Reasonable distribution of electric vehicle charging stations

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Abstract. With the depletion of petroleum resources and people's attention to environmental issues, the electric vehicle industry is booming. However, due to limitations in charging technology, the charging speed of electric vehicles cannot be completely satisfactory. Therefore, this paper proposes an innovative charging station location model to meet the needs of consumers. Due to the shortcomings of the traditional coverage model, the proposed location model consists of a loop model and a generalized maximum coverage model combined with a cellular network model. After the simulation experiment of the model in Ireland, we obtained the experimental results. Our proposed location model improves the utilization of EV charging stations.

1. Introduction
In recent years, the traditional automotive industry has faced enormous challenges as people become aware of the deterioration of the ecological environment and the exhaustion of limited energy. At the same time, electric vehicles have entered the field of vision. For example, Toyota's Camry, Highlander and Prius are popular with consumers because of the use of the synergic drive system [1]. After analyzing the sales situation and development plan of the automobile company, we found that due to the continuous support policies of the electric vehicles and the continuous efforts of many R&D teams, the technology of electric vehicles and even the whole industry are relatively mature, and sales are always leading [2]. It is estimated that by 2020, China's electric vehicle ownership will reach 5 million units [3]. With the rise in oil prices and people's attention to environmental issues, the market for electric vehicles continues to expand. In addition to the traditional car companies that develop electric vehicles, new companies focused on electric-powered vehicles represented by Tesla was born in the 21st century.

However, the popularity of electric vehicles does not represent the practicality of electric vehicles. Most users now prefer fuel cars, and the reasons given by them are similar, mostly because of charging problems. Therefore, the rational allocation of charging station construction has been forced.

Many scholars have conducted research on the distribution of charging stations. For example, (ZH Zhu, ZY Gao, JF Zheng, HM Du, 2012) proposed a model for solving the location problem of a plug-in electric vehicle charging station, and minimizing the travel cost of the user from the charging station to the destination as the target of the model [7]. Based on the predecessors' research and theoretical basis, the purpose of this paper for the rational allocation of scarce resources is:

Building an innovative charging station location model. The significance is to layout the charging station to meet the needs of consumers with the least cost, and give advice on the construction of electric vehicle charging station.

This article has the following innovations:
1. Use a loop model to construct a charging station location model for a larger area (country or city). Such a model allows for better modeling and better charging station planning for cities that are built...
with loops or roads that are diverging from the center, while increasing the utilization of charging stations.

2. Integrate the cellular network model based on the traditional maximum generalized coverage model. The model is more suitable for non-loop and non-divergent cities. In this paper, the model is applied in Ireland for simulation experiments, and the most preferable location of the charging station is obtained.

2. Background

2.1. Related Work

In 1909, the German economist Weber first proposed the concept of location theory. However, due to the immature theory and the limitations of research, the site selection theory has not been widely applied. It was not until the 1960s that people began to study the location problem more systematically [8]. In 1964, Hakimi proposed two typical siting issues, including p-median and p-center. The p-median problem shows how to minimize the distance from all demand points to the device. The p-center problem refers to how to minimize the distance between each demand point and the nearest equipment after a certain number of equipment is put into use [9]; C.A.Rogers first proposed the definition of coverage location [10]; in 1971, Set Covering Location Model was proposed by Toregas et al. to solve the problem of location of emergency service facilities such as ambulances, with the goal of minimizing construction costs [11]; for demand flow problems, FCLM (Flow Capturing Location Models) proposed by MJ Hodgson opened up new research methods in many fields and provides a new method for site selection [12]; based on Flow Capturing Location Models, Flow Refueling Location Model proposed by Kuby et al. used driving time as the basis to achieve the goal of maximizing the flow of refueling vehicles [13, 14]; in 2015, Polo et al. established a location model to improve the distribution of public health service facilities [15]; in 2016, Caunhye et al. proposed Two-stage location model for disaster risk management under uncertain conditions [16]; in 2018, AMF Fard and M Hajaghaei-Keshteli used a new nested approach to build a new tri-level location model [17].

