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Mathematical modeling of COVID-19 pandemic in India using Caputo-Fabrizio fractional derivative

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ARTICLE INFO
Keywords:
Infectious diseases
COVID-19
Caputo-fabrizio fractional derivative
Model prediction
Pandemic slow down
Collocation technique
Genocchi polynomial

ABSTRACT
The range of effectiveness of the novel corona virus, known as COVID-19, has been continuously spread worldwide with the severity of associated disease and effective variation in the rate of contact. This paper investigates the COVID-19 virus dynamics among the human population with the prediction of the size of epidemic and spreading time. Corona virus disease was first diagnosed on January 30, 2020 in India. From January 30, 2020 to April 21, 2020, the number of patients was continuously increased. In this scientific work, our main objective is to estimate the effectiveness of various preventive tools adopted for COVID-19. The COVID-19 dynamics is formulated in which the parameters of interactions between people, contact tracing, and average latent time are included. Experimental data are collected from April 15, 2020 to April 21, 2020 in India to investigate this virus dynamics. The Genocchi collocation technique is applied to investigate the proposed fractional mathematical model numerically via Caputo-Fabrizio fractional derivative. The effect of presence of various COVID parameters e.g. quarantine time is also presented in the work. The accuracy and efficiency of the outputs of the present work are demonstrated through the pictorial presentation by comparing it to known statistical data. The real data for COVID-19 in India is compared with the numerical results obtained from the concerned COVID-19 model. From our results, to control the expansion of this virus, various prevention measures must be adapted such as self-quarantine, social distancing, and lockdown procedures.

1. Introduction

The first case of corona virus outbreak was reported in the Wuhan city of Hubei province in China in December 2019. This disease was initiated as a result of human’s experimental failure [36]. In spite of a large effort to prevent this infectious disease transmission outside of China, the range of effectiveness for novel corona virus has been continuously spread worldwide. Till today, the number of cases of COVID-19 is still increasing rapidly worldwide [32,33]. The vaccines, drugs, and antidote of novel corona virus have not been found yet at its initial stages. However, due to excellent efforts of medical professionals, various COVID-19 vaccines have been available now to everyone [9,13,23].

An emergency for public health was initiated globally by the World Health Organization (WHO) in which this disease is dealt with as a pandemic because of its global exponential growth [1] causing a global threat to society. As a result, this disease is classified as a global threat to our health [14] which affects human life and countries’ economy. The most common symptoms of this virus in almost all cases are fever and dry cough, while in certain cases, some other symptoms have been appeared such as sore headache, throat, shortness of the breath, fatigue, and joint/muscle pain. Various reports on COVID-19 cases indicate that many cases of infected people turned out to be infectious cases without any noticeable common symptoms. These reports also indicate that without any interventions, the doubling period of this pandemic is nearly one week [21,29]. For a regular analysis of this pandemic’s
situation, countries have planned some essential measures at various stages to be adopted [22]. Due to the unavailability of accurate medical treatments for coronavirus, some limited preventive measures have been adopted by most governments such as the complete isolation of infectious people, social distancing, and quarantining people with high potentiality to be infected [35] in order to reduce the spread of this infectious disease globally. Many research works strongly indicate that lockdown procedures and other social distancing methods are effective to reduce the expansion of various respiratory viral diseases [18,37]. The quarantine strategy has been adopted by countries involving full lockdowns and other social distancing strategies. The uncontrolled cases of infected coronavirus cases move towards acute respiratory distress syndrome, severe pneumonia, and mortality. Commonly, the expansion of COVID-19 is found within a large extent among the close contacts of people within approximately 2 m. In most cases, the average incubation time which is the time between infection’s exposure and first symptoms’ appearance, is approximately 1–14 days. There are four fundamental COVID-19 transmission’s stages concerning places/countries where.

1. The 1st stage is achieved when no cases are recorded in particular places/countries.
2. The 2nd stage is achieved when some sporadic cases are recorded.
3. The 3rd stage is achieved when we have clusters’ cases at any place.
4. The 4th stage is sensitive in which some cases begin to be recorded as a result of community transmission.

