Higher Curvature Brane Corrections to the DGP Model

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We investigate the Dvali-Gabadadze-Porrati (DGP) model corrected by higher curvature brane terms. We show that these corrections have a dramatic impact on the spectrum of the model at the linearized level. Owing to the presence of higher derivatives in the field equations very massive ghost excitations with mass of order of Planck mass are generated in the ordinary branch of the model. These excitations describe an instability of Minkowski vacuum with time-scale of order of the Hubble time $H_0^{-1}$. At large distances these tachyonic excitations are expected to decouple from brane-localized matter. Our modified DGP model represents therefore a very promising framework for solving of the cosmological constant problem, in which Planck-scale physics is responsible for the elementary excitations driving the accelerated expansion of the universe, but the time-scale of the instability is settled by gravitational physics at large scales.

Gravity models that allow for large-distance deviations from standard General Relativity (GR), the so-called Modified theories of gravity [1, 2, 3, 4], have been widely investigated in recent years. The main motivation behind this interest is the hope that the accelerated expansion of the universe [5, 6, 7, 8] could be explained and the cosmological constant problem solved by very large distance, of order $r \gg r_c = H_0^{-1}$, modifications of General Relativity without postulating nonbaryonic forms of matter such as dark energy. From a purely theoretical point of view Modified Gravity represents a way to circumvent Weinberg’s no-go theorem on the “old” cosmological constant problem [3, 10] and to shift it from the realm of short distance particle physics to that of large distance gravitational physics. Moreover, from the phenomenological point of view there is plenty of room for such infrared modifications of gravitational physics. Our experimental knowledge of gravity is in agreement with GR but is limited to distances between say $10^{-3} cm - 100 Mpc$.

One of the most promising scenarios for modifications of GR in a relativistic and general covariant framework is the Dvali-Gabadadze-Porrati (DGP) model [1]. The DGP model works in the context of a brane-world scenario (see Ref. [11] for a early proposal), in which bulk non-local effects modify the large distance behavior of gravity on the brane. In particular, the DGP model allows in the so-called self-accelerating branch for tachyonic excitations describing an instability of Minkowski vacuum with a time-scale of order $\tau_I = r_c$. Such an instability could represent a solution to the cosmological constant problem. Unfortunately, it was soon realized that the tachyonic excitation is a light ghost of mass $m \sim m_D = r_c^{-1}$ gravitationally coupled to brane-localized matter [12, 13]. From particle physics point of view the presence of such a light ghost is simply disastrous.

There is no compelling reason for having $\tau_I \sim m_D^{-1}$. We would rather expect the cosmological constant problem to be solved by a conspiracy between short-distances and large-distances physics. Motivated by these considerations, in this letter we explore the possibility of a brane-world scenario in which the time-scale of the tachyonic instability is still of order $r_c$ (i.e. it is determined by large-distance gravitational physics), but its mass is of order of 4D Planck-mass $M_P$. Introducing terms quadratic in the curvatures in the brane part of the DGP action we show that at the linearized level the model allows, in the ordinary branch, for tachyonic excitations describing an instability of Minkowski vacuum with timescale $\sim r_c$. Although these excitations are ghosts, they are heavy with mass $\sim M_P$ and at large distances they are expected to decoupled from brane-localized matter.

In the DGP model all known interactions, except gravity, are confined on a zero-tension $(1 + 3)$-brane that is embedded in a five-dimensional (5D) infinite-volume space-time (the bulk). Although 5D brane-induced gravity cannot solve the old cosmological constant problem [14], it can be used to deepen our understanding about modified gravity theories. The DGP action is

$$S = M^3 \int d^3 x \sqrt{G} \left( (5) R + M_P^3 \int d^4 x \sqrt{-g} R + \text{bound. terms} \right), \quad (1)$$

where $M$, $X$, $G$ and $(5) R$ are bulk quantities, while $M_P$, $x$, $g$ and $R$ are referred to the brane. In Eq. (1) boundary, Gibbons-Hawking, terms should be taken in account to warrant the correct equations in the bulk. Matter fields are also localized on the brane and they are omitted here for simplicity.

