LARGE SCALE QUANTIZATION AND THE PUZZLING COSMOLOGICAL PROBLEMS

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Abstract

We have investigated the implications of Quantum Mechanics to macroscopic scale. Evaporation of Black Holes and evolution of pulsars may be one of the consequences of this conjecture. The two equations $GM = Rc^2$ and $GM^2 = \hbar c$ where $R$ and $M$ are the radius and the mass of the universe, are governing the evolution of the universe throughout its entire cosmic expansion, provided that the appropriate Planck constant is chosen. The existence of very large values of physical quantities are found to be due to cosmic quantization. We predict a constant residual acceleration of the order $10^{-7}\text{cms}^{-2}$ acting on all objects at the present time.

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1. INTRODUCTION

The general theory of relativity (GTR), which attributes the structure of space-time to the gravitating mass, is incompatible with the theory of quantum mechanics (QM). While quantum mechanics is a linear theory the GTR is highly nonlinear so that the two theories are dissimilar. Attempts to linearize the GTR do not give all manifestation of the theory. Initially the intention of the development of GTR was to describe the large scale structure of the universe, i.e., solar and galactic scales, whereas QM deals with the microscopic scales. The unification of electromagnetic, weak and strong was successful and gave a lot of hints to further unification with the hitherto unachievable gravitational interaction. Particle physicists are ambitious to get the unification of all interactions at an energy scale of $10^{19}\text{GeV}$!

However, these disparate worlds can meet in some cases. In these situations, phenomena in atomic physics appear to be written in the language of gravity and Einstein's GTR. By knowing the working
of one subject, we can thus make educated guesses about the other. 

One therefore should not look for a paradigm in which gravity is manifested as a force, but we should treat gravity as a background (framework) on which other interactions occur. Thus gravity provides only the shape of the space-time membrane on which other interactions are carried. We therefore, expect that gravitational effects are always present and manifested in the way space-time affects our physical phenomena. We remark that one should treat gravity not as an independent interaction but rather as a framework in which all interactions take place and consider only the strong, weak and electromagnetic interactions to be independent. We therefore expect to observe quantum effects at microscopic as well as macroscopic scale depending on the dimension of the system under consideration. This would imply that quantum effect at cosmic scale to be similar to those at microscopic scale, they only differ in the magnitude of the quantization. This is plausible since the masses and distances are enormous for macroscopic system.

After making these arrangement we would end up with a cosmic Planck constant having a huge value for macroscopic scales. The relative errors in determining the physical properties of the macroscopic system would have the same value as for microscopic one. One would also expect that the same laws governing the microscopic scale to be still functioning but with scaled ones. This might require some physical quantities to evolve in order to satisfy these laws.

Based on Tiftt’s [1] experimental data on galactic red-shift DerSarkissian [2] suggested a cosmic quantum mechanics (CQM) characterized by the Planck constant [3]

\[ h_g = 10^{102}h = 10^{68} \text{ J.s} \] (1)

where \( h \) is the ordinary Planck constant of the quantum mechanics. The quantum state of a galaxy is described by a wave function \( \psi(x,t) \) so that galaxies become the elementary particles of CQM. For a gravitational system and according to Einstein equivalence principle a body with observable inertial mass \( (m_i) \) has observable active mass \( (m_a) \) and hence an observable gravitational field. The observability of the this field requires that the total energy (self-gravitational energy) stored in it is measurable and, therefore, greater than \( h/2t \) (according to the uncertainty principle \( \Delta E \Delta t > h/2 \) and \( \Delta t \sim t, \Delta E \sim E \)). For a spherical body of mass \( M \) and radius \( R \) we have

\[ \frac{GM^2}{R} > h/2t. \] (2)

But we know, from general relativity, that \( \frac{GM}{Rc^2} \leq 1 \) so that \( Mc^2 > h/2t. \)

In QM a body is observable only if its self-gravitational energy \( E_G \) is greater than \( h/2t(E_G > h/2t) \). All bodies in QM obey this inequality. By supposing that the CQM obey the same rules as QM except that the corresponding Planck constant is different we can write,

\[ E_G > h_c/2t \] (3)

which is obeyed by all cosmic bodies. For galaxies this formula gives

\[ \frac{GM^2}{R} > h_c/2t. \] (4)

This implies that, for \( t = 10^{17} \text{ sec} \), \( R = 10^{19} \text{ m} \), and

\[ h_c = 10^{68} \text{ J.s}, \] (5)
$M > 10^{40}$ kg. This represents a constraint for galaxies obeying CQM (however all galaxies obey this).

