Magnetic properties of rotational bands in $^{160}$Dy and $^{170-174}$Yb isotopes

A A Okhunov$^{1,3}$, A A M Al-Sammarrae$^2$, and H Abu Kassim$^2$

$^1$ Department of Science in Engineering, International Islamic University Malaysia, 57528, P.O. 10, Kuala Lumpur, Malaysia
$^2$ Department of Physics, Faculty of Science, University of Malaya, 50603, Kuala Lumpur, Malaysia
$^3$ Institute of Nuclear Physics, Academy of Science of Uzbekistan, 100132, Tashkent, Uzbekistan

E-mail: abdurahimokhun@iium.edu.my, aaokhunov@gmail.com

Abstract. In this work, non-adiabatic effects manifested in the magnetic properties of low-lying states of even-even deformed nuclei are studied. A simple phenomenological model which takes into account the Coriolis mixing of states of the ground and $K^\pi = 2^+ + n$ and $K^\pi = 1^+ + n$ bands are proposed. The calculations for isotopes $^{160}$Dy and $^{170,172,174}$Yb, are carried out. The calculated $g_{H^-}$ factors for the states of ground band which are compared with experimental data. Decreases of $g_{H^-}$ factor with increasing angular momentum $I$ have been discussed. The probability of $M1$ transition from $1^+_1$ and coefficients of multipole mixture $\delta(E2/M1)$ from $0^+_2$, $0^+_1$, $2^+_1$, and $2^+_2$ bands are calculated and compared with the experimental data. Decreases of $g_{H^-}$ factor with increasing angular momentum $I$ have been discussed.

That these is an obvious inverse relation between $g_{H^-}$ factor and angular momentum $I$ of the ground band states. This has been explained by a mixing ground and $K^\pi = 1^+$ bands which have a strong $B(M1)$ to ground state.

1. Introduction

The interest to investigation in the properties of deformed nuclei, especially increased in recent years with the opening of a new collective isovector magnetic dipole mode [1, 2].

The measured values of energies of the excited states such as modes indicate that they are not so high in the excitation spectrum, and consideration of mixing the isovector magnetic modes with low-lying exciting states can lead to interesting physical phenomena [3, 4].

Using $^{85}$Kr ions with energy of 350 MeV by Coulomb excitation has been studied ground band state in the flesh to the spin $I = 16\bar{h}$ isotopes $^{160}$Dy and $^{170,174}$Yb in paper [5]. Non-adiabaticity of magnetic characteristics of low-spin states of ground bands has been observed. It is shown that, with an increase of angular momentum $I$, $g_{H^-}$ factor decreases in these three nuclei.

The nuclei $^{170,172,174}$Yb are among the most well studied. These isotopes are studied experimentally in a number of ways: such as radioactive decay of $^{170,172,174}$Lu, and also different nuclear reactions. In the study of nuclear reactions of bands with the small $K$ were supplemented bands with high spin [6]-[8]. In these isotopes detected many $1^+$ states and also bands with $K^\pi = 0^+_n$, $2^+_n$, and some of these bands include the levels with rather high spins.
The aim of the present paper is applied to investigating the properties of positive parity low-lying states of $^{160}Dy$ and $^{170,172,174}Yb$ isotopes. To studying the properties of positive parity low-lying states of these isotopes used phenomenological model [4] which takes into account the Coriolis mixing all of the experimentally known low-lying rotational bands with $K^π \leq 2^+$. The behavior of $g_\mu$– factor of ground band has been discussed by the growth of angular momentum. The energy spectra and the reduced probability of $M1$– transition and the value of multipole-mixture $\delta(E2/M1)$ are calculated and compared with the experimental data.

2. The Model

To analyze the properties of low-lying positive parity states in $Dy$ and $Yb$ isotopes, the phenomenological model of [4] is exploited. This model takes into account the mixing of states of the $K^π = 0^+, 2^+$ and $1^+$ bands. The Hamiltonian model is

$$H = H_{\text{rot}}(I^2) + H_{K,K'}$$

$$H_{K,K'}^{\pi}(I) = \omega_K \delta_{K,K'} - \omega_{\text{rot}}(I)(j_x)_{K,K'} \zeta(I,K) \delta_{K,K'\pm 1}$$

where $\omega_K$– bandhead energy of rotational band, $\omega_{\text{rot}}(I)$– an angular frequency of rotational nucleus, $(j_x)_{K,K'}$– matrix elements which describe Coriolis mixture between rotational bands and

