Born-Infeld-type phantom on the brane world

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ABSTRACT: We study the evolution of Born-Infeld-type phantom in the second Randall-Sundrum brane scenario, and find that there exists attractor solution for the potential with a maximum, which implies a cosmological constant at the late time. Especially, we discuss the BI model of constant potential without and with dust matter. In the weak tension limit of the brane, we obtain an exact solution for the BI phantom and scale factor and show that there is no big rip during the evolution of the brane.

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The result of the recent astronomical observations [1], including WMAP [2], indicate that about seventy percent of the energy density in our universe is in the form of dark energy that has negative pressure and can drive the accelerating expansion of the universe. Many candidates for dark energy have been proposed so far to fit the current observations. Among these models, the most important ones are cosmological constant and a time varying scalar field evolving in a specific potential, referred to as "quintessence" [3]. The major difference among these models are that they predict different equation-of-state parameter \( w \) of the dark energy and thus different cosmology. Another way to distinguish the nature of dark energy is to measure its sound speed \( c_s \) which affects the perturbations in the energy distribution and thus detectable through the observation of cosmic microwave background(CMB) and large scale structure(LSS) [4]. For cosmological constant and quintessence with canonical Lagrangian, the sound speed is equal to unity (the speed of light). But the sound speed of dark energy with non-canonical Lagrangian can differ from unity and vary with time.

It is worth noting that recent observations do not exclude, but in fact suggest a value even less than \(-1\), indeed, they can lie in the range \(-1.38 < w < -0.82\) [5]. Especially, the new results from SN-Ia alone are suggesting \( w < -1 \) at 1 \( \sigma \) [6]. So far, no fundamental justification is given for the negative kinetic (phantom energy) term whose use is hence motivated by the phenomenology. Phantom energy, a term coined by Caldwell [7] for matter with \( w < -1 \), certainly has some strange properties. For example, its energy density increases with time. Phantom field also violates the dominant-energy condition [8] that might allow the existence of astrophysical or cosmological wormholes and a striking consequence that the universe will undergo a catastrophic "big rip" in a finite cosmic time. However, it is avoidable that the universe will undergo a catastrophic big rip [8]. Matter with \( w < -1 \) has received increased attention among theorists, some phantom models that possess negative kinetic energy are investigated by many authors [8, 10]. Recently, Hao and Li [11] proposed an interesting model of phantom with a Born-Infeld (BI) type Lagrangian and show that current universe is not a stable stage in such a model while is in its way to the stable stage, at which the universe is dominated by the vacuum energy like dark energy with a equation of state of \(-1\).

BI phantom model was also studied in Ref. [12].

On the other hand, there are intensive interest in the brane world scenario during the past several years [13]. In contrast to the conventional Kaluza-Klein (KK) picture where the extra-dimensions are compacted on a small enough radius to evade detection in the form of KK modes, the extra dimensions could be large in the brane world scenario. Particles in the standard model are expected to be confined to the brane, whereas the gravitons propagate in the entire bulk spacetime, which gives an interesting feature in the brane world scenario. In the Randall-Sundrum (RS) second model [14], 4D Newtonian gravity is recovered at low energies, because gravity is confined in a single positive-tension brane even if the extra dimension is not compact. For this model, many authors discussed the geometrical aspects [15] as well as cosmology [16]. The purpose of this work is to study the evolution of a BI type dark energy with \( w < -1 \) on the brane world. We shall firstly discuss the general property of the BI phantom, then analyze a concrete model numerically.

We consider the second RS brand world scenario on a 3-brane, in which the bulk is invariant under \( Z_2 \) reflection along the brane and described by 5-dimensional Einstein gravity with a negative cosmological constant and the brane motion is described by Israel’s junction condition. In this scenario, the bulk geometry is AdS-Schwarzschild spacetime and the evolution of the brane is governed by

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the following effective Friedmann equation:

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa_5^2}{36} \rho_b^2 - \frac{K}{a^2} + \frac{\mu}{a^4} - \frac{1}{H^2}, \]  

