Universal accelerating cosmologies from 10d supergravity

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Based on:

* Paul Marconnet & DT, JHEP 01 (2023) 033
Introduction

There has been a lot of recent effort in obtaining realistic 4d cosmologies from the 10d/11d supergravities that capture the low-energy limit of string/M-theory.

In the early 21st century accelerating 4d cosmologies from compactification were thought to be as difficult as 4d Sitter.

The famous no-go excludes acceleration, provided:
- absence of sources, no (or mild) singularities
- compactness
- two-derivative actions
- the Strong Energy Condition is obeyed by the 10d/11d theory

* Gibbons, 1984
* Maldacena & Nuñez, 2000
Introduction

Consider a compactification of the form
\[ \hat{g}_{MN} dX^M dX^N = \Omega^2(y) \left( g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \right) \]
where the 4d factor is of FLRW form,
\[ g_{\mu\nu}(x) dx^\mu dx^\nu = -dT^2 + S^2(T) \gamma_{ij} dx^i dx^j ; \quad R(\gamma)_{ij} = 2k \gamma_{ij} \]
In particular we have
\[ \hat{R}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \left( \nabla^2 \ln \Omega + 8(\partial \ln \Omega)^2 \right) \]
The SEC
\[ \hat{R}_{00} = \kappa^2 \left( T_{00} - \frac{1}{8} \hat{g}_{00} T_L^L \right) \geq 0 \]
then implies
\[ \ddot{S}(T) \leq 0 \]
Introduction

- Time-dependent compactifications, however, can evade the no-go!
  \[ \Omega = \Omega(y; T) ; \quad g_{mn} = g_{mn}(y; T) \]
  * Townsend & Wohlfarth, 2003

- Transient acceleration is in fact generic in flux compactifications!

- de Sitter space is still ruled out by the SEC (if the 4d Newton’s constant is time-independent in the conventional sense)

- Late-time acceleration is not ruled out by the SEC (although no known examples from 10d/11d compactifications, if we require non-vanishing acceleration asymptotically)
  * Russo & Townsend, 2018; 2019
Introduction

- Reexamine these statements within the framework of universal 10d/11d compactifications
- Type II supergravity 10d solutions with a 4d FLRW factor
- Compactification on 6d Einstein, Einstein-Kähler, or CY
- Solutions independent of the compactification details
- All solutions are obtainable from a 1d action (consistent truncation) of 3 time-dependent scalars (the dilaton and 2 warp factors). All fluxes appear as constant coefficients in the potential.
- In certain cases there is a 4d consistent truncation to 2 scalars
Introduction

- Many analytic solutions
  - Always possible if a single excited species of flux.
  - Examples with up to four excited species of flux.
- Autonomous dynamical system if 2 excited species of flux
  - Intuitive description of the cosmological features of the cosmologies (trajectories), in particular the condition of accelerated expansion
  - Fixed points and trajectories on phase-space boundary correspond to analytic cosmological solutions
- Several novel (top down) examples of (semi-)eternal inflation; cosmologies with parametric control of e-foldings; rollercoaster cosmologies
Type IIA supergravity

Action

\[ S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left( - R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2 \cdot 2!} e^{3\phi/2} F^2 \right. \]
\[ + \left. \frac{1}{2 \cdot 3!} e^{-\phi} H^2 + \frac{1}{2 \cdot 4!} e^{\phi/2} G^2 + \frac{1}{2} m^2 e^{5\phi/2} \right) + S_{\text{CS}} \]

Bianchi identities

\[ \text{d}F = mH \; ; \; \text{d}H = 0 \; ; \; \text{d}G = H \wedge F \]
The 10d Einstein-frame metric

\[ ds_{10}^2 = e^{2A(t)} \left[ e^{2B(t)}(-dt^2 + d\Omega_k^2) + g_{mn}(y)dy^m dy^n \right] \]

where

\[ d\Omega_k^2 = \gamma_{ij}(x)dx^i dx^j ; \quad R^{(3)}_{ij} = 2k \gamma_{ij} \]

