Power of Finite Memory and Finite Communication Robots under Asynchronous Scheduler

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Abstract. In swarm robotics, a set of robots has to perform a given task with specified internal capabilities (model) and under a given adversarial scheduler. Relation between a model $M_1$ under scheduler $S_1$, and that of a model $M_2$ under scheduler $S_2$ can be of four different types: not less powerful, more powerful, equivalent and orthogonal. In literature there are four main models of robots with lights: LUMI, where robots have the power of observing the lights of all the robots, FSTA, where each robot can see only its own light, FCOM, where each robot can observe the light of all other robots except its own and OBLOT, where the robots do not have any light. In this paper, we investigate the computational power of FSTA and FCOM model under asynchronous scheduler by comparing it with other model and scheduler combinations. Our main focus is to understand and compare the power of persistent memory and explicit communication in robots under asynchronous scheduler.

Keywords: Look-Compute-Move, Oblivious Mobile Robots, Robots with Lights, Memory versus Communication

1 Introduction

A large amount of research in the area of distributed computing has been devoted to the study of computational capabilities of autonomous mobile entities called robots operating in an Euclidean plane. Each of the robots can freely move through out the space, and each are endowed with its individual co-ordinate system and operate in Look – Compute – Move (LCM) cycles. One of the main research focus has been to understand the extent of computational power of robots with minimum possible internal capabilities such as memory or communication. Some of the most notable tasks are Gathering [4,7,9], Pattern Formation [9–11], etc.

An external factor which plays a fundamental role in determining the computational power of a swarm of mobile robots is the activation schedule. With respect to this factor there are three different settings: fully – synchronous or FSYNCH, semi-synchronous or SSYNCH, and asynchronous or ASYNCH. In asynchronous setting there is no common notion of time, while synchronous
setting time is divided into discrete intervals called rounds. Among the synchronous schedulers if in each round all the robots are activated then the scheduler is said to be fully-synchronous. If there is no such constraint, the scheduler is said to be semi-synchronous.

In addition to external capabilities the robots can be further differentiated based on the presence and absence of internal capabilities such as memory or communication. Our main focus in this paper are robots with light, where the lights act as a basis of memory or communication. There are four main models based on different labels of internal capabilities of memory and communication of the robots: \textit{OBLOT}, \textit{LUMI}, \textit{FSTA}, and \textit{FCOM}. In \textit{OBLOT} model robots neither have persistent memory nor have the power of communication. In the \textit{LUMI} model the robots are equipped with a constant-sized memory (called light), whose value can be set in the Compute phase. The light is visible to all the robots and is persistent. Hence, the robots are able to remember and communicate a constant number of bits through the light. Now \textit{FSTA} and \textit{FCOM} are two sub-models of \textit{LUMI} where in the former each robot can see only its own light and in the latter, the robot can see the lights of all the other robots except its own. Thus in \textit{FSTA} the robots are silent but has a constant amount of persistent memory, while in \textit{FCOM} the robots are oblivious but can communicate a constant number of bits. \(X^Y\) denotes model \(X\) under scheduler \(Y\). \(A > B\) indicates that model \(A\) is computationally more powerful than model \(B\), \(A \equiv B\) denotes that they are computationally equivalent, \(A \perp B\) denotes that they are computationally incomparable.

\textbf{Previous Works} It has been shown in [9] that within \textit{OBLOT}, robots in \textit{FSYNCH} are strictly more powerful than those in \textit{SSYNCH}. In [1] it has been shown that within \textit{LUMI}, robots in \textit{ASYNCH} has the computational power as the robots in \textit{SSYNCH}, and that asynchronous luminous robots are strictly more powerful than oblivious synchronous robots. Recently in [6], the authors provided a complete exhaustive map of computational relationships among various models given in the tables [1, 2, 3] but only under \textit{FSYNCH} and \textit{SSYNCH} scheduler. In addition to that they considered the question that which of the internal capabilities is more important, memory or communication. The authors examined this question through the lens of various computational results they obtained and came to the conclusion that the answer depends on the type of scheduler, i.e., communication is more powerful than persistent memory if the scheduler is fully synchronous and are incomparable under the semi-synchronous. In [8] the author further expanded the work of [9]. The author investigated how the models relate to each other in a single robot system and also showed that the results obtained for rigid robots with same chirality in tables [1] and [2] are the same if non-rigid robots are considered. The author also investigated the point between \textit{SSYNCH} and \textit{FSYNCH} where the strict dominance of \textit{LUMI} over \textit{FCOM} turns into equivalence.
| LUMI | FCOM | FSTA | OBLOT |
|------|------|------|-------|
| ≡    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |

Table 1: Relationships within $F_{synch}$ [6]

| LUMI | FCOM | FSTA | OBLOT |
|------|------|------|-------|
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |

Table 2: Relationships within $S_{synch}$ [6]

| LUMI | FCOM | FSTA | OBLOT |
|------|------|------|-------|
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |
| ＞    | ＞    | ＞    |       |

Table 3: Relationship between $F_{synch}$ and $S_{synch}$ [6]

Our Contributions In this paper, we partially extend the work of [6] under ASYNCH scheduler as well. We first observe that, when we consider the four models under asynchronous scheduler, some of the results follow in a straightforward manner from the previous results established in the papers [1, 5, 6]. For, some of the remaining cases which requires a deeper investigation we give non-trivial proofs. For example we prove that $F_{STA}^A \perp OBLOT^F$, and we prove that $FCOM^A \perp OBLOT^F$, which is a clear refinement of the recently established result that $LUMI^A \perp OBLOT^F$ [6]. To prove $F_{STA}^A \perp OBLOT^F$ we introduce a new problem Oscillating Configurations while to prove $FCOM^A \perp OBLOT^F$, we provide a new asynchronous algorithm for the problem $\neg IL$ introduced in [6]. By designing an algorithm for $\neg IL$ in $FCOM^A$ we also prove that $FCOM^A \perp FSTA^F$, which is in contrast to the fact that $FCOM^F > FSTA^A$, which is easily deduced from some previous results. We introduce a new problem Independent Oscillating Problem and prove that $FCOM^S \perp FSTA^A$ which in turn implies that $FCOM^A \perp FSTA^A$. In [6], the question was raised that whether “it was better to remember or to communicate?” Thus we have shown here that in the case of asynchronous scheduler, finite memory model is incomparable to finite communication model.

2 Model and Technical Preliminaries

The model and setting in this paper is same as that of [6]. The System considered in this paper consists of a team $R$ of computational entities called robots moving and operating in the Euclidean plane $\mathbb{R}^2$. The robots are viewed as points in the Euclidean plane and has its own local co-ordinate system which may not agree with other robots. It always perceives itself at the origin. The robots are identical (indistinguishable) and autonomous (lacks a central control). At any point in time, a robot is either active or inactive. When active, a robot executes a Look–Compute–Move (LCM) cycle performing the following three operations:
1. **Look**: The robot activates its sensors to obtain a snapshot of the positions occupied by robots with respect to its own co-ordinate system.

2. **Compute**: The robot executes its algorithm using the snapshot as input. The result of the computation is a destination point.

3. **Move**: The robot moves to the computed destination, i.e., we assume a rigid mobility. If the destination is the current location, the robot stays still.

When inactive, a robot is idle. All robots are initially idle. The amount of time to complete a cycle is assumed to be finite, and the **Look** operation is assumed to be instantaneous. We assume that the robots agree on the same circular orientation of the plane, i.e., there is chirality.

In the **OBLOT** model, the robots are silent and oblivious. By silent we mean that the robots have no explicit means of communication, and by oblivious we mean that, at the start of a cycle, a robot has no memory of observations and computations performed in previous cycles.

In the **LUMI** model, each robot $r$ is equipped with a persistent visible state variable $\text{Light}[r]$, called light, whose values are taken from a finite set $C$ of states called colors. The colors of the lights can be set in each cycle by $r$ at the end of its **Compute** operation. A light is persistent from one computational cycle to the next and can be seen by all the robots. The robot is otherwise oblivious forgetting all other information from previous cycles. In **LUMI**, the **Look** operation produces the set of pairs $(\text{position}, \text{color})$ of the other robots. It can be clearly understood that the lights simultaneously provide persistent memory and direct means of communication.

**FSTA** is a sub-model of **LUMI** having only the property of persistent memory. Here the robots can only see the color of its own light i.e. the light is an internal light. As a result the robots are silent as in **OBLOT** but finite-state.

**FCOM** is a sub-model of **LUMI** having only the property of external communication. Here the lights of the robot are external light i.e. a robot cannot see the color of its own light but can see the color of the lights of the other robots. As a result the robots are oblivious as in **OBLOT** but are finite-communication.

With respect to the activation schedule of the robots, there are three kind of schedulers. In the asynchronous (ASYNCH) scheduler, there is no common notion of time, each robot is activated independently of others, the duration of each phase is finite but unpredictable and might be different in different cycles. In the semi-synchronous (SSYNCH) setting, time is divided into discrete intervals, called rounds, in each round some robots are activated simultaneously, and perform their LCM cycles in perfect synchronization. The fully-synchronous (FSYNCH) setting, is same as SSYNCH except the fact that each robot is activated in every round. In all of these settings, the selection of which robots are activated at a round is made by an adversarial scheduler, which must be fair, i.e., every robot must be activated infinitely often.

