Andreev Spectroscopy for Superconducting Phase Qubits

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We propose a new method to measure the coherence time of superconducting phase qubits based on the analysis of the magnetic-field dependent dc nonlinear Andreev current across a high-resistance tunnel contact between the qubit and a dirty metal wire and derive a quantitative relation between the subgap I-V characteristic and the internal correlation function of the qubit.
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1. INTRODUCTION

While new algorithms are pushing the frontier of quantum computation, their practical implementation requires novel ideas for the design of qubits which can cope with the contradictory constraints of scalability and long decoherence time. Solid state electronics offers scalability but involves macroscopic elementary blocks which usually interact strongly with the environment and, thus, suffer from a short decoherence time. In contrast, atomic scale designs (trapped atoms, photons in cavities, nuclear spins) can be easily decoupled from the environment but are difficult to combine into large and complex devices. The ideal system for a qubit implementation is a macroscopic device with two quantum states decoupled from the environment which can be manufactured and combined with others by conventional techniques. This ideal is closely approximated by the superconducting phase qubit (SCPQ) that consists of a frustrated Josephson junction circuit with two states distinguished mostly by their superconducting phases.

The simplest example of such a circuit is a low inductance SQUID loop
consisting of three (or more) Josephson junctions frustrated by a magnetic field that creates a flux $\Phi = \Phi_0/2$ through the loop, see Fig. 1. The two states of the qubit are defined by the phase changing clockwise or counter-clockwise around the loop. The advantage of an “all phase” design is that the superconducting phase is not directly coupled to the classical environment; furthermore, superconductors themselves have a gap such that the environment has a low density of low energy modes potentially leading to decoherence and the charge fluctuations are screened such that the electric coupling to the environment is negligible. The magnetic coupling to the environment cannot be completely avoided as the phase changing around the frustrated loop is coupled to an induced current, however, the latter can be made small if the Josephson junctions are weak and the loop inductance is small. In particular, we shall show that the interaction of these currents with the environment leads to a small decoherence even if the environment includes potentially dangerous normal wires. Finally, we note that the coupling with the frustrating magnetic field present in the simplest design can be eliminated in designs involving $d$-wave superconductors or $\pi$-junctions.

The first problem to be addressed in the development of a SCPQ is the implementation of a convenient probe which tests whether the device produces coherent Rabi oscillations. It is obviously difficult to test a system which is decoupled from the environment and it is even more difficult to check that it remains in the phase coherent state without disturbing it. Here we suggest an experiment to probe the state of such qubits and to determine their decoherence time. The idea is to measure the Andreev conductance...
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between the ground and a normal wire in the geometry of a fork with the two prongs connecting to two separate islands of the qubit (Fig. 1). If the qubit phase is fixed (classical regime), the low-$T$ conductance exhibits a contribution from processes in which electrons from the normal metal are reflected as holes from the qubit boundary, diffuse to the bulk superconductor boundary and are reflected from it as electrons. Such a phase coherent electron diffusion in the normal metal prongs of the “fork” leads to periodic conductance oscillations (with period $\Phi_0$) as a function of the magnetic flux penetrating through the $S_f$ region of the fork. This is due to the fact that the magnetic field controls the superconducting phase difference between the two NS contacts of the fork [(1) and (2) in Fig. 1] and thus influences the electron interference picture. A similar mechanism controls the experiment discussed in the quantum regime when the phase of the qubit fluctuates. Qualitatively, at small bias voltage the quantum phase acquired by the reflected electrons fluctuates and the resulting contribution to the conductivity averages out. At larger voltages the phase fluctuations become slow compared with the electron tunneling time and the contribution to the conductivity is restored. Below we describe the specific device parameters and derive quantitative formulas describing this effect. A similar proposal — to probe the Josephson splitting of levels in a superconducting single Cooper-pair transistor by measuring the Andreev conductance — was put forward in Ref. 9.

