Simulation of turbulent wakes in model wind farm with arbitrary location for wind turbines

S V Strijhak, K B Koshelev and A S Kryuchkova
ISP RAS, 109004, Russia, Moscow, Solzhenitsyna str., 25
strijhak@yandex.ru, koshelevkb@mail.ru, kryuchkova.arina.s@gmail.com

Abstract. Wind energy is an important part of renewable energy sources in the Russian Federation. A new modern wind farm with 28 wind turbines was built on the territory of the Russian Federation, in the Ulyanovsk region in 2017-2018. When designing and operating a wind farm, it is necessary to assess correctly the power of each individual wind turbine. In this paper we used the open SOWFA library based on OpenFOAM 2.4.0 and URANS approach with k-e model of turbulence. The Actuator Line Model for the calculation of the forces from the wind turbines was used in pisoFoamTurbine.ALM solver. To set the boundary conditions at the input, the Precursor method was used. The unstructured grids with different number of cells were built. As a result of the calculation for 14 simulators of wind turbines, the values of the power coefficients $C_p$ and thrust coefficients $C_t$ were also determined. The maximum $C_p$ value was in agreement with the Bets-Zhukovsky law for an ideal wind turbine. The new numerical case with 28 wind turbines was prepared for future simulations. The calculations were performed on a cluster of the UNICFD web-laboratory of ISP RAS. Each calculation was performed on 48-72 cpu cores.

1. Introduction
Wind energy is an important part of renewable energy sources in the Russian Federation. The turbulent wakes dynamics and analysis of wind turbines performance in wind farms are the questions of the great interest for the scientific community and industry now. A new modern wind farm with 28 wind turbines with power capacities of 2.5 MW and 3.6 MW was built on the territory of the Russian Federation, in the Ulyanovsk region in the village Krasny Yar, in 2017-2018. A new wind farm has been under construction in the Republic of Adygea since 2019. The wind farm in the Republic of Adygea with 60 wind turbines is the most ambitious construction project of wind energy of Russia. The power capacity will be 150 MW, the 2nd and 3rd stages are planned.

The problems of modeling the physical processes in the wind farm, modeling of vortex turbulent wakes from wind turbines, calculating the values of wind power are relevant due to the fact that the wind is an extremely unstable source and is subject to daily and seasonal changes [1]. When designing and operating a wind farm, it is necessary to assess correctly the power of each individual wind turbine and power deficit depending on the location of the wind farm, take into account the influence of the atmospheric boundary layer and the complex terrain.

2. Mathematical model
Large-eddy simulation (LES) has been recently well applied in the context of numerical simulation of a flow over wind turbines on flat and complex terrains [1]. The LES requires very detailed computational grids and therefore the URANS approach can be a good alternative [2]. In this paper we
used the open SOWFA library based on OpenFOAM 2.4.0 and the ABLSolver with LES model and pisoFoamTurbine.ALM solver with URANS model. The Actuator Line Model (ALM) for calculation of the forces from the wind turbines was used in pisoFoamTurbine.ALM solver [3]. SOWFA library includes several incompressible solvers and utilities, it was developed in NREL (Colorado, USA) and the library is of the active use by the research community now.

The calculation was carried out in the formulation of URANS with the k-ε turbulence model on different grids. To set the boundary conditions at the input of the numerical domain, the Precursor method was used [2].

The mathematical model includes the equations for mass conservation, momentum conservation, equations for kinetic turbulent energy, dissipation energy for incompressible flow:

\[ \frac{\partial \bar{u}_i}{\partial x_i} = 0 \]  
(1)

\[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + f_i \]  
(2)

\[ f_i = f_x = -\frac{1}{2} C_T U_{free}^2 \delta(x-x_h)\Theta(R^2 - (y-y_h)^2 - (z-z_h)^2) \]  
(3)

\[ \tau_{ij} = 2(\nu + \nu_t) S_{ij} \]  
(4)

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \]  
(5)

\[ \frac{\partial k}{\partial t} + \frac{\partial (k \bar{u}_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ (\nu + \nu_t) \frac{\partial k}{\partial x_i} \right] + P - \varepsilon \]  
(6)

\[ \frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon \bar{u}_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ (\nu + \nu_t) \frac{\partial \varepsilon}{\partial x_i} \right] + C_1 \frac{\varepsilon}{k} P - C_2 \frac{\varepsilon^2}{k} \]  
(7)

\[ \nu_t = C_\mu \frac{\nu_t^2}{\varepsilon} \]  
(8)

\[ P = 2\nu_t S_{ij} S_{ij} \]  
(9)

