Mathematical model of heating a layer polymer dielectrics

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Abstract. The use of modern polymeric materials as dielectrics makes it possible to increase operational characteristics of the elements of various electric power and electronic devices. The necessary combination of interconnected values of permissible electric field strength and maximum operating temperature is crucial when choosing a particular polymer material. The relationship of these parameters is nonlinear due to the nonlinear (in general case) dependence the electrical resistivity and the thermal conductivity coefficient on temperature of the polymer material. If the dielectric layer is under a constant different electric potential, then there is a dependence on the temperature of electrical resistance and thermal conductivity coefficient of the polymer material. In this case the relationship can be represented in a closed analytical form of integral relations in which the variable limit of the integrals is the desired function describing the temperature distribution in the dielectric layer. In the work, a quantitative analysis of these ratios was carried out for some polymeric materials used as dielectrics with known electrothermal characteristics. The results of calculations of the temperature state for the dielectric layer and the distribution of the absolute value of the electric field intensity are presented. The presented results can be used to justify the choice of a particular polymer material as a dielectric in the designed devices.

1. Introduction

In modern electric power and electronic devices, various polymeric materials are widely used as dielectrics. The main requirements for the used polymeric materials are a combination of interrelated values of the limiting operating temperature and the allowable electric potential difference on the surfaces of the dielectric layer. In devices of constant voltage, the interrelation of these characteristics is influenced by the change with temperature of the electrical resistance of the polymer material and its thermal conductivity.

At a temperature of about 300 K, polymeric materials used as dielectrics have a very high electrical resistance of the order of $10^{14} - 10^{18}$ Ohm $\cdot$ m [1-3], that limiting the strength electric current passing through the dielectric layer. However, with a large difference in electric potentials, an increase in current increases the release of Joule heat in the dielectric and causes an increase in its temperature. The electrical resistance of most polymer dielectrics decreases with increasing temperature. Therefore, at a fixed electric potentials difference, this leads to a further intensification of heat release. The relatively low thermal conductivity coefficient of polymeric materials [1, 4, 5] and the insufficient efficiency of heat removal of released energy into the external environment create prerequisites for the occurrence of positive feedback causing
an additional temperature rise, which may lead to the so-called thermal breakdown of the dielectric [2, 6, 7].

The choice of a polymeric material as a dielectric with a lower decrease in electrical resistivity with increasing temperature can weaken the negative effect of this positive feedback. The thermal conductivity of some polymer dielectrics increases with increasing temperature [1, 4]. It also helps reduce the risk of thermal breakdown of the dielectric. Finally, for reliable functioning of a polymeric dielectric with a high electric potentials difference, it is necessary to effectively discharge the released Joule heat.

The factors noted above that affect the performance of a polymeric dielectric are interconnected, with quite complex non-linear dependencies. Therefore, a quantitative analysis of this relationship, which is necessary for assessing the performance of a polymer dielectric, requires the use of modern methods of mathematical modeling [8–11]. In the case under consideration, with a constant difference of electric potentials on the surfaces of a flat layer of a polymer dielectric, a nonlinear problem of steady-state thermal conductivity was formulated. This task forms the basis of the differential form of a mathematical model that describes the temperature state of a dielectric layer with a given temperature of the cooled surface and an ideally thermally insulated opposite surface. The information necessary for the quantitative analysis of the temperature state of this layer on the dependences of the electrical resistivity on temperature and the thermal conductivity coefficient for several polymer dielectrics is presented. Due to the absence of a closed analytical solution of the formulated problem, the differential form of the model is transformed to two integral relations. The variable limit of the integrals in these relations is the unknown function describing the temperature state of the dielectric layer. The definition of this function allows to compare the maximum operating temperature and the allowable electric potentials difference in the layer under consideration.

2. Problem statement

In the case of homogeneous heat transfer conditions on each of the two surfaces bounding a polymer dielectric flat layer with a thickness of $h = \text{const}$, the temperature distribution of $T(z, t)$ changing in time $t$ will be one-dimensional and will depend only on one coordinate $z$, measured in the direction of the normal to the surfaces of the layer. In the case of temperature dependence of the volumetric heat capacity $c$ and the thermal conductivity $\lambda$ of the dielectric material, this distribution should satisfy the nonlinear nonstationary differential equation

$$
c(T) \frac{\partial T}{\partial t} = \frac{d}{dz} \left( \lambda(T) \frac{dT}{dz} \right) + q_V(T, z, t) = 0,
$$

where $q_V$ is volumetric power of energy release, caused by the transition of some of the electrical energy into Joule heat and depending both on temperature and on the coordinate.

