The heavy gravitino, naturalness, and sizable anomaly mediation

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We consider the situation in which \( m_{3/2} \sim O(100 \text{ TeV}) \) for solving the gravitino problem and the other supersymmetry (SUSY) breaking parameters are \( O(1 \text{ TeV}) \) for the naturalness. We point out that the anomaly mediation cancels out the renormalization group contribution to the gaugino and sfermion masses other than the stop masses at a scale known as the mirage scale. The situation is similar to mirage mediation, in which special boundary conditions for the SUSY breaking parameters are required, though, for the stop masses and the up-type Higgs mass, such cancellation at the mirage scale does not happen. Despite no cancellation for the up-type Higgs mass, we show that the little hierarchy problem becomes less severe in this situation. One advantage of this situation over mirage mediation is that the stop mixing parameter \( A_t \) can be larger and, therefore, a smaller stop mass is sufficient for the 125 GeV Higgs. When the mirage scale is around the TeV scale, the SUSY breaking parameters induced by gravity mediation on the grand unification scale can be observed directly by TeV-scale experiments.

Subject Index B12, B13

1. Introduction

The minimal supersymmetric (SUSY) Standard Model (MSSM) is still one of the most promising candidates for physics beyond the Standard Model (SM). The MSSM can solve the gauge hierarchy problem and provide a dark matter candidate for the lightest supersymmetric particle (LSP). Moreover, the SUSY grand unified theory (GUT) is experimentally supported by the remarkable coincidence of three SM gauge coupling constants around \( 10^{16} \text{ GeV} \). However, many SUSY models suffer from a tuning problem, called the SUSY little hierarchy problem. This problem arises from a tension between naturalness, which requires lightness of several SUSY particles, and the Higgs mass \( m_h = 125 \text{ GeV} \) [1,2], which forces these particles to be heavy. Cosmologically, it has been pointed out that the decay of the gravitino spoils the success of the Big Bang Nucleosynthesis (BBN). This is called the gravitino problem [3–11].

One of the simplest solutions to the gravitino problem is to assume that the gravitino decays before BBN begins. For example, if the gravitino is heavier than 100 TeV, then the lifetime of the gravitino becomes of the order of \( 10^{-2} \text{ sec} \). At this time in the history of the universe, the proton–neutron ratio has not yet been fixed by freezing out the weak decay process. In the literature, the high-scale SUSY breaking scenario [12–15], in which the scalar fermion masses are taken to be of the same order as the heavy gravitino mass, has been studied because such a scenario can realize the Higgs mass \( m_h \sim 125 \text{ GeV} \) without the large stop mixing parameter \( A_t \) [16–21]. Such a high-scale SUSY...
breaking scenario has various advantages, e.g., it has no SUSY flavor problem, no SUSY CP problem, etc. Unfortunately, the fine-tuning problem on the Higgs mass becomes much worse in this scenario.

For the fine-tuning problem, it is preferable that the stop masses and the gaugino masses are of the order of 1 TeV. These two requirements, the gravitino mass $m_{3/2} \geq 100 \text{ TeV}$ and the sfermion masses $\tilde{m} \sim O(1 \text{ TeV})$, are not inconsistent with each other. Actually, both requirements are satisfied in the mirage mediation scenario [22–26], in which the moduli [27–30] and anomaly [31,32] contributions to the SUSY breaking parameters become comparable. One of the most important features in mirage mediation is that the effective SUSY mediation scale can be lower because the renormalization group effects can be canceled by the anomaly-mediation effect. As a result, the little hierarchy problem may be solved [33–35]. Unfortunately, in mirage mediation, very specific boundary conditions for the SUSY breaking parameters are required. What happens if we take more generic boundary conditions for the SUSY breaking parameters? If the contribution of anomaly mediation dominates that of gravity mediation, then the mass squares of the right-handed slepton become negative. Therefore, we have an upper bound for the gravitino mass, which is nothing but $O(100 \text{ TeV})$.

In this paper, we will examine a scenario in which the gravitino mass is of the order of 100 TeV, to solve the gravitino problem, and the other SUSY breaking parameters, which are induced by gravity mediation, are around the TeV scale, to stabilize the weak scale. We will not discuss how to realize such a situation. Here we simply note that it could be possible, at least in the mirage mediation scenario.

Let us examine the little hierarchy problem in more detail, because it is one of the main purposes of this paper to improve the fine-tuning in the Higgs sector. In supersymmetric models, a quantum correction of the up-type Higgs squared mass $\Delta m_{H_u}^2$ strongly depends on the stop mass $m_t$:

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{8\pi^2} \left( m_{i_1}^2 + m_{i_2}^2 + A_t^2 \right) \ln \frac{\Lambda}{m_t},$$

(1)

where $\Lambda$ is the messenger scale; here we consider $\Lambda = 2 \times 10^{16} \text{ GeV}$. In order to realize electroweak symmetry breaking without fine-tuning, one can expect that $m_t$ is of the order of 100 GeV. On the other hand, the lightest CP-even Higgs mass $m_h$ is also linked to the stop mass:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2} \pi^2} \left[ \ln \frac{m_t^2}{m_h^2} + \frac{A_t^2}{m_t^2} \left( 1 - \frac{A_t^2}{12 m_t^2} \right) \right].$$

(2)

The Higgs mass $m_h = 125 \text{ GeV}$, which was discovered by ATLAS and CMS, implies a heavy stop mass such as several TeV. Therefore, it is difficult to obtain a realistic Higgs mass without destroying the naturalness.

One of the solutions for avoiding the little hierarchy problem is to move beyond the MSSM. For instance, one may add an extra singlet as in the next-to MSSM [36–44]. On the other hand, we can also reduce the fine-tuning within the MSSM by lowering the messenger scale $\Lambda$, such as in the low-scale gauge mediation model [45–55]. One can also lower the messenger scale effectively in the case where several SUSY breaking contributions cancel the renormalization group (RG) evolution, as in the TeV-scale mirage mediation model [22–25,33–35]. Note that the large stop mass spoils the naturalness even if the messenger scale is small. The value of $m_t$ while realizing a 125 GeV Higgs depends on the value of $A_t$. It is minimized when $|A_t/m_t| = \sqrt{6}$ [56]. It is, however, difficult to realize a large $A_t$ in the low-scale messenger models. In the TeV-scale mirage mediation, the model fixes the ratio $A_t/m_t = \sqrt{2}$ at the mirage scale, which is considered to be around the TeV scale. The
gauge mediation model also fails to give a large $A_t$ because it does not appear at the leading order. The value of $A_t$ in these models is not sufficient to obtain the Higgs mass naturally.

