Gender by racial/ethnic intersectionality in the patterns of Adolescents’ math motivation and their math achievement and engagement

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ABSTRACT

Individuals’ math motivational beliefs are theorized to shape their STEM achievement and engagement in high school and beyond. Combining situated expectancy-value theory and intersectionality framework, the goals of this study were to (a) identify the unique patterns of U.S. high school students’ math motivational beliefs, (b) examine differences in the patterns based on the intersection of gender and race/ethnicity, and (c) test the extent to which these patterns predicted differences in students’ math achievement and classroom behavioral engagement for each of the gender by racial/ethnic groups. The current study included 16,120 high schoolers (50% female; 63% White, 17% Latina/o, 11% Black, and 9% Asian Americans; Mage = 14.46 at Grade 9) from the High School Longitudinal Study. There were six unique patterns of students’ math motivational beliefs: Overall High, Above Average but not Identified, Identified but Average Value, Average, Low Identity, and Overall Low. Pattern membership at the intersection of gender and race/ethnicity showed nuances that could not be represented by gender or race/ethnicity alone; for example, male and female Asian American adolescents had similar patterns, but many male and female adolescents of other racial/ethnic groups had different patterns. Adolescents’ math motivational belief patterns were associated with their Grade 11 math achievement and behavioral engagement even after controlling for prior math achievement and family socioeconomic status, and the associated varied by the gender and racial/ethnic groups.

1. Introduction

Math is central to a variety of science, technology, engineering, and math (STEM) domains (National Mathematics Advisory Panel, 2008; Watt et al., 2017a). However, only one in four U.S. high schoolers are performing at or above proficiency in math (NCES, 2015). Furthermore, individuals’ persistence into math-intensive STEM fields is both gendered and racialized in the U.S., with disproportionately more men, Whites, and Asian Americans than women, Latinas/os, and Blacks. For example, women account for 46% of the U.S. labor force, but only 25% of all computer/mathematical scientists1 (NSF, 2019). Similarly, Latinas/os and Blacks account for 17% and 12% of the labor force respectively, but they each account for only 5% of all computer/mathematical scientists (NSF, 2019). Racial/ethnic disparities in math achievement emerge with Latinas/os and Blacks on average scoring 10% lower on standardized tests than their White and Asian American peers by the time they graduate high school (NCES, 2015), which is a pivotal developmental period in the STEM pipeline because of its implications for later persistence (Ceci et al., 2009; Sadler et al., 2012; Wigfield and Eccles, 2000).

Recent studies on the gender and racial/ethnic disproportions in math have drawn attention to psychological factors, such as adolescents’ motivational beliefs, to help understand these gaps (Else-Quest et al., 2013; Wang & Degol, 2017; Watt, 2006). In the current study, we tested tenets of situated expectancy-value theory (Eccles & Wigfield, 2020) by examining the patterns of adolescents’ math motivational beliefs. Then, we took an intersectionality approach (Crenshaw, 1989; Harris & Patton, 2019) to examine how membership in the patterns varied by both gender and simultaneously race/ethnicity. Lastly, we tested how those math motivational belief patterns corresponded to adolescents’ math

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1 According to the National Science Foundation definition of computer/mathematical scientists, which excluded professions such as math teachers.

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achievement and classroom behavioral engagement within each of the gender and racial/ethnic groups.

2. Situated expectancy-value theory and the patterns of adolescents’ motivational beliefs

Widely used in studies on academic motivation, the situated expectancy-value theory posits that individuals are motivated to pursue a domain and perform better if they think the domain is valuable and if they expect it is within their ability to succeed in it (Eccles, 2011; Eccles & Wigfield, 2020). According to the theory, value and expectancy beliefs function in combination with one another in predicting performance and choices. Value beliefs include intrinsic value (i.e., how interesting individuals find math to be), utility value (i.e., how useful individuals find math to be), and attainment value (i.e., how important is math to individuals’ identity; e.g., Boaler & Greer, 2000; Shanahan, 2009).

Expectancy beliefs, also known as ability self-concepts, pertain to how good individuals think they are currently in a domain and how well they expect themselves to do in the future.

Studies on math intrinsic value, utility value, attainment value, and expectancy beliefs have shown that each of these four math motivational beliefs independently predicts math achievement (Denissen et al., 2007; Else-Quest et al., 2013; Guo et al., 2015; Köller et al., 2001; Marsh et al., 2005; Seo et al., 2019; Valentine et al., 2004). Similarly, math expectancy and value beliefs independently predict students’ math engagement (Fredricks, Holfken, Wang, Mortensen, & Scott, 2018; Shanahan, 2009; Simpkins, Fredricks, & Eccles, 2015; Zeldin & Pajares, 2000).

Although existing research provides support for the situated expectancy-value theory, most prior studies are based on analytic models in which one motivational belief is analyzed independently or while holding other beliefs constant. Eccles & Wigfield (2020) argued that individuals’ motivational beliefs, however, do not function in isolation even though they are often analyzed that way. In fact, studies focused on the interrelations between multiple expectancy and value beliefs showed that leveraging one belief might not be enough (Durik, Schechter, Noh, Rozek, & Harackiewicz, 2015; Guo et al., 2016; Guo, Parker, Marsh, & Morin, 2015; Nagengast et al., 2011; Trautwein et al., 2012). For example, Lauermann, Tsai, and Eccles (2017) found that math expectancy beliefs were predictive of math-related career attainment only when individuals’ math intrinsic values were moderate or high, but not when math intrinsic values were low. These studies analyzed the interrelations between multiple motivational beliefs by creating interaction terms, which becomes challenging to interpret and less parsimonious once three or more motivational beliefs are simultaneously examined.

A more efficient analytic method that addresses the interrelations among multiple motivational beliefs as posited by situated expectancy-value theory is a person-centered approach (Eccles & Wigfield, 2020). Person-centered approaches examine multiple indicators simultaneously and identify the most prevalent and parsimonious patterns. As such, person-centered approaches, including cluster analysis, identify more complex and ecologically valid patterns among multiple interrelated constructs than what can be uncovered by variable-centered approaches including researcher-defined interactions (Conley, 2012). Person-centered approaches have been increasingly utilized in educational psychology in recent years to examine individuals’ motivational beliefs (Andersen & Chen, 2016; Chittum & Jones, 2017; Ng et al., 2016; Perez et al., 2018; Van Soom & Donche, 2014). For example, Snodgrass Rangel and colleagues (Snodgrass Rangel et al., 2020) used the same dataset as the current study and identified four patterns of math and science motivational beliefs: high math and science beliefs, low math and science beliefs, high math-low science beliefs, and low math-high science beliefs. Although their results address the cross-domain tenets of the theory concerning patterns of expectancies and various value beliefs. Thus, our study focuses on one domain and explored the nuances among the motivational beliefs, such as the correlates for adolescents with high expectancy but low value beliefs in math.

