Analytical Methods for Interpretable Ultradense Word Embeddings

Philipp Dufter, Hinrich Schütze
Center for Information and Language Processing (CIS)
LMU Munich, Germany
philipp@cis.lmu.de

Abstract
Word embeddings are useful for a wide variety of tasks, but they lack interpretability. By rotating word spaces, interpretable dimensions can be identified while preserving the information contained in the embeddings without any loss. In this work, we investigate three methods for making word spaces interpretable by rotation: Densifier (Rothe et al., 2016), linear SVMs and DensRay, a new method we propose. While DensRay is very closely related to the Densifier, it can be computed in closed form, is hyperparameter-free and thus more robust than the Densifier. We evaluate the methods on lexicon induction and set-based word analogy and conclude that analytical methods such as DensRay and SVMs are preferable. For word analogy we propose a new method to solve the task which outperforms the previous state of the art by large margins.

1 Introduction
Distributed representations for words have been of interest in natural language processing for many years. Word embeddings have been particularly effective and successful. On the downside, embeddings are generally not interpretable. This is desirable for several reasons. i) Semantically or syntactically similar words can be extracted: e.g., for lexicon induction. ii) Interpretable dimensions can be used to evaluate word spaces by examining which information is covered by the embeddings. iii) Computational advantage: for a high-quality sentiment classifier only a couple of dimensions of a high-dimensional word space are relevant. In addition, by removing interpretable dimensions one can remove unwanted information (e.g., gender bias). iv) Most importantly, interpretable embeddings support the goal of interpretable deep learning models.

Orthogonal transformations have been of particular interest in the literature. The reason is twofold: under the assumption that existing word embeddings are of high-quality one would like to preserve the original embedding structure by using orthogonal transformations (i.e., preserving original distances). Further, Park et al. (2017) show that rotating existing dense word embeddings achieve the best performance across a range of interpretability tasks.

In this work we propose marginal modifications to the objective function of the Densifier (Rothe et al., 2016) such that a closed form solution becomes available. We call this method DensRay. As additional baseline inspired by Amir et al. (2015) we compute simple linear SVMs, which we find to perform surprisingly well. We compare these methods on the task of lexicon induction.

Further we show how interpretable word spaces can be applied to other tasks. For the word analogy task we propose a new set-based method called ClaCo to solve it. We find ClaCo to outperform the previous best method LRCos by Drozd et al. (2016) by large margins up to 100%.

Our contributions are: i) We show that one can derive an analytical solution for the Densifier with a slightly modified objective function. ii) We show that the analytical solution performs as well as the Densifier but is more robust. ii) We provide evidence that simple linear SVMs are best suited for the task of lexicon induction. iii) We introduce “ClaCo” a new method for solving the set-based word analogy problem, which outperforms LRCos by a large margin. The source code is available.

2 Methods
2.1 Notation
We consider a vocabulary \( V := \{v_1, v_2, \ldots, v_n\} \) together with an embedding matrix \( E \in \mathbb{R}^{n \times d} \) (e.g., generated by word2vec) where \( d \) is the embedding

\[\text{https://github.com/pdufter/densray}\]
dimension. The \(i\)th row of \(E\) is the vector \(e_i\). We require an annotation for a specific linguistic feature (e.g., sentiment) and denote this annotation by \(l : V \rightarrow \{-1, 1\}\). The objective is to find an orthogonal matrix \(Q \in \mathbb{R}^{d \times d}\) such that \(EQ\) is interpretable, i.e., the values of the first \(k\) dimension should correlate well with the linguistic feature. We refer to the first \(k\) dimensions as ultrade dense interpretable word space. We interpret any vector \(x \in \mathbb{R}^n\) as a column vector and \(x^T\) as row vector.

