Kramers map approach for stabilization of hydrogen atom in a monochromatic field

D.L. SHEPELYANSKY (a)

Laboratoire de Physique Quantique, Université Paul Sabatier,
118, route de Narbonne, 31062 Toulouse Cedex, France

Abstract: The phenomenon of stabilization of highly excited states of hydrogen atom in a strong monochromatic field is discussed. Approximate description of dynamics by the introduced Kramers map allows to understand the main properties of this phenomenon on the basis of analogy with the Kepler map. Analogy between the stabilization and the channeling of particles in a crystal is also discussed.

Submitted to Physica D, February/March 1993
1 Introduction

During the last years the phenomenon of stabilization of atom in a strong laser field attracted a great deal of attention [1]. While the existence of the stabilization of atom has been clearly demonstrated in the numerical experiments the clear analytical criterion of stabilization is still absent. Usually it is assumed that stabilization condition is satisfied if the energy of the laser photon is larger than the electron coupling energy and the amplitude of electron oscillations in the field is large in comparison with Bohr radius [2]. However, the recent investigations of the corresponding classical problem demonstrated that stabilization remains also in the classical atom [3, 4], where the above conditions are violated. The physical explanation of this phenomenon and the condition of stabilization were given in [3, 4] but the detailed explanation of the effect still remains an open problem.

For a better understanding of this stabilization I introduce here a one-dimensional atom model which I will call Kramers model (having in mind that it arose from the Kramers - Henneberger transformation). Numerical analysis of this model allowed to construct an approximate Kramers map which describes the process of energy excitation and gives conditions of classical ionization. In some sense the obtained Kramers map is quite close to the Kepler map [5] which describes the motion in the limit of relatively small field. Indeed, even in the strong field the change of the electron energy happens only when the electron passes near the nucleus while far from it the electron follows the Kepler orbit.

The paper is constructed as follows. In the section 2 a brief description of the Kepler map is given since analogy with this map can be useful in the stabilization regime. In the section 3 a qualitative explanation of the stabilization is presented. The numerical
analysis of the introduced one-dimensional Kramers model and the derivation of the Kramers map are carried out in the section 4. In the section 5 I discuss analogy between the stabilization and the channeling of electrons in the crystal. In the conclusion the possibilities of experimental observation of stabilization of Rydberg atoms are discussed.

2 Kepler Map

After the pioneer experiments of Bayfield and Koch in 1974 the problem of microwave ionization of highly excited states of the hydrogen atom has been investigated by many groups (see and Refs. therein). The fast ionization observed in the experiments was really surprising since about 100 photons were required to ionize the atom. The typical experimental conditions were the following: $n_0 \approx 70$, $\epsilon_0 = \epsilon n_0^4 \approx 0.05$, $\omega_0 = \omega n_0^3 \approx 1$ where $n_0$ is the principal quantum number of initially excited state, $\epsilon$ and $\omega$ are the strength and the frequency of microwave field (here and below we use atomic units). The classical dynamics depends only on the rescaled values $\epsilon_0$ and $\omega_0$.

For the understanding of the process of ionization in linearly polarized field it is convenient to use the one-dimensional atom model. The investigations of one-dimensional model showed that for high microwave frequency ($\omega n^3 > 1$) the dynamics of the system, which originally is ruled by the continuous Hamiltonian equations, can be described by the Kepler map:

$$\tilde{N} = N + k \sin \phi, \quad \tilde{\phi} = \phi + 2\pi \omega (-2\omega \tilde{N})^{-3/2}$$

(1)

Here $k = 2.58\epsilon/\omega^{5/3}$, $N = E/\omega$ has the meaning of the number of absorbed or emitted...
photons ($E$ is the energy of the electron), $\phi$ is the phase of microwave field at the moment when the electron passes near the nucleus. The bar denotes the new values of the variables after one orbital period.

The physical reason due to which the motion can be quite accurately described by the simple area-preserving map is the following: when the electron is far from the nucleus microwave field leads only to a small fast oscillations which doesn’t modify the average energy and the Coulomb trajectory of the electron. The change of energy happens only at perihelion where the Coulomb singularity leads to a sharp increase of the electron velocity. Ionization takes place when the energy of the electron becomes positive after a pass near the nucleus $N > 0$. Then the electron goes to infinity and never returns back. Therefore for the map (1) ionization is equivalent to absorption of trajectories with $N > 0$.

