FAST DYNAMOS IN WEAKLY IONIZED GASES
ELLEN G. ZWEIBEL and FABIAN HEITSCH
Received 2008 February 15; accepted 2008 May 1

ABSTRACT

The turnover of interstellar gas on $\sim 10^9$ yr timescales argues for the continuous operation of a galactic dynamo. The conductivity of interstellar gas is so high that the dynamo must be “fast”; i.e., the magnetic field must be amplified at a rate nearly independent of the magnetic diffusivity. Yet all the fast dynamos so far known, and all direct numerical simulations of interstellar dynamos, yield magnetic power spectra that peak at the resistive scale, while galactic magnetic fields have substantial power on large scales. In this paper we show that in weakly ionized gas the limiting scale may be the ion-neutral decoupling scale, which, although still small, is many orders of magnitude larger than the resistive scale.

Subject headings: ISM: magnetic fields — methods: numerical — MHD

1. INTRODUCTION

Despite many theoretical and observational advances, our understanding of galactic magnetic fields is still incomplete. Although there is evidence for magnetic fields in young galaxies (Kulsrud & Zweibel 2008), it is likely that dynamo processes still operate continuously in galaxies today. Perhaps the most compelling argument for ongoing dynamo activity is that the turnover time of interstellar gas due to loss and replacement of material, $\sim 10^9$ yr in the case of the Milky Way, is much less than the ages of galaxies. As interstellar gas is added, whether by infall from the intergalactic medium or through stellar mass loss, its magnetic field must be brought to the galactic value so as to maintain the field in a steady state.

In contrast to the primordial situation at early times, when a magnetic field had to be built up from nearly zero, it is likely that all the material on which galactic dynamos now act is at least somewhat magnetized. Under these conditions, the primary functions of the dynamo are to increase magnetic energy to the observed value, to generate and maintain a component of the field that is coherent over at least several kiloparsecs, as observed, and to transport magnetic flux into the gas that has newly arrived, which, especially if it is intergalactic in origin or was shed by stars, may be undermagnetized. These processes are also necessary in theories such as that of Rees (1987), in which galactic magnetic fields are seeded by random fields injected by many small-scale sources.

Dymanos are considered “fast” if the rate at which the field is amplified tends to a finite limit as the magnetic diffusivity $\eta$ approaches zero (Childress & Gilbert 1995). It may seem paradoxical that magnetic diffusivity, which removes energy from the magnetic field, is required for dynamos at all. But without diffusion the magnetic topology is fixed, and this places strong constraints on magnetic field amplification (Moffat 1978). The ohmic diffusion time in galaxies is so much longer than any dynamical time that it seems that galactic dynamos must be fast.

Fast dynamo theory has followed two approaches. One involves computation and analysis of the action of prescribed flows on magnetic fields under nearly nondiffusive conditions (e.g., Galloway & Proctor 1992; Ott 1998; Gilbert 2002; Courvoisier et al. 2006). These so-called kinematic studies have demonstrated the role of chaotic flows in magnetic field amplification and have elucidated important relationships between the topological properties of the flows and their dynamo properties (Klapper & Young 1995). The modification of some of these flows by magnetic forces has also been considered and has provided some insight into how dynamos might saturate (Cattaneo et al. 1996; Tanner & Hughes 2003; Cameron & Galloway 2005).

The second approach to fast dynamos involves analysis and direct numerical simulations of driven magnetohydrodynamic (MHD) turbulence, again with the lowest magnetic diffusivity $\lambda$ possible. In particular, $\lambda$ is chosen to be less than the viscous diffusivity $\nu$ (i.e., the large Prandtl number case). These models are self-consistent in the sense that magnetic forces are fully included in the dynamics, and theory and simulation are in good agreement on how the field evolves (Schekochihin et al. 2004).

