Highly anisotropic $g$-factor of two-dimensional hole systems

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Coupling the spin degree of freedom to the anisotropic orbital motion of two-dimensional (2D) hole systems gives rise to a highly anisotropic Zeeman splitting with respect to different orientations of an in-plane magnetic field $B$ relative to the crystal axes. This mechanism has no analogue in the bulk band structure. We obtain good, qualitative agreement between theory and experimental data, taken in GaAs 2D hole systems grown on $(113)$ substrates, showing the anisotropic depopulation of the upper spin subband as a function of in-plane $B$.

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Since the early days of two-dimensional (2D) carrier systems in semiconductors it has been commonly assumed that the Zeeman energy splitting, $\Delta E = g^*\mu_B B$, with $g^*$ the effective $g$-factor and $\mu_B$ the Bohr magneton, is independent of the direction of the external magnetic field $B$. Recently, however, calculations and experiments have shown that $g^*$ can have different values for $B$ applied in the direction normal to the plane of the 2D system compared to in-plane. Here we report calculations and experimental data for 2D holes occupying the heavy hole subband, demonstrating that, even for a purely in-plane $B$, $g^*$ can depend strongly on the orientation of $B$ with respect to the crystal axes. We show that the mechanism giving rise to this remarkable $g$-factor anisotropy is fundamentally different from the one responsible for the in-plane/out of plane anisotropy of $g^*$ discussed in previous work: the latter is basically a consequence of the bulk Zeeman Hamiltonian whereas the in-plane anisotropy we are reporting stems from the combined effect of the bulk $g^*$ and the anisotropic orbital motion of 2D hole states in semiconductor quantum wells.

In bulk semiconductors the motion of electrons and holes in the presence of spin-orbit interaction gives rise to a $g^*$ which is significantly modified compared to the free particle $g$-factor $g_0 = 2$ (Ref. [3]). The resulting $g^*$ of holes (and electrons) is nearly isotropic. Commonly, the isotropic part of the hole $g^*$ is denoted by $\kappa$. The anisotropic part, $q$, is typically two orders of magnitude smaller than $\kappa$ and, in the present discussion, is neglected completely. The smallness of $q$ is in sharp contrast to the orbital motion of holes for which we have highly anisotropic effective masses $m^*$ reflecting the spatial anisotropy of the crystal structure. In a 2D hole system (2DHS) we have heavy hole (HH) subbands ($z$ component of angular momentum $M = \pm 3/2$) and light hole (LH) subbands ($M = \pm 1/2$). In the presence of an in-plane $B$, $\kappa$ couples the two LH states, and the HH states to the LH states $\pm_2$. But there is no direct coupling between the HH states proportional to $\kappa$. Therefore, the authors of Refs. [3–6] concluded that the Zeeman splitting of HH states due to an in-plane $B$ is suppressed. However, this result is correct only for quantum wells (QW’s) grown in the crystallographic high-symmetry directions [001] and [111] as for the other growth directions we show that a new mechanism gives rise to a large and highly anisotropic Zeeman splitting with respect to different orientations of the in-plane magnetic field $B$ relative to the crystal axes. In the following we will discuss QW’s grown in the crystallographic [mnn] direction (with $m, n$ integers). For this purpose we use the coordinate system shown in Fig. [4] with $\theta$ denoting the angle between [mnn] and [001]. We remark that recently QW’s for 2DHS’s have often been grown in [113] direction as this yields particularly high hole mobilities.

