The Thermal Field Theory methods are applied to calculate the dispersion relation of the photon propagating modes in a strictly one-dimensional ideal plasma. The electrons are treated as a gas of particles that are confined to a one-dimensional tube or wire, but are otherwise free to move, without reference to the electronic wave functions in the coordinates that are transverse to the idealized wire, or relying on any features of the electronic structure. The relevant photon dynamical variable is an effective field in which the two space coordinates that are transverse to the wire are collapsed. The appropriate expression for the photon free-field propagator in such a medium is obtained, the one-loop photon self-energy is calculated and the (longitudinal) dispersion relations are determined and studied in some detail. Analytic formulas for the dispersion relations are given for the case of a degenerate electron gas, and the results differ from the long-wavelength formula that is quoted in the literature for the strictly one-dimensional plasma. The dispersion relations obtained resemble the linear form that is expected in realistic quasi-1D plasma systems for the entire range of the momentum, and which have been observed in this kind of system in recent experiments.

1 Introduction

The application of thermal field theory (TFT)\textsuperscript{1,2,3} to study lower dimensional systems is of interest for several reasons. Some of them have to do with the intrinsic interest and potential applications that they may have in condensed-matter and other related branches of physics. On the other hand, there has
recently been suggestions that lower dimensional systems may be relevant at a fundamental level. One idea is that the effective dimensionality of the space we live in depends on the length scale being probed and in particular at short scales the space is lower dimensional\cite{1, 5, 6}, and lower dimensional systems may also emerge as some extension of the standard model of particle physics\cite{7}. In such contexts, the TFT formulation for lower dimensional systems or models can have phenomenological applications for particle physics and cosmology as well.

Here we use the TFT methods to study the electromagnetic properties of a one-dimensional plasma, in which the electrons are confined to a line (i.e., an ideal one-dimensional wire which can be taken to be the $z$ axis). These systems have been studied in recent experiments\cite{8}, and earlier from a theoretical point of view\cite{9, 10}. The analogous non-abelien systems have also been considered\cite{11}. The quantities of interest are the dispersion relations and damping of the propagating photon modes and, from the TFT point of view, the quantities to be determined are the free-field propagator for the photon effective field in the medium and the self-energy, from which the dispersion relations can be obtained.

The same method has been applied to the 2D plane sheet in Ref. \cite{12}. As emphasized there, this is not the same thing as what is usually called QED$_3$ (or QED in 2+1 dimensions), which had been studied previously in the literature\cite{13, 14}. In QED$_3$, the system, with regard to the space coordinates, has cylindrical symmetry, and the physics, being independent of the $z$ coordinate, can be studied by considering a two-dimensional cross section. An important consequence of this difference is that the propagation of the photon in the layer is described by an effective field which has a corresponding free-field propagator that is very different from the usual one. The photon propagator is an important quantity because its inverse determines the bilinear part of the effective action or, equivalently, the equation of motion for the effective field, from which the dispersion relations and wave functions of the propagating modes can be obtained.

We follow here a similar procedure for the wire. As we will see, the photon free-field propagator in the present case is ultraviolet logarithmically divergent. The existence of this kind of divergence in the strictly 1D plasma has been known for some time\cite{9}. The workaround has been to introduce a cut-off parameter and evaluate the divergent integrals in terms of it. However, this procedure leads to a long-wavelength dispersion relation that is valid only for a very small range of the momentum $\kappa$. The alternative has been to consider more realistic models using the fact that a real quasi-1D plasma has some finite radial width, and then take into account the electronic bound state wave functions in the directions perpendicular to the wire. Since the calculations of this type are typically numerical ones, and they involve some knowledge or modeling of the electronic structure of the system under consideration, they are not applicable to more general situations such as, for example, the relativistic and high energy limits. Thus, there is no treatment of the strictly 1D ideal plasma system that models the results predicted by studying realistic quasi-1D plasma systems.
This work fills this gap. Here we insist in considering the strictly 1D ideal system of free electrons. We determine the appropriate expression for the photon free-field propagator and, using the TFT rules, we obtain the 1-loop formula for the photon self-energy in the medium. The formula is given, as usual as an integral over the electron momentum distribution function. The general expression for the dispersion relation is obtained and explicit formulas are given by considering specifically the case of a degenerate electron gas. The result differs from the cut-off dependent long-wavelength formula that is quoted in the literature for the strictly 1D system. The dispersion relation obtained resembles the linear form that has been observed in the recent experiments in systems of this type\[8\] and which is expected on the basis of the numerical calculations that take into account the electronic structure and finite-width of a real quasi-1D system. The result obtained here for the degenerate, strictly 1D system, has a leading term $v_F \kappa$, where $v_F$ is the Fermi velocity of the electrons, with logarithmic corrections proportional to $e^2 v_F^2 / 2 \pi^2$.