Both approaches yield magnetic fields that are dominated by structure on the resistive scale, which is far smaller than any other characteristic scale in the interstellar medium. This follows from the fact that in the absence of magnetic diffusion, any divergence-free flow that amplifies the field lengthens the field lines in the same proportion. Therefore, as the field is amplified, it becomes folded or tangled. The large random field is in stark contrast to the observed structure of galactic magnetic fields, which display considerable long-range order, and thus presents a serious challenge to dynamo theory. In particular, the magnetic field is amplified on scales far below the minimum velocity scale, whether this scale is prescribed (the kinematic case) or created by viscous effects (the full turbulent simulation).

Most of the mass, and a considerable part of the volume, of the interstellar medium is weakly ionized, with the plasma and neutral fluids coupled by collisions and by ionization and recombination. On long length scales and timescales, the medium behaves like a single conducting fluid, but on short length scales and timescales, the two fluids decouple. For present interstellar medium parameters, the magnetic field is in approximate energy equipartition with the bulk fluid and dominates the plasma component. The question therefore arises as to whether a dynamo in a weakly ionized medium could be quenched on scales below the plasma-neutral decoupling scale while continuing to amplify the field on larger scales. On the basis of the analysis and computations presented in this paper, the answer seems to be that it can.
This is far from the first study of the effects of partial ionization on dynamos. There have been a number of studies of large-scale dynamos in weakly ionized gases that show that the nonlinearity introduced by ion-neutral drifts has important consequences for mean field theory (Zweibel 1988; Proctor & Zweibel 1992) and for the general problem of how large-scale fields are amplified in a medium with small-scale turbulence (Kulsrud & Anderson 1992; Subramanian 1997, 1999; Brandenburg & Subramanian 2000). These studies all used the strong coupling approximation (Shu 1983), according to which the plasma drift relative to the neutrals is found by balancing the Lorentz force by ion-neutral drag. This approximation breaks down at small scales. In this work, we treat the plasma and neutrals as two frictionally coupled fluids, which is more appropriate at the small scales, and also more general.

In order to see how ion-neutral friction might affect the dynamo, consider the equation for magnetic energy, which in a domain $V$ with periodic or infinitely distant boundaries can be written as

$$\frac{\partial}{\partial t} \int_V \frac{B^2}{8\pi} \, d^3x = - \int_V \left[ v_i \cdot \left( \nabla \times B \right) \times B - \frac{\nabla \times B^2}{4\pi} \right] \, d^3x,$$

(1)

where $v_i$ is the plasma velocity and $\lambda$ is the ohmic diffusivity. The first term in the integral on the right-hand side represents the work done by the field on the plasma; the second term is the resistive dissipation.

The plasma velocity can always be expressed in terms of the neutral velocity $v_n$ and the drift velocity $v_D = v_i - v_n$: $v_i = v_n + v_D$. If the ionization fraction is low, $v_n$ in turn is very nearly the center-of-mass velocity $v$. In the strong coupling approximation, $v_D$ can be expressed as

$$v_D = \frac{(\nabla \times B) \times B}{4\pi c \rho n},$$

(2)

where $c \equiv (\sigma T)_{m/n}(n_i + n_n)$ is the ion-neutral friction coefficient. Replacing $v_i$ by $v + v_D$ in equation (1) and using equation (2), we see that when the strong coupling approximation holds, the magnetic energy equation is

$$\frac{\partial}{\partial t} \int_V \frac{B^2}{8\pi} \, d^3x = - \int_V \left[ v \cdot \left( \nabla \times B \right) \times B + \frac{\nabla \times B^2}{4\pi} \right] \, d^3x + \frac{\gamma}{4\pi}\frac{\rho_i n_i}{\rho n} v_D^2 + \frac{\nabla \times B^2}{4\pi} \, d^3x.$$  

(3)

Equation (3) shows, as expected, that ion-neutral friction is a dissipative effect. The work term, however, can have either sign, and it is possible that in the presence of ambipolar drift, the magnetic field and flow are modified so as to increase the rate at which energy flows to the field. Equation (1), on the other hand, is valid whether the strong coupling approximation holds or not, is closer to the magnetic energy equation for a plasma, and does not attempt to split the inductive and dissipative effects.

