Transfer and storage of qubits in the presence of decoherence

KELLY R. PATTON and UWE R. FISCHER

Seoul National University, Department of Physics and Astronomy, Center for Theoretical Physics
151-747 Seoul, Korea

received 11 March 2013; accepted in final form 8 April 2013
published online 23 April 2013

PACS 03.65.Yz – Decoherence; open systems; quantum statistical methods
PACS 03.67.-a – Quantum information

Abstract – The effects of decoherence on the transfer and storage of coherent quantum states in hybrid systems are studied within the Caldeira-Leggett approach. In general, we find that a high transfer fidelity can be achieved even if the decoherence time is less than an order of magnitude larger than the transfer time, which is approximately half a Rabi period and determined by the qubit-qubit coupling strength. Finally, we apply our results to assess the feasibility of a hybrid quantum memory system, comprised of the hyperfine qubit states of an ultracold atomic Bose-Einstein condensate and the flux qubit of a SQUID.

The implementation of a quantum analog of random-access memory for qubits, qRAM, is currently one of the major goals of quantum information processing [1]. As with their classical counterpart, which can be found in all modern-day computers, the qRAM should be both fast and robust. Ideally, arbitrary coherent quantum states can be quickly passed to, stored, and retrieved from the qRAM with high fidelity. Unlike in classical systems, these processes have to be performed with an unavoidable continuous loss of information, caused by the detrimental influence of decoherence. The minimization of the effects of decoherence on transfer and storage of quantum information is therefore at the forefront of implementing a qRAM architecture and of quantum computing in general. To realize this demanding goal, quantum-hybrid systems represent some of the most interesting and promising candidates [2–7].

Within a hybrid architecture one subsystem, or qubit, works as part of the quantum central processing unit (CPU), while the other component operates as the qRAM. One of the major benefits of combining seemingly disparate systems is that they can each reinforce various advantages, while compensating for deficiencies, of either subsystem. For example, recent experiments involving the nuclear and electron spin states of nitrogen-vacancy (NV) centers in diamond, and a sophisticated dynamical decoupling of qubits, have produced record decoherence times for a solid-state system [8], on the order of a second. Along with constructing a qRAM architecture from either a single NV center [9] or an ensemble of such centers [10,11], a variety of other hybrid systems have been experimentally realized, such as superconducting qubits or atomic/molecular quantum gases coupled to electromagnetic and nanomechanical resonators [12–16].

By their very nature, each component of a hybrid system can have vastly different coherence times. Frequently, as is the case in many of the previous examples, the CPU element originates from a solid-state system, where the qubit states, $|0\rangle$ and $|1\rangle$, span the low-energy Hilbert space of some effective quasiparticle. While solid-state qubits have many advantages, such as scalability and controllability, they typically have small coherence times. This is a major hurdle for quantum information processing. In contrast, the memory qubits may not be as easily constructed on a large scale, but they potentially have much longer decoherence times, making them ideal for storage of quantum information.

The transfer of a quantum state from one qubit to the other is commonly done in experimentally realized hybrids via the Rabi process, cf., e.g., [8,13], dynamically controlling one qubit’s level spacing, bringing the two subsystems into and out of resonance for half a Rabi cycle. The Rabi period is determined by the qubit-qubit interaction. Ideally this coupling should be strong enough to enable a fast transfer, but weak enough to not excite states outside of the qubit Hilbert space. Although there has been previous work on the decay of Rabi oscillations between two qubits caused by decoherence [17] and on qubit state transfer in the presence of decoherence in the perturbative regime [18], a general study on the
effects of decoherence during a complete qRAM memory operation, *i.e.*, state transfer and storage, is however missing. In what follows, we undertake such a study, determining a practically applicable lower bound of the transfer fidelity which can be obtained as a function of the ratio of the degree of coherence to transfer time. This bound is of fundamental importance for the development of current and future hybrids, when the latter ratio becomes of order unity.

We simulate the transfer and storage of a state from one qubit to another in the presence of decoherence and determine the transfer fidelity. Within the Caldeira-Leggett model of dissipation and generalized spin-boson coupling [19] considered here, we find that the limited decoherence time is determined by the energy-relaxation time $T_1$. Naturally, one would expect that to obtain a high fidelity transfer, the transfer time should be much smaller than the coherence time. However, we find that to obtain a fidelity greater than 95%, a $T_1$ time of only about three times larger than the qubit-qubit coupling Rabi period, which in turn is larger than the transfer time by a factor between two and five (see also below), is required. As a direct application of these results to a concrete system, we propose a hybrid qRAM system, composed of a flux qubit (SQUID) and the hyperfine states of a trapped atomic Bose-Einstein condensate (BEC). With current technology, the relevant time scales of decoherence and Rabi period in this hybrid can be of the same order. Therefore, the inclusion of decoherence effects in a state transfer simulation are needed to verify the feasibility of this qRAM architecture.

