Optical Costas loop: pull-in range estimation and hidden oscillations

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Abstract: In this work we consider a mathematical model of the optical Costas loop. The pull-in range of the model is estimated by analytical and numerical methods. Difficulties of numerical analysis, related to the existence of so-called hidden oscillations in the phase space, are discussed.

Keywords: Optical Costas loop, nonlinear model, simulation, hidden oscillation, global stability, Lyapunov function, pull-in range.

1. INTRODUCTION

The Costas loop is a special modification of the phase-locked loop, which is widely used in telecommunication for the data demodulation and carrier recovery. The optical Costas loop is used in intersatellite communication, see e.g. European Data Relay System (EDRS) [Rosenkranz and Schaefer, 2016; Schaefer et al., 2015; Heine et al., 2014]. In the optical intersatellite communication the lasers are used instead of radio signals for the data transmission between satellites. Both lasers, used in the transmitter and the receiver, have a frequency mismatch due to the Doppler shift, natural frequency drift, and phase noise. The optical Costas loop adjusts the frequency and phase of the receiver to the incoming signal, which allows one to demodulate received data. Operating principles of this circuit are similar to those of conventional Costas loop circuit (homodyne detection).

In Section 2 a mathematical model of the optical Costas loop is derived. In Section 3 the pull-in range of the optical Costas loop with active proportional-integral (PI) and lead-lag filters are discussed.

2. MATHEMATICAL MODEL OF OPTICAL COSTAS LOOP

Consider a nonlinear mathematical model of the optical Costas loop model in the signal space (see, e.g. [Schaefer and Rosenkranz, 2015])

The input signal is a BPSK signal, which is the product of the transferred data \( m(t) = \pm 1 \) and the harmonic high frequency carrier \( \sqrt{P_1} \sin(\theta_1(t)) \) \( (\omega_1(t) = \dot{\theta}_1(t) \) is the carrier frequency). A local oscillator (LO - voltage-controlled oscillator VCO) signal is sinusoidal signal \( \sqrt{P_2} \sin(\theta_2(t)) \) with the frequency \( \omega_2(t) = \dot{\theta}_2(t) \). The block 90° Hybrid combines inputs shifting phases by 90° as follows:

\[
\begin{align*}
E_1 &= \frac{1}{2} m(t) \sqrt{P_1} \cos(\theta_1(t)) + \frac{1}{2} \sqrt{P_2} \cos(\theta_2(t)), \\
E_2 &= \frac{1}{2} m(t) \sqrt{P_1} \cos(\theta_1(t)) - \frac{1}{2} \sqrt{P_2} \cos(\theta_2(t)), \\
E_3 &= \frac{1}{2} m(t) \sqrt{P_1} \cos(\theta_1(t)) + \frac{1}{2} \sqrt{P_2} \cos(\theta_2(t) + \frac{\pi}{2}), \\
E_4 &= \frac{1}{2} m(t) \sqrt{P_1} \cos(\theta_1(t)) - \frac{1}{2} \sqrt{P_2} \cos(\theta_2(t) + \frac{\pi}{2}).
\end{align*}
\]

Fig. 1. Optical BPSK Costas loop in the signal space

Four outputs of the receivers are the following
model of the classical PLL [Viterbi, 1966; Gardner, 1966; Shakhgildyanyan and Lyakhovkin, 1966], shown in Fig. 2.

\[ I_1(t) = \frac{R}{8}(P_1 + P_2 + 2m(t)\sqrt{P_1P_2}\cos(\theta_1(t) - \theta_2(t))), \]
\[ I_2(t) = \frac{R}{8}(P_1 + P_2 - 2m(t)\sqrt{P_1P_2}\cos(\theta_1(t) - \theta_2(t))), \]
\[ I_3(t) = \frac{R}{8}(P_1 + P_2 + 2m(t)\sqrt{P_1P_2}\cos(\theta_1(t) - \theta_2(t) - \frac{\pi}{2})), \]
\[ I_4(t) = \frac{R}{8}(P_1 + P_2 - 2m(t)\sqrt{P_1P_2}\cos(\theta_1(t) - \theta_2(t) - \frac{\pi}{2})). \]

(2)

The block TIA multiplies its inputs by certain factor \( A \) and then subtracts two pairs of signals:
\[ I_1(t) = A|I_1(t)| - A|I_2(t)| = \frac{m(t)RA\sqrt{P_1P_2}}{2}\cos(\theta_1(t) - \theta_2(t)), \]
\[ I_Q(t) = A|I_3(t)| - A|I_4(t)| = \frac{m(t)RA\sqrt{P_1P_2}}{2}\cos(\theta_1(t) - \theta_2(t) - \frac{\pi}{2}). \]

(3)

After multiplication the loop filter input becomes
\[ v(t) = I_1(t)I_Q(t) = \frac{R^2A^2P_1P_2}{4}\cos(\theta_1(t) - \theta_2(t))\sin(\theta_1(t) - \theta_2(t)), \]
\[ = \frac{R^2A^2P_1P_2}{8}\sin(2\theta_1(t) - 2\theta_2(t)). \]

(4)

It should be noted that unlike the classical PLL circuit the filter input of the considered model (see (4)) depends on the difference of the phases only (see also two-phase phase-locked loop model in [Best et al., 2014, 2016]). This allows one to derive a mathematical model of the optical Costas loop in a signal’s phase space without additional engineering assumption on the complete filtration of high-frequency signal component [Leonov et al., 2015b; Best et al., 2015].

