Spectrum and scaling in a strongly coupled fermion-gauge-scalar model

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The strongly coupled lattice gauge models show an interesting mechanism of dynamical mass generation. If a suitable continuum limit can be found, one may think of it as an alternative to the Higgs mechanism. We present data on the spectrum, obtained in the model with U(1) gauge symmetry with dynamical fermions. They indicate that the fermion mass scales in the vicinity of the whole chiral phase transition line. In contrast to this, the composite scalar boson mass seems to get small only in the region near the endpoint E of the Higgs phase transition. Thus this point is the most interesting candidate for approaching the continuum limit. The masses of fermion–antifermion bound states are also discussed.

1. A Fermion-Gauge-Scalar model

It seems to be a generic feature of strongly coupled gauge-theories to give fermions a mass due to dynamical chiral symmetry breaking. Some models use this fact to explain a heavy fermion mass generation (see \cite{1} for references). In our current work, we try to consider such a possibility by examining a model with U(1) gauge symmetry.

We use three fields in our HMC simulation:

- a compact U(1) symmetric gauge-field $U_{x,\mu}$ with gauge coupling $\beta = \frac{1}{g^2}$,
- a charged staggered fermion field $\chi_x$, describing 4 fermions in the continuum and
- a charged complex scalar field $\phi_x$ with fixed length and hopping parameter $\kappa$.

For technical reasons we introduce a bare fermion mass $m_0$, although the model is meant in the chiral limit $m_0 = 0$. The precise action and a schematic phase diagram can be found in \cite{1}. All calculations presented in this paper are obtained with dynamical fermions.

As described in some detail in \cite{1}, this model has for strong gauge coupling a chiral-phase-transition ($\chi$PT) line. Part of this line also includes a Higgs phase transition and is of first order. On the left of the endpoint $E$ of this Higgs phase transition (at small $\beta$) the line is of second order (NE line). The chiral condensate vanishes on the whole line.

In this paper we want to discuss the spectrum. In the 2\textsuperscript{nd} section we will explain the basic properties of the spectrum. Some more details can be found in \cite{1}. In the 3\textsuperscript{rd} section we then show our first numerical results. More results will be included in a forthcoming paper \cite{1}.

2. Spectrum

2.1. Fermions

Because we consider the model in the confinement region, only neutral particles survive in the spectrum. To shield the charge we construct a composite fermion $F = \phi^+\chi$. Due to the strong gauge coupling it is strongly bounded. Here one sees the important role of the scalar field. It protects the fermions from getting confined. Think of those fermions being a heavy quark (top).

2.2. Mesons

As in QCD there is a large number of fermion–antifermion bound states which we call by analogy mesons. Up to now we only looked for those described by local operators, which are the first four entries in the famous Golterman-tables \cite{5}:

$$O^{ik}(t) = \sum_{\vec{x}, \vec{t}} s_{\vec{x}, \vec{t}} \bar{\chi}_{\vec{x}, \vec{t}} \chi_{\vec{x}, \vec{t}}$$  \hspace{1cm} (1)
The sign factors $s^\mu_\nu$ are composed of the standard staggered phase factors $\eta_{\mu x} = (-1)^{x_1+\cdots+x_{\mu-1}}$, $\xi_{\mu x} = (-1)^{x_{\mu+1}+\cdots+x_4}$ and $\varepsilon_x = (-1)^{x_1+\cdots+x_4}$.

The same continuum particles show up in different operators. Within large error bars we did not observe up to now any contradiction to the flavour symmetry restoration.

If one replaces the Higgs-sector of the standard model, the $\sigma$-meson would be the candidate for a composite Higgs-boson. The $\pi$-meson would be the Goldstone boson, which would later be eaten by the electro-weak gauge bosons.

### 2.3. Other Bosons

We are further interested in the scalar and vector bosons, which are also present in the $U(1)$ sector without fermions. The corresponding operators are

\[
O^{(S)}(t) = \frac{1}{L^3} \sum_{x} \text{Re} \left\{ \sum_{i=1}^{3} \phi_{x,t}^i U(x,t), \phi_{x+\hat{t}}^i \right\},
\]

\[
O^{(V)}(t) = \frac{1}{L^3} \sum_{x} \text{Im} \left\{ \phi_{x,t}^1 U(x,t), \phi_{x+\hat{t}}^i \right\}, \quad i = 1, 2, 3.
\]

Because of the fixed length of the scalar field, $\phi_{x}^i$ is a trivial observable. Therefore one uses the suitable link products even in the scalar case.

