Stable charged radiating systems associated with tilted observers

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Abstract This paper is aimed to study the influence of electromagnetic field and tilted congruences on the dynamical features of self-gravitating system. We shall explore the stability of homogeneous energy density in the background of Maxwell–Palatini $f(R)$ gravity. In this respect, we have considered an irrotational non-static planar geometry which is assumed to have two different types of gravitating sources. The role of tilted congruences and the geodesic motion of an evolving system is studied through the divergence of the entropy vector field. The condition for the emergence of Minkoskian cavity is also explored. In order to connect tilted and non-tilted reference frames with inflationary and inverse Ricci scalar corrections in the charged medium, few well-consistent relations are presented. It is concluded that effective electric charge is trying to increase the stability of regular energy density of the planar system.

1 Introduction

General Relativity (GR) was the most influential gravitational theory of the last era, widely understood as a theory explaining geometrical attributes of the space and time on macroscopic scales. The implication of GR provides Friedmann equations for a regular and perfect fluid configurations that could accurately explain the astrophysical transition of radiation and then matter dominated cosmic epochs. Indeed, the current advancement of observational cosmology accompanied by the highly precise experimentation, like supernovae observations [1–3], has revealed that our cosmos is in a state of accelerated expansion. This phenomenon can not be explained by GR provided with the conventional gravitating source.

In addition, GR does not take into consideration the cosmological period known as inflation [4], which was thought to have arisen before the radiation stage and which could mitigate some of the challenges of standard cosmology such as the flatness and horizon issues [5]. Furthermore, GR with the conventional baryonic matter could not address the observed fluid density estimated by fitting the standard cosmic model with the observations of WMAP7 [6].

Moreover, in order to study inflationary cosmic era in an Einstein–$\Lambda$ gravity, one needs to add inflaton (slow roll scalar field) by hand, thereby indicating that the cosmological constant $\Lambda$ can not accommodate inflationary period of our universe. Indeed, other interpretations for

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the aforementioned acceleration can be given through theories which generalize GR by adopting action function, dissimilar to the action of Einstein–Hilbert. Nojiri and Odintsov [7] addressed the relevance and the need for these theories in depth. The \( f(R) \) (where \( R \) is the Ricci scalar), \( f(T) \) (where \( T \) is the trace of energy momentum tensor), \( f(R, □ R, T) \) (\( □ \) is the de Alember’s operator and \( T \) is the trace of energy momentum tensor) etc., are among the appealing models of modified theories (for further reviews on such models, see, for instance, [7–15]). Numerous aspects of these theories have also been extensively studied in gravitation and astrophysics [16].

There are three different versions of \( f(R) \) gravity. In this paper, we are considering Palatini \( f(R) \) gravity. In this theory while calculations, the metric and connections are considered as independent quantities. The Palatini \( f(R) \) approach may lead to explaining several unusual phenomenological GR modifications the study of supermassive compact objects, structure formation and evolution of the universe [17–21]. This scheme of gravity provides singularity free second order field equations instead of fourth order, thus making them comparatively easy to solve and handle mathematically [22]. Many scholars have addressed the comprehensive analysis on the feasibility of this theory, including Olmo and his collaborators [23–27]. Recently, Olmo et al. [28] described the challenges as well as the achievements of modified gravity theories in explaining the structure and evolution of stellar objects, while junction condition for the matching of interior and exterior metrics are found in [29].

Ilyas et al. [30] studied the stellar evolution by taking into account isotropic relativistic spheres and calculated some feasible and stable models in modified gravity. Recently, Moraes et al. [31] investigated the dynamics of compact objects after exploring modified versions of equations of motion. Bhatti et al. [32] after applying numerical techniques calculated stable epochs of few strange stars in modified gravity. Recently, few researchers have calculated. The astrophysical properties of various stellar bodies, like wormholes [33,34], gravastars [35,36] and cosmic models [37–39] are also evaluated by many researchers in modified gravity.

Research to investigate the explanation behind the phenomenon of inhomogeneous energy density (IED) has motivated several theorists not only in GR, but also in modified theories. To explore the factors evolved throughout in the development of IED over the stellar structures, Hawking and Israel [40] found a physical relationship between the tidal forces and the matter parameters. Herrera et al. [41] studied the construction of naked singularity using IED and locally anisotropic pressure for the spherically symmetric fluid configurations. Herrera et al. [42] studied radiating spherical compact objects in GR and predicted a remarkable connection between tidal forces, IEDs, and anisotropic pressure. Furthermore, Raychaudhuri evolution equation for the irrotational relativistic spheres was found by Herrera et al. [43] via well-known structural scalars. These scalars can be determined from the orthogonal breaking down of the Riemann curvature tensor. The role of structural variables on the dynamical instability and irregularity factors of relativistic geometric populations with matter distribution have been investigated by [44]. They concluded that the factors which control the stability and inhomogeneity of the corresponding systems are energy density, modified curvature terms and pressure.

Di Prisco et al. discussed the solutions having thin shell and also some solutions which satisfy the Darmois conditions on the boundary \( Σ \) [45]. Yousaf et al. [16, 46–48] taken into consideration an/isotropic structure having the influence of heat dissipation and analyzed its basic properties through structure scalars. They formulated the field and dynamical equations in terms of such scalars and described the importance of these scalars in the modeling stellar bodies. Recently, Herrera [49–51] and Yousaf et al. [52–54] described the significance of tilted and non-tilted observers in order to explain the some distinct physical properties of the same
matter configuration. Sussman and Jaime [55] studied the class of inhomogeneous model of non-interacting particles through LTB geometry in $f(R)$ gravity. Yousaf [56] calculated some constraints describing inflationary and late time acceleratory universe with the help of planar dissipative models with tilted and non-tilted congruences.

This paper aims to describe the role of tilted congruences and Maxwell-$f(R)$ gravity on the dynamical properties of planar relativistic systems. We shall also present some relationship connecting the matter variables of tilted and non-tilted observers in the presence of electromagnetic field. The coming section is devoted to present two different set of structural variables corresponding to comoving and non-comoving frame of references. Sect. 3 describes the Bianchi identities and few well-known kinematical variables in the presence of Maxwell-$f(R)$ gravity. Furthermore, the factor involved in the emergence of IED is also explored in the same section. The results are summarized in the last section.

