Deep-PANTHER: Learning-Based Perception-Aware Trajectory Planner in Dynamic Environments

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Abstract—This paper presents Deep-PANTHER, a learning-based perception-aware trajectory planner for unmanned aerial vehicles (UAVs) in dynamic environments. Given the current state of the UAV, and the predicted trajectory and size of the obstacle, Deep-PANTHER generates multiple trajectories to avoid a dynamic obstacle while simultaneously maximizing its presence in the field of view (FOV) of the onboard camera. To obtain a computationally tractable real-time solution, imitation learning is leveraged to train a Deep-PANTHER policy using demonstrations provided by a multimodal optimization-based expert. Extensive simulations show replanning times that are two orders of magnitude faster than the optimization-based expert, while achieving a similar cost. By ensuring that each expert trajectory is assigned to one distinct student trajectory in the loss function, Deep-PANTHER can also capture the multimodality of the problem and achieve a mean squared error (MSE) loss with respect to the expert that is up to 18 times smaller than state-of-the-art (Relaxed) Winner-Takes-All approaches. Deep-PANTHER is also shown to generalize well to obstacle trajectories that differ from the ones used in training.

Index Terms—UAV, Imitation Learning, Perception-Aware Trajectory Planning, Optimization.

I. INTRODUCTION AND RELATED WORK

Trajectory planning for UAVs in unknown dynamic environments is extremely challenging due to the need for gaining information about the obstacles while avoiding them at the same time. Perception-aware planning has emerged as one promising approach for this, where the translation and/or rotation of the UAV are optimized to maximize the presence of the obstacles in the FOV of the onboard camera while flying towards the goal [1]–[7]. The dynamic nature of these environments requires very fast replanning times, which are usually achieved by simplifying the optimization problem by fixing some variables (such as the time allocation or the planes that separate the UA V from the obstacles) beforehand or by ignoring the multimodality of the problem [7]. While these simplifications help reduce the computation time, that is often achieved at the expense of more conservative planned trajectories. This leaves open the question of whether or not it is possible to obtain faster computation times while achieving less conservative trajectories.

Towards this end, Imitation Learning (IL) has recently gained interest due to its ability to train a computationally-cheap neural network (the student) to approximate the solution of a computationally-expensive algorithm (the expert). IL has been successfully used to compress MPC policies [12]–[15] and/or to learn path planning policies [11], [16], [17]. Compared to other IL-based trajectory planning works, which typically either assume static worlds or do not take into account perception awareness, our work proposes to use IL to obtain perception-aware trajectories that perform obstacle avoidance in dynamic environments.

When performing obstacle avoidance, capturing the multimodality of the trajectory planning problem is crucial to reduce the conservativeness. Indeed, for a given scenario, there may be $n_e \geq 1$ locally-optimal expert trajectories that avoid the obstacle(s) (e.g., see Fig. 1), where $n_e$ may change between different scenarios. The use of a unimodal student that produces a single trajectory either introduces an artificial bias towards a specific direction of the space, or averages together the different expert trajectories, which can be catastrophic in obstacle avoidance scenarios. The challenge is then how to design and train a neural network capable of generating a multimodal trajectory prediction.

One possible approach is to use Mixture Density Networks to learn the parameters of a Gaussian mixture
Figure 2: Comparison between the assignment matrix $A$ obtained by the WTA [8], [9], RWTAr [9], [10], WTAc [11], RWTAc [11], and LSA approaches. This matrix $A$ is then the one used to weigh each (target, prediction) pair in the loss. In the figure, $\epsilon \geq 0$, $n_s = 3$ and $\text{obs}_i$ denotes the observation of the training sample $i$. In WTA and RWTAr, each row of $A$ sums up to 1, while in WTAc and RWTAc, each column of $A$ sums up to 1. We propose instead to obtain $A$ as the solution of the linear sum assignment (LSA) problem, which minimizes the total assignment cost, and guarantees that all the target labels have one distinct prediction assigned to them (i.e., all the rows sum up to 1, $n_e$ columns sum up to 1, and $(n_s - n_e)$ columns sum up to 0). More visualizations of the WTA and RWTAr assignments are available at [9] and [8].

