Learning obstacles on the concept of function: a hermeneutic phenomenological study

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Abstract. The meaning of a mathematical concept especially function, which formed by students, is subjective in nature and has its own interpretation. Therefore, this reality is important to examine further. This study aims to explore the learning obstacles on the concept of function based on the students’ meaning of the concept and how students acquire as well as construct the meaning related to their experiences. This is a qualitative study with uses a hermeneutic phenomenological approach involving 65 10th grade students who have learned about function. The data collected through a test and interview. The interviews were transcribed and then conducted in-depth analysis to identify the learning obstacles possibility. The results of this study showed that the concept of function is meant actually as a correspondence between two elements; there is the meaning of function inconsistency; and the meaning of function that tends to be partial. On the other hand, there is a tendency that students acquired and constructed the concept of function are limited on what they’ve learned during the classroom. Based on the results of this study, it can be concluded that in general there are learning obstacles on the concept of function includes epistemological, ontogenic, and didactical obstacles.

1. Introduction
The complexity of mathematics learning can lead the students’ understanding process of a mathematical concept being not easy. One of these is the concept of function. The concept of function is an important unifying idea in mathematics [1]. The concept of function is associated with variety of real-life situations and not easy to understand for students mainly because it also pertains to a lot of concepts in mathematics [2,3].

A lot of students’ difficulty in solving various problems on the concept of function are indicated that there were difficulties in students’ understanding on the concept of function. In general, a lot of students in various countries have difficulties to solve the concept of function problems that manifested in diverse representation [1,4,5]. The problem-solving questions of the concept of function are also became a challenge for students that is not easy to be solved [4,5]. The existence of cognitive conflict between students’ concept image and concept definition of function are also identified to be one of the causes of students’ difficulty to understand the concept of function [6].

Considering to those facts, it is necessary to notice the existence of fundamental problem in students’ conception of function, more precisely in students’ meaning about it. This is due to the meaning of a mathematical concept especially function, which formed by students, is subjective in nature and has its own interpretation [7]. On the other hand, the students’ meaning of function is also very closely related to their experiences when learned the concept of function. This is due to the students’ meaning of the
concept of function is influenced by variety of viewpoints, such as the meaning of the other students, teachers, books, etc. [8]. Thus, the students’ meaning-making process about the concept of function becoming an important thing that should be considered in mathematics learning. If it not facilitated properly, the students’ meaning of the concept of function that have constructed, precisely can be lead students to be unable to understand the concept of function as whole. Therefore, all of this obviously very potential to give rise for the existence of learning obstacles on the concept of function.

Based on the background that have outlined, the issues in this study can be formulated into 3 main questions: (1) what the meaning of function according to students; (2) how students acquired and constructed the meaning of function; and (3) based on the students meaning and experiences, are there learning obstacles on the concept of function. Thus, this study aims to explore the learning obstacles on the concept of function based on the students’ meaning of the concept and how students acquire as well as construct the meaning related to their experiences.

2. Methods
This is a qualitative study with uses a hermeneutic phenomenological approach. Hence, interpretative paradigm is used in this study [7,9]. This study focuses to explore the reality of the students’ conception of function; students’ experiences on constructing the concept of function; and also, their implication to the possibility of learning obstacles on the concept of function. Therefore, we need hermeneutic that used to explore the students’ meaning, and phenomenology that used to explore the students’ experiences, regarding the concept of function.

This study was involved to 65 10th grade students (two classes which consist of 32 and 33 students) who have learned about function, in one of senior high school at Cimahi, Indonesia. The data collected in triangulation through a test and interview. The test consists of five problems about function that presented in subjective question form, and conducted to each student during 45 minutes. The problems include the concept of function in real and formal context, representation, application, and definition. Whereas the interview consists of some open questions and conducted to seven students selected. Students selection was used purposive sampling based on review result of the test and also adapted the concept definition of function according to Vinner and Dreyfus [6]. The interview questions were drafted in semi-structured form that includes students’ test answer deepening and students’ meaning as well as experiences when learning the concept of function.

The interviews were transcribed and then with overall the data were conducted in-depth analysis. The process in general by reduced the data into several units of students’ meaning and experience on the concept of function; present the data in descriptive; and draw conclusions. Furthermore, multi-perspective identification was conducted over the reality of students’ meaning and experiences to obtain the existence possibility of learning obstacles on the concept of function.

3. Result and Discussion
This study has 2 assumptions relate to the meaning of the concept of function, i.e. (1) for each student response, conceive a certain unit of meaning associated to the concept of function; and (2) to obtain the actually students’ meaning of the concept of function is by observing the consistencies of the students’ meaning employed, either through a concept definition of function wrote as well as the solutions of the test question solved, and through what students’ revealed on the interview. Following are the results and discussion of this study findings analysis.

