A periodic review integrated inventory model with controllable setup cost, imperfect items, and inspection errors under service level constraint

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Abstract. This paper presents an integrated inventory model which consists of single vendor and buyer. The buyer managed its inventory periodically and orders products from the vendor to satisfy the end customer’s demand, where the annual demand and the ordering cost were in the fuzzy environment. The buyer used a service level constraint instead of the stock-out cost term, so that the stock-out level per cycle was bounded. Then, the vendor produced and delivered products to the buyer. The vendor had a choice to commit an investment to reduce the setup cost. However, the vendor’s production process was imperfect, thus the lot delivered contained some defective products. Moreover, the buyer’s inspection process was not error-free since the inspector could be mistaken in categorizing the product’s quality. The objective was to find the optimum value for the review period, the setup cost, and the number of deliveries in one production cycle which might minimize the joint total cost. Furthermore, the algorithm and numerical example were provided to illustrate the application of the model.

Keywords: fuzzy demand; imperfect inspection; inventory model; periodic review; service level constraint

1. Introduction
Along with the changing of times, many companies compete with each other in satisfying the customer’s requirement by producing better products and services in a minimum cost. During the recent years, most inventory problem that focuses on the integration between vendor-buyer has become more important because the vendor and the buyer want to increase their mutual benefit.

The integrated vendor-buyer inventory model was introduced for the first time by Goyal [1]. His research has encouraged other researchers to develop various types of integrated inventory system. Banerjee [2] generalized Goyal [1] model and proposed an integrated model with a joint economic-lot-size where the vendor produces for buyer’s order on a lot-by-lot basis. From that time on, there are more researchers developing the integrated inventory system, such as Goyal and Nebebe [3], Ouyang, Wu, and Ho [4], and many more.

Even at this time, the literature about the integrated inventory system with a periodic review policy is rather rare compared to the continuous review policy. However, this periodic review model is considered easier to be applied for many cases. Montgomery et al. [5] studied deterministic and stochastic models under backorders and lost sales. Donaldson [6] developed a periodic review
inventory model with normal demand, and obtained the optimal order-up-to level and review period. Ouyang and Chuang [7]) proposed a periodic review inventory with variable lead time. Furthermore, Lin [8]) proposed a periodic review inventory model with fuzzy expected demand short and backorder rate.

In the earlier literature, many researchers have assumed that the process from beginning to the end is error free. Whereas in reality, neither the production process nor inspection process are perfect due to unreliable process or human error. Lin [9] developed periodic review integrated model with imperfect production process and backorder discount. Salamah [10] developed an integrated inventory model with stochastic demand, imperfect quality, and inspection errors. Jauhari et al. [11] developed Lin [9] by considering adjustable production rate and variable lead time. Jauhari [12] also developed integrated inventory model with defective items, inspection error, and stochastic demand.

In real situation, everything including inventory management will face an uncertainty. This can cause difficulties in building an inventory model as it would be more complex. Hence, we can use fuzzy number to resolve this problem. Yao and Wu [13] developed an inventory model with triangular fuzzy number approach. Kurdhi et al. [14] developed an inventory model considering fuzzy demand, transportation cost, setup cost reduction, and partially backorder. Furthermore, Jauhari et al. [15] developed an inventory model with fuzzy demand, imperfect production process, and inspection error.

Researches concerning stochastic integrated inventory model in a single vendor-buyer can be divided into two broad groups, the full cost model, and the service level approach model. Service level approach model is more popular than full cost model. This approach introduces a service level constraint (SLC) instead of the shortage cost term in the model. It is because the determination of the stock-out cost is considered more difficult. Ouyang and Wu [16], Chen and Krass [17] replaced the stock-out cost with SLC in order to prevent unacceptable stock-out. Lin and Lin [18] established an integrated inventory model with inspection process, ordering cost reduction, and SLC. Priyan and Uthayakumar [19] developed an inventory model with variable lead time and SLC in a fuzzy cost environment.

