The one-loop NLO radiative corrections (RC) to the observables in polarized DIS using assumption that a quark is an essential massive particle are considered. If compared with classical QCD formulae the obtained results are identical for the unpolarized and different for polarized sum rules, that can be explained as the influence of the finite quark mass effects on NLO QCD corrections. The explicit expression for one-loop NLO QCD contribution to the structure function $g_2$ is presented.

1 Introduction

Notice that QCD and QED RC having different origins possess on the one-loop level some common features. If to restrict our consideration to so-called QCD Compton process then both these corrections will be described by the identical set of Feynman graphs (see fig.1). Nevertheless it has to be emphasized that different methods of calculations are used within the framework of these theories.

![Fig. 1. Full set of Feynman graphs for one-loop QCD or QED correction to the hadronic current](image-url)

One of the standard approach to the calculation of electromagnetic RC to the hadronic current is based on the cancellation of the infrared divergences by Bardin-Shumeiko method [1, 2] developed to the case of polarized particles in refs. [3, 4]. Later this approach will be called as the "massive" scheme that supposes the presence of a quark as an essential massive particle.

At the same time the "classical" calculations of RC in the perturbative QCD are performed in the chiral limit i.e. when a quark mass is equal to zero [5–8].
Later that method of calculation will be called as the "massless" scheme. However there is no fundamental reason to consider QCD RC within the "massless" scheme especially when we deal with such fine effects as polarized ones.

As an example let us consider RC to the lepton current in polarized DIS. The polarized part of the cross section of the process is proportional to the polarization vector of the scattering lepton that has the form \[4\]:

\[\xi_L = \frac{k}{m} - \frac{m}{pk},\] (1)

where \(k\) (\(p\)) is a momentum of an initial lepton (target). As a rule the second part of the polarization vector can be dropped. But, as it was shown in \[9–11\], when the radiation of a real photon is considered even in NLO approximation the second term of (1) gives non-zero contribution to the polarized observables. It has to be noted that the same situation appears when QED RC to the hadronic current \[9\] are calculated and has to be expected in QCD.

Unfortunately the "massless" scheme can not take into account the second term in the polarization vector of the target by definition. So the main aim of this report is application of the "massive" scheme usually used in QED to the calculation of QCD RC within a naive quark-parton model in order to estimate the value of the finite quark mass effect in polarized DIS.

2 Method of Calculation and Main Results

The lowest-order one-loop RC to the hadronic current consist of two parts whose contribution are calculated in a different way:

\[W^{1-loop}_{\mu\nu} = W^R_{\mu\nu} + W^V_{\mu\nu}.\]

The first one appears from a gluon emission and requires the integration over its phase space:

\[
W^R_{\mu\nu} = \frac{\alpha_s}{12\pi^2} \sum_q e_q^2 \int \frac{d^3k}{k_0} \frac{1}{(p_{2q} - p_{1q})^2} \left[ S\rho\Gamma_{\mu\alpha}(\hat{p}_{1q} + m_q)\tilde{\Gamma}_{\alpha\nu}(\hat{p}_{2q} + m_q)f_q \\
+ S\rho\Gamma_{\mu\alpha}(\hat{p}_{1q} + m_q)\gamma_5\tilde{\eta}\tilde{\Gamma}_{\alpha\nu}(\hat{p}_{2q} + m_q)\Delta f_q \right],
\] (2)

where \(p_{1q}\) (\(p_{2q}\)) in an initial (final) 4-momentum of the quark and

\[\Gamma_{\mu\alpha} = 2\Omega_q^\mu\gamma_\alpha - \frac{\gamma_\mu\hat{k}\gamma_\alpha}{2kp_{1q}} - \frac{\gamma_\alpha\hat{k}\gamma_\mu}{2kp_{2q}}, \quad \tilde{\Gamma}_{\alpha\nu} = 2\Omega_q^{\alpha\nu} - \frac{\gamma_\alpha\hat{k}\gamma_\nu}{2kp_{1q}} - \frac{\gamma_\nu\hat{k}\gamma_\alpha}{2kp_{2q}},\]

\[\Omega_q = \frac{p_{1q}}{2kp_{1q}} - \frac{p_{2q}}{2kp_{2q}}.\]
The second part appearing from a gluon exchange graph reads

\[ W_{\mu\nu}^V = \frac{4}{3} \alpha_s \sum_q \left[ -2 \left( \mathcal{P}^{IR} + \log \frac{m_q}{\mu} \right) (l_q - 1) - \frac{1}{2} l_q^2 + \frac{3}{2} l_q - 2 + \frac{\pi^2}{6} \right] W_{\mu\nu}^{0q}. \]  

(3)

where \( W_{\mu\nu}^{0q} \) is a contribution of \( q \)-quark to the hadronic tensor on the Born level.

