We discuss an exact time dependent $O(3)$ symmetric solution with a horizon of the 5d AdS classical gravity equations searching for a 4d boundary theory which would correspond to expanding gauge theory matter. The boundary energy-momentum tensor and entropy density are computed. The boundary metric is the flat Friedmann one and any time dependence on the boundary is incompatible with Minkowski metric. At large times when curvature effects are negligible, perfect fluid behavior arises in a natural way.
1 Introduction

It is of interest to search for 5d gravity duals \[1\] of time dependent phenomena in 4d gauge theories \[2,3,4,5,6\]. These could, for example, serve as prototypes of the dynamics of heavy ion collisions. For quasi-static phenomena in strongly coupled matter gauge-gravity duality picture has already produced lots of interesting results, for viscosity \[7,8,9\], jet energy loss \[10,11,12,13\] and for photon production \[14\].

One can model collisions of large nuclei at very high energies by taking the transverse size and collision energy to be effectively infinite so that the dynamics will be invariant under longitudinal \((x^1)\) Lorentz transformations. Natural variables then are \(\tau = \sqrt{t^2 - x_1^2}\), \(\eta = 1/2 \log(x_1/t)\) and there will be no dependence on \(x_2, x_3\). The 5d gravity dual of this 4d physics then has the metric \[2\]

\[
\begin{align*}
    ds^2 &= \ell^2 z^2 \left[ -a(\tau, z) d\tau^2 + \tau^2 b(\tau, z) d\eta^2 + c(\tau, z)(dx_2^2 + dx_3^2) + dz^2 \right].
\end{align*}
\]

The unknown functions \(a, b\) and \(c\) would be determined as solutions of 5d AdS gravity equations and 4d physics on the boundary at \(z = 0\) could be computed. Several interesting and suggestive results have been obtained by studying large-\(\tau\) behavior, without using an exact solution. In particular, perfect fluid behavior for the energy density \(\epsilon(\tau) \sim 1/\tau^{4/3}\) is singled out by absence of curvature singularities \[2\], also evidence for the viscosity/entropy density ratio \(\eta/s = \hbar/4\pi\) has been found by studying corrections to the leading large-\(\tau\) behavior \[6\].

To clarify the issue we will here restore the 3d spherical symmetry broken by the metric \[1\]. For this case we found an exact solution, which, in fact, proved to be a known solution \[15,16,17\] in different coordinates. The boundary theory is the isotropic and homogeneous cosmological FRW metric with an arbitrary scale factor \(r(t)\), it is thus rather big than little bang. In big bang energy density decreases because space expands. Here we have no 4d gravity to determine \(r(t)\), but we shall fix it so that the comoving energy density decreases as that in the center of a spherical little bang. Little bang takes place in Minkowski space and the energy density in the center, in the rest frame, decreases because matter flows outwards. To permit one to go away from the rest frame or the comoving frame would require a more complicated ansatz for the 5d metric than what we use in this paper.

2 Similarity expansion in relativistic hydrodynamics

Experimental measurements of QCD matter in relativistic heavy ion collisions have shown that the matter flows like a nearly ideal fluid. As discussed above, for large nuclei and energies the expansion can be approximated by a 1+1 dimensional longitudinal boost invariant similarity flow of 3-dimensionally thermalised matter,

\[
    v^i(t, x^1, x^2, x^3) = \frac{x^i}{t} \delta_{i1}. \quad i = 1, 2, 3.
\]

Let us now generalise this to radial flow in \(d - 1\) spatial dimensions, keeping \(d\) arbitrary for the moment.
A radial similarity flow is defined by the velocity field
\[ v(t, x) = \frac{x}{t}, \quad u^\mu = (\gamma, \gamma v) = \frac{x^\mu}{\sqrt{-x^2}}, \quad \mu = 0, 1, \ldots, d - 1, \quad -x^2 = -\eta_{\mu\nu}x^\mu x^\nu = t^2 - x^2 \equiv \tau^2. \]

