One Loop Chiral Corrections to Hard Exclusive Processes: I. Pion Case

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Abstract

We computed the leading non-analytic chiral corrections to the generalized parton distributions (GPDs) of the pion and to the two-pion distribution amplitudes. This allows us to obtain the corresponding corrections for the hard exclusive processes, such as $\gamma^*\gamma \rightarrow \pi\pi$, $\gamma^*N \rightarrow 2\pi N'$ and deeply virtual Compton scattering on the pion target.

1 Introduction

Hard processes are known to provide us with valuable information about the quark and gluon structure of hadrons in terms of parton distributions and parton distribution amplitudes. The generalized parton distributions (GPDs) \cite{1, 2, 3, 4}, entering the QCD description of the hard exclusive processes, interpolate, in a sense, between usual parton distribution, distribution amplitudes and elastic hadron form factors (for a review see e.g. \cite{5}). GPDs are determined by the low energy physics, therefore their dependence on the quark mass, small momentum transfer, etc. can be studied with help of chiral perturbation theory (ChPT).

In the present paper we develop the ChPT for the simple case of the GPDs in the pion and two pion distribution amplitudes (2DA). We present the results at the one loop level of the ChPT. In this way we compute the leading non-analytic corrections of the type $p^2 \ln(p^2)$ (where $p^2 \sim m^2_\pi \sim t$) to GPDs. Such corrections are universal and allow us to get an insight into structure of the GPDs. Additionally the leading non-analytic chiral correction to GPDs can be immediately translated to corresponding correction for the exclusive hard processes such as $\gamma^*\pi \rightarrow \gamma\pi$, $\gamma^*N \rightarrow 2\pi N'$, $\gamma^*\gamma \rightarrow \pi\pi$, etc. Such chiral corrections to the hard exclusive processes are computed in the present paper for the first time.

2 Chiral expansion for the light-cone matrix elements

In this section we discuss the matching of the light-cone quark-gluon operators to the operators in the effective field theory.

The generalized parton distributions (GPDs) and distributions amplitudes are defined as various matrix elements of the quark-gluon operators on the light cone. Let us introduce left and right twist-2 quark operators on the light cone:
\[ O_{fg}^{L}(\lambda) = \bar{\psi}_g \left( \frac{\lambda n}{2} \right) \gamma \frac{1 + \gamma_5}{2} \psi_f \left( -\frac{\lambda n}{2} \right), \]
\[ O_{fg}^{R}(\lambda) = \bar{\psi}_g \left( \frac{\lambda n}{2} \right) \gamma \frac{1 - \gamma_5}{2} \psi_f \left( -\frac{\lambda n}{2} \right). \]  

(1)

Here the vector \( n^\mu \) is the light-cone vector \( n^2 = 0 \), \( f, g \) stand for flavour indices. It is always assumed the colour gauge link along a straight line between the points \( \lambda n/2 \) and \(-\lambda n/2\). In the effective field theory the operators (1) are matched to the operators formulated in terms of effective degrees of freedom:

\[ O^{L,R}(\lambda) = F \otimes O^{L,R}_{\text{eff}}(\lambda), \]

(2)

where \( O^{L,R}_{\text{eff}}(\lambda) \) is an effective hadronic operator with the same quantum numbers (but not necessarily with the same twist) as the quark operators (1) and \( F \) is the generating function for the c-number coefficients which are input for effective field theory. In order to make sense out of decomposition (2) we need to have systematic power counting rules for construction of the hadronic operator \( O^{L,R}_{\text{eff}}(\lambda) \).

As usually we are going to use the Goldstone bosons of spontaneous chiral symmetry breaking as degrees of freedom for the construction of the effective operators. The standard power counting of the chiral perturbation theory (ChPT) uses the fact that the Goldstone bosons do not interact at zero momentum. Therefore on the level of this effective field theory, the expansion amounts to a derivative expansion of the effective Lagrangian [6] (see [7] for introduction to ChPT). The lowest order term reads:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \ldots \sim \frac{F_\pi^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle + \ldots \]  

(3)

where the Goldstone boson fields \( U(x) = \exp(i\pi^a(x)\tau^a/F_\pi) \) and \( \chi = 2B\text{diag}(m_u,m_d)\pi \) are subject to the following chiral counting rules:

\[ U \sim O(p^0), \quad \partial_\mu U \sim O(p^1), \quad \chi \sim O(p^2) \]  

(4)

where \( p \) is small momentum, i.e. small parameter of chiral expansion. From (3) one finds that the leading order effective Lagrangian (3) is of order \( p^2 \). We would like to emphasize, that locality together with condition \( UU^\dagger = 1 \) plays important role in derivation of the chiral expansion [3].

On the other hand, the description of the many hadron hard reactions grounds on the QCD collinear factorization. In such approach the non-perturbative part associated with a soft physics is parametrized by the matrix elements of some non-local light-cone operators. These objects appear as natural constructing blocks and it is convenient to

\footnote{We use standard notation for quark condensate \( \langle \bar{\psi}\psi \rangle = -F_\pi^2 B + O(p^2 \ln p) \)}
keep these operators non-local without transition to a series of the local operators. Such approach is useful in the higher energy phenomenology and we would like to follow this philosophy in the effective theory.

Therefore, to perform the matching (2) in terms of the effective fields the standard counting rules (4) should be slightly extended. The point is that although after the QCD factorization the soft part of the hard processes does not contain the hard momenta, it still “remembers” about them. In the operators (4) such “memory” is reflected by the dependence on the light-cone vector \( n^\mu \) and we have to specify the chiral order of this parameter.

