Some studies on dark energy related problems

Fei Wang\textsuperscript{1}, Jin Min Yang\textsuperscript{2,1}

\textsuperscript{1} Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China
\textsuperscript{2} CCAST (World Laboratory), P. O. Box 8730, Beijing 100080, China

Abstract

In this work we perform some studies related to dark energy. Firstly, we propose a dynamical approach to explain the dark energy content of the universe. We assume that a massless scalar field couples to the Hubble parameter with some Planck-mass suppressed interactions. This scalar field develops a Hubble parameter-dependent (thus time-dependent) vacuum expectation value, which renders a time-independent relative density for dark energy and thus can explain the coincidence of the dark energy density of the universe. Secondly, we assume the dark matter particle is meta-stable and decays very lately into the dark energy scalar field. Such a conversion of matter to dark energy can give an explanation for the starting time of the accelerating expansion of the universe. Thirdly, we introduce multiple Affleck-Dine fields to the landscape scenario of dark energy in order to have the required baryon-asymmetry universe.

96.35.+d, 04.65.+e, 12.60.Jv
I. INTRODUCTION

The nature of the content of the universe is a great mystery in today’s physical science. The Wilkinson Microwave Anisotropy Probe (WMAP) collaboration gives fairly accurate values on the content of the universe \[ \Omega_m = 0.27_{-0.04}^{+0.04}, \quad \Omega_b = 0.04_{-0.004}^{+0.004}, \quad \Omega_{\Lambda} = 0.73_{-0.04}^{+0.04}, \quad \eta = (6.14 \pm 0.25) \times 10^{-10}, \] \hspace{1cm} (1)
where \( \Omega_m, \Omega_b \) and \( \Omega_{\Lambda} \) denotes the density of total matter, baryonic matter and dark energy, respectively. \( \eta \) denotes the baryon to photon ratio. We see that, coincidentally, the dark matter density is comparable to the dark energy density as well as to the baryonic matter density. Such a coincidence needs to be understood.

For the explanation of such a coincidence, some phenomenologically dynamical approaches have been proposed, such as the quintessence [2,3], phantom [4] and k-essence [5]. In this note we propose a new dynamical approach to explain the dark energy coincidence in the content of the universe. In this approach a massless scalar field is assumed to couple to the Hubble parameter with some Planck-mass suppressed interactions. This scalar field develops a Hubble parameter-dependent (thus time-dependent) vacuum expectation value, which renders time-independent dark energy density and thus can explain the coincidence of the dark energy density of the universe. We further assume the dark matter particle is meta-stable and decays very lately into the dark energy scalar field. Such a conversion of matter to dark energy can give an explanation for another puzzle, namely the starting time of the accelerating expansion of the universe.

Another puzzle related to the dark energy is the smallness of dark energy (or cosmological constant). Weinberg used anthropic principle [6] to argue that fine-tuning is needed by the existence of human beings. Such an approach is based on the hypothesis of multiple vacua, each of which has identical physical properties but different value of vacuum energy. Motivated by such an approach, landscape from string theory is used to provide the vast amount of vacuum. In such a landscape scenario, the vast amount (\( \sim 10^{120} \)) of vacua of the potential arise from a large number of fields (say 100 \( \sim 300 \)), which ensure the statistical selection to give a plausible vacuum energy. However, the baryon content may be over-washed out by sphaleron effects even at temperature moderately beneath \( \Lambda_{QCD} \) and give a small baryon to photon ratio to be consistent with baryon-symmetry universe [7]. In order to give the required baryon-asymmetry universe, we propose to use multiple Affleck-Dine fields. We find that with multiple Affleck-Dine fields the baryon content can be much higher, which has a large range to ensure the present asymmetric baryon abundance.

This article is organized as follows. In Sec. II we elucidate the new dynamical approach to dark energy. In Sec. III we discuss the conversion of dark matter to dark energy, which cause the accelerating expansion of the universe. In Sec. III we introduce multiple Affleck-Dine fields to the landscape scenario in order to give the required baryon-asymmetry universe. The conclusion is given in Sec. V.