To find the chaos border in the Kepler map we can linearize the second equation in (1) near the resonant (integer) values of $\omega n^3$ obtaining the Chirikov standard map:

$$\bar{N} = N + k \sin \phi, \quad \bar{\phi} = \phi + T \bar{N}$$

with $T = 6\pi \omega^2 n^5$. The global chaos appears for $K = kT > 1$ that determines the critical field strength above which the classical atom is ionized. In this regime excitation goes in a diffusive way with the diffusion rate $D = (\Delta N)^2 / \Delta \tau = k^2/2$ where $\tau$ measures the number of orbital periods of the electron.

The first numerical and analytical investigations of the quantum one-dimensional atom model showed that quantum effects leads to the suppression of classical diffusion. Indeed, in the quantum case the variables $(N, \phi)$ becomes the operators with the commutation rule $[N, \phi] = -i$ and the system is locally described by the quantum kicked rotator
The photon number is analogous to the level number in the kicked rotator and the
excitation probability decreases exponentially with the number of absorbed photons so
that the ionization rate is proportional to $W_I \sim \exp(-2N_I/l_\phi)$. Here $N_I = n_0/2\omega_0$ is
the number of photons required for ionization, $l_\phi = D = 3.33\epsilon^2/\omega^{10/3}$ is the localization
length. For $l_\phi << N_I$ quantum ionization is exponentially small in comparison with
the classical value. However, for $l_\phi > N_I$ the diffusion is delocalized and the process of
ionization is close to the classical one.

In the 3-dimensional atom the Coulomb degeneracy leads to a slow motion along
energy surface that allows to describe the excitation in energy also by the Kepler map with
a small change of constant $k$. The motion along the energy surface has some additional
integral of motion that explains the existence of localization in 3-dimensional atom \[5, 11\].
Recently the existence of localization in the 3d case was reconfirmed in \[12\].

Quantum localization of classical chaotic ionization has been observed in the mi-
crowave experiments with hydrogen \[13, 14\] and rubidium \[15\] atoms. Numerical simula-
tions with the quantum Kepler map \[16\] reproduce the 10% ionization threshold obtained
in the laboratory \[13\]. The theoretical prediction for the quantum delocalization border
was also observed in the skilful numerical simulations \[12\].

Being very successful in the description of energy excitation the Kepler map, however,
cannot be applied for the case of very strong field. Indeed, in its derivation it was
assumed that the change of energy after one kick $k\omega$ is much larger than the energy
of free oscillations $\epsilon^2/2\omega^2$. This gives the condition of applicability of the Kepler map
\[ \epsilon \ll \epsilon_{ATI} \approx 5\omega^{4/3} \]  

(3)

Let us note that this condition is independent on the initial state since \( n_0 \) doesn’t enter directly in the expression for \( \epsilon_{ATI} \).

In the one-dimensional case for \( \epsilon >> \epsilon_{ATI} \) a collision with the nucleus, being unavoidable, goes in a fast way like with an elastic wall leading to a prompt ionization. In the two-dimensional case for zero magnetic quantum number \( m \) such collision also always takes place if the amplitude of free oscillations \( \epsilon/2\omega^2 \) is larger than the unperturbed distance between the electron and the nucleus in perihelion \( l^2/2 \) (\( l \) is the orbital momentum). This gives the condition of prompt ionization for \( l > (3/\omega)^{1/3} \) \[17\]:

\[ \epsilon > \omega^2 l^2/4 \]  

(4)

where it was assumed that \( l \) is few times less than \( n \). For \( l < (3/\omega)^{1/3} \) ionization is ruled by the Kepler map and for \( \epsilon_0 > \omega_0^{2/3}/2.6 \) prompt ionization takes place after one orbital period (see (1)). Therefore, there is no stabilization of classical atom in the strong field for \( m = 0 \). However, for high orbital momentum the atom remains stable up to very high field values.

3 Stabilization Border

While for the magnetic number \( m = 0 \) ionization always takes place in a sufficiently strong field the case of nonzero \( m \) is much more interesting. Indeed, for the linear polarization of the field the projection \( m \) is an exact integral of motion and the created by it
centrifugal potential gives a possibility to avoid a collision with the nucleus. To analyze the motion in the strong field it is convenient to use the oscillating Kramers - Henneberger frame \([1]\) and cylindrical coordinates in which the Hamiltonian has the form:

\[
H = \frac{p_z^2}{2} + \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{(\rho^2 + (z - \frac{\epsilon}{\omega^2} \sin(\omega t))^2)^{1/2}}
\]  

(5)

If the frequency of the nuclear oscillations is large enough (the condition will be given later) then in first approximation the nucleus can be considered as a charged thread with a linear charge density \(\sigma\) slowly dependent on \(z\): \(\sigma(z) = \omega^2/\pi \epsilon (1 - (z\omega^2/\epsilon)^2)^{1/2}\). Then, for small \(z\) and \(\rho\) the Hamiltonian of averaged motion takes the form \([3, 4]\):

\[
H_{\text{ave}} = \frac{p_z^2}{2} + \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} + 2\sigma(z) \ln(\frac{\rho\omega^2}{\epsilon})
\]  