A system with spatial dimension $R$, mass $M$, spin $S$ and a positive energy density has $R > \frac{S}{M^3}$ [4]. This together with eq.(4), yield the inequality

$$M^3 > \frac{Sh_c}{2Gct}$$

(6)

that holds for cosmic bodies obeying CQM with intrinsic spin angular momentum. The quantization of spin requires that $S = nh$ ($n$ is an integer). Hence,

$$M^3 > \frac{h^2}{Gct}$$

(7)

valid for spinning cosmic bodies. By considering a cluster made by many galaxies with quantized spin Ruffini and Bonazzola [5] have shown that there exists no equilibrium configuration for a cluster of more that $N \sim (M_P/M)^A$ galaxies of mass $M$ in their ground state, where $M_P$ is the cosmic Planck mass.

$$M_P = \left(\frac{h_c}{G}\right)^{1/2} \sim 10^{43} \text{kg},$$

(8)

where $A = 2$ for bosons, $A = 3$ for fermions. Thus CQM applies well to galaxies. Is there other CQM that applies at planetary, solar or universal scale with different quantum of action ($h_c$)?

Caldirola et al [6] suggested in a framework of unified theory of strong and gravitational interaction that a quantum of action for the universe is given by

$$h_c = MRC$$

(9)

where $M$, $R$ are the mass and the radius of the universe. Equation (9) gives

$$h_c = 10^{87} \text{ J.s}$$

(10)

with $M = 10^{53}$ kg and $R = 10^{26}$ m [6].

A possible connection between $h$ and $h_c$ for the universe exists where $\frac{h}{r^3} \sim \frac{h_u}{R^3}$ where $r$ is the size of a typical hadron and $h_u$ regarded as nonvanishing total angular momentum of the universe due to torsion field [7]. Massa [3] enlarged the above relation to become

$$\frac{h}{r^3} \sim \frac{h_u}{R^3} \sim \frac{h_g}{R_g^3}$$

(11)

where $R_g$ is a typical radius of a galaxy ($R_g \sim 10^{19}$m). It was suggested by Caldirola et al [6] that “these forms are special case of a more general form still unknown”. In fact, the CQM for planetary scale is introduced by Pierucci [8] in which

$$h_c = 10^{-13} \text{J.s}$$

(12)

which explains the Titius-Bode law and other regularities of the solar system. Massa considered the possibility of the growth of $h_c$ with cosmic time ($t$). He, according to the Large Number hypothesis (LNH) [9], suggested that for a flat universe $h_c \sim t^{5/2}$, and he concluded that the CQM is incompatible with this LNH. Carneiro [10] investigated the extension of scale invariance to quantum behavior. The price that paid was the scaling of Planck constant ($\hbar$) leading to quantization of large structure that
treated till now as classical systems. He used the LNH in his theory and concluded that the angular momentum for a rotating universe is of the order of magnitude of $10^{87}$ J.s.

So far there is yet no evidence that the universe is rotating but if it does it should do that with this value! However, Kuhne [11] claimed an observation of a rotation of the universe. We remark that an order of magnitude for the universe angular momentum is within the limit for the global rotation obtained from the cosmic microwave background radiation (CMBR) anisotropy obtained by Kogut et al [12]. Carneiro [10] suggested that “one of the possible explanation of the large scale quantization could be based on an evoluntional point of view: the quantum nature of the universe during its initial times has molded its-apparently quantized nowadays large scale structure”.

