Thinking development and aesthetic education of students in the process of teaching mathematics by example solutions for one problem

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Abstract: The article discusses one of the leading goals of teaching mathematics - the development of students' thinking by means of mathematics, it is noted that this requires the creation of special situations, the resolution of which leads to the development of thinking. The possibility of developing logical and figurative thinking of students of secondary schools in the process of aesthetic education using the subject of mathematics is investigated. The beauty of problem solving allows you to increase interest in the study of mathematics. The development of intellectual and creative capabilities, their imagination, as well as broadening the horizons in the application of methodological programs in teaching and evaluating students' knowledge and skills. The present work is presented on the example of solving one problem by several mathematical methods. Using a mathematical model to solve various kinds of problems with the greatest number of solutions contributes to aesthetic education, allowing you to develop a culture and logical thinking, to form in students the optimal analysis of the task in solving it. The work is presented in an experiment on organizing aesthetic education by knowing the beauty of problem solving, which will fully reveal the criteria for applying the individual approach to learning through the development of thinking in solving various kinds of mathematical problems. The organization of the intensification of the process of mastering knowledge and applying aesthetic beauty is shown by the example of solving problems in mathematics of grades 7-9. We carried out a number of experimental works on the development of thinking and aesthetic education, relying on the activity-based approaches of educational and developmental teaching of mathematics.

Key words: development of students' thinking, aesthetic education, mathematical education, assessment, knowledge, skills.

1. Introduction

Among the main directions of modernization of education, one can single out the personal orientation of the content of education, the active nature of education, the focus of education on the formation of generalized methods of various types of activities, the formation of key competencies, the focus of education on the development of the student’s personality, his cognitive and creative abilities. The development of a person’s personality involves the development of his thinking, and in this regard, mathematics has great potential.
2. Main part

V.M. Tikhomirov writes about this: “The goal of mathematical education, in my humble opinion, should be, first of all, development. The development of skills in operating with numbers and figures, spatial imagination, logical thinking - in a word, the development of intelligence. Nothing can teach this better than mathematics, - the whole experience of mankind speaks of this” [1].

A.V. Borovsky and N.Kh. Rozov [2], sharing the point of view expressed above, note that academic subjects should be viewed as means, tools of training, education and development.

The same idea was expressed in a resolution of the Congress of Mathematics Teachers, which was convened again in October 2010 after a hundred-year break: “... mathematical education is the most important and necessary component of personality development ...” [3].

Note that the development of intelligence refers mainly to the development of thinking. It is necessary to search for the bases of the development of thinking of students not only in the content and technologies of teaching mathematics, but also in psychology. Thinking should be developed through various types of educational activities performed by students in learning mathematics, and this development should go through such stages as visual-effective thinking, visual-figurative thinking, and verbal-logical thinking.

We, following N.N. Pospelov, we will mean by the development of thinking, students in the learning process "the formation and improvement of all types, forms and operations of thinking, the development of skills for applying the laws of thinking in cognitive and educational activities, as well as the ability to carry out the transfer of methods of mental activity from one area of knowledge to another" [4].

Here are examples of situations that are created in order to organize the educational and cognitive activities of students with the aim of developing their thinking.

1. The challenge. Point M moves along the sides of the square (Figure 1).

![Figure 1. Location of a material point in the square boundaries](image1)

By analogy with the known trigonometric functions, we introduce new functions: sanα = x – y (the sine of the angle α); casα = x + y (the casanus of the angle α); tigα = x² – y² (tigent of angle α). Complete the following tasks:

- establish links between these functions;
- plot the graphs of these functions;
- set the set of values of each of the square trigonometric functions;
- define the signs of the square trigonometric functions in the quarters marked is shown in Figure 2;
- establish a connection between classical and square trigonometric functions;
- set the algorithm for solving square trigonometric equations: sant = ɑ, cast = ɑ, tigt = ɑ, etc.