To summarize the above description, an integrated periodic review inventory model has been much developed, but none has considered fuzzy annual demand and ordering cost, setup cost reduction, imperfect items, and inspection error under service level constraint, which is proposed in this study. Therefore, an algorithm and numerical example are provided to establish the model’s optimal solution.

2. Notations and assumptions

2.1. Assumptions

1. This periodic review inventory model considers one vendor-buyer with a single-product.

2. The demand in buyer side is normally distributed with mean $D$ and standard deviation $\sigma$. 

| Notation | Description |
|----------|-------------|
| $T$      | review period (year) (decision variable) |
| $S$      | setup cost ($/setup) (decision variable) |
| $n$      | number of deliveries (decision variable) |
| $D$      | demand rate (units/year) |
| $V$      | transportation cost ($/shipment) |
| $I(S)$   | the capital investment required to reduce the setup cost per setup |
| $c_p$    | unit purchasing cost ($/unit) |
| $S_0$    | original setup cost ($/setup) |
| $g$      | transportation time (year) |
| $A$      | ordering cost ($/order) |
| $P$      | production rate (units/year) |
| $a_1$    | fixed production cost ($) |
| $a_2$    | variable production cost ($/unit) |
| $c_{df}$ | post-sale defective item cost for vendor ($/unit) |
| $c_{db}$ | post-sale defective item cost for buyer ($/unit) |
| $c_w$    | warranty cost ($/unit) |
| $c_i$    | inspection cost ($/unit) |
| $h_B$    | buyer’s holding cost ($/unit/year) |
| $c_r$    | false rejection of non-defective item cost ($/unit) |
| $h_V$    | vendor’s holding cost ($/unit/year) |
| $e_1$    | inspection error type I probability |
| $e_2$    | inspection error type II probability |
| $\gamma$ | probability of defect |
| $X$      | the protection interval demand |
| $R$      | buyer’s target level |
| $\xi$    | proportion of demands which are not met from stock, i.e. $1-\xi$ is the service level |
3. There is a choice to do an investment in reducing the setup cost. The investment required to reduce the setup cost from initial setup cost $S_0$ to a target level $S$ is denoted by $I(S)$.

4. To satisfy the end customer demand’s, the buyer orders $DT$ units each time and vendor produces $nDT$ items at one set-up with a finite production rate $P$ and delivers each lot in $n$ equally batch sized shipment. The transportation cost $V$ is charged to the vendor for each delivery of $DT$ units.

5. The demand during lead-time follows normal distribution with mean $D(T+L)$ and standard deviation $\sqrt{T+L}$, where the length of lead-time $L$ is depending on production time $DT/P$ and transportation time $g$.

6. A production process is imperfect in a way that each time the shipment lot has arrived at the buyer, and it contains some defective products with rate $\gamma$. All products will be inspected by the buyer, but the inspection process is also imperfect. The inspector may classify the product’s quality incorrectly.

7. Instead of considering stock-out cost, the buyer sets the target service level corresponding to the proportion of demands which are not met from stock. This service level constraint sets the limit of the stock-out level per cycle, where it should not exceed the value of $\xi$.

3. **Model development**

The vendor’s total cost per year is composed of production cost, setup cost, investment in setup cost reduction, holding cost, cost for rejecting non-defective item (type I error cost), post-sale failure cost (type II error), and warranty cost. The formulation of production cost adopted from Khouja and Mehrez [20] is $(\frac{a_1}{P} + a_2P)D$ and the setup cost is $\frac{S_0}{nT}$.

