No $Z_N$ - bubbles in hot Yang-Mills theory

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Abstract

Pure Yang-Mills theory at high temperature is considered. We show that no distinct $Z_N$- phases separated by domain walls do exist in the physical Minkowski space. That means the absence of the spontaneous breaking of $Z_N$- symmetry in the physical meaning of this word.

1 Introduction.

It was shown some time ago that the pure YM theory undergoes a phase transition at some temperature $T_c \sim \Lambda_{QCD}$ [1, 2]. This phase transition exhibits itself in a radical change of the behaviour of the correlator

$$C(x) = \langle P(x) P^*(0) \rangle$$

where $P(x)$ is the Polyakov line

$$P(x) = \frac{1}{N_c} \text{Tr}\{\exp[ig\beta \hat{A}_0(x)]\}$$

(we choose the gauge where $\hat{A}_0$ is time-independent; $\beta = 1/T$). Physically, this gauge transition corresponds to deconfinement: at low $T$, the interaction part of free energy of a test heavy quark-antiquark pair at distance $R$ grows linearly with $R$ whereas, for high $T$, it tends to zero at large distances.

There were scores of papers published since 1978 where it was explicitly or implicitly assumed that one can use the cluster decomposition for the correlator [3] at large $T$ and attribute the meaning to the temperature average $\langle P \rangle_T$. Under this assumption, the

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phase of this average can acquire $N_c$ different values: $< P >_T = C \exp\{2\pi i k/N_c\}$, $k = 0, \ldots, N_c - 1$ which would correspond to $N_c$ distinct physical phases and to the spontaneous breaking of the discrete $Z_N$-symmetry. In recent [3], the surface energy density of the domain walls separating these phases has been evaluated.

We show, however, that the standard interpretation is wrong. In particular:

1. Only the correlator (1) has the physical meaning. The phase of the expectation value $< P >_T$ is not a physically measurable quantity. There is only one physical phase in the hot YM system.

2. The "walls" found in [3] should not be interpreted as physical objects living in Minkowski space but rather as Euclidean field configurations, kind of "planar instantons" appearing due to nontrivial $\pi_1(G) = Z_N$ where $G = SU(N)/Z_N$ is the true gauge symmetry group of the pure YM system.

3. The whole bunch of arguments which is usually applied to nonabelian theories can be transferred with a little change to hot QED. The latter also involves planar instantons appearing due to nontrivial $\pi_1[U(1)] = Z$. These instantons should not, however, be interpreted as Minkowski space walls.

It is impossible to present an adequate discussion of this issue in this short note. The reader is referred to [4] where such a discussion is given. We can only briefly mention here some crucial points of our reasoning.

2 Continuum Theory.

A preliminary remark is that the situation when the symmetry is broken at high temperatures and restores at low temperatures is very strange and unusual. The opposite is much more common in physics. We are aware of only one model example where spontaneous symmetry breaking survives and can even be induced at high temperatures [3]. But the mechanism of this breaking is completely different from what could possibly occur in the pure Yang-Mills theory.

Speaking of the latter, we note first that there is no much sense to speak about the spontaneous breaking of $Z_N$-symmetry because such a symmetry is just not there
in the theory. As was already mentioned, the true gauge group of pure YM theory is \( SU(N)/Z_N \) rather than \( SU(N) \). This is so because the gluon fields belong to the adjoint colour representation and are not transformed at all under the action of the elements of the center \( Z_N \) of the gauge group \( SU(N) \).

\(< P >_T \) as such is not physical because it corresponds to introducing a single fundamental source in the system: \(< P >_T = \exp\{-\beta F_T\} \) where \( F_T \) is the free energy of a single static fundamental source \( \int \). But one cannot put a single fundamental source in a finite spatial box with periodic boundary conditions \( \int \). This is due to the Gauss law constraint: the total colour charge of the system “source + gluons in the heat bath” should be zero, and adjoint gluons cannot screen the fundamental source. This observation resolves the troubling paradox: complex \(< P >_T \) would mean the complex free energy \( F_T \) which is meaningless.

