Estimating The Mean Linewidth And Lifetime of Solar-like Oscillations of Stars

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ABSTRACT

Scaling formulas were deduced to describe the relations between the fundamental stellar parameters and the mean linewidth and lifetime of solar-like oscillations of stars. The mean linewidth and lifetime of solar-like oscillations are dependent on the large frequency separation, the effective temperature, and the acoustic impedance ($\rho c$) in the photosphere of stars. The mean lifetime can be approximate to the lifetime of the mode with $\nu \sim \nu_{\text{max}}$. We compared the results of the scaling relations with the mean lifetimes of solar-like oscillations of stars observed by Kepler and CoRoT, which shows that the observed mean lifetimes are reproduced well by the scaling relations. The dependence of the mean lifetime on the large frequency separation, the effective temperature, and the acoustic impedance of stars indicates that lifetimes of solar-like oscillations rely on the mass and evolutionary phase of stars. Moreover, our calculations show that the mean lifetimes of $p$-modes of stars can be affected by metallicity abundances.

Subject headings: asteroseismology — stars: fundamental parameters — stars: oscillations (including pulsations).

1. INTRODUCTION

By matching the oscillation frequencies of models with the observed ones, asteroseismology is used to determine the fundamental parameters of stars, such as mass, radius, and age, and diagnose internal structures of stars (Roxburgh & Vorontsov 2003, Christensen-Dalsgaard & Houdek 2010; Deheuvels et al. 2010; Chaplin et al. 2014; Gueuther et al. 2014; Liu et al. 2014; Yang et al. 2015; Guggenberger et al. 2016; Rodrigues et al. 2017; Silva Aguirre et al. 2017). Asteroseismology plays an irreplaceable role in studying the structure and evolution of stars. The measurable characteristics of solar-like oscillations of stars mainly include individual frequencies $\nu_{n,l}$, the large frequency separation $\Delta \nu$, the frequency of maximum power $\nu_{\text{max}}$, and so on.

Individual frequencies are widely used in asteroseismology. In order to determine the fundamental parameters of stars, the observed frequencies $\nu_{n,l}$ are directly compared with those calculated from models by the chi-squared method or the maximum likelihood function method. By calculating the ratios of the small separations to the large separations, $r_{01}$ and $r_{10}$, from individual frequencies, individual frequencies are also used to extract the information about the internal structure of stars (Roxburgh & Vorontsov 2003, 2007; Cunha & Metcalfe 2007; Deheuvels et al. 2010; Liu et al. 2014; Yang 2016).

The large frequency separation, $\Delta \nu$, and the frequency of maximum power, $\nu_{\text{max}}$, are considered to be more easily-measured than individual frequencies. The value of $\Delta \nu$ is proportional to the mean density of stars (Ulrich 1986). The value of $\nu_{\text{max}}$ scales with the acoustic cutoff frequency ($\nu_{\text{ac}}$) (Brown et al. 1991). Basing on these results, Kjeldsen & Bedding (1995) obtained the well-known scaling relations:

$$\Delta \nu = (M/M_\odot)^{1/2}(R/R_\odot)^{-3/2}\Delta \nu_\odot, \quad (1)$$

$$\nu_{\text{max}} = (M/M_\odot)^{1/2}(R/R_\odot)^{-3/2}\nu_{\text{ac}}_\odot. \quad (2)$$
The lifetime \( \tau \) is considered to be related to the linewidth \( \Gamma \) by \( \Gamma = 1/(\pi \tau) \). Chaplin et al. (2009) is the first to try to find a simple scaling relation to describe the average lifetime, \( \langle \tau \rangle \), of solar-like oscillations and obtain \( \langle \tau \rangle \propto T_{\text{eff}}^{-4} \). Hereafter, we throw out the average symbol \( \langle \cdot \rangle \). Baudin et al. (2011) obtain \( \tau \propto T_{\text{eff}}^{-s} \), where the value of the \( s \), however, is 14 \( \pm \) 8 for main-sequence (MS) stars and \(-0.3 \pm 0.9 \) for red giants. The difference between the result of Chaplin and that of Baudin is very significant. The lifetime is potentially an important diagnostic of near-surface convection in stars, affects the detectability of modes, and aids in better understanding the excitation and extinction mechanisms of modes, so that the lifetime predictions play an important role in asteroseismology (Chaplin et al. 2009).

