Electromagnetic Fields in a Thermal Background

Per Elmfors\textsuperscript{1}\textsuperscript{,} and Bo-Sture Skagerstam\textsuperscript{2}\textsuperscript{,}
\textsuperscript{a}CERN, TH-Division, CH-1211 Geneva 23, Switzerland
\textsuperscript{b}Institute of Theoretical Physics, Chalmers University of Technology and University of Göteborg, S-412 96 Göteborg, Sweden

Abstract

The one–loop effective action for a slowly varying electromagnetic field is computed at finite temperature and density using a real-time formalism. We discuss the gauge invariance of the result. Corrections to the Debye mass from an electric field are computed at high temperature and high density. The effective coupling constant, defined from a purely electric weak–field expansion, behaves at high temperature very differently from the case of a magnetic field, and does not satisfy the renormalization group equation. The issue of pair production in the real–time formalism is discussed and also its relevance for heavy–ion collisions.

\textsuperscript{1}Email address: elmfors@surya11.cern.ch.
\textsuperscript{2}Email address: tfebss@fy.chalmers.se. Research supported by the Swedish National Research Council under contract no. 8244-311.
1 Introduction

The formulation of quantum field theory in an external field is interesting because of the many applications where the background field is strong and cannot be treated perturbatively. Some relativistic examples are the magnetic field in neutron stars, white dwarfs and heavy-ion collisions. More extreme situations are given by the electroweak phase transition and cosmic strings. Strong (colour) electric fields furthermore appear in some models of hadronization in heavy-ion collisions (for some recent discussions see e.g. [1, 2]). In most of the examples above the fields exist in a thermal heat bath or some non-equilibrium background which is very different from the vacuum. In the case of hadronization in heavy-ion collisions it has actually been argued (see e.g. Ref. [3]) that there is a time-interval during which local thermal equilibrium has been achieved but the external (colour) electric field has not yet been depleted due to particle production. For such a time-interval the results of the present paper apply.

In this letter the one-loop effective action for a constant (or slowly varying) electromagnetic field is generalized to finite temperature and chemical potential, $(T, \mu)$. Or, to put it differently, the thermal effective action for a constant magnetic field is generalized to arbitrary constant electromagnetic fields. The case of a pure magnetic field has been treated earlier in [8, 9, 7] and the incomplete result of [9] was corrected in [7]. The case of a general electromagnetic field has been studied in [10, 11] and some corrections to the expressions for the effective action in those papers are presented here. For constant external fundamental fields there are various methods of calculating the effective action to all orders in the field. Schwinger calculated the effective action for a constant background field in QED, for which the gauge fields are non-constant. The proper-time method used in [6] relies on the fact that the solution of the quantum mechanical equations of motion for a particle in the background field can be found explicitly, and an extension to finite $(T, \mu)$ is possible using real-time thermal propagators. The case of a magnetic field has a rather clear physical interpretation at finite $(T, \mu)$ with the particles in equilibrium occupying the time-independent Landau levels. This situation has been studied in detail in [7]. In the presence of a slowly varying electric field the particles in equilibrium screen the field over a distance determined by the Debye screening length. Schwinger’s calculation for a general slowly varying electromagnetic field can be extended to finite $(T, \mu)$ in a rather straightforward manner without finding the particle spectrum explicitly. The constant field approximation is satisfied if the relative gradient of the field is smaller than any other scale, i.e. $|\partial_\alpha F_{\mu\nu}|/|F_{\mu\nu}| \ll |eF_{\mu\nu}|^{1/2}$; $m_e$, $T$ or $n^{1/3}$, where $n$ is the particle density. In
this letter we consider only the non–interacting $e^+e^−$–gas in a time–independent and slowly varying background where the conditions above are satisfied. For instance, the result applies to a shallow potential well with an arbitrary constant magnetic field and low density. It has been strongly argued against the possibility of having an electric field and a net charge density at equilibrium \[12\]. We want to stress that this may be true for a strictly constant electric field, but it is perfectly physical to have a slowly varying field at thermal equilibrium and consider an expansion in derivatives in the field, at least for a stable system. A deeper analysis is required to determine whether the result can be used for more general situations. We shall limit the discussion here to the thermal corrections since the vacuum part is easily added when needed.

