Energy partition and distribution of excited species in direction-sensitive detectors for WIMP searches

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Abstract. The Bragg-like curve for compounds is introduced for directional detection of galactic dark matter. The slow ion collisions are discussed in relation to direct dark matter searches. The Coulomb effect and the threshold effect in stopping power theory are examined. Ionization via molecular orbit (MO) is suggested for an additional contribution to the electronic stopping power at very slow energy.

1. Introduction
Detectors with directional capability, such as gas TPC, can observe daily fluctuations of the WIMP window. The knowledge of the fraction of projectile kinetic energy $E$ into the electronic excitation is a key issue for the direct detection of galactic dark matter. The information on spatial distribution of the electronic energy in the detector medium is essential for the 3D track reconstruction. The Bragg-like curve, $-d\eta/dR_{PRJ}$, where $\eta$ is the electronic energy given to the target medium and $R_{PRJ}$ is the projection range, has been introduced for practical purpose in gas TPC for directional dark matter searches [1] and may be basically applied also for other directional detection media.

There are criticisms against Lindhard theory [2] that the Coulomb effect and the threshold effect may reduce the electronic stopping power, and consequently the numbers of ionization and excitation produced will be quite small in slow collisions. Also, the recoil nucleus/gamma ratio, $R_{N}/\gamma$, ratio may be significantly small at low energy. The stopping mechanism in slow ion collisions will be discussed in relation to those effects. Also the partition of the electronic energy between ionization and excitation in low energy collisions will be considered.

2. Stopping power and LET
Only a part of the energy $\eta$ goes to the electronic excitation can be used as charge or scintillation signals. The nuclear quenching factor $q_{nc}$ is defined as $\eta/E$ and was given by Lindhard for recoil ions in single-element media [2] and was later fitted by Lewin and Smith [3]. It is used for quenching calculation for slow recoil ions in condensed phase [4]. For fast particles, contribution to stopping power is exclusively electronic stopping power $S_e$. Then, the linear energy transfer LET is simply $-d\eta/dx$. For slow particles the nuclear stopping power $S_n$ becomes the same order of magnitude as $S_e$. The secondary ions go to collisions again and can give their energy to the electronic excitation. The stopping power, $dE/dx$, is a quantity belongs to the projectile. The contribution of the secondary electrons and ions are not included. On the other hand, LET is a quantity belongs to the target medium. All the cascade processes has to be counted. The electronic LET (LET$_{el}$ = $-d\eta/dx$), the specific
electronic energy deposition along the track of charged particle, can be a good measure for scintillation quenching. For example, Birks low is written as \[5\],

\[
\frac{dL}{dx} = \frac{C_1(-dE/dx)}{1+C_2(-dE/dx)},
\]

for fast particles. However, \(dE/dx\) in Eq. (1) should be replaced by \(dn/\eta dx\) for slow recoil ions. It is an averaged light yield that is often measured, therefore, an averaged LET_{el}, \(<\text{LET}_{el}>) \approx \eta/R = -\eta/q_{nc}E/R,\)

where \(R\) is the range, may be used for a crude approximation.

\(^{57}\text{Co} 122 \text{ keV} \gamma\) is used as a standard to compare the quenching theory with experimental results. However, the efficiency \(L_\eta\) for 122 keV \(\gamma\) is necessary to compare measurements with theories in rare gases and inorganic scintillators, since \(L_\eta\) is less than 1 because of the existence of escaping electrons.

2.1. Nuclear quenching factor for compounds

Detector media for such TPCs are mostly compounds. The charge distribution is given by the distribution of the electronic energy. However, the evaluation of Lindhard factor, the nuclear quenching factor \(q_{nc}\), becomes extremely difficult for the medium contains more than one element. The Lindhard calculation of \(\eta\) mentioned above is valid for \(Z_1 = Z_2\), single element media. A simple model, the independent element approach, has been proposed to obtain \(q_{nc}\) values for compounds \[1\]. In binary gases, e.g., taking C ions in C instead of C ions in CF\(_4\), and F ions in F instead of F ions in CF\(_4\). The results calculated for C and F recoils in CF\(_4\) are compared with experimental results in Fig. 1a \[6\]. The simple model shows a good agreement for F recoils in CF\(_4\). However, the experimental results for C recoils are considerably smaller than the estimated values. One reason may due to the compound ratio \([\text{C}]:[\text{F}] = 1:4\). C recoils see F atoms 4 out of 5. The Bragg-like curve is given for C and F recoils in CF\(_4\) in Fig. 1b. The axis on the right has been changed using better value of \(W = 34\) eV.

