Application of fractal analysis methods in the study of deformation mechanisms and composite building materials fracture

Nizina T A, Selyaev V P, Nizin D R, Balykov A S, Korovkin D I and Kanaeva N S
National Research Mordovian State University named after N.P. Ogarev, Russia, 430005, Saransk, Bolshevistskaya, 68

nizinata@yandex.ru

Abstract. Approach for the study of tensile deformation mechanism of composite materials based on the fractal analysis use of the curves “σ-ε” is proposed in the paper. The methods for calculating fractality index using the method of the minimum coverage, its changes depend on the load duration; allow to quantitatively determining stress and strain levels corresponding to the moments of micro and macro defects formation in samples. Samples of epoxy composites produced on the basis of the modified resin, Etal-247 cured with Etal-45M, were applied for confirmation of possibility to provide quantitative determination of the “critical” points on tensile deformation curves of the polymers.

1. Introduction
Analysis of composite materials deformation and fracture mechanisms is one of the essential tasks in the field of building material science, which relevance, due to development of new composite materials based on mineral binders and polymeric binders, grows every year. It is known that the destruction of composite building materials is a multiple origins process, development, and aggregation of various defects and micro cracks until the macro cracks appear [1]. Heterogeneity of the composite materials structure leads to the weak zones formation, where loosening and fracture of composites start.

Reaching the inclusions (pores or filler) by a micro crack leads to discharge of the critical density energy to the crack mouth and transition of a system to unstable state (bifurcation point). The bifurcation point may feature branching, change of the mechanism and fracture crack direction development [2, 3]. Thus, the fracture is of probabilistic nature and the process of damage accumulation is self-similar, which makes the application of fractal analysis methods as a promising approach [2, 4, 5].

Different methods for the fractal structure dimension determination of the real composite materials with cement-based binders and polymer binders can be found in scientific literature [1, 6–12]. It is revealed that the use of the quantitative evaluation fractal analysis of the structural heterogeneity of polymeric materials and pore structure of cement composites enables to find a compact way for describing these objects [2, 7, 9].
In the work [12], the authors proposed a method for determining the fractal dimension of deformation curves on the minimum coverage method basis allowing to obtain integrated quantitative assessment of the destruction process in the building composites at compression and to locate the destruction curve parametric point. The proposed method was compared with the algorithms for determining Hurst exponent and fractal dimension by the square coating method. The methods advantage based on the determination of the fractal dimension using the method of minimum coverage is shown.

The proposed method is based on the data use obtained with the help of modern testing equipment allowing recording the stress change and deformation in the process of applying high frequency mechanical stress to the sample. In particular, hardware and software complex WilleGeotechnik® used in [12] enables to record the results with increments of second fractions.

The indexes of fractality and fractal dimension of stress increase and strain in deformation curves using the minimum coverage method were determined by the example of disperse compositions and reinforced fine-grained concretes [12]. On the basis of time series fractal analysis, the point of the studied samples transition from state of rest to the pronounced trend state was located and the area around it was analyzed. The change of the parametric point position and the fractal dimensions values depending on the kind of applied fiber was revealed.

Further, the proposed method use at analysis of the deformation curves in fine-grained concrete samples with different additives by the authors is shown the relevance and applicability for obtaining valuable information about destruction process in the composite materials of different nature. In the present work, the developed approach was used for fractal analysis of deformation and fracture mechanism of polymer composite materials under tensile stresses.

2. Equipment and materials used
Tensile machine of AGS–X series with TRAPEZIUM X software was used for tensile mechanical tests execution of the polymer composite structures. Frequency of recording stress and strain values made 0.01 sec. The tests were carried out in accordance with GOST 11262 "Plastic. Tensile test method" at temperature of 23±2 °C and relative humidity of 50±5 %. Clamps separation rate at tensile testing machine made up 2 mm/min. Not less than 6 samples having the shape of "eight" were tested simultaneously (type 2 according to GOST 11262).

Polymer composite obtained on the basis of modified epoxy resin Etal-247 (TU 2257-247-18826195-07), cured with hardener Etal-45M (TU 2257-045-18826195-01) which is a mixture of aromatic and aliphatic di- or polyamines modified with salicylic acid was chosen as the research object. Specifications of Etal-247 resin: mass fraction of epoxy groups – min. 21.4-22.8%; Brookfield viscosity at 25°C – 650-750 GPA.

