Hawking Radiation from Black Holes Formed During Quantum Tunneling

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Abstract

We study the behaviour of scalar fields on background geometries which undergo quantum tunneling. The two examples considered are a moving mirror in flat space which tunnels through a potential barrier, and a false vacuum bubble which tunnels to form a black hole. WKB approximations to the Schrödinger and Wheeler-DeWitt equations are made, leading one to solve field equations on the Euclidean metric solution interpolating between the classically allowed geometries. The state of the field after tunneling can then be determined using the method of non-unitary Bogolubov transformations developed by Rubakov. It is shown that the effect of the tunneling is to damp any excitations initially present, and, in the case of the black hole, that the behaviour of fields on the Euclidean Kruskal manifold ensures that the late time radiation will be thermal at the Hawking temperature.
1. Introduction

The radiation of particles from matter evolving along a classical trajectory has been heavily studied in recent years. Less well studied is the radiation accompanying quantum tunneling from one classically allowed trajectory to another. The following question is of interest: if a matter system impinges upon a potential barrier with a radiation field in a certain state, what is the state of the field given that the matter is subsequently observed to be on the other side of the barrier? A method to answer this question in the context of false vacuum decay in flat space was developed by Rubakov [1] and has been generalized to include gravity as well as topology changing processes [2, 3]. The spectrum of radiation is found by solving an imaginary time Schrödinger equation, the occurrence of which leads to novel features. Instead of solving field equations in real time, one is naturally led to consider propagation on the Euclidean solution interpolating between the two classical trajectories. As phase factors in real time are converted into exponential damping factors in imaginary time, the resulting particle creation can be distinctly different and is accompanied by the systematic suppression of excitations present before tunneling.

Given this situation, it is natural to ask how the radiation from black holes might be affected by the presence of tunneling. If we consider a distribution of matter, initially outside its Schwarzschild radius, which tunnels through a potential barrier to form a black hole, the conventional calculation [4] of the radiation does not apply. On the other hand, it would be shocking if the same answer was not obtained for the radiation at late times, as this is thought to depend only on the hole’s late time geometry and not on its history at early times. Here we compute the radiation for this process and show that while the Euclidean time evolution has an effect at early times, it has none at late times so that the standard result is in fact obtained.

In order illustrate the technique of Ref. [1] in a simpler setting, we first study the effect of tunneling on another well known radiating system — the moving mirror [7]. We show in Sect. (2) how an imaginary time Schrödinger equation emerges from a Born-Oppenheimer approximation, and use this result to calculate the shift in the spectrum of radiated particles as a result of the tunneling. It is shown that the initial spectrum is shifted to favor low energy excitations, as is understood by realizing that the probability to tunnel is increased if energy is transferred from the radiation to the mirror.

In Sect. (3) this approach is extended to include gravity in asymptotically flat space. A WKB approximation to the Wheeler-DeWitt equation, as considered in [3, 4], is used
to obtain an imaginary time Schrödinger equation which can then be solved as before. In Sect. (4) we use this result to examine the radiation from a black hole which is formed by tunneling. In particular, we consider the tunneling of a false vacuum bubble, a system extensively studied in Refs. [8] — [12]. This example involves a complication due to the peculiar structure that arises; Refs. [13, 14] show that the sequence of three-geometries encountered during tunneling can not be stacked together to form a manifold. Employing a slight modification of the standard approach, we show how the behaviour of fields on the Euclidean Schwarzschild manifold protects the late time radiation from being affected by tunneling. An intuitive reason for this is that the bubble’s tunneling probability is unchanged by the presence of Hawking radiation, which involves the creation of pairs of particles with zero total energy.

2. Tunneling Mirror

Consider a mirror moving in a one dimensional potential in the presence of a massless scalar field. The Schrödinger equation for this system is

\[ [\hat{H}_m + \hat{H}_\phi]\Psi[\phi, x_m; t] = i \frac{\partial}{\partial t} \Psi[\phi, x_m; t] \]  

(2.1)

where

\[ \hat{H}_m = -\frac{1}{2m} \frac{\partial^2}{\partial x_m^2} + V(x_m) \]  

(2.2)

and

\[ \hat{H}_\phi = \frac{1}{2} \int_{x_m}^{\infty} dx \left[ -\frac{\delta^2}{\delta \phi(x)^2} + \left( \frac{d \phi}{dx} \right)^2 \right]. \]  

(2.3)

Note that \( \Psi \) is a function of the mirror coordinate \( x_m \), and a functional of the field configuration \( \phi(x) \). The mirror boundary condition is imposed by demanding that the field vanish at \( x_m \),

\[ \Psi[\phi, x_m; t] = 0 \quad \text{if} \quad \phi(x_m) \neq 0. \]  

(2.4)

The system is solved by assuming that the backreaction of the field on the mirror is a small perturbation of the mirror’s motion, and that the mass and momenta of the mirror are large enough that it can be described by a well localized wave packet. In this domain
the system admits a Born-Oppenheimer approximation, which amounts to an expansion in \(1/m\). In particular, we seek a solution to the time independent Schrödinger equation

\[
\hat{H}_m + \hat{H}_\phi \Psi[\phi, x_m] = E \Psi[\phi, x_m]
\]  

valid to zeroth order in \(1/m\). Following Refs. [1, 6] the Born-Oppenheimer approximation is implemented by writing \(\Psi\) in the form

\[
\Psi[\phi, x_m] = \psi_{VV}(x_m) e^{iS(x_m)} \chi[\phi, x_m]
\]

where \(\psi_{VV}\) is a slowly varying function to be identified with the Van Vleck determinant. To lowest order in \(1/m\), (2.5) reduces to the Hamilton-Jacobi equation.

\[
\frac{1}{2m} \left( \frac{dS}{dx_m} \right)^2 + V(x_m) = E
\]

since \(dS/dx_m, V(x_m)\) and \(E\) are all of order \(m\).