The logarithmic ultraviolet divergence of the free-field photon propagator is treated by noticing that its derivative is finite and calculable. The propagator, and the physical quantities, then depend on an unknown mass scale parameter $\mu$ that appears as an integration constant. Our point of view is that this leads to an effective theory, and the results obtained for the physical quantities of interest are applicable to a wide range of real quasi-1D systems, in the situations in which they can be idealized as a 1D plasma of free electrons, independently of the particular electronic structure, width or geometry in the perpendicular directions of such systems. Those details are parametrized by the single parameter $\mu$. While $\mu$ cannot be determined, this strategy is very economical since it is the only unknown free parameter of the model.

This paper is organized as follows. In Section 2 we summarize the notation for the kinematic variables used throughout. In Section 3 the assumptions that define the model are stated precisely. In particular, the photon effective field that will be treated as the relevant dynamical variable is identified, along with its interaction with the electron field. In Section 4 the photon free field propagator is determined and the equation for the dispersion relation of the propagating modes is written in terms of its inverse and the photon self-energy. In Section 5, 2 the one-loop calculation of the photon self-energy is carried out and the dispersion relations are studied in some detail by considering specifically the case of a degenerate electron gas.

2 Notation and kinematics

We denote by $u^\mu$ the velocity four-vector of the medium. Adopting the frame in which the medium is at rest, we set

$$ u^\mu = (1, 0), $$

and from now on all the vectors refer to that frame. We introduce the four-vector

$$ n^\mu = (0, \bar{n}), $$

\[2\]
where $\vec{n}$ is the unit vector along the wire, and denote the momentum four-vector of a photon that propagates in the wire by

$$k_\mu^\nu = (\omega, \vec{k}_\parallel),$$

(3)

where

$$\vec{k}_\parallel = \kappa \vec{n},$$

(4)

and its square

$$k_\parallel^2 = \omega^2 - \kappa^2.$$  

(5)

Since neither $u^\mu$ nor $n^\mu$ is orthogonal to $k_\parallel^\mu$, it is useful to define the combination

$$\tilde{u}_\mu \equiv u_\mu - \frac{\kappa k_\parallel \mu}{k_\parallel^2},$$

(6)

which satisfies

$$k_\parallel \cdot \tilde{u} = 0.$$  

(7)

Any given four dimensional vector $a^\mu = (a^0, \vec{a})$ can be decomposed in the form

$$a^\mu = a_\parallel^\mu + a_\perp^\mu,$$

(8)

where

$$a_\parallel^\mu = (a^0, \vec{a}), \quad a_\perp^\mu = (0, \vec{a}_\perp),$$

(9)

with

$$\vec{a}_\parallel = (\vec{a} \cdot \vec{n}) \vec{n}, \quad \vec{a}_\perp = \vec{a} - \vec{a}_\parallel.$$  

(10)

Without loss of generality, we can take the $z$ axis to point along $\vec{n}$, and with that convention the components of $\vec{a}_\parallel$ and $\vec{a}_\perp$ are

$$a_\parallel^i = (0, 0, a^3), \quad a_\perp^i = (a^1, a^2, 0).$$

(11)

It is useful to introduce the tensors

$$Q_{\mu\nu} = \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}^2},$$

$$R_{\mu\nu} = g_{\mu\nu} - \frac{k_\parallel^\mu k_\parallel^\nu}{k_\parallel^2} - Q_{\mu\nu},$$

(12)

which are transverse to $k_\parallel^\mu$, and define

$$g_{i\mu\nu} = g_{\mu\nu} - R_{\mu\nu}.$$  

(13)

Noticing that $R$ satisfies

$$u^\mu R_{\mu\nu} = n^\mu R_{\mu\nu} = 0,$$

(14)
the decomposition in Eq. (8) can be accomplished by writing

\[
\begin{align*}
    a_\perp \mu &= R_{\mu \nu} a^\nu, \\
    a_\parallel \mu &= g_{\mu \nu} a^\nu. 
\end{align*}
\]

