A Global Analysis of DIS Data at Small-x with Running Coupling BK Evolution

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We present a global fit to the structure function $F_2$ measured in lepton-proton experiments at small values of Bjorken-$x$, $x \leq 0.01$, for all experimentally available values of $Q^2$, $0.045 \text{ GeV}^2 \leq Q^2 \leq 800 \text{ GeV}^2$, using the Balitsky-Kovchegov equation including running coupling corrections. Using our fits to $F_2$, we reproduce available data for $F_L$ and perform predictions, parameter-free and completely driven by small-$x$ evolution, to the kinematic range relevant for the LHeC.

Deep Inelastic lepton-hadron Scattering (DIS) experiments constitute one of the main sources of information about the partonic structure of hadrons, as well as a testing ground of pQCD techniques. The DIS program carried out in HERA has explored an unprecedented kinematic regime in the DIS variables $Q^2$ and Bjorken-$x$. At small values of $x$, the emission of additional soft gluons in the hadron wavefunction is enhanced by factors of $\alpha_s \ln 1/x \sim O(1)$, resulting in a fast growth of the gluon density of the hadron with decreasing Bjorken-$x$. Such growth has been observed in data and, at small enough values of $x$, or, equivalently, for large gluon densities, it is expected to be tamed by repulsive gluon-gluon self-interactions. The latter mechanism is known as \textit{saturation} of the gluon distribution. It is a non-linear phenomenon, closely related to unitarity of the theory, which is not included in the more standard linear approaches to DIS based on the DGLAP or BFKL pQCD evolution equations. Here we present an analysis of available DIS data at small-$x$ based on the non-linear Balitsky-Kovchegov (BK) equation including the recently calculated running coupling corrections to the evolution kernel \cite{1, 2, 3, 4}.

The starting point of our analysis is the dipole model of DIS, which stems from the $k_t$-factorization theorems in the high-energy limit. It states that, at small-$x$, the virtual photon-proton cross section can be written as a convolution of the light-cone wavefunction for a virtual photon to split into a quark-antiquark dipole, $|\Psi_{T,L}|^2$, times the cross section for dipole-proton interaction. For a hadron homogeneous in the transverse plane, one has:

$$\sigma_{T,L}^\gamma p(x, Q^2) = \sigma_0 \int_0^1 dz \, dr \, |\Psi_{T,L}(z, Q^2, r)|^2 \, N(r, x),$$

where the subscripts T, L stand for the polarization of the virtual photon (transverse or longitudinal), $r$ is the dipole transverse size and $Q^2$ the photon virtuality. Explicit expressions for $\Psi_{T,L}$ in the case of any number of active flavors can be found in \cite{5}. In our analysis we consider only the three flavor case, i.e., with no charm quark. All the information about the strong interactions, and also all the $x$ dependence of the absorption cross section, is encoded in the imaginary part of the dipole scattering amplitude, $N(r, x)$ (related to the dipole cross
section through the optical theorem). The normalization $\sigma_0$ in Eq. (1) originates from integration over the transverse plane, trivial in the case of a cylindrical proton considered here, and will be one of the free parameters in the fit. The inclusive DIS structure function is then given by

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T + \sigma_L), \quad (2)$$

whereas the longitudinal structure function $F_L(x, Q^2)$ is obtained by just considering the longitudinal component.

General arguments in QCD indicate that the dipole scattering amplitude is small for small dipole sizes $r$ (color transparency), whereas large dipoles suffer a large absorption. Phenomenological works in the context of saturation parametrize the transition between these two regimes, dilute and saturated, in terms of a $x$-dependent saturation scale $Q_s^2(x)$, such that $\mathcal{N}(r, x) \sim r^2$ for $r \ll 1/Q_s(x)$, whereas $\mathcal{N}(r, x) \sim 1$ for $r \gg 1/Q_s$. The details of the parametrization depend on the physical input considered in each case. The leading $x$-dependence of the structure functions is thus encoded in that of the saturation scale, commonly assumed to have a power-law behavior: $Q_s^2(x) \sim (x_0/x)^\lambda$ GeV$^2$, with the evolution speed $\lambda$ being a free parameter fitted to experimental data. The results from different groups, albeit using slightly different parametrizations of the dipole amplitude, report $\lambda \sim 0.2 \div 0.3$.

