Space-time evolution induced by spinor fields with canonical and non-canonical kinetic terms

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Abstract

We study spinor field theories as an origin to induce space-time evolution. Self-interacting spinor fields with canonical and non-canonical kinetic terms are considered in a Friedman-Robertson-Walker universe. The deceleration parameter is calculated by solving the equation of motion and the Friedman equation, simultaneously. It is shown that the spinor fields can accelerate and decelerate the universe expansion. To construct realistic models we discuss the contributions from the dynamical symmetry breaking.

1 Introduction

The acceleration periods of the universe evolution are one of the most important problems to be explored in astroparticle physics today. Though the cosmological constant gives a simple solution to accelerate the universe expansion, a time dependent source is necessary to stop the inflation at the early universe and restart it at the present low energy scale. There is no simple candidate to derive the accelerated expansion of the universe in the Standard Model contents for particle physics. The problem calls for considerations to new field theory models which may describe the acceleration periods of the cosmic expansion.

Various models beyond the Standard Model have been studied to include candidates which accelerate the expansion of the universe. The simplest model can be constructed by an inflaton like scalar field which is easy to treat in a curved spacetime. A non-vanishing potential energy for the scalar field can induce the exponential expansion of the universe. On the other hand, a spinor field also has played an important role as a gravitational source. An ordinary free fermion behaves as a matter field and decelerates the universe expansion. Much interest has been demonstrated in order to extend the model for the spinor field in recent years.

The study of spinor fields in curved spacetime has a long history. The Dirac equation was investigated for massless spinor fields in curved space-time more than 50 years ago [1]. The Einstein-Dirac equations were solved for a massive spinor field in a special anisotropic spacetime, Bianchi type I universe with a cosmological constant [2]. The possibility to induce a primordial inflation [3]
and the current expansion in a self-interaction spinor field [4] has recently been pointed out. Similar self-interacting fields have been discussed as a kind of Inflaton [5] and Quintom [6, 7]. They have been studied in a Bianchi type I framework [8, 9, 10] and other anisotropic space-time [11, 12]. Non-standard spinors are considered as a source for the current expansion in Refs. [13, 14, 15, 16, 17, 18, 19, 20, 21]. There is a possibility to cause late-time cosmic acceleration by a spinor field through a coupling with the Brans-Dicke scalar field [22, 23], a form invariance transformation [24] and a non-minimal curvature coupling [25].

The class of dark energy models has been proposed in a scalar field with non-canonical kinetic terms, such as models named k-inflation [26, 27] or k-essence [28, 29, 30]. A similar extension is possible for a spinor field. The spinor counterparts of k-essence for scalar field are called f-essence [31, 32] and g-essence [33]. We would like to continue the study of such models on self-interacting spinor fields with non-canonical kinetic terms as well. It should be noted that models with the square of an usual Dirac Lagrangian may be special cases of f-essence scenario [34, 35].

The scalar invariant constructed from two spinor fields dynamically develops a non-vanishing value in QCD like theories [36, 37]. Then the chiral symmetry for the spinor fields is broken. It has played an essential role for the evolution of the universe. Only a little work has been done to evaluate the dynamical symmetry breaking in a non-static spacetime. In de-Sitter space it is possible to calculate the expectation value for the scalar invariant based on the maximal symmetry of the space [38, 39]. However, we would like to find solution with early- and late-time acceleration periods. For this purpose it is not avoidable to develop different procedure.

In this paper a spinor field is assumed as a gravitational source. We confine ourselves, for simplicity, in a Friedman-Robertson-Walker (FRW) metric and evaluate the corresponding space-time evolution. In Sec. 2 we review the space-time evolution induced by a spinor field with an ordinary canonical kinetic term. A self-interacting spinor field is considered in curved space-time. Evaluating the equation of motion for the spinor field and Einstein’s field equations, we show typical behaviors of the Hubble and deceleration parameters. It is quite general to introduce a non-canonical kinetic term in an effective model. We assume a simple form for the non-canonical kinetic term and evaluate the influences on the space-time evolution in Sec. 3. In Sec. 4 we set the critical scale where the dynamical symmetry breaking takes place by hand and discuss the critical behavior caused by the dynamical symmetry breaking. The space-time evolution may be suddenly modified by a first order phase transition. Finally, some concluding remarks are given in Sec. 5.

