Comparative analysis of the dependence of the bulk elastic modulus of the liquid on pressure and gas factor

A S Lunev, A A Nikitin, Y F Kaizer, A V Lysyannikov, D A Sokolov and V Y Obvintseva

Siberian Federal University, 660041, Krasnoyarsk, 79, Svobodny Avenu, Russia

E-mail: Allynev@mail.ru

Abstract. This paper considers the dependence of the influence of the gas factor of a power fluid on the physical properties of hydraulic oils. When describing the processes, the hydraulic drive should be considered as a system with lumped parameters. The work is important because the hydraulic fluid is a mixture of liquid and undissolved gas. This mixture can arise when tanking and during dynamic processes because of the different dissolution rates of gas and flow rate of the liquid in a pressure fall in some spots of power fluid flow.

1. Introduction

The compressibility of the liquid as a whole has a negative influence on the energetics and dynamics of the hydraulic drive. The compressibility of the liquid causes a degradation of the flow and volume efficiency of the pump and impairs the processing speed of the hydraulic motor. It can cause unstable motion of the servo drive with a large mass of the working body.

The calculation of the dynamics of the hydraulic drive under a large mass load without taking into account the liquid compressibility cannot be considered even approximate because such a calculation is fundamentally incorrect. Therefore, an in-depth theoretical study of liquid compressibility is of paramount importance. [8]

Papers [1, 3] give fuller information about the dynamics of the hydraulic drive. Paper [4] shows the dependence of the bulk elastic modulus of the hydraulic liquid on the pressure, but the dependence of the bulk modulus on the gas factor is not taken into account.

Papers [1, 2, 7, 8] contain the theoretical dependence of the bulk elastic modulus of the mixture (liquid and undissolved gas) on the pressure and gas factor. The bulk modulus of the mixture is given the following expression

\[ B_{\text{mix}} = -V_{\text{mix}} \frac{dp}{dV_{\text{mix}}}, \]  

(1)

where \( B_{\text{mix}} \) – the bulk elastic modulus of the mixture under unrestricted pressure \( p \), \( V_{\text{mix}} \) – the volume of the mixture under unrestricted pressure \( p \); \( dp \) – an infinitesimal increment of pressure; \( dV_{\text{mix}} \) – an infinitesimal increment of the volume of a mixture.

The volume of a mixture \( V_{\text{mix}} \) is defined as the sum of the volumes of the liquid component \( V_l \) and the gas one \( V_g \)

\[ V_{\text{mix}} = V_l + V_g. \]  

(2)
The infinitesimal increment of the volume of a mixture is interpreted as the sum of increments of the volumes of the liquid component \(dV_l\) and the gas one \(dV_g\):

\[
dV_{\text{mix}} = dV_l + dV_g. \tag{3}
\]

After substituting \(V_{\text{mix}}\) and \(dV_{\text{mix}}\) from equations (2) and (3) into equation (1), formula is deduced

\[
B_{\text{mix}} = - \frac{V_l}{V_l + V_g} dV_l - \frac{V_g}{V_l + V_g} dV_g dp. \tag{4}
\]

Formulas for detecting the bulk elastic modulus of the liquid \(B_l\) and gas \(B_g\) under unrestricted pressure \(p\) are used to find increments of the volumes of the liquid component \(dV_l\) and the gas one \(dV_g\).

\[
B_l = -V_l \frac{dp}{dV_l}, \tag{5}
\]

\[
B_g = -V_g \frac{dp}{dV_g}. \tag{6}
\]

Formulas (7) and (8) are deduced using equations (4) and (5)

\[
dV_l = -\frac{V_l}{B_l} dp, \tag{7}
\]

\[
dV_g = -\frac{V_g}{B_g} dp. \tag{8}
\]

After placing \(dV_l\) and \(dV_g\) from equations (6) and (7) into equation (4), formula is obtained

\[
B_{\text{mix}} = \frac{\frac{V_l}{V_l + V_g}}{\frac{1}{B_l} \frac{1}{B_g}}. \tag{9}
\]

Expressing parameters included in the formula (9) is more convenient for practical use in terms of the value of the parameters under atmospheric pressure \(p_0\). When the pressure changes from \(p_0\) to \(p\), the process of gas-phase compression is considered to occur by polytrope in papers [1, 2, 7, 8]

\[
V_g = V_{g,0} \left(\frac{p_0}{p}\right)^\frac{1}{n}, \tag{10}
\]

where \(V_{g,0}\) – the volume of gas under atmospheric pressure \(p_0\); \(n\) – polytropic exponent.

