Enhancing the performance of coupled quantum Otto thermal machines without entanglement and quantum correlations

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Abstract

We start with a revision study of two coupled spin-1/2 under the influence of Kaplan–Shekhtman–Entin-Wohlman–Aharony interaction and a magnetic field. We first show the role of idle levels, i.e. levels that do not couple to the external magnetic field, when the system is working as a heat engine as well as when it is a refrigerator. Then we extend the results reported in Altintas and Müstecaplıoğlu (2015 Phys. Rev. E 92 022142) by showing that it is not necessary to change both the magnetic field as well as the coupling parameters to break the extensive property of the work extracted globally from two coupled spin-1/2 as has been demonstrated there. Then we study the role of increasing the number of coupled spins on efficiency, extractable work, and coefficient of performance (COP). First, we consider two- and three-coupled spin-1/2 Heisenberg XXX-chain. We prove that the latter can outperform the former in terms of efficiency, extractable work, and COP. Then we consider the Ising model, where the number of interacting spins ranges from two to six. We show that only when the number of interacting spins is odd the system can work as a heat engine in the strong coupling regime. The enhancements in efficiency and COP are explored in detail. Finally, this model confirms the idea that entanglement and quantum correlations are not the reasons behind the enhancements observed in efficiency, extractable work, and COP, but only due to the structure of the energy levels of the Hamiltonian of the working substance. In addition to this, the extensive property of global work as well, is not affected by entanglement and quantum correlations.

Keywords: coupled quantum Otto thermal machines, entanglement, quantum correlations, quantum thermodynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

Classical thermodynamics [1] was developed in the 19th century to enhance the efficiency of heat engines [2]. On the other hand, quantum thermodynamics [3–18], is an emergent field that aims to extend thermodynamics to the regime where the working substance is quantum in nature and quantum effects are no longer negligible. In this field, particular attention has been devoted to the study of thermal machines fabricated from tiny quantum systems to improve their performance. This upsurge of interest started with the first work of Scovil and Schulz-DuBois when they showed that a three-level system is equivalent to a quantum heat engine (QHE) [19]. Since then, a lot of work has been developed to understand thermal machines at the quantum scale. Furthermore, in 2016, the first
experimental realization of a QHE was demonstrated using a trapped ion as a working fluid [20–22]. In 2017, the same platform was used to fabricate a quantum absorption refrigerator made of only three trapped ions [23]. Other platforms to realize quantum thermal machines experimentally could be found in [24–31], and more recently, all of these platforms have been collected in an article review [32]. When the baths are not thermal [33–43], it has been shown that the efficiency of these quantum thermal machines could surpass the thermodynamic Carnot bound. However, there is no violation of the laws of thermodynamics since in this case we have used resources, e.g. coherence, squeezing, and quantum correlations.

Many studies have been done since the first work done by Kieu [44, 45] on a QHE composed of one qubit undergoing an Otto cycle composed of two adiabatic and two isochoric stages in which he clarifies some important aspects of the second law of thermodynamics. First, Zhang et al have extended the same work to two coupled spin-1/2 Heisenberg XXX chain [46] where the exchange coupling J is the parameter changed in the adiabatic stages, and then in [47] where the external magnetic field h is the parameter changed in the adiabatic stages. The studies in this direction could be stated as follows: in [48] the authors investigated the case when one of the composite system is a spin-1/2 particle and the other is a qudit, in [49] when both interacting spins are qubits where the role of negative temperature on the efficiency has been investigated in detail, in [50] the role of degeneracy, in [51–53] when a thermal bath being replaced by a quantum measurement as a new source of heat, in [54–56] the role of Dzyaloshinski–Moriya interaction, in [57] the authors examine the performance of a two-coupled spins of arbitrary magnitudes (where majorization [58] has been used to find the upper bound of efficiency, which was an open question to be answered), and last but not least, the role of non-adiabaticity (i.e. when the unitary strokes are done in finite time) for one spin and two coupled spin-1/2 particles [59–63].

Recently, it has been shown in [47] that the efficiency of two coupled spin-1/2 particles is greater than the one of the Otto system. And since quantum systems are known for strange phenomena, e.g. quantum coherence, entanglement and quantum correlations, special attention has been devoted to understanding the role of entanglement and quantum correlations in efficiency and extractable work. An investigation of the role of entanglement and quantum correlations has been done in [46, 64–66]. However, even with these works, there was no sufficient and clear answer until recently, when Oliveira and Jonathan [67] provided an explanation by demonstrating that the enhancement observed in efficiency is solely due to the presence of idle levels. Moreover, in [53] the same enhancement was proved even for a three-level system, a system that cannot be divided into subsystems. However, in our opinion, a revisited study and more clarifications are needed to gain more insights in this direction.

Motivated by these works, we use the Ising model (with and without Kaplan–Shekhtman–Entin–Wohlmann–Aharony (KSEA) interaction) and the Heisenberg model to strengthen and demonstrate the correctness of their reasoning [53, 63, 67]. First, we use as a working fluid two coupled spin-1/2 particles under the influence of KSEA interaction [68–72] and a magnetic field along the z-direction. The KSEA interaction arises from spin–orbit coupling. We will see that when we have only quantum entanglement and working levels (i.e. levels that do couple to the external magnetic field), there is no enhancement in efficiency as it is expected. However, when the z-component of J is not equal to zero, two idle levels emerge, and we see that there is an enhancement in the efficiency which is due to heat passing through these levels from the cold to the hot bath. Then we show how idle levels affect the coefficient of performance (COP) when the system is working as a refrigerator. We show that they shift the enhancement in COP to large values of \( \Gamma_2 \). When the system is working as a refrigerator, it has been studied in [56, 73], but in contrast to them, we explore the role of idle levels as well as the role of increasing the number of interacting spins on the COP, two situations that have not been explored there. Moreover, taking advantage of this model, we generalize the results reported in [48], by showing that it is not necessary to change both the external magnetic field and the coupling parameters to break the extensive property of the global extractable work.

Secondly, we will increase the number of interacting systems to more than two coupled spin-1/2 up to six. We first compare the efficiency, extractable work, and COP of two- and three-coupled Heisenberg XXX chain. Our results show that three coupled spins could harvest more work than two coupled spin-1/2. Even more, the former could work as a heat engine in the strong coupling regime, which is not the case for the latter. When it comes to the COP, three coupled spins could surpass that of the Otto, which is not the case with two-coupled spin-1/2. Then we use the Ising model in which the number of interacting spins ranges from two to six spin-1/2 particles. We show that, in contrast to [48], when we increase the number of coupled spins, we see a remarkable enhancement in the extractable work. The efficiency will be enhanced only for small values of the coupling parameter J. Even more, a new conclusion has been drawn, which is that only when the number of interacting spins is odd, the system can work as a heat engine in the strong coupling regime. When it comes to COP, we found that when the system is ferromagnetic, there is a remarkable enhancement in the COP even though for three-to-six coupled spin-1/2 they are nearly coincidable. When the number of interacting spins ranges from two to six, we only consider the z-component of J to ensure that no entanglement or quantum correlations will build up along the cycle. More precisely, the coupled system will be only in a statistical mixture of factorized states. Therefore, the enhancements in efficiency, extractable work, and COP are only due to the structure of the energy levels of the system. Note that for all models chosen and studied in this work, we have degenerate eigenvalues, where in [52] the role of degeneracy on efficiency and extractable work has been explored in detail.

The rest of the article is organized as follows: in section 2 a brief review of the quantum Otto cycle and the relevant thermodynamic quantities to characterize the Otto heat engine and refrigerator is given. In section 3 we study the role of KSEA interaction on the efficiency, local and global extractable works and COP. The comparison between the efficiency,
COP and extractable work from two and three up to six-coupled spin-1/2 is done in the section 4. Then in section 5, we clarify some very important points. And finally, in section 6 we give a summary of our results with future directions.

2. Quantum Otto cycle

Before we start presenting our results, we should provide some necessary definitions and expressions used in this paper. Suppose we have a quantum system $S$ described by the state $\rho$ and a Hamiltonian $H$. The expectation value of the measured energy of $S$ is

$$U = \langle E \rangle = \text{Tr}(\rho H) = \sum_i p_i E_i,$$

where $E_i$ are the energy levels and $p_i$ are the corresponding occupation probabilities. The derivative of $U$ gives

$$dU = \sum_i E_i dp_i + p_i dE_i,$$

which can be divided into heat and work given, respectively, as follows: $\delta Q = \sum_i E_i dp_i$ and $\delta W = \sum_i p_i dE_i$. From these definitions, the average heat and work are: $Q = \sum_i p_i E_i$ and $W = \sum_i E_i p_i$. Therefore, the first law of thermodynamics reads

$$dU = \delta Q + \delta W.$$

Mathematically speaking, $dU$ is an exact differential, however, $\delta Q$ and $\delta W$ are not total differentials but are path dependent.

The Otto cycle is composed of four stages: two adiabatic stages in which an external controlled parameter is varied, and two isochoric stages in which the system is in contact with a heat bath. These stages are given as follows. Stage 1: The occupation probabilities of each level are $p_i'$, and the system is in contact with a hot bath at temperature $T_h$ until it reaches thermal equilibrium. Then it is described with the new occupation probabilities $p_i$. Since at this stage we have only a change in the occupation probabilities, only heat is exchanged between the working substance and the hot bath. Stage 2: The system is isolated from the hot bath and the magnetic field is changed from $h$ to $h'$ (with $h > h'$). This transformation will be done slowly to ensure the holding of the quantum adiabatic theorem. At this stage, only work is performed since the system is in contact with a cold bath at temperature $T_c$ (which is $T_h > T_c$ until it reaches equilibrium with it. In this case we have a change in the occupation probabilities from $p_i$ to $p_i'$. At this stage, only heat is exchanged. Stage 4: The system is again isolated from the cold bath and the magnetic field is changed back from $h'$ to $h$. At this stage, only work is performed. After this stage, the system is again attached to the hot bath at temperature $T_h$ to complete the cycle and return the system to its initial state. Furthermore, note that the Otto cycle is the most commonly used cycle to study quantum thermal machines. The reason is that in this cycle, the system at each stage exchanges either heat or work, not both of them, which leaves no ambiguity in the identification of them correctly. In addition, the definitions of heat and work used in this paper are only valid in the weak coupling regime.

