Convergence of daily mean coordinates of precise positioning methods

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Abstract. As part of the research of modern movements of the Earth's crust, an analysis of 7 high-precision methods for calculating GNSS positions was carried out for the convergence of their daily mean coordinates. Based on Euclidean distances, regular and maximal discrepancies between coordinates of different methods are given. According to the coordinates in the ITRF, 5 methods are stood out with regular coordinate discrepancies <1 mm, and individual maximum discrepancies up to 30 mm. The other two methods have regular discrepancies in coordinates up to 2 cm, and the maximum differences reach 1 m. For a group of stations global coordinates transformation into a local reference frame leads to the effect of coordinate stabilization and increases their relative precision in the time series. As a result of such procedure, the level of maximum coordinate discrepancies between the methods decreased to 46%. Moreover, one of the methods of calculating coordinates has improved its convergence with the other methods by 80%. Based on the Euclidean distance method, the quality of the raw data for each station was evaluated. Thus, there is a group of 8 stations, for which the convergence of coordinates in different methods are approximately at the same level, and 2-3 times better than for the other 2 stations.

1. Introduction

Geodynamic researches based on space-geodetic methods allows us to study the current movements of the near-surface part of the Earth's crust through calculations of high-precision point positions based on many years of repeated or continuous measurements. Obtained in this way coordinate time series allows us to calculate velocity vectors, kinematic and deformation parameters of the research area. Almost all such calculations are based on daily mean coordinates. Modern movements of the Earth's crust globally and regionally are caused by endogenous energy sources, and in most cases have a level of displacement not always exceeding the a priori precision of instrumental methods of positioning. Therefore, for correct generalizations and conclusions based on the positioning data of Global Navigation Satellite Systems (GNSS) it is important to know the real positioning precision and convergence of coordinates at different methods of their obtaining.

This work provides a comparative analysis of the daily mean geocentric cartesian XYZ coordinates obtained by high-precision positioning methods, mainly based on GPS signals. The choice of coordinate system XYZ due to the fact that the basic software packages provide results mainly in this system, based on its referencing to the world system International Terrestrial Reference Frame (ITRF), orthogonal location of the axes gives a simplification of subsequent operations for comparison. Other coordinate systems can be obtained by recalculating from XYZ while maintaining similar spatial discrepancies from one positioning method to another.
2. Software packages and methods of GNSS positions calculation

To obtain high-precise daily mean positions we have the opportunity to use three sources of processing the measured GNSS data: GAMIT/GLOBK software package, Bernese GNSS Software package and online-tools.

The GAMIT/GLOBK software package allows high-precision positioning of the stations being processed using a number of additional input data (satellite clocks and orbits corrections, ionospheric and tropospheric signal delays, tidal loading effects, reference to ITRF etc.). Of the many options for settings and implementations of data processing within this software package, we have chosen three significantly distant from each other methods of calculations on the daily mean values of coordinates XYZ.

The first calculation method is based on the default program settings and using the necessary files to handle earthquakes or other discontinuities in the observation series [1]. In addition, data about the processed local stations and external IGS points with their a priori coordinates are used for stabilization. We refer to the method as GAMIT/GLOBK Standard Processing (GSP).

In the second method, the information on the global reference frame and stabilization points (IGS stations) is excluded, and the solutions are based on the local reference frame, consisting of the 10 points included in the processing. We refer to this method as GAMIT/GLOBK No Global Data (GNG) and use it to estimate the contribution to coordinate precision of the ITRF use and stabilization points.

As part of the GAMIT/GLOBK package, there is a separate TRACK (GTR) program, which allows you to obtain intraday coordinates for each recorded epoch, then you can independently calculate the daily mean position. This program has a minimal set of input information including RINEX files of the stations being processed and an auxiliary file of satellite orbit corrections for a day. One of the disadvantages of TRACK is the tendency for positioning precision to decrease as distances between stations increase. This method of positioning is comparable with a series of programs for calculating coordinates in kinematic mode in geodetic works.

