Entanglement protection in higher-dimensional systems

Ashutosh Singh* and Urbasi Sinha
Light and Matter Physics Group, Raman Research Institute, Sadashivanagar, Bangalore—560080, India
* Author to whom any correspondence should be addressed.
E-mail: asinghrri@gmail.com and usinha@rri.res.in

Keywords: decoherence, entanglement sudden death, entanglement protection

Abstract
The inevitable dissipative interaction of an entangled quantum system with its environment causes degradation in quantum correlations present in the system. This can lead to a finite-time disappearance of entanglement, which is known as Entanglement Sudden Death (ESD). Here, we consider an initially entangled qubit–qutrit system and a dissipative noise which leads to ESD, and propose a set of local unitary operations, which when applied on the qubit, qutrit, or both subsystems during the decoherence process, cause ESD to be hastened, delayed, or avoided altogether, depending on its time of application. Delay and avoidance of ESD may find practical application in quantum information processing protocols that would otherwise suffer due to short lifetime of entanglement. The physical implementation of these local unitaries is discussed in the context of an atomic system. The simulation results of such ESD manipulations are presented for two different classes of initially entangled qubit–qutrit systems. A prescription for generalization of this scheme to a qutrit–qutrit system is given. This technique for entanglement protection in the noisy environment is compared with other related techniques such as weak measurement reversal, dynamic decoupling, and quantum Zeno effect.

1. Introduction

Quantum decoherence [1] is a ubiquitous and unavoidable phenomenon arising because of entanglement between quantum systems and their environment. Entanglement, on the other hand, is a fundamentally important phenomenon in the studies of quantum foundations and has great significance in quantum technologies. It is now seen as an indispensable resource in Quantum Information Processing (QIP) for various tasks such as quantum computation, teleportation, superdense coding, cryptography, etc., which are either impossible or less efficient using classical correlations [2–4].

The inevitable dissipative interaction of an entangled quantum system with its environment leads to an irreversible loss of single particle coherence as well as degradation of entanglement present in the system [5]. For some initial states, in the presence of an Amplitude Damping Channel (ADC), entanglement degrades asymptotically, whereas for others it can disappear in finite time, also known as early stage disentanglement or Entanglement Sudden Death (ESD), in literature [6–8]. Soon after its theoretical prediction, ESD was experimentally demonstrated in atomic [9] and photonic [10] systems. The real world success of several quantum information, communication, and computation tasks depends on the resilience of entanglement from noises present in the environment, and longevity of the entanglement. Thus, ESD poses a practical limitation on QIP tasks. Therefore, strategies which make entanglement robust against the detrimental effects of the noise are of practical interest in QIP.

Owing to the simplicity of two-qubit entangled systems and its usefulness as a resource in QIP, there have been several theoretical proposals to combat decoherence and finite-time disentanglement in these systems [2–11, 11–24]. Some of these proposals have been experimentally demonstrated in atomic, photonic, and solid state systems [25–34]. One of the entanglement protection schemes [21] considered a class of two-qubit entangled states which undergo ESD in the presence of an ADC. For such systems, a Local Unitary Operation (LFO), in this case the
Pauli $\sigma_y$ operator also known as NOT operation, has been proposed. When LUO is applied on one or both the subsystems during the process of decoherence, it can hasten, delay, or completely avoid the ESD, all depending on the time when NOT operation is applied. This proposal was later transformed by our group into an all-optical experimental setup to study the effect of LUOs on the disentanglement dynamics in a photonic system [22]. This work proposed an experimentally feasible architecture for the implementation of ADC and the local NOT operation to suitably manipulate the ESD of a two-qubit system in a controlled manner.

Higher-dimensional entangled quantum systems (qudits, $d \geq 3$) can offer practical advantages over the canonical two-qubit entangled systems in QIP protocols. These systems are more resilient to errors than their qubit counterparts in quantum cryptography, and they offer practical advantages; for example, increased channel capacity in quantum communication, enhanced security in QIP protocols, efficient quantum gates, and in the tests of the foundations of quantum mechanics [35]. It is therefore important to study the effects of decoherence on these systems. Entanglement evolution has been studied in higher-dimensional systems present in the noisy environment, and despite the resilience of these systems to noise [36], ESD is established to be ubiquitous in all dimensions of the Hilbert spaces [37, 38].

As a first step towards generalization to higher dimensional systems, a qubit-qutrit system is the natural choice as it is intermediate in complexity to a qubit-qubit and qudit-qudit system. Given the availability of full-fledged separability criteria for pure as well as mixed state qubit-qutrit systems, experimental feasibility of state preparation, and the significance of higher-dimensional systems in QIP, they become an important architecture towards understanding and controlling the effects of decoherence in higher-dimensional systems. Some attempts have been made to locally manipulate the disentanglement dynamics in qubit-qutrit [39, 40], and qutrit-qutrit [41–43] systems to retain the entanglement for longer duration. It is found that the quantum interference between two upper levels of the qutrit can also be used to control the disentanglement dynamics in higher-dimensional systems [39, 44].

Consider a practical scenario where Charlie prepares a bipartite entangled state for some QIP task and he has to send the entangled particles to Alice and Bob through a quantum channel which is noisy and can potentially cause disentanglement before the particles reach the two parties. In this scenario, we ask the following question: given a higher-dimensional bipartite entangled state which would undergo ESD in the presence of an ADC, can we alter the time of disentanglement by some suitable LUOs during the process of decoherence? As a possible answer to this question, we explore the generalization of the proposal in [21] for higher-dimensional systems, say qubit-qutrit or qutrit–qutrit system, in the presence of an ADC. If such systems undergo ESD, we propose a set of LUOs, such that when they are applied on the subsystems during the process of decoherence, they manipulate the disentanglement dynamics, in particular, delay the time at which ESD occurs. Such a study was partially done in [39], and here we generalize their scheme and propose a more general class of LUOs, which can always suitably manipulate ESD for an arbitrary initially entangled state. In the case of qubits, [21] found it to be NOT operation which is an operation corresponding to the Pauli $\sigma_y$ operator. In this work, for qutrits, we propose a set of LUOs, which allow flipping the population between different levels of the qutrit. Depending on the combination of LUOs, and the time of their application, this method is shown to be able to hasten, delay or completely avoid the ESD in qubit-qutrit as well as qutrit-qutrit systems.

