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Hadronic decays of charmed mesons offer an ideal testbed to investigate strong and weak interactions. Remarkable progress in studies of hadronic D decays involving $K^\pm$ and $K^0_S$ has been achieved to date. However, experimental knowledge of hadronic $D$ decays involving a $K^0_L$ is still very poor [1] mainly due to the difficulty in $K^0_L$ reconstruction. It is often assumed (or taken as a good approximation) that the branching fractions (BFs) of $D$ decays into hadronic final states containing $K^0_L$ meson(s) are equal to those for the corresponding final states with $K^0_S$ meson(s). However, as clarified in Refs. [2–7], the interference between Cabibbo-Favored (CF) and Doubly-Cabibbo-Suppressed (DCS) amplitudes can lead to a significant asymmetry between the BFs of $D^0 \to K^0_S X$ and $D^{0*} \to K^0_L X$ ($X = \pi^0, \eta, \eta', \omega, \rho$, or $\phi$).

$$\mathcal{R}(D^0, X) = \frac{B(D^0 \to K^0_S X) - B(D^0 \to K^0_L X)}{B(D^{0*} \to K^0_S X) + B(D^{0*} \to K^0_L X)} = -2r \cos \delta + y_D,$$  

(1)

where $r$ and $\delta$ are the relative strength and phase between the DCS and CF amplitudes, respectively, and $y_D$ is the $D^0$-$D^{0*}$ mixing parameter [8]. One has $\mathcal{R}(D^0, P) = 2\tan^2 \theta_C (+y_D) = 0.113 \pm 0.001$ for $P = \pi^0, \eta$, or $\eta'$ naively [2–6], where $\theta_C$ is the Cabibbo mixing angle [9].

Using the factorization-assisted topological (FAT) amplitude approach and assuming $E_P = E_V$, Ref. [6] stated that the $\mathcal{R}(D^0, V)$ for $V = \rho, \omega$, or $\phi$ can also be simplified as $2\tan^2 \theta_C + y_D = 0.113 \pm 0.001$, where $E_P$ and $E_V$ are the $W$-exchange amplitudes for $D \to PP$ and $D \to VP$ decays, respectively. Here, $P$ and $V$ denote pseudoscalar and vector mesons, respectively. It is independent of $X$ because the ratio of DCS and CF amplitudes only depends on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The large asymmetry for $\mathcal{R}(D^0, \pi^0)$ has been confirmed by a previous measurement of the CLEO experiment [10]. Measurements of the BFs of $D^0 \to K^0_L \phi, D^0 \to K^0_L \eta$, $D^0 \to K^0_L \omega$, and $D^0 \to K^0_L \eta'$ are crucial to test theoretical calculations and help understand the CKM mechanism. Study of the $K^0_S$-$K^0_L$ asymmetry, $\mathcal{R}(D^0, X)$ is also important to improve the understanding of quark U-spin [11, 12] and SU(3)-flavor symmetry breaking effects and can benefit theoretical predictions of CP violation in $D$ decays [13–21]. These decays are all CP+ eigenstates and can be used to extract the strong phase differences of neutral $D$ decays [22, 23].

Studies of CP violation of the weak decays of $D$ mesons are important for exploring physics within and beyond the Standard Model. The size of CP violation in various $D$ decays is predicted to be in the order of $10^{-3}$ [19, 24–29]. In 2019, LHCb reported the first observation of CP violation in neutral $D$ decays [30]. Currently, the knowledge of CP violation in the charm sector is still limited and further measurements are highly desirable.

This paper reports the first measurements of the BFs of $D^0 \to K^0_L \phi, D^0 \to K^0_L \eta$, $D^0 \to K^0_L \omega$, and $D^0 \to K^0_L \eta'$ as well as the BF asymmetries between $D^0 \to K^0_S X$ and $D^0 \to K^0_L X$. In addition, the CP asymmetries in these decays are also determined. Throughout this paper, charge conjugate channels are implied, unless noted otherwise.

