Simple method of modelling of digital holograms registering and their optical reconstruction

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Abstract. The technique of modeling of digital hologram recording and image optical reconstruction from these holograms is described. The method takes into account characteristics of the object, digital camera’s photosensor and spatial light modulator used for digital holograms displaying. Using the technique, equipment can be chosen for experiments for obtaining good reconstruction quality and/or holograms diffraction efficiency. Numerical experiments were conducted.

1. Introduction
Reconstruction quality of images and the diffraction efficiency (DE) are the major characteristics in classical (analog) holography [1-3]. Problem of estimation of DE and quality of reconstructed from analog holograms images were in detail described for various recording media. In digital holography [4-6], digital camera’s photosensor can be considered as "recording medium".

There are two types of reconstruction of images from digital holograms: numerical [4-11] and optical [11-19] ones. In the case of numerical reconstruction, only reconstruction images quality is important and DE makes only speculative sense. However in case of optical reconstruction, DE has practical meaning. The most popular method of optical image reconstruction is displaying of the holograms on spatial light modulator (SLM) [11-19]. Optical reconstruction of images from holograms is used for practical applications in remote display of the static and dynamic 2D- and 3D-scenes including recorded in non-visible radiation [12], for projective systems [13-14], 3-dimensional vision, etc.

The problem of estimation of DE and quality of the reconstructed images can be solved by numerical modeling of process of recording of digital holograms and optical reconstructed of images from holograms. In this paper simple method of modeling of digital holograms registering and their optical reconstruction is described.
2. Description of the method of modeling of digital holograms registering and their optical reconstruction

The used scheme for numerical experiments on modeling of recording of digital holograms and optical reconstruction of images consists of 4 stages [20]:

1. creation of a model object,
2. modeling of hologram recording (modeling of wave propagation from the object plane to the hologram plane, synthesis of the hologram),
3. quantization of the hologram and modeling of displaying hologram to SLM,
4. modeling of optical reconstruction of the image (modeling of wave propagation from the SLM plane to the plane of the focused reconstructed image).

The technique considers the following values:

- parameters of the used object:
- characteristics of the digital camera, used for displaying digital holograms:
  - resolution,
  - spatial and temporal noises,
  - radiometric parameters (dynamic range, maximum linear signal).
- characteristics of the SLM, used for displaying digital holograms:
  - resolution,
  - quantity of gradations,
  - SLM type (phase or amplitude).

Distribution of complex amplitudes of the object wave $B(u, v, z)$ in the plane of registration of the hologram by the photosensor of the digital camera located at $z$ distance from the plane of object is calculated by direct calculation of Fresnel diffraction method [4]:

$$B(u, v, z) = \frac{\exp(ikz)}{i\lambda z} \cdot \exp \left( \frac{i\pi(u^2 + v^2)}{\lambda z} \right) \cdot \text{FFT} \left\{ A(x, y, 0) \cdot \exp \left( \frac{i\pi(x^2 + y^2)}{\lambda z} \right) \right\}.$$

where

- FFT( ) — fast Fourier transform,
- $k = \frac{2\pi}{\lambda}$ — wave number,
- $i$ — imaginary unit,
- $\lambda$ — wavelength,
- $(x, y)$ are discrete coordinates in the object plane.

Then intensity distribution of the registered hologram $H(u, v)$ is calculated:

$$H(u, v) = |B(u, v, z) + C|^2$$

$$C = \sqrt{p \cdot \langle|B(u, v, z)|^2\rangle}$$

where

- $\langle * \rangle$ — average value of the intensities of the object beam in hologram plane,
- $C$ is amplitude value of each pixel of the reference beam in the hologram plane,
- $p$ is ratio of average intensities of reference and object beams.
Then the values of the hologram pixels are quantized:

\[ H_Q(u,v) = \begin{cases} 
Q - 1, & \text{if } H(u,v) = \max\{H(u,v)\} \\
\text{fix}\left(\frac{H(u,v)}{\max\{H(u,v)\}} \times Q\right), & \text{in other cases}
\end{cases} \]  \hspace{1cm} (4)

where

- \( H_Q(u,v) \) – array of elements of the quantized hologram,
- \( \text{max}\{\ast\} \) – maximum value of intensity of pixels of the hologram,
- \( Q \) – number of gradations of the hologram (2 or 256),
- \( \text{fix}\{\ast\} \) – function of rejection of fractional part.

