Non-Fickian Diffusion Affects the Relation between the Salinity and Hydrate Capacity Profiles in Marine Sediments

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Abstract

On-site measurements of water salinity (which can be directly evaluated from the electrical conductivity) in deep-sea sediments is technically the primary source of indirect information on the capacity of the marine deposits of methane hydrates. We show the relation between the salinity (chlorinity) profile and the hydrate volume in pores to be significantly affected by non-Fickian contributions to the diffusion flux—the thermal diffusion and the gravitational segregation—which have been previously ignored in the literature on the subject and the analysis of surveys data. We provide amended relations and utilize them for an analysis of field measurements for a real hydrate deposit.

Keywords:
non-Fickian diffusion, hydrate deposits, brine salinity

1. INTRODUCTION

Since being discovered methane hydrates have attracted significant attention. First estimates of their amount on the Earth, especially in the marine sediments, and their importance were extremely exaggerated. Today’s assessment of their amount and role are more moderate and well underpinned by field data and results of numerical modelling and, thus, may be treated as realistic. Even with this “moderate” evaluation, research on natural hydrates in marine sediments is considered to be important.

In particular, methane hydrates present a potential hazard under anthropogenic climate change. The sensitivity of hydrate stability to changes in local pressure-temperature conditions and their existence beneath relatively shallow marine environments, mean that submarine hydrates are vulnerable to changes in bottom water conditions (e.g. warming). The potential climate impact of methane release following dissociation of hydrate in the past has been compared to climate feedbacks associated with the terrestrial biosphere and identified as a possible trigger of abrupt climate change (e.g., [1, 2]). The role of hydrate disassociation as a trigger for submarine landslides has also been investigated [4], with reports of known hydrate occurrences that coincide with slumping and submarine landslides being common [4]. It is therefore imperative to improve our understanding of the global hydrate inventory. Studies [5, 6] highlight that this improvement requires also certain revision of the physical and mathematical models of the marine deposits of methane hydrates employed in the literature [7, 8, 9, 10].
Acquiring samples with methane hydrates from sediments beneath deep water bodies is a costly procedure which is not practically employed for large scale surveys [11]. Instead, the presence of hydrate is typically inferred from seismic data (e.g., [12]) or on-site salinity measurements in boreholes (e.g., [7]). The seismic data are (i) the presence of the “bottom-simulating reflector” which appears when hydrate deposit touches the bottom boundary of the hydrate stability zone and, therefore, is underlain by a free gas horizon [11, 7, 10] (Fig. 1), and (ii) sound speed increase in sediments with hydrates owing to the sediments cementation by hydrate in pores [12]. Both seismic techniques have significant limitations. For instance, the bottom simulating reflector appears only when hydrate deposit reaches the bottom edge of the hydrate stability zone, and the velocity increase owing to the sediment cementation does not allow accurate estimation of the amount of hydrate. Hence, on-site salinity measurements become an important source of information.

To date, the mathematical models reconstructing hydrate deposit parameters by means of fitting the measured salinity profiles disregard non-Fickian contributions to the diffusion flux of salt in sediments. In the present paper we (i) derive relations between the profiles of the hydrate volumetric fraction in pores and the measured salinity and (ii) demonstrate the non-Fickian contributions to be important.

The paper is organized as follows. In Sec. 2 we describe the transport processes in carbon-rich marine sediments and derive the relation between the salinity (chlorinity) and hydrate profiles. In Sec. 3 this relation is employed for reconstruction of the hydrate profile for a real natural hydrate deposit in the Blake Ridge hydrate province. In the concluding section we discuss importance of the non-Fickian diffusion and implementation of our reconstruction procedure.

