Road Traffic Offences in Nigeria: An Empirical Analysis using Buys-Ballot Approach

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Authors’ contributions

This paper was carried out in collaboration between both authors. Author KCND designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author CCI managed the analyses and literature searches of the study. Both authors read and approved the final manuscript.

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Abstract

Road traffic offences in time series analysis when trend-cycle component is quadratic is discussed in this study. The study is to investigate the variance stability, trend pattern, seasonal indices and suitable model for decomposition of study data. The study shows that, the series is seasonal with evidence of upward trend or downward trend. There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. The periodic standard deviations are stable while the seasonal standard deviations differ, suggesting that the series requires transformation to make the seasonal indices additive.

Keywords: Decomposition model; buys-ballot table; successful transformation; seasonal mean; standard deviation; suitable model.

1 Introduction

Descriptive method of time series analysis is a set of observations taken at different time period especially equal time interval, this periods can be daily, weekly, monthly, quarterly etc. Generally, a time series may

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usefully considered as a mixture of four components include, the trend, seasonal, cyclical and irregular components. Cyclical variation is regarded as long term oscillations. However, for short series, the cyclical component is jointly estimated into the trend [1] and the observed time series $\{X_t, t=1, 2, ..., n\}$ can be decomposed into the trend-cycle component $(M_t)$, seasonal component $(S_t)$ and the irregular $(e_t)$. Therefore, the decomposition models are

**Additive Model**

$$X_t = M_t + S_t + e_t$$  \(1\)

**Multiplicative Model**

$$X_t = M_t \times S_t \times e_t$$  \(2\)

and **Mixed Model**

$$X_t = M_t \times S_t + e_t.$$  \(3\)

On when to apply each of these three models, Linde [2] stated that when the seasonal indices is independent of the absolute level of the time series, but it takes appropriately the same magnitude each period then is said to be additive model and shown in equation (1) may be employed. If the seasonal indices takes the same relative magnitude each period, then it is said to be multiplicative model obtained in equation (2).

Iwueze and Ohakwe [3] observed the Buys-Ballot procedure to the case in which the trend cycle component is quadratic. In their summary, they proposed that the estimation of the slope of the curve is as in Iwueze and Nwogu [4]. The difference in procedure lies in the calculation of $c$ which is easily calculated from differences in the periodic means.

Dozie [5] discussed the expression for estimation of trend parameters and seasonal indices using periodic, seasonal and overall averages for the mixed model in time series. He observed that the estimate of trend parameters and seasonal indices for mixed model, when there is no trend and $(b = 0)$.

### 2 Methodology

2.1 Buys-Ballot procedure for time series is employed in this study. For details of this procedure see Wei [6], Iwueze and Ohakwe [4], Dozie, et al. [7], Dozie and Ihekuna [8].

#### 2.1 Quadratic trend cycle and seasonal components

The expression of the quadratic trend is given by

$$\bar{X}_t = a + bt + ct^2$$  \(4\)

Iwueze and Nwogu [4] discussed estimation of the trend and seasonal indices for an additive model when trend-cycle component is quadratic as;
Table 1. Buys-Ballot tabular arrangement of time series data

