Chiral Perturbation Theory for $B \to D^*$ and $B \to D$
Semileptonic Transition Matrix Elements at Zero Recoil* 

Lisa Randall†

Massachusetts Institute of Technology, Cambridge, MA 02139

Mark B. Wise

California Institute of Technology, Pasadena, CA 91125

Abstract

Heavy quark symmetry predicts the value of $B \to D$ and $B \to D^*$ transition matrix elements of the current $\bar{c}\gamma_\mu(1-\gamma_5)b$, at zero recoil (where in the rest frame of the $B$ the $D$ or $D^*$ is also at rest). We use chiral perturbation theory to compute the leading corrections to these predictions which are generated at low momentum, below the chiral symmetry breaking scale.

* Work supported in part by the U.S. Dept. of Energy under Contract numbers DEAC-03-81ER40050 and DE-AC02-76ER03069 and by the Texas National Research Laboratory Commission under Grant no. RGFY26C6.

† National Science Foundation Young Investigator Award, Alfred P. Sloan Foundation Research Fellowship, Department of Energy Outstanding Junior Investigator Award.
The interactions of a heavy quark $Q$ (i.e., $m_Q \gg \Lambda_{QCD}$) are simplified by going over to an effective theory where the heavy quark mass goes to infinity with its four-velocity fixed. The effective theory reveals a heavy quark spin-flavor symmetry$^{[1,2]}$ that is not manifest in the full theory of QCD. Heavy quark symmetry has been used to predict many properties of hadrons containing a single heavy quark. For example, it implies that all the form factors for $B \to D e \bar{\nu}_e$ and $B \to D^* e \bar{\nu}_e$ can be expressed in terms of a single universal function$^{[1]}$ and that the value of this function at zero recoil (where in the rest frame of the $B$ the $D$ or $D^*$ is also at rest) is known.$^{[1,3,4]}$

One of the most significant applications of these results will be determining the value of the Cabibbo–Kobayashi–Maskawa matrix element $V_{cb}$.

It has been shown$^{[5]}$ that at zero recoil the matrix elements which are necessary to determine $V_{cb}$ are known up to corrections at order $1/m_Q^2$. These corrections have been estimated in various models; dimensional analysis implies the corrections will be of order $(\Lambda/m_Q)^2$, where $\Lambda$ is determined by the chiral symmetry breaking scale$^{[6]}$. In addition to these “matching” corrections to the current, there are also corrections which are generated at low momentum that have a nonanalytic dependence on the pion mass and on the heavy quark suppressed mass difference between heavy meson vector and pseudoscalar states. It is these we compute in this letter. We first review the heavy meson chiral lagrangian and the standard analysis of the heavy quark currents. We use chiral perturbation theory to compute the dominant corrections at zero recoil. Because the inclusion of kaons in the heavy meson chiral lagrangian is suspect$^{[7]}$, we focus on pion loop corrections.

The ground state heavy mesons with $Q \bar{q}_a$ flavor quantum numbers (Here $a = 1, 2$ and $q_1 = u, q_2 = d$) have $s^\pi_\ell = \frac{1}{2}^-$, for the spin parity of the light degrees of freedom. Combining the spin of the light degrees of freedom with the spin of the heavy quark gives (in the $m_Q \to \infty$ limit) two degenerate doublets consisting of spin zero and spin-one mesons that are denoted by $P_a$ and $P^*_a$ respectively. In the case $Q = c, P_a = (D^0, D^+) \text{ and } P^*_a = (D^{*0}, D^{*+})$ while for $Q = b, P_a = (B^-, \bar{B}^0) \text{ and } P^*_a = (B^{*-}, B^{*0})$. It is convenient to combine the fields $P_a$ and $P^*_a$ that destroy
these mesons \( (v^\mu P_{a\mu}^* = 0) \) into a 4 × 4 matrix \( H_a \) given by\[^8\]

\[
H_a = \left( \frac{1 + \not{v}}{2} \right) \left( P_{a\mu}^* \gamma^\mu - P_a \gamma_5 \right). \tag{1}
\]

