Exclusive production of $B_c$ mesons in $e^+e^-$ colliders

Yan-Bing Wei$^{a,b}$, Run-Hui Li$^c$, Dan Luo$^d$, Cai-Dian Lü$^{a,b}$ and Yue-Long Shen$^d$

$^a$Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, P.R. China
$^b$School of Physics, University of Chinese Academy of Sciences, Beijing 100049, P.R. China
$^c$School of Physical Science and Technology, Inner Mongolia University, Huhhot 010021, P.R. China
$^d$College of Information Science and Engineering, Ocean University of China, Qingdao, Shandong 266100, P.R. China

Abstract

Within the framework of the perturbative QCD approach, we calculate the time-like $B_c B_c(B_c^*)$ form factors $F(Q^2)$ and $A_2(Q^2)$. We include relativistic corrections and QCD corrections, either of which can give about 20% correction to the leading-order contribution, but there are cancellation effects between them. We calculate the cross sections of the $e^+e^- \to B_c^- B_c^+(B_c^{*+})$ processes. The cross sections are enhanced at the $Z$ pole to be $\sigma^{PP}(Q = m_Z) \sim 1.3 \times 10^{-5}$pb and $\sigma^{PV}(Q = m_Z) \sim 2.5 \times 10^{-5}$pb, which are still too small to be detected by proposed $e^+e^-$ colliders such as the Circular Electron Positron Collider.

PACS numbers:
I. INTRODUCTION

The study of the $B_c$ meson is of special interest, since it is the ground state of the doubly heavy-flavored ($\bar{b}c$) system, which is unique in the Standard Model. Both production and decay processes of the $B_c$ meson contain rich heavy quark dynamics, so they are worthy of systematic study. Experimentally, observations of $B_c$ mesons are so far available only at hadron colliders [1]. Hadron colliders only provide limited knowledge of the production of $B_c$ mesons because the total collision energy cannot be well controlled. For $B_c$ production in $e^+e^-$ colliders, the total collision energy can be well controlled and many meaningful observables, such as the angular distributions of final states, could be measured. Despite these advantages, the $B_c$ meson has not yet been observed in $e^+e^-$ colliders [2]. The production rate is too small to have been observed at LEP. Nowadays, several high luminosity $e^+e^-$ facilities are proposed, such as the International Linear Collider (ILC), Circular Electron Positron Collider (CEPC), and Future Circular Collider (FCC-ee). These $e^+e^-$ colliders could provide us with the opportunity to produce $B_c$ mesons, especially at the Z-pole.

There are many studies of $B_c$ meson decays [3–18]. The semi-inclusive production of the $B_c$ meson has also been investigated extensively [19–23]. However, the knowledge of $B_c$ production in exclusive processes is very limited. To deal with hard exclusive processes involving heavy quark-anti-quark systems, two kinds of factorization approaches have been proposed in the literature. One is the non-relativistic QCD (NRQCD) factorization approach [24–27], in which the amplitude of the process can be factorized into the product of the short-distance coefficient and NRQCD matrix-element. In the other approach (called collinear factorization) the amplitude of the production process can be expressed as a convolution of the hard-kernel and the universal light-cone distribution amplitude (LCDA) [28, 29]. The LCDA of a large boosted $B_c$ meson is defined by sandwiching the gauge invariant non-local quark bilinear operators between the vacuum and the $B_c$ state. For the leading-twist LCDA, we have:

$$\langle B_c(1S_0, P)|\langle \bar{b}W_c(\omega n_+)|\gamma_5(W_c^\dagger c)(0)(\omega)|0\rangle = -if_P n_+ \int_0^1 dx \, e^{i\omega n_+ P x} \hat{\phi}_P(x; \mu), \quad (1)$$

$$\langle B_c(3S_1, P, \varepsilon^*)|\langle \bar{b}W_c(\omega n_+)|\gamma_5(W_c^\dagger c)(0)(\omega)|0\rangle = f_V m_V n_+ \varepsilon^* \int_0^1 dx \, e^{i\omega m_+ P x} \hat{\phi}_V(x; \mu), \quad (2)$$

$$\langle B_c(3S_1, P, \varepsilon^*)|\langle \bar{b}W_c(\omega n_+)|\gamma_5(W_c^\dagger c)(0)(\omega)|0\rangle = f_V f_\perp n_+ \varepsilon^* \int_0^1 dx \, e^{i\omega n_+ P x} \hat{\phi}_V(x; \mu), \quad (3)$$
where the Wilson-line
\[
W_c(x) = \text{P} \exp \left[ i g_s \int_{-\infty}^{0} ds \, n_+ A(x + sn_+) \right],
\]
(4)
and \(\hat{\phi}_P(x), \hat{\phi}_V(x)\) and \(\hat{\phi}_T(x)\) are LCDAs of pseudo-scalar, longitudinally polarized and transversely polarized vector \(B_c\) mesons, respectively. Using the “re-factorization” scheme \[30\], the \(B_c\)-meson LCDA has been calculated to next-to-leading order with respect both to the coupling constant and the velocity of the heavy quark \[31–34\].

Collinear factorization breaks down if there exists an endpoint singularity. A solution is to introduce a transverse momentum to the parton, which is called \(k_T\) factorization. The PQCD approach based on \(k_T\) factorization has been employed to study a large number of exclusive processes \[35–39\], including pair production of light mesons \[40–43\]. In the present paper we will employ the PQCD approach to study the exclusive \(B_c\) meson pair production processes, i.e. \(e^+e^- \rightarrow B_c^-B_c^+(B_c^{(*)+})\). The amplitude can be divided into the leptonic part and the hadronic part. The hadronic part is actually the time-like \(B_cB_c\) or \(B_cB_c^*\) form factors and evaluating these form factors is the main object of this work. The form factors will be calculated at the leading power in \(1/Q\) (\(Q\) being the energy of the \(e^+e^-\) pair in the center-of-mass frame), while power corrections from higher-twist LCDAs and quark mass effects will be considered in the future.

This paper is organized as follows. In the next section we calculate time-like \(B_cB_c(B_c^{(*)})\) form factors and cross sections of \(e^+e^- \rightarrow B_c^-B_c^+(B_c^{(*)+})\) processes using PQCD approach. Numerical analysis is presented in the third section. The last section gives a summary.