The relevant new ingredient in Eq. (1) is the Einstein-Hilbert term on the brane. This 4D kinetic term, as well as all the possible Lorentz invariant terms, can be induced via loop corrections by brane-localized matter.

Using the notation $\eta_{AB} = (+ - - - -)$, with capital indices running on 0, 1, 2, 3, 5 and Greek indices running on
where \( G_{AB}^{(5)} \) is the Einstein tensor built from the 5D metric, \( g_{AB}^{(5)} \), and \( G_{\mu\nu} \) is the 4D Einstein tensor built from the induced 4D metric, \( g_{\mu\nu} \). Eqs. (2) are extremely difficult to solve, only symmetric solutions are known. Deffayet found the modified Friedman-Robertson-Walker cosmological solutions [12], while some approximation for the Schwarzschild solution has been also derived [10, 17].

Relevant physical information about the model (spectrum of excitations, stability) can be gained at the linearized level by expanding the metric around the Minkowski vacuum: \( g_{AB}^{(5)} = \eta_{AB} + h_{AB} \). The \( \{\mu\nu\} \) component of Eq. (2) in the harmonic gauge, \( \partial^a h_{AB} = \frac{1}{2} \partial_B h^A_{\mu\nu} \), becomes

\[
[M^3 \partial_C \partial^C + M_p^2 \delta(y) \partial_\mu \partial^\mu] h_{\mu\nu} = -\delta(y) (\partial_\mu - \frac{1}{3} \eta_{\mu\nu} T) + \delta(y) M_p^2 \partial_\mu \partial_\nu h_{55}. \tag{3}
\]

The structure of the propagator in the Fourier space is

\[
\tilde{h}_{\mu\nu}(p, y = 0) \tilde{T}_{\mu\nu}(p) = \frac{1}{M_p^2} \frac{\tilde{T}_{\mu\nu} \tilde{T}_{\mu\nu} - \frac{1}{3} \tilde{T}^2}{p^2 \pm m_D p}, \tag{4}
\]

where \( p = \sqrt{p^2} \) is the square root of the Euclidean 4-momentum, and \( m_D = 2 M^3 / M_p^2 \) is the DGP crossover parameter. The different signs in Eq. (4) are related to different boundary conditions at \( y \to \infty \) and to the brane embedding in the 5D bulk. They describe different branches of solutions. The “+” sign refers to the ordinary branch and the “−” sign to the self-accelerating branch. The spectrum of linear excitations near the Minkowski vacuum can be read out from the pole structure of the propagator (4). In the ordinary branch we have a massless excitation, which does not propagate physical degrees of freedom (corresponding to a pole with zero residue) and a spin 2 resonance of mass \( m = m_D \). In the self-accelerating branch we always have the massless excitation and a tachyon of mass \( m_D \).

The tensorial part of Eq. (2) is the same as that of Pauli-Fierz (PF) theory for a massive graviton. In fact, from a 4D point of view, gravity in the DGP model is mediated by a continuum of massive Kaluza-Klein modes, with no normalizable massless graviton in the spectrum. This is a consequence of the presence of a flat and infinite size extra-dimension. We have therefore a 5D (or a massive 4D) graviton propagating 5 on-shell degrees of freedom: a helicity-2 state which is the analog of the GR massless graviton, a helicity-1 state which does not contribute to the extra-dimension. We have therefore a 5D (or a massive 4D) graviton propagating 5 on-shell degrees of freedom: a helicity-2 state which is the analog of the GR massless graviton, a helicity-1 state which does not contribute to the extra-dimension.

The static potential of the DGP model behaves therefore like \( V(r) \sim 1/r \) for \( r \ll r_D = 1/m_D \), while it behaves as a truly five dimensional Newton potential, \( V(r) \sim 1/r^2 \), for \( r \gg r_D \). It is usually required \( r_D \sim H_0^{-1} \) in order to allow for IR modifications of GR.

Although the vDVZ discontinuity makes the linear approximation of the DGP model problematic, it has been argued that nonlinearities may play a fundamental work, restoring solutions sufficiently close to those of GR at solar system scales [13, 20, 21, 22].