(6)

The constant under the logarithm takes into account that for \(\rho >> \epsilon/\omega^2\) the coupling energy becomes much less than \(\omega^2/\epsilon\). From (6) one easily finds the position of the potential minimum \(\bar{\rho} = \sqrt{\pi \epsilon/2m/\omega}\) and the frequency of small oscillations \(\Omega = 2\sqrt{2\omega^2/\pi \epsilon m}\) (for \(z << \epsilon/\omega^2\)). The depth of the potential or the energy required for ionization of atom is approximately \(I \approx 2\omega^2 L/\pi \epsilon\) with \(L = \ln(2\epsilon/(e\pi \omega^2 m^2))/2\). The minimal distance between the nucleus and electron is determined by the condition \(I = m^2/2(\rho_{\text{min}})^2\) giving:

\[
\rho_{\text{min}} = \frac{m}{2\omega} \sqrt{\frac{\pi \epsilon}{L}}
\]  

(7)

The physical reason for the growth of the minimal distance with the field strength is the following: with the increase of the field the amplitude of the field oscillations grows leading to the decrease of attractive Coulomb force while the centrifugal potential remains the same.
The averaged description of the motion (6) is correct if the frequency of field oscillations $\omega$ is much larger than the frequency $\Omega$ of oscillations in $\rho$. In that case the averaged Hamiltonian (6) is the constant of the motion with adiabatic accuracy and ionization of atom doesn’t take place. This gives the stabilization border [3, 4]:

$$\epsilon > \epsilon_{stab} = \frac{\beta \omega}{m}$$  \hspace{1cm} (8)

where $\beta$ is some numerical constant. The same estimate can be obtained from the condition that the change of energy $\Delta E$ during the collision between the electron and the nucleus is smaller than $I$. Indeed, the change of the momentum is $\Delta p \approx \Delta t/\rho_{\min}^2 \approx \omega/(\epsilon \rho_{\min})$ and the change of the energy $\Delta E \approx \omega^2/(\epsilon^2 \rho_{\min}^2)$ is less than $I$ if (8) is satisfied. It is interesting to note that the stabilization border (8) can be written as $v_n = \epsilon/\omega > v_{\max}$ where $v_n$ is the typical velocity of the nucleus and $v_{\max} = 2/m$ is the maximal velocity of the electron in the atom without the external field.

Another condition intrinsically used in the derivation of (6) and (8) is $\rho_{\min} < \epsilon/\omega^2$ which gives $\epsilon > m^2\omega^2$. Also, there are two qualitatively different situations depending on the ratio between $m$ and $(3/\omega)^{1/3}$. In the case $m < (3/\omega)^{1/3}$ (stabilized atom regime) we have $m^2\omega^2 < 5\omega^{4/3} < \beta \omega/m = \epsilon_{stab}$. For small field amplitude (3) the excitation is described by the Kepler map and the complete ionization after one orbital period of the electron takes place for $\epsilon_0 > \omega_0^{2/3}/2.6^4$. Between this border and above chaos border $\epsilon_0 = 1/49\omega_0^{1/3}$ ionization goes in the diffusive way which is also relatively fast. However, for the more strong field (8), when the Kepler map picture is not valid (see (3)), atom becomes stable. The case of opposite inequality is less impressive. Indeed, for $m > (3/\omega)^{1/3}$ (stable atom regime) we have $\beta \omega/m < 5\omega^{4/3} < m^2\omega^2$ and atom
remains stable (nonionized) up to $\epsilon \sim m^2 \omega^2$ as it was in (4) ($l \sim m$). Above this value
a significant portion (order of half) of atoms will remain stable since condition (8) is
satisfied. Finally, ionization takes place only when the value of $\rho_{\text{min}}$ (7) becomes larger
than the size of the atom $2n_0^2$ and the electron cannot be captured in the stable region
during the switching of the field. This gives the destabilization border

$$\epsilon_{\text{destab}} \approx \frac{16L\omega^2n_0^2}{\pi m^2}$$

(9)

This border is also valid for the case $m \ll (3/\omega)^{1/3}$. Of course, in that case the stabiliza-
tion can be observed only for the time of field switching $T_{sw}$ less or order of one orbital
period of the electron. Otherwise a collision with nucleus will take place at field strength
$\epsilon < \epsilon_{\text{stab}}$ and atom will be ionized.

