Non-negative Factorization of the Occurrence Tensor from Financial Contracts

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Abstract

We propose an algorithm for the non-negative factorization of an occurrence tensor built from heterogeneous networks. We use $\ell_0$ norm to model sparse errors over discrete values (occurrences), and use decomposed factors to model the embedded groups of nodes. An efficient splitting method is developed to optimize the nonconvex and nonsmooth objective. We study both synthetic problems and a new dataset built from financial documents, resMBS.

1 Introduction

Tensor factorization is a powerful approach to a myriad of unsupervised learning problems [1,14,17]. We propose an efficient algorithm called non-negative occurrence tensor factorization (NOTF) for analyzing heterogeneous networks. As an application of NOTF, we study the resMBS dataset [5,19], which consists of the relationships that financial institutions (FIs, e.g., Bank of America) play roles (e.g., issuer) in financial contracts (FCs). ResMBS is automatically extracted from a collection of public financial contracts. We can represent resMBS as a three-mode (FC, FI, Role) occurrence tensor with non-negative discrete values corresponding to the confidence of the relationship extraction.

The occurrence tensor has several characteristics. First, the tensor has positive discrete values. Second, the observed tensor contains sparse noise corresponding to errors caused by inaccurate extraction. Most importantly, we expect that FIs will play specific roles across multiple FCs to create a community. The observed occurrence tensor could then be decomposed into a low-rank tensor together with sparse noise. This decomposition can be used to clean and complete the observation, and also allow us to understand the underlying (FC, FI, Role) communities.

The proposed NOTF extends the CANDECOMP/PARAFAC (CP) decomposition [6,12] of a tensor. After decomposing the occurrence tensor, each rank-one tensor component represents a (FC, FI, Role) community. Hence the decomposed factors are constrained to be non-negative. Instead of the $\ell_2$ norm for factorization of real valued tensors with Gaussian noise, the $\ell_0$ norm is considered for the discrete values and the sparse errors. We develop an algorithm based on the alternating direction method of multipliers (ADMM) [4,22] to optimize the nonconvex and nonsmooth objective of NOTF.

We briefly discuss some related research. While tensor decomposition dates back to the 1920s, robust tensor decomposition solutions have been presented in some recent papers [14,2,8,11]. Standard CP decomposition has been applied to heterogenous networks [16]. Discrete valued tensors have been studied in [13]. ADMM has been extensively used as a solver for robust tensor recovery [8,11], and also for standard non-negative tensor factorization [15]. Non-$\ell_2$ norms have been used for CP decomposition of real valued tensors in computer vision [13,7]. ADMM for $\ell_0$ norm was empirically studied in [21]. Probabilistic communities were discussed for financial documents in [19].

2 Non-negative Occurrence Tensor Factorization

We use notations similar to [14]. Vectors and matrices are denoted by lowercase and capital letters, respectively. Higher mode tensors are denoted by Euler script letters. Three mode tensors $\mathbf{X} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ are used as an example in this paper. Fibers are column vectors extracted from tensors by fixing every index but one, e.g., $x_{ij}$, $x_{i:k}$, $x_{i:j}$. Mode-d fibers are arranged to get the mode-d
unfolding matrix \( X_{(d)} \), e.g., \( X_{(1)} = [x_{ijk}] \in \mathbb{R}^{N_1 \times N_2 \times N_3} \). Denote the vector outer product by \( \circ \), then the CP decomposition factorizes a tensor into a sum of rank-one tensors as \( \mathcal{X} = \sum_{r=1}^R a_r \circ b_r \circ c_r \). If we denote \( A = [a_r] \in \mathbb{R}^{N_1 \times R} \), \( B = [b_r] \in \mathbb{R}^{N_2 \times R} \) and \( C = [c_r] \in \mathbb{R}^{N_3 \times R} \), the Kronecker product \( A \otimes B \in \mathbb{R}^{N_1 \times N_2 \times R} \), and the Khatri-Rao product \( A \odot B = [a_r \odot b_r] \in \mathbb{R}^{N_1 \times N_2 \times R} \), then we can compactly represent CP as \( \mathcal{X} = \sum_{r=1}^R a_r \circ b_r \circ c_r = [A, B, C] \), which is equivalent to the unfolding representation \( X_{(1)} = A(C \odot B)^T \), \( X_{(2)} = B(C \odot A)^T \), and \( X_{(3)} = C(B \odot A)^T \).

