A numerical investigation of electric current conservation associated with solar wind plasma under GES-model approach

P. K. Karmakar¹
Department of Physics, Tezpur University, Napaam, Tezpur-784028, Assam, India.
E-mail: pkk@tezu.ernet.in

Abstract. A plasma-based Gravito-Electrostatic Sheath (GES) model for theoretical description of the basic physics of the surface emission mechanism of the quasi-neutral Solar Interior Plasma (SIP) on a bounded scale, and subsequently, its transonic transition into Solar Wind Plasma (SWP) on an unbounded scale with a simplified field-free fluid model approach has already been proposed. An autonomous closed set of self-consistently coupled nonlinear eigenvalue equations for analyzing the dynamical stability of the steady GES model on both the scales is developed. The focussed aim of the present contribution is only to study the electric current profiles associated with the SIP as well as SWP, and hence to investigate conservative dynamical features thereof. Applying the developed set of coupled nonlinear dynamical evolution equations, the profiles of electric currents associated with both SIP as well as SWP have been obtained numerically under a wide range spectrum of initial values of relevant physical parameters (by nonlinear stability analysis). Interestingly, it is conjectured that the dynamical evolution of solar electric currents is found fairly to be conservative (divergence-free) in nature except some deviations near the defined solar surface boundary. Results, discussions and conclusions on the basis of the obtained numerical results are presented in brief.

1. Introduction
A plasma-based Gravito-Electrostatic Sheath (GES) model [1] has already been proposed with a simplified field-free fluid model approach. It deals with the basic physical mechanism behind the surface origin of the quasi-neutral flow of Solar Interior Plasma (SIP) and subsequently, its transonic transition into Solar Wind Plasma (SWP), i.e., steady supersonic outflow of ionized gas from the sun. According to this analysis, the GES divides into two scales: one bounded (solar interior scale) and the other unbounded (solar exterior scale). The former includes the steady state equilibrium description of the SIP dynamics bounded by the solar self-gravity. This extends from the solar centre to the self-consistently defined Solar Surface Boundary (SSB) lying at a radial position specified with $\xi \sim 3.5$ (in units of the Jeans length). The unbounded scale encompasses the SWP dynamics extending from the SSB to infinity. The SIP thermal electrons can easily escape from the SSB. On the contrary, the SIP ions cannot cross the gravitational potential barrier of the solar mass at the cost of their thermal energy alone. However, surface leakage of the SIP electrons is bound to create an ambipolar electrostatic field.

¹To whom any correspondence should be addressed.
by virtue of surface charge polarization. This, in turn, provides an additional source of acceleration to act on the SIP ions to further energize and promote them cross over the solar self-gravitational potential barrier.

We, nevertheless, arrived at some valuable results of a fundamental importance for describing the SWP in particular [1] and stellar wind plasma in general. For example, however, we qualitatively reported that a net negative electric current flows towards the heliocentre. It is dominated by the electric current due to the thermal motion of the SWP electrons. The positive electric current carried by the solar plasma ions as already mentioned, however, is actually produced due to the space charge polarization $E$-field acceleration through the gravito-electrostatic coupling processes.

A coupled set of required autonomous nonlinear eigenvalue evolution equations to describe the evolution of electric currents under different observation scalings is derived in the present contribution. The closed system is solved numerically to obtain a number of current profiles over a wide range of initial values obtained with the standard methodology of nonlinear stability analysis [2]. Efforts are made to explain the obtained numerical results physically. Possible physical results, future scopes and conclusions are presented. It provides an integrated theoretical outlook of the law of conservation of electric current associated with the GES equilibrium and fluctuation as well under different situations. This is important to add that efforts have been made in the past to produce solar wind-like plasma under laboratory conditions of laser-target interaction processes [3]. There are obvious differences in the scales of length, time and density of space plasma as well as laboratory-produced plasmas. Laser produced plasmas, of course, are unique realistic situations for investigating many space and astrophysical processes of electric current conservation of the current interest of a similar nature.

2. Physical model
A very simplified ideal plasma fluid model approach is adopted to study electric current profiles under GES model under a steady plasma flow pattern. Gravitationally bounded quasi-neutral field-free plasma by a spherically symmetric surface boundary of non-rigid nature under a hydrostatic kind of equilibrium is considered for simplicity. Zeroth order boundary surface is defined by an exact hydrostatic condition of the gravito-electrostatic force balancing of the enclosed plasma mass at some arbitrary radial position from the center of the mean solar gravitational mass. For simplicity, we consider spherical symmetry of the inertially confined SIP mass, which helps to reduce the three dimensional problem of describing the GES into a simplified one-dimensional problem in the radial direction. Only a single radial degree ($\xi$) of freedom is therefore sufficient for describing the three dimensional SIP and SWP under the assumed spherical symmetry. The effects of solar rotation, viscosity, non-thermal energy transport (wave dissipation), and magnetic field are neglected.