Moreover, if \( a^\mu \) is transverse to \( k_\parallel^\mu \), that is

\[
a \cdot k_\parallel = 0,
\]

we then have

\[
a_\parallel \mu = Q_{\mu \nu} a^\nu,
\]

and in particular

\[
a_\parallel \mu \propto \tilde{u}_\mu.
\]

3 The Model

We wish to emphasize that these two systems, to which we refer as the layer and the wire, are not the same thing as what are usually called \( QED_3 \) (or \( QED \) in 2+1 dimensions) and \( QED \) in 1+1 dimensions, respectively. For example in \( QED_3 \), which has been studied previously in the literature\[13, 14\], the system has cylindrical symmetry in the spatial coordinates, and therefore the physics, being independent of the \( z \) coordinate, can be studied by considering a two-dimensional cross section. Thus, for example, the electron in \( QED_3 \) is really a line of charge in the three-dimensional world, and the Coulomb potential between two such electrons is logarithmic. In contrast, in the system that we considered, the electron is an ordinary point charge, which is confined to the \( z = 0 \) plane, but the Coulomb potential between two electrons is the usual \( 1/r \) potential.

3.1 The electron field

We envisage the system as a tube along the \( z \) axis of length \( L \) and cross sectional area \( L_\perp^2 \) in which the electrons are confined but otherwise free to move, and eventually the limits \( L \to \infty \) and \( L_\perp \to 0 \) are taken. In the Furry picture, the one-particle electron wavefunction is a product of a plane wave in the \( z \) direction times some function \( H(\vec{x}_\perp) \) in the perpendicular directions. While in principle the function \( H \) is characterized by some quantum number, the assumption is that in the \( L_\perp \to 0 \) limit only the lowest state survives. Specifically, the model is based on the assumption that the electron wavefunctions are such that, when that limit is taken, the electron current density operator reduces to

\[
    j^\mu(x) = \delta^{(2)}(\vec{x}_\perp) \hat{\psi}(x_\perp) \gamma^\mu \hat{\psi}(x_\perp),
\]

where, in the free field case,

\[
    \hat{\psi}(x_\perp) = \int \frac{dp}{(2\pi)^2} \left[ a(\vec{p}_\perp, s) u(\vec{p}_\parallel, s) e^{-ip_\parallel \cdot x_\parallel} + b^*(\vec{p}_\parallel, s) v(\vec{p}_\parallel, s) e^{ip_\parallel \cdot x_\parallel} \right],
\]

5
with
\[
\begin{align*}
p_{\parallel}^u &= (E, \vec{p}_{\parallel}), \\
\vec{p}_{\parallel} &= \vec{p} n, \\
E &= \sqrt{p^2 + m^2}.
\end{align*}
\]
(21)
The spinors \(u\) are the standard Dirac spinors normalized such that
\[
u \bar{u} = 2m,
\]
and the creation and annihilation operators satisfy
\[
\{ a(\vec{p}', s), a^*(\vec{p}'', s') \} = (2\pi)^2 E \delta(p - p') \delta_{s,s'},
\]
(23)
with analogous relations for the spinors \(v\) and the \(b\) operators.

The thermal propagators for the field \(\psi\) are given by the familiar formulas[1, 2, 3],
\[
\begin{align*}
S_{11}(p_{\parallel}) &= (\gamma_0 + m_e) \left[ -\frac{1}{p_{\parallel}^2 - m_e^2 + i\epsilon} + 2\pi i \delta(p^2 - m_e^2) \eta_e(p_{\parallel}) \right], \\
S_{22}(p_{\parallel}) &= (\gamma_0 + m_e) \left[ -\frac{1}{p_{\parallel}^2 - m_e^2 - i\epsilon} - 2\pi i \delta(p^2 - m_e^2) \eta_e(p_{\parallel}) \right], \\
S_{12}(p_{\parallel}) &= (\gamma_0 + m_e) 2\pi i \left[ \eta_e(p_{\parallel}) - \theta(p_{\parallel} \cdot u) \right], \\
S_{21}(p_{\parallel}) &= (\gamma_0 + m_e) 2\pi i \left[ \eta_e(p_{\parallel}) - \theta(-p_{\parallel} \cdot u) \right],
\end{align*}
\]
(24)
where
\[
\eta_e(p) = \theta(p \cdot u) f_e(p \cdot u) + \theta(-p \cdot u) f_{\bar{e}}(-p \cdot u),
\]
(25)
with
\[
\begin{align*}
f_e(x) &= \frac{1}{e^{\beta_e(x - \mu_e)} + 1} \\
f_{\bar{e}}(x) &= \frac{1}{e^{\beta_e(x + \mu_e)} + 1}
\end{align*}
\]
(26)
and \(\theta(x)\) is the step function. Here \(\beta_e\) and \(\mu_e\) are the inverse temperature and the chemical potential of the electron gas, respectively.

