Hydraulic fracturing is the state-of-the-art development technique to optimize the productivity of low permeability hydrocarbon reservoirs and is performed both in production and injection wells. Depending on the proppant volumes placed during a hydraulic fracturing operation, fracture half-lengths of 150–200 m and even further are desired in order to tap oil and gas resource plays. Development of hydraulic fractures mainly occurs along the axes of the regional stress field assuming fracturing operations are conducted in areas lacking a significant alteration of pressure and temperature, i.e. new exploration wells [1].

Modern pumping units which are employed for hydraulic fracturing operations achieve wellhead pressures of up to 20 MPa during the injection of fracturing fluid into the reservoir. Spontaneous hydraulic-fracturing growth in the horizon of interest is initiated as soon as injection pressure exceeds yield strength of the particular reservoir rock [2–8]. Real-time data acquisition of wellhead pressures from active and observation wells and the results of interference testing confirm that in some cases hydraulic fractures may achieve a length of up to 1 000 m [8].

S. Ekie et al. [9] present a mathematical model for the fracture formation for two vertical wells — one active well and one observation well. Conducting pulse-tests for the tracking of registration response pressures in several vertical observation wells can be used to determine the orientation of the fracture originating from the active fracture well. N. Mousli et al. [10], D. Meehan et al. [11] presented analytical solutions of modeling interference test for wells with parallel. D. Tiab and E. Abobise [12] discussed the design of pulse testing of a fractured active well and vertical observation well (without fractures), presenting correlations for fracture orientation, average permeability and other parameters of the reservoir on the response amplitude and the response time in the observation well. In the same year K. Cooper and R. Collins [13] applied of a mathematical model which is based on interference test of a two-well system with a fractured (one fracture) and another non-fracture well that was used to interpret the results of a field case and to estimate underlying parameters of the system. The article of H. Najurieta et al. [14] presents a two-dimensional model for heterogeneous anisotropic reservoirs; demonstrating the mathematical modeling for all wells. It can be used to calculate 2D pressure maps, transmissivity and diffusivity between active and observation wells estimated. Reservoir case studies with conductive faults were also presented in the article [15].

Contrary to the abovementioned papers we elaborate a case that consists of the active fractured well and the observation fractured well (fractures are oriented along the regional stress) either having a low-permeability porous bridge between the fracture tips, or two wells which are being directly connected through intersecting fracture planes without a porous media bridge. Furthermore, we want to highlight the results of interfe-
rence test from the work [8] by interpreting the chosen data model and providing an estimation of the fracture and reservoir characteristics. Formulating the abovementioned problem, injection pressure is assumed to surpass the fracture closure pressure, i.e. its geometry (height, length, width) is treated constant.

**Mathematical model.** In our approach we consider a symmetric geometry of two hydraulic fractured vertical wells each with one fracture as shown in Figure 1.

The wells are located in the same row and have same orientation along the lines of the regional stress. Fractures are symmetrical and parallel to the Ox axis, \( x_{f_1} \) and \( x_{f_2} \) are the length of the active well and the observation well respectively, both fractures have same width \( w_f \) and permeability \( k_f \). \( L \) is the distance between the boundaries of layer and fractures. In our model the distance \( d \) between the fracture tips acts as a so called porous medium bridge.

![Model geometry with distance d > 0 between the fracture tips](image)

**Fig. 1. Model geometry with distance \( d > 0 \) between the fracture tips**

We assume that the reservoir is isotropic, i.e. \( k_{w_x} = k_{w_y} \), and formation thickness \( h \) is constant, coinciding with the fracture height. Top and bottom of the reservoir are no-flow boundaries. Here \( x \) and \( y \) is the distance along the \( x \) axis and along \( y \) axis respectively, \( p \) is the reservoir pressure, which is described by the Laplace equation. With respect to the view from the problem’s symmetry \( Ox \) axis or along the fracture planes, half of the computational domain is considered. Fluid flow in the fractures behaves only one-dimensional. We suppose that viscosity of reservoir liquid differs a little from viscosity of the injected liquid. Wherein injected agent is water.

Distribution of pressure within the fractures varies according to their length and is constant in each vertical section, as described by equation (1)

\[
\frac{\partial^2 p_f}{\partial x^2} = \frac{\varphi_f \mu c_{f_1}}{k_f} \frac{\partial p_f}{\partial t},
\]

\( L \leq x \leq L + 2x_{f_1}, L + 2x_{f_1} + d \leq x \leq L + 2x_{f_1} + d + 2x_{f_2}, \quad \) (1)

\( 0 \leq y \leq w_f, \)

where is in the formation the pressure is given by equation (2)

\[
\frac{\partial^2 p_m}{\partial x^2} + \frac{\partial^2 p_m}{\partial y^2} = \frac{\phi_m \mu c_{m_1}}{k_m} \frac{\partial p_m}{\partial t}.
\]

(2)

