NGC 6397: A case study in the resolution of post-collapse globular cluster cores

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ABSTRACT
Model surface brightness profiles based on Fokker-Planck simulations have been used to assess the high resolution surface brightness profile of the globular cluster NGC 6397 of Lauzeral et al. (1992). The profile is well fitted by the model in the maximum-expansion phase of a gravothermal oscillation with a core radius of 0.06 pc (6′′), but an unresolved core cannot be ruled out. A core this size is also expected in highly evolved clusters including significant numbers of primordial binaries. A more massive cluster will provide better statistics, as would star counts. The greater distance of the other globular clusters and the requirements of star counting will demand very high resolution observations to definitively resolve cores this size.

Key words: globular clusters: individual: NGC 6397—globular clusters: general

1 INTRODUCTION

In globular clusters which have passed through core collapse, the central density profile takes the form of a power-law. At some point the profile must flatten out, forming a core of approximately constant density, but whether this flattening is observable will depend on the underlying stellar distribution function and the sampling of this distribution function by the finite number of stars. From a theoretical perspective, the size of the post-collapse core of a cluster will depend on the nature of the heating mechanism which drives the re-expansion. Models which depend on binaries resulting from three-body encounters (Lee 1987) require much higher central densities before contraction is reversed than do models using tidally-formed binaries (Statler, Ostriker & Cohn, 1987) or with primordial binaries (Gao et al. 1991). Further, if gravothermal oscillations, which are observed in statistical numerical models such as gas sphere or Fokker-Planck codes, also occur in nature, then the post-collapse core would be much larger than is expected from a steady expansion. In the case of gravothermal oscillations the core radius is of order 1% of the half-mass radius of the cluster at maximum expansion (Murphy, Cohn & Hut 1990); from a few hundredths to a tenth of a parsec. A core this large may be observable under some conditions and its detection or non-detection would help in guiding further theoretical work. The finite number of stars in the region of interest places a fundamental restriction on the precision with which the comparisons can be made.

Lauer et al. (1991) analyzed a U-band HST image of the high-concentration cluster M15. After removing the bright stars, they claimed to have resolved a 2′′ core in the residual light profile. Grabhorn et al. (1992) fitted a Fokker-Planck model to the HST profile and stated that the observations were best matched by the model when it was in the state of maximum expansion. However, further consideration of the relationship between the models and the observations by Grabhorn et al. (1993) has demonstrated that for these M15 data it is in practice impossible to tell the difference between the two extreme phases of gravothermal oscillations. This is because there are so few stars in the region of interest that sampling variations mask the difference. Monte Carlo representations of the light profile drawn from the distribution functions of the two model phases overlap to the extent that stellar distributions drawn from the maximum-expansion distribution can look cuspy, and vice versa.

Yanny et al. (1993) have done photometry on new HST V and I observations of the core of M15. They find that the star counts are consistent with both a power-law cusp and with a small core of radius less than 1′′5 (0.09 pc). They showed that given the large wings of the HST point spread function, the residual light profile is an unreliable guide to the cluster’s structure. A nearer cluster is required to resolve the question of the observability of such a cluster’s core.

The recent high resolution observation by Lauzeral et al. (1992; hereafter LOAM) of the surface brightness profile of the nearby cluster NGC 6397 goes some way towards clarifying this issue. NGC 6397 is in many ways the key cluster in this discussion. First, it has a central cusp in its surface brightness and surface density. This is generally taken as signifying that a cluster has passed through core collapse. Second, at the distance of NGC 6397 (2.2 kpc; Drukier et al. 1993), an arc second corresponds to 0.01 pc, and a maximum-expansion-phase core should be easily resolvable. Third, there is an extensive set of other observations available to help constrain models of the cluster (Fahlman et al. 1989, Drukier et al. 1993).
One of the problems which have arisen in understanding the cores of cusp-profile clusters is the origin of the colour gradients observed in them. Djorgovski et al. (1991) have argued that the colour gradients are due to a deficiency of red giants in the cluster cores. It is this hypothesis that LOAM have investigated with their new observations. These are based on data taken with the Danish 1.5 m telescope at ESO under conditions of 0.95 seeing. What LOAM demonstrate is that the colour gradient in NGC 6397 can be eliminated if the light from the brighter red giants and from the blue stragglers is removed. The resulting surface brightness profile then loses its power-law shape in the central few arc seconds and flattens out. In the final profile LOAM find the core radius to be 6". With the removal of the brightest stars the surface brightness is dominated by the less evolved stars and so should better reflect the underlying density distribution.