What happens if we do not impose the specific condition $A_t/m_{\tilde{t}} = \sqrt{2}$ in mirage mediation? To answer this question, we have to know what happens when the specific boundary conditions in the mirage mediation scenario are not imposed. This is one of our motivations for the work in this paper.

The paper proceeds as follows. In Sect. 2, we recall that the anomaly mediation contribution can cancel the RG evolution of the gravity mediation contribution by analytic solutions of the one-loop RG equations of the MSSM. In Sect. 3, we study what happens if the gravity mediation produces $O(1\text{ TeV})$ SUSY breaking parameters while the gravitino mass is $O(100\text{ TeV})$. In particular, we show that the little hierarchy problem becomes less severe, as in mirage mediation. Section 4 is for the discussion and summary.

2. Cancellation property of anomaly mediation

It is known that anomaly mediation [31,32] has the property of canceling the RG evolution of gravity mediation. In this section, we will review this property by solving the one-loop RG equations for the SUSY breaking parameters in the MSSM.

2.1. Small Yukawa case

Let us see this cancellation property in the case where the Yukawa coupling can be neglected. The results in this subsection can be applied to all sfermion masses except stop masses and the up-type Higgs mass $m_{H_u}$, when the bottom and tau Yukawa couplings can be neglected, i.e., $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \ll 50$.

First we consider the gaugino mass $M_a$ ($a = 1, 2, 3$). It satisfies the RG equation

$$\frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a$$

at the one-loop level. Here the gauge coupling $g_a$ obeys the RG equation

$$\frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3,$$

where $(b_1, b_2, b_3) = (33/5, 1, -3)$ in the MSSM. Then the anomaly mediation solution is written as

$$M_a(\mu)|_{\text{anomaly}} = \frac{1}{16\pi^2} b_a g_a^2 m_{3/2},$$

where $m_{3/2}$ is the gravitino mass. There is also a gravity mediation solution as follows:

$$M_a(\mu)|_{\text{gravity}} = \tilde{M}_a + \frac{1}{8\pi^2} b_a g_a^2 \tilde{M}_a \ln \frac{\mu}{\Lambda},$$

where $\tilde{M}_a$ is the mass from the gravity mediation at the cutoff scale $\Lambda$. Note that, in this paper, “gravity mediation” does not include anomaly mediation. Hereafter we assume that $\tilde{M}_a$ is universal as

$$\tilde{M}_1 = \tilde{M}_2 = \tilde{M}_3 = M_{1/2},$$

which is imposed if the GUT is assumed at the cutoff scale $\Lambda = \Lambda_G = 2 \times 10^{16}$ GeV. One can easily check that these two expressions satisfy the RG equation (3), respectively. These two contributions
can coexist because the sum $M_a|_{\text{anomaly}} + M_a|_{\text{gravity}}$ also satisfies the same RG equation. It can be rewritten as

$$M_a(\mu) = M_{1/2} + \frac{1}{8\pi^2} b_a g_a^2 M_{1/2} \ln \frac{\mu}{M_{\text{mir}}},$$

(8)

where the mirage scale $M_{\text{mir}}$ is defined as

$$\ln \frac{M_{\text{mir}}}{\Lambda} = -\frac{m_{3/2}}{2M_{1/2}}.$$

(9)

At the mirage scale, the anomaly mediation contribution cancels the quantum corrections of the gravity mediation contribution and we get $M_a(M_{\text{mir}}) = M_{1/2}$.

Second, we see the trilinear coupling $A_{ijk}$. The one-loop RG equation is

$$\frac{d}{dt} A_{ijk} = -\frac{1}{4\pi^2} \sum_a (C_i^a + C_j^a + C_k^a) g_a^2 M_a.$$

(10)

where $C_i^a$ is the quadratic Casimir coefficient for the field $i$ and $C_i^a = (N^2 - 1)/(2N)$ for a fundamental representation of the gauge group $SU(N)$; $C_i^a = q_i^2$ for the $U(1)$ charge $q_i$. It is related to the anomalous dimension $\gamma_i$ as $\gamma_i = 2 \sum_a C_i^a g_a^2$. Then the anomaly mediation

$$A_{ijk}(\mu)|_{\text{anomaly}} = -\frac{1}{16\pi^2} (\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

and the gravity mediation

$$A_{ijk}(\mu)|_{\text{gravity}} = \tilde{A}_{ijk} - \frac{1}{8\pi^2} (\gamma_i + \gamma_j + \gamma_k) M_{1/2} \ln \frac{\mu}{\Lambda}$$

(12)

satisfy the RG equation when they are combined with $M_a|_{\text{anomaly}}$ and $M_a|_{\text{gravity}}$, respectively. Here $\tilde{A}_{ijk}$ are also the gravity mediation contributions at the cutoff scale. The sum of the two contributions ($A_{ijk}|_{\text{anomaly}} + A_{ijk}|_{\text{gravity}}$, $M_a|_{\text{anomaly}} + M_a|_{\text{gravity}}$) also obeys the same RG equation. As a result,

$$A_{ijk}(\mu) = \tilde{A}_{ijk} - \frac{1}{8\pi^2} (\gamma_i + \gamma_j + \gamma_k) M_{1/2} \ln \frac{\mu}{M_{\text{mir}}}. $$

(13)

One can see that the RG evolution of the trilinear coupling also vanishes at $M_{\text{mir}}$.

Lastly, we see the scalar mass $m_i^2$. The one-loop RG equation is

$$\frac{d}{dt} m_i^2 = -\frac{1}{2\pi^2} \sum_a C_i^a g_a^2 |M_a|^2 + \frac{3}{40\pi^2} g_i^2 \gamma_i S,$$

(14)

where the quantity $S$ is defined as

$$S = \sum_i Y_i m_i^2 = m_H^2 - m_d^2 + \text{Tr} \left[ m_Q^2 - 2m_{u_R}^2 + m_{d_R}^2 - m_L^2 + m_{e_R}^2 \right].$$

(15)

The scalar mass is generated from the anomaly and gravity mediations as

$$m_i^2(\mu)|_{\text{anomaly}} = -\frac{1}{32\pi^2} \gamma_i m_{3/2}^2$$

(16)

$$m_i^2(\mu)|_{\text{gravity}} = \tilde{m}_i^2 - \frac{1}{4\pi^2} \gamma_i M_{1/2} \ln \frac{\mu}{\Lambda} - \frac{1}{8\pi^2} \gamma_i M_{1/2} \left( \ln \frac{\mu}{\Lambda} \right)^2 + \frac{3}{40\pi^2} g_i^2 S \ln \frac{\mu}{\Lambda},$$

