Developing a Mathematical Model for Order Quantity Optimization by Considering Discount

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Abstract:
This work is aimed at presenting a mathematical model for determining the optimal order in-extenuating conditions (All Unit Discount, Incremental Discount). The expected results are producing several kinds of multi-supplier and multi-time periods with regards to inventories, demand, suppliers’ capacity, storage space, and budget constraints in order. It is desirable to minimize the purchasing costs, storage and ordering while considering the lack of needed materials at specified times. To confirm the efficiency of the proposed model in solving purchase planning problems in real world, it has been applied in Asian Oil Turbo Compressor Design Corporation and received approval from the obtained results and experts’ comments. To solve the designed model, the GAMS software is used. Also, model sensitivity analysis and usable results in management decision making based on the importance of proposed matters (shortage reduction in each period and demand increase effect on final cost) are then presented.

Keywords: Optimizing Order Quantity, Discount, Mathematical Modeling, Inventory

Introduction:
The direct and indirect impacts of inaccurate decision makings regarding the choice of suppliers and the order quantity are observed more strongly and vitally with the ever increasing dependence of organizations on their suppliers. On the other hand, making the right decision in this regard and strategic partnership with the suppliers with better performance can lead to reduced total production and delivery time, increased production, enhanced quality, reduced purchase and quality costs, reduced waste and continuous improvement and thus increased customer satisfaction [1]. Raw materials make up approximately 70% of production costs in manufacturing industries. Therefore, purchase process plays a significant part in the reduction of costs. The main objective of this paper is the development of an appropriate model to determine the optimal order quantity and minimize the shortage, total material purchase, storage and ordering costs considering the terms of the whole unit and incremental discounts.

In most companies and manufacturing industries, the ordering system is such that several materials are ordered to different suppliers in different periods and in multiple packages. The suppliers may offer special discounts for their products including all unit discounts for some goods and incremental discounts for others. In addition, companies may not well use discount terms due to such constraints as low budget. Therefore, all the above items can be included as constraints in the model.
Much research has been carried out in this field. Kim et al. have adopted the two items of all unit discount and credit sale in their model [2]. This model has been investigated under five different conditions including determination of customer price, buyer’s order quantity, accumulated production size, all unit discount and credit sale for suppliers. Manerba and Mansini have analyzed the purchase problem in the situation where suppliers offer the all unit discount [3]. Their objective was the selection of a collection of suppliers to meet their requirements at minimum cost. Due to the NP-hardness of the problem, the hybrid algorithm model has been used to solve it. Considering such factors as accumulated size for several suppliers, multi-period ordering and all unit discounts, Amy et al. have developed a mixed, integrated planning model to minimize total expenses such as ordering, storage, purchase and transportation costs and used the genetic algorithm to solve it [4]. The basic condition in this model is the storage of an acceptable level of the inventory. Esfandiar and Seifbarghy have proposed a multi-objective model and considered factors such as minimization of purchase cost, returned products and delays in delivery to evaluate the corresponding suppliers [5]. The objective of the model was cost reduction in the supply chain level and obtaining the most ideal ordering times and part delivery. The program used in this work to solve the problem has been written in the Visual Basic language. Paydar et al. have developed a dual objective mathematical model with integrated approach to determine the flow of the products throughout the dynamic supply chain in the production, distribution and market levels. The objective of the proposed model was the minimization of the purchase, transportation and storage costs on the one hand and maximization of the purchase of higher quality products on the other. Ultimately, in order to evaluate the proposed model, Epsilon limit method was used as an optimization approach for solving multi-objective problems of an integrated example from a case study in a military organization. Mafakheri et al. have proposed a two-stage, dynamic planning for the supply chain management [6]. In the first stage, the suppliers are ranked using AHP method and the order assignment model has been designed and implemented to maximize the desirability and minimize the total price with demand, capacity and inventory level constraints. Another supply network is reverse logistic that acts pretty much like a supply chain in which the seller and customer have been exchanged. Jafarzadeh et. al. [7] design such a model and solve it by GAMS as a MILP for small size problem and Genetic Algorithms for big problems and compare the results. Kamali et al. have proposed a mixed integer, non-linear planning model for coordination of single and multi-seller systems under the condition of discounting all the parts. The objectives of this research included the optimization of cost, quality, on time delivery and annual production quantity [8]. Given the model complexity, two metaheuristic algorithms were used to solve it. Tsai and Wang have used a mixed integer planning approach to solve the problem and assign the order for a multi-source and multi-product case in the supply chain [9]. The objective of this model was minimization of cost, returned products and the number of parts delivered with delays and the metaheuristic algorithm has been used to solve the problem.