2. MARINE SEDIMENTS HOSTING HYDRATE DEPOSITS: TRANSPORT PROCESSES

Real geological systems are much more uniform along two directions (say horizontal) than along the third direction (say vertical). Hence, we consider a one-dimensional problem with vertical spatial coordinate \( z \) (Fig. 1). On the field scale, such systems are featured by the temperature growth with depth

\[
T = T_{sf} + Gz, \tag{1}
\]

where \( T_{sf} \) is the temperature of the water-sediment interface (or seafloor) and \( G \) is the geothermal gradient.
Methane is generated from the sediments by anaerobic bacteria. If the temperature is low enough and the pressure is high enough, methane forms hydrate. However, the critical pressure for the thermodynamic stability of hydrate depends on temperature nearly exponentially, and the hydrostatic pressure, which grows linearly with depth, cannot compensate the linear growth of temperature \( T = T_d + Gz \) below a certain depth \( \text{BSR} \). This depth \( \text{BSR} \) is the bottom boundary of the hydrate deposit: below this depth, hydrate is dissociated into water and methane-gas bubbles (Fig. 1).

The major part of natural hydrates of hydrocarbons on the Earth is a structure I clathrate of methane (> 99%). The elementary cell of an “ideal” structure I clathrate is formed by 8 molecules of CH\(_4\) and 46 molecules of H\(_2\)O, i.e., the mass fraction of water in the clathrate is 95% \([13]\), i.e. very close to the ideal value, 100%. Hence, we assume the “ideal” structure of hydrate, \( K_{\text{H}_2\text{O}} \approx 0.866 \).

Hydrate forming in pores consumes water from the brine while salts remain in the brine. Therefore, salt concentration increases and diffusion drives redistribution of the salt mass. This process determines the formation of the salinity (chlorinity) profile.

Additionally, one should distinguish the on-site chlorinity, \( \omega_s \), and the measured chlorinity, \( \tilde{\omega}_s \), because the drilling procedure results in dissociation of hydrate and the release of the hydrate water into the brine at the measurement site \([11]\). Given the volumetric fraction of hydrate in pores is \( h(z) \), the mass of NaCl in the unit volume of pores before the dissociation of hydrate, \( \omega_s \rho_w (1 - h) \), equals the mass after dissociation, \( \tilde{\omega}_s (\rho_w (1 - h) + K_{\text{H}_2\text{O}} \rho_h h) \), where \( \rho_w = 1000 \text{ kg/m}^3 \) is the water density, \( \rho_h = 930 \text{ kg/m}^3 \) is the hydrate density.

\[
\tilde{\omega}_s = \omega_s \left( 1 - \frac{K_{\text{H}_2\text{O}} \rho_h h}{\rho_w} \right) + O\left( \left( 1 - \frac{K_{\text{H}_2\text{O}} \rho_h h}{\rho_w} \right)^2 h^2 \right) \approx \omega_s \left( 1 - \frac{K_{\text{H}_2\text{O}} \rho_h h}{\rho_w} \right) \equiv \omega_s (1 - \rho_w h),
\]

where

\[
k \equiv \frac{K_{\text{H}_2\text{O}} \rho_h}{\rho_w} \approx 0.805,
\]

the correction \( O(1 - k)^2 h^2 \) can be neglected for real systems, where \( h \) rarely exceeds 7% \([13]\), is less than 0.002 \( \cdot \) \( h \) and can be neglected.

Since the salt is transported with pore water, we have to describe the water mass transport in the system. The water mass is transported as a part of hydrate, with sediments, and with the brine, by the pore water flux. The downward transport of sediments is significantly affected by the sediments compaction with depth \([12,11]\). The porosity \( \phi \) significantly decreases with depth according to the empirical law

\[
\phi(z) = \phi_0 \exp(-z/L),
\]

where \( L \) is the depth of \( e \)-folding of porosity. The mass conservation law for sediments yields

\[
\nu_s(z) = \frac{1 - \phi_0}{1 - \phi(z)} \nu_0,
\]

where \( \nu_s(z) \) is the sediment motion velocity. This relation is nearly not affected by the conversion of part of sediments into methane. Indeed, the defect of the solid matrix volume owing to methane generation is approximately \( \Delta V_s = \Delta m_{\text{CH}_4}/\rho_s \), whereas the production of hydrate \( \Delta V_h \) from this mass of methane is \( \Delta m_{\text{CH}_4}/K_{\text{H}_2\text{O}} \rho_h \). The ratio \( \Delta V_s/\Delta V_h = K_{\text{H}_2\text{O}} \rho_h/\rho_s \approx 0.04 \) is small and thus \( \Delta V_h \) related to methane generation is negligible.