We also note that Nottale obtained, on a different ground, a scale Planck constant of order $10^{42}$ J.s for a QM model for the solar system [13].

Carneiro [10] applied the Bohr quantization condition to the motion of Earth around the Sun and found that this is consistent provided that Planck constant has the value $10^{42}$ J.s. Muradian [14] found an angular momentum of stars around $10^{42}$ J.s which is close to Kerr limit of a rotating black hole with a mass of $10^{30}$ kg.

In this work we try to explain the origin of large numbers occurring in nature by appealing to this conjecture. The hierarchical nature of mass, cosmological constant, entropy, temperature and angular momentum are found to be one of the manifestation of this conjecture. The evolution and characteristics of astrophysical objects like, black holes and binary pulsars are investigated in the framework of this conjecture. The huge quantum number is absorbed in the definition of the cosmic Planck constant.

We present our model in sect.2 and discuss its implication in a framework in which $G$ and $\Lambda$ vary with cosmic time in sec.3. In sect.4 we apply our conjecture to Black Holes and Binary Pulsars. In sec.4 we constrain the maximal acceleration and force appearing in the universe, according to our model. We wind up our paper with a conclusion.

2. THE MODEL

The usual definition of Planck mass ($m_P$) can be inverted to give

$$h = \frac{G m_P^2}{c}.$$  (13)

In a similar fashion, we suggest a cosmic Planck constant ($h_c$) with a cosmic Planck mass $M_P$ as

$$h_c = \frac{G M_P^2}{c}.$$  (14)

Another possibility, from a dimensional point of view, is

$$h_c = \frac{c^3}{G \Lambda},$$  (15)

where $\Lambda$ is the cosmological constant. This is equivalent to writing it as $\Lambda = \frac{1}{L_P^2}$ where $L_P$ is the Planck length. By supposing that the energy density of the vacuum is caused by the gravitational
interaction of the neighboring particles with mass $m$, Lima and Carvalho [15] obtained

$$\Lambda = \frac{G^2 m^6}{\hbar^4}.$$  \hspace{1cm} (16)

from which one can write

$$h_c = \frac{G^{1/2} M_p^{3/2}}{\Lambda^{1/4}}.$$ \hspace{1cm} (17)

Hence, we have three different forms of cosmic Planck constant to be tested with observation.

2. A MODEL WITH VARIABLE $G$ AND $\Lambda$

We have shown in an earlier work that [16] $G \sim t^2$ and $\Lambda \sim t^{-2}, m \sim t^{-1}$ in the early universe. Therefore, all three forms reduce to the ordinary Planck constant in the early universe (radiation dominated era). Thus one has $h_c = h$ in the early universe. Hence CQM and QM are equal in the early universe. However, in the matter dominate universe [16] $G \sim t, \Lambda \sim t^{-2}$ and $m = \text{constant}$. Therefore $h_c \sim t$. This asserts that the Planck constant evolves with time and that why it has different values for different systems. Thus QM applies to macroscopic system in the same way as in microscopic system except Planck constant is replaced by the cosmic Planck constant.

Equation (15) shows that the quantum of action for the universe is related to the vacuum energy density ($\Lambda$) of the universe, so that one obtains

$$\frac{\Lambda_p}{\Lambda_0} = \frac{h_c}{h} = 10^{120}.$$ \hspace{1cm} (18)

This gives a present value for $\Lambda$ (i.e., $\Lambda_0$) a value of $10^{-52} \text{m}^{-2}$ as expected from present observations. Equations (15) and (17) yield

$$\Lambda = \frac{c^4}{G^2 M_p^2},$$ \hspace{1cm} (19)