2 One–loop effective action

Schwinger’s equation for the one–loop effective action, generalized to finite temperature and density, can be written as

$$\frac{\partial \Gamma[A]}{\partial m} = i \text{Tr} \langle x | \frac{1}{\not\! p - m + i\epsilon} - f_F(p_0, A_0) \left( \frac{1}{\not\! p - m + i\epsilon} - \frac{1}{\not\! p - m - i\epsilon} \right) | x \rangle, \quad (1)$$

where Tr is the trace over spin and $x$, $\not\! p = \gamma^\mu (p_\mu - eA_\mu)$, and $f_F(p_0, A_0)$ is the thermal distribution function. Under the gauge transformation $A_\mu(x) \mapsto A_\mu(x) + \partial_\mu \Lambda(x)$, the zero temperature part of Eq.(1) is transformed to

$$i \text{Tr} \langle x | e^{-ie\Lambda(x)} \frac{1}{\not\! p - m + i\epsilon} e^{ie\Lambda(x)} | x \rangle, \quad (2)$$

and is thus gauge invariant. Since the thermal part of Eq.(1) is not invariant under a general gauge transformation we have to explain the apparent problem of gauge dependence. The density matrix for the whole system of fermions and the electromagnetic field is given by

$$\rho_{\text{QED}} = \frac{\exp[-\beta \int d^3x (T_{\text{QED}}^{00}(x) - \mu e \Psi \gamma^0 \Psi)]}{\text{tr} \exp[-\beta \int d^3x (T_{\text{QED}}^{00}(x) - \mu e \Psi \gamma^0 \Psi)]}, \quad (3)$$

where $T_{\text{QED}}^{\mu\nu}$ is the energy–momentum tensor

$$T_{\text{QED}}^{\mu\nu} = F^{\mu\alpha} F_\alpha{}^\nu + (\partial_\alpha F^{\mu\alpha}) A^\nu + i \bar{\Psi} \gamma^\rho \partial_\rho \Psi + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - g^{\mu\nu} \bar{\Psi} (i\not\! \partial - e A - m) \Psi. \quad (4)$$

One can easily verify that $T_{\text{QED}}^{\mu\nu}$ is gauge invariant when Maxwell’s equation $\partial_\alpha F^{\alpha\mu} = e \bar{\Psi} \gamma^\mu \Psi$ is satisfied. It is, however, common to consider the fermions in the background
field $F^{\mu\nu}$ separately and only use the fermionic part of $\rho_{QED}$ to define the density matrix. The energy defining $\rho_{e^{+}e^{-}}$,

$$P_0 = \int d^3 x T^{00}_{e^{+}e^{-}} = \int d^3 x [\bar{\Psi}(i\gamma_i \partial_i - e\gamma_i A_i + m)\Psi + e\bar{\Psi}\gamma_0 A_0 \Psi] = \int d^3 x \bar{\Psi}i\gamma_0 \partial_0 \Psi ,$$  

is then gauge dependent. It is not even a conserved quantity in a general gauge, in contrast to the total energy of the particles and the field. The equilibrium $\rho_{e^{+}e^{-}}$ can also be defined as the state with maximal entropy for a given expectation value of $P_0$.

It is then clear that the separation of the background field and the fermions is only meaningful in a gauge where $P_0$ is a conserved quantity. Only for time independent $A_\mu(x)$ is $P_0$ separately conserved (assuming $F^{\mu\nu}$ to be constant) and that determines the gauge we have to use, namely $\partial_0 A_\mu(x) = 0$, up to time independent gauge transformations which, anyway, leave the final result invariant. To be more precise, $\Lambda(x)$ can have time dependence of the form $\Lambda(x) = c \cdot t + \lambda(x)$, where $c$ is a constant and $\lambda(x)$ is an arbitrary function. This transformation shifts the potential $A_0$ by the constant $c$. Such a constant is actually absorbed into the definition of the chemical potential, which is not an independent physical parameter but is to be determined by a given charge density. Only the difference $\mu - eA_0$ is physically meaningful for $\rho_{e^{+}e^{-}}$.

Note that this discussion is equally relevant when $F^{\mu\nu} = 0$ but then one normally puts $A_\mu = 0$.

