Real-time correlators in warped AdS/CFT correspondence

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ABSTRACT: We study real-time correlators in the warped AdS/CFT correspondence. We apply the prescription used in the usual AdS/CFT correspondence and obtain the retarded Green’s functions for the scalar and vector fields in the spacelike warped and the null warped black hole backgrounds. We find that the retarded Green’s functions and the cross sections are well consistent with the predictions from dual CFT. Our results not only support strongly the conjectured warped AdS/CFT correspondence, but also show that the usual relativistic AdS/CFT prescription of obtaining the real-time correlators remain effective in more general backgrounds with anisotropic conformal infinity.
1. Introduction

During the past few years, the AdS/CFT correspondence[1] has been widely applied to study the physics in various many-body strong coupling systems, ranging from quark-gluon-plasma in RHIC, superfluid, ultra-cold atoms to superconductor. From the computations in the gravity side, one may obtain qualitative or even half-quantitative information on some strong coupling issues in these systems, which otherwise is difficult, if not completely impossible to get with the traditional methods. In particular, the AdS/CFT correspondence has the advantage to deal with real-time process, such as the transport behaviors of the systems. This usually requires the calculation of real-time correlators of composite operators of boundary quantum field theory from dual gravity.

Compared to its Euclidean counterpart, the calculation of real-time correlators is much subtler. For the zero-temperature quantum field theory, real-time correlators could be obtained via analytically continuation of Euclidean Green’s functions[2]. For the finite temperature case, the dual geometry involves the black holes[3], which requires choosing appropriate boundary condition at the horizons of the black holes. This is the first subtle point. Different Green’s functions correspond to different boundary conditions at the horizon. It is now agreed that the retarded Green’s function corresponds to the ingoing
boundary condition, while the advanced Green’s function corresponds to the outgoing one. However, even after fixing the boundary condition, one cannot obtain the Green’s function by naively using the prescription in Euclidean version of AdS/CFT correspondence. In [4], a simple prescription was proposed to compute real-time correlators from gravity. This prescription has been instrumental to the study of strongly interacting system at finite temperature during the past few years. It has also been justified from different points of view in [5, 6, 7, 8, 9, 10]. In particular, it was observed that the prescription proposed in [4] could be recast in terms of the boundary values of the canonical conjugate momentum of the bulk fields by treating the AdS radial direction as “time” direction[7]. And furthermore, this reformulation was shown to be able to follow directly from the analytic continuation of Euclidean AdS/CFT correspondence[10].

In this paper, we would like to apply the prescription to calculate real-time Green’s functions in the warped AdS/CFT correspondence. The warped AdS/CFT correspondence conjectures[13] that \( v > 1 \) quantum topological massive gravity in three dimension is holographically dual to a two-dimensional conformal field theory with central charges \((c_L, c_R)\)

\[
\begin{align*}
    c_L &= \frac{l G}{G} \frac{4v}{v^2 + 3}, \\
    c_R &= \frac{l G}{G} \frac{5v^2 + 3}{v(v^2 + 3)}
\end{align*}
\]  

(1.1)

More precisely, suitable asymptotically spacelike stretched AdS\(_3\) boundary condition on the gravity configurations needs to be imposed in this case. Moreover, there is another conjecture[15] stating that even at \( v = 1 \) quantum topological massive gravity with null warped AdS\(_3\) boundary could be dual to a two-dimensional CFT with central charges \((1.1)\). These two conjectures are somehow intriguing in the sense that the warped AdS\(_3\) spacetimes have very different conformal boundaries from the one of AdS\(_3\). Therefore, the naive expectation that the holographic CFT resides on the asymptotic boundary seems not true any more. Nevertheless, the discussion on black hole thermodynamics and the study of the quasi-normal modes of the warped black holes[14, 15] support the conjecture. Moreover, for the spacelike stretched case, the physical asymptotic boundary condition has been discussed in [18, 20], from which the central charges were derived from the Virasoro algebra and current algebra. And the stability of stretched spacelike AdS\(_3\) was investigated in [19, 21], which paves the ground of the correspondence.

The motivation of our study on real-time correlators in warped AdS/CFT correspondence is two-fold. On one hand, the prescription to get real-time correlators has been focused on AdS backgrounds, with the dual field theory resides at the conformal boundary. It would be very valuable to apply the prescription to more general backgrounds which have nontrivial conformal boundaries and check its effectiveness, especially considering the recent trend in AdS/CMT. On the other hand, in the warped AdS\(_3\) black hole backgrounds, since the wave functions of the perturbations could be calculated exactly, the retarded Green’s functions could be obtained analytically. As the Green’s functions in two-dimensional CFT is well determined by the conformal invariance, this gives another nontrivial check of the warped AdS/CFT correspondence. We will show that real-time correlators from gravity are well consistent with the CFT predictions. And from the retarded
correlators, we can read the quasi-normal modes and also the cross section of the fields scattering the black hole, both of which are in good agreement with the CFT prediction.

The remaining parts of this paper are organized as follows. In section 2, we give a brief introduction to the prescription for computing real-time Green’s functions from the gravity side in AdS/CFT correspondence. In section 3, we outline the various kinds of Green’s functions in two-dimensional conformal field theory. In section 4, we discuss the retarded Green’s functions for the scalar and vector perturbations in the spacelike stretched AdS$_3$ black holes. In section 5, we turn to the null warped case. In section 6, we investigate the extremal spacelike stretched warped black hole, which has different eigenfunctions from the non-extremal ones. We end with some discussions in section 7.

2. Retarded Green’s functions from gravity: prescription

In this section, we give a brief review of the prescription for computing two-point Green’s functions from gravity. The detailed discussion could be found in [4, 7, 10].

In Euclidean space, the core relation of AdS/CFT correspondence is  
\[
\langle e^{\int_{\partial M} \phi_0(x) O(x)} \rangle_{QFT} = e^{-S_{\text{grav}}[\phi_0]},
\]
(2.1)

where $O$ is a boundary CFT operator coupled to the bulk field $\phi$. $S_{\text{grav}}[\phi_0]$ is the bulk action for $\phi$ evaluated at the classical solution $\phi_E$ which is regular in the interior and asymptotical to $\phi_0$ at the boundary. The two-point Green’s function $\langle O(x) O(0) \rangle$ could be obtained from taking the second functional derivative of $S_{\text{grav}}$ with respect to $\phi_0$. However there is an equivalent way to compute the Green’s function. From the above relation (2.1), one can get the one-point function of $O$ in the presence of the source $\phi_0$ as  
\[
\langle O(x) \rangle_{\phi_0} = -\frac{\delta S_{\text{grav}}}{\delta \phi_0(x)} = -\lim_{r \to \infty} \Pi_E(r, x) |_{\phi = \phi_E}.
\]
(2.2)

Here $\Pi_E$ is the canonical momentum conjugate to $\phi$ with respect to a foliation in the $r$-direction. Namely, $\Pi_E$ is the canonical momentum, taking $r$ as “time”. Note also that $\Pi_E$ should be evaluated at the classical solution $\phi_E$. Transforming to momentum space, we have  
\[
\langle O(\omega, \vec{k}) \rangle_{\phi_0} = -\lim_{r \to \infty} \Pi_E(r, \omega, \vec{k}) |_{\phi = \phi_E}.
\]
(2.3)

Since $O(\omega, \vec{k})$ is the response of the system to external perturbations generated by adding the term $\int_{\partial M} \phi_0(x) O(x)$ to the boundary theory, we have  
\[
\langle O(\omega, \vec{k}) \rangle_{\phi_0} = G_E(\omega, \vec{k}) \phi_0(\omega, \vec{k}).
\]
(2.4)

This allows us to read the Green’s function in Euclidean signature:  
\[
G_E(\omega, \vec{k}) = -\left( \lim_{r \to \infty} \frac{\Pi_E(r, \omega, \vec{k}) |_{\phi = \phi_E}}{\phi_E(r, \omega, \vec{k})} \right) |_{\phi_0=0}.
\]
(2.5)

This prescription has no ambiguity since in Euclidean space, $\phi_E$ is uniquely determined by regularity and the boundary condition.
In Minkowski spacetime, one can define the Green’s function by doing analytic continuation directly in the zero-temperature case, as the Euclidean and Minkowski Green’s functions are closely related to each other. However, for the finite temperature case, the things become much subtler. In Minkowski space, the retarded propagator is analytic in the upper half complex-ω plane and is related to the Euclidean propagator by the relation

\[ G_E(\omega_E, \vec{k}) = -G_R(i\omega_E, \vec{k}), \quad \omega_E > 0. \]  

This means that the value of the retarded propagator along the upper imaginary ω-axis gives the Euclidean propagator. The relation (2.6) applies for generic cases, not only for zero temperature but also for finite temperature, and for both bosonic and fermionic propagators. For the finite temperature case, \( \omega_E \) take discrete values, and

\[ G_E(2\pi T n, \vec{k}) = -G_R(i2\pi T n, \vec{k}). \]  

On the other hand, for \( \omega_E < 0 \), the Euclidean propagator is related to the advanced propagator:

\[ G_E(\omega_E, \vec{k}) = -G_A(i\omega_E, \vec{k}), \quad \omega_E < 0. \]  

It seems that one can obtain real-time retarded Green’s function from the inverse relation of (2.6):

\[ G_R(\omega, \vec{k}) = \lim_{r \to \infty} \frac{\Pi(r, \omega, \vec{k})}{\phi_R(r, \omega, \vec{k})}. \]  

For the finite temperature case, this analytic continuation is quite tricky since \( \omega_E \) takes only discrete values. One may ignore this subtlety and just do analytic continuation and wish the best. This turns out to be the right way. From (2.5) and (2.9), one may have the retarded correlators from gravity.

