A Comprehensive Survey on Curriculum Learning
Xin Wang, Member, IEEE, Yudong Chen, and Wenwu Zhu, Fellow, IEEE

Abstract
Curriculum learning (CL) is a training strategy that trains a machine learning model from easier data to harder data, which imitates the meaningful learning order in human curricula. As an easy-to-use plug-in tool, the CL strategy has demonstrated its power in improving the generalization capacity and convergence rate of various models in a wide range of scenarios such as computer vision and natural language processing, etc. In this survey article, we comprehensively review CL from various aspects including motivations, definitions, theories, and applications. We discuss works on curriculum learning within a general CL framework, elaborating on how to design a manually predefined curriculum or an automatic curriculum. In particular, we summarize existing CL designs based on the general framework of Difficulty Measurer + Training Scheduler and further categorize the methodologies for automatic CL into four groups, i.e., Self-paced Learning, Transfer Teacher, RL Teacher, and Other Automatic CL. Finally, we present brief discussions on the relationships between CL and other methods, and point out potential future research directions deserving further investigations.

Index Terms
Curriculum Learning, Machine Learning, Training Strategy, Example Reweighting, Self-Paced Learning.

I. INTRODUCTION
Human learning has inspired various algorithm designs throughout the development of machine learning. As an outstanding feature of human learning, curriculum, or learning in a meaningful order, has been exploited and transferred to machine learning, which forms the subdiscipline named curriculum learning (CL). In essence, human education is highly organized as curricula, by “starting small” and gradually presenting more complex concepts. For example, to learn calculus at college, a student should first learn basic arithmetic at primary school, abstract function at middle school, and then derived function at high school. However, in traditional machine learning algorithms, all the training examples are randomly presented to the model, ignoring the various complexities of data samples and the learning status of the current model. Therefore, an intuitive question is: “could the curriculum-like training strategy ever benefit machine learning?” According to the extensive experiments from early work [6], [41], [106] to recent efforts [15], [25], [29], [68] in various applications of machine learning, we may summarize the answer as: “yes, but not always.” As we will demonstrate in this survey, the power of introducing curriculum into machine learning depends on how we design the curriculum for specific applications and datasets.

Fig. 1. Illustration of the Curriculum Learning (CL) concept (The fruit images are from [84]). CL is a training strategy for machine learning that trains from easier data to harder data, imitating human curricula. Specifically, CL initially trains the model on a small and easy subset. With the progress of the training, CL gradually introduces more harder examples into the subset, and finally trains the model on the whole training dataset. This CL strategy can improve both model performance and convergence rate, compared with direct training on the whole training dataset. $Q_t$ here stands for a reweighting of the training data distribution $P$ at the $t$-th training epoch (See details in Sec. II).

Xin Wang, Yudong Chen, Wenwu Zhu are with the Department of Computer Science and Technology, Tsinghua University, Beijing, China.
E-mail: xin_wang@tsinghua.edu.cn, cyd18@mails.tsinghua.edu.cn, wwzhu@tsinghua.edu.cn.
The concept of CL is first proposed in [6]. In short, curriculum learning means “training from easier data to harder data”. More specifically, the basic idea is to “start small” [13], train the machine learning model with easier data subsets (or easier subtasks), and then gradually increase the difficulty level of data (or subtasks) until the whole training dataset (or the target task(s)). An illustration of CL is demonstrated in Fig. 1 where we take the task of image classification as an example. Initially, CL trains the model on a small subset of “easy” images, i.e., the images of apples and oranges are clear, typical, and easily recognizable. With the progress of model training, CL adds more “harder” images (i.e., harder to recognize) to the current subset, which is akin to the increasing difficulty of learning materials in human curricula. Finally, CL leverages the whole training dataset for training.

As the idea of CL serves as a general training strategy beyond specific machine learning tasks, scholars have been exploiting its power in considerably wide application scopes, including supervised learning tasks within computer vision (CV) [27], [32], natural language processing (NLP) [68], [90], healthcare prediction [12], etc., various reinforcement learning (RL) tasks [18] as well as other applications such as network embedding [70] and neural architecture search (NAS) [28]. Different CL algorithms have been proposed in the past decade, including predefined CL, which predefines the curricula by human prior, and automatic CL, which automatically derives the curricula from the model and dataset. The CL algorithms in numerous applications provide evidence for its strong generalization capacity and practicability in miscellaneous real-world scenarios. The advantages of CL training strategies can be mainly summarized as improving the model performance on target tasks and accelerating the training process, which cover the two most significant requirements in major machine learning research. More importantly, in many CL algorithms, the CL acts as a flexible plug-and-play submodule which is independent of the original training algorithms. However, to the best of our knowledge, little effort has been made to systematically summarize the methodologies and applications of CL.

In this paper, we fill this gap by comprehensively reviewing CL and summarizing its methodologies. To be more specific, we hope to provide the readers with an overall picture of CL, which includes the comprehensible and elaborate answers to the following questions:

- What is the definition of CL (Sec. II)? We briefly introduce the development history, formalized definition, and extension of the CL concept.
- Why is CL effective, and why should researchers use CL (Sec. III)? We summarize existing analysis about the reasons why CL helps to improve model performance and training efficiency. Based on these analyses, we categorize the existing application scenes of CL by two main motivations for applying CL: to guide and to denoise, as shown in Table I.
• How to design a curriculum? A general framework for curriculum design, i.e., “Difficulty Measurer + Training Scheduler”, is shown in Fig 2(a) (Sec. IV). We introduce the common techniques for manually designing the two core components in this framework, i.e., Difficulty Measurer and Training Scheduler, with some representative examples. These CL methods are denoted as predefined CL (Sec. IV-B).

• How can we automatically design a curriculum (Sec. IV-C)? The main obstacle to the wider application of CL is that it is often hard to accurately predefine the Difficulty Measurer and Training Scheduler for a specific task or dataset by human priors. Thus, a growing trend of CL is to design automatic CL algorithms that could let the machine design the most suitable curricula to a specific task and data. We demonstrate and compare four major methodologies for automatic CL with cutting-edge representative literature, including Self-Paced Learning (SPL), Transfer Teacher, RL Teacher, and Other Automatic CL.

We conclude the paper with a comparison of “easier first” and “harder first” training strategies and presenting a relation graph connecting CL and other related machine learning concepts in Sec. V. We also summarize some open questions and future directions for CL to inspire future researchers in Sec. VI.

Scope. There are two surveys related to our paper, both of which focus on the CL for reinforcement learning (RL). In particular, Narvekar et al. [64] elaborate introduce the formalized definition, evaluation measures, dimensions of categorization of CL, as well as methodologies for the three central elements of the CL framework for RL tasks and open tasks. Portelas et al. [69] focus mainly on automatic CL (ACL) for deep RL, by analyzing the motivation, controlled aspects, and optimization targets of ACL. We refer readers who are interested in RL domains to these two surveys for more details. Basically, we differ from these works in that we systematically and comprehensively review the CL literature mainly in supervised/unsupervised learning domains instead of RL domains, and provide a complete picture of the CL concepts, applications, general framework, and methodologies.

II. DEFINITION OF CL

The idea of introducing a curriculum into the training strategy of machine learning algorithms can be traced back at least to Selfridge et al. [79]. The authors proposed to train a cart pole controller, a classic problem in robotics, first on long and light poles and then gradually on shorter and heavier poles. Later related work [75, 78] in RL and robotics domains also discussed how to organize the presenting order of tasks from easy to hard. The first attempt of the curriculum-like idea on supervised learning is made by Elman [13] in the NLP task of grammar learning with recurrent networks. The author highlighted the importance of “starting small”; restricting the range of data exposed to neural networks during initial training. This strategy is also revisited in [74] and [39], the latter of which provides evidence for faster convergence.

Based on all these previous works, the concept of CL was first proposed by Bengio et al. [6] with experiments on supervised visual and language learning tasks, exploring when and why a curriculum could benefit machine learning. The original definition of CL by Bengio et al. [6] is as follows.

Definition 1: Curriculum. A curriculum is a sequence of training criteria over $T$ training steps: $C = \langle Q_1, \ldots, Q_t, \ldots, Q_T \rangle$. Each criterion $Q_t$ is a reweighting of the target training distribution $P(z)$:

$$Q_t(z) \propto W_t(z)P(z) \quad \forall \text{example } z \in \text{training set } D,$$

such that the following three conditions are satisfied:

1. The entropy of distributions gradually increases, i.e., $H(Q_t) < H(Q_{t+1})$. It means the diversity and information of training set should gradually increase.

2. The weight for any example increases, i.e., $W_t(z) \leq W_{t+1}(z) \quad \forall z \in D$. This means to gradually add more training examples during training.

3. In the final steps, all examples are uniformly sampled, i.e., $Q_T(z) = P(z)$.

Definition 2: Curriculum Learning. Curriculum learning is the training strategy that trains a machine learning model with a curriculum defined above.

The above definition is illustrated in Fig 1. As shown in the figure, the curriculum learning strategy determines the training data subset of each training step, such that the size and difficulty of the subsets are gradually increasing throughout the training process.

Since the concept of CL was formally proposed, the academic community follows and further extends the definition of CL. In a broad sense, most of the CL literature follows the spirit of “training from easier data to harder data”, while the subject of easiness may vary among different works. For example, in multi-task learning settings [65, 76], the scholars train the model by each time selecting the easiest or most correlated task(s) to the previous task(s) in domain adaption settings, some authors [111] propose to gradually expands the training set from a purely-in-domain subset (which is easier) to a not-so-in-domain larger dataset. In RL settings, it becomes a widely adopted strategy to train the RL agent from easy (sub)tasks to harder tasks (until target) [17]. Moreover, other works also extend the definition of CL by adding more dimensions of data characteristics to the “easy to hard” strategy for different purposes in applications. For instance, Jiang et al. [33] propose to train “from easy & diverse to hard” to avoid overfitting to the same sample group in multi-group event detection tasks. In
addition, Wang et al. [98] train the model “from easy & imbalanced to hard & balanced” data to alleviate the severe class imbalance in the task of human attribute analysis.

At a more abstract level, a curriculum can actually be seen as a sequence of data selection or example reweighting along the training process to achieve faster convergence or better generalization, which is beyond the “easy to hard” or “starting small” principles. This perspective inspires the academic community to bring more connotations to CL definition with new methodologies. As we will discuss in Sec. IV-C, the automatic CL methodologies in “RL Teacher” and “Other Automatic CL” (mostly) categories could learn to automatically and dynamically select the most suitable examples or tasks (with adjustable loss weights) for the current training step. Interestingly, in some of the works, the best curriculum found by the algorithm is the opposite of traditional CL, i.e., “hard to easy” [15], [96] or “starting big” (from full dataset to informative subset) [96], [97], [112]. A discussion on this seemingly paradoxical phenomenon will be made in Sec. V.

To even further broaden the scope of CL, some scholars jump from data level to strategy level, to regard a curriculum as a sequence of training strategies during the training process. The training strategies here include, but are not limited to loss function [77], [101], dropout operation [63], input scheme [4], and hypothesis space [28]. For example, in Curriculum Dropout [63], the algorithm gradually reduces the ratio of active units in dropout operation from 1 to a predefined $\theta_0 \in (0, 1)$ to achieve adaptive regularization during training. In Curriculum NAS [28], the algorithm starts from a small search space and gradually incorporates the learned knowledge to guide the search in larger spaces, which significantly improves the search efficiency and also finds better neural architectures. These works broaden the extension of CL and exploit the potentialities of the human curriculum idea for machine learning at a higher level, leaving room for imagination for future work.

III. ANALYSIS ON EFFECTIVENESS OF CL AND SUITABLE APPLICATION SCENES

In the last two sections, we have mentioned some application examples of CL in various fields including CV, NLP, RL, etc. As reported in most of the works, CL algorithms help improve the model performance on target tasks and/or accelerate the training process. Before applying CL to their own studies, researchers might be curious about a fundamental question: why on earth does this human-curriculum-like training strategy work? To explain why CL could lead to generalization improving and convergence speedup, scholars have provided hypotheses and proofs from different perspectives. Basically, existing analysis uncover the essence of CL from the perspectives of optimization problem and data distribution, based on which we can further summarize the 2 main motivations for applying CL: to guide and to denoise.

A. Theoretical Analysis on CL

To begin with, from the perspective of optimization problem, Bengio et al. [6] initially point out that CL can be seen as a particular continuation method. Intuitively, continuation methods are optimization strategies for non-convex criteria which first optimize a smoother (and also easier) version of the problem to reveal the “global picture”, and then gradually consider less smoothing versions, until the target objective of interest. This strategy also shares the same spirit with simulated annealing. As illustrated in Fig 3, continuation methods provide a sequence of optimization objectives, starting with a heavily smoothed objective for which it is easy to find a global minimum, and tracking the local minima throughout the training. In this way, continuation methods guide the training towards better regions in parameter space, i.e., as shown in Fig 3, the local minima learned from easier objectives have better generalization ability and are more likely to approximate global minima. Moreover, from the view of transfer learning, this continuation strategy can be also regarded as a sequence of unsupervised pre-training: training on the preceding objectives could act as a pre-training process which both helps optimization and provides regularization on succeeding objectives.