2 Spinor fields with a canonical kinetic term

Before we study special features of the non-canonical kinetic term for a spinor field, it is more instructive to discuss the space-time evolution induced by a spinor field with an ordinary canonical kinetic term. The space-time evolution is evaluated by solving the equation of motion for the spinor field and Einstein’s field equations, simultaneously. The variations of the action give rise to these equations.
Here we start from the action, 

\[ S = \int d^4 x \sqrt{-g} (\mathcal{L}_g + \mathcal{L}_\Lambda + \mathcal{L}_D), \]  

where \( \mathcal{L}_g \) is the Einstein-Hilbert Lagrangian and \( \mathcal{L}_\Lambda \) describes a cosmological constant term 

\[ \mathcal{L}_g = \frac{1}{16\pi G} R, \quad \mathcal{L}_\Lambda = -\frac{1}{8\pi G} \Lambda, \] 

with \( R \) the curvature scalar and \( G \) the Newtonian constant. The Dirac Lagrangian \( \mathcal{L}_D \) is given by 

\[ \mathcal{L}_D = \frac{i}{2} \left[ \bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \psi \right] - m \bar{\psi} \psi - V(\bar{\psi} \psi), \]  

where the covariant derivatives for spinor fields, \( \psi \) and \( \bar{\psi} \), are defined by 

\[ D_\mu \psi \equiv \partial_\mu \psi - \Omega_\mu \psi, \]  

and 

\[ D_\mu \bar{\psi} \equiv \partial_\mu \bar{\psi} - \bar{\psi} \Omega_\mu. \]  

The spinor connection, \( \Omega_\mu \), is written by the vierbein, \( e_\mu^a \), 

\[ \Omega_\mu = -\frac{1}{4} g_{\rho\sigma} (\Gamma^\rho_\mu \delta - e_\rho^b \partial_\mu e_\delta^b) \Gamma^\sigma \Gamma^\delta, \]  

and the Dirac matrices, \( \Gamma^\mu \), are generalized in a curved spacetime, 

\[ \Gamma^\mu \equiv e_\mu^a \gamma^a. \]  

Here we suppose that the main contribution as a gravitational source comes from a non-vanishing expectation value for a scalar invariant, \( \bar{\psi} \psi \). The potential \( V(\bar{\psi} \psi) \) can be written as a function of a composite operator \( \bar{\psi} \psi \). We noticed that only scalar invariants can develop non-vanishing vacuum expectation values to keep the Lorentz structures in Minkowski spacetime. It is not valid in a curved space-time. Thus the above assumption should be modified in a strongly curved space-time.

The equations of motion for \( \psi \) and \( \bar{\psi} \) are given by the variations of the action (1) with respect to \( \bar{\psi} \) and \( \psi \), respectively, 

\[ -i(D_\mu \bar{\psi}) \Gamma^\mu - m \bar{\psi} - \frac{dV}{d\bar{\psi}} = 0, \]  

and 

\[ i \Gamma^\mu D_\mu \psi - m \psi - \frac{dV}{d\psi} = 0. \]  

The variation of the action (1) with respect to the vierbein gives the Einstein’s field equations, 

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G \langle T_{\mu\nu} \rangle, \]  

where the energy-momentum tensor, \( T_{\mu\nu} \), is given by 

\[ T_{\mu\nu} = \frac{i}{4} [\bar{\psi} \Gamma_\mu D_\nu \psi + \psi \Gamma_\nu D_\mu \psi - (D_\nu \bar{\psi}) \Gamma_\mu \psi - (D_\mu \bar{\psi}) \Gamma_\nu \psi] - g_{\mu\nu} \mathcal{L}_D. \]
The space-time evolution is found by solving Eqs. (8), (9) and (10) with the help of the above energy-momentum form.