In the decoherence process, pure states evolve to mixed states. Therefore, a computable measure of entanglement for mixed state is required to quantify the entanglement. Negativity is such a measure, based on the Positive Partial Transpose (PPT) criterion due to Peres and Horodecki [45, 46]. For $2 \otimes 2$ and $2 \otimes 3$ dimensional systems, all PPT states are separable and Negative Partial Transpose (NPT) states are entangled. Negativity is defined as the sum of the absolute values of all the negative eigenvalues of the partially transposed density matrix with respect to one of the subsystems. While the enigmatic features of entanglement were predicted back in 1935, its witnessing and quantification continues to be at the forefront of current research. For a specific class of higher dimensional states $\rho$ which are supported on $d \otimes D$ Hilbert space ($d < D$) and with rank $r(\rho) < D$, PPT criterion has been found to be necessary and sufficient condition for separability [47]. But for more general higher-dimensional systems ($d < D$, $\forall d, D \geq 3$ and $r(\rho) > D$), PPT criterion is only a necessary but not sufficient condition for separability.

For these systems, we study Negativity Sudden Death (NSD), whose non-occurrence guarantees Asymptotic Decay of Entanglement (ADE). But for $3 \otimes 3$ dimensional PPT states with $r(\rho) > 3$ which have zero Negativity, we cannot comment on the separability and we need some other measure. For this purpose, we have used matrix realignment method for detecting and quantifying entanglement [48–50] after state is found to have zero negativity using PPT criterion. Realignment criterion states that for any separable state $\rho$, the trace norm of the realignment matrix $(m \otimes |m\rangle \langle m| \otimes |\nu\rangle = |m\rangle \otimes (|m\rangle \langle m| \otimes |\nu\rangle) never exceeds one. Therefore, if the realigned negativity given by $R(\rho) = \max(0, ||\rho^R|| - 1)$, is non-zero then state $\rho$ is entangled, where $\rho^R_{ik\ell j} = \rho_{ik\ell ij}$. This criterion can detect some of the bound-entangled states which may not be detected by PPT criterion. We use PPT as well as realignment criterion for entanglement detection.
The rest of the paper is organized as follows: In section (2), we briefly discuss the physical model in which qubit-qutrit entangled system can be realized in an atomic system and the natural presence of ADC in such a system. Next, Kraus operators governing the evolution of quantum system in the presence of ADC are given. We then propose a set of LUOs for qubit, and qutrit for manipulating the ESD, and their physical implementation for an atomic system is discussed. In section (3) and (4), we present our calculations implementing the proposed LUOs on two different class of qubit-qutrit system and their results. In section (5), we briefly comment on the generalization of this proposal to higher dimensions, taking an example of a two-qutrit system. In section (6), the results of ESD manipulations are compared and contrasted for a given initial state with respect to the choice of various LUOs. In the end, we conclude with the advantages and limitations of our proposal for ESD manipulation with other existing schemes.

2. Physical model

Consider a hybrid qubit-qutrit entangled system in the presence of an ADC. The qubit and qutrit systems can be realized by a two-level, and a three-level atom in V-con

Figure 1. Two-level atom (qubit), and a three-level atom (qutrit) in V-configuration. The parameters $p, p_1, p_2$ denote the decay probabilities between the levels $|1\rangle_S \rightarrow |0\rangle_S$ of the qubit, and $|1\rangle_S \rightarrow |0\rangle_S$ and $|2\rangle_S \rightarrow |0\rangle_S$ levels of the qutrit, respectively. The qutrit is taken to be in V-configuration such that the dipole transitions are allowed only between the levels $|1\rangle_S \leftrightarrow |0\rangle_S$ and $|2\rangle_S \leftrightarrow |0\rangle_S$.

The evolution of a qubit in the presence of an ADC is given by the following quantum map:

$$
\begin{align*}
|0\rangle_S & |0\rangle_E \rightarrow |0\rangle_S |0\rangle_E, \\
|1\rangle_S |0\rangle_E & \rightarrow \sqrt{1 - p} |1\rangle_S |0\rangle_E + \sqrt{p} |0\rangle_S |1\rangle_E, \\
\end{align*}
$$

(1)

where $|0\rangle_S$ and $|1\rangle_S$ are the levels of qubit, and $|0\rangle_E$ and $|1\rangle_E$ are vacuum state and single-photon Fock state of the environment (cavity), respectively. In the Born-Markov approximation: $p = 1 - \exp(-\Gamma t)$, which is the probability of de-excitation of the qubit from the higher level $|1\rangle_S$ to the lower level $|0\rangle_S$. The subscripts 'S' and 'E' refer to the system and environment, respectively.

The quantum map of a qutrit (in V-configuration) in the presence of an ADC is given by

$$
\begin{align*}
|0\rangle_S & |0\rangle_E \rightarrow |0\rangle_S |0\rangle_E, \\
|1\rangle_S |0\rangle_E & \rightarrow \sqrt{1 - p_1} |1\rangle_S |0\rangle_E + \sqrt{p_1} |0\rangle_S |1\rangle_E, \\
|2\rangle_S |0\rangle_E & \rightarrow \sqrt{1 - p_2} |2\rangle_S |0\rangle_E + \sqrt{p_2} |0\rangle_S |1\rangle_E, \\
\end{align*}
$$

(2)

where $|0\rangle_S, |1\rangle_S$, and $|2\rangle_S$ are three levels of the V-type qutrit. There are two decay probabilities for such a qutrit corresponding to the transitions $|1\rangle_S \rightarrow |0\rangle_S$ and $|2\rangle_S \rightarrow |0\rangle_S$ and given by $p_1 = 1 - \exp(-\Gamma_1 t)$ and $p_2 = 1 - \exp(-\Gamma_2 t)$, respectively. Here, $\Gamma_1$ and $\Gamma_2$ represent the decay rate of levels $|1\rangle_S$ and $|2\rangle_S$, respectively.