The Computational Relationships we now define below are exactly the same as [6]. For the sake of completion of the paper we re-write it.

Let $M = \{\text{LUMI}, \text{FSTA}, \text{FCOM}, \text{OBLOT}\}$ be the set of models, and $S = \{\text{FSYNCH}, \text{SSYNCH}, \text{ASYNCH}\}$ the set of schedulers. $R$ denotes the
set of all team of robots satisfying core assumptions (i.e., they are identical, autonomous and operate in $LCM$ cycles) and $R \in R$, a team of robots having identical capabilities. By $R_n \in R$ we denote the set of all teams of size $n$.

Given a model $M \in M$, a scheduler $S \in S$, and a team of robots $R \in R$, let $Task(M, S; R)$ denote the set of problems solvable by $R$ in $M$ under adversarial scheduler $S$.

Let $M_1, M_2 \in M$ and $S_1, S_2 \in S$. We define the relationships between model $M_1$ under scheduler $S_1$ and model $M_2$ under scheduler $S_2$:

- computationally not less powerful ($M_1^{S_1} \geq M_2^{S_2}$), if $\forall R \in R$ we have $Task(M_1, S_1; R) \supseteq Task(M_2, S_2; R)$;
- computationally more powerful ($M_1^{S_1} > M_2^{S_2}$), if $M_1^{S_1} \geq M_2^{S_2}$ and $\exists R \in R$ such that $Task(M_1, S_1; R) \setminus Task(M_2, S_2; R) \neq \emptyset$;
- computationally equivalent ($M_1^{S_1} \equiv M_2^{S_2}$), if $M_1^{S_1} \succeq M_2^{S_2}$ and $M_1^{S_1} \preceq M_2^{S_2}$;
- computationally orthogonal or incomparable, ($M_1^{S_1} \perp M_2^{S_2}$), if $\exists R_1, R_2 \in R$ such that $Task(M_1, S_1; R_1) \setminus Task(M_2, S_2; R_1) \neq \emptyset$ and $Task(M_2, S_2; R_2) \setminus Task(M_1, S_1; R_2) \neq \emptyset$.

For simplicity of notation, for a model $M \in M$, let $M^F$ and $M^S$ denote $M^{FSnch}$ and $M^{SSnch}$, respectively; and let $M^F(R)$ and $M^S(R)$ denote the sets $Task(M, FSnch; R)$ and $Task(M, SSnch; R)$, respectively.

Trivially, for any $M \in M$, $M^F \geq M^S \geq M^A$; also for any $S \in S$, $LUMI^S \geq FSTA^S \geq OBLOT^S$ and $LUMI^S \geq FCOM^S \geq OBLOT^S$.

## 3 Obvious Deductions from previous algorithms

A number of results has been established previously in the papers [1][2][3][4][5] but a most of them are considering $FSnch$ and $SSnch$ schedulers only. When we change the scheduler to asynchronous many new results follow easily:

- (D1) $FSTA^F > OBLOT^A$
- (D2) $FCOM^F > OBLOT^A$
- (D3) $LUMI^F > OBLOT^A$
- (D4) $LUMI^S > OBLOT^A$
- (D5) $FCOM^F > FSTA^A$
- (D6) $FSTA^F > FSTA^A$
- (D7) $LUMI^A > FSTA^A$
- (D8) $LUMI^S > FSTA^A$
- (D9) $LUMI^F > FSTA^A$
- (D10) $FCOM^F > FCOM^A$
- (D11) $LUMI^A > FCOM^A$
- (D12) $LUMI^S > FCOM^A$
- (D13) $LUMI^F > FCOM^A$
- (D14) $FCOM^F > LUMI^A$
- (D15) $LUMI^F > LUMI^A$
- (D16) $FSTA^S < LUMI^A$
– (D17) $\text{FCOM}^S < \text{LUMI}^A$

