Self-consistent calculation of parton distributions at low normalization point in the chiral quark–soliton model

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Abstract

We calculate the isosinglet unpolarized quark– and antiquark distributions at a low normalization point in the large–$N_c$ limit. The nucleon is described as a self-consistent soliton solution of the effective chiral theory. The ultraviolet cutoff is implemented by a Pauli–Villars subtraction. The quark and antiquark distributions satisfy the momentum as well as the baryon number sum rule, corresponding to “constituent” quarks whose structure is not yet resolved at the low scale. Already at this level a sizable number of antiquarks is present.

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An essential ingredient in the theory of deep–inelastic scattering and other hard processes are the leading–twist parton distribution functions of the nucleon \([1, 2]\). Their evolution in the asymptotic region is governed by perturbative QCD and well understood. However, the initial conditions for the perturbative evolution, \(i.e.,\) the parton distributions at a relatively low normalization point, belong to the field of non-perturbative physics, and at present one has to rely on model calculations to estimate them.

Recently, an approach has been formulated to calculate the leading–twist parton distributions in the large–\(N_c\) limit, where the nucleon can be described as a chiral soliton \([3, 4]\). It was shown that this field–theoretic approach allows to preserve all general requirements on parton distributions such as positivity and the partonic sum rules which hold in QCD. Results for the leading quark– and antiquark distribution function in the \(1/N_c\)-expansion (isosinglet unpolarized and isovector polarized) show reasonable agreement with the parametrizations of the data at a low normalization point \([2]\); the approach has been extended to transverse polarized distributions \([5]\). First calculations of the subleading distributions have also been reported \([6]\).

The description of the nucleon as a chiral soliton is based on the dynamical breaking of chiral symmetry, which can be encoded in an effective theory, valid at low energies. The effective theory can be derived from the instanton vacuum of QCD, in which case the ultraviolet cutoff is determined by the inverse average instanton size, \(\bar{\rho}^{-1} = 600\) MeV, the scale at which the dynamically generated “constituent” quark mass drops to zero \([7]\). In actual calculations the fall–off of the dynamical quark mass is usually simulated by applying some external UV regularization. Hadronic observables, such as the nucleon mass, axial coupling constant \(\text{etc.}\) are, as a rule, insensitive to the details of the UV cutoff. When one turns to the calculation of parton distribution functions, however, there are very strong restrictions on how one should introduce the UV cutoff in the effective theory. The point is that one has to preserve certain general properties of parton distributions such as positivity, sum rules \(\text{etc.}\), which can easily be violated by an arbitrary UV regularization. Specifically, the regularization should preserve the completeness of the set of quark single–particle wave functions in the soliton pion field. One possible regularization method which fulfills all requirements is a Pauli–Villars subtraction \([3, 4]\).

An important aspect which has not been taken into account in the calculation of parton distribution functions so far, related to the constraints on the UV regularization, is the self–consistent description of the chiral soliton. The classical pion field describing the nucleon should be determined as a minimum of the static energy, suitably regularized in a way that ensures consistency with the regularization applied to the parton distributions. The point where the two regularizations meet is the momentum sum rule for the isosinglet
distribution, which is satisfied only if the classical pion field is a solution of the equations of motion of the effective chiral theory \[3\]. The calculations of parton distributions in refs.\[3, 4\] were performed with a particular variant of the Pauli–Villars regularization and using a fixed soliton profile, and the momentum sum rule was satisfied only approximately.

In the present letter we compute the isosinglet unpolarized distributions in the approach formulated in refs.\[3, 4\] with a self–consistent description of the nucleon. In fact, a class of regularizations exist which preserve all general requirements on parton distributions (positivity, sum rules \emph{etc}.) and simultaneously lead to a stable minimum of the energy. With such a regularization the momentum sum rule is explicitly preserved (along with other sum rules). We also investigate the effects of possible finite regularizations on the valence and, particularly, the antiquark distribution.

