Some analytical methods used to process research drawing - Part I

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Abstract. Of the multitude analytic methods use for reserches the deep-drawing process, in this paper is present the classical method and their application.

Keywords: analytical methods, Tresca’s plasticity condition, Huber-Mises-Hencky plasticity condition, Hooke's law, Holomon's law, Hill's plasticity criterion

Introduction

The research of the drawing process is done using the analytical and experimental methods. This paper presents one of the analytical methods, namely the method of solving equilibrium equations

1. Resolving method of equilibrium equations

Analytical resolving methods of plastic deformation problems are based on resolving equilibrium equations of an element in the deformed body; for this it will be used: equilibrium equations; deformation equations; compatibility equations; the condition of plasticity; relations between stresses and deformations ([1], [2], [3]).

As most of the equations of the mathematical model are differentiated, the finding of solutions obliges to integrations, which implies the introduction of certain constants. Therefore, in addition to the set of equations above, the complete mathematical model will also have to contain the initial and on contour conditions, which are expressed by the stress or deformation values at the beginning of the deformation and/or on the edge (contour) of the piece. From this system will result the relationships corresponding to the stresses and the specific deformations. Knowing the values of normal stresses on the contact surface between the workpiece and the active element that produces the deformation, the forces required for processing will be determined.

A. Equations of equilibrium after radial and tangential directions

It is considered a plate of constant thickness equal to the unit from which an infinitesimal element is separated and on which sides the normal and tangential stresses are applied (Fig.1 [1]).

The influence of mass forces is neglected, so that the considered element is in equilibrium only under the action of the forces generated by the tensions which acting on its faces.

To determine the equilibrium equations of the considered element, the equations resulting from the radial (O'r) and tangential (O'θ) design of forces and the moment equation in relation to the center O' of the element [1] shall be written.
1) Equation of equilibrium by radial direction
\[
\left( \sigma_r + \frac{\partial \sigma_r}{\partial r} dr \right) (r + dr) d\theta - \sigma_r \cdot r \cdot d\theta - \left( \sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta \right) \sin \frac{d\theta}{2} \cdot dr - 
- \sigma_\theta \sin \frac{d\theta}{2} \cdot dr + \left( \tau_\theta + \frac{\partial \tau_\theta}{\partial \theta} d\theta \right) \cos \frac{d\theta}{2} \cdot dr - 
- \tau_\theta \cos \frac{d\theta}{2} \cdot dr = 0 \tag{1}
\]

2) Equation of equilibrium by tangential direction
\[
\left( \tau_{r0} + \frac{\partial \tau_{r0}}{\partial r} dr \right) (r + dr) d\theta - \tau_{r0} \cdot r \cdot d\theta + \left( \sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta \right) \sin \frac{d\theta}{2} \cdot dr - 
- \sigma_\theta \cos \frac{d\theta}{2} \cdot dr + \left( \tau_\theta + \frac{\partial \tau_\theta}{\partial \theta} d\theta \right) \sin \frac{d\theta}{2} \cdot dr + \tau_\theta \sin \frac{d\theta}{2} \cdot dr = 0 \tag{2}
\]

3) Equation of moments relative to the center O' of the element
\[
\left( \tau_{r0} + \frac{\partial \tau_{r0}}{\partial r} dr \right) \cdot (r + dr) \cdot d\theta \cdot \frac{dr}{2} + \tau_{r0} \cdot r \cdot d\theta \cdot \frac{dr}{2} - 
- \left( \tau_{\theta0} + \frac{\partial \tau_{\theta0}}{\partial \theta} d\theta \right) \cdot dr \left( r + \frac{dr}{2} \right) \tan \frac{d\theta}{2} - \tau_{\theta0} \cdot r \cdot dr \left( r + \frac{dr}{2} \right) \tan \frac{d\theta}{2} = 0 \tag{3}
\]
\[
d\theta \Leftrightarrow \sin \frac{d\theta}{2} \approx \frac{d\theta}{2}; \quad \tan \frac{d\theta}{2} \approx \frac{d\theta}{2}; \quad \cos \frac{d\theta}{2} \approx 1
\]