Proof. 1. (D1) and (D2) follows from the fact that $\text{FSTA}^F > \text{OBLOT}^F$ and $\text{FCOM}^F > \text{OBLOT}^F$ [6] and $\text{OBLOT}^F > \text{OBLOT}^A$ [6];
2. (D3) and (D4) follows from the fact that $\text{LUMI}^A > \text{OBLOT}^A$ [1];
3. (D5) follows from the fact that $\text{FCOM}^F > \text{FSTA}^S$ [6]. (D6) follows from the fact that $\text{FSTA}^F > \text{FSTA}^S$ [6];
4. (D7) follows from $\text{LUMI}^S \equiv \text{LUMI}^A$ and $\text{LUMI}^S > \text{FSTA}^S$ [6]. (D8) and (D9) follows from (D7).
5. (D10) follows from the fact that $\text{FCOM}^F > \text{FCOM}^S$ [6].
6. (D12) follows from $\text{LUMI}^S > \text{FCOM}^S$ [6]. (D11) follows from $\text{LUMI}^S \equiv \text{LUMI}^A$ and (D12). (D13) follows from (D11) and (D12).
7. (D14), (D15) follows from $\text{FCOM}^F \equiv \text{LUMI}^F$ and $\text{LUMI}^F > \text{LUMI}^S$ [6].
8. (D16), (D17) follows from the fact that $\text{LUMI}^S \equiv \text{LUMI}^A$ and $\text{LUMI}^S > \text{FSTA}^S$, $\text{LUMI}^S > \text{FCOM}^S$ [6].

There are still some non-trivial relationships whose nature still remained unresolved and which cannot be deduced from results of the previous papers. In the next section, in sub-section 4.1 we prove that $\text{FSTA}^A$ is orthogonal to $\text{OBLOT}^F$, and strictly powerful than $\text{OBLOT}^A$. In sub-section 4.2 we prove that $\text{FCOM}^A$ is orthogonal to $\text{OBLOT}^F$, $\text{FSTA}^F$ and $\text{FSTA}^S$.

4 Computational Power of $\text{FSTA}$ and $\text{FCOM}$ in Asynch

4.1 Problem Oscillating Configurations

We first investigate the computational relationship between $\text{OBLOT}^F$ and $\text{FSTA}^A$. It has already been shown in [6] that the SRO (Shrinking Rotation) problem can be solved in $\text{OBLOT}^F$ but not in $\text{FSTA}^S$. Hence SRO problem cannot be solved in $\text{FSTA}^A$. Now we show that, there exists a problem which can be solved in $\text{FSTA}^A$ but not in $\text{OBLOT}^F$. We now define the problem:

**Definition 1.** Problem OC (Oscillating Configurations): Four robots $x, y, z$ and $t$ are placed initially in Configuration-I represented by Figure (I). The problem OC requires the robots to move to continuously alternate the configuration of the robots in the sequence I-II-III-II-III-...., i.e. the robots start at Configuration-I, form Configuration-II, then forms Configuration-III, then again forms Configuration-II,..... and the process is continuous.
Algorithm 1: Algorithm AlgOC for OC in $FSTA^A$ executed by each robot $r$, initially $r.light \leftarrow NIL$

We claim that the problem can be solved in $FSTA^A$ but cannot be solved in $OBLOT^F$. We prove our claim by providing an algorithm AlgOC to solve OC in $FSTA^A$ and then proving that OC cannot be solved in $OBLOT^F$.

Description of the Algorithm AlgOC

The internal light used by the robots can have three colors: $NIL, RED, BLUE$. Initially all the robots start from Configuration-I and have their internal lights set to $NIL$. A robot $r$ when activated takes a snapshot of its surrounding in the Look phase and tries to understand the current configuration. The robot $r$ then performs the following actions based on the dual information about its position in the current configuration and the color of its internal light:

1. If the color of the internal light of $r$ is $NIL$, and the visible configuration is not I, then it does nothing in that phase, i.e., there is no change of internal
light or movement on the part of \( r \) in that phase. If the configuration is \( I \), but the location of \( r \) is not at \( C \) then also it does nothing in that phase. But if the location of \( r \) is at \( C \), then it changes the color of its internal light to \( RED \) and moves to the point \( C' \).

2. If the color of the internal light of \( r \) is \( RED \), the visible configuration is \( II \) and it is at \( C' \), then \( r \) changes the color of its internal light to \( BLUE \), and moves to the point \( C'' \).

3. If the color of the internal light of \( r \) is \( BLUE \), then if the location of \( r \) is at \( C'' \) then \( r \) moves to the point \( C' \), and if the location of \( r \) is at \( C' \) then \( r \) changes the color of its internal light to \( NIL \) and moves to the point \( C \).