Another software package Bernese GNSS Software also provides the ability to calculate high-precision station coordinates using a number of auxiliary input data, but differs from GAMIT/GLOBK by having several approaches for calculating daily mean coordinates. In this work, we used two basic methods in Bernese to calculate the coordinates of stations per day.

The first method Precise Point Positioning (B3P) in its calculations uses signal data between one satellite and one station, which is called zero-difference [2]. This method is recommended for calculating both daily mean and intraday positions.

Another method in Bernese is called Rinex2Sinex (BRS), which is configured by default to calculate daily mean coordinates, but with additional settings is capable of providing positions for each record or measurement epoch. The algorithm «two satellites – two stations» or the double-difference is used here.

In addition to the large standalone software packages (GAMIT/GLOBK and Bernese), we used two other online services. The first of these, the Canadian Spatial Reference System Precise Point Positioning (W3P) is designed for standard GNSS data processing using a large set of correction information needed to determine the most precise station position anywhere in the world [3]. By means of automatic e-mail services, RINEX field observation files are transformed into daily mean point positions.

Another online service The Automatic Precise Point Service of the Global Differential GPS System (WPP) does not differ much from the previous one in the calculation methodology [4]. However, more quickly in real time uploads the source files to the server and returns the result without the mediation of e-mail.

3. Source data for the comparative analysis of positioning methods

Based on the same GPS field data, different positioning methods provide different daily mean XYZ coordinates. A priori, we have no good reason to state unequivocally the degree of superiority of one
or another method of determining the position. However, we can estimate these discrepancies in the calculations of the same positions obtained by the above mentioned methods.

For the validity of our comparative estimates, it is necessary to determine the volume of test processed GNSS measurements. On the one hand, the more data processed, the more accurately we can define the conclusions about the convergence of positioning methods. But it also increases the number of positions for comparison. Data processing time increases in the above methods, cumbersome material becomes difficult to comprehend. In this regard, it was decided to opt for 10 permanent stations of the Central Asian GPS network. A priori the number of 3 coordinates XYZ is defined. The choice of several days for comparison is due to the presence of a number of negative factors within a day and day to day, but also the positioning can be influenced by the seasonal factor. Therefore, it was decided to use for the calculations measurements for two days falling at the end of each season in the year.

Thus, the following regional continuous observation stations were selected for comparative analysis: IATA, CHUM, KAZA, KRTV, POL2, POL3, POL7, POLY, SUMK, TALA. Defined 8 days of their measurements falling at the end of each season of the year: November 23-24, 2019; February 11-12, May 21-22, August 22-23, 2020. As a result, each of the 7 methods provides 3 coordinates for each of the 8 days for each of the 10 stations. A total of 1680 unique individual coordinates will be used for analysis, among which it makes sense to make 5040 pairwise comparisons. Therefore, the seemingly small volume of the input data we have selected requires considerable costs for analysis. With a further increase in the input information, the number of analytical manipulations will increase many times over. Moreover, the comprehension and visual representation of the results becomes cumbersome.

4. Convergence of XYZ coordinates in the ITRF from different calculation methods
The purpose of the work is to compare the coordinates obtained by the above 7 methods. Each method contains 240 units of information (3 coordinates × 8 days × 10 stations). In order to obtain the most general result of comparison of methods, we chose Euclidean distance (ED) as a measure of similarity-difference, which uses multidimensional space. Therefore, we arrange the coordinates of each method in a given order and get a d-dimensional space (d = 240), in which all methods will be located further or closer to each other. The spatial series for one method was formed in one column; first all X ordered 10 stations from all 8 days are entered. Then the Y and Z values are added in the column below. A fragment of such a data array and the results of calculating Euclidean distances and their derivatives are presented in table 1.