![Fig. 3. Illustration of the continuation method from [5], which is the essence of the CL strategy [6]. It starts from optimizing a heavily smoothed version of the objective, and gradually moves to the target objective. Tracking the local minima throughout the training actually guides the model to better parameter space and makes it more generalizable.](image-url)
Additionally, recent studies provide more theoretical evidence for the convergence speedup in CL from the optimization perspective. Weinshall et al. [100] prove a theorem based on the linear regression problem with SGD training, which states that the expected rate of convergence of gradient descent is monotonically decreasing with the ideal difficulty score of examples, i.e., easier training data leads to convergence speedup. An intuitive observation is that the variance in the gradient directions derived from easier examples is significantly smaller (especially at the initial training steps), which contributes to a higher convergence rate. A follow-up work [29] further proves that CL changes the optimization landscape by the induced prior probability and make the gradients (in the direction of the optimal parameters) overall steeper in the new landscape. The precondition for this theorem is that the prior probability should be positively correlated with the optimal parameters, and more so than with any other parameters, which requires careful curriculum design.

On the other hand, researchers also analyze the CL mechanism from the perspective of data distribution. With the prominence of deep learning, large-scale data sources are required for training deep networks, which are collected and annotated by company users, the web, and crowd-sourcing systems. This kind of big data collection process brings the problem of noisy or “weakly-supervised” datasets, where the noisy data is less cognizable and more likely to be wrongly annotated than other cleaner data. In the CL setting, the noisy data corresponds to harder examples in the datasets while the cleaner data forms the easier part. Since CL strategy encourages training more on the easier data, an intuitive hypothesis is that CL learner wastes less time with the harder and noisy examples to achieve faster training [6]. This hypothesis reveals the denoising efficacy of CL on big/noisy data.

To have a closer look at this denoising mechanism, Gong et al. [23] provide a theory based on the assumption that there exists deviation between training and testing distributions caused by noisy/wrongly-annotated training data. Intuitively, training and target/testing distributions share a common high-confidence annotated region with large density, which corresponds to the easier examples in CL. Therefore, to start training from easier examples by CL strategy actually simulates learning from this high-confidence common region (as an approximation to the target distribution), which guides the learning towards the expected target while reduces the negative impacts from low-confidence noisy examples. This data distribution perspective of CL is illustrated in Fig 4. The common density peak (at the center of the x-axis) of training and target distributions $P_{\text{train}}(x)$ and $P_{\text{target}}(x)$ in the left part refers to the common high-confidence area, while the heavy tail of $P_{\text{train}}(x)$ demonstrates the relatively more noisy data in training distribution. The right part illustrates the sequence of weight functions in CL, which initially assigns small values to the noisy tails and much larger values in the common easy area, and gradually moves to equal weights for all examples. Based on the above analysis, the authors formulate $P_{\text{target}}(x)$ as the weighted expression of $P_{\text{train}}(x)$. A follow-up theory clarifies that CL essentially minimizes an upper bound of the expected risk under target distribution, and this bound shows that we could approach the task of minimizing the expected risk on $P_{\text{target}}(x)$ by taking the core idea of CL: gradually taking relatively easy examples according to the curriculum and minimizing the empirical risk on these examples.

![Fig. 4. Illustration of the CL from the data distribution perspective [23]. The left part demonstrates the data distribution shift from the easy subset (the black curve, which is assumed to approximate the testing distribution $P_{\text{target}}(x)$ well) to the full training set $P_{\text{train}}(x)$ (the red dashed curve). The right part shows the corresponding weighting scheme to enable this distribution shift. The center peak of curves refers to the high-confidence clean data, while the tails refer to the noisy data in the distributions. As shown in the left part, $P_{\text{target}}(x)$ is cleaner than $P_{\text{train}}(x)$.

B. Suitable Application Scenes of CL

Based on the above analysis on why CL is effective, we can categorize the motivations for applying CL into 2 groups: to guide, regularizing the training towards better regions in parameter space (with steeper gradients) as from the perspective of the optimization problem, and to denoise, focusing on high-confidence easier area to alleviate the interference of noisy data as from the perspective of data distribution. Not surprisingly, most of the existing application scenes of CL can be classified into these 2 motivation groups, as demonstrated in Table [1].

The application scenes based on the “to guide” motivation often involves difficult target tasks where direct training on these tasks results in poor performance or slow convergence. CL strategies are adopted to guide the training from easier tasks or smoother versions of objectives to the target tasks. For instance, in sparse-reward RL settings, direct training on the final tasks rarely gets any positive rewards, which obviously hinders agent learning. To alleviate this problem, researchers propose to take the CL strategy and manually [59] or automatically [18] design a sequence of auxiliary (sub)tasks/goals from easy to hard to guide the training. In multi-task learning settings, learning all the tasks simultaneously or in random order often leads to unsatisfactory performance. To yield performance gains, CL strategies are adopted to automatically choose the easier tasks which are more related to the previous one [60] or bring more learning progress to the current model [25], [59].
Scheduler, which is illustrated in Fig 2(a). To begin with, all the training examples are sorted by the Difficulty Measurer from judgment of the Difficulty Measurer. This measurement of example difficulty is predefined by the relative “easiness” of each data example. In predefined CL methods, this measurement of example difficulty is predefined directly from the whole training set. This schedule sometimes also depends on the training loss feedback from the Model Trainer (the dashed arrow in Fig 2(a)), e.g., Training Scheduler presenting more harder data when the current model converges. Note that CL for domain adaption is also related with the “to denoise” motivation, if we regard the less in-domain data as a kind of noisy data. Another example is imbalanced classification problems, where the training distribution on different classes is extremely imbalanced (e.g., bald v.s. not bald in human attribute analysis). Different studies adopt curricula either from balanced subset to more imbalanced full dataset [32] or from easy and imbalanced subset to harder and more balanced subset [98] to improve the generalization capacity of the classifier.

On the other hand, the application scenes based on the “to denoise” motivation often has a noisy or heterogeneous (large-scale) training dataset, and CL strategies could help denoise, making the training faster, more robust, and more generalizable. A popular application of CL with this motivation is neural machine translation (NMT), the dataset of which is highly heterogeneous in quality, difficulty, and noise [40]. This is because the translation of a sentence could be both long and short with different vocabulary and grammar structures, and different annotators always provide translations with different qualities due to the fact that translation requires more expert knowledge than other tasks. Additionally, the training of NMT models (e.g., RNNs) is often time-consuming. Therefore, CL is naturally suitable for NMT tasks to denoise during training and to achieve both performance boost and faster convergence. Similarly, CL is also adopted in other NLP tasks with noisy or heterogeneous data, including natural language understanding [102], relation extraction [30], reading comprehension [90], etc. Moreover, CL is also effective in weakly-supervised CV tasks [27], [51] where the dataset is often large-scale, collected from the web, and thus noisy.

### IV. CL Design: A General Framework

Since we have understood why CL is effective and why researchers apply CL to different scenes, a following and important question should be: how could I design an appropriate curriculum for my specific learning task? In this section, we provide a general framework of “Difficulty Measurer + Training Scheduler” (Sec. IV-A), which unifies most of CL methodologies. Based on this framework, we categorize the existing CL methods into predefined CL (Sec. IV-B) and automatic CL (Sec. IV-C) and introduce the representative designs in each category. Fig. 2 illustrates the typology of CL methods introduced in this section.

#### A. The General Framework of Difficulty Measurer + Training Scheduler

Recall the definition of CL of “training from easier data to harder data”. In essence, to design a such curriculum, we need to decide two things: 1) What kind of training data is supposed to be easier than other data? 2) When should we present more harder data for training, and how much more? The issue 1) can be abstracted to a Difficulty Measurer, which decides the relative “easiness” of each data example. In predefined CL methods, this measurement of example difficulty is predefined by human experts, while automatic CL methods let the machine to measure the difficulty. On the other hand, the issue 2) can be abstracted to a Training Scheduler, which decides the sequence of data subsets throughout the training process based on the judgment of the Difficulty Measurer.

Therefore, a general framework for curriculum design consists of these two core components: Difficulty Measurer + Training Scheduler, which is illustrated in Fig 2(a). To begin with, all the training examples are sorted by the Difficulty Measurer from the easiest to the hardest and passed to the Training Scheduler. Then, at each training epoch $t$, the Training Scheduler samples a batch of training data from the relatively easier examples, and send it to Model Trainer for training. With the progressing of training epochs, Training Scheduler will decide when to sample from more harder data, (usually) until uniform sampling from the whole training set. This schedule sometimes also depends on the training loss feedback from the Model Trainer (the dashed arrow in Fig 2(a)), e.g., Training Scheduler presenting more harder data when the current model converges. Note that in [29], the authors conclude the two core components as scoring function and pacing function, which share the same spirit with Difficulty Measurer and Training Scheduler, respectively, while the latter names are chosen to be more abstract and clearer.

Let us take the experiment in Fig 1 as an instantiation example for this CL framework. In that experiment, Difficulty Measurer is the annotation by a human expert, who decides that some fruit images in the dataset are easier than other images.

| Motivation | Effect | Scene | Examples |
|------------|--------|-------|----------|
| To guide   | make training possible / better and faster | the target task is hard or has a different distribution | sparse reward RL, multi-task learning, GAN training, NAS; domain adaption, imbalanced classification |
| To denoise | make training faster, more robust and generalizable | tasks with noisy, uneven quality, heterogeneous data (often large-scale, cheaply collected) | NLP tasks (neural machine translation, natural language understanding, etc.), large-scale image classification |

**TABLE I: Suitable Application Scenes of CL.**

### IV. CL Design: A General Framework

In addition, the “to guide” application scenes also include the tasks where the target distribution is quite different from the training distribution, and a curriculum helps to guide the training for adaption to the target distribution. A representative scene is domain adaption, which task aims at improving prediction on unlabeled target domain data by knowledge transfer from richly annotated source domain data with a distribution drift. Recent studies [83], [11] propose to train from more in-domain data (similar to target domain) to less in-domain data, guiding the model to adapt to the target domain while adequately exploiting the source domain data. Note that CL for domain adaption is also related with the “to denoise” motivation, if we regard the less in-domain data as a kind of noisy data. Another example is imbalanced classification problems, where the training distribution on different classes is extremely imbalanced (e.g., bald v.s. not bald in human attribute analysis). Different studies adopt curricula either from balanced subset to more imbalanced full dataset [32] or from easy and imbalanced subset to harder and more balanced subset [98] to improve the generalization capacity of the classifier.

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according to their recognizability and complexity. In addition, Training Scheduler can be, for example, a linear scheduler (see Sec. IV-B2 for details) that starts with 40% of easiest examples in each class, and increase this proportion by 5% each epoch until 100%. In this way, an effective curriculum is successfully designed by filling the general CL framework with appropriate Difficulty Measurer and Training Scheduler according to the specific task of image classification.

According to this general framework, we could also clarify the scopes of predefined CL and automatic CL in the next two sections. Specifically, when both the Difficulty Measurer and Training Scheduler are totally designed by human prior knowledge with no data-driven models or algorithms involved, we call the method **predefined CL**. On the other hand, if any (or both) of the Difficulty Measurer and Training Scheduler are learned by data-driven models or algorithms, then we denote the CL method as **automatic CL**. In some of the automatic CL methods (e.g., MentorNet [35] in Sec. IV-C4), the Difficulty Measurer and Training Scheduler can also be learned as one model, which simultaneously measures the example-wise difficulty and determines the data selection/reweighting. We will elaborate on the common techniques of predefined CL in Sec. IV-B and popular methodologies for automatic CL in Sec. IV-C.

B. Predefined CL

In this section, we discuss the common types of manually predefined Difficulty Measurers (Sec. IV-B1) and Training Schedulers (Sec. IV-B2) under our general CL framework with some representative examples and insights. We also conclude the main limitations of the predefined CL in Sec. IV-B3.

1) Common Types of Predefined Difficulty Measurer: Researchers have manually designed various Difficulty Measurers mainly based on the data characteristics of specific tasks. We summarize common types of Difficulty Measurer in Table II. Most of the predefined Difficulty Measurers are designed for image and text data in various CV and NLP scenarios, while other data types include audio data, programs, tabular data, etc. Interestingly, we find that except some domain knowledge based measurement (marked as “Domain”), most of the predefined Difficulty Measurers are designed from the angles of complexity, diversity, and noise estimation, which are separate but also related.

| Difficulty Measurer* | Angle | Data Type | ∝Easy |
|----------------------|-------|-----------|-------|
| Sentence length [68], [85] | Complexity | Text | - |
| Number of objects [99] | Complexity | Images | - |
| # conj. [38], #phrases [91] | Complexity | Text | - |
| Parse tree depth [91] | Complexity | Text | - |
| Nesting of operations [106] | Complexity | Programs | - |
| Shape variability [6] | Diversity | Images | - |
| Word rarity [38], [68] | Diversity | Text | - |
| POS entropy [91] | Diversity | Text | - |
| Mahalanobis distance [12] | Diversity | Tabular | - |
| Cluster density [106] | Noise | Images | + |
| Data source [9] | Noise | Images | / |
| SNR / SND [7], [71] | Noise | Audio | - |
| Grammaticality [53] | Domain | Text | + |
| Prototypicality [91] | Domain | Text | + |
| Medical based [36] | Domain | X-ray film | / |
| Retrieval based [16], [65] | Domain | Retrieval | / |
| Intensity [24] / Severity [89] | Intensity | Images | + |
| Image difficulty score [84], [92] | Annotation | Images | - |
| Norm of word vector [55] | Multiple | Text | - |

* Abbreviations: POS = Part Of Speech, SNR = Signal to Noise Ratio, SND = Signal to Noise Distortion, Domain = Domain knowledge, # conj. = number of coordinating conjunctions.

