Cyclotron dynamics of interacting bosons in artificial magnetic fields

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We study theoretically quantum dynamics of interacting bosons in artificial magnetic fields as engineered in recent ultracold atomic experiments, where quantum cyclotron orbital motion has been observed. With exact numerical simulations and perturbative analyses, we find that interactions induce damping in the cyclotron motion. The damping time is found to be dependent on interaction and tunneling strengths monotonically, while its dependence on magnetic flux is non-monotonic. Sufficiently strong interactions would render bosons dynamically localized inhibiting the cyclotron motion. The damping predicted by us can be construed as an interaction-induced quantum decoherence of the cyclotron motion.

I. INTRODUCTION

Cyclotron orbits and Landau levels formed by electrons moving in magnetic fields play an essential role in the emergence of several novel phenomena in solid state systems. Semiclassical cyclotron orbital motion in two dimensional electron gases give rise to the Hall conductance and eventually to quantized Hall conductance in high enough magnetic fields. In quantum Hall insulators, chiral edge states mediating dissipationless edge current [1, 2] can be understood as quantum cyclotron orbits bounded by the edges in a strip-like geometry. In the current work, we study the cyclotron motion of interacting bosonic neutral atoms in an optical lattice subjected to artificial gauge fields which act like effective external magnetic fields in the lattice leading to novel physics [3].

Ultracold atomic gases confined in optical lattices, because of their unprecedented controllability, allow for quantum simulations of various lattice Hamiltonians, e.g., Bose-Hubbard models [4, 5], where both equilibrium many-body physics [6, 7] and non-equilibrium dynamics have been extensively studied theoretically [8] and experimentally [9, 10]. Recent experiments created a two dimensional square lattice pierced by magnetic flux (“artificial gauge fields”) by engineering laser assisted tunneling [11–13]. Due to non-trivial Berry curvatures in such a system, charge neutral atoms, e.g. $^{87}$Rb, loaded into this flux lattice behave like “charged” bosons experiencing strong magnetic fields, and the consequent effective Lorentz force results in atomic cyclotron motion, which has been experimentally observed [12]. While these interesting experimental developments are largely motivated by considerations of observing fascinating new equilibrium many-body phases such as atomic quantum spin Hall insulators [14–20], the observed non-equilibrium cyclotron dynamics of bosons itself [12] is extremely interesting and requires theoretical understanding. In particular, the interaction effect on the dynamical cyclotron motion is obviously of great interest, and is the main topic of study in the current work.

In this article, we study cyclotron dynamics of interacting bosons with both exact numerical simulations and perturbative analyses. Weak interactions are found to induce damping effects (i.e., quantum decoherence) in the dynamics. We find that while the damping time (or the decoherence time) monotonically decreases with increasing tunneling and interaction, it has non-monotonic behavior with varying magnetic flux. With the perturbative analyses, the damping effect is attributed to specific scattering processes, and such physics is established to be generic for interacting bosons in artificial gauge fields, i.e., not relying on the model Hamiltonian used in our numerical simulations. With sufficiently strong interactions, cyclotron dynamics is completely suppressed and the bosons form a dynamically localized state, analogous to self-trapping effects observed in Bose-Einstein condensates in double-well potentials [21–24]. Our finding of the cyclotron damping effect suggests importance of interactions and many-body physics in quantum transport of bosons in artificial gauge fields, which is of great interest in recent atomic gases [18, 25, 26]. Quantum simulations of this damping effect in such controllable systems would help understand relaxations in Hall transport experiments in complex electronic materials where the decoherence could be attributed to various origins.

![FIG. 1. An optical lattice plaquette with magnetic flux $\Phi$ and its spectra. (a) The plaquette system where the sites 1, 2, 3 and 4 are located at (0, 0), (0, −1), (1, −1), and (1, 0) in our coordinate choice. (b) The single-particle energy spectra with varying $\Phi$.](image-url)
II. SYSTEM AND MODEL HAMILTONIAN

