Optimal Model of Fire Fighting and Rescue Operational Plan Based on Utility Function

Wanguo He², Hao Xie¹, ², *
¹University of Science and Technology Beijing, China
²Fire Rescue Detachment of Qiannan Prefecture, China
*Corresponding author e-mail: xh389611202@163.com

Abstract. The fire brigade was a specialized and militarized team that fought fires and other disasters, whose main duties were to dispose of fires and other disasters in order to protect people's lives and property. Under the new situation, with the increasing complexity of disaster accidents and the status quo of the equipment of the fire brigade, firefighting and rescue work presented the characteristics of complexity, diversity and arduousness. Therefore, in order to complete the firefighting rescue faster and better, and avoid the disadvantages of the command, it was particularly important to carry out the plan optimization. In view of this, an optimization model of firefighting and rescue operational plan was proposed by this paper to determine the good and bad of the plan by using the utility function model to do quantitative calculation. The construction principles and specific calculation steps of the utility function model were firstly introduced by the paper. Then the actual case was combined with to establish the index system of a firefighting and rescue optimal plan to solve the utility function value. Finally, the weights and utility function values were used to determine the comprehensive evaluation value, in order to select the best operational plan. The feasibility of the plan waiting for the selection was transformed by this model into the utility function, making it more comparable and easy to optimize the plan. The determination of the optimal plan improved the decision-making ability of the commander, and the operation mechanism to guide and improve the firefighting rescue, thereby to effectively.

1. Construction of a utility function model
The utility function was a substantial function for calculating the good and bad of each plan waiting for selection and a mapping from the firefighting and rescue scene setting to the utility range. It reflected the commander's view on the risk and consequences of firefighting and rescue. The utility function value reflected the satisfaction of the evaluator of the plan optimization with the optimal indexes [1]. Different optimal indexes had different dimensions, and each index was transformed by referring to the utility function thus to make different indexes comparable, so that the commander could analyze and use the utility function qualitatively to quantify. At the scene of the accident, especially the heavy and extraordinary accident scenes, it was very important for firefighting and rescue to build the optimal model of the operational plan. It could provide a quantitative basis for the commander to choose the operational plan, which was not only beneficial to improve the decision-making ability of the...
commander, improve the operating mechanism of the firefighting rescue, improve the overall combat capability of the forces, decrease the unnecessary losses caused by the accidents to the people furthest, but also reduced the political negative impact of the accidents and alleviated the various pressures brought by the accidents to the officers and soldiers [2].

1.1. Building an evaluation matrix

Assume that there were no plans waiting for selection for one firefighting and rescue site, for each plan waiting for selection, the expert judged and graded for the optimal index on the spot, and the scores formed an nx3 rank matrix B:

\[
\begin{bmatrix}
    b_{11} & b_{12} & b_{13} \\
    b_{21} & b_{22} & b_{23} \\
    b_{31} & b_{32} & b_{33} \\
    b_{41} & b_{42} & b_{43} \\
    \vdots & \vdots & \vdots \\
    b_{n1} & b_{n2} & b_{n3}
\end{bmatrix}
\]

In the formula, \( b_{ij} \) was the jth index value of the ith plan waiting for selection.

1.2. Determine the utility function value of the optimal indexes

According to the utility function theory, when the optimization of the operational plan was carried out, for the positive index (the larger the value, the more favorable the combat, such as operational feasibility, operational economy, etc.), the jth utility function value \( U_{ij} \) of the ith plan waiting for selection could be expressed by the following formula [3]:

\[
U_{ij} = \frac{b_{ij}}{\max\{b_{ij}\}}
\]

Among which, \( i=1, 2, 3, 4 \cdot n; \ j=1, 2, 3 \)

While for the reverse index (the smaller the value, the more favorable the combat, such as the operational timeliness, etc.), the jth utility function value \( U_{ij} \) of the ith plan waiting for selection could be expressed by the following formula:

\[
U_{ij} = 1 + \frac{\min\{b_{ij}\}}{\max\{b_{ij}\}} - \frac{b_{ij}}{\max\{b_{ij}\}}
\]

Among which, \( i=1, 2, 3, 4 \cdot n; \ j=1, 2, 3 \)

Among which, \( \max\{b_{ij}\} \) was the maximum value of the jth optimal index of n plans waiting for selection; \( \min\{b_{ij}\} \) was the minimum value for the jth optimal index of the plan waiting for selection; \( U_{ij} \) was the index value of the jth optimal index of i plans waiting for selection.

1.3. Determine the comprehensive evaluation value of the optimal plan

In the optimization of the operational plan, the comprehensive evaluation value of the plan waiting for election should be calculated according to the determined optimal index weight value and the utility
function value, and then the maximum value thereof was selected, that is, this plan waiting for election was the optimal plan which was finally used for the scene. Its calculation formula was as follows:

$$Y_1 = \sum_{i=1}^{n} \omega_j \times \mu_j$$  \hspace{1cm} (3)

Where \(j=1, 2, 3\)

The calculation steps were as follows:

The first step: the utility function value of the optimal index was calculated by using the formula (1) and the formula (2);

The second step: the comprehensive evaluation value of the optimal plan was calculated by bringing the utility function value to the formula (3);

The third step: the optimal operational plan was determined based on the size of the comprehensive evaluation value.