After reducing the terms, neglecting small infinitesimal order of the two, the equations become:
\[
\sigma_r \cdot dr \cdot d\theta + r \cdot \frac{\partial \sigma_r}{\partial r} dr \cdot d\theta - \sigma_\theta \cdot r \cdot d\theta + \frac{\partial \tau_\theta}{\partial \theta} d\theta \cdot dr = 0 ; \tag{4}
\]
\[
\tau_{r0} \cdot dr \cdot d\theta + r \cdot \frac{\partial \tau_{r0}}{\partial r} dr \cdot d\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta \cdot dr + \tau_\theta \cdot r \cdot d\theta \cdot dr = 0 ; \tag{5}
\]
\[
r \cdot \tau_{r0} \cdot d\theta \cdot \frac{dr}{2} + r \cdot \tau_{r0} \cdot d\theta \cdot \frac{dr}{2} - \tau_{\theta0} \cdot r \cdot dr \cdot \frac{d\theta}{2} - \tau_{\theta0} \cdot r \cdot dr \cdot \frac{d\theta}{2} = 0 . \tag{6}
\]

From the last equation we get the principle of the duality of the tangential forces:
\[
\tau_{r0} = \tau_{\theta0} . \tag{7}
\]

Dividing equations (4) and (5) with \((r \cdot dr \cdot d\theta)\) results equilibrium equations:
\[
\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial \tau_{\theta0}}{\partial \theta} + \frac{1}{r} (\sigma_r - \sigma_\theta) = 0 ; \tag{8}
\]
\[
\frac{\partial \tau_{r0}}{\partial r} + \frac{1}{r} \cdot \frac{\partial \sigma_\theta}{\partial \theta} + \frac{2}{r} \tau_{r0} = 0 .
\]

B. Deformation equations

In Fig. 2 [1] are the positions in which the points considered are reached, after the displacements imposed by the deformations, i.e:
\[
A(r, 0) \xrightarrow{u} A'' \xrightarrow{v'} A'
\]
\[
B(r + dr, 0) \xrightarrow{\dot{u} + \frac{\partial u}{\partial r} dr} B'' \xrightarrow{\dot{v} + \frac{\partial v}{\partial r} dr} B'
\]
Fig. 1. The stress variation on an infinitesimal element in cylindrical coordinates [1]

Fig. 2. Deformation of an infinitesimal element in the plane [1]
\[ D(r, \theta + d\theta) \xrightarrow{\gamma_u} D^* \xrightarrow{\gamma_v} D' \]

The specific deformation in radial direction \((\varepsilon_r)\) is ([1], [14], [17]):

\[
\varepsilon_r = \frac{A'B' - AB}{AB} \approx \frac{A'\theta^* - AB}{AB} = \frac{A'B'* - A'A* - AB}{AB} = \frac{\partial u}{\partial r} dr + u \frac{\partial u}{\partial r} dr - u - dr = \frac{\partial u}{\partial r} dr.
\]

The specific tangential strain \((\varepsilon_0)\) is ([1], [15], [18]):

\[
\varepsilon_0 = \frac{A'D' - AD}{AD} \approx \frac{A'\theta^* - AD}{AD} = \frac{A'\theta^* - A'A^* + D'D^* - AD}{AD} \approx \frac{r + u \partial v}{r \partial \theta} = \frac{u + \frac{\partial v}{\partial \theta}}{r}.
\]