When the system is working as a heat engine (for more details see, [44–46]), we must have $Q_h > 0$, $Q_c < 0$ and $W > 0$. In this case, in Stage 1 the system will absorb heat from the hot bath. Its expression is given by

$$Q_h = \sum_i E_i (p_i - p_i'),$$

where $E_i$ are the eigenvalues of the system at the hot bath side, $p_i$ and $p_i'$ are the populations of the system when it is in contact with the hot bath and the cold bath, respectively. In Stage 3 the system will release heat to the cold bath, which is given as follows

$$Q_c = \sum_i E_i' (p_i' - p_i).$$

From the first law of thermodynamics, we have

$$W = Q_h + Q_c = \sum_i (E_i - E_i') (p_i - p_i').$$

Therefore, the expression of efficiency is

$$\eta = \frac{W}{Q_h} = 1 + \frac{Q_c}{Q_h}.$$

When the system is working as a refrigerator, we must have $Q_h < 0$, $Q_c > 0$ and $W < 0$, in other words, the system is running in the reverse of the Otto heat engine cycle. In this case, the system is characterized by its COP, which is given as follows

$$\text{COP} = \frac{Q_c}{|W|} = \frac{Q_c}{Q_h + Q_c}.$$

We should note that the efficiency of the Otto cycle when the working medium is either a single or (multiple but uncoupled) spin(s) or harmonic oscillator(s) is given by $\eta_0 = 1 - h'/h$. In the same manner, when the system is working as a refrigerator COP $= \frac{h'}{h}$, see [6, 7]. In this paper, the units are chosen such that $k_B = h = 1$.

3. Revisited study of a coupled quantum Otto heat engine and refrigerator

Our working fluid in this section is a two-coupled spin-1/2 1D Ising model with a $z$-component KSEA interaction parameter $\Gamma_z$ under the influence of a magnetic field $h$ in the $z$-direction. The expression of the Hamiltonian describing this system is [74]

$$H = J_x \sigma_x^1 \sigma_x^2 + \Gamma_z (\sigma_x^1 \sigma_z^2 + \sigma_y^1 \sigma_y^2) + h(\sigma_z^1 + \sigma_z^2),$$

where $\sigma_{x,y,z}^i$ are the standard Pauli matrices acting on the site $i \in \{1,2\}$. The eigenvalues of $H$ are: $E_1 = E_2 = -J_z$, $E_3 = J_x - 2\sqrt{h^2 + \Gamma_z^2}$ and $E_4 = J_x + 2\sqrt{h^2 + \Gamma_z^2}$. The expression of the magnetic field is $h = \frac{\gamma}{g \mu_B}$, where $\gamma$, $g$, and $\mu_B$ are the gyromagnetic ratio, the Landé g-factor, and the Bohr magneton, respectively.

The correctness of the mathematical expressions and the accuracy of the numerical calculations are confirmed, for instance, with the given expressions of the eigenvalues and the eigenfunctions. The obtained results, in particular the calculated values of COP and efficiency, are also compared with the previous results in the literature.
\[ E_4 = J_z + 2\sqrt{\hbar^2 + \Gamma_z^2} \]. Their corresponding eigenstates in the standard basis \{\{11\}, \{10\}, \{01\}, \{00\}\} are: |
\[ |\psi_1\rangle = |10\rangle, \quad |\psi_2\rangle = |01\rangle, \quad |\psi_3\rangle = \frac{1}{\sqrt{\alpha_1^2 + 1}}(\alpha_1|11\rangle + |00\rangle) \] and \[ |\psi_4\rangle = \frac{1}{\sqrt{\alpha_2^2 + 1}}(\alpha_2|11\rangle + |00\rangle) \] with \( \alpha_1 = \left(-h + \sqrt{\hbar^2 + \Gamma_z^2}\right)/\Gamma_z \) and \( \alpha_2 = \left(h + \sqrt{\hbar^2 + \Gamma_z^2}\right)/\Gamma_z \). The KSEA interaction has been chosen for two reasons. First, when the z-component of \( J \) is equal to zero, we have entanglement-(between the two coupled spins described by equation (9) when they are in thermal equilibrium with either the hot bath or the cold bath, see equation (10))-but no idle levels. More precisely, here we want to test if the presence of only entanglement or quantum correlations could boost the efficiency beyond that of the Otto.

Second, for this Hamiltonian, we see that the eigenstates are dependent on the magnetic field \( h \) and \( \Gamma_z \); and that the eigenvalues are non-linear in \( h \). This situation is different from the cases considered in [46, 47, 49, 52]. Below, we see that this influences the extensive property of the work extracted globally. Moreover, as far as we know this interaction has not been considered in the previous works to the authors’ knowledge.

When the system is in a thermal equilibrium with a heat bath at inverse temperature \( \beta \), it can be described by the density operator \( \rho_\text{th} = \sum_i \rho_i |\psi_i\rangle \langle \psi_i| \). In the computational basis \{\{11\}, \{10\}, \{01\}, \{00\}\}, this matrix is given as follows:

\[
\rho_\text{th} = \begin{pmatrix}
|\alpha_1|^2|p_3 + |\alpha_2|^2|p_4 |0 0 |n_1^2|p_3 + |n_2^2|p_4 |0 0 |n_1^2|p_3 + |n_2^2|p_4 \\
|\alpha_1|^2|p_3 + |\alpha_2|^2|p_4 |0 0 |0 |0 0 |0 \\
|\alpha_1|^2|p_3 + |\alpha_2|^2|p_4 |0 0 |1 |0 |1 |0 \\
|\alpha_1|^2|p_3 + |\alpha_2|^2|p_4 |0 0 |0 |1 |0 |1 |0
\end{pmatrix}
\]

(10)

with \( p_i = e^{-\beta E_i}/Z \). The partition function is \( Z = 2(e^{\beta J_z} + e^{-\beta J_z}) \). The parameter values for the plot are \( T_b = 4, T_c = 1, h = 4 \) and \( h' = 3 \), on the other hand, when it is working as a refrigerator, the parameter values for the plot are \( T_b = 2, T_c = 1, h = 5 \) and \( h' = 2 \). These parameters have been chosen to ensure that \( h > h' \) and \( h/T_b < h'/T_c \) for the system to function as a heat engine [44, 45], \( h > h' \) and \( h/T_b > h'/T_c \) for the system to function as a refrigerator. This is for uncoupled systems, however, for coupled ones, the conditions become more involved because of the coupling.

3.1. Global description

Shifting the eigenvalues of the Hamiltonian \( H \) will not alter heats and work [6]. They become: \( E_1 = E_2 = -2J_z, \ E_3 = -2\sqrt{\hbar^2 + \Gamma_z^2} \) and \( E_4 = 2\sqrt{\hbar^2 + \Gamma_z^2} \). Therefore, the expressions of the global thermodynamical quantities, i.e. heats and work, are

\[
Q_h = \frac{2\sqrt{\hbar^2 + \Gamma_z^2} \sinh(2\beta\sqrt{\hbar^2 + \Gamma_z^2}) + 2J_z e^{2\beta J_z}}{e^{2\beta J_z} + \cosh(2\beta\sqrt{\hbar^2 + \Gamma_z^2})}
\]

\[
= \frac{-2\sqrt{\hbar^2 + \Gamma_z^2} \sinh(2\beta\sqrt{\hbar^2 + \Gamma_z^2}) - 2J_z e^{2\beta J_z}}{e^{2\beta J_z} + \cosh(2\beta\sqrt{\hbar^2 + \Gamma_z^2})},
\]

(13)

\[
Q_c = \frac{-2\sqrt{\hbar^2 + \Gamma_z^2} \sinh(2\beta\sqrt{\hbar^2 + \Gamma_z^2}) + 2J_z e^{2\beta J_z}}{e^{2\beta J_z} + \cosh(2\beta\sqrt{\hbar^2 + \Gamma_z^2})}
\]

\[
= \frac{2\sqrt{\hbar^2 + \Gamma_z^2} \sinh(2\beta\sqrt{\hbar^2 + \Gamma_z^2}) + 2J_z e^{2\beta J_z}}{e^{2\beta J_z} + \cosh(2\beta\sqrt{\hbar^2 + \Gamma_z^2})},
\]

(14)
and vertical quantities are unchanged when substituting not write them here. Moreover, since here the thermodynamic COP. However, since their expressions are too long, we will
levels and should be distinguished from the local heats given by, qh from the hot bath as a function of Γz for Jz = 0, 0.5 and 2.6. Note that qh here is the heat absorbed globally from the hot bath by idle levels and should be distinguished from the local heats given by, qh1 and qh2. The parameter values for the plots are Tk = 4, Tc = 1, \( h = 4 \) and \( h^* = 3 \). Note that if one plot \( \eta \) and qh versus Jz for different values of \( \Gamma_z \) will get the same plots.

\[
W = 2 \left( \sqrt{h^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2} \right) \frac{\sinh(2\beta h \sqrt{h^2 + \Gamma_z^2})}{\cosh(2\beta h \sqrt{h^2 + \Gamma_z^2})} - \frac{\sinh(2\beta h \sqrt{h^2 + \Gamma_z^2})}{\cosh(2\beta h \sqrt{h^2 + \Gamma_z^2})}.
\]

(15)

From these formulas and equations (7) and (8), it is straightforward to get the formulas of the efficiency as well as the COP. However, since their expressions are too long, we will not write them here. Moreover, since here the thermodynamical quantities are unchanged when substituting \( \Gamma_z \) by \( -\Gamma_z \) we only plot them for positive values of \( \Gamma_z \). For negative values of \( J_z \) we found no enhancement, so we plotted them only for positive values of \( J_z \).

In figure 1(a), the efficiency is plotted as a function of \( \Gamma_z \) for three different values of \( J_z = 0, 0.5, \) and 2.6. For \( J_z = 0 \), we see that the efficiency could not surpass that of the Otto \( \eta_o = 1 - h^*/h \), since in this case \( q_h = 0 \). However, for \( J_z > 0 \), e.g. \( J_z = 0.5 \) and 2.6, we see that the efficiency could surpass \( \eta_o \). This could be explained from figure 1(b). When \( J_z \neq 0 \), we can have \( q_h < 0 \) depending on \( \Gamma_z \). This is why efficiency can be enhanced. For example, when \( \Gamma_z = 0 \), \( q_h \) is a monotonically increasing function as we increase \( J_z \) and it reaches its maximal value(\( \approx 0.435 \)) when \( J_z \approx 1.788 \). However, for \( \Gamma_z = 0 \) and after \( J_z \) exceeding 1.788, \( \ln|q_h| \) starts decreasing again, and if \( J_z \) is kept increasing, then \( q_h \) can even become positive. This is why we see that \( |q_h| \) for \( J_z = 2.6 \) is less than \( |q_h| \) for \( J_z = 0.5 \) for \( \Gamma_z < \approx 0.61 \). The explicit expression of \( q_h \) in terms of \( J_z \) and \( \Gamma_z \) is given as follows,

\[
q_h = -2J_z \frac{e^{-J_z/4}}{2(e^{J_z/4} + e^{-J_z/4} \cosh(\sqrt{16 + \Gamma_z^2}/2))} = -\frac{e^{-J_z}}{2(e^{J_z} + e^{-J_z} \cosh(\sqrt{9 + \Gamma_z^2}/2))}.
\]

(16)

Figure 2 displays the contour plot of \( q_h \) as a function of \( J_z \) and \( \Gamma_z \). For an arbitrary value of \( \Gamma_z \), there is a critical value of \( J_z \) above which \( q_h \) would become positive, and it is given by,

\[
J_z \leq J_z^* = 2\log \left( \frac{\cosh(2\sqrt{9 + \Gamma_z^2})}{\cosh(\sqrt{16 + \Gamma_z^2}/2)} \right)/3.
\]

(17)

For example, when \( \Gamma_z = 0 \), we have \( J_z^* \approx 2.65 \).