In this case, the formula for the bulk elastic modulus of gas can be deduced using the equation (6)

\[
B_g = np. \tag{11}
\]

When the pressure changes from \(p_0\) to \(p\), the dependence of the bulk elastic modulus of the hydraulic fluid on the pressure becomes linear [2, 7, 8]

\[
B_l = B_{l,0} + Ap, \tag{12}
\]

where \(B_{l,0}\) – the bulk elastic modulus of the hydraulic fluid under atmospheric pressure \(p_0\); \(A\) – coefficient that depends on the type of liquid and temperature.

Figure 1 shows that empirical formula (12) is not enough to describe changes of the bulk modulus of the hydraulic fluid, especially under low-pressure conditions.
Figure 1. The linear dependence of the bulk elastic modulus of the hydraulic fluid $B_{l,0}$ on the pressure $p$.

2. Analysis of the proposed dependencies for determination the bulk modulus of the liquid.

The approximate formula is used to define the volume of the liquid component

$$V_l = V_{l,0} - V_{l,0} \frac{p - p_0}{B_{l,med}}$$  \hspace{1cm} (13)

where $V_{l,0}$ – the volume of liquid under atmospheric pressure $p_0$; $B_{l,med}$ – mean value of the bulk modulus of the hydraulic fluid in the pressure interval $p_0$ to $p$.

Formula (14) in paper [8] estimates the mean value of the bulk modulus of the hydraulic fluid $B_{l,med}$

$$B_{l,med} = B_{l,0} + \frac{1}{2} \Delta p.$$  \hspace{1cm} (14)

After substituting $V_g$, $V_l$, $B_{g,n}$ and $B_{l}$ from equations (13), (10), (12) and (11) into equation (9) it is possible to deduce an approximate formula [8] for determining the bulk modulus of the mixture can be deduced.

$$B_{mix} = (B_{l,0} + \Delta p) \frac{1 - \frac{p - p_0}{B_{l,med} + \Delta p}}{B_{l,0} + \frac{V_g p (p_0)}{V_l (p) B_{l,med}}}.$$  \hspace{1cm} (15)

The volume of liquid component $V_l$ is found after substituting $B_{g}$ from equation (12) into equation (9) and after integrating the developed expression within the limits of $p_0$ to $p$ by a formula that is used to determine the bulk modulus of the mixture[2].

$$V_l = V_{l,0} \frac{A}{(B_{l,0} + \Delta p) / (B_{l,0} + \Delta p)}.$$  \hspace{1cm} (16)

After substituting $V_g$, $V_l$, $B_{l}$ and $B_{g}$ from equations (16), (10), (12) and (11) into equation (9) the bulk modulus of the mixture can be determined[2].

$$B_{mix} = \frac{V_{l,0} A (B_{l,0} + \Delta p) / (B_{l,0} + \Delta p) + V_g \frac{p_0}{p} \frac{A}{(B_{l,0} + \Delta p) / (B_{l,0} + \Delta p) + V_g \frac{p_0}{p} \frac{A}{(B_{l,0} + \Delta p) / (B_{l,0} + \Delta p)}}}{V_{l,0} A (B_{l,0} + \Delta p) / (B_{l,0} + \Delta p) + V_g \frac{p_0}{p} \frac{A}{(B_{l,0} + \Delta p) / (B_{l,0} + \Delta p) + V_g \frac{p_0}{p} \frac{A}{(B_{l,0} + \Delta p) / (B_{l,0} + \Delta p)}}}.$$  \hspace{1cm} (17)

The formula [1, 2] determines the volume content of gas under atmospheric pressure $p_0$

$$\alpha_g = \frac{V_{g,0}}{V_{mix,0}},$$  \hspace{1cm} (18)

where $V_{mix,0}$ – the volume of the mixture under atmospheric pressure $p_0$, $V_{mix,0} = V_{l,0} + V_{g,0}$. 