The results of numerical simulation of ionization process of system (5) are presented
on the Fig.1. The stabilization probability $W_{\text{stab}} = 1. - W_{\text{ion}}$ is given for different field
strengths $\epsilon_0$ and frequencies $\omega_0$. The ionization probability $W_{\text{ion}}$ was defined as the
relative part of the trajectories with positive energies after field pulse. The initial distri-
bution of 100 trajectories (25 for $\omega_0 = 1000$.) corresponded to a quantum state with fixed
spherical quantum numbers (fixed actions and equipartition in conjugated phases). The
initial value of orbital momentum was equal to $l/n_0=0.3$ and its projection was equal to
$m/n_0=0.25$. The time of field switching (on/off) measured in the number of field periods
was chosen to be equal to $T_{sw} = \omega_0$ (one unperturbed orbital period of the electron).
The pulse duration of the field was $T_{\text{int}} = 500\omega_0$ (500 orbital periods). The data clearly
demonstrate the stabilization of atom for field strength larger than some critical value. It
is convenient to define the stabilization border as the field strength $\epsilon_{\text{stab}0}(20\%)$ for which
$W_{\text{stab}} = 0.2$. The dependence of $\epsilon_{\text{stab}}(20\%)$ on $\omega_0$, extracted from the data of Fig.1 can be well fitted by the theoretical expression (8) with $\beta = 12$ in the wide frequency range (see Fig.2 of [3]). This dependence continues up to $\omega_0 = 1000$ where we enter in the stable atom regime with $m > (3/\omega)^{1/3}$ and where stabilization disappears in agreement with above theoretical arguments (see Fig. 1). However, the stability of atom in that case is of the other nature than it was in [17] since the condition (4) is strongly violated. So, for such strong fields the stability of atom is based on the same physical grounds (8) as in the stabilized atom regime for $m << (3/\omega)^{1/3}$. The numerical check of the dependence of stabilization border $\epsilon_{\text{stab}}(20\%)$ on $m$ as well as the destabilization border (9) on $\omega_0$ demonstrates good agreement with the theory (8)-(9) [3].

4 Kramers Map

It is important to stress that according to (8) the stabilization can take place even when the size of electron oscillations $\alpha = \epsilon/\omega^2$ is much less than the unperturbed size of the atom $n_0^2$. An example of the motion in this case is presented on the Fig.2. In such a case the electron follows the usual Kepler elliptic orbit and his energy (the size of the orbit) can be changed only during his fast passage near the nucleus. In this sense we can expect that the motion can be effectively described by some map analogous to the Kepler map.

To construct such kind of map let’s introduce a simplified one-dimensional Kramers
model given by the Hamiltonian
\[
H = \frac{p^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{(\rho^2 + \frac{\epsilon^2}{\omega^2}(\sin \gamma + \sin(\omega t))^2)^{1/2}} \tag{10}
\]

This model is obtained from the Hamiltonian (5) by neglecting the changes of \(z\) and considering \(z = -\epsilon/\omega^2 \sin \gamma\) as a constant. In other words electron collides with the line \(\rho = 0\) always at the same \(z\) value. The physical reason for that is the following. The collision of the electron with the nucleus is analogous to a collision of a fast heavy particle with the light electron. In such collision the change of energy (velocity) takes place mainly in the perpendicular \(\rho\)-direction, while the velocity in \(z\)-direction remains practically the same. According to this physical picture the model (10) mainly presents the changes in \(\rho\)-direction. In that sense it is quite different from the well known one-dimensional atom model of Eberly [1] which implicitly takes into account the change of energy (velocity) only in \(z\)-direction. Also in [18] the authors considered the velocity change only in \(z\) that has led them to a higher stabilization border than (8), while the estimate for \(\rho_{\text{min}}\) has been found correctly (see (7)).

According to the analogy with the Kepler map we can expect that the change of the electron energy in the model (10) will take place only when the electron passes near the nuclear and that it will depend only on the phase of the field \(\phi = \omega t\) at that moment. If also the size of the orbit is much larger than the size of the nucleus oscillations (\(\alpha = \epsilon/\omega^2 << n_0^2\)) then the change of the phase is given by the Kepler law and is the same as in (1). Basing on these arguments we can assume that the dynamics of energy excitation is governed by the Kramers map of the following form:
\[
\bar{E} = E + Jh(\phi), \quad \bar{\phi} = \phi + 2\pi\omega(-2E)^{-3/2} \tag{11}
\]
with $E = \omega N$, where as in (10) $N$ is the photon number, the maximum change of the energy is given by a constant $J$ and the unknown function of the kick $h(\phi)$ varies in the interval [-1,1].

