Scalar mesons in radiative $\phi \to K^0\overline{K}^0\gamma$ decay

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Abstract

We study the radiative $\phi \to K^0\overline{K}^0\gamma$ decay within a phenomenological framework by considering the contributions of the $f_0(980)$ and $a_0(980)$ scalar resonances. We calculate the branching ratio $B(\phi \to K^0\overline{K}^0\gamma)$ by employing the coupling constants $g_{f_0K+K-}$ and $g_{a_0K+K-}$ as determined by different experimental groups.

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I. INTRODUCTION

The radiative decays of φ meson into a single photon and a pair of neutral pseudoscalar mesons are valuable sources of information in hadron physics. These days can provide insight into the structure and the properties of low-mass scalar resonances. In particular, the radiative decay $\phi \rightarrow K^0\overline{K}^0\gamma$ where the scalar resonances $f_0(980)$ and $a_0(980)$ are known to provide the dominant contribution to the amplitude allows for a direct measurement of the couplings of the $K\overline{K}$ system to $f_0$ and $a_0$ scalar resonances and thus it can yield information on the properties of these scalar resonances. Moreover, the study of the radiative decay process $\phi \rightarrow K^0\overline{K}^0\gamma$ is important because it provides a background to the reaction $\phi \rightarrow K^0\overline{K}^0$. This latter process was proposed as a way to study CP violation and measuring the ratio $e'/e$ [1]. Since this involves seeking for very small effects, if the branching ratio of the decay $\phi \rightarrow K^0\overline{K}^0\gamma$ is of the order of $10^{-6}$, or more precisely if $B(\phi \rightarrow K^0\overline{K}^0\gamma) \geq 10^{-6}$, then this background decay $\phi \rightarrow K^0\overline{K}^0\gamma$ will limit the scope of CP violation measurements in $\phi \rightarrow K^0\overline{K}^0$ decay at DAΦNE. Therefore, the study of the reaction $\phi \rightarrow K^0\overline{K}^0\gamma$ and the calculation of the branching ratio $B(\phi \rightarrow K^0\overline{K}^0\gamma)$ is crucial for the measurements of CP violation and small CP violating parameters in $\phi \rightarrow K^0\overline{K}^0$ decay.

The decay $\phi \rightarrow K^0\overline{K}^0\gamma$ was first considered by Achasov et al. [2] using the one-loop mechanism where the decay proceeds through the chain of reactions as $\phi \rightarrow K^+K^- \rightarrow (f_0 + a_0)\gamma \rightarrow K^0\overline{K}^0\gamma$. They noted the negative interference between the contributions of $f_0$ and $a_0$ resonances, and they obtained the value $BR(\phi \rightarrow (f_0 + a_0)\gamma \rightarrow K^0\overline{K}^0\gamma) = 1.3 \times 10^{-8}$ for the branching ratio of the $\phi \rightarrow K^0\overline{K}^0\gamma$ decay for some set of $f_0$ and $a_0$ masses and the values of the coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$.

The radiative $\phi$-meson decays, among other radiative decay processes of the type $V^0 \rightarrow P^0P^0\gamma$, where $V$ and $P$ belong to the lowest multiplets of vector (V) and pseudoscalar (P) mesons, were studied by Bramon et al. [3] in the framework of vector meson dominance (VMD) mechanism using standard Lagrangians obeying SU(3)-symmetry. In this framework, the radiative $\phi \rightarrow K^0\overline{K}^0\gamma$ reaction proceeds through the decay chains $\phi \rightarrow K^0V \rightarrow K^0\overline{K}^0\gamma$ where the intermediate vector mesons are $V = K^0$ and $V = \overline{K}^0$. They obtained the branching ratio for the decay as $B(\phi \rightarrow K^0\overline{K}^0\gamma) = 2.7 \times 10^{-12}$. Bramon et al. [4] later considered the radiative vector meson decays in Chiral Perturbation Theory (ChPT) using chiral effective Lagrangians enlarged to include on-shell vector mesons, and they calculated the branching ratios for various decays of type $V^0 \rightarrow P^0P^0\gamma$ at the one-loop level, including both $\pi\pi$ and $K\overline{K}$ intermediate loops. In this approach the decay $\phi \rightarrow K^0\overline{K}^0\gamma$ proceeds through charged-kaon loops, and they obtained the contribution of charged-kaon loops to the branching ratio as $B(\phi \rightarrow K^0\overline{K}^0\gamma) = 7.6 \times 10^{-9}$. They concluded that due to smallness of the VMD contribution, this decay is dominated by charged-kaon loops.