Theory and prior empirical work suggest that at least five unique patterns of math motivational beliefs should emerge. First, the positive correlations between expectancies and values will likely yield patterns of overall high and overall low motivational beliefs (Denissen, Zarrett, & Eccles, 2007; Eccles, 2009; Snodgrass Rangel, Vaval, & Bowers, 2020; Wigfield and Eccles, 2000). Recent studies also suggested patterns in which individuals’ motivational beliefs differ on their relative level (Durik et al., 2015; Jacobs et al., 2005; Lauermann et al., 2017). For example, the combination of high math expectancy but low value beliefs (“I can, but I don’t want to”) was identified and further postulated to be more common in girls than boys (Jacobs et al., 2005). The opposite pattern may exist where individuals feel “math is interesting and important, but too hard for me,” which is a combination where math values are high despite low expectancies (Durik et al., 2015; Lauermann et al., 2017). Finally, some individuals might find math useful and within their ability, but still do not identify with math-intensive fields or have a low attainment value if the field is discriminatory toward or incompatible with their social identities (McGee, 2013; Shanahan, 2009).

In their recent review of the situated expectancy-value theory, Eccles and Wigfield (2020) pointed out that attainment values may be more salient during periods when individuals are actively exploring their options and identities. Because high school and adolescence more broadly is a time when individuals actively explore who they are and who they want to be, we expect that there will be patterns driven by adolescents’ attainment values, such as adolescents who believe math is useful and they are good at it, but just do not see themselves as a math person (Amschauer, Li, & Roth, 2018; Erikson, 1972; Tan, Calabrese-Barton, Kang, & O’Neil, 2013).

According to situated expectancy-value theory, these unique patterns of adolescents’ math motivational beliefs should map onto their math achievement and classroom engagement (Eccles & Wigfield, 2020). Following the theory and prior studies (Durik et al., 2015; Lauermann et al., 2017; Snodgrass Rangel et al., 2020), the combination of both high expectancy and value beliefs should be associated with the highest math achievement and engagement. In addition, the combination of low expectancy and value beliefs should be associated with the lowest math achievement and engagement. Patterns characterized by high expectancy but low value beliefs (e.g., “I can, but I don’t want to”) should be associated with better math achievement than patterns characterized by low expectancy despite high value beliefs (e.g., “math is interesting and important, but too hard for me”) because typically achievement is more strongly predicted by expectancy than value beliefs (Rosenzweig et al., 2019).

3. Intersectionality framework: gender and race/ethnicity

The situated expectancy-value theory argues that individuals’ motivational beliefs are shaped by their social identities, including gender and race/ethnicity (Eccles & Wigfield, 2020). On average, White male youth report higher math motivational beliefs than White female youth, which aligns with the stereotypical notion of math as a male discipline (Sax et al., 2015; Su et al., 2009; Umari et al., 2018). In terms of racial/ethnic differences, studies suggest that White and Asian American youth report higher math expectancies than their Latina/o and Black peers (Andersen & Ward, 2014; Bouche & Harter, 2005; Brown & Lephar, 2010; Wenner, 2003), but the findings are mixed concerning math intrinsic values (e.g., Riegel-Crumb et al., 2011). Prior studies also suggest racial/ethnic differences in math identity or attainment values that reflect the systemic inequities regarding who is perceived and given resources/supports to be math people (McGee, 2013; Nasir & Cobb, 2002; Shanahan, 2009). A limitation to most studies, however, is that researchers often examined gender and race/ethnicity separately (Else-Quest et al., 2013). A recent review on STEM motivational beliefs concluded that the intersection of gender and race/
ethnicity is particularly important in the U.S. context but remains understudied (Parker et al., 2020).

Coined by legal scholar Kimberle Crenshaw (1989), the intersectionality framework argues that power dynamics defined by both gender and race/ethnicity function in more complex ways than what can be accounted by gender or race/ethnicity alone. Specifically, relevant aspects of individuals’ social identities should be examined simultaneously with particular attention to those identities associated with existing power dynamics in that area or field (Cole, 2009; Harris & Patton, 2019). As reviewed, math is a gendered and racialized field where men as well as White and Asian Americans hold more power than their peers (e.g., McGee, 2013; NSF, 2019; Parker et al., 2020). Accordingly, gender representations and dynamics in math will be better understood by simultaneously attending to the power dynamics based on race/ethnicity. For example, even though men on average are expected to hold more power in math than women, this pattern may not be universal across race/ethnicities. It may emerge for Asian American men whose race/ethnicity is also stereotyped to hold power in math, but perhaps not Black men whose race/ethnicity is marginalized in math. In other words, being a member of multiple groups who hold more power in a domain may serve to promote individuals’ achievements in and pursuits of that domain, as in the case of Asian American men in math (Armenta, 2010; Walton & Cohen, 2003). However, what happens for individuals who are simultaneously part of a group with more power in a domain and also a group that is marginalized in a domain, such as Asian American women or Latino men in math, is often overlooked (Gibson et al., 2014; Shapiro & Williams, 2012). Such nuanced differences can only be uncovered by examining the intersection of gender and race/ethnicity.

Examining the intersection of gender and race/ethnicity also allows the possibility of identifying underrepresented groups and developmental processes for those groups that remain hidden when they are examined separately. Black women, for example, are severely underrepresented in math-intensive fields; however, this pattern is obscured when they are only seen in terms of their gender (women) or only their race/ethnicity (Black) instead of in terms of their joint membership in these two groups (Ong et al., 2011; Patton et al., 2016). As another example, Hazari et al. (2013) found that although female adolescents, on average, reported lower science identity than male adolescents, the gender difference was particularly pronounced among Latina/o students. Not only is an intersectional approach the only way to identify the most vulnerable groups defined by both gender and race/ethnicity, but it also affords the examination of whether the developmental processes or correlates of individuals’ motivational patterns are similar within each group. That is, the associations between patterns of adolescents’ math motivational beliefs and their math outcomes can be more precisely examined when adolescents are grouped by both their gender and race/ethnicity.

### 4. Current study

The current study extends the existing literature on math motivational beliefs by using a person-centered approach to (a) identify prevalent, unique patterns of high school students’ math motivational beliefs, (b) examine gender by racial/ethnic differences in these patterns, and (c) examine how these patterns were associated with adolescents’ math achievement and behavioral engagement for each of the gender by racial/ethnic groups. Specifically, three research questions and corresponding hypotheses were tested.

For Research Question 1, we asked what are the most prevalent patterns of adolescents’ Grade 11 math intrinsic value, utility value, attainment value, and expectancy beliefs? At least five unique patterns were hypothesized to emerge: overall high, overall low, high expectancy but low value, low expectancy but high value, and generally high motivational beliefs but low attainment value.

In Research Question 2, we asked what are the differences in pattern membership defined by both gender (male or female adolescents) and race/ethnicity (Asian, Black, Latina/o, or White adolescents)? As prior studies suggested and aligning with gender and racial/ethnic stereotypes in math, adolescents whose gender and race/ethnicity are associated with more power in STEM (e.g., White and Asian male adolescents) might have more positive motivational patterns than those who are members of one advantaged and one marginalized group (e.g., Black and Latino male adolescents, and White and Asian female adolescents), followed by adolescents who are members of two marginalized groups (e.g., Black and Latina female adolescents).

