Remarks on Inflation

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Abstract. It has been shown that sub-Planckian models of inflation require initial
homogeneity on super-Hubble scales under certain commonly held assumptions. Here
I remark on the possible implications of this result for inflationary cosmology.

The observed homogeneity of the universe on superhorizon scales can be ex-
plained if there was a period of accelerated expansion (inflation) in the early uni-
verse that took a relatively small homogeneous patch of space and blew it up to
encompass a volume larger than what we observe today. In a recent paper [1], Mark
Trodden and I addressed the issue of how large the initial homogeneous patch has
to be. The result we obtained is that there is a lower bound on the inflationary
horizon, \( H_{\text{inf}}^{-1} \), which depends on the pre-inflationary cosmology. In particular, if
the pre-inflationary cosmology is a Friedman-Robertson-Walker (FRW) universe,
we must have \( H_{\text{inf}}^{-1} > H_{\text{FRW}}^{-1} \). If we further assume that, to get inflation with Hubble constant \( H_{\text{inf}}^{-1} \), the appropriate inflationary conditions (homogeneity, vacuum
domination etc.) must be satisfied over a region of physical size \( L \) which is larger
than \( H_{\text{inf}}^{-1} \), then we obtain \( L > H_{\text{FRW}}^{-1} \). Hence the conditions for inflation need to
be satisfied on cosmological scales specified by the pre-inflationary epoch. In the
particular case of a radiation dominated FRW, the problem seems to be even more
severe since the causal horizon coincides with \( H_{\text{FRW}}^{-1} \). Hence, it is clear that to solve
the homogeneity problem, inflationary models in which inflation emerges from a
non-inflationary, classical epoch, must assume large-scale homogeneity as an initial
condition.

Perhaps more important than the result itself is the fact that we have identified
the conditions under which such a derivation is possible. The key assumptions
are that Einstein’s equations and the weak energy conditions are valid, and that
spacetime topology is trivial. (We also assume that singularities apart from the big
bang are absent.) If these conditions hold, one would conclude that sub-Planckian
inflation alleviates the large-scale homogeneity problem but does not solve it.

\footnote{The word “solve” may hold different meanings for different individuals. My view is that a
solution should not assume the result it purports to obtain.}
Note that there are two related issues that are brought to the forefront in the above discussion. The first is - do inflationary models (within the stipulated assumptions) solve the homogeneity problem? - and my answer to this question is in the negative. The second is - can inflation occur, given that it seems to require unlikely initial conditions? Here the answer is in the positive - inflation can indeed occur as long as we have a suitable physical theory. However, one may wish to further consider the likelihood of having no inflation or the probabilities with which various kinds of inflation can occur, and this is where the questions become difficult. Hopefully some of the issues involved will become clearer by the end of this article.

A way around the result in [1] is to consider inflationary models in which inflation starts at the Planck epoch (for example, chaotic inflation [2]) since these do not contain a classical pre-inflationary epoch. Hence, at least as far as classical physics goes, inflation is imposed as an initial condition in these models. These initial conditions are sometimes justified by using quantum cosmology – if the wavefunction of the universe is “highly peaked” around the inflationary initial conditions, one might say that these conditions are favored. However, what if the wavefunction only has a small tail around the inflationary initial conditions? I do not think that that would exclude an inflating epoch of our universe since, it could be argued, that most of the other non-inflating universes (where the peak of the wavefunction is) would not be able to harbor observers such as us. Hence the argument appears to be inconclusive at present. On the other hand, anthropic arguments are essential to any theory in which the creation of universes is probabilistic. As our understanding of astrophysics and gravity improves, the arguments are likely to get sharpened. As a very simple example of possible forthcoming refinements, if we assume that life can only exist on planets, then an understanding of the cosmological conditions leading to maximal planetary formation will help in narrowing down a measure for calculating probabilities.

Guth [3] has given a persuasive argument for believing that inflation took place in the early universe in spite of any required unlikely initial conditions. He likens inflationary cosmology to the evolution of life. Today we observe a rich variety of life forms, and also a large homogeneous universe. It is hard to imagine how all the miraculous forms of life could have been created directly. Similarly, it is hard to explain the direct creation of our universe. These wonders are easier to comprehend in terms of evolutionary theory - life started out in the shape of some very simple molecules, which then inevitably evolved into the present forms of life. Similarly, a small patch of the universe that satisfied some suitable properties underwent inflation and inevitably evolved into our present universe. So Guth makes the correspondence shown in Fig. 1.

I find this to be a very compelling argument for the existence of an inflationary phase of the universe. The discussion in Ref. [1] impacts on this correspondence in the first stage - what is the chance of getting a small patch with the correct conditions? This is the same question that biologists may ask - what is the chance of getting the first few molecules from which life can follow?
In the theory of evolution, the formation of the right kind of molecules would depend on the geological and climatic conditions at the time. For example, the occurrence of lightening storms could facilitate chemical reactions that could enhance the probability of forming the molecules. Similarly we have found that the chance of getting a small patch in the universe with all the right conditions for inflation is very low, but that there are conditions under which this probability can be enhanced. So I would like to propose a further extension to the correspondence as shown in Fig. 2.

Our philosophy in adopting inflation as a paradigm is that it greatly enhances the probability for the creation of the universe that we see, even though it is at the expense of invoking a fundamental scalar field - the inflaton. Then, since the introduction of the other factors in the extended correspondence can further enhance the chances of creating an inflating universe, this same philosophy guides us to include them as part of the paradigm.

I should add that the preceding argument is not completely obvious, as Guth and Linde explain, since inflation washes out the initial conditions and a scheme to obtain more likely initial conditions may only provide an infinitesimal increase in the final probability of observing a homogeneous universe. However, if we consider a physical theory in which direct creation of the universe, sub-horizon inflation, as well as super-horizon inflation are all possible, and in which the inflationary phases occur with the same expansion rates, it seems clear that the most spatial volume will be produced by the sub-horizon inflation. This will be true even in the case when the inflation is “eternal”.

At this conference Koffman has raised the practical question of how these developments should affect the way inflationary cosmology is studied. I would first of all suggest recognition of the fact that current inflationary models do not “solve” the homogeneity problem but only alleviate it. To solve the problem, one must resort to relaxing at least one of the common assumptions. Linde has been advocating the path of Planckian inflation for a variety of reasons. The path of quantum cosmology has also been taken by a number of researchers but, based on what I discussed above, these approaches appear to leave open a number of difficult questions that
Lightening storms, volcanic eruptions, aliens, etc. \[ \longleftrightarrow \] Non-minimal scalars, quantum effects, spatial topology, etc.

**FIGURE 2.** A proposed extension to Guth’s correspondence.

need to be answered before we can claim to have understood the homogeneity of the universe. To me two other approaches seem more promising. The first is to study the conditions under which quantum effects can give rise to violations of the weak energy condition (such as pursued by Ford and collaborators [4]), and subsequently to creation of baby universes (for an early related paper see [5]). The second approach is to explore modifications of the classical Einstein equations. This also ties in with the fact that essentially any high energy theory that is being considered these days does include such modifications.

If quantum effects or extensions to Einstein’s equations lead to the conclusion that inflation can begin from a microphysical patch, profound consequences will follow since it would open the possibility of creating universes in the laboratory. One could further imagine that stellar collapse, for example, may lead to baby universe creation, and that there may be other universes lurking in the centers of galaxies.

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