B\bar{B} Mixing and CP Violation in SU(2)_{L} \times SU(2)_{R} \times U(1) Models

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Abstract
We reexamine the mass mixing and CP violation in the B\bar{B} system in general SU(2)_{L} \times SU(2)_{R} \times U(1) models related to the recent measurements. The right-handed contributions can be sizable in B\bar{B} mixing and CP asymmetry in B decays for a heavy W' even with a mass about 3 TeV. On the other hand the lower bound on the mass of W' can be taken down to approximately 300 GeV.
1 Introduction

The Standard $SU(2)_L \times U(1)$ model ($SM$) has been very successful in describing the known weak interaction phenomena. But the consistency of the present experimental results with the general scheme of charged weak interactions and $CP$ violation in the $SM$ is non-trivial so the model is challenged both experimentally and theoretically in its prediction of large $CP$ violation effects in the B meson system $[1]$. As one of the simplest extensions of the Standard model gauge group and so the complement of the purely left-handed nature of the $SM$, the left-right theory with the group $SU(2)_L \times SU(2)_R \times U(1)$ has been widely studied. In this model, even with two generations of quarks one could get $CP$ violation. With three generations of quarks, this model contains many parameters and many sources of $CP$ violation $[2]$. One of the main source is the relative phase $\alpha$ between the two vacuum expectation values (VEVs) $k$ and $k'$ of the Higgs bidoublet $\Phi$. The other sources are the complex phases in the left and right-handed quark mixing matrices $U_L$ and $U_R$ respectively. Here it would be convenient to regard $U_L$ as the usual Cabbibo-Kobayashi-Maskawa ($CKM$) matrix and shift all phases except one to $U_R$. Using the Wolfenstein parameterization $[3]$, we can express the $CKM$ matrix approximately as

$$U_L = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4), \quad (1)
$$

where $\lambda (\approx 0.22)$ is a real expansion parameter, and $A$, $\rho$ and $\eta$ are also real quantities. From the above expression, the elements $U_{ub}^L$ and $U_{td}^L$ can be parameterized in terms of two phases $\gamma$ and $\beta$ respectively which form a unitary triangle (Fig.1) given by the orthogonality condition $\sum_{i=u,c,t} U_{id} U_{id}^\ast = 0$. The recent significant measurements of $\beta_{\exp}$ give $[4]$

$$\sin 2\beta_{\exp} = \begin{cases}
0.59 \pm 0.14 \pm 0.05 & (BABAR) \\
0.99 \pm 0.14 \pm 0.06 & (Belle)
\end{cases} \quad (2)
$$

If there are new physics effects involved, the experimental value $\beta_{\exp}$ can be expressed through other parameters representing the new physics as well as the phase of $U_{td}^L$ in the $SM$.

In addition to the phases mentioned above, the masses ($M_{W_R}$) of the right-handed gauge bosons, the mixing angle $\xi$ between the left- and right-handed gauge bosons $W_L$ and $W_R$, and the right-handed gauge coupling constant $g_R$ play an important role in new physics effects as fundamental input parameters in the left-right model ($LRM$). The success of the $SM$ in the low-energy phenomenology requires that the masses ($M_{W_R}$) of the right-handed
gauge bosons are significantly larger than those \((M_{W_L})\) of left-handed gauge bosons. The first lower bound on \(M_{W_R}\) comes from a study of the low energy charged current sector allowing \(M_{W_R} \gtrsim 3M_{W_L} \approx 240\) GeV \([4]\). Soon after, many theoretical limits have been presented on \(M_{W_R}\) and \(\xi\) under various assumptions \([1]\). The recent experimental limits are obtained by DØ and CDF from the direct search of the decay channels of the extra gauge bosons \(W'^+ \to \ell^+_R \nu_R\). DØ found \(M_{W'} > 720\) GeV for \(m_{\nu_R} \ll M_{W'}\) or \(M_{W'} > 650\) GeV for \(m_{\nu_R} = M_{W'}/2\) \([7]\). CDF has a limit of \(M_{W'} > 652\) GeV for \(m_{\nu_R} \ll M_{W'}\) if \(\nu_R\) is stable \([8]\). All of these limits were obtained assuming manifest (\(U^R = U^L\)) or pseudo-manifest (\(U^R = U^L K\)) left-right symmetry (\(g_L = g_R\)), where \(K\) is a diagonal phase matrix \([9]\). In this paper, we will not impose the discrete left-right symmetry which can cause troubles in explaining the cosmological baryon asymmetry and may lead to cosmological domain-wall problems \([10]\). However we will also consider the possibility of the left-right symmetric case among other possibilities.