Lastly, in research Question 3, we asked to what extent are the patterns of math motivational beliefs associated with adolescents’ math achievement and behavioral engagement? The overall high pattern was hypothesized to be associated with higher math achievement and engagement than clusters with some high and some low motivational beliefs. Patterns that are low on both expectancy and value beliefs were hypothesized to be associated with the lowest math achievement and engagement. Aligning with the intersectionality perspective, we tested these associations within each of the eight gender by racial/ethnic groups to understand the extent to which these patterns replicated within each group.

### 5. Method

#### 5.1. Dataset and participants

The High School Longitudinal Study (HSLS) surveyed a nationally representative sample of ninth graders from 944 high schools during the years 2009–2010 and followed them three, four, and eight years later. The dataset employed a complex sampling design to ensure representativeness of indicators such as race/ethnicity. In this study, we used the first two waves of HSLS when participants were in high school. We intentionally focused on identifying patterns of math motivational beliefs in Grade 11 instead of Grade 9, so that we can control for prior (Grade 9) math achievement when we examine how the patterns were associated with math achievement and behavioral engagement. HSLS was designed to study adolescents’ trajectories in and transitions beyond high school with a specific focus on science, technology, engineering, and math (STEM), making it ideal for the current study (see Ingels et al., 2014 for more details). All sample sizes presented were rounded to the nearest tenth place per NCES data use agreement requirements.

Three exclusion criteria were applied to the full HSLS sample in the following order (see Appendix 1 for comparisons of the analytic and excluded samples): (a) participants who were enrolled in the HSLS study but did not participate in Grade 9 or 11 data collection (n = 6580), (b) participants who were Native Hawaiian or Pacific Islanders (n = 90), American Indian (n = 120), or multi-racial (n = 1650) because their group sizes were too small or heterogeneous for analysis, and (c) participants who had any missing data on math motivational beliefs in Grade 11 (n = 640; <5% of the participants). The third exclusion criteria ensured that no missing data needed to be imputed for the person-centered analysis.

The resulting analytic sample included 16,120 adolescents (M_{age} = 14.46 at Grade 9) and was 63% White, 17% Latina/o, 11% Black, and 9% Asian American with an even distribution of male and female adolescents. Specifically, there were 5050 White female adolescents, 5060 White male adolescents, 1430 Latina adolescents, 1380 Latino adolescents, 890 Black female adolescents, 910 Black male adolescents, 690 Asian American female adolescents, and 710 Asian American male adolescents.

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2 Among the 640 participants excluded due to missing math motivational belief items, 230 were missing on one motivational belief item, 250 on two, 30 on three, and 130 on all four items.
5.2. Measures

Math motivational beliefs. In Grade 11, participants self-reported four math motivational beliefs using scales based on the expectancy-value theory and prior studies (Jacobs, Davis-Kean, Bleeker, Eccles, & Malanchuk, 2005; Shanahan, 2009; Simpkins, Fredricks, & Eccles, 2015; Snodgrass Rangel, Vaval, & Bowers, 2020; Eccles & Wigfield, 2020). Math intrinsic value was a composite of 3 items (α = 0.78; e.g., “you enjoy math classes very much”). Math utility value was a composite of 3 items (α = 0.82; e.g., “math is useful for everyday life”). Math attainment value was a composite of 2 items (α = 0.88; e.g., “you see yourself as a math person”). Math expectancies were a composite of 4 items (α = 0.89; e.g., “you are certain that you can master math skills”). See Appendix 2 for all items. Items were reverse coded so that higher scores reflected higher math motivational beliefs (1 = strongly disagree, 4 = strongly agree). Confirmatory factor analysis including all four motivational beliefs and their respective items showed acceptable model fit among the full analytic sample (CFI = 0.96; RMSEA = 0.08; SRMR = 0.04). All the loadings were equal or larger than 0.60 on their own factor, suggesting the convergent validity of these constructs. Measurement invariance tests suggested that the math motivational belief constructs evidenced full configural, weak, and strong invariance (ΔCFI = 0.001 to 0.006; Appendix 3) across the eight groups defined by both gender and race/ethnicity (e.g., Black female adolescents, Black male adolescents). These results suggest that differences across groups are unlikely to be attributable to measurement bias. See Table 1 for descriptive statistics for all main variables.

Math achievement and engagement. Students’ Grade 11 and math achievement were measured with an Item Response Theory-based standardized assessment that was developed for HSLS and norm-referenced (see Ingels et al., 2014 for detailed documentation). The assessment was computer-based and consisted of 40 algebraic questions that encompassed six algebraic content domains (language of algebra; proportional relationships and change; linear equations, inequalities, and functions; nonlinear equations, inequalities, and functions; systems of equations; sequences and recursive relationships) and four algebraic processes (demonstrating algebraic skills; using representations of algebraic ideas; performing algebraic reasoning; solving algebraic problems). Scores on the assessment were standardized (to a mean of 0 with a standard deviation of 1) to ease interpretation for this study. Standardized math achievement in Grade 9 was used as a control, whereas Grade 11 math achievement was examined as an outcome.

Students’ self-reported behavioral engagement in math class in Grade 11 was a standardized composite of 4 items (Fredricks et al., 2011; e.g., “how often did/do you paid attention to the teacher in math class”; α = 0.74; 1 = never, 4 = always; see Appendix 2 for all items). Measurement invariance test suggested that adolescents’ math engagement evidenced partial strong invariance (ΔCFI = <0.001–0.009; Appendix 3) across the eight groups defined by both gender and race/ethnicity.

Demographic factors and covariates. Adolescents reported their gender and race/ethnicity. Two additional indicators were included as covariates in the analyses. The first was the standardized math achievement in Grade 9 described above. The second was a composite measure of socioeconomic status that encompasses parent education (1 = less than high school, 7 = Ph.D./M.D./Law/other high-level professional degrees), family income (1 = less than or equal to $15,000, 13 = greater than $235,000), and school urbanicity (city, suburb, town, or rural).

5.3. Analytical plan

Research Question 1 addressed the prevalent patterns of adolescents’ math intrinsic value, utility value, attainment value, and expectancy beliefs. The four math motivational beliefs were cluster analyzed using Ward’s method on average squared Euclidean distances, which is more appropriate than the correlation-distance method given that the four math motivational beliefs are all on the same 4-point Likert scale instead of different scales. After a cluster solution was identified, K-mean relocation was used to move cases to optimize the fit between the case and the cluster patterns if the change can reduce variation within clusters. K-means clustering has been shown to perform as well or better than mixture-model based clustering such as latent profile analysis when the structure of the solutions (e.g., whether the variances of each variable are held constant across individuals within a cluster) is not set a priori (Steinley & Brusco, 2011). It should be noted that cluster analysis was estimated on the six random subsamples instead of on the racial/ethnic by gender-specific subgroups because the latter would have only been appropriate if we had specific a priori hypotheses for potential cluster solutions for each of the subgroups. Sampling weights were not incorporated at this stage because the software program used (as well as other software programs) currently does not have options to fully account for complex sampling design features (sampling weights, strata, primary sampling unit) in cluster analysis. The complex sampling design features were incorporated in later research questions where the patterns’ membership as well as association with math outcomes were estimated.