**Correctness of the Algorithm AlgOC**

**Lemma 1.** \( \forall R \in R_4, OC \in FSTA^A(R) \).

*Proof.* A robot’s internal light can take three possible colors: \( NIL, RED, BLUE \). Initially each robot has its initialized to \( NIL \). In our algorithm, we ensure that the robot placed at position \( C \) in the initial configuration, i.e \( z \), is the only robot that makes any movement throughout the execution. We denote the position where robot \( z \) must move to make the configuration \( II \) and \( III \) as \( C' \) and \( C'' \) respectively. Initially a robot wakes up to find its colour \( NIL \), and if it is at \( C \) it can clearly distinguish its position from other robots, since it is the only robot in the whole configuration which is equidistant from the other three robots. So, at that point the robot changes its colour to \( RED \) and move to position \( C' \) such that the resulting configuration is \( II \). At this point \( z \) is the only robot with colour \( RED \). When again \( z \) is activated by the adversary it wakes to find its internal light having colour \( RED \) and configuration \( II \), so it changes the color of its internal light to \( BLUE \) and moves to the position \( C'' \) such that the resulting configuration becomes \( III \). Next again when \( z \) is activated it finds it internal light having the colour \( BLUE \), and visible configuration to be \( III \). It then moves to the position \( C'' \) forming configuration \( II \). Now when again \( z \) is activated it finds the visible configuration to be configuration \( II \) and internal light \( BLUE \), it then changes the color of its internal light to \( NIL \) and moves to the point \( C \) forming configuration \( I \). When \( z \) is activated again, in this position, it finds its internal light having colour \( NIL \) and configuration \( I \), then \( r \) repeats its moves and the algorithm continues like this.

Note that, after the execution of the algorithm has started, if any robot other than \( z \) is activated, they do not do anything, as a robot with light \( NIL \) makes a movement only when the visible configuration is \( I \) and it is in \( C \), and according to our algorithm only robot \( z \) satisfy this criteria, throughout the execution of the algorithm. \( \square \)

**Lemma 2.** \( \exists R \in R_4, OC \notin OBLOT^F(R) \).

*Proof.* Let there exists an algorithm \( A \) which solves problem \( OC \) in \( OBLOT^F \). Now let us consider two different scenarios:

**Case-1:** The robots have just changed their configuration to \( II \) from \( I \), and the
configuration is currently II.

**Case-2:** The robots have just changed their configuration to II from III, and the configuration is currently II.

In Case-1 the next configuration that the robots must form is III, and in Case-2 the next configuration robots have to form is I. But in both the cases, the view of the robots are same. And as in $\text{OBLOT}^F$ setting the robot has to decide its next destination based on its present view as the robots neither have any internal memory nor they have the power of external communication. As a result, the robots cannot distinguish between Case-1 and Case-2, hence they will perform the same move in both the cases. Hence, problem $OC$ cannot be solved in $\text{OBLOT}^F$.  

**Theorem 1.** $\text{OBLOT}^F \perp \text{FSTA}^A$

**Proof.** Problem $OC$ can be solved in $\text{FSTA}^A$, but cannot be solved in $\text{OBLOT}^F$, and in [6] it has been proved that the problem $SRO$ or Shrinking Rotation can be solved in $\text{OBLOT}^F$ but cannot be solved in $\text{FSTA}^S$, hence not in $\text{FSTA}^A$. Hence our theorem is proved.  

**Theorem 2.** $\text{OBLOT}^A \leq \text{FSTA}^A$

**Proof.** Problem $OC$ cannot be solved in $\text{OBLOT}^F$, hence obviously not in $\text{OBLOT}^A$. But problem $OC$ as proved in Theorem 1 can be solved in $\text{FSTA}^A$. Also, trivially $\text{OBLOT}^A \leq \text{FSTA}^A$. Hence, the theorem follows.

4.2 An Asynchronous Algorithm in $\text{FCOM}$ for Problem $-IL$

We consider the problem $-IL$ defined in [6].

![Diagram](image)

**Definition 2.** Three robots $a$, $b$, and $c$, starting from the initial configuration shown in Figure 2(a), must form first the pattern of Figure 2(b) and then move to form the pattern of Figure 2(c).
It has been already proved in [6] that $-IL$ cannot be solved in $FSTA^F$. In this paper, we provide an algorithm for $-IL$ in $FCOM^A$. We re-name Figure-2(a),2(b),2(c) in our Figure-2 as Configuration I,II and III respectively. Also we call the point where robot $a$ must move from configuration-I to form configuration-II to be $P_1$, and the point where the robot $b$ must move from configuration-II to form configuration-III to be $P_2$.