(1)

where \( \kappa_5 \) is the 5-dimensional gravitational coupling constant, \( \rho_b \) is the energy density on the brane, \( K \) is the curvature constant with positive, zero and negative value corresponding to spherical, flat and hyperbolic brane, respectively, \( l = \sqrt{\frac{\kappa_5}{3\kappa_4}} \) is the length scale of the bulk (negative) cosmological constant \( \Lambda_5 \), and \( \mu \) is related to the mass parameter of the bulk black hole. The term \( \frac{\mu}{a^4} \) is due to Weyl tensor in the bulk and can be understood as dark radiation. This equation relates the Hubble parameter to the energy density but it is different from the usual Friedmann equation. The most remarkable feature of Eq.(1) is that the first term in the right hand side of this equation is proportional to \( \rho_b^2 \) which is different from that of the standard Friedmann equation which the energy density enters linearly. If one considers a brane with the total energy density

\[ \rho_b = \rho + \sigma, \]  

(2)

where \( \sigma \) is the tension of the brane which is constant in time, and \( \rho \) the energy density of ordinary cosmological matter, then one obtains

\[ H^2 = \frac{\kappa^2}{3} \rho \left( 1 + \frac{\rho}{\sigma} \right) + \frac{\mu}{a^4} - \frac{K}{a^2}, \]  

(3)

where \( \kappa^2 = 8\pi G = \frac{\kappa_4^2}{\kappa_5^2} = \kappa_2^2 l^{-1} \). It is clear that the standard cosmology is recovered at low energy if the dark radiation is neglected.

The BI phantom with Lagrangian as following [11]:

\[ L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}. \]  

(4)

where \( g_{\mu\nu} \) is the induced brane metric and \( V(\phi) \) is the potential of the model. On the homogeneous and isotropic brane, we can rewrite the above Lagrangian as

\[ L = -V(\phi)\sqrt{1 + \dot{\phi}^2} \]  

(5)

for the spatially homogeneous phantom field. The equation of motion reads

\[ \ddot{\phi} + 3H\dot{\phi}(1 + \dot{\phi}^2) - \frac{V'(\phi)}{V(\phi)}(1 + \dot{\phi}^2) = 0 \]  

(6)

where \( H \) is the Hubble parameter as that in Eq.(1), the over dot represents the differentiation with respect to \( t \) and the prime denotes the differentiation with respect to \( \phi \). The density \( \rho_\phi \) and the pressure \( p_\phi \) are defined as following:

\[ \rho_\phi = \frac{V(\phi)}{\sqrt{1 + \dot{\phi}^2}}, \]  

(7)

\[ p_\phi = -V(\phi)\sqrt{1 + \dot{\phi}^2}. \]  

(8)

Therefore, the equation of state can be written as

\[ w = \frac{p_\phi}{\rho_\phi} = -\dot{\phi}^2 - 1, \]  

(9)

and the sound speed

\[ c_s^2 = \frac{p_\phi X}{\rho_\phi X} = -w, \]  

(10)

where \( X = \frac{1}{2}\dot{\phi}^2 \). It is obvious that the equation of state \( w \) will be less than \(-1 \) and \( c_s^2 \) greater than the speed of light (unity), unless the kinetic energy term \(-\dot{\phi}^2 = 0 \).

Before discussing the property of general model, let us first consider a simple model that \( V = V_0 \) where \( V_0 \) is a positive constant. In this case, BI phantom behaves as Chaplygin gas with \( p_\phi = -\frac{V_0^2}{\rho_\phi^2} \) which is now a popular candidate for dark energy. Meanwhile, one can find the obviously difference with respect to the 4-dimensional case considered in Ref.[11]. There is a potentially observable difference between the two scenarios for the specific potential. In the specific case of \( V_0 >> \sigma \), the evolution of the brane is determined by

\[ \frac{da}{dt} = \frac{\kappa V_0}{\sqrt{3\sigma(1 + \dot{\phi}^2)}}, \]  

(11)

\[ \frac{d(\dot{\phi}^2)}{dt} = \frac{\kappa V_0}{3\sigma}\phi^2 \sqrt{1 + \dot{\phi}^2}. \]  

(12)

From Eqs.(11) and (12), we obtain the solution of the scale factor that

\[ \frac{\sqrt{c_0 a^3} + \sqrt{c_0 a_0^3} - 1}{\sqrt{c_0 a_0^3} + \sqrt{c_0 a^3} - 1} = \exp \left[ -\frac{3}{\sigma} \kappa V_0 (t - t_0) \right], \]  

