On a new class of non-Abelian expanding waves

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Abstract

We study non-Abelian expanding waves that could be radiated from cosmic sources of Yang–Mills fields. This investigation generalizes results of our earlier paper in which a class of non-Abelian transverse waves was obtained. We find a new class of wave solutions to the Yang–Mills equations describing expanding waves with not only transverse but also longitudinal components. These solutions could be applied to detect cosmic sources of Yang–Mills fields.

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Consider Yang–Mills fields in the case of an $N$-parameter gauge group. These fields are described by the following equations [1,2]:

$$\partial^\mu F^{a,\mu\nu} + f_{abc} A_b^\mu F^{c,\mu\nu} = j^{a,\nu},$$

$$F^{a,\mu\nu} = \partial^\mu A^{a,\nu} - \partial^\nu A^{a,\mu} + f_{abc} A^b A^c, \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3, a, b, c = 1, 2, \ldots, N$, $A^{a,\nu}$ and $F^{a,\mu\nu}$ are potentials and strengths of a Yang–Mills field, respectively. $j^{a,\nu}$ are field sources, $f_{abc}$ are the structure constants of an $N$-parameter gauge group and $\partial^\mu \equiv \partial/\partial x^\mu$, where $x^\mu$ are orthogonal space–time coordinates of the Minkowski geometry.

One of the important problems is a search for non-Abelian wave solutions to the Yang–Mills equations. In Refs. [3–11] non-Abelian plane waves and their generalizations were studied and a number of interesting results were obtained. The case of non-Abelian expanding waves was considered in our work [12] where a class of transverse wave solutions to the Yang–Mills equations (1)–(2) was found. In the present work we study non-Abelian expanding waves with not only transverse but also longitudinal components.

Let us consider an expanding wave radiated from a star and caused by movements of its electric charges. By convention, this wave is described by the Maxwell equations. However, such a big field source could generate not only photons but also $Z^0$ and $W^\pm$ bosons. In this case the Maxwell equations may be inapplicable since they describe fields for which only photons are their carriers. On the other hand, the Yang–Mills equations with $SU(2)$ symmetry, which present a reasonable nonlinear generalization of the Maxwell equations, play a leading role in various models of electroweak interactions caused by photons and $Z^0$ and $W^\pm$ bosons [1,2,13,14].

That is why in our papers [15,16] a nonlinear electrodynamics based on the Yang–Mills equations with $SU(2)$ symmetry was studied. In these papers we considered the Yang–Mills equations (1)–(2) with $N = 3$ and classical sources $j^{1,\nu} \neq 0$ and $j^{2,\nu} = j^{3,\nu} = 0$ in the spherically symmetric case and found a class of their non-trivial exact solutions. As follows from these solutions, the field strengths $F^{1,\nu}$ $(\nu \neq 0)$ contain an arbitrary function and hence cannot be uniquely determined from the considered Yang–Mills equations. Besides, as shown in Refs. [17–20], the classical Yang–Mills equations (1)–(2) represent a chaotic system containing unstable solutions.
This means that the nonlinear electrodynamics based only on the classical Yang–Mills equations cannot be a complete theory. That is why in Refs. [15,16] we posed the problem to find a new equation additional to Eqs. (1)–(2) in order to get rid of unnecessary solutions. Let us now consider the additional equation suggested in Refs. [15,16].

First, note that the Yang–Mills equation (1) can be represented as

\[ \partial_\mu F^{a,\mu\nu} = J^{a,\nu}, \]

(3)

\[ J^{a,\nu} = j^{a,\nu} - f_{abc} A^b_\nu F^{c,\mu\nu}. \]

(4)

As easily follows from (3), the components \( J^{a,\nu} \) satisfy the differential equations of charge conservation

\[ \partial_\nu J^{a,\nu} = 0, \quad a = 1, 2, \ldots, N. \]

(5)

From (4) and (5) we find that the components \( j^{a,\nu} \) can be interpreted as \( N \) four-dimensional vectors of full current densities which are the sum of the source components \( f^{a,\nu} \) and the field components generated by the source.

Let us find a correlation between the current densities \( j^{a,\nu} \) and \( j^{a,\nu} \). For this purpose, consider a small part of a field source and let \( \Delta Q^a \) be its intrinsic charges corresponding to the current densities \( j^{a,\nu} \) and \( \Delta Q^a \) be its full charges corresponding to \( j^{a,\nu} \) and including, besides \( \Delta Q^a \), the charges of field virtual particles created inside it.