For the coupled qubit-qubit system, the initial state is prepared in a parent qubit ($p$) and passed to a memory qubit ($m$). The Hamiltonian is taken to be ($\hbar = 1$)

$$
\hat{H}_{q-m}(t) = \frac{\omega_{\text{m}}}{2} \sigma_z \otimes \sigma_z + \frac{\omega_{\text{p}}(t)}{2} \sigma_0 - \begin{pmatrix} 0 & \Omega \\ \Omega^* & 0 \end{pmatrix} \otimes \sigma_x,
$$

where $\hat{A} \otimes \hat{B} = \hat{A} \otimes I \oplus I \otimes \hat{B}$ with $I$ being the identity matrix, $\sigma_i$ are Pauli matrices, and the time dependence of the level spacing of the parent qubit, $\omega_p(t) = \omega_n W(t)$, is used to bring the two qubits into and out of resonance\(^1\). The Rabi coupling $\Omega$ is assumed complex, due to the symmetrical $\sigma_z \otimes \sigma_z$ and $\sigma_y \otimes \sigma_x$ interaction terms. Although the form of the qubit-qubit coupling is chosen to coincide with the hybrid system proposed below, the following method and results can be easily generalized to include other forms.

There can be many sources of decoherence, originating both internally and externally. These sources will be collectively modeled phenomenologically, by the common

$$
\hat{V} = \sum_k \lambda_k \cdot \sigma \otimes (\hat{b}_k + \hat{b}_k^\dagger),
$$

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ and $\lambda_k = (\lambda^x_k, \lambda^y_k, \lambda^z_k)$ is the coupling constant to each bath mode, labeled by $k$. The total Hamiltonian of the qubit-qubit-bath system is then

$$
\hat{H}(t) = \hat{H}_{q-m}(t) \oplus \sum_k \epsilon_k \hat{b}_k \hat{b}_k^\dagger + I \otimes \hat{V},
$$

where $\epsilon_k$ determines the dispersion of the bath bosons.

To accurately determine the fidelity of a state transferred in the presence of decoherence, we numerically solve the time-dependent master equation for the reduced density matrix of the two-qubit system $\rho(t)$. If at time $t = 0$ the density matrix of the qubit-qubit-bath system is given by $\hat{\rho}(0) = \rho(0) \otimes e^{-\beta H_b}/Z_b$, with $\beta = (k_B T)^{-1}$ (the inverse temperature), $\hat{H}_b = \sum_k \epsilon_k \hat{b}_k \hat{b}_k^\dagger$, and $Z_b = \text{Tr} e^{-\beta H_b}$, then the full density matrix at a later time is given by $\hat{\rho}(t) = U_H(t) \hat{\rho}(0) U_H^\dagger(t)$, where the time-evolution operator $U_H(t)$ is the time-ordered exponential of the full Hamiltonian (3). Changing to the interaction representation, $\hat{\rho}_I(t) = U^\dagger_H(t) \hat{\rho}(t) U_H(t)$, where $\hat{H}_I(t) = \hat{H}_{q-m}(t) \oplus \hat{H}_b$ and $U_H(t) = U_{H_{q-m}}(t)e^{-\beta H_b}$. Within the Markov approximation [20] the equation of motion can be written

$$
\partial_t \hat{\rho}_I(t) = -i [\hat{V}_I(t), \hat{\rho}_I(t)] - \int_0^t dt' [\hat{V}_I(t), \hat{V}_I(t'), \hat{\rho}_I(t')].
$$