The analysis was conducted on ROPstat (ROPstat, 2018) in accordance with the developers’ guidelines (Bergman et al., 2003; Vargha et al., 2015). Replication is critical in person-centered approaches as a

### Table 1

Bivariate Correlations and Descriptive Statistics.

| Variable | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
|          |       |       |       |       |       |       |       |       |
| Grade 11 mathematical beliefs |       |       |       |       |       |       |       |       |
| Math intrinsic value | 1.00  |       |       |       |       |       |       |       |
| Math utility value | 0.42*** | 1.00  |       |       |       |       |       |       |
| Math attainment value | 0.52*** | 0.42*** | 1.00  |       |       |       |       |       |
| Math expectancies | 0.56*** | 0.38*** | 0.59*** | 1.00  |       |       |       |       |
| Grade 11 math outcomes |       |       |       |       |       |       |       |       |
| Math achievement | 0.22*** | 0.16*** | 0.42*** | 0.31*** | 1.00  |       |       |       |
| Math class behavioral engagement | 0.49*** | 0.28*** | 0.33*** | 0.45*** | -0.08*** | 1.00  |       |       |
| Covariates |       |       |       |       |       |       |       |       |
| Grade 9 math achievement | 0.18*** | 0.13*** | 0.28*** | 0.26*** | 0.06*** | 0.15*** | 1.00  |       |
| Socioeconomic status | 0.05*** | 0.02*  | 0.13*** | 0.12*** | 0.11*** | 0.07*** | 0.10*** | 1.00  |
| Mean | 2.65  | 3.28  | 2.43  | 2.78  | 51.90 | 0.07  | 51.86  | 0.08  |
| Standard Deviation | 0.74  | 0.49  | 0.91  | 0.71  | 10.13 | 0.97  | 9.97   | 0.80  |
| Min | 1.00  | 1.00  | 1.00  | 1.00  | 22.24 | -4.10 | 24.07  | -1.92 |
| Max | 4.00  | 4.00  | 4.00  | 4.00  | 84.91 | 1.18  | 82.19  | 2.98  |
| Skewness | -0.23 | -0.57 | -0.01 | -0.30 | 0.04  | -1.12 | -0.07  | 0.32  |

Note: Sample size is 16,120 except math behavioral engagement, which has 1,610 cases of legitimate missing data.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Longitudinal Study of 2009 (HSLS:09), Base Year and First Follow-Up.

* p < .05; ** p < .01; *** p < .001.
tool to enhance validity and guide identification of the solution (Breckenridge, 2000). To validate the cluster solution, the analysis sample was randomly divided into six subsamples. We tested the differences across the six subsamples in terms of adolescents’ math motivational beliefs, math outcomes, demographic indicators, and covariates with ANOVA and chi-square tests. There were no statistical differences in any of these indicators across the subsamples (Appendix 4). The cluster analysis was replicated on each of the six subsamples. The optimal number of clusters was selected from the range 3 to 8 based on (a) minimum error sum of squares (i.e., the extent to which cluster solutions deviation from the actual data), (b) homogeneity coefficient (H. C.; i.e., the extent to which cases within each cluster are similar to one another), and (c) theoretical meaning of the cluster solution (Vargha et al., 2015). The ideal cluster solution was hence a balance between being accurate and parsimonious in describing the data and including theoretically meaningful patterns.

Research Question 2, guided by intersectionality framework, examined representation defined by both gender and race/ethnicity in adolescents’ math motivational belief patterns. Over-and under-representation of each pattern was analyzed using chi-square tests with adjusted standardized residuals (ASRs), which are the standard error corrected (i.e., accounting for sample size) tests of the difference between observed and expected cell counts (Agresti, 2007). Adjusted standardized residuals with absolute values greater than 2.58 corresponded to a statistically significant 2-tailed test at $p = .01$ and were regarded as statistically significant under-/over-representation. Analysis for this research question was also done in STATA version 14 with sampling weights, primary sampling unit, and strata incorporated.

Missing data. Participants in the analytic sample had complete data except for 2460 participants who had missing data for math behavioral engagement. Among them, 1620 participants were missing legitimately because they were not enrolled in any math class during Grade 11.3 For the remaining 840 students who were in a Grade 11 math class and were missing behavioral engagement, multiple imputation was estimated with 20 imputed datasets using a set of auxiliary variables to enhance the imputation process (Enders, 2010; see Appendix 5 for how the imputed participants compared with participants with complete data). Because the multiple imputation analyses are only relevant for the regression predicting behavior engagement (Research Question 3), all other analyses were estimated on our analytic sample ($N = 16,120$) with no need to impute data.

6. Results

6.1. Six distinct patterns of Grade 11 math motivational beliefs

Six unique patterns were identified as the most optimal cluster solution for adolescents’ math intrinsic value, utility value, attainment value, and expectancy beliefs. In each of the six random subsamples, the error sum of squares dropped with the 6-cluster solution and leveled off after that, meaning that the 6-cluster solution better fit the data than solutions with a fewer number of clusters and that adding a seventh cluster did not yield as much of an improvement (see Appendix 6 for the follow-up analysis to the regressions; these pairwise comparisons are statistically identical to an analysis of variance test with all possible contrasts. Because there were numerous pairwise comparisons, we adjusted the p-value with a Bonferroni correction. Analysis for this research question was also done in STATA version 14 with sampling weights, primary sampling unit, and strata incorporated.

Fig. 1. Clusters by Raw Means of Math Motivational Beliefs. SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Longitudinal Study of 2009 (HSL:09), Base Year and First Follow-Up.
cluster solution was chosen over the 7-cluster solution because the theoretically unique clusters (of each of the six subsamples). Furthermore, the 6-cluster solution was reported by adolescents in this pattern, in contrast, were average relative to students in other patterns (Appendix 8). The fourth pattern, with the lowest motivational beliefs across all patterns.

Across all six patterns, utility values were mostly higher in terms of the means and had a smaller variance than intrinsic values, attainment values, and expectancy beliefs (with only a few exceptions). The relative levels of the other motivational beliefs, however, varied by pattern. In other words, the level of adolescents’ intrinsic values, attainment values, and expectancy beliefs did not always go hand in hand. For example, the Average and Low Identity patterns were almost identical on their level of expectancy beliefs, but the Average pattern had higher attainment values than intrinsic values, whereas the Low Identity pattern had the opposite pattern with lower attainment values than intrinsic values.

### 6.2. The intersection of gender and race/ethnicity in the patterns of Adolescents’ motivational beliefs

In addition to identifying patterns of adolescents’ math motivational beliefs, another goal of the current study was to explore each pattern’s gender by racial/ethnic representation given the existing gender and racial/ethnic composition (\(\chi^2(35) = 414.80, p < .001, \phi = 0.07\)).

Note. Dark gray cells denote overrepresentation (adjusted standardized residual corresponds to \(p < .01\)), light gray cells denote underrepresentation (\(p < .01\)), uncolored cells denote fair representation than expected by chance. Percentages sum to 100% across the rows. The six math motivational belief patterns varied in their gender (\(\chi^2(5) = 130.81, p < .001, \phi = 0.09\)), racial/ethnic (\(\chi^2(15) = 245.68, p < .001, \phi = 0.07\)), and gender-by-racial/ethnic composition (\(\chi^2(35) = 414.80, p < .001, \phi = 0.07\)).

