Lambda the Ultimate SSA:
Optimizing Functional Programs in SSA

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Abstract—Static Single Assignment (SSA) is the workhorse of modern optimizing compilers for imperative programming languages. However, functional languages have been slow to adopt SSA and prefer to use intermediate representations based on minimal lambda calculi due to SSA’s inability to express higher-order constructs. We exploit a new SSA construct — regions — in order to express functional optimizations via classical SSA-based reasoning. Region optimization currently relies on ad-hoc analyses and transformations on imperative programs. These ad-hoc transformations are sufficient for imperative languages as regions are used in a limited fashion. In contrast, we use regions pervasively to model sub-expressions in our functional IR. This motivates us to systematize region optimizations. We extend classical SSA reasoning to regions for functional-style analyses and transformations. We implement a new SSA+regions based backend for LEAN4, a theorem prover that implements a purely functional, dependently typed programming language. Our backend is feature-complete and handles all constructs of LEAN4’s functional intermediate representation $\lambda rc$ within the SSA framework. We evaluate our proposed region optimizations by optimizing $\lambda rc$ within an SSA+regions based framework implemented in MLIR and demonstrating performance parity with the current LEAN4 backend. We believe our work will pave the way for a unified optimization framework capable of representing, analyzing, and optimizing both functional and imperative languages.

Index Terms—Optimizing compilers, Functional programming

I. INTRODUCTION

Many optimizing compilers for imperative programming languages use Static Single Assignment (SSA) as their intermediate representation (IR) \cite{1,2}. Such IRs impose structural and semantic rules on values to aid the compiler’s reasoning. Traditionally, such IRs express control flow using a Control Flow Graph (CFG) – a flat collection of sequences of instructions (basic blocks) that transfer control amongst one another. On the other hand, functional intermediate representations use an expression-based IR, where control flow is represented via particular expressions such as case statements. An IR for functional constructs necessarily needs the ability to manipulate sub-expressions.

The famous slogan “SSA is continuations” \cite{3} is not entirely true as stated; the correspondence between SSA and continuation-passing-style (CPS) is established between SSA and a syntactically restricted fragment of CPS, which is further annotated with information about which continuations represent intra-procedural control flow and which represent inter-procedural control flow. Thus, the above translation is not practically useful in order to design an SSA-based intermediate representation for functional programming languages, and most functional IRs continue to use expression-based intermediate representations. This has resulted in a schism within the compiler community, where the infrastructure built around optimizing compilers for imperative languages does not get reused for functional languages and vice versa.

In this work, we set out to heal this schism by providing
convenient, easily analyzable and optimizable encodings of core functional IR constructs within SSA. Our key innovation is to use regions to represent functional sub-expressions as SSA values. This allows us to extend SSA’s core strength, the ability to reason about values via sophisticated algorithms, to also cover reasoning about sub-expressions. This enables us to mostly reuse SSA algorithms, and in some cases extend SSA algorithms, to recover key functional language optimizations within SSA. We also gain the ability to use the tooling provided by MLIR’s compiler infrastructure for (a) testing, (b) having a stable textual and in-memory representation, (c) sophisticated infrastructure for parallel peephole rewriting, and (d) support for parallelization and vectorization.

We contribute this novel encoding of a functional intermediate representation, and an implementation of the encoding within the MLIR compiler framework. We also design and implement new algorithms and modifications to existing algorithms to reason about regions as sub-expressions. We test our design by building a fully-functional backend for the LEAN4 \([4]\), functional programming language. Our new backend uses the MLIR compiler framework to implement SSA+regions. We embed LEAN4’s intermediate representation, λrc, within MLIR and show how to perform common functional programming language optimizations on λrc via SSA-based reasoning. Concretely, our contributions are:

- A minimal intermediate representation lp which allows functional constructs to be lowered, analyzed, and optimized within an SSA setting.
- A novel use of regions to encode functional constructs within an imperative SSA-style IR, called rgn.
- The design of functional-style optimizations for rgn, which are analogues of classical SSA optimizations, applied to regions (Figure 1).
- An evaluation of our feature-complete backend for the LEAN4 compiler and its λrc intermediate representation which demonstrates the soundness of our approach towards representing functional programs in SSA via lp and rgn.

II. BACKGROUND

We provide background on SSA-based IRs, LEAN4, and functional programming constructs.