Then we modeled the displaying of the holograms on SLM. We should consider amplitude \((H_{\text{AMP}}^Q(u,v))\) and phase \((H_{\text{PH}}^Q(u,v))\) SLM separately:

\[ H_{\text{AMP}}^Q(u,v) = H_Q(u,v) \cdot \exp(i \cdot 0) \]  \hspace{1cm} (5)

\[ H_{\text{PH}}^Q(u,v) = 1 \cdot \exp\left(i \left[ H_Q(u,v) \cdot 2\pi \left(\frac{Q - 1}{Q}\right)\right]\right) \]  \hspace{1cm} (6)

Final step is modeling of the optical reconstruction of the digital holograms displayed on the amplitude and phase SLM by the direct calculation of Fresnel diffraction method:

\[ D_{\text{AMP}}^{Q}(x', y', 2z) = \frac{\exp(ikz)}{i\lambda z} \cdot \exp\left(-\frac{i\pi(x'^2 + y'^2)}{\lambda z}\right) \cdot \text{FFT}\left\{ H_{\text{AMP}}^Q(u,v) \cdot \exp\left(\frac{i\pi(u^2 + v^2)}{\lambda z}\right)\right\} \]  \hspace{1cm} (7)

\[ D_{\text{PH}}^{Q}(x', y', 2z) = \frac{\exp(ikz)}{i\lambda z} \cdot \exp\left(-\frac{i\pi(x'^2 + y'^2)}{\lambda z}\right) \cdot \text{FFT}\left\{ H_{\text{PH}}^Q(u,v) \cdot \exp\left(\frac{i\pi(u^2 + v^2)}{\lambda z}\right)\right\} \]  \hspace{1cm} (8)

where

- \((x', y')\) are discrete coordinates in the reconstruction plane.

3. Numerical application of the method of modeling of digital holograms registering and their optical reconstruction

Different amplitude-phase two-dimensional objects were as the test objects. Examples of intensity distribution of the initial objects are shown in Fig. 1:

- standard grayscale image with size of 256 \times 256 pixels (Fig. 1a),
- binary contour image with size of 128 \times 128 pixels (Fig. 1b).

Amplitude values of pixels of object were equal to the square root of the intensity. Phase values of the object pixels had uniform distribution between 0 and 2\pi.
Figure 1. Examples of objects used in numerical experiments.

Fragments of synthesized digital hologram with 2 gradations (binary hologram) and with 256 gradations are shown in Fig. 2a,b correspondingly. Fragments have 128×128 pixels.

Figure 2. Fragments of synthesized digital hologram with 2 gradations (a) and with 256 gradations (b).

After digital holograms synthesis, we simulated optical reconstruction of images from the holograms using spatial light modulators. Two basic types of such modulators were considered: phase and amplitude SLMs [21-23]. Quantity of amplitude levels (in amplitude SLM) and phase levels (in phase SLM) are equal to 2 and 256.

Results of modeling of optical reconstruction of images object are shown in Fig. 3,4. Reconstruction of images using amplitude SLM are shown in Fig. 3a, 4a (binary digital hologram with 2 gray levels) and Fig. 3b, 4b (digital hologram with 256 gray levels). Reconstruction using phase SLM are shown in Fig. 3c, 4c (halftone image with uniform histogram) and Fig. 3d, 4d (binary contour image).

As can be seen, the reconstructed images from the amplitude SLM with 256 levels has a significantly better quality than that from the phase modulator. This is due to the fact that digital hologram is intensity that was registered by digital camera. In result amplitude SLM should provide better quality of reconstruction that phase one.
**Figure 3.** Reconstructed images of the halftone object in the cases of simulation of the amplitude (a, c) and phase (b, d) spatial light modulators. Modulators (holograms) with 256 (a and b) or 2 (c and d) levels are considered.

**Figure 4.** Reconstructed images of the binary object in the cases of simulation of the amplitude (a, c) and phase (b, d) spatial light modulators. Modulators (holograms) with 256 (a and b) or 2 (c and d) levels are considered.
4. Conclusion
A simple technique of digital holograms registering and their optical reconstruction was described. This technique makes possible to select possible conditions of experiments, parameters of the digital camera and the objects, the SLM type, and etc. This technique can be useful for optical information processing, for registration and displaying of volume scenes in real time.

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