In § 2 we introduce the basic setup. In § 3 we show how plasma-neutral friction can allow the field to saturate at small scales while still being amplified at large scales. Section 4 is a summary.

2. FORMULATION

2.1. Important Timescales and Length Scales

In a turbulent cascade, the velocity at scale $l$, $v_l$, typically decreases with decreasing $l$, but slowly enough that the turnover time, $\tau_l \equiv l/v_l$, decreases with $l$ as well. For example, in Kolmogorov turbulence, or in the magnetohydrodynamic turbulence model of Sridhar & Goldreich (1994) and Goldreich & Sridhar (1995, 1997), $v_l$ is related to $l$ and the scale $l_d$ at which the turbulence is driven by $v_l \sim \sqrt{l/d(l)$). The turnover time $\tau_l$ is then equal to $(l_d/v_l)(l/d)^3$. The cascade terminates at the scale $l_K$ at which the dissipation rate becomes faster than the turnover rate.

In diffuse, weakly ionized interstellar gas, $\tau_K$, the turnover time at $l_K$ is comparable to the neutral-ion collision time $\tau_{ni}$ and much longer than the ion-neutral collision time $\tau_{in}$. Only for motions on timescales of less than $\tau_{in}$ is friction with the neutrals unimportant for plasma dynamics. Resistive effects are typically unimportant for motions with turnover times $\tau_{in}$, but they become important on much smaller scales. The ion viscous scale probably terminates the ion flow on scales much larger than the resistive scale (the suppression of viscous stresses perpendicular to $B$ even for a weak magnetic field makes the actual value difficult to assess). The limited computational resources available to us preclude the modeling of all these scales in a realistic manner. Our main focus is on separating the ion-neutral decoupling scale and the resistive scale. Therefore, instead of assigning a third, still larger scale to the neutral motions, we allow the neutral flow to take place on the ion-neutral decoupling scale. Furthermore, we rely entirely on numerical viscosity, which allows the flow in the ions to extend below the resistive scale.

The consequences of these aspects of our calculations are discussed in the next two sections (§§ 3 and 4).

2.2. Case Study

We assume that the neutral velocity $v_n$ is of a form that would give fast dynamo action if it were the plasma velocity $v$. On timescales much longer than the ion-neutral collision time $\left(\rho_i n_i \gamma_{in}\right)^{-1}$, the plasma should move with the neutrals ($v_l \sim v_n$), and the magnetic field should grow at the fast dynamo rate, at least while Lorentz forces are unimportant. On short length scales, inertial and Lorentz forces compete with ion-neutral friction to drive $v_l$ away from $v_n$. We investigate the dynamo properties of the resulting plasma flow. The setup is somewhat similar to the two-fluid study of Kim (1997), although that work focused on diffusion of a large-scale field and the computations were two-dimensional and incompressible.

We simplify the problem by assuming that the ionization and recombination timescales are much shorter than the advective timescales, so that ionization equilibrium holds. This assumption allows us to bypass the plasma continuity equation, but it is unrealistic in the diffuse gas that is the primary application for this study (although it is much better for molecular gas; see Table 3 of Heitsch & Zweibel 2003a). The neutral velocity we choose is divergence-free, and so by taking the neutral density $\rho_n$ to be initially uniform, we force it, and the plasma density $\rho_p \sim \rho_n$ to remain so. Therefore, there are no thermal pressure gradient forces in the problem. Under these conditions, the momentum equation for the plasma is

$$\rho_p \left( \frac{\partial}{\partial t} + v_i \cdot \nabla \right) v_i = \frac{(\nabla \times B) \times B}{4\pi} - \gamma \rho_p n_i (v_i - v_n),$$

