Lepton Universality in the $\nu$MSM

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Abstract

We consider the $\nu$MSM which is an extension of the Standard Model by three right-handed neutrinos with masses below the electroweak scale, in which the origins of neutrino masses, dark matter, and baryon asymmetry of the universe are simultaneously explained. Among three heavy neutral leptons, $N_2$ and $N_3$, which are responsible to the seesaw mechanism of active neutrino masses and the baryogenesis via flavor oscillation, can induce sizable contributions to various lepton universality in decays of charged mesons. It is then investigated the possible deviations of the universality in the $\nu$MSM. We find that the deviation in kaon decay can be large as $O(10^{-3})$, which will be probed in near future experiments.
1 Introduction

The $\nu$MSM (neutrino Minimal Standard Model) \cite{1, 2} is a simple extension of the Standard Model (SM), explaining the origins of neutrino masses, dark matter and baryon asymmetry of the universe at the same time. Three right-handed neutrinos are introduced with Majorana masses below the electroweak scale $O(100)$ GeV, which realize the seesaw mechanism \cite{3} for neutrino masses with very suppressed Yukawa couplings. The model predicts three heavy neutral leptons $N_I$ ($I = 1, 2, 3$) in addition to ordinary active neutrinos $\nu_i$ ($i = 1, 2, 3$).

The lightest heavy neutral lepton $N_1$ with $O(10)$ keV mass is a candidate for dark matter (see, for example, a review \cite{4}). The others $N_2$ and $N_3$ with quasi-degenerate masses can generate baryon asymmetry of the universe through the mechanism given in \cite{5, 2}. Enough baryon asymmetry can be generated even if the degenerate mass $M_N$ of $N_2$ and $N_3$ is small as $O(1)$ MeV \cite{6, 7}. However, the lower bound on masses is further restricted to avoid constraints from direct search and cosmology \cite{8}. The recent analysis \cite{7} shows that $M_N > 163$ MeV for the normal hierarchy (NH), while $M_N = 188 - 269$ MeV and $M_N > 285$ MeV for the inverted hierarchy (IH) of active neutrino masses. It is remarkable that, thanks to the smallness of masses, the heavy neutral leptons in the $\nu$MSM, especially $N_2$ and $N_3$, can be directly tested by a variety of experiments and/or observations \cite{9, 8, 10, 11, 12}.

These heavy neutral leptons mix with flavor neutrinos and their mixing elements are given by the ratios between Dirac and Majorana masses. It is then possible to produce $N_I$ by decays of various mesons through the mixing as the production of ordinary active neutrinos. As an example, when they are sufficiently lighter than charged kaon, the decays $K^+ \rightarrow e^+ N_I$ and $K^+ \rightarrow \mu^+ N_I$ are possible. In fact, these channels are good targets for direct search of heavy neutral leptons by using the technique of so-called the peak search experiment \cite{13}.

Furthermore, such decays may spoil lepton universality of charged meson decay \cite{14, 15}. For instance, it is possible that the ratio of decay rates ($M = \pi, K, \cdots$)

$$R_M = \frac{\Gamma (M^+ \rightarrow e^+ \nu)}{\Gamma (M^+ \rightarrow \mu^+ \nu)},$$

is significantly different from the SM prediction. Although each partial decay width receives considerable hadronic uncertainties, the theoretical prediction can be very precise by taking the ratio, and thus $R_M$ offers a promising test for physics beyond the SM. The general expression for the contribution to $R_M$ from heavy neutral leptons had already been presented in Ref. \cite{14}.

Recently, Refs. \cite{16, 17} had revisited the importance of this issue and violations of various universality including $R_M$ had been extensively studied. Especially, the numerical estimation of $R_M$ in the inverse seesaw model had been performed. In addition, they had also pointed out that $R_M$ can be applied in the $\nu$MSM.

In this letter, following to these developments, we estimate the possible deviation of $R_M$ induced by heavy neutral leptons in the $\nu$MSM. The deviation strongly depends on masses and
mixing elements of \( N_I \). The mixing elements \( \Theta_{\alpha 1} \) of \( N_1 \) must be very suppressed in order to avoid various constraints of dark matter. We then find that the contribution of \( N_1 \) to \( R_M \) is negligible. Thus, \( N_2 \) and \( N_3 \) for the seesaw mechanism and baryogenesis give the dominant contributions to lepton universality.