An additional contribution to the decay of the type $V^0 \rightarrow P^0P^0\gamma$ is provided by the amplitude involving scalar mesons as an intermediate state. These radiative decays were studied by Marko et al. [5] in the framework of unitarized chiral perturbation theory. They used the techniques of chiral unitary theory developed earlier to include the final state interactions of two pseudoscalars by summing the pseudoscalar loops through the Bethe-Salpeter equation. A review is given by Oller et al. [6]. In this approach the scalar resonances
are generated dynamically by utilizing the one-loop pseudoscalar amplitudes. The reaction $\phi \to K^0\bar{K}^0\gamma$ was studied by Oller [7] within this framework. The amplitude of this radiative decay has contributions coming from the scalar mesons $f_0(980)$ and $a_0(980)$, and Oller obtained for the branching ratio the value $B(\phi \to K^0\bar{K}^0\gamma) = 5 \times 10^{-8}$. The radiative vector meson decays $V^0 \to P^0 P^0\gamma$ were also studied using Linear Sigma Model (LσM), which is a $U(3) \times U(3)$ chiral model that incorporates the pseudoscalar and scalar meson nonets [8]. The decay $\phi \to K^0\bar{K}^0\gamma$ was studied within the framework of LσM by Escribano [9] who obtained the result $B(\phi \to K^0\bar{K}^0\gamma) = 7.5 \times 10^{-8}$.

In this work, we attempt to calculate the branching ratio $B(\phi \to K^0\bar{K}^0\gamma)$ using a phenomenological approach employed earlier in the studies of radiative $\phi \to \pi^0\pi^0\gamma$ [10] and $\phi \to \pi^0\eta\gamma$ [11] decays. In our calculation we use the kaon-loop model [2] where the initial vector meson $\phi$ decays into a pair of charged kaons, which after the emission of a photon couple to the neutral kaon pair $K^0\bar{K}^0$ through the scalar resonances $a_0$ and $f_0$. Moreover, we consider the vertices involving the scalar mesons as point like, therefore the effects of the structure are reflected in the coupling constants.

II. FORMALISM

The mechanism of the radiative decay process $\phi \to K^0\bar{K}^0\gamma$ in the kaon-loop model is provided by the reactions $\phi \to K^+K^-\gamma \to K^0\bar{K}^0\gamma$ where the last reaction proceeds by a two-step mechanism with the charged-kaon loop coupling to the final $K^0\bar{K}^0$ state through the scalar resonance $f_0(980)$ or $a_0(980)$. In Fig. 1 we show the processes contributing to the $\phi \to K^0\bar{K}^0\gamma$ amplitude diagramatically where the diagram in Fig. 1(c) results from the minimal coupling for gauge invariance. We do not make any assumption about the structure of the scalar mesons $S = f_0$ or $a_0$. We note that the $\phi$ meson and the scalar mesons $f_0$ and $a_0$ both couple strongly to the $K^+K^-$ system, we therefore describe the $\phi K^+K^-$ and $SK^+K^-$ vertices phenomenologically by effective Lagrangians. The $\phi K^+K^-$ vertex in the diagrams shown in Fig. 1 is described by the phenomenological Lagrangian

$$L_{\phi K^+K^-} = -ig_{\phi K^+K^-}\phi^\mu(K^+\partial_\mu K^- - K^-\partial_\mu K^+) \quad (1)$$