Algorithm 2: Algorithm $COMIL$ for $-IL$ in $FCOM^A$ executed by each robot $r$, initially $r.$light $\leftarrow$ $NIL$

```
1 if there exists a robot with light $F$ then
2 do nothing
3 else
4 if there exists a robot with light $M$ then
5   if Visible Configuration is not II then
6     do nothing
7   else
8     if position is in the line segment joining the other two robots then
9        r.light $\leftarrow$ $F$
10       Move to the $P_2$
11     else
12       do nothing
13 else
14 if Visible Configuration is II then
15   do nothing
16 else
17   if I can form II with least clockwise rotation then
18      r.light $\leftarrow$ $M$
19      Move to $P_1$
20   else
21     do nothing
```

Description of Algorithm COMIL

Here the robots are equipped with an external light whose color can only be seen by other robots. The external light can take one of the following three colors: $NIL$, $M$, $F$. Initially, all robots start at Configuration-I and have their external light set to $NIL$. A robot $r$ when activated takes a snapshot of its surrounding in the Look phase. The snapshot provides $r$ with the location of the robots according to its own local co-ordinate system and the current color of the light of the robots except itself. The robot $r$ then performs the following actions based on the dual information about its position in the current configuration and the colors of the external lights of the robots except itself:

1. If $r$ observes any robot or robots in the snapshot obtained having external light $F$ it does nothing, i.e, it neither changes the color of its external light nor it executes any movement, irrespective of the present configuration and any other light attained in the snapshot.
2. If $r$ does not observe any robot having external light $F$, but observes a robot having external light $M$, then there can be two cases: either the visible configuration is Configuration-II, or the visible configuration is not Configuration-II. If the configuration is not Configuration-II $r$ does not do anything. Further
if the current configuration is Configuration-II, there can be two cases. If \( r \) does not lie in the line segment joining the other two robots it does nothing. Otherwise if \( r \) lies in the line segment joining the location of the other two robots, then \( r \) changes the color of the external light to \( F \) and moves to the point \( P_2 \).

3. If \( r \) does not observe any visible light, then there can be two cases, either the observed configuration is Configuration I or the observed configuration is Configuration II, i.e. \( r \) is \( a \). If the observed configuration is Configuration I, then if \( r \) is \( a \) (the robot which can form Configuration II with least clockwise angular movement), then \( r \) changes the color of the external light to \( M \) and moves to the point \( P_1 \). Otherwise if the observed configuration is Configuration II, \( r \) does nothing.

**Correctness of the Algorithm COMIL**

**Lemma 3.** \( \forall R \in \mathcal{R}_3, -IL \in FCOM^A(R) \).

**Proof.** Initially all robots have colour NIL and are arranged in Configuration-I. Here the robot \( a \) can uniquely identify its position, as it is the only robot whose ninety degree clockwise rotation around the robot \( b \) results in Configuration-II. According to our algorithm, at this position only robot \( a \) is allowed to move, \( a \) changes its colour to \( M \) and moves to position \( P_1 \) forming Configuration-II.

While \( a \) is moving towards the point \( P_1 \), if \( b \) or \( c \) is activated, their snapshot in the Look returns a configuration where there is one robot with light set to \( M \), and the configuration is not yet Configuration II. Hence at this point neither \( b \) nor \( c \) makes any movement. When \( a \) is at the point \( P_1 \), if \( a \) or \( c \) is activated neither of them changes their color or make any movement. If \( a \) is activated \( a \) observes the color of the other two robots to be set to \( NIL \) and the visible configuration is Configuration II. Hence \( a \) does not do anything. If \( c \) is activated it observes one robot (\( a \)) having external light set to \( M \) and the visible configuration to be Configuration II, but \( c \) does not lie in the line segment joining the other two robots, i.e., \( a \) and \( b \). Hence in this case \( c \) also does not do anything. If \( b \) is activated, it observes one robot with its external light set to \( M \), visible configuration to be Configuration II, and also it lies on the line segment joining the other two robots. As the external light of \( a \) is still set to \( M \), \( b \) clearly identifies the position \( P_2 \) where it must move to form Configuration III. Now it turns its light to \( F \) and moves to \( P_2 \) which results in Configuration III. Now while \( b \) is moving towards the \( P_2 \), if either \( a \) or \( c \) is activated they observe a robot with its external light set to \( F \) and hence does not do anything. Once \( b \) reaches \( P_2 \) the resulting configuration is Configuration III. If again at this point if \( a \) or \( c \) is activated they observe a robot with its external light set to \( F \), and hence again, neither of them do anything. If \( b \) is activated, \( b \) observes a robot having its external light set to \( M \) and the configuration is not Configuration II. Hence \( b \) does not make any movement. As a result the robots do not make any further movement and remain at Configuration III permanently according to our algorithm and the problem is solved. It must be noted that as we have assumed
rigid movement of the robots, so the robots $a$ and $b$ do not stop in between their movement. Hence our algorithm solves problem $-IL$.

**Theorem 3.** $FCOM^A \perp FSTA^F$

*Proof.* It has been proved earlier in [6] that Problem $-IL$ cannot be solved in $FSTA^F$, and we have proved in Lemma 3 that the Algorithm $COMIL$ solves $-IL$ in $FCOM^A$. Also in [6], it has been proved that there exists a problem, i.e., Problem $CGE$ or Center of Gravity Expansion which can be solved in $FSTA^F$ but cannot be solved in $UMI^S$.

