Data-Driven Predictive Control for Multi-Agent Decision Making With Chance Constraints

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Abstract—In the recent literature, significant and substantial efforts have been dedicated to the important area of multi-agent decision-making problems. Particularly here, the model predictive control (MPC) methodology has demonstrated its effectiveness in various applications, such as mobile robots, unmanned vehicles, and drones. Nevertheless, in many specific scenarios involving the MPC methodology, accurate and effective system identification is a commonly encountered challenge. As a consequence, the overall system performance could be significantly weakened in outcome when the traditional MPC algorithm is adopted under such circumstances. To cater to this rather major shortcoming, this paper investigates an alternate data-driven approach to solve the multi-agent decision-making problem. Utilizing an innovative modified methodology with suitable closed-loop input/output measurements that comply with the appropriate persistency of excitation condition (rigorously specified in the work here), a non-parametric predictive model is suitably constructed. This non-parametric predictive model approach in the work here attains the key advantage of alleviating the rather heavy computational burden encountered in the optimization procedures typical in alternative methodologies requiring open-loop input/output measurement data collection and parametric system identification (and where also, extremely large signals in the target system could be encountered arising from situations involving the injection of an input signal to, say, an open-loop unstable system). Then with a conservative approximation of probabilistic chance constraints for the MPC problem, a resulting deterministic optimization problem is formulated and solved efficiently and effectively. In the work here, this intuitive data-driven approach is also shown to preserve good robustness properties (even in the inevitable existence of parametric uncertainties that naturally arise in the typical system identification process). Finally, a multi-drone system is used to demonstrate the practical appeal and highly effective outcome of this promising development in achieving very good system performance.

Index Terms—Data-driven control, multi-agent system, path planning, decision making, collision avoidance, probabilistic chance constraint.

I. INTRODUCTION

Highly efficient and effective decision making (as a key element of autonomous systems such as mobile robots, unmanned vehicles, and drones) has certainly been the subject of significant and substantial efforts in the research literature, attracting increasing attention from researchers and engineers [1], [2]. Some core decision-making frameworks include route planning (which plans the best route between the origin and the destination of a trip), and motion planning (which dynamically and adaptively adjusts the control inputs to plan feasible trajectories and avoid collisions). In the literature, different decision-making approaches have indeed been presented and explored. Likely the most common approach would be those that are rule-based, with a purpose to provide the system with a feasible and collision-free path along with traffic rules and boundaries. Additionally, there are also search-based (such as A*, Hybrid A*, D* Lite) and sampling-based (such as RRT, RRT*, PRM, PRM*) approaches that have also received considerable research attention [3], [4]. Nevertheless, a significant shortcoming of these approaches is that they are typically not suitable for complex environments and dynamics, and they are computationally expensive. As a consequence, alternate competing optimization-based methods have also been actively developed (for example, the well-known curve optimization approach of [5], [6]); and with these optimization-based methods, multiple feasible curves are generated, such as Bezier curves, polynomial curves, etc., and then the optimal curve among them is selected. Furthermore, due to the fact that autonomous systems are generally subjected to physical constraints and nonlinear characteristics, the model predictive control (MPC) methodology has been widely used to tackle such resulting constrained sequential decision problems [7], [8]. It is noteworthy that this is likely so, as the MPC approach addresses the trajectory generation problem in a highly effective manner; and system requirements can be explicitly expressed as equality and inequality constraints as part of a control synthesis problem.

In the MPC problem, an optimization problem is solved at each time-step to obtain a sequence of control inputs over a prediction horizon, and this highly relies on the future states predicted by the system dynamic model [9]. In view of its appropriate applicability, various research works present the use of MPC-based methods to generate a feasible solution in trajectory generation problems [10], [11]. Generally, most of these MPC-based works focus on the motion tasks with the system model obtained from appropriate system identification.
processes. However, in certain scenarios where the system identification procedure is rather costly (i.e., computationally) or the parameters cannot be identified accurately (such as in typical applications on robotics [12] and multi-axis coupling systems [13], [14] where there are various practical uncertainties and imperfections), the challenges involved could result in the overall system performance being significantly weakened in outcome. Often-times, such inevitable existence of parametric inaccuracies (from practical uncertainties and imperfections) in the system identification process forms a rather significant impediment to being able to attain highly stringent control precision. Also too, in the interest of robustness, it is noteworthy that several variants of the MPC methodology have been developed to address the decision-making problem in the presence of parametric uncertainties and sensor noises; and thus for instance, appropriate robust model predictive control schemes are extensively studied and exploited in [15], [16]. In most of these robust control approaches, the most prevalent design goal is to find a control policy such that the state of the uncertain dynamic system complies with the predefined constraint set. In other words, the worst-case scenarios are considered therein to determine the control policy [17]; though along this line, such over-conservatism in the robust optimization procedure can unfortunately become a barrier to further enhance the system performance. In contrast to the robust optimization procedure considering the worst-case scenarios, one possible alternative is to instead consider the system state constraints in specific scenarios with stochastic dynamics, wherein probabilistic chance constraints could be appropriately formulated (such as in [18]). In this case then, the state constraints need to be ensured only within a specified confidence level. However, the very involved and detailed probabilistic characteristics required for the comprehensive formulation of probabilistic chance constraints render it difficult to derive a computationally tractable solution.