2.2 DensRay
Throughout this section \(k = 1\). Consider \(L_\mathcal{E} := \{(v, w) \in V \times V \mid l(v) = l(w)\}\), and analogously \(L_{\mathcal{E}^C}\). We call \(d_{vw} := e_v - e_w\) a difference vector. The Densifier (Rothe et al., 2016) solves the following optimization problem

\[
\max_Q \sum_{(v, w) \in L_{\mathcal{E}^C}} \alpha_{\mathcal{E}^C} ||P^*Q (e_v - e_w)||_2 - \sum_{(v, w) \in L_\mathcal{E}} \alpha_{\mathcal{E}} ||P^*Q (e_v - e_w)||_2,
\]

subject to \(Q\) being an orthogonal matrix. Further \(\alpha_{\mathcal{E}^C}, \alpha_{\mathcal{E}} \in [0, 1]\) and \(P^* \in \mathbb{R}^{k \times d}\) is an identity matrix for dimensions up to \(k\). For \(k = 1\) we get

\[
\max_q \sum_{(v, w) \in L_{\mathcal{E}^C}} \alpha_{\mathcal{E}^C} ||q^T d_{vw}||_2 - \sum_{(v, w) \in L_\mathcal{E}} \alpha_{\mathcal{E}} ||q^T d_{vw}||_2
\]

subject to \(q^T q = 1\) and \(q \in \mathbb{R}^d\). Note that \(q^T d_{vw}\) is a scalar. We now propose a slight modification to the objective function: we use the squared euclidean norm instead of the euclidean norm, something that is frequently done in optimization to simplify the gradient. The problem becomes then

\[
\max_q \sum_{(v, w) \in L_{\mathcal{E}^C}} \alpha_{\mathcal{E}^C} ||q^T d_{vw}||_2^2 - \sum_{(v, w) \in L_\mathcal{E}} \alpha_{\mathcal{E}} ||q^T d_{vw}||_2^2\]  

(1)

Using the fact that \(||x||_2^2 = x^T x\) together with associativity of the matrix product we can simplify as follows.

\[
\max_q q^T \left( \sum_{(v, w) \in L_{\mathcal{E}^C}} d_{vw} d_{vw}^T - \alpha_{\mathcal{E}} \sum_{(v, w) \in L_\mathcal{E}} d_{vw} d_{vw}^T \right) q
\]

\[
= \max_q q^T A q \quad \text{subject to} \quad q^T q = 1
\]

(2)

Thus we aim to maximize the Rayleigh quotient of \(A\) and \(q\). Note that \(A\) is a real symmetric matrix. Then it is well known that the eigenvector belonging to the maximal eigenvalue of \(A\) solves the above problem (cf. (Horn et al., 1990, Section 4.2)). We call this analytical solution DensRay.

A second dimension, which is orthogonal to the first dimension and encodes the linguistic features second strongest is given by the eigenvector corresponding to the second largest eigenvalue. The matrix of \(k\) eigenvectors of \(A\) ordered by the the corresponding eigenvalues yields the the desired matrix \(Q\) (cf. (Horn et al., 1990, Section 4.2)) for \(k > 1\). Due to \(A\) being a real symmetric matrix, \(Q\) is always orthogonal.

2.3 Comparison to Densifier
While we have shown that DensRay is a closed form solution to a marginally modified version of the Densifier, there remain a few differences.

Case \(k > 1\). While both methods, Densifier and DensRay yield ultradense \(k\) dimensional subspaces and are equivalent for \(k = 1\), we leave it to future work to examine how the subspaces differ for \(k > 1\). Multiple linguistic signals. Given multiple linguistic features the Densifier can obtain a single orthogonal transformation simultaneously for all linguistic features with chosen dimensions reserved for different features. DensRay can encode multiple linguistic features in one transformation only by iterative application. Optimization. Densifier is based on solving an optimization problem using stochastic gradient descent with iterative orthogonalization of \(Q\). DensRay, in contrast, is an analytical solution. Thus we expect DensRay to be more robust, which is confirmed by our experiments.