For water, the mass conservation law reads

\[
\frac{\partial}{\partial t}(\phi(1 - h - b)\rho_w + \phi h K_{\text{H}_2\text{O}} \rho_h) = \frac{\partial}{\partial z}(\rho_w u_w) - \frac{\partial}{\partial z} (\phi h K_{\text{H}_2\text{O}} \rho_h \nu_s),
\]
where \( b \) is the volumetric fraction of bubbles in the pore volume (\( h = 0 \) beyond the hydrate stability zone, \( b = 0 \) within it), \( u_w \) is the brine filtration velocity. Following Davie and Buffett [3–8], we consider a steady-state situation and set time derivatives to zero. Hence,

\[
\rho_w u_w(z) + \phi(z) h(z) K_{H_2O} \rho_w v_z(z) = \rho_w u_w(0)
\]

(hydrate is not present close to the water-sediment interface, \( h(0) = 0 \), and, substituting \( v_z(z) \) from Eq. (4), we find

\[
u_w = u_{w0} - k\phi h - \frac{1 - \phi}{1 - \phi} v_{z0}.
\]  

Given the brine filtration velocity \( u_w \) is known (Eq. (6)), one can evaluate the salt transport from the mass conservation law:

\[
\frac{\partial}{\partial h} (\phi(1 - h - b) \rho_w \omega_s) = - \frac{\partial}{\partial z}(\rho_w \omega_s u_w) - \frac{\partial}{\partial z} J_{\text{diff}}.
\]  

(7)

The diffusive flux \( J_{\text{diff}} \) of the NaCl mass is contributed by the Fickian molecular diffusion flux and non-Fickian diffusion fluxes—the thermal diffusion and gravitational segregation (importance of which for the gas transport in geological systems under consideration was previously demonstrated [5]). The diffusive flux reads [14, 5]

\[
J_{\text{diff}} = -\chi \phi(1 - h - b) D_s \rho_w \omega_s \left( \frac{\partial}{\partial z} \ln \omega_s + \alpha_s \frac{\partial}{\partial z} \ln T - \frac{\tilde{\mu}_s g}{RT} \right).
\]  

(8)

The following notations are introduced here:

- \( \chi \) is the tortuosity factor, which characterize the effect of the pore morphology on the effective diffusivity of species in pore fluid. For our system \( \chi = 0.75 \) [15].
- \( D_s \) is the molecular diffusion coefficient in bulk brine.
- \( \alpha_s \) is the thermodiffusion constant.
- \( g = 9.8 \text{ m/s}^2 \) is the gravity.
- The universal gas constant \( R = 8.314 \text{ J/(mol K)} \).
- \( \tilde{\mu}_s = \mu_{\text{NaCl}} - N \mu_{\text{H}_2O} \) is the effective molar mass of the pair of ions Na\(^+\) and Cl\(^-\) in the aqueous solution, \( N \) is the number of water molecules in the volume occupied by this pair in the solvent, which can be evaluated from the dependence of the solution density on its concentration (see, e.g., Appendix A in [5]):

\[
\tilde{\mu}_s = \frac{\mu_{\text{NaCl}}}{\rho_w} \frac{\partial \rho_{\text{solution}}}{\partial \omega_s} \bigg|_{\omega_s = 0} \approx 42 \text{ g/mol}.
\]