In this work, we give some simple scaling relations to describe the average linewidth \( \Gamma \) and lifetime \( \tau \) of solar-like oscillations. The paper is organized as follows: in Section 2, we deduce the scaling relations and compare the results of the scaling relations with observations, then discussion and summary are given in Section 3.

2. SCALING RELATIONS OF MEAN LINEWIDTH AND LIFETIME OF SOLAR-LIKE OSCILLATIONS

2.1. Scaling Relation of Mean Linewidth of Oscillations

The energy transported per unit time across a spherical surface by propagating acoustic waves with angular frequency \( \omega \) is

\[
F = 4\pi r^2 \frac{1}{2} \rho |v|^2 c,
\]

where \( v \) is the velocity of oscillations, \( c \) the adiabatic sound speed, \( \rho \) the density, \( r \) the radius.

The total energy of the oscillations in a star is (Balmforth & Gough 1990)

\[
E_{\text{osc}} = 2 F \int_0^R r^2 \rho |v|^2 \frac{2\pi}{4} dr
= 2 F \int_0^R r^2 \frac{c}{\rho} dr = F \Delta \nu^{-1}.
\]

The growth rate \( \omega_i \) of amplitude of an oscillation, which is the imaginary part of \( \omega(\omega = \omega_r + i\omega_i) \), is related to the work integral \( W \), i.e.

\[
\omega_i = \frac{1}{2} \frac{W/E_{\text{osc}}}{\Pi},
\]

where the period \( \Pi = 2\pi \omega_r^{-1} \), and the work integral \( W \) is defined as (see equations 25.16, 25.17 and 25.18 of Unno et al. 1989 for more details of \( W \) and \( E_{\text{osc}} \))

\[
W = \int dt \frac{dE_{\text{osc}}}{dt}.
\]

Therefore, we obtain

\[
\omega_i = -\frac{\omega_r}{4\pi} \frac{W \Delta \nu}{F}.
\]

The work integral \( W \) can be decomposed into \( W_N \) and \( W_E \), which are related to the perturbations of nuclear energy generation rate and energy transfer, respectively. Let us neglect \( W_N \). We only consider radial oscillations in this work. The work integral \( W_E \) is given as (see Equations 26.3 and 26.4 of Unno et al. 1989, and Equation 3 of Goldreich & Kumar 1991)

\[
W_E = \frac{n}{\omega_r} \int_0^R dr (-\frac{\partial L}{\partial r}) \frac{\Delta T}{T},
\]

where \( L \) is luminosity and \( T \) temperature of a star. Let us estimate the order of magnitude of the integral of Equation (8) from adiabatic oscillations, For the adiabatic radial oscillations, Goldreich & Kumar (1991) gave

\[
\frac{\Delta T}{T} = -\frac{\Gamma_3 - 1}{\Gamma_3 - 1} \frac{\partial \xi}{\partial r} \sim -\frac{\Gamma_3 - 1}{\Gamma_3 - 1} \frac{\omega^2 \xi}{g},
\]

where \( \xi \) is radial displacement eigenfunction, \( g \) gravity, and

\[
\Gamma_3 - 1 = \frac{\partial \ln T}{\partial \ln \rho} |_s.
\]
Neglecting the change in \((\Gamma_3 - 1)\), Goldreich & Kumar (1991) gave
\[
\frac{\Delta T}{T} \sim -\frac{\omega^2 \xi}{g}, \tag{11}
\]
and
\[
\frac{\partial \Delta L}{\partial r} \sim -\frac{\Delta L}{H} \sim -\frac{L\omega^2 \xi}{gH}, \tag{12}
\]
where \(H\) is the local pressure scale height. Then Goldreich & Kumar (1991) obtained
\[
\int_0^R dr \left( -\frac{\partial \Delta L}{\partial r} \frac{\Delta T}{T} \right) \sim -\frac{L\omega^4 \xi^2}{g^2}. \tag{13}
\]

Therefore, we can obtain
\[
W_T \sim -\frac{\pi}{\omega_r} \frac{L\omega^4 \xi^2}{g^2}. \tag{14}
\]