2. SUPERCONDUCTING PHASE QUBIT AND ENVIRONMENT

Consider the SQUID loop with four junctions shown in Fig. 1. Two degenerate states naturally appear in such a loop if the flux $\Phi_q$ of the external field through the qubit loop is exactly $\Phi_0/2 = hc/4e$. Indeed, in a gauge with $A_x = 0, A_y = Bx$ the classical minima of the Josephson energy correspond to the phase drops 0 or $\pi$ around the loop; in these states the phase of the island $G$ (see Fig. 1) is $\phi^\pm = \pm\pi/2$ (relative to the phase at point 0); below we refer to these states as $|\uparrow\rangle$ and $|\downarrow\rangle$. In order to reduce the parasitic coupling to the environment, the inductance $L$ of the loop should be sufficiently small so that $LI_c/c \leq 10^{-5}\Phi_0$, $I_c$ is the critical current. In the absence of any charging energy in the junctions the system prepared in one of these states will stay put forever; quantum effects are due to the small but finite charging energy $E_C = e^2/2C$ determined by the capacitances $C$ of the junctions, which should be smaller than the Josephson energy $E_J \equiv hI_c/2e \gg E_C$. The tunnelling frequency between the two classically degenerate ground states is estimated as $\Omega_0 = \omega_0 \exp(-\sqrt{\hbar I_c C} / e^2)$, where $\hbar\omega_0 = \sqrt{8E_CE_J}$ is the Josephson plasma frequency of the contacts; the coefficient $a \approx 1.6$ was
found by a numerical evaluation of the saddle-point trajectory in the space of the three phase variables $\phi_2, \phi_3 \equiv \phi, \phi_4$. We assume values of $I_c$ in the range $10^{-7}$ A, and capacitances of order of a few fF, resulting in $\omega_0/2\pi \sim 100$ GHz and a tunneling frequency $\Omega_0/2\pi$ in the range of few GHz. Our choice of parameters is limited by the following constraints: (a) the system should be sufficiently small to fulfill the condition $L I_c/c \leq 10^{-3}$, but not too small, since (b) we need $E_C \ll E_J$; finally, (c) $\Omega_0$ should not be too small, in order to keep the decoherence time $t_{dc}$ much longer than $\Omega_0^{-1}$.

Once tunnelling is taken into account, the true eigenstates become $|0\rangle = [(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ and $|1\rangle = [|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$ and are separated by the energy gap $\bar{\hbar}\Omega_0$ (here and below we shall use the “spin” notation to describe the classically degenerate states). The deviation of the magnetic flux $\Phi_q = BS_q$ through the loop from the point $\Phi_q = \Phi_0/2$ removes the degeneracy of the states $|\uparrow\rangle$ and $|\downarrow\rangle$. Thus the intrinsic Hamiltonian of our qubit written in the basis $|\uparrow\rangle, |\downarrow\rangle$ is $H = h_z\sigma_z + h_x\sigma_x$, where $h_z = (I_c/\sqrt{2}c)(\Phi_q - \Phi_0/2)$ and $h_x = \hbar\Omega_0$. All operations on the qubit can be performed if one is able to vary the effective fields $h_z$ and $h_x$. The variation of $h_z$ can be achieved by changing the flux $\Phi_q$ in the loop, while the variation of $h_x$ can be implemented by two means: the variation of the gate potential applied to the island $G$ changes this field smoothly (c.f., Ref. 10), while switching a capacitor $C_{ext} \sim 10C$ in parallel blocks the tunneling abruptly.