Now, $UMI^S \equiv UMI^A > FCOM^A$ (Section 3), and hence Problem $CGE$ cannot be solved in $FCOM^A$ as well. Hence, the theorem follows.

**Theorem 4.** $FCOM^A \perp FSTA^S$

*Proof.* Problem $-IL$ can be solved in $FCOM^A$, but cannot be solved in $FSTA^F$ and as we know that, $FSTA^S \leq FSTA^F$, hence $-IL$ cannot be solved in $FSTA^S$. Also in [6] it has been shown that the Problem $TAR(d)$ or Triangle Rotation can be solved in $FSTA^S$, but not in $FCOM^S$. Now as $FCOM^A \leq FCOM^S$, so Problem $TAR(d)$ cannot be solved in $FCOM^A$. Hence the theorem follows.

**Theorem 5.** $FCOM^A \perp OBLOT^F$

*Proof.* We know that $OBLOT^F < FSTA^F$ [6]. Now it has been proved in [6] that the problem $-IL$ cannot be solved in $FSTA^F$. Hence the problem $-IL$ cannot be solved in $OBLOT^F$ as well. Hence $-IL$ can be solved in $FCOM^A$ but not in $OBLOT^F$.

Again in [6], the authors gave a problem $SRO$ or Shrinking Rotation which is solvable in $OBLOT^F$, but not solvable in $FCOM^S$ hence in $FCOM^A$. Hence our theorem is proved.

### 4.3 Independent Oscillating Problem

It has already been proved that there exists a problem which is not solvable in $FSTA^F$, hence in $FSTA^A$ but solvable in $FCOM^S$, i.e., $-IL$ problem. In this section we introduce the *Independent Oscillating Problem* defined in Definition [3] which is solvable in $FSTA^A$ but not in $FCOM^S$.

![Fig. 3: Initial configuration of Independent Oscillating Problem](image)

**Definition 3.** *Independent Oscillating Problem (IOP):* Let three robots be placed initially on a straight line $L$ at different positions on the plane arbitrarily.
Let $r_m$ be the robot that lie in between the remaining two terminal robots $r_1$ and $r_2$ (Figure 3). For $i=1,2$, let $x_i$ be the initial distance between $r_i$ and $r_m$. The problem requires that for each $i=1,2$, whenever $r_i$ is activated, then if it is at distance $x_i$ from $r_m$, it moves away $x_i$ distance from $r_m$ along $L$ and, if it is at distance $2x_i$ from $r_m$, it moves closer $x_i$ distance towards $r_m$ along $L$. More precisely, for $i=1,2$, let $d_i(t)$ denote the distance between robots $r_m$ and $r_i$ at time $t$. The Independent Oscillating Problem requires each robot $r_i$, starting from an arbitrary distance $d_i(t_0) > 0$ at time $t_0$, to move so that there exists a monotonically increasing infinite sequence of time instances $t_0, t_1, t_2, \ldots$ such that $d_i(t_{2k}) = d_i(t_0)$ and $d_i(t_{2k+1}) = 2d_i(t_0)$ for all $k = 1, 2, 3, \ldots$, and $\forall h', h'' \in [t_{2k}, t_{2k+1}]$ and $h' < h''$, $d_i(h') \leq d_i(h'')$ and $\forall h', h'' \in [t_{2k-1}, t_{2k}]$ and $h' < h''$, $d_i(h') \geq d_i(h'')$.

Algorithm 3: Algorithm AlgoIOP for IOP in $\text{FSTA}_A^4$ executed by each robot $r$, initially $r$.light $\leftarrow$ NIL;

1. $M =$ position of the robot $r_m$;
2. $C =$ position of the robot $r$;
3. if $r = r_m$ then do not do anything
4. else
5. if $r$.light = NIL then
6. Set the r.light to RED;
7. Move away $CM$ distance along $L$ from the closest robot;
8. if $r$.light = RED then
9. Set the r.light to NIL;
10. Move closer $CM$ distance along $L$ to the closest robot;