As is well known, the classical intrinsic electric energy of a homogeneous body with charge \( q \) is proportional to \( q^2 \). That is, applying the law of energy conservation, let us require the invariance of the value \( \sum_{a=1}^N (\Delta Q^a)^2 \) which is proportional to the energy of the source small part associated with its full charges \( \Delta Q^a \). Then, since the value \( \sum_{a=1}^N (\Delta Q^a)^2 \) should be the same in both the trivial case when \( \Delta Q^a = \Delta q^a \) and the non-trivial case when \( \Delta Q^a \neq \Delta q^a \), we come to the following correlation:

\[ \sum_{a=1}^N (\Delta Q^a)^2 = \sum_{a=1}^N (\Delta q^a)^2 \]

which expresses the conservation of the energy in a small part of a field source.

Using the components \( j^{a,\nu} \) of full current densities and the source components \( j^{a,\nu} \), this correlation for a small part of a field source can be represented as

\[ \sum_{a=1}^N j^{a,\nu} j^{0,\nu} = \sum_{a=1}^N j^{a,\nu} j^{a,\nu}. \]

(7)

From (3) and (7) we derive the following correlation:

\[ \sum_{a=1}^N \partial_\mu F^{a,\mu\nu} \partial_\nu F^{a,\lambda\nu} = \sum_{a=1}^N j^{a,\nu} j^{a,\nu}. \]

(8)

Correlation (8) was suggested in our papers [15,16] as a new equation that should be added to the Yang–Mills equations (1)–(2) to get rid of unnecessary solutions.

Consider now expanding wave solutions to the Yang–Mills equations (1)–(2) and the additional equation (8) in the region outside field sources where

\[ j^{a,\nu} = 0. \]

(9)

Let us seek field potentials \( A^{a,\nu} \) satisfying the Yang–Mills equations (1)–(2) and Eqs. (8) and (9) in the following form which was proposed in Ref. [12]:

\[ A^{a,0} = u^a(y_0, y_1, y_2, y_3), \quad A^{a,n} = \left( \frac{x^n}{r} \right) A^{a,0}, \quad y_0 = x^0 - r, \quad y_n = x^n, \]

\[ n = 1, 2, 3, \quad a = 1, 2, \ldots, N, \quad r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}. \]

(10)

where \( u^a \) are some functions of the wave phase \( y_0 = x^0 - r \) and of the spatial coordinates \( y_n = x^n \).

We will further consider gauge groups with compact semi-simple Lie algebras which have totally antisymmetric structure constants \( f_{abc} \). Then substituting expressions (10) into formula (2) for the field strengths \( F^{a,\mu\nu} \), we readily find

\[ F^{a,0n} = \frac{\partial u^a}{\partial y_n}, \quad F^{a,in} = \left( \frac{1}{r} \right) \left( \frac{\partial u^a}{\partial y_n} - x^i \frac{\partial u^a}{\partial y_i} \right), \quad y_n = x^n, \quad i, n = 1, 2, 3. \]

(11)

As will be shown below, these field strengths satisfy Eq. (8) in the considered case (9). Let us now substitute expressions (10) and (11) for \( F^{a,\mu\nu} \) and \( F^{a,\mu\nu} \) into the Yang–Mills equation (1).

Then when the index \( \nu = 0 \) from (1) and (9) we obtain [12]

\[ \sum_{i=1}^3 \left( \frac{\partial^2 u^a}{\partial y_i^2} - \frac{y_i}{r} \frac{\partial u^a}{\partial y} \frac{\partial y_i}{\partial y} - \frac{y_i}{r} f_{abc} u^b \frac{\partial u^c}{\partial y_i} \right) = 0, \]

(12)

where \( y_i = x^i \) and \( y_0 = x^0 - r \). From this point on we shall label \( x^i \) by \( y_i \) when \( i = 1, 2, 3 \).