We utilize the experimental value for the branching ratio $B(\phi \to K^+K^-)$ [12] and determine the coupling constant $g_{\phi K^+K^-}$ as $g_{\phi K^+K^-} = (4.47 \pm 0.05)$. The $SK^+K^-$ vertex, where $S$ denotes the scalar meson $f_0$ or $a_0$, is described by the phenomenological Lagrangian

$$L_{SK^+K^-} = -g_{SK^+K^-}K^+K^- S \quad (2)$$

which is usually considered to define the coupling constant $g_{SK^+K^-}$ [13]. Similarly, we describe the $SK^0\bar{K}^0$ vertex by the effective Lagrangian

$$L_{SK^0\bar{K}^0} = -g_{SK^0\bar{K}^0}K^0\bar{K}^0 S \quad (3)$$

Furthermore, isotopic spin invariance implies that the coupling constants $g_{SK^+K^-}$ and $g_{SK^0\bar{K}^0}$ are related by the equations $g_{f_0K^+K^-} = g_{f_0K^0\bar{K}^0}$ and $g_{a_0K^+K^-} = -g_{a_0K^0\bar{K}^0}$ [2]. In our approach we assume isotopic spin invariance and use the coupling constants $g_{SK^+K^-}$ and $g_{SK^0\bar{K}^0}$ satisfying this requirement.
These coupling constants have been determined by theoretical calculations and from experimental analysis. In our phenomenological calculation we use the values of these coupling constants which are determined from the experimental studies of the radiative decay processes $\phi \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$.

We then obtain the amplitude for the radiative decay reaction $\phi \rightarrow K^0\overline{K}^0\gamma$ following from the diagrams shown in Fig. 1 as

$$
\mathcal{M}(\phi \rightarrow K^0\overline{K}^0\gamma) = -\frac{e g_{0KK}}{i2\pi \xi M_{K^0}^2} [(p \cdot k)(\epsilon \cdot u) - (p - \epsilon)(k \cdot u)] I(a, b)
\times \mathcal{M}(K^+K^- \rightarrow K^0\overline{K}^0)
$$

where $(u, p)$ and $(\epsilon, k)$ are the polarizations and four-momenta of the $\phi$ meson and the photon, respectively. The invariant function $I(a, b)$ has been calculated in different contexts [13,14], and it is given by

$$
I(a, b) = \frac{1}{2(a - b)} - \frac{2}{(a - b)^2} \left[ f \left( \frac{1}{b} \right) - f \left( \frac{1}{a} \right) \right] + \frac{a}{(a - b)^2} \left[ g \left( \frac{1}{b} \right) - g \left( \frac{1}{a} \right) \right]
$$

where $a = M_{\phi}/M_{K^+}$ and $b = M_{K^0}/M_{K^+}$ with $M_{KK}^2$ being the invariant mass of the final $K^0\overline{K}^0$ system given by $M_{K^0\overline{K}^0}^2 = (p - k)^2 = q^2$. The amplitude $\mathcal{M}(K^+K^- \rightarrow K^0\overline{K}^0)$ contains the scalar $f_0$ and $a_0$ resonances and in the approach we adopted it is given by

$$
\mathcal{M}(K^+K^- \rightarrow K^0\overline{K}^0) = -ig_{SKK^+} - g_{SK\overline{K}^0} \frac{1}{q^2 - M_S^2}.
$$

Since the scalar resonances $f_0$ and $a_0$ are unstable and they have a finite lifetime we use Breit-Wigner propagators with an energy dependent width for these resonances. We therefore in the scalar meson propagator make the replacement $q^2 - M_S^2 \rightarrow q^2 - M_S^2 + i\sqrt{q^2}\Gamma_S$ where

$$
\Gamma_{f_0}(q^2) = \frac{g_{0KK}^2}{16\pi \sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^+}}{q^2}} \theta(\sqrt{q^2} - 2M_{K^+})
+ \frac{g_{a_0KK^0}^2}{16\pi \sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^0}}{q^2}} \theta(\sqrt{q^2} - 2M_{K^0})
+ \frac{2}{3}\frac{M_{a_0}}{\sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^0}}{M_{a_0}^2}} \theta(\sqrt{q^2} - 2M_{a_0}),
$$

