Coulomb’s law modification driven by a logarithmic electrodynamics

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Abstract – We examine physical aspects for the electric version of a recently proposed logarithmic electrodynamics, for which the electric field of a point-like charge is finite at the origin. It is shown that this electrodynamics displays the vacuum birefringence phenomenon in the presence of external magnetic field. Afterwards we compute the lowest-order modification to the interaction energy by means of the gauge-invariant but path-dependent variables formalism. These are shown to result in a long-range \((1/r^3)\)-type correction, in addition to a linear and another logarithmic correction, to the Coulomb potential.

The subject of quantum vacuum nonlinearities has been of great interest since the work of Heisenberg and Euler [1], who showed one of the most astonishing predictions of quantum electrodynamics (QED), namely, the light-by-light scattering in vacuum arising from the interaction of photons with virtual electron-positron pairs. From then on, the physical consequences of this fundamental result have been intensely studied from different perspectives [2–5]. It is to be specially recalled, at this stage, that only very recently the experimental detection of light-by-light scattering has been reported in the ATLAS Collaboration [6,7]. It is of interest also to notice experiments related to photon-photon interaction physics have suggested that electrodynamics in vacuum is a nonlinear theory [8–12]. With this in view, different nonlinear electrodynamics of the vacuum may have significant contributions to photon-photon scattering such as Born-Infeld [13] and Lee-Wick [14,15] theories.

In addition, nonlinear electrodynamics have also attracted considerable attention because they emerge naturally in string theories. As is well known, the low-energy dynamics of D-branes is described by a Born-Infeld–type action [16,17]. In this perspective, recently nonlinear electrodynamics have been the object of intensive investigations in the context of phase transitions of black-hole physics [18].

With these considerations in mind, in previous works [19–22], we have studied the physical effects presented by different models of \((3 + 1)\)-D nonlinear electrodynamics in vacuum. In fact, it was shown that for Generalized Born-Infeld, and logarithmic electrodynamics the field energy of a point-like charge is finite. We also point out that generalized Born-Infeld, exponential, logarithmic and massive Euler-Heisenberg–like electrodynamics exhibit the vacuum birefringence phenomenon.

Given the ongoing experiments related to light-by-light scattering, it is of interest to understand better the phenomenological consequences presented by vacuum electromagnetic nonlinearities. Seen from such a perspective, the present work is an extension of our previous studies. To do this we consider the electric version of a recently proposed logarithmic electrodynamics and investigate aspects of birefringence, as well as the computation of the static potential along the lines of [19–22]. In our conventions the signature of the metric is \((+1, -1, -1, -1)\).

Before going ahead, we would like to mention that solutions to nonlinear (non-Abelian) gauge theories which lead to linear and/or logarithmic static potentials similar to those obtained in this work (eq. (34)) have also appeared before in [23,24]. Also, the form of the modified electric field (eqs. (7) and (8) below) has a denominator which is reminiscent of classical solutions of nonlinear Yang-Mills equations [25–28].
Let us start off our considerations with a brief description of the model under consideration (logarithmic electrodynamics). In this case, the gauge theory we are considering is described by the Lagrangian density:

$$\mathcal{L} = -2\beta^2 \ln \left[ 1 + \frac{1}{\beta} s \mathcal{F} \right] + 2\beta \sqrt{s \mathcal{F}},$$

(1)

where $\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$. Here, $s = -1$ for $|E| \geq |B|$ and $s = 1$ for $|E| < |B|$. Furthermore, the $\beta$ constant has (mass)$^2$ dimension in natural units. We also note that, in a purely electric case, the $\beta$ constant could be identified by a background electric field.

With this, we can write the corresponding equations of motion as

$$\nabla \cdot \mathbf{D} = 0, \quad \frac{\partial \mathbf{D}}{\partial t} - \nabla \times \mathbf{H} = 0,$$

(2)

$$\nabla \cdot \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0,$$

(3)

where the $\mathbf{D}$ and $\mathbf{H}$ fields are given by

$$\mathbf{D} = \frac{\sqrt{2\beta} \mathbf{E}}{\sqrt{2\beta} + \sqrt{-s(\mathbf{E}^2 - \mathbf{B}^2)}},$$

(4)

and

$$\mathbf{H} = \frac{\sqrt{2\beta} \mathbf{B}}{\sqrt{2\beta} + \sqrt{-s(\mathbf{E}^2 - \mathbf{B}^2)}},$$

(5)

Next, we readily see that for an external point-like charge, $q$, at the origin, the $\mathbf{D}$-field lies along the radial direction and is given by $\mathbf{D} = \frac{q}{4\pi r^2} \hat{r}$. Hence, for $s = -1$ and $\beta > 0$, the electrostatic field assumes the form

$$|\mathbf{E}| = \frac{\sqrt{2\beta} |q|}{4\sqrt{2\pi} |\beta r^2 - q|}.$$  

(6)

It is important to observe that this solution is valid for $r > \sqrt{\frac{|q|}{2\pi|\beta r^2 - q|}}$.