Size of the supply chain models can be problematic most of the time and make it unsolvable by conventional optimization methods. Nasab et. al. [10] presented a variable and constraint reduction method called Bender’s Decomposition to deal with a 5-layer petroleum supply chain model, and they applied it into a case study. Mohammad Ebrahim et al. have proposed a model for different types of discounts including all unit, incremental and purchase volume discounts by the multi-objective formulation for single product purchase [11]. The supplier’s capacity and demand constraints have been considered in this model. Scatter search algorithm has been proposed for solving this problem given its complexity. Amid and Ghodsypour have proposed a multi-objective, weighty model in a fuzzy environment for assigning purchase orders to the suppliers in the supply chain [12]. This model is a multi-objective problem in which the objectives conflict with each other and are of different significance and preference depending on the purchase situation. Considering the uncertainty and vagueness of the information corresponding to the constraints and parameters of the problem, fuzzy technique has been used to eliminate the vagueness and uncertainty of the information. In this problem, the three objective functions of total cost minimization, quality maximization and
optimization of the service level of all purchased materials have been used. Burke et al. have investigated the effects of different types of pricing models using different discount models in a supplier selection problem when the buyer intends to buy only one product [13]. They have proposed non-linear mathematical models for each discount type. Haksever and Moussourakis have developed a mixed integer, planning model for ordering products in multi-constraint and multi-product inventory systems in which the suppliers offer incremental discount for pricing [14]. The model has been obtained based on a linear approximation of the order quantity. The constraints include organization policy, constraints of resources such as those in investment for storage of the inventory, space and purchase orders. In addition, the results of solving random problems generated by the workers show that problems with 150 products can be quickly solved using the proposed model even on PCs. Goosens et al. have also investigated the multi-product purchase problem to minimize expenses [15]. Contrary to Haksever and Moussourakis, they have assumed that the suppliers would use the all unit discount for pricing to the customer and constraints such as meeting the demands and suppliers’ capacity have been considered in modeling this problem. Thus, they have developed a mixed integer linear planning model. Four specific situations, which may affect the problem, have been studied as follows:

- The suppliers declare minimum and maximum allowable order quantities to the buyer.
- The buyer is allowed to purchase more than the demand.
- The buyer can sign the contract with a limited number of suppliers, according to his policies.
- There are limited suppliers for each material.

Chang et al. have proposed a mixed integer approach with the purpose of cost minimization for the problem of selection of single product suppliers [16]. They have assumed that the suppliers would use the all unit discount system. Their model not only determines the order quantity for each supplier, but it also allows the decision makers to consider the resource constraints. Kokangula et al. have used a combination of AHP model, integer, linear and multi-objective planning to determine the order quantity for each of the suppliers by considering constraints such as all unit discount, capacity and budget [17]. In addition, the model has been defined non-linearly, which requires an algorithm for solving the model, in the case of incremental discount. The non-linear planning model determines the optimal order quantity. The objectives of this model are minimization of total purchase value and minimization of purchase cost.

A different stream of research on dynamic order quantity optimization, develops a concave-cost network flow model of the inventory system as that exhibit scale economies. The network structures arising from this approach of modeling, optimizes the inventory system for scheduling orders, inventories and backorders of a product over periods where the planner minimizes the average cost per period of satisfying given demands. Therefore, we can decompose the directed graph model into its connected components and solve the network-flow problem on each such connected component. [18, 19]

The research works reviewed mostly use one type of discount in the process of ordering and consider a maximum of one or two multiple situations of single or multi-product, single or multi-period, single or multi-source and single or multi-objective forms.

The objective of the present work is the development of a mathematical model for determination of optimal order quantity under the conditions where both types of discounts (all unit and incremental) are available and multi-product, multi-dimensional, and multi-period modes govern. Furthermore, the model has been defined as a linear model, which can be easily solved by coding. In the second part of the paper, the model assumptions, parameters, problem modeling and a numerical model and solving it will be discussed. The results and future suggestions will be presented in the third part.

2- Model introduction and problem solution:
2-1- Model introduction:
This research is carried out under conditions in which the company plans to purchase several materials from a number of suppliers in the global market, but some of the suppliers are not able to provide all the company’s
demands and have limited capacities. Thus, a planning model has been developed to minimize the costs of purchase under discount conditions, storage, order and expense shortage, considering the problem objectives and order assignment to suppliers. In addition, other constraints, which the company encounters when providing the materials such as limitation of budget and storage space are added to the model. In the fourth section, the model developed is solved using GAMS software and the data collected from the company are solved to ensure the appropriate performance of the model. In addition, the precision of the suggested solution is studied with the opinion of company’s experts while carrying out the sensitivity analysis and comparison of the results.

2-1-1- Model assumptions:

The type of demand for each material is independent on the others and its quantity is known and fixed in each period.

The planning horizon is six months.

The suppliers offer special discounts for each material such that all unit discounts are implemented for some materials and incremental discounts are applied for others.

Order and purchase quantities are integer values.

The inventory level for each material in the beginning of the year (beginning of the first period) is zero and inventory storage for subsequent periods is allowed.

Shortages are allowed in this model.

Shipping expenses (similar to real conditions) are paid by the supplier.

2-1-2- Indices and sets applied in the model:

I: Set of all materials
i: Material index and \( i \in I \)
J: Set of all suppliers
j: Supplier index and \( j \in J \)
K: Set of discount periods
k: Index of discount period \( k \in K \)
T: Set of time periods
t: Index of time period and \( t \in T \)
J_{ij}: Set of suppliers of material \( i \)
I_{ij}: Set of the materials, which supplier \( j \) can produce.
J_{Uij}: Set of suppliers, which offer all unit discount for material \( i \)
J_{Qij}: Set of suppliers, which offer incremental discount for material \( i \)
K_{ijt}: Set of discount periods offered by supplier \( j \) for material \( i \) during period \( t \)
a_{ijt}: First discount period offered by supplier \( j \) for material \( i \) during period \( t \)
z_{ijt}: Last discount period offered by supplier \( j \) for material \( i \) during period \( t \)

2-1-3- Model parameters:

