Introduction. -The concept of topological defects has been one of the cornerstones in the study of phase transitions in two-dimensional (2D) systems. The known example is the Berezinskii-Kosterlitz-Thouless (BKT) phase transition \[1\] where unbinding of integer vortices occurs in the absence of spontaneous breaking of continuous \(U(1)\) symmetry. The followed Kosterlitz-Thouless-Halperin-Nelson-Young (KTHNY) theory \[2-4\] characterizes a two-stage melting process of 2D crystals. The first stage is the transition from a crystal phase to a hexatic phase resulting from the unbinding of dislocations. Upon further heating, the second transition is driven by unbinding of disclinations. When multiple types of topological defects are considered in liquid crystals \[1\] and superfluid/superconducting systems \[7-10\], the interaction between distinct topological excitations leads to enriched physical phenomena.

One prominent prototype exhibiting these fascinating features is a coupled 2D hexatic-nematic XY model, which is used to understand the unusual melting process \[11,15\] and a hidden-order phase transition of isotropic liquid-crystal films \[16-18\]. Most previous studies were based on large-scale Monte Carlo simulations and focused on the strongly coupled model, where the nematic and hexatic fields favor a parallel relative alignment. However, when the coupling of nematic fields is larger than that of hexatic fields, it cannot unambiguously determine how the inter-component \(Z_3\) Potts long-range order (LRO) exists, indicating that the hexatic and nematic fields are locked together and their vortices exhibit quasi-LRO. Particularly, when the coupling of nematic fields is larger than that of hexatic fields, the inter-component Potts LRO undergoes a two-stage melting process. As increasing the temperature, there emerges an intermediate Potts liquid phase with an algebraic correlation, and the Potts quasi-LRO occurs simultaneously with the formation of charge-neutral pairs of hexatic and nematic vortices. These two-stage phase transitions are associated with the proliferation of the domain walls and vortices of the relative \(Z_3\) Potts variable, respectively. Our results thus reveal the hidden structure of the phase transitions driven by multiple topological defects.

In this work, we apply a new efficient numerical method to the weakly coupled 2D hexatic-nematic XY model and resolve those open issues. Tensor network method has recently become a powerful tool to characterize various phase transitions of quantum many-body systems \[19,20\]. Since the partition function of a 2D statistical model can be represented as a product of 1D quantum transfer operator \[21\], the eigen-equation of the 1D quantum transfer operator can be solved by the algorithm of variational uniform matrix product states (MPS) in the thermodynamic limit \[22-24\]. According to the singularity displayed by the entanglement entropy for the 1D quantum analogue, a global phase diagram can be precisely determined \[25-27\]. At low temperatures, an inter-component Potts LRO always exists, indicating that the hexatic and nematic fields are locked. When the coupling of nematic fields is dominant, the inter-component Potts LRO undergoes a two-stage melting process. An intermediate Potts liquid phase emerges simultaneously with the formation of charge-neutral pair of hexatic and nematic vortices. Such unconventional two-stage phase transitions are associated with the separated proliferations of the domain walls and vortices of the Potts variable.

Model and Method. -As shown in Fig\[1(a)\], the hexatic degrees of freedom describe a six-fold bond-orientational
field represented by $\Theta = |\Theta| \exp(i\theta \vartheta)$ with $\vartheta$ as the bond angle linking the centers of mass of neighboring molecules, while the nematic degrees of freedom arising from the herringbone order in the crystalline phase are described by $\Phi = |\Phi| \exp(i2\varphi)$. The nematic order can also originate from the nematic bond-orientational ordering due to the rod-like shape of the molecules. Three inequivalent herringbone patterns are displayed in Fig. 1(b), from which a different herringbone pattern with the same orientation can be obtained by translating over a lattice vector. Accordingly the 2D hexatic-nematic XY model can be defined as a coupled bilayer model

$$
H = -J_2 \sum_{(i,j)} \cos(2\varphi_i - 2\varphi_j) - J_6 \sum_{(i,j)} \cos(6\vartheta_i - 6\vartheta_j)
- K \sum_i \cos(6\vartheta_i - 6\varphi_i),
$$