as a cosmological law governing the evolution of the universe. This implies that in the early universe [16] ($G \sim t^2, m \sim t^{-1}$) $\Lambda \sim t^{-2}$. Similarly in the matter dominated phase ($G \sim t, m = \text{const.}$) $\Lambda \sim t^{-2}$. Hence $\Lambda \sim t^{-2}$ during both phases. To reproduce the result in eqs. (1), (5) and (10) one requires $M_p^u = 10^{53}\text{kg}, M_p^g = 10^{43}\text{kg}$ and $M_p^s = 10^{30}\text{kg}$, representing the corresponding Planck masses at universal, galactic and solar scales, respectively. We remark that cosmic Planck constant for the planetary system is $10^{20}\text{kg}$. This represents the average mass of the planets in the solar system. Hence,

$$h_c = \frac{G M_p^u}{c} = 10^{34}\text{J.s}.$$ \hspace{1cm} (20)

This value is different from the Planck constant for the planetary system quoted in eq.(12). Thus eqs.(14), (15) and (17) provide a bridge connecting macroscopic and microscopic phenomena through a simple formula. It is found that the universe satisfies the equation

$$GM = Rc^2.$$ \hspace{1cm} (21)

This is an statement of the equality of the rest mass energy of the universe to its gravitational energy (i.e., $Mc^2 = \frac{GM^2}{R}$). We observe that eq.(9) is same as eq.(14), if we use the Mach relation (eq.(21)) [25].
We have shown in an earlier work [16] that eq.(21) gives \( R \sim t \), so that eq.(21) yields
\[
M \leq \frac{c^3 t}{G}.
\] (22)
which is similar to eq.(7). This inequality is obtained by [17] by different approaches. One may define a maximal mass for a bound system, at time \( t \), to be gravitational stable with
\[
M_{\text{max}} = \frac{c^3 t}{G}.
\] (23)
So that the above equation becomes
\[
M \leq M_{\text{max}}.
\] (24)
This formula is obtainable from Friedman cosmology \((3H^2 = 8\pi G \rho)\) with \( R \sim t, H \sim t^{-1} \) giving \( M = \frac{c^3 t}{G} \).
If we consider the stars to be the atoms of the universe we will observe that the universe is a typical one solar mole, consisting of \( 10^{23} \) stars(suns).
The \( M_{\text{max}} \) defined above would have a significance for the time \( t = 1 \) sec and \( t = 10^8 \) sec. The former defines the Planck mass for the stars and the latter for galaxies. Specifically,
\[
M_{\text{max}} = 10^{35} \text{kg} \quad \text{and} \quad M_{\text{max}} = 10^{43} \text{kg},
\] (25)
respectively. Hence the stars and globular galaxies today were once the actual states of our past states of our universe. That is why one need not go back in time to see how was the past. Thus our universe may contain information about the past with it at all times and not just cosmic background radiation!
For these system the Planck constant evolves as
\[
\hbar_c = \frac{c^5}{G} t^2
\] (26)
During this time the gravitational constant has remained unchanged. Hence, stars and globular galaxies would have the following cosmic Planck constants:
\[
10^{52} \text{J.s} \quad \text{and} \quad 10^{68} \text{J.s},
\] (27)
respectively. However, these values are reported by Capozziello et al.[27].
We now turn to calculate the Compton wavelength of the universe, i.e., \( \lambda_U = \frac{\hbar}{m_U} = 10^{26} \text{m} \), which is of the same order of magnitude of the present radius of the universe. Thus the use of CQM for the present universe is also logical and plausible. Thus all known bounded systems are characterized by their cosmic Planck constant.
We would like to remark that the magnetic dipole moment of the Earth (E) is found to be \( 7.98 \times 10^{25} G \text{ cm}^3 \). This value can be obtained from the definition of the magnetic dipole moment, i.e.,
\[
\mu_E = \frac{e \hbar}{2mc}
\] for an atomic system. Here we make the following replacement: \( e^2 = G m^2 \) and \( \hbar = 10^{34} \text{J.s} \), \( m = M = 6 \times 10^{24} \text{kg} \). This gives the same order of magnitude for the magnetic dipole moment of the Earth. Thus, as we remarked in the beginning, having known the atomic system parameters their gravitational analog can be obtained by the generalization above. We also note that \( \mu \propto m^2 \). Hence, one can conclude that for Mercury (M) this gives \( \mu_M = \mu_E \left( \frac{m}{m_M} \right)^2 = 7.98 \times 10^{25} (0.0553)^2 = 2.4 \times 10^{22} G \text{ cm}^3 \).
This in fact is the presently known value for Mercury. We observe that there is a new large numbers associated with our physical world. These numbers are \((10^{34})^a\), where \( a = -1, 2, 2.5, 2.75, 3, 3.5 \). These
3. APPLICATION OF CQM TO BLACK HOLES