The inflow of liquid at the boundary of reservoir and fractures in the active well \(( L \leq x \leq L + 2x_{f_1}, \quad 0 \leq y \leq w_f / 2 )\) described by equation (3)
\[ q_a = -2 \left( (x_{f1} \cdot h) \frac{k_m \partial p_m}{\mu \partial y} \bigg|_{y=w/2} + (w_j/2 \cdot h) \left( \frac{k_m \partial p_m}{\mu \partial x} \bigg|_{x=L} + \frac{k_m \partial p_m}{\mu \partial x} \bigg|_{x=L+2x_{f1}} \right) \right), \]  
\[ q_o = -2 \left( (x_{f2} \cdot h) \frac{k_m \partial p_m}{\mu \partial y} \bigg|_{y=w/2} + (w_j/2 \cdot h) \left( \frac{k_m \partial p_m}{\mu \partial x} \bigg|_{x=k-L+2x_{f1}+2x_{f2}} + \frac{k_m \partial p_m}{\mu \partial x} \bigg|_{x=k-L+2x_{f1}+2x_{f2}} \right) \right). \]  

and in the observation well \((L + 2x_{f1} + d \leq x \leq L + 2x_{f1} + 2x_{f2}, 0 \leq y \leq w_j/2)\), described by equation (4)

Boundary conditions at time \(t = 0\) in the «fracture — formation» and at the outer boundary of the reservoir suggest a constant initial pressure

\[ p_m \bigg|_{t=0} = p_j \bigg|_{t=0} = p_i, \]
\[ p_m \bigg|_{y=L} = p_m \bigg|_{x=0} = p_m \bigg|_{x=2L+2x_{f1}+2x_{f2}} = p_i. \]

At the boundary of the «fracture — formation» pressures and fluid flow are equalized by equation (6)

\[ p_f \bigg|_{y=w/2} = p_m \bigg|_{y=w/2}, \]
\[ \frac{k_f}{\mu} \frac{\partial p_f}{\partial y} \bigg|_{y=w/2} = \frac{k_m}{\mu} \frac{\partial p_m}{\partial y} \bigg|_{y=w/2}. \]

The condition of symmetry across the Ox axis, i.e. along fractures, is given in equation (7)

\[ \frac{\partial p_f}{\partial x} \bigg|_{y=0} = \frac{\partial p_m}{\partial x} \bigg|_{y=0} = 0. \]

The boundary conditions in the wells are determined by setting the pressure or flow rate of injected fluid as follows (equation 8)

\[ p_f \bigg|_{y=L+x_{f1}} = p_{wA}, \]
\[ \frac{\partial p_f}{\partial x} \bigg|_{y=0} = -\frac{Q_{wA} \mu}{2k_f w_j h}. \]

Here, \(p_m\) and \(p_f\) represent the pressure, \(k_m\) and \(k_f\) are the permeability, \(\phi_m\) and \(\phi_f\) are the porosity, and \(c_{nf}\) and \(c_{ff}\) are the total compressibility of the fracture and

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formation (matrix) respectively. $p_i$ is initial reservoir pressure, $p_{w|A}$ is pressure in the well, $Q_{w|A}$ is the flow rate at the active well.

**Effect of the porous medium bridge between the fractures on the pressure response:** $d > 0$. The system of equations (1)-(4) and the corresponding boundary conditions (5)-(8) for the geometry, as shown in Figure 1, were solved using the finite difference method for the Newton iteration scheme on a non-uniform rectangular difference grid [16]. The accuracy of the approximation was tested against an analytical solution for the case of wells with a single vertical fracture of finite conductivity [17]. An example of calculation was performed using the following parameters of the system:

- $k_m = 1 \times 10^{-15}$ $m^2$;
- $\mu = 0.3$ mPa·s;
- $L = 500$ m;
- $h = 21.23$ m;
- $\phi_m = 0.17$;
- $\phi_f = 0.414$;
- $c_m = 3.6687 \times 10^{-9}$ $1/Pa$;
- $c_f = 9.4845 \times 10^{-9}$ $1/Pa$;
- $d = 1, 3, 5, 10, 30, 100, 200, 500, 800$ m;
- $x_{f1} = 900 - d$ m;
- $x_{f2} = 100$ m;
- $k_f w_f = 10 \times 10^{-12}$ $m^2$/m;
- $p_i = 27 \times 10^6$ Pa.

Subsequently, we began modeling of interference test to study the effect of a porous medium between the fractures. In the active wells we prescribed a variety of cycles: constant pressure on the injection cycle $p_f \big|_{y=0} = p_w$ and a rate equal to 0 at the stop cycle $\partial \frac{\partial p_f}{\partial y} \big|_{y=0} = 0$, thereby gaining reproducible responses of the pressure in observation wells. Figure 2 shows the pressure evolution in the active well (Fig. 2 a) and the pressure response in observation wells for different values of the distance between the fracture tips: $d = 100, 200, 500, 800$ m (Fig. 2 b) and $d = 3, 5, 10, 30$ m (Fig. 2 c). The outcome shows that for $d > 30$ m a poor pressure response would be seen in the observation well. Diagnosing the pressure response of field measurements might become a difficult task when data are noisy in case of low accuracy measurement equipment. However, if $d \leq 30$ m (Fig. 2 c), a clear signal from of reaction pressure can be obtained. Furthermore, the computed pressure amplitude in the observation well is increasing when the distance between the fracture tips becomes more narrow.