The comparison of these data with a Fokker-Planck model which includes gravothermal oscillations, will both investigate whether NGC 6397 is in a maximum expansion phase (as is expected) and address the question of the limits placed on the observability of post-collapse cores by the finite number of stars. A distinction should be made between a surface brightness profile, which is derived from the integrated light of the individual stars, and a surface density profile, which comes from star counts. When a surface brightness profile is to be compared with a model, the mass-luminosity relation is needed to convert between the observed surface brightness profile and the model profile which is expressed in terms of the number of stars per unit area.

2 MODEL COMPARISON

As part of a project to compare Fokker-Planck models with observations of globular clusters (Drukier 1993), I discovered one model in particular for NGC 6397 which gave a good match to mass functions at three radii as well as the surface density profile of the cluster’s bright stars. Due to the crowding of stellar images in the core of the cluster a relatively bright magnitude cutoff had to be used (Drukier et al. 1993), hence the central region of the observed surface density profile has too few stars to resolve anything within the central 10". The LOAM surface brightness profile allows the extension of this comparison into the very centre of the cluster.

The models are based on the numerical integration of the isotropic, orbit-averaged, Fokker-Planck equation, as originally described in Cohn (1980). This is a statistical approach with the physical state of the cluster being given by a distribution function in energy space. The stellar mass spectrum is represented by a series of mass bins, each with its own distribution function.

A tidal boundary has been applied in energy space following the method of Lee & Ostriker (1987). Given an initial tidal radius, the mean density of the cluster within that radius is easily determined. At subsequent times, the tidal boundary in energy is given by the potential at the radius enclosing the same mean density. A fraction of the distribution function at energies greater than this “tidal energy” is removed at each time step. The fraction removed at a given energy is determined by the tidal stripping timescale and by the energy difference with respect to the tidal energy. The stripping rate is proportional to the cube of this energy difference.

In order to drive the gravothermal oscillations, an energy source is needed to oppose the gravitational collapse. In a star the energy comes from nuclear fusion. In a globular cluster it is expected that the energy comes from the hardening of binary stars. As interactions with the rest of the stars in the cluster cause the binary to become more tightly bound, energy is liberated to reverse the collapse. In the models used here, the binaries are assumed to form via three-body interactions. This energy source is represented statistically by calculating the binary creation rate and the average amount of energy released by each binary.

These models are much the same as those in Drukier, Fahlman, & Rich (1992), but two additions have been made. First, the effects of stellar mass loss (due to stellar evolution) have been included by allowing the masses of the various mass bins to vary with time in much the same manner as in Chernoff & Weinberg (1990). This leads to a much more realistic handling of the mass spectrum and remnant population than when no stellar evolution is included (as, for example, in Drukier et al. 1992). It also restricts the choice of initial mass function to ones which do not result in so much mass loss that the cluster would not survive to the present. This class of model is discussed further in Drukier (1993). At any given time, there is one mass bin with a changing mass. This bin, which I will refer to as the “evolving bin”, represents the stars which currently are near to the turn-off or have evolved past it onto the giant and horizontal branches or their final states as white dwarfs or neutron stars. A non-evolving bin represents stars which are still somewhere on the main sequence or which have finished evolving and are now degenerate remnants.