where $\varphi_i$ and $\vartheta_i \in [0, 2\pi]$ are two $U(1)$ phase fields, $J_2$ and $J_6$ are their respective nearest-neighbour intra-field couplings, and the inter-component coupling $K$ denotes the minimal hexatic-nematic coupling allowed by the relative symmetry. With $\phi = 2\varphi$ and $\theta = 6\vartheta$, the model Hamiltonian is simplified as

$$
\frac{H}{J} = -\Delta \sum_{(i,j)} \cos(\phi_i - \phi_j) - (2 - \Delta) \sum_{(i,j)} \cos(\theta_i - \theta_j)
- \lambda \sum_i \cos(\theta_i - 3\phi_i),
$$

where $\lambda = K/J$, $J = (J_2 + J_6)/2$, and $\Delta = J_2/J$ as the relative interacting strength of the hexatic and nematic fields. In the absence of the hexatic-nematic coupling, the model has $U(1) \times U(1)$ symmetry and exhibits two independent BKT phase transitions. For $\lambda \neq 0$, the model is invariant under a $U(1) \times Z_3$ transformation, $\phi_i \to \phi_i + \alpha/3 + 2\pi k_i/3$ and $\theta_i \to \theta_i + \alpha$, where $k_i = 0, 1, 2$ correspond to a $Z_3$ degrees of freedom. Specially, a sufficient large $\lambda$ tends to lock the hexatic and nematic fields, $\theta_i = 3\phi_i$, and the model is reduced to a generalized XY model.

In the tensor network framework, after a duality transformation, the partition function is expressed as a tensor contraction over all auxiliary links,

$$
Z = \text{tTr} \prod_i O_{n_1m_1,n_2m_2}^{n_3m_3,n_4m_4}(i),
$$

which forms a double-layer tensor network shown in Fig. 1(c). Each local tensor $O$ is given by

$$
O_{n_1m_1,n_2m_2}^{n_3m_3,n_4m_4} = \sum_k \left( \prod_{\lambda=1}^{4} I_{\lambda} \left( \beta \Delta \right) I_{\lambda} \left( \beta \left( 2 - \Delta \right) \right) \right)^{1/2}
\times I_k \left( \beta \lambda \right) \delta_{n_3+n_4+3k} \delta_{m_3+m_4} \delta_{m_1+m_2+k},
$$

where $I_\lambda(x)$ is the modified Bessel function of the first kind, $n$ and $m$ are integers. The global $U(1) \times Z_3$ invariance is encoded in the local tensor: $O_{n_1m_1,n_2m_2}^{n_3m_3,n_4m_4} \neq 0$ only if $(n_2 + 3m_1 + n_2 + 3m_2) = (n_3 + 3m_3 + n_4 + 3m_4)$. In the thermodynamic limit, the partition function is determined by the dominant eigenvalues of the 1D quantum transfer operator $T$, whose eigen-equation (Fig. 1(d)) $T |\Psi(A)\rangle = \Lambda_{\text{max}} |\Psi(A)\rangle$ can be accurately solved by the algorithm of variational uniform MPS. The leading eigenvector $|\Psi(A)\rangle$ is represented by a MPS, whose precision is controlled by the auxiliary bond dimension $D$ of the local tensors.

From the leading eigenvalue and eigenvector of the 1D quantum transfer operator, various physical quantities can be estimated accurately. As the phase transitions are concerned, the quantum entanglement entropy is the most efficient measure, which can be directly determined via the Schmidt decomposition: $S_E = -\sum_{n=1}^{D} s_n \ln s_n$ and $s_n$ are the singular values. Various two-point correlation functions of the local observable can be evaluated by the trace of an infinite sequence of channel operators containing two local impurity tensors. The details are given in Supplemental Materials.

The hexatic-nematic XY model has a rich physics, and the most intriguing results are expected in the weakly coupled case. For a typical value of $\lambda = 0.1$, a global phase diagram is derived in Fig. 2. All phase boundaries are determined by the singularities displayed in the entanglement entropy $S_E$ for the 1D quantum transfer operator. Since the succession of phases crucially depends on the intra-component coupling ratio, our results are discussed in $\Delta < 1$ and $\Delta > 1$, separately.

