Risk analysis of five stocks indexed by LQ45 using credible value at risk and credible expected tail loss

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Abstract. Value at Risk (VaR) and Expected Tail Loss (ETL) are two risk measures that are used frequently to measure the investment risk. Even though VaR can estimate maximum loss when the investor holds a single asset in a particular period and interval confidence, the investor frequently develops a portfolio of assets. This condition can create shared risk among assets in the portfolio so that there will be a chance of an asset for getting loss caused by the other assets developing the portfolio. On the other hand, there is a fact that VaR cannot provide loss information at the tail loss part so that we also need ETL that can overcome this problem. Because of that reason, this paper uses Credible Value at Risk (CredVaR) and Credible Expected Tail Loss (CredETL), which are formulated based on the Buhlman credibility concept. Both methods can estimate an investment risk that can overcome the shortcoming of VaR and ETL that do not consider the risk among assets inside the portfolio. The application of both methods was utilized to evaluate the individual risk of each asset in a portfolio comprised of five stocks in the LQ-45 Index (period of February 2019 until July 2019). The data divided into ten periods of risk analysis comprises of ten-year daily data of each stock from June 2009 to May 2019. According to the result of the analysis, it can be concluded that both methods are powerful in measuring the risk.

1. Introduction

The risk, a part of an investment, sometimes cannot be eliminated and avoided [1]. There are eight types of financial investment risks, namely business risk, financial risk, inflation risk, liquidity risk, country risk, currency risk, market risk, and interest risk [2]. A good risk measure is a tool that can evaluate, manage, and monitor all types of risks faced by investors.

There is an enormous development of risk measurement in both financial and actuarial science [3]. Value at Risk (VaR) and Expected Tail Loss (ETL) or Conditional Value at Risk (CoVaR), two popular tools for risk measurement consorted with an asset or a portfolio asset, have been developed by numerous researchers [3,4]. A portfolio VaR at a confidence level $a$ is defined as a loss threshold such that the loss probability which exceeds the threshold is not more than $1 - a$ [5]. It can also be defined as a requirement of capital sufficiency to cover portfolio losses for a specified holding period [6]. Then, in statistical terms, a portfolio VaR is regarded to provide the quantile of the estimation of return distribution [7]. Meanwhile, CoVaR of a portfolio at a confidence level $a$ is the loss expectation of the portfolio conditioned on the event in which loss greater than VaR [5]. To sum up, VaR considers the...
maximum loss, which can occur in the absence of a disaster tail event, whilst ETL provides the expected loss provided by the occurrence of the tail event [5].

Making a decision to determine an appropriate risk measure among numerous methods proposed by several researchers of an asset or asset portfolio is a challenging problem [8]. In the following paragraph, there will be some brief summaries of the previous measurement risk research.

Karimalis and Nomikos [8] developed VaR and ETL formulas in terms of options, while Martins-Filho and Yao [9] proposed nonparametric estimators for ETL of the financial asset returns. Pitselis [11] developed Credible VaR and credible ETL by mixture Bühlmann credible concept and VaR/ETL classic. The new risk measures latter were called by credible VaR and credible ETL. Then, in 2017, Karimalis and Nomikos [8] formulated a new methodology based on copula functions for ETL estimation of an institution’s financial system. Meanwhile, Kolla, Prashant, Bhat, and Jagannathan [5] considered a problem of ETL estimation from i.i.d samples of a sub-Gaussian or sub-exponential random variable that is unbounded. Among the researches mentioned in this paragraph, we considered focusing on the methodology of risk measurement from the credibility theory perspective utilized widely in the insurance field, as proposed by Pitselis [11].

Credible Value at Risk (CredVaR) and Credible Expected Tail Loss (CredETL), introduced by Pitselis [11], are novel types of risk measures. Both of them are a mixture between credibility methodology and a widespread risk measure, namely VaR chosen by the Basel Committee on Banking Supervision as a benchmark to measure the risk for capital requirements [12] and Expected Tail Loss (ETL). CredVaR and CredETL were claimed to provide more information than the previous methods, namely VaR and ETL, because they can capture both individual asset risk and portfolio risk comprised of some similar but not identical asset returns that are assembled in sharing the risk [3].

