Study on acc/dec algorithm based on piece-wise polynomial in CNC

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Abstract. In order to improve the stability of feed movement in high speed CNC system, the feedrate planning algorithm based on piece-wise polynomial function was proposed. The flexible transition of feedrate was realized through maintaining linear continuous jerk. The principle of the proposed algorithm was introduced and the method to generate smooth motion profile based on the proposed algorithm was presented. The proposed algorithm is simple and it can be applied in acceleration/deceleration before interpolation in high speed feed movement to improve the stability of it. The proposed algorithm was applied in multi-contour high speed processing and the result indicated that it could improve the stability of large-scale parts motion.

Introduction

In the CNC processing, the feedrate of the controlled device can not be adjusted abruptly because of the mechanical and electrical inertia from the CNC system, driving system and controlled device. In order to avoid shock, losing step, over travel, vibration of the controlled device, the feedrate of it should be modified smoothly according to acceleration and deceleration (acc/dec) mode. The conventional CNC has the linear and exponential acc/dec function only, which result in abrupt acceleration change during the CNC operation and cause excessive vibration in the processing. The advanced CNC has the S curve acc/dec ability [1-3] currently resulting the continuous acceleration but discontinuous jerk change during CNC processing [4]. The jerk change of the CNC system reflects the change of its kinetic energy with processing time [5,6]. In CNC processing especially high speed machining, in order to reduce the moving parts’ especially the large-scale moving parts’ shock and impact, the kinetic energy of the whole CNC system should be continuous change with time. When the feedrate of moving parts is high, its jerk besides its acceleration should be continuous change with processing time [7-9]. Intense vibration can be found during the large-scale parts moving in S curve acc/dec mode. The acc/dec algorithm based on the trigonometric series was proposed by Guo et al. to realize the continuous change of jerk during CNC processing, but the complicated calculation of it can not meet the real time requirement of servo control. So the acc/dec process should be pre-treated, and it is realized with look-up table and interpolation calculation in operation, moreover, the Guo’s algorithm has large truncation error, and it will effect the smoothness of acc/dec process. A novel acc/dec algorithm based on piece-polynominal was proposed in this paper to realize the continuous change of jerk during CNC operation, and the detail of it was given.

Acc/dec Algorithm Based on Piece-polynominal

Smooth feedrate change, continuous acceleration, the start and end feedrate equalling to the desired value, the start acceleration equalling to zero should be meet on constructing acc/dec algorithm [10]. The whole acc/dec course includes acceleration, constant feedrate and deceleration phase. As indicated in figure 1, the proposed acc/dec algorithm include 15 phases, of which from 1st to 7th phases are acceleration, from 9th to 15th phases are deceleration and the left one is constant feedrate phase.

For brevity, all the variables in the proposed algorithm were explained here. ①j,a,v,s indicates jerk, acceleration, feedrate and displacement respectively; ②t1…t15 represents the absolute time of the end of each acc/dec phase respectively; ③T1…T15 indicates the run time of each acc/dec phase...
respectively; \( j_1 \ldots j_{15} \) were the jerk of each end of acc/dec step respectively; \( a_1 \ldots a_{15} \) were the acceleration of each end of acc/dec step respectively; \( v_1 \ldots v_{15} \) were the feedrate of each end of acc/dec step respectively; \( l_1 \ldots l_{15} \) were displacement of each acc/dec phase respectively; \( \tau, \tau \) were absolute time variable and relative time variable; \( v_s, v_e \) were the start and end feedrate of the whole acc/dec course, \( R, J, A, v_f \) indicates change rate of jerk, acceleration limit of machine tool, the acceleration maximum and maximal feedrate respectively.

As shown in figure 1, the jerk, acceleration, feedrate and displacement of each acc/dec phase can be written as the expression (1) to (4) respectively.

\[
j(\tau) = \begin{cases} 
R\tau_1 & 0 \leq \tau_1 \leq \tau_i \\
J & \tau_i \leq \tau_2 \leq \tau_5 \\
J - R\tau_3 & \tau_5 \leq \tau_3 \leq \tau_8 \\
0 & \tau_8 \leq \tau_4 \leq \tau_4 \\
-R\tau_5 & \tau_4 \leq \tau_6 \leq \tau_6 \\
-J & \tau_6 \leq \tau_7 \leq \tau_7 \\
-J + R\tau_9 & \tau_7 \leq \tau_8 \leq \tau_8 \\
0 & \tau_8 \leq \tau_9 \leq \tau_9 \\
-J & \tau_9 \leq \tau_{10} \leq \tau_{10} \\
-J + R\tau_{11} & \tau_{10} \leq \tau_{11} \leq \tau_{11} \\
0 & \tau_{11} \leq \tau_{12} \leq \tau_{12} \\
R\tau_{13} & \tau_{12} \leq \tau_{13} \leq \tau_{13} \\
J & \tau_{13} \leq \tau_{14} \leq \tau_{14} \\
J - R\tau_{15} & \tau_{14} \leq \tau_{15} \leq \tau_{15} 
\end{cases}
\]

