Permittivity Measurement Using the Resonance Circuits

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Permittivity Measurement Using the Resonance Circuits

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Abstract. A pair of LC circuits has been used to demonstrate the resonance condition of electromagnetic waves. The first circuit comprises of an inductor, a capacitor and a power supply; while the second pair employs an inductor, a variable capacitor and a neon bulb. The same inductance is applied in both circuits. The variable capacitor is a parallel plate capacitor. By varying the capacitance in the second circuit, we achieve the resonance condition as indicated by the lit neon bulb. By doing so, we observe the equal oscillation frequency in both, the first and second circuit. The condition is then used to obtain the permittivity of air to be \((6.1\pm0.2) \times 10^{-11} \text{ F/m}\).

1. Introduction
Permittivity of free space is widely known as one of the universal physical constants. In general the permittivity relies on the medium, which is indicated by their dielectric constants (relative permittivity). Students encounter the permittivity in various parts of electromagnetism courses e.g. The Coulomb law, capacitance, and the speed of electromagnetic wave. Thus it is important to understand all aspects related to the permittivity or dielectric constant by employing different methods of measurements.

Many experiments used the parallel plate capacitor to obtain the dielectric constant of the medium located in between the capacitor plates. In this case the capacitance should be measured and subsequently its dielectric constant. A direct measurement of the capacitance can be done using an LCR meter. Riaz and Kanwal used an LCR meter to determine the capacitance of an acrylic filled plate capacitor, and Munguía and Madonaldo used a professional capacitance meter to measure the dielectric constant of the overheated edible oil [1, 2]. The low cost digital multimeters are also capable of measuring capacitance [3]. Although this method of measurement is relatively simple, due to its low resolution, the typical digital multimeter possesses its own limitation.

One can incorporate the flat plate capacitor in a series RLC circuit. The capacitance is determined from the measured resonance frequency [1]. Mohsen-Nia et al measured both input voltage and the voltage on the capacitor of a RC circuit [4]. By comparing these voltages they were able to determine the dielectric constant of water, ethanol, methanol, butanol and acetone. Similar to this, Nogi et al measured the voltage across a resistor in the RC circuit to obtain the dielectric constant of the material inserted in a parallel plate capacitor, e.g. Soda-lime glass, Bakelite, acrylic glass, and Teflon [5]. Another setup used an LC circuit to determine the dielectric constant of gases by measuring its resonance frequency [6].

Mak used a pair of LC circuits with identical resonance frequencies in both the transmitter and receiver. He demonstrated the generation and transmission of electromagnetic wave by adjusting the frequency of the signal generator [7]. Similarly, the generation of electromagnetic wave has also been
demonstrated using a simple RLC circuit. A digital oscilloscope is used for a quantitative analysis of the electromagnetic emission [8].

In this paper we show the measurement of capacitance of the parallel plate capacitor based on the transmission of electromagnetic wave. Unlike Mak’s experiment, here we tune the capacitance of the receiver to achieve a resonance condition. By graphical analysis we then determine the permittivity of the air which is the medium in between the capacitor plates. For teaching purposes this experiment also demonstrates the principle of a radio receiver.

**Theory**

Figure 1 shows a pair of LC circuit used in the experiment. The first LC circuit comprises of an inductor L1, a capacitor C1 and power supply. Similarly in the second circuit employs an inductor L2, a variable capacitor C2 and a neon bulb as an indicator.

![Figure 1](image)

**Figure 1.** The experiment setup, PS is a power supply and B is a neon bulb.

The oscillation in the first circuit occurs with the frequency \( f_1 \) that is given by

\[
f_1 = \frac{1}{2\pi} \sqrt{\frac{L_1}{C_1}}.
\]

(1)

Where \( L_1 \) and \( C_1 \) are the inductance and capacitance.

Due to the oscillation, this circuit generates an electromagnetic wave and behaves as the transmitter. This wave is then to be captured by the second circuit which has an inductance of \( L_2 \) and a capacitance of \( C_2 \). The frequency oscillation \( f_2 \) of the second circuit (receiver) follows

\[
f_2 = \frac{1}{2\pi} \sqrt{\frac{L_2}{C_2}}.
\]

(2)

By tuning the variable capacitor C2 the resonance condition is achieved as indicated by the lit neon bulb. In this condition the frequency oscillation is the same for both the first and the second circuit, and hence

\[
L_1 C_1 = L_2 C_2.
\]

(3)

In the experiment we use the same inductor in both circuits, therefore equation (3) becomes

\[
C_1 = C_2.
\]

(4)

Our variable capacitor is a parallel plate capacitor, its capacitance \( C_2 \) is given by
\[ C_2 = \varepsilon \frac{A}{d} \]  

(5)

where \( \varepsilon \): permittivity

\( A \): surface area of the plate

\( d \): the distance between the plates.

Using equation (4) and (5), in the resonance condition the equation applies

\[ C_1 = \varepsilon \frac{A}{d} \]  

(6)

Our variable capacitor consists of two sets of semicircular metal plates, see figure 2. The first set is a stationary plate or stator, while the second set (rotor) is placed in between the plates of the first set that can be rotated against its axis. The area of overlapping can be varied by rotating the shaft.

![Figure 2](image)

**Figure 2.** The variable capacitor used in the experiment. The rotor is rotated against the axis to vary the overlapping area.