The situation described above is artificially created, but according to A.S. Krygovskoy: “The significance of creative activity itself is not what it has created” [5].

2. In the school course of mathematics, only arithmetic progressions with constant differences d = const are considered. The concept of arithmetic progression allows a completely natural generalization, if we assume that the difference of the arithmetic progression itself will be a function of the natural argument, that is, dn = d(n). In this case, we will deal with arithmetic progressions with variable differences. The study of such progressions should be carried out in the same way as the study of ordinary arithmetic progressions: the formula of the general term, the formula of the sum of the n first terms of the progression, a characteristic property, etc.
3. It is known that inside any triangle ABC (see Figure 3) there exists a point P (and there are two) such that.

![Figure 3. Krell-Brokar’s triangle point composition](image)

Points P and P1 are called Krell-Brokar points.

a) Perform the following research task: Find ways to build Krell-Brokar points using a computer. (We give suggestive hints: similar triangles are built on the sides of the original triangle or formulas are used for the coordinates of the Krell-Brokar points.)

4. Solve the problem: “A trapezoid is given (see Figure 4) with bases of 2 cm and 5 cm. The lateral side of the trapezoid is divided into three equal parts. Through the points of division drawn straight parallel to the bases of the trapezoid. Find the lengths of the obtained segments.”

The lengths of the segments MN and KP can be found by solving the following system of equations:

\[
\begin{align*}
MN & = \frac{2 + KP}{2}, \\
KP & = \frac{5 + MN}{2}.
\end{align*}
\]

Solve the same problem for the case when the side is divided into 6 equal parts. It is clear that this problem should not be solved in the same way as the previous one (we will have a system of five equations with five unknowns). This problem should be solved (through the points D1, D2, D3, D4, D5, C, segments parallel to the side of the trapezoid AB are drawn) is shown in Figure 5.

![Figure 5. Trapezoid with five’s midlines](image)

Summarize these two tasks in the case when the bases of the trapezoid are equal, a and b, and the side is divided into n equal parts.

In the works of V.A. Dalinger [6-8], the reader will find many tasks of this nature.

We examined one aspect of the problem raised in the article, but another aspect is stated in the title of the article - the aesthetic education of students in the process of teaching mathematics. Let us dwell on this issue more systematically.

In the process of creating a harmoniously developed personality, the aesthetic education of students is of paramount importance. The role of mathematics as one of the important subjects of the school is difficult not to notice in aesthetic education, the potential of mathematics in this regard is huge. Mathematics is very rich in beautiful formulas, proofs, multi-type methods for solving problems and you can specify entire sections
where the multifaceted elements of beauty and magnificence are hidden, such as “Trigonometry”, “Golden Section”, “Symmetry”, “Algebra and Number Theory”, “Geometry in space”, excellent for aesthetic education.

The effective development of the wide potential of mathematics presupposes a full-fledged perception of mathematical literacy, the cultivation of aesthetic feelings, taste and ideal through imaginative thinking and logical culture, the formation of a person’s value orientation in his quest for the beauty of original problem solving.

In various pedagogical literature, aesthetic education is considered as a system of measures for the development of good artistic tastes in a person, the ability to correctly and truly judge the beautiful in art. According to scientists authors, “... education through beauty is, on the one hand, an important means of developing educational motivation and, on the other hand, a source of emotionality of a person as one of the leading components of aesthetic culture” [9-23].

One of the founders of the development of the educational process problem is the outstanding teacher Yu.K. Babansky, he had and has a theory in conjunction with the educational process. The author notes that the aforementioned components of the aesthetic culture at the same time act as criteria for the aesthetic education of students. They determine the tasks and content of the aesthetic education of schoolchildren [9].

In the late 80s of the twentieth century V.P. Bespalko [10, p.31] introduced the learning process by the formula:

\[ D_{pr} = M + P_D + Y \]

where \( D_{pr} \) - is the didactic process; \( M \) - motivation student for learning; \( P_D \) - his cognitive activity; \( Y \) - student activity management.

