INTERSECTION RULES AND OPEN BRANES

RICCARDO ARGURIO
Service de Physique Théorique
Université Libre de Bruxelles, Campus Plaine, C.P.225
Boulevard du Triomphe, B-1050 Bruxelles, Belgium

Abstract. A general rule determining how extremal branes can intersect in a configuration with zero binding energy is presented. It is derived in a model independent way and without explicit use of supersymmetry, solving a set of classical equations of motion. When specializing to M and type II theories, it is shown that some intersection rules can be consistently interpreted as boundary rules for open branes ending on other branes.

1. Introduction

Classical solutions of various supergravities are very interesting to study in the context of string theories and M-theory, because they are often essential in establishing or corroborating the existence of dualities relating (some compactified versions of) the above-mentioned theories [1, 2, 3]. These p-brane solutions (for some reviews on p-branes, see e.g. [4, 5, 6]) provide us with informations on the long-range, low-energy fields produced by objects which live in a more complete theory, i.e. a theory of superstrings or the 'would-be' M-theory. Solutions involving several (classical) branes are thus useful in determining some characteristics of the interactions between the quantum objects, and in putting forward conjectures about the quantum dynamics of the underlying theory.

The problem of studying the interactions between different branes in string and M-theory will be addressed in this contribution considering supergravity solutions which involve intersecting branes. More precisely, we will be concerned with orthogonal intersections of extremal branes. This means that each constituent brane saturates a BPS bound, its mass being equal to its charge in the relevant units. Considered on its own, such a single brane solution would preserve half of the space-time supersymmetries.
The intersecting brane solutions that we will consider are such that the full solution still preserves some (lower) fraction of supersymmetry. This is related to the fact that the binding energy of these configurations vanishes. As we will show hereafter, these solutions are relevant to the study of black hole physics, since they will provide, in the reduced space-time, black holes with non-vanishing Bekenstein-Hawking entropy, but which are nevertheless still supersymmetric and thus much more tractable.

Historically, solutions allowing for a single $p$-brane were presented most generally in [7]. The most important feature of these solutions, in their extreme limit, is that they are characterized entirely by a single harmonic function, which depends on the coordinates transverse to the brane. In spring ’96, Papadopoulos and Townsend [8] reinterpreted some 11 dimensional supergravity solutions found by Güven [9] as intersecting branes and then used this interpretation to build new solutions. Soon after, Tseytlin further generalized in [12] these solutions to include an independent harmonic function for each (non-parallel) brane in the solution. The application of dualities to these particular solutions then predicted a lot of new configurations involving all sorts of branes. For D-branes, these new solutions were compatible with the supersymmetric intersections derived in string theory (see [10, 11]). The main common feature, besides the appearance of the harmonic function associated to each brane, was the vanishing of the binding energy. The rule to build such solutions was simply to ‘superpose’ the single brane solutions. This led to the formulation by Tseytlin of the ‘harmonic superposition rule’ for orthogonally intersecting branes. Still, the dimension of the intersection had to be determined case by case, from supersymmetry arguments and/or by duality. The nature of the argument strongly depended on the type of brane considered (NS-, D- or M-brane).

The outline of the rest of my contribution is as follows. We will first show how to derive the harmonic superposition rule from the equations of motion of a general theory, which models supergravity. Supersymmetry will not be an ingredient of this derivation, though it will (remotely) motivate some of the ansätze made in order to solve the equations. Almost as a byproduct, some of the equations of motion will reduce to a set of algebraic equations determining the dimension of the pairwise intersections of the branes in the configurations (i.e. the ‘intersection rules’). We will then proceed to the tentative deduction of some brane dynamics from these solutions. In the case at hand we will consider the possibility for some branes to open, with boundaries tied to some other brane. For this to work, we have to check that the charge of the open brane is still conserved. The mechanism by which this is done sheds some light on the world-volume effective theory of the brane on which the open brane ends. In the end we speculate on the relevance of closed brane emission by other branes. This talk is based on
the two papers [13, 14], written in collaboration with F. Englert, L. Houart and P. Windey.