The angular specific deformation \(\gamma_0\) represents the change of the right angle from A and consists of \(\gamma_1\), with which the tangential segment AD has rotated, plus \(\gamma_2\), with which the radial segment AB has rotated. Taking into account the \(\tan \gamma \approx \gamma\) approximation, one can write:

\[
\gamma_1 \approx \frac{D'D^*}{A'D^*} = \frac{D'D^*}{A'\theta^* + D'D^* - A'A^*} = \frac{\partial u}{\partial \theta} \frac{d\theta}{r \cdot d\theta + u \cdot d\theta + \partial v}{\partial \theta} d\theta.
\]

Considering \(r \gg u \approx \frac{\partial v}{\partial \theta} \Rightarrow \gamma_1 \approx \frac{\partial u}{\partial \theta} \frac{d\theta}{r \cdot d\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta}.
\]

\[
\gamma_2 = \gamma'_2 - \gamma''_2 = \frac{B'B'}{A'B'} - \frac{B'B}{A'B} = \frac{B'B}{A'B} - \frac{A'A^*}{OA^*} = \frac{\partial v}{\partial \theta} \frac{dr}{dr + u + \frac{\partial u}{\partial r} dr} - \frac{v}{r + u}
\]

Considering \(\frac{\partial u}{\partial r} \ll 1 \Rightarrow \gamma_2 \approx \frac{\partial v}{\partial \theta} \frac{dr}{r} - \frac{v}{r} = \frac{\partial v}{\partial \theta} - \frac{v}{r} \Rightarrow \gamma_2 \approx \frac{1}{\partial \theta} - \frac{v}{r}.
\]

Result:

\[
\gamma'_{r\theta} = \gamma_1 + \gamma_2 = \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial \theta} - \frac{v}{r}.
\]

C. Equivalence equation

The specific deformations given by relationships (9 – 11) are not independent, but are linked by
compatibility (continuity) relationships. Angular deformation $\gamma_\theta$ can not be chosen arbitrarily but is correlated with the other two, by equation ([1], [14],[15]):

$$\frac{\partial^2 \varepsilon_r}{\partial \theta^2} + r \frac{\partial^2}{\partial r^2} \left( r \cdot \varepsilon_\theta \right) - r \frac{\partial \varepsilon_r}{\partial r} = \frac{\partial^2}{\partial r \partial \theta} \left( r \cdot \gamma_\theta \right).$$

(12)

D. Plasticity conditions

In the study of deformation processes it is important to know the conditions of material transition from elastic state to plastic state. In the case of plastic deformations, the relationship between the main stresses is a function of the form, which geometrically represents a smooth and convex surface called the flowing (loading) surface. For any point inside this surface (when $F < 0$) the stresses are elastic, and for those on the surface (when $F = 0$) the stresses are plastic. For plane stresses, the surface $F$ becomes a curve called the flow (loading) curve. The explicit determination of the $F$ function, which defines the plasticity condition, is based on experimentally determined criteria, most often used by Tresca and Huber-Mises-Hencky ([1], [2],[3]).

a) Tresca’s plasticity condition (maximum tangential effort)

The material moves from the elastic state to the plastic when the maximum tangential stress reaches a certain critical value, independent of the state of stress type, ie $\tau_{\text{max}} = K$. As $\tau_{\text{max}} = (\sigma_1 - \sigma_3)/2$, where $\sigma_1, \sigma_3$ represents the maximum or minimum main stress, it results :

$$\sigma_1 - \sigma_3 = 2K.$$  

(13)

Discussion of the equation (13):

- for the uniaxial stress state ($\sigma_2 = \sigma_3 = 0$) ⇒ the achievement of plasticity is produced for $\sigma_1 = \sigma_c$, so that : $\tau_{\text{max}} = \tau_c = \sigma_c/2$. As a result, the plasticity condition becomes :

$$\sigma_1 - \sigma_3 = \sigma_c = 2K.$$  

(14)
for the plane state stress:

when \( \sigma_\tau \) and \( \sigma_\theta \) are oriented along the main directions, the plasticity condition becomes:

\[
\sigma_\tau - \sigma_\theta = \pm \sigma_c = \pm 2K, \text{ if } \sigma_\tau > \sigma_\theta \quad \text{or} \quad \sigma_\theta - \sigma_\tau = \sigma_c = 2K, \text{ if } \sigma_\tau < \sigma_\theta
\]