| Rows (i) | 1    | 2    | ... | j    | ... | s    | T_j | X_i | \( \hat{\sigma}_i \) |
|----------|------|------|-----|------|-----|------|-----|-----|-------------------|
| 1        | \( X_1 \) | \( X_2 \) | ... | \( X_j \) | ... | \( X_s \) | \( T_1 \) | \( \bar{X}_1 \) | \( \hat{\sigma}_1 \) |
| 2        | \( X_{s+1} \) | \( X_{s+2} \) | ... | \( X_{s+j} \) | ... | \( X_{2s} \) | \( T_2 \) | \( \bar{X}_2 \) | \( \hat{\sigma}_2 \) |
| 3        | \( X_{2s+1} \) | \( X_{2s+2} \) | ... | \( X_{2s+j} \) | ... | \( X_{3s} \) | \( T_3 \) | \( \bar{X}_3 \) | \( \hat{\sigma}_3 \) |
| ...      | :    | :    | ... | :    | ... | :    | :   | :   | :                |
| i        | \( X_{(i-1)s+1} \) | \( X_{(i-1)s+2} \) | ... | \( X_{(i-1)s+j} \) | ... | \( X_{is} \) | \( T_i \) | \( \bar{X}_i \) | \( \hat{\sigma}_i \) |
| ...      | :    | :    | ... | :    | ... | :    | :   | :   | :                |
| m        | \( X_{(m-1)s+1} \) | \( X_{(m-1)s+2} \) | ... | \( X_{(m-1)s+j} \) | ... | \( X_{ms} \) | \( T_m \) | \( \bar{X}_m \) | \( \hat{\sigma}_m \) |
| \( T_j \) | \( T_1 \) | \( T_2 \) | ... | \( T_j \) | ... | \( T_s \) | \( T_s \) | \( \bar{X}_j \) | \( \bar{X}_s \) | \( \bar{X}_j \) | \( \bar{X}_s \) |
| \( \bar{X}_j \) | \( \bar{X}_1 \) | \( \bar{X}_2 \) | ... | \( \bar{X}_j \) | ... | \( \bar{X}_s \) | \( \bar{X}_s \) | \( \bar{X}_j \) | \( \bar{X}_s \) | \( \bar{X}_j \) | \( \bar{X}_s \) |
| \( \hat{\sigma}_j \) | \( \hat{\sigma}_1 \) | ... | \( \hat{\sigma}_j \) | ... | \( \hat{\sigma}_s \) | \( \hat{\sigma}_s \) | \( \hat{\sigma}_j \) | \( \hat{\sigma}_s \) | \( \hat{\sigma}_j \) | \( \hat{\sigma}_s \) |

Where, \( m \) = number of periods, \( s \) = length of periodic interval and \( n \) = length of the series

\[
\hat{a} = a' + \left( \frac{s-1}{2} \right) b - \left( \frac{s-1}{6} \right) c
\]

(5)

\[
\hat{b} = \frac{b}{s} + \hat{c}(s-1)
\]

(6)

\[
\hat{c} = \frac{c}{s^2}
\]

(7)

\[
\hat{S}_j = \hat{X}_j - d_j
\]

(8)

\[
d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}j^2
\]

(9)

2.2 Choice of model

2.2.1 Cochran’s test for constant variance

To test the null hypothesis that the variances are equal, that is

\[
H_0 = \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2
\]

Against the alternative

\[
H_1 \neq \sigma_1^2 \neq \sigma_2^2 \neq \ldots \neq \sigma_k^2 \quad (Atleast \ one \ variance \ is \ different \ from \ others)
\]

The statistic is given as

\[
C = \frac{\max \left( S_j^2 \right)}{\sum_{j=1}^{k} S_j^2}
\]

(10)
Where, \( \max \left( S_j^2 \right) \) is the maximum variance among all column variances \( \sum_{j=1}^{k} S_j^2 \) is the sum of the variances \( S_j \) has the range \( j = 1, 2, \ldots, s \), which are the variances of the \( j \)th sub-group.

Using the parameters of the Buys-Ballot table: \( S_j^2 = \sigma_j^2 \), the statistic in (10) is then given as;

\[
C = \frac{\max \left( \sigma_j^2 \right)}{\sum_{j=1}^{k} \sigma_j^2}
\]

(11)

Where, \( \sigma_j^2 = (j = 1, 2, \ldots, s) \) is the column variance of the Buys-Ballot table.

3 Real Life Example

Empirical example is given in this section to demonstrate the application of the Buys-Ballot method of time series decomposition. The series is listed in the Buys-Ballot table, with its row, column and overall means and standard deviations. The associated graphs are obtained in Figs. 3.1, 3.2 and 3.3. The graphs show that the series is seasonal with evidence of upward or downward trend.