The Kraus operators governing the evolution of the system in the presence of ADC, for qubit ($M_q$) and qutrit ($M_s$), are obtained by tracing over the degrees of freedom of the environment from equations (1) and (2), respectively, and given by

$$
M_q = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1 - p} \end{pmatrix}, \quad M_s = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}.
$$

(3)
Let us label another set of Kraus operators \( M_{ij} \) with the parameter \( p \) replaced by \( p' \) \((t \text{ replaced by } t')\), \( t' = t - t_m \). These Kraus operators are applied to

\[
\rho(p') = (U_1 \otimes U_2) \rho(p) (U_1 \otimes U_2)^\dagger,
\]

where \( U_1 = \sigma_x \text{ or } \mathbb{1}_S \), and \( U_2 = F_{01}, F_{02}, F_{102}, F_{201} \text{ or } \mathbb{1}_S \).

Let us label another set of Kraus operators \( \{ M'_{ij} \} \) with the parameter \( p \) replaced by \( p' \) \((t \text{ replaced by } t')\), \( t' = t - t_m \); \( p' = 1 - \exp(-\Gamma t') \), and of the form similar to equation (5). These Kraus operators are applied to
the state (8) to see the evolution of the system when it undergoes ADC after the application of LUOs as follows:

$$\rho(p', p_n) = \sum_{i,j} M_{ij}^l \rho(p_n) M_{ij}^l.$$  \hspace{1cm} (9)

When both the LUOs are identity operations, i.e., $U_1 = I_2$ and $U_2 = I_3$, we have the uninterrupted system evolving in the ADC The state of the system in this case is given by

$$\rho(p', p) = \sum_{i,j} M_{ij}^l \rho(p) M_{ij}^l.$$ \hspace{1cm} (10)

Our aim is to investigate whether the phenomenon of hastening, delay and avoidance of ESD also occurs in higher-dimensional entangled systems as seen in $2 \otimes 2$ systems [21] or not. For this purpose, we compare the Negativity of the system manipulated using LUOs (9) with that of the uninterrupted system (10). For the purpose of current analysis, we choose the decay probabilities for the qutrit as $p_1 = 0.8$ and $p_2 = 0.6$, where $p$ is the decay probability of the qubit.

3. X-type qubit-qutrit entangled system: state-I

Let us consider the one-parameter qubit-qutrit entangled state given by,

$$\rho(0) = \frac{x}{2} \left[ \langle 00| + \langle 01| + \langle 11| + \langle 12| \langle 02| \right]$$

$$+ \frac{1 - 2x}{2} \left[ \langle 02| + \langle 02| \langle 02| \langle 10| + \langle 02| \langle 02| + \langle 10| \langle 10| \right],$$ \hspace{1cm} (11)

where $0 \leq x < 1/3$.

Assuming that both the subsystems, qubit as well as qutrit, undergo local and independent ADC, evolution of the system is given by equation (6). For state (11), the mathematical form of Negativity in the presence of ADC and in terms of $x$ and $p$ is given as

$$N(x, p) = \frac{1}{4} \left[ 2p^2x - 4px + 16p + 2x - (0.64p^2x^2 - 1.28p^3x^2 + 10.24p^4x^2 + 2.56p^5x^2 - 12.16p^2x + 4.96p^2x - 25.6px^2 + 25.6px - 6.4p + 16x^2 - 16x + 4)^{1/2} \right].$$ \hspace{1cm} (12)

Evolution of the entanglement versus ADC parameter $(p)$ for $0 \leq x < 1/3$ is shown in figure 3. The entangled state (11) undergoes ADE for $0 \leq x \leq 0.2$, and ESD for $0.2 < x < 1/3$. This can be easily verified by equation (12).

3.1. ESD in the presence of ADC

We choose $x = 0.25$ such that the initial state (11) undergoes ESD at $p = 0.6168$. The mathematical form of Negativity in terms of $p$ and $p'$ is given as

\[ Figure 3. Plot of Negativity versus ADC probability for $0 \leq x < 1/3$ for the entangled state (11). The system undergoes ADE for $0 \leq x \leq 0.20$, and ESD for $0.20 < x < 1/3$. \]
Using equation (13), the plot of Negativity versus ADC probability \((p, p')\) for the state (11) is shown in figure 4. For \(p = 0\), ESD occurs at \(p' = 0.6168\) and for arbitrary values for \(p\), ESD occurs along the non-linear curve in \(pp'\) plane as shown in figure 4.

For entanglement protection, we apply different combinations of LUOs and their effects are discussed below.

3.2. \(\sigma_x\) applied to qubit and \(F_{01}\) applied to qutrit

A NOT operation \((\sigma_x)\) is applied to the qubit and trit-flip operation \(F_{01}\) applied to qutrit part of the state (11) at \(p = p_n\) as in equation (9).