\( D_{it} \): Number of demands for material \( i \) during period \( t \)
\( B_t \): Maximum available budget during period \( t \)
\( F_{jt} \): Fixed order cost from supplier \( j \) during period \( t \)
\( c_{ijt} \): Capacity of supplier \( j \) for material \( i \) during period \( t \)
\( l_{ijkt} \): Lower limit of discount period \( k \) offered by supplier \( j \) for material \( i \) during period \( t \)
\( u_{ijkt} \): Upper limit of discount period \( k \) offered by supplier \( j \) for material \( i \) during period \( t \)
\( p_{ijkt} \): Unit price of material \( i \) with discount period \( k \) offered by supplier \( j \) during period \( t \)
The determination of which the model tries to minimize the objective function is as follows:

\[ \min z = \sum_{i \in I} \sum_{j \in J_i} \sum_{t \in T} \sum_{k \in K_{ij}} P_{ijkt} x_{ijkt} + \sum_{i \in I} \sum_{t \in T} h_{it} I_n v_{it} + \sum_{j \in J} \sum_{t \in T} F_{jt} S_{jt} + \sum_{i \in I} \sum_{t \in T} \pi_{it} b_{it} \]  

(1)

2-1-4 Decision variables:

Decision variables are variables, by the determination of which the model tries to minimize the objective function (cost minimization) and are as follows:

- \( x_{ijkt} \): Order quantity of material \( i \) from supplier \( j \) with discount period \( k \) during period \( t \)
- \( Y_{ijkt} = 1 \) if the material is supplied from supplier \( j \) with discount period \( k \) during period \( t \) and 0 otherwise
- \( S_{jt} = 1 \) if the material is supplied from supplier \( j \) during period \( t \) and 0 otherwise
- \( b_{it} = \) Shortage quantity for material \( i \) during period \( t \)
- \( I_n v_{it} = \) Initial inventory level for material \( i \) in the beginning of period \( t \)

2-1-5 Objective function:

The objective of the model is minimization of the total costs of purchase, order, storage and shortage of materials. These costs, in the order presented in the objective function, are as follows:

- Total purchase costs from different suppliers during different periods and at different discount levels
- Total storage costs of materials during different periods
- Total ordering costs from different suppliers during different periods
- Total costs of shortages due to lack of materials during different periods

2-1-6 Model constraints:

The constraints of the mathematical model are as follows:

\[ \sum_{j \in J_i} \sum_{k \in K_{ij}} x_{ijkt} + I_n v_{it} (t-1) + b_{it} = D_{it} + I_n v_{it}, \forall i \in I, t \in T \]  

(2)

\[ \sum_{k \in K_{ij}} x_{ijkt} \leq C_{ij}, \forall i \in I, j \in J_i, t \in T \]  

(3)

\[ L_{ijkt} Y_{ijkt} \leq x_{ijkt} \leq U_{ijkt} Y_{ijkt}, \forall i \in I, j \in J_N_i, t \in T, k \in K_{ijt} \]  

(4)

\[ \sum_{i \in I} \sum_{j \in J_{ii}} x_{ijkt} \leq M \times S_{jt}, \forall j \in J, t \in T \]  

(5)

\[ U_{ijkt} Y_{ijkt} - L_{ijkt} \leq x_{ijkt} \leq \begin{cases} U_{ijkt} - L_{ijkt} & \text{if } i \in J_N_i, j \in J_q_i, t \in T, k \in K_{ijt}, k = 1 \bigcup \{a_{ij}, z_{ijt}\} \\ 0 & \text{otherwise} \end{cases} \]  

(6)

\[ x_{ijkt} \leq \begin{cases} U_{ijkt} - L_{ijkt} Y_{ij(t-1)} & \text{if } i \in J_N_i, j \in J_q_i, t \in T, k = z_{ijt} \bigcup \{a_{ij}, z_{ijt}\} \\ 0 & \text{otherwise} \end{cases} \]  

(7)

\[ \sum_{i \in I} \sum_{j \in J_i} \sum_{k \in K_{ij}} p_{ijkt} x_{ijkt} \leq B_t, \forall t \in T \]  

(8)

\[ \sum_{t \in T} I_n v_{it} v_{i} \leq W, \forall t \in T \]  

(9)

\[ Y_{ijk}, S_{ij} \in \{0, 1\}, \forall i, j, k \]  

(10)

\[ x_{ijk} \in N \cup \{0\}, \forall i, j, k \]  

(11)
In the following, the explanations of the constraints are given:

Constraint (2): This constraint expresses the demand and order quantities of material i during period t. If the demand quantity is larger than the order, there is a shortage problem. Otherwise, there is a surplus inventory.

Constraint (3): This constraint is associated with the capacity of supplier j regarding material i. The total order quantity from supplier j during each period t should not exceed the supplier’s capacity.

Constraint (4): This constraint corresponds to the all unit discount issues and shows the total purchase from a supplier during a given period. Therefore, at least one discount period must be active. In other words, this constraint ensures that the purchase from a supplier is one during an offered discount period and zero in other discount levels.

Constraint (5): This constraint shows the contribution of the supplier (supplier’s share) in the supply of a material. The value of M shows this constraint and the sum of the values assigned to each supplier is assumed to be not unlimited. If a purchase is performed from supplier j during period t, $S_{jt}$ equals one.

Constraints (6), (7) and (8): These constraints show the incremental discount periods. The total amount of the purchase from a supplier is in the various intervals of the supplier's price. Therefore, several discount periods must be considered with regards to the purchase volume. Relationship (6) is associated with placing the order quantity on the first level of discount (k = 1). Relationship (7) corresponds to the discount levels between the first and last price within the discount domain and relationship (8) is related to the last level of the discount defined by the supplier.