We describe the hole subband states by means of the $4 \times 4$ Luttinger Hamiltonian [5]. For the in-plane $B = (B_x, B_y, 0)$ we use the vector potential $A = (zB_y, -zB_x, 0)$. Treating $A$ and $B$ by means of degenerate perturbation theory we obtain in second order for $g^*$ at the bottom of the HH subbands in an infinitely deep rectangular QW

\[
g_{[mm(2mm)]}^{HH} = 6\left[2 - 3\sin^2(\theta)\right] \sin(\theta) \sqrt{4 - 3\sin^2(\theta)} \frac{\kappa(\gamma_3 - \gamma_2)}{\gamma_2^{HH} - \gamma_2^{LH}}
\]

\[
g_{[110]}^{HH} = -6\left[2 - 3\sin^2(\theta)\right] \sin^2(\theta) \frac{\kappa(\gamma_3 - \gamma_2)}{\gamma_2^{HH} - \gamma_2^{LH}}
\]
with
\[
\begin{align*}
\gamma_{z}^{HH} &= -\gamma_1 + 2[1 - \alpha \gamma_2 + \alpha \gamma_3] \quad (2a) \\
\gamma_{z}^{LH} &= -\gamma_1 - 2[1 - \alpha \gamma_2 + \alpha \gamma_3] \quad (2b) \\
\alpha &= \sin^2(\theta) \left[ 3 - \frac{2}{3} \sin^2(\theta) \right]. \quad (2c)
\end{align*}
\]

Here \(\gamma_1, \gamma_2, \) and \(\gamma_3,\) are the Luttinger parameters \(\text{and} \) \(\gamma_{z}^{HH} \) and \(\gamma_{z}^{LH} \) are the reciprocal effective masses in \(z\)-direction in the axial approximation for the HH and LH subbands, respectively \(\text{and} \) dashed-dotted lines) are in very good agreement with Eq. (1).

Yet, this implies a remarkable difference \([3–6]\) compared to when \(B \parallel \{110\}\). Moreover, the sign of \(g_{HH}^{[\mu B(2\mu)]} \) is opposite to the sign of \(g_{HH}^{[\mu B(4\mu)]}\).

Equation (1) is applicable to a wide range of cubic semiconductors with results qualitatively very similar to Fig. 2. In particular, the relative anisotropy
\[
\frac{g_{HH}^{[\mu B(100)]}}{g_{HH}^{[\mu B(2\mu)]}} = -\frac{\sin(\theta)}{\sqrt{4 - 3 \sin^2(\theta)}}
\]

is independent of the material-specific parameters \(\gamma_i\) and \(\kappa\). This remarkable result can be traced back to the fact that the anisotropy for different directions \(\theta\) in \(k\)-space is always characterized by the single parameter \((\gamma_3 - \gamma_2)\) (Ref. [12]). Note that for QW’s based on narrow-gap semiconductors we have a larger \(\kappa\) and smaller effective masses so that for these materials the absolute values of \(g^\ast\) are significantly larger than \(g^\ast\) of GaAs shown in Fig. 2 but the \(g^\ast\) anisotropy is still given by Eq. (3) and depends only on \(\theta\). The Zeeman splitting can be even further enhanced if one uses semimagnetic semiconductors containing, e.g., Mn. For these materials the structure of the Hamiltonian is identical to the conventional Luttinger Hamiltonian in the presence of a magnetic field with \(\kappa\) replaced by the effective \(g\)-factor due to the paramagnetic exchange interaction \(\frac{\mu B}{k}\). Therefore Eq. (1) and the \(g^\ast\) anisotropy [Eq. (3)] are readily applicable to semimagnetic materials, also.

In a Taylor expansion of the Zeeman splitting, \(\Delta E(B)\), \(g^\ast \) (times \(\mu_B\)) is the prefactor for the lowest order term linear in \(B\). Often terms of higher order in \(B\) are neglected because of their relevant insignificance. An interesting feature of Fig. 2 is the vanishing of \(g^\ast\) for the high-symmetry growth directions [001] and [111] (Ref. [4]). For the 2DHS discussed in Fig. 2 this results in a splitting \(\Delta E\) which at \(B = 1 \text{T}\) is more than 2 orders of magnitude smaller than \(\Delta E\) for growth directions [113] and [110]. For these high-symmetry directions \(\Delta E\) is proportional to \(B^3\) (Ref. [13]). In second order perturbation theory we obtain for the HH1 subband of an infinitely deep rectangular QW of width \(d\) grown in [001] direction
\[
\Delta E = (\mu_B B)^3 \left( \frac{m_0 d^2}{\pi^2 h^2} \right)^2 \sqrt{\gamma_2^2 \cos^2(2\phi) + \gamma_3^2 \sin^2(2\phi)} \left[ \frac{4\kappa (\pi^2 - 6)}{\gamma_2^{HH} - \gamma_2^{LH}} + \frac{27\gamma_3}{\gamma_2^{HH} - \gamma_2^{LH}} \right] . \quad (4)
\]