| Training sample 1 | WTAr | RWTAr | WTAc | RWTAc | LSA (Ours) |
|-------------------|------|-------|------|-------|------------|
| Plot              | ![Plot](image1) | ![Plot](image2) | ![Plot](image3) | ![Plot](image4) | ![Plot](image5) |
| $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ | $\begin{bmatrix} 1 - \epsilon & \epsilon/2 & \epsilon/2 \\ \epsilon/2 & 1 - \epsilon & \epsilon/2 \\ \epsilon/2 & \epsilon/2 & \epsilon \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \epsilon & 1 - \epsilon & 1 - \epsilon \end{bmatrix}$ | $\begin{bmatrix} 1 - \epsilon & \epsilon \\ \epsilon & 1 - \epsilon \\ \epsilon & \epsilon \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ \epsilon & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ |

| Training sample 2 | WTAr | RWTAr | WTAc | RWTAc | LSA (Ours) |
|-------------------|------|-------|------|-------|------------|
| Plot              | ![Plot](image6) | ![Plot](image7) | ![Plot](image8) | ![Plot](image9) | ![Plot](image10) |
| $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} \epsilon/2 & \epsilon/2 & \epsilon/2 \\ \epsilon/2 & 1 - \epsilon/2 & \epsilon/2 \\ \epsilon/2 & \epsilon/2 & \epsilon \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ \epsilon/2 & \epsilon/2 & \epsilon/2 \end{bmatrix}$ | $\begin{bmatrix} \epsilon/2 & \epsilon/2 & \epsilon/2 \\ \epsilon/2 & 1 - \epsilon/2 & \epsilon/2 \\ \epsilon/2 & \epsilon/2 & \epsilon \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ |

Model [18]. Mixture Density networks are however known to suffer from numerical instability and mode collapse [9], [10]. Another option is to design multimodal losses \(^1\) that are able to compare a set of predicted trajectories with a set of target trajectories. For example, the Winner-Takes-All (WTA or WTAc) losses [8], [9], [19] (see Fig. 2) use an binary assignment matrix $A$ that weighs the contribution of each (target, prediction) pair in the loss. In WTA [8], [9] (Winner-Takes-All-row), each target label is assigned to the closest prediction, while in WTAc (Winner-Takes-All-column), each prediction is assigned to the closest target label. Other works propose instead the use of the relaxed losses RWTAr [9], [10] (Relaxed-Winner-Takes-All-row) and RWTAc [11] (Relaxed-Winner-Takes-All-column), where the constraint of $A$ being a binary matrix is relaxed (see Fig. 2). These relaxed costs typically address the mode collapse problem (which happens when all the predictions of the network after training are close to the same target label), but due to the nonzero weights between all the predictions and all the target labels, the predictions of these relaxed costs may reach an equilibrium position that does not represent any of the target labels [20].

In contrast to these approaches, and inspired by the multi-object detection and tracking algorithms [21]–[23], we propose to use (in the loss) the optimal assignment matrix $A$ found by solving the linear sum assignment (LSA) problem, which minimizes the total assignment cost and guarantees that all the target labels are assigned to a distinct prediction (see Fig. 2). This ensures that a target label is not assigned to multiple predictions (reducing therefore the mode collapse problem) and that each prediction is not assigned to multiple target labels (being therefore less prone to equilibrium issues).

The contributions of this work are therefore summarized as follows:

- Novel multimodal learning-based trajectory planning framework able to generate collision-free trajectories that avoid a dynamic obstacle while maximizing its presence in the FOV.
- Computation times two orders of magnitude faster than a multimodal optimization-based planner, while achieving a similar total cost (as defined in Section II-E).
- Multimodal loss that achieves an MSE of the predicted trajectories with respect to the expert trajectories up to 18 times smaller than the (Relaxed) Winner-Takes-All approaches.
- Deep-PANTHER also presents a very good generalization to environments where the obstacle is following a different trajectory than the one used in training.

II. DEEP-PANTHER

Deep-PANTHER is a multimodal trajectory planner able to generate a trajectory that avoids a dynamic obstacle, while trying to keep it in the FOV. To achieve very fast computation times, we leverage imitation learning, where Deep-PANTHER is the student (a neural network) that is trained to imitate the position trajectories generated by an optimization-based expert (Section II-A). Both the student and the expert have an observation as input and an action.