(4)
where $v_n$ is to be prescribed. Equation (4) is to be solved together with the magnetic induction equation,
\[
\frac{\partial B}{\partial t} = \nabla \times (\psi B) + \chi \nabla^2 B. \tag{5}
\]

For $v_n$, we choose the 2.5D flow shown by Galloway & Proctor (1992, hereafter GP) to be a fast dynamo. This flow, which we will refer to as $v_{GP}$, can be written as
\[
v_n = \hat{x}(2\pi k_{GP} \psi) + \hat{y} \frac{\partial \psi}{\partial z} - \hat{z} \frac{\partial \psi}{\partial y}, \tag{6}
\]
where the stream function $\psi$ is
\[
\psi = \sqrt{\frac{3}{2}} \frac{V_0}{2\pi k_{GP}} \left\{ \sin[2\pi(k_{GP}z + \epsilon \sin(2\pi \omega_t))] + \cos[2\pi(k_{GP}y + \epsilon \cos(2\pi \omega_t))] \right\}, \tag{7}
\]
and $\epsilon$ is a constant. This flow has periodic, cellular structure on a single spatial scale, $k^{-1}$, and temporal structure at frequency $\omega$ and all of its harmonics. This can readily be seen by rewriting equation (7) in the form
\[
\psi = \sum_{n=0}^{\infty} \psi_n \left\{ \sin[2\pi(k_{GP}z + n\omega t)] + \cos[2\pi(k_{GP}y + n(\omega t + \frac{1}{4}))] \right\}, \tag{8}
\]
where
\[
\psi_n = \sqrt{\frac{3}{2}} \frac{V_0}{2\pi k_{GP}} J_n(2\pi \epsilon), \tag{9}
\]
and the $J_n$ are ordinary Bessel functions of the first kind. However, bearing in mind that $J_n(x) \sim (x/2)^n/n!$ for $x < n$, and that $x = \pi/2$ in our problem, we see that only the first few harmonics of the series are important in the flow. The cell pattern drifts back and forth with temporal frequency $\omega$ and amplitude $\epsilon$. The flow has chaotic regions along the cell edges. In this paper, we choose units of length and time such that $k_{GP} = V_0 = \omega = 1$. The eddy turnover time and the period over which the pattern oscillates are then of the same order. We set $\epsilon \equiv 0.25$, near the value at which chaos is maximized (Brummell et al. 2001). The ranges of $y$ and $z$ are $-1 \leq y \leq 1$ and $-1 \leq z \leq 1$, so only one GP cell fits into the domain. We choose our value of $\lambda$ such that $\lambda^2 k_{GP}^2$, the resistive decay rate at the GP scale, is between $10^{-2}$ and $10^{-4}$, and the friction coefficient, $\gamma_{\text{in}} \rho_0$, is between 0.03 and 0.60. We set $\rho_0 = 1$. The magnetic field is initialized by choosing a vector potential that is a Fourier series in modes of the computational domain, with random amplitudes and random phases selected from uniform distributions.

The numerical scheme, Proteus, is based on the conservative gas-kinetic flux splitting method introduced by Xu (1999) and Tang & Xu (2000) and is second order in time and space. Resistivity is included through dissipative fluxes (Heitsch et al. 2004, 2007), but viscosity is not (there is a small amount of numerical viscosity). The frictional force is implemented through a drag term (Heitsch et al. 2004).

### 3. RESULTS

The complete set of models that we considered is summarized in Table 1. We first verified that our computational scheme recovers the previously known dynamo behavior of the GP flow and assessed numerical convergence. The left panel of Figure 1 shows the magnetic energy against time for three resolutions and five diffusivities. Once the short-wavelength magnetic power present in the random initial conditions has decayed, the magnetic energy grows at an exponential rate. The growth rates are plotted in the right panel of Figure 1. With our choice of units, $Re_m = \lambda^{-1}$. While some flattening of the curves is apparent, the growth rate has evidently not converged to the $Re_m \rightarrow \infty$ limit. Nevertheless, the evident lack of convergence at the largest value of $Re_m$ and highest numerical resolution practical for us shows that it would not be meaningful to set $\lambda$ to any value smaller than what we used here. This series of models will be referred to as KI (for model names, see Table 1). All the models discussed subsequently have $Re_m = 10^4$.