II. TIME-LIKE FORM FACTORS IN THE PQCD APPROACH

The differential cross section of \(e^+(k')e^- (k) \rightarrow \gamma^*, Z^0 \rightarrow B_c^- (P)B_c^+(P')(B_c^{(*)+})\) is
\[
\frac{d\sigma}{d\Omega} = \frac{|k|}{32\pi^2 Q^3} \frac{1}{4} \sum_{s,s'} |A_\gamma + A_Z|^2,
\]
(5)
where $s$ and $s'$ are the spins of the ingoing electron and positron respectively. The momentum transfer $q = k + k' = P + P'$ and the energy scale $Q = \sqrt{q^2}$. The amplitudes are

$$A_{\gamma}^{PP} = \sum_{q=b,c} \frac{-ie^2 e_q}{Q^2} \bar{v}(k') \gamma_\mu u(k) (P' - P'_\mu) F^q(Q^2),$$

$$A_{\gamma}^{PP} = \frac{i \sqrt{2} G_F m_W^2}{\cos^2 \theta_W} \sum_{q=b,c} \frac{T_3^q - 2e_q \sin^2 \theta_W}{Q^2 - m_Z^2 + i \Gamma_Z m_Z} \bar{v}(k') \gamma_\mu \left(-\frac{1}{2} + 2 \sin^2 \theta_W + \frac{1}{2} \gamma_5\right) u(k)
\times \left(g^{\mu\nu} - \frac{q^{\mu} q^{\nu}}{m_Z^2 - i \Gamma_Z m_Z}\right) (P' - P)_\nu F^q(Q^2),$$

$$A_{\gamma}^{PV} = \sum_{q=b,c} \frac{-ie^2 e_q}{Q^2} \bar{v}(k') \gamma_\mu u(k) \epsilon_{\mu\nu\rho\sigma} \epsilon_{B_Z^c}^{*\nu} P^\rho P^\sigma V(Q^2),$$

$$A_{\gamma}^{PV} = \frac{i \sqrt{2} G_F m_W^2}{\cos^2 \theta_W} \sum_{q=b,c} \frac{1}{Q^2 - m_Z^2 + i \Gamma_Z m_Z} \bar{v}(k') \gamma_\mu \left(-\frac{1}{2} + 2 \sin^2 \theta_W + \frac{1}{2} \gamma_5\right) u(k)
\times \left(g^{\mu\lambda} - \frac{q^{\mu} q^{\lambda}}{m_Z^2 - i \Gamma_Z m_Z}\right) \left\{[T_3^q - 2e_q \sin^2 \theta_W] \epsilon_{\lambda\mu\rho\sigma} \epsilon_{B_Z^c}^{*\nu} P^\rho P^\sigma V^q(Q^2)
- T_3^q [A_1^q(Q^2) Q^2 \epsilon^{*\lambda} + A_2^q(Q^2) \epsilon^{*} \cdot q (P' - P)_\lambda]\right\},$$

where $T_3^b = -1/2$ and $T_3^b = 1/2$. The time-like form factors which appear in the above amplitudes are defined by:

$$\langle B^+_c(P') B^-_c(P) | \bar{q} \gamma_\mu q | 0 \rangle = (P' - P)_\mu F^q(Q^2),$$

$$\langle B^{++}_c(P') B^-_c(P) | \bar{q} \gamma_\mu q | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} \epsilon_{B_Z^c}^{*\nu} P^\rho P^\sigma V^q(Q^2),$$

$$\langle B^{*+}_c(P') B^-_c(P) | \bar{q} \gamma_\mu \gamma_5 q | 0 \rangle = A_1^q(Q^2) (Q^2 \epsilon^{*}_\mu - \epsilon^{*} \cdot q q_\mu) + A_2^q(Q^2) \epsilon^{*} \cdot q (P' - P)_\mu,$$

where the equation of motion has been employed. Furthermore, form factors $V^q(Q^2)$ and $A_1^q(Q^2)$ vanish at leading power at tree level, and will not be considered in this work.

Substituting the above amplitudes into Eq. (3), we have

$$\frac{d\sigma^{PP}}{d\Omega} = \frac{1}{64 \pi^2 Q^2} \left\{ \sum_{q=b,c} \left[ \frac{e^2 e_q}{Q^2} + Z_1^q(Q^2 - m_Z^2) \right] \text{Re} F_q + Z_1^q m_Z \Gamma_Z \text{Re} F_q \right\}^2
+ \left\{ \sum_{q=b,c} \left[ \frac{-e^2 e_q}{Q^2} - Z_1^q(Q^2 - m_Z^2) \right] \text{Re} F_q + Z_1^q m_Z \Gamma_Z \text{Im} F_q \right\}^2
+ \left\{ \sum_{q=b,c} Z_2^q(Q^2 - m_Z^2) \text{Im} F_q - Z_2^q m_Z \Gamma_Z \text{Re} F_q \right\}^2
+ \left\{ \sum_{q=b,c} Z_2^q(Q^2 - m_Z^2) \text{Re} F_q + Z_2^q m_Z \Gamma_Z \text{Im} F_q \right\}^2 \right\} \frac{1}{2} Q^4 \sin^2 \theta,$$
where $Z_1^q = \sqrt{G_F m_W^2} \frac{T^2_q - 2e_q \sin^2 \theta_W}{(Q^2 - m_Z^2)^2 + m_Z^2 T^2_q}$, $Z_2^q = \sqrt{G_F m_W^2} \frac{T^2_q - 2e_q \sin^2 \theta_W}{(Q^2 - m_Z^2)^2 + m_Z^2 T^2_q}$. For the $B_c B_c^*$ channel, we have

$$\frac{d\sigma^{PV}}{d\Omega} = \frac{1}{32\pi^2} \left[ \frac{G_F m_W^2}{\cos^2 \theta_W} \left( \frac{1}{2} + 2 \sin^2 \theta_W \right) \right]^2 \left[ \sum_{q=b,c} [(Q^2 - m_Z^2) \text{Im} A^q_2 - m_Z \Gamma_Z \text{Re} A^q_2] T^q_3 \right]^2$$

$$+ \left[ \sum_{q=b,c} [(Q^2 - m_Z^2) \text{Re} A^q_2 + m_Z \Gamma_Z \text{Im} A^q_2] T^q_3 \right]^2 \left( \frac{1}{2} + 4 \sin^4 \theta_W - 2 \sin^2 \theta_W \right)$$

$$\times \frac{Q^2 \sin^2 \theta}{4m_{B_c^*}^2} (\frac{Q^2}{4m_{B_c}^2} - 1).$$

(10)

In the proposed forthcoming accelerators, the collision energy in the center-of-mass frame is much larger than the masses of the $b$ quark and $c$ quark. In this paper, we concentrate on the leading power result of the relevant time-like form factors. Power suppressed contributions, such as quark mass effects and corrections from higher-twist LCDA, which may be important when $Q^2$ is not large enough, will be neglected in our calculations. In the PQCD approach, form factors are factorized into a convolution of the transverse momentum dependent (TMD) wave function and the hard scattering kernel. The tree-level hard kernel can be obtained by computing the Feynman diagrams plotted in Fig. 11. In the calculation, we choose the momentum fractions of charm quarks in the $B_c^-$ meson and $B_c^{+(s)}$ meson to be $x$ and $y$ respectively. For form factors $F^q$, we have

$$F^b(Q^2) = \frac{2\pi f_{B_c}^2 C_F}{N_c} Q^2 \int_0^1 dx dy \int_0^\infty b_1 b_2 b_2 b_2 \alpha_s(t) \left[ -x \mathcal{P}_{PI}(x, b_1) \mathcal{P}_{PII}(y, b_2) \right]$$

$$\times \mathcal{H}(x, y, Q, b_1, b_2) \exp [-S(x, y, b_1, b_2, Q)] ,$$

$$F^c(Q^2) = \frac{2\pi f_{B_c}^2 C_F}{N_c} Q^2 \int_0^1 dx dy \int_0^\infty b_1 b_2 b_2 b_2 \alpha_s(t) \left[ -\bar{x} \mathcal{P}_{PI}(x, b_1) \mathcal{P}_{PII}(y, b_2) \right]$$

$$\times \mathcal{H}(\bar{x}, \bar{y}, Q, b_1, b_2) \exp [-S(x, y, b_1, b_2, Q)] ,$$