To begin with, complexity stands for the structural complexity of a particular data example, such that examples with higher complexity have more dimensions and are thus harder to be captured by models. For instance, sentence length, the most popular Difficulty Measurer in NLP tasks (e.g., dependency parsing [35], reading comprehension [90], machine translation [68], etc.), intuitively expresses the complexity of a sentence/paragraph. Therefore, a longer sentence is often supposed as a harder training example. Other examples include the number of objects in images in the task of semantic segmentation [99]; the number of coordinating conjunctions (e.g., “and”, “or”) [38] or phrases (e.g., prepositional phrases) [91]; the parse tree depth [91] (i.e., the depth of the grammar tree of a sentence), which measures the sentence complexity in the view of grammar; and the nesting of operations in program text [106], which measures the complexity of the instruction set of program text in program execution tasks.

In addition, the angle of diversity here stands for the distributional diversity of a group of data (e.g., regular or irregular shapes [6]) or the elements (e.g., words) of a data point (e.g., sentence). A larger value of diversity means the data is more various, including more (rare) types/styles of data or elements, and is thus more difficult for model learning. For example, a
sentence with more rare words is usually considered harder for learning \cite{68}. A popular measure of diversity is information entropy, which is exploited both in text data as the Part-Of-Speech (POS) entropy \cite{91} and in tabular data as the Mahalanobis distance of feature vectors \cite{12}. Note that intuitively, both high complexity and high diversity brings more degrees of freedom to the data examples, which needs a model with larger capacity and bigger effort of training.

Larger diversity sometimes also makes the data noisier. Therefore, another angle of Difficulty Measurer is noise estimation, which estimates the noise level of data examples and defines cleaner data as easier. A quite intuitive method is taken in \cite{9} to judge the noise level by the source of image data on the web: images retrieved by a search engine like Google are supposed to be cleaner, and images posted on photo-sharing website like Flickr are more realistic and noisier. In CurriculumNet \cite{27}, the authors map images to vectors by CNNs and suppose that the cleaner images often have similar visual appearance, and thus have larger values of local density. Therefore, examples with lower local density are supposed to be noisier and harder to predict. Moreover, the Signal to Noise Ratio/Distortion (SNR/SND) \cite{7}, \cite{71} is widely adopted to estimate the noise in audio data.

Other interesting Difficulty Measurers include signal intensity \cite{26}, \cite{89} and human annotation based Image Difficulty Scores \cite{84}, \cite{92}, both designed for image data. Signal intensity can be regarded as a measurement for the informativeness of data features. For example, in the task of facial expression recognition \cite{26}, more intense/exaggerated faces are supposed to be easier data than poker faces. In the task of thoracic disease diagnosis \cite{89}, more severe symptoms provide more information and are easier to recognize. Moreover, Image Difficulty Score \cite{92} is proposed to measure the difficulty of an image by collecting the response times of human annotators in the following protocol: (i) ask the annotator “Is there an \{object class\} (e.g., elephant) in the next image?” and (ii) record the time spent by the annotator to answer “Yes” or “No” and use this response time to estimate Image Difficulty Score: intuitively, longer response time corresponds to harder image example. After collecting the annotation, the authors train a regression model to map the CNN features of new images to the difficulty score.

2) Common Types of Predefined Training Scheduler: While predefined Difficulty Measurers vary among different data types and tasks, the existing predefined Training Schedulers are usually data/task agnostic, i.e., the majority of CL literature in various scenarios leverages similar types of Training Schedulers. Generally, Training Schedulers can be divided into discrete and continuous schedulers. The difference is: discrete schedulers adjust the training data subset after every fixed number (> 1) of epochs or convergence on the current data subset, while continuous schedulers adjust the training data subset at every epoch.

Discrete schedulers are widely adopted owing to its simplicity and effectiveness. The most popular discrete scheduler is named as Baby Step \cite{6}, \cite{85} (Algorithm 1), which first distributes the sorted data into buckets from easy to hard and starts training with the easiest bucket. After a fixed number of training epochs or convergence, the next bucket is merged into the training subset. Finally, after all the buckets are merged and used, the whole training process either stops or further continues several extra epochs. Note that at each epoch, the scheduler usually shuffles both the current buckets and the data in each bucket and then sample mini-batches for training (instead of using all data at once).

Algorithm 1 The Baby Step Training Scheduler \cite{11}.

\begin{verbatim}
Input:  \( D \): training dataset; \( \mathcal{C} \): the Difficulty Measurer;
Output: \( M^\ast \): the optimal model.
1: \( D' = \text{sort}(D, \mathcal{C}) \);
2: \( \{D^1, D^2, \ldots, D^k\} = D' \text{ where } \mathcal{C}(d_a) < \mathcal{C}(d_b), \ d_a \in D^i, d_b \in D^j, \forall i < j \);
3: \( D^{train} = \emptyset \);
4: for \( s = 1 \cdots k \) do
5: \( D'^{train} = D'^{train} \cup D^s \);
6: while not converged for \( p \) epochs do
7: \( \text{train}(M, D'^{train}) \);
8: end while
9: end for
\end{verbatim}

Another discrete scheduler called One-Pass \cite{6} takes a similar strategy of data bucketing from easy to hard and starting training from the easiest bucket. However, after a fixed number of training epochs or convergence, One-Pass scheduler discards the current bucket and switches to the next harder bucket. One-Pass is less used than Baby Step in CL literature (see \cite{90}, \cite{92}, \cite{95}, \cite{106} for One-Pass examples), probably due to the lower performance in many tasks. Intuitive reasons might include: 1) The complexity/diversity of the training data is gradually increasing in Baby Step scheduler, which helps improve generalization capacity; 2) The One-Pass scheduler is like training on a sequence of independent tasks as in multi-task learning and continual learning \cite{44}, which faces the problem of catastrophic forgetting even though the early tasks are easier. Please refer to the \cite{11} for the comparison experiments of the two schedulers on LSTMs.

Other discrete schedulers are also based on data bucketing, but take different sampling strategies. For example, in \cite{38}, the authors modify the Baby Step to unevenly divide the examples into buckets such that easier buckets have more data examples,
which is natural to reach in the case of machine translation corpora. Then they sample examples without replacement from the easiest bucket only until there remain the same number of examples as in the second most easy bucket. Afterward, they uniformly sample from the first two buckets until the size is the same as that of the third bucket. In an empirical study of CL on NMT tasks [110], the authors also test other extensions of Baby Step, including 1) “boost”: to copy the hardest bucket for further training; 2) “reduce and add-back”: to gradually remove one easiest bucket from training set once all buckets have been used, and then add them back and repeat the removing until convergence; 3) “no-shuffle”: to discard inter-bucket shuffling and always present from easier to harder buckets to the model. A conclusion is, including Baby Step, no single scheduler consistently outperforms others.

Continuous schedulers, on the other hand, can be mostly regarded as a function \( \lambda(t) \) to map training epoch number \( t \) to a scalar \( \lambda \in (0, 1] \), which means \( \lambda \) proportion of easiest training examples are available at the \( t \)-th epoch. According to the formal definition of CL in Sec. II, this function \( \lambda(t) \) must be a nonmonotonically decreasing function, starting at \( \lambda(0) > 0 \) and ending at \( \lambda(T) = 0 \) (i.e., all examples are visible and uniformly sampled). This function of continuous scheduler is also called pacing function [29] or competence function [68] in CL literature.

Existing \( \lambda(t) \) functions are various, while researchers could design new functions for their specific tasks. The most intuitive function is the linear function, where \( \lambda_0 \) is the initial proportion of available easiest examples, and \( T_{\text{grow}} \) denotes the epoch when the function reaches 1 for the first time.

\[
\lambda_{\text{linear}}(t) = \min\left(1, \lambda_0 + \frac{1 - \lambda_0}{T_{\text{grow}}} \cdot t\right)
\]  

(2)

Root function is later proposed in [68] according to the observation that in linear function, the newly added examples are less likely to be sampled as the training data subset grows in size. Therefore, to give the model sufficient time to learn the newly added examples, the authors reduce the number of newly added examples as training progresses by defining the rate of adding examples to be inversely proportional to the size of the current training subset: \( \frac{d\lambda(t)}{dt} = \frac{P}{\lambda(T)} \), where \( P \geq 0 \) is a constant. Then we get:

\[
\lambda_{\text{root}}(t) = \min\left(1, \sqrt[2]{\frac{1 - \lambda_0^2}{T_{\text{grow}}} \cdot t + \lambda_0^2}\right).
\]

(3)

To make the curve even sharper, a more general form root-p function is also considered as follows, where \( p \geq 1 \):

\[
\lambda_{\text{root-p}}(t) = \min\left(1, \sqrt[2^p]{\frac{1 - \lambda_0^p}{T_{\text{grow}}} \cdot t + \lambda_0^p}\right).
\]

(4)

Interestingly, in [65] the authors oppositely propose to give easier examples more training time, by taking the following geometric progression function:

\[
\lambda_{\text{geom}}(t) = \min\left(1, 2^{-\frac{\log_2 1 - \log_2 \lambda_0}{T_{\text{grow}}} \cdot t + \log_2 \lambda_0}\right).
\]

(5)

The above continuous scheduler functions are illustrated in Figure 5. Note that training without CL (“baseline”) and Baby Step are also regarded as special cases of continuous schedulers. The experiments in [68] and [65] on NLP tasks show that the root-p function \( (p \geq 2) \) is the most beneficial predefined Training Scheduler for CL, though the relative improvement to other schedulers is not dramatic.

Moreover, there is also a special group of continuous schedulers which do not follow the formal definition of CL, but perform as a sequence of data selection as introduced in Sec. II. We name these schedulers as distribution shift, which start training on an initial distribution and gradually move to a target distribution. For example, in [55], all the examples are divided into 2 groups: Common (lower quality and simpler) and Target (higher quality and more complex). The sampling weights are initially distributed on the Common and gradually shifted to the Target. In [31], to alleviate extreme data imbalance in the lung nodule detection task, the scheduler starts sampling purely from images with nodules to learn to represent nodules, and then gradually decreases the proportion of examples with nodules until the extremely imbalanced data distribution (rare nodule).

3) Limitations of predefined CL: Despite the simplicity and effectiveness of the predefined CL, there are some essential limitations as follows.

- It is difficult to find the most suitable combination of Difficulty Measurer and Training Scheduler for a specific task and its dataset. There are no existing methodologies for selecting Difficulty Measurer and Training Scheduler other than exhaustive trials.
- Both the predefined Difficulty Measurers and Training Schedulers stay fixed during the training process, which is not flexible enough and to some extent ignores the feedback of the current model.
Functions of continuous schedulers

- Expert domain knowledge is often necessary for designing a predefined Difficulty Measurer. Moreover, in some cases where the dimension of example features is large, it is hard to predefine a computable Difficulty Measurer even with expert knowledge.
- Easy examples for humans are not always easy for models, due to the fact that the decision boundaries of models and humans are basically different [105].
- The best hyperparameters of Training Scheduler are hard to find. Additionally, a basic problem in Baby Step scheduler is to decide the number of buckets and how to divide the bucket [3].
- Both the absolute and relative performance of various predefined Training Schedulers are sensitive to the initial learning rate (in NMT task) [110].

These limitations of predefined CL have prevented CL from being explored in more various applications. A natural and critical question is: how can we design more automatic Difficulty Measurers and Training Schedulers, which are more data-driven and model-driven instead of human-driven, more dynamically adaptive to the current training status, and need fewer or even no hyperparameters to fine-tune? We will discuss this question in the next section.

C. Automatic CL

In this section, we take a further step on the curriculum design by introducing automatic CL methods to break through the limits of predefined CL. A general comparison of predefined CL and automatic CL is presented in Table III.

| Issues                  | Predefined CL                      | Automatic CL                    |
|-------------------------|------------------------------------|---------------------------------|
| Applicability           | Need expert domain knowledge       | General, domain agnostic        |
| Difficulty Measurer     | Human defined, fixed               | Model decided, dynamic          |
| Training Scheduler      | Ignore model feedback, fixed       | Consider model feedback, dynamic|

We summarize the four major methodologies for automatic CL: Self-Paced Learning (SPL), Transfer Teacher, RL Teacher, and Other Automatic CL. In predefined CL, the teacher designing the curriculum is a human expert, and the student getting trained by the curriculum is the machine learning model. To reduce the need for human teachers, the four methodologies take different ideas, which can be intuitively summarized in the bullets. Note that while SPL and Transfer Teacher methods are semi-automatic with automatic Difficulty Measurer and predefined Training Scheduler, the RL Teacher and Other Automatic CL methods are fully automatic and adopt the extended definition of CL as a sequence of data selection, example reweighting, or training criteria (Sec. [II]).

