A Gentle Introduction to Cognitive Map Based on Input Output Linguistic Variables

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Abstract

This paper studies on feedback graph model of linguistic variables which is generated from Hedge algebra. We also introduce a visual graphic model for input - output cognitive maps.

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Keywords: Fuzzy logic, Linguistic variable, feedback Graphs, Cognitive Maps.

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1 Introduction

In everyday life, people use natural language (NL) for analyzing, reasoning, and finally, make their decisions. Computing with words (CWW) [2, 9, 11–14, 20] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy set is fuzzy graph [3, 10, 17, 18], combined fuzzy set with graph theory. Fuzzy graph (FG) has a lots of applications in both modeling and reasoning fuzzy knowledge such as Human trafficking, in ternet routing, illegal immigration [16] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain, for example, linguistic summarization problems [13]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra (HAlg) as a tool for computing with words. The remainder of paper is organized as follows: Section 2 reviews some main concepts of computing with words based on HAlg. Important section 3 studies a graph database modeling with words using HAlg and its properties. Section 4 outlines conclusions and future work.

2 Preliminaries

This section presents basic concepts of HAlg and some important knowledge used in the paper.

2.1 Hedge algebra

In this section, we review some HAlg knowledges related to our research paper and give basic definitions. First definition of an HAlg is specified by 3-Tuple $HAlg = (X, H, \leq )$ in [9]. In [8] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $HAlg = (X, G, C, H, \leq )$ where $H \neq \emptyset$, $G = \{c^+, c^-\}$, $C = \{0, W, 1\}$. Domain of X is $L = Dom(X) = \{c| c \in G, \delta \in P^*(\text{hedge string over } H)\}, \{L, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \ldots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X is Age, $G = \{c^+ = \text{young}; c^- = \text{old}\}$, $H = \{\text{less; more; very}\}$ so term-set of linguistic variable Age X is $L(X)$ or L for short:

$L = \{\text{very less young}; \text{less young}; \text{young}; \text{more young}; \text{very young}\}$

Fuzziness properties of elements in HAlg, specified by $fm$ (fuzziness measure) [8] as follows:

Definition 2.1. A mapping $fm : L \rightarrow [0, 1]$ is said to be the fuzziness measure of L if:

1. $\sum_{c \in \{c^+, c^-\}} fm(c) = 1, \ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h \in H} fm(h x) = fm(x), \ x = h_n h_{n-1} \ldots h_1 c$, the canonical form.

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3. \( fm(h_nh_{n-1} \ldots h_1c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x) \).

The truth and meaning are fundamental important concepts in fuzzy logic, artificial intelligence and machine learning. In RCT (restriction-centered theory) [13], truth values are organized as a hierarchy with ground level or first-order and second-order. First order truth values are numerical values whereas second order ones are linguistic truth values. A linguistic truth value, say \( \ell \), is a fuzzy set. We study linguistic truth values on POSET \( \mathbb{L} \) whose elements are comparable [8, 9].

**Definition 2.2.** A \( \mathbb{L} \) STRUCT[\( \rho \)] on relational signature \( \rho \) is a tuple:

\[
\mathbb{L} = \langle \mathbb{L}, f_{a_i}^\mathbb{L}, c_j^\mathbb{L} \rangle
\]

(1)

Consists of a universe \( \mathbb{L} \neq \emptyset \) together with an interpretation of:

- each constant symbol \( c_j \) from \( \rho \) as an element \( c_j^\mathbb{L} \in \mathbb{L} \)
- each \( a_i \)-ary function symbol \( f_{a_i} \) from \( \rho \) as a function:

\[
f_{a_i}^\mathbb{L} : \mathbb{L}^{a_i} \to \mathbb{L}
\]

(2)

In \( \mathbb{H} \mathbb{A} \), \( \ell \in \mathbb{L} \) and there are order properties:

**Theorem 2.1.** In [9], let \( \ell_1 = h_n \ldots h_1u \) and \( \ell_2 = k_m \ldots k_1u \) be two arbitrary canonical representations of \( \ell_1 \) and \( \ell_2 \), then there exists an index \( j \leq \wedge \{m, n\} + 1 \) such that \( h_i = k_j \), for \( \forall i < j \), and:

1. \( \ell_1 \prec \ell_2 \) iff \( h_jx_j < k_jx_j \) where \( x_j = h_{j-1} \ldots h_1u \);
2. \( \ell_1 = \ell_2 \) iff \( m = n = j \) and \( h_jx_j = k_jx_j \);
3. \( \ell_1 \) and \( \ell_2 \) are incomparable iff \( h_jx_j \) and \( k_jx_j \) are incomparable;

**Example 2.** Consider linguistic variables: \( \{V\ \text{true}, \ P\ \text{true}, \ L\ \text{true}\} \in H \), in which \( \{V\ \text{true}, \ P\ \text{true}, \ L\ \text{true}\} \) stand for very true, possible true and less true, which are linguistic truth values generated from variable truth. Assume propositions \( p = "Lucie \ldots \)
is young is \( \forall \text{true} \) and \( q = \text{Lucie is smart is true} \), interpretations on \( H \) are:

- truth\( (p) = \forall \text{true} \in H \), truth is a unary function.