(17)

where $\gamma_i = \frac{d}{dt} \gamma_i$, $S = \sum_i Y_i m_i^2$, and $\tilde{m}_i^2$ is the mass from the gravity mediation at the cutoff scale. They satisfy the RG equation when they are combined with $M_a|_{\text{anomaly}}$ and $M_a|_{\text{gravity}}$, respectively.
However, the combination \( (m_i^2)_{\text{anomaly}} + m_i^2 \gamma_i \) does not satisfy the same RG equation. This is not a problem because the scalar mass has interference terms

\[
(m_i^2(\mu))_{\text{interference}} = -\frac{1}{8\pi^2} \gamma_i M_{1/2}^2 m_{3/2} - \frac{1}{8\pi^2} \gamma_i M_{1/2}^2 m_{3/2} \ln \frac{\mu}{\Lambda}
\]

(18)

when there are different SUSY breaking sources. This guarantees the coexistence of the two contributions. Finally, the scalar mass under the anomaly and gravity mediations is

\[
m_i^2(\mu) = \tilde{m}_i^2 - \frac{1}{4\pi^2} \gamma_i M_{1/2}^2 \ln \frac{\mu}{M_{\text{mir}}} - \frac{1}{8\pi^2} \gamma_i M_{1/2}^2 \left( \ln \frac{\mu}{M_{\text{mir}}} \right)^2 + \frac{3}{40\pi^2} Y_i g_i^2 \tilde{S} \ln \frac{\mu}{\Lambda}.
\]

(19)

Note that the RG evolution of the scalar mass also cancels at \( M_{\text{mir}} \) if \( \tilde{S} = 0 \) because it is satisfied in the GUT models where \( H_d \) and \( H_d \) are unified into a single multiplet, such as \( SO(10) \).

We have seen that all the RG evolution effects of gaugino mass, trilinear coupling, and scalar mass vanish at the same scale \( M_{\text{mir}} \) in the small Yukawa case. Therefore we can see that anomaly mediation effectively lowers the cutoff scale \( \Lambda \) to \( M_{\text{mir}} \). In the case of \( m_{3/2}/M_{1/2} \sim 60 \), the mirage scale is around the TeV scale. Note that the value \( m_{3/2}/M_{1/2} \sim 60 \) is consistent with the assumption \( m_{3/2} \sim 100 \text{TeV} \), which is used for solving the gravitino problem, and that the SUSY breaking scale is around 1 TeV.

2.2. Effect of top Yukawa coupling

We have shown in the previous subsection that the anomaly mediation cancels the RG evolution of the gravity mediation if there is no Yukawa coupling. However, the expressions for \( m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2, m_{\tilde{b}_L}^2, \) and \( A_t \) should be modified because top Yukawa coupling makes a sizable contribution. Here we consider the case where the bottom and tau Yukawa coupling contributions can be neglected.

Let us see the effect of top Yukawa coupling in more detail. First, the RG equation for top Yukawa coupling is

\[
dt \gamma_i = \frac{1}{16\pi^2} \gamma_i \left( 6y_i^2 - 2 \sum_a C_t^a s_a^2 \right)
\]

(20)

with \( C_t^a = C_{t_L}^a + C_{t_R}^a + C_{\tilde{t}}^a \). The running top Yukawa coupling is given as

\[
y_i^2(\mu) = \frac{y_i^2(\Lambda) E(\mu)}{1 - \frac{3}{4\pi^2} \gamma_i^2(\Lambda) F(\mu)}.
\]

(21)

where the function \( E(\mu) \) and \( F(\mu) \) are defined as

\[
E(\mu) = \prod_a \left( 1 - \frac{b_a}{8\pi^2} \frac{s_a^2}{\ln \frac{\mu}{\Lambda}} \right)^{2C_t^a/b_a}
\]

(22)

\[
F(\mu) = \int_\Lambda^\mu \frac{d\mu'}{\mu'} E(\mu').
\]

(23)

The RG equations for \( A_t \) and \( m_i^2 (i = \tilde{t}_L, \tilde{t}_R, H_u) \) become

\[
\dt A_t = \frac{1}{4\pi^2} \left( 3|y_t|^2 A_t - \sum_a C_t^a s_a^2 M_a \right)
\]

(24)

\[
\dt m_i^2 = -\frac{1}{8\pi^2} \left( k_i |y_t|^2 (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) + k_i |y_t|^2 |A_t|^2 + 4 \sum_a C_t^a s_a^2 |M_a|^2 \right).
\]

(25)
With top Yukawa coupling, $A_t$, the up-type Higgs mass, and the stop masses generated by gravity mediation are given as

$$A_t(\mu) = \tilde{A}_t + 6\rho(\tilde{A}_t - M_{1/2}) - \frac{1}{8\pi^2} (\gamma H_u + \gamma L + \gamma R) M_{1/2} \ln \frac{\mu}{\Lambda},$$  \hspace{1cm} (26)

$$m^2_\chi(\mu) = \tilde{m}^2_i - k_i \rho \left[ (\tilde{A}_t - M_{1/2})^2 (1 + 6\rho) + \tilde{\Sigma}_t - M_{1/2}^2 \right]$$

$$- \frac{M_{1/2}}{8\pi^2} \left[ \gamma_i M_{1/2} + k_i (\tilde{A}_t - M_{1/2}) (1 + 6\rho) \gamma_i^2 \right] \ln \frac{\mu}{\Lambda} - \frac{1}{8\pi^2} \gamma_i M_{1/2}^2 \left( \ln \frac{\mu}{\Lambda} \right)^2,$$  \hspace{1cm} (27)

where $\tilde{\Sigma}_t = \tilde{m}^2_{H_u} + \tilde{m}^2_{h_L} + \tilde{m}^2_{t_R}$. The anomalous dimension $\gamma_i$ is written as

$$\gamma_i = 2 \sum_a C^a_i \frac{\gamma^2}{E(\mu)}.$$

Note that if the function $E(\mu)$ is just a constant, $\rho$ can be estimated as $\rho \sim \ln(\mu/\Lambda)$.