2 Palatini $f(R)$ formalism

The action function for the derivation of field equation in $f(R)$ gravity can be given as follows

$$S_{f(R)} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(\hat{R}) + S_M,$$

where $f(\hat{R})$, $S_M$ and $\kappa$ represent the generic function of the Ricci scalar, action for matter and coupling constant, respectively. Here $\hat{R} = g^{\gamma\delta} R_{\gamma\delta}$, comes from the contraction of the $R_{\gamma\delta}$ and metric tensors $g^{\gamma\delta}$ associated with the connection symbol. One should keep in mind that the Ricci scalar as a function of the connection symbols (please see for details [23–27]). The variations of the above action with respect to $g^{\gamma\delta}$ and $\Gamma^\mu_{\gamma\delta}$, respectively provides

$$f_R(\hat{R}) \hat{R}_{\gamma\delta} - \left( g_{\gamma\delta} f(\hat{R}) \right)/2 = \kappa T_{\gamma\delta},$$

$$\hat{\nabla}_\mu (g^{\gamma\delta} \sqrt{-g} f_R(\hat{R})) = 0,$$

where $f_R = \frac{df(R)}{dR}$ and $T_{\gamma\delta}$ is an energy-momentum tensor that is not dependent on connections. It expression can be casted as

$$T_{\alpha\delta} = -2(-g)^{-1/2} \frac{\delta S_M}{\delta g^{\alpha\delta}},$$

One can get the following equation after taking the trace of Eq. (2) as

$$\hat{R} f_R(\hat{R}) - 2 f(\hat{R}) = \kappa T,$$

thereby describing $R$ as dependent on $T$ and $T \equiv g^{\gamma\delta} T_{\gamma\delta}$. The consideration of vacuum case, i.e., $T = 0$ in the above equation makes $R$ as a constant entity, thereupon demonstrating it as function of $f(\hat{R})$ parameter. In this way, one can see an auxiliary metric function $h_{\gamma\delta}$ to be proportional to a metric tensor $g_{\gamma\delta}$, whence $f(R)$ theory would describe the dynamical properties of $\Lambda$-dominated epoch. Under these circumstances, the Levi–Civita connection will become the connection associated with $h_{\gamma\delta}$ given as follows

$$\Gamma^v_{\gamma\delta} = \frac{1}{2} h^{va} (\partial_\gamma h_{a\delta} + \partial_\delta h_{a\gamma} - \partial_a h_{\gamma\delta}).$$

We are now focusing on a metric equation of motion, which would of the second order. To this end, we find the value of connection from Eq. (2). If the same is used in Eq. (3), a single
The field equation can be found as under

\[
\frac{1}{f_R} \left( \hat{\nabla}_\gamma \hat{\nabla}_\delta - g_{\gamma\delta} \hat{\Box} \right) f_R + \frac{1}{2} g_{\gamma\delta} \hat{R} + \frac{\kappa}{f_R} T_{\gamma\delta} + \frac{1}{2} g_{\gamma\delta} \left( \frac{f}{f_R} - \hat{R} \right) \\
+ \frac{3}{2f_R} \left[ \frac{1}{2} g_{\gamma\delta} (\hat{\nabla} f_R)^2 - \hat{\nabla}_\gamma f_R \hat{\nabla}_\delta f_R \right] - \hat{R}_{\gamma\delta} = 0,
\]

(7)

Alternatively, this can be expressed via the Einstein tensor \( \hat{G}_{\gamma\delta} \) as.

\[
\hat{G}_{\gamma\delta} = \frac{\kappa}{f_R} (T_{\gamma\delta} + \hat{T}_{\gamma\delta}),
\]

(8)

where \( \hat{\Box} \) is an operator of d’Alembertian, and

\[
T_{\gamma\delta} = \frac{1}{\kappa} \left( \hat{\nabla}_\gamma \hat{\nabla}_\delta - g_{\gamma\delta} \hat{\Box} \right) f_R - \frac{f_R}{2\kappa} g_{\gamma\delta} \left( \hat{R} - \frac{f}{f_R} \right) \\
+ \frac{3}{2\kappa f_R} \left[ \frac{1}{2} g_{\gamma\delta} (\hat{\nabla} f_R)^2 - \hat{\nabla}_\gamma f_R \hat{\nabla}_\delta f_R \right].
\]

It is worthy to mention that here the operator \( \hat{\nabla}_\gamma \) describes the covariant derivative for \( g_{\gamma\delta} \).

We now model our problem with the following plane symmetric metric

\[
ds^2 = -dt^2 + B^2(t, z)(dx^2 + dy^2) + C^2(t, z)dz^2.
\]

(9)

We assume that the gravitational source of the above planar metric as seen by an observer having a locally Minkowkain frame (LMF) resting in a comoving congruences is given by

\[
T_{\gamma\delta} = \hat{\rho} u_{\gamma} u_\delta
\]

(10)

where \( \hat{\rho} \) is the dust energy density. In this environment, the fluid velocity takes the form

\[
u^\gamma = (1, 0, 0, 0).
\]

(11)

We assume that our systems has a relativistic matter content in the presence of electromagnetic field. The role of electric charge can be analyzed through the following tensor

\[
E_{\mu\nu} = \frac{1}{4\pi} (F_{\mu}^\gamma F_{\gamma\nu} - \frac{1}{4} F_{\gamma\delta} F_{\gamma\delta} g_{\mu\nu}),
\]

(12)

where \( F_{\mu\nu} \) is a Maxwell tensor that can be expressed through 4-potential \( (\psi_\mu) \) as \( F_{\mu\nu} = -\psi_{\mu,v} + \psi_{v,\mu} \). It satisfy the equations of motion for the charged medium given follows

\[
F_{\hat{\gamma}v} = \mathcal{M} J^\mu, \quad F_{[\mu\nu;\gamma]} = 0,
\]

where \( J^\mu \) is the 4-current with its magnetic permeability \( \mathcal{M} \). For the current reference frame, we take

\[
\Psi_v = \psi_v^0, \quad J^\mu = \Gamma u^\mu,
\]

where \( \Gamma(t, r) \) stands for the density associated with the charged medium. In an environment of the non-tilted planar configuration, we get

\[
\frac{\partial^2 \psi}{\partial r^2} - \frac{\partial \psi}{\partial r} \left( \frac{C'}{C} - \frac{2B'}{B} - \frac{2f_R'}{f_R} \right) = 4\pi \Gamma B^2,
\]

(13)

\[
\frac{\partial^2 \psi}{\partial r \partial t} - \frac{\partial \psi}{\partial r} \left( \frac{\dot{B}}{B} - \frac{2\dot{C}}{C} - \frac{2\dot{f}_R}{f_R} \right) = 0,
\]