For uninterrupted system, end of entanglement \(p'\) depends on \(p\) as follows:

\[
p' = \frac{1}{2(0.0625p^2 - 0.15p + 0.0875)}[0.15p^2 + 0.035p - 0.25 + ((-0.15p^2 - 0.035p + 0.25))^2] - 4(0.0625p^2 - 0.15p + 0.0875)(0.0875p^2 + 0.25p - 0.1875)^{1/2}.
\] (14)

When LUOs are applied at \(p = p_n\), the end of entanglement \(p'\) depends on \(p_n\) as follows:

\[
p' = \frac{(p_n - 1)(0.0875p_n^2 + 0.25p_n - 0.1875)}{(0.25p_n + 0.5)(0.35p_n^2 + p_n + 0.25)}.
\] (15)

The figure 5 shows the non-linear curvature in \(p'\) versus \(p\) or \(p_n\) for ESD (red curve) and its manipulation (green curve). The manipulation leads to avoidance of ESD for \(0 \leq p_n \leq 0.0615\), delay for \(0.0615 < p_n < 0.1641\), and hastening of ESD for \(0.1641 < p_n < 0.6168\) as the green curve dips below red curve in this range. This can be easily verified through equations (14) and (15).

3.3. \(F_{01}\) applied to qutrit only

The trit-flip operation \(F_{01}\) is applied to the qutrit part of the state (11) at \(p = p_n\) as in equation (9). The figure 6 shows the non-linear curvature in \(p'\) versus \(p\) or \(p_n\) in ESD (red curve) and its manipulation (green curve). The manipulation leads to avoidance for \(0 \leq p_n \leq 0.2941\), delay for \(0.2941 < p_n < 0.6168\), and hastening of ESD does not occur in this case.

The LUOs \(\sigma_x \otimes F_{02}\) and \(I_2 \otimes F_{02}\) applied on the state (11) lead to same effect as \(\sigma_x \otimes F_{01}\) and \(I_2 \otimes F_{01}\), respectively. The other combination of LUOs such as \(\sigma_x \otimes F_{02}, \sigma_x \otimes F_{201}, \sigma_x \otimes I_2, I_2 \otimes F_{02}, I_2 \otimes F_{201}\) applied on the state (11) give rise to only hastening of ESD in the entire range \(0 < p_n < 0.6168\).

Let us now intuitively understand the disentanglement dynamics of the qubit–qutrit system and the occurrence of ESD. The state (11) is entangled due to the coherence terms \(\rho_{34}(|02\rangle\langle10|)\) and \(\rho_{43}(|10\rangle\langle02|)\) of the
The separability condition depends on the instantaneous value of the quantity
\[ N = \frac{1}{2} \left( \rho_{11} + \rho_{66} - \sqrt{\left( \rho_{11} - \rho_{66} \right)^2 + 4 \rho_{34} \rho_{43}} \right). \]
If \( N \) is negative, the state (11) is entangled else separable. In the presence of an ADC, coherence terms \( \rho_{34} \) and \( \rho_{43} \) decay as \( \sim (1 - p) \) and the term \( \rho_{66} \) decays as \( \sim (1 - p)(1 - p_2) \). The population of qubit-qutrit ground state \( \rho_{11} \) changes as \( \rho_{11} + \rho_{44} p + \rho_{22} p_1 + \rho_{55} p_1 + \rho_{33} p_2 + \rho_{66} pp_2 \). The terms \( \rho_{34} \) and \( \rho_{43} \) decrease with time and \( \rho_{11} \) increases. Due to the cumulative evolution of all these terms, the time at which \( N \) becomes zero, is known as the time of sudden death and the state (11) becomes separable afterwards. For \( p = 1 \) or \( p_2 = 1 \) system in equation (11) loosres the coherence completely and for \( p_1, p_2 = 1 \) qubit-qutrit system is found in the ground state \( |00 \rangle \).

The physical reason behind the action of different LUOs resulting in hastening, delay, or avoidance of ESD for this class of state can be understood as follows: when we apply the trit-flip operation \( \Phi_{01} \) (for example) on the system (11) after it has evolved in the ADC, it changes the instantaneous population between different levels of the qutrit in such a way that the elements of the density matrix (i) \( \rho_{11} \) and \( \rho_{22} \), (ii) \( \rho_{34} \) and \( \rho_{55} \) get swapped, and (iii) the coherence term \( \rho_{34} (p_{43}) \) change their position to \( \rho_{35} \) (855). When this flipped state evolves in the ADC, new coherence terms \( \rho_{35}^{(n)} \) and \( \rho_{53}^{(n)} \) (where terms with superscript ‘(n)’ indicate the density matrix elements after the application of LUOs) now decay as \( \sim (1 - p)(1 - p_1)(1 - p_2) \), and new \( \rho_{66}^{(n)} \) term decays as \( \sim (1 - p)(1 - p_2) \). The term \( \rho_{35}^{(n)} \) evolves as \( \rho_{11}^{(n)} + \rho_{44}^{(n)} p + \rho_{22}^{(n)} p_1 + \rho_{55}^{(n)} p_1 + \rho_{33}^{(n)} p_2 + \rho_{66}^{(n)} pp_2 \). After the application of LUOs,
condition for ESD now becomes:

\[ N = \frac{1}{2} (\rho^{(n)}_{11} + \rho^{(n)}_{66}) - \sqrt{(\rho^{(n)}_{11} - \rho^{(n)}_{66})^2 + 4 (\rho^{(n)}_{16} \rho^{(n)}_{61})}, \]

which looks similar to the earlier condition for ESD but it depends on new terms of the density matrix after the LUOs. The instantaneous population of different levels of the qubit-qutrit system depends on the decay rate of the different levels of the qubit-qutrit system, the time when LUO (trit-flip operation) is applied, and time elapsed after the application of LUO. Again, due to cumulative evolution of different terms of the density matrix, when \( N \) becomes zero, system becomes separable. The basic idea behind this entanglement protection scheme is to choose correct combination of LUOs depending on the decay rate of different levels of the qubit-qutrit system and the time of application of LUOs such that the state after the application of LUOs results in the delay or avoidance of ESD.

