Rethinking the Properties of the Quark-Gluon Plasma at $T \sim T_c$

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We argue that although at asymptotically high temperatures the QGP in bulk behaves as a gas of weakly interacting quasiparticles (modulo long-range magnetism), at temperatures up to a few times the critical temperature $T_c$ it displays different properties. If the running of the QCD coupling constant continues in the Coulomb phase till the screening length scale, it reaches the strong coupling treshold $\alpha_s(m_D) \sim 1$. As a result, the Coulomb phase supports weakly bound Coulombic s-wave $\bar{c}c$, light quark and even $gg$ states. The existence of shallow bound states dramatically increases the quasiparticle rescattering at low energies, reducing viscosity and thereby explaining why heavy ion collisions at RHIC exhibit robust collective phenomena. In conformal gauge theories at finite temperature the Coulomb binding persists further in the strong coupling regime, as found for $N = 4$ SUSY YM in the Maldacena regime.

Soon after the discovery of QCD, it has been found that at high temperature $T$ the color charge is screened [1] (rather than anti-screened in the vacuum), and the corresponding phase of matter was named the Quark Gluon Plasma (QGP). Also, it has been shown, both analytically and on the lattice, that at high temperature bulk quarks and gluons exist in it as relatively free propagating quasiparticles, modulo color-magnetic effects that are known to be non-perturbative [2]. Although explicit perturbative series for thermodynamical quantities were found to be badly divergent, it was still hoped that some kind of re-summation will make the weak-coupling quasiparticle picture work, as the screening would keep the effective coupling weak anywhere at $T > T_c$. The earliest suggested QGP signal was a disappearance of familiar hadronic peaks -- $\rho, \omega, \phi$ mesons -- in the dilepton spectra [3]. Moreover, even small-size deeply-bound $\bar{c}c$, light quark and even $gg$ states, the electric effects are still strong, causing perturbation theory to fail. But before we go into our reasoning, let us point out that a motivation comes from two recent developments, the experimental discovery of robust collective effects at RHIC, known as radial and elliptic flows, as well as recent advances in lattice simulations at $T = (1-3)T_c$. They have led to two related questions we are going to address: i. Are there bound states of quasiparticles above the QCD phase transition? ii. Why does the Quark-Gluon Plasma behave like a good liquid rather than a dilute gas of quasiparticles?

The former question should not be confused either with the non-perturbative magnetic interaction [2] or with the issue of "screening masses" for space-like correlators [6]. For earlier discussion of real bound states above $T_c$ in the framework of high temperature QCD sum rules see [7]. Based on chiral dynamics driven by instanton it was found that the pi-sigma chiral multiplet should still exist as a resonance in the QGP [8].

Recent numerical lattice studies have found that, contrary to earlier expectations, charmonium states remain bound at least up to $T = 1.5T_c$ [9]. Moreover, the two-direction correlators at finite $T$ found significant deviations from free behavior at $T \sim 3T_c$ [10], in quantitative agreement with [8]. Their analysis by the minimal entropy method have suggested that pseudoscalars and even vector resonances exist above $T_c$. Below, we will show why these states exist, and not just for quarks but for gluonic plasmons as well.

The main idea [11] is that after deconfinement and chiral symmetry restoration at $T_c$, nothing prevents the QCD coupling from running to larger values at lower momentum scale until it is stopped at the screening mass scale. Although at high $T$ the coupling is always small [1], at $T \approx T_c$ it can reach large value $\alpha_s \sim 1$. As we show below, this makes Coulomb binding of the pertinent $s$-wave levels possible, in spite of screening.

Let us start with charmonium problem first, using the non-relativistic Schrodinger equation (and ignoring collisional broadening). For the standard radial wave function $\chi(r) = \psi/r$ it has the usual form with the reduced mass $m = m_c/2$. We will use throughout this work the same in-QGP screened Coulomb potential of the Debye form $V = (4\alpha_s(r)/3r)\exp(-MD/r)$ and use the $(T$-dependent) screening mass as a definition of the length unit to have its shape fixed for fixed coupling. In such units the equation to be discussed reads

$$\frac{d^2\chi}{dx^2} + \left(\kappa^2 + \frac{4m_c\alpha_s(x)}{3MD}e^{-x}\right)\chi = 0$$

with $\kappa^2 = m_cE/M_D^2$. The appearance of a bound level corresponds to zero binding $E = 0, \kappa = 0$. If the coupling constant $\alpha_s$ does not run and is a constant, all parameters of the problem appear in a single combination. Solving the equation, one can find the condition for the bound state to exist

$$\frac{4m_c}{3MD} \alpha_s > 1.68$$

For example, using $4/3\alpha_s = 0.471$ from the vacuum charmonium potential and $m_c = 1.32\text{ GeV}$, as Karsch et al [5] did long ago, one finds a restriction on the screening mass $M_D < M_D^{\text{crit}} = 0.37\text{ GeV}$. Lattice studies of the
screening masses have been carried in [13] and found that for the near-critical QGP $M_D = (2.25 \pm 0.25)T$. So, for $T \approx T_c = 170$ MeV one expects $M_D = 0.34 - 0.42$ GeV. So, the condition (2) is satisfied marginally if at all, and these authors concluded that $\eta_c, J/\psi$ may hardly exist inside the QGP phase.