We consider the case that the spinor field has a homogeneous and isotropic distribution and look for the solution in a FRW universe with a flatly spatial part. The FRW universe is defined by the metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2).$$

(12)

The space-time evolution is described by the time development of the scale factor $a(t)$. Under the FRW metric, (12), the generalized Dirac matrices are given by

$$\Gamma^0 = \gamma^0, \quad \Gamma^i = \frac{1}{a(t)}\gamma^i,$$

(13)

and the spinor connection reads

$$\Omega^0 = 0, \quad \Omega^i = \frac{1}{2}a(t)\gamma^i\gamma^0.$$

(14)

Since the spinor field is assumed to be homogeneous and isotropic, it is written as only a function of time. In this case Eqs. (8) and (9) are simplified to be

$$\partial_0 \bar{\psi} + \frac{3}{2}H \bar{\psi} + im\bar{\psi}\gamma^0 - \frac{i}{\bar{\psi}}\frac{dV}{d\bar{\psi}}\gamma^0 = 0,$$

(15)

and

$$\partial_0 \psi + \frac{3}{2}H \psi + i\gamma^0 m \psi + i\gamma^0 \frac{dV}{d\psi} = 0,$$

(16)

where $H$ is the Hubble parameter, $H \equiv \dot{a}(t)/a(t)$. The derivative of the potential, $V(\bar{\psi}\psi)$, can be written as

$$\frac{dV}{d\bar{\psi}} = V' \cdot \bar{\psi}, \quad \frac{dV}{d\psi} = V' \cdot \psi.$$

(17)

From Eqs. (15), (16) and (17) the potential dependence can be eliminated. Therefore the composite operator, $\bar{\psi}\psi$, satisfies a simple equation of motion,

$$(\partial_0 + 3H)\bar{\psi}\psi = 0.$$

(18)

This equation has a trivial solution, $\bar{\psi}\psi = 0$. A non-trivial solution of Eq. (18) is

$$\bar{\psi}\psi = \frac{C}{a^3(t)},$$

(19)

where $C$ is an arbitrary constant parameter fixed by an initial condition [3, 4, 6]. The composite operator, $\bar{\psi}\psi$, develops as an ordinary matter field. We noticed that the solution (19) is obtained at the classical limit. It may be modified by radiative corrections for the potential and the energy-momentum tensor. It should be also noticed that the solution is found for a spatially constant, $\bar{\psi}\psi$. However, we consider that the solution has something essential for the development of the composite operator and discuss the evolution of the space-time under the solution (19).

In the FRW universe the energy-momentum tensor, $T_{\mu}^\nu$, has diagonal form. It can be parametrized as

$$\langle T_{\mu}^\nu \rangle \equiv \text{diag}(\rho, -p, -p, -p),$$

(20)
where $\rho$ corresponds to energy density and $p$ pressure. Thus the Einstein’s field equations (10) read

$$3 \left( \frac{\dot{a}(t)}{a(t)} \right)^2 - \Lambda = 8\pi G \rho,$$

(21)

and

$$- 2 \left( \frac{\ddot{a}(t)}{a(t)} \right) - \left( \frac{\dot{a}(t)}{a(t)} \right)^2 + \Lambda = 8\pi G p.$$

(22)

From Eq. (11) with the equation of motion (15) and (16) the energy density, $\rho$, and the pressure, $p$, can be expressed as functions of the scalar invariant, $\bar{\psi}\psi$,

$$\rho = m \langle \bar{\psi}\psi \rangle + \langle V(\bar{\psi}\psi) \rangle,$$

(23)

and

$$p = -\langle V(\bar{\psi}\psi) \rangle + \langle V'(\bar{\psi}\psi) \cdot \bar{\psi}\psi \rangle.$$

(24)

