An Approximate Analysis of Insulation Thickness of Elliptic Pipe

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Abstract. The thermal insulation characteristics of elliptic pipe were analyzed preliminarily. Thermal resistances between elliptic pipe line to surrounding ambient were studied in dimensionless form. The expression for critical insulation thickness was obtained. The lower bound thickness and heat transfer rate were researched. Influences of different factors were discussed. It is found that for cases of small Biot number, the lower bound thickness is obviously greater than the critical thickness. And the addition of insulation material upon elliptic pipe line should be conducted enough so that the insulation layer exceeds the lower bound thickness in order the insulation investment play a positive role.

Keywords. Elliptic pipe; thermal insulation; critical thickness.

1. Introduction
In thermal insulation, Biot number is a subtle dimensionless criterion defined as the production of the convective heat transfer coefficient between the pipe outer wall and the ambient surrounding times the pipe radius and divided by the insulation thermal conductivity. For cases of larger Biot number, insulation demand could be met simply according to conventional method. However, when Biot number is small, the insulation measurement should be considered carefully. In this situation, the amount of insulating material should be sufficiently added upon the outer wall of bare pipe so that the insulation radius exceeds not only the critical radius, but a lower bound radius as well which is an insulation radius larger than its critical radius and the heat transfer rate equals to that of bare pipe. Only in that, thermal insulation really reduces heat transfer between the pipe and ambient surrounding in the interest of investment.

Small Biot number often appear in engineering such as refrigeration, cryogenics, compact heat exchanger where small cross sectional pipes, small environmental convective heat transfer coefficient or somewhat high insulating material conductivity might be encountered occasionally [1-5].

The fundamental insulation characteristics of pipe or sphere apparatus have been investigated extensively. For pipes of other cross sectional shape, however, there are few studies reported. With development of serious innovative technology in thermal engineering, pipes or apparatus with cross sections of various shapes might be confronted more and more often. Thereafter, understanding of their insulation characteristics is important and necessary.

In this paper the thermal insulation of elliptic pipe is analyzed. Expression for the critical insulation thickness is presented, the lower bound insulation thickness is researched, and the variables effects are discussed. The objective of this study is to provide valuable reference to relevant thermal engineering.
2. Analysis

It is well known that the criterion Biot number is 1 for circular pipe and 2 for sphere apparatus. When the considered Biot number is larger than the relative value, then the outer radius of the bare pipe or apparatus is definitely larger than its critical radius and any insulation will bring decrease of its heat loss or heat gain. When the Biot number is less than its criterion value, then the insulation must be carried out to ensure the actual outer radius of the insulation layer be larger than the lower bound radius.

As to elliptic pipe or apparatus, there is no constant radius. So, critical insulation thickness and lower bound thickness were used. Next, elliptic pipe will be studied in this regard.

2.1. Critical Insulation Thickness

The considered model is depicted in figure 1. An elliptic pipe of length $L$ coated with insulating material is exposed to ambient environment. The outer wall of the elliptic pipe has a semi major axis $a$ and semi minor axis $b$, which are also the related parameters of the inner wall of the insulation layer. The insulating material with a thermal conductivity $\lambda$ is coated around the bare pipe with a uniform thickness $\delta$. The convective heat transfer coefficient between the outer wall of the insulation layer and surrounding is $h$. Perfect contact between the pipe and insulation layer is assumed and radiation heat transfer between the outer surface of the insulation layer and ambient surrounding is neglected.

![Figure 1. Elliptic insulation layer.](image)

When the material and geometry of the pipe, the flowing medium inside the pipe and its hydrodynamic and thermodynamic states were given, then the convection resistance between the inside wall surface of the pipe and the inside medium and conduction resistance of the bare pipe thickness might be fixed. Then only the conduction resistance of the insulation layer and convection resistance between the outer surface of insulation layer and surrounding would vary with the choice of the insulation material and its thickness so that the two latter resistances become the dominant factors of the thermal insulation. Summation $R$ of the two latter resistances is

$$ R = \frac{1}{2\pi L \lambda} \ln \frac{a + b + 2\delta}{a + b} + \frac{1}{\pi L h(a + b + 2\delta)} $$

(1)

where the term of conduction resistance for elliptic pipe is written from Ref. [6]. In the term of convection resistance, the elliptic surface area is from Ref. [7]. Derivation of equation (1) upon $\delta$ and set it to be zero, we have

$$ \frac{dR}{d\delta} = \frac{1}{\pi L \lambda(a + b + 2\delta)} - \frac{2}{\pi (a + b + 2\delta)^2 h} = 0 $$

$$ a + b + 2\delta_{cri} = \frac{2\lambda}{h} $$

(2)
Here δ reaches its critical value at which the insulation thermal resistance being the minimum value, the critical thickness is

\[ \delta_{cri} = \frac{\lambda}{h} \frac{a + b}{2} \]  

(3)

Introducing dimensionless parameters,

\[ A = \frac{\delta}{(a + b)/2}, \quad Bi = \frac{h}{\lambda} \left( \frac{a + b}{2} \right) \]

Equation (3) becomes

\[ A_{cri} = \frac{1}{Bi} - 1 \]  

(4)