The energy-momentum tensor is
\[ T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p \eta_{\mu\nu} = (\epsilon + p)\frac{x_\mu x_\nu}{\tau^2} + p \eta_{\mu\nu} \]
and the conservation equation \( \partial_\mu T^{\mu\nu} = 0 \), split in components parallel and perpendicular to \( u^\mu \), can be written in the form
\[ x^\mu \partial_\mu \epsilon + (d - 1)(\epsilon + p) = 0, \]
\[ \left( \eta^{\mu\nu} - \frac{x^\mu x^\nu}{x^2} \right) \partial_\mu p = 0. \]

The second is solved by \( p(t, x) = p(\tau) \) and, if the conformally invariant equation of state \( p = \epsilon/(d - 1) \) is assumed, the first one becomes
\[ \tau \epsilon'(\tau) + d \epsilon(\tau) = 0 \quad \Rightarrow \quad \epsilon(\tau) = \frac{\epsilon_0}{\tau^d} = \frac{\epsilon_0}{(t^2 - x^2)^{d/2}}. \]

The constant \( \epsilon_0 \) here is given by the initial conditions of the flow. Formally, it could also be fixed by including a source term on the RHS of (5).

At any fixed time \( t \) the flow thus consists of a spherical shell with infinite energy density moving radially with light velocity. In the interior the flow pattern is of the similarity type (3) with energy density only depending on the proper time. In the local rest frame \( x = 0 \)
\[ \epsilon(t) = \epsilon(\tau) = \frac{\epsilon_0}{\tau^d}. \]

Our goal thus is to find the gravity dual of radially expanding thermalised matter with the energy-momentum tensor (4) and comoving energy density (8).

One may try to express \( T_{\mu\nu} \) in terms of radial rapidity coordinates \( (\tau, \eta_r, \theta, \phi) \), \( t = \tau \cosh \eta_r, r = \tau \sinh \eta_r \); \( \theta, \phi \) are the usual spherical angles. Then \( x^\mu \rightarrow (\tau, 0, 0, 0) \) but \( \eta_{\mu\nu} \rightarrow g_{\mu\nu} \) with complicated non-diagonal terms.

In the above matter is flowing in Minkowski space; compare it now with space expanding as in the flat FRW metric diag\((-1, r^2(t), \ldots, r^2(t))\). Then
\[ \epsilon'(t) + \frac{(d - 1)r'(t)}{r(t)}(\epsilon + p) = 0. \]

One observes that if \( r(t) \sim t \) this coincides with (5) evaluated in the local rest frame \( v = 0, x = 0 \). For this behavior of the radius the energy density decreases in the same way for radial flow in Minkowski space and for Hubble flow in the FRW metric. We shall later in Section 5 show how gauge-gravity duality selects just this power. This is in analogy to the selection (2) of the adiabatic power \( \epsilon \sim 1/\tau^4/3 \) for the flow (2).
For completeness, the standard dissipative tensor for the flow (3) is

$$\Delta T_{\mu\nu} = -\zeta \frac{d - 1}{\sqrt{-x^2}} \left( \eta_{\mu\nu} - \frac{x_{\mu} x_{\nu}}{x^2} \right).$$  \hspace{1cm} (10)$$

The shear part of the dissipative tensor vanishes and thus studying this flow does not give a handle on the shear viscosity. Eq.(7) becomes

$$\tau \epsilon' (\tau) + d \epsilon (\tau) - \zeta (d - 1)^2 / \tau = 0,$$

but for a massless conformal fluid further \(\zeta = 0\). 