4. X-type qubit-qutrit entangled system: state-II

Let us consider another class of one-parameter qubit-qutrit entangled state given by,

\[
\rho(0) = \frac{x}{2} [\ket{00}\bra{00} + \ket{01}\bra{01} + \ket{11}\bra{11} + \ket{12}\bra{12} + \ket{00}\bra{12} + \ket{12}\bra{00}] \\
+ \frac{1 - 2x}{2} [\ket{02}\bra{02} + \ket{10}\bra{10}],
\]

where \( 1/3 < x \leq 1/2 \).

Assuming that both the subsystems; qubit as well as qurit, undergo local and independent ADC, evolution of the system is given by equation (6). For state (16), evolution of the entanglement versus ADC parameter \( p \) for \( 1/3 \leq x \leq 1/2 \) is computed. The initial entangled state (16) undergoes ESD in the entire range \( 1/3 < x \leq 1/2 \). For \( p = 0 \), ESD occurs at \( p' = 0.8452 \) and for arbitrary values for \( p \), ESD occurs along the non-linear curve in pp' plane similar to figure 4. We choose \( x = 0.5 \) such that ESD occurs at \( p = 0.8452 \), and study the effect of NOT operation \((\sigma_x)\) applied to the qubit, and/or trit-flip operations \( F_{01}, F_{02}, F_{10} \) applied to the qutrit, in manipulating the ESD. The effect of different combinations of LUOs in entanglement protection are discussed below.

4.1. \( \sigma_x \) applied to qubit and \( F_{01} \) applied to qutrit

The NOT operation \((\sigma_x)\) is applied to the qubit and trit-flip operation \( F_{01} \) is applied to the qutrit part of the state (16) at \( p = p_n \) as in equation (9). The figure 7 shows the non-linear curvature in \( p' \) versus \( p \) or \( p_n \) in ESD (red curve) and its manipulation (green curve). The manipulation leads to avoidance for \( 0 \leq p_n \leq 0.3586 \), delay for \( 0.3586 < p_n < 0.4177 \), and hastening of ESD for \( 0.4177 < p_n < 0.8452 \) as green curve lies below the red curve in this range.

4.2. \( \sigma_x \) operation applied to qubit only

The NOT operation \((\sigma_x)\) is applied to the qubit part of the state (16) at \( p = p_n \) as in equation (9). The figure 8 shows the non-linear curvature in \( p' \) versus \( p \) or \( p_n \) in ESD (red curve) and its manipulation (green curve). The
manipulation leads to avoidance for $0 \leq p_n \leq 0.2309$, delay for $0.2309 < p_n < 0.2964$, and hastening of ESD for $0.2964 < p_n < 0.8452$ as green curve lies below the red curve in this range.

4.3. $F_{01}$ applied to qutrit only
The trit-flip operation ($\mathcal{F}_{01}$) is applied only to the qutrit part of the state (16) at $p = p_n$ as in equation (9). The figure 9 shows the non-linear curvature in $p'$ versus $p$ or $p_n$ in ESD (red curve) and its manipulation (green curve). The manipulation of ESD leads to avoidance for $0 \leq p_n \leq 0.7143$, and delay of ESD for $0.7143 < p_n < 0.8452$ as green curve lies above the red curve but less than one in this range. The hastening of ESD does not occur in this case.

4.4. $F_{02}$ applied to qutrit only
The trit-flip operation $\mathcal{F}_{02}$ is applied to the qutrit part of the state (16) at $p = p_n$ as in equation (9). The figure 10 shows the non-linear curvature in $p'$ versus $p$ or $p_n$ in ESD (red curve) and its manipulation (green curve). The manipulation leads to avoidance for $0 \leq p_n \leq 0.2032$, delay for $0.2032 < p_n < 0.2693$, and hastening of ESD for $0.2693 < p_n < 0.8452$ as green curve lies below the red curve in this range.
4.5. $F_{201}$ applied to qutrit only

The trit-flip operation $F_{201}$ is applied to the qutrit part of the state (16) at $p = p_n$ as in equation (9). The figure 11 shows the non-linear curvature in $p'$ versus $p$ or $p_n$ in ESD (red curve) and its manipulation (green curve). The manipulation leads to avoidance for $0 \leq p_n \leq 0.2059$, delay for $0.2059 < p_n < 0.2676$, and hastening of ESD for $0.2676 < p_n < 0.8452$ as green curve lies below the red curve in this range.

5. Generalization to qutrit-qutrit system

In the framework of ADC, the dissipative interaction of system and environment causes the flow of population of the system from excited state to ground state. Therefore, any LUO which reverses this effect will change the disentanglement time. Thus, generalization of the above proposal to higher-dimensions is reasonably straightforward. For example, the form of unitary operations for two-qutrits will be same as in equation (7). However, lack of a well defined universal entanglement measure for mixed entangled states of dimension greater than six, makes it difficult to study the disentanglement dynamics in these systems because even an initial pure entangled state becomes mixed during the evolution.
For the purpose of our study, we use Negativity as a witness for entanglement, as in general, qutrit-qutrit entanglement is not known to be characterized fully, and that negativity is a sufficient but not necessary condition for entanglement. Thus, if negativity undergoes asymptotic decay then this implies that ESD does not happen. However, if negativity undergoes sudden death (NSD), this may be suggestive (but does not imply) ESD. Here, we take $a = 1$ ($p_1 = ap$) and $b = 0.75$ ($p_2 = bp$).

Let us consider an initially entangled two-qutrit system in the presence of ADC as given below.

\[
\rho(0) = \frac{x}{3} (|00\rangle \langle 00| + |02\rangle \langle 02| + |10\rangle \langle 10| + |12\rangle \langle 12| + |20\rangle \langle 20| + |21\rangle \langle 21|)
\]

\[
+ \frac{1 - 2x}{3} (|00\rangle \langle 00| + |11\rangle \langle 11| + |22\rangle \langle 22| + |22\rangle \langle 00| + |00\rangle \langle 02|),
\]

where $0 \leq x < 1/3$.