The 4d Einstein-frame metric

\[ ds_{4E}^2 = -S^6 d\tau^2 + S^2 d\Omega_k^2 \]

where

\[ S = e^{4A+B} ; \quad \frac{dt}{d\tau} = S^2 \]

The cosmological time

\[ \frac{dT}{d\tau} = S^3 ; \quad ds_{4E}^2 = -dT^2 + S^2 d\Omega_k^2 \]
Flux Ansätze examples

- **Calabi-Yau**

\[ m = 0 \; ; \quad F = 0 \; ; \quad H = h + d\chi \wedge J + \frac{1}{2} b_0 \text{Re}\Omega \; ; \]
\[ G = \varphi \text{vol}_4 + \frac{1}{2} c_0 J \wedge J - \frac{1}{2} d\xi \wedge \text{Im}\Omega - \frac{1}{2} d\xi' \wedge \text{Re}\Omega \]

solution of form equations and Bianchi identities
\[ \varphi = e^{-\phi/2-A+B} c_\varphi \; ; \quad h = c_h \text{vol}_3 \; ; \]
\[ dt\chi = c_\chi e^{\phi-2A-2B} \; ; \quad (dt\xi)^2 + (dt\xi')^2 = 2c_{\xi\xi'}^2 e^{-\phi-2A-2B} \]

- **Einstein-Kähler with internal 2-form**

\[ m = 0 \; ; \quad H = 0 \; ; \quad G = \varphi \text{vol}_4 \; ; \]
\[ F = c_f J \; ; \quad R_{mn} = \lambda g_{mn} \]

solution of form equations and Bianchi identities
\[ \varphi = e^{-\phi/2-A+B} c_\varphi \]
The 1d consistent truncation

The remaining equations of motion (Einstein & dilaton)

\[ d_\tau^2 A = -\frac{1}{48} (\partial_A U - 4\partial_B U) \]
\[ d_\tau^2 B = \frac{1}{12} (\partial_A U - 3\partial_B U) \]
\[ d_\tau^2 \phi = -\partial_\phi U \]

Constraint

\[ 72(d_\tau A)^2 + 6(d_\tau B)^2 + 48d_\tau A d_\tau B - \frac{1}{2} (d_\tau \phi)^2 = U \]
The 1d consistent truncation

They are derivable from

\[ S_{1d} = \int d\tau \left\{ \frac{1}{N} \left( -72(d_\tau A)^2 - 6(d_\tau B)^2 - 48d_\tau A d_\tau B + \frac{1}{2} (d_\tau \phi)^2 \right) - NU(A, B, \phi) \right\} \]

where

\[ U = \left\{ \begin{align*}
\frac{1}{2} c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2} c_h^2 e^{-\phi+12A} + \frac{3}{2} c_\chi^2 e^{\phi+4A} + c_\xi^2 e^{-\phi/2+6A} - 6k e^{16A+4B} & \quad \text{CY} \\
72b_0^2 e^{-\phi+12A+6B} + \frac{3}{2} c_0^2 e^{\phi/2+10A+6B} & \quad \text{CY} \\
\frac{1}{2} c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2} m^2 e^{5\phi/2+18A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \quad \text{E} \\
\frac{1}{2} c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2} c_h^2 e^{-\phi+12A} + \frac{3}{2} c_\chi^2 e^{\phi+4A} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \quad \text{EK} \\
\frac{3}{2} c_0^2 e^{\phi/2+10A+6B} + \frac{1}{2} m^2 e^{5\phi/2+18A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \quad \text{EK} \\
\frac{1}{2} c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{3}{2} c_f^2 e^{3\phi/2+14A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \quad \text{EK} \\
\end{align*} \right\} \]
The 1d consistent truncation

- The 1d origin of the constants

| $m$ | zero-form (Romans mass) |
|-----|-------------------------|
| $c_f$ | internal two-form |
| $c_h$ | external three-form |
| $b_0$ | internal three-form |
| $c_\chi$ | mixed three-form |
| $c_\varphi$ | external four-form |
| $c_0$ | internal four-form |
| $c_{\xi\xi'}$ | mixed four-form |
| $k$ | external curvature |
| $\lambda$ | internal curvature |
The 1d consistent truncation