The coefficients for standard k-ε model are:

\[ f_p = 1.0 \quad C_{1\varepsilon} = 1.44 \quad C_{2\varepsilon} = 1.92 \quad C_\mu = 0.09 \quad \sigma_k = 1.0 \quad \sigma_\varepsilon = 1.3 \]  
(10)

Where \( \bar{u}_i \) is gas (air) velocity vector component, \( x_i \) is Cartesian system coordinate, \( p \) is gas (air) pressure, \( k \) is kinetic energy of turbulence, \( \varepsilon \) is dissipation of energy, \( \nu_t \) is turbulent viscosity.

The Gauss linear Scheme was used for approximation of the convective terms, the Gauss linear corrected scheme was used for approximation of Laplacian terms. To solve linear system of equations the PBiCG method with DILU preconditioner was used for velocity, temperature, and the GAMG method was used for pressure. The tolerance was set to \( 10^{-6} \).

3. Definition of the problem
The calculation for a model wind farm with 14 simulators of wind turbines was performed according to data for wind farm in Ulyanovsk region in the village Krasny Yar. Fourteen simulators of wind turbines were located in accordance with the plan of the first stage of wind farm construction in the Ulyanovsk region of the Russian Federation. The wind farm has geographic coordinates N54°17' E48°08'. The location of each wind turbines was defined using satellite images from Google Earth Pro program (Fig.1, Fig.2a). The data on velocity profile and wind direction were taken from the weather station and the free report of Lahmeyer International Company in Internet for the period of time from 26.05.2012 till 25.05.2013 (Fig. 2b). The numerical domain was defined in the form of a
parallelepiped with dimensions of 9 meters on the axis OH, 5 meters on the axis OY, 1 meter on the axis OZ (Fig. 3, Fig. 4).

![Figure 1](image1.png)

**Figure 1.** The wind farm near the Volga River from satellite image.

![Figure 2](image2.png)

**Figure 2.** a) the location of wind turbines; b) the velocity profile.

A preliminary calculation was performed for the atmospheric neutral boundary layer using the ABLSolver solver to get data for Precursor method. To set the profile of the velocity and direction of the wind at the input of the computational domain, the data obtained during the wind monitoring conducted in 2013 were used. In URANS calculations, an unstructured grid with different number of cells was used: 1.7 million, 3.3 million and 6.2 million cells (Fig.3).
Figure 3. The domain and grid for wind farm with 14 model wind turbines in axonometria.

The diameter of rotor for wind turbine was equal to D=416 mm. The reference velocity was set to Uref=1.5 m/s. The Atmospheric Boundary Layer (ABL) model was introduced to represent experimental conditions. The parameters of Neutral ABL, used in our simulation, are listed in Table 1 of work [5].

Each of the wind turbine prototypes had 3 blades with the same cross section and with the same airfoil NREL S826 from root to tip. The wind blade was made of carbon fiber material with a shape of a twisted thin flat plate of 0.8 mm thickness, without using any aerofoil cross-section [5,6]. The operating tip-speed ratio (TSR) was set to 6. The 3D geometry for model wind turbine with tower and nacelle is shown in Figure 5. The mesh was initially created with uniform resolution using OpenFOAM blockMesh tool, and then OpenFOAM snappyHexMesh solver was used to create the 3D unstructured grid representing the real geometry of wind turbines (Fig.3, Fig.4). The snappyHexMesh solver was run on 48-64 cores of high performance cluster to speed up the process of grid generation.

