Proton lifetime, Yukawa couplings and dynamical SUSY breaking in SU(5) GUT

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Abstract

We study the influence of messenger Yukawa couplings and top, bottom and \( \tau \) Yukawa couplings on the proton lifetime in SU(5) Supersymmetric GUT with dynamical supersymmetry breaking mechanism due to Dine and Nelson.
1 Introduction.

Recently, there was a great increase of interest in the study of the physics beyond the standard model, in particularly in the study of the supersymmetric grand unification theories (SUSY GUTs). This interest was in particular stimulated by the new data due to LEP1 and LEP1.5 experiments. If one believes literally in all this data, especially in the $R_b$ measurements, one comes to the conclusion that neither the standard model (SM) nor the minimal supersymmetric standard model (MSSM) are sufficient to explain the experimental results \[1, 2, 3\]. One naturally has to turn to the other models. One of the most popular directions in the search for the new physics is the study of SUSY GUTs.

Unfortunately, one immediately encounters the problem of how to break supersymmetry (SUSY). The most popular way is to use a hidden sector through which SUSY breaking is transferred to the visible world by means of supergravity ( See e.g. ref.\[4\] for a review). Recently a new approach to break SUSY was developed\[5, 6, 7\]. In this approach SUSY is broken dynamically and the effects of SUSY breaking are transferred from hidden sector to the visible world by means of gauge interactions. The natural question is whether one can use this mechanism to construct realistic SUSY GUT theories \[8\].

There are several tools one can use at present to decide whether the GUT theory is realistic or not. One of the most powerful among them is the experimental limit on the proton lifetime. It was shown in ref.\[8\] that simple extension of minimal SU(5) SUSY GUT with the mechanism of dynamical SUSY breaking due to refs.\[5, 6, 7\] leads to the too small lifetime of proton. Consequently, this model is ruled out.

The purpose of this note is to return once again to the analysis of the proton lifetime in the minimal SU(5) SUSY GUT with dynamical supersymmetry breaking due to Dine and Nelson , and to include the Yukawa couplings in the analysis of ref. \[8\]. We study the influence of Yukawa couplings, both of the 3rd generation and of the hidden sector, on the proton lifetime and masses of the color triplet Higgs boson $M_{H_c}$. We shall see that the inclusion of Yukawa couplings increases the proton lifetime but not significantly. However, the model cannot be ruled out at $2\sigma$ level. The model is not ruled out in a small window
at small tan $\beta$ if one takes into account the uncertainties in the LEP data for $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ – QED and weak coupling constants at $m_Z$ scale.

The note is organised in the following way: First we use renormalization group (RG) equations for gauge couplings taking Yukawa couplings into account. We then find the masses of color triplet bosons $M_{H_c}$, depending on $\alpha_3(m_Z)$ (QCD coupling) for different values of $\alpha_1, \alpha_2$. We then use the formula for proton lifetime from ref.[12] to find the proton lifetime for different allowed by LEP data values of $\alpha_1(m_Z)$ and $\alpha_2(m_Z), \alpha_3(m_Z)$. Our final results are depicted in figs.1-6, thus giving the bounds on the possibility of using mechanism of refs.[5, 6, 7] to break SUSY in SU(5) GUT model.

2 Calculation.

Let us briefly describe the model we are going to study (we refer to the reader to refs.[5, 6, 7, 8] for details).

The model is the minimal SU(5) SUSY GUT plus the messenger superpotential

$$\Delta W_m = \lambda_L SLL + \lambda_D SDD.$$  \hspace{1cm} (1)

Here L and D, that carry quantum numbers of SM gauge groups and form together into $5 + \bar{5}$ of SU(5), are the messenger lepton and quark chiral superfields added to the model. S is a scalar chiral superfield singlet under the SM gauge groups. S transfers the supersymmetry breaking from hidden sector to the fields of the standard model. At the messenger scale $\Lambda_m \sim 100$ TeV the singlet S and its F-component get vacuum expectation values and SUSY breaks down. The heavy superfields L and D transfer the information of SUSY breaking to the fields of SM through their couplings to the SM gauge groups. The analysis of proton decays in this note is similar to the work by Carone and Murayama[8]. However, these authors did not take into account the influence of the Yukawa couplings on the evolution of gauge couplings. We use below the RG equations for Yukawa couplings that can be derived using ref.[9, 10]. First, we evolve the RG equations from $m_Z$ to $m_{top}$ using the SM $\beta$-functions and then from $m_{top}$ to $\Lambda_m$ (taken as 100 TeV) using the MSSM ones. Above $\Lambda_m$ we take into
account the effects of the messenger fields:

\[
\begin{align*}
\frac{d g_1}{d \mu} &= \frac{1}{16\pi^2} \frac{38}{5} g_1^3 + \left( \frac{1}{16\pi^2} \right)^2 g_1^3 \left( \frac{632}{75} g_1^2 + \frac{36}{5} g_2^2 + \frac{296}{15} g_3^2 - \frac{4}{5} Y_D^2 - \frac{6}{5} Y_L^2 - \frac{26}{5} Y_t^2 - \frac{14}{5} Y_b^2 - \frac{18}{5} Y_\tau^2 \right) \\
\frac{d g_2}{d \mu} &= \frac{1}{16\pi^2} 2 g_2^3 + \left( \frac{1}{16\pi^2} \right)^2 g_2^3 \left( \frac{12}{5} g_1^2 + 32 g_2^2 + 24 g_3^2 - 2 Y_L^2 - 6 Y_t^2 - 6 Y_b^2 - 2 Y_\tau^2 \right) \\
\frac{d g_3}{d \mu} &= -\frac{1}{16\pi^2} 2 g_3^3 + \left( \frac{1}{16\pi^2} \right)^2 g_3^3 \left( \frac{37}{15} g_1^2 + 9 g_2^2 + \frac{76}{3} g_3^2 - 2 Y_D^2 - 4 Y_t^2 - 4 Y_b^2 \right)
\end{align*}
\]

The evolution of Yukawa couplings themselves are taken into account in one loop approximation and are carried out using the following RG equations (for scales larger than \( \Lambda_m \)):

\[
\begin{align*}
\mu \frac{d Y_D}{d \mu} &= \frac{Y_D}{16\pi^2} \left( -\frac{4}{15} g_1^2 - \frac{16}{3} g_3^2 + 5 Y_D^2 + 2 Y_L^2 \right) \\
\mu \frac{d Y_L}{d \mu} &= \frac{Y_L}{16\pi^2} \left( -\frac{6}{5} g_1^2 - 6 g_2^2 + 6 Y_D^2 + 8 Y_L^2 \right) \\
\mu \frac{d Y_t}{d \mu} &= \frac{Y_t}{16\pi^2} \left( -\frac{13}{15} g_1^2 - 3 g_2^2 - \frac{16}{3} g_3^2 + 6 Y_t^2 + Y_b^2 \right) \\
\mu \frac{d Y_b}{d \mu} &= \frac{Y_b}{16\pi^2} \left( -\frac{7}{15} g_1^2 - 3 g_2^2 - \frac{16}{3} g_3^2 + Y_t^2 + 6 Y_b^2 + Y_\tau^2 \right) \\
\mu \frac{d Y_\tau}{d \mu} &= \frac{Y_\tau}{16\pi^2} \left( -\frac{9}{5} g_1^2 - 3 g_2^2 + 3 Y_b^2 + 4 Y_\tau^2 \right)
\end{align*}
\]

Our notations are self-evident: \( Y_t, Y_b \) and \( Y_\tau \) are Yukawa couplings of top, bottom, and \( \tau \) to the corresponding Higgs doublets in the minimal SU(5) SUSY model. \( Y_D \) and \( Y_L \) are the messenger Yukawa coupling constants to the singlet S. We proceed then in the same way as in ref. [8]. We use the evolution of \( \alpha_1 \) and \( \alpha_2 \) to determine the GUT scale \( M_{GUT} \) where \( \alpha_1 \) and \( \alpha_2 \) unify into \( \alpha_5 \), and then get the mismatch between \( \alpha_5(M_{GUT}) \) and \( \alpha_3(M_{GUT}) \), which should be attributed to the GUT threshold effects, that are assumed to originate from the color triplet \( H_c \). We get the mass of the color-triplet Higgs \( M_{H_c} \) from its dependence on the threshold effects (the detailed formulae can be found in [8]). Once \( M_{H_c} \) is determined, we use the standard formulae for the proton lifetime(see ref. [9]) to find the partial width and to compare them with the current experimental bounds.