The terms in the potential are of the form

$$\text{const} \times e^{\alpha A + \beta B + \gamma \phi}$$

where (for RR forms)

$$\alpha = 18(1 - n_t) - 2(-1)^{n_t}(n_s + n_i)$$;
$$\beta = 6(1 - n_t) - 2(-1)^{n_t}n_s$$;
$$\gamma = (-1)^{n_t} \frac{5 - (n_t + n_s + n_i)}{2}$$

with $n_t, n_s, n_i$ the number of legs along the time, 3d space, internal directions
Minimal solution (zero flux)

- Warp factors and dilaton
  \[ A = c_A \tau + d_A ; \quad B = c_B \tau + d_B ; \quad \phi = c_\phi \tau + d_\phi \]

- Constraint
  \[ \frac{c_A}{c_B} \leq -\frac{1}{2} \quad \text{or} \quad \frac{c_A}{c_B} \geq -\frac{1}{6} \]

  with constant dilaton if either inequality is saturated

- 4d Einstein metric
  \[ ds_{4E}^2 = -dT^2 + T^{\frac{2}{3}} d\vec{x}^2 \]

- \( e^A \) may collapse, decompactify or stay constant as \( T \to 0, \infty \)
The 4d consistent (cosmological) truncation

- The equations of motion are derivable from

\[ S_{4d} = \int d^4x \sqrt{g} \left( R - 24 g^{\mu\nu} \partial_\mu A \partial_\nu A - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(A, \phi) \right) \]

where

\[
V = \begin{cases}
72b_0^2 e^{-\phi-12A} + \frac{3}{2} c_0^2 e^{\phi/2-14A} & \text{CY with internal 3- and 4-form fluxes} \\
\frac{1}{2} c_\phi^2 e^{-\phi/2-18A} + \frac{1}{2} m^2 e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{E with external 4-form flux} \\
\frac{3}{2} c_0^2 e^{\phi/2-14A} + \frac{1}{2} m^2 e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{EK with internal 4-form flux} \\
\frac{1}{2} c_\phi^2 e^{-\phi/2-18A} + \frac{3}{2} c_f^2 e^{3\phi/2-10A} - 6\lambda e^{-8A} & \text{EK with internal 2-form, external 4-form flux} \\
\end{cases}
\]

- In the CY case: a sub-truncation, to the metric and two scalars, of the consistent truncation to the universal sector

* Robin Terrisse & DT, 2019; DT, 2020
Dynamical system analysis

Consider the case of a two-term potential

\[ U = \sum_{i=1}^{2} c_i e^{E_i} ; \quad E_i := \alpha_i A + \beta_i B + \gamma_i \phi \]

The eom’s become a dynamical system, with \( d\sigma := e^{E_1/2} d\tau \)

\[
\begin{align*}
\sigma v &= - \left( \frac{1}{8} \alpha_2 - \frac{1}{2} \beta_2 \right) u^2 - \left( \alpha_2 + \frac{1}{2} \beta_1 - 4 \beta_2 \right) uv - \left( \frac{1}{2} \alpha_1 + \frac{3}{2} \alpha_2 - 6 \beta_2 \right) v^2 - \frac{1}{2} \gamma_1 vw \\
&\quad - \left( - \frac{1}{96} \alpha_2 + \frac{1}{24} \beta_2 \right) w^2 + \frac{1}{48} c_1 \left[ \alpha_2 - \alpha_1 + 4 (\beta_1 - \beta_2) \right] \\
\sigma u &= - \frac{1}{2} \left( - \alpha_2 + b_1 + 3 \beta_2 \right) u^2 - \left( \frac{1}{2} \alpha_1 - 4 \alpha_2 - 12 \beta_2 \right) uv - \left( -6 \alpha_2 + 18 \beta_2 \right) v^2 - \frac{1}{2} \gamma_1 uw \\
&\quad - \left( \frac{1}{24} \alpha_2 + \frac{1}{8} \beta_2 \right) w^2 + \frac{1}{12} c_1 \left[ \alpha_1 - \alpha_2 + 3 (\beta_2 - \beta_1) \right] \\
\sigma w &= - 6 \gamma_2 u^2 - 48 \gamma_2 uv - 72 \gamma_2 v^2 - \frac{1}{2} \beta_1 uw - \frac{1}{2} \alpha_1 vw - \frac{1}{2} (\gamma_1 - \gamma_2) w^2 + c_1 (\gamma_2 - \gamma_1)
\end{align*}
\]