It should be further noted that, for $\beta < 0$ and $(\sqrt{2\beta} + |\mathbf{E}|) > 0$, the corresponding electrostatic field becomes

$$|\mathbf{E}| = -\frac{\sqrt{2\beta} |q|}{4\sqrt{2\pi} |\beta r^2 + |q|},$$

(7)

which is restricted to the domain $(4\sqrt{2\pi} |\beta r^2 + |q|) > 0$, that is, for $r > \sqrt{\frac{|q|}{4\sqrt{2\pi}|\beta r^2|}}$.

Whereas for $\beta < 0$ and $(\sqrt{2\beta} + |\mathbf{E}|) < 0$, the electrostatic field reads

$$|\mathbf{E}| = \frac{\sqrt{2\beta} |q|}{4\sqrt{2\pi} |\beta r^2 - |q||},$$

(8)

which is valid for $r \geq 0$. Evidently, as $r \to 0$, we get $|\mathbf{E}| = -\sqrt{2\beta}$.

In order to explore the optical properties of the model under consideration, we shall concentrate as the $s = -1$ and $\beta > 0$ case. In such a case

$$\mathbf{D} = \frac{\mathbf{E}}{1 + \frac{1}{\beta \sqrt{2}} \sqrt{\mathbf{E}^2 - \mathbf{B}^2}},$$

(9)

and

$$\mathbf{H} = \frac{\mathbf{B}}{1 + \frac{1}{\beta \sqrt{2}} \sqrt{\mathbf{E}^2 - \mathbf{B}^2}}.$$  

(10)

At this point, it is interesting to recall that the complicated field problem can be simplified to a large extent if the previous equations are linearized. In this case, we consider a weak electromagnetic wave ($\mathbf{E}_p, \mathbf{B}_p$) propagating in the presence of a strong constant external field ($\mathbf{E}_0, \mathbf{B}_0$). Furthermore, for computational simplicity, we will only consider the case of a purely external magnetic field, namely, $\mathbf{E}_0 = 0$. We thus find that

$$\mathbf{D} = \mathbf{E}_p, \quad \mathbf{H} = \mathbf{B}_0,$$

(11)

and

$$\mathbf{H} = \Gamma \left[ \mathbf{B}_p - \frac{(\mathbf{B}_p \cdot \mathbf{B}_0)}{\beta^2 (1 - \frac{\mathbf{B}_p}{\mathbf{B}_0})} \mathbf{B}_0 \right],$$

(12)

with

$$\Gamma = \left( 1 - \frac{\mathbf{B}_0^2}{\beta^2} \right) \left( 1 - i \frac{\sqrt{2}}{\sqrt{2} \beta} \right),$$

(13)

where we have kept only linear terms in $\mathbf{E}_p, \mathbf{B}_p$. From these expressions we readily deduce that

$$\epsilon_{ij} = \Gamma \delta_{ij},$$

(14)

and

$$\left( \mu^{-1} \right)_{ij} = \Gamma \left( \delta_{ij} - \frac{1}{\beta^2 (1 - \frac{\mathbf{B}_0}{\mathbf{B}_p})} \mathbf{B}_0 \mathbf{B}_0 \right).$$

(15)

Next, we make a plane-wave decomposition for the fields $\mathbf{E}_p$ and $\mathbf{B}_p$:

$$\mathbf{E}_p (x, t) = \mathbf{E} e^{-i(wt - k \cdot x)}, \quad \mathbf{B}_p (x, t) = \mathbf{B} e^{-i(wt - k \cdot x)}.$$  

(16)

Assuming then that the external magnetic field is in the direction $z$, $\mathbf{B}_0 = B_0 \mathbf{e}_3$, and the light wave moves along the $x$-axis, the corresponding Maxwell equations assume the form

$$\left( \frac{k^2}{w^2} = \varepsilon_{32} \mu_{33} \right) E_2 = 0,$$

(17)

and

$$\left( \frac{k^2}{w^2} = \varepsilon_{33} \mu_{22} \right) E_3 = 0.$$  

(18)

In passing we note that the preceding equations were obtained in the limit $\mathbf{B}_0 \gg \mathbf{B}_p$ and $\beta \gg |\mathbf{B}_0|$.

Accordingly, two interesting situations arise from the foregoing equations.