$$\zeta(I,0) = 1 \quad \zeta(I,2) = \left[1 - \frac{2}{I(I+1)}\right]^{1/2}$$

The eigenfunction of Hamiltonian model (1) is

$$|IMK> = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2} \Psi_{\text{gr},K}^I D_{MK}^I(\theta) + \sum_K \frac{\Psi_{K,K'}^I}{\sqrt{1 + \delta_{K',0}}} \left[ D_{M,K'}^I(\theta) b_{K'}^+ + (-1)^I b_{M,K'}^I D_{-K,K'}^I(\theta) b_{K'}^+ \right] \right\} |0>$$

here $\Psi_{K',K}^I$ is the amplitude of mixture of basis states.

Solving the Shrödinger equation

$$H_{K,K'}^{\pi}\Psi_{K,K'}^I = \mathcal{E}_K(I)\Psi_{K,K'}^I,$$

we define eigne function and energy of a Hamiltonian. The total energy of state is defined by

$$E_K(I) = E_{\text{rot}}(I) + \mathcal{E}_K(I)$$

In figure 1,2,3 and 4 the comparison of calculation values of energy with experimental dates for isotopes $^{160}Db$ and $^{170-174}Yb$ are presented. From figure we can see that in high spin values $I \geq 12\hbar$ noticeable deviation has been observed in calculated values of energy from experimental data, and this deviation increases with growth of angular momentum $I$. 


3. Magnetic Characteristics

The equation for the reduced probability of $M1\,$– transitions from the states $I_i K_i$ to the level $I_f\,$ 0$^+_1$ band by phenomenological model is as follows [4]:

$$ B_{\,0^+_1} (M1; I_i K_i \rightarrow 0^+_1 I_f) = $$

$$ = \frac{3}{4\pi} \sum_{n=1}^{\ell+\nu} |\Psi_{K_n}^{I_i} \Psi_{I_f K_n,0_1}^{I_f} K_n C_{I_i,0_1,0_1}^{I_f K_n} (g_{K_n} - g_R)|^2 $$

where $g_R \approx Z/A$, $g_{K}$– intrinsic $g$– factor of $K \neq 0$ bands. Here $m_n^{(0)} = \langle 0^+_1 \vert \hat{n}(M1) \vert 0^+_1 \rangle$ matrix elements between intrinsic wave functions of 0$^+_1$ and $K^\pi = 1^+_1$ bands, the value which is estimated from the experimental data. In adiabatic approximation of the equation (6) yields

$$ B(M1; 11^+_1 \rightarrow 00^+_1) = \frac{3}{4\pi} \times 0.02 \left( m_n^{(0)} \right)^2 \mu_n^2 $$

Figure 1. Comparison of the calculated and experimental energy spectra of positive-parity states for $^{160}\text{Dy}$.

Figure 2. Comparison of the calculated and experimental energy spectra of positive-parity states for $^{170}\text{Yb}$.

Figure 3. Comparison of the calculated and experimental energy spectra of positive-parity states for $^{172}\text{Yb}$.

Figure 4. Comparison of the calculated and experimental energy spectra of positive-parity states for $^{174}\text{Yb}$.
From the known experimental value of probability such as $M1^-$ transitions $m_{1^+}'$ can be calculate. Using experimental values of $B(M1)$ [12], $m_{1^+}'$ are determined by formula (7) which are presented in Table 1. However, formula (7) does not allow to define sign of the parameters $m_{1^+}'$.

Usually, the coefficient of multipole mixture $\delta$ is experimentally investigated which is found by the following ratio.

$$
\delta (I_iK_i \rightarrow I_fK_f) = \frac{0.834E_{\gamma}(MeV) \times <I_fK_f||\hat{m}(E2)||I_iK_i>|}{<I_fK_f||m(M1)||I_iK_i>} (\mu_N).
$$

Note, that $M1^-$ transitions from the $K^\pi = 0^+$ and $2^+$ to the level ground bands are equal to the zero (absent) in adiabatic approximation. Thus, $M1^-$ transitions appear due to $1^+_\nu$ components $\Psi_{L\nu,K_i}^I$ in wave functions of this states.