To zeroth order:

\[
- \frac{i}{2m} \frac{d^2 S}{dx_m^2} \psi_{VV} \chi[\phi, x_m] - \frac{i}{m} \frac{dS}{dm} \frac{d\psi_{VV}}{dx_m} \chi[\phi, x_m]
\]

\[- \frac{i}{m} \psi_{VV} \frac{dS}{dx_m} \frac{\partial}{\partial x_m} \chi[\phi, x_m] + \psi_{VV} \hat{H}_\phi \chi[\phi, x_m] = 0.
\]

\(\psi_{VV}\) is chosen so that the first two terms cancel, leaving

\[
\hat{H}_\phi \chi[\phi, x_m] = \frac{i}{m} \frac{dS}{dx_m} \frac{\partial}{\partial x_m} \chi[\phi, x_m].
\]

This can be put in a familiar form by defining the time variable \(\tau(x_m)\). In a classically allowed region, where \(E - V(x_m) > 0\) and \(dS/dx_m\) is real, \(\tau\) is defined by

\[
\frac{d\tau}{dx_m} = \frac{m}{dS/dx_m} \quad \text{allowed regions}
\]

whereas in a classically forbidden region with \(dS/dx_m\) imaginary,

\[
\frac{d\tau_E}{dx_m} = \frac{m}{dS/dx_m} \quad \text{forbidden regions.}
\]

The resulting zeroth order equations for \(\phi\) are:

\[
\hat{H}_\phi \chi[\phi, \tau] = i \frac{\partial}{\partial \tau} \chi[\phi, \tau] \quad \text{allowed regions}
\]
\[-\hat{H}_\phi \chi[\phi, \tau_E] = \frac{\partial}{\partial \tau_E} \chi[\phi, \tau_E] \quad \text{forbidden regions.} \quad (2.13)\]

These are the fundamental equations governing the evolution of the scalar field in the presence of the mirror. In the allowed regions we have recovered the time-dependent Schrödinger equation with the position of the mirror playing the role of a clock, whereas in the forbidden regions we have obtained a diffusion equation, which we shall refer to as the Euclidean Schrödinger equation, with the Euclidean time \( \tau_E \) measuring the position of the mirror in the potential barrier.

Now, choose the potential to be of the form illustrated in Fig. 1 and let the mirror come from right to left. In the allowed region to the right of \( x_m^i \) the state \( \chi[\phi, \tau] \) obeys the normal Schrödinger equation, and so standard methods can be used to find \( \chi[\phi, \tau^i] \). Between \( x_m^i \) and \( x_m^f \) the mirror is in a forbidden region, so the state evolves according to

\[-\frac{1}{2} \int_{x_m(\tau_E)}^{\infty} dx \left[ -\frac{\delta^2}{\delta \phi(x)^2} + \left( \frac{d\phi}{dx} \right)^2 \right] \chi[\phi, \tau_E] = \frac{\partial}{\partial \tau_E} \chi[\phi, \tau_E] \quad (2.14)\]

with \( \chi[\phi, \tau_E^i] = \chi[\phi, \tau^i] \). We wish to solve this equation in order to find the state at the final turning point \( x_m^f \). It is useful to transform the mirror to rest by defining the coordinate

\[y(x, \tau_E) = x - x_m(\tau_E)\]

in terms of which the Euclidean Schrödinger equation is

\[-\frac{1}{2} \int_0^{\infty} dy \left[ -\frac{\delta^2}{\delta \phi(y)^2} + 2 \frac{dx_m}{d\tau_E} \frac{d\phi}{dy} \frac{\delta}{\delta \phi(y)} + \left( \frac{d\phi}{dy} \right)^2 \right] \chi[\phi, \tau_E] = \frac{\partial}{\partial \tau_E} \chi[\phi, \tau_E] \quad (2.16)\]

or

\[-\hat{H}_E^E(\tau_E) \chi[\phi, \tau_E] = \frac{\partial}{\partial \tau_E} \chi[\phi, \tau_E]. \quad (2.17)\]

The solution is

\[\chi[\phi, \tau_E] = T \exp \left[ -\int_{\tau_E^i}^{\tau_E} \hat{H}_E^E(\tau_E') d\tau_E' \right] \chi[\phi, \tau_E^i] = \hat{U}_E(\tau_E, \tau_E^i) \chi[\phi, \tau_E^i]. \quad (2.18)\]

Here \( T \) represents time ordering with respect to \( \tau_E' \). The crucial point is that the Euclidean time evolution operator, \( \hat{U}_E \), is non-unitary. This is natural since we know that wavefunctions decay exponentially during tunneling. If \( \hat{U}_E \) was unitary, the easiest way to calculate it would be to transform to the Heisenberg picture, solve the field equations mode by mode, and compute Bogolubov coefficients. However, as emphasized in Ref. [1] the non-unitarity of \( \hat{U}_E \) implies that the Schrödinger and Heisenberg pictures are
inequivalent, making the standard method inapplicable. Instead, one can use the method developed in Ref. [1] which closely resembles the standard one but is more general. We first describe the state right before tunneling. For convenience, set $x_m^i = \tau^i = \tau^i_E = 0$. Let $\xi_\omega(x, \tau)$ be a complete set of positive norm solutions to the Klein-Gordon equation which vanish vanish at the mirror:

$$\left[-\frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial x^2}\right] \xi_\omega(x, \tau) = 0$$

(2.19)

$$i \int dx \left[ \xi_\omega^*(x, \tau) \frac{\partial}{\partial \tau} \xi_\omega'(x, \tau) - \frac{\partial}{\partial \tau} \xi_\omega^*(x, \tau) \xi_\omega'(x, \tau) \right] = \delta_{\omega' \omega}$$

(2.20)

$$\xi_\omega(x_m(\tau), \tau) = 0.$$  

(2.21)

The set of allowed frequencies $\omega$ is taken to be discrete, and $\sum_\omega$ represents summation over this set. The field operators can then be expanded in terms of these modes:

$$\hat{\phi}(x, \tau) = \sum_\omega \left[ a_\omega \xi_\omega(x, \tau) + \hat{a}_\omega^\dagger \xi_\omega^*(x, \tau) \right]$$

(2.22)

$$\hat{\pi}_\phi(x, \tau) = \frac{\partial}{\partial \tau} \hat{\phi}(x, \tau) = \sum_\omega \left[ \hat{a}_\omega \frac{\partial}{\partial \tau} \xi_\omega(x, \tau) + \hat{a}_\omega^\dagger \frac{\partial}{\partial \tau} \xi_\omega^*(x, \tau) \right]$$

(2.23)

with $[\hat{a}_\omega, \hat{a}_\omega^\dagger] = \delta_{\omega' \omega}$.

