalized by the Einstein timescale of the planet, \( t_{E,p}. \) The inset shows the part of the light curves near the peaks.
To see the variation of anomaly pattern depending on the planet’s mass ratio, we also construct maps for planets with different mass ratios and present them in Figure 3. For these maps, the planet-primary separation is fixed as $d = 7$ ($l = 14$ AU). The tested planets have mass ratios of $q = 0.003$, 0.001, and 0.0001, which correspond to a Jupiter-, Saturn-, and $10 M_\odot$ mass planet, respectively, around a $0.3 M_\odot$ star. The dot on the upper right corner of each left panel represents source size relative to the Einstein ring of the planet. We note that since all lengths are scaled by $\theta_{\text{E, p}}$, which is proportional to $\sqrt{q}$, the source size appears to be different although the actual size is the same. From the maps, we find that finite-source effect does not seriously affect the anomaly patterns for planets with $q \gtrsim 10^{-3}$. One also finds that for planets in this mass regime, the maps have similar patterns because both the size of the caustic and the planet’s Einstein ring radius have the same dependency on the mass ratio, i.e., $\propto \sqrt{q}$, and thus, they decrease by the same scale as $q$ decreases.

As the mass ratio further decreases, however, the finite-source effect becomes important. We find that for the case of planets with $q \lesssim 10^{-4}$, the source size becomes bigger than the size of the caustic even for main-sequence source stars, and the signatures of the central star is seriously washed out by the finite-source effect.

4. DETECTION PROBABILITY

By using the excess map, we then estimate the probability of distinguishing events caused by wide-orbit planets from those produced by free-floating planets. For this estimation, we assume that events with $A \gtrsim A_{\text{th}}$ are continuously monitored continuously.

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Fig. 3.—Excess magnification maps of planets with different mass ratios. The planet-primary separation is fixed as $d = 7$ ($l = 14$ AU) and the tested planets have mass ratios of $q = 0.003$, 0.001, and 0.0001, which correspond to a Jupiter-, Saturn-, and $10 M_\odot$ mass planet, respectively, around a $0.3 M_\odot$ star. The filled circle on the right upper corner of each left panel represents the source size, which is assumed to be $R_s = 1 R_\odot$. Notations and scales of lengths are similar to those in Fig. 1. Note that since all lengths are scaled by $\theta_{\text{E, p}}$, the sizes of the individual Einstein rings appear to be the same, although the actual size is proportional to $\sqrt{q}$, and the source sizes appear to be different, although they are the same.
monitored with a frequency of 3 times per hour. Here $A_{th}$ is the threshold magnification, which is required for event identification. For each planet separation, we produce 900 light curves resulting from source trajectories with random orientations relative to the planet-primary axis and uniform impact parameters to the planet within the range of $0 \leq u_p \leq u_{th}$, where $u_{th}$ is the impact parameter corresponding to $A_{th}$. With the excess map, light curves are produced by the one-dimensional cut through the map. Since the map is constructed by considering finite-source effect, the light curves produced in this way automatically incorporate the effect. To be identified as a bound planet, it is assumed that the light curve should have at least one data point with deviations greater than a threshold value of $\epsilon_{th} = 5\%$. We note that the photometric precision of the GEST mission will be $\leq 1\%$ even for main-sequence source stars, and thus the adopted detection threshold is very conservative choice.

With this detection criteria, the probability is determined as the ratio of the number of events with noticeable deviations to the total number of tested events.

In Figure 4, we present the resulting probabilities as a function of planetary separation for planets with different mass ratios, which are distinguished by different line colors. The sets of curves with different line types are the probabilities for different threshold magnifications. When the threshold is fixed, the probability becomes smaller as $q$ decreases because of the combination of shorter duration of perturbations and larger effect of extended sources. For a given planet, the probability increases as the threshold magnification increases. This is because the deviation region is confined to the central region around the effective planet position and thus the chances to detect deviations are greater for higher magnification events. From the figure, we find that the signatures of central stars can be detected with significant probabilities ($\geq 60\%$) for planets with separations $l \approx 10$ AU, which roughly corresponds to the distance of Saturn from the Sun. Although the probability drops rapidly for planets with $q \leq 10^{-4}$, the probability is still substantial for giant planets with separations up to $\sim 20$ AU, corresponding to the separation between Uranus and the Sun.

We investigate the dependency of the probability on the applied detection criteria. For this, we estimate the probabilities under different values of the threshold excess and number of data points, $N_{th}$, required to distinguish bound and free-floating planets. We test four cases, $(\epsilon_{th}, N_{th}) = (3\%, 1), (3\%, 3), (5\%, 1)$, and $(5\%, 3)$. Since we assume that the observational frequency is 3 times hr$^{-1}$, $N_{th} = 3$ implies that to be identified as a bound planet, the signal should last for more than an hour. Figure 5 shows the resulting probabilities. From the figure, one finds that the probability is sensitive to the applied threshold excess, implying that precise photometry is essential for efficient discrimination of bound planets from free-floating planets. On the other hand, the probability is less sensitive to $N_{th}$.

5. SUMMARY AND CONCLUSION

Under the current microlensing planet search strategy of monitoring events caused by stellar-mass lenses, only planets located within a narrow region of separations from central stars can be effectively detected. However, with the dramatic increase of the monitoring frequency, two additional populations of free-floating and wide-orbit planets can be detected. We investigated the lensing properties of events caused by wide-orbit planets and found that a significant fraction of these events could be distinguished from those caused by free-floating planets. We determined that even with moderate detection criteria the probability to detect signatures of central stars for events caused by wide-orbit planets would be $\geq 80\%$ for giant planets with separations $\leq 10$ AU, and the probability is still substantial for
planets with separations up to \(~20\) AU. Detecting a large sample of these events will be important because they can provide useful information about the spatial distribution of extrasolar planets, which cannot be obtained by the sample to be acquired under the current microlensing planet search strategy or from other planet search methods such as the radial velocity and transit methods. In addition, proper estimation of the probability of distinguishing events caused by the two populations of planets will be important for the correct determination of the frequency of free-floating planets, whose microlensing sample will be contaminated by wide-orbits planets.

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