To find thermal one–particle distribution function $f_F(p_0, A_0)$ we argue that at high temperature it must be reduced to the classical Boltzmann distribution for electrons and positrons, $\exp[-\beta(\sqrt{p^2 + m^2} \pm eA_0 \mp \mu)]$, and it is the sign of $(p_0 - eA_0)$ that distinguishes between particles and anti–particles. We therefore write

$$f_F(p_0, A_0) = \frac{\theta(p_0 - eA_0)}{e^{\beta(p_0 - \mu)} + 1} + \frac{\theta(-p_0 + eA_0)}{e^{\beta(-p_0 + \mu)} + 1} .$$  

The distribution function does not have to be chosen to represent an equilibrium distribution. Other choices may be appropriate when the electric field drives the system out of equilibrium. It has been emphasized in the literature (see e.g. Ref.[12]) that the distribution in Eq.(6) has a non–trivial limit when $T \to 0$, and describes a Dirac sea filled up to the energy $\mu$.

From Eq.(1) one can follow the calculations in [6] to obtain the thermal part of the effective action. The only difference is that the trace should be taken in the basis

3 Notice that such a gauge was not used in [10].

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We find
\[
\frac{\partial L_\text{eff}^{\beta,\mu}(x)}{\partial m^2} = -\frac{1}{2\pi^{3/2}} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} f_F(p_0, A_0) \text{Im} \left\{ \int_0^\infty ds e^2 ab \cot(esa) \coth(esb) \right. \\
\times (h(s) - i\epsilon)^{-1/2} \exp \left[ -i(m^2 - i\epsilon)s + \frac{1}{h(s) - i\epsilon} - i\pi \right] \left. \right\},
\]
where
\[
h(s) = (eF \coth eF s)_{00},
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,
\]
\[
a^2 - b^2 = B^2 - E^2 \equiv 2F = \frac{1}{2} F_{\mu\nu} F^{\mu\nu},
ab = E \cdot B \equiv G = \frac{1}{4} F_{\mu\nu}^{\ast} F^{\mu\nu},
\]
\[
B = |B|, \quad E = |E|.
\]
We have added \(-i\epsilon\) to \(h(s)\) in Eq.(7) in order to get the correct value using the principle branch when \(h(s) < 0\). It is determined by the \(x_0\)–integration of Schwinger’s formula. From the \(\cot(esa)\) factor we find that the \(s\)–integral goes through a number of poles on the real axis, \(s = k\pi/ea\), which were not apparent in the original expression. In the case of a pure magnetic field it has been shown \[7\] that the poles are absent if we only include a finite number of Landau levels, and to get the correct result after summing over all Landau levels the \(s\)–integration contour has to go slightly below the real axis. We assume that the same prescription is valid even for non–zero \(E\) field since the poles are related to the \(B\) field. The expression in Eq.(7) can be directly integrated with respect to \(m^2\). The singularity at \(s = 0\) should be cured by subtracting the \(F_{\mu\nu} = 0\) part or using a \(\zeta\)–function regularization as in \[7\] (or using Eq.(15)). We notice that in the limit \(T \to \infty\), and \(f_F(p_0, A_0) \to 1/2\), Eq.(7) equals the negative of the real part of the vacuum contribution up to divergent terms which are only quadratic in the field.

It is possible to find an explicit expression for \(h(s)\) using the following observations. The characteristic equation for the fieldstrength tensor \(F \equiv F_{\mu\nu}^{\ast} F_{\mu\nu}\) is
\[
\det((\lambda I - F) = \lambda^4 + 2F \lambda^2 - G^2 = 0,
\]
with the eigenvalues
\[
\lambda = \pm i(\sqrt{F^2 + G^2} + F)^{1/2}, \quad \pm(\sqrt{F^2 + G^2} - F)^{1/2}.
\]
We know that \((F \coth F)\) has a formal power series expansion involving only even powers of \(F\). The Cayley–Hamilton theorem states that \(F\) satisfies its own characteristic
equation, which can be used to reduce the powers of $F$ down to at most $F^2$, so the matrix structure must be

$$ F \coth F = \gamma \mathbb{1} + \delta F^2. \quad (11) $$

Taking the trace and determinant on both sides of Eq.\((11)\) gives two equations to determine $\gamma$ and $\delta$. We find

$$ h(s) = e^{\frac{E^2 + B^2}{2 \sqrt{F^2 + G^2}}} (\sqrt{F^2 + G^2} + F)^{1/2} \cot [es(\sqrt{F^2 + G^2} + F)^{1/2}] $$

$$ + \frac{e}{2} \left( 1 - \frac{(E^2 + B^2)}{2 \sqrt{F^2 + G^2}} \right) (\sqrt{F^2 + G^2} - F)^{1/2} \cot [es(\sqrt{F^2 + G^2} - F)^{1/2}] \right). \quad (12) $$