Now define Euclidean fields $\hat{\phi}^E(y, \tau_E)$, $\hat{\pi}_\phi^E(y, \tau_E)$ which agree with $\hat{\phi}(x, \tau)$, $\hat{\pi}_\phi(y, \tau)$ at $\tau = \tau_E = 0$, but evolve according to

$$\hat{\phi}^E(y, \tau_E) = \hat{U}_E^{-1}(\tau_E, 0) \hat{\phi}(y, 0) \hat{U}_E(\tau_E, 0)$$

(2.24)

$$\hat{\pi}_\phi^E(y, \tau_E) = \hat{U}_E^{-1}(\tau_E, 0) \hat{\pi}_\phi(y, 0) \hat{U}_E(\tau_E, 0).$$

(2.25)

We will calculate $\hat{U}_E(\tau_E, 0)$ by first finding $\hat{\phi}^E(y, \tau_E)$, $\hat{\pi}_\phi^E(y, \tau_E)$. The field equations for these operators are

$$\frac{\partial \hat{\phi}^E}{\partial \tau_E} = -\left[ \hat{\phi}^E, \hat{H}_E^E \right] = -i \hat{\pi}_\phi^E + \frac{dx_m}{d\tau_E} \frac{\partial \hat{\phi}^E}{\partial y}$$

(2.26)

$$\frac{\partial \hat{\pi}_\phi^E}{\partial \tau_E} = -\left[ \hat{\pi}_\phi^E, \hat{H}_E^E \right] = -i \frac{\partial^2 \hat{\phi}^E}{\partial y^2} + \frac{dx_m}{d\tau_E} \frac{\partial \hat{\pi}_\phi^E}{\partial y}.$$  

(2.27)

So

$$\hat{\pi}_\phi^E = i \left( \frac{\partial \hat{\phi}^E}{\partial \tau_E} - \frac{dx_m}{d\tau_E} \frac{\partial \hat{\phi}^E}{\partial y} \right)$$

(2.28)
\[
\frac{\partial^2 \hat{\phi}^E}{\partial \tau_E^2} + \left[ 1 + \left( \frac{dx_m}{d\tau_E} \right)^2 \right] \frac{\partial^2 \hat{\phi}^E}{\partial y^2} - 2 \frac{dx_m}{d\tau_E} \frac{\partial^2 \hat{\phi}^E}{\partial y \partial \tau_E} - \frac{d^2 x_m}{d\tau_E^2} \frac{\partial \hat{\phi}^E}{\partial y} = 0. \tag{2.29}
\]

Equation (2.29) can be obtained by varying the action
\[
S = \frac{1}{2} \int \! dy \! d\tau_E \sqrt{g_E} \ g_E^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \tag{2.30}
\]
with the Euclidean metric
\[
ds_E^2 = g_E^{\mu \nu} dx_\mu dx_\nu = d\tau_E^2 + 2 \frac{dx_m}{d\tau_E} dx \! d\tau_E + dx^2. \tag{2.31}
\]

\[\hat{\phi}^E, \hat{\pi}_\phi^E\] can be expanded in terms of modes \(f_\omega\) which satisfy the Euclidean Klein-Gordon equation (2.29) and which vanish at \(y = 0\),
\[
\hat{\phi}^E(y, \tau_E) = \sum_\omega \hat{b}_\omega f_\omega(y, \tau_E) \tag{2.32}
\]
\[
\hat{\pi}_\phi^E(y, \tau_E) = i \sum_\omega \hat{b}_\omega \left( \frac{\partial}{\partial \tau_E} f_\omega(y, \tau_E) - \frac{dx_m}{d\tau_E} \frac{\partial}{\partial y} f_\omega(y, \tau_E) \right). \tag{2.33}
\]

As the Euclidean Klein-Gordon equation is elliptic, one cannot in general impose Cauchy boundary conditions at \(\tau_E = 0\) on \(f_\omega\). The resulting solutions would not satisfy the mirror boundary condition. With the appropriate boundary conditions, either Dirichlet or Neumann, imposed at \(\tau_E = 0\) and \(\tau_E = \tau_f\), a detailed calculation is, of course, required to find \(f_\omega\) for a generic mirror trajectory. We shall take the solutions as given and only use their specific forms in a region far from the mirror, where they are simple.

Now, using the condition that the two sets of operators \(\hat{\phi}, \hat{\pi}_\phi\) and \(\hat{\phi}^E, \hat{\pi}_\phi^E\) are equal at \(\tau = \tau_E = 0\), and taking inner products, the operators \(\hat{b}_\omega\) can be expressed as a linear combination of \(\hat{a}_\omega, \hat{a}_\omega^\dagger\):
\[
\hat{b}_\omega = \sum_{\omega'} \left[ \alpha_{\omega \omega'} \hat{a}_{\omega'} + \beta_{\omega \omega'} \hat{a}_{\omega'}^\dagger \right]. \tag{2.34}
\]

Then using
\[
\hat{\phi}^E(y, \tau_E) = \hat{U}_E^{-1}(\tau_f, 0) \hat{\phi}_E(y, 0) \hat{U}_E(\tau_f, 0) = \hat{U}_E^{-1}(\tau_f, 0) \hat{\phi}(y, 0) \hat{U}_E(\tau_f, 0) \tag{2.35}
\]
and the analogous expression for \(\hat{\pi}_\phi^E\), the following equations for \(\hat{U}_E\) are obtained:
\[
\sum_\omega \sum_{\omega'} \left[ \alpha_{\omega\omega'} \hat{a}_{\omega'} + \beta_{\omega\omega'} \hat{a}_{\omega'}^\dagger \right] f_\omega(y, \tau_E^f) \\
= \sum_\omega \left[ \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_\omega \hat{U}_E(\tau_E^f, 0) \xi_\omega(y, 0) + \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_{\omega'}^\dagger \hat{U}_E(\tau_E^f, 0) \xi_{\omega'}^*(y, 0) \right] 
\]

(2.36)

and

\[
\sum_\omega \left[ \alpha_{\omega\omega'} \hat{a}_{\omega'} + \beta_{\omega\omega'} \hat{a}_{\omega'}^\dagger \right] \frac{\partial}{\partial \tau_E} f_\omega(y, \tau_E^f) \\
= \sum_\omega \left[ \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_\omega \hat{U}_E(\tau_E^f, 0) \frac{\partial}{\partial \tau} \xi_\omega(y, 0) + \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_{\omega'}^\dagger \hat{U}_E(\tau_E^f, 0) \frac{\partial}{\partial \tau} \xi_{\omega'}^*(y, 0) \right]. 
\]