In other words:

\[
(\sigma_\tau - \sigma_\theta)^2 = \sigma_c^2.
\]

- when the directions of tensions \( \sigma_i \) and \( \sigma_0 \) do not coincide with the directions of the main deformations, then the relation (15) becomes:

\[
(\sigma_i - \sigma_0)^2 + 4r_{ij}^2 = \sigma_c^2.
\]

The function F is defined in this case by the relation: \( F(\sigma_i, \sigma_0) = \sigma_i - \sigma_0 \pm \sigma_c = 0 \).

The graphical representation of the plasticity condition Tresca (15) in the plane \( (\sigma_i, \sigma_0) \) is in the form of a hexagon (Fig. 3 [1]).

b) **Huber-Mises-Hencky** plasticity condition (shape change energy)

The material moves from the elastic state to the plastic state when the shape change energy reaches a certain critical value, independent of the state of effort. Potential deformation energy \( (W_p) \) comprises the potential volume change energy \( (W_v) \) and the potential shape change energy \( (W_f) \), i.e:

\[
W_p = W_v + W_f;
\]

\[
W_p = \frac{1}{2} (\sigma_1 \cdot \varepsilon_1 + \sigma_2 \cdot \varepsilon_2 + \sigma_3 \cdot \varepsilon_3);
\]

\[
W_v = \frac{3}{2} \sigma_{med} \cdot \varepsilon_{med} = \frac{3}{2} \left[ \frac{\sigma_i + \sigma_j + \sigma_k}{3} \cdot \varepsilon_i + \varepsilon_j + \varepsilon_k \right];
\]

\[
e_i = \frac{1}{E} \left[ \sigma_i - \mu (\sigma_j + \sigma_k) \right], \text{ i, j, k take successive values 1, 2, 3.}
\]

Taking into account the relationships (17 - 20) result ([1], [2], [3]):

\[
W_f = \frac{1 + \mu}{6E} \left[ (\sigma_i - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right].
\]

Discussion of the equation (21):

- for uniaxial stress state \( (\sigma_2 = \sigma_3 = 0) \Rightarrow \text{the achievement of plasticity is produced for } \sigma_1 = \sigma_c, \)

such as \( W_{pl} = (1+\mu)2\sigma_c^2/6E \). Equaling expressions for \( W_{pl} \), results:

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_c^2
\]

- for the plane state of stress \( \sigma_1 = \sigma_\tau, \sigma_2 = \sigma_\theta, \sigma_3 = 0 \). From equation (22) results:

\[
(\sigma_\tau - \sigma_0)^2 + \sigma_\tau^2 + \sigma_\theta^2 = 2\sigma_c^2 \text{ or } \sigma_\tau^2 + \sigma_\theta^2 - \sigma_\tau \sigma_\theta = \sigma_c^2 \text{ or }
\]

\[
3 \left( \frac{\sigma_\tau - \sigma_0}{\sigma_c} \right)^2 + \left( \frac{\sigma_\tau + \sigma_0}{\sigma_c} \right)^2 = 4.
\]

Function F is defined by the relationship: \( F(\sigma_i, \sigma_0) = \sigma_i^2 + \sigma_0^2 - \sigma_\tau \sigma_\theta - \sigma_c^2 \).

The plasticity condition Huber-Mises-Hencky (23) represents the equation of an ellipse in the plane \( (\sigma_i, \sigma_0) \) (Fig. 3 [1]).

