EXTREMELY CLOSE-IN GIANT PLANETS FROM TIDAL CAPTURE

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ABSTRACT

Planets that form around stars born in dense stellar environments such as associations, open clusters, and globular clusters are subject to dynamical perturbations from other stars in the system. These perturbations will strip outer planets, forming a population of free-floating planets, some of which will be tidally captured before they evaporate from the system. For systems with velocity dispersion \( \sigma \sim 1 \) km s\(^{-1}\), Jupiter-mass planets can be captured into orbits with periods of \( 0.1 - 0.4 \) days, which are generally stable over \( \sim 10^9 \) years, assuming quadratic suppression of eddy viscosity in the convective zones of the host stars. Under this assumption, and the assumption that most stars form several massive planets at separations \( 5 - 50 \) AU, I estimate that \( \sim 0.03\% \) of stars in rich, mature open clusters should have extremely close-in tidally captured planets. Approximately \( 0.005\% \) of field stars should also have such planets, which may be found in field searches for transiting planets. Detection of a population of tidally-captured planets would indicate that most stars formed in stellar clusters. In typical globular clusters, the fraction of stars with tidally-captured planets under these assumptions rises to \( \sim 0.1\% \) – in conflict with the null result of the Hubble Space Telescope transit search in 47 Tuc. This implies that, if the quadratic prescription for viscosity suppression is correct, planetary formation was inhibited in 47 Tuc: on average \( \lesssim 1 \) planet of mass \( \gtrsim M_{\text{Jup}} \) (bound or free-floating) formed per cluster star. Less than half of the stars formed solar-system analogs. Brown dwarfs can also be captured in tight orbits; the lack of such companions in 47 Tuc in turn implies an upper limit on the initial frequency of brown dwarfs in this cluster. However, this upper limit is extremely sensitive to the highly uncertain timescale for orbital decay, and varies by four orders of magnitude depending on the choice of prescription for the suppression eddy viscosity. Therefore, it is difficult to draw robust conclusions about the low-mass end of the mass function in 47 Tuc.

Subject headings: planetary systems – binaries: close, eclipsing – stars: low-mass, brown dwarfs, mass function – globular clusters: general, individual: 47 Tuc – open clusters and associations: general

1. INTRODUCTION

The discovery of an extrasolar planetary companion orbiting a mere 0.05 AU from its host star 51 Peg (Mayor & Queloz 1995) originally came as a surprise, given the seeming impossibility of forming a giant planet so near to its parent star. However, the theorists quickly recovered, and, drawing on older works which studied interactions between protoplanets and their natal disks (Goldreich & Tremaine 1980, Wart 1980), invoked planetary migration as the mechanism for delivering the companion to 51 Peg from its birthplace to its current observed position (e.g., Lin, Bodenheimer, & Richardson 1996).

Since this time, considerable progress in the theory of planetary migration, combined with the detection of a large number of additional giant planets on short-period orbits, has left little doubt that this process is important in shaping the architecture of planetary systems. However, many questions remain. For example, the mechanisms by which planetary migration is stopped are uncertain (Kuchner & Lecar 2002, Lin, Bodenheimer, & Richardson 1996, Trilling et al. 1998). These mechanisms have the additional burden that they should explain both the observed ‘pile-up’ of planetary companions at periods of \( P = 3 \) d (i.e., Kuchner & Lecar 2002), and the newest discovery of a planet at \( P = 1.2 \) d (Konacki et al. 2003, Sasselov 2003). In addition, it is clear from examples of extrasolar giant planets orbiting close to their supposed birthplaces, as well as our own solar system, that orbital migration is not always so efficient. A satisfactory explanation for the causes of these variations in the efficiency of migration is lacking.

It seems plausible that many, if not the majority, of disk stars were initially formed in stellar systems – loose associations or open clusters that have since dissolved. Evidence for this comes in part from studies of the nearby moving groups (de Zeeuw et al. 1999). Planets orbiting stars in stellar systems are subject to numerous effects resulting from the dense stellar environment. Planets orbiting at larger distances from their parent stars are generally more sensitive to these effects. Therefore, the structure of planetary systems will be the result of a complex interplay between the local properties of the system that drive planetary formation and migration, and the non-local effects that arise from the star’s environment.

Thus, if it is indeed the case that most stars formed in stellar clusters, planetary systems cannot be understood as isolated systems: a complete picture of planetary formation and evolution must consider both local and non-local effects.

Several authors have considered the effects of dense stellar environments on the formation
and survival of planetary systems (Armitage 2000). Finally, in §47 Tuc transit search and planetary companions to stars in globular clusters, where tidal capture of free-floating planets. In globular clusters will occasionally be tidally captured, resulting in a detectable population of extremely close-in BD companions to main-sequence stars if the frequency of freely-floating BDs is large. Similar considerations can also be applied to planets in stellar systems.

Here I consider the formation and stability of extremely close-in giant planetary companions to stars via tidal capture of free-floating planets. In §2 I summarize the dynamical processes of ionization, evaporation, capture, and tidal decay, and their relevant timescales. In §3 I give a crude estimate for the frequency of extremely close-in giant planets as a function of the parameters of the stellar system. In §4 I consider tidally-captured BD and planetary companions to stars in globular clusters, and interpret the null result of the 47 Tuc transit search (Gilliland et al. 2000). Finally, in §5 I discuss possible implications and prospects for the detection of such a population of planetary companions.