Constraint (9): This constraint shows the budget in different time periods.

Constraint (10): This constraint shows the storage space for all the materials. The entire storage space available is assumed to be W. $v_i$ is the space allocated to material i and $Inv_{it+1}$ of the materials enter the storage in each period t.

Constraint (11): These are structural constraints and show the zero and one control of the decision variables in the problem, which is associated with issuing or not issuing material purchase from the jth supplier in different discount levels.

(12): This constraint shows the control of non-negativity of decision making parameters. Constraint

2-2- Mathematical model solving method:

According to what was stated in the problem description, the general methodology of the research is of mathematical kind. The views of the company’s experts and pure literature were used to determine model assumptions. Mathematical modeling was carried out under definitive conditions (defining variables, parameters, and determination of objective and constraints). Coding in GAMS environment was used to solve the model. Sensitivity analysis has also been used based on the minor variations in the program code.

2-3- Case study:

OTEC Company obtains its monthly orders for five types of materials from suppliers A, B and C, as shown in Tables 1-6 (six-month planning horizon). Monthly demands for each material, suppliers’ capacities and prices and sale conditions are shown in this table. Table 7 shows the amount of available budget in each period and the cost of ordering suppliers. Table 8 shows the cost of material shortage in each period and Table 9 shows the corresponding storage costs. The optimal order from each supplier is determined considering the constraints below:

Explanations:

Transportation cost of the material from the supplier’s site to the Company production facility is paid by the suppliers.
Each supplier’s capacity is considered as the limit of failure level.

Company A has offered all unit discounts for Mat1 and Mat2 and incremental discounts for Mat3 and Mat4.

Company B has offered all unit discounts for Mat2, Mat3 and Mat5 and incremental discounts for Mat1 and Mat4 in a six-month period.

Company C has offered all unit discounts for Mat1, Mat2, Mat3 and Mat4 and incremental discounts for Mat5 in a six-month period.

Companies may not simultaneously use both types of discounts for the sale of their products.

Table 1: Information about monthly demand and suppliers’ capacity and sale terms in the first period

| Material | Monthly Consumption | Supplier’s Capacity | Supplier | A | B | C |
|----------|---------------------|---------------------|----------|---|---|---|
|          |                     |                     |          |   |   |   |
|          |                     | A       | B       | C |   |   |
|          |                     | Failure Level | Price (€) | Number | Price (€) | Number | Price (€) | Number |
| Mat1     | 705                 | 250     | 350     | 150 | 1 | 90 | 1-85 | 86-170 | 70 | 121-240 | 88 | 1-50 | 51-100 |
|          |                     | 2       | 75      |     | 2 | 55 | 1-70 | 71-140 | 53 | 41-80  | 52 | 1-45 | 46-90 |
|          |                     | 3       | 43      |     | 3 | 43 | 141-200 | -       | -    | -     | 50 | 91-130 |
| Mat2     | 150                 | 200     | 80      | 130 | 1 | 68 | 1-350 | 68      | 1-350 | 72 | 1-250 |
|          |                     | 2       | 60      |     | 2 | 50 | 351-700 | 58      | 351-700 | 62 | 251-500 |
|          |                     | 3       | 50      |     | 3 | 50 | 701-1000 | 55      | 501-800 |
| Mat3     | 630                 | 700     | 1000    | 800 | 1 | 50 | 1-80  | 55      | 1-90   | -    | -     |
|          |                     | 2       | 45      |     | 2 | 40 | 81-160 | 43      | 91-180 | -    | -     |
|          |                     | 3       | 40      |     | 3 | 40 | 161-240 | 38      | 181-270 | -    | -     |
| Mat4     | 500                 | 240     | 270     |     | 1 | 80 | 1-150 | 75      | 1-200 |
|          |                     | 2       | 75      |     | 2 | 75 | 151-300 | 73      | 201-400 |
|          |                     | 3       | 70      |     | 3 | 70 | 301-500 | 71      | 401-600 |
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

**Table 2: Information about monthly demand and suppliers’ capacity and sale terms in the second period**

| Material | Monthly Consumption | Supplier’s Capacity | Supplier | Failure Level | Price (€) | Number | Price (€) | Number | Price (€) | Number |
|----------|---------------------|---------------------|----------|---------------|----------|--------|----------|--------|----------|--------|
| Mat1     | 710                 | 300 350 160         | A        | 1             | 90       | 1-75   | 85       | 1-130  | 100      | 1-70   |
|          |                     |                     | B        | 2             | 75       | 76-150 | 70       | 131-220| 88       | 71-110 |
|          |                     |                     | C        | 3             | 70       | 151-260| 65       | 221-360| 80       | 111-160|
| Mat2     | 170                 | 210 85 135          | A        | 1             | 55       | 1-70   | 54       | 1-45   | 54       | 1-40   |
|          |                     |                     | B        | 2             | 47       | 71-130 | 53       | 46-90  | 52       | 41-85  |
|          |                     |                     | C        | 3             | 43       | 131-210| -        | -      | 50       | 86-140 |
| Mat3     | 620                 | 690 1000 810        | A        | 1             | 68       | 1-350  | 68       | 1-300  | 72       | 1-250  |
|          |                     |                     | B        | 2             | 60       | 351-710| 58       | 301-700| 62       | 251-450|
|          |                     |                     | C        | 3             | -        | -      | 50       | 701-1010| 55       | 451-810|
| Mat4     | 490                 | 245 270 -           | A        | 1             | 50       | 1-70   | 55       | 1-80   | -        | -      |
|          |                     |                     | B        | 2             | 45       | 71-150 | 43       | 81-190 | -        | -      |
|          |                     |                     | C        | 3             | 40       | 151-250| 38       | 191-280| -        | -      |
| Mat5     | 905                 | 250 510 590         | A        | 1             | -        | -      | 80       | 1-200  | 75       | 1-150  |
|          |                     |                     | B        | 2             | -        | -      | 75       | 201-380| 73       | 151-500|
|          |                     |                     | C        | 3             | -        | -      | 70       | 381-510| 71       | 501-610|
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