(2.37)

Again taking inner products, this leads to relations of the form

\[
\hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_\omega \hat{U}_E(\tau_E^f, 0) = \sum_{\omega'} \left[ u_{\omega\omega'} \hat{a}_{\omega'} + v_{\omega\omega'} \hat{a}_{\omega'}^\dagger \right] 
\]

(2.38)

\[
\hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_{\omega'}^\dagger \hat{U}_E(\tau_E^f, 0) = \sum_{\omega'} \left[ w_{\omega\omega'} \hat{a}_{\omega'} + z_{\omega\omega'} \hat{a}_{\omega'} \right]. 
\]

(2.39)

Then it can be shown that [1]

\[
\hat{U}_E(\tau_E^f, 0) = \text{const.} \times \exp \sum_\omega \sum_{\omega'} \left[ \frac{1}{2} D_{\omega\omega'} \hat{a}_{\omega'}^\dagger \hat{a}_{\omega'} + F_{\omega\omega'} \hat{a}_{\omega} \hat{a}_{\omega'} + \frac{1}{2} G_{\omega\omega'} \hat{a}_{\omega} \hat{a}_{\omega'} \right]. 
\]

(2.40)

where the matrices \( D, F, \) and \( G \) are defined by

\[
D = vz^{-1} ; \quad F = (z^T)^{-1} - 1 ; \quad G = -z^{-1}w. 
\]

(2.41)

The state after tunneling is then determined,

\[
\left| \chi(\tau_E^f) \right> = \hat{U}_E(\tau_E^f) |\chi(0)\rangle
\]

(2.42)

and is expressed in terms of occupation numbers with respect to the modes \( \xi_\omega(y, 0) \), where now \( y = x - x_m^f \). All of the information about the final state is contained in the matrices \( D, F, \) and \( G \), which are in turn given in terms of inner products between the modes \( f_\omega \) and \( \xi_\omega \).

As a simple application of these formulæ we will calculate the shift in the spectrum of outgoing particles which are far from the mirror at the time of tunneling. It is assumed that the mirror was initially at rest and the field in its ground state. The mirror subsequently accelerates in the potential \( V(x_m) \) until it reaches the classical turning point \( x_m^i \). It is well known that as a result of the mirror’s acceleration, a flux of outgoing particles
is created whose spectrum is calculable by standard methods [7]. Outgoing particles far from the mirror are wavepackets composed of superpositions of plane waves,

\[ \xi_\omega(x, \tau) = \frac{1}{2\sqrt{\omega}} e^{-i\omega(\tau-x)} \]  

(2.43)

The spectrum of outgoing particles located at \( x = \bar{x} \gg \omega^{-1} \) at \( \tau = 0 \) is written as

\[ \sum_{\{n_\omega\}} S_\bar{x}(\{n_\omega\}) \{n_\omega\} \]  

(2.44)

where \( \{n_\omega\} \) is a set of occupation numbers and \( S_\bar{x}(\{n_\omega\}) \) is the amplitude for the set to occur.

Far from the mirror, the modes \( f_\omega \) are easy to calculate since the mirror boundary condition is irrelevant. They are of two types,

\[ f^-_\omega = \frac{1}{2\sqrt{\omega}} e^{-\omega \tau_E + i\omega x} = \frac{1}{2\sqrt{\omega}} e^{-\omega \tau_E + i\omega (y + x_m(\tau_E))} \]

\[ f^+_\omega = \frac{1}{2\sqrt{\omega}} e^{\omega \tau_E + i\omega x} = \frac{1}{2\sqrt{\omega}} e^{\omega \tau_E + i\omega (y + x_m(\tau_E))} \]  

(2.45)

Then \( \hat{b}_\omega \) and \( \hat{b}^E \), \( \hat{\pi}^f \), \( \hat{\pi}_E \) are equal at \( \tau = \tau_E = 0 \) if

\[ \hat{b}_\omega = \hat{a}_\omega \; ; \; \hat{b}^+_\omega = \hat{a}^+_\omega. \]  

(2.46)

Equation (2.38) gives:

\[ \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}_\omega \hat{U}_E(\tau_E^f, 0) = e^{-\omega \tau_E^f + i\omega x_m^f} \hat{a}_\omega \]

\[ \hat{U}_E^{-1}(\tau_E^f, 0) \hat{a}^+_\omega \hat{U}_E(\tau_E^f, 0) = e^{\omega \tau_E^f + i\omega x_m^f} \hat{a}^+_\omega \]  

(2.47)

leading to

\[ D = G = 0 \; ; \; F_{\omega\omega'} = \left(e^{-\omega \tau_E^f - i\omega x_m^f} - 1\right) \delta_{\omega\omega'} \]  

(2.48)

and

\[ \hat{U}_E(\tau_E^f, 0) = \text{const.} \times \exp \sum_\omega \left[e^{-\omega \tau_E^f - i\omega x_m^f} - 1\right] \hat{a}^+_\omega \hat{a}_\omega : \\
= \text{const.} \times \exp \sum_\omega \left[e^{-i\omega x_m^f} - 1\right] \hat{a}^+_\omega \hat{a}_\omega : \exp \sum_\omega \left[e^{-\omega \tau_E^f - 1}\right] \hat{a}^+_\omega \hat{a}_\omega : \]  

(2.49)

The first factor is a translation operator which expresses the state in terms of the modes \( \xi_\omega(x, 0) \) instead of \( \xi_\omega(x + x_m^f, 0) \), and the second factor acts on a state \( \{n_\omega\} \) to give
\[ e^{-E(n,\omega)\tau_E} |\{n,\omega\}\rangle, \] 
where \( E(\{n,\omega\}) = \sum n\omega \) is the energy of the state. Therefore, the state after tunneling is

\[ \text{const.} \times \sum_{\{n,\omega\}} e^{-E(n,\omega)\tau_E} S_{\bar{x}}(\{n,\omega\}) |\{n,\omega\}\rangle. \quad (2.50) \]

The result of the tunneling is simply to shift the spectrum from \( S_{\bar{x}} \) to \( e^{-E(n,\omega)\tau_E} S_{\bar{x}} \).