The main purpose of this letter is to identify the possible deviations of lepton universality in the \( \nu \)MSM. Hereafter, we first summarize the constraints on heavy neutral leptons \( N_2 \) and \( N_3 \) and present the allowed region of their mixing elements in Sec. 2. We then consider in Sec. 3 lepton universality in decays of light mesons, \( R_K \) and \( R_\pi \), in the \( \nu \)MSM and estimate the deviations from the SM. Current status and future perspective of experiments of lepton universality are also discussed. Finally, Sec. 4 is devoted for conclusion.

2 Heavy Neutral Leptons in the \( \nu \)MSM

First of all, we explain briefly the \( \nu \)MSM. Three right-handed neutrinos are introduced with Lagrangian

\[
\mathcal{L} = \overline{\nu_{RI}} \gamma^\mu \partial_\mu \nu_{RI} - F_{\alpha I} \overline{L_\alpha} \Phi \nu_{RI} - \frac{M_I}{2} \nu_{RI}^c \nu_{RI} + \text{h.c.}.
\]  

(2)

Here and hereafter, we follow the notation presented in Ref. [13]. The seesaw mechanism works when Dirac masses \( F_{\alpha I} \langle \Phi \rangle \) are much smaller than Majorana masses \( M_I \). In this case mass eigenstates of neutrinos are three active neutrinos \( \nu_i \) with masses \( m_i \) and three heavy neutral leptons \( N_I \) with masses \( M_I \). Then, the neutrino mixing is given by

\[
\nu_{La} = U_{\alpha i} \nu_i + \Theta_{\alpha I} N^c_I,
\]  

(3)

where \( U_{\alpha i} \) are elements of the PMNS matrix \([19, 20]\), and \( \Theta_{\alpha I} = F_{\alpha I} \langle \Phi \rangle / M_I \) are mixing elements of heavy neutral leptons.

Heavy neutral lepton \( N_1 \) with \( M_1 = \mathcal{O}(10) \) keV plays a role of dark matter. The mixing elements of \( N_1 \) must be suppressed enough since too large \( |\Theta_{\alpha 1}| \) would lead to the overclosure of the universe due to too much present abundance and also would provide too much X-rays from its radiative decay \( N_1 \to \nu \)\(^\#1\) (see Ref. [4]). It is then found that \( N_1 \) can only give negligible contribution to the seesaw mass matrix of active neutrinos and can essentially play no role in baryogenesis to avoid these difficulties. In addition, as will be discussed later, \( N_1 \) contribution to the ratio \( R_M \) in Eq. (1) can be neglected compared with those from \( N_2 \) and \( N_3 \). Therefore, we shall take \( |\Theta_{\alpha 1}| = 0 \) in this analysis for simplicity.

Heavy neutral leptons \( N_2 \) and \( N_3 \) are then responsible to the mass matrix for active neutrinos via the seesaw mechanism and also the baryogenesis via flavor oscillation. In this case, to

\(^\#1\)Recently, the unidentified line spectrum is observed \([21, 22, 23]\), which can be interpreted by X-ray lines emitted by sterile neutrino dark matter (i.e., \( N_1 \) in the considering model).
realize the seesaw mechanism Yukawa coupling constants $F_{\alpha I}$ of $N_2$ and $N_3$ can be expressed as follows [24]:

$$F_{\alpha I} = \frac{i}{\langle \Phi \rangle} \left[ U D_\nu \Omega D_N^\dagger \right]_{\alpha I}.$$  \hspace{1cm} (4)