An explicit form of the potential is necessary to find the typical behavior of the space-time. It is assumed that the potential can be expanded in terms of the scalar invariant,

$$\langle V(\bar{\psi}\psi) \rangle = \sum_{n=1}^{\infty} \alpha_n \langle \bar{\psi}\psi \rangle^{2n},$$

(25)

where $\alpha_n$ corresponds to a coupling constant for multi-fermion interactions. To avoid an unexpected divergence we consider only a positive $n$ in Eq. (25). Since the scalar invariant, $\bar{\psi}\psi$, is a dimension 3 operator, the potential (25) is not renormalizable. We regard the potential as a low energy effective one coming from a more fundamental theory at a high energy scale. It is expected that the higher order terms of $\bar{\psi}\psi$ are suppressed at low energy. Below, we cut the summation at $n_{max}$. We noted that the model is a kind of four-fermion interaction models for $n_{max} = 1$ which is often used as a low energy effective model of QCD [36, 37].

Hence, the energy density, $\rho$, and the pressure, $p$, read

$$\rho = m \langle \bar{\psi}\psi \rangle + \sum_{n=1}^{n_{max}} \alpha_n \langle \bar{\psi}\psi \rangle^{2n},$$

(26)

and

$$p = \sum_{n=1}^{n_{max}} \alpha_n (2n - 1) \langle \bar{\psi}\psi \rangle^{2n}.$$

(27)

For a trivial solution of Eq. (19), $\langle \bar{\psi}\psi \rangle = 0$, the right hand sides of the Einstein’s field equations, (21) and (22) vanish, which simply describes a well known de Sitter universe. We can recognize that the solution corresponds to lepton and heavy quark fields. In our universe finite vacuum expectation values are not observed for the lepton and the heavy quark bilinears. These spinor fields have no role for the evolution of the universe. However, it is known that the composite operator, $\bar{\psi}\psi$, develops non-vanishing value for up and down quarks in the current universe. Thus we evaluate the space-time evolution under the solution (19). Inserting the solution (19) into Eqs. (26) and (27), we rewrite the Einstein’s field equations, (21) and (22), as

$$3 \left( \frac{\dot{a}(t)}{a(t)} \right)^2 - \Lambda = 8\pi G \left[ m \frac{C}{a^4(t)} + \sum_{n=1}^{n_{max}} \alpha_n \frac{C^{2n}}{a^{2n}(t)} \right],$$

(28)
and
\[ -2 \left( \frac{\ddot{a}(t)}{a(t)} \right) - \left( \frac{\dot{a}(t)}{a(t)} \right)^2 + \Lambda = 8\pi G \sum_{n=1}^{n_{\text{max}}} \alpha_n (2n - 1) \frac{C^{2n}}{a^{6n}(t)} \]  

(29)

As is known, these equations are not independent. Equation (29) can be derived from Eq. (28) plus the Bianchi identity. These equations are called Friedman equations.

The Hubble parameter, \( H \), then is given by
\[ H^2 = \frac{\Lambda}{3} + \frac{8\pi G}{3} \left[ m \frac{C}{a^3(t)} + \sum_{n=1}^{n_{\text{max}}} \alpha_n \frac{C^{2n}}{a^{6n}(t)} \right] \]  

(30)

The cosmological constant and the mass terms mainly contribute to the right-hand side of Eq. (30) at the low energy limit, \( a(t) \to \infty \). As is well-known, the composite operator, \( \bar{\psi} \psi \), develops a negative expectation value for up and down quarks in the current hadronic phase. It means that the parameter, \( C \), has to be negative for the light quarks. Thus the mass term in Eq. (30) is negative in the standard model of particle physics. On the other hand, the left-hand side of Eq. (30) has possessed a non-negative value. The cosmological constant is necessary to take the low energy limit as, \( a(t) \to \infty \). Upper limit for the scale factor exists without the cosmological constant. It should be noted that the parameter \( C \) is assumed to be positive in Refs. [4, 6].