However, it was proposed in [10] that there is an intrinsic way to obtain the retarded correlators. Instead of working in Euclidean space, one can work with the classical solution in Lorentz-signature spacetime. More precisely, one has the prescription:

\[ G_R(\omega, \vec{k}) = \lim_{r \to \infty} \frac{\Pi(r, \omega, \vec{k})}{\phi_R(r, \omega, \vec{k})}. \]  

where \( \Pi \) is the canonical momentum conjugate to \( \phi \), taking \( r \) as the “time” direction. Now \( \phi_R \) is the classical solution, which should be purely in-falling at the black hole horizon and turns to \( \phi_0(\vec{k}) \) asymptotically. For the advanced correlators, one has to instead choose out-going boundary condition at the horizon.

There is another subtlety in applying the above prescription (2.10) to get a finite retarded correlators. In fact, the naive application of (2.10) leads to a divergent result. In order to cancel the divergence, an extra factor, being powers of \( r \), has to be taken into account. The exact value of the power depends on what kind of source we consider. For example, if the asymptotic behavior of the scalar is

\[ \phi \sim A r^{h_R-1} + B r^{-h_R}, \]  

where...
where \( h_R \) is the conformal weight, and both terms are renormalizable if \( h_R < 1 \) such that both \( A \) and \( B \) can be the source, then the extra factor could be \( r^{2(h_R - 1)} \) or \( r^{-2h_R} \), up to which term is taken to be the source.

For the scalar perturbation, the conjugate momentum of \( \phi \) is

\[
\Pi_\phi = -\sqrt{-g}g^{\tau \tau}\partial_\tau \phi. \tag{2.12}
\]

For the Maxwell field \( A_\mu \), its conjugate momentum is

\[
\Pi^\mu = -\sqrt{-g}F^{\tau \mu}. \tag{2.13}
\]

For the tensor perturbation, its conjugate momentum is the Brown-York stress tensor:

\[
\Pi^{\mu \nu} = \frac{\sqrt{-\gamma}}{16\pi G_N} (K^{\mu \nu} - \gamma^{\mu \nu} K^\lambda_{\lambda}), \tag{2.14}
\]

where \( \gamma^{\mu \nu} \) and \( K^{\mu \nu} \) are the induced metric and extrinsic curvature on constant \( r \)-slice. For the spinor perturbation, the prescription is quite similar but the expression of the conjugate momentum depends on the choice of Gamma matrices.

For the scalar and vector case, the above prescription \((2.10)\) has turned out to be equivalent to the ones proposed in [4]. And they have been applied to the study the transport properties of finite temperature \( \mathcal{N} = 4 \) super-Yang-Mills.

3. Green’s functions in 2D CFT

In general, it is hard to compare the retarded correlators obtained from gravity with the ones in CFT. However, in our case this is feasible. This is because on one hand the computations in gravity could be done analytically as we will show, and more importantly on the other hand the dual two-dimensional CFT is much more restricted by the symmetries. In a two-dimensional CFT, there are two independent sectors: left-moving one and right-moving one. This requires us to analyze them separately. The relation between the retarded Green’s function and the Euclidean Matsubara propagator is modified to be

\[
G_R(i\omega_L, i\omega_R) = G_E(\omega_{L,E}, \omega_{R,E}), \tag{3.1}
\]

at

\[
\omega_{L,E} = 2\pi n_L T_L, \quad \omega_{R,E} = 2\pi n_R T_R \tag{3.2}
\]

with \( n_L, n_R \) being integers.

In a 2D conformal field theory (CFT), one can define a two-point function as

\[
G(t^+, t^-) = \langle \mathcal{O}_\phi(t^+, t^-) \mathcal{O}_\phi(0) \rangle, \tag{3.3}
\]

where \( t^+, t^- \) are the left and right moving coordinates of 2d worldsheet, and \( \mathcal{O}_\phi \) is the operator corresponding to the field perturbing the black hole. For our later use, let us consider an operator of conformal dimensions \((h_L, h_R)\), right charge \( q_R \), at temperature \((T_L, T_R)\) and chemical potential \( \Omega_R \). Its two-point function is decided by conformal invariance \([11]\):

\[
G(t^+, t^-) \sim (-1)^{h_L + h_R} \left( \frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R} e^{iq_R \Omega_R t^-}. \tag{3.4}
\]
For the left mover, it contributes

$$G_E(\omega_{L,E}) \sim \int_0^{1/T_L} e^{i\omega_{L,E} \tau_L} \left( \frac{\pi T_L}{\sin(\pi T_L \tau_L)} \right)^{2h_L} d\tau_L,$$

where $\tau_L$ is Euclidean time. In fact, the above function is only defined at the discrete frequency in (3.2). Through analytic continuation, we have

$$G_E(\omega_{L,E}) \sim \frac{T^{2h_L-2} e^{i\omega_{L,E}/2T_L} \Gamma(1-2h_L)}{(1-h_L + \omega_{L,E}/2\pi T_L)\Gamma(1-h_L - \omega_{L,E}/2\pi T_L)}.$$

(3.6)

For the right mover, it has similar contribution, taken into account the extra dependence on the chemical potential:

$$G_E(\omega_{R,E}) \sim \frac{T^{2h_R-2} e^{i\omega_{R,E}+q_R \Omega_R}/2T_R \Gamma(1-2h_R)}{(1-h_R + \omega_{R,E}/2\pi T_R)\Gamma(1-h_R - \omega_{R,E}/2\pi T_R)}.$$

(3.7)

The total contribution is the product of the left-mover’s (3.6) and the right-mover’s (3.7):

$$G_E(\omega_{L,E},\omega_{R,E}) \sim \frac{T^{2h_L} T^{2h_R} e^{i(\omega_{L,E}+q_R \Omega_R)/2T} \Gamma(1-2h_L) \Gamma(1-2h_R)}{(1-h_L + \omega_{L,E}/2\pi T_L)\Gamma(1-h_L - \omega_{L,E}/2\pi T_L)\Gamma(1-h_R + \omega_{R,E}/2\pi T_R)\Gamma(1-h_R - \omega_{R,E}/2\pi T_R)}.$$

(3.8)

The CFT absorption cross section could be defined with the two-point functions, following Fermi’s golden rule:

$$\sigma_{abs} \sim \int dt^+ dt^- e^{-i\omega_{L}t^- - i\omega_{L}t^+} [G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)].$$

(3.9)

Then after being changed into momentum space, the absorption cross section is

$$\sigma \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh(\frac{\omega_L}{2T_L} + \frac{\omega_R - q_R \Omega_R}{2T_R}) \Gamma(h_L + i\frac{\omega_L}{2\pi T_L})^2 \Gamma(h_R + i\frac{\omega_R - q_R \Omega_R}{2\pi T_R})^2.$$

(3.10)

4. Spacelike warped case

The spacelike stretched $AdS_3$ spacetime is the vacuum solution of three-dimensional topological massive gravity [10, 11]. It could be described by the metric of the form

$$ds^2 = \frac{\ell^2}{v^2 + 3} [- (1 + r^2) dt^2 + \frac{dr^2}{1 + r^2} + \frac{4v^2}{3} (dx + r d\tau)^2],$$

(4.1)

where $-\ell^{-2}$ is a negative cosmological constant and the parameter $v$ is defined to be $v \equiv \mu l/3$ with $\mu$ being the mass of the graviton. It turns out that only when $v > 1$, the spacetime is free of pathology [13] and could be a stable vacuum with appropriate boundary conditions [19]. This spacetime has $SL(2)_R \times U(1)_L$ isometry group. Just as the BTZ black hole [12] could be constructed as discrete quotient of the $AdS_3$ spacetime, the black hole asymptotic to spacelike warped $AdS_3$ could be constructed from discrete identification as well, c.f. [13].
The metric of the spacelike stretched warped $\text{AdS}_3$ black hole takes the following form in terms of the Schwarzschild coordinates:

$$ds^2 = l^2(dt^2 + 2M(r)dt\theta + N(r)d\theta^2 + D(r)dr^2) \tag{4.2}$$

where

$$M(r) = vr - \frac{1}{2}\sqrt{r_+r_-(v^2 + 3)}, \tag{4.3}$$

$$N(r) = \frac{r}{4}\left(3(v^2 - 1)r + (v^2 + 3)(r_+ + r_-) - 4v\sqrt{r_+r_-(v^2 + 3)}\right), \tag{4.4}$$

$$D(r) = \frac{1}{(v^2 + 3)(r - r_+)(r - r_-)}. \tag{4.5}$$

Just like the BTZ black hole, there are two horizons located at $r = r_+$ and $r = r_-$. We will focus on the physical black holes without any pathology, which requires $v > 1$. When $v = 1$, there is no stretching and the above black hole becomes the usual BTZ black hole, in a rotational frame.