- SPL methods let the student himself act as the teacher and measure the difficulty of training examples according to its losses on them. This strategy is analogous to the self-study of human students: one decides his/her own learning pace based on his/her current status.

---

2The hyperparameters include $\lambda_0$, $T_{grow}$ and $p$ (in root-$p$ function) in continuous schedulers, and the number of steps, the number of epochs in each step in Baby Step based schedulers.

3Division by thresholds on difficulty scores makes it hard to assign each bucket with roughly the same number of examples, while division by size may result in fluctuations in difficulty within a bucket or not enough difference between different buckets [110]. An alternative is the Jenks Natural Breaks classification algorithm, as adopted in [110].
Transfer Teacher methods invite a strong teacher model to act as the teacher and measure the difficulty of training examples according to the teacher’s performance on them. Since the teacher model is pretrained and transfers its knowledge to measure example difficulty for student model training, we denote this strategy as Transfer Teacher.

RL Teacher methods adopt reinforcement learning (RL) models as the teacher to play dynamic data selection according to the feedback from the student. This strategy is the most ideal scene in human education, where the teacher and the student improve together through benign interactions: the student makes the biggest progress based on the tailored learning materials that the teacher selects for him, while the teacher also effectively adjusts her teaching strategy to teach better.

Other Automatic CL methods include various automatic CL strategies except for the above-mentioned. The works take different optimization techniques to automatically find the best curriculum for model training, including Bayesian Optimization, meta-learning, hypernetworks, etc. The curriculum in these methods often refers to a sequence of loss weights or even loss functions on data batches.

The comparison of these automatic CL methodologies are in Table IV. Note that as aforementioned, automatic CL is broadly applied to Deep RL tasks, and we refer readers to the recent surveys [64], [69] for further reading. The automatic CL methods discussed in this section are mostly designed for supervised learning settings, though some of them are also shown to be effective in RL tasks [37], [59].

### Table IV

| Characteristic       | Self-Paced Learning | Transfer Teacher | RL Teacher                  |
|----------------------|---------------------|------------------|-----------------------------|
| Difficulty Measurer  | Student-driven difficulty | Teacher-driven difficulty | Teacher select data according to student feedback |
| Training Scheduler   | Predefined          | Predefined       | Automatic                   |
| Strength             | Efficient, robust   | Reliable difficulty | Flexible                   |
| Weakness             | Fixed strategy      | Extra pretraining | Costly (Deep RL)            |

1) Self-Paced Learning: Self-paced Learning (SPL) is a primary branch of CL which automates the Difficulty Measurer by taking the example-wise training loss of the current model as criteria. The concept of “self-paced learning” originates from human education, where the student is able to control the learning curriculum, including what to study, how to study, when to study, and how long to study [93]. Under machine learning settings, SPL refers in particular to a training strategy initially proposed by Kumar et al. [41], which trains the model at each iteration with the easiest proportion of training set according to the model’s current performance, i.e., the examples with the lowest training losses. With the progress of training, this proportion of easiest examples gradually grows to the whole training set, which essentially takes a predefined Training Scheduler as we discussed in Sec. IV-B2. Note that in the literature of SPL, CL and SPL are usually mentioned as two different strategies, where the CL actually refers to the predefined CL in Sec. IV-B. However, in this paper, SPL is regarded as a branch of automatic CL, since it shares the same spirit with CL and fits perfectly with our general CL framework, as shown in Fig 2(b). The most valuable advantages of SPL over predefined CL is mainly two-fold: 1) SPL is semi-automatic CL with loss-based automatic Difficulty Measurer and dynamic curriculum, which makes it more flexible and adaptive for various tasks and data distributions. 2) SPL embeds the curriculum design into the learning objective of the original machine learning tasks, which makes it widely applicable as a plug-in tool.

1-a) The Original Version of SPL: The original SPL algorithm [41] is formally defined as follows. Let \( D = \{x_i, y_i\}_{i=1}^N \) denotes the training set, where \( x_i \) and \( y_i \) is the feature and label of example \( i \) respectively. The machine learning model \( f_w \) with parameters \( w \) maps each \( x_i \) to the model prediction \( f_w(x_i) \), and gets a loss \( l_i = L(f_w(x_i), y_i) \), where \( L \) is the learning objective. The original goal is then to minimize the empirical loss on the whole training set:

\[
\min_{w} \mathbb{E}(w; \lambda) \sum_{i=1}^{N} l_i + R(w),
\]

where \( R(w) \) is a regularizer to encode prior knowledge on \( w \) to avoid overfitting. SPL introduces example weight \( v_i \) into the above learning objective with an SP-regularizer \( g(v; \lambda) \), where \( v = [v_1, v_2, ..., v_N]^T \in [0, 1]^N \) is a vector of weights, and \( \lambda \) is the age parameter, a hyperparameter which controls the learning pace (i.e., as Training Scheduler) and determines the proportion of the easiest selected examples at each training epoch. The new learning objective becomes:

\[
\min_{w;v: v \in [0,1]^N} \mathbb{E}(w, v; \lambda) \sum_{i=1}^{N} v_i l_i + g(v; \lambda).
\]

In the original SPL, \( g(v; \lambda) \) is a negative \( l_1 \)-norm:

\[
g(v; \lambda) = -\lambda \sum_{i=1}^{N} v_i.
\]

\(^4\text{For brevity, we ignore } R(w) \text{ in the following discussion.}\)
The above learning objective is often optimized with the Alternative Optimization Strategy (AOS)\footnote{AOS is also called ASS (Alternative Search Strategy), ACS (Alternative Convex Search)\cite{34}, or CCM (Cyclic Coordinate Method)\cite{32} in SPL literature.}. Concretely, we alternatively optimize $w$ and $v$ while fix the other. With the fixed $w^*$, we calculate the global optimum $v^*$ by solving:

$$v_i^* = \arg \min_{v_i \in [0,1]} {v_i l_i + g(v_i; \lambda), \quad i = 1, 2, \ldots, n}$$

Then, with fixed $v^*$, we learn the global optimum $w^*$:

$$w^* = \arg \min_w \sum_{i=1}^{N} v_i^* l_i.$$  

The two optimization steps are iteratively conducted, while the value of $\lambda$ is gradually increased to add more harder examples. The overall SPL algorithm is shown in Algorithm 2. Note that the time overhead is mainly in the weighted loss minimization step for finding $w^*$, and therefore the SPL algorithm will not increase the time complexities. Moreover, SPL could even accelerate training on noisy datasets, according to the CL principles in Sec. III-A.

Algorithm 2 Self-Paced Learning

```
Input: $D = \{x_i, y_i\}_{i=1}^{N}$: training dataset; $f$: the machine learning model; $T$: the maximum number of iterations;
Output: $w$: the optimal parameters of $f$.
1: Initialize $w$, $v$, $\lambda = \lambda_0$, $t = 0$.
2: while $t \neq T$ do
3: \hspace{0.5cm} $t = t + 1$;
4: \hspace{0.5cm} Update $v^*$ by Eq. 9\footnote{Meng et al.\cite{61} first prove that the AOS strategy in SPL intrinsically accords with the majorization minimization (MM) algorithm\cite{43} on a minimization problem of the above latent SPL objective. Therefore, one could leverage theories of MM to provide analyses of the properties of SPL (e.g., convergence). Additionally, they found\cite{12} $v_i^*$ is the solution in Eq. 9.}
5: \hspace{0.5cm} Update $w^*$ by Eq. 10\footnote{While the solution for Eq. 10 is provided by machine learning algorithms (e.g., gradient descent) for the original task, the solution for Eq. 9 is simple. In fact, in the original SPL, since $g(v; \lambda)$ in Eq. 8 is a convex function of $v$, the global minimum can be easily derived by setting the partial derivative of $\mathbb{E}(w, v; \lambda)$ to $v_i$ as zero. Formally, we have:

$$\frac{\partial \mathbb{E}(w, v; \lambda)}{\partial v_i} = l_i - \lambda = 0.$$}
6: \hspace{0.5cm} Update $\lambda$ to a larger value; // to include harder data
7: end while
```

While the solution for Eq. 10 is provided by machine learning algorithms (e.g., gradient descent) for the original task, the solution for Eq. 9 is simple. In fact, in the original SPL, since $g(v; \lambda)$ in Eq. 8 is a convex function of $v$, the global minimum can be easily derived by setting the partial derivative of $\mathbb{E}(w, v; \lambda)$ to $v_i$ as zero. Formally, we have:

$$\frac{\partial \mathbb{E}(w, v; \lambda)}{\partial v_i} = l_i - \lambda = 0.$$ 

Therefore, considering $v_i \in [0,1]$, we get the following close-formed optimal solution for $v^*$ with the fixed $w^*$:

$$v_i^* = \begin{cases} 1, & l_i < \lambda \\ 0, & \text{otherwise} \end{cases}$$

This solution can be intuitively explained: if an example has a training loss $l_i$ less than the threshold $\lambda$, then it is regarded as an easy example for the current model, and should be selected at the current training epoch (i.e., $v_i^* = 1$). Otherwise, it is hard and should not be selected (i.e., $v_i^* = 0$). When the model becomes more mature, $\lambda$ gets increased and more harder examples get involved in training.

Another remaining issue is how to adjust the threshold $\lambda$, or the so-called age parameter, throughout the training. Initially, $\lambda$ should be set as $\lambda_0$ to ensure that a small proportion of easy examples are selected. Later on, a simple method is to multiply or add a constant at each epoch, i.e., $\lambda_{t+1} = \eta \cdot \lambda_t$ ($\eta > 1$) or $\lambda_{t+1} = \lambda_t + \mu$ ($\mu > 0$), to gradually increase $\lambda$. Finally, $\lambda$ becomes large enough so that all the examples are selected (i.e., $v_i^* = 1 \ \forall i$) for training. This strategy of adjusting $\lambda$ is analogous to predefined continuous Training Scheduler, which plans to increase the size of the training set at each epoch. More methods for the adjustment of $\lambda$ will be discussed later.

1-b) Theories for SPL. Before we discuss variant SPL versions enhanced from different aspects, we briefly summarize some existing theories on SPL. In short, sound theories have been established for the convergence, robustness, and essence of SPL, which supports the wide applications of SPL.

To begin with, the new learning objective Eq. 7 in SPL is equivalent to the following latent objective function:

$$\sum_{i=1}^{N} F_\lambda(l_i) = \sum_{i=1}^{N} \int_{0}^{l_i} v_i^*(\tau, \lambda) d\tau$$

where $v_i^*$ is the solution in Eq. 9. Meng et al.\cite{61} first prove that the AOS strategy in SPL intrinsically accords with the majorization minimization (MM) algorithm\cite{43} on a minimization problem of the above latent SPL objective. Therefore, one could leverage theories of MM to provide analyses of the properties of SPL (e.g., convergence). Additionally, they found
that this latent objective \( \sum_{i=1}^{N} F_{i}(l_{i}) \) is also closely related to the non-convex regularized penalty (NCRP), a well-known machine learning methodology with attractive properties in sparse estimation and robust learning, which provides evidence on the robustness of SPL. Based on this work, the authors further proved that the optimization of \( \sum_{i=1}^{N} F_{i}(l_{i}) \) converges to critical points of the original SPL problem under mild conditions [58].

Moreover, Liu et al. [54] establish a systematic framework for SPL under concave conjugacy theory, which completely tallies with the requirements of SPL models. Based on this framework, they provided proof for the derived relationship among the SP-regularizer \( g(v; \lambda) \), latent objective \( \sum_{i=1}^{N} F_{i}(l_{i}) \), and the example weights \( v \). This result also inspired two general approaches for SPL designs.

1-c) Soft SP-regularizers. As a weighting strategy on the learning objective, the core design of SPL is the SP-regularizer \( g(v; \lambda) \), which directly determines the optimal weights \( v^* \) at each training epoch. Therefore, most of the existing improvements on SPL have been focused on SP-regularizers. Recall that in the original version of SPL, \( g(v; \lambda) \) leads to a hard/binary weighting on the examples, assigning 1 to easy examples and 0 to hard examples. However, this style of hard weights tend to lose flexibility, since any two “easy” (or “hard”) examples are unlikely to be strictly equally important and learnable [113]. Therefore, an intuitive choice is to design new SP-regularizers which could result in soft weights \( v^* \). We call such group of SP-regularizers soft regularizers. A list of existing SP-regularizers \( g(v; \lambda) \) and the corresponding close-formed solutions of \( v^* \) is shown in Table V. In addition, the \( l-v^* \) functions (i.e., the function of example weight \( v_i^* \) w.r.t. losses \( l_i \)) of these solutions are visualized in Fig 6.