A. Static Single Assignment (SSA) and Regions

An intermediate representation (IR) is in SSA form if each variable is assigned exactly once and no variable remains undefined. SSA has gained popularity in imperative compilers such as GCC \([6]\), LLVM \([1]\), and many others, as data-flow information is explicitly expressed through dependencies from the definition of a value to its uses (def-use chains). SSA-based IRs typically use basic blocks that hold lists of sequentially executed operations, each taking a list of argument values and returning a tuple of return values. Terminator operations at the end of each basic block, which either branch to another basic block or return from a function, combine these basic blocks into a control flow graph (CFG). While IRs typically use a flat CFG, MLIR \([7]\) recently introduced nested control flow as a first-class concept to support abstractions that require control over scoping. Operations can now receive regions, nested single-entry sub-CFGs, as additional arguments. Regions make it easy to express concepts such as loop bodies, branches of an if-statement, and case statements in functional programs.

B. LEAN, λpure, and λrc

LEAN\([8]\) is an open source theorem prover based on a minimal dependently typed \([9]\) kernel. After type checking the LEAN program, the compiler compiles the program to λpure, a minimal, pure, strict, higher order intermediate representation that is suitable for further lowering into assembly. λpure is lowered to λrc, an extension of λpure with reference counting. The current LEAN backend then lowers λrc to C. Runtime features such as primitives for I/O, numerics, reference counting, and task-based parallelism are provided by a custom runtime library, libleanrt.

C. MLIR

MLIR \([7]\) is a new compiler infrastructure that aims to simplify the development of domain-specific compilers. For this, MLIR provides a minimal SSA-based intermediate representation (IR), with the ability to easily instantiate extensions to the core IR (known as dialects) which follow SSA conventions. Having a customizable IR allows compiler developers to model domain-specific concerns by introducing custom types, operations, and attributes. Here, we briefly describe the relevant aspects of MLIR that are used in our compiler.

1) Modules: Each program in MLIR is called a Module. A Module consists of several global functions. Function names such as @foo are global and allow for linking function calls across modules. A function consists of basic-blocks. Each basic-block is a sequence of operations, ending with a terminator operation. SSA values are local to the scope of the function and have names of the form %bar.

2) Operations: SSA values are produced by Operations, such as addi, which stands for the integer addition operation. Each operation takes zero or more SSA operands that are defined before it and returns zero or more SSA values. Operations can also have compile time constants attached to them, such as (phase = 90.0 : f64). These are called attributes.

3) Dialects: A Dialect in MLIR is a collection of operations and types. The type system in MLIR consists of either primitive types or user defined custom types to encode more complex type systems. In this paper, we use two existing dialects, std and scf. The std dialect contains all basic operations such as constant declarations, arithmetic operations and memory manipulation. The Structured Control Flow Dialect scf contains if-else and for loop constructs.

\[\text{We refer to LEAN 4 when using the versionless expression LEAN.}\]
III. EXPRESSING FUNCTIONAL PROGRAMS VIA SSA

In this section, we survey \texttt{\lambda rc}, LEAN’s intermediate representation and discuss our embedding of \texttt{\lambda rc} into MLIR’s SSA-based compiler IR.

LEAN4’s \texttt{\lambda pure} IR is a minimal functional language with support for expressing data constructors, pattern matching, function applications, and closure creation. A lower level IR, known as \texttt{\lambda rc}, extends \texttt{\lambda pure} with operational concerns such as reference counting within the same IR. The current LEAN4 compiler compiles \texttt{\lambda rc} down to C, followed by invoking a C compiler to link with the runtime and generate binaries. Our MLIR backend generates an MLIR dialect (\texttt{lp}) from \texttt{\lambda rc}, and then continues compilation using the MLIR compilation pipeline (Figure 3).

We introduce the \texttt{lp} dialect (which stands for \texttt{\lambda pure}, though we also support \texttt{\lambda rc} reference counting instructions) within MLIR, with the following set of high-level operations (Figure 2):  

- \texttt{lp.int} to create machine integers and GMP-based big-integers.  
- \texttt{lp.switch} to pattern-match on integers.  
- \texttt{lp.construct} to create algebraic data types.  
- \texttt{lp.getlabel} to extract the tag of an algebraic data type.  
- \texttt{lp.project} to extract out fields from an algebraic data type.  
- \texttt{lp.pap, lp.papextend}, to express partial function applications (closure creation).  
- \texttt{lp.joinpoint, lp.jump}, for representing structured control flow using join points.  
- \texttt{lp.inc, lp.dec} for reference counting.  
- \texttt{lp.return} to return values from \texttt{lp} control flow.

We note that the dialect is \textit{feature complete} to represent a functional programming language. It supports all the core constructs necessary, including closures and partial applications, data constructors and pattern matching on data constructors, as well as support for reference counting. This completeness is a strength of \texttt{lp}, since we are able to compile a realistic functional programming language (LEAN4) with our compiler pipeline. Like \texttt{\lambda rc}, the \texttt{lp} dialect uses a single type, denoted by \texttt{lp.t}, to represent values that live on the heap, i.e., boxed values. We also use standard integer types such as \texttt{i32} and \texttt{i8} as necessary to express interactions with machine integers. \texttt{lp} is \textit{type erased}, as we have erased most typing information from the LEAN source program. We are left with just enough of the type information to generate code that is aware of types such as \texttt{int}, \texttt{float}, pointers, as well as integer bit widths.