In this work, rather than resorting to phenomenological models, we calculate the $x$-dependence of the dipole amplitude in terms of the pQCD non-linear BK evolution equation including the recently calculated running coupling corrections. Importantly, the original, leading-log (LL) in $\alpha_s \ln 1/x$ BK equation yields an evolution speed $\lambda \sim 4.8 \alpha_s$, with $\alpha_s$ the bare coupling [6]. Such value is too large to attempt a good description of data. Importantly, the running coupling corrections to the LL kernel obtained through resummation of $\alpha_s N_f$ terms bring the evolution speed down to values compatible with those extracted from data, among other interesting dynamical effects [3]. The BK equation reads

$$\frac{\partial \mathcal{N}(r, Y)}{\partial \ln x_0/x} = \int d\mathbf{r}_1 K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \times [\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x) \mathcal{N}(r_2, x)], \quad (3)$$

where $\mathbf{r}_2 = \mathbf{r} - \mathbf{r}_1$ and $x_0$ is the starting point for the evolution. In our case $x_0 = 10^{-2}$ is the largest experimental value of $x$ included in the fit. The precise shape of the running coupling kernel $K$ depends on the scheme chosen to single out the ultra-violet divergent contributions from the finite ones that originate after the resummation of quark loops. It was found in [4] that the prescription suggested by Balitsky in [2] minimizes the role of the subleading contributions, and is therefore better suited for phenomenological applications. The corresponding kernel is

$$K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_1^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_2^2)} - 1 \right) \right]. \quad (4)$$

Details about the numerical method used to solve the BK equation can be found in [4]. We evaluate the coupling according to the standard one-loop QCD expression. In order to regularize it in the infrared, we freeze it to a constant value for $r > r_{fr}$, with $\alpha_s(r_{fr}^2) = 0.7$. Thus

$$\alpha_s(r^2) = \frac{12 \pi}{(11N_c - 2N_f) \ln \left( \frac{4C_s^2}{r^2 \Lambda_{QCD}} \right)} \text{ for } r < r_{fr},$$

$$\alpha_s(r_{fr}^2) = 0.7.$$
Table 1: Values of the fitting parameters from the fit to available $F_2(x, Q^2)$ data with $x \leq 10^{-2}$ and for all available values of $Q^2$, $0.045 \text{ GeV}^2 \leq Q^2 \leq 800 \text{ GeV}^2$.

| Initial condition | $\sigma_0$ (mb) | $Q_{s0}^2$ (GeV$^2$) | $C^2$ | $\gamma$ | $\chi^2$/d.o.f. |
|------------------|-----------------|------------------|------|---------|-----------------|
| GBW              | 31.59           | 0.24             | 5.3  | 1 (fixed) | 916.3/844=1.086 |
| MV               | 32.77           | 0.15             | 6.5  | 1.13    | 906.0/843=1.075 |

Figure 1: Comparison between a selection of experimental data and the results from the fit for $F_2(x, Q^2)$. Solid red lines correspond to GBW i.c., and dotted blue ones to MV i.c.

and $\alpha_s(r^2) = 0.7$ for $r > r_f$. The parameter $C$ in Eq. (5) is a free parameter in the fit. We take $\Lambda_{QCD} = 0.241$ GeV. A final ingredient in our analyses is the initial condition for the evolution, i.e. the specific form of the dipole amplitude at $x = 10^{-2}$. We considered two different initial conditions, GBW:

$$N^{GBW}(r, x = 10^{-2}) = 1 - \exp \left[ - \left( \frac{r^2Q_{s0}^2}{4} \right)^\gamma \right], \quad (6)$$

and MV:

$$N^{MV}(r, x = 10^{-2}) = 1 - \exp \left[ - \left( \frac{r^2Q_{s0}^2}{4} \right)^\gamma \ln \left( \frac{1}{r\Lambda_{QCD} + e} \right) \right], \quad (7)$$

where $Q_{s0}^2$ is the initial saturation scale and $\gamma$ is another free parameter which controls the small $r$ behavior of the dipole amplitude. For the fit with GBW i.c. $\gamma$ is kept fixed to 1.