The total number of particles in the gas can be calculated from
\[
N = \int d^3 x j^0(x) = L \int \frac{dp}{(2\pi)^3} \text{Tr} [S_{11}(p_{\parallel}) \gamma^0],
\]
(27)
where we have used Eq. (19) and we have set \(\int dz \to L\). Using formulas given above for the propagator, this yields
\[
N/L = n_e + n_{\bar{e}},
\]
(28)
where
\[
n_{e,\bar{e}} = 2 \int \frac{dp}{(2\pi)^3} \frac{1}{e^{\beta_e(E + \mu_e)} + 1}
\]
(29)
represent the linear density of electrons and positrons, respectively.
3.2 The photon effective field

Using Eq. (19) the usual interaction Lagrangian term \( j \cdot A \) yields the following term in the action

\[
S_{\text{int}} = -e \int d^2x_{\|} \bar{\psi} \gamma_{\mu} \psi \hat{A}_{\mu},
\]

(30)

where

\[
\hat{A}_{\mu} \equiv A_{\mu}|_{x_\perp = 0}.
\]

(31)

This indicates in particular that the transverse component \( A_{\perp} \) decouples. Thus, regarding \( \hat{A}_{\mu} \) as the effective field for the photon, our goal is to determine its effective action, or equivalently its equation of motion, including the thermal corrections. Formally, this involves integrating out all the dynamical field variables except \( \hat{A}_{\mu} \) itself.

Adapting the functional method of quantization of the electromagnetic field\cite{15} to the present model, a convenient way to proceed is to introduce in the action an external current of the form

\[
J_{\mu}(x) = \delta^{(2)}(\vec{x}_\perp) J_{\|}(x_{\|}),
\]

(32)

with \( J_{\parallel}(x_{\|}) \) satisfying

\[
\partial_{\parallel} \cdot J_{\parallel}(x_{\|}) = 0,
\]

(33)

where \( \partial_{\parallel} = (\partial_{x_{\|}}, -\partial_{\vec{x}_{\perp}}) \). Eq. (33), which in turns implies,

\[
\partial \cdot J = 0.
\]

(34)

ensures that the source term is selecting the gauge invariant (i.e., transverse to the photon momentum four-vector) part of the longitudinal component \( A_{\parallel} \).

The classical field \( A_{\mu}^{(J)} \), in the presence of both the external current \( J_{\parallel} \) and the interaction given by \( S_{\text{int}} \) in Eq. (30), is then defined by

\[
A_{\mu}^{(J)} = \frac{1}{Z} \frac{i \delta Z}{\delta J_{\parallel}},
\]

(35)

where \( Z \) is the generating functional.

4 Photon propagation in the wire

4.1 Photon free-field propagator

The question here is, what is the propagator associated with the photon effective field \( \hat{A}_{\mu} \)? Following the usual argument, the generating functional for the photon free field in the wire is

\[
Z \propto \exp \left\{ -\frac{i}{2} \int d^2x_{\|} d^2x'_{\|} J_{\|}(x_{\|}) \hat{A}_{\mu}(x_{\|} - x'_{\|}) J_{\|}(x'_{\|}) \right\},
\]

(36)
where $\hat{\Delta}_{F\mu\nu}(x - x')$ is obtained from the standard photon propagator $\Delta_{F\mu\nu}(x - x')$ by setting the coordinates normal to the wire ($\vec{x}_\perp$ and $\vec{x}'_\perp$) equal to zero, as implied by the delta function in Eq. (32). Therefore, taking into account Eq. (33) and remembering Eq. (18), the free-field propagator in the wire is given, in momentum space, by

$$\hat{\Delta}_{F\mu\nu}(k) = Q^{\mu\alpha}Q^{\nu\beta} \left( \int \frac{d^2k}{(2\pi)^2} \Delta_{F\alpha\beta}(k) \right),$$  

where $Q_{\mu\nu}$ has been defined in Eq. (12) and, in the integrand, the momentum vector $k$ is decomposed in the form

$$k_\mu = k_\perp + k_\parallel,$$

with $k_\parallel$ as given in Eq. (3) and $k_\perp = (0, \vec{\kappa}_\perp)$. Writing

$$\Delta_{F\mu\nu}(k) = \frac{-g_{\mu\nu}}{k^2 + i\epsilon} + \text{gauge-dependent terms},$$

we then obtain

$$\hat{\Delta}_{F\mu\nu}(k) = -\hat{\Delta}(k)Q_{\mu\nu},$$

where

$$\hat{\Delta}(k) = \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - \kappa^2 + i\epsilon}.$$