Let us mention that the light-cone decomposition of any four-vector \( V^\mu \) reads:

\[
V^\mu = V^+ \tilde{n}^\mu + V^- n^\mu + V^\perp_\perp.
\]

Here are \( n^\mu \) and \( \tilde{n}^\mu \) are light-cone vectors \( n^2 = \tilde{n}^2 = 0 \) which we normalize as \( n \cdot \tilde{n} = 1 \). These two vectors define two-dimensional plane, the perpendicular plane is called transverse plane. The vectors from the transverse plane \( V^\perp_\perp \) by definition satisfy \( n \cdot V^\perp = \tilde{n} \cdot V^\perp = 0 \). The physical observables are obviously invariant under rescaling of the vector \( n \), i.e. under transformation \( n^\mu \rightarrow c \, n^\mu \) where \( c \) is an arbitrary nonzero constant. This invariance corresponds to the boost invariance of the physical observables. It is convenient to fix the normalization of the light-cone vector \( n^\mu \) by condition like \( n \cdot p = 1 \) where \( p \) is one of the small external momenta entering the soft part of the amplitude. Such condition implies that the light-cone vectors \( n^\mu \sim O(1/p) \) and \( \tilde{n}^\mu \sim O(p) \), where \( p \) assumed to be a generic soft momentum as in power counting (4).

To summarize, we have to construct the effective hadronic operator in eq. (2) using as building blocks chiral fields \( U(x) \) and their derivatives with counting rules:

\[
n \cdot \partial U(x) \sim O(p^0), \quad \tilde{n} \cdot \partial U(x) \sim O(p^2), \quad \partial_\perp U(x) \sim O(p).
\]

Using these building blocks one can derive that in effective field theory the operators (4) are matched to the operators in terms of Goldstone degrees of freedom with the same quantum numbers:

\[
O_{fg}^L(\lambda) = \frac{i F_\pi^2}{4} \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, F(\beta, \alpha) \left[ U \left( \alpha + \beta \Lambda n \right) n \cdot \partial U \left( \alpha - \beta \Lambda n \right) \right]_{fg} + \ldots,
\]

\[
O_{fg}^R(\lambda) = \frac{i F_\pi^2}{4} \int_{1}^{-1} d\beta \int_{1+|\beta|}^{-1-|\beta|} d\alpha \, F(\beta, \alpha) \left[ U^\dagger \left( \alpha + \beta \Lambda n \right) n \cdot \partial U \left( \alpha - \beta \Lambda n \right) \right]_{fg} + \ldots.
\]

Here \( F(\beta, \alpha) \) is the generating function of the tower of low-energy constants and \( \partial \) denotes \( \partial_\perp = \hat{\partial} - \hat{\partial} \). The low-energy constants are characteristics of the structure of the pion, they are not determined in the effective field theory. The ellipsis in eqs. (7) stands for operators which do not contribute to the one and two pion matrix elements of the operators \( O_{fg}^{L,R} \) or which are of higher orders in the chiral counting. Note that if one would consider the chiral corrections say for three pion distribution amplitudes one would need to add additional operators to eq. (7). In the next section we consider chiral expansion of some light-cone matrix elements at the leading order using formulae (7).
Let us note that for the gluon light-cone operators the construction of the corresponding effective operators is actually the same as for the singlet quark operators. The only difference that for gluon operators we have to introduce an independent generating function $F_G(\beta, \alpha)$. We do not discuss the gluon distributions in the present paper because their discussion repeats almost word by word that for the singlet quarks distributions.

3 Light-cone matrix elements in the leading order of chiral expansion

In order to see the physical meaning of the generating function $F(\beta, \alpha)$ in (7) we compute the GPDs of the pion and $2\pi$DAs in the leading order of chiral expansion. GPDs of the pion are defined as:

$$
\int \frac{d\lambda}{2\pi} e^{-i(n-P)\lambda} \langle \pi^b(p') | \text{tr} \left[ \hat{T} O^{L+R}(\lambda) \right] | \pi^a(p) \rangle = \begin{cases} 2i\varepsilon^{abc} H^I_1(x, \xi, t), & \text{for } \hat{T} = \tau^c \\ 2\delta^{ab} H^I_0(x, \xi, t), & \text{for } \hat{T} = 1 \end{cases} \quad (8)
$$

Here we introduced the nonsinglet $H^I_1$ and singlet $H^I_0$ GPDs of the pion. In what follows we use the standard notations for kinematical variables: $P = \frac{1}{2}(p + p')$, $\xi = -\frac{n \cdot (p' - p)}{n \cdot (p + p')}$ and $t = (p' - p)^2$. Now substituting the expressions for the light-cone operators in the effective theory (7) into the definition of GPDs (8) and computing the corresponding matrix elements at the tree level we obtain the following expressions for GPDs in the leading order of ChPT (denoted $\hat{\mathcal{H}}$):

$$
H^I_\alpha(x, \xi) = \int [d\alpha d\beta] F^I(\beta, \alpha) \left[ \delta(x - \xi \alpha - \beta) - (1 - I) \xi \delta(x - \xi(\alpha + \beta)) \right]. \quad (9)
$$

Here we introduce the notations $F^{1,0}(\alpha, \beta) = \frac{1}{2}(F(\beta, \alpha) \pm F(-\beta, \alpha))$ and $[d\alpha d\beta]$ stands for the integration over the rombus $|\alpha| + |\beta| \leq 1$, see Fig. 1. In the first term of eq. (9) one recognizes immediately the double distribution representation for the GPD $\hat{\mathcal{H}}_1$. The second term in eq. (9) corresponds to the D-term contribution $\hat{\mathcal{H}}$ to the GPDs. Note