The coupling of a two-level quantum system to the environment is usually described by the Caldeira-Leggett “spin-boson” model where the “spin” (i.e., two-level system, TLS) is linearly coupled to an ensemble of oscillators. Actually, our TLS originates from the dynamics of a continuous phase variable $\phi$ with a potential energy $U(\phi)$ strongly favoring values $\phi$ around $\pm\phi_0/2$. Then the term that describes the coupling to the environment can be written, upon integration over environmental modes, in the form of a non-local imaginary-time action $S_{\text{diss}} = \frac{1}{2}\int dt dt' K(t-t')[\dot{\phi}(t) - \dot{\phi}(t')]^2$. While in a superconducting system we can hope the low energy spectrum of the environment to be gapped providing favorable conditions for a long coherenece time, here, we analyze the (worst case) situation where the coupling of low lying modes survives to produce a finite ohmic dissipation with $K(t) = \eta/2\pi t^2$, where $\eta$ is the generalized “friction coefficient” in the equation of motion $\eta\dot{\phi} = -\partial U(\phi)/\partial \phi$. The strength of the environmental coupling is determined by the dimensionless parameter $\alpha = \eta\phi_0^2/2\pi\hbar$; consider the symmetric system with $h_z = 0$: for $\alpha > 1$ the tunnelling between the states $|\uparrow\rangle$, and $|\downarrow\rangle$ is suppressed and the system possesses two classically degenerate ground states. For intermediate coupling $1/2 < \alpha < 1$ tunnelling is finite but incoherent: at time scales $t > \Omega^{-1} = \hbar/h_x$ (with $\Omega$ the effective tunneling frequency depending on
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α) the TLS is described by the classical master equation for the occupation probabilities \( P_{\uparrow,\downarrow} \) of the two states with a transition rate \( \Gamma(\alpha) \) vanishing at \( \alpha \to 1 \). At \( \hbar \omega = 0 \) this equation takes the form \( \dot{m}(t) = -\Gamma m \) and the “magnetization” \( m(t) = \langle \sigma_z(t) \rangle = P_\uparrow(t) - P_\downarrow(t) \) monotonically decays to zero. Here, we are interested in the case of very weak coupling \( \alpha \ll 1 \), where the TLS dynamics is weakly perturbed by the environment, i.e., the decoherence time \( t_{dc} = \Gamma^{-1} \) shall be much longer than the period of coherent oscillations \( 2\pi/\Omega^{-1} \). In this case, the magnetization exhibits damped oscillations \( m(t) = \exp(-\Gamma t)\cos(\Omega t) \) with a damping rate \( \Gamma \) and a frequency \( \Omega \) determined by the parameter \( \alpha < 1/2 \) via:

\[
\Omega = (\Omega_r/\pi\alpha)\sin[\pi\alpha/(1 - \alpha)], \quad \Gamma/Q = \Omega \tan[\pi\alpha/2(1 - \alpha)], \quad (1)
\]

with the renormalized tunnelling frequency \( \Omega_r = \Omega_0(\Omega_0/\omega_c)^{\alpha/1-\alpha} \); here, \( \omega_c \) is an upper cutoff for the frequencies of the environmental modes. A measurement of \( m(t) \), particularly its quality factor \( Q \), then allows us to test the degree of quantum coherence in the device.

The direct determination of \( m(t) \) in a time-domain experiment like the one reported in Ref.\[1\] (where a superconducting “single Cooper-pair” device was studied) is a demanding task. As an alternative one may attempt to find the correlation function \( D(t) = \langle \sigma_z(t)\sigma_z(0) \rangle \) through a measurement\[2\] of the rf absorption; indeed, the latter provides the imaginary part \( \chi''(\omega) \) of the response function \( \chi(\omega) = 2\int_0^\infty dt D^\alpha(t) \exp(i\omega t) \), where \( D^\alpha(t) = \langle [\sigma_z(0),\sigma_z(t)] \rangle/2i \) is the antisymmetric part of the spin-spin correlation function. Standard relations then allow us to extract the correlator \( D(t) \) from the measured absorption \( \chi''(\omega) \): the symmetrized correlation function \( D^s(t) = \langle [\sigma_z(0),\sigma_z(t)] \rangle/2 = \coth(h\omega/2T) \chi''(\omega) \) relates to the correlator via \( D(\omega) = 2D^s(\omega)/[1 + \exp(-\omega/T)] \) (note that \( D(\omega < 0) = 0 \) at \( T = 0 \)). Below we discuss yet another experiment giving us access to \( D(t) \): we will see that the nonlinear Andreev conductance \( dI_A/dV \) through the qubit’s island \( G \) can be expressed via the Fourier-transform \( D(\omega) \) of its correlator. Scanning magnetic field and voltage in a transport measurement then appears to give a much easier and direct access to the operation of the qubit than the more usual real-time and rf-techniques.