Furthermore, we consider the logarithmic investment function $I(S)$, as used in Lin [21], which is

$$I(S) = d \ln \frac{S_0}{S}, \quad \text{for } 0 < S_0 \leq S$$

(1)

where $d = \frac{1}{\delta}$ and $\delta$ is a percentage decrease in setup cost $S$ per dollar increase in $I(S)$. Thus, the investment cost per unit per year is $rI(S)$, where $r$ is fractional opportunity cost of capital per unit time. The vendor’s holding cost can be expressed as

$$Hold_{CV} = h_v \frac{DT}{2} \left[\frac{D(2-n)}{P} + (n - 1)\right]$$

(2)

As explained in the previous section, the model developed in this study considers that the production process and the inspection process are imperfect. Therefore, the vendor bears the type I error cost $c_{el}(1 - \gamma)DE_1$ that occurs because the inspector is incorrectly categorizing non-defective products as defective products, and the type II error cost $c_{el} \gamma DE_2$ that occurs because the inspector is incorrectly categorizing defective products as non-defective products, and warranty cost $c_{w} \gamma D$ that occurs because of the vendor’s consequences for producing defective products.

Therefore, the total cost of vendor can be expressed as:

$$TC_V = \left(\frac{a_1}{P} + a_2P\right)D + \frac{S_0}{nT} + \frac{r}{\delta} \ln \frac{S_0}{S} + h_v \frac{DT}{2} \left[\frac{D(2-n)}{P} + (n - 1)\right] + c_{el}(1 - \gamma)DE_1 + c_{el} \gamma DE_2 + c_{w} \gamma D$$

(3)

The buyer’s total cost per year is composed of ordering cost, purchasing cost, transportation cost, inspection cost, cost of type II error, and holding cost where the formulation can be expressed as ordering cost $\frac{A}{T}$, purchasing cost $c_P D$, and transportation cost $\frac{V}{T}$. As mentioned before, an inspection process $c_{el}D$ will be charged by the buyer for every time the shipment lot arrived at its place where the lot always contains $\gamma$ defective products. Hence, the buyer also bears the type II error cost $c_{el} \gamma DE_2$.

The formulation of buyer’s holding cost developed in this study is adopted from Lin [9] and Hsu and Hsu [22] which consists of holding cost for not only defective and non-defective items found by inspector but also the returned products from end customer. The buyer’s holding cost formulation is

$$Hold_{CB} = h_B \left(\frac{DT}{2} + DT(1 - \gamma)e_1 + DT \gamma (1 - e_2) + \frac{DT \gamma e_2}{2}\right)$$

(4)
We assume that the demand during protection period $X$ follows a normal distribution with the mean $D(T + L)$ and the standard deviation $\sigma \sqrt{T + L}$. The value of lead time $L$ is $\frac{DT}{p} + g$ and the target level is

$$R = \left(T + \frac{DT}{p} + g\right) + k \sigma \sqrt{T + \frac{DT}{p} + g}.$$ 

Thus, the expected of demand shortage at the end of the cycle is

$$E(X - R)^+ = \int_R^\infty (X - R)f_x(x)dx = \sigma \psi(k) \sqrt{T + \frac{DT}{p} + g} \tag{5}$$

where $\psi(k) = f_s(k) - k[1 - F_s(k)]$.

As mentioned earlier, we do not use a stock-out cost term but a service level constraint in the objective function in this study. Thus, the service level constraint can be set as

$$\frac{\sigma \psi(k) \sqrt{T + \frac{DT}{p} + g}}{D(T + \frac{DT}{p} + g)(1 - \gamma)(1 - e_1)} \leq \xi \tag{6}$$

Hence, the total cost of buyer can be expressed by the equation below:

$$TC_B = \frac{A}{T} + \frac{F}{T} + c_pD + c_lD + c_{dB}qD + h_B \left(\frac{DT}{2} + DT(1 - \gamma)e_1 + DT\gamma(1 - e_2) + \frac{DT\gamma e_2}{2}\right) \tag{7}$$

subject to

$$\frac{\sigma \psi(k) \sqrt{T + \frac{DT}{p} + g}}{D(T + \frac{DT}{p} + g)(1 - \gamma)(1 - e_1)} \leq \xi$$