In the experimental setup to observe the cyclotron motion [12], a plaquette of four lattice sites is isolated in a two-dimensional optical lattice by suppressing interplaquette tunnelings with superlattice techniques. To study the cyclotron motion, we look at one isolated plaquette threaded by magnetic flux as illustrated in Fig. 1. The model Hamiltonian describing bosons loaded into this plaquette is ($\hbar = 1$ throughout)

$$H = H_0 + V$$

$$H_0 = -K \left[ e^{i\Phi/2} (b_j^\dagger b_1 + b_3^\dagger b_3) + h.c. \right]$$

$$-J \left[ b_j^\dagger b_{j+1} + b_j b_{j+1}^\dagger + h.c. \right]$$

$$V = \frac{U}{2} \sum_j b_j^\dagger b_j^\dagger b_j b_j,$$  \hspace{1cm} (1)

where $b_j$ is a bosonic annihilation operator for the $j$th site. This engineered Hamiltonian connects to charged bosons in magnetic fields through Peierls substitution [27-28]. The tunneling strength $J$ can be fixed to be 0.5 × 2πkHz following the experimental situation. The free part of the Hamiltonian can be written as $H_0 = \sum_{j,j'} \mathcal{H}_{jj'}^{(0)} b_j^\dagger b_{j'},$ with $\mathcal{H}^{(0)}$ the single-particle Hamiltonian matrix. We study the quantum dynamics assuming an initial state

$$|\Psi\rangle = \frac{1}{\sqrt{N}} [\psi^\dagger]^N |0\rangle,$$  \hspace{1cm} (2)

with $\psi^\dagger = \frac{1}{\sqrt{2}} \left( b_3^\dagger + b_4^\dagger \right),$ which describes $N$ bosons prepared in a superposed state of sites 3 and 4. The physics described here is otherwise robust against the choice of $\psi$ as long as it is not fine-tuned.

To characterize the cyclotron motion, the time dependent occupation numbers $n_j$ and an average position vector $\overline{X}(t) = (x(t), y(t))$ are defined as

$$n_j(t) = \frac{1}{N} \langle \Psi(t) | b_j^\dagger b_j | \Psi(t) \rangle,$$  \hspace{1cm} (3)

$$\overline{X}(t) = \sum_j \overline{R}_j n_j(t),$$  \hspace{1cm} (4)

where $|\Psi(t)\rangle$ is the time evolved many-body state, and $\overline{R}_j$ is the position of the $j$th site (see Fig. 1(a)). The initial state is not an eigenstate of the Hamiltonian and is thus not stationary. For non-interacting bosons, we have $|\Psi(t)\rangle = \frac{1}{\sqrt{N}} [\psi^\dagger]^N |0\rangle,$ with $\psi^\dagger = e^{-iH_0 t} \psi |e^{-iH_0 t} = \sum_j \psi_j(t) b_j^\dagger,$ where the coefficients $\psi_j(t)$ are determined by the single-particle Schrödinger equation $i\hbar \psi_j(t) = \sum_{j,j'} \mathcal{H}_{jj'}^{(0)} \psi_j(t).$ In this state, bosons actually rotate in the plaquette when the engineered magnetic flux is non-zero (see Fig. 2), which is a quantum analogue of classical charged particles moving in a magnetic field, and this quantum cyclotron motion is undamped. The density inhomogeneity among the four sites oscillates without any relaxation. One useful quantity in this dynamical process is the occupation fraction $P_\psi(t) = N_\psi(t)/N$ with

$$N_\psi(t) = \langle \Psi(t) | \psi^\dagger(t) \psi(t) | \Psi(t) \rangle,$$

where $N_\psi(t)$ can be thought as the occupation number of the initially occupied single-particle mode $\psi(t).$ Although the quantum state is fully dynamical and involves fast oscillations on the tunneling time scale, $J^{-1},$ (around one millisecond), non-interacting bosons remain in the single-particle state $\psi(t),$ and the occupation fraction $P_\psi(t)$ remains unity, indicating a perfectly coherent bosonic cyclotron motion in the non-interacting optical lattice.

III. WEAKLY INTERACTING BOSONS

A. Numerical simulations

We first simulate the dynamics (Figs. 3-4) with an exact treatment of the many-body Schrödinger equation $i\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle,$ where the Hamiltonian $H$ and the time evolved state $|\Psi(t)\rangle$ are represented in a complete basis

$$|M_1 M_2 M_3 M_4\rangle = \prod_j \frac{1}{\sqrt{M_j}} \left( b_j^\dagger \right)^{M_j} |0\rangle.$$

In our numerical simulations, the total particle number is fixed to be 8, i.e., the mean filling is two particles per site. In Fig. 4 the cyclotron motion illustrated by oscillations in the average position $\overline{X}(t) = (x(t), y(t))$ shows damping in the presence of repulsive interactions. After several (quasi-)periods of cyclotron motion, $\overline{X}(t)$
coherence should be observable within the experiment. Thus, our predicted interaction-induced cyclotron decoherence is well captured by our perturbative analysis presented below.