In figure 3, we plot COP as a function of \( J_z \) for different values of \( J_z \). We see that, as it is expected, idle levels diminish the COP of the system when it is working as a refrigerator. More precisely, for \( J_z = 0 \) we have no idle levels which makes the COP surpass that of the Otto \( \eta_o \approx 0.435 \). However, when \( J_z \neq 0 \) the COP get enhanced only after certain value of \( \Gamma_z \), i.e. the enhancement of COP shift to big values of \( \Gamma_z \). This is because for small values of \( \Gamma_z \) and when \( J_z \neq 0 \) we found that idle levels are taking heat from the hot to the cold bath which diminishes the COP. This is due to the fact

\( J_z \leq J_z^* = 2\log \left( \frac{\cosh(2\sqrt{9 + \Gamma_z^2})}{\cosh(\sqrt{16 + \Gamma_z^2}/2)} \right)/3. \)
These equations could be expressed as well as follows:

\[
Q_h = q_1 + q_{wh}, \quad Q_c = -q_1 - q_{wc} \quad \text{and} \quad W = q_{wh} - q_{wc}. \quad \text{Here } q_1 \text{ and } q_{wh} \text{ are the heats absorbed from the hot bath by idle and working levels respectively. The efficiency is,}
\]

\[
\eta = \left( 1 - \frac{\sqrt{h^2 + \Gamma_z^2}}{\sqrt{h^2 + \Gamma_z^2}} \left( 1 - \frac{1}{2\sqrt{h^2 + \Gamma_z^2}(p_2 - p'_2)} \right) \right),
\]

which could be written as well as follows, \( \eta = \eta_1 \left( 1 + \frac{1}{q_{wh}} \right) \), with \( \eta_1 = 1 - \frac{\sqrt{h^2 + \Gamma_z^2}}{\sqrt{h^2 + \Gamma_z^2}} \).

One can see that for \( \Gamma_z = 0 \), \( \eta \) will be \( \eta_0 \) times another term (which has to be bigger than one for \( \eta > \eta_0 \)). Thus, the condition for \( \eta > \eta_0 \) is that, \( 0 < \frac{4J_c(p_2 - p'_2) - \Gamma_z p_2 - \Gamma_z p'_2}{2\Gamma_z^2(p_2 - p'_2) + p_2 - p'_2} < 1 \), or equivalently \( -2h(p_3 - p'_3 + p'_1 - p_1) < q_1 < 0 \). This condition shows that as long as the heat channelled from the hot to the cold bath by idle levels is not positive and not less than \(-q_{wh}\), then efficiency could be improved. However, if \( q_1 < -q_{wh} \), \( Q_0 \) will be negative, thus the system will not work as a heat engine. The same thing could be said when \( \Gamma_z \neq 0 \) even though things get complicated because the eigenvalues are not linear in the magnetic field. Thus, different from \( \Gamma_z = 0 \), in this case we must have \( 1 + \frac{\eta_1}{q_{wh}} < \frac{\eta_0}{\eta_0} \). Adding the next condition, \( \eta_0 < \eta < \eta_c \) (even though \( \eta \) may not be able to reach the Carnot bound, but let us take the biggest upper bound allowed by the second law of thermodynamics), in this case the condition on heat absorbed by idle level from the hot bath is,

\[
-q_{wh} \frac{\eta_k - \eta_0}{\eta_k} < q_1 < -q_{wh} \frac{\eta_k - \eta_0}{\eta_0} < 0. \quad (22)
\]

We see that \( q_1 \) not only has to be less than 0 as in the case when \( \Gamma_z = 0 \), but only when it is less than \(-q_{wh}\eta_0/\eta_0\), the efficiency can be enhanced, and this is due to the fact that the eigenvalues are nonlinear in the magnetic field.

The expression of the COP is,

\[
\text{COP} = \frac{\sqrt{h^2 + \Gamma_z^2}}{\sqrt{h^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2}} - \frac{4J_c(p_2 - p'_2)}{2(\sqrt{h^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2})(p_4 - p'_4 + p'_3 - p_3)}. \quad (23)
\]

From this equation, for \( J_z = 0 \) we have, \( \text{COP} = \frac{\sqrt{h^2 + \Gamma_z^2}}{\sqrt{h^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2}} \), this term is always bigger than \( \text{COP}_0 \) as long as \( \Gamma_z \neq 0 \). Moreover, if we continue increasing \( \Gamma_z \) one can see that this term could reach the Carnot bound. In this case \( (J_z = 0) \) we have no idle levels, but we have two working levels and two degenerate levels with energy 0, that neither contribute to heats nor work. Thus, in this case, our system is equivalent to a two-level system with an energy gap \( 4\sqrt{h^2 + \Gamma_z^2} \). However, when \( J_z \) is not equal to zero things will be different and complicated. For example, when \( J_z > 0 \), the enhancement in this case is dependent on the sign of \( p_2 - p'_2 \). More precisely, if \( p_2 - p'_2 < 0 \), in this case we see that the second term will be negative thus it will diminish.
the COP. However, if $p_2 - p_1' > 0$, in this case the COP will be enhanced. Therefore, in this case, for $\text{COP} > \text{COP}_b$ the term $q_e = 4J_z(p_2 - p_1')$ has to be positive, i.e. idle levels has to take heat in the right direction from the cold bath to the hot bath. When $J_z = 0$ it does not matter if $p_2 - p_1' < 0$ or $p_2 - p_1' > 0$, since in this case idle levels will channel no heat. However, when $J_z > 0$ then in this case we must have $p_2 - p_1' > 0$, i.e. the probability of occupation of the degenerate idle level $-2J_z$ at the hot bath side has to be equal or bigger than the one at the cold side.

Now we give the theory used to describe the thermodynamics of the global and the local cycles. Let’s give the expressions of the global as well as the local work and heats. We start with the global ones.

$$W_1 = \text{Tr}(\rho(H_g - H'_g)), \quad (24)$$

this is the work performed during the adiabatic stage 2. $\rho$ is the state of the system when it is in equilibrium with the hot bath, $H_g$ and $H'_g$ are the global Hamiltonians when the magnetic field is equal to $h$ and $h'$ respectively.

$$Q_h = \text{Tr}(H_g(\rho - \rho'))), \quad (25)$$

this is the heat absorbed from the hot bath, $\rho'$ is the state of the system when it is in equilibrium with the cold bath.

$$W_2 = -\text{Tr}(\rho'(H_g - H'_g)), \quad (26)$$

this is the work performed during the adiabatic stage 4. Finally,

$$Q_c = -\text{Tr}(H'_g(\rho - \rho'))), \quad (27)$$

this is the heat released to the cold bath. The first law of thermodynamics reads as $W_1 + W_2 + Q_1 + Q_2 = 0$, from which we have $W = -(W_1 + W_2) = T(\rho(\rho - \rho')(H_g - H'_g))$. The local ones are given in the same way as follows,

$$w_\alpha = \text{Tr}(\rho_\alpha(H_1 - H'_1)), \quad (28)$$

$$q_{h\alpha} = \text{Tr}(H_1(\rho_\alpha - \rho'_\alpha)), \quad (29)$$

$$w'_\alpha = -\text{Tr}(\rho'_\alpha(H_1 - H'_1)), \quad (30)$$

$$q_{c\alpha} = -\text{Tr}(H'_1(\rho_\alpha - \rho'_\alpha)), \quad (31)$$

with $\alpha = 1, 2$, $H_1 = \text{diag}\{h_1, -h_1\}$ is the local Hamiltonian of the spins. The definitions of the global and local works will be compared below for the Ising+KSEA model. Since the reduced state of both spins is the same $\rho_1 = \rho_2$, the total work extracted locally is,

$$w = 2\text{Tr}((\rho_1 - \rho'_1)(H_1 - H'_1)), \quad (32)$$

and if $\rho_1 \neq \rho_2$, $w = \sum_\alpha \text{Tr}((\rho_\alpha - \rho'_\alpha)(H_1 - H'_1))$. Another definition of local heats and work which one can use as well is the case when the local heats and work are defined with respect to the global Hamiltonian $H_g$ and local state $\rho_\alpha$. In this case we have,

$$w_\alpha = \text{Tr}(\rho_\alpha(H_g - H'_g)), \quad (33)$$

$$q_{h\alpha} = \text{Tr}(H_g(\rho_\alpha - \rho'_\alpha)), \quad (34)$$

$$w'_\alpha = -\text{Tr}(\rho'_\alpha(H_g - H'_g)), \quad (35)$$

$$q_{c\alpha} = -\text{Tr}(H'_g(\rho_\alpha - \rho'_\alpha)). \quad (36)$$

If $\rho_1 = \rho_2$, the total work extracted locally is,

$$w = 2\text{Tr}((\rho_1 - \rho_1)(H_g - H'_g)). \quad (37)$$

In general, we have four cases with respect to which we can compute the average of work and heats: 1) with respect to the global state and the global Hamiltonian, 2) with respect to the local state and the global Hamiltonian, 3) with respect to the global state and the local Hamiltonian, and finally 4) with respect to the local state and the local Hamiltonian. 3 and 4 are equivalent. Furthermore, 1, 2 and 3 may be equal for the total work extracted depending on the model. However, in terms of heats will not be equal. Thus, we would obtain different efficiencies with these definitions.