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First, if \( \mathbf{E} \perp \mathbf{B}_0 \) (perpendicular polarization), from (18) \( E_3 = 0 \), and from (17) we get \( \frac{k}{w} = \varepsilon_{22} \mu_{33} \). Therefore, in this case, the refractive index has the form

\[
n_\perp \equiv \frac{k}{w} = \sqrt{\frac{1 - \frac{B_3^2}{2\beta^2}}{1 - \frac{B_3^2}{2\beta^2}}}.
\]  

(19)

Second, if \( \mathbf{E} \parallel \mathbf{B}_0 \) (parallel polarization), from (17) \( E_2 = 0 \), and from (18) we get \( \frac{k}{w} = \varepsilon_{33} \mu_{22} \). The refractive index reduces to

\[
n_\parallel \equiv \frac{k}{w} = 1.
\]  

(20)

Thus, we have the vacuum birefringence phenomenon, that is, electromagnetic waves with different polarizations have different velocities. It is of interest also to notice that for the \( s = -1 \) and \( \beta < 0 \) case, we obtain the same dispersion equations as expressed by eqs. (19) and (20).

We shall now compute the interaction energy between static point-like sources for this logarithmic electrodynamics. The starting point is the Lagrangian density \( s = -1, \beta > 0 \):

\[
\mathcal{L} = (-\mathcal{F}) - \frac{2}{3\beta}(-\mathcal{F})^{3/2}.
\]  

(21)

To get the last line we used \( \beta \gg \sqrt{-\mathcal{F}} \), according to our preceding development. We remark that the new feature of the present model is the nontrivial presence of the exponent \( 3/2 \) in expression (21). Thus, the purpose of analyzing this model here is to investigate the impact of this exponent on a physical observable.

As was explained in [19–22], an alternative way of writing the above equation is to make use of an auxiliary field \( v \), such that its equation of motion gives back the original equation. We thus find that eq. (21) can be brought to the form

\[
\mathcal{L} = \left(1 - \frac{2}{\beta}v\right)(-\mathcal{F}) + \frac{8}{3\beta}v^3.
\]  

(22)

It is also convenient to rewrite this equation as

\[
\mathcal{L} = -\frac{1}{4V}F_{\mu\nu}F^{\mu\nu} - \frac{\beta^2}{3} \left(V-1\right)^3 V^{-3},
\]  

(23)

where we have used \( \hat{V} = \left(1 - \frac{2}{\beta}v\right) \). Here, the quantization is carried out using Dirac’s procedure. The canonically conjugate momenta are \( \Pi^\mu = -\frac{1}{\beta} F^{0\mu} \). In this manner we have two primary constraints \( \Pi^0 = 0 \) and \( p \equiv \frac{2}{\beta} \lambda = 0 \). Furthermore, \( \Pi_i = \frac{1}{V} \mathcal{E}_i \). The canonical Hamiltonian is then

\[
H_C = \int d^3x \left\{ \Pi_i \partial^i A_0 + \frac{V}{2} \mathcal{H}^2 + \frac{1}{2V} \mathbf{B}^2 \right\} - \frac{\beta^2}{3} \int d^3x \frac{(V-1)^3}{V^3}.
\]  

(24)

Temporal conservation of the primary constraint, \( \Pi^0 \), leads to the secondary constraint \( \Pi = 0, \Pi^0 = 0 \). Whereas for the constraint \( p \), we obtain the auxiliary field \( V \)

\[
V = -\frac{\sqrt{2} \beta}{2\sqrt{\Pi}} \left(1 + \sqrt{1 + \frac{4}{\sqrt{2} \beta \Pi^2}}\right),
\]  

(25)

which will be used to eliminate \( V \). It is worth mentioning that to get this last expression we have ignored the magnetic field in eq. (24), because it adds nothing to the static potential calculation.

By proceeding in the same way as in [19–22], we obtain the extended Hamiltonian as

\[
H = \int d^3x \left\{ w(x) \partial^i \Pi_i + \frac{V}{2} \Pi^2 \right\} - \frac{\beta^2}{3} \int d^3x \frac{(V-1)^3}{V^3},
\]  

(26)

where \( w(x) \) is an arbitrary Lagrange multiplier and \( V \) is given by (25).

We can now compute the interaction energy for the model under consideration. To accomplish this task, we shall recall that the interparticle potential energy can be calculated through the expression [19–22]

\[
\mathcal{V} \equiv \varepsilon (\mathcal{A}_0(0) - \mathcal{A}_0(L)),
\]  

(27)

where the physical scalar potential is given by

\[
\mathcal{A}_0(t, \mathbf{r}) = \int_0^1 d\lambda \mathcal{E}_i(t, \lambda \mathbf{r}).
\]  

(28)

This equation follows from the vector gauge-invariant field expression

\[
\mathcal{A}_\mu(x) \equiv \mathcal{A}_\mu(x) + \partial_\mu \left(-\int_\xi^0 dz^\mu \mathcal{A}_\mu(z)\right),
\]  

(29)

where the line integral is along a space-like path from the point \( \xi \) to \( x \), on a fixed slice time. It should again be stressed here that the gauge-invariant variables (29) commute with the sole first constraint (Gauss law). We have skipped all the technical details and refer to [19–22] for them.