2. DYNAMICAL PROCESSES

It is not clear if planets can form around stars in stellar systems at all. In particular, in rich clusters with $N \gtrsim 10^5$ members, high-mass stars can generate an ultraviolet radiation field that is sufficiently intense to photoevaporate protoplanetary discs in a few hundred thousand years (Armitage 2000). However, planet formation might still be possible if there exists a substantial delay between high and low-mass star formation. This fact, combined with our incomplete understanding of the efficiency of planetary migration, make it difficult to predict a priori the frequency and distribution of planets orbiting stars in stellar systems. For the purposes of discussion, I will sweep all of these uncertainties under the rug, and simply consider the evolution and consequences of a substantial population of long-period planets around stars in stellar systems.

Binaries in stellar systems evolve under random encounters with other stars in the system. This evolution proceeds according to Heggie’s law (Heggie 1975), such that soft binaries tend to get disrupted by encounters. A binary is soft when its binding energy is equal to the mean kinetic energy of the stars in the system. Thus the planet must pass within two stellar radii to be captured (Fabian, Pringle, & Rees 1975). This leads to the requirement that the stellar component of the system dominates the mass, the characteristic timescale for this process is the relaxation timescale (Binney & Tremaine 1987; Spitzer 1987),

$$t_{\text{rel}} = 0.34 \frac{\sigma^3}{\nu(GM)^2 \ln \Lambda}.$$  

For the parameters adopted above, this yields $t_{\text{rel}} \sim 0.03$ Gyr.

Free-floating planets will occasionally pass sufficiently close to another star to raise a significant tide. If the energy required to raise this tide is larger than the relative energy of the planet and star at infinity, then the encounter will lead to a bound system (Fabian, Pringle, & Rees 1975; Lee & Ostriker 1980; Press & Teukolsky 1977). This leads to the requirement that the planet must pass within a minimum distance $a_{\text{cap}}$ to be captured (Fabian, Pringle, & Rees 1975),

$$a_{\text{cap}} \sim R_\star \left[ \frac{GM}{R_\star \sigma} q(1 + q) \right]^{1/6}.$$  

For $q = 10^{-3}$, $M_\star \sim M_\odot$, $R_\star \sim R_\odot$, $a_{\text{cap}}/R_\star \sim 2$. Thus the planet must pass within two stellar radii to be captured.

The timescale for disruption of a planet is approximately the time it takes for the random encounters to change the planet’s energy by an amount equal to its binding energy (Binney & Tremaine 1987),

$$t_{\text{dis}} = (1 + q) \frac{M_\star}{\langle M \rangle} \frac{\sigma}{16\sqrt{3}G(M)va \ln \Lambda}.$$  

Here $q \equiv m_p/M_\star$, $\nu$ is the stellar number density, and $\ln \Lambda \approx 0.4N$ is the Coulomb logarithm. The number density and velocity dispersion within a stellar system can vary by several orders of magnitude. For simplicity, here I will adopt these properties averaged over the region interior to $r_{hm}$, the half-mass radius of the cluster. The mean stellar density in this area is simply

$$\nu = \frac{1}{3} \frac{N(M)}{\pi r_{hm}^3}.$$  

For a wide range of equilibrium stellar systems, the half-mass radius can be related to the total mass and velocity dispersion of the system by (Spitzer 1987),

$$r_{hm} = \frac{0.4G(M)N}{3\sigma}.$$  

I will adopt this relation throughout unless stated otherwise. For an average stellar density $\nu \sim 10^3$ pc$^{-3}$ and velocity dispersion $\sigma \sim 1.5$ km s$^{-1}$, the above relation yields $N \sim 7600$. For the stellar parameters given above, $t_{\text{dis}} \sim 0.5$ Gyr($a/2.5$AU)$^{-1}$. Thus most planets with $a \gtrsim 2.5$AU in an open cluster will be disrupted over the cluster’s lifetime ($\sim 0.5$ Gyr).

Since planets are typically liberated due to the accumulated effects of distant encounters, free-floating planets will initially have velocities similar to the stars, and will not escape the system immediately (Hurley & Shara 2002). However, equipartition of energy will slowly drive the velocity dispersion of free-floating planets to the point where many of the planets have sufficient velocities to evaporate from the system entirely. Assuming that the stellar component of the system dominates the mass, the characteristic timescale for this process is the relaxation timescale (Binney & Tremaine 1987; Spitzer 1987),

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The timescale for tidal capture is,

$$t_{\text{enc}} \simeq \frac{\sigma}{8\sqrt{\pi GM_\ast \varpi a_{\text{cap}}}},$$