**Table 3: Information about monthly demand and suppliers’ capacity and sale terms in the third period**

| Material | Monthly Consumption | Supplier’s Capacity | Supplier | Failure Level | Price (€) | Number | Price (€) | Number | Price (€) | Number |
|----------|---------------------|---------------------|----------|---------------|-----------|--------|-----------|--------|-----------|--------|
| Mat1     | 730                 | 250 350 165         | A        | 1             | 90        | 1-70   | 85        | 1-100  | 100       | 1-45   |
|          |                     |                     |          | 2             | 75        | 71-150 | 70        | 101-230| 88        | 46-90  |
|          |                     |                     |          | 3             | 70        | 151-240| 65        | 231-340| 80        | 91-140 |
| Mat2     | 160                 | 220 90 140          | A        | 1             | 55        | 1-50   | 54        | 1-35   | 54        | 1-40   |
|          |                     |                     |          | 2             | 47        | 51-120 | 53        | 36-70  | 52        | 41-80  |
|          |                     |                     |          | 3             | 43        | 121-190| -         | -      | 50        | 81-120 |
| Mat3     | 625                 | 680 1000 820        | A        | 1             | 68        | 1-300  | 68        | 1-280  | 72        | 1-200  |
|          |                     |                     |          | 2             | 60        | 301-690| 58        | 281-650| 62        | 201-480|        |
|          |                     |                     |          | 3             | -         | -      | 50        | 651-990| 55        | 481-790|        |
| Mat4     | 490                 | 250 260 -           | A        | 1             | 50        | 1-75   | 55        | 1-70   | -         | -      |
|          |                     |                     |          | 2             | 45        | 76-150 | 43        | 71-160 | -         | -      |
|          |                     |                     |          | 3             | 40        | 151-230| 38        | 161-260| -         | -      |
| Mat5     | 850                 | 270 520 580         | A        | 1             | -         | -      | 80        | 1-120  | 75        | 1-150  |
|          |                     |                     |          | 2             | -         | -      | 75        | 121-280| 73        | 151-350|        |
|          |                     |                     |          | 3             | -         | -      | 70        | 281-490| 71        | 351-590|        |
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

**Table 4: Information about monthly demand and suppliers’ capacity and sale terms in the fourth period**

| Material | Monthly Consumption | Supplier’s Capacity | Supplier |
|----------|---------------------|---------------------|----------|
|          |                     | A  | B  | C  | A | B   | C    | Number | Price (£) | Number | Price (£) | Number | Price (£) | Number |
| Mat1     | 720                 | 300| 340| 170| 1 | 90  | 1-70 | 85     | 1-100    | 100   | 1-45    |        |
|          |                     | 2  |    |    | 75 | 71-150 | 70   | 101-220 | 88       | 46-80 |
|          |                     | 3  |    |    | 70 | 151-230 | 65   | 221-330 | 80       | 81-130|
| Mat2     | 155                 | 230| 85 | 145| 1 | 55  | 1-50 | 54     | 1-40     | 54    | 1-30    |        |
|          |                     | 2  |    |    | 47 | 51-120 | 53   | 41-60  | 52       | 31-80 |
|          |                     | 3  |    |    | 43 | 121-180 | -    | -      | 50       | 81-110|
| Mat3     | 630                 | 670| 1000| 830| 1 | 68  | 1-300 | 68     | 1-310    | 72    | 1-220   |        |
|          |                     | 2  |    |    | 60 | 301-680 | 58   | 311-600 | 62       | 221-450|
|          |                     | 3  |    |    | -  | -   | 50   | 601-980 | 55       | 451-780|
| Mat4     | 480                 | 255| 250| -  | 1 | 50  | 1-60 | 55     | 1-70     | -     | -       |        |
|          |                     | 2  |    |    | 45 | 61-140 | 43   | 71-150 | -        | -      |
|          |                     | 3  |    |    | 40 | 141-220 | 38   | 151-250 | -        | -      |
| Mat5     | 750                 | 290| 530| 570| 1 | -   | -   | 80     | 1-120    | 75    | 1-160   |        |
|          |                     | 2  |    |    | -  | -   | 75   | 121-280 | 73       | 161-370|
|          |                     | 3  |    |    | -  | -   | 70   | 281-480 | 71       | 371-580|
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