This integral is ultraviolet logarithmically divergent and whence it cannot be computed using the above formula literally without modification. This is of course due to our insistence on considering the strictly infinitely thin wire. However, rather than give up and resort to the fact that a realistic wire has some finite width and incorporating the electronic wave functions would soften the integral, we proceed as follows.

We notice that its derivative

$$\frac{\partial \hat{\Delta}(k)}{\partial k_\parallel^2} = -\int \frac{d^2k}{2\pi} \frac{1}{(k^2 - \kappa^2 + i\epsilon)^2},$$

is finite and therefore, by a straightforward evaluation,

$$\frac{\partial \hat{\Delta}(k)}{\partial k_\parallel^2} = \frac{1}{4\pi k_\parallel^2 + i\epsilon}.$$

Thus, by direct integration this implies that

$$\hat{\Delta}(k) = \frac{1}{4\pi} \log \left( \frac{k_\parallel^2 + i\epsilon}{\mu^2} \right),$$

where $\mu^2$, which appears is a constant of integration, is a parameter that we cannot determine further.
The existence of the divergences of the strictly 1D plasma has been known in the plasma physics literature for some time. The workaround has been to resort to the fact that a real quasi-1D plasma has some finite (radial) width, and then take into account the electronic bound state wave functions in the directions perpendicular to the wire[9]. Thus, since the calculations of this type have been numerical ones, and they involve some knowledge or modeling of the electronic structure of the system under consideration, they are not applicable to more general situations such as, for example, the relativistic and high energy limits.

Our point of view is that this approach leads us to an effective theory that is valid for energy scales \((\omega, \kappa, \mu)\) less than some mass scale \(\Lambda\) which is of the order of the inverse length of a radial dimension of the wire. The results obtained for the physical quantities of interest using the effective theory can then be applicable to a wide range of real quasi-1D systems, in the situations in which they can be idealized as a 1D plasma of free electrons, independently of the particular electronic structure, width or geometry in the perpendicular directions of such systems. All those details are hidden and parametrized by the single parameter \(\mu\), which we cannot determine, reflecting our ignorance of those details. On the other hand, this strategy is very economical when we take into account the fact that \(\mu\) is the only unknown parameter of the model.

### 4.2 Photon self-energy and equation of motion

Denoting the photon self-energy in the medium by \(\hat{\pi}_{\mu\nu}\), the bilinear part of the effective action for \(A^{(J)}_{\mu}\) is then given, in momentum space, by

\[
S^{(2)} = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} A^{(J)*}_{\mu}(k) \left[ D^{\mu\nu}(k) + \hat{\pi}^{\mu\nu}(k) \right] A^{(J)}_{\nu}(k) - A^{(J)*}_{\mu}(k) \cdot \hat{J}(k) \right\},
\]

(45)

where \(D^{\mu\nu}(k)\) is defined by

\[
\hat{\Delta}^{\mu\lambda}(k) D_{\lambda\nu}(k) = Q^\nu_{\mu}.
\]

(46)

The equation of motion for the classical field in the absence of the external current is then

\[
[D^{\mu\nu}(k) + \hat{\pi}^{\mu\nu}(k)] A^{(0)}_{\nu}(k) = 0.
\]

(47)

Since on one hand Eq. (44) implies that

\[
D_{\mu\nu}(k) = -\hat{\Delta}^{-1}(k) Q_{\mu\nu},
\]

(48)

where \(\hat{\Delta}(k)\) is given in Eq. (44), while in the other hand, as we will verify, \(\hat{\pi}_{\mu\nu}\) is of the form

\[
\hat{\pi}_{\mu\nu}(k) = \hat{\pi}(k) Q_{\mu\nu},
\]

(49)
\[ i\pi^{(ab)}_{\mu\nu}(k_\parallel) = \int \frac{d^2p_\parallel}{(2\pi)^2} \text{Tr} \gamma_{\mu} iS_{11}(p_\parallel + k_\parallel) \gamma_{\nu} iS_{11}(p_\parallel) \]

Figure 1: One-loop diagram for the photon thermal self-energy matrix.