Here \(\phi\) is the angle between the in-plane \(B\) and the [100] axis. The first term in the square bracket stems from \(k \cdot p\) coupling between the subbands HH1 and LH1, and the second term is due to coupling between HH1 and LH3. We get similar, though somewhat longer expressions for growth direction [111]. It is remarkable that we have a nonzero \(\Delta E\) even in the limit \(\kappa = 0\). This can be understood as follows: The 4 \(\times\) 4 Luttinger Hamiltonian \([\text{at} \) which is underlying our calculations corresponds to an

infinitely large spin-orbit splitting between the topmost valence band \(\Gamma_6\) and the split-off band \(\Gamma_7\). Therefore spin-orbit interaction is not explicitly visible in our results, though, similar to Zeeman splitting in bulk semiconductors \([\text{at} \) Eqs. (1) and (3) are a consequence of spin-orbit interaction. In 2D systems the motion of electrons and holes in the presence of this interaction can give rise to a Zeeman splitting even without a bulk \(g^\ast\). We remark that in a parabolic QW the in-plane \(g^\ast\) of the
HH subbands also contains such terms independent of \( \kappa \). We have here a 2D analogue of Roth’s famous formula \[ \rho(B) = \rho(0) \] for the electron bulk \( g^* \). Finally we note that, unlike Eq. (1), \( \Delta E \) in Eq. (4) increases proportional to \( d^4 \), i.e., Zeeman splitting is most efficiently suppressed in narrow QW’s.

Now we will show that the anisotropy of \( g^* \) can be probed experimentally by measuring the magnetoresistance of a high-mobility 2DHS as a function of in-plane \( B \). The samples are 200 Å wide Si-modulation doped GaAs QW’s grown on (113)A GaAs substrates. These samples exhibit a mobility anisotropy believed to be due to an anisotropic surface morphology \[ [16] \]. They are patterned with an L-shaped Hall bar to allow simultaneous measurements of the resistivity along the [332] and [\( \overline{1}10 \)] directions. Front and back gates are used to control the 2D density in the QW and the perpendicular electric field which characterizes the asymmetry of the sample \[ [10] \].

The left two panels of Fig. 3 show the resistivity \( \rho \) measured as a function of in-plane \( B \) for three different densities and different relative orientations of \( B \) and current \( I \). For easier comparison we have plotted the fractional change \( \rho(B)/\rho(B = 0) \). Apart from an overall positive magnetoresistance these curves show a broad feature consisting of an inflection point followed by a reduction in slope followed by another inflection point. In Fig. 3 we have placed arrows between the two inflection points at a value of \( B \) we call \( B^* \). Similar, though sharper features have been observed in systems with several occupied confinement subbands when a subband is depopulated by means of an in-plane \( B \) \[ [17] \]. We propose that the magnetoresistance feature at \( B^* \) in Fig. 3 is related to a spin-subband depopulation and the resulting changes in subband mobility and intersubband scattering as the in-plane \( B \) is increased. Note that in each panel of Fig. 3, \( B^* \) is the same for both current directions even though the magnetoresistance is very different. This implies that \( B^* \) depends on parameters which do not depend on current direction. This supports our hypothesis, as spin-subband depopulation should not depend on the direction of current in the sample.