The damping of cyclotron motion is thus well described (see Fig. 4) by a two-parameter fitting formula

$$P_{\text{fit}}(t) = \frac{1}{1 + \gamma - \gamma e^{-(t/\tau)\pi}},$$

where $\tau$ and $\gamma$ are the fitting parameters. This fitting formula is proposed from extending our perturbative results (to present below) to longer time. After the cyclotron motion relaxes, i.e., $t > \tau_{\text{damp}}$, and $\overline{X}(t)$ collapses to the plaquette center, $P_{\text{fit}}(t)$ no longer captures the dynamics of $P_{\psi}(t)$ (see Fig. 4). We note that the decoherence process in Eq. (5) is not a simple temporal exponential relaxation phenomenon.

We have also studied the dependence of $\tau_{\text{damp}}$ on the (complex) tunneling strength $K$ and the applied magnetic flux $\Phi$. We find that $\tau_{\text{damp}}$ decreases with increasing $K$. The dependence of $\tau_{\text{damp}}$ on magnetic flux exhibits a non-monotonic behavior, having a minimum around $\pi/2$ (Fig. 5). When $\Phi$ reaches $\pi$, the spectra of $\mathcal{H}^{(0)}$ become degenerate (Fig. 5(b)) and the cyclotron dynamics changes dramatically. Actually even with the flux value close to $\pi$, the $\psi$-mode occupation fraction $P_{\psi}(t)$, as well as the oscillation amplitudes of $\overline{X}(t)$, yield long-time oscillations, which we can attribute to the small energy scale in the single-particle spectrum near the degeneracy point. Also $P_{\psi}(t)$ is then no longer well-described by the

FIG. 3. Cyclotron motion with various interaction strengths. The average position $(x(t), y(t))$ (see the main text) illustrates the rotation of bosons in the plaquette. (a) The cyclotron motion of non-interacting bosons, where damping does not occur. (b) show the interacting case with varying interaction strength $U$. Interaction effects make $(x(t), y(t))$ collapse into the center of the plaquette after several periods of rotation. The periods it costs for the rotation to collapse decrease with increasing interaction strength. Here we use the parameters $K = 0.25 \times 2\pi kHz$, and $\Phi = 0.735 \times \pi/2$ as realized in the experiment [12].

collapses to the regime around the center of the plaquette (Fig. 3). In this case, the $\psi$-mode occupation fraction $P_{\psi}(t)$ no longer remains unity, nonetheless it still remains quasi-static, namely, does not exhibit fast oscillations. The damping of oscillation amplitudes in $\overline{X}(t)$ is found to coincide with the decrease in $P_{\psi}(t)$ (Fig. 3). The damping of cyclotron motion is thus well captured by $P_{\psi}(t)$. Physically, the decrease in $P_{\psi}(t)$ is caused by interaction processes where bosons are scattered out of their originally occupied $\psi$ mode (this physical picture is borne out by our perturbative analysis presented below). Its coincidence with the cyclotron motion damping implies that the scattered bosons do not contribute to the cyclotron motion coherently, thus contributing to quantum decoherence.

The strength of damping can be quantified by a damping time (decoherence time) $\tau_{\text{damp}}$ which we define to be the time it takes for half of the bosons in the $\psi$ mode to be scattered into other single-particle states, namely the time when $P_{\psi}(t = \tau_{\text{damp}})$ reaches $1/2$. The damping time $\tau_{\text{damp}}$ is found to be inversely proportional to the interaction strength when it is sufficiently weak. For the parameters used in experiments [12], $\Phi \approx 0.735 \times \pi/2$, and $K \approx 0.25 \times 2\pi kHz$, the damping time is around 10ms for an interaction strength of $U = 0.05 \times 2\pi kHz$ (Fig. 5).

Thus, our predicted interaction-induced cyclotron decoherence should be observable within the experimen-
empirical fit \( P_{\text{fit}}(t) \) (Eq. 5).