This analysis is performed with a 2.93 fb$^{-1}$ [31] sample of $e^+e^-$ annihilation data taken at a center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector. Details about the design and performance of the BESIII detector are given in Ref. [32]. Simulated samples, produced with the GEANT4-based [33] Monte Carlo (MC) package including the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate background contributions. The simulation includes the beam-energy spread and initial-state radiation in the $e^+e^-$ annihilations mod-
eled with the generator KKMC [34]. An inclusive MC sample, containing the production of $D\bar{D}$ pairs, the non-$D\bar{D}$ decays of the $\psi(3770)$, the initial-state radiation production of the $J/\psi$ and $\psi(3686)$ states, and the continuum processes, is used in this analysis. Known decay modes are modeled with EVTGEN [35] using the BFs taken from the Particle Data Group (PDG) [1] and the remaining unknown decays from the charmonium states are modeled with LUNDCHARM [36]. Final state radiation from charged final-state particles is incorporated using photos [37].

At $\sqrt{s} = 3.773$ GeV, $D^0$ and $\bar{D}^0$ mesons are produced in pairs without accompanying hadrons, and hence the environment is ideal to investigate $D^0$ decays with the double-tag (DT) method [38]. In this method, the $D^0$ meson, later referred as single-tag (ST), is first reconstructed through the hadronic decay $D^0 \to K^+\pi^-$, $K^+\pi^-\pi^0$, and $K^+\pi^-\pi^+\pi^-$, which have large BFs and small background contamination. If a signal $D^0 \to K^0_L\phi, K^0_L\eta, K^0_L\rho_0$, or $K^0_L\eta'$ decay can be reconstructed in the rest of the event, the event is then considered as a DT event. The BF of the signal decay is determined by

$$B_{\text{sig}} = N_{\text{DT}}/(N_{\text{ST}} \cdot \epsilon_{\text{sig}}),$$

where $N_{\text{ST}}$ and $N_{\text{DT}}$ are the yields of the total ST and DT candidates in data, respectively, and $\epsilon_{\text{sig}} = \Sigma_i[(\epsilon_{\text{ST}} \cdot N_{\text{ST}}^i)/(\epsilon_{\text{ST}} \cdot N_{\text{ST}})]$ is the effective signal efficiency of finding the signal decay in the presence of the ST $D^0$ meson, where $\epsilon_{\text{ST}}$ and $\epsilon_{\text{DT}}$ are the detection efficiencies of the ST and DT candidates, respectively, and the index $i$ runs over all ST modes.

In the work described in this paper, candidates for $K^\pm$, $\pi^\pm$, $\gamma$, and $\pi^0$ are selected by using the same selection criteria as in Ref. [39]. The two-body ST mode $D^0 \to K^+\pi^-$ suffers from background contributions from cosmic rays and Bhabha scattering events. These background contributions are rejected by using the same requirements as in Ref. [40]. For the $D^0 \to K^+\pi^-\pi^+\pi^-$ ST mode, the $D^0 \to K^0_LK^\pm\pi^-$ decays are rejected if the mass of the $\pi^+\pi^-\pi^-$ pair falls within $(0.483, 0.513)$ GeV/c$^2$.