Our interpretation of \( B^* \) is obviously consistent with \( B^* \) in Fig. 3 becoming larger with increasing density. It is remarkable that \( B^* \) for the \( B \parallel [332] \) traces is about 4 T smaller than for the \( [\overline{1}10] \) traces, regardless of the \( I \) direction. We associate this with the anisotropy of the in-plane \( g^* \). This interpretation is validated by our self-consistently calculated \[ [4,11,13] \] results for the density \( N_+ \) of the upper spin subband as a function of \( B \), shown in the right panel of Fig. 3. The density \( N_+ \) decreases much faster for \( B \parallel [332] \) than for \( B \parallel [\overline{1}10] \), in agreement with Fig. 3. We have further support for our interpretation of \( B^* \) from experiments where we increase the asymmetry of the confining potential by means of the front and back gates while keeping the 2D density fixed. We observe an increase in \( B^* \), in agreement with the results of our self-consistent calculations.

One might ask whether the data in Fig. 3 actually could be summarized by a single value of \( g^* \) for each trace. Unfortunately, this is not possible because, due to the complicated band structure of holes, \( g^* \) depends on energy \( E \), and we are averaging over \( g^*(E) \) for \( E \) between the subband edge and the Fermi energy. The importance of this effect can be readily deduced from the right panel of Fig. 3, as we would have straight lines for \( N_+(B) \) if \( g^* \) (and the effective mass \( m^* \)) were not dependent on \( E \).

In Fig. 3 the measured \( B^* \) is significantly smaller than the calculated \( B \) for a complete depopulation of the upper spin-subband. We note that for our low-density samples it can be expected that \( g^* \) is enhanced due to the exchange interaction \[ [11,13] \]. This effect was not taken into account in our self-consistent calculations. The qualitative agreement between the experimental data and the calculations, however, implies that these many-particle effects do not affect the anisotropy of \( g^* \).

The large anisotropy of the Zeeman splitting in 2DHS’s offers many possible device applications. In a polycrystalline material, e.g., one could alter the degree of spin polarization in different domains by changing the direction of the external \( B \). As can be seen in Fig. 2 because of the sign reversal of \( g^* \) it is even possible to have different domains with opposite spin polarization for a given direction of \( B \). Recently, there has been a growing interest in controlling the spin degree of freedom for quantum computing and spin electronics. In Ref. \[ [16] \] the authors have sketched a quantum device which makes use of the spatial variation of \( g^* \) in layered semiconductor structures made of, e.g., Al\(_x\)In\(_y\)Ga\(_{1-x-y}\)As. However, the authors have estimated that a substantial change in \( g^* \) requires a fairly large electric field of the order of 100 kV/cm. Oestreich et al. \[ [21] \] and Fiederling et al. \[ [21] \] have suggested a spin aligner based on semimagnetic semiconductors. Here the \( g \)-factor anisotropy of 2DHS’s provides a powerful additional degree of freedom for engineering such devices.

In conclusion, we have shown that coupling the spin degree of freedom to the anisotropic orbital motion of 2D hole systems gives rise to a highly anisotropic Zeeman splitting with respect to different orientations of an in-plane magnetic field relative to the crystal axes.

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FIG. 1. Coordinate system for QW’s grown in [mmn] direction (z direction). Here $\theta$ is the angle between [mmn] and [001], i.e., we have $\theta = \arccos(n/\sqrt{2m^2 + n^2})$. The axes for the in-plane motion are [nn(2m)] ($x$) and [110] ($y$).

FIG. 2. Anisotropic effective $g$-factor $g^*$ of the HH1 subband for a 200 Å wide GaAs/Al$_0$.3Ga$_0$.7As QW as a function of $\theta$, the angle between [001] and the growth direction. Results are shown for the in-plane $B$ along the [nn(2m)] and [110] directions. The solid and dashed lines were obtained by means of a numerical diagonalization of the Luttinger Hamiltonian. The dotted and dashed-dotted lines were obtained by means of Eq. (1).

FIG. 3. Left and central panels: Fractional change in resistivity $\rho(B)/\rho(0)$ due to an in-plane $B$, measured at $T = 0.3$ K in a GaAs 2D hole system grown on a (113) substrate, for different 2D densities as indicated. The arrows mark $B^*$ as defined in the text. Right panel: Calculated density $N_s$ in the upper spin subband as a function of $B$.  

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