MAGNETOROTATIONAL INSTABILITY IN CORE-COLLAPSE SUPERNOVAE*

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We discuss the relevance of the magnetorotational instability (MRI) in core-collapse supernovae (CCSNe). Our recent numerical studies show that in CCSNe, the MRI is terminated by parasitic instabilities of the Kelvin–Helmholtz type. To determine whether the MRI can amplify initially weak magnetic fields to dynamically relevant strengths in CCSNe, we performed three-dimensional simulations of a region close to the surface of a differentially rotating proto-neutron star in non-ideal magnetohydrodynamics with two different numerical codes. We find that under the conditions prevailing in proto-neutron stars, the MRI can amplify the magnetic field by (only) one order of magnitude. This severely limits the role of MRI channel modes as an agent amplifying the magnetic field in proto-neutron stars starting from small seed fields.

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1. Introduction

The magnetorotational instability (MRI) was suggested in [1] to be the physical mechanism responsible for the redistribution of angular momentum required for the accretion process in Keplerian discs orbiting compact objects. Reference [2] pointed out the potential importance of the MRI for rapidly-rotating core-collapse supernovae (CCSNe) where it may amplify the weak pre-collapse fields to a dynamically relevant strength, thereby generating magnetohydrodynamic (MHD) turbulence and making rotational

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energy available for launching an explosion. Their simplified simulations as well as multi-dimensional models (e.g. [3, 4]) showed that in such CCSNe, proto-neutron stars (PNSs) possess regions where the MRI can grow on time scales shorter than the time between the bounce and the successful explosion. However, due to limitations of the above mentioned numerical simulations, the question remained open as to whether the MRI can amplify the initial magnetic field to dynamically important field strengths so that it can tap the rotational energy of the core and power MHD turbulence.

An upper limit for the magnetic field amplification caused by the MRI can be given by assuming that the MRI ceases to grow once the magnetic field energy comes close to equipartition with the energy of the differential rotation. In a CCSN, this corresponds to dynamically important field strengths of up to $10^{15}$ G. However, [5] studied the phase of exponential growth of the MRI which is characterised by channel modes, i.e. layers of radial and azimuthal magnetic field with alternating polarity and velocity. They found that the channel modes are unstable against secondary (or parasitic) instabilities of Kelvin–Helmholtz (KH) or tearing-mode (TM) type. Hence, according to the model of parasitic instabilities (further developed and studied analytically in [6, 7]), the MRI channel modes can be disrupted by secondary instabilities before the equipartition of the energy of the magnetic field and of the differential rotation is reached.

The goal of numerical studies of [8–10] was to answer the question whether MRI channel modes can amplify the magnetic field to relevant strengths in CCSNe, and to test the theoretical predictions of the parasitic model. We summarise the findings of those papers in the next section.

2. Numerical simulations

In [8], following [11], we performed two-dimensional (2D) and three-dimensional (3D) shearing-disc (semi-global) simulations which focus on a small representative region of a PNS. The simulations were done with the finite volume code Aenus [12] in resistive-viscous magnetohydrodynamics. According to the estimates of [13], close to the surface of PNS, the contribution of neutrinos to viscosity is low and, therefore, the flow is characterised by high Reynolds numbers. Therefore, in our studies [8], we mainly focused on the regime of high Reynolds numbers, which required using very high resolution. We point out that it is very important to distinguish the effects of numerical viscosity and resistivity from their physical counterparts. Therefore, we studied numerical errors giving rise to artificial dissipation in detail [14], finding that the use of numerical methods of very high convergence order is crucial for resolving the small-scale features of the parasitic instabilities [10].
Our main result is that in 3D simulations with high Reynolds numbers, the MRI growth is, in accordance with the parasitic model, terminated by secondary parasitic KH instabilities, whose properties are in a good agreement with the theoretical predictions. 2D simulations, because of the axisymmetry constraint, give a qualitatively wrong result, i.e. the MRI is terminated by TMs (as already observed by [11]).

In [9], we used two numerical codes, i.e. Aenus and the pseudo-spectral code Snoopy ([15]; using the shearing box and incompressible approximations) to test the prediction of the parasitic model for the maximum amplification of the magnetic field by MRI channel modes (see Fig. 1). We found a disagreement between the theoretical predictions and scaling laws for the termination field strength obtained from our simulations. With the help of our scaling laws, we estimate that under the conditions found in PNSs a realistic value for the magnetic field amplification is of the order of 10.

![Graph](image-url)

Fig. 1. (Colour on-line) Left: magnetic field energy of the MRI channel modes (solid black and dash-three dotted black/blue) and of the parasitic instabilities (dotted grey/red and dash-dotted grey/orange) as a function of time in a simulation performed with Aenus, with an initial (seed) magnetic field strength of \(1.22 \times 10^{13}\) G (i.e. magnetic field energy density of \(7.44 \times 10^{25}\) G\(^2\); marked with a grey/green horizontal line). When the energy of both instabilities is comparable, the MRI is terminated. Right: magnetic field amplification (roughly defined as the ratio of the amplitude of the initial seed magnetic field to the amplitude of the MRI channel at termination) as a function of initial magnetic field strength in simulations performed with Aenus and Snoopy. See [8] for more details.
This result casts doubt on the viability of MRI channel modes as an agent amplifying the magnetic field in proto-neutron stars starting from small seed fields. A further amplification should, therefore, rely on other physical processes, such as for example an MRI-driven turbulent dynamo (for numerical studies in the presence of buoyancy, see [16]) or the standing accretion shock instability.

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