(13)

where \( c_0, a_0 \) and \( t_0 \) are three positive constants, and the equation-of-state parameter \( w = -\frac{1}{\sqrt{3}\kappa V_0} - 1 < -1 \). As above mentioned, it has been pointed out by some authors that in the \( w < -1 \) case the fate of the universe may be a big rip [3]. However, we notice that \( a \to \infty \) when and only when \( t \to \infty \), and there is, therefore, no such a doomsday for this solution.
In the 4-dimensional case with the constant potential which corresponds to the limit case $V_0 << \sigma$ in brane scenario, we have a solution as follows

$$t - t_0 = \frac{2}{3a} \left( \frac{\rho_0}{c_0} \right)^{1/4} F \left[ \frac{1}{4}, \frac{5}{4}, \frac{3}{4}, -\frac{\rho_0}{c_0} \right],$$

(14)

where $F$ denotes Hypergeometric Function, $c_0$ and $t_0$ are two integral constants. It is easy to find that $a \sim t^{2/3}$ for early time and $a \sim e^{\sqrt{M_*}t}$ for late time. But in Eq. (13), $a(t)$ expands exponentially not only in the late time but also in the early time. Therefore, in the brane scenario, the universe comes into the accelerating phase earlier, which is a potentially observable effect, and $\sigma$ is an impressive and potentially observable parameter. Furthermore, this argument can be generalized to the model with other reasonable potentials.

In general case, according to the equation of energy conservation, we have

$$\dot{\rho} = \sqrt{3\kappa} \left( \rho_0 + \frac{\rho_d^2}{\sigma} \right)^{1/2} \left( \frac{V_0^2}{\rho_0} - \rho_0 \right).$$

(15)

Because $\rho_0$ is always not greater than $V_0$, the energy of BI phantom will grow increasingly until $\rho_0$ is equal to $V_0$ at that time $\dot{\phi} = 0$ and hence $w$ becomes $-1$.

In a more realistic model we consider, including pressureless dust case, the conservation equation and Friedmann equation can be written as

$$\rho_d' + \rho_d = 3 \left( \frac{V_0^2}{\rho_0} - \rho_0 - \rho_d \right),$$

(16)

$$N^2 = \frac{\kappa^2}{3} \left[ \dot{\rho}_d + \rho_0 + (\rho_d + \rho_0)^2 \sigma^{-1} \right],$$

(17)

where $N \equiv \ln a$ and dot and prime denote derivatives with respect to $t$ and $N$, respectively. We assume that there is no interaction between dust and phantom,

$$\rho_d = \rho_d e^{3(N_i - N)} ,$$

(18)

$$\rho_0 = [V_0^2 - (V_0^2 - \rho_0^2)e^{6(N_i - N)}]^{1/2} ,$$

(19)

where $N_i$, $\rho_d$ and $\rho_0$ denote the value of $N$, energy densities of dust and phantom at $t_i$ respectively. Therefore, with the growth of the scale factor, the cosmic density parameter for dust $\Omega_d = \frac{\rho_d}{\rho_{tot}}$ decreases to zero, while the cosmic density for BI phantom $\Omega = \frac{\rho_0}{\rho_{tot}}$ increases to unity, where $\rho_{tot}$ denotes the total energy density on the brane.

Now, we turn to BI phantom with general potential on the brane. For simplicity, we consider the spatially flat brane filled with BI phantom, the energy density of non-relativistic matter and radiation can be neglected, i.e. $\rho \simeq \rho_0$. In this case, omitting the effect of dark radiation ($\frac{\rho}{\sigma}$ term), we have

$$\ddot{\phi} + \sqrt{3\kappa} \dot{\phi} (1 + \phi^2)^{3/4} V^{1/2} (\phi) \left( 1 + \frac{V(\phi)}{\sqrt{1 + \phi^2}} \right)^{1/2} - V' (\phi) (1 + \phi^2) = 0. $$

(20)

Introducing the new variables

$$x = \phi, \quad y = \dot{\phi},$$

(21)

then Eq. (20) becomes

$$\frac{dx}{dt} = y,$$

(22)

$$\frac{dy}{dt} = (1 + y^2) \frac{V'(x)}{V(x)} - \sqrt{3\kappa} y (1 + y^2)^{3/4} V(x)^{1/2} (1 + \sigma^{-1} V(x) (1 + y^2)^{-1/2})^{1/2}.$$

Linearizing the above system around its critical point $(x_c, 0)$ where the value of $x_c$ is determined by $V'(x_c) = 0$, one obtain the following linear system

$$\frac{dx}{dt} = y,$$

(23)

$$\frac{dy}{dt} = \frac{V''(x_c)}{V(x_c)} x - \kappa \sqrt{3V_c (1 + \sigma^{-1} V_c)} y,$$

where $V_c$ is the value of potential at the critical point which is a stable node when $V'' < 0$ and a saddle when $V'' > 0$. Therefore, the system permits attractor solutions when the potential of the Born-Infeld scalar field have a maximum.