From a phenomenological point of view the self-accelerating branch is the most interesting one, because it allows for cosmological de Sitter solutions even without a 4D cosmological constant [13]. In fact the tachyon corresponds to an instability of the Minkowski vacuum with an exponential growth time, \( e^{m_D t} \). This is a very appealing feature, since the background is modified with a curvature term of order \( \sim m_D^2 \). The new parameter, \( m_D \), has to be fine-tuned to describe the present acceleration of the universe, thus it may seem that there is no advantage in using the DGP model instead of introducing a cosmological constant. But, in the DGP case the acceleration of the universe is a truly dynamical effect, which is a consequence of the modified field equations.

Unfortunately, the DGP model has been found to suffer from different problems. There is a strong coupling problem emerging when one tries to extend the results beyond the linear approximation [12, 23]. At the first non linear order the propagator gets divergent contributions. But the most dangerous issue is the presence of ghosts in the spectrum. It has been shown [12, 13] that the tachyon in the self-accelerating branch is a ghost of mass \( m_D \), i.e. it has a negative kinetic energy term. The presence of a light gravitational ghost excitation that couples to brane-localized matter makes the 4D description meaningless and seems to rule out at least the self-accelerating branch of the DGP model as explanation for cosmic acceleration [13].
On the other hand the ordinary branch is ghost-free, but in this case the Minkowski vacuum is stable and no de Sitter solutions exist without introducing a positive cosmological constant on the brane. This is however strictly true only if we consider a brane action of the Einstein-Hilbert type. The situation could change if we allow for other general covariant terms in the brane action, e.g terms depending non-linearly on the 4D curvature tensors. Motivated by this arguments, we consider a DGP model with also induced second order curvature terms on the brane [28]:

\[
S = M^3 \int d^5 x \sqrt{G} R^{(5)} + \int d^4 x \sqrt{-g} \left( M_P^2 R + \alpha R_{\mu \nu} R^{\mu \nu} - \beta R^2 \right),
\]

where \( \alpha, \beta \) are dimensionless real parameters.

The DGP model is recovered for \( \alpha = \beta = 0 \). It is worth to notice that, due to the 4D Gauss-Bonnet relation the quadratic terms in \([5]\) are the most general second order terms that can be added on a \((1 + 3)\)-brane. There are several motivations for considering these corrections to the DGP model. First, the same quantum corrections which induce massive spin 2 modes. Nevertheless, it is well known that in general such higher derivatives theories are pathological, due to the unavoidable presence of ghosts. In Stelle’s model the ghost has a mass \( \sim M_P/\sqrt{\alpha} \) and can be therefore exited at high energy scale. This makes the theory inconsistent at least if one requires unitarity.

In the framework of frame-world models the unitarity requirement can be reformulated in a different way. Actions like \([1] \) or \([5] \) have to be considered as low-energy effective descriptions of some fundamental unitary quantum theory of gravity, such as string theory. At short distance, say of order \( \alpha \)Planck length, actions \([1] \) and \([5] \) are not longer valid and the infinite series of higher order terms should be taken into account. For instance, in the complete fundamental theory higher curvature terms can contribute in some form protected by topological invariance (as in the Gauss-Bonnet case) such that unitarity is restored.

Thus, as far as we are interested in the large distance, IR behavior of gravity described by the model \([5] \), we can neglect the presence of ghosts as long as they have a mass \( \sim M_P \) which is above the UV cutoff of our effective theory and they do not couple to brane-localized matter.