We are seeking to recover a tensor \( \mathcal{X} = [A, B, C] \) that is at most rank \( R \) from the noisy observations \( \mathcal{O} \). Each rank-one tensor \( a_r \circ b_r \circ c_r \) captures a community and \( a_r \geq 0, b_r \geq 0, c_r \geq 0 \) represent the weights of nodes (e.g., FL, FC, and roles) in the community. For discrete valued tensors, we minimize the sparse error measured with the \( \ell_0 \) norm rather than the \( \ell_2 \) loss in standard decomposition,

\[
\min_{A, B, C} \| [A, B, C] - \mathcal{O} \|_0, \text{ subject to } A \geq 0, B \geq 0, C \geq 0,
\]

where \( \| \mathcal{X} \|_0 = \sum_{i,j,k} \mathbb{I}_{\{z_{ijk} > 0\}}(x_{ijk}) \) is the counts of nonzero values in a tensor, \( \mathbb{I}_S \) is the indicator function of the set \( S \); \( \mathbb{I}_S(v) = 1 \), if \( v \in S \), and \( \mathbb{I}_S(v) = 0 \), otherwise.

We minimize the NOTF objective \([11]\) by introducing an intermediate variable \( \mathcal{U} \),

\[
\min_{u, A, B, C} \| \mathcal{U} \|_0 + \epsilon \| \mathcal{U} | \mathcal{O} = [A, B, C] - \mathcal{O},
\]

where \( \epsilon \) is the characteristic function of the set \( \mathcal{S} \); \( \epsilon_S(v) = 0 \), if \( v \in S \), and \( \epsilon_S(v) = \infty \), otherwise. We then apply alternating direction method of multipliers (ADMM) \([9, 4, 10, 22]\) by introducing dual variables \( \lambda \) and alternatively solving subproblems of \( \mathcal{U} \) and \( A, B, C \), with \( p \) indexing iterations,

\[
\mathcal{U}^{p+1} = \arg \min \| \mathcal{U} \|_0 + \frac{\tau}{2} \| \mathcal{U} - [A^p, B^p, C^p] + \mathcal{O} + \lambda^p \|_F^2
\]

\[
A^{p+1}, B^{p+1}, C^{p+1} = \arg \min_{A, B, C} \epsilon \| z_{ijk} \|_0 (A, B, C) + \frac{\tau}{2} \| \mathcal{U}^{p+1} - [A, B, C] + \mathcal{O} + \lambda^p \|_F^2
\]

\[
\lambda^{p+1} = \lambda^p + \mathcal{U}^{p+1} - [A^{p+1}, B^{p+1}, C^{p+1}] + \mathcal{O},
\]

where \( \| \mathcal{X} \|_F = \sqrt{\sum_{i,j,k} x_{ijk}^2} \), and \( \tau \) is a hyperparameter called the penalty parameter.

Subproblem \( [3] \) can be solved by the proximal operator of the \( \ell_0 \) norm, known as hard-thresholding,

\[
\mathcal{U}^{p+1} = \text{hard}([A^p, B^p, C^p] - \mathcal{O} - \lambda^p, 1/\tau),
\]

where hard(\( \mathcal{Z}, t \)) = \( \arg \min_{\mathcal{U}} \| \mathcal{Z} \|_0 + 1/2 \| \mathcal{Z} - \mathcal{Z} \|_F^2 = \mathcal{Z} + I_{\{z_{ijk} > \sqrt{\tau}\}}(\mathcal{Z}) \), with * representing the element-wise Hadamard product, and \( I_S(\mathcal{X}) = \mathbb{I}_S(\mathcal{x}_{ijk}) \) the element-wise indicator function.

Subproblem \( [4] \) is nonnegative tensor factorization, which can be solved by alternatively optimizing one of \( A, B, \) or \( C \) when the other two are fixed, with \( q \) indexing iterations,

\[
A^{q+1} = \arg \min_{A} \epsilon \| z_{ijk} \|_0 (A) + \frac{\tau}{2} \| \mathcal{U}^{p+1} + \mathcal{O} + \lambda^p \|_F^2 - A(C^{q,q} \circ B^{q,q})^T
\]

\[
B^{q+1} = \arg \min_{B} \epsilon \| z_{ijk} \|_0 (B) + \frac{\tau}{2} \| \mathcal{U}^{p+1} + \mathcal{O} + \lambda^p \|_F^2 - B(C^{q,q} \circ A^{q+1})^T
\]

\[
C^{q+1} = \arg \min_{C} \epsilon \| z_{ijk} \|_0 (C) + \frac{\tau}{2} \| \mathcal{U}^{p+1} + \mathcal{O} + \lambda^p \|_F^2 - C(B^{q,q} \circ A^{q+1})^T
\]