When the index \( \nu = n = 1, 2, 3 \) from Eqs. (1) and (9) we obtain after reductions [12]

\[ \frac{y_n}{r} \sum_{i=1}^3 \left( y_i \frac{\partial^2 u^a}{\partial y_i \partial y} - \frac{\partial^2 u^a}{\partial y_i^2} + \frac{y_i}{r} \frac{\partial u^a}{\partial y} + f_{abc} y_i u^b \frac{\partial u^c}{\partial y_i} + \frac{\partial}{\partial y_n} \left( \sum_{i=1}^3 \frac{\partial u^a}{\partial y_i} \right) \right) = 0. \]

(13)
It should be noted that Eqs. (12) and (13) can be represented in the form

\[ \partial_\mu F^{a,\mu 0} = -\sum_{i=1}^{3} \frac{y_i}{r} f_{abc} u_i^b \partial u_i^a \partial y_i, \quad \partial_\mu F^{a,\mu n} = -\frac{y_n}{r} \sum_{i=1}^{3} \frac{y_i}{r} f_{abc} u_i^b \partial u_i^a \partial y_i. \]  

(14)

From (14) we readily find that the field strengths \( F^{a,\mu \nu} \) of the form (11) satisfy Eq. (8) in the considered case (9). Let us denote

\[ p^a = \sum_{i=1}^{3} y_i \frac{\partial u_i^a}{\partial y_i}, \quad q^a = \sum_{i=1}^{3} \frac{\partial^2 u^a}{\partial y_i}. \]  

(15)

Then from (12) and (13) we find

\[ q^a = \frac{1}{r} \left( \frac{\partial p^a}{\partial y_0} + f_{abc} u^b p^c \right), \quad r = \sqrt{y_1^2 + y_2^2 + y_3^2}, \]  

(16)

\[ y_n \left( \frac{1}{r} \frac{\partial p^a}{\partial y_0} - q^a + \frac{p^a}{r^2} + \frac{1}{r} f_{abc} u^b p^c \right) + \frac{\partial p^a}{\partial y_n} = 0, \quad n = 1, 2, 3. \]  

(17)

As follows from (11) and (15), in the case \( p^0 = 0 \), which was investigated in Ref. [12], the considered expanding waves are transverse. We will further study the case \( p^0 \neq 0 \) corresponding to expanding waves with longitudinal components.

Let us substitute expression (16) for \( q^a \) into Eqs. (17). Then we easily obtain

\[ y_n p^a + \frac{\partial p^a}{\partial y_n} = 0, \quad n = 1, 2, 3. \]  

(18)

As can be readily verified, these equations have the following solution:

\[ p^a = \frac{s^a(y_0)}{r}. \]  

(19)

where \( s^a \) are arbitrary differentiable functions of the argument \( y_0 \).

From (15), (16) and (19) we get

\[ \sum_{i=1}^{3} y_i \frac{\partial u_i^a}{\partial y_i} = \frac{s^a(y_0)}{r}, \]  

(20)

\[ \sum_{i=1}^{3} \frac{\partial^2 u^a}{\partial y_i} = \frac{1}{r^2} \left[ s^a(y_0) + f_{abc} u^b s^c(y_0) \right], \quad \frac{\partial s^a}{\partial y_0} = \frac{ds^a}{dy_0}. \]  

(21)

As can be readily verified, Eq. (20) has the following solution:

\[ u^a(y_0, y_1, y_2, y_3) = -\frac{s^a(y_0)}{r} + g^a(y_0, \xi_1, \xi_2, \xi_3), \quad \xi_i = \frac{y_i}{r}, \quad r = \sqrt{y_1^2 + y_2^2 + y_3^2}. \]  

(22)

where \( g^a \) are arbitrary differentiable functions.

Actually, from (22) we derive

\[ \frac{\partial u_i^a}{\partial y_i} = \frac{s^a(y_0) y_i}{r^2} + \frac{1}{r} \frac{\partial g^a}{\partial \xi_i} - y_i \frac{3}{r^2} \sum_{n=1}^{3} \frac{y_n}{\partial \xi_n} \frac{\partial g^a}{\partial \xi_n}, \quad i = 1, 2, 3. \]  

(23)

From (23) we get the identity \( \sum_{i=1}^{3} y_i \frac{\partial u_i^a}{\partial y_i} = s^a(y_0)/r \).

Therefore, formula (22) gives solutions to Eq. (20).