In addition to the model-based approaches, there have also been several rather promising preliminary studies on data-driven approaches [19]–[21]. Essentially, these data-driven approaches avoid the necessity for very precise system identification; and thus they can rather effectively accommodate the existence of parametric uncertainties [22], [23]. Suitably representative non-parametric predictive control approaches are reported in [24]–[26]. It is also remarkable that the most recent advances involve the development of a so-called data-enabled predictive control (DeePC) algorithm [27], [28], where input/output measurement data are collected by drawing the input sequence from a uniformly distributed random variable. However, a direct (and adverse) consequence that is possibly incurred is the extremely large output signals that are caused in instances where the open-loop is an unstable system; and this indeed further aggravates the computational burden and challenges to derive a reliable solution. These challenges notwithstanding, it is noteworthy that (along similar lines) a data-driven model-free adaptive predictive control method has also been shown in [29], where it can be employed in a class of discrete-time single-input and single-output nonlinear systems. Furthermore, another piece of interesting work in [30] introduces a robust data-driven model predictive control for linear time-invariant (LTI) systems, and the approach suitably assures the exponential stability of the closed-loop system with respect to the noise level. Meanwhile, tremendous efforts have likewise been dedicated to reinforcement learning, which is considered as a paradigm that trains an agent to take optimal actions (as measured by the total cumulative reward achieved) in an environment through interactions [31]–[33]. This reinforcement learning paradigm presents substantial potential to solve some suitably difficult and hard problems, often in an unprecedented way (some of which has been demonstrated in scenarios involving appropriate intelligent transportation systems). However, reinforcement learning is known to be prone to suffer from fatal failure (because trial-and-error is required), and also a large number of data samples are required for training processes.

This paper investigates an alternate data-driven approach to solve the multi-agent decision-making problem. By incorporating probabilistic chance constraints of the system output into the decision-making problem formulation, a constrained optimization problem is thus appropriately constructed. Then, with an adroitly invoked relaxation technique, the probabilistic chance constrained optimization problem is further transformed into a deterministic constrained optimization problem. With this innovation and transformation, a non-parametric routine is thus presented for decision making of multi-agent systems without explicit knowledge of the system model. Compared with the model-based counterpart, it circumvents the necessity of precise system modeling, and thus can effectively accommodate the inevitable presence of parametric uncertainties (arising from, say, various practical uncertainties and imperfections) such that iterative improvements in system performance are possible. With an innovative modified methodology with suitable closed-loop input/output measurements that comply with the appropriate persistency of excitation condition, this non-parametric predictive model approach in the work here also attains the key advantage of alleviating the rather heavy computational burden encountered in the optimization procedures typical in alternative methodologies requiring open-loop input/output measurement data collection and parametric system identification (and where also, extremely large signals in the target system could be encountered arising from situations involving the injection of an input signal to, say, an open-loop unstable system).

The remainder of this paper is organized as follows. Section II formulates the decision-making problem for a multi-agent system with probabilistic chance constraints, and subsequently, a deterministic constrained optimization problem is given. Section III proposes the data-driven predictive control methodology for solving the multi-agent decision-making problem. In Section V, a multi-agent system comprising a group of drones serves as an illustrative example to demonstrate the effectiveness of the proposed method. Finally, pertinent conclusions of this work are drawn in Section VI.

Notations: The following notations are used in this work. $\mathbb{R}^{m \times n}$ denotes the real matrix with $m$ rows and $n$ columns (real column vector with the dimension $n$). $\mathbb{S}^n_+$ denotes the real positive semi-definite symmetric matrix with the dimension $n$. The norm operator based on the inner product
operator is defined by \( \|x\| = \sqrt{x^T x} \) for all \( x \in \mathbb{R}^n \). The operators \( \|\cdot\|_1, \|\cdot\|_2, \) and \( \|\cdot\|_\infty \) denote the \( \ell_1 \)-norm, \( \ell_2 \)-norm, and \( \ell_\infty \)-norm, respectively. \( \Pr(\cdot) \) denotes the probability of an expression. \( \Re(\cdot) \) returns the real part of a number. \( \text{eig}(\cdot) \) denotes the eigenvalues of a matrix. \( \text{diag}\{a_1 \ldots a_n\} \) represents a diagonal matrix with numbers \( a_i, \forall i = 1 \ldots n \) as diagonal entries. The operator \( \otimes \) represents the Kronecker product. \( I_n \) represents the identity matrix with dimensions \( n \times n \).