Phases transitions in the hexatic regime. -For a typical value $\Delta = 0.8$, our numerical results show two singular peaks in the entanglement entropy at $T_{c1} \approx 0.805 J$ and $T_{c2} \approx 1.145 J$, respectively, as shown in Fig. 3(a). The peak positions are nearly unchanged under the bond dimensions $D = 100, 110, 120$, so the transition points are
determined with high accuracy. The corresponding specific heat is shown in Fig. 3(b), exhibiting a sharp divergence at $T_{c1}$ and a small rounded bump around $T_{c2}$. The maximum of the hump is above $T_{c2}$, a typical feature of the BKT transition$^{[2][3]}$, while the low-$T$ singular behavior is fitted by $C_V \propto |T - T_{c1}|^\alpha$ with $\alpha \approx 1/3$. Further evidence is provided by a $Z_3$ order parameter $M_\sigma = \langle \cos(\sigma_i) \rangle$ with $\sigma_i = \theta_i/3 - \phi_i$. As shown in Fig. 3(c), $M_\sigma$ becomes finite at $T_{c1}$, suggesting that a true LRO is established and the relative phase between the $\phi$ and $\theta$ fields is fully locked. The magnetization satisfies the scaling form $M_\sigma \propto (T_{c1} - T)^\beta$ with the critical exponent $\beta \approx 1/9$. These critical exponents are perfectly in agreement with the $Z_3$ Potts transition$^{[3]}

The superfluid response is achieved by the spin stiffness defined by the second derivative of the free energy density with respect to a twist $\nu$ along a reference direction: $\rho_s = \frac{\partial^2 f}{\partial \nu^2}|_{\nu=0}$. The twist needs to be imposed in a way that respects the joint $U(1)$ invariance of the hexatic and nematic fields as

$$\langle \phi_i, \theta_i \rangle \rightarrow \langle \phi_i + \vec{\nu} \cdot \vec{r}_i, \theta_i + 3\vec{\nu} \cdot \vec{r}_i \rangle,$$

(5)

where $\vec{r}_i$ is the position vector for the lattice site $i$. In this way, the jump of spin stiffness should be altered from the BKT predictions when the unbinding of the $\theta$ and $\phi$ vortices happens separately$^{[3]}$. As shown in Fig. 3(d), the spin stiffness starts to dramatically increase from zero at the BKT transition $T_{c2}$, and the detailed calculations are given in the Supplementary Materials. When the temperature further decreases, however, a small jump appears to enhance the spin stiffness at the Potts transition $T_{c1}$ precisely. Such a small increase of spin stiffness is resulted from bindings of integer $\phi$ vortices$^{[1][3]}$. When cooling the system from the disordered phase, the correlation function $G_\phi(r) = \langle \cos(\theta_i - \theta_{i+r}) \rangle$ exhibits a power law decay at $T_{c2}$, indicating the binding of hexatic vortices with charge $q_\phi = 1$. In the intermediate temperature regime ($T_{c1} < T < T_{c2}$), we found that the correlation function $G_{3\phi}(r) = \langle \cos(\phi_i - \phi_{i+r}) \rangle$ decays exponentially, but $G_{3\phi}(r)$ exhibits an algebraic correlation. In Fig. 3(e) and (f), a direct comparison at $T = 1.0 J$ suggests that the integer vortices are fractionalized into fractional vortices with $q_\phi = 1/3$. These are not point-like defects around which the phase angles wind by $2\pi/3$, and each fractional vortex should be connected with a

![FIG. 2: The global phase diagram of the weakly coupled hexatic-nematic XY model with a typical coupling $\lambda = 0.1$. (a) Schematic pictures of different topological excitations. (b) In the Potts liquid phase, the inter-component Potts variable has quasi-LRO, the vortex-antivortex pairs in $\phi$ fields are formed, and the dominant topological defects of the $\theta$ fields are composite vortex pairs with charge $q_\theta = \pm 3$. (c) In the fractional vortex paired phase, the $\theta$ vortices have quasi-LRO, and the $\phi$ vortices are fractionalized as paired vortices with charge $q_\phi = 1/3$. (d) In the Potts ordered phase, vortices in both types are bound in pairs, accompanying the inter-component Potts LRO.](image)

