Preheating in Bubble Collisions

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In a landscape with metastable minima, the bubbles will inevitably nucleate. We show that during the bubbles collide, due to the dramatically oscillating of the field at the collision region, the energy deposited in the bubble walls can be efficiently released by the explosive production of the particles. In this sense, the collision of bubbles is actually highly inelastic. The cosmological implications of this result are discussed.

When the universe is initially set in a metastable minimum of certain landscape of scalar fields, it will undergo a dS expansion, bubbles with lower energy minima will inevitably nucleate in this background\[1\]. When the radius of bubble is larger than its critical radius, the bubble will expand outwards, and eventually collide with other expanding bubbles. In general, it is expected that during the bubbles collide, the energy deposited in the bubble walls will be released, e.g.\[2\]. This release of energy is significant, e.g. in old inflation \[3\], extended inflation \[4\], and \[5\],\[6\],\[7\],\[8\]. In general, it is thought that during the bubbles collision the energy deposited in the bubble walls will be released, e.g.\[2\]. This release of energy is efficient released by the explosive production of the particles.

The bubble collision has been studied earlier in \[3\],\[10\]. In general, when the bubbles collide, the walls of bubbles will pass through each other or be reflected. The region between outgoing walls remains in high energy metastable minimum, while other region is not affected. However, there is a net force, which will compel the walls to rest, and then back and move towards each other. Thus the collision of walls will inevitably occur again and again. This oscillation of walls has been displayed in the numerical simulations for bubble collision \[3\],\[11\],\[12\],\[13\]. In general, it is thought that during the bubble collision the energy deposited in the walls will be released by the direct decaying of scalar wave into other particles, e.g.\[11\],\[14\], or the gravitational radiation, e.g.\[12\],\[15\],\[16\],\[17\]. However, this release of the energy might be more dramatic than expected. In this paper, we show that due to the oscillation of the background field at the collision region, the energy can be efficiently released by the explosive production of the particles.

We begin with a brief review of the numerically simulation of the collision of bubbles nucleating in a given potential in Fig.1, in which the high energy metastable minimum is $\phi_F$ and the low energy minimum is $\phi_I$. We only care the numerical results of the evolution of field at the collision region during the bubble collision. Thus the details of the equations of the nucleation and evolution of bubble are neglected, see e.g.\[18\],\[19\].

The field $\phi$ is initially in $\phi_F$. Thus the universe is inflating. Then the bubbles with $\phi_F$ will be expected to nucleate. The radius of bubble is determined by the instanton equation of $\phi$. We have numerically solved this instanton equation and will use the data obtained as the initial state of bubbles which will collide. Their initial distance is defined as $D_0$. Here we choose $D_0 \sim \frac{375}{1/\eta}$, where $H_F$ is the Hubble rate of the false vacuum which can be estimated by $H_F \approx \sqrt{V(\phi_F)}/M_P$ and in this paper $H_F \approx \eta^2/100$. $M_P = 1$ is set for whole paper and $\eta$ is the normalized parameter with mass dimension, see Fig.1. The nucleation radius of the bubbles is $R_0 = 3\sigma/V_F$, which can be given by

$$R_0 = \frac{3}{\phi_F} \sqrt{2V(\phi)\phi} \frac{d\phi}{V_F} \approx 0.3 \frac{\sqrt{2\eta}}{\sqrt{V_F}}$$

where $\sigma$ is the surface tension of bubble wall, which means that $H_F R_0 \sim 3\sqrt{2}\eta \ll 1$, since $\eta \ll 1$ and in the simulations $\eta = 0.01$ is actually used.

In the simulation of the evolution and collision of bubbles, we will neglect the gravitational effect, which is not important since $V(\phi) \ll 1$, the nucleation radius of bubbles $R_0 H_F \ll 1$, their initial separation $D_0 H_F \ll 1$ and the time per oscillation, which can be found in Fig.2

50/$M_F \approx 125/\eta \ll 1$.

We solve the evolving equation $\Box \phi = \partial_\phi V$ of field with the flat space metric in 3+1D by using a modified version of LATTICEEASY \[20\] and 2048 lattices, in which the initial data of bubbles is given by the instanton equation, and at $t = 0$, $x = \pm 0.5 D_0$.