II. DYNAMICAL APPROACH TO DARK ENERGY

The smallness of the dark energy (cosmological constant) may imply the existence of some new fundamental law of nature. It is possible for dark energy to have dynamical
behaviors. We know that dark energy can be related to the Hubble constant $H_0$ and Planck scale $M_{pl}$ by a see-saw mechanism $^1$

\[ \frac{\Lambda}{H_0} \sim \frac{M_{pl}}{\Lambda}, \]  

(2)

where $\Lambda$ is related to the dark energy density by $\rho_{DE} \sim \Lambda^4$. We can attribute the varying of the dark energy to a massless scalar field $\phi$. Such a massless scalar can be the Nambu-Goldstone boson from the broken of the global $U(1)$ R-symmetry $^8$ by gravity effects. We can phenomenologically adopt the potential of the form $^9$

\[ V(\phi) = H^2 \phi^2 f\left(\frac{\phi^2}{M_{pl}^2}\right). \]  

(3)

It is quite possible for the second derivative of the potential to be negative. As an effective theory, the flatness of the potential for the massless scalar can be lifted by higher order gravitational force. We introduce the Planck scale-suppressed terms which preserve $-\phi \leftrightarrow \phi$ symmetry

\[ V(\phi) \approx -H^2 \phi^2 + \frac{\lambda}{M_{pl}^2} \phi^6, \]  

(4)

where $\lambda$ is a dimensionless constant or variable of $O(1)$, characterizing the coupling of $\phi$.

The vacuum expectation value of $\phi$ is then given by

\[ \langle \phi \rangle^4 \sim \frac{1}{3\lambda} H^2(t) M_{pl}^2. \]  

(5)

Here we can see two features for our approach:

(1) The dark energy density $\rho_{DE} \sim \langle \phi \rangle^4$ is time-dependent, which makes the relevant density $\Omega_\Lambda = \rho_{DE}/(\rho_m + \rho_{DE})$ time-independent.

$^1$Our universe can be described by the Robertson-Walker metric

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \]

Here $a(t)$ is the scale factor, $k$ can be chosen to be $+1, -1,$ or $0$ for spaces of constant positive, negative or zero curvature, respectively. The see-saw relation can also be seen in the Friedman equation

\[ H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE}). \]

The gravitational constant is related to the planck scale by $G = \hbar c^5/M_{pl}^2$. $H$ is the Hubble parameter defined by $H(t) = \dot{a}(t)/a(t)$ where $t = t_0$ gives the Hubble constant.
When $t = t_0$ (present time), $\rho_{DE} \sim \langle \phi \rangle^4$ can naturally take the required value $\sim H_0^2 M_{pl}^2$. So in this way, the dark energy coincidence in the content of the universe can be understood.

We also note that the coincidence of dark matter density and baryonic matter density can be understood in the Affleck-Dine mechanism for baryogenesis. In this mechanism, baryonic number can be generated dynamically. The oscillation of the field is generically unstable with spatial perturbation and can condenses into non-topological solitons called Q-balls [10–13]. The late decays of these Q-balls into dark matter relate baryonic matter density to dark matter density [14,15].

### III. CONVERSION OF DARK MATTER TO DARK ENERGY

For the dynamical scalar field $\phi$ introduced in the preceding section, we can also introduce some subdominate terms for its interaction with the dark matter particle, which is assumed to be a scalar $\tilde{f}$ (say the super-partner of sterile neutrino)

$$\frac{1}{M_{pl}^3} \phi^6 \tilde{f}. \quad (6)$$

It will not cause any phenomenological problems in particle physics since it is much suppressed. Through this interaction the scalar $\tilde{f}$ decays into $\phi$ and its lifetime $\tau$ can be estimated as

$$\tau^{-1} \sim \left( \frac{m_f}{M_{pl}} \right)^6 m_f. \quad (7)$$

Suppose $m_f$ is large as the $\sim 10^9$ GeV, such decay occurs at the time scale

$$\tau \sim 10^{18} s, \quad (8)$$

which is of the order of the age of the universe. Thus such meta-stable particles can be a component of the relic dark matter (for some extensive studies on the cosmology of meta-stable sfermions, see [16]).

Through such decays, dark matter particles are being converted to dark energy field particles, which can explain the starting time ($z \sim 1$) of the accelerating expansion of the universe, as explained in the following.

From the Friedman equation

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3} (\rho + 3P) \quad (9)$$

we know that the accelerating expansion of the universe starts when $\rho + 3P$ becomes negative. Here $\rho = \rho_m + \rho_{DE}$ is the total energy density and $P$ is the pressure. The equation of state is given by

$$\omega = \frac{P}{\rho} = \frac{T-V}{T+V} \quad (10)$$
where $T, V$ are the kinematic and potential energy, respectively. $\omega$ is vanishing for matter. Generally, the equation of state for dark energy is similar to that of the quintessence model and $\omega > -1$. The decay of the dark matter can alter the state of equation for dark energy by kinematic terms.