The basic idea of such a simplified model for the GES formation [1] can be well justified with quantitative estimates of the gravito-thermal coupling constants for the SIP species. The estimations [1], however, are carried out under the condition of an isothermal SIP system. It is, for instant reference, re-stated that the tiny SIP electrons can very easily overcome the gravitational potential barrier at the SSB, whereas the massive SIP ions fail to do so because of self-solar gravity. This makes us understand the basic physical reason behind why a surface polarization-induced space charge (electrostatic) field (leading to GES structure) is likely to appear due to thermal leakage of the tiny electrons from the SSB in a radially outward direction.

3. Mathematical formulations
In our mathematical model, for simplification as in our earlier contribution [1], the plasma electrons are assumed to obey a Maxwellian velocity distribution in an idealistic situation. In reality, of course, deviations exist from such an idealistic velocity distribution. Again the solar plasma ions are described by their full inertial response of dynamical evolution. This includes the ion fluid momentum equation as well as the ion continuity equation. The first describes the change in ion momentum under the action of the heliocentric gravito-electrostatic field due to potential gradient and forces induced by
thermal gas pressure gradients. The latter is considered a gas dynamic analog of plasma flowing through a spherical chamber of radially varying surface area with macroscopic uniformity. This needs to be clearly mentioned that the same set of coupled dynamical evolution equations [1] are being reproduced in a modified form for investigation of electric current conservation and stability by a numerical technique as a coupled dynamical evolution system in GES-model.

3.1. Bounded scale calculation scheme
A basic set of differential evolution equations [1] constituting a closed dynamical system of our governing hydrostatic model is developed to study the equilibrium character, linear fluctuation and correspondingly, self-consistent current flow of a bounded GES structure on the order of the Jeans scale length ($\lambda_\text{J}$). The normalized form of the basic autonomous closed system of coupled nonlinear dynamical evolution equations under the quasi-neutral plasma approximation is enlisted as follows

$$
\left(M^2 - (1 + \varepsilon_\tau) \right) \frac{1}{M} \frac{dM}{d\xi} = \frac{2(1 + \varepsilon_\tau)}{\xi} - g_s, \quad (1)
$$

$$
\frac{d\theta}{d\xi} + \frac{1}{M} \frac{dM}{d\xi} + 2 + \frac{2}{\xi} = 0, \quad (2)
$$

$$
\frac{dg_s}{d\xi} + \frac{2}{\xi} g_s = e^\theta. \quad (3)
$$

The normalizations applied in the basic development of the governing equations (1) – (3) with all usual notations [1] are defined as follows

$$
\theta = \frac{e\phi}{T_e}, \quad \eta = \frac{\psi}{C_s^2}, \quad N_e = \frac{n_e}{n_0}, \quad N_i = \frac{n_i}{n_0}, \quad M = \frac{v_i}{C_i}, \quad \xi = \frac{r}{\lambda_j}, \quad \lambda_j = \frac{C_s}{\omega_j}, \quad C_j = \left(\frac{T_e}{m_j}\right)^{1/2},
$$

$$
\omega_j = (4\pi n_0 G)^{1/2}, \quad \varepsilon_\tau = \frac{T_i}{T_e}.
$$

For convenience, let us re-define the various notations. The symbol $n_0$ here denotes the constant bulk SIP density corresponding to the mean solar mass density. In addition, the notations $\phi$ and $\psi$ respectively stand for the dimensional (unnormalized) values of the plasma electrostatic potential and the solar self-gravitational potential as variables associated with the GES. The dimensional values of the electron and ion density variables are respectively denoted by $n_e$ and $n_i$. Likewise, the dimensional ion fluid velocity is represented by $v_i$. The notation $\eta$ stands for the normalized value of the solar self-gravitational potential. The notation $N_j$ denotes the normalized ion density. The quantity $M$ , defined as the ratio of ion flow speed to sound phase speed, stands for the ion flow Mach number. We define the value of the solar self-gravitational acceleration as $g_s = d\eta/d\xi$.

The notations $r$ and $\xi$ stand for the unnormalized and normalized radial distance respectively from the heliocentric origin in spherical polar co-ordinates. The other notations $\lambda_j$, $C_s$ and $\omega_j$
defined above stand for the Jeans length, sound speed and Jeans frequency respectively. Finally, \( \varepsilon_r \) stands for the ratio of ion-to-electron temperatures.

Let us now apply a linear perturbation technique for analyzing the fluctuations \( M_1, g_{s1}, \theta_1 \) of solar parameters \( M(\xi), g_s(\xi) \) and \( \theta(\xi) \) around the corresponding GES equilibrium set of values \( M, g_s, \theta \) respectively. The linear perturbative transformations are \( M(\xi) = M + M_1, g_s(\xi) = g_s + g_{s1} \) and \( \theta(\xi) = \theta + \theta_1 \). The linearized closed set of coupled differential dynamical equations from equations (1), (2) and (3) for the description of the SIP fluctuations is given as follows

\[
\begin{align*}
\left[M^2 - (1 + \varepsilon_r) \right] \frac{1}{M} M_1 + \left[M^2 + (1 + \varepsilon_r) \right] \frac{1}{M} g_{s1} = 0, \\
\frac{d\theta_1}{d\xi} + \frac{1}{M} M_1 - \frac{1}{M} \frac{dM}{d\xi} M_1 = 0, \\
\frac{dg_{s1}}{d\xi} + \frac{2}{\xi} g_{s1} = e^\theta \theta_1.
\end{align*}
\]