From discrete identification which leads to the black hole, one can define two temperatures

$$T_L = \frac{(v^2 + 3)}{8\pi l} \left( r_+ + r_- - \frac{\sqrt{(v^2 + 3)r_+r_-}}{v} \right), \tag{4.6}$$

$$T_R = \frac{(v^2 + 3)(r_+ - r_-)}{8\pi l}, \tag{4.7}$$

which are identified to be the temperatures of left-moving and right moving sectors in the dual two-dimensional CFT\(^\text{[13]}\). In terms of them, the temperature of the black holes were rewritten as

$$\frac{1}{T_H} = \frac{4\pi vl}{v^2 + 3} \frac{T_L + T_R}{T_R}. \tag{4.8}$$

The entropy of the black hole could be recovered from dual CFT by applying Cardy formula as well. This help us to determine the central charges of dual CFT

$$c_L = \frac{l}{G} \frac{4v}{v^2 + 3}, \quad c_R = \frac{l}{G} \frac{5v^2 + 3}{v(v^2 + 3)}. \tag{4.9}$$

It has been shown in \(^\text{[18, 20]}\) that the above central charges could be obtained from central extended Virasoro algebra, based on the fact that the asymptotic symmetries of the geometries form a semi-product of a Virasoro algebra and a current algebra.

The spacelike warped AdS/CFT correspondence states that $v > 1$ quantum topological massive gravity with asymptotical spacelike stretched AdS$_3$ geometry is holographically dual to a two-dimensional conformal field theory with central charges $(c_L, c_R)$.

\section*{4.1 Scalar correlators}

The scalar perturbation about the warped black hole background obeys the equation of motion:

$$(\nabla_\mu \nabla^\mu - m^2)\Phi = 0. \tag{4.10}$$
Since the background (4.12) has the translational isometry along $t$ and $\theta$, we may make the following ansatz

$$\Phi = e^{-i\omega t + ik\theta}. \quad (4.11)$$

With the variable

$$z = \frac{r - r_+}{r - r_-}, \quad (4.12)$$

the equation of motion on $\phi$ turns out to be

$$z(1 - z)\frac{d^2 \phi}{dz^2} + (1 - z)\frac{d\phi}{dz} + \frac{1}{(v^2 + 3)^2} \left( \frac{A}{z} + B + \frac{C}{1 - z} \right) \phi = 0, \quad (4.13)$$

where

$$A = \frac{1}{(r_+ - r_-)^2} (2k + \omega \sqrt{r_+}(2v\sqrt{r_+} - \sqrt{v^2 + 3\sqrt{r_+}}))^2, \quad (4.14)$$

$$B = -\frac{1}{(r_+ - r_-)^2} (2k + \omega \sqrt{r_-}(2v\sqrt{r_-} - \sqrt{v^2 + 3\sqrt{r_-}}))^2, \quad (4.15)$$

$$C = 3(v^2 - 1)\omega^2 - m^2 l^2(v^2 + 3). \quad (4.16)$$

The solutions to the equation (4.13) take the form of hypergeometric function. Near the horizon, there are two independent solutions. Only one of them satisfies the purely ingoing boundary condition at the horizon, which is necessary to get the retarded Green’s function. It is

$$\phi = z^\alpha (1 - z)^\beta F(a, b, c, z), \quad (4.17)$$

where

$$\alpha = -i\sqrt{A} \frac{\sqrt{v^2 + 3}}{v^2 + 3},$$

$$\beta = \beta_\pm = \frac{1}{2} \left( 1 \pm \sqrt{1 - \frac{4C}{(v^2 + 3)^2}} \right), \quad (4.18)$$

and

$$c = 2\alpha + 1,$$

$$a = \alpha + \beta + i\sqrt{-B}/(v^2 + 3),$$

$$b = \alpha + \beta - i\sqrt{-B}/(v^2 + 3).$$

For a scalar of mass $m$ propagating in spacelike warped AdS$_3$, its conformal weight is [13, 22]

$$h_R^\pm = \Delta_s^\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} + s_s}, \quad (4.19)$$

with

$$s_s = \frac{3(1 - v^2)}{4v^2} \tilde{k}^2 + \frac{l^2}{v^2 + 3} m^2. \quad (4.20)$$

Here $\tilde{k}$ is the quantum number with respect to the translational symmetry along $x$ in the background (1.1). The presence of quantum number $\tilde{k}$ in the conformal weight has interesting physical implications. Obviously, $s_s$ could be negative in a natural way.
Note that in order to have a well-behaved asymptotic behavior, we have $\Delta^\pm_s > 0$. More precisely, if $s_s > 0$, we can only choose
\[ \Delta^+_s > 1. \]  
(4.21)

On the other side, if $-\frac{1}{4} < s_s \leq 0$, we are free to choose both $\Delta^\pm_s$ with
\[ 0 \leq \Delta^-_s < \frac{1}{2}, \quad \frac{1}{2} < \Delta^+_s \leq 1. \]  
(4.22)

It is remarkable that after taken into account of the subtle identification of the quantum numbers, $\beta$ is directly related to the conformal weight $\Delta^\pm_s = \Delta^\pm_s$. But note that $\beta^\pm$ do not need to be always positive.

Actually, there is a subtlety in choosing $\beta^\pm$ in the above solution. If $s_s$ is positive, we take the conformal dimension of the scalar to be $h^+_R > 1$, then we have to choose $\beta^-$ in (4.17). Correspondingly, the extra factor plugged in (2.10) should be $r^{2(h^+_R - 1)}$. However, if $s_s < 0$ we have to choose $\beta^+$ in the solution. In this case, there are two normalizable sources, which need special care as we show soon.

Let us first consider the source of a dimension $h^+_R > 1$. In this case, the momentum is
\[ \Pi_s = -\frac{1}{2}(r_+ - r_-)(v^2 + 3)z\partial_z\phi \]  
(4.23)

where $\phi = z^a(1 - z)^{\beta^-}F(a, b, c; z)$. Note that now $\beta^- < 0$. From the relation
\[ F(a, b, c; z) = \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)}F(a, b, a + b - c + 1; 1 - z) \]
\[ + (1 - z)^{c - a - b}\frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)}F(c - a, c - b, c - a - b + 1; 1 - z), \]  
(4.24)

asymptotically the dominant contribution in $\phi$ is proportional to $r^{-\beta^-}$ since the other term proportional to $r^{\beta^- - 1}$ runs to zero. On the other hand, in the momentum, there are various terms asymptotically proportional to
\[ r^{-\beta^-}, \quad r^{\beta^- - 1}, \quad r^{1 - \beta^-}, \quad r^{\beta^-}. \]  
(4.25)

The terms proportional to $r^{\beta^-}$ should be picked out. Then the retarded correlator is just
\[ G_R \sim \frac{\Gamma(a + b - c + 1)\Gamma(c - a)\Gamma(c - b)}{\Gamma(c - a - b)\Gamma(a)\Gamma(b)} \]
\[ = \mathcal{N}\frac{\Gamma(2\beta^-)}{\Gamma(1 - 2\beta^-)}|\Gamma(c - a)\Gamma(c - b)|^2, \]  
(4.26)

where
\[ \mathcal{N} = \frac{1}{2\pi^2}(\cosh\left(\frac{\sqrt{-B\pi}}{v^2 + 3}\right) - \cos(2\pi\beta^-)\cosh(2i\alpha\pi) + i\sin(2\pi\beta^-)\sinh(2i\alpha\pi)). \]  
(4.27)

In order to compare with the CFT result, we have to apply the identification of quantum numbers to simplify the above relation. The identification is essential to set up the dictionary of warped AdS/CFT correspondence. It was suggested in [15] that
\[ \tilde{k} = \frac{2v}{v^2 + 3}\omega, \quad \tilde{\omega} = \frac{2}{v^2 + 3}k, \]  
(4.28)
where \( \tilde{k}, \tilde{\omega} \) are the quantum numbers of global warped AdS spacetime. \( \tilde{\omega} \) is the quantum number with respect to the translational symmetry along \( \tau \) in the background (4.1).

