Neutrinoless Double Beta Decay with R-parity Violation

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Abstract

We consider recently observed neutrinoless double beta decay in the context of the minimal supersymmetric standard model with R-parity violating couplings $\lambda'$. We observe that most of the current experimental bounds on the R-parity violating couplings do not exclude the possibility that the neutrinoless double beta decay is caused by R-parity violation. But if we consider $K - \bar{K}$ oscillation, we observe that we have to make the R-parity violating couplings generation-dependent to accommodate with the observed neutrinoless double beta decay. And furthermore, we need some mechanism to cancel the contribution to $K - \bar{K}$ mixing from a large R-parity violating coupling. We realized this cancellation by assuming that the first- and the second-generation of quark sector do not couple with the first-generation lepton sector by R-parity violating couplings except the term $W = \lambda'_{111}L_1Q_1D^c_1$, which is responsible for the observed neutrinoless double beta decay.
Recently evidence for neutrinoless double beta decay has been found by the HEIDELBERG-MOSCOW double beta decay experiment \[1\]. The half-life of \(^{76}\text{Ge}\) is reported to be:

\[
T_{1/2}^{0\nu} = (0.8 - 18.3) \times 10^{25}\text{yr}.
\] (1)

This means that, lepton number is broken in nature. In the Standard Model (SM), lepton number is conserved, and this evidence becomes signature for physics beyond the SM.

We can realize lepton number violation in the R-parity violating Minimal Supersymmetric Standard Model (MSSM) (for reviews, see \[2\]). The R-parity violating couplings are:

\[
W = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k
\] (2)

These terms violates lepton number and baryon number simultaneously, and thus lead to rapid proton decay. So we must forbid some or all of these terms. Usually, to achieve that, a \(Z_2\)-symmetry called as “R-parity” is imposed. R-parity is defined as:

\[
R_p = (-1)^{3B+L+2S},
\] (3)

where B is the baryon number of the particle, L is the lepton number of the particle and S is the spin of the particle. If we impose R-parity, all the couplings in equation (2) are forbidden, and no dangerous phenomena occur.

But there is another possibility. \(Z_3\)-symmetry is anomaly-free discrete gauge symmetry, and can protect proton from rapid decay \[3\]. This symmetry forbids baryon number violation, but allows lepton-number violation. So it is worthwhile to investigate the lepton-number violating phenomena as resultants of a \(Z_3\)-symmetry \[4\]. The charge assignment of this \(Z_3\)-symmetry is shown in table \[4\].

Neutrinoless double beta decay was considered in the context of the MSSM with lepton-number violating R-parity \[5, 6\]. Detailed calculations for the neutrinoless double beta decay rate including nuclear matrix elements was done in \[6\]. When
Table 1: Charge assignment under the discrete gauge symmetry. $\alpha^3 = 1$.

| particle | $Q$ | $U^c$ | $D^c$ | $L$ | $E^c$ |
|----------|-----|-------|-------|-----|-------|
| charge   | 1   | $\alpha^2$ | $\alpha$ | $\alpha^2$ | $\alpha^2$ |

only $\lambda'$ couplings are considered, the Feynman diagrams contributing to the neutrinoless double beta decay are drawn in figure 1. Since squark- and gluino-mediated process dominates, we drop the contribution from neutralino- and slepton-exchange diagrams.

Following reference [6], the recent result yields following constraints:

$$1.6 \times 10^{-4} \left( \frac{m_{\tilde{q}}}{100 \text{GeV}} \right)^2 \left( \frac{m_{\tilde{g}}}{100 \text{GeV}} \right)^{1/2} < \lambda'_{111} < 3.6 \times 10^{-4} \left( \frac{m_{\tilde{q}}}{100 \text{GeV}} \right)^2 \left( \frac{m_{\tilde{g}}}{100 \text{GeV}} \right)^{1/2},$$

(4)

where we assume $m_{\tilde{d}_R} = m_{\tilde{u}_L} \equiv m_{\tilde{q}}$.

By scanning the parameter region $100 \text{GeV} < m_{\tilde{q}} < 2000 \text{GeV}$, $200 \text{GeV} < m_{\tilde{g}} < 2000 \text{GeV}$, we make a contour plot of the allowed values of $\lambda'_{111}$. It is shown in figure 2. Here we have conservatively adopted $m_{\tilde{g}} > 200 \text{GeV}$. This figure shows the allowed region of $m_{\tilde{q}}$ and $m_{\tilde{g}}$ for given values of $\lambda'$. We can see that as $\lambda'$ couplings become smaller, the allowed region of $m_{\tilde{q}}$ and $m_{\tilde{g}}$ is lowered. This is because if squark and gluino masses are heavy, their contribution to the neutrinoless double beta decay becomes small.