The above set of autonomous equations describes the fluctuations of solar parameters for the SIP flow dynamics with suffix ‘1’ representing respective perturbations as already defined. Now, in order to explain the SIP electric current flow as a coupled aspect of the GES evolution, we first write down the expression for the net SIP current density as follows

\[
\mathbf{J}_{\text{SIP}} = \mathbf{J}_{\text{SIPB}} - \sqrt{2 g_s \xi} + \sqrt{2(-\theta)} - \left(\sqrt{m_i/m_e}\right) e^\theta \chi^1_{\text{SSB}}.
\]

Here \( \mathbf{J}_{\text{SIPB}} = n_0 e c_{s1} \) is defined as the usual Bohm current density where \( c_{s1} = (T_{ei}/m_i)^{1/2} \) specifies the sound speed in the bulk SIP. The notation \( T_{ei} \) specifies the SIP electron temperature. The other relevant normalized notations will recast themselves as \( \theta = e^{-\phi}/T_{ei} \) and \( \xi = r/\lambda_{ei} \) with \( \lambda_{ei} = c_{s1}/\omega_\theta \).

Near the SSB, electron current will exist, as one can see from equation (7) for the SSB values of the GES potentials [1]. According to convention, the electron dominated current will be flowing toward the heliocenter. Furthermore, it is also inferred from the numerical analysis [1] that the electron temperature differs from SIP to SWP. Consequently, the normalization factors for all the physical variables of the GES problem of current interest are bound to differ. Let us note that the first two terms on the right-hand side equation (7) respectively define the ion current density driven by free fall of the ion fluid under the independent actions of the self-gravitational and the self-electrostatic force fields associated with the GES. That is, although the current components are linearly summed, the GES associated potentials show nonlinear effects. The sign associated with each term on the right-hand side of equation (7) specifies the direction of the current component. Now, the expression for \( \nabla \cdot \mathbf{J}_{\text{SIP}} \) can be derived from equation (7) as follows

\[
\nabla \cdot \mathbf{J}_{\text{SIP}} = \mathbf{J}_{\text{SIPB}} \left[-(2 g_s \xi)^{1/2} \left(g_s + \frac{dg_{s1}}{d\xi} \right) - 2(-\theta)^{1/2} \frac{d\theta}{d\xi} \left(1 + \sqrt{m_i/m_e\sqrt{2(-\theta)}} e^\phi \right) \right].
\]

Now one may, of course, deduce the mathematical condition analytically for the conservative nature of the SIP electric current (in absence of any additional source or sink) from equation (8) using
\[ \text{div}\mathbf{J}_{\text{SIP}} = 0. \] The condition for the existence of a divergence-free electric current (conservative current) on the bounded scale of the SIP distribution flowing helio-centrically can therefore be derived as follows in the form of potential gradient

\[
\frac{\partial \vartheta}{\partial \xi} = \left[ \frac{-\vartheta}{g_s \xi} \right]^{1/2} \left( g_s + \frac{d g_s}{d \xi} \xi \right) \left( 1 + \frac{m_s}{m_e} \sqrt{2(-\vartheta)} e^{\vartheta} \right). \tag{9}
\]

In order to assess the validity of \( \vartheta \) – variation over the SIP scale, one should check the values of the electrostatic potential gradient as equation (9) at all radial points including the initial radial position and the SSB so as to compare with the earlier numerical results [1]. The SSB values may, however, not match with them. It is therefore apparent that the condition for divergence-free current is not satisfied on the bounded scale. Thus there exists a source of current from the SSB, as \( \nabla \cdot \mathbf{J}_{\text{SIP}} > 0 \) near the SSB for \( d\vartheta/d\xi < 0 \). Of course, this poses the problem of finding some appropriate analytical expression for the \( E \)-field profile to maintain a self-consistent plasma current with non-zero divergence. Now applying the linear perturbation technique, the expression for the net perturbed SIP current density is given as follows

\[
\mathbf{J}_{\text{SIP}} = J_{\text{SIPB}} \left[ - \sqrt{2(g_s + g_{s1})} \xi - \sqrt{2(-\vartheta - \vartheta_t)} - \left( \sqrt{m_s/m_e} \right) e^{\vartheta + \vartheta_t} \right]. \tag{10}
\]

In order to obtain the equilibrium as well as perturbed profiles of electric current of SIP flowing in the bounded scale, a numerical technique is to be applied to solve the coupled set of nonlinear dynamical evolution equations (1) – (7) and (8) self-consistently with the same set of initial values obtained by the method of nonlinear stability analysis [2].