The relation $s \cdot h(s) \geq 0$ is thus not valid for general electromagnetic field, contrary to the claim in \([11]\). The quantity $\sum_{\mu=1,2,3} [\ln(\sinh(eF/s)/eF)]_{\mu}$ appearing in the effective action in \([10]\), can be calculated using the same method, and we do not agree on their result either. The standard way to proceed from Eq.\((7)\) is to deform the $s$–contour to the imaginary axis. Here, it is not always useful, except in special cases, since $h(s)$ has zeros on the imaginary axis which give essential singularities from the exponential. On the other hand, the same problem occurs also on the real axis. To be more specific we can consider the following special cases

I. $\mathbf{E} \parallel \mathbf{B}$: $h(s) = eE \coth esE$,

II. $\mathbf{E} \perp \mathbf{B}$, $E > B$: $h(s) = e^{\frac{E^2 \sqrt{E^2 - B^2}}{es \sqrt{E^2 - B^2} - B^2}} \left( \frac{E^2 \sqrt{E^2 - B^2}}{es \sqrt{E^2 - B^2} - B^2} \right)$,

III. $\mathbf{E} \perp \mathbf{B}$, $B > E$: $h(s) = e^{\frac{B^2}{B^2 - E^2}} \left( \frac{B^2}{es \sqrt{B^2 - E^2} \cot es \sqrt{B^2 - E^2}} \right)$.

(13)

In the cases I and II $h(s)$ has no zeros for real $s$ so one can keep the contour as in Eq.\((7)\). A special case of III is when $E \to 0$ and we get back the result in \([7]\) by deforming the contour to the imaginary axis. Another interesting special case of $\mathbf{E} \perp \mathbf{B}$ is when $E = B$ in which case we have $a = b = 0$, $h(s) = 1/s$, just like in absence of the external field, but with a potential $A_0$. We then find

$$ \mathcal{L}_{\text{eff}}^{\beta \mu}(F_{\mu \nu} = 0, A_0) = \mathcal{L}_{\text{eff}}^{\beta \mu}(\mathbf{E} \perp \mathbf{B}, E = B) = \frac{1}{3 \pi^2} \int dp_0 f_F(p_0, A_0) \theta((p_0 - eA_0)^2 - m^2)((p_0 - eA_0)^2 - m^2)^{3/2}, \quad (14) $$

a result similar to the absence of quantum corrections in a propagating plane wave at zero temperature \([3]\). Notice that, because of the lack of Lorentz invariance, this
observation is non–trivial since $L^{β,µ}_{\text{eff}}$ does not only depend on $F$ and $G$. To obtain Eq.(14) we have used dimensional regularization as in [7] and the relation
\begin{equation}
\int_{0}^{∞} ds s^\nu \sin(as - \frac{π}{4}) = \frac{\Gamma(\nu + 1)}{|a|^{\nu+1}} \begin{cases} 
\sin \frac{π}{4}(\nu + \frac{1}{2}) & \text{if } a > 0 , \\
-\sin \frac{π}{4}(\nu + \frac{3}{2}) & \text{if } a < 0 . 
\end{cases}
\end{equation}

3 Weak $E$–field expansion

The $θ$–function in Eq.(14) arises from the non–analytic behaviour of Eq.(13). If we take $B = 0$ and expand the integrand of Eq.(7) in powers of $E$, using Eq.(13), we find that the $s$–integral can be performed term by term using Eq.(15), resulting in the same $θ$–functions. However, the remaining $p_0$–integral becomes infra–red divergent at $(p_0 - eA_0)^2 = m^2$. This problem occurs already at $O(E^2)$ in contrast to the case of a $B$–field [4] where the $O(B^2)$ term can be calculated by a direct expansion of the integrand. It is not a priori clear that a power expansion in $E^2$ exists, and it is also plausible that the expansion is only asymptotic just like at zero temperature. Such an expansion would still be useful in many physical situations where the field is not too strong. To find the leading weak field expansion we have to be more careful than simply expanding the integrand. Using the notation from Eqs.(13,14), we derive from Eq.(7)
\begin{equation}
L^{β,µ}_{\text{eff}}(E) - L^{β,µ}_{\text{eff}}(0) = -\frac{1}{4π^{5/2}} \int_{−∞}^{∞} dω f_F(ω + eA_0) \int_{0}^{∞} \frac{ds}{s^{5/2}} \left\{ \left[ (h(s)s)^{1/2} \cos \left( \frac{ω^2}{h(s)s} - \frac{1}{h(s)s} \right) - 1 \right] \cos \left( (ω^2 - m^2)s - \frac{π}{4} \right) \right. \\
- \left( (h(s)s)^{1/2} \sin \left( \frac{ω^2}{h(s)s} - \frac{1}{h(s)s} \right) \sin \left( (ω^2 - m^2)s - \frac{π}{4} \right) \right) \right\} \\
≡ L_1(E) + L_2(E) ,
\end{equation}

