Guided Modes in Uniaxial Chiral Waveguide of Circular Cross-Section under PEC Boundary

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Abstract. Propagation of electromagnetic waves through a circular waveguide of uniaxial anisotropic chiral medium is studied with the emphasis on the energy flux patterns. The outer surface of the guide is assumed to be bounded by a perfect electric conductor (PEC) medium. The dispersion relation of guide is derived by applying suitable boundary conditions, and the allowed values of propagation constants are computed. Energy flux patterns corresponding to three different types of uniaxial anisotropic chiral metamaterials are taken into account. The propagation of negative energy flux is observed, and attributed to the presence of backward waves in the waveguide structure.

1. Introduction

Chiral objects possess the property that their mirror images are not superimposable by any set of translation or rotation. In terms of chemistry, such objects are also known as enantiomers; their molecules have similar molecular formula but different structural formula. Chiral mediums possess the remarkable property that, when a plane wave is made to incident on it, the wave decomposes into two circularly polarized plane waves, one is left circularly polarized (LCP,−) and the other is right circularly polarized (RCP,+) [1].

Chiral metamaterials possess the property of negative reflection and refraction, which made these to be of attractive field of interest among the researchers. It has been investigated that the phenomenon of negative refraction can be achieved from these materials by increasing their chirality (or the coupling) parameter enough [2−6]. In such a case, one of the eigenwaves, i.e. either LCP or RCP, acquires negative refraction. Materials having negative refractive index possess many fabulous technological applications. For instance, many novel devices can be fabricated by the use of such materials, such as perfect lens, cloaks, oscillators, etc. [5−7].

Within the context, circular waveguides of chiral metamaterials have attracted the interest of many researchers, and the investigation of the propagation behavior of electromagnetic waves through such guides has been reported in the literature [8−13]. In this stream, uniaxial chiral metamaterials are of special kind in which chirality appears only in one direction. It has been reported that these materials can easily be fabricated by using miniature spirals wires or conducting springs in ordinary dielectric host mediums [14,15]. It is to be noted in this connection that chiral waveguides with unconventional shapes have also been reported in the literature [16,17], and also, the property of chirality may be introduced by the use of helical clad (or twisted clad) optical fibers [18−21].
In the present communication, we make an attempt to present the investigation of the propagation behavior of energy flux patterns through a circular waveguide consisting of uniaxial anisotropic chiral metamaterials; the outer surface of the guide being bounded by PEC (perfect electric conductor) medium. The results reveal that the energy flux remains confined near the central region of the guide, and greatly depends on the type of metamaterial used. The existence of backward waves is also observed.

2. Analytical Treatment
We consider a circular waveguide composed of homogeneous anisotropic chiral medium having its radius as \( a \), as shown in Fig. 1. The outer surface of the guide is assumed to be coated with a PEC medium. The constitutive relations corresponding to uniaxial anisotropic medium can be described as [22]

\[
D = \varepsilon \mathbf{I}_t + \varepsilon_z \mathbf{u}_z \mathbf{u}_z \cdot \mathbf{E} - j \kappa \sqrt{\varepsilon_0 \mu_0} \mathbf{u}_z \mathbf{u}_z \cdot \mathbf{H} \\
B = \mu \mathbf{I}_t + \mu_z \mathbf{u}_z \mathbf{u}_z \cdot \mathbf{H} + j \kappa \sqrt{\varepsilon_0 \mu_0} \mathbf{u}_z \mathbf{u}_z \cdot \mathbf{E}
\]

(1a)

(1b)

In these equations, \( \mathbf{u}_z \) is the unit vector along the axis of the guide, and \( \mu_z \) and \( \varepsilon_z \) are, respectively, permeability and permittivity of the waveguide medium along the optical axis. Also, \( \mu_t \) and \( \varepsilon_t \) are those along the transverse directions, and \( \kappa \) is the chirality parameter. Further, \( \mathbf{I}_t \) is the unit dyadic defined as

\[
\mathbf{I}_t = \mathbf{\hat{x}} \mathbf{\hat{x}} + \mathbf{\hat{y}} \mathbf{\hat{y}}.
\]

Figure 1. Circular waveguide made of anisotropic chiral metamaterial with a PEC boundary.