| No. | B3P       | BRS       | GSP       | \((B3P - BRS)^2\) | \((B3P - GSP)^2\) | ED      | \(ed\) | \(edm\) |
|-----|-----------|-----------|-----------|-------------------|-------------------|--------|--------|--------|
| 1   | 1228955159.1 | 1228955156.3 | 1228955155.5 | 7.84              | 12.96             | 55.49  | 0.23   | 16.07  |
| 2   | 1228950393.2 | 1228950387.7 | 1228950387.3 | 30.69             | 35.28             |        |        |        |
| 3   | 1325807748.0 | 1325807749.1 | 1325807744.0 | 1.14              | 15.92             |        |        |        |
| 238 | ...       | ...       | ...       | ...               | ...               | ...    | ...    | ...    |
| 239 | ...       | ...       | ...       | ...               | ...               | ...    | ...    | ...    |
| 240 | ...       | ...       | ...       | ...               | ...               | ...    | ...    | ...    |

Table 1. Example of arrangement of elements of 240-dimensional space and calculation of Euclidean distances and their derivatives
In columns \((B3P - BRS)^2\), \((B3P - GSP)^2\) etc the squares of the differences between the corresponding method coordinates are calculated. Finding the root of their sum will give us the ED value between the two methods. Comparing all 7 methods with each other will produce \(\frac{7 \times 6}{2} = 21\) unique combinations of methods pairs and their corresponding Euclidean distances.

The Euclidean distance between the two methods gives a generalizing parameter to estimate the total discrepancy between all pairs of 240 coordinates and is defined:

\[
ED = \sqrt{\sum_{i=1}^{d} (M1_i - M2_i)^2},
\]

where \(M1, M2\) – ordered XYZ coordinates of the selected calculation methods.

The value of the Euclidean distance depends not only on the distances of objects per each dimension of space, but also on the number of \(n\) dimensions. Therefore, to better understand the difference between the methods, let us calculate the specific Euclidean distance (ed) averaged over one pair of corresponding coordinates of the two methods. In addition, of interest is the maximum value of the discrepancy in the coordinates of the two methods. These criteria are calculated by the formulas:

\[
ed = \frac{ED}{d} = \frac{ED}{240}, \quad edm = \sqrt{\max((M1_i - M2_i)^2)}
\]

These parameters characterize the level of regular and maximum discrepancy in the coordinates obtained by the two compared methods. The results of such calculations are presented in Table 2.

**Table 2.** Specific (ed) and maximum (edm) values between pairs of methods

| ed, mm | B3P | BRS | GSP | GNG | GTR | W3P | WPP |
|--------|-----|-----|-----|-----|-----|-----|-----|
| edm, mm |     |     |     |     |     |     |     |
| B3P    | 0.23 | 0.63 | 14.71 | 18.13 | 0.18 | 0.37 |     |
| BRS    | 16.07 | 0.63 | 14.66 | 18.09 | 0.23 | 0.42 |     |
| GSP    | 35.59 | 37.61 | 14.90 | 18.08 | 0.61 | 0.58 |     |
| GNG    | 462.14 | 454.84 | 454.72 | 17.82 | 14.71 | 14.81 |     |
| GTR    | 992.93 | 995.11 | 980.24 | 1027.53 | 18.11 | 18.14 |     |
| W3P    | 9.40 | 15.04 | 34.38 | 460.69 | 990.22 | 0.34 |     |
| WPP    | 21.94 | 25.10 | 26.30 | 463.60 | 988.82 | 22.70 |     |

Table 2 shows that according to the ed criterion (upper diagonal half of the table), the regular discrepancies in coordinates between the researched positioning methods vary from 0.2 to 18.1 mm and on average are 8.5 mm. According to the edm criterion, the maximum discrepancies vary from 9.4 to 1027.5 mm, averaging 405.5 mm. Obviously, the variations of ed and edm are proportional, with an increase in ed values correspondingly increasing edm. Different groups of methods with minimal level of variation in ed and edm are clearly distinguished, which can be shown in the results of clustering of methods when they are combined into a cluster by the principle of smallest value (Table 3).