FIG. 5. Damping time with different tunnelings, interactions and magnetic flux. (a) The dependence of damping time \( \tau_{\text{damp}} \) on \( K \) and \( \Phi \), where \( U \) is fixed to be \( 0.02 \times 2\pi \text{kHz} \). The lobe structure in (a) implies that \( \tau_{\text{damp}} \) decreases monotonically with increasing \( K \) and that it has non-monotonic behavior with increasing \( \Phi \). The minima of \( \tau_{\text{damp}} \) locates around \( \Phi = \pi/2 \). (b) shows its dependence on \( U \), where we choose \( K = 0.25 \times 2\pi \text{kHz} \), and \( \Phi = 0.735 \times \pi/2 \).

FIG. 6. Long time behavior of the cyclotron decoherence. In this plot we use \( U = 0.02 \times 2\pi \text{kHz} \), \( K = 0.25 \times 2\pi \text{kHz} \) and \( \Phi = 0.735 \times \pi/2 \).

B. Perturbative analysis

To better understand the cyclotron damping found in the numerics, we carry out a perturbative analysis with the standard time-dependent perturbation theory (see the Appendix). Here it is useful to introduce single particle modes \( \psi_{i=1,2,3}(t)|0\rangle \), which are orthogonal to \( \psi^\dagger(t)|0\rangle \). These modes \( \{\psi^\dagger(t), \chi^\dagger_{ij}(t)\} \) form an instantaneous complete basis for the single-particle states. Similar to \( \psi^\dagger(t) \), we have \( \chi^\dagger_{ij}(t) = e^{-iH_0t}\chi^\dagger_{ij}(0) e^{iH_0t} \). The operators \( b^\dagger_{ij} \) are then expanded as \( b^\dagger_{ij} = \psi^\dagger_{ij}(t)\psi^\dagger(t) + \chi^\dagger_{ij}(t)\chi^\dagger(t) \). The occupation fraction of \( \psi(t) \) is obtained as

\[
P_{\psi}(t) = 1 - U^2(N - 1)^2 \sum_l |I_l|^2 - U^2(N - 1) \sum_{l_1,l_2} |I_{l_1,l_2}|^2, \tag{6}
\]

with

\[
I_1 = \int_{t_0}^{t} dt' \sum_j |\psi_j(t')|^2 \psi_j(t') \chi_j(t'), \\
I_{l_1,l_2} = \int_{t_0}^{t} dt' \sum_j |\psi_j(t')|^2 \chi_{l_1,j}(t') \chi_{l_2,j}^*(t'). \tag{7}
\]

Expanding the single-particle wavefunctions \( \psi_j(t) \) and \( \chi_{ij}(t) \) in the eigen-basis of \( \mathcal{H}^{(0)} \) as

\[
\psi_j(t) = \sum_\alpha \varphi_\alpha \lambda^j_\alpha e^{-i\epsilon_\alpha t}, \\
\chi_{ij}(t) = \sum_\alpha \kappa_{i\alpha} \lambda^\alpha_{ij} e^{-i\epsilon_\alpha t},
\]

\( \lambda^j_\alpha \) is the \( \alpha \)th eigenstate of \( \mathcal{H}^{(0)} \) with energy \( \epsilon_\alpha \), we get

\[
I_1 = C^{(1)}_1(t) + \mathcal{O}(t^0), \quad I_{l_1,l_2} = C^{(2)}_{l_1,l_2}(t) + \mathcal{O}(t^0),
\]

with

\[
C^{(1)}_1 = 2 \sum_{j \alpha \alpha'} |\lambda^j_\alpha|^2 |\lambda^j_{\alpha'}|^2 |\varphi_\alpha|^2 \varphi_{\alpha'} \kappa_{i\alpha}^* \kappa_{j\alpha'}, \\
C^{(2)}_{l_1,l_2} = 2 \sum_{j \alpha \alpha'} |\lambda^j_\alpha|^2 |\lambda^j_{\alpha'}|^2 |\varphi_\alpha|^2 \varphi_{\alpha'} \kappa_{i\alpha} \kappa_{j\alpha'}, \tag{8}
\]

provided that there is no fine-tuned degeneracy in the spectrum of \( \mathcal{H}^{(0)} \). Then \( P_{\psi}(t) \) simplifies to

\[
P_{\psi}(t) \approx 1 - U^2 t^2 \times \left[ (N - 1)^2 \sum_l |C^{(1)}_l|^2 + (N - 1) \sum_{l_1,l_2} |C^{(2)}_{l_1,l_2}|^2 \right]. \tag{9}
\]

