Recent status of polarized parton distributions

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Abstract. We study an influence of precise data on uncertainty of polarized parton distribution functions. This analysis includes the SLAC-E155 proton target data which are precise measurements. Polarized PDF uncertainties are estimated by using the Hessian matrix. We examine correlation effect between the antiquark and gluon uncertainties. It suggests that reducing the gluon uncertainty is needed to determine the polarized antiquark distribution clearly.

INTRODUCTION

Polarized parton distribution functions (polarized PDF’s) have so far been optimized from polarized deep inelastic scattering (polarized DIS) world data [1, 2]. We could obtain only a slight piece of information about polarized antiquark and gluon distributions. At this stage, the antiquark SU(3)$_f$ flavor symmetry is assumed in most of the polarized PDF analyses. The SU(3)$_f$ symmetry breaking is already known as the Gottfried sum rule violation in the unpolarized case. In principle, the polarized PDF analysis should take account of the symmetry breaking. However, we must not only determine a shape but also a sign of each polarized antiquark distribution. It needs more precise data to improve the current status. Semi-inclusive DIS experiments [3] are also expected to separate antiquark flavor distributions. However, the separated distributions may not be credible due to ambiguity of the fragmentation functions. Then, antiquark flavor distributions cannot be decomposed clearly. The current knowledge of the polarized gluon distribution is still poor. The polarized gluon distribution is suggested as the positive distribution; however, there is large difference between various parameterization results.

We would like to know ambiguity of polarized PDF’s quantitatively. PDF uncertainty plays an important role in illustrating the ambiguity. Furthermore, it is important to show the phenomenological uncertainty of predicted physical quantities (e.g., scattering cross-sections and spin asymmetries) with parameterized PDF’s and their uncertainties in our work. A purpose of this analysis is to clarify the current knowledge about the polarized PDF’s from the polarized DIS world data by using their PDF uncertainty. In this analysis, the polarized PDF’s are optimized including precise SLAC-E155 proton target data [4]. Then, we examine an influence of the precise data on the polarized PDF uncertainty, which is estimated by the Hessian method.

1 A fortran program of the AAC PDF library is available from [http://spin.riken.bnl.gov/aac](http://spin.riken.bnl.gov/aac).
PARAMETERIZATION OF THE POLARIZED PDF’S

The polarized PDF’s are determined by using spin asymmetry $A_1$ of the polarized DIS experiments from the EMC, SMC, SLAC-E130, E142, E143, E154, E155, and HERMES:

$$A_1(x,Q^2) = \frac{2x[1 + R(x,Q^2)]}{F_2(x,Q^2)}g_1(x,Q^2),$$

where $F_2$ is the unpolarized structure function. The function $R(x,Q^2) = \sigma_L/\sigma_T$ is the ratio of absorption cross sections for longitudinal and transverse virtual photons, and it is determined from experimental data in reasonably wide $Q^2$ and $x$ ranges in the SLAC experiments [5]. The polarized structure function $g_1$ is expressed with polarized PDF’s:

$$g_1(x,Q^2) = \frac{1}{2} \sum_{i=1}^{n_f} e_i^2 \left\{ \Delta C_q(x,\alpha_s) \otimes [\Delta q_i(x,Q^2) + \Delta \bar{q}_i(x,Q^2)] + \Delta C_g(x,\alpha_s) \otimes \Delta g(x,Q^2) \right\},$$

where $e_i$ is the electric charge of quarks, and $\Delta C_q, \Delta C_g$ are Wilson’s coefficient functions. The convolution $\otimes$ is defined by $f(x) \otimes g(x) = \int_x^1 dy f(x/y)g(y)$. The polarized PDF’s $\Delta f(\equiv f_\uparrow - f_\downarrow)$ are defined as helicity distributions in the nucleon. In the AAC analysis, the polarized PDF $\Delta f(x)$ is defined at initial $Q^2$ by the weight function form:

$$\Delta f(x) = Ax^\alpha(1 + \lambda x^\gamma)f(x),$$

where $f(x)$ is the unpolarized PDF, and $A$, $\alpha$, $\lambda$, and $\gamma$ are free parameters. Optimized PDF’s are four distributions; $\Delta u_\uparrow(x)$, $\Delta d_\uparrow(x)$, $\Delta \bar{q}(x)$, and $\Delta g(x)$, and these are evolved from the initial $Q^2(=1$ GeV$^2$) to the same $Q^2$ of experimental data by the DGLAP equation [7]. In particular, the gluon distribution $\Delta g(x)$ contributes to the structure function with the non-zero coefficient function $\Delta C_g$ in the NLO case.