In the formula, \(\omega_j\) was the weight of the optimal index and \(\mu_j\) was the utility function value.

1.4. Determine the optimal operational plan

The optimal index system table for the firefighting and rescue operational plan was constructed to determine the optimal index and the optimal index value and the analytic hierarchy process was utilized to determine the weighted value of the optimal index; then the utility function model was optimized by constructing the operational plan, and the utility function value of the positive index was calculated by using formula (1) and the utility function value of the reverse index was calculated by formula(2); finally, the comprehensive evaluation value was calculated by using formula (3) according to the optimal index weighted value and the utility function value, and the plan waiting for selection with the largest comprehensive evaluation value was the optimal operational plan. Therefore, at the firefighting and rescue site, the optimal operational plan was determined according to the comprehensive evaluation value of the plan.

2. Application examples

At 14:55 on July 13, 1990, the 0201 train consisting of the aviation petrol tank car with 46 sections and the truck with 9 sections departed from Ankang Station, Shaanxi province and exploded and burned when this train travelled to the "Pear Garden" tunnel in Wanyuan County, Sichuan province. 4 people were killed and 14 people were injured. 18 sections of the tank car and 5 sections of the truck were damaged to varying degrees. The explosion and burning caused the southwestern traffic life—the Xiang Yu railway to stop for 24 days, causing a rare traffic accident in the history of railway. When the explosion and burning occurred, a large amount of oil was sprayed out. The vegetation within 24 meters of the north hole was ignited and the rock was burst. The fire rescue team which firstly showed up used two foam guns and two multi-jet squirts to eliminate the flowing flame. They stacked the sandbag dam about more than one meter at the entrance of the hole, which prevented the oil from overflowing and controlled the fire to spread outside the hole. However, at about 2 o'clock in the morning of the next day, a tank car explosion occurred one after another. The oil flowed to the tunnel entrance and the flame broke out of the hole more than 30 meters high. The arched stone continued to explode. The strong radiation made people unbearable beyond 100 meters. After a large number of firefighters and rescuers arrived at the scene, in the face of this situation, they quickly set up the on-site command and conducted fire detection and judgment, and learned the scene. Soon afterwards, the site command then set three sets of operational options.

Option 1: Concentrate on strength and actively defend against the spread of fire outside the hole;
Option 2: Dispatch the engineering corps to blow up the whole entrance to suffocate the fire;
Option 3: Adopt the method of sealing the hole and smothering.
In the process of this accident, it was the key to the accident disposal to determine the optimal operational plan from the three options. It was also possible to verify whether the disposal of the plan was reasonable, scientific and efficient. If the optimal utility function model of the above-mentioned plan was used, the optimal operational plan could be determined, to make the on-site command and decision more scientific, standardized and efficient thus to avoid the drawbacks and errors generated by the commander at the scene of the accident. At the same time, it could also reduce the risk of accident handling, enhance the timeliness of accident handling, improve the decisiveness of accident handling, minimize the loss of property and casualties, and minimize the political negative impact of the accident. In addition, it could fully demonstrate the fire brigade's ability to handle major accident and extra serious accident, which was beneficial to improve the overall operational capability of the fire brigade, improve the ability of the forces to handle emergent accidents, improve the operation mechanism of firefighting and rescue, and improve the political influence of the fire brigade and the fighting capacity of the forces.

In view of the significant impact of this accident, the on-site command fully analyzed the possible consequences. In view of the large political influence, the large economic loss, the large number of fighters, the large scale of the fire scene, the difficulty of firefighting and the difficulty of rescue of the accident scene, the on-site command determined the evaluation and analysis criteria of the plan waiting for selection. (Excellent, good, medium, poor) were regarded as the result of the possible judgment.

Based on the investigation of the site conditions, according to the progress of the firefighting rescue [4], the following calculation steps were taken to determine which plan was optimal. Firstly, the on-site experts made a reasonable assignment of the operational feasibility, operational timeliness, and operational economics of the three options waiting to be selected. Each of the optimal index was evaluated, and the expert evaluation table of the optimal index for the operational plan was established, as shown in Table 1 below.

| Plan waiting for selection | operational feasibility | operational timeliness | operational economics |
|----------------------------|-------------------------|------------------------|----------------------|
| Option one                 | Medium                  | Medium                 | Poor                 |
| Option two                 | Good                    | Excellent              | Good                 |
| Option three               | Excellent               | Good                   | Good                 |

Then the optimal evaluation matrix $B$ of the operational plan was

$$
B = \begin{bmatrix}
0.6 & 0.7 & 0.5 \\
0.7 & 0.9 & 0.8 \\
0.9 & 0.8 & 0.8
\end{bmatrix}
$$

