On the Success Rate of Crossover Operators for Genetic Programming with Offspring Selection

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Abstract. Genetic programming is a powerful heuristic search technique that is used for a number of real world applications to solve amongst others regression, classification, and time-series forecasting problems. A lot of progress towards a theoretic description of genetic programming in form of schema theorems has been made, but the internal dynamics and success factors of genetic programming are still not fully understood. In particular, the effects of different crossover operators in combination with offspring selection are largely unknown. This contribution sheds light on the ability of well-known GP crossover operators to create better offspring when applied to benchmark problems. We conclude that standard (sub-tree swapping) crossover is a good default choice in combination with offspring selection, and that GP with offspring selection and random selection of crossover operators can improve the performance of the algorithm in terms of best solution quality when no solution size constraints are applied.

1 Genetic Programming

Genetic programming (GP) is a generalization of genetic algorithms first studied at length by John Koza [5]. Whereas the goal of genetic algorithms is to find a fixed length vector of symbols that encodes a solution to the problem, the goal of genetic programming is to find a variable-length program that solves the original problem when executed. Common practice is to use a tree-based representation of computer programs similar to so called symbolic expressions of functional programming languages such as LISP.

Genetic programming is a powerful heuristic search method that has been used successfully to solve real world problems from various application domains, including classification, regression, and forecasting of time-series [9,10].

Offspring selection [11] is a generic selection concept for evolutionary algorithms that aims to reduce the effect of premature convergence often observed...
with traditional selection operators by preservation of important alleles \[2\]. The main difference to the usual definition of evolutionary algorithms is that after parent selection, recombination and optional mutation, offspring selection filters the newly generated solutions. Only solutions that have a better quality than their best parent are added to the next generation of the population. In this aspect offspring selection is similar to non-destructive crossover \[21\], soft brood selection \[3\], and hill-climbing crossover \[13\]. Non-destructive crossover compares the quality of one child to the quality of the parent and adds the better one to the next generation, whereas offspring selection generates new children until a successful offspring is found. Soft brood selection generates \(n\) offspring and uses tournament selection to determine the individual that is added to the next generation, but in comparison to offspring selection the children do not compete against the parents. Hill-climbing crossover generates new offspring from the parents as long as better solutions can be found. The best solution found by this hill-climbing scheme is added to the next generation. The recently described hereditary selection concept \[11,12\] also uses a similar offspring selection scheme in combination with parent selection that is biased to select solutions with few common ancestors.

2 Motivation

Since the very first experiments with genetic programming a lot of effort has been put into the definition of a theoretic foundation for GP in order to gain a better understanding of its internal dynamics. A lot of progress \[9,17,18,20\] towards the definition of schema theorems for variable length genetic programming and sub-tree swapping crossover, as well as homologous crossover operators \[19\] has been made. Still, an overall understanding of the internal dynamics and the success factors of genetic programming is still missing. The effects of mixed or variable arity function sets or different mutation operators in combination with more advanced selection schemes are still not fully understood. In particular, the effects of different crossover operators on the tree size and solution quality in combination with offspring selection are largely unknown.

In this research we aim to shed light on the effects of GP crossover operators regarding their ability to create improved solutions in the context of offspring selection. We apply GP with offspring selection to three benchmark problems: symbolic regression (Poly-10), time series prediction (Mackey-Glass) and classification (Wisconsin diagnostic breast cancer). The same set of experiments was also executed for the 4-bit even parity problem, but because of space constraints the results of those experiments are not reported in this paper.