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FIG. 1: The potential with two minima, which will be used in the numerical simulations of the bubble collision in Figs.2 and 4, and the production of $\chi$-particle in collision region in Fig.3. $\eta \equiv |\phi_T - \phi_F|$ is a normalized parameter with mass dimension. The parameter we used is $\eta = 0.01$ for whole paper.
where $g$ is the coupling constant, and $\phi$ is a parameter between 0 and 1, see the renormalization in Fig. 11.

We will neglect the expansion of space. However, the result is not altered qualitatively by the inclusion of expansion. The evolution of $\phi$ at the collision region is given by $\phi C$ in Fig. 2. This coupling means that when $|\phi C - \phi_s| \sim 0$, the adiabatic condition becomes violated, the parametric resonance will inevitably occur, which will result in the explosive production of $\chi$-particles at corresponding region, similar to the preheating after inflation [22, 23], which has been intensively studied, e.g. [24, 25].

The initial width $d_0$ of bubble wall can be estimated as $d_0 \sim \frac{g}{\sqrt{V_{bar}}}$, in which $V_{bar}$ is the effective height of the potential barrier. This width will become $d = d_0/\gamma$ at the time of bubbles collision, in which $\gamma$ can be given by $\gamma = \frac{D_0}{2D_0} + 1$, where it is noticed that $D_0$ is the distance between the bubble walls at the time of their nucleations.

When the parameters in Fig.1 are considered, $d_0 \sim 132/\eta$ and $\gamma \sim 2.5$ are obtained. Thus $d \sim 53/\eta$. Then, in the linear order the velocity that $\phi C$ passes through $\phi_s$ can be estimated as $\dot{\phi}_s \simeq \sqrt{\frac{g\phi C}{d}}$. Thus in a very short interval $\Delta t \sim 1/\sqrt{g\phi_s} \simeq \sqrt{\frac{\gamma}{gd\Phi C}} d$, the adiabatic condition is broken. Thus the characteristic momentum is given by

$$k_s \sim (\Delta t)^{-1} \simeq \sqrt{g\phi_s} \simeq \sqrt{\frac{g\Phi C}{d}},$$

which means $\Delta t \sim 0.35d$ and $k_s \sim 3/d$. This is consistent with general assumption that the energy of particles produced by the bubbles collision is about $1/d$, e.g. [14].

During the bubbles collide, the collision of the bubble walls is periodical, which induces $|\phi C - \phi_s| \sim 0$ twice at each time of the collisions of bubble walls, since $\phi C$ overshoots to $\phi T$ and then backs to $\phi F$, and oscillates around $\phi_F$. The lower panel is that of $\phi C$ in color panel. Here $M_F \sim 0.04\eta$.

is considered, since the collided bubbles can be boosted into a particular frame in which they are nucleated at the same time. The program is ran with double precision. The time step is small enough to guarantee the Courant stability condition. In the meantime, we checked the results by using higher precision, and found that the results are same. The numerical result is plotted in Fig. 2 in which the width in space is actually far larger than that in time, and the most of regions irrelevant with the bubbles collision is cut out to make the colliding region in figure clearer.

The color panel in Fig. 2 is that of the evolution and collision of bubbles in position space. We define the value of $\phi$ field in a small region around the collision center as $\phi C$. The lower panel of Fig. 2 is the evolution of $\phi C$, which can be explained as follows. $\phi C$ before collision rests with $\phi C = \phi F$. However, when the collide occurs, the gradient energy of this region will become large, which will induce $\phi C$ get cross the potential barrier, overshoot $\phi T$. Then it backs to $\phi F$ and oscillates. This behavior is repeated during the following collisions of bubble walls. However, when there is the third lower energy minimum $\phi_s$, after overshooting $\phi T$, the field at the collision region might not back to $\phi F$, and straightly run into $\phi_3$. Thus a new bubbles will be generated [21].