3.2. Local description

Now let us see how the coupled spin-1/2 particles are undergoing the cycle locally. The reduced density matrix of the subsystems 1 and 2 in the standard basis $\{|1\rangle, |0\rangle\}$ when they are in thermal equilibrium with the hot bath is

$$\rho_1 = \rho_2 = \left(\begin{array}{cc}
\frac{1}{2} & \frac{1}{2} - \frac{h(p_1 - p_1')}{\sqrt{h^2 + 1}} \\
0 & \frac{1}{2} + \frac{h(p_1 - p_1')}{\sqrt{h^2 + 1}}
\end{array}\right). \quad (38)$$

The reduced density matrix of the sub-systems 1 and 2 in the standard basis $\{|1\rangle, |0\rangle\}$ when they are in thermal equilibrium with the cold bath is

$$\rho'_1 = \rho'_2 = \left(\begin{array}{cc}
\frac{1}{2} & \frac{1}{2} - \frac{h'(p'_1 - p'_1')}{\sqrt{h'^2 + 1}} \\
0 & \frac{1}{2} + \frac{h'(p'_1 - p'_1')}{\sqrt{h'^2 + 1}}
\end{array}\right). \quad (39)$$

Note that equations (38) and (39), has been obtained using the normalization constraint $\sum_\alpha \rho_\alpha = 1$. Following [47], the local Hamiltonians for the spins are $H = \text{diag}(h, -h)$ and $H' = \text{diag}(h', -h')$ at Stages 1 and 3, respectively. And using the formulas given in the section above, the local heat absorbed from the hot bath $q_{h1,2}$, the local heat released to the cold bath $q_{c1,2}$, and the amount of work extracted locally $w_{1,2}$ for each spin are expressed as follows.
In figure 4, values of and global works become more pronounced. This is in contrast to what was suggested that to violate the extensive property depends as well on the different parameters entering the eigenstates of the Hamiltonian describing the system. However, when the results are in agreement with each other.

At this point we want to give some clarifications about the extensive property is not related to any type of classical or quantum correlations. To make it clear, let us take the above example and divide the expression of the global work into two terms. The expression of the total work extracted locally is given as follows:

$$W = \sum_i (E_i - E'_i)(p_i - p'_i).$$

(44)

From this expression we have,

$$W = \sum_i (E_i - E'_i)p_i + \sum_i (E'_i - E_i)p'_i = -(W_1 + W_2).$$

(45)

The first work \((-W_1)\) is the work performed when we change the magnetic field from \(h\) to \(h'\) with respect to the state \(\rho\). The second term \((-W_2)\) is the work performed when we change back \(h'\) to \(h\). Now let us consider these two works separately and examine the extensive property. We want to compare \(-W_1\) and \(-W_2\) with their corresponding works invested \(-w_1\) and extracted \(-w_2\) locally.

$$w = \sum_i ((E_i - E'_i)p_i + (E'_i - E_i)p'_i) = -(w_1 + w_2).$$

(46)

with \(E_i = \{h, -h\}\), and \(p_{ia}\) are the local probabilities. Let’s now compare \(-W_1\) with \(-w_1\). More precisely, we want to compare the works performed globally and locally with each other during stage 2. In this case we have,

$$-W_1 = 2\left(\sqrt{h'^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2}\right)(p_4 - p_3),$$

(47)

and,

$$-2w_1 = 2(h - h')\left(\frac{h(p_4 - p_3)}{\sqrt{h'^2 + \Gamma_z^2}}\right).$$

(48)

Since \((p_4 - p_3) < 0\), thus \(-W_1\) and \(-2w_1\) are both negative, therefore they are the works invested globally and locally, respectively. Their difference is given as follows:

$$-(W_1 - 2w_1) = 2\left(\sqrt{h'^2 + \Gamma_z^2} - \sqrt{h^2 + \Gamma_z^2}\right) - \frac{(h - h')h}{\sqrt{h'^2 + \Gamma_z^2}}(p_4 - p_3).$$

(49)

One can see clearly that if \(\Gamma_z = 0\) we get \(-W_1 = -2w_1\), in this case the model reduces to the Ising model. Since \(p_4 - p_3 < 0\) and that the first term is as well negative when \(\Gamma_z \neq 0\), thus \(-(W_1 - 2w_1) > 0\). Thus, the total work invested locally during the adiabatic stage 2 is not equal to the one invested globally. In the same way, one can show
that \(-(W_2 - 2w_2) > 0\). Note that because \(-W_2\) and \(-2w_2\) are positive, thus they are the works extracted during stage 4. And because the total works invested (extracted) globally and locally are not equal in stage 2 (stage 4), therefore, the global work will not be extensive. The explanation for this gap is that the eigenvalues of Ising+KSEA model are not linear in the magnetic field and that the eigenstates are parameter dependent.

Let’s now explain why \(W - w \geq 0\) has nothing to do with quantum correlations. If one studies the work extracted by two coupled spins in the Heisenberg, Ising and Ising+KSEA models, one will find that only the first and second models preserve the extensive property, even though both Heisenberg and Ising+KSEA models are quantum mechanically correlated. Thus, the extensive property of the work has nothing to do with quantum correlations, it only depends on the structure of the Hamiltonian. For example, in [64] the eigenvalues were nonlinear in the magnetic field and the eigenstates were parameter dependent. However, there they explain the gap \(W - w > 0\) by quantum correlations. In contrast to their conclusion, ours show that quantum correlations are not responsible for this gap. And thus it is difficult to give an interpretation of the extracted work in terms of quantum correlations, since there is no clear relationship between them in our study. However, note that if one includes measurement and feedback in the protocol, in this case, quantum correlations will affect the work output. More precisely, suppose we apply a measurement on one of the correlated spins and then apply some operations on the unmeasured one depending on the result of the measurement (feedback). In this case, the correlations between the spins will be important. However, in our study we were only interested in running the Otto cycle on each spin locally.

Another point that should be clarified is the one concerning the local and global descriptions. More precisely, one may think that the reason behind \(W - w > 0\) is that the global cycle is Otto and the local one is not. Let’s explain this in detail. This is because from equations (38) and (39) one may notice that the local probabilities at both sides are magnetic field dependent. Then, one may directly conclude that the cycle is globally Otto and that the quantum adiabatic theorem is valid. However, the adiabatic stages of the local cycle may not be adiabatic and, therefore, the quantum adiabatic theorem is no longer satisfied locally. However, this conclusion is not true. Because according to this conclusion, even the global probabilities will not stay the same in the adiabatic stages. And this is because the global probabilities as well are magnetic field dependent either through 
\[ e^{-\beta H} \text{ (if } E_i \text{ is magnetic field dependent) or through } Z = 2(e^{\beta \mu} + e^{-\beta \mu} \cos(2\beta \sqrt{h^2 + T_\gamma^2})), \]
however, if the transformation is done slowly there will be no transitions between the states and thus the cycle would satisfy the quantum adiabatic theorem, i.e. there will be no changes in the populations. Therefore, using the same reasoning, the local adiabatic stages would be adiabatic if the transformation is done slowly, thus the system will only exchange work and no heat \(q_m = 0\), and the adiabatic theorem is still satisfied. Therefore, the local cycle is also Otto. Furthermore, this model makes it difficult to show that correlations are not necessary to surpass the efficiency of the Otto. To show this, below we use the Ising model, keeping only the \(z\)-components which exhibit no quantum correlations at all. The system would be along the cycle in a separable state.

4. Comparison between multi-coupled spin-1/2 particles

Here we first compare the efficiency, the extractable work, and the COP of two- and three-coupled spin-1/2 1D Heisenberg XXX-chain. Then we do the same comparison between two coupled and the systems corresponding up to six spins in the Ising model. The latter has been chosen for two reasons: first to ensure that no entanglement and quantum correlations will be created during the cycle between the interacting spins, as our purpose is to show and confirm that the enhancement is only due to the structure of the energy levels of the system. Secondly, we want to study the role of increasing the number of interacting spins on efficiency, extractable work and COP.

4.1. Heisenberg model

Here, the working fluid is a three-coupled spin-1/2 Heisenberg XXX-chain under the influence of a magnetic field \(h\) along the \(z\)-axis. We call that the previous works [46–49, 57] have considered only two coupled spin-1/2. The expression of the Hamiltonian is given by

\[
H = J \sum_{i} (\sigma_i^x\sigma_i^{x+1} + \sigma_i^y\sigma_i^{y+1} + \sigma_i^z\sigma_i^{z+1}) + h \sum_{i} \sigma_i^z, \tag{50}
\]

where \(\sigma_i^{x,y,z}\) are the standard Pauli matrices acting on the site \(i = 1, 2, 3\). Note that the periodicity is presumed. \(J\) and \(h\) are the exchange coupling and the strength of the external magnetic field, respectively. When \(J < 0\) the model is ferromagnetic, while \(J > 0\) corresponds to an antiferromagnetic system. Here, we have the isotropic situation \(J_1 = J_2 = J\). The eigenvalues of \(H\) are given by:

\[
E = \{-h + 3J, -(h + 3J), h - 3J, h - 3J, -3(h - J), 3(h + J), -h + 3J, h + 3J\}. \tag{51}
\]

Their associated eigenstates will not be reported here, since we will not use them.

In figure 5(a), we plot the work as a function of \(J\) for \(N = 2\) and \(N = 3\). We see that when \(N = 3\), the work extracted globally is bigger than the one from two-coupled spin-1/2. Even more, the three coupled spins could still harvest work in the strong coupling regime in contrast to the two-coupled spin-1/2. In figure 5(b), the efficiency \(\eta\) is plotted as a function of \(J\). We see that \(N = 3\) outperforms \(N = 2\) in terms of the highest value riched of the efficiency. More precisely, for the considered value of the parameters, the maximum value riched when \(N = 3\) is \(\approx 0.36\), and when \(N = 2\) it is \(\approx 0.34\). We should add that for \(J \leq 0.52\), the efficiency of two coupled spins is higher than that of \(N = 3\), as figure 5(b) shows. In figure 6 we plot their COP as well as the ones of Otto COP, and Carnot COP. We see that only three-coupled spin-1/2 COP could surpass the COP of the Otto COP, however, the COP of two-coupled spin-1/2 is always equal or less than it.
spins is always bigger than that of \( N = 2 \) since, as figure 6 shows, the latter can be higher than that of \( N = 3 \) for broad values of \( J \), but its maximum value does not exceed the one of \( N = 3 \). In addition to this, notice that when the system is working as a heat engine for three-coupled spin-\( 1/2 \), its efficiency could surpass that of the Otto \( \eta_o \), only when it is anti-ferromagnetic. And when it is a refrigerator, the enhancement is seen only when the system is ferromagnetic. We have also calculated the work extracted locally by the three coupled spins, and we found that it is equal to the one extracted globally. As we said before, this is due to the fact that the eigenstates of the system are parameter independent and that the eigenvalues are linear in the magnetic field [47]. However, we did not include its plot here for brevity.

Now let us explain the reasons behind the enhancement in the work extracted. First, we expect that more levels will help extract more work, e.g. a harmonic oscillator could extract more work than a single spin. However, their efficiencies are equal if all energies are shifted by the same amount of energy. More precisely, we see from the figure 5 that when the coupling is turned off, i.e. \( J = 0 \), then as we increase the number of spins (i.e. when \( N = 2 \) and \( N = 3 \) for \( J = 0 \)) more work will be extracted. When turning on the interaction, we see that the work extracted by coupled spins enhances the one from uncoupled ones. Thus we see that both the number of spins \( N \) as well as the interaction \( J \) between them enhances the work extracted globally \( W \).