Figure 4. The model wind turbine with tower and nacelle.
4. Main results of simulation

The fields of velocity, pressure, temperature, turbulent viscosity and stream functions were obtained during the calculation and data analysis, using the pisoFoamTurbine.ALM solver. The flow turbulent structures around four turbines, aligned to the first row of the array, were studied to determine the general behavior of the resulting flow in the model wind farm. It was noted that the turbulent wakes behind the first turbines row are more stable, but with the second and the third turbine rows the wake turbulent behavior becomes more unstable and turbulent (Fig.5). The pisoFoamTurbine.ALM solver, together with the ALM model, was previously verified on the well-known Blind Test problem for two model wind turbines in a wind tunnel [7,8] and for the case with 12 wind turbines in a wind tunnel [5,9]. During verification a comparison was made with the results of the experiment on the value of the dimensionless velocity in different sections behind the wind turbine for the values of the power coefficients $C_p$ and the axial thrust force coefficients $C_t$ [8,9]. As a result of our calculations for 14 simulators of wind turbines, the values of power coefficients $C_p$ and thrust coefficients $C_t$ were also determined (Table 1, Table 2). A range of values for $C_p$ from 0.2 to 0.58 was obtained. The wind turbine, located in the wake of other wind turbines, had the smallest value of $C_p$.

| Wind Turbine | Thrust | Power | $C_p$ | $C_t$ |
|--------------|--------|-------|-------|-------|
| 1            | 0.126  | 0.080 | 0.673 | 0.285 |
| 2            | 0.109  | 0.057 | 0.580 | 0.205 |
| 3            | 0.143  | 0.103 | 0.762 | 0.367 |
| 4            | 0.126  | 0.081 | 0.674 | 0.289 |
| 5            | 0.150  | 0.115 | 0.801 | 0.411 |
| 6            | 0.157  | 0.128 | 0.839 | 0.455 |
| 7            | 0.172  | 0.153 | 0.921 | 0.546 |
| 8            | 0.166  | 0.145 | 0.889 | 0.516 |
| 9            | 0.154  | 0.121 | 0.820 | 0.432 |
| 10           | 0.128  | 0.084 | 0.686 | 0.300 |
| 11           | 0.113  | 0.062 | 0.605 | 0.222 |
| 12           | 0.128  | 0.083 | 0.686 | 0.294 |
| 13           | 0.159  | 0.133 | 0.851 | 0.473 |
| 14           | 0.119  | 0.071 | 0.634 | 0.253 |

| Wind Turbine | Thrust | Power | $C_p$ | $C_t$ |
|--------------|--------|-------|-------|-------|
| 1            | 0.127  | 0.083 | 0.677 | 0.294 |
| 2            | 0.108  | 0.058 | 0.579 | 0.206 |
| 3            | 0.145  | 0.108 | 0.775 | 0.384 |
| 4            | 0.127  | 0.083 | 0.677 | 0.296 |
| 5            | 0.154  | 0.124 | 0.824 | 0.440 |
| 6            | 0.160  | 0.134 | 0.857 | 0.478 |
| 7            | 0.176  | 0.163 | 0.941 | 0.582 |
| 8            | 0.170  | 0.150 | 0.909 | 0.534 |
| 9            | 0.156  | 0.127 | 0.833 | 0.452 |
| 10           | 0.130  | 0.087 | 0.695 | 0.310 |
| 11           | 0.114  | 0.064 | 0.607 | 0.226 |
| 12           | 0.130  | 0.085 | 0.692 | 0.301 |
| 13           | 0.164  | 0.140 | 0.874 | 0.499 |
| 14           | 0.119  | 0.071 | 0.633 | 0.252 |
Figure 5. The U magnitude field and stream functions.

The maximum $C_p$ value was in agreement with the Bets-Zhukovsky law for an ideal wind turbine [10]. The analysis of changes in the values of dimensionless velocity in different sections behind wind turbines for the 1st, 2nd and 3rd rows has been carried out. The dimensions of the turbulent vortex wakes were estimated from the vorticity field. The calculations were performed on a cluster of the UNICFD web-laboratory of ISP RAS. Each calculation was performed on 48-72 computing cores.

The numerical domain, grid and data for numerical case with 28 model wind turbines were prepared in accordance with the plan of the second stage of wind farm construction in the Ulyanovsk region of the Russian Federation (Fig. 6).

Figure 6. Wind farm with 28 model wind turbines.

Conclusions
More detailed information about wind turbines is needed primarily for quantitative coincidence of results, as also the SCADA data from industrial wind turbines. More fine computational grids should be used for numerical simulations for URANS and LES models. The numerical case with 28 wind turbines was prepared for numerical simulations and it will be studied in future.

Acknowledgments
This work was supported by the Russian Foundation for Basic Research (Grant No. 17-07-01391).
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