3.2. Unbounded scale calculation scheme

In the unbounded scale, the use of the equation for \( g \)-evolution may be avoided because the sun now behaves as a point source for external gravity. In other words, the constant SIP mass acts as an external object to offer a source of gravity for monitoring the outgoing SWP flow with the initially subsonic speed specified at the SSB. Let the quantity \( a_0 = GM_{\odot}/c_s^2 \dot{A}_j \) (a normalization coefficient) is treated as a free parameter, which eventually, provides indirectly an analytical trick to estimate the SWP electron temperature. The value of this parameter is determined by the condition that a transonic solution for the SWP exists for a given set of initial values of the required physical GES variables of current interest. The unperturbed GES dynamics, as in the case of bounded scale calculation (of the SIP), are now described by the following autonomous set of coupled nonlinear differential equations

\[
\left[ M^2(1 + \varepsilon) \right] \frac{1}{M} \frac{dM}{d\xi} = \frac{2(1 + \varepsilon)}{\xi} - \frac{a_0}{\xi^2}, \tag{11}
\]

\[
\frac{d\vartheta}{d\xi} + \frac{1}{M} \frac{dM}{d\xi} + \frac{2}{\xi} = 0. \tag{12}
\]

The above pair of differential equations (11) – (12) is allowed to undergo a linear perturbative technique under the defined GES–equilibrium background. Therefore the closed autonomous set of nonlinear dynamical evolution equations under the linear perturbative treatment for describing the SWP flow dynamics becomes
\[
\left[ M^2 - (1 + \varepsilon_r) \right] \frac{1}{M^2} \frac{dM}{d\xi} + \left[ M^2 + (1 + \varepsilon_r) \right] \frac{1}{M^2} \frac{dM}{d\xi} M_1 = 0, \quad (13)
\]
\[
\frac{d\theta}{d\xi} + \frac{1}{M} \frac{dM}{d\xi} - \frac{1}{M^2} \frac{dM}{d\xi} M_1 = 0. \quad (14)
\]

On the unbounded scale of SWP dynamics, as a coupled nonlinear dynamical GES evolution, the expression for the net electric current density of SWP can be written as
\[
J_{\text{SWP}} = J_{\text{SWPB}} \left[ -\sqrt{\frac{2a_0}{\xi}} + \sqrt{2(1 - \theta)} - \left( \sqrt{m_1/M_r} \right) e^\theta \right]. \quad (15)
\]

Here \( J_{\text{SWPB}} = n_e e c_{i2} \) is the Bohm current density on the SWP scale and \( c_{i2} = (T_{e2}/m_i)^{1/2} \) specifies the corresponding plasma sound speed. The notation \( T_{e2} \) specifies the SWP electron temperature. Moreover, the new normalized quantities are \( \xi = r/\lambda_{i2} \) with \( \lambda_{i2} = c_{i2}/a_0 \). The first two terms on the right-hand side of equation (15) respectively define the ion current density driven by the free fall of ion fluid under the solar gravity and electrostatic force fields associated with the GES on the solar exterior scale. Now, the expressions for \( \nabla \cdot J_{\text{SWP}} \) can be written as
\[
\nabla \cdot J_{\text{SWP}} = J_{\text{SWPB}} \left[ - (2a_0/\xi)^{1/2} \left( \frac{a_0}{\xi^2} \right) - 2(1 - \theta)^{1/2} \frac{d\theta}{d\xi} \left( 1 + \sqrt{\frac{m_1}{m_r}} \sqrt{2(1 - \theta)} e^\theta \right) \right]. \quad (16)
\]

It is obvious that the condition for divergence-free current (conservative current) for the SWP, similar to that in the case of the SIP description, can be derived from equation (16) as follows
\[
\frac{d\theta}{d\xi} = \left( \frac{\xi^2}{\xi^2 - a_0} \right)^{1/2} \left( \frac{a_0}{\xi^2} \right) \left( 1 + \sqrt{\frac{m_1}{m_r}} \right)^2 \left( 2(1 - \theta)^{1/2} e^\theta \right). \quad (17)
\]

One interesting thing that is quite clear from the expression itself is that in the defined asymptotic limit of \( \xi \to \infty \), the electrostatic potential gradient approaches zero, which means the potential is constant. This scaling behavior is quite good and justifiable in absence of any external electrostatic source or sink. Applying some asymptotic scaling transformations of relevant physical variables [1] in equation (17), one can directly write down the mathematical expression for the electrostatic potential gradient at the SWP base (at our defined SSB) for conservative current flow as follows
\[
\left. \frac{d\theta}{d\xi} \right|_{\xi_0} = - \left( \frac{2(1 - \theta_0 \xi_0)}{a_0} \right)^{1/2} \frac{a_0}{\xi_0^2}. \quad (18)
\]

For some known values [1] of \( \theta_0 \sim -1, \xi_0 \sim 3.5, \) and \( a_0 \sim 3 \), equation (18) yields \( d\theta/d\xi |_{\xi_0, -0.4} \sim -0.4 \) which may be treated as a reasonably good one. One can thus conclude that a divergence-free current indeed exists in a limited zone on the unbounded scale under the condition that...
the electrostatic potential and its gradient profiles satisfy equation (17). Furthermore, in the asymptotic
distance limit \( \xi \to \infty \), the ion current under the free fall of only a self-consistent space charge-limited
force field action dominates in the radially outward direction. This is carried out at the cost of total
electron thermal energy flux density (in the form of electrostatic potential energy) that can accelerate
the plasma ions to supersonic speed. Now the net perturbed electric current density of the SWP is as

\[
\tilde{J}_{SWP} = J_{SWP} \left[ -\frac{2a_e}{\xi} + \sqrt{2}\left(e^{-\theta - \theta_l} - \sqrt{m_i/m_e}e^{\theta + \theta_l}\right) \right].
\]

Now as in the case of the description of the SIP current, in order to obtain the equilibrium as well
as perturbed profiles of electric current of the SWP flowing in the unbounded scale, a numerical
technique is applied to solve the coupled set of nonlinear dynamical evolution equations (11) – (15)
and (19) self-consistently as a coupled dynamical evolution system.