Being not aware of exact results available for \( \chi(\omega) \) or \( D^s(t) = \langle [\sigma_z(0),\sigma_z(t)] \rangle/2 \), we make use of an approximate analysis\[3\] to arrive at an idea of the relevant dependencies in these quantities: for \( Q \gg 1 \) the absorption characteristic is close to a combination of Lorentzians: \( \chi''(\omega) = \Gamma/[\Gamma^2 + (\omega - \Omega)^2] - \Gamma/[\Gamma^2 + (\omega + \Omega)^2] \), implying a symmetrized correlator of the form \( D^s(\omega) = 4\omega\Gamma\Omega/[\Gamma^2 + (\omega^2 + \Omega^2)^2 - 4\omega^2\Omega^2] \). In the time domain, \( D^s(t)\big|_{T=0} \approx \cos(\Omega t) \exp(-\Gamma t) + 4/[\pi Q(\Omega t)^2] \), where the last term is due to the non-analyticity of \( D^s(\omega) \) at \( \omega = 0 \).
3. ANDREEV CONDUCTANCE

We first concentrate on the subgap conductance through a NIS contact (#1 in Fig. 1) with a low normal-state conductance $\sigma = G_t e^2/\hbar \ll e^2/\hbar$ and discuss the effect of fluctuations in the phase $\phi(t)$ on the island $G$. Consider an ideal SCPQ operating at the degeneracy point ($h_z = 0$) with the ground and excited states $|0\rangle$ and $|1\rangle$ separated in energy by $\hbar \Omega$. The operator $C^\dagger = \exp(-i\phi)$ creating Cooper pairs changes the relative sign of the states $|\uparrow\rangle$ and $|\downarrow\rangle$, thus $C^\dagger |0\rangle = |1\rangle$ and $C^\dagger$ has no matrix element in the ground state. Therefore $|0\rangle$ is a state with a well-defined number of Cooper-pairs modulo 2, implying that the usual Andreev process transmitting a single Cooper pair through the interface is forbidden at low voltage $V < \hbar \Omega / 2e$, while at voltages above the gap $\hbar \Omega / 2$ the Andreev conductance will be large. Higher-order processes with simultaneous tunnelling of two Cooper pairs are allowed, but their rate is of the order of $G_t^4$ and thus is negligible at $G_t \ll 1$. Overall, we then expect an Andreev conductance $dI_A/dV$ similar to the one of a usual NIS junction but with the superconducting gap $\Delta$ replaced by the two-level separation $\hbar \Omega$. On the other hand, for a fixed phase on the island we can expect to observe the usual Andreev current across a NIS junction.

The above consideration defines the simplest example of a situation where the quantum fluctuations of the superconducting phase suppress the Andreev conductance of the SIN contact. General expressions describing these effects have been derived in Ref. 21 (c.f., Appendix B). Here, we discuss the result for the nonlinear subgap current $I_A(V)$ for the case where the normal metal is relatively clean (with a dimensionless conductance $g \gg 1$) such that all corrections of relative order $1/g$ can be neglected,

$$ I_A(V) = \frac{eG_t^2}{4\hbar} \int_{-\infty}^{\infty} C(E) dE \left[ \frac{dE'}{2\pi\hbar} D\phi(E') \right] \frac{1 - e^{-2eV/T}}{1 - e^{(E' - 2eV)/T}} \times \left[ \tanh[(E - E'/2 + eV)/2T] - \tanh[(E + E'/2 - eV)/2T] \right] . $$

Here, $C(E)$ is the real part of the Cooperon amplitude in the normal metal (see below) and $D\phi(E) = \int dt \exp(iEt/\hbar)D\phi(t)$ is the Fourier-transformed autocorrelation function $D\phi(t) = \langle \Psi(t)\Psi^+(0) \rangle$ of the superconducting order parameter $\Psi(t) = \exp[i\phi(t)]$ on the island $G$. On long time scales $t \gg \Omega^{-1}$ the oscillatory modes near the minima of the Josephson potential are irrelevant; the values of $\phi$ in these minima differ by $\pi$, thus $D\phi(t)$ is equivalent to the (sought for) TLS correlation function $D(t)$ discussed in the previous section.