Sometimes the Huber-Mises-Hencky plasticity condition is also expressed in form:

\[
\sigma_i - \sigma_0 = \pm \beta \cdot \sigma_c, \text{ unde } \beta \in [1, 2/\sqrt{3}].
\]
Plasticity condition for anisotropic materials (Hill)

It is known that the mechanical properties in the plane of the sheet differ from those in its normal plane. To characterize the degree of normal anisotropy, the anisotropy coefficient R is used, which is defined by the relationship:

$$R = \frac{\ln b_0/b}{\ln g_0/g},$$

(25)

where the degree of deformation appears in the numerator in the direction of the width of a rectangular section specimen, and the denominator shows the degree of deformation in the direction of thickness. The values of this coefficient are determined by the traction test [17].

Huber-Misses-Hencky plasticity condition was generalized by Hill for anisotropic materials in the form [1]:

$$\sigma_r^2 + \sigma_\theta^2 - \frac{2R}{1+R} \sigma_r \cdot \sigma_\theta = \sigma_e^2$$

or

$$(1 + 2R) \left( \frac{\sigma_r - \sigma_\theta}{\sigma_e} \right)^2 + \left( \frac{\sigma_r + \sigma_\theta}{\sigma_e} \right)^2 = 2(1 + R).$$

(26)

These last two relationships represent the equations of elongated ellipses whose axes are dependent on R.

However, there are materials such as titanium, zinc, aluminum, which are not subject to the condition (26) and for these Hill expressed the plasticity condition in an unquadratic form [1]:

$$(1 + 2R) \left( \frac{\sigma_r - \sigma_\theta}{\sigma_e} \right)^p + \left( \frac{\sigma_r + \sigma_\theta}{\sigma_e} \right)^p = 2(1 + R),$$

(27)

where p is determined experimentally, $p \geq 1$.

Researchers such as Bassani, Budiansky, Hosford, Barlat and Richmond, Barlat and Lian, Gotoh, Drucker, Yamaguchi, Gurson have come to different forms of plasticity, but with a more limited use [1].

E. Relationships between stresses and deformations (physical or constitutive)

In the domain of elasticity, these relations are linear (expressed by Hooke's law), and in the plastic domain are non-linear, the most used being the exponential type ([1], [16]):

$$\sigma = K \cdot \varepsilon^n,$$

(28)

where: K - constant material; n - coefficient of ecrusion, which represents the slope of the line represented in the system (ln $\varepsilon$, ln $\sigma$) [17]).

Experimentally, it was observed that the relation (28) is also valid for multiaxial stresses, thus resulting:

$$\sigma_e = K \cdot \varepsilon_e^n,$$

(29)

where $\sigma_e$ is the equivalent stress and $\varepsilon_e$ is the equivalent specific deformation.

For the case of plane load state, $\sigma_e$ and $\varepsilon_e$ are defined as follows [1]:

$$\sigma_e = \frac{\sqrt{2}}{2} \sqrt{(\sigma_r - \sigma_\theta)^2 + \sigma_\theta^2 + \sigma_r^2},$$

$$\varepsilon_e = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + \varepsilon_\theta^2 + \varepsilon_r^2}.$$
\[ \text{de}_e = \frac{\sqrt{2}}{3} \sqrt{(\text{de}_r - \text{de}_0)^2 + (\text{de}_0)^2 + (\text{de}_r)^2}. \]

in which:
\[ \text{de}_r = \int \text{de}_r. \]

It can be noticed that the relation (28) (Holomon’s law) (Figure 4) only takes into consideration the elastic cruising of the material, but other parameters such as deformation velocity \( \dot{\varepsilon} \), elasticity, temperature and others influence the plastic behavior of the materials. Under these conditions, it is possible to discuss laws for materials that are hardened to processing and which may be visco-plastic, thermo-visco-plastic or plastic materials. [1].

![Fig. 4. Relationships between tension and deformations in the elastic (Hooke's law) and plastic domain (Holomon's law) [1]](image)

From the point of view of approximating the relations between \( \sigma_e \) and \( \varepsilon_e \) or the relations between the stress and deformation components, several theories have developed. The most commonly used are the theory of deformations (or the theory of small deformations) and the theory of flow (or large plastic deformations).