It is not difficult to understand this result. Since the total energy is fixed, the state before tunneling is given by a superposition of the various ways of distributing the energy between the mirror and the radiation. As the mirror’s probability to tunnel depends exponentially on its energy, we expect an inverse exponential correlation between tunneling and energy in radiation. Thus an observer measuring the spectrum of radiation, conditional on the mirror tunneling, finds the result (2.50). Far from the mirror the shift in the spectrum depends only on \( \tau_E \), the amount of Euclidean time spent during tunneling. This is because the tunneling amplitude in the WKB approximation is \( e^{-S} \), and the derivative of \( S \) with respect to energy is just the Euclidean time.

If we were to identify the Euclidean time with an inverse temperature, the shift would become a Boltzmann factor. This makes it easy to generate thermal distributions of radiation. Specifically, if the distribution before tunneling was a constant, then after tunneling tracing over the states of the mirror would yield a thermal density matrix for the radiation. A number of authors have been led by this fact to seek a connection between the thermal radiation that arises in the contexts of cosmology and black holes and an occurrence of tunneling [3, 18, 19]. Such a connection relies upon assumptions about what is on the other side of the barrier and what the spectrum of radiation is there. In this work we only consider situations where there is a well defined classical trajectory on either side of the barrier; we are interested in the case in which there is collapsing matter on one side of the barrier and a black hole on the other. The treatment of this process requires an extension of the previous method to include gravity.

### 3. Application to Gravity

In this section we make a WKB approximation to gravity in a manner which directly parallels that for the moving mirror. The starting point for the canonical quantization of
gravity is to write the metric as

\[ ds^2 = -\left(N^i dt\right)^2 + h_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right). \]  

(3.1)

With this definition, the action for gravity plus matter takes the form

\[ S = \frac{m_p^2}{16\pi} \int d^4x \sqrt{-g} \left(R - 2\Lambda\right) + S_M + \text{boundary terms} \]

\[ = \int d^4x \left(\pi_{\phi_i} \phi^i + \pi_{ij} \dot{h}^{ij} - N^t \mathcal{H}_t - N_i \mathcal{H}^i\right) + \text{boundary terms} \]  

(3.2)

with

\[ \mathcal{H}_t = \frac{8\pi}{m_p^2} h^{-\frac{1}{2}} \left(h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}\right) \pi_{ij} \pi^{kl} - \frac{m_p^2}{16\pi} h^\frac{3}{2} \left(3R - 2\Lambda\right) + \mathcal{H}_M \]

\[ = \frac{16\pi}{m_p^2} G_{ijkl} \pi^{ij} \pi^{kl} - \frac{m_p^2}{16\pi} h^\frac{3}{2} \left(3R - 2\Lambda\right) + \mathcal{H}_M \]

\[ \mathcal{H}^i = -2\pi_{ij} \dot{h}^i_{\mid j} + \mathcal{H}_M^i \]  

(3.3)

(3.4)

Covariant differentiation and the raising and lowering of indices are performed with respect to the spatial metric \(h_{ij}\). \(\phi_i\) refer to arbitrary matter fields.

The boundary term in the action is determined by requiring that no such term arise in the variation of the Hamiltonian \[13\]. Restricting to asymptotically flat metrics with \(N^t \to 1\), \(N_i \to 0\) as \(r \to \infty\), but allowing for time translations at spatial infinity, the appropriate boundary term is numerically equal to the mass of the system. Thus the Hamiltonian is

\[ H = \int d^3x \left[N^t \mathcal{H}_t + N_i \mathcal{H}^i\right] + M. \]  

(3.5)

Varying the action with respect to \(N^t\) and \(N_i\) yields the constraints

\[ \mathcal{H}_t = \mathcal{H}^i = 0. \]  

(3.6)

The system is quantized by making the replacements

\[ \pi_{ij} \to -i \frac{\delta}{\delta h_{ij}}; \quad \pi_{\phi_i} \to -i \frac{\delta}{\delta \phi_i} \]  

(3.7)

and demanding that the state satisfy the constraints \(\hat{\mathcal{H}}_t \Psi = \hat{\mathcal{H}}^i \Psi = 0\) as well as the Schrödinger equation \(H \Psi = i\partial \Psi / \partial t\).
The constraints
\[
\hat{H}_i \Psi = \left[ 2i \left( \frac{\delta}{\delta h_{ij}} \right) + \hat{\mathcal{H}}_M^i \right] \Psi = 0 \tag{3.8}
\]
enforce invariance of the state under spatial reparameterizations, and the constraint
\[
\hat{H}_t \Psi = \left[ -\frac{16\pi}{m_p^2} G_{ijkl} \frac{\delta}{\delta h_{ij}} \frac{\delta}{\delta h_{kl}} - \frac{m_p^2}{16\pi} h^{\frac{1}{2}} \left( 3\mathcal{R} - 2\Lambda \right) + \hat{\mathcal{H}}_t M \right] \Psi = 0 \tag{3.9}
\]
is the Wheeler-DeWitt equation. Proceeding as before, we seek a semiclassical solution of the form
\[
\Psi \left[h_{ij}, \phi_i\right] = \psi_{VV} \left[h_{ij}\right] e^{im_p^2 S[h_{ij}]} \chi \left[\phi_i, h_{ij}\right]. \tag{3.10}
\]
At first order the Einstein-Hamilton-Jacobi equation is obtained:
\[
\frac{16\pi}{m_p^2} G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta S}{\delta h_{kl}} - \frac{m_p^2}{16\pi} h^{\frac{1}{2}} \left( 3\mathcal{R} - 2\Lambda \right) = 0. \tag{3.11}
\]
Zeroth order yields
\[
-\frac{16\pi}{m_p^2} i G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta \chi}{\delta h_{kl}} + \hat{\mathcal{H}}_t M \chi = 0 \tag{3.12}
\]
provided \(\psi_{VV}\) satisfies
\[
G_{ijkl} \frac{\delta^2 S}{\delta h_{ij} \delta h_{ij}} \psi_{VV} + G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta \psi_{VV}}{\delta h_{kl}} = 0. \tag{3.13}
\]
The momentum constraints at first order are
\[
\left( \frac{\delta S}{\delta h_{ij}} \right)_{ij} = 0 \tag{3.14}
\]
and at zeroth order are
\[
2i \left( \frac{\delta \chi}{\delta h_{ij}} \right)_{ij} + \hat{\mathcal{H}}_M^i \chi = 0. \tag{3.15}
\]