(14)
where over dots and over primes are the notations for the time and radial partial differentiations, respectively. Equation (13) provides

$$\psi' = \frac{C\hat{s}}{B^2 f_R^2},$$

having

$$\hat{s} = 4\pi \int_0^r \Gamma CB^2 f_R^2 dr,$$

to be the total amount of charge within the manifold of the dust cloud. From Eq. (12), we obtain

$$E_{00} = \frac{\hat{s}^2}{8\pi \Lambda^4 f_R^4}, \quad E_{01} = 0, \quad E_{11} = -\frac{C^2 \hat{s}^2}{8\pi \Lambda^4 f_R^4}, \quad E_{22} = \frac{E_{33}}{\sin^2 \theta} = \frac{\hat{s}^2}{8\pi \Lambda^2 f_R^4}.$$

The application of a Lorentz boost from the dust source accompanying LMF to an auxiliary LMF associated with a fluid radial velocity ($\chi$) provides a radiating and locally anisotropic (having pressure components $P_z$ and $P_\perp$) stress energy tensor as follows

$$T_{\gamma\delta} = (\rho + P_\perp) U^\gamma U_\delta + \epsilon l^\gamma l_\delta - P_\perp g_{\gamma\delta} + q_\gamma U_\delta,$$

where

$$U^\gamma = \left( \frac{1}{\sqrt{1 - \chi^2}}, \frac{\chi}{B\sqrt{1 - \chi^2}}, 0, 0 \right)$$

(16)

describes the vector field for the tilted reference frame. In Eq. (15), the structural variables $\epsilon$, $\rho$ explain radiation and energy densities, respectively, while the vector corresponding to the heat conduction is given by $q_\gamma$. We further assume that an observer resting in a tilted congruences spotted the radiation transmissions through the gravitating source in both diffusion and streaming out approximations. Therefore, we have taken two distinct factors of dissipation in Eq. (15). We define the vector fields $S^\eta$, $q^\eta$ and $l^\eta$ having tilted backgrounds as follows

$$S^\gamma = \left( \frac{\chi}{\sqrt{1 - \chi^2}}, \frac{1}{B\sqrt{1 - \chi^2}}, 0, 0 \right), \quad q^\gamma = q S^\gamma,$$

$$l^\gamma = \left( \frac{1 + \chi}{\sqrt{1 - \chi^2}}, \frac{1 + \chi}{B\sqrt{1 - \chi^2}}, 0, 0 \right),$$

(17)

(18)

which obey the following constraints

$$U^\gamma U_\gamma = -1 = l_\gamma U^\gamma, \quad S^\gamma S_\gamma = 1 = l_\gamma S^\gamma, \quad l^\gamma l_\gamma = 0 = S^\gamma U_\gamma = U^\gamma q_\gamma.$$

The equations of motion for the planar tilted observer provide

$$\frac{\partial^2 \psi}{\partial r^2} - \frac{\partial \psi}{\partial r} \left( \frac{C'}{C} - \frac{2B'}{B} - \frac{2f'_R}{f_R} \right) = \frac{4\pi \Gamma C^2}{\sqrt{1 - \chi^2}},$$

$$\frac{\partial^2 \psi}{\partial r \partial t} - \frac{\partial \psi}{\partial r} \left( \frac{\dot{C}}{C} - \frac{2\dot{B}}{B} - \frac{2\dot{f}_R}{f_R} \right) = -\frac{4\pi \Gamma \omega C}{\sqrt{1 - \chi^2}}.$$

(19)

(20)

On solving Eq. (19), we obtain

$$\psi' = \frac{C\hat{s}}{B^2 f_R^2},$$
in which $\tilde{s}$ describes the total amount of charge in the non-comoving relativistic matter for the planar geometry. Its expression is found as under

$$\tilde{s} = 4\pi \int_0^r \frac{\Gamma CB^2 f_R^2}{\sqrt{1 - \chi^2}} dr.$$  

From Eq. (12), we get

$$E_{00} = \frac{\tilde{s}^2}{8\pi B^4 f_R}, \quad E_{01} = 0, \quad E_{11} = -C^2 \tilde{s}^2 \frac{1}{8\pi B^4 f_R}, \quad E_{22} = \frac{E_{33}}{\sin^2 \theta} = \frac{\tilde{s}^2}{8\pi B^2 f_R}.$$  

These are the non-vanishing components of the electromagnetic energy momentum tensor for the planar charged medium whose observations are noticed by a tilted observer.

2.1 Relations between comoving and non-comoving reference frames with an electromagnetic field

From Eq. (8), the corresponding Palatini $f(R)$ field equations for the charged anisotropic plane symmetric systems can be written as follows

$$G_{00} = \frac{\kappa}{f_R} \left[ \frac{1}{1 - \chi^2} \left\{ \tilde{P}_\pi \chi^2 + \tilde{\rho} + 2\tilde{q} \chi \right\} + \frac{1}{\kappa} \left\{ -\left( \frac{f}{R} - f_R \right) \frac{R}{2} - \tilde{f}_R \left( \frac{\dot{C}}{C} + \frac{9}{4} \right) \right\} \right. \right.$$  

$$\left. \left. \times \left( \frac{\dot{f}_R}{f_R} + \frac{2\tilde{B}}{B} \right) - \left( \frac{C'}{C} - \frac{2B'}{B} + \frac{1}{4} \frac{f_R'}{f_R C^2} + \frac{f_R''}{C^2} \right) \right\} + \frac{\tilde{s}^2}{8\pi B^4 f_R^4} \right], \quad (21)$$

$$G_{11} = \frac{\kappa}{f_R} \left[ P_\perp + \frac{1}{\kappa} \left\{ \left( \frac{f}{R} - f_R \right) \frac{R}{2} - \frac{f_R''}{C^2} \right\} + \tilde{f}_R + \left( \frac{4}{C} - \frac{\tilde{f}_R}{f_R C^2} + \frac{4\tilde{B}}{B} \right) \frac{\tilde{f}_R}{4} \right.$$  

$$\left. + \left( \frac{4C'}{C} + \frac{f_R'}{f_R - 4B^2 \tilde{C}_R} \right) \frac{f_R'}{4C^2} \right\} + \frac{\tilde{s}^2}{8\pi B^4 f_R^4} \right], \quad (22)$$

$$G_{03} = \frac{\kappa}{f_R} \left[ -\frac{C}{1 - \chi^2} \left\{ \left( \tilde{P}_\pi + \tilde{\rho} \right) \chi + \left( 1 - \chi^2 \right) \tilde{q} \right\} + \frac{\tilde{f}_R}{\kappa} \left\{ \tilde{f}_R - \frac{5}{2} f_R - \frac{f_R' \tilde{C}_R}{\kappa} \right\} \right], \quad (23)$$

$$G_{33} = \frac{\kappa}{f_R} \left[ \frac{1}{1 - \chi^2} \left\{ \tilde{P}_\pi + \tilde{\rho} \chi^2 + 2\tilde{q} \chi \right\} + \frac{1}{\kappa} \left\{ \left( \frac{f}{R} - f_R \right) \frac{R}{2} - \frac{f_R'}{f_R C^2} \left( \frac{9 f_R'}{4C^2} \right) \right\} \right.$$  

$$\left. + \frac{8B'}{B} \right\} + \tilde{f}_R - \left( \frac{\tilde{f}_R}{f_R - \frac{8\tilde{B}}{B} \frac{\tilde{f}_R}{4} \right) - \frac{\tilde{s}^2}{8\pi B^4 f_R^4} \right], \quad (24)$$

where the tilde over the quantities describes that the corresponding quantities are evaluated after adding $\epsilon$ in them, for instance, $\tilde{y} \equiv y + \epsilon$, while the values of Einstein tensor $G_{ij}$s can be found at [49].

