Finite element analysis of the dynamic interaction between a single abrasive grain and a glass surface

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Abstract. The article presents the results of 2D and 3D formulation finite element analysis of glass surface scratching process using single abrasive grain. Johnson-Holmquist model JH-2 was chosen to describe the brittle behavior of the glass material. The cracked layer depth for a given depth penetration of grain and scratching speed were obtained.

Existing technological processes of optical glass machining are very laborious due to the special chemical, mechanical, physical properties of the material. It should also be borne in mind that the machining of optical glass is accompanied by the formation of a defective surface layer. When grinding brittle materials, surface irregularities representing the roughness $h_r$ are always accompanied by a volumetric network of cracks propagating into the depth of the glass from the level of the cavities (Fig. 1).

Fig. 1. The surface structure of brittle material formed during grinding.

This $h_{cr}$ zone can be called the “cracked” layer. The ratio of the depth of the damaged layer $H$ to the height of the roughness $h_r$, does not depend on the grade of glass, the abrasive grain size, and for this method of machining is constant: 4.0 and 2.7 - when grinding is loose and bonded abrasive grain, respectively[1]. Minimizing the $H$ layer in grinding operations leads to a significant reduction in the complexity of manufacturing and, as a result, cost reduction.

An experimental study of this process requires a large investment of time and resources, and also gives a rather large scatter of the data obtained. Therefore, numerical modeling of the glass grinding process is of great interest.

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In the article, the problem of modeling the process of scratching of a glass sample with a single abrasive grain at a speed of 5 m/s and penetration depth of 10 μm was considered. The schematic for model is shown in Fig.2.

**Fig. 2.** Schematic for model.

One of the main tasks in modeling is the choice of material model, since it should reflect aspects such as brittle fracture of the material and the propagation of cracks. To model the reaction of the glass material, the Johnson-Holmquist material model JH-2 is used.

The material model JH-2 [4] is intended to describe the reaction of brittle materials (glass, ceramics) under large and high-speed deformations. Due to the presence of a damage parameter in this model and the dependence on the strain rate, the model has found wide application in the study of such problems as impact on ceramics [6], glass grinding [2, 3], etc.

The mathematical formulation of the Johnson-Holmquist model is represented by expression (1). In the initial state, the material is assumed to be isotropic and elastic, so that the stresses $\sigma_{ij}$ are associated with strains $\varepsilon_{ij}$ by Hooke's law:

$$\sigma_{ij} = p(\xi) \delta_{ij} + 2\mu \varepsilon_{ij}, \quad (1)$$

where $\delta_{ij}$ is the Kronecker symbol, $\mu$ is the shear modulus, and pressure $p$ is a function of volume compression:

$$p(\xi) = \frac{\rho}{\rho_0} - 1, \quad (2)$$

where $\rho_0$ and $\rho$ are the initial and current density of the material, respectively.

The specific form of this function is determined by the equation of state, which in the Johnson-Holmquist model has a polynomial form:

$$p(\xi) = \begin{cases} K_1 \xi + K_2 \xi^2 + K_3 \xi^3 + \Delta p, & p > 0 \text{ (compression)} \\ K_1 \xi, & p < 0 \text{ (tension)} \end{cases}, \quad (3)$$

where $\Delta p$ is the increment of pressure calculated at each time step associated with the loss of internal (elastic) energy during the increment of the damage parameter introduced below, $K1$, $K2$, and $K3$ of the material constant.

The equivalent stress for a ceramic-type material is given by:

$$\sigma^* = \sigma^*_i - D(\sigma^*_i - \sigma^*_f), \quad (4)$$
where the parameter $D$ is determined by the formula:

$$D = \sum \Delta \varepsilon \frac{p}{\varepsilon_f^p},$$

and represents the accumulation of damage due to the increment of plastic deformation in one cycle of calculations. Plastic deformation during cracking is equal to:

$$\varepsilon_f^p = D_i (p^* + T^*)^{D_1},$$

where $p^*$ and $T^*$ are the normalized values of pressure and maximum pressure (tensile strength $T_{\text{max}}$ under hydrostatic tension), respectively. Parameter $D_1$ controls the rate of damage accumulation. If it is equal to zero, complete destruction occurs in one counting step, i.e. instantly.

The normalized equivalent stresses for the initial (intact) and completely damaged materials are represented by the formulas:

$$\sigma_i^* = A (p^* + t^*)^N (1 + C \ln \dot{\varepsilon});$$

$$\sigma_f^* = B (p^*)^M (1 + C \ln \dot{\varepsilon}),$$

where $A$, $B$, $C$, $N$ and $M$ are material parameters; the asterix '*' indicates a normalized value, the stresses are normalized using the equivalent stress and the elastic Hugoniot limit (HEL):

$$\sigma^* = \frac{\sigma}{\sigma_{\text{HEL}}}, \quad p^* = \frac{p}{p_{\text{HEL}}}, \quad T^* = \frac{T}{T_{\text{HEL}}}. $$

The value $\dot{\varepsilon}$ is the ratio of the equivalent strain rate to the reference speed (usually unit), i.e. . As can be seen, the Johnson-Holmquist model provides an explicit dependence of the deformation parameters, including the strain rate.