The ”states” with different \(< P >_T \) could be associated with different minima of the effective potential \( \int \)

\[
V_{eff}^\pi(A_0^3) = \frac{\pi^2 T^4}{12} \left\{ 1 - \left[ \frac{gA_0^3}{\pi T} \right]_{mod.2} - 1 \right\}^2 
\]

(3)

For simplicity, we restrict ourselves here and in the following with the \( SU(2) \) case.

This potential is periodic in \( A_0^3 \). The minima at \( A_0^3 = 4\pi n T/g \) correspond to \( P = 1 \) while the minima at \( A_0^3 = 2\pi (2n + 1) T/g \) correspond to \( P = -1 \). There are also planar (independent of \( y \) and \( z \)) configurations which interpolate between \( A_0^3 = 0 \) at \( x = -\infty \) and \( A_0^3 = 2\pi T/g \) at \( x = \infty \). These configurations contribute to Euclidean path integral and are topologically non-equivalent to the trivial configuration \( A_0^3 = 0 \) (Note that the configuration interpolating between \( A_0^3 = 0 \) and \( A_0^3 = 4\pi T/g \) is topologically equivalent to the trivial one. Such a configuration corresponds to the equator on \( SU(2)/Z_2 \) ). Actually, such configurations were known for a long time by the nickname of ’t Hooft fluxes \( \int \).

Minimizing the surface action density in a nontrivial topological class, we arrive at the configuration which is rather narrow (its width is of order \( (gT)^{-1} \)) and has the action
density
\[ \sigma^{su(2)} = \frac{4\pi^2 T^2}{3\sqrt{3}g} + C g T^2 \]  
(4)

(the constant \( C \) cannot be determined analytically in contrast to the claim of [3] due to infrared singularities characteristic for thermal gauge theories [10]). These topologically nontrivial Euclidean configurations are quite analogous to instantons. Only here they are delocalized in two transverse directions and thereby the relevant topology is determined by \( \pi_1 [\mathcal{G}] \) rather than \( \pi_3 [\mathcal{G}] \) as for usual localized instantons. But, by the same token as the instantons cannot be interpreted as real objects in the Minkowski space even if they are static (and, at high \( T \), the instantons with the size \( \rho \gg T^{-1} \) become static), these planar configurations cannot be interpreted as real Minkowski space domain walls.

I want to elucidate here the analogy between nonabelian and abelian theories. The effective potential for standard QED at high temperature has essentially the same form as (3):
\[ V_{\text{eff}}^\text{T}(A_0) = -\frac{\pi^2 T^4}{12} \left\{ 1 - \left[ \left( \frac{\epsilon A_0}{\pi T} + 1 \right) \mod 2 - 1 \right]^2 \right\}^2 \]  
(5)

It is periodic in \( A_0 \) and acquires minima at \( A_0 = \frac{2\pi n T}{e} \). Here different minima correspond to the same value of the standard Polyakov loop \( P_1(x) = \exp \{ i e A_0(x) \} \). One can introduce , however, the quantity \( P_{1/N}(x) = \exp \{ i e A_0(x) / N \} \) which corresponds to probing the system with a fractionally charged heavy source : \( \epsilon_{\text{source}} = e / N \). Note that a fractional heavy source in a system involving only the fermions with charge \( e \) plays exactly the same role as a fundamental heavy source in the pure YM system involving only the adjoint colour fields. A single fractional source would distinguish between different minima of the effective potential. If \( N \to \infty \), all minima would be distinguished, and we would get infinitely many distinct "phases".

But this is wrong. One cannot introduce a single fractional source and measure \( < P >_T \) as such due to the Gauss law constraint. What can be done is to introduce a pair of fractional charges with opposite signs and measure the correlator \( < P_{1/N}(x) P_{1/N}^*(0) >_T \). The latter is a physical quantity but is not sensitive to the phase of \( P \). The same concerns the correlator \( < P_{1/N}(x_1) \ldots P_{1/N}(x_N) >_T \) which corresponds to putting \( N \) fractional same-sign charges at different spatial points.

