In-plane resistivity of hole doped cuprates: role of pseudogap and quantum criticality

M Afsana Azam\textsuperscript{1,3}, M Borhan Uddin\textsuperscript{1,2} and S H Naqib\textsuperscript{1}

\textsuperscript{1}Department of Physics, University of Rajshahi, Rajshahi-6205, Bangladesh
\textsuperscript{2}Department of CSE, International Islamic University Chittagong, Bangladesh
\textsuperscript{3}Department of Physics, DUET, Gazipur, Dhaka, Bangladesh

E-mail: salehnaqib@yahoo.com

Abstract. The anomalous behaviour of dc charge dynamics in high-$T_c$ cuprates cannot be explained by the prevalent Boltzman transport theory. In high-$T_c$ cuprates the in-plane and out-of-plane dc resistivities exhibit high anisotropy and unconventional temperature dependence. In this study we have modeled the temperature ($T$) and hole content ($p$) dependent in-plane resistivity, $\rho_a(T, p)$, of pure and Ca doped Y123 (Y(Ca)Ba$_2$Cu$_3$O$_{7-\delta}$). We have adapted and extended the formalism developed by Naqib et al. (Physica C 471 1598 (2011)) by taking account of two generic features present in all hole doped cuprates, namely - (i) the presence of the pseudogap in the quasiparticle spectrum and (ii) high-$T$ linear behavior, to elucidate non-Fermi liquid $\rho_a(T, p)$ of Y(Ca)Ba$_2$Cu$_3$O$_{7-\delta}$ over a wide range of temperature and hole contents. The characteristic pseudogap energy scale, $\epsilon_g(p)$, extracted from the analysis of the resistivity data was found to agree well with those found in a variety of earlier studies. Other extracted parameters from the analysis of $\rho_a(T, p)$ data showed methodical variations with the variation of hole content. Important features of the analysis are briefly discussed.

1. Introduction

The phenomenon of superconductivity in copper oxides presents an outstanding problem to the condensed matter research community. It is widely perceived that the mystery lies within the anomalous normal state (NS) properties from which superconductivity emerges as temperature is lowered [1–3]. In hole doped high-$T_c$ cuprates (HTCs), non-Fermi liquid dc resistivity in the NS provides with a challenging issue and has generated substantial research interest [4–7]. The temperature dependent in-plane resistivity shows many unusual features which in turn depend primarily on the number of added holes, $p$, in the CuO$_2$ planes. The NS charge carrier scattering mechanism is intimately linked with charge-boson coupling that leads to Cooper pairing. Therefore, understanding the temperature- and $p$-dependent resistive behavior can help us to understand the phenomenon of superconductivity in cuprates itself [6, 8–10].

A detailed and empirically relevant description of the $ab$-plane resistivity in HTCs is still not available. Experimental results have showed that the generic behaviors of the $T$- and $p$-dependent resistivity are qualitatively identical in different families of HTCs, even though structural and electronic features vary over a large extent. This implies that the dominant electronic ground states are

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\* Presenter
responsible for the unusual features of the resistivity and the genre specific details (e.g., structural and electronic anisotropy, spin-charge stripe orders [11, 12] etc.) are not very important. With the exception of high-$T_c$ itself, the pseudogap (PG) correlation in the NS is the single most robust characteristic of all HTCs. Along with a large number of electronic and magnetic properties, the NS resistivity of cuprates is also affected by the presence and magnitude of the PG, particularly in the underdoped (UD) regime [13, 14].

A large number of theoretical studies have been done to investigate the dc charge transport of hole doped cuprates [15 - 20]. All these investigations have limited success. A common shortcoming of most of the formalisms developed is that the theoretical framework used to describe the $p$- and $T$-dependent resistivity does not hold when wider bodies of other experimental findings are considered.

In an earlier work [21], we have modeled the c-axis resistivity of bi-layer HTCs by considering the effect of PG on the interlayer charge tunneling matrix element. In this short communications, we intend to extend the model for the description of in-plane dc charge transport.

The rest of the paper has been organized as follows. In Section 2, the outline of the proposed model is presented. In Section 3, the experimental in-plane resistivity, $\rho_p(T)$, of Y(Ca)123 single crystals have been shown and analyzed. Discussion on the important findings and conclusions constitute sections 4 and 5, respectively.