Because the spacelike warped AdS/CFT correspondence is between the spacelike stretched warped AdS and its holographically dual 2D CFT, we need to use the quantum numbers in global warped AdS spacetime rather than the ones in the black holes to set up the dictionary. In terms of the quantum numbers \( \tilde{k}, \tilde{\omega} \), we have

\[
\begin{align*}
    c - a &= h_R^+ - i\frac{1}{2\pi T_R} \left( \frac{\nu^2 + 3}{2} \tilde{\omega} + 2\pi T_L \tilde{k} \right), \\
    c - b &= h_R^+ - i\tilde{k}, \\
    a &= 1 - h_R^+ - i\tilde{k}, \\
    b &= 1 - h_R^+ - i\frac{1}{2\pi T_R} \left( \frac{\nu^2 + 3}{2} \tilde{\omega} + 2\pi T_L \tilde{k} \right). 
\end{align*}
\]

(4.29)

To compare with the CFT result, it is better to rewrite the correlator in the following form:

\[
G_R \sim N' \Gamma(1 - h_R^+ + \frac{\omega_R - q_R \Omega_R}{2\pi T_R}) \Gamma(1 - h_L^+ + \frac{\omega_L - q_L \Omega_L}{2\pi T_L}) \Gamma(1 - h_R^+ - i(\omega_R - q_R \Omega_R) / 2\pi T_R). 
\]

(4.30)

with

\[
N' = \frac{4\pi \sin((2h_R^+ - 1)\pi)}{\cosh \left( \frac{\omega_R - q_R \Omega_R}{2\pi T_R} \right) - \cos \left( 2h_R^+ - i \left( \frac{\omega_R - q_R \Omega_R}{2\pi T_R} \right) \right)}. 
\]

(4.31)

Here \( h_L^+ = h_R^+ \) and

\[
\omega_L = 2\pi T_L \tilde{k}, \quad \omega_R = \frac{\nu^2 + 3}{2l} \tilde{\omega}, \quad q_R = -\tilde{k}, \quad \Omega_R = 2\pi T_L. 
\]

(4.32)

Obviously the retarded correlator (4.30) is proportional to (3.8), up to a normalization factor, taken the relation (3.1) into account. This fact shows that real-time scalar correlator in the spacelike stretched AdS black hole is well consistent with the prediction of CFT.

The quasi-normal modes of the black hole correspond to the poles of the retarded Green’s function. From (4.26), we obtain that

\[
\begin{align*}
\omega_L &= -i2\pi T_L(n_L + h_L) \\
\omega_R &= q_R \Omega_R - i2\pi T_R(n_R + h_R) 
\end{align*}
\]

(4.33)

(4.34)

where \( n_L \) and \( n_R \) are the non-negative integers. This is in precise match with the result found in [15].

Moreover, the imaginary part of real-time retarded correlators could be identified as the cross section. The cross section reads

\[
\sigma \sim \frac{1}{(\Gamma(1 - 2\beta_-))^2} \Gamma(c - a)\Gamma(c - b)^2 \sinh(2i\alpha\pi). 
\]

(4.35)
It could be rewritten as
\[
\sigma \sim \sinh \left( \frac{\omega_L}{2\pi T_L} + \frac{\omega_R - qR\Omega_R}{2\pi T_R} \right) \pi \left[ \Gamma(h_L^+ + i\frac{\omega_L}{2\pi T_L}) \right]^2 \left[ \Gamma(h_R^- + i\frac{\omega_R - qR\Omega_R}{2\pi T_R}) \right]^2, \tag{4.36}
\]

The relation (4.36) is reminiscent of the absorption cross section in a CFT. It is actually proportional to (3.10), up to a normalization factor. The situation is very similar to the one in Kerr/CFT correspondence [25]. The normalization factor depends on the temperatures as
\[
(T_L)^{h_L}(T_R)^{h_R}. \tag{4.37}
\]

Such a factor may arise from the nontrivial coordinate transformation, which redefine the temperature, as in BTZ case [10].

There is another interesting issue on the cross section of a scalar scattering the black hole. It has been argued that for a massless scalar, the low energy limit of the cross section is proportional to the horizon area [27]. For the spacelike stretched black hole, this has been studied in [28, 29]. In our study, since we cannot fix the normalization factor exactly, we can not check this statement precisely. However, we can check if the low energy limit of the cross section has the right dependence on frequency. It is straightforward to investigate the \( \omega \to 0 \) limit, with \( m = k = 0 \), which reads \( \sigma \sim \omega \). This linear dependence on \( \omega \) is in consistent with the universal statement.

It is more tricky if \( s_s < 0 \) so the scalar field can pick both the conformal weight \( h_R^+ \). In this case, note that one has to choose \( \beta_+ \) in the scalar solution. The asymptotic behavior of the scalar field is like
\[
\phi \sim A r^{-\beta_+} + B r^{-1+\beta_+}. \tag{4.38}
\]

Since \( 0 < \beta_+ < 1 \), both terms are normalizable. We are free to choose either one as the source: \( A \) is the source of a dimension \( 1 - \beta_+ = h_R^- \), while \( B \) is the source of a dimension \( \beta_+ = h_R^+ \). In other words, if we want to study the correlator of the operators of dimension \( h_R^- \), then we should find the term proportional to \( r^{\beta_+} \) in the canonical momentum \( \pi_s \). Consequently, the retarded correlator take the same form as (4.26), with \( \beta_- \) being replaced by \( \beta_+ \). As a result of the replacement, the conformal dimension appeared in the retarded correlator (4.30) and the cross section (4.36) should be \( h_R^- \) rather than \( h_R^+ \).

On the other hand, in order to study the correlator of the operator of dimension \( h_R^+ \), we have to find the term proportional to \( r^{1-\beta_+} \) in the canonical momentum \( \Pi_s \). Consequently, the retarded Green’s function is now
\[
G_R \sim \frac{\Gamma(a)\Gamma(b)}{\Gamma(c - a - b)\Gamma(c - b)\Gamma(a + b - c)} \frac{\Gamma(1 - 2\beta_-)}{\Gamma(2\beta_- - 1)} |\Gamma(a)\Gamma(b)|^2, \tag{4.39}
\]

where
\[
\mathcal{N}_\infty = \frac{1}{2\pi^2} \left( \cosh(\frac{\sqrt{-B}}{v^2 + 3}) - \cos(2\pi\beta_-) \cosh(2i\alpha\pi) + i \sin(2\pi\beta_-) \sinh(2i\alpha\pi) \right). \tag{4.40}
\]
Taking into account of the identifications of quantum numbers (4.28), we find that

\[ b = h_R^+ - i \frac{1}{2\pi T_R l} \left( \frac{v^2 + 3}{2} \tilde{\omega} + 2\pi T_L \tilde{k} \right), \]

\[ a = h_R^+ - i \tilde{k}, \]

\[ c - b = 1 - h_R^+ - i \tilde{k}, \]

\[ c - a = 1 - h_R^+ - i \frac{1}{2\pi T_R l} \left( \frac{v^2 + 3}{2} \tilde{\omega} + 2\pi T_L \tilde{k} \right). \] (4.41)

Note that it is different from (4.29). The difference stems from the choice of \( \beta_\pm \) in the scalar solutions. Nevertheless, the retarded correlator takes exactly the same form as (4.30), even though the scalar solution now is different. In this case, the poles of the real-time correlator is simply

\[ a = -n_L, \quad \text{or} \quad b = -n_R, \] (4.42)

with \( n_L, n_R \) being non-negative integer. This is the same as the ones found in [15].

Similarly the cross section is

\[ \sigma = \text{Im}(G_R) \sim \frac{1}{\Gamma(2\beta_+)\Gamma(2\beta_+ - 1)} |\Gamma(a)\Gamma(b)|^2 \sinh(2i\alpha\pi). \] (4.43)

It could be written as (4.36). And the comparison with the prediction of CFT is along the similar line. The low energy cross section for massless scalar could be obtained by taking \( m \to 0, \omega \to 0 \) limit and is found to be proportional to \( \omega \) as well.

In short, no matter what kind of scalar source we choose, we end up with the same real-time retarded correlator (4.30) and the cross section (4.36). These results are in good match with the prediction (3.8, 3.10), up to a normalization factor.

Before ending this section, we would like to discuss another interesting case when the black hole has super-radiance. This happens when

\[ S = \frac{1}{4} + \frac{3(1 - v^2)}{4v^2} \tilde{k}^2 + \frac{\ell^2}{v^2 + 3} m^2 < 0. \] (4.44)

This is possible now due to the presence of the quantum number \( \tilde{k} \) in the conformal weight. Therefore even though the mass-square of the scalar field satisfies the Breitenlohner-Freedman bound for three-dimensional AdS spacetime, the perturbation could still be unstable. As a result, superradiance may happen in the spacelike stretched AdS\(_3\) black holes[22], just like the superradiance in Kerr black hole[23]. Superradiance happens in the null warped AdS\(_3\) spacetime as well.