Substituting \(W\) in Equation (7) by \(W_T\), we obtain that the thermal damping rate can be given as
\[
\omega_i \sim \frac{1}{4} \frac{\Delta \nu \omega \xi^2}{\pi \rho c g^2} \Delta \nu T^4, \tag{15}
\]
where \(\sigma\) is Stefan constant. For turbulent stresses, the work integral is given as (Goldreich & Kumar 1991)
\[
W_T = \frac{\pi}{\omega_r} 4\pi \omega^2 \int_0^R dr r^2 \nu_H \left( \frac{\partial \xi}{\partial r} \right)^2 \sim \frac{\pi}{\omega_r} \frac{L\omega^4 \xi^2}{g^2}, \tag{16}
\]
where \(\nu_H\) is turbulent viscosity. Therefore, mechanical damping rate also has the expression of Equation (15).

Solar-like oscillations are considered to be stable. The value of \(F\) can be estimated as
\[
F \sim 4\pi \omega^2 \frac{1}{2} \rho c \omega^2 \xi^2. \tag{17}
\]

Substituting \(F\) in Equation (15) by Equation (17), we obtain
\[
\omega_i \approx \frac{\sigma}{2pc} \frac{\omega^2}{g^2} \Delta \nu T^4, \tag{18}
\]
where \(\rho\), \(c\), \(g\), and \(T\) take their values at \(r = R\).

The linewidth is considered to be related with the damping rate by \(\Gamma_\omega = \omega_i/2\pi\), i.e.
\[
\Gamma_\omega \approx \frac{\sigma}{4\pi \rho c g^2} \frac{\omega^2}{\Delta \nu T^4}. \tag{19}
\]

Due to the fact that frequencies \(\nu_{n,l}\) have an approximately equal \(\Delta \nu\), the mean linewidth \(\Gamma\) is approximate to \(\Gamma_{\omega_{\max}}\), i.e.
\[
\Gamma \approx \Gamma_{\omega_{\max}} \approx \frac{\sigma}{4\pi \rho c g^2} \frac{\omega_{\max}^2}{\Delta \nu T^4}. \tag{20}
\]

The mean linewidth of the solar \(p\)-modes is about \(0.95 \pm 0.08\) \(\mu\)Hz (Baudin et al. 2011) or around \(1.15 \pm 0.07\) \(\mu\)Hz (Chaplin et al. 2009). The value of \(\Gamma\) of Equation (20) is about \(2.83\) \(\mu\)Hz at \(\nu = 3090\) \(\mu\)Hz for the Sun. This indicates that Equations (19) and (20) overestimate the linewidth of the Sun by about 3 times. Equation (19) divided by 3, we obtained that the linewidths of solar-like oscillations can be estimated as
\[
\Gamma_\omega \approx \frac{\sigma}{12\pi \rho c g^2} \frac{\omega^2}{\Delta \nu T^4}. \tag{21}
\]

Figure 1 shows the \(\Gamma_\omega\) of the Sun as a function of frequency calculated by using Equation (21), which indicates that the order of magnitude of linewidths of \(p\)-modes with \(\nu \sim \nu_{\max}\) can be properly estimated by Equation (21).

Equation (11) neglects the effect of \((\Gamma_3 - 1)\). Calculations show that the value of \((\Gamma_3 - 1)\) is less than 1 at \(r = R\). The more massive the star, the closer to 1 the value of \((\Gamma_3 - 1)_R\). The lower the metallicity, the larger the value of \((\Gamma_3 - 1)_R\). The value of \((\Gamma_3 - 1)_R\) of MS models with \(M \lesssim 1.1 M_\odot\) is approximate to that of the Sun. Therefore, linewidths of oscillations of these stars can be estimated by Equation (21). But the value of \((\Gamma_3 - 1)_R\) of MS models with masses between 1.1 and 1.5 \(M_\odot\) is about 1–5 times as large as that of the Sun, which is dependent on the mass, chemical compositions, and evolutionary stage of stars. Thus linewidths of oscillations of these stars can be estimated by Equations (19) and (20). For these stars, Equation (19) can be rewritten as
\[
\Gamma_\omega \approx f \frac{\sigma}{12\pi \rho c g^2} \frac{\omega^2}{\Delta \nu T^4}, \tag{22}
\]
where the value of \(f\) is mainly in the range of 1–5. For the models with masses between 1.2 and 1.5 \(M_\odot\), the value of \((\Gamma_3 - 1)_R\) is
mainly around 3 times greater than that of the Sun. As a consequence, the mean value of \( f \) is around 3. Equations (11), (12), and (17) only give an estimation of the order of magnitude. Thus the value of \( f \) is not only affected by \((\Gamma_3 - 1)_R\). The value of \( f \) can be determined from detailed asteroseismic analyses of some F-type stars with solar-like oscillations.