3. Method for determining the fractal dimension of the polymer composites tensile deformation curves
Analysis of tensile deformation curves of the epoxy composites showed that the loading process is accompanied by discrete acts of strain rises and falls (Figure 1). Taking into account the fact that strain and stress change in polymer samples at mechanical tests over time with a certain increments (0.01 sec), a theory of time series fractal analysis can be applied for deformation curve analysis.

Let's consider the time series \( y(t) \), defined on a certain site \([a, b]\). To calculate the fractal dimension we use the minimal coverage method, which main provisions are set out in [12–15], as this method is more accurate compared to cell dimensions method. The essence of the method consists in a uniform splitting of a line

\[
\omega_m = \left[ a = t_0 < t_1 < \cdots < t_m = b \right]
\]

into \( m \) parts and counting the function \( y = f(t) \) in the coats class consisting of rectangles with the base \( \delta = \frac{b - a}{m} \) (Figure 2 [13]). Then the height of the rectangle on \([t_{i-1}, t_i]\) line is equal to the
The difference between the maximum and minimum function $f(t)$ on this segment is $K_i(\delta)$. By introducing the value of amplitude variation of $f(t)$ function corresponding to the splitting scale $\delta$ on the $[a, b]$ section

$$V_f(\delta) = \sum_{i=1}^{m} K_i(\delta),$$

we get the dependence for determining the full coverage area:

$$S_\mu(\delta) = V_f(\delta) \times \delta.$$  \hspace{1cm} (1)

Then, according to [4, 13], fractality index $\mu$ can be determined from linearized dependence

$$V_f(\delta) \sim \delta^{-\mu} \text{ at } \delta \to 0.$$  \hspace{1cm} (3)

Thus, the fractal dimension depends on the fractal dimensions inde determined by the minimal coverage method as

$$D_\mu = \mu + 1.$$  \hspace{1cm} (4)

![Figure 1](image1.png)

**Figure 1.** General view (a) and a fragment (b) of tensile deformation curve in polymer composite material samples based on the epoxy binder (maximum stress level is shown as a solid line in "a" figure, maximum strain - as dashed line).
4. Fractal analysis of tensile deformation curves of polymer composite materials

To determine the fractal dimensions index $\mu$ a series of nested segmentations where $m = 2^n$, where $n = 0,1,2,3,4$ was used in this work. Every segmentation consisted of $2^n$ intervals containing $2^{4-n}$ experimental points. For every segmentation $\omega_m$ we have calculated the amplitude variation $V_f(\delta)$ according to the formula (1), where $K_f(\delta)$ was determined as a difference between the maximum and the minimum tensile stress increase in the time interval $[t_{i-1}, t_i]$. The index of fractality and fractal dimensions of the minimum coverage was calculated by $\beta$ ratio of $\delta$ amplitude variation $V_f(\delta) = \alpha_0 + \beta \times \text{lg}(\delta)$, regression equation determined using the Least Squares Method:

$$\mu = -\beta; D_\mu = 1 + \mu.$$

Analysis of data, reflecting the fractality index change depending on the stress duration determined with displacement of the analyzed site with point increments showed (Figure 3) that the destruction process features a sharp decrease of this index, ranging during the stress from 0.35÷0.8 to 0.2. Thus, from the time series analysis [12–13], the higher $\mu$, value is the more stable the series is. If $\mu < 0.5$, series is considered as a "trend" (a period of sharp upward or downward movement which usually proves the emergence of the critical state in the system under study); if $\mu > 0.5$, then it is "flat" (a period of relative rest). $\mu \approx 0.5$ shows compliance of the occurring changes to Brownian motion.