2.4 Geometric Interpretation
Assuming we normalize the vectors \(d_{vw}\) one can interpret Eq. 1 as follows: we search for a unit vector \(q\) such that the square of the cosine similarity with \(d_{vw}\) is large if \((v, w) \in L_{\mathcal{E}^C}\), small if \((v, w) \in L_\mathcal{E}\). Thus, we identify dimensions that are parallel/orthogonal to difference vectors of words belonging to different/same classes. It seems reasonable to consider the average cosine similarity. Thus if \(n_{w}, n_{\mathcal{E}^C}\) is the number of elements in \(L_{\mathcal{E}}, L_{\mathcal{E}^C}\) one can choose \(\alpha_{\mathcal{E}^C} = n_{\mathcal{E}^C}^{-1}\) and \(\alpha_{\mathcal{E}} = n_{\mathcal{E}}^{-1}\).

3 Lexicon Induction
We show that DensRay and Densifier indeed perform comparable using the task of lexicon induc-
tion. We adopt the same experimental setting as in (Rothe et al., 2016). Given a word embedding space and a sentiment/concreteness dictionary (binary or continuous scores) we identify a one-dimensional interpretable subspace. Subsequently we use the values along this dimension to predict a score for unseen words and report Kendall’s Tau rank correlation with the gold scores.

To ensure comparability across methods we have redone all experiments in the same setting: we deduplicated lexicons, removed a potential train/test overlap and ignored neutral words in the lexicons. We set $\alpha_\ell = \alpha_m = 0.5$ to ensure comparability between Densifier and DensRay.

Additionally we report results created by linear SVM/SVR inspired by Amir et al. (2015). While they did not use linear kernels, we require linear kernels to obtain interpretable dimensions. Naturally the normal vector of the hyperplane in SVMs/SVRs reflects an interpretable dimension. An orthogonal transformation can be computed by considering a random orthogonal basis of the null space of the interpretable dimension.

Table 1 shows results. As expected the performance of Densifier and DensRay is comparable (macro mean deviation of 0.001). In 5/8 tasks DensRay is better than the Densifier. We explain deviations between the results with the unstable training process of the Densifier. This hypothesis is supported by Figure 1. Figure 1 assesses of the stability by reporting mean and standard deviation for the concreteness task (BWK lexicon). We varied the size of the training lexicon as depicted on the x-axis and sampled 40 subsets of the lexicon with the prescribed size. This shows that Densifier is less stable – presumably because of the instability of the complex optimization procedure that is used for Densifier in contrast to a closed form solution in DensRay.

Surprisingly simple linear SVMs perform best in the task of lexicon induction. SVR is slightly better when continuous lexica are used for training (line 8).

---

3 For Densifier we used code by Rothe et al. (2016).
4 Continuous scores are binarized using the median.
A line of work that uses sRay considers differences vectors within the left/right class. For LRCos and ClaCo precision values are shown. Full table with all individual categories listed in the appendix. GA: Google Analogy Dataset; BATS: GN: Google News Embeddings; FT: FastText subword embeddings.

We propose the method ClaCo: we train DensRay or an SVM to obtain interpretable embeddings $E' = EQ$ using the class information as reasoned above. We now denote as $E_{v,1}$ the first column of $E'$ (i.e., the most interpretable dimension) and $E_{v,-1}$ the remaining columns. We minimize $E_{v,1}$ such that words belonging to the right class have a high value. For a query word $a$ we now want to identify the corresponding $a'$ by solving

$$a' = \arg\max_{v \in V} \text{sim}(E_{v,1}) \cdot \text{sim}(a, -1, E_{v,-1})$$

where \text{sim} computes the cosine similarity.

We evaluate this method across two analogy datasets and two wordspaces. Table 2 shows the results. ClaCo performs significantly better than LRCos. Especially for the challenging derivational categories of BATS, the performance is doubled. Clearly analogies which have low similarity in either Interclass or the right class (IntraR) are harder. DensRay outperforms SVMs on GN/BATS: here the classes are widespread and exhibit low cosine similarity. Given that DensRay considers differences vectors within classes as well - in contrast to SVMs -, this seems to be of advantage here.