Consider a spinning ($S$) black hole (or neutron star) with frequency $\omega$. We have

$$S = I \omega,$$  \hspace{1cm} \text{(28)}

with $I = MR^2$, $M$ and $R$ are the mass and radius of the object. For an object with a gravitational radius $R = \frac{2GM}{c^2}$ and spin $S \sim h_c$, eq.(28) yields

$$\omega = \frac{c^3}{GM},$$ \hspace{1cm} \text{(29)}

Pulsars are believed to be rotating neutron stars and a newborn pulsar formed in supernova may be rotating with a frequency of $10^3 s^{-1}$ emitting a gravitational radiation with a rate of $10^{48} J s^{-1}$. For a pulsar of a mass $M = M_\odot$ one gets, from CQM, a period of $10^{-3}$ sec. One of the most promising source is the pulsar NP0532 in the Crab nebula [18]. This pulsar is observed to emit pulses of electromagnetic radiation, at optical, X-ray and radio frequencies with a period of 33 msec. Thus our model, though based on rough estimates, is in a good agreement with observations.

A black hole emitting radiation as a black body with a temperature $T$ given by

$$k_B T = \hbar \omega.$$ \hspace{1cm} \text{(30)}

Using eqs.(29) one gets

$$T = \frac{\hbar c^3}{GMk_B}$$ \hspace{1cm} \text{(31)}

in comparison with Hawking [19] formula obtained from QM treatment for a non-rotating black hole, viz.,

$$T = \frac{\hbar c^3}{4\pi GMk_B}.$$ \hspace{1cm} \text{(32)}

A rotating galaxy of mass $10^{43} \text{kg}$ would have a frequency of $10^{-8} s^{-1}$ or a period of 10 years. We can compare this value with the presently observed rotation of galaxies. The time for the evaporation of the black hole can be estimated from the uncertainty relation

$$\Delta E \Delta t \sim \hbar c$$ \hspace{1cm} \text{(33)}

with $\Delta E = Mc^2$ and $\Delta t = \tau$. This upon using eq.(14) becomes

$$\tau = \frac{GM}{c^3}.$$ \hspace{1cm} \text{(34)}
Since $\frac{GM^2}{\hbar} = 1$, one can write eq.(34) as

$$\tau = \frac{GM}{c^3} \cdot \frac{GM^2}{\hbar} = \frac{G^2 M^3}{c^4 \hbar}.$$  \hspace{1cm} (35)

Which is obtained by Hawking [19] from a quantum mechanical treatment. Hence, we may write for the evaporation of Black holes the formula

$$\tau = \frac{G^2 M^3}{c^4 \hbar_c}$$ \hspace{1cm} (36)

as a CQM analogue.

We see that a black hole of one Planck mass $m_P$ evaporate during Planck time ($10^{-43}$s). A galactic rotating black mass evaporates during a time of 1 year while a solar rotating black hole evaporates during a time of $\mu$ sec. A rotating black hole of size of the universe evaporates during a time of $10^{10}$ years.