In the analysis of stellar systems with tilted or non-tilted congruences, the selection of $f(R)$ models is of significant importance. Here, we would like to study the influence of $f(R)$ gravity on bringing the importance of congruences of the observers in the explanations of physical phenomena under certain feasible theoretical backgrounds. It is worthy to stress that the use of quadratic Ricci curvature corrections is among the viable attempts to renormalize GR as an alternative to the most conventional gravitational theory. The inclusion of these corrections in the EHA occupies key relevance in the field of theoretical cosmology. These
could assist one to analyze the dynamical features of galaxies, cluster, their evolutions as well as compact objects in an arena of self-consistent inflationary environment. Such terms are induced as an approximation for the study of DE models.

There is growing evidence that the Universe at the present epoch is undergoing an exponential expansion process. The preferred reason for this phenomenon is that a sort of dark energy is actually dominating the Universe. It is worthy to stress that, the dynamics of the early universe can be well discussed through the higher derivative corrections containing the positive curvature terms. The terms with negative powers of curvature could act as a gravitational alternative to allow the interstellar acceleration to become consistent with DE [7]. In order to study such epochs in a unified way, we take, with the non-tilted gravitational source (10), the following model

$$f(R) = R + \psi R^2,$$

in which $\psi$ is a constant [57] which may equals $\frac{1}{6M^2}$ (with $M$ as $2.7 \times 10^{-12}$ GeV along with $\psi \leq 2.3 \times 10^{22} GeV^2$) for the study of the exotic matter, that appear precisely at the present times. We take the $f(R)$ model associated with the non-comoving reference frame (15) as follows [58]

$$f(R) = R + \rho_1 \frac{\delta^4}{R},$$

where $\delta \in (0, \infty)$ and $\rho_1 = +1$. The choice $\delta^{-1} \sim (10^{33} eV)^{-1} \sim 10^{18}$ sec may helps to understand the evolution of our cosmos at present times.

The gravitating sources connected with the both types of tilted and non-tilted systems are producing the same kind of geometric structure. Therefore, the connections between matter variables along with the effective curvature terms can be calculated. The same relations are being found as under

$$\frac{\kappa \dot{\rho}}{1 + 2\psi R} + \frac{\kappa \tilde{\delta}^2}{8\pi B^4(1 + 2\psi R)^3} + \frac{\psi R^2}{2(1 + 2\psi R)} + \frac{R\delta^2}{R^2 - \delta^4} = \frac{\kappa \tilde{\delta}^2 R^{10}}{8\pi B^4(R^2 - \delta^4)^5}$$

$$\frac{\kappa R^2(\dot{\rho} + \dot{\tilde{\rho}} + 2\tilde{\phi} \chi)}{(R^2 - \delta^2)(1 - \chi^2)} + \frac{\kappa \tilde{\delta}^2}{8\pi B^4(1 + 2\alpha R)^5} = \frac{-\psi R^2}{2(1 + 2\psi R)} - \frac{\kappa \tilde{\delta}^2 R^{10}}{8\pi B^4(R^2 - \delta^4)^5}$$

$$P_\perp = \frac{-\psi (R^2 - \delta^4)}{2(1 + 2\psi R)\kappa} - \frac{\tilde{\delta}^2 R^8}{8\pi B^4(R^2 - \delta^4)^4} - \frac{\delta^4}{R\kappa} + \frac{\tilde{\delta}^2(R^2 - \delta^4)}{8\pi B^4 R^2(1 + 2\alpha R)^5}.$$

These relations are connecting gravitating variables of non-interacting particles found in inflation field with the fluid variables of non-ideal radiating locally anisotropic matter resting in the cosmic speed up epoch. The electromagnetic field and Palatini $f(R)$ terms are making the role of $P_\perp$ to be non-zero that was zero in the background of Einstein gravity [49]. This shows that these two forces are producing special effects (equal to tangential pressure of the non-tilted gravitating source) on the dynamics of the system. Some more relations among the variables of the charged fluid configurations of the above mentioned systems can be found by making use of Eqs. (10) and (15) as
\[ \epsilon = \frac{(R^2 - \delta^4)}{1 + 2\psi R} \left\{ \frac{\hat{\rho}}{R^2(1 - \chi^2)} + \frac{\hat{\delta}^2}{8\pi B^4 R^2(1 + 2\psi R)^4(1 - \chi^2)} + \frac{\psi}{2\kappa} \right\} + \frac{\delta^4}{R\kappa} - \rho, \]

\[ P_r = \rho - \frac{(R^2 - \delta^4)\hat{\rho}}{R^2(1 + 2\psi R)} - \frac{\hat{\delta}^2(R^2 - \delta^4)}{8\pi B^4 R^2(1 + 2\psi R)^5} - \frac{(R^2 - \delta^4)\psi}{\kappa(1 + 2\psi R)} - \frac{2\delta^4}{R\kappa} \]

\[ q = \frac{-\hat{\rho}(R^2 - \delta^4)\chi}{R^2(1 + 2\psi R)(1 - \chi^2)} - \frac{\hat{\delta}^2(R^2 - \delta^4)\chi}{8\pi B^4 R^2(1 + 2\psi R)^5(1 - \chi^2)} + \frac{\tilde{\delta}^2 R^8}{(R^2 - \delta^4)^4} \]

\[ \times \frac{\chi}{8\pi B^4(1 - \chi^2)} - \epsilon, \]

\[ = \rho - \frac{\hat{\rho}(R^2 - \delta^4)}{R^2(1 - \chi)(1 + 2\psi R)} - \frac{\hat{\delta}^2(R^2 - \delta^4)}{8\pi B^4 R^2(1 + 2\psi R)^5(1 - \chi)} - \frac{\delta^4}{R\kappa} \]

\[ + \frac{8\pi B^4(R^2 - \delta^4)(1 - \chi)}{2\kappa(1 + 2\chi R)}, \]

\[ = P_r - \rho \chi - \frac{\psi(R^2 - \delta^4)(1 + \chi)}{2\kappa(1 + 2\psi R)(1 - \chi)} + \frac{\delta^4(1 + \chi)}{R\kappa(1 - \chi)}. \]

It is clear from Eq. (33) that the effects of energy dissipation stemming from the streaming out and diffusion approximations could only be seen if the tilted system is able to retain its four velocity. On the contrary the conditions \( \epsilon = q = 0 \), would keep the value of tilted four velocity to be zero, even though we are observing this analysis in the background of Maxwell–Palatini \( f(R) \) theory. Thus, one can state that the observations of heat dissipation from the charged relativistic matter is only possible if a system is able to retain the configurations of non-comoving reference frame. Taking this into consideration, we now provide few relationships under some particular constraints.