(11)

where $\bar{x} = 1 - x$. The hard function is:

$$\mathcal{H}(x, y, Q, b_1, b_2) = \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(\sqrt{xy} Qb_2) \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{x} Qb_1) J_0(\sqrt{x} Qb_2) \right.$$

$$+ \theta(b_2 - b_1) H_0^{(1)}(\sqrt{x} Qb_2) J_0(\sqrt{x} Qb_1) \right].$$

(12)

The TMD wave function is:

$$\mathcal{P}_P(x, b) = \hat{\phi}_P(x) \Sigma(x, b),$$

(13)
where $\Sigma(x, b)$ is the transverse momentum dependent part. The LCDAs can be expressed in the following expansions,

\begin{align}
\hat{\phi}^a_P(x) &= \hat{\phi}^{(0,0)}_P(x) + \hat{\phi}^{(1,0)}_P(x) + \hat{\phi}^{(0,1)}_P(x), \\
\hat{\phi}^{|l|}_V(x) &= \hat{\phi}^{(|l|,0)}_V(x) + \hat{\phi}^{(|l|,1)}_V(x) + \hat{\phi}^{(|l|,1)}_V(x),
\end{align}

where the superscript $(i, j)$ denotes the order of $\alpha_s$ and $v^2$-expansion. In order to express the form factors in terms of $\alpha_s$ expansion, we write

\begin{align}
\mathcal{P}(x, b) &= \mathcal{P}^{(0)}(x, b) + \mathcal{P}^{(1)}(x, b) + \ldots, \\
\mathcal{P}^{(0)}(x, b) &= [\hat{\phi}^{(0,0)}_V(x) + \hat{\phi}^{(0,1)}_V(x)]\Sigma(x, b), \\
\mathcal{P}^{(1)}(x, b) &= \hat{\phi}^{(1,0)}_V(x)\Sigma(x, b).
\end{align}

A typical value of $x$ is $x \sim x_0 = \frac{m_c}{m_b + m_c}$. We have $xQ^2 \gg k_T^2$ when the energy scale $Q$ is large, thus we can drop the transverse momentum in the quark propagator. Then form factors can then be simplified as

\begin{align}
F^{b}_{(LO)}(Q^2) &= i\frac{\pi^2 f_B^2 C_F \alpha_s(\mu)}{N_c} \int_0^1 dxdy \int_0^\infty ddb \mathcal{P}_{P1}(x)\mathcal{P}_{P1I}(y) \\
&\quad \times \exp[-S_1(x, y, b, Q, \mu)]H_0^{(1)}(\sqrt{xy}Qb), \\
F^{c}_{(LO)}(Q^2) &= i\frac{\pi^2 f_B^2 C_F \alpha_s(\mu)}{N_c} \int_0^1 dxdy \int_0^\infty ddb \mathcal{P}_{P1}(x)\mathcal{P}_{P1}(y) \\
&\quad \times \exp[-S_1(x, y, b, Q, \mu)]H_0^{(1)}(\sqrt{xy}Qb),
\end{align}

where $S_1(x, y, b, Q, \mu) \equiv S(x, b, y, b, Q, \mu)$ and the Hankel function $H_0^{(1)}(x) = J_0(x) + iY_0(x)$. Form factors $A^{b,c}_{2,(LO)}$ are related to $F^{b,c}$ at tree level, as shown in Eqs. (15a,15b):

\begin{align}
A^{b}_{2,(LO)}(Q^2) &= -i\frac{\pi^2 f_B^2 f_V^2 m_V C_F \alpha_s(\mu)}{Q^2 N_c} \int_0^1 dxdy \int_0^\infty ddb \mathcal{P}_{P1}(x)\mathcal{P}_{V||II}(y) \\
&\quad \times \exp[-S_1(x, y, b, Q, \mu)]H_0^{(1)}(\sqrt{xy}Qb), \\
A^{c}_{2,(LO)}(Q^2) &= i\frac{\pi^2 f_B^2 f_V^2 m_V C_F \alpha_s(\mu)}{Q^2 N_c} \int_0^1 dxdy \int_0^\infty ddb \mathcal{P}_{P1}(x)\mathcal{P}_{V||II}(y) \\
&\quad \times \exp[-S_1(x, y, b, Q, \mu)]H_0^{(1)}(\sqrt{xy}Qb).\n\end{align}

The QCD corrections to form factors are of great importance in theoretical analysis. In the PQCD approach, NLO corrections to the time-like pion electromagnetic form factors have been calculated by Li et al [42]. In this paper, we consider NLO corrections to the form factors $F^V$. The calculation of NLO corrections in the PQCD approach is tediously.
FIG. 1: Feynman diagrams for $e^+e^\rightarrow B^+_c B^{+(\ast)}_c$.

complicated. To simplify the PQCD calculation, the hierarchy $xQ^2, yQ^2 \gg xyQ^2 \sim k_T^2$ is postulated. As the quark masses have been neglected, the NLO hard kernel can be borrowed directly from [42]:

$$H_F^{(NLO)}(x, k_{1T}, y, k_{2T}, Q^2, \mu) = h(x, y, \delta_{12}, Q, \mu)H_F^{(LO)}(x, k_{1T}, y, k_{2T}, Q^2),$$  \tag{19}

where

$$h(x, y, \delta_{12}, Q, \mu) = \frac{\alpha_s(\mu)C_F}{4\pi} \left[ -\frac{3}{4} \ln \frac{\mu^2}{Q^2} - \frac{17}{4} \ln^2 x + \frac{27}{8} \ln x \ln y - \frac{13}{8} \ln x + \frac{31}{16} \ln y 
- \ln^2 \delta_{12} + \left( \frac{17}{4} \ln x + \frac{23}{8} + i2\pi \right) \ln \delta_{12} + \frac{\pi^2}{12} + \frac{1}{2} \ln 2 + \frac{53}{4} - \frac{3\pi}{4} \right], \tag{20}
$$

and

$$\ln \delta_{12} \equiv \ln \frac{||k_{1T} + k_{2T}|^2 - xyQ^2|}{Q^2} + i\pi \Theta \left( ||k_{1T} + k_{2T}|^2 - xyQ^2 \right). \tag{21}
$$

Including the NLO hard kernel and performing a Fourier transform of Eq. (19), we derive the TMD factorization formula of form factors $F^q$ at leading power,

$$F_{NLO}^b(Q^2) = i\frac{\pi f_{B_c}^2 C_F^2}{4N_c} \int_0^1 dx dy \int_0^\infty db \alpha_s^2(\mu)P^{(0)}_{pl}(x)P^{(0)}_{pl}(y) \exp[-S_1(x, y, b, Q, \mu)] \times \left[ \tilde{h}(x, y, b, Q, \mu) H_0^{(1)}(\sqrt{xy}Qb) + H_0^{(1)\prime}(\sqrt{xy}Qb) \right],$$

$$F_{NLO}^c(Q^2) = i\frac{\pi f_{B_c}^2 C_F^2}{4N_c} \int_0^1 dx dy \int_0^\infty db \alpha_s^2(\mu)P^{(0)}_{pl}(x)P^{(0)}_{pl}(y) \exp[-S_1(x, y, b, Q, \mu)] \times \left[ \tilde{h}(x, y, b, Q, \mu) H_0^{(1)}(\sqrt{xy}Qb) + H_0^{(1)\prime}(\sqrt{xy}Qb) \right]. \tag{22}$$
where

\[
\tilde{h}(x, y, b, Q, \mu) = -\frac{3}{4} \ln \frac{\mu^2}{Q^2} - \frac{1}{4} \ln^2 \frac{4xy}{Q^2b^2} + \left( \frac{17}{8} \ln x + \frac{23}{16} + \gamma_E + i \frac{\pi}{2} \right) \ln \frac{4xy}{Q^2b^2} \\
- \frac{17}{4} \ln^2 x + \frac{27}{8} \ln x \ln y - \left( \frac{13}{8} + \frac{17\gamma_E}{4} - i \frac{17\pi}{8} \right) \ln x + \frac{31}{16} \ln y \\
- \frac{\pi^2}{2} + (1 - 2\gamma_E)\pi + \frac{1}{2} \ln 2 + \frac{53}{4} - \frac{23}{8} \gamma_E - \gamma_E^2 + i \left( \frac{171}{16} + \gamma_E \right) \pi.
\]

