General fractional financial models of awareness with Caputo–Fabrizio derivative

Amr MS Mahdy\textsuperscript{1,2}, Yasser Abd Elaziz Amer\textsuperscript{2}, Mohamed S Mohamed\textsuperscript{1,3} and Eslam Sobhy\textsuperscript{2}

Abstract
A Caputo–Fabrizio (CF) form a fractional-system mathematical model for the fractional financial models of awareness is suggested. The fundamental attributes of the model are explored. The existence and uniqueness of the suggest fractional financial models of awareness solutions are given through the fixed point hypothesis. The non-number request subordinate gives progressively adaptable and more profound data about the multifaceted nature of the elements of the proposed partial budgetary models of mindfulness model than the whole number request models set up previously. In order to confirm the theoretical results and numerical simulations studies with Caputo derivative are offered.

Keywords
Fractional financial models of awareness, Caputo–Fabrizio, fixed point theorem, Lipschitz condition, numerical simulation

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Introduction
Numerical models in the study of disease transmission are utilized generally so as to see major the elements of an irresistible illness.\textsuperscript{1,2} The utilization of the scientific models isn’t constrained to just human maladies, however, they are likewise generally applied in other wonders of organic sciences, for example, nature, timberland, and so on. In human life, timberland has a significant job, accordingly, it is important to guarantee the security methodologies to shield it from being tainted with ailments. The woods give greenery to the earth and a wonderful climate for people.

It is notable that the point of the publicizing is to persuade the shoppers to buy the items, that relying upon the featuring the poverty of the items all in all and by showing the separation of a particular brand over different items to en-boldness buyers to get it. There are a few strategies to change crowd sentiment on items or administrations. One of these strategies is publicizing missions. These missions can be by means of body media like TV, radio, papers, and magazines, additionally, these missions can be through delicate media, for example, instant missions, sites, and so on.\textsuperscript{3} Concentrating of the commercial techniques is significant so as to expand the deals and to show signs of improvement in the organization’s acquiring. In this way, it is extremely helpful to build and study a legitimate powerful notice model to depict the deals that rely upon the time and on the crowd populace.\textsuperscript{4} There are a ton of proposed models to depict the promotions undertaking that set these issues from the

\textsuperscript{1}College of Science, Department of Mathematics, Taif University, Taif, Saudi Arabia
\textsuperscript{2}Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
\textsuperscript{3}Department of Mathematics, Faculty of Science, Al-Azhar University, Cairo, Egypt

Corresponding author:
Amr MS Mahdy, Department of Mathematics, College of Science, Taif University, P. O. Box 11099, Taif, 21944, Saudi Arabia.
Email: amattaya@tu.edu.sa

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perspective of showcasing, financial, and activities the board,\textsuperscript{3,5} where dissecting the publicizing approaches is completed over time utilizing dynamical models.\textsuperscript{6,7} Dynamical models have portrayed by differential conditions, where the piece of the pie, deals, subpopulations, and all the basic state factors are thought to be changed inconsistent structure as for the time. The reasons for each publicizing are very surprising. For example, some of them are intended to analyze between two, or among at least three trademarks. Another is to acquaint another item with the market. Contingent upon these reasons the promoting models will be developed.

Fractional order models\textsuperscript{8–43} are progressively solid and supportive in the genuine wonders than the old-style models because of innate properties and the portrayal of memory.\textsuperscript{44,45} Additionally, in reality, clarification, the whole number request subordinate doesn’t investigate the elements between two unique focuses. To manage such disappointments of traditional neighborhood separation, various ideas on separation with non-nearby or partial requests have been created in the current writing. For example, Riemann and Liouville presented the idea of fragmentary requests separation in Samko et al.\textsuperscript{45} Recently, CF\textsuperscript{46} presented another subsidiary with partial request dependent on the exponential portion. The novel CF has been utilized effectively in the demonstrating of different genuine wonders. A fractional Adams–Bashforth technique via the CF derivative has given in Owolabi et al.\textsuperscript{47} An investigation of the magneto hydrodynamic electro osmotic stream of Maxwell liquids with CF subordinates was completed by Abdullah et al.\textsuperscript{48} In Firoozjaee et al.,\textsuperscript{49} the CF fragmentary subordinate was utilized for the numerical methodology of the Fokker–Planck condition utilizing Ritz guess. A scientific relative examination of RL and RC electrical circuits utilizing AB and CF fragmentary subordinates was as of late done in Abro et al.\textsuperscript{50} Mustafa et al.\textsuperscript{51} investigated the elements of the malignant growth treatment model with the CF fragmentary subordinate. As of late, another partial model of hepatitis B infection in the CF subordinate sense has introduced in Ullah et al.\textsuperscript{52} Hence, roused by the above work, in this paper, we plan to broaden the as of late distributed partial monetary models of mindfulness\textsuperscript{53} to a fragmentary case by utilizing the recently settled subordinate known as CF subsidiary of request \(\alpha \in (0,1]\).