The real part of the Cooperon amplitude is given by

$$ C(E) = \frac{1}{\nu V} \Re \sum_q \frac{1}{Dq^2 - 2iE/\hbar + \tau_\phi^{-1}}, \quad \quad \quad (3) $$
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where the sum goes over the Cooperon eigenmodes $q_n$, $V$ denotes the volume, $
u$ the density of states (for a single projection of spin), and $D$ the diffusion constant of the normal metal. Assuming a wire geometry with $w, L \gg \min(L_E \equiv \sqrt{hD/4E}, L_\varphi \equiv \sqrt{D\tau_\varphi}, w)$, and $d \leq w$ denoting its length, width and thickness, Eq. (3) reduces to the expression $C(E) \approx \rho L E / \left[ 1 + \left( \bar{\hbar}/4 \right)^2 \right]^{1/4}$, where the resistance $\rho = (2\nu D w d)^{-1}$ per unit length of the wire is measured in units of $(\bar{\hbar}/e^2)/\text{Length}$. The term $\propto \tau_\varphi^{-1}$ describes the electron dephasing present at all $T > 0$ due to electron-electron interactions and external noise.

For a short wire, $C(E) = \rho L$; in the end we arrive at the simple result $C(E) \approx \rho \min(L_E, L_\varphi, L)$. An expression for $I_A(V)$ analogous to Eqs. (2) has been derived earlier for a similar physical problem.

Let us analyze the result (2) in the limit $T \to 0$ which applies to the regime $T \ll (eV, \bar{\hbar}\Omega)$. At the same time we keep the electron dephasing rate $\tau_\varphi^{-1}$ nonzero, since experimentally an apparent saturation at $T \lesssim 0.1 - 1K$ has been found. Differentiating Eq. (2) by $V$ at $T = 0$ we find

$$\frac{dI_A}{dV} = \frac{e^2 G_t^2}{2\pi \bar{\hbar}^2} \int_0^{2eV} dE D\varphi(E) C(eV - E/2)$$  \hspace{1cm} \text{(4)}$$

(values $E < 0$ are excluded from the integral since $D\varphi(E < 0) = 0$ at $T = 0$).

Consider first of all the degenerate case with $h_z = 0$ in the absence of any dephasing within the qubit, i.e., $\varGamma = 0$ and $D\varphi(E) = 2\pi \delta(E - \bar{\hbar} - \Omega)$. The evaluation of the integral in (4) is trivial and we obtain the result

$$\frac{dI_A}{dV} = \begin{cases} 0, & eV < \hbar\Omega/2, \\ (e^2/\hbar) G_t^2 C(eV - \hbar\Omega/2), & h\Omega/2 < eV, \end{cases}$$  \hspace{1cm} \text{(5)}$$
in agreement with the above qualitative arguments. In the following we restrict the discussion to long wires $L \gg \sqrt{D/\Omega}$; then the conductivity above threshold shows a square-root singularity which is smeared by a nonzero dephasing rate $\tau_\varphi^{-1}$. The nonzero conductance below threshold is due either to a finite dissipation rate $\varGamma > 0$ or a finite temperature. At zero temperature, a finite $\varGamma$ produces the sub-threshold $(eV < \hbar\Omega/2)$ conductance

$$\frac{dI_A}{dV} = \frac{e^2}{\hbar} G_t^2 \rho \sqrt{\frac{D}{\varGamma}} \left\{ \begin{array}{ll} \frac{a}{\hbar} \left( \frac{\hbar \varGamma}{\hbar \Omega} \right)^3, & eV \ll \hbar\Omega/2, \\ 1/2 \left[ \frac{\hbar \varGamma}{\delta} \sqrt{1 + \frac{h^2 \varphi^2}{\delta^2} - 1} \right]^{1/2}, & \delta \equiv \hbar \Omega - 2eV \ll \hbar\Omega/2, \end{array} \right.$$  \hspace{1cm} \text{(6)}$$