![Fig. 3.1. Plot of road traffic offences between 2007-2017](image)

There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. There is a reasonably seasonal pattern over the period, suggesting that the series need transformation to make the seasonal effect additive.
The estimates of the parameters of quadratic trend cycle and seasonal components are obtained here, using (5), (6) and (7), we have

\[ \hat{X}_i = 9.985 + 0.3642 t - 0.0379 t^2 \]

\[ c = \frac{-0.0379}{144} = -0.0003 \]

\[ b = \frac{0.3642}{12} - 0.0003(12 - 1) = 0.0271 \]

\[ a = 9.985 + \left( \frac{12 - 1}{2} \right)(0.0271) - \left( \frac{(12 - 1)(24 - 1)}{6} \right)(-0.0003) \]

\[ = 10.1467 \]

\[ \hat{S}_j = \hat{X}_j - 10.2877 + 0.036 j + 0.0003 j^2 \]

Fig. 3.2. Transformed series of road traffic offences between 2007-2017
### Table 2. Estimates of seasonal effect

| $j$ | $\hat{X}_j$ | $S_j$ |
|-----|-------------|-------|
| 1   | 10.353      | -0.290|
| 2   | 10.460      | -0.146|
| 3   | 10.544      | -0.025|
| 4   | 10.423      | -0.108|
| 5   | 10.483      | -0.009|
| 6   | 10.375      | -0.078|
| 7   | 10.410      | -0.003|
| 8   | 10.511      | 0.139 |
| 9   | 10.369      | 0.038 |
| 10  | 10.386      | 0.097 |
| 11  | 10.499      | 0.252 |
| 12  | 10.338      | 0.134 |

\[ \sum_{j=1}^{12} S_j = 0.0000 \]

### Table 3. Estimates of quadratic trend parameters and seasonal indices

| Parameter | Quadratic trend and seasonal indices |
|-----------|--------------------------------------|
| $\hat{a}$ | 10.147                               |
| $\hat{b}$ | 0.027                                |
| $\hat{c}$ | -0.0003                              |
| $\hat{S}_1$ | -0.2902                              |
| $\hat{S}_2$ | -0.1463                              |
| $\hat{S}_3$ | -0.0248                              |
| $\hat{S}_4$ | -0.1077                              |
| $\hat{S}_5$ | -0.009                               |
| $\hat{S}_6$ | -0.0777                              |
| $\hat{S}_7$ | -0.0028                              |
| $\hat{S}_8$ | 0.1387                               |
| $\hat{S}_9$ | 0.0378                               |
| $\hat{S}_{10}$ | 0.0965                              |
| $\hat{S}_{11}$ | 0.2518                              |
| $\hat{S}_{12}$ | 0.1337                              |
Fig. 3.3. Seasonal means and standard deviations

Table 4. Seasonal means and standard deviations

| $j$ | $\bar{X}_j$ | $\sigma_j$ |
|-----|-------------|------------|
| 1   | 10.353      | 0.382      |
| 2   | 10.460      | 0.487      |
| 3   | 10.544      | 0.477      |
| 4   | 10.423      | 0.468      |
| 5   | 10.483      | 0.501      |
| 6   | 10.375      | 0.508      |
| 7   | 10.410      | 0.500      |
| 8   | 10.511      | 0.629      |
| 9   | 10.369      | 0.576      |
| 10  | 10.386      | 0.573      |
| 11  | 10.499      | 0.712      |
| 12  | 10.338      | 0.580      |

3.1 Choice of model

Cochran’s statistic in (11) is used to determine if the series accepts additive model. The null hypothesis that the series accepts additive model is rejected, if $C$ is greater than the tabulated value $C_{tab} \{k, V, 1 - \alpha\}$ level of significance, or do not reject null hypothesis otherwise.

From Appendix A and Table 5.

$$m=12, \max\left(\hat{\sigma}_j^2\right) = 896929395.00, \sum_{j=1}^{k} \hat{\sigma}_j^2 = 3458064838$$

hence,

$$C = \frac{896929395}{3458064838} = 0.2594$$
Table 5. Seasonal means ($\bar{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$) of the actual series

| $j$ | $\bar{X}_j$ | $\hat{\sigma}_j^2$ |
|-----|-------------|---------------------|
| 1   | 33152.73    | 97808446.00         |
| 2   | 37868.82    | 158792463.40        |
| 3   | 41744.45    | 349906360.10        |
| 4   | 36594.00    | 195739139.40        |
| 5   | 39142.73    | 214664393.80        |
| 6   | 35326.91    | 199927172.50        |
| 7   | 36601.64    | 222625161.90        |
| 8   | 42528.45    | 437170557.50        |
| 9   | 35918.09    | 233406043.70        |
| 10  | 36606.55    | 255221781.10        |
| 11  | 44553.73    | 896929395.00        |
| 12  | 34715.45    | 196479923.30        |
| Total| 454753.55   | 3458064838.00       |