We next set $v_n = v_{GP}$ and solved equation (4) with the Lorentz force and Reynolds stress terms dropped. These models, which we denote by KN, demonstrate the interplay between inertia and friction in determining the ion flow. Using equation (8), we can then reduce equation (4) to a series of equations for the $j$th Fourier harmonic of $v_i$:
\[
\left( \frac{\partial}{\partial t} + \rho_0 \gamma_{\text{in}} \right) v_{ji} = \rho_0 \gamma_{\text{in}} \psi_j \left( \hat{x} + \hat{y} \frac{\partial}{\partial z} - \hat{z} \frac{\partial}{\partial y} \right) \times \left\{ \sin[2\pi(k_{GP}z + j\omega t)] + \cos[2\pi(k_{GP}y + j(\omega t + \frac{1}{4}))] \right\}, \tag{10}
\]
which can be solved analytically. For the parameters chosen here, $\rho_0 \gamma_{\text{in}} \ll j \omega$ for $j > 0$, and $v_{ji}$ is nearly $\pi/2$ out of phase with the frictional forcing term and reduced in amplitude by a factor of $\rho_0 \gamma_{\text{in}} / j \omega$. Thus, inertial effects force the plasma away from the GP flow (this is an artifact of the overlap in our models between the turnover rate of the neutral eddies and the ion-neutral collision rate; in the interstellar medium, the latter is much larger). However, even though the plasma flow departs significantly from the GP flow, it does amplify the magnetic field. This is shown in Figure 2. The amplification rate decreases with decreasing

| Name   | Mnemonic              | Physics                                |
|--------|-----------------------|----------------------------------------|
| KI     | Kinematic, ions       | Single-fluid GP flow in ions           |
| KN     | Kinematic, neutrals   | Two-fluid GP flow in neutrals          |
| DL     | Dynamic, Lorentz      | Two-fluid GP flow in neutrals, Lorentz force |
| DR     | Dynamic, Reynolds     | Two-fluid GP flow in neutrals, Reynolds stress |
| DA     | Dynamic, all          | DL combined with DR                    |

**TABLE 1**

**Key to Model Names**

| Name   | Mnemonic | Physics                                |
|--------|----------|----------------------------------------|
| DA     | Dynamic, all | DL combined with DR                    |
| DR     | Dynamic, Reynolds | Two-fluid GP flow in neutrals, Reynolds stress |
| DL     | Dynamic, Lorentz | Two-fluid GP flow in neutrals, Lorentz force |
| KN     | Kinematic, neutrals | Two-fluid GP flow in neutrals          |
| KI     | Kinematic, ions | Single-fluid GP flow in ions           |
This is caused primarily by the reduced amplitude of the ion flow, but also by its structure, which, as $\rho_n/\gamma_{in}$ decreases, departs more and more from $v_{GP}$. The dissipative nature of ion-neutral friction may also be playing a role here—see equation (3)—although equation (2) is not completely accurate in this case.

It can be seen from equation (5) that magnetic fluctuations at wavenumber $k$ interacting with flow at the GP wavenumber $k_{GP}$ are coupled to fields at wavenumber $k \pm k_{GP}$. This sets off a magnetic cascade in wavenumber, and therefore, both the GP flow and the modified flow present in the KI models drive magnetic field at all spatial scales.