**Table 3.** Specific (ed) and maximum (edm) values between method pairs (mm)

| ed, mm | B3P-W3P(9.4)-BRS(15.55)-WPP(23.71)-GSP(31.3) | GNG | GTR |
|--------|-------------------------------------------------|-----|-----|
| edm, mm | B3P-W3P(0.18)-BRS(0.23)-WPP(0.39)-GSP(0.60) |     |     |
| B3P    | 14.82 | 18.10 |
| B3P    | 457.79 |     |
| B3P    | 985.66 |     |
| B3P    | 1027.53 |     |
Obviously, the order of clustering by \( ed \) and \( edm \) is identical and proportional in the values of the clustering methods, B3P-W3P(0.18)-BRS(0.23)-WPP(0.39)-GSP(0.60) and B3P-W3P(9.4)-BRS(15.55)-WPP(23.71)-GSP(31.3). The selected cluster at the level of regular discrepancy of 0.6 mm and maximum discrepancy of 31.3 mm includes 5 positioning methods: B3P, W3P, BRS, WPP, GSP. The remaining 2 methods from the GAMIT/GLOBK software package clearly have significant up to 2 cm in \( ed \) and up to 1.03 m in \( edm \) systematic discrepancies in coordinates, both with the complex cluster and among themselves. From the insignificant regular discrepancies (<1 mm) in the coordinates for the 5 methods of the complex cluster with a high degree of probability can follow the conclusion about the high level of their precision of XYZ coordinate calculations.

5. Convergence of coordinates from different methods in the local reference frame

For different methods, the daily mean coordinates are determined taking into account a different number of external corrections for the current day in the ITRF. Therefore, the total movements and ITRF errors are also superimposed on the coordinates of distant regions. Therefore, the level of increment of ITRF coordinates for stations on different days is greater than the level of increments of line lengths between these stations on the same days. Consequently, the positioning method may provide less precise coordinates in the global reference frame (RF), but in the local RF it will be more precise. The transformation of ITRF coordinates for a group of stations into a local RF with the effect of their stabilization and increasing accuracy over time is described in detail in [5]. A similar procedure for converting ITRF coordinates into local RF gave the effect of reducing the level of coordinate variations by ~17% [6].

To transform the source daily mean coordinates XYZ (table 1) into a local RF, it is necessary to put the observer at the common center of mass of the data for 10 stations studied by us (C10) for every day within the framework of each method. For 10 stations of one day and one method, the x coordinate for C10 will be calculated:

\[
x_{C10}^i = \frac{1}{10} (x_{\text{IATA}}^i + x_{\text{CHUM}}^i + x_{\text{KAZA}}^i + \ldots + x_{\text{POLY}}^i + x_{\text{SUMK}}^i + x_{\text{ALTA}}^i)
\]  

(3)

Similarly, the average y and z positions are calculated for all methods for each day. The coordinates in the local RF will be calculated as:

\[
x_L = x - x_{C10}, \quad y_L = y - y_{C10}, \quad z_L = z - z_{C10}
\]  

(4)

By analogy with the structure of table 1, let's place the local coordinates in table 4 to calculate \( ED \), \( ed \), and \( edm \) for each pair of compared methods.

Table 4. Locating elements of 240-dimensional space based on coordinates in the local reference frame, calculating Euclidean distances and their derivatives