2 which contributes only for the singlet GPD
that the D-term for the pion GPD is also fixed in terms of the double distribution due to the soft pion theorem of ref. [10].

\[ \hat{H}^{I=0}(x, \xi = \pm 1) = 0. \]  

(10)

From this simple exercise we learned that the generating function \( F(\beta, \alpha) \) for the low-energy chiral constants coincides in the leading order of ChPT with the double distribution for the pion in the chiral limit and at zero momentum transfer squared. This implies that the function \( F(\beta, \alpha) \) is related to the quark distributions in the pion in the chiral limit \( (m_\pi = 0) \):

\[
\int_{-1}^{1} d\alpha F^{I=0}(\beta, \alpha) = \frac{1}{2} \left[ \theta(\beta) \hat{q}(\beta) - \theta(-\beta) \hat{q}(-\beta) \right], \tag{11}
\]

\[
\int_{-1}^{1} d\alpha F^{I=1}(\beta, \alpha) = \theta(\beta) \hat{q}(\beta) + \theta(-\beta) \hat{q}(-\beta). \tag{12}
\]

The first moment of these distributions is related to the forward matrix elements of the energy momentum tensor and vector current respectively. This gives:

\[
\int [d\alpha d\beta] F^{I=0}(\beta, \alpha) = M_2^Q \tag{13}
\]

\[
\int [d\alpha d\beta] F^{I=1}(\beta, \alpha) = 1. \tag{14}
\]

where we introduce notation for the fraction of the pion momentum carried by quarks and antiquarks \( M_2^Q = \int_0^1 dx x (q(x) + \bar{q}(x)) \). Using these equations and (10) one can easily obtain first moment for GPD \( \hat{H}^{I=1,0} \):

\[
\int_{-1}^{1} dx x \hat{H}^{I=0}(x, \xi) = (1 - \xi^2)M_2^Q \tag{15}
\]

\[
\int_{-1}^{1} dx \hat{H}^{I=1}(x, \xi) = 1. \tag{16}
\]

The interpretation of the generating functions \( F^I(\beta, \alpha) \) as DD’s means that these functions depend on the factorization scale \( \mu \). The functional dependence from this parameter is described by the evolution equations [3]. For the sake of simplicity we do not write this argument explicitly but imply it.

Because the matching of the operators (11) is universal, the same effective operators (7) can be used to define the pion and two-pion distribution amplitudes in the leading order of ChPT.

The pion DA is defined as:

\[
\int \frac{d\lambda}{2\pi} e^{-i(n-p)u\lambda} \langle \pi^a(p) | \text{tr} \left[ \tau^b O^{L-R}(\lambda) \right] | 0 \rangle = i\delta^{ab} F_{\pi} \phi_\pi(u). \tag{17}
\]

In the leading order of ChPT we obtain for the pion DA in the chiral limit (referred to as \( \phi_\pi \)):

5
\[ \phi_\pi (u) = \int [d\alpha d\beta] \ F^{I=1} (\beta, \alpha) \ \delta(u - \alpha - \beta). \] (18)

Comparing this expression with eq. (9) we recover that

\[ \phi_\pi (u) = \pm \check{H}^{I=1} (u, \xi = \pm 1). \] (19)

This is again in an agreement with the soft pion theorem of ref. [10].

The two-pion DAs (2\pi DAs) are defined as:

\[ \int \frac{d\lambda}{2\pi} e^{-iu\lambda(n\cdot(p+p'))/2} \pi^a(p)\pi^b(p') \text{tr} \left[ \hat{T} \ O^{L+R}(\lambda) \right] |0\rangle = \left\{ \begin{array}{l} i\varepsilon^{abc} \Phi_{2\pi}^{I=1}(u, \eta, m_{\pi\pi}), \text{ for } \hat{T} = \tau^c \\ \delta^{ab} \Phi_{2\pi}^{I=0}(u, \eta, m_{\pi\pi}), \text{ for } \hat{T} = 1 \end{array} \right. \] (20)

with the kinematical variables \( \eta = \frac{n \cdot (p-p')} {n \cdot (p+p')} \) and \( m_{\pi\pi}^2 = (p+p')^2 > 0 \). The leading ChPT order expression for 2\pi DAs (referred to as \( \phi_{2\pi} \)) has the form:

\[ \phi_{2\pi}^{I=1}(u, \eta) = \int [d\alpha d\beta] \ F^{I}(\beta, \alpha) \ \eta \ \delta(u - \alpha - \eta - \beta) - (1 - I) \ \delta(u - \alpha - \beta). \] (21)

We see that the 2\pi DAs are expressible in terms of the same double distribution. In Fig. 1 we illustrated how to obtain pion GPDs, D-term and pion and 2\pi DAs from the double distribution \( F(\beta, \alpha) \). From eq. (13) we obtain

\[ \int_{-1}^{1} du \ \phi_{2\pi}^{I=0}(u, \eta) = -(1 - \eta^2)M_2^Q \] (22)

\[ \int_{-1}^{1} du \ \phi_{2\pi}^{I=1}(u, \eta) = 1. \] (23)

Again, using (21) and (18) one easily obtain soft pion theorems [10, 11]:

\[ \phi_{2\pi}^{I=0}(u, \eta = \pm 1) = 0, \] (24)

\[ \phi_{2\pi}^{I=1}(u, \eta = \pm 1) = \pm \phi_\pi (u). \] (25)