\(^1\)In this paper we use the term multimodal to refer to the fact that the set of predicted trajectories can contain more than one trajectory. Intuitively, this means that the planned trajectories capture the fact that we can go right, left, up... (see Fig. 1). Unimodal approaches, on the contrary, are able to generate only one trajectory.
Table I: Notation used in this paper.

| Symbol | Meaning |
|--------|---------|
| $S_{\alpha}^{d,m}$ | Set of clamped uniform splines with dimension $d$, degree $m$, and $m+1$ knots. |
| $\varphi$ | Quaternions. |
| $\mathcal{U}(a,b)$ | Uniform distribution in $[a,b]$. |
| $(\alpha)_{jk}A$ | $\alpha$ normalized: $(\alpha)_{jk}A = \frac{\alpha_{jk}A}{\|\alpha_{jk}A\|}$. |
| $p^\alpha$ | Point expressed in the frame $\alpha$. For the definitions of this table that include the sentence “expressed in the world frame”, the notation of the frame is omitted. |
| $T^\alpha_{1} = \left[ \begin{array}{cc} R^\alpha & t^\alpha \\ 0 & 1 \end{array} \right]$ | Transformation matrix. |
| $\psi$ | Angle such that $q^\psi = \frac{1 + \xi}{\sqrt{1 + \xi^2}}$. |
| $\xi$ | Relative acceleration, expressed in the world frame. |
| $\xi_{u}, \xi_{v}, \xi_{w}$ | Components of a unit quaternion. |
| $\vec{R}^\alpha_{b} = [b_1, b_2, b_3]$ | B-Spline control points of the planned trajectory expressed in frame $\alpha$. |
| $\mathbf{Theta}$ | Angle in the plane spanned by $w_{ab}$, $\psi$ is the derivative of $\mathbf{Theta}$. |

as output (Section II-B). The multimodality is captured through the design of the loss function (Section II-C), and the trajectories for the extra degree of freedom of the rotation ($\psi$) can then be obtained from the position trajectories (Section II-D). The final trajectory chosen for execution is obtained according to the cost and the constraint satisfaction (Section II-E). This paper uses the notation shown in Table I.

A. Expert and Student

Our prior work [7] developed PANTHER, an optimization-based perception-aware trajectory planner able to avoid dynamic obstacles while keeping them in the FOV. However, and as discussed in Section I, real-time computation was achieved at the expense of conservative solutions. Hence, we design PANTHER$^*$ (the expert) by reducing the conservativeness of PANTHER as follows:

- The planes that separate the trajectory of the UAV from the obstacles [7] and the total time of the planned trajectory $T$ are included as decision variables. To ensure that $T$ does not go beyond the prediction horizon, the constraint $0 \leq T \leq T_{\text{pred}}$ is imposed for both the expert and the student. Here, $T_{\text{pred}}$ is the total time of the future predicted trajectory of the obstacle, and it is a user-chosen parameter.
- The future predicted trajectory of the obstacle is a spline whose control points are $Q_{\text{p,obst}}$.
- The optimization problem runs $n_{\text{trans}}$ times (with different initial guesses obtained by running the OSA [25]), and $n_{\text{sol}} \leq n_{\text{trans}}$ distinct trajectories are obtained.

The student (Deep-PANTHER) consists of a fully connected feedforward neural network with two hidden layers, 64 neurons per layer, and with the ReLU activation function. The student produces a total of $n_s$ trajectories, where $n_s$ is a user-chosen parameter. Note that the trajectories produced by the expert are then the best $n_e = \min(n_{\text{sol}}, n_s)$ trajectories obtained in the optimization.

B. Observation and Action

We use the observation $(v^f, a^f, g^f, \psi, Q_{\text{p,obst}}, s_{\text{obst}})$, where, as defined in Table I, $v^f \in \mathbb{R}^3$, $a^f \in \mathbb{R}^3$, $g^f \in \mathbb{R}^3$, and $Q_{\text{p,obst}}$ are, respectively, the velocity of the UAV, the acceleration of the UAV, the projection of the terminal goal $g_{\text{term}}$, and the control points of a spline fit to the future predicted trajectory of the obstacle. All of these quantities are expressed in the frame $f$. $\psi \in \mathbb{R}$ is the derivative of $\psi(t)$. $s_{\text{obst}} \in \mathbb{R}^3$ contains the length of each side of the axis-aligned bounding box of the obstacle. In this work, we use a spline in $\mathbb{R}^3_{13}$ for the predicted trajectory of the obstacle, which means that $Q_{\text{p,obst}}$ contains 10 position control points, each one in $\mathbb{R}^3$. This leads to an observation size of 43.

