Comparison of optimization methods for hybrid seven-bend-achromat lattice design

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Abstract. Generally, for a hybrid multi-bend-achromat (MBA) lattice with fixed linear optics, there is little potential to further optimize the nonlinear dynamics due to limited free knobs. To obtain a hybrid MBA lattice with better nonlinear dynamics performance, it is better to consider some indicators of nonlinear dynamics as objective functions in designing the linear optics using an optimization algorithm. In this paper, integral strengths of sextupoles and natural chromaticities are used as the nonlinear dynamics indicators, and different optimization methods with both or either of the two indicators are carried out and compared. As an example, a hybrid 7BA lattice with an energy of 2.4 GeV is designed towards an emittance of less than 70 pm·rad.

1. Introduction
In the hybrid multi-bend-achromat (MBA) lattice [1], there are three families of sextupoles located in the pair of dispersion bumps. Since two families are reserved to correct chromaticities to desired values, there is only one free knob left. Thus, for a hybrid MBA lattice with fixed linear optics, the potential for further nonlinear dynamics optimization is limited, though the sextupoles can be grouped into more families within some lattice cells. To obtain a hybrid MBA lattice with better nonlinear dynamics performance, we can explore the diversity of linear optics solutions and take some factors related to nonlinear dynamics into consideration during the linear optics design. So in designing the linear optics using multi-objective genetic algorithm (MOGA) or multi-objective particle swarm optimization (MOPSO), nonlinear dynamics indicators can be included as objective functions so as to benefit the following nonlinear optimization.

Natural chromaticity can be taken as a nonlinear dynamics indicator, since a very large natural chromaticity usually means serious nonlinear dynamics. For example, in a lattice design for ALS-U, the sum of natural chromaticities was used as an objective function [2]. Sextupole strength can also be taken as a nonlinear dynamics indicator, and a weaker value is usually preferred. For the hybrid MBA lattice, the sum of the integral strengths of three families of sextupoles can be considered as an objective function in the linear optics design. This paper will study the effectiveness of these two indicators in designing a hybrid 7BA lattice, and compare different optimizations including both or either of the two indicators. The designed lattice has the same energy as that of HALS [3, 4], a new diffraction-limited storage ring (DLSR) light source proposed by NSRL.
2. Comparison of optimization methods

We will apply MOGA to the linear lattice design of a hybrid 7BA lattice, and carry out optimizations with different objective functions. In the objective functions of the optimizations, both or either of two nonlinear dynamics indicators, natural chromaticities and integral strengths of sextupoles, will be considered. The decision variables in the optimizations include strengths of quadrupoles and combined-function bends, lengths of drifts and bends, bending angles of bends and dipole field gradients of longitudinal gradient bends. To study and compare the optimizations, a 2.4 GeV DLSR, consisting of 24 identical hybrid-7BA lattice cells, will be designed with a circumference of 576 m.

2.1. Method 1: both natural chromaticities and integral strengths of sextupoles considered

In the first optimization method for the hybrid 7BA lattice design, both natural chromaticities and integral strengths of sextupoles are considered as nonlinear dynamics indicators. There are three objective functions to be optimized:

- the natural emittance $\epsilon_{nat}$,
- the sum of the absolute values of natural chromaticities, $|\xi_{sum}| = |\xi_x| + |\xi_y|$,
- the sum of the integral strengths of three families of sextupoles, $|I_{sum}| = |I_{SD1}| + |I_{SF}| + |I_{SD2}|$.

Since there are three families of chromatic sextupoles in the lattice, $|I_{sum}|$ can not be directly calculated. Inspired by the chromatic sextupole pair optimization method [5], the chromaticity correction can be divided into two parts, contributed by two pairs of sextupoles, (SF, SD1) and (SF, SD2). If the two contributions are determined, then $|I_{sum}|$ can be calculated. We assume that 2/3 of the chromaticity correction is contributed by (SF, SD1) and 1/3 by (SF, SD2), because we found that a larger contribution from (SF, SD1) was better for nonlinear dynamics performance. To obtain desired solutions, two constraints are set: (1) the horizontal and vertical phase advances between the pair of dispersion bumps are roughly equal to $(3\pi, \pi)$; (2) the integer parts of transverse tunes are set to $(57, 20)$.

![Figure 1. Distribution of the objective function values of solutions of the last generation.](image)

A MOGA with a population of 10,000 was run for 500 generations, and the objective function values of solutions of the last generation were obtained, as shown in Fig. 1. Tens of solutions with different objective function values were taken from these solutions obtained, and then their
Figure 2. Linear optical functions and magnet layout of the selected lattice. In the magnet layout, bends are in blue, quadrupoles in red, sextupoles in yellow and octupoles in brown.

dynamic apertures (DAs) and dynamic momentum apertures (MAs) were optimized. After that, several better solutions were selected. We found that solutions with very small $|I_{sum}|$ or very small $|\xi_{sum}|$ did not necessarily have both good DAs and good dynamic MAs.

Fig. 2 shows the linear optical functions and magnet layout of one selected lattice solution with $\epsilon_{nat}$ of 63 pm-rad. For the selected lattice, the DA and dynamic MA were further optimized using MOPSO, where three families of sextupoles and one family of octupole adjacent to the focusing sextupole were employed. The chromaticities were corrected to (4, 3). The optimized DA is shown in Fig. 3, where the part with $y > 5$ mm is not presented. We can see that the horizontal DA is large, about 15 mm. Fig. 4 shows momentum dependent tune footprints, with on-momentum transverse tunes of (57.19, 20.19). The horizontal tune crosses the half-integer resonance line at up to -4.3%.