where we have assumed $\tau_\varphi \to \infty$ and have made use of the form of $D\varphi(E)$ discussed at the end of Sec. 2. The numerical coefficient $a$ depends on the shape of the correlator $D\varphi(E)$ and is of order unity; for the Lorentzian shape
discussed above we find $a = 32/3\pi$. A finite phase breaking time $\tau_\varphi$ in the normal wire changes this result at low voltages $eV < \hbar/\tau_\varphi$: $dI_\Lambda/dV \approx (e^2/\hbar)G_0^2 \rho L_{\varphi}[\hbar \Gamma(eV)^2/(\hbar \Omega)^3]$. Finally, a finite temperature also produces a finite sub-threshold conductance which, however, can be well separated (at $T \ll (2eV, \hbar \Omega - 2eV)$) from the above behavior due to its exponential $\propto \exp[(2eV - \hbar \Omega)/T]$ rather than algebraic dependence $\propto (\sqrt{\hbar \Gamma eV/\hbar \Omega})^3$ and $\propto \hbar \Gamma/(\hbar \Omega - 2eV)$ at low and high voltages. Note, that these results for the suppression in the conductance $dI_\Lambda/dV$ due to phase fluctuations refer to terms of order $G_0^2$ and hence our analysis is valid only for low $G_t$.

The above discussion clarifies the appearance of a threshold behavior in the degenerate case $\hbar_0 = 0$, where the differential conductance $dI_\Lambda/dV$ sharply drops below the gap $\hbar \Omega/2$ and vanishes at $V, T \to 0$. For $\hbar_0 \geq \hbar_0 = \hbar \Omega$ the correct ground state is close to a state with a well-defined phase $\phi$ and the conductance attains its semiclassical value $dI_\Lambda/dV = (e^2/2\hbar)G_0^2 \rho \sqrt{\hbar D/eV}$. As a consequence, a sharp minimum (at $\Phi_q = \Phi_0/2$ corresponding to $\hbar_0 = 0$) is expected in $I_\Lambda(V)$ when changing the magnetic flux $\Phi_q$ through the qubit. This qualitative conclusion survives the presence of a finite electron decoherence rate $\tau_\varphi^{-1}$ in the normal-metal, however, the precise form of $dI_\Lambda/dV$ is modified. A short electron decoherence time $\tau_\varphi \leq \hbar/T$ is frequently observed in thin wires at lowest temperatures as relevant for the experiments on qubits (c.f., Ref. 23). These observations are poorly understood theoretically and thus it is more practical to measure $\tau_\varphi$ independently in the same experimental setup. Such a measurement can be carried out via an analysis of the magnetic field dependent Andreev conductance in the “fork” geometry as we are now going to discuss.

Consider the interference experiment for the device shown in Fig. 1. The first stage of the proposed experiment is to measure the amplitude $I_\Lambda^{(12)}$ in the oscillations of the Andreev current $I_\Lambda = I_\Lambda^{(1)} + I_\Lambda^{(2)} + I_\Lambda^{(12)}$ as a function of magnetic field $B$ with the aim to extract the electron decoherence time $\tau_\varphi$ in the normal metal (the superscripts $^{(i)}$ refer to the contributions from the contacts $i = 1, 2$, see Fig. 1). The total phase determining the interference current $I_\Lambda^{(12)}$ sums up to $\phi_\Lambda = \phi + 2\pi BS_f/\Phi_0$. Away from the degeneracy field $B_n \equiv (n + 1/2)\Phi_0/S_q$, the (semiclassical) phase $\phi$ in our device is determined by the minimization of the Josephson energy and we obtain $\phi = \pi\{BS_q/\Phi_0 + 1/2\} - 1/2$, where $\{x\}$ is the fractional part of $x$. Choosing a geometry with $S_q = S_f$, we find for $BS_q/\Phi_0 \in [n, n + 1/2]$ an interference current $I_\Lambda^{(12)} = 2j_{12}\cos\phi_\Lambda = 2j_{12}\cos(3\pi BS_q/\Phi_0)$, whereas for $BS_q/\Phi_0 \in [n + 1/2, n + 1]$ the sign changes: $I_\Lambda^{(12)} = -2j_{12}\cos(3\pi BS_q/\Phi_0)$. The amplitude of the interference component $j_{12}$ depends on the precise geometry of the “fork” and on the electron dephasing length $L_\varphi$ which should be longer than the distance $L_{12}$ between the contacts $\#1,2$ (otherwise $j_{12}$ will
Fig. 2. Schematic view of the Andreev current through the “fork” enclosing the qubit versus magnetic flux for the special case $S_q = S_f$. The fork serves two purposes: the interference pattern gives access to $\tau_\phi$ through measuring $I_A^{(12)}$ and allows for the determination of the exact point of degeneracy.