In this case, the conformal weight is just

\[ h_R^\pm = \frac{1}{2} \pm i\sqrt{-S} \] (4.45)

No matter which conformal weight and the corresponding eigenfunction we choose, the asymptotic behavior of the scalar field is always of the form

\[ \phi \sim S_1(1 - z)^{\frac{1}{2} + i\sqrt{-S}} + S_2(1 - z)^{\frac{1}{2} - i\sqrt{-S}}. \] (4.46)
The first term could be taken as the ingoing wave and the second term as the outgoing wave. The ratio $S_2/S_1$ measures the response of the black hole to an incoming wave, and is related to the retarded two-point function of dual CFT. Therefore, we have

$$G_R \sim \frac{S_2}{S_1} = \begin{cases} \frac{\Gamma(c-a)\Gamma(c-b)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)\Gamma(c-a-b)} & \text{if } h_R = h_R^+ \\ \frac{\Gamma(c-a)\Gamma(c-b)\Gamma(a+b-c)}{\Gamma(c-a)\Gamma(c-b)\Gamma(a+b-c)} & \text{if } h_R = h_R^- \end{cases}$$

(4.47)

Note that since even with a complex conformal weight the relations (3.6,3.7) always make sense, the retarded correlator (4.47) is still consistent with (3.1,3.8).

### 4.2 Vector correlator

The equation of motion for the massive vector field in three dimension could be cast into a first-order differential equation of the form

$$\epsilon_{\lambda}^{\alpha\beta} \partial_{\alpha} A_{\beta} = -mA_{\lambda},$$

(4.48)

where $\epsilon_{\lambda}^{\alpha\beta}$ is the Levi-Civita tensor with $\epsilon^{r\theta} = 1/\sqrt{-g}$. The Killing symmetry of the background allows us to make the following ansatz:

$$A_{\mu} = e^{-i\omega t + ik \phi_{\mu}}.$$  

(4.49)

The complete solution could be found in [15]:

$$\phi_t = z^{a_v}(1 - z)^{b_v+1} F(a_v + 1, b_v + 1, c_v, z),$$

$$\phi_{\theta} = \tilde{A}_v \phi_t + \frac{B_v}{1 - z} \phi_t + \tilde{C}_v z \frac{d\phi_t}{dz},$$

$$\phi_r = -\frac{2D(r)}{ml} (ik\phi_t + i\omega\phi_{\theta}),$$

(4.50)

where $\alpha_v = -i\sqrt{A_v}$, $\beta_v = (-1 + \sqrt{1 - 4C_v})/2$ and

$$\alpha_v = 1 + 2\alpha_v, \quad \beta_v = \alpha_v + \beta_v + i\sqrt{-B_v}, \quad b_v = \alpha_v + \beta_v - i\sqrt{-B_v},$$

(4.51)

with

$$A_v = \frac{1}{(r_+ - r_-)^2(v^2 + 3)^2} \left(2k + \omega \sqrt{r_+}(2v\sqrt{r_+} - \sqrt{v^2 + 3}\sqrt{r_-})\right)^2,$$

$$B_v = -\frac{1}{(r_+ - r_-)^2(v^2 + 3)^2} \left(2k + \omega \sqrt{r_-}(2v\sqrt{r_-} - \sqrt{v^2 + 3}\sqrt{r_+})\right)^2,$$

$$C_v = \frac{1}{(v^2 + 3)^2} \left(3(v^2 - 1)\omega^2 - (m^2 l^2 + 2ml)(v^2 + 3)\right),$$

(4.52)

$$\tilde{A}_v = \frac{1}{2\omega^2 + 2m^2 l^2} (-2\omega k + 2m^2 l^2 v_{r_-} - m^2 l^2 \sqrt{r_+ r_-}(v^2 + 3)),$$

$$\tilde{B}_v = \frac{ml^2 v (r_+ - r_-)}{\omega^2 + m^2 l^2},$$

$$\tilde{C}_v = \frac{-ml(v^2 + 3)(r_+ - r_-)}{2\omega^2 + 2m^2 l^2}.$$
The conjugate momentum is the conserved current $J^\mu$
\[
J^\mu = - \lim_{r \to \infty} \sqrt{-g} F^\mu. \tag{4.53}
\]

Using the equation of motion, we find that the current is simply related to the fields
\[
J^t = mA_\theta, \quad J^\theta = -mA_t \tag{4.54}
\]

In order to use the prescription (2.10) for the retarded Green’s function, we have to analyze the asymptotic behavior of the vector fields. It can be read directly from the solutions as the following
\[
\phi_t = A_1(1 - z)^{1+\beta_v} + A_2(1 - z)^{-\beta_v}, \tag{4.55}
\]
\[
\phi_\theta = (\tilde{B}_v - \tilde{C}_v(\beta_v + 1))A_1(1 - z)^{\beta_v} + (\tilde{B}_v + \tilde{C}_v\beta_v)A_2(1 - z)^{-\beta_v-1} \tag{4.56}
\]

where
\[
A_1 = \frac{\Gamma(c_v)\Gamma(c_v - a_v - b_v - 2)}{\Gamma(c_v - a_v - 1)\Gamma(c_v - b_v - 1)} A_0, \quad A_2 = \frac{\Gamma(c_v)\Gamma(a_v + b_v + 2 - c_v)}{\Gamma(a_v + 1)\Gamma(b_v + 1)} A_0, \tag{4.57}
\]

with $A_0$ being a constant. If the dominant term in $\phi_t$ is the source, then the retarded Green’s function describing the response is given by
\[
G_{tt} = \frac{mA_1(\tilde{B}_v - \tilde{C}_v(\beta_v + 1)) A_1}{A_2}. \tag{4.58}
\]

If the $\phi_\theta$ is the source, the corresponding retarded Green’s function is
\[
G_{\theta\theta} = -\frac{mA_1}{(\tilde{B}_v + \tilde{C}_v\beta_v)A_2}. \tag{4.59}
\]

Up to a normalization factor, we have in both cases
\[
G_R \sim \frac{A_1}{A_2} = \frac{\Gamma(a_v + 1)\Gamma(b_v + 1)\Gamma(c_v - a_v - b_v - 2)}{\Gamma(c_v - a_v - 1)\Gamma(c_v - b_v - 1)\Gamma(a_v + b_v + 2 - c_v)}
\]
\[
= \mathcal{N}_v \frac{\Gamma(1 - 2h^v_R)}{\Gamma(2h^v_R - 1)}\frac{\Gamma(a_v + 1)\Gamma(b_v + 1)^2}{\Gamma(c_v - a_v - 1)\Gamma(c_v - b_v - 1)\Gamma(a_v + b_v + 2 - c_v)}, \tag{4.60}
\]

with
\[
\mathcal{N}_v = \frac{1}{2\pi^2} \left( \cosh(2\sqrt{-B_v\pi}) - \cos(2\pi h^v_R) \cosh(2i\alpha_v\pi) + i \sin(2\pi h^v_R) \sinh(2i\alpha_v\pi) \right). \tag{4.61}
\]

Here $h^v_R$ is the conformal weight of a massive vector field.

\[
h^v_R = \frac{1}{2} + \sqrt{\frac{1}{4} + s_v} \tag{4.62}
\]

with
\[
s_v = \frac{3(1 - v^2)}{4v^2} k^2 + \frac{(m^2 l^2 + 2vml)}{v^2 + 3}. \tag{4.63}
\]
Taking into account of the fact
\[
c_v - a_v - 1 = 1 - h^v_R - i \frac{1}{2\pi T_R l} \left( \frac{\nu^2 + 3}{2} \bar{\omega} + 2\pi T_L \bar{k} \right),
\]
\[
c_v - b_v - 1 = 1 - h^v_R - i \bar{k},
\]
\[
a_v + 1 = h^v_R - i \bar{k},
\]
\[
b_v + 1 = h^v_R - i \frac{1}{2\pi T_R l} \left( \frac{\nu^2 + 3}{2} \bar{\omega} + 2\pi T_L \bar{k} \right),
\]
we find that just like the scalar case, the vector correlator is well consistent with (3.8), up to a normalization factor.

However, strictly speaking there is a discrepancy from the CFT prediction. Now for the vector field, the conformal weights of left-mover and right-mover are different:
\[
h^v_L = h^v_R \pm 1. \tag{4.65}
\]
Correspondingly the contribution from left-mover is slightly different from the right-mover one. However, this difference could not be seen in (4.60), recalling the fact that both the real parts of \((1 + a_v)\) and \((1 + b_v)\) give the same \(h^v_R\). Nevertheless, using \(\Gamma(1 + z) = z\Gamma(z)\), one can absorb the extra \(a_v\) factor into the normalization factor such that one has
\[
G_R \sim N^0_v \frac{1}{|\Gamma(c_v - a_v)\Gamma(c_v - b_v - 1)|^2}. \tag{4.66}
\]
Then the parts involving the Gamma functions have the right dependence consistent with the prediction of CFT. It is remarkable that the normalization factor including the contribution of \(a_v\) is independent of \(\bar{\omega}\) so that the retarded correlator has the correct frequency dependence for the Green’s function of a finite temperature CFT.

The cross section can be read directly
\[
\sigma \sim \frac{1}{\Gamma(2h^v_R)\Gamma(2h^v_R - 1)} \left| \Gamma(a_v + 1)\Gamma(b_v + 1) \right|^2 \sinh(2i\alpha_v \pi)). \tag{4.67}
\]

It is similar to (4.36) in the scalar case, with \(h^v_R\) replacing \(h^+_R\). The discussions on the quasi-normal modes, the low energy limit of massless vector cross section and super-radiance are very similar to the scalar case. We just omit them here.