- \( p \land q = \forall \text{true} \land \mathcal{P} \text{true} = \mathcal{P} \text{true} \in H \). \( \land \) is a binary function.

- \( p \lor q = \forall \text{true} \lor \mathcal{P} \text{true} = \forall \text{true} \in H \). \( \lor \) is a binary function.

2.2 New model fuzzy cognitive map

The first Fuzzy Cognitive Maps (FCM) was introduced in \([10, 15]\) and fast developed in many applications \([3, 17]\). Fig. 1 is a simple FCM with its matrix in Fig. 3. In \([1, 19]\), that is a new FCM model with input signals.

**Example 3.** Fig. 2 shows a new model together with state transform equation which presents in Fig. 4.
\[
\begin{bmatrix}
    x_1(t + 1) \\
    x_2(t + 1) \\
    x_3(t + 1) \\
    x_4(t + 1) \\
    x_5(t + 1)
\end{bmatrix}
= f(
\begin{bmatrix}
    0 & 0 & 0 & W_{41} & W_{51} \\
    W_{12} & 0 & W_{32} & 0 & 0 \\
    0 & W_{23} & 0 & 0 & 0 \\
    0 & 0 & W_{34} & 0 & 0 \\
    0 & 0 & 0 & W_{45} & 0
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
+ \begin{bmatrix}
    W_1 & 0 & 0 & 0 & 0 \\
    W_2 & 0 & W_3 & 0 & 0 \\
    0 & 0 & W_4 & 0 & 0 \\
    0 & 0 & 0 & W_5 & 0
\end{bmatrix}
\begin{bmatrix}
    u_1(t) \\
    u_2(t) \\
    u_3(t) \\
    u_4(t) \\
    u_5(t)
\end{bmatrix}
)\]

Fig. 4. Matrix equation for new FCM model

\begin{center}
\begin{tikzpicture}[node distance=2cm, auto]
  \node [circle, draw, minimum size=2cm] (A1) {A1} ;
  \node [circle, draw, minimum size=2cm, below of=A1] (A2) {A2} ;
  \node [circle, draw, minimum size=2cm, below of=A2] (A3) {A3} ;
  \node [circle, draw, minimum size=2cm, left of=A1, xshift=-3cm] (A4) {A4} ;
  \node [circle, draw, minimum size=2cm, right of=A1, xshift=3cm] (A5) {A5} ;
  \node [circle, draw, minimum size=2cm, right of=A2, xshift=3cm] (A6) {A6} ;

  \path [->] (A1) edge node [above] {LCM$^1$} (A2);
  \path [->] (A1) edge node [above] {f(E,C)} (A3);
  \path [->] (A1) edge node [above] {f(E,C)} (A4);
  \path [->] (A1) edge node [above] {f(E,C)} (A5);
  \path [->] (A2) edge node [above] {f(E,C)} (A3);
  \path [->] (A2) edge node [above] {f(E,C)} (A4);
  \path [->] (A2) edge node [above] {f(E,C)} (A5);
  \path [->] (A2) edge node [above] {f(E,C)} (A6);
  \path [->] (A3) edge node [above] {f(E,C)} (A4);
  \path [->] (A3) edge node [above] {f(E,C)} (A5);
  \path [->] (A3) edge node [above] {f(E,C)} (A6);
  \path [->] (A4) edge node [above] {f(E,C)} (A5);
  \path [->] (A4) edge node [above] {f(E,C)} (A6);
  \path [->] (A5) edge node [above] {f(E,C)} (A6);

\end{tikzpicture}
\end{center}

Fig. 5. Diagram of new LCM-computing model

3 Input-Output Linguistic Cognitive maps

Linguistic cognitive maps LCM\(^1\) have been applying and studying in many areas of artificial intelligence [4–7]

Definition 3.1. A linguistic cognitive map (LCM) is a 4-Tuple:

\[
\text{LCM} = \{C, E, C, f\} \quad (3)
\]

In which:

1. \(C = \{C_1, C_2, \ldots, C_n\}\) is the set of \(N\) concepts forming the nodes of a graph.

2. \(E : (C_i, C_j) \rightarrow e_{ij} \in L; \ e_{ij} = \text{weight of edge directed from} \ C_i \text{ to} \ C_j.\) The connection matrix \(E(N \times N) = \{e_{ij}\}_{N \times N} \in L^{N \times N}\)

3. The map: \(C : C_i \rightarrow C_i^t \in L, t \in N\)

4. \(C(0) = [C_1^0, C_2^0, \ldots, C_n^0] \in L^N\) is the initial vector.