Recently we have analyzed the success rate of GP crossover operators with offspring selection with strict solution size constraints \[6\]. In the paper at hand we report results of similar experiments with the same set of crossover operators and benchmark problems, but without strict solution size constraints.
3 Configuration of Experiments

The crossover operators used in the experiments are: standard (sub-tree swapping) [5] [20], one-point [9], uniform [15], size-fair, homologous, and size-fair [7]. Additionally, the same experiments were also executed with a crossover variant that chooses one of the five crossover operators randomly for each crossover event [6]. Except for the crossover operator, the problem specific evaluation operator, and the function set all other parameters of the algorithm were the same for all experiments. The random initial population was generated with probabilistic tree creation (PTC2) [10] and uniform distribution of tree sizes in the interval [3; 50]. A single-point mutation operator was used to manipulate 15% of the solution candidates by exchanging either a function symbol (50%) or a terminal symbol (50%). See Table 1 for a summary of all GP parameters.

To analyze the results, the quality of the best solution, average tree size in the whole population as well as offspring selection pressure were logged at each generation step together with the number of solutions that have been evaluated so far. Each run was stopped as soon as the maximal offspring selection pressure or the maximal number of solution evaluations was reached.

Offspring selection pressure of a population is defined as the ratio of the number of solution evaluations that were necessary to fill the population to the population size [1]. High offspring selection pressure means that the chance that crossover generates better children is very small, whereas low offspring selection pressure means that the crossover operator can easily generate better children.

3.1 Symbolic Regression – Poly-10

The Poly-10 symbolic regression benchmark problem uses ten input variables $x_1, \ldots, x_{10}$. The function for the target variable $y$ is defined as $y = x_1 x_2 + x_3 x_4 + x_5 x_6 + x_1 x_7 x_9 + x_3 x_6 x_{10}$ [8,14]. For our experiments 100 training samples were generated randomly by sampling the values for the input variables uniformly in the range $[-1,1]$. The usual function set of $+,-,\times,\%$ (protected division) and the terminal set of $x_1 \ldots, x_{10}$ without constants was used. The mean squared errors function (MSE) over all 100 training samples was used as fitness function.

3.2 Time Series Prediction – Mackey-Glass

The Mackey-Glass ($\tau = 17$) chaotic time series is an artificial benchmark data set sometimes used as a representative time series for medical or financial data sets [8]. We used the first 928 samples as training set, the terminal set for the prediction of $x(t)$ consisted of past observations $x_{128}, x_{64}, x_{32}, x_{16}, x_8, x_4, x_2, x_1$ and integer constants in the interval $[1;127]$. The function set and the fitness function (MSE) were the same as in the experiments for Poly-10.

Data set available from: http://neural.cs.nthu.edu.tw/jang/benchmark/
3.3 Classification – Wisconsin Diagnostic Breast Cancer

The Wisconsin diagnostic breast cancer data set from the UCI Machine Learning Repository [4] is a well known data set for binary classification. Only a part (400 samples) of the whole data set was used and the values of the target variable were transformed to values 2 and 4. Before each genetic programming run the whole data set was shuffled, thus the training set was different for each run.

Again the mean squared errors function for the whole training set was used as fitness function. In contrast to the previous experiments a rather large function set was used that included functions with different arities and types (see Table 1). The terminal set consisted of all ten input variables and real-valued constants in the interval \([-20;20]\).

| General parameters for all experiments | Population size | 1000 |
|----------------------------------------|-----------------|------|
| Initialization                         | PTC2 (uniform [3..50]) |
| Parent selection                       | fitness-proportional (50%), random (50%) |
| Mutation rate constraints              | strict offspring selection, 1-elitism |
|                                        | 15% single point (50% functions, 50% terminals) |
|                                        | unlimited tree size and depth |
| Poly-10                                 | Function set    | ADD, SUB, MUL, DIV (protected) |
|                                        | Terminal set    | \(x_1 \ldots x_{10}\) |
|                                        | Fitness function| Mean squared errors |
|                                        | Max. evaluations| 1,000,000 |
| Mackey-Glass                           | Function set    | ADD, SUB, MUL, DIV (protected) |
|                                        | Terminal set    | \(x_{128}, x_{64}, \ldots, x_2, x_1, \text{ constants: 1..127}\) |
|                                        | Fitness function| Mean squared errors |
|                                        | Max. evaluations| 5,000,000 |
| Wisconsin                              | Function set    | ADD, MUL, SUB, DIV (protected), LOG, EXP, SIGNUM, SIN, COS, TAN, IF-THEN-ELSE, LESS-THAN, GREATER-THAN, EQUAL, NOT, AND, OR, XOR |
|                                        | Terminal set    | \(x_1, \ldots, x_{10}, \text{ constants: } [-20..20]\) |
|                                        | Fitness function| Mean squared errors |
|                                        | Max. evaluations| 2,000,000 |

Table 1. General parameters for all experiments and specific parameters for each benchmark problem.