In [16] the thermodiffusion constant \( \alpha_s \) and the molecular diffusion coefficient \( D_s \) were measured for a seminormal aqueous solution of NaCl. For the thermodiffusion constant a sign inversion was observed near \( T_s = 12^\circ \text{C} \). In the temperature range typical for our system, \( T \in (275 \text{ K}, 305 \text{ K}) \), the temperature dependencies of \( \alpha_s \) and \( D_s \) are strong and well represented by expressions

\[
\alpha_s \approx 0.0246 \text{ K}^{-1}(T - T_s) \quad \text{and} \quad D_s \approx 6.1 \cdot 10^{-10} \exp[0.0371 \text{ K}^{-1}(T - 273.15 \text{ K})] \text{ m}^2/\text{s}.
\]

Hydrostatic pressure in marine sediments, which is up to several hundreds atmospheres, is not strong enough to affect the diffusion constant of chemicals in water.

For a steady state, Eqs. (7) and (8) yield

\[
\omega_s u_w - \chi \phi(1 - h - b) D_s \omega_s \left( \frac{\partial}{\partial z} \ln \omega_s + \alpha_s \frac{\partial}{\partial z} \ln T - \frac{\tilde{\mu}_s g}{RT} \right) = \omega_s u_{ws} + J_{\text{diff}},
\]  

(9)

where \( \omega_s \equiv \omega_s(z_s) \), \( u_{ws} \equiv u_w(z_s) = u_{w0} \), \( J_{\text{diff}} \equiv J_{\text{diff}}(z_s) \), and \( z_s \) is a certain depth deep below the hydrate stability zone (see Fig. 1). Hereafter, the sign “s” indicates the value at depth \( z_s \).

Since \( h \ll 1 \), we restrict our consideration to the linear in \( h \) (and \( b \)) approximation. Substituting Eqs. (2) and (5) into Eq. (9) we can find

\[
\frac{\partial h}{\partial z} + \gamma(z) h = f(z),
\]  

(10)
of the hydrate profile.

Table 1: Geophysical properties for the Ocean Drilling Program site 997

| $T_{sd}$ | water-sediment interface temperature, Eq. (1) | 2°C | Refs. [11, 7] |
| $G$ | geothermal gradient, Eq. (1) | 35°C/km | Refs. [11, 7] |
| $\phi(0)$ | seafloor porosity, Eq. (3) | 0.69 | Ref. [7] |
| $L$ | $e$-folding depth of porosity, Eq. (3) | 2 km | Ref. [7] |

where
\[
\gamma(z) = \frac{1}{\chi D_s} \left( \frac{1 - \phi_0}{1 - \phi} \nu_s - \frac{u_{s0}}{\phi} \right), \quad f(z) = -\frac{1}{k} \left( \frac{1}{\omega_s} \frac{\partial \omega_s}{\partial z} + \beta \frac{G_s}{T_s} \right) + \frac{\phi_s D_s}{\kappa \phi D_s} \left( \frac{1}{\omega_s} \frac{\partial \omega_s}{\partial z} + \beta \frac{G_s}{T_s} \right) + \frac{u_{s0}}{k \chi D_s} \left( 1 - \frac{\omega_s}{\omega_s} \right);
\]

and the parameter
\[
\beta \equiv \alpha_s(T) - \frac{\hat{\mu}_L g}{RG}
\]
characterizes the strength of the non-Fickian flux.

With Eq. (10), one can reconstruct the hydrate profile $h(z)$ from the measured chlorinity profile $\hat{\omega}_s(z)$. Although one can write down an analytical solution to the problem (10)
\[
h(z) = \int_0^\infty f(z_1) e^{-\int_0^{z_1} \chi(z) dz_2} dz_1,
\]
numerical integration of Eq. (10) is more convenient for data analysis in practice. Remarkably, the relation between $h(z)$ and $\hat{\omega}_s(z)$ does not involve quantitative data on the process and history of the formation of hydrate deposit and the process of generation of methane from sediments.