In the presence of an external current \( J^0 \), we first observe that Gauss’ law (obtained from the Hamiltonian formulation above) reduces to

\[
\partial_\mu \Pi^\mu = J^0,
\]  

(30)

where \( E^i = \Pi^\mu \) and \( V \) is given by eq. (25). For \( J^0(\mathbf{r}) = e\delta^{(3)}(\mathbf{r}) \), the electric field is then

\[
\mathbf{E} = -\frac{\sqrt{2} \beta}{2} \left(1 + \frac{1}{\pi \sqrt{2} \beta V^2} \right) \hat{r}.
\]  

(31)

With the aid of eq. (31), eq. (28) can be written as

\[
\mathcal{A}_0(t, \mathbf{r}) = \frac{\sqrt{2} \beta}{2} \int_0^r dz \left(1 + \frac{1}{\pi \sqrt{2} \beta V^2} \right).
\]  

(32)
From this last equation it follows that

\[ A_0(t, r) = \frac{\sqrt{2}\beta r}{2} + \frac{\sqrt{2}\beta}{2} \left\{ \sqrt{p+r^2} + \beta \ln \left[ \frac{r}{\sqrt{p+r^2}} \right] \right\}, \]

(33)

with \( p = \frac{e^2}{\pi \sqrt{2} \beta} \).

Making use of the foregoing equation, we finally obtain the potential (to the lowest order in \( \beta \)) for a pair of static point-like opposite charges located at 0 and \( L \),

\[ \mathcal{V} = -\frac{e^2}{4\pi r} + \frac{e^3\sqrt{2}}{32\beta\pi^2 r^3} - e\sqrt{2}\beta r - \sqrt{\frac{\beta}{\pi \beta}} \ln \left[ \frac{r}{\sqrt{\frac{e}{2\pi \beta}}} \left( r^2 \right) \right], \]

(34)

after subtracting a self-energy term, and \( r = |L| \). Here, an interesting matter comes out. Although the third term in the previous equation is proportional to \( r \), such a term has the wrong sign. Therefore, we cannot speak about confinement. This is a consequence of the nontrivial presence of the exponent \( 3/2 \) in expression (21). It is also important to observe that for the \( s = -1 \) and \( \beta < 0 \) case, we obtain the same static potential profile as expressed by eq. (34).

In order to illustrate this last point, we shall consider the model defined by the following Lagrangian density:

\[ \mathcal{L} = -2\beta^2 \ln \left[ 1 + \frac{1}{\beta} \sqrt{-\mathcal{F}} \right]. \]

(35)

Following our earlier procedure, we first observe that for a point-like charge, \( e \), at the origin, the electrostatic field is given by

\[ |E| = \sqrt{2}\beta \left( 1 - \frac{4\pi}{\sqrt{2} e^2} \right), \]

(36)

hence, as \( r \to 0 \), we get \( |E| = \sqrt{2}\beta \).

Again, to leading order in \( \beta \), eq. (35) becomes

\[ \mathcal{L} = (-\mathcal{F}) - 2\beta \sqrt{-\mathcal{F}}. \]

(37)

By proceeding in the same way as before, we obtain the static potential for two opposite charges located at 0 and \( L \) turns out to be

\[ \mathcal{V} = -\frac{e^2}{4\pi r} + \frac{\beta e}{2\pi} r. \]

(38)

Expression (38) immediately shows the effect of the exponent being now 1/2 (expression (37)) on the static potential, which is the sum of a Coulomb and a linear potential, leading to the confinement of static charges. Interestingly enough, the above static potential profile is analogous to that encountered in a new nonlinear electrodynamics governed by the Lagrangian density

\[ \mathcal{L} = \beta^2 \left\{ 1 - \left[ 1 + \frac{2\sqrt{2}}{\beta} \sqrt{-\mathcal{F}} \right]^p \right\}, \]

(39)

with \( 0 < p < 1 \). For \( p = 1/2 \) we obtain the same potential as the one given by eq. (38).

We conclude by putting our work in its proper perspective. This paper is a sequel to [19–22], where once again we have exploited a correct identification of field degrees of freedom with observable quantities. It was shown that in this new electrodynamics the phenomenon of birefringence takes place in the presence of external magnetic fields. Subsequently we have studied the interaction energy. Our analysis reveals that the static potential profile contains a long-range \( (1/r^4) \)-type correction, in addition to a linear and another logarithmic correction, to the Coulomb potential.

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