(23)

III. NUMERICAL ANALYSIS

The most important nonperturbative input is the LCDA of the $B_c$ meson, which can be studied with non-relativistic QCD effective theory as both internal quarks are heavy. At tree level and at leading order in the expansion over the relative velocity, the quark and the antiquark simply share the momentum of the meson according to their masses,

\[
\hat{\phi}^{(0,0)}_P(x) = \hat{\phi}^{(0,0)}_V(x) = \delta(x - x_0).
\]

(24)

If we include the gluon exchange effect and the power correction relevant to the quark velocity, the parton configuration will be changed. Explicit expressions of $\hat{\phi}^{(0,1)}(x)$ are given by

\[
\hat{\phi}^{(0,1)}_P(x) = -\frac{\langle q^2 \rangle_P}{M^2} \left[ \frac{2(1 - x_0)}{3x_0^0} \delta'(x - x_0) - \frac{1}{6} \delta''(x - x_0) \right],
\]

(25)

\[
\hat{\phi}^{(0,1)}_V(x) = \hat{\phi}^{(0,1)}_V(x) = -\frac{\langle q^2 \rangle V}{M^2} \left[ \frac{2(1 - x_0)}{3x_0^0} \delta'(x - x_0) - \frac{1}{6} \delta''(x - x_0) \right],
\]

(26)

where $M = m_b + m_c$ and $\langle q^2 \rangle_{P,V}$ are the mean values of $q^2$ in scalar and vector $B_c$ mesons respectively. For QCD corrections to the LCDAs, we have

\[
\hat{\phi}^{(1,0)}_P(x; \mu) = \frac{\alpha_s}{4\pi} C_F \left\{ \Phi_1(x_0) \right\},
\]

(27)

\[
\hat{\phi}^{(1,0)}_V(x; \mu) = \frac{\alpha_s}{4\pi} C_F \left\{ \Phi_1(x_0) - 4 \left[ \frac{1}{x_0} \theta(x_0 - x) + (x \leftrightarrow \bar{x}, x_0 \leftrightarrow \bar{x}_0) \right] \right\},
\]

(28)

with

\[
\Phi_1(x, x_0) = 2 \left[ \left( \ln \frac{\mu^2}{M^2(x_0 - x)^2} - 1 \right) \left( \frac{x_0 + \bar{x}}{x_0 - x} \frac{x}{x_0} \theta(x_0 - x) + (x \leftrightarrow \bar{x}, x_0 \leftrightarrow \bar{x}_0) \right) \right] + \left[ \frac{4x\bar{x}}{(x_0 - x)^2} \right]_{++} + \left[ 4x_0\bar{x}_0 \ln \frac{x_0}{\bar{x}_0} + 2(2x_0 - 1) \right] \delta'(x - x_0).
\]

(29)
Here we adopt the NDR scheme of $\gamma_5$, as the NLO hard kernel is also obtained in this scheme. The ++- and +-distributions are defined as

$$\int_0^1 dx [f(x)]^{++} g(x) = \int_0^1 dx f(x)(g(x) - g(x_0) - g'(x_0)(x - x_0)), \quad (30a)$$

$$\int_0^1 dx [f(x)]^{+-} g(x) = \int_0^1 dx f(x)(g(x) - g(x_0)), \quad (30b)$$

where $g(x)$ is a smooth test function. Because LCDA with $\nu$ and $\alpha_s$ corrections contains a Dirac-$\delta$ function or plus distribution, we need to integrate over the momentum fraction $x$ first. The results are given in the Appendix.

For the TMD wave function, we use

$$\Sigma(x, b) = \exp \left( -\frac{b^2}{4\beta^2} \right). \quad (31)$$

The input parameters are listed in Table I:

| parameter | value         | parameter | value         |
|-----------|---------------|-----------|---------------|
| $m_b$     | 4.8 GeV       | $m_c$     | 1.6 GeV       |
| $m_W$     | 80.425 GeV    | $m_Z$     | 91.1876 GeV   |
| $m_{B_c}$ | 6.2749        | $\langle q^2 \rangle$ | 1.59 GeV$^2$ |
| $\Gamma_Z$ | 2.4952 GeV    | $\sin^2 \theta_W$ | 0.23129       |
| $n_f$     | 5             | $\Lambda$ | 0.217 GeV     |
| $\gamma_E$ | 0.57721566    | $G_F$     | $1.166391 \times 10^{-5}$ GeV$^{-2}$ |
| $\beta$   | 2 GeV$^{-1}$  | $f_{B_c}$ | 0.489$\pm$0.005 GeV |

TABLE I: Parameters used for numerical analysis.

We are now ready to evaluate the numerical results of the form factors and cross sections. We first concentrate on the $Q$ dependence of the time-like form factors. In the PQCD approach the time-like form factors are complex, as the internal quark line may be on-shell. The absolute values of form factors $F^c$ and $F^b$ are plotted in Fig. 2. We do not show form factors $A^c_2$ and $A^b_2$, since they are related to $F^c$ and $F^b$. From the left-hand plot, we can see that the absolute value of form factor $F^c$ (the dot-dashed blue curve) is about an order or magnitude smaller than that of $F^b$ (the solid black curve) because the wave function of $B_c$ meson is not symmetrical between charm quark and bottom quark, so the invariant mass of the internal gluon propagator is much larger in $F^c$. To illustrate the effects of relativity and
FIG. 2: Shape of $Q^2|F_{B_c}|$ and $Q^2|f_{B_c}|$. Left: the total results of $F^b$ and $F^c$ are presented by the solid black curve and the dot-dashed blue curve respectively. Right: the solid black, dotted black, dot-dashed red and dashed blue line are the total result, leading order results, NLO correction and relativistic correction of $F^b$, respectively.