In addition, we also make the Born approximation, which neglects correlations that develop between the qubit subsystem and the bath. In other words, at later times the density matrix is taken to be of the form $\hat{\rho}(t) \approx \rho(t) \otimes e^{-\beta H_b}/Z_b$. An equation of motion for the reduced density matrix can then be found from (4) by tracing over the bath; $\rho(t) = \text{Tr}_b \hat{\rho}_I(t)$. Assuming an isotropic qubit-bath coupling $\lambda_k = \lambda_k (e_x + e_y + e_z) \equiv \lambda_k n$ and $\text{Tr}_b e^{-\beta H_b} \hat{\rho}(t) = 0$, the equation of motion for the reduced density matrix is given by the equation

$$
\partial_t \rho(t) = -\int_0^t dt' \int_0^\infty dt'' \frac{dJ(\epsilon)}{2 \pi} \left\{ \text{coth}(\epsilon/2) \cos(\epsilon(t-t')) \times \left[ I \otimes n \cdot \sigma(t), I \otimes n \cdot \sigma(t'), \rho(t) \right] - i \sin(\epsilon(t-t')) \times \left[ I \otimes n \cdot \sigma(t), I \otimes n \cdot \sigma(t'), \rho(t) \right] \right\},
$$

where the time dependence of the operators is now given by $U_{H_{q-m}}(t)$, and $J(\epsilon) = \sum_k \lambda_k^2 \delta(\epsilon - \epsilon_k)$ is a coupling
weighted density of states or spectral density of the bath. A common choice of \( J(\epsilon) \) is the so-called ohmic bath, with \( J(\epsilon) \propto \epsilon \) up to some upper cutoff \( \omega_c \). The form used here is \( J(\epsilon) = \eta \epsilon e^{-\epsilon/\omega_c} \), where \( \eta \) is the dimensionless dissipation or friction constant. The energy integral over \( \epsilon \) can be done analytically, but is not important for the following results.

Next, we numerically solve the 16 coupled differential equations of (5) for the reduced density matrix elements using a fourth-order Runge-Kutta method with a time step of \( \omega_0 t = 1 \) and parameters: \( \omega_0 = 100 \omega_m, \beta = 100 \omega_m^{-1}, \Omega = 0.01 \omega_m \). The temperature \( \beta^{-1} \) is set to be much smaller than any qubit energy spacing. This is, first of all, necessary to ensure a well-defined qubit. Furthermore, in this regime the dephasing due to thermal effects is negligibly small; as long as \( \beta^{-1} \ll \omega_m \), our results will therefore not significantly depend on the value of \( \beta \).

Additionally, the nontrivial time-evolution operator \( \hat{U}_{\text{He}}(t) \), which determines the time dependence of the operators in (5), and the final time integral have to be numerically found at each Runge-Kutta time step. The Rabi frequency used for the simulations allows for the calculations to be done in a reasonable time frame, while still within the weak coupling regime. Furthermore, the physically relevant quantity is the dimensionless ratio of the decoherence time to the time needed to transfer a state, approximately half a Rabi period \( T_{R/2} \approx \pi \Omega/2 \).

Figure 1 shows the obtained decoherence times of the parent qubit as a function of the ohmic friction constant \( \eta \). These were found by first decoupling the parent qubit from the memory one, by setting \( \Omega = 0 \) in (1). At time \( t = 0 \) the system is then put in the pure state \( |\Psi_t\rangle = 2^{-1/2}(|00\rangle + |01\rangle) \), where the computational basis states \( |ij\rangle = |i\rangle_m \otimes |j\rangle_p \) are the energy eigenstates of the uncoupled system. The energy-relaxation time \( T_1 \) and phase-decoherence time \( T_2 \) are then found by fitting the decay of the diagonal (\( T_1 \)) and off-diagonal (\( T_2 \)) components of \( \rho(t) \) to a decaying exponential \(^3\) of the form \( e^{-t/T_i} \). As can be seen from fig. 1, within this model \( T_2 \approx 2T_1 \); therefore, the \( T_1 \) energy relaxation time is the major limiting factor of any state transfer. The \( T_1 \) time can also be found by calculating the lifetime of the excited state \( \tau \) from diagrammatic many-body perturbation theory. The lifetime is then defined as \( \tau^{-1} = -2i\text{Im} \Sigma(\omega_p) \) \(^2\), where \( \Sigma(\omega) \) is the parent qubit’s self-energy. To second order in the qubit-bath coupling, which contains the leading term describing the relaxation of the qubit and excitation of a boson in the bath, \( T_1 \) is \( \tau \approx \frac{4\pi \eta \omega_p}{\sigma} e^{-\sigma/\omega_c} \), which agrees well with the numerically calculated values from our model, as shown in fig. 1.