For two-qutrit system, Kraus operators are given by

\[
M_{ij} = M_i \otimes M_j ; \quad i, j = 0, 1, 2.
\]

Assuming that both the qutrits suffer identical but independent ADC, evolution of the system is given by,

\[
\rho(p) = \sum_{i,j} M_{ij} \rho(0) M_{ij}^\dagger.
\]

Evolution of the entanglement versus ADC parameter ($p$) for $0 \leq x < 1/3$ is computed. It is found that the entangled state (17) undergoes NSD in the entire range of $x$-parameter for $0 < x < 1/3$.

Let us label another set of Kraus operators ($M'_{ij}$) with the parameter $p$ replaced by $p'$ ($t$ replaced by $t'$, $t' = t - t_0$); $p' = 1 - \exp(-1/t')$, and of the form similar to (18), and apply it to the state (17) to get the state of the uninterrupted system evolving in the ADC.

\[
\rho(p', p) = \sum_{i,j} M'_{ij} \rho(0) M'_{ij}^\dagger.
\]

We choose $x = 0.25$ such that the initial state (17) undergoes NSD at $p = 0.3636$. For $p = 0$, NSD occurs at $p' = 0.3636$ and for arbitrary values for $p$, NSD occurs along the non-linear curve in $pp'$ plane similar to figure 4.

For protecting entanglement, trit-flip operation ($F_{01}$) is applied on only one of the qutrits at $p = p_n$ as follows,

\[
\rho^{(1)}(p_n) = (F_{01} \otimes I_y) \rho(p)(F_{01} \otimes I_y)^\dagger.
\]

Evolution of the system afterwards in ADC is given by,

\[
\rho^{(1)}(p', p_n) = \sum_{i,j} M'_{ij} \rho^{(1)}(p_n) M'_{ij}^\dagger.
\]

The figure 12 shows the non-linear curvature in $p'$ versus $p$ or $p_n$ for NSD (red curve) and its manipulation (green curve). The action of LUO ($F_{01}$) gives rise to avoidance of NSD for $0 \leq p_n \leq 0.0238$, delay for $0.0238 < p_n < 0.3636$, and hastening of NSD does not occur for this choice of parameter.
Table 1. Different combination of local unitary operations applied on the initial state-I (equation (11)) and state-II (equation (16)) resulting in avoidance (A), delay (D), and hastening (H) of ESD. Due to symmetry in the population of initial entangled state, LUOs $\sigma_x \otimes F_{01}$ and $I_z \otimes F_{01}$ applied on the either states lead to same effect as $\sigma_x \otimes F_{02}$ and $I_z \otimes F_{02}$, respectively.

| Operation     | State-I   | State-II  |
|---------------|-----------|-----------|
| $\sigma_x \otimes F_{02}$ | A, D, and H | A, D, and H |
| $\sigma_x \otimes F_{01}$ | only H    | only H    |
| $\alpha \otimes F_{01}$   | A, D, and H | A, D, and H |
| $\alpha \otimes F_{02}$   | only H    | Only H    |
| $\alpha \otimes I_z$      | only H    | A, D, and H |
| $I_z \otimes F_{01}$      | only A and D | only A and D |
| $I_z \otimes F_{02}$      | only H    | A, D, and H |
| $I_z \otimes F_{10}$      | only A and D | only A and D |

6. Summary and discussion

We have proposed a set of Local Unitary Operations (LUOs) for qubit-qutrit system undergoing Entanglement Sudden Death (ESD) in the presence of an amplitude damping channel (ADC), such that when they are applied locally on one or both subsystems, then depending on the initial state, choice of the operation, and its time of application, one can always suitably manipulate the ESD. We have considered two different classes of initially entangled qubit-qutrit systems which undergo ESD, and we find that for a given initial state, one can always find a suitable combination of LUOs, such that when applied at appropriate time it can always delay the time of ESD, and therefore facilitate the tasks which would not have been possible under shorter entanglement lifetime.

The results of different combinations of LUOs applied on the qubit-qutrit system on the manipulation of ESD for two different initially entangled states are summarized in the table 1 below. In some cases, ESD can be hastened, delayed, as well as avoided, whereas in other cases, it can be only delayed and avoided, or ESD can be only hastened. Due to symmetry in the population of initial entangled state, the NOT operations $\sigma_x \otimes F_{01}$ and $I_z \otimes F_{01}$ applied on the either states lead to same effect as $\sigma_x \otimes F_{02}$ and $I_z \otimes F_{02}$, respectively. Based on the results in table 1, the following LUOs are advisable: for example, for state-II, $\sigma_x$ on the first qubit and no action on the qutrit suffices to guarantee avoidance of ESD provide it is applied sufficiently early. Since this is the simplest of all possible combination of operations, this may be called the optimal in terms of gate operations. For state-I, all operations allowing avoidance are two-sided. It is worth noting here that although the noise is acting on both the subsystems, still even LUO applied on only one of the subsystems can suitably delay or avoid the ESD as in the case of a two qubit system.

Such a scheme will find application where two parties, say Alice and Bob, share an entangled pair for some quantum information processing task and they know a-priori that ADC is present in the environment, and therefore they can decide whether they are faced with the prospect of ESD. Then, they can locally apply suitable LUOs at appropriate time to delay or avoid the ESD. Our proposed scheme for preserving entanglement longer will also find application in entanglement distillation protocols.

Furthermore, these LUOs can also be effective in protecting higher-dimensional entangled quantum systems in the presence of generalized amplitude damping channel as in the case of two-qubit system [24]. Applying LUOs in the presence of generalized ADC towards the entanglement protection of higher dimensional systems could be an interesting work for future study. However, unlike ADC or generalized ADC, LUOs are not effective in protecting entanglement in the presence of bit-flip or phase-flip channel or both and the argument is as follows: under the action of ADC, population of a qubit (or qutrit) system flows from excited state to the ground state. The action of NOT operation (or more general LUO) is to reverse this effect by pumping the population from the ground to an excited state. While LUOs don’t change the amount of entanglement present in the system, they change the subsequent entanglement dynamics leading to hastening, delay or avoidance of ESD. This is the reason for effectiveness of LUOs in controlling the disentanglement dynamics in the presence of ADC. However, same reasoning doesn’t hold true for the bit-flip or phase flip channel as these are non-dissipative noises.