In the training manual P.I. Obraztsov and V.M. Kosukhin refers to the statement of V.D. Dyachenko: “... the content of training is always some kind of activity that, to one degree or another, is owned by the teacher and is not fully or partially owned by the student” [11].

3. Research methods

Teacher G.P. Bursa notes that aesthetic education in a comprehensive school is to instill in students good tastes, correct concepts, views and opinions in the field of music, painting, literature, etc. [12].

In the dictionary of pedagogy G.M. Kojaspirova, A.Yu. Kojaspirova pointed out that aesthetic education is the development and improvement in a person of the ability to perceive, correctly understand, value and create beauty in life and art, actively participate in creativity, creation according to the laws of beauty [13].

In addition, I.F. Kharlamov emphasizes that aesthetic education is organically linked to the term “aesthetics”, which means the science of beauty. The very word aesthetics comes from the Greek aesthesis, which in translation into Russian means sensation, feeling. Therefore, in general terms, aesthetic education denotes the process of forming feelings in the field of beauty [14].

N.I. Firstova emphasizes that aesthetic education should be considered as an integral part of the comprehensive development of the individual. Through aesthetic education, it is possible to expand and deepen the knowledge and ideas of schoolchildren about reality, the formation of their views [15].

G.I. Sarantsev notes that much has been written about the beauty of mathematics. Authors see the beauty of mathematics in:

- harmony of numbers and forms;
- geometric expressiveness;
- harmony of mathematical formulas;
- the grace of mathematical evidence;
- order;
- a wealth of applications;
- universality of mathematical methods.

According to G.I. Sarantseva, the peculiarity of mathematics lies not only in that, as in art, it has a huge aesthetic potential, but also in the fact that mathematical activity obeys the laws of beauty [16]. Therefore, mathematics, along with art, is considered the most important means of introducing students to beauty, the formation of their aesthetic taste. Opinion, W.U. Sawyer, speaking of the importance of mathematical theory, calls beauty, harmony, “so attractive to the mind” as one of its indicators [17].
So, D. von Neumann noted that mathematics, like art, is driven almost exclusively by aesthetic motives. J. Hadamard argued that a scientist, seeing a structurally imperfect, asymmetric, “crooked” mathematical construction, begins to feel the need for active work to harmoniously supplement it [18]. According to V.G. Boltiansky, the beauty of a mathematical object can be expressed through isomorphism between the object and its visual model, the simplicity of the model and the unexpectedness of its appearance. This statement can be supported by the formula of “mathematical aesthetics” from his article “Mathematical Culture and Aesthetics” [19]:

\[ \text{BEAUTY} = \text{VISIBILITY} + \text{UNEXPECTED} = \text{ISOMORPHISM} + \text{SIMPLICITY} + \text{UNEXPECTED}. \]

The beauty of objects reveals their property, which exists independently of consciousness, says A.V. Shcherva: “The sense of beauty is interpreted by the author as a product of the reflection in the human mind of the really existing aesthetic properties of the surrounding world” [20]. The psychological basis of this interpretation is presented in the intuitive attraction of the human psyche to grace and harmony, comprehended by feelings.

From here, from all the theses examined, it is possible to derive methodological recommendations on the formation of the aesthetic education of the subject of mathematics among schoolchildren.

One of the most important tasks in the formation of a person’s worldview is the problem of the formation of an aesthetic attitude to mathematics, as is clear from the above quotes. When teaching mathematics, school students can and must learn to perceive, feel the beauty of mathematical expressions, theoretical constructions, evaluate the wide possibilities of a mathematical culture from an aesthetic point of view. To reveal to students the beauty of the content of mathematics, which will use special methods to develop creativity in the classroom, use these methods to assess students' knowledge, prepare them for life in modern conditions - an important factor in aesthetic education.