| Method mark, coordinates in mm | ED calculation element |
|--------------------------------|------------------------|
| No. | B3P | BRS | GSP | ... | \((B3P - BRS)^2\) | \((B3P - GSP)^2\) | ... |
|-----|-----|-----|-----|-----|----------------|----------------|-----|
| 1   | 4270159.0 | 4270156.2 | 4270156.5 | ... | 8.01 | 6.47 | ... |
| 2   | 4265393.2 | 4265387.6 | 4265388.3 | ... | 31.02 | 23.85 | ... |
| 3   | 101122748.0 | 101122749.0 | 101122745.0 | ... | 1.08 | 8.61 | ... |
| ... | ... | ... | ... | ... | ... | ... | ... |
| 238 | -63096043.0 | -63096040.5 | -63096039.8 | ... | 6.30 | 10.58 | ... |
| 239 | 59710040.8 | 59710042.9 | 59710026.4 | ... | 4.33 | 209.58 | ... |
| 240 | -82436028.9 | -82436030.0 | -82436029.1 | ... | 1.12 | 0.05 | ... |
|     | **ED** | **45.53** | **89.91** | ... | **ed** | **0.19** | **0.37** | ... |
|     | **edm** | **9.31** | **18.47** | ... |
Parameters of the regular and maximal discrepancy in the local coordinates obtained by the two compared methods will be placed in Table 5.

### Table 5. Values of specific Euclidean distances ($ed$) and maximum values ($edm$) between pairs of methods in the local RF,

| ed, mm | B3P | BRS | GSP | GNG | GTR | W3P | WPP |
|--------|-----|-----|-----|-----|-----|-----|-----|
| edm, mm |     |     |     |     |     |     |     |
| B3P    | 0.19 | 0.37 | 0.43 | 15.28 | 0.18 | 0.35 |
| BRS    | 9.31 | 17.95 | 18.65 | 15.19 | 15.26 | 15.22 |
| GSP    | 18.47 | 17.95 | 0.22 | 34.78 | 18.97 | 17.62 |
| GNG    | 793.92 | 794.80 | 789.35 | 787.89 | 15.23 | 15.23 |
| GTR    | 790.31 | 790.31 | 790.31 | 790.31 | 790.31 | 790.31 |
| W3P    | 10.28 | 14.29 | 17.22 | 34.78 | 790.31 | 0.26 |
| WPP    | 18.19 | 17.18 | 19.77 | 37.79 | 789.87 | 17.96 |

It follows from the data in Table 5 that the criterion $ed$ (upper diagonal half of the table) has regular discrepancies in local coordinates between the studied positioning methods from 0.2 to 15.3 mm and on the average is 4.6 mm. According to the $edm$ criterion, the maximum discrepancies range from 9.3 to 800 mm, with an average of 241 mm. Thus, transformation of ITRF coordinates to the local RF led to a decrease in the level of coordinate discrepancy between methods by Table 5 to 46%.

Noteworthy is the GNG calculation method, which improved its convergence rates with other methods by about 80% compared to ITRF coordinates. The procedure of recalculation of the source coordinates in the minimizing local RF resulted in a significant stabilization of the coordinates. This indicates that the absence of input information on ITRF and IGS stabilization points leads to significant errors in determining the ITRF coordinates in the GNG method. However, the relative positions between the stations are determined quite well. This is proved by the results of clustering in the local RF (Table 6), where only two clusters are distinguished.

### Table 6. Results of clustering of specific Euclidean distances and maximum discrepancies between methods in the local RF

| ed, mm | B3P-BRS(9.31)-W3P(12.29)-GSP(17.71)-W3P(31.80) |
|--------|-------------------------------------------------|
| edm, mm | GTR(789.13)                                      |
| B3P-W3P(0.18)-BRS(0.20) | GSP(17.71)-W3P(18.80)-GNG(31.80) |
| GSP-WPP(0.26)-GNG(0.29) | 15.23 |
| GTR    | 789.13                                          |

In this case, the final set of methods in the complex cluster by $ed$ and by $edm$ is identical, but the order of clustering is somewhat different. As for the ITRF coordinates, the GTR method stands alone here, with much worse local coordinate discrepancy rates.

### 6. Quality estimation of initial ITRF coordinates for each station

During the processing of GNSS observation data and the calculation of XYZ coordinates using different methods, it was noticed that different observation stations have different positioning errors. This may be due to the specific geometric configuration of the satellites orbits over a given point, limited visibility of the satellites (due to topography, buildings or trees, and signal refraction), unstable foundation or mast under the antenna, and other reasons. Using this circumstance, we have made an attempt to estimate the quality of the information provided by different GNSS stations through the
convergence of the positioning results obtained by the above seven methods of calculation of ITRF coordinates. The less regular discrepancy of coordinates at one station in different methods, the more qualitative the information provided by GNSS from this station should be.