The deceleration parameter is written as
\[ q \equiv -\frac{\ddot{a}(t)}{H^2 a(t)} = \frac{1}{2} (1 + 3\omega), \]  

(31)

where \( \omega \) shows the equation of state of the universe,
\[ \omega \equiv \frac{-\Lambda + 8\pi G p}{\Lambda + 8\pi G p} \]
\[ = \frac{-\Lambda + 8\pi G \left[ \sum_{n=1}^{n_{\text{max}}} \alpha_n (2n - 1) \frac{C^{2n}}{a^{6n}(t)} \right]}{\Lambda + 8\pi G \left[ m \frac{C}{a^3(t)} + \sum_{n=1}^{n_{\text{max}}} \alpha_n \frac{C^{2n}}{a^{6n}(t)} \right]} \]  

(32)

An acceleration period is realized for a negative \( q \). At the low energy limit, \( a(t) \to \infty \), it diverges for \( \Lambda = 0 \) and approaches a negative unity for a non-vanishing \( \Lambda \).

Evaluating Eqs. (30) and (31) numerically, we show features of the spinor field as a gravitational source. In numerical calculations all the mass scales are normalized by the fermion mass \( m \). In Fig. 1 typical behavior for the Hubble and the deceleration parameters are shown as a function of the scale factor \( a \) for a four-fermion interaction model without cosmological constant. An eight-fermion interaction is introduced in Fig. 2. The parameter, \( C \), in Eq. (30) is negative for QCD-like theories. The equation (30) has a real solution for
The Hubble parameter, only if the coupling constant for the higher dimensional operator, \( \alpha_{n_{\text{max}}} \), takes a positive value. As is seen in Fig. 1 (a), the right-hand side of Eq. (30) develops a negative value for a larger \( a \). Since the Friedman equation has no real solution, the parameter range for the dashed line is ruled out. There is an upper limit for the scale factor. In Fig. 1 (b) we observe that the deceleration parameter, \( q \), blows up at the upper limit.

In Fig. 2 we introduce a cosmological constant term for a four-fermion interaction model. Since the cosmological constant term makes the right-hand side of Eq. (30) shift, it is possible to cancel a negative contribution from the fermion self interaction terms. In other words, a cosmological constant like gravitational source is necessary to realize an open universe with a non-vanishing \( \bar{\psi} \psi \). In Fig. 2 (b) the deceleration parameter converges to unity for a large \( a \) limit. Thus the late time universe can be accelerated by the cosmological constant.

For a small \( a \) higher dimensional operators modify the behavior of the Hubble and the deceleration parameters. Interesting behavior is found if one of \( \alpha_n \) is negative. In Fig. 3 we draw the behavior of the Hubble parameter for a negative...
eight-fermion coupling, $\alpha_2 < 0$. We also introduce a positive twelve-fermion coupling, $\alpha_3 > 0$ to stabilize the potential. A negative $\alpha_2$ modifies the behavior of Hubble parameter in Fig. 3 (a). As is shown in Fig. 3 (b) the square of the Hubble parameter becomes negative without cosmological constant. Thus the Friedman equation has no real solution on the parameter range for the dashed line. This again means that the universe has an upper limit. The deceleration parameter blows up at the scale in Fig. 4 (a). In the case $a_2 m^8 = -1$ we find an acceleration period, $q < 0$, without any cosmological constant. In Fig. 4 (b) it is found that the late time universe can also be accelerated by introducing cosmological constant.