The development of modern textbooks did not eradicate the need for creativity, but, on the contrary, demanded an ever higher level of general cultural development, education, creativity and activity from a person. Modern educational methods using technology open up new didactic opportunities in realizing the goals of aesthetic education in mathematics, which should be used to familiarize oneself with beauty, nurturing aesthetic tastes and experiences, including through methods related to computer graphics and animation, and the development of multimedia tools etc. [21].

The use of technological tools in training in solving various types of problems can have a positive impact on the formation of aesthetic features, the growth of interest in the study of mathematics, as well as the increase in the level of fundamental knowledge using digital technologies, while improving traditional methods of various levels of education and social development of students 7-9 classes.

To improve the knowledge and effectiveness of teaching a subject of mathematics, we decided to create an electronic textbook intended for students as well as mathematics teachers using innovative information and communication technologies and its main goal is to provide methodological assistance in a better use of technologies that allow teaching on The various stages are interesting, accessible and specific [22].

Research hypothesis: the formation of logical thinking in students by means of aesthetic education in the learning process in mathematics will be effective if:

1. provide a solution to problems with the aim of developing creative activity, striving for the beauty of the originality of tasks
2. integrate various tasks, including exercises, tasks that require several methods in their solution to ensure the integration of the process of teaching mathematics, this will purposefully provide educational and developmental functions, then in the final stage, the cognitive interest of students will be formed by ensuring conscious assimilation and development of personally significant qualities aimed at aesthetic development.

The goals and objectives of the electronic textbook:

1. identification of the relationship of mathematics with various fields of human activity and phenomena occurring in nature.
2. broadening the horizons in the field of application of the subject of mathematics for secondary schools.
3. the formation of a general and mathematical culture of personality.
4. aesthetic development of personality.
5. the development of logical and imaginative thinking among students in schools.
6. development of skills in working with electronic technologies.
7. Development of modern electronic textbooks in the field of natural sciences.
The assimilation of the material was not limited to the properties and relationships of the basic concepts of mathematics. For the assimilation of the theory, the formation of the skills and its application to solving problems was important to us.

In our opinion, it is most advisable to use the above goals in the process of illustrative demonstration work, integration in mathematics lessons, as well as in the disciplines of the natural sciences. In modern science, an interdisciplinary approach is one of the priority areas [23]. According to Kazakh and Russian scientists, the mathematical apparatus and mathematical methods can be used to study qualitatively different fragments of reality, they contribute to the disclosure of their unity and thereby indicate new ways of integrating new knowledge [20; 22-23]. For example, students will receive information on the use of these methods in formative assessment in mathematics using digital technologies.

This approach will allow students to fully understand the subject-matter relationship, learn how to use technological capabilities, and experience the aesthetic appeal of mathematics problems.

Strengthening the role of the electronic textbook is aimed at fulfilling practical tasks as a determining component in studying the theoretical information of a mathematics course, helping to bring students to understanding internal logic, deploying their ability to experiment and conduct modeling experiments with various options for solving problems, as well as analyze and synthesize any kind of information. This makes it possible to determine quick and complete information about the work done by students, which makes feedback more effective and productive in assessment.

Each practical and experimental task is accompanied by a lecture (using presentation and full text) on the topic of the lesson. For example, when studying a specific topic before performing practical tasks, students are invited to get acquainted with the concepts introduced in the lesson, the history of the occurrence of these concepts and brief explanations for them, using them in various fields of science to compare, analyze and reveal their beauty in nature and life.

Then it is proposed to perform several practical exercises on the topic with various solution methods, build various images, as well as their compositions, design the image to formulate a character that meets the requirements of the problem. Thus, when performing these tasks, one can comprehensively consider the possibilities of students of any link defined at the present stage and including an important component as a process of aesthetic education through the beauty of illustrativeness.