II. Problem Statement

In this section, a multi-agent decision-making problem is formulated under the MPC with probabilistic chance constraints, and with a relaxation technique, this problem is further transformed into a deterministic constrained optimization problem.

A. Model Predictive Control with Probabilistic Chance Constraints

For a multi-agent decision-making problem, if precise system models of all the agents are available, it is highly attractive to use the MPC controller to realize the control objective. In the classical MPC controller, for all \( \tau = t, t + 1, \ldots, t + T - 1 \), the LTI system dynamics can be expressed as

\[
\begin{align*}
x(\tau + 1) &= Ax(\tau) + Bu(\tau) \\
y(\tau) &= Cx(\tau) + Du(\tau),
\end{align*}
\]

where \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{q \times n}, D \in \mathbb{R}^{q \times m} \) are the state matrix, input matrix, output matrix, and feedthrough matrix, respectively; \( x(\tau) \in \mathbb{R}^n, u(\tau) \in \mathbb{R}^m, y(\tau) \in \mathbb{R}^q \) are the state vector, input vector, and output vector, respectively; \( t \) is the initial time; \( T \) is the prediction horizon.

If the weighting parameters in terms of the state variables and control input variables are chosen to be time-invariant, and a path tracking problem is considered in the objective function, the MPC optimization problem in terms of each agent with a given quadratic objective function can be formulated as

\[
\begin{align*}
\min_{\tau = t} & \sum_{\tau = t}^{t+T} \left( (x(\tau) - r(\tau))^T \hat{Q} (x(\tau) - r(\tau)) + u(\tau)^T \hat{R} u(\tau) \right) \\
\text{subject to} & \quad x(\tau + 1) = Ax(\tau) + Bu(\tau) \\
& \quad \tau = t, t + 1, \ldots, t + T - 1,
\end{align*}
\]

where \( \hat{Q} \in \mathbb{S}_+^q \) and \( \hat{R} \in \mathbb{S}_+^m \) are the weighting matrices for the state variables and control input variables, respectively; \( r(\tau) \) is the reference signal at the time \( \tau \).

To denote the optimization variables in a compact form, we define the optimization variables \( x \) and \( u \) in terms of the state variables and control input variables, respectively, where

\[
\begin{align*}
x &= (x(t + 1), x(t + 2), \ldots, x(t + T)) \in \mathbb{R}^{nT} \\
u &= (u(t), u(t + 1), \ldots, u(t + T - 1)) \in \mathbb{R}^{mT}.
\end{align*}
\]

Next, define the reference vector \( r \) and the weighting matrices \( Q, R \) with respect to the whole prediction horizon as

\[
\begin{align*}
r &= (r(t + 1), r(t + 2), \ldots, r(t + T)) \in \mathbb{R}^{nT} \\
Q &= \text{diag} \{\hat{Q}, \hat{Q}, \ldots, \hat{Q}\} \in \mathbb{R}^{nT \times nT} \\
R &= \text{diag} \{\hat{R}, \hat{R}, \ldots, \hat{R}\} \in \mathbb{R}^{mT \times mT}.
\end{align*}
\]

Then the optimization problem can be equivalently expressed as

\[
\begin{align*}
\min_{\tau = t} & \sum_{\tau = t}^{t+T} \left( (x(\tau) - r(\tau))^T Q (x(\tau) - r(\tau)) + u(\tau)^T R u(\tau) \right) \\
\text{subject to} & \quad x = Gx^t + Hu,
\end{align*}
\]

where \( x^t \) is the initial state variables of the system, \( G \) and \( H \) are the matrices for system dynamic constraints.

Considering the box constraints on the state variables and control input variables, we generalize the single-agent MPC tracking problem to a multi-agent MPC tracking problem with \( N \) agents, which yields

\[
\begin{align*}
\min_{\tau = t} & \sum_{i=1}^N \left( (x_i(t) - r_i(t))^T Q_i (x_i(t) - r_i(t)) + u_i(t)^T R_i u_i(t) \right) \\
\text{subject to} & \quad x_i = G_ix^t + H_iu_i \\
& \quad x_i \in X_i, \\
& \quad u_i \in U_i, \\
& \quad \forall i = 1, 2, \ldots, N,
\end{align*}
\]

where \( X_i \) and \( U_i \) denote the box constraints with respect to the input vector \( u_i \) and state vector \( x_i \).