The calculation of parton distributions from the effective chiral theory relies on the parametric smallness of the ratio of the dynamical quark mass, \(M\), to the UV cutoff, \(\Lambda\). [In the instanton vacuum this ratio, \((M\bar{p})^2\), is proportional to the small packing fraction of instantons \[7\].] The basic expressions for the twist–2 parton distributions in refs.\[3, 4\] were derived in the limit \(M/\Lambda \to 0\). Differences between various UV regularizations are parametrically of order \((M/\Lambda)^2\). Since the existence of a minimum of the energy depends on the details of the UV regularization, \emph{i.e.}, on terms of order \((M/\Lambda)^2\), one does not, from a principal point of view, achieve a higher accuracy of the parton distributions by making the description of the soliton self–consistent. Nevertheless, since the existence of a stable soliton is an important qualitative issue — not only for the momentum sum rule — it makes sense to “improve” the regularization at level \((M/\Lambda)^2\) in order to ensure soliton stability, and this is the point of view we take here.

**Self-consistent soliton with Pauli–Villars regularization.** The classical pion field characterizing the nucleon in the effective chiral theory at large \(N_c\) is of “hedgehog” form \[8\]:

\[
U(x) = \exp \left[ i (n \cdot \tau) P(r) \right] = \cos P(r) + i (n \cdot \tau) \sin P(r)
\]

\[
r = |x|, \quad n = \frac{x}{r}.
\]

The profile function, \(P(r)\), is determined by minimizing the static energy of the pion field,

\[
E_{\text{tot}}[U] = N_c E_{\text{lev}} + N_c \sum_{\text{neg. cont.}} (E_n - E_n^{(0)}),
\]

where \(E_n\) are the quark single–particle energies, given as the eigenvalues of the Dirac Hamiltonian in the background pion field (\(M\) is the dynamical quark mass),

\[
H\Phi_n = E_n \Phi_n, \quad H = -i\gamma^0 \gamma^k \partial_k + M\gamma^0 U^\gamma_5, \quad U^\gamma_5(x) = \frac{1 + \gamma_5}{2} U(x) + \frac{1 - \gamma_5}{2} U^\dagger(x).
\]
The spectrum of $H$ includes a discrete bound–state level, whose energy is denoted by $E_{\text{lev}}$, as well as the positive and negative Dirac continuum, which are polarized by the presence of the pion field. The sum in Eq.(2) runs over all occupied quark single–particle levels in the soliton, that is, the bound–state level and the negative continuum. The $E_n^{(0)}$ denote the energy levels of the vacuum Hamiltonian given by Eq.(3) with $U = 1$.

The contribution of the Dirac continuum to the energy, Eq.(2), is logarithmically divergent and requires UV regularization. We consider here regularizations of the form of a Pauli–Villars subtraction, which can be implemented consistently with the Pauli–Villars regularization of the parton distribution functions below. A possible regularization of Eq.(2) is to subtract from the continuum contribution a suitable multiple of the corresponding sum computed with the quark mass replace by a regulator mass, $M_{PV}$:

$$E_{\text{tot,reg}}[U] = N_c E_{\text{lev}} + N_c \sum_{\text{neg.cont.}} (E_n - E_n^{(0)}) - N_c M_{PV} \sum_{\text{neg.cont.}} (E_{PV,n} - E_{PV,n}^{(0)}),$$  \hspace{1cm} (4)

where $E_{PV,n}$ are the eigenvalues of the Hamiltonian, Eq.(3), with $M$ replaced by $M_{PV}$ and the same pion field. The subtraction coefficient follows from the fact that the logarithmic divergence of the unregularized sum, Eq.(2), is proportional to $M^2$. The value of the regulator mass can be fixed from the (also logarithmically divergent) pion decay constant, $F_\pi = 93 \text{ MeV}$. The regulator mass now plays the role of the UV cutoff of the theory, $\Lambda$.