In recent years, many Chinese scholars have also studied and discussed the problem of site selection. Zhou and Li used game theory to construct a reasonable distribution model of EV charging stations based on traditional location problems [18]; Feng et al. combined Delphi and GAHP as a reasonable basis for the location problem of EV charging stations [19]; Xia et al. constructed the location model with the goal of minimizing the operating cost of the charging station and minimizing the user's cost [20].

2.2. Location-allocation network model

2.2.1. Traditional coverage model. Traditional coverage models include the Set Covering Model [11] proposed by Toregas et al. and the Maximum Covering Location Model [21] proposed by Church and Velle.

The Set Covering Model studies the cost of facility construction without limiting the number of facilities. The goal is to minimize construction costs while meeting the needs of all demand points.

| Parameter | Significance | Parameter | Significance |
|-----------|--------------|-----------|--------------|
| i         | demand point | j         | candidate point |
| N         | demand point set | M         | candidate point set |
| d_{ij}    | distance between i and j | c_{j}    | cost of building station in j |
| R         | acceptable maximum coverage distance | X_{j}    | if a station is built in j, X_{j}=1; otherwise, X_{j}=0 |
| Y_{ij}    | If 0≤d_{ij}≤R, Y_{ij}=1; otherwise, Y_{ij}=0 |          |              |
Model

\[
\text{Minimize} \sum_{j \in M} c_j X_j
\]  \hspace{1cm} (1)

s.t.

\[
\sum_{j \in M} Y_{ij} X_j \geq 1 \quad \forall \ i \in N
\]  \hspace{1cm} (2)

\[
Y_{ij} \in \{0,1\} \quad \forall \ i \in N, j \in M
\]  \hspace{1cm} (3)

\[
X_j \in \{0,1\} \quad \forall \ j \in M
\]  \hspace{1cm} (4)

Formula (1) minimizes the construction cost of the point to be selected; the limiting factor (2) means that each demand point can find a candidate facility within an acceptable distance; limiting factors (3) and (4) define the binary values of the decision variables.

Because the model does not limit the number of facilities and ignores the number of demand points, even if there is only one demand point in an area, stations will be built in this area. This leads to waste of resources and unreasonable layout of the facilities.

Since the number of facilities is bound to be limited in real life, it is impossible to cover every demand point. Based on the traditional Set Covering Model, Church and Velle have made some improvements [21]. The goal of the new model is to use the limited facility resources to best meet the needs of the demand point.

### Table 2. Parameter and significance of the maximum coverage model

| Parameter | Significance | Parameter | Significance |
|-----------|--------------|-----------|--------------|
| i         | demand point | j         | candidate point |
| N         | demand point set | M         | candidate point set |
| \(d_{ij}\) | distance between i and j | \(R\) | acceptable maximum coverage distance |
| \(X_j\)  | if a station is built in j, \(X_j=1\); otherwise, \(X_j=0\) | \(Y_{ij}\) | If \(0 \leq d_{ij} \leq R\), \(Y_{ij}=1\); otherwise, \(Y_{ij}=0\) |
| \(H_i\)  | if i is satisfied, \(H_i=1\); otherwise, \(H_i=0\) |           |              |

Model

\[
\text{Minimize} \sum_{i \in N} H_i Z_i
\]  \hspace{1cm} (5)

s.t.

\[
\sum_{j \in M} X_j = p
\]  \hspace{1cm} (6)

\[
\sum_{j \in M} Y_{ij} X_j \geq Z \quad \forall \ i \in N
\]  \hspace{1cm} (7)

\[
Y_{ij} \in \{0,1\} \quad \forall \ i \in N, j \in M
\]  \hspace{1cm} (8)

\[
X_j \in \{0,1\} \quad \forall \ j \in M
\]  \hspace{1cm} (9)

\[
H_i \in \{0,1\} \quad \forall \ i \in N
\]  \hspace{1cm} (10)

Formula (5) maximizes the demand for the covered area; limiting factor (6) means that a total of \(p\) facilities are laid out; and limiting factor (7) means that the covered demand point can find a candidate for the facility within an acceptable distance; Factors (8), (9) and (10) define the binary values of the decision variables.