Consider Eq. (21). For the functions \( g^a(y_0, \xi_1, \xi_2, \xi_3) \) we have [12]

\[ \frac{\partial g^a}{\partial y_i} = \frac{1}{r} \sum_{k=1}^{3} \frac{\partial g^a}{\partial \xi_k} (\delta_{ik} - \xi_k \delta_{ik}), \quad i = 1, 2, 3, \quad \xi_i = \frac{y_i}{r}, \quad \delta_{ik} = 1, \quad \delta_{ik} = 0 \text{ when } k \neq i, \]

\[ \frac{\partial^2 g^a}{\partial y_i} = \frac{1}{r^2} \sum_{k,n=1}^{3} \frac{\partial^2 g^a}{\partial \xi_k \partial y_i} (\delta_{ik} - \xi_k \delta_{ik}) (\delta_{in} - \xi_n \delta_{in}) - \frac{1}{r^2} \sum_{k=1}^{3} \frac{\partial g^a}{\partial \xi_k} \left[ \xi_k (1 - 3 \xi_i^2) + 2 \xi_i \xi_k \right]. \]  

(24)

Let us substitute expression (22) for the function \( u^a \) into Eq. (21) and take into account that the function \( 1/r \) is harmonic and the considered constants \( f_{abc} \) are antisymmetric. Then using formulæ (24) and the evident equality \( \xi_i^2 + \xi_2^2 + \xi_3^2 = 1 \), we obtain

\[ \sum_{i=1}^{3} \left[ (1 - \xi_i^2) \frac{\partial^2 g^a}{\partial \xi_i^2} - 2 \xi_i \frac{\partial g^a}{\partial \xi_i} \right] - \sum_{i \neq k} \xi_i \xi_k \frac{\partial^2 g^a}{\partial \xi_i \partial \xi_k} = s^a(y_0) + f_{abc} u^b s^c(y_0). \]  

(25)

The arguments \( \xi_i = y_i/r \) of the functions \( g^a \) are not independent, since \( \xi_1^2 + \xi_2^2 + \xi_3^2 = 1 \). That is why instead of \( \xi_1, \xi_2, \xi_3 \), we can choose two independent arguments related to them.
As is shown in Ref. [12], it is convenient to choose the following two arguments $\theta$ and $\sigma$:

$$g^a(y_0, \xi_1, \xi_2, \xi_3) = h^a(y_0, \theta, \sigma), \quad \theta = \frac{1}{2} \ln \left( \frac{1 + \xi_1}{1 - \xi_1} \right), \quad \sigma = \arctan \left( \frac{\xi_2}{\xi_3} \right).$$

(26)

Then we have [12]

$$\begin{align*}
\frac{\partial^2 g^a}{\partial \xi_1^2} &= \gamma \frac{\partial^2 h^a}{\partial \sigma^2} + \gamma \frac{\partial^2 h^a}{\partial \sigma \partial \xi_1}, \\
\frac{\partial^2 g^a}{\partial \xi_2^2} &= \gamma^2 \xi_1 \left( \frac{\partial^2 h^a}{\partial \sigma^2} - 2 \frac{\partial^2 h^a}{\partial \sigma \partial \xi_2} \right), \\
\frac{\partial^2 g^a}{\partial \xi_3^2} &= \gamma^2 \xi_1 \left( \frac{\partial^2 h^a}{\partial \sigma^2} + \frac{1}{\xi_2^2 + \xi_3^2} \frac{\partial^2 h^a}{\partial \sigma \partial \xi_3} \right).
\end{align*}$$

(27)

and as can be readily verified, the left-hand side of (25) acquires the form

$$\sum_{i=1}^{3} \left[ (1 - \xi_i^2) \frac{\partial^2 g^a}{\partial \xi_i^2} - 2 \xi_i \frac{\partial^2 g^a}{\partial \xi_i \partial \xi_j} \right] - \sum_{i,k=1 \atop i \neq k} \xi_i \xi_k \frac{\partial^2 g^a}{\partial \xi_i \partial \xi_k} = \frac{1}{1 - \xi_1^2} \frac{\partial^2 h^a}{\partial \sigma^2} + \frac{1}{\xi_2^2 + \xi_3^2} \frac{\partial^2 h^a}{\partial \sigma \partial \xi_3}.$$

(28)