The action is given by $(T_k)_{k \in \{0, ..., \beta - 1\}}$, where $\beta = n_s$ for the student, and $\beta = n_e$ for the expert, and where $T_k := (Q_{\text{p}})_{k \in \{0, ..., \beta - 1\}}), T_k$. As defined in Table I, $Q_{\text{p}}$ contains all the B-Spline control points of the planned trajectory expressed in frame $f$ except the first three and the last two, while $T$ is the total time of the planned trajectory. Note that the first
Figure 3: Multimodal training in Deep-PANTHER. The student outputs a fixed number of trajectories, denoted as $n_s$. The expert (PANTHER*) produces $n_e$ trajectories, where $n_e \leq n_s$. Then, the cost matrix in position space ($D_p$) and in time space ($D_T$) are computed. Using $D_p$, the linear sum assignment (LSA) problem is solved to find the assignment matrix $A$, which is then used in the loss to penalize the expert-student assigned pairs.

Figure 4: Optimal $\psi$ trajectory (--- in the right plots) given the position trajectory defined by $(Q_p)_k, T_k)$. A spline is then fit to this $\psi$ trajectory found. The sphere $\bullet$ denotes the position of the obstacle $p_{obst}$. For visualization purposes, we show here a static obstacle, but this method is also applicable when the obstacle is dynamic.

The key advantage of using $\hat{Q}_p$ instead of $Q_p$ is that every trajectory generated by the student will satisfy by construction the initial and final conditions for any given observation. It also helps reduce the action size. Moreover, the advantage of using the B-Spline position control points, instead of sampled future positions as in [11], is that every trajectory generated by the student is smooth by construction ($C^2$-continuous in our case), and it also avoids the need of a post-projection step into polynomial space.

C. Loss: Capturing Multimodality

As discussed in Section I and Fig. 1, the number of trajectories found by the expert changes depending on the specific observation. To train a neural network with a fixed-size output ($n_s$ trajectories) to predict the varying-size output of the expert ($n_e$ trajectories), we propose to use the approach shown in Fig. 3. The observation is passed through the neural network of the student to generate $n_s$ trajectories, and through the expert to produce $n_e$ trajectories. We then define $D_p$ as a matrix whose element $(i,j)$ is the mean squared error (MSE) between the position control points of the $i$-th trajectory of the expert and the position control points of the $j$-th trajectory of the student. A similar definition applies to $D_T$, but using the total time of the trajectory instead of the control points.

Letting $A$ denote the assignment matrix (whose $(i,j)$ element is 1 if the $i$-th trajectory of the expert has been assigned to the $j$-th trajectory of the student, and 0 otherwise), we then find the optimal $A$ that minimizes the assignment cost $1^T (A \odot D_p + \beta T A \odot D_T) 1$, and that assigns a distinct student trajectory to every expert trajectory. Here, $\odot$ denotes the element-wise product and $1$ is a column vector of ones. This is an instance of the linear sum assignment (LSA) problem, and the optimal $A$ can be obtained leveraging the Jonker-Volgenant algorithm [26] (a variant of the Hungarian algorithm [27]). As we have that $n_e \leq n_s$, all the rows of $A$ sum up to 1, $n_e$ columns sum up to 1, and $(n_s-n_e)$ columns sum up to 0. To penalize only the MSE of the optimally-assigned student-expert pairs, the loss is then computed as

$$L = 1^T (\beta_p A \odot D_p + \beta T A \odot D_T) 1,$$

where $\beta_p$ and $\beta T$ are user-chosen weights.

Our approach ensures that all the expert trajectories have exactly one distinct student trajectory assigned to them, see Fig 2. Compared to WTA$\bar{r}$, RWT$\bar{a}$r, WTAc, and RWTAc, our LSA loss prevents the same student trajectory from being assigned to several expert trajectories (reducing therefore the equilibrium issues), guarantees that all the trajectories
of the expert are captured in every training step, and also prevents the same expert trajectory from having several student trajectories assigned to it (being therefore less prone to the mode collapse problems).

**D. Generation of \( \psi \) given the Position Trajectory**

Each \( T_k \), together with the initial and final conditions contained in the observation, defines the position trajectory. Since \( b_3 := R_b^w e_z = (\xi)_n \) (see Fig. 4 and Table I), this position trajectory determines part of the rotation, but leaves \( \psi \) free. We now derive a closed-form expression for \( \psi(t) \) that maximizes the presence of the obstacle in the FOV given the position trajectory. Let \( p := t_w^b \) be the position of the UAV, and let \( p_{\text{obs}} \) denote the position of the obstacle (both expressed in the world frame). Let us also define \( b_1 := R_b^w e_x \). Using a cone with opening angle \( \theta \) to model the FOV, the obstacle is in the FOV if and only if \( \cos(\theta/2) \leq b_1^T (p_{\text{obs}} - p)_n \). We can therefore maximize the presence of the obstacle in the FOV by solving the following optimization problem:

\[
\min_{b_1} -b_1^T (p_{\text{obs}} - p)_n \quad \text{subject to} \quad b_1^T \xi = 0 \quad \text{and} \quad \|b_1\|^2 = 1
\]

where the two constraints guarantee that \( b_1 \) is a unit vector perpendicular to \( \xi \). Computing the Lagrangian and solving the Karush-Kuhn-Tucker (KKT) conditions [29], [30] yields the optimal solution:

\[
b_1 = \left( (p_{\text{obs}} - p) - \frac{(p_{\text{obs}} - p)^T \xi}{\|\xi\|^2} \right)_n \tag{1}
\]

Given that \( p_{\text{obs}} \), \( p \), and \( \xi \) are functions of time, Eq. 1 gives the evolution of \( b_1 \) that maximizes the presence of the obstacle in the FOV (see Fig. 4). \( \psi(t) \) can then be easily obtained from \( b_1 \) and \( b_3 \), and a spline is fit to it to obtain the control points \( \mathcal{Q}_\psi \).

Note that, in PANTHER*, position and rotation are coupled together in the optimization [7]. This coupling helps reduce the conservativeness that arises when they are optimized separately [1]–[3]. Deep-PANTHER (the student) learns to predict the position trajectory resulting from this coupled optimization problem, and then the closed-form solution is leveraged to obtain \( \psi \) from this position trajectory. In other words, Deep-PANTHER benefits from the coupling (since it is learning one of the outputs of the coupled optimization problem), while leveraging the closed-form solution for \( \psi \).

**E. Testing**

In testing time the procedure is as follows (see Fig. 5): The observation is fed into the neural network, which produces \( \mathcal{Q}_\psi \) for each \( T_k \). The optimal \( \psi \) control points \( (\mathcal{Q}_\psi)_k \) are then obtained as explained in Section II-D. Then, and using the observation, each triple \(( (\mathcal{Q}_p)_k, (\mathcal{Q}_\psi)_k, T_k \) is ranked according to the cost and the constraint satisfaction. The trajectory chosen for execution is then the one that is collision-free and achieves the smallest augmented cost, which is defined as \( c_{\text{obj}} + \lambda c_{\text{dyn}} \), where \( c_{\text{obj}} \) is the cost of the objective function, \( c_{\text{dyn}} \) is a soft cost that penalizes the velocity, acceleration, and jerk violations, and \( \lambda > 0 \). If none of the trajectories generated by the student are collision-free, the UAV will continue executing the trajectory it had in the previous replanning step (which is collision-free) and will replan again.

**III. RESULTS AND DISCUSSION**

To better compare the different aspects of the proposed framework, Section III-A first focuses on a stopped UAV that needs to plan a trajectory from the start location to the goal (without moving along that planned trajectory) while avoiding a static obstacle. Then, Section III-B studies the more general case where a UAV is flying and constantly replanning in a dynamic environment.

We use \( n_s = 6, n_{\text{runs}} = 10, T_{\text{pred}} = 6 \text{ s}, \text{ and } \beta_p = \beta_T = 1.5 \). To train the neural network we use the Adam optimizer [31] and a learning rate of \( 10^{-3} \). In all these simulations, and for all the algorithms tested, \( \mathcal{Q}_{p,\text{obs}} \) is obtained by simply fitting a spline to the ground-truth future positions of the obstacle. In real-world applications, this future predicted trajectory of the obstacle can be obtained from past observations [7].

Note that the selection of \( n_s \) and \( n_{\text{runs}} \) sufficiently high helps reduce a potential bias problem that could appear if the expert generated very few (2 or 3) trajectories. Moreover, we also randomize the training environments to help reduce this potential bias (see following subsections).

**A. Static Obstacle**

In this section, the task is to plan once from the starting location to the goal (i.e., the UAV does not move along the planned trajectory and/or replan again). We collect 2K (observation, expert action) pairs, and use 75% of these pairs to train the student offline (the rest of the pairs are used as the evaluation dataset in the MSE comparisons of Section III-A2). Section III-A1 first compares the cost vs replanning time, and then Section III-A2 analyzes how well the multimodality is captured.