We introduce a coupling of the background field $\phi$ with $\chi$ as follows

$$\mathcal{L}_{int} = -\frac{1}{2}g^2(\phi - \phi_s)^2\chi^2,$$

where $g$ is the coupling constant, and $\phi_s$ is a parameter

![Figure 2: The numerical simulation of bubbles collision. The color panel describes the evolution and collision of bubbles after their nucleations, where the green region denotes the region in $\phi_T$ while the blue region denotes that in $\phi_F$, and the light blue region between them denotes the bubble wall. We can see that during the bubbles collision the bubble walls will be oscillating around the center of bubbles collision, which induces that at the collision region the field $\phi$ initially in $\phi_F$ overshoots to $\phi_T$ and then backs to $\phi_F$, and oscillates around $\phi_F$. The lower panel is that of $\phi C$ in color panel. Here $M_F \sim 0.04\eta$.](image-url)
energy can be quite efficient \cite{22}. In this case, the release of

\[ \phi \]  

can be considered. When the energy deposited in the

\[ X \]  

case in \cite{26}. This case is not efficient for the release

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G. The larger \( g \) is, the larger

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is insensitive to the concrete value of \( \phi_* \). The reason is

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In above estimate, the backreaction of

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the speed of the walls decrease after each collision and this will make the peaks

\[ \phi \]  

In general, the explosive production of particles

\[ \phi \]  

the resonant behavior are
different for different \( \phi_* \). The intensity of the parametric

\[ \phi \]  

and the modes shown are \( k = 0 \) and \( k = 0.1 \eta \), respectively.

We also show the evolution of \( \phi_C \) in red lines.

for different values of \( \phi_* \) by using the data of the bubbles

collision in Fig.3. The red lines in this figure denote

the evolution of \( \phi_C \) during the bubbles collision. When

the bubble walls collide, the red line has a sharp peak

which shows the rapid change of \( \phi_C \) overshooting to \( \phi_T \) then

backing to \( \phi_T \) at the time of each collision. Here

the backreaction from particle production is not taken

into account. However, since during the collisions a little

of energy of the walls will transfer into the energy of

oscillation of the false vacuum, the speed of the walls
decrease after each collision and this will made the peaks

of \( \Phi_C \) drop slightly.

The energy density of \( \chi \)-particles produced after the

\[ N \]  

Nth collision is \( \rho_T = n_{\chi} M_{\chi} \), where \( M_{\chi} \sim k_* \simeq \sqrt{g^2 \Phi_C / d} \)
can be considered. When the energy deposited in the

bubble walls is completely released, \( \rho_T \) should be approxi-
mately equal to the energy of the corresponding bubble

walls in the collision region. Thus this requires

\[ n_{\chi} \sqrt{g^2 \Phi_C / d} \sim \gamma \sigma \], \hspace{1cm} (6) \]

where \( R_0 \) and the surface tension \( \sigma \) are given in Eq.11. \( \gamma \) has been mentioned, \( l \) describes the effective width of

collision region, which is generally \( l \sim 2d \). Thus substituting

Eq.(5) into Eq.(6), \( N \) can be estimated as

\[ N \sim \frac{1}{4 \pi \mu} \ln \left( \frac{64 \pi^3 \sqrt{\gamma \eta} \ln \mu}{\ln \Phi_{\mathrm{C}}^2} \right) \], \hspace{1cm} (7) \]

where all \( \mu \) is adopted. Eq.(7) means the inelastic

degree of the bubbles collision is dependent on the

coupling \( g \) and the parameters of bubbles at the time of

collision, which is expected. The larger \( g \) is, the larger

the inelastic degree is. When the potential given in Fig.1

is introduced, for \( g^2 \simeq 0.5 \) used in Fig.3, at least \( N \sim 6 \) is

required for the complete release of the energy of bubble

walls.

It can be significantly noticed that \( N \) given by (7) is

insensitive to the concrete value of \( \phi_* \). The reason is

that the release of energy is proportional to the velocity of

\( \phi_C \) passing through \( \phi_* \), which is about \( \sqrt{\Phi_C / d} \) irre-

sitive with \( \phi_* \). The simulation of the evolution of field

value at the collision region, in which the loss of walls

energy is considered, is interesting to grasp the physics

of the collision of bubbles. We used the same initial data

and parameters with that in Fig.2, and \( \phi_* = 0.4 \) with-

out loosing generalization, to perform it in Fig.4. This

performance is explained as follows.