The main purpose of this paper is to investigate the \(CP\) violation in the \(B^0\bar{B}^0\) system in the \(LRM\) related to the recent experiments, since \(B^0\bar{B}^0\) mixing has recently been advocated as a very sensitive probe for the \(CP\) violation and the presence of right-handed current. The \(SM\) contribution to \(K^0\bar{K}^0\) mixing was previously computed for any internal quark mass by Inami and Lim \([11]\). The right-handed contribution in the \(LRM\) has been done first by Beall, Bander, and Soni assuming the discrete left-right symmetry \([12]\) and again by many authors \([13]\) under various assumptions. But we notice that the contributions of the mixing angle \(\xi\) to \(B^0\bar{B}^0\) mixing and \(CP\) asymmetry can be large due to the heavi ness of the top quark mass and the possibility of the enhancement in the right-handed quark mixing matrix in the general \(LRM\). After reviewing the structure of the \(LRM\) in Sec.2, we will discuss \(B^0\bar{B}^0\) mixing in Sec.3 and \(CP\) asymmetry in \(B^0\) decay in Sec.4 in detail.
2 \( SU(2)_L \times SU(2)_R \times U(1) \) models

We briefly review here some of the main features of the \( LRM \), which are needed to obtain our results. As the simplest extension of the \( SM \), the gauge group of the \( LRM \) breaks down to that of the \( SM \) and it finally cascades down to \( U(1)_{EM} \). The covariant derivative for the fermions \( f_{L,R} \) with respect to the gauge group of the \( LRM \) appears as

\[
D^\mu f_{L,R} = \partial^\mu f_{L,R} + ig_{L,R} W_{L,R}^a T^a_{L,R} f_{L,R} + ig_1 B^\mu S f_{L,R}. \tag{3}
\]

The electric charge which is the unbroken \( U(1) \) generator is given as

\[
Q = T^3_L + T^3_R + S. \tag{4}
\]

The quarks and leptons transform under the gauge group of the \( LRM \) \( (T_L, T_R, S) \) as

\[
q'_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \sim \left( \frac{1}{2}, 0, -\frac{1}{6} \right), \quad q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \sim \left( 0, \frac{1}{2}, \frac{1}{6} \right),
\]

\[
l'_L = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L \sim \left( \frac{1}{2}, 0, \frac{1}{2} \right), \quad l'_R = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_R \sim \left( 0, \frac{1}{2}, -\frac{1}{2} \right), \tag{5}
\]

where the primes indicate that the fermions are gauge rather than mass eigenstates.

In order to generate masses for the fermions and implement the symmetry breaking, we need to include scalar fields into our theory. The simplest choice is to introduce one Higgs multiplet and two doublets,

\[
\Phi = \begin{pmatrix} \phi_0^0 \\ \phi_1^+ \\ \phi_0^- \\ \phi_2^0 \end{pmatrix} \sim \left( \frac{1}{2}, -\frac{1}{2}, 0, 0 \right), \quad \chi_L = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix}_L \sim \left( \frac{1}{2}, 0, \frac{1}{2} \right), \quad \chi_R = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix}_R \sim \left( 0, \frac{1}{2}, \frac{1}{2} \right), \tag{6}
\]

which acquire the vacuum expectation values (VEVs)

\[
\langle \Phi \rangle = \begin{pmatrix} k \\ 0 \end{pmatrix}, \quad \langle \chi_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \tag{7}
\]

where \( k \) and \( k' \) are complex, and \( v_L \) and \( v_R \) are real. \( \chi_R \) is needed to generate a large \( M_{W_R} \) if \( v_R \gg |k|, |k'|, v_L \). But \( \chi_L \) is not essential unless we impose an left-right symmetry. It is also possible to adopt other choice of Higgs such as Higgs triplets instead \[14\]. The Lagrangian for the scalar field is

\[
L_{scalar} = Tr[D^\mu \Phi]^\dagger D_\mu \Phi + (D^\mu \chi_L)^\dagger D_\mu \chi_L + (D^\mu \chi_R)^\dagger D_\mu \chi_R - V(\Phi, \chi_L, \chi_R). \tag{8}
\]
For the Higgs fields described above, the kinetic terms in the Lagrangian generate the charged $W$ boson matrix

$$M_{W^\pm} = \begin{pmatrix} g_L^2(v_L^2 + K^2)/2 & -g_L g_R k^* k' \\ -g_L g_R k k' & g_R^2(v_R^2 + K^2)/2 \end{pmatrix} \equiv \begin{pmatrix} M_{W_L}^2 & M_{W_{LR}}^2 e^{i\alpha} \\ M_{W_{LR}}^2 e^{-i\alpha} & M_{W_R}^2 \end{pmatrix}, \quad (9)$$

where $K^2 = |k|^2 + |k'|^2$ and $\alpha$ is the phase of $k^* k'$. After the mass matrix is diagonalized by a unitary transformation the eigenvalues can be expressed in terms of a mixing angle as

$$M_{W_L}^2 = M_{W_{LR}}^2 \cos^2 \xi + M_{W_R}^2 \sin^2 \xi, \quad M_{W_R}^2 = M_{W_{LR}}^2 \sin^2 \xi + M_{W_R}^2 \cos^2 \xi - M_{W_{LR}}^2 \sin 2\xi. \quad (10)$$