be exponentially small. Detailed calculations and measurements of $j_{12}$ have been presented by the Saclay group; they found a discrepancy by a factor 1.8 between the best fit (with $\tau_\phi$ as fitting parameter) to their data and the calculated expression for $j_{12}$ (c.f., Ref. 20, p. 190; this discrepancy may be due to the use of a simplified description of the electron decoherence using a factor $\exp(-t/\tau_\phi)$ in the real time expression for the Cooperon). Second, we test for the coherent time evolution of the qubit: In the geometry with $S_q = S_f$, the semiclassical current amplitude $I^{(12)}_\Lambda(B)$ is periodic with a period $\Phi_0/S_q$, showing upward cusps at $B = B_n$, see Fig. 2, dotted line. These cusps are due to the penetration of single flux quanta $\Phi_0$ into the qubit loop at $B = B_n$. At these fields, the interference contribution $I^{(12)}_\Lambda$ vanishes and the semiclassical Andreev current is given by the sum $I^{(1)}_\Lambda + I^{(2)}_\Lambda$ (in the generic case $S_q \neq S_f$ the component $I^{(2)}_\Lambda$ does not vanish at $B = B_n$ and the flux penetration leads to a jump in the measured $I_\Lambda$). In addition, at $B = B_n$ the quantum fluctuations in the phase $\phi$ become large and suppress the Andreev current $I^{(1)}_\Lambda$ to the “active” island G of the SCPQ, see Eq. (4). Hence, close to the points $B = B_n$ a narrow dip is superimposed upon the above-mentioned cusp in the $I_\Lambda(B)$ dependence, see Fig. 2, solid line. The relative width $\Delta B/B_0 \sim e\Omega/I_c$ of this dip is determined by the relation
$h_z = I_c(\Phi_q - \Phi_0/2)/\sqrt{2}c \leq h_x = h\Omega$ and provides the coherence gap of the qubit. In addition, we can measure the decoherence rate $\Gamma$ via an analysis of the conductance $dI_\Lambda/dV = \sigma_\Lambda(V)$ at the degeneracy points $B = B_n$. In the limit $T \ll eV$ and $eV \rightarrow 0$ the entire conductance is determined by the current through the contact #2, $\sigma_\Lambda^{(2)}(V) = (e^2/h)G_t^2 C(eV)$. The variation of the full differential conductance $\sigma_\Lambda(V) = \sigma_\Lambda^{(1)}(V) + \sigma_\Lambda^{(2)}(V)$ at low voltages $eV \leq h\Omega$ then can be used to extract the value of $\Gamma$. Note that $C(E)$ for the fork differs from the form used in the expression for the subgap conductance, c.f., Ref. [19]. The detailed analysis of $\sigma_\Lambda(V)$ for the different relations between $\Omega$, $\Gamma$, and $\tau_\varphi$ will appear in a future publication.