$$
\Gamma_{a_0}(q^2) = \frac{g_{0KK}^2}{16\pi \sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^+}}{q^2}} \theta(\sqrt{q^2} - 2M_{K^+})
+ \frac{g_{a_0KK^0}^2}{16\pi \sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^0}}{q^2}} \theta(\sqrt{q^2} - 2M_{K^0})
+ \frac{2}{3}\frac{M_{a_0}}{\sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^0}}{M_{a_0}^2}} \theta(\sqrt{q^2} - 2M_{a_0}).
$$
and
\[
\Gamma_{ao}(q^2) = \frac{g_{aoK^+K^0}^2}{16\pi\sqrt{q^2}} \sqrt{1 - \frac{4M_{K^+}^2}{q^2}} \theta(\sqrt{q^2} - 2M_{K^+}) + \frac{g_{aoK^0K^0}^2}{16\pi\sqrt{q^2}} \sqrt{1 - \frac{4M_{K^0}^2}{q^2}} \theta(\sqrt{q^2} - 2M_{K^0}) + \Gamma_{ao} M_{ao} \sqrt{q^2} \sqrt{1 - \frac{(M_{ao} + M_{\eta})^2}{q^2}} \left[ 1 - \frac{(M_{ao} - M_{\eta})^2}{M_{ao}^2} \right] \theta(\sqrt{q^2} - (M_{\eta}^2 + M_{\eta}^2)) ,
\]

and we use the experimental values for the widths \(\Gamma_{f_0}\) and \(\Gamma_{ao}\) [12] in the above expressions. Then the differential decay probability for the radiative decay \(\phi \to K^0\bar{K}^0\gamma\) for an unpolarized \(\phi\) meson at rest is given as
\[
\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M_\phi} |\mathcal{M}|^2 ,
\]
where \(E_\gamma\) and \(E_1\) are the photon and \(K^0\) meson energies respectively. We perform an average over the spin states of \(\phi\) meson and a sum over the polarization states of the photon. The decay width \(\phi \to K^0\bar{K}^0\gamma\) is then obtained by integration
\[
\Gamma = \int_{E_\gamma,min.}^{E_\gamma,max.} dE_\gamma \int_{E_1,min.}^{E_1,max.} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} ,
\]
where the minimum photon energy is \(E_{\gamma,min.} = 0\) and the maximum photon energy is given as \(E_{\gamma,max.} = (M_\phi^2 - 4M_{K^0}^2)/2M_\phi\). The maximum and minimum values for the energy \(E_1\) of \(K^0\) meson are given by
\[
\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \left\{ -2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 \pm E_\gamma \sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_{K^0}^2)} \right\} .
\]

III. RESULTS AND DISCUSSION

The branching ratios \(BR(\phi \to f_0(980)\gamma)\) and \(BR(\phi \to a_0(980)\gamma)\) are listed as \(BR(\phi \to f_0(980)\gamma) = (4.40 \pm 0.21) \times 10^{-4}\) and \(BR(\phi \to a_0(980)\gamma) = (7.6 \pm 0.6) \times 10^{-5}\) in the Particle Physics Data Tables [12]. Achasov [15] showed in detail that the existing data on radiative \(\phi\) meson decays support the charged-kaon loop mechanism for \(BR(\phi \to f_0(980)\gamma)\) and \(BR(\phi \to a_0(980)\gamma)\) radiative decay reactions. From the above branching ratios within the framework of charged-kaon loop mechanism for these decays we determine the coupling constants \(g_{SK^+K^-}\) as \(g_{f_0K^+K^-} = (5.14 \pm 0.12)\) GeV and \(g_{a_0K^+K^-} = (2.26 \pm 0.08)\) GeV. Using these values for the coupling constants and the values of the relevant masses taken from the Particle Physics Data Tables [12], if we include the contribution of \(f_0\) resonance only in the
decay mechanism of the radiative decay $\phi \to K^0\bar{K}^0\gamma$, we obtain the result for the branching ratio as $BR(\phi \to f_0\gamma \to K^0\bar{K}^0\gamma) = 2.25 \times 10^{-7}$. On the other hand, if the contribution of $a_0$ resonance is considered only the branching ratio is $BR(\phi \to a_0\gamma \to K^0\bar{K}^0\gamma) = 2.64 \times 10^{-8}$.

