Trapped-Ion Quantum Logic Utilizing Position-Dependent ac Stark Shifts

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We present a scheme utilizing position-dependent ac Stark shifts for doing quantum logic with trapped ions. By a proper choice of direction, position and size, as well as power and frequency of a far-off-resonant Gaussian laser beam, specific ac Stark shifts can be assigned to the individual ions, making them distinguishable in frequency-space. In contrast to previous all-optical based quantum gates with trapped ions, the present scheme enables individual addressing of single ions and selective addressing of any pair of ions for two-ion quantum gates, without using tightly focused laser beams. Furthermore, the decoherence rate due to off-resonant excitations can be made negligible as compared with other sources of decoherence.

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In recent years, physical realizations of quantum computers have received growing interest. Several very different physical implementations have been considered, and schemes based on a string of trapped ions, as first introduced by Cirac and Zoller, are among the most promising and popular candidates for demonstrating large scale quantum logic. In these schemes two internal levels of an ion represent a quantum bit (qubit), which can be manipulated through laser interactions. Two key requirements are individual addressing of the ions for single-qubit manipulations, and the ability to make gate-operations between any pair of ions. Although multi-ion entanglement was demonstrated in a recent experiment, individual and selective addressing remains a major experimental challenge for making an ion-trap quantum computer. The difficulties of addressing originates from the need for high trap-frequencies, to ensure efficient motional ground-state cooling and high gate-speeds, which leads to a small spatial separation of the ions. In addition, the ion separation decreases with an increasing number of ions. In current experimental setups where ground-state cooling has been demonstrated, typical trap-frequencies are \( \omega_z = 2\pi \times 0.7 \text{ MHz} \) in Innsbruck \(^m\) and \( \omega_z = 2\pi \times 10 \text{ MHz} \) at NIST \(^n\), which yields minimum spacings of 7.1 \( \mu \text{m} \) (two \(^{40}\text{Ca}^{+}\)-ions) and 3.2 \( \mu \text{m} \) (two \(^{9}\text{Be}^{+}\)-ions), respectively.

The most obvious method for individual addressing is simply to focus a laser beam onto a single ion. This was demonstrated by the Innsbruck-group \(^m\), but it is extremely demanding with the much stronger traps used at NIST or when more ions are involved. A few more complex methods for individual addressing of ions have been presented, which use either position-dependent micromotion \(^n\) or a position-dependent magnetic field \(^n\), but they are either technically demanding or hard to generalize beyond two ions.

In this Letter, we propose to use a position-dependent energy-shift, an ac Stark shift, of the qubit-levels, to obtain a unique resonance-frequency of each ion, such that the ions can be addressed individually just by tuning the frequency of a laser beam illuminating the whole ion string. In addition, we demonstrate how a position-dependent ac Stark shift can be used for selecting any pair of ions in a multi-ion string for implementing a two-ion quantum gate, e.g., a Mølmer-Sørensen gate \(^n\). Our scheme is technically not very demanding, since it relies on applying a far-off-resonant ac Stark-shifting laser beam focussed to a spot size larger than the ion spacing (see Fig. 1). This feature makes the scheme applicable even in experiments with relatively tightly confining traps (\( \omega_z/2\pi \sim 10 \text{ MHz} \)).

First, we consider the criteria for performing single qubit operations between states of the type \( |\downarrow, n\rangle \) and \( |\uparrow, n'\rangle \), where \( |\downarrow\rangle \) and \( |\uparrow\rangle \) are the two eigenstates of the ions and \( n \) is the vibrational quantum number for one of the motional modes of the ions. To selectively manipulate such two states of a single ion in a string, the spectral resolution \( \gamma \) of the laser performing the qubit operation has to much better than the trap-frequency \( \omega_z \) and sufficiently high that transitions in any other ion are prohibited. For simplicity, in the following, we consider a two-ion string, with one motional mode having the oscillation frequency \( \omega_z \), and with the ac Stark shift induced energy-difference between the two ions being \( E_{1,2} \) (see Fig. 1). First, we treat the situation where \( h\omega_z > E_{1,2} \gg \gamma \) as sketched in Fig. 1b (Case A). In this case, \( E_{1,2} = \hbar \omega_z/2 \) is the optimum choice, since a laser resonant with a specific transition \( |\downarrow, n\rangle \leftrightarrow |\uparrow, n'\rangle \) in one ion, is maximally off-resonant with all the transitions of the type \( |\downarrow, n\rangle \leftrightarrow |\uparrow, n'\rangle , |\downarrow, n\rangle \leftrightarrow |\uparrow, n' + 1\rangle , |\downarrow, n\rangle \leftrightarrow |\uparrow, n' - 1\rangle \) in the other ion, leading to the highest possible gate-speed. In the case \( E_{1,2} \gg h\omega_z \gg \hbar \gamma \) (Case B in Fig. 1), a laser resonant with a transition \( |\downarrow, n\rangle \leftrightarrow |\uparrow, n'\rangle \) in one ion, is only resonant (or near-resonant) with a transition \( |\downarrow, n\rangle \leftrightarrow |\uparrow, n' + m\rangle \) in the