Finally, any method measuring the SCPQ coherence time is useful only if the presence of the measurement circuit does not, by itself, lead to a decoherence rate comparable to the “intrinsic” one. In our setup there are two sources of additional phase $\phi$ decoherence due to the presence of the normal-metal wire nearby: (i) the direct coupling of the qubit to the environment via the non-zero Andreev conductance, and (ii) the inductive coupling between ac supercurrents in the qubit loop and normal currents in the Andreev fork. We consider them one by one: (i) Comparing the dissipative action for the Andreev conductance (c.f., Ref. [24] for example) with the general relation for $S_{\text{diss}}$ and the definition of the parameter $\alpha = \eta_\phi^2/2\pi\hbar$ one finds the contribution $\alpha_\Lambda = (\pi\hbar/8e^2)\sigma_{\Lambda}^{\text{cl}}$, where $\sigma_{\Lambda}^{\text{cl}} = (e^2/h)G_t^2 \rho \min(L, L_\varphi)$ is the semiclassical Andreev conductance via the contact #1. Thus, a classical subgap resistance of the SN contacts in the M$\Omega$ range will produce a value $\alpha_\Lambda \leq 10^{-3}$. (ii) We calculate the friction coefficient $\eta_\text{ind}$ via the energy dissipation rate $W_{\text{ind}} = \eta_\text{ind}(d\phi/dt)^2$. The main contribution to $W_{\text{ind}}$ is due to the $ac$ magnetic field applied to the conducting wire (with thickness $d < 50$ nm, width $w \sim 100$ nm and a few microns length $L$), which gives $W_{\text{ind}} \approx \omega(w L d)\chi''(\omega)B_\omega^2$, where $\chi''(\omega) \sim \omega w^2/c^2$ is the imaginary part of the magnetic susceptibility of a thin-film wire in a magnetic field $B_\omega$ in the low-frequency limit (for frequencies $\omega \sim 10^{10}$ and a conductivity $\sigma \sim 10^{-5}$ $\Omega$cm of the wire the effective skin depth is much longer than the wire’s width $w \sim 100$ nm). The magnetic field outside the SCPQ loop (of size $L_q \sim 1\mu$m) is less than $\Phi_q/L_q^2$, thus $(\omega B_\omega)^2 < (\omega \phi_\omega)^2(h/e)^2(\Phi_q/\Phi_0)^2L_q^{-4}$. Combining all factors, one finds $\alpha_{\text{ind}} < (w^2L/\rho L_q^3)(\Phi_q/\Phi_0)^2$. For a SCPQ with $\Phi_q/\Phi_0 < 10^{-3}$ and $w/L_q \sim 0.1$ this results in a negligible value $\alpha_{\text{ind}} < 10^{-7}$.

In conclusion, we have derived expressions for the subgap $I(V)$ characteristics, Eqs. (2) and (4), which can be used for the determination of the intrinsic correlation function $D_\phi = \langle \exp[i\phi(t)] \exp[-i\phi(0)] \rangle$ of a superconducting phase qubit (SCPQ), once the electron dephasing time $\tau_\varphi$ that enters the Cooperon, Eq. (3), is known. As the latter is difficult to control theoretically at the ultralow temperatures of the experiment we propose an
interference experiment probing the SCPQ with an “Andreev fork”, allowing

to determine \( \tau_\phi \) within the same setup and then use its value to extract \( t_{dc} \).

In the absence of decoherence in the SCPQ, \( \Gamma = 0 \), the current \( I_A(V) \) is zero

(at \( T = 0 \)) below the threshold \( eV = \hbar \Omega/2 \), irrespective of the value of \( \tau_\phi \).

Thus the SCPQ decoherence time \( t_{dc} = \Gamma^{-1} \) can be measured even if it is

longer than \( \tau_\phi \). In practice, the most important limitation of this technique

seems to be due to the non-vanishing temperature of experiment which pro-
duces a nonzero sub-threshold conductance even at \( \Gamma = 0 \). However, in the

range \( T \ll \hbar \Omega \) and at voltages such that \( T \ll (2eV, \hbar \Omega - 2eV) \) the behavior

of \( dI_A/dV \) is exponentially close to the zero-\( T \) one; therefore even values of \( \Gamma \ll T/\hbar \) can be extracted from the measured \( dI_A/dV \) curves.

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