An Efficient Structural Analysis of Super Tall Mega Frame Buildings
Using a Multi-level Condensation Method

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Abstract
The mega frame system is considered to be suitable for skyscrapers, because it offers the structural efficiency of providing high rigidity against lateral loads, with a minimum amount of structural materials. Since the super tall mega frame building has a large number of elements and nodes, dynamic analyses of this system require significant computational resources. Therefore, a matrix condensation technique can be employed to efficiently predict the structural behavior of a mega frame structure. However, the computational resources required for the matrix condensation process of a super tall mega frame structure by a conventional condensation technique are significant, because hundreds of thousands of degrees of freedoms (DOFs) should be condensed. To overcome this difficulty, the purpose of this study is set as the development of an efficient matrix condensation method with accuracy for a mega frame structure. Mega elements, sub-mega elements and multi-level condensation techniques are developed for saving the computing resources required for matrix condensation. A special purpose computer program for the analysis of mega frame structures has been developed, and structural analyses of a 100-story mega frame structure were performed to verify the effectiveness of the proposed method. Based on the analysis results, it was confirmed that the proposed method could provide an efficient analytical model with outstanding accuracy, requiring significantly reduced computational efforts for both matrix condensation and structural analysis. Therefore, the developed computer program is expected to provide an efficient means for the preliminary design of mega frame structures.

Keywords: mega frame system; mega element; multi-level matrix condensation; super tall building; structural analysis

1. Introduction
To satisfy social and economical needs, building structures have recently become larger and higher. Therefore, the development and application of efficient structural systems for high-rise buildings has been widely performed (Kowalczyk et al., 1995; Schueller, 1990; Taranath, 1998). Although the appropriate structural system for a high-rise building structure may vary with different conditions, the outrigger and belt truss system is commonly used for 40-60 story buildings, and the tubular system is expected to be an economic system for 50-100 story buildings. There has been much progress in structural analysis, design and construction of high-rise buildings, with remarkable advances in materials, geotechnical engineering and information technology in recent years. Such advances continue to increase the height of super tall buildings, so that buildings as high as 1000 m may become a reality in the near future.

The mega frame system is expected to be effective for the super tall buildings of the future, because it has the structural efficiency of providing high rigidity against lateral loads, with minimum amount of structural materials (Feng and Mita, 1995). This structural system has sufficient lateral stiffness to resist wind or earthquake loads, with the combined action of mega columns and mega girders, which consist of many columns, girders, trusses and/or slabs. In general, parts of a structure with the same configuration and/or structural properties, referred to as mega columns and mega girders, are repeatedly used in many locations of a mega frame structure. Thus, the application of the substructuring technique may be very effective in the preparation of an analytical model for mega frame structures. A well-known building constructed by the mega frame system is the HSBC (Hongkong and Shanghai Bank Corporation) headquarters building, as shown in Fig.1.(a) (Foster et al., 1986). Even though there are not very many mega frame structures, the
Mega frame system is frequently used for future super tall building projects. Holonic Tower, which is planned by the Takenaka Corporation (2003), adopted the mega frame system for super tall buildings that are more than twice as high as present high-rise buildings. Moreover, Tuntex Sky Tower (Lee et al., 1997), an 85-floor skyscraper located in Lingya District, Kaohsiung, Taiwan, adopted the mega frame system as well.

Since a mega frame structure consists of a large number of structural members, a complicated finite element model with hundreds of thousands of DOFs would normally be used in the analysis. Using an ordinary finite element model, it would take a significant amount of computational time and computer memory for the analysis of mega frame structures. In preliminary design, repetitive structural analyses are required to select a suitable structural system, and thus it would be a lengthy task using a complicated finite element model. To overcome this problem, the condensation method of stiffness and mass matrices was proposed by Guyan (1965). Many researchers have applied this method to the structural analysis of high-rise building structures (Archer, 2001; Kim et al., 2005; Bouhaddi and Fillod, 1996; Weaver, 1987). To evaluate the effectiveness of the structural system, since only global structural behavior is investigated in preliminary design, a simple condensed model with several tens of DOFs can be used, provided it can predict global structural behavior. However, when a conventional matrix condensation method is employed to make a condensed model with several tens of DOFs, from the complicated model with hundreds of thousands of DOFs, significant computing resources would be required for the matrix condensation itself. If too many DOFs are condensed at once, using the conventional Guyan reduction technique, the required computer memory may exceed the capacity of an ordinary personal computer. Therefore, an efficient matrix condensation method for mega frame structures is proposed in this study, which takes advantage of the geometric configuration of mega frame systems. In order to accurately predict the global structural behavior of the mega frame structure using a condensed model, it is essential to use an appropriate criterion to select the active DOFs to be retained in the analysis. Thus, an efficient analytical model that can accurately predict the structural behavior with minimal DOFs was investigated. To improve the efficiency of the matrix condensation technique, mega elements, sub-mega elements and multi-level condensation techniques are proposed in this study. A computer program that includes pre- and post-processors was developed, based on the proposed method. To verify the effectiveness and accuracy of the proposed method, static and dynamic analyses of a 100-story mega frame structure were performed using the developed computer program.