This paper is organized as follows. In section two, there will be an explanation about CredVaR and CredETL proposed by Pitselis [11]. In section three, CredVaR and CredETL will be applied to a portfolio made up of five stocks included in LQ 45 Index. Then, in the last section, there will be a conclusion.

2. Credible Value at Risk and Credible Expected Tail Loss

Even though risk measurement of an individual asset using VaR or ETL commonly only involves its individual asset as a count base, it is seldom to find an investor who invests only in one asset in the market. Investors generally create a portfolio in order to diversify their assets. The diversification of financial industries is regarded to give some advantages in financial systems [13]. Asset diversification enables a sharing risk among assets constructing the portfolio so that there will be a chance that an individual’s loss risk is affected by other assets in the portfolio. Therefore, a risk measure covering joint utilization of asset information and other relevant information in terms of the risk of other assets in the portfolio to estimate a single asset’s risk is needed. CredVaR and CredETL are the risk measures that can provide the mentioned proposes.

According to [14], credibility is regarded as a prominent estimation method for a group of insurance contracts when it has several claim experience for the group and numerous more experience for a greater group of contracts which are similar but not exactly the same. The credibility models proposed in [14] will be discussed previously before CredVaR because it will be a basic theory to derive CredVaR and CredETL. Bühlman’s credibility model focused on VaR is developed by some assumptions as follows [12].

1. A portfolio comprising of $m$ assets is given. Then, a random vector representing Value at Risk (VaR) of the $j^{th}$ assets at period $i = 1, 2, \ldots, n$ where $j = 1, 2, \ldots, m$ defined by $w_i = (N_{1,j}, ..., N_{n,j})$.
2. Next, a random variable $N_{1,j}, ..., N_{n,j}$ is assumed identically distributed with mean $E(N_{i,j}) = \mu$ and variance $Var(N_{i,j}) = \sigma^2$.
3. Then, the risk of each asset constructing a portfolio is represented by a random variable $S$ assumed unknown distributed. Meanwhile, a random variable $N_{i,j}, ..., N_{n,j}$ is assumed conditional i.i.d for a fixed $\theta$ with $E(N_{i,j}|S=s) = \mu(s)$ and $Var(N_{i,j}|S=s) = \tau(s)$ for $i = 1, 2, \ldots, n$. 
Then, adopting Bühlman Credibility in risk estimation concept, VaR of the $j^{th}$ assets in the next period that latter will be known as CredVaR will be estimated.

**Theorem 1.** In accord with several assumptions as noted earlier, the linear estimator of CredVaR of $j^{th}$ asset can be expressed by [11]

$$ N_{i,j}^{\text{CredVaR}}(s) = \bar{N}_j Z_{\text{CredVaR}} + (1 - Z_{\text{CredVaR}}) \mu (s) $$

(1)

where $Z_{\text{CredVaR}}$ is a risk factor of CredVaR defined as follows.

$$ Z_{\text{CredVaR}} = \frac{n \text{Var}(\mu (s))}{E(\tau(s)) + n \text{Var}(\mu (s))} $$

(2)

Asset risk estimation using CredVaR requires some information, namely $\mu (s)$, $E[\text{Var}(N_{i,j}|S = s)] = E[\tau(s)]$, and $\text{Var}(\mu (s))$. According to the previous assumptions $\mu (s)$, $E[\tau(s)]$, and $\text{Var}(\mu (s))$ will be estimated by the mean sample formula of the data. The unbiased estimator of $\mu (s)$, $E[\tau(s)]$, and $\text{Var}(\mu (s))$ successively are provided in Equation (3).

$$ \mu (s) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} N_{i,j}, \quad E[\tau(s)] = \frac{1}{m(n-1)} \sum_{j=1}^{m} \sum_{i=1}^{n} (N_{i,j} - \bar{N}_j)^2, $$

$$ \text{Var}(\mu (s)) = \frac{1}{m-1} \sum_{j=1}^{m} (\bar{N}_j - \mu (s))^2 - \frac{E(\tau(s))^2}{n}. $$

(3)