(This is a compressed notation. In situations where the type of heavy quark \( Q \) and its four-velocity \( v \) are important the 4 × 4 matrix is denoted by \( H_a^{(Q)}(v) \). It transforms under the heavy quark spin symmetry group \( SU(2)_v \) as

\[
H_a \to S H_a, \tag{2}
\]

where \( S \in SU(2)_v \) and under Lorentz transformations as

\[
H_a \to D(\Lambda) H_a D(\Lambda)\!^{-1}, \tag{3}
\]

where \( D(\Lambda) \) is an element of the 4 × 4 matrix representation of the Lorentz group. It is also useful to introduce

\[
\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0
\]

\[
= (P_{a\mu}^* \gamma^\mu + P_a \gamma_5) \frac{(1 + \not{v})}{2}. \tag{4}
\]

For \( \bar{H}_a \) the transformation laws corresponding to those in eqs. (2) and (3) become \( \bar{H}_a \to S \bar{H}_a S^{-1} \) and \( \bar{H}_a \to D(\Lambda) \bar{H}_a D(\Lambda)^{-1} \).

The strong interactions also have an approximate \( SU(2)_L \times SU(2)_R \) chiral symmetry that is spontaneously broken to the vector \( SU(2)_V \) isospin subgroup. This symmetry arises because the light up and down quarks have masses that are small compared with the typical scale of the strong interactions. (If the strange quark is also treated as light the chiral symmetry group becomes \( SU(3)_L \times SU(3)_R \). Associated with the spontaneous breaking of \( SU(2)_L \times SU(2)_R \) chiral symmetry are the pions. The low momentum strong interactions of these pseudo Goldstone bosons are described by a chiral Lagrangian that contains the most general interactions consistent with chiral symmetry. The effects of the up and down quark masses are included by adding terms that transform in the same way under chiral symmetry as the quark mass terms in the QCD Lagrangian.

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The pions are incorporated in a $2 \times 2$ unitary matrix

$$\Sigma = \exp \left( \frac{2iM}{f} \right)$$

(5)

where

$$M = \begin{bmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{bmatrix},$$

(6)

and $f \simeq 132$ MeV is the pion decay constant. Under a chiral $SU(2)_L \times SU(2)_R$ transformation

$$\Sigma \rightarrow L\Sigma R^\dagger,$$

(7)

where $L \in SU(2)_L$ and $R \in SU(2)_R$. It is convenient when discussing the interactions of the $\pi$ mesons with the $P_a$ and $P^*_a$ mesons to introduce

$$\xi = \exp \left( \frac{iM}{f} \right).$$

(8)

Under a chiral $SU(2)_L \times SU(2)_R$ transformation

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger,$$

(9)

where typically the special unitary matrix $U$ is a complicated nonlinear function of $L$, $R$ and the pion fields. However, for transformations $V = L = R$ in the unbroken subgroup $U = V$. We assign the heavy meson fields the transformation law

$$H_a \rightarrow H_b U_{ba}^\dagger,$$

(10)

under chiral $SU(2)_L \times SU(2)_R$ (In eq. (10) and for the remainder of this paper repeated subscripts $a$ and $b$ are summed over 1, 2).
The low momentum strong interactions of pions with heavy $P_a$ and $P_a^*$ mesons are described by the effective Lagrange density$^{[9,10,11]}$

$$
\mathcal{L} = -iT r \bar{H}_a v_\mu \partial^\mu H_a + \frac{1}{2} iT r \bar{H}_a H_b \gamma^\mu (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba} + \frac{1}{2} ig T r \bar{H}_a \gamma_\nu \gamma_5 H_b (\xi^\dagger \partial_\nu \xi - \xi \partial_\nu \xi^\dagger)_{ba} + \ldots ,
$$

(11)

where the ellipsis denote terms with more derivatives. This Lagrange density is the most general one invariant under $SU(2)_L \times SU(2)_R$ chiral symmetry, heavy quark spin symmetry, parity and Lorentz transformations. Heavy quark flavor symmetry implies that $g$ is independent of the heavy quark mass. Note that in eq. (11) factors of $\sqrt{m_P}$ and $\sqrt{m_{P^*}}$ have been absorbed into the $P_a$ and $P_a^{*\mu}$ fields so they have dimension $\frac{3}{2}$.