3. SALINITY PROFILE ANALYSIS AND HYDRATE PROFILE RECONSTRUCTION

We demonstrate application of our results to the analysis of one of the most important marine hydrate provinces—the Blake Ridge. For the Ocean Drilling Program site 997, on the Blake Ridge, extensive data have been acquired, including hydrate samples and the on-site salinity (chlorinity) measurements [11] (Fig. 2(a)). The reported parameters for this site are presented in Table 1. We have two parameters which are not imposed by the results of direct measurements: sedimentation rate $v_{s0}$ and filtration velocity $u_{s0}$.

In natural systems, the hydrate deposit cannot be in touch with the water-sediment interface, because aqueous methane concentration in the water body above the sediments is zero and hydrate must dissociate. Moreover, in seas, methane is oxidized by sulfates, which are present in sea water, and its concentration is zero within the so-called sulfate reduction zone, which typically expands approximately 20 m below the water-sediment interface [8]. Hence, hydrate should not be present in a quiet extended upper part of the hydrate stability zone. With Eq. (10), the absence of hydrate, $h = 0$, requires $f(z) = 0$ next to $z = 0$. The function $f(z)$ is independent of $v_{s0}$ and we can set it to zero for small $z$ by tuning $u_{s0}$. With the chlorinity profile plotted in Fig. 2(a), this procedure yields $u_{s0} = -(8 \pm 0.5)$ cm/kyear (the flux is negative, i.e. ascending). One can see that for $u_{s0} = -8$ cm/kyear, $h = 0$ down to depths slightly over 100 m, while for different filtration velocity it deviates from zero next to $z = 0$ m. Furthermore, with fixed $u_{s0}$, the reconstructed amount of hydrate below the hydrate stability zone (for $z > 450$ m in Fig. 2) depends on $v_{s0}$ monotonically; it vanishes for $v_{s0} = (9 \pm 0.5)$ cm/kyear (Fig. 2(b)). The hydrate profile plotted in Fig. 2(b) with the red solid line is the final result of the reconstruction of the hydrate profile from the measured chlorinity profile plotted in Fig. 2(a).

It is noteworthy that our reconstruction procedure is free of uncertainties in parameters: all but two parameters are available from direct measurements and these two parameters are strictly imposed by two inevitable inherent features of the hydrate profile.
Figure 2: (Color online) (a): Measured chlorinity profile (solid circles) and smoothed chlorinity profile used for calculation of the hydrate profile (red solid line) for the site 997 of the Ocean Drilling Program [11]. (b): Hydrate profile reconstructed from the chlorinity data with Eq. (10) is plotted with the red solid line (parameters are specified in Table 1 and $u_{w0} = -8$ cm/kyear, $v_{s0} = 9$ cm/kyear). For demonstration, we plot a formal hydrate profile for purely Fickian diffusion flux of the same strength (blue dashed line). Non-Fickian contributions are obviously non-negligible.

Indeed, the latter profile significantly deviates from the former one and possesses unphysical features: negative values of the hydrate volumetric fraction and non-zero (negative) amount of hydrate beyond the zone of the thermodynamic stability of hydrate.

4. CONCLUSION AND DISCUSSION

In this paper the transport of water and salt have been considered for marine sediments hosting natural hydrate deposits. The mathematical description employed accounts for
- non-Fickian diffusion of NaCl, and
- temperature dependence of the molecular diffusivity.

We have demonstrated the crucial importance of the both, whereas they are disregarded in the literature on the modelling of marine hydrate deposits (e.g., [7, 8, 9, 10]). Based on this consideration, we have derived the relation between the measured salinity (chlorinity) profile and the hydrate profile. Application of this relation has been demonstrated for a real hydrate deposit (Fig. 2).