Then we get

$$\rho + 3P = \rho_m + \rho_{DE} + 3(P_m + P_{DE}) = \rho_m + (1 + 3\omega)\rho_{DE}. \tag{11}$$

As the decay of $\tilde{f}$ into $\phi$ proceeds, $\rho_m$ is getting smaller and $\rho_{DE}$ is getting larger, and at some point $\rho_m + (1 + 3\omega)\rho_{DE}$ becomes negative since $1 + 3\omega$ is negative. Such a point is called the critical point, at which $\rho_m + \rho_{DE} = \Omega\rho_{critical} \ (\Omega \equiv \rho/\rho_{critical})$ and

$$\frac{\rho_{DE}}{\Omega\rho_{critical}} = \frac{1}{-3\omega}. \tag{12}$$

Since in our scenario such a critical point happens as a result of the decay of the meta-stable dark matter particle and the decay occurs at the time scale of $10^{18}$ s, we get an understanding why the universe starts accelerating expansion quite lately ($z \sim 1$).

Note that in our scenario the dynamical field $\phi$ may couple to graviton. The radiation of gravitons can slowly decrease the kinematic energy. So the transition of dark matter to dark energy cause the slow loss of universe content. In this way our scenario predicts that the universe is evolving toward an Anti-de-Sitter universe. If the universe is flat till now ($\Omega = 1$), it will evolve to be open ($\Omega < 1$).

**IV. MULTIPLE AFFLECK-DINE FIELDS IN LANDSCAPE**

In Affleck-Dine mechanism, a complex scalar field has U(1) symmetry, which corresponds to a conserved current and is regarded as baryon number. It has potential interactions that violate CP. It can develop a large vacuum expectation value and when oscillation begins, it can give a net baryon number. Supersymmetry provides the natural candidates for such scalar fields. The vast number of flat directions [17] that carry baryon or lepton number can have vanishing quartic terms. Nonrenormalizable higher-dimensional terms can lift the flat directions which then can give a large vacuum expectation value. Here we propose to use multiple flat directions (each of which denoted by $\Phi_i$) to generate the net baryon number in our universe.

We consider a superpotential, which can lift such flat directions from supersymmetry breaking terms, to have the following leading form

$$W_{n}^i = \frac{1}{M_n} \Phi_i^{n+3}, \tag{13}$$

\(^{2}\)At the critical point, if we naively use $\omega = -1$, we find that the dark energy constitutes about 1/3 of the total content of the universe. However, such a portion can be increased when $\omega$ is larger than $-1$, which is highly justified.
FIG. 1. The illustration plot of different vacuum expectation values in the flat direction potential which can generate different baryonic contents.

where $M$ is the scale of new physics and $n$ is some integer. So the corresponding potential takes the form

$$V = -H^2|\Phi_i|^2 + \frac{1}{M^{2n}}|\Phi_i|^{2n+4}. \quad (14)$$

The leading sources of $B$ and $CP$ violations come from supersymmetry breaking terms (by gravity)

$$am_{3/2}W_i^i + bHW_i^n, \quad (15)$$

where $a$ and $b$ are complex dimensionless constants and $m_{3/2}$ is the gravitino mass. The relative phase in these two terms, $\delta = \tan^{-1}(ab^*/|ab|)$, violates $CP$. We can chose $n$ and $a, b$ to ensure each potential to have several meta-stable vacuum expectation values with very different magnitudes as illustrated in Fig. 1 (Acceptable selection in natural consideration may require that the magnitudes be different by $10^3 \sim 10^4$). The two vacuum expectation values of $\Phi_i$ are given as

$$\Phi_{i,0} \approx M \left(\frac{H}{M}\right)^{1/(n+1)} \quad (16)$$

and

$$\Phi_{i,0} \approx M \left(\frac{2|\text{Re}(a)m_{3/2} + \text{Re}(b)H|}{M}\right)^{1/(n+1)}. \quad (17)$$

So we can get more than $10^{120}$ vacua for $100 \sim 300$ Affleck-Dine fields.
The evolution of the baryon number is

$$\frac{dn_B}{dt} = \frac{\sin(\delta) m_3/2}{M^n} \Phi_i^{n+3}. \quad (18)$$

Naive estimation gives (we assume $H \sim 1/t$):

$$n_B = \sum_i \frac{\sin(\delta)}{M^n} \Phi_i^{n+3}. \quad (19)$$

As each of the two metastable vacua differs significantly, the combination of multiple fields can give a large range for baryonic content. The rate of washing out baryon asymmetry is given by [7]

$$\frac{dn_B}{dt} = -\Gamma. \quad (20)$$

Here $\Gamma$ is given by

$$\Gamma = \alpha_W^4 T \left( \frac{M_W(T)}{\alpha_W T} \right)^7 e^{-\frac{M_W(T)}{\alpha_W T}}, \quad (21)$$

where at zero temperature $M_W$ is given by

$$M_W \sim g_W f \sim g_W \frac{\Lambda_{QCD}}{4\pi}. \quad (22)$$

The residue abundance in our multiple fields case can be several orders higher than ordinary approach, which can greatly enhance the residue value for baryon content and thus make it possible to be consistent with baryon-asymmetry universe.