**Table 5: Information about monthly demand and suppliers’ capacity and sale terms in the fifth period**

| Material | Monthly Consumption | Supplier’s Capacity | Supplier | A | B | C | Failure Level | Price (€) | Number | Price (€) | Number | Price (€) | Number |
|----------|---------------------|---------------------|----------|---|---|---|---------------|-----------|--------|-----------|--------|-----------|--------|
| Mat1     | 600                 | 250 340 175         | A         | 1 | 90 | 1-60 | 85 | 1-100 | 100 | 1-40 |
|          |                     |                     | B         | 2 | 75 | 61-150 | 70 | 101-230 | 88 | 41-90 |
|          |                     |                     | C         | 3 | 70 | 151-220 | 65 | 231-320 | 80 | 91-120 |
| Mat2     | 140                 | 240 80 150          | A         | 1 | 55 | 1-50 | 54 | 1-30 | 54 | 1-40 |
|          |                     |                     | B         | 2 | 47 | 51-110 | 53 | 31-60 | 52 | 41-80 |
|          |                     |                     | C         | 3 | 43 | 111-170 | - | - | 50 | 81-120 |
| Mat3     | 520                 | 660 1010 840        | A         | 1 | 68 | 1-300 | 68 | 1-300 | 72 | 1-200 |
|          |                     |                     | B         | 2 | 60 | 301-650 | 58 | 301-650 | 62 | 201-450 |
|          |                     |                     | C         | 3 | -  | - | 50 | 651-1000 | 55 | 451-800 |
| Mat4     | 450                 | 260 280 -           | A         | 1 | 50 | 1-75 | 55 | 1-70 | - | - |
|          |                     |                     | B         | 2 | 45 | 76-150 | 43 | 71-150 | - | - |
|          |                     |                     | C         | 3 | 40 | 151-250 | 38 | 151-250 | - | - |
| Mat5     | 700                 | 300 540 560         | A         | 1 | -  | - | 80 | 1-130 | 75 | 1-150 |
|          |                     |                     | B         | 2 | -  | - | 75 | 131-280 | 73 | 151-380 |
|          |                     |                     | C         | 3 | -  | - | 70 | 281-490 | 71 | 381-600 |
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

| Material | Monthly Consumption | Supplier’s Capacity | Supplier | Failure Level | Price (€) | Number | Price (€) | Number | Price (€) | Number |
|----------|---------------------|---------------------|----------|--------------|-----------|--------|-----------|--------|-----------|--------|
| Mat1     | 650                 | 300 340 180         | A        | 1            | 90        | 1-70   | 85        | 1-90   | 100       | 1-60   |
|          |                     |                     | B        | 2            | 75        | 71-160 | 70        | 91-250 | 88        | 61-110 |
|          |                     |                     | C        | 3            | 70        | 161-260| 65        | 251-360| 80        | 111-160|
| Mat2     | 150                 | 250 75 155          | A        | 1            | 55        | 1-55   | 54        | 1-45   | 54        | 1-50   |
|          |                     |                     | B        | 2            | 47        | 56-130 | 53        | 46-90  | 52        | 51-95  |
|          |                     |                     | C        | 3            | 43        | 131-210| -         | -      | 50        | 96-140 |
| Mat3     | 700                 | 650 1050 850        | A        | 1            | 68        | 1-400  | 68        | 1-350  | 72        | 1-300  |
|          |                     |                     | B        | 2            | 60        | 401-710| 58        | 351-800| 62        | 301-550|
|          |                     |                     | C        | 3            | -         | -      | 50        | 801-1050| 55        | 551-810|
| Mat4     | 350                 | 265 280 -           | A        | 1            | 50        | 1-100  | 55        | 1-100  | -         | -      |
|          |                     |                     | B        | 2            | 45        | 101-180| 43        | 101-190| -         | -      |
|          |                     |                     | C        | 3            | 40        | 181-250| 38        | 191-280| -         | -      |
| Mat5     | 800                 | 320 550 550         | A        | 1            | -         | -      | 80        | 1-120  | 75        | 1-170  |
|          |                     |                     | B        | 2            | -         | -      | 75        | 121-290| 73        | 171-380|
|          |                     |                     | C        | 3            | -         | -      | 70        | 291-510| 71        | 381-600|
Gray and white colors indicate all unit and incremental discounts offered by the suppliers, respectively.

Table 7: Information about the company's budget limit and the cost of the order from each supplier in each period

| Period Number | T1   | T2   | T3   | T4   | T5   | T6   |
|---------------|------|------|------|------|------|------|
| Budget (€)    | 200000 | 180000 | 230000 | 210000 | 190000 | 200000 |
| Ordering Cost (F) |      |      |      |      |      |      |
| A             | 200  | 250  | 200  | 250  | 200  | 250  |
| B             | 210  | 200  | 220  | 210  | 200  | 220  |
| C             | 180  | 190  | 200  | 210  | 220  | 230  |

Table 8: Information about the cost of material shortages in each period

| Material/Period | T1 | T2  | T3  | T4  | T5  | T6  |
|-----------------|----|-----|-----|-----|-----|-----|
| Mat1            | 85 | 85  | 85  | 85  | 85  | 85  |
| Mat2            | 50 | 50  | 50  | 50  | 50  | 50  |
| Mat3            | 65 | 65  | 65  | 65  | 65  | 65  |
| Mat4            | 45 | 45  | 45  | 45  | 45  | 45  |
| Mat5            | 75 | 75  | 75  | 75  | 75  | 75  |

Table 9: Information about the storage cost of materials in each period

| Material/Period | T1 | T2 | T3 | T4 | T5 | T6 |
|-----------------|----|----|----|----|----|----|
| Mat1            | 9  | 8.5| 8.5| 9  | 8  | 8  |
| Mat2            | 5  | 4  | 5  | 5  | 5  | 4  |
| Mat3            | 6  | 6.5| 7  | 6.5| 6  | 7  |
| Mat4            | 4.5| 5  | 4  | 4.5| 5  | 4.5|
| Mat5            | 7.5| 8  | 7.5| 7.5| 8  | 8  |