Two kinematic variables, the energy difference $\Delta E = E_{D^0 \to \bar{E}_{\text{beam}} - E_{\text{beam}}}$ and the beam-constrained mass $M_{\text{BC}} = \sqrt{E_{\text{beam}}^2/c^2 - [p_{D^0}]^2/c^2}$ are used to separate the ST $D^0$ mesons from combinatorial backgrounds. Here, $E_{\text{beam}}$ is the beam energy and $E_{D^0}$ and $p_{D^0}$ denote the total energy and momentum of the ST $D^0$ candidate in the $e^+e^-$ center-of-mass frame, respectively. If there are multiple combinations in an event, only the combination with the smallest $|\Delta E|$ is accepted. To suppress combinatorial backgrounds, the $\Delta E$ of any ST candidate is required to be within $(-0.055, 0.040)$ GeV for $D^0 \to K^+\pi^-\pi^0$ and within $(-0.025, 0.025)$ GeV for $D^0 \to K^+\pi^-$ and $D^0 \to K^+\pi^-\pi^+\pi^-$. The $M_{\text{BC}}$ distributions of the accepted ST $D^0$ candidates are shown in Fig. 1. To extract the yield of ST $D^0$ mesons for each ST mode, a binned maximum-likelihood fit is performed on the corresponding $M_{\text{BC}}$ distribution. The signal is modeled by the MC-simulated shape convolved with a double-Gaussian function to take into account the resolution difference between data and MC simulation. In the fits, the Gaussian means and widths are free parameters whose ranges are $(0.04, 0.20)$ MeV/c$^2$ and $(0.73, 3.19)$ MeV/c$^2$, respectively. The combinatorial background is described by the ARGUS function [41]. The associated fit results are shown in Fig. 1. The candidates with $M_{\text{BC}} < (1.859, 1.873)$ GeV/c$^2$ are kept for further analyses. Integrating the fitted signal shape in the aforementioned $M_{\text{BC}}$ interval gives the yield of the ST $D^0$ mesons for each ST mode. Summing over all ST modes, the total yield of ST $D^0$ mesons is obtained to be $N_{\text{ST}}^0 = 2266311 \pm 1842$.

The $D^0 \to K^0_L\chi$ candidates are reconstructed with charged and photon candidates which have not been used in the ST side. Candidates for $\phi$ and $\omega$ are reconstructed from $K^+K^-$ and $\pi^+\pi^-\pi^0$ combinations with $M_{K^+K^-} \in (1.005, 1.035)$ GeV/c$^2$ and $M_{\pi^+\pi^-\pi^0} \in (0.752, 0.812)$ GeV/c$^2$, respectively. Candidates for $\eta$ are reconstructed from $\gamma\gamma$ pairs with $M_{\eta\gamma} \in (0.510, 0.570)$ GeV/c$^2$ or $\pi^+\pi^-\pi^0$ combinations with $M_{\pi^+\pi^-\pi^0} \in (0.535, 0.560)$ GeV/c$^2$. Candidates for $\eta'$ are reconstructed from $\pi^+\pi^-\eta$ ($\eta \to \gamma\gamma$) combinations with $M_{\pi^+\pi^-\eta} \in (0.945, 0.970)$ GeV/c$^2$ or $\gamma\gamma$ ($\rho^0 \to \pi^+\pi^-$) combinations with $M_{\rho^0} \in (0.938, 0.978)$ GeV/c$^2$. To improve resolution of $\eta \to \gamma\gamma$, a mass constrained (1C) fit is performed, constraining the selected $\gamma\gamma$ pair invariant mass to the known $\eta$ mass [1]. If there are multiple combinations of $\pi^0$ or $\eta\gamma\gamma$, the one with the least $\chi^2/2$ is kept for further analyses. For $\eta' \to \gamma\rho^0$ ($\rho^0 \to \pi^+\pi^-$), the $\pi^+\pi^-$ system is required to satisfy $M_{\pi^+\pi^-} \in (0.57, 0.97)$ GeV/c$^2$. For $D^0 \to K^0_L\eta'_{\rho\rho}$, the background events from $D^0 \to K^0_L\pi^+\pi^-$ are suppressed by requiring that the recoil mass of the $\pi^+\pi^-$ pair from the signal combined with the ST particles is greater than 0.53 GeV/c$^2$. This requirement suppresses 90% of background events from $D^0 \to K^0_L\pi^+\pi^-$ at the cost of losing 0.5% of the signal. If there are multiple $\gamma\rho^0$ combinations for $D^0 \to K^0_L\eta'_{\rho\rho}$, the one with $M_{\rho^0}$ closest to the known $\eta'$ mass [1] is kept for further analyses. Throughout this paper, the subscripts of $\phi$, $\eta$, $\omega$, and $\eta'$ denote the corresponding reconstruction modes.