5 Related Work

Identifying Interpretable Dimensions. The most relevant to our method is a line of work that uses transformations of existing word spaces to obtain interpretable subspaces. Rothe et al. (2016) compute an orthogonal transformation using shallow neural networks. Park et al. (2017) apply exploratory factor analysis to embedding spaces to obtain interpretable dimensions in an unsupervised manner. Their approach relies on solving complex optimization problems, while we focus on closed form solutions and simple optimization problems. Senel et al. (2018) use SEMCAT categories in combination with the Bhattacharya distance to identify interpretable directions. Also, oriented PCA (Diamantar and Kung, 1996) is closely related to our method. However, both methods yield non-orthogonal transformation. Faruqui et al. (2014) use semantic lexicons to retrofit embedding spaces. Thus they do not fully maintain the structure of the word space, which is in contrast to this work.

Interpretable Embedding Algorithms. Another line of work modifies embedding algorithms to yield interpretable dimensions (Koc et al., 2018; Luo et al., 2015; Shin et al., 2018; Zhao et al., 2018). There is also much work that generates sparse embeddings that are claimed to be more interpretable (Murphy et al., 2012; Faruqui et al., 2015; Fyshe et al., 2015; Subramanian et al., 2018). Instead of learning new embeddings, we aim at making dense embeddings interpretable.

Lexicon Induction, Analogy Generation

There are many prior publications that address the above tasks. We list the ones that are most closely related to our methodology. Amir et al. (2015) use word embeddings and regression to induce sentiment lexica. Drozd et al. (2016) compare a range of methods to solve the word analogy problem. Their best method, LRCos, is used as baseline.

6 Conclusion

We have investigated analytical methods for obtaining interpretable word dimensions in embedding spaces. We introduced a closed form solution of the popular Densifier method. In addition we introduced a new method for solving the set-based word analogy task, which outperforms previous methods by up to 100%.

We gratefully acknowledge funding through a Zentrum Digitalisierung.Bayern fellowship awarded to the first author. This work was also supported by the European Research Council (#740516).
References

Amine Abdaoui, Jérôme Azé, Sandra Bringay, and Pascal Poncelet. 2017. Feel: a french expanded emotion lexicon. Language Resources and Evaluation, 51(3).

Silvio Amir, Ramón Astudillo, Wang Ling, Bruno Martins, Mario J Silva, and Isabel Trancoso. 2015. Inesc-id: A regression model for large scale twitter sentiment lexicon induction. In Proceedings of the 9th International Workshop on Semantic Evaluation (SemEval 2015).

Tolga Bolukbasi, Kai-Wei Chang, James Y Zou, Venkatesh Saligrama, and Adam T Kalai. 2016. Man is to computer programmer as woman is to homemaker? debiasing word embeddings. In Advances in Neural Information Processing Systems.

Marc Brysbaert, Amy Beth Warriner, and Victor Kuperman. 2014. Concreteness ratings for 40 thousand generally known english word lemmas. Behavior research methods, 46(3).

Konstantinos I Diamantaras and Sun Yuan Kung. 1996. Principal component neural networks: theory and applications, volume 5. Wiley New York.

Aleksandr Drozd, Anna Gladkova, and Satoshi Matsuoka. 2016. Word embeddings, analogies, and machine learning: Beyond king-man+ woman= queen. In Proceedings of COLING 2016, the 26th International Conference on Computational Linguistics: Technical Papers.

Manaal Faruqui, Jesse Dodge, Sujay K Jauhar, Chris Dyer, Eduard Hovy, and Noah A Smith. 2014. Retrofitting word vectors to semantic lexicons. arXiv preprint arXiv:1411.4166.

Manaal Faruqui, Yulia Tsvetkov, Dani Yogatama, Chris Dyer, and Noah Smith. 2015. Sparse overcomplete word vector representations. arXiv preprint arXiv:1506.02004.

Jean-Philippe Fauconnier. 2015. French word embeddings.