Theoretically, when \( \phi \) passes through \( \phi_* \), the preheating

leaded by the coupling at \( \phi_* \) will make the kinetic en-

ergy of \( \phi \) decreasing, and the loss of energy is proportion

to the velocity of \( \phi \) at this time. To simulate this effect, we

multiplys a factor \( 1 - \beta^2 < 1 \) on \( \beta^2 \) in the correspond-

ing program when \( \phi \) pass through \( \phi_* \) each time. \( \beta \) can be

calculated by letting the loss of kinetic energy of \( \phi \) around

\( \phi_* \) equal to the energy of particles produced at this time,

which is given by \( \beta \simeq \sqrt{2 n_{\chi} M_{\chi} / \Phi_C} \), where \( n_{\chi} \) denotes

the number density of \( \chi \)-particles produced at each time

when \( \phi_C \simeq \phi_* \). In actually simulation, this process is

carried out by the cumulation of the time steps around

\( \phi_* \). We can see that after about \( N = 4 \), the blue region

denoting \( \phi_T \) disappears, \( \phi_C \) at the collision region will

oscillate around \( \phi_T \). The reason is that with the gradual

release of the energy in the bubble walls, the oscillation

of bubble walls will have smaller and smaller amplitude,

which will lead that during the \( N \)th collision, instead of

backing to \( \phi_T \) during previous collision of walls, after getting
cross the potential barrier, \( \phi_C \) will oscillate around

\( \phi_T \).

In above estimate, the backreaction of \( \chi \)-particles

produced to the evolution of \( \phi \) at the collision region has

been neglected. However, with the increase of \( \chi \)-

particles, its backreaction will be enhanced gradually,

which will inevitably shut off the resonance at certain
be drained rapidly. In the example given, the time that the energy is completely released is $60/M_F < 1/H_F$, and the collision times $N = 4$ is smaller than $N > 13$ when the preheating is not taken into account which is usual expected.

This result has interesting applications in extended inflation and other relevant inflation models, e.g.\[5.\] It not only helps to obtain the enough reheating temperature in these models, but enrich the phenomenological studies on the reheating of these models. In principle, the preheating in these models could be nearly similar to that in slow roll inflation models. Here, however, the results are dependent on the parameters of bubbles at the time of collision, which are mainly determined by the structure of potential landscape. Thus the preheating in these models could have different predictions from those in slow roll inflation models.

In this paper, we only consider a simple model. In general, dependent on the parameters of bubbles at the time of collision, the oscillation of field at the collision region can be different. However, as long as the coupling of background field $\phi$ to a light scalar field $\chi$ is considered, the explosive production of the particles at the region of bubbles collision will be general. In a landscape with multiple dimensions, such couplings can be actually expected, see \[24,30\] for the studies of cosmological models in such a landscape. This means that the collision of bubbles is generally highly inelastic. In principle, the classical $\chi$-wave might be also important for such a coupling. We left the detailed studies and the discussion on its correlation with the $\chi$-particles produced, and the production of particles induced by the collision of bubbles in different potential landscapes in coming works.

The production of particles makes the bubble collision highly inelastic. However, it seems not remarkable impact on classical transition of bubbles \[21\], since in general the field excursion is nearly unchanged during the first oscillation.

In principle, it might be possible that for a given potential and the coupling $g$, the energy can be released completely after a single collision of bubble walls. The condition that this occurs is obvious. When the residual energy of the wall after one collision is smaller than the potential energy between the barrier and the true vacuum, the field in the collision region do not have enough energy to cross the barrier between the false vacuum and the true vacuum, therefore it will stay at the true vacuum and oscillate around the minimum. In this sense, the collision of bubbles will be extremely inelastic, and the energy deposited in the bubble wall can be rapidly drained. Here, the effect of the coupling on the moving of the bubble walls before the bubble collision is neglected, since it only slows down the acceleration of the bubble walls.

We might live inside a bubble universe in eternally inflating background. Recently, the observable signals of the collision of bubble universes have been discussed \[18,19,27,28\]. The resonant production of particles
during the bubbles collision might bring some distinct observable signals or impacts on the CMB, which will be explored in the future.

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