![Fig. 2](image)

The pressure profiles along the Ox axis with fractures at $d = 800$ m (Fig. 3 a), $d = 30$ m (Fig. 3 b), $d = 1$ m (Fig. 3 c) at time 1 and 30 days is shown in Figure. 3. In the case of $d = 800$ m (Fig. 3 a) the pressure is increasing only along the fractures in the active...
well, the pressure in observation wells remains equal to the initial reservoir pressure \( p_i \).

Comparing the profiles with \( d = 30 \text{ m} \) (Fig. 3 b) and \( d = 1 \text{ m} \) (Fig. 3 c) we can infer that the greater the distance between the fractures, the greater the pressure loss between the active and the observation wells.

Figure 4 shows the pressure distribution in the «wells — fractures — formation» system in 2D for a range of \( d = 500 \text{ m} \) (Fig. 4 a), \( d = 30 \text{ m} \) (Fig. 4 b) and \( d = 1 \text{ m} \) (Fig. 4 c) at a time of 30 days. It is seen that for \( d = 500 \text{ m} \) high pressure located just around the active well (along the fracture and around the reservoir) and therefore the response of pressure is not diagnosed in the observation well. However, for \( d = 1 \text{ m} \) and \( d = 30 \text{ m} \) high pressure regimes are distributed along the two wells and their attached fracture.

The presence of a small porous medium bridges having low filtration properties leads to a significant loss of pressure response between the active and the observation wells.

The field case which are presented by A. Davletbaev et al. [8] show that pressure is responding almost instantaneous between the active and observation wells. Furthermore, the pressure difference between the wells is much smaller than in the results which are shown in Figures 2 – 4. Thus, the case of two fractures without the porous medium bridge must be considered for the interpretation of the interference test with \( d = 0 \).

The case without the porous medium bridge between the fractures: \( d = 0 \). For the interpretation of the field test we have chosen the case of a single fracture intersecting both the active and the observation well. In this geometry the fracture has a length \( 2x_{f1} + 2x_{f2} \) and an absent porous medium bridge between the fractures, i. e. \( d = 0 \). In this system the response time of pressure and differential pressure between the active and observation wells is given by the dimensionless group for fracture conductivity.
In Figure 5 we plot sensitivity curves for different pressures in the active and observation well for the calculations performed with a range of fracture conductivities, i.e., $F_{CD} = 10$ (Fig. 5a), $F_{CD} = 25$ (Fig. 5b), $F_{CD} = 100$ (Fig. 5c). The Figure emphasizes that higher fracture conductivities result in a better match for both pressure changes and absolute values of pressures in the active and the observation wells.

Field case interpretation. The mathematical model, which is presented by A. Davletbaev et al. [8], was used to interpret interference-test pressure data for wells pair (active well and observation well) and to acquire refined fracture parameters. History of injected water volumes into the active well was included in the mathematical model. A genetic algorithm is applied to obtain the best combination of the measured pressure and the computed (theoretical) pressure in the observation well. The following model parameters were used in the interpretation of measured data: $\mu = 0.33$ mPa·s; $L = 200$ m; $h = 9.4$ m; $\phi_f = \phi_m = 0.16$; $c_{f1} = c_{mf} = 9.48 \times 10^{-9}$ 1/Pa; $d = 0$ m; $x_{f1} = 846$ m; $x_{f2} = 100$ m, the other parameters ($k_m, p_i, F_{CD}$) were determined by simulation.

Figure 6 shows the best combination of calculated and measured curves resulting after 100 iterations performed. The best combination was obtained for a matrix rock permeability to the water phase $k_{mw} = 0.293 \times 10^{-15}$ m² (if phase relative permeability of water $k_{mrw} = 0.2$, then absolute matrix permeability $k_m = 1.465 \times 10^{-15}$ m²), resulting in a dimensionless fracture conductivity $F_{CD}$ of 58.5, and a reservoir pressure $p_i = 33.13$ MPa.
Conclusions

- By modeling the injection of water into the «well — fracture — formation» geometry we demonstrate effects for several arrangements of a porous medium bridge and fracture conductivity on the distribution of the pressure field and the pressure response between the active and the observation wells. In presence of a low permeability porous medium bridge between the active and the observation wells we found that fast pressure pulses in the observation well cannot be produced, which could also be seen in field tests. Therefore, the field pulse test was simulated by two fractures without porous medium bridge.

- According to the hydraulic fracturing report the initial dimensionless fracture conductivity and half-length in the active well was 14.4, and 130 m respectively. However, the analysis of a field pulse test identifies a dimensionless fracture conductivity of 58.5 and fracture half-length of 946 m. Thus, the injection above the formation fracturing pressure leads to spontaneous growth of fracture length and increase of the conductivity in injection wells.

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