To transfer a quantum state from the parent qubit to the other, one first initializes the system by bringing the two qubits’ level spacings out of resonance. Then, one prepares the parent qubit in an arbitrary initial state \( |\Psi_t\rangle = \alpha|00\rangle + \beta|01\rangle \). Next, the two subsystems are brought into resonance for half a Rabi period \( T_{R/2} \). Finally, one takes the two qubits out of resonance, leaving the parent qubit in its ground state and its original state in the memory qubit \( |\Psi_f\rangle = \alpha|00\rangle + \beta|10\rangle \). The transfer fidelity is defined to be \( F(t) = \text{Tr}[\rho(t)|\Psi_f(t)\rangle \langle \Psi_f(t)|] \), where \( \rho(t) \) is the reduced density matrix of the qubit system transformed back to the Heisenberg picture, and \( |\Psi_f(t)\rangle \) is the final target state, time evolved under the time-dependent but decoupled qubit-qubit Hamiltonian \( \hat{H}_{\text{He}} \), eq. (1).

Figure 2 shows the obtained time evolution of the fidelity and the final fidelities immediately after the transfer has completed, for various \( T_1 \) times, and for a direct \( |01\rangle \rightarrow |10\rangle \) (left column) and superposition state (right column) transfer. As one can see, the fidelity is strongly dependent on the initial state. In particular to transfer the superposition state \( 2^{-1/2}(|00\rangle + i|10\rangle) \rightarrow 2^{-1/2}(|00\rangle + i|10\rangle) \) a fidelity greater than 90% can readily be achieved for \( T_1 \approx T_{R/2} \); while for the transfer \( |01\rangle \rightarrow |10\rangle \) a fidelity of only about 50% is found. This is primarily due to the fact that for the superposition state the initial wave function has already a significant overlap with the final transferred one. Thus, even before any transfer steps are taken, it starts with a 50% fidelity. Generally, we find that to obtain a fidelity of at least 95%, \( T_1 \) should

---

\(^2\)We have verified that, within numerical accuracy, the data points shown in the bottom parts of figs. 1 and 2 are left unchanged when the temperature is increased from \( \beta^{-1} = \omega_m/100 \) to \( \beta^{-1} = \omega_m/10 \).

\(^3\)In the literature one can find several definitions of decoherence times, which can differ slightly from the ones used here, e.g., by factors of two.
be no smaller than about three times the Rabi period. Additionally, because of the bath, after the transfer the state can still slowly dissipate from the memory qubit—even when the two subsystems are out of resonance. This occurs on a time scale on the order of \( \omega_m/|\Omega|^2 T_1 \). This finite lifetime comes from a process where the memory qubit state \( |1\rangle \) can relax, exciting the parent qubit’s state from its ground state, which then decays into the bath. In diagrammatic many-body theory, this comes from a fourth-order term in the expansion of the memory qubit’s self-energy. We conclude that in general the decoherence, due to the bath in which the parent system is immersed, must be taken into account after the qubit has been transferred to the qRAM. However, in certain physical realizations, see, e.g., ref. [8], parent and memory qubits can be dynamically decoupled, \( \Omega \to 0 \), eliminating such decoherence effects.

The immediate applicability of our results can be seen in the following quantum-hybrid qRAM architecture. The memory qubits consist of the hyperfine split ground state of a trapped ultracold atomic BEC, e.g., \(^{87}\text{Rb}\). These states have significant potential for a qRAM, as their \( T_1 \) times are essentially infinite, on the order at least of seconds, and potentially even much longer than the lifetime of the BEC itself. Recent experimental data for a BEC cloud trapped near a superconducting waveguide resonator geometry indeed indicates phase-coherence times \( T_2 \approx 7 \text{s} \) [22], so that the use of our presently proposed hybrid for long-term storage of quantum information is promising.4

The BEC states are magnetically coupled to the parent flux qubit of a SQUID [23], which during a Rabi cycle excites a single atom from the macroscopically occupied ground state containing \( N \) atoms \((\downarrow)\) to an unoccupied excited state \((\uparrow)\), see fig. 3. The qubit subspace of the BEC is spanned by

\[
|0\rangle_B = (N!)^{-1/2}(\hat{a}_\uparrow^\dagger)^N|\text{vac}\rangle,
\]

\[
|1\rangle_B = [(N-1)!]^{-1/2}(\hat{a}_\uparrow^\dagger)^{N-1}\hat{a}_\downarrow|\text{vac}\rangle.
\]