(7)

where I have assumed that $GM_\ast/2\sigma^2 \simeq a_{\text{dis}}/q \gg a_{\text{cap}}$, which is valid for the cases considered here. I find $t_{\text{enc}} \simeq 500 \text{Gyr}(a_{\text{cap}}/2R_\odot)^{-1}$. Assuming every star forms $N_p$ planets that are liberated from their parent stars essentially immediately, the fraction of stars with tidally captured planet can be crudely estimated as

$$\sim N_p(t_{\text{enc}}/t_{\text{enc}}) \sim 0.01\%N_p.$$  

Planets passing within $a_{\text{coll}} = R_p + R_\ast$ of a star will physically collide with it, resulting in a merger. This therefore sets an absolute lower limit on the period of a tidally captured planet. For gaseous objects supported only by degeneracy pressure (including giant planets and brown dwarfs), $R_p \sim 0.1R_\odot$, roughly independent of mass. For $R_\ast \sim R_\odot$, $a_{\text{coll}} \simeq 0.005\text{AU}$, or a period of $P \simeq 0.13\text{d}$. This also sets a lower limit on the mass of a planet that can be captured, as very low-mass planets must pass within the collision separation in order to raise a sufficient tide to capture. This limit is found by equating $a_{\text{coll}}$ with $a_{\text{cap}}$:

$$q \gtrsim \frac{R_* \sigma^2}{GM_\ast} \left(1 + \frac{R_p}{R_*}\right)^6,$$

(8)

where I have assumed $q \ll 1$. For the fiducial parameters, this is $q \gtrsim 2 \times 10^{-5}$.

Low-mass planets may also be tidally captured sufficiently close to their parent stars that their radii exceed the Roche limit. The limiting separation for Roche lobe overflow is,

$$a_R = \left(\frac{3}{q}\right)^{1/3} R_p \simeq 1.44 \left(\frac{q}{10^{-5}}\right)^{-1/3} R_\odot,$$

(9)

where, for the last relation, I have assumed $R_p = 0.1R_\odot$. The Roche limit separation exceeds $a_{\text{coll}}$ for $q \gtrsim 2 \times 10^{-3}$. Planets that are captured within their Roche limit will lose mass to their parent star. This mass loss is accompanied by a transfer of angular momentum, resulting in an outward migration of the planet. Thus a planet initially captured inside its Roche limit can still result in a stable system. The lower the mass of the planet, the farther it must migrate to halt mass transfer. Very low mass captured planets may not survive at all.

Planets captured into very close orbits will quickly circularize, on a timescale of $\sim 10^3\text{yr}$ (Rasio et al. 1996), and thereafter be subject to tidal decay. The timescale for tidal decay is (e.g. Rasio et al. 1996),

$$t_{\text{dec}} = \frac{f M_\ast}{t_c M_\ast} q(1 + q) \left(\frac{R_\ast}{a}\right)^8,$$

(10)

where $M_\ast$ is the mass of the convective zone; for solar-type stars, $M_\ast \sim 0.02M_\odot$. Here $t_c$ is the convective eddy turnover timescale, which I will approximate as $t_c \sim [M_\ast0.2R_*^2/3L_*]^{1/3}$, where $L_*$ is the luminosity of the star (see, e.g., Rasio et al. 1996). For solar-type stars, $t_c \sim 20\text{d}$. Thus for tidally-captured planets, $P \ll t_c$, and the largest eddies can no longer contribute to the total viscosity, resulting is a suppression of the viscous dissipation. Therefore $f$ is less than unity. However, the correct form for $f$ remains controversial (see Goodman & Oh 1997 for a discussion). Following Sasselov (2003), I will consider both linear suppression of the eddy viscosity (Zahn 1989),

$$f_L = \left(\frac{P}{2t_c}\right)^2, \quad P \ll t_c,$$

(11)

as well as quadratic suppression (Goldreich & Keeley 1977),

$$f_Q = \left(\frac{P}{2\pi t_c}\right)^2, \quad P \ll t_c.$$

(12)

Although quadratic suppression is theoretically better motivated, linear suppression appears to be in better agreement with the observed timescale of tidal circularization in close binaries (Goodman & Oh 1997).

For tidally-captured planets with $a_{\text{cap}} \sim 2R_\odot$, the decay timescale for linear suppression is $t_{\text{dec}} \sim 0.1\text{Gyr}$, whereas for quadratic suppression, $t_{\text{dec}} \sim 80\text{Gyr}$. Therefore the choice of prescription has an enormous effect on the number of surviving tidally-captured planets: one expects a negligible number of surviving planets for linear suppression, whereas planets are stable over much longer than $\sim 10^9\text{Gyr}$ for quadratic suppression.

The angular momentum lost from the tidal decay of the planet’s orbit will be transferred to the star, spinning it
The exact number will depend on the balance between the timescales of these various dynamical effects. This will in turn depend on the dynamical properties of the cluster at all times in its evolution, and thus the processes of relaxation, mass segregation, mass loss, etc., should all be considered. Ideally, the most robust way to accomplish this is through detailed N-body simulations. However, such a study is outside of the scope of this paper. Furthermore, such a detailed study is perhaps not warranted, as the number of tidally captured planets will also depend critically on the initial frequency and distribution of planets formed around stars in the cluster. Given that a general theory of planet formation is lacking, this can only be a wild guess at best. I therefore simply provide only a crude estimate for the frequency of tidally-captured planets. This estimate should be good to an order-of-magnitude, and should serve to elucidate the dependence of the frequency on the gross parameters of the stellar system and the input assumptions.

I consider only average properties of the stars and planets in the stellar system. The dynamical properties of the stellar system are specified by the average number density $\nu$ and velocity dispersion $\sig$ inside the half-mass radius, and the average mass of the stars, $\langle M \rangle$. The total number is then set by equations (3) and (4). Unless otherwise stated, I assume $\langle M \rangle = 0.5M_\odot$, which is roughly appropriate for mass functions observed in the field, young clusters, open clusters, and globular clusters (see Chabrier 2003). Here and throughout, I adopt radii and bolometric luminosities from Allen (1976) and convective zone masses from Pinsonneault, DePoy, & Coffee (2001).