The Reynolds stress $\nu \cdot \nabla v$, because of its nonlinearity, drives higher spatial harmonics of the GP wavenumber. In our parameter regime, it is of the same order as the inertial term, at least if $v_1$ is of the same magnitude as $v_{GP}$. Because in our calculation the resistivity exceeds the (numerical) viscosity, the ion flow extends to smaller scales than the magnetic field does. It is known that dynamo activity is often suppressed in these so-called low magnetic Prandtl number situations unless the magnetic Reynolds number is large (Boldyrev & Cattaneo 2004; Schekochihin et al. 2007). Because the ion density remains constant, the compressibility of the medium is exaggerated; suppression of the dynamo has also been associated with compressibility effects (Haugen et al. 2004). For both these reasons, it is perhaps unsurprising that including the Reynolds stress obliterates the dynamo; see the DR models in Figure 2.

We also considered the effect of dropping the Reynolds stress term but including the Lorentz force (models DL). Because it is nonlinear in $B$, and because magnetic power is driven at all scales, the Lorentz force also drives higher spatial harmonics in the flow. The feedback from the magnetic field eventually saturates the dynamo (Fig. 2). At the time of saturation, the volume-integrated plasma kinetic energy is about 100 times larger than the magnetic energy. However, magnetic forces along the cell walls, where the field is concentrated (see Fig. 3), are locally strong and inhibit the exponentially fast stretching needed for further amplification. This is consistent with the results of Cattaneo et al. (1996), who showed that magnetic forces suppress chaos, which is characterized by exponential stretching, in the Galloway-Proctor flow even when the field is too weak to modify the flow kinetic energy.

Models with the most complete physics, namely, both Reynolds stress and Lorentz forces in addition to inertia and ion-neutral friction, fail to be dynamos (Fig. 2; models DA). This is to be expected, since the initial magnetic field is too weak to affect the dynamics, and nonlinear advection alone suppresses the dynamo.

It is revealing to look at the structure of the magnetic field, as well as its growth rate. Figure 3 shows the logarithm of the magnetic energy density from three views (along the $x$-, $y$-, and $z$-axes) for the KI, KN, DL, DR, and DA models at $t = 20$. The top set of panels represents the canonical kinematic GP dynamo

---

**Fig. 1.** Left: Natural logarithm of the normalized magnetic energy against time for all kinematic models (series KI), at linear resolutions of $N = 64$ (thin lines), 128 (medium lines), and 256 (thick lines). The higher the value of $\lambda$, the stronger the initial decay of the (noise-like) seed field. Right: Exponential growth rates derived from fits to the curves in the left panel, for all resolutions and magnetic Reynolds numbers.
Fig. 3.—Logarithm of magnetic energy density at midplane, in the $(y,z)$-plane (left), the $(x,z)$-plane (middle), and the $(x,y)$-plane (right). The models, from top to bottom, are KI, KN, DL, DR, and DA. For a key to the model names, see Table 1.
In this paper we investigated the role of ion-neutral friction in eliminating the smallest scales. We solved a simple model problem in which the neutral flow was known to be a dynamo and was coupled to the plasma through frictional drag. The plasma flow was determined by friction, inertia, and Lorentz forces. Ionization equilibrium was assumed. We used a spatially constant ohmic resistivity, and we relied on viscous dissipation at the resolution scale.

This study is, as far as we know, the first dynamo model in which the plasma flow is calculated from the full equation of motion, rather than assuming the strong coupling approximation (Shu 1983). The strong coupling approximation breaks down at small scales, and it leads to an equation for magnetic energy growth that differs somewhat from the standard equation in a plasma (compare eqs. [1] and [3]). However, our models differ from the interstellar medium in several significant respects, due to the prohibitively large computational resources required for a realistic simulation. In the models, the neutral forcing occurs on timescales comparable to the ion-neutral collision time, whereas in the diffuse, weakly ionized interstellar medium, the shortest timescale in neutral turbulence is probably 2–3 orders of magnitude longer. This introduces an artificially large difference between $v_i$ and $v_n$. By taking the magnetic diffusivity to be larger than the viscous diffusivity (small magnetic Prandtl number $Pr_m$), we reverse the interstellar ordering ($Pr_m \gg 1$). By assuming ionization equilibrium, an isothermal equation of state, and a uniform density of neutrals, we preclude the possibility of plasma pressure gradients. And finally, the resistive scale is only about 2 orders of magnitude less than the dynamical scale, instead of 5–6, as in the interstellar medium.