The entropy of a black hole is given by [19]

$$S = \frac{GM^2}{c \hbar} k_B$$ \hspace{1cm} (37)

which upon using eq.(14) yields

$$S = \frac{\hbar_c}{\hbar} k_B = \left(\frac{k_B}{\hbar_c}\right) \hbar_c.$$ \hspace{1cm} (38)

This entropy is independent of the mass of the object in question as long as $\hbar_c$ describes that object. We see that for a black hole formed in the early universe $\hbar_c = \hbar$, and therefore irrespective of its mass the black hole will have one unit of entropy, i.e., $S = k_B$. Black holes forming during solar and galactic time will have entropy that is multiple of $\hbar_c$. We thus conclude that the entropy of black holes is quantized. A galactic mass black hole will have an entropy of $10^{102} k_B$, while a solar mass black hole will have an entropy of $10^{76} k_B$. However, a black hole of the mass of the universe has an entropy of $10^{120} k_B$.

4. THE VACUUM ENERGY DENSITY

The Planck energy density is defined as

$$\rho_P = \frac{c^5}{G^2 \hbar}$$ \hspace{1cm} (39)

which represents the maximum density of the universe at Planck time. We now employ the CQM and evaluate the present maximum energy density of the universe, i.e.,

$$\rho_0 = 5.4 \times 10^{-28} \text{gcm}^{-3}.$$ \hspace{1cm} (40)

Present observations set a limit on the present energy density as

$$10^{-30} \text{gcm}^{-3} < \rho_0 < 10^{-29} \text{gcm}^{-3}.$$ \hspace{1cm} (41)

We have thus obtained a constraint on the density of the universe viz., $\rho_0 \leq \rho_P$. It is evident that the Planck energy density of the universe indeed evolves with time.  

\footnote{see ref.[26] for a review about vacuum energy}
We argue that the Planckian energy density ($\rho_P$) is equal to the vacuum energy density of the universe at all times. In this sense the universe is still governed by quantum mechanics (cosmic). We have, from Einstein de Sitter model, the relation

$$3H^2 = 8\pi G\rho$$

(42)

Upon using eq.(39) this becomes

$$GH^2\hbar = \frac{8\pi}{3}c^5 = 2 \times 10^{43}$$

(43)

This equation may be taken to define a cosmic Planck constant. Thus

$$\hbar_c = \frac{2 \times 10^{43}}{GH^2}.$$ 

(44)

In the early universe [16] we have $G \propto t^2$, $R \propto t$, so that the cosmic Planck constant coincides with the ordinary Planck constant $\hbar$. At the present time, we see that the cosmic Planck constant, $h_c = 1.087\text{J.s}$. In the present universe we have [16] $G \propto t$, $R \propto t$, so that eq.(44) implies $h_c \propto t$. This confirms our earlier findings of cosmic Planck constant for the whole universe. Therefore (44) represents an additional definition of cosmic Planck constant to those already found in eqs.(14), (15), and (17).

The vacuum energy density is defined as

$$\rho_v = \frac{\Lambda c^2}{8\pi G} = \frac{c^5}{G^2\hbar}.$$ 

(45)

During the radiation dominated phase this yields

$$\rho_v = \rho_v P\left(\frac{t_P}{t}\right)^4,$$

(46)

where $\rho_v P$ is the vacuum energy density at Planck time. This shows that $\rho_v$ behaves like radiation in the early universe. During the matter dominated phase one gets

$$\rho_v = \rho_v 0\left(\frac{t_0}{t}\right)^3,$$

(47)

where $\rho_v 0$ is the vacuum energy density at present time. Similarly $\rho_v$ scales like matter in the matter dominated phase. This would mean that the vacuum contribution to the energy density of the university has been very significant in the past and in the present time. It is therefore not surprising to find out that the vacuum contribute 66% to the total energy density of the present universe. Thus one may resolve the dark matter energy that is postulated to circumvent some astrophysical problems. We remark that since the vacuum energy density was equal to the Planck energy density in the early universe it still governed by this quantum energy. This is plausible since the Planck energy density energy with time. One thus could say that our universe today is a typical quantum system. Moreover, the Planck length has become today our physical radius of the universe. We also see that the Planck length $L_P = \sqrt{G\hbar/c^3}$ has the same time evolution in both the early phase and the present matter phase, viz.