### TABLE V

| Regularizers              | \( g(v; \lambda) \)                                                                 | \( v_i^* (l_i; \lambda) \)                                                                 |
|--------------------------|----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| Hard [41]                | \(-\lambda \sum_{i=1}^{N} v_i \)                                                | \( \begin{cases} 1, & l_i < \lambda \\ 0, & \text{otherwise} \end{cases} \)              |
| Linear [32]              | \( \frac{1}{2} \lambda \sum_{i=1}^{N} (v_i^2 - 2v_i) \)                        | \( \begin{cases} 1 - l_i/\lambda, & l_i < \lambda \\ 0, & \text{otherwise} \end{cases} \) |
| Logarithmic [32]         | \( \sum_{i=1}^{N} \left( \zeta v_i - \frac{\zeta}{\ln \zeta} v_i \right) \)    | \( \begin{cases} \log(l_i + \zeta), & l_i < \lambda \\ \log \zeta, & \text{otherwise} \end{cases} \) |
| Mixture [32]             | \( -\zeta \sum_{i=1}^{N} \log \left( \frac{v_i + \zeta}{\lambda \gamma} \right) \) | \( \begin{cases} 1, & l_i \leq \lambda_2 \\ 0, & l_i \geq \lambda_1 \\ \left( \frac{1}{l_i} - \frac{1}{\lambda_1} \right), & \text{otherwise} \end{cases} \) |
| Mixture2 [113]           | \( \frac{v_i^2}{v_i + \pi}, \quad \gamma > 0 \)                                | \( \begin{cases} 1, & l_i \leq \left( \frac{\lambda \gamma}{\lambda + \gamma} \right)^{\frac{1}{\gamma}} \\ 0, & l_i \geq \lambda^2 \\ \gamma \left( \frac{1}{l_i} - \frac{1}{\lambda} \right), & \text{otherwise} \end{cases} \) |
| Logistic [103]           | \( \sum_{i=1}^{N} \ln(v_i)^\mu + \ln(v_i)^\lambda \) \quad v_i > 0, \quad \mu_i = 1 + e^{-\lambda - v_i} \) | \( \begin{cases} 1 + e^{-\lambda} - \lambda \ln(1 + e^{-\lambda}/\lambda), & \text{otherwise} \end{cases} \) |
| Polynomial [22]          | \( \lambda \left( \frac{1}{2} \sum_{i=1}^{N} \sum_{i=1}^{N} v_i \right) \) \quad \lambda > 0, \quad t \in \mathbb{N}^+ \) | \( \begin{cases} \left( 1 - \frac{l_i}{\lambda} \right)^{\frac{v_i}{(1 + e^{-\lambda})}}, & l_i < \lambda \\ 0, & \text{otherwise} \end{cases} \) |

As in Fig 6 compared to the hard regularizer, the solutions of various soft regularizers assign soft weights to reflect example importance in finer granularity, which helps soft regularizers achieve better performance in various applications. However, one needs to choose suitable regularizers for specific scenarios. For example, the logarithmic regularizer is more prudent than the linear, while the mixture regularizers tolerates small losses, compared with other regularizers [32]. The polynomial regularizer extends the linear to arbitrary orders (when \( t = 2 \), it is identical to linear), and Li et al. [48] further propose to dynamically adjust the order \( t \) during training to improve flexibility.

To allow more possibility on SP-regularizer designs, a general and formal definition is taken as follows [32], [113]:

**Definition 3: SP-regularizer.** Suppose that \( v \) is a weight variable, \( l \) is the loss, and \( \lambda \) is the age parameter. \( g(v; \lambda) \) is called a self-paced regularizer, if:

1. \( g(v; \lambda) \) is convex w.r.t. \( v \in [0, 1] \);
2. \( v^*(l; \lambda) \) is monotonically decreasing w.r.t. \( l \), and \( \lim_{l \to 0} v^*(l; \lambda) = 1 \), \( \lim_{l \to \infty} v^*(l; \lambda) = 0 \);
3. \( v^*(l; \lambda) \) is monotonically increasing w.r.t. \( \lambda \), and \( \lim_{\lambda \to 0} v^*(l; \lambda) \leq 1 \), \( \lim_{\lambda \to \infty} v^*(l; \lambda) = 0 \);

where \( v^*(l; \lambda) \) is defined in Eq. 9.

It is not difficult to verify that all the regularizers in Table V conform to Definition 3. Based on this definition, Li et al. [46] propose a general framework for designing SP-regularizers. Specifically, they proved that \( g(v; \lambda) \) conforms with Definition 3 if and only if \( \frac{\partial g(v; \lambda)}{\partial v} = -s(v; \lambda) \) and \( \frac{\partial^2 g(v; \lambda)}{\partial v^2} \leq 0 \) for \( v \in [0, 1] \), where \( s(v; \lambda) \) is the inverse function of any S-shaped function.
While the SP-regularizers in Definition 3 have explicit form, Fan et al. [14] further introduce implicit regularizers into SPL, which method is called SPL-IR. Based on the convex conjugacy theory, a group of implicit SP-regularizers, whose analytic form can be unknown, are deduced from some well-studied robust loss functions (e.g., Huber loss function), and the corresponding best weights \( v^*(l; \lambda) \) can be directly derived from these loss functions. The weights thus inherit the good robustness properties, which helps SPL-IR to outperform explicit SP-regularizers. The \( l-v^* \) functions of implicit regularizers derived from four types of robust loss functions, i.e., Huber, Cauchy, L1-L2, and Welsch loss functions, are visualized in Fig. 6.

The last thing worth mentioning is the relationship between soft-regularizer-based SPL and other loss weights optimization methods in Sec. IV-C4 (e.g., MentorNet [35]). In fact, both of them are trying to find the best example weights \( v^* \in \mathbb{R}^N \) for each training epoch. However, while the soft SP-regularizers always encourage to assign larger weights to easier examples, the approaches in Sec. IV-C4 may dynamically choose to focus on easier or harder examples at different training states. In addition, SPL methods leverage current training loss as Difficulty Measurer and adopt predefined Training Scheduler. However, in other methods, both the Difficulty Measurer and Training Scheduler are data-driven and dynamically learned.

1-d) Prior-embedded SPL. In SPL methods, given fixed SP-regularizers \( g(v; \lambda) \), the example weights \( v^* \) are entirely determined by the example-wise losses and the age parameter \( \lambda \). However, in some cases, we hope to introduce some loss prior knowledge into this learning scheme. For example, we may want to compulsively assign some outliers with \( v_i = 0 \) to improve robustness, or assign pre-known high-quality examples with \( v_i = 1 \). Such prior knowledge is closely related to the predefined Difficulty Measurer in Sec. IV-B1.

Fortunately, the AOS algorithm naturally decomposes SPL into two problems of optimizing \( w \) and \( v \), which makes it feasible to embed the loss prior knowledge into SPL by encoding it as a part of SP-regularizer or a constraint on \( v \). Four typical types of priors are summarized in [61] as follows: a) Outlier prior: Some outliers in the datasets show extremely large losses. b) Spatial/temporal smoothness prior: Spatially or temporally adjacent examples tend to have similar losses. c) Sample importance order prior: Some examples are pre-known to be more important than others. d) Diversity prior: Important examples should be scattered across the data range to help learn global data knowledge.

A famous representative of Prior (d) is SPL with diversity (SPLD) [33], which incorporates a negative \( l_{2,1} \)-norm into the hard SP-regularizer to avoid overfitting to a data subset while ignoring easy examples in other groups:

\[
g(v; \lambda, \gamma) = -\lambda \sum_{i=1}^{N} v_i - \gamma \sum_{j=1}^{b} \| v^{(j)} \|_2
\]

where \( \gamma > 0 \) is a balance hyperparameter between easiness and diversity, \( b \) is the assumed number of groups (e.g., themes in the video event detection task) in the training set, and the \( v^{(j)} \) is a vector of corresponding example weights \( v_i \) in group \( j \). Since the \( l_{2,1} \)-norm is well-known to lead to group-wise sparse representation, its opposite term should then encourage diversity of non-zero \( v_i \) across groups. Moreover, SPLD also leads to a close-formed solution. An alternative method for Prior

\[\text{(14)}\]

For clearer comparison with other explicit \( l-v^* \) functions, we divide the weights \( v^* \) by 2 in Cauchy and Welsch. This linear scaling does not influence the model learning if we accordingly amplify the learning rates in SGD.
(d) is to adopt $-l_{0.5,1}$-norm $\| \cdot \|_{0.5,1}$, i.e., $-\sum_{j=1}^{b} \sqrt{\sum_{i=1}^{n_j} v_i^{(j)}}$, where $n_j$ is the size of group $j$. This diversity term makes the whole $g(v; \lambda, \gamma)$ conform with Definition 3.

For Prior (c), a representative work is self-paced curriculum learning (SPCL) [34], which introduces a curriculum region $\Psi$ with formal definition as a convex feasible region constraint on $v$. SPCL combines the power of SPL and predefined CL, whose objective is as follows:

$$\min_{w,v \in [0,1]^N} E(w, v; \lambda, \Psi) \sum_{i=1}^{N} v_i l_i + g(v; \lambda). \quad \text{s.t. } v \in \Psi$$

(15)

An example of $\Psi$ is the linear curriculum region, i.e., $\{v | a^T v \leq c\}$, where $c$ is a constant and $a$ is an $N$-dimensional vector derived from the total order relationship among the $N$ examples [7]. Theoretical analysis on SPCL is provided in [54].

Another method for Prior (c) is proposed in [107], which is helpful when the precise total order knowledge is hard to obtain. Similar to the $-l_{2,1}$-norm for Prior (d), this method encodes the prior knowledge about image difficulty by adding a regularization term $h(v; \eta, p) = -\eta \sum_{i=1}^{N} p_i v_i$ to the objective, where $p_i$ indicates the priority values of each image. A larger $p_i$ means the example $i$ is easier and should be assigned a larger weight $v_i$. To generate such $p_i$, all the Difficulty Measurements discussed in Sec. [4-V-B] can be adopted.

It is worth mentioning that when the above kinds of convex constraint on $v$ is applied, we could no longer use the close-solved forms of $v^*$ in Table [V]. Instead, we can calculate $v^*$ by applying gradient-based methods [34] or other off-the-shelf techniques like CVX toolbox [107] due to the convexity.

I-e) Other enhancements of SPL. Besides the various enhanced versions of SP-regularizers, there still remains some other aspects to be carefully considered in SPL. A key element in SPL is the age parameter $\lambda$. As aforementioned, traditional SPL takes a naive strategy to add/multiply $\lambda$ with a constant at the end of each epoch. However, with the model making progress, the losses on all the examples are expected to become smaller and smaller, and thus an monotonic increasing threshold $\lambda$ may add much more hard examples in the early epochs. For some SP-regularizers it would be more effective to gradually decrease $\lambda$.

Another interesting variant of SPL is proposed in [116], who changes the scope of SPL objective from min-min to minimax. The insight is that with the min-min objective, the SPL algorithm is actually learning the $w$ to minimize a lower bound of loss, i.e., $\min_{w \in [0,1]^N} E(w, v; \lambda)$. To improve the generalization capacity and robustness of SPL, the authors propose to minimize the “worst case loss” by setting the objective as $\min_{w} \max_{v \in [0,1]^N} E(w, v; \lambda)$. By this minimax objective, the $w$ is optimized to minimize the upper bound of loss at each epoch. Although this is almost opposite to the spirit of SPL, it inspires deeper thinking about the SPL designs.

1-f) Applications of SPL. We conclude the review on SPL by summarize the existing applications of SPL. SPL has been widely applied to many practical problems, including CV tasks of visual category discovery [45], segmentation learning [42], image classification [87], object detection [86], reranking in multimedia retrieval [32], person ReID [115], etc., and traditional machine learning tasks of matrix factorization [113], feature selection [114], cross-modal matching [20], co-training [57], clustering [20], [103], [104], etc. As a primary branch of CL, SPL has the same application motivations as CL, i.e., to guide and to denoise (see Sec. [4-V-B]). Besides, SPL is also effective for a group of applications where the algorithm needs
to assign pseudo-labels by models, including reranking [32], co-saliency detection [108], and other weakly or unsupervised learning tasks [20]. Additionally, some works also extend SPL by introducing group-wise weights to improve the performance on multiple data groups, e.g., multi-modal [21], multi-view [103], multi-instance [108], multi-label [46], multi-class [73], multi-task [47], etc. Finally, SPL is also combined with complementary data-selection-based training strategies like boosting [67] and active learning [52], [88] to benefit both schemes.

2) Transfer Teacher: SPL takes the current student model as an automatic Difficulty Measurer. However, this strategy has a risk of uncertainty at the beginning of training, when the student model is not mature enough (i.e., not sufficiently trained). For example, at the first epoch, the example-wise training loss \( l_i \) relies heavily on the random initialization of the student model, which results in random weights \( v_i \) (i.e., random easiness). This is analogous to human education: if a student understands little about the learning materials, it would be hard for him/her to measure the difficulty of the materials and find out the easy ones. Thus, a natural idea is to invite a mature teacher to help the student assess the materials and form an easy-to-hard curriculum. This idea leads to the CL approaches that we denote as Transfer Teacher.