A. Integers & Switch Cases

LEAN supports arbitrarily sized natural numbers and integers with the \texttt{Nat} and \texttt{Int} types. LEAN also guarantees that small integers are represented by a machine word. To support this,


---

def intUsage : Nat -> Nat | 42 => 43 | _ => 99999999
def eval(%x: i32, %y: i32, %z: i32) -> i32 {
  @default -> {
    %out = lp.int 43 : !lp.t
    lp.return %out : !lp.t
  },
  @default -> {
    %out = lp.bigint "99999999" : !lp.t
    lp.return %out : !lp.t
  }
}

lp introduces the lp.bigint operation to construct arbitrary sized integers and the lp.int operation to construct machine word sized integers. The LEAN4 compiler lowers lp.bigint to runtime calls to create big-integers, while lp.ints are lowered to machine integers.

Note that the runtime does not distinguish between naturals and integers — they lower to the same signed machine level representation, and are only distinguished at the LEAN level by the dependently typed theory.

As LEAN integers can be both machine integers and big-integers, pattern matching on a LEAN integer must anticipate the scrutinee as being either a machine integer or a big-integer. Arc hides this complexity by staging the pattern match: (1) First, we check whether one LEAN integer is equal to another using a runtime call to @lean_nat_dec_eq. This function handles equality checking between machine-machine, machine-bigint, and bigint-bigint integers uniformly. (2) If the integers are equal, we execute the pattern match.

In more detail, the @lean_nat_dec_eq function receives two integers of type !lp.t as input, and returns either a 0 or a 1 of type i8 to indicate whether the inputs are equal. We switch-case on the return value with the lp.switch operation which dispatches control to a matching switch arm. If no switch arm matches, control is dispatched to the @default arm.

For concreteness, consider the example program in Figure 4. To compile the first arm of the pattern match 42 =>, we call lean_nat_eq_dec to check if the argument %arg equals 42. If this is true, we proceed to execute the right hand side of the pattern match and return 43. The other arm _ => of the switch becomes the @default branch of the lp.switch which returns 99999999.

---

Fig. 4: Program demonstrating integer manipulation with lp.int, lp.bigint, and lp.switch. We use lp.switch to pattern match on raw integer values. LEAN integers are type-erased to create a uniform representation for small and large integers. Integers are compared using the runtime call @lean_nat_dec_eq.

---

B Code Duplication

\[
\begin{align*}
| 0, 2, _ & \rightarrow 40 \\
| 0, _ , 2 & \rightarrow 50 \\
| _, _, _ & \rightarrow 60
\end{align*}
\]

---

A Pseudocode example

def eval(%x: i32, %y: i32, %z: i32) -> i32 {
  @default -> {
    %out = lp.bigint "99999999" : !lp.t
    lp.return %out : !lp.t
  }
}

---

C Deduplication via join-points

def eval(%x: i32, %y: i32, %z: i32) -> i32 {
  @default -> {
    %out = lp.bigint "99999999" : !lp.t
    lp.return %out : !lp.t
  }
}

---

B Join Points

The naive lowering of case statements to switch operations can result in code duplication across the right hand sides of nested cases. We wish to avoid such duplication to ensure efficient compilation of pattern matching. Consider a case statement that matches on two arguments (Figure 5). For the first two cases 1 and 2, we expect that the first argument is
0, and the second or the third argument is 2. If this is not the case, we execute the default case 3.

As \texttt{lp.switch} can only match on a single integer at a time, we introduce two nested \texttt{lp.switch} statements when lowering this pattern match. The outer switch analyzes the first argument and the inner one analyzes the second argument. If either case 1 or case 2 fails, we must execute the code for the default case 3. When directly lowering to nested switch operations, the expression that computes the result of the default case is duplicated into multiple branches of the switch operations (duplication indicated by arrows in Figure 5 A). In case of complex pattern matches with many variables, this duplication can be very costly.