Our work does not address the growth of the field at the largest scales. Whether ambipolar diffusion acting on small-scale turbulence can drive a large-scale field, either by quasi-linear (Zweibel 1988; Proctor & Zweibel 1992) or nonlinear (Subramanian 1997, 1999; Brandenburg & Subramanian 2000) effects, cannot be probed by our study because of the large range of scales that would be required, and also, perhaps, because the neutral velocity is already a dynamo.

Our main results are as follows. First, friction successfully imprints a dynamo flow on the ions, despite the importance of inertial terms (models KN; Fig. 2). That is, computing $v_i$ from equation (10) leaves a flow that amplifies a weak magnetic field. Second, adding nonlinear advection ($v_i \cdot \nabla v_i$) to equation (10) results in an ion flow that does not amplify the magnetic field, except in thin regions with large vorticity or negative divergence (models DR; Fig. 3). This may be a manifestation of the difficulty of driving a dynamo when $Pr_m \ll 1$, or in a highly compressible medium. It would not necessarily carry over to the diffuse interstellar medium, in which $Pr_m$ is large$^4$ and the recombination time is longer than the eddy turnover time. Third, when the Lorentz force is added but the Reynolds stress is dropped, the dynamo saturates when the magnetic energy is about 2 orders of magnitude less than the plasma kinetic energy (models DL; Fig. 2). According to conventional wisdom, the magnetic energy saturates only close to equipartition; the difference here is due to the spatial intermittency of the magnetic field, which allows Lorentz forces to become strong just where the field is being amplified. Finally, adding the Lorentz force suppresses small-scale structure (Fig. 3).

$^4$ The viscosity tensor in interstellar plasma is highly anisotropic because of the large ratio of the ion gyrofrequency to the Coulomb collision rate. Therefore, viscous transport across the magnetic field is strongly suppressed (Braginskii 1965), which makes the actual value of $Pr_m$ somewhat unclear.
The saturation reported here for models without nonlinear advection (models DL) has some features in common with the study by Tanner & Hughes (2003), although these authors considered fully ionized systems. They studied a superposition of the GP flow (which they call the CP [circularly polarized flow] and a steady flow known to be a slow dynamo, restricted the dynamics by omitting the Reynolds stress term, and averaged the Lorentz force over what in our case would be the \( x \)-direction. They found that when the GP flow dominates, saturation occurs through suppression of exponential field line stretching by Lorentz forces, rather than by enhanced dissipation brought about by an increase in small-scale structure.

Taken at face value, our computations suggest that in weakly ionized gases, the efficiency of dynamos is reduced at scales below the neutral-ion coupling length. The main point is that saturation of the field at small scales is determined by the kinetic energy in the plasma at these scales rather than in the bulk medium. The inertia of the neutrals contributes, and presumably increases the saturation level, only for motions with turnover times \( \tau_l \) that are greater than the neutral-ion collision time \( \tau_{ni} \equiv (\rho_i \gamma_{ii}^{-1})^{-1} \). If we take \( \tau_i \sim 2 \times 10^{-3} \, \text{cm}^{-3} \), representative of both cold and warm neutral interstellar gas, then \( \tau_{ni} \sim 3 \times 10^{11} \, \text{s} \). Taking the expressions for Kolmogorov turbulence given in §2.1, and assuming that the driving scale and velocity are 10 pc and 10 km \( \text{s}^{-1} \), respectively, we find that \( \tau_l \sim \tau_{nl} \) at \( l \sim 10^{-2} \, \text{pc} \). The equipartition magnetic field strength at this scale relative to the ions is only \( \sim 0.02 \, \mu \text{G} \), or about 1% of the Galactic value. Thus, saturation of the dynamo below the neutral-ion decoupling scale could prevent the amplification of the field at very small scales.