To check the validity of this map I integrated the continues equations of motion for the model (10) and plotted the change of energy as a function of the field phase at the moment when the value of $\rho$ took one of its minimal values ($p_\rho = 0$). Such approach allows to find the kick function $h(\phi)$ the examples of which are presented on Figs.3,4. The numerical results clearly demonstrate that the function $h$ exists. However, it has a quite unusual property. Indeed, some values of $\phi$ never appear (even if the number of periods was increased in 20 times). These values of $\phi$ are approximately equal to $\pi + \gamma, 2\pi - \gamma$ and correspond to that values of the field at which the nucleus passes via the point of collision $z = -\epsilon/\omega^2 \sin \gamma$. A more close consideration of motion near these special $\phi$-values shows that the electron remains during some small time interval (within corresponding phase interval $\Delta\phi$) near the nucleus making one (Fig.3) or two (Fig.4) oscillations in $\rho$ of very small amplitude so that the value of $\rho$ remains practically (but not exactly) the same. This gives correspondently two (or three) values of the phase $\phi$ with the same change of $\Delta E$ since the value of $E$ was determined in the aphelion. This of course puts the question about the derivation of the Kramers map in some other synonymous form. However, the main properties of the motion can be derived already from the approximate representation (11) where we will define the function $h$ in the empty intervals by connecting the last points at the ends of the interval by straight line.

Defined in such a way the Kramers map has the properties quite similar to the Kepler
map. Indeed, the function $h$ is close to $\cos \phi$ and the approximate chaos border in (11) can be defined by the linearization of the second equation giving:

$$K = 6\pi \omega J n^5 > 1$$

(12)

where we used substitution $E = -1/2n^2$. According to this criterion and in agreement with the numerical data the motion is chaotic for the cases of Figs. 3,4. If to introduce $k = J/\omega$, which will give the number of absorbed photons after an orbital period, we will get the same formulas for the diffusion rate $D = k^2/2$, the localization length ($l = D$) and the ionization time $\tau_{ion} = N_I^2/D$ as in the Kepler map. In this sense the most important problem is the definition of the dependence of $J$ on the parameters of the system.

According to the results of the previous section the amplitude of the kick $J$ must decrease exponentially with the increase of the stabilization (adiabatic) parameter $S = \epsilon m/\omega \sim \omega/\Omega$ (see (7),(8)). This expectation is in agreement with the results presented on Fig.5. Indeed, the exponential decrease of $J$ with the field strength, and therefore stabilization, are evident. Let us at first discuss the properties of $J$ for nonzero values of $\gamma$. Even though the value of energy for the cases of Fig.5 was quite small nevertheless there is some dependence of $J$ on energy. An example of such dependence is presented on the Fig. 6. We see that $ln J$ depends on energy $E$ approximately in a linear way and goes to a constant value for $E = 0$. This means that in the limit $n_0^2 >> \epsilon/\omega^2$ the value of $J$ is independent from $n_0$. This result is consistent with the above arguments that the change of the energy takes place only in the small vicinity of the nucleus. However, in the difference from the Kepler map it is necessary to have quite strong inequality $-\alpha E << 1$ to neglect the dependence of $J$ on $E$. We will try to explain this fact later. In this regime
of small energies the main change of the phase of the field between collisions (the second equation in (11)) is obviously given by the Kepler law.

To determine the dependence of $J(E = 0)$ on the parameters it is convenient to fix the stability parameter $S$ that allows to eliminate the strong exponential dependence and to find the factor before the exponent. The numerical results are presented on Fig.7. The values of $J(E = 0)$ were obtained from nonzero energies by linear extrapolation to $E = 0$ (see Fig.6). The numerical data clearly show that for the fixed $S$ the value of $J(E = 0)$ is independent on the frequency and is inversely proportional to $m^2$. In principle the factor $1/m^2$ gives simply the correct dimensionality however the independence on the other dimensionless parameter $\nu = m\omega^{1/3}$ is not so obvious.

Combining all the obtained numerical results we can present the dependence of the kick amplitude $J$ on the parameters for $|E|\epsilon/\omega^2 << 1$ in the following form:

$$J = \frac{g_1 \sin \gamma}{m^2} \exp\left(-(g_2 - g_3 \epsilon E/\omega^2)\epsilon m/\omega\right)$$

(13)

where $g_{1,2,3}$ are some functions weakly dependent on $\gamma$. For $\gamma = 0.6$ we have from Figs. 5-7 that $g_1 \approx 0.13$, $g_2 \approx 0.19$, $g_3 \approx 0.08$. The numerical data for other values of $\gamma$ show that the fitting parameters vary not more than in 2 times for practically the whole interval of $\gamma$. For example, $g_1 = 0.1$ and 0.2, $g_2 = 0.13$ and 0.21, $g_3 = 0.045$ and 0.1 for $\gamma = 1.2$ and 0.3 correspondingly.