To remedy such code duplication, as well as to increase the expressiveness of the IR to encode complex control flow, LEAN uses \texttt{join-points} \cite{9, 10}. These \texttt{join-points} introduce the ability to create labels within the IR that control flow can jump to. Intuitively, a label is a \textit{point} in the program, and jumping to a label \textit{joins} control flow across the program at that point. The code duplication that we observed in Figure 5 A is eliminated via join-points (Figure 5 B). We create a common region 4 that performs the default action of returning 60. Code duplication is avoided by introducing two \texttt{lp.jumps} as indicated by the arrows. Conceptually, the combination of \texttt{lp.joinpoint} and \texttt{lp.jump} allows us to represent local, named closures within the IR, since the join-point region can refer to values that were defined before the join-point, as well as the arguments that are passed by \texttt{lp.jump}. The difference between the join-point closures and regular closures is that join-point closures are known to not escape, and all of their call sites are within the function body. This allows join-points to be efficiently lowered to jumps, unlike regular closures which require us to build heap objects that represent the closure. Together, \texttt{lp.joinpoint} and \texttt{lp.jump} provide a powerful tool to decrease code duplication incurred by complex pattern matching.

\textbf{C. Data Constructors & Pattern Matching}

In this section, we describe \texttt{lp}'s support for constructing data types via data constructors, and destructuring data types via pattern matching. A data constructor is conceptually a tagged union. Thus, we construct data using \texttt{lp.construct}, which receives a tag and a sequence of arguments. The tag denotes which variant of a given data type is being built, and the argument list denotes the fields of the data constructor.

For example, if we wish to build linked lists (Figure 6), we would need two tags, one for the empty list (\texttt{Nil / tag 0}) and one for a cons-cell (\texttt{Cons / tag 1}). The function \texttt{@singleton} constructs a list with a single cons-cell by constructing a nil value and linking a cons-cell to the nil value.

To compute the length of the linked list, we pattern match on the tag of the data constructor. If the tag is 0 (the object is \texttt{Nil}), we return zero. Otherwise, the tag must be 1 (the object is \texttt{Cons}) and so we recursively call length on the rest of the list and add one. We extract the tag via \texttt{lp.getlabel}, pattern match on the tag using \texttt{lp.switch}, and extract the list pointer from the cons-cell using \texttt{lp.project}.

\textbf{D. Closures}

LEAN supports two types of function application: (1) regular (saturated) function applications where the caller provides as many arguments as the arity of the function, (2) partial function applications, where the caller provides fewer parameters than the function's arity. In the case of a regular function application, we call the function eagerly, and reuse MLIR's function call infrastructure to lower these. For partial function applications, we build a \textit{closure} that holds onto the arguments that are supplied, and waits for further arguments to be provided before the call can be made. Once a closure receives all its arguments, the function held by the closure is invoked with all the arguments that are held by the closure.

Partial applications require some care. First, to build a closure, we introduce the \texttt{lp.pap} (partial application) operation, which partially applies a function to some arguments and builds a closure. To \textit{extend} a closure with more arguments, we introduce the \texttt{lp.papextend} (partial application extend) operation, which takes as inputs a closure and further arguments.
def k (x: Nat) (y: Nat): Nat := x
def k10: Nat -> Nat := k 10
def ap42 (f: Nat -> Nat -> Nat): Nat -> Nat := f 42
def k42: Nat -> Nat := ap42 k

global @kslot : !lp.t
def @init() {
  %k = lp.pap @k
  store @kslot, %k // initialize @kslot
}
def @entrypoint(%argc: i64, %argv: i8*) {
  call @init() // setup top-level closures
  call @entrypoint(%argc, %argv) // run program
}

def @main(%argc: i64, %argv: i8*) {
  call @init() // initialize @kslot
  call @entrypoint(%argc, %argv) // run program
}

We first run init() which initializes closures, followed by entrypoint which is the entry point to the LEAN program.

Finally, we draw attention to subtlety when invoking ap42 with the argument k. ap42, which obeys LEAN’s calling convention, expects a closure as an argument. But what is the closure associated to k? We don’t have one, as k is a top-level, raw function, not a LEAN closure. This mismatch requires us to be able to create empty closures for top-level functions. This is performed by the runtime, where we have an initialization phase which creates top-level closures. In this case, the closure associated to k is initialized in the global variable @kslot by the function @init.

As for lowering, lp.pap and lp.papextend are lowered into runtime calls which manipulate closures. A closure is represented in-memory by a function pointer and a list of pointers to the arguments which are held by the closure.

E. Tail Calls

λrc explicitly keeps track of calls which must be tail calls for memory consumption guarantees. To respect this, we generate an LLVM call annotated with a musttail attribute, which enforces that these calls must be tail calls. If a call that is annotated as a musttail cannot be guaranteed to be a tail call, then the LLVM optimizer provides an error. This is critical as tail-call-optimization is guaranteed by the LEAN language semantics, but is not guaranteed by the lowering to C as the C standard does not require guaranteed tail call elimination.