According to Figure 3, we observe a systematic sharp decrease of the fractality index below $\mu = 0.5$ level in the stress process until the fracture, which occurred in this sample 113.16 seconds after the test start, in our opinion, it can be explained with micro damages formation under tensile stresses in the polymer composite structure. Thus, the functionality of a sample, until the fractured composite is provided mainly by transfer of stress to zones that are free of microdefects, which is expressed on the chart as increase of $\mu$ values above 0.5 level. Therefore, the analysis of Figure 3 allows determining the critical moments of stresses, which the fractality index will have the lowest values for. In this case, the time coordinates corresponding to bifurcation points occurred (at $\mu < 0.4$) in the analyzed polymer samples in: 10.3; 18.44; 19.39; 23.12; 35.5; 44.18; 56.98; 65.93; 65.94; 68.12; 71.54; 78.05; 89.44; 95.47; 102.22 seconds after the stress start. The lowest fractality index values corresponding to $\mu \approx 0.36$, were recorded in 10.3 and 71.54 sec (marked with dots in Figure 3). Thus, the analysis of stress and strain changes in the first time interval (Figure 4 (a)) has shown that the greatest reduction of fractality index is caused by abrupt increase, and for the second interval

![Figure 2. A fragment of the cell (squares) and minimal (rectangle) coverage of the fractal function graph on $[t_{i-1}, t_i]$ section.](image-url)
(Figure 4 (a)) with intensive decrease of the perceived stress level. Analysis of all other above-mentioned time points has shown that all of them, except for the first one (10.3 sec), are also associated with discharge of stresses accumulated in the sample at uniform deformation increase, which is explained by reduction of perceived stress level due to microdefects occurring in the composite structure.

Figure 3. Change of fractality index of tensile deformation curves of the epoxy composites, depending on the stress duration (displacement of 16 points).

Figure 4. Fragments of tensile deformation curve of the epoxy composites for the selected time "critical" levels (a - 10.3 sec; b – 71.54 sec) corresponding to the minimum values of fractality index ($\mu \approx 0.36$).

The proposed approach to the tensile deformation mechanism study of composite materials implemented on the basis of deformation curve fractal analysis registered with the help of the modern high frequency test equipment allows receiving valuable information about the accumulation process of micro- and macro defects in its structure, leading to fracture of composites. In our opinion, such
studies execution on polymer composite samples of different composition, as well as after aging under exposure to various aggressive factors including climate ones will provide valuable information about the processes occurring in the structure of composite materials under tensile stresses.

Acknowledgements
The work has been executed at financial support of the Russian Foundation for Basic Researches grant No. 18-08-01050.

References
[1] Ivanova V S, Balankin A S, Bunin I Z and Oksogoev A A 1994 Synergetics and Fractals in Materials Science (Moscow: Science) p 384
[2] Selyaev V P, Kupryashkina L I, Neverov V A and Selyaev P V 2015 Reg. Arch. Constr. 1 11–22
[3] Selyaev V P, Selyaev P V and Kechutkina E L 2016 Bull. Volga Reg. Bran. Rus. Acad. Arch. Constr. Sci. 19 258–68
[4] Feder E 1991 Fractals (Moscow: Mir) p 254
[5] Mandelbrot B B 1983 The Fractal Geometry of Nature (New York: Freeman) p 480
[6] Selyaev V P, Nizina T A and Lankina Yu A 2007 News of High Schools. Constr. 4 43–48
[7] Khakhardin A N and Khodykin E I 2007 Build. Mater. 8 62–63
[8] Selyaev V P, Nizina T A, Lankina Yu A and Tsyganov V V 2006 Achievements, Problems and Promising Directions for Development Building Material Science Theory and Practice. Proc. 10th Academic Readings of RAASN 73–76
[9] Pertsev V T, Kozodaev S P, Ledenev A A and Bobryshev A N 2015 Sci. Bull. Voronezh State Univ. Archit. Constr. Series: Phys. Chem. Iss. Modern Tech. Build. Mater. Sci. 1 21–28
[10] Khamidulina D D and Shishkin I V 2015 Topical Iss. Mod. Sci. Tech. Educ. 2 5–8
[11] Bannikov M V, Fedorova A Yu, Terekhina A I and Plekhov O A 2013 Bull. Perm National Res. Polytech. Univ. Mech. 2 21–36
[12] Selyaev V P, Nizina T A, Balykov A S, Nizin D R and Bablilin A V 2016 Bull. Perm National Res. Polytech. Univ. Mech. 1 129–46
[13] Starchenko N V 2005 Fractality Index and Local Analysis of Chaotic Time Series Thes. Cand. Tech. Sci. (Moscow: Moscow Engineering and Physical Institute) p 122
[14] Dubovikov M M and Starchenko N S 2003 Sci. Almanac Gordon 1 1–30
[15] Dubovikov M M, Starchenko N S and Dubovikov M S 2004 Physica A 339 591–608