where \( v = e^{-E_1/2} d\tau A \); \( u = e^{-E_1/2} d\tau B \); \( w = e^{-E_1/2} d\tau \phi \)

The constraint takes the form

\[ 72 v^2 + 6 u^2 + 48 vu - \frac{1}{2} w^2 = c_1 + c_2 e^{E_2-E_1} \]
Dynamical system analysis

The phase space can be compactified using

\[ x = \frac{2v}{4v + u} \; ; \; y = \frac{w}{2\sqrt{3}(4v + u)} \; ; \; z = \frac{\sqrt{c_1}}{\sqrt{6}(4v + u)} \]

The eom's become an autonomous dynamical system

\[
\begin{align*}
x' &= \frac{1}{4} \left( [\alpha_2 + 2\beta_2(-2 + x)](-1 + x^2 + y^2 + z^2) + [-\alpha_1 - 2\beta_1(-2 + x)]z^2 \right) \\
y' &= \frac{1}{2} \left( (2\sqrt{3}\gamma_2 + \beta_2y)(-1 + x^2 + y^2 + z^2) - (2\sqrt{3}\gamma_1 + \beta_1y)z^2 \right) \\
z' &= \frac{1}{4}z\left( \alpha_1 x + 4\sqrt{3}\gamma_1 y - 2\beta_1(-1 + 2x + z^2) + 2\beta_2(-1 + x^2 + y^2 + z^2) \right)
\end{align*}
\]

where \( f' = d_\omega f \) and \( d_\omega := \frac{\sqrt{c_1}}{\sqrt{6}z} \, d\sigma \)

Relation to the other time parameters

\[
\begin{align*}
d_\omega &= \frac{\sqrt{c_1}}{\sqrt{6}z} e^{E_1/2} \, d\tau \; ; \; dT = \frac{\sqrt{6}z}{\sqrt{c_1}} e^{12A+3B-E_1/2} \, d_\omega
\end{align*}
\]
Dynamical system analysis

The constraint takes the form
\[ c_1 (1 - x^2 - y^2 - z^2) = c_2 z^2 e^{E_2 - E_1} \]
restricting to either the interior or the exterior of the unit sphere.

The unit sphere is an invariant surface
\[ \frac{1}{2} (x^2 + y^2 + z^2)' = \frac{1}{4} (-1 + x^2 + y^2 + z^2) \times \left( \alpha_2 x + 4\sqrt{3} \gamma_2 y - 2\beta_1 z^2 + 2\beta_2 \left[ (-2 + x)x + y^2 + z^2 \right] \right) \]

The equatorial disc (at \( z = 0 \)) is an invariant surface.

The equator (at \( x^2 + y^2 = 1 \) and \( z = 0 \)) is a circle of fixed points.

The plane \( ax + by + c = 0 \) is an invariant surface, where
\[ (\alpha_2 - 4\beta_2) a + 4\sqrt{3} \gamma_2 b - 2\beta_2 c = 0 \]
\[ [\alpha_2 - \alpha_1 - 4(\beta_2 - \beta_1)] a + 4\sqrt{3}(\gamma_2 - \gamma_1) b - 2(\beta_2 - \beta_1) c = 0 \]
Dynamical system analysis