The other difference is that for the model described here the time step has been taken to be two central relaxation times. Such a small time step allows gravothermal oscillations to take place. By way of comparison, a model with large time steps (typically several thousand central relaxation times in the post-collapse phase) was also run. The evolution of the core radius for both models is shown in Fig. 1. The curve for the non-oscillating model has been shifted 0.16 Gyr earlier to match the time of core collapse. Since the details of the evolution are dependent on the initial conditions and the choice of time step, these models should not be taken as claiming a dynamical age for NGC 6397.

The initial model contained 25 bins covering the initial mass range between 0.1 and 20M⊙. The initial numbers of stars in each bin were distributed as two power-laws, one, for stars with $M < 0.4M_\odot$, with mass spectral index $x = 1.5$ (where $x = 1.35$ for a Salpeter mass function), the other, for $M > 0.4M_\odot$, had $x = 0.9$. The mass function was made continuous at 0.4M⊙. The stellar evolution times and final masses were the same as those used by Chernoff & Weinberg (1990) except that stellar evolution was assumed to stop at 12 Gyr. This was done to keep the mean mass of the stars in the bin which was to start evolving at that time constant for the purposes of comparison. By doing this, the question of the appropriate light-to-mass ratio to use for comparison to the surface brightness profile need only be addressed once and not for each time of comparison. One limitation of the approach used here to handle stellar evolution is that while a bin is evolving it is held to contain a mixture of...
There are several concerns which must be addressed in choosing these magnitudes. The first is the importance of the contribution of the low mass stars to the overall light of the cluster. For these low mass stars, $\log L \sim 2.5 \log M + \text{constant}$ (Bergbusch & VandenBerg 1992). If the local mass function has $x < 1.5$, then the cumulative contribution of these stars converges. If $x$ is much smaller then the total light from the faint stars is unimportant. In the model presented here, mass segregation, the enhancement of higher mass stars with respect to lower mass stars in the centres of clusters due to equipartition of energy, reduces the mass spectral index of the low mass stars from its initial value of $x = 1.5$ to $x = 0.8$ in the region we are interested in. Therefore we should be able to safely neglect the lowest mass bins.

How many bins are actually needed was determined in the following manner. LOAM removed the blue stragglers and the stars with magnitudes brighter than $V = 14.0$. I estimated the mean magnitude for the stars between $V = 14.0$ and $V = 16.4$ using the luminosity function of Alcaino et al. (1987). This came out to be 15.5, but it is unclear whether the $V = 16.4$ magnitude cut off is appropriate for the turn-off bin. The reason for this is the mismatch mentioned above between the mass of this bin and that of the turn-off stars in the cluster. Further, there is a mismatch in age and metallicity between the isochrone used to assign the evolutionary time scale of the mass bins in the cluster model and the observed age and metallicity of NGC 6397. In any event, first estimates for the the mean magnitudes of the next three mass bins were take from the isochrone which is from VandenBerg and Bell (1985).

The gravothermal oscillations in this model affect only the inner 0.1 pc, so I attempted to improve these magnitude estimates by fitting the two model profiles to the eight outermost LOAM data points. These fits did not provide a strong constraint on the magnitudes. In the case of the least massive of the four bins a limit of $V > 18$. was found. This suggests that there are an insufficient number of stars in this bin to contribute significant light to the surface brightness profile. This mass bin has an upper mass limit of $0.58 M_\odot$. Given the initial mass function for the model and the $Z = 10^{-4}$ isochrone from Bergbusch & VandenBerg (1992), the total light contributed from stars less massive than $0.58 M_\odot$ is only 19% of the light of all the stars up to $V = 14.0$. Mass segregation reduces this fraction further, to less than 12%, so only the turn-off bin and the next two were used in the comparisons. Based on these fits mean magnitudes were adopted for the the three remaining bins, with $V = 16.$ being used for the bin covering the upper main sequence and more evolved stars. The estimated uncertainty on these magnitudes is 0.3.