2. Mega Elements for Efficient Modeling of Mega Frame Structures

Based on the geometrical characteristics of a mega frame structure, the mega element concept is introduced in this study, to improve the conventional matrix condensation method. The modeling procedure of a mega frame structure using mega elements is illustrated in Fig. 2. In general, some parts of the mega frame structure shown in Fig. 2.(a) have the same configuration and/or structural properties, and they are repeatedly used in many locations. These parts are referred to as mega members, and include mega columns, mega girders and mega joints. For the efficiency of matrix condensation, this example mega frame structure can be divided into 5 mega story structures, and the matrix condensation procedure can be applied to each mega story structure. One of the mega story structures, shown in Fig. 2.(b), can be subdivided into mega members, such as mega beams, mega columns and mega joints, as shown in Fig. 2.(c). The matrix condensation technique is applied to these mega members to derive mega elements, as shown.
in Fig.2.(d), and the mega elements are assembled to obtain a mega story model, as shown in Fig.2.(e).

As shown in Fig.3., mega elements have several DOFs only at the interfaces between mega elements, for the purpose of connecting them to each other. Finally, the analytical model of a mega frame structure can be obtained by combining 5 mega story models, as illustrated in Fig.2.(e). This mega element concept is similar to the super element or substructure technique, which is effective in the matrix condensation of large-scale structures. However, since an ordinary finite element model of a mega frame structure has too large a number of DOFs, the use of mega elements alone cannot sufficiently reduce the computational efforts for matrix condensation. Therefore, this study proposes a multi-level matrix condensation method.

3. Multi-level Matrix Condensation Method

3.1 Development of a mega element using sub-mega elements

When a mega element has a large number of DOFs, computational efforts for the development of a mega element may be considerable. Thus to effectively develop mega elements, a sub-mega element was introduced. If all the DOFs in the inner area of a mega element shown in Fig.4.(a) are condensed in one step, the number of inactive DOFs is 378 and the number of active DOFs is 108. Therefore, the inversion of a $378 \times 378$ stiffness matrix is necessary, which is the most time consuming procedure in the matrix condensation. The computational time required for the development of the simple mega element shown in Fig.4.(a) was 1.5sec by the 'one-step' condensation procedure. For the efficient development of mega elements, the matrix condensation is carried out in two steps, as illustrated in Fig.4., by the introduction of sub-mega elements. In the first step, a mega element is divided into four sub-mega elements, as shown in Fig.4.(b), and 54 DOFs in the inner area of a sub-mega element are eliminated, leaving 108 DOFs at the interfaces, as shown in Fig.4.(c). The computational time for the generation of each sub-mega element was 0.1sec. Then, sub-mega elements are assembled into a mega element, as shown in Fig.4.(d), and 162 inactive DOFs are condensed out, leaving 108 active DOFs, as shown in Fig.4.(e), in 0.3sec. Therefore, for the development of mega elements, the use of sub-