Since both $f_0$ and $a_0$ resonances make a contribution to the decay $\phi \to K^0\bar{K}^0\gamma$, when considering their contribution to the decay rate we have to note that the amplitudes involving $f_0$ and $a_0$ resonances interfere destructively due to isotopic spin invariance as reflected in the relations between the coupling constants $g_{f_0K^+K^-} = g_{f_0K^0\bar{K}^0}$ and $g_{a_0K^+K^-} = -g_{a_0K^0\bar{K}^0}$. Thus, if we consider that the interference between the contributions of $f_0$ and $a_0$ resonances is destructive, we obtain for the branching ratio the value $BR(\phi \to (a_0 + f_0)\gamma \to K^0\bar{K}^0\gamma) = 9.85 \times 10^{-8}$, which is somewhat higher than the previous estimations. However, it is small enough, we therefore conclude that this reaction will not provide a significant background to the measurements of $\phi \to K^0\bar{K}^0$ decay for testing CP violation.

In Fig. 2, we plot the distribution $dBR/dM_{KK}$ for the radiative decay $\phi \to K^0\bar{K}^0\gamma$ in the phenomenological approach that we adopted, where we also indicate the contributions coming from $f_0$ resonance, $a_0$ resonances, and the contribution resulting from the destructive interference of these mechanisms for the coupling constants used above. Furthermore, in Fig. 3, we show the photon spectrum $d\Gamma/dE_\gamma$ for the process $\phi \to K^0\bar{K}^0\gamma$ for the same amplitudes as in Fig. 2.

We note, however, that our result for the branching ratio $B(\phi \to K^0\bar{K}^0\gamma)$ depends sensitively on the values of the coupling constants $g_{SK^0\bar{K}^0}$ and the masses $M_S$ of the scalar resonances $f_0$ and $a_0$. We, therefore, repeat our calculation using the values of the coupling constants $g_{SK^+K^-}$ and the masses $M_S$ determined by different experimental groups by performing fits to their data of the radiative $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\eta\gamma$ decays. SND Collaboration [16,17] obtained the values $M_{a_0} = (985.51 \pm 0.8)$ MeV, $g_{a_0K^+K^-}/4\pi = (0.6 \pm 0.015)$ GeV$^2$, $M_{f_0} = (996 \pm 1.3)$ MeV, $g_{f_0K^+K^-}/4\pi = (1.29 \pm 0.017)$ GeV$^2$ from their fit to SND data considering $f_0$ and $\sigma$ mixing. If we use these values, we obtain the results $BR(\phi \to f_0\gamma \to K^0\bar{K}^0\gamma) = 1.79 \times 10^{-7}$ and $BR(\phi \to a_0\gamma \to K^0\bar{K}^0\gamma) = 5.02 \times 10^{-8}$ and $BR(\phi \to (f_0 + a_0)\gamma \to K^0\bar{K}^0\gamma) = 4.39 \times 10^{-8}$. They also performed a fit to their data without $f_0$ and $\sigma$ mixing in which case they obtained $M_{a_0} = 994_{-33}^{+33}$ MeV, $g_{a_0K^+K^-}/4\pi = 1.05_{-0.25}^{+0.36}$ GeV$^2$, $M_{f_0} = 969.8 \pm 4.5$ MeV and $g_{f_0K^+K^-}/4\pi = 2.47_{-0.51}^{+0.73}$ GeV$^2$. Using these values in our calculation we obtain the branching ratios $BR(\phi \to f_0\gamma \to K^0\bar{K}^0\gamma) = 2.60 \times 10^{-7}$, $BR(\phi \to a_0\gamma \to K^0\bar{K}^0\gamma) = 1.03 \times 10^{-7}$ and $BR(\phi \to (f_0 + a_0)\gamma \to K^0\bar{K}^0\gamma) = 4.47 \times 10^{-8}$.

Achasov et al. [18] also calculated the branching ratio for the decay $\phi \to K^0\bar{K}^0\gamma$ using the above sets of parameters describing the SND data [16,17], and for the first set they obtained $BR(\phi \to (f_0 + a_0)\gamma \to K^0\bar{K}^0\gamma) = 4.36 \times 10^{-8}$ while for the second set their result was $BR(\phi \to (f_0 + a_0)\gamma \to K^0\bar{K}^0\gamma) = 1.29 \times 10^{-8}$.