A surface brightness profile from a model is based on a continuous distribution. Assuming that the observed profile has the same underlying distribution, the observations represent a discrete sampling of that distribution. A proper comparison between the observations and the models should take into account the sampling uncertainties in the model "data points". Figure 2 presents the comparison between the observed surface brightness profile and those of the model at the two extrema. The points are the LOAM data and the two lines central to the hashed regions are the model profiles at the two extrema. The width of the hashed regions indicates the uncertainties. The closer-spaced hashing

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**Figure 1.** Core radius as a function of time for the model described here. The inset shows the core radius for the whole run, while the main part of the figure shows the post-core-collapse epoch to emphasize the gravothermal oscillations. The dashed line corresponds to the evolution of the same model when large time steps, which suppress gravothermal oscillations, are used. The curve for the model without oscillations has been shifted 0.16 Gyr earlier in order to match the core collapse times. The two vertical bars indicate the times of the two profiles shown in Fig. 2.

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within the bounding lines represent $1\sigma$ Poisson errors on the integrated model points, while the wider shadings, extending beyond the bounding lines, include the 0.3 magnitude uncertainties in adopted magnitudes added in quadrature.

To the eye, the maximum expansion profile provides a much better fit to the observations than does the core collapse profile. In order to quantify this comparison, I used Monte Carlo techniques to produce model surface brightness profiles consistent with Poisson statistics in the densities. For each mass species contributing to the surface brightness profile, and for each observing annulus, the number of stars of that mass “observed” in that bin was randomly drawn. These numbers were then used to calculate the surface brightness in that bin. This surface brightness profile was then compared with the parent profile and a $\chi^2$ value was calculated using the same uncertainties as for the observations. The fraction of representations which had $\chi^2$ greater than or equal to that observed was then used to give the quality of the fit.

The first column of Table 1 gives the range of LOAM data points (counting from the centre) used in each comparison. The full data set contains 20 points, of which the outermost eight were used in estimating the magnitudes. The reason for starting some comparisons at the second data point is coupled with the reason for the numbers listed in column 2. While, as was argued above, the fainter stars, which have been neglected in producing these profiles, do not contribute a significant amount of light on average, the Monte Carlo sampling can occasionally introduce anomalies. For the innermost point of maximum-expansion profile there less than three stars in each of the mass classes. Quite frequently, the simulated counts in this bin would be zero, giving an undefined surface brightness. To alleviate this problem, and to test the assumption that the faint stars are relatively unimportant, a background surface brightness for the centre was assumed in these cases and these are listed in column 2. The value of 16.89 is the observed surface brightness in the bin. The choice for a background surface brightness did not have any effect on the probabilities. Hence the neglect of the faint stars is justified. Alternatively, we could just ignore the central bin and start at the second, since the innermost data point only reinforces the trend already established by those further out. What this demonstrates is that the poor fit of the core-collapse profile is not exclusively due to the central datum. There is little effect except to make the core-collapse profiles somewhat more probable than the case where the central point is included; an unsurprising result.

The rightmost two columns of Table 1 repeat these comparisons, but include the uncertainty in the stellar magnitudes. Since the comparison is being made in terms of surface brightness, and since I have adopted the magnitudes based on fits to the outer part of the observed surface brightness profile, any systematic uncertainties in either the distance to NGC 6397 or the magnitudes will balance out. On the other hand, the fits do allow for some tradeoff in magnitude between the three mass bins. This set of comparisons allows for these uncertainties by assuming that the probability that a given magnitude is the correct magnitude for each bin is normally distributed about my adopted magnitudes with a $1\sigma$ dispersion of 0.3 magnitudes. With this assumption, both model profiles are more probably fit, but the maximum-expansion phase is still the more strongly favoured. The reason for the differences in the probabilities is that outermost points contribute much less strongly to the $\chi^2$ distribution when the magnitude errors are included. The high probabilities assigned to the maximal-expansion model when the magnitude uncertainty is included suggests that 0.3 magnitudes are a conservative estimate of these errors. The two sets of comparisons provide limiting cases on the magnitude uncertainties.