3.2 Mega joint by mega joint condensation procedure for a mega story structure

The mega elements are assembled to form a mega story model, as shown in Fig.5.(a). Then, all the DOFs used for the connections of mega elements are eliminated. Therefore, a mega story model has nodes only at the interfaces between mega story models, and at the center of the mega joint. The nodes at the interfaces are used for the connection of mega story models, and the node at the center of the mega joint is retained to represent the global behavior of a mega frame structure, as presented in Fig.5.(d). To select the center node of the mega joint, the X coordinate of the center point is calculated to be the mean value of the minimum and maximum X coordinates of all nodes in the mega joint. The Y and Z coordinates of the center point are calculated in the same way for each axis, respectively. When X, Y and Z coordinates of the center point of the mega joint are calculated, the
When the conventional 'one-step' matrix condensation method is employed, the condensed mega story model, as in Fig.5.(d), is directly obtained from the assembled model, shown in Fig.5.(a). However, when a mega frame structure is large, the computations for this procedure may not be trivial. Therefore, a mega joint by mega joint condensation technique, as illustrated in Fig.5., is proposed in this study for the efficiency of matrix condensation for a mega story model. The solid circles indicate active nodes, and the nodes to be eliminated are marked by white circles. In the mega joint by mega joint condensation procedure, the inactive DOFs on the surfaces of each mega joint are condensed out consecutively, as illustrated in Fig.5.(a). Table 1. shows a comparison of the number of DOFs between the 'one-step' condensation and the mega joint by mega joint condensation, for the structural example shown in Fig.5. Because there are four mega joints in this example, one fourth of the total inactive DOFs are eliminated in each condensation step. When the number of DOFs to be eliminated is \( N \), the amount of calculation for the inversion of the stiffness matrix will be approximately proportional to \( N^3 \). This procedure is considered to be the most time-consuming procedure in the matrix condensation. This is true in the case of full matrix. Because a skyline matrix method is used in every computation for the structural analysis, the amount of the inversion calculation of the stiffness matrix will be proportional to the value less than \( N^3 \). However, the exact value cannot be easily obtained because it is changed depending on how the mega frame structure is modeled. Therefore, the value of \( N \) is directly used in this study in order to see the changing tendency of the calculation amount only. If the mega joint by mega joint condensation procedure is used for a mega story model having \( J \) mega joints, the number of DOFs to be condensed in each step is reduced to \( N/J \). Therefore, the calculation for the inversion of the matrix for a mega story model will be reduced to \( J \times (N/J)^3 = N^3/J^3 \). Since the number of mega joints is four in this structural example, as shown in Fig.5., the amount of calculation for the inverse matrix will be approximately reduced to 1/16 by using the proposed condensation method.

Table 1. Number of DOFs for Each Condensation Method

| Eliminated DOFs | One-step condensation | Mega joint 1 | Mega joint 2 | Mega joint 3 | Mega joint 4 |
|-----------------|------------------------|-------------|-------------|-------------|-------------|
|                 | 336                    | 84          | 84          | 84          | 84          |
| Active DOFs     | 456                    | 222         | 336         | 450         | 456         |

3.3 Mega story by mega story condensation procedure for a mega frame structure

Mega story models developed in the previous section are assembled to obtain the analytical model of a mega frame structure. In this procedure, all the DOFs used for the connection between mega story models are eliminated, to reduce the number of DOFs to be used in the analysis. In the development of the final analytical model, if the number of DOFs to be eliminated is large, the 'one-step' condensation may require considerable computational efforts. Fig.6. illustrates the mega story by mega story condensation procedure proposed in this study, for the reduction of condensation time for the development of the final analytical model. If all of the inactive DOFs shown in Fig.6.(a) are condensed in one step, 1080 DOFs are eliminated, leaving 120 active DOFs. On the other hand, when the mega story by mega story condensation procedure is used, 216 DOFs, which is only one fifth of the total number of inactive DOFs, are eliminated in each condensation step. If the number of DOFs to be eliminated is \( N \), and a mega frame structure has \( S \) mega stories, the amount of calculation for the inversion of the stiffness matrix will be proportional to \( N^3/S \) in the 'one-step' condensation procedure. However, in the mega story by mega story condensation procedure, the number of DOFs to be eliminated in each step is reduced to \( N/S \), and thus the amount of calculation for the inverse matrix will be proportional to \( S \times (N/S)^3 = N^3/S^2 \). As the number of mega stories increases, the efficiency of the mega story by mega story condensation will become more significant.

Fig. 6. Mega Story by Mega Story Condensation Procedure for Modeling of a Mega Frame Structure

4. Computer Program using Multi-level Condensation Method

The efficient condensation method proposed in this study requires a special purpose computer program that includes pre- and post-processors. The pre-processor developed for the reduction of engineers' efforts for the preparation of the analytical model is shown in Fig.7. The pre-processor has three modeling steps in accordance with the proposed condensation method, i.e. development of mega elements, mega story models, and a mega frame structure. When the computer program developed in this study is used, the engineer decides which DOFs of each step model should be retained to make the next step model and assembles
mega joint, mega element, and mega story models to make a final analytical model of a mega frame structure. Then, the developed computer program automatically makes an efficient analytical model with a small number of DOFs by using a multi-level condensation method.

Although in a mega frame structure a mega story model with an identical geometrical configuration is repeatedly used, material properties or section sizes of structural members can vary with their locations in the structure. Therefore, the pre-processor introduces a member group, as well as a geometry group. The concept of the geometry group and member group is illustrated in Fig.8.(a). Since the mega frame structural example has 5 mega stories, its structural configuration can be repeatedly used as the geometry group entitled ‘G1’. However, the section sizes or material properties of structural members will vary, because of the difference in member forces. Therefore, different member groups can be assigned to structural members in the same geometry group. If member sizes in a specific member group are to be changed, those of all structural members belonging to the corresponding member group are automatically changed. A tree view of a pre-processor using geometry and member groups is presented in Fig.8.(b).