KLOE collaboration at the DAΦNE collider also studied the decays $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\eta\gamma$ [19,20]. They performed two different fits in order to measure the parameters of the scalar states. In the first fit, they included the contribution of a possible broad scalar $\sigma$ state as well as the intermediate $f_0(980)$ state interfering destructively in the analysis of the data of the $\phi \to \pi^0\pi^0\gamma$ reaction. In the second fit only the contribution of the intermediate $f_0(980)$ state was considered. In the first case they obtained $M_{f_0} = (973 \pm 1)$ MeV, $g_{f_0K^+K^-}/4\pi = (2.79 \pm 0.12)$ GeV$^2$ and in the second case $M_{f_0} = (962 \pm 4)$ MeV,
\[ g_{f_0 K^+ K^-}^2/4\pi = (1.29 \pm 0.14) \text{ GeV}^2. \] They also reported the values \( M_{a_0} = 984.8 \text{ MeV}, \) \( g_{a_0 K^+ K^-}^2/4\pi = (0.40 \pm 0.04) \text{ GeV}^2. \) If we use the values of the coupling constants \( g_{S' K^0\bar{K}^0} \) and the masses \( M_S \) that the KLOE Collaboration obtained in the first fit, we obtain the results \( BR(\phi \to f_0 \gamma \to K^0\bar{K}^0 \gamma) = 2.59 \times 10^{-7} \) and \( BR(\phi \to a_0 \gamma \to K^0\bar{K}^0 \gamma) = 2.56 \times 10^{-8} \) and \( BR(\phi \to (f_0 + a_0) \gamma \to K^0\bar{K}^0 \gamma) = 1.35 \times 10^{-7}. \) On the other hand, using the values they obtained in their second fit without considering the contribution of intermediate \( \sigma \) state in our calculation results in the branching ratios \( BR(\phi \to f_0 \gamma \to K^0\bar{K}^0 \gamma) = 1.32 \times 10^{-7} \) and \( BR(\phi \to a_0 \gamma \to K^0\bar{K}^0 \gamma) = 2.56 \times 10^{-8} \) and \( BR(\phi \to (f_0 + a_0) \gamma \to K^0\bar{K}^0 \gamma) = 4.50 \times 10^{-8}. \) However, it was pointed out by Achasov et al. \cite{21,22} that the KLOE data also allow \( g_{a_0 K^+ K^-}^2/4\pi = (0.82^{+0.81}_{-0.27}) \text{ GeV}^2 \) \cite{21} and \( g_{f_0 K^+ K^-}^2/4\pi = 0.62 \text{ GeV}^2 \) \cite{22}. If we employ these values of the coupling constants with the values of the masses \( M_{a_0} = (1003^{+32}_{-13}) \text{ MeV}, \) \( M_{f_0} = 984.2 \text{ MeV} \) and in these analyses \cite{21,22} in our calculation we obtained the results \( BR(\phi \to f_0 \gamma \to K^0\bar{K}^0 \gamma) = 7.69 \times 10^{-8}, \) \( BR(\phi \to a_0 \gamma \to K^0\bar{K}^0 \gamma) = 7.25 \times 10^{-8}, \) and \( BR(\phi \to (f_0 + a_0) \gamma \to K^0\bar{K}^0 \gamma) = 1.25 \times 10^{-8}. \)

We can, therefore, conclude that within the formalism we consider in this work the branching ratio \( BR(\phi \to K^0\bar{K}^0 \gamma) \) of the radiative decay \( \phi \to K^0\bar{K}^0 \gamma \) is small enough so that this decay will not cause any serious background problem for the studies of CP violation in the \( \phi \to K^0\bar{K}^0 \) decay. Other effects such as structure \cite{23,24} and the finite widths of scalars \cite{24} on the radiative \( \phi \) decays have also been investigated. Oller \cite{24} showed that the inclusion of these contribution does not change the conclusions of the charged-kaon loop model.
FIG. 1. Diagrams for the decay $\phi \rightarrow K^0\bar{K}^0\gamma$ where S denotes the scalar meson resonance $f_0$ or $a_0$.

FIG. 2. The distribution $d\text{BR}/dM_{KK}$ for the radiative decay $\phi \rightarrow K^0\bar{K}^0\gamma$.
FIG. 3. The photon spectrum $d\Gamma/dE_\gamma$ for the process $\phi \to K^0\overline{K}^0\gamma$
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