The subtleties of the rest of the departments of this paper have as following: the essential definition and consequences of partial request subordinate are expressed in department 2. In department 3, we investigate the model definition, balance, and the essential generation number. Department 4, arrangements with the presence of fragmentary monetary models of mindfulness. Likewise, the uniqueness of a model arrangement has acquired. Numerical reenactments are introduced in department 5. At long last, the closing comments are given in department 6.

### Preludes

Here, we allow a few essential meanings of partial analytics which can be utilized in the forward investigation of the system.

**Definition 1.** Put \( f \in H^1(a,b) \), with \( b \) major than \( a, \alpha \in [0,1] \), then the Caputo–Fabrizio\textsuperscript{46,54} is given as:

\[
\mathcal{D}^\alpha_a \{ f(t) \} = \frac{M(\alpha)}{1-\alpha} \frac{t}{\sigma} \int_0^t \frac{f(t) - f(\theta)}{(t-\theta)^\alpha} d\theta, \quad \theta \leq t.
\]

**Remark 1.** If \( \sigma = \frac{1-\alpha}{\alpha} \in [0,\infty), \alpha = \frac{1}{1-\sigma} \in [0,1] \), then equation (2) allows as follows style:

\[
\mathcal{D}^\alpha_a \{ f(t) \} = \frac{N(\sigma)}{\alpha} \int_0^t f(\theta) d\theta, \quad \theta \leq t.
\]

**Definition 2.** The order fractional integral \( \alpha, (0<\alpha \leq 1) \) of \( f(t) \) is defined as:

\[
\mathcal{I}^\alpha_a \{ f(t) \} = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} f(t)
+ \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t f(x) dx, \quad t \geq 0.
\]

**Remark 2.** From Definition 2, we have:

\[
\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha}{(2-\alpha)M(\alpha)} = 1,
\]

which implies \( M(\alpha) = \frac{2}{\alpha}, 0 < \alpha < 1 \). In Losada and Nieto,\textsuperscript{55} the equation (5), novel CF as follows:

\[
\mathcal{D}^\alpha_a \{ f(t) \} = \frac{1}{1-\alpha} \int_0^t f'(\theta) exp\left[\frac{\alpha}{1-\alpha} (t-\theta)\right] d\theta,
\]
the CF subsidiary, given in the above definitions, has been as of late utilized in the numerical demonstrating of HBV, Maxwell liquid with slip impacts, and diabetes model. For more details for CF (see literatures).

Formulation of the model

Here, we expand the financial models of awareness employ a CF derivative of order \( \alpha \in [0, 1] \). The integer order financial models of awareness is planned by the accompanying nonlinear arrangement of differential conditions:

\[
\begin{align*}
\frac{dx}{dt} &= -ux(t) - \frac{k}{N(t)} x(t)[N(t) - x(t)] + \mu_y N(t) - \mu_x x(t), \\
\frac{dy}{dt} &= wx(t) - \frac{k}{N(t)} y(t)[N(t) - y(t)] -(a + \nu)y(t) + \delta z(t) - \mu_y y(t), \\
\frac{dz}{dt} &= (a + \nu)y(t) + \delta z(t) - \mu_d z(t),
\end{align*}
\]

where \( x(t) \) is the number of gathering of people who don’t think about the presence of the item, \( y(t) \) is the quantity of gathering of people who think about the item however have not yet bought it, \( z(t) \) Number of the gathering of individuals who have bought the item, \( N(t) \) Size populace, \( N(t) = x(t) + y(t) + z(t) \). We employ a CF derivative of order \( \alpha \) to reformulate the old-style money-related models of mindfulness with initial conditions:

\[
x(0) = x_0, y(0) = y_0, z(0) = z_0.
\]

