Double Compton and cyclo-synchrotron in super-Eddington discs, magnetized coronae and jets

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ABSTRACT
Black hole accretion discs accreting near the Eddington rate are dominated by bremsstrahlung cooling, but above the Eddington rate, the double Compton process can dominate in radiation-dominated regions, while the cyclo-synchrotron can dominate in strongly magnetized regions like a corona or a jet. We present an extension to the general relativistic radiation magnetohydrodynamical code HARMRAD to account for emission and absorption by thermal cyclo-synchrotron, double Compton, bremsstrahlung, low-temperature OPAL opacities, as well as Thomson and Compton scattering. The HARMRAD code and associated analysis and visualization codes have been made open-source and are publicly available at the github repository website. We approximate the radiation field as a Bose–Einstein distribution and evolve it using the radiation number–energy–momentum conservation equations in order to track photon hardening. We perform various simulations to study how these extensions affect the radiative properties of magnetically arrested discs accreting at Eddington to super-Eddington rates. We find that double Compton dominates bremsstrahlung in the disc within a radius of \( r \sim 15 r_g \) (gravitational radii) at hundred times the Eddington accretion rate, and within smaller radii at lower accretion rates. Double Compton and cyclo-synchrotron regulate radiation and gas temperatures in the corona, while cyclo-synchrotron regulates temperatures in the jet. Interestingly, as the accretion rate drops to Eddington, an optically thin corona develops whose gas temperature of \( T \sim 10^5 \text{K} \) is \( \sim 100 \) times higher than the disc’s blackbody temperature. Our results show the importance of double Compton and synchrotron in super-Eddington discs, magnetized coronae and jets.

Key words: accretion, accretion discs – black hole physics – MHD – radiation: dynamics – galaxies: jets – X-rays: binaries.

1 INTRODUCTION
Black hole (BH) accretion flows become radiation-dominated and geometrically thick once the luminosity \( L \gtrsim 0.3 L_{\text{Edd}} \), where \( L_{\text{Edd}} \approx 1.3 \times 10^{38}[M/(10 M_\odot)] \text{erg s}^{-1} \) is the Eddington luminosity for solar mass \( M_\odot \) and BH mass \( M \). Such accretion flows tend to have temperatures of \( T \gtrsim 10^5 \text{K} \) (Abramowicz et al. 1988), where bremsstrahlung (BR, free–free, \( ep \rightarrow e'p' \gamma \)) is important.

Double (radiative) Compton (DC, \( \gamma e \rightarrow \gamma' e' \)) (Lightman 1981; Thorne 1981; Pozdnyakov, Sobol & Syunyaev 1983; Svensson 1984) can dominate free–free while regulating temperatures so that pairs remain sub-dominant (see fig. 1 in Thorne 1981). While DC has received attention in cosmology to determine distortions to the cosmic microwave background (Chluba & Sunyaev 2012; Chluba 2014) and been applied to gamma-ray bursts (Vurm, Lyubarsky & Piran 2013; Bégué & Pe’er 2015), it has received little attention in accretion theory except by Poutanen & Svensson (1996).

For photon number density \( n_\gamma \), ion number density \( n_i \), electron mass \( m_e \), speed of light \( c \), Boltzmann’s constant \( k_b \) and temperature \( T \), the rates of each process show that DC dominates free–free if

\[
\frac{n_\gamma}{n_i} \gtrsim 1.3 \left( \frac{m_e c^2}{k_b T} \right)^{5/2}
\]

(Thorne 1981; Svensson 1984), which has the correction by Svensson (1984) of 1.3 instead of the value 0.1 by Thorne (1981). In this
paper, we find DC dominates free–free for the mean opacity when 
\[ T > 7 \times 10^{7} \text{ K} \rho^{2/11}, \]
where \( \rho \) is mass density in cgs units. For BH X-ray binaries accreting 
at 10 times the Eddington rate, one has \( \rho \approx 10^{-3} \text{ g cm}^{-3} \) giving equality at \( T \approx 2 \times 10^{7} \text{ K} \), while simulations show that \( T \approx 5 \times 10^{7} - 10^{8} \text{ K} \) (Sadowski & Narayan 2015a) indicating that DC dominates free–free even if not yet included. For supermassive BHs relevant 
for tidal disruption events (TDEs) accreting at the Eddington rate, one has \( \rho \approx 10^{-3} \text{ g cm}^{-3} \), giving equality at \( T \approx 1.4 \times 10^{9} \text{ K} \), which is reached within tens of gravitational radii.

Still, it remains uncertain that which opacity dominates the mean flow behaviour. The Novikov–Thorne solution (Novikov & Thorne 1973) or slim disc solution (Abramowicz et al. 1988) could be used as a guide to how important DC is, but we find that the temperature sensitivity makes the conclusions not robust – especially as the slim disc solution is modified by winds, vertical structure and magnetic fields. Also, unlike free–free, DC is entirely dependent upon the radiation temperature for non-relativistic electrons, so distinct gas and radiation temperatures are important to treat. This motivates using simulations to consider both opacities with separate gas and radiation temperatures.

In addition, cyclo-synchrotron (hereafter, synchrotron) is expected to be an important source of opacity or emission for low-luminosity accretion flows or in regions where non-thermal electrons are present. In super-Eddington accretion flows, one expects synchrotron to be unimportant in the disc. However, the atmosphere above the disc (i.e. corona) and jet can contain a strong magnetic field at low densities where synchrotron can dominate (Di Matteo, Celotti & Fabian 1997; Uzdensky & McKinney 2011). In addition, magnetically arrested accretion discs (MADs) are much more strongly magnetized than weakly magnetized discs that are usually considered (Igumenshchev, Narayan & Abramowicz 2003; Narayan, Igumenshchev & Abramowicz 2003; Tchekhovskoy, Narayan & McKinney 2011). Magnetically supported atmospheres can harden the spectrum by producing a more extended vertical disc (Blaes et al. 2006), and synchrotron can provide an abundant source of low-energy soft photons that can undergo inverse Compton before being absorbed. So the magnetic field strength can regulate photon hardening in disc atmospheres and magnetized jets.

In this paper, we consider the competition between free–free, bound-free bound–bound opacities from OPAL (Iglesias & Rogers 1996), DC, and synchrotron in MAD-type BH accretion flows. We also consider separate absorption and emission mean opacities to focus on emission rates in the energy and momentum equations, rather than the Rosseland mean that only applies in the diffusion limit for the momentum equation. The absorption mean handles the effect of irradiation and how the absorption of radiation is affected by the photon distribution being different than that given by Planck at the local temperature (Sadowski et al. 2017). Such effects lead to a significant correction to the mean opacities and can qualitatively change the outcome (Hubeny, Burrows & Sudarsky 2003). Additionally, we evolve the photon number separately from the photon energy (Sadowski & Narayan 2015a), with separate number and energy opacities, in order to track photon hardening due to inverse Comptonization (IC). The photon distribution is assumed to be a Bose–Einstein (BE) distribution, but with a low-energy transition to Planck when absorption is faster than inverse Compton. Unlike a Planck distribution, a BE distribution can handle how the photon distribution evolves towards a Wien distribution in a scattering-dominated regime.

These physical effects are included within our general relativistic radiative magnetohydrodynamics (GRRMHD) code HARMSRAD (McKinney et al. 2014) that uses the M1 closure. Similar general relativistic (GR; Fragile et al. 2012; Takahashi et al. 2013; Sadowski et al. 2014; Fragile, Olejar & Anninos 2014; McKinney et al. 2014; Ryan, Dolence & Gammie 2015; Takahashi et al. 2016) and non-GR (Jiang, Stone & Davis 2014b) schemes have been developed. These codes have been used to study BH X-ray binaries and their discs, coronae and jets (Jiang, Stone & Davis 2014a; McKinney, Dai & Avarai 2015; Sadowski & Narayan 2015a), growth of 10 solar-mass BH seeds (McKinney et al. 2014), low-luminosity active galactic nuclei (AGN; Sadowski et al. 2017), high-luminosity AGN (Jiang, Davis & Stone 2016), AGN feedback (Sadowski et al. 2016a), TDEs (Sadowski & Narayan 2015b; Sadowski et al. 2016b), ultraluminous X-ray sources (ULXs; Narayan et al. 2016), massive stellar envelopes (Jiang et al. 2015), neutron star atmospheres (Wielgus et al. 2016), etc.

In Section 2, we outline our equations of motion; in Section 3, we present our GRRMHD simulations, and in Section 4, we summarize our results. For the appendices: in Appendix A, we discuss our use of the BE distribution, in Appendix B, we discuss how we compute the mean opacities, in Appendix C, we discuss free–free and related low-temperature opacities, in Appendix D, we discuss synchrotron opacities, and in Appendix E, we discuss Compton scattering and DC opacities.

2 EQUATIONS OF MOTION

The conservation laws are
\[ (\rho u^\mu)_{,\mu} = 0, \]
\[ (T^\mu_\nu)_{,\mu} = G_{\nu}, \]
\[ (R^\mu_\nu)_{,\mu} = -G_{\nu}, \]
where \( \rho \) is the gas density in the comoving fluid frame, \( u^\mu \) is the gas four-velocity as measured in the 'lab frame' and \( T^\mu_\nu \) is the magnetohydrodynamics (MHD) stress–energy tensor in this frame,
\[ T^\mu_\nu = (\rho + p_\gamma + p_\beta + b_\gamma^2) u^\mu u_\nu + \left( p_\gamma + \frac{b_\gamma^2}{2} \right) \delta^\mu_\nu - b_\gamma^\mu b_\nu. \]
\( R^\mu_\nu \) is the stress–energy tensor of radiation, \( G_{\nu} \) is the radiative four-force describing the interaction between gas and radiation, \( u_\nu \) and \( p_\gamma \) are, respectively, the internal energy and pressure of the gas in the comoving frame and \( b_\gamma \) is the magnetic field four-vector (Gammie, McKinney & Tóth 2003). The magnetic pressure is \( p_\beta = b_\gamma^2/2 \) in our Heaviside–Lorentz units. The ideal induction equation and entropy equations are also used (McKinney et al. 2014). We assume collisions keep gas and electron temperatures similar (Sadowski et al. 2017).

2.1 Radiative four-force

We use a covariant formalism for computing the interaction due to absorption, emission and scattering (McKinney et al. 2014; Sadowski et al. 2014) via a four-force between the gas and radiation of
\[ G^\mu = -(\kappa_\gamma + \kappa_s) R^{\mu_\nu} u_\nu - (\kappa_s R^{\mu_\nu} u_\nu u_\mu + \lambda) u^\mu, \]
where \( \kappa_\gamma \) is the energy absorption mean opacity of any emission in the fluid frame (in which ions and electrons are isotropic) in units...
of inverse length, \( \lambda \) is the fluid-frame total energy density loss rate before absorption (which includes changes in energy while conserving photon number) and \( \kappa_s \) is the fluid-frame energy scattering mean opacity. Self-absorption is treated directly by having separate total absorption and total emission.

### 2.2 Photon number evolution

If the radiation is Planckian, then the chemical potential \( \mu = 0 \) and the radiation temperature is derived from only the fluid-frame radiation energy density via \( T_s = (E/\nu)^{1/4} \), where \( E = R^{\text{th}} u_\nu u_\mu \), which can differ from the gas temperature \( T_g \) or electron temperature \( T_e \).

A more general photon distribution, like our choice of a BE distribution (see Appendix A), can be considered by simultaneously evolving the number density of photons in the radiation frame (\( n_\nu \)) for radiation isotropic in the frame with four-velocity \( u_\nu \)) via

\[
(n u_\nu^2)_{\nu} = \dot{n}_\nu, \tag{8}
\]

where the fluid-frame number density of photons is \( n = n_\nu (-u_\nu^\mu u_\mu) \) and \( n_\nu \) by Lorentz invariance, with

\[
\dot{n}_\nu = (-\kappa_{\text{abs}}) n + \lambda_\nu, \tag{9}
\]

for a number absorption mean opacity \( \kappa_{\text{abs}} \) and total number emission rate \( \lambda_\nu \) before absorption. This photon number conservation equation approximates the Kompaneets equation most accurately when thermal Comptonization (TC) is included and when gas and radiation temperatures are similar (Sadowski & Narayan 2015a).

### 2.3 Opacities and rates

We obtain fits to several different energy-weighted and number-weighted opacities (computed in Appendix B), including OPAL opacities for solar abundances (energy mean \( \kappa_{\text{eff}} \) and number mean \( \kappa_{\text{abs}, \text{eff}} \)) in Appendix C, synchrotron (energy mean \( \kappa_{\text{syn}} \) and number mean \( \kappa_{\text{abs}, \text{syn}} \)) in Appendix D, and Compton scattering (scattering energy mean \( \kappa_s \), TC energy exchange rate \( \lambda_s \), and DC energy mean opacity \( \kappa_{\text{abs}, \text{dc}} \)) in Appendix E.

Then, the energy mean energy absorption opacity is

\[
\kappa_s = \kappa_{\text{eff}} + \kappa_{\text{syn}} + \kappa_{\text{abs}, \text{dc}}, \tag{10}
\]

and the number mean number absorption opacity is

\[
\kappa_{\text{abs}} = \kappa_{\text{abs}, \text{eff}} + \kappa_{\text{abs}, \text{syn}} + \kappa_{\text{abs}, \text{dc}}. \tag{11}
\]

From these opacities, the energy and number emission rates before any absorption are obtained in Appendix B5, giving energy emission rate \( \lambda_\nu \) and number emission rate \( \lambda_\nu \), such that

\[
\lambda = \lambda_\nu + \lambda_s \tag{12}
\]

and \( \lambda_\nu \) includes all number emission.

We do not include thermal pairs relevant when \( k_B T_s \gtrsim m_e c^2 \), as only the jet can be that hot and one requires pair production there (see, e.g. appendix B in McKinney & Uzdensky 2012).

### 3 SIMULATIONS

Our GRRMHD simulations evolve accretion flows around BHs with BH mass of \( M = 10 M_\odot \). All models have identical initial conditions, except some have different initial density values due to an overall rescaling of the density in order to vary the final mass accretion rate \( \dot{M} \). For different models, we vary the choices of opacity and how the temperature of radiation is evolved, and we investigate how the opacity and radiation temperature evolution affect the accretion flow behaviour and properties.

#### 3.1 Diagnostics

For various quantities \( R \), we consider time-averages \( \langle |R| \rangle \), spatial averages and their spatial distributions. Diagnostics are computed from snapshots produced every \( \sim 4r_g/c \) for \( r_g \equiv GM/c^2 \) with gravitational constant \( G \).

##### 3.1.1 Fluxes

The stress energy tensor \( T^\nu_\mu \) includes both matter (MA) and electromagnetic (EM) terms:

\[
T^{\text{MA}}_{\nu \mu} = (\rho + u_\nu u_\mu) u_\nu u_\mu + p_b \delta^{\nu}_\mu, \tag{13}
\]

Here, \( u_\nu \) is the internal energy density and \( p_b = (\Gamma - 1) u_\nu \) is the ideal gas pressure with adiabatic index \( \Gamma \). The contravariant-fluid-frame magnetic four-field is given by \( B^\mu \), which is related to the lab-frame three-field via \( b^\mu = B^\nu u_\nu / u_\mu \), where \( h^\mu = u^\mu u_\nu + \delta^\mu_\nu \) is a projection tensor and \( \delta^\mu_\nu \) is the Kronecker delta function. The magnetic energy density \( (u_b) \) and pressure \( (p_b) \) are \( u_b = b^\mu b_\mu / 2 = b^2 / 2 \). The total pressure is \( p_\text{tot} = p_b + p_\text{g} \), and plasma \( \beta \equiv p_b / p_\text{g} \).

The gas rest-mass flux, specific energy flux and specific angular momentum flux are, respectively, given by

\[
\dot{M} = \left| \int \rho u^\mu dA_{\mu \phi} \right|, \tag{14}
\]

\[
e \equiv \frac{\dot{E}}{[M]_h}, \tag{15}
\]

\[
J \equiv \frac{\int \left( T^\phi_\phi + R^\phi_\phi \right) dA_{\mu \phi}}{[M]_h}, \tag{16}
\]

where \( dA_{\mu \phi} = \sqrt{-g} dx^\mu dx^\phi \) for metric determinant \( g = \text{Det}(g_{\mu \nu}) \) and uniform code coordinates with spacing \( dx^1 \), \( dx^2 \) and \( dx^3 \) in the radial-like, \( \theta \)-like and \( \phi \)-like directions.

The net flow efficiency is given by

\[
\eta \equiv \frac{E - \dot{M}}{[\dot{M}]_h}. \tag{17}
\]

Positive values correspond to an extraction of positive energy from the system at some radius. For a jet, which we define as the region within which \( b^\mu / \rho > 1 \), the jet efficiency is \( \eta_j \) and includes all non-radiative terms in \( E \). We often measure the jet efficiency at \( r = 10 r_g \) and call that the inner jet efficiency \( \eta_{j\text{in}} \). We measure this at small radii before much of the jet energy is lost to the surroundings at larger radii, while we measure it off the horizon so that much of the mass is outgoing inside the jet. The efficiency measured at the horizon over all angles is given by \( \eta_H \).