\[
a(\tau) = \begin{cases} 
\frac{R\tau_1^2}{2} & 0 \leq \tau_1 \leq \tau_i \\
RT_1^2/2 + J\tau_2 & \tau_i \leq \tau_2 \leq \tau_5 \\
RT_3^2/2 + JT_2 + JT_3 - R\tau_3^2/2 & \tau_5 \leq \tau_3 \leq \tau_8 \\
A & \tau_8 \leq \tau_4 \leq \tau_4 \\
A - R\tau_5^2/2 & \tau_4 \leq \tau_6 \leq \tau_6 \\
A - RT_5^2/2 - J\tau_6 & \tau_6 \leq \tau_7 \leq \tau_7 \\
A - RT_9^2/2 - JT_6 - J\tau_7 + R\tau_7^2/2 & \tau_7 \leq \tau_8 \leq \tau_8 \\
0 & \tau_8 \leq \tau_9 \leq \tau_9 \\
-A & \tau_9 \leq \tau_{10} \leq \tau_{10} \\
-A - RT_{10}^2/2 + J\tau_{10} & \tau_{10} \leq \tau_{11} \leq \tau_{11} \\
-A + RT_{13}^2/2 + JT_{13} + J\tau_{13} - R\tau_{13}/2 & \tau_{13} \leq \tau_{14} \leq \tau_{14} \\
-A + RT_{15}^2/2 + JT_{14} + J\tau_{15} - R\tau_{15}/2
\end{cases}
\]
Realization of the Proposed Algorithm

The $J$, $A$ value were determined according to the dynamical performance of CNC machine tool. The feedrate $v_{\text{max}}$ was selected based on dynamical performance of CNC machine tool, geometrical property of pathway to be traveled and requirements for processing accuracy, then $t_v = v_{\text{max}} / A$ and $t_a = A / J$ were calculated. When $t_v > t_a$, select the change rate of jerk $R$, and $t_j = J / R$ was calculated to enable $t_v > t_j$; When $t_v \leq t_a$, select the change rate of jerk $R$, and $t_j = J / R$ was calculated to enable $t_v < t_j$, the smooth feedrate profile can be generated under two kinds of case mentioned above.

For simplifying discussion, the start and end feedrate of the CNC operation were set to zero, in this case, the acc/dec curve exhibits symmetry.
Fig. 2 Motion profile for case \( l=12\text{mm} \)

Fig. 3 Motion profile for case \( l=8.5\text{mm} \)

Fig. 4 Motion profile for case \( l=6\text{mm} \)

Fig. 5 Motion profile for case \( l=0.4\text{mm} \)

Fig. 6 Motion profile for case \( l=20\text{mm} \)

Fig. 7 Motion profile for case \( l=14\text{mm} \)

\( T_v > t_a > t_j \) Case. \( t_v > t_a > t_j \) indicate it is possible to attain acceleration maximum with constant jerk and to achieve maximal feedrate with constant acceleration, in this case, there exists three critical value of distance of run \( s_{cr1}, s_{cr2}, s_{cr3} \) corresponding to no constant feedrate phase just right; no constant feedrate and acceleration phase; no constant feedrate, acceleration and jerk phases respectively, \( s_{cr1}, s_{cr2}, s_{cr3} \) can be calculated according to formula (1) to (4) and the corresponding motion conditions.

For instance, in the CNC operation, the \( v_{fmax}=100\text{mm/s}, A=2000\text{mm/s}^2, J=60000\text{mm/s}^3 \) and \( R=6000000\text{mm/s}^4 \) were specified, the \( s_{cr1}=9.13\text{mm}, s_{cr2}=7.51\text{mm}, \) and \( s_{cr3}=0.48\text{mm} \) can be obtained. The smooth moving profile for \( l=12\text{mm}, 8.5\text{mm}, 6\text{mm}, 0.4\text{mm} \) were generated in figure 2 to 5.

\( T_v \geq t_a \leq t_j \) Case. The CNC operation can reach attainable maximal acceleration without constant jerk and reach attainable maximal feedrate without constant acceleration in the case. Further more, it can be divided into the case with and without phase 8th according to the length of travel of CNC operation great than or less than the critical value \( s_{cr4} \). The critical value \( s_{cr4} \) corresponds to the CNC acc/dec process without constant feedrate just right, and can be calculated combining with formula (1) to (4). For instance, in the CNC operation, the feedrate \( v_{fmax}=100\text{mm/s}, \) acceleration \( A=4000\text{mm/s}^2, \) jerk \( J=60000\text{mm/s}^3, \) change rate of jerk \( R=6000000\text{mm/s}^4 \), the critical value \( s_{cr4}=17.47\text{mm} \) was solved with these conditions. The smooth moving profile for \( l=20\text{mm}, 14\text{mm} \) were given in figure 6 to 7.
Summary
The continuous evolution of jerk can be realized with the proposed acc/dec algorithm based on piece-wise polynomial. The proposed algorithm needs simple calculation and can realize the flexible evolution of feedrate during the acc/dec operation. So the proposed one can be applied in the acc/dec procedure before interpolation in high speed machining, and the vibration or shock resulting from high speed cutting can be reduced greatly. The proposed algorithm has been applied in acc/dec before interpolation in one kind of the home-made large-scale multi-contour cutting machine tool, the corresponding feedrate stationarity was improved significantly.

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