### 2.1.1 \( \epsilon \neq 0 \) in the background of zero diffusion approximations

In this scenario, the structural quantities of plane symmetric model in the presence of electromagnetic field in view of Eqs. (31)–(35) provide

\[ P_r = \rho \chi - \frac{\psi(R^2 - \delta^4)(1 + \chi)}{2\kappa(1 + 2\psi R)} - \frac{\delta^4(1 + \chi)}{\kappa R}, \]

\[ \rho = \frac{(R^2 - \delta^4)}{(1 + 2\psi R)} \left\{ \frac{\hat{\rho}}{R^2(1 - \chi)} + \frac{\psi}{2\kappa} + \frac{\hat{\delta}^2}{8\pi B^4 R^2(1 + 2\psi R)^4(1 - \chi)} \right\} \]

\[ - \frac{8\pi B^4(R^2 - \delta^4)(1 - \chi)}{\tilde{\delta}^2 R^8} + \frac{\delta^4}{R\kappa}, \]

\[ \epsilon = \frac{\delta^2(R^2 - \delta^4)\chi}{R^2(1 + 2\psi R)(1 - \chi^2)} - \frac{\delta^2(R^2 - \delta^4)\chi}{8\pi B^4 R^2(1 + 2\psi R)^3(1 - \chi^2)} \]

\[ + \frac{8\pi B^4(R^2 - \delta^4)(1 - \chi^2)}{\tilde{\delta}^2 R^8\chi} \].
2.1.2 $q \neq 0$ in an environment of zero streaming out approximations

In this end, the planar charged matter variables are found to be related through Eqs. (31)–(35) as follows

$$P_r = \rho \chi^2 - \frac{\psi (R^2 - \delta^4)(1 + \chi^2)}{2\kappa (1 + 2\psi R)} - \frac{\delta^4 (1 + \chi^2)}{\kappa R},$$

$$\rho = \frac{(R^2 - \delta^4)}{(1 + 2\psi R)} \left\{ \frac{\hat{\rho}}{R^2 (1 - \chi^2)} + \frac{\hat{s}^2}{8\pi B^4 R^2 (1 + 2\psi R)^4 (1 - \chi^2)} + \frac{\psi}{2\kappa} \right\}$$

$$- \frac{8\pi B^4 (R^2 - \delta^4)^4 (1 - \chi^2)}{\kappa R},$$

$$q = -\frac{\hat{\rho} (R^2 - \delta^4) \chi}{R^2 (1 + 2\psi R) (1 - \chi^2)} - \frac{\hat{s}^2}{8\pi B^4 R^2 (1 + 2\psi R)^5 (1 - \chi^2)} + \frac{\hat{s}^2}{8\pi B^4},$$

$$\times \frac{R^8 \chi}{(R^2 - \delta^4)^4 (1 - \chi^2)}.$$  

2.1.3 $P_r = 0$

It becomes from Eqs. (31)–(35) as

$$\rho = \frac{\hat{\rho} (R^2 - \delta^4)}{R^2 (1 + 2\psi R)} + \frac{\hat{s}^2 (R^2 - \delta^4)}{8\pi B^4 R^2 (1 + 2\psi R)^5} + \frac{\psi (R^2 - \delta^4)}{\kappa (1 + 2\psi R)}$$

$$- \frac{8\pi B^4 (R^2 - \delta^4)^4 (1 - \chi^2)}{\kappa R},$$

$$q = -\frac{\rho \chi}{1 - \chi} + \frac{\psi (R^2 - \delta^4) (1 + \chi)}{2\kappa (1 + 2\psi R) (1 - \chi)} + \frac{\delta^4 (1 + \chi)}{R\kappa (1 - \chi)},$$

$$\epsilon = \frac{\rho \chi^2}{(1 - \chi^2)} - \frac{\psi (R^2 - \delta^4)(1 + \chi^2)}{2\kappa (1 + 2\psi R) (1 - \chi^2)} - \frac{\delta^4 (1 + \chi^2)}{R\kappa (1 - \chi^2)}.$$  

3 Kinematical quantities

This section deals with the computation of kinematical quantities of the charged non-static plane symmetric model in $f(R)$ theory with Palatini formalism. The investigation of such terms with $f(R)$ model (26) could help us to understand physical features of planar irrotational locally anisotropic radiating interiors. We shall evaluate such properties one by one as follows.

The generic formula to calculate the four acceleration of the collapsing matter configurations can be specified as

$$a^\gamma = U^\gamma_{\delta} \ddot{U}^\delta.$$  

This with the help of $f(R)$ extra curvature corrections is found as under

$$a^\gamma = aS^\gamma - g^{\gamma\nu} \frac{\partial \nu f_R}{2f_R}.$$  

where
\[
a = \frac{1}{\sqrt{1 - x^2}} \left[ \dot{x} + \frac{\chi x'}{C} + (1 - x^2) \left( \frac{\dot{C}}{C} + \frac{2\chi \delta^4 R^{-3} \dot{R}}{(1 - \delta^4 R^{-2})} + \frac{2\delta^4 R^{-3} R'}{C(1 - \delta^4 R^{-2})} \right) \right]. \tag{47}
\]

Since we have figured out the non-zero contribution of the 4-acceleration, therefore, the observer residing in the non-comoving framework identified the non-geodesic nature of the plane geometric structure. This happened due to the inclusion of Palatini \( f(R) \) gravity in an environment of tilted congruences. From Eq. (47), it can be noticed that the radial velocity \( \chi \) and the modifications in Einstein gravity implied by Palatini \( f(R) \) theory force the process to allow non-geodesic motion of the test particles.