4. Results and discussions
The GES-formation occurs as a consequence of solar surface leakage of solar plasma thermal electrons
thereby producing an appreciable space charge polarization to occur near the defined SSB. The depth
of the electrostatic potential well for the ions, so formed, is such as to allow the incoming ions from
the bulk SIP to acquire the directed kinetic energy of ion flow to overcome the maximum gravitational
potential barrier height of its own creation near the SSB. Accordingly, however, the electric current
distributions of the SIP in the solar interior scale and SWP in solar exterior scale may depict the solar
plasma flow dynamics under a coupled dynamical system in association with the law of conservation
of solar electric current locally as well as globally. It may be conjectured clearly that there are,
however, some loss factor of solar electric current through some dissipative channels producing
thereby heating effect, noise formation, intermittency generation, etc.
Figure 1. Profile of (a) solar internal electric current (normalized by equilibrium Bohm current) and (b) net GES-potential (formed due to gravito-electrostatic interaction) associated with SIP. The various lines correspond to the special cases of $\epsilon_f = 0.0$ (line 1), 0.2 (line 2), 0.4 (line 3), 0.6 (line 4), 0.8 (line 5), and 1.0 (line 6), respectively. Figure 1 shows the profiles of (a) solar internal electric current (normalized by equilibrium Bohm current) and (b) net potential (effective potential formed due to gravito-electrostatically coupled interaction) associated with the SIP with normalized radial position $(\xi)$ from the heliocentre $(\xi = 0)$ on a bounded scale. The unperturbed values of the initial position $\xi_0 = 0.01$ with step-size $(\xi_i - \xi_{i-1}) = 0.01$, and initial electrostatic potential $\theta_i = -0.001$ are held fixed. The perturbed initial values of $g_{i1}$, $\theta_i$ and $m_i$ arbitrarily are $10^{-3}$, $10^{-4}$, and $10^{-3}$, respectively. The various lines correspond to the cases $\epsilon_f = 0.0$ (line 1), 0.2 (line 2), 0.4 (line 3), 0.6 (line 4), 0.8 (line 5), and 1.0 (line 6), respectively. The defined zeroth order SSB lies at a radial position $\xi_0 \sim 3.5$ (implying $R_0 \sim 3.5 \lambda_s$). All the current lines obtained in figure 1 (a) overlap to a single one, but all the net potential curves do not as in figure 1 (b). The net potential is responsible for electric current flow. The net potential is very small near the SSB which agrees with already reported results [1]. Again it is clear that the distribution of solar electric current associated with SIP is not sensitive due to above set of the initial values. The encircled points indicate the respective magnitudes of the solar surface values. It is clear that the magnitude of electric current associated with the flow of the SIP near the SSB is 5.74 (in units of equilibrium Bohm current) which is electron-dominated current. For some mean values [1] of the SIP parameters like $T_s = 15 \times 10^6$ K $- 15 \times 10^7$ eV and $\rho_s = 1410$ Kg m$^{-3}$, one can estimate that the Bohm current density $J_{sIP} = 2.62 \times 10^{27}$ A m$^{-2}$. It is also observed that the GES-layer is a source of electric current and the total negative current flows toward the centre of the SIP mass distribution. It is therefore both analytically and numerically clear that the SSB-portion of the
SIP mass distribution is a source of a negative current flowing toward the centre of the spherical chamber containing the SIP mass distribution on a bounded scale.

**Figure 2.** Same as figure 1, but with $\varepsilon_i = 0.40$ and $\theta_i = -0.001$ held fixed. The various lines correspond to the special cases of $\xi_i = 10^{-3}$ (line 1), $2.5 \times 10^{-3}$ (line 2), $1.5 \times 10^{-2}$ (line 3), $7.5 \times 10^{-2}$ (line 4), $10^{-2}$ (line 5), and $10^{-1}$ (line 6), respectively.
Figure 3. Same as figure 1, but with $\xi = 0.01$ and $\xi = 0.40$ held fixed. The various lines are the special cases of $\theta = -0.10$ (line 1), -0.30 (line 2), -0.50 (line 3), -0.70 (line 4), -0.90 (line 5), and -1.3 (line 6), respectively.
Figure 2 shows the same as figure 1, but with the ion-to-electron temperature ratio $\epsilon_{ei} = 0.40$ and initial unperturbed electrostatic potential $\theta_i = -0.001$ held fixed. The various lines correspond to the cases with initial positions $\xi_i = 10^{-1}$ (line 1), $2.5 \times 10^{-2}$ (line 2), $1.5 \times 10^{-2}$ (line 3), $7.5 \times 10^{-2}$ (line 4), $10^{-2}$ (line 5), and $10^{-1}$ (line 6), respectively. It shows that the profiles of the SIP current distribution are sensitive to the input values of the initial position relative to the centre of the entire SIP mass distribution. For the above set of initial values of position, the corresponding electric current variation occurs by 0.69% on the defined SSB.

