Chiral dynamics and pion-nucleon scattering around the $N^*(1535)$ resonance

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(Received: November 21, 2018)

We study here the S-wave interaction of mesons with baryons in the strangeness $S = 0$ sector in a coupled channel unitary approach. See ref. 3 for details.

1 Introduction

The various meson-baryon scatterings are studied in a chiral unitary model with the Bethe-Salpeter equation, where the on-shell amplitudes are factorized and the BS equation is reduced to an algebraic equation 3-5. The scattering matrix at the total center of mass energy $\sqrt{s}$, is given by

$$T(\sqrt{s}) = [1 - V(\sqrt{s})G(\sqrt{s})]^{-1}V(\sqrt{s})$$

(1)

where $V$ is a transition potential matrix and $G$ is a diagonal matrix representing the loop integral of a meson and a baryon. The matrix $V$ is taken from the lowest order chiral Lagrangian involving mesons and baryons. For example, in the case of S-wave, $V$ is written as

$$V_{ij}(\sqrt{s}) = -C_{ij} \frac{1}{4f^2} \bar{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu)$$

(2)

with the meson weak decay constant $f$, and the initial(final) meson momentum $k$ ($k'$). The coefficients $C_{ij}$ which reflect the flavor symmetry of the problem, are obtained from the Lagrangian. The divergent loop integral $G$ defined by

$$G_i(\sqrt{s}) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_i}{(P - q)^2 - M_i^2 + i\epsilon} \frac{1}{q^2 - m_i^2 + i\epsilon}$$

(3)

with $P \equiv (\sqrt{s}, 0)$ and the baryon(meson) mass $M_i(m_i)$, is done with some regularization. The finite contribution of the renormalization which appears in the real part of $G_i(\sqrt{s})$, is treated as an unknown parameter and determined through the fitting to the data. The imaginary part of $G_i(\sqrt{s})$ is proportional to the phase space and ensures unitarity. For example, with the dimensional regularization, the integral is calculated as

$$G_i(\sqrt{s}) = \frac{2M_i}{(4\pi)^2} \left\{ a_i(\mu) + \log \frac{m_i^2}{\mu^2} + \frac{M_i^2 - m_i^2 + s}{2s} \log \frac{M_i^2}{m_i^2} \right\}$$

(4)

where $a_i(\mu)$ is the contribution of the higher order counter terms and $Q_i(\sqrt{s})$ is the on-shell center of mass momentum of $i$-th meson-baryon system 3.

To study the pion-nucleon scattering, the six coupled meson-baryon systems, \{ $\pi^-p$, $\pi^0n$, $\eta n$, $K^+\Sigma^-$, $K^0\Sigma^0$, and $K^0\Lambda$ \}, are considered. It is found that qualitatively good S-wave scattering amplitudes are obtained with appropriate values of $a_i(\mu)$ 3. However, in this simple model, quantitative agreement is not achieved, particularly in the isospin 3/2 sector. In this paper, we improve this model and try to reproduce the $\pi N$ scattering in a wide energy range.
2 VMD inspired chiral coefficients and the $\pi\pi N$ channels

First, we recall that the S-wave meson-baryon amplitudes from the lowest order chiral Lagrangian are equivalent to the amplitudes of vector meson exchange in the t-channel, in the vector meson dominance (VMD) hypothesis. This indicates that the Lagrangian is the effective manifestation of the vector meson exchange mechanism. According to this consideration, we introduce the following correction to the chiral coefficient to account for the momentum transfer dependence of the vector meson propagator

$$
C_{ij} \rightarrow C_{ij} \times \int \frac{d^4k'}{4\pi} \frac{-m^2_{n}}{(k' - k)^2 - m^2_{v}} \quad \text{at} \quad \sqrt{s} > \sqrt{s^0_{ij}} \tag{5}
$$

where $\sqrt{s^0_{ij}}$ is the energy where the integral of (5) is unity, and which appears in between the thresholds of the two $i, j$ channels. At low energies, this correction is negligible but this is not the case at the intermediate energies studied here. In fact the $\rho$ meson tail, with $m_{\rho} = 770$ MeV, reduces the $\pi^-p \leftrightarrow \pi^-p$ element about 25% at energies around 1500 MeV.