![FIG. 3: The numerical results for $\Delta = 0.8$ and $\lambda = 0.1$. (a) The entanglement entropy. (b) The specific heat. (c) The magnetization of the Potts variable. (d) The superfluid stiffness. (e) and (f) At $T = 1.0 J$, the correlation function $G_{3\phi}(r)$ for $q_\phi = 1$ vortices decays exponentially, but the correlation function $G_{3\phi}(r)$ for $q_\phi = 1/3$ vortices decays in power law.](image)
Phase transitions in the nematic regime. -For a typical value $\Delta = 1.2$, our numerical calculations show two peaks in the entanglement entropy at $T_{c1} \approx 0.915J$ and $T_{c2} \approx 1.15J$, respectively, as seen in Fig. 4(a). Unlike the strongly coupled case\cite{18}, the specific heat $C_V$ exhibits a pronounced peak at $T_{c1}$ and a bump at $T_{c2}$, as displayed in Fig. 4(b). The bump appears slightly above $T_{c2}$ as the usual BKT transition, but the peak locates below $T_{c1}$. Surprisingly, the magnetization of the relative Potts variable displays a two-step feature below $T_{c2}$, as shown in Fig. 4(c). To figure out the transition at $T_{c1}$, we calculate the superfluid stiffness, which also has a two-step jump, as displayed in Fig. 4(d). In contrast to the hexatic regime, the stiffness drops more dramatically at $T_{c1}$ than at $T_{c2}$. Since the phase twist $\vartheta$ applied to the hexatic $\theta$ field is three times larger than that to the nematic $\phi$ field, the first drop of the stiffness is nearly six times larger than the second drop upon increasing temperature.

Moreover, the abrupt drop of spin stiffness at $T_{c1}$ is rather rare, and the binding of charge-neutral vortex pairs of $\theta$ field can be ruled out by calculating the correlation function $G_\theta(r) = \langle \cos(\theta_i - \theta_{i+r}) \rangle$. On two sides of $T_{c1}$, the $G_\theta(r)$ correlations have the power law behavior as shown in Fig. 5(a) and (c), indicating that the quasi-LRO persists through the transition. As displayed in Fig. 5(e), a direct comparison between the amplitudes of $G_\theta(r)$ at $r = 100$ indicates that the correlation above $T_{c1}$ is greatly suppressed by three orders of magnitude, and the exponent $\eta_\theta$ varies with temperature depicted in the inset. The extremely weak correlation above $T_{c1}$ may account for the dramatic jump in total stiffness at $T_{c1}$. Below $T_{c2}$, the onset of algebraic correlations of the $\phi$ vortices also induces the quasi-LRO in the $\theta$ vortices via the coupling $\cos(\theta_i - 3\phi_i)$. So the dominant topological excitations between $T_{c1}$ and $T_{c2}$ are composite vortex pairs of $q_\theta = 3$ with an internal structure of three bound $q_\phi = 1$ vortices, as depicted in Fig. 2(b).

The most interesting physics is found by calculating the correlations of the relative $Z_3$ Potts variable that the inter-component Potts order-disorder phase transition of the weakly coupled hexatic-nematic XY model splits into two different transitions separated by an intermediate liquid phase with quasi-LRO correlation. As shown in Fig. 5(b), (d) and (f), the correlation function $G_\phi(r)$ exhibits a LRO in the Potts ordered phase, an algebraic decay in the intermediate phase, and an exponential decay in the Potts disordered phase, respectively. The intermediate phase is referred as the inter-component Potts liquid phase. Actually the Potts quasi-LRO occurs simultaneously with the formation of charge-neutral pairs of hexatic and nematic vortices, in sharp contrast to the strongly coupled hexatic-nematic model, where the Potts LRO is established together with the quasi-LRO in both hexatic and nematic fields via a single BKT transition\cite{16,17}. It is emphasized that the similar intermediate phase was argued in the $p$-state ($p > 4$) clock model\cite{10,13}.