To minimize the impact of a $K^0_L$ shower in electromagnetic calorimeter on an extra $\pi^0$ and $\eta$ vetoes, the opening angle between any remaining photon and the missing momentum is required to be greater than 15°. Events with extra charged tracks, $\pi^0$ or $\eta\gamma\gamma$, are rejected to suppress background contributions from $D^0 \to K^0_S(\to \pi^+\pi^-)X$, $D^0 \to K^0_S(\to \pi^0\pi^0)X$, and $D^0 \to \eta\gamma\gammaX$. To separate signal events from background contributions, the variable $\text{MM}^2 = E^2_{\text{miss}}/c^4 - [p_{\text{miss}}]^2/c^2$ is defined, where $E_{\text{miss}}$ and $p_{\text{miss}}$ are the missing energy and
momentum of the DT event in the $e^+e^-$ center-of-mass frame, respectively. They are calculated as $E_{\text{miss}} \equiv E_{D^0} - E_X$ and $\vec{p}_{\text{miss}} \equiv \vec{p}_{D^0} - \vec{p}_X$, where $E_{D^0}$, $\vec{p}_{D^0}$, $E_X$ and $\vec{p}_X$ are the measured energy and momentum of the $D^0$ and $X$ candidates, respectively. The MM$^2$ resolution is improved by constraining the energy of $D^0$ to the beam energy and $\vec{p}_{D^0} \equiv -\vec{p}_{D^0} \cdot \sqrt{E^2_{\text{beam}}/c^4 - m_{D^0}^2}$, where $\vec{p}_{D^0}$ is the unit vector in the momentum direction of the ST $D^0$ meson and $m_{D^0}$ is the known $D^0$ mass [1].

The signal yields ($N_{\text{sig}}$) are extracted by fitting the MM$^2$ distributions of selected events. Background events are divided into four categories. The first (BKGI) contains $D^0 \to K^0_S(\to \pi^0\pi^0)X$ events. The second (BKGII) contains $D^0 \to \eta\gamma\phi, \eta\gamma\gamma, \eta\gamma\eta'$ events. The third (BKGIII) is from all the remaining peaking background channels. The fourth (BKGIV) is from combinatorial background components. In the fits, the signal is modeled by the MC-simulated shape convolved with a double-Gaussian function. The means and widths of signal mode dependent Gaussian functions are in the intervals (-0.58, 1.97) MeV$^2/\text{c}^4$ and (0.16, 3.70) MeV$^2/\text{c}^4$, respectively. The BKGI and BKGII are described by the corresponding MC-simulated shapes, and their sizes are fixed to the values estimated using the BFs from the PDG [1] and the corresponding misidentification rates. The shape and size of BKGIII are fixed to those obtained from the inclusive MC sample. BKGIV for $D^0 \to K^0_L\eta'_{\gamma\rho\rho}$ is not smooth and is modeled by the MC-simulated shape; it is modeled by a linear function for the other signal decays. Figure 2 shows the results of the fits to the MM$^2$ distributions of the accepted candidates in data.

There are combinatorial backgrounds in the $\phi \to K^+K^-$, $\eta \to \pi^+\pi^-\pi^0$, $\omega \to \pi^+\pi^-\pi^0$, and $\eta' \to \pi^+\pi^-\eta$ signal regions, which can form a peak in the MM$^2$ distributions. This kind of background is estimated by the corresponding sideband regions, defined as $M_{K^+K^-} \in (0.985, 1.000) \cup (1.045, 1.060)$ GeV/$c^2$, $M_{\pi^+\pi^-\pi^0} \in (0.495, 0.515) \cup (0.580, 0.600)$ GeV/$c^2$, $M_{\pi^+\pi^-\pi^0} \in (0.712, 0.732) \cup (0.832, 0.852)$ GeV/$c^2$, $M_{\pi^+\pi^-\eta} \in (0.913, 0.938) \cup (0.978, 1.003)$ GeV/$c^2$. Their yields ($N_{\text{sig}}$) are obtained from similar fits to individual MM$^2$ distributions. Table 1 summarizes the fitted yields of $N_{\text{sig}}$ and $N_{\text{sid}}$, the normalization factors of background events in the signal and sideband regions ($S_{\text{co}}$), the net signal yield ($N_{\text{net}} = N_{\text{sig}} - N_{\text{sid}} \cdot S_{\text{co}}$), the signal efficiencies ($\varepsilon_{\text{sig}}$), and the obtained BFs ($B_{\text{sig}}$).