Fig.7. Pre-processor for Modeling of Mega Frame Structures

Fig.8. Modeling of Mega Frame Structure using Geometry and Member Groups

Fig.9. Mode Shapes from Commercial Software

Fig.10. Mode Shapes from the Developed Program

Four important mode shapes of the structural example obtained from the conventional analytical model are presented in Fig.9. When all of the DOFs are used in the analytical model, it may be difficult to clearly display the structural behavior, since a mega frame structure usually consists of a large number of structural members.

To overcome this problem, post-processors for a mega frame structure have been developed in this study. As shown in Fig.10., the mode shapes of a mega frame structure can be clearly presented by the developed post-processor, which is the mode shape viewer shown in Fig.11.
5. Analysis of Structural Example

A typical 100-story mega frame structure of 400m in height is used as a structural example, to verify the accuracy and efficiency of the proposed method. Static and dynamic analyses of the structural example are performed by using equivalent static lateral loads, and the ground acceleration record of the El Centro earthquake (1940, NS). The structural example is divided into five mega story structures, and a mega story structure has 20 stories, as shown in Fig.12.

The five analytical models shown in Fig.13 are employed in this study to find the optimal model that can accurately provide the global structural behavior of the mega frame structural example using the minimal DOF. Because the mega story model in Fig.13. has 20 stories, each mega story is used five times to make a 100-story example mega frame structure. As shown in Fig.13.(a), Model A has only 40 horizontal translational DOFs, associated with the center nodes of 20 mega joints, as indicated by the white circles. This model uses the minimum number of DOFs to represent the global behavior of the structural example. In addition to these 40 DOFs, Model B uses 100 DOFs associated with the nodes at the center of mega columns and mega girders, to represent their local deformation. Models C and D use more nodes in mega columns and mega girders as shown in Figs.13.(c) and (d). Consequently, the total numbers of DOFs of Model C and Model D are 340 and 1000, respectively. Model F is a conventional finite element model which uses 6 DOFs per node.

The lateral displacements of each model due to equivalent static lateral loads were compared in Fig.14. The results of four condensed models are almost identical to that of Model F, which is considered to be the most accurate. Thus, any model appears to be suitable for the static analysis of mega frame structures.

As presented in Fig.15., the natural frequencies of all of the analytical models turned out to be almost identical, up to the 15th mode; after that the discrepancy between Model A and the other models increases. This is because Model A does not have any DOFs in the inner area of mega columns and mega girders, and thus Model A cannot present their local behavior. As shown in Fig.15., 20 Eigenmodes are considered in this study for dynamic analysis of the 100-story mega frame structure. The cumulative mass participation factor of Model F with 20 modes is approximately 97%. The cumulative mass participation
factor of 97% is considered to be big enough for accurate predictions of dynamic responses of the example mega frame structure.

Since all of the condensed models provided natural frequencies very similar to those of Model F up to the 15th mode, it could be expected that any condensed model can be effectively used for the analysis of a mega frame structure. The peak displacements at three points marked in Fig.16. are in good agreement with those of Model F, and the error is less than 1%, as shown in Table 2. Based on static and dynamic analyses results, any of the analytical models used in this study can be accepted for the global structural behavior used in preliminary design. Accordingly, when only two DOFs per mega joint of the mega frame structure are used in the analysis, the structural responses of the mega frame structure can be most efficiently predicted without meaningful loss in the accuracy.

The five condensation methods (CM) listed in Table 3. are used to investigate the efficiency of each condensation procedure proposed in this study. CM1 uses only mega elements, while CM2 uses sub-mega elements in addition. The mega joint by mega joint condensation technique employed in CM3 and CM4 uses all of the condensation techniques proposed in this study. CM5 uses the ordinary condensation technique, which eliminates all of the inactive DOFs in one step.