### 3 The exact solution

In this article we shall search for gravity duals of spherically expanding systems; the perfect fluid solution was presented in detail in the previous section. The gravity solution thus has to be of the type

$$ds^2 = \frac{\mathcal{L}^2}{z^2} \left[ -a(t, z) dt^2 + b(t, z) (dx_1^2 + dx_2^2 + dx_3^2) + dz^2 \right]$$

$$= g_{MN} dx^M dx^N = \frac{\mathcal{L}^2}{z^2} \left[ g_{\mu\nu}(x, z) dx^\mu dx^\nu + dz^2 \right],$$  \hspace{1cm} (11)$$

where the metric is determined from the 5d AdS gravity equations

$$\mathcal{R}_{MN} - \frac{1}{2} \mathcal{R} g_{MN} - \frac{6}{\mathcal{L}^2} g_{MN} = 0, \quad x^M = (t, x^1, x^2, x^3, z).$$  \hspace{1cm} (12)$$

Here the AdS radius \(\mathcal{L}\) is related to the 5d Newton’s constant by

$$\frac{\mathcal{L}^3}{G_5} = \frac{2N^2_e}{\pi}. \hspace{1cm} (13)$$

The functions \(a(t, z)\) and \(b(t, z)\) in (11) can now be solved from (12) as follows \cite{18}. Using the \(tz\)-component of (12), one can solve \(a(t, z)\) in terms of an arbitrary function \(F_1(t)\):

$$a(t, z) = \frac{F_1(t) |\partial b(t, z)|^2}{b(t, z)}. \hspace{1cm} (14)$$

Using this solution for \(a(t, z)\), one can solve the function \(b(z, t)\) from the \(tt\)-equation in terms of two more arbitrary functions:

$$b(t, z) = F_2(t) z^2 + \frac{F_3(t)}{z^2} - \frac{1}{8F_1(t)}. \hspace{1cm} (15)$$

Finally, using the \(zz\)-equation one gets a differential equation for \(F_1(t)\), \(F_2(t)\) and \(F_3(t)\), from which one of the functions can be eliminated. We specify the remaining two arbitrary functions \(r(t)\) and \(h(t)\) so that the 4d boundary metric is

$$g^0_{\mu\nu} = \text{diag}(-h^2(t), r^2(t), r^2(t), r^2(t)). \hspace{1cm} (16)$$
and the constant of integration $\sqrt{2} z_0$ so that it is the position of the horizon in $z$ when \( r = h = 1 \). One can trivially set $h = 1$ by a choice of time coordinate and the boundary metric then is a flat cosmological FRW metric.

The solution is, abbreviating $r = r(t), h = h(t), r' = r'(t), r'' = r''(t), h' = h'(t)$,

$$a(t, z) = \frac{h^2 r^2}{b(t, z)} \left(1 + \left(\frac{h'r'}{2h^3r} - \frac{r''}{2h^2r}\right) z^2 + \left(\frac{r'^2 r''}{8h^4 r^3} - \frac{r'^4}{16 h^4 r^4} - \frac{h' r'^3}{8h^5r^3} - \frac{1}{4r^4 z_0^4}\right) z^4 \right)^2$$

$$b(t, z) = \frac{r^2}{b(t, z)} \left[\left(1 - \frac{r'^2}{4r^2} z^2\right)^2 - \left(\frac{r''}{4r} - \frac{r'^2}{4r^2}\right) z^4 - \frac{1}{4r^4 z_0^4} z^4 \right]^2, \quad (17)$$

For $r(t) = h(t) = 1$ the solution reduces to the metric [2]

$$ds^2 = \frac{\mathcal{L}^2}{z^2} \left[-\left(1 - \frac{z^4}{4z_0^4}\right) dt^2 + \left(1 + \frac{z^4}{4z_0^4}\right) \left(dx_1^2 + dx_2^2 + dx_3^2 + dz^2\right)\right] \quad (19)$$

which, choosing a new variable $\tilde{z} = z^2/(1 + z^4/4z_0^4)$, can be brought to the standard AdS black hole form

$$ds^2 = \frac{\mathcal{L}^2}{\tilde{z}^2} \left[-\left(1 - \frac{\tilde{z}^4}{z_0^4}\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + \frac{1}{1 - \tilde{z}^4/z_0^4} dz^2\right]. \quad (20)$$

The metrics have a horizon at $z = \sqrt{2} z_0, \tilde{z} = z_0$, with Hawking temperature $T_H = 1/(\pi z_0)$ and with entropy density $A/(4G_5 V_3) = \mathcal{L}^3/(4G_5 z_0^3) = \pi^2 N^2 T_H^3/2$.