To understand the numerically obtained formula (13) for $J$ it is possible to make the following estimate. Taking the partial time derivative from the Hamiltonian (10) we obtain the expression for the change of the energy after one orbital period:

$$\Delta E = \frac{\epsilon^2}{\omega^4} \int \frac{\cos(\eta + \phi)(\sin \gamma + \sin(\eta + \phi))d\eta}{\rho^{3/2}(\eta)}$$

(14)
where \( \eta = \omega t \) and in the denominator we neglected the term with \( \epsilon^2/\omega^4 \) in comparison with \( \rho^2 \). We can assume that near the nucleus the time dependence of \( \rho \) is the same as for a free electron with momentum \( m \) that gives \( \rho^2(t) = \rho_0^2 + v^2t^2 \) where \( \rho_0 \) is the minimal distance from the center and \( v \) is the velocity of the electron far from the center. For this free motion with the fixed momentum \( m \) we have the relation: \( \rho_0^2 = m^2/v^2 \). For the velocity it is possible to use the following expression: \( v^2 = \omega^2/C\epsilon + 2E \). Where the first term takes into account the fact that the energy must be measured in respect to the minimum of the effective potential (see (6)) and \( C \) is some unknown constant. It is easy to see that \( C \) determines the minimal distance \( \rho_0^2 = C\epsilon m^2/\omega^2 \) for \( E = 0 \). In principal the value of \( C \) depends on \( \gamma \).

After substitution of all these expressions in (14) we obtain the following estimate

\[
J \sim \frac{S^2}{m^2(m^{3/2}\omega^{1/2})} \sin \gamma exp(-CS/(1 + 2C\epsilon E/\omega^2)), \quad h(\phi) = \cos(\phi), \quad S = \epsilon m/\omega
\]

Of course, the presented derivation is not exact. However, it reproduce quite well the exponential dependence (13) (while the factor before the exponent is not in agreement with the dependence obtained from the numerical simulation). Indeed, numerically \( h(\phi) \) has maxima near 0 and \( \pi \). Comparison of (15) with (13) gives \( g_2 = C \) and \( g_3 = 2C^2 \). The value of \( C \) can be defined directly from the numerical simulation of the one-dimensional Kramers model for different \( \gamma \). The comparison of the \( g_2 \) with \( C \) is presented on the Fig.8 showing the good agreement with the prediction. The ratios of the numerical value of \( g_3 \) (see above) to the theoretical value \( 2g_2^2 \) are equal to 1.13, 1.1, 1.33 correspondingly for \( \gamma = 0.3, 0.6, 1.2 \) and are also in good agreement with the theoretical estimate. In the future estimates we will use the expression (13) with the theoretical substitution for \( g_2 \).
and $g_3$. Let us also mention that for $\gamma = 0$ we get from (14) that $g_2 = 2C$ (from Fig.5 the ratio to the theoretical value is approximately 1.1) and $h(\phi) = \sin(2\phi)$ that is quite close to the numerical data. Further theoretical analysis is required to obtain the factor before the exponent in (13).

While still there are some unclear questions with the construction of the Kramers map the approximate consideration made above and the analogy with the Kepler map allow to understand the main properties of motion. If the number of photons required for the ionization is large then, as it was for the quantum Kepler map, it is possible to have diffusive excitation and quantum localization of chaos. However, due to high values of the frequency it is also quite easy to have a situation when one photon can already lead to ionization. In this case for $k = J/\omega < 1$ the one-photon ionization rate (per unit of time) is given by the perturbation theory and as for the Kepler map (see [5]) it is equal:

$$\Gamma \approx \frac{J^2}{8\pi n^3 \omega^2} \tag{16}$$

For $J > w$ approximately a half of probability is ionized after one orbital period (in (11) as in the Kepler map the orbit is ionized if after a kick $E > 0$). From (16) it is clear that we may have long living states if the field is sufficiently strong. From the quantum viewpoint one of the most interesting cases is the case of small $m$. In this case we need to make substitution $m \to m + 1$ since as it is well known the correct quasiclassical quantization leads to the appearance of the effective centrifugal potential even for zero orbital momentum. That gives the stabilization border $\epsilon > 10\omega$ for $m = 0$.

Finally let us mention that in (11) we assumed that $J$ is independent on energy. To take into account this dependence we need to put in the first equation $J = J(\tilde{E})$ and in
the second equation to add the phase shift $\Delta \phi = dJ/d\hat{E}f(\phi)$ with $h(\phi) = -df(\phi)/d\phi$. In that way the map will remain canonical.