The goal of any material model is to adequately describe the response of the material under various types of loading. But in developing material models for numerical methods, it is also necessary to seek a compromise between modeling a plausible response and the computational efficiency of its application. For example, it is well known that damage in the glass begins in the form of small cracks that grow and merge, forming faults or crushed material [5]. However, numerical modeling of the onset and development of damage is very costly and to some extent not necessary. Damage in the JH-2 model is presented as a state parameter corresponding to the average damage of the material in the volume of the finite element. This damage changes with deformation and leads to a decrease in stiffness. Thus, material stiffness and damage are functions of pressure at a particular point in the material.

It is believed that at first the response of the material is elastic and its stress state is completely described by the elastic properties of the material (shear modulus) and the equation of state. When compressed, damage begins to accumulate when the deviator stresses exceed a critical value. Damage accumulation is monitored through a special parameter that varies from 0 to 1. The stiffness of the material for each element is determined by the curves of undamaged and damaged material through the value of the current damage. When tension occurs, the material behaves elastically up to brittle fracture at a certain value of stress intensity.

The material model takes into account the effects of the strain rate, but it was noted that these effects are, as a rule, secondary in comparison with the effects of pressure [6].
has been observed experimentally and is reflected in typical values for constants in material models.

Since severe deformation of the finite elements can lead to a sharp decrease in the time step, the condition for the erosion of elements in which the plastic deformation exceeds 200% was set.

The parameters of the JOHNSON_HOLMQUIST_CERAMICS material model for glass, experimentally determined in [6], are presented in the Table 1.

Table 1. Johnson - Holmquist parameters for glass.

| Parameter | \(\rho\) (kg/m\(^3\)) | \(G\) (GPa) | A | B | C | M | N | T (GPa) |
|-----------|----------------|-------------|---|---|---|---|---|---------|
| Value     | 2530           | 30.4        | 0.93 | 0.088 | 0.003 | 0.35 | 0.77 | 0.15    |

The finite element method (FEM) in the Lagrange formulation was used due to the fact that it allows to take into account a wide range of parameters that affect the result of simulation, so it has found quite wide application in solving such problems [2, 3]. The main disadvantage of this method is its cost in computing resources when modeling processes at the microscopic level [3].

The results of two-dimensional modeling are presented in Figure 3. To check their adequacy, three-dimensional modeling was performed. In this case, the assumption is made that in the 3D model the material located at a distance of more than 0.4 mm from the surface of the workpiece is practically not loaded and does not affect the process of crack formation.

![Fig. 3. Distribution of the damage parameter in the glass scratched by abrasive grain in 2D modeling.](image)

To reduce CPU time, and also considering that the workpiece is most loaded only in the contact zone, the workpiece model was meshed by three regions:

1. The region of direct contact of the workpiece and the grain in which cracks form (the minimum element size is 1 \(\mu\)m);
2. The intermediate region of low loading (the largest element size is 12 \(\mu\)m);
3. The outer region with a minimum element size of 70 \(\mu\)m.

Since geometry and loads have a plane of symmetry, half of the workpiece and grain are considered. A part of finite elements mesh for abrasive grain and workpiece is shown in Fig. 4. In general, the model contains 3 837 540 nodes.

Between the grain and the workpiece contact is established taking into account the element erosion. At the bottom of the workpiece, the fixed condition is specified.
plane of symmetry the boundary condition of symmetry is given. It is accepted that the abrasive grain is a rigid body.

**Fig. 4.** Part of finite elements mesh for abrasive grain and workpiece.

Figure 5 shows distribution of the damage parameter in the glass scratched by abrasive grain in 3D modeling.

**Fig. 5.** Distribution of the damage parameter in the glass scratched by abrasive grain in 3D modeling.

The maximum depth of the damaged layer according to the results of numerical simulation was 37 μm. The results of numerical simulations show good agreement with the experimental results and theoretical data, according to which, with a grain penetration depth of 10 μm, the depth of the damaged layer of glass is 30–40 μm.

The approach based on numerical modeling allows a deeper understanding of the process of crack formation, depending on the conditions of interaction of abrasive grains with the surface of glass material. The accumulated results of simulation will allow to offer optimal processing conditions and improve the technological process of grinding brittle materials (in particular optical glass), which will lead to lower costs and higher product quality.

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