Thus the mass eigenstates are written as

$$\begin{pmatrix} W^+ \\ W'^+ \end{pmatrix} = \begin{pmatrix} \cos \xi & e^{-i\alpha} \sin \xi \\ -\sin \xi & e^{-i\alpha} \cos \xi \end{pmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}, \quad (11)$$

where $\xi$ is a mixing angle defined by

$$\tan 2\xi = -\frac{2M_{W_{LR}}^2}{M_{W_R}^2 - M_{W_L}^2}. \quad (12)$$

For $v_R \gg |k|, |k'|, v_L$, the mass eigenvalues and the mixing angle reduce to

$$M_{W_L}^2 \approx \frac{1}{2} g_L^2(v_L^2 + K^2), \quad M_{W_R}^2 \approx \frac{1}{2} g_R^2 v_R^2, \quad \xi \approx \frac{2 g_L |k^* k'|}{g_R v_R^2}. \quad (13)$$

Here, the Schwarz inequality requires that $\zeta \equiv M_{W_R}^2/M_{W_L}^2 \geq \xi_q \equiv (g_L/g_R) \xi$. From the limits on deviations of muon decay parameters from the V-A prediction, the lower bound on $M_{W'}$ can be obtained as follows $[15]$

$$(g_R/g_L)^2 \zeta < 0.033 \quad \text{or} \quad M_{W'} > (g_R/g_L) \times 440 \text{ GeV}. \quad (14)$$

We will use this number for our numerical analysis.

As well as the above charged gauge bosons, the charged would-be Goldstone bosons corresponding to the longitudinal components of the physical bosons take part in the charged current interactions. The coupling of the Goldstone fields to the fermions can be found from the detailed structure of the Higgs potential $V(\Phi, \chi_L, \chi_R)$ and the Yukawa couplings. However one can directly determine the Goldstone couplings in terms of the gauge couplings without considering the Higgs potential, but using the Ward identities that ensure that the
unphysical poles in the two diagrams shown in Fig. 2 should cancel each other \[16\]. The charged interaction Lagrangian is then given by

\[
L_{CC} = -\frac{1}{\sqrt{2}} \bar{\psi} \gamma^\mu \left\{ [U^L g_L c_\xi L + U^R g_R s_\xi^+ R] W^+_\mu + [-U^L g_L s_\xi L + U^R g_R c_\xi^+ R] W'^+_\mu \\
+ [ (U^L M_P g_L c_\xi - U^R M_N g_R s_\xi^+) L + ( - U^L M_N g_L c_\xi + U^R M_P g_R s_\xi^+) R ] \frac{\varphi^+_\mu}{M_W} \\
+ [ - (U^L M_P g_L s_\xi + U^R M_N g_R c_\xi^+) L + (U^L M_N g_L s_\xi + U^R M_P g_R c_\xi^+) R ] \frac{\varphi'^+_\mu}{M_{W'}} \right\} N + h.c. + \ldots,
\]

where \( c_\xi \) (\( s_\xi \)) \equiv \cos \xi \) (\( \sin \xi \)), \( s_\xi^\pm \equiv e^{\pm \alpha} \sin \xi \), \( L, R \equiv (1 \mp \gamma^5)/2 \) denote left- and right-handed projection operators, \( M_P = \text{diag}(m_u, m_c, m_t) \) and \( M_N = \text{diag}(m_d, m_s, m_b) \) are the diagonalized quark mass matrices, \( P \) (\( N \)) is the mass eigenstate corresponding to its eigenvalue \( M_P \) (\( M_N \)), and \( U^L \) (\( U^R \)) is the left (right)-handed quark mixing matrix.

Figure 3: Box diagrams for \( B^0 \bar{B}^0 \) mixing with the gauge-bosons \( (W, W') \) and the Goldstone bosons \( (\varphi, \varphi') \).