4 Results

Figure 1 shows the quality progress (MSE, note log scale), average tree size, and offspring selection pressure for each of the six crossover operators over time (number of evaluated solutions). The first row shows the best solution quality, the second row shows average tree size over the whole population and the third row shows offspring selection pressure.

Size-fair, homologous, and mixed crossover are the most successful operators, whereas onepoint and uniform crossover show rather bad performance. The average tree size grows exponentially in the experiments with standard and mixed crossover, whereas with onepoint, uniform, size-fair and homologous crossover
the average tree size stays at a low level. The most interesting result is that offspring selection pressure stays at a low level over the whole run when standard or mixed crossover are used. Offspring selection pressure rises gradually over the whole run when standard crossover is used with size constraints [6]. The different behavior when no size constraints are applied indicates that larger offspring solutions are more likely to be better than their parent solutions than offspring solutions of equal or smaller size. The offspring selection pressure charts for one-point, uniform, size-fair and homologous crossover show the usual effect, namely that it becomes increasingly harder for crossover to produce successful children.

![Fig. 1. Best solution quality (MSE, note log scale), average tree size, and offspring selection pressure for 20 runs with each crossover operator for the Poly-10 problem.](image)

Figure 2 shows the results for the Mackey-Glass problem. Standard crossover and mixed crossover show good performance in terms of solution quality and the expected exponential growth of solution size. Size-fair crossover had similar behavior as homologous crossover. Onepoint and uniform crossover are the least effective operators. The offspring selection pressure charts show that with onepoint and uniform crossover the offspring selection pressure rises quickly. The runs with standard crossover and mixed crossover again have low offspring selection pressure over the whole run.

Figure 3 shows the results for the Wisconsin classification problem. Mixed crossover performs better than standard crossover for this problem. Onepoint, uniform, size-fair, and homologous crossover reached similar solution quality, except for one outlier with homologous crossover. The offspring selection pressure curves of onepoint and uniform crossover show that offspring selection pressure remains at a low level until a point of convergence is reached where the offspring selection pressure rapidly increases to the upper limit. The explanation for this
is that one-point and uniform crossover cause convergence to a fixed tree shape. When all solutions have the same tree shape it becomes very hard to find better solutions. Only the runs with size-fair crossover show the usual pattern of gradually increasing offspring selection pressure. An interesting result is that offspring selection pressure also remains low for homologous crossover even though it doesn’t show the exponential growth in solution size as standard and mixed crossover. The flat offspring selection pressure curve could be caused by either the extended function set or the structure of the data set. Further investigations are necessary to fully explain this observation.

5 Conclusion

Based on the results for the benchmark problems it can be concluded that standard (sub-tree swapping) crossover is a good default choice. The results also show that one-point and uniform crossover operators do not perform very well on their own. They also have the tendency to quickly freeze the tree shape, and should be combined with mutation operators which manipulate tree shape.

The aim of the experiments with the mixed-crossover variant was to find out if a combination of all five crossover operators in one GP run has a beneficial effect either in terms of achievable solution quality or efficiency. For two of the three benchmark problems the runs with mixed crossover found better solutions than runs with standard crossover. This result is in contrast to the results of experiments with strict size constraints where runs with mixed crossover did not find better solutions than runs with standard crossover [6].
Fig. 3. Best solution quality (MSE, note log scale), average tree size, and offspring selection pressure for 20 runs with each crossover operator for the Wisconsin classification problem.

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