3+1 dimensional viscous hydrodynamics at high baryon densities

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Abstract. We apply a 3+1D viscous hydrodynamic + cascade model to heavy ion collision reactions with $p_{\text{SN}} = 6.3\ldots39$ GeV. To accommodate the model for a given collision energy range, the initial conditions for the hydrodynamic phase are taken from UrQMD, and the equation of state at finite baryon density is based on a Chiral model coupled to the Polyakov loop.

We study the collision energy dependence of pion and kaon rapidity distributions and $m_T$-spectra, as well as charged hadron elliptic flow and how shear viscosity affects them. The model calculations are compared to the data for Pb-Pb collisions at CERN SPS, as well as for Au-Au collisions in the Beam Energy Scan (BES) program energies at BNL RHIC. The data favours the value of shear viscosity $\eta/s \gtrsim 0.2$ for this collision energy range.

1. Introduction
Hydrodynamic approaches are well established for the description of hot and dense matter created in ultrarelativistic heavy ion collisions at BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC). Its success is based on the reproduction of bulk observables such as radial flow, elliptic and higher order flow harmonics [1], femtoscopy [2] etc. in so-called hybrid models coupling a viscous (or inviscid) hydrodynamic phase to a hadron cascade.

In this paper we report on the application of the viscous+cascade model tuned to the maximum RHIC energy of $\sqrt{s_{NN}} = 200$ GeV to heavy ion collisions at lower collision energies. To accommodate to the different collision energy regimes, we change the equation of state (EoS) and the initial conditions, and keep all the other parameters fixed.

2. Viscous hydro+cascade model
The approximation of boost-invariant scaling flow is well-justified for the hydrodynamic evolution of matter around midrapidity at top RHIC energies and above. However one should not pursue this assumption at lower energies, since at $\sqrt{s_{NN}} \approx 20$ GeV and below it becomes invalid because of the absence of a plateau in the rapidity distribution of produced particles.

We avoid this issue by using rapidity-dependent initial conditions together with a 3D hydrodynamic evolution. The model employed in the present studies consists of components,
Figure 1. Left: Sketch of space-time picture of evolution, the different phases are marked by different colour. Right: starting time for the hydrodynamic phase as a function of $\sqrt{s_{NN}}$ according to Eq. 1.

Each corresponding to a certain stage of the nucleus-nucleus collision. The space-time picture is sketched on Fig. 1.

**Pre-thermal phase.** The Ultrarelativistic Quantum Molecular Dynamics (UrQMD) model [3, 4] is used for the description of initial stage dynamics. The two nuclei are initialized according to Woods-Saxon distributions and the binary interactions are taken into account until a hypersurface at constant $\tau = \sqrt{R^2 - z^2}$ is reached. We generate a large enough set of UrQMD events to get smooth averaged 3-dimensional distribution of particles which cross this hypersurface. The energy and momentum of particles is then converted to energy and momentum densities of the fluid. In addition to energy/momentum densities, initial baryon and charge densities are non-zero and obtained from UrQMD, evolved in the hydro stage and accounted for in the particlization procedure. On the other hand, net strangeness density is set to zero.

**Hydrodynamic phase.** We start the hydrodynamic phase at

$$\tau_0 = 2R/\sqrt{(\sqrt{s}/2m_N)^2 - 1}$$  \hspace{1cm} (1)

where $R$ is a radius of nucleus and $m_N$ is a nucleon mass. This corresponds to the time when the two nuclei have passed through each other. The values of the starting time for different collision energies are shown in Fig. 1, right. The hydrodynamic equations are solved in Milne $(\tau - \eta - r_x - r_y)$ coordinates.

Another important ingredient of the model relevant to lower collision energies is the equation of state (EoS) for finite baryon density. We employ the chiral model based EoS [8], which features correct asymptotic degrees of freedom, i.e. quarks and gluons at high temperature and hadrons in the low-temperature limits, crossover-type transition between confined and deconfined matter for all values of $\mu_B$ and qualitatively agrees with lattice QCD data at $\mu_B = 0$.