Existence and uniqueness of fractional financial models of awareness

Here portrays the presence of model arrangements by utilizing a fixed point hypothesis. We utilize the fundamental administrator in Losada and Nieto on (7) to acquire:

\[
x(t) - x(0) = \frac{CF}{t^\alpha} \left\{ -u x \frac{k}{N} x[N - x] + \mu_y N - \mu_x x \right\}, \\
y(t) - y(0) = \frac{CF}{t^\alpha} \left\{ u x + \frac{k}{N} x[N - x] - (a + \nu)y + \delta z - \mu_y y \right\}, \\
z(t) - z(0) = \frac{CF}{t^\alpha} \left\{ (a + \nu)y - \delta z - \mu_d z \right\}.
\]

Applying the thought utilized in Losada and Nieto, we get:

\[
\begin{align*}
x(t) - x(0) &= \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \left\{ -u x - \frac{k}{N} x[N - x] + \mu_y N - \mu_x x \right\} \\
y(t) - y(0) &= \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \left\{ u x + \frac{k}{N} x[N - x] - (a + \nu)y + \delta z - \mu_y y \right\} \\
z(t) - z(0) &= \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \left\{ (a + \nu)y - \delta z - \mu_d z \right\} + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \left\{ (a + \nu)y - \delta z - \mu_d z \right\} ds,
\end{align*}
\]

for effortlessness, we supplant as follows:

\[
\begin{align*}
s_0 &= \left\{ -u x - \frac{k}{N} x[N - x] + \mu_y N - \mu_x x \right\} \\
s_0 &= \left\{ u x + \frac{k}{N} x[N - x] - (a + \nu)y + \delta z - \mu_y y \right\}, \\
s_0 &= \left\{ (a + \nu)y - \delta z - \mu_d z \right\}.
\end{align*}
\]
Theorem 1. The kernels $F_1, F_2$ and $F_3$ fulfill the Lipschitz condition and withdrawal if the accompanying imbalance holds:

$$0 \leq (a^2 + v^2) + \mu_2^2 < 1.$$  

Proof. Here, we start from $F_2$. Let $y$ and $y_1$ are two functions, then we locate the following:

$$\|F_2(t,y) - F_2(t,y_1)\|$$

$$= \left\| u^x x - u^x x + k^x x[N-x] - k^x x[N-x] - (a^2 + v^2)\right\|$$

$$\|a''(a'' + v'')y_1 + \delta^z z - \mu_2^2 y + \mu_2^2 y_1\|.$$

(12)

Employing the triangular inequality about equation (13), we fulfill:

$$\|F_2(t,y) - F_2(t,y_1)\| \leq \| (a^2 + v^2)(y - y_1)\|$$

$$+ \| -\mu_2^2(y - y_1)\|,$$

$$\|F_2(t,y) - F_2(t,y_1)\| \leq \left[ (a^2 + v^2) + \mu_2^2 \right]\|y - y_1\|,$$

by taking that $(a^2 + v^2) + \mu_2^2 = \mu_1$, we get:

$$\|F_2(t,y) - F_2(t,y_1)\| \leq \mu_1 \|y - y_1\|.$$  

(14)

The Lipschitz hypothesis is utilized for $F_2$ and if add $0 \leq (a^2 + v^2) + \mu_2^2 < 1$ then it is also a contraction. For the rest of the cases, likewise, the Lipschitz conditions are given as follows:

$$\|F_1(t,x) - F_1(t,x_1)\| \leq \mu_2 \|(x - x_1)\|,$$

$$\|F_3(t,z) - F_3(t,z_1)\| \leq \mu_3 \|(z - z_1)\|.$$  

(15)

utilizing documentation for parts, equation (10) becomes:

$$x(t) = x(0) + \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_1(t,x) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_1(s,x)ds,$$

$$y(t) = y(0) + \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_2(t,y) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_2(s,y)ds,$$

$$z(t) = z(0) + \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_3(t,z) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_3(s,z)ds,$$