The cylindrical structure of the guide allows to consider the longitudinal component of the excited electromagnetic field as

\[
E_z = J_n(kr) e^{j\phi}
\]

with \( k = \sqrt{k_0^2 - \beta^2} \); \( k_0 \) and \( \beta \), respectively, being the propagation constants in the free-space and the unbounded medium. As stated in the introductory part, the excited electromagnetic field in the chiral medium gets decomposed into two circularly polarized plane waves, viz. the LCP and the RCP [1], given as

\[
E_{z+} = A_n J_n(k r) e^{j\phi}
\]

(2a)
In the above Eqs. (2), $A_n$ and $B_n$ are unknown coefficients, and $J_n(\bullet)$ is Bessel function of the first kind with integer order $n$ (i.e. $n = 0, 1, 2, \ldots$). Now, the total electromagnetic field inside the chiral medium is the sum of the RCP and the LCP waves [1], and can be written as

$$E = E_+ + E_- \quad (3a)$$

$$H = \frac{j}{\eta} (E_+ - E_-) \quad (3b)$$

The electromagnetic field can be broken into transverse and longitudinal components as

$$E = \left( E_t + \frac{z}{\eta} E_z \right) e^{-j\beta z} \quad (4a)$$

$$H = \left( H_t + \frac{z}{\eta} H_z \right) e^{-j\beta z} \quad (4b)$$

with $\beta$ as the propagation constant. The longitudinal and transverse field components are related to each other [13] as

$$E_t = -j \frac{\beta}{\lambda^2} \nabla_t E_z - j \frac{\omega \mu_t}{\lambda^2} \nabla_z H_z \times \hat{z} \quad (5a)$$

$$H_t = -j \frac{\beta}{\lambda^2} \nabla_t H_z - j \frac{\omega \epsilon_t}{\lambda^2} \nabla_z E_z \times \hat{z} \quad (5b)$$

with $\nabla_t = \nabla - \frac{\partial}{\partial z}$, and $\lambda^2 = \omega^2 \mu_t \epsilon_t - \beta^2$.

Now, the longitudinal components must satisfy the wave equation [13], and it can be shown that the eigenvalues will be of the form

$$k_z^2 = \lambda^2 \left[ \frac{\epsilon_t}{\epsilon_t} + \frac{\mu_z}{\mu_t} \pm \sqrt{\left( \frac{\epsilon_z}{\epsilon_t} - \frac{\mu_z}{\mu_t} \right)^2 + 4K^2 \frac{\epsilon_0 \mu_0}{\epsilon_t \mu_t}} \right]$$

(6)

Also, the corresponding eigenfunctions are given by [13]

$$\left( E_z, H_z \right) = \left( E_z, j \frac{\alpha}{\eta_t} E_z \right)$$

(7)

where $\alpha = \left( \frac{k^2}{\lambda^2} - \frac{\epsilon_z}{\epsilon_t} \right) \sqrt{\epsilon_t \mu_t} / \left( k \sqrt{\epsilon_0 \mu_0} \right)$ and $\eta_t = \sqrt{\frac{\mu_t}{\epsilon_t}}$.

The total longitudinal component of the electromagnetic field can be written as
\[ E_{z1} = \left[ A_n J_n(k_r) + B_n J_n(k_r) \right] e^{jkz} e^{-j\beta z} \]  
\[ H_{z1} = \frac{j}{\eta_l} \left[ A_n \alpha_1 J_n(k_r) + B_n \alpha_1 J_n(k_r) \right] e^{jkz} e^{-j\beta z} \]  
\( (8a) \)  
\( (8b) \)

and the transverse components can be derived from the longitudinal components by using Eq. (5). These transverse field components can be explicitly written as

\[ E_{r1} = \left\{ A_n \left[ \frac{jnk}{\lambda^2} \alpha_1 J_n(k_r) - \frac{j\beta k}{\lambda^2} J_n'(k_r) \right] \right\} + B_n \left[ \frac{jnk}{\lambda^2} \alpha_1 J_n(k_r) - \frac{j\beta k}{\lambda^2} J_n'(k_r) \right] \} e^{jkz} e^{-j\beta z} \]  
\[ E_{\phi_1} = \left\{ A_n \left[ \frac{n\beta}{\lambda^2} J_n(k_r) - \frac{k k}{\lambda^2} J_n'(k_r) \right] \right\} + B_n \left[ \frac{n\beta}{\lambda^2} J_n(k_r) - \frac{k k}{\lambda^2} J_n'(k_r) \right] \} e^{jkz} e^{-j\beta z} \]  
\[ H_{r1} = \left\{ A_n \frac{1}{\lambda^2 \eta_l} \left[ -\frac{nk}{r} J_n(k_r) + \beta k \alpha_1 J_n(k_r) \right] \right\} + B_n \frac{1}{\lambda^2 \eta_l} \left[ -\frac{nk}{r} J_n(k_r) + \beta k \alpha_1 J_n(k_r) \right] \} e^{jkz} e^{-j\beta z} \]  
\[ H_{\phi_1} = \left\{ A_n \frac{1}{\lambda^2 \eta_l} \left[ \frac{j\beta n}{r} J_n(k_r) - k k J_n(k_r) \right] \right\} + B_n \frac{1}{\lambda^2 \eta_l} \left[ \frac{j\beta n}{r} J_n(k_r) - k k J_n(k_r) \right] \} e^{jkz} e^{-j\beta z} \]  
\( (9a) \)  
\( (9b) \)  
\( (9c) \)  
\( (9d) \)