The modified DGP field equations derived from the action \([5] \) are,

\[
G_{AB}^{(5)} = \frac{1}{2M^3} \delta(y) \delta_{AB} \left( T_{\mu \nu} - 2M_P^2 G_{\mu \nu} + 2S_{\mu \nu} \right),
\]

with

\[
S_{\mu \nu} = (\alpha - 2\beta) \nabla_\mu \nabla_\nu R - \alpha \nabla_\beta \nabla_\nu R_{\mu \nu} +
- \frac{\alpha}{2} g_{\mu \nu} \nabla_\beta \nabla_\beta R + 2\alpha R^{\alpha \beta} R_{\mu \alpha \nu \beta} + 2\beta RR_{\mu \nu} + \frac{1}{2} g_{\mu \nu} (\alpha R^{\mu \nu} R_{\mu \nu} - \beta R^2)
\]

Following the original computation of \([1] \), the linearized equations for the \( \{\mu \nu\} \) component read

\[
\left[ M^3 \partial_C \partial^C + M_P^2 \delta(y) \partial_\alpha \partial^\alpha + \alpha \partial_\beta \partial_\delta \partial_\beta \partial_\alpha \partial^\alpha \right] h_{\mu \nu} = -\delta(y)(T_{\mu \nu} - \frac{1}{3} h_{\mu \nu} T) + \delta(y) \left[ M_P^2 \partial_\mu \partial_\nu + \alpha \partial_\mu \partial_\nu \partial_\alpha \partial^\alpha \right] h_{55}.
\]

Notice that the linearized field equations \([8] \) do not depend on the parameter \( \beta \). This is due to the fact that in the harmonic gauge the 4D linearized Ricci scalar is identically zero. This means that apart from the \( R \) term, only the \( (R_{\mu \nu})^2 \) term contributes to the propagator. In particular the introduction of \( f(R) \) terms on the brane does not change the DGP propagator \([4] \).

For \( \alpha, \beta \neq 0 \) the propagator turns out to be

\[
h_{\mu \nu}(p, y = 0) \tilde{T}^{\mu \nu}(p) = \frac{1}{M_P^2} \left( \frac{\tilde{T}^{\mu \nu} \tilde{T}_{\mu \nu} - \frac{1}{3} \tilde{T}^2}{p^2 + m_D p^2} + \frac{\alpha}{M_P^2} p^2 \right).
\]
The tensorial part of the propagator is identical to that of the DGP model and to that of massive gravity. But in our case the excitation spectrum of the theory is much richer and it depends not only on the two branches of the theory, but also on the parameter space \((m_D, \alpha, M_P)\). Moreover, owing the the higher derivative terms in Eq. (8) we expect the presence of ghosts in the spectrum.

As usual, the spectrum of linear excitations can be inferred from the pole structure of the propagator \([9]\). In the \([\text{Re}(p), \text{Im}(p)]\) plane the propagator \([9]\) has in general 4 complex poles. One is located at \(p^2 = 0\). As in the DGP model the residue of this pole is zero, consistently with the absence of a normalizable 4D massless state in the spectrum. For \(\alpha \neq 0\) the other three poles can be found solving the algebraic third order equation

\[
p^3 + \frac{M_P^2}{\alpha}p \pm \frac{m_D M_P}{\alpha} = 0.
\]  

(10)

Defining \(\alpha_0 = \frac{4}{27} \left( \frac{m_D}{M_P} \right)^2\) we find two different regions in the parameter space: (i) for \(\alpha > 0\) and \(\alpha < -\alpha_0\) there is one real solution and two complex conjugate solutions, (ii) for \(-\alpha_0 \leq \alpha < 0\) there are three real solutions. Notice that the case \(\alpha < -\alpha_0\) is phenomenologically highly suppressed since \(\alpha_0 \gg 1\). In any realistic situation we are left with \(\alpha > 0\) and a small \(\alpha < 0\). These two regions are discussed in detail below.

In the following we will focus on the ordinary branch of the theory corresponding to the + sign in equation \((9)\). Note that, due to the absence of \(p^2\) terms in Eq. (10) the sum \(x_1 + x_2 + x_3\) of the poles is identically zero. For \(\alpha > 0\) the solutions of Eq. (10) are

\[
x_1 = -m_1, \quad x_{2,3} = \frac{m_1}{2} \pm m_2 i,
\]  

(11)

where we have defined the “effective” masses

\[
m_1 = -\frac{M}{\alpha} \left[ \left( -1 + \sqrt{\frac{1 + x}{x}} \right)^\frac{3}{2} - \left( 1 + \sqrt{\frac{1 + x}{x}} \right)^\frac{3}{2} \right],
\]  