## 5 Channeling Analogy

Here I would like to discuss the analogy between the phenomenon of stabilization of atom in strong field and the channeling of particles in a crystal (see for example [19] and Refs. there in). Let’s consider the electron moving in the crystal with the velocity $v \approx c = 137$ (we will consider nonrelativistic case). Then in the frame of the moving electron its interaction with the protons in the crystal lattice will have approximately the form (5) if to take into account the interaction only with a nearest proton. On the grounds of that analogy we find that the effective distance between atoms in the crystal $a$ and the velocity of the electron are equal to:

$$a = \frac{\epsilon}{\omega^2}, \quad v = \frac{\epsilon}{\omega}$$

(17)

The frequency of perturbation is $\omega = v/a$ so that $\epsilon = v^2/a$. Since in the crystal the distance between the atoms is approximately the same in all directions the analogy is valid for $\epsilon/\omega^2 > n_0^2$. The necessary condition of channeling is that the critical injection angle $\theta$ must be much less than one that implies: $\theta \approx v_\perp/v \approx 1/(v\sqrt{a}) \approx (\omega^{4/3}/\epsilon^{3/2}) < < 1$. This is the condition of unapplicability of the Kepler map (3). From the stabilization condition (8) it follows that channeling takes place for electrons with momentum $m > 10/v$. This is always satisfied for fast electrons with $v \approx 137$. The existence of channeling for very energetic electrons (that corresponds to strong field for stabilization problem) gives one
more evidence for existence of stabilization of atom in strong field in the regime when one photon frequency is larger then the ionization energy.

6 Conclusion

Basing on the Kramers map (11) and using its analogy with the Kepler map we obtained the estimate for the one-photon ionization rate (16). This ionization rate sharply decreases with the stabilization parameter \( S = \epsilon m / \omega \). Such stabilization for excited states has some interesting advantages in comparison with the stabilization of atom in the ground state. Indeed, in this case stabilization can take place with \( \epsilon << 1 \) and \( \omega << 1 \). This leads to a large energy difference \( \delta E \) between the excited states and the ground state. So, the energy in an exited state is approximately \( (\epsilon / \omega)^2 / 2 >> 1 \) while the energy of the ground state remains as in the unperturbed atom (it is not the case for \( \epsilon, \omega >> 1 \) when the ground state is also stabilized since there the electron has the same energy of free oscillations). Due to that in the case of Rydberg stabilization it is possible to have radiative transitions to the ground state with the radiation of X-ray photons. For the frequency of \( CO_2 \) laser \( \omega \approx 1/300 \) (0.1 ev) and \( m=0 \) (or 1) the stabilization will take place for \( \epsilon \approx 1/30 \) (1.6 \( 10^8 \) V/cm). The size of the atom will be larger than the size of the field oscillations \( \alpha = \epsilon / \omega^2 \) for \( n > 40 \). According to (13) and (16) for the field \( \epsilon = 5 \ 10^8 \) V/cm and \( n=60 \) the life time of the atom will be about \( 5 \ 10^5 \) orbital periods or \( 10^{-6} \) seconds (we take for the estimate the case with \( \gamma = 0.6 \)). Of course, to obtain such states the time of field switching must be less than the time of orbital period as we discussed above. Since recently it was predicted that the Rydberg atoms can form long
living states (bands) in the solid state \[20\] (giving very high density of excited atoms) it will be interesting to consider a possibility of stabilization not only for a separate atom but also for such Rydberg solid state.

I had started to be interested in the problem of microwave ionization of hydrogen atom in far cold 1980 during the first visit of Jeff Tennyson to the group of Boris Chirikov at Novosibirsk. Now, after many years of researches by different groups, this problem still continues to live by its own life as well as the memory about Jeff continues to live among the people who met him in Siberia.
References

[a] also Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

[1] M.Pont, N.R.Walet, M.Gavrila, C.W.McCurdy, Phys. Rev. Lett. 61 (1988) 939; M.Dorr, R.M.Potvlledge, R.Shakeshaft, Phys. Rev. Lett. 64 (1990) 2003; Q.Su, J.H.Eberly, J.Javanainen, Phys. Rev. Lett. 64 (1990) 862; K.C.Kulander, K.J.Schafer, J.L.Krause, Phys. Rev. Lett. 66 (1991) 2601.

[2] R.J.Vos, M.Gavrila, Phys. Rev. Lett. 68 (1992) 170.

[3] D.L.Shepelyansky, "Hydrogen in monochromatic field: stabilization and channeling vs. chaos", Int. Conf. on Atomic Physics, Munich (1992), to be published by AIP.