Scale from the solar value to estimate the mean linewidth and use

\[
\omega_{\text{max}} = \frac{g/g_\odot}{\sqrt{T_{\text{eff}}/5777}} \omega_{\text{max},\odot},
\]

(23)

one can obtain

\[
\begin{align*}
\Gamma &\approx \frac{(pc)_\odot \Delta \nu}{\rho c} (\frac{T_\odot}{5777})^3 \Gamma_\odot, & \text{for } M \lesssim 1.1M_\odot; \\
\Gamma &\approx f(\frac{(pc)_\odot \Delta \nu}{\rho c} \frac{T_\odot}{5777})^3 \Gamma_\odot, & \text{for } M \gtrsim 1.2M_\odot.
\end{align*}
\]

(24)

where the value of \( \Delta \nu_\odot \) is 134.6 \( \mu \)Hz, and \((pc)_\odot \approx 0.12 \text{ g cm}^{-2} \text{ s}^{-1} \). The value of \( \Gamma_\odot \) is about 1.15 \( \mu \)Hz (Chaplin et al. 2009) or 0.95 \( \pm \) 0.08 \( \mu \)Hz (Baudin et al. 2011). The value of \( f \) is mainly in the range of 1 – 5, which is dependent on the mass and evolutionary stage of stars. Its mean value is around 3.

2.2. Scaling Relation of Mean Lifetime of Oscillations

The lifetime of a mode is considered to be relevant to the linewidth of the mode by \( \tau = 1/(\pi \Gamma_\nu) \). The average lifetime, \( \tau \), of low-degree \( p \)-modes of the Sun given by Chaplin et al. (2009) is 3.2 \pm 0.2 days, which is close to that calculated from the \( \Gamma_\nu \) of the mode with \( \nu \approx \nu_{\text{max}} \). Therefore, we obtain that the mean lifetime of \( p \)-modes can be estimated as

\[
\tau \approx \tau_{\nu_{\text{max}}} \simeq \left\{ \begin{array}{ll}
\frac{12pc}{\sigma} \omega_{\text{max}}^2 \Delta \nu^{-1} T^{-4}, & \text{for } M \lesssim 1.1M_\odot; \\
\frac{12pc}{f \sigma} \omega_{\text{max}} \Delta \nu^{-1} T^{-4}, & \text{for } M \gtrsim 1.2M_\odot.
\end{array}\right.
\]

(25)

In order to estimate the mean lifetime of \( p \)-modes of stars, we scale from the solar value to calculate the mean lifetime:

\[
\tau \approx \left\{ \begin{array}{ll}
\frac{(pc)_\odot}{(pc)_\odot} \frac{\Delta \nu}{\Delta \nu_\odot} (\frac{T_\odot}{T_{\text{eff}}})^3 \tau_\odot, & \text{for } M \lesssim 1.1M_\odot; \\
\frac{f(\rho c)_\odot}{\rho c} \frac{\Delta \nu}{\Delta \nu_\odot} (\frac{T_\odot}{T_{\text{eff}}})^3 \tau_\odot, & \text{for } M \gtrsim 1.2M_\odot.
\end{array}\right.
\]

(26)

where the value of \( \tau_\odot \) is about 3.2 days (Chaplin et al. 2009) or 3.8 days (Baudin et al. 2011). This relation shows that the mean lifetime of solar-like oscillations increases with a decrease in \( \Delta \nu \) and \( T_{\text{eff}} \).