The magnetic flux is given in a normalized form as

\[
\gamma_{\text{BH}} \approx 0.7 \frac{0.5 B^\mu dA_{\mu \phi}}{\sqrt{[M]_h}}, \tag{18}
\]

which accounts for \( B^\mu \) being in Heaviside–Lorentz units (Gammie 1999; Penna et al. 2010).
3.1.2 Inflow equilibrium

Inflow equilibrium is defined as when the flow is in a complete quasi-steady state and the accretion fluxes are constant (apart from noise) versus radius and time. The inflow equilibrium time-scale is

\[ t_{ie} = N \int_{r_i}^{r_{ie}} dr \left( \frac{-1}{(v_t \rho)_|_{r_{ie}}} \right), \tag{19} \]

where \((v_t)_|_{r_{ie}}\) is the \(\rho\) weighted radial velocity, and \(N \approx 3\) inflow times from an inflow equilibrium radius of \(r = r_{ie}\) down to \(r_i = r_{ith}\) at the horizon.

Viscous theory gives a GR \(\alpha\)-viscosity estimate for \(v_t\) of \(v_{\text{visc}} \sim \frac{Q}{\sqrt{H/R}} |v_{\text{col}}|\) for rotational velocity \(v_{\text{rot}}\), disc thickness \(H/R\) and GR correction \(Q\) (Page & Thorne 1974; Penna et al. 2010), so we can define an effective \(\alpha\) viscosity as

\[ \alpha_{\text{eff}} \equiv \frac{v_t}{v_{\text{visc}}/\alpha}. \tag{20} \]

All our models have \(\alpha_{\text{eff}} \approx 1\) in the quasi-steady state, as expected for MADs (McKinney, Tchekhovskoy & Blandford 2012).

3.1.3 Optical depth and radiative quantities

The scattering optical depth is computed as

\[ \tau_{\text{sca}} \approx \int k_s d l, \tag{21} \]

while the effective optical depth for absorption is computed as

\[ \tau_{\text{eff}} \approx \int \sqrt{3 k_s (k_s + k_a)} d l. \tag{22} \]

For the radial direction, \(d l = -f_r dr, f_r = u_t (1 - (v/c) \cos \theta), (v/c) \approx 1 - 1/(u^2)\) (as valid at large radii), \(\theta = 0\), and the integral is from an outer radius of \(r_0 = 4000r_g\) (chosen to be inside the outer grid’s radial boundary, but still far outside the scattering photosphere) to \(r\) to obtain \(\tau_s(r)\). For the angular direction, \(d l = f_r r d \theta, \theta = \pi/2,\) and the integral is from each polar axis towards the equator to obtain \(\tau_s(\theta)\). Note that the photospheres in the polar jet region are strongly determined by the numerical density floors assumed.

To scale \(Mc^2\) or a luminosity \(L\), one can use the Eddington luminosity

\[ L_{\text{Edd}} = \frac{4 \pi GM c}{k_{es}} \approx 1.3 \times 10^{46} \frac{M}{10^8 M_\odot} \text{ erg s}^{-1}, \tag{23} \]

for Thomson electron scattering opacity \(k_{es}\). One can also choose to normalize \(\dot{M}\) by \(M_{\text{Edd}} = (1/\eta_{\text{NT}}) L_{\text{Edd}} / c^2\), where \(\eta_{\text{NT}}\) is the nominal accretion efficiency for the Novikov–Thorne thin disc solution (Novikov & Thorne 1973) (commonly, a fixed \(\eta_{\text{NT}} = 0.1\) is used, but we include the spin dependence).

The radiative luminosity is computed as

\[ L_{\text{rad}} = - \int dA_{\text{sh}} R_{\gamma}^\prime, \tag{24} \]

which is measured just beyond the scattering photosphere to give the quantity we call \(L_{\text{rad}, 0}\). The radiative efficiency is \(\eta_{\text{rad}, 0} = L_{\text{rad}, 0} / [M\dot{M}]_0\). From any cumulative luminosity \(L(\theta)\), we can compute the isotropic equivalent luminosity

\[ L_{\text{iso}}(\theta) = \frac{\partial L(\theta)}{-\partial_{\cos \theta}} \tag{25} \]

and the corresponding beaming factor

\[ b = \frac{L_{\text{iso}}}{L}. \tag{26} \]

where \(L = L_{\text{rad}}\) as computed over all angles.

The fluid-frame radiation temperature is

\[ T_\gamma \approx \frac{E}{n E_0}, \tag{27} \]

with \(E_0 \approx 2.7016 \text{keV}\), which is within 10 per cent of the full BE formula given by equation (A5) that we actually use. The lab-frame radiation temperature is

\[ T_\gamma \approx \frac{-R_k^\prime}{n_i u_i E_0^{1/4}}. \tag{28} \]

The blackbody assumption gives a fluid-frame temperature of

\[ T_{\text{BB}} = \left( \frac{E}{a_{\text{rad}}} \right)^{1/4}. \tag{29} \]

for radiation constant \(a_{\text{rad}}\), while in the lab frame, the assumption of Planck gives

\[ T_{\text{BB}} = \left( \frac{-R_k^\prime}{a_{\text{rad}}} \right)^{1/4}. \tag{30} \]

From these, one can compute the lab-frame photon hardening factor

\[ f_{\text{col}} = \frac{T_\gamma}{T_{\text{BB}}} \tag{31} \]

and the fluid-frame hardening factor

\[ f_{\text{col}} = \frac{T_\gamma}{T_{\text{BB}}} \tag{32} \]

3.1.4 Numerical diagnostics

The magneto-rotational instability (MRI) is a linear instability with fastest growing wavelength of

\[ \lambda_{\text{MRI}} \approx 2 \pi \frac{|v_t A|}{|\Omega_{\text{col}}|}, \tag{33} \]

for \(x = \theta, \phi\), where \(|v_t A| = \sqrt{b^2 + \rho + u_2 + p_g}\) and \(\Omega_{\text{col}} = v_{\text{col}}, \Omega_{\text{col}}\), which are separately angle-volume-averaged at each \(r, t\).

The MRI is resolved for grid cells per wavelength (equation 33),

\[ Q_{\lambda_{\text{MRI}}} \approx \frac{\lambda_{\text{MRI}}}{\Delta_x}, \tag{34} \]

of \(Q_{\lambda_{\text{MRI}}} \geq 6\), for \(x = \theta, \phi\), where \(\Delta_x \approx d x (dr / d x^{(1)})\), \(\Delta_\phi \approx r d x (d \phi / d x^{(2)})\) and \(\Delta_\theta \approx r \sin \theta d x (d \phi / d x^{(3)})\). Volume-averaging is done as with \(S_{\lambda_{\text{MRI}}}\), except \(v_{\text{col}, A} / \Delta_x\) and \(\Omega_{\text{col}}\), which are separately \(\theta, \phi\)-volume-averaged before forming \(Q_{\lambda_{\text{MRI}}}\). At \(t = 0\), all our models have \(Q_{\lambda_{\text{MRI}}} \approx 100\) and 20 (except model M13 that is quite thin and has \(Q_{\lambda_{\text{MRI}}} \approx 6\) and so probably has somewhat underdeveloped MRI turbulence).

The MRI suppression factor corresponds to the number of MRI wavelengths across the full disc:

\[ S_{\lambda_{\text{MRI}}} \approx \frac{2 \pi (R / H)}{\lambda_{\phi_{\text{MRI}}}}. \tag{35} \]

Wavelengths \(\lambda < 0.5 \lambda_{\phi_{\text{MRI}}}\) are stable, so the linear MRI is suppressed for \(S_{\lambda_{\text{MRI}}} < 1/2\) when no unstable wavelengths fit within the full disc (Balbus & Hawley 1998; Pessah & Psaltis 2005). \(S_{\lambda_{\text{MRI}}}\) uses averaging weight \(w = (b^2 \rho)^{1/2}\), condition \(\beta > 1\) and excludes regions where density floors are activated. When computing the averaged \(S_{\lambda_{\text{MRI}}}, v_A\) and \(\Omega_{\text{col}}\) are separately \(\theta, \phi\)-volume-averaged.
within \( \pm 0.2r \) for each \( t, r \). The \( \delta_{\text{MAD}} \sim 0.5 \) at \( t = 0 \), while in quasi-steady state, \( \delta_{\text{MAD}} \sim 0.1 \). So the field strength has increased considerably due to magnetic flux accumulation. All models are MAD with \( \delta_{\text{MAD}} < 1/2 \) out to \( r \sim 30r_g \).

The flow structure can also be studied by computing the correlation length-scale and then computing how many grid cells cover each self-correlated piece of turbulence. We follow our prior works (McKinney et al. 2012; McKinney et al. 2014) and compute this. One would desire to have at a minimum six grid cells per correlation length-scale, since otherwise uncorrelated parts of turbulence are not independently resolved by our piece-wise parabolic monotonicity-preserving scheme that needs six grid cells to resolve a structure well. All our models have \( \approx 12 \) grid cells per vertical and radial correlation length for density and magnetic field strength, while our moderate resolution models have \( \approx 6 \) cells in the \( \phi \)-direction per correlation length-scale. Our survey models have only \approx 3 cells in the \( \phi \)-direction per correlation length. This means that the survey models should be considered not resolved enough to demonstrate fully resolved turbulence, but they are still sufficiently interesting to identify what physical effects (being switch on/off) could be important in fully resolved simulations. In addition, we compare some survey models against moderate resolution versions to confirm the survey models are reasonable.

### 3.2 Initial conditions

The initial disc is Keplerian with a rest-mass density that is Gaussian in angle with a height-to-radius ratio of \( H/R \approx 0.2 \) and radially follows a power law of \( \rho \propto r^{-0.6} \). The solution near and inside the inner-most stable circular orbit (ISCO) is not an equilibrium, so near the ISCO, the solution is tapered to a smaller density (\( r \rightarrow \rho(r/15)^2 \), within \( r = 15r_g \)) and a smaller thickness (\( H/R \rightarrow 0.2(r/10)^{0.5} \), within \( r = 10r_g \) based upon a low-resolution simulation). The total internal energy density \( u_{\text{int}} \) is estimated from vertical equilibrium of \( H/R \approx c_s/\sqrt{\Gamma} \) for sound speed \( c_s \approx \sqrt{\Gamma u_{\text{tot}}/\rho} \) with \( \Gamma_{\text{tot}} \approx 4/3 \) and Keplerian speed \( \sqrt{\Gamma} \approx (r/r_g)/(r/\Gamma^{1/2} + a/M) \). The total ideal pressure \( P_{\text{tot}} = (\Gamma_{\text{tot}} - 1)u_{\text{tot}} \) is randomly perturbed by 10\% to seed the MRI. The disc gas has \( \Gamma_{\text{gas}} = 5/3 \), as used during the simulation. The disc has an atmosphere with \( \rho = 10^{-2}(r/r_g)^{-1/4} \) and gas internal energy density \( u_g = 10^{-5}(r/r_g)^{-3/2} \). The disc’s radiation energy density and flux are set by flux-limited diffusion and are having the same temperature for both gas and radiation (McKinney et al. 2014).

The initial magnetic field is large scale and poloidal. For \( r < 300r_g \), the coordinate basis \( \phi \)-component of the vector potential is

\[
A_\phi = \text{MAX}(r^{-1}0^{10} - 0.02, 0)(\sin \theta)^{1+h},
\]

(36)

with \( v = 0.75 \) and \( h = 4 \). For \( r \geq r_g = 300r_g \), the field transitions to monopolar using \( A_\phi = \text{MAX}(r_g^{-1}0^{10} - 0.02, 0)(\sin \theta)^{1+h}(r/r_g) \). The field is normalized with \( \sim 1 \) MRI wavelength per half-height \( H \) giving a ratio of average gas+radiation pressure to average magnetic pressure of \( \beta \approx 40 \) for \( r < 100r_g \).

### 3.3 Numerical setup

The numerical grid mapping equations and boundary conditions used here are identical to that given in McKinney et al. (2012) and McKinney et al. (2014). The grid focuses on the disc at small radii and on the jet at large radii.

Models M14h and M15h are moderate resolution models at \( N_g \times N_g \times N_g = 128 \times 64 \times 64 \), while the rest are survey models at \( 128 \times 64 \times 32 \) in order to resolve the lowest order \( m \) modes to allow for accretion in MADs. For MADs, such lower resolution models have shown reasonable convergence (McKinney et al. 2012; McKinney et al. 2014). The grid aspect ratio is roughly 1:1:1 for radii \( 5r_g \leq r \leq 30r_g \) for our moderate resolution models.

Models M4 and M12 are very low-resolution \((128 \times 64 \times 16)\) test models that otherwise match models M3 and M11, respectively. These models and others at different resolutions exhibit all of the same radiative properties we will discuss, which shows that resolution is unlikely a dominant factor in controlling our results. We do not discuss models M4 and M12 further.

The rest-mass and internal energy densities are driven to zero near the BH within the jet and near the axis, so we use numerical ceilings of \( b^2/\rho = 300, b^2/\rho = 10^{11} \) and \( u_g/\rho = 10^{10} \). The value of \( b^2/\rho \) is at the code’s robustness limit for the chosen resolution, while the value of \( b^2/\rho \) is chosen to ensure that an artificial temperature floor is not introduced.

At early times in all simulations, we ramp-up the chemical potential factor from Planck towards its desired target. This avoids difficult-to-resolve opacity changes due to synchrotron within the first \( 1000r_g/c \) in time. Also, the sharp changes in synchrotron opacity for high \( \phi \) (equation D5) near Planck (i.e. \( \exp(-\xi) = 1 \) with dimensionless chemical potential \( \xi = 0 \)) is difficult to handle. For synchrotron only, we enforce \( \exp(-\xi) < 0.99 \) in order to smooth out these opacity changes (Seaton 1993).

### 3.4 Models

Our goal is to consider the physical effect of various choices for the radiative transfer opacities. We simulate several low-resolution ‘survey’ runs over a time period of about \( 10^4 \) \( r_g/c \), allowing the simulations to reach a single inflow time out to \( r \sim 50r_g \) and infall equilibrium out to \( r \sim 20r_g \). We check these survey runs with a couple moderate resolution runs, and we consider even lower resolution runs to see if resolution plays a dominate or sub-dominant role compared to the opacity effects.

Table 1 shows the physical choices made for each model, including the BH spin, choices for opacities, whether the radiation photon number density is evolved, how the radiation temperature is computed and whether the chemical potential is varied.

The ‘OPAL’ opacity refers to the opacity \( \kappa_{\text{eff}} \) given by equation (C16). This is physically equivalent to the OPAL-based opacity used in Jiang et al. (2016), except ours is more approximate, but we also account for radiation temperatures being different from gas temperatures. ‘DC’ refers to double Compton, which no simulations have yet accounted for. ‘Syn’ refers to synchrotron, which has been accounted for in sub-Eddington accretion cases (Fragile & Meier 2009; Ryan et al. 2015; Sadowski et al. 2017). All models with photon number evolution include a separately computed number mean opacity not yet accounted for in simulations (except only synchrotron number opacity in Sadowski et al. 2017). The simplified fluid-frame radiation temperature given by \( E_{\gamma}(\delta \delta_{\text{d}}) \) with \( E_{g} \approx 2.701k\.b \) is the dominant factor in equation (A5) (Sadowski & Narayan 2015a). We consider both Planck and BE photon distributions.

All models include TC except model M6 in order to study how turning that off affects gas temperatures. Model M6 would be like many existing radiative transfer simulations (Fragile et al. 2012; Fragile et al. 2014; Jiang et al. 2014b; Takahashi et al. 2016) except those by Fragile & Meier (2009), Kawashima et al. (2009) and Jiang et al. (2014a) and ourselves (Sadowski et al. 2014; McKinney et al. 2014).
Models M9, M11, M12, M13, M14, M14h, M15 and M15h are used to explore how varying $M$ affects the results while using our full opacity physics, with M9 and M11 not behaving much differently due to having similar $M$.

Throughout our discussion of model results, we focus on specific models, often showing more details for model M15 because it has an intermediate mass accretion rate and demonstrates properties of all our models.

### 3.5 Initial and final state

Fig. 1 shows the initial and final state of the accretion flow for model M15, which is typical of all models with rotating BHs. The initial magnetic field structure is large scale and poloidal, and threads the disc that is initially a Gaussian disc with $H/R \approx 0.2$ and a power-law radial behaviour. The rotating BH and disc have launched a jet that starts out magnetically dominated but converts its energy into kinetic energy at large radii.

Table 2 shows results for dynamical quantities (like fluxes and efficiencies) for all our models as time-averaged from $4000\, r_g/c$ till the end of the simulation. This table can be used to compare results for models with different opacity physics and different $M$. The models vary in mass accretion rate through the horizon with $\dot{M}/\dot{M}_{\text{Edd}} = 1–140$, and the radiative luminosity is given by $L_{\text{rad},\, 0}$. Other quantities are measured on the horizon (e.g. $\eta_\text{H}$), at an inner radius of $10r_g$ for inner ('i') quantities, or at large radii for outer ('o') quantities.