Transfer Teacher, illustrated in Fig 2(c), is a semi-automatic CL method, which takes a pretrained teacher model as the Difficulty Measurer with a predefined Training Scheduler. Particularly, this method first pretrain a teacher model on the training dataset or an external dataset (e.g., ImageNet), and then transfer its knowledge to calculate the example-wise difficulty, based on which a predefined Training Scheduler can be applied to finish the CL design. Transfer Teacher reduces the burden of artificial Difficulty Measurer designs and thus could be easily applied to the tasks where the example-wise easiness is hard to measure.

Some representatives of Transfer Teacher are presented in Table VI. The most general Transfer Teachers are the loss-based methods (the first three rows), which do not need any domain knowledge and are closely related to SPL. Concretely, these methods take the example-wise losses calculated by a teacher model as the example difficulty and assume that the lower the loss, the easier the example. The teacher model can either be different from the student model and have greater model capacity (i.e., more complex) [100], or actually share the same structure with the student model [29], [102]. For instance, in [100], a strong teacher classifier pretrained on ImageNet is taken to transfer its knowledge to calculate the example-wise losses on the training dataset. On the other hand, the authors in [29] adopts a bootstrapping (or “self-taught”) strategy, which uses a teacher classifier with the same network structure as the student classifier, and pretrains it on the training dataset. This pretrained teacher can be regarded as a mature version of the student to calculate loss-based difficulty. Note that the difference between bootstrapping and SPL is that the former’s Difficulty Measurer is mature and fixed, while the latter’s is the current student model which gradually grows up. Another example of loss-based Transfer Teacher is the Cross Review strategy [102], which alleviates the fluctuation of the difficulty measurement. Concretely, the authors uniformly divide the trainset into \( N \) subsets and share the same structure with the student model [29], [102]. For instance, in [100], a strong teacher classifier pretrained on ImageNet is taken to transfer its knowledge to calculate the example-wise losses on the training dataset. On the other hand, the authors in [29] adopts a bootstrapping (or “self-taught”) strategy, which uses a teacher classifier with the same network structure as the student classifier, and pretrains it on the training dataset. This pretrained teacher can be regarded as a mature version of the student to calculate loss-based difficulty. Note that the difference between bootstrapping and SPL is that the former’s Difficulty Measurer is mature and fixed, while the latter’s is the current student model which gradually grows up. Another example of loss-based Transfer Teacher is the Cross Review strategy [102], which alleviates the fluctuation of the difficulty measurement. Concretely, the authors uniformly divide the trainset into \( N \) subsets and share the same structure with the student model [29], [102].

Moreover, in NLP literature, there exist some typical methods that adopt a “teacher” model to measure example-wise difficulty for training data selection. These methods can be naturally incorporated into CL as Transfer Teacher. For example, some works [110], [117] leverage the following model-based data uncertainty \[ u_{\text{data}}(s) = \frac{1}{|s|} \sum_{i=1}^{|s|} \log P(s_i|s_{<i}) \] to measure sentence-wise difficulty in NMT tasks, where \( P(s_i|s_{<i}) \) is the confidence of the pretrained language model (LM) for its prediction about the \( i \)-th word in sentence \( s \), and \(|s|\) is the length of \( s \). The lower of this uncertainty score, the easier the sentence according to the teacher LM. In addition, Moore et al. [62] propose to use two LMs to measure how much a sentence \( s \) is related to a specific domain (e.g., news, talks, patents, etc.) and select domain sentences. Specifically, let \( \theta_{ge} \) be an LM pretrained on general-domain corpus and \( \theta_{in} \) an LM pretrained on an external in-domain corpus. Then the domain score of \( s \) is calculated as \[ \phi_{\text{domain}}(s; \theta_{ge}, \theta_{in}) = \log P(s; \theta_{ge}) - \log P(s; \theta_{in}) \] which is actually the cross entropy difference between \( \theta_{ge} \) and \( \theta_{in} \), the lower value of which refers to more in-domain data. This measurement is leveraged in [96], [111] as Transfer Teacher according to the specific scenarios (e.g., in-domain data can be seen as easier for domain adaption). Moreover, Wang et al. [96] also use two NMT models to measure the noise level of a sentence pair \( \{x, y\} \). Suppose \( \theta_{\text{noisy}} \) is a baseline NMT model pretrained on the noisy training set, and \( \theta_{\text{clean}} \) is obtained by fine-tuning the \( \theta_{\text{noisy}} \) on a small and clean external dataset, then the quality (or “noise score”) of \( \{x, y\} \) is defined as \[ \phi_{\text{quality}}(\{x, y\}; \theta_{\text{noisy}}, \theta_{\text{clean}}) = \log P(y|x; \theta_{\text{clean}}) - \log P(y|x; \theta_{\text{noisy}}) \] where

| Representatives | Teacher model | Teacher pretraining dataset | Difficulty |
|----------------|---------------|-----------------------------|------------|
| Transfer learning [100] | Diff. structure with student | ImageNet | Loss |
| Bootstrapping [29] | Same structure as student | The training dataset | Loss |
| Cross Review [102] | Same structure as student | \( N \) training subset | Loss |
| Uncertainty [110], [117] | Language model | The training dataset | Cross entropy |
| Domain score [96], [111] | Language model | General- and in-domain datasets | Cross entropy difference |
| Noise score [96] | Same NMT models as student | Noisy and clean datasets | Cross entropy difference |

TABLE VI

Representatives of Transfer Teacher. Diff. = different.
$\log P(y|x; \theta)$ is the predicted probability by NMT models. A higher $\phi_{\text{quality}}$ refers to cleaner (and also easier) data, which makes it helpful in Transfer Teacher based CL.

3) RL Teacher: The SPL and Transfer Teacher only automate the Difficulty Measurer and still use predefined Training Scheduler, and they only consider one side of the “curriculum” or teaching scenario: SPL takes the student feedback (i.e., losses) to adjust the curriculum, while Transfer Teacher leverages the teacher’s knowledge to determine the order of presenting learning materials. A common sense in human education is that an ideal teaching strategy should involve both the teacher and the student, where the student could interactively provide feedback to the teacher, and the teacher could then adjust the teaching action accordingly. In this way, both the teacher and student will make progress together.

To this end, RL Teacher methods are proposed, which involve a student model and a reinforcement-learning-based teacher model. At each training epoch, the RL teacher will dynamically select examples/tasks for training according to the student feedback. Concretely, the data selection is taken as the action in the RL schemes, and the student feedback is taken as the state and reward. From the view of the general CL framework in Sec. IV the RL Teacher actually sets the teacher model as both the Difficulty Measurer and Training Scheduler by dynamically considering the student feedback. The illustration of RL Teacher is shown in Fig 2(d). It is clear to see that, with this teacher-student interactive strategy, RL Teacher achieves the fully-automated CL design.

Some representatives of the RL Teacher are listed in Table VII. Both traditional RL and deep RL models are leveraged in these designs, where the deep RL models are stronger in performance but more time-consuming and harder to train. It is worth mentioning that RL Teacher methods make it possible to set different student feedback according to different goals, e.g., training efficiency or generalization performance, which brings great flexibility and applicability to various scenarios. Additionally, RL Teacher is typically suitable for multi-task learning, where the teacher model selects the most valuable tasks for the student training.

| Representatives | RL Algorithm | Reward/Student Feedback | Main Goal |
|-----------------|--------------|--------------------------|-----------|
| AutoCL [25]     | Multi-armed bandit | Loss/Complexity-driven learning progress | Efficiency |
| TSCL [59]       | Multi-armed bandit | Absolute value of slope of learning curve | Efficiency |
| L2T [15]        | REINFORCE    | How fast the student achieve valid accuracy | Efficiency |
| RL-based CL [40] | Q-Learning | Log-likelihood on valid set | Performance |
| RCL [112]       | Discriministic Actor-Critic | Perplexity difference on valid set | Performance |

AutoCL [25] and TSCL [59] are two RL Teacher methods designed for multi-task settings, where the goal is to learn a student model that achieves high performance on all the tasks. In both the works, bandit-based RL models are adopted as the teacher model, whose job is to receive the reward signals from the student model and select one training task for student learning in the next epoch. Specifically, the RL teachers learn the mapping from history reward sequences $r = \{r_t\}_{t=1}^N$ (of different tasks) to the probability vector $\pi$ of sampling the $N$ training tasks. As both the works aim to design a CL algorithm to improve the training efficiency, various reward measurements from different perspectives are proposed. In AutoCL [25], the authors define a group of learning progress as the reward, which includes loss-driven and complexity-driven measurements. The intuition is, if a decrease in some loss or an increase in the student model’s complexity is observed after training on the $t$-th task, then this task is helpful to the student model for making big progress and should be assigned larger sampling probability. An example of loss-driven reward is the Prediction Gain: $L(x; \theta) - L(x; \theta')$, where $\theta$ and $\theta'$ is the model before and after training on the training batch $x$ (of the current task), and $L$ is the loss function. In addition, an example of complexity-driven awards is the Variational Complexity Gain underpinned by the Minimum Description Length principle. On the other hand, in TSCL [59], the authors set the reward as the absolute value of the slope of the learning curve (the absolute difference between the performance scores of two successive epochs) on a specific task. This is an elegant design: when the slope is a large positive value, it means the student is making progress on this task; and when the slope is a negative value, it implies that the student is forgetting this task. Both conditions should lead to a larger sampling probability on this task to achieve faster and more generalizable student training.

L2T (Learning to Teach) [15] adopts the REINFORCE algorithm as the RL teacher. Given a random mini-batch $D_t$ in the $t$-th supervised training epoch, the goal of the teacher model is to dynamically determine which data examples are used and which are abandoned. To this end, the action $a_t = \{d_t^{(m)}\}_{m=1}^M$ is a hard selection on each of the $M$ examples in this mini-batch. The state $s_t = (D_t, f_t)$ is defined as the concatenation of various features of the current mini-batch $D_t$ and the current state of student model $f_t$. Note that this design of state/observation is quite general and applicable to most learning scenarios. Moreover, aiming at fast convergence, the reward $r_t$ is set as a terminal reward (i.e., $r_t = 0, \forall t < T$) to be related with how fast the student model learns. In particular, $r_t = -\log(i_t/T^*)$, where $i_t$ is the iteration number for the student model achieving an accuracy threshold $\tau \in [0, 1]$ on the valid set, and $T^*$ is a predefined maximum iteration number. With all

\textsuperscript{a}For example, data features include the predefined Difficulty Measurer features in Table II and model features include iteration number, average historical training loss / validation accuracy, etc.
the definition above, L2T trains the teacher model by maximizing the expected reward $J(\theta) = \mathbb{E}_{\phi(\theta)}[R(s, a)]$, where $R(s, a)$ is a state-action value function to estimate the reward, and $\phi(\theta)$ is the data selection policy parameterized by $\theta$, which can be any binary classification model. Through this dynamic data selection by the teacher model, the student model is expected to converge faster to a better optima.

Beyond traditional RL algorithms, recent works also leverage deep RL models, e.g., Q-learning [40] and Deterministic Actor-Critic [112], to design RL Teacher methods for automatic data selection, sharing the same spirit with L2T. Both the two works focus on the NMT task, a typical application for CL discussed in Sec. III-B. RL-based CL first sorts the examples according to a predefined measurement and divide them into $M$ bins of equal sizes, and then defines the action as selecting one bin for NMT training. The reward and state are related to the log likelihood on the valid set and a prototype batch sampled from all bins, respectively. Moreover, in RCL [112], the state $s$ is similarly defined as L2T, including feature embeddings from data and the student model. Given $s$, the actor network $\mu$ is optimized to select examples from a mini-batch (i.e., action $a = \mu(s)$) to form the training set at each epoch, such that the estimated reward $Q(s, a)$ by critic network $Q$ is maximized. The critic network, on the other hand, is optimized to estimate the reward $r$ more accurately, where $r$ is defined as the performance improvement of the student model on valid set after trained. Compared with traditional RL methods like REINFORCE, Actor-Critic is supposed to help reduce the update variance and accelerate convergence.

4) Other Automatic CL: Besides RL Teacher, there exists some other fully-automatic CL designs. Intuitively, these designs should require the generation of the curriculum to rely only on the dataset, the student model, and the goal of the task. According to the CL definition in Sec. [1], we can regard this curriculum as a sequence of training criteria or objectives. Thus, from the optimization perspective, at each training epoch, we hope to optimize the following mapping to improve performance: {data, current state of student model, task goal} $\mapsto$ training objective. To this end, RL Teacher methods typically adopt an RL framework to learn the policy for training data selection. Additionally, more optimization methods, such as Bayesian Optimization (BO), Stochastic Gradient Descent (SGD), Meta-learning, and Hypernetwork, are also demonstrated to have great potential to learn this mapping. Note that these methods can also be regarded as a “teacher” searching for the best curriculum according to the student state/feedback, although this “teacher-student interaction” is not as explicit as that under the RL Teacher framework. What is more, while RL Teacher methods only learn the data selection policy, other automatic CL methods might learn more aspects of training criteria, including the loss weights, loss function, etc. Since the methodologies and focuses of optimization are diverse in these works, we conclude them in this subsubsection as “Other Automatic CL”, the methods of which are summarized in Table VIII.