**Deformation theory** is an extrapolation of Hooke’s law for the plastic domain. According to this theory we determine the average values of the parameters defining the deformation process in its temporal deployment and link the total plastic deformation to the final stress [1]. Deformation theory can be applied when:

a) loading material is proportional: \( \frac{d\sigma_r}{\sigma_r} = \frac{d\sigma_0}{\sigma_0} = \text{ct.} \)

b) the stress component’s directions do not change and coincide with those of the deformations. Relationships of deformation theory (Madai-Hencky-Ilyushin relations) have the same structure as Hooke’s relations with the difference that plasticity is considered \( E_p = \sigma_0/\varepsilon_0 \), and the transverse contraction coefficient \( \mu = 0.5 \cdot E_p \) is no longer a constant of material, but depends on the degree of loading.
Determination is done using the load curve (Fig. 5 [1])

\[ E_p = \tan \alpha = \frac{\sigma_0}{\varepsilon_0}, \]  

(Hooke's generalized law is ([2], [3]))

\[ \varepsilon_i = \frac{1}{E} \left[ \sigma_i - \mu (\sigma_j + \sigma_k) \right]; \]  
\[ \gamma_{ij} = \frac{\tau_{ij}}{G} = \frac{2(1+\mu)}{E} \tau_{ij}, \]

where i, j, k take the notations successively r, \( \theta \), z (for cylindrical coordinates).

In the case of the plane of stress from the plasticity theory, the following customizations are made:

\[ \sigma_z = \tau_{rz} = \tau_{\theta z} = 0; \]  
\[ \varepsilon_z = \gamma_{rz} = \gamma_{\theta z} = 0; \]  
\[ E \rightarrow E_p; \mu = 1/2 \]

According to the above, relations of the deformation theory for the plan case become:

\[ \varepsilon_r = \frac{1}{E_p} \left( \sigma_r - \frac{1}{2} \sigma_0 \right); \]
\[ \varepsilon_0 = \frac{1}{E_p} \left( \sigma_0 - \frac{1}{2} \sigma_r \right); \]
\[ \gamma_{r0} = \frac{3}{E_p} \cdot \tau_{r0}. \]
\[ \varepsilon_r - \varepsilon_0 = \frac{3}{2E_p} (\sigma_r - \sigma_0) \]

Despite the restrictive conditions for the application of the theory of deformations, the equations have a wide applicability, because it eliminates the integration assumed by the relationship (32).

The flow theory operates with the strain and instantaneous stress, while the deformation theory that operates with the total deformation and the final stress. This allows the application of this theory to any form of variation in charge.

The total deformation determined by relation (32) assume calculating of the integral on the load curve taking into account the initial and final stress state and its variation between the two states. In the case of a proportional load, the contour integral is reduced to a defined one between the initial and the final state and the theory of flow coincides with that of the deformations. The high degree of generalization of the flow theory makes it applicable to plastic deformations.

Because the axes of the strain increments \((d\varepsilon)\) must coincide with those of the stress variation, it means the primes must be normal to the flow curve. This normal to the curve is given by the gradient of the function, which is the plastic potential \(G(\sigma_i)\), i.e [1]:

\[ d\varepsilon_{ij} = d\lambda \left( \frac{\partial G}{\partial \sigma_{ij}} \right), \quad (i, j = r, \theta), \]  (38)

where: \(d\lambda\) - proportionality factor; \(\frac{\partial G}{\partial \sigma_{ij}}\) - gradient components.