### Table 2

Motivation Patterns by Gender, Race/Ethnicity, and the Intersection of Gender and Race/Ethnicity with Overrepresentation in Dark Gray Cells and Underrepresentation in Light Gray Cells.

| Group | Overall high | Above Average but not identified | Identified but average value | Average | Low identity | Overall low | Total |
|-------|--------------|----------------------------------|-----------------------------|---------|--------------|------------|-------|
| **Gender** | | | | | | | |
| Females | 13% | 21% | 18% | 19% | 16% | 13% | 100% |
| Males | 16% | 19% | 21% | 21% | 12% | 10% | 100% |
| **Race/ethnicity** | | | | | | | |
| Asian | 21% | 21% | 26% | 20% | 7% | 6% | 100% |
| Black | 18% | 24% | 18% | 15% | 16% | 9% | 100% |
| Latina/o | 12% | 22% | 19% | 20% | 16% | 11% | 100% |
| White | 15% | 18% | 20% | 21% | 13% | 13% | 100% |

### Note

For the presentation and results from this point on, we merged the six other sub-samples into the full analytic sample. Fig. 1 presents the six clusters by their raw means on the four motivational beliefs, and Appendix 8 presents their standardized means. The cluster analysis was based on raw scores to preserve the scale (average 62%, range 61–63%; the convention is to be higher than 0.20, Vargha et al., 2015) and explained sufficient error sum of squares (average 62%, range 61–63%; the convention is to be higher than 50%) across all six subsamples after the k-means relocation.

- **Students with legitimate missing data on math outcome (n = 1620) were distributed across clusters: Overall High (n = 120; 12% of the legitimate missing), Above Average but not Identified (n = 310, 19%), Identified but Average Value (n = 300, 19%), Average (n = 350, 21%), Low Identity (n = 210, 13%), Overall Low (n = 250, 16%).**
racial/ethnic disparities in STEM (NSF, 2019; Parker et al., 2020).

Intersectionality framework posits that the intersection of gender and race/ethnicity can uncover nuances that are not seen when gender and race/ethnicity are examined separately (Crenshaw, 1989; Harris & Patton, 2019). As shown in Table 2, the six math motivational belief patterns varied in their gender-by-racial/ethnic composition (\(\chi^2 = 35\) = 414.80, \(p < .001, \phi c = .07\)). To highlight what was gained by taking an intersectionality approach, we present gender composition (\(\chi^2 = 130.81, p < .001, \phi c = .09\)) and racial/ethnic composition (\(\chi^2 = 245.68, p < .001, \phi c = .07\)) separately in the upper half of Table 2 and the gender-by-racial/ethnic composition in the lower half. To test over-/under-representation of each gender-by-racial/ethnic composition, adjusted standardized residuals (ASR) are estimated with 2.58 as the cutoff, which corresponds to statistical significance at the \(p < .01\) level. The percentages in Table 2 represent the percentage of individuals from each demographic group in each of the six patterns. Thus, the percentages across each row sum to 100%.

Male adolescents’ representation in the math motivational belief patterns varied across the full analytic sample and the various racial/ethnic groups. In the full analytic sample, male adolescents on average were more likely than chance to be in the Overall High (16% of male adolescents; ASR = 6.77) and Identified but Average Value (21%; ASR = 5.17) patterns and less likely than chance to be in the Low Identity (12%; ASR = 6.94) and Overall Low (10%; ASR = 8.07) patterns. However, these differences across patterns on the full analytic sample did not emerge in every racial/ethnic group but instead was primarily driven by certain racial/ethnic groups. The overrepresentation in the Overall High pattern among male adolescents was actually only the case for Asian American male adolescents (24%; ASR = 8.77), but not for White male adolescents (16%; ASR = 2.39), Latino male adolescents (14%; ASR = 0.98), or Black male adolescents (17%; ASR = 0.89) adolescents. Similarly, male adolescents on average were more likely than chance to be in the Identified but Average Value pattern, but that overrepresentation was true only among White male adolescents (22%; ASR = 4.74), but not for male adolescents of the other three racial/ethnic groups (17%-24%; ASR = 0.34-1.99). Examining pattern membership at the intersection of race and gender/ethnicity also provided nuances that were otherwise masked in the full analytic sample. For example, Black male adolescents were more likely than expected to exhibit the Above Average but not Identified pattern (25%; ASR = 4.31), but White male adolescents (18%; ASR = 7.32) were less likely than expected to exhibit this pattern. Such differences were cancelled out when we combined the different racial/ethnic groups and examined group differences on the full analytic sample (19% male adolescents; ASR = 0.08). In sum, our results suggest that the representation in math motivational belief patterns for male adolescents on the full analytic sample (i.e., without intersecting with race/ethnicity) did not generalize to male adolescents in each racial/ethnic group.

Examining female adolescents as one group masked how female adolescents of different races/ethnicities evidenced conflicting representation for four of the six math motivational beliefs patterns. For example, female adolescents on average were less likely than expected to show the Overall High pattern (13% ASR = 6.77); however, this underrepresentation was only true for White female adolescents (13% ASR = 6.31) and Latina female adolescents (10% ASR = 3.95) adolescents. Asian American female adolescents, in contrast, were actually more likely than expected to show the Overall High pattern (17%; ASR = 3.80). Similarly, underrepresentation of Asian American female adolescents in the Low Identity pattern (9%; ASR = 1.20) was not the case when they were grouped with female adolescents from the other three racial/ethnic groups (16% ASR = 6.94), all of whom showed overrepresentation in this pattern (15%-18%; ASR = 3.15-4.63). As another example, White female adolescents were less likely than expected to show the Above Average but not Identified pattern (18%; ASR = 4.00), which is the opposite pattern than that for Latina (24% ASR = 2.72) and Black female adolescents (23%; ASR = 2.77) adolescents, who showed more representation in this pattern than expected. These results suggested that looking at female adolescents as one group often failed to capture the divergent patterns for females of different racial/ethnic groups.

Another way to interpret results guided by the intersectionality framework is to look at gender differences within certain racial/ethnic groups (Table 2). For example, White male adolescents were overrepresented (22%; ASR = 4.74) whereas White female adolescents were underrepresented (18%; ASR = 3.55) in the Identified but Average Value pattern—such differences would have been canceled out if they were examined just as White adolescents regardless of gender (20%; ASR = 1.15). As another example, Latina (female) adolescents were underrepresented in the Overall High pattern (10%; ASR = 3.95) and overrepresented in the Low Identity (18%; ASR = 3.15) and Above Average but not Identified (24%; ASR = 2.77) patterns; Latina (male) adolescents, in contrast, were neither over- nor under-represented in any of the patterns (14%-21%; ASR = 0.98-1.94). It is important to point out that these differences among male and female adolescents within each group did not emerge for Asian American adolescents. Both male and female Asian American adolescents were overrepresented in the Overall High pattern (24%, 17%; ASR = 8.77, 3.80) and underrepresented in the Low Identity (5%, 9%; ASR = 5.94, 2.79) and Overall Low patterns (4%, 8%; ASR = 6.55, 3.82).