2.2.2. Generalized maximal covering model. After the Maximum Covering Location Model was proposed, research scholars believe that it still has defects. First, there are only two cases of demand points in the model: completely covered and completely uncovered. This is unreasonable in real life. Second, in practice, people usually regard the center point of the demand area as a demand point. This has led to the fact that parts of the demand area are covered and cannot be reflected.
Therefore, Berman and Krass proposed the generalized maximal coving model, which considers that all facilities near the demand point will have an impact on the coverage level of the demand point [22]. Each demand point $i$ has multiple coverage level parameters and multiple coverage radius parameters, the value of which depends on the distance from the demand point to the facility to be selected. If the facility to be selected is within the coverage radius $R_i$ of the demand point, the coverage of the facility to the demand point is $a_i(R_i)$. As the distance from the demand point to the facility to be selected increases, the level of coverage decreases. That is, the degree of coverage is inversely proportional to the distance from the demand point to the facility. The purpose of the generalized maximal coving model is to maximize the weighted coverage of all demand points.

2.2.3. Cellular network. Cellular networks are widely used in the communications industry because of its mathematical nature: in the case where a plane is covered by a circle of the same radius, when the center of the circle is at the center of each regular hexagon of the regular hexagonal grid (i.e., the center of the circle is located in the grid of the regular triangle), the number of circles used is the least. Circles are commonly used during communication. Considering cost issues, a simple hexagonal grid is considered as the best choice. Since many networks of simple hexagonal grids resemble cells, they are called Cellular networks.

In this paper, the Cellular network is applied to the rational allocation of scarce resources by “transfer learning”, and the Cellular network is combined with the generalized largest coverage model to model the location of EV charging stations.

3. The model of Ireland
In this section, the loop model and the cellular network model fused with the generalized largest coverage model are applied in Ireland for simulation experiments, and the location suggestions for the EV charging station are given.

3.1. Loop model
The cities of Ireland are diverse, with Dublin, Waterford, Cork and Limerick in the Middle East of Ireland, which is considered as Ireland's main transportation hub. Therefore, we will establish a loop model centered on these four transportation hub cities.

Taking Dublin as an example, we set up a loop model on the premise of setting up stations at road intersections, as shown in Figure 1. We built two loops in the urban of Dublin with a radius of 5 kilometers and 9.4 kilometers. The total number of EV charging stations on the two loops is five. Among them, three EV charging stations are located on the M50 Ring Expressway, and the other two are located at the intersection of R805 and R102 and the intersection of R137 and R112. Since the urban traffic congestion problem cannot be ignored, we choose the road conditions from 8:00-9:00 in the morning to carry out simulation experiments. The distance between the charging stations we set is between 13 km and 20 km, the driving time between charging stations is between 25 and 50 minutes, and the actual tram mileage is between 33.25 km and 50 km.

At the same time, the suburbs of Dublin also need to properly distribute EV charging stations. We applied the loop model again and built the third and fourth loops. The radius of these two loops is 23km and 36km respectively. We built 10 charging stations on two loops, located on N7, M1, M3, M4, M7, M9, M50 Road. The distance between each charging station is between 28km and 50km, the driving time between charging stations is between 25 and 50 minutes, and the actual tram mileage is between 28km and 60km.

In summary, we used the loop model to build 15 EV charging stations in Dublin with a coverage area of 4071.5 square kilometers. In Dublin, each demand point is within 30km from the nearest EV charging station, so this charging station layout can meet the users’ charging needs in Dublin driving.
3.2. **Cellular network model and generalized largest coverage model**

According to the analysis of the distribution of roads in Ireland, the main roads are distributed in the central and southern regions, while the northern regions have no highways and national roads, and most of the towns are connected by ordinary roads. On this issue, scholars usually use the traditional generalized coverage model with a circle as the core. In this paper, we apply the cellular network model combined with the generalized maximum coverage model to these non-trunk areas to get the most preferred location of the EV charging station in these areas.