Finally, one can consider the configurations \( A_0(x) \) interpolating between different minima of (5). They are topologically inequivalent to trivial configurations and also have the
meaning of planar instantons \[^3\]. But not the meaning of the walls separating distinct physical phases. The profile and the surface action density of these abelian planar instantons can be found in the same way as it has been done in Ref.\[^3\] for the nonabelian case. For configurations interpolating between adjacent minima, one gets

\[ \sigma^{u(1)} = \frac{2\pi^2(2\sqrt{2} - 1)T^2}{3\sqrt{6}e} + CeT^2 \ln(e) \]

(6)

where \( C \) is a numerical constant which \textit{can} in principle be analytically evaluated.

There is a very fruitful and instructive analogy with the Schwinger model. Schwinger model is the two-dimensional \( QED \) with one massless fermion. Consider this theory at high temperature \( T \gg g \) where \( g \) is the coupling constant (in two dimensions it carries the dimension of mass). The effective potential in the constant \( A_0 \) background has the form which is very much analogous to (3,5):

\[ V_{\text{eff}}(A_0) = \frac{\pi T^2}{2} \left[ \left( 1 + \frac{gA_0}{\pi T} \right)^2 - 1 \right]^2 \]

(7)

It consists of the segments of parabola and is periodic in \( A_0 \) with the period \( 2\pi T/g \). Different minima of this potential are not distinguished by a heavy integerly charged probe but could be distinguished by a source with fractional charge. Like in four dimensions, there are topologically nontrivial field configurations which interpolate between different minima. These configurations are localized (for \( d = 2 \) there are no transverse directions over which they could extend) and are nothing else as high-\( T \) instantons. The minimum of the effective action in the one-instanton sector is achieved at the configuration \[^4,11\]

\[ A_0(x) = \begin{cases} \frac{\pi T}{g} \exp \left\{ \frac{g}{\pi} (x - x_0) \right\}, & x \leq x_0 \\ \frac{\pi T}{g} \left[ 2 - \exp \left( \frac{g}{\pi} (x_0 - x) \right) \right], & x \geq x_0 \end{cases} \]

(8)

the instanton (8) is localized at distances \( x - x_0 \sim g^{-1} \) and has the action \( S_I = \pi^{3/2}T/g \). But, in spite of that it is time-independent, it is the essentially Euclidean configuration and should not be interpreted as a "soliton" with the mass \( M_{sol.?} = TS_I \) living in the physical Minkowski space.

\section{3 Lattice Theory}

The most known and the most often quoted arguments in favour of the standard conclusion of the spontaneous breaking of \( Z_N \)-symmetry in hot Yang-Mills theory come from lattice

\[^2\]In the abelian case, there are infinitely many topological classes: \( \pi_1[U(1)] = Z \).
considerations. Let us discuss anew these arguments and show that, when the question is posed properly, the answer is different.

Following Susskind [2], consider the Hamiltonian lattice formulation where the theory is defined on the 3-dimensional spatial lattice and the time is continuous. In the standard formulation, the dynamic variables present the unitary matrices $V(r, n)$ dwelling on the links of the lattice (the link is described as the vector starting from the lattice node $r$ with the direction $n$). The Hamiltonian is

$$H = \sum_{\text{links}} \frac{g^2(E^a)^2}{2a} - \frac{2}{ag^2} \sum_{\text{plaq.}} \text{Tr}\{V_1 V_2 V_3 V_4\}$$

(9)

where $a$ is the lattice spacing, $g$ is the coupling constant and $E^a$ have the meaning of canonical momenta $[E^a(r, n), V(r, n)] = t^a V(r, n)$. Not all eigenstates of the Hamiltonian (9) are, however, admissible but only those which satisfy the Gauss law constraint. Its lattice version is

$$G^a(r) = \sum_n E^a(r, n) = 0$$

(10)

It is possible to rewrite the partition function of the theory (9, 10) in terms of the dual variables $\Omega_r \in SU(2)$ which are defined not at links but at the nodes of the lattice. $\Omega_r$ are canonically conjugate to the Gauss law constraints (10) and have the meaning of the gauge transformation matrices acting on the dynamic variables $V(r, n)$. In the strong coupling limit when the temperature is much greater than the ultraviolet cutoff $\Lambda_{\text{ultr}} \sim 1/a$, the problem can be solved analytically. The effective dual Hamiltonian has 2 sharp minima at $\Omega_r = 1$ and $\Omega_r = -1$ and this has been interpreted as the spontaneous breaking of $Z_2$-symmetry.