To do this, we arrange the coordinates of each individual station for each method in a given order and obtain a 24-dimensional space (3 coordinates × 8 days), by analogy with Table 1. The result will be 10 such tables corresponding to each station. Then, in each table we calculate \( ED, \) \( ed, \) \( edm \) for pairs of methods by formulas (1-2). As a result, each station will correspond to 21 values of \( ed \) and \( edm \), the mean of which will give a characteristic for comparing stations with each other:

\[
\overline{ed} = \frac{1}{21} \sum_{i=1}^{21} ed_i, \quad \overline{edm} = \frac{1}{21} \sum_{i=1}^{21} edm_i
\]  

(5)

Thus, the meant information about the levels of discrepancy ITRF coordinates of different methods for one station will be presented in Table 7.

Table 7. Mean values of specific Euclidean distances and maximum discrepancies between methods for each station

|        | IATA | CHUM | KAZA | KRTV | POL2 | POL3 | POL7 | POLY | SUMK | TALA |
|--------|------|------|------|------|------|------|------|------|------|------|
| ed, mm | 17.21| 15.17| 16.54| 15.22| 14.61| 15.43| 36.72| 16.36| 17.24|      |
| edm, mm| 158.37| 146.65| 153.43| 151.70| 141.92| 399.32| 146.85| 280.11| 156.18| 161.10|

Table 7 shows that station POL2 has the lowest averaged by regular discrepancies in the coordinates of different methods <15 mm, with a maximum discrepancy of < 142 mm. Slightly worse \( \overline{ed} \) and \( \overline{edm} \), but the same level have 7 stations IATA, CHUM, KAZA, KRTV, POL7, SUMK and TALA. The worst indicators in the convergence of coordinates in different methods have POLY and POL3, indicating the worst quality of the signal they receive and the positioning corresponding to it.

7. Conclusion

For geodynamic researches of modern movements of the Earth’s crust based on GNSS measurements the positioning precision of the stations observed for many years is critically important. As a result of the analysis of the positions by 7 high-precision calculation methods, the convergence of their daily mean coordinates in the global and local reference systems is evaluated. On the basis of Euclidean distances the regular and maximum discrepancies between coordinates of different methods are given. A group of 5 methods (B3P, W3P, BRS, WRP, GSP) is distinguished in the framework of ITRF positioning. For them the regular discrepancies for individual coordinates are <1 mm, and individual maximum discrepancies are up to 30 mm. This can indicate the quality of ITRF coordinates obtained by these 5 methods. The GNG and GTR methods from GAMIT/GLOBK have more significant regular discrepancies up to 2 cm and up to 1 m in the maximum differences in coordinates.

Daily mean ITRF coordinates are calculated in conditions of daily corrections and significantly distant from us reference system, which leads to the phenomenon of excess of the level of ITRF coordinate increments for stations in different days over the level of line length changes between these stations in the same observation days. In this regard, the transformation of global coordinates for a group of stations into a local RF leads to the effect of stabilizing the coordinates and increasing their relative precision in the time series. As a result of this procedure, the level of maximum coordinate discrepancies between the methods decreased to 46%. In addition, the method for calculating GNG coordinates in the local RF improved its convergence to other methods by about 80%. This indicates a significant stabilization of the coordinates of all stations due to the procedure of transforming ITRF coordinates into local RF.
A useful addition to the analysis based on the Euclidean distances method was the estimation of the quality of raw data for each station. The less the regular discrepancies of coordinates for one station in different methods, the better the GNSS information from this station will be. Thus, a group of 8 stations was identified whose convergence indices are 2-3 times better than those of the other two stations.

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