All in all, there are four (three when $\gamma$ is fixed) free parameters in the fit: The initial saturation scale $Q_{s0}$, the overall normalization $\sigma_0$, the infrared parameter $C$ and the
anomalous dimension $\gamma$. With this setup we obtain an excellent fit to available experimental data for the inclusive DIS structure function $F_2(x, Q^2)$ with $x < 10^{-2}$ and all existing values for $Q^2$, $0.045 \leq Q^2 \leq 800 \text{ GeV}^2$, which adds up to 847 data points. In order to smoothly go to photoproduction, we have used the following redefinition of the Bjorken variable: $\tilde{x} = x (1 + 4 m_f^2/Q^2)$, with $m_f = 0.14 \text{ GeV}$ for the three light flavors considered in this work. The experimental data included in the fit has been obtained by the H1 and ZEUS Collaborations (HERA) and by the E665 (Fermilab) and NMC (CERN) Collaborations. The parameters from the fit are presented in Table 1. We have checked that the quality of the fit and the values of the parameters are stable under the restriction of the data range to the region $Q^2 < 50 \text{ GeV}^2$ (which leaves 703 data points for the fit). A comparison between a selection of experimental data and the results from the fit for $F_2(x, Q^2)$ is shown in Fig. 1. The two initial conditions GBW and MV offer equally good fits to data. With the parameters values yielded by the fit, we obtain a good description of the longitudinal structure function, as shown in Fig. 2.

The analyses of available data, including this work, offer compelling evidence for the presence of gluon saturation effects. However, a clear distinction from alternative (linear) approaches cannot yet be established unambiguously. Indeed, a good description of all data for $Q^2 \geq 1$ can be achieved using DGLAP fits, even though the very application of DGLAP at such low $Q^2$ is questionable. On the other hand, the observables like $F_L$ which are more sensitive to the gluon distribution are poorly determined experimentally. The proposed LHeC may have the potential to disentangle different physical scenarios through a deeper exploration of the small-$x$ kinematic region. The question rises of which observable is best suited to pin down the saturation of the proton gluon distribution. A partial answer is given in Fig. 3 courtesy of Juan Rojo. It presents a NNPDF1.2 DGLAP fit to pseudodata for proton $F_2$ and $F_L$ in the kinematic regime relevant for the LHeC (down to $x = 10^{-6}$ approx.). The pseudodata are generated by extrapolation of the fit to $F_2$ presented here using BK evolution, and thus include saturation effects.

A detailed list of references to the publications where data is presented can be found in [7].
Figure 3: NNPDF1.2 DGLAP fits (green stars) to pseudodata (red bars) for proton $F_2$ (left plot) and $F_L$ (right plot) generated by extrapolation of our fits down to $x = 10^{-6}$ for $Q^2 = 2, 5, 10$ and 20 GeV$^2$.

$F_2$ can be well fitted by DGLAP, the best DGLAP fit to pseudodata underestimate $F_L$ at small $Q^2$ and overshoots it at the largest $Q^2$ considered, yielding a large $\chi^2$/d.o.f.. One concludes that a precise experimental determination of $F_L$ at the LHeC over a large enough $Q^2$ range might suffice to pin down the kinematic region where departure between DGLAP and non-linear (or, maybe, linear resummed small-$x$ evolution) takes place, while $F_2$ does not offer such discrimination power.

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