In the above Eq. (9), the prime represents the differentiation of Bessel function of the first kind with respect to the argument.

The outer surface of the guide is bounded with a PEC medium. The dispersion relation of the guide is derived by applying suitable boundary conditions at the interface of the uniaxial anisotropic and PEC mediums. According to the nature of PEC, the tangential components of the electric field vanish at the surface of the guide which, in our case, can be given as below:

\[ E_{z1} = 0 \]  
\[ E_{\phi_1} = 0 \]  
at \( r = a \)

By employing the above boundary conditions, a set of two homogeneous equations can be obtained, and the determinant formed by the coefficients in those equations should be zero, in order to have a non-trivial solution to that set of equations. Thus, the characteristic eigenvalue equation for the structure can be written as equating the aforesaid determinant to zero. After some mathematical steps, the dispersion relation of the guide can finally be given as

\[ \beta \alpha_1 k^2 J_n(k,a) \left\{ J_{n-1}(k,a) - J_{n+1}(k,a) \right\} - \beta \alpha_1 k^2 J_n(k,a) \left\{ J_{n-1}(k,a) - J_n(k,a) \right\} = 0 \]  
\( (10) \)
Further, the energy flux flowing through the guide can be derived by using Eq. (9). It can be shown that the equation corresponding to the energy flux finally assumes the form

\[
S_z = \frac{A_n}{2\lambda^2 n_r} \left\{ \frac{n^2 k^2_j}{\lambda^2 r^2} J_n^2(k,r) - \frac{n\beta^2 k^2_j}{2\lambda^2 r} \alpha_n J_n(k,r) (J_{n-1}(k,r) - J_{n+1}(k,r)) \right\}
+ \frac{B_n}{2\lambda^2 n_r} \left\{ \frac{n^2 k^2_j}{\lambda^2 r^2} J_n^2(k,r) - \frac{n\beta^2 k^2_j}{2\lambda^2 r} \alpha_n J_n(k,r) (J_{n-1}(k,r) - J_{n+1}(k,r)) \right\}
+ \frac{A_n B_n}{2\lambda^2 n_r} \left\{ n\beta k J_n(k,r) (nk\alpha J_{n-1}(k,r) - 0.25\beta k J_{n-1}(k,r) - J_{n+1}(k,r)) \right\}
- 0.25 \times n\beta^2 k^2 \alpha_n J_n(k,r) (J_{n-1}(k,r) - J_{n+1}(k,r)) \right\}
\]  
(11)

3. Results and Discussion

We are now in a position to investigate the features related to the propagation of energy flux through the guide of interest. As described before, the waveguide is loaded with a PEC medium. In order to perform computations of the sustained flux, we consider three different types of uniaxial anisotropic chiral mediums – Type I, Type II and Type III. These three types of material structures assume the parametric values as follows:

Type I: \( \mu_z = \mu_t = \mu_0, \epsilon_z = 2\times\epsilon_0 \) and \( \epsilon_t = 1.5\times\epsilon_0 \),

Type II: \( \mu_z = \mu_t = \mu_0, \epsilon_z = 4\times\epsilon_0 \) and \( \epsilon_t = -1.5\times\epsilon_0 \),

and Type III: \( \mu_z = \mu_t = \mu_0, \epsilon_z = -\epsilon_0 \) and \( \epsilon_t = 1.5\times\epsilon_0 \).

We first obtain the allowed values of propagation constants of the guide corresponding to the above stated three types of material mediums by the use of the dispersion relation, as given in Eq. (10). For this purpose, we assume the operating wavelength of the excited electromagnetic field as 1.5 µm. Also, the radius of the guide is taken to be 15 µm.

\[ r \mu m \]

Figure 2. Energy flux in the guide with Type I material composition.