The efficiency $\eta_\text{H}$ is the total efficiency of the system as measured at the horizon, which is constant to within 30 per cent out to the scattering photosphere where outer quantities are measured. The total and radiative efficiency at large radii ($\eta_{\text{RAD, 0}}$) are comparable to the NT standard thin disc efficiencies ($\eta_{\text{NT}}$). The total non-radiative jet efficiency $\eta_{\text{J, in}}$ is the efficiency at $r = 10r_g$ in the jet with $b^2/\rho > 1$. This tracks each model’s total efficiency, but the jet energy is progressively lost to the surrounding material that heats up and radiates (McKinney et al. 2015), leading to relatively low gas jet efficiencies by $r = 1000r_g$. The total radiative efficiency is therefore dependent upon the mass-loading physics, which is controlled partially by numerical floor injection in our simulations. For non-rotating BH models, the jet efficiency is low at 1 per cent, except for the model without TC with jet efficiency at 4 per cent due to the thermal energy content of the jet (this also leads to a slightly higher radiative efficiency).

**Table 1.** Spin, mass accretion rate, opacity and temperature choices.

| Model | $a/M$ | $\dot{M}/\dot{M}_{\text{Edd}}$ | Opacities | Radiation number density | Radiation temperature | Chemical potential |
|-------|-------|-------------------------------|-----------|-------------------------|-----------------------|-------------------|
| M1    | 0.8   | 140                           | OPAL+Syn+DC | Evolved                | $E/(n\bar{E}_0)$     | 1                 |
| M2    | 0.8   | 140                           | OPAL       | Evolved                | $E/(n\bar{E}_0)$     | 1                 |
| M3    | 0.8   | 80                            | OPAL       | Evolved                | Bose–Einstein        | Bose–Einstein     |
| M5    | 0.8   | 120                           | OPAL       | Planck at $\tilde{T}$  | Planck at $\tilde{T}$ | 1                 |
| M6    | 0     | 5.9                           | OPAL (no TC)| Planck at $\tilde{T}$  | Planck at $\tilde{T}$ | 1                 |
| M7    | 0     | 4.8                           | OPAL       | Evolved                | $E/(n\bar{E}_0)$     | 1                 |
| M8    | 0     | 5                             | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M9    | 0.8   | 50                            | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M10   | 0.8   | 27                            | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M11   | 0.8   | 36                            | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M13   | 0.8   | 1.2                           | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M14   | 0.8   | 3.5                           | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M14h  | 0.8   | 2.4                           | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M15   | 0.8   | 14                            | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |
| M15h  | 0.8   | 31                            | OPAL+Syn+DC| Evolved                | Bose–Einstein        | Bose–Einstein     |

**Figure 1.** Model M15, showing the initial condition (top panel) and final state (bottom panel). The initial disc is shown as rest-mass density (scaled by an Eddington density value inferred from $M_{\text{Edd}}$, $r_g$, and $c$, shown in colour with legend) threaded by magnetic field lines (green lines). The final disc has become much higher density near the BH and the rotating BH, and disc have launched a magnetically dominated jet due to the accumulation of magnetic flux into the MAD state. The BH spin-driven jet emerges and has a boundary that can be seen where $b^2/\rho = 1$ (red line). At large radii at late times, the jet has become kinetically dominated with Lorentz factor $\gamma \sim 2$ by $r = 500r_g$, so that $b^2/\rho < 1$ there.
Table 2. Accretion rates, luminosities, efficiencies (per cent) and magnetic fluxes.

| Model | $\dot{M}/\dot{M}_{\text{Edd}}$ | $L_{\text{rad}}/L_{\text{Edd}}$ | $\eta_H$ | $\eta_{\text{j, in}}$ | $\eta_{\text{RAD}}$ | $\eta_{\text{NT}}$ | $\Upsilon_H$ |
|-------|-------------------------------|---------------------------------|---------|-----------------|-----------------|----------------|-----------|
| M1    | 140                          | 67                              | 54.7    | 5.68            | 12.2            | 9.8           |           |
| M2    | 140                          | 110                             | 58.2    | 9.76            | 12.2            | 9             |           |
| M3    | 80                           | 81                              | 114     | 84.6            | 12.2            | 13            |           |
| M5    | 120                          | 81                              | 59.2    | 8.39            | 12.2            | 9.5           |           |
| M6    | 5.9                          | 6                               | 9.83    | 5.86            | 5.72            | 3.9           |           |
| M7    | 4.8                          | 2.4                             | 7.98    | 2.84            | 5.72            | 3.9           |           |
| M8    | 5                            | 1.9                             | 8.39    | 2.21            | 5.72            | 5             |           |
| M9    | 50                           | 240                             | 124     | 59.4            | 12.2            | 14            |           |
| M10   | 27                           | 160                             | 267     | 71.8            | 12.2            | 20            |           |
| M11   | 36                           | 290                             | 185     | 95.5            | 12.2            | 18            |           |
| M13   | 1.2                          | 1.1                             | 18.5    | 8.2             | 12.2            | 3.6           |           |
| M14   | 3.5                          | 2.8                             | 23.7    | 9.79            | 12.2            | 5.1           |           |
| M14h  | 2.4                          | 2.5                             | 21.8    | 11.3            | 12.2            | 4.7           |           |
| M15   | 14                           | 18                              | 32.3    | 14.2            | 12.2            | 6.7           |           |
| M15h  | 31                           | 11                              | 30.9    | 17.7            | 12.2            | 7.1           |           |

The normalized magnetic flux on the horizon $\Upsilon_H \sim 10$ for relatively thick discs at higher super-Eddington rates, while lower $\dot{M}$ lead to down to $\Upsilon_H \sim 4$ as seen in thin MAD simulations (Avara, McKinney & Reynolds 2016). A few models (M9, M10 and M11) show up to $\Upsilon_H \sim 20$ and have quite high efficiencies apparently due to radiative suppression of the magnetic Rayleigh–Taylor modes due to opacity effects at their intermediate $\dot{M}/\dot{M}_{\text{Edd}} \sim 30–50$, but one suspects that higher resolutions would show no such effect (which M15h approaches and does show more moderate $\Upsilon_H$ and efficiencies).

Fig. 2 shows a snapshot from the simulation and shows various fluxes and efficiencies versus time, whose constancy indicates that the flow has reached a quasi-steady state in which the total efficiency is $\eta_H \sim 30$ per cent, almost three times the NT thin disc efficiency, and the radiative efficiency is $\eta_{\text{rad, o}} \sim 5$ per cent. The disc is in a MAD state out to about $r \sim 50r_g$ with evident magnetic Rayleigh–Taylor instabilities in the $y$–$x$ plane. The dimensionless magnetic flux $\Upsilon_H \approx 7$, comparable with non-radiative discs with $H/R \approx 0.3$ (McKinney et al. 2012). The effective photosphere reaches close to the disc, except where the jet has relatively high densities.
Table 3. Radiative properties and radiation/jet opening angles in radians.

| Model | $\frac{L_r}{T_{\text{iso}}}$ | $T_{\gamma r, \theta=100r_g}$ (K) | $T_{\gamma r, \theta=100r_g, \text{rad beam}}$ (K) | $\gamma r, \theta=100r_g, \text{rad beam}$ | $\gamma r, \theta=100r_g, \text{disc}$ | $\gamma r, \theta=100r_g, \text{disc}$ | $\gamma r, \theta=100r_g, \text{beaming}$ | $\theta_r=1000r_g$ | $\theta_r=1000r_g$ |
|-------|-----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|-----------------|-----------------|
| M1    | 0.96            | 4.2e7                           | 1.7e7                           | 1                               | 1.4                            | 1                               | 1                               | 0.12            | 0.067           |
| M2    | 0.93            | 1.3e8                           | 7.7e8                           | 3.9                             | 55                             | 1                               | 1                               | 0.089           | 0.0038          |
| M3    | 1               | 1.1e8                           | 6e8                             | 3.2                             | 42                             | 0.88                            | 0.36                            | 0.084           | 0.021           |
| M5    | 0.97            | 3.8e7                           | 9.1e6                           | 1                               | 0.77                           | 1                               | 1                               | 0.092           | 0.025           |
| M6    | 9.8e-5          | 1.5e7                           | 3.3e6                           | 1                               | 0.58                           | 1                               | 1                               | 0.6             | 0.12            |
| M7    | 0.87            | 1.4e8                           | 2.5e7                           | 8.9                             | 5.4                            | 1                               | 1                               | 0.6             | 0.19            |
| M8    | 0.63            | 1.9e7                           | 6.7e6                           | 1.1                             | 1.6                            | 1                               | 1                               | 0.6             | 0.15            |
| M9    | 0.98            | 4.1e7                           | 1.5e7                           | 1                               | 1.1                            | 0.97                            | 0.96                            | 0.13            | 0.16            |
| M10   | 0.99            | 4.9e7                           | 2.1e7                           | 1.3                             | 1.6                            | 0.87                            | 0.69                            | 0.13            | 0.041           |
| M11   | 0.99            | 4.3e7                           | 1.5e7                           | 1                               | 1                              | 0.99                            | 0.97                            | 0.098           | 0.21            |
| M13   | 0.12            | 3e7                             | 9.2e6                           | 4.5                             | 2.6                            | 0.43                            | 0.45                            | 0.28            | 0.0091          |
| M14   | 0.23            | 1.5e7                           | 2.1e6                           | 1.3                             | 0.48                           | 0.79                            | 0.94                            | 0.26            | 0.011           |
| M14h  | 0.2             | 1.3e7                           | 2e6                             | 1.2                             | 0.45                           | 0.76                            | 0.99                            | 0.47            | 0.055           |
| M15   | 0.85            | 2.4e7                           | 7.3e6                           | 1.1                             | 1.2                            | 0.86                            | 0.76                            | 0.16            | 0.028           |
| M15h  | 0.97            | 2.5e7                           | 7.2e6                           | 1                               | 1.1                            | 0.99                            | 0.92                            | 0.37            | 0.090           |

Table 3 shows results for our radiative diagnostics, which can be used to deduce how the radiative properties are affected by different opacity choices and different $M$. This table includes the fluid-frame radiation temperature per unit gas temperature at $r = 10r_g$ in the disc, fluid-frame radiation temperature in the disc, lab-frame radiation temperature in the radiation beam at $r = 100r_g$, fluid-frame hardening factor $f_{\text{col}}$ in the disc, lab-frame hardening factor $f_{\text{col}}$ in the radiation beam at $r = 100r_g$, chemical potential factors in the disc and radiation beam, radiation beam half-opening angle ($\theta_r$) at $r = 1000r_g$, and jet (kinetic+enthalpy+electromagnetic) half-opening angle ($\theta_j$) at $r = 1000r_g$. Disc quantities are computed as weighted by volume of the grid cell times square of density, while radiation or electromagnetic beam quantities are measured at the peak in the luminosity per unit angle ($\partial_\delta L(\theta)$). This does not give a representation of what an observer sees (as would come from the luminosity per unit solid angle), and instead shows what opening angles have the most efficiency associated with them. This can be useful when considering how an ambient medium would be affected by the jet or radiation and lead to secondary emission due to that interaction, like in TDE afterglow or ULX ionization cones.

The low $M \sim M_{\text{Edd}}$ models have higher gas temperatures in the disc, but gas temperatures are at most about 10 times the radiation temperatures with TC. Only model M6 without TC shows very low $T_{\gamma r} / T_{\text{gas}}$. The radiation beam and disc have hardening with a Wien spectrum in models without DC or in models with $M \sim M_{\text{Edd}}$.

The photon distributions tend to be somewhat Wien in the coronae for models with chemical potential evolution. Models without DC and synchrotron (like M2, M3 and M7) show significant photon hardening, which becomes much more limited when including these opacities. Radiation beam lab-frame temperatures are comparable to the disc core, except for models without DC and synchrotron. Models without DC and synchrotron (e.g. M7) have much higher radiation temperatures than otherwise identical models (e.g. M8) with DC and synchrotron. This shows that DC and synchrotron are crucial to include in order to obtain accurate observer-frame radiation temperatures for flows with $M \gtrsim M_{\text{Edd}}$.

The half-opening angles in radians identify the maximum in $L_{\text{iso}}$ within the radiation beam or gas jet. We also computed (not in table) the beaming factor ($b = L_{\text{iso}} / L$, i.e. isotropic equivalent luminosity per unit total luminosity) measured at $r = 1000r_g$. The electromagnetic jet is beamed by factors up to $b = 15$ for rotating BH models and up to $b = 6$ for non-rotating BH models.

3.6 Magnetic and radiative fluxes

Fig. 3 shows the magnetic flux lines (with electromagnetic efficiency) and lab-frame radiation flux stream lines (with radiative efficiency). The radiation is broadly distributed, but has an enhanced beamed region whose opening angle is about twice larger than the electromagnetic jet’s opening angle. Models with zero BH spin (not plotted) show the peak EM luminosity per unit angle emerging from cylindrical radius near the ISCO (Tchekhovskoy, McKinney & Narayan 2012), instead of the rotating BH models where the peak power per unit angle emerges from near the equatorial region of the BH horizon (McKinney & Gammie 2004; McKinney 2006; McKinney & Blandford 2009).

3.7 Effective energy photospheres

Fig. 4 shows the effective photospheres for model M15. At large radii, the total effective photosphere sits above the disc and disc wind. Sitting inside the total effective photosphere is the free–free photosphere, the DC photosphere, and the synchrotron photosphere. The free–free, DC and synchrotron opacities all merge within some radius, showing that they become comparably important. While free–free and DC are clearly important in metal-free plasmas, bound-free and bound–bound contributions are important with solar abundances, which leads to an effective photosphere far beyond the free–free photosphere. For higher $M$ models, these different opacities become more comparable at larger radii than in this lower $M$ model. The scattering photosphere is at $r \sim 800r_g$ in this model.

3.8 Gas overheated regions

Fig. 5 shows the lab-frame radiation temperature and fluid-frame gas temperature for model M15. The radiation temperature reaches up to $T_r \sim 10^9$ K, while the gas temperature reaches up to $T_{\text{gas}} \sim 10^9$ K.
in the jet region. Given our discussion in the Introduction, this suggests that DC should be important throughout the flow, while synchrotron is likely important in the jet region. Note that as $\dot{M}$ drops, the disc becomes thinner, although such MAD type discs are also magnetically compressed by the large-scale poloidal and toroidal fields threading the BH and disc (McKinney et al. 2012).

Fig. 6 shows the fluid-frame radiation temperature per unit gas temperature. TC acts to regulate gas temperatures towards the radiation temperature in radiation-dominated plasmas. We show several model’s poloidal plane temperature ratios in order to present how
Figure 5. Model M15, showing lab-frame radiation temperature in Kelvin (top panel) and fluid-frame gas temperature in Kelvin (bottom panel). Green, blue, yellow and purple lines are, if present, as in Fig. 3. Radiation temperatures are high in the equatorial and polar regions, while gas temperatures are high in the jet region due to insufficient Comptonization.

Figure 6. Model M1 at high $M \sim 100M_{\text{Edd}}$ (top panel), model M15 at lower $M \sim 10M_{\text{Edd}}$ (middle panel), model M14 at low $M \sim 3M_{\text{Edd}}$ (next panel) and model M13 at low $M \sim 1M_{\text{Edd}}$ (bottom panel), showing fluid-frame $\hat{T}_γ/T_{\text{gas}}$. Black line has $\hat{T}_γ/T_{\text{gas}} = 1$. Green, blue, yellow and purple lines are, if present, as in Fig. 3. Models M13 and M14 have a photosphere near the disc, except the jet that is launched, which keeps the density high at large radii. At high or low $M$, the value of $\hat{T}_γ/T_{\text{gas}}$ is order unity due to TC. The disc is evidently thinner and cooler at lower mass accretion rates of $M \sim 10M_{\text{Edd}}$ even though the inflow is still quite super-Eddington. Only for the lowest $M \sim M_{\text{Edd}}$ model M13, does $\hat{T}_γ/T_{\text{gas}} \sim 0.1$ in the central disc, but progressively more coronal material has higher gas temperatures as $M$ drops.

This effect works at high $\dot{M} \sim 100\dot{M}_{\text{Edd}}$ to low $\dot{M} \sim 1\dot{M}_{\text{Edd}}$. Note that model M14 has a long-lived hemispherical asymmetry, leading to scattering photosphere far away on the upper hemisphere. The figure shows fluid-frame $\hat{T}_γ/T_{\text{gas}}$ (numerator and denominator separately time-$\phi$-averaged). While lower $M$ models have a slightly overheated gas region within some radius, the core disc region has at most 10 per cent higher gas temperatures than radiation temperatures for $M \sim 10\dot{M}_{\text{Edd}}$ models.

Model M13 with $M \approx 1\dot{M}_{\text{Edd}}$ shows an optically thin corona with gas temperatures about 100 times larger than the disc’s blackbody temperature and about 20 times larger than the disc’s radiation temperature that is hardened by $\hat{f}_{\text{col}} \approx 4.5$ (see related data from Fig. 9). The gas pressure is up to a tenth of the radiation pressure and the disc thickness $H/R \sim 0.1$ in this model.