The scalar for the describing expansion is
\[
\Theta = U^\mu_{\cdot \mu}, \tag{48}
\]
which for the relativistic charged planar system is found as follows
\[
\Theta = \frac{1}{\sqrt{1 - x^2}} \left[ x \dot{x} + \frac{x' x'}{C} + (1 - x^2) \left( \frac{\dot{C}}{C} + \frac{4 \delta^4 R^{-3} \dot{R}}{(1 - \delta^4 R^{-2})} + \frac{2\dot{B}}{B} \right) \right]. \tag{49}
\]
The condition \( \Theta = 0 \), would reduce the above equation as follows
\[
\frac{\delta^4 \dot{R}}{R^3} = \frac{(1 - \delta^4 R^{-2})}{4(1 - \delta^4 R^{-2})} \left\{ x \dot{x} + \frac{x' x'}{C} + (1 - x^2) \left( \frac{\dot{C}}{C} + \frac{2\dot{B}}{B} \frac{2\chi x'}{BC} + x \frac{2\delta^4 R^{-3} R'}{C(1 - \delta^4 R^{-2})} \right) \right\}. \tag{50}
\]
This equation would help to understand those systems which are having Minkowskian core.

Here we are dealing with the relativistic model consisting of a fluid with shear. Therefore, the tensorial quantity for describing shear fluid is described through projection tensor \( h_{\gamma \delta} \) by
\[
\sigma_{\gamma \delta} = U_{(\gamma; \delta)} + a_{(\gamma} U_{\delta)} - \frac{1}{3} \Theta h_{\gamma \delta}, \tag{51}
\]
whose non-zero values are found as under
\[
\sigma_{00} = \frac{2\chi^2}{3(1 - \chi^2)} \left[ \sigma + \frac{3\delta^4 R^{-3} \dot{R}}{\chi^2(1 - \delta^4 R^{-2}) \sqrt{1 - \chi^2}} + \frac{3\delta^4 R^{-3} R'}{2C(1 - \delta^4 R^{-2}) \sqrt{1 - \chi^2}} \right], \tag{52}
\]
\[
\sigma_{11} = \frac{2C^2}{3(1 - \chi^2)} \left[ \sigma + \sqrt{1 - \chi^2} \left\{ \frac{3\delta^4 R^{-3} \dot{R}}{2(1 - \delta^4 R^{-2}) \sqrt{1 - \chi^2}} - \frac{3\chi \delta^4 R^{-3} R'}{C f_R} \right\} \right], \tag{53}
\]
\[
\sigma_{22} = -\frac{B^2}{3} \left[ \sigma + \frac{1}{\sqrt{1 - \chi^2}} \left\{ \frac{3\delta^4 R^{-3} \dot{R}}{B(1 - \delta^4 R^{-2}) \sqrt{1 - \chi^2}} - \frac{\chi \delta^4 R^{-3} R'}{B(1 - \delta^4 R^{-2})} \right\} \right], \tag{54}
\]
where
\[
\sigma = \frac{1}{\sqrt{1 - x^2}} \left[ x \dot{x} + \frac{x' x'}{C} + (1 - x^2) \left( \frac{\dot{C}}{C} - \frac{2\delta^4 R^{-3} \dot{R}}{(1 - \delta^4 R^{-2})} - \frac{\dot{B}}{B} - \frac{\chi x'}{BC} \right) \right]. \tag{55}
\]
On making $\chi = 0$, the value of the shear scalar $\sigma$ reduces to
\[
\sigma = \frac{\dot{C}}{C} - \frac{2\delta^4 R^{-3} \dot{R}}{(1 - \delta^4 R^{-2})} - \frac{\dot{B}}{B}
\] (56)
that clearly indicates the inclusion of $\delta$ terms in the analysis. The condition $\sigma = 0$ yields the following components of the shear tensor
\[
\sigma_{00} = \frac{2\chi^2 \delta^4}{3(1 - \chi^2)(1 - \delta^4 R^{-2})} \left[ \frac{3R^{-3} \dot{R}}{\chi^2 \sqrt{1 - \chi^2}} + \frac{3\dot{R} - 3R^{-3} R'}{2C \chi} \sqrt{1 - \chi^2} \right],
\]
(57)
\[
\sigma_{11} = \frac{2C^2 \delta^4}{3(1 - \chi^2)(1 - \delta^4 R^{-2})} \left[ \sqrt{1 - \chi^2} \left( \frac{3\dot{R}}{2R^3} - \frac{3\chi R^{-3} R'}{C} \right) \right],
\]
(58)
\[
\sigma_{22} = \frac{-B^2 \delta^4}{3(1 - \delta^4 R^{-2})} \left[ \frac{1}{\sqrt{1 - \chi^2}} \right],
\]
(59)
which formulate that Palatini $\delta$ corrections explicitly tend to regulate the shear-free phases of the locally anisotropic dissipative planar metric.

### 4 Equations of motion

The purpose of this section is to compute dynamical, collapse and Weyl scalar equations. We shall calculate dynamical and collapse equations from the contracted form of modified Bianchi identities, while the Weyl scalar equation will be computed through the procedure presented firstly by Ellis [59] and then by Herrera [60]. These mathematical expressions will help us to explain the appearance of irregularities in the charged planar celestial population which is assumed to be initially homogeneous in nature. The equations for the description of radial and temporal fluctuations in the collapsing interiors in the presence of Maxwell–Palatini $f(R)$ corrections can be calculated from the following identities
\[
Y^\gamma_{\delta,\gamma} = 0, \quad \text{where } Y^\gamma_{\delta} = E^\gamma_{\delta} + T^\gamma_{\delta} + T^\gamma_{\delta}.
\]
Now, we define two operators $\dagger$ and $\ast$ as follows
\[
y^\dagger = y, v S^v, \quad y^\ast = y, v U^v.
\]
After using Eqs. (21)–(23), (47), (49) and (55), we get
\[
\bar{\rho}^\ast + \bar{\rho} \Theta + \bar{q}^\dagger + \bar{q} \left\{ \chi \Theta + \frac{\sqrt{1 - \chi^2}}{C} \left( \frac{2B'}{B} + \frac{f_R}{f_R} \right) + \frac{2\chi}{\sqrt{1 - \chi^2}} \right\} + \bar{\rho} \frac{f^*_R}{f_R}
\]
\[
+ \frac{q f^*_R}{f_R} + \frac{\chi P'_\perp}{C \sqrt{1 - \chi^2}} + \frac{\bar{s}^2 f_R}{4\pi B^4 f^5_R \sqrt{1 - \chi^2}} + P_{\perp} \left( \Theta + \frac{\dot{f}_R}{f_R \sqrt{1 - \chi^2}} + \frac{\chi f'_R}{f_R \sqrt{1 - \chi^2}} \right)
\]
\[
+ \frac{\kappa_0}{\sqrt{1 - \chi^2}} = 0. \tag{60}
\]
\[
\bar{P}_z^\dagger + a(\bar{\rho} + \bar{P}_z) + \frac{2\bar{q}}{3} \left[ 2\Theta + \sigma - 3\chi (\ln B)^\dagger \right] + \bar{q}^\ast + \frac{\chi f^*_R}{f_R} (\bar{\rho} + P_{\perp}) - \bar{q} \sqrt{1 - \chi^2}
\]
\[
\times \left( \frac{\dot{C}}{C} + \frac{\dot{B}}{B} \right) + \frac{1}{f_R \sqrt{1 - \chi^2}} \left( \bar{q} \chi^2 \dot{f}_R - \frac{\bar{\rho} f^*_R}{C} - \frac{P_{\perp} f^*_R}{C} \right) - \frac{\bar{s}^2}{4\pi B^4 f^4_R} + \frac{\bar{s}^2 f'_R}{4\pi B^4 f^5_R}
\]
\[-\frac{x^2P_1'}{C\sqrt{1-x^2}} + \frac{\chi}{\sqrt{1-x^2}}[\dot{\mu} + (\chi \dot{q})] + \frac{\mathcal{K}_1 \sqrt{1-x^2}}{C} - \sqrt{1-x^2}(P_\perp \dot{\chi}) = 0, \tag{61}\]