Anomaly mediation changes the expressions (26) and (27) as follows:

$$A_t(\mu) = \tilde{A}_t + 6\rho(\tilde{A}_t - M_{1/2}) - \frac{1}{8\pi^2} (\gamma H_u + \gamma L + \gamma R) M_{1/2} \ln \frac{\mu}{M_{\text{mir}}},$$  \hspace{1cm} (30)

$$m^2_\chi(\mu) = \tilde{m}^2_i - k_i \rho \left[ (\tilde{A}_t - M_{1/2})^2 (1 + 6\rho) + \tilde{\Sigma}_t - M_{1/2}^2 \right]$$

$$- \frac{M_{1/2}}{4\pi^2} \left[ \gamma_i M_{1/2} + k_i (\tilde{A}_t - M_{1/2}) (1 + 6\rho) \gamma_i^2 \right] \ln \frac{\mu}{M_{\text{mir}}} - \frac{1}{8\pi^2} \gamma_i M_{1/2}^2 \left( \ln \frac{\mu}{M_{\text{mir}}} \right)^2.$$  \hspace{1cm} (31)

These analytic formulas are given by Ref. [26]. One can see that the cancellation at the mirage scale is spoiled by the top Yukawa contribution. Moreover, a large logarithmic factor appears because $\rho \sim \ln(\mu/\Lambda)$. However, if we impose the special boundary conditions $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$, as in the mirage mediation scenario, then the cancellation at the mirage scale can be restored. In the literature [33–35], researchers have discussed the tuning improvement in mirage mediation if the mirage scale is around the weak scale.

What happens in the more general case in which the special boundary conditions are not satisfied? We will discuss this subject in the next section.

### 3. More general cases

In the usual mirage mediation, special boundary conditions for the gravity contribution to the SUSY breaking parameters are imposed, i.e., universal sfermion masses to satisfy the flavor-changing neutral current (FCNC) constraints, vanishing Higgs masses, and $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$. In this section, we study more general cases in which the anomaly mediation contribution is sizable.

#### 3.1. Generalization of mirage mediation: Natural SUSY

Before studying completely general cases, we discuss the cases in which the cancellation is complete, as in the mirage mediation scenario. In these cases, the little hierarchy problem can be improved, as
discussed in the usual mirage mediation. It is obvious that, for the cancellation, only the conditions \( \tilde{A}_t = M_{1/2} = \sqrt{\Sigma_t} \) are important. For the cancellation, basically no additional condition is required for the other sfermion masses except vanishing \( \tilde{S} \).

As an example, we mention the natural SUSY-type boundary conditions [57,58], in which the sfermion masses \( m_3 \) for the third generation 10 of \( SU(5) \) are around the TeV scale to stabilize the weak scale, and the other sfermion masses \( m_0 \) are taken to be much larger than \( m_3 \) to suppress the SUSY contributions to the FCNC and CP violation processes. These boundary conditions are consistent with \( \tilde{A}_t = M_{1/2} = \sqrt{\Sigma_t} \) and \( \tilde{S} \) can vanish. For example, we can adopt the conditions \( \tilde{m}_{H_d}^2 = \tilde{m}_{H_u}^2 = 0 \) and \( \tilde{m}_{\tilde{t}^c_R}^2 = \tilde{m}_{\tilde{\tau}^c_R}^2 = \tilde{m}_{\tilde{\nu}^c_R}^2 = \Sigma_t/2 \). Similar boundary conditions in the mirage mediation, in which only stop masses are taken to be different from the others, have been discussed in the literature [59,60], though \( \tilde{S} = 0 \) is not satisfied in their boundary conditions. We think this possibility interesting because the \( E_6 \) GUT with the family symmetry \( SU(2)_F \) predicts such natural SUSY-type sfermion masses [61–67].

The most important point is that if the mirage scale is around the SUSY breaking scale, we may directly obtain the signatures of GUT scenarios by observing the sfermion mass spectrum. For example, if the rank of the unification group is larger than the rank of the SM gauge groups, the \( D \)-term contribution is non-vanishing generically. We may observe the magnitude of the \( D \)-term contribution directly. In the usual arguments, by calculating the RG equations from the SUSY breaking scale to the GUT scale, we can obtain the signatures for the GUT scenarios from the observed sfermion mass spectrum [68]. However, in our cases, we do not always have to calculate the RG equations; if necessary, it is sufficient to partly calculate the RG flow. We will return to this point later.

### 3.2. Upper bound for \( m_{3/2} \) from stability conditions

First of all, we explain the gravitino mass range that we would like to study in our scenario. The lower bound of the gravitino mass is about 50 TeV [8–11], to solve the gravitino problem. Strictly speaking, the lower bound is dependent on the reheating temperature of the inflation. If a low reheating temperature is considered, lower \( m_{3/2} \) becomes possible. But if thermal leptogenesis is adopted for the baryogenesis, the lower bound of \( m_{3/2} \) is not so different from 50 TeV.

If \( m_{3/2} \) is so large that the anomaly mediation contribution becomes dominant, the right-handed sleptons must have negative mass squares [31,32]. Therefore, we have an upper bound for the gravitino mass. The upper bound for the ratio \( m_{3/2}/M_{1/2} \) can be obtained by requiring the positivity of the stop and stau mass squares at the SUSY breaking scale, or at the GUT scale. From Eq. (19), the positivity condition for the right-handed stau mass square at \( \mu \) can be written as

\[
\ln \frac{\mu}{M_{\text{mir}}} \leq \frac{10\pi}{33\alpha_1(\mu)} \left( 1 + 5.5 \frac{\tilde{m}_{\tilde{\tau}^c_R}^2}{M_{1/2}^2} - 1 \right). \tag{32}
\]

Since \( \ln \frac{\mu}{M_{\text{mir}}} = \ln \frac{\mu}{\Lambda_1} + \frac{m_{3/2}}{2M_{1/2}} \), this gives the upper bound for \( m_{3/2} \). If we take \( \tilde{m}_{\tilde{\tau}^c_R} = M_{1/2} \), the upper bound for the gravitino mass becomes \( 222M_{1/2} \) for \( \mu = 1 \) TeV and \( 76M_{1/2} \) for \( \mu = \Lambda_G \). For the stop masses, numerical upper bounds are given in Fig. 1 for \( \tilde{A}_t/M_{1/2} = -1, 1, 2 \). In the calculation, we assume that \( \tilde{m}_{\tilde{t}^c_R} = \tilde{m}_{\tilde{\tau}^c_R} = \tilde{m}_{\tilde{\nu}^c_R} = 0 \) and \( \tilde{m}_{Hu} = 0 \). All sfermion mass squares must be positive, at least at the SUSY breaking scale. For this minimal requirement, roughly \( m_{3/2} \) could be less if \( \tilde{m} < M_{1/2} \), and \( m_{3/2} < 500M_{1/2} \) if \( \tilde{m} < 2M_{1/2} \). If positivity at the GUT scale is required (though this is not necessary for the theory to be consistent), \( m_{3/2} < 200M_{1/2} \) when \( \tilde{m} < 2M_{1/2} \). In the numerical
Fig. 1. Allowed region for the stability conditions at 1 TeV and at the GUT scale $\Lambda_G$ in the $(\tilde{m}/M_{1/2}, m_{3/2}/M_{1/2})$ plain, where $\tilde{m} = \tilde{m}_{\tilde{t}L} = \tilde{m}_{\tilde{t}R} = \tilde{m}_{\tilde{t}L}$. The shaded region is forbidden by the stability conditions at 1 TeV, and the upper side of the dotted line is the region where the mass square is negative at the GUT scale. The upper left figure is for $\tilde{A}_{\tilde{t}}/M_{1/2} = -1$, the upper right figure is for $\tilde{A}_{\tilde{t}}/M_{1/2} = 1$, and the lower figure is for $\tilde{A}_{\tilde{t}}/M_{1/2} = 2$. The star denotes the mirage point. Note that, at the mirage point, the GUT scale stability cannot be satisfied.