The COVID-19 disease’s first case in India was recorded on January 30, 2020. Since then there are many ups and downs that have been seen in the trajectories of coronavirus cases in India. Initially, certain cases were recorded in students coming from Wuhan. New additional cases were recorded in India in March 2020 as a result of contact with COVID-19 reported cases [2]. The Indian government has applied certain measures to slow down COVID-19 spread. A deep analysis of average cases of coronavirus among the affected countries shows that the trajectories of corona positive cases in India are steeper than most countries in Asia. A full country lockdown for twenty-one days was announced by the Indian government on March 24, 2020. Later on, this lockdown was extended to May 03, 2020. This lockdown has reduced the COVID-19 cases to some extent. Based on the total infected cases, the Indian government has decided to divide the area of cases into three zones such as red zone, orange zone, and green zone. In this work, authors have studied the effect of this virus at its initial stages including various parameters. This study depicts the starting nature and behavior of COVID-19 spreading characteristics under the influence of quarantined parameters.

This investigation of the effects of various factors related to COVID-19 is recently studied by many researchers from the biological viewpoints [11,20]. The mathematical modeling of the biological processes has become a very important tool. Life scientists with a strong background in applied mathematical techniques have modeled many biological phenomena. Formulated biological models with the help of mathematical modeling via classical integer derivatives have certain limitations and most likely fail to correctly interpret biological phenomena. For efficient biological models, fractional models have been studied [17,25]. Due to the important property of memory impact of fractional differential operators, the fractional calculus is an effective tool to provide the mathematical demonstration of nature-related truths, natural processes, and processes involving non-local dynamics behaviors. Researchers have developed new efficient models via fractional calculus [3,6,7,12,26,27].

In this scientific work, we have proposed SIR model with a quarantined parameter. This constructed infectious disease model was used to analyze the spread of COVID-19 virus and predict the number of exposed cases, recovered cases, and death population. The predicted model is a dynamical system that is associated with seven coupled fractional differential equations under Caputo-Fabrizio sense. The pandemic slows down is the stage when the globalized number of COVID-19 cases and mortality population will become downwards. Presently, it is hard to predict the end of COVID-19, but there are few points to claim the slowing down of this pandemic.

In modeling COVID-19 dynamics, we start from a set of early infected peoples and some classes of society with different immune systems and properties. The basic parameters in the transmission of novel coronavirus are the average quarantine time, average latent time, and contact rate; these parameters are very significant in modeling COVID-19 pandemic. This model calculates the number of infectious cases, latently infected cases. In this work, we formulate the COVID-19 dynamics in a human society using a systemic way via Caputo-Fabrizio fractional derivative. This model explains the transmission rate of disease, disease’s impact on the susceptible cases, exposed cases, and infectious cases’ rates with various parameters including contact rate and average latent time. In this formulation, the important interventions of the government to minimize the coronavirus spread is also incorporated as a parameter. Our proposed model’s simulations are examined, and the model’s sensitive analysis are investigated to conclude with efficient model outcomes. This fractional mathematical model of COVID-19 is useful in many clinical trials, evaluations, and control and preventive programs within limited specified resources. The Genocchi fractional operational matrix of Caputo-Fabrizio derivative is used for the numerical analysis and simulations of the formulated mathematical model. Authors have used the Caputo-Fabrizio fractional derivative during the mathematical formulation of COVID-19 containing parameters like susceptible, exposed parameter, and infectious. Earlier, many authors used classical derivatives including Caputo fractional derivatives to study these kinds of epidemiological models. The modeling of various physical models and complex phenomena under Caputo-Fabrizio derivative is more efficient than classical derivatives such as Caputo fractional derivatives [8]. Many researchers have used Finite difference method, Runge-Kutta fourth order method, Hermite Wavelet Technique, and multilevel gradient methods [16,34,38]. In addition, a recent study proposed a novel stochastic modeling of COVID-19 under the environmental white noise [15].

This work is outlined as: Some important concepts of Caputo-Fabrizio fractional derivative and integral are discussed in Section 2. The proposed fractional model is provided in Section 3 via Caputo-Fabrizio fractional derivative. A brief outline of Genocchi polynomial and approximated real state variables is provided in Section 4. The novel operational matrix is investigated in Section 5. Numerical results and simulations are provided in Section 6. Our outcomes are provided in section 7.

2. Fractional differentiation and integration

In this section, the fractional order Caputo-Fabrizio derivatives and integrals notions are discussed.

2.1. Caputo-Fabrizio fractional integral

The definition of this new fractional order integration of order η > 0 of a given function k(t) is expressed as [10,24]:

$$\text{CF}_\eta^\nu f_k(t) = \frac{2(1-\eta)}{(2-\eta)M(\eta)} k(t) + \frac{2\nu}{(2-\eta)M(\eta)} \int_0^t k(p)dp,$$

where the function $M(.)$ is known as the normalization function and 0 < η < 1.