The coupling $g$ determines the $D^{*+} \to D^0 \pi^+$ decay width

$$
\Gamma(D^{*+} \to D^0 \pi^+) = \frac{1}{6\pi} \frac{g^2}{f^2} |\vec{p}_\pi|^3 .
$$

(12)

Using the measured branching ratio, for this decay$^{[12]}$ and the recent limit on the $D^*$ width$^{[13]}$ gives the bound $g^2 \leq 0.5$.

It is possible to include the symmetry breaking effects of order $m_q$ and $1/m_Q$ into the effective Lagrangian for pion heavy meson strong interactions. Explicit chiral symmetry breaking effects are suppressed relative to the leading correction which we calculate. We include terms of order $m_q$ only through the nonzero pion mass.

The $1/m_Q$ terms that break the spin-flavor heavy quark symmetry give rise to the additional term

$$
\delta \mathcal{L}^{(2)} = \frac{\lambda_2}{m_Q} Tr \bar{H}_a \sigma^{\mu \nu} H_a \sigma_{\mu \nu} + \frac{\lambda_2'}{m_Q} Tr \bar{H}_a H_a + \ldots
$$

(15)

in the Lagrange density. The second term in eq. (15) violates the heavy quark flavor symmetry but not the spin symmetry and the first term violates both the heavy quark
spin and flavor symmetries. The ellipsis denote terms with derivatives. Included in these are, for example, the $1/m_Q$ correction to $g$. The second term in eq. (15) can be removed by phase transformations on the heavy meson fields. Therefore, at the leading order in chiral perturbation theory, it is only the first term in eq. (15) that produces violations of heavy quark spin-flavor symmetry in the low energy heavy meson lagrangian. The effect of $\lambda_2$ is to shift the mass of the pseudoscalar relative to the vector meson by

$$\Delta^{(Q)} = m_{P^*(Q)} - m_{P(Q)}$$

which distinguishes the heavy meson propagators. Explicitly, $\Delta^{(Q)} = -2\lambda_2/m_Q$, which determines $\lambda_2 \approx 300\text{MeV}$. Heavy quark mass suppressed operators in which the pion couples can be neglected at leading order in chiral perturbation theory.

Semileptonic decays $B \to D e\bar{\nu}_e$ and $B \to D^* e\bar{\nu}_e$ can be studied using chiral perturbation theory. It is the hadronic matrix elements of $\bar{c}\gamma_\mu(1 - \gamma_5)b$ that are needed for these decays. This operator is a singlet under $SU(2)_L \times SU(2)_R$ and in chiral perturbation theory its $B(v) \to D(v')$ and $B(v) \to D^*(v')$ hadronic matrix elements are given by those of

$$\bar{c}\gamma_\mu(1 - \gamma_5)b = -\beta(v \cdot v') Tr H^{(c)}_a(v')\gamma_\mu(1 - \gamma_5)H^{(b)}_a(v) + \ldots ,$$

where the ellipsis denote terms with derivatives, factors of $m_q$ and factors of $1/m_Q$. Heavy quark symmetry implies the normalization

$$\beta(1) = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25},$$

at zero recoil. The deviation of $\beta(1)$ from unity arises from calculable perturbative QCD corrections associated with momentum scales between the bottom and charm quark masses. In eq. (18) these perturbative corrections have been summed using the leading logarithmic approximation$^{[14,15]}$
The zero recoil matrix elements

\[ < D^*(v, \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(v) > = 2 \epsilon_\mu \beta(1) \]  \hspace{1cm} (19a)

and

\[ < D(v) | \bar{c} \gamma_\mu b | B(v) > = 2 v_\mu \beta(1) \]  \hspace{1cm} (19b)

receive no corrections at order \( 1/m_Q \). This result (which is often referred to as Luke’s theorem) is important for extracting a precise value of \( V_{cb} \) from semileptonic \( B \)-decays.