Table 3 includes a sequence of $a/M = 0$ models M6, M7 and M8 that have no TC and no photon hardening (M6), have TC and photon hardening but without DC or synchrotron (M7), and have both along with all our opacities (M8). This shows that the lack of TC leads to unphysically high gas temperatures similar to those...
3.9 Photon hardening

Fig. 7 shows the lab-frame photon hardening factor \( f_{\text{col}} = T_\gamma / T_{\text{BB}} \) for the \( a/M = 0 \) models M6, M7 and M8 that we discussed above. For an observer in the lab-frame, this would correspond to the colour correction factor assuming the observers fit spectra with Planck or BE distributions. We find that DC and synchrotron play a crucial role in limiting photon hardening. These opacities generate much more radiation at high temperatures, unlike free-free, and result in the primary radiation beam changing from \( f_{\text{col}} = 5 \) down to \( f_{\text{col}} = 1.5 \). Our model M7 is similar to the model in Sadowski & Narayan (2015a), who find up to \( f_{\text{col}} = 7 \) for comparable, but slightly higher, accretion rates. This shows that photon hardening in Eddington to super-Eddington flows must account for these additional opacities.

Table 3 includes two models, M10 and M11, where M10 has no synchrotron while M11 has all our opacities. These models have comparable \( \dot{M} \) and fluxes as shown in Table 2. The goal here is to see if DC, by itself, regulates photon hardening and to what extent synchrotron goes beyond DC. In model M10, the radiation beam is only moderately hardened due to the sensitivity of DC to radiation temperature, and synchrotron in M11 provides soft photons that lead to essentially no hardening of the radiation beam. This means DC, by itself, regulates photon hardening away from large values seen in other simulations with no DC (e.g. M7).

Table 3 includes two models, M3 and M5, where M3 has a BE chemical potential, while M5 has a Planck chemical potential. The goal here is to see if chemical potential evolution has an effect in the case where neither DC nor synchrotron is included. These models are dynamically quite similar as shown in Table 2. Table 3 shows significantly more photon hardening in the disc and radiation beam when using the BE chemical potential. This shows that the underlying assumption about the photon distribution has a strong effect on the photon hardening.

Fig. 8 shows how Wien the spectrum is, with the primary radiation beam in model M15 being down from Planck \((\exp(-\xi) = 1)\) to \(\exp(-\xi) \approx 0.85\). This distribution with radius and angle is typical for models with DC and synchrotron.

3.10 Radial and angular dependencies for all models

Here we consider how radiative quantities are affected by photon conservation, opacity choices, etc. Quantities like rest-mass density, internal energy density, radiation–energy density, velocity, magnetic...
Figure 9. All models, showing fluid-frame density ($\rho$ in g cm$^{-3}$, top panel), fluid-frame gas temperature ($T_{\text{gas}}$ second panel) and ratio of fluid-frame radiation to gas temperatures ($\hat{T}_r/T_{\text{gas}}$, last panel) measured at $r = 10r_g$. Legend shows model types, where models are grouped by colour for similar $\dot{M}$, $a/M$, or resolution changes for otherwise same initial density. M1–M5 are high $\dot{M} \sim 100\dot{M}_{\text{Edd}}$ models, M6–M8 are $a/M = 0$ models, M9–M11 are intermediate $\dot{M} \sim 50\dot{M}_{\text{Edd}}$ models, M13 is our lowest $\dot{M} \approx \dot{M}_{\text{Edd}}$ model, M14–M14h are our slightly higher $\dot{M} \approx 3\dot{M}_{\text{Edd}}$ models at different resolutions and M15–M15h are intended to be $\dot{M} \approx 10\dot{M}_{\text{Edd}}$ models (but M15h ended up with much higher $\dot{M}$). Lines are drawn in order from M1 to M15h, so that latter lines may overlap earlier lines. High $\dot{M}$ models have a broader density profile compared to the more sharply peaked low $\dot{M}$ models. Models with high $\dot{M} \sim 100\dot{M}_{\text{Edd}}$ without DC and synchrotron (M2,M3) or models without TC (M6) have unphysically hot coronae and jets by three orders of magnitude in temperature, showing that these processes are crucial to include in order to obtain accurate thermodynamical and radiative properties in flows with $\dot{M} \gtrsim \dot{M}_{\text{Edd}}$.

Figure 10. All models, showing fluid-frame $\hat{T}_r/T_{\text{gas}}$ (top panel) and fluid-frame $\hat{T}_r$ (bottom panel) in the disc. Even for $\dot{M} \sim \dot{M}_{\text{Edd}}$, gas temperatures reach no more than 10 times radiation temperatures at $r \sim 10r_g$ due to efficient TC. Higher $\dot{M}$ tend to have higher radiation temperatures. Model M7 with photon number density evolution, but without DC, has much higher radiation temperatures than model M8 that has DC and is otherwise identical. Model M6 is like M7 but also has no TC, and as a consequence, it has extremely high gas temperatures. These results demonstrate the importance of TC and DC as thermostats that, respectively, lower gas and radiation temperatures in the disc.

field, etc., behave like power laws versus radius within $r \sim 20r_g$ within the inflow equilibrium region. Such radial power laws are typical of MADs (McKinney et al. 2012), so we do not discuss the radial behaviour here.

Fig. 9 shows rest-mass density, gas temperature and radiation to gas temperature ratio versus $\theta$. The gas temperatures are artificially high in models without TC or in models with photon number evolution that do not include DC and synchrotron. For example, models M2 and M10 do not include DC and synchrotron, while comparable models M1 and M9 do. Models with DC and synchrotron have much lower gas and radiation temperatures in the corona and jet. So, the thermodynamical and radiative properties of the disc, corona and jet are only accurate with DC and synchrotron.

Fig. 10 shows the fluid-frame $\hat{T}_r/T_{\text{gas}}$ (numerator and denominator separately time-φ-averaged) as weighted by rest-mass density times each grid cell volume size. This focuses the measurement of temperature on the core of the disc at the highest densities. Models at high $\dot{M}$ have $\hat{T}_r \sim T_{\text{gas}}$, and progressively lower $\dot{M}$ down to $\dot{M} \sim \dot{M}_{\text{Edd}}$ have gas temperatures as high as 10 times the radiation temperatures. Resolution should modify the results the most for the lowest $\dot{M}$ models, but the models like M14 and M14h (at twice the $\phi$ resolution of M14) only show up to 30 per cent
relative differences. So, the corona region’s temperatures are roughly converged.

Fig. 10 also shows the fluid-frame $\dot{T}_y$ (weighted like $\dot{T}_y/T_{\text{gas}}$ above). Lower $M \sim M_{\text{Edd}}$ are radiatively cooler than higher $M$ – even for super-Eddington rates. Compared to the temperatures in Sadowski & Narayan (2015a), which were for weakly magnetized models at $M \sim 10 M_{\text{Edd}}$, our results are comparable. However, our model M7, without DC and synchrotron, demonstrates that our more magnetized discs would be much hotter if it were not for DC and synchrotron as included in model M8. So DC and synchrotron are required to regulate both gas and radiation temperatures in magnetized flows in general. Model M6, without TC, shows extremely high gas temperatures, similar to those seen in Takahashi et al. (2016), but TC severely limits the difference between gas and radiation temperatures by driving them towards equilibrium.

Fig. 11 shows the fluid-frame $\dot{\gamma}_\text{col} = \dot{T}_y/T_{\text{gas}}$, weighted like $\dot{T}_y/T_{\text{gas}}$. The photon distribution within the disc is essentially Planckian at large radii, but for low $M \sim M_{\text{Edd}}$, the distribution becomes harder at intermediate radii of $r \sim 5 r_g$. Model M7 (without DC and synchrotron, as compared to otherwise identical model M8 with DC and synchrotron) has an excessive unphysical hardening even in the disc due to photon conserving Comptonization driving the photon distribution towards Wien.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure11.pdf}
\caption{All models, showing fluid-frame $\dot{\gamma}_\text{col}$ (top panel) and chemical potential factor exp (− $\xi$) (bottom panel) in the disc. Lower $M$ (e.g. M13) tend to have regions in the disc around $r \sim 5 r_g$ where the disc photon distribution is hardened, while at high $M$, the disc photons are essentially Planckian as long as DC and synchrotron are included. Model M2, M3 and M7 have no DC or synchrotron, leading to an unphysically large photon hardening in the disc. This shows that DC and synchrotron are required in order to accurately track photon hardening.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure12.pdf}
\caption{All models, showing fluid-frame DC opacity divided by free–free opacity in the disc (top panel) and across the disc at $r = 10 r_g$ (bottom panel). Some models, like M2, M3, M5, M7 and M10, have no DC and show excessive DC opacity because it was not present to regulate the radiation temperature to lower values. For $M \sim M_{\text{Edd}}$, DC dominates free–free inside the disc within $r < 3 r_g$, for $M \sim 10 M_{\text{Edd}}$, DC dominates free–free inside the disc within $r < 8 r_g$, and for $M \sim 100 M_{\text{Edd}}$, DC dominates free–free inside the disc within $r < 15 r_g$ (and potentially further if simulation duration was extended, as inflow equilibrium is only out to $r \sim 20 r_g$). Comparing non-rotating BH models M7 (no DC) and M8 (with DC), DC acts as a thermostat that keeps the plasma in a state where free–free and DC are comparable in the corona. Since significant radiation emerges from within $r = 10 r_g$, and the temperature of the corona affects the radiation that reaches large radii, DC is an important process to include for highly magnetized super-Eddington accretion flows.}
\end{figure}

Fig. 11 also shows the fluid-frame chemical potential factor exp (− $\xi$) given by equation (A6), weighted like $\dot{T}_y/T_{\text{gas}}$. The photon distribution within the disc is essentially Planckian at large radii, but for low $M \sim M_{\text{Edd}}$ (e.g. model M13), the distribution becomes more Wien as densities become small enough such that there is an inefficient photon production.

Fig. 12 shows the ratio of DC to free–free opacities (in units of $cm^{-1}$, which includes a density scale) with no additional weighting, as averaged over a density scaleheight for each simulation at each radius. Higher $M$ lead to progressively more importance of DC relative to free–free within the equatorial disc region. For $M \sim 10 M_{\text{Edd}}$, DC dominates free–free within $r < 8 r_g$, within which significant radiation is generated.

Table 3 includes models M1 and M2 that primarily differ, as M1 includes DC and synchrotron while M2 does not. DC and synchrotron provide plentiful soft photons that lead to a much lower photon hardening than seen in M2, so this shows that DC and
synchrotron are crucial to include in highly magnetized MAD-type super-Eddington flows with \( M \sim 100M_{\text{Edd}} \).

Fig. 13 shows the synchrotron opacity divided by the free–free opacity in the disc as well as \( b^2/\rho \) (each \( b^2 \) and \( \rho \) weighted with density, as done for \( T_g/T_{\text{cor}} \), above) in the disc. At small radii, where magnetic energy density approaches rest-mass energy density, synchrotron dominates free–free in the disc for rapidly rotating BH models. This is due to the jet and its interaction with the disc in a MAD state, where at \( r = 10r_g \), the orthonormal \( B^\phi \) is 10 times higher at \( a/M = 0.8 \) than at \( a/M = 0 \) outside the disc.

Fig. 14 shows the synchrotron energy opacity across the disc as well as \( b^2/\rho \) (with \( \rho \) evaluated at the equator) across the disc at \( r = 10r_g \). Synchrotron is not important as an energy opacity in the disc at such radii, but, in the jet, is the dominant process due to the large magnetic energy density.

By contrast, the synchrotron number opacities in the disc, corona and jet dominate the free–free (and even OPAL) opacities out to \( r \sim 20r_g \) for \( M \gtrsim 100M_{\text{Edd}} \), out to \( r \sim 5r_g \) for \( M \gtrsim 3M_{\text{Edd}} \) and only in the jet for nearly sub-Eddington accretion rates. This is due to the relatively large \( \phi \gtrsim 10^3 \) (equation D5), while also having relatively stronger magnetic fields at higher accretion rates.

![Figure 13](image)

**Figure 13.** All models, showing fluid-frame synchrotron opacity divided by free–free opacity in the disc (top panel) and \( b^2/\rho \) in the disc (bottom panel). Synchrotron dominates free–free in the disc at small radii as temperatures rise to \( T \sim 10^9 \) K and \( b^2/\rho \sim 1 \). Low \( M \sim M_{\text{Edd}} \) models have only comparable synchrotron and free–free quite close to the horizon, but higher \( M \) have progressively important synchrotron that dominates free–free to larger radii – out to \( r \sim 5r_g \) for \( M \sim 100M_{\text{Edd}} \). Model M6 has no TC or synchrotron, so gas temperatures and synchrotron emission opacities are unphysically high. For models with TC, synchrotron is not important in the central disc except for quite high \( M \).

![Figure 14](image)

**Figure 14.** All models, showing fluid-frame synchrotron opacity divided by free–free opacity as well as \( b^2/\rho \) (using density at equator for \( b^2/\rho \) across the disc at \( r = 10r_g \). Synchrotron becomes as important as free–free in the corona in models with \( M \sim 100M_{\text{Edd}} \) and becomes much more important than free–free in the jet for most models. So synchrotron is crucial to include for the corona and jet thermodynamics in highly magnetized MAD models with \( M \gtrsim M_{\text{Edd}} \).

Model M6 without TC shows an artificial dominance of synchrotron due to the artificially high gas temperatures, which are suppressed by TC.

### 4 SUMMARY

We have incorporated several new opacity effects within HARMRAD\(^1\) as applicable to black hole (BH) accretion flows accreting at Eddington to super-Eddington rates. We investigated a range of accretion rates from \( M \sim M_{\text{Edd}} \) up to \( M \sim 100M_{\text{Edd}} \) in order to consider how thermal Comptonization (TC), double Compton, synchrotron and other opacity effects control the thermodynamic and radiative properties of highly magnetized arrested accretion disc (MAD) type accretion flows.

We found that DC dominates free–free in the central core of the disc even out to fairly large radii of \( r \sim 15r_g \) (and potentially much further if we considered longer duration simulations) in highly super-Eddington accretion flows with \( M \sim 100M_{\text{Edd}} \). We also found for these MAD models that synchrotron dominates free–free within \( r \sim 5r_g \) for highly super-Eddington accretion flows, and synchrotron provides plentiful soft photons to regulate radiation temperatures.

\(^1\) The HARMRAD code and associated analysis and visualization codes have been made open-source and are publicly available at the github repository website: https://github.com/pseudotensor/
throughout the flow for $M \gtrsim 10M_{\text{Edd}}$. Progressively lower accretion rates are dominated by DC and synchrotron within progressively smaller radii until, for sub-Eddington accretion, these processes do not dominate in the central core of the disc.

For our MAD models, we found that the coronal gas and radiation temperatures are regulated by DC and synchrotron, while jet temperatures are regulated by synchrotron. Of our more realistic models with all our opacity physics, our model with $M \approx M_{\text{Edd}}$ shows the highest disc gas temperatures compared to higher accretion rate models. The surrounding coronal gas has temperatures of $T_{\text{c}} \approx 7 \times 10^9 \, \text{K}$ corresponding to 60 keV, which is $\approx 100$ times higher than the disc’s blackbody temperature and $\approx 20$ times higher than the disc’s radiation temperature (hardened by $f_{\text{rad}} \approx 4.5$). This compares to other recent local shearing box simulations that studied coronae and saw only up to $T \sim 10^9 \, \text{K}$ in the corona and a factor of 10 cooler disc blackbody temperatures (Jiang et al. 2014a). The polar jet region near the BH has $T_{\text{j}} \sim 4 \times 10^8 \, \text{K}$ corresponding to 350 keV, about 10 times higher than models with $M \approx 30M_{\text{Edd}}$, which should also contribute to Comptonized emission (O’Riordan, Pe’er & McKinney 2016a,b). The gas heating in the corona is plausibly driven by the strong highly dynamic magnetic fields threading the BH and MAD-type disc, but more detailed analysis of this state will be presented in future work.

TC with DC and synchrotron were crucial to include, otherwise gas temperatures were excessively high by factors of 10 (with TC, but without DC or synchrotron) and up to factors of a thousand (when no TC is included) as seen in Takahashi et al. (2016). However, their suggestion that hot gas regions could lead to the very high state (Kubota & Done 2004) might still apply to our model with $M \approx M_{\text{Edd}}$, which has disc gas temperatures 10 times higher than disc radiation temperatures.

We also found that for rotating BH models, the isotropic equivalent luminosity is enhanced compared to the total luminosity by a beaming factor of up to $b = 15$ for the jet and up to $b = 15$ for the radiation. Non-rotating models show only a moderate beaming of the jet by $b = 6$ and radiation by factor $b \approx 3$, which agrees with Sadowski & Narayan (2015b). The rotating BH models with significant beaming of radiation could help explain ULXs.

None of our models show signs of thermal instability, as consistent with our prior studies of super-Eddington flows (McKinney et al. 2014; McKinney et al. 2015). The stability of radiation-dominated discs is uncertain, but affected by numerical method details, opacities used, the degree of magnetization and how these lead to enhanced advection (Hirose, Krolik & Blaes 2009; Jiang, Stone & Davis 2013; Jiang et al. 2016; Mishra et al. 2016; Sadowski 2016). Free–free does not limit the thermal instability for increasing temperatures due to its opacity dropping, but processes like DC increase with temperature and so might ultimately limit such runaways.

