Spin-Orbit-Torque Field-Effect Transistor (SOTFET): A New Magnetoelectric Memory

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Spin-based memories are attractive for their non-volatility and high durability but provide modest resistance changes, whereas semiconductor logic transistors are capable of large resistance changes, but lack memory function with high durability. The recent availability of multiferroic materials provides an opportunity to directly couple the change in spin states of a magnetic memory to a charge change in a semiconductor transistor. In this work, we propose and analyze the spin-orbit torque field-effect transistor (SOTFET), a device with the potential to significantly boost the energy efficiency of spin-based memories, and to simultaneously offer a palette of new functionalities.

Introduction - The understanding of transport of electron spin in heterostructures [1] led to the realization of magnetic memories based on giant magnetoresistence (GMR) [2–4] and spin-transfer torque (STT) [5–8]. Current research aims to make the writing process for magnetic memories more efficient using spin-orbit torques (SOTs) [9, 10]. STT and SOT magnetic random access memories (MRAMs) offer the virtues of non-volatility, infinite endurance, and good write speeds [8]. Nonetheless, the change in resistance between the magnetic ‘0’ and ‘1’ states of STT- and SOT-MRAMs is modest (<600% at room temperature [11]). This necessitates a substantial current to obtain acceptable readout voltages, impairing read energies and read times, and also limits the types of circuit architectures into which MRAM devices can be incorporated for logic or search functions.

In contrast, non-magnetic semiconductor field-effect transistors (FETs) achieve many orders of magnitude change in resistance in each switching event. The field effect converts a linear change in the voltage $V_g$ on the gate metal into an exponential change in the mobile carrier density $n \sim \exp(qV_g/k_BT)$ in the band of the semiconductor, and consequently modulates its resistance. Here $k_B$ is the Boltzmann constant, and $T$ the temperature. Thus, a material that can transduce the change in the spin/magnetic state in a SOT structure into the charge of a semiconductor channel could significantly boost the change in resistance of a magnetic memory.

This requirement can be met by recently developed magnetoelectric multiferroic materials, which simultaneously possess magnetic order and ferroelectricity in a manner that these order parameters are coupled: changing one also changes the other due to the magnetoelectric effect [12–14]. Exchange coupling of spins in a ferromagnetic layer to the magnetic order of a multiferroic layer across ferromagnet/multiferroic heterointerfaces has been experimentally demonstrated [15, 16].

Inspired by these recent advances in SOT and multiferroic materials, we propose a new magnetoelectric memory device, the spin-orbit-torque field-effect transistor (SOTFET). This device aims to combine the virtues of magnetic memories with the large resistance change of FETs, providing both memory and logic functionalities. Analysis of the memory aspect indicates that the SOTFET can offer orders of magnitude increase in the on-off resistance ratio compared to existing magnetic memories, which can potentially lower the operation energy significantly. The logic aspect of the SOTFET also enables new circuit architectures for efficient logic or search functions [17]. In this paper, we present the physical operation of the SOTFET along with a device model established with BiFeO$_3$ as the magnetoelectric multiferroic layer, and will mainly focus on the memory aspect.

Device structure - Figure 1 shows the structure of a SOTFET. It resembles an ordinary metal-oxide-semiconductor FET (MOSFET), but with a unique gate stack. The SOTFET gate stack comprises three layers (from top to bottom): a spin-orbit (SO) layer, a ferromagnetic (FM) layer, and a multiferroic (MF) layer, adjacent to a semiconductor channel to which source and drain contacts are made. By flowing current in the SO layer, we desire to gate the semiconductor channel by switching the magnetization and electric polarization in the gate stack.

The working principle of the SOTFET is illustrated in Fig. 1. The state of magnetization $\mathbf{M}$ of the FM layer is the memory component. When a charge current $J_{SO}$ flows in the SO layer, transverse spin-polarized currents are generated due to spin-momentum locking [18–23]. Therefore, spins of opposite orientation accumulate at the surfaces of the SO layer. Spin absorption at the SO/FM interface exerts a spin-orbit-torque that switches the magnetization $\mathbf{M}$ of the FM [8, 24, 25], as illustrated in Fig. 1(a) and qualitatively plotted in Fig. 1(b). Flow...
Stable states, \(P\), indicated in its pseudo-cubic unit cell. At thermodynamically stable states, \(P\) points to one of the \((111)\) directions. The \(P\) component determines the charge in the semiconductor channel between the source and drain by shifting the surface potential, similar to the effect in ferroelectric-gate FETs [30–33]. The current \(I_D\) flowing in the semiconductor channel switches the electric polarization \(P\), as indicated in Fig. 1(a). Coupling the magnetic dipole of the MF is also switched from ‘1’ to ‘0’, identical to the conventional writing mechanism in SOT-MRAMs.