Then the formula [2] characterizes the ratio of the volume of the liquid component to the volume of the mixture under atmospheric pressure $p_0$

$$\frac{V_{L0}}{V_{mix.0}} = 1 - \alpha_g.$$  \hspace{1cm} (19)

The formula (17) can be reduced to the form below using equations (18) and (19)

$$B_{mix} = \frac{(1-\alpha_g)^{\frac{1}{2}}}{(B_{L0}+Ap_0)/(B_{L0}+Ap) + \alpha_g(p_0)^{\frac{1}{2}}} \cdot \frac{(1-\alpha_g)A_p}{(B_{L0}+Ap_0)/(B_{L0}+Ap) + \alpha_g(p_0)^{\frac{1}{2}}}. \hspace{1cm} (20)$$

The dependence of the bulk elastic modulus of the mixture on pressure with different values of dissolved gas is plotted.

Figure 2. The dependence of the bulk elastic modulus of the mixture on pressure at different values of dissolved gas (in percentage terms): $B_{2m}$ (p, $\alpha_0$) – quantity of gas is 0%, $B_{2m}$ (p, $\alpha_1$) – quantity of gas is 1%, $B_{2m}$ (p, $\alpha_2$) – quantity of gas is 5%, $B_{2m}$ (p, $\alpha_3$) – quantity of gas is 10%.

Figure 2 shows the dependence of the bulk modulus of the mixture on pressure is nonlinear. Moreover, the higher the gas content is, the more a discordance of obtained data with the empirical formula.

For clarity, the dependence with formulas (17), (20) and (9) is plotted.

Figure 3 shows that the curves obtained from formulas (17) and (19) practically are in line with each other, and the curve obtained from formula (9) has not much difference, although its rate is almost the same.
Figure 3. Curves of dependence of the bulk modulus of the mixture on the pressure under equal conditions: $B_{1m}(p, \alpha_2)$ – the graph is obtained by the formula (17), $B_{2m}(p, \alpha_2)$ – the graph obtained by the formula (20), $B_{m}(p, \alpha_2)$ – the graph obtained by the formula (9).

3. Conclusion

According to the obtained results, it can be concluded that the bulk modulus varies widely depending on the type of liquid, temperature, pressure, strain rate and the nature of the thermodynamic compression process.

The presence of undissolved gas and air in the hydraulic liquid complicates the problem of compressibility in the dynamic theory of the drive.

References

[1] Popov D N 2002 Mechanics of Fluid and Pneumatic Actuators (Moscow: Bauman Moscow State Technical University Press)

[2] Metlyuk N F, Avtushko V P 1980 Dynamics of Fluid and Pneumatic Actuators of Vehicles (Moscow: Machinery Construction)

[3] Mandrakov E A, Nikitin A A 2014 Dynamics of Hydraulic Systems (Moscow: INFRA-M; Krasnoyarsk: Siberian Federal University Press)

[4] Korobochkin B L 1976 Dynamics of Machine Hydraulic Systems (Moscow: Machinery Construction)

[5] Prokopjeva V N 1972 Dynamics of Fluid Actuator (Moscow: Machinery Construction)

[6] Nikitin A A, Mandrakov E A 2014 Influence of undissolved gas in the hydraulic liquid on the dynamics of logger’s hydraulic drive Proc. of Tomsk Polytechnic University. Math. and mech. Phys. 325 2 65–71

[7] Popov D N, Panaiotti S S and Ryabinin M V 2002 Hydromechanics ed D N Popov (Moscow: Bauman Moscow State Technical University Press)

[8] Gamynin N S 1972 Hydraulic Drive of Control Systems (Moscow: Machinery Construction)

[9] Danilov Y A, Kirillovskii Y L and Kolpakov Y G 1990 Equipment of Fluid Power Drive: Work Processes and Characteristic (Moscow: Machinery Construction)

[10] Gorbeshko M V 1997 Development of Mathematical Models for the Hydraulic Machinery of Systems Controlling the Moving Components of Water Development Works Hydrotechnical construction 31 (12) 745–50

[11] Parr E A 1998 Hydraulics and Pneumatics: a technician’s and engineer’s guide 2nd ed (Oxford: Butterworth-Heinemann)

[12] Mobley R K 1999 Fluid Power Dynamics (Oxford: Butterworth Heinemann)

[13] Rabie M G 2009 Fluid Power Engineering (New York City: McGraw Hill Professional)