In principle we expect, that in the meson- and boson-operators with the same quantum numbers also the same particles show up, but we couldn’t confirm this up to now.

### 2.4. Theoretical Considerations

For $\beta = 0$ the Lee-Shrock-transformation shows that this model is equivalent to the Nambu-Jona-Lasinio model. It is known to have a $\chi$PT (point $N$) and dynamical mass generation in the chirally broken phase. On the other hand this model is non renormalizable and a continuum limit is not possible.

In the case $\kappa \to 0$ states with $\phi$, like the fermion $F$, become infinitely heavy, whereas in the case $\kappa = \infty$ the fermion is free and $m_F = 0$. It is very likely that $m_T = 0$ in the whole chirally symmetric phase. Therefore it is of great interest to look for the scaling near the $\chi$PT-line. On the right side of $E$, where the phase transition is first order, no scaling can be achieved. On the left hand side of $E$ the question is open. We expect that the $\chi$PT of the NJL model continues to $\beta > 0$. So it may also be in the same universality class and the model non renormalizable. But at the point $E$ the universality class probably changes, and thus the renormalizability might improve. Therefore our greatest interest concerns the region near $E$.

### 3. Numerical results

The calculations have been performed on $6^3 \cdot 16$ and $8^3 \cdot 24$ lattices for different $\beta$ and $\kappa$ near the endpoint $E$ and for some intermediate $\beta$ on the NE line.

#### 3.1. Fermions

We observe in the whole chirally symmetric phase small fermion masses, which can be linearly extrapolated in the bare fermion mass to very small values. We expect that those small values are finite size effects and the values are consistent with $0$.

In the broken phase the masses increase very fast with decreasing $\kappa$, the closer to the point $E$ the faster. In the first order regime the fermion mass jumps to a nonzero value. In figure the measured fermion masses near the endpoint $E$ are shown.

#### 3.2. Bosons

In contrast to fermions and mesons, the bosons have only little dependence on the bare mass, but large finite size effects can be observed.

The mass of the scalar boson $m_S$ shows a minimum at the point of the phase transition. For large $\beta$ this is as a signal that chiral and Higgs phase transitions coincide. For small $\beta$, where no Higgs phase transition is present, this might be
the effect of a cross over. The smaller $\beta$ is, the larger the mass $m_\sigma$ and the less pronounced the minimum is.

We see a global minimum for the boson mass at $(\beta, \kappa) \approx (.65, .31)$. Its value on the $6^3\cdot16$ is 0.6(1) and on the $8^3\cdot24$ lattice 0.4(1). Measurements on larger lattices are required to show, whether this mass vanishes on an infinite lattice, what might be expected. The coordinates of this point coincides within good precision with those of the endpoint $E$ determined by the local observables.

Figure 2 shows scalar and vector boson mass for $\beta = .64$, where we have most data. In this figure it can also be seen that the vector boson doesn’t scale.

3.3. Mesons

Until now we’d payed our main interest to $\sigma$- and $\pi$-meson. The $\sigma$ is very hard to measure. We don’t have enough statistics and sufficiently good understanding of the first operator to present conclusive results.

In the chirally broken phase the $\pi$-meson should behave like a Goldstone boson. This can be checked by $(am_\pi)^2$ linearly going to zero. In we show a figure, which demonstrates this.

4. Summary

We have shown, that the model has in the broken chiral symmetry phase essentially the expected properties. The results for the fermion and boson masses look very promising. Also the $\pi$-meson seems to behave like a Goldstone boson in the broken phase. The very interesting mass of the $\sigma$-boson, which would be the composite Higgs, couldn’t be determined up to now.

The calculation have been performed on the HLRZ Cray Y-MP8/864 and the ‘Landesvektorrechner’ of NRW SNI/Fujitsu VPP 500.

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