Eq. (47) implies the condition
\[ \Delta^{-1}(k_\parallel) - \hat{\pi}(k_\parallel) = 0, \]
which determines the dispersion relations of the propagating modes.

Furthermore, since in this work we are concerned only with the real part of the dispersion relation, in order to determine \( \hat{\pi} \) we need to calculate only the 11 element of the thermal self-energy matrix \( \pi^{(11)}_{\mu\nu} \), in terms of which
\[ \text{Re} \hat{\pi}_{\mu\nu}(k_\parallel) = \text{Re} \pi^{(11)}_{\mu\nu}(k_\parallel). \]

Taking the real part in Eq. (44), the real part of the dispersion relation is then obtained from
\[ 4\pi \left( \log \frac{k^2_\parallel}{\mu^2} \right)^{-1} - \text{Re} \hat{\pi} = 0. \]

5 Self-energy and dispersion relations

5.1 One-loop formula for the self-energy

Referring to Fig. 1, the one-loop formula for the the 11 element of the photon self-energy matrix is
\[ i\pi^{(11)}_{\mu\nu}(k_\parallel) = e^2 \int \frac{d^2p_\parallel}{(2\pi)^2} \text{Tr} \gamma_{\mu} iS_{11}(p_\parallel + k_\parallel) \gamma_{\nu} iS_{11}(p_\parallel). \]

When the formula for \( S_{11} \) given in Eq. (24) is substituted in Eq. (53), there are three types of terms. The term that contains two factors of \( \eta \) contributes only to the imaginary part of the self-energy and, since we restrict ourselves here to the real part, we do not consider it further. The remaining terms then yield
\[ \text{Re} \pi^{(11)}_{\mu\nu} = \pi^{(0)}_{\mu\nu} + \pi^{(m)}_{\mu\nu}, \]

where \( \pi^{(0)}_{\mu\nu} \) is the vacuum polarization term, which is neglected, while the background dependent contribution is given by
\[ \pi^{(m)}_{\mu\nu} = -4e^2 \int \frac{dp}{(2\pi)^2E} (f_e(E) + f_\nu(E)) \left[ \frac{L_{\mu\nu}}{k^2_\parallel + 2p_\parallel \cdot k_\parallel} (k_\parallel \rightarrow -k_\parallel) \right]. \]
In this formula,

\[ L_{\mu\nu} = 2p_\parallel\mu p_\parallel\nu + p_\parallel^{\mu}k_\parallel^{\nu} + k_\parallel^{\mu}p_\parallel^{\nu} - g_{\parallel\mu\nu}p_\parallel \cdot k_\parallel, \]  

(56)

where \( g_{\parallel\mu\nu} \) has been defined in Eq. (13), \( f_{\pm} \) denote the particle and antiparticle number density distributions defined in Eq. (26), while \( k_\parallel^{\mu} \) and \( p_\parallel^{\mu} \) are parametrized as indicated in Eqs. (3) and (21), respectively. Furthermore, the integral in Eq. (55) is to be interpreted in the sense of its principal value part.

It is easily verified that, besides being symmetric and transverse to \( k_\parallel^{\mu} \), \( \pi_{\mu\nu}^{(m)} \) satisfies

\[ R^{\mu\nu}_{\lambda\lambda'} \pi_{\lambda\nu}^{(m)} = 0. \]  

(57)

These properties imply that it is of the form given in Eq. (49), and the coefficient of \( Q_{\mu\nu} \) can be found by projecting Eq. (55) with \( Q_{\mu\nu} \). This procedure then yields

\[ \hat{\pi} = 4e^2 k_\parallel^2 B, \]  

(58)

with

\[ B = \int \frac{dp}{4\pi E} (f_+ (E) + f_-(E)) \times \left[ \frac{2(p_i \cdot u)^2 + 2(p_i \cdot u)(k_\parallel \cdot u) - p_\parallel \cdot k_\parallel + (k_\parallel \rightarrow -k_\parallel)}{k_\parallel^2 + 2p_\parallel \cdot k_\parallel} \right]. \]  

(59)

Substituting Eq. (58) in Eq. (52), we obtain

\[ \left( \log \frac{k_\parallel^2}{\mu^2} \right)^{-1} - \frac{e^2}{\pi} \left( \frac{k_\parallel^2}{\kappa^2} \right) B = 0, \]  

(60)

which is the equation to be solved for \( \omega(\kappa) \).