### 4 Energy-momentum tensor

To study the dynamics of the boundary theory, we have to find its covariantly conserved energy-momentum tensor. We present two methods for doing this.

For the first [19], expand the $g_{\mu\nu}$ in (11), (17) and (18) near the boundary:

$$g_{\mu\nu}(t, z) = g_{\mu\nu}^{(0)}(t) + g_{\mu\nu}^{(2)}(t) z^2 + g_{\mu\nu}^{(4)}(t) z^4 + ..., \quad (21)$$

where $g^{(0)}$ is the FRW metric in (11) and the rest easy to work out from (17) and (18). Then the energy-momentum tensor can be evaluated using

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} \left[g_{\mu\nu}^{(4)} - \frac{1}{8} g_{\mu\nu}^{(0)} (\text{Tr} g^{(2)})^2 - \frac{1}{2} g^{(2)g_{\mu\nu}^{(2)}} - \frac{1}{4} (\text{Tr} g^{(2)} g_{\mu\nu}^{(2)}) \right]$$

with the result

$$T_{\nu} = g_{\mu\nu}^{(0)} T_{\alpha\nu} = \text{diag}(-\epsilon(t), T_1(t), T_1(t), T_1(t)) \quad (22)$$

$$T_{\nu} = g_{\mu\nu}^{(0)} T_{\alpha\nu} = \text{diag}(-\epsilon(t), T_1(t), T_1(t), T_1(t))"
where, using (13),

\[
\epsilon(t) = T_{tt} = \frac{3N^2_c}{8\pi^2} \left( \frac{1}{z^4_{0}r^4} + \frac{r^4}{4h^4r^4} \right),
\] (24)

\[
T_1^1(t) = \frac{1}{3} \epsilon(t) + \frac{N^2_c}{8\pi^2} \frac{r^2(h' r' - h r'')}{h^5 r^3}.
\] (25)

The trace of \( T_{\mu\nu} \) is

\[
T_\mu = g^{\mu\nu} T_{\mu\nu} = -\epsilon + 3T_1^1 = \frac{3N^2_c}{8\pi^2} \frac{r^2(h' r' - h r'')}{h^5 r^3}
\] (26)

which is just the standard trace anomaly \[20\]. Further, the tensor \( T_{\mu\nu} \) is covariantly conserved, \( \nabla^\mu T_{\mu\nu} = 0 \), leading for \( \nu = 0 \) to

\[
\epsilon'(t) + \frac{3r'}{r} [\epsilon(t) + T_1^1(t)] = 0.
\] (27)

For the second, the same result can be obtained without the expansion of \( g_{\mu\nu}(x, z) \) by noting that, (a) due to the imposed symmetries \( T_{\mu\nu} \) must have the form (23), (b) \( T_{\mu\nu} \) must be conserved as in (27), (c) the trace anomaly is \( L^3/(64\pi G_5) (R_{\mu\nu} R_{\mu\nu} - R^2/3) \), which leads to (26). Eliminating \( T_1^1 \) using the anomaly equation leads to an equation for \( \epsilon(t) \) which can be solved to again give the result in (24) and (25).

The boundary energy-momentum tensor can be naturally interpreted to describe massless fluid in a curved background metric. The energy density has two components: \( \epsilon(t) = \epsilon_0(t) + \Delta \epsilon(t) \). The temperature dependent first part \( \epsilon_0(t) \sim r(t)^{-4} \) is the standard behavior of homogeneous radiation in expanding spacetime and the temperature independent part \( \Delta \epsilon(t) \sim r'(t)^4/r(t)^4 \) describes quantum corrections to the matter due to the curved background.