It was shown in ref.[9] that the energy functional regularized according to Eq.(4) possesses a minimum with respect to the profile function, i.e., that a stable soliton exists. The minimum is determined by the stationarity condition,

$$\frac{\delta E_{\text{tot,reg}}}{\delta P(r)} = -\sin P(r) S(r) + \cos P(r) P(r) = 0,$$  \hspace{1cm} (6)

where

$$S(r) = N_c S_{\text{lev}}(r) + N_c \sum_{\text{neg.cont.}} S_n(r) - N_c \frac{M^2}{M_{PV}^2} \sum_{\text{neg.cont.}} S_{PV,n}(r),$$  \hspace{1cm} (7)

$$P(r) = N_c P_{\text{lev}}(r) + N_c \sum_{\text{neg.cont.}} P_n(r) - N_c \frac{M^2}{M_{PV}^2} \sum_{\text{neg.cont.}} P_{PV,n}(r),$$  \hspace{1cm} (8)

$$S_n(r) = M \int d^3x \, \Phi_n^\dagger(x) \delta(|x| - r) \gamma^0 \Phi_n(x),$$  \hspace{1cm} (9)

$$P_n(r) = M \int d^3x \, \Phi_n^\dagger(x) \delta(|x| - r) \gamma^0 \gamma_5 i(n \cdot \tau) \Phi_n(x),$$  \hspace{1cm} (10)
and similarly for $S_{PV,n}(r), P_{PV,n}(r)$. The profile function can be found by iterative solution of Eq.(3), see ref.[9], where also a number of hadronic nucleon observables have been calculated with this regularization. In the leading order of the $1/N_c$–expansion the nucleon mass is given simply by the value of the energy, Eq.(4), at the minimum; with standard values for the constituent quark mass, $M = 350$ MeV ($M_{PV}^2/M^2 = 2.25$) and $M = 420$ MeV ($M_{PV}^2/M^2 = 1.90$), we find, respectively, $M_N = 1140$ MeV and $M_N = 1040$ MeV.

The regularization of the energy, Eq.(4), is different from the one used in the calculations of refs.[3, 4], where in addition to the Dirac continuum also the contribution of the bound–state level of the Hamiltonian with the regulator mass was subtracted. The latter regularization does not lead to stable solitons. The two regularizations differ by a finite contribution of order $M^2/M_{PV}^2$. From the point of view of computing parton distributions both regularizations belong to the class of physically acceptable ones, see below.

**Isosinglet unpolarized quark distribution function.** The theoretical framework for the calculation of quark distribution functions with the effective chiral theory has been given in refs.[3, 4]. The isosinglet unpolarized distribution appears in the leading order of the $1/N_c$–expansion and can be represented as a sum of contributions of quark single–particle levels in the background pion field:

$$u(x) + d(x) = N_c f_{lev}(x) + N_c \sum_{\text{neg.cont.}} [f_n(x) - f_n^{(0)}(x)],$$

$$f_n(x) = M_N \int \frac{d^3k}{(2\pi)^3} \Phi_n^+(k) (1 + \gamma^0 \gamma^3) \delta(k^3 + E_n - xM_N) \Phi_n(k).$$

This expression has been derived from the QCD representation of the parton distribution as nucleon matrix element of a light–ray (non-local) operator [3]; it is equivalent to the definition of the parton distribution as the number of particles carrying a fraction $x$ of the nucleon momentum in the infinite–momentum frame [4]. The function Eq.(11) describes the quark distribution for positive values of $x$, and minus the antiquark distribution for negative $x$. In Eq.(11) $f_n^{(0)}$ denotes the matrix element Eq.(12) between levels of the vacuum Hamiltonian ($U = 1$), with $f_n^{(0)}(x) = 0$ for $x > 0$, $E_n < 0$ on kinematical grounds.

When computing the isosinglet distribution function in the effective theory we must consider separately the even and odd part of the function defined by Eq.(11), since they exhibit different behavior with respect to the UV cutoff. The valence quark distribution is given by the even part of Eq.(11),

$$u(x) + d(x) - \bar{u}(x) - \bar{d}(x) = N_c[f_{lev}(x) + f_{lev}(-x)]$$

$$+ N_c \sum_{\text{neg.cont.}} [f_n(x) + f_n(-x) - f_n^{(0)}(x) - f_n^{(0)}(-x)].$$
Here the continuum contribution is finite and does not require an UV cutoff. This distribution is normalized by the baryon number sum rule,

$$\int_0^1 dx \left[ u(x) + d(x) - \bar{u}(x) - \bar{d}(x) \right] = N_c,$$

which holds “level by level” in the chiral soliton model, as can be seen by integrating Eq. (12) over \( x \), upon which the \( \gamma^0 \gamma^3 \)-term drops out.