(a) When the internal light of the terminal robots are NIL

(b) When the internal light of the terminal robots are RED

Fig. 4: Illustration of Algorithm AlgoIOP

Description and Correctness of the Algorithm AlgoIOP The internal light used by the robots can take two colors: NIL, RED. Initially all robots have their internal lights set to NIL. A robot $r$ when activated takes a snapshot of its surrounding in the LOOK phase and recognises itself either as the middle robot $r_m$ or as a terminal robot $r_i$. If it recognises itself as $r_m$ then it does nothing. Moreover, at any time instance the this robot stays in the middle according to the algorithm. So whenever $r_m$ gets activated, it neither changes its position nor its internal color. When a terminal robot gets activated for the first time
then it recognises itself as a terminal robot from the snapshot taken in $LOOK$ phase. From the internal light it decides whether to move closer to the middle robot or move away from the middle robot. Let $C, M$ be the position of the current terminal robot and $r_m$ in a snapshot. If the internal light is $NIL$ then it determines that in this round it has to move away $CM$ distance from the middle robot along $L$, and if the internal light is $RED$ then it determines that in this round it has to move closer $\frac{CM}{2}$ distance towards the middle robot along $L$.

From the above discussion it is easy to observe that $IOP$ is solvable by algorithm $AlgoIOP$ in $FSTA^A$. We record this result in Lemma 4.

**Lemma 4.** \( \forall R \in R_3, IOP \in FSTA^A(R) \).

**Lemma 5.** \( \exists R \in R_3, IOP \notin FCOM^S(R) \).

**Proof.** If possible let there be an algorithm $A$ which solves the $IOP$ in $FCOM^S$. If both of the terminal robots do not move ever, then it contradicts the correctness of the algorithm $A$. Let at $k^{th}$ round $r_1$ makes the first non null movement, according to the problem that is, to move $x_1$ distance away from $r_m$ along $L$. If the scheduler activates only $r_1$ at both the $k^{th}$ and $(k+1)^{th}$ round, then $r_1$ will not be able to differentiate between the situation in $k^{th}$ round and $(k+1)^{th}$ round and in round $k+1$, it moves $2x_1$ distance away from $r_m$ along $L$, because the distances of the robots from each other at any particular round is arbitrary and external lights of $r_2$ and $r_m$ are same in the both $k$ and $(k+1)^{th}$ round. This contradicts the correctness of $A$. \( \Box \)

**Lemma 6.** \( \exists R \in R_3, -IL \notin FSTA^F(R) \).

**Lemma 7.** \( \forall R \in R_3, -IL \in FCOM^S(S) \).

Next in [6] Lemma 6 and 7 are shown regarding the $-IL$ problem which prove that $-IL$ problem can not be solved in $FSTA^F$, hence in $FSTA^A$ but can be solved in $FCOM^S$. Also in this paper we have shown that $-IL$ is solvable in $FCOM^A$. Therefore from Lemmas 4, 6 and 7 we can conclude:

**Theorem 6.** $FCOM^S \parallel FSTA^A$.

Also, given the fact that $IOP$ cannot be solved in $FCOM^S$ implies that $IOP$ cannot be solved in $FCOM^A$, we get the following result:

**Theorem 7.** $FCOM^A \parallel FSTA^A$.

### 5 Conclusion

In this paper we have extended the study of computational relationship between models of [6] under asynchronous scheduler. It has only been recently proved in [6] that $LUMTA \parallel OBLOT^F$, which had been an long-standing open problem. In this paper we have further refined that result by proving that $FCOM^A \parallel OBLOT^F$, and $FSTA^A \parallel OBLOT^F$, as $FCOM^A$ and $FSTA^A$ are sub-models
of $LUMI^A$, with either only the power of communication or memory compared to the power of both communication and memory of $LUMI^A$. In fact we have gone one step further and even proved that $FCOM^A \perp FSTA^F$. This is in contrast to the result $FCOM^F > FSTA^A$. Perhaps the most significant contribution of our paper is to find out the exact computational relationship between $FSTA^A$ and $FCOM^A$. In this paper we have proved that, $FSTA^A \perp FCOM^A$, proving that the finite memory model and the finite communication model are incomparable under asynchronous scheduler. Still there are many relations whose nature is still yet to be resolved. The relations that are yet to be resolved are:

1. $OBLOT^S \iff OBLOT^A$
2. $FSTA^S \iff FSTA^A$
3. $FSTA^A \iff OBLOT^S$
4. $FCOM^S \iff FCOM^A$
5. $FCOM^A \iff OBLOT^S$

The first of these questions is answered as $OBLOT^S > OBLOT^A$ in graph domain [3]. But in the background of continuous setting, where the robots can detect multiplicity, the question is yet to be resolved. The rest of the questions remain open for future investigations. Also we assumed in our setting rigid movements and the presence of chirality. It would be interesting to explore the nature of the relations by changing one or both the assumptions. In the papers [2] and [3] characterization of some of the relations have been done in graph environment or discrete setting. Also, it would be interesting to characterize the remaining relations of the model in discrete setting.

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