(12)

\[
m_2 = \frac{\sqrt{3} M}{2 \alpha^\frac{3}{2}} \left[ \left( -1 + \sqrt{\frac{1 + x}{x}} \right)^\frac{3}{2} + \left( 1 + \sqrt{\frac{1 + x}{x}} \right)^\frac{3}{2} \right],
\]  

(13)

and \(x = \frac{m_1}{m_D}\). In any phenomenologically viable model we have naturally \(\alpha = \mathcal{O}(1)\) and \(x\) is an extremely small quantity \((x \sim 10^{-120})\) with reasonable values \(M_P = 10^{19} \text{GeV}, m_D \sim H_0 \sim 10^{-42} \text{GeV}\) and \(\alpha = 1\). With this approximation the masses become

\[
m_1 \sim m_D \left( 1 - \alpha \left( \frac{m_D}{M_P} \right)^2 \right) \sim m_D,\]

(14)

\[
m_2 \sim \frac{M_P}{\sqrt{\alpha}} \left( 1 + \alpha \frac{3\sqrt{3}}{16} \left( \frac{m_D}{M_P} \right)^2 \right) \sim \frac{M_P}{\sqrt{\alpha}}.
\]  

(15)

As expected there are two independent energy scales in the model: \(m_D\), which is the scale arising in the DGP model, and \(m_S = \frac{M_P}{\sqrt{\alpha}}\), which is exactly the mass of the massive 4D graviton found by Stelle in the framework of gravity theories with higher derivatives. The pole structure of the propagator is shown in Fig. (1).

Interesting novel features arises from the investigation of pole structure presented in Fig. (1). Similarly to the ordinary branch of the DGP model, there is a resonance state, corresponding to the pole \(p = -m_1 \sim -m_D\). This particle couples to brane-localized matter and does not lead to any instability of the Minkowski vacuum. The new feature is the presence of two new poles located at \(p \sim m_D/2 \pm m_S i\) corresponding to a tachyon-like state with mass \(m_S = M_P/\sqrt{\alpha}\). It is easy to prove that this tachyon is a ghost. The residue of the pole is complex and thus the norm of the state is not positive. But this ghost is very different from that arising in the self-accelerating branch of the usual DGP model. In the latter case the ghost couples gravitationally to brane-localized matter and has mass \(m_D \sim H_0\), so it can be excited in the IR, exactly at the scale where we expect significant modifications from the self-accelerating branch solution.

Conversely in the DGP model with higher curvature brane terms the ghost is very heavy, it has mass \(\sim M_P/\sqrt{\alpha}\). Despite of its large mass, the tiny real part of tachyon pole leads to an instability of the Minkowski spacetime with time-scale of order \((m_D)^{-1}\). Self-accelerating solutions exist even in the ordinary branch of the DGP model if one
FIG. 1: Schematic view of the Euclidean pole structure for the DGP model with higher curvature brane terms in the ordinary branch and for $\alpha > 0$. The massless particle does not propagate any degree of freedom. This is a consequence of the absence of normalizable massless 4D states in the model. The pole on the real axis corresponds to a resonance with a typical mass $\sim m_D \sim 10^{-42}\text{GeV}$. The resonance does not lead to any instability of the Minkowski spacetime. The two complex poles correspond to a massive tachyon, $m \sim M_P/\sqrt{\alpha} \sim 10^{19}\text{GeV}$.

considers higher curvature terms on the brane. Furthermore, since in the ordinary branch the gravitational interaction is mediated by the resonance of mass $m_D$, we naturally expect that at large distances the heavy particle decouples from brane-localized matter.

The physical explanation of this behavior involves both 4D brane and 5D bulk physics. The tiny real (in the Euclidean space) contribution, of order $m_D$, to the pole describing the ghost has to be explained in terms of 5D nonlocal effects, hence as a IR effect. On the other hand the huge imaginary contribution of order $M_P/\sqrt{\alpha}$ has to be explained as a short distance 4D effects which modifies only the UV behavior of the propagator. The net result is a very massive tachyon which has a very small and positive real part that, as in the self-accelerating branch of the usual DGP model, leads to an instability of the Minkowski spacetime. This suggests that in the DGP model with higher curvature brane terms even the ordinary branch is self-accelerating but without some of the pathologies arising in the self-accelerating branch.