It is interesting to compare the combined constraint on $\lambda'$ v.s. squark and gluino masses obtained here, with those from other experimental results. There are many experimental results which can constrain $\lambda'$. Hereafter, we study them in detail.

For example, the existence of R-parity violation leads to a violation of the universality of quark and lepton couplings to the W boson. In the quark sector, the R-parity violating couplings $\lambda'_{ijk} L_i Q_j D^c_k$ gives an additional contribution to the quark semileptonic decay (e.g., in nuclear $\beta$ decay) like muon decay. The effective
coupling becomes:

\[
g^2 \left[ V_{ud} + r'_{11k} (\bar{d}_R^k) \right],
\]

(5)

where \( r'_{11k} \) is defined as:

\[
r'_{11k}(\bar{l}) = \frac{m_W^2 |\lambda'_{ijk}|^2}{g^2 m_l^2}.
\]

(6)

The CKM matrix elements are experimentally determined from the ratio of the \( Q \to qe\nu_e \) to \( \mu \to \nu_\mu e\nu_e \) partial widths. The experimental value is related to theoretical quantities by

\[
|V_{ud}|^2_{\text{exp}} = \frac{|V_{ud} + r'_{11k}(\bar{d}_R^k)|^2}{1 + r_{12k}(\bar{e}_R^k)|^2},
\]

(7)

where \( r_{12k} \) is defined like \( r'_{11k} \). A comparison with the experimental value:

\[
\sum_j |V_{udj}|^2_{\text{exp}} = 0.9979 \pm 0.0021
\]

(8)

yields the limit [7]:

\[
|\lambda'_{11k}| < 0.03 \left( \frac{m_{d_R}}{100\text{GeV}} \right).
\]

(9)

at the 2\( \sigma \) level.

This does not exclude the possibility that R-parity violation is responsible for the neutrinoless double beta decay. For example, for \( m_{d_R} = m_{\bar{q}} = 500\text{GeV} \), this limit becomes \( |\lambda'_{11k}| < 0.15 \), which is compatible with the allowed values of \( \lambda'_{111} \) shown in figure 2.

The decay rate of pion into electron and muon is also changed in the presence of the R-parity violating couplings. The ratio \( R_\pi \equiv \Gamma(\pi \to e\nu) / \Gamma(\pi \to \mu\nu) \) is

\[
\frac{R_\pi(\text{expt})}{R_\pi(\text{SM})} = 0.991 \pm 0.18.
\]

(10)

R-parity violation gives an effective contribution to \( R_\pi \) [7]:

\[
R_\pi = R_\pi(\text{SM}) \left[ 1 + \frac{2}{V_{ud}} [r'_{11k}(\bar{d}_R^k) - r'_{21k}(\bar{d}_R^k)] \right].
\]

(11)
The experimental value \( \langle 10 \rangle \) set upper limit on the R-parity violating couplings as:

\[
|\lambda'_{11k}| < 0.05 \left( \frac{m_{\tilde{d}_R}}{100 \text{GeV}} \right).
\]  

(12)

This is weaker limit compared to the equation \( \langle 9 \rangle \), thus we can neglect this limit in this study.

The decay \( K^+ \to \pi \nu \bar{\nu} \) is also modified in the presence of the R-parity violating couplings \( \langle 8 \rangle \). We obtain:

\[
\frac{\Gamma[K^+ \to \pi^+ \nu \bar{\nu}_i]}{\Gamma[K^+ \to \pi^0 \nu e^+]} = \left( \frac{|\lambda'_{ijk}|^2}{4G_F m_{\tilde{d}_R}^2} \right) \left( \frac{|V_{j1} V_{j2}^*|}{|V_{12}^*|} \right)^2.
\]  

(13)

So using \( B(K^+ \to \pi^+ \nu \bar{\nu}) \lesssim 4.4 \times 10^{-10} \) \( \langle 9 \rangle \) and \( B(K^+ \to \pi^0 \nu e^+) = 0.0482 \) \( \langle 10 \rangle \), we obtain the constraint \( \langle 8, 9 \rangle \):

\[
|\lambda'_{ijk}| < 0.0056 \left( \frac{m_{\tilde{d}_R}}{100 \text{GeV}} \right).
\]  

(14)

This constraint is stronger. For example, take \( m_{\tilde{d}_R} = 900 \text{GeV} \), then \( \lambda'_{111} < 0.05 \).

