Research of crack formation in inseparable double-span reinforced concrete elements

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Abstract. According to the results of the conducted, experimental researches empirical dependences of crack formation and width of opening of normal and inclined cracks of inseparable double-span reinforced concrete beams. A reliable quantitative and qualitative assessment of the impact of structural factors, as well as external factors on the crack resistance of inseparable elements, both separately and in interaction with each other. An improved deformation method (implemented in the PC MATLAB) for calculating the crack resistance of reinforced concrete elements. Based on the comparative analysis of the obtained empirical dependences and calculated values according to normative methods: DSTU B B.2.6-156: 2010 (Ukraine), previously acting SNiP 2.03.01-84 *, SR 63.13330.2012 (SET OF RULES, actualized) Russian standard, as well as the deformation model of Scientific Research Institute of Building Constructions (SRIBC), town of Kyiv, the coefficient of variation of crack resistance and width of opening of normal and inclined cracks is determined. The comparative analysis is carried out and informative graphic drawings are presented.

1. Introduction

The peculiarity of the work of inseparable double-span reinforced concrete beams is the redistribution of internal efforts in the sections, so the appearance of primary and secondary cracks, their development, the opening of normal and inclined cracks has a great impact on their performance. In the process of loading the experimental elements, the first normal cracks appear above the middle support in the area of maximum bending moment, and then or simultaneously - under the average forces and in the span of the stretch of the stretched area. This feature of crack formation is due to the work of beams as once statically indeterminate elements, in which the bending moment over the middle support is several times greater than the moments under concentrated forces.

The purpose of the research is to reveal the nature of crack formation, both normal and inclined cracks above the middle support and under the concentrated forces of inseparable double-span beams, to investigate the influence of structural factors.
Perform a comparative analysis of the experimental values of the studied parameters with the calculated ones performed according to normative methods and the improved deformation model.

A large number of scientific works are devoted to crack resistance, deformability and strength of reinforced concrete elements [1-5]. The strength of normal and inclined sections was studied by T.N. Azizov H.S. Aliev, A.Y. Babynch, A.Y. Barashykov, L.V. Baykov, Z.Y. Blikharskii, O.O. Hvozdev, O.S. Horodetskii, B.H. Demchyna, S.A. Dmytriev, L.O. Doroshkevych, V.S. Dorofeev, O.S. Zaliesov, V.M. Karpiuk, P.M. Koval, R.L. Mailyan, A.M. Pavlikov, O.I. Storozhenko, O.V. Andriichuk, Y.O. Shkola, O.F. Yaremenko, M.M. Petrov etc. Among foreign works can be noted the work of R. Walter, A. Vista, L. Cany, M.F. Collins, F. Leongardt, J. Mitchell, K. Modi, J. Morrow, J. Regan, J. Taub, D. Hanson etc.

2. Research Methodology

2.1 Experimental research

In accordance with the developed methodology of experimental research have been tested two-span continuous beams, respectively, b×h =100×180 mm with h₀ =155 mm, l=8h₀, a/h₀=1, 2, 3, made of heavy concrete of classes C12/15, C20/25, C30/35, reinforced by two plain frames with the lower rebar 2Ø10, 12, 14 500S, with the top rebar 2Ø10, 12, 14 A500S and with transverse rebar 2Ø3, 4, 5 Vr-I. Experiments were performed on five-factor, close to properties of the D-optimal plan of type H5, which provides the same accuracy of the prediction of the experimental value in the area described by a radius equal to 1 relative to the central "0" point (Experiment № 27).

2.2 The mechanism of cracking, deformation and fracture of a continuous double-span beam

The initial stage of crack development is characterized by the concentration of concrete deformations, especially at the level of the location of the stretched reinforcement above the middle support.

In the process and after the formation of a plastic hinge over the middle support, which is characterized by the fluidity of the upper longitudinal reinforcement of the beam and (or) crushing of the concrete of the compressed zone, there is a redistribution of internal forces with a sharp increase in bending moment under concentrated forces and holding its maximum value above the specified support, which confirmed by studies [3-7].

Before the appearance of normal cracks, the distribution of internal forces corresponds to the rules of structural mechanics. After the appearance of the first cracks above the middle support (stage I, Fig. 1), inelastic properties appear and the forces are found in the diagram σ-ε. Upon reaching the yield strength over the middle support, i.e. the formation of a plastic hinge, the beam is divided into two split with concentrated moments above the middle support (stage III, Fig. 1).