(16)

the accompanying recursive recipe is introduced:

$$x_n(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_1(t,x_{n-1}) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_1(s,x_{n-1})ds,$$

$$y_n(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_2(t,y_{n-1}) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_2(s,y_{n-1})ds,$$

$$z_n(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)}F_3(t,z_{n-1}) + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t F_3(s,z_{n-1})ds,$$

(17)

with the initial conditions:

$$x^0(t) = x(0),$$

$$y^0(t) = y(0),$$

$$z^0(t) = z(0),$$

the contrast between the progressive terms is determined as follows:

$$\omega_{1n}(t) = x_n(t) - x_{n-1}(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s,x_{n-1}) - F_1(s,x_{n-2})]ds,$$

$$\omega_{2n}(t) = y_n(t) - y_{n-1}(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \int_0^t [F_2(s,y_{n-1}) - F_2(s,y_{n-2})]ds,$$

$$\omega_{3n}(t) = z_n(t) - z_{n-1}(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \int_0^t [F_3(s,z_{n-1}) - F_3(s,z_{n-2})]ds,$$

notice that

$$x_n(t) = \sum_{i=1}^n \omega_{1i}(t),$$

$$y_n(t) = \sum_{i=1}^n \omega_{2i}(t),$$

$$z_n(t) = \sum_{i=1}^n \omega_{3i}(t),$$

(20)

on proceeding with a similar procedure, we survey

$$\|\omega_{1n}(t)\| = \|x_n(t) - x_{n-1}(t)\|$$

$$= \left\| \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s,x_{n-1}) - F_1(s,x_{n-2})]ds \right\|,$$

(21)

using the triangular inequality, equation (21) is simplified to

$$\|x_n(t) - x_{n-1}(t)\|$$

$$\leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \left\| \int_0^t [F_1(s,x_{n-1}) - F_1(s,x_{n-2})]ds \right\|,$$

(22)

as the kernel realizes the Lipschitz hypothesis, then we give
Now we state the theorem below.

**Theorem 2.** The fractional financial models of awareness (7) has precise coupled arrangements if the conditions underneath hold. That is, we can discover $t_0$ with the end goal that

\[
\frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \mu_2 t_0 < 1.
\]

**Proof.** Since all the capacities $x(t)$, $y(t)$, and $z(t)$ is limited, we are demonstrated which pieces satisfy the Lipschitz hypothesis, hence on utilizing equations (24) and (25) and by utilizing the recursive technique, we get the achieving connection as follows:

\[
\|x_n(t) - x_{n-1}(t)\| \leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 \|x_{n-1} - x_{n-2}\| + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s, x_{n-1}) - F_1(s, x_{n-2})] ds,
\]

then we have

\[
\|\omega_{1n}(t)\| \leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 \|\omega_{1(n-1)}(t)\| \quad \text{and (25)}
\]

\[
\|\omega_{2n}(t)\| \leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_1 \|\omega_{2(n-1)}(t)\| + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s, x_n) - F_1(s, x_{n-1})] ds,
\]

then, the existence and continuity of the tell solutions is assured. Also, to include that the above function is a solution of equation (9), we advanced:

\[
x(t) - x(0) = x_n(t) - B_n(t),
\]

\[
y(t) - y(0) = y_n(t) - C_n(t),
\]

\[
z(t) - z(0) = z_n(t) - D_n(t),
\]

therefore, we have:

\[
\|B_n(t)\| = \left\| \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} [F_1(t, x_n) - F_1(t, x_{n-1})] + \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s, x_n) - F_1(s, x_{n-1})] ds \right\|
\]

Using the process in a recursive manner gives:

\[
\|B_n(t)\| \leq \left( \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} + \frac{2\alpha}{(2 - \alpha)M(\alpha)} t \right)^n \mu_2^{n+1} a.
\]