3.1. Students’ conception of function
There are 3 main study findings related to the students’ conception of function, i.e. (1) The meaning of function as a correspondence between two elements; (2) The meaning of function inconsistency; and (3) The meaning of function that tends to be partial.

Based on some students’ response, we can notice that in several situations, a student used the meaning of function consistently as a correspondence between two elements. Therefore, uses the study assumptions that has been pointed out, obtained that the meaning of function according to students is
function as a correspondence between two elements. The meaning is revealed with the existence of a relation or connection between two sets, the first set namely domain and the second set namely codomain. It is assigns to every element in the domain exactly one element in the codomain. Here is a few student’s responses (S1) regarding the meaning of function as a correspondence between two elements in several situations.

Figure 1. One of the students (S1) answer about the meaning of function as definition.

R : Well, what do you think or reflect in your mind firstly, if you hear the word of function?
S1 : Aaa, the relationship, relationship or more to mapping. So, the relationship between one point with relation to other set. So, there are two sets that the first set was connected into another set.
R : Can you explain your answer? (problem number 2 from test instrument)
S1 : Because return to my first explanation, sir, use the domain and codomain concept. So, the codomain in graph number 1 is parallel with y axis where x axis as the domain, the domain is one but generate a lot of codomain, so this graph is no function. Then, in graph number 5, that is no function because same with previous, there are some points which have two range, one domain has two range and this is more certainly the inverse of quadratic functions.

Figure 1 shows the meaning that used by S1 to describe the concept of function as definition (concept definition), is a correspondence between two elements. The word “an operation” is wrote but the notion of domain and codomain as well as their property could be examining as a powerful meaning that used to wrote the definition of function. In fact, the student’s concept definition, which showed in figure 1, actually is the most common concept definition of function that usually expressed by students [6]. Furthermore, in general, it uses in a lot of student’s mathematics book at various countries [6,10].

The first conversation between researchers (R) and a student (S1), shows a student first thought about the concept of function. The meaning, which used by S1 to describe directly the concept of function, still a meaning that he wrote previously namely function as correspondence between two elements. The significant statement that used by S1 is relationship or mapping between two sets.

While the second conversation, shows the verbal explanations that used by S1 to solve the problem number 2 from test instrument. The problem presents a various form of graph where students were asked to choose and give the reason, which of the following graph is a graph of function and not a function. The conversation shows that the meaning, which used by S1 to determine where is a graph of function from following graphs, also still similar from the meaning of function that revealed previously. The significant idea, which used by S1, is observed the condition that every element in domain (x axis) must not be generate a lot of element in codomain (y axis). In fact, the concept of function, which showed in last conversation, actually is the most common concept image of function that usually expressed by students as a correspondence between two elements with the condition of “exactly one” [6]. It simplified as an idea of one-valuedness on the concept of function [6].

On the other hand, based on the study findings analysis and still use the study assumptions, there is also some inconsistency in the meaning of function that used by students. Here is a few student’s responses (S2) regarding the meaning of function that used by S2 in several situations.
Figure 2. One of the students (S2) answer about the meaning of function as definition.

R : Well, what do you think or reflect in your mind firstly, if you hear the word of function?
S2 : Function as a way to enter, suppose we want to create an item, and now we use a machine. The machine will process an item called input become a new output, which has the better value than previously, Well, here’s how it’s called function.
R : Ok, can you explain your answer for b? (problem number 1b from test instrument)
S2 : Em… (for a moment thinking), the function here I think in Indonesian meaning view. Well for example, students and her table, like I have been doing exam, we have been sitting beside 11-grade students so we don’t have any chance to cheat, its function (smiling). Well additionally, it is done to simplify when teacher check out our presence.

Figure 2 shows the meaning that used by S2 to describe the concept of function as definition (concept definition) is a function as an equation. It indicated from student’s significant statements at the beginning of definition that the function is an equation. This student’s concept definition of function is completely can be meaning as a formula, an algebraic expression, or an equation [6].

The first conversation between researchers (R) and a student (S2), shows a student first thought about the concept of function. The conversation shows the meaning that used by S2 to describe directly the concept of function is quite different from previously. The existence of the word “way”, “process”, and the role of machine to change the input become the certain output were indicated that there is a tendency of student to use the meaning of function as a rule. It is also indicating that the function is expected to have some regularity. Furthermore, it is one of the concept definitions of function that views the function as a rule of correspondence [6].

While the second conversation, shows the verbal explanations that used by S2 to solve the problem number 1b from the test instrument. The problem presents the real context of function where students were asked to determine and give the reason, whether the case is a function or not a function. The conversation shows that S2 used the meaning of function differently. It is very clear where the function is meant from the Indonesian Language meaning as “usability” or “benefits”. Related to this, we observe that students encountered unfamiliar problems on the concept of function in a real context.