The primary limitation of our study is the use of the $M_1$ closure approximation because rays cannot intersect in the optically thin regime. This limits applications to cases where most of the radiation is in the scattering-dominated regime (true for all of our models in the disc, and true for many of our models out to large radii) or where radiation emerges primarily from a single region (as common in accretion discs, for which radiation comes from near the BH; Tsang & Milosavljević 2015). Multifrequency transport could also be important (Roberts et al. 2016).

Our study is most relevant to observations related to TDEs (Dai, McKinney & Miller 2015), super-Eddington BH X-ray binary systems like GRS1915+105 and SS433 (Kulkarni et al. 2011), ULXs and numerous quasars that accrete near or above Eddington. Our results show that radiation beaming and photon hardening effects are strong, and that Eddington to super-Eddington models require DC and synchrotron in order to obtain accurate temperatures in the corona and jet. DC may be an important thermostat (which enforces slightly lower temperatures than pair creation seen in Fabian et al. 2015) in quasars and other radiatively efficient supermassive BHs. Either improvements to our radiative transfer scheme or multifrequency radiative transfer post-processing (Narayan et al. 2016) could allow these simulations to be compared directly with spectra and timing from these systems (Castelli-Mor, Netzer & Kaspi 2016).

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\[ \gamma(A4b) \]

respective. This corresponds to two equations and two unknowns for \( T_e \) and \( \xi \) (or equivalently \( e^{-\xi} \)).

Solving these equations, one obtains \( e^{-\xi} = n_i^0 I_3/(CE E) \) and \( k_b T_y = E/n(E) \). Noting that the behaviour of \( T_e \) and \( e^{-\xi} \) is like exponentials, and searching for a simple fitting function to avoid an inversion of a polylogarithmic function, we find a fit for radiation temperature of

\[ k_b T_y = \frac{E/n}{0.3333 + 0.060725/(0.646756 + 0.121982 CE^3/n^2)} \]  
(A5)

with a corresponding equation for the chemical potential (\( \mu \)) written in terms of a dimensionless \( \xi = \mu/(k_b T_y) \) as

\[ e^{-\xi} = \frac{1.64676}{0.646756 + 0.121982 CE^3/n^2}. \]  
(A6)

These fits agree with the full solution to \(<2\) per cent from Planck (\( \mu = 0 \)) to Wien (\( \mu \to \infty \)) limits. To match the BE condensate limit \( \mu = 0 \) when \( T_e \to 0 \), we restrict \( e^{-\xi} < 1 \), while \( T_e \) naturally tends to zero as \( n \to \infty \) for fixed \( E \). Now, given the equations of motion value for \( E \) and \( n \), we can obtain \( T_e \) and \( e^{-\xi} \) that are used to compute any opacity or emission rate.

Note that for a Planck distribution, the integration over the \( BE \) distribution function gives \( B = B_0(\xi/(4\pi)) \) with photon energy density of \( u_0 = aT^3 \) (the radiation constant), and integration over the \( BN \) distribution function gives \( N = n_0(\xi/4\pi) \) with photon number density of \( n_0 = (30\xi^3/(\pi^2))(a/k_b) T^3 \approx (1/2.7)(u_0/(k_b T)) \), where \( \xi \) is the Riemann zeta function.

More generally, a BE distribution can be considered as a rough fit to the frequency position of the peak and the shape of the soft photon portion for the general distribution \( n_\gamma \). This fit would break down when the emission is optically thin and absorption occurs far from the peak in emission.

\section*{APPENDIX B: MEAN EMISSION AND ABSORPTION}

Here we discuss how the mean opacity should be computed for Boltzmann moment methods like the M1 closure method. Here, the emission mean intensity (or mean emission coefficient) is \( j_e = e_c/(4\pi) \) and \( e_c \) is the emissivity (energy loss per unit volume per unit time per unit frequency). The energy loss rate per unit volume is then \( \lambda_c = \int j_e \, dv \).

\section*{B1 Integrated evolution equations}

We obtain the form of Kirchhoff’s law and the correct mean opacity from the Boltzmann equation in the fluid frame in a homogeneous plasma without Compton scattering (not required to obtain our result). The Boltzmann equation for the photon occupation number \( n_\gamma \) with \( x = \nu/(k_b T_y) \) and \( x_0 = \nu/(k_b T_e) \), caused by BR and DC emission without the Kompaneets operator, reads

\[ \partial_t n_\gamma = \frac{\Delta g_\nu(x, T_y, T_e)}{x^3} \left[ 1 - n_\gamma(e^\nu - 1) \right] \]

\[ + \frac{\Delta g_{he}(x, T_y, T_e)}{x^3} \left[ 1 - n_\gamma(e^\nu - 1) \right] , \]  
(B1)

in the fluid frame for a fluid element with proper time \( t \), where \( \Delta g_\nu(x, T_y, T_e)/x^3 = d n_\gamma/d \nu \) of emission, describes the respective emission rates (excluding stimulated emission). The BR process drives the photon occupation number towards a blackbody at temperature \( T_e \), i.e. \( n_\gamma = 1/(e^\nu - 1) \), while DC pushes towards a blackbody at

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APPENDIX A: BOSE–EINSTEIN PHOTON DISTRIBUTION

The fluid-frame quantities \( E \) and \( n \) described in Section 2 provide two parameters that describe the photon distribution function. All quantities in these appendices are in the fluid frame. A reasonable assumption for general optical depths is a BE distribution (Miller et al. 2011), which correctly captures the behaviour of the photon distribution in a scattering-dominated atmosphere and how it goes from Planck to Wien as radiation and gas temperatures equilibrate. A diluted Planck might be reasonable in some cases (Sadowski et al. 2017). We assume that BE holds in the fluid frame, which will be valid in the optically thick regime or when the radiation is fairly isotropic in the fluid frame, and energy exchange with the electrons drives the photon distribution towards kinetic equilibrium. This neglects Doppler effects, because the radiation should be isotropic in its own frame according to the M1 closure.

Let us define \( x = \nu/(k_b T_y) \) and \( \xi = \mu/(k_b T_y) \). Then, the BE distribution function is

\[ n_\gamma = \frac{e^{-\xi}}{e^{-\xi} - 1}. \]  
(A1)

The angle-integrated distribution of photons is given by the number distribution

\[ BN dx = C(k_b T_y)^3 e^{-\xi} \frac{x^2 dx}{e^{-\xi} - 1} = C(k_b T_y)^3 e^{-\xi} dI_n, \]  
(A2)

with \( C = (8\pi)/(h^3 \nu^3) \). The corresponding energy distribution is

\[ BE dx = C(k_b T_y)^4 e^{-\xi} \frac{x dx}{e^{-\xi} - 1} = C(k_b T_y)^4 e^{-\xi} dI_E. \]  
(A3)

Then, the radiation number and energy densities are

\[ n(\xi) = C(k_b T_y)^3 e^{-\xi} I_n(\xi), \]  
(A4a)

\[ E(\xi) = C(k_b T_y)^4 e^{-\xi} I_E(\xi), \]  
(A4b)

\[ (\text{or equivalently } e^{-\xi}). \]
temperature $T_\nu$, i.e. $n = 1/(e^\nu - 1)$ (see Appendix E3). Neither of these two solutions make the Boltzmann collision term vanish for $T_\nu \neq T_e$ and the general equilibrium solution with respect to the BR and DC processes is given by

$$n_\nu = \left( \frac{\Lambda_{BR\nu} + \Lambda_{DC\nu} + 1}{\Lambda_{BR\nu} + \Lambda_{DC\nu} - 1} \right)^{-1}. \quad (B2)$$

As expected, for $T_\nu = T_e$, this reduces to $n_\nu = 1/(e^\nu - 1)$. So assuming a BE-type distribution is only accurate in the equilibrium limit when a single process dominates at some point in time and space or if $T_\nu \sim T_e$, the latter being true in equilibrium because we include TC.

From equation (B1), we wish to obtain evolution equations for the moments $\Lambda_j = \int x^j n_\nu \, dx$ and $\rho_j = \int x^j n_\nu \, dx$ of the photon distribution. We furthermore want to express the equations in a form $\partial_t \Lambda_j = \Lambda_{j+1} - \Lambda_{j-1}$, where the coefficients $\Lambda_j$ and $\rho_j$ are functions of the two temperatures and possible momentum conservation, which would introduce a radiation flux (and so radiation frame velocity in the M1 closure) as a weight. We do not try to model the angular distribution and frequency dependence on the M1 radiation velocity, and, in general, the M1 flux mean and DC processes is given by

$$\Lambda_j = \int_{\nu_\nu}^{\nu_c} \Lambda_{BR\nu} + \Lambda_{DC\nu} \, d\nu \quad \text{and} \quad \rho_j = \int_{\nu_\nu}^{\nu_c} \rho_{BR\nu} + \rho_{DC\nu} \, d\nu,$$

so that one has the emission and absorption coefficients

$$\Lambda_N = \int_{\nu_\nu}^{\nu_c} \frac{\Lambda_{BR\nu} + \Lambda_{DC\nu}}{\nu} \, d\nu \quad \text{(4a)}$$

and

$$\kappa_N = \frac{\nu}{N_\nu(\nu_c, \nu)} \left( \int_{\nu_\nu}^{\nu_c} \frac{\Lambda_{BR\nu} + \Lambda_{DC\nu}}{\nu (\nu^2 + 1)} \, d\nu \right). \quad (4b)$$

Similarly, for $\rho_N$, we find

$$\Lambda_{\rho} = \int_{\nu_\nu}^{\nu_c} \Lambda_{BR\nu} + \Lambda_{DC\nu} \, d\nu \quad \text{(5a)}$$

and

$$\kappa_{\rho} = \frac{1}{\rho_N(\nu_c, \nu)} \left( \int_{\nu_\nu}^{\nu_c} \frac{\Lambda_{BR\nu} + \Lambda_{DC\nu}}{\nu (\nu^2 + 1)} \, d\nu \right). \quad (5b)$$

One can also obtain the moment equation associated with momentum conservation, which would introduce a radiation flux (and so radiation frame velocity in the M1 closure) as a weight. We do not try to model the angular distribution and frequency dependence on the M1 radiation velocity, and, in general, the M1 flux mean and DC processes behaves in a more complicated way as a function of optical depth (Struchtrup 1997; Christen & Kassubek 2014).

These equations shows how one must construct $\alpha_{\nu}$ via the correct Kirchhoff’s law (i.e. which temperature enters) and the related mean opacity (i.e. how to weight the emission rate), where $\lambda = C(k_\nu T_\nu)^2 \Lambda_{\nu}$, $\lambda_{\nu} = C(k_\nu T_\nu)^2 \Lambda_N$, $\kappa_\nu = \kappa_{\nu}/c$, and $\kappa_{\mu} = \kappa_{\mu}/c$.

### B2 Kirchhoff’s Law

The opacity for each process, $\alpha_{\nu}$ (in cm$^{-1}$, where $\kappa$, in the same units, is reserved for the mean opacity), for a thermal electron distribution, is given by Kirchhoff’s law:

$$\alpha_{\nu} = \frac{j_{\nu}}{B_{\nu}(T_{eq}, \mu_{eq})}, \quad (B6)$$

where, as mentioned in the last section, $T_{eq} = T_e$ for a process like BR and $T_{eq} = T_\nu$ for a process like DC, where $\mu_{eq} = 0$.

### B3 Mean opacities

We use an approximation to the absorption mean, flux mean and emission mean that enter the four-force. For a more general discussion, see section 82 of Mihalas & Mihalas (1984), section 2.3 of Huebner & Barfield (2014), page 38 in Sturrock et al. (1986), page 174 in Castor (2004), or section 15.2 in Modest (2013).

The Planck or emission mean is

$$\kappa_{\nu} = \frac{\int_0^\nu \, d\nu \, \nu \, j_{\nu}}{\int_0^\nu \, d\nu \, B_{\nu}(T_{eq}, \mu_{eq})} = \frac{\int_0^\nu \, d\nu \, j_{\nu}}{\int_0^\nu \, d\nu \, B_{\nu}(T_{eq}, \mu_{eq})} = j/B = e/(4\pi B), \quad (B7)$$

for optically thin emission rate $e$. The Planck mean would be what is required if the radiation and gas were the same temperature and one wanted to ensure that the optically thin emission rate was correct if using our four-force in the comoving frame with an energy loss rate of

$$\frac{du_e}{dt} = \kappa_{\nu}(e - 4\pi B), \quad (B8)$$

such that in the limit that $E \to 0$ as $\tau = \kappa_{\nu}L \to 0$ for some length $L$ of the region, then the gas would lose energy at a rate of $4\pi\kappa_{\nu}B = e \equiv \kappa_{\nu}$ as required.

In the diffusion limit, the Rosseland mean opacity $\kappa_R$ with

$$\kappa_R^{-1} = \frac{\int_0^\nu \, d\nu \, \nu \, j_{\nu} \, dB_{\nu}/dT}{\int_0^\nu \, d\nu \, dB_{\nu}/dT}, \quad (B9)$$

is a good approximation to the flux mean (Davis et al. 2014). The Rosseland mean should not be generally used in the energy equation as it would fail to give the correct emission rate in the optically thin limit. Another approach is to use the Planck or absorption mean for the energy equation while using the Rosseland mean to approximate the flux mean for the momentum equation (Sadowski et al. 2017), but this only gives a correct momentum exchange in the diffusive limit for each grid cell.

Section B1 shows that the opacity that enters the energy and number equations is the absorption mean given by

$$\kappa_{\nu} = \frac{\int_0^\nu \, d\nu \, \nu \, J_{\nu}}{\int_0^\nu \, d\nu \, J_{\nu}} \quad (B10)$$

for a mean intensity $J_{\nu} = \int_0^\nu I_{\nu} \, d\Omega/(4\pi)$ of radiation. Nominally, one might not know or it may be hard to model $J_{\nu}$, but we model the distribution function as BE. The absorption mean has also been called the two-temperature Planck mean, and it has been used to handle irradiated planetary atmospheres (Hube et al. 2003; Heng et al. 2012) and circumstellar atmospheres (Malygin et al. 2014).

For the momentum equation and its flux mean, the mean absorption opacity should vary from the full Rosseland mean (including every emission process and scattering in a single integration) in the diffusion limit to the above absorption mean in the streaming limit. This could be approximately achieved by interpolation (Sampson 1965; Patch 1967; Ludwig, Jordan & Steffen 1994; Vögel, Bruls & Schüssler 2004), or by splitting the trapped and streaming
radiation (e.g. Rosdahl & Teyssier 2015) with different opacities for each component. For simplicity, we use the same way of computing the absorption mean opacity for the energy, number and momentum equations. This may be reasonable for accretion discs, whose global photospheres are often optically thin across each grid cell where source terms are applied. In the deep optically thick limit within the disc, the diffusion times are often slower than an inflow or outflow time. Then, radiation is trapped in the flow, so that the opacity just needs to be high enough to maintain trapping.

The energy absorption mean-opacity is given by

$$\kappa_a = \frac{\int \, d\nu \alpha_r \nu^a T_{BE}(T_{abs}, \mu_{abs})}{\int \, d\nu \nu^a T_{BE}(T_{abs}, \mu_{abs})}, \quad (B11)$$

for a BE energy distribution $BE$. The corresponding number absorption-mean-opacity is given by

$$\kappa_{an} = \frac{\int \, d\nu \nu \, \alpha_r \nu^a N_v(T_{abs}, \mu_{abs})}{\int \, d\nu \nu^a N_v(T_{abs}, \mu_{abs})}, \quad (B12)$$

for a BE number distribution $BN$. For the distribution $BE$ and $BN$, one must choose $T_{abs} \rightarrow T_e$ and $\mu_{abs} \rightarrow \mu$ for an ambient BE radiation field at temperature $T_e$ and chemical potential $\mu$ as consistent with equation (B1). The lower integration range over frequency is assumed to be set by the Rasin effect for each process.

Note that if one used $\kappa_a$ instead of $\kappa_{an}$ in the number evolution equation, it would not give back a consistent optically thick thermal equilibrium density of photons. From equation (9), one would have obtained $\kappa_{an} = \kappa_a$. However, the denominator of $\kappa_a$ integrates to $B = \mu_{abs} \nu^4 / (4\pi)$, while the numerator of $\kappa_a$ integrates to the energy density loss rate per solid angle $\lambda_{a\nu} / (4\pi)$, which, while dimensionally correct, is not the required Planck answer of $\nu_{0\nu}$. However, by using the number absorption-mean-opacity, one obtains an equilibrium number density from $\kappa_{an} = \kappa_a$, giving $n = \kappa_{an} \nu / (c \kappa_{an}) = n_0$ as required.