When bending a beam with a moderate content of transverse reinforcement in the zones of action of bending moments and transverse forces as the load increases and the development of inclined cracks, the beam is divided into parts connected by the concrete of the compressed zone and the reinforcement intersecting by cracks. The destruction of the beam is characterized by a sharp opening of one of the inclined cracks - the so-called critical with the following physical destruction of concrete over this crack due to the achievement of longitudinal deformations in the direction of action of the main compressive stresses of the limiting values that manifest themselves either as external crushing of concrete or as crushing with a puncture. The stress state of the compressed zone at the approach to fracture is complex, since shear stresses act to the compressed ones.
Graph of bending moments $M_x$ before the appearance of the «plastic hinge»

I

Graph of bending moments $M_x$ after the appearance of the «plastic hinge» over the middle support

II

III

IV

V

Figure 1. The mechanism of cracking, deformation and fracture of a continuous double-span beam. (I - the appearance of normal cracks above the middle support, node $A$ - dynamometer; II - the development of normal cracks above the middle support, the appearance of normal cracks under the load and (or) inclined slots in the span; III - the development of normal and inclined cracks, the appearance of a “plastic hinge” above the middle support; IV - development of normal and inclined cracks with a predominant opening of inclined cracks, up to failure along inclined sections; V - destruction along inclined sections with the formation of a “plastic hinge” under concentrated loads in beams with large span sections).
3. Results

Processing of the experimental data presented in [6-9] on the resistance to cracking, deformation and fracture of integral double-span beams, allowed to obtain mathematical models (1-8) by the method [9], which can be used to assess the impact of relative shear span (experimental factor $X_1$, $a/h_0$=1.2,3), concrete class (experimental factor $X_2$, C, MPa: C12/15; C20/25; C30/35), number of transverse reinforcement (experimental factor $X_3$: $\rho_w$ (BpI) = 0.0018: 0.0032; 0.0050 (2Ø3.4.5 at $S$=7.75 cm)), the number of lower and upper longitudinal reinforcement (experimental factor $X_4$: $\rho_f$,lower (A500C) = 0.010; 0.0146; 0.0146 (2Ø10, 12, 14), $X_5$: $\rho_f$,upp. (A500C) = 0.010; 0.0146; 0.0146 (2Ø10, 12, 14) on their crack formation:

$$Y_{V_{\text{mid, sup}}} = 16.6 - 6.0X_1 + 4.6X_2 + 0.9X_3 + 3.7X_1^2 - 1.3X_2X_3, \ kH,$$

$$\Sigma = (Y - \bar{Y})^2 = 22.50; \ \sigma = \sqrt{\frac{(Y - \bar{Y})^2}{n-1}} = 0.9; \ \nu = \frac{\sigma}{b_{\text{avg}}} \cdot 100\% = 5.6\%;$$

(1)

$$Y_{M_{\text{mid, sup}}} = 3.4 + 0.8X_2 + 0.16X_3, \ kH_m,$$

$$\Sigma = 0.75; \ \sigma = 0.17; \ \nu = 5.0\%;$$

(2)

$$Y_{V_{\text{fore, sup}}} = 62 - 38.8X_1 + 11.9X_2 + 14.4X_3 + 17.4X_1^2 - 9.4X_1X_3, \ kH,$$

$$\sigma=6.4; \ \nu = 10.4\%;$$

(3)

$$Y_{M_{\text{fore, sup}}} = 3.4 + 0.8X_2 + 0.14X_3, \ kH_m,$$

$$\sigma=0.2; \ \nu = 5.5\%;$$

(4)

$$Y_{V_{\text{fore, l}}} = 40.9 + 9.6X_2, \ kH,$$

$$\sigma=2.3; \ \nu = 5.6\%;$$

(5)

$$Y_{M_{\text{fore, l}}} = 8.4 + 2.2X_1 + 1.8X_2 - 0.9X_1^2 + 0.6X_1X_2, \ kH_m,$$

$$\sigma=0.5; \ \nu = 5.5\%;$$

(6)

$$Y_{M_{\text{fore, l}}} = 3.2 + 2.5X_1 + 0.8X_2 + 0.14X_1^2 + 0.6X_1X_2, \ kH_m,$$

$$\sigma=0.2; \ \nu = 5.4\%;$$

(7)

$$Y_{Q_{\text{fore, l}}} = 37.4 - 3.4X_1 + 8.6X_2 - 0.7X_1^2 - 0.6X_1X_2, \ kH,$$

$$\sigma=2.0; \ \nu = 5.4\%;$$

(8)

Adequate mathematical models of external load, which causes the appearance of normal cracks above the support (1) and under concentrated forces (3) in the rungs, indicate that the greatest influence is exerted by the relative shear of the cut, concrete class and the number of upper longitudinal reinforcement.