6.3. Associations between patterns of math motivational beliefs and math outcomes

Lastly, we tested the associations between the patterns of adolescents’ math motivational belief and their math outcomes for each of the eight gender by racial/ethnic groups, controlling for Grade 9 standardized math achievement and socioeconomic status through regression analyses (Table 3 and 4). We followed up these regressions with all pairwise comparisons among the patterns. Given the number of pairwise comparisons, the statistical significance of pairwise comparisons was adjusted with a Bonferroni correction. We focus our discussion below on the more conservative pairwise comparison tests, which were Bonferroni-corrected.

As shown in Table 3, the math motivational belief patterns were associated with changes in math achievement for White male and female adolescents as well as Asian and Latino male adolescents; once the Bonferroni correction was applied, the associations were not significant for Black male adolescents, as well as Asian, Black, and Latina female adolescents. For the groups that showed significant associations between math motivational belief patterns and achievement, adolescents in the Overall High pattern generally had the largest gains in math achievement, followed by adolescents in the Above Average but not Identified, Identified but Average Value, and Average patterns, whereas adolescents in the Low Identity and Overall Low patterns tended to show the lowest math achievements. For example, among White male adolescents, those who showed the Overall High pattern had greater increase in math achievement than those showing all other patterns (B = 0.29-58; p < .001). Then, White male adolescents who showed the Identified but Average Value pattern had the second highest level of math achievement which was similar to those who showed the Above Average but not Identified pattern (B = 0.05, ns) and higher than those who showed the Average pattern (B = 0.12, p = .01). Finally, White male adolescents who showed the Low Identity and Overall Low patterns had similar math achievements (B = 0.10, ns), which were lower than that of those who showed all the other patterns (B = 0.15-0.58, p < .05). The exact rank order and magnitude of difference in math achievement varied for each of the gender and racial/ethnic groups as detailed in Table 3.

As shown in Table 4, adolescents’ math motivational belief patterns were related to different levels of math behavioral engagement for all eight racial/ethnic by gender groups. Similar to the comparisons for math achievement, adolescents who showed the Overall High pattern had the highest level of math behavioral engagement, whereas those who showed the Overall Low pattern had the lowest levels. Several of the
| Motivation patterns          | Math achievement | Asian female adolescents | Asian male adolescents | Black female adolescents | Black male adolescents | Latina adolescents | Latino adolescents | White female adolescents | White male adolescents |
|-----------------------------|------------------|--------------------------|------------------------|--------------------------|------------------------|---------------------|---------------------|------------------------|------------------------|
|                             | Mean B (SE)      | Mean B (SE)              | Mean B (SE)            | Mean B (SE)              | Mean B (SE)            | Mean B (SE)         | Mean B (SE)         | Mean B (SE)            | Mean B (SE)            |
| Overall High                | Overall low      |                          |                        |                          |                        |                     |                     |                        |                        |
| Above average but not identified | 0.01^a          | 0.12 (0.18)              | 0.09^b                 | 0.09 (0.29)              | 0.01^a                 | 0.03 (0.09)         | 0.06^b               | 0.06 (0.22)             | 0.25 (0.20)             |
| Identified but average value | 0.13             | 0.46                     | 0.27 (0.13)           | 0.19 (0.15)              | 0.09^b                 | 0.43 (0.06)         | 0.43 (0.22)         | 0.26 (0.22)             | 0.25 (0.15)             |
| Average                     | 0.09             | 0.04 (0.15)              | 0.03^b                 | 0.19 (0.14)              | 0.25 (0.14)           | 0.28 (0.15)         | 0.26^b               | 0.15 (0.11)             | 0.14 (0.11)             |
| Low identity                | 0.22^a           | 0.09 (0.18)              | 0.02^b                 | 0.25 (0.14)              | 0.18^b                 | 0.02 (0.09)         | 0.15 (0.11)         | 0.15 (0.11)             | 0.14 (0.11)             |
| Overall low                 | 0.01^a           | 0.12 (0.18)              | 0.09^b                 | 0.53 (0.29)              | 0.40^*                 | 0.43 (0.22)         | 0.26 (0.22)         | 0.20 (0.22)             | 0.25 (0.22)             |

Covariates

|                     | Grade 9 standardized math | Socioeconomic status | Intercept | R-square |
|---------------------|---------------------------|----------------------|-----------|----------|
|                     | 0.74***                   | 0.13*                | 0.13 (0.09) | 0.65     |
|                     | (0.04)                    | (0.06)               | 0.46      | 0.69     |

Note. Different letters denote statistical difference within column based on post-hoc pairwise comparison at p < .05 with Bonferroni correction; thus, the regression coefficients that do not have a Bonferroni correction (with Overall High as the reference group) and the post-hoc comparisons with a Bonferroni correction can vary in statistical significance. SE = Standard Error. Grade 9 math achievement and socioeconomic status as controls. Sampling weight, primary sampling unit, strata, and clustered standard error adjusted.

^ p < .05. ** p < .01. *** p < .001.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Longitudinal Study of 2009 (HSLS:09), Base Year and First Follow-Up.
Table 4

Grade 11 Math Motivation Clusters Predicting Math Behavioral Engagement Including Regression Results (B[SE]) and Bonferroni Corrected Post-hoc Pairwise Comparisons (under the Mean).

| Math behavioral engagement | Asian female adolescents | Asian male adolescents | White male adolescents | White female adolescents | Latina adolescents | Latino adolescents | Black female adolescents | Black male adolescents |
|---------------------------|-------------------------|------------------------|------------------------|--------------------------|-------------------|-------------------|-------------------------|------------------------|
|                           | Mean                    | B (SE)                 | Mean                   | B (SE)                   | Mean              | B (SE)            | Mean                    | B (SE)                 |
| Overall high              | 0.68^                   | 0.20^                  | 0.45^                  | 0.70^                    | 0.75^             | 0.51^             | 0.73^                   | 0.34^                  |
| Above average but not identified | -0.44***             | -0.24 (0.13)          | 0.19^                  | -0.25***                 | -0.32***          | 0.48^-             | -0.35**                 | 0.22^-                  |
| Identified but average value | -0.19b               | -0.17 (0.20)          | -0.10f                 | -0.55***                 | 0.07f             | 0.05b             | -0.27b                  | -0.78***                |
| Average                   | -1.08***                | -0.40f                 | -0.84***               | -0.72***                 | -0.77***          | -0.23b            | -1.11***                | -0.65***                |
| Low identity              | -1.61***                | -1.14f                 | -1.59***               | -1.45***                 | -1.13f            | -1.31f            | -1.63***                | -1.50f                  |
| Overall low               | -0.94b                  | -0.10f                 | -0.130***              | -0.76f                   | -0.13f            | -0.90f            | -1.65***                | -1.85***                |

Covariates

| Grade 9 standardized math achievement | 0.04 (0.06) | 0.10 (0.07) | 0.08*** | 0.05** | 0.12* | 0.15** | 0.05 (0.05) | 0.08 (0.07) |
| Socioeconomic status           | 0.09 (0.06) | 0.02 (0.10) | 0.08** | 0.06* | 0.18** | 0.02 | -0.06 | 0.16 (0.10) |
| Intercept                     | 0.68*** | 0.20 | 0.45*** | 0.70*** | 0.75*** | 0.51*** | 0.73*** | 0.34*** |
| R-square                      | 0.21 | 0.14 | 0.22 | 0.20 | 0.28 | 0.25 | 0.22 | 0.26 |

Note. Different letters denote statistical difference within column based on post-hoc pairwise comparison at \( p < .05 \) with Bonferroni correction; thus, the regression coefficients that do not have a Bonferroni correction (with Overall High as the reference group) and the post-hoc comparisons with a Bronferroni correction can vary in statistical significance. SE = Standard Error. Grade 9 math achievement and socioeconomic status as controls. Sampling weight, primary sampling unit, strata, and clustered standard error adjusted.