### 3 Non-canonical kinetic term

In the previous section we only consider scalar type multi-fermion interactions. It is assumed that such interactions are introduced in a field theory as low energy effective model stemming from a more fundamental theory at high energy scales.
Varieties of interactions which keep the actual symmetry for the particle physics can be generated. A kinetic term for fermion may also be modified at low energy scale. Here we investigate a class of fermion field theories which are described by the Dirac Lagrangian with non-canonical kinetic terms,

\[ \mathcal{L}_D = \frac{i}{2} f(\bar{\psi}\psi) \left[ \bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \right] + m\bar{\psi}\psi - V(\bar{\psi}\psi), \]  

(33)

where the function \( f(\bar{\psi}\psi) \) modifies the kinetic term. If we set \( f(\bar{\psi}\psi) = 1 \), the Lagrangian (3) is reproduced. Here we consider a more general case, where the function \( f(\bar{\psi}\psi) \) is given by a function of only a scalar invariant \( \bar{\psi}\psi \).

The non-canonical kinetic terms modify the equations of motion for fermion fields, \( \bar{\psi} \) and \( \psi \), to

\[ \frac{i}{2} f' \left[ \bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \right] - \frac{i}{2} \left( \partial_\mu f \right) \Gamma^\mu \bar{\psi} + m\bar{\psi} - V' \bar{\psi} = 0, \]  

(34)

and

\[ \frac{i}{2} f' \left[ \bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \right] + \frac{i}{2} \left( \partial_\mu f \right) \Gamma^\mu \bar{\psi} + i f \Gamma^\mu D_\mu \psi + m\psi - V' \bar{\psi} = 0, \]  

(35)

where we write

\[ f' \equiv \frac{df(\bar{\psi}\psi)}{d(\bar{\psi}\psi)}. \]  

(36)

Derivatives of the potential, \( V(\bar{\psi}\psi) \), and the function \( f(\bar{\psi}\psi) \) can be written as

\[ \frac{dV}{d\bar{\psi}} = V' \cdot \bar{\psi}, \quad \frac{dV}{d\psi} = V' \cdot \psi, \quad \frac{df}{d\bar{\psi}} = f' \cdot \bar{\psi}, \quad \frac{df}{d\psi} = f' \cdot \psi. \]  

(37)

From Eqs. (34) and (35) with Eqs. (37) we obtain a simple equation of motion for the composite operator, \( \bar{\psi}\psi \),

\[ \left[ (f + \bar{\psi}\psi f') \partial_0 + 3f H \right] \bar{\psi}\psi = 0. \]  

(38)

There is a trivial solution, \( \bar{\psi}\psi = 0 \). To find a non-trivial solution we expand the function \( f(\bar{\psi}\psi) \) in terms of the scalar invariant,

\[ f(\bar{\psi}\psi) = \sum_n \beta_n (\bar{\psi}\psi)^{2n}. \]  

(39)

Thus the non-trivial solution of Eq. (35) is found to be

\[ f(\bar{\psi}\psi)\bar{\psi}\psi = \sum_n \beta_n (\bar{\psi}\psi)^{2n+1} = \frac{C}{a'(t)}, \]  

(40)

where \( C \) is a constant parameter.

We numerically solve Eq. (40) for a fixed \( a(t = t_0), \langle \bar{\psi}\psi \rangle(t = t_0) \) and \( \beta_n \). We draw typical behavior of \( \langle \bar{\psi}\psi \rangle \) in Fig. 5 with \( \langle \bar{\psi}\psi \rangle(t = t_0) = m^3 \) and \( a(t = t_0) = m^{-1} \). The term, \( \beta_1 (\bar{\psi}\psi)^2 \), changes the slope for a small \( a \). If the coefficient, \( \beta_1 \),
We set $\beta_0 = 1$ and $\beta_n = 0$ for $n \neq -1, 0, 1, 2$.

In Fig. 5 (c) a negative $\beta_{-1}$ raises up a value, $|\bar{\psi}\psi|$, for a large $a$.