Finally, to ensure the requirement of collision avoidance in the multi-agent system, probabilistic chance constraints are introduced. Given the safe distance among the agents \( d_{\text{safe}} \) and the confidence level \( \varphi_{ij} \) for the \( i \)th and \( j \)th agents, we have the following MPC problem:

\[
\begin{align*}
\min_{\tau = t} & \sum_{i=1}^N \left( (x_i(t) - r_i(t))^T Q_i (x_i(t) - r_i(t)) + u_i(t)^T R_i u_i(t) \right) \\
\text{subject to} & \quad x_i = G_ix^t + H_iu_i \\
& \quad \text{Pr}(\|M(x_i - Mx_j)\|_2 \leq d_{\text{safe}}) \leq \varphi_{ij} \\
& \quad x_i \in X_i, \\
& \quad u_i \in U_i, \\
& \quad \forall i = 1, 2, \ldots, N, \\
& \quad \forall j = 2, 3, \ldots, N, j > i,
\end{align*}
\]

where \( M \) is a matrix for the purpose of extracting the position state variables from the state vector.

B. Relaxation of Probabilistic Chance Constraints

Due to the fact that probabilistic chance constraints are hard to handle in an optimization problem, the following lemma is given first to convert a general probabilistic chance constraint to a deterministic linear constraint.

Lemma 1. Given a vector \( a \) with appropriate dimensions and a scalar \( b \), for a multivariate random variable \( X(t) \) with the
mean $\mu(t)$ and the covariance matrix $\Sigma(t)$, the probabilistic chance constraint
\[
\Pr(a^TX(t) \leq b) \leq \varphi,
\] (8)
can be equivalently converted to a deterministic linear constraint as
\[
a^T\mu(t) - b \geq \eta,
\] (9)
with
\[
\eta = \sqrt{2a^T\Sigma(t)a}\erf^{-1}(1 - 2\varphi),
\] (10)
where $\varphi$ is the predefined confidence level, and the function $\erf$ represents the standard error function defined as
\[
\erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt.
\] (11)

**Proof of Lemma I.** Given a multivariate Gaussian random variable $X(t) \sim \mathcal{N}(\mu(t), \Sigma(t))$ at time $t$, we define a univariate random variable $Y(t)$ as the perpendicular distance from the plane $a^TX(t) = b$ to the random variable $X(t)$, which means $Y(t) \leq 0$ is equivalent to $a^TX(t) \leq b$.

Besides, we have $Y(t) \sim \mathcal{N}(\mu_Y, \sigma_Y)$ with the mean $\mu_Y = a^T\mu(t) - b$ and the covariance matrix $\Sigma_Y = a^T\Sigma(t)a$. Therefore, $\Pr(a^TX(t) \leq b) \leq \varphi$ can be converted to $\Pr(Y(t) \leq 0) \leq \varphi$. Based on the definition of the probabilistic density function of a Gaussian random variable, it is obvious that $\Pr(Y(t) \leq 0) \leq \varphi$ is equivalent to $\mu_Y \geq \eta$. Then, the proof of Lemma [I] is completed.

Along with the development above, the following theorem is presented, where collision avoidance constraints in the multi-agent decision-making problem are suitably relaxed to deterministic linear constraints. With this technique, it facilitates the use of many numerical tools and solvers to deal with probabilistic chance constraints effectively.

**Theorem 1.** Given $y_i \sim \mathcal{N}(\mu_i, \Sigma_i)$ and $y_j \sim \mathcal{N}(\mu_j, \Sigma_j)$, probabilistic chance constraints in terms of collision avoidance
\[
\Pr(\|y_i - y_j\|_2 \leq d_{\text{safe}}) \leq \varphi_{ij},
\] (12)
can be relaxed to deterministic linear constraints given by
\[
k_{ij}^T(\mu_i - \mu_j) - d_{\text{safe}} \geq \eta_{ij},
\] (13)
with
\[
k_{ij} = \frac{\|\mu_i - \mu_j\|_2}{\|\mu_i - \mu_j\|_2},
\] (14)
and
\[
\eta_{ij} = \sqrt{2k_{ij}^T(\Sigma_i + \Sigma_j)k_{ij}}\erf^{-1}(1 - 2\varphi_{ij}).
\] (15)

**Proof of Theorem I.** To represent the probabilistic chance constraints (12), we define a set
\[
C_{ij} = \{(y_i, y_j) \|y_i - y_j\|_2 \leq d_{\text{safe}}\}.
\] (16)
Then, (12) can be rewritten as
\[
\Pr((y_i, y_j) \in C_{ij}) \leq \varphi_{ij}.
\] (17)
The collision probability of the agent $i$ with the agent $j$ is
\[
\Pr((y_i, y_j) \in C_{ij}) = \int\int_{R^3} \delta_{C_{ij}}(y_i, y_j)p(y_i)p(y_j)dy_idy_j
\] (18)
where $\delta_{C_{ij}}(y_i, y_j)$ is an indicator function on set $C_{ij}$ and $p(y_i)$ and $p(y_j)$ denote the probability of locating at the position $y_i$ and $y_j$, respectively.