From the figure 2 we can see that gluino mass is constrained in the region:

\[
m_{\tilde{g}} \lesssim 1100 \text{GeV}.
\]  

(15)

Other experiments, like \( K - \bar{K} \) oscillation, and \( B - \bar{B} \) oscillation give stronger limits on the lepton number violating couplings \( \langle 11 \rangle \). But their limit always contain products of two \( \lambda' \). Thus we cannot state strongly that we can derive upper limit on \( \lambda'_{111} \). For example, \( K - \bar{K} \) oscillation gives \( \langle 11 \rangle \):

\[
\text{Re} \left[ \sum_{i,j,j'} \left( \frac{100 \text{GeV}}{m_{\tilde{\phi}}} \right)^2 \lambda'_{ij2} \lambda'_{j'1} V_{j1}^* V_{j'2} \right] < 4.5 \times 10^{-9}.
\]  

(16)

So we cannot extract the information of \( \lambda'_{111} \) from \( K - \bar{K} \) oscillation. Of course if we assume generation-independence of the \( \lambda' \) couplings, we can estimate:

\[
\lambda'_{111} \lesssim 10^{-4} \left( \frac{m_{\tilde{\phi}}}{100 \text{GeV}} \right).
\]  

(17)

As we can see from figure 2, this is so strong constraint that we cannot explain the observed neutrinoless double beta decay if we impose this constraint. So we
can say that if the observed neutrinoless double beta decay is truly the result of R-parity violation, the $\lambda'$ couplings are not generation-independent.

But there still exists a non-trivial problem that how such a large $\lambda'_{111}$ coupling can be consistent with the stringent bound from $K - \bar{K}$ mixing (equation (16)). We should make such a large coupling be cancelled by some mechanism to accomodate with the stringent bound from $K - \bar{K}$ mixing. One way is to assume that

$$\lambda'_{112} = \lambda'_{121} = \lambda'_{122} = 0.$$  \hspace{1cm} (18)

In this case, the contribution from $\lambda'_{111}$ to the equation (16) becomes

$$\text{Re}[\lambda'_{111} \lambda'_{132}^* V_{31}^* V_{12}] < 4.5 \times 10^{-9}.$$  \hspace{1cm} (19)

We substitute $|V_{31}| \sim 0.003$, $|V_{12}| \sim 0.22$ and $\lambda'_{111} \sim 0.005$ into (19), then we get

$$\lambda'_{132}^* < 1.4 \times 10^{-3},$$  \hspace{1cm} (20)

which is moderate value compared to $\lambda'_{111} = 5 \times 10^{-3}$. So we conclude that the large value of $\lambda'_{111}$ can be consistent with the stringent bound from $K - \bar{K}$ mixing.

To summarize, we consider the neutrinoless double beta decay in the context of the Minimal Supersymmetric Standard Model with the lepton-number violating R-parity couplings. We observe that most of the current experiments do not exclude the possibility that R-parity violation is the source of the observed neutrinoless double beta decay. But if the R-parity violating couplings are generation-independent, the constraint on $K - \bar{K}$ oscillation excludes this possibility. Generation-dependency and some mechanism to cancel a large $\lambda'_{111}$ coupling contribution to $K - \bar{K}$ oscillation is needed. We realized this cancellation by assuming $\lambda'_{112} = \lambda'_{121} = \lambda'_{122} = 0$, namely the first- and the second-generation of quark sector do not couple with the first-generation lepton sector by the R-parity violating couplings, except the coupling which is responsible for the neutrinoless double beta decay, $\lambda'_{111}$.

**Note Added**

After the submittion of this letter, we learned from Dr.Liu that they considered within the framework of R-parity violating supersymmetry, the sleptons play...
a partial role in electroweak symmetry breaking. The scalar neutrinos get non-vanishing vacuum expectation values (vevs). These non-zero vevs break the family symmetry (say, $Z_3$) naturally. This breaking of the family symmetry may result in the realistic pattern of the fermion masses [12, 13]. To be specific, Refs. [12, 13] proposed that the muon mass originates from the sneutrino vevs, whereas tau from Higgs. Neutrino masses are discussed in Ref. [14]. Especially in [14], they have obtained an electron-neutrino Majorana mass to be around 0.1 eV.

And we also learned from Dr. Dedes that the bounds on all R-parity violating couplings have been collated and updated in their paper [15].

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Figure 1: The processes relevant for neutrinoless double beta decay.
Figure 2: The allowed region for given values of $\lambda'_{111}$. Here we take $\lambda'_{111} = 0.005, 0.05, 0.1$. 