3 \( B^0 \bar{B}^0 \) mixing

The effective Hamiltonian in the \( B^0 \bar{B}^0 \) system is obtained by integrating out the internal loop in the box diagrams in Fig. 3 just as in the \( SM \). We neglect external momenta and
where $B$ to the Feynman-’t Hooft gauge, the charged gauge boson and Goldstone boson contributions with $B$ being to choice of scalar representation, the Ward identities require that the box diagrams contribute. Although the form of the charged interactions in eqs.(17-19) is independent of our particular flavor-changing Higgs boson is much heavier than $M$ has been known that their contributions, even at the tree-level as long as the mass of the $B$-quark mass, but the result is valid for general internal quark masses. One finds, using (17) the manifest invariance into our theory, we need to involve flavor-changing neutral Higgs bosons, but it is found using $g_{LR} \equiv U_{id}^A U_{jd}^B$, $x_i \equiv m_i / M_W$ $(i = u, c, t)$, and

\[
f(x_i, x_j; \zeta) = \frac{\ln(1/\zeta)}{(1 - \zeta)(1 - x_i \zeta)(1 - x_j \zeta)} + \frac{x_i x_j \ln x_i}{(x_i - x_j)(1 - x_i)(1 - x_j \zeta)} + (i \rightarrow j),
\]

\[
g(x_i, x_j; \zeta) = \frac{\zeta \ln(1/\zeta)}{(1 - \zeta)(1 - x_i \zeta)(1 - x_j \zeta)} + \frac{x_i \ln x_i}{(x_i - x_j)(1 - x_i)(1 - x_j \zeta)} + (i \rightarrow j).
\]

Although the form of the charged interactions in eqs. is independent of our particular choice of scalar representation, the Ward identities require that the box diagrams contributing to $B^0 \bar{B}^0$ mixing in the LRM are not gauge invariant. In order to impose gauge invariance into our theory, we need to involve flavor-changing neutral Higgs bosons, but it has been known that their contributions, even at the tree-level as long as the mass of the flavor-changing Higgs boson is much heavier than $M_W$, are suppressed by approximately $^3\text{The tree-level flavor-changing neutral Higgs contributions with masses } M_H \text{ of order } M_W \text{ in the manifest or pseudo-manifest left-right symmetric model were discussed in Ref. [13].}$
a factor of $\zeta$ compared to the above gauge boson contributions \cite{18}. Therefore the above results, in the approximation of neglecting external momenta and $d$-quark mass, provide the complete effective Hamiltonian contributing to $B^0\bar{B}^0$ mixing.

At this stage, in order to analyze the obtained effective Hamiltonian quantitatively, we need to consider specific forms of the right-handed quark mixing matrices $U^R$. If the model has manifest or pseudo-manifest left-right symmetry, $W_R$ mass has stringent bound $M_{W_R} \geq 1.6$ TeV \cite{12}, and the $W_R$ boson contributions to $B^0\bar{B}^0$ mixing and tree level $b$ decay are very small. But, in general, the form of $U^R$ is not necessarily to be restricted to manifest or pseudo-manifest symmetric type, so the $W_R$ mass limit can be lowered to approximately $300$ GeV by taking the following forms of $U^R$ \cite{19}:

$$U^R_1 = \begin{pmatrix} e^{i\omega} & \sim 0 & \sim 0 \\ \sim 0 & c_R e^{i\alpha_1} & s_R e^{i\alpha_2} \\ \sim 0 & -s_R e^{i\alpha_3} & c_R e^{i\alpha_4} \end{pmatrix}, \quad U^R_{11} = \begin{pmatrix} \sim 0 & e^{i\omega} & \sim 0 \\ c_R e^{i\alpha_1} & \sim 0 & s_R e^{i\alpha_2} \\ -s_R e^{i\alpha_3} & \sim 0 & c_R e^{i\alpha_4} \end{pmatrix},$$

(22)

where $c_R (s_R) \equiv \cos \theta_R (\sin \theta_R)$ ($0^\circ \leq \theta_R \leq 90^\circ$). Here the matrix elements indicated $\sim 0$ may be $\lesssim 10^{-2}$ and the unitarity requires $\alpha_1 + \alpha_4 = \alpha_2 + \alpha_3$. From the $b \to c$ semi leptonic decays of the $B$ mesons, we can get an approximate bound $\xi_9 \sin \theta_R \lesssim 0.013$ by assuming $|U^R_{cb}| \approx 0.04$ \cite{20}.