2. The Harmonic Superposition and the Intersection Rules

In this section we will derive the harmonic superposition rule and the dimension of the intersection of extreme branes simply solving a set of bosonic equations of motion, provided some particular ansätze are made. A more detailed and step-by-step derivation can be found in [13]. As a starting point we take a general action in $D$ dimensions, which can be the bosonic part of a supergravity action:

$$I = \frac{1}{16\pi G_N^{(D)}} \int d^D x \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \sum_I \frac{1}{2n_I!} e^{a_I \phi} F^2_{n_I} \right), \quad I = 1 \ldots M.$$  

(1)

$M$ is the number of antisymmetric tensor fields, and we take $n_I \leq D/2$ for all field strengths. The metric is written in the Einstein frame, and the coupling of the forms to the dilaton in this frame is entirely governed by the constants $a_I$. Note that since there is only one scalar, this theory is most suitable to model 10 or 11 dimensional supergravities. Generalizations to include several scalars can be found in the literature. Also, we did not include for the moment Chern-Simons terms. A posteriori, they can be shown to play no role in the determination of the classical solutions considered here, but their presence will be crucial when we will discuss the opening of the branes.

We begin now to simplify the problem taking the metric to be of a particular, diagonal, form:

$$ds^2 = -B^2 dt^2 + C_1^2 dy_1^2 + \ldots + C_p^2 dy_p^2 + G^2 dx_a dx_a, \quad a = 1 \ldots D-p-1.$$  

(2)

We have an $SO(D-p-1)$ symmetry left in the ‘overall transverse’ space of the $x$’s, which are taken to be the non-compact directions. Note that there is no a priori $SO(p,1)$ symmetry, and that we will not necessarily have a $p$-dimensional brane in the solution. The $y$’s directions will be eventually compactified. Since the metric is diagonal, we exclude for the moment solutions involving KK waves and monopoles. All functions in the problem depend only on the $x_a$’s.

For the $n$-form field strengths, we have the choice between two different ansätze:

$$\text{Electric} \quad F_{ty_1 \ldots y_n A}^a = \partial_a E_A, \quad (3)$$

$$\text{Magnetic} \quad \tilde{F}_{ty_1 \ldots y_n A}^a = \partial_a E_A, \quad (4)$$
where we have defined the dual field strength by:

\[ \tilde{F}_{\mu_1...\mu_D-n} = \sqrt{-g} e^{a\phi} \epsilon_{\mu_1...\mu_D} F^{\mu_D-n+1...\mu_D}. \]

The space-time charges are thus respectively defined by:

\[ Q^el_A \sim \int \ast F_{qA+2}, \quad Q^{mag}_A \sim \int F_{D-qA-2}. \quad (5) \]

\( A = 1...N \), where \( N \) is the total number of different (non-parallel) branes, electric and magnetic, in the solution. This number can of course exceed the number of different \( n \)-forms.

We can now take the key steps which will enable us to solve quite straightforwardly the equations of motion derived from the theory above. These are the following two ansätze:

- Extremality, which (by experience) is enforced on the metric by the condition:
  
  \[ BC_1...C_p G^{D-p-3} = 1. \quad (6) \]

- No-force condition between the constituent branes (in other words, the requirement that the branes form a BPS marginal bound state). This is translated in our problem in the statement that to each brane is associated one independent harmonic function, and that the solution is completely characterized by these \( N \) harmonic functions.

These two conditions could in principle be found asking the solution to preserve some supersymmetries, i.e. demanding that the equation \( \delta_{\text{SUSY}} \psi = 0 \) has non-trivial solutions. However this cannot be done in this generic set up, i.e. for arbitrary \( D \).