### 4 Leading non-analytic chiral corrections to GPDs and DAs

In what follows we consider the next-to-leading chiral corrections of the type \( p^2 \ln(p^2) \) with \( m_{\pi}^2 \sim t \sim m_{\pi\pi}^2 \sim p^2 \). These leading non-analytic contributions to the pion GPDs and DAs are universal in the sense that they are expressible completely in terms of the leading order generating function \( F(\beta, \alpha) \). The corresponding corrections are obtained computing the loop diagrams shown on Figs. 2, 3. The divergencies contained in the one loop graphs can be removed by renormalization of the coupling constants which appear in the NLO of ChPT. Then loop contributions depends on the chiral renormalization scale \( \mu_\chi \). A change in this scale however only adds a constants which will shift running couplings. Because we compute only the leading non-analytic terms the detailed discussion of this subject is outside of our consideration.
Figure 2: Diagrams contributing to GPD matrix elements. Arrows denote the directions of external momenta. Tree level (a) and loop corrections (b), (c). The four pion vertex in the diagram (b) originates from expansion of effective Lagrangian (3).

4.1 Generalized Parton Distributions

Computing diagrams on Fig. 2 for the case of pion GPDs we obtain the following expressions for them with leading non-analytic corrections included:

\[ H^{l=0}(x, \xi, t) = \hat{H}^{l=0}(x, \xi) + \frac{\theta(|x| \leq \xi)}{2(4\pi F_\pi)^2} \ln \left[ m_\pi^2 - (1 - \eta^2) \frac{t}{4} \right] \frac{\partial}{\partial \eta} \Phi_{2\pi}^{l=0} \left( \frac{x}{\xi}, \eta \right) \]  \hspace{1cm} (26)

\[ H^{l=1}(x, \xi, t) = \hat{H}^{l=1}(x, \xi) \left( 1 - \frac{m_\pi^2 \ln m_\pi^2}{(4\pi F_\pi)^2} \right) + \frac{\theta(|x| \leq \xi)}{2(4\pi F_\pi)^2} \ln \left[ m_\pi^2 - (1 - \eta^2) \frac{t}{4} \right] \frac{\partial}{\partial \eta} \Phi_{2\pi}^{l=1} \left( \frac{x}{\xi}, \eta \right) \]  \hspace{1cm} (27)

In both equations \( \hat{H}^l \) and \( \Phi_{2\pi}^l \) are the leading order expressions given by eq. (8) and eq. (21) correspondingly. For convenience we do not write explicitly the dependence on \( \mu_\chi \) assuming it in all logarithms:

\[ \ln \left[ m_\pi^2 - (1 - \eta^2) \frac{t}{4} \right] \equiv \ln \left[ m_\pi^2 / \mu_\chi^2 \right] + \ln \left[ 1 - (1 - \eta^2) \frac{t}{4m_\pi^2} \right] \]  \hspace{1cm} (28)

Nontrivial contributions with two pion DAs \( \Phi_{2\pi}^l \) originated from the diagram Fig. 2(b). Appearance of two pion DAs is a direct consequence of the presence of the two pion state in the \( t \)-channel. From eq. (26) and eq. (27) we see that in both cases the leading non-analytic in \( t \) chiral corrections are nonzero only in the so-called ERBL-region \(|x| \leq \xi\). This shows that the \( t \)-dependence of GPDs can not be reduced to popular factorized ansatz \( H(x, \xi, t) = H(x, \xi) F(t) \).
At the points $x = \pm \xi$ chiral corrections nullifies (except trivial $t$-independent term in the isovector GPD), because it is expected that the two-pion distribution amplitudes nullify at the end points:

$$\hat{\phi}^{I=1}_{2\pi}(u = \pm 1, \eta) = 0. \quad (29)$$

Such behaviour at the points $x = \pm \xi$ is very important for validity of the factorization theorem in different hard exclusive reactions.

Let us mention that DD’s $F^I(\beta, \alpha)$ depend on the factorization scale $\mu$. From physical point of view, the evolution in $\mu$ and computing chiral corrections are independent operations and hence must commute with each other. Using formulae (26) and (27) one can easily see that this property takes place. Consider, for simplicity, the leading logarithmic approximation for the evolution in factorisation scale $\mu$. To this accuracy one can construct multiplicatively renormalizable moments:

$$H^{I=1}_n(\xi, t) = \int_{-1}^{1} dx \; H^{I}(x, \xi, t) \; C_n^{3/2}(x/\xi) \quad (30)$$

$$\Phi^{I=1}_n(\eta, m_{\pi\pi}) = \int_{-1}^{1} du \; \Phi^I_{2\pi}(u, \eta, m_{\pi\pi}) \; C_n^{3/2}(u) \quad (31)$$

where $C_n^{3/2}(u)$ is a Gegenbauer polynomial. Their evolution is given by simple equations$^3$

$$H^{I=1}_n(\xi, t|\mu^2) = L(\mu, \mu_0)^{\gamma_n/\beta_0} H^{I=1}_n(\xi, t|\mu_0^2) \quad (32)$$

$$\Phi^{I=1}_n(\eta, m_{\pi\pi}|\mu^2) = L(\mu, \mu_0)^{\gamma_n/\beta_0} \Phi^{I=1}_n(\eta, m_{\pi\pi}|\mu_0^2) \quad (33)$$

where $L(\mu, \mu_0) = \alpha(\mu^2)/\alpha(\mu_0^2)$, $\gamma_n$ and $\beta_0$ is leading order anomalous dimension and QCD $\beta$-function respectively.