\* \( p < .05 \), ** \( p < .01 \), *** \( p < .001 \).
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Longitudinal Study of 2009 (HSLS:09), Base Year and First Follow-Up.
comparisons of the four other patterns suggest they did not vary significantly in terms of their behavioral engagement though there are a few patterns worth noting. For example, Latina and Latino adolescents who showed the Overall High or the Above Average but not Identified patterns had similarly high levels of math behavioral engagement that were higher than that of adolescents who showed the other four patterns \( (B = 0.43–1.88 \text{ for Latina adolescents, } 0.43–1.82 \text{ for Latino adolescents; } p < .01) \); behavioral engagement was the next highest for Latina and Latino adolescents in the Identified but Average Value, Average, and Low Identity patterns. Finally, Latina and Latino adolescents who showed the Overall Low pattern had the lowest math engagement compared to all other patterns \( (B = 0.97–1.88 \text{ for Latina adolescents, } 0.71–1.82 \text{ for Latino adolescents, } p < .01) \). Though the patterns were similar by gender for Latinx students, sometimes they varied for other groups. For Asian adolescents for instance, Asian female adolescents in the Overall High pattern had higher engagement than all other patterns \( (B = 0.44–1.61, p < .05) \), which were similar to each other \( (B = 0.18–1.17, ns) \). In contrast, several comparisons were statistically significant for Asian male adolescents; Overall High was the highest in terms of their engagement, followed by Above Average but not Identified but Average Value, followed by Average and Low Identity, and finally Overall Low. Overall, adolescents’ math motivational belief patterns were associated with different levels of math behavioral engagement, and the exact associations varied for each gender by racial/ethnic groups.

7. Discussion

This study was guided by three research goals. Under our first goal, we identified six patterns of adolescents’ math motivational beliefs; some patterns demonstrated Overall High and Overall Low beliefs, whereas other patterns demonstrated different levels across the beliefs, such as the Identified but Average Value pattern. Under our second goal, we found that the demographic representation at the intersection of gender and race/ethnicity across the math motivational patterns captured deeper nuances than gender and race/ethnicity did separately. Under our third and final goal, we found that the patterns of adolescents’ math motivational beliefs were associated with their math achievement and classroom engagement though some of the associations varied within each gender by racial/ethnic group.

7.1. Patterns of Adolescents’ Math Motivational Beliefs

One strength of the current study was the use of a person-centered approach to capture the complexity of multiple motivational beliefs by allowing them to differ in relative levels while maintaining a parsimonious presentation. We identified six unique patterns of adolescents’ math motivational beliefs. The Overall High and Overall Low patterns emerged as expected and aligned with the situated expectancy-value theory such that adolescents in those two patterns had the highest and lowest math outcomes respectively \( (Ecce, 2009; Nagengast et al., 2011) \).

Extending prior studies, we identified additional nuanced, theoretically aligned patterns of adolescents’ motivational beliefs that evidenced within-person differences in the patterns of endorsements of the various beliefs. For example, students in the Above Average but not Identified group felt they had strong math skills and also agreed more often than not that math is interesting and important, but being a math person was not central to how they or others saw them. This pattern could speak to the body of literature on math identity, for example how even highly competent adolescents may still not identify as math people in order to preserve their other social identities (e.g., McGee, 2013), and how social structure and interactions foster math identity more easily for certain groups of people than others \( (e.g., Shanahan, 2009) \).

7.2. Intersectional Perspective

Intersectionality is a fitting framework to examine psychological processes such as motivational beliefs where multiple aspects of a person do not function in isolation. We found that gender differences in adolescents’ math motivation patterns and the correlates of those patterns did not hold across racial/ethnic groups. For instance, male adolescents’ under- or over-representation in math motivational patterns were largely driven by White and Asian male adolescents. Among female adolescents, the patterns in different racial/ethnic groups often conflicted with each other such that female adolescents from one racial/ethnic group were overrepresented and female adolescents from another racial/ethnic group were underrepresented. These patterns suggest that although gender stereotypes appear to marginalize White females in math, there are additional variations explained by race/ethnicity that influence female adolescents’ math motivational belief patterns.

By intersecting gender and race/ethnicity, we also showed that examining only at the level of race/ethnicity would often overlook how female and male adolescents of the same race/ethnicity showed different patterns of math motivational beliefs. For example, Black and Latina/o students were overrepresented in the Low Identity pattern, but that was actually only the case for female but not male adolescents. Although being Black and Latina/o is stereotypically marginalized in STEM, our results suggest that there is also a substantial power dynamic on the basis of gender within these two groups. Black female and Latina adolescents were more likely than chance to show the Low Identity pattern, but this overrepresentation was not evident for their same-race/ethnicity male peers (i.e., Black male and Latino adolescents), whose race/ethnicity stereotypically marginalizes but gender favors them in math. In contrast, the lack of gender differences among Asian American adolescents opposed our expectation that Asian American female adolescents would have lower motivational beliefs than Asian American male adolescents as they were hypothesized to benefit from stereotype boost in terms of both ethnicity and gender \( (Armenta, 2010) \). A possible explanation is that the cultural norm for Asian Americans to thrive in math, which is a defining feature of the model minority label \( (Trytten et al., 2012) \), might be so strong that it renders gender relatively less influential at least in terms of their math motivational beliefs. That is, for Asian American female adolescents, the stereotype for them to excel in math because they are Asian American might have more strongly influenced their math motivational beliefs than the stereotype of them being female adolescents \( (Gibson et al., 2014) \). Overall, our results support the essence of the intersectionality framework such that the intersection of gender and race provides a richer description than gender or race/ethnicity alone \( (Crenshaw, 1989; Harris & Patton, 2019; Parker et al., 2020) \). In fact, we show that failure to do so misrepresents certain groups.

The importance of an intersectional perspective was also evident when we examined the associations between the math motivational belief patterns and adolescents’ math achievement and engagement. For most groups, adolescents in the Overall High pattern had the highest achievement and engagement whereas adolescents in the Overall Low pattern had the lowest, which supports situated expectancy-value theory \( (Ecce, 2009; Nagengast et al., 2011) \). Our results suggest that the associations outside of these two trends did not hold uniformly for all eight gender by racial/ethnic groups. For example, adolescents’ Grade 11 math achievement significantly varied across the motivational patterns for four groups: (a) White and Latina/o adolescents (b) White male and Latino adolescents, and (c) Asian, Black, and Latina female adolescents and Black male adolescents. Why did math motivational belief patterns explain significant variations in math achievement largely for male adolescent groups but not for three of the four female adolescent groups? It is possible, for example, that female adolescents experience overpowering or dominating pressure such as gender stereotype threat \( (Starr & Simpkins, 2021) \) that limits the extent to which their motivational beliefs relate to their math.
achievement. Our findings might also be a manifestation of the trend that men’s math ability tends to exhibit greater variance than women’s (Lindberg, Hyde, Petersen, & Linn, 2010), which means there might be fewer differences in female than male adolescents’ math achievement to be explained by any factor, including math motivational belief patterns.