The obtained effective Hamiltonians in eqs.\cite{17,19} are then further simplified using the Glashow-Iliopoulos-Maiani (GIM) cancellation $\sum_{i=u,c,t} \lambda_i = 0$ and neglecting the $u$-quark mass :

$$H^{SM}_{eff} = \frac{G^F M_W^2}{4\pi^2} (\lambda_t^{LL})^2 S(x_t^2) (\bar{d}_L \gamma_\mu b_L)^2,$$

$$H^{LR}_{eff} = \frac{G^F M_W^2}{2\pi^2} \left\{(\frac{g_R}{g_L})^2 \left\{ \lambda_e^{LR} \lambda_t^{RL} x_c x_t \zeta A_1(x_t^2, \zeta) + \lambda_e^{LR} \lambda_t^{RL} x_c^2 x_t \zeta A_2(x_t^2, \zeta) \right\} (\bar{d}_L b_R)(\bar{d}_R b_L) \\
+ \lambda_t^{LL} \lambda_t^{RL} x_b \xi_9 [x_t^3 A_3(x_t^2)(\bar{d}_L \gamma_\mu b_L)(\bar{d}_R \gamma_\mu b_R) + x_t A_4(x_t^2)(\bar{d}_L b_R)(\bar{d}_R b_L)] \right\},$$

(23)

(24)

where

$$S(x) = \frac{x(4 - 11x + x^2)}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3},$$

$$A_1(x, \zeta) = \frac{4 - x}{(1-x)(1-x\zeta)} + \frac{(1-\zeta)(1-x\zeta)}{(1-\zeta)(1-x\zeta)},$$

$$A_2(x, \zeta) = \frac{(4 - 2x + x^2(1-3\zeta)) \ln x}{(1-x)^2(1-x\zeta)^2} + \frac{(1-\zeta) \ln \zeta}{(1-\zeta)(1-x\zeta)^2},$$

$$A_3(x) = \frac{7 - x}{4(1-x)^2} + \frac{(2 + x) \ln x}{2(1-x)^3},$$

(25)
\[ A_4(x) = \frac{2x}{1-x} + \frac{x(1+x)\ln x}{(1-x)^2}. \]

Note that \( S(x) \) is the usual Inami-Lim function, \( A_1(x, \zeta) \) is obtained by taking the limit \( x_c^2 = 0 \), and \( H_{eff}^{RR} \) is suppressed because it is proportional to \( \zeta^2 \). Also, in the case of \( U_1^{R} \), one can see that there is no significant contribution of \( H_{eff}^{LR} \) to \( B^0 \bar{B}^0 \) mixing, so we will concentrate on the second type \( U_1^{R} \) in this section.

The dispersive part of the \( B^0 \bar{B}^0 \) mixing matrix element can then be written as

\[ M_{12} = M_{12}^{SM} + M_{12}^{LR} = M_{12}^{SM} \left\{ 1 + \left( \frac{g_R}{g_L} \right)^2 r_{LR} \right\}, \quad (26) \]

where

\[ \left( \frac{g_R}{g_L} \right)^2 r_{LR} \equiv \frac{M_{12}^{LR}}{M_{12}^{SM}} = \frac{\langle B^0|H_{eff}^{LR}|B^0 \rangle}{\langle B^0|H_{eff}^{SM}|B^0 \rangle}. \quad (27) \]

For specific phenomenological estimates one needs the hadronic matrix elements of the operators in eq.\((23,24)\) in order to evaluate the mixing matrix element. We use the following parametrization :

\[ \langle B^0|(\bar{d}_L\gamma\mu b_L)^2|B^0 \rangle = \frac{1}{3} B_1 f_B^2 m_B, \]
\[ \langle B^0|(\bar{d}_L\gamma\mu b_L)(\bar{d}_R\gamma\mu b_R)|B^0 \rangle = \frac{-5}{12} B_2 f_B^2 m_B, \quad (28) \]
\[ \langle B^0|(\bar{d}_L b_R)(\bar{d}_R b_L)|B^0 \rangle = \frac{7}{24} B_3 f_B^2 m_B, \]

where

\[ \langle 0|\bar{d}_\beta\gamma^\mu b_\alpha|B^0 \rangle = -\langle B^0|\bar{d}_\beta\gamma^\mu b_\alpha|0 \rangle = -\frac{i f_B p_\mu^\alpha}{\sqrt{2m_B}} \delta_{\alpha \beta}. \quad (29) \]

and where \( f_B \) is the \( B \) meson decay constant and \( B_i (i = 1, 2, 3) \) are the bag factors. In the vacuum-insertion method \([21]\), \( B_i = 1 \) in the limit \( m_b \approx m_B \). We will use \( f_B B_i^{1/2} = (210 \pm 40) \) MeV for our numerical estimates \([22]\). Using the standard values of the quark masses and \( |U_{\text{cd}}^L| \approx 0.222 \), one can express \( r_{LR} \) in terms of the mixing angle and phases in \([22]\) as