Joint total cost for this system is established by adding the vendor’s total cost and buyer’s total cost. While $D$ and $A$ is fuzzified to be $\tilde{D}$ and $\tilde{A}$, the joint total cost for the system is expressed by

$$fTC = (a_1 + a_2P)\tilde{D} + \frac{S}{nT} + \frac{\tau}{\delta} \ln \frac{S_0}{S} + h_V \frac{\tilde{D}(2-n)}{p} \left((\tilde{D}, \tilde{B})\right) + (n - 1) + c_r(1 - \gamma)\tilde{D} e_1 + c_{av}\tilde{D} e_2 + c_{av}\tilde{B} e_2 +$$

$$c_{av}\tilde{D} e_2 + c_{av}\tilde{B} e_2 + h_B \left(\frac{DT}{2} + \tilde{D}(1 - \gamma)e_1 + \tilde{D}\gamma(1 - e_2) + \frac{\tilde{D}\gamma e_2}{2}\right) \tag{8}$$

We defuzzify $JT_C$ by using the signed distance method. The signed distance of $JT_C$ is expressed by

$$d(\tilde{JTC}, \tilde{A}) = \frac{a_1}{p} + a_2P)\left(\tilde{D}, \tilde{B}\right) + \frac{S}{nT} + \frac{\tau}{\delta} \ln \frac{S_0}{S} + h_V \left(\frac{\tilde{D}, \tilde{B}}{2} \right) \left(D, B\right) + (n - 1) + c_r(1 - \gamma)\tilde{D} e_1 + c_{avr}\tilde{B} e_2 + c_{avr}\tilde{D} e_2 + \left(\frac{\tilde{D}, \tilde{B}}{2} \right) \left(D, B\right)\gamma(1 - e_2) + \frac{(\tilde{D}, \tilde{B})e_2}{2} \tag{9}$$

As adopted from Jauhari et al. [15], for any $a$ and $0 \in R$, the signed distance between the real numbers $a$ and $0$ is $d_p(a, 0) = a$. The signed distance of $\tilde{A}$ to $\tilde{B}$ is

$$d(A, B) = \frac{1}{4}(a + 2b + c) = \frac{1}{4}(A - p_1) + 2A + (A + p_2) = A + \frac{1}{4}(p_2 - p_1) \tag{10}$$

The joint total cost for the integrated vendor and buyer system can be rewritten as

$$d(\tilde{JTC}, \tilde{B}) = \frac{a_1}{p} + a_2P)\left(D, B\right) + \frac{S}{nT} + \frac{\tau}{\delta} \ln \frac{S_0}{S} + h_V \left(\frac{D, B}{2} \right) \left(D, B\right) + (n - 1) + c_r(1 - \gamma)\left(D, B\right) + c_{avr}\left(D, B\right)\gamma(1 - e_2) + \frac{(D, B)e_2}{2} \tag{11}$$
$$\frac{1}{4}(q_2 - q_1) + \left(\frac{A + \frac{3}{4}(p_2-p_1)}{T}\right) + V^T + c_p(D + \frac{1}{4}(q_2 - q_1)) + c_i\left(D + \frac{1}{4}(q_2 - q_1)\right) + c_B\gamma(D + \frac{1}{4}(q_2 - q_1)) e_2 + h_B\left(\frac{(D + \frac{1}{4}(q_2 - q_1))T}{2} + (D + \frac{1}{4}(q_2 - q_1))T(1 - \gamma)e_1 + \gamma(D + \frac{1}{4}(q_2 - q_1)) T(1 - \gamma)e_2\right)$$

subject to

$$\sigma_k \left(\frac{(D + \frac{1}{4}(q_2 - q_1))^1}{T} + g, 1 - (1 - e_1)\right) \leq \xi$$

(11)

4. Solution methodology

To find the optimal solution of problem (11), we temporarily ignore the service level constraint and consider a fixed value of \(n\), then find the first-order partial derivatives of \(JTC(T, S, n)\) with respect to \(T\) and \(S\). Next, by setting the equations to zero, rearranging and simplifying we obtain

$$T^* = \sqrt{\frac{2P\left(A + \frac{3}{4}(p_2-p_1)\right)n + Vn + S}{(D + \frac{1}{4}(q_2 - q_1))^2\left(2h_n + h_B(2e(1-\gamma))\right)}}$$