Figure 3 shows the same as figure 1, but with the initial position $\xi_i = 0.01$ and ion-to-electron temperature ratio $\epsilon_{ei} = 0.40$ held fixed. The various lines correspond to the special cases of $\theta_i = -0.10$ (line 1), -0.30 (line 2), -0.50 (line 3), -0.70 (line 4), -0.90 (line 5), and -1.3 (line 6), respectively. The circled points as before indicate the respective magnitudes of the SSB values. It is seen that the numerical profiles of the SIP current distribution are sensitive to the input values of the initial electrostatic potential. For the above scaling set of initial values of electrostatic potential, the corresponding electric current variation occurs by 0.34% on the defined SSB.

Figure 4 shows the profile of (a) solar equilibrium current (all the lines overlapping to a single one), (b) solar perturbed current, and (c) solar net current associated with the SWP flow dynamics with normalized position ($\xi_i$) on an unbounded scale. The unperturbed values of initial position $\xi_i = 2.50$ with step-size $(\xi_i - \xi_i) = 0.01$, $a_0 = GM_p/e^2\lambda_i = 95$, electrostatic potential $\theta_i = -0.001$, and Mach No. $M_i = 10^{-4}$ are held fixed. The perturbed initial values of $\theta_i$ and $M_i$ arbitrarily are $-10^{-3}$, and $10^{-8}$, respectively. The various lines correspond to the cases $\epsilon_{ei} = 0.00$ (line 1), 0.2 (line 2), 0.4 (line 3), 0.6 (line 4), 0.8 (line 5), and 1.0 (line 6), respectively. All the electric currents are normalized by equilibrium Bohm current. It is observed that the equilibrium electric current is not affected due to any change in $\epsilon_{ei}$ values. The perturbed part of electric current and the total electric current are jointly found to be sensitive due to variation in $\epsilon_{ei}$ values. In fact, at an asymptotically large distance $\xi \geq 100\lambda_i$ on an unbounded scale length under this situation, the equilibrium electric currents associated with the SWP flow dynamics become saturated with an average magnitude on the order of 1.50 (in units of equilibrium Bohm current). Now for some mean values [1] of the SWP-base parameters like $T_e = 1.0 \times 10^6$ K $\sim 1.0 \times 10^6$ eV and $n_i = 10^{15}$ m$^{-3}$, one can estimate that the equilibrium Bohm current density $J_{SWP} = 2.22 \times 10^{11}$ A m$^{-2}$. This is well justified under the basic background that the magnitude of the net current almost co-moving decreases in space due to the flow-flow interaction of the SWP electrons and ions on an unbounded scale length.

Figure 5 shows the same as figure 4, but with initial unperturbed electrostatic potential $\theta_i = -0.001$ and ion-to-electron temperature ratio $\epsilon_{ei} = 0.40$ held fixed. The various lines correspond to the cases $\xi_i = 2.00$ (line 1), 2.25 (line 2), 2.50 (line 3), 2.75 (line 4), 3.0 (line 5), and 3.25 (line 6), respectively. It is seen that the equilibrium electric current associated with the SWP is not affected due to any change in initial position $\xi_i$ values. The perturbed part of the electric current and the total electric current, however, are altogether found to be sensitive for $\xi_i$ variation. In fact, at an asymptotically large distance $\xi \geq 100\lambda_i$ on an unbounded scale length under this situation, the electric currents associated with the SWP flow dynamics become saturated with a magnitude on the order of 4.50 (in units of equilibrium Bohm current).

Figure 6 shows the same as figure 4, but with the initial position $\xi_i = 2.50$ and ion-to-electron temperature ratio $\epsilon_{ei} = 0.40$ held fixed. The various lines correspond to the special cases with initial electrostatic potential $\theta_i = -0.10$ (line 1), -0.20 (line 2), -0.30 (line 3), -0.40 (line 4), -0.50 (line 5), and
-0.60 (line 6), respectively. It is seen that all the numerical evolutionary profiles of the SWP electric current are appreciably affected due to changes in $\theta_i$ – value. The magnitude of each of the electric current counterpart increases due to an increase in $\theta_i$ – value over any spatial scale. The increasing nature of electric current is observed to be a uniformly small one and becomes almost divergence-free at an asymptotically large distance ($\zeta \geq 100\lambda_i$).