F. Reference Counting

Since LEAN is a functional language, one does not explicitly refer to memory allocation or de-allocation. Hence, LEAN implements automatic memory management via reference counting. To incorporate reference counting for objects that are created on the heap, we expose the operations lp.inc and lp.dec to increment and decrement reference counts. These are lowered to corresponding LEAN runtime calls.

G. Linking against the Lean Runtime

We link against the LEAN runtime which is written in C, by compiling to LLVM, and then linking using llvm-link. We perform this step as many performance critical runtime routines, including recounting, are written in C, and are inlined by the leanc backend. Thus, for performance parity, we compile the C code down to LLVM and link the runtime, thereby providing LLVM visibility to LEAN’s runtime symbols.

IV. THE RGN DIALECT

In this section, we introduce the rgn dialect to represent and optimize control flow within λrc. We describe the semantics of the rgn dialect, the lowering from λ’s control-flow constructs (lp.switch, lp.joinpoint, lp.jump) to rgn constructs, and the adaptation of classical SSA optimizations to rgn. The rgn dialect has two instructions:

- The rgn.val instruction creates an SSA value which names a region definition. These named values declare regions prior to being called. This is conceptually a continuation, where the region represents a computation that is to be performed when invoked.
• The \texttt{rgn.run} terminator instruction transfers control flow to a region with the supplied arguments. This is conceptually invoking a continuation. We branch to the region that is to be executed and continue execution from this region.

We allow \texttt{rgn.val} values to be passed as operands to MLIR’s \texttt{select} and \texttt{switch} instructions. These instructions allow us to pick a value based on a boolean or an integer respectively. We use this to express choosing which region is to be executed for control flow. We do not allow \texttt{rgn.val} operations to interact with other operations; in particular, they may not be passed to other functions as arguments and may not be returned. This ensures that all uses of \texttt{rgn.val} can be statically analyzed by our compiler, while still allowing us to leverage the analyses and optimizations provided for \texttt{select} and \texttt{switch}.

A. Lowering \texttt{lp} to \texttt{rgn}

A 2-Way Switch
\begin{verbatim}
lp.switch %<tag>
  <alt-0> -> {
    // rhs-0
  },
  <alt-1> -> {
    // rhs-1
  },
  ...
  @default -> {
    // rhs-default
  }
\end{verbatim}

\begin{verbatim}
%r0 = rgn.val {
  // rhs-0
}
%r1 = rgn.val {
  // rhs-1
}
%r = select %<tag>, %r0, %r1
rgn.run %r
\end{verbatim}

B N-Way Switch
\begin{verbatim}
lp.switch %<tag>
  <alt-0> -> {
    // rhs-0
  },
  <alt-1> -> {
    // rhs-1
  },
  ...
  @default -> {
    // rhs-default
  }
\end{verbatim}

\begin{verbatim}
%r0 = rgn.val {
  // rhs-0
}
%r1 = rgn.val {
  // rhs-1
}
...
%rd = rgn.val {
  // rhs-default
}
%r = switch %<tag>
  <alt-0> -> %r0,
  <alt-1> -> %r1,
  ...
  default -> %rd
rgn.run %r
\end{verbatim}

C Join Points
\begin{verbatim}
lp.joinpoint @<lbl> {
  // after-jump
},
  // pre-jump
  ...
  lp.jump @<lbl> (%arg)*
}
\end{verbatim}

\begin{verbatim}
%lbl = rgn.val {
  // after-jump
}
  ...
  rgn.run %lbl (%arg)*
\end{verbatim}

Fig. 8: Lowering the control flow of \texttt{lp} to \texttt{rgn}. \texttt{lp.switch} with two cases is lowered via \texttt{select} (A). \texttt{lp.switch} with many cases is lowered via \texttt{switch} (B). \texttt{lp.joinpoint} is lowered to a combination of \texttt{rgn.val} and \texttt{rgn.run} (C).

The lowering from \texttt{lp} to \texttt{rgn} is straightforward (Figure 8). \texttt{lp} has two control flow constructs, \texttt{lp.switch} & \texttt{lp.joinpoint}. \texttt{lp.switch} is lowered by converting every right hand side of a pattern match to a \texttt{rgn.val}, then selecting the correct right hand side that needs to be executed, and finally executing the selected right hand side via \texttt{rgn.run}. We lower the selection to either a \texttt{select} or a \texttt{switch} depending on the number of cases. We lower \texttt{lp.joinpoint} by converting the jump target to a \texttt{rgn.val}, the \texttt{lp.jump} to a \texttt{rgn.run}, replacing the joinpoint by the region that is to be executed before the jump.

B. Optimization Passes for \texttt{rgn}

In this section, we adapt classical optimization passes to \texttt{rgn}’s region-values. We explore optimizations that are made possible by these adapted transformations.