In this paragraph, we attempt to give a recursive process comprised of several steps to derive CredVaR and its credible risk factors. The process is adapted from [15] with adjustment in the Bühlman’s concept used in CredVaR. The recursive process comprises of several steps, namely, specify the number of assets ($m$) constructing a portfolio, specify the observed period $n$, $n \geq 2$, compute the VaR for the $j^{th}$ assets at period $i = 1, 2, ..., n$ where $j = 1, 2, ..., m$, $N_{i,j}$. Then, we continue the steps by computing the estimated mean of VaR for the $j^{th}$ assets during $n$ observed period, $\bar{N}_j$, computing the average estimation of VaR mean of $m$ asset during the entire observed period, computing the estimated mean of VaR variance of $m$ asset during the entire observed period. Next, we have to compute the estimated variance of VaR mean of $m$ asset during $n$ observed period, compute the estimated value of $n \text{Var}(\mu (s))$, compute the estimated $Z_{\text{CredVaR}}$, and finally, compute the CredVaR.

The concept utilized in CredVaR derivation will be used analogously to formulate CredETL of $n$ assets for analyzed periods. The formula of CredETL is derived based on several assumptions that are similar to the assumptions in the derivation formula of CredVaR. The assumptions and the CredETL formula are explained in the following sentences and in Equation 4 and Equation 5 [12] as follows.

1. A portfolio comprising of $m$ assets is given. Then, a random vector representing ETL of the $j^{th}$ assets at period $i = 1, 2, ..., n$ where $j = 1, 2, ..., m$ defined by $w'_j = (Q_{1,j}, ..., Q_{m,j})$.
2. A random variable $Q_{1,j}, ..., Q_{m,j}$ is assumed identically distributed with mean $E(Q_{i,j}) = \mu^*$ and variance $E(Q_{i,j}) = \sigma^{**}$.
3. Moreover, the risk of each asset in a portfolio is represented by a random variable $S$ assumed unknown distributed. Meanwhile, a random variable $Q_{1,j}, ..., Q_{n,j}$ is assumed conditional i.i.d for a fixed $S$ with $E(Q_{i,j}|S = s) = \mu^*(s)$ and $\text{Var}(Q_{i,j}|S = s) = \tau^*(s)$, for $i = 1, 2, ..., n$.

**Theorem 2.** According to several assumptions mentioned previously, the linear estimator of CredETL of $j^{th}$ asset can be expressed by [12].

$$ Q_{i,j}^{\text{CredETL}}(s) = \bar{Q}_j Z_{\text{CredETL}} + (1 - Z_{\text{CredETL}}) \mu^*(s) $$

(4)
where $Z_{CredETL}$ is a risk factor of CredETL defined as follows.

$$Z_{CredETL} = \frac{nVar(\mu'(s))}{E(\tau'(s)) + nVar(\mu'(s))}$$  \hspace{1cm} (5)

Based on the results of the recursive process to get CredETL of $j^{th}$ asset joined in a portfolio and Theorem 2, we can obtain CredETL for the corresponding CredVaR of $j^{th}$ asset using the recursive process of CredETL, which are having analog stages as stages to obtain CredVaR.

3. Application of Credible Value at Risk and Credible Expected Tail Loss in Real Data

In this section, the theory in the previous section will be applied to analyze a portfolio comprised of five stocks in the LQ 45 Index (period of February 2018 until July 2018), namely PT. Aneka Tambang Tbk (ANTAM), PT. Bank Central Asia Tbk (BBCA), PT. Indofood Sukses Makmur Tbk (INDF), PT. Telekomunikasi Indonesia Tbk (TLKM), and PT. Semen Indonesia (SMGR) Tbk. The daily-close-price data (period February 2019 until July 2019) of the five stocks accessed from https://finance.yahoo.com/ was divided into ten risk analysis periods. The risk analysis covers ten-year-daily data of each stock from June 2008 to May 2018. The ten periods of data are 1 June 2008-31 May 2009 (period 1), 1 June 2009-31 May 2010 (period 2), 1 June 2010-31 May 2011 (period 3), 1 June 2011-31 May 2012 (period 4), 1 June 2012-31 May 2013 (period 5), 1 June 2013-31 May 2014 (period 6), 1 June 2014-31 May 2015 (period 7), 1 June 2015-31 May 2016 (period 8), 1 June 2016-31 May 2017 (period 9), and 1 June 2017-31 May 2018 (period 10).

Before conducting the analysis, data of the close stock prices over the ten-period was transformed priorly into log return data. After that, the estimation of historical VaR for each asset with a confidence level of 0.95 for the ten periods was calculated. The computation process involves a long process to be completed, so that we constructed an R program to ease the computation.