| Points | $SB_{bgd}$ | $P(\chi^2_{cc})^c$ | $P(\chi^2_{cc})^d$ | $P(\chi^2_{exp})^c$ | $P(\chi^2_{exp})^d$ |
|--------|-----------|-----------------|-----------------|-----------------|-----------------|
| 1–20   | 16.89     | 0.18            | 0.96            | 0.81            | 1.00            |
| 1–20   | 18        | 0.19            | 0.97            | 0.80            | 1.00            |
| 1–20   | 20        | 0.17            | 0.97            | 0.81            | 1.00            |
| 2–20   | 16.89     | 0.21            | 0.95            | 0.84            | 0.999           |
| 2–20   | 18        | 0.24            | 0.95            | 0.86            | 0.9995          |
| 2–20   | 20        | 0.22            | 0.95            | 0.86            | 1.00            |
| 1–12   | 16.89     | 0.21            | 0.97            | 0.57            | 0.998           |
| 1–12   | 18        | 0.21            | 0.97            | 0.57            | 0.995           |
| 1–12   | 20        | 0.23            | 0.96            | 0.57            | 0.996           |
| 2–12   | 16.89     | 0.25            | 0.97            | 0.61            | 0.996           |
| 2–12   | 18        | 0.25            | 0.96            | 0.62            | 0.997           |
| 2–12   | 20        | 0.23            | 0.96            | 0.62            | 0.993           |

$^a$Counting from centre
$^b$Central background surface brightness, see text
$^c$Sampling error only
$^d$Sampling and magnitude errors
On the basis of Table 1, the core-collapsed profile cannot be ruled out, but the maximum expansion phase, with a core radius of $6'' (0.06 \, \text{pc})$, is to be preferred.

### 3 DISCUSSION

Given this probable resolution of a $6''$ core in NGC 6397, what can be said about its dynamical state? One possibility is that NGC 6397 had a population of primordial binaries which have given the cluster a large core (Gao et al. 1992). Alternatively, if NGC 6397 is undergoing gravothermal oscillations, then it is not so surprising that we have caught it in the maximum expansion phase, as can be seen in Fig. 1.

However, due to the sampling problems involved, an unresolved core cannot be ruled out even for this nearby cluster. Even with the repaired HST PSF, surface brightness data are unlikely to help in clarifying the matter. The surface brightness profile is dominated by the brightest stars and there are too few of them to give a good statistical sample. To get around this, more stars are required. This could be achieved by using star counts which go further down the main sequence, but high resolution data will be needed to overcome the crowding in the core. If the distorting effects of the colour gradient can be ironed out (and the number of stars involved is few enough that overall star counts should not be affected) then a thorough set of high resolution star counts should unambiguously discriminate between the core-collapse and maximum-expansion phases and set firm bounds on the core radius of NGC 6397.

There are two main points to consider in extending this comparison to other globular clusters. On the positive side, more populous clusters have more stars. This reduces the noise associated with sampling. On the negative side, all the other clusters are further away from us than is NGC 6397, most considerably further. The greater distance and the higher crowding associated with higher densities increase the resolution requirements for resolving the core and limits how far down the main sequence we can count stars. M15 is 2.7 magnitudes brighter than NGC 6397, and all other things being equal, this corresponds to a improvement in the signal to noise of the counting by a factor of 3.5. On the other hand, it is 4.4 times further from us than NGC 6397, giving an expected core radius of order $1''$, and the extra gain in sampling is lost in the relatively larger area of the cluster we are required to average over. This core radius is consistent with the limit set by Yanny et al. (1993). With a repaired HST the large wings of the current point spread function will disappear and the diffuse light would once again be available for measuring the surface brightness profile at high resolution. In this case discrimination on this basis may be possible for M15. What is required to definitively resolve these very small cores is a combination of a large number of stars in the core of the cluster of interest and high resolution. The defect of surface brightness profiles is that they are hostage to the brightest stars, which are proportionately few in number. Much better in terms of counting statistics would be extensive star counts probing the main sequence. Even higher resolution observations are then needed due to the high degree of crowding. And even with the counts in hand, there may just be too few stars in any given cluster. These requirements highlight the difficulty of observing post-collapse cores in all but the nearest clusters.

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