![Table VIII](image)

Tsvetkov et al. [91] make one of the earliest attempts on automatic CL by leveraging BO to learn the best curricula for word representation learning. The curriculum here is determined by the scalar product of a learned weight vector $w$ and an example-wise difficulty feature vector $x$, according to which the examples are scored and sorted for later representation learning. While $x$ is manually engineered, the weight vector $w$ learned by BO provides the possibility for different curriculum according to different downstream tasks. Specifically, BO in this work is a sequential approach to performing a regression from $w$ to the performance on the downstream task. At the $t$-th iteration, the algorithm first sort the examples by the $w_t \cdot x$, learn word representations $V$ (i.e., student model) with this curriculum, and then train extrinsic models on downstream task and evaluate the performance $val_t$. Finally, $val_t$ is collected by BO algorithm to generate the $w_{t+1}$. Through this process, BO learns to predict a better $w$ and thus a better curriculum.

While SPL methods in Sec. IV-C1 optimize the example-wise loss weights $v$ by solving the new objective with SP-regularizers, existing works have made further efforts to optimize $v$ throughout training by different approaches. One idea is to predict the loss weight $v_i$ of example $(x_i, y_i)$ by a teacher model, which is adopted in MentorNet [35] and ScreenerNet [37]. The MentorNet $h$ is a teacher model with parameters $\Theta$ which maps the example-wise feature $z_i = \phi(x_i, y_i, w)$ to the corresponding loss weight $v_i$. Here, $z_i$ includes the loss, loss difference to the moving average, label, and epoch percentage, and $w$ denotes the parameters of the student model. Given fixed $w$, the MentorNet is trained on a trusted small dataset $D_{\text{eval}}$ by SGD:

$$\Theta^* = \arg \min_{\Theta} \sum_{i \in D_{\text{eval}}} \text{CE}(h(z_i; \Theta), v_i^*),$$

where $\text{CE}(h(z_i; \Theta), v_i^*)$ is the cross-entropy loss function.
where \( v_i \) is manually annotated as 1 if \( y_i \) is a correct label and 0 otherwise, and CE stands for cross-entropy. During the mini-batch training of the student model, the MentorNet is only updated a fixed number of times (with student fixed). Besides the data-driven curriculum learned on \( D_{val} \), we could also train the MentorNet to approximate a predefined curriculum, e.g., by setting \( v_i \) as the loss weights derived from some SPL objectives. The convergence and robustness of student learning are also theoretically proved.

Similar to MentorNet, ScreenerNet [37] also learns a mapping from example-wise feature \( x_i \) to loss weight \( v_i \). However, the ScreenerNet and the student model are jointly trained with block-coordinate descent (similar to the AOS algorithm in SPL). Concretely, on each mini-batch \( D \), the optimal ScreenerNet parameters \( \Theta \) is optimized as:

\[
\Theta^* = \arg \min_{\Theta} \sum_{i \in D} ((1 - v_i)^2 l_i + v_i^2 \max(M - l_i, 0)) + \alpha ||\Theta||_1
\]

where \( l_i \) is the example-wise loss, the second term is a regularizer of \( \Theta \), and \( M \) is a margin hyperparameter. With this perspective, the ScreenerNet encourages to increase \( v_i \) if the loss \( l_i \) is high. Although it is opposite to CL, ScreenerNet is shown effective in deep RL tasks and image classification.

Ren et al. [22] further propose a meta-learning perspective for optimizing loss weights \( v \). Meta-learning methods [24] aim at learning the hyperparameters of training algorithms, e.g., initialization, learning rates, etc., and the target hyperparameter here is \( v \). Akin to MentorNet, a clean unbiased valid set is adopted to guide the meta-learning. Specifically, at the \( t \)-th epoch, they first locally update the student model (with parameters \( w_t \)) by one gradient step on a training mini-batch \( D_{train} \), where the example weights \( v_i \) are perturbed by \( \epsilon_i \):

\[
\tilde{w}_{t+1}(\epsilon) = w_t - \alpha \nabla \sum_{i \in D_{train}} \epsilon_i l_i(w_t),
\]

where \( l_i(w) \) is the loss and \( \alpha \) is the local learning rate. To estimate the best loss weights \( v \) according to the clean valid set, they take a meta-gradient step on a validation mini-batch \( D_{val} \) w.r.t. \( \epsilon \), and force the weights to be non-negative:

\[
\tilde{v}_{i,t} = \max \left( 0, -\frac{\partial}{\partial \epsilon} \frac{1}{|D_{val}|} \sum_{j \in D_{val}} \tilde{w}_{t+1}(\epsilon) \right),
\]

where \( \eta \) is the meta learning rate. The \( \tilde{v}_t \) is then normalized to obtain the final new weights \( v_t \). Finally, they meta update the model parameters to \( w_{t+1} \) with the new objective weighted by \( v_t \), i.e., \( \sum_{i \in D_{train}} v_i l_i(w_t) \). This meta-learning mechanism would lead the student model to converge to an appropriate distribution favored by the clean and balanced valid set and thus become more generalizable and robust.

Beyond loss weights, some other works [77], [101] also focus on learning dynamic loss function as a whole, which complies with the most general definition of CL in Sec. [4]. As argued in L2T [15], while data selection is analogous to human teacher determining the examination criteria, which is another significant issue in a “curriculum”. In [101], the scholars propose to leverage a two-layer perceptron as the teacher hypernetwork \( \mu_{\Theta} \) to predict the parameters of the loss function \( l_\Phi(y, y) \). In other words, the loss function is assumed to be itself a neural network with coefficients \( \Phi \), and at the \( t \)-th epoch, \( \Phi_t = \mu_{\Theta}(s_t) \), where \( s_t \) is the state vector of the student model \( f_w \). Akin to MentorNet, the goal of the teacher model is to maximize the performance of induced student model on a valid set \( D_{val} \): \( \Theta^* = \max_{\Theta} M(f_{w^*}, D_{val}) \), where \( f_{w^*} = F(D_{val}, \mu_{\Theta}) \) stands for the student model trained on the training set with the loss function predicted by \( \mu_{\Theta} \), and \( M \) is the performance measurement on \( D_{val} \). Novel algorithms are also proposed to make this optimization of teacher hypernetwork possible.

Moreover, Saxena et al. [77] provide an imaginative solution for learning the loss function, which is called Data Parameters. Concretely, each example and class in a dataset is equipped with a learnable parameter (data parameter) \( \sigma \) to scale the logits \( z_{y} \) of example \( \{x, y\} \) in the task of image classification. For example, class-level data parameters \( \sigma_{\text{class}} \in \mathbb{R}^k \) are inserted before the softmax layer:

\[
p_y = \text{softmax}_{w \sigma} (z_y) = \frac{\exp(z_y / \sigma_{\text{class}})}{\sum_j \exp(z_j / \sigma_{\text{class}})},
\]

where \( p_{y}, z_{y}, \sigma_{\text{class}} \) are the probability, logit and data parameter of the target class \( y \) of the \( i \)-th example. The data parameters \( \sigma_{\text{class}} \) are then updated by gradient descent, such that if the example \( \{x, y\} \) is misclassified, \( \sigma_y \) will increase, and the gradient w.r.t. logits will be decayed, i.e., learning more slowly and carefully on this class. In addition, instance-level data parameters would also help. Different from loss weights optimization, this data parameter mechanism essentially learns the loss function specific to each data point and class. Note that during inference, the data parameters are discarded.
V. DISCUSSIONS

In this section, we present more discussions on CL. In particular, we first compare the “easier examples first” strategy of CL with its opposite strategy, i.e., “harder examples first”. Then we summarize a relation graph connecting CL and other related concepts.

A. Easier First v.s. Harder First

A fundamental question for the CL strategy is: does this “easy to hard” training strategy always help, given all of these works and theories? In some literature of CL, the answer to this question is “No”. For example, Avramova [3] finds that convolutional neural networks derive most learning values from the hardest examples, and the damage of excluding those easiest examples is minor. Zhang et al. [110] also test a reverse version of CL (i.e., a copy of baseline CL reversing the difficulty ranking to “hard to easy”, also called anti-curriculum) on NMT tasks, which shows that in some cases, anti-curriculum may even achieve best performance among various Training Scheduler designs. In addition, Hacohen et al. [29] demonstrate that SPL will hurt the performance and significantly delay learning in their experiments. Other works [96], [116] also design “harder examples first” curricula that benefit the model learning.

Besides CL literature, hard example mining (HEM) [81] serves as another well-studied and popular data selection strategy, which is opposite to CL. Concretely, in each training batch, HEM selects the hardest examples for training (or assign them with higher weights), assuming that the harder examples are more informative. The difficulty in HEM is often defined according to the current model losses on examples [56], [81] or the gradient magnitude [1], [24]. Akin to CL, HEM also has various applications, and the famous boosting algorithm [19] in ensemble learning also takes the same strategy by upweighting the wrongly-classified examples.

So which strategy should we apply to our own scenario, “easier first” as CL or “harder first” as HEM? It remains an unsolved problem to be carefully considered. Theoretically, under different settings, both CL and HEM strategies can benefit the learning as long as the “curriculum” is positively correlated with the optimal utility. However, this criterion is very hard to verify. More intuitively, Chang et al. [8] point out that CL is more suitable for the scenarios with more noisy labels or outliers to improve the model robustness and convergence rate, while HEM is more beneficial for cleaner datasets and leads to faster and more stable SGD. One should also note that if the target task is very difficult, CL will be more preferred than HEM, since CL is able to result in a more effective training process through the easier/smallerer versions.

An alternative is to combine the two strategies together with a tradeoff policy. For example, Pi et al. [67] embed the self-paced regularizers into the objective of boosting algorithm, which simultaneously enhances the learning effectiveness (by boosting) and robustness (by SPL). In addition, Chang et al. [8] propose to select the most uncertain examples according to the prediction history, which is consistent with the variance reduction strategies in active learning [80]. The uncertain examples are predicted both incorrectly and correctly in history and are thus neither too easy (always correct) nor too difficult (always incorrect). It is worth mentioning that the fully automatic CL methods (e.g., RL Teacher in Sec. IV-C3) would also be an ideal choice when it is hard to choose between “easier first” CL and “harder first” HEM.

B. Relation Graph Connecting CL and Other Concepts

We conclude the relation graph connecting CL and other related concepts in Fig 7. To begin with, SPL (Sec. IV-C1) is a well-studied branch of automatic CL, and both CL and HEM belong to example reweighting strategies. CL has a mutual benefit with RL and transfer learning (TL) since both TL and RL are adopted in automatic CL frameworks (Sec. IV-C2 and IV-C3), and CL could help training in deep RL tasks and domain adaption tasks in TL. Additionally, CL can also be regarded as a special case for TL. What is more, multi-task learning is an important application of CL.

The relationships between meta-learning and automatic CL is interesting. As discussed in Sec IV-C4, meta-learning enriches automatic CL methods, where the best example-wise loss weights can be meta-learned. Other hyperparameters like age parameter λ in SPL could also be optimized through meta-learning. In addition, CL strategies can also benefit meta-learning schemes. A recent attempt is by Mehta et al. [60], who take a dynamic curriculum to adjust the task distribution for Meta-RL, thus avoiding meta-underfitting or meta-overfitting. Another perspective to consider the relations between meta-learning and automatic CL lies in the essence of their goals. Specifically, automatic CL is a special case of learning to teach, whose goal is to find the hyperparameters of teaching, including data selection and loss functions. On the other hand, meta-learning or automated machine learning (AutoML) is about learning to learn, where the goal is to find the hyperparameters of learning, including parameter initialization, learning rate, etc. Therefore, meta-learning and automatic CL together imitate both sides of human education and automate the process of learning and teaching.

Another concept closely related to CL is active learning (AL) [80]. AL is well-motivated in scenarios where data annotation is costly. An active learner aims at achieving greater performance with fewer labeled data, via generating queries to ask an oracle (e.g., human) to annotate several unlabeled instances for further training. Selecting query instances in AL is analogous to selecting easy examples/tasks in CL, and they share some common measurements such as uncertainty and density. Although

9These works comply with the generalized CL definition as data selection sequence in Sec. II.

10The optimal utility is $\sum_{i \in D} e^{-l_i(\theta^*)}$, where $D$ is the training dataset, $l_i(\theta^*)$ is the loss on the $i$-th example calculated by the optimal model $\theta^*$. 
both AL and CL involve dynamic data selection, the main differences between them lie in their goals and settings: CL is adopted in (weakly) supervised settings to improve performance and convergence rate, while AL is designed for training with fewer labels in semi-supervised settings. Based on different goals, selection in AL focuses on the informativeness and representativeness of instances, while mainly concentrating on the easiness in CL. Recent works [52], [88] also make efforts to combine SPL with AL, utilizing the complementariness between them.

VI. FUTURE DIRECTIONS OF CL

We conclude this paper with some ongoing or future directions of CL, which are worthy of discussion:

- **Evaluation benchmarks.** Although various CL methods have been proposed and demonstrated effective, few works have made efforts on evaluating them with general benchmarks. Such benchmarks are supposed to include representative datasets, well-defined evaluation methods, and corresponding metrics. Specifically, a benchmark may incorporate datasets for different applications (e.g., CV, NLP, recommendation, etc.) with different noise levels (e.g., clean, weakly-supervised, etc.). For the evaluation methods, the comparable baselines (e.g., ordinary training, classical predefined CL and their anti-curriculum versions, etc.) should be appropriately defined. Accordingly, evaluation metrics on the relative performance boost, convergence speedup, or other aspects should also be designed. With such benchmarks, new CL designs can then be evaluated on one or more tasks and compared with existing methods.

