A graphical environment to express the semantics of computer-controlled systems

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Abstract. We present the concept of a unified graphical environment for expressing the semantics of control systems. The graphical control system design environment in Simulink already allows engineers to insert a variety of assertions aimed at verification and validation of the control software. We propose extensions to a Simulink-like environment’s annotation capabilities to include formal control system stability, performance properties and their proofs. We provide a conceptual description of a tool, that takes in a Simulink-like diagram of the control system as the input, and generates a graphically annotated control system diagram as the output. The annotations can either be inserted by the user or generated automatically by a third party control analysis software such as IQCβ or µ-tool. We finally describe how the graphical representation of the system and its properties can be translated to annotated programs in a programming language used in verification and validation such as Lustre or C.

1 Introduction

Embedded control systems are ubiquitous in present day safety-critical applications. The aerospace and medical fields are filled with examples of such systems. The verification and validation (V&V) of their software implementation has always been a major preoccupation given the dire consequences of any potential malfunction. It has been the endeavor of the formal methods community to provide tools that facilitate and rationalize this process. However, currently there is little communication between the engineers who design the control system and the engineers who do the V&V. It has been noted in [5] that the former could potentially provide valuable inputs for the latter in regards to finding the relevant invariants. We believe that inputs from the control engineer can be helpful to the V&V community if the following can be provided:

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An environment for the control engineers to easily insert stability and performance proofs into their designs.

Automatic translation of the information provided by the control engineers into a form that is familiar to the V&V community.

In this paper, we present an extension to the current block diagram representations of control systems (Simulink, Xcos) that include fundamental control systems proof information such as a Lyapunov function, which establishes stability, and the plant model with respect to which stability was established at the time the controller was designed. These extensions resemble, but are different from, Simulink’s current diagram annotation capability.

1.1 Challenges

The following are the challenges that we like to address:

- Provide a coherent set of new blocks in a Simulink-like environment that enable a wide array of systems and types of stability proofs to be handled.
- Provide a formal semantics for the new graphical environment. The semantics can be inherited from a Simulink-like environment that has a formal semantics such as Scade.
- Develop a tool to perform translation from the Simulink-like environment to an industrial programming language such as Lustre or C.

1.2 Background

Simulink being the de-facto industry tool for embedded controllers, has been the subject of many research efforts in the validation and verification community. There have been numerous attempts at the translation of Simulink into other languages for the purpose of formal analysis. See [1], [10], [2] and [12]. The ideas presented in this paper are not specifically confined to Simulink, but rather they form an overall concept that can be implemented in any graphical modeling language tool. This work is closely related to the ideas presented in [5], as it explores autocoding with proof at the interface with the control engineer.

2 Conceptual Overview

We begin by proposing a tool represented by the illustration in figure 1. The front-end of the tool provides a unified graphical environment for the design of control systems as well as the insertion of proofs about the control systems. The back-end translates the visual design with proofs into annotated code in an industrial programming language, such as Lustre or C which can then be analyzed using V&V tools developed by the formal methods community. We describe the top-level components that make up the tool with the focus on the graphical environment for the expression of control systems semantics. The demonstrations in this paper use existing annotative capabilities of the graphical modeling platform Simulink.
2.1 Front-end

The front-end of the tool takes in an input of a Simulink-like model and feeds to the verification block generator (the green block in figure 1). Let us consider the model of the double integrator system in figure 2 as the input. The verification block generator produces an output model that includes additional blocks and wires (see the red portion in figure 3), which are arranged to express an assertion on the states of the input model. The Simulink environment gives the user great flexibility in expressing a variety of stability and performance criteria from control theory literature. In this particular example the verification block checks the stability of the double integrator system in the presence of noise that is bounded in power. However, note that the stability proof annotation in figure 3 was constructed by concatenating two signals together, and then feeding the output through several mathematical and logic blocks from the Simulink library. This can become a very cumbersome process to the control engineer. To simplify this as much as possible we propose an extension to the current Simulink block library to allow a more direct way of expressing control systems properties and their proofs. For example the bounded noise stability property in figure 3 can be captured more succinctly by a new annotation block type denoted stability with the following three parameters: the positive-definite matrix $P$, the characteristics of the noise input and the states of the control system. More examples and detailed descriptions of the variety of control systems properties and proofs can be found in section 3.
Fig. 2. Model Input