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other ion, where $|m| \gg 1$. In the so-called Lamb-Dicke limit such a transition is strongly suppressed. Case B is particularly interesting when more than two ions are present, since in such cases the gate-time will only be limited by the vibrational frequency $\omega_z$ instead of a fraction thereof as in Case A.

An experimental realization of the above situation can be achieved, e.g., by a string of two $^{40}$Ca$^+$, $^{88}$Sr$^+$, or $^{138}$Ba$^+$-ions, with the qubit states $|\downarrow\rangle$ and $|\uparrow\rangle$ represented by the $^2S_{1/2}(m_J = +1/2)$ ground state and the $^2D_{5/2}(m_J = +5/2)$ metastable state, respectively. The far-off-resonant Stark-shifting laser beam is set to propagate perpendicular to the ion string, as indicated in Fig. 2, and its polarization is assumed to be linear along the inter-ion axis. Assuming a Gaussian intensity profile with a waist $W$, a maximum difference in the ac Stark shift of the ions is obtained by displacing the laser beam by $W/2$ with respect to the center of the ion string. The relevant internal levels of the considered ions, with respect to the Stark-shifting laser beam, are shown in Fig. 3. For simplicity, we assume that the Stark-shifting laser beam is so far red detuned from any transition-frequency that fine-structure splitting can be neglected.

The ac Stark shift $\varepsilon_\uparrow - \varepsilon_\downarrow$ of the $|\uparrow\rangle - |\downarrow\rangle$ transition of a single ion can be calculated by summing the contributions from all relevant dipole-allowed couplings.

In Fig. 3a, the laser power required to achieve an ac Stark shift difference $\varepsilon_{1,2} = \Delta \varepsilon_1 = \hbar \omega_z/2$ in the case of $\omega_z = 2\pi \times 1 \text{ MHz}$ is presented for $^{40}$Ca$^+$, $^{88}$Sr$^+$, and $^{138}$Ba$^+$ as a function of the laser wavelength [21].
of the laser beam is taken to be 30 \mu m, which is much larger than the equilibrium spacing of 5.6 \mu m, 4.3 \mu m and 3.7 \mu m for the \( ^{40}Ca^+, ^{88}Sr^+, \) and \(^{138}Ba^+\)-ions, respectively. The required power, which approaches a constant in the long-wavelength limit, is well within reach of commercial lasers, e.g., a CO\(_2\) laser (\( \lambda = 10.6 \mu m \)), a Nd:YAG laser (\( \lambda = 1064 \) nm), or a frequency-doubled Nd:YAG laser (\( \lambda = 532 \) nm).

Another very important parameter to consider in the present scheme is the spontaneous scattering rate, \( \Gamma_{sc} \), of light from the Stark-shifting laser beam, since it will limit the ultimate coherence time. Under the assumptions made above in calculating the ac Stark shifts, the sum of the scattering rates of both ions can be expressed as:

\[
\Gamma_{sc} = \frac{E_{1,2}}{\kappa \psi} \times \frac{e^{-1/2}3\pi c^2 \omega^3}{\hbar} \times \left[ \frac{1}{\omega_P^6} \left( \frac{\Gamma_P}{\omega_P - \omega} + \frac{\Gamma_P}{\omega_P + \omega} \right)^2 \right.
\]

\[
+ \sum_{n}\frac{1}{\omega_{n'-F}^6} \left( \frac{\Gamma_{n'-F}}{\omega_{n'-F} - \omega} + \frac{\Gamma_{n'-F}}{\omega_{n'-F} + \omega} \right)^2 \right],
\]

where \( \Delta z/W \ll 1 \), as obeyed by the parameters used in Fig. 3, is assumed [4].