Notations are made: \(d\varepsilon_{rr} = d\varepsilon_r; \quad d\varepsilon_{\theta\theta} = d\varepsilon_\theta; \quad \sigma_{rr} = \sigma_r; \quad \sigma_{\theta\theta} = \sigma_\theta; \quad \sigma_{r\theta} = \tau_{r\theta}. \) (39)

Parameters \(d\lambda\) are obtained from equality: \(dW_p = \sigma_{ij} \cdot d\varepsilon_{ij} = \sigma_e \cdot d\varepsilon_e.\) (40)

If the plastic potential \(G\) coincides with the \(F\) function due to the Huber-Misses-Hencky plasticity condition, the relationship (38) becomes:

\[ d\varepsilon_r = d\lambda \left( \frac{\partial (\sigma_r^2 + \sigma_\theta^2 - \sigma_r \cdot \sigma_\theta - \sigma_e^2)}{\partial \sigma_r} \right) = d\lambda (2\sigma_r - \sigma_\theta) = d\lambda \left( \sigma_r - \frac{\sigma_\theta}{2} \right) = \frac{d\varepsilon_e}{\sigma_e} (\sigma_r - \frac{\sigma_\theta}{2}); \]  (41)

\[ d\varepsilon_\theta = d\lambda \left( \frac{\partial (\sigma_r^2 + \sigma_\theta^2 - \sigma_r \cdot \sigma_\theta - \sigma_e^2)}{\partial \sigma_\theta} \right) = d\lambda (2\sigma_\theta - \sigma_r) = d\lambda \left( \sigma_\theta - \frac{\sigma_r}{2} \right) = \frac{d\varepsilon_e}{\sigma_e} (\sigma_\theta - \frac{\sigma_r}{2}), \]  (42)

in which: \(d\lambda^* = 2d\lambda = \frac{d\varepsilon_e}{\sigma_e}\) (v. Fig. 6).

\[ d\gamma_{r\theta} = 3 \frac{d\varepsilon_e}{\sigma_e} \cdot \tau_{r\theta}. \]  (43)

By recording the average effort \(\sigma_{med} = \frac{\sigma_r + \sigma_\theta + 0}{3}\)

and \(d\lambda^* = \frac{3}{2} d\lambda = \frac{3}{2} \frac{d\varepsilon_e}{\sigma_e}\), relationships (41 - 43) become:

\[ d\varepsilon_r = \frac{3}{2} \frac{d\varepsilon_e}{\sigma_e} (\sigma_r - \sigma_{med}); \]

\[ d\varepsilon_\theta = \frac{3}{2} \frac{d\varepsilon_e}{\sigma_e} (\sigma_\theta - \sigma_{med}); \]
\[ d\gamma_{\rho\phi} = 3 \frac{d\epsilon_{\rho}}{\sigma_e} \tau_{\rho\phi}, \]

from which the Lévy-Misses equations are obtained:

\[ \frac{d\epsilon_r}{\sigma_r - \frac{\sigma_0}{2}} = \frac{d\epsilon_\theta}{\sigma_r - \frac{\sigma_0}{2}} = \frac{3d\gamma_{r\theta}}{d\epsilon_e} = \frac{d\epsilon_\lambda}{\sigma_e}. \quad (44) \]

Fig. 6. Determination of the proportionality factor \( d\lambda' \) [1]

**Relationships between strains and deformations for anisotropic materials**

For the Hill’s plasticity criterion given by the relation (26), relationships defining the equivalent stress and the equivalent deformation become:

\[ \sigma_e = \sqrt{\sigma_r^2 - \frac{2R}{1+R} \sigma_r \cdot \sigma_0 + \sigma_0^2}; \quad (45) \]
\[ \epsilon_e = \frac{1+R}{\sqrt{1+2R}} \sqrt{\epsilon_r^2 + \frac{2R}{1+R} \epsilon_r \cdot \epsilon_\theta + \epsilon_\theta^2}; \quad (46) \]
\[ d\epsilon_e = \frac{1+R}{\sqrt{1+2R}} \sqrt{d\epsilon_r^2 + \frac{2R}{1+R} d\epsilon_r \cdot d\epsilon_\theta + d\epsilon_\theta^2}. \quad (47) \]