We employ the Israel-Stewart framework for relativistic viscous hydrodynamics [5]. Several variants of Israel-Stewart equations exist in literature, the differences between them coming from slightly different assumptions in the derivation [7]. In particular we stick to the following choice for equations for the shear stress tensor:

$$\langle u^\gamma \partial_\gamma \pi^{\mu\nu} \rangle = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} \partial_\gamma u^\gamma$$  \hspace{1cm} (2)
where \( \partial_{\lambda} \) denotes covariant derivative and the brackets \( \langle A^{\mu \nu} \rangle \) denote the symmetric, traceless and orthogonal to \( u^\nu \) part of \( A^{\mu \nu} \). For the current study we consider only the effects of shear viscosity, fixing bulk viscosity to zero, \( \zeta/s = 0 \). We do not include the baryon/electric charge diffusion either. For viscous hydro simulations, we initialize the shear stress tensor to zero. The relaxation time for shear, \( \tau_s \), is taken as \( \tau_s = 3\eta/(sT) \), as in [6].

**Particlization and hadron corona.** Motivated by previous studies at \( \sqrt{s_{NN}} = 200 \) A GeV RHIC and LHC energies [2], we fix the transition from fluid to particle description (so-called particlization) to take place at a constant energy density \( \epsilon_{sw} = 0.5 \) GeV/fm\(^3\), when the relevant degrees of freedom are hadrons. The Cornelius subroutine [9] is used to calculate the 3-volume elements \( d\sigma_\mu \) of the transition hypersurface. It is important to note that while we use fixed energy density as transition criterion for all collision energies, the average net baryon density increases with decreasing collision energy. This corresponds to the increase of average temperature and decrease of baryon chemical potential with increasing collision energy, resembling the results for the collision energy dependence of chemical freezeout parameters from thermal model studies [10]. Since the EoS used has a mean field included, which somewhat shift the values of thermodynamic quantities from those in a free hadron-resonance gas (HRG) approximation, we switch to the free HRG EoS to sample the hadrons according to Cooper-Frye prescription. We recalculate the energy density, pressure, flow velocity and corresponding thermodynamical quantities from the energy-momentum tensor using a free HRG EoS, and employ them when sampling. To account for the effect of shear viscosity, we use the same corrections to the local equilibrium distribution functions for all hadron species:

\[
 f_i(x,p) = f_{i,eq}(x,p) \left[ 1 + \left( 1 + \frac{p_{\mu}p_{\nu}\pi^{\mu\nu}}{2T^2(\epsilon + p)} \right) \right] 
\]

Finally, UrQMD is employed to calculate the further evolution of the hadron corona.

### 3. Results

We fix most of the parameters of the model as described above. The only free parameter left is the shear viscosity to entropy density ratio \( \eta/s \) in the hydro stage. Finally we perform simulations corresponding to various collision energy and \( \eta/s \) values. Since a bulk of data from SPS exists for different observables (\( dN/dy \), \( p_T \)-distributions, pion femtoscopy) one can first check how well the overall collective matter dynamics is described in the model. Thus we simulate Pb-Pb collisions at energies \( E_{lab} = 158, 80, 40, 30 \) and 20 A GeV (i.e. \( \sqrt{s_{NN}} = 17.3, \ldots, 6.3 \) GeV), and compare \( m_T \) and \( dN/dy \) distributions with the data from the NA49 collaboration [11, 12].

With decreasing beam energy, the passage time calculated according to (1) increases, whereas the average initial energy density for hydrodynamic expansion becomes smaller. The latter leads to a shorter duration of the hydro phase, provided that the criterion for fluid to particle transition is kept the same. Consequently, at the lowest SPS energy (\( \sqrt{s_{NN}} = 6.3 \) GeV) the lifetimes of pre-hydro (4.1 fm/c) and hydro stages are comparable, and the pre-hydrodynamic stage gives a significant contribution to the development of transverse flow.

In the case of zero shear viscosity in the hydrodynamic phase we underestimate the particle yields and radial flow at midrapidity. However, the inclusion of shear viscosity in the hydro phase increases the yields at midrapidity due to viscous entropy production, see Fig. 2. One observes that shear viscosity makes the \( dN/dy \) profile narrower. The longitudinal expansion is weaker, and the expansion tends to be more spherical, as seen in the comparison of the \( p_T \)-spectra, Fig. 3: \( p_T \)-spectra become flatter, which is due to stronger transverse expansion and larger radial flow.