The mathematical implementation of the second ansatz can be motivated as follows. We know that a single brane solution is entirely determined by only one harmonic function. If there are \( N \) branes in the configuration, but there is no binding energy, nothing prevents us from pulling one of the branes apart from the others. Then the fields near that brane should be a good approximation to the fields in the single brane solution. Thus we see that we should expect exactly \( N \) independent functions in the solution.\(^1\)

These \( N \) independent functions are taken to be \( H_A \) such that the \( E_A \)'s in (3) and (4) satisfy:

\[ E_A \sim H_A^{-1}. \quad (7) \]

\(^1\)See [15] for a derivation of the intersection rules based on the application of the no-force ansatz on the effective brane actions, and see [16] for a detailed discussion of the second ansatz and its extension to non-extreme intersecting branes.
Then the equations of motion impose (see [13] for the details) \( \partial_a \partial_a H_A = 0 \), which gives:

\[
H_A = 1 + \sum_k \frac{c_{A,k} Q_{A,k}}{|x^a - x^a_k|^{D-p-3}}. \tag{8}
\]

Solving for the Einstein and the dilaton equations gives the following metric and dilaton:

\[
ds^2 = -\prod_A H_A^{-\frac{2-D-q_A}{2D}} dt^2 + \sum_i \prod_A H_A^{\frac{\delta_A^{(i)}}{2D}} dy_i^2 + \sum_i \prod_A H_A^{\frac{2q_A+1}{2D}} dx_a dx_a, \tag{9}
\]

\[
e^\phi = \prod_A H_A^{\varepsilon_A a_A \frac{2-D-q_A}{2D}}, \tag{10}
\]

where \( \Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2} q_A^2 (D - 2) \), \( \varepsilon_A = +(-) \) if the corresponding brane is electrically (magnetically) charged and \( \delta_A^{(i)} = D - q_A - 3 \) or \(- (q_A + 1)\) depending on whether the direction of \( y_i \) is parallel or perpendicular to the \( q_A \)-brane. Note that \( \Delta_A = 16 \) and 18 for all the branes of, respectively, 10 and 11 dimensional supergravities. To recapitulate, in order to build up a metric according to the harmonic superposition rule, we have to include a factor of \( H_A^{-\frac{2-D-q_A}{2D}} \) in front of each coordinate longitudinal to the \( q_A \)-brane (including the time direction), and a factor of \( H_A^{\frac{2q_A+1}{2D}} \) in front of each transverse coordinate, and this has to be done for each brane in the configuration.

In the process of finding (9) and (10), we did not use the \( R^a_b \) off-diagonal components of the Einstein equations. These have by now reduced to a set of algebraic conditions, that for consistency impose the following pairwise intersection rule for \( q = \dim(\cap) \):

\[
\bar{q} + 1 = \frac{(q_A + 1)(q_B + 1)}{D - 2} - \frac{1}{2} \varepsilon_A a_A \varepsilon_B a_B. \tag{11}
\]

We now point out some remarks.

The formulae above (9), (10) and (11) hold for \( D - p > 3 \), in which case the space is asymptotically flat, as well as for \( D - p = 2 \) or 3, where the equation (8) does not hold any more, i.e. the \( H_A \)'s do not tend to a finite value at infinity. In that cases the solutions have to be considered rather formally. Also for a Euclidean signature the same formulae hold, without the obligation for the time coordinate to be always longitudinal to all the branes (however the electric fields have to be imaginary).

The total mass of these configuration is, as expected, the sum of the masses of each constituent brane, which are equal to the charges: \( M = \sum M_A = \sum Q_A. \)
All the solution above have a functional dependence restricted to the overall transverse space. Some configurations which exist in string theory, as the two D5-branes intersecting over a string (the $\nu = 8$ configurations in [11]) and their duals, are thus excluded since they correspond to the solutions discussed in [17], where the functions depend on the ‘relative transverse’ coordinates.

As already stated above, we did not consider for simplicity non-diagonal metrics. One can nevertheless find the solutions involving KK travelling waves and KK monopoles applying some duality transformation on the solutions above, since all KK charges are related by U-duality to the RR and NSNS charges. A classification of the intersections involving also KK branes can be found in [18].

