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ABSTRACT
Interactions between two chains consisted of micro-sized magnetic beads are experimentally investigated. Driven by a rotating field, three distinct modes of magnetic interactions are identified, referred to as chaining, locking and transit. The two chains gradually approach to form a longer chain in the chaining mode. The locking mode refers the chains undergo an interesting orbital-like circular motion, which mimics a binary star system. The chains could also evolve from locking to chaining in the transit mode. A necessary criterion for a locking mode is derived, which agrees excellently with the correspondent experiments.

I. INTRODUCTION AND EXPERIMENTAL METHODS
Micro-chain consisted of magnetic beads have been the interests of researches in the past decades because of the potential applications in MEMS, e.g., mixer, rotor, valve, swimmer and biomicrofluidics. Most of the early studies mainly focused on the dynamics of a single micro-chain or the interaction between a single chain and non-magnetic objects. Nevertheless, multiple chains often present when the dispersed load of magnetic beads in the solvent fluid is high. In the present paper, we experimentally study the interactions between a pair of magnetically-driven rotating chains. The interactions between the pair chains are carefully analyzed and classified based on their motion. Furthermore, a theoretical criterion for possible determination of distinct mode of interaction is derived and validated by the experimental results.

The rotating field is generated by two perpendicularly placed pairs of coils powered by AC sources with phase difference $\pi/2$. Two micro-sized chains, consisted of superparamagnetic beads whose diameters and susceptibility are $a = 4.5 \mu m$ and $\lambda = 1.6$, respectively, are first formed in a solvent fluid with viscosity $\eta = 0.00175$ Pa·s, and then driven to rotate synchronously with the external field. Details about generation of the rotating field and the formation of bead-chain are referred to Refs. 7 and 9. The field strength ($H_0$) and rotating frequency ($f$) applied in the experiments is fixed at $H_0 = 1540$ A/m and $f = 1$ Hz, respectively. It has been well concluded that chain rupture occurs if the length (or number of beads) of chain is sufficiently long. To ensure a stable chaining formation, we restrain our experiments to the pair of chains consisted with 3, 4 or 5 beads, respectively referred to as P3, P4 or P5 chain.

II. RESULTS AND DISCUSSION
A. Modes of interaction
Shown in Fig. 1 are representative experiments for three P3-P4 chain pairs undergoing distinct modes of interaction. Shown in top row of Fig. 1 is the most commonly expected interaction, such that the pair of chains undergo two rotational motions, i.e. (i) self-rotation along the center of the individual chain, and (ii) rotational motion around the center of mass of the two chains, which mimic the dual rotations observed for ferrofluid drops subjected to similar rotational field. In the meantime, the two magnetized chains also interact to each other, and gradually approach to form a longer P7 chain. Afterward, the long chain will proceed self-rotation along its center. This interaction to form a longer chain is referred to as the “chaining” mode. On the other hand, an unusual interaction is also observed as shown in the middle row of Fig. 1 under an identical magnetic field. For this pair of magnetic chains, the two chains...
not just self-rotate, but also move circularly while keeping a constant distance. Actually, as it will be confirmed in Fig. 2, the centers of two chains undergo a revolutionary motion around their center of mass, whose movement behaves like a system of binary stars. The revolutionary motion remains stable, and the chains seem being locked by each other, so that this type of interaction is referred to as the “locking” mode. Transition from the locking mode to chaining mode is also observed as a P3-P4 pair shown in the bottom row. The chains initially show revolutionary motion and last for tens of revolutions before the locking mode is broken to proceed chaining mode. This transition from locking to chaining is termed as the “transit” mode.