In Fig. 4 we show \( p_T \)-differential charged hadron elliptic flow at collision energies \( \sqrt{s_{NN}} = 7.7, 27 \) and 39 A GeV compared with the results from the RHIC BES. An inviscid hydrodynamic phase leads to an overestimate of the data, while choosing \( \eta/s = 0.2 \) greatly improves agreement.
Figure 2. Rapidity distributions for $\pi^-, K^+$ and $K^-$ for Pb-Pb collisions at $E_{\text{lab}} = 158$ and $40$ A GeV (corresponding to $\sqrt{s_{NN}} = 17.3$ and $6.3$ GeV). Model calculations are compared to NA49 data [12]. The figures are taken from [13].

Figure 3. $m_T$-spectra for $\pi^-, K^+$ and $K^-$ for Pb-Pb collisions at $E_{\text{lab}} = 158, 80, 40, 20$ A GeV ($\sqrt{s_{NN}} = 17.3, 12.3, 8.8, 6.3$ GeV, respectively). Model calculations are compared to NA49 data [11, 12]. Dashed lines on bottom right plot: model calculations for $\pi^+$. The figures are taken from [13].
\( \eta/s = 0, 0.1 \) and 0.2. So one can conclude that a consistent description of \( v_2 \), \( dN/dy \), and \( p_T \)-spectra requires a value of \( \eta/s \) which is somewhat larger than 0.2, especially for the lowest collision energy points. \( \eta/s = 0.2 \) also makes \( v_2(p_T) \) almost independent of the collision energy.

We conclude that the introduction of shear viscosity in the hybrid model consistently improves the description of the data in the low collision energy region. With the averaged initial state from UrQMD used, the suggested value of the effective shear viscosity is \( \eta/s \geq 0.2 \) for both Pb-Pb collisions with \( E_{\text{lab}} = 20 \ldots 158 \) GeV at SPS and Au-Au collisions with \( \sqrt{s} = 7.7 \ldots 39 \) GeV in the BES program at RHIC. For comparison, the typical \( \eta/s \) value obtained from the fit to elliptic flow at \( \sqrt{s_{NN}} = 200 \) GeV RHIC energy is 0.08 for the case of a Monte Carlo Glauber initial state [15].

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**References**

[1] Hirano T, Huovinen P, Murase K and Nara Y 2013, *Prog. Part. Nucl. Phys.* 70 108-158
[2] Karpenko Iu A, Sinyukov Yu M, Werner K 2013, *Phys. Rev. C* 87 24914
[3] Bass S A et al. 1998, *Prog. Part. Nucl. Phys.* 41 255-369
[4] Bleicher M et al. 1999, *J. Phys. G* 25 1859
[5] Israel W and Stewart J M 1979, *Ann. Phys.* 118 341
[6] Song H and Heinz U 2008, *Phys. Rev. C* 77 064901
[7] Denicol G S, Molnár E, Niemi H, Rischke D H, *Eur. Phys. J. A* 48 11 (2012) 170
[8] Steinheimer J, Schramm S and Stocker H 2011, *J. Phys. G* 38 35001
[9] Huovinen P and Petersen H 2012, *Eur. Phys. J. A* 48 171
[10] Andronic A, Braun-Munzinger P, Stachel J 2006, *Nucl. Phys. A* 772 167-199
[11] Alt C et al. (NA49 Collaboration) 2008, *Phys. Rev. C* 77 24093
[12] Afanasiev S V et al. (NA49 Collaboration) 2002, *Phys. Rev. C* 66 54902
[13] Karpenko Iu A, Bleicher M, Huovinen P, Petersen H 2013, Preprint arXiv:1310.0702 [nucl-th]
[14] Adamczyk L et al. (STAR Collaboration) 2012, *Phys. Rev. C* 86 54908
[15] Song H, Bass S A, Heinz U, Hirano T, Shen C 2011 *Phys. Rev. Lett.* 106 192301