The total distribution of quarks plus antiquarks is given by the odd part of Eq. (11). The continuum contribution to this function is logarithmically divergent for each value of \( x \) and requires regularization. (Again, the divergence is proportional to \( M^2 \).) We regularize it by a Pauli–Villars subtraction, analogous to the energy, Eq. (4):

$$u(x) + d(x) + \bar{u}(x) + \bar{d}(x) = N_c \left[ f^{\text{lev}}_n(x) - f^{\text{lev}}_n(-x) \right]$$

$$+ N_c \sum_{\text{neg. cont.}} \left[ f_n(x) - f_n(-x) - f^{(0)}_n(x) + f^{(0)}_n(-x) \right]$$

$$- N_c \frac{M^2}{M_{PV}^2 \sum_{\text{neg. cont.}}} \left[ f^{PV, n}_{PV, n}(x) - f^{PV, n}_{PV, n}(-x) - f^{(0)}_{PV, n}(x) + f^{(0)}_{PV, n}(-x) \right].$$

Note that, again, this regularization differs from the one used in refs. [3, 4] in that only the Dirac continuum is subtracted here but not the level contribution. We emphasize that such regularization preserves the general properties of parton distributions just as the one used in refs. [3, 4]; in particular, it satisfies all criteria for a “good” regularization established in ref. [4], such as absence of anomalies, uniform logarithmic cutoff dependence, etc.

The total distribution of quarks plus antiquarks regularized according to Eq. (15) satisfies the momentum sum rule, if the classical pion field is a stationary point of the regularized energy, Eq. (4):

$$\int_0^1 dx x \left[ u(x) + d(x) + \bar{u}(x) + \bar{d}(x) \right] = 1.$$

A general proof of the momentum sum rule has been given in ref. [3], which can straightforwardly be carried over to the Pauli–Villars regularized energy, Eq. (4), and distribution function, Eq. (15). Note that the momentum sum rule differs from the baryon number sum rule in that it is a statement only about the sum over all occupied levels but does not hold “level by level”.

Results. We compute the distribution functions Eqs. (13, 15) for the self-consistent soliton profile using the numerical method developed in ref. [4]. For the numerical calculations it is convenient to make use of the fact that with Pauli–Villars regularization

\[\text{We note that for Eq. (16) to be satisfied it is also necessary that } M_N \text{ in Eq. (13) be the minimum value of the static energy, i.e., the nucleon mass in the leading order of the } 1/N_c \text{-expansion quoted above.}\]
one may equivalently compute the distribution function at negative $x$ by summing over non-occupied states, thus avoiding the need of vacuum subtraction:

$$f_{\text{lev}}(-x) + \sum_{\text{neg. cont.}} f_n(-x) \rightarrow - \sum_{\text{pos. cont.}} f_n(x), \quad (17)$$

and similarly for the sum with $M \rightarrow M_{PV}$; see ref. [4] for details.

The distribution functions receive contributions from the bound–state level as well as from the polarized Dirac continuum of quarks. To understand the nature of these contributions it is instructive to consider for a moment the “universal” function, Eq.(11), which describes the quark distribution at positive and minus the antiquark distribution at negative values of $x$. Fig.1 shows the contributions of the bound–state level and the Dirac continuum to this function ($M = 350$ MeV), the latter being regularized by a Pauli–Villars subtraction as in Eq.(13). The distribution resulting from the bound–state level (dashed line) is positive for all values of $x$; hence it makes a negative contribution to the antiquark distribution. The correct sign of the antiquark distribution is obtained by including the contribution of the Dirac continuum (dot–dashed line), which is positive for $x > 0$ and negative for $x < 0$, in agreement with the fact that the logarithmic divergence of the distribution function is an odd function $x$. We note that the so-called valence quark approximation advocated in ref.[10], when applied to parton distribution functions, would violate the positivity of the antiquark distributions.

The total distribution of quarks plus antiquarks, $u(x) + d(x) + \bar{u}(x) + \bar{d}(x)$, is obtained by taking the odd part in $x$ of the function shown in Fig.1, or, equivalently, performing the sum Eq.(15). The resulting distribution is shown in Fig.2 (solid line). It receives a sizable contribution from the Dirac continuum (dot–dashed line). We emphasize that the calculated distribution explicitly satisfies the momentum sum rule, thanks to the self–consistency of the classical pion field.