5. Null warped case

The null warped \(AdS_3\) spacetime is another vacuum solution of three-dimensional topological massive gravity. It is only well defined at \(v = 1\). Similar to other warped \(AdS_3\) spacetimes, it also has isometry group \(SL(2,R) \times U(1)_{null}\). The null warped black holes could be obtained as the quotient of the null warped \(AdS_3\). The metric of the null warped black hole is of the form
\[
\frac{ds^2}{l^2} = -2r d\theta dt + (r^2 + r + \alpha^2) d\theta^2 + \frac{dr^2}{4r^2}. \tag{5.1}
\]
where $1/2 > \alpha > 0$ in order to avoid the naked causal singularity. The horizon is located at $r = 0$. From the thermodynamics of this extremal black hole, it was argued that there exist only non-vanishing right-moving temperature

$$ T_R = \frac{\alpha}{\pi l}. \quad (5.2) $$

There is a conjecture on the null warped background \cite{13}: $v = 1$ quantum topological massive gravity with asymptotical null warped $AdS_3$ geometry is holographically dual to a 2D boundary CFT with the left-moving central charge $c_L = \frac{L}{4G^{v_2+3}}$ and the right-moving central charge $c_R = \frac{(5v^2+3)L}{4G^{v_2+3}}$. From the black hole entropy, it seems that it is not necessary to have left-moving central charge since $T_L = 0$. However, the diffeomorphism anomaly requiring that $c_L - c_R = - \frac{L}{4G^v}$ asks for the existence of the left-moving sector.

### 5.1 Scalar correlator

For a scalar field in the null warped black holes background (5.1), its solution can be expressed by Kummer function as

$$ \phi_{\pm} = e^{-i\omega t + ik \theta} e^{-\frac{i}{2} z^2 \pm \tilde{m}_s} F\left(\frac{1}{2} \pm \tilde{m}_s - \kappa, 1 \pm 2\tilde{m}_s, z\right) \quad (5.3) $$

where $z = -i\omega \frac{1}{2}$, $\tilde{m}_s = \frac{1}{2} \sqrt{1 + m^2 l^2 - \omega^2}$ and $\kappa = \frac{i}{4\alpha} (\omega - 2k)$. The wave function should be a composition

$$ \phi = \tilde{A}_1 \phi_+ + \tilde{A}_2 \phi_- \quad (5.4) $$

In order to obtain the retarded Green’s function, the solutions should satisfy the purely ingoing boundary condition at the horizon. This requirement helps us to fix the normalization factors relatively

$$ \tilde{A}_1 = - \frac{\Gamma(1 - 2\tilde{m}_s)}{\Gamma(\frac{1}{2} - \tilde{m}_s - \kappa)} (-i\omega \alpha)^{\frac{1}{2} + \tilde{m}_s} \tilde{A}_0, \quad \tilde{A}_2 = \frac{\Gamma(1 + 2\tilde{m}_s)}{\Gamma(\frac{1}{2} + \tilde{m}_s - \kappa)} (-i\omega \alpha)^{\frac{1}{2} - \tilde{m}_s} \tilde{A}_0, \quad (5.5) $$

where $\tilde{A}_0$ is a constant, and $-i = e^{-\frac{i\pi}{2}}$.

The conformal weight of the scalar field of mass $m$ in the null warped $AdS_3$ spacetime is

$$ h_\pm = \frac{1}{2} \left(1 \pm \sqrt{1 + s_n}\right) \quad (5.6) $$

with

$$ s_n = m^2 l^2 - 4\tilde{k}^2, \quad (5.7) $$

where $\tilde{k}$ is the $U(1)$ quantum number of the global null warped background. As usual, the conformal weight should be positive for stable perturbation. Similar to the spacelike warped case, if $s_n > 0$, there is only one possible conformal weight $h^+ > 1$, while if $-1 < s_n < 0$ there are two possible choices $h^\pm$ with $0 < h^- < \frac{1}{2}$, $\frac{1}{2} < h^+ < 1$.

The asymptotic behavior of the solution is

$$ \phi \sim \tilde{A}_1 z^h + \tilde{A}_2 z^{1-h} \quad (5.8) $$
where \( s_n > 0 \) and \( h > 1 \). The conjugate momentum is

\[
\Pi_{null} \propto \partial_z \phi
\]  

(5.9)

In this case, the source term is \( \tilde{A}_2 \) and correspondingly in the conjugate momentum the relevant terms should be proportional to \( z^{h-1} \). Then the retarded Green’s function is

\[
G_R \sim \frac{\tilde{A}_1}{\tilde{A}_2} \propto \frac{\Gamma(1 - 2\tilde{m}_s) \Gamma(\frac{1}{2} + \tilde{m}_s - \kappa)}{\Gamma(1 + 2\tilde{m}_s) \Gamma(\frac{1}{2} - \tilde{m}_s - \kappa)},
\]  

(5.10)

If \( s_n < 0 \), just as usual AdS/CFT case, there could be two kinds of renormalizable sources. We are free to choose one of them, then we have

\[
G_R \sim \begin{cases} 
\frac{\tilde{A}_1}{\tilde{A}_2}, & \text{taking } \tilde{A}_2 \text{ as the source} \\
\frac{\tilde{A}_2}{\tilde{A}_1}, & \text{taking } \tilde{A}_1 \text{ as the source.}
\end{cases}
\]  

(5.11)

In the following discussion, we will focus on the case with conformal weight \( h = \frac{1}{2} + \tilde{m}_s > 1 \) without losing generality.

As the spacelike case, we need to take into account of the identification of quantum numbers, which was discussed in [15]

\[
k = -\tilde{\omega},
\]  

(5.12)

\[
\omega = 2\tilde{k}.
\]  

(5.13)

Then we have

\[
\frac{1}{2} + \tilde{m}_s - \kappa = h - \frac{i}{2\pi T_R l}(\tilde{k} + \tilde{\omega}),
\]  

(5.14)

\[
\frac{1}{2} - \tilde{m}_s - \kappa = 1 - h - \frac{i}{2\pi T_R l}(\tilde{k} + \tilde{\omega}).
\]  

(5.15)

Now it is easy to see that the correlator is in good match with (3.7), once the frequency, the charge and the chemical potential in the right-moving sector are identified to be

\[
\omega_R = \frac{\tilde{\omega}}{l}, \quad q_R = -\tilde{k}, \quad \Omega_R = \frac{1}{l}.
\]  

(5.16)

Note that in the null warped case, the absence of the left temperature make the theory to be “chiral”, so it matches with the right-moving sector of dual CFT.

The quasi-normal modes correspond to the poles of the retarded correlator, which is just

\[
\frac{1}{2} + \tilde{m}_s - \kappa = -n.
\]  

(5.17)

Taking into account of (5.12), we have

\[
\tilde{\omega}_R = -\tilde{k}_n - i2\pi T_R l(n + h_R).
\]  

(5.18)

This is exactly we have found in [15].
The cross section is now
\[
\sigma \sim \text{Im} G_R \sim \frac{1}{(\Gamma(2h_R))^2 \sin(h_R \pi)} \sinh \left( \frac{k + \tilde{\omega}}{2T_R \pi} \right) |\Gamma(h + i(k + \tilde{\omega}))|^2.
\] (5.19)

It looks quite similar to the prediction (3.10) of dual 2D CFT, up to a prefactor being the power of the temperature. In the low energy limit for the massless scalar field, we have \( \text{Im} G_R \sim -l \omega \alpha \). So the cross section \( \sigma \) is proportional to \( \omega A_H \), where \( A_H = 2\pi l \alpha \) is the area of the horizon.

### 5.2 Vector correlation

After making the ansatz \( A_\mu = e^{-i\omega t + ik\theta} \phi_\mu(r) \), we also have two independent solutions of Eq.(4.48) as
\[
\phi_{t \pm} = e^{-\frac{\pi}{2} z \frac{1}{2} \pm \tilde{m}_v} F\left(\frac{1}{2} \pm \tilde{m}_v - \kappa, 1 \pm 2\tilde{m}_v, z\right)
\] (5.20)

and \( \phi_\theta, \phi_r \) can be read from
\[
\phi_\theta = \frac{2mlr^2}{\omega^2} \left( \frac{d\phi_t}{dr} + \left( \frac{\omega k}{2mlr^2} + \frac{ml}{2r} \right) \phi_t \right), \quad \phi_r = -\frac{1}{2mlr^2} (ik\phi_t + i\omega \phi_\theta),
\] (5.21)

where \( z = -i\omega \alpha \frac{1}{2}, \tilde{m}_v = \pm \frac{1}{2} \sqrt{(ml - 1)^2 - \omega^2} \) and \( \kappa = \frac{i}{4\alpha} (\omega - 2k) \).