This 2nd order perturbative result is checked against exact numerics (see Fig. 4). The fitting formula \( P_{\text{fit}}(t) \) (Eq. 5) can be thought as an empirical extension of this perturbative result to longer time. The physical picture that emerges is \( |C^{(1)}_l|^2 \) describes one-particle loss rate and \( |C^{(2)}_{l_1,l_2}|^2 \) two-particle loss rate (Fig. 7). The damping time is estimated from our perturbative analysis to be

\[
\tau_{\text{damp}} \propto \frac{U^{-1}}{\sqrt{(N - 1)^2 \sum_l |C^{(1)}_l|^2 + (N - 1) \sum_{l_1,l_2} |C^{(2)}_{l_1,l_2}|^2}}. \tag{10}
\]

Carrying out the summations in Eq. 10 numerically, we find that two particle processes dominate over single particle ones, when the particle number is not too large, say \( N < 10 \). With bosons scattered into the \( \chi \) modes, the depletion of \( N_\psi \) causes the damping of cyclotron motion. The dependence of \( \tau_{\text{damp}} \) on tunneling, interaction, and magnetic flux found in numerical simulations is reproduced in the perturbative analysis, and in particular, the
non-monotonic dependence on the magnetic flux is reproduced. The long-time oscillations in $P_\psi(t)$ show up naturally in the integrals of Eq. (7) near $\pi$-flux, where the spectral degeneracy actually invalidates Eq. (8).

Given the perturbative analysis, the damping phenomena in cyclotron motion are expected to be generic for interacting bosons in artificial magnetic fields. Despite the used specific model Hamiltonian (Eq. (1)) in numerical simulations, the described damping physics is rather model independent.

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IV. STRONG INTERACTIONS AND DYNAMICAL LOCALIZATION

We further look at stronger interactions, which are potentially accessible in experiments, for example by implementing deep lattices. The perturbative analysis would no longer be reliable in the strongly interacting limit. Our numerical simulations show that bosons tend to localize for strong interaction, suppressing cyclotron motion completely. The quantity characterizing the localization phenomenon is the number imbalance among the four sites

$$\Delta n(t) = (n_3(t) + n_4(t) - (n_1(t) + n_2(t)),$$  

whose time average

$$\overline{\Delta n} = \frac{1}{T} \int_0^T dt \Delta n(t)$$

distinguishes localized and delocalized states. In our simulations, we choose $T$ to be 2 seconds and convergence is checked for longer time. As shown in Fig. 8 in a delocalized state with weak interactions, the number imbalance $\Delta n$ oscillates fast (at tunneling time scale) in time and the time average $\overline{\Delta n}$ vanishes. In a localized state with strong interactions, $\Delta n$ still oscillates in time but is otherwise always positive, and thus $\overline{\Delta n}$ is finite, meaning that bosons are localized on sites 3 and 4. The particle transfer from sites 3 and 4 to other two sites is suppressed. An intuitive picture to understand this localization is that the tunneling probability, with large repulsion, is greatly suppressed because bosons have to tunnel all together in order to preserve energy. In the intermediate/crossover regime, the dynamics in $\Delta n$ yields fluctuations at very long time scale, which makes it challenging to determine a precise transition point in numerics. Another property of the localized state is that the $\psi$-mode occupation fraction $P_\psi(t)$ yields fast oscillations, i.e., is no longer quasi-static. This peculiar dynamical localization of strongly repulsive bosons is a generalization of self-trapping in double-wells to the plaquette system, and is an important testable prediction of our theory.

V. DISCUSSION AND CONCLUSION

Although this work focused on a specific model Hamiltonian as motivated by the recent experiments [12, 13], the studied interaction induced damping in atomic cyclotron motion is expected to be a generic phenomenon. In particular, the damping mechanism as shown in Fig. 7 and the derived damping time in Eq. (10) are actually model-independent and directly applicable to more generic magnetic Hamiltonians as well. For example, the neglected trap effects as in the experimental setup [12, 13] could be easily included within our developed framework. The presence of the shallow trap in principle generates self-trapping in double-wells to the plaquette system, and is an important testable prediction of our theory.
that our analytical theory agrees with our direct numerical simulations and that we see no revival in the simulation indicates that such a revival, even if it happens, will occur at an unphysically long time of little interest to laboratory experiments.