The other study findings analysis is the meaning of function that tends to be partial. In this case, there is a student who revealed the meaning of function in diverse meanings as a definition of function. Here is a student’s response regarding the meaning of function that tends to be partial.

Figure 3. One of the students (S3) answer about the meaning of function as definition.

Figure 3 shows the meaning that used by S3 to describe the concept of function as definition (concept definition). The definition of function that wrote by S3 actually contains some the meaning of function
categories. There is a significant meaning, namely function as an operation or manipulation where S3 wrote that function as well as substitute or enter a value x into \( f(x) \) thus will obtain specific results. There is a meaning, namely function as a rule of correspondence where the regularity of input-output principal that implicitly appeared. In addition, there is also a meaning namely function as a representation where S3 wrote that function as a math lessons, function \( f(x) = x \), and also graphs.

Related to student’s response in figure 3, we can observe that the students’ meaning of function is incomplete or tends to be partial. This because every meaning of function that appeared, actually are complementary. Thus, the meaning categories that appeared can be views as an elaboration of what is students known related to the concept of function.

3.2. Students’ experiences on constructing the concept of function

The overall reality of students’ conception of function who have been previously revealed, is not immediately be formed without through some experiences in meaning within the students. Therefore, it is important to explore for another reality behind the meaning of the concept of function, i.e. the students’ experiences on constructing the meaning of the concept of function. The results of this study findings analysis show a tendency that students acquire the meaning of function as long as sourced from the teachers and the books. The teachers and the books are actually inter-related in coloring the meaning of function that constructed by students.

Two students revealed same understanding that the meaning of function as a correspondence between two elements was acquired since 8-grade and learned again in 10-grade. Here is one of the conversations between researchers (R) and a student (S1), associated with his learning experience on the concept of function.

R : Well, how the process you can acquire this definition?
S1 : If the function material may have a few years ago, from 8-grade was taught and then repeated again in 10-grade first semester. In the theories are less understood but the work procedures are more understandable. So, let’s say there is a formula of the function \( f(x) = ax + b \), well because it is was often repeated, become memorized, and if there is a problem of function, oh it have definitely the function that relate to \( f(x) \) again, and will not be far like that.

The students explained that the meaning of function was constructed firstly by introduce the concept of relation. The arrow diagram between two sets was used as introduction until developed become a relation between domain and codomain with condition that for each element in the domain must not be have more than one element in the codomain. However, based on some students’ responses, obtained that the construction of students’ meaning of function tends to be oriented in question examples and the certain existence of procedural ways. It can be observed in the last conversation that students better understand if the question of function has a formula and the solving procedure.

While on the other hand, in this study were also obtained that five from seven students’ participant are mentioned the machine illustration on their meaning process about the concept of function. Some students were explained that the machine illustration acquired not only from the teacher but also from the students’ mathematics book. Relate to this, one of the students were revealed that function as a method or process like a machine that convert a certain input and produce a certain output. But in this case, we had observing that through the machine illustration, students tend to meant the concept of function in partial as a number processor or number changer. Here is one of the conversations between researchers (R) and a student (S4), associated with her meaning of function using the machine illustration.

R : Well, here you wrote that function as a calculation or operation, why?
S4 : The function usually called a calculation because can be use in a machine, suppose we enter a function formula in to the machine that a number which entered to it, will be change become another number. So, the function belongs to calculation.
Based on the conversation between researchers (R) and a student (S4), this certain seems to give indirectly a limited impression for students in the process of constructing the meaning of function. Whereas in fact, the function machine illustration is an empirical notion that play a role to be a cognitive root and can be uses to embodies the whole students’ meaning of the concept of function [11]. On the other hand, the results of this study findings are also obtained that actually some students have less interested in learning the concept of function. Here is one of the conversations between researchers (R) and a student (S5), associated with her insight when learned the concept of function.

S5 : In my opinion, function is less interesting material.
R : Why?
S5 : Well, just less interested, not too like function from the first. So, when learn about function, I feel lazy, too hard to remember it.

Based on the conversation, S5 revealed explicitly that she was not too interested in learning the concept of function. It is important to diagnosis because we observe that the interest aspect was constituted as impact of the previous students’ experiences and will be influence to the further learning experiences on the concept of function. Furthermore, this study also obtained that some students were revealed either implicitly or explicitly, the lack of integration regarding the concept of function benefits in daily life on learning process. This was revealed on a student (S3) when responding to one of the researchers (R) question, as follows.

R : Is there anything that felt less during this time in learning the concept of function?
S5 : Em is there, like what the examples of the concept of function in daily life. Usually in the questions, function is certain interrelated with the formula, most of which are given in the question is about the formula or only just the numbers, but examples of function in daily life is rarely, so that is who I felt less.

Related to the conversation, based on the results of this study findings analysis, there is four from seven student participant were revealed the same hope that there would be an integration of benefits and problems of the concept of function in daily life during the learning process.