The solution, we found for this “reverse engineering” problem, does not involve quantitative data on the process and history of the formation of hydrate deposit and the process of generation of methane from sediments. This is an important feature of our results because previously, in the literature, closed models of hydrate formation involve particular assumptions on the generation process (e.g., [7, 8]). In these studies the entire model is tested against the measured salinity profile, while we can see that only the current hydrate profile determines the salinity profile. Moreover, the sedimentation rate is unambiguously imposed by features of one of these profiles, whereas it has been previously indirectly inferred from geological data.
Importantly, our reconstruction procedure is free of uncertainties in model parameters: all but two parameters—sedimentation rate \( v_s \) and filtration velocity \( u_{w0} \)—are available from direct measurements. These two parameters are strictly imposed by two inherent features of the hydrate profile: the absence of hydrate (i) close to the water-sediment interface and (ii) beneath the hydrate stability zone.

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**References**

[1] M. Maslin, et al., Gas hydrates: past and future geohazard? *Phil. Trans. Roy. Soc. A*, **368** (2010) 2369–2393.

[2] J. Maclennan, S.M. Jones, Regional uplift, gas hydrate dissociation and the origins of the Paleocene-Eocene Thermal Maximum, *Earth and Planetary Science Letters*, **245** (2006) 65–80.

[3] D.R. Caldwell, Measurements of negative thermal diffusion coefficients observing onset of thermodaline convection, *J. Phys. Chem. B*, **109** (2005) 9468–9475.

[4] J.-P. Henriet, J. Mienert, Gas Hydrates: Relavance to world margin stability and climate change, Special Publication of the Geological Society of London, **137** (1998).

[5] D.S. Goldobin, N.V. Brilliantov, Diffusive counter dispersion of mass in bubbly media, *Phys. Rev. E*, **84** (2011) 056328.

[6] D.S. Goldobin, et al., Scaling of transport coefficients of porous media under compaction, *Europhys. Lett.*, **95** (2011) 64004.

[7] M.K. Davie, B.A. Buffett, A numerical model for the formation of gas hydrate below the seafloor, *J. Geophys. Res.*, **106** (2001) 497-514;

[8] M.K. Davie, B.A. Buffett, A steady state model for marine hydrate formation: Constraints on methane supply from pore water sulfate profiles, *J. Geophys. Res.*, **108** (2003) 2495;

[9] M.K. Davie, B.A. Buffett, Sources of methane for marine gas hydrate: inferences from a comparison of observations and numerical models, *Earth and Planetary Science Letters*, **206** (2003) 51-63.

[10] D. Archer, Methane hydrate stability and anthropogenic climate change, *Biogeosciences*, **4** (2007) 521-544;

[11] K.A. Kvenvolden, Potential effects of gas hydrate on human welfare, *PNAS*, **96** (1999) 3420–3426;

[12] R.B. Bird, W.E. Stewart, E.N. Lightfoot, *Dissipating Late Cenozoic development of the western Svalbard margin from ODP 986 results*, *Marine Geology*, **169** (2000) 373–390;

[13] D.R. Caldwell, Thermal and Fickian diffusion of sodium chloride in a solution of oceanic concentration, *Deep Sea Research and Oceanographic Abstracts*, **20** (1973) 1029-1039;

[14] T. Dunkley Jones, et al., A Palaeogene perspective on climate sensitivity and methane hydrate instability, *Phil. Trans. Roy. Soc. A*, **368** (2010) 2395–2415;

[15] J.P. Kennett, et al., Methane Hydrates in Quaternary Climate Change: The Clathrate Gun Hypothesis AGU, Washington D.C. (2003);

[16] E.G. Nisbet, The end of the ice age, *Canadian Journal of Earth Sciences*, **27** (1990) 148–157;

[17] C.K. Paull, W. Ussler, W.P. Dillon, Is the extent of glaciation limited by marine gas hydrates? *Geophys. Res. Lett.*, **18** (1991) 432–434.

[18] R.D. McIver, Role of naturally occurring gas hydrates in sediment transport, *American Association of Petroleum Geologist Bulletin*, **66** (1982) 789–792.