In case, the calculations use identical signs for parameters $m_{1^+}'$, the calculated values of coefficients of multipole mixture $\delta$ in many cases have smaller values than experimental data. In paper [12], $1^+_\nu$ states have been investigated and illustrated values of ratio $R_{11_\nu} = B(M1; 11 \rightarrow 20h_i)/B(M1; 11 \rightarrow 00h_i)$ for $\gamma^-$ transitions from $1^+_\nu$ states to ground state in which adiabatic approximation is $\approx 0.5$. As an axiom, for the $\gamma^-$ transitions from $1^+_\nu$ states having $R_{11_\nu} > 0.5$ identical sings of $m_{1^+}'$ are taken, and for $R_{11_\nu} < 0.5$, opposite signs are taken (see Table 1).

Comparison of measured experimental data [10, 11] and calculated value of coefficients of multipole mixture $\delta$ for $^{172,174}Yb$ are illustrated in Table 2. Table 2, shows that calculated values have good correspondence with experimental data and precisely reproduces a sign of $\delta$. The values of $\delta$ are decreasing with the growth of spin $I$, i.e. with a growth the probability of $B(M1)$ is increasing.

Experimental data [9] for the probability of $M1^-$ transitions from $K^\pi = 0^+_2, 0^+_1, 2^+_1$ and $2^+_2$ bands has been compared with calculated values in Table 3. Experimental values of $B(M1)$ for transitions from $I = 2$ and $I = 4$ states of $K^\pi = 2^+_1$ increases with rise of $I$. Here, it is worth mentioning that this finding confirms our calculated results.

In [5], ground band states up to spin $I = 16h$ of $^{170,174}Yb$ nuclei are investigated by Coulomb excitation through $350 MeV 85Kr$ ions. In these nuclei, it is observed that $g_{R^-}$ factor of ground band states decreases with an increase of angular momentum $I$.

A $g_{R^-}$ factor of rotational $K$ band determined within our model by the following formula:

$$
g_R^K(I) = g_R + \sum_n |\Psi_{n,K}^I|^2 \left( g_{K_n} - g_R \right) \frac{K_n^2}{I(I+1)} + \frac{\sqrt{3}}{5} \frac{\Psi_{gr,K}^I}{\sqrt{I(I+1)}} \sum_\nu m_{1^+_\nu}' \Psi_{1^+_\nu,K_i}^I.
$$

The first non-adiabatic correction is negligible in the calculation of the $g_R$ factors of ground state. But, the correction is considerable for $g_{R^-}$ factor for states with $K \neq 0$.

Theoretical values of magnetic characteristics excited states are obtained for the case $m_1 = m_{1^+_\nu} = 14 \mu_N$.

4. Conclusion
In the present work, non-adiabatic effects in energies and electromagnetic characteristics of excited states are studied within the phenomenological model which taking into account Coriolis mixing of all experimentally known rotational bands for isotopes $^{160}Dy$ and $^{170,172,174}Yb$, with $K^\pi \leq 2^+$. 
Table 1. Magnetic characteristics of $1^+_\nu$ states in isotopes $^{172,174}$Yb.

| $^{172}$Yb [12] | $\nu$ | $E_{1\nu}(MeV)$ | $R_{11\nu}$ | $B(M1) \uparrow (\mu_N^2)$ | $m_{1\nu}'(\mu_N)$ |
|----------------|-----|-----------------|-----------|-----------------|------------------|
| 1              | 2.010 | 1.06±0.14      | 1.27·10^{-2} | +1.63           |
| 2              | 2.573 | 0.51±0.09      | 0.93±0.10   | -8.06±0.87      |
| 3              | 2.612 | 0.70±0.13      | 0.33±0.09   | -4.80±1.31      |
| 4              | 3.002 | 0.51±0.10      | 0.34±0.09   | -4.87±1.29      |
| 5              | 3.096 | 0.46±0.12      | 0.11±0.04   | +2.77±1.01      |
| 6              | 3.253 | 0.46±0.12      | 0.09±0.03   | +2.51±0.84      |
| 7              | 3.604 | 0.76±0.13      | 0.49±0.12   | -5.85±1.43      |
| 8              | 3.863 | 1.14±0.24      | 0.45±0.14   | -5.61±1.75      |