Fig. 1. a) The nuclear quenching factor \(q_{nc}\) estimated for C and F recoil ions and recoil Pb ions in \(\alpha\)-decay in CF\(_4\), see ref \[1\]. The closed circles (C recoil ions) and squares (F recoil ions) are measurements \[6\]. b) The Bragg-like curve estimated for C and F recoil ions in CF\(_4\). The area below each curve expresses the number of ions produced, \(N_i\). A value of 34.3 eV was taken for the \(W\) value for CF\(_4\).

2.2. The energy partition

It has been mentioned that measured \(q_{nc}\) values for heavy recoil ions in \(\alpha\)-decay in CH\(_4\), C\(_2\)H\(_2\) and C\(_3\)H\(_4\) seem to be too small \[1\]. Also, the experimental results for He-He collisions in 5-50 keV was reported about 40-80 % calculated values based on Lindhard model \[6\]. The result, however, does not necessary mean that calculations by Lindhard model is not adequate. They measured the charge produced in He-C\(_3\)H\(_4\)(0) mixture. The number of ions produced par unit electronic energy deposit for recoil ions can be considerably smaller than those for electrons or \(\alpha\)-particles. The average electronic
energy required to produce an electron-ion pair can be different for slow particles in light elements and molecules. The energy balance [7] gives the partition of the electronic energy between the ionization and the excitation,

\[
W/I = (E_i/I) + (E_{ex}/I)(N_{ex}/N_i) + (\varepsilon/I),
\]

where, \(I\) is the ionization potential, \(N_{ex}\) and \(N_i\) are numbers of excitation and ionization produced, \(E_{ex}\) and \(E_i\) are average energy expenditure of excitation and ionization, \(\varepsilon\) is average energy of sub-excitation electrons. For the fast particles, the optical approximation is good. The resonance states \(^1P\) are excited. The ratio of \(N_{ex}/N_i\) can be estimated from the oscillator strength. Eq. (2) gives a constant \(W/I\) values for fast particles such as \(\beta\)’s and \(\alpha\)’s within a few % for rare gases. However, \(^1S\) and \(^3S\) states are excited by slow ions and those states are ‘true’ metastable in He. The energy spent for the metastable states does not give ionization in pure He. When some molecules are introduced in He, the energy transfer from metastable states to the molecules will take place. Those include Penning ionization, however, considerable portion will be wasted for dissociation and non-radiative processes. Triplet states are excited by slow ions also in CH\(_4\), C\(_2\)H\(_4\) and C\(_3\)H\(_6\) and the energy partition can be different from that by fast ions.

3. Collision theory for slow ions

The Coulomb effect is the deflection and the deceleration of the projectile in the field of the nucleus [8, 9]. The threshold effect states that the energy delivered to the electron must be as large as the ionization energy. We discuss those effects in this section in relation to the heavy ion track, and the molecular orbit (MO) theory.

3.1. Heavy ions

Heavy ions produces co-axial cylindrical track [10]. Direct interaction, the Coulomb interaction, with the primary particle create the core, the inner zone of relatively high-energy density. Bohr’s impulse principle considers the incident particle of velocity \(v\) interacting only for a duration \(2b/v\), where \(b\) is the classical impact parameter, defined as the distance of closest approach. The uncertainty between the energy \(\Delta E\) and the collision time \(\Delta t\) gives:

\[
\Delta E \cdot \Delta t \approx \Delta E \cdot \frac{2b}{v} \leq \hbar. \tag{3}
\]

If the energy difference of two states \(E_1 - E_0\) is less that \(\Delta E\), the system cannot be decided between the two states in the collision, then the electronic excitation can take place. Then the radius of the track core \(b_{max}\) is given as \(b_{max} = \hbar v/2E_1\), where \(E_1\) is the lowest excitation energy. The Coulomb effect prevents the projectile to come close to the target atom in the field of the nucleus when collision velocity \(v\) is not fast enough.