Computing the Gegenbauer moments of eq. (27) we obtain:

$$H^{I=1}_{2n}(\xi, t|\mu^2) = \hat{H}^{I=1}_{2n}(\xi|\mu^2) \left( 1 - \frac{m_{\pi}^2 \ln m_{\pi}^2}{(4\pi F_{\pi})^2} \right) +$$

$$\frac{1}{2(4\pi F_{\pi})^2} \int_{-1}^{1} d\eta \left[ m_{\pi}^2 - (1 - \eta^2)^t \right] \ln \left[ m_{\pi}^2 - (1 - \eta^2)^t \right] \frac{\partial}{\partial \eta} \hat{\Phi}^{I=1}_{2n}(\eta|\mu^2). \quad (34)$$

Using (32) and (33) we obtain that above equation remains unchanged. In other words, chiral corrections are decoupled from evolution as it should be.

Chiral loops generate corrections to the soft pion theorem (10). Taking limit $\xi \to \pm 1, t \to 0$ we obtain:

$$H^{I=0}_{I=0}(x, \xi = \pm 1, t = 0) = \frac{m_{\pi}^2 \ln(m_{\pi}^2)}{2(4\pi F_{\pi})^2} \int_{-1}^{1} d\eta \; \hat{\phi}^{I=0}_{2\pi}(x, \eta). \quad (35)$$

Whereas the soft pion theorem (19) does not get corrections of the order $m_{\pi}^2 \ln(m_{\pi})$

$$H^{I=1}(x, \xi = \pm 1, t = 0) = \hat{\phi}_{\pi}(u) + O(m_{\pi}^2). \quad (36)$$

$^3$We consider only isovector case $I=1$ for simplicity
Now we consider various limiting cases of the obtained results for the chiral expansion of the pion GPDs.

**Sum rules**

The first Mellin moment of the non-singlet GPD \( H_{I=1} \) is related to the pion electromagnetic form factor

\[
\int_{-1}^{1} dx \ H_{I=1}^1(x, \xi, t) = F_{\pi e.m}^1(t). \tag{37}
\]

Integrating the one-loop result (27) we obtain the well-known result for the leading non-analytic contribution to the pion e.m. form factor\[6\]

\[
F_{\pi e.m}^1(t) = 1 - \frac{m_\pi^2 m_{\pi}^2}{(4\pi F_\pi)^2} + \frac{1}{(4\pi F_\pi)^2} \int_0^1 d\eta \left[ m_\pi^2 - (1 - \eta^2) \frac{t}{4} \right] \ln \left[ m_\pi^2 - (1 - \eta^2) \frac{t}{4} \right]. \tag{38}
\]

The second moment of the singlet GPD \( H_{I=0} \) is related to the form factors of the quark part of the energy momentum tensor:

\[
\int_{-1}^{1} dx \ x \ H_{I=0}^1(x, \xi, t) = 2 \theta_2(t) - 2 \xi^2 \theta_1(t), \tag{39}
\]

where \( \theta_{1,2}(t) \) are the pion form factors of the quark part of the energy momentum tensor \[12, 13\]:

\[
\langle p'|T_{\mu\nu}^Q|p \rangle = 2 \ P^\mu P^\nu \theta_2(t) + \frac{1}{2} (g_{\mu\nu} \Delta^2 - \Delta^\mu \Delta^\nu) \theta_1(t). \tag{40}
\]

Computing the second Mellin moment of our results for the singlet GPD (23) we obtain the leading non-analytic corrections for the energy momentum tensor:

\[
\theta_2(t) = \frac{1}{2} M_2^Q \left( 1 + O(p^2) \right) \tag{41}
\]

\[
\theta_1(t) = \frac{1}{2} M_2^Q \left( 1 + \frac{m_{\pi}^2 - 2t}{(4\pi F_\pi)^2} \int_0^1 d\eta \ (1 - \eta^2) \ln \left[ m_{\pi}^2 - (1 - \eta^2) \frac{t}{4} \right] \right), \tag{42}
\]

This result coincides with that of ref. \[13\].

**Forward limit and parton distributions in the transverse plane**

In the forward limit, i.e., \( \xi \to 0, t \to 0 \) the GPDs \( H_{I=0}^1 \) and \( H_{I=1}^1 \) are reduced to the singlet and non-singlet quark distributions in the pion correspondingly. It is easy to see from eq. (26) that the singlet quark distribution in the pion does not receive non-analytical chiral corrections. The same conclusion was reached in refs. \[14, 15\]. As to nonsinglet quark distribution, the forward limit of eq. (27) gives:
\[ q(x) = \tilde{q}(x) \left( 1 - \frac{m_\pi^2 \ln m_\pi^2}{(4\pi F_\pi^2)^2} \right) + \delta(x) \frac{m_\pi^2 \ln m_\pi^2}{(4\pi F_\pi^2)^2}. \] (43)

This is exactly the result obtained recently in refs. [14, 15]. Note that two limits \( \xi \to 0 \) and \( m_\pi \to 0 \) do not commute. The above result is obtained by taking the limit \( \xi \to 0 \) before the limit \( m_\pi \to 0 \).