[4] F.Benvenuto, G.Casati, D.L.Shepelyansky, "Classical stabilization of hydrogen atom in monochromatic field", Phys. Rev. A (1993) (to be published).

[5] G. Casati, I. Guarneri and D.L. Shepelyansky, IEEE J. Quant. Elec. 24 (1988) 1420.

[6] J.E.Bayfield and P.M.Koch, Phys. Rev. Lett. 33 (1974) 258.

[7] R.V.Jensen, S.M.Susskind and M.M.Sanders, Phys. Rep. 201 (1991) 1.

[8] D.L.Shepelyansky, Preprint INP 83-61 (Novosibirsk, 1983); Proc. Int. Conf. on Quantum Chaos (Como 1983), Ed. G.Casati (Plenum, N.Y., 1985) p.187.

[9] B.V.Chirikov, Phys. Rep. 52 (1979) 263.

[10] B.V.Chirikov, F.M.Izrailev and D.L.Shepelyansky, Sov. Scient. Rev. 2C (1981) 209; Physica 33D (1988) 77.
[11] G. Casati, B.V. Chirikov, I. Guarneri, D.L. Shepelyansky, Phys. Rev. Lett. 59 (1987) 2927.

[12] A. Buchleitner, D. Delande, Phys. Rev. Lett. 70 (1993) 33.

[13] E. J. Galvez, B. E. Sauer, L. Moorman, P. M. Koch, D. Richards, Phys. Rev. Lett. 61 (1988) 2011.

[14] J. E. Bayfield, G. Casati, I. Guarneri, D. W. Sokol, Phys. Rev. Lett. 63 (1989) 364.

[15] M. Arndt, A. Buchleitner, R. N. Mantegna, H. Walther, Phys. Rev. Lett. 67 (1991) 2435.

[16] G. Casati, I. Guarneri and D. L. Shepelyansky, Physica 163A (1990) 205.

[17] F. Benvenuto, G. Casati, D. L. Shepelyansky, Phys. Rev. A, 45 (1992) R7670.

[18] R. V. Jensen, B. Sundaram "Classical Theory of Intense Field Stabilization" (1992) (to be published).

[19] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Sov. Phys. JETP 65 (1987) 686.

[20] E. A. Manikin, M. I. Ozhovan, P. P. Poluektov, JETP 75, N 3 (1992) 440.
Figure captions

Fig. 1: Stabilization probability $W_{stab} = 1 - W_{ion}$ is given for different field strengths $\epsilon_0$ and frequencies ($\omega_0$=0.3 (○), 1. (★), 3. (+), 10. (◇), 30. (△), 100. (◇), 300. (△), 1000 (●)).

Fig. 2: Example of trajectory for $\omega_0 = 300$, $\epsilon_0 = 20000$ with initial $l/n_0 = 0.3$ and $m/n_0 = 0.25$; $10^5$ field periods are shown.

Fig. 3: Example of numerically obtained kick function $h(\phi)$ in Kramers map (11) for $\epsilon = 3 \times 10^4$, $\omega = 125$, $m=0.2$, $\gamma=0.6$, $E=-0.125$ (so that effective $n_0=2$), $J=1.1 \times 10^{-4}$. Near 200 orbital periods (points) are shown.

Fig. 4: The same as Fig.3 with $\epsilon = 410^4$, $\gamma=1.2$ and $J = 5.810^{-4}$.

Fig. 5: Dependence of the kick amplitude $J$ in (11) on stabilization parameter $S = \epsilon m/\omega$ for $\omega = 125$, $m = 0.2$, $\gamma = 0$ (○); $\omega = 1000$, $m = 0.1$, $\gamma = 0$ (+); $\omega = 125$, $m = 0.2$, $\gamma = 0.3$ (open squares); $\omega = 125$, $m = 0.2$, $\gamma = 0.6$ (points); $\omega = 125$, $m = 0.2$, $\gamma = 1.2$ (full squares). For all cases $E=-0.125$. Lines are drawn to adopt an eye.

Fig. 6: Example of dependence of $J$ on energy $|E|$ for $\epsilon = 2.2 \times 10^4$, $\omega = 125$, $m = 0.2$, $\gamma = 0.6$ (points).

Fig. 7: Dependence of $J$ on $m$ for fixed stabilization parameter $S = 35.2$ and $E=0$; 9 cases are shown for $\omega$ in the interval [10,1000] and $m$ in the interval [0.05,0.6]. The straight line shows the dependence $J \sim 1/m^2$. 
Fig. 8: Dependence of $C = \rho_0 \epsilon / S^2$ on $\gamma$ (full line). Points give values of $g_2$ to demonstrate theoretical relation $C = g_2$. 