Unlike the 2D Ising model where a phase transition is driven by loop-like domain walls, the $Z_3$ Potts model allows the loop-like domain walls as well as $Z_3$ vortices, and the usual single $Z_3$ Potts transition is resulted from a coupled proliferation of these two topological defects\cite{12}. Actually, a quasi-LRO phase can emerge in a generalized $Z_3$ Potts model by artificially raising the vortex core energy\cite{14}. We notice that the effective core energy of $Z_3$ vortices in the coupled hexatic-nematic model is related to the hexatic-nematic coupling. A larger $\lambda$ makes the excitations of free vortices in the $\theta$ field more costly, and the domain wall energy increases dramatically to forbid the pre-excitations of domain walls, leading to one phase transition only. An estimation from an effective long-wavelength model\cite{18} gives rise to the energy of domain walls proportional to $\sqrt{\lambda\Delta}$, while the core energy of the $Z_3$ vortices is proportional to $\Delta$. So a small $\lambda$ is required to achieve a relatively high ratio of the core energy of the $Z_3$ vortices. The dependence of these two transitions on the hexatic-nematic coupling strength $\lambda$ is discussed in detailed in the Supplemental Materials.
**Conclusion and Outlook.** We have studied a weakly coupled heaxtic-nematic XY model by using the tensor-network method. It is found that, the structure of the phase diagram in the haxtic regime remains the same as the strongly coupled model [15-16], but a fractionalized vortex paired phase occurs as an intermediate phase between an upper BKT transition and a lower hybrid Potts-BKT transition. However, in the nematic regime, an inter-component Potts liquid phase emerges as the intermediate phase of the two-stage melting of the inter-component Potts order. Two continuous phase transitions are driven by a decoupled proliferations of domain walls and vortices of the relative \(Z_3\) degrees of freedom between the haxtic and nematic phase fields.

The existence of such an emergent Potts liquid phase opens up a promising route towards better understanding the hidden structure of the phase transitions driven by different topological defects. Our work enriches the physics of multi-stage melting process of 2D liquid crystal films [15-17]. Moreover, the unconventional Potts liquid phase might also be realized in condensed atom-molecular mixtures [18-19] and multi-component superconductors [20].