Fig. 3. Verification Block Generator Output
2.2 Automatic Generation of Control System Proofs

The most important parameter in the proposed stability block type is the positive-definite matrix $P$ since stability proofs for many control systems boils down to computing this matrix [4], and the proof flows down to the code-level in a nice fashion [5]. To automate this process as much as possible we propose linking the verification block generator with automatic control system analysis tools (see the pink block in figure 1). Several third party tools exist which can adapted for this role. For linear control systems the $P$ can always be generated automatically by a robust control toolbox such as the IQC β or μ-tool (see [9],[11] and [3]). For nonlinear control systems such as adaptive controllers it is likely that manual input will be needed. For example the user might be expected to insert the Lyapunov function by using the appropriate Simulink blocks and connecting them with the wires. For these cases where the automation fails, the model output from the verification block generator is provided to the user as the interface for the manual insertions of the stability proofs (see the gray block in figure 1).

2.3 Formalism

We propose a new type of wire that elevates signals to the abstract level to allow easy differentiation between signals that represent the states of the control system i.e. $x(t)$ in figure 3 and signals that are not the states of the control system. The rest of the newly proposed annotation block types can use the existing blocks and wires structure, with some indication that they are not to be used for code generation, but for annotation generation. For this we also propose an intermediate language representation of the Simulink model that makes this distinction clearly. The existing labeling options that exist in Simulink can be useful here to keep track of names.

2.4 Back-end

The back-end of the tool is the translator from the graphical environment to a programming language such as C or Lustre. The majority of work here is will be formalizing the semantics of the graphical environment. The translation process must also deal with time discretization of the stability proof annotations, which can be quite difficult depending on the type of control system that the user put into the tool.

3 Simulink Examples

We present several examples of annotating stability proofs and performance criteria for control systems in the Simulink environment. These examples can obtained from control engineering books such as [8] or [6].
3.1 Semantics of the Simulink blocks and wires

The mathematical operational blocks such as sum, multiply, divide, dot product are polymorphic just like every other Simulink blocks. They take input arguments of many different types: scalar, vector of arbitrary dimensions, and matrices. Semantically the blocks can change either due to different input arguments or user specification. For example the ”Product” block can be a product of scalars, matrix multiplication, or element-wise multiplication of the entries of the vector depending on the input argument and user choice. The wires can carry all numerical data types available in Simulink. The two relevant types in the control system diagrams are scalars and vectors that are assumed to be either real or complex. For expressing most annotations of stability proofs and performance criteria, we can use the existing blocks in Simulink. A small amount of annotations may require the convenience of functions defined outside of Simulink environment i.e. MATLAB for example.

3.2 Lyapunov Stability

We start with Lyapunov stability, since it is the simplest stability result in time-domain that we can express using the graphical environment of Simulink. The essential part of the Lyapunov stability is the quadratic Lyapunov function $V(x)$ in (1) where $x$ is the state of the control system and $P$ is a positive-definite matrix.

$$V(x) = x^T P x$$

The noise block is set to 0. The verification block shown in figure 4 expresses the following assertion: the lie derivative of the Lyapunov function $V(x(t))$ is less than or equal to zero. The matrix $P$ can be computed using either one of the two robust control toolboxes mentioned.

3.3 $\mathcal{L}_2$ Gain Stability

For input-to-output stability, the annotation diagram becomes more complex. In this case both the storage and supply functions need to be constructed [6]. Figure 5 has an example of a proof annotation showing finite $\mathcal{L}_2$-gain stability of the double integrator system. The annotation expresses the following assertion:

$$\dot{V}(x) - \alpha^2 w^T w + y^T y \leq 0$$

where $w$ is the unit-peak uniform noise input, $V(x) = x^T P x$ is a quadratic Lyapunov-like function called the storage function, and $y$ is the output signal.