Then at $t_0$ we have:

\[
\|B_n(t)\| \leq \left( \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} + \frac{2\alpha}{(2 - \alpha)M(\alpha)} t_0 \right)^n \mu_2^{n+1} a.
\]

by putting the limit on equation (30) as $n$ tends to infi-

\[
\|C_n(t)\| \rightarrow 0, \|D_n(t)\| \rightarrow 0,
\]

similarly, we obtain

\[
\|C_n(t)\| \rightarrow 0, \|D_n(t)\| \rightarrow 0,
\]

to prove the uniqueness system (7), we select on the reverse that there work out the second solution of (7) given by $x_1(t), y_1(t)$ and $z_1(t)$. Then

\[
x(t) - x_1(t) = \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} [F_1(t, x) - F_1(t, x_1)]
\]

\[
+ \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t [F_1(s, x) - F_1(s, x_1)] ds.
\]
Taking norm on equation (31), we get

\[ \|x(t) - x_1(t)\| \leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \left\| F_1(t, x) - F_1(t, x_1) \right\| 
+ \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t \left\| F_1(s, x) - F_1(s, x_1) \right\| ds. \]

(32)

By putting the Lipschitz condition of kernel, we have:

\[ \|x(t) - x_1(t)\| \leq \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 \left\| F_1(t, x) - F_1(t, x_1) \right\| 
+ \frac{2\alpha}{(2 - \alpha)M(\alpha)} \int_0^t \mu_2 \left\| F_1(s, x) - F_1(s, x_1) \right\| ds. \]

(33)

It gives that:

\[ \|x(t) - x_1(t)\| \left(1 - \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 - \frac{2\alpha}{(2 - \alpha)M(\alpha)} \mu_2 t \right) \leq 0. \]

(34)

**Theorem 3.** The model (7) solution will be unique if

\[ \left(1 - \frac{2(1 - \alpha)}{(2 - \alpha)M(\alpha)} \mu_2 - \frac{2\alpha}{(2 - \alpha)M(\alpha)} \mu_2 t \right) > 0. \]

(35)

**Proof.** If condition (35) holds, then (34) implies that

\[ \|x(t) - x_1(t)\| = 0, \]

hence, we get

\[ x(t) = x_1(t), \]

(36)

on employing the same procedure, we get:

\[ y(t) = y_1(t), z(t) = z_1(t). \]

(37)

\[ y(t) = y_1(t), z(t) = z_1(t). \]

\[ y(t) = y_1(t), z(t) = z_1(t). \]

(38)

**Numerical experiments**

In the following, numerical simulations for the model (8) and (9) are presented. Two schemes (27), (31) are presented to solve the proposed model by the improved Adams-Bashforth-Moulton predictor-corrector numerical integration methods.

**Simulation results**

In this segment, we give mathematical outcomes that demonstrate the presence of the proposed plot. We have actualized the improved Adams-Bashforth-Moulton calculation for mathematical simulation (see literature36).

![Figure 1](image1.png) The numerical simulations of the model at alpha = 0.95, a = 0.02, N = 1000, delta = 0.2, v = 0.05, k = 0.01, x(0) = 30, and y(0) = 60.

![Figure 2](image2.png) The numerical simulations of the model at alpha = 0.95, a = 0.02, N = 1000, delta = 0.2, v = 0.05, k = 0.01, x(0) = 30, and z(0) = 10.

![Figure 3](image3.png) The numerical simulations of the model at alpha = 0.95, a = 0.02, N = 1000, delta = 0.2, v = 0.05, k = 0.01, y(0) = 60, and z(0) = 10.
From Figures 1 to 3, show the approximate solutions of NFFMA by using numerically at $\alpha = 0.95$, and from Figures 4 to 6, show the illustrates the phase spaces and the behavior of parameters.

Conclusion

In the present work, we extended the financial models of awareness model to fractional order using the Caputo–Fabrizio. The presence and uniqueness of the answer for the fragmentary budgetary models of the mindfulness model with CF subordinate are demonstrated in detail. From numerical recreations, one can see that when the fragmentary request of subordinate $a_{15,18}$ diminishes, the CF subsidiary gives all the more organically doable conduct about the dynamic of pine shrivel infection. Along these lines, we reasoned that the recently partial subsidiary is valuable for demonstrating such marvels. Additionally, from the graphical social we infer that the proposed partial request model gives more extravagant and increasingly adaptable outcomes when contrasted and the comparing whole number request monetary models of mindfulness model. Moreover, numerical simulations are shown to verify the effectiveness of the proposed scheme.

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ORCID iD

Amr MS Mahdy https://orcid.org/0000-0003-2218-5408

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