**V. CONCLUSION**

We performed some studies related to dark energy. Firstly, we proposed a dynamical approach to explain the dark energy content of the universe. We assumed that a massless scalar field couples to the Hubble parameter with some Planck-mass suppressed interactions. Such a scalar field develops a Hubble parameter-dependent (thus time-dependent) vacuum expectation value, which renders a time-independent relative density for dark energy and thus can explain the coincidence of the dark energy density of the universe. Secondly, we assumed the dark matter particle is meta-stable and decays very lately into the dark energy scalar field. Such a conversion of matter to dark energy can give an explanation for the starting time of the accelerating expansion of the universe. Finally, we introduced multiple Affleck-Dine fields to the landscape scenario of dark energy in order to have the required baryon-asymmetry universe.

$^3n_{\text{total}} \sim n_{\text{field}} \times n_{\text{differ}}$ can be as higher as $10^3 \sim 10^{3(n+3)}$. Here $n_{\text{field}}$ is the number of Affleck-Dine fields and $n_{\text{differ}}$ is the order of difference between the meta-stable vacuum values.
ACKNOWLEDGEMENT

We are grateful to Dr Zongkuang Guo, Dr Wei Hao and Dr Dingfang Zeng for enlightenment discussions. This work is supported in part by National Natural Science Foundation of China.
REFERENCES

[1] WMAP Collaboration, Astrophys. J. Suppl. 148, 1 (2003); 148, 175 (2003); 248, 195 (2003).
[2] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988); C. Wetterich, Nucl. Phys. B302, 668 (1988).
[3] I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999); P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
[4] R.R.Caldwell, Phys. Lett. B 545, 23 (2002).
[5] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000); C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D 63, 103510 (2001); T. Chiba, T. kobe and M. Yamaguchi, Phys. Rev. D 62, 023501 (2000); J. M. Aguirregabiria, L. P. Chimento and R. Lazkoz, Phys. Rev. D 70, 023509 (2004).
[6] S. Weinberg, Phys. Rev. Lett. 59, 2607 (1987); T. Banks, Nucl. Phys. B249, 332 (1985); A. D. Linde, in “300 Years of Gravitation” (Editors: S. Hawking and W. Israel, Cambridge University Press, 1987), 604; A. Vilenkin, Phys. Rev. Lett. 74, 846 (1995).
[7] N. Arkani-Hamed, S. Dimopoulos, S. Kachru, hep-th/0501082.
[8] G. Farrar and A. Masiero, hep-ph/9410401; A. E. Nelson and N. Seiberg, Nucl. Phys. B416, 46 (1994); A. H. Chamseddine, H. Dreiner, Nucl. Phys. B458, 65 (1996).
[9] M. Dine, L. Randall, S. Thomas, Phys. Rev. Lett. 75, 398 (1995).
[10] K. Enqvist, J. McDonald, Phys. Lett. B 425, 309 (1998); K. Enqvist, A. Mazumdar, Phys. Rept. 380, 99 (2003).
[11] S. Kasuya, M. Kawasaki, Phys. Rev. D 61, 041301 (2000); Phys. Rev. D 62, 023512 (2000).
[12] M. Fujii, K. Hamaguchi, Phys. Lett. B 525, 143 (2002); Phys. Rev. D 66, 083501 (2002).
[13] K. Ichikawa, M. Kawasaki, F. Takahashi, Phys. Lett. B 597, 1 (2004).
[14] F. Wang, J. M. Yang, Nucl. Phys. B709, 409 (2005).
[15] K. Jedamzik, Phys. Rev. Lett. 84, 3248 (2000); astro-ph/0402344.
[16] J. L. Feng, A. Rajaraman, F. Takayama, Phys. Rev. Lett. 91, 011302 (2003); Phys. Rev. D 68, 063504 (2003); J. L. Feng, B. T. Smith, Phys. Rev. D 71, 015004 (2005); J. L. Feng, S. Su, F. Takayama, hep-ph/0404198; hep-ph/0404231; hep-ph/0410178; hep-ph/0405215; hep-ph/0410119; hep-ph/0503117; F. Wang, J. M. Yang, Eur. Phys. J. C 38, 129 (2004).
[17] T. Gherghetta, C. Kolda, S. P. Martin, Nucl. Phys. B468, 37 (1996).