In the chiral perturbation theory, the corrections to eqs. (19) arise from the tree level matrix elements of \( 1/m_Q^2 \) suppressed operators. The coefficients of these operators can be estimated, through models or dimensional analysis. In addition there are one-loop Feynman diagrams that give nonanalytic dependence on the pion mass. These one-loop diagrams depend on the subtraction point \( \mu \) and this dependence is canceled by the coefficient of an operator whose symmetry structure is that of the spin symmetry breaking operator inserted twice

\[ O_s = \frac{1}{m_c^2} Tr \sigma^{\alpha\beta} \sigma^{\mu\nu} \bar{H}_{a}^{(c)}(v) \sigma_{\mu\nu} \left( \frac{1 + \gamma_5}{2} \right) \sigma^{\alpha\beta} \left( \frac{1 + \gamma_5}{2} \right) \gamma_\mu (1 - \gamma_5) H_{a}^{(b)}(v). \]  \hspace{1cm} (20)

For simplicity in this letter we neglect effects suppressed by powers of the bottom quark mass.

Including the effects of the Feynman diagram in Fig. 1, one loop wave function renormalization and a tree level “counter term” eq. (19b) becomes

\[ < D(v) | \bar{c} \gamma_\mu b | B(v) > = 2 v_\mu \beta(1) \left\{ 1 + C(\mu)/m_c^2 - \frac{3g^2}{2} \left( \frac{\Delta^{(c)}}{4\pi f} \right)^2 \left[ \ln(\mu^2/m_{\pi}^2) + f(\Delta^{(c)}/m_{\pi}) \right] \right\}, \]  \hspace{1cm} (21)
where
\[
 f(x) = 2 \int_0^\infty dq \frac{q^4}{(q^2 + 1)^{3/2}} \left\{ \frac{1}{[(q^2 + 1)^{1/2} + x]^2} - \frac{1}{q^2 + 1} \right\}.
\] (22)

The function \( f(x) \) is given by
\[
 f(x) = \frac{2}{3} + \pi \left[ -2ix^{-3}(x^2 - 1)^{3/2} - 2x^{-3} + 3ix^{-1}(x^2 - 1)^{1/2} \right]
\]
\[
 + 4x^{-2} + 2x^{-3}(x^2 - 1)^{3/2} \log \left( \frac{(x^2 - 1)^{1/2} - x}{(x^2 - 1)^{1/2} + x} \right)
\]
\[
 - 3x^{-1}(x^2 - 1)^{1/2} \log \left( \frac{(x^2 - 1)^{1/2} - x}{(x^2 - 1)^{1/2} + x} \right).
\] (23)

The expression for \( f(x) \) is valid for all \( x \). However, for \( x < 1 \), it is more sensible to replace the logarithm by an arctangent and to rewrite the square roots so they are real. In the limit of small \( x \), \( f(x) \) can be expanded in \( x \) beginning at order \( x \).

Notice that the loop contribution is proportional to \( \Delta^{(c)} \) because we are calculating a quantity which is protected at order \( 1/m_Q \). Had we calculated wave function or decay constant renormalization for example, there are in general contributions of order \( \Delta^{(c)} \).

In eq. (21), \( C(\mu) \) is the contribution of a tree level “counter term” of order \( 1/m_c^2 \). Its coefficient has subtraction point dependence that cancels that in the logarithm. For \( \mu \) of order the chiral symmetry breaking scale \( C(\mu) \) contains no large logarithms and is, at least formally, smaller than the term which is enhanced by the logarithm of the pion mass. The function \( f \) takes into account the effects of corrections of order \( (1/m_c)^{2+n}, n = 1, 2, \ldots \). It is enhanced by powers of \( (1/m_\pi) \) over terms we have neglected and so should provide a reliable estimate of the \( (1/m_c)^{2+n}, n = 1, 2, \ldots \) effects. Experimentally, \( \Delta^{(c)} = m_\pi \) and the expression in eq. (23) gives \( f(1) = 2(\frac{7}{3} - \pi) \). Numerically, for \( g^2 = 0.5 \) and \( \mu = 1GeV \), the correction from the logarithmically enhanced term is \(-2.1\%\) and the correction from \( f \) is \( 0.9\%\). Although the amplitude appears to be complex, it can be checked that the imaginary part cancels.
Including the effects of the Feynman diagram in Fig. 1, one loop wave function
renormalization and a tree level “counter term” eq. (19a) becomes