Also the \( T_{ij} \) can be suggestively composed to two parts: \( T_{ij} = p(t) \delta_{ij} - \Delta T_{ij} \), where \( p(t) = 1/3 \epsilon(t) \) is the pressure of the fluid. One may try to interpret \( \Delta T_{ij} \) in terms of bulk viscosity: comparing (25) with the dissipative part (Eq. (10) for \( x = 0 \)) of the energy-momentum tensor, \( \Delta T_{ij} = -\zeta \delta_{ij} \nabla \cdot \mathbf{v} = -3\zeta r'/r \delta_{ij} \) we can identify a time dependent bulk viscosity (take \( h = 1 \))

\[
\zeta(t) = \frac{N_c^2}{24\pi^2} \frac{r' r''}{r^2}.
\] (28)

Bulk viscosity vanishes for a conformal massless system, but here it is precisely the anomaly which gives rise to it. A positive \( \zeta \) implies that entropy increases as can be explicitly verified from our solution.

5 Entropy, gravity dual

The coefficient \( a(t,z) \) in (17) can be written in the form (for \( h = 1 \))

\[
a(t,z) = \frac{r^2}{b(t,z)} \left[ \left( 1 - \frac{z^2}{z^2_{H+}} \right) \left( 1 - \frac{z^2}{z^2_{H-}} \right) \right]^2,
\] (29)
expression for that \( r \) = 4. For a general powerlike behavior \( d \) and
malised supersymmetric Yang-Mills matter. Also, for the value \( R \) of
the equations of motion, we automatically have to study whether the bulk solution is singular or not [2]. Because (11) is a solution to
the standard result

thermore, the metric then is (20) with the Hawking temperature

We have now a bulk theory with a dynamical horizon at \( z_H(t) \). The entropy density \( s(t) \) then is given by the area of the horizon:

\[
s(t)r^3(t) = \frac{S}{V_3} = \frac{\text{Area}}{4G_5V_3} = \frac{1}{4G_5} \int d^3x \sqrt{\gamma} = \frac{\mathcal{L}^3b^3/2}{4G_5z_H^2} = \frac{N_c^2b^3/2(t, z_H)}{2\pi z_H^3},
\]

where \( \gamma \) is the determinant of the metric of the \((x^1, x^2, x^3)\) subspace in (11) and

\[
\frac{b(t, z_H)}{z_H^2} = \frac{4/z_0^4 + |rr'' - r^2 + \sqrt{4/z_0^4 + (r'' - rr'')^2}|^2}{4(rr'' + \sqrt{4/z_0^4 + (r'' - rr'')}}).
\]

For arbitrary \( r(t) \) we thus have the entropy, but the issue of defining a temperature for a dynamical horizon is a very complicated one [21]. It should be defined so as to satisfy the thermodynamic relations \( \epsilon + p = Ts \), \( s(T) = dp/dT \), for thermal energy density and pressure. Further, it is of interest to compute the curvature invariants for the solution (18) in order to study whether the bulk solution is singular or not [2]. Because (11) is a solution to the equations of motion, we automatically have \( \mathcal{R} = -20/\mathcal{L}^2 \), \( \mathcal{R}^{MN}\mathcal{R}_{MN} = 80/\mathcal{L}^4 \). The expression for \( \mathcal{R}_{MNPQ}\mathcal{R}_{MNPQ} \) simplifies to the form

\[
\mathcal{R}_{MNPQ}\mathcal{R}_{MNPQ} = \frac{1}{\mathcal{L}^4} \left\{ 40 + 72 \left[ \frac{z^2}{z_0^2} b(t, z) \right]^4 \right\},
\]

where \( b(t, z) \) is given by (18). The 40 here is the maximally symmetric part \( 2\mathcal{R}^2/(d^2 - d) \) of \( \mathcal{R}_{MNPQ}^2 \).

To calibrate the above expressions note that for \( r = 1 \) we have \( z_H^2 = 2z_0^2 \) and \( b(z_H)/z_H^2 = 1/z_0^2 \), which, using (31), gives \( s = N_c^2/(2\pi z_0^3) \), the standard value for a static horizon. Furthermore, the metric then is (20) with the Hawking temperature \( T = 1/(\pi z_0) \) and one obtains the standard result \( s = \pi^2 N_c^2 T^3/2, p = \pi^2 N_c^2 T^4/8 \) for the pressure of strongly coupled thermalised supersymmetric Yang-Mills matter. Also, for \( r = 1 \) the curvature invariant (33) has the value \( \mathcal{R}_{MNPQ}^2 = 112/\mathcal{L}^4 \) at the horizon.