Equations (3.12) and (3.13) describe how the matter wave function evolves as the spatial geometry changes. Quantum field theory in curved space can be recovered by writing \(\chi\)’s dependence on \(h_{ij}\) in terms of a time functional \(\tau[x; h_{ij}]\), and by reintroducing a lapse \(N^\tau\) and shift \(N_i\), demanding that they obey
\[
G_{ijkl} \frac{\delta S}{\delta h_{ij}} = \frac{m_p^2}{16\pi N^\tau} \left( \int dy \frac{\delta h_{kl}}{\delta \tau[y; h_{ab}]} - N_{|ij} - N_{j|i} \right). \tag{3.16}
\]
Then
\[-i \frac{16\pi}{m_p^2} \int \left( N^\tau G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta \chi}{\delta h_{kl}} + 2i N_i \left( \frac{\delta \chi}{\delta h_{ij}} \right) \right) = -i \int \frac{\delta h_{ij}}{\delta \tau} \frac{\delta \chi}{\delta h_{ij}} \right) = -i \int \frac{\delta h_{ij}}{\delta \tau} \frac{\delta \chi}{\delta h_{ij}} \right) \tag{3.17}\]
so that the equation for \( \chi \) becomes
\[\int d^3x \left[ N^\tau \hat{H}_{tM} + N_i \hat{H}_M^i \right] \chi[\phi_i; \tau] = i \frac{\partial}{\partial \tau} \chi[\phi_i; \tau]. \tag{3.18}\]
The condition (3.16) agrees with the classical relation between \( \pi_{ij} \) and \( h_{ij} \), demonstrating that \( \tau[x; h_{ij}] \) is the classical time and that (3.18) is the Schrödinger picture version of quantum field theory in curved space.

As with the mirror example, \( \tau \) becomes imaginary during tunneling so we define a Euclidean time \( \tau_E \) along with a Euclidean lapse \( N^\tau_E = i N^\tau \), in terms of which \( \chi \) obeys
\[-\int dx [N^\tau_E \hat{H}_{tM} + i N_i \hat{H}_M^i] \chi[\phi_i, \tau_E] = \frac{\partial}{\partial \tau_E} \chi[\phi_i, \tau_E]. \tag{3.19}\]
For a massless scalar field with action
\[S = -\frac{1}{2} \int d^4x \sqrt{-g} g_{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi}, \tag{3.20}\]
we have
\[\hat{H}_{tM} = \frac{1}{2} \left( h^{-\frac{1}{2}} \hat{\pi}_{\phi}^2 + h^\frac{1}{2} h^{ij} \partial_i \hat{\phi} \partial_j \hat{\phi} \right) \tag{3.21}\]
\[\hat{H}_M^i = \partial_i \hat{\phi} \hat{\pi}_{\phi}. \tag{3.22}\]
To evolve \( \chi \) through the tunneling region one is required to calculate the Euclidean time evolution operator
\[\hat{U}_E(\tau_E^f, \tau_E^i) = T \exp \left[ -\int_{\tau_E^i}^{\tau_E^f} \hat{H}^E_\phi d\tau_E \right] \tag{3.23}\]
with
\[\hat{H}^E_\phi = \int d^3x \left[ N^\tau_E \left( -\frac{1}{2} h^{-\frac{1}{2}} \frac{\delta^2}{\delta \phi^2} + \frac{1}{2} h^\frac{1}{2} h^{ij} \partial_i \phi \partial_j \phi \right) + N_i \partial_i \phi \frac{\delta}{\delta \phi} \right]. \tag{3.24}\]
As before, one proceeds by defining Euclidean fields obeying (2.24,2.25). In the present case the resulting field equations are:
\[\left( \sqrt{g_E} g_E^{\mu\nu} \partial_\mu \hat{\phi}^E_\phi \right)_\nu = 0 \]
\[\hat{\pi}^E_{\phi} = i \frac{h^\frac{1}{2}}{N^\tau_E} \left( \frac{\partial \hat{\phi}^E_\phi}{\partial \tau_E} - N_i \partial_i \phi \right) \tag{3.25}\]
with
\[
ds_E^2 = g^E_{\mu\nu} dx^\mu dx^\nu = (N^\tau E d\tau_E)^2 + h_{ij} \left( dx^i + N_i d\tau_E \right) \left( dx^j + N_j d\tau_E \right).
\] (3.26)

The evolution operator, and therefore the state after tunneling, is determined by solving the field equations mode by mode, and repeating the steps leading from (2.32) to (2.42).

4. Black Hole Radiation in the Presence of Tunneling

We can now apply this method to determine how the radiation from a black hole is affected by tunneling. It is well known that a black hole formed classically from collapsing matter radiates in a complicated manner at early times due to the time dependent geometry, but at late times will inevitably radiate as a black body at the Hawking temperature. Is this scenario altered if the black hole is formed while tunneling? We shall show that it is not. The form of the late time radiation is insensitive to the hole’s unconventional history in a way that is consistent with the intuitive picture of Hawking radiation being caused by pair production near the horizon.

We consider the behaviour of a scalar field on the background of a false vacuum bubble which tunnels leading to the formation of a black hole. The action for a false vacuum bubble in the thin wall approximation is
\[
S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} \ R - \frac{\Lambda_I}{8\pi} \int_{\text{bubble}} d^4 x \sqrt{-g} - \frac{\mu}{4\pi} \int_{\text{wall}} d^3 A
\] (4.1)
where \(\Lambda_I\) is the cosmological constant of the false vacuum, and \(\mu\) is the energy density of the bubble wall. The classical solutions for this action have been derived in Refs. [8]-[12]. In what follows we refer to the treatment of Ref. [12]. The spherically symmetric solutions are characterized by three parameters: \(\Lambda_I\), \(\mu\), and the total mass \(M\). In addition, for given \(\Lambda_I\) and \(\mu\) there is a critical mass \(M_{cr}\) below which there are two solutions: type (a), where the bubble emerges from a singularity with zero radius, subsequently expands to a maximum radius, and then recollapses; type (b), where the bubble initially collapses from infinite radius, reaches a minimum radius, and then reexpands. Using the results of Refs. [13, 14], we focus on an expanding solution of type (a) which tunnels to an expanding solution of type (b). We confine our interest to the region outside the bubble where the metric, written in terms of Schwarzschild time \(t\) and \(r_s = r + 2M \ln(r/2M - 1)\), is
\[
ds^2 = \left( 1 - \frac{2M}{r} \right) \left( -dt^2 + dr_s^2 \right) + r_s^2 d\Omega^2.
\] (4.2)
As $t$ and $r_*$ cover only part of the complete manifold, we introduce Kruskal-Szekeres coordinates,

$$ds^2 = \frac{32M^3e^{-r/2M}}{r} (-dT^2 + dX^2) + r^2d\Omega^2. \quad (4.3)$$

The two sets of coordinates are related by

$$\left(\frac{r}{2M} - 1\right) e^{r/2M} = X^2 - T^2$$

$$t = \begin{cases} 
4M \tanh^{-1}(T/X) & \text{if } |T/X| < 1 \\
4M \tanh^{-1}(X/T) & \text{if } |T/X| > 1 
\end{cases} \quad (4.4)$$

Using these coordinates the type (a) and (b) solutions of interest are depicted in Fig. 2.