$$L_P(t) = L_P\left(\frac{t_P}{t}\right).$$
and
\[ L_P(t) = L_P \left( \frac{t_0}{t} \right) \]  
Equation shows that the vacuum's quanta shifts like photon according to the Planck law. Thus if they had different background temperature the former would cool (at the present time and in the early universe) as
\[ T_v(t) = T_{vP} \left( \frac{t_P}{t} \right) \]  
where \( T_{vP} \) is the vacuum temperature during Planck time (see eq.(57)). We observe that in the early universe we have
\[ m \propto t^{-1} \quad \text{and} \quad T \propto t^{-1} \propto R^{-1} \]  
so that if these relations hold throughout the cosmic expansion, one would obtain the relation
\[ m = m_P \left( \frac{t_P}{t} \right) \]  
where \( m_P \) is the Planck mass at Planck time and \( T \) is the temperature. We note that De Sabbata and Sivaram [20] relate the temperature \( (T) \) to curvature \( (\kappa) \) and showed that \( T \propto \sqrt{\kappa} \), but the time \( (t) \) scales as \( t \propto \frac{1}{\sqrt{\kappa}} \). For a maximal curvature \( \kappa_{\text{max.}} = \frac{c^3}{\bar{h}G} \), which implies
\[ G \propto t^2, \quad \text{and} \quad T \propto t^{-1}. \]  
Comparison with eq.(15) immediately yields
\[ \kappa_{\text{max.}} = \Lambda. \]  
Thus one may connect the smallness of the present value of the cosmological constant to the flatness of our present universe. Using eq.(52) one would obtain
\[ m = 10^{-5} \left( \frac{10^{-43}}{10^{18}} \right) = 10^{-66} \text{g}. \]  
which represents a minimal mass scale at the present time. Equation (50) gives
\[ T_v = 10^{32} \left( \frac{10^{-43}}{10^{18}} \right) = 10^{32} \times 10^{-61} = 10^{-29} \text{K}, \]  
where \( T_P = 10^{32} \text{K} \). We remark that De Sabbata and Sivaram obtained a similar value by considering a time-temperature uncertainty relation \( (\Delta t \Delta T = \hbar/k_B) \) and relate the maximum time to Hubble time. They suggested that they could obtain such a value by considering a black hole of a mass of the universe using the formula outlined in eq.(31). Or by considering the maximal possible entropy of \( 10^{120} k_B \) which would imply this minimal temperature. They also found a similar value and noted that this minimal temperature corresponds to the quantum fluctuations of cosmological torsion background. Massa [21] has obtain a similar value and relates this to the mass of graviton. He argued that in an expanding universe this mass depends on time. Rosati [22] found the quantum field today has typically a mass of the order of \( 10^{-66} \text{g} \). Larionov [23] attributed a similar mass term to an effective mass associated with the vacuum energy density (or \( \Lambda \)). He assigned this extremely low value of this effective mass to a quantum with wavelength equal to the present radius of the universe. One may also add to
this conjecture the cosmological constant problem (rooted in the enormous value, i.e., $\Lambda_P = 10^{120} \Lambda_0$) as due to the cosmic quantization, as is evident from eq.(18)! In fact, Zizzi [28] has brought an idea of quantized cosmological constant that is in the same line as our reasoning here. According to Massa assertion the graviton has a mass; this would mean that the range of the gravitational interaction is not infinite but limited by this mass scale. Hence, the maximum possible interaction distance between any two gravitating objects has to be at a maximum distance of $10^{26}$ m. A similar assertion would also hold for electromagnetic interaction if it turned out that a photon is not massless! We have so far shown that the two formulae (eqs.(14) and (21))

$$GM = c^2 R \quad \text{and} \quad GM^2 = \hbar c$$

hold throughout the cosmic evolution of the universe, provided we consider the CQM to be a valid principle.