Using the left-right current states of the SQUID [23], the uncoupled BEC-SQUID Hamiltonian then reads

\[
\hat{H}_{\text{NS}} = \frac{\omega_{\text{hf}}}{2}\sigma_z \otimes \left(\frac{\varepsilon}{2}\sigma_z - \frac{\Delta(t)}{2}\sigma_x\right),
\]

where \( \omega_{\text{hf}} \) (see eq. (1)) is the hyperfine splitting of the BEC states (for \(^{87}\text{Rb}, \omega_{\text{hf}} \approx 6.8 \text{GHz}\)), and \( \varepsilon \) is the energy difference of the \(|L\rangle\) and \(|R\rangle\) current states with tunneling amplitude \( \Delta(t) \), which can be dynamically modulated to bring the two systems into and out of resonance [24]. In eq. (7), the effective qubit-BEC Hamiltonian, \( \hat{H}_B = \frac{2\varepsilon}{\sigma_z} \), is expressed in the qubit (i.e., energy eigenstate) basis \((\hat{\sigma}_z)\). While the SQUID Hamiltonian is in the more commonly used \(|L\rangle\), \(|R\rangle\) basis. On the other hand, for \( \varepsilon = 0 \), in the qubit basis the SQUID Hamiltonian is simply given by \( \hat{H}_S = \frac{\Delta(t)}{2}\sigma_z \), and \( \omega_{\text{hf}} = \Delta(t) \) in (1), then.

4The natural decoherence times of the memory qubit, the BEC, can potentially compete with the coherence times of NV centers [8–11].
In the left-right current state basis of the SQUID, a current operator can be defined as follows; \( I \approx I \sigma_x \), where \( I = (|I| I(\Phi)|) \approx -i (|R| I(\Phi)|) \) (exact equality holding when \( \epsilon = 0 \)). The current operator is related to the flux operator by \( I = (\Phi - \Phi_x)/L \) with \( \Phi_x \) the classical externally applied flux and \( L \) the loop inductance. The magnetic vector potential\(^5\) of a loop of radius \( R \) in the dipole approximation and polar coordinates reads \( A \approx -\mu_0 R^2 I \sin \theta/(4r^2) \phi_\theta \phi_\sigma \). The SQUID’s magnetic field operator is then \( \mathbf{B} = \mathbf{V} \times \mathbf{A} = \mathbf{B}(r) \sigma_x \).

For an atom with total magnetic moment \( \mu \), (including orbital, nuclear, and electronic spin contributions) the magnetic dipole coupling of the BEC and SQUID is given by \( H_{\text{int}} = -\int d\mathbf{r} \mathbf{m}(\mathbf{r}) \cdot \mathbf{B}(\mathbf{r}) \), where \( \mathbf{m}(\mathbf{r}) \) is the second quantized magnetic moment density of the BEC; \( \mathbf{m}(\mathbf{r}) = \sum_{\sigma, \sigma'} \hat{\Psi}_{\sigma}^\dagger(\mathbf{r}) [\mu]_{\sigma \sigma'} \hat{\Psi}_{\sigma'}(\mathbf{r}) \). In the qubit basis and neglecting Zeeman energy shifts, which simply renormalize the energies of the flux qubit \( \varepsilon \rightarrow \varepsilon' \), the coupling takes the form of that given in eq. (1), with

\[
\Omega = \sqrt{N} g_{\sigma \sigma'} \phi_{\sigma} \cdot \mu_{\sigma \sigma'} \quad g_{\sigma \sigma'} = \int d\mathbf{r} \phi_{\sigma}^\dagger(\mathbf{r}) \mathbf{B}(\mathbf{r}) \phi_{\sigma'}(\mathbf{r});
\]

here \( \phi_{\sigma}(\mathbf{r}) \) is the center-of-mass atomic wave function for the trapped atoms. For a SQUID of radius \( 1 \mu m \) carrying a 1 mA current, and a center-to-center BEC-SQUID separation of \( 50 \mu m \), \( g_{\sigma \sigma'} \mu_{\sigma \sigma'} \sim 100 \) Hz. Therefore, given a typical BEC has \( N = 10^6 \) atoms, \( T_{\text{R}2/3} \) is on the order of \( 10 \mu s \), while current SQUIDs have decoherence times of the order of \( 1 \mu s \) [25].