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[1] V. Berezinsky, Sov. Phys. JETP 32, 493 (1971).
[2] J. M. Kosterlitz and D. J. Thouless, Journal of Physics C: Solid State Physics 6, 1181 (1973), URL https://doi.org/10.1088
[3] J. M. Kosterlitz, Journal of Physics C: Solid State Physics 7, 1046 (1974), URL https://doi.org/10.1088
[4] B. I. Halperin and D. R. Nelson, Phys. Rev. Lett. 41, 121 (1978), URL https://link.aps.org/doi/10.1103/PhysRevLett.41.121
[5] D. R. Nelson and B. I. Halperin, Phys. Rev. B 19, 2457 (1979), URL https://link.aps.org/doi/10.1103/PhysRevB.19.2457
[6] A. P. Young, Phys. Rev. B 19, 1855 (1979), URL https://link.aps.org/doi/10.1103/PhysRevB.19.1855
[7] L. Radzihovsky, P. B. Weichman, and J. I. Park, Annals of Physics 323, 2376 (2008), ISSN 0003-4916, URL https://www.sciencedirect.com/science/article/pii/S0003491608000742
[8] L. d. F. de Parny, A. Rançon, and T. Roscilde, Phys. Rev. A 93, 023639 (2016), URL https://link.aps.org/doi/10.1103/PhysRevA.93.023639
[9] P. Serna, J. T. Chalker, and P. Fendley, Journal of Physics A: Mathematical and Theoretical 50, 424003 (2017), URL https://doi.org/10.1088
[10] M. Kobayashi, M. Eto, and M. Nitta, Phys. Rev. Lett. 123, 075303 (2019), URL https://link.aps.org/doi/10.1103/PhysRevLett.123.075303
[11] K. J. Strandburg, Rev. Mod. Phys. 60, 161 (1988), URL https://link.aps.org/doi/10.1103/RevModPhys.60.161
[12] E. P. Bernard and W. Krauth, Phys. Rev. Lett. 107, 155704 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.107.155704
[13] S. C. Chae, N. Lee, Y. Horibe, M. Tanimura, S. Mori, B. Gao, S. Carr, and S.-W. Cheong, Phys. Rev. Lett. 108, 167603 (2012), URL https://link.aps.org/doi/10.1103/PhysRevLett.108.167603
[14] R. Bruinsma and G. Aeppli, Phys. Rev. Lett. 48, 1625 (1982), URL https://link.aps.org/doi/10.1103/PhysRevLett.48.1625
[15] G. Aeppli and R. Bruinsma, Phys. Rev. Lett. 53, 2133 (1984), URL https://link.aps.org/doi/10.1103/PhysRevLett.53.2133
[16] I. M. Jiang, S. N. Huang, J. Y. Ko, T. Stoebbe, A. J. Jin, and C. C. Huang, Phys. Rev. E 48, R3240 (1993), URL https://link.aps.org/doi/10.1103/PhysRevE.48.R3240
[17] I. M. Jiang, T. Stoebbe, and C. C. Huang, Phys. Rev. Lett. 76, 2910 (1996), URL https://link.aps.org/doi/10.1103/PhysRevLett.76.2910
[18] V. Drouin-Touchette, P. F. Orth, P. Coleman, P. Chand, and T. C. Lubensky, Phys. Rev. X 12, 011043 (2022), URL https://link.aps.org/doi/10.1103/PhysRevX.12.011043
[19] F. Verstraete, V. Murg, and J. Cirac, Advances in Physics 57, 143 (2008), https://doi.org/10.1080/00018730801912366
[20] R. Orús, Annals of Physics 349, 117 (2014), ISSN 0003-4916, URL http://www.sciencedirect.com/science/
[21] J. Haegeman and F. Verstraete, Annual Review of Condensed Matter Physics 8, 355 (2017), https://doi.org/10.1146/annurev-conmatphys-031016-025507, URL https://doi.org/10.1146/annurev-conmatphys-031016-025507.

[22] V. Zauner-Stauber, L. Vanderstraeten, M. T. Fishman, F. Verstraete, and J. Haegeman, Phys. Rev. B 97, 045145 (2018), URL https://link.aps.org/doi/10.1103/PhysRevB.97.045145.

[23] M. T. Fishman, L. Vanderstraeten, V. Zauner-Stauber, J. Haegeman, and F. Verstraete, Phys. Rev. B 98, 235148 (2018), URL https://link.aps.org/doi/10.1103/PhysRevB.98.235148.

[24] L. Vanderstraeten, J. Haegeman, and F. Verstraete, SciPost Phys. Lect. Notes p. 7 (2019), URL https://scipost.org/10.21468/SciPostPhysLectNotes.7.

[25] L. Vanderstraeten, B. Vanhecke, A. M. Läuchli, and F. Verstraete, Phys. Rev. E 100, 062136 (2019), URL https://link.aps.org/doi/10.1103/PhysRevE.100.062136.

[26] Z.-Q. Li, L.-P. Yang, Z. Y. Xie, H.-H. Tu, H.-J. Liao, and T. Xiang, Phys. Rev. E 101, 060105 (2020), URL https://link.aps.org/doi/10.1103/PhysRevE.101.060105.

[27] F.-F. Song and G.-M. Zhang, Phys. Rev. Lett. 128, 165301 (2022), URL https://link.aps.org/doi/10.1103/PhysRevLett.128.165301.

[28] M. J. P. Gingras, P. C. W. Holdsworth, and B. Berge Lena, Europhysics Letters (EPL) 9, 539 (1989), URL https://doi.org/10.1209/0295-5075/9/6/008.

[29] E. Granato and J. M. Kosterlitz, Phys. Rev. B 33, 4767 (1986), URL https://link.aps.org/doi/10.1103/PhysRevB.33.4767.

[30] F. C. Poderoso, J. J. Arenzon, and Y. Levin, Phys. Rev. Lett. 106, 067202 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.106.067202.