3. Intersections in String and M-theory and Black Hole Entropy

We can now specialize the formula above (11) to the case of $D = 10$ and 11 maximal supergravities. Actually, this is done straightforwardly specifying $D$ and the dilaton couplings $a_A$. For $D = 11$ we simply have $a = 0$ since there is no dilaton. For $D = 10$ IIA and IIB theories, we have $a = -1$ for the NSNS 3-form field strength and $\varepsilon a = \frac{1}{2}(3 - q)$ for a $q$-brane carrying RR electric or magnetic charge.

As a first application, we will use the metric (9) to derive the number of charges one needs to build up a (supersymmetric) extreme black hole with non-vanishing horizon area in a definite number of non-compact dimensions. Let us define $\bar{D} = D - p$ ($D = 10, 11$). Then in the Einstein frame the Bekenstein-Hawking entropy is proportional to the horizon area defined as follows:

$$S \sim V(\text{compact space})A(S^{D-p-2})_{r=0}.$$ 

Using (9) in the case where all the harmonic functions are centered at the same point $r = 0$ (the horizon), we have:

$$S \sim \prod_A H^{1/2}_A r^{\bar{D}-2} \\
\sim \prod_A Q^{1/2}_A r^{\frac{1}{2}N(D-3)+\bar{D}-2} \\
\sim \prod_A Q^{1/2}_A,$$

the last relation being true only provided the following relation between $N$ and $\bar{D}$ holds:

$$N = 2\frac{\bar{D} - 2}{\bar{D} - 3}.$$ 

This relation has only two integer solutions, which are:

$$\bar{D} = 5, \quad N = 3 \quad \text{and} \quad \bar{D} = 4, \quad N = 4.$$ 

(12)
This also proves that there are no (stringy) extreme black holes with non-zero entropy in $D \geq 6$.

All the solution described in (9)–(11) can be shown to be supersymmetric, and the fraction of preserved supersymmetry is in general at least $1/2^N$. For instance, the $\mathcal{N} = 3$ and $\mathcal{N} = 4$ solutions discussed just above both preserve 1/8 of supersymmetry, thus providing an example and a counterexample to the ‘$1/2^N$ rule’.

We can now summarize all the possible pairwise intersections between the branes which appear in string/M-theory. We use the notation $q_A \cap q_B = \bar{q}$. This rules appeared case by case in the literature, following from rather different arguments, in [19, 20, 15].

In $D = 11$, we have for the M-branes:

\[ 2 \cap 2 = 0, \quad 2 \cap 5 = 1, \quad 5 \cap 5 = 3. \]  
(14)

In $D = 10$, for the intersections between D-branes we have generically $q_1 \cap q_2 = \frac{1}{2}(q_1 + q_2 - 4)$, which gives the following three cases:

\[ q \cap q = q - 2 \]  
(15)
\[ (q - 2) \cap q = q - 3 \]  
(16)
\[ (q - 4) \cap q = q - 4 \]  
(17)

The last case (17) can be interpreted as a D($q - 4$)-brane within a D$q$-brane as in [21].

The intersections involving NSNS branes are:

\[ 1_F \cap 5_S = 1 \]  
(18)
\[ 1_F \cap q_D = 0 \]  
(19)
\[ q_D \cap 5_S = q - 1, \quad 1 \leq q \leq 6 \]  
(20)

where the subscripts $F$, $S$ and $D$ denote respectively fundamental strings, solitonic 5-branes and D-branes.

It is interesting to see how all these intersections come on an equal footing in this framework, while they have a very different origin in the underlying theories.

4. When the Intersection is Actually a Boundary

The second case in (14) and the cases (16), (19) and (20) all have a common feature: the intersection has the same dimension as the (would-be) boundary of one of the two branes. Are we allowed to consider each $p - 1$ dimensional intersection as the boundary of an open $p$-brane tied to the world-volume of the other brane? The case (19) is effectively consistent
with the picture of fundamental strings ending on D-branes, but for the other cases we do not have any quantum description of the phenomenon.