The SOTFET differs from the conventional SOT-MRAM in the read mechanism. Coupling the \(M\) of the FM with the semiconductor channel is achieved by the magnetoelectric multiferroic layer because of its coupled electric polarization and magnetization [12–14]. Due to the exchange coupling between the FM and the MF [15, 16], the magnetic dipole of the MF is also switched with the \(M\) in the FM. Within the MF material, the Dzyaloshinskii-Moriya interaction (DMI) [26, 27] effectively couples electric and magnetic dipoles, since the weak canted magnetic moment \(M_{C}\) originates from the DMI [15, 25] [29]. When the \(M_{C}\) switches polarity, the electric polarization \(P\) in the MF switches in tandem, all in response to \(J_{SO}\), as indicated in Fig. 1(c).

The circuit symbol for the SOTFET is shown in Fig. 1(e): it is bi-stable, and provides the desired large resistance ratio for efficient readout. The circuit symbol for the SOTFET is shown in Fig. 1(e).

**Quantitative Analysis** - How realistic is the SOTFET? To answer this question, we quantitatively analyze the dynamical coupling across each interface, and across the entire device. The analysis to follow shows that the SOTFET behavior is achievable, but requires magnetoelectric multiferroics of specific magnetism and polarization, along with an appropriate hierarchy of strengths for the exchange coupling, DMI, and anisotropy forces within the gate stack. Because the FM/MF heterointerface is the least explored, instead of a generic case, we use experimental results of the CoFe/BiFeO₃ heterostructure in this initial exploration. Other materials candidates are discussed in [34]. The aim of the model is to guide experiments by pointing towards desired heterointerface choices.

The magnetization \(M\) of the FM layer is switched by spin-orbit torque (SOT). For simplicity we assume single-domain macrospin behavior. The switching dynamics of this process are captured by the Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation [6] [8] [33] [50]:

\[
\frac{d\hat{m}}{dt} = -\gamma \mu_0 \hat{m} \times H_{eff} + \alpha \hat{m} \times \frac{d\hat{m}}{dt} + \left(\frac{\gamma}{M_S}\right)\tau_{SOT},
\]

where \(\hat{m}\) is the normalized magnetization of the FM, \(H_{eff}\) is the effective magnetic field acting on \(\hat{m}\), \(\gamma\) is the electron gyromagnetic ratio, \(\mu_0\) is the vacuum permeability, \(\alpha\) is the Gilbert damping factor, \(M_S\) is the saturation magnetization and \(\tau_{SOT}=\tau_{AD}+\tau_{FL}\) is the spin-orbit torque, the sum of the anti-damping torque \(\tau_{AD}\) and field-like torque \(\tau_{FL}\), which are given by [8, 35]:

\[
\tau_{AD} = (\frac{\mu_0}{2e})(\frac{1}{2})j\theta_{AD}\hat{m} \times (\hat{m} \times \hat{m}_p), \quad (2)
\]

\[
\tau_{FL} = (\frac{\mu_0}{2e})(\frac{1}{2})j\theta_{FL}\hat{m} \times \hat{m}_p. \quad (3)
\]

Here \(h\) is the reduced Planck constant, \(e\) is electron charge, \(\tau\) is the thickness of ferromagnetic (FM) material, \(j=J_{SO}\) is the charge current density in the SO layer, \(\theta_{AD(FL)}\) is the spin Hall angle of the anti-damping (AD) or field-like (FL) torque from the SO layer, and \(\hat{m}_p\) is the normalized spin polarization. We assume a value of \(\theta_{AD}=\theta_{FL}=3.5\), as reported for the Bi₉Se₃ [24].