1) Dead Region Elimination: Dead code elimination requires no changes to work with region values. If a region value is never referenced, then it is never executed. It is thus dead and can safely be removed.

\begin{verbatim}
A
out =
  case True of
    True -> 3
    False -> 7
A
\end{verbatim}

\begin{verbatim}
B
%b = <external>
%w = rgn.val { return 7 }
%z = select %b, %w, %w
rgn.run %z
\end{verbatim}

\begin{verbatim}
C
%t = constant True
%x = rgn.val { return 3 }
%y = rgn.val { return 5 }
rgn.run %x
\end{verbatim}

\begin{verbatim}
D
%b = <external>
%w = rgn.val { return 7 }
%z = select %b, %w, %w
rgn.run %w
\end{verbatim}

Figure A above is the original program. We show the naive translation to \texttt{rgn} in B. The optimizations for \texttt{select} on a constant value \texttt{true} kick in and replace \texttt{z} by \texttt{x} in C. Finally, the running of a known region is replaced by the constant value that corresponds to the region in D.

2) Global Region Numbering: We introduce global value numbering for regions.\footnote{MLIR does not perform global value numbering as it is unclear how to define value numbers for instructions with regions, as in general arbitrary regions do not yet have a prescribed semantics.} For \texttt{straight line regions}, the value number of the region is defined as a rolling hash of the value numbers of all instructions within the region. Two regions are defined to have the same value number if and only if the sequence of instructions in the two regions have the same value numbers in identical order. The restriction that a region has a single basic block is not as restrictive as it first appears, since languages with high level control flow can always be lowered to regions with a single basic block, with control flow expressed via \texttt{rgn} and \texttt{select}. Global value numbering of regions identifies redundant computations across branches of control flow. This allows us to fold away equivalent computations.

\begin{verbatim}
A
out =
  case b of
    True -> 7
    False -> 7
A
\end{verbatim}

\begin{verbatim}
B
%b = <external>
%w = rgn.val { return 7 }
%z = select %b, %w, %w
rgn.run %z
\end{verbatim}

\begin{verbatim}
C
%b = <external>
%w = rgn.val { return 7 }
%z = select %b, %w, %w
rgn.run %z
\end{verbatim}

\begin{verbatim}
D
%b = <external>
%w = rgn.val { return 7 }
%z = select %b, %w, %w
rgn.run %z
\end{verbatim}
Figure A above is the original program. We show the naive translation to rgn in B, where the value \( %b \) is external to the current scope being analyzed. The Region-CSE algorithm fuses \( %x \) and \( %y \) into one region, relabelled \( %w \) in C. Finally, the select of two equal values (\( %w, %w \)) on both branches is folded away in D.

C. Lowering rgn to std

In this section, we describe how to lower rgn to a traditional SSA-based IR without regions. For concreteness, we describe a lowering to MLIR’s std dialect. We anticipate no changes necessary to lower to another purely SSA based IR such as LLVM.

Since the semantics of rgn is given entirely by adding extra structure to flat CFGs, rgn can be lowered by forgetting this extra structure. The lowering is driven entirely by rgn.run. First, we lower rgn.run by matching on the argument — (1) A rgn.run of a known rgn.val is compiled to a branch of the region that is run. (2) A rgn.run of a switch (or select) is compiled to a jump-table. Finally, dead rgn.val instructions are entirely dropped.

V. Evaluation

We evaluate our work by analyzing the correctness, performance, and memory usage of the proposed SSA-based LEAN backend. Our evaluation was run on an Intel Xeon CPU @ 2.20GHz with 126GB memory. We first discuss the correctness of our compiler. We then consider the end-to-end performance, by comparing runtimes of programs compiled by our compiler versus the baseline Lean compiler on LEAN’s benchmark suite, and then perform an evaluation of the rgn optimizations on the benchmark suite. Finally, we have a holistic discussion about the costs and benefits of our new backend in comparison to the current C backend of LEAN4.

A. Correctness

We test our compiler for correctness against the LEAN test suite, which consists of 648 test cases, out of which we pass 648 (100%) of tests. This shows that our compiler correctly implements the semantics of \( \lambda rc \) and interfaces properly with the LEAN runtime system. This ensures that our evaluation is representative of functional programming workloads.

B. Performance Analysis

We characterize the performance of our backend in comparison to LEAN’s default backend (commit hash be4cf60) using LEAN’s benchmark suite. The programs in the LEAN benchmark suite represent workloads commonly encountered by functional programming languages:

- binarytrees and binarytrees-int implement a purely functional binary tree lookup, insert, and delete benchmark.
- const_fold implements constant folding on an expression based language.
- deriv benchmarks derivative computations on expression trees.
- filter implements filtering values from a linked list based on a predicate.
- qsort implements real in-place quicksort using LEAN’s arrays.
- rbmap_checkpoint implements red-black tree insertion and lookup.
- unionfind implements Tarjan’s union-find algorithm.