Thus, a number of measures were taken to identify the logical development of aesthetic potential in teaching mathematics. The problem of aesthetic forms of learning as a means of educating the educational process at school yielded its results in experimental work - to reveal the potential of the essence, to determine the significance of introducing aesthetics into the process of mastering the subject “Mathematics”, to discover the beauty of solving problems, and to identify ways to implement practical work in secondary schools.

By the example of solving one math problem carried out within the walls of the profile school-gymnasium of multilingual education No. 3 named after M. Gabdullin, secondary school No. 2, the regional boarding school for gifted children “Daryn”, as well as the AES NIS and KTL schools (school data the cities of Kokshetau, Akmola Oblast, Kazakhstan assisted in carrying out experimental work with students in grades 7–9 in mathematics) we were able to determine the criteria by aesthetic education of students in the process of general disciplinary education.

By forming aesthetic forms of learning, a link can be constructed to generalize and consolidate knowledge, skills and abilities using the example of one task. Thus, according to the practical activities of students, it is possible to identify certain links in understanding mathematical simplicity, that is, the aesthetic potential in solving one problem in various ways.

4. Practical part

Let us show, as an example, the task for independent work, where the student applies various methods to solve and, justifying, forms the essence of the same answers of the same task.

Task. Two cars left one city in one direction. The first was at a speed of 60 km / h, and the second - 90 km / h. The second car came out at 2h. later than the first. After how many hours and at what distance from the city will the car with the highest speed catch up with the car with the lowest speed?

Solution: 1-way. Arithmetic method. We give a brief record of the problem is shown in Figure 6.
1) - the first car will pass in 2 hours;
2) - the difference in speed;
3) - after so many hours after its release, the second will catch up with the first car;
4) - at such a distance from the city, the second one will catch up with the first car.

2-way. The geometric method. Consider the solution to the problem on a schedule, that is, we will characterize a visual schedule of the movement of fire engines.

When solving geometrically, we will arbitrarily take the length of one segment vertically for 30 km, and the length of one segment horizontally for 1 hour. We set aside from 0 to 6 hours on a horizontal line, vertically we will lay off the segments of the path traveled by each vehicle in 1 hour, 2 hours, 3 hours, etc. (Figure 7).

First, we postpone the segments of the path traveled by the first to the exit of the second car, and then the segments of the path that characterize the distances traveled by the first and second car (table 1):

| Distance (km) | Time (h) |
|--------------|----------|
| 180          | 3; 1     |
| 240          | 4; 2     |
| 300          | 5; 3     |
| 360          | 6; 4     |

The segments of the found paths turned out to be equal, which means that the second will catch up with the first after 4 hours, that is, after the exit of the first car. As you can see, this will happen at a distance of 360 km from the city is shown in Figure 7.

3-way. The method of geometric construction of the problem consists in dividing into intervals in accordance with the difference in speeds of two cars. Assuming that the length of one segment is 30 km, we will postpone segments of the path traveled by each of the vehicles. The interval of the path corresponds to 360 km from the beginning of their movement (Figure 8).
Figure 8. Interval path segments

4 way. Algebraic method. Let's make a linear equation:

\[
\frac{5}{60} - \frac{5}{90} = z
\]  \hspace{1cm} (1)

And so where \( x \) is the path to go. Substituting into equation (1) we obtain the following equality by shown in Figure 9:

\[
\frac{120+x}{60} - \frac{120+x}{90} = z
\]  \hspace{1cm} (2)

From equality (2), it is easy to calculate that \( x = 240 \) km, this is the distance of the distance traveled by the first car 4 hours after 120 km.

5 way. Algebraic method. Finding time using NOC speeds of two cars:

1) \( 2 \times \text{NOC (60; 90)} = 2 \times 180 = 360 \) km - from the beginning of their movement;
2) \( 360 : 90 = 4h. \) - after so many hours after its release, the second will catch up with the first car;

In each of the proposed solution methods, the same result was obtained.