Other schemes which aim to protect entanglement in a noisy environment include dynamic decoupling [56], weak measurement (partial-collapse measurement) and quantum measurement reversal [57], and quantum Zeno effect [58]. Dynamic decoupling uses a sequence of $\pi$-pulses to protect the quantum states from noise. This
scheme can potentially freeze the initial state and thereby preserve the quantum coherence stroboscopically to infinite time. The weak measurement and reversal scheme is based on applying a weak measurement prior to decoherence and then probabilistically reversing this operation on the decohered state. The measurement induced quantum Zeno effect [58] for entanglement protection in cavity QED architecture, on the other hand, utilizes a very simple method of monitoring the population of the cavity modes that results in the entanglement protection exceeding its natural life-time. In our proposed method, LUOs alter the state in such a way that the decoherence effect on entanglement is minimized, i.e., for the states which were initially undergoing ESD, either time of ESD is delayed or it is averted altogether. Resource-wise, our scheme is simpler compared to the aforementioned schemes as it requires just one time intervention using LUOs but unlike the weak measurement and reversal protocol it does not restore the initial state after the decoherence.

To conclude, we have presented a scheme based on local unitary operations to protect entanglement from undergoing sudden death in the presence of amplitude damping channel for higher-dimensional systems. We have also compared and contrasted our entanglement protection scheme with other existing schemes. An interesting future line of theoretical study could be the entanglement protection in the presence of generalized amplitude damping channel in higher-dimensional systems. Our scheme will also attain more practical significance through commensurate future experiments.

Acknowledgments

We thank Prof. A R P Rau and Prof. R Srikanth for useful discussions. US would like to thank the Indian Space Research Organisation for support through the QuEST-ISRO research grant.

Data availability statement

No new data were created or analysed in this study.