The recurring transformation function \(f\) is defined as:

\[
C_{i}^{t+1} = f \left( \sum_{i=1}^{N} e_{ij}C_j^t \right) \in L \quad (4)
\]

In the paper, we modify LCM to have a new LCM with input-output linguistic variables. Fig.5 illustrates a new abstract LCM and Fig.6 shows a simple new LCM with 5 inputs \(I = \{I_1, I_2, I_3, I_4, I_5\}\) and 5 outputs \(O = \{O_1, O_2, O_3, O_4, O_5\}\)

Property 3.1. A new LCM runs faster than LCM if it has the same concept space vertices.
4 Conclusions and future work

We have introduced a visual model so-called new LCM with the following two advantages

1. Inputs and Outputs use linguistic variable by applying hedge algebra

2. Computing with words on linguistic variable is not converting to numeric values therefore reducing number of operators for computation phases.

Our next study will investigate algorithms to construct and compute state space for new LCM.

References

[1] Chrysostomos D. Stylios E and Peter P. Groumpos. Mathematical formulation of fuzzy cognitive maps. Proceedings of the 7th Mediterranean Conference on Control and Automation, pages 28–30, 1999.

[2] Mabel. Frias, Filliberto Yaima, Gonzalo Nápoles, Koen. Vahoof, and Rafael. Bello. Fuzzy cognitive maps reasoning with words: An ordinal approach. In ISFUROS, 2017.

[3] Michael Glykas. Fuzzy Cognitive Maps, Advances in Theory, Tools and Applications. Springer, 2010.

[4] Nguyen Van Han and Phan Cong Vinh. Modeling with words based on hedge algebra. 7th EAI International Conference, ICCASA 2018 and 4th EAI International Conference, ICTCC 2018, 266:211–217, 2018.
[5] Nguyen Van Han and Phan Cong Vinh. Toward modeling and reasoning with words based on hedge algebra. EAI Endorsed Trans. Context-aware Syst. & Appl, 5(15):e5, 2018.

[6] Nguyen Van Han and Phan Cong Vinh. Reasoning with words: A hedge algebra linguistic cognitive map approach. Concurrency and Computation Practice and Experience, 33(2):e5711, Feb 2020.

[7] Nguyen Van Han and Phan Cong Vinh. Toward computing linguistic fuzzy graphs and applying to illegal immigration problem. EAI Endorsed Trans. Context-aware Syst. & Appl, 7(22):166361, 2020.

[8] Nguyen Cat Ho, Tran Thai Son, Tran Dinh Khang, and Le Xuan Viet. Fuzziness measure, quantified semantic mapping and interpolative method of approximate reasoning in medical expert systems. Journal of Computer Science and Cybernetics, 18(3):237–252, 2002.

[9] Nguyen Cat Ho and W.Wechler. Hedge algebras: An algebraic approach to structure of sets of linguistic truth values. Fuzzy Sets and Systems, 35(3):281–293, 1990.

[10] B. Kosko. Fuzzy cognitive maps. Internation Journal of Man-Machine Studies, 24:65–75, 1986.

[11] L.A.Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. Information Sciences, 8(3):199–249, 1975.

[12] L.A.Zadeh. Fuzzy-set-theoretic interpretation of linguistic hedges. Journal of Cybernetics, 2:4–34, 1977.

[13] L.A.Zadeh. Computing with words - Principal Concepts and Ideas. Studies in Fuzziness and Soft Computing. Springer, 2012.

[14] E. H. Mmdani. Application of fuzzy logic to approximate reasoning using linguistic synthesis. IEEE Transactions on Computers, C-26(12):1182–1191, 1977.

[15] J.N. Mordeson and P.S. Nair. Fuzzy Graphs and Fuzzy Hypergraphs. Physica-Verlag, Heidelberg, 2000.

[16] John N. Mordeson and Sunil Mathew. Advanced Topics in Fuzzy Graph Theory. Springer Nature Switzerland AG, 2019.

[17] Elpiniki I. Papageorgiou. Fuzzy Cognitive Maps for Applied Science and Engineering From Fundamentals to Extensions and Learning Algorithms. Springer-Verlag Berlin Heidelberg, 2014.

[18] A Rosenfeld. Fuzzy graphs. Fuzzy Sets and Their Applications, pages 77–95, 1975.

[19] C.D. Stylis, V. C. Georgopoulos, and P. P. Groumpos. Mathematical formulation of fuzzy cognitive maps. IEEE Med. Conf. on Control & Systems, 1997.

[20] Lotti A. Zadeh and Janusz Kacprzyk. Computing with Word in Information Intelligent System 1. Springer-Verlag BBerlin Heidelberg GmbH, 1999.