For the brane with strong tension, $\sigma >> V_0$, the Eq. (20) recovers the case of standard Einstein frame which have been considered in Ref. [11], and for the brane with weak tension, $\sigma << M^4_p$, we have

$$\ddot{\phi} + \sqrt{3\kappa} \dot{\phi} \sqrt{1 + \phi^2} - V'(\phi) V \left( \frac{1 + \phi^2}{V} \right) = 0.$$  

(24)

Especially, we choose a potential $V(\phi) = \frac{2\sqrt{\sigma}}{\sqrt{3\kappa}} (1 + \phi^2)^{-1/2}$ with an maximum at $\phi = 0$. There exists an exact solution $\phi \sim e^{-t}$, which implies that phantom will decay into a cosmological constant exponentially, meanwhile the scale factor of the brane goes with $a \sim (1 + e^{2t})^{1/2}$. We notice that $a \rightarrow \infty$ when and only when $t \rightarrow \infty$, and hence there is also no ‘big rip’ for this solution.

Next, we consider another potential as follows

$$V(\phi) = V_0 (1 + \frac{\phi}{\phi_0}) e^{-\phi/\phi_0}$$

(25)
Clearly, this potential have also a maximum value \( V_0 \) at \( \phi = 0 \). By rescaling, Eq. (20) can be reduced

\[
\ddot{\phi} + \alpha \dot{\phi}(1 + \dot{\phi}^2)^{3/4}(1 + \phi)^{1/2}e^{-\phi/2} \left(1 + \beta \frac{(1 + \phi)e^{-\phi}}{\sqrt{1 + \phi^2}}\right)^{1/2} = 0,
\]

where the dimensionless parameters \( \alpha = \sqrt{3V_0/\kappa} \phi_0 \) and \( \beta = \frac{\kappa}{\rho_c,i} \), respectively.

The numerical results with different brane tension are shown in FIG.1, from which we can find that the BI phantom behaves as a cosmological constant at the late time (\( c_s^2 \rightarrow 1 \) and \( w \rightarrow -1 \) when \( t \rightarrow \infty \)) and the weaker the brane tension is, the faster the BI phantom comes into cosmological constant.

When matter and radiation are also considered, we have

\[
\frac{dx}{dN} = \frac{yH_i^{-1}E(N)^{-1/2}}{1 + \frac{\beta}{\alpha}E(N)^{-1/2}},
\]

\[
\frac{dy}{dN} = -3y(1 + y^2)
\]

\[
- \frac{x(1 + y^2)H_i^{-1}E(N)^{-1/2}}{1 + x} \left[1 + \frac{\beta}{\alpha}E(N)^{-1/2}\right]^{1/2}
\]

where \( E(N) = \Omega_{m,i} e^{-3N} + \Omega_{r,i} e^{-4N} + \frac{\alpha(1+z)e^{-x}}{\sqrt{1 + y^2}}, \) \( H_i \), \( \Omega_{m,i} \) and \( \Omega_{r,i} \) are Hubble parameter, density parameter of matter and radiation at \( t_i \), respectively. The parameter \( \alpha = \frac{\kappa}{\rho_c,i} \), where \( \rho_c,i \) is the critical density at \( t_i \).

The FIG 2 plots the evolution of density parameter of BI phantom \( \Omega_\phi \) with the different value of \( \beta = \frac{\kappa}{\rho_c,i} \). We find again that the weaker the brane tension is, the earlier the BI phantom becomes dominant, which is potentially observable effect.

In summary, the behavior of the BI phantom on the brane will attract to a cosmological constant for the model with a potential having a maximum and the brane will be dominated by the BI phantom at the late time. Even though these models are all asymptotically de Sitter, in the sense that they are indistinguishable from dS space at very late time, the tension of the brane \( \sigma \) is a potentially observable quantity which determines sensitively the evolution of the scale factor \( a \). We obtain an exact solution for the BI phantom and the scale factor of the brane for a model satisfying above condition in weak tension limit and find that the brane will not trend to a big rip. It is of importance to point out that the BI phantom, which can be treated as a realization of generalized Chaplygin gas model for dark energy, may exist not only nowadays, but also in the very early period of the universe while the effect of the brane might be leading.