The penumbra, the outer zone of low-energy density, is formed by \(\delta\)-rays produced by close collisions of hard sphere. The maximum energy given to recoil electron is \(4(m_e/M)E\), where \(m_e\) is the mass of the electron and \(M\) is the atom mass, \((m_e \ll M\)). The energy of the electron must be higher than \(E_1\) in this simple picture. The experimental evidence shows that the electronic excitation takes place even below the threshold. However, they argue that the threshold effect reduces \(S_3\) [8].

Some pointed out [8] that the introduction of the coulomb effect reduces the stopping cross section only by a few % and other stated the reduction is significant [9]. There has been numbers of nuclear stopping power calculation using more refined potentials. They also find that the Lindhard theory gives considerably larger values at low energy for both \(S_3\) and \(S_e\). It is the ratio of stopping powers, rather than absolute values, which influences most the nuclear quenching factor \(q_{nc} = \eta/E\). The ratio of \(S_3\) and \(S_e\) by Lindhard theory can be good even if the absolute values are too large. Therefore, the Lindhard theory is still useful at low energy when no other convenient models are available and indeed explains a wide range of experimental results. The Coulomb effect and the threshold effect in the
stopping power theory are not important in very low energy since those effects are valid for collisions of low and intermediate, or higher energy. Refinements or corrections of Lindhard theory or Firsov theory are not enough. The real issue in very slow collisions is that separation of the scattering and stopping at very low energy. As Lindhard has pointed out, the nuclear scattering and stopping becomes somewhat uncertain, because the Thomas-Fermi treatment, which most stopping theories for low energy collisions based on, becomes a crude approximation at very low energy [2]. A new approach is necessary as discussed in the following section.

The measurement of stopping power for heavy ions of low velocity is quite difficult. Usually, the total stopping power $S_T = S_n + S_e$ is measured and often Lindhard theory is assumed for either $S_n$ or $S_e$. Fukuda has measured $S_e$ using a sophisticated apparatus and concluded that Lindhard theory tend to give large $S_e$ values [11]. However, the measurement did not take contributions from $\theta > 0$.

3.2. Molecular orbit

When atomic projectile goes hard (wide deflection angle small impact parameter) collision with atom, the large inelastic energy losses occur at characteristic internuclear distances. Showers of fast-electrons are thrown out [12]. When the collision velocity $v_c$ become much slower than the orbital electron velocity $v_e$, the electron can follow to the colliding nuclear axis. The transient formation of molecular orbitals (MO’s) takes place. The coordinate system for a quasimolecule is composed of an electron and the two nuclei. A significant part of the ion energy can be transferred into the electronic excitation and electrons can be released effectively by very slow ions, such as:

$$\text{Ar}^+ + \text{Ar} \rightarrow \text{Ar}^{(m+n)} + \text{Ar}^{(m+n)} + (m+n-1)\text{e}^-.$$ (4)

An electron occupies a MO and become excited during the collision to a higher MO at smaller separation. The system contains vacancies after the collision. Then, energy losses, multiple ionization, ejection of fast electrons and photons follows. Those associated with a large scattering angle $\theta$. The MO theory [13] has been applied mainly to the inner shell excitation and there are no applications for the low energy stopping theory.

References

[1] Hitachi A 2008 Rad Phys Chem 77 1311
[2] Lindhard J et al 1963 Mat Fys Medd Dan Vid Selesk 33 (10) 1
[3] Lewin J D and Smith P F 1996 Astropart Phys 6 87
[4] Hitachi A 2005 Astropart Phys 24 247
[5] Birks J B 1964 The Theory and Practice of Scintillation Counting (Oxford: Pergannon)
[6] Guillaudin O et al 2012 EAS Pub Ser 53, 119
[7] Platzman R L 1961 Int J Appl Rad Iso 10 116
[8] Semrad D 1986 Phys Rev A 33 1646
[9] Tilinin I S 1995 Phys Rev A 51 3058
[10] Mozumder A 1999 Fundamentals of Radiation Chemistry (San Diego: Academic Press)
[11] Fukuda A 1981 J Phys B: At Mol Phys 14 4533
[12] Kessel Q C and Everhart E 1966 Phys Rev 146 16
[13] Fano U and Lichten W 1965 Phys Rev Lett 14 627