\[ r_{LR} \approx \left\{ l \left\{ 18.1l \left( \frac{1 - \zeta - (3.49 - 14.0\zeta) \ln(1/\zeta)}{1 - 5.68\zeta} \right) \right\} \right\} \zeta_s R e^{i\delta_1} \\
- 739 \left\{ \frac{1 - 5.04\zeta - (0.483 - 1.93\zeta) \ln(1/\zeta)}{1 - 10.4\zeta + 31.3\zeta^2} \right\} \zeta_s R e^{i\delta_2} - 7.68\zeta s R e^{i\delta_3}, \quad (30) \]

where \( l = 0.009/|U_{\text{cd}}^L|, \delta_1 = -2\beta + \alpha_2 - \alpha_3, \delta_2 = -\beta - \alpha_3 + \alpha_4, \delta_3 = -\beta - \alpha_3 \) and the mixing phase \( \alpha \) was absorbed in \( \alpha_i \) by redefining \( \alpha_i + \alpha \rightarrow \alpha_i \).
Figure 4: Behavior of the ratio $|r_{LR}|$ as $\delta_{1,2}$ are varied.

Figure 5: Allowed region for $|r_{LR}|$ and $\theta_R$.

Now we investigate numerically the behavior of the ratio $|r_{LR}|$, which is the deviation of $M_{12}$ from the $SM$, under variation of $M_{W^\prime}$, $\xi_g$, $\theta_R$ and the phases in $U^R$, assuming $l = 1$. Although we use the average value of $|U_{td}|$ which might be different from the actual value of $|U_{td}|$, it should not affect the order of magnitude in our estimates. First, in order to see the dependence of $|r_{LR}|$ on the phases, we fix $M_{W^\prime} = 800$ GeV, $\xi_g = 0.005$, $\theta_R = 15^\circ$, and set $\delta_3 = \pi$ because its effect is relatively much smaller than that of $\delta_1$ and $\delta_2$. The plot is shown in Fig.4. From eq.(30) and Fig.4, one can see that $|r_{LR}|$ becomes maximal when $\delta_{1,3} = \pi$ and $\delta_2 = 0$, and minimal when $\delta_{1,2,3} = \pi$ if $\theta_R \lesssim 70^\circ$ (or $\delta_{1,2} = \pi$ and $\delta_3 = 0$ if $\theta_R \gtrsim 70^\circ$). This behavior also holds for other values of $M_{W^\prime}$ and $\xi_g$. Since $|r_{LR}|$ is the continuously varying function of the phases, we can probe the allowed region for $|r_{LR}|$ with respect to
the parameters $M_{W'}$, $\xi_g$ and $\theta_R$. Next, we fix $M_{W'} = 800$ GeV, $\xi_g = 0.005$, and evaluate $|r_{LR}|$ by varying $\theta_R$. Note that $|r_{LR}|$ can approach zero at a non-zero $\theta_R$ near $73^\circ$ as shown in Fig.4. Otherwise, it is larger than 1, which means that generally it is possible to have $|M^L_{12}| \gg |M^S_{12}|$. In Fig.6, we consider the behavior of $|r_{LR}|$ for $g_R/g_L \geq 0.5$, $\xi_g = 0.0004$ and $\theta_R = 14^\circ$, $70^\circ$ as $M_{W'}$ is varied. The behavior of $|r_{LR}|$ exhibits a substantial dependence on $M_{W'}$, and $|r_{LR}|$ can be larger than 1 even for $M_{W'} \sim 2$ TeV. Moreover, it can be noticed that $|r_{LR}|$ falls near $M_{W'} \sim 300$ GeV at certain angles and phases in the mixing matrices. This reflects a possibility of relatively light masses of $W'$ compared to the previously known bound. We will return to this point in Sec.4. The dependence of $|r_{LR}|$ on $\xi_g$ satisfying $\xi_g \sin \theta_R \lesssim 0.013$ at fixed $M_{W'} = 700$ GeV and $\theta_R = 14^\circ$, $70^\circ$ is shown in Fig.7. As one can see, $|r_{LR}|$ can be enhanced up to 10% of the SM contribution for the given inputs. Although its effect is smaller than that of other parameters, it is not negligible and can be dominant in $|r_{LR}|$ if the first two $\zeta$ dependent terms in eq. (30) cancel each other.

Figure 6: Allowed regions for $|r_{LR}|$ and $M_{W'}$ for $g_R/g_L \geq 0.5$. The dotted lines correspond to the lower bounds on $M_{W'}$ in eq.(14) for the ratio $g_R/g_L = 1,2,3$ and 4 respectively.