The effective field \(H_{eff} = H_{ext} + H_a + H_{demag} + H_{DMI}\), where \(H_{ext}\) is any external magnetic field and \(H_a\) is the anisotropy field with perpendicular magnetic anisotropy (PMA) calculated by \(H_a = \frac{2K}{\mu_0 M_p} m_z \hat{z} \equiv H_p m_z \hat{z}\) [36], where \(K\) is the anisotropy constant. \(H_{demag}\) is the demagnetization field as calculated by Beleggia et al. [37]. The last term \(H_{DMI}\) in \(H_{eff}\) is the effective magnetic field arising from the exchange coupling of the magnetic order of the MF (BiFeO₃) with that of the FM (CoFe), and from the DMI within the MF BiFeO₃. We discuss this term further below.

Switching of \(M\) in the FM switches the electric polarization \(P\) of the MF due to the exchange coupling and is shown in Fig. 1(d): it is bi-stable, and provides the desired large resistance ratio for efficient readout. The circuit symbol for the SOTFET is shown in Fig. 1(e).
The dynamics of \( \mathbf{P} \) are captured by the Landau-Khalatnikov (LK) equation [33, 35, 39]:

\[
\gamma_{FE} \frac{\partial \mathbf{P}}{\partial t} = -\frac{\partial \mathbf{F}}{\partial \mathbf{P}},
\]

where \( \gamma_{FE} \) is the viscosity coefficient, \( P_i (i = x, y, z) \) is the \( x/y/z \) component of \( \mathbf{P} \). \( \mathbf{F} \) is the total ferroelectric free energy [39, 40]:

\[
F(\mathbf{P}, \mathbf{u}) = \alpha_1 (P_x^2 + P_y^2 + P_z^2) + \alpha_{11} (P_x^4 + P_y^4 + P_z^4) + \alpha_{12} (P_x P_y^2 P_z + P_x^2 P_y P_z + P_x P_y P_z^2) + K_{strain}(\mathbf{P} \cdot \mathbf{u})^2 - \mathbf{P} \cdot (\mathbf{F}_{ext} + \mathbf{F}_{DMI}),
\]

(5)

where \( \alpha_1, \alpha_{11}, \alpha_{12} \) are the phenomenological Landau expansion coefficients, \( K_{strain} \) is the strain energy. \( \mathbf{u} \) is the axis of substrate strain, \( \mathbf{F}_{ext} \) is the external electric field and \( \mathbf{F}_{DMI} \) is the electric field from DMI. The strain term \( K_{strain}(\mathbf{P} \cdot \mathbf{u})^2 \), in Eq. (5) arises from the substrate-induced strain [39, 40], which dictates the energy-favorable planes for the lowest-energy (equilibrium) states of \( \mathbf{P} \), thereby reducing the degeneracy of \( \mathbf{P} \) orientations in the specific case of the MF BiFeO\(_3\). This phenomenon is also shown in previous studies [15, 41, 42] of this particular multiferroic.

In our model, the exchange coupling, which couples \( \mathbf{M}_C \) in BiFeO\(_3\) and \( \mathbf{M} \) in CoFe, and the DMI, which couples \( \mathbf{P} \) and \( \mathbf{M}_C \) in BiFeO\(_3\), are merged into one effective DMI field that directly captures the interaction between the \( \mathbf{M} \) in CoFe and \( \mathbf{M}_C \) in BiFeO\(_3\). The magnitudes of effective magnetic field \( \mathbf{H}_{DMI} \) acting on the CoFe due to the BiFeO\(_3\) and the effective electric field \( \mathbf{F}_{DMI} \) acting on the BiFeO\(_3\) are related via an effective Hamiltonian [15, 29]:

\[
E_{DMI} = -E_{DMI,0} \mathbf{P} \cdot (\hat{\mathbf{N}} \times \mathbf{M}),
\]

(6)

where \( E_{DMI,0} \) is the energy coefficient of DMI, \( \mathbf{P} \) is the polarization of BiFeO\(_3\), \( \hat{\mathbf{N}} \) is the Neel vector, and \( \mathbf{M} \) is the magnetic moment in CoFe. All vectors in the equation are normalized vectors. The effective fields that enter into the equations of motion are then:

\[
\mathbf{H}_{DMI} = -\frac{1}{\mu_0 M_S} \frac{\partial E_{DMI}}{\partial \mathbf{M}} = H_{DMI,0}(\mathbf{P} \times \hat{\mathbf{N}}),
\]