The surface of the dielectric layer, from which the coordinate $z$ is counted, is assumed to be ideally thermally insulated, and on the opposite cooled surface with the coordinate $z = h$ we set the temperature $T_1 = \text{const}$. The electric field potential at $z = 0$ put equal to zero, and the absolute value of the potential on the opposite surface denote by $U_1 = \text{const}$. The formulated conditions are applicable to the dielectric layer doubled thickness with an absolute value of the potential difference $2U_1$ and given temperature values $T_1$ on both surfaces of such layer.

Under given conditions, the temperature distribution over the layer thickness will approach a stationary state determined by the function $T(z)$, which depends only on the coordinate $z$. This function will satisfy the following non-linear stationary differential equation from the equation (1)

$$
\frac{d}{dz} \left( \lambda(T) \frac{dT}{dz} \right) + q_V(T, z) = 0
$$

(2)
with boundary conditions
\[ \lambda(T) \frac{dT(z)}{dz} \bigg|_{z=0} = 0, \quad T(h) = T_1. \] (3)

In the steady state, the module \( j \) of the density vector of the electric current passing through a fragment of a dielectric layer will be constant and equal to \([12]\)
\[ j = E(z)/\rho(T), \] (4)
where \( E \geq 0 \) is the modulus of the electric field strength vector, \( \rho > 0 \) is temperature-dependent electrical resistance of a dielectric at constant voltage. Then the power of volumetric heat release in a dielectric in a stationary state can be represented with regard to equality (4) in the form \([13]\)
\[ q_v(T, z) = (E(z))^2/\rho(T) = j^2\rho(T). \] (5)

Equation (2) with regard to equality (5) and boundary conditions (3) constitute the differential form of a mathematical model describing the steady state temperature of the dielectric layer under consideration. However, the nonlinearity of this form of the model does not allow us to represent the desired function \( T(z) \) as an analytical dependence of temperature on the coordinate \( z \). But with the known dependences \( \lambda(T) \) and \( \rho(T) \) it is possible to construct integral relations in which this function or its individual values at fixed points are variable limits of the integrals.

3. Integral relations
The substitution \( \lambda(T) \frac{dT(z)}{dz} = p \) and the equality (5) allows to represent equation (2) in the form:
\[ p \frac{dp}{dT} + j^2 \lambda(T)\rho(T) = 0 \]
and then for this equation, using the second equality of (3), we write the first integral
\[ \left( \lambda(T) \frac{dT(z)}{dz} \right)^2 = C_1 - 2j^2 \int_{T_1}^{T(z)} \lambda(T')\rho(T') \, dT'. \]

If \( T_0 \) denotes the unknown temperature on the ideally heat-insulated surface of the dielectric layer and use the first equality (3), which determines the boundary condition on this surface, then we obtain
\[ C_1 = 2j^2 \int_{T_1}^{T_0} \lambda(T')\rho(T') \, dT'. \]

After substituting \( C_1 \) in the first integral the ordinary differential equation of the first order can be obtained
\[ \lambda(T) \frac{dT(z)}{dz} = \pm j \left( 2 \int_{T}^{T_0} \lambda(T')\rho(T') \, dT' \right)^{1/2} \]. (6)

Proceeding from the physical meaning of the problem, the derivative in the left part of equation (6) is nonpositive \([13]\). Therefore, on the right-hand side of this equation, the minus sign should be chosen and, after integration, written
\[ z = C_2 - \frac{1}{j} \int_{T_0}^{T(z)} \left( 2 \int_{T}^{T'} \lambda(T'')\rho(T'') \, dT'' \right)^{-1/2} \lambda(T') \, dT'. \]
Using the second equality (5), we find

\[ C_2 = h + \frac{1}{j} \int_{T_0}^{T_1} \left( 2 \int_T^{T_0} \lambda(T') \rho(T') \, dT' \right)^{-1/2} \lambda(T) \, dT \]

and in the end

\[ h - z = \frac{1}{j} \int_{T_1}^{T(z)} \left( 2 \int_T^{T_0} \lambda(T') \rho(T') \, dT' \right)^{-1/2} \lambda(T') \, dT'. \]  

Equation (7) includes unknown values of \( j \) and \( T_0 \). In the considered dielectric layer, the losses per unit time of electrical energy in the form of Joule heat per unit surface area of the layer are equal \( jU_1 \). Therefore, by taking into account the formula (6), we can write

\[ -\lambda(T) \frac{dT(z)}{dz} \bigg|_{z=h} = j \left( 2 \int_{T_1}^{T_0} \lambda(T) \rho(T) \, dT \right)^{1/2} = jU_1. \]