Since $y_i \sim \mathcal{N}(\mu_i, \Sigma_i)$ and $y_j \sim \mathcal{N}(\mu_j, \Sigma_j)$, we have
\[
y_i - y_j \sim \mathcal{N}(\mu_i - \mu_j, \Sigma_i + \Sigma_j).
\] (19)
The probability of $(y_i, y_j) \in C_{ij}$ is equivalent to the probability of set intersection between the ellipsoid of the probabilistic position of $y_i - y_j$ and the sphere of the potential collision region with a radius of $d_{\text{safe}}$. Therefore, this collision region can be relaxed as a half space $\hat{C}_{ij}$, which is perpendicular to the radial from origin to the point $y_i - y_j$.

Therefore, it follows that
\[
\hat{C}_{ij} = \{(y_i, y_j) | k_{ij}^T(y_i - y_j) \leq d_{\text{safe}}\},
\] (21)
where
\[
k_{ij} = \frac{\mu_i - \mu_j}{\|\mu_i - \mu_j\|_2}.
\] (22)
Obviously, $C_{ij} \subset \hat{C}_{ij}$, and thus it yields
\[
\Pr((y_i - y_j) \in C_{ij}) \leq \Pr((y_i - y_j) \in \hat{C}_{ij}) \leq \varphi_{ij}.
\] (23)
Then, it gives
\[
\Pr((y_i - y_j) \in \hat{C}_{ij}) = \Pr(k_{ij}^T(y_i - y_j) \leq d_{\text{safe}}).
\] (24)
According to Lemma [1] (24) can be equivalently expressed as deterministic linear constraints
\[
k_{ij}^T(\mu_i - \mu_j) - d_{\text{safe}} \geq \eta_{ij},
\] (25)
with
\[
\eta_{ij} = \sqrt{2k_{ij}^T(\Sigma_i + \Sigma_j)k_{ij}}\erf^{-1}(1 - 2\varphi_{ij}).
\] (26)
This completes the proof of Theorem [I].

In the prediction horizon, the covariance matrix $\Sigma_i$ of variable $y_i$ is generally increasing with time, and this phenomenon is due to the decrease of accuracy in prediction and measurement. Therefore, a scaling factor could be imposed to the covariance matrix, such that the magnitude of the entries in the covariance matrix is increasing with time. Also, several methods available in the literature can be used to predict the covariance matrix [24, 25].

III. DATA-DRIVEN PREDICTIVE CONTROL FOR MULTI-AGENT DECISION MAKING

In this section, the data-driven predictive control approach is presented with detailed analysis. On the basis of input/output measurement data in a closed-loop control scheme, the algorithm is summarized.
A. Input/Output Measurement Data Collection

To find a suitable controller in a data-driven manner, the terminology persistently exciting is commonly used, which reveals that the input signal is sufficiently rich to excite the output signal such that the system’s information can be characterized using the measurement data \([35, 37]\). As follows, the pertinent definition of a sequence which is stated as being persistently exciting is given.

**Definition 1.** A sequence \( u = \{u_k\}^T \) with \( u_k \in \mathbb{R}^m \) is persistently exciting of order \( \lambda \) if the Hankel matrix

\[
\mathcal{H}_\lambda(u) = \begin{bmatrix}
u_1 & \cdots & u_{T-L+1} \\
\vdots & \ddots & \vdots \\
u_L & \cdots & u_T
\end{bmatrix}
\]

has full row rank.

In the remaining text, \( T_p \) denotes the length of the past data, \( T_f \) denotes the length of the future data, \( T_{\text{num}} \) denotes the number of input/output measurements, \( L \) represents the exciting order. Recall that \( n, m, \) and \( q \) represent the number of the state variables, input variables, and output variables, respectively.

To generate the input/output measurement data, we propose the injection of a signal to the input channels. In this work, we consider the system input signal as the noise generated from the injection of a signal to the input channels. In this work, we propose the use of closed-loop input/output measurement data to overcome this shortcoming, where a prescribed controller is designed to make sure the closed-loop system is stabilizing.

Typically, injecting a random signal to an unstable open-loop system leads to a significantly large output signal, which unpleasantly increases the condition number of the Hankel matrix. Essentially, a large condition number can dramatically slow down the convergence of the optimization process in the following numerical procedures, and cause inexact solution due to the existence of numerical errors. Thus, in this paper, we propose the use of closed-loop input/output measurement data to stabilize the closed-loop system, where a prescribed controller is designed to make the closed-loop system stabilizing.

**Theorem 2.** Assume the persistency of excitation assumption holds and there exists a controller \( K \) stabilizing the closed-loop system, the system input during the data measurement process can be denoted by \( u_d = K y_d + u_r \), where \( u_r \) is a random system input vector to ensure the full row rank of the Hankel matrix. To facilitate the development of the data-driven approach, the following theorem is introduced.

\[
y_d = G_c u_r \\
u_d = K y_d + u_r
\]

where \( u_r \) is a random system input vector to ensure the full row rank of the corresponding Hankel matrix. Then the Hankel matrix with respect to the measurement data \( (u_d, y_d) \) spans the whole behavior space.