Let us now consider more general limit when \( \xi \to 0 \) but \( t \neq 0 \). In such limit the GPDs can interpreted as the Fourier transform of the probability distribution of partons in the transverse plane [16]. Taking this particular limit we again obtain that the singlet GPDs has no the non-analytic chiral corrections, whereas the result for the non-singlet GPDs can be written as:

\[ H^{t=1}(x,0,t) \equiv q(x,t) = \tilde{q}(x) \left( 1 - \frac{m_\pi^2 \ln m_\pi^2}{(4\pi F_\pi^2)^2} \right) + \delta(x) \frac{1}{(4\pi F_\pi^2)^2} \int_0^1 d\eta \left[ m_\pi^2 - (1 - \eta^2)^2 \right] \ln \left[ m_\pi^2 - (1 - \eta^2)^2 \right]. \] (44)

The probability distribution of partons in the transverse plane is obtained from the above result by the Fourier transformation [16]:

\[ f(x,b) = \int \frac{d^2 \Delta}{(2\pi)^2} H(x,0,-\Delta) e^{ib \cdot \Delta}. \] (45)

As the expression (44) is valid only for small values of \( t \) we are able to derive only the behaviour of \( f(x,b) \) at large values of the impact parameter \( b \):

\[ f(x,b) \sim \delta(x) \frac{1}{(4\pi F_\pi^2)^2} \frac{2}{3\pi b} \quad \text{for} \quad \frac{1}{4\pi F_\pi} \ll b \ll \frac{1}{m_\pi}. \] (46)

From eq. (44) we can also obtain the leading chiral contribution to the average width of the \( b \) distribution

\[ \langle b^2 \rangle = \int d^2 b b^2 f(x,b) = -\frac{2}{3} \delta(x) \frac{\ln m_\pi^2}{(4\pi F_\pi^2)^2}. \] (47)

We see that the width of the parton distribution in the transverse plane is divergent in the chiral limit. This divergent piece arises from the contribution of the long-range pion cloud. Note that partons which are “responsible” for the divergent chiral contribution are “concentrated” in the region of small \( x \).

**Extrapolation to \( t = 0 \)**

Generalized parton distributions can be probed in the hard exclusive reactions. Usually

\(^4\text{More precisely at } x \ll m_\pi^2/(4\pi F_\pi^2) \)
the point \( t = 0 \) is not accessible directly in these processes. Our results for the leading non-analytical corrections for the GPDs allow us to study the extrapolation of the GPDs to the point \( t = 0 \). Exactly at \( t = 0 \) the expression for the pion GPDs with leading non-analytic chiral corrections included has the form:

\[
H^{I=0}(x, \xi, t = 0) = \frac{\partial}{\partial t} H^{I=0}(x, \xi)
\]

(48)

\[
+ \frac{\theta ||x| \leq \xi|}{2(4\pi F_\pi)^2} m_\pi^2 \ln(m_\pi^2) \int_{-1}^{1} d\eta \Phi^{I=0}_{2\pi}(x, \xi, \eta) \]

\[
H^{I=1}(x, \xi, t = 0) = \frac{\partial}{\partial t} H^{I=1}(x, \xi)
\]

(49)

\[
+ \frac{\theta ||x| \leq \xi|}{(4\pi F_\pi)^2} m_\pi^2 \ln(m_\pi^2) \phi^{I=1}(x, \xi, \eta).
\]

Also from eqs. (26,27) we can easily obtain the leading non-analytic contributions to the “slopes” of the \( t \)-dependence of the GPDs. The result is:

\[
\frac{\partial}{\partial t} H^{I=0}(x, \xi, t) \bigg|_{t=0} = -\frac{\theta ||x| \leq \xi|}{(4\pi F_\pi)^2} \ln(m_\pi^2) \int_{-1}^{1} d\eta \Phi^{I=0}_{2\pi}(x, \xi, \eta),
\]

(50)

\[
\frac{\partial}{\partial t} H^{I=1}(x, \xi, t) \bigg|_{t=0} = -\frac{\theta ||x| \leq \xi|}{4(4\pi F_\pi)^2} \ln(m_\pi^2) \int_{-1}^{1} d\eta \eta \Phi^{I=1}_{2\pi}(x, \xi, \eta).
\]

Such relations can be useful for extrapolation of the experimental data from nonzero \( t \) to the point \( t = 0 \). Also these relations can be useful for interpretation of the lattice results.

4.2 Two pion Distribution Amplitudes

Diagrams for the chiral expansion of the \( 2\pi \)DAs are depicted in Fig.3. Their contribution reads:

\[
\Phi^{I=0}_{2\pi}(u, \eta, m_\pi) = \frac{\partial}{\partial t} \Phi^{I=0}_{2\pi}(u, \eta) + \frac{1}{2(4\pi F_\pi)^2}
\]

\[
\times \left[ m_\pi^2 - 2m_\pi^2 \right] \int_{-1}^{1} d\eta' \ln \left[ m_\pi^2 - (1 - \eta'^2) m_\pi^2 \right] \phi^{I=0}_{2\pi}(u, \eta'),
\]

(51)
\[ \Phi_{2\pi}^{(I)}(u, \eta, m_{\pi\pi}) = \frac{\phi}{2\pi} \frac{1}{(4\pi F_{\pi})^2} \left( 1 - \frac{m_{\pi}^2 \ln m_{\pi}^2}{4} \right) \]

These equations can be easily obtained from \((26)\) and \((27)\) using crossing symmetry. One can investigate all basic properties of ChPT corrections in the same way as for GPD. We shall not repeat this discussion.

As we can see from \((51)\) and \((52)\) chiral corrections have a simple behaviour in parameter \(\eta\). Recall that

\[ \eta = v \cos \theta_{cm}, \]

where \(\theta_{cm}\) is polar angle of the pion momentum in the CM frame with respect to the direction of the total momentum \(P\) and \(v\) is the velocity of produced pions in the center of mass frame:

\[ v = \sqrt{1 - \frac{4m_{\pi}^2}{m_{\pi\pi}^2}} \]

In other words, chiral corrections contribute only to lowest partial wave because outgoing pions can not be produced in state with higher orbital momentum from the corresponding diagram Fig. 2(b).