Note, however, that the same arguments could be repeated in a much simpler and the very well known two-dimensional Ising model. Being formulated in terms of the physical spin variables $\sigma$, the theory exhibits the spontaneous breaking of $Z_2$-symmetry at low temperatures, and at high $T$ the symmetry is restored. But the partition function of the Ising model can also be written in terms of the dual variables $\eta$ defined at the plaquette centers [12]. Dual variables are ordered at high rather than at low temperatures. This obvious paradox is resolved by noting that the dual variables $\eta$ are not measurable and have no direct physical meaning. The ”domain wall” configurations interpolating between $\eta = 1$ and $\eta = -1$ do contribute in the partition function formulated in dual terms. But
one cannot feel these configurations in any physical experiment.

And the same concerns the lattice pure YM theory. There are configurations interpolating between different $\Omega r \in Z_2$ and contributing to the partition function, but they do not correspond to any real-time object and cannot be felt as such in any physical experiment.

Up to now we discussed the system with a standard lattice hamiltonian (9). Note, however, that one can equally well consider the lattice theory with the hamiltonian having the same form as (9) but involving not the unitary but the orthogonal matrices $V^{adj}(r, n) \in SO(3)$. Both lattice theories should reproduce one and the same continuous Yang-Mills theory in the limit when the inverse lattice spacing is much greater than all physical parameters (As far as I understand, there is no unique opinion on this issue in the lattice community. If, however, lattice hamiltonia involving unitary and orthogonal matrices would indeed lead to different field theories in the continuum limit, it would mean that the Yang-Mills field theory is just not defined until a particular procedure of ultraviolet regularization is specified. This assertion seems to me too radical, and I hesitate to adopt it.).

But in the strong coupling limit $T \gg \Lambda_{ultr.}$ the two lattice theories are completely different. The theory with orthogonal matrices has the same symmetry properties as the continuum theory, and there is no $Z_2$-symmetry whatsoever. The effective dual hamiltonian depending on the gauge transformation matrices $\Omega^{adj} \in SO(3)$ also has no such symmetry and there is nothing to be broken.

Earlier the lattice studies of the deconfinement phase transition have been performed exclusively with the standard lattice lagrangian involving unitary matrices. These studies suggest that the deconfinement phase transition occurs simultaneously with the spontaneous symmetry breaking in the dual hamiltonian $H^{eff}(\Omega^{fund.}_r)$ (We repeat that such a breaking is not a physical symmetry breaking because it does not lead to the appearance of domain walls detectable in experiment.). In our opinion, however, the additional $Z_2$-symmetry which the hamiltonian (9) enjoys is a nuisance rather than an advantage. It is a specifically lattice feature which is not there in the continuum theory. We strongly suggest to people who can do it to perform a numerical study of the deconfinement phase transition for the theory involving orthogonal matrices. In that case, no spontaneous
\[ Z_N \] breaking can occur. Probably, for finite lattice spacing, one would observe kind of crossover rather than the phase transition. The crossover is expected to become more and more sharp as the lattice spacing (measured in physical units) would become smaller and smaller.

It would be interesting also to try to observe the "walls" (i.e. the planar Euclidean instantons) for the orthogonal lattice theory. They should "interpolate" between \( \Omega^\text{adj}_r = 1 \) and \( \Omega^\text{adj}_r = 1 \) along a topologically nontrivial path. Like any other topological effect, these instantons should become visible only for a small enough lattice spacing (much smaller than the characteristic instanton size), and to detect them is definitely not an easy task. But using the orthogonal matrices is the only way to separate from lattice artifacts. The only available numerical study \cite{14} was done for the theory with unitary matrices and too close to the strong coupling regime where these artifacts are desisive. Thereby, it is not conclusive.

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