![Profile of solar equilibrium electric current in SWP](image1)

![Profile of solar perturbed electric current in SWP](image2)
Figure 4. Profile of (a) solar equilibrium current (all the lines overlapping to a single one), (b) solar perturbed current, and (c) solar net current associated with the SWP flow dynamics with normalized position. The unperturbed values of initial position $\xi = 2.50$ with step-size $(\xi - \xi_{in}) = 0.01$, $a_0 = GM/a^2\lambda_e = 95$, electrostatic potential $\theta = -0.001$, and Mach No. $M_e = 10^4$ are held fixed. The various lines correspond to the special cases of $\epsilon_T = 0.0$ (line 1), 0.2 (line 2), 0.4 (line 3), 0.6 (line 4), 0.8 (line 5), and 1.0 (line 6), respectively.
Figure 5. Same as figure 4, but with $\theta = -0.001$ and $\varepsilon_r = 0.40$ held fixed. The various lines correspond to the special cases of $\xi = 2.00$ (line 1), 2.25 (line 2), 2.50 (line 3), 2.75 (line 4), 3.0 (line 5), and 3.25 (line 6), respectively.
It is observed from our numerical analyses on GES-model that a finite electron-dominated current with a positive finite divergence exists on the solar interior scale for $\beta/\alpha^2 < 0$. Immediately beyond the SSB, that is, on the unbounded scale of the SWP, a divergence-free current exists. This seems to exhibit a discontinuous behavior. In order to resolve this, it will require further research. In reality, electron temperature has profile variations on both the bounded and unbounded scale lengths. It is probable that a self-consistent consideration of two distinct electron temperature profiles in the two regions of bounded and unbounded scale could resolve the interface transition problem of the proposed two-scale theory of the GES-associated solar plasma current system.

![Profile of solar equilibrium electric current in SWP](image1)

![Profile of solar perturbed electric current in SWP](image2)
Figure 6. Same as figure 4, but with $\xi = 2.50$ and $\zeta = 0.40$ held fixed. The various lines correspond to the special cases of $\theta = -0.10$ (line 1), -0.20 (line 2), -0.30 (line 3), -0.40 (line 4), -0.50 (line 5), and -0.60 (line 6), respectively.

In other words, at an asymptotically large distance on an unbounded scale length, the electric currents associated with the SWP flow dynamics become uniform with an average magnitude on the order of 5.00 (in units of Bohm current). It seems that the electron-dominated electric current asymptotically dissipates mainly through a channel of inertial resistance of the plasma ions due to solar gravity as a barrier. The other dissipation channels of the electric current may be through the SWP heating of non-Ohmic type, generation of fluctuations in thermal noise level, propagation of plasma sheath-induced collective oscillations (GES-oscillations), etc. on the scale lengths of local as well as global importance. These processes of creation of dissipations, of course, result at the cost of energy consumption from the flow dynamics of solar plasma on the SIP scale in particular and SWP scale in general. The net current resulting from the coupled flow-flow interaction of nearly co-moving electrons and ions on the SWP-scale exhibits a uniform behavior at asymptotically large distance with respect to the SWP-base. The GES-layer, however, plays the role of a reservoir of electric current dominated by electrons only, because the flow velocity of plasma ions around this layer lies in a lower subsonic range. The net current in this SIP-scale is contributed mainly through electronic flow. The current-current coupling processes to be understood from the obtained numerical results do not show any characteristic channel signature of dissipative mechanisms by which it can be identified. The obtained numerical profiles are in fair agreement with expected results (of electrodynamical aspects in the Sun) in GES-model analysis [[1] and references therein]. In totality apart from this, however, there is no noteworthy violation of the law of conservation of the solar electric current associated with solar plasma flow dynamics under a current-current coupled system in our GES-model analyses with a bounded-unbounded two-scale theory.
5. Conclusions
In the present piece of my contribution, systematic efforts are made to understand the flow dynamics of the SIP on a bounded scale and SWP on an unbounded scale under electric current conservation principle under the GES model with a very simplified approach. Both the solar scales, however, have an integrated outlook as a closed inter-connected dynamical evolution system. It is quite simplified in the sense that it does not include magnetic forces and the role of the interplanetary medium or any other complications such as rotations, viscosity, or non-Maxwellian distributions under kinetic regime.

Under such a model approach, the basic autonomous set of governing differential equations for the description of the SIP current as well as SWP current are developed applying a simple perturbative analysis. Asymptotically, the current distribution on the bounded scale (figures 1-3) fairly matches with that on the unbounded scale (figures 4-6) in accord with the law of conservation of electric current as a coupled dynamical evolution system. But over a distance comparable to the SSB, primarily near the SSB, they get mismatching violating thereby the law of electric current conservation. The numerical results of the flow dynamics of electric current associated with the SIP are shown in figures 1-3 under a wide range set of initial values of relevant physical parameters. The SIP current distribution is found to be insensitive to any variation in temperature (figure 1). It is observed that the profiles of the SIP current distribution are also sensitive to the input values of the initial position relative to the centre of the entire SIP mass distribution (figure 2). It is also further seen that the numerical evolution of the SIP current distribution are sensitive to the input values of the initial electrostatic potential (figure 3).