A useful formula for \( B \), that holds when the photon momentum is such that \( \omega, \kappa < E_e \), is obtained as follows. In this case Eq. (59) can be approximated by the form

\[ B(\omega, \kappa) = \frac{1}{2} \int \frac{dp}{(2\pi)} \left( \frac{vk}{\omega - v\kappa} \right) \frac{d}{dE} (f_e + f_\pi), \]  

(61)

where \( E_e \) is a typical energy of an electron in the gas, is obtained as follows. In this case Eq. (59) can be approximated by the form

\[ B(\omega, \kappa) = -\frac{1}{2} \int \frac{dp}{(2\pi)} \left( \frac{vk}{\omega - v\kappa} \right) \frac{d}{dE} (f_1 + f_\pi), \]  

(62)

where \( v = p/E \) is the velocity of the particles in the background and we have indicated explicitly the dependence of \( B \) on the photon momentum variables. The form given in the second line can be obtained from the first line by inserting in the integrand the factor \( (\omega - v\kappa + v\kappa) \). Eq. (62) is obtained from Eq. (61) by expanding the integrand in terms of \( k_\parallel/E \) and retaining only the dominant terms when the limit \( k_\parallel/E \) is taken. If the gas in non-relativistic, Eq. (62) holds for \( \omega, \kappa < m_e \). For a relativistic gas, Eq. (62) holds also for \( \omega, \kappa > m_e \), subject to Eq. (61). Thus, Eq. (62) is a useful formula that can be employed to find the dispersion relations from Eq. (60) in many situations of interest.
5.2 Dispersion relations

The integral in Eq. (62) cannot be reduced any further in general, but it can be evaluated for specific cases of the distribution functions or by making further approximations that depend on the kinematic regime being considered and the conditions of the electron gas.

We consider for definiteness a completely degenerate electron gas. A simple evaluation of Eq. (62) then yields

$$B = \frac{v_F}{2\pi} \frac{\kappa^2}{\omega^2 - v_F^2 \kappa^2},$$  \hspace{1cm} (63)

where $v_F$ is the Fermi velocity of the electrons. Substituting this in Eq. (60), the dispersion relation, which we denote by $\omega = \omega(\kappa)$ is then obtained by solving

$$\omega^2 - v_F^2 \kappa^2 = \omega_p^2 (\omega^2 - \kappa^2) \log \left( \frac{\mu^2}{\kappa^2 - \omega^2} \right),$$  \hspace{1cm} (64)

with

$$\omega_p^2 = \frac{2\alpha v_F}{\pi},$$  \hspace{1cm} (65)

where we have introduced the fine structure constant $\alpha = e^2/4\pi$. This equation has a solution for $\omega > \kappa$, but it is not a physical one since it lies outside the range given in Eq. (61) for which Eq. (62) is valid.

In order to consider the solution with $\omega < \kappa$, we rewrite Eq. (64) in the form

$$\omega^2 - v_F^2 \kappa^2 = \omega_p^2 (\kappa^2 - \omega^2) \log \left( \frac{\mu^2}{\kappa^2 - \omega^2} \right),$$  \hspace{1cm} (66)

The solution to this equation can be represented in parametric form as

$$(\omega/\mu)^2 = \gamma_F^2 v_F^2 t + \gamma_p^2 \omega_p^2 \log(1/t),$$  

$$\kappa/\mu)^2 = t + \gamma_F^2 v_F^2 t + \gamma_p^2 \omega_p^2 \log(1/t).$$  \hspace{1cm} (67)