ORCID iDs

Ashutosh Singh @ https://orcid.org/0000-0001-9628-6127

References

[1] Zurek W H 2003 Decoherence, einselection, and the quantum origins of the classical Rev. Mod. Phys. 75 715–75
[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 Quantum entanglement Rev. Mod. Phys. 81 865
[3] Bennett C H and DiVincenzo D P 2000 Quantum information and computation Nature 404 247
[4] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
[5] Breuer H P and Petruccione F 2002 The Theory of Open Quantum Systems (Oxford: Oxford University Press)
[6] Yu T and Eberly J H 2004 Finite-time disentanglement via spontaneous emission Phys. Rev. Lett. 93 140404
[7] Yu T and Eberly J H 2006 Quantum open system theory: bipartite aspects Phys. Rev. Lett. 97 140403
[8] Yu T and Eberly J H 2009 Sudden death of entanglement Science 323 598
[9] Laurat J, Choi K S, Deng H, Chou C W and Kimble H J 2007 Heralded entanglement between atomic ensembles: preparation, decoherence, and scaling Phys. Rev. Lett. 99 140504
[10] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H and Davidovich L 2007 Environment-induced sudden death of entanglement Science 316 555
[11] Lidar D A, Chuang I L and Whaley K B 1998 Decoherence-free subspaces for quantum computation Phys. Rev. Lett. 81 2594
[12] Viola Lorenza, Knill Emanuel and Lloyd Seth 1999 Dynamical decoupling of open quantum systems Phys. Rev. Lett. 82 2417
[13] Shor P W 1995 Scheme for reducing decoherence in quantum computer memory Phys. Rev. A 52 R2493
[14] Steane A M 1996 Error correcting codes in quantum theory Phys. Rev. Lett. 77 793
[15] Facchi P, Lidar D A and Pascazio S 2004 Unification of dynamical decoupling and the quantum Zeno effect Phys. Rev. A 69 032314
[16] Naokī Yamamoto Hendra I, Nurdin Matthew R, James and Petersen Ian R 2008 Avoiding entanglement sudden death via measurement feedback control in a quantum network Phys. Rev. A 78 042339
[17] Maniscalco S, Francica F, Zaffino R L, Gullo N L and Plastina F 2008 Protecting entanglement via the quantum Zeno effect Phys. Rev. Lett. 100 090503
[18] Oliveira I G Jr+ , Rossi R Jr+ and Nemes M C 2008 Protecting, enhancing, and reviving entanglement Phys. Rev. A 78 044301
[19] Sun Q, Al-Amri M, Davidovich L and Zubairy M S 2010 Reversing entanglement change by a weak measurement Phys. Rev. A 82 052323
[20] Korotkov A N and Keane K 2010 Decoherence suppression by quantum measurement reversal Phys. Rev. A 81 040103 R
[21] Rau A R P, Ali M and Alber G 2008 Fastening, delaying or averting sudden death of quantum entanglement EPL 82 40002
[22] Singh Ashutosh, Pradynuma Siva, Rau A R P and Sinha Urbasi 2017 Manipulation of entanglement sudden death in an all-optical setup J. Opt. Soc. Am. B 34 681–90
[23] Hussain Mahmood Irtiza, Tahira Rabia and Ikram Manzoor 2011 Manipulating the sudden death of entanglement in two-qubit atomic systems J. Korean Phys. Soc. 59 2901–4
[24] Ali M, Rau A R P and Alber G 2009 Manipulating entanglement sudden death of two-qubit X-states in zero- and finite-temperature reservoirs J. Phys. B: At. Mol. Opt. Phys. 42 025501
[25] Kwiat P G, Berglund A J, Altepeter J B and White A G 2000 Experimental verification of decoherence-free subspaces Science 290 498
[26] Kielpiński D, Meyer V, Rowe M A, Sackett C A, Itano W M, Monroe C and Wineland D J 2001 A decoherence-free quantum memory using trapped ions Science 291 1013
[27] Viola L, Fortunato E M, Pravia M A, Knill E, Laflamme R and Cory D G 2001 Experimental realization of noiseless subsystems for quantum information processing Science 293 2059
[28] Biercuk Michael J, Uys Hermann, VanDevender Aaron P, Shiga Nobuyasu, Itano Wayne M and Bollinger John J 2009 Optimized dynamical decoupling in a model quantum memory Nature 458 990
[29] Du J, Rong X, Zhao N, Wang Y, Yang J and Liu R B 2009 Preserving electron spin coherence in solids by optimal dynamical decoupling Nature 461 1265
[30] Lee J C, Jeong Y C, Kim Y S and Kim Y H 2011 Experimental demonstration of decoherence suppression via quantum measurement reversal Opt. Express 19 16309–16
[31] Kim Y S, Lee J C, Kwon O and Kim Y-H 2012 Protecting entanglement from decoherence using weak measurement and quantum measurement reversal Nature Phys. 8 117
[32] Lim H T, Lee J C, Hong K H and Kim Y H 2014 Avoiding entanglement sudden death using single-qubit quantum measurement reversal Opt. Express 22 19055
[33] Lee Jong-Chan, Lim Hyang-Tag, Hong Kang-Hee, Jeong Youn-Chang, Kim M S and Kim Yoon-Ho 2014 Experimental demonstration of delayed-choice decoherence suppression Nat. Commun. 5 4522
[34] Xu J-S, Li C-F, Gong M, Zou X-B, Shi C-H, Chen G and Guo G-C 2010 Experimental demonstration of photonic entanglement collapse and revival Phys. Rev. Lett. 104 100502
[35] Erhard M, Krenn M and Zeilinger A 2020 Advances in high-dimensional quantum entanglement Nat. Rev. Phys. 2 365–81
[36] Collins Daniel, Gisin Nicolas, Linden Noah, Massar Serge and Popescu Sandu 2002 Bell inequalities for arbitrarily high-dimensional systems Phys. Rev. Lett. 88 040404
[37] Kevin A and Gregg J 2008 Entanglement sudden death in qubit-qutrit systems Phys. Lett. A 372 579–83
[38] Ali M, Rau A R P and Ranade K 2007 Disentanglement in qubit-qutrit systems arXiv:0710.2238
[39] Ali M 2009 Quantum control of finite-time disentanglement in qubit-qubit and qutrit-qutrit systems Ph.D. Thesis
[40] Xing Xiao 2008 Protecting qubit-qutrit entanglement from amplitude damping decoherence via weak measurement and reversal Phys. Scr. 89 065102
[41] Xiao X and Li Y L 2013 Protecting qutrit-qutrit entanglement by weak measurement and reversal Eur. Phys. J. D 67 204
[42] Liao X-P, Fang M-F, Rong M-S and Zhou X 2017 Protecting free-entangled and bound-entangled states in a two-qutrit system under decoherence using weak measurements J. Mod. Opt. 64 1184–91
[43] Ali M 2010 Distillability sudden death in qutrit-qutrit systems under amplitude damping J. Phys B 43 045504
[44] Derkacz L and Jakobczyk L 2006 Quantum interference and evolution of entanglement in a system of three-level atoms Phys. Rev. A 74 032313
[45] Peres A 1996 Separability criterion for density matrices Phys. Rev. Lett. 77 1413
[46] Horodecki M, Horodecki P and Horodecki R 1996 Separability of mixed states: necessary and sufficient conditions Phys. Lett. A 223 1
[47] Horodecki P, Lewenstein M, Vidal G and Cirac I 2000 Operational criterion and constructive checks for the separability of low-rank density matrices Phys. Rev. A 62 032310
[48] Rudolph O 2004 Computable cross-norm criterion for separability Lett. Math. Phys. 70 57–64
[49] Rudolph O 2005 Further results on the cross norm criterion for separability Quant. Inf. Proc. 4 219–39
[50] Chen K and Wu L-A 2003 A matrix realignment method for recognizing entanglement Quantum Inf. Comput. 3 193–202
[51] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimond JM and Haroche S 1997 Generation of einstein-podolsky-rosen pairs of atoms Phys. Rev. Lett. 79 1
[52] Lanyon B P, Weinhold T J, Langford N K, O’Brien J L, Resch K J, Gilchrist A and White A G 2008 Manipulating biphotonic qutrits Phys. Rev. Lett. 100 060504
[53] Fickler R, Lapkiewicz R, Plick W N, Krenn M, Schaeff C, Ramelow S and Zeilinger A 2012 Quantum entanglement of high angular momenta Science 338 640–3
[54] Galvez E J, Nomoto S M, Schubert W H and Novenstern M D 2011 Polarization-Spatial-Mode Entanglement of Photon Pairs International Conference on Quantum Information, OSA Technical Digest (CD) (Optical Society of America) paper QMI18
[55] Zheng Shi-Biao 2006a Production of entanglement of multiple three-level atoms with a two-mode cavity Commun. Theor. Phys. 45 539–41
[56] Viola L, Knill E and Lloyd S 1999 Dynamical decoupling of open quantum systems Phys. Rev. Lett. 82 2417
[57] Kim Y-S, Lee J-C, Kwon O and Kim Y-H 2012 Protecting entanglement from decoherence using weak measurement and quantum measurement reversal Nat. Phys. 8 117–20
[58] Maniscalco S, Francica F, Zaffino R L, Gullo N L and Plastina F 2008 Protecting entanglement via the quantum Zeno effect Phys. Rev. Lett. 100 090503