For my fiducial calculations, I assume that every star in the system originally has four companions with mass $M_p = M_{\text{Jup}}$, with a distribution that is uniform in $\log a$ between 5 and 50AU. This choice is primarily motivated by the distribution of massive planets in our own solar system, but is not inconsistent with the distribution of extrasolar planets detected via radial velocity surveys. These surveys are only just becoming sensitive to planets around solar-type stars at 5AU; however extrapolations based on current samples indicate that the fraction of stars with Jupiter-mass planets at periods longer than this may be quite large (e.g., Lineweaver & Grether 2003). For the same distribution of $a$ but other choices for the frequency of massive planets, one can simply scale all results by $\langle N_p/4 \rangle$, where $N_p$ is the average number of planets per star.

Free-floating planets in stellar stellar are depleted via evaporation and replenished by disruption of bound planets. The rate of change in the number $N_{\text{ff}}$ of free-floating planets per star is thus

$$\frac{dN_{\text{ff}}}{dt} = \frac{dN_{\text{dis}}}{dt} - \frac{dN_{\text{evap}}}{dt}. \tag{15}$$

Here $dN_{\text{dis}}/dt$ is the rate at which planets are ionized,

$$\frac{dN_{\text{dis}}}{dt} = \frac{d}{dt} \left[ \int_0^\infty \frac{dN_p}{d\log a} P_{\text{dis}}(a) d\log a \right]. \tag{16}$$

Here $dN_p/d\log a$ is the initial distribution of planetary companions, which is constant between 5 and 50 AU and

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2 I am indebted to I. Bonnell for pointing this out.

3 Free-floating planets are also depleted via captures, but the rate of captures is generally so small that this can safely be ignored.
zero otherwise, and \( P_{\text{dis}} = 1 - e^{-t/t_{\text{dis}}(a)} \) is the probability that a planet at separation \( a \) will be disrupted. The rate at which planets are evaporated is

\[
\frac{dN_{\text{vap}}}{dt} = \frac{\xi_c N_{\text{ff}}}{t_{\text{rlx}}},
\]

where \( \xi_c = 0.156 \) is the evaporation probability appropriate for test particles [Spitzer 1957]. The number of free-floating planets as a function of time can be found by integrating eq. (17) from \( t = 0 \).

Tidally-captured planets are depleted via orbital decay and replenished via new captures of free-floating planets. The rate of change in the number \( N_{\text{sur}} \) of tidally-captured planets per star is thus

\[
\frac{dN_{\text{sur}}}{dt} = \frac{dN_{\text{cap}}}{dt} - \frac{dN_{\text{dec}}}{dt} = \frac{N_{\text{ff}}}{t_{\text{enc}} - t_{\text{dec}}} - \frac{N_{\text{sur}}}{t_{\text{dec}}},
\]

I determine the mean capture cross section by averaging the width of the region between \( a_{\text{cap}} \) and \( a_{\text{coll}} \), which depends on the relative velocity between the planet and star, over a Maxwellian (relative) velocity distribution with dispersion \( \sigma \). I then evaluate \( t_{\text{enc}} \) for this cross section, and \( t_{\text{dec}} \) for the average semi-major axis of the captured planets. The number of surviving free-floating planets is then found by integrating eq. (18).

Note that I have ignored the effects of mass segregation. Mass segregation will decrease the rate at which planets are captured, as it will partially decouple the dynamical interactions between the stellar and free-floating planetary populations. This will generally be a relatively small correction, as most planets are tidally captured during the first few relaxation times of the cluster.

Figure 2 shows the resulting evolution of the planetary population for three types of stellar systems, characterized by the parameters \( \nu \) and \( \sigma \). These are \( \nu = 10^4 \, \text{pc}^{-3} \) and \( \sigma = 10 \, \text{km s}^{-1} \), appropriate to a globular cluster, \( \nu = 10^3 \, \text{pc}^{-3} \) and \( \sigma = 1.5 \, \text{km s}^{-1} \), appropriate to a rich open cluster, and \( \nu = 10^2 \, \text{pc}^{-3} \) and \( \sigma = 0.6 \, \text{km s}^{-1} \), appropriate to a loose association. These systems have a total number of stars and half-mass radii equal to \( N \sim 7 \times 10^5, r_{\text{hm}} \sim 2 \, \text{pc} \) (globular cluster), \( N \sim 7600, r_{\text{hm}} \sim 1 \, \text{pc} \) (rich open cluster) and \( N \sim 1500, r_{\text{hm}} \sim 1.2 \, \text{pc} \) (association). These values are not meant to be definitive, but merely span the interesting range of possible systems. Note that \( r_{\text{hm}} \) does not enter into the calculations directly, and \( N \) enters only logarithmically via the Coulomb logarithm.

Due to their high stellar density and velocity dispersions, globular clusters generally have relatively small disruption timescales and long relaxation timescales. Therefore, planets are liberated from their parent stars very quickly, and evaporation is slow, making the dynamics particularly simple: the number density of free-floating planets is roughly constant, and \( d \log N_{\text{sur}} / d \log t \) is approximately constant when tidal decay is negligible. At the typical globular cluster lifetime of 10 Gyr, \( N_{\text{sur}} \sim 0.1 \% \).