At $\sqrt{s} = 3.773$ GeV, the $D^0\bar{D}^0$ pairs are produced coherently. The measurements of BFs with the DT method are affected by the quantum correlation (QC) effect. Following Ref. [42], this effect is considered as a tag-mode-dependent correction factor, $f_{\text{QC}} = 1 - C^i_{D^0} = \frac{1 - C^i_{D^0}}{1 + C^i_{D^0}}$, where $C^i_{D^0}$ is the strong-phase factor calculated as $C^i_{D^0} = 2r_ie^{-i\delta^i_{D^0}}$, $R_i$ is the coherence factor, $\delta^i_{D^0}$ is the strong-phase difference between the CF and DCS amplitudes, $r^i_{+}$ is defined as $r^i_{+}e^{-i\delta^i_{D^0}} \equiv \frac{F^i_{CP+}}{F^i_{D^0}}$, for the tag mode $i$; and $F^i_{CP+}$ is the CP+ fraction for the signal decay and it equals to 1 for all the studied decays. With necessary parameters quoted from Refs. [43, 44], the $f_{\text{QC}}$ factors are determined to be $0.898 \pm 0.007$, $0.935 \pm 0.007$, and $0.972 \pm 0.019$ for $D \to K^-\pi^+$, $D \to K^-\pi^+\pi^0$, and $D \to K^-\pi^-\pi^+$, respectively. All signal decay final states studied are CP+ eigenstates. The averaged QC correction factor, which has been weighted by the ST yields in data, is determined to be $f_{\text{QC}} = 0.937 \pm 0.007$. Multiplying the directly-measured BFs by this factor yields the reported BFs. After this correction, the residual uncertainty of $f_{\text{QC}}$ will be assigned as a systematic uncertainty. For

![Fig. 1. Fits to the $M_{\text{BC}}$ distributions of the ST $D^0$ candidates. Data are shown as dots (error bars are not visible at this scale). The blue solid and red dashed curves are the total fit results and the fitted backgrounds, respectively. Pairs of red arrows show the $M_{\text{BC}}$ signal region.](image_url)
$D^0 \rightarrow K^0_L \eta$ and $D^0 \rightarrow K^0_L \eta'$, the BFs measured by two different $\eta$ or $\eta'$ decay modes have been weighted by the combined statistical and independent uncertainties and the obtained results are shown in Table 1.

In the measurements of the BFs for $D^0 \rightarrow K^0_L X$ using the DT method, the systematic uncertainties associated with the ST selection are canceled. The major sources of systematic uncertainties related to the measured BFs are described below.

The tracking and particle identification (PID) efficiencies of charged kaons and pions are studied by analyzing DT hadronic $D\bar{D}$ events [46]. The data/MC differences in various momentum intervals are re-weighted by the corresponding momentum distributions of the signal decays. We correct the MC efficiencies to data by signal mode dependent factors of (0.2 – 5.5)% where the larger difference between data and MC simulation comes from the tracking efficiencies for low momentum $K$ in $D^0 \rightarrow K^0_L \phi_{K^+K^-}$ decay. The residual systematic uncertainties are (0.2 – 0.6)% for tracking and PID efficiencies per $K^\pm$ or $\pi^\pm$.

The systematic uncertainty due to the photon detection in $D^0 \rightarrow K^0_L \rho_{0,\gamma}$ decay is 1.0% per photon, as estimated from a $J/\psi \rightarrow \rho^0 \pi^0$ control sample [47].