2.2. Caputo-Fabrizio fractional derivatives

The definition of this new fractional derivative of order η > 0 of a
given function $k(t)$ is given by [10].

\[ c^\eta D^\eta k(t) = \frac{M(\eta)}{1 - \eta} \int_0^t \exp\left(-\frac{\eta(t-\rho)}{1-\eta}\right) \hat{k}(\rho) d\rho. \quad (2) \]

Let $A$ be a constant function then its Caputo-Fabrizio derivative (CpFab) of order $\eta$ is zero i.e.,

\[ c^\eta D^\eta A = 0. \quad (3) \]

If we choose the normalization function $M(\eta) = 1$, then the other representation of Caputo-Fabrizio derivative (CpFab) of order $\eta$ in convolution form is given as

\[ c^\eta D^\eta k(t) = \exp\left(-\frac{\eta t}{1-\eta}\right) \ast \hat{k}(t), \quad (4) \]

where $\ast$ denote the convolution.

**Lemma:** [30] For any function $k(t)$ and arbitrary fractional order $\eta$, we have:

\[ c^\eta D^\eta k(t) = k(t) - k(0). \quad (5) \]

The Laplace transform of Caputo-Fabrizio derivative can be written as:

\[ \mathcal{L}\{ c^\eta D^\eta k(t) \} = \frac{s^{\eta+1} \mathcal{L}\{k(t)\} - k(0)s^\eta - \hat{k}(0)s^{\eta-1} - \cdots - k^{(\eta)}(0)}{s + (1-s+1)s^{\eta}}. \quad (6) \]

Moreover, the Caputo-Fabrizio fractional derivative satisfies the following equations for function $k(t) \in H[a, b]$ as:

\[ \lim_{\eta \to 0} c^\eta D^\eta k(t) = k'(t), \quad (7) \]

\[ \lim_{\eta \to 0} c^\eta D^\eta k(t) = k(t) - k(a). \quad (8) \]

More details and properties of the Caputo-Fabrizio fractional derivative can be found in the work [4,5,28].

3. Model formulation for COVID-19 with Caputo-Fabrizio derivative

In the formulation of the present disease model, we have opted some basic rules. These rules could also be significant to many fundamental research problems. The first one is to have a clear knowledge of research problem according to the available statistical data. We may adopt SIR model for the prediction of the growth of infectious disease or for the general idea of disease in response to some specific environments. The construction of multiple disease models is also very significant as we are mainly not sure about the efficient numerical scheme and variables required for our research problems.

A new fractional model is constructed in this section. This mathematical model includes susceptible parameter, exposed parameter, infectious parameter, recovered parameter, and mortality parameter with a quarantined parameter. The effect of the quarantined parameter is very significant as it could reduce the number of infected people. Here, the effect of transmission rate on exposed cases is also very significant as exposed cases increase with the transmission rate. In this work, a behavior-changed parameter is also included which strives to minimize the COVID-19 transmission rate using social distancing and other prevention measures in India. The governments strategies bring lockdowns in effect to prevent every avoidable death and expansion of virus, but it may cause some unequal impacts on society. Assume that $N(t)$, $S(t)$, $S_P(t)$, $E(t)$, $I(t)$, $Q(t)$, $D(t)$, and $R(t)$ represent the total India population in current time, total number of susceptible cases at time $t$, behavior-changed susceptible cases, total number of exposed cases at time $t$ such that the total number of infected cases in which they have not indicated yet to be infectious, total number of infectious cases which is not quarantined, total number of the quarantined cases, death cases, and recovered cases, respectively. Thus, $N(t) = S(t) + S_P(t) + E(t) + I(t) + Q(t) + D(t)$ at time $t$ in India. The following Fig. 1 represents the flow diagram of interactions of various parameters presented in the transmission of COVID-19 disease in our epidemic fractional mathematical model.