- The condition for expansion, $\dot{S}(T) > 0$, is equivalent to $z > 0$
- The flow is invariant under $(z, \omega) \rightarrow -(z, \omega)$ so trajectories in the northern and southern hemispheres are paired
- The condition for acceleration, $\ddot{S}(T) > 0$, is equivalent to

$$
(\beta_1 - \beta_2)z^2 - \beta_2 (x^2 + y^2) + \beta_2 - 4 > 0
$$

| $\beta_1$ | $\beta_2$ | 0   | 4   | 6   |
|-----------|-----------|-----|-----|-----|
| 0         | 0         | $\emptyset$ | $\emptyset$ |       |
| 4         | 0         | $\emptyset$ | $\emptyset$ |       |
| 6         | 0         |       |       |       |
Dynamical system analysis

- The flow parameter is related to the scale factor via

\[ \omega = \ln \frac{S}{S_0} \]

so the number of e-foldings is given by

\[ N = \int d\omega \]

- The cosmological time reads

\[ T(\omega) = \sqrt{\frac{6}{c_1}} \int^{\omega} d\omega' z(\omega') \exp \left[ \left( 12 - \frac{\alpha_1}{2} \right) A(\omega') + \left( 3 - \frac{\beta_1}{2} \right) - \frac{\gamma_1}{2} \phi(\omega') \right] \]

- This can be inverted to obtain the scale factor \( S(T) \) via

\[ \omega(T) = \ln S \]

and similarly for all other cosmological parameters.
Recap

- Many analytic solutions
  - Always possible if a single excited species of flux.
- Autonomous dynamical system if 2 excited species of flux
  - 3 first-order equations and a constraint
  - Solutions correspond to trajectories in phase-space
  - Compactification of phase-space to (the interior of) a 3d ball
- Sonner & Townsend, hep-th/0608068
  - The equatorial disc and the 2d sphere boundary are invariant surfaces of the dynamical flow
  - There is always an additional invariant plane
  - Fixed points and trajectories on the sphere boundary or on the disc correspond to analytic solutions
Recap

- Rephrasing the question of accelerated expansion
- Expanding cosmologies correspond to trajectories in the northern hemisphere (interpolating between two fixed points)
- Acceleration is possible whenever there is a non-empty *acceleration region* (determined by the type of excited fluxes)
- This explains why transient accelerated expansion is generic: it corresponds to trajectories in the northern hemisphere, passing through the accelerated region.
Results

- Fixed points correspond to scaling cosmologies: \( S(T) \sim T^a \)
- The equator is a circle of fixed points with \( a = \frac{1}{3} \)
- Fixed points on the boundary of the acceleration region have \( a = 1 \)
  They correspond to a regular (singular) Milne universe if the fixed point is (not) the origin of the sphere.
- There are no eternally accelerating scaling cosmologies, \( i.e. \ a \leq 1 \)
- There are fixed points with \( a = \frac{3}{4}, \frac{19}{25}, \frac{9}{11} \)
Results

- Many examples of (semi-)eternal inflation, and cosmologies with a parametric control of the number of e-foldings
- They have $k = -1$ and non-vanishing $\lambda, m, c_f, c_0$ or $c\varphi$
- They have a fixed point on the boundary of the accelerated region
Results

- Examples of **semi-eternal** inflation, and cosmologies with a parametric control of the number of e-foldings
- An example of **eternal** inflation without Big-Bang singularity

- Accelerated contraction (expansion) for $T < 0$ ($T > 0$)
- de Sitter in the neighborhood of $T = 0$
Results

- Several examples of solutions with infinite cycles of accelerated and decelerated expansion (rollercoaster cosmology)
- Example without Big-Bang singularity
Conclusions

- We confirm that transient acceleration is generic in flux compactifications (universal, top-down models).
- Cosmologies featuring (semi-)eternal acceleration, or a parametric control on the number of e-foldings also seem generic!
  - They have $k = -1$ and asymptotically vanishing acceleration.
- Examples of spiraling cosmologies with an infinite number of cycles alternating between accelerated and decelerated expansion (rollercoaster cosmology).
- Comparison with the effective 4d approach, swampland.
- Extend the dynamical system analysis to more than 2 fluxes.
- Inclusion of sources (orientifolds), higher derivatives.
- Realistic cosmologies?