where \(\mathcal{K}_i\)'s describe the part of the role emerging from the Palatini \(f(R)\) gravity. These are mentioned in an “Appendix”. Upon substitution of Eqs. (50), (61) with expansion-free condition provides

\[
\begin{align*}
\tilde{P}_z &+ a(\tilde{\rho} + \tilde{P}_z) + \frac{2\tilde{\eta}}{3} \left[ \sigma - 3\chi (\ln B) \right] + \tilde{q}^* + \frac{\chi f_R^*}{f_R} (\tilde{\rho} + P_\perp) - \tilde{q} \sqrt{1-x^2} \\
\times &\left( \frac{\dot{C}}{C} + \frac{2\dot{B}}{B} \right) + \frac{1}{f_R \sqrt{1-x^2}} \left( \tilde{q} \chi^2 \Psi f_R - \frac{\tilde{\rho} f_R^*}{C} - \frac{P_\perp f_R^*}{C} \right) - \frac{\tilde{g}^*}{4\pi B^4 f_R^4} + \frac{\tilde{g}^2}{4\pi B^4 f_R^5} \\
- &\frac{x^2P_1'}{C\sqrt{1-x^2}} + \frac{\chi}{\sqrt{1-x^2}}[\dot{\mu} + (\chi \dot{q})] - \sqrt{1-x^2}(P_\perp \dot{\chi}) + \frac{\mathcal{K}_2 \sqrt{1-x^2}}{C} = 0, \tag{62}\end{align*}
\]

where

\[
\Psi = \frac{1}{2(\chi^2 - 1)} \left\{ \chi \dot{\chi} + \frac{\chi'}{C} + (1 - \chi^2) \left( \frac{\dot{C}}{C} + \frac{2\dot{B}}{B} + \frac{2\chi B'}{BC} + \frac{\chi f_R^*}{C f_R} \right) \right\}. \tag{63}
\]

while \(\mathcal{K}_2\) is being calculated by putting zero expansion condition in the expression of \(\mathcal{K}_1\). This equation describes the radial variations on the matter variables of those relativistic objects who are able to create empty core during evolution.

A relativistic self-gravitating fluid may reach at the collapsing phase after experiencing energy density irregularities in its energy density. Based about how much massive a celestial structure is, the phenomenon of gravitational collapse may produce various types of compact objects, like black hole, neutron star or white dwarf. Therefore, the research for the elements of inhomogeneity in the initially regular spacetimes is of considerable importance. Here, in an atmosphere of tilted anisotropic configuration, we determine those variables that are involved in producing disturbances on the homogeneous energy density of the planar relativistic interiors evolving an environment of electromagnetic field. A mathematical combinations of Weyl scalar and effective form of the charged plane symmetric fluid variables can be devised to obtain such factor [16]. For non-comoving congruences, it follows that

\[
\begin{align*}
\begin{bmatrix}
\mathcal{E} - \frac{\kappa}{2f_R} \left( \tilde{\rho} - \tilde{P}_z + P_\perp + T_{00} - \frac{T_{11}}{C^2} + \frac{T_{33}}{B^2} - \frac{\tilde{g}^2}{8\pi B^4 f_R^4} \right) \\
\times \left( \tilde{P}_z - P_\perp + \frac{T_{11}}{C^2} + \frac{\tilde{g}^2}{8\pi B^4 f_R^4} - \frac{T_{33}}{B^2} \right) + 3\kappa \frac{\dot{B}}{2B f_R} \left\{ \frac{\tilde{\rho} + \tilde{P}_z}{\chi} + \tilde{q} (1 + \chi^2) \right\} \\
- \frac{T_{03}}{C} \end{bmatrix}
\end{align*}
\]

This is a non-linear partial differential equation. The solution to this equation is very tough and complicated. However, the analytical approach for getting solution of the above equation may be done by considering certain viable explanations. The model that could help to explain the current cosmic acceleration in the realm of an inhomogeneous background is LTB. The considerations of tilted and non-tilted congruences in LTB will not produce hindrances in the description of inhomogeneity picture. But the situation about the appearances of irregularities in the energy density is quite different for the tilted framework with planar LTB-like spacetime. In the following, we wish to calculate the factors that are participating in the
evolution of the charged planar system from the homogeneous phase to the inhomogeneous window with Maxwell–Palatini $f(R)$ corrections.

In this direction, we assume that our plane symmetric relativistic model develops in a such a way that the entire emission and absorption through the matter distribution is null and thereby making the system to retain its only isotropic pressure effects. Therefore, the conditions $P_z = P_\perp = P$ and $q = \epsilon = 0$ would reduce Eq. (64) to

$$E' + \frac{3B'}{B}E = \frac{\kappa}{2} \left[ \frac{1}{f_R} \left( \dot\rho + T_{00} \right) - \frac{T_{11}}{C^2} - \frac{\tilde{g}^2}{8\pi B^4 f_R^4} + T_{33}B^2 \right] + \frac{3\kappa \dot{B}}{2Bf_R} \left( \dot\rho + \dot{\tilde{P}} \right)(1 - \chi^2)^2. \tag{65}$$

It has been well recognized that the scalar that governs the impact of energy density irregularities of the galactic and inter-galactic celestial population surface of an object is the Weyl tensor. Here, we want the same type of variable to be determined for the plane symmetric spacetime with Palatini $f(R)$ gravity in the presence of electromagnetic field. After calculating the solution of the above partial differential equation, we notice that three elements are forcing the system to persist in the homogenous phase. These are effective electric charge $s$, Palatini $f(R)$ corrections and velocity $\psi$ mediating from vector field of the non-comoving congruences. These three quantities are striving to sustain the planar system to be remain in the initial homogeneous state. The constraints $\psi = 0$, $f(R) = R$ reduce the above the above to

$$\tilde{\rho}' = 0 \Leftrightarrow E = 0. \tag{66}$$

The stability of gravitational collapse as well as the comoving congruences for the relativistic geometric system has been studied by many researchers [35,61–64].