For MS stars and red giants, the value of \( \rho(R) \) generally decreases with an increase in \( R \). The increase in \( M \) can result in an increase in radius, i.e. can lead to a decrease in \( \rho(R) \). Therefore, the acoustic impedance \( \rho c \) generally decreases with an increase in mass and radius of stars. But for the same type stars, they are expected to have an approximate \( \rho c \). The parameter \( f \) in Equation (26) can be roughly replaced by the mean value 3. As a consequence, for stars extracted \( \Delta \nu \) and \( T_{\text{eff}} \), their \( \tau \) can be estimated from Equation (26).

For theoretical calculations, the value of \( \rho c \) can be obtained from stellar models. But for observations, the value of \( \rho c \) of stars is hard to be obtained. The density \( \rho(R) \) and sound speed \( c \) decrease with mass and age of stars, i.e. decrease with an increase in mass and radius. Assume \( \rho c \propto 1/(MR) \), Equation (26) can be rewritten as

\[
\tau \approx f_2 \frac{\Delta \nu_\odot (5777K)^3 M_\odot R_\odot}{MR \tau_\odot},
\]

(27)

where \( f_2 \) is a free nondimensional parameter. It is more convenient to estimate \( \tau \) of stars by using Equation (27).

If the mass of a star is unknown, Equation (27) is also hard to be used in observations. The masses of stars with solar-like oscillations are mainly between about 1 and 1.5 \( M_\odot \). The change of \( R \) in Equation (27) can be much larger than that of \( M \). Therefore, for stars whose radius is determined, their \( \tau \) can be approximately estimated as

\[
\tau \approx f_3 \frac{\Delta \nu_\odot (5777K)^3 M_\odot}{R \tau_\odot},
\]

(28)

where \( f_3 \) is a free parameter. Our calculations show that the value of \( f_3 \) is 1 for MS stars with \( M \lesssim 1.1M_\odot \) and red giants (\( \Delta \nu \lesssim 40 \mu \text{Hz} \)) and 1/3 for MS stars with \( M \gtrsim 1.2 \ M_\odot \). For stars observed \( R, \Delta \nu, \) and \( T_{\text{eff}} \), it is convenient to estimate \( R \) or \( \Gamma \) by using Equation (28). But Equation (28) is an approximation of Equation (27).
2.3. Comparison with Observations

Lund et al. (2017) determined the values of FWHM ($\Gamma_\alpha$) at $t_{\text{max}}$ of 66 stars (LEGACY Sample) from the observations of Kepler. Moreover, Hekker et al. (2010) studied in detail the lifetimes of $p$-modes of four red giants from the observations of CoRoT and gave the values of radii of the four red giants. The values of $\tau$ of LEGACY Sample and Hekker et al. (2010) sample are obtained by $\tau = 1/(\pi \Gamma)$. In addition, Chaplin et al. (2009) and Bandin et al. (2011) also gave $\tau$ or $\Gamma$ of some stars. Thanks to these works, we have a large enough sample to test Equations (26), (27), and (28).

The observed $\tau$ of the sample is shown in panel a of Figure 2. The results of Equation (26) are also shown in the panel. Due to the fact that the values of $\rho c/(\rho c)_\odot$ of the stars are unknown, we assumed that the value of $\rho c/(\rho c)_\odot$ is a constant in the calculations. The sample can be divided into three subsamples: one has $\tau > 2$ days and $\Delta \nu > 50 \mu$Hz, labeled as ‘low-mass’ sample; one has $\tau < 2$ days, labeled as ‘more massive’ sample; four red giants of Hekker et al. (2010) are labeled as ‘red’ sample.

Panel b of Figure 2 shows the mean lifetimes of the ‘low-mass’ sample. Part of the sample is reproduced by Equation (26), with $\rho c/(\rho c)_\odot = 1$; but part cannot be reproduced correctly. This can be due to the assumption of $\rho c/(\rho c)_\odot = 1$. For stars with mass less than $1 M_\odot$, their $\rho c/(\rho c)_\odot$ may be larger than 1; but for stars with mass larger than $1 M_\odot$, their $\rho c/(\rho c)_\odot$ may be less than 1.