Taking advantage of the form factors computed with the PQCD approach, the total cross sections of $e^+e^- \rightarrow B_c^- B_c^{+(*)}$ are depicted in Fig. 3. The factorization scale $\mu$ is taken to be $\mu = Q$ for the central value and $\mu \sim [Q/2, 2Q]$ for the error estimate. The solid black curves correspond to central values and the dashed blue and dot-dashed red lines represent the errors of the cross sections. Considering $f_{B_c}$ only causes tiny errors because of its high accuracy, the cross sections at our kinematic region are not sensitive to the variation of factorization scale. As expected, the cross section has a significant peak at the $Z$ pole ($Q = m_Z$) and the maximum values of the cross sections are $\sigma^{PP}(Q = m_Z) \sim 1.3 \times 10^{-5}$ pb.
FIG. 3: Cross sections of $e^+e^- \rightarrow B_c^-B_c^{+(*)}$ with errors from variations of the factorization scale $\mu$ and $B_c$-meson decay constant $f_{B_c}$. The solid black, dashed blue and dot-dashed red curves correspond to central, upper and lower values of cross sections, respectively. The kinematic region of the left two diagrams is $50 - 250$ GeV and that of the right two diagrams is around the Z pole.

and $\sigma^{PV}(Q = m_Z) \sim 2.5 \times 10^{-5}$pb. They are so small that it is almost hopeless to detect a $B_c^-B_c^{+(*)}$ pair in an $e^+e^-$ collider at the Z pole. This result is quite different from $B\bar{B}$ production, which is enhanced by a low energy resonance. For single $B_c$ production in semi-inclusive processes, the cross section is much larger [23], and can be reached by a high luminosity $e^+e^-$ collider.

IV. SUMMARY

$B_c$ production processes are important because the $B_c$ meson has unique properties and is worthy of a thorough study. Within the framework of the PQCD approach, we calculated
the time-like form factors $F_q(Q^2)$ and $A_q^2(Q^2)$ at leading power in $1/Q$. The form factors are factorized into a convolution of the transverse momentum dependent wave function and the hard kernel. The wave functions employed in this work, including QCD corrections and relativistic corrections, have recently been studied using NRQCD. We evaluate the effects of relativistic corrections and QCD corrections of the form factors numerically. Both relativistic and QCD corrections can give about 20% correction to the LO contribution, but there are cancellation effects between them. We further worked out the cross sections of $e^+e^- \rightarrow B^-_c B^+_c(B^{*+}_c)$ processes. The cross sections are enhanced at the $Z$ pole, but they are still too small to be detected in future accelerators such as the CEPC.

Acknowledgement

We are grateful to Q. A. Zhang for useful discussions and comments. This work was supported in part by Natural Science Foundation of Shandong Province, China under Grant No. ZR2015AQ006 and by National Natural Science Foundation of China (Grants No. 11521505, No. 11621131001, No. 11447009, No. 11505098, and No. 11765012). R. H. Li is partly supported by the plan of Young Creative Talents under the Talent of the Prairie project of the Inner Mongolia.

Appendix A: Results after momentum fraction integration

1. Tree level

   a. Leading power in $v$

   \[
   F_{(LO)}^{b}(Q^2) = i f_P P_c \int_0^\infty db \exp[-S_{1}(x_0, x_0, b, Q, \mu)] H_0^{(1)}(x_0Qb) \phi_T^2(b), \tag{A1}
   \]
   \[
   F_{(LO)}^{c}(Q^2) = i f_P P_c \int_0^\infty db \exp[-S_{1}(x_0, x_0, b, Q, \mu)] H_0^{(1)}(\bar{x}_0Qb) \phi_T^2(b), \tag{A2}
   \]
   \[
   A_{2,(LO)}^{b}(Q^2) = -\frac{f_V M_V P_c}{Q^2} \int_0^\infty db \exp[-S_{1}(x_0, x_0, b, Q, \mu)] H_0^{(1)}(x_0Qb) \phi_T^2(b), \tag{A3}
   \]
   \[
   A_{2,(LO)}^{c}(Q^2) = f_V M_V P_c \int_0^\infty db \exp[-S_{1}(x_0, x_0, b, Q, \mu)] H_0^{(1)}(\bar{x}_0Qb) \phi_T^2(b), \tag{A4}
   \]
where $P_c = \pi^2 f_P C_F \alpha_s(\mu)/N_c$, $x_0 = 1 - x_0$, $S_1(x, y, b, Q, \mu) \equiv S(x, b, y, b, Q, \mu)$ and $H_n^{(1)}(x) = J_n(x) + iY_n(x)$. We use the LQCD decay constant $f_B = 0.489$ GeV. The Sudakov factor is

$$S(x, b_1, y, b_2, Q, \mu) = s(x, b_1, Q) + s(x, b_1, Q) + 2sc(\mu, b_1) + s(y, b_2, Q) + s(\bar{y}, b_2, Q) + 2sc(\mu, b_2),$$

$$sc(\mu, b) = -\frac{1}{2\beta_1} \ln \left[ \frac{\ln(\mu^2/A^2)}{\ln[1/(b^2 A^2)]} \right],$$

where

$$s(\xi, b) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln \left( \frac{\hat{q}}{\hat{b}} \right) - \frac{A^{(1)}}{2\beta_1} \left( \hat{q} - \hat{b} \right) + \frac{A^{(2)}}{4\beta_1^2} \left( \frac{\hat{q}}{\hat{b}} - 1 \right) - \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[ \frac{\ln(2\hat{b}) + 1}{\hat{b}} - \frac{\ln(2\hat{q}) + 1}{\hat{q}} \right]
- \left[ \frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \left( \frac{e^{2\gamma_E-1}}{2} \right) \right] \frac{\ln(\hat{q})}{\frac{\hat{b}}{\hat{q}}} + \frac{A^{(1)}\beta_2}{8\beta_1^3} \left[ \ln^2(2\hat{q}) - \ln^2(2\hat{b}) \right],$$

where

$$\hat{q} \equiv \ln \left[ \frac{\xi Q}{\sqrt{2\Lambda_{QCD}}} \right], \quad \hat{b} \equiv \ln \left[ \frac{1}{b\Lambda_{QCD}} \right],$$

$$A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln \left( \frac{e^{\gamma_E}}{2} \right).$$

### b. NRQCD corrections

The $\mathcal{O}'$ term $-\frac{(q^2)^2}{(mb + mc)^2} \frac{2(1 - x_0)}{3x_0x_0} \delta'(x - x_0)$

$$F_{(LO)}^{b, NR, I}(Q^2) = 2i f_P \left( \frac{q^2}{mb + mc} \right) \frac{2(1 - x_0)}{3x_0x_0} \int_0^\infty db \exp[-S_I(x_0, x_0, b, Q, \mu)]
\times \left[ s_p(\bar{x}_0, b, Q) - s_p(x_0, b, Q) \right] H_0^{(1)}(\bar{x}_0 Qb) - \frac{H^{(1)}(x_0 Qb)}{2} \phi_T^2(b),$$

$$F_{(LO)}^{c, NR, I}(Q^2) = 2i f_P \left( \frac{q^2}{mb + mc} \right) \frac{2(1 - x_0)}{3x_0x_0} \int_0^\infty db \exp[-S_I(x_0, x_0, b, Q, \mu)]
\times \left[ s_p(\bar{x}_0, b, Q) - s_p(x_0, b, Q) \right] H_0^{(1)}(\bar{x}_0 Qb) + \frac{H^{(1)}(x_0 Qb)}{2} \phi_T^2(b),$$

$$A_{2, (LO)}^{b, NR, I}(Q^2) = \frac{1}{f_P Q^2} F_{(LO)}^{b, NR, I}(Q^2),$$

$$A_{2, (LO)}^{c, NR, I}(Q^2) = \frac{i}{f_P Q^2} F_{(LO)}^{c, NR, I}(Q^2),$$

$$s_p(\xi, b, Q) = \frac{ds(\xi, b, Q)}{d\xi}
= \frac{1}{4\xi \beta_1} \left\{ A^{(1)} \left( 2 \ln \left( \frac{\hat{q}}{\hat{b}} + \frac{1}{\hat{q}} \right) e^{2\gamma_E-1} \right) + \frac{A^{(1)}\beta_2}{\beta_1^3} \left[ \ln(2\hat{q}) + 1 - \ln(2\hat{b}) + 1 \right] + \frac{A^{(2)}(1 - 1/\hat{q})}{\beta_1} \right\},$$