Here and hereafter we shall follow the notation in Ref. [18]: $U$ represents the PMNS matrix,

$$U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -c_{23}s_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{23}s_{12} - c_{23}s_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \times \text{diag}(1, e^{i\eta}, 1),$$  \hspace{1cm} (5)

with $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, $D_\nu = \text{diag}(m_1, m_2, m_3)$ and $D_N = \text{diag}(M_2, M_3)$. The matrix $\Omega$ is given by

$$\Omega = \begin{pmatrix}
  0 & \cos \omega & 0 \\
  \cos \omega & 0 & -\sin \omega \\
  \xi \sin \omega & \xi \cos \omega & 0
\end{pmatrix} \text{ for the NH case, } \quad \Omega = \begin{pmatrix}
  \cos \omega & -\sin \omega \\
  \xi \sin \omega & \xi \cos \omega \\
  0 & 0
\end{pmatrix} \text{ for the IH case}. \hspace{1cm} (6)$$

The couplings are written in terms of parameters of active neutrinos and heavy neutral leptons. The former ones consist of masses $m_i$ as well as mixing angles $\theta_{ij}$, Dirac phase $\delta$ and Majorana phase $\eta$ in the PMNS matrix.\footnote{Since $N_1$ essentially decouples from the seesaw mechanism, the lightest active neutrino obtains a mass smaller than $O(10^{-5})$ eV [11]. The number of Majorana phases in the PMNS matrix is effectively reduced to be one (rather than two in the usual case with three massive active neutrinos).} The latter ones are a complex parameter $\omega$, masses $M_{2,3}$ and the sign parameter $\xi$. As for the masses, the successful baryogenesis requires that $N_2$ and $N_3$ are quasi-degenerate in mass, and we write them in the form $M_3 = M_N + \Delta M/2$ and $M_2 = M_N - \Delta M/2$ with $\Delta M \ll M_N$. The imaginary part of $\omega$ is important to determine the typical size of the mixing elements since $|\Theta_{\alpha I}| \propto X_\omega \equiv \exp(\text{Im}\omega)$. In fact, as shown in Ref. [18], $|\Theta_{\alpha I}|$ can be large by taking $X_\omega \gg 1$ without changing masses of active neutrinos.

The mixing elements $\Theta_{\alpha I}$ characterize the strength of interactions for heavy neutral leptons, and then receive constraints from direct searches and cosmology. Interestingly, as pointed out in Ref. [3], the former ones place the upper bounds on $|\Theta_{\alpha I}|$ while the latter one gives the upper bound on lifetimes $\tau_{N_2}$ and $\tau_{N_3}$ leading to the lower bounds on $|\Theta_{\alpha I}|$. Consequently, we may obtain the allowed range of the mixing elements. Such regions have already been evaluated in Refs. [3, 7]. Here we reconsider this issue, especially taking into account for the first time the preliminary result from the BNL-E949 experiment [25]. Notice that we shall restrict ourselves for the case when $M_N < m_K - m_e$ because such heavy neutral leptons, as we will show later, induce a significant deviation of the lepton universality in kaon decay.

In deriving the allowed region, we construct Yukawa couplings of $N_2$ and $N_3$ by using the central values of $\theta_{ij}$ and $\Delta m^2_{ij}$ from the global analysis of neutrino oscillations in Ref. [26] and by varying all the possible ranges for other free parameters. We then show the allowed range
for the combination of $\Theta_{\alpha I}$

$$|\Theta|^2 \equiv \sum_{l=2,3} \sum_{\alpha=e,\mu,\tau} |\Theta_{\alpha l}|^2,$$

(7)

for a given $M_N$. In our parameterization of Yukawa couplings, it is written as

$$|\Theta|^2 = \frac{\sum_{i=1,2,3} m_i}{2M_N} (X_\omega + X_\omega^{-2}).$$

(8)

As for the bounds from direct search experiments, we first consider the case when $M_N < 450$ MeV and use the results from the peak search experiments [27, 28, 29, 25] as well as the beam-dump experiments [30, 31, 32]. (See the discussion later for the case in which $m_K - m_e > M_N > 450$ MeV.) Following to Refs. [9, 33] we have taken into account the corrections applying the bounds from PS191 experiment [30, 31, 32] to the $\nu$MSM, i.e., the targets are two heavy neutral leptons $N_2$ and $N_3$ which are Majorana particles (the target is one Dirac particle in the original analysis), and the neutral current contributions for decays of heavy neutral leptons are added (such a contribution is neglected in the original analysis).