Our goal is to find the gravity dual of the flow in (11) with a powerlike \( \epsilon(t) \) in (8) with \( d = 4 \). For a general powerlike behavior \( r = (t/t_0)^n \) we have \( r' = nr/t, r'' = n(n - 1)r/t^2 \) so that

\[
\epsilon = \frac{3a}{4t^4} \left( \frac{4t^4}{z_0^4r^4} + n^4 \right), \quad T^1 = \frac{a}{4t^4} \left( \frac{4t^4}{z_0^4r^4} - 3n^4 + 4n^3 \right),
\]

\[
z_H^2(t) = \frac{4t^2}{n^2 - n + \sqrt{4t^4/(z_0^4r^4) + n^2}}
\]

\[
s = \frac{4\pi a}{8t^2} \left[ \frac{4t^4/(z_0^4r^4) + \left( -n + \sqrt{4t^4/(z_0^4r^4) + n^2} \right)^2}{n^2 - n + \sqrt{4t^4/(z_0^4r^4) + n^2}} \right]^{3/2},
\]

where

\[
z_{H+} = \frac{4r^2}{rr'' + \sqrt{4/z_0^4 + (r'' - rr'')^2}}, \quad z_{H+} \equiv z_H(t) < z_{H-}.
\]
where \( a \equiv N_c^2/(8\pi^2) \). All the quantities depend essentially on the combination \( 4t^4/(z^4_0 r^4(t)) \).

The required behavior \( \epsilon = \epsilon_0/t^4 \) is thus obtained when \( n = 1 \), \( r(t) = t/t_0 \). Then

\[
    z_H = \frac{2t}{(4t_0^4/z_0^4 + 1)^{1/4}}, \quad \epsilon = \frac{3a}{4t^4} \left( \frac{4t_0^4}{z_0^4} + 1 \right) = \frac{3\pi^2 N_c^2}{8} \left( \frac{\sqrt{2}}{\pi z_H(t)} \right)^4 = 3p, \quad (37)
\]

\[
    s = \frac{\sqrt{2} \pi a}{t^3} \left( \frac{4t_0^4}{z_0^4} + 1 - 1 \right)^{3/2} = \frac{\pi^2 N_c^2}{2} \left( \frac{\sqrt{2}}{\pi z_H(t)} \right)^2 - \frac{1}{2\pi^2 t^2} \right)^{3/2}. \quad (38)
\]

The gravity dual we searched for thus is given by the metric (17), (18) with \( r(t) = t/t_0 \). This result has the following properties:

- For the boundary flow \( \# \) the scale factor \( \epsilon_0 \) is given by initial conditions. Here this is related to the parameter \( t_0 \) of the dual metric. Writing \( 1/z_0 = \pi T_H \) it appears in the combination \( 4\pi^4 t_0^4 T_H^4 + 1 \), where the factor +1 represents the effect of the curvature of the boundary metric. We want this to be negligible, since the little bang takes place in a flat space. Thus we have to demand

\[
    \pi T_H t_0 = \frac{t_0}{z_0} \gtrsim 1. \quad (39)
\]

Then also the scale factor in \( \{8\} \) has to satisfy \( \epsilon_0 \gtrsim 3a = 3N_c^2/(8\pi^2) \).

