Generalized optimality criteria of energy-efficient composites

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Abstract. The article discusses generalized optimality criteria of energy-efficient materials. The task of choosing an optimality criteria of a composite is considered as a step-by-step, hierarchical transition from individual criteria of the first level - operational characteristics to combined ones, related to subsequent levels and then to the general one. At each hierarchical level, the criteria become more complicated as it considers the criteria of the previous levels. The criteria constructed in this way are classified into structural, relative and conditional. It is shown that the softening coefficient and the water resistance index correspond to the criteria of the second and third levels, respectively. Some of the new combined criteria, having the character of efficiency were reviewed. As an example of a combined criteria, the criterion of the effectiveness of the realization of the strength-thermal conductivity ratio was considered. Examples of its use for evaluating and comparing the effectiveness of materials with significantly different properties and areas of use were given. It is shown that the efficiency criterion has similar values for materials with equal properties combination. On the basis of the proposed criterion, the problem of optimizing the efficiency of the plaster material is considered. As a result of numerical optimization, compositions with the maximum efficiency of the considered type were obtained. The possibility of further application of this and other combined optimality criteria in the problems of quality assessment and optimization of composite materials was shown.

1. Introduction
The main modern trend in the area of building materials science is the creation of multifunctional materials that simultaneously satisfy a set of requirements: energy-saving properties, high strength, adhesion and sound insulation, optimal environmental and economic characteristics. The current stage of the research development in the area of building materials science is associated with the development of multicomponent composite materials, which presupposes a preliminary assessment of their quality and a compromise optimization of their properties [1]. Due to this trend, it seems rational to consistently complicate the necessary optimality criteria of building composites.

2. The purpose of the research
This process can be based on a sequential grouping of individual characteristics, which ultimately leads to a single generalized criterion fig.1. In this case, the characteristics usually used for optimization (for example, thermal conductivity $\lambda$, compressive strength $R_c$ and bending strength $R_b$, density $d$) can be represented as first level individual criteria.
Figure 1. Hierarchy of optimality criteria for composite materials. I, II, III – hierarchical levels of criteria, \( K_s \) – softening coefficient, \( K_w \) – water resistance index, \( R_c(w) \) and \( R_c(d) \) – strength in water-saturated and dry states, respectively.

At each hierarchical level, the criteria are complicated by taking into account the criteria of the previous levels. Such complicated criteria of II, III and higher levels will be called combined. In this case, the combined criteria in a number of cases can be of independent interest and be useful for the research and optimization of materials of different classes.

3. Methods and Results of the Research

The most reasonable is the use of expressions in the form of products of criteria degrees of the previous level in a normalized form \( \bar{C}_i = \frac{C_i}{C_{\text{max}}} \) (normalization is made for the maximum value of a property in a group or for an interval of property change \( \bar{C}_i = \frac{C_i - C_{\text{min}}}{C_{\text{max}} - C_{\text{min}}} \)). Combinations of criteria are carried out according to the rule:

\[
    C_{\text{comb}} = \prod_i \bar{C}_i^{n_i}
\]

This introduction of combined criteria ensures its maximum value while simultaneously fulfilling the optimality conditions for all the criteria of the previous level. For example, the combined criterion - the water resistance index is equal to the product of the water-saturated strength (one criterion) by the softening coefficient (the second criterion) [2]:

\[
    \dot{E}_w = \left( \frac{R_{w_s}}{R_d} \right)^2 = R_w \cdot K_p
\]

The use of this combined criterion is quite effective in optimizing heat and sound insulating building composites with increased water resistance.

The proposed combined criteria can be classified as follows.

3.1. Structural criteria

They describe the relation between the structure and properties of the material, indicate the effectiveness of the implementation of the specified properties on the basis of the structure of the material organized using the control of recipe-technological factors. They have a general form \( C_{SP} = P^n S^{-1} \), where \( P \) – desired property, \( n=1,-1 \), \( S \) – a measure of the number of structural units per volume, in the simplest case - density \( d \). Examples of structural criteria:
a) \[ C_{dl} = \frac{1}{d \lambda} \] – one of the criteria for the thermal insulation capacity of a material. Great for lightweight heat-insulating composites. By varying the structure of the material, one can solve the problems of structural optimization and, in particular, the problem of a conditional maximum \( C_{dl} \) - to search the least dense material with a given thermal conductivity \( \lambda \), or to minimize the thermal conductivity at a known density \( d \).

b) \[ C_{dr} = \frac{R}{d} \] – effective strength criteria (bending, compression, adhesive). Shows how effectively the structure of the composite is organized for the implementation of the specified strength characteristics. Great for lightweight but durable materials. Such criteria can be used to solve optimization problems similar to those discussed above.