Panel c of Figure 2 shows the mean lifetimes of the ‘more massive’ sample. Most of the sample are reproduced well by Equation (26) with $\rho c/[3(\rho c)_\odot] = 1/6$, which indicates that the acoustic impedance ($\rho c$) of these stars is lower than that of the Sun. Because the more massive the stars, the smaller the $\rho c$, masses of these stars may be larger than $1 M_\odot$ and that the difference in the masses may not be significant, especially for the F-like stars.

Panel d of Figure 2 shows the mean lifetimes of ‘red’ sample of Hekker et al. (2010). In order to reproduce the sample, the value of $\rho c/[3(\rho c)_\odot]$ decreases from $1/6$ for ‘more massive’ sample to $1/15$. However, the results are unsatisfied. This may be due to neglecting the variation in $\rho c$ of red giants. The variation in radii of red giants is significant, which leads to the fact that there is a significant difference in $\rho c$ of red giants. This hints to us that Equation (28) could be more suitable for red giants. Figure 3 shows that the values of $\tau$ of the red giants are reproduced well by Equation (28) with $f_3 = 1$. Three of the four are reproduced within a standard error. This indicates that scaling relations (26) and (28) work well.

2.4. Mean Lifetimes Calculated from Stellar Models

Equation (26) provides us with an opportunity to fastly calculate the mean lifetimes of $p$-modes from stellar models. By using the Yale Rotation Evolution Code (Pinsonneault et al. 1989; Yang & Bi 2007, YREC) in its nonrotation configuration, we calculated a serial stellar models with different masses and metallicities. Mean lifetimes of oscillations of the models were calculated by using Equation (26).

Calculated results are represented in Figures 4 and 5. The mean lifetimes of oscillations of F-like stars of LEGACY sample can be reproduced well by those of MS and MS-turnoff models with masses between about 1.2 and 1.5 $M_\odot$. But the masses of Simple stars of the sample change from about 1.0 to 1.2 $M_\odot$. The F-like stars are more massive, thus they have a higher effective temperature. The more massive the stars, the smaller the $\tau$. The higher the metallicity, the larger the $\tau$ of MS stars. This is because the higher the metallicity, the lower the luminosity of a star, and the lower the effective temperature. For MS models with a given mass and radius, they have a given $\Delta \nu$; the higher the metallicity, the larger the value of $\rho c$, and the lower the effective temperature. As a consequence, the higher the metallicity, the larger the $\tau$.

Figures 4 and 5 show that post-MS models with $M \lesssim 1.1$ have a $\tau$ larger than about 4 days between $\Delta \nu = 80$ and $\Delta \nu = 60 \mu$Hz. These stars could not have evolved into the post-MS stage. Thus we could hardly find the stars with $\tau > 4$ days and $\Delta \nu$ between 80 and 60 $\mu$Hz. The lower the mass, the larger the $\tau$; the higher the metallicity, the larger the $\tau$. This explains why the Simple stars with $\tau \sim 5$ days have a higher metallicity and a lower
effective temperature (see panels a and b of Figure 4). Moreover, Figures 4 and 5 show that the effect of metallicity on lifetimes is significant for MS stars. Thus determining metallicity of solar-like stars, especially for the ‘low-mass’ type stars, aids us in understanding their oscillations.

Figure 4 compares the values of \( \tau \) of \( \text{Hekker et al. (2010)} \) sample and those calculated from models by using Equation (26) with \( \tau_0 = 3.8 \) days. The mean lifetimes of \( \text{Hekker et al. (2010)} \) sample can be reproduced by models with masses between about 1.2 and 1.5 \( M_\odot \).

The mean lifetimes of oscillations calculated by using Equation (27) are shown in Figure 7. Compared with the results shown in Figure 4, Equation (27) seems to underestimate the values of \( \tau \) of models with \( Z = 0.03 \) and 0.02. This indicates that the acoustic impedance is dependent on metallicity. Moreover, in the calculations for Figure 4 we adopted \( f = 3 \), which may overestimate the values of \( \tau \) for more massive stars. For these stars, the value of \( f \) might be larger than 3 and should be determined from detailed asteroseismic analyses.

For red giants with a given mass and radius, the higher the metallicity, the lower the effective temperature and \( \rho c \). Therefore, compared with Equation (26), Equation (27) overestimates \( \tau \) of stars with a higher metallicity (see Figures 6 and 8).