### B4 Modifying the BE distribution with inverse Compton and self-absorption

An ambient photon distribution as a BE with a frequency-dependent $\mu_{abs} \neq 0$ is only a reasonable assumption if the absorption mean opacity for the energy, number and momentum equations is larger than a typical flow time-scale $t_{flow}$ and longer than the time-scale for IC to upscatter photons (and so avoid absorption). Otherwise, if absorption is effective, then $\mu_{abs} \rightarrow 0$ (Sunyaev & Zeldovich 1970; Lightman 1981; Burigana & de Zotti 1991; Burigana, de Zotti & Danese 1995). Because DC and BR emission become increasingly effective at low frequencies, this condition is always fulfilled at very low frequencies.

When $\kappa_{an} T_e \lesssim m_e c^2$, the time-scale for IC to photons to higher energies is

$$t_{IC} \sim \frac{m_e c^2}{\kappa_{an} T_e}, \quad (B13)$$

where $\tau_{c} = (n_e c \sigma_T)^{-1}$ is the electron scattering time-scale. (This is not the energy redistribution time between gas and the electrons, which would be given by $t_{c\nu} \sim \nu / \kappa_{an}$ from equation E2.) The time-scale for absorption is given by the absorption term for $dn_\nu / dt$ in Boltzmann’s equation and computing

$$t_{abs} \sim \frac{n_\nu}{[dn_\nu / dt]_{abs}} = \frac{1}{c \alpha_\nu}, \quad (B14)$$

where $\alpha_\nu = \alpha_\nu$. From the condition $\nu_{IC} \geq \nu_{abs}$ one can thus determine those frequencies for which the distribution is Planck because those low-energy photons are absorbed before being upscattered.

So, a significant improvement to the BE distribution assumption is to force $\mu_{abs} \rightarrow 0$ when $\nu_{abs} < \nu_{flow}$ and $\nu_{abs} < \nu_{IC}$ for a typical flow time or simulation time-step $t_{flow} \sim 10^3 (t/\gamma/c)$. Otherwise, IC avoids self-absorption, or insufficient time has elapsed to assume that absorption has had time to force $\mu_{abs} \rightarrow 0$. This is how we modify the BE distribution in this paper. We do not try to model the transition and just assume that it is a discontinuous change in the distribution. This modified BE distribution acts as a sub-time-step model for the photon distribution function. We apply this modification to $\mu$ for each process, which assumes that the dominant process is the only one we need to treat accurately.

#### B5 Absorption balancing emission

The energy emission rate is

$$\lambda_e = 4\pi \int \, d\nu j_\nu = 4\pi \int \, d\nu \alpha_r \nu^4 E_\nu = \kappa_a 4\pi \int \, d\nu \nu^4 E_\nu = \kappa_a 4\pi B, \quad (B15)$$

with the last equation defining the so-called energy emission mean opacity $\kappa_a$. The number emission rate is

$$\lambda_n = 4\pi \int \, d\nu (j_\nu / \nu^4) = 4\pi \int \, d\nu \alpha_r \nu^a N_v \equiv \kappa_{an} 4\pi n, \quad (B16)$$

where $\kappa_a$ and $\kappa_{an}$ are from the generalization of absorption mean opacity expressions equation (B11) and equation (B12). For these emission processes, in the distribution functions $BE$ and $BN$ that appear in $\kappa_a$ and $\kappa_{an}$, one replaces $T_{abs} \rightarrow T_{eq}$ ($T_{eq} = T_e$ for BR-like processes and $T_{eq} = T_f$ for DC-like processes) and $\mu_{abs} \rightarrow \mu_{eq} = 0$.

This construction of $\kappa_a$ and $\kappa_{an}$ just ensures that we treat the integration limits consistently so that in the optically thick thermal equilibrium limit, the absorption balances emission. It also defines the so-called emission mean opacity that is just a way to write the expressions equation (B11) and equation (B12). For these emission processes, in the distribution functions $BE$ and $BN$ that appear in $\kappa_a$ and $\kappa_{an}$, one replaces $T_{abs} \rightarrow T_{eq}$ ($T_{eq} = T_e$ for BR-like processes and $T_{eq} = T_f$ for DC-like processes) and $\mu_{abs} \rightarrow \mu_{eq} = 0$.

### APPENDIX C: FREE–FREE EMISSION

Following the discussion in Rybicki & Lightman (1986) that follows Novikov & Thorne (1973), the electron–ion free–free emissivity $j_\nu^{ffei}$ is obtained from

$$j_\nu^{ffei} = \frac{8}{3 \pi^2} \frac{m_n \nu^4}{e^2} \int_{\gamma_{\nu 0}}^{\gamma_{\nu 1}} \frac{e^{-\gamma_{\nu 0}}}{\sqrt{\gamma_{\nu 1} - \gamma_{\nu 0}}} \tilde{g}_{\nu 0} \tilde{R}_{ei}(\theta_e), \quad (C1)$$

where $\theta_e = \theta_{eq} (m_e c^2)$, and where we assume no pairs so $(n_{-} + n_{+}) \rightarrow n_e$, and where the relativistic correction factor is

$$R_{ei}(\theta_e) = \begin{cases} 1 + 1.76 e^{-\theta_e^2}, & \theta_e \leq 1 \\ 1.4 e^{0.5 \ln(1.12 \theta_e + 0.48) + 1.5}, & \theta_e \geq 1 \end{cases} \quad (C2)$$

(Svensson 1982; Esin et al. 1996). We assume that temperatures are $T_e \gtrsim 10^5$ K, which is in the small-angle uncertainty principle regime, so that $\gamma_{\nu 0} = (3 k_\nu T_e / (\pi \hbar \nu))^{1/2}$ for $\nu > (k_\nu T_e)^{1/2}$ and $\nu > (k_\nu T_e)^{1/2}$, which we interpolate as $\gamma_{\nu 0} = \gamma_{\nu 0}^{1.2} \exp(-1/x_e) + \gamma_{\nu 0}^{2.3}(1 - \exp(-1/x_e))$. More accurate analytical fitting formulae can be used (Svensson 1984;
Shu 1991; Itoh et al. 2000; Sakamoto et al. 2001), although this level of accuracy is not required.

For abundances with mass fractions of hydrogen, helium and ‘metals’, respectively, $X$, $Y$ and $Z$ in a mostly ionized gas $n_i/\rho = 1/(\mu_m \mu_e) = (1 + X)/(2 m_e)$, while $\sum n_i Z_i = (\mu/m_e) \sum_i (X_i Z_i^2 / A_i) = (\rho/m_e) (X + Y + B)$ for mass fraction of species $i$ of $X$, where $B = \sum_{i\neq 2} (X_i Z_i^2 / A_i)$ with $X + Y + B \approx 1 - Z$ for $B \ll \{X, Y\}$. We often assume solar abundances (mass fractions of hydrogen, helium and ‘metals’, respectively, $X = 0.7$, $Y = 0.28$, $Z = 0.02$) with electron fraction $\zeta = (1 + X)/2$ giving mean molecular weight $\bar{\mu} \approx 1/(2 X + 0.75 Y + 0.5 Z) \approx 0.62$ for ionized gas, which enters the gas entropy, pressure and temperature $T_\theta = T_e$.

For free–free, $\kappa_{\text{ff},i} \approx 1 \times 10^{24} \rho^2 T_e^{-7/2}(1 + X)(1 - Z) R(\theta_i)$, for near-solar abundances for the Planck mean. A Rosseland mean gives $\kappa_{\text{RS},i} \approx 3.8 \times 10^{22} \rho^2 T_e^{-7/2}(1 + X)(1 - Z) R(\theta_i)$, which is about 30 times smaller than the Planck mean. These calculations correct the formulae used in McKinney et al. (2015), where the free–free and free–bound coefficients were for Rosseland, while the functional form was for Planck.

One can then compute the energy and number absorption mean from $\alpha_{\text{en}} = j_{\text{ff},i} / B(T_e)$ from Kirchhoff’s law with a distribution based upon $T = T_e$.

C1 Razin effect for BR

The well-known infrared divergence for free–free leads to a formally infinite number of photons generated. Without any adjustment, the number absorption mean opacity diverges (Ascoli & Bussetti 1956; Yennie, Frautschi & Suura 1961). For a given experiment size, this divergence can be removed by adding in other non-BR processes that cannot be distinguished and that can occur (Akhiezer & Berestetskii 1953; Kaku 1993). The divergence is logarithmic, so any physical effect that would limit the presence of low-frequency radiation would likely lead to a sufficient removal of the divergence to order unity or so.

Several processes can destroy the coherence of the emission process over the formation length, while the electron and photon continue to distinguish themselves quantum mechanically. This includes Comptonization, Razin–type collective plasma effects, finite temperature effects (Weldon 1994), Debye–length limit of the Coulomb potential (Gould 1990), and Landau–Pomeranchuk–Migdal (LPM) effect of multiple scatterings (Akopyan & Tsytovich 1976; Chen & Klein 1993; Shul’Ga & Fomin 1998; Klein 1999, 1997; Fortmann et al. 2003; Fortmann, Roepke & Wiering 2007).

Collective plasma effects at the plasma frequency scale lead to dielectric suppression of the thermal BR rate that is the lowest frequency and strongest cut-off of radiation compared to the LPM or other effects (Scheuer 1960; Dawson & Oberman 1962; Mercier 1964; Bekfki 1966; Melrose 1972; Ichimaru 1973; Weldon 1994; Anthony et al. 1996). This can be accurately represented by multiplicative suppression factor of $S = \alpha_{\text{en}}/\omega^2 = (1 + \alpha^2_{\text{LPM}}/\omega^2)^{-1}$ (Melrose 1972) for electron plasma frequency $\omega_{\text{pe}}$. As a contribution to the mean opacity, a sufficiently accurate suppression factor is given by

$$S = (1 - \alpha^2_{\text{LPM}}/\omega^2)^{1/2} = (1 - x_p / x^2)^{1/2}$$

(Bekfki 1966), for which the frequency integrals then start at $\omega = \omega_{\text{pe}}$, where $x_p = h \nu/(k T_e)$ and $\nu_{\text{pe}} = \omega_{\text{pe}} / (2 \pi)$, such that

$$x_p = 4.3 \times 10^{-7} \sqrt{n_e / T_e} = 3.3 \times 10^3 \sqrt{T_e / \rho / \mu / T_e}.$$  

So $S$ can be treated as a suppression factor that simply depends upon the integral’s lower cut-off, i.e. $x_p$. Note that this suppression is not an absorption process, and instead, the light wave is evanescent below plasma frequencies. This cut-off operates even when the system is not in equilibrium. In principle, this means that we need to tabulate/fit our opacities versus the dimensionless lower frequency cut-off $x_{\text{cut}} = x_p$. Instead, for a given system, because the effect of the cut-off is logarithmic, we use a fixed cut-off for expected densities and temperatures.

C2 Tabulating/fitting the free–free mean opacity

After substituting $v \rightarrow x_b T_e / h$ and using $x, e^{-\theta} \text{ and } \zeta = T_e / T$ as independent variables, and given a dimensionful term set as

$$f(n_e, n_i, T_e) \equiv 1.2 \times 10^{24} T_e^{-7/2} \rho^2 (1 + X)(1 - Z) R(\theta_i),$$

then one can tabulate/fit the residual dimensionless factor $\kappa_{\text{ff},i} / f(n_e, n_i, T_e)$. The mean opacity is quite linear in log–log space, so such a direct look-up table can be generated from small dimensions that covers all dimensionless space of $\zeta = 10^{-10} - 10^{10}$ and $0 \leq e^{-\theta} \leq 1$.

For $\text{HARMREAD}$, we obtain a fitting function

$$\frac{\kappa_{\text{ff},i}}{f(n_e, n_i, T_e)} = a_{\text{ff}} e^{-b_{\text{ff}} \ln(1 + c_{\text{ff}} \zeta)}$$

where, in general, $a(e^{-\theta}), b(e^{-\theta}), c(e^{-\theta})$ are functions fitted for in that separate dimension. Similarly, the residual for the number opacity $\kappa_{\text{en},i} / f(n_e, n_i, T_e)$ can be fitted with the same form of the expression with different constants.

The final fitted constants $a, b, c$ for the energy opacity are

$a = 0.188(e^{-\theta})^{13.9} - 0.2(1 - (e^{-\theta}))^{0.555} + 0.356$

$b = 0.0722(e^{-\theta})^{1.36} + 0.255(1 - (e^{-\theta}))^{0.113} + 3.06$

$c = -1.41(e^{-\theta})^{1.08} - 1.44(1 - (e^{-\theta}))^{0.128} + 5.99,$

and the constants $a_n, b_n, c_n$ for the number opacity are

$a_n = 21.0(e^{-\theta})^{5} - 2.06(1 - (e^{-\theta}))^{1/4} + 4$

$b_n = -0.412(e^{-\theta})^{5.91} + 0.000894(1 - (e^{-\theta}))^{10.2} + 3.15$

$c_n = 5.27(e^{-\theta})^{0.82} + 2.39(1 - (e^{-\theta}))^{0.552}.$

These fits use Razin and modified BE distribution based upon densities and temperatures for an Eddington-accreting BH X-ray binary. This gives a fit accurate for the energy opacity to less than 20 per cent relative error for $10^{-5} < \zeta < 10^{5}$ and $0 < e^{-\theta} < 1$ with careful attention near $e^{-\theta} = 1$, where significant changes occur. The number opacity has a similar error except at $e^{-\theta} = 1$, where the results are accurate to factors of 2 as related to sharp changes right at $e^{-\theta} = 1$ for $\zeta \gtrsim 10^{10}$. The fitting coefficients at only $e^{-\theta} = 1$ are $\{a, b, c\} = \{0.532, 3.14, 4.52\}$ for the energy opacity and $\{a_n, b_n, c_n\} = \{20.0, 2.67, 5.00\}$ for the number opacity. These Planck coefficients can be used for the emission mean opacity when one also enforces $\zeta = 1$.

C3 Electron–electron opacity

The electron–electron opacity is the same as the electron–ion opacity, except $(n_e + n_i)n_i$ is replaced by $(n_e^n + n_i^n) \approx n_e^n$ and $R_{\text{el}}$ is replaced by

$$R_{\text{el}}(\theta_e) = \begin{cases} 1.70 \theta_e(1 + 1.1 \theta_e + \theta_e^2 - 1.06 \theta_e^{2.5}) & \theta_e \leq 1 \\ 1.70^{0.5}(1.46(1.28 + \ln(1.12 \theta_e)) & \theta_e > 1 \end{cases}$$

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κff = κffenc + κfffree, \hfill (C10)
for both energy and number mean opacities.

\section*{C4 Effective opacity including lower temperature opacities}

The bound-free (number and energy) opacity κbf is the same as free–free with \((1 + X)/(1 - Z)\) replaced by \(\sim 750Z(1 + X + 0.75Y)\) (Rybicki & Lightman 1986), where the \(1 + X + 0.75Y\) term is roughly accurate near solar abundances. This assumes mostly ionized hydrogen and helium. One should also drop the high-temperature \(R(\theta_e)\) factor.

Rosseland means are often provided for lower temperature opacities due to the expectation of applications to stellar interiors with temperature transport as the primary study. These opacities also tend to assume \(T_e = T_s\). Direct integration to obtain absorption means is not easy because the required frequency-dependent opacity data are not readily available. Rosseland means also include the electron scattering opacity.

We approximately convert these Rosseland means into absorption means (that allow \(T_e \neq T_s\)) by removing the electron scattering and assuming that the opacity has a generic free–free-like form of \(\alpha \propto (1 - \exp(-\gamma x))/x^3\). This allows us to obtain the generic ratio of absorption mean to Rosseland mean of \(\sim 30\) at \(T_e = T_s\), which is consistent with the free–free calculations above. This factor of 30 already appears in the below opacities.

At intermediate temperatures of \(10^7\)–\(10^9\) K and low densities, the free–free and bound-free opacity change their behaviour for solar abundances to become the ‘Chianti opacity’ of

\[\kappa_{\text{Chianti}}/\rho \sim 30 \times 10^{32} \rho(1 + Z/Z_{\text{Solar}})X(1 + X)T_e^{-4.7}, \hfill (C11)\]

This behaves similarly to the free–free or free–bound opacity, but with a steeper temperature dependence. This accounts for the assumed \(Z = Z_{\text{Solar}} = 0.02\) for fig. 34.1 in Draine (2011), most applicable for baryon densities of \(n_b \sim 1\) cm\(^{-3}\). This gives a kink in the opacity at \(T \sim 10^9\) K. To capture the drop in opacity at even lower temperatures and obtain a peak opacity at \(T \sim 10^7\) K for solar abundances as in that fig. 34.1, we use the H\(^-\) opacity of

\[\kappa_{\text{H}}/\rho \approx 33 \times 10^{-25} Z^{0.5} \rho^{0.5} T_e^{7.7}, \hfill (C12)\]

and use the molecular opacity of \(\kappa_m/\rho \approx 3Z\). The H\(^-\) and molecular opacities should have a absorption mean that will change with \(T_s\) (e.g. Malagin et al. 2014), but, in our case, these low-temperature opacities most often occur when \(T_e \approx T_s\).

The Chianti opacity works for low densities at intermediate temperatures, but at higher densities and lower temperatures, additional effects modify the opacity. Realistic opacities for \(T \lesssim 10^9\) K at higher densities can be obtained from the \(\kappa_{\text{OPAL}}\) opacity tables that include bound–bound lines and other physics (Iglesias & Rogers 1996).