This analysis uses two constraint conditions. First, the positivity condition is used to restrict large-$x$ behavior of the polarized PDF’s. This condition corresponds to the probabilistic interpretation of the parton distributions in the LO: $|\Delta f(x)| \leq f(x)$. It needs not to be satisfied strictly in the NLO analysis. However, the polarized antiquark and gluon distributions tend to badly break the positivity limit: $|\Delta f(x)| \gg f(x)$. Such excessive behavior is due to the large experimental errors in the large-$x$ region. Hence, this behavior should be limited by this condition.

Next, the SU(3)$_f$ flavor symmetry is assumed: $\Delta u(x) = \Delta d(x) = \Delta \bar{q}(x) = \Delta s(x)$. Using this assumption, one can fix the first moments of the valence quarks with hyperon decay constants, then $\Delta u_v = 0.926$ and $\Delta d_v = -0.341$ are obtained. Note that the Bjorken sum rule is satisfied automatically by fixing first moments. Furthermore, the spin content $\Delta \Sigma$ is obtained by $\Delta \Sigma_{N_f=3} = \Delta u_v + \Delta d_v + 6\Delta \bar{q}$. Since, the antiquark contribution is emphasized, then the spin content determination is susceptible to the antiquark behavior.

In the analysis, we choose the modified minimal subtraction ($\overline{\text{MS}}$) scheme, and the GRV parameterization for the unpolarized PDF’s at the NLO analysis [8]. The total $\chi^2$ is minimized by the CERN subroutine MINUIT.
UNCERTAINTY ESTIMATION

Fortunately PDF uncertainty estimation method has been developed in the last several years (see a brief review [6]). The polarized PDF uncertainty comes from several error sources, e.g., experimental errors, unpolarized PDF, $\Lambda_{QCD}$, and so on. However, it is difficult to incorporate these errors into uncertainty estimation simultaneously. In the present analysis, the polarized PDF uncertainty is estimated from experimental errors by using the Hessian matrix $H_{ij}$ which is defined as a second order derivative matrix in the expanded $\chi^2(a_i)$ function around its minimum point. The PDF uncertainty $\delta \Delta f(x)$ can be obtained easily by the inverse matrix of the Hessian and linear error propagation:

$$[\delta \Delta f(x)]^2 = \Delta \chi^2 \sum_{i,j} \frac{\partial \Delta f(x)}{\partial a_i} H^{-1}_{ij} \frac{\partial \Delta f(x)}{\partial a_j}, \quad (4)$$

where $\Delta \chi^2 (= \chi^2(a_i) - \chi^2_{\text{min}})$ is defined as the difference from the minimum $\chi^2$. It determines a confidence level of the PDF uncertainty, and it depends on the $\chi^2$ distribution $K(s)$ with $N$ degrees of freedom. Here, $N$ is the number of optimized parameters. In our estimation, the value of $\Delta \chi^2$ is obtained by the following equation: $\int_0^\infty K(s) \, ds = \sigma$, where $\sigma (= 0.683)$ corresponds to 1 $\sigma$ error of a standard distribution in order to compare with general experimental errors. The statistical and systematic errors are added in quadrature, so that it could be overestimation. The proper estimation exists between the overestimated uncertainty and the uncertainty from only the statistical error.