Since the variables $\xi_i = y_i/\sigma$ satisfy the equality $\xi_2^2 + \xi_3^2 = 1 - \xi_1^2$, from (25), (26) and (28) we come to the following equation:

$$\frac{\partial^2 h^a}{\partial \sigma^2} + \gamma \frac{\partial^2 h^a}{\partial \sigma \partial \xi_1} = (1 - \xi_1^2) \left[ s^a(y_0) + f_{abc} h^b s^c(y_0) \right], \quad \xi_1 = \tanh \theta.$$

(29)

Let us put

$$h^a = v^a(y_0, \theta, \sigma) + k(y_0) s^a(y_0) \ln(\cosh \theta) + d^a(y_0),$$

(30)

where $v^a(y_0, \theta, \sigma)$, $k(y_0)$ and $d^a(y_0)$ are some functions.

Then substituting (30) into (29) and taking into account that $f_{abc}$ are antisymmetric, we get

$$\frac{\partial^2 v^a}{\partial \sigma^2} + \frac{\partial^2 v^a}{\partial \sigma \partial \xi_1} = (1 - \tanh^2 \theta) \left[ s^a(y_0) - k(y_0) s^a(y_0) + f_{abc} (v^b + d^b(y_0)) s^c(y_0) \right].$$

(31)

Let us require that the $N + 1$ functions $k(y_0)$ and $d^a(y_0)$ should satisfy the following system of $N$ algebraic equations which are linear with respect to them:

$$s^a(y_0) - k(y_0) s^a(y_0) + f_{abc} d^b(y_0) s^c(y_0) = 0.$$

(32)

Then from (31) we get

$$\frac{\partial^2 v^a}{\partial \sigma^2} + \frac{\partial^2 v^a}{\partial \sigma \partial \xi_1} = (1 - \tanh^2 \theta) f_{abc} v^b s^c(y_0), \quad v^a = v^a(y_0, \theta, \sigma).$$

(33)

After multiplying (32) by $s^a(y_0)$, summing it over the index $a$ and taking into account the antisymmetry of $f_{abc}$, we derive the following simple formula for the function $k(y_0)$:

$$k(y_0) = \frac{s}{s}, \quad s^2 = \sum_{a=1}^{\infty} s^a, \quad s = s(y_0), \quad \dot{s} = \frac{ds}{dy_0}.$$

(34)

From (32) and (34) we find that in the case $s = \text{const}$ the function $k = 0$ and $d^a$ are proportional to $s^a$.

Consider Eq. (33) in the region $\theta > 0$. The case $\theta \leq 0$ can be treated quite analogously. Let us seek solutions of Eq. (33) when $\theta > 0$ in the form

$$v^a(y_0, \theta, \sigma) = \text{Re} \int_0^\infty V^a(y_0, \theta, \omega) \exp(-\omega(\theta + i\sigma)) d\omega, \quad \theta > 0,$$

(35)

where $V^a(y_0, \theta, \omega)$ are some complex functions for which the integrals (35) are finite. Then substituting (35) into Eq. (33), we come to the following equation:

$$\frac{\partial^2 V^a}{\partial \sigma^2} - 2\omega \frac{\partial V^a}{\partial \sigma} = (1 - \tanh^2 \theta) f_{abc} V^b s^c(y_0), \quad \theta, \omega > 0.$$

(36)

which provides the fulfillment of (33).

Let us choose the variable $\varphi = \tanh \theta$ instead of $\theta$ and put

$$V^a = V^a(y_0, \varphi, \omega), \quad \varphi = \tanh \theta.$$
Then Eq. (36) acquires the following form, since \(d\varphi/d\theta = 1 - \tanh^2 \theta = 1 - \varphi^2\):

\[
(1 - \varphi^2)^2 \frac{d^2\varphi}{d\varphi^2} - 2(\omega + \varphi) \frac{d\varphi}{d\eta} = f_{abc} V^b s^c(y_0), \quad \omega \geq 0, \quad 0 \leq \varphi \leq 1.
\]  

(38)

Putting

\[
\eta = (1 - \varphi)/2, \quad 0 \leq \eta \leq 1/2, \quad 0 < \varphi \leq 1,
\]

from (38) and (39) we get

\[
\eta(\eta - 1) \frac{d^2\varphi}{d\eta^2} - (\omega + 1 - 2\eta) \frac{d\varphi}{d\eta} + f_{abc} V^b s^c(y_0) = 0, \quad \omega \geq 0.
\]  