5 Summary

After recent developments in the study of modern Physics, (CMB and supernova-type Ia surveys), interest in the study of plane symmetrical geometry has been established in order to investigate few mysterious features of our evolutionary universe. This paper is aimed to study the effects of Maxwell–Palatini $f(R)$ terms on the stability of tilted frame for the plane symmetric model. We shall also try to connect inflationary and late time cosmic speed up eras with the help of tilted and non-tilted observers. After the rotation-free transformation of a LMF corresponding to Eq. (11), a scenario of tilted congruences with a gravitating source possessing a certain velocity in the radial direction can be created. In this respect, we assume that LTB-like plane symmetric spacetime is filled observer’s dependent two different types of fluid distributions. An observer resting in a comoving congruence observe that LTB-like plane symmetric model is generated by a dust fluid, while the anisotropic radiating matter is a gravitating source of the same geometry as noticed by a titled observer whose reference frame is moving with a specific radial velocity $\psi$.

We shall explore the Palatini $f(R)$ and Maxwell field equations for the plane symmetric collapsing model. As we have taken two different physical interpretation of congruences therefore, it is worth while to expect that quantities whose calculations are based on congruences should play an essential fundamental role in our study. For the irrotational relativistic study, the congruence of an observer is linked with three kinematical variables. We investigated the non-zero components of such variables in the Maxwell-$f(R)$ gravity with Palatini formalism.
We also established relationships between field variables in these subsequent frames with such contexts to see the influence of observers on the interpretations of planar geometry dynamics. This has helped us to study cosmological aspects of a non-tilted model with a quadratic Ricci inflationary model with the tilted late time cosmic speed up terms in a unified way with an electromagnetic field. The effects of electric charge and Palatini $f(R)$ terms are extensively studied.

As shown in Eq. (46), a particular form of the 4-acceleration is evaluated with Palatini $f(R)$ gravity corrections. We have determined the non-geodesic nature of LTB-like plane symmetric model. This happened due to the presence of Maxwell- $f(R)$ terms. The non-zero values of the shear tensor and a scalar associated with the expansion scalar is calculated. We have also developed a relation that could help to understand the cavity production within the matter distribution of tilted framework. This scenario could be helpful in the study of cosmological voids. Herrera et al. [65] stated that the zero expansion condition can only be applicable to those relativistic fluids who have pressure anisotropy in their gravitating sources. Therefore, we can not apply the condition of expansion-free to the matter source mentioned in Eq. (11). Thus the the phenomenon of cavity evolution is expected to appear in the tilted charged plane symmetric congruences during the inflationary cosmic eras. Thus the tilted framework is likely to host structures of cosmological voids.

The assumption of non-comoving frame of reference could play a significant role in the explanation of many secrets and hidden facets of our evolving universe. This concept is sufficiently versatile to accommodate and describe many celestial systems of various distributions of matter. We have also studied the stability of regular energy density of the homogeneous relativistic plane symmetric model in the presence of electromagnetic field. It is concluded that the presence of effective charge is trying to increase the stability of homogeneous energy density of the system. This finally lead us to find the state of conformal flatness within the charged planar model in Palatini $f(R)$ gravity. This result has been obtained after computing the modified version of Ellis equations from the second Bianchi identity of the charged plane symmetric metric.

There are a collection of structures with well recognized physical properties and thermal history persists besides supernovae, that could provide valuable and compatible tests for...
modified gravity theories. The ultimate objective behind this research is to recognize the
most significant changes in the astrophysical predictions of self-gravitating non-rotational
stellar structures in contrast with GR frameworks. We have noticed some very interesting
results because of the modification of gravity. We have noticed that modification of gravity
mediated by \( f(R) = R + \delta R^4 \) corrections is likely to host shearing stellar models than
that in GR. The stellar models having stronger shearing effects and unit radii are likely to
exist in more abundance in an environment created by \( \delta = 1 \) parameter. The subsistence of
the self-gravitating systems with relatively less shear remain in a sequence for the region
\( \delta \in (0.5, 1.0) \). After that, the tilted stellar objects with higher shearing motion can likely be
observed near the parametric values \( \delta = 1 \). However, in GR, stellar objects with relativity
less shearing effects within the system are noticed. One can observe this kind of structure
formation from the Fig. 1.

On the other hand, we observed that for the parametric choices of \( \delta \in (0.4, 1.0) \), the
system having unit radius undergoes imploding and exploding phases due to the negative
and positive values of \( \Theta \). However, the same object enters into the stable window on taking
\( \delta = 0 \), i.e., GR case. This also indicates the occurrences of irregular distribution of matter
due to inclusion of Palatini \( f(R) \) factor \( \delta \) as seen by Fig. 2. All our findings and observations
would reduce to GR [49] under the limit \( f(R) = R \).

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Appendix

The expressions \( K_0 \) and \( K_1 \) arising in Eqs. (60) and (61) are given below

\[
K_0 = \frac{\mathcal{T}_{01}}{C^2} \left( \frac{2B'}{B} + \frac{2f_R'}{f_R} + \frac{C'}{C} \right) - \mathcal{T}_{00} - \mathcal{T}_{00} \left( \frac{\dot{C}}{C} + \frac{3f_R}{2f_R} + 2\frac{\dot{B}}{B} \right) - \frac{\mathcal{T}_{11}}{C^2} \\
\times \left( \frac{\dot{C}}{C} + \frac{f_R'}{2f_R} \right) + \left( \frac{\mathcal{T}_{10}}{C^2} \right) - \frac{2\mathcal{T}_{22}}{B^2} \left( \frac{\dot{B}}{B} + \frac{f_R}{2f_R} \right),
\]

(A1)
\[ K_1 = -\dot{T}_{10} + T_{00} \frac{f'_R}{2f_R} + \left( 2B + \frac{3f'_R}{2f_R} \right) \frac{T_{11}}{C^2} - T_{10} \left( \frac{\dot{C}}{C} + \frac{2f'_R}{f_R} + \frac{2B}{B} \right) \\
\quad + \left( \frac{T_{11}}{C^2} \right)' - \frac{2}{B^2} \left( \frac{B'}{B} + \frac{f'_R}{2f_R} \right) T_{22} . \]  

(A2)

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