The tunneling amplitude for this process has been computed by two different methods. In Ref. [14] the solution to the Wheeler-DeWitt equation is found in the WKB approximation by solving the Einstein-Hamilton-Jacobi equation (3.11). Since the solution behaves as $e^{-S}$, and the tunneling amplitude is given by the ratio of the wavefunction evaluated at the initial and final geometries, the tunneling amplitude is

$$\exp \left(S[h^\text{initial}\,_{ij}] - S[h^\text{final}\,_{ij}]\right) \quad (4.5)$$

No difficulties arise in this approach; the calculation of tunneling amplitude proceeds in a straightforward fashion.

In Ref. [13] the calculation is performed using the functional integral. In this formalism one looks for a manifold which interpolates between the initial and final surfaces and which is a solution to the Euclidean Einstein equations. The tunneling amplitude is $e^{-S}$, where $S$ is the action of the solution. It is found, however, that solving the field equations leads to a sequence of three geometries which do not form a manifold. To see this, first note that the geometry outside the bubble is Euclidean Schwarzschild space, obtained by $t \rightarrow it_E$, $T \rightarrow iT_E$,

$$ds^2_E = \left(1 - \frac{2M}{r}\right) \left(dt^2_E + dr^2ight) + r^2d\Omega^2 = \frac{32M^3e^{-r/2M}}{r} \left(dT^2 + dX^2\right) + r^2d\Omega^2 \quad (4.6)$$

with

$$\left(\frac{r}{2M} - 1\right) e^{r/2m} = X^2 + T^2_E \ ; \ t_E = 4M \tan^{-1}(T_E/X). \quad (4.7)$$

It remains to describe the motion of the bubble wall. Solving the equations of motion leads to the trajectory in Fig. 3. It is seen that the bubble wall crosses the initial surface during the course of its motion, creating a situation in which it is impossible to identify
a region which is swept out by the evolving hypersurface. Some regions of the manifold are crossed twice by the hypersurface, some once, and some not at all. The authors of Ref. [13] call this object a pseudomanifold and give a prescription to calculate its action by assigning covering numbers to the various regions, but this is not needed for what follows.

With these results in hand, the technique of Sect. (3) can be used to calculate the state of the scalar field after tunneling. It was seen that once the solution of the Einstein-Hamilton-Jacobi equation is given, the field wave functional $\chi$ is fully determined by equations (3.12) and (3.15). Since $S[h_{ij}]$ is calculated in Ref. [14], we have all that we need to find $\chi$. This would, however, require finding the solution to an unfamiliar functional differential equation. To cast it in in the form of the Schrödinger equation a lapse $N^E\tau$, shift $N_i$ and time $\tau_E$ were reintroduced leading to the appearance of the Euclidean metric $g^{E}_{\mu \nu}$. In the present case there is no true interpolating Euclidean manifold, so that any choice of $N^E\tau$ and $N_i$ which define a well behaved $g^{E}_{\mu \nu}$ will lead to a bubble trajectory that is a multivalued function of time. Alternatively, a choice of time functional which gives a single valued bubble trajectory will necessarily lead to a Euclidean metric with vanishing determinant at some point. In either case, it is not clear that the resulting Schrödinger equation is well defined. This is apparent from Fig. 3, where it can be seen that boundary conditions imposed on the initial surface and on the bubble wall may contradict each other. These difficulties arise as a result of trying to compute the final state of the field in one step, which requires a Euclidean manifold interpolating all the way from the initial surface to the final surface, and can be avoided by calculating the state on a series of intermediate hypersurfaces. In this approach, it does not matter that the bubble wall eventually crosses the initial surface since once the state is calculated at some intermediate point we can forget about what preceded it.

For simplicity, we will consider only the s-wave component of the scalar field and frequencies high enough such that the geometrical optics approximation is valid. This means that the field equation is taken to be

$$g^{E}_{\mu \nu} \partial_\mu \partial_\nu \phi = 0. \tag{4.8}$$

The state of the field on the initial surface, $t = T = 0$, is most conveniently expressed in terms of the coordinates $r_*$ and $t$. We divide the modes into ingoing and outgoing,

$$\xi^{\text{in}}_\omega (r_*, t) = C_\omega e^{-i\omega (t + r_*)}$$

$$\xi^{\text{out}}_\omega (r_*, t) = C_\omega e^{-i\omega (t - r_*)} \tag{4.9}$$
and write the field operator as

\[ \hat{\phi}(r_*, t) = \sum_\omega \left[ \hat{a}^\omega_{\text{in}} \xi^\omega_{\text{in}}(r_*, t) + \hat{a}^\omega_{\text{in}} \xi^\omega_{\text{in}}^*(r_*, t) \right] \]  \tag{4.10} \]

\( C_\omega \) are normalization constants whose values will not be important. We shall only consider the \( \text{in} \) modes as the treatment of the \( \text{out} \) modes is exactly the same. We also suppress the \( \text{in} \) superscript.