When we compared the efficiency of 3 and 2 coupled Heisenberg spins we found that the latter cannot work as a heat engine in the strong coupling regime, and the reason is that the positive work condition is not satisfied, since in this regime the system consumes work instead of outputting it, thus the system no longer works as a heat engine. More precisely, the work extracted \( -W_2 \) during the adiabatic stage 4 is less than the one invested \( -W_1 \) in the adiabatic stage 2, thus \( W = -(W_1 + W_2) \leq 0 \). When it comes to the three coupled spins in the strong coupling regime, the eigenvalues of the system become equidistant, thus with efficiency equal to the one of the Otto \( \eta_o = 1 - \frac{W}{H} \).

Concerning the local and global thermodynamics of the three spins Heisenberg model, we have the preservation of the extensive property of the work, i.e.

\[
W = w_1 + w_2 + w_3, \tag{51}
\]

with the local work extracted by each spin is \( w_\alpha = Tr((H_\alpha - H_\alpha')(\rho_\alpha - \rho_\alpha')) \), with \( \alpha = 1, 2, \text{ and } 3 \). The local states are, \( \rho_1 = (a, 1 - a) \), \( \rho_2 = (b, 1 - b) \) and \( \rho_3 = (c, 1 - c) \). The parameters a, b and c are functions of the populations and they are different, not like the case of two coupled spins, because in this case the state is no longer symmetric with respect to the partial trace. Therefore, the local extractable work is,

\[
w_1 + w_2 + w_3 = 2(h - h')(a - a' + b - b' + c - c'). \tag{52}
\]

If we replace \( a, a', b, b', c, \text{ and } c' \) with their expressions in terms of populations, we get that the global work \( W \) is equal to the total local one. It would be very important to compute the

![Figure 5. Plot of (a) the extracted work when \( N = 2 \) (orange dashed line) and \( N = 3 \) (blue solid line) as a function of \( J \). (b) the efficiency for both \( N = 2 \) and \( N = 3 \) as a function of \( J \). The parameter values are the same as in figure 1.]

![Figure 6. The plot of COP for \( N = 2 \) (red large-dashing line) and \( N = 3 \) (blue dotted line) as a function of \( J \). The parameter values are the same as in figure 3. The green dot-dashed line and the solid orange line correspond to the COP of Otto and Carnot, respectively.]
upper bound of the efficiency when we are no longer interested in two interacting systems but \( N \) \( s \)-spins, and to investigate if the number of coupled spins could affect the upper bound of the efficiency in addition to the coupling and the spin of the particles (see, [57]), i.e. if \( \eta_{\text{up}} \) is a function of \( N \).

### 4.2. Ising model

The Hamiltonian of the \( N \) spin-\( \frac{1}{2} \) Ising model under the influence of a magnetic field \( h \) is

\[
H = J \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + h \sum_{i=1}^{N} \sigma_i^z, \tag{53}
\]

where \( \sigma_i^z \) is the \( z \)-component of the Pauli spin matrices acting on the site \( i \in [1, N] \). The periodicity is here as well as assumed, i.e. \( \sigma_{i+1}^z = \sigma_i^z \). \( N \) is the total number of sites, and here it will range from 2 to 6. The eigenvalues of this Hamiltonian are given in the appendix. Note that this Hamiltonian has been used in [67], to show that the enhancement is only a matter of the structure of the system and it is not quantum correlations. Here, we also study the case when the system is working as a refrigerator which was not considered in [67].

In figure 7(a), we plot the work extracted from two coupled spins up to six. We see that as we increase their number, more work can be extracted. As explained before, it is the number of spins \( N \) and the interaction between them \( J \) that enhances the work extracted. In figure 7(b), we plot the efficiency as a function of \( J \) for \( N = 2 \) up to \( N = 6 \). We see that increasing the number of coupled spins enhances the efficiency, but only for small values of \( J \). However, the efficiency of three up to six coupled spins is nearly coincidable. In [48] the authors studied a coupled spin-1/2 with another \( s \) spin where \( s \) ranges from 1 to 3, and they found that all of these interacting spins can work as heat engines in the strong coupling regime. However, in our case, we see from figure 8, that only three and five interacting spins could work as heat engines in the strong regime. Furthermore, in our case, in contrast to them, we see a remarkable enhancement in the extractable work, as in [52], since in our case as well, we have degeneracy. In figure 9 we plot work versus efficiency for \( J \) in the interval [0, 10]. We can see that the efficiency at maximum work is not altered by the number of interacting spins. However, in contrast to [48], the work at maximum efficiency is affected by the number of interacting spins.

Now let us study the case when our coupled spins are working as a refrigerator. For this, the COP is plotted in figure 10. It is seen that when the system is ferromagnetic, the COP of 3 to 6 coupled spins is above that of two-coupled spin-1/2. However, all of them surpass the COP of the Otto engine. In addition to this, notice that when the system is working as a heat engine (figures 7 and 8), its efficiency can surpass Otto one only when the system is anti-ferromagnetic. And when it is a refrigerator, the enhancement is seen when the system is ferromagnetic (figure 10). We now briefly compare our results with the ones reported in [46, 48, 54]. It was shown in [46, 54] that entanglement between the two-coupled spins at the end of the hot isochoric stage is not necessary for harvesting work. They show that we only need to have entanglement at the end of the cold isochotic stage. In contrast, in [48], it was shown that we can harvest work even if we have no entanglement on the cold isochoric side and the hot isochoric side, but quantum discord was present. In our case, we have neither entanglement nor quantum correlations, and even more, the system could harvest work. To resume, from figures 7–10 we see that the extractable work, the efficiency, as well as the COP of these coupled but not correlated systems could surpass the corresponding ones of the Otto without exploiting entanglement and quantum correlations.

Here, when the system is working as a heat engine, we plot work and efficiency only for positive values of \( J \). This is because only when the system is anti-ferromagnetic do we have an enhancement in work as well as efficiency. When the system is working as a refrigerator, the enhancement is observed in the COP only when the system is ferromagnetic. One can explain this just from the eigenvalues and the
Figure 8. Plot of (a) the extracted work from \( N \) coupled spin-1/2 as a function of \( J \), where \( N \) ranges from 2 to 6 (b) the efficiency of coupled spin-1/2 as a function of \( J \). Here, in contrast to figure 7 the work and efficiency are plotted in the weak as well as in the strong coupling regimes. The parameter values are the same as in figure 1. \( N = 6 \) (blue dashed line), \( N = 5 \) (orange dotted line), \( N = 4 \) (green dot-dashed line), \( N = 3 \) (red large-dashing line), \( N = 2 \) (purple solid line).

Figure 9. The work extracted versus the efficiency, where \( J \) is in the interval \([0, 10]\), and \( N \) ranges from 2 to 6. The parameter values are the same as in figure 1. \( N = 6 \) (blue dashed line), \( N = 5 \) (orange dotted line), \( N = 4 \) (green dot-dashed line), \( N = 3 \) (red large-dashing line), \( N = 2 \) (purple solid line).

Figure 10. The COP of \( N \) coupled spin-1/2 as a function of \( J \). It is only in the interval \([-0.5, 0]\) that we have an enhancement in the COP for three to six-coupled spin-1/2. The parameter values are the same as in figure 3. \( N = 6 \) (blue dashed line), \( N = 5 \) (orange dotted line), \( N = 4 \) (green dot-dashed line), \( N = 3 \) (red large-dashing line), \( N = 2 \) (purple solid line).

As for the Ising model, the eigenvalues are linear in the magnetic field and the eigenstates are parameter independent, thus one can easily demonstrate that we have the preservation of the extensive property of the work, i.e. \( W_g = Nw_{\text{local}} \) which means that the work extracted from \( N \) coupled spins globally will be equal to \( N \) times the one extracted locally from one spin \( w \). This is due to the fact that, for the Ising model considered here, all the local spins have the same reduced state independently on \( N \), thus \( W = Nw \). We conclude that as long as the eigenvalues are linear in the magnetic field and the eigenstates are parameter independent, the global work would be extensive.

We recall that the eigenvalues of this system are \( \{E_1 = 2h, E_2 = E_3 = -2J, E_4 = -2h\} \) and their corresponding probabilities are, \( \rho = \{p_1, p_2, p_3, p_4\} \). The local reduced states of the subsystems are equal, \( \rho_1 = \rho_2 = \text{diag}\{p_1, p_2, p_4, p_2\} \). The expression of the global work in terms of the energies and the populations is,

\[
W = (E_1 - E_1') (p_1 - p_1' + p_4 - p_4').
\]
The local work extracted from each subsystem is,

$$w_1 = (E_1 - E'_1)(p_1 - p'_1 + p'_4 - p_4)/2.$$  

(55)

Thus, the global work is extensive. In addition, one should note that even though the state is only classically correlated, the local effective temperatures are not equal to the ones of the baths, and this is because of the coupling $J$. The effective temperatures [47] are given by,

$$T_{\text{eff},1} = T_{\text{eff},2} = 2\hbar \left( \log \left( \frac{p_2 + p_4}{p_1 + p_2} \right) \right)^{-1},$$  

(56)

at the hot bath side and,

$$T'_{\text{eff},1} = T'_{\text{eff},2} = 2\hbar' \left( \log \left( \frac{p'_2 + p'_4}{p'_1 + p'_2} \right) \right)^{-1},$$  

(57)

at the cold bath side. This tells us that even though the global extracted work is the same as the local one and that the spins are only classically correlated, the heats absorbed from and released to the heat baths are not equal. Therefore, the operation modes will not be the same. This is because the local temperatures are not equal to the ones of the heat baths. The global heats are,

$$Q_h = E_1(p_1 - p'_1 + p'_4 - p_4) + 2E_2(p_2 - p'_2),$$  

(58)

$$Q_c = -E'_1(p_1 - p'_1 + p'_4 - p_4) - 2E'_2(p_2 - p'_2),$$  

(59)

and the total local ones are,

$$q_{h,12} = E_1(p_1 - p'_1 + p'_4 - p_4),$$  

(60)

$$q_{c,12} = -E'_1(p_1 - p'_1 + p'_4 - p_4).$$  

(61)

We see that they are not equal, and are different by the quantity $2E_2(p_2 - p'_2)$, which is the responsible for the enhancement of the efficiency. More precisely, this term, even though makes no difference between the work extracted by a global agent and a local one, the global $\eta_0$ and COP$_v$ can exceed the ones of the Otto $\eta_0$ and COP$_v$, which is not the case for the local efficiency ($\eta_l = 1 - h'/h$) and local COP (COP$_1 = h'/h - h'$) which are equal to the ones of the Otto. This means that a global agent can extract the same work as a local one, but by absorbing less amount of heat from the hot bath. And for refrigeration, for the same amount of consumed work, the global agent can absorb a bigger amount of heat than the local one. Thus, the global agent will cool more efficiently than a local one, and this is independently if the system is classically or quantum mechanically correlated.