- **More advanced theories.** Existing theoretical analyses in Sec. III-A provide different angles for understanding CL. Nevertheless, more theories are still required to help us reveal why typical CL (Definition 2 in Sec. II) is effective. For example, if the dataset has no noise, are there any bounds for the effectiveness of CL? What is the actual effect of each condition in Definition 2, i.e., increasing dataset size/variance and increasing difficulty? In addition, the fully automatic CL methods in Sec. IV-C3 and IV-C4 also need more theoretical guarantees on their effectiveness. Moreover, a remaining fundamental question is to theoretically reveal the relations between the data distribution, task objective and the best training strategy among “easier first” (CL), “harder first” (HEM), and other strategies. Theories on this topic shall provide the basis for the application of CL in a specific task.

- **More CL algorithms and various applications.** Automatic CL (Sec. IV-C) provides the potential application values for CL in wider research areas and has become a cutting-edge direction. Therefore, one promising direction is to design more automatic CL methodologies with different optimizations (e.g., bandit algorithms, meta-learning, hyperparameter optimization, etc.) and different objectives (e.g., data selection/reweighting, finding the best loss function or hypothesis space, etc.). Moreover, as shown in [52], [67], [88], CL methods can be incorporated with other strategies like boosting and AL to achieve improvement. In addition to methodologies, more efforts should be made to explore the power of CL in more various applications, including both cutting-edge research areas (e.g., meta-learning, continual learning, NAS, graph neural network, self-supervised learning, etc.) and traditional machine learning topics (e.g., clustering, regression, etc.). Although the directions mentioned above may adopt the extended definition of CL as a sequence of training criteria in Sec. II (instead of Definition 2), the spirit of imitating the human curriculum shall drive more breakthroughs in the machine learning community.

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Fig. 7. The relation graph among CL and other related concepts.
## TABLE IX

An overview of the CL methods introduced in this paper. ML = Machine Learning.

| Papers | Type | Task | Domain | Curriculum | Difficulty Measurer | Training Scheduler |
|--------|------|------|--------|------------|---------------------|--------------------|
| Bengio et al. (ICML 2009) | Methodology | Shape recognition, language modeling | CV, NLP | Predefined CL | Shape variability, vocabulary size | Baby Step, Leap Frog |
| Spivakovsky et al. (NAACL 2010) | Methodology | Unsupervised dependency parsing | NLP | Predefined CL | Sentence length | One task each period |
| Zaremba et al. (ICLR 2015) | Application | Executing programs by LSTM | CV | Predefined CL | Length, nesting | One task each period |
| Pentina et al. (CVPR 2015) | Methodology | Multi-task learning | Multi-task | Predefined CL | Upper bound of expected error | One task each period |
| Chen et al. (ICCV 2015) | Application | Image classification and detection | CV | Predefined CL | Noise estimate | One-Pass |
| Tudor et al. (CVPR 2016) | Application | Image classification and detection | NLP | Predefined CL | Image difficulty | One-Pass, Baby Step |
| Kocmi et al. (IARAA 2017) | Application | Neural machine translation | CV | Predefined CL | Linguistical features | Uniform Baby Step |
| Moreno et al. (ICCV 2017) | Application | Image classification | CV | Predefined CL | None | No dropout → dropout |
| Cirik et al. (AAA 2017) | Analysis | Digit sum, sentiment analysis | NLP | Predefined CL | Correlation coefficients | Baby Step |
| Sarrabini et al. (ICCV 2017) | Application | Executing programs by LSTM | NLP | Predefined CL | Intensity | Baby Step |
| Gu et al. (APGR 2017) | Application | Facial expression recognition | CV | Predefined CL | Signal to noise ratio (SNR) | One-Pass |
| Braun et al. (EUSIPCO 2017) | Application | Speech recognition | Audio | Predefined CL | Proportion of nodule samples | Distribution shift |
| Jeon et al. (MICCAI 2017) | Analysis | Image detection or segmentation | CV-Medical | Predefined CL | Multiple types | Multiple types |
| Zhang et al. (arXiv 2018) | Application | Image classification | NLP | Predefined CL | Cluster density | Baby Step |
| Guo et al. (ECCV 2018) | Application | Natural question answering | NLP | Predefined CL | Term frequency, grammar | Distribution shift |
| Liu et al. (IJCAI 2018) | Application | Image classification and localization | NLP | Predefined CL | Baby Step |
| Tang et al. (arXiv 2018) | Application | Speaker recognition | Audio | Predefined CL | Baby Step |
| Ranjan et al. (TASLP 2018) | Application | Neural machine translation | NLP | Predefined CL | Baby Step |
| Plataniotis et al. (NAACL-HLT 2019) | Analysis | Reading comprehension | CV | Predefined CL | Baby Step |
| Hay et al. (ACL 2019) | Application | Human attribute analysis | CV | Predefined CL | Baby Step |
| Wang et al. (ICCV 2019) | Application | Unsupervised domain adaptation | CV, DA | Predefined CL | Baby Step |
| Jeon et al. (MICCAI 2019) | Application | Image classification | CV-Medical | Predefined CL | Baby Step |
| Wang et al. (ACL 2020) | Application | Speech translation | Multimedia | Predefined CL | Difficulty of subtasks | One-Pass |
| Liu et al. (ACL 2020) | Application | Neural machine translation | NLP | Predefined CL | Baby Step |
| Guo et al. (ICML 2020) | Application | Natural language architecture search | NAS | Predefined CL | Baby Step |
| Penha et al. (ECIR 2020) | Application | Conversation response ranking | IR | Predefined CL | Baby Step |
| Soviany et al. (WACV 2020) | Application | Image generation | CV | Predefined CL | Baby Step |
| Kumar et al. (NeurIPS 2010) | Methodology | Latent variable model learning | ML | Predefined Learning | Baby Step |
| Kumar et al. (ICCV 2011) | Application | Segmentation | CV | Predefined Learning | Baby Step |
| Lee et al. (CVPR 2011) | Application | Visual category discovery | CV | Predefined Learning | Baby Step |
| Tang et al. (MM 2012) | Application | Image classification | CV | Predefined Learning | Baby Step |
| Jiang et al. (NeurIPS 2013) | Methodology | Event detection, action recognition | CV, MM | Predefined Learning | Baby Step |
| Jiang et al. (MM 2014) | Methodology | Ranking in multimedia search | MM | Predefined Learning | Baby Step |
| Zhao et al. (AAAI 2015) | Methodology | Matrix factorization | ML | Predefined Learning | Baby Step |
| Jiang et al. (AAAI 2015) | Methodology | Image classification | CV | Predefined Learning | Baby Step |
| Zhang et al. (ICCV 2015) | Application | Co-saliency detection | CV | Predefined Learning | Baby Step |
| Xu et al. (IJCAI 2015) | Application | Clustering | ML | Predefined Learning | Baby Step |
| Liang et al. (IJCAI 2015) | Methodology | Video concept detection | CV | Predefined Learning | Baby Step |
| Li et al. (AAAI 2016) | Methodology | Matrix factorization | ML | Predefined Learning | Baby Step |
| Li et al. (AAAI 2016) | Methodology | Multi-task learning | Multi-task | Predefined Learning | Baby Step |
| Pi et al. (IJCAI 2016) | Methodology | Image classification | CV | Predefined Learning | Baby Step |
| Liang et al. (SIGIR 2016) | Methodology | Cross-modal matching | ML | Predefined Learning | Baby Step |
| Li et al. (IJCAI 2017) | Methodology | Image classification | CV | Predefined Learning | Baby Step |
| Ren et al. (IJCAI 2017) | Methodology | Multi-class classification | CV | Predefined Learning | Baby Step |
| Zhang et al. (CVPR 2018) | Application | Segmentation | CV | Predefined Learning | Baby Step |
| Ma et al. (ICML 2017) | Methodology | Co-training | ML | Predefined Learning | Baby Step |
| Lin et al. (TPAMI 2018) | Methodology | Incremental face recognition | CV | Predefined Learning | Baby Step |
| Zhou et al. (ICLR 2018) | Methodology | Image classification | CV | Predefined Learning | Baby Step |
| Zhou et al. (PR 2018) | Methodology | Person re-identification | CV | Predefined Learning | Baby Step |
| Li et al. (TNNLS 2018) | Methodology | Multi-label image classification | CV | Predefined Learning | Baby Step |
| Papers                        | Type             | Task                              | Domain | Curriculum                | Difficulty Measurer                  | Training Scheduler |
|------------------------------|------------------|-----------------------------------|--------|---------------------------|--------------------------------------|-------------------|
| Ghasedi et al. (CVPR 2019)   | Methodology      | Clustering                        | ML     | Self-paced Learning       | Model loss                           | Baby Step         |
| Shu et al. (AAAI 2019)       | Methodology      | Weakly-supervised domain adaption | CV, DA | Self-paced Learning       | Model loss, domain similarity         | Baby Step         |
| Zhang et al. (ICCV 2019)     | Methodology      | Weakly supervised object detection| CV     | Self-paced Learning       | Model loss                           | Baby Step         |
| Tang et al. (AAAI 2019)      | Application      | Active learning                   | ML     | Self-paced Learning       | Model loss                           | Baby Step         |
| Gong et al. (TEC 2019)       | Methodology      | Matrix factorization              | ML     | Self-paced Learning       | Model loss                           | Baby Step         |
| Zheng et al. (PRL 2020)      | Application      | Unsupervised feature selection    | ML     | Self-paced Learning       | Model loss                           | Baby Step         |
| Gong et al. (TIP 2016)       | Application      | Semi-supervised image classification| CV     | Transfer Teacher          | Reliability, discriminability         | Continuous        |
| Weinschall et al. (ICML 2018)| Methodology      | Image classification              | CV     | Transfer Teacher          | Teacher’s loss                       | Baby Step         |
| Hacohen et al. (ICML 2019)   | Methodology      | Image classification              | CV     | Transfer Teacher          | Teacher’s loss                       | Baby Step         |
| Xu et al. (ACL 2020)         | Application      | Natural language understanding    | NLP    | Transfer Teacher          | Uncertainty                          | Baby Step         |
| Zhou et al. (ACL 2020)       | Application      | Neural machine translation        | NLP    | Transfer Teacher          | Uncertainty                          | Multiple types    |
| Zhang et al. (arXiv 2018)    | Application      | Neural machine translation        | NLP    | Transfer Teacher          | Domain score, noise score            | Starting Big      |
| Wang et al. (ACL 2019)       | Application      | Neural machine translation        | NLP    | Transfer Teacher          | Domain score                         | Baby Step         |
| Zhang et al. (NAACL 2019)    | Application      | Neural machine translation        | NLP    | Transfer Teacher          | Domain score                         | Baby Step         |
| Graves et al. (ICML 2017)    | Methodology      | Sequence to sequence tasks        | NLP    | RL Teacher                | RL reward                            | RL action         |
| Mattis et al. (NeurIPS 2018)| Methodology      | Program execution, navigation     | NLP, RL| RL Teacher                | RL reward                            | RL action         |
| Fan et al. (ICLR 2018)       | Methodology      | Image classification, text understanding| CV, NLP| RL Teacher                | RL state, reward                     | RL action         |
| Qu et al. (WSDM 2018)        | Methodology      | Heterogeneous star network embedding| NE     | RL Teacher                | RL state, reward                     | RL action         |
| Kumar et al. (NAACL-HLT 2019)| Methodology      | Neural machine translation        | NLP    | RL Teacher                | RL state, reward                     | RL action         |
| Zhao et al. (AAAI 2020)      | Application      | Neural machine translation        | NLP    | RL Teacher                | RL state, reward                     | RL action         |
| Tsvetkov et al. (ACL 2016)   | Methodology      | Word representation learning       | NLP    | Other Automatic CL        | Bayesian Optimization                 | Baby Step         |
| Jiang et al. (ICML 2018)     | Methodology      | Image classification              | CV     | Other Automatic CL        | MentorNet                            | MentorNet         |
| Huang et al. (EMNLP)         | Application      | Relation extraction               | NLP    | Other Automatic CL        | MentorNet, collaborative conflict    | MentorNet         |
| Kim et al. (arXiv 2018)      | Methodology      | Deep RL, image classification     | CV, RL | Other Automatic CL        | ScreenerNet                          | ScreenerNet       |
| Ren et al. (ICML 2018)       | Methodology      | Image classification              | CV     | Other Automatic CL        | Meta-learner                          | Meta-learner       |
| Wu et al. (NeurIPS 2018)     | Methodology      | Image classification, NMT         | CV, NLP| Other Automatic CL        | Hypernetwork                         | Hypernetwork      |
| Saxena et al. (NeurIPS 2019) | Methodology      | Image classification and detection| CV     | Other Automatic CL        | Data parameters                      | Data parameters   |
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