2. Essential features of the model

Linear temperature dependence ($T$-linear) of dc resistivity over an extended range in the NS and the effect of PG on the interlayer transport properties are two features common to all hole doped HTCs. Unconventional $T$-linear resistivity demands a non-Fermi liquid scheme. Among different frameworks, quantum criticality seems to be the most promising [22].

In general, conventional phase transitions are induced by thermal energy and lead to a transformation from an ordered to a disordered state with lower symmetry. At 0 K temperature, thermal fluctuations are absent and an entirely new type of change in phase can occur - a quantum phase transition (QPT). A QPT is triggered by quantum fluctuations associated with the zero-point energy. QPT involves no change in entropy. QPT can be probed only by varying some non-thermal parameter, for example the doped holes in copper oxide superconductors. The specific value of this non-thermal parameter that demarcates the two distinct quantum states at 0 K is known as the quantum critical point (QCP). It is interesting to note that a $T$-linear resistivity originates from the quantum criticality quite naturally. The mechanism can be described as follows.

From critical scaling analysis, one finds that the thermal equilibrium time scale, $\Gamma_{qcp}$, at a QCP can be expressed as [22, 23],

$$\Gamma_{qcp} = \frac{ch}{2\pi k_B T}$$

(1)

Here, $h$ denotes the Planck’s constant, $k_B$ the Boltzmann’s constant, and $C$ is a constant determined by the effective dimensionality of the system of interest. It is readily seen from Eqn. 1 that the relaxation time depends only on $T$. Since various transport coefficients depend on the same process that establish local thermal equilibrium inside the system, this relaxation time is the same as the scattering time relevant to charge transport.

At high temperatures above the QCP, the thermal timescale is much shorter than the quantum timescale; the physical properties at finite $T$ are, therefore, significantly influenced by the presence of the QCP in the electronic phase diagram. Various physical properties around the QCP cannot simply be described by the ground state wave function. In this quantum critical regime, the $T$ dependence of the physical quantities often displays striking deviation from the conventional Fermi-liquid character. In this situation the charge carrier scattering rate is given by the inverse of Eqn. 1, and the dc resistivity shows a completely $T$-linear behavior.

The dc electrical resistivity of a metal can be expressed via the Drude formula,

$$\rho = \frac{m^*}{ne^2\tau}$$

(2)
where $m^*$ is the carrier effective mass, $n$ is the carrier concentration, $e$ is electronic charge, and $\tau$ is the scattering time. For metallic compounds the $T$ dependence of $\rho$ comes from the $T$-dependent carrier scattering rate, $1/\tau$.

The net effect of the PG in the electronic energy density of states (EDOS) is a reduction in the scattering rate and thereby reducing the resistivity [24]. A relation between the pseudogap in the quasiparticle (QP), EDOS and the scattering rate can be formed employing the time-dependent perturbation theory. The result can be expressed as,

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} N(\varepsilon_F) \langle |i|V|f\rangle^2$$  \hspace{1cm} (3)

Here, $N(\varepsilon_F)$ is the EDOS at the Fermi level and $V$ is the potential responsible for carrier scattering. The symbols $i$ and $f$ signify the initial and final quantum states of the scattered charge carrier. In practice, due to the broadening of the Fermi function at finite temperature the scattering rate will be dependent on the thermally averaged EDOS centered at the Fermi-level, $\langle N(\varepsilon_F) \rangle_T$. Previously, $\langle N(\varepsilon_F) \rangle_T$ has been expressed quite effectively via the following formula in the presence of a PG [25],

$$\langle N(\varepsilon_F) \rangle_T = N_0 \left[ 1 - \left( \frac{2T}{\varepsilon_g} \right) \ln \left[ \cosh \left( \frac{\varepsilon_g}{2T} \right) \right] \right]$$  \hspace{1cm} (4)