The loophole in this traditional argument is the assumption that the gauge coupling constant remains frozen in the QGP, at the same value it had in the vacuum charmonium potential. Our main idea is that it does not have to be so: in the QGP, after deconfinement and chiral symmetry restoration, non-perturbative effects are smaller and the charge continues to run to larger values. In Fig.1(a) we compare two such potentials: although the version with the running coupling leads to a smaller potential at smaller distances, it is larger at larger $r$. One cannot of course rely on perturbative formulae in a strong coupling regime (the alternative will be discussed elsewhere [14]), in this work we have decided to simply freeze $\alpha_s$ when it reaches 1. The cusp in the potential reflects on that, and it occurs at about the screening length $r M_D \approx 1$.

The modified potential with the running coupling leads to a more liberal condition for charmonium binding, namely $M_D < 0.62$ GeV. This translates into charmonium remaining bound at

$$T < T_{cc} \approx 1.6 T_c.$$  

This value agrees well with recent lattice measurements [9] mentioned in the introduction, so the modified color potential had passed its first test.

Are there bound states of $\bar{q}q$, and $gg$? For simplicity, in this work we ignore quark masses, as well as instantons and the $U(1)$ axial anomaly, so one can view the chirality of light fermions to be permanently conserved in the QGP. Chiral symmetry excludes the usual mass from being developed, and $L, R$-handed quarks propagate independently. Nevertheless, the propagating quark modes in the QGP have some dispersion curves with the so-called chiral mass, defined as the energy of the mode at zero momentum $M_q = \omega (\vec{p} = 0)$, perturbatively $M_q = g T/\sqrt{6}$ to the lowest order [15]. In weak coupling there are two fermionic modes: i. with the same chirality and helicity the dispersion curve at small $p$ is $\omega = M_q + p^2/3 + \ldots$; ii. with the opposite chirality and helicity the mode is often called a “plasmino”, its dispersion curve has a shallow minimum at $p = 0.17 g T$ with the energy $E_{\min} = 0.38 g T$ slightly below $M_q$. The latter is related to a new in-matter gluonic mode, the plasmon [1].

For a general analysis of these modes see [16], where in particular both modes should have the same effective mass. Lattice data on the dispersion curves for quasiparticles remains rather crude, and restricted to essentially Coulomb gauge [17]. Here we will assume that $\omega^2 = p^2 + M^2$, with the following values (at $T = 1.5T_c$)

$$\frac{m_{\bar{q}q}}{T} = 3.9 \pm 0.2 \quad \frac{m_{gg}}{T} = 3.4 \pm 0.3$$

Note that at such $T$ masses are large and their ratio is very different from the weak coupling prediction $1/\sqrt{6}$, while at $T \approx 3T_c$ and higher they are reduced toward perturbative values (which we would not include in this work, for simplicity).

If the dispersion law is known, the effective equation of motion suitable for discussion of the bound state problem can be obtained by standard substitution of the covariant derivatives in the place of momentum and frequency. It has the form of the Schrodinger equation if the dispersion curve can be parameterized as $\omega = M_q + p^2/2M' + \ldots$, in general with two different constants $M, M'$. Both for weak coupling and lattice data, such approximation seem to be accurate withing several percents.

Now, to address the issue of binding, we first note that if all effective masses grow linearly with $T$, including the screening mass, the explicit $T$ dependence drops out of (2) to the exception of the logarithmic dependence in $T$ left out in $\alpha_s \sim 1/\ln(T/\Lambda_{QCD})$. This is why the region of “strongly coupled QGP” turns out to be relatively substantial. For a qualitative estimate, let us set the coupling to its maximum, $\alpha_s = 1$. The combination of constants is $(4 \times 3.9 T)/(3 \times 2.25 T) = 2.3$, larger than the critical value (2), so one should expect the occurrence of (strong) Coulomb bound states. Although the (plasmon) gluon modes are somewhat lighter than quarks in (4), their Coulomb interaction has a larger coefficient due to a different Casimir operator for the adjoint representation, 3 instead of 4/3. As a result the effective combination in the potential is $3m_{\bar{q}q} \alpha_s/M_D$, which is about twice larger than for quarks, and thus the gluons are bound even stronger (modulo collisional broadening).

Solving the equations one can make a quantitative analysis, using the same potential as above. We found that the highest temperature $T$ at which light quark states are Coulomb bound is somehow lower than that of charmonium,

$$T_{\bar{q}q} \approx 1.45 T_c \approx 250 \text{ MeV},$$

while the $s$-wave $gg$ gluonium states remain bound till higher temperatures $T_{gg} \approx 4 T_c$. As an example, in Fig. 1(b) we show the binding energy of two gluons. (In absolute units it reaches about 100 MeV, while it is much smaller MeV for quarks.) The wave functions of all loosely bound states are similar to that of a deuteron, with the usual $\chi \approx e^{-r} r$ behavior outside the potential.