![Table 1. Historical VaR of each stock](image)

| Period | ANTM | BBCA | INDF | TLKM | SMGR |
|--------|------|------|------|------|------|
| 1      | 0.076961 | 0.059089 | 0.065241 | 0.057570 | 0.047467 |
| 2      | 0.047069 | 0.037388 | 0.039609 | 0.026668 | 0.031952 |
| 3      | 0.031086 | 0.028779 | 0.032261 | 0.029853 | 0.029501 |
| 4      | 0.027103 | 0.031749 | 0.035268 | 0.027029 | 0.039221 |
| 5      | 0.028574 | 0.023203 | 0.024491 | 0.027399 | 0.024491 |
| 6      | 0.042199 | 0.034486 | 0.044125 | 0.045985 | 0.045462 |
| 7      | 0.029724 | 0.015748 | 0.019048 | 0.022908 | 0.023375 |
| 8      | 0.051578 | 0.028118 | 0.046189 | 0.028537 | 0.041008 |
| 9      | 0.028655 | 0.014185 | 0.025533 | 0.023287 | 0.030600 |
| 10     | 0.031499 | 0.015412 | 0.023770 | 0.026540 | 0.036192 |
| Mean of VaR | 0.039445 | 0.028816 | 0.035554 | 0.031581 | 0.034927 |

According to Table 1, it can be concluded that the mean of maximum potential loss with confidence level 95 percent within one day holding period over ten years of ANTM, BBCA, INDF, TLKM, and SMGR successively are 3.9445 percent, 2.8816 percent, 3.5554 percent, 3.1581 percent, and 3.4927 percent, relative to each asset price on a preceding day. Then, the results provided in Table 1 would be used to estimate $\mu(s), E[\tau(s)],$ and $Var(\mu(s))$ required in CredVaR computation for each asset in the portfolio. Estimators of the three parameters are given in Table 2.

![Table 2. Estimation parameter of $\mu(s), E[\tau(s)],$ and $Var(\mu(s))$](image)

| Parameter | $\mu(s)$ | $E[\tau(s)]$ | $Var(\mu(s))$ |
|-----------|----------|--------------|---------------|
| Estimated Parameter | 0.034064 | 0.000163 | 0.000000 |
Based on the results in Table 2, it can be summed up that the mean of maximum potential loss at confidence level 95 percent for each asset is 3.4064 percent relative to asset price at the previous one day. Moreover, the mean of VaR variance at the specified confidence level for each asset in the portfolio is 0.000163, and the variance of VaR mean for this case for each asset is 0.000000. The estimators of $\mu(s), E[r(s)],$ and $Var(\mu(s))$ will be used to compute a risk factor of CredVaR. Using Equation 5, we obtain $Z_{CredVaR}$ at 0.005446.

Next, using the precedence information, CredVar and its risk factor of $j^{th}$ asset can be calculated directly using Equation 4 and Equation 5, and the results are written in Table 3.

Table 3. CredVaR of each Stock

| Parameter | ANTM | BBCA | INDF | TLKM | SMGR |
|-----------|------|------|------|------|------|
| CredVaR   | 0.034094 | 0.034036 | 0.034073 | 0.034051 | 0.034069 |

As seen from Table 3, the maximum potential losses for the holder of ANTM stock, BBCA stock, INDF stock, TLKM stock, and SMGR stock in the next period of investment when the stock is held for one day at a confidence level of 95 percent are 3.4094, 3.4036 percent, 3.4073 percent, 3.4051 percent, and 3.4069 percent, respectively. Table 3 gives information that there is a relatively small discrepancy of CredVaR of the five stocks. Moreover, when we compared the estimation of loss for each asset using conventional VaR given in Table 1 and $\mu(s)$, it can be noted that the estimation of loss using CredVaR for the five assets is tended to approach $\mu(s) = 0.034064$. It is caused by a risk factor of CredVaR, $Z_{CredVaR}$, at 0.005446, which is relatively small, so based on Equation (1), the weight given to $\mu(s)$ is greater than the weight given to the mean of VaR for each asset over the ten periods.

Then, CredETL will be calculated in order to measure information of tail loss that is not provided by CredVaR. Firstly, the ETL of the five log-returns are calculated and summarized in Table 4.