This implies the connection of the functions \( \lambda(T) \) and \( \rho(T) \) and the required temperature \( T_0 \) with a given value of \( U_1 \) in the form of the integral relation

\[ U_1^2 = 2 \int_{T_1}^{T_0} \lambda(T) \rho(T) \, dT. \]  

Determining the value of \( T_0 \) from relation (8) and putting \( z = 0 \) and \( T(0) = T_0 \) in the formula (7), we can find the magnitude of the current density vector

\[ j = \frac{1}{h} \int_{T_1}^{T_0} \left( 2 \int_T^{T_0} \lambda(T') \rho(T') \, dT' \right)^{-1/2} \lambda(T) \, dT \]  

and then get the integral relation

\[ h - z = \frac{1}{j} \int_{T_1}^{T(z)} \left( 2 \int_T^{T_0} \lambda(T') \rho(T') \, dT' \right)^{-1/2} \lambda(T') \, dT', \]

which allow to calculate the temperature distribution \( T(z) \) over the thickness of the dielectric layer.

4. Quantitative analysis of integral relations

To carry out a quantitative analysis of the obtained integral relations and to establish the relationship between the limit values \( T_0 \) and \( U_1 \), it is necessary in this case to have reliable information about the electrophysical characteristics of the dielectric material. In this case, such analysis requires dependencies the electrical resistivity \( \rho \) and the thermal conductivity \( \lambda \) on temperature.

There are graphs of dependences of the decimal logarithm of the ratio \( \rho/\rho_s \) in figure 1, using the data from work [1], where \( \rho_s \), Ohm-m is the electrical resistance value at temperature \( T = 300 \) K for each of the five most widely used polymeric materials: \( 10^{16} \) — polyimide (1); \( 1.58 \cdot 10^{15} \) — polyethylene terephthalate (2), \( 6.31 \cdot 10^{16} \) — polychlorotrifluoroethylene (3).
3.16 \cdot 10^{14} — polycarbonate (4), 10^{17} — polytetrafluoroethylene (5). The numbers in brackets correspond to the numbers of the curves in this and all subsequent figures.

One of the features of polymer dielectrics is that for them the coefficient of thermal conductivity, as a rule, increases with increasing temperature. In addition, the coefficient of thermal conductivity may depend on the microstructure of the polymer, also dependent on temperature. In the case of an amorphous structure corresponding to the chaotic arrangement of polymer macromolecules, the coefficient of thermal conductivity usually has the smallest value and increases as the arrangement of macromolecules increases due to the increase in polymer density [1, 4].

For the polymeric materials listed above figure 2 shows temperature-dependent boundaries of changes in the coefficient of thermal conductivity upon transition from an amorphous structure (lower boundary) to a structure with the highest density (upper boundary) [1, 4]. For polypiromellitimide (position 1), the given curves are not related to the structural features of the polymer and refer to different modifications of the material produced by DuPont (USA) with the Capton trademark (the lower curve is Kapton H, and the upper one is Kapton HN [14]). Other materials also have different industrial brands along with the names defined by the chemical composition: for example, 2 — dacron, lavsan, mylar, 3 — fluoroplast-3, 4 — macrolon, merlon, 5 — fluoroplast-4, teflon.

![Figure 1](image1.png)  
**Figure 1.** Dependence on the temperature of the electrical resistance for some polymeric materials.

![Figure 2](image2.png)  
**Figure 2.** Upper and lower bounds of temperature variation of the thermal conductivity of some polymeric materials.

The value of $T_1$ temperature of the cooled surface of the dielectric layer significantly depends on the cooling intensity. With the chosen value of $T_1 = 300$ K in figure 3a, in semi-logarithmic coordinates, graphs, that illustrating the relationship between the value of $U_1$ and temperature $T_0$ on a perfectly thermally insulated surface of a dielectric layer of the above-mentioned polymeric materials, are given. The calculations are performed using the integral relation (8). For polypiromelimitimide, polyethylene terephthalate and polychlorotrifluoroethylene in the calculations of the dependence of the $\lambda(T)$ thermal conductivity on temperature (see figure 2) are extrapolated to the value of $T = 400$ K. For polytetrafluoroethylene at $T > 350$ K, the corresponding value of $U_1$ can be assumed constant. With an increase in the temperature $T_1$ of the cooled surface for all considered polymeric materials, the values of $U_1$ decrease (figure 3b).
From a comparison of the graphs in figures 3a, and 3b, it can be seen that in the case of a layer of polytetrafluoroethylene, an increase in the value of $T_1$ only by 20 K leads to a decrease in $U_1$ about two times.