And also considered some of the practical problems of composites:

Continuous reinforced composite strength fibers

In the general case, the tension diagram of a unidirectional fiber composite (Figure 8) should consist of three main sections:

I - matrix and fibers are deformed elastically;
II - the matrix goes into an elastic-plastic state, fibers continue to deform elastically;
III - both components of the system are in a plastic state deformation.

Depending on the properties of the components of the composite, sections II and III on the curve may be absent.
The external load is equal to the sum of the loads on the matrix and the fibers, provided that the bond strength at the fiber-matrix interface is sufficient to ensure joint deformation of the components up to failure, i.e. Then, the tensile strength of the composite along the fibers, depending on the volume fraction of VB fibers for a typical composite reinforced with continuous unidirectional fibers, varies in direct proportion to the volume fraction of fibers:

\[
\frac{\sigma_f}{E_f}V_f = \frac{\sigma_m}{E_m}V_m
\]

where the average value of the tensile strength of the fibers in tension (compression, shear); - voltage in the matrix at the time of rupture of the fibers.

The value depends on the complex relationships between the properties of the matrix and the fibers and for engineering purposes is usually determined by the measured strength of the composite material with a known volume fraction of fibers.

The strains of the fibers and the matrix are rarely equal, so for brittle fibers in an elastic matrix, for example, for a ceramic-ceramic composite material, the value is given by the ratio: where EM, EB are the elastic moduli of the matrix and fibers. For a matrix capable of being plastically deformed, for example, metal, a more suitable parameter in the calculation is the yield strength.

When compressed along the fibers, the destruction of the composite material occurs due to the loss of stability of the fibers, similar to destruction during the longitudinal bending of the rod.

In the case of the fracture mechanism, when the main type of matrix deformation is tensile directed perpendicular to the fiber axis, the compressive strength is determined by the expression

\[
\sigma_f = 2V_f \left[ V_f E_f E_m \frac{3(1-V_f)}{V_f} \right]
\]

In the case of failure by type of shear, the tensile strength is determined expression

\[
\sigma_f = \left( \frac{\sigma_f}{E_f} \right)_M \left( \frac{1-V_f}{V_f} \right)
\]

Minimum and critical fiber concentration

The concept of the minimum concentration (volume fraction) of Vmin fibers was introduced in relation to composites in which the matrix is more plastic than fibers. At values of VB <Vmin, the destruction of the fibers does not lead to the immediate destruction of the entire composite, since the undamaged cross-section of the matrix has the ability to bear a higher load than the total cross-section of the destroyed fibers. The growing load will lead to the destruction of the fibers into smaller parts - multiple destruction. If VB <Vmin, then at an external load corresponding to the strength of the fibers, they are destroyed. The stress that acted on the fibers is redistributed to the matrix, the strength of which is insufficient to withstand this stress - there is a catastrophic destruction of the entire composite.

The value of the minimum fiber concentration V min is determined from

Conditions
\[(\sigma_f)_{V_{\text{min}}} + \sigma_M (1 - V_{\text{min}}) = (\sigma_f)_{V_c} = (\sigma_f)_{M} (1 - V_{\text{min}})\]

where from

\[V_{\text{min}} = \left[\left(\sigma_f\right)_M - \sigma_m \right] / \left(\sigma_f \right)_f - \left(\sigma_f\right)_M - \sigma_m \]

The critical volume fraction of fibers \(V_{\text{cr}}\) is called such, at where the strength of the composite becomes equal to the strength of unreinforced matrices. The value is calculated from the condition

\[(\sigma_f)_{V_{\text{cr}}} + \sigma_M^* (1 - V_{\text{cr}}) = (\sigma_f)_{V_c}\]

from here

\[V_{\text{cr}} = \left[\left(\sigma_f\right)_M - \sigma_m^* \right] / \left(\sigma_f \right)_f - \sigma_m^* \]

The concentration dependence of the strength of composites in which the matrix is more plastic than fibers. If the fibers have a greater plasticity margin than the matrix, then the transition from single to multiple fracture occurs at a fiber concentration \(C_v\), determined from the condition

\[(\sigma_f)_{V_f} = (\sigma_f)_{M} (1 - V_f) + \sigma_f^* V_f\]

where from

\[V_f = (\sigma_f)_{M} / \left(\sigma_f \right)_f + \left(\sigma_f\right)_M - \sigma_f^* \]

where \(\sigma\) is the stress in the fibers during deformation of the destruction of the matrix.

