Fermion Sector of Little Higgs

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Abstract

The little Higgs mechanism provides an alternative solution to the hierarchy problem, arguably fitting better into the phenomenological hint of the "little hierarchy" which may cause some fine-tuning for the case of supersymmetry. We discuss an aspect of little Higgs physics lacking proper attention — the construction of an interesting and consistent chiral fermionic sector and its phenomenological implications. At least for the kind of example models to be discussed, the gauge and top sector structure of a model largely dictates, through gauge anomaly cancellation conditions, a specific chiral fermion spectrum. The spectrum has interesting, family non-universal, flavor structure. The implications for flavor physics are specially interesting. We also add a brief comment of little Higgs versus supersymmetry.

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ABSTRACT
The little Higgs mechanism provides an alternative solution to the hierarchy problem, arguably fitting better into the phenomenological hint of the “little hierarchy” which may cause some fine-tuning for the case of supersymmetry. We discuss an aspect of little Higgs physics lacking proper attention — the construction of an interesting and consistent chiral fermionic sector and its phenomenological implications. At least for the kind of example models to be discussed, the gauge and top sector structure of a model largely dictates, through gauge anomaly cancellation conditions, a specific chiral fermion spectrum. The spectrum has interesting, family non-universal, flavor structure. The implications for flavor physics are specially interesting. We also add a brief comment of little Higgs versus supersymmetry.

1. Introduction

The SM is a model of interactions dictated by an $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, with a anomaly free chiral fermion spectrum and a Higgs multiplet responsible for the spontaneous breaking of the electroweak (EW) symmetry $SU(2)_L \times U(1)_Y$. Supersymmetry with or without grand unification is the most popular candidate theory beyond the SM. There, the beautiful boson-fermion symmetry to tackle the hierarchy problem, essentially extending the chiral nature of the fermions to fix the problem for the scalar sector. The approaches do not provide any new insight into the difficult problem of the origin of flavor structure. Why there are three families of SM fermions is still a fundamental problem that we have no credible approach to handle. The so-called little Higgs mechanism [1] comes as an alternative solution to the hierarchy problem, with an extended EW symmetry. Here, one can also use gauge anomaly cancellation constraints to ‘predict’ the fermion spectrum. The later may even provide an understanding of why three SM families, together with specific implications on flavor physics [2].

2. The Simplest Model

We focus on the simplest model here. A little Higgs model with a $SU(3)_L \times U(1)_X$ extended EW symmetry is available [3]. We draw attention to the gauge anomaly considerations and present the solution spectrum, as given in the table. Note that the embedding of the SM doublets is not family universal. This is necessary to avoid accumulation of $SU(3)_L$ anomalies. Cancellation of the latter is here achieved by exploring the equality of the number of family and the number of color, as done in Ref.[4]. The $U(1)$ related gauge anomaly contributions are illustrated explicitly. The full fermion spectrum is essentially fixed by the anomaly cancellation scheme and the little Higgs requirement of having the extra heavy top quark $T$ living in a $SU(3)$ multiplet with the SM $(t, b)$ doublet.
The SU(3)_C × SU(3)_L × U(1)_X spectrum with little Higgs. Electroweak doublets are put in [.]’s.

| Gauge anomalies | \( tX \) | \( LLL \) | \( LLX \) | \( CCX \) | \( X^3 \) | \( U(1)_Y \) states |
|-----------------|-------|-------|-------|-------|---------|------------------|
| (3_c, 3_L, \frac{1}{3}) | 3     | 3     | 1     | 1     | \( \frac{1}{3} \) | 2 \( \frac{1}{6} \)[Q] |
| 2 (3_c, 3_L, 0)     | 0     | -6    | 0     | 0     | \( -\frac{1}{3} \) | 2 \( \frac{1}{3} \)[T] |
| 3 (1_c, 3_L, \frac{1}{3}) | -3    | 3     | -1    | 0     | \( -\frac{1}{3} \) | 3 \( \frac{1}{2} \)[L] |
| 4 (3_c, 1_L, \frac{2}{3}) | -8    | 3     | 3     | \( \frac{1}{3} \) | \( \frac{1}{3} \)[N] |
| 5 (3_c, 1_L, 0)     | 5     | 3     | 3     | \( \frac{1}{3} \) | \( \frac{1}{3} \)[N] |
| 3 (1_c, 1_L, 1)     | 3     | 0     | 1     | 3     | 3 \( \frac{1}{3} \)[N] |
| **Total**           | 0     | 0     | 0     | 0     | 0       | 0               |