**Proof of Theorem 2** To prove Theorem 2 the following lemma is introduced, which serves as the basis to derive many typical data-driven predictive control methodologies.

**Lemma 2.** \([38]\) Consider a controllable system \( B \in \mathcal{L}^w \). Let \( \tilde{u} : [1, T] \rightarrow \mathbb{R}^{m(B)}, \tilde{y} : [1, T] \rightarrow \mathbb{R}^{q(B)}, \) and \( \tilde{w} = [\tilde{u}^T \quad \tilde{y}^T]^T \). Assume that \( \tilde{w} \in \mathbb{B}[[1, T]] \). Then, if \( \tilde{u} \) is persistently exciting of order \( \lambda + n(B) \), it yields that \( \text{colspan}(\mathcal{H}_\lambda(\tilde{w})) = \mathcal{B}[[1, L]] \), where \( n(\cdot), m(\cdot), \) and \( q(\cdot) \) denote the state cardinality, input cardinality, and output cardinality of a system, respectively.

From Lemma 2 under the persistency of excitation assumption, the whole behavior space can be expressed as the product space of two sub-behavior spaces in terms of the input signals and output signals, respectively. Furthermore, it means that all trajectories can be constructed from a finite number of past trajectories. Then it is straightforward to prove Theorem 2.

Notably, even though the precise system model is not available, there are still various approaches that can be readily implemented to stabilize an unstable system, such as the well-known proportional-integral-derivative (PID) controller and
many robust controller design approaches. It is straightforward
to see that the output $y_d$ of the resulting closed-loop system
$(A + BK C, B, C, D)$ is bounded because $u_r$ is bounded and\n$\text{Re}(\text{eig}(A + BK C)) < 0$. In this work, we will use the
controller $u_d = Ky_d + u_r$ for illustrative purposes.

**Remark 2.** The data included in the Hankel matrix with
respect to the original open-loop system is from the mea-

sured data. Hence, the columns of the Hankel matrix are linearly independent.

**B. Data-Driven Predictive Control Algorithm**

As the main results of the data-driven predictive control
approach, the following theorem is given.

**Theorem 3.** Under the persistency of excitation assumption,
and with $W = \begin{bmatrix} U_p^T & Y_p^T & U_f^T & Y_f^T \end{bmatrix}^T$, the following
input/output relationship can be established:

$$ Wg = \begin{bmatrix} u_p^T & y_p^T & u^T & y^T \end{bmatrix}^T, \quad (33) $$

where $u_p$ and $y_p$ denote the most recent input signal and
output signal, respectively, $u$ and $y$ represent the optimal
control input sequence and the corresponding output sequence,
respectively. Then there exists a unique decision variable $g \in \mathbb{R}^{T_{nn} - T_p - T_f + 1}$
such that (33) holds, under the condition that no measurement noise exists in the input/output measurement data.

**Proof of Theorem 3.** First, the following lemma is introduced
to complete the proof.

**Lemma 3.** Suppose $\begin{bmatrix} u^d, y^d \end{bmatrix} = \begin{bmatrix} u_k^d, y_k^d \end{bmatrix}^T$ is a trajectory
of an LTI system $G_o$, where $u^d$ is persistently exciting of order
$L + n$. Then, $\begin{bmatrix} \bar{u}, \bar{y} \end{bmatrix} = \begin{bmatrix} \bar{u}_k, \bar{y}_k \end{bmatrix}^T$ is a trajectory of $G_o$ if and
only if there exists $\alpha \in \mathbb{R}_{T_{nn} - T_p - T_f + 1}$ such that

$$ \begin{bmatrix} \mathcal{H}_L(u^d) \\ \mathcal{H}_L(y^d) \end{bmatrix} \alpha = \begin{bmatrix} \bar{u} \\ \bar{y} \end{bmatrix}. \quad (34) $$

Under the condition that no measurement noise exists in the
input/output measurement data, it is straightforward that
the column span of the Hankel matrix with respect to the
input/output measurement data is the whole behavior space,
and then it follows from Lemma 3 that the vector $g$ always
exists. Since the signal $u_d$ is persistently exciting of order
$T_p + T_f + n$, it is also straightforward that the vector $g$
is unique. This completes the proof of Theorem 3.