Similarly we have to make a combination of the two solutions to satisfy the boundary condition that there are only purely ingoing modes at the horizon. We get
\[
\phi_\mu = C_1 \phi_{\mu +} + C_2 \phi_{\mu -}
\] (5.22)

where
\[
C_1 = -\frac{\Gamma(1 - 2\tilde{m}_v)}{\Gamma(\frac{1}{2} - \tilde{m}_v - \kappa)} C, \quad C_2 = \frac{\Gamma(1 + 2\tilde{m}_v)}{\Gamma(\frac{1}{2} + \tilde{m}_v - \kappa)} C
\] (5.23)

with \( C \) being a constant. The asymptotic solution for the vector field in the null black hole background is
\[
\phi_t = C_1 z^{\frac{1}{2} + \tilde{m}_v} + C_2 z^{\frac{1}{2} - \tilde{m}_v}, \quad \phi_\theta = \frac{iml\alpha}{\omega} (-1 - 2\tilde{m}_v + ml) C_1 z^{\frac{1}{2} + \tilde{m}_v} + \frac{iml\alpha}{\omega} (-1 + 2\tilde{m}_v + ml) C_2 z^{\frac{1}{2} - \tilde{m}_v}.
\] (5.24)

If we choose \( \phi_t \) as the source, then we have the retarded Green function
\[
G_{tt} = \frac{im^2 l\alpha}{\omega C_2} (-1 - 2\tilde{m}_v + ml) C_1.
\] (5.26)

Similarly taking the \( \phi_\theta \) as the source, we have
\[
G_{\theta\theta} = \frac{i\omega C_1}{al(-1 + 2\tilde{m}_v + ml)C_2}.
\] (5.27)
In any case, the retarded Green’s function is proportional to $C_1/C_2$. Similarly, from

$$\frac{1}{2} - \tilde{m}_v - \kappa = 1 - h_R^v - \frac{i}{2\pi T_R^v}(\tilde{k} + \tilde{\omega})$$  \hspace{1cm} (5.28)

with $h_R^v$ being the conformal weight of massive vector fields in the null warped AdS$_3$

$$h_R^v = \frac{1}{2} + \frac{1}{2} \sqrt{(ml - 1)^2 - 4\tilde{k}^2_n},$$ \hspace{1cm} (5.29)

we see that the retarded Green’s function is consistent with (3.7). The quasi-normal modes could be read directly from the poles:

$$\frac{1}{2} + \tilde{m}_v - \kappa = -n,$$ \hspace{1cm} (5.30)

which gives

$$\tilde{\omega}_R^v = -\tilde{k}_n - i2\pi T_R^v(n + h_R^v).$$ \hspace{1cm} (5.31)

Similarly, one can read out the cross section from the retarded Green’s function, which is consistent with the prediction of dual CFT.

6. Extremal stretched space-like warped black hole

In this section, we consider the retarded Green’s function for the extremal stretched space-like warped black hole. The metric of the black hole is still of the form (4.2), but with $r_+ = r_0$. In this case, the right moving temperature is just zero. However, the eigenfunctions of the perturbations can not be obtained simply from the non-extremal ones by taking the limit $r_+ = r_0$ since the variable $z$ defined in (4.12) becomes a constant and $A, B$ given in (4.14) are divergent. One has to solve the equations of motion anew. In fact, we will see soon that the eigenfunctions are given by Kummer functions rather than hypergeometric functions. It is very much like the null black holes which are also extremal black holes.

Certainly, the warped AdS/CFT correspondence still make sense in the extremal limit. The only thing one has to take care is that there is only non-vanishing left-moving temperature, but there are still central charges in both left-moving and right-moving sectors. This is not a surprise since the asymptotical geometry, the global warped AdS$_3$ is intact and the black hole changes only the local geometry.

6.1 Scalar correlator

Let us start from the scalar field case. Making the ansatz $\Phi = e^{-i\omega t + ik\theta} \phi$ and introducing the variable $z = \frac{\hat{a}}{r - r_0}$ with

$$\hat{a} \equiv -\frac{2i(2\omega vr_0 - \omega \sqrt{v^2 + 3r_0^2} + 2k)}{(v^2 + 3)},$$ \hspace{1cm} (6.1)

we find the equation of motion

$$\frac{d^2\phi}{dz^2} + \left(\frac{1}{4} - \tilde{m}_v^2 \frac{1}{z^2} + \frac{\kappa}{z} - \frac{1}{4}\right)\phi = 0,$$ \hspace{1cm} (6.2)
where \( \hat{m}_s = \sqrt{\frac{1}{4} + \frac{m_s^2 l^2}{v^2 + 3} - \frac{3(v^2 - 1)\omega^2}{(v^2 + 3)^2}} \) and \( \hat{\kappa} = \frac{2\kappa v}{v^2 + 3} \). It has two independent solutions

\[
\phi_\pm = e^{-\frac{1}{2}z^2 \pm \hat{m}_s} F\left(\frac{1}{2} \pm \hat{m}_s - \hat{\kappa}, 1 \pm 2\hat{m}_s, z\right) \tag{6.3}
\]

Obviously the situation is quite similar to the scalar field in the null black hole background. The retarded Green’s function can be found in the similar way. First, we should combine the two solution into a solution with purely ingoing mode at the black hole horizon. We find

\[
\phi = \hat{A}_1 \phi_+ + \hat{A}_2 \phi_- \tag{6.4}
\]

where

\[
\hat{A}_1 = -\frac{\Gamma(1 - 2\hat{m}_s)}{\Gamma\left(\frac{1}{2} - \hat{m}_s - \hat{\kappa}\right)} \hat{A}, \quad \hat{A}_2 = \frac{\Gamma(1 + 2\hat{m}_s)}{\Gamma\left(\frac{1}{2} + \hat{m}_s - \hat{\kappa}\right)} \hat{A}, \tag{6.5}
\]

with \( \hat{A} \) being a constant. The conjugate momentum is

\[
\Pi = \frac{1}{2} l(v^2 + 3) \hat{A} \partial_z \phi \tag{6.6}
\]

Taking into the identification (4.28), we see that the conformal weights of the scalar field of mass \( m \) are

\[
h^\pm_R = \frac{1}{2} \pm \hat{m}_s. \tag{6.7}
\]

Certainly, it is exactly the same as the ones in the non-extremal case. If we choose \( s_s > 0 \) and \( h^+_R > 1 \), then we get real-time retarded correlator

\[
G_R \propto \frac{\hat{A}_1}{\hat{A}_2}. \tag{6.8}
\]

If we have \( s_s < 0 \), then we can choose either \( \hat{A}_1 \) or \( \hat{A}_2 \) as source, and we find that

\[
G_R \propto \begin{cases} 
\frac{\hat{A}_1}{\hat{A}_2}, & \text{taking } \hat{A}_2 \text{ as the source} \\
\frac{\hat{A}_2}{\hat{A}_1}, & \text{taking } \hat{A}_1 \text{ as the source}.
\end{cases} \tag{6.9}
\]

In either case, the Green’s function is consistent with the prediction (3.6).

Let us focus on the case with the conformal weight \( h^+ > 1 \), then

\[
G_R \propto \frac{A_1}{A_2} = \frac{\Gamma(2 - 2h)}{\Gamma(2h)} \frac{\Gamma(h^+ - i\tilde{k})}{\Gamma(1 - h^+ - i\tilde{k})}. \tag{6.10}
\]

Note that since \( \tilde{k} = \frac{\omega l}{2\pi T} \), the correlator is actually well consistent with (3.6).

It is interesting to read out the quasi-normal modes. In either case, they are characterized by the relation

\[
\tilde{k} = -i(n + h_L), \tag{6.11}
\]

with \( n \) being a non-negative integer and \( h_L = h_R \). This relation is exactly the same as the one (4.33) in the left-moving sector of the non-extremal spacelike stretched black hole. As we expected, there is no quasi-normal modes in the right-moving sector.
The cross section is
\[ \sigma = ImG_R \sim \frac{\Gamma(2-2h)|\Gamma(h-i\kappa)|^2}{\Gamma(2h)} \cos(h\pi) \sinh(\kappa\pi). \] (6.12)

It is proportional to \( \frac{\omega A_H}{2\pi} \) in the low energy limit for the s wave, where \( A_H = \pi l(2v - \sqrt{v^2 + 3}) \) is the area of the horizon.