Second, we extend our model to include the $\pi\pi N$ channels and $\pi N \leftrightarrow \pi\pi N$ transitions. We consider the BS equation (1) with the eight coupled channels including the six meson-baryon channels and two $\pi\pi N$ channels, $\pi^0\pi^-p$ and $\pi^+\pi^-n$. The $\pi^0\pi^0 n$ channel is not included because it does not couple to the S-wave $\pi N$. We treat the transition potentials of S-wave $\pi N \leftrightarrow \pi\pi N$ as free inputs and determine them so that they account for the data. We introduce a two loop integral $\tilde{G}(\sqrt{s})$ for the intermediate $\pi\pi N$ state

$$
\tilde{G}(\sqrt{s}) = \frac{1}{i} \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{2M_N}{(P - q_1 - q_2)^2 - M^2_N + i\epsilon} \frac{1}{q_1^2 - m^2_{\pi} + i\epsilon} \frac{1}{q_2^2 - m^2_{\pi} + i\epsilon} \tag{6}
$$

which includes the vertex structure for the factorization. The real part of $\tilde{G}(\sqrt{s})$ in the renormalized model has several subtraction terms to be fixed by the data and we find it to be compatible with zero.

3 Results and discussions

By varying the input $\pi N \leftrightarrow \pi\pi N$ potential and the subtractions in the real part of loop integrals, we find a reasonable set of them which reproduce both the elastic $\pi N$ scattering and the pion production cross sections simultaneously. The subtraction parameters $a_i(\mu)$ for the meson-baryon loop integrals which we obtain are

$$
\mu = 1200 \text{ MeV}, \quad a_{\pi N}(\mu) = 2.0, \quad a_{\eta N}(\mu) = 0.1, \quad a_{K\Lambda}(\mu) = 1.5, \quad a_{K\Sigma}(\mu) = -2.8 . \quad (7)
$$

The determined S-wave $\pi N \leftrightarrow \pi\pi N$ amplitudes $a_{11}$ (isospin 1/2) and $a_{31}$ (isospin 3/2) are shown in Fig.1 where we compare the real part of them with two empirical ones [7,8]. The lower energy part of the our $a_{11}(\sqrt{s})$ agrees with that of the paper [8] but it is quite different from the one of [7].

Figure 1: The S-wave $\pi N \leftrightarrow \pi\pi N$ amplitudes $a_{11}$ and $a_{31}$. The solid lines are the real part obtained in the present model. The dashed and dotted lines are those of [7] and [8] respectively.
Figure 2: Scattering amplitudes, phase-shifts and inelasticities for the $S_{11}$ and $S_{31}$ $\pi N$ partial waves. The solid(dashed) lines in amplitudes are the real(imaginary) parts. The dotted lines in the phase-shifts and inelasticities correspond to the data analysis of ref. [9].

On the other hand our $a_{31}(\sqrt{s})$ amplitude is different from both [7] and [8]. While it is possible to reproduce the $\pi N \rightarrow \pi\pi N$ cross sections with the three set of amplitudes, a simultaneous description of the $\pi N \rightarrow \pi\pi N$ cross sections and the $\pi N \rightarrow \pi N$ scattering date is not possible with the amplitude of [7] and [8].

The resulting scattering amplitudes, phase-shifts and inelasticities for the $S_{11}$ (isospin 1/2) and $S_{31}$ (isospin 3/2) $\pi N$ partial waves, are shown in Fig. 2. One can see that results agree with the data in the energy range from threshold to 1600 MeV. We cannot obtain agreement in such a broad range without the correction of the chiral coefficient. It shows the importance of the correction. Another important thing to note is that, as seen in this figure, the inelasticities are well reproduced even at low energies in both $S_{11}$ and $S_{31}$. These are provided by the $\pi\pi N$ channels. If $\pi\pi N$ channels are not included, the inelasticities of $S_{11}(S_{31})$ below 1487 (1690) MeV are zero and do not agree with data.

Acknowledgments: This work has been partly supported by the Spanish Ministry of Education in the program “Estancias de Doctores y Tecnólogos Extranjeros en España”, by the DGICYT contract number BFM2000-1326 and by the EU TMR network Eurodaphne, contact no. ERBFMRX-CT98-0169.

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