Figure 3. DA of the selected lattice, tracked for 1024 turns.

2.2. Method 2: only natural chromaticities considered

Then we present the second optimization method, where only $|\xi_{sum}|$ is considered as a nonlinear dynamic indicator. The objective functions include $\epsilon_{nat}$, $|\xi_{sum}|$ and the absolute value of the dispersion function at the long straight section. The third one is required in this optimization.
Figure 4. Momentum dependent tune footprints of the selected lattice.

Figure 5. Distribution of $\epsilon_{\text{nat}}$ and $|\xi_{\text{sum}}|$ of solutions of the last generation. Blue circles are the studied solutions.

Table 1. Values of $\epsilon_{\text{nat}}$, $|\xi_{\text{sum}}|$ and $|I_{\text{sum}}|$ for the solutions of the first and second methods.

| Lattices | $\epsilon_{\text{nat}}$ (pm-rad) | $|\xi_{\text{sum}}|$ | $|I_{\text{sum}}|$ (m$^{-2}$) |
|----------|-------------------------------|-----------------|-------------------|
| M1       | 63.0                          | 140.0           | 77.2              |
| M2-1     | 51.7                          | 138.1           | 96.6              |
| M2-2     | 53.7                          | 131.9           | 102.1             |
| M2-3     | 58.9                          | 128.9           | 102.6             |
| M2-4     | 64.4                          | 128.4           | 105.8             |

The optimization was also carried out using MOGA, and the distribution of $\epsilon_{\text{nat}}$ and $|\xi_{\text{sum}}|$ of solutions of the last generation is shown in Fig. 5. Four solutions with different $\epsilon_{\text{nat}}$ and $|\xi_{\text{sum}}|$, marked with blue circles in the figure, were studied and compared.

Table 1 lists the values of $\epsilon_{\text{nat}}$, $|\xi_{\text{sum}}|$ and $|I_{\text{sum}}|$ for the four solutions in Fig. 5 (denoted
as M2-1, M2-2, M2-3 and M2-4) as well as the selected lattice of the first method (denoted as M1). Although $|I_{\text{sum}}|$ was not employed in the second method, we still calculated the $|I_{\text{sum}}|$ for the four solutions for a better comparison. Fig. 6 shows the linear optical functions and magnet layout of the M2-1 lattice. MOPSO was also used to optimize the nonlinear dynamics of the four solutions, and the optimization results showed that the strengths of sextupoles and octupoles of these four lattices were larger than that of the M1 lattice.

Fig. 7 and Fig. 8 show the optimized DAs and momentum dependent tune footprints of the four lattices as well as the M1 lattice. For the second method, the DA and local dynamic MA of the M2-1 lattice are larger than those of the other three lattices, though the M2-1 lattice has the lowest emittance. A possible reason is that the $|I_{\text{sum}}|$ of the M2-1 lattice is weaker than that of the other three lattices, as shown in Table 1. So $|I_{\text{sum}}|$ is a better nonlinear dynamics indicator than $|\xi_{\text{sum}}|$ for the hybrid 7BA lattice. Besides, the DA of the M1 lattice is larger than that of the M2-1 lattice, but the latter has a lower emittance.

We also carried out a third optimization method with only $|I_{\text{sum}}|$ considered as the nonlinear dynamics indicator. The lattice solutions obtained by the third method generally have larger DAs and dynamic MAs than those of the second method. Compared to the first method, the third method generally have smaller DAs, but the difference is small. The first and the third methods have almost the same dynamic MAs. So the first optimization method is better than the second and the third ones for the hybrid 7BA lattice design.
Figure 8. Momentum dependent tune footprints for the four lattices of the second method compared to the M1 lattice.

3. Conclusion
To explore the potential for improving the nonlinear dynamics performance of the hybrid 7BA lattice, nonlinear dynamics indicators are considered as objective functions in designing the linear optics using an optimization algorithm. Different optimization methods with both or either of the two indicators, $|\xi_{sum}|$ and $|I_{sum}|$, have been carried out and compared. The results show that if only one indicator is considered, the method with $|I_{sum}|$ can obtain larger DA and dynamic MA than that with $|\xi_{sum}|$. And the method with both $|\xi_{sum}|$ and $|I_{sum}|$ can obtain better nonlinear dynamics performance than that with $|\xi_{sum}|$ or $|I_{sum}|$, which is recommended for the hybrid 7BA lattice design. During the study of the optimization methods, a 2.4 GeV DLSR was designed with a natural emittance of 63 pm-rad, and the DA was large and the dynamic MA at the long straight section was larger than 4% without crossing the half-integer resonance line.

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References
[1] Farvacque L et al. 2013 A low-emittance lattice for the ESRF Proc. IPAC’13 (Shanghai) p 79
[2] Steier C et al. 2013 Lattice studies for a potential soft X-ray diffraction limited upgrade of the ALS Proc. IPAC’13 (Shanghai) p 258
[3] Wang L et al. 2018 Hefei advanced light source: a future soft X-ray diffraction-limited storage ring at NSRL Proc. IPAC’18 (Vancouver) p 4598
[4] Bai Z, Yang P, Yang Z, Li W and Wang L 2018 Design of the second version of the HALS storage ring lattice Proc. IPAC’18 (Vancouver) p 4601
[5] Bai Z, Jia Q, Li W, Wang L and Zhang Q 2013 Chromatic sextupole pair optimization methods for enlarging dynamic aperture Proc. IPAC’13 (Shanghai) p 2555