How many bound states are there? Stating with quarks, we note that if one ignores current quark masses and instanton effects, the chiralities (left and right) are conserved. Furthermore, there are two different modes of quarks depending on helicity (“particle” and “plasmino”), times antiquarks, times flavors. All in all $4N_f^2$ states, connected by continuity to the pseudoscalar, scalar, vector and axial vector nonets in the vacuum (see Fig.2).
has a peak when there is both a shallow level and a virtual state, as we know from NN scattering in the triplet and singlet channels [21]. With the denominator $E + |E_b|$ ranging from 100 to 10 MeV, the corresponding $gg$ cross section is in the range 50-500 mb. These huge cross sections make quasi-free propagation of quasiparticles impossible.

We stress that our analysis is not based on the in-vacuum effective potential [22], but on the in-matter screened one. The bound states emerge in a screened but still strongly coupled Coulomb phase. Still, not all of the many-body effects are included in the screening. Indeed, the range of distances in the weakly bound states is large, and by including the $1/N_c^2$ factor needed for absolute matching of the gluon colors, we may even have more than one matching gluon inside the available bound-state volume. More detailed many-body studies should be made, such as the ones carried for “excitonic matter” in semiconductors and insulators. We recall that the Coulomb parameter in QED plasmas is $e^2/\epsilon v$ where $e^2 = 1/137$ is the usual fine structure constant, $\epsilon$ is the dielectric constant of the background substance and $v$ is a typical particle/hole velocity. It can in principle be tuned to be around 1, resulting in bound excitons (particle-hole pairs). The latter occur despite Debye-screening, a situation much alike ours. Depending on a number of parameters, including the density of excitons and temperature, the system exhibits various phases, ranging from an ideal gas of excitons to a liquid or plasma, or even a Bose-condensed gas. On its way from a gas to a liquid, clustering with 3- and 4-body states play an important role. Although one cannot directly relate these two problems (quarks and gluons have $N_c$ and $(N_c^2 - 1)$ colors respectively, while particles and holes have simply charges $\pm e$), one may think that in the QGP at $T \sim T_c$ some of these phenomena may well be there.

More generally, strongly coupled many-body problems, with large or divergent binary scattering length, are studied in at least two other settings: (i) a gas of neutrons, with $a = 18$ fm due to a virtual level; (ii) trapped atomic $Li^6$, $Li^7$ in which the scattering length can be tuned. Its hydrodynamic properties are of great interest.

**Discussion and Outlook.** The main assumption made in this work is that at $T > T_c$ the gauge coupling is allowed to run till the screening scale. As a result, we found that the QCD phase above $T_c$ supports Coulomb bound states. In this work we were mainly interested in the location of the point at which the $s$-wave state disappears. However, more generally, one may ask how these states are related to known hadrons at $T = 0$. How it supposed to happen is shown in Fig.2 (for 2 massless flavors, when the phase transition is second order and all lines must be continuous). Very close to $T_c$, the quark (chiral) mass is expected to become large, so here mesons are rather deeply bound states in this region and both relativistic and non-perturbative effects become important: we plan to study this region elsewhere. It would also be interesting to study whether annihilation of vector states

$$\sigma(E) = \frac{2\pi}{m} \frac{1}{E + |E_b|}$$
can be seen in the dilepton spectra, and how the survival of charmonium above $T_c$ affects its evolution at RHIC.

![Figure 2](image)

**FIG. 2.** Schematic dependence of hadronic masses on temperature $T$ (in units of the critical one $T_c$), for 2-flavor QCD in the chiral limit. The dash-dotted line corresponds to twice the (chiral) effective mass of a quark. Black dots marked $s, p, d$ correspond to the points where the binding vanishes for states with orbital momentum $l = 0, 1, 2,...$

In QCD, as soon as the lowest state hits zero there is a phase transition. This is however impossible for conformal gauge theories (CFTs), such as $\mathcal{N}=4$ supersymmetric gauge theory. In CFT the gauge coupling is allowed to become *supercritical* or even large $\lambda \equiv g^2 N_c \gg 1$. Maldacena’s AdS/CFT duality [23] has opened a way to study this *strong coupling limit* using classical gravity. At finite $T$ it was recently actively discussed, for Debye screening [25], bulk thermodynamics [24] and kinetics [26]. Although the thermodynamical quantities are only modified by an overall factor of 3/4 in comparison to the black-body limit, kinetics is changed dramatically. In our separate paper [14] we show that in this regime the matter is made of very deeply bound binary composites, in which the supercritical Coulomb can be balanced by centrifugal force. a re-summation of a class of diagrams, in vacuum and at finite $T$. Specific towers of such bound states can be considered as a continuation of Fig.2 to the left, toward stronger and stronger coupling.

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