The dependence of the strength of the composite on the volume fraction of fibers for this case is presented in Figure 11.

From Fig. Figure 11 shows that composites with a plastic matrix are destroyed by a single destruction mechanism at high fiber concentrations, and with a brittle matrix, at low fiber concentrations.

**Figure 11.** A typical view of the dependence of the tensile strength of the composite on the volume fraction of fibers.
fraction of fibers: a - for a unidirectional composite with a plastic matrix and brittle fibers; b - s
Fragile matrix and plastic fibers (I-region of multiple fracture, II - region of single fracture)

The influence of fiber orientation on the destruction of the composite.
A description of the influence of fiber orientation is based on the theory of maximum principal stresses.
Under the action of tensile stresses a applied at an angle $\Theta$ to the direction of laying of the fibers (Figure 12),
depending on the value of $\tau_{pq}$, three types of destruction are possible.

![Figure 12. Scheme of loading unidirectional composites at an angle $\Theta$ to the wave axis](image)

5. Approbation
When solving a problem in several ways, the knowledge of students in mathematics, the ability to apply
them to problem solving, and the ability to build mathematical models were tested.
After instilling skills in solving one problem in several ways, we conducted an experiment. During the experiment, in the process of checking the assimilation of knowledge in the choice of methods for solving one problem, two stages were outlined: initial and final.
The experiment involved a student in grades 7–9 of the specialized school-gymnasium for multilingual education No. 3 named after M. Gabdullin, secondary school No. 2, the regional boarding school for gifted children “Daryn”, NIS (Nazarbayev Intellectual School) and KTL (Kazakh-Turkish Lyceum).
Students were assigned to a control group (CG) and an experimental group (EG). Students in these groups were asked to solve one problem in five ways. On the basis of monitoring and summarizing the results of the experiment, the quality of knowledge was determined for students finding several ways to solve one problem, which made it possible to identify the flexibility of thinking, capable of multi-level cognition and comprehensive understanding when performing certain types of tasks.
Testing of knowledge, skills of students in grades 7-9 was carried out on the basis of the methodology proposed by A.A. Kyveryalg and is defined by the following formula [20]:

$$C_{ak} = \frac{n}{N}$$

$C_{ak}$ - coefficient of assimilation of knowledge.
$n$ – is the number of students who correctly used the methods of solving one problem.
$N$ – is the number of all students participating.

| Table 2. The levels of students' knowledge acquisition were divided into a control group (CG) and an experimental group (EG) in the choice of ways to solve one problem at the initial stage (school-gymnasium of multilingual education No. 3 named after M. Gabdullin, Grade 7 - 126 students, Grade 8 - 98 students, Grade 9 - 84 students) |
| Problem solving by methods | CG | EG |
| 7 Gr | 8 Gr | 9 Gr | 7 Gr | 8 Gr | 9 Gr |

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The results of the experiment conducted with students in grades 7-9 of the above school from this table will be shown in the form of diagrams (Figure 13):

**Figure 13.** The skill levels of students in the CG and the EG in choosing ways to solve one problem at the initial stage of the pedagogical experiment (based on grades 7-9 of the school-gymnasium of multilingual education No. 3 named after M. Gabdullin)

Based on the table and the corresponding diagrams (Figure 13), we will draw some conclusions: it is necessary to conduct consistent work with EG students to choose various methods for solving one problem, since at the initial stage the knowledge, skills of students in grades 7-9 are still insufficient. Students need to be brought to an understanding of the essence of aesthetic education by developing their logical thinking in solving one problem in several ways. It should be noted that in this case, you can use an electronic textbook, which stimulates the activities of students and is an additional tool in the course of students learning.