(7)

and

\[
\mathbf{F}_{DMI} = -\frac{1}{P_S} \frac{\partial E_{DMI}}{\partial \mathbf{P}} = F_{DMI,0}(\hat{\mathbf{N}} \times \mathbf{M}),
\]

(8)

where \( H_{DMI,0} \) is the effective DMI magnetic field magnitude and \( F_{DMI,0} \) is the effective DMI electric field magnitude. Both fields have constant magnitudes for specific material combinations, because they originate from the energy and material parameters:

\[
E_{DMI,0} = \mu_0 M_S; H_{DMI,0} = P_S; F_{DMI,0}.
\]

The ratio of \( H_{DMI,0} \) and \( F_{DMI,0} \) is also determined by the material properties, defined here as the DMI transconductance \( \sigma_{DMI} \):

\[
\sigma_{DMI} \equiv \frac{H_{DMI,0}}{F_{DMI,0}} = \frac{P_S}{\mu_0 M_S},
\]

(9)

similar in spirit to the converse magnetoelectric coefficient [15]. It will be seen shortly that \( E_{DMI,0} \) and \( \sigma_{DMI} \) are critical parameters that dictate the feasibility of achieving SOTFET action.

![FIG. 2. Modeling procedure flow of the SOTFET model.](image)

With the direction of \( \mathbf{N} \) defined as \( \hat{\mathbf{N}} = -\mathbf{P} \times \mathbf{M} \), all vectors in the CoFe/BiFeO\(_3\) FM/MF system (\( \mathbf{P}, \mathbf{N} \) and \( \mathbf{M} \)) are connected by the DMI. The dynamic evolution of the \( \mathbf{M} \) and \( \mathbf{P} \) vectors is therefore governed by the above set of equations. The method of implementing this dynamic evolution is shown schematically in Fig. 2. The initial state of the SOTFET is defined by a set of vectors: \( \mathbf{M} \) in the FM, and \( \mathbf{M}_C \), \( \mathbf{P} \), and \( \mathbf{N} \) in the MF. When a current \( J_{SO} \) flows in the SO layer, all 4 vectors (\( \mathbf{M}, \mathbf{M}_C, \mathbf{P}, \) and \( \mathbf{N} \)) can switch to new states, with dynamics dictated by the LLGS and the LK equations in each loop. Finally, a new set of the 4 vectors in a new equilibrium state will be reached by iteration. The switching behavior of \( \mathbf{P} \) and \( \mathbf{M} \) is assumed to be purely rotational, with no change in their magnitudes, consistent with experimental studies of BiFeO\(_3\) [15].

**Results and discussion** - Key parameters used in the numerical evaluation of the SOTFET are provided in Appendix A. The model is validated by comparing to the micromagnetic simulation tools OOMMF [43] and MuMax3 [44], and other theoretical calculations and experimental results; this information is shown in Appendix B. For the SOTFET gate stack to controllably gate the semiconductor channel, a deterministic switching of the polarization \( \mathbf{P} \) in the MF in \( z \)-direction is needed.

For the CoFe/BiFeO\(_3\) FM/MF heterostructure, we assume that the DMI transconductance \( \sigma_{DMI} \) is determined by the saturation polarization \( P_S \) of BiFeO\(_3\) and saturation magnetization \( M_S \) of CoFe. Taking \( P_S = 100 \, \mu C/cm^2 \) of BiFeO\(_3\) [45] and \( M_S = 1.6 \times 10^6 \, A/m \) of CoFe [46], switching behavior for a range of DMI energies is shown in Fig. 3(a). Upon applying a current...
$$J_{SO} = -30 \text{ MA/cm}^2$$
different switching behavior of the $x$-component of the magnetization ($M_z$) is observed for different DMI energies. For these values, however, the $z$-component of the polarization $P_z$ in the MF layer does not follow the motions of $M$. This is because, given the large $P_S$, a moderate DMI energy is not sufficient to overcome the anisotropy energy in $P$ to switch it. For a high DMI energy, with $P$ held in place, $M$ also does not switch because the DMI field $H_{DMI}$ then functions as an effective unidirectional anisotropy acting back on $M$. This is therefore a situation when the SOTFET does not achieve the desired functionality of switching $P_z$ to gate the transistor.