The unbounded profiles of the flow dynamics of electric current associated with the SWP are numerically obtained as shown in figures 4-6 under a wide range set of initial values of relevant physical parameters for the SWP-base too. It is observed, on the unbounded scale as well, that the equilibrium current is not affected due to any change in $\varepsilon_T$ values (figure 4 (a)). Thus, as shown in the numerical results, the SSB behaves as a source of electric current flowing toward the heliocentre. The perturbed part of electric current and the total electric current associated with the SWP are found to be sensitive to variation in $\varepsilon_T$ values (figures 4 (b), 4 (c)). Furthermore, it is seen that the equilibrium current associated with the SWP is not affected due to any change even in $\xi_i$ values (figure 5 (a)). The perturbed part of electric current and the total electric current, however, are found to be sensitive to $\xi_i$ variation (figures 5 (b), 5 (c)). It is also seen that all the profiles of electric current associated with the SWP are appreciably affected due to changes in $\theta_i$ value (figure 6). The magnitude of each of the current counterpart increases due to increase in $\theta_i$ value over any spatial scale. In brief, the some subjective philosophical conclusions of physical interest are given as follows:

1. The flow dynamics of the SIP on a bounded scale and SWP on an unbounded scale under the light of electric current conservation principle under the GES model is a coupled dynamical process. The current associated with SIP on the solar interior scale is mainly dominated by the SIP electrons (due to very low subsonic ion flow), and that associated with the SWP flow dynamics on the solar exterior scale, by the SWP ions (due to very high supersonic ion flow).

2. Asymptotically, the current distribution on the bounded scale (figures 1 (a), 2 (a), 3 (a)) fairly matches with that on the unbounded scale (figures 4-6) in accord with the law of conservation of electric current as a coupled dynamical evolution system with a very simplified approach. The current perturbations on the SIP-scale are not shown due to their negligible effects relative to the GES-equilibrium ones.

3. Parker’s model and exospheric model discuss electric current dynamics neither in the SIP scale nor in SWP scale [1] quantitatively. This piece of present contribution, however, makes efforts to explain the solar electric current profiles on either scale in accordance with the law of conservation of solar electric current by a numerical method. It is clear from the numerical results that divergence-free
current (mainly SWP proton-dominated with supersonic speed) flows remarkably beyond the SSB and not before that. This is the result of the interaction of the SWP thermal electrons at comparable speed.

(4) The net gravito-electrostatic potential increases with increase in thermal energy of the SIP electrons (figure 1 (b)). The same is also true for an increase in initial GES-inputs (figure 3 (b)). But the net gravito-electrostatic potential does not change due to variation in the initial position value (figure 2 (b)). The net potential is found to be very small near the SSB.

(5) The GES surface is a potential boundary formed due to nearly gravito-electrostatic balancing of hydrostatic type. The net potential has higher magnitudes (figures 1(b), 2 (b), 3 (b)) beyond the SSB compared with those near it (figures 1 (a), 2 (a), 3 (a)). Once the GES is developed, negative currents flow increasingly toward the SIP centre. But positive currents flow conservatively beyond the SWP base with negligible fluctuations in them (figures 4-6). This, in turn, is responsible for the electro-dynamical evolution of the net solar electric current on both the SIP and SWP scales. The little deviations, whatsoever, observed on the interfacial layer near the defined SSB of the two scales may be attributed to some dissipative mechanisms like non-thermal heat conduction (wave dissipation), viscosity, forced deformation of solar plasma fluid, noise generation, etc. As a whole, on a time scale on the order of the Jean’s time scale, the net flow of electric current near the interfacial GES-layer is fairly conserved. But how in-flowing net current is negative and out-flowing net current is positive with respect to the SWP-base which get biased negatively (reverse biased) with 1.00 kV? The current-current interaction near the vicinity of the GES-layer may be studied in future as another interesting work with a multi-temperature electron distribution in our solar plasma system.

(6) As numerically already discussed, the solar electric current evolutions are abruptly sensitive to the input values of some of the relevant physical parameters under GES model. The detailed picture of the sensitivity of the solar electric current on the SIP scale as well as SWP scale on different sets of the initial values of the GES – parameters are computationally shown in figures 1-6.

(7) Lastly, the electric currents dominated by the solar plasma electrons (in SIP-scale), and ions (in SWP scale) get coupled around the GES-layer; and they are of opposite dynamical sense of evolution. At asymptotically large distance, net current acquires a lower saturated value due to relative motion of electrons and ions (almost co-moving species) in SWP in accordance with current conservation principle. The SSB-oscillations (due to the consequences of the GES-perturbation and electric current fluctuation) may be useful to a further structural study of the solar internal properties as a GES-mode (p- & g-modes) stability analysis of more astrophysical interest.

These results may be useful to study the flow dynamics of electric current in the SWP in particular, of stellar wind plasma in general and of inter-stellar nebulae as a whole. The flow dynamics of electric current, moreover, is observed to be a uniform one and hence conservative too (divergence-free) at an asymptotically large distance $\xi \geq 100\lambda_i$. In this asymptotic SWP flow region with global quasi-neutrality at supersonic speed as well as in similar situations of gravitationally sensitive multi-ion colloidal plasma system [4], an application of inertia-induced acoustic excitation theory [[1-3] in 4] may be made for a further study of the GES-associated solar plasma current system, plasma sheath-induced global oscillations and various electro-dynamical conservation principles thereof under the analyses of linear and nonlinear stabilities of the solar collective dynamics [5] in the unique form of wave kinetics of internal origin. This, however, is in progress and expected to appear elsewhere.

6. References

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