In the above expression $N_0$ denotes the flat EDOS outside the pseudogap and $\varepsilon_g$ is the characteristic PG energy expressed in temperature scale. Therefore, the in-plane resistivity of HTCs can be thought of as dominated by two different terms, the first one, due to quantum criticality, is given by,

$$\rho_{QCP} = \alpha_p T$$  \hspace{1cm} (5)

and the second one proportional to the thermal average of the depleted EDOS due to PG is,

$$\rho_{PG} = \beta_p N_0 \left[ 1 - \left( \frac{2T}{\varepsilon_g} \right) \ln \left[ \cosh \left( \frac{\varepsilon_g}{2T} \right) \right] \right]$$  \hspace{1cm} (6)

In Eqns. 5 and 6, $\alpha_p$ is a hole content dependent constant measuring the strength of the critical scattering of the mobile holes and $\beta_p$ is a factor taking into consideration of the effect of the overall momentum dependence of $\langle N(\varepsilon_F) \rangle_T$ and the scattering matrix elements on in-plane dc resistivity, respectively. Combining all these effects and the $T$-independent residual resistivity, $\rho_{0p}$, the total in-plane dc resistivity can be written as,

$$\rho_p(T) = \rho_{0p} + \alpha_p T + \beta_p N_0 \left[ 1 - \left( \frac{2T}{\varepsilon_g} \right) \ln \left[ \cosh \left( \frac{\varepsilon_g}{2T} \right) \right] \right]$$  \hspace{1cm} (7)

this, on rearrangement becomes,

$$\rho_p(T) = \rho_{0p} + \alpha_p T + b_p \left( \frac{2T}{\varepsilon_g} \right) \ln \left[ \cosh \left( \frac{\varepsilon_g}{2T} \right) \right]$$  \hspace{1cm} (8)

with $\alpha_p = (\rho_{0p} + \beta_p N_0)$ and $b_p = \beta_p N_0$.

Experimental dc resistivity data for Y(Ca)123 single crystals have been fitted to Eqn. 8 over a wide range of hole contents and temperatures. Eqn. 8 excludes the possible $T$-dependence of the square of the matrix element for scattering of charge carriers (see Eqn. 3). Within the proposed scheme, the $T$-dependence of $\rho_{PG}$ arises from the thermally averaged $N(\varepsilon_F)$ (given by Eqn. 4).

3. Experimental samples and analysis of the resistivity data

High-quality pure (Y123) and Ca substituted (Y(Ca)123) single crystals were synthesized using the self-flux method. Information regarding sample preparation, characterization, and measurements of in- and out-of-plane dc resistivities can be found elsewhere [21, 26]. In-plane carrier concentrations were changed by annealing the crystals at different temperatures under different oxygen partial pressures [21, 26]. The $p$-values quoted herein are accurate within $\pm 0.004$. We show the in-plane dc resistivities in Figs. 1. Experimental $\rho_p(T)$ with respective fits to Eqn. 8 are shown in Figs. 2. $T$-range from $T_c + 25$ K to 300 K has been used for these fits. The lower $T$-limit has been selected to avoid temperature regions with significant superconducting fluctuations [27 – 30]. The extracted fitting parameters are tabulated (Table 1). The characteristic PG energy, $\varepsilon_g$ (in degree Kelvin) and $T_c$ are plotted against hole concentration in Fig. 3.
Figure 1. $\rho_p(T)$ of (a) $Y_{0.94}Ca_{0.06}Ba_2Cu_3O_{7-\delta}$ and (b) $YBa_2Cu_3O_{7-\delta}$ single crystals. Hole contents are shown in the plots.

Figure 2. Experimental $\rho_p(T)$ and respective fits to equation 8 (full black lines) for (a) $Y_{0.94}Ca_{0.06}Ba_2Cu_3O_{7-\delta}$ and (b) $YBa_2Cu_3O_{7-\delta}$ single crystals. Hole contents are shown in the plots. To enhance clarity one in ten experimental data points are shown only.