In our numerical evolution, we use the following boundary conditions. First, we made a standard choice of the masses of b-quark, \( \tau \)-lepton and the top-quark as: \( m_b = 4.5 \text{ GeV} \), \( m_{\tau} = 1.78 \text{ GeV} \) and \( m_t = 175 \text{ GeV} \). We do not depict uncertainties in these boundary conditions due to masses, since their influence on the results is not large. For the messenger
Yukawa couplings we take the largest possible values:

\[ Y_D(\mu = \Lambda_m) = 0.9, \]
\[ Y_L(\mu = \Lambda_m) = 0.452. \]  \hspace{1cm} (4)

If we choose the boundary conditions to be given by eq. (4), \( Y_D \) and \( Y_L \) unify into a common value at the GUT scale. If inputs larger than those in (4) is taken, the messenger Yukawa couplings will blow up at the GUT scale. The boundary conditions for gauge couplings are determined from LEP data[11]:

\[ \alpha_1^{-1}(m_Z) = 58.96 \pm 0.05, \]
\[ \alpha_2^{-1}(m_Z) = 29.63 \pm 0.05. \] \hspace{1cm} (5)

For the QCD coupling we use \( \alpha_3(m_Z) = 0.116 \pm 0.005 \) \([11, 8]\) — the number that incorporates both low energy data and LEP1 data. All the gauge couplings are taken within \( 2\sigma \) variations in our estimations.

3 The Results.

Our main results are depicted in Figs. 1-6. First, we depict the mass of the color-triplet, \( M_{H_C} \), as a function of \( \alpha_3(m_Z) \) for different \( \tan \beta \). These results are depicted in Fig. 1-3 for different values of \( \alpha_1(m_Z) \) and \( \alpha_2(m_Z) \). We take the lowest bound of \( \tan \beta \) as 0.85. Below this bound the Yukawa couplings will blow up at the GUT scale. This lowest bound can be achieved only when the Yukawa couplings are included in the RG analyses. We see that for reasonable values of \( \tan \beta \), Yukawa couplings of messenger quarks and leptons actually do not influence \( M_{H_C} \) greatly. However, when we consider the different values of \( \alpha_1 \) and \( \alpha_2 \) within \( 2\sigma \) variations, we see that \( M_{H_C} \) can change quite significantly as a function of \( \alpha_1(m_Z) \) and \( \alpha_2(m_Z) \). The most favored \( M_{H_C} \) is gained when \( \alpha_1(m_Z) \) is taken at its upper bound while \( \alpha_2(m_Z) \) the lower one.

Next, we consider the decay rate of a typical process \( n \rightarrow K^0\bar{\nu} \). This mode was argued in ref. \([8]\) to be the most appropriate one in analysing proton decay bound in SUSY GUT.
The results are shown in Figs. 4-6. We see that the allowed region of $\alpha_3$ and tan $\beta$ depends on the inputs $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$. Inclusion of the usual Yukawa couplings in RG equations improves the situation with the proton lifetime. On the other hand, inclusion of the messenger Yukawa couplings even worses the situation slightly. As the current experimental bound\cite{13} of proton decay lifetime is concerned, the only window where the model survives is the region of small tan $\beta$ combined with large $\alpha_1(m_Z)$ and small $\alpha_2(m_Z)$ as input.

Our general conclusions are that, first, the inclusion of Yukawa couplings does not lead to significant changes in the mass of color triplet and the allowed parameter space for the model we consider. Second, the allowed parameter space, color triplet mass and proton lifetime seem to be quite sensitive to exact values of $\alpha_1$ and $\alpha_2$ at $M_Z$ scale.

**Acknowledgments**

The authors thank M. Shifman for useful discussion.

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**Figure Captions**

Fig. 1 The dependence of the color triplet boson mass on tan $\beta$ and $\alpha_3(m_Z)$, when $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ are both taken as central values of experiment results. The dotted and the dash-dotted lines correspond to the results without any Yukawa couplings and with only messenger Yukawa couplings, respectively. The dashed and solid lines correspond to results with all Yukawa couplings included for tan $\beta = 0.85$ and tan $\beta = 50$, respectively.

Fig. 2 Same as Fig.1 except that both $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ are taken 2$\sigma$ smaller than the experimental central values.

Fig. 3 Same as Fig.1 except that $\alpha_1(m_Z)$ is taken 2$\sigma$ larger and $\alpha_2(m_Z)$ is taken 2$\sigma$ smaller than the experimental central values.

Fig. 4 $\alpha_3(m_Z)$ and tan $\beta$ plane excluded by the proton decay constraint, when $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ are both taken their central values. The dash-dotted, dashed and solid lines correspond to the cases without Yukawa couplings, with only messenger Yukawa couplings, and with all Yukawa couplings included, respectively.

Fig. 5 Same as Fig.4 except that both of $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ are taken 2$\sigma$ smaller than their central values.

Fig. 6 Same as Fig.4 except that $\alpha_1(m_Z)$ is taken 2$\sigma$ larger and $\alpha_2(m_Z)$ is taken 2$\sigma$ smaller than their central values.
Fig. 4