13
\[
\frac{ds(\xi, b, Q)}{d\xi} = -s_p(\xi, b, Q), \quad (A14)
\]
\[
\frac{dH_n^{(1)}(x)}{dx} = \frac{nH_n^{(1)}(x)}{x} - H_{n+1}^{(1)}(x). \quad (A15)
\]

The \( \delta'' \) term \( \int_0^{(q^2)} \delta''(x - x_0) \)

\[
F_{(LO)}^{b,NR,II}(Q^2) = 2i f_P \frac{(q^2)}{b(m_b + m_c)^2} \int_0^\infty db \exp[-S_f(x_0, x_0, b, Q, \mu)]
\times \left\{ [s_p(\bar{x}_0) - s_p(x_0)] \left\{ [s_p(\bar{x}_0) - s_p(x_0)] H_0^{(1)}(x_0Qb) - \frac{Qb}{2} H_1^{(1)}(x_0Qb) \right\} - [s_{pp}(\bar{x}_0) + s_{pp}(x_0)] H_0^{(1)}(x_0Qb) - [s_p(\bar{x}_0) - s_p(x_0)] \frac{Qb}{2} H_1^{(1)}(x_0Qb) \right\} \phi_T^2(b), \quad (A16)
\]
\[
F_{(LO)}^{c,NR,II}(Q^2) = 2i f_P \frac{(q^2)}{b(m_b + m_c)^2} \int_0^\infty db \exp[-S_f(x_0, x_0, b, Q, \mu)]
\times \left\{ [s_p(\bar{x}_0) - s_p(x_0)] \left\{ [s_p(\bar{x}_0) - s_p(x_0)] H_0^{(1)}(x_0Qb) + \frac{Qb}{2} H_1^{(1)}(x_0Qb) \right\} - [s_{pp}(\bar{x}_0) + s_{pp}(x_0)] H_0^{(1)}(x_0Qb) + [s_p(\bar{x}_0) - s_p(x_0)] \frac{Qb}{2} H_1^{(1)}(x_0Qb) \right\} \phi_T^2(b), \quad (A17)
\]
\[
A_{2,(LO)}^{b,NR,II}(Q^2) = \frac{i f_P^{MV}}{f_P Q^2} F_{(LO)}^{b,NR,II}(Q^2), \quad (A18)
\]
\[
A_{2,(LO)}^{c,NR,II}(Q^2) = -\frac{i f_P^{MV}}{f_P Q^2} F_{(LO)}^{c,NR,II}(Q^2), \quad (A19)
\]

\[
s_{pp}(\xi, b, Q) = \frac{ds_p(\xi, b, Q)}{d\xi}
= \frac{1}{4\xi^2\beta_1} \left\{ A^{(1)} \left[ 2 - \frac{\ln(e^{2\gamma}-1)}{\hat{q}} \right] - \frac{A^{(1)}\beta_2 \ln(2\hat{q})}{\beta_1^2 \hat{q}^2} + \frac{A^{(2)}}{\beta_1 \hat{q}^2}
\right.
\]
\[
- \left[ A^{(1)} \left( 2 \ln \frac{\hat{q}}{b} + \frac{1}{\hat{q}} \ln e^{2\gamma-1} - \frac{1}{2} \right) + \frac{A^{(1)}\beta_2}{\beta_1^2} \left[ \frac{\ln(2\hat{q}) + 1}{\hat{q}} - \frac{\ln(2\hat{b}) + 1}{b} \right] + \frac{A^{(2)}}{\beta_1} \left( \frac{1}{\hat{b}} - \frac{1}{\hat{q}} \right) \right\}. \quad (A20)
\]
2. One-loop level

The NLO correction from the hard kernel is:

\[
F_{(NLO)}^{b,I}(Q^2) = \int_0^\infty db \exp[-S_1(x, x_0, b, Q, \mu)] \phi_\tilde{h}^2(b) \\
\times \left[ \tilde{h}(x_0, x_0, b, Q, \mu) H_0^{(1)}(x_0 Q b) + \frac{\ln^2(x_0 Q b)}{3} H_0^{(1)}(x_0 Q b) \right],
\]

(A21)

\[
F_{(NLO)}^{c,I}(Q^2) = \int_0^\infty db \exp[-S_1(x, x_0, b, Q, \mu)] \phi_\tilde{h}^2(b) \\
\times \left[ \tilde{h}(x_0, x_0, b, Q, \mu) H_0^{(1)}(x_0 \bar{Q} b) + \frac{\ln^2(x_0 \bar{Q} b)}{3} H_0^{(1)}(x_0 \bar{Q} b) \right].
\]

(A22)

We add the one-loop corrections from the two wave functions together. We rewrite the wave function as:

\[
\phi^{(1,0)}_p(x) = \frac{\alpha_s C_F}{4\pi} [\hat{\phi}^I(x) + \hat{\phi}^{II}(x)],
\]

(A23)

\[
\hat{\phi}^I(x) = 2 \left[ \frac{\mu^2}{(m_b + m_c)^2(x_0 - x)^2 - 1} \frac{x_0 + \bar{x}}{x_0 - x} \theta(x_0 - x) \right] + 2 \left[ \ln \frac{\mu^2}{(m_b + m_c)^2(x_0 - x)^2} - 1 \right] \frac{x_0 + \bar{x}}{x_0 - x} \theta(x_0 - x) + \left[ \frac{4x\bar{x}}{(x_0 - x)^2} \right]_+, \quad \hat{\phi}^{II}(x) = 4x_0 \bar{x}_0 \ln \frac{x_0}{\bar{x}_0} + 2(2x_0 - 1) \delta'(x - x_0).
\]

(A24)

(A25)

The NLO corrections from \( \hat{\phi}^I \) are:

\[
F_{(NLO)}^{b,II}(Q^2) = 2i \int_0^\infty db \exp[-S_1(x, x_0, b, Q, \mu)] H_0^{(1)}(\sqrt{x_0 \bar{Q} b}) \\
\times \left[ s_p(x_0, b, Q) - s_p(x_0, b, Q) \right] H_0^{(1)}(x_0 \bar{Q} b) - \frac{H_0^{(1)}(x_0 \bar{Q} b)}{2} \phi_\tilde{h}^2(b),
\]

(A26)

\[
F_{(NLO)}^{c,II}(Q^2) = 2i \int_0^\infty db \exp[-S_1(x, x_0, b, Q, \mu)] H_0^{(1)}(\sqrt{x_0 \bar{Q} b}) \\
\times \left[ s_p(x_0, b, Q) - s_p(x_0, b, Q) \right] H_0^{(1)}(x_0 \bar{Q} b) + \frac{H_0^{(1)}(x_0 \bar{Q} b)}{2} \phi_\tilde{h}^2(b),
\]