The natural next step is to explore reduced $P_S$ in the MF layer. Reducing $P_S$ in BiFeO$_3$ is experimentally feasible, for example, by La-substitution of BiFeO$_3$ [40, 41]. Qualitatively, this implies that the multiferroic layer should have a relatively weak ferroelectricity, a strong magnetization, and strong coupling between the two order parameters. The calculated results with a reduced $P_S=10 \mu C/cm^2$ and other parameters unchanged are shown in Fig. 3(b) for a range of $E_{DMI,0}$. For the same current $J_{SO}$, a critical $E_{DMI,0}$ is observed. Above the critical $E_{DMI,0}$, $M_z$ and $P_z$ concomitantly switch, signalling the required materials parameters for successful SOTFET operation.

Reducing $P_S$ of the BiFeO$_3$ helps the switching of $P$ successfully for two reasons. First, as shown in Eq. 5 for a fixed $H_{DMI,0}$ and $M$, lowering $P_S$ for the same $E_{DMI,0}$ implies an enhanced $F_{DMI,0}$ to switch the polarization. Second, a reduced $P_S$ leads to a weaker polarization anisotropy as described in the free energy equation Eq. 5. This lowers the energy barrier between polarization equilibrium states, making it easier to switch.

Figures 3(c) and 3(d) show that the desired stable switching behavior of the SOTFET is achieved by choosing the CoFe/BiFeO$_3$ heterostructure with a reduced $P_S=10 \mu C/cm^2$ of the MF (BiFeO$_3$) and an above-critical $E_{DMI,0}=0.8 \text{ pJ/} \mu \text{m}^2$, which corresponds to DMI fields of $H_{DMI,0}=5 \text{ kOe}$ and $F_{DMI,0}=80 \text{ kV/cm}$. It is seen that switching the current direction in the SO layer successfully switches the direction of $P_z$. The current density used, 30 MA/cm$^2$, is about 1 order of magnitude lower than heavy-metal based SOT-MRAMs [48, 49] due to the assumed large spin Hall angle of Bi$_2$Se$_3$, and can be further reduced by using larger spin Hall angle materials such as BiSb [50]. $M$ is observed to switch within the $x$-$y$ plane and $P$ is switched out-of-plane. The switching trajectories of $P$ are shown in the spherical plot in Fig. 3(d). Clear set and reset processes between State 0 and 1 are observed, proving feasibility of the SOTFET operation for the chosen material parameters.

The switching of $P$ with $P_S=10 \mu C/cm^2$ shown in Figs. 3(c) and 3(d) results in a charge difference $\Delta Q = 2F_z \approx 12 \mu C/cm^2$ in the semiconductor channel, assuming the absence of traps at the interface between the MF
and the semiconductor channel. For example, for the choice of a silicon channel, this will lead to a surface potential change approximately at $\Delta \psi \approx 1.3$ V by a simple calculation [51], accessing the entire operating regime of a MOSFET from strong inversion to accumulation. Thus, by estimation, at least an on/off ratio of $10^8$ in $I_D$ can be achieved due to the resistance change of the channel, which in practice will be limited by gate leakage and interfacial trap states rather than the intrinsic capability of a SOTFET. The high on/off ratio in $I_D$ as the read-out component brings the read energy of a SOTFET down to the same level as a conventional semiconductor transistor. The choice of the semiconductor channel will be determined by the quality of the integrated material stack.

An electrically insulating magnetic (FM) layer is more desirable for SOTFET application in order to reduce the shunting current from SO layer, and boost the spin torque efficiency [52]. Besides, the insulating FM layer should potentially reduce the charge injection in the MF layer, thus alleviating the fatigue that is often confronted by ferroelectric materials. The fatigue issue is also addressed by the fact that the polarization switching is driven by coupling to the magnetic layer rather than an external electric field, which should reduce the tendency for long-distance atom motion.

To highlight the difference between a SOTFET and a magnetoelectric (ME) device, where an external electric field is applied to switch $P$ that subsequently switches $M$, we refer to the $\sigma_{DMI}$ introduced in Eq. 10. For a given $E_{DMI,B}$, a high $\sigma_{DMI}$ is desired for a ME device (to maximize $H_{DMI}$ for eventually switching $M$) while a low $\sigma_{DMI}$ is necessary for a SOTFET. That is because in a SOTFET, the switching energy for $P$ should be the lowest, which is also locked by the switching of $M$. Since $P$ cannot switch unless $M$ is switched by $J_{SO}$, a moderate depolarization field will not unintentionally flip $P$.