Moreover, the successful baryogenesis also gives the upper bounds on the mixing elements in order to avoid the strong washout of the produced asymmetry [6]. However, as shown in Ref. [6], such bounds are much weaker than those from PS191 experiment in the considering mass range.

In this analysis, we also consider the recent bound from BNL-E949 experiment [25]. It is the peak search experiment in $K^+ \rightarrow \mu^+ \nu$ decay giving the upper bound on $|\Theta_{\mu I}|^2$. Finally, to avoid the cosmological difficulty we impose the lifetime bound $\tau_{N_{2,3}} < 0.1$ sec [34, 35]. Unfortunately, the analysis in Refs. [34, 35] has been done in the different situation from the $\nu$MSM. We then also discuss the case when the lifetime bound is relaxed as $\tau_{N_{2,3}} < 1$ sec to make the most conservative analysis. To evaluate $\tau_{N_{2,3}}$, we use the formulae of the partial decay widths of heavy neutral leptons given in Ref. [8].

The results are summarized in Fig. 1. We find that BNL-E949 experiment gives the more stringent bound for $M_N \simeq 180 - 260$ MeV compared with the bounds from PS191 experiment, which is seen by the hatched regions in Fig. 1. (See also the result in Ref. [7] for comparison.) Especially, in the IH case, the lower bound on $M_N$ changes a lot by the inclusion of such a bound. We then find that the allowed mass region when $\tau_{N_{2,3}} < 0.1$ sec is

$$M_N > \begin{cases} 
173 \text{ MeV} & \text{for the NH case} \\
264 \text{ MeV} & \text{for the IH case}
\end{cases}.$$

(9)

#3 The mass difference $\Delta M$ gives negligible corrections to all the results in the present analysis, and hence we take $\Delta M = 0$ for simplicity.

#4 Ref. [7] had used the data of global analysis of the neutrino oscillations in Ref. [36] rather than Ref. [26] used in this analysis.

#5 For the NH case the small mass region $M_N = 208 - 211$ MeV is excluded.
Figure 1: *Allowed region in the $M_N$-$|\Theta|^2$ plane for the NH case (left panel) and IH case (right panel). Allowed regions are shown by the shaded regions with red-solid line or red-dashed line for the case with the cosmological lifetime bound $\tau_{N_{2,3}} < 0.1$ sec or $\tau_{N_{2,3}} < 1$ sec, respectively. The hatched regions are excluded by the bounds from BNL-E949 experiment* [25].

It should be noted that, if the cosmological upper bound of the lifetime is relaxed as $\tau_{N_{2,3}} < 1$ sec, the lower bound on $M_N$ becomes smaller as $M_N > 122$ MeV and 136 MeV for the NH and IH cases, respectively. See also Fig. 1. Therefore, the cosmological bound on the lifetime is crucial for determining the lower bound of the masses of $N_2$ and $N_3$. 

It is seen that the allowed range in Fig. 1 is very limited for both NH and IH cases. In practice, all such regions can be verified if the sensitivity of $|\Theta|^2$ by future experiments will be improved by a factor of $O(10^2)$ or $O(10^3)$ when applying the lifetime bound $\tau_{N_{2,3}} < 0.1$ or 1 sec, respectively. Such experiments will be not only the peak search and beam-dump experiments, but also the precision measurements of lepton universality of light meson decays as shown below.

3 Lepton Universality in the $\nu$MSM

Let us discuss lepton universality of charged meson decays shown in Eq. (1) in the context of the $\nu$MSM. We first consider the universality in charged kaon decay $R_K$. The SM prediction

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#6 The lifetime bound for the case when $M_N < m_\pi$ had also been discussed in Ref. [37] and had shown that the mass region $M_N < m_\pi$ is excluded. To make a very conservative analysis, however, we also consider the case where the lifetime of $N_{2,3}$ is longer than the limit in [37].
of $R_K$ is

$$R^K_{\text{SM}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m^2_K - m^2_\mu}{m^2_K - m^2_e}\right)^2 (1 + \delta R_K),$$