The systematic uncertainty of $\pi^0$ reconstruction has been studied by using the DT events of $D^0 \rightarrow K^+\pi^-$, $K^+\pi^-\pi^-\pi^+$ vs. $D^0 \rightarrow K^-\pi^+\pi^0$ and $K^0_S\pi^0$ [45]. After correcting the MC efficiencies to agree with data using the momentum-weighted difference (0.5 – 0.6)% the residual systematic uncertainty, 0.7%, is assigned as the systematic uncertainty in $\pi^0$ reconstruction. Due to the limited size of the $\eta$ sample, the uncertainties of the $\eta$ reconstruction in $D^0 \rightarrow K^0_L \eta\gamma$ and $D^0 \rightarrow K^0_L \eta'\pi^+\pi^-\eta$ decays are assigned to be 0.5% and 0.7% by referring to the $\pi^0$ reconstruction.

The systematic uncertainties due to the mass windows of $\rho_{K^+K^-}$, $\eta_{\pi^+\pi^-\pi^0}$, $\omega_{\pi^+\pi^-\pi^0}$, $\eta'_{\pi^+\pi^-\pi^0}$, and $\eta'_{\rho\gamma}$ candidates are studied using control samples of $D^0 \rightarrow K^0_S\phi_{K^+K^-}$, $K^0_S\eta_{\pi^+\pi^-\pi^0}$, $K^0_S\omega_{\pi^+\pi^-\pi^0}$, $K^0_S\eta'_{\pi^+\pi^-\pi^0}$, and $K^0_S\eta'_{\rho\gamma}$, respectively. The relative differences of (0.2 – 0.5)% in the acceptance efficiencies between data and MC simulation are taken as individual systematic uncertainties.

The systematic uncertainty due to requiring no extra charged track, $\pi^0$ and $\eta$ is studied using the control sample of $D^0 \rightarrow K^0_S\pi^0$. The relative difference in efficiencies between data and MC simulation, 0.8%, is assigned as the systematic uncertainty.

The systematic uncertainties arising from the $M^2$ fits are evaluated by varying the signal shape, the background shape, and the size of peaking backgrounds within their uncertainties. The relative changes of various re-measured BFs are added in quadrature and these totals, (0.9 – 2.4)% are taken as the corresponding system uncertainty.
The quantities used for BF determinations and the obtained BFs. The signal efficiencies include the BFs for all possible sub-decays and necessary correction factors mentioned later. The listed BFs have been corrected by the QC factor $f_{QC}$ and $B$ denotes the weighted average BFs for $D^0 \rightarrow K_S^0 \eta$ and $D^0 \rightarrow K_S^0 \eta'$. The first and second uncertainties for $B$ are statistical and systematic, respectively; uncertainties for other variables are statistical only.

| Decay                  | $N_{\text{sig}}$ | $N_{\text{sig}}$ | $S_{\text{col}}$ | $N_{\text{net}}$ | $\epsilon_{\text{sig}}(\%)$ | $B_{\text{sig}}(\%)$ | $B_{\text{sig}}(\%)$ |
|------------------------|-------------------|-------------------|------------------|-------------------|-----------------------------|------------------------|------------------------|
| $D^0 \rightarrow K_S^0 \phi_{K^+K^-}$ | 1271$\pm$ 39 | 276$\pm$ 19 | 1.33$\pm$ 0.17 | 904$\pm$ 46 | 9.02$\pm$ 0.10 | 0.414$\pm$ 0.021$\pm$ 0.010 | ... |
| $D^0 \rightarrow K_S^0 \eta \gamma$ | 2132$\pm$ 71 | ... | ... | 2132$\pm$ 71 | 20.46$\pm$ 0.15 | 0.431$\pm$ 0.014$\pm$ 0.013 | 0.433$\pm$ 0.012$\pm$ 0.010 |
| $D^0 \rightarrow K_S^0 \eta_{\pi^+\pi^-}$ | 565$\pm$ 29 | 36$\pm$ 10 | 0.61$\pm$ 0.10 | 543$\pm$ 30 | 5.11$\pm$ 0.07 | 0.439$\pm$ 0.024$\pm$ 0.015 | ... |
| $D^0 \rightarrow K_S^0 \omega_{\pi^+\pi^-\pi^0}$ | 6692$\pm$ 100 | 368$\pm$ 39 | 1.58$\pm$ 0.07 | 6110$\pm$ 118 | 21.70$\pm$ 0.18 | 1.164$\pm$ 0.022$\pm$ 0.028 | ... |
| $D^0 \rightarrow K_L^0 \eta_{\pi^+\pi^-\eta}$ | 688$\pm$ 29 | 8$\pm$ 6 | 0.47$\pm$ 0.08 | 684$\pm$ 29 | 3.30$\pm$ 0.10 | 0.857$\pm$ 0.037$\pm$ 0.022 | ... |
| $D^0 \rightarrow K_L^0 \eta_{\rho\gamma}$ | 2002$\pm$ 61 | ... | ... | 2002$\pm$ 61 | 10.55$\pm$ 0.15 | 0.785$\pm$ 0.024$\pm$ 0.023 | ... |