2-4 Problem solution:

This numerical example is solved using GAMS software to give an objective function of \( Z = 1013400 \) € and optimal order quantity shown in Table 10.
| Time Period | Material | Total Number of Purchases | Failure Level | Supply Quantity from Each Supplier |
|-------------|----------|---------------------------|---------------|-----------------------------------|
|             |          |                           | A  | B  | C  |
| T1          | Mat1     | 705                       | 1  | -  | 120|
|             |          |                           | 2  | -  | 119|
|             |          |                           | 3  | 250| 109| 107|
|             | Mat2     | 150                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | 150| -  |
|             | Mat3     | 701                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 701|
|             | Mat4     | -                         | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | -  |
|             | Mat5     | 900                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 500|
|             |          |                           |     |    | 1  |
| T2          | Mat1     | 721                       | 1  | -  | 130|
|             |          |                           | 2  | -  | 89 |
|             |          |                           | 3  | 260| 131| 111|
|             | Mat2     | 170                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | 170| -  |
|             | Mat3     | 701                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 701|
|             | Mat4     | 245                       | 1  | 70 | -  |
|             |          |                           | 2  | 79 | -  |
|             |          |                           | 3  | 96 | -  |
|             | Mat5     | 905                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 510|
|             |          |                           |     |    | -  |
| T3          | Mat1     | 718                       | 1  | -  | 100|
|             |          |                           | 2  | -  | 129|
|             |          |                           | 3  | 240| 109| 140|
|             | Mat2     | 160                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | 160| -  |
|             | Mat3     | 651                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 651|
|             | Mat4     | 486                       | 1  | 75 | 70 |
|             |          |                           | 2  | 74 | 89 |
|             |          |                           | 3  | 79 | 99 |
|             | Mat5     | 840                       | 1  | -  | -  |
|             |          |                           | 2  | -  | -  |
|             |          |                           | 3  | -  | 490|
|             |          |                           |     |    | 11 |
| T4 | Mat1 | 720 | 1 | - | 100 | - |
|    |      |     | 2 | 71 | 119 | - |
|    |      |     | 3 | 229 | 109 | 92 |
| Mat2 | 155 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | 155 | - | - |
| Mat3 | 672 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | - | 672 | - |
| Mat4 | 466 | 1 | 60 | 70 | - |
|      |      | 2 | 79 | 77 | - |
|      |      | 3 | 79 | 99 | - |
| Mat5 | 750 | 1 | - | - | 160 |
|      |      | 2 | - | - | 110 |
|      |      | 3 | - | - | 480 |

| T5 | Mat1 | 568 | 1 | - | 100 | - |
|    |      |     | 2 | 61 | 129 | - |
|    |      |     | 3 | 189 | 89 | - |
| Mat2 | 140 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | 140 | - | - |
| Mat3 | 1000 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | - | 1000 | - |
| Mat4 | 450 | 1 | 75 | 70 | - |
|      |      | 2 | 74 | 79 | - |
|      |      | 3 | 53 | 99 | - |
| Mat5 | 490 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | - | 490 | - |

| T6 | Mat1 | 640 | 1 | - | 90 | - |
|    |      |     | 2 | 71 | 159 | - |
|    |      |     | 3 | 229 | 91 | - |
| Mat2 | 150 | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | 150 | - | - |
| Mat3 | - | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | - | - | - |
| Mat4 | - | 1 | - | - | - |
|      |      | 2 | - | - | - |
|      |      | 3 | - | - | - |
| Mat5 | 800 | 1 | - | - | 170 |
|      |      | 2 | - | - | 120 |
|      |      | 3 | - | 510 | - |

Considering the results obtained by solving the model (based on real values from the Company) and comparison of the results with real values indicate that the results from the model have a better status than the real values.
The inventory level in the real situation is by far higher than that obtained from the model. This accumulation increases inventory costs and the total model cost, resulting in the increased product price.

The shortage quantity in the real situation is by far higher than that obtained from the model. This adversely affects the total model cost. Therefore, the implementation of the results obtained from the model gives a more optimal situation from the cost reduction aspect, which confirms the efficiency of the research model.

2-5- Analysis of results:

2-5-1- Analysis of variations of increased shortage cost during the shortage in each period and the total cost

Given the strong dependency of the Company’s timetable on the timely delivery of materials at specified times and, possibly, the lack of shortages in periods, a situation is considered in which the supplier pays a higher fine in the case of failure to provide timely delivery. Diagram 1 shows the percent effect of shortage expense on the total model cost. A 0.23% increase of shortage expense indicates zero shortage.

2-5-2- Analysis of effects of increased material demand on purchase expenses and other costs

In this part, the results obtained by variation of the quantity of material demand on the model costs will be analyzed. As observed in diagram 2, a 5% increase in the quantity of material demand in each period increases the total model cost and other expenses (except for the ordering expense) by 4-5%.
3- Results and conclusions:

In this work, a mathematical model has been developed to design and present a solution to determine the optimal order quantity under discount terms at OTEC Company in order to achieve the main objective of the research. In fact, this seemed necessary, having studied the situation in the Company and the absence of a systematic and scientific system to determine the optimal order quantity. This generated a double motivation for the researchers to provide a model to determine the optimal order quantity under discount terms to respond to the problem under study and overcome the present vacancy in the company. Having designed and solved the problem, the results obtained have been presented and analyzed. The demand quantity has been assumed constant in this problem and each supplier can provide several kinds of materials, but has limited capacities. Given that the ordering quantities are expressed as integers; integer type decision variables have been considered. The model efficiency in the improvement of decisions on ordering quantity was confirmed, according to the results obtained and the comparison of these results with real values. Overall, there are many positive outcomes for the application of this model, as discussed below:

1. Since the planning for supplying materials is carried out using a mathematical model considering the complex situations of the suppliers (all unit and incremental discounts during different periods), the ordering will be demand based and attempt will be made to decrease the costs. Thus, the inventory of materials can be transferred to the warehouse and shortage of materials will be based on the decision of the Chief Executive.
2. The mathematical model includes different situations such as increased demand and reduction of shortage costs and the optimal decisions will be made based on the total cost.
3. This model uses a combination of all unit and incremental discount situations offered by the suppliers during different periods in making decisions. The objective of designing this model is to consider the real conditions, which exist when making decisions.
4. The mathematical model designed is capable of being applied in other industrial institutions with minor variations. Based on the results obtained, the companies will be able to make optimal decisions to reduce their costs and increase their purchasing power.

References:

1. Burke G., Carrilo J., Vakharia A., "Heuristic For Sourcing From Multiple Suppliers With Alternative Quantity Discounts.", European Journal Of Operational Research, (2008), 186(1), 317-329.
2. Du, Ruo, Banerjee, Avijit, Kim, Seung-Laee, "Coordination Of Two-Echelon Supply Chains Using Wholesale Price Discount", International Journal Production Economics 143, (2013), 327–334.
3. Manerba, Daniele, Renata, Mansini, "An Exact Algorithm For The Capacitated Total Quantity Discount Problem"Scatter Search Algorithm For Supplier Selection & Order Lot Sizing Under Multiple Price Discount Environment", European Journal Of Operational Research 222,(2012),287–300.
4. Amy H.I., He-Yau K., Chun-Mei L., Wan-Yu Hong, "An Integrated Model For Lot Sizing with Supplier Selection and Quantity Discounts", Applied Mathematical Modelling 37 , (2013),4733–4746.
5. Esfandiari N., Seibfarghry M., "Modeling A Stochastic Multi-Objective Supplier Quota Allocation Problem with Price-Dependent Ordering", Applied Mathematical Modelling37 , (2013), 5790–5800.
6. Mafakheri F., Breton M., Gholiemi A., "Supplier Selection–Order Allocation: A two – Stage Multiple Oriteria Dynamic Programming Approach", International Journal Of Production Economics, (2011),132,52-57.
7. Jafarzadeh, Hassan, N. Moradinasab, H. Eskandari, and S. Gholami. "Genetic Algorithm for A Generic Model of Reverse Logistics Network." International Journal of Engineering Innovation & Research 6, no. 4 (2017): 174-178.
8. Kamali A., Ghomi F., Jolai F., “A Multi–Objective Quantity Discount & Joint Optimization Model For Coordination Of A Single- Buyer Multi–Vendor Supply Chain”, Computers & Mathematics with Application, (2011),62,3251-3269.

9. Tsai W.C., Wang Ch., "Decision Making Of Sourcing & Order Allocation With Price Discounts", Journal Of Manufacturing Systems ,(2010),29,47-54.

10. Nasab, N. Moradi, M. R. Amin-Naseri, and Hassan Jafarzadeh. "A Benders’ Decomposition Method to Solve a Multi-period, Multi-echelon, and Multi-product Integrated Petroleum Supply Chain." Process Integration and Optimization for Sustainability 2, no. 3 (2018): 281-300.

11. Mohammad Ebrahim R., Razmi J., Haleh H., "Scatter Search Algorithm For Supplier Selection & Order Lot Sizing Under Multiple Price Discount Environment" Advances In Engineering Software , (2009),40,766-776

12. Amid A., Ghodsypour S.H., O'Brien C., "A Weighted Additive Fuzzy Multi Objective Model For The Supplies Selection Problem Under Price Breaks In A Supply Chain.", International Journal Of Production Economics, (2009), 121, 323-332

13. Burke G., Carrilo J., Vakharia A., "Heuristic For Sourcing From Multiple Suppliers With Alternative Quantity Discounts.", European Journal Of Operational Research, (2008), 186(1), 317-329.

14. Haksever C., Moussourakis J., "Determining Order Quantities In Multi Product Inventory Systems Subject To Multiple Constraints & Incremental Discounts.", European Journal of Operational Research, (2008), 184(3), 930-945

15. Goosens D.R., Maas A.J.T., Spieksma F.C.R., Klundert J.J., "Exact Algorithms For Procurement Problems Under A Total Quantity Discounts Atructure.", European Journal Of Operational Research, (2007), 178, 603-626.

16. Chang Ch., Chin Ch., Lin M., "On the Single Item Supplier System With Variable Lead Time, Price Quantity Discount & Resource Constraints.", Applied Mathematics And Computation, (2006), 182, 89-97

17. Kokangula A., Susuzb Z., "Integrated analytical hierarch process and Mathematical programming to supplier selection problem with quantity discount", Applied Mathematical Modelling, (2009),33,1417-1429.

18. Hosseininia, Mahtab, and Faraz Dadgostari. "Hamiltonian Paths and Cycles." In Graph Theory for Operations Research and Management: Applications in Industrial Engineering, pp. 96-105. IGI Global, 2013.

19. Hosseininia, Mahtab, and Faraz Dadgostari. "Connectivity." In Graph Theory for Operations Research and Management: Applications in Industrial Engineering, pp. 37-47. IGI Global, 2013.