In the first stage of the evolution the hypersurface is pivoted around \( r_* = r_*^b \) by 180°, where \( r_*^b \) is the position of the bubble wall on the initial surface. The solutions to the Euclidean field equations are most conveniently obtained by choosing Cauchy boundary conditions on the initial surface, (clearly a valid procedure in this case)

\[ \begin{align*}
  f^+_\omega(r_*, 0) &= \xi_\omega(r_*, 0) ; \quad \partial / \partial t_E f^+_\omega(r_*, 0) = -i \partial / \partial r^* \xi^*_\omega(r_*, 0) \\
  f^-_\omega(r_*, 0) &= \xi^*_\omega(r_*, 0) ; \quad \partial / \partial t_E f^-_\omega(r_*, 0) = -i \partial / \partial r^* \xi^\omega(r_*, 0)
\end{align*} \]  \tag{4.11} \]

It is also easiest to use the \( X, T \) coordinates as they are well behaved everywhere. Since the evolution of the hypersurface is simply a reflection about the point \( X = X^b \), a mode which has the form \( f(X, T_E) \) on the initial surface has the form \( f(-X + 2X^b, T) \) on the new surface. Using the relations

\[ \begin{align*}
  r_* &= 4M \ln \sqrt{X^2 + T_E^2} ; \quad t_E = 4M \tan^{-1}(T_E/X) \\
  f^\pm_\omega(r_*, t_E) &= C_\omega e^{\pm \omega t_E - i\omega r_*}
\end{align*} \]  \tag{4.12} \]

and that

\[ f^\pm_\omega(X, T_E) = C_\omega \exp \left( \mp 4iM \omega T_E \overline{-X + 2X^b} - 4iM \omega \ln(-X + 2X^b) \right). \]  \tag{4.14} \]

Since on the new surface, \( f^+_\omega = (f^-_\omega)^* \) and \( \partial f^+_\omega / \partial t_E = -(\partial f^-_\omega / \partial t_E)^* \), the evolution operator \( \hat{U}_E \) is unitary. This means that the state on the new surface has the same form as it did on the initial surface, but is now expressed in terms of the modes

\[ \xi_\omega(X, T) = C_\omega \exp \left( -4iM \omega T \overline{-X + 2X^b} + 4iM \omega \ln(-X + 2X^b) \right). \]  \tag{4.15} \]

These modes can be approximated near \( T = 0 \) as

\[ \xi_\omega = \begin{cases} 
  C_\omega e^{i\omega(t-r_*)} & \text{if } |X| \gg X^b \\
  C_\omega e^{-(2iM \omega/X^b)(T-X)} & \text{if } |X| \ll X^b
\end{cases} \]  \tag{4.16} \]
Now it is useful to express the state in terms of modes which are nonzero only inside or outside the horizon,

\[ \eta_\omega^< = \begin{cases} D_\omega e^{i\omega(t-r_*)} & \text{if } X < 0 \\ 0 & \text{if } X > 0 \end{cases} \]

\[ \eta_\omega^> = \begin{cases} 0 & \text{if } X < 0 \\ D_\omega e^{-i\omega(t+r_*)} & \text{if } X > 0. \end{cases} \] (4.17)

A fundamental result [14, 16] in the derivation of black hole radiance is that the vacuum state with respect to modes which have a time dependence \( e^{-i\omega T} \) is the state

\[ \text{const.} \times \sum_{\{n_\omega\}} e^{-E(\{n_\omega\}) / 2T_H} |\{n_\omega\}_<\rangle |\{n_\omega\}_>\rangle \] (4.18)

with respect to the modes \( \eta_\omega^< \) and \( \eta_\omega^> \). The sum runs over all sets of occupation numbers, \( E = \sum n_\omega \omega \), and \( T_H = 1 / 8\pi M \) is the Hawking temperature. Further, near the horizon, any deviation of \( |\chi\rangle \) from the vacuum state can be ignored because of the arbitrarily large redshift as \( r_* \to -\infty \). Far from the horizon \( \xi_\omega \) and \( \eta_\omega^< \) agree so the form of the state is unchanged there.

Now the hypersurface can be evolved the remainder of the way. If we restrict our attention to the region \( X < X^b \), then the motion of the hypersurface is simply a translation, \( t_E \to t_E - \Delta t_E \). This causes states with time dependence \( e^{i\omega t} \) to be damped by a factor \( e^{-\omega \Delta t_E} \), and states with time dependence \( e^{-i\omega t} \) to be amplified by a factor \( e^{\omega \Delta t_E} \). Near the horizon, the state \( |\chi\rangle \) consists of pairs of positive and negative frequency states according to (4.18). One member of the pair is damped but the other is amplified by a compensating amount so as to leave the state \( |\chi\rangle \) unchanged. The final state of the field can then be summarized as follows. Far from the hole, where there is no pairing, the initial state is damped:

\[ \sum_{\{n_\omega\}} S(\{n_\omega\}) |\{n_\omega\}\rangle \longrightarrow \text{const.} \times \sum_{\{n_\omega\}} e^{-E(\{n_\omega\})/2T_H} S(\{n_\omega\}) |\{n_\omega\}\rangle . \] (4.19)

Near the horizon the final state is given by (4.18). This is true for both the in and out modes, so an observer stationed on either side of the horizon would observe a thermal distribution of both ingoing and outgoing particles. As time passes, all of the ingoing particles will eventually cross the horizon and be swallowed by the hole, whereas the outgoing particles will propagate out to infinity where they can be detected at arbitrarily late times as a flux of thermal radiation at the Hawking temperature.
5. Conclusion

It was shown that the standard picture of black hole radiance is unchanged by tunneling. At late times, the hole radiates just as it would have had it been formed from a classical collapse. This makes sense if one thinks of Hawking radiation as pair production. The probability of tunneling is not affected by the creation of a pair, since the pair has zero total energy. From this point of view it is also clear that what happens at early times cannot possibly affect the late time radiation, since the produced pairs only see the late time geometry. The conventional derivation of radiance obscures this point somewhat and it seems desirable to find an approach which makes this feature manifest from the outset. For the two systems considered here, and presumably this is true in general, the effect of the tunneling was to shift the distribution of any particles that were present before tunneling. In the present case initial excitations were damped because the final surface is rotated clockwise relative to the initial surface. A counterclockwise rotation would have led to amplification. In [13] numerical investigations are quoted which show that the rotation is always clockwise for the false vacuum bubble. One is led to speculate whether this is a general phenomenon — whether all tunneling transitions lead to damping.

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Figure Captions

1. A generic mirror potential. The turning points for energy E are indicated.

2. The type (a) and (b) solutions. The heavy lines represent the bubble trajectory, and the dashed lines are the initial and final surfaces of the tunneling solution. In these figures, only the regions to the right of the trajectory are of interest, as they are outside of the bubble.

3. Bubble trajectory in Euclidean Schwarzschild space.
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