The uncertainties arising from the finite sizes of the signal MC samples are (0.3 – 0.6)%. Systematic uncertainties from other selection criteria are found to be negligible.

For each signal decay, the total systematic uncertainty is obtained by summing individual contributions in quadrature and is shown in Table 2. The uncertainties of the quoted BFs [1] of $\phi \rightarrow K^+K^-$, $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $\omega \rightarrow \pi^+\pi^-\pi^0$, $\eta' \rightarrow \pi^+\pi^-\eta$, and $\eta' \rightarrow \rho^0\gamma$ are assigned as individual systematic uncertainties.

The QC effect on the measured BFs has been corrected by the factor $f_{QC}$ and the residual error of $f_{QC}$ is assigned as the systematic uncertainty, which is 0.7%.

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The QC effect on the measured BFs has been corrected by the factor $f_{QC}$ and the residual error of $f_{QC}$ is assigned as the systematic uncertainty, which is 0.7%.
Table 2. Systematic uncertainties (%) in the measurements of the BFs.

| Source                                      | $K_L^0\phi K^+K^-$ | $K_L^0\eta\gamma$ | $K_L^0\eta\pi^+\pi^-\pi^0$ | $K_L^0\omega\pi^+\pi^-\pi^0$ | $K_L^0\eta^{\prime}\pi^+\pi^-\pi^0$ | $K_L^0\eta^{\prime}\phi\pi^0$ |
|---------------------------------------------|--------------------|-------------------|-----------------------------|--------------------------------|-------------------------------------|---------------------------------|
| ST yield $N_{\text{tag}}$                  | 0.5                | 0.5               | 0.5                         | 0.5                            | 0.5                                 | 0.5                             |
| $K^{\pm}/\pi^{\pm}$ tracking              | 0.4                | -                 | 0.3                         | 0.2                            | 0.6                                 | 0.2                             |
| $K^{\pm}/\pi^{\pm}$ PID                    | 0.3                | -                 | 0.2                         | 0.2                            | 0.2                                 | 0.2                             |
| $\gamma$ reconstruction                    | -                  | -                 | -                           | -                              | 1.0                                 | -                               |
| $\pi^0/\eta$ reconstruction                | -                  | 0.5               | 0.7                         | 0.7                            | 0.7                                 | -                               |
| Mass window requirement                    | 0.2                | -                 | 0.2                         | 0.1                            | 0.5                                 | 0.1                             |
| $N_{\text{extra}}^{\text{charged}/\pi^0/\eta}$ | 0.8                | 0.8               | 0.8                         | 0.8                            | 0.8                                 | 0.8                             |
| MM$^2$ fit                                  | 0.9                | 2.3               | 2.4                         | 1.1                            | 1.4                                 | 1.2                             |
| Opening angle                               | 1.4                | 1.4               | 1.4                         | 1.3                            | 0.9                                 | 1.6                             |
| Quoted BFs                                  | 1.0                | 0.5               | 1.2                         | 0.7                            | 1.2                                 | 1.3                             |
| MC statistics                               | 0.6                | 0.3               | 0.5                         | 0.5                            | 0.6                                 | 0.4                             |
| Strong phase                                | 0.7                | 0.7               | 0.7                         | 0.7                            | 0.7                                 | 0.7                             |
| **Total**                                   | **2.4**            | **3.0**           | **3.4**                     | **2.4**                        | **2.6**                             | **2.9**                         |