Finally, let us explain the enhancement observed in the COP of the Ising model. The COP is given as follows:

$$\text{COP} = \frac{h'}{h - h'} + \frac{-4J_c(p_2 - p'_2)}{2(h - h')(p_4 - p'_4 + p'_3 - p_3)}.$$  

(62)

$q_c = 4J_c(p_2 - p'_2)$ this is the heat absorbed from the cold bath by the degenerate idle level $-2J_c$. In this case, one sees that if $J_c = 0$ the COP is equal to the one of the Otto. And to exceed it, the second term has to be positive, which is equivalent to $-4J_c(p_2 - p'_2) < 0$, since the denominator is negative. This means that idle levels have to absorb heat from the cold bath in addition to working ones, otherwise the COP will not be degraded. Thus, if $J_c > 0$ then $p_2 - p'_2$ has to be positive. However, for $J_c > 0$, we have $p_2 - p'_2 < 0$ thus the second term will degrade the COP. This explains why the enhancement in the COP is observed only when the system is ferromagnetic. Note that in contrast to Ising+$KSEA$ model in which idle levels are not needed to enhance the COP, here we need them, but they have to channel heat in the same direction as working levels. Finally, following the same arguments, we can explain the enhancement in the efficiency.

5. Clarifying some important points

At this point, before we conclude this paper, we should clarify some important points:

First, in the previous works [47–49] we have seen that the eigenvalues were linear in $h$ for the local as well as for the global Hamiltonian. However, we see here (section 3) that the eigenvalues of the global Hamiltonian are not linear in the $h$ in contrast to the local one. Therefore, the extensive property of the work is dependent on the eigenvalues as well as on the eigenstates of the Hamiltonian. Moreover, we can safely say that the extensive property of the work is not related to any type of quantum or classical correlations. It is only related to the coupling parameters. More precisely, if the chosen interaction between the spins does still preserve the linearity of the eigenvalues in the magnetic field, then in this case the extensive property of the work would still be preserved. However, if it violates the linearity, then we can no longer ensure that the global work is equal to the total local one. This is because the local Hamiltonian is linear in $h$.

Note that one may think that the preservation of the extensive property has to do with the assumption that the system is in equilibrium with a heat bath a positive temperature $\beta > 0$, thus the state is passive, i.e. higher levels of energy are occupied with less probabilities (for $E_i \geq E_j$ we have $p_i \leq p_j$, see [75]), which means that state ergotropy is zero. However, this is not the case. More precisely, we can let the system thermalizes with a heat bath at negative temperature $\beta < 0$, thus the state in this case is not passive because higher levels will be occupied with high populations, i.e. for $E_i \geq E_j$ we have $p_i \geq p_j$, thus the state would have non-zero ergotropy. However, even in this case, the extensive property is still preserved for the Heisenberg model but not for the Ising+$KSEA$ model. And this is because the temperature of the bath will only affect the value of populations, but not the expression of the global as well as the local works in terms of energies and populations. Therefore, $W = w \geq 0$ is neither related to quantum correlations nor to ergotropy, i.e. is the state being passive or not, it is only dependent on the Hamiltonian $H$.

Second, actually one may ask that even though if entanglement is not present, however there will be quantum discord which could affect the thermodynamical quantities, i.e. heats,
work and efficiency. However, the Ising model shows us that this enhancement in efficiencies, i.e. $\eta > \eta_0$ and COP > COP$_0$ could be observed as well in the absence of discord.

Third, note that when one sees the eigenvalues of the interacting spins, either for the Heisenberg or the Ising model, one may still not be convinced if it is the presence of idle levels or something else that is the reason behind this enhancement. In this case, one can simply shift the eigenvalues and will get two sets of levels, i.e. working and idle levels. Let’s take two examples: the first is the case of three coupled spins XXX chain. In this case, shifting the eigenvalues $E = \{- (h + 3J), -(h + 3J), h - 3J, h - 3J, -3(h - J), 3(h + J), -h + 3J, h + 3J\}$ by $+ h$ one gets the new eigenvalues: $E = \{- 3J, -3J, 2h - 3J, 2h - 3J, -2h + 3J, 4h + 3J, -3J, 2h + 3J\}$. Thus, we obtain three idle levels. These levels would not contribute to work, but only to heat. The other example is the Ising model of two interacting spins which are, $\{- 2h, -2J, -2J, 2h\}$. And if one plots the heat absorbed by the idle levels, one sees that they are channeling heat in the wrong direction of the one by the working levels. However, it is not necessary that all idle levels take heat in the opposite direction, but only the average of heat should be in the opposite direction of the working levels. Furthermore, one should note that even though there is a part of working levels that does not depend on the magnetic field, i.e. J, this part only contributes to $Q_h$ and $Q_c$, but work is only dependent on the working part, i.e. the part that depends on the magnetic field. Thus, it is not necessary to divide the levels into two sets: idle and working levels to explain the enhancements.

Let’s take the case when the eigenvalues are linear in the $h$ and $J$, i.e. $E_i = a_i h + b_i J$ and $E'_i = a_i h' + b_i J$. $a_i$ and $b_i$ are just numbers, they can be positive or negative. Then the heats and work expressions are,

$$Q_h = \sum_i (a_i h + b_i J) \left( \frac{e^{-\beta_i (a_i h + b_i J)}}{Z_h} - \frac{e^{-\beta_i (a_i h' + b_i J)}}{Z_c} \right), \quad (63)$$

$$Q_c = -\sum_i (a_i h' + b_i J) \left( \frac{e^{-\beta_i (a_i h + b_i J)}}{Z_h} - \frac{e^{-\beta_i (a_i h' + b_i J)}}{Z_c} \right), \quad (64)$$

and work is,

$$W = \sum_i a_i (h - h') \left( \frac{e^{-\beta_i (a_i h + b_i J)}}{Z_h} - \frac{e^{-\beta_i (a_i h' + b_i J)}}{Z_c} \right). \quad (65)$$

With $Z = \sum e^{-\beta E_i}$. We see that the heats are affected by both, the part of eigenvalues that depend on $h$ and the one of $J$. However, work is affected by $J$ only through the populations, i.e. the idle part of $E_i$ disappears because of the difference $E_i - E'_i$. Thus, if $J$ is tuned in the right way, more work will be extracted than in the case $J = 0$. The same thing can be said when the eigenvalues are linear in $h$ but not in $J$. However, when $E_i$ are nonlinear in $h$ things become more complicated.

The above expressions of heats and work can be rewritten as,

$$Q_h = a h + b J, \quad Q_c = -a h' - b J, \quad \text{and} \quad W = a (h - h'). \quad (66)$$

With $a = \sum_i a_i (e^{-\beta_i (a_i h + b_i J)}/Z_h - e^{-\beta_i (a_i h' + b_i J)}/Z_c)$ and $b = \sum_i b_i (e^{-\beta_i (a_i h + b_i J)}/Z_h - e^{-\beta_i (a_i h' + b_i J)}/Z_c)$. In this case the efficiency will be,

$$\eta = \eta_0 \frac{1}{1 + \frac{b J}{a (h - h')}}. \quad (67)$$

For $\eta > \eta_0$ the second term has to be bigger than one. From $W > 0$ we have $a > 0$ (we have assumed that $h > h'$), thus depending on the sign of $b J$, $\eta$ either can be enhanced or degraded. More precisely, if $b J > 0$ then $\eta < \eta_0$ and if $b J < 0$ then $\eta > \eta_0$. Thus, when the term of global heat $Q_h$ absorbed from the hot bath multiplied by $h$ takes heat from the hot bath to the cold bath, the heat multiplied by $J$ has to take it in the other direction. Now, let us explain the enhancement in the COP. The expression of the COP is,

$$\text{COP} = \text{COP}_0 + \frac{b J}{a (h - h')}. \quad (68)$$

Since $a (h - h') < 0$, in this case $b J$ has to be negative as well. And because $-b J$ is the amount of heat absorbed from the cold bath, thus it will enhance the COP. Therefore, it is necessary in this case that idle levels take heat from the bath we want to cool otherwise the COP will be degraded.

Third, in [64] the authors found a nice relation between the work extracted and entropies. This relationship is given as follows:

$$W = (T_h - T_c) (S(\rho) - S(\rho')) - T_c H[p_i|p_i] - T_c H[p_i'|p_i']. \quad (69)$$

$H$ is the relative entropy and it is positive. $\rho$ and $\rho'$ are the equilibrium states of the system at the hot and cold baths, and $p$ and $p'$ are the corresponding populations, respectively. This relationship can be easily proven starting from,

$$W = \sum_i (E_i - E'_i) (p_i - p'_i). \quad (70)$$

Then using $E_i = -T_h \log(p_i|Z_h)$ and $E'_i = -T_c \log(p'_i|Z_c)$, we get this relation. Therefore, for $W > 0$ we have to ensure not only that $T_h > T_c$ but as well $S(\rho) > S(\rho')$, and an addition to this $(T_h - T_c) (S(\rho) - S(\rho')) > T_c H[p_i|p_i] + T_c H[p_i'|p_i']$. More precisely, this relationship tells us that for the positive work condition to be satisfied the entropy of the system at the hot bath side has to be greater than the one at the cold bath side. And according to the second law, this is true, as heat will flow spontaneously from a high entropy system to a less entropy one. Moreover, this relation is useful either for a coupled or only one system, and it is difficult to relate it to quantum correlations, since it tells us for $W > 0$ we have only to compare the entropies of the system at the hot and cold bath sides independently of the system being correlated or just a one system. In the same manner, if one is interested in the local thermodynamics of correlated systems, one can have an expression of the local work in terms of entropies and relative entropies.
In this case, the expression of the local work in terms of entropies is,
\[ \omega_\alpha = (T_h - T_c) (S(\rho_\alpha) - S(\rho'_\alpha)) - T_c H[p_{1,\alpha} | p'_{1,\alpha}] \]
with \( \alpha = 1 \) and 2. \( \rho_\alpha \) is the local reduced state of the system, \( p_{1,\alpha} \) and \( p'_{1,\alpha} \) are the local populations of the system when it is in thermal equilibrium with a hot bath and cold bath, respectively. Therefore, according to our study, the equations (69) and (69) tell us that the condition for a positive work either globally or locally can be explained only from the entropies of the system at the hot and cold bath sides without mentioning quantum correlations and entanglement. Furthermore, one should note that equations (69) and (69) are not used to explain efficiency enhancement, but they are only related to the positive work condition.