**Figure 3.** The relationship of temperature $T_0$ of an ideally heat-insulated surface of a dielectric layer made of various polymeric materials and electric potential $U_1$ for a given value of $T_1$ of the temperature of the layer surface to be cooled: a $T_1 = 300$ K; b $T_1 = 320$ K

We choose the same values $T_1 = 300$ K and $T_0 = 400$ K to compare the temperature distributions in the dielectric layer from the polymeric materials under consideration. Figure 4 shows the graphs of the dependence of $T(\zeta)$ on the dimensionless coordinate $\zeta = z/h$, calculated using the integral relation (10). At the same time, for each of the materials, the above-mentioned range of variation with temperature of the thermal conductivity coefficient is taken into account. But for all the materials considered, the difference in temperature distributions corresponding to the two limiting dependences $\lambda(T)$ (see figure 2) does not exceed the value of $0.005(T_0 - T_1)$ for $\zeta < 0.999$. Therefore, the two calculated temperature distributions related to a fixed material are presented in figure 4 with one general graph. In the accepted temperature range, the electrical resistance of polycarbonate changes only in 10 times (see figure 1). This leads to the most gentle temperature variation across the thickness of the layer of this material. On the contrary, the most uneven is the temperature distribution across the thickness of the polytetrafluoroethylene layer, caused by a change in electrical resistance of almost four orders of magnitude in this temperature range.

The calculated dependences $T(\zeta)$ allow us to find the distribution over the thickness of the dielectric layer of the modulus of the electric field intensity vector. Using the formula, we write

$$E_s(\zeta) = \frac{E(\zeta)h}{U_1(T_0)} = \frac{jh}{U_1(T_0)} \rho(T(\zeta)),$$

where $U_1(T_0)$ is the value of $U_1$ corresponding to the selected value of $T_0$. With a uniform distribution of electric field strength $E_s = 1$. In figure 4, graphs of the dependence $E_s(\zeta)$ for all the polymer materials under consideration at the chosen values of $T_1 = 300$ K and $T_0 = 400$ K are presented in semilogarithmic coordinates. The most non-uniform distribution of the electric field strength arises in the layer of polytetrafluoroethylene: the lower boundary of the thermal conductivity coefficient corresponds to the values of $E_s(0) \approx 0.0050$ and $E_s(1) \approx 250.4$
(curve 5 with dark circles), and its upper boundary is $E_\star(0) \approx 0.0051$ and $E_\star(1) \approx 262.2$ (curve 5 with light circles). The main reason for this unevenness is the significant change in the electrical resistivity noted above for this material in this temperature range. However, the effect of a change in the dependence of thermal conductivity on temperature is negligible. This effect is most significant for a dielectric layer made of polychlorotrifluoroethylene: for the lower border $E_\star(0) \approx 0.0678$ and $E_\star(1) \approx 17.04$, and for the upper boundary — $E_\star(0) \approx 0.0709$ and $E_\star(1) \approx 17.80$. In the case of the other three polymeric materials this effect is insignificant and for each of them the dependences $E_\star(\zeta)$ are shown in figure 5 one common schedule. The smallest change in the electric field strength is in the polycarbonate layer: $E_\star(0) \approx 0.326$ and $E_\star(1) \approx 3.27$, which is also associated with a relatively small change in the electrical resistivity in the temperature range under consideration.

![Figure 4](image_url)  
**Figure 4.** The temperature distribution over the thickness of the dielectric layer from various polymer materials.

![Figure 5](image_url)  
**Figure 5.** The distribution of the absolute value of the electric field strength along the thickness for various polymeric materials.

5. Conclusion
A nonlinear mathematical model of steady-state thermal conductivity in a flat dielectric layer was used for a quantitative analysis of the relationship between the limiting values of temperature and the difference in electrical potentials on the surfaces of this layer, made of various polymeric materials. The presented integral ratios take into account the change with temperature of the electrical resistance of the dielectric and its coefficient of thermal conductivity of the polymer material and allow us to calculate the distribution of temperature and electric field intensity over the layer thickness. Comparison of the results of the quantitative analysis showed that the highest values of the allowable working temperature and the maximum difference of electric potentials can be realized by using polytetrafluoroethylene as the dielectric material.
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