Each subproblem is a constrained least squares problem that recovers the mode-\( d \) unfolding matrix \( \mathcal{U}^{p+1} + \mathcal{O} + \lambda^p \), starting from \( A^{0+1} = A^0, B^{0+1} = B^0, C^{0+1} = C^0 \), and updating \( A, B, C \) according to \([7, 12]\) until convergence, then \( A^{p+1} = A^{p,\text{end}}, B^{p+1} = B^{p,\text{end}}, C^{p+1} = C^{p,\text{end}} \). The updates \([7, 12]\) for the subproblems \( [4] \) usually converge in less than ten iterations when warm started from the previous iteration. Relative “residuals” are used to monitor the convergence of \([3]-[5] \), and are defined by

\[
\text{res}_1 = \frac{\| [A^{p,q}, B^{p,q}, C^{p,q}] - [A^{p,q-1}, B^{p,q-1}, C^{p,q-1}] \|_F}{\| [A^{p,q-1}, B^{p,q-1}, C^{p,q-1}] \|_F}
\]
We test the NOTF algorithm on a synthetic dataset constructed as follows: (1) We create random sparse matrices \( A \in \mathbb{R}^{50 \times 3}, B \in \mathbb{R}^{20 \times 3}, C \in \mathbb{R}^{10 \times 3} \), where the sparse ratios (ratio of zero to non-zero values) are 70.67%, 55% and 30%, respectively. Each nonzero value is uniformly sampled from range \((0, 1)\). (2) Create the ground truth low rank matrix \( X = \mathcal{I}_{\{x:|x|>0\}} \{ A, B, C \} \in \mathbb{R}^{50 \times 20 \times 10} \); it is the indicator tensor of nonzero values in the CP reconstruction from \( A, B, C \); \( [A, B, C] \) has CP rank 3. Note that \( X \) is a discrete (binary) valued tensor with sparsity ratio 75.75%. (3) Create the observation tensor \( O \) by adding noise that flips a small portion of the binary values in \( X \).

NOTF reconstructs a low rank matrix \( [\hat{A}, \hat{B}, \hat{C}] \) with at most CP rank \( R \), where \( A \in \mathbb{R}^{50 \times R}, B \in \mathbb{R}^{20 \times R}, C \in \mathbb{R}^{10 \times R} \). We vary the ratio of noise, i.e., the percentage of flips in \( O \), up to 10%, and the rank parameter \( R \) up to 10. We report on convergence iterations, false positive and false negative counts when reconstructing the binary values in tensor \( X \) and \( O \), and the mean square error for reconstructing \( X \) and \( O \). We compare NOTF with \( \ell_0 \) norm in \( \ell_0 \) with non-negative tensor factorization (NTF) baselines using the \( \ell_1 \) norm and \( \ell_2 \) norms. Both NOTF and baseline methods are initialized with a CP decomposition of the observation tensor \( O \), \( \lambda^0 = 0 \), with penalty parameter \( \tau = 10 \), and implemented in Matlab using the Tensor toolbox \[4\].

Fig. 1 presents the results of varying the noise ratio up to 10% (top) with rank parameter \( R = 3 \), and varying the rank parameter \( R \) up to 10 (bottom) with noise of 10%, for the synthetic dataset. We observe that the proposed NOTF solution with \( \ell_0 \) norm performs well on the discrete measures (false positive and false negative counts in (b) and (c)); we consider non-zeros as positives and zeros as negatives in a tensor. NOTF achieves zero false positives and a relatively low false negative count over all noise ratios. The \( \ell_2 \) baseline achieves zero false negatives, but the false positive counts are quite large. The \( \ell_1 \) baseline achieves larger errors than NOTF on both positives and negatives, and is slower to converge.

Fig. 1(d) shows that NOTF with \( \ell_0 \) norm does not outperform the baselines for mean square error; this is not surprising. The discrete measurements are more important when reconstructing an occurrence.
We present non-negative occurrence tensor factorization (NOTF) for analyzing heterogeneous networks that could be represented with an occurrence tensor. The error between the recovered low rank tensor \([\hat{A}, \hat{B}, \hat{C}]\) and the observation \(O\) grows with the noise ratio, while the error between \([A, B, C]\) and the groundtruth \(X\) is relatively stable. This suggests that the recovered tensor \([\hat{A}, \hat{B}, \hat{C}]\) can be used to de-noise the observation.