In addition, by the virtue of simultaneously being a FET, the SOTFET also provides logic functionality by a gate voltage controlling the channel. As a merger of memory and logic, the SOTFET is capable of process-in-memory (PiM) functionalities that significantly lower the energy consumption and physical size of computation, comparing to a von Neumann architecture where logic and memory are separated. Some examples are explored in [17].

The experimental realization, and various modes of operation of the SOTFET are currently being investigated.

**Conclusions** - The SOTFET, a new magnetoelectric memory device is proposed, in which a change in magnetization of a SO/FM layer is transduced to control the resistance of a semiconductor channel by using a magnetoelectric multiferroic layer. The switching of the semiconductor channel provides a readout with several orders of magnitude change in resistance, much larger than in conventional magnetic tunnel junctions. We establish a quantitative model of the dynamics of the magnetization and polarization of the layers of the SOTFET. From the model, the materials needs for the successful operation of the device are identified, via a strong dependence on the DMI transconductance parameter.

In a properly designed CoFe/BiFeO$_3$ gate stack, we predict that the SOTFET can achieve fast switching within 5 ns, with deterministic out-of-plane switching of the polarization $P_z$ at a current density of $J_{SO}$ =30 MA/cm$^2$. Using $I_D$ as the read-out component, a >$10^8$ on/off ratio is achieved due to the resistance change of the semiconductor channel. This suggests that the SOTFET could be feasible as a low-power memory, while the embedded logic offers a palette of functionalities in circuits design and PiM architecture.

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**Appendix A: Key parameters in the model**

Key parameters used in the LLGS equation to describe $M$ dynamics are listed in Table I.

| Parameter                  | Symbol | Value | Unit |
|----------------------------|--------|-------|------|
| Electron gyromagnetic ratio| $\gamma$ | $1.76\times10^4$ | s$^{-1}$T$^{-1}$ |
| Spin Hall angle            | $\theta_{AD(FL)}$ | 3.5 | [24] |
| Gilbert damping factor     | $\alpha$ | 0.01 | [53] |
| Saturation magnetization   | $M_s$ | $1.6\times10^6$ | A/m |
| Anisotropy constant        | $K$ | $1.5\times10^4$ | J/m$^3$ |
| External field             | $H_{ext}$ | 0 | A/m |
| Ferromagnetic thickness    | $t$ | 3 | nm |
| Device length/width        | $L_x/L_y$ | 30 | 30 nm |

Key parameters used in the LK equation to describe $P$ dynamics are shown in Table II.

| Parameter | Symbol | Value | Unit |
|-----------|--------|-------|------|
| Viscosity coefficient | $\gamma_{FE}$ | $5\times10^{-3}$ | m$^3$/F |
| Landau coefficients | $\alpha_1$ | $-4\times10^8$ | C$^{-2}$m$^2$N |
| | $\alpha_{11}$ | $6.5\times10^8$ | C$^{-2}$m$^6$N |
| | $\alpha_{12}$ | $1\times10^8$ | C$^{-2}$m$^6$N |
| Strain energy | $K_{strain}$ | $6 \times 10^6$ | J/m$^3$ |
| Strain axis | $u$ | $[0, 1, 1]$ | - |

**Appendix B: Validation of the model**

For the magnetic dynamics described by the LLGS equation, comparisons with existing magnetic simulation tools such as OOMMF [43] and MuMax3 [44] are used to validate the model developed in this work. The responses of the ferromagnetic layer to spin-orbit torques that we
calculate match well with the results from OOMMF and MuMax3, as shown by selected results in Fig. 4(a). Our results also agree with switching behavior calculated analytically. 

For the ferroelectric dynamics described by the LK equation, we performed test simulations as a function of applied electric field for the BiFeO$_3$ material system. With the inclusion of a depolarization term to model the effect of domain walls, we find two-step P switching in agreement with previous theoretical and experimental works in [15], with the trajectory of P switching shown in Fig. 4(b).

![FIG. 4. Validation of the SOTFET model. (a) The $xyz$ components versus time plot when $M$ of FM is switched by spin-orbit torque from the SO layer. Results from MuMax3 are shown by the dots and results from this model are shown by the line. (b) A two-step switching trajectory of P in BiFeO$_3$ with an applied electric field in the $-z$ direction. Red dots represent the 8 stable states of P.](image)

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