Table 6. Seasonal means ($\bar{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$) of the transformed series

| $j$ | $\bar{X}_j$ | $\hat{\sigma}_j^2$ |
|-----|-------------|---------------------|
| 1   | 10.35       | 0.15                |
| 2   | 10.46       | 0.24                |
| 3   | 10.54       | 0.23                |
| 4   | 10.42       | 0.22                |
| 5   | 10.48       | 0.25                |
| 6   | 10.37       | 0.26                |
| 7   | 10.41       | 0.25                |
| 8   | 10.51       | 0.40                |
| 9   | 10.37       | 0.33                |
| 10  | 10.39       | 0.33                |
| 11  | 10.50       | 0.51                |
| 12  | 10.34       | 0.34                |
| Total| 125.15      | 3.49                |

As shown in Table 5, the critical value (0.2353) is greater than the test statistic in (11), suggesting that the series does not accept additive model. Having verified that the actual series is not additive. We transformed the series to meet the constant variance and normality assumptions in the distribution. When the transformed series listed in Table 6 are subjected to test for constant variance, the test statistic (0.1461) is less than the tabulated (0.2353) at $C_{tab}$ {k, V, 1 – α} level significant. Therefore, transformed series accepts additive model.

From Appendix B and Table 6.

\[
m = 12, \quad \max \left( \hat{\sigma}_j^2 \right) = 0.51, \quad \sum_{j=1}^{4} \hat{\sigma}_j^2 = 3.49
\]

\[
C = \frac{0.51}{3.49} = 0.1461
\]

Decision rule: Reject $H_0$ if $C > C_{tab}$ {11,12:0.05}
4 Conclusion

We have discussed the road traffic offences in time series analysis. The emphasis is to investigate the variance stability, trend pattern, seasonal effect and suitable model for decomposition. The study indicates that, the series is seasonal with evidence of upward or downward trend. There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. There is a reasonably seasonal pattern over the period, suggesting the additive model. Also, the suitable model that best describe the pattern in the transformed series shown in the summary table (Table 6) is additive.

Competing Interests

Authors have declared that no competing interests exist.

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### Appendix A. Actual data on number of road traffic offences in Nigeria (2007-2017)

| Year | Jan.   | Feb.   | Mar.   | Apr.   | May    | Jun.   | Jul.   | Aug.   | Sept.  | Oct.   | Nov.   | Dec.   | $\overline{X}_j$ | $\sigma_j^2$ |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----------------|------------|
| 2007 | 27905  | 32732  | 42104  | 31288  | 40529  | 30727  | 42742  | 43807  | 41146  | 38152  | 121049 | 47113  | 44941.2        | 610818258.7 |
| 2008 | 26613  | 35623  | 36655  | 41333  | 31288  | 40529  | 30727  | 42742  | 36793  | 25334  | 31170  | 34728.3        | 268434131   |
| 2009 | 38268  | 47134  | 30474  | 34365  | 31714  | 30673  | 31584  | 34008  | 36479  | 43294  | 40956  | 36710  | 36304.9        | 28174340.8  |
| 2010 | 41683  | 46299  | 50404  | 30008  | 55681  | 52653  | 52107  | 56362  | 43392  | 47885  | 39863  | 36878  | 49186.7        | 57679005.33 |
| 2011 | 36223  | 45036  | 50404  | 50404  | 55681  | 52653  | 52107  | 56362  | 43392  | 47885  | 39863  | 36878  | 49186.7        | 57679005.33 |
| 2012 | 39294  | 50635  | 84320  | 35797  | 46285  | 43715  | 50989  | 49011  | 42607  | 37393  | 57381  | 42619  | 48327.4        | 166439233.7 |
| 2013 | 44724  | 39486  | 38661  | 58112  | 52004  | 48191  | 36777  | 31262  | 43449  | 51205  | 49543  | 56472  | 45823.8        | 67562322.7  |
| 2014 | 42376  | 49697  | 43837  | 44665  | 49674  | 48038  | 50812  | 56405  | 52633  | 45569  | 51234  | 41617  | 48046.4        | 20260646.3  |
| 2015 | 32164  | 39025  | 36687  | 29488  | 30114  | 25486  | 27264  | 30927  | 26211  | 25224  | 26201  | 24772  | 29388.6        | 19895997.2  |
| 2016 | 23421  | 22098  | 23919  | 23348  | 21468  | 21871  | 19224  | 17262  | 20173  | 15726  | 17288  | 16709  | 20175.6        | 8142224.6   |
| 2017 | 12009  | 9692   | 13609  | 10729  | 10384  | 9582   | 11242  | 8986   | 7943   | 9066   | 8139   | 7328   | 9909.1         | 3307375.2   |