From the supergravity point of view, all these branes appear to be closed. The intersection is not localized in the compact space, rather all the branes are ‘smeared’ over the transverse compact directions. In this sense there is little distinction between closed branes and open branes with both ends joined.

If we want to go deeper into the consideration of open branes, we need some additional input. For the opening of the branes to be consistent, we need a conservation law for the charge carried by the open brane. We will now see that such a conservation law exists, provided the boundaries of the open brane are constrained to live on another brane.

In words, the mechanism goes as follows. The charge carried by the brane is conserved when the brane is open if the boundary carries itself a charge in the effective theory on the world-volume of the brane on which it is constrained. In this way, each brane which can act as a D-brane for other branes (including strings) has an effective world-volume theory whose field content is determined by this mechanism.

It has to be noted that branes ending on other branes were used in [22, 23] and all the following literature to study field theory phenomena, such as dualities, from the dynamics of brane configurations.

There are two complementary approaches to the charge conservation. One is due to Townsend [24] and crucially makes use of the Chern-Simons terms in the supergravity equations of motion. The other is based on the gauge invariance of the open brane world-volume action. We work out here both approaches for a definite example, a D2-brane ending on a D4-brane in IIA string theory. The general case is described in [14].

The equation of motion for the 4-form field strength has to be supplemented by the Chern-Simons term present in the full IIA supergravity action, and by the source due to the presence of the D2-brane. The equation thus reads, neglecting the dilaton and all numerical factors:

\[ d * F_4 = F_4 \wedge H_3 + Q_2 \delta_7. \]  \hspace{1cm} (21)

Since there is also a D4-brane in the configuration, the Bianchi identity for \( F_4 \) is also modified by a source term:

\[ dF_4 = Q_4 \delta_5. \]  \hspace{1cm} (22)

On the other hand, due to the absence of NS5-branes, \( H_3 \) can be globally defined as \( H_3 = dB_2 \). The equation (21) can be rewritten as:

\[ d(*F_4 - F_4 \wedge B_2) = Q_2 \delta_7 + Q_4 \delta_5 \wedge B_2. \]  \hspace{1cm} (23)
We can now integrate both sides of this equation over a 7-sphere $S^7$ which intersects the D2-brane only once (this is possible only if the D2-brane is open). The result is:

$$0 = Q_2 + Q_4 \int_{S^2} \hat{B}_2,$$

where the hat denotes the pull-back to the world-volume of the D4-brane of a space-time field.

We see that the Chern-Simons term indicates the presence on the world volume of the D4-brane of a 2-form field strength, for which the (string-like) boundaries of the D2-branes act as magnetic charges. As we will discuss shortly, the gauge invariant combination is $F_2 = dV_1 - \hat{B}_2$. The presence of the Chern-Simons term in (21) ensures the consistency between charge conservation of the open D2-brane and gauge invariance of the world-sheet action of the open fundamental string. It is essential to note that a CS term exists for each case mentioned above which could lead to the opening of one brane.

Considering now the world-volume action of the D2-brane, we know that there is a minimal coupling to the RR 3-form potential:

$$I_{D2} = Q_2 \int_{W_3} \hat{A}_3 + \ldots$$

When the D2-brane is open, the gauge transformation $\delta A_3 = d\Lambda_2$ becomes anomalous:

$$\delta I_{D2} = Q_2 \int_{(\partial W)_2} \hat{\Lambda}_2.$$ 

The standard way to cancel this anomaly is by constraining the boundary $(\partial W)_2$ to lie on the D4-brane world-volume where a 2-form gauge potential $V_2$, transforming as $\delta V_2 = \hat{\Lambda}_2$, couples to it. The boundary of the D2-brane is now an electric source for the 3-form field strength built out from this potential. Again, the gauge invariant combination is given by $G_3 = dV_2 - \hat{A}_3$.