Cellular network is widely used in construction, communication and other fields due to its structural advantages. Similarly, the cellular network can also help each charging station reach the maximum coverage area in the location of the charging station. As shown in Figure 3, in the conventional case, the charging station is located at the center of each regular hexagon. R is the circumradius of the cell in the Cellular network model, and it has a formula (11) relationship with the area S of the cell.

\[ R = \frac{2}{\sqrt{3} \cdot \sqrt{3}} \cdot S \]  

(11)

Depending on the type of city and regional economic development, we need to adjust the size of the honeycomb model appropriately. In developed cities with large population density and dense vehicles, the value of R is set smaller; in sparsely populated areas, the value of R is set larger. Moreover, in the actual operation process, we found that due to the different distribution of roads, the charging station cannot be built in the center of the regular hexagonal area, and it is necessary to avoid natural landscapes such as mountains and rivers. Taking the northern region as an example, as shown in Fig. 4, a cellular network model with equal R values for each cellular structure is used, and a total of 15 EV charging stations are provided. The cellular structures are adjacent to one another such that within the area covered by the cellular network model, each demand point is within R from the nearest EV charging station, maximally meeting the charging needs of EV users.
At the same time, we combined the cellular model with the generalized maximum coverage model to model the candidate EV charging station in Ireland.

Table 3. Parameter and significance of the generalized maximum coverage model

| Parameter | Significance | Parameter | Significance |
|-----------|--------------|-----------|--------------|
| i, j      | Node         | N         | set of all nodes |
| M         | set of all driving routes | m         | vehicle on the route m |
| d_{ij}    | distance between i and j | w_i     | requirement of i |
| c_i       | cost of building station in i | C         | cost budget |
| Y_{im}    | if m charges in node i, Y_{im}=1; otherwise, Y_{im}=0 | β         | maximum capacity of batteries |
| l_i       | acceptable minimum coverage distance of i | u_i     | acceptable maximum coverage distance of i |
| b_{im}    | residual electricity of m at point i | r_{im}   | supplementary electricity of m at point i |
| y_{ij}    | if j is the nearest station for i, y_{ij}=1; otherwise, y_{ij}=0 | x_i     | if a station is built on i, x_i=1; otherwise, x_i=0 |

Model

\[
\text{Max} \sum_{i \in N} \sum_{j \in N_j} w_i f_i y_{ij} \\
\text{s.t.}
\]

\[
x_j \geq y_{ij} \quad \forall i, j \in N
\]

\[
\sum_{j \in N_i} y_{ij} \leq 1 \quad \forall i \in N
\]

\[
b_{im} = (b_{jm} + r_{jm}) - d_{ij} \quad \forall i, j \in N, \forall m \in M
\]

\[
r_{im} \leq \beta - b_{im} \quad \forall i \in N
\]

\[
r_{im} = Y_{im} \times \beta - A_{im} \quad \forall i \in N, \forall m \in M
\]

\[
\sum_{m \in M} Y_{im} \leq k x_i \quad \forall i \in N
\]

\[
\sum_{i \in N} c_i x_i \leq C
\]

\[
Y_{im} \in \{0,1\} \quad \forall i \in N, \forall m \in M
\]

\[
x_i \in \{0,1\} \quad \forall i \in N
\]

\[
A_{im} \geq 0, r_{im} \geq 0, b_{im} \geq 0 \quad \forall i \in N, \forall m \in M
\]

And which \( f_i = \begin{cases} 
1 & d_{ij} \leq l_i \\
(u_i - d_{ij}) / (u_i - l_i) & l_i < d_{ij} < u_i \\
0 & d_{ij} \geq u_i 
\end{cases} \)

The goal of equation (12) is to cover the demand point to the greatest extent; the limiting condition (13) indicates that if the charging station is established at j, \( y_{ij}=1 \); the limiting condition (14) indicates...
that the model only considers that the demand point \( i \) is covered by the nearest EV charging station; the limiting condition (15) gives the calculation method of the EV remaining power; the limiting condition (16)(17) explains that the electric vehicle cannot exceed the battery capacity after charging; the limiting condition (18) indicates that the EV can be charged at point \( i \) only if there is a charging station at point \( i \); the limiting condition (19) constrains the cost calculation; the limiting condition (20)(21)(22) defines the value of the decision variable. The value of the function \( f_i(d) \) is inversely proportional to the distance, between 0 and 1.

The model we built in northern Ireland can be simplified to the structure shown in Figure 5.