By measuring the rotated angle \( \theta \) using a protractor we can determine the overlapping area \( A \):

\[ A = \frac{\theta}{180} A_{\text{max}} \]  

(7)

Where \( A_{\text{max}} \) is the maximum area of the semicircular plate.

2. Method

Experimental setup is depicted in figure 3. It consists of two LC circuits i.e. Transmitter and receiver. A transmitter circuit comprises of a capacitor, an inductor and power supply. Ceramic capacitors were used in the transmitter circuit. In the receiver circuit we used a variable capacitor, an inductor and a small neon lamp as an indicator. The variable capacitor as seen figure 4 is equipped with a protractor for measuring the rotor angle.

In this experiment the inductor used in the transmitter was the same as that in the receiver. Both inductors were mounted close each other. The oscillation frequency of the transmitter was set of mounting a ceramic capacitor. During the experiment the variable capacitor was tuned to achieve a resonance condition. As a result, this process will light the neon indicator lamp. The angle of the rotor position was then measured by observing the scale in the protractor.
3. Result and Discussion
The experiment is based on the resonance conditions that incorporate the transmitter and receiver circuits. In the receiver circuit we use a variable capacitor with the maximum area of the semi-circular plate $A_{\text{max}} = 5443$ mm$^2$. The average distance between plates is $8 \times 10^{-4}$ m. By carefully rotating the variable capacitor of the second circuit the neon bulb becomes lit when the stator and rotor from a certain angle. It has been observed that the neon bulb is lit only within a short range of angles. This event reveals that the oscillation occurring in the second circuit is generated by the oscillation in the first circuit. Overall, here we have demonstrated a particular behaviour known as a resonance condition, i.e. the oscillation frequency of the second circuit (receiver) is equal to the oscillation frequency of the first circuit (transmitter). The rotated angle for several transmitters’ capacitances is tabulated in Table 1. Equation (7) is used to obtain the overlapping area $A$ from the measured rotated angle $\theta$. The complete result is depicted in figure 5.
Table 1. Experimental results in the resonance condition.

| $C_1$ (pF) | $\theta$ (°) | $A$ (mm$^2$) |
|------------|--------------|--------------|
| 50         | 14           | 423          |
| 100        | 46           | 1391         |
| 150        | 66           | 1996         |
| 200        | 86           | 2600         |
| 250        | 109          | 3296         |
| 300        | 127          | 3840         |
| 350        | 149          | 4506         |
| 400        | 166          | 5019         |

Figure 5. The relation between the capacitance of the transmitter and area of the receiver capacitor. Solid line is the best fit.

Figure 5 shows a clear linear relation between the capacitance in the transmitter and the area of the overlapping area $A$. The overlapping area $A$ determines the capacitance of the receiver capacitor, thus we can deduce that figure 5 also shows a linear relation of the transmitter’s capacitance and the receiver’s capacitance. This behaviour is based on the resonance condition that is formulated in equation (6). The graph gives a gradient of $(7.6 \pm 0.1) \times 10^{-8}$ F/m and an offset of $(2 \pm 4)$ pF. In accordance with equation (6) the gradient is $(\varepsilon / d)$. Thus the gradient yields the permittivity of medium $\varepsilon$ to be $(6.1 \pm 0.2) \times 10^{-11}$ F/m.

This result is quite high compared to the reported air permittivity i.e. Ranging from 4 to $9 \times 10^{-12}$ F/m [9]. In the experiment, for the transmitter we used a common ceramic capacitor available in our lab. This capacitor has a tolerance of about 20% of the nominal value. The most critical quantity is the distance between plates in the variable capacitor. The irregularities of the capacitor plates caused the nonlinearity in the experiment result [3]. The distance between plates should be the same for all [1]. When we look into the variable capacitor in detail (figure 4), it shows a set of 30 plate capacitors.
We assume the distance is the same for all the plates; we do not account for the individual capacitor. For the calculation we take the average distances. As seen in figure 4 some plates closer than others and the spacing becomes irregular. Another factor that contributes to the result is the fact that the measured capacitance is in order of Pico Farad. In this very small order of capacitance, a stray capacitance will give a significant contribution.

In general our experiment is intended for pedagogical purposes, which is suitable for teaching the topic of electromagnetic wave. Although the students are familiar with various wireless communications devices, they do not understand the underlying physics [10]. Here students learn the basic of electromagnetic wave and the resonance frequency. During this experiment students are amazed and excited when observing the neon bulb lit up albeit not being directly connected to the power supply. A parallel situation happens when students operate a radio as a common device. Students have a basic understanding that when selecting a radio station, they dial the capacitor within the radio transmitter located in the radio; which is acting like a black box.

4. Conclusion
This simple experiment demonstrates the resonance condition for electromagnetic waves; it incorporates the transmitter and receiver circuits. A pair of LC circuits is in resonance when the oscillation frequency arises from the first circuit is equal to the second. A variable capacitor is used to tune the resonance. The variable capacitor’s capacitance depends on the overlapping area; it is determined by measuring the rotated angle of two sets of semicircular metal plates. In this experiment this resonance condition is indicated by the lit neon bulb. For the same inductance the condition is presented by a linear relation of the transmitter’s capacitance in the area of the receiver’s capacitor. The permittivity of medium $\varepsilon$ is obtained to be $(6.1 \pm 0.2) \times 10^{-11}$ F/m. The uncertainty of this measurement is contributed mainly by the irregularities of the capacitor plates.

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