The reported timing results (Figure 9) are gathered as the geometric mean of runtimes over ten runs. We report a geometric speedup of 1.09x with the baseline LEAN compiler, which validates our claim that we achieve performance parity with the LEAN4 compiler.

Finally, we compare the effectiveness of the rgn dialect to that of the rc dialect (Figure 10) by running three versions of the pipeline: (a) a baseline of our MLIR pipeline which receives optimized code from the \( \lambda rc \) compiler, which we compare against (b) unoptimized \( \lambda rc \) code which is left unoptimized by rgn before passing to LLVM. We find that the performance is almost identical across the three variants. We conjecture that LLVM’s sophisticated control flow analysis is capable of optimizing away the unoptimized IR we pass it.

In summary, we show that our SSA-based optimization pipeline matches the hand-written LEAN optimization pipeline, while reasoning about functional constructs purely via SSA.
The \texttt{rgn} dialect allows us to cleanly express \texttt{\lambda rc}'s semantics with SSA, thereby making SSA+regions an attractive choice to represent functional programming languages in traditional imperative compiler tooling.

C. Qualitative Analysis

In this section, we compare and contrast our backend to LEAN's backend from the perspective of a compiler engineer. We first survey a list of salient differences between LEAN’s \texttt{\lambda rc} backend and our MLIR-based backend (Figure 11).

| Feature                | \texttt{\lambda rc} + C | \texttt{lp} + \texttt{rgn} |
|------------------------|-------------------------|---------------------------|
| Backend                | C                       | MLIR                      |
| Vectorization          | No                      | affine, linalg            |
| Testing harness        | makefile                | FileCheck, llvm-lit      |
| Constant folding       | Hand-written            | MLIR rewriter             |
| CSE                    | Hand-written            | MLIR builtin              |
| DCE                    | Hand-written            | MLIR builtin              |
| Inliner                | Hand-written            | MLIR builtin              |
| Test minimization      | None                    | mlir-reduce               |
| Debug information      | None                    | Possible                  |
| IDE support            | None                    | LSP                       |
| Tail call optimization | Heuristic               | Guaranteed                |

Fig. 11: Ecosystem differences between our MLIR-based backend and the current LEAN4 tooling. Note that MLIR provides us access to a rich ecosystem of tooling for compiler development.

Notice the availability of rich tooling due to our use of the MLIR compiler framework — we are able to reuse existing optimization infrastructure for constant folding, common subexpression elimination (extended by \texttt{rgn}), dead code elimination, and inlining. Similarly, we use LLVM’s test harness infrastructure for parallel test running and failure reporting. The \texttt{\lambda rc} pipeline generates C code from which crashes and miscompiles are difficult to debug as there is no association between the generated C program and the source LEAN program. One can potentially teach the LEAN frontend to preserve debug information to be passed to MLIR. The MLIR compiler framework makes retaining debug information a top priority that enables the compiler engineer to correlate miscompiles to LEAN program source locations accurately. Finally, as indicated before, LEAN requires guaranteed tail call elimination which the current C based backend cannot. Our MLIR-based compiler guarantees this by using LLVM’s \texttt{musttail} annotations.

In summary, our compiler lays the foundation for LEAN to access an extensive suite of analyses and optimizations. It also provides the compiler engineer tooling for debugging and unit testing. Lastly, it allows us to provide deterministic tail call elimination, which is necessary for the semantics of \texttt{\lambda rc}.

VI. RELATED WORK

In this section, we survey related work on functional intermediate representations and uses of SSA in compilers for functional languages.
encode algebraic data types and case analysis. They perform whole program compilation, and use aggressive whole program analyses reminiscent of GRIN to analyze and eliminate overhead. Furthermore, their adaptations to SSA do not involve regions. They choose to encode special terminator operations using and extending SSA algorithms to operate on rgn. Finally, we implement our suggestions and demonstrate that our new whole program compilation, and use aggressive whole program C. Description

B. Artifact Check-List (Meta-Information)

and the table come with the docker container. needed to run all the experiments. Scripts to generate the figures consists of a docker container with accompanying scripts to create an MLIR-based backend for LEAN which achieves performance parity with the LEAN4 compiler. The artifact to introduce rgn, an intermediate representation that is designed for region analysis and optimization. We adapt classical functional language transformations to the SSA setting by using and extending SSA algorithms to operate on rgn. Finally, we implement our suggestions and demonstrate that our new LEAN4 backend based on lp and rgn passes all regression tests and achieves performance parity with the existing LEAN4 compiler. This foundation is a force multiplier, as we can now express the LEAN4 semantics in a compiler framework that has been designed to be analyzed, optimized, and offers mature support for performance and regression testing. We envision lp and rgn together acting as the bedrock for SSA-based optimizing compilers for many more functional programming languages.