Substituting the relation (26) in (38) is obtained:

\[ d\epsilon_r = d\lambda \frac{\partial}{\partial \sigma_r} \left( \sigma_r^2 + \sigma_0^2 - \frac{2R}{1+R} \sigma_r \cdot \sigma_0 - \sigma_0^2 \right) \]
\[ = d\lambda \left( 2\sigma_r - \frac{2R}{1+R} \sigma_0 \right) = \frac{2d\lambda}{1+R} \left[ (1+R)\sigma_r - R\sigma_0 \right] = \frac{1}{1+R} \frac{d\epsilon_e}{\sigma_e} \left[ (1+R)\sigma_r - R\sigma_0 \right]. \quad (48) \]

Analogously, the other relationships are determined so that the Lévy-Misses equations become:
\[
\frac{d\varepsilon_r}{(1+R)\sigma_r - R\sigma_\theta} = \frac{d\varepsilon_\theta}{(1+R)\sigma_\theta - R\sigma_r} = \frac{d\varepsilon_\phi}{(1+R)\sigma_\phi}.
\] (49)

2. Applications of the classical method of research of the drawing process

With the help of the relations between the main stresses (strain) and the maximum shear stresses, the number of possible wrinkles in the case of conical drawing part as well as other relations between the diameter of the blank (or the appearance of thinness) and the minimum pressure of the retaining ring, at cylindrical drawing \[15\].

On the basis of the equations of plasticity theory and minimax theory, a methodology was developed to calculate the number of successive deformations required for the drawing of a cylindrical piece and range of values for the drawing coefficient for each deformation processing operation, taking into account the mechanical properties of the material \[9\].

The theoretical analysis of the stress state according to the applied force on the deformed material is made by Doroşko \[7\]. The admissible degree of deformation is determined in case of use the conical and cylindrical punch in one or more successive die. The method of guiding the deformation is determined by adjusting the frictional conditions on the die surface and the angles of inclination of the die. The concordance of the results of the calculations with the experimental data led to the design of the technological processes of drawing the deep cave parts.

Combining the elasto-plastic theory (Mises-Hill criterion) with the use of an artificial neural network (ANN), a method has been obtained that has allowed the identification of material properties and lubrication conditions during the anisotropic drawing processes when applying a process self-regulation of the restraining force. The method is experimentally confirmed by high-speed rapid control systems \[14\].

A model (\[12\], \[13\]) has been established with the help of the classic method of research of the drawing process, which allows calculation of the variation of the thickness of the part wall taking into account the hardening and the frictional force between the blank and the die. This allows estimation of the critical thickness at which breakage is initiated.

The appearance of flange waves in the Swift test was analyzed using deformation and flow theory \[19\]. It has been noticed that critical stresses and displacements corresponding to the beginning of the crease increase with decreasing the coefficient of drawing, and the stresses, deformations and the admissible coefficient of drawing increase with the coefficient of anisotropy.

Starting from Hill’s plasticity criterion for anisotropic materials, a theoretical relationship was established to evaluate the maximum draw force \(F_{\text{max}}\) in the case of a flat bottom cylindrical piece. \(F_{\text{max}}\) increases linearly by increasing the friction coefficient, thickness, respectively by decreasing the die radius and the hardening module; \(F_{\text{max}}\) increases nonlinear with decreasing anisotropy coefficient (\[6\], \[10\], \[11\]).

The relationships between stresses and deformations allowed the analysis of the drawing process in the case of small cylindrical pieces made of strip type blank, for which 3 simulation programs were used \[4\]:

- when using the LS-DYNA program, it was demonstrated that for the intact strip, the hemispherical bottom parts would break when passing from the wall to the flange;
- when using the COSMOS program, it was demonstrated that for the intact tape, the flat base pieces were broken when switching from the wall to the flange;
- when using the MARC-Mentat program, it was noticed that for the nicked band, the parts were broken when the wall was connected to the flat bottom.
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