On the other hand, an explicit approximate solution of Eq. (66) can be obtained as follows. Since, according to Eq. (65), $\omega_p^2$ is not larger than about $10^{-2}$, we can consider an expansion in terms of $\omega_p^2$. Thus, to the zeroth order,

$$\omega^{(0)} = v_F \kappa,$$  \hspace{1cm} (68)

and substituting this term in the right-hand side of Eq. (66) yields the solution

$$\omega^{(1)} = \left[ v_F^2 \kappa^2 + \frac{\omega_p^2 \kappa^2}{\gamma_F^2} \log \left( \frac{\mu^2}{\kappa^2} \right) \right]^{1/2},$$  \hspace{1cm} (69)

where

$$\gamma_F = (1 - v_F^2)^{-1/2}.$$  \hspace{1cm} (70)
The function \( \omega_{\kappa}^{(1)} \) is plotted in Fig. 2 for various values of \( v_F \). Fig. 3 shows the plots of \( \omega_{\kappa}^{(1)} \) as well as the exact solution \( \omega_{\kappa} \) of Eq. (66) together with the plot of the zeroth order term \( \omega_{\kappa}^{(0)} \), for \( v_F = 0.3 \). For comparison, Fig. 3 includes the plot of the long-wavelength approximation formula

\[
\omega_{\kappa}(\ell) = \omega_p\kappa \left[ \log \frac{\mu}{\kappa^2} \right]^{1/2}.
\]

which is quoted in the literature for the strictly 1D plasma.

Notice that Eq. (71) can be obtained formally from Eq. (69) when \( v_F \) and \( \kappa \) are small, which evidences why that formula has a very restricted range of validity. On the other hand, as can be seen from Fig. 3, the function \( \omega_{\kappa}^{(1)} \) approximates very well the exact solution of Eq. (66), represented by Eq. (67). Their difference becomes smaller or even negligible for higher values of \( v_F \), as illustrated in Fig. 4. The plots in Fig. 2 exhibit the property that the slope of the dispersion relation (the group velocity) is almost constant for the entire range of \( \kappa \), which are some of the general and unique characteristics that have been observed in recent experiments in this type of system.[8]

The main result here is that a proper treatment of the strictly 1D plasma leads to a dispersion relation which, (1) is very different from the long-wavelength formula of Eq. (71) that is quoted in the literature for this system, and (2) reproduces the expected characteristics that have been observed in this kind of system. It is appealing that this result has been obtained while insisting in considering the strictly 1D plasma. This contrasts with previous treatments in the literature that abandon the strictly 1D plasma and instead use the fact that a semi realistic 1D plasma has some finite width and therefore the calculations must necessarily be numerical model calculations involving the details of the electronic wave functions in the transverse directions in the wire.
Figure 3: Plot of the exact solution $\omega_\kappa$ of Eq. (66), the analytic formula $\omega_\kappa^{(1)}$ given Eq. (69), the zeroth order term $\omega_\kappa^{(0)}$ defined in Eq. (68) and the long-wavelength limit formula defined in Eq. (71), all for $v_F = 0.3$.

Figure 4: Plot of the exact solution $\omega_\kappa$ of Eq. (66), the analytic formula $\omega_\kappa^{(1)}$ and the zeroth order term $\omega_\kappa^{(0)}$, for $v_F = 0.6$. 
6 Conclusions

The Thermal Field Theory methods have been used to study the propagation of photons in the model of the strictly 1D plasma, that is, a system in which the electrons that are free to move but are confined to an infinitely thin tube, or wire. An important step was to identify the appropriate photon effective field and to determine the corresponding free-field propagator. We performed the one-loop calculation of the photon self-energy in that medium, and we considered the photon dispersion relations.

The dispersion relation was studied in some detail for the case of a degenerate electron gas, and analytic formulas were obtained. The dispersion relation has a leading term $v_F \kappa$, where $v_F$ is the Fermi velocity of the electrons, with logarithmic corrections proportional to $\varepsilon^2 v_F^2 / 2 \pi^2$. This result is very different from the long-wavelength formula that is usually quoted in the literature for this type of system. In particular the formulas obtained here are valid for all the range of values of $\kappa$ and they resemble the linear form that have been observed in recent experiments this type of system\cite{8}, and which are expected on the basis of the numerical calculations that take into account the electronic structure and finite-width of a real quasi-1D system\cite{9}.

While we envisaged the simplest situation of an ordinary gas of electrons, which are confined to a wire but are otherwise free, the same method can be used to consider variations of the model in a systematic way, such as the effects of external fields, which have been studied by other means\cite{16}.

The application of TFT that we have described here, and to the layer in Ref. \cite{12}, can be useful in the context of the recently proposed “vanishing dimensions”\cite{6} and “layered structure of space”\cite{4} ideas. The methods can be also useful in astrophysical\cite{17}, plasma physics\cite{18} and condensed matter\cite{19,20,8} and other systems of current interest in which a plasma is confined to a layer\cite{21} or a wire\cite{22}.

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