Calculations in this paper, we take $m_{t}(\text{pole}) = 173.07$ GeV and the unified gauge coupling $g_{GUT}^2 = 0.48$. We are interested in the region $30 < m_{3/2}/M_{1/2} < 200$ in this paper.

Note that, under the special boundary conditions $M_{1/2} = \tilde{A}_{\tilde{t}} = \sqrt{2}\tilde{m}_{\tilde{t}}$ in mirage mediation, some of the sfermion mass squares become negative at the GUT scale, as seen in the figure. However, under the general boundary conditions, the positivity at the GUT scale can be satisfied.

3.3. Improvement in general cases

In this subsection, we show that, even in the general cases, the fine-tuning can be improved by using a numerical calculation.

First, we explain the improvement in the mirage mediation. Let us evaluate the quantum correction for the Higgs mass $m_{H_u}^2(\mu = 1 \text{ TeV})$ from Eq. (27), obtained in gravity mediation. We express the quantum correction $\Delta m_{H_u}^2 = m_{H_u}^2 - \tilde{m}_{H_u}^2$ as

$$\Delta m_{H_u}^2(1 \text{ TeV}) = c_0 M_{1/2}^2 + c_1 \tilde{\Sigma} + c_2 \tilde{A}_{\tilde{t}}^2 + c_3 \tilde{A}_{\tilde{t}} M_{1/2},$$

where the constants $c_i$ are numerically calculated as

$$c_0 = -1.601, \quad c_1 = -0.396, \quad c_2 = -0.082, \quad c_3 = -0.260.$$
If we set $M_{1/2} = \tilde{A}_t = \sqrt{\Sigma_t}$, we obtain $\Delta m_{H_u}^2 = -2.34 M_{1/2}^2$. In order to obtain the quantum correction for the Higgs mass in mirage mediation, we re-evaluate $c_i$ under the anomaly mediation from Eq. (31). If we set $m_{3/2}/M_{1/2} = 60.0$, we obtain

$$c_0 = 0.291, \quad c_1 = -0.396, \quad c_2 = -0.082, \quad c_3 = 0.156.$$  \hspace{1cm} (35)

If we take the boundary conditions in the mirage mediation as $M_{1/2} = \tilde{A}_t = \sqrt{\Sigma_t}$, we obtain $\Delta m_{H_u}^2 = -0.031 M_{1/2}^2$. These calculations show that the mirage mediation requires more than one order less tuning than the gravity mediation without anomaly mediation. The essential points of this improvement are that the coefficients $c_i$ become small and the cancellation happens because of the different signatures of $c_i$.

These points are also applicable to the more general cases. Therefore, it is obvious that, even for the general cases, some improvement in the tuning can be expected, at least when the ratio $m_{3/2}/M_{1/2} = 60$. Is this improvement realized only in this special value for the ratio? Note that $c_1$ and $c_2$ do not change by including the anomaly mediation. On the other hand, $c_0$ and $c_3$ depend on $M_{\text{mir}}$, namely, $m_{2/3}/M_{1/2}$. Figure 2 shows this dependence. One can see that the absolute values of $c_0$ and $c_3$ are reduced by the anomaly mediation with a wide range of values for $m_{2/3}/M_{1/2}$ within the range we are interested in. Actually, if $29 < m_{2/3}/M_{1/2} < 73$, the condition $|c_i| < 0.5$ is satisfied for $i = 0, 1, 2, 3$. Therefore, we conclude that, even in general cases, some improvements in the tuning problem are expected in our scenario.

Here we numerically check whether the quantum correction of the Higgs mass $\Delta m_{H_u}^2$ can be small. In our scenario, $\Delta m_{H_u}^2$ depends on four parameters: $M_{1/2}$, $\tilde{A}_t$, $\Sigma_t$, and $M_{\text{mir}}$. Hereafter we use $m_{3/2}/M_{1/2}$ instead of $M_{\text{mir}}$. Figure 3 shows $m_h^2/|2\Delta m_{H_u}^2(\mu = m_{\text{SUSY}})|$ in the $(\tilde{A}_t, M_{1/2})$ plain with $\sqrt{\Sigma_t} = 2$ TeV, which corresponds to $\tilde{m}_{i_L} = \tilde{m}_{i_R} = \sqrt{2}$ TeV. Therefore, roughly, $m_{\text{SUSY}} \sim \sqrt{2}$ TeV. The dark gray, gray, and light gray regions correspond to $(m_h^2/2)/|\Delta m_{H_u}^2| > 0.1, 0.02$ and $0.01$, respectively, where $m_h$ is the Higgs mass measured at the LHC as $m_h \sim 125$ GeV. One can see that tuning weaker than 1% is realized in a wide range of parameters. (Strictly speaking, we have to
Fig. 3. $m_{\tilde{H}_u}^2/[2\Delta m_{H_u}^2(m_{\text{SUSY}})]$ in the $(\tilde{A}_t, M_{1/2})$ plain for $m_{3/2}/M_{1/2} = 50$ (upper left), 60 (upper right), 70 (middle left), 80 (middle right), 90 (lower left), 100 (lower right). We take $\sqrt{\Sigma} = 2$ TeV. The shaded region is forbidden by the stability conditions at $m_{\text{SUSY}}$. For reference, the Higgs mass, which is calculated by taking $\mu = m_A = 500$ GeV, $\tan \beta = 10$, and $m_0 = 3$ TeV, is shown by lines for 124 GeV, 125 GeV, and 126 GeV. The mirage point is shown by a star. If we require that $M_3(1$ TeV$) > 1$ TeV, $M_{1/2}$ must be larger than 813 GeV, 971 GeV, 1.22 TeV, 1.64 TeV, 2.44 TeV, and 5.0 TeV for $m_{3/2}/M_{1/2} = 50, 60, 70, 80, 90,$ and 100, respectively. The stability condition $m_{\tilde{e}}^2 \geq 0$ at the GUT scale leads to an upper bound for $M_{1/2}$ as 1.91 TeV, 1.69 TeV, 1.51 TeV, 1.38 TeV, 1.26 TeV, and 1.17 TeV for $m_{3/2}/M_{1/2} = 50, 60, 70, 80, 90,$ and 100, respectively. For large $\tilde{A}_t$, all sfermion mass squares can be positive up to the GUT scale if this condition is satisfied.