The essential features of the model discussed here seem quite general and in principle are not limited to braneworld scenarios. They could be present, at linearized level, in any theory of gravity whenever the pole structure of the Euclidean propagator of some excitation is determined by both IR (positive real part) and UV (imaginary part) effects. The two mass regimes are so disentangled that one can have a heavy excitation with mass of order $\sqrt{\text{Re}(p)^2 + \text{Im}(p)^2} \sim \text{Re}(p)$ leading to a tiny instability of the Minkowski background.

The main difficulty that we have to face in order to give a consistent physical explanation of the above-described instability of Minkowski vacuum is the ghost nature of the excitation. The corresponding pole has a complex residue and this leads to a non-unitary theory. Obviously this unitarity problem cannot be solved in the framework of a low-energy effective theory of gravity described by the action (5). One can naturally assume the UV completion of our theory of gravity to be ghost free. The ghost could be reabsorbed in the (infinite) series of terms of the fundamental theory. Whether the instability due to the ghost does or does not remain in the fundamental theory is a subtle issue that is not addressed here.

The presence of higher curvature term on the brane seems to lead quite generically to an instability of the Minkowski spacetime in the ordinary branch. There are indications that adding a $f(R)$-term on the brane leads to self-accelerating solutions even in the ordinary branch [26]. Interestingly enough, there is no trace of this cosmological instability at the linearized level because the propagator remains unchanged by adding $f(R)$ brane terms. In this case instabilities of the Minkowski spacetime arise at the full non-linear level only. This is another example in which nonlinear effects in the DGP model have a strong impact on the dynamics. The study of cosmological solutions arising from action (5) will be presented elsewhere.

Although it is not a central issue, let us we briefly discuss the case $-\alpha_0 < \alpha < 0$. Now the solutions of Eq. (10) (with the $+$ sign) are

$$x_1 = m_0 = \frac{M_P}{\sqrt{-3\alpha}} \cos \left( \frac{1}{3} \arctan \sqrt{\frac{1-y}{y}} \right) > 0, \quad x_{2,3} = -\frac{m_0}{2} \left[ 1 \pm \sqrt{3} \tan \left( \frac{1}{2} \arctan \sqrt{\frac{1-y}{y}} \right) \right] < 0,$$

(16)
where \( y = -\alpha/\alpha_0 < 1 \). In this region we find two resonances and a tachyon, all of them with zero imaginary part. The pole structure is richer than in the usual DGP case, but no new features arise in the ordinary branch. The behavior in the self-accelerating branch is qualitatively the same as that described above. In particular, the poles structure in the Euclidean plane is \( y \)-reflected. This means that, going from one branch to the other, we just need to change resonances into tachyons and vice versa. This general result holds in the usual DGP case too. Always a very light tachyon-like state appears in this branch and it is a ghost with a mass of the order \( m_D \).

Higher curvature brane terms do not change the large distance behavior of the static potential of the usual DGP model. Nevertheless they can give short distance contributions, at energy scales of order \( M_\alpha/\sqrt{\alpha} \). The modified DGP model \([5]\) corrects GR both in the IR and in the UV. Neglecting the tensorial structure of the propagator \([\mathbb{9}]\), the static potential is

\[
V(r) = \int dt \int \frac{dp}{(2\pi)^4} e^{i p \sigma} \phi(p, y = 0),
\]

where \( \phi(p, y = 0) \) is the Fourier-transformed scalar part of the propagator \([\mathbb{9}]\) on the brane. Focusing on the phenomenologically interesting case \( \alpha > 0 \) the potential reads

\[
V(r) = -\frac{1}{m_2^2 \pi^2 (4 + 9 \eta^2)} \left( 2 V_1(r) - \text{Re} \left( (2 + 3i \eta) V_2(r) \right) \right),
\]