In the FRW universe the Einstein’s field equations are given by Eqs. (21) and (22). For the Lagrangian (33) the energy-momentum tensor, $T_{\mu\nu}$, is modified to be

$$T_{\mu\nu} = \frac{i}{4} f(\bar{\psi}\psi) \left[ \bar{\psi} \Gamma_{\mu} D_{\nu} \psi + \bar{\psi} \Gamma_{\nu} D_{\mu} \psi - (D_{\mu} \bar{\psi}) \Gamma_{\nu} \psi \right] - g_{\mu\nu} L_D. \tag{41}$$

From Eq. (41) with the equation of motions (34) and (35) we find the same expression for the energy density, $\rho$, and the pressure, $p$, with Eqs. (26) and (27), respectively. Thus the Friedman equations read

$$3 \left( \frac{\dot{a}(t)}{a(t)} \right)^2 - \Lambda = 8\pi G \left( m(\bar{\psi}\psi) + \sum_{n=1}^{\text{max}} \alpha_n (\bar{\psi}\psi)^{2n} \right), \tag{42}$$
As mentioned in the previous section, the Eq. (43) can be derived from the Eq. (42) with the help of continuous equation for the homogenous pressure and density from the cosmic fermionic components.

\[-2 \left( \frac{\ddot{a}(t)}{a(t)} \right) - \left( \frac{\dot{a}(t)}{a(t)} \right)^2 + \Lambda = 8\pi G \left( \sum_{n=1}^{n_{\text{max}}} \alpha_n (2n-1)(\bar{\psi}\psi)^{2n} \right).\] (43)

We again assume that the non-trivial solution is realized for some of spinor fields and numerically solve the Friedman equation (42) under the non-trivial
The results depend on the parameters $\alpha_n$ and $\beta_n$. Varieties of evolution can be reproduced by choosing these parameters. Here we set finite values for $\alpha_1$ and $\beta_n$ with $n \in \{-1,0,1,2\}$. To avoid the upper limit for the scale factor we introduce a cosmological constant.

As is seen in Figs. 6 and 7 the acceleration period, $q < 0$, appears later for a larger $\beta_1$ and $\beta_2$. If we introduce a negative $\alpha_2$ with a positive $\alpha_3$, a similar behavior with Fig. 4 is observed for a small $a$. Therefore an early-time acceleration can be induced by the negative $\alpha_2$. Non-canonical kinetic terms can control when the late-time acceleration period starts with the model parameters tuning properly.

![Figure 8](image-url)

Figure 8: The solid lines show the behavior of the Hubble and the deceleration parameters for $\Lambda/(8\pi G m^4) = 0.1$, $\langle \bar{\psi} \psi \rangle(t = 0) = m^3$, $a(t = 0) = m^{-1}$, $\alpha_1 m^2 = 10$, $n_{\max} = 1$, $\beta_0 = 1$, $\beta_{-1} m^{-6} = -0.05, -0.1, -0.15$ and $\beta_m = 0$ for $m \neq -1, 0$. The dashed lines represent the behavior for $\Lambda = 0$.

The term, $\beta_{-1}(\bar{\psi} \psi)^{-2}$, raises up the Hubble parameter. As is shown in Fig. 8, an open universe can be realized without any cosmological constant like gravitational source for a negative $\beta_{-1}$.

## 4 Condensation of spinor fields

In QCD like theory the gauge interaction becomes stronger at low energy. The strong gauge interaction induces the condensation of spinor fields below a critical scale, $a_{cr}$ or a critical temperature $T_{cr}$, see for example Ref. [40]. After the condensation the expectation value for the composite operator constructed by fermion and anti-fermion, $\bar{\psi} \psi$, develops a non-vanishing value. It is known as the spontaneously chiral symmetry breaking result in QCD [36]. Thus it is natural to use the trivial solution $\langle \bar{\psi} \psi \rangle = 0$ before the universe arrives the critical scale, $a(t) < a_{cr}$.

In a static background metric the multi-fermion interaction is evaluated in Refs. [41, 42, 43]. In the FRW universe the critical scale have to be fixed by observing the effective action of the theoretical model. For this purpose the quantum field theory in an unstable background metric is necessary. It seems to have many difficulties [44, 45]. We take the remaining problem to fix the critical
scale as future works. However, the symmetry breaking changes the solution for the equation of motion. It has played a decisive role for the evolution of the universe. Here we set the critical scale by hand and discuss the contribution from the symmetry breaking effects at certain scales.