4. Discussion and conclusions
It has been demonstrated here that the simple phenomenological model developed accounts for the $p$- and $T$-dependent NS in-plane plane dc resistivity remarkably well for double layered pure and Ca substituted Y123 single crystals. The obtained values of $\epsilon_g(p)$ from the fits agree very well with other studies [3, 13, 31].
Table 1. Extracted values of $a_p(p)$, $\alpha_p(p)$, $b_p(p)$, and $\varepsilon_g(p)$ from the fits to $\rho_p(T)$ data.

| Sample                          | Hole content ($p$) | $a_p(p)$ (m$\Omega$-cm) | $\alpha_p$ (m$\Omega$-cm/K) | $b_p$ (m$\Omega$-cm) | $\varepsilon_g$ (K) |
|---------------------------------|-------------------|--------------------------|----------------------------|----------------------|---------------------|
| YBa$_2$Cu$_3$O$_{7-\delta}$    | 0.123             | 0.2010                   | 0.00101                    | 0.307                | 262                 |
|                                 | 0.148             | 0.0251                   | 0.00097                    | 0.088                | 171                 |
|                                 | 0.164             | 0.0031                   | 0.00081                    | 0.039                | 122                 |
| Y$_{0.94}$Ca$_{0.06}$Ba$_2$Cu$_3$O$_{7-\delta}$ | 0.118             | 0.2730                   | 0.00140                    | 0.508                | 266                 |
|                                 | 0.122             | 0.2481                   | 0.00100                    | 0.405                | 254                 |
|                                 | 0.131             | 0.1450                   | 0.00096                    | 0.306                | 214                 |
|                                 | 0.149             | 0.0692                   | 0.00090                    | 0.230                | 142                 |
|                                 | 0.169             | 0.0096                   | 0.00086                    | 0.102                | 107                 |
|                                 | 0.187             | 0.0008                   | 0.00060                    | 0.046                | 62                  |

Fig. 3 demonstrates that Ca substitution has no noticeable effect on the magnitude of the PG. This agrees completely with previous studies [28, 32–34], where the insensitiveness of the characteristic PG energy scale on in- and out-of-plane disorder has been firmly established.

Table 1 shows that, just like $\varepsilon_g$, Ca doping has no significant influence on $\alpha_p$. It is worth noticing that the parameters $a_p$ and $b_p$ are not independent - both contain the term $\beta_pN_0$. Somewhat larger values of $a_p$ and $b_p$ for the Ca substituted samples are possibly due to the disordering effect of Ca atoms. Ca atoms are located quite close to the CuO$_2$ planes and may affect the in-plane charge dynamics.

By definition, the difference between $a_p$ and $b_p$ gives $\rho_{0p}$, the in-plane residual resistivity. It is seen from Table 1 that $\rho_{0p}$ is negative for all the HTCs under study. The magnitude of this unphysical negative residual resistivity rises with decreasing hole content. This, we believe, is a consequence of the presence of the pseudogap in the quasiparticle energy spectrum. Pseudogap induces a downturn in the in-plane resistivity and any fit incorporating this effect should yield a negative residual resistivity when extrapolated to 0 K of temperature. We will explore this in detail in a future communication.

![Figure 3](image-url)

Figure 3. Characteristic pseudogap energy scale, $\varepsilon_g$, (expressed in K) and superconducting transition temperature of Y$_{0.94}$Ca$_{0.06}$Ba$_2$Cu$_3$O$_{7-\delta}$ and YBa$_2$Cu$_3$O$_{7-\delta}$. The dashed-dotted line shows the $T_c(p)$ trend for Y$_{0.94}$Ca$_{0.06}$Ba$_2$Cu$_3$O$_{7-\delta}$ and YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals. $\varepsilon_g(p)$ values extracted from the analysis of $\rho_p(T)$ data are also plotted (taken from Ref. 21).
The extrapolated value of extracted pseudogap energy scale goes to zero at \( p \approx 0.195 \) (Fig. 3), close to the critical doping level as found by variety of experiments on different families of hole doped HTCs \([13, 25, 28, 32, 35, 36]\). Such behavior strongly suggests that the pseudogap is not directly linked to the superconducting correlations.

To summarize, we have developed a minimalistic phenomenological scheme to model the \( p \)- and \( T \)-dependent in-plane dc resistivity for hole doped HTCs. The proposed model fits the experimental dc resistivity data quite well and the extracted PG energy scale from the fits to the resistivity data is in very good agreement with previous findings.