The little Higgs mechanism as a solution to the hierarchy problem only alleviates the quadratic divergent quantum correction to the SM Higgs states and admits a natural little hierarchy between the EW scale and a higher scale of so-called UV-completion at around the 10 TeV order, beyond which further structure would be required. The idea is a rather humble bottom-up approach then; but experimental hints at the existence of such a little hierarchy has been discussed[5].

The little Higgs mechanism is to be implemented here with two scalar multiplets, \( \Phi_1 \) and \( \Phi_2 \), having the right quantum number to couple to the chiral parts of the \( T \) quark. This is illustrated by top-sector Yukawa couplings

\[
\mathcal{L}_{\text{top}} = \lambda^t_t \bar{t} \Phi_1 Q + \lambda^l_t \bar{T} \Phi_2 Q
\]

\[
= f (\lambda^t_t \bar{t} + \lambda^l_t \bar{T}) T + \frac{i}{\sqrt{2}} (\lambda^t_t \bar{t} - \lambda^l_t \bar{T}) h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots
\]

where \( Q \) denotes the \((T, t, b)\) triplet (contrary to notation in the table above). The SM Higgs doublet \( h \) is retrieved as the pseudo-Nambu-Goldstone boson from a \([SU(3)/SU(2)]^2\) nonlinear sigma model parametrization of \( \Phi_1 \) and \( \Phi_2 \). The multiplets are required to have aligned \( SU(3) \) breaking VEVs, from a scalar potential with the \([SU(3)]^2\) global symmetry.

### 3. General Relevance of the Anomaly Cancellation Considerations

We are here talking about an extended EW symmetry model as a TeV scale effective field theory. This leads to the thinking that may be one needs not be asking for cancellation of the gauge anomalies among the chiral fermionic states. The are some good reasons, however, that make us consider the issue very relevant.

First, let us take a look at the SM itself, which is in this case an effective field theory below the scale of the breaking of part of the extended EW symmetry. The SM has three families of chiral fermionic states each consists of a perfect unique set of 15 states with all gauge anomalies well canceled. Indeed, we have argued in earlier works [6] that one can essentially derive the spectrum by simply imposing the anomaly cancellation constraints. This is the best we come to in terms of understanding why there is what there is, though it still begs the question of why three families \(^a\). It seems very difficult to convince oneself that this is rather some sort of accident. If the SM fermion spectrum is a guideline,

\(^a\)Similar considerations have been used to derive minimal chiral spectra of bigger gauge symmetries,
gauge anomaly cancellation is relevant. While consistent models with the anomaly or its cancellation implemented beyond simply the chiral fermionic contributions are possible, one with an anomaly free fermion spectrum looks far more attractive. Besides, giving up the requirement, one loses an control on what are the plausible extra fermionic states. Any spectrum may then look as good as another from a pure theoretical point of view.

From our perspective, little Higgs models of extended EW symmetries can be constructed to be essentially unique and very predictive so long as the symmetry is chosen. The gauge symmetry fixes the gauge sector, as well as the fermionic sector through the anomaly cancellation constraints. Implementation of the little Higgs mechanism to stabilize the (little) hierarchy helps to fix the scalar or Higgs sector. Then, one arrives at a very definite model of TeV scale physics, without much room for simple modifications, to be checked with phenomenological studies. That is a very solid approach for model-building, with hardly a competing alternative.