(10)

where $\delta R_K$ denotes the radiative correction. Notice that $K^+ \rightarrow e^+ \nu_e$ and $K^+ \rightarrow \mu^+ \nu_\mu$ occur through charged current interaction and their rates are helicity-suppressed. It should be mentioned that both decay rates receive the hadronic uncertainties, e.g., through the decay constant of parent meson, such uncertainties cancel to a large extent by taking the ratio. The theoretical prediction of the SM is thus very precise as $[38, 39]$

$$R^K_{\text{SM}} = (2.477 \pm 0.001) \times 10^{-5}.$$  (11)

In addition, the measurements at high precision have been done $[40, 41, 42, 43]$. The recent NA62 experiment provides $[43]$

$$R^K_{\text{exp}} = (2.488 \pm 0.010) \times 10^{-5}.$$  (12)

It is seen that the observational data agrees with the SM value at the $1\sigma$ level. Consequently, the deviation

$$\Delta r_K = \frac{R_K}{R^K_{\text{SM}}} - 1,$$

(13)

is small as

$$\Delta r_K = (4 \pm 4) \times 10^{-3},$$

(14)

and thus it provides a powerful probe for physics beyond the SM.

In the $\nu$MSM, $K^+$ is possible to decay into not only active neutrinos $\nu_i$ but also heavy neutral leptons $N_I$ depending on $M_I$. Then, the ratio $R_K$ is given by

$$R_K = \frac{\sum_{i=1,2,3} \Gamma(K^+ \rightarrow e^+ \nu_i) + \sum_{I=1,2,3} \Gamma(K^+ \rightarrow e^+ N_I)}{\sum_{i=1,2,3} \Gamma(K^+ \rightarrow \mu^+ \nu_i) + \sum_{I=1,2,3} \Gamma(K^+ \rightarrow \mu^+ N_I)}.$$  (15)

The general expression of $R_M$ in the presence of heavy neutral leptons has been given by Ref. $[14]$. (See Eq. (3.2) in Ref. $[14]$.) By neglecting the masses of active neutrinos and the experimental energy thresholds of charged leptons in kaon decays, the deviation is $[14]$

$$\Delta r_K = \frac{\sum_{i=1,2,3} |U_{ei}|^2 + \sum_{I=1,2,3} |\Theta_{eI}|^2 G_{eI}}{\sum_{i=1,2,3} |U_{\mu i}|^2 + \sum_{I=1,2,3} |\Theta_{\mu I}|^2 G_{\mu I}} - 1,$$  (16)

where $G_{\alpha I} = 0$ if $M_I > m_K - m_{\ell_\alpha}$; otherwise

$$G_{\alpha I} = \frac{r_\alpha + r_I - (r_\alpha - r_I)^2}{r_\alpha(1 - r_\alpha)^2} \sqrt{1 - 2(r_\alpha + r_I) + (r_\alpha - r_I)^2},$$  (17)
with \( r_\alpha = m_{\ell_\alpha}^2 / m_K^2 \) and \( r_I = M_I^2 / m_K^2 \). (See Ref. [8] for the expressions of \( \Gamma(K^+ \to l_\alpha^+ N_I) \).)

The physical importance of \( \Delta r_K \) (and also \( \Delta r_\pi \) in the later discussion) had been readdressed in Refs. [16, 17]. The main origins of such deviations are (i) the additional contributions to the kaon decay from heavy neutral leptons and (ii) the deviation from the unitarity of the PMNS mixing matrix of active neutrinos [14, 15, 16, 17]. Refs. [16, 17] had presented the possible kaon decay from heavy neutral leptons and (ii) the deviation from the unitarity of the PMNS mixing elements \( \Theta_{\alpha I} \) in Refs. [16, 17]. The main origins of such deviations are (i) the additional contributions to the mixing matrix of active neutrinos and heavy neutral leptons satisfy the unitarity condition

\[
\sum_{i=1,2,3} |U_{\alpha i}|^2 + \sum_{I=1,2,3} |\Theta_{\alpha I}|^2 = 1. \tag{18}
\]

It is seen that the violation of the unitarity in the PMNS matrix \( U \) is very suppressed at \( O(|\Theta_{\alpha I}|^2) \) in this framework (see Fig. 1). From the above condition \( \Delta r_K \) in Eq. (16) can be written as

\[
\Delta r_K = \frac{1 + \sum_{I=1,2,3} |\Theta_{e I}|^2 [G_{e I} - 1]}{1 + \sum_{I=1,2,3} |\Theta_{\mu I}|^2 [G_{\mu I} - 1]} - 1. \tag{19}
\]