3.2. Relative criteria
Shows how much the desired property will increase with decreasing undesirable property by one, and vice versa. They are effective when taking into account the properties that change symbatically (for example, strength \( R \) – thermal conductivity \( \lambda \)). General view \( C_r = \frac{P_d}{P_{ud}} \) – ratio of indicators of the desired property \( P_d \) to the indicator of undesirable \( P_{ud} \). Example: \( C_{R,\lambda} = \frac{R}{\lambda} \) – efficiency indicator of the strength - thermal conductivity ratio. Great for energy-saving materials with increased strength. Since both of these properties are structurally determined, this criterion is combined with respect to structural.

3.3. Conditional criteria
Determine the degree of change in the properties of the material with a change in the technological factors of its manufacture or in case of extreme exposure (humidification, freezing, high temperature). This is the ratio of the conditional (for example, in extreme conditions) characteristics to the initial \( C_c = \frac{P_c}{P} \times 100\% \); great for materials that are resistant to impact. One of the used criteria of this type is the softening coefficient \( K_p = \frac{R_p}{R_d} \), equal to the ratio of the strengths of the material in the water-saturated state \( R_p \) and dry state \( R_d \). So, the softening coefficient for dry gypsum \( K_p = 0.4 - 0.5 \), and in gypsum composites with loose hygroscopic fillers (for example, perlite, vermiculite) - even less, up to spontaneous destruction in a wet state. Creation of multicomponent materials based on gypsum (for example, gypsum-cement-slag binders - GCSB) and the use of hydrophobization makes it possible to obtain waterproof materials based on gypsum (\( E_\delta = 0.7 - 0.8 \)).

The degree of usefulness of the combined optimality criteria is determined, first of all, by the possibilities and results of their application in the practice of building materials science. The advantages of using them are as follows: criteria of the second and higher levels (fig.1) make it possible to reduce several operational properties to one value.

These values can be used to compare materials from different areas of application that have significantly different properties, but similar (comparable) values of the corresponding criteria. So materials with different sets of performance properties can be combined due to similar values of such criteria. Their general nature and interpretation can be made on the basis of the concept of the implementation effectiveness of the given relations. Example could be \( C_{R,\lambda} \). Consider the implementation of the comparison of the effectiveness of this type for composites of different nature and purpose. Compare efficiencies of 4 different composite materials type - gypsum perlite plaster composition [3] (hereinafter - the material samples 1, 18), heat-insulating polystyrene-gypsum...
concrete [4] (hereinafter - the material samples 2, 15) heat-insulating composition based on gypsum concrete [5] (hereinafter - the material samples 3, 15), lightweight spherogypsum concrete for tongue-and-groove slabs [6] (hereinafter - the material samples 4, 18). Their compressive strength $R_c$ and thermal conductivity ($\Lambda$), measured by normative methods, are shown in fig.2.

Figure 2 shows that materials 1 and 2 are characterized by similarity in the properties under consideration, while materials 3 and 4 differ significantly from the first group and differ from each other. Statistical researches of the combined group of materials are difficult. One option is to use an effectiveness parameter $\lambda_{RC}$. The values of this parameter for different samples of the corresponding materials 1–4 are shown in fig.3.

For materials substantially different in their initial properties $R_c$, $\Lambda$ of the materials in fig.3 regions of intersection of values $\lambda_{RC}$ are observed. Some samples from series 1–4 have similar values $\lambda_{RC}$. If we use $\lambda_{RC}$ to evaluate materials only by two properties, then the efficiency of materials 2 and 4 turns out to be higher than 1 and 3.

From the point of view of practical application, similar values of the considered efficiency and other similar criteria combine materials with the same efficiency. The choice of the material to use from such a group is carried out according to other characteristics (sound absorption, cost, etc.).
Figure 3. Comparison of values \( C_{R\lambda} \) for composite materials 1–4.

The intersection area \( C_{R\lambda} \) for different materials is highlighted.

Another way to apply the combined criteria is to use them as an indicator of the optimality of composites. They are easily calculated on the basis of already existing experimental data; regression (experimental-statistical) models can be built for them.