Let us also note that in the physical region \(m_{\pi\pi} > 2m_{\pi}\) chiral logarithm generate imaginary part due to two pion intermediate state:

\[ \frac{1}{\pi} \mathrm{Im} \Phi_{2\pi}^{(I=0)}(u, \eta, m_{\pi\pi}) = \Theta(m_{\pi\pi} > 2m_{\pi}) \left[ m_{\pi}^2 - \frac{2m_{\pi}^2}{2(4\pi F_{\pi})^2} \right] \int_{-\nu}^{\nu} d\eta' \frac{\phi}{2\pi} \Phi_{2\pi}^{(I=0)}(u, \eta'), \]

\[ \frac{1}{\pi} \mathrm{Im} \Phi_{2\pi}^{(I=1)}(u, \eta, m_{\pi\pi}) = \frac{\eta \Theta(m_{\pi\pi} > 2m_{\pi})}{2(4\pi F_{\pi})^2} \int_{-\nu}^{\nu} d\eta' \left[ m_{\pi}^2 - (1 - \eta'^2) \frac{m_{\pi\pi}^2}{4} \right] \frac{\partial}{\partial \eta'} \Phi_{2\pi}^{(I=1)}(u, \eta'), \]

This imaginary part does not depend on the renormalization scale \(\mu_{\chi}\) because to a given accuracy analytical contributions can not develop imaginary part of the matrix element.

## 5 Chiral corrections to the amplitudes of the hard exclusive processes

From the expressions for the chiral expansion of the pion GPDs and DAs we can obtain the chiral expansion of the amplitudes of various hard exclusive processes.

### 5.1 \(\gamma^*\gamma \to \pi\pi\) and \(\gamma^* N \to 2\pi N'\) near pions threshold

We start with the reaction \(\gamma^*\gamma \to \pi\pi\) near the threshold and in the hard regime, \(i.e.\) the virtuality of the photon \(\gamma^*\) is much larger than the typical hadronic scale \(Q \gg \Lambda_{QCD}\).
To the leading twist approximation and leading order of QCD perturbation theory the amplitude of the reaction is dominated by the helicity amplitude describing scattering of transversely polarized photons \[17\]. The answer is expressible in terms of the isoscalar 2πDA:

\[
A_{++}(\eta, m_{\pi\pi}) = \int_{-1}^{1} \frac{d\eta}{1-u} \Phi_{2\pi}(u, \eta, m_{\pi\pi}).
\]

(55)

Substituting here the results for the chiral corrections to the 2πDA (see eq. (51)) we obtain the chiral expansion of the corresponding amplitude

\[
A_{++}(\eta, m_{\pi\pi}) = \hat{A}(\eta) + \frac{1}{2(4\pi F_\pi)^2} \left[ m_{\pi}^2 - 2m_{\pi\pi}^2 \right] \int_{-1}^{1} d\eta' \ln \left[ m_{\pi}^2 - (1 - \eta'^2) \frac{m_{\pi\pi}^2}{4} \right] \hat{A}(\eta').
\]

(56)

Here we used an obvious notation \(\hat{A}(\eta)\) for

\[
\hat{A}(\eta) = A(\eta, m_{\pi\pi} = 0) \bigg|_{m_{\pi\pi} = 0} = \int_{-1}^{1} \frac{d\eta}{1-u} \Phi_{2\pi}(u, \eta).
\]

(57)

We note that the amplitude of the hard \(\gamma^* \gamma \rightarrow \pi\pi\) reactions, in contrast to its soft counterpart (low photon virtuality), is the same for the \(\pi^0\pi^0\) and \(\pi^+\pi^-\). The difference between these final states is the higher twist effect.

From eq. (56) we see immediately that the \(O(p^2 \ln p^2)\) corrections affect only the S-wave partial amplitude and hence the whole amplitude is shifted by the \(\eta\) independent “partonic” form factor \(F(m_{\pi\pi})\):

\[
F(m_{\pi\pi}) = \frac{1}{2(4\pi F_\pi)^2} \left[ m_{\pi}^2 - 2m_{\pi\pi}^2 \right] \int_{-1}^{1} d\eta' \ln \left[ m_{\pi}^2 - (1 - \eta'^2) \frac{m_{\pi\pi}^2}{4} \right] \hat{A}(\eta').
\]

(58)

From (58) we obtain that imaginary part of the amplitude is given by imaginary part of that formfactor:

\[
\text{Im}A_{++} = \frac{\pi \theta(m_{\pi\pi} > 2m_{\pi})}{2(4\pi F_\pi)^2} \left[ m_{\pi}^2 - 2m_{\pi\pi}^2 \right] \int_{-\gamma}^{\gamma} d\eta' \hat{A}(\eta').
\]

(59)

At the twist three level there is contribution from the amplitude describing scattering of longitudinally polarized virtual photon. In the so-called Wandura-Wilczek approximation the answer can be expressed in terms of the same isoscalar DA:

\[
A_{0+}(\eta, m_{\pi\pi}) = \partial_\eta \int_{-1}^{1} d\eta \Phi_{2\pi}(u, \eta, m_{\pi\pi}) \frac{2}{1-u} \ln \left( 1 - \frac{1-u}{2} \right). \]

(60)