4. MAXIMAL ACCELERATION AND FORCE

The self-gravitational force of a system of mass $M$ and radius $R$ is given by

$$F = \frac{GM^2}{R^2}.$$ \hspace{1cm} (58)

Using eq.(57)

$$F = \frac{c^4}{G},$$ \hspace{1cm} (59)

for the universe. This force is clearly independent of the mass of the object into consideration. It is thus a universal force, and since it depends on $G$ inversely it defines a maximal self-gravitational force. There corresponds to this maximal force a maximal acceleration ($F_{\text{max.}} = Ma_{\text{max.}}$) defined by

$$a_{\text{max.}} = \frac{c^4}{GM}.$$ \hspace{1cm} (60)

If we consider a variable gravitational constant as suggested in [16], one gets a minimal gravitational force in the universe during the nuclear (or hadronic) epoch as

$$F_{\text{min.}} = 10^{-2} N,$$ \hspace{1cm} (61)

since during nuclear (or hadronic) epoch the gravitational constant was $G_N = 10^{40} G_0$. The smallness of this force may account for the fact that quarks are asymptotically free inside hadrons, according to the theory of quantum chromodynamics (QCD). A maximal force in the universe during the present or Planck time is

$$F_{\text{max.}} = 10^{44} N.$$ \hspace{1cm} (62)

We remark that the factor $\frac{c^4}{8\pi G}$ appearing in the Einstein’s equation may be interpreted as the force per area required to give space-time unit curvature, that is $10^{43} N m^{-2}$ for a curvature of $1 m^{-2}$. Space-time is therefore an extremely stiff medium. De Sabbata and Siviram noted that there exists a maximal acceleration given by

$$a_{\text{max.}} = \frac{c^{7/2}}{G^{1/2} \hbar^{1/2}},$$ \hspace{1cm} (63)
originated as quantum effect due to torsion. Their formula reduces to our formula upon using eq.(14) into (60). Therefore, we have, according to CQM

\[ a_{\text{max.}} = \frac{c^{7/2}}{G^{1/2} \hbar^{1/2}}. \]  

(64)

We now turn to calculate the maximal acceleration at a universal scale, according to CQM.

\[ a_{\text{max.}} = 10^{-9} \text{ms}^{-2}. \]  

(65)

However, De Sabbata obtained a value of $10^{-10}\text{ms}^{-2}$ on different grounds that agrees with [24]. Note that the maximal acceleration at Planck time was $a_{\text{max}} = 10^{51}\text{ms}^{-2}$. Very recently, Anderson et al.[29] have found an unmodelled acceleration towards the Sun of $8.09 \times 10^{-8} \text{cm s}^{-2}$ for Pioneer 10 and Pioneer 11. Kuroda and Moi [30] have measured the relative acceleration in free fall of pairs of test bodies (aluminium/copper-aluminium/carbon) and found it to be of the order of $3 \times 10^9 \text{ms}^{-2}$. One may therefore attribute this acceleration to some residual quantum background filling the whole universe having a Planckian temperature of the order $10^{-29}\text{K}$, as shown before.

5. CONCLUSION

We have extended the implication of quantum mechanics form microscopic scale to include macroscopic scale. This extension resulted in a lot of interesting consequences concerning the evolution and characteristics of black holes and binary pulsars. We have found that quantities like entropy, cosmological constant and time are quantized for macroscopic scales. Limiting values for temperature, entropy, angular momentum ($\hbar c$), force, acceleration are obtained due to this conjecture. The universe is found to have an energy density less that Planckian density at the present time. We have also shown that we have different Planckian parameters for the universe for different time. The relation $GM^2 = \hbar c$ which was found to apply in the early universe is still valid during other phases, provided that we replace the ordinary Planck constant with the cosmic Planck constant, whose value depends on the properties of the macroscopic entity. We have provided a physical justification for the Large Number Hypothesis advocated by Dirac. We have made a correspondence between atomic and gravitational systems and that all atomic parameters have their gravitational analogue. The comparison between the atomic parameters and the large scale parameters always involves numbers of orders of $10^{20}, 10^{40}, 10^{60}, 10^{80}, 10^{100}, 10^{120}$ times the atomic parameters.
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