The evolution of rich open clusters is more complicated due to their short relaxation times. From inspection of Figure 2, it is clear that the majority of planets are tidally captured within a time \( t_{\text{rlx}}/\xi_c \sim 0.2 \, \text{Gyr} \), after which the number of surviving planets is approximately constant at \( N_{\text{sur}} \approx 0.03 \% \). The lifetime of open clusters is uncertain, but rich clusters have been observed with ages of a Gyr or more [Burke et al. 2003; Kalirai et al. 2001]. Less massive clusters are probably dissolved on shorter timescales.

Loose associations have even shorter relaxation times, but are otherwise similar to open clusters. However one important difference is that, because of their smaller velocity dispersions, planets are captured at larger separations. Due to the extremely strong dependence of the tidal decay rate on the semi-major axis, this has a profound effect on the survival of tidally-captured planets for the linear prescription for the suppression of eddy viscosity: even under such a prescription, one expects a significant number of surviving tidally-captured planets at a Gyr or less. Loose associations are likely to disperse fairly rapidly, on timescales of a few hundred million years or less. At 10 Gyr, \( N_{\text{sur}} \approx 0.005 \% \).

4. Brown dwarfs and planets in 47 tuc

Because of their high encounter rates and large numbers of stars, globular clusters are expected to contain a large number of systems formed via tidal capture. Using the results from the previous section, a system with \( \nu \sim 10^4 \, \text{pc}^{-3} \) and \( \sigma \sim 10 \, \text{km s}^{-1} \) should have
\( N_{\text{sur}} = 7 \times 10^3 \times 0.001 \sim 700 \) systems of tidally-captured planets. This process is not limited to planets stripped from their parent stars, of course: any planets and brown dwarfs formed in isolation will also be tidally captured by stars in globular clusters. The initial mass function of single objects is observed to be approximately flat down to well below the hydrogen burning limit in many young clusters (Chabrier 2003, but see Briceño et al. 2002). Such a mass function implies a similar number density of stars and brown dwarfs. If this mass function is universal then globular clusters should also contain a large number of stars with tidally-captured BD companions (Bonnell et al. 2003).

Because of their extremely close orbits, tidally-captured planetary and BD companions have a very high probability (\( \gtrsim 25\% \)) of transiting their parent star, making photometric monitoring an ideal way of detecting such companions. Gilliland et al. (2000) monitored the globular cluster 47 Tuc continuously for \( \sim 8 \) d using the Hubble Space Telescope, but found no transits. This null result can be used to place constraints on the initial frequency of planetary companions as well as freely-floating planets and BDs formed in isolation (Bonnell et al. 2003).

Figure 3 shows the expected number of tidally-captured companions per star in 47 Tuc as a function of the mass of the companion, for both linear and quadratic prescriptions for viscosity suppression.

I have adopted parameters appropriate to 47 Tuc, namely \( \nu = 10^4 \) pc\(^{-3}\), \( \sigma = 12 \) km s\(^{-1}\), \( r_{\text{hm}} = 3.9 \) pc, \( \langle M \rangle = 0.56 M_\odot \) and an age of 13 Gyr (Djorgovski 1993, Gebhardt, Pryor, Williams, & Hesser 1993, Paresce & De Marchi 2000). Note that here I do not assume eq. (4). Figure 3 assumes an initial frequency of one object (planet or brown dwarf) per star. Due to the extremely high disruption rate, these results are essentially independent of whether the objects are initially free-floating, or bound on orbits with \( a \gtrsim 1 \) AU. For the quadratic prescription for viscosity suppression, the expected frequency of tidally-captured BD (0.05\( M_\odot \)) companions is \( N_{\text{sur}} \sim 0.002 \) companions per star, whereas the frequency of tidally-captured Jupiter-mass planets is \( N_{\text{sur}} \sim 0.0002 \).

Gilliland et al. (2000) claim that, if \( f = 0.8 - 1.0\% \) of the 34,091 main-sequence stars they monitored had 1.3\( R_{\odot} \) companions at periods of 3.5 d, then they should have seen 17 planets. This implies a 95\% confidence level (c.l.) upper limit of \( 3f/17 = 0.14 - 0.18\% \) to the fraction of main-sequence stars with such companions. This null result can also combined with the prediction for the number of tidally-captured companions \( N_{\text{sur}} \) to determine an upper limit to the initial frequency \( N_p \) of objects per star of

\[
N_p \leq \frac{3f}{17 N_{\text{sur}}} \frac{\mathcal{P}_T(3.5d)}{\mathcal{P}_T(3.5d)}. \tag{19}
\]

Here \( \mathcal{P}_T = (R_p + R_*)/a \) is the transit probability. Figure 3 shows the resulting 95\% c.l. upper limit to the initial frequency of objects as a function of the mass of the object. The shaded region comes from varying \( f \) in the range \( 0.8\% \leq f \leq 1.0\% \).

The null result implies that, on average, each star in 47 Tuc originally formed \( \leq 1 \) planet with mass \( M_p \geq 5M_{\text{Jup}} \), or \( \leq 0.4 \) planets with \( M_p \geq 5M_{\text{Jup}} \). Less than 50\% of stars in 47 Tuc could have formed solar-system analogs.