(A27)

\[
\Psi(x) = 2 \left[ \frac{\mu^2}{(m_b + m_c)^2(x_0 - x)^2 - 1} \frac{x_0 + \bar{x}}{x_0 - x} \theta(x_0 - x) \right] + 2 \left[ \ln \frac{\mu^2}{(m_b + m_c)^2(x_0 - x)^2} - 1 \right] \frac{x_0 + \bar{x}}{x_0 - x} \theta(x_0 - x) + \frac{4x\bar{x}}{(x_0 - x)^2}.
\]

(A28)
The NLO corrections from $\hat{\chi}^{IH}$ are:

$$F_{(NLO)}^{b,II}(Q^2) = \frac{\alpha_s C_F}{4\pi} \left[ 4x_0\bar{x}_0 \ln \frac{\bar{x}_0}{x_0} + 2(2x_0 - 1) \right] / \left[ -\frac{(q^2)}{(m_b + m_c)^2} \frac{2(1 - 2x_0)}{3x_0\bar{x}_0} \right] F_{(LO)}^{b,NRI}(Q^2), \quad (A29)$$

$$F_{(NLO)}^{c,II}(Q^2) = \frac{\alpha_s C_F}{4\pi} \left[ 4x_0\bar{x}_0 \ln \frac{\bar{x}_0}{x_0} + 2(2x_0 - 1) \right] / \left[ -\frac{(q^2)}{(m_b + m_c)^2} \frac{2(1 - 2x_0)}{3x_0\bar{x}_0} \right] F_{(LO)}^{c,NRI}(Q^2), \quad (A30)$$

The NLO corrections from the decay constant and Sudakov factor are:

$$F_{(NLO)}^{b,IV}(Q^2) = i(2f_P^{NLO}) f_P f_P \int_0^\infty dbb \exp[-S_1(x_0, x_0, b, Q, \mu)] \eta_0^{(1)} (\bar{x}_0 Qb) \phi_2^2(b)$$

$$- i f_P f_P \int_0^\infty dbb \exp[-S_1(x_0, x_0, b, Q, \mu)] S_1^{NLO}(x_0, x_0, b, Q, \mu) H_0^{(1)} (\bar{x}_0 Qb) \phi_2^2(b), \quad (A31)$$

$$F_{(NLO)}^{c,IV}(Q^2) = i(2f_P^{NLO}) f_P f_P \int_0^\infty dbb \exp[-S_1(x_0, x_0, b, Q, \mu)] \eta_0^{(1)} (\bar{x}_0 Qb) \phi_2^2(b)$$

$$- i f_P f_P \int_0^\infty dbb \exp[-S_1(x_0, x_0, b, Q, \mu)] S_1^{NLO}(x_0, x_0, b, Q, \mu) H_0^{(1)} (\bar{x}_0 Qb) \phi_2^2(b), \quad (A32)$$

where $S_1^{NLO}(x, y, b, Q, \mu) \equiv S^{NLO}(x, y, b, Q, \mu)$.

$$f_P^{NLO} = \frac{\alpha_s (m_{B_c}) C_F}{4\pi} \left( 3 \ln \frac{m_{B_c}}{m_c} - 4 \right),$$

$$S^{NLO}(x, b_1, y, b_2, Q, \mu) = s_{NLO}(x, b_1, Q) + s_{NLO}(\bar{x}, b_1, Q) + 2 s_{E_{NLO}}(\mu, b_1)$$

$$+ s_{NLO}(y, b_2, Q) + s_{NLO}(\bar{y}, b_2, Q) + 2 s_{E_{NLO}}(\mu, b_2),$$

$$s_{E_{NLO}}(\mu, b) = \frac{\beta_2}{\beta_1^{1/6}} \left\{ \ln \ln [1/(b^2 \Lambda^2)] + 1 - \frac{\ln \ln (\mu^2 / \Lambda^2)}{\ln (\mu^2 / \Lambda^2)} \right\}, \quad (A33)$$

$$s_{NLO}(\xi, b) = \frac{A^{(1)}(\beta_2)}{8 \beta_1^{3/4}} \frac{1}{\ln (2q) - \ln (2\bar{b})} \left[ \ln (2q) - \ln (2\bar{b}) \right] - \frac{A^{(1)}(\beta_2)}{16 \beta_1^2} \frac{1}{\bar{b}} \left[ 2 \ln (2\bar{b}) \right]$$

$$+ \frac{A^{(2)}(\beta_2)}{1728 \beta_1^6} \frac{1}{\ln (2\bar{q}) - \ln (2\bar{b})} \left[ 18 \ln (2\bar{q}) - 30 \ln (2\bar{b}) + 19 \right] - \frac{A^{(2)}(\beta_2)}{1728 \beta_1^6} \frac{1}{\ln (2\bar{q}) - \ln (2\bar{b})} \left[ 18 \ln (2\bar{q}) - 30 \ln (2\bar{b}) + 19 \right], \quad (A34)$$

$$F^b(Q^2) = F^b_{(LO)} + F^b_{NRI} + F^b_{NRHI} + F^b_{(LO)} + F^b_{NLO} + F^b_{(NLO)}; \quad (A35)$$

$$F^c(Q^2) = F^c_{(LO)} + F^c_{NRI} + F^c_{NRHI} + F^c_{(LO)} + F^c_{NLO} + F^c_{(NLO)}; \quad (A36)$$

$$A^b_2(Q^2) = A^b_2(LO) + A^b_{2,NRI} + A^b_{2,NRI}; \quad (A37)$$

$$A^c_2(Q^2) = A^c_2(LO) + A^c_{2,NRI} + A^c_{2,NRI}.$$

(A38)
[1] F. Abe, et al. (CDF Collaboration), Observation of the $B_c$ Meson in $p\bar{p}$ Collisions at $\sqrt{s} = 1.8$TeV, Phys. Rev. Lett. 81, 2432 (1998); Observation of $B_c$ mesons in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$TeV, Phys. Rev. D 58, 112004 (1998).

[2] P. Abreu, et al (DELPHI Collaboration), Search for the $B_c$ meson, Phys. Letts. B 398, 207 (1997); R. Barate, et al (ALEPH Collaboration), Search for the $B_c$ meson in hadronic $Z^0$ decay, Phys. Letts. B 402, 213 (1997); K. Ackerstaff, et al (OPAL Collaboration), Search for the $B_c$ meson in hadronic $Z^0$ decay, Phys. Letts. B 420, 157 (1998).

[3] X. Liu, Z. J. Xiao and C. D. Lü, Phys. Rev. D 81, 014022 (2010) doi:10.1103/PhysRevD.81.014022 [arXiv:0912.1163 [hep-ph]].

[4] X. Liu and Z. J. Xiao, J. Phys. G 38, 035009 (2011) doi:10.1088/0954-3899/38/3/035009 [arXiv:1003.3929 [hep-ph]].

[5] X. Liu and Z. J. Xiao, Phys. Rev. D 81, 074017 (2010) doi:10.1103/PhysRevD.81.074017 [arXiv:1001.2944 [hep-ph]].

[6] X. Liu and Z. J. Xiao, Phys. Rev. D 82, 054029 (2010) doi:10.1103/PhysRevD.82.054029 [arXiv:1008.5201 [hep-ph]].

[7] Z. J. Xiao and X. Liu, Phys. Rev. D 84, 074033 (2011) doi:10.1103/PhysRevD.84.074033 [arXiv:1111.6679 [hep-ph]].