Next, the utility function value of each index was calculated based on the optimal evaluation matrix. According to formula (1), the operational feasibility and operational economics belonged to the positive index; according to formula (2), the operational timeliness was the reverse index. The utility function value of each optimal index in option one, option two and option three was calculated respectively. The calculation results were as follows:

$$
\mu_{i1} = \frac{b_y}{\max \{b_y\}} = \frac{0.6}{0.9} = 0.667
$$
In a similar way:  
\[ \mu_{21} = 0.778, \quad \mu_{31} = 1.0 \]
\[ \mu_{12} = 1 + \frac{\min \{b_{ij}\}}{\max \{b_{ij}\}} \cdot \frac{b_{ij}}{\max \{b_{ij}\}} = 1 + \frac{0.7}{0.9} \cdot \frac{0.7}{0.9} = 1.0 \]

In a similar way:  
\[ \mu_{22} = 0.778, \quad \mu_{32} = 0.889 \]
\[ \mu_{13} = \frac{b_{ij}}{\max \{b_{ij}\}} = \frac{0.5}{0.8} = 0.625 \]

In a similar way:  
\[ \mu_{23} = 1.0, \quad \mu_{33} = 1.0 \]

The final result was:
\[ \mu_{11} = 0.667, \quad \mu_{12} = 1.0, \quad \mu_{13} = 0.625 \]
\[ \mu_{21} = 0.778, \quad \mu_{22} = 0.778, \quad \mu_{23} = 1.0 \]
\[ \mu_{31} = 1.0, \quad \mu_{32} = 0.889, \quad \mu_{33} = 1.0 \]

Finally, the comprehensive evaluation value was determined based on utility function value. According to formula (3), the comprehensive evaluation values of the three plans waiting for selection were calculated:

\[ Y_1 = \sum_{i=1}^{n} \omega_j \times \mu_{ij} = \omega_1 \times \mu_{11} + \omega_2 \times \mu_{12} + \omega_3 \times \mu_{13} \]
\[ = 0.527 \times 0.667 + 0.333 \times 1.0 + 0.14 \times 0.625 = 0.772 \]
\[ Y_2 = \sum_{i=1}^{n} \omega_j \times \mu_{ij} = \omega_1 \times \mu_{21} + \omega_2 \times \mu_{22} + \omega_3 \times \mu_{23} \]
\[ = 0.527 \times 0.778 + 0.333 \times 0.778 + 0.14 \times 1.0 = 0.809 \]
\[ Y_3 = \sum_{i=1}^{n} \omega_j \times \mu_{ij} = \omega_1 \times \mu_{31} + \omega_2 \times \mu_{32} + \omega_3 \times \mu_{33} \]
\[ = 0.527 \times 1.0 + 0.333 \times 0.889 + 0.14 \times 1.0 = 0.963, \]

\[ Y_1 = 0.772, \quad Y_2 = 0.809, \quad Y_3 = 0.963 \]

In view of this firefighting and rescue site, the plan waiting for selection must be calculated mathematic quantitatively and the plan optimization must be carried out to determine which plan was reasonable, scientific and efficient. The steps of the plan optimization were as follows: Firstly, the weight value of the optimal index was calculated by using the analytic hierarchy process. Then the formula (1) was used to calculate the utility function value of the positive index and the formula (2) was used to calculate the utility function value of the reverse index. Then for the calculated optimal index weighted value and utility function value, the formula (3) was used to get the comprehensive evaluation value of each plan. For this case, by using the mathematical theorem to calculate, the comprehensive
optimal evaluation value of option three was calculated to be the largest, so this plan was optimal. So option three should be determined as the best firefighting rescue operation plan.

Finally, based on the calculation results of the utility function, the on-site command analyzed and compared the plan waiting for selection, weighed the advantages and disadvantages, and adopted the third option—the method of sealing the hole and smothering. They made the detailed implementation steps. The whole sealing task was finally finished successfully by large formation joint operation of the army, police and the people.

3. Conclusion

(1) At the fire fighting and rescue site, the operational plan optimization was a very complex activity. The utility function model was built in this paper to transform different optimal indexes by using the utility function model, making it more comparative and easy to implement plan optimization.

(2) The fire fighting and rescue operation plan optimization was built through the utility function model in this paper. This plan was relatively reasonable, objective and reliable, which was beneficial to improve the decision-making ability of the commander and increase the success rate of accident handling. Applying the mathematical model to the fire fighting and rescue site was a beneficial attempt and harvest for the plan optimization, which provided some experience for the future plan optimization and was of great significance.

References
[1] GUO KUNQI, Sun LIXIN&JIA SHILOU. UTILITY function based fair data scheduling algorithm for OFDM wireless net work [J]. 2007. 731-738.
[2] RIGUI Zhou, DUAN BIN&AIQIU NIE. A NEURUAL Networks Based on Hermits Polynomial Functions [N]. 2007. 2636-2638.
[3] Zhang Daoyan, Wu Jun, Cao Yong. The optimal method of artillery operational decision plan [J]. Military Station Office for Shenyang Military Bureau in Shenyang district, Military Artillery Academy, 2007.112-114.
[4] Chen Jiaqiang. Fire Fighting and Rescue [M]. Chinese People's Public Security University Press, 2002. 364-366.