Fourth, our results do not claim that quantum correlations should be excluded at all to enhance the performance of thermal machines. But it shows that surpassing the Otto efficiencies could be possible even without using them. Therefore, one may wonder if the Carnot efficiency could be surpassed using idle levels, and our answer would be no. And the reason is that even though some levels take heat in the opposite direction to the average of heat flow, the effective temperature associated with each of the two levels of the system is equal to the heat baths, i.e., \( T_h \) and \( T_c \), thus \( \eta_i \leq \eta_C = 1 - \frac{T_c}{T_h} \).

In addition to this, one should note that the enhancement in efficiency comes at a cost which is due to creating such non-uniform eigenvalues.

Finally, our studies show that neither the enhancement in work, efficiency, COP nor the extensive property of work are related to the presence of quantum correlations. We have seen as well that idle levels are needed to take heat in the wrong direction to enhance efficiency. However, when the system is working as a refrigerator, we saw that depending on the model, they may not be necessary to contribute to the heats to enhance the COP or if their contribution is necessary they have to take heat in the same direction as the working levels, otherwise they will degrade the heat absorbed from the cold bath, thus COP \( \leq \) COP$_o$.

6. Conclusions

By studying two coupled spin-1/2 Ising model under the influence of KSEA interaction in a magnetic field along the z-axis, we show that the enhancement observed in the efficiency is only due to the structure of the energy levels of the system and not entanglement or quantum correlations. Furthermore, we reexamine the results reported in [48] according to which to break the extensive property of the work extracted globally, we have to change the coupling parameters next to the magnetic field. In our case, we show that this is not necessary and we can break it even when only the magnetic field is changed in the adiabatic stages. Then we study the effect of increasing the number of interacting spin-1/2 on efficiency, extractable work, and COP. In contrast to [48], when they extend the dimension of one spin, they have a little enhancement in efficiency and the amount of extractable work, in our case we see a remarkable enhancement in the extractable work, though the maximum of the efficiencies is still bounded by the one of two-coupled spin-1/2. In addition, we see that only when the number of the interacting spins is odd, the system could work as a heat engine in the strong coupling regime. The enhancement in COP is seen as well when the system is working as a refrigerator. Moreover, note here that we did not consider the XY components of J, only the z-component of J to ensure that no entanglement or quantum correlations will build up between the interacting spins along the cycle. Our results support the expectation, put forward in [53, 63, 67], that the structure of the Hamiltonian is a resource and that this is the reason behind the enhancement observed in the efficiency and the extractable work, as well as the COP.

It is worth noting that the enhancement observed in efficiency and COP was also observed in [76, 77]. In [77] the role of anharmonicity on COP has been investigated. The authors there showed that the COP of a quantum harmonic oscillator after introducing anharmonicity could surpass that of the Otto. Moreover, this was shown even for a two-level system, as in our case, when we have two coupled spin-1/2 under the influence of KSEA interaction in the absence of idle levels, i.e., when \( J_z = 0 \). In [76], the role of Kerr-nonlinearity on efficiency and COP has been studied in detail, showing that Kerr-nonlinearity could boost them above the ones of Otto. To resume, our work supports the idea that the enhancement in the performance of coupled spins is only a matter of the structure of the Hamiltonian. It would be useful to look at the role of thermal fluctuations [78–80], since here and in the previous works [46–50, 53, 55–57, 73] the interest was only in the average of the thermodynamical quantities, i.e., heat, work, efficiency, and COP. Furthermore, our work and the previous ones will pave the way for efficient quantum thermal machines in which the structure of the system is used as a resource. Inspired by other quantum resource theories, [81–86], a resource theory of the structure of the energy levels of quantum systems, will also be interesting to study [87].

Our results go in the same way as in [88–92], which show the effect of the coupling on the performance of QHEs. We hope to investigate further our study concerning the global and the local thermodynamics more thoroughly, or in general, when both the unitary and the thermalization strokes are in finite time.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors declare no conflicts of interest or personal relationships that could have appeared to influence this work, and no data associated in the manuscript.

Appendix. Eigenvalues of the working medium

Here we give the eigenvalues of the Hamiltonian (53). Their corresponding eigenstates are the elements of the computational basis. The eigenvalues of the Hamiltonian when \( N = 2 \) are \( \{2h, -2J, -2J, -2h\} \) \([47, 53]\).

The eigenvalues of the Hamiltonian when \( N = 3 \) are \( \{3h + 3J, h - J, -h - J, -h - J, -h - J, -h - J, -3h + 3J\} \). The partition function of the system when it is in equilibrium with a heat bath at inverse temperature \( \beta \) is:

\[
Z = 2e^{-3J\beta} \cosh(3\beta h) + 6e^\beta \cosh(\beta h).
\]

The eigenvalues of the Hamiltonian when \( N = 4 \) are \( \{4h + 4J, 2h, 2h, 0, 2h, -4J, 0, -2h, 2h, 0, -4J, -2h, 0, -2h, -2h, -4h + 4J\} \). The partition function of the system when it is in equilibrium with a heat bath at inverse temperature \( \beta \) is:

\[
Z = 2(e^{4J\beta} + e^{-4J\beta} \cosh(4\beta h)) + 4(1 + 2\cosh(2\beta h)).
\]

The eigenvalues of the Hamiltonian when \( N = 5 \) are \( \{5h + 5J, 3h + J, h + J, h + J, -3J, h + J, -h + J, 3h + J, h + J, -3J, h + J, -h + J, -3h + J, h + J, -J\} \). The partition function of the system when it is in equilibrium with a heat bath at inverse temperature \( \beta \) is:

\[
Z = 2e^{-5J\beta} \cosh(5\beta h) + 10e^{-3\beta} \cosh(3\beta h) + 10e^{3\beta} \cosh(\beta h).
\]

The eigenvalues of the Hamiltonian when \( N = 6 \) are \( \{6h + 6J, 4h + 2J, 4h + 2J, 4h + 2J, 4h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J, 2h + 2J\} \). The partition function when the system is in thermal equilibrium with a heat bath at inverse temperature \( \beta \) is:

\[
Z = 2(e^{-6J\beta} \cosh(6\beta h)) + e^{6\beta}\cosh(4\beta h) + 12e^{-2\beta} \cosh(2\beta h) + e^{2\beta} \cosh(\beta h) + 12 \cosh(2\beta J).
\]