To better illustrate the distinct modes of interaction, trajectories of the individual chains, represented by their centers in a moving reference frame with the center of mass of the two chains (C.M.), for the three representative cases shown above are plotted in Fig. 2. It is noticed that, when the two chains form a longer P7 chain, the distance between the centers of the original two chains, denoted as d, would be half length of the P7 chain, i.e., d = 15.75 μm. For the case of chaining mode shown in Fig. 2(a), whose initial distance between the centers of two chains is d > 15.75 μm, the chains are mutually attracted and gradually approach to form a longer P7 chain. Revolutionary movements along C.M. is hardly observed. The near linear trajectories indicate the magnetic interaction mainly acts radially but tangentially. On the other hand, trajectories for the case of locking mode, shown in Fig. 2(b), appear completely opposite. The
trajectories follow circular orbits nicely, in which the orbiting radius is inversely proportional to the number of beads. The distance between the centers of two chains remains constant, and is much shorter, i.e. \( d \approx 10.25 \, \mu \text{m} \). The circular orbits suggest that only net azimuthal force acts on the chains. Fig. 2(c) demonstrates the trajectories of pair undergoing transit mode. The trajectories first appear similar with Fig. 2(b) for several revolutions with a slightly larger \( d \approx 11.5 \, \mu \text{m} \). Afterward, the chains expel each other radially to increase the distance. As time proceeds, the distance reaches \( d \approx 15.75 \, \mu \text{m} \) to form a P7 chain. From these observations, it can be reasoned that the relative position between the chains plays an important role to the determination of distinct modes of interaction.

**B. Criterion of modes**

To illustrate the forces act on the chain pair, sketches of the two representative positions resulting in the distinct modes of motion: chaining (left) and locking (right) are demonstrated in Fig. 3. The phase lag between the alignment of chain’s centers and external field \((H_o)\) is denoted by \(\Delta \theta_l\). The chains rotate separately due to the local magnetic torque \(T_m\) generated by the external field. In the meantime, presence of magnetic field induces magnetic interactions between the two chains, including radial (\(F_r\)) and tangential (\(F_\theta\)) force. Noted the orientations of these magnetic interactions depend on the relative positions, e.g., radial attraction and repulsion for the cases shown in left and right of Fig. 3, respectively. It had been concluded that the torque \(T_m\) depends on the local phase lag between the individual chain, noted not the alignment of chains, and external field \(\Delta \theta_l\) and number of bead consisted of the chain \((N)\). This torque can be approximated by\(^5\)

\[
T_m = \frac{\mu_0 m^2 N^2}{4\pi r^3} \left(\sin 2\Delta \theta_l\right)
\]  

(1)

Since the individual rotations of both chains remain constant, which indicates these local torques are balanced by the induced hydrodynamic torques. As a result, the local torques are believed not important to affect the mode of motion.

The other two important forces generated by the magnetic field are radial force (\(F_r\)) and tangential force (\(F_\theta\)). Noted the orientations of these magnetic interactions depend on the relative positions which can be explained by the well concluded models of two ideal magnetic dipoles as\(^6\)

\[
F_r = \frac{3\mu_0 m^2}{4\pi d^4} (1 - 3 \cos^2 \Delta \theta_l)
\]  

(2)

\[
F_\theta = \frac{3\mu_0 m^2}{4\pi d^4} \sin 2\Delta \theta_l
\]  

(3)

These models can shed lights to understand the magnetic interactions of chains. For the case of small \(\Delta \theta_l\) shown in the left of Fig. 3, the models result in significant attractive radial force with relatively smaller tangential force. These suggest a chaining mode, in which the two chains gradually approach without apparent tangential motion as the trajectory confirmed in Fig. 2(a). By applying the dipole model to the case with large \(\Delta \theta_l\) shown in the right plot of Fig. 3, the direction of radial force would be repulsive with weaker magnitude comparing with its tangential counterpart. The weaker repulsion could not possibly exceed the significant form drag since the alignment of chain is perpendicular to the repulsion. As a result, the chains are restrained from radial movement and distance between them remains constant, which is consistent with the trajectories of locking mode shown in Fig. 2(b). Nevertheless, the periodically circular trajectory of locking mode indicates a state of force balance tangentially, so that the magnitudes of \(F_\theta\) are desired for detailed analysis.