1) **Cost vs Replanning Time**

We compare the cost vs replanning time of these three different approaches: PANTHER (Ref. [7]), PANTHER* (the

\[4\]

For simplicity, here we focus on the case where \( n_b^w = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \) and \( t_b^w = 0 \). A similar derivation applies to more general cases. See also [28].

\[2\]

Note that Eq. 1 presents a singularity when \( (p_{\text{obs}} - p) \) is parallel to \( \xi \). In that case, we can choose any \( b_1 \), since all of them are perpendicular to \( (p_{\text{obs}} - p) \) and therefore achieve the same (zero) cost in the objective function.

\[3\]

The code contains the details of the randomization performed.
• MSE with respect to the trajectories of the expert. For each of the trained policies, we obtain the optimal assignment between the trajectories of the expert and the student using the cost matrix $D_p$ [26]. The trajectories of the student are then ranked according to the position MSE loss with respect to their assigned expert trajectory, and the index of this ranking is denoted as $\kappa$. For instance, the case $\kappa = 0$ corresponds to the trajectory of the student that best predicts an expert trajectory. The results are shown in Fig. 8, where values above 1 represent cases where LSA (our approach) performs better. Compared to RWTAr, our approach achieves an average MSE between 1.09 and 18.02 times smaller. Compared to RWTAc, our approach achieves an average MSE between 2.35 and 2.68 times smaller.

• Number of collision-free trajectories obtained. Using the same testing scenario as in Section III-A1 (Fig. 6), Fig. 9 shows the number of collision-free trajectories produced by each algorithm. Our approach is able to produce at least one collision-free trajectory for all the $\theta_{term}$ tested, while RWTAr-\(\epsilon\) \((\epsilon \in \{0.25, 0.35\})\) and RWTAc-\(\epsilon\) \((\epsilon \in \{0, 0.05, 0.15, 0.25, 0.35\})\) fail to generate a collision-free trajectory for some of the goals, especially for the ones that are directly behind the obstacle.

B. Replanning with a Dynamic Obstacle

We train the student in an environment that consists of a dynamic obstacle flying a trefoil-knot trajectory [32]. The position, phase, and scale of this trefoil-knot trajectory, together with the terminal goal, are randomized. We use the Dataset- Aggregation algorithm (DAgger) [33] to collect the data and train the student. DAgger is an iterative dataset collection and policy training method that helps reduce covariate shift issues by querying actions of the expert while executing a partially trained policy. The total number of observation, expert action) pairs collected is approximately 23K.

To test this trained policy, we deploy a dynamic obstacle following a trefoil-knot trajectory with a random phase, and
C. Generalization to other Obstacle Trajectories

To evaluate how well the student in Section III-B (trained using trefoil-knot obstacle trajectories) generalizes, we test it with different obstacle trajectories: static, square, eight and epitrochoid (see Fig. 11). During 45 seconds, the UAV must fly back and forth between two goals separated 10 m, with the trajectory of the obstacle lying between these goals. The number of collision-free trajectories generated is shown in Table II. Despite being trained with a different obstacle trajectory, the policy succeeded in generating at least one collision-free trajectory in all the approximately 740 replanning steps. In all the cases the UAV reached 8 goals during the total simulation time.

D. Several Obstacles

In these simulations, the task is to fly from \( x = 0 \) m to \( x = 15 \) m avoiding multiple randomly-deployed obstacles that follow epitrochoid trajectories. The policy used is the one of Section III-B, which was trained with only one obstacle that followed a trefoil-knot trajectory. For the input of the neural network, Deep-PANTHER then chooses the obstacle that has the highest probability of collision [7]. The results,
in terms of the safety ratio,⁷ are available in Fig. 12. Note how even though Deep-PANTHER has been trained with only one obstacle, it is able to succeed at all times when the number of obstacles is 1 or 2. When the number of obstacles is 3, 4, or 5, Deep-PANTHER is able to succeed on average. The failures could be addressed by incorporating multiple obstacles in the training (instead of only one obstacle), which is left as future work.

IV. CONCLUSION AND FUTURE WORK

This work derived Deep-PANTHER, a learning-based perception-aware trajectory planner in dynamic environments. Deep-PANTHER is able to achieve a similar cost as the optimization-based expert, while having a computation time two orders of magnitude faster. The multimodality of the problem is captured by the design of a loss function that assigns a distinct student trajectory to each expert trajectory. This leads to MSE losses with respect to the expert up to 18 times smaller than the (Relaxed) Winner-Takes-All approaches. Deep-PANTHER also performs well in environments where the obstacle follows a different trajectory than the one used in training. Future work includes the extension to multiple dynamic obstacles, the inclusion of the camera images directly in the observation, and real-world experiments.

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