After focused work with students of the experimental group, the results of mastering the knowledge in choosing ways to solve one problem have changed for the better. These results are given in table 3 and in the corresponding diagrams (Figure 14).

**Table 3.** Learning levels of students in choosing ways to solve one problem at the final stage (school-gymnasium of multilingual education No. 3 named after M. Gabdullin, Grade 7 - 126 students, Grade 8 - 98 students, Grade 9 - 84 students)

| Problem solving by methods | CG | EG |
|---------------------------|----|----|
|                           | 7 Gr | 8 Gr | 7 Gr | 8 Gr | 7 Gr | 8 Gr |
| 1                         | 0.84 | 0.90 | 0.81 | 0.84 | 0.92 | 0.83 |
| 2                         | 0.81 | 0.88 | 0.88 | 0.86 | 0.90 | 0.93 |
| 3                         | 0.75 | 0.76 | 0.74 | 0.81 | 0.84 | 0.81 |
| 4                         | 0.79 | 0.80 | 0.90 | 0.92 | 0.90 | 0.93 |
| 5                         | 0.92 | 0.79 | 0.71 | 0.98 | 0.92 | 0.98 |
Figure 14. Learning levels of students in the CG and the EG in the choice of ways to solve one problem at the final stage of the pedagogical experiment (based on grades 7-9 of the school-gymnasium of multilingual education No. 3 named after M. Gabdullin)

A similar experiment was conducted on the basis of schools: secondary school No. 2, a regional boarding school for gifted children “Daryn”, NIS and KTL (Figures. 15 and 16).

Figure 15. The levels of assimilation of students in the CG and EG in the choice of ways to solve one problem at the initial stage of a pedagogical experiment

Figure 16. The levels of assimilation of students in the CG and EG in the choice of ways to solve one problem at the final stage of the pedagogical experiment

6. Conclusion
The results of the experiment confirmed the hypothesis of the study and made it possible to draw some conclusions:

1. The essence and content of the development of thinking in the aesthetic education of students are determined by choosing several methods for solving one problem in several ways.

2. The analysis of the compiled electronic textbook on the example of various tasks with numerous ways and methods of solving it. Thus, a significant difference in the levels of formation of students' ability to solve problems of the same type by several methods based on mathematical calculation makes it possible to develop aesthetic education and logical thinking at a professional level. The assimilation of knowledge increases interest, makes it possible to see aesthetic beauty in solving one problem. This means that the upbringing of a mathematics lesson is just as important as the upbringing of the mind, thought and independence of the choice of a solution path, thereby ensuring the conscious assimilation and development of personally significant qualities.

Based on the aforementioned studies of scientists, in particular, our research, we can state that aesthetic education occupies an important place in the process of personality formation. No less significant is the fact that the knowledge of the world from the standpoint of its beauty, aesthetic value contributes to the formation of the concepts of student culture, which is one of the most important components in the educational process of students in secondary schools.

Using an electronic textbook, as an additional tool in learning, you can show the beauty of solving mathematical problems, the harmony of their solutions not only of one example, but also of many other tasks in mastering quite fundamental knowledge. About even greater manifestations of aesthetics in mathematical education, in revealing all the intellectual and creative possibilities of a person, in developing imagination, as well as broadening the horizons of students at each stage, according to Academician B.V. Rauschenbakh, "... penetrate the essence of the object ... ".

So, solving the same problem in different ways shows that the essential component of the forms of aesthetic teaching as a means of educating the educational process in mathematics is the mastery of schoolchildren by knowledge related to understanding the art of a creative approach.

In conclusion of this article, I would like to note that solving the same problem in different ways (if necessary, using an electronic textbook) increases the level of general development of students, contributes to their intellectual growth, that is, affects the formation of personal and mental actions in the formation of personality.

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