APPENDIX

A. Abstract

The artifact’s goal is to show how regions+SSA allows us to create an MLIR-based backend for LEAN which achieves performance parity with the LEAN4 compiler. The artifact consists of a docker container with accompanying scripts to replicate figure 9, 10. The docker container is the only piece needed to run all the experiments. Scripts to generate the figures and the table come with the docker container.

B. Artifact Check-List (Meta-Information)

- Program: A custom LEAN4 backend based on the MLIR compiler toolchain, along with LEAN4’s test suite for testing, and LEAN4’s benchmark suite for performance analysis.
- Compilation: A C++ compatible compiler to bootstrap LLVM/MLIR as well as LEAN4.
- Run-time environment: Any operating system that supports Docker.
- Hardware: Any x86 machine.
- Output: PDF files replicating Figure 9 and Figure 10 and a successful run of the entire LEAN4 test suite.
- How much disk space required (approximately)?: 10GB.
- How much time is needed to prepare workflow (approximately)?: 2 hours.
- How much time is needed to complete experiments (approximately)?: 1 hour.
- Publicly available?: Yes.

C. Description

1) How Delivered: The artifact is delivered as a Docker container and is available at http://doi.org/10.5281/zenodo.5786074.

2) Hardware Dependencies: None.

3) Software dependencies: The Docker image has dependencies needed to compile MLIR, Lean4, and our compiler tooling to run the test suite.

D. Experiment Workflow

Access the docker image cgo22.docker from [http://doi.org/10.5281/zenodo.5786074](http://doi.org/10.5281/zenodo.5786074), then run:

```
$ docker load -i cgo22.docker
$ docker run -it siddudruid/cgo21-v4
$ docker run -it siddudruid/cgo21-v4
$ su nonroot # switch to non-root user
$ export PATH=/code/lz/build/bin:$PATH
$ export PATH=/code/lean4/build/release/stagel/bin:$PATH
$ export PATH=/code/ll/build/bin:$PATH
$ export LEANLIB=/code/lean/build/release/stagel/lib
$ export LEANLIB=/code/lean/build/release/stagel/lib
$ export LEANLIB=/code/lean/build/release/stagel/lib
$ export LEANLIB=/code/lean/build/release/stagel/lib
$ cd /code/lean4/build/release && make -j4 test
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-time.py --data --plot --nruns 10
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-time.py --data --plot --nruns 10
```

Upon running make -j4 test, the test output is printed to stdout. The scripts speedup-time.py and speedup-rgn-time.py, produce PDFs speedup-time.pdf and speedup-rgn-time.pdf in the directory /code/lz/test/lambdapure/compile/bench/:

```
$ cd /code/lz/test/lambdapure/compile/bench/speedup-time.pdf
$ cd /code/lz/test/lambdapure/compile/bench/speedup-rgn-time.pdf
```

To open the pdf file, keep the container running, and in another shell instance, use the docker cp command to copy files from within the container out to the host:

```
$ docker cp <CONTAINERID>:<PATH/INSIDE/CONTAINER> /path/to/destiny
```

For more about docker cp, please see: [https://docs.docker.com/engine/reference/commandline/cp/](https://docs.docker.com/engine/reference/commandline/cp/)

E. Evaluation and Expected Result

On running the test suite with:

```
$ cd /code/lean4/build/release && make -j test
```

We find that the output is:

```
100% tests passed, 0 tests failed out of 648
```

See that we pass all tests. The test script that is run can be found at /code/lean4/test/common.sh which can be seen to invoke our compiler pipeline.

To generate performance plots, we run:

```
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-time.py --data --plot --nruns 10
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-rgn-time.py --data --plot --nruns 10
```

For speedup-time.pdf, we expect a geomean speedup of 1.0x with the baseline lean4, and for speedup-rgn-time.pdf, we expect a geomean speedup of 1.0x between region optimizations and no region optimizations.

F. Full Workflow Example

Grab docker image from http://doi.org/10.5281/zenodo.5786074:

```
$ curl https://zenodo.org/record/5786074/files/cgo22.docker?download=1 > cgo22.docker
```

Access the docker image cgo22.docker:

```
$ docker run -it siddudruid/cgo21-v4
```

5281/zenodo.5786074), then run:

```
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-time.py --data --plot --nruns 10
$ cd /code/lz/test/lambdapure/compile/bench && ./speedup-time.py --data --plot --nruns 10
```

Total Test time (real) = 399.28 sec

100% tests passed, 0 tests failed out of 648

```
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