Therefore, we find that the deviation \( \Delta r_K \) in the \( \nu \)MSM is determined by the masses \( M_I \) and mixing elements \( \Theta_{\alpha I} \) of heavy neutral leptons. Note that \( \Delta r_K \) does not depend explicitly on the PMNS matrix elements, but it depends on them implicitly through \( \Theta_{\alpha I} \). (See the parametrization of Yukawa couplings of \( N_I \) in Eq. (16).) Since the mixing elements of dark matter \( N_1 \) must be very small, we can safely neglect its contribution to \( \Delta r_K \).

First, we consider the case when \( M_N < m_K - m_\mu \), i.e., both \( K^+ \to \mu^+ N_I \) and \( K^+ \to e^+ N_I \) are kinematically allowed. In this case one might expect that the deviation \( \Delta r_K \) is very suppressed as \( O(10^{-9}) - O(10^{-7}) \) since \( |\Theta|^2 \) should be in such a range as shown in Fig. 1. Decay rate of \( K^+ \to \ell_\alpha^+ N_I \) (and then \( G_{\alpha I} \)) is, however, enhanced by \( (M_I/m_\epsilon)^2 \) compared with \( K^+ \to \ell_\alpha^+ \nu_\alpha \) due to the helicity suppression [14]. Interestingly, since this enhancement factor is much larger for the decay into \( e^+ \) than that into \( \mu^+ \), the \( \nu \)MSM predicts a positive \( \Delta r_K \) in this mass region as

\[
\Delta r_K \approx \sum_{I=2,3} |\Theta_{e I}|^2 \left( \frac{M_N^2}{m_\epsilon^2} \right) \left( 1 - \frac{M_N^2}{m_K^2} \right)^2. \tag{20}
\]

Moreover, the upper limit of \( \Delta r_K \) is then derived from the upper bounds on the mixing elements \( |\Theta_{\alpha I}| \). Such elements are severely restricted by PS191 experiment looking for the production and decay modes \( K^+ \to e^+ N_I \) and \( N_I \to e^+ \pi^-, e^- \pi^+ \), e.g., \( |\Theta_{e I}|^2 < O(10^{-9}) - O(10^{-8}) \) for \( M_N \approx 200-400 \text{ MeV} \). Therefore, we expect \( \Delta r_K < O(10^{-4}) - O(10^{-3}) \) by taking into account the enhancement factor of \( (M_N/m_\epsilon)^2 \sim 10^5 \).
When \( m_K - m_e > M_N > m_K - m_\mu \), \( K^+ \to \mu^+ N_I \) is forbidden, but the behavior of the correction \( \Delta r_K \) is very similar to the above case. On the other hand, when \( M_N > m_K - m_e \), the situation is changed. We should note that, even if \( K^+ \to \mu^+ N_I \) and \( K^+ \to e^+ N_I \) are kinematically forbidden, the correction of \( \Delta r_K \) is induced due to the non-unitarity of the PMNS matrix (see Eq. (18)) as
\[
\Delta r_K \simeq \sum_{I=2,3} (|\Theta_{\mu I}|^2 - |\Theta_{e I}|^2).
\]
In this case the sign of \( \Delta r_K \) is determined according to the relative sizes of \( |\Theta_{\mu I}|^2 \) and \( |\Theta_{e I}|^2 \) and the magnitude is \( |\Delta r_K| \leq |\Theta|^2 = O(10^{-9}) - O(10^{-7}) \).