Consequently, the optimization problem is formulated as

$$ \min \sum_{i=1}^{N} \left( (\mu_i - r_i)^T Q_i (\mu_i - r_i) + u_i^T R_i u_i \right) $$

subject to

$$ Wg = \begin{bmatrix} u_p^T & y_p^T & u^T & y^T \end{bmatrix}^T $$

$$ k_{ij}^T (\mu_i - \mu_j) - d_{\text{safe}} \geq \eta_{ij} $$

$$ k_{ij} = \frac{\|\mu_i - \mu_j\|_2}{\|\|d_{\text{safe}}\|_2} $$

$$ \eta_{ij} = \sqrt{2k_{ij}^T (\Sigma_i + \Sigma_j) k_{ij}} \cdot \text{erf}^{-1} (1 - 2\varphi_{ij}) $$

$$ \mu_i \in \mathcal{Y}_i $$

$$ u_i \in \mathcal{U}_i $$

$$ \forall i = 1, 2, \cdots, N $$

$$ \forall j = 2, 3, \cdots, N, j > i, \quad (35) $$

where the feasible set $\mathcal{Y}_i$ denotes the box constraints on the
system output.

**Remark 3.** To ensure the existence of the vector $g$ in the
presence of real-time measurement noise in the input/output
measurement data, the objective function in (33) can be modified by adding a weighted 1-norm of a slack variable,
i.e., $\lambda_o \|g\|_1$, where more details are discussed in [27].

To summarize the above descriptions and discussions, Al-
grithm 1 is given. In this algorithm, three major steps are
necessary to carry out. The first step aims at designing a
simple controller to stabilize the original open-loop system.
The second step focuses on the construction of the matrix
$W$ based on offline input/output measurement data. The third
step completes the predictive controller design and realizes the
control objectives.

**IV. ILLUSTRATIVE EXAMPLE**

To clearly demonstrate the effectiveness of the proposed
approach, an illustrative example of a multi-drone system is
used. In this example, the number of drones is set as $N = 8$,
and each drone is controlled by 4 motors. First, the state-space
model of each drone is given by

$$ x(t + 1) = Ax(t) + Bu(t) $$

$$ y(t) = Cx(t) + Du(t), \quad (36) $$

where $x = [p_x, p_y, p_z, \dot{p}_x, \dot{p}_y, \dot{p}_z, \omega_x, \omega_y, \omega_z, \dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z]^T$,
$u = [u_1, u_2, u_3, u_4]^T$. In the state vector $x$, $p_x, p_y, p_z$ represent
the spatial coordinates, $\omega_x, \omega_y, \omega_z$ represent the angular
coordinates, $u_1, u_2, u_3, u_4$ denote the thrust of 4 motors. It
is assumed that all the state variables are measurable. $C$ is an
identity matrix, $D$ is a zero matrix, and the details of matrices
**Algorithm 1** Proposed Algorithm on Data-Driven Predictive Control for Multi-Agent Decision Making

**Require:** For all \(i = 1, 2, \ldots, N\), \(j = 2, 3, \ldots, N\), \(j > i\), initialize the parameters \(Q_i, R_i, \varphi_{ij}, \Sigma_i, d_{s\text{afe}}, T_{\text{num}}\). Given the initial system input \(u_{p0}\) and initial system output \(y_{p0}\).

1. **Step 1:** Design a controller \(K\) that stabilizes the closed-loop system.
2. **Step 2:** Construct the matrix \(W\).
3. for \(k = 1, 2, \ldots, T_{\text{num}} - T_p - T_f + 1\)
4. Inject random signal \(u_r\) into the closed-loop system.
5. Measure the system output \(y_d\).
6. Compute the input of the original open-loop system \(u_d = K y_d + u_r\) and partition \(u_d = [u_p^T, u_f^T]^T, y_d = [y_p^T, y_f^T]^T\).
7. Construct the matrix \(W\), where the \(k\)th column of the matrix \(W\) is \(W_k = [u_p^T, y_p^T, u_f^T, y_f^T]^T\).
8. end for
9. **Step 3:** Implement the iterative predictive control.
10. for \(\tau = t, t + 1, \ldots, t + T\)
11. if \(\tau = t\)
12. Set \(u_p = u_{p0}\) and \(y_p = y_{p0}\).
13. else
14. Set \(u_p\) to be the most recent system input and \(y_p\) to be the most recent system output.
15. end if
16. Solve the optimization problem \([35]\), determine the optimal \(g^*, u^*, y^*\).
17. Inject the first group of system input in \(u^*\) to the original open-loop system, and measure the system output \(y\).
18. end for

\[A, B \text{ are given as follows:}
\begin{align*}
A &= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.049 & 0 & 0 & 0.0016 & 0 \\
0 & 1 & 0 & 0 & 0.049 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\end{align*}
\begin{align*}
B &= 
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\end{align*}
\] (37)

During the input/output measurement procedures, a feedback controller is implemented to stabilize the closed-loop system, and then the matrix \(W \in \mathbb{R}^{496 \times 184}\) is effectively derived for each agent. Because the dimension of \(W\) is quite large, the results will not be displayed in this paper. However, to show the effectiveness of closed-loop input/output measurements, a comparison is carried out by computing \(\|W\|_2\) and \(\|W\|_{\infty}\), where the detailed results are depicted in Table I. In this table, our proposed approach is denoted as \(\|W\|_2\) and \(\|W\|_{\infty}\) obtained in our approach are apparently smaller, and it is certainly because the system is stabilized in the input/output measurement procedure, and this phenomenon aligns well with our claims in Sec. III.