In Fig. 3b, the coherence time (or rather \( \Gamma_{sc}^{-1} \)) is plotted as a function of laser wavelength, and we see that in the long-wavelength limit, the coherence time grows as the wavelength to the third power, which is also readily deduced from Eq. (4). Hence, at first, a CO\(_2\)-laser seems to be favorable. However, since the lifetime of the \( ^2D_{5/2} \)-level is only 1.0 s, 345 ms, and 47 s for \( ^{40}Ca^+, ^{88}Sr^+, \) and \(^{138}Ba^+\), respectively, the use of the fundamental wavelength of a Nd:YAG laser might be more attractive, since this will be much easier to focus to the required spot size. Actually, in current experimental setups the maximal coherence time is limited by heating of the ions on a timescale of 1–100 ms [3 4], hence even a continuously operated frequency-doubled Nd:YAG laser can be used without introducing significant additional decoherence.

There are several reasons for not choosing \( W \) too large compared with \( \Delta z \). First, the required power to achieve a certain energy difference \( E_{1,2} \) grows as \( W^3 \). Second, the total scattering rate for a fixed \( E_{1,2} \) also increases with \( W \) (\( \Gamma_{sc} \propto W \)). Furthermore, it should be noted that although a large \( E_{1,2} \) implies a short coherence time, it also allows a high gate-speed.

The effect of the (internal state dependent) gradient-force exerted on the ions by the Stark-shifting laser beam has to be considered. The maximal gradient-force will be on the order of \( F_{grad} = -\partial \psi/\partial z \approx E_{1,2}/\Delta z \). Taking the example of \( ^{40}Ca^+ \), and using the same parameters as above, the maximal gradient-force will be \( \sim 10^5 \) times smaller than the confining force exerted by the trap, and the associated change in the equilibrium distance between the ions, \( \delta z \), is \( \sim 300 \) times smaller than the spread of the vibrational wavefunction. This displacement is totally negligible. Nevertheless, when the Stark-shifting laser beam is turned on, an ion obtains a speed \( v \approx \delta z/t_{rise} \), where \( t_{rise} \) is the “rise-time” of the Stark-shifting laser beam. The associated kinetic energy must be much smaller than \( \hbar \omega_z \), which is fulfilled if \( t_{rise} \gg 1 \) ns. In practice, this is no limitation.

Above we considered in detail the simple case of two ions and one motional mode. If we take both motional modes, i.e., the so-called center-of-mass mode at frequency \( \omega_z \) and the stretch mode at frequency \( \sqrt{3}\omega_z \) [4], into account, the optimal value of \( E_{1,2} \) is slightly changed, but our conclusions remain valid. Further, we can generalize Case A and Case B of Fig. 3b to more than two ions. In Case A, the additional energy-levels will make it difficult to address individual ions, but it should be possible with a few ions, particularly in a relatively tightly confining trap. Case B works just as well.
with more than two ions. Only, \( E_{1,2}/\hbar \) should not coincide with the frequency of one of the higher motional modes.

It should be noted that the different ionic transition-frequencies, while applying the Stark-shifting laser beam, leads to a differential phase-development of the various ions. Since the frequency-differences are known, this can be accounted for by controlling the phase of the addressing light-field.

In addition to individual addressing, a Stark-shifting laser beam can be used for realizing two-ion quantum logic operations with a single bichromatic laser pulse, as proposed by Molmer and Sorensen [11, 12], between any two ions in a string. As an example, we show in Fig. 4, how one can make two ions in a three-ion string have the same unique resonance-frequency, needed for making a Molmer-Sorensen gate between these two ions.