Now, we are at the point to present the numerical prediction of \( \Delta r_K \) in the \( \nu \)MSM. As explained in Sec. 2, we impose the constraints from direct search experiments and cosmological lifetime bound. The possible range of \( \Delta r_K \) by varying all the free parameters is shown in Fig. 2. It is found that \( \Delta r_K = O(10^{-7}) - O(10^{-3}) \) for the NH case, and \( \Delta r_K = O(10^{-6}) - O(10^{-3}) \) for the IH case, where we have considered \( M_N < 450 \) MeV and \( \tau_{N_{2,3}} < 0.1 \) sec. The predicted region becomes wider if the lifetime bound is relaxed as \( \tau_{N_{2,3}} < 1 \) sec. The search bounds place the upper limit while the lifetime bound places the lower limit of \( \Delta r_K \), and hence the \( \nu \)MSM predicts \( \Delta r_K \) in certain range. We find that the predicted range is indeed consistent with the current upper bound at 3 \( \sigma \) level, \( \Delta r_K < 1.2 \times 10^{-2} \).

We have considered the mass range \( M_N < 450 \) MeV so far. When \( M_N > 450 \) MeV, there is
no stringent constraint on the mixing elements from PS191 experiment. So, we expect a large $\Delta r_K$ for $450 \text{ MeV} < M_N < m_K - m_e$. In such a case, the upper bounds on $|\Theta_{\alpha I}|$ are placed from CHARM and CHARM II [44, 45, 46], IHEP-JINR [47] and NuTeV [48] experiments. When $M_N$ is just above 450 MeV, the most stringent bound on $|\Theta_{ei}|^2$ is obtained from IHEP-JINR and the bound in Fig. 4 of Ref. [47] is weaker than that of PS191 [32] by a factor of $\sim 40$. This means that $\Delta r_K$ can be $\sim 4 \times 10^{-3}$ in such a value of $M_N$.

In addition, the successful scenario of baryogenesis also puts the important bound of the mixing elements for such mass regions. We, however, find that such a bound on $|\Theta|^2$ in Ref. [49] is slightly weaker than the above IHEP-JINR bound on $|\Theta_{ei}|^2$. This shows that search for heavy neutral leptons with $M_N$ just above 450 MeV is very interesting since the present bounds from beam-dump experiments, baryogenesis and also lepton universality are very competitive and may be possible to be cross-checked in various ways. More precise estimation of these bounds as well as $\Delta r_K$ in this case will be done elsewhere [50].

Near future experiments (such as NA62 at CERN [51], ORKA at FNAL [52] and TREK/E36 at J-PARC [53]) will achieve the sensitivity $\Delta r_K = 10^{-3}$. Therefore, it is very interesting that these experiments will start to probe the predicted region in the $\nu$SM. In particular, large $\Delta r_K$ are obtained when $M_N \sim 180 \text{ MeV}$ and just above 450 MeV. Such mass regions will also be tested by experiments using different search techniques, like the peak search and/or beam-dump experiments, in decays of kaon and charmed-mesons, respectively.

Next, we turn to consider lepton universality in pion decay. The theoretical prediction of the SM is [39]

$$R_{\pi}^{\text{SM}} = (1.2352 \pm 0.0001) \times 10^{-4}, \quad (22)$$

while the experimental value is $[54]^\#7$

$$R_{\pi}^{\text{exp}} = (1.230 \pm 0.004) \times 10^{-4}. \quad (23)$$

The deviation is then given as

$$\Delta r_\pi = (-4 \pm 3) \times 10^{-3}. \quad (24)$$

In the considering model, $\pi^+ \rightarrow \mu^+ N_{2,3}$ are impossible even if the lifetime bound is relaxed as $\tau_{N_{2,3}} < 1 \text{ sec}$ and then we restrict ourselves to the case with $M_N > m_\pi - m_\mu$. When $\pi^+ \rightarrow e^+ N_{2,3}$ are available, the sizable correction to $R_\pi$ is expected due to the enhancement factor of $(M_N/m_e)^2$ and its maximal value is determined by the upper bounds of $|\Theta_{ei}|^2$. Then, the approximate form of $\Delta r_\pi$ is given by

$$\Delta r_\pi \simeq \sum_{I=2,3} |\Theta_{ei}|^2 \frac{M_N^2}{m_e^2} \left(1 - \frac{M_N^2}{m_\pi^2}\right)^2, \quad (25)$$