3.3. Learning obstacles on the concept of function

Based on reality of the students’ conception of function obtained, the students’ meaning of function inconsistency is one of the significant indications of the cognitive conflict that called as compartmentalization phenomenon [6]. It formed when students’ meaning of function become divided as the consequences of something [6,12]. It can be caused because the students’ meaning of function that constructed is still at the stage of information and not yet reached knowledge [13]. Since still at the stage of information, the students’ meaning of function is still mutually independent and depends heavily on the contexts [13]. The most often occurs is students usually have difficulty to solve some problems in other context and form that commonly encountered. In this case, the students actually experiencing limitations on the context and form in the process of constructing the meaning of function. Therefore, all of this affirms that there are epistemological obstacles on the concept of function [14].

Furthermore, the students’ meaning of function inconsistency marks that the meaning of function not yet being a part of the students. It is related to the study finding of the meaning of function that tends to be partial. The students are actually not fully convinced over their own meaning of function in the form of definition. Thus, it appeared the diverse meaning in a frame of the definition of function. This case can be also views cause the meaning of function not yet reached to the internalization and institutionalization process [13,14]. Explicit knowledge regarding the concept of function, which acquired by students from a various source, have not been fully precipitated into each own tacit knowledge. While the certain situation has not been created where students should be able to understand and further develop the meaning of function to solve any problems. This indicate that actually students not fully ready to learn the concept of function further. Therefore, this affirms that there are ontogenic obstacles in conceptual on the concept of function [7].
Based on reality of the students’ experiences on constructing the concept of function, in general the students’ meaning of function not reached to the whole meaning of function as structurally or objects approach [15]. This revealed on a student’s response previously, that students better understand if the concept of function has a formula and solving procedure, without perhaps to know advanced the actual meaning of the concept of function. This is confirmed with a statement that normally, students sure to be understand the concept of function based on the work process like numerical procedural [15]. But important to notice that it a necessary condition but not sufficient to support the optimization understanding of the concept of function [15]. This condition causes a lack of sufficient understanding on the fundamental concept that revealed previously in the students’ conception of function. Therefore, all of this more affirms that there are ontogenic obstacles in conceptual on the concept of function.

While, this study also obtained that some students felt not too motivated and enthusiastic when learned the concept of function. This reasonably affirms that there are ontogenic obstacles in psychologist on the concept of function [7].

Since of limited impression for students, we observe the existence of a less optimum utilization of machine illustration in developed the students’ meaning of function. Furthermore, the machine illustration should be a good medium for the meaning process of function because it opens to all possibilities of meaning and representation [11]. Thus, the function machine illustration should be made as a medium in the beginning of the students’ meaning construction up to the meaning of function in definitively and its further application. Therefore, this quietly as an indication of the existence of didactical obstacles on the concept of function [14].

On the other hand, the nature of mathematics learning is preceded by a problem and aim to resolve the problem [16]. Based on this study findings analysis, this seems has not been well developed in the students learning process of the concept of function. Some students expressed the same hope that there would be an integration of the concept of function benefits and problems in daily life. This can be views as an indication of a lack of mental action, or may have suffered, on the students that causes students’ ways of thinking related to the concept of function becomes limited [7,16]. The limitation of the students’ ways of thinking related to the concept of function also causes the students’ ways of understanding related it, being not facilitated properly [7,16]. Therefore, this affirms that there are didactical obstacles on the concept of function related to the learning phases of the concept of function.

4. Conclusion
This study aims to explore the learning obstacles on the concept of function based on the students’ meaning of the concept and how students acquire as well as construct the meaning related to their experiences. Based on the result and discussion in this study, it can be concluded that there are learning obstacles on the concept of function includes epistemological, ontogenic, and didactical obstacles.

Epistemological obstacles on the concept of function are main form in the existence of the students’ meaning of function inconsistency. In several situations, students were expressed the meaning of function are differently. This occurs basically because the students have limitations on the context and form in the process of constructing the meaning of function. Ontogenic obstacles on the concept of function includes conceptual and psychologist aspect. Ontogenic obstacles in conceptual are main form in the existence of the students’ meaning of function that tends to be partial. Students actually not fully ready to learning the concept of function further, especially in the insufficient mastery of the fundamental concept of function such as the definition. Whereas, ontogenic obstacles in psychologist are main form in the existence of a students’ less attraction in learning the concept of function. While didactical obstacles on the concept of function are main form in the less optimum role of the machine illustration in developed the students’ meaning of function and the existence of the insufficient mental action such as integration of the concept of function benefits as well as problems in daily life.

According to this study results, future studies are therefore necessary to explore more deeply learning obstacles on the concept of function with consider the teacher and scientific conception. Thus, it can be obtained the overall learning obstacles on the concept of function as the previously consideration to develop a proper didactical design of the concept of function.
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