As we mentioned previously, the average value of $|U_{td}|$ might be different from the actual
value of $|U_{td}^2|$, and there is also ambiguity from errors in $f_B B_1^{1/2}$. Therefore the mass mixing $\Delta M_{B'}^{SM}$ can be either much larger or smaller than $\Delta M_{B}^{exp}$. However, if we assume that $0.5 \lesssim |\Delta M_{B'}^{SM}/\Delta M_{B}^{exp}| \lesssim 2$, we can get specific bounds on the mass $M_{W'}$ and the angle $\theta_R$ using the experimental value $\Delta M_{B}^{exp} \simeq 0.472 \times 10^{12} s^{-1}$. We will estimate the lowest possible bound on $M_{W'}$ with respect to $\theta_R$ in their parameter space with the numerical consideration of $\sin 2\beta$ in the next section.

4 CP asymmetry in $B^0$ decay

The CP angle $\beta$ in CKM matrix can be measured in $B \to J/\psi K_S$ decays. In $B$ decays into a final CP eigenstate $J/\psi K_S$, $\beta$ is related to a parametrization invariant quantity $\lambda$ as follows [1]:

$$\sin 2\beta_{eff} = \text{Im}\lambda(B^0 \to J/\psi K_S),$$

where

$$\lambda \equiv -\left(\frac{q}{p}\right)_B \frac{A(\bar{B}^0 \to J/\psi K_S)}{A(B^0 \to J/\psi K_S)} \simeq \frac{M_{12}^*}{|M_{12}|}.$$  \hspace{1cm} (32)

The minus sign in the above expression is coming from the fact that $J/\psi K_S$ is CP-odd. As mentioned earlier, $\beta_{eff} = \beta$ in the SM.

In the LRM, the two types of $U^R$ give us two distinct results. In the case of $U^R$, the $W'$ contribution to the mixing parameter $(q/p)_B$ is negligible so that $(q/p)_B \simeq (q/p)_{SM} = e^{-2i\beta}$. Then the CP angle $\beta_{eff}$ can be expressed by

$$\sin 2\beta_{eff}^I \simeq -\text{Im} \left( e^{-2i\beta} \frac{U_{cs}^* U_{cb}^* (g_R/g_L)^2 (-2U_{cs}^* U_{cb}^* \xi^+ + U_{cs}^* U_{cb}^* \xi^-)}{U_{cs}^* U_{cb}^* (g_R/g_L)^2 (-2U_{cs}^* U_{cb}^* \xi^+ + U_{cs}^* U_{cb}^* \xi^-)} \right) \simeq -\text{Im} \left( e^{-2i\beta} \frac{1 + 25(g_R/g_L)^2 (-2s_R \xi g e^{i\alpha_2} + c_R s_R \xi e^{i(\alpha_2 - \alpha_1)})}{1 + 25(g_R/g_L)^2 (-2s_R \xi g e^{-i\alpha_2} + c_R s_R \xi e^{-i(\alpha_2 - \alpha_1)})} \right),$$

where the mixing angle $\alpha$ is absorbed in $\alpha_i$ again, and we ignored the $K\bar{K}$ mixing and assumed that

$$<J/\psi K_s|\bar{c}_L \gamma_\mu s_L \bar{b}_L \gamma_\mu c_L|B^0> \simeq <J/\psi K_s|\bar{c}_R \gamma_\mu s_R \bar{b}_R \gamma_\mu c_R|B^0> \simeq -\frac{1}{2} J/\psi K_s|\bar{c}_L \gamma_\mu s_L \bar{b}_L \gamma_\mu c_L|B^0>.$$ \hspace{1cm} (34)

As one can easily see in eq.(33), $\sin 2\beta_{eff} = \sin 2\beta$ if $\alpha_{1,2} = 0$ or $\pi$.

For illustration of the possible effect of the new interaction on the effective value of $\sin 2\beta$ : $\sin 2\beta_{eff}$, we assume that the SM contribution produces $\sin 2\beta = 0.60$, and show the region
Figure 8: Behavior of $\sin 2\beta_{\text{eff}}$ as $\alpha_{1,2}$ are varied.

Figure 9: Contour plots corresponding to $\sin 2\beta_{\text{eff}} = 0.99$ for $\sin 2\beta = 0.60$, (a) $\xi_g = \zeta/2$ and $g_R = g_L$, (b) $\xi_g = \zeta$ and $g_R = g_L$, and (c) $\xi_g = \zeta/2$ and $g_R = 2g_L$. The dotted line corresponds to the lower bound on $M_{W'}$ for $g_R = 2g_L$.

of parameters, where the effective value is shifted to $\sin 2\beta_{\text{eff}} \sim 1$. We first plot $\sin 2\beta_{\text{eff}}$ in Fig. 8 for the typical values $M_{W'} = 800$ GeV, $\xi_g = 0.005$, $\theta_R = 15^\circ$ and $g_R = g_L$ as $\alpha_{1,2}$ are varied. In the figure, $\sin 2\beta_{\text{eff}}$ has a maximum variation from $\sin 2\beta$ near $\alpha_2 \approx \pi/2$ and $\alpha_1 = \pi$, and this behavior holds for other values of $M_{W'}$, $\xi_g$ and $\theta_R$. Next, we plot the
Figure 10: Contour plots of $\Delta M_B$ (dashed) for $|U_{td}^L| = 0.009$ and those corresponding to $\sin 2\beta_{\text{eff}} = 0.99$ (dotted) for $\sin 2\beta = 0.60$.