Figures 9 and 10 represent the results calculated by using Equation (28), which shows that Equation (28) is a good approximation of Equation (27). However, Equation (28) slightly overestimates the values of \( \tau \) of stars with \( M \gtrsim 1.2 \ M_\odot \). Moreover, it slightly overestimates the \( \tau \) of red giants with \( Z = 0.03 \).

3. DISCUSSION AND SUMMARY

3.1. Discussion

For some stars whose \( \Delta \nu \), \( T_\text{eff} \), and \( \tau \) have been determined, the acoustic impedance \( \rho c \) at the surface of stars can be determined by Equation (26). Thus determining the linewidths of modes with \( \nu \sim \nu_{\text{max}} \) play an important role in asteroseismology.

In the calculations of the results shown in Figure 4, we adopted the mean value of \( f \). This could overestimate the values of \( \tau \) of more massive stars and lead to a difference between the theoretical mean lifetimes and the observed ones. The dependence of \( f \) and \( f_2 \) on mass could be obtained from detailed asteroseismic analyses of some F-type stars with solar-like oscillations.

The metallicity of a star can affect the energy transfer in the star. Thus the work integral of Equation (8) can be affected by metallicity, i.e. \( \Gamma \) or \( \tau \) of modes can be affected by metallicity. The mean lifetime of CoRoT 102767771 in \( \text{Hekker et al. (2010)} \) sample is overestimated by Equation (28) (see Figure 3). This could be related to the effect of metallicity. Compared with the results of Equation (26), calculations show that Equation (28) actually overestimates the values of \( \tau \) of red giants with \( Z_1 = 0.03 \) (see Figure 10). The change in metallicity for a star with a given mass can lead to a variation in the effective temperature and radius of the star or a variation in \( \rho c \). This effect has been considered by the scaling relations. Therefore, the effect of metallicity cannot significantly change the results of Equation (26).

The mean lifetime of oscillations of a star is dependent on the acoustic impedance. However, the acoustic impedance is hard to be estimated in observation. The value of \( \rho c/(\rho c)_\odot \) is 1/2 for ‘more massive’ sample and 1/5 for ‘red’ sample. This indicates that the value of the acoustic impedance of stars decreases with mass and radius. Thus we can assume \( \rho c \propto 1/(MR) \). The calculations show that the effect of \( \rho c \) on \( \tau \) can be approximated to the effect of 1/(\( MR \)). Parameters \( f_2 \) and \( f_3 \) derive from the assumption of \( \rho c \propto 1/(MR) \) and the dependence of \( f \) on the mass of stars. The calculations show that the values of \( f_2 \) and \( f_3 \) are about 1 for MS stars with \( M \lesssim 1.1 \ M_\odot \) but are about 1/3 for MS stars with \( M \gtrsim 1.2 \ M_\odot \). This can be partly due to the fact that the value of \( (\Gamma_3 - 1)_R \) of MS stars with \( M \gtrsim 1.2 \ M_\odot \) is about 3 times greater than that of the Sun. The values of \( f_2 \) and \( f_3 \) are 1 for red giants. In red-giant phase, the radius of a star with \( M = 1.2 \ M_\odot \)
can increase by more than 10 times. The rapid change in radius of red giants is the main factor affecting the $\tau$ of red giants.

Figures 4 and 5 show that there are three stars whose $\tau$ is hard to be explained by stellar models. The values of their $\Delta \nu$ are larger than $\sim 150 \mu$Hz, which indicates that their masses might be less than $1 M_\odot$, i.e. the values of their $\rho c$ could be larger than that of the Sun. But their $\tau$ can be well reproduced by Equation (26) with $\rho c/[3(\rho c)\odot] = 1/6$ (see panel c of Figure 2).