The analysis of the Goldstone modes of broken supersymmetry and of broken translation invariance, and the requirement that these bosonic and fermionic modes still fit into a representation of the unbroken supersymmetries, forces us to identify the two field strengths $F_2$ and $G_3$ by an electric-magnetic duality on the D4-brane world-volume:

$$F_2 = \ast G_3.$$ 

Moreover, we could have analyzed instead the (more familiar) configuration of a fundamental string ending on the D4-brane. We would have found that its end point behaves like an electric charge for the 2-form field strength and like a magnetic charge for the 3-form field strength. Thus
we conclude that the boundaries of the string and of the membrane are electric-magnetic dual objects on the world-volume of the D4-brane.

Let us now review the outcome of the analysis above when applied to all the cases discussed at the beginning of this section.

− All the $D_p$-branes have a world-volume effective theory which can be formulated in terms of a 2-form field strength $F_2$. The electric charges are the end points of the fundamental strings, while the magnetic charges are the boundaries of the $D(p - 2)$-branes (as in [19]). Note the interesting case of the $D3$-brane on the world-volume of which the $S$-duality between fundamental strings and $D$-strings becomes electric-magnetic duality between their end points. The presence of the 2-form field strength is in this case supported by the quantum stringy computation which gives super-Yang-Mills as the low energy effective action of the $D$-branes.

− On the world-volume of the IIA NS5-brane, we can have the boundaries of the $D2$- and $D4$-brane. The boundary of the $D2$-brane is self-dual and thus couples to a self dual 3-form field-strength, while the boundary of the $D4$-brane couples magnetically to a scalar potential. This scalar potential is nothing else than the 11th direction which remains after reduction of the $M5$-brane action. There is also a limiting case here: from (20) we can see that a $D6$-brane can end on a NS5-brane. In this case however the NS5-brane is the boundary of the $D6$-brane, much in the same way as the $D0$-branes are the end points of fundamental strings. The charge conservation in these cases has to be treated in a somewhat different way, e.g. one needs the presence of $D8$-branes resulting in a non-vanishing cosmological constant, see [25].

− For the IIB NS5-brane, again all the IIB $D$-branes can have boundaries on it. The $D1$- and the $D3$-brane boundaries are respectively the electric and magnetic charge related to a 2-form field strength $\tilde{F}_2 = d\tilde{V}_1 - \tilde{A}_{RR}^2$ which can be considered the $S$-dual of the $F_2$ field on the $D5$-brane. The boundary of the $D5$-brane couples electrically to a (non-propagating) 6-form field strength $G_6$. This 6-form should be related to the mass term in the IIB NS5-brane action as discussed in [26], and could play a role in the definition of an $SL(2, Z)$ invariant IIB 5-brane action. Indeed, by $S$-duality we should also have the possibility of a NS5-brane ending on a D5-brane.

Let us conclude this contribution with some speculations about the emission of closed branes. The idea is to revert to argument which leads, from the intersecting configurations, to the open brane configurations.

Suppose that we have an open $q$-brane with both boundaries tied to the same closed $p$-brane. The world-volume of the open $q$-brane wraps some transverse compact directions in order to have a definite space-time charge.
The two boundaries are, from the point of view of the world-volume of the closed $p$-brane, two opposite charges. This has as a consequence that the configuration is not BPS, and therefore not supersymmetric, since there is a force between these two charges. Now the two opposite charges can meet and annihilate, which from the $q$-brane point of view means that the two boundaries meet and reconnect. The $q$-brane is now closed, and moreover the bound state it constitutes with the closed $p$-brane has vanishing binding energy. Nothing then prevents it to leave the $p$-brane (there is some energy left due to the attraction of the two opposite charges). This is thus a very rough picture of how a $p$-brane could emit other closed branes.

This mechanism has been shown for the emission of closed fundamental strings by D-branes [27]. To describe in a more detailed way the emission of higher branes, a quantum theory of the latter is still lacking. Matrix theory [28] could be a suitable framework to treat this problem (see e.g. [29] for a proposal on open membranes in Matrix theory).

The interest to study these processes is certainly very high. Brane emission could be the dominant process when Hawking radiation is considered at strong coupling, $g_s \gg 1$, or for the emission from black NS 5-branes.

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