Alona Fyshe, Leila Wehbe, Partha P Talukdar, Brian Murphy, and Tom M Mitchell. 2015. A compositional and interpretable semantic space. In Proceedings of the 2015 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies.

Roger A Horn, Roger A Horn, and Charles R Johnson. 1990. Matrix analysis. Cambridge university press.

Minqing Hu and Bing Liu. 2004. Mining and summarizing customer reviews. In Proceedings of the tenth ACM SIGKDD international conference on Knowledge discovery and data mining.

Aykut Koç, Ihsan Ulu, Lutfi Kerem Senel, and Haldun M Ozaktas. 2018. Imparting interpretability to word embeddings. arXiv preprint arXiv:1807.07279.

Hongyin Luo, Zhiyuan Liu, Huanbo Luan, and Maosong Sun. 2015. Online learning of interpretable word embeddings. In Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing.

Tomas Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781.

Saif M Mohammad and Peter D Turney. 2013. Crowd-sourcing a word–emotion association lexicon. Computational Intelligence, 29(3).

Brian Murphy, Partha Talukdar, and Tom Mitchell. 2012. Learning effective and interpretable semantic models using non-negative sparse embedding. Proceedings of COLING 2012.

Sungjoon Park, JinYeong Bak, and Alice Oh. 2017. Rotated word vector representations and their interpretability. In Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing.

Veronica Perez-Rosas, Carmen Banea, and Rada Mihalcea. 2012. Learning sentiment lexicons in spanish. In LREC.

Sascha Rothe, Sebastian Ebert, and Hinrich Schütze. 2016. Ultradense word embeddings by orthogonal transformation. In Proceedings of the 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies.

Lutfi Kerem Senel, Ihsan Ulu, Veyes Yuceso, Aykut Koç, and Tolga Cukur. 2018. Semantic structure and interpretability of word embeddings. IEEE/ACM Transactions on Audio, Speech, and Language Processing.

Jamin Shin, Andrea Madotto, and Pascale Fung. 2018. Interpreting word embeddings with eigenvector analysis.

Anant Subramanian, Danish Pruthi, Harsh Jhamtani, Taylor Berg-Kirkpatrick, and Eduard Hovy. 2018. Spine: Sparse interpretable neural embeddings. In Thirty-Second AAAI Conference on Artificial Intelligence.

Kateřina Veselovská and Ondřej Bojar. 2013. Czech sublex 1.0.

Ulli Waltinger. 2010. Germanpolarityclues: A lexical resource for german sentiment analysis. In LREC.
Theresa Wilson, Janyce Wiebe, and Paul Hoffmann. 2005. Recognizing contextual polarity in phrase-level sentiment analysis. In Proceedings of Human Language Technology Conference and Conference on Empirical Methods in Natural Language Processing.

Jieyu Zhao, Yichao Zhou, Zeyu Li, Wei Wang, and Kai-Wei Chang. 2018. Learning gender-neutral word embeddings. arXiv preprint arXiv:1809.01496.
7 Appendix

7.1 Continuous Lexicon

In case of a continuous lexicon \( l : V \rightarrow \mathbb{R} \) one can extend Eq. 2 by defining:

\[
A := \sum_{(v,w) \in V \times V} -l(v)l(w) d_{vw} d_{vw}^T
\]

In the case of a binary lexicon Eq. 2 is recovered for \( \alpha \neq \alpha_\pm = 1 \). We found to gain slight performance increases when using the continuous vs. the binary version for continuous lexicons.

7.2 Lexicon Induction Resources

For lexicon induction we use the same setup as in (Rothe et al., 2016). See Table 3.