At early times, when the magnetic field might have been several orders of magnitude or more weaker than it is today, the decoupling effect would have been less important. Furthermore, the interstellar medium was probably almost fully ionized until metallic coolants had mixed into it. Thus, the mechanisms addressed here are probably more relevant to the maintenance of galactic magnetic fields than to their ultimate origin.

We are happy to acknowledge supports by NSF grants AST-0507367 and PHY-0215581 to the University of Wisconsin. Parts of the computations were performed at the National Center for Supercomputing Applications under projects DAC AST-040026 and MRAC AST-060031. We are grateful to an anonymous referee for useful suggestions.

**REFERENCES**

Boldyrev, S., & Cattaneo, F. 2004, Phys. Rev. Lett., 92, 144501
Braginskii, S. I. 1965, Rev. Plasma Phys., 1, 205
Brandenburg, A., & Subramanian, K. 2000, A&A, 361, L33
Brummell, N. H., Cattaneo, F., & Tobias, S. M. 2001, Fluid Dynamics Res., 28, 237
Cameron, R., & Galloway, D. 2005, MNRAS, 358, 1025
Cattaneo, F., Hughes, D. W., & Kim, E.-J. 1996, Phys. Rev. Lett., 76, 2057
Childress, F., & Gilbert, A. D. 1995, Stretch, Twist, Fold: The Fast Dynamo (Berlin: Springer)
Courvoisier, A., Hughes, D. W., & Tobias, S. M. 2006, Phys. Rev. Lett., 96, 034503
Galloway, D. J., & Proctor, M. R. E. 1992, Nature, 356, 691
Gilbert, A. D. 2002, Physica D, 166, 167
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
Haugen, N. E., Brandenburg, A., & Mee, A. J. 2004, MNRAS, 353, 947
Heitsch, F., Slyz, A. D., Devriendt, J. E. G., Hartmann, L. W., & Burkert, A. 2007, ApJ, 665, 445
Heitsch, F., & Zweibel, E. G. 2003a, ApJ, 583, 229
———. 2003b, ApJ, 590, 291
Kim, E.-J. 1997, ApJ, 477, 183
Klapper, I. M., & Young, L. S. 1995, Commun. Math. Phys., 173, 623
Kulsrud, R. M., & Anderson, S. W. 1992, ApJ, 396, 606
Kulsrud, R. M., & Zweibel, E. G. 2008, Rep. Prog. Phys., 71, 046901
Moffatt, H. K. 1978, Magnetic Field Generation in Electrically Conducting Fluids (Cambridge: Cambridge Univ. Press)
Ott, E. 1998, Phys. Plasmas, 5, 1636
Proctor, M. R. E., & Zweibel, E. G. 1992, Geophys. Astrophys. Fluid Dyn., 64, 145
Rees, M. J. 1987, QJRAS, 28, 197
Schekochihin, A. A., Cowley, S. C., Taylor, S. F., Maron, J. L., & McWilliams, J. C. 2004, ApJ, 612, 276
Schekochihin, A. A., Iskakov, A. B., Cowley, S. C., McWilliams, J. C., Proctor, M. R. E., & Yousef, T. A. 2007, New J. Phys., 9, 300
Shu, F. H. 1983, ApJ, 273, 202
Sridhar, S., & Goldreich, P. 1994, ApJ, 432, 612
Subramanian, K. 1997, preprint (astro-ph/9708216)
———. 1999, Phys. Rev. Lett., 83, 2957
Tang, H.-Z., & Xu, K. 2000, J. Comput. Phys., 165, 69
Tanner, S. E. M., & Hughes, D. W. 2003, ApJ, 586, 685
Tanner, S. E., & Hughes, D. W. 2003, ApJ, 586, 685
Xu, K. 1999, J. Comput. Phys., 153, 334
Zweibel, E. G. 1988, ApJ, 329, 384