\[
\overline{X}_j = \frac{\sum_{i=1}^{12} X_{ij}}{12}, \quad \sigma_j^2 = \frac{\sum_{i=1}^{12} (X_{ij} - \overline{X}_j)^2}{12}
\]

- $\overline{X}_j$ represents the mean of the data for each month $j$.
- $\sigma_j^2$ represents the variance of the data for each month $j$.

\[
j = 77
\]
Appendix B. Transformed data on number of road traffic offences in Nigeria (2007-2017)

| Year | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | $\bar{x}$ | $\sigma^2$ |
|------|------|------|------|------|-----|------|------|------|-------|------|------|------|---------|---------|
| 2007 | 10.24 | 10.40 | 10.65 | 10.35 | 10.61 | 10.33 | 10.66 | 10.69 | 10.62 | 10.55 | 11.70 | 10.76 | 10.63 | 0.14 |
| 2008 | 10.19 | 10.48 | 10.51 | 10.63 | 10.51 | 10.19 | 10.12 | 11.33 | 9.99 | 10.20 | 10.14 | 10.35 | 10.39 | 0.13 |
| 2009 | 10.55 | 10.76 | 10.32 | 10.44 | 10.36 | 10.33 | 10.36 | 10.43 | 10.50 | 10.68 | 10.62 | 10.51 | 10.49 | 0.02 |
| 2010 | 10.64 | 10.74 | 10.99 | 10.98 | 10.93 | 10.87 | 10.86 | 10.94 | 10.68 | 10.78 | 10.59 | 10.52 | 10.79 | 0.03 |
| 2011 | 10.50 | 10.72 | 10.83 | 10.46 | 10.93 | 10.84 | 10.92 | 10.95 | 10.99 | 11.04 | 10.88 | 10.61 | 10.80 | 0.04 |
| 2012 | 10.58 | 10.83 | 11.34 | 10.49 | 10.74 | 10.69 | 10.84 | 10.80 | 10.66 | 10.53 | 10.96 | 10.66 | 10.76 | 0.05 |
| 2013 | 10.71 | 10.58 | 10.54 | 10.97 | 10.86 | 10.78 | 10.51 | 10.35 | 10.68 | 10.84 | 10.81 | 10.94 | 10.72 | 0.03 |
| 2014 | 10.65 | 10.81 | 10.69 | 10.71 | 10.81 | 10.78 | 10.84 | 10.94 | 10.87 | 10.73 | 10.84 | 10.64 | 10.78 | 0.01 |
| 2015 | 10.38 | 10.55 | 10.51 | 10.29 | 10.31 | 10.15 | 10.21 | 10.34 | 10.17 | 10.14 | 10.17 | 10.12 | 10.28 | 0.02 |
| 2016 | 10.06 | 10.00 | 10.07 | 10.06 | 9.97 | 9.99 | 9.86 | 9.76 | 9.91 | 9.66 | 9.76 | 9.72 | 9.90 | 0.03 |
| 2017 | 9.39 | 9.18 | 9.52 | 9.28 | 9.27 | 9.17 | 9.33 | 9.10 | 8.98 | 9.11 | 9.00 | 8.90 | 9.19 |         |
| $\bar{x}$ | 10.35 | 10.46 | 10.54 | 10.42 | 10.48 | 10.37 | 10.41 | 10.51 | 10.37 | 10.39 | 10.50 | 10.34 | 10.43 |      |
| $\sigma^2$ | 0.15 | 0.24 | 0.23 | 0.22 | 0.25 | 0.26 | 0.25 | 0.40 | 0.33 | 0.33 | 0.51 | 0.34 | 0.91 | 0.01 |

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