In the above diagram, various coefficients denote the various factors in the transmission of infection. The constant $\beta$ denotes the transmission rate of corona virus disease, $\beta_P$ is the transmission rate of the fear and awareness of the disease, $1/\tau$ denotes the number of confirmed cases reported, $\delta$ is the reduction-ratio of transmission in behavior-changed persons, $u$ is the rate of progression, $\alpha$ is isolation rate, $\gamma$ denotes cure rate, $\zeta$ is the mortality rate, $\sigma$ is the cure rate of $I$, and $\omega$ denotes death rate of $I$. The number of cases of susceptible peoples had been shifted towards the behavior-changed susceptible class as the total number of corona positive cases increased. This occurs as the society strives for prevention and increasing their immune system by adopting social-distancing or using masks. The term $\rho(1 - e^{-\omega})\mathcal{S}$ in the above diagram denotes this transmission. Both cases: Behavior-changed susceptible cases and susceptible cases join the class of exposed people through a contact with infected persons, and $\beta S(t)I(t)$ represents the number of susceptible people who have been infected (but not infectious yet) through a contact with infected people. Here, we also assume that the rate of transmission among behavior-changed cases is less because of their fear/awareness with a transmission reduction ratio $\delta$ which is denoted by $\delta S(t)I(t)\mathcal{R}$. After an average incubation time, the exposed cases $E(t)$ move to infectious class $I(t)$ with a progression rate $\rho$, and this term is given by $\rho E(t)$. Once the symptoms have been seen, it is assumed that some people get hospitalized or quarantined, some persons die and some cases recover. The transmission of these cases are denoted by $\alpha I(t)$, $\alpha I(t)$ and $\delta I(t)$, respectively. At the end, the cure and mortality rates for quarantined cases are given by $\rho Q$ and $\rho Q$, respectively. Here, it is assumed that after being quarantined (i.e., isolation), these cases can not infect other persons.

The proposed experimental fractional model of corona virus transmission via Caputo-Fabrizio derivative is expressed as:

![Fig. 1. Relation of interaction among parameters of COVID-19 model in India.](image-url)
$D_0^\alpha S(t) = -\beta \frac{S(t)I(t)}{N} - \beta_f (1 - e^{-\gamma t})S(t)$.

$D_0^\alpha D_0^\beta S(t) = \beta \frac{S(t)I(t)}{N} - \delta \frac{S(t)I(t)}{N} + kE(t)$,

$D_0^\alpha D_0^\beta E(t) = \beta \frac{S(t)I(t)}{N} - \delta \frac{S(t)I(t)}{N} + kE(t)$.

(9)

$\int_0^t G_i(t)G_j(t)dt = \frac{(-1)^{2k}}{(l + k)}g_{i+k,l}, k \geq 1$.

(12)

$\zeta_i(t) = \sum_{k=1}^{r} b_k G_i(t) = B^T \sigma(t)$,

(13)

where the constant coefficients are given by the inner product $b_k = \langle \zeta(t), G_i(t) \rangle$, the column vector $B^T$ is given by $B = \{b_1, b_2, \ldots, b_r\}$, and the term $\sigma(t)$ is a column vector known as Genocchi vector which is expressed as: $\sigma(t) = [G_1(t), G_2(t), \ldots, G_r(t)]^T$.

$H^{\eta} = \left( \frac{2(1-\eta)}{(2-\eta)\Gamma(\eta)} (I_0 + \frac{\eta}{1-\eta} ADE) \right)$.

(15)

In the above equation, $I_d$ is the identity matrix, $C'$ is Genocchi coefficient matrix in the approximation, and $A, D$ are the matrix of order $r \times r$ as

$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \frac{1}{7} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix}$,

(16)

and

$D = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \frac{1}{r} \end{bmatrix}$.

(17)

6. Numerical simulations and results

The Caputo-Fabrizio operational matrix (CpFabOM) and spectral collocation (SCo) techniques are applied to our proposed model [30]. From equation (13), the function $\zeta(t)$ can be constructed in terms of initial r GPols as:

$\zeta_i(t) = \sum_{k=1}^{r} b_k G_i(t) = B^T \sigma(t)$,

(18)

where $b_k$ are the unknown coefficients to be found later, and $\sigma(t)$ is a Genocchi vector.

From equation (18), the NNICs (10) can be expressed as:

$B^T_1 \sigma(0) = a$,

$B^T_2 \sigma(0) = b$,

$B^T_3 \sigma(0) = c$,

$B^T_4 \sigma(0) = d$,

$B^T_5 \sigma(0) = e$,

$B^T_6 \sigma(0) = f$,

$B^T_7 \sigma(0) = g$.

The investigated numerical scheme is utilized for our model (9) with NNICs (10) which are estimated/collected for the total India population: $N(0) = 1352604385$. The total susceptible case for is $S(0) = 11391904385$, behavior-changed susceptible cases $S_p = 200000000$, exposed cases $E(0) = 12000000$, infectious cases $I_0(0) = 12000$, death cases $D(0) = 450$, quarantine cases $Q(0) = 1400000$ and the recovered cases $R(0) = 1500$. The other coefficients parameter’s values are:

Transmission rate of COVID-19 disease $\beta = 3.8101$,

Transmission rate of the fear of awareness of disease $\beta_f = 2.9418$,

Characteristic number of confirmed cases $1/\gamma = 10000$,

Transmission reduction ratio $\delta = 0.02$,

Progression Rate $k = 1/4.1 = 0.24390$,

Isolation rate $\sigma_1 = 1/4 = 0.25$,

Cure Rate $\gamma = 1/14 = 0.07143$,

Death Rate $\zeta = 0.5$.