Other opacity calculations focused on stellar atmospheres give similar results (Seaton et al. 1994), while modern opacity tables use the \(\kappa_{\text{OPAL}}\) opacity tables in some regimes (Paxton et al. 2011). Particularly, the opacity is enhanced between \(T \approx 10^4\) and \(10^9\) K at high densities due to H and He as well as enhanced around \(10^8\) K at low densities due to metals such as Fe. Such opacities have been approximately fit by Iben (1975) with corrections for the \(\kappa_{\text{OPAL}}\) opacities by Guzik & Cox (1995), but those fits do not capture the low-temperature or Fe line features. For \(T \approx 10^5\) K, this opacity is significantly enhanced beyond the electron scattering opacity and can introduce driving of winds (Proga, Stone & Drew 1998).

We obtain a rough fit to the \(\kappa_{\text{OPAL}}\) opacity table 73 that is relevant for solar abundances. We subtract out the electron scattering opacity from their Rosseland mean to obtain a Rosseland absorption mean. The \(\kappa_{\text{OPAL}}\) fit is given as extra terms

\[\kappa_{\text{COPAL}} \sim 3 \times 10^{-13} \kappa_{\text{Chianti}} T_e^{1.6} \rho^{-0.4} \hfill (C13)\]

and

\[\kappa_{\text{HOPAL}} \sim 10^4 T_e^{-1.2} \kappa_{\text{H}}^{-1}. \hfill (C14)\]

The Fe line at \(T \approx 1.5 \times 10^5\) K can be approximated by adding (to the overall opacity) the Gaussian

\[\kappa_{\text{Fe}}/\rho \approx 0.3 \left( \frac{Z}{Z_{\text{Solar}}} \right) \exp(-12+\ln(T_\gamma)g^2), \hfill (C15)\]

This gives an iron line bump, which can affect the structure of discs and stars with substantial metals (Jiang et al. 2015; Jiang et al. 2016). For the Fe line, we do not modify this Rosseland mean into a Planck mean, because the line sits at a narrow band of frequencies and does not follow the free–free frequency dependence.

Our conversion from Rosseland to absorption means assuming the frequency-dependence is roughly free–free-like, which also allows us to apply the general free–free \(\zeta\) dependence to these opacities, as well estimate the energy and number mean opacities. We take the ratio of a given low-temperature \(\kappa\) (that applies to \(T_e = T_s\)) to \(\zeta^0\) at \(\zeta = 1\), and then we set the final lower temperature opacity as \(\kappa_{\text{eff}}\) times that ratio. This is done for both energy and number mean opacities separately. Only for the temperature-independent molecular opacity and narrow-band Fe line opacity, do we assume a \(\zeta\)-independent opacity that is the same for number and energy mean opacities.

The final effective opacity \(\kappa_{\text{eff}}\) that bridges between the various opacities is given by

\[(\kappa_{\text{eff}} - \kappa_{\text{Fe}})^{-1} \sim (\kappa_m + \kappa_{\text{HOPAL}})^{-1} + \kappa_{\text{COPAL}}^{-1} \nonumber + (\kappa_{\text{Chianti}} + \kappa_{\text{eff}} + \kappa_{\text{Fe}})^{-1}. \hfill (C16)\]

This gives a fit to the \(\kappa_{\text{OPAL}}\) opacities, which is accurate to less than 30 per cent in most cases and factors of 10 for \(T \ll 10^5\) K. This defines both the energy opacity and number opacity by assuming \(\kappa_m\), \(\kappa_{\text{HOPAL}}\), \(\kappa_{\text{COPAL}}\) and \(\kappa_{\text{Chianti}}\) scale with free–free (and bound-free) for energy and number opacities.

\section*{APPENDIX D: CYCLO-SYNCHROTRON EMISSION}

Cyclo-synchrotron is important when magnetic fields are strong in diffuse plasmas. We follow the discussion in Mahadevan, Narayan & Yi (1996) (see also Ginzburg & Syrovatskii 1965), where fits are given to cyclo-synchrotron at several discrete temperatures from \(5 \times 10^3\) to \(3.2 \times 10^8\) K as well as the ultrarelativistic limit. Other fits are considered for higher temperatures in Leung, Gammie & Noble (2011). Planck mean opacities have been considered before in the context of black hole accretion flows (Mason & Turolla 1992).

We consider thermal electrons with temperature \(T_e\). Following Mahadevan et al. (1996) and Esin et al. (1996), the emissivity is

\[j_{\text{cycle}}(v) = 4.43 \times 10^{-30} \nu_T^2 \frac{\nu_M (\nu_M)}{K_2 (1/\theta_e)} \text{erg cm}^{-3} \text{s}^{-1} \text{Hz}^{-1}. \hfill (D1)\]
where $K_2$ is the second modified Bessel function, and
\[
x_M = \frac{v}{v_M}, \quad v_M = \frac{eB}{2\pi n_ec} \theta_c^2 = 1.19 \times 10^7 BT_0^2 \text{Hz}, \tag{D2}
\]
where $T_0 = T_e/10^9$ K. The function $I(x_M)$ is defined in Mahadevan et al. (1996), who provide the following fitting function for it,
\[
I(x_M) = \frac{4.0505a}{x_M^{1/6}} \left(1 + \frac{0.40b}{x_M^{1/4}} + \frac{0.5316c}{x_M^{1/2}}\right) \exp(-1.8999x_M^{1/3}). \tag{D3}
\]

In the ultrarelativistic limit, $\alpha = \beta = \gamma = 1$ and $K_2(1/\theta_c) \to 2\theta_c^2$, while table 1 in Mahadevan et al. (1996) gives these coefficients for several other temperatures.

For synchrotron, a primary limitation of our opacities is the lack of non-thermal electrons, which would greatly modify the opacity in optically thin regions.

We use the low-frequency ($h\nu \ll kT_e$) (e.g. Rayleigh–Jeans for Planck) expansion for the distribution that appears in Kirchhoff’s law within the opacity. Equivalently, we assume that synchrotron emission at $h\nu \gtrsim k_b T_e$ is sub-dominant. This approximation is reasonable when $(kT_e)/(h\nu_M) \gg 1$, which will be valid for simulations we consider. One can then compute the energy and number absorption mean from $\alpha_n^\nu = f e^{\phi}/B(T_e)$.

D1 Razin effect for synchrotron
Collective plasma effects, like the Razin effect, occur for synchrotron (Hornby & Williams 1966; McCray 1967; Simon 1969). This has been applied in the astrophysical case recently by Dougherty et al. (2003). The characteristic cut-off occurs at $v_{ck} \approx v_{pe}^2/v_{ck} \approx 19n_e/B$,
\[
v_{ck} \approx v_{pe}^2/v_{ck} \approx 19n_e/B, \tag{D4}
\]

or in terms of our dimensionless synchrotron integration variable in $x_M^{1/3} \approx 0.01(n_e/B)^{1/3}$. Below $v_{ck}$, the synchrotron spectrum varies as $v^{3/2} \exp(-v_{ck}/v)$, independent of the electron spectrum as long as there is no high-energy cut-off in the electron spectrum (in which case, the $v_{ck}$ is even higher at $v_{ck} \approx 500\gamma_B n_e^{3/4}/B^{1/2}$ for electron energy cut-off at $E = \gamma_B n_e e^2$). This cut-off can be approximated quite accurately by multiplying $\alpha_n^\nu$ by $e^{-\phi}/e^{-\phi}$ (Dougherty et al. 2003), which is the approximation we apply to the integral. Like with free–free, the cut-off is small enough that changes in the dependence of opacity versus $\zeta = T_e/T_e$ only occur for $\zeta \ll 1$, where the BE assumption is unlikely accurate and the simulation very rarely accesses. Hence, like with free–free, we only tabulate/fit the region above this change in character.

D2 Tabulating/fitting cyclo-synchrotron
Defining
\[
\phi = \frac{k T_e}{B v_M}, \tag{D5}
\]
and using a dimensionful factor of
\[
g(n_e, B, T_e) = 5.85374 \times 10^{-14} n_e \phi \theta_c^{-3} T_y^{-1}, \tag{D6}
\]
the residual factor $\kappa_{\text{res}}/g(n_e, B, T_e)$ and $\kappa_{\text{res}}/g(n_e, B, T_e)$ can be tabulated. This residual is quite linear in log–log space, so not much resolution is required. In order to handle regions with weak or zero magnetic field strengths or radiation temperatures, we limit $\phi$ to no smaller than $10^{-20}$ and no larger than $10^{20}$.

For HARMRAD, we fit the energy and number opacity residual by
\[
\left(\frac{\kappa_{\text{res}}}{g(n_e, B, T_e)}\right)^{-1} \approx (a\phi^{-b}\ln(1+c\phi))^{-1} + (d\phi^{-e})^{-1}, \tag{D7}
\]

with different coefficients $a, b, c, d, e$ for the energy and number opacities. As with the free–free opacity, $a, b, c, d, e$ are independently fitted as functions of $\phi$ for each $e^{-\phi}$. We obtain fits accurate to less than 20 per cent relative error for any temperature $T_e$ case given in Mahadevan et al. (1996) over the range $10^{-3} \leq \phi \leq 10^{-1}$.

For densities and temperatures relevant for Sgra*, which enters the Razin cut-off and modification of the BE distribution, the ultrarelativistic temperature limit has coefficients for the energy opacity of
\[
a = -0.0295(e^{-\phi})^{2.29} - 0.143(1 - (e^{-\phi}))^{0.251} + 0.236 \tag{D8}
b = 0.00977(e^{-\phi})^{3.29} + 0.0291(1 - (e^{-\phi}))^{0.48} + 2.58 \tag{D8}
c = 1.29(e^{-\phi})^{1.59} + 3.46(1 - (e^{-\phi}))^{0.234} + 2.15 \tag{D8}
d = -78.1(e^{-\phi})^{6.64} - 40.3(1 - (e^{-\phi}))^{0.399} + 87.4 \tag{D8}
e = 0.415(e^{-\phi})^{0.399} + 1.04(1 - (e^{-\phi}))^{0.252} + 2.68, \tag{D8}
\]

and the constants for the number opacity are
\[
a_n = -0.0189(1 - (e^{-\phi}))^{0.252} + 2.59(1 - (e^{-\phi}))^{0.69} + 29.2 \tag{D9}
b_n = -0.18(e^{-\phi})^{3.19} + 0.425(1 - (e^{-\phi}))^{0.179} + 2.76 \tag{D9}
c_n = 0.0207(e^{-\phi})^{0.69} + 0.0506(1 - (e^{-\phi}))^{0.804} + 0.0314 \tag{D9}
d_n = (1.51 \times 10^6) (e^{-\phi})^{0.283} \tag{D9}
-1.4 \times 10^7(1 - (e^{-\phi}))^{0.66} + 1.4 \times 10^5 \tag{D9}
e_n = 0.1(e^{-\phi})^{0.95} + 1.57(1 - (e^{-\phi}))^{0.124}. \tag{D9}
\]

These fits are accurate to 20 per cent error over most of the domain $10^{-4} \leq \phi \leq 10^{-1}$ and $0 \leq e^{-\phi} \leq 1$. For $e^{-\phi} = 1$, the coefficients for the energy opacity are $\{a, b, c, d, e\} = \{0.206, 2.59, 3.44, 9.33, 3.09\}$ and for the number opacity are $\{a_n, b_n, c_n, d_n, e_n\} = \{40.0, 2.58, 0.0522, 1.65 \times 10^6, 0.100\}$. These Planck coefficients can be used for the emission mean opacity when one also enforces $\phi = (kT_e)/(h\nu_M)$. For $\phi \leq 10^{-4}$, these opacities becomes constant for each $e^{-\phi}$ versus $\phi$ due to the Razin cut-off, so for such values of $\phi$, one should replace the opacities with their values at $\phi = 10^{-5}$, thus overriding the above fits.

For densities and temperatures relevant for an Eddington-accreting BH X-ray binary, we only obtain fitting accurate to order unity since other processes usually dominate and finding fitting functions can be difficult. The ultrarelativistic temperature limit has coefficients for the energy opacity of
\[
a = -2.31 \times 10^{-8} (e^{-\phi})^{34} \tag{D10} - (8.24 \times 10^{-6}) (1 - (e^{-\phi}))^{2.42} + 1.27 \tag{D10}
b = -0.0261(1 - (e^{-\phi}))^{1.55} + 0.00475(1 - (e^{-\phi}))^{1.55} + 1.06 \tag{D10}
c = 0.00179(e^{-\phi})^{0.332} + 0.000441(1 - (e^{-\phi}))^{0.332} + 0.000584 \tag{D10}
d = -17.4(e^{-\phi})^{0.64} - 3.33(1 - (e^{-\phi}))^{0.76} + 18.3 \tag{D10}
e = 0.427(e^{-\phi})^{0.214} + 1.23(1 - (e^{-\phi}))^{0.214} + 2.49, \tag{D10}
\]

and the constants for the number opacity are
\[
a_n = -0.000359(e^{-\phi})^{1.31} - 0.000552(1 - (e^{-\phi}))^{0.135} + 0.00209 \tag{D11}
b_n = 0.035(e^{-\phi})^{5.43} + 0.0433(1 - (e^{-\phi}))^{1.159} + 0.948 \tag{D11}
c_n = -0.122(e^{-\phi})^{0.371} - 0.0685(1 - (e^{-\phi}))^{2.8} + 1.04 \tag{D11}
d_n = -8.59(e^{-\phi})^{1.55} - 6.47(1 - (e^{-\phi}))^{0.436} + 8.71 \tag{D11}
e_n = -0.447(e^{-\phi})^{0.394} + 0.506(1 - (e^{-\phi}))^{1.155} + 2.45. \tag{D11}
\]
For $e^{-\xi} = 1$, the coefficients for the energy opacity are $\{a, b, c, d, e\} = \{1.27, 1.03, 0.000763, 0.616, 2.91\}$, and for the number opacity are $\{a_n, b_n, c_n, d_n, e_n\} = \{0.00173, 0.983, 0.921, 0.123, 2.00\}$, to which the fits reduce to in this limit. These Planck coefficients can be used for the emission mean opacity when one also enforces $\phi \to (kT_e)/(\hbar \nu \gamma)$. However, the simulations reach to much lower temperatures where there are no accurate fits. The fits are not too dissimilar from the ultrarelativistic mean opacities, except when $\phi$ is far from unity. So, we assume the ultrarelativistic opacity in all temperature regimes, as has been done by others (Fragile & Meier 2009). In the future, we can use more accurate cyclo-synchrotron calculations (Leung et al. 2011; Pandya et al. 2016) to compute the mean opacities.

APPENDIX E: COMPTON SCATTERING

Comptonization modifies the spectra and cools accretion flows (Kawashima et al. 2009; Xie et al. 2010; Kawashima et al. 2012; Schnittman, Krolik & Noble 2013), and Comptonization may generate useful observational signatures of super-Eddington accretion (Sutton, Roberts & Middleton 2013). So it is an important process to consider.

E1 Thomson Scattering with Klein–Nishina

The electron scattering opacity is

$$\kappa_s \approx \kappa_{s0} \kappa_{s1},$$

(E1)

where the Klein–Nishina (KN) correction for thermal electrons is $\kappa_{s0} \approx (1 + (T_e/(4.5 \times 10^5))^3) 0.5^{1.5}$ (Buchler & Yueh 1976; Paczynski 1983) and $\kappa_{s1} = 0.2 (1 + X)\rho$. This is applicable as a Rosseland mean for a Planck distribution of photons in non-degenerate matter (with KN correction applicable when $T_e \sim T_\gamma$). However, the Rosseland mean and streaming limit of the flux mean are similar to within 20 per cent (Poutanen 2017), so a fixed scattering opacity as a function of optical depth is reasonable. A Wien distribution leads to a faster drop in scattering opacity as $\theta_e > 1$ (Svensson 1984), but this is a small correction as we do not end up with solutions having large regions with $\theta_e > 1$ or $\theta_\gamma > 1$.

E2 Thermal Comptonization

We account for energy exchange via Comptonization in the soft-photon limit of Kompaneets equation, which we implement in a similar way to that described in Kawashima et al. (2009). For a general temperature, using the result given in equation 2.43 in Pоздняков et al. (1983), Sadowski et al. (2015) obtained a TC term of

$$\lambda_C = -c_k e^{-E} \left[ \frac{4k(T_e - T_\gamma)}{m_c c^2} \right] \left[ 1 + 3.683 \left( \frac{kT_e}{m_c c^2} \right) + 4 \left( \frac{kT_e}{m_c c^2} \right)^2 \left[ 1 + \left( \frac{kT_e}{m_c c^2} \right) \right]^{-1},$$

(E2)

which is consistent with the frequency-integrated energy-weighted Kompaneets equation and is valid for a BE distribution with any chemical potential. We assume that the Compton-scattered radiation is emitted isotropically in the fluid frame, which is a good approximation in the soft photon limit or for when there are numerous scatterings.