### 6.2 Vector correlator

Now let us move to the correlators of the vector fields. Taken the ansatz
\[ A_\mu = e^{-i\omega t + ik\theta} \phi_\mu, \] (6.13)
then the equations of motion can be given explicitly
\[
\frac{d\phi_t}{dr} = 2D(r) \left( \frac{-\omega k}{ml} + mlM(r) \right) \phi_t - \left( \frac{\omega^2}{ml} + ml \right) \phi_\theta,
\] (6.14)
\[
\frac{d\phi_\theta}{dr} = 2D(r) \left( \frac{k^2}{ml} + mlN(r) \right) \phi_t - \left( \frac{-\omega k}{ml} + mlM(r) \right) \phi_\theta,
\] (6.15)
\[
\phi_r = - \frac{2D(r)}{ml} (ik\phi_t + i\omega \phi_\theta).
\] (6.16)

The two independent solutions of \( \phi_t \) are
\[ \phi_{t\pm} = e^{-\frac{\pi}{2} z} \frac{1}{2} \pm \hat{m}_v F \left( \frac{1}{2} \pm \hat{m}_v - \kappa, 1 \pm 2\hat{m}_v, z \right), \] (6.17)
where \( \hat{m}_v = \sqrt{\frac{1}{4} + \frac{m^2 l^2 + 2mlv}{(v^2 + 3)^2} - \frac{3(v^2 - 1)\omega^2}{(v^2 + 3)^2}} \) and \( \kappa, z \) are the same as the ones in the scalar field case. Now we should also combine these two solutions into a solution with purely ingoing boundary condition at the black hole horizon
\[ \phi = \hat{C}_1 \phi_+ + \hat{C}_2 \phi_- \] (6.18)
where
\[ \hat{C}_1 = \frac{\Gamma(1 - 2\hat{m}_v)}{\Gamma \left( \frac{1}{2} - \hat{m}_v - \kappa \right) \hat{C}_0}, \quad \hat{C}_2 = \frac{\Gamma(1 + 2\hat{m}_v)}{\Gamma \left( \frac{1}{2} + \hat{m}_v - \kappa \right) \hat{C}_0}, \] (6.19)
with \( \hat{C}_0 \) being a constant.

\( \phi_\theta, \phi_r \) can be obtained directly from the equations (6.14)(6.16). If \( \omega^2 + m^2 l^2 \neq 0 \),
\[ \phi_\theta = \hat{A} \phi_t + \hat{B} \frac{1}{z} \phi_t + \hat{C} \partial_z \phi_t, \] (6.20)
where
\[
\hat{A} = \frac{m^2 l^2 (2v - \sqrt{v^2 + 3})r_0 - 2wk}{2\omega^2 + 2ml^2},
\] (6.21)
\[
\hat{B} = \frac{m^2 l^2 v\hat{a}}{2\omega^2 + 2ml^2}, \quad \hat{C} = \frac{ml(v^2 + 3)\hat{a}}{2\omega^2 + 2ml^2}.
\] (6.22)
The asymptotic behavior of the fields are

\[ \phi_t = \hat{C}_1 z^{\frac{1}{2} + \hat{m}_v} + \hat{C}_2 z^{\frac{1}{2} - \hat{m}_v}, \]  
(6.23)

\[ \phi_\theta = (\hat{B} + \hat{C}(\frac{1}{2} + \hat{m}_v))\hat{C}_1 z^{-\frac{1}{2} + \hat{m}_v} + (\hat{B} + \hat{C}(\frac{1}{2} - \hat{m}_v))\hat{C}_2 z^{-\frac{1}{2} - \hat{m}_v}. \]  
(6.24)

So if the dominant term in \( \phi_t \) is the source, then the retarded Green’s function describing the response is given by

\[ G_{tt} = \frac{m(\hat{B} - \hat{C}(\frac{1}{2} + \hat{m}_v))\hat{C}_1}{\hat{C}_2} \]  
(6.25)

If the \( \phi_\theta \) is the source, the corresponding retarded Green’s function is

\[ G_{\theta\theta} = -\frac{m\hat{C}_1}{(\hat{B} + \hat{C}(\frac{1}{2} - \hat{m}_v))\hat{C}_2} \]  
(6.26)

Up to a normalization factor, we have in both cases

\[ G_R \sim \frac{\hat{C}_1}{\hat{C}_2} = N'_v \frac{\Gamma(1 - 2\hat{m}_v)}{\Gamma(1 + 2\hat{m}_v)} |\Gamma(\frac{1}{2} + \hat{m}_v - \tilde{\kappa})|^2, \]  
(6.27)

with

\[ N'_v = \cosh(i\pi \tilde{\kappa}) \sin \pi(\frac{1}{2} + \hat{m}_v) - i \sinh(i\pi \tilde{\kappa}) \cos \pi(\frac{1}{2} + \hat{m}_v). \]  
(6.28)

The conformal weight of the massive vector field is just \( h^v = \frac{1}{2} + \hat{m}_v \), where the identification \( (5.12) \) has to be taken into account in \( \hat{m}_v \). It is easy to see that the correlator is consistent with the CFT prediction (3.6).

The quasi-normal modes could be read easily and are characterized by exactly the same relation (6.11).

The cross section is just

\[ \sigma \sim \frac{\Gamma(2 - 2h^v)}{\pi \Gamma(2h^v)} |\Gamma(h^v - i\tilde{k})|^2 \cos(h^v \pi) \sinh(i\pi \tilde{k}), \]  
(6.29)

which is very similar to the one of the scalar field.

In short, for the extremal spacelike warped AdS\(_3\) black hole, the retarded Green’s function is well consistent with (3.6). Since the right-moving temperature is zero, it behaves as a “chiral” theory, similar to the null warped black hole.

7. Conclusions and discussions

In this paper, we computed real-time correlators of the scalar and vector operators from the warped AdS/CFT correspondence. We discussed both the spacelike stretched and the null warped case. In these cases, we can solve the equations of motion and obtain the correlators analytically such that we can compare in a precise way the results got from gravity calculation with the CFT prediction which are much restricted by 2D conformal invariance. We found that the retarded correlators were in good match with the prediction.
of dual CFT, up to a renormalization factor. This strongly supports the warped AdS/CFT correspondence.

From the retarded correlators, we read out the scalar and vector quasi-normal modes in the warped black hole background. They correspond to the poles of the retarded Green’s correlators in the momentum space. The results are the same as the ones found in [13]. Moreover, we obtained the cross section of the scalar and vector field scattering the black holes from the imaginary part of the retarded real-time correlators. Once again, the cross sections are well consistent with the prediction of dual CFT.

In our study, we noticed that the extremal spacelike warped black hole behaves quite similarly to the null warped black hole. In both cases, there exist only one temperature in dual CFT, which seems suggest that the dual CFT is “chiral”. However, this differs from the chiral gravity [24] in which case there does exist only one sector. We know that from the warped AdS/CFT correspondence on the spacelike case, the dual CFT actually has two independent sectors. The absence of one sector in our study comes from the extremality of the black hole. This indicates that in the null warped case, there may exist two sectors with nonvanishing central charges, in support of the diffeomorphism anomaly argument.

For the global warped AdS$_3$ backgrounds, the presence of the angular momentum quantum number $\tilde{k}$ has very interesting physical implications. Firstly it appears in the conformal weight of various kinds of the operators. It may make the conformal weight not real, in which case there exist superradiance in the black hole backgrounds. It is remarkable that even in this case, the warped AdS/CFT correspondence still make sense. This is very similar to the situation in Kerr/CFT correspondence [25, 26]. Another interesting implication of the angular momentum is that it induce a charge with respect to a chemical potential in the right-moving sector of dual CFT. This is important to set up the correspondence.

It would be interesting to compare our study on the warped AdS black hole with Kerr/CFT correspondence [23, 25, 26]. In the latter case, one cannot read the real-time correlators directly. Instead one can compute the cross section of the scattering and compare with the CFT prediction. This is feasible since the asympotical geometry of Kerr black hole is flat and there is no ambiguity in defining the ingoing and outcoming waves. In practice, one has to divide the spacetime into several regions, with one of them being the near horizon region, and study the perturbations in different regions and try to glue them together. Since the Kerr/CFT correspondence is actually the duality between the quantum gravity on near horizon geometry of (near-)extremal Kerr black hole (NHEK) with a 2D CFT, one has to figure out the contribution at the NHEK region and compare them to CFT. While for the warped AdS spacetime, it is hard to well define the ingoing and outcoming waves to get the absorption cross section, as shown in the study of [28, 29]. Nevertheless, one may bypass this obstacle by calculating directly the real-time correlators from the prescription developed in usual AdS/CFT correspondence. One can compare the Green’s functions on both sides. Furthermore, one may read the cross section from the real-time correlator easily, and find the agreement with CFT prediction. Considering the fact that the near horizon geometry of extremal Kerr black hole is a warped AdS spacetime, one would not be surprised to find the similarity in the dictionaries of two correspondences.
From our study, we have shown that both in real-time correlators and absorption cross sections, the nontrivial dependence on Gamma functions are in perfect match in warped AdS/CFT correspondence. However, there is a discrepancy on normalization factor. From CFT prediction, such a normalization factor should have nontrivial temperature dependence, which is closely related to the conformal weights of the operators. Our calculation failed to recover such factors. It would be interesting to investigate this issue further.

On the other hand, our investigation also suggests that the prescription to get real-time correlator is still effective even for the warped AdS spacetime which has a nontrivial conformal boundary, even though it was developed in the usual AdS/CFT correspondence. This may have profound implications in the recent study of AdS/CMT correspondence. In fact, the null warped backgrounds appear in the study of non-relativistic AdS/CFT of cold atoms\cite{30, 31, 32, 33, 34} and also as the gravity dual of a Lifshitz field theory with anisotropic scaling\cite{35, 36, 37}. Our result gives strong support to apply the prescription in these areas.

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