3.2. Summary

In this work, basing on the works of Balmforth & Gough (1990) and Goldreich & Kumar (1991) and the definition of Unno et al. (1989), we deduced a formula to describe the linewidth of the mode with $\nu \sim \nu_{\text{max}}$. Due to the fact that the frequencies of $p$-modes have an approximately equal separation, the mean linewidth is approximate to $\Gamma_{\omega_{\text{max}}}$. By using $\tau = 1/(\pi \Gamma)$, we obtained a scaling relation to describe the average lifetime of solar-like oscillations of stars. The mean linewidth and lifetime are determined by the large frequency separation, the effective temperature, and the acoustic impedance $\rho c$. Furthermore, the dependence of the mean lifetime on the acoustic impedance can be roughly reduced to a dependence on the mass and radius of stars. However, this will introduce a free parameter into the formula of the mean lifetime. The calculations show that the mean lifetimes of $p$-modes of stars decrease with an increase in mass of stars. The mean lifetimes also decrease with a decrease in metallicity for MS stars.

We compared the results of the scaling relations with the observational results of Kepler (Lund et al. 2017) and CoRoT (Hekker et al. 2010). Most of the observational results are well reproduced. This indicates that the scaling relations work well. Our calculations show that the lifetime of modes can be affected by stellar metallicity. The effects of metallicity cannot be fully described by the change in radius and effective temperature. Therefore, the effects of metallicity on $\tau$ could lead to the fact that $\tau$ of some stars deviates from the results of the scaling relations, such as CoRoT 102767771.

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Fig. 1.— The linewidth of radial modes of the Sun as a function of frequency, calculated from GS98M (Yang 2016a) by using Equation (21). The values of $\rho(R)$ and $c(R)$ of GS98M are $1.516 \times 10^{-7}$ g cm$^{-3}$ and $7.908 \times 10^5$ cm s$^{-1}$, respectively.
Fig. 2.— Average lifetimes of $p$-modes as a function of $\Delta \nu$. The red dots represent the observations of Lund et al. (2017), Hekker et al. (2010), Chaplin et al. (2009), and Baudin et al. (2011). The triangles, circles, and squares refer to the results of Equation (26) with $\tau_\odot = 3.2$ days. The value of $f$ is 3 for ‘more massive’ and ‘red’ samples.

Fig. 3.— Mean lifetimes of red giants of Hekker et al. (2010) as a function of $\Delta \nu$. The squares represent the results of Equation (28) with $\tau_\odot = 3.2$ days and $f_3 = 1$. Three of the four are reproduced within a standard error.
Fig. 4.— Comparison between the mean lifetimes of LEGACY sample (Lund et al. 2017) and the mean lifetimes calculated by using Equation (26) with $\tau_\odot = 3.8$ days and $f = 3$. The large circles represent F-like stars of LEGACY sample, while the small circles refer to Simple stars of LEGACY sample. The colorbars of panels a, c, d, and f are proportional to [Fe/H] of LEGACY sample, while those of panels b and e are proportional $T_{\text{eff}}$ of the sample.
Fig. 5.— Same as Figure 4 but for the models with different masses and metallicities. The symbol α is the mixing-length parameter.

Fig. 6.— Comparison between the mean lifetimes of the Hekker et al. (2010) sample and the mean lifetimes computed by using Equation (26) with $\tau_\odot = 3.8$ days and $f = 3$. The colorbar is proportional to $T_{\text{eff}}$ of the sample.
The theoretical mean lifetimes were calculated by using Equation (27) with \( \tau_0 = 3.8 \) days. The value of \( f_2 \) is 1 for models with \( M \leq 1.1 \, M_\odot \) and 1/3 for models with \( M \geq 1.2 \, M_\odot \).

Fig. 7.— Same as Figure 4 but the theoretical mean lifetimes calculated by using Equation (27) with \( \tau_0 = 3.8 \) days. The value of \( f_2 \) is 1 for models with \( M \leq 1.1 \, M_\odot \) and 1/3 for models with \( M \geq 1.2 \, M_\odot \).

Fig. 8.— Same as Figure 7 but for red giants. The theoretical mean lifetimes were calculated by using Equation (27) with \( f_2 = 1 \).
Fig. 9.— Same as Figure 4 but the theoretical mean lifetimes calculated by using Equation (28) with $\tau_\odot = 3.8$ days. The value of $f_3$ is 1 for models with $M \lesssim 1.1 M_\odot$ and $1/3$ for models with $M \gtrsim 1.2 M_\odot$.

Fig. 10.— Same as Figure 9 but for red giants. The theoretical mean lifetimes were calculated by using Equation (28) with $f_3 = 1$. 