RESULTS AND DISCUSSIONS

The best fitting result is $\chi^2(/d.o.f.) = 346.33(0.90)$. The first moments of new results and the AAC pervious results (AAC00, NLO set2) [1] are shown in Table 1. A correlation coefficient $\rho_{\bar{q}g}$ between the first moment of the antiquark and gluon distributions is $\rho_{\bar{q}g} = -0.836$, and there is strong correlation between two distributions. The uncertainties of the new results become smaller than those of the previous results. The gluon first moment and spin content $\Delta \Sigma$ still have large uncertainty. The fixed first moments $\Delta u$ and $\Delta d$ do not have uncertainty, then the $\Delta \Sigma$ uncertainty is six times as large as the antiquark uncertainty. Thus, the spin content is subject to the uncertainty of the antiquark distribution. Figure 1 shows the uncertainty of the new antiquark distribution. The antiquark uncertainty becomes rather large in the region $x < 0.01$, however the experimental data scarcely exist. The polarized DIS spin asymmetries $A^{p,d}_1(x)$ approaches rapidly to zero in the range $x < 0.004$. It is insufficient to clarify small-$x$ behavior of the antiquark distribution. Therefore, the antiquark determination has extrapolating ambiguity in small-$x$ region. It is needed tight constraint condition or other experiment.

In addition, Figure 1 shows comparison between the PDF uncertainties of new results and the previous results. There are no significant improvements of the valence quark uncertainties. On the SU(3)$_f$ symmetry assumption, the fixing first moments strongly restricts the behavior of valence quark distributions. In contrast, the antiquark and gluon uncertainties are reduced in the range $0.01 < x < 0.5$, where the E155 proton data exist.
The precise polarized DIS data can reduce the antiquark uncertainty mainly. On the other hand, the gluon uncertainty changes in response to antiquark uncertainty reduction due to a strong correlation between two distributions. Since the gluon contribution to the structure function $g_1(x)$ is smaller than the quark and antiquark contributions, we can extract only a little information of the gluon distribution in spite of the NLO analysis. Actually, the gluon uncertainty is still large. It indicates the difficulty of determining the gluon distributions from the polarized DIS data. Therefore, the uncertainty reduction of the gluon distribution is due to the strong correlation rather than the NLO contribution.

In order to examine the correlation effect on the parameterization, we re-analyzed the $\Delta g(x) = 0$ case in which the fixed gluon distribution does not have uncertainty. The polarized PDF uncertainties of the $\Delta g(x) = 0$ case are compared to those of the $\Delta g(x) \neq 0$ case in Figure 2. The gluon distribution slightly exists at high-$Q^2$ due to $Q^2$ evolution of the singlet type DGLAP equation. The valence quark uncertainties scarcely change. Drastic improvement of the antiquark uncertainty is due to vanished the large gluon uncertainty. the obscure gluon distribution brings about the larger antiquark uncertainty by the complementary relation.

![Diagram](attachment:diagram.png)

**FIGURE 1.** Polarized PDF’s with their uncertainties at $Q^2 = 1 \text{ GeV}^2$. Dashed curves are the uncertainties of previous results (AAC NLO-2)
FIGURE 2. Polarized antiquark and gluon distributions with their uncertainties at $Q^2 = 1 \text{ GeV}^2$. The shaded portion shows the uncertainty of $\Delta g(x) = 0$ results, and the dashed curves are the uncertainties of new results ($\Delta g(x) \neq 0$).

SUMMARY

By this analysis, the polarized PDF’s were optimized from the polarized DIS world data which included the SLAC-E155 proton target data. The polarized PDF uncertainties were estimated by the Hessian method. The E155 precise measurements scarcely improve the valence quark uncertainties, but they can reduce the antiquark and gluon uncertainties. These, however, are still wrapped in large uncertainty. The SU(3)$_f$ symmetry, which we are obliged to assume, restricts strongly the valence quark behavior by fixing first moments, and the spin content determination depends on the antiquark behavior. Additionally, there is the strong correlation between the antiquark and gluon distributions. If the gluon distribution is clarified by RHIC-Spin at BNL, the uncertainty of the antiquark distribution can be reduced to some extent. Similarly, the complementary relation can reduce the large uncertainty of the spin content which comes from the extrapolating issue of the antiquark behavior. Then, we will be able to investigate the antiquark flavor dependence in detail.

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