The proposed extensions to the interface will reduce drastically the number of blocks and wires necessary to construct this stability proof annotation. Despite the increase in complexity one of the essential component of the annotation is still a positive-definite matrix $P$. This $P$ can also be obtained using the two robust control solvers mentioned in section 2, therefore the diagram in figure 5 can be generated automatically.
3.4 Plant Model as Annotation

Most controller stability proofs are based on some form of model for the plant that is being controlled. Thus the plant needs to be introduced somehow in the annotation framework. This is actually relatively straightforward at the graphical level, since most graphical modeling tools are not only meant for implementing controllers, but also for testing them, and in the latter case a model for the plant must be present. Figure 6 shows how we go about displaying plant model information at an abstract level, which will not interfere with the executable code that will be generated. We take the example of the following plant from [7]

\[ m\ddot{x} + c_1 (x^2 - c_2) \dot{x} + (k_1 + k_2x^2) x = u \quad (3) \]

with the adaptive controller

\[ u = \psi x \]

\[ \dot{\psi} = -D_0^T P \begin{bmatrix} x^2 & x\dot{x} \\ x\dot{x} & \dot{x}^2 \end{bmatrix} \]

(4)
The Lyapunov stability of the feedback interconnection of the plant and the controller is established by the Lyapunov function

$$V(x, \dot{x}, \psi) = \frac{1}{2} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}^T P \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + p_2 \int_0^x \sigma \hat{K}(\sigma) d\sigma + p_{12} \int_0^x \sigma \hat{C}(\sigma) d\sigma + \frac{1}{2} \psi \psi^T$$

(5)

This complex example is mainly introduced to show the almost unlimited expressive power of the proposed interface extension.

### 3.5 $L_1$ Adaptive Controller Performance

To further demonstrate the expressive power of the graphical environment, we show the following $L_1$ adaptive controllers (see figure 7) and an example of annotating a performance bound. For $L_1$ controllers there exists a proof of not only closed-loop Lyapunov stability but also bounds on the transient performances. For example the bound on the state prediction error $\hat{x}$ of the controller is a function of the uncertainty of the plant "theta_max", the Lyapunov function matrix $P$ and the adaptive gain "Gamma". Note that the uncertainty parameter of
plant can be part of the plant model or separate by itself as in figure 7.

$$\| \dot{x} \|_{\infty} - \sqrt{\frac{\theta_{\max}}{\lambda_{\min}(P) I}} \leq 0 \quad (6)$$

### 3.6 Discretization during Translation

Thus far we have shown several examples of continuous-time proofs of stability for control systems. Now how do these proofs translate down to the code-level? The main issue as mentioned before is the time-discretization that is applied to the model when it is translated into code. Fortunately for all linear control systems, the discrete-time stability proofs can be easily and systematically obtained. However for the adaptive controllers, it is very impractical to look for Lyapunov stability proofs in discrete-time hence the problem remains that we must use the continuous-time proofs as invariants for the discretized system.

### 4 Integration with Third Party Tools

For the integration of third party control system analysis tools, we’ll need to build a component that extracts the necessary control system parameters and characteristics from the input Simulink model. For this procedure to be automated it is again necessary to have a formal semantics of the proposed graphical environment. System information such as the state-space model, the system states, the input noise disturbance are examples of the required inputs needed for IQC/μ and β-tool.
Fig. 7. $L_1$ Transient Performance Bound
5 Conclusion and Future Work

In this paper we have proposed a new graphical environment that allows the easy expression of the semantics of computer-controlled systems. We believe this environment simplifies the process by which the control engineer can provide domain knowledge for the deductive verification of the controller implementation. We have provided several examples of how control stability proofs and performance criteria can be expressed in a current graphical modeling environment i.e. Simulink. For the new graphical environment, we have proposed several extensions designed to enhance the proof annotative capabilities of the current environment. We are currently in the process of formalizing the new unified graphical environment. This is a proposed research orientation that still requires much effort: formalizing the graphical annotation "language", integrating it in an existing graphical modeling environment, interfacing it with third party tools, and lastly implementing the translation tool that will generate the annotated code from the extended diagrams.

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