The relation between the input \( v(t) \) and the output \( g(t) \) of the filter is as follows
\[ \dot{x} = Ax + b\varphi(t), \quad g(t) = c^*x + h\varphi(t), \]

(5)

where \( A \) is a constant \( n \times n \) matrix, the vector \( x(t) \in \mathbb{R}^n \) is a filter state, \( b, c \) are constant vectors, and \( x(0) \) is initial state of the filter. The control signal \( g(t) \) is used to adjust the VCO frequency to the frequency of input carrier signal:
\[ \dot{\theta}_2(t) = \omega_2(t) = \omega_2^{\text{free}} + K_{VCO}g(t). \]

(6)

Here \( \omega_2^{\text{free}} \) is a free-running frequency of the VCO and \( K_{VCO} \) is the VCO gain. Similarly, various nonlinear models of the VCO can be considered (see, e.g., [Bianchi et al., 2016a]). Note that the initial VCO frequency (at \( t = 0 \)) has the form
\[ \omega_2(0) = \omega_2^{\text{free}} + K_{VCO}a_0(0) + K_{VCO}h\varphi(\theta_1(0) - \theta_2(0)). \]

(7)

Let \( \theta_\Delta = \theta_1 - \theta_2 \). If the frequency of the input carrier is constant:
\[ \dot{\theta}_1(t) = \omega_1(t) \equiv \omega_1, \]

then equations (5)-(6) give the following mathematical model of the optical Costas loop in the signal’s phase space:
\[ \dot{x} = Ax + b\varphi(\theta_\Delta), \]
\[ \dot{\theta}_\Delta = \omega_2^{\text{free}} - K_{VCO}c^*x - h\varphi(\theta_\Delta), \]

(9)

where \( |\omega_2^{\text{free}}| = |\omega_1 - \omega_2^{\text{free}}| \) is called a frequency deviation. This model corresponds to the classical signal’s phase

![Fig. 2. Optical BPSK Costas loop in signal’s phase space.](image)

Fig. 2. Optical BPSK Costas loop in signal’s phase space. \( \theta_2(0) \) is initial phase of the VCO, \( x(0) \) is initial state of the filter.

3. THE PULL-IN RANGE COMPUTATION

One of the main engineering characteristics of the PLL-based circuits is the pull-in range which corresponds to the global stability of the model. The pull-in range is a widely used engineering concept (see, e.g. [Gardner, 1966, p.40], [Best, 2007, p.61]). The following rigorous definition is from [Kuznetsov et al., 2015; Leonov et al., 2015b; Best et al., 2016]: the largest interval of frequency deviations \( 0 \leq |\omega_2^{\text{free}}| < \omega_{\text{pull-in}} \) such that the model in the signal’s phase space acquires lock for arbitrary initial phase difference and filter state (i.e. any trajectory tends to a stationary point) is called a pull-in range, \( \omega_{\text{pull-in}} \) is called a pull-in frequency.

3.1 Lead-lag filter

Consider the mathematical model of the optical Costas loop with lead-lag filter in the signal’s phase space.

![Fig. 3. Phase portrait with stable and unstable cycles.](image)

Fig. 3 shows phase portrait corresponding to the signal’s phase model where the carrier frequency is 10000 rad/s,
the VCO free-running frequency is 10089.5 rad/s, the lead-
lag filter transfer function is \( H_f(s) = \frac{\omega_s + \beta}{s + \alpha} \) \( \alpha = 63.1565, \beta = 63.1565, \omega_s = 0.2922 \). The solid blue line in Fig. 3 corresponds to the trajectory with the filter initial state \( x(0) = 0.6304 \) and the VCO phase shift 0.3975 rad. This line tends to a periodic trajectory. Thus in this case the model cannot acquire lock.

The solid red line corresponds to the trajectory with the filter initial state \( x(0) = -0.1373 \) and the VCO initial phase 24.3161. This trajectory lies just under an unstable periodic trajectory (unstable limit cycle) and tends to a stable equilibrium. In this case the model acquires lock.

All the trajectories between the stable and unstable periodic trajectories (see, e.g., a solid green curve) tend to the stable one which is a stable limit cycle. Therefore, if the gap between the stable and unstable trajectories is smaller than the discretization step, then the numerical procedure may slip through the stable trajectory. Thus, in this case we have coexistence of stable periodic trajectory (which is a local hidden attractor\(^2\) — locally attracting, closed, and bounded set in the cylindrical phase space) and unstable periodic trajectory (repeller) close to each other; the merge of these periodic trajectories leads to the birth of semistable trajectory (semistable limit cycle) [Gubar’, 1961; Shakhtarin, 1969; Belyustina et al., 1970; Leonov and Kuznetsov, 2013; Kuznetsov et al., 2014]. In this case numerical methods are limited by the errors on account of the linear multistep integration methods (see [Biggio et al., 2013, 2014]). The corresponding difficulties of simulation in SPICE and MATLAB are discussed in [Leonov et al., 2015b; Bianchi et al., 2016b; Kuznetsov et al., 2017].