As one can see, the upper bound of $M_{W'}$ goes down with decreasing $\xi g$ and $g_R/g_L$. Therefore, under the given assumption, $g_R \ll g_L$ is disfavored, and so is $M_{W'} \gg 1$ TeV unless $g_R \gg g_L$.

In the case of $U_{II}^R$, one has $U_{cs}^R \sim 0$ so that the $\zeta (M_{W'})$ dependent term in eq.(33) is very small. However the ratio $(q/p)_B$ depends on $M_{W'}$. Thus the $W'$ contribution enters in a somewhat different way.

$$\sin 2\beta_{\text{eff}}^{II} \sim -\text{Im} \left( e^{-2i\beta} \frac{(1 + (g_R/g_L)^2 r_{LR}^*) (1 - 50(g_R/g_L)^2 s_R \xi g e^{i\alpha_2})}{(1 + (g_R/g_L)^2 r_{LR}) (1 - 50(g_R/g_L)^2 s_R \xi g e^{-i\alpha_2})} \right).$$

(35)

Unlike in the previous case, we need to consider here the mass mixing of $B^0 \bar{B}^0$ in order to analyze $\sin 2\beta_{\text{eff}}$ numerically. Assuming that $|U_{td}^L| = 0.009$ and $\sin 2\beta = 0.60$, we plot the contours corresponding to $\sin 2\beta_{\text{eff}} = 0.99$ and $\Delta M_{B}^{LR} = \Delta M_{B}^{\exp}$ in the parameter space of

\[g_R/g_L = 0.5\] but there was no allowed region.

\[\text{We did the same analysis for} \quad g_R/g_L = 0.5\]
δ_{1,2} for δ_3 = \pi, \xi_g = \zeta/4 and g_R/g_L = 0.5, 1, 2 by varying \theta_R and M_{W'} in Fig.10. Because of the non-triviality of the behavior of sin 2\beta_{eff} on \delta_i, we repeated this analysis until the two contours have overlapped by varying M_{W'} from 350 GeV to 8 TeV, and found that it appeared where 350 GeV \lesssim M_{W'} \lesssim 1.3 TeV if g_R/g_L = 0.5, 440 GeV \lesssim M_{W'} \lesssim 3.1 TeV if g_R/g_L = 1 and 880 GeV \lesssim M_{W'} \lesssim 7.1 TeV if g_R/g_L = 2. Even though the existence of a heavy W' with the mass M_{W'} > 7 TeV may be allowed by the numerical analysis of \Delta M_{LR}^B, it is excluded by that of sin 2\beta_{eff} under the given assumption. For the different values of \xi_g, we also have similar results.

5 Conclusion

In the LRM, if one does not impose the manifest or pseudo-manifest left-right symmetry, the W' contributions to \bar{B}_0 - B_0 mixing and CP asymmetry in B_0 decays are highly dependent upon the phases in the mass mixing matrix U^{L,R}. For certain phases, the contribution of W' with a heavy mass about a few TeV to \bar{B}_0 - B_0 mixing can be sizeable. On the other hand, there is also a possibility of the existence of W' with a light mass about a few hundred GeV, whose contribution can be either very large or small, and so the contribution of the mixing angle \xi is not negligible. Since the existence of a light W' requires a small g_R, g_R \lesssim g_L, one can see from eq.(23) and Fig.8 that its contribution is limited. Therefore even assuming that \Delta M_{B}^{LR} \lesssim \Delta M_{B}^{SM}, we find that there is a possibility of a light W' with a mass M_{W'} \sim 300 GeV.

Also this possibility arises from the numerical analysis of the CP asymmetry in B_0 decay. Since U^{L}_{td} is not known with sufficient accuracy, estimates of the pure right-handed current contributions to \Delta M_B and sin 2\beta are somewhat uncertain. But, for the certain values of the parameter sets, one can see from Fig.8 and Fig.10 that the CP asymmetry parameter sin 2\beta can be as large as almost 1, and the mass of W' can be as small as about 350 GeV. Therefore, the existence of the light W' can be tested once future experiments confirm the value of sin 2\beta and |U^{L}_{td}|.

6 Acknowledgements

The author would like to thank M. B. Voloshin for helpful comments.
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