Thus, from the results presented in the above, for the BEC to be a viable element of qRAM, the Rabi coupling, the SQUID decoherence times, or some combination thereof would have to be increased by a factor of approximately 100, according to the above parameter estimates. The simplest way to achieve this would be to move the BEC closer to the SQUID. For instance, at a distance of \( 10 \mu m \) the Rabi coupling would increase by almost two orders of magnitude\(^6\). There are, however, potentially detrimental effects caused by having the BEC in such close proximity to a (comparatively hot) surface, such as increased heating or spin flips caused by thermal emission from the SQUID [26,27].

In summary, we find that the quantum memory in a hybrid system can be functional even if the time to transfer the state is of the same order as the smallest decoherence time. Ideally coherence times or couplings would be large enough to perform a large number of qubit operations. But as quantum computing remains in its infancy, identifying the applicable limits of current technology to manipulate quantum information in hybrid systems is of major importance. We further expect our results to remain generally true, with only minor quantitative differences, for systems with different qubit-qubit or qubit-bath couplings than those considered here (for example dissipation into a sub-ohmic bath), because the ratio of transfer to decoherence times should be relatively insensitive to the details of the model. A detailed confirmation of this is left for future work. As an interesting and currently feasible application, a hybrid BEC-SQUID qRAM system was introduced. We have demonstrated that with the present coherence times of SQUIDs, the derived effects of decoherence on the transfer fidelity need to be taken into account, to assess the functionality of the BEC-SQUID qRAM as a quantum memory device.

***

This research was supported by the NRF of Korea, grant Nos. 2010-0013103 and 2011-0029541.

REFERENCES

[1] BLENCOE M., Nature, 468 (2010) 44.
[2] BLAIS A., HOANG R-S., WALLRAFF A., GIRVIN S. M. and Schoelkopp R. J., Phys. Rev. A, 69 (2004) 062320.
[3] VERDÓ J. et al., Phys. Rev. Lett., 103 (2009) 043603.
[4] LUKIN M. D., Rev. Mod. Phys., 75 (2003) 457.
[5] ARMOUR A. D., BLENCOE M. P. and SCHWAB K. C., Phys. Rev. Lett., 88 (2002) 143001.
[6] NAKAMURA Y., PASHKIN Yu. A. and TSAI J. S., Nature, 398 (1999) 786.
[7] WALLRAFF A. et al., Nature, 431 (2004) 162.
[8] VAN DER SAR T. et al., Nature, 484 (2012) 82.
[9] FUCHS G. D., BURKARD G., KLMOV P. V. and AWSCHALOM D. L., Nat. Phys., 7 (2011) 789.
[10] ZHU X. et al., Nature, 478 (2011) 221.
[11] WU H. et al., Phys. Rev. Lett., 105 (2010) 140503.
[12] MAJER J. et al., Nature, 449 (2007) 443.
[13] O’CONNELL A. D. et al., Nature, 464 (2010) 697.
[14] CLELAND A. N. and GELLER M. R., Phys. Rev. Lett., 93 (2004) 070501.
[15] RABL P. et al., Phys. Rev. Lett., 97 (2006) 033003.
[16] BRENNER S. et al., Nature, 450 (2007) 268; BAUMANN K., GUERLIN C., BRENNER S. and ESSLINGER T., Nature, 464 (2010) 1301.
[17] AMENT L. J. P. and BEENAKKER C. W. J., Phys. Rev. B, 73 (2006) 121307(R).
[18] ESCHER B. M. et al., J. Phys. B: At. Mol. Opt. Phys., 44 (2011) 154015.
[19] LEGGETT A. J. et al., Rev. Mod. Phys., 59 (1987) 1.
[20] BREUER H.-P. and PETRUCCIONE F., The Theory of Open Quantum Systems (Oxford University Press, New York) 2002.
[21] MAHAN G. D., Many-Particle Physics (Plenum Publishers, New York) 2000.
[22] Bernon S. et al., arXiv:1302.6610.
[23] Makhlin Y., Schön G. and Shnirman A., Rev. Mod. Phys., 73 (2001) 357.
[24] Paauw F. G., Fedorov A., Harmans C. J. P. M. and Mooij J. E., Phys. Rev. Lett., 102 (2009) 090501.
[25] Kakuyanagi K. et al., Phys. Rev. Lett., 98 (2007) 047004.
[26] Cano D. et al., Eur. Phys. J. D, 63 (2011) 17.
[27] Fermani R., Müller T., Zhang B., Lim M. J. and Dumke R., J. Phys. B: At. Mol. Opt. Phys., 43 (2010) 095002.