Table 3. $CP$-conjugate BFs ($B^+_{\text{sig}}$ and $B^-_{\text{sig}}$) and their asymmetries ($A^\text{CP}_{\text{sig}}$). The first and second uncertainties are statistical and systematic, respectively.

| Decay          | $B^+_{\text{sig}}$ (%) | $B^-_{\text{sig}}$ (%) | $A^\text{CP}_{\text{sig}}$ (%) |
|----------------|------------------------|------------------------|-------------------------------|
| $D^0 \rightarrow K_L^0\phi$ | 0.428 ± 0.029 | 0.405 ± 0.034 | 2.7 ± 5.4 ± 0.7 |
| $D^0 \rightarrow K_L^0\eta$  | 0.445 ± 0.018 | 0.421 ± 0.017 | 2.8 ± 2.9 ± 0.4 |
| $D^0 \rightarrow K_L^0\omega$ | 1.200 ± 0.030 | 1.121 ± 0.031 | 3.4 ± 1.9 ± 0.6 |
| $D^0 \rightarrow K_L^0\eta'$ | 0.789 ± 0.028 | 0.826 ± 0.028 | -2.2 ± 2.5 ± 0.4 |

11822506, No. 11835012, No. 11935015, No. 11935016, No. 11935018, No. 11961141012, No. 12022510, No. 12025502, No. 12035013, No. 12061310003, No. 12192260, No. 12192261, No. 12192262, No. 12192263, No. 12192264, and No. 12192265; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Grants No. U1732263 and No. U1832207; CAS Key Research Program of Frontier Sciences under Grant No. QYZDJ-SSW-SLH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Grant No. 758462; European Union Horizon 2020 research and innovation programme under Marie Skłodowska-Curie Grant Agreement No. 894790; German Research Foundation DFG under Grant No. 443159800, Collaborative Research Center CRC 1044, FOR 2399, GRK 214; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Grant No. DPT2006K-124070; National Science and Technology fund; Olle Engkvist Foundation under Grant No. 200-0605; STFC (United Kingdom); The Knut and Alice Wallenberg Foundation (Sweden) under Grant No. 2016.0157; The Royal Society, UK under Grants No. DH140054 and No. DH160214; The Swedish Research Council; and U. S. Department of Energy under Awards No. DE-FG02-05ER41374 and DE-SC-0012069.

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Table 4. Comparison of measured BF$s and $K_S^0-K_L^0$ asymmetries with theoretical calculations of Ref. [6]. $B_{\text{exp}}$ ($B_{\text{FAT}}$) and $\mathcal{R}(D^0)_{\text{exp}}$ ($\mathcal{R}(D^0)_{\text{FAT}}$) are the BF$s and $K_S^0-K_L^0$ asymmetries of the experimental measurements (theoretical calculations).

| Decay                  | $B_{\text{exp}}$ (%) | $B_{\text{FAT}}$ (%) | Difference | $\mathcal{R}(D^0)_{\text{exp}}$ | $\mathcal{R}(D^0)_{\text{FAT}}$ | Difference |
|------------------------|-----------------------|-----------------------|------------|-------------------------------|--------------------------------|------------|
| $D^0 \to K_S^0 \phi$   | 0.414 ± 0.021 ± 0.010 | 0.33 ± 0.03           | 2.2σ       | −0.001 ± 0.047                | 0.113 ± 0.001                  | 2.4σ       |
| $D^0 \to K_L^0 \eta$   | 0.433 ± 0.012 ± 0.010 | 0.40 ± 0.07           | 0.5σ       | 0.080 ± 0.022                 |                                | 1.5σ       |
| $D^0 \to K_L^0 \omega$ | 1.164 ± 0.022 ± 0.028 | 0.95 ± 0.15           | 1.4σ       | −0.024 ± 0.031                |                                | 4.4σ       |
| $D^0 \to K_L^0 \eta'$  | 0.809 ± 0.020 ± 0.016 | 0.77 ± 0.07           | 0.5σ       | 0.080 ± 0.023                 |                                | 1.6σ       |

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