### Table 2. Comparison of Peak Displacements (Unit: cm)

| Model | Comparison Position |  |  |
|-------|---------------------|---|---|
|       | P1                  | P2 | P3 |
| F     | 24.42               | 27.72 | -25.92 |
| A     | 24.19               | 27.69 | -25.41 |
| B     | 24.61               | 27.85 | -25.93 |
| C     | 24.56               | 27.82 | -25.99 |
| D     | 24.63               | 27.82 | -26.01 |

The computational time ratios for each condensation method are compared with respect to analytical Model D in Table 4. The results for CM5 are not included in this table, because CM5 requires 33.5GB of computer memory, which far exceeds the capacity of the conventional RAM of a personal computer. As shown in Table 4., the computational time of CM1 is the longest, because only mega elements are used. In the case of CM2, the computational time required for the development of mega elements can be reduced by about 79.2% (13% → 0.2% + 2.5%) compared to CM1, by using sub-mega elements. CM3 of Model D shows that the mega joint by mega joint condensation technique can reduce the computational time for the development of a mega story model by approximately 67.3%, compared to CM2. Moreover, it can be seen that the mega story by mega story condensation technique can reduce the computational time required for the development of a mega frame structure by about 84.8%, by using CM4. Accordingly, it can be seen that the multi-level condensation methods proposed in this study.
study can significantly improve the efficiency of the conventional matrix condensation procedure. Computer memory may be a critical factor in the matrix condensation of a mega frame structure, because this kind of structure may have hundreds of thousands of inactive DOFs. When all of the proposed condensation methods are used, the computer memories required for the stiffness and mass matrices of each analytical model are compared in Table 5. As shown in Table 5, the condensed models use only tens of megabytes (MB), while Model F requires more than 1 gigabyte (GB) of memory. Therefore, when the proposed multi-level condensation method is used for the matrix condensation of a mega frame structure, the savings in required computing resources can be significant. A personal computer with Pentium IV 2.6 GHz processor and 2.0 GB RAM was employed in this study.

Table 5. Computer Memory for Each Model (Unit: MB)

| Model Purpose | Sub-mega Element | Mega Element | Mega Story | Mega Frame | Subtotal | Total | Ratio (%) |
|---------------|------------------|--------------|------------|------------|----------|-------|-----------|
| F             | N/A              | N/A          | 1,041,240  | 1,041,240  | 1,041,240| 100.0 |           |
| A             | 1.374            | 17.490       | 22.266     | 0.012      | 41.142   | 68.368| 6.6       |
| K&M           | 3.090            | 16.622       | 1.524      | 5.790      | 27.226   | 69.858| 6.7       |
| B             | 1.374            | 17.652       | 23.010     | 0.150      | 42.186   | 69.858| 6.7       |
| K&M           | 3.090            | 16.632       | 1.622      | 6.328      | 27.672   | 73.434| 7.1       |
| C             | 1.374            | 18.000       | 24.534     | 0.884      | 44.792   | 73.434| 7.1       |
| K&M           | 3.090            | 16.250       | 1.826      | 7.476      | 28.642   | 73.434| 7.1       |
| D             | 1.544            | 19.134       | 29.088     | 7.638      | 58.224   | 92.306| 8.9       |
| K&M           | 2.888            | 15.504       | 2.590      | 13.140     | 34.082   | 100.0 |           |

K&M: memory for stiffness and mass matrices, respectively. K&M,: memory for condensation of stiffness and mass matrices, respectively.

6. Conclusions

A multi-level condensation method was developed in this study to improve the efficiency of the conventional matrix condensation procedure, and an efficient analytical model for a super tall mega frame building has been proposed. The main features of this study are summarized as follows:

1. In the preliminary design of a super tall mega frame building, what is required is a simple model that can present global structural behavior, rather than a complicated finite element model. If the conventional Guyan reduction method is used for a condensed simple model, the computational resources for the matrix condensation can be considerable. Therefore, a more efficient matrix condensation method is required.

2. The improved matrix condensation method using mega elements, sub-mega elements and the multi-level condensation techniques proposed in this study takes advantage of the geometric characteristics of mega frame systems. The proposed matrix condensation method can significantly reduce the computational efforts required for the reduction of stiffness and mass matrices of mega frame structures, compared to the conventional condensation method.

3. A condensed analytical model with minimal DOFs was proposed in this study, to efficiently predict the global structural behavior of mega frame structures. Reliable static and dynamic responses could be obtained from the proposed model with significantly reduced computational efforts. Therefore in the preliminary design of a mega frame structure, the proposed method is expected to reduce the lengthy task undertaken by a complicated finite element model.

4. A structural analysis program including a pre- and post-processor was developed, using the proposed method in this study. When this computer program is employed in practical engineering, especially in preliminary design, it can be expected to enhance the productivity and efficiency of the structural analysis of mega frame structures.

5. It is usual to use a sparse matrix storage format for large-scale computation. In this study, a skyline matrix storage format is used instead. Therefore, if the multi-level condensation method proposed in this study is used with a sparse matrix, the computation efficiency of matrix condensation may be greatly improved.

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