where $ω = p_0 - eA_0$. In the first part of the curly bracket in Eq.(16), called $L_1$, we can use the expansion $h(s)s \simeq 1 + (seE)^2/3$ to find the finite $E^2$ term
\begin{equation}
L_1(E) \simeq -\frac{(eE)^2}{24π^2} \int_{−∞}^{∞} \frac{dω}{(ω^2 - m^2)^{1/2}} f_F(ω + eA_0)θ(ω^2 - m^2) .
\end{equation}
The infra–red problems arise when trying to expand $L_2$ in $E$. After introducing a new integration variable $x = ω^2$, and performing a partial integration with respect to $x$, we get
\begin{equation}
L_2(E) = \frac{1}{8π^{5/2}} \int_{0}^{∞} dx \int_{0}^{∞} \frac{ds}{s^{7/2}} (h(s)s)^{1/2} \cos(xs - m^2s - \frac{π}{4}) \\
d \left[ \frac{f_F(ω + eA_0) + f_F(-ω + eA_0)}{ω} \sin \left( xs \frac{1 - h(s)s}{h(s)s} \right) \right] \\
\end{equation}
which can be expanded to $O(E^2)$. To this order in the field we then have that

$$
\mathcal{L}_{\text{eff}}^{\beta,\mu}(E) - \mathcal{L}_{\text{eff}}^{\beta,\mu}(0) = \frac{(eE)^2}{24\pi^2} \int_m^\infty \frac{d\omega \omega}{(\omega^2 - m^2)^{1/2}} \frac{d}{d\omega} \left( \frac{1}{e^{\beta(\omega+A_0-\mu)} + 1} + \frac{1}{e^{\beta(\omega-A_0+\mu)} + 1} \right).$$

(19)

Eq. (19) leads to

$$
\mathcal{L}_{\text{eff}}^{\beta,\mu}(E) = \mathcal{L}_{\text{eff}}^{\beta,\mu}(0) - \frac{(eE)^2}{24\pi^2} \frac{|\mu - eA_0|}{\sqrt{(\mu - eA_0)^2 - m^2}} \theta (|\mu - eA_0| - m),
$$

(20)

if $T = 0$ and

$$
\mathcal{L}_{\text{eff}}^{\beta,\mu}(E) = \mathcal{L}_{\text{eff}}^{\beta,\mu}(0) - \frac{(eE)^2}{24\pi^2} \left( 1 - 2\pi \sum_{l=0}^{\infty} \frac{1}{((m/T)^2 + (2l + 1)^2\pi^2)^{3/2}} \right),
$$

(21)

if $\mu - eA_0 = 0$. The Debye mass can be extracted as the second derivative of $\mathcal{L}_{\text{eff}}^{\beta,\mu}(E)$ with respect to $A_0$. From the zero field part in Eq. (14) we get $m_\gamma^2(\mu) \simeq e^2(\mu - eA_0)^2/\pi^2$ and $m_\gamma^2(T) \simeq e^2T^2/3$ in the high density and temperature limit. This agrees with [13, 14] but not with [11]. Corrections from finite $E$–field, to lowest order in the field, can also be calculated making use of Eq. (19) and we find for high temperature and high density, respectively

$$
m^2_\gamma(\mu, E) = m^2(\mu) - \frac{\alpha (eE)^2}{2\pi m^2 (\mu - eA_0)^4} \left( 1 + \mathcal{O} \left( \frac{m}{\mu - eA_0} \right)^2 \right),
$$

$$
m^2_\gamma(T, E) = m^2(T) - \frac{\alpha (eE)^2}{2\pi m^2} \frac{31}{4\pi^2} \frac{\zeta(5)}{T^4} \left( 1 + \mathcal{O} \left( \frac{m}{T} \right)^2 \right),
$$

(22)

where $\zeta(n)$ is the Riemann zeta-function and $\alpha = e^2/4\pi$ is the fine–structure constant.

We notice that the $E$–field tends to decrease the screening mass.