\[ < D^*(v, \epsilon)|\bar{c}\gamma_{\mu}\gamma_5 b|B(v)> = 2\epsilon^*_\mu\beta(1) \left\{ 1 + C'(\mu)/m_c^2 - \frac{g^2}{2} \left( \frac{\Delta^{(c)}}{4\pi f} \right)^2 \right\} \left[ \ell_n(\mu^2/m_\pi^2) + f(-\Delta^{(c)}/m_\pi) \right] \].

Here, the scale dependence of \( C'(\mu) \) is 1/3 that of \( C(\mu) \) but because there are two independent counterterms, the “matching” contribution is different from that of \( C(\mu)/3 \). For this current, the one–loop calculation gives an imaginary part to the amplitude because the intermediate pion and \( D \) meson states can be simultaneously on shell. However, because \( \Delta^{(c)} \) is very close to \( m_\pi \), this imaginary part is negligible and it is a good approximation is to use eq. (23) with \( \Delta^{(c)} \) set equal to the pion mass in \( f \). The expression in eq. (23) gives \( f(-1) = 2(\frac{7}{3} + \pi) \). Numerically for \( g^2 = 0.5 \) and \( \mu = 1GeV \) the correction form the “large logarithm” is about \(-0.7\%\) and the correction from \( f \) is about \(-2.0\%\).

In this paper we have used chiral \( SU(2)_L \times SU(2)_R \). The leading order contribution of kaon and eta loops is absorbed into \( C(\mu) \) and \( C'(\mu) \). These constants are independent of \( m_c \) and \( m_\pi \) and are coefficients of two independent \( 1/m_c^2 \) operators, one of which can be taken to be that in eq. (20). If we had used chiral \( SU(3)_L \times SU(3)_R \) symmetry, the kaon and eta loops would have given a smaller contribution than the pion loops.

In this paper we have shown that corrections to the heavy quark symmetry relations in eqs. (19) are computable using chiral perturbation theory. For corrections of order \((1/m_Q)^{2+n}, n = 1, 2, \ldots\) the terms we have kept are enhanced over those that were neglected by a factor of \( 1/m_\pi \) and we have confidence that our calculation accurately takes into account physical effects of this order. Because the pion mass occurs in the denominator the expansion in powers of \( 1/m_Q \) breaks down in the limit where
the pion mass goes to zero. Since $m_\pi$ is about equal to $\Delta^{(c)}$ for $Q = c$ all terms of order $(1/m_c)^{2+n}, n = 1, 2, \ldots$ are of comparable importance.

The factors of the pion mass in the denominator arise from low momentum in the one loop Feynman diagrams. If one matches the heavy quark theory onto the heavy meson chiral Lagrangian then (even for massless pions) the coefficients of operators in the chiral Lagrangian (evaluated at a subtraction point $\mu \simeq 1 GeV$) have a well defined expansion in powers of $1/m_Q$.

It is encouraging from the standpoint of the validity of the heavy quark effective theory that the contribution we calculate is not very large. This is due in part to the factor of $g^2$ and in part due to the partial cancellation between diagrams. However, because the one–loop result is small, the contributions of $C(\mu)$ and $C'(\mu)$, although not logarithmically enhanced, might nevertheless be comparable in magnitude. These other effects need to be estimated in phenomenological models like QCD sum rules and the nonrelativistic constituent quark model.\[16\] The very low momentum effects we have explicitly computed are not represented in such models. Therefore, our results should be added to their predictions.

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Figure Caption

Fig. 1 One loop Feynman diagram contributing to $B \to D^*$ and $B \to D$ transition matrix elements at zero recoil. The dashed line is a pion propagator and the shaded square is an insertion of the current in eq. (17).