The values of the moments above the average support (2) and the concentrated forces (4), at which normal cracks are formed, increase in relation to their average value:

- with an increase in the class of concrete from C12/15 to C30/35 - up to 48%;
- with an increase in the longitudinal upper $\rho_{f,\text{upp}}$ (A500C) from 0.010 to 0.0146 (lower $\rho_{f,\text{lower}}$ from 0.010 to 0.0146) rebar, in accordance, - on 9.4% and 8.3%, that is, insignificant.

An analysis of the mathematical model of the transverse force at which inclined cracks appear (5) indicates that only the concrete class has the greatest influence. When it increases from minimum to maximum values $V_{\text{fore, l}}$ increases by 47% relative to its average value.
The moments of occurrence of inclined cracks over the middle support (6) and average forces (7) are influenced by the relative run of the cut and the class of the concrete.

The transverse force corresponding to the appearance of inclined cracks (8) will increase in relation to their average values at:
- reduction of the relative run of the slice \( a/h_0 \) from 3 to 1 by 18%;
- increased concrete grades from C12/15 to C30/35 by 46%.

The presence of a negative sign of the quadratic effect \( X_1^2 \) indicates that with decreasing slice run, the experimental parameter will increase more rapidly.

Below are the results (Fig. 2) of the comparative analysis of the obtained empirical dependences of crack resistance, width of opening of normal and inclined cracks with calculated values according to the improved deformation technique, normative methods (National standard of Ukraine DSTU B V.2.6-156:2010, previously acting SNiP 2.03.01-84 *, SR 63.13330.2012 (SET OF RULES, actualized) Russian standard, as well as the deformation model of Scientific Research Institute of Building Constructions (SRIBC), town of Kyiv [11].

![Figure 2. Comparison of the results of calculations of the coefficient of variation.](image)

The coefficient of variation, \( \nu \)

| Parameter                         | Method                                  | Value |
|----------------------------------|-----------------------------------------|-------|
| The moment of appearance of normal cracks, Merc | Advanced deformation method (PC MATLAB) | 7     |
| The moment of appearance of inclined cracks, Merc/ | Ukrainian standard                       | 21    |
| The width of the opening of normal cracks above the middle support, W | Deformation method SRIBC (A.M. BAmbura) | 18    |
| The width of the opening of normal cracks under concentrated forces, W | Previously acting SNiP2.03.01-84*       | 36    |
| The width of the opening of inclined cracks, Wcrc/ | SR 63.13330.2012 (SET OF RULES, actualized) Russian standard | 55    |

Among the important criteria chosen for the comparative analysis, the coefficient of variation in strength was selected (Fig. 2):

\[
\nu = \frac{\sigma}{b_{0,avr}} \cdot 100\% ,
\]

where \( \sigma = \sqrt{\frac{\sum (V_{exp} - V)^2}{n - 1}} \) ; \( \sum (V_{exp} - V)^2 \) - is average square deviation; \( b_{0,avr} \) - is arithmetic average experimental value of bearing capacity \( V_{exp} \); \( n \) - number of test specimens.
4. Appendix
Photographic recording of experimental samples is shown below (Fig. 3)

a) 

b) 

c) 

Figure 3. The nature of cracking and destruction of experimental two-span continuous beams with small (a), middle (b) and large (c) shear span.

5. Conclusions
Experimental-theoretical studies allowed to perform a reliable quantitative and qualitative assessment due to the action of structural factors, as well as factors of external action on the deformability, stability of inseparable elements, both separately and in interaction with each other. With a decrease in the run of the cut, the crack resistance of normal sections increases by 18%; with the increase of the concrete class, the crack resistance of normal and inclined increases to 48%, the amount of transverse, longitudinal lower and upper reinforcement is insignificant - up to 9.5%.

Comparison of experimental and calculated values $V_{cr.\perp}$, $V_{cr.\perp}$, $M_{cr.\perp}$, $M_{cr.\perp}$ above the middle support and under concentrated forces, as well as the width of normal and inclined forces $W_\perp$, $W_\perp$, showed that the use of deformation model SRIBC (Kyiv) [11] and the proposed deformation model using PC MATLAB, allows to achieve satisfactory convergence of experimental and calculated values of loads that correspond to the appearance of normal ($\nu=6.0…6.5\%$) and inclined cracks ($\nu=6.6…17\%$).
Comparison of experimental and calculated values of the opening width of normal and inclined cracks (Fig. 2) showed generally unsatisfactory convergence (υ to 58%).

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