This scheme is tested for accuracy as in Ryan et al. (2015) section 4.2, except they started with a delta function for the photon distribution and evolved the photons in a scattering region to see the redistribution towards Wien. Since we assume BE, we start with Planck at $T_e = 500 061$ K that gives their energy density for our Planck initial distribution. We assume their $T_e = 5 \times 10^5$ K, baryon number density $n_b = 2.5 \times 10^{-6} cm^{-3}$ and ideal gas constant $\gamma = 5/3$. We evolve the system with only non-relativistic electron scattering and TC terms. We check the time-scale for equilibration as well as the final distribution and temperature, which can be computed from the conditions of thermal equilibrium and photon conservation. Analytically, we obtain a final temperature of $T_e \approx 2.778 \times 10^6$ K and $e^{-\xi} = 0.0070$. We use HARMRAD and evolve for $t = 20$ s. If we evolve for this time in a single time-step, as possible with our implicit method, we find that after this time, $T_e \approx 2.786 \times 10^6$ K, $T_\gamma \approx 2.779 \times 10^6$ K and $e^{-\xi} \approx 0.0070$ (i.e. quite Wien). If we evolve the system for about 48000 time-steps, then we find $T_e = T_\gamma \approx 2.779 \times 10^6$ K (with temperatures similar to machine precision) and $e^{-\xi} \approx 0.0070$. In this case, the temperatures approach each other on a time-scale of 0.02 s, as is the equilibration time-scale. Errors in these results are dominated by the temperature and chemical potential fits for the BE distribution that are only accurate to, at the worst, 2 per cent, as well as by the first-order time error in the implicit method when taking only a single large time-step.

Other extensions to Kompaneets can be found elsewhere that account for relativistic effects (Nagirner, Loskovut & Grachev 1997; Challinor & Lasenby 1998; Sazonov & Sunyaev 1998; Nozawa & Kohyama 2015), high temperatures (Sampson 1959; Xie et al. 2010; Garain & Chakrabarti 2013; Niedzwiecki, Stepniski & Xie 2015), and evolve the electron temperature (Procopio & Burigana 2009; Chluba & Sunyaev 2012).

E3 Double Compton

An important source of soft photons is the double (or radiative) Compton effect, which dominates BR in radiation-dominated diffuse plasmas (Lightman 1981; Thorne 1981; Svensson 1984; Chluba, Sazonov & Sunyaev 2007). These photons are therefore an important source for TC.

The statistical factor for the DC emission process, $e^{-\gamma_0} \leftrightarrow e^{-\gamma_1} \leftrightarrow e^{-\gamma_2}$, is given by

$$F = \frac{f(E) n_0(1 + n_1)(1 + n_2) - f(E^0) n_1 n_2(1 + n_0)}{f(E) n_0(1 + n_1)} \times \left[ 1 + n_2 - e^{-\langle E' - E \rangle/kT_e} \frac{(1 + n_0)}{n_0} \frac{n_1}{(1 + n_1)} n_2 \right].$$

(E3)

where a relativistic Maxwell–Boltzmann, $f(E) \propto e^{-E/kT_e}$, was assumed for the electrons. Inserting BE distributions for the photons around energies $h\nu_0$ and $h\nu_1$, and using $E' = E = h(\nu_0 - \nu_1 - \nu_2)$, one has

$$F = \frac{f(E) n_0(1 + n_1)}{f(E) n_0(1 + n_1)} \left[ 1 + n_2 - e^{-\langle \nu_0 + \nu_1 - \nu_2 \rangle} e^{\nu_0 + \nu_1} e^{-\nu_1 + \mu_1} n_2 \right],$$

(E4)

where, in the last step, we assumed constant chemical potential, $\mu_1 = \mu_0$. Let us look at the average change in the energy of the
\[ \Delta x = x_0 - x_1 - x_2. \]  

(E5)

In the approximations that are used for the derivations, in fact, there is no direct energy exchange, so that all the energy comes from the scattering photons and \( x_0 - x_1 - x_2 \approx 0 \). Thus, we have

\[
F \approx f(E) n_0 (1 + n_1) (1 + n_2) \left( 1 + e^\gamma n_2 \right)
\]

(E6)

This shows that without direct energy exchange in the scattering event (no recoil and Doppler boosting), the spectrum is driven towards an equilibrium at the temperature of the photon field with \( T = T_\gamma \). Of course, electron recoil and Doppler do occur during the DC process and Compton scattering of those photons leads to evolution of the photon spectrum. However, because the derivation did not include these effects, and we assume a strict BE distribution of photons, we assume that energy exchange occurs as the emitted photons are Compton-scattered into the assumed BE distribution. Once the new BE distribution has been established, only then can we use our standard TC formula. This two-stage way of computing the DC process is required because the derivation of DC without electron recoil or Doppler require Kirchoff’s law have \( T = T_\gamma \), else Kirchoff’s law with \( T = T_\gamma \) would lead to divergences in \( \alpha \) due to the slower decay rate at high frequencies compared to, e.g. BR.

For photon occupation number \( n_x \), the number density of photons is

\[
n = (8\pi k_b T_\gamma)^3 / \hbar^3 \int_0^\infty n_x \rho^2 \, dx,
\]

(E7)

where we set the dimensionless energy \( x = \nu / (k_b T_\gamma) \).

The change in the occupation number per unit time due to emission of DC photons with energy \( x \) is

\[
dn_{DC} / dt = \frac{4\alpha}{3\pi} \frac{1}{x} \frac{1}{E_{DC}}
\]

(Lightman 1981; Chluba et al. 2007), where \( \gamma_x = k_b T_\gamma / (m_e c^2) \), \( \gamma_c = c / \nu \), and \( \alpha \) is the fine structure constant. Note that the DC rates are dependent upon the radiation temperature \( T_\gamma \), unlike the other emission processes. Note that Lightman (1981) did not distinguish \( T_\gamma \) from \( T \), when obtaining their equation 10, a but their derivation only depends explicitly upon the radiation temperature and just assumed \( \theta_x \ll 1 \) in order to drop frequency shift terms. The dimensionless DC emission-Gaunt factor is

\[
I_{DC}(x, \nu, \gamma_x, \theta_x, \theta_y) = \int_{2x}^\infty y^4 (1 + n_{y-1}) \rho_y
\]

\[
\times \left[ \frac{x}{y} H_G \left( \frac{x}{y} \right) \right] G_m(y \theta_x, \theta_y) \rho_y \, dy,
\]

(E9)

where \( y = \nu / (k_b T_\gamma) \) is a dummy variable for \( x \) and \( n_{y-1} \) means to substitute \( y = x \) as \( x \) in \( n_x \), and dimensionless

\[
G_m(x \theta_x, \theta_y) = \int_0^\infty G_m(\nu_0, \rho_0) f(E_0, \theta_x) \rho_0^2 \, d\rho_0
\]

(E10)

for momentum \( \rho_0 = m_e \gamma_0 \rho_0 \) and \( \gamma_0 = 1 / \sqrt{1 - \beta_0^2} \), photon frequency per electron rest-mass energy \( \nu_0 = h \nu_0 / (m_e c^2) = x \theta_x \), three-velocity \( \beta_0 \), and energy \( E_0 = m_e \gamma_0 \) or \( E_0 = \rho_0^2 + (m_e c^2)^2 \). The relativistic Maxwell–Boltzmann distribution per \( n_x \) is

\[
f(E, \theta_x) = \left( \frac{1}{4\pi m_e^2 K_1(1/\theta_x)} \right) e^{-E / m_e \theta_x},
\]

(E11)

where \( K_1(1/\theta_x) \) is the modified Bessel function of the second kind, with \( \theta_x = k_b T_\gamma / (m_e c^2) \), where \( n_e \) is the electron density number, such that \( 1 = \int f(E) \, d^3 p \), and

\[
G_{\nu 0}(\nu_0, \rho_0) = \frac{\gamma_0^2 (1 + \beta_0^2)}{1 + \sum_{k=1}^\infty f_k(\rho_0) \gamma_0^2 \rho_0^k},
\]

(E12a)

with the functions \( f_k(\rho_0) \)

\[
f_k(\rho_0) = \frac{1}{1 + \rho_0^2} \left[ \frac{21}{5} + \frac{42}{5} \beta_0^2 + \frac{21}{25} \beta_0^4 \right]
\]

(E12b)

and

\[
f_1(\rho_0) = \frac{1}{1 + \rho_0^2} \left[ \frac{84}{25} \beta_0^2 + \frac{1967}{125} \beta_0^4 \right]
\]

(E12c)

\[
f_0(\rho_0) = - \frac{1}{1 + \rho_0^2} \left[ \frac{2041}{875} \beta_0^2 + \frac{1306}{125} \beta_0^4 \right]
\]

(E12d)

\[
f_0(\rho_0) = \frac{9663}{4375}
\]

(E12e)

(Chluba et al. 2007; Chluba & Sunyaev 2012) (with typo fixed in \( f_1 \)). As compared to previous expressions that require \( \theta_x \ll 1 \) and \( \theta_y \ll 1 \) and \( x \ll 1 \) (i.e. cold electrons, cold photons and soft photons) (Lightman 1981), where \( I_{DC} \rightarrow I_{DC} \) with

\[
I_{DC} = \int_0^\infty x^4 (1 + n_x) n_x \, dx,
\]

(E8)

Our version from Chluba et al. (2007) is accurate for moderately relativistic electrons and photons (i.e. \( \theta_x \lesssim 1 \) and \( \theta_y \lesssim 1 \) and \( x \lesssim 1 \)). For \( x \gtrsim 1 \), DC is suppressed as \( e^{-2x} \), and high-energy photons are more readily generated by single Comptonization off the DC photons (Thorne 1981).

\[ d\nu_{DC} = (8\pi k_b T_\gamma)^3 / \hbar^3 \int_0^\infty dx \frac{dn_{DC}}{dx}, \]

(E14)

\[ dE_{DC} = (k_b T_\gamma) x d\nu_{DC}, \]

(E15)

such that

\[ d_E_{DC} = (k_b T_\gamma) x d\nu_{DC}. \]

(E16)

The energy and number mean opacities can then be computed from \( a_{DC} = j_{DC} / B_{\nu 0}(T_\gamma) = j_{DC}(x) / B(x, T_\gamma) \), where \( B(x) = B_{\nu 0}(dx/dx) = \nu_0 / (k_b T_\gamma) / h \). As shown in Appendix E3, Kirchoff’s law is based upon a distribution at \( T = T_\gamma \), because these DC expressions include no recoil or Doppler shifting, and they presume energy balance between the incoming photon and the two outgoing photons. That is, DC here includes no energy exchange with electrons and so absorption only drives the photon distribution to Planck at its own radiation temperature. Energy exchange from single Comptonization dominates that one would obtain from recoil/Doppler for DC, so the energy exchange from equation (E2) is sufficient as an independent (non-DC) mechanism to drive thermal equilibrium between electrons and photons. Unlike free–free, electrons can be completely cold and DC emission still occurs if there are seed photons (Lightman 1981).

DC leads to a small correction to the TC process for the high-frequency photons (\( \gamma_0 \gamma_1 \)), which is neglected here (for more discussion, see Chluba 2005). We are not aware of work done to establish the Razin effect with DC, so we apply the same approach used for BR.

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The dimensionless factor for the DC opacity is
\[ h(n_e, T_y, e^{-\xi}) = 7.360 \times 10^{-4} n_e T_y^2 e^{-\xi}, \]  
(E17)
where the \( e^{-\xi} \) factor was pulled from the dimensionless integrals. Now the residual factor \( \kappa_{\alpha, \beta}/(p(\theta_e)h(n_e, T_y, e^{-\xi})) \) and \( \kappa_{\gamma, \delta}/(p(\theta_e)h(n_e, T_y, e^{-\xi})) \) can be tabulated, where \( p(\theta_e) \) is an electron temperature dependence that is factored out separately as discussed below. The DC emission opacity is obtained by choosing \( T_{eq} = T_y \) and \( \mu_{eq} = 0 \), but that still leaves the emission opacity dependent upon the radiation \( \mu \) from the DC \( j_\gamma \).

For \( \text{HARMRAD} \), we fit the opacity residuals. For DC, our approach is to first fit the regime \( \theta_e \ll 1 \), where the residual is constant versus \( \theta_e \), and then find a rough fit versus \( \theta_e \geq 0.01 \) when the opacity begins to change by more than 20 per cent versus \( \theta_e \).

For any \( \theta_e \), the residual factors \( \kappa_{\alpha, \beta}/(p(\theta_e)h(n_e, T_y, e^{-\xi})) \) and \( \kappa_{\gamma, \delta}/(p(\theta_e)h(n_e, T_y, e^{-\xi})) \) can be fit by the form
\[ \left( \frac{\kappa_{\alpha, \beta}}{p(\theta_e)h(n_e, T_y, e^{-\xi})} \right)^{-1} \approx a^{-1} + (b \theta_e)^{-1} + (d \theta_e^{-3/2})^{-1}, \]  
(E18)
with a relative error less than 20 per cent. For DC, we choose densities and temperatures relevant for an Eddington-acreting BH X-ray binary, which enters the Razin cut-off and modification of the BE distribution. DC for diffuse systems like SgrA* is much lower than synchrotron due to the very low radiation temperature of thermal synchrotron, so it is not considered.

For \( \theta_e \ll 1 \), we obtain fits are accurate to less than 20 per cent relative error for all \( 10^{-3} \leq \theta_y \leq 10^2 \) and \( 0 \leq e^{-\xi} \leq 1 \). The energy absorption opacity coefficients are
\[ a = 4.16(1 - (e^{-\xi}))^{1.69} + 6.7(e^{-\xi})^{0.942} + 3.1 \times 10^{-8} \]
\[ b = -0.0334(1 - (e^{-\xi}))^{0.469} - 0.0021(e^{-\xi})^{0.0217} + 0.042 \]
\[ c = -0.18(e^{-\xi})^{33} + 0.201(1 - (e^{-\xi}))^{0.258} + 3.8 \]
\[ d = 0.0169(e^{-\xi})^{35.4} - 0.0626(1 - (e^{-\xi}))^{0.35} + 0.118. \]  
(E19)
The energy emission opacity coefficients are
\[ a = -0.0589(1 - (e^{-\xi}))^{10.7} + 0.488(e^{-\xi})^{1.75} + 6.34 \]
\[ b = 0.0282(e^{-\xi})^{1.56} + 0.0142(1 - (e^{-\xi}))^{0.361} + 0.00875 \]
\[ c = -0.16(e^{-\xi})^{15.4} + 0.184(1 - (e^{-\xi}))^{0.366} + 3.78 \]
\[ d = 0.015(e^{-\xi})^{26.3} - 0.0256(1 - (e^{-\xi}))^{0.398} + 0.119. \]  
(E20)
The Planck limit gives coefficients \( \{a, b, c, d\} = \{6.83, 0.0374, 3.63, 0.134\} \) for the energy absorption and emission opacities.

The number absorption opacity coefficients are
\[ a_n = 29.4(e^{-\xi})^{28.7} - 76.4(1 - (e^{-\xi}))^{0.136} + 87.5 \]
\[ b_n = 0.196(e^{-\xi})^{1.51} - 1.12(1 - (e^{-\xi}))^{0.134} + 1.16 \]
\[ c_n = 0.0427(1 - (e^{-\xi}))^{21.6} - 0.8(e^{-\xi})^{21.6} + 3.93 \]
\[ d_n = 1.87(e^{-\xi})^{100} - 2.72(1 - (e^{-\xi}))^{0.106} + 2.86. \]  
(E21)
The number emission mean opacity coefficients are
\[ a_n = -81.7(e^{-\xi})^{0.01} - 94.8(1 - (e^{-\xi}))^{0.925} + 198. \]
\[ b_n = 1.31(e^{-\xi})^{1.12} + 1.05(1 - (e^{-\xi}))^{0.249} + 4.8 \times 10^{-11} \]
\[ c_n = -0.418(e^{-\xi})^{14.8} + 0.442(1 - (e^{-\xi}))^{0.361} + 3.44 \]
\[ d_n = 1.37(e^{-\xi})^{31.3} - 1.38(1 - (e^{-\xi}))^{0.316} + 3.38. \]  
(E22)
The Planck limit gives coefficients \( \{a_n, b_n, c_n, d_n\} = \{116., 1.34, 3.03, 4.72\} \).

To obtain a rough fit that will be accurate for \( \theta_e \geq 0.01 \), we fit the ratio of the higher \( \theta_e \) opacities to the \( \theta_e \ll 1 \) opacity. This gives a fit to the ratio \( p(\theta_e) \) of the general opacity to the \( \theta_e \ll 1 \) opacity:
\[ p(\theta_e) \approx (1 + \theta_e)^{-3}, \]  
(E23)
which does not change the accuracy at low \( \theta_e \) but improves the accuracy for \( \theta_e \leq 1 \) to be within a factor of 3 or better and \( \theta_e \leq 0.1 \) to be within a factor of 2 or better. This factor \( p(\theta_e) \) gives the same correction and accuracy for all opacities (energy absorption, number absorption, energy emission and number emission). The DC calculation is only accurate for \( \theta_e \ll 1 \), beyond which pair production processes (not included) become important.