Our predictions of interaction induced damping, decoherence, and dynamical localization (i.e. complete suppression) of the recently reported bosonic cyclotron motion in optical lattices in the presence of an artificial magnetic flux should be directly experimentally observable since all our results presented in this work use reasonable parameters easily achieved in the laboratory. The observation of our predicted novel dynamical phenomena will be a direct manifestation of interaction effects on the quantum dynamics of Bose-Hubbard model in an effective magnetic field.

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Appendix A: Details of perturbative analysis for cyclotron damping

The details of perturbative analysis of the cyclotron damping dynamics are given here. With standard perturbation theory, the time-dependent quantum state in the interaction picture reads

\[ \Psi_I(t) = A(t)\Psi_I^{(0)} + |\Psi_I^{(1)}(t)\rangle + |\Psi_I^{(2)}(t)\rangle + \mathcal{O}(U^3) \]  

with the leading part \( |\Psi_I^{(0)}\rangle = |\Psi(t = 0)\rangle \), the renormalization factor

\[ A(t) = 1 - i \int_{t_0}^{t} dt' \langle \Psi_I^{(0)} | V_I(t') | \Psi_I^{(0)} \rangle - \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle \Psi_I^{(0)} | V_I(t') V_I(t'') | \Psi_I^{(0)} \rangle \]

the first order correction

\[ |\Psi_I^{(1)}(t)\rangle = -i \int_{t_0}^{t} dt' \mathcal{P} V_I(t') |\Psi_I^{(0)}\rangle, \]

and the second order correction

\[ |\Psi_I^{(2)}(t)\rangle = - \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \mathcal{P} V_I(t') V_I(t'') |\Psi_I^{(0)}\rangle, \]

where the projection operator is \( \mathcal{P} = 1 - |\Psi_I^{(0)}\rangle \langle \Psi_I^{(0)} | \) and the interaction term \( V_I(t) = e^{-iH_0 t} V e^{iH_0 t} \). Then the occupation number of the \( \psi(t) \) mode, \( N_\psi(t) \), is given by

\[ N_\psi(t) = N |A(t)|^2 + \langle \Psi_I^{(1)}(t) | \psi^\dagger(t) \psi(t) | \Psi_I^{(1)}(t) \rangle + \mathcal{O}(U^3). \]  

The state \( |\Psi_I^{(2)}\rangle \) does not contribute to this order because \( \langle \Psi_I^{(0)} | \psi^\dagger(t) | \Psi_I^{(2)} \rangle = 0 \).

It is useful to introduce single particle modes \( \chi_{l=1,2,3}(t) \), which are orthogonal to \( \psi^\dagger(t) |0\rangle \). These modes \( \{ \psi^\dagger(t), \chi_{l}(t) \} \) form an instantaneous complete basis for the single-particle states. Similar to \( \psi^\dagger(t) \), we have \( \chi^\dagger_{l}(t) = e^{-iH_0 t} \chi^\dagger_{l}(0) e^{iH_0 t} \), and \( \chi_{l}(0) \) will be shortened as \( \chi_l \) in the following. The operators \( b^\dagger_{l} \) are then expanded as

\[ b^\dagger_{l} = \psi^\ast_{l}(t) \psi^\dagger_{l}(t) + \chi^\ast_{l}(t) \chi_{l}(t). \]

The renormalization factor \( A(t) \) is given by

\[ 1 - |A(t)|^2 = U^2 N (N - 1)^2 \sum_l |I_l|^2 + \frac{1}{2} U^2 N (N - 1) \sum_{l_1 l_2} |I_{l_1 l_2}|^2, \]

with \( I_l \) and \( I_{l_1 l_2} \) given in Eq. [7]. The perturbed state \( |\Psi^{(1)}_I(t)\rangle \) is

\[ |\Psi^{(1)}_I(t)\rangle = -i (N - 1) \sqrt{N U} \sum_l I_l \chi^\dagger_l \frac{\psi^\dagger N - 1}{\sqrt{(N - 1)!}} |0\rangle - i \frac{1}{2} \sqrt{N (N - 1) U} \sum_{l_1 l_2} I_{l_1 l_2} \chi^\dagger_{l_1} \chi_{l_2} \frac{\psi^\dagger N - 2}{\sqrt{(N - 2)!}} |0\rangle. \]

The obtained occupation fraction of the \( \psi(t) \) mode is given in Eq. [9].
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