The pole term which corresponds to the infrared divergence is contained in \( \mathcal{P}^{IR} \).

Both of these contributions include the infrared divergences, which have to be carefully considered in order to be canceled. Like QED we use the identity:

\[ W_{\mu\nu}^{R} = W_{\mu\nu}^{IR} - W_{\mu\nu}^{IR} = W_{\mu\nu}^{IR} + W_{\mu\nu}^{IR}. \]

Here \( W_{\mu\nu}^{IR} \) is finite for \( k \to 0 \), and \( W_{\mu\nu}^{IR} \) is the infrared divergent part of (3). Using the dimensional regularization scheme the latter can be given in the form

\[ W_{\mu\nu}^{IR} = \frac{4}{3} \alpha_s \sum_q \left[ 2 \left( \mathcal{P}^{IR} + \log \frac{m_q}{\mu} \right) (l_q - 1) + l_q l_v + \frac{l_q^2}{2} - \frac{3}{4} l_q - \frac{7}{4} l_v + \frac{3}{2} - \frac{\pi^2}{3} \right] W_{\mu\nu}^{0q}, \]

where

\[ l_q = \log \frac{Q^2}{m_q^2}, \quad l_v = \log \frac{1 - x}{x}. \]

The sum of \( W_{\mu\nu}^{IR} \) and \( W_{\mu\nu}^{V} \)

\[ W_{\mu\nu}^{IR} + W_{\mu\nu}^{V} = \frac{2}{3} \alpha_s \sum_q \left[ 2 l_q l_v - l_q^2 + \frac{3}{2} l_q - \frac{7}{2} l_v - \frac{5}{2} - \frac{\pi^2}{3} \right] W_{\mu\nu}^{0q} = \frac{2}{3} \alpha_s \sum_q \delta_{q} W_{\mu\nu}^{0q}. \]  

(4)

is infrared free.

In order to extract some information about QCD contribution to the polarized structure function \( g_2 \), the integration in \( W_{\mu\nu}^{F} \) over the gluon phase space should be performed without any assumptions about the polarization vector \( \eta \). So the technique of tensor integration have to be applied in this case. Since the result of the analytical integration has the same tensor structure as the usual hadronic tensor in polarized DIS, the coefficients in front of the corresponding tensor structures (like \( g_{\mu\nu}, p_{\mu}p_{\nu} \ldots \)) can be interpreted as one-loop QCD contributions to the corresponding structure functions.

Thus the QCD-improved structure functions read:

\[ F_1(x, Q^2) = \frac{1}{2x} [F_2(x, Q^2) - F_L(x, Q^2)], \]

\[ F_2(x, Q^2) = x \sum_q e_q^2 f_q(x, Q^2), \]
\[ F_L(x, Q^2) = \frac{4\alpha_s}{3\pi x} \sum_q e_q^2 \int \frac{dz}{x} f_q(x/z), \]
\[ g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta f_q(x, Q^2), \]
\[ g_2(x, Q^2) = -\frac{\alpha_s}{6\pi} \sum_q e_q^2 \left\{ (l_q + \log(1-x) - 1) f_q(x) + \int \frac{dz}{x} \left[ (4l_q - 4 \log z + 7) - \frac{1}{1-z} \Delta f_q(x/z) - \frac{\Delta f_q(x)}{1-z} \right]\right\}, \]

where the \(Q^2\)-dependent unpolarized and polarized parton distributions are defined as

\[ f_q(x, Q^2) = (1 + \frac{2\alpha_s}{3\pi} \delta_q) f_q(x) + \frac{2\alpha_s}{3\pi} \int \frac{dz}{x} \left[ \frac{1 + z^2}{1-z} (l_q - \log z(1-z)) - \frac{7}{2} - \frac{1}{1-z} \right] f_q(x), \]
\[ \Delta f_q(x, Q^2) = (1 + \frac{2\alpha_s}{3\pi} \delta_q) \Delta f_q(x) + \frac{2\alpha_s}{3\pi} \int \frac{dz}{x} \left[ \frac{1 + z^2}{1-z} (l_q - \log z(1-z)) - \frac{7}{4} \right] \Delta f_q(x), \]

and \(\delta_q\) can be found in (4).

Now it is interesting to compare the one-loop QCD contribution to the structure functions presented above with the classical results obtained within massless scheme.

It is clear that the explicit expressions found in both of the discussed schemes have the same structure. However the results obtained by us in spite of the standard ones do not require any renormalization (see [8] for details). The another interesting issue of our approach is the explicit finite expression for \(g_2\), which cannot be obtained in the "massless" scheme since the first moment of \(g_2\) includes the divergence.