(20)

The variation of computed susceptible cases and behavior-changed susceptible cases for various fractional order’s values: $\mu = 0.6, 0.8, 1$ are shown through Figs. 2–3. In Figs. 4–5 the variation for exposed and
Fig. 2. Plot of susceptible cases for various values of fractional order $\mu$.

Fig. 3. Plot of behavior-changed susceptible cases for various values of fractional order $\mu$.

Fig. 4. Plot of exposed cases for various values of transmission rate $\beta$. 

infected cases’ numbers are represented for various transmission rate’s values: $\beta$ at $\mu = 0.8$. The variation of quarantined cases for various orders: $\mu = 0.6, 0.8, 1$ is shown in Fig. 6.

Now, the graph of infected cases is plotted for both of the experimental infected cases and the estimated/collected data fitting in India between April 15, 2020 and April 21, 2020 as shown in Fig. 7 for fractional order $\mu = 1$. From this figure, the model fitting curve is in agreement with the experimental outcomes. Fig. 8 is the plot for recovered cases between the data fitting and experimental recovered cases for fractional order $\mu = 1$. A good agreement between the fitting curve and experimental outcomes can be clearly seen in Fig. 8. The mortality cases for experimental and reported cases are shown in Fig. 9.

In Figs. 2–9, the behavior of individuals is plotted for various values of the constant parameter and fractional order. The sensitiveness and effect of various parameters presented in the experimental model are analyzed numerically for different variables. The dynamics of pictorial presentation of the susceptible cases and behavior-changed susceptible cases for the fractional order cases advances in comparison to the plot for integer order ones as we move the fractional order system to the integer one which justifies the modeling of our model’s dynamics in Figs. 2–3. As we increase the value of transmission rate of COVID-19 disease, the exposed cases and infectious cases are increased to a large extent as shown in Figs. 4–5. In our daily life, the real behavior of transmission rate can be easily seen, as we gather, avoid precautions, or break lockdowns, the transmission rate increases, and hence, the infected and exposed cases will also be increased very rapidly. The quarantined cases’ graph is represented in Fig. 6 for the alternative system. The data fitting curve for infected cases, recovered cases, and death cases vs. the numerical results obtained from the formulated mathematical model are shown in Figs. 7–9.

7. Conclusion

This scientific work has provided an efficient modeling of our proposed model’s dynamics via CpFab derivative. The Genocchi operational matrix has been derived to investigate various constant parameters’ effect in the concerned fractional mathematical model. The numerical results obtained from our concerned model are compared with the real data from Indian COVID-19 cases. The model fitting curve and numerical outcomes have been represented graphically for $\mu = 1$. It can also be noticed that the memory system of CpFab derivative describes the technical dynamics of transmission of the infectious disease COVID-19 more accurately than the classical derivative [31]. In the absence of efficient medical treatments of the infectious disease, it is observed that the prevention measures such as social distancing, self-quarantine system, and lockdowns must be applied to the community to slow down the
Fig. 7. Plot of no. of infected case between reported case vs. experimental result of COVID-19 in India.

Fig. 8. Plot of no. of recovered case between reported case vs. experimental result of COVID-19 in India.

Fig. 9. Plot of no. of death case between reported case vs. experimental result of COVID-19 in India.
expansion of COVID-19.

Date statement

Data are available upon request.

Authors contributions

Prashant Pandey: Actualization, methodology, initial draft, and investigation; José Francisco Gómez Aguilar: Validation, investigation, formal analysis, and initial draft; Mohammed K.A. Kaabar: Validation, formal analysis, investigation, initial draft, and supervision of the original draft and editing; Zailan Siri: Investigation, validation, and initial draft; Abd Allah A. Mousa: Investigation, validation, and initial draft. All authors read and approved the final version.

Funding

There is no funding to declare for this research study.

Declaration of competing interest

All authors confirm that they do not have any conflict of interests.

Acknowledgments

José Francisco Gómez Aguilar acknowledges the support provided by CONACYT: Cátedras CONACYT para jóvenes investigadores 2014 and SNI-CONACYT.

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