[31] G. A. Canova, Y. Levin, and J. J. Arenzon, Phys. Rev. E 89, 012126 (2014). URL https://link.aps.org/doi/10.1103/PhysRevE.89.012126.

[32] G. A. Canova, Y. Levin, and J. J. Arenzon, Phys. Rev. E 94, 032140 (2016), URL https://link.aps.org/doi/10.1103/PhysRevE.94.032140.

[33] V. Zauner-Stauber, L. Vanderstraeten, M. T. Fishman, F. Verstraete, and J. Haegeman, Phys. Rev. B 97, 045145 (2018), URL https://link.aps.org/doi/10.1103/PhysRevB.97.045145.

[34] F.-F. Song and G.-M. Zhang, Phys. Rev. B 103, 024518 (2021), URL https://link.aps.org/doi/10.1103/PhysRevB.103.024518.

[35] F.-F. Song and G.-M. Zhang, Phys. Rev. B 105, 134516 (2022), URL https://link.aps.org/doi/10.1103/PhysRevB.105.134516.

[36] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. 90, 227902 (2003), URL https://link.aps.org/doi/10.1103/PhysRevLett.90.227902.

[37] F. Pollmann, S. Mukerjee, A. M. Turner, and J. E. Moore, Phys. Rev. Lett. 102, 255701 (2009), URL https://link.aps.org/doi/10.1103/PhysRevLett.102.255701.

[38] F. Y. Wu, Rev. Mod. Phys. 54, 235 (1982), URL https://link.aps.org/doi/10.1103/RevModPhys.54.235.

[39] D. M. Hubscher and S. Wessel, Phys. Rev. E 87, 062112 (2013), URL https://link.aps.org/doi/10.1103/PhysRevE.87.062112.

[40] J. V. José, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Phys. Rev. B 16, 1217 (1977), URL https://link.aps.org/doi/10.1103/PhysRevB.16.1217.

[41] S. Elitzur, R. B. Pearson, and J. Shigemitsu, Phys. Rev. D 19, 3698 (1979), URL https://link.aps.org/doi/10.1103/PhysRevD.19.3698.

[42] M. B. Einhorn, R. Savit, and E. Rabinovici, Nuclear Physics B 170, 16 (1980), ISSN 0550-3213, URL https://www.sciencedirect.com/science/article/pii/0550321380904733.

[43] C. M. Lapilli, P. Pfeifer, and C. Wexler, Phys. Rev. Lett. 96, 140603 (2006), URL https://link.aps.org/doi/10.1103/PhysRevLett.96.140603.

[44] S. Bhattacharya and P. Ray, Phys. Rev. Lett. 116, 097206 (2016), URL https://link.aps.org/doi/10.1103/PhysRevLett.116.097206.

[45] R. Pindak, D. E. Moncton, S. C. Davey, and J. W. Goodby, Phys. Rev. Lett. 46, 1135 (1981), URL https://link.aps.org/doi/10.1103/PhysRevLett.46.1135.

[46] C. C. Huang, J. M. Viner, R. Pindak, and J. W. Goodby, Phys. Rev. Lett. 46, 1289 (1981), URL https://link.aps.org/doi/10.1103/PhysRevLett.46.1289.

[47] C.-F. Chou, A. J. Jin, S. W. Hui, C. C. Huang, and J. T. Ho, Science 280, 1424 (1998), URL https://www.science.org/doi/abs/10.1126/science.280.5368.1424.

[48] E. A. Donley, N. R. Claussen, S. T. Thompson, and C. E. Wieman, Nature 417, 529 (2002), URL https://doi.org/10.1038/417529a.

[49] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010), URL https://link.aps.org/doi/10.1103/RevModPhys.82.1225.

[50] R. H. Fernandes, L. H. VanBebber, S. Bhattacharya, P. Chandra, V. Keppens, D. Mandrus, M. A. McGuire, B. C. Sales, A. S. Sefat, and J. Schmalian, Phys. Rev. Lett. 105, 157003 (2010), URL https://link.aps.org/doi/10.1103/PhysRevLett.105.157003.