$^\#7$ Here we have cited the averaged value of Particle Data Group [54]. The recent measurements at TRIUMF and PSI give $R_\pi = (1.2265 \pm 0.0034 \pm 0.0044) \times 10^{-4}$ [55] and $R_\pi = (1.2346 \pm 0.0035 \pm 0.0036) \times 10^{-4}$ [56], respectively.
Figure 3: $\Delta r_\pi$ in the $\nu$MSM. Possible region is shown by the shaded region with red-solid line or blue-dashed line for the NH or IH case, respectively. Here we impose the cosmological lifetime bound $\tau_{N_{2,3}} < 1$ sec. The horizontal (black dotted) line is $\Delta r_\pi = 5 \times 10^{-4}$ (which will be reached by the near future experiments).

similar to Eq. (20). Moreover, the sign of $\Delta r_\pi$ is positive as in the kaon decay, and thus, even if $N_2$ and $N_3$ were allowed to be lighter than pion (to be precise $m_\pi - m_e$), they would be conflict with $R_\pi^{\text{exp}}$ at 1 $\sigma$ level (see Eq. (24)). As shown in Fig. 3 we find numerically that the predicted range is $\Delta r_\pi < \mathcal{O}(10^{-4})$, and then it is consistent with $R_\pi^{\text{exp}}$ at 2 $\sigma$ level.

Notice that the experiments like PIENU at TRIUMF [57] and PEN at PSI [58] will improve the sensitivity at the level $\Delta r_\pi \simeq 0.05 - 0.06$ % (see also Ref. [59]), which is slightly above the predicted range. Thus, the further improvement may be required to probe $\Delta r_\pi$ in the $\nu$MSM.

When $M_N$ becomes larger than $m_\pi - m_e$, the non-unitarity of the PMNS mixing matrix for active neutrinos induces the correction

$$\Delta r_\pi \simeq \sum_{I=2,3} (|\Theta_{\mu I}|^2 - |\Theta_{e I}|^2),$$

(26)

as in the kaon decay (21). In this case the magnitude of $\Delta r_\pi$ is too small to be probed in near future experiments. It is, however, interesting to notice that $\Delta r_\pi$ and $\Delta r_K$ become the same when $M_N > m_K - m_e$.

We have so far discussed the corrections to lepton universality in kaon and pion decays. It should be noted that heavy neutral leptons $N_2$ and $N_3$ may lead to violations of lepton universality in decays of charmed mesons, beauty mesons and tauon. See the recent analysis in Ref. [17]. The comprehensive study for the test of the $\nu$MSM by lepton universality will be discussed elsewhere [50].
4 Conclusions

We have discussed lepton universality of charged meson decays in the $\nu$MSM. Among three heavy neutral leptons, $N_2$ and $N_3$, which explain the seesaw mechanism for active neutrino masses and the baryogenesis via their flavor oscillation, may induce the violations of such universality due to the non-unitarity of the mixing matrix of active neutrinos and the additional contributions to meson decays.

The deviation of lepton universality in kaon decay $R_K$ has been found to be large as $\Delta r_K = \mathcal{O}(10^{-3})$ when applying the cosmological bound on lifetime as $\tau_{N_{2,3}} < 0.1$ sec. Such a large $\Delta r_K$ is possible when $M_N \sim 180$ MeV and just above 450 MeV. Further, if the cosmological bound on the lifetime is weak as $\tau_{N_{2,3}} \lesssim 1$ sec, $\Delta r_K$ can be larger as $\mathcal{O}(10^{-2})$. Notice that the sign of $\Delta r_K$ is always positive in the case when $K^+ \to e^+ N_{2,3}$ are open. Furthermore, we have also discussed lepton universality in pion decay. When $\pi^+ \to e^+ N_{2,3}$ are allowed by relaxing the lifetime bound, the deviation can be large as $\Delta r_\pi = \mathcal{O}(10^{-4})$.

Such regions of the model will begin to be explored by near future experiments; the experiments of lepton universality in kaon decay as NA62, ORKA and TREK/E36 experiments and those in pion decay as PIENU and PEN experiments. It should be noted that such regions are also good targets of direct search experiments using the different methods (the peak search experiments, the beam-dump experiments, and so on). These facilities might reveal physics of $N_2$ and $N_3$, namely the origins of neutrino masses and baryon asymmetry of the universe.

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