The parameters for the optimization problem \([35]\) are given as follows. For all \(i = 1, 2, \ldots, N\), the weighting matrix \(Q_i\) is chosen as \(Q_i = I_T \otimes \text{diag}(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0)\), and \(R_i\) is chosen as a zero matrix, which means only the tracking error is penalized in the objective function. \(d_{s\text{afe}}\) is chosen as \(0.3\) m. \(\Sigma_i\) is chosen as \(0.01 I_{12}\) for all agents \(i = 1, 2, \ldots, N\). \(\varphi_{ij}\) is chosen as \(0.1\) for all agents \(i = 1, 2, \ldots, N\). \(j > i\). Moreover, there is no constraint imposed on the system output. The lower bound and upper bound of the control input are chosen as \(-0.7007\) N and \(0.2993\) N, respectively. For this problem, \(T_p = 1, T_f = 30\), and thus the input is persistently exciting of order \(L = T_p + T_f + n = 43\). Then, the minimum number of input/output measurements is \(T_{\text{min}} = (m + 1) \times L - 1 = 214\). Here, we choose \(T_{\text{num}} = 214\). The sampling time is chosen as \(0.1\) s. For demonstrative purposes, the initial positions of 8 agents are located at different vertices of a cube, and each agent aims to move towards its corresponding diagonal vertex without making any collision. For example, the destination of the agent that initially located at \((-1\text{ m}, -1\text{ m}, -1\text{ m})\) is considered as \((1\text{ m}, 1\text{ m}, 1\text{ m})\).

Table I

| Agent | \(\|W\|_2\) | \(\|W\|_{\infty}\) |
|-------|---------|---------|
| Method 1 | Method 2 | Method 1 | Method 2 |
| 1     | 110.75  | 56406.18 | 273.59  | 110198.05 |
| 2     | 111.82  | 51756.98 | 275.64  | 112595.50 |
| 3     | 111.29  | 83346.61 | 275.10  | 132569.32 |
| 4     | 110.41  | 54450.31 | 271.37  | 116890.37 |
| 5     | 111.07  | 69652.93 | 273.48  | 114865.41 |
| 6     | 107.79  | 71311.66 | 267.50  | 115348.90 |
| 7     | 107.01  | 88463.50 | 265.75  | 149047.45 |
| 8     | 112.06  | 65681.44 | 279.10  | 117736.91 |

The simulation is implemented in the environment of Python 3.7 with two processors Intel(R) Xeon(R) CPU E5-2695 v3 @ 2.30GHz. The constrained data
driven predictive control optimization problem is solved by GUROBI. By implementing the controller determined from the proposed approach, real-time trajectories of all the agents are illustrated in Fig. 1. The 3D illustration clearly shows that the control objective in terms of path tracking is achieved, and no collision occurs in the whole process. To clearly visualize the paths of the agents, Fig. 2(a) and Fig. 2(b) present the 2D views of the real-time trajectories, where the top view and side view are depicted separately. Furthermore, Fig. 3 presents the distances between each pair of agents, and it is obvious that all the agents are strictly constrained within the prescribed safe distance. Additionally, Fig. 4 and Fig. 5 present the thrust of the 4 motors for all the agents. It can be observed that the box constraints in terms of the control input are well guaranteed. With the descriptions and pertinent analysis above, the effectiveness of the proposed approach is appropriately demonstrated.

V. CONCLUSION

In this work, a data-driven predictive control approach is presented and investigated to solve the multi-agent decision-making problem. The resulting MPC problem is formulated; and by incorporating probabilistic chance constraints of the system output into the decision-making problem formulation, a constrained optimization problem is thus appropriately constructed. Then, with an adroitly invoked relaxation technique, the probabilistic chance constrained optimization problem is further transformed into a deterministic constrained optimization problem. With this innovation and transformation, a non-parametric routine is thus presented for decision making of multi-agent systems without explicit knowledge of the system model. In the approach here, a finite data set is collected offline within a closed-loop control framework, and an optimization problem is iteratively solved (without the requirement of knowledge of the system model). With this framework, the optimal solution to the decision-making problem is efficiently and effectively determined. Finally, a multi-drone system example is introduced for validation purposes, and the simulations results clearly demonstrate that the data-driven approach in the work here is rather effective in the decision-making problem. It is also worthwhile to mention that applications of the proposed methodology are not merely limited to the multi-drone system only; and certainly, it can be further suitably deployed to other decision-making problems (such as unmanned vehicles and mobile robots) where accurate system models are difficult or costly to obtain.
Fig. 3. Distances between each pair of agents.

Fig. 4. Control inputs of the first four agents.

Fig. 5. Control inputs of the last four agents.

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