We take advantages of the well-developed dipole model, and modify the magnitudes of magnetic moments by multiplying the number of beads \(N\). The modified expression, after substituting all the relevant properties to calculate magnetic moment \(m\), is

\[
F_\theta = N^2 \times \frac{4\mu_0 m^2 \pi d^4}{3d^4} \left(\sin 2\Delta \theta_l\right)
\]  

(4)

This tangential force needs to be balanced by the hydrodynamic drag, denoted as \(D_\theta\). The hydrodynamic drag for the present chain with multiple beads is always greater than the Stokes drag of a single bead, such as

\[
D_\theta > 3\pi \eta d \omega
\]  

(5)

By balancing these two forces, i.e. \(F_\theta = D_\theta\), we obtain a necessary condition for the locking mode as

\[
d < \left[ E N^2 \times \frac{4\mu_0 m^2 \pi d^4}{9\eta \omega} \left(\sin 2\Delta \theta_l\right)^{1/3} \right]
\]  

(6)

This condition can be plotted as a closed envelope by varying \(d\) and \(\Delta \theta_l\) for fixed number of \(N\), such as envelopes for combinations...
of P3-P4, P4-P4 and P4-P5 pairs shown in Fig. 4. Numerous data points obtained at different times for several cases whose interactions appear distinct modes are also marked in Fig. 4. We refer to the plot as a diagram of trajectory positions since the data represent positions of the chains at consequent times. The trajectory positions for cases of locking mode distribute closely and generally inside their correspondent envelopes. On the other hand, the trajectory positions of chaining modes are far outside the envelopes to scatter widely for both d and $\Delta \theta$. We like to emphasize the particular P3-P4 case of transit mode, whose trajectory positions are around the boundary of envelope at early time stage, which represents a nearly locking mode. Nevertheless, the distance d starts to increase associated with significant change of $\Delta \theta$. The criterion is supported by experiments in the system, a locking mode requires all the trajectory positions fall within the envelope. The criterion is supported by experiments for all the three modes. Particularly, for the case of transit mode, the trajectory positions are initially located at the border of envelope, so that the chains undergo motion of locking mode. After several rounds of revolutionary motion, the trajectory positions move far away from the envelope, and chains evolve to the motion of chaining mode.

III. CONCLUDING REMARKS

We experimentally study the interaction of two magnetic chains subjected to a rotating external field. Driven by the external field, both chains undergo self-rotation synchronized with the external field. Nevertheless, three distinct modes of interaction between the two chains are identified, such as chaining, locking and transit. In the motion of chaining mode, the two chains gradually approach, without visible tangentially revolutionary motion, to form a longer chain. On the other hand, the chains follow circular orbits along the center of mass of two chains, and behave like a system of binary stars. The diameters of the orbits are inversely proportional to the number of bead consisted of the chain, and remain unchanged without visible radial movements. There also exists a transit mode, which the trajectories of the chains at consequent times. The trajectory positions appear distinct modes are also marked in Fig. 4. Numerous data points obtained at different times for several cases whose interactions appear distinct modes are also marked in Fig. 4. We refer to the plot as a diagram of trajectory positions since the data represent positions of the chains at consequent times. The trajectory positions for cases of locking mode distribute closely and generally inside their correspondent envelopes. On the other hand, the trajectory positions of chaining modes are far outside the envelopes to scatter widely for both d and $\Delta \theta$. We like to emphasize the particular P3-P4 case of transit mode, whose trajectory positions are around the boundary of envelope at early time stage, which represents a nearly locking mode. Nevertheless, the distance d starts to increase associated with significant change of $\Delta \theta$. The criterion is supported by experiments in the system, a locking mode requires all the trajectory positions fall within the envelope. The criterion is supported by experiments for all the three modes. Particularly, for the case of transit mode, the trajectory positions are initially located at the border of envelope, so that the chains undergo motion of locking mode. After several rounds of revolutionary motion, the trajectory positions move far away from the envelope, and chains evolve to the motion of chaining mode.

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