| $^{174}$Yb [12] | $\nu$ | $E_{1\nu}(MeV)$ | $R_{11\nu}$ | $B(M1) \uparrow (\mu_N^2)$ | $m_{1\nu}'(\mu_N)$ |
|----------------|-----|-----------------|-----------|-----------------|------------------|
| 1              | 1.625 | 2.31±0.28      | 3.1·10^{-2} | -0.80           |
| 2              | 2.037 | 0.64±0.40      | 0.15±0.11  | -3.24±2.38      |
| 3              | 2.068 | 0.67±0.34      | 0.20±0.12  | -3.83±2.30      |
| 4              | 2.338 | 0.74±0.20      | 0.28±0.10  | -4.34±1.55      |
| 5              | 2.500 | 0.60±0.16      | 0.35±0.11  | -5.01±1.58      |
| 6              | 2.581 | 0.46±0.14      | 0.21±0.08  | +3.83±1.46      |
| 7              | 2.815 | 0.90±0.38      | 0.16±0.009 | -3.24±1.62      |
| 8              | 2.920 | 0.41±0.07      | 0.44±0.11  | +5.61±1.40      |
| 9              | 3.050 | 1.06±0.42      | 0.14±0.07  | -3.24±1.62      |
| 10             | 3.122 | 0.50±0.27      | 0.10±0.06  | -2.51±1.51      |
| 11             | 3.145 | 0.76±0.29      | 0.13±0.06  | -2.89±1.33      |
| 12             | 3.349 | 0.58±0.18      | 0.33±0.14  | -4.80±2.04      |
| 13             | 3.485 | 0.68±0.09      | 0.24±0.08  | +4.09±1.36      |
| 14             | 3.562 | 0.47±0.08      | 0.41±0.10  | -5.42±1.32      |
| 15             | 3.695 | 0.48±0.16      | 0.33±0.13  | +4.58±1.80      |

The energy and structure of wave functions of excited states are calculated. The reduced probabilities of $M1^-$ transitions and coefficients of multipole mixture $\delta(E2/M1)$ from $K^\pi = 0^+_m; 2^+_\ell$ and $1^+_\nu$ bands are also calculated and compared with experimental data which gives the satisfactory result are also calculated.

Thus within our theoretical analysis it is possible to explain the $M1^-$ transition from the state $\beta_1^-, \beta_2^-, \gamma_1^-$ and $\gamma_2^-$ bands to the ground bands level, which is forbidden by adiabatic approximation.

The experimentally observed $K$ forbidden $M1^-$ transitions from state $2^+_\ell$ bands are explained by the presence of $K^\pi = 1^+_\ell$ components in wave functions of these bands. With the growth of the angular moment $I$, the coefficient of $\delta(E2/M1)$ for transitions from $0^+_m$ and $2^+_\ell$ bands decrease. Experimental data of $B(M1)$ for transitions $I = 2$ and $I = 4$ state $K^\pi = 2^+_\ell$ bands shows an increase of $B(M1)$ with the of growth $I$. This confirms our results.

The result of calculation show that the decrease of $g_R^-$ factor with an increase of angular momentum $I$ of rotational states of ground band is associated with the mixing in these states with $1^+_\nu$ bands which have strong $B(M1)$ to the ground state in isotope $^{170}$Yb.
Table 2. Multipole mixture coefficients of $\delta(E2/M1)$ in isotopes $Yb$.