Table 3: Overview on resources for lexicon induction. The setup is identical to (Rothe et al., 2016).

| Embeddings | Description | Citation |
|------------|-------------|----------|
| CZ         | CZ          | (Rothe et al., 2016) |
| DE         | DE          | (Rothe et al., 2016) |
| ES         | ES          | (Rothe et al., 2016) |
| FR         | FR          | (Rasooli et al., 2015) |
| EN         | GoogleNews Embeddings | (Mikolov et al., 2013) |
| EN(c)      | Twitter Embeddings | (Rothe et al., 2016) |

7.3 Full Analogy Results

In this section we present the detailed results of the word analogy task per category. For each embedding space - analogy dataset combination we present one individual table. The format and numbers presented are equivalent to the corresponding table in the main paper. See Table 4.

7.4 Gender Debiasing

Word embeddings are well-known for encoding prevalent biases and stereotypes (cf. (Bolukbasi et al., 2016)). We now show that by identifying an interpretable gender dimension and subsequently removing this dimension from the word space, one can remove large parts of gender information that would have caused biased processing. We do not necessarily achieve a debiased word space, but a gender-free word space. Using the notation from the main paper we denote by \( E \) the original word space and consider the interpretable space \( E' := EQ \), where \( Q \) is computed using DensRay. The gender-free word space by removing the interpretable dimension is then \( E_{-1} \in \mathbb{R}^{n \times (d-1)} \). Obviously one can debias the embeddings even more by removing more dimensions.

To examine this approach qualitatively we use a list of occupation names by (Bolukbasi et al., 2016) and examine the cosine similarities with the vectors of “man” and “woman”. Figure 2 provides an overview on the similarities in \( E \) and \( E_{-1} \). Table 5 further lists the occupations that exhibit the greatest bias.

Table 5: We show cosine similarities of occupation vectors with the vector of man and woman respectively. The table lists top 5 occupation that exhibit the greatest bias (measured by difference in cosine similarity). One can clearly see a debiasing effect.

Table 5: We show cosine similarities of occupation vectors with the vector of man and woman respectively. The table lists top 5 occupation that exhibit the greatest bias (measured by difference in cosine similarity). One can clearly see a debiasing effect.

9 https://github.com/tolga-b/debiaswe/blob/master/data/professions.json
## Table 4: Detailed results for all combinations of FastText/Google News embeddings and Google Analogy and BATS analogies.

| FastText | Google News |
|----------|-------------|
| **Mean Cosine Sim** | **Precision** |
| **ClaCos** | **LRCos** | **Interclass** | **Intraclass** | **IntraR** | **Interclass** | **Intraclass** | **IntraR** |
| **Lexicographic** | 0.62 0.37 0.80 | 0.24 0.26 0.15 | 0.45 0.17 0.18 | 0.22 0.20 0.16 | 0.62 0.37 0.80 | 0.24 0.26 0.15 | 0.45 0.17 0.18 | 0.22 0.20 0.16 |
| **Infinitival** | 0.75 0.48 0.55 | 0.95 0.97 0.95 | 0.81 0.83 0.80 | 0.75 0.48 0.55 | 0.95 0.97 0.95 | 0.81 0.83 0.80 | 0.75 0.48 0.55 | 0.95 0.97 0.95 |
| **Infinitival** | 0.75 0.48 0.55 | 0.95 0.97 0.95 | 0.81 0.83 0.80 | 0.75 0.48 0.55 | 0.95 0.97 0.95 | 0.81 0.83 0.80 | 0.75 0.48 0.55 | 0.95 0.97 0.95 |

| **Mean Cosine Sim** | **Precision** |
| **ClaCos** | **LRCos** | **Interclass** | **Intraclass** | **IntraR** | **Interclass** | **Intraclass** | **IntraR** |
| **Lexicographic** | 0.54 0.36 0.84 | 0.23 0.21 0.09 | 0.45 0.17 0.18 | 0.22 0.20 0.16 | 0.54 0.36 0.84 | 0.23 0.21 0.09 | 0.45 0.17 0.18 | 0.22 0.20 0.16 |
Figure 2: Similarities of occupation vectors with the vectors of man and woman. Top shows the original word space and bottom the word space with removed gender dimension. While there is some bias left, one can clearly see a debiasing effect.