The criterion under consideration can, in particular, be used in experimental and statistical modeling of gypsum-perlite heat and sound insulating composition [3].

To study the properties and optimization of the material, a four-factor optimal experimental plan was used, the main factors and the levels of their variation were determined and shown in Table 1.

Table 1. Main factors and levels of their variation

| i | Factors and levels of variation | \( X_i = -1 \) | \( X_i = 0 \) | \( X_i = +1 \) |
|---|---------------------------------|---------------|---------------|---------------|
| 1 | Perlite sand amount (volumetric parts per 1 part of gypsum), P/G | 10            | 15            | 20            |
|   | The amount of microspheres (Mks) and metakaolin (Mk) in their mixture (5% of the gypsum volume), MksMk | 5% Mks       | no 2.5% Mks and Mk | 5% Mk |
| 2 | Superplasticizer dosage (% of gypsum volume), Plast | 0.5          | 0.75          | 1.0          |
| 3 | Latex content (% to the volume of the liquid mixture), Lat | 1            | 1.5           | 2            |

On the basis of the values of compressive strength \( R_c \) and thermal conductivity \( \lambda \) (Lambda) obtained for 18 samples, the values of \( C_{R\lambda} (R_c / \lambda) \) were calculated. For this characteristic an experimental-statistical (ES) model was built [7] fig.4.
The resulting model can be displayed as follows (fig.5.)

\[
\text{Rc}_i \text{Lambda} = +1.152 \cdot \text{MksMk} \cdot \text{Plast} \\
+9.949 \quad -1.653 \cdot \frac{P}{G} \\
-3.553 \cdot \frac{P}{G} \quad +2.510 \cdot \text{MksMk}^2 \\
+0.473 \cdot \text{MksMk} \quad \text{R-Squared} \quad 0.933 \\
+0.302 \cdot \text{Plast} \quad \text{Adj R-Squared} \quad 0.886 \\
+0.850 \cdot \text{Lat} \quad \text{Pred R-Squared} \quad 0.819
\]

**Figure 4.** ES-model of the efficiency indicator and its statistical characteristics

**Figure 5.** Graphical representation of the efficiency factor model \( C_{Ri} \)

ES-model of efficiency (fig. 4,5) allows to specify the values of the factors, necessary to achieve maximum efficiency \(- \frac{P}{G} = -1, \text{MksMk} = 1, \text{Plast} = 1, \text{Lat} = 1\). As the perlite-gypsum ratio decreases, the efficiency increases. It is characterized by the maximum values at the boundary points of the factor MksMk (1, -1), the ratio of the efficiency levels at the boundary points varies depending on the factors Plast, Lat.

The obtained efficiency criterion can be used in the numerical optimization procedure. The following structure of optimization tasks is proposed: individual operational properties (the first level criteria, fig. 1) are set within the framework of regulatory constraints, and the performance criteria are maximized. The optimization goals and the corresponding boundary values and importance are shown in Table 2.
Sound permeability is a value equal to the ratio of the energy of the transmitted radiation to the energy of the incident radiation, $\frac{E_{\text{transmitted}}}{E_{\text{incident}}}$. It shows the relative decrease in the force of sound when it passes through the thickness of the building material.

As a result of efficiency optimization, the compositions were obtained (table 3.)

| Properties          | Goal       | Min. | Max. |
|---------------------|------------|------|------|
| Sound*              | in interval| 0,3  | 0,5  |
| Rec/Lambda          | maximum    | 8    | 20   |

4. Conclusions

Thus, the proposed strategy of using combined (generalized) optimality criteria of energy-efficient composites makes it possible to assess the effectiveness of multicomponent materials of increasing complexity based on individual operational characteristics. These performance criteria are useful for comparing materials with different sets of operational properties. "Equivalent" materials are characterized by similar values of the combined criteria. Some of them (water resistance index $K_w$, “strength-thermal conductivity” efficiency) can serve as the main part of the objective function for optimizing the structure of composites. For them, as well as for conventional operational characteristics, it is possible to build ES models. Application of optimization according to combined criteria leads to a decrease in the proportion of arbitrarily introduced auxiliary information of a subjective nature, which, in particular, improves the reproducibility of the optimization of the compositions of composite materials.

Thus, the development of combined optimality criteria seems to be a promising direction in building materials science, leading to the improvement of design processes, optimization and evaluation of the effectiveness of composite materials.

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