2.2. Lower Bound Insulation Thickness

For small Biot number, the insulation thermal resistance attains its minimum value at the critical insulation thickness, and the heat transfer rate reaches the maximum value. Therefore, the insulation should be conducted adequately so that the insulation resistance becomes larger than that of the bare pipe. This insulation thickness is defined as lower bound thickness at which the two resistances are equivalent, that is

\[ \frac{1}{h \pi L (a + b)} = \frac{1}{2 \pi L \lambda} \ln \frac{a + b + 2 \delta_{low}}{a + b} + \frac{1}{h \pi L (a + b + 2 \delta_{low})} \]  

(5)

Dividing the right hand side with the left, one gets

\[ 1 = Bi \ln (1 + A_{low}) + \frac{1}{1 + A_{low}} \]  

(6)

Although the lower bound thickness \( A_{low} \) could not be obtained straightforward from equation (6) due to the implicity, it could be easily solved numerically.

2.3. Heat Transfer Rate

How much reduction of heat transfer was achieved due to the addition of insulating material needs special case study. Above all, a comparison of heat transfer rate between the piping systems with and without insulation could be carried out first. The heat transfer ratio is

\[ Q_t = \frac{1}{Bi \ln (1 + A) + 1/(1 + A)} \]  

(7)

3. Results and Discussion

It is illustrated from equation (3) that critical thickness increases with the increment of the heat conductance of the insulation material, and with the decrement of the heat convection coefficient between the insulation layer and the ambient surrounding. Besides, critical thickness decreases with the growing of the cross section of the pipeline. When the pipe cross section gets large enough, critical thickness may become to be zero or even minus, meaning that the cross sectional dimensions of the pipe has exceeded its critical scales. In this case, covering insulation material upon the outer surface of the pipe could eventually diminish heat transfer from or to the pipeline, no matter how thick or thin is the insulation layer.

It could also be aware of by examining equation (3) that the critical thickness of insulation layer for elliptic pipe is similar to that of circular pipe, if we consider the half of the sum of the semi major and minor axis of the elliptic pipe as a characteristic radius of the elliptic pipe.
Equations (4) and (6) correlate Biot number with critical and lower bound thicknesses respectively in dimensionless form. Figure 2 plots their evolution tendencies against Biot number. It is shown that the lower bound thickness is thoroughly larger than the critical thickness in the range of Bi<1. Their difference being large at small Biot number, decreases apparently with the increment of Biot number. When the Biot number gets to be 1, the two thicknesses become zero.

Figure 3 shows the comparison of heat transfer rate $Q_r$ between pipe systems with and without insulation versus dimensionless insulation thickness $\Delta$ in different Biot numbers. As it is shown, when $Bi<1$, the heat transfer rate between the insulated pipe and ambient surrounding is definitely greater than that of bare pipe system at small insulation thickness. As the insulation thickness grows larger, the heat transfer ratio of the insulated pipe to bare pipe reaches a maximum value, then decreases along way. As Biot number increases, curves of $Q_r$ go down with the maximum value of the heat transfer ratio decreases. At $Bi=1$, the maximum diminished completely meaning that in this situation, utility of any amount of insulation would cause reduction of heat transfer between the pipe system and surrounding compared to the bare pipe system.

![Figure 2. Variation of critical and lower bound insulation thicknesses.](image)

![Figure 3. Ratio of heat transfer rate with and without insulation.](image)

To eliminate heat transfer rate as minimal as possible in cases of Biot number less than 1, the insulation coverage layer should be larger not only than the critical thickness, but also larger than the lower bound thickness. Besides above mentioned demands, practical coverage thickness should be eventually decided as well in the base of comprehensive considerations of government policies, enterprise desires, investment expense and running cost.

4. Concluding Remarks
This paper is devoted to the significance of the coverage thickness of the thermal insulation layer upon elliptic pipe line of small Biot number. Expression of critical thickness of thermal insulation coverage layer is obtained. Lower bound thickness is solved numerically that is the minimal insulation thickness at which the heat loss or heat gain of the insulated pipe equals to that of the uninsulated pipe. Heat transfer ratio of insulated pipe to uninsulated pipe is discussed with respect to the variation of the insulation thickness and Biot number. The present result might play a referent role in thermal engineering.

References
[1] Kulkami M R 2004 Critical radius for radial heat conduction: A necessary criterion but not always sufficient Appl. Thermal Engineering 24 967.
[2] Sahin A Z and Kalyon M 2005 Maintaining uniform surface temperature along pipes by insulation Energy 30 637.
[3] Kayfeci M 2014 Determination of energy saving and optimum insulation thicknesses of the heating piping systems for different insulation materials Energy and Buildings 69 278.

[4] Ersöz M A and Yıldız A 2016 Effect of refrigerants on the economical optimum insulation thickness for indoor pipelines of split air conditioning systems Int. J. Refrigeration 64 51.

[5] Merkin J 1997 Free convection boundary layers on cylinders of elliptic cross section ASME J. Heat Transfer 99 453.

[6] Qian B J, Wu Y W and Chang J F 1983 Brief Handbook of Heat Transfer (Higher Education Press of China) p 41 (in Chinese).

[7] Li H X 2009 One method calculating the ellipse perimeter and the applications Henan Mechanical and Electrical Engineering College 17 99 (in Chinese).