address how strong tuning is required for model parameters to be included in these areas. From these figures, we can see that $O(1\%)$ tuning is required in this scenario. Since this value is better than $O(0.1\%)$ tuning for the usual minimal SUGRA boundary conditions, we can conclude that the tuning problem becomes less severe. Note that the amount of tuning to realize small $\Delta m_{H_u}^2$ increases as $M_{1/2}$ becomes large, as seen in Fig. 4. Therefore, the masses of the gauginos should not be much larger than TeV scale if we expect not so large tuning with model parameters to obtain a small $\Delta m_{H_u}^2$.  

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Fig. 4. $m_t^2/|2 \Delta m^2_{H_u}(m_{\text{SUSY}})|$ in the $(M_{1/2}, \sqrt{\Sigma_t})$ plane. We take $\tilde{A}_t = 2$ TeV and $m_{3/2}/M_{1/2} = 60$. If we require $M_3(1 \text{ TeV}) > 1$ TeV, $M_{1/2}$ must be larger than 971 GeV.

One more important feature in general cases is that $A_t$ can be as large as $\sqrt{6} \tilde{m}_t$, which results in the maximal Higgs mass. This is an advantage in the general cases. One can check from Fig. 3 that $|\Delta m^2_{H_u}|$ is not greatly influenced by the value of $\tilde{A}_t$. On the other hand, a large $A_t$ is important to obtain a heavier Higgs. In Fig. 3, we calculate the lightest Higgs mass by using the program FeynHiggs-2.9.5 [69–72] under the additional assumptions that are adopted in Ref. [35]. Namely, we assume that $\tilde{m}^2_{t_L} = \tilde{m}^2_{t_R} = \tilde{\Sigma}_t/2$ and the parameters $\mu$, $\tan \beta$, $m_0$, and the mass of the CP-odd Higgs $m_A$ are fixed by hand at the SUSY breaking scale. (The latter assumption can be adopted if the unknown GUT threshold corrections to the Higgs mass parameters are taken into account, as noted in Ref. [35].) Therefore, we can realize the 125 GeV Higgs with small $\tilde{m}_t$ by setting $A_t/m_t \simeq \sqrt{6}$. Actually, when $50 \leq m_{3/2}/M_{1/2} \leq 90$, a 125 GeV Higgs mass can be realized with a reasonable value for $\tilde{A}_t$, as seen in Fig. 3. On the other hand, if $m_{3/2}/M_{1/2}$ is 100, no line for the 125 GeV Higgs appears because the stop masses are too small when $\tilde{m}_t = \sqrt{2}$ TeV. Note that these numerical results, except for the Higgs mass, can be basically obtained from only four parameters, $M_{1/2}$, $\tilde{\Sigma}_t$, $\tilde{A}_t$, and $m_{3/2}$. Once we fix the other parameters, we can discuss the other phenomenological constraints from the LHC etc. Though it is important to show the allowed region for all parameters, it is beyond the scope of this paper. Here, we just discuss the constraint from the gaugino masses that are determined by $M_{1/2}$ and $m_{3/2}$ as

$$M_a(1 \text{ TeV}) \sim M_{1/2} \left[ 1 + \frac{h_a \alpha_a}{2\pi} \left( -30 + \frac{m_{3/2}}{2M_{1/2}} \right) \right]. \tag{36}$$

This is important since the gluino mass can be strongly constrained by the LHC experiments. We explicitly show $M_a/M_{1/2}$ at 1 TeV for various $m_{3/2}/M_{1/2}$ in Table 1. Note that the gluino mass $M_3$ is vanishing around $m_{3/2}/M_{1/2} \sim 110$. This means that the LHC constraints from the gluino mass can be severe if $m_{3/2}/M_{1/2}$ is around 110, though this cancellation is quite accidental. Actually, requiring $M_3 > 1$ TeV, we have no allowed region for $m_{3/2}/M_{1/2} = 90$ and $\tilde{m}_t = \sqrt{2}$ TeV in Fig. 3. However, we should mention that this result is strongly dependent on the value of $\tilde{m}_t$, because the stability condition is essential. If we take a larger $\tilde{m}_t$, a larger $M_{1/2}$ is allowed and therefore the allowed region must appear, though the fine-tuning must be severer. However, we have shown from

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this numerical calculation that we have a sizable parameter region in which $O(1\%)$ tuning is realized. We do not have to take a special value for $m_{3/2}/M_{1/2}$ to obtain $O(1\%)$ tuning. This result is important. Is it a general feature of this scenario that $\Delta m^2_H$ is dependent quite weakly on $\tilde{A}_t$? This interesting feature can also be understood from the numerical formula (33), which is rewritten as

$$\Delta m^2_H(1\text{ TeV}) = \tilde{c}_0 M_{1/2}^2 + c_1 \tilde{\Sigma}_t + c_2\left(\tilde{A}_t + \frac{c_3}{2c_2} M_{1/2}\right)^2,$$

(37)

where $\tilde{c}_0 \equiv c_0 - c_2^2/(4c_2)$. Since, without the anomaly mediation contribution, all parameters $\tilde{c}_0, c_1$, and $c_2$ are negative, $\Delta m^2_H$ can be neither zero nor small and therefore the tuning becomes worse. However, if the anomaly mediation contribution is sizable, $\tilde{c}_0$ can be positive and, therefore, $\Delta m^2_H$ can vanish. How large an anomaly mediation contribution is needed for positive $\tilde{c}_0$? Numerically, $m_{3/2} \geq 47 M_{1/2}$ is needed. What is important here is that $\Delta m^2_H$ is quite weakly dependent on $\tilde{A}_t$ when $\tilde{A}_t \sim -c_3 M_{1/2}/(2c_2)$, which is derived from $\partial \Delta m^2_H/\partial A_t = 0$. The scale of $M_{1/2}$ for vanishing $\Delta m^2_H$ can be determined by the cancellation condition for the first two terms in Eq. (37) as $M_{1/2} \sim \sqrt{-c_1 \tilde{\Sigma}_t}/\tilde{c}_0 = \sqrt{-2c_1/\tilde{c}_0 m_t}$. Note that the ratio $m_t/M_{1/2} = \sqrt{-\tilde{c}_0/(2c_1)}$ is important in deriving the stability conditions, as in Fig. 1. These values for various $m_{3/2}/M_{1/2}$ are found in Table 2.