where \( \eta = m_1/m_2 \) and \( V_1, V_2 \) are obtained from the standard DGP potential \([\mathbb{1}]\)

\[
V_{DGP}(r) = \frac{1}{r} \left[ \sin(m_D r) \text{Ci}(m_D r) + \frac{1}{2} \cos(m_D r) (\pi - 2 \text{Si}(m_D r)) \right],
\]

substituting \( m_D \to m_1 \) and \( m_D \to m_1/2 + m_2 i \) respectively. In the large-\( r \) limit \( (r \gg 1/m_1, 1/m_2) \) the potential \([\mathbb{18}]\) becomes

\[
V(r) \sim -\frac{1}{m_1 \alpha^2 (4 + 9 \eta^2)} \left[ \frac{2 - 3\eta}{(m_2 r)^2} - \eta \frac{e^{-m_2 r}}{m_2 r} \left( 2 \cos \frac{m_1 r}{2} - 3 \eta \sin \frac{m_1 r}{2} \right) \right].
\]

As expected, corrections due to higher curvature brane terms decay exponentially. As in the usual DGP model the gravitational potential is five dimensional and its behavior can set a limit on the DGP parameter, \( m_D \). In the intermediate region, \( 1/m_2 \ll r \ll 1/m_1 \) the potential is

\[
V(r) \sim -\frac{1}{m_2 \alpha^2 (4 + 9 \eta^2)} \left( \frac{1}{m_2 r} - \frac{2\eta}{\pi} (-1 + \gamma - \log(m_1 r)) + \frac{3\eta}{\pi} \frac{1}{(m_2 r)^2} + \frac{2e^{-m_2 r}}{m_2} - \frac{3\eta^2}{2} e^{-m_2 r} \right),
\]

where \( \gamma \) is the Euler constant. In this region one recovers Newton potential corrected by subleading logarithmic terms arising from the DGP bulk and by exponential and by \( r^{-2} \) corrections arising from the higher curvature brane terms. In the small-\( r \) region, \( r \ll 1/m_2, 1/m_1 \), the potential behaves as \( \sim r \). This, together with submillimeter table-desk experiments can put constraints on the value on the parameter \( \alpha \).

In this paper we have investigated the DGP model corrected by higher curvature brane terms. We have found that these corrections have a dramatic effect on the linearized spectrum of the model. Ghost excitations with mass of order \( M_P \), which describe an instability of Minkowski vacuum with time-scales of order \( H_0^{-1} \) are generated. In the IR the tachyonic excitation is expected to decouple from brane-localized matter. Our modified DGP model represents therefore a very promising framework for solving the cosmological constant problem, in which Planck scale physics is responsible for the elementary excitations driving the accelerated expansion of the universe, but the timescale of the instability is settled by the gravitational physics at large scales.

Our result gives a strong hint that short distance corrections to the DGP model may hold the key for solving some of the open problem of the model. There are still some issues and consistency checks, which have been not addressed in detail in this paper. The first is the particle physics meaning of the ghost excitation driving the instability of Minkowski space. The main issue here is not the ghost nature of the excitation - it is rather natural to assume that this problem will be solved by the UV completion of the theory, because any consistent quantum theory of gravity, such as string theory, is expected to be unitary - but the reliability of our low-energy approximation given by the action \([\mathbb{5}]\). It is not clear to what extent the truncation is consistent and if in an alternative treatment the ghost excitation would still be present. An other, related, issue is the reliability of perturbation theory. Our results have been derived in the linear approximation. It is well known that the linear approximation of the DGP model is
strongly limited because of the vDVZ discontinuity and of the strong coupling effect. Nonlinear effects are therefore crucial for making the model consistent. It is not clear whether the features we have found at the linearized level still persist at a full nonlinear level. A very important check in this context is represented by the investigation of the cosmological solutions of our model. It is also important to stress that the introduction of higher-derivatives interactions -also in the form of higher curvature terms- in the DGP model may be also very useful for solving the vDVZ discontinuity problem. These terms can support a robust implementation of the Vainshtein effect [19] and still allow for self-accelerating de Sitter solutions with no light ghost instabilities [27].

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