4. A $SU(4)$ Analog

The basic construction strategy of the anomaly free spectrum may actually be generalized to the case of any $SU(N)_L \times U(1)_X$ extended EW symmetries. In the case of $N = 4$, it looks like they could be more choice for little Higgs model-building. A simple extension of the above $SU(3)$ case with one more $T$ quark fits the similarly extended Higgs sector structure. The latter may have a better behaved Higgs quartic coupling $^{[3]}$. One other fermion spectrum we find intriguing is presented in the table below. It has a kind of duplicated fermion list at the QCD/QED level. Each SM fermion get a heavy singlet partner. A little Higgs model based on the spectrum would likely have a few special and phenomenologically appealing features. We have to refrain from elaborating here though.

| The $SU(4)_L \times U(1)_X$ Spectrum |
|-------------------------------------|
| $U(1)_Y$-states | $\frac{1}{6}[Q]$ | $\frac{2}{3}(T)$ | $\frac{1}{2}(B)$ |
| $2(3C, 4L, \frac{1}{6})$ | $2\frac{1}{6}[2Q]$ | $2\frac{1}{3}(D, S)$ | $2\frac{1}{3}(U, C)$ |
| $3(1C, 4L, \frac{1}{2})$ | $3\frac{1}{2}[3L]$ | $3\frac{1}{3}(3N)$ | $3\frac{1}{3}(3E^-)$ |
| $6(3C, 1L, \frac{1}{3})$ | $4\frac{2}{3}(\bar{u}, \bar{c}, \bar{t}, \bar{T})$ | $2\frac{2}{3}(\bar{U}, \bar{C})$ |
| $6(3C, 1L, \frac{2}{3})$ | $5\frac{1}{3}(d, \bar{s}, \bar{b}, \bar{D}, \bar{S})$ | $\frac{1}{3}(B)$ |
| $6(1C, 1L, 1)$ | $3\frac{1}{3}(e^+, \mu^+, \tau^+)$ | $3\frac{1}{3}(3E^+)$ |

5. Remarks

We observe that little Higgs models typically have extended EW symmetries with extra chiral fermionic states. The gauge anomaly issue comes in. Imposing the requirement that all gauge anomalies be canceled among the chiral fermionic states, as in the SM case, has such as $SU(4) \times SU(3) \times SU(2) \times U(1)$, with admissible spontaneous symmetry breaking to that of the SM giving rise to exact the three families as the only remaining chiral states $^{[6]}$. 

the power to essential dictate the full spectrum. Such a complete model has a specific
flavor structure which, at least in the case of $SU(N)_L \times U(1)_X$ symmetries, put the
three SM families into one whole anomaly free set. This may shred new light into the
origin of the families. The full gauge quantum numbers may then be used to extract
admissible Yukawa couplings. As illustrated for the simplest cases of the $SU(3)$ model
discussed above [2], while the extra singlet quark states can all obtain consistent TeV scale
masses, the generic admissible mass mixings with the SM quarks pose interesting FCNC
constraints. The latter should be an important aspects of phenomenological explorations
of such little Higgs models.

A conceptual comparison of little Higgs versus supersymmetry is in order here. The
former uses (global) bosonic symmetries instead of (local) supersymmetry to achieve the
stability of the SM Higgs mass, hence EW scale. The basic quantum field content of
the SM is almost perfection, apart from the strange triplication of the unique anomaly
free fermion spectrum. But the scalar/Higgs field bears the major short-coming — the
hierarchy problem. In the way, supersymmetry cures the sick scalar sector by pairing it
with the healthy (chiral) sector of fermions. However, the $\mu$-problem makes exactly the
conceptual incompleteness of the task. The little Higgs scheme uses less specific bosonic
symmetries with only a little accomplishment — stabilizing only a little hierarchy. It
looks like it has less intuitive beauty. While supersymmetry has nothing to provide on
improving our understanding of the flavor problems, our perspective of complete little
Higgs model construction does better in the aspect. The fermionic spectrum, though not
as beautiful as that of one SM family, does tied the three families together in a unique
framework.

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7. References

[1] N. Arkani-Hamed et.al., Phys. Rev. Lett. 86, 4757 (2001).
[2] O.C.W. Kong, hep-ph/0307250 (see also NCU-HEP-k015); hep-ph/0308148; hep-
ph/0312060, talk given at ICFP II.
[3] D.E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003); M. Schmaltz, hep-
ph/0407143.
[4] P.H. Frampton, Phys. Rev. Lett. 69, 2889 (1992).
[5] R. Barbieri, hep-ph/0312253.
[6] O.C.W. Kong, Mod. Phys. Lett. A11, 2547 (1996); Phys. Rev. D55, 383 (1997).