In analogy with [13, 4] we can define an effective fine structure constant by

$$
\frac{1}{\alpha(T, \mu)} = \frac{1}{\alpha} + \frac{1}{\alpha E} \left. \frac{\partial \mathcal{L}_{\text{eff}}^{\beta,\mu}(E)}{\partial E} \right|_{E \to 0}.
$$

(23)

In the high density or temperature limit we then have that $\alpha(T, \mu) \to \alpha/(1 - \alpha/3\pi)$, showing a completely different behaviour from the $\alpha$ defined using a magnetic field [4], which satisfies a zero temperature renormalization group equation.

## 4 Pair production

One issue that has been discussed in the literature is pair production at finite $T$ [10, 11], which we find to be absent from the thermal part of the one–loop contribution, in the real–time formalism. This can be seen directly from Eq. (21) in which the thermal part
is manifestly real. It is illustrative to compare with the standard calculation of the effective potential $V(\phi)$ for a spontaneously broken $\lambda \phi^4$-model where one may find an imaginary part when the effective mass, $M^2(\phi) = \lambda \phi^2/2 - m^2$, is negative. The expression

$$\frac{dV(\phi)}{d\phi} = \frac{\lambda \phi}{2} \int \frac{d^4p}{(2\pi)^4} f(p_0) \left( \frac{i}{p^2 - M^2 + i\epsilon} - \frac{i}{p^2 - M^2 - i\epsilon} \right), \quad (24)$$

is obviously real for any real $M^2$. An imaginary part of $V(\phi)$ is found when first calculating Eq.(24) for a positive $M^2$ in the limit $\epsilon \to 0$ and then performing an (ambiguous) analytic continuation in $M^2$. This new function corresponds to, for negative $M^2$, a $p_0$-integration contour which does not follow the real axis but goes above (below) the poles for negative (positive) $p_0$, or vice versa. The procedure of obtaining an imaginary part in this way is thus both ambiguous and does not correspond to the $\epsilon$-prescription in Eq.(24). The conclusion we can draw from the example above is that the standard real-time calculation of the one-loop effective potential gives no imaginary part. One should, however, remember that the standard real-time rules are usually derived under the assumptions of certain factorization properties (see [16] for a discussion) that may not be fulfilled in presence of unstable modes. An imaginary part can be obtained in the imaginary-time formalism, but it is not clear that a consistent calculation of the pair production rate can be performed assuming equilibrium.

We conclude with a remark on heavy quark production in ultra-relativistic nuclear collisions, showing the physical importance of pair production. This example serves as a motivation for finding a solution to the problem with the imaginary part. If $R_{g \to q\bar{q}}$ is the production rate of thermal gluon decay into quark-antiquarks one finds e.g. that

$$\frac{R_{g \to q\bar{q}}}{T^4} \simeq 0.01, \quad (25)$$

for quark masses $m$ such that $m/T \leq 1$. Taking the imaginary part of the effective action $\text{Im} \mathcal{L}_\text{eff}^{\beta/\mu}(E)$ found in [14, 1] seriously, which we question in our paper, we would find for $T \gg m$ and $\mu = 0$ a rate

$$\frac{\text{Im} \mathcal{L}_\text{eff}^{\beta/\mu}(E)}{T^4} \simeq \left( \frac{0.3}{T_{\text{GeV}}} \right)^2, \quad (26)$$

which can be as important as perturbative production rates. Here we have used that $eE \simeq 1 \text{GeV}^2$. It turns out that perturbative production of heavy quark-antiquarks due to the time-variation in the background electric field can be as large as the thermal gluon decay rate Eq.(25). Since quark-antiquarks are spontaneously produced by Schwinger’s mechanism, the background electric field will be depleted, as mentioned in
the introduction. The time-scale, $t_d$, for the depletion of the electric field we estimate using Schwinger’s expression for the rate of pair production, $w_s$, and $mt_d w_s \simeq E^2/2$. Approximatively, this amounts to assuming an exponential decay of the background electric field, i.e. $eE(t) \simeq eE \exp(-t/t_d)$. The explicit time-dependence in the electric field leads to pair production already at the one-loop level with a rate $R_E$. A straightforward calculation leads to the result

$$\frac{R_E}{T^4} \simeq \left( \frac{0.2}{T_{GeV}} \right)^4,$$

(27)

where we have used $\alpha_s = 0.32$ and considered a quark mass $m = 0.2 \text{ GeV}$. If the pair-produced particles are assumed to be in thermal equilibrium, one can expect a decrease of this production rate from Pauli blocking factors.

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