The position-dependent ac Stark shift method discussed above is also applicable to other qubit-levels and ions. For example the two Zeeman-sublevels of the ground state in \(^{40}\text{Ca}^+, \ ^{88}\text{Sr}^+, \ 138\text{Ba}^+\) can be used as qubit-levels ([\(\downarrow\) = \(2S_{1/2}(m_J = -1/2)\), \(\uparrow\) = \(2S_{1/2}(m_J = +1/2)\)] with qubit-operations performed by two-photon stimulated Raman transitions. An ac Stark shift can be induced by a circularly polarized Stark-shifting laser beam with wavelength \(\lambda\) tuned in between the two fine-structure levels of the excited \(P\)-state. If we take \(\omega_0 = 2\pi \times 1 \text{ MHz}\) and \(W = 30 \mu\text{m}\), as in the previous discussion, an ac Stark shift difference \(E_{1,2} = \hbar \omega_0/2\) can be obtained for \(^{40}\text{Ca}^+\) with a minimum scattering rate of 161 Hz using a laser with \(\lambda = 395.2 \text{ nm}\) and a power of 64 mW. This scattering rate allows only for a very limited number of gate-operations, even in the case where the Stark-shifting laser beam is only present during the quantum logic processing. For \(^{88}\text{Sr}^+\) and \(^{138}\text{Ba}^+\) somewhat lower scattering rates can be obtained, owing to their larger fine-structure splitting. Applying the same approach to \(^{25}\text{Mg}^+\) or \(^{4}\text{Be}^+\) (with hyperfine-levels of the ground state as qubit-levels [8]) is impracticable, due to their relatively small fine-structure splitting.

In conclusion, we have shown that individual or selective addressing of trapped ions can be achieved, without introducing significant decoherence, by utilizing a Stark-shifting laser beam with modest focusing- and power-requirements. The presented scheme readily makes it possible to perform single- as well as multi-qubit gates.

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[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995).
[3] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, et al., Nature 404, 256 (2000).
[4] D. James, Appl. Phys. B 66, 181 (1998).
[5] D. Leibfried, C. Roos, P. Barton, H. Rohde, S. Gulde, A. Mundt, G. Reymond, M. Lederbauer, F. Schmidt-Kaler, J. Eschner, et al., in *Atomic Physics 17*, edited by E. Arimondo, P. D. Natale, and M. Inguscio (AIP Conference Proceedings vol. 551, New York, 2001).
[6] Q. A. Turchette, D. Kielpinski, B. E. King, D. Leibfried, D. M. Meekhof, C. J. Myatt, M. A. Rowe, C. A. Sackett, C. S. Wood, W. M. Itano, et al., Phys. Rev. A 61, 063418 (2000).
[7] H. C. N"{a}gerl, D. Leibfried, H. Rohde, G. Thallhammer, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. A 60, 145 (1999).
[8] D. Leibfried, Phys. Rev. A 60, R3335 (1999).
[9] Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. 81, 3631 (1998).
[10] F. Mintert and C. Wunderlich, Phys. Rev. Lett. 87, 257904 (2001).
[11] A. Sorensen and K. Molmer, Phys. Rev. Lett. 82, 1971 (1999).
[12] K. Molmer and A. Sorensen, Phys. Rev. Lett. 82, 1835 (1999).
[13] S. Stenholm, Rev. Mod. Phys. 58, 699 (1986).
[14] R. Grimm, M. Weidem"{u}ller, and Y. B. Ovchinnikov, Adv. At. Mol. Opt. Phys. 42, 95 (2000).
[15] C. Roos, T. Zeiger, H. Rohde, H. C. N"{a}gerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. 83, 4713 (1999).
[16] NIST, *Atomic spectra database*, http://physics.nist.gov.
[17] C. E. Moore, *Atomic Energy Levels*, vol. II (National Bureau of Standards, Washington, 1952).
[18] C. E. Moore, *Atomic Energy Levels*, vol. III (National Bureau of Standards, Washington, 1958).
[19] A. Lindg"{a}rd and S. E. Nielsen, Atomic Data and Nuclear Data Tables 19, 533 (1977).
[20] Indeed the \(2S_{1/2}\) and \(2D_{5/2}\) states in \(^{40}\text{Ca}^+\) are used as qubit-levels in Innsbruck [36].
[21] In Fig. 3 we do not take the fine-structure splitting and the different couplings to the fine-structure levels into account. For the \((n - 1)D \rightarrow n'F\) transitions we sum over \(n' = 4 \rightarrow 10\). The data used are from Ref. 10 (\(^{40}\text{Ca}^+\)) and Refs. 14, 15, 36 (\(^{88}\text{Sr}^+\) and \(^{138}\text{Ba}^+\)). For a good
overview of the S-, P- and D-levels, see Ref. [4].