While this limit is considerably weaker than the original limit placed by Gilliland et al. (2000) of \( \sim 0.1\% \), it is important to emphasize that this original limit applies only to planets in closed (\( \sim 3 \) d) orbits. Since planets in such small orbits were not formed in situ, and must be the result of migration, drawing conclusions about the formation of planets at larger separations based on the lack of planets at small separations is subject to uncertainties in the timescale for orbital migration. If the migration timescale were greater than or of order the disruption timescale, the planets would have been stripped from their parent stars before they had a chance to migrate sufficiently close to be detectable via transits. In contrast, the limit on the frequency of planets implied by the lack of tidally-captured planets, although weaker, is essentially independent of where they were initially formed. In fact, one can conclude the initial number of free-floating planets (planets formed in isolation via gas fragmentation and collapse) is also less than or equal to the number of stars.

Under a similar set of assumptions as adopted here,
but using a more sophisticated simulation and neglecting orbital decay, [Bonnell et al. (2003)] found an upper limit to the BD frequency in 47 Tuc of ~ 15%. Figure 3 shows the upper limit derived using the formalism in [Bonnell et al. (2003)] but neglecting orbital decay. I find 12%, in good agreement with Bonnell et al. (2003). This suggests that, in the absence of orbital decay, the predicted frequency of tidally-captured BD companions should be reasonably robust.

For quadratic suppression of eddy viscosity, the timescale for orbital decay of tidally-captured BD (q ~ 0.1) companions is $t_{\text{dec}} \sim 9$ Gyr. Therefore, approximately $\exp(-9/13) \sim 50\%$ of the captured BD will still be present today. Thus tidal decay is significant at the current age of 47 Tuc for tidally-captured BD companions. Figure 3 shows the upper limit including orbital decay. The limit is revised upward by ~ 2 relative to the estimate of Bonnell et al. (2003). The lack of transiting BD companions in 47 Tuc implies that the initial frequency of BDs relative to stars is $\lesssim 25\%$.

The upper limit of ~ 25% on the relative frequency of BDs in 47 Tuc is inconsistent with the observed mass functions of most young clusters and the field, for which BDs and stars exist in equal numbers [Chabrier 2003], although it is consistent with the frequency of BDs in the Taurus star-forming region [Briccio et al. 2002]. At face value, this is an indication that the initial mass function compact objects is not universal [Bonnell et al. 2003]. Unfortunately, this conclusion relies heavily on the assumed value for the tidal decay timescale. Different assumptions for the mechanisms for tidal dissipation will give rise to radically different conclusions. For example, in Figure 3 I show the predicted frequency of tidally-captured companions under the assumption of linear suppression of eddy viscosity. The frequencies are always $\lesssim 3 \times 10^{-6}$, i.e. $\lesssim 5$ in the entire cluster. This low number results from the fact that the tidal decay timescale under linear suppression is smaller by a factor of $\sqrt{Q} \sim 10^{-4}$. Thus $t_{\text{dec}} \approx 4 \times 10^{6}$ yrs, and the frequency of tidally-captured objects is only $\sim t_{\text{dec}}/t_{\text{enc}} \sim 10^{-6}$. Obviously the resulting upper limits are not very interesting. Even if one accepts that the quadratic suppression of eddy viscosity is correct, there are still some ambiguities. For example, there exist discrepancies in the quoted form for $f_{Q}$: Goodman & Oh (1997) and Sasselov (2003) use the form adopted here, namely $f_{Q} = (P/2\pi t_{e})^{2}$, but Rasio et al. (1996) adopt $f_{Q} = f_{Q}\pi^{2}$, i.e. larger by almost an order of magnitude. This factor translates directly to the implied upper limit on the relative frequency, weakening the constraint considerably (see Fig. 3). Note that this would also weaken the constraint on planets in 47 Tuc, but by a smaller factor than the constraint on BDs.

Thus, given the current uncertainties in the physics of tidal dissipation, I conclude that no robust inferences can be made about the frequency of brown dwarf companions in 47 Tuc, and any conclusions regarding the universality of the initial mass function are probably premature.

5. Implications and Prospects for Detection

It is clear that, under certain sets of assumptions, extremely close-in tidally-captured massive planets should exist around stars formed in dense stellar systems. What are the prospects for the detection of such companions, and what would be the implications of any such detections?

Given their extremely close orbits, the most promising method for detecting companions is transits. For a planet captured at an orbit of $a \lesssim 2R_{\odot}$, the transit probability is $P_{T} \gtrsim 50\%$. The duration of the transit is

$$t_{T} = \frac{P}{\pi} \arcsin \left( \sqrt{\frac{P^{2}}{2} - \cos^{2} i} \right),$$

(20)

where $i$ is the inclination. Thus for a central transit, the duty cycle (fraction of time in transit) is quite large: $t_{T}/P \sim 15\%$. For a solar-type primary, the transits will last ~ 50 min and recur every ~ 5 hours. The transit depth will be ~ 1%. Given the extremely close proximity of the planet to the star, one might worry that the planet will induce additional photometric variations on the star. The reflected light from the companion will induce an fractional flux variation of order $\Delta F/F \sim p(R_{p}/a)^{2} \sim 2 \times 10^{-3}$, where $p$ is the geometric albedo, and I have adopted $p = 2/3$ as appropriate for Lambert sphere scattering. Although five times lower than the signal from the transit, a signal of this magnitude may be large enough to be detected. In fact, depending on the selection criterion used to choose targets for the transit search in 47 Tuc, this may be cause for concern in interpreting this result. The planet will also induce photometric variations of the star from tidal and Doppler effects, but these will likely be small for planetary companions [Loeb & Gaudi 2003]. They may, however, be significant for more massive companions. Thus, barring unforeseen effects that induce additional photometric variability, it should be possible to cleanly detect tidally-captured planets via transits.