- Since we are looking for the gravity dual of a thermalised system, we also have to define the temperature. This is defined by the two conditions \( T = (\epsilon + p)/s = 4p/s \) and \( s = dp/dT \). These can be integrated to give \( p = c T^4, s = 4c T^3 \), where \( c = (s/4)^{1/3}/p^3 \) is a constant, given by Eqs. (38). For the temperature one obtains from \( T = 4p/s \) and Eqs. (38) that

\[
    \sqrt{2} \pi t = \frac{4\pi^4 t_0^4 T_H^4 + 1}{(\sqrt{4\pi^4 t_0^4 T_H^4 + 1 - 1})^{3/2}} \rightarrow \sqrt{2} \pi T_H t_0 \quad \text{if } \pi T_H t_0 \gtrsim 1. \quad (40)
\]

Inserting this to \( s = 4c T^3, \ p = c T^4 \), one sees that the equation of state \( s(t) = \frac{1}{2} \pi^2 N_c^2 T^3(t) \) and \( p(t) = \frac{1}{8} \pi^2 N_c^2 T^4(t) \), appropriate for the matter we are considering, is obtained if \( \pi T_H t_0 = t_0/z_0 \gtrsim 1 \). Thus matter with correct equation of state at a temperature satisfying \( T t = T_H t_0 \) is defined in the boundary theory when (39) holds.

The behaviour \( T t \sim \text{constant} \) is the analogue of the \( T t^{1/3} = \text{constant} \) behaviour for a longitudinally expanding system.

- Eq. (39) is also a natural condition for the initial temperature of a matter system thermalised at \( t_0 \) at the initial temperature \( T_H \), just from the uncertainty principle. For \( t_0 \leq z_0 = 1/\pi T_H \) one is in a pre-thermalisation region in which \( T, \epsilon = 3p \) and \( s \) are formally defined by the above equations but are not related by the correct equation of state. In this pre-thermalisation region curvature effects are large.
• The adiabatic power $\epsilon \sim 1/t^{4/3}$ for the flow in [2] was singled out in [2] by demanding regularity of the invariant $R^2_{MNPQ}$ at the horizon. The same argument works out here, too. Evaluating (33) at the horizon for $r(t) \sim t^n$ shows that it is constant for $n = 1$, grows without bounds $\sim t^{8(n-1)}$ for $t \gg z_0$ when $n > 1$ and grows without bounds $\sim 1/t^{8(1-n)}$ for $t \ll z_0$ when $n < 1$. Thus, if one demands that $R^2_{MNPQ}$ at the horizon be regular when $t \to \infty$ or $t \to 0$, one again finds that $n = 1$, corresponding to $\epsilon(t) = \epsilon_0/t^4$, is the only option.

6 Conclusions

The goal of this work was to develop the framework for studying time dependent systems in the gauge/gravity duality picture in [2, 3, 4, 5, 6] by analysing an exact solution of the 5d AdS gravity equation. The price we had to pay was that more symmetry had to be assumed: we had as the goal 3d radial expansion. It turns out that the boundary metric is the standard cosmological FRW metric with an unknown time dependent radial function $r(t)$. In cosmology one, of course, can determine $r(t)$ from Einstein’s equations when $T_{\mu\nu}$ is given. Since we want to have adiabatically expanding matter in the boundary theory, we determine $r(t) = t/t_0$ so that the correct $T_{\mu\nu}$ for the expanding system is obtained. The uncertainty in $t_0$ corresponds to arbitrariness of the initial conditions of the radial flow, for $t_0 T_0 \gtrsim 1$ the system is thermalised and curvature effects of the boundary metric are negligible.

Conversely, by demanding regularity of $R^2_{MNPQ}$ at the horizon, one can derive the adiabatic exponent in $\epsilon \sim 1/t^4$.

It would be most interesting to also obtain information on the transport coefficients. However, with spherical symmetry, $v = r/t$, shear viscosity does not contribute, Eq. (10), and for the perfect fluid case $r(t) \sim t$ the anomaly, interpreted as bulk viscosity, vanishes.

There are clearly many issues requiring further study. For the first, an exact solution with the symmetries [1] would be very useful. The boundary metric would then be of the form diag($-1, g^2(t), r^2(t), r^2(t)$) [5]. For the second, consequences of the time dependent horizon and its effects on the determination of entropy and temperature should be understood better. The passage from known energy density and pressure, with non-thermal components, and entropy to temperature is the key issue here. For the third, exact solutions with scalar and form fields could possibly give more information.

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