From both relations $\tilde{A}_t \sim -c_3 M_{1/2}/(2c_2)$ and $M_{1/2} \sim \sqrt{-c_1 \tilde{\Sigma}_t}/\tilde{c}_0 = \sqrt{-2c_1/\tilde{c}_0 m_t}$, an interesting relation $\tilde{A}_t/\tilde{m}_t = \sqrt{-c_1 c_3^2/(2\tilde{c}_0 c_2^2)}$ is obtained. Surprisingly, over a very wide range of $m_{3/2}/M_{1/2}$, the coefficient $\sqrt{-c_1 c_3^2/(2\tilde{c}_0 c_2^2)}$ is around 2, as in Table 2. This means that an interesting feature is generally realized in this scenario, that $\Delta m^2_H$ is quite weakly dependent on $\tilde{A}_t$ around $\tilde{A}_t \sim 2$, and therefore we can obtain the 125 GeV Higgs more easily by taking a large $\tilde{A}_t$.

The lower bound for the ratio $m_{3/2}/M_{1/2}$, which realizes $\Delta m^2_H = 0$, is also shown in Fig. 5, in which $\tilde{A}_t = \sqrt{\tilde{\Sigma}_t} = 2$ TeV. This lower bound is consistent with the above arguments from the numerical formula (37). Even the upper bound for the ratio $m_{3/2}/M_{1/2}$ is seen in Fig. 5. The upper bound becomes lower than the value discussed above, because $\tilde{A}_t$ is fixed at 2 TeV in the numerical calculation in Fig. 5. Interestingly, the lower bound for $M_{1/2}$ is seen in the figure.

**3.4. Strategy for testing GUT in general cases**

In this subsection, we discuss how to obtain the signatures for GUT scenarios in general cases from the mass spectrum of SUSY particles, which is assumed to be observed by experiments here. As noted in the previous section, the top Yukawa contribution spoils the cancellation between the RG and anomaly mediation contributions for the up-type Higgs mass and stop masses at $M_{\text{mir}}$. However, for

| $m_{3/2}/M_{1/2}$ | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 180 | 200 |
|-------------------|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|
| $M_1/M_{1/2}$     | 0.90 | 0.99 | 1.08 | 1.17 | 1.26 | 1.35 | 1.44 | 1.53 | 1.62 | 1.71 | 1.80 | 1.89 | 2.07 | 2.25 |
| $M_2/M_{1/2}$     | 0.97 | 1.00 | 1.02 | 1.05 | 1.07 | 1.10 | 1.12 | 1.15 | 1.18 | 1.20 | 1.23 | 1.25 | 1.30 | 1.36 |
| $|M_3|/M_{1/2}$     | 1.23 | 1.03 | 0.82 | 0.61 | 0.41 | 0.20 | 0.01 | 0.21 | 0.42 | 0.63 | 0.83 | 1.04 | 1.45 | 1.87 |
Table 2. Coefficients $c_0$, $c_3$, etc. for various $m_3/2/M_{1/2}$. $\Delta m_{H_d}^2 = 0$ and $\partial \Delta m_{H_d}^2/\partial A_t = 0$ lead to $\sqrt{-c_0/(2c_1)} = m_t/M_{1/2}$, $-c_3/(2c_2) = \tilde{A}_t/M_{1/2}$, respectively, and therefore $\sqrt{-c_1c_3^2/(2c_0c_2^2)} = \tilde{A}_t/m_t$.

| $m_3/2$ | 10 | 30 | 50 | 60 | 70 | 80 | 90 | 100 | 120 | 150 | 200 |
|-------|----|----|----|----|----|----|----|-----|-----|-----|-----|
| $c_0$ | -1.177 | -0.458 | 0.085 | 0.291 | 0.453 | 0.572 | 0.646 | 0.677 | 0.608 | 0.176 | -1.418 |
| $c_3$ | -0.191 | -0.052 | 0.087 | 0.156 | 0.225 | 0.295 | 0.364 | 0.433 | 0.572 | 0.780 | 1.127 |
| $\tilde{c}_0$ | -1.066 | -0.450 | 0.108 | 0.365 | 0.607 | 0.837 | 1.050 | 1.249 | 1.606 | 2.031 | 2.454 |
| $\sqrt{c_0/2c_1}$ | - | - | 0.369 | 0.679 | 0.876 | 1.023 | 1.152 | 1.256 | 1.425 | 1.603 | 1.761 |
| $c_3/2c_2$ | -1.165 | -0.317 | 0.530 | 0.951 | 1.372 | 1.799 | 2.220 | 2.640 | 3.488 | 4.756 | 6.872 |
| $\sqrt{c_1c_3^2/2\tilde{c}_0c_2^2}$ | - | - | 1.435 | 1.401 | 1.567 | 1.750 | 1.927 | 2.101 | 2.449 | 2.968 | 3.903 |

Fig. 5. $m_h^2/2\Delta m_{H_d}^2(m_{SUSY})$ in $(M_{1/2}, m_3/2/M_{1/2})$ plain. We take $\tilde{A}_t = \sqrt{\Sigma_t} = 2$ TeV.

the other sfermion masses and the gaugino masses, the cancellation at $M_{\text{mir}}$ is still valid. Therefore, from the mass spectrum of the gauginos, we can obtain the mirage scale $M_{\text{mir}}$ by calculating the RG equations for gaugino masses. Once the mirage scale is known, we can obtain the gravity contribution to the masses of the sfermions other than two stops by calculating the RG equations from the SUSY breaking scale to the mirage scale. The method of testing a concrete GUT scenario by this strategy is beyond the scope of this paper. We will study this subject in the future.