We suppose that the first order phase transition takes place, the solution for the equation of motion suddenly jumps from the trivial to the non-trivial one. Under the trivial solution the Hubble parameter is constant, which corresponds to the well known de Sitter universe phase. A static universe realized without any cosmological constant like terms. The cosmological constant can exponentially expand the universe. After the universe arrives at the critical scale, the expectation value for the composite operator $\bar{\psi}\psi$ decelerates the cosmic expansion.

![Figure 9: Behavior of the Hubble and the deceleration parameters for $a_{cr}=1.5$, $\Lambda/(8\pi G m^4) = 0.1$, $C = -1$, $\alpha_1 m^2 = 5$ and $n_{max} = 1$.](image)

We set the critical scale at $a_{cr} = 1.5$ by hand and draw a typical behavior of the Hubble and the deceleration parameters for the same parameter setting with a line for $\alpha_1 m^2 = 5$ in Fig. 2. As is seen in Fig. 3 the Hubble and the deceleration parameters are constant, $H^2 = \Lambda/3$ and $q = -1$ below the critical scale $a < a_{cr}$. We observe the same solution with Fig. 3 above the critical scale. In this case the spinor field contributes in the intermediate scale. The cosmological constant dominated universe can be realized at the early and the late stage of the universe evolution. We note that the cosmological parameters moves smoothly at the critical scale if the phase transition is of the second order.

## 5 Conclusions

We have investigated the self-interacting spinor fields as a gravitational source for space-time evolution via the Einstein’s gravity. It has been assumed that the background metric is homogeneous and isotropic in the large scale. We have solved the Einstein-Dirac equations for the spinor bilinear, $\bar{\psi}\psi$, in the FRW metric. There are two solutions. One of the solutions, $\bar{\psi}\psi = 0$, has played no significant role for the space-time evolution.

First, we have evaluated the non-trivial solution and showed some behaviors of the cosmological parameters. It has been observed that a self-interacting
potential can induce the early time cosmic acceleration. It is consistent with result in Ref. [3]. The expectation value for the mass term dominates the vacuum energy at a late stage of the universe evolution. Because of a negative expectation value for the spinor bilinear operator, $\bar{\psi}\psi$, mass term decelerates cosmic expansion. Only a closed universe can be realized without another gravitational source. In this work we have introduced a cosmological constant term. The cosmological constant can promote the accelerated expansion of the Universe at a late stage. It is shown that the non-canonical kinetic term controls when the late time acceleration starts.

Non-vanishing expectation value for the spinor bilinear breaks the chiral symmetry for the spinor field. It is believed that high temperature, high density and strong curvature may restore the broken symmetry at the early universe. Thus it is not always valid to adopt the non-trivial solution of the Einstein-Dirac equations. The trivial solution is realized before the critical scale, $a < a_{cr}$. Here we have set a critical scale, $a_{cr}$, by hand and demonstrate the possibility that the spinor fields can decelerate the cosmic expansion in an intermediate scale.

It should be noted that the late time accelerated expansion or the dark energy phenomena can be produced without a cosmological constant. An interesting mechanisms is found in a modified gravity. For general review of the later time accelerated expansion in the modified gravity, see Ref. [4]. It is not hard to take account of spinor source in above scheme.

In this paper we restrict the potential as a function of only the scalar invariant operator, $\bar{\psi}\psi$, and consider isotropic universe evolution for large scale. Other types of spinor bilinear may generate an angular momentum. It is known that an anisotropic universe is generally realized due to the spinor angular momentum. Hence, it is interesting to include other types of spinor bilinear and study the model in an anisotropic universe. We have not calculated the critical scale, $a_{cr}$, starting from the action. It should be fixed by observing the stationary point of the effective action. To evaluate the effective action we should take into account the quantum and thermal effects in an expanding universe background. We will continue our work further and hope to report on these problems.

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