Table 4. ETL of each stock

| Period | ANTM | BBCA | INDF | TLKM | SMGR |
|--------|------|------|------|------|------|
| 1      | 0.070303 | 0.058284 | 0.061039 | 0.059765 | 0.080283 |
| 2      | 0.051034 | 0.047321 | 0.045885 | 0.050521 | 0.052025 |
| 3      | 0.045188 | 0.047183 | 0.047902 | 0.048891 | 0.047472 |
| 4      | 0.059571 | 0.046457 | 0.050717 | 0.052647 | 0.043518 |
| 5      | 0.044014 | 0.053554 | 0.047208 | 0.044197 | 0.047952 |
| 6      | 0.052058 | 0.046551 | 0.053054 | 0.049036 | 0.056388 |
| 7      | 0.046800 | 0.048410 | 0.061502 | 0.038915 | 0.049730 |
| 8      | 0.056221 | 0.045572 | 0.056755 | 0.040665 | 0.050350 |
| 9      | 0.051106 | 0.040477 | 0.064359 | 0.041758 | 0.041731 |
| 10     | 0.041502 | 0.036902 | 0.040234 | 0.043170 | 0.040544 |
| Mean of ETL | 0.051780 | 0.047071 | 0.052866 | 0.046956 | 0.050999 |

According to the result of the analysis, it can be summarized that the mean of the worst loss in which larger than CredVaR of each asset for the ten observed periods is between 4.6956 percent until 5.2866 percent when the assets were held for one day and confidence interval of 95 percent. Those results would be used to count the three-parameter estimation required in the CredETL calculation for each asset in the portfolio. The estimators of $\mu^*(s), E[r^*(s)],$ and $Var(\mu^*(s))$ are listed in Table 5.

Table 5. Estimation parameter of $\mu^*(s), E[r^*(s)],$ and $Var(\mu^*(s))$

| Parameter | $\mu^*(s)$ | $E[r^*(s)]$ | $Var(\mu^*(s))$ |
|-----------|------------|-------------|-----------------|
| Estimated Parameter | 0.049934 | 0.000068 | 0.000001 |
From the three estimated parameters in Table 5, it can be interpreted that the loss average that is greater than CredVaR from each asset is about 4.9934 percent relative to the close-price asset in the one previous day. In addition, the expectation of the ETL variance from each asset is about 0.000068, and the variance of the average ETL of each asset is 0.000001. The estimators of $\mu^*(s), E[\tau^*(s)],$ and $Var(\mu^*(s))$ will be utilized to count a risk factor of CredETL based on Equation 5. From the latter calculation, we get a risk factor, $Z_{\text{CredETL}}$, at 0.096456.

Next, using the precedence information, CredETL and its risk factor of $j^{th}$ asset can be calculated directly using Equation 4 and Equation 5, and the results are written in Table 6.

Table 6. CredETL of each Stock

|       | ANTM  | BBCA  | INDF  | TLKM  | SMGR  |
|-------|-------|-------|-------|-------|-------|
| CredETL | 0.050112 | 0.049658 | 0.050217 | 0.049647 | 0.050037 |

According to the result of the analysis, it can be summarized that the mean of loss in which is larger than CredVaR of asset ANTM, BBCA, INDF, TLKM, and SMGR when the stock is held for one day at a confidence level of 95 percent successively are 5.0112 percent, 4.9658 percent, 5.0217 percent, 4.9647 percent, and 5.0037 percent. Moreover, when we compared the estimation of loss for each asset using conventional ETL given in Table 4 and the loss average that is greater than CredVaR from each asset, namely $\mu^*(s)$, it can be noted that the estimation of loss using CredETL for the five assets is tended to approach $\mu^*(s) = 0.049934$. It is caused by a risk factor of CredETL, $Z_{\text{CredETL}} = 0.096456$, which is relatively small, so based on Equation (4), the weight given to $\mu^*(s)$ is greater than the weight given to the mean of ETL for each asset.

4. Conclusion

Based on the analysis in the previous section, it can be concluded that Credible Value at Risk and Credible Expected Tail Loss are both risk measures that can be utilized as an alternative for the investors and also financial managers in order to analyze the risk of assets. The risk measures are giving an easiness to users because there is no specified distribution of return assets required in their calculations.

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