References

[1] Leggett A J 2006 *Nature Physics* **2** 134
[2] Norman M R, Pines D and Kallin C 2005 *Advances in Physics* **54** 715
[3] Hashimoto M, Vishik I M, He R-H, Devereaux T P and Shen Z-X 2014 *Nature Physics* **10** 483
[4] Pines D 1997 *Z. Phys. B* **103** 129
[5] Littlewood P and Varma C 1992 *Phys. Rev. B* **46** 405
[6] Anderson P W 1987 *Science* **235** 1196
[7] Alexandrov A S, Kabanov V V and Mott N F 1996 *Phys. Rev. Lett* **53** 2863
[8] Anderson P W 1997 in *The Theory of Superconductivity in High-\( T_c \) Cuprates* (Princeton University Press) and the references therein
[9] Dzhumanov D, Ganiev O K and Djumanov Sh S 2014 *Physica B* **440** 17
[10] Su Y H, Luo H G and Xiang T 2006 *Phys. Rev. B* **73** 134510
[11] Kivelson S A, Bindloss I P, Fradkin E, Oganesyan V, Tranquada J M, Kapitulnik A and Howald C 2003 *Rev. Mod. Phys.* **75** 1201
[12] Naqib S H 2012 *Physica C* **476** 10
[13] Tallon J L and Loram J W 2001 *Physica C* **349** 53 and the references therein
[14] Kokanovic I, Cooper J R, Naqib S H, Islam R S and Chakalov R A 2006 *Phys. Rev. B* **73** 184509
[15] Rojo A G and Levin K 1993 *Phys. Rev. B* **48** 16861
[16] Lal R, Ajay, Hota R L and Joshi S K 1998 *Phys. Rev. B* **57** 6126
[17] Turlakov M and Leggett A J 2001 *Phys. Rev. B* **63** 064518
[18] Levchik M, Mikkelsen T, Norman M R and Paul I 2010 *Phys. Rev. B* **82** 060502
[19] Luo H G, Su Y H and Xiang T 2008 *Phys. Rev. B* **77** 014529
[20] Prelovsek P, Ramsak A and Sega I 1998 *Phys. Rev. Lett.* **81** 3745
[21] Naqib S H, Borhan Uddin M and Cole J R 2011 *Physica C* **471** 1598
[22] Shibauchi T, Carrington A and Matsuda Y 2014 *Anu. Rev. Condens. Matter Phys.* **5** 113
[23] Sachdev S 2000 *Science* **288** 475
[24] Hussey N E, Cooper R A, Xiaofeng Xu, Wang Y, Mouzopoulou I, Vignolle B and Proust C 2011 *Phil. Trans. R. Soc.* **369** 1626
[25] Naqib S H and Islam R S 2008 *Supercond. Sci. Technol.* **21** 105017 and references therein
[26] Cole J R 2004 *Ph.D. thesis*, University of Cambridge, UK (unpublished)
[27] Naqib S H, Cooper J R, Tallon J L, Islam R S and Chakalov R A 2005 *Phys. Rev. B* **71** 054502
[28] Corson J, Mallozzi R, Orenstein J, Eckstein J N and Bozovic I 1999 *Nature* **398** 221
[29] Naqib S H and Islam R S 2015 *Supercond. Sci. Technol.* **28** 065004
[30] Grbic M S, Pozek M, Paar D, Hinkov V, Raichele M, Haug D, Keimer B, Barisic N and Dulcic A 2011 *Phys. Rev. B* **83** 144508
[31] Lee P A 2008 *Rep. Prog. Phys.* **71** 012501
[32] Naqib S H, Cooper J R, Tallon J L and Panagopoulos C 2003 *Physica C* **387** 365
[33] Naqib S H, Cooper J R and Loram J W 2009 Phys. Rev. B 79 104519
[34] Naqib S H, Chakalov R A, Cooper J R 2004 Physica C 407 73
[35] Walker D J C, Mackenzie A P and Cooper J R 1995 Phys. Rev. B 51 15653
[36] Cooper R A, Wang Y, Vignolle B, Lipscombe O J, Hayden S M, Tanabe Y, Adachi T, Koike Y, Nohara M, Takagi H, Proust C and Hussey N E 2009 Science 323 603