4. Summary and discussion

We have shown that, if we require that $m_3/2 \sim O(100 \text{ TeV})$ for solving the gravitino problem and the other SUSY breaking parameters are $O(1 \text{ TeV})$ for the naturalness, the little hierarchy problem becomes less severe. The essential point is that, in such a situation, the anomaly mediation contribution becomes sizable, which can generically lower the messenger scale of the gravity mediation effectively.
If the Yukawa coupling is negligible, all the RG evolutions of gaugino mass, scalar mass, and trilinear coupling are canceled at the same scale $M_{\text{mir}}$ by the anomaly mediation contribution. However, the Yukawa contribution breaks the complete cancellation at $M_{\text{mir}}$ for the scalar and trilinear coupling. In practice, the large top Yukawa coupling spoils the cancellation at $M_{\text{mir}}$ for the stop masses, up-type Higgs mass, and $A_t$. One possibility to allow the top Yukawa contribution to vanish is that special boundary conditions are adopted, such as mirage mediation. These special boundary conditions are applied only for the stop masses and up-type Higgs masses, and, therefore, we have no constraints for the other sfermion masses. First, we discussed the generalization of mirage mediation. It is interesting that the natural SUSY mass spectrum is consistent with the mirage-type boundary conditions. Second, we considered another possibility in which we do not have special boundary conditions for the gravity contributions. We have shown that, even in such general cases, the tuning is improved over the wide range of parameter spaces in which we are interested. An attractive feature of this scenario is that it has the flexibility of the mass parameters at the cutoff scale because we need not exactly cancel the top Yukawa contribution. We can get large values such as $A_t/m_{\tilde{t}} \simeq \sqrt{6}$, which is important for realizing a 125 GeV Higgs with smaller $m_{\tilde{t}}$.

One of the disadvantages of gravity mediation is that the universality of the sfermion masses, which are important in solving the SUSY FCNC problem, is not guaranteed generically. One interesting possibility is to introduce flavor symmetry to realize the universality. One of the most interesting symmetries is $E_6 \times SU(2)^F$, which realizes the modified universality in which the third generation $10$ of $SU(5)$ can have a different mass $m_3$ than the other sfermion mass $m_0$. If we take $m_0 \gg m_3 \sim 1$ TeV, this is nothing but the natural SUSY-type SUSY breaking parameters.

Our new scenario has several interesting features. First, the mirage scale $M_{\text{mir}}$, where the quantum corrections for the gaugino and scalar masses that do not couple with the top vanish, need not just be on the TeV scale. The scale $M_{\text{mir}}$ can be smaller than the weak scale, so long as the correction of the Higgs mass is not so large. Then the lightest gaugino may be the gluino, unlike the TeV-scale mirage mediation.

Second, this model predicts that the mass difference of two stop masses is around the weak scale, even if these masses are around the TeV scale. Suppose two stop masses from gravity mediation unify at the cutoff scale:

$$ \tilde{m}_{\tilde{t}_L}^2 = \tilde{m}_{\tilde{t}_R}^2. \tag{38} $$

This is expected from the GUT models such as $SU(5)$. The top Yukawa contribution splits these masses even at the mirage scale $M_{\text{mir}}$. However, these masses nearly degenerate if $\Delta m_{\tilde{H}_u}^2$ is small because the relation

$$ \Delta m_{\tilde{t}_L}^2 - \Delta m_{\tilde{t}_R}^2 = -\frac{1}{3} \Delta m_{\tilde{H}_u}^2 + \frac{M_{\tilde{t}}^2}{2\pi} (-2\alpha_2 + \frac{2}{3}\alpha_1) \ln \frac{\mu}{M_{\text{mir}}} + \frac{M_{\tilde{t}}^{1/2}}{8\pi^2} (-4\alpha_2^2 + \frac{132}{25}\alpha_1^2) \left( \ln \frac{\mu}{M_{\text{mir}}} \right)^2 \tag{39} $$

can be found. Note that the QCD and top Yukawa contributions cancel between the two stop masses; therefore, the mass difference is approximately proportional to the correction of the Higgs mass.

In this paper, we have focused on the physics that can be discussed by considering the specific parameters, $M_{\tilde{t}}$, $\Sigma_t$ (or $m_3$), $\tilde{A}_t$, and $M_{\text{mir}}$ (or $m_{3/2}$). Actually, all figures in this paper are based on these parameters, except in calculating the Higgs mass. However, in some cases, other parameters can be important. For example, it has been pointed out that, when
$m_0$ is much larger than $m_3$, two loop RG effects give sizable negative contributions to the stop mass square, which makes the constraints in Fig. 1 severer. Additionally, of course, in order to discuss phenomenological constraints from the LHC, or FCNC processes, the other parameters must be fixed. For example, if we take $M_{1/2} = 2$ TeV, $\tilde{\Sigma}_t = (2$ TeV$)^2$ ($\tilde{m}_t = \sqrt{2}$ TeV), $m_{3/2}/M_{1/2} = 70$, $\tilde{A}_0 = 3.5$ TeV, $\tilde{m}_0 = 3$ TeV, $\tan \beta = 10$, and $\mu = m_A = 0.5$ TeV, then we can obtain the parameters at the scale $m_{\text{SUSY}} = 1130$ GeV as $M_3 = 1630$ GeV, $M_2 = 2046$ GeV, $M_1 = 2162$ GeV, $m_{\tilde{g}_1} = m_{\tilde{g}_2} = 2743$ GeV, $m_{\tilde{\nu}_R} = m_{\tilde{e}_R} = 2784$ GeV, $m_{\tilde{\mu}_R} = m_{\tilde{\tau}_R} = 2784$ GeV, $m_{\tilde{\tau}_L} = m_{\tilde{\tau}_3} = 2948$ GeV, $m_{\tilde{\mu}_L} = m_{\tilde{\tau}_3} = 1012$ GeV, $m_{\tilde{\tau}_R} = 1252$ GeV, $m_{\tilde{\nu}_R} = 1370$ GeV, $A_u = A_c = 2981$ GeV, $A_t = 2095$ GeV, $A_d = A_s = A_b = 2693$ GeV, $A_e = A_\mu = A_\tau = 3317$ GeV, and $m_h = 126.0$ GeV. Phenomenological constraints from the LHC can be satisfied in this example. The constraint from $b \to s\gamma$ may be sizable [73] but must be weaker because the stop and the chargino are heavier than in Ref. [73]. (Also, the final allowed region is dependent on the SUSY mixing parameters, which have not yet been fixed.) Though it is also important to show the allowed region with all the SUSY breaking parameters, most of which have not been fixed here, this subject is beyond the scope of this paper.

If naturalness is required, the Higgsino mass $\mu$ must not be much larger than the weak scale. Therefore, the lightest SUSY particle (LSP) can be expected to be the Higgsino. If it is additionally required that the thermally produced Higgsino abundance is consistent with the observed abundance of dark matter, we can obtain further constraints on the SUSY parameters. We do not discuss this direction in detail.

One of the most important features in our scenario is that, if the mirage scale is around the SUSY breaking scale, the signatures of the GUT scenarios can be observed directly by observing the mass spectrum of SUSY particles. It is difficult to reach the GUT scale directly by experiments, while the SUSY GUT is the most promising candidate for physics beyond the SM. Therefore, it becomes quite important that future experiments can observe the signature of the SUSY GUT, e.g., through the $D$-term contributions to the sfermion masses.

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