![Figure 5. 15 candidate sites](image)

Mark each cell individually with 1-15, and the distance between any two candidate sites is a straight distance (according to the Google Map). The error between the actual driving distance and the straight distance is not more than 10km, so it can be ignored. The distance between each candidate site is shown in Table 1. At the same time, we have set 15 simulated driving routes. Assuming that the O-D flow matrix is symmetrical, the distance from point \( i \) to point \( j \) is equal to the distance from point \( j \) to point \( i \).

Table 4. The distance between any two nodes

| Nodes | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1     | 0   | 158 | 140 | 38  | 100 | 78  | 135 | 169 | 204 | 66  | 116 | 67  | 36  | 175 | 101 |
| 2     | 158 | 0   | 39  | 112 | 65  | 79  | 75  | 69  | 106 | 98  | 39  | 101 | 141 | 44  | 102 |
| 3     | 140 | 39  | 0   | 99  | 39  | 65  | 39  | 41  | 89  | 101 | 39  | 81  | 130 | 35  | 70  |
| 4     | 38  | 112 | 99  | 0   | 65  | 40  | 99  | 133 | 176 | 39  | 79  | 42  | 39  | 140 | 79  |
| 5     | 100 | 65  | 39  | 65  | 0   | 39  | 39  | 68  | 105 | 78  | 39  | 39  | 100 | 72  | 44  |
| 6     | 78  | 79  | 65  | 40  | 39  | 0   | 78  | 102 | 156 | 39  | 39  | 40  | 65  | 99  | 69  |
| 7     | 135 | 75  | 39  | 99  | 39  | 78  | 0   | 37  | 71  | 117 | 65  | 67  | 145 | 60  | 42  |
| 8     | 169 | 69  | 41  | 133 | 68  | 102 | 37  | 0   | 42  | 150 | 79  | 97  | 170 | 33  | 77  |
| 9     | 204 | 106 | 89  | 176 | 105 | 156 | 71  | 42  | 0   | 196 | 130 | 150 | 222 | 71  | 111 |
| 10    | 66  | 98  | 101 | 39  | 78  | 39  | 117 | 150 | 196 | 0   | 65  | 67  | 39  | 134 | 106 |
| 11    | 116 | 39  | 39  | 79  | 39  | 65  | 79  | 130 | 65  | 0   | 66  | 99  | 70  | 83  |     |
| 12    | 67  | 101 | 81  | 42  | 39  | 40  | 67  | 97  | 150 | 67  | 66  | 0   | 79  | 114 | 42  |
| 13    | 36  | 141 | 130 | 39  | 100 | 65  | 145 | 170 | 222 | 39  | 99  | 79  | 0   | 181 | 131 |
| 14    | 175 | 44  | 35  | 140 | 72  | 99  | 60  | 33  | 71  | 134 | 70  | 114 | 181 | 0   | 99  |
| 15    | 101 | 102 | 70  | 79  | 44  | 69  | 42  | 77  | 111 | 106 | 83  | 42  | 131 | 99  | 0   |

Using MATLAB, we import the parameters, the objective function and the data from the above table into the cplex solver in the yalmip and cplex libraries. We use a computer to automatically simulate the driving route of the vehicle. The number of users and the weight of demand are reasonably allocated according to the total population of Ireland. The results are solved as shown in Table 5.

Table 5. The results of model coverage

| Charging station point | Fully covered node | Partially covered node |
|------------------------|--------------------|------------------------|
| 2                      | 2, 3, 11, 14       | 5, 6, 7, 8             |
4. Conclusion
For the rational distribution of scarce resources, this paper briefly introduces the loop model (for cities with loop-building cities and roads diverging from the center) and the generalized maximum coverage model combined with the cellular network model (for non-loop and non-divergent cities). In the background section, we introduce the predecessors’ location theory and population density counting research and queuing theory. In the most important modeling section (Part 3), we applied the model to parts of Ireland (the capital Dublin and the northern region) to obtain experimental results. The new model we propose will increase the coverage of the charging station and make the distribution of the EV charging station more reasonable. In addition, the number of charging stations that should be set in the charging station also need to be given reasonable advice. Therefore, in the following research, we will have a reasonable distribution of charging piles in the charging station.

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