In Fig. 1 (bottom), we set a 10% noise ratio and vary the rank parameter \(R\) for the recovered tensor \([A, B, C]\). Note that \(R\) is an upper bound of the CP rank of \([A, B, C]\). NOTF could achieve zero for both false positives and false negatives when \(R = 6\), which means that the ground truth can be completely recovered from the noisy observation. However, NOTF becomes unstable when larger \(R\) and leads to large false negative counts. A possible reason is that the initialization by CP decomposition of \(O\) becomes less stable when large \(R\) is used, and the least squares in (7)-(12) are often ill-posed and hard to solve. We finally observe that modeling the sparse error by the \(\ell_0\) norm brings an additional benefit in that the recovered \(\hat{A}, \hat{B}, \hat{C}\) are sparse; this leads to a clearer interpretation of each rank-one tensor as a community.

4 Experiments on the resMBS dataset

We explore the roles played by financial institutions (FIs) across multiple contracts (FCs) using NOTF with the \(\ell_0\) norm. ResMBS\([5, 20, 19]\) contains extracted relationship of FI (e.g., Bank of America) playing a role (e.g., issuer) for a specific financial contract. The discrete values of occurrence tensor \(O \in \mathbb{R}^{971 \times 85 \times 27}\) indicate the counts of extractions of the specific (FC, FI, Role) occurrence from documents issued in 2005. \(O\) is sparse (1.02% non-zero values) and extraction noise is estimated to be \(\leq 0.2\%\). We describe some observations here and present the relevant figures in Section 6 due to space limitations.

We vary the CP rank parameter \(R\) and reconstruct tensors for both the discrete observation \(O\) and its binary version. Performance is similar for both while it is notably slower to reconstruct discrete values (Fig. 2). We note that resMBS is challenging as the tensor is sparse. The false positives are relatively stable while the false negatives decrease as \(R\) increases. With \(R = 20\), the total error count between the reconstructed tensor and the noisy observation is 3002; this roughly matches the expected errors of the information extractor. The histogram (Fig. 3 (left)) shows that errors for each FC is in a reasonable range (0, 20) with a mean of 3.

At last, we examine the discovered communities by NOTF for resMBS. Each rank-one tensor \(a_r \circ b_r \circ c_r, r = 1, \ldots, R\) represents a community. Fig. 3 (right) presents the nonzero ratio and Fig. 4 presents the distribution of the reconstructed tensor component \(A = [a_r], B = [b_r], C = [c_r]\). An interesting observation is that the communities are “centered” around FIs, i.e., each community only contains one or two FIs. Some FIs could play various roles and appear in various FCs, while some FIs only play a limited number of roles in a limited number of FCs.

5 Discussion and future work

We present non-negative occurrence tensor factorization (NOTF) for analyzing heterogeneous networks. CP tensor decomposition is adapted to discover the embedded communities. The \(\ell_0\) norm is used to model the discrete tensor values and sparse errors, and the objective is solved with an efficient splitting optimization algorithm. NOTF is applied to both synthetic data and a new heterogeneous bipartite graph, resMBS, representing financial role relationships extracted from financial contracts. Preliminary results are promising and suggest that NOTF can be used to de-noise the occurrence tensors and identify communities in resMBS.

There are several directions for future work. The \(\ell_0\) norm is known to be difficult to optimize. The \(\ell_p\) norm (\(0 < p < 1\)) satisfies the KL inequality, is often used as a surrogate, and may provide a theoretical convergence guarantee. The penalty parameter \(\tau\) is crucial for both convergence speed and solution quality for nonconvex problems; adaptive ADMM\([22, 21]\) which automates the selection of \(\tau\), achieves promising practical performance. To deal with the high sparsity of the resMBS tensor, domain-specific constraints (e.g., each FC should contain an FI play role “Issuer”) may boost performance. Finally, it is interesting to apply NOTF for analyzing some other heterogeneous networks that could be represented with an occurrence tensor.
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6 Appendix: experimental results for resMBS

Figure 2: (a) Convergence iteration, (b) false positive counts, and (c) false negative counts when vary the tensor CP rank $R$ for the reconstruction of resMBS dataset. Both binary and discrete tensor of resMBS are tested. Note that reconstruction errors presented in (b)(c) are based on noisy observation $O$ as the groundtruth is unknown.

Figure 3: (left) Histogram of errors when constructed resMBS tensor with $R = 20$. (right) Nonzero ratio for the constructed tensor component $\hat{A}, \hat{B}, \hat{C}$. Each rank-one tensor $a_r \circ b_r \circ c_r, r = 1, \ldots, R$ represents a community.

Figure 4: The distribution of FC, FI, and roles in each community.