Because of their short periods and relatively large duty cycles, the observational requirements of a transit search for tidally-captured planets are generally much less severe than transit searches for more distant planets. In particular, complete coverage of several phases can be achieved in less than a few days. This is especially important for ground-based transit searches, as the coherence time of weather patterns is typically several days. This proves to be devastating when searching for planets with periods of order the coherence time, but nearly optimal for very short periods. Furthermore, aliasing is minimized because the period of the planet is typically less than the duration of a typical clear observing night.

Tidally-captured planets will also induce radial velocity variations in the host star, of amplitude $K \approx 340$ m s$^{-1}$ (5 hr/$P$)($M_{p}/M_{\odot}$). For reasonably bright sources, this is well within reach of current instrumentation. However, the presence of such a close planet may induce additional radial velocity variability, making detection of the planetary signal difficult. Furthermore, the extremely low expected frequency of tidally-captured planets makes this an extremely inefficient method of detection, unless multi-object spectrographs capable of precision radial velocity measurements become available, so that many stars can be monitored simultaneously.

The systems that offer the best chance of observing tidally-captured planets are globular clusters, because they have the highest frequencies of captured planets ($\sim 0.025 - 0.1\%$), and the largest total number of systems. Furthermore, due to the large velocity dispersions,
planets are captured into very close orbits. This increases the a priori probability that the planet will transit its parent stars to nearly unity. In §3 I considered in detail frequency and detectability of tidally-captured planets in 47 Tuc. Although no transiting planets were found in this cluster, it may still be worthwhile to search in other clusters. An observing campaign with the Hubble Space Telescope need not be as ambitious as that toward 47 Tuc.

The expected number of transiting, tidally-captured planets in a rich open cluster is only of order unity. Therefore, the prospects of detecting such planets in transit searches toward open clusters (Burke et al. 2003, Street et al. 2003) are poor, unless many massive planets are formed per star.

Loose clusters are subject to fairly rapid disruption, after which their stars disperse into the general disk population. It is possible that a large fraction of stars in the disk were initially born in such environments. Assuming a typical lifetime of a loose cluster of ~ 300 Myr, then ~ 0.005% of stars in the field should have a tidally-captured planet, if all stars in the field were born in associations, and planet formation were ubiquitous. Accounting for a binary fraction of 50%, and assuming a transit probability of ~ 40% for planets captured in loose clusters, I estimate that ~ 100,000 stars need to be monitored to have a chance of detecting even one transiting tidally-captured planet. OGLE monitored 5 million stars toward the Galactic bulge over 45 nights to search for transiting planets to disk stars (Udalski et al. 2002). They only analyze a subset of ~ 52,000 of these stars with photometry better than 1.5%. Similarly, the EXPLORE project has monitored ~ 350,000 stars in the Galactic plane for 11 nights (Mall´ en-Ornelas et al. 2003). They achieved better than 1% photometry for a subset of 37,000 stars, which they visually inspected for transits. In both cases, an insufficient number of stars were searched to place interesting constraints on the fraction of tidally-captured planets. However, because of the large number of expected transits, it should be possible to detect transiting planets even when the scatter is larger than the expected signal. Therefore, the optional (but time-consuming) approach would be to search all light curves for transiting planets. For such a photon-limited survey, the detection probability scales as $P^{-5/3}$ (Pepper, Gould & DePoy 2003), favoring extremely close-in tidally-captured planets.

It is important to reiterate that all of these frequency estimates rely on the assumption that the tidal decay timescale is longer than ~ 1 Gyr, and thus that the quadratic prescription for the suppression of eddy viscosity is approximately correct. The only exception is planets captured in loose associations. Such planets are captured sufficiently far from their parent stars that they are stable over ~ Gyr timescales for both linear and quadratic prescriptions. However, for ages much larger than 1 Gyr, the orbits will rapidly decay under the linear prescription (see Fig. 2). The detection of a planet on an extremely close orbit would therefore have important implications for the tidal dissipation theory.

How can tidally-captured planets be distinguished from planets that migrated to such close orbits? The period distribution of radial-velocity planets shows a pile-up at $P \sim 3$ d, with a significant lack of planets with smaller periods. Tidally-captured planets would be easily identified in such a distribution. Kuchner & Lecar 2002 suggest that this distribution arises from the fact that the centers of protoplanetary disks are evacuated interior to separations corresponding to periods of 6 d, and that planetary migration thus halts when the planet’s outer Lindblad resonance reaches the inner disk edge. Although this seems to provide a compelling explanation for the observed period distribution of radial velocity planets, the recent detection of a transiting planet at $P = 1.2$ days (Konacki et al. 2003) generally suggests that this picture cannot be universal (Sasselov 2003). Indeed, Trilling et al. 1998 suggest another mechanism for halting migration: planets migrate until they reach their Roche limit, at which point they lose enough mass to the parent star to balance inward migration. If disk dispersal occurs in a sufficiently short time, the planets will be left intact in short-period orbits. Such planets would generally have similar periods as tidally-captured planets. Therefore, additional diagnostics are needed.