The valence quark distribution, $u(x) + d(x) - \bar{u}(x) - \bar{d}(x)$, is shown in Fig.3. This distribution one obtains by taking even part of the function shown in Fig.1 and removing the Pauli–Villars regularization, $M_{PV}/M \rightarrow \infty$, or, equivalently, by evaluating Eq.(13). In this distribution the logarithmic divergence of the Dirac continuum cancels; the finite continuum contribution obtained in the limit $M_{PV}/M \rightarrow \infty$ is shown in Fig.3 (dot–dashed line); it is numerically small. The calculated distribution satisfies the baryon number sum rule, Eq.(14). Also shown in Figs.2 and 3 (triangles) are the distributions calculated in ref.[4] with Pauli–Villars subtraction of the level contribution and a fixed soliton profile.

The valence quark distribution receives a non-zero contribution from the Dirac continuum (Fig.3, dot–dashed line). The contribution of the Dirac continuum to the baryon number, Eq.(14), however, is exactly zero. This is in fact necessary for the nucleon
to have baryon number unity, since the baryon number of the bound–state level is already unity. A Pauli–Villars regularization of the Dirac continuum contribution as in \( u(x) + d(x) + \bar{u}(x) + \bar{d}(x) \), Eq.(15), would not violate the baryon number sum rule — the contributions of the true and the Pauli–Villars continuum to the baryon number would be individually zero — and thus constitute an admissible, if “unnecessary”, finite regularization. Since aside from preserving sum rules and other general requirements the UV regularization of the effective theory is fraught with some uncertainty, it is instructive to explore the consequences of such finite regularization on the distribution functions. For the valence distribution, \( u(x) + d(x) - \bar{u}(x) - \bar{d}(x) \), the influence of a regularization of the Dirac continuum is negligible, since this distribution is dominated by the bound–state level contribution, see Fig.3. The antiquark distribution, however, which is defined as the difference of Eq.(15) and Eq.(13), receives a large contribution from the Dirac continuum and is potentially more sensitive to finite regularizations of \( u(x) + d(x) - \bar{u}(x) - \bar{d}(x) \).

In Fig.4 we show the antiquark distribution obtained as the difference between the same distribution of quarks plus antiquarks (Fig.2, \( M = 350 \text{ MeV} \)) and a valence distributions computed with \( i) \) unregularized continuum contribution (solid line) and \( ii) \) continuum Pauli–Villars regularized with \( M_{PV}^2/M^2 = 2.52 \) as in the total distribution (short–dashed line). One sees that the effect of the finite regularization on the antiquark distribution is not dramatic. In particular, the antiquark distribution is positive in both cases. Of course, it is guaranteed to be positive in the limit \( M_{PV}/M \to \infty \), because of the logarithmic divergence of \( u(x) + d(x) + \bar{u}(x) + \bar{d}(x) \). The fact that it is positive also for finite \( M_{PV}/M \) shows that the finite, regularization–dependent terms do not upset this asymptotic statement for the particular regularization used here, as it should be.

Figs.2 and 3 also show the distributions obtained for the soliton with \( M = 420 \text{ MeV} \) (dotted lines). We see that the distributions calculated with the self–consistent soliton profile are rather insensitive to the value of the constituent quark mass. This parallels the behavior of most hadronic observables in this model [11].

The numerical method employed here does not allow for an accurate determination of the limiting behavior of the parton distributions for \( x \to 0 \). The limiting behavior of the Dirac continuum contribution in Fig.1 has been obtained by extrapolation, see ref.[4] for details; for the distributions multiplied by \( x \), Figs.2, 3 and 4, this limitation of the present method is irrelevant. We note also that the “small–x” behavior of the parton distributions in the effective chiral theory, \( \text{i.e.} \), the behavior in the parametrically small region \( |x| \leq (M/\Lambda)^2/N_c \), depends on the details of the UV regularization, as has recently been emphasized in connection with off–forward parton distributions [12].

The parton distributions studied here pertain to the large–\( N_c \) limit, where the nucleon
is heavy, $M_N \propto N_c$. The calculated distributions therefore do not go to zero at $x = 1$, rather, they are exponentially small at large $x$, as discussed in ref. $[3]$. 