The above example shows that, in general, the pull-in of the model with a lead-lag filter is bounded and demonstrates the difficulties of numerical estimation of the pull-in range. In this case the pull-in range estimation can be done by analytical-numerical methods based on phase-plane analysis, however, it is a challenging task due to hidden oscillations (see, e.g. [Shakhtarin, 1969; Belyustina et al., 1970; Shalfeev and Matrosov, 2013; Leonov and Kuznetsov, 2013]). Corresponding diagram for pull-in range is on Fig. 4.

\[ 1 \] In engineering literature filter transfer function is usually defined as \( H(s) = \frac{c_n(sI - A)}{s^2 + 2\xi_s s + \omega_n^2} \) [Best, 2007], at the same time in the control theory [Leonov and Kuznetsov, 2014] it is defined as \( c_n(sI - A)^{-1}b - h \).

\[ 2 \] Attractor is called a self-excited attractor if its basin of attraction intersects any arbitrarily small open neighborhood of an equilibrium, otherwise it is called a hidden attractor [Leonov et al., 2011, 2012; Leonov and Kuznetsov, 2013; Leonov et al., 2015a; Kuznetsov, 2016]. For self-excited attractors there is a transient process from a small vicinity of an unstable equilibrium to an attractor, which allows to find the attractor easily. If there is no such a transient process for an attractor, it is called a hidden attractor. For example, hidden attractors are attractors in systems without equilibria or with only one stable equilibrium (a special case of multistability and coexistence of attractors). Some examples of hidden attractors can be found in Shahzad et al. [2015]; Brezetskyi et al. [2015]; Jafari et al. [2015]; Zhusubaliyev et al. [2015]; Saha et al. [2015]; Semenov et al. [2015]; Feng and Wei [2015]; Li et al. [2015]; Feng et al. [2015]; Sprott [2015]; Pham et al. [2015]; Vaidyanathan et al. [2015]; Chen et al. [2015]; Menacer et al. [2016]; Danca [2016]; Zelinka [2016]; Dudkowski et al. [2016]; Danca et al. [2017]; Kiseleva et al. [2016].

Consider, e.g., parameters \( a = 0.3 \) and corresponding curve in Fig. 5. Here normalized pull-in frequency \( \frac{\omega_{\text{pull-in}}}{K_{\text{VCO}}} \)

\[ \omega_{\text{pull-in}} \]

\[ K_{\text{VCO}} \]

\[ a=0.3 \]

is plotted for various values of the loop gain \( K_{\text{VCO}} \). For all points on the diagram in Fig. 5 below the curve (filled with grey) system (9) is globally asymptotically stable. Pull-in frequency diagram for different parameters of the loop filter is in Fig. 4 for sinusoidal PD characteristic.

### 3.2 PI filter

The filter – perfect integrator is preferred (if it can be implemented in the considered architecture) since it allows us to prove analytically that the pull-in range is infinite.

Since PLL-based circuits are nonlinear control systems for their global analysis it is essential to apply the stability criteria, which are developed in control theory, however their direct application to analysis of the PLL-based models is often impossible, because such criteria are usually not adapted for the cylindrical phase space\(^3\); The corresponding modifications of classical stability criteria for the

\[ 3 \] For example, in the classical Krasovskii-LaSalle principle on global stability the Lyapunov function has to be radially unbounded (e.g. \( V(x, \theta_A) \to -\infty \) as \( ||(x, \theta_A)|| \to +\infty \)). While for the application of this principle to the analysis of phase synchronization systems there are usually used Lyapunov functions periodic in \( \theta_A \) (e.g. \( V(x, \theta_A) \) is bounded for any \( ||(0, \theta_A)|| \to +\infty \), and the discussion of this gap is often omitted (see, e.g. patent [Abramovitch, 2004] and works [Bakaev, 1963; Abramovitch, 1990, 2003]). Rigorous discussion can be found, e.g. in [Gelig et al., 1978; Leonov and Kuznetsov, 2014].
nonlinear analysis of control systems in cylindrical phase space were developed in the second half of the 20th century (see, e.g. [Gelig et al., 1978; Leonov et al., 1992, 2009; Leonov and Kuznetsov, 2014]).

In this work we consider a mathematical model of the optical Costas loop and study its pull-in and lock-in ranges. For the lead-lag filter we show that numerical estimation of the pull-in range is a challenging task due to the existence of hidden oscillations. For the active PI filters we prove that the pull-in range is infinite by application of special modification of the direct Lyapunov method for the cylindrical phase space.

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