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[1] P. de Bernardis et al., Nature 404 955 (2000); S. Hanany et al. Astrophys. J. 545 1 (2000); N. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science 284
5

1481 (1999); S. Perlmutter et al., Astrophys. J. 517 565 (1999); A. G. Riess et al., Astron. J. 116 1009 (1998).

[2] C. Bennett et al., astro-ph/0302207; G. Hinshaw et al., astro-ph/0302217; A. Kogut et al., astro-ph/0302021

[3] B. Ratra and P. J. E. Peebles, Phys. Rev. D37 3406 (1988); R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80 1582 (1998); P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D59 123504 (1999); I. Zlatev, L. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82 896 (1999); K. Coble, S. Dodelson, J. A. Frieman, Phys. Rev. D55 1851 (1997); X. Z. Li, J. G. Hao, D. J. Liu, Class. Quantum Grav. 19 6049 (2002); X. Z. Li, D. J. Liu and J. G. Hao, Chin. Phys. Lett. 19, 1584 (2002).

[4] J. K. Erickson et al, Phys. Rev. Lett. 88 121301 (2002); S. DeDeo, R. R. Caldwell and P. J. Steinhardt, Phys. Rev. D67, 103509 (2003).

[5] A. Melchiorri, L. Mersini, C. J. Odmann and M. Trodden, astro-ph/0302207

[6] J. L. Tonry et al., astro-ph/0305008

[7] R. R. Caldwell, Phys. Lett. B545, 23 (2002).

[8] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, astro-ph/0302021; G. W. Gibbons, hep-th/0302199

[9] J. G. Hao and X. Z. Li, hep-th/0306033

[10] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D9 373 (2002); L. Parker and A. Raval, Phys. Rev. D60 063512 (1999); T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D62 023511 (2000); B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. 85 2236 (2000); A. E. Schulz, Martin White, Phys. Rev. D64 043514 (2001); V. Faraoni, Int. J. Mod. Phys. D11 471 (2002); I. Maor, R. Brustein, J. McMahon and P. J. Steinhardt, Phys. Rev. D65 123003 (2002); V. K. Onemli and R. P. Woodard, Class. Quant. Grav. 19 4607 (2002); D. F. Torres, Phys. Rev. D66 043522 (2002); S. M. Carroll, M. Hoffman, M. Trodden, astro-ph/0301273; P. H. Frampton, hep-th/0302007; J. G. Hao and X. Z. Li, Phys. Rev. D67, 107303 (2003); X. Z. Li and J. G. Hao, hep-th/0303093; A. Feinstein and S. Jhingan, hep-th/0304069; L. P. Chimento and A. Feinstein, astro-ph/0305007; P. Singh, M. Sami and N. Dadhich, hep-th/0305110; S. Nojiri and S. D. Odintsov, Phys. Lett. B562 147 (2003); hep-th/0303117; S. Nojiri and S. D. Odintsov, Phys. Lett. B565 1 (2003); hep-th/0304131

[11] J. G. Hao and X. Z. Li, Phys. Rev. D68, 0435XX (2003), in press, hep-th/0305207

[12] S. Nojiri and S. D. Odintsov, hep-th/0306212

[13] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436, 257 (1998); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D59, 086004 (1999); T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. D62, 024012 (2000); G. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D64, 084004 (2001).

[14] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); ibid, 83, 4690 (1999).

[15] C. Deffayet, G. Dvali, G. Gabadadze and A. Vainshtein, Phys. Rev. D65, 044026 (2002); C. Deffayet, Phys. Rev. D66, 103504 (2002); G. Kofinas, E. Papantonopoulos and I. Pappa, Phys. Rev. D66, 104014 (2002); G. Kofinas, E. Papantonopoulos and V. Zamarias, Phys. Rev. D66, 104028 (2002).

[16] C. Deffayet, Phys. Lett. B502, 199 (2001); C. Deffayet, G. Dvali and G. Gabadadze, Phys. Rev. D65, 044023 (2002); C. Deffayet, S. J. Landau, J. Raux, M. Zaldarriaga and P. Astier, Phys. Rev D66, 024019 (2002); N. J. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B504, 323 (2001).

[17] A. A. Gerasimov and S.L. Shatashvili, J.High Energy Phys. 0010, 034 (2000); D. Kutasov, M. Marino and G. W. Moore, J.High Energy Phys. 0010, 045 (2000).