The additional derivative in \(\eta\) ensures correct symmetrical properties of the amplitude. Because of of that derivative the chiral corrections \(O(p^2 \ln p^2)\) does not contributue to this amplitude and we obtain:

\[
A_{0+}(\eta, m_{\pi\pi}) = \partial_\eta \int_{-1}^{1} d\eta \frac{\hat{A}(\eta)}{\Phi_{2\pi}(u, \eta)} \frac{2}{1-u} \ln \left( 1 - \frac{1-u}{2} \right) + O(p^2).
\]

(61)

\(^5\text{For brevity we do not write trivial factors related to the quark charges}\)
In the hard exclusive process of two-pion production off the nucleon $\gamma^* N \rightarrow 2\pi N'$ the dependence of the amplitude (at the leading order) on the di-pion mass $m_{\pi\pi}$ is governed by the integral (55) (for the pion pairs in the $C = +1$ channel) and by the integral:

$$B(\eta, m_{\pi\pi}) = \int_{-1}^{1} du \frac{\Phi_{2\pi}^{I=1}(u, \eta, m_{\pi\pi})}{1 - u}, \quad \text{(62)}$$

in the $C = -1$ channel [10, 20, 21]. Substituting here the results for the chiral corrections to the $2\pi$DA (see eq. (52)) we obtain the chiral expansion of the corresponding amplitude

$$B(\eta, m_{\pi\pi}) = \mathcal{B}(\eta) \left( 1 - \frac{m_{\pi}^2 \ln m_{\pi}^2}{(4\pi F_\pi)^2} \right)$$

$$+ \frac{\eta}{2(4\pi F_\pi)^2} \int_{-1}^{1} d\eta' \left[ \int_{-1}^{1} m_{\pi}^2 - (1 - \eta'^2) m_{\pi\pi}^2 \frac{\partial}{\partial \eta'} \mathcal{B}(\eta') \right] \ln \left[ \frac{m_{\pi}^2 - (1 - \eta'^2) m_{\pi\pi}^2}{4} \right]. \quad \text{(64)}$$

5.2 DVCS on the pion target

The amplitude of the deeply virtual Compton scattering (DVCS) in the leading order of pQCD is expressible in terms of the Compton form factors, which are defined as [1, 2]

$$H(\xi, t) = \int_{-1}^{1} dx \frac{H^{I=0}(x, \xi, t)}{x - \xi + i\varepsilon}. \quad \text{(64)}$$

The imaginary part of the amplitude is defined by the residue at $x = \xi$:

$$\text{Im} H(\xi, t) = -\pi H^{I=0}(\xi, \xi, t), \quad \text{(65)}$$

and sensitive to the diagonal value of the GPD, therefore it is not affected by the non-analytic chiral corrections.

Chiral expansion of the Compton form factors can be written as

$$H(\xi, t) = \mathcal{H}(\xi) + \frac{1}{2(4\pi F_\pi)^2} \int_{-1}^{1} d\eta \ln \left[ m_{\pi}^2 - (1 - \eta^2) \frac{t}{4} \right] \mathcal{A}(\eta). \quad \text{(66)}$$

We see again that the amplitude is shifted by the $\xi$-independent form factor $F(t)$ (see eq. (58)) in the space-like region.

We want to emphasize that the 1-loop chiral corrections affect only the real part of the Compton amplitude because chiral correction nullifies at the points $x = \pm \xi$ according to eq. (26). Hence we see that real and imaginary parts have a quite different behaviour at the small values of $t$. This true not only for isoscalar GPD which contains the D-term but also for the isovector GPD. This qualitative difference in $t$-behaviour of the real and the imaginary parts of the Compton amplitude leads to qualitatively different $t$-behaviour of, say, beam charge and beam helicity asymmetries. The former is sensitive to the real part and the latter to the imaginary part of the DVCS amplitude. Note the naive factorization ansatz used for modelling of GPD at small $t$ suggests the same $t$-dependence of the real and imaginary parts of the Compton amplitudes.
6 Conclusions

We calculated the leading non-analytic chiral corrections to the generalized parton distributions and to the two pion distribution amplitudes. In this paper we restricted ourselves to the case of the pion. First we established chiral counting rules for the leading twist operators on the light-cone in the effective field theory. On the basis of these counting rules we constructed the corresponding operators in the effective field theory and further computed the leading non-analytic corrections to GPDs, $2\pi$DAs and as consequence to the hard exclusive processes like $\gamma^*\gamma \to \pi\pi$, $\gamma^*N \to 2\pi N'$, $\gamma^*\pi \to \gamma\pi$, etc.

Obtained results show that the leading non-analytic chiral corrections to the GPDs affect only in the region $|x| \leq \xi_6$ and as consequence only the real part of the leading order DVCS amplitude receives the non-analytical chiral corrections. This implies that the $t$-dependencies of such observables as charge and helicity beam asymmetries could be qualitatively different even at small momentum transfer. Hence, the studying of the $t$-dependence of these asymmetries can provide us important information about the $t$-dependence of the GPDs.

We checked our results comparing them in the specific limiting cases with known chiral corrections to form factors and to parton distributions. Additionally we derived the form of the parton distributions in the transverse plane at large values of the impact parameter.

Our results for the leading non-analytic chiral corrections to GPDs and $2\pi$DAs can be used for extrapolation \cite{22} of the lattice and experimental data in $m_\pi$ and $t$. Also the data on the hard exclusive processes confronted with our chiral perturbation theory predictions will be a new, complementary to the low energy soft reactions, test of the chiral dynamics.

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\footnote{For isovector GPD there is a $t$-independent correction “living” on the whole interval of $x$.}
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