As the waveguide medium is assumed to be chiral in nature, the electric/magnetic fields are coupled with each other through the chirality parameter, resulting thereby the existence of hybrid modes in the guide with right- and left-circular polarizations. We make an attempt to investigate the
propagation of energy flux, as depicted in Figs. 2, 3 and 4, corresponding to the three different low-order hybrid modes – viz. $H_{01}$, $H_{11}$ and $H_{-11}$ – considering the aforesaid material compositions. In Figs. 2, 3 and 4, the dotted lines correspond to the situations of the $H_{01}$ mode, and the dashed and solid lines, respectively, represent those of the $H_{11}$ and $H_{-11}$ modes.

In the attempt to evaluate the allowed values of propagation constants, we found that when the Type I medium is in use, the propagation constant $\beta$ corresponding to the $H_{01}$ mode becomes $5.52 \times 10^6$ m$^{-1}$. Further, the $H_{11}$ and $H_{-11}$ modes exhibit the similar value of $\beta$, which is $5.82 \times 10^6$ m$^{-1}$. Figure 2 illustrates the energy flux characteristics under this situation. We observe that the flux is mostly bound near the central region of the guide, and it goes on decreasing with the increase of radial distance. Near the interface of the uniaxial chiral and PEC mediums, it almost disappears. Further, corresponding to the $H_{01}$ and $H_{-11}$ modes, the flux characteristics remain negative, though it is more negative for the hybrid $H_{01}$ mode, which is more prominent near the central region of the guide, and gets attenuated as it reaches the waveguide interface. The negative flow of energy flux can be interpreted as the support of backward wave propagation in the guide of interest.

While using the Type II material for the waveguide, the propagation constant corresponding to the $H_{01}$ mode is found to be $4.29 \times 10^6$ m$^{-1}$, and that for the $H_{11}$ and $H_{-11}$ modes as $4.43 \times 10^6$ m$^{-1}$ ($\beta$-values are the same for both of these modes). We observe from Fig. 3 that, under the Type II kind of material composition, the $H_{01}$ mode exhibits similar characteristics as seen while the Type I material was in use. The flux corresponding to this mode primarily remains negatively bounded near the central region of the guide, and goes on negatively decreasing while moving toward the waveguide interface. However, the $H_{11}$ and $H_{-11}$ modes show typical features that the flux remains independent of the radial variation in the waveguide dimension. As such, the distribution of energy flux due to these two modes becomes uniform throughout the guide. Interestingly, the features of flux corresponding to the $H_{11}$ and $H_{-11}$ modes become reversed (as compared to the situation of the waveguide with Type I material) in this case as the $H_{11}$ mode exhibits negative flux whereas the $H_{-11}$ mode positive.

![Figure 3](image-url)

**Figure 3.** Energy flux in the guide with Type II material composition.
In the case of Type III medium, we evaluated the propagation constants corresponding to the $H_{01}$ mode as $5.65 \times 10^6$ m$^{-1}$, and that for the $H_{11}$ and $H_{-11}$ modes as $5.73 \times 10^6$ m$^{-1}$. We find from Fig. 4 in this case that, for all the three low-order modes of interest, the flux characteristics are almost similar to that observed in the case of waveguide with Type I material composition in use. Thus, the flux remains more confined in the central region of the guide, and goes on decreasing with the increase in radial direction. The only difference from the situation of Fig. 2 is that the flux in the case of Type III material composition remains a little loosely bounded near the central region of the guide as compared to that under the usage of Type I material composition. As such, the flux characteristics of the guide remain greatly dependent on the material composition used for waveguide fabrication.

4. Conclusion

The paper presents numerical investigations of the energy flux patterns corresponding to the propagating modes considering three different types of uniaxial anisotropic chiral waveguides. It has been found that most of the energy flux remains confined near the central region of the guide, and it varies upon changing the type of medium through its constitutive parameters. Backward waves are also observed to be supported by the PEC loaded uniaxial anisotropic chiral medium, which is evident from the propagation of negative energy flux in the guide.

Acknowledgements

The authors are thankful to Prof. Burhanuddin Yeop Majlis, the Director of IMEN (Universiti Kebangsaan Malaysia), for constant encouragement and help. This work is partially supported by the Fundamental Research Grant Project (FRGS/1/2011/TK/UKM/01/16) by the Ministry of Higher Education (Malaysia); the authors are thankful to the Ministry. Also, the constructive criticisms by an anonymous reviewer are gratefully acknowledged.

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