**Summary and Conclusions.** We have shown, by a specific example, that the general requirements on the UV regularization in the calculation of parton distributions in the effective chiral theory can be reconciled with the requirements of a stable soliton solution, which depends on $O(M^2/\Lambda^2)$--terms in the energy functional. The calculated quark and antiquark distributions explicitly satisfy the baryon number and momentum sum rules.

The distributions obtained with the self-consistent soliton profile largely support the results of the calculations of refs. $[3, 4]$ using a fixed soliton profile. Except for the momentum sum rule the differences are not significant, given the limited theoretical accuracy of this approach. We note that the regularization used in the present work is only an example of a class of possible regularizations leading to a stable soliton; for instance, one may perform multiple Pauli–Villars subtraction of the Dirac continuum with different masses, etc. However, we see at present no need to explore such possibilities.

The basic expressions for the parton distributions in refs. $[3, 4]$ have been derived in the leading order of $M/\Lambda$, in which the “constituent” quarks of the effective theory can be regarded as pointlike. This is why the momentum sum rule is saturated by the quark and antiquark distributions computed here. If one knew the precise form of the effective theory at order $M^2/\Lambda^2$, one could “resolve” the structure of the constituent quark and recover the distribution of quarks and gluons inside the constituent quark. Such a picture is in fact suggested by the instanton vacuum, where the parameter $M^2/\Lambda^2$ is related to the small packing fraction of the instanton medium $[13, 14]$; its concrete realization is the subject of current investigations. It is not unlikely that the “resolution” of the constituent quark structure could be represented in the form of a convolution in Bjorken–$x$ of the constituent quark distributions with parton distributions inside the constituent quark. This general idea is in fact rather old $[15]$; it has recently been taken up again $[16]$. Since the “constituent” quark and antiquark distributions computed here are correctly normalized to the total nucleon momentum, they can serve as a starting point for investigations assuming a structure inside the constituent quark.

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Figure 1: The isosinglet distribution function, Eq. (11), corresponding to \( u(x) + d(x) \) for \( x > 0 \) and \(-\bar{u}(-x) - \bar{d}(-x)\) for \( x < 0 \), for the self–consistent soliton with \( M = 350 \text{ MeV} \). Dashed line: contribution of the bound–state level, \( f_{\text{lev}}(x) \), giving a negative contribution to the antiquark distribution. Dot–dashed line: contribution of the Pauli–Villars regularized Dirac continuum. Solid line: total result (bound–state level plus Dirac continuum). The total antiquark distribution is positive.
Figure 2: The isosinglet distribution of quarks plus antiquarks, $\frac{1}{2}x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$. Shown are the total result (solid line) and the contribution of the Dirac continuum (dot–dashed line) for the self–consistent soliton with $M = 350$ MeV. Also shown is the total result for the soliton with $M = 420$ MeV (dotted line). Triangles: the distribution calculated in ref.[4] with an analytic soliton profile and regularized level contribution.
Figure 3: The isosinglet valence quark distribution, $\frac{1}{2}x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$. Shown are the total result (solid line), and the contribution of the unregularized Dirac continuum (dot–dashed line) for the self–consistent soliton with $M = 350\text{MeV}$. This distribution is obtained from the symmetric part of the function in Fig.[1] after removing the Pauli–Villars cutoff, $M_{PV}/M \to \infty$. Also shown is the total result for the soliton with $M = 420\text{MeV}$ (dotted line). Triangles: the distribution calculated in ref.[4] with an analytic soliton profile.
Figure 4: The isosinglet antiquark distribution, \(x[\bar{u}(x) + \bar{d}(x)]\), for the self-consistent soliton with \(M = 350\) MeV, obtained as difference between \(u(x) + d(x) + \bar{u}(x) + \bar{d}(x)\), Fig.2, and \(u(x) + d(x) - \bar{u}(x) - \bar{d}(x)\), Fig.3, with different regularizations of the continuum contribution to \(u(x) + d(x) - \bar{u}(x) - \bar{d}(x)\). **Solid line:** unregularized. **Short-dashed line:** regularized with \(M_{PV}^2/M^2 = 2.25\).