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References

[1] Kondepudi D and Prigogine I 2015 Modern Thermodynamics: From Heat Engines to Dissipative Structures 2nd edn (Chichester: Wiley)
[2] Carnot S 1824 Réflexions sur la Puissance Motrice du feu et sur les Machines Propres a Développer Cette Puissance (Paris: Bachelier)
[3] Alicki R 1979 The quantum open system as a model of the heat engine J. Phys. A: Math. Gen. 12 L103
[4] Kosloff R 1984 A quantum mechanical open system as a model of a heat engine J. Chem. Phys. 80 1625
[5] Allahverdyan A E, Balian R and Nieuwenhuizen T M 2004 Quantum thermodynamics: thermodynamics at the nanoscale J. Mod. Opt. 51 2703
[6] Quan H T, Liu Y-X, Sun C P and Nori F 2007 Quantum thermodynamic cycles and quantum heat engines Phys. Rev. E 76 031105
[7] Quan H T 2009 Quantum thermodynamic cycles and quantum heat engines II Phys. Rev. E 79 041129
[8] Glimmer J, Michel M and Mahler G 2009 Quantum Thermodynamics 2nd edn (Berlin: Springer)
[9] Brunner N, Linden N, Popescu S and Skrzypczyk P 2012 Virtual qubits, virtual temperatures and the foundations of thermodynamics Phys. Rev. E 85 05111
[10] Kosloff R 2013 Quantum thermodynamics: a dynamical viewpoint Entropy 15 2100
[11] Kosloff R and Levy A 2013 Quantum heat engines and refrigerators: continuous devices Annu. Rev. Phys. Chem. 65 365
[12] Godd J, Huber M, Riera A, del Rio L and Skrzypczyk P 2016 The role of quantum information in thermodynamics—a topical review J. Phys. A: Math. Theor. 49 143001
[13] Vinjanampathy S and Anders J 2016 Quantum thermodynamics Contemp. Phys. 57 545
[14] Millen J and Xuereb A 2016 Perspective on quantum thermodynamics New J. Phys. 18 011002
[15] Kosloff R and Rezek Y 2017 The quantum harmonic Otto cycle Entropy 19 136
[16] Binder E, Correa I A, Gogolin C, Anders J and Adesso G 2018 Thermodynamics in the Quantum Regime (Fundamental Theories of Physics vol 195) (Berlin: Springer) pp 1–2
[17] Defnner S and Campbell S 2019 Quantum Thermodynamics: An Introduction to the Thermodynamics of Quantum Information (San Rafael, CA: Morgan & Claypool Publishers)
[18] Mitchison M T 2019 Quantum thermal absorption machines: refrigerators, engines and clocks Contemp. Phys. 60 164
[19] Scovil H E D and Schulz-Dubois E O 1959 Three-level masers as heat engines Phys. Rev. Lett. 2 262
[20] Abah O, Roß nagel J, Jacob G, Defnner S, Schmidt-Kaler F, Singer K and Lutz E 2012 Single-ion heat engine at maximum power Phys. Rev. Lett. 109 203006
[21] Roß nagel J, Abah O, Schmidt-Kaler F, Singer K and Lutz E 2014 Nanoscale heat engine beyond the Carnot limit Phys. Rev. Lett. 112 030602
[22] Roß nagel J, Dawkins S T, Tolazzi K N, Abah O, Lutz E, Schmidt-Kaler F and Singer K 2016 A single-atom heat engine Science 352 325
[23] Maslennikov G, Ding S, Hablutzel R, Jan J, Roulet A, Nimmtich S, Dal J, Scarani V and Matsukevich D 2017 Quantum absorption refrigerator with trapped ions Nat. Commun. 10 202
[24] Quan H T, Zhang P and Sun C P 2006 Quantum-classical transition of photon–Carnot engine induced by quantum decoherence Phys. Rev. E 73 036122
[25] Sothmann B and Böttiker M 2012 Magnon-driven quantum-dot heat engine Europhys. Lett. 99 27001
[26] Venturaelli D, Fazio R and Giovanni V 2013 Minimal self-contained quantum refrigeration machine based on four quantum dots Phys. Rev. Lett. 110 256801
[27] Zhang K, Bariani F and Mysest© 2014 Quantum optomechanical heat engine Phys. Rev. Lett. 112 150602
[28] Altintas F, Hardal A U C and MÜstekapipoğlu Ö E 2015 Rabi model as a quantum coherent heat engine: from quantum
biology to superconducting circuits Phys. Rev. A 91 023816
[29] Peterson J P S, Batalhão T B, Herrera M, Souza A M, Sarthour R S, Oliveira I S and Serra R M 2019 Experimental characterization of a spin quantum heat engine Phys. Rev. Lett. 123 240601
[30] Klatzow J, Becker J N, Ledingham P M, Weinzetl C, Kaczmarek T K, Saunders D J, Nunn J, Walmsley I A, Uzdin R and Poem E 2019 Experimental demonstration of quantum effects in the operation of microscopic heat engines Phys. Rev. Lett. 112 116001
[31] von Lindenfels D, Gräb O, Schmiegelow C T, Kaushal V, Schulz J, Mitchison M T, Goold J, Schmidt-Kaler F and Poschinger U G 2019 Spin heat engine coupled to a harmonic-oscillator flywheel Phys. Rev. Lett. 123 080602
[32] Myers N M, Abah O and Defner S 2022 Quantum thermodynamic devices: from theoretical proposals to experimental reality AVS Quantum Sci. 4 027201
[33] Scully M O 2001 Extracting work from a single thermal bath via quantum negentropy Phys. Rev. Lett. 87 220601
[34] Scully M O, Zubairy M S, Agarwal G S and Walther H 2003 Extracting work from a single heat bath via vanishing coherence Science 299 826
[35] Dillenschneider R and Lutz E 2009 Energetics of quantum heat engines Phys. Rev. Lett. 88 50003
[36] Huang X L, Wang T and Yi X X 2012 Effects of reservoir squeezing on quantum systems and work extraction Phys. Rev. E 86 051105
[37] Abah O and Lutz E 2014 Efficiency of heat engines coupled to nonequilibrium reservoirs Europhys. Lett. 106 20001
[38] Roßnagel J, Abah O, Schmidt-Kaler F, Singer K and Lutz E 2014 Efficiency of heat engines coupled to nonequilibrium reservoirs Phys. Rev. Lett. 112 030602
[39] Hardal A U C and Müstecaplolu Ö E 2015 Superradiant quantum heat engine Sci. Rep. 5 12953
[40] Niedenzu W, Gelbwaser-Klimovsky D, Kofman A G and Kurizki G 2016 On the operation of machines powered by quantum non-thermal baths New J. Phys. 18 083012
[41] Manzano G, Galve F, Zambrini R and Parrondo J M R 2016 Entropy production and thermodynamic power of the squeezed thermal reservoir Phys. Rev. E 93 052120
[42] Klaers J, Fach S, Imamoglu A and Togan E 2017 Squeezed thermal reservoirs as a resource for a nanomechanical engine beyond the Carnot limit Phys. Rev. X 7 031044
[43] Agarwalla B K, Jiang J-H and Segal D 2017 Quantum efficiency bound for continuous heat engines coupled to noncanonical reservoirs Phys. Rev. B 96 104304
[44] Kieu T D 2004 The second law, Maxwell’s demon and work derivable from quantum heat engines Phys. Rev. Lett. 93 140403
[45] Kieu T D 2006 Quantum heat engines, the second law and Maxwell’s demon Euro. Phys. J. D 39 115
[46] Zhang T, Liu W-T, Chen P-X and Li C-Z 2007 Four-level quantum thermal engines Phys. Rev. A 75 050302(R)
[47] Thomas G and Johal R S 2011 A coupled quantum Otto cycle Phys. Rev. E 83 031135
[48] Altintas F and Müstecaplolu Ö E 2015 General formalism of local thermodynamics with an example: quantum Otto engine and refrigerator in Heisenberg model Eur. Phys. J. B 87 166
[49] Çakmak B and Müstecaplolu Ö E 2019 Spin quantum heat engines with shortcuts to adiabaticity Phys. Rev. E 99 032108
[50] Abah O and Lutz E 2014 Efficiency of heat engines coupled to nonequilibrium reservoirs Europhys. Lett. 106 20001
[51] Roßnagel J, Abah O, Schmidt-Kaler F, Singer K and Lutz E 2014 Efficiency of heat engines coupled to nonequilibrium reservoirs Phys. Rev. Lett. 112 030602
[52] Das A and Ghosh S 2019 Measurement based quantum heat engine with coupled working medium Entropy 21 1131
[53] Anka M F, de Oliveira T R and Jonathan D 2021 Measurement-based quantum heat engine in a multilevel system Phys. Rev. E 104 5
[54] Zhang G-F 2008 Entangled quantum heat engines based on two two-spin systems with Dzyaloshinski-Moriya anisotropic antisymmetric interaction Eur. Phys. J. D 49 123
[55] Zhao L-M and Zhang G-F 2017 Entangled quantum Otto heat engines based on two-spin systems with the Dzyaloshinski–Moriya interaction Quantum Inf. Process. 16 216
[56] Ahadpour S and Mirmasoudi F 2021 Coupled two-qubit engine and refrigerator in Heisenberg model Quantum Inf. Process. 20 63
[57] Johal R S and Mehta V 2021 Quantum heat engines with complex working media, complete Otto cycles and heuristics Entropy 23 1149
[58] Marshall A W, Oliko and Arnold B C 2011 Inequalities: Theory of Majorization and Its Applications (Springer Series in Statistics) (Berlin: Springer)
[59] Thomas G and Johal R S 2013 Friction due to inhomogeneous driving of coupled spins in a quantum heat engine Eur. Phys. J. B 87 166
[60] Çakmak B and Müstecaplolu Ö E 2019 Spin quantum heat engines with shortcuts to adiabaticity Phys. Rev. E 99 032108
[61] Solfanelli A Falsetti M and Campisi M 2020 Nonadiabatic single-qubit quantum Otto engine Phys. Rev. B 101 054513
[62] Çakmak B 2021 Finite-time two-spin quantum Otto engines: shortcuts to adiabaticity vs. irreversibility Turk. J. Phys. 45 59
[63] Cherubin C, de Oliveira T R and Jonathan D 2022 Nonadiabatic coupled-qubit Otto cycle with bidirectional operation and efficiency gains Phys. Rev. E 105 044120
[64] Yeo Y and Kwong C C Quantum heat engines and information (arXiv:0708.2480v1)
[65] Altintas F, Hardal A U and Müstecaplolu Ö E 2014 Quantum correlated heat engine with spin squeezing Phys. Rev. E 90 032102
[66] Hewgill A, Ferraro A and De Chiara G 2018 Quantum correlations and thermodynamic performances of two-qubit engines with local and common baths Phys. Rev. A 98 042102
[67] de Oliveira T R and Jonathan D 2020 Efficiency gain and bidirectional operation of quantum engines with decoupled internal levels Phys. Rev. E 104 044133
[68] Kaplan T A 1983 Single-band Hubbard model with spin-orbit coupling Z. Phys. B 49 313
[69] Shekhtman L, Entin-Wohlman O and Aharony A 1992 Moriya’s anisotropic superexchange interaction, frustration and Dzyaloshinsky’s weak ferromagnetism Phys. Rev. Lett. 69 836
[70] Shekhtman L, Aharony A and Entin-Wohlman O 1993 Bond-dependent symmetric and antisymmetric superexchange interactions in La2CuO4 Phys. Rev. B 47 174
[71] Yildirim T, Harris A B, Aharony A and Entin-Wohlman O 1995 Anisotropic spin Hamiltonians due to spin-orbit and Coulomb exchange interactions Phys. Rev. B 52 10239
[72] Yurichev M A 2020 On the quantum correlations in two-qubit XYZ spin chains with Dzyaloshinskii–Moriya and Kaplan–Shekhtman–Entin–Wohlman–Aharony interactions Quantum Inf. Process. 19 336
[73] Türkpençe D and Altintas F 2019 Coupled quantum Otto heat engine and refrigerator with inner friction Quantum Inf. Process. 19 255
[74] Moriya T 1960 New mechanism of anisotropic superexchange interaction Phys. Rev. Lett. 4 228
[75] Allahverdyan A E, Balian R and Nieuwenhuizen T M 2004 Maximal work extraction from finite quantum systems Europhys. Lett. 67 565
[76] Mendes U C, Sales J S and de Almeida N G 2021 Quantum Otto thermal machines powered by Kerr nonlinearity J. Phys. B: At. Mol. Opt. Phys. 54 175504
[77] Karar S, Datta S, Ghosh S and Majumdar A S Anharmonicity can enhance the performance of quantum refrigerators (arXiv:1902.10616 [quant-ph])
[78] Jarzynski C 1997 Nonequilibrium equality for free energy differences Phys. Rev. Lett. 78 2690
[79] Crooks C E 1999 Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences Phys. Rev. E 60 2721
[80] Campisi M, Hanggi P and Talkner P 2011 Quantum fluctuation relations: foundations and applications Rev. Mod. Phys. 83 771
[81] Gour G and Spekkens R W 2008 The resource theory of quantum reference frames: manipulations and monotones New J. Phys. 10 035023
[82] Brandao F, Horodecki M, Oppenheim J, Renes J M and Spekkens R W 2013 Resource theory of quantum states out of thermal equilibrium Phys. Rev. Lett. 111 250404
[83] Baumgratz T, Cramer M B and Plenio M 2014 Quantifying coherence Phys. Rev. Lett. 113 140401
[84] Chitambar E, Leung D, Mancinska L, Ozols M and Winter A 2014 Everything you always wanted to know about LOCC (but were afraid to ask) Commun. Math. Phys. 328 303
[85] Brandao F, Horodecki M, Ng N, Oppenheim J and Wehner S 2015 The second laws of quantum thermodynamics Proc. Natl Acad. Sci. 112 3275
[86] Chitambar E and Gour G 2019 Quantum resource theories Rev. Mod. Phys. 91 025001
[87] Albarelli F, Ferraro A, Paternostro M and Paris M G A 2016 Nonlinearity as a resource for nonclassicality in anharmonic systems Phys. Rev. A 93 032112
[88] Niedenzu W and Kurizki G 2018 Cooperative many-body enhancement of quantum thermal machine power New J. Phys. 20 113038
[89] Watanabe G, Venkatesh B P, Talkner P, Hwang M-J and del Campo A 2020 Quantum statistical enhancement of the collective performance of multiple bosonic engines Phys. Rev. Lett. 124 210603
[90] Latune C L, Sinayskiy I and Petruccione F 2020 Collective heat capacity for quantum thermometry and quantum engine enhancements New J. Phys. 22 083049
[91] Kamimura S, Hakoshima H, Matsuaki Y, Yoshida K and Tokura Y 2022 Quantum-enhanced heat engine based on superabsorption Phys. Rev. Lett. 128 180602
[92] Souza L D S, Manzano G, Fazio R and Iemini F 2022 Collective effects on the performance and stability of quantum heat engines Phys. Rev. E 106 014143