(40)

Let us seek solutions \(V^a(y_0, \eta, \omega)\) to Eq. (40) in the form

\[
V^a = \sum_{l=0}^{\infty} \lambda^a_l \eta^l, \quad \lambda^a_0 = \lambda^a_0(y_0, \omega),
\]

(41)

where \(\lambda^a_l\) are some complex functions of the arguments \(y_0\) and \(\omega\). Then substituting (41) into Eq. (40), we obtain the recurrence relation for \(\lambda^a_1, \lambda^a_2, \lambda^a_3, \ldots\):

\[
\lambda^a_{l+1} = \frac{l(l+1)\lambda^a_l + f_{abc}\lambda^c_l s^b}{(l+1)(l+1+\omega)}, \quad \omega \geq 0, \quad l = 0, 1, 2, \ldots.
\]  

(42)

where the complex values \(\lambda^a_0 = \lambda^a_0(y_0, \omega)\) may be assigned arbitrarily.

From (42) we can easily derive that the sequence \(|\lambda^a_l|\) is bounded for any \(y_0\) and \(\omega \geq 0\).

Actually, let us denote

\[
L(y_0) = \max_{0 \leq l \leq N} \left|f_{abc} s^b(y_0)\right|
\]

(43)

and consider (42) when \(l > L(y_0) - 1\) for an arbitrary \(y_0\). Then we find

\[
\max_{1 \leq a \leq N} \left|\lambda^a_{l+1}\right| \leq \frac{l(l+1) + L(y_0)}{(l+1)(l+1+\omega)} \max_{1 \leq a \leq N} \left|\lambda^a_l\right|, \quad l > L(y_0) - 1, \quad \omega \geq 0.
\]  

(44)

Formula (44) precisely proves that the sequence \(|\lambda^a_l|\) is bounded for any \(y_0\) and \(\omega \geq 0\).

From (44) we also get that the values \(\max_{1 \leq a \leq N} |\lambda^a_0|\), \(0 \leq l < \infty\), are bounded by their maximum when \(0 \leq l \leq L(y_0)\).

As indicated in (39), we have \(0 \leq \eta \leq 1/2\). Besides, as shown above, the sequence \(|\lambda^a_l|\) is bounded for any \(y_0\) and \(\omega \geq 0\). Therefore, the considered power series (41) is absolutely convergent and one can determine the functions \(V^a(y_0, \eta, \omega)\).

After finding \(\lambda^a_l\) and then using (35), (37), (39) and (41) we can determine the functions \(V^a(y_0, \eta, \sigma)\) in the region \(\theta \geq 0\). The case \(\theta < 0\) can be easily brought to the considered case \(\theta \geq 0\) by the substitution \(\theta \rightarrow -\theta\) in (33). After determining the functions \(V^a(y_0, \theta, \sigma)\) and then using (22), (26), (30), (32) and (34) we can find the functions \(V^a(y_0, y_1, y_2, y_3)\) describing non-Abelian expanding waves by means of formulas (10) and (11). These functions are determined by the \(N\) arbitrary complex functions \(\lambda^a_0(y_0, \omega)\).

Thus we have found a class of exact wave solutions in the vacuum to the Yang–Mills equations (1)–(2) and the additional equation (8) expressing the conservation of the energy in a small part of a field source.

Let us note the importance of the additional equation (8) to provide stability of non-Abelian solutions. Due to Eq. (8), the obtained class of wave solutions does not contain terms with chaotic behaviour which can appear in solutions of the pure Yang–Mills equations [17–20]. As stated above, the considered power series (41), which determines the obtained wave solutions, is absolutely convergent for any values of \(\lambda^a_0\). Therefore, this series varies only slightly when the values \(\lambda^a_0\) are subject to small accidental variations.

In the case of \text{SU}(2) symmetry, the found wave solutions describe radiations of big charged sources within the framework of the nonlinear electrodynamics proposed in Refs. [15,16] and based on the Yang–Mills equations with the additional equation (8).

As indicated above, the considered waves can have longitudinal components when the functions \(\rho^a\) of the form (19) are nonzero.

It is worth noting that the obtained wave solutions to the Yang–Mills equations could be applied to detect cosmic sources of Yang–Mills fields. This could be realized by a search for longitudinal components in waves radiated from stars.

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