Another example of the nuances in the relations is that adolescents in the Overall Low pattern had lower behavioral engagement in math class than adolescents in the Average and Low Identity patterns, with the exception of Asian female and male adolescents. Moreover, the only difference among Asian female adolescents was that the Overall High pattern had the highest engagement compared to all the other patterns. It is possible that math behavioral engagement can be better explained by factors other than motivational belief patterns for Asian adolescents, such as the expectations and stereotypes from being a model minority in math (Trytten et al., 2012). Although being in the Overall Low pattern for other gender and racial/ethnic groups was associated with relatively low math behavioral engagement, perhaps a significant proportion of Asian adolescents who showed patterns like Overall Low nonetheless showed high engagement which contributed to greater variation within Asian adolescents who showed the same pattern and thus less distinguishable from those of other patterns. Future studies could examine the mediating pathways between the math motivational belief patterns and outcomes within each gender by racial/ethnic groups to better understand why the associations did not hold uniformly across groups. Overall, our nuanced results regarding the associations between math motivational belief patterns and outcomes extended our understanding beyond results from variable-centered studies and again highlighted the importance of intersecting gender and race/ethnicity.

### 7.3. Implications

Our findings offer several implications for educational professionals in terms of both practice and research. Regarding practice, the person-centered patterns of math motivational beliefs suggest that isolating dimensions of motivational beliefs may be insufficient. It is not enough to just focus on promoting one dimension of motivation now that there is accumulating evidence that students’ math outcomes depend on where individuals simultaneously stand on multiple dimensions of their motivational beliefs (e.g., Durik et al., 2015). Moreover, our findings provide support to move beyond ‘one size fits all’ interventions to tailor intervention components based on the current pattern of students’ motivational beliefs. For example, adolescents showing the Overall Low pattern might benefit most from interventions that comprehensively promote the various aspects of math motivational beliefs or interventions focused on better math instruction so that they can experience more success. In contrast, adolescents showing the Above Average but not Identified pattern might benefit from interventions that target attainment value. Much like the call for personalized medicine in the field of health care, our findings provided empirical support for educational interventions to move beyond changing specific math motivational beliefs one at a time, but rather to more holistically examine the patterns of multiple motivational beliefs simultaneously for each individual. Similarly, our finding that pattern membership varied based on the intersection of race/ethnicity and gender suggest that motivational interventions based on only one of these characteristics, such as gender, will likely be insufficient because the intersection of these characteristics matter for the patterns of adolescents’ motivation and their correlates.

Regarding research, our findings suggest that researchers need to use analytic techniques compatible with addressing the interrelatedness of multiple other motivational beliefs. If data for more than two motivational beliefs are available, a person-centered approach can be more succinct than a variable-centered approach that requires multiplicative interaction terms. Additionally, our finding that the intersection of gender and race provided a richer description than gender or race/ethnicity alone in explicating the membership and associated outcomes of math motivational belief patterns offers a clear implication for educational researchers to examine gender in combination with race/ethnicity instead of using them as separate independent variables. This implication is particularly important for research on STEM motivation as the intersection of gender and race/ethnicity is pointed out as a critical yet understudied topic (Parker et al., 2020). Overall, our approach speaks to the call for more studies to focus on within-group variability to complement the existing research on between-group comparisons (Causadias et al., 2018). For example, although studies on the female underrepresentation in math are helpful at a broad level, they might perpetuate a deficit narrative that treats all female individuals as marginalized. Our findings highlighted that, yes, some female adolescents are underrepresented in the strong motivational patterns, but other female adolescents are overrepresented in those patterns depending on their race/ethnicity. Similarly, the association between math motivational belief patterns and outcomes looked differently among female adolescents depending on their race/ethnicity.

### 7.4. Limitations and future directions

The current study incorporated four math motivational beliefs that, according to expectancy-value theory, should promote individuals’ pursuit of and achievement in math (Eccles & Wigfield, 2020). Future studies could also incorporate motivational beliefs theorized to deter individuals from a domain, such as cost (Flake et al., 2015; Rosenzweig et al., 2019). Furthermore, we focused on the within-domain interrelations between multiple motivational beliefs which addresses critical aspects of within-domain processes of situated expectancy-value theory (Eccles & Wigfield, 2020); the theory also addresses cross-domain processes. As a result, future studies could incorporate motivational beliefs from multiple domains, such as math and English or science, in order to account for people making decisions across multiple domains (e.g., Snodgrass Rangel et al., 2020; Umarji et al., 2018).

The current study took advantage of HSLS’s longitudinal nature by controlling for prior math adjustment. Another way to leverage the longitudinal nature of these data is to examine the change in the patterns of adolescents’ motivational beliefs. Change in patterns could identify changes in the levels of math motivation over time similar to the findings from growth curves, but the interrelations among the unique dimensions might also shift over time, which cannot be discerned from growth curves. Additionally, one could examine whether changes in individuals’ math motivation patterns predict math outcomes above and beyond individuals’ math motivation patterns at any particular time point (e.g., Petersen & Hyde, 2017; Musu-Gillette et al., 2015). For example, two students who have overall low motivational beliefs might have variant outcomes if one student was always low over time compared to a student who dropped from high to low over time. It is also possible that students’ motivation might dramatically change if they encounter a challenging math course or fail a class. Thus, it will be important to consider not only changes by grade level but also more dynamic changes based on the types of math courses and their experiences in those math courses. From a developmental perspective, the period covered by HSLS (i.e., adolescence) is critical, but only one period in the STEM pipeline (e.g., Sadler et al., 2012; Wigfield and Eccles, 2000). Situated expectancy-value theory, in fact, emphasizes stage-environment fit (Eccles & Wigfield, 2020), so it would be worth examining the patterns of math motivational beliefs during different developmental stages (e.g., earlier in development when identity formation is not as mature).

Another note regarding the study design is temporal precedence, specifically, the fact that both the predictor (motivational beliefs) and outcome variables (achievement and engagement) were measured in Grade 11. This choice was intentional considering that when participants responded to the math motivational beliefs items, they were most likely thinking about the math they were immersed in. Thus, it is more meaningful to look at how those motivational beliefs predict students’ engagement in their corresponding class, instead of a class two years earlier.
8. Conclusion

The current study takes a step further into the nuances of adolescents’ math motivation by identifying six prevalent patterns of math motivational beliefs. These patterns differed not only in terms of their gender by racial/ethnic representation but also their associated math achievement and behavioral engagement. Our results suggest that examining the interrelations among multiple math motivational beliefs is a meaningful analytical and empirical contribution. Our findings also underscored the importance of intersectionality between gender and race/ethnicity in math.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found在线上at https://doi.org/10.1016/j.cedpsych.2021.101974.

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