One possible method of distinguishing tidally-captured planets from migrating planets is a radius measurement. Both known transiting planets have radii significantly larger than that of Jupiter. It has been suggested that these large radii are due to the fact that these planets migrated on a sufficiently short timescale that their gravitational contraction was retarded, leaving them in a permanently inflated state (Burrows et al. 2001). If this process is universal, then we can expect migrated planets to have significantly larger radii than tidally-captured planets, which could not have migrated far from their birthplaces. This assumes that there are no processes that can inflate planets once they have been tidally captured. In principle, variable tidal forces from the star on a planet in an eccentric orbit can deposit enough energy to inflate the planet (Gu, Lin, & Bodenheimer 2003), but to have a significant effect, there must be some external force continuously pumping the eccentricity, since the circularization timescale for extremely close-in planets is very short.

6. Summary

Planets which form around stars born in dense stellar environments are subject to dynamical perturbations from other stars in the system. These perturbations will generally serve to strip outer planets from their parent stars, leading to a significant population of freely-floating planets in the system. Further interactions with stars in the system drive this planetary population toward equipartition, thus raising the velocity dispersion of the planets, ultimately resulting in escape from the system. However, some planets will survive long enough to be pass sufficiently close to another star to be tidally captured.

The stability of tidally-captured planets against orbital decay depends critically on the physics of viscous dissipation of the tide in the convective envelope of the star. This processes is extremely uncertain, and different prescriptions lead to decay timescales that differ by four orders of magnitude. For quadratic suppression of eddy viscosity, planets on tidally-captured orbits will generally be stable for $\gtrsim$ Gyr, whereas for linear suppression, planets will decay very quickly.
The frequency of tidally-captured planets depends on the competing effects of dynamical stripping, evaporation, capture, and orbital decay. The timescales for these effects depend in turn on the properties of the stellar systems, particularly the velocity dispersion $\sigma$ and the number density $\nu$. Under the assumption that all stars form four Jupiter-mass planets with a uniform logarithmic distribution in semi-major axis between $5 - 50$ AU, and assuming quadratic suppression of eddy viscosity, I have estimated the frequency of tidally-captured planets in a stellar system as a function of the age, velocity dispersion, and number density of the system.

For loose associations with $\sigma \sim 0.6$ km s$^{-1}$ and $\nu \sim 10^2$ pc$^{-3}$, the frequency of tidally-captured planets after a typical cluster lifetime of $10^8$ years is $\sim 0.005\%$. If most stars formed in such systems, this is roughly the expected frequency of extremely close-in planetary companions to stars in the field. These planets may be found by deep, wide-angle searches for transiting planets around disk stars [Mallén-Ornelas et al. 2003; Udalski et al. 2002]. Detection of such a population would imply both that most stars were formed in dense stellar environments, and that the timescale for orbital decay is $\gtrsim$ Gyr.

For rich open clusters ($\sigma \sim 1.5$ km s$^{-1}$ and $\nu \sim 10^4$ pc$^{-3}$), the frequency of tidally-captured planets is $\sim 0.03\%$. Unfortunately, the prospects for detecting such a population of planets in a given cluster are poor, as $\lesssim 1$ transiting planet per cluster is expected.

Due to the long relaxation times and high encounter rates, the frequency of tidally-captured planets in a typical globular cluster ($\sigma \sim 10$ km s$^{-1}$ and $\nu \sim 10^4$ pc$^{-3}$) is $\sim 0.1\%$. Therefore, one expects several hundred transiting planets in each globular cluster.

The relatively high frequency of tidal capture in globular clusters makes them excellent targets to search for such systems. The null result of the search for transiting planets in 47 Tuc can be used to place interesting constraints on the initial frequency of planets and brown dwarfs (either initially free-floating or bound) relative to stars. I find that $\lesssim 1$ planet with mass $\gtrsim M_{\text{Jup}}$ originally formed per star in 47 Tuc. Less than 50% of stars formed solar-system analogs. The initial frequency of brown dwarfs relative to stars is 25%. All of these conclusions are under the assumption of quadratic suppression of eddy viscosity. If viscosity suppression is much less efficient, no interesting constraints can be placed.

Tidally-captured planets will have orbital periods of $\sim 0.1 - 0.4$ d, and can be distinguished from migrating planets from their distribution of orbital periods provided the mechanism that is observed to halt planetary migration at $P \sim 3$ d is robust. However, if the recent discovery of a planet at $P \sim 1$ d is indeed indicating that there exist multiple mechanisms for halting migration, then one cannot rule out the hypothesis that an observed extremely close-in planet was the result of orbital migration. However, the radii of tidally-captured planets are expected to be similar to their radius at their sites of formation, and thus considerably smaller than planets that migrated quickly to close orbits. Thus a radius measurement might provide a diagnostic for tidally-captured planets.

If the majority of stars is formed in dense stellar environments, then most planetary systems will be subject to a variety of dynamical effects that can alter planetary formation, migration, and survival. Therefore, planetary systems cannot be understood as isolated systems, and the interpretation of the observed properties of extrasolar planets will require the consideration of these effects. This study highlights one example of how such considerations can lead to new observable diagnostics of planetary formation.

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