| $A$ | $I_iK_i \rightarrow I_fK_f$ | $E_i$ (MeV) | $E_\gamma$ (MeV) | $\delta$ [10, 9] | $\delta_{\text{theor.}}$ |
|-----|-----------------------------|-------------|------------------|----------------|------------------|
| $^{172}Yb$ | $22^+ \rightarrow 20^+$ | 1.4659 | 1.3871 | $-5.1^{+1.6}_{-1.4}$ | $-8.8$ |
| | $32^+ \rightarrow 40^+$ | 1.5492 | 1.2888 | $+2.8^{+0.7}_{-1.0}$ | $-9.0$ |
| | $\rightarrow 20^+$ | 1.4704 | $-7.6^{+1.9}_{-1.6}$ | $-2.1$ |
| | $42^+ \rightarrow 40^+$ | 1.6578 | 1.3975 | $-1.1^{+1.2}_{-0.5}$ | $-1.9$ |
| | $22^+ \rightarrow 20^+$ | 1.6085 | 1.5297 | $+10(3)$ | $+12.6$ |
| | $32^+ \rightarrow 40^+$ | 1.7006 | 1.4404 | $+6.5^{+2.2}_{-1.4}$ | $+7.8$ |
| | $\rightarrow 20^+$ | 1.6219 | $+17.0(4)$ | $+11.3$ |
| | $42^+ \rightarrow 40^+$ | 1.8031 | 1.5429 | $+9.0^{+1.1}_{-1.0}$ | $+8.0$ |
| | $52^+ \rightarrow 40^+$ | 1.9270 | 1.6664 | $+6.9^{+1.2}_{-1.1}$ | $+6.8$ |
| | $20^+ \rightarrow 20^+$ | 1.1179 | 1.0393 | $+2.3^{+0.5}_{-0.3}$ | $+1.0$ |
| | $40^+ \rightarrow 40^+$ | 1.2868 | 1.0265 | $+0.87(13)$ | $+0.49$ |
| | $60^+ \rightarrow 60^+$ | 1.5375 | 0.9977 | $+0.63(7)$ | $+0.30$ |
| $^{174}Yb$ | $22^+ \rightarrow 20^+$ | 1.6340 | 1.1456 | $-0.48(8)$ or $-32^{+29}_{-28}[11]$ | $-15.0$ |
| | $32^+ \rightarrow 20^+$ | 1.7094 | 1.6329 | $-3.8^{+1.3}_{-1.1}$ | $-17.5$ |
| | $20^+ \rightarrow 20^+$ | 1.5610 | 1.4845 | $+1.7(4)$ or $+0.12^{+0.17}_{-0.16}[11]$ | $+0.82$ |
| | $40^+ \rightarrow 40^+$ | 1.7160 | 1.4630 | $-0.03^{+0.08}_{-0.06}[11]$ | $+0.40$ |
| | $20^+ \rightarrow 20^+$ | 1.9582 | 1.8823 | $+0.01(8)$ or $+2.2(5)[11]$ | $-0.16$ |
| | $21^+ \rightarrow 20^+$ | 1.6750 | 1.5983 | $-0.23(5)$ or $+5.2^{+1.5}_{-1.3}[11]$ | $-0.21$ |
| | $31^+ \rightarrow 20^+$ | 1.7338 | 1.6573 | $+0.50(5)$ or $+4.6^{+2.1}_{-1.9}[11]$ | $+0.65$ |

Table 3. Probability of $M1$– transitions of $^{172}Yb$.

| $I_iK_i \rightarrow I_f0_f$ | $B(\text{M}1; I_iK_i \rightarrow I_f0_f)$ | $\mu_N^2$ |
|-----------------------------|---------------------------------|-----------|
| exp. [9] | theor. | theor. |
| $20^+ \rightarrow 20^+$ | $0.6 \cdot 10^{-3}(2)$ | $1.8 \cdot 10^{-3}$ |
| $20^+ \rightarrow 20^+$ | $1.2 \cdot 10^{-4}(7)$ | $1.1 \cdot 10^{-3}$ |
| $22^+ \rightarrow 20^+$ | $1.7 \cdot 10^{-2}(2)$ | $0.2 \cdot 10^{-3}$ |
| $42^+ \rightarrow 40^+$ | $9.0 \cdot 10^{-2}(6)$ | $3.1 \cdot 10^{-3}$ |
| $22^+ \rightarrow 20^+$ | $5.2 \cdot 10^{-5}(27)$ | $4.0 \cdot 10^{-5}$ |

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References
[1] Mikhailov I N, Neergor K, Pashkevich V V, Frauendorf C 1977 Fiz. Elem. Chastits At. Yadra 8, No.6 1338
[2] Jungclaus A, Binder B, Dietrich A, et al. 2002 Phys.Rev. C 66 014312.
[3] Usmanov P N, Adam I, Salikhbaev U S, and Solnyshkin A A 2010 *Physics of Atomic Nuclei* **73** No.12 1990
[4] Usmanov Ph N, I.N.Mikhailov IN 1997 *Phys. Part. Nucl. Lett.* **28**, 348
[5] Andrews H R, Hauss O, Ward D and et al. 1980 *Phys.Rev.Lett.* **43**, No.23 1835
[6] Riech C W 2005 *Nucl. Data Sheets* **105** 557
[7] Baglin M 2002 *Nucl. Data Sheets* **96** 611
[8] Bengtsson R, Frauendorf S 1979 *Nucl. Phys. A* **327** No.1 139
[9] Singh B 1995 *Nucl.Data Sheets*. Vol **75** 199
[10] Kracikova T I, Davaa S, Finger M and et al. 1984 *J. Phys.* Vol. G**10** 1115
[11] Youhana H M, Al-Obeidi S R, Al-Amili M A and et al. 1986 *Nucle. Phys.* Vol. A**458** 51
[12] Zilges A, Brentano P von, C.Wesselborg C and et al 1990 *Nucl.Phys. A* **507** 399