[8] Z. Rui, W. F. Wang, G. X. Wang, L. h. Song and C. D. Lü, Eur. Phys. J. C 75, no. 6, 293 (2015) doi:10.1140/epjc/s10052-015-3528-0 [arXiv:1505.02498 [hep-ph]].

[9] W. F. Wang, X. Yu, C. D. Lü and Z. J. Xiao, Phys. Rev. D 90, no. 9, 094018 (2014) doi:10.1103/PhysRevD.90.094018 [arXiv:1401.0391 [hep-ph]].

[10] X. Liu, R. H. Li, Z. T. Zou and Z. J. Xiao, arXiv:1703.05982 [hep-ph].

[11] Z. J. Xiao and X. Liu, Chin. Sci. Bull. 59, 3748 (2014) doi:10.1007/s11434-014-0418-z [arXiv:1401.0151 [hep-ph]].

[12] N. Wang, Adv. High Energy Phys. 2016, 6314675 (2016) doi:10.1155/2016/6314675 [arXiv:1602.07027 [hep-ph]].

[13] Y. L. Yang, J. F. Sun and N. Wang, Phys. Rev. D 81, 074012 (2010) doi:10.1103/PhysRevD.81.074012 [arXiv:1004.2825 [hep-ph]].

[14] C. F. Qiao and R. L. Zhu, Phys. Rev. D 87, no. 1, 014009 (2013)
doi:10.1103/PhysRevD.87.014009  [arXiv:1208.5916 [hep-ph]].

[15] C. F. Qiao, P. Sun, D. Yang and R. L. Zhu, Phys. Rev. D 89, no. 3, 034008 (2014) doi:10.1103/PhysRevD.89.034008  [arXiv:1209.5859 [hep-ph]].

[16] J. M. Shen, X. G. Wu, H. H. Ma and S. Q. Wang, Phys. Rev. D 90, no. 3, 034025 (2014) doi:10.1103/PhysRevD.90.034025  [arXiv:1407.7309 [hep-ph]].

[17] W. Wang and R. L. Zhu, Eur. Phys. J. C 75, no. 8, 360 (2015) doi:10.1140/epjc/s10052-015-3583-6  [arXiv:1501.04493 [hep-ph]].

[18] R. Zhu, Y. Ma, X. L. Han and Z. J. Xiao, [arXiv:1703.03875 [hep-ph]].

[19] C.-H Chang and Y.-Q Chen, The Production of $B_c$ or $\bar{B}_c$ associated with two heavy quark jets in $Z^0$ boson decay, Phys. Rev. D 46, 3845 (1992); Erratum, Phys. Rev. D 50, 6013 (1994); The $B_c$ and $\bar{B}_c$ mesons accessible to experiments by $Z^0$ boson decay, Phys. Letts. B 284, 127-132 (1992).

[20] X. G. Wu, C. H. Chang, Y. Q. Chen and Z. Y. Fang, Phys. Rev. D 67, 094001 (2003) doi:10.1103/PhysRevD.67.094001  [hep-ph/0209125].

[21] C. H. Chang, C. F. Qiao, J. X. Wang and X. G. Wu, Phys. Rev. D 71, 074012 (2005) doi:10.1103/PhysRevD.71.074012  [hep-ph/0502155].

[22] C. H. Chang, J. X. Wang and X. G. Wu, Phys. Rev. D 77, 014022 (2008) doi:10.1103/PhysRevD.77.014022  [arXiv:0711.1898 [hep-ph]].

[23] X. C. Zheng, C. H. Chang, T. F. Feng and Z. Pan, Sci. China Phys. Mech. Astron. 61, no. 3, 031012 (2018) doi:10.1007/s11433-017-9121-3  [arXiv:1701.04561 [hep-ph]].

[24] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125 (1995) Erratum: [Phys. Rev. D 55, 5853 (1997)] doi:10.1103/PhysRevD.55.5853, 10.1103/PhysRevD.51.1125  [hep-ph/9407339].

[25] N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005) doi:10.1103/RevModPhys.77.1423  [hep-ph/0410047].

[26] N. Brambilla et al. [Quarkonium Working Group], [hep-ph/0412158].

[27] N. Brambilla et al., Eur. Phys. J. C 71, 1534 (2011) doi:10.1140/epjc/s10052-010-1534-9  [arXiv:1010.5827 [hep-ph]].

[28] G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980). doi:10.1103/PhysRevD.22.2157

[29] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. 112, 173 (1984). doi:10.1016/0370-1573(84)90126-1
[30] J. P. Ma and Z. G. Si, Phys. Lett. B 647, 419 (2007) doi:10.1016/j.physletb.2007.02.040 [hep-ph/0608221].

[31] G. Bell and T. Feldmann, JHEP 0804, 061 (2008) doi:10.1088/1126-6708/2008/04/061 [arXiv:0802.2221 [hep-ph]].

[32] X. P. Wang and D. Yang, JHEP 1406, 121 (2014) doi:10.1007/JHEP06(2014)121 [arXiv:1401.0122 [hep-ph]].

[33] J. Xu and D. Yang, JHEP 1607, 098 (2016) doi:10.1007/JHEP07(2016)098 [arXiv:1604.04441 [hep-ph]].

[34] W. Wang, J. Xu, D. Yang and S. Zhao, JHEP 1712, 012 (2017) doi:10.1007/JHEP12(2017)012 [arXiv:1706.06241 [hep-ph]].

[35] H. n. Li and G. F. Sterman, Nucl. Phys. B 381, 129 (1992). doi:10.1016/0550-3213(92)90643-P.

[36] H. n. Li and H. L. Yu, Phys. Rev. Lett. 74, 4388 (1995) doi:10.1103/PhysRevLett.74.4388 [hep-ph/9409313].

[37] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001) doi:10.1016/S0370-2693(01)00247-7 [hep-ph/0004004].

[38] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001) doi:10.1103/PhysRevD.63.054008 [hep-ph/0004173].

[39] C. D. Lü, K. Ukai and M. Z. Yang, Phys. Rev. D 63, 074009 (2001) doi:10.1103/PhysRevD.63.074009 [hep-ph/0004213].

[40] C. D. Lü, W. Wang and Y. M. Wang, Phys. Rev. D 75, 094020 (2007) doi:10.1103/PhysRevD.75.094020 [hep-ph/0702085].

[41] J. W. Chen, H. Kohyama, K. Ohnishi, U. Raha and Y. L. Shen, Phys. Lett. B 693, 102 (2010) doi:10.1016/j.physletb.2010.05.082 [arXiv:0908.2973 [hep-ph]].

[42] H. C. Hu and H. n. Li, Phys. Lett. B 718, 1351 (2013) doi:10.1016/j.physletb.2012.12.006 [arXiv:1204.6708 [hep-ph]].

[43] C. D. Lü, W. Wang, Y. Xing and Q. A. Zhang, arXiv:1802.09718 [hep-ph].

[44] T. Kurimoto, Phys. Rev. D 74, 014027 (2006) doi:10.1103/PhysRevD.74.014027 [hep-ph/0605112].

[45] T. W. Chiu et al. [TWQCD Collaboration], Phys. Lett. B 651, 171 (2007) doi:10.1016/j.physletb.2007.06.017 [arXiv:0705.2797 [hep-lat]].