(12)

and

$$S^* = \frac{nT_0}{\delta}$$

(13)

Therefore, we can develop an iterative procedure to solve the proposed problem and obtain the optimal value of all decision variables. The proposed algorithm is as follows

1. Fuzzify \(D\) and \(A\) to be a triangular fuzzy number, \(\tilde{D} = (D-q_1, D, D+q_1)\) and \(\tilde{A} = (A-p_1, A, A+p_2)\), where \(p_i\) and \(q_i\) for \(i = 1, 2\) are decided by decision makers.
2. Use the signed distance method to defuzzify the fuzzy number \(\tilde{D}\) and \(\tilde{A}\).
3. Set \(n = 1\) and \(JTC(T_{n-1}, S_{n-1}, n-1) = \infty\).
4. Perform (i)-(v)
   (i) Start with \(S = S_0\).
   (ii) Substituting the value of \(S\) into Equation (12) evaluates \(T\).
   (iii) Using \(T\) in Equation (13) and determines \(S\).
   (iv) Repeat (ii) and (iii) until no change occurs in the values of \(T\) and \(S\).
5. Compare \(S\) and \(S_0\)
   (i) If \(S < S_0\), \(S\) is feasible, denote the solution by \((T^*, S^*)\) then go to Step 6.
   (ii) If \(S > S_0\), \(S\) is not feasible. Set \(S = S_0\) and calculate the corresponding value of \(T\) from Equation (12), then go to Step 6.
6. Compute \(JTC(T^*_n, S^*_n, n^*)\) from Equation (11).
7. For a given safety factor \(k\), check the Equation (6), if it satisfied, the solutions satisfy the service level constraint, and the solution \(JTC(T^*_n, S^*_n, n^*)\) is the optimal solution for fixed \(n\).
   Otherwise, find another solution until it satisfies the service level constraint. If all solutions do not satisfy the service level constraint, this inventory model has no feasible solution.
8. If \(JTC(T^*_n, S^*_n, n^*) \leq JTC(T_{i-1}^*, S_{i-1}^*, n-1)\) and the service level constraint is satisfied, repeat Steps 3-7 with \(n = n + 1\), otherwise go to Step 9.
9. Set $JTC(T^*_n, S^*_n, n^*) = JTC(T^*_n, S^*_n, n^*)$, then $T^*_n, S^*_n$, and $n^*$ are the solutions of the model.

5. Numerical example

The numerical example given below is for illustrating the feasibility of the above solution procedure.

Let us consider an integrated inventory model with the following data:

- $q_1 = 50, A = $100/order, $p_1 = 20, p_2 = 50, S_0 = $1,500/setup, V = $25/shipment, $c_p = $65/unit, g = 1/12 year, $P= 2500 units/year, $a_1 = $2,000/batch, $a_2 = $1/2,000/unit, $c_{id} = $200/unit, $c_{dl} = $270/unit, $c_f = $1/unit, $c_o = $100/unit, $h_g = $8/unit/year, $h_{1i} = $6/unit/year, $e_1 = 0.04, e_2 = 0.04, r = 0.01%, we take an assumption that one year = 52 weeks.

Using the solution procedure explained above, the optimal value for each variable can be obtained.

Therefore, the optimal value of $n^*, T^*$, and $S^*$, satisfying the proposed solution procedure are 5 deliveries, 0.1850 years, and $1,500/setup which can minimize the value of $JTC$ at $174,650.

6. Conclusion

This paper proposed an integrated periodic review inventory model involving a vendor and a buyer under fuzzy annual demand and ordering cost, setup cost reduction, imperfect items, inspection error, and service level constraint. The review period $T$, the setup cost $S$, and the number of deliveries $n$ were considered as decision variables. An algorithm was developed to determine the model’s optimal solutions. The numerical example’s results showed us that the higher the service level, the higher joint total cost. This paper can help managers to decide whether investing in reducing the setup cost in this inventory model is necessary or not.

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