After integration of the QCD-improved structure functions over the scaling variable \(x\) the explicit QCD contributions to the sum rules can be presented by the following expressions:

\[ \int_0^1 dx F_1(x, Q^2) = (1 + C_{f_1} \frac{\alpha_s}{\pi}) \int_0^1 dx F_1^0(x), \]
Radiative corrections to the structure functions and sum rules in polarized DIS. . .

\begin{table}[h]
\begin{center}
\begin{tabular}{c|cccc}
\hline
 & $C_{f1}$ & $C_{f2}$ & $C_{g1}$ & $C_{g2}$ \\
\hline
$m_q = 0$ & $-2/3$ & $0$ & $-1$ & $\infty$ \\
$m_q \neq 0$ & $-2/3$ & $0$ & $-5/3$ & $-l_q/2 - 4/3$ \\
\hline
\end{tabular}
\end{center}
\caption{Corrections to sum rules in polarized DIS}
\end{table}

\[ \int_0^1 \frac{dx}{x} F_2(x, Q^2) = (1 + \frac{C_{f2} \alpha_s}{\pi}) \int_0^1 \frac{dx}{x} F_2^0(x), \]

\[ \int_0^1 dx g_1(x, Q^2) = (1 + C_{g1} \frac{\alpha_s}{\pi}) \int_0^1 dx g_1^0(x), \]

\[ \int_0^1 dx g_2(x, Q^2) = \frac{\alpha_s}{\pi} \sum_q e_q^2 C_{g2} \int_0^1 dx \Delta f_q(x), \]

where the structure functions with index "0" are defined in the naive parton model, and the coefficient $C_{f1, f2, g1, g2}$ both to the well-known classical ($m_q = 0$) and our results ($m_q \neq 0$) in the table are presented.

Notice that in the both of the discussed schemes the QCD corrections to the unpolarized sum rules have the identical values. However the difference between the QCD-corrections to the polarized sum rules are existed. The origin of the disagreement for $C_{g1}$ can be visualized in the case of longitudinal polarized DIS: the additional contribution comes from that part of the polarization vector which is proportional to the quark mass. The similar situation for QED corrections to the leptonic current has been already discussed above. The different value of the $C_{g2}$ can be explained in a simple way: $l_q \rightarrow \infty$ when $m_q \rightarrow 0$.

As it was mentioned above our calculation was performed within a naive parton model when the quark mass is defined as $m_q = xM$. Another possibility is to consider it as a constant. However an additional contribution appearing during the integration of the QCD-improved structure functions over $x$ is completely cancels due to DGLAP equations and the final results for these two cases are identical.

The additional contributions to the spin dependent sum rules influence on the experimentally measured first moment of $g_1$

\[ \Gamma_1^{\text{meas}} = (1 + C_{g1} \frac{\alpha_s}{\pi}) \Gamma_1^0, \]

where $\Gamma_1^{\text{meas}}$ and $\Gamma_1^0$ are measured and extracted quantities respectively. It could be seen from the follow relation:

\[ \frac{\Gamma_1^{0 \ m_q \neq 0}}{\Gamma_1^{0 \ m_q = 0}} = \left( 1 - \frac{\alpha_s}{\pi} \right) / \left( 1 - \frac{5\alpha_s}{3\pi} \right) \sim 1.07, \]
that the influence of the finite quark mass on QCD RC to the first moment of $g_1$
is approximately 7% for $\alpha_s = 0.27$.

Note that within our approach we can estimate the first moment of $g_2$. As can be seen from the table 3 the sign of the first moment of $g_2$ calculated within our scheme

$$\int_0^1 dx g_2 < 0,$$

is in agreement with the result of SLAC experiment [12].

References

[1] D. Bardin, N. Shumeiko: Nucl. Phys. B127 (1977) 242
[2] A. Akundov, D. Bardin, L. Kalinovskaya, T. Rieman: Fortsch. Phys. 44 (1996) 373
[3] T. Kukhto and N. Shumeiko: Nucl. Phys. B219 (1983) 412
[4] I. Akushevich and N. Shumeiko: J. Phys. G20 (1994) 513
[5] G. Alterelli and G. Parisi: Nucl. Phys. B126 (1977) 298
[6] M. Anselmino, A. Efremov and E. Leader: Phys. Rept. 261 (1995) 1
[7] M. Glück, E. Reya, M. Stratmann and W. Vogelsang: Phys. Rev. D53 (1996)4775
[8] B. Lampe, E. Reya: MPI-PHT-98-23,(1998)
[9] I. Akushevich, A. Ilyichev and N. Shumeiko: Phys. At. Nucl. 58 (1995) 1919
[10] D. Bardin, J. Blumlein, P. Christova, L. Kalinovskaya: Nucl. Phys. B506 (1997) 295
[11] I. Akushevich, A. Ilyichev and N. Shumeiko: J. Phys. G24 (1998) 1995
[12] K. Abe, et al.: Phys. Rev. Lett. 76 (1996) 587