Entropy spectrum of charged BTZ black holes in massive gravity’s rainbow

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Regarding the significant interests in massive gravity’s rainbow and also BTZ black holes, we apply the formalism introduced by Jiang and Han in order to investigate the quantization of the entropy of black holes. We show that the entropy of BTZ black holes in massive gravity’s rainbow is quantized with equally spaced spectra and it depends on the value of the parameters of this black hole such as; massive parameters, electrical charge, the cosmological constant and also rainbow functions.

I. INTRODUCTION

General relativity (GR) is a successful theory of gravity, however there are some evidences which show this theory cannot explain them. For example: accelerated expansion of the Universe and also massive gravitons. For this purpose, GR was modified. There are some modified theories such as: F(R) gravity [1–6], Lovelock gravity [7–11], Horava-Lifshitz gravity [12,13], brane world cosmology [14,16], scalar-tensor theories [17–25], gravity’s rainbow [26–32] and also massive gravity [33–49].

In order to understand the UV behavior of GR, various attempts have been made to obtain the UV completion of GR such that it reduces to GR in the IR limit. The first attempt in this field is related to Horava-Lifshitz gravity [12,13]. In this gravity, space and time are made to have different Lifshitz scaling and it reduces to GR in the IR limit. However, behavior of Horava-Lifshitz gravity in the UV limit is different from that of GR. It is notable that, Horava-Lifshitz gravity is based on a deformation of the usual energy-momentum dispersion relation in the UV limit, in which it reduces to the usual energy-momentum dispersion relation in the IR limit. Another approach for extracting the UV completion of GR is called gravity’s rainbow [26], which is based on the deformation of the usual energy-momentum dispersion relation in the UV limit, and it reduces to GR in the IR limit. In this theory, the geometry of spacetime is made energy dependent and this energy dependence of the spacetime metric is incorporated through the introduction of rainbow functions. The standard energy-momentum relation in gravity’s rainbow is given as

\[ E^2 f^2(E/E_P) - p^2 g^2(E/E_P) = m^2, \]

in which \( E \) and \( E_P \) are the energy of test particle and the Planck energy, respectively. For the sake the simplicity, we will use \( \varepsilon = E/E_P \). Also, \( f(\varepsilon) \) and \( g(\varepsilon) \) are energy functions which are restricted as \( \lim_{\varepsilon \to 0} f(\varepsilon) = 1 \) and \( \lim_{\varepsilon \to 0} g(\varepsilon) = 1 \), in the IR limit and could be used to build energy dependent spacetime metric with following recepie

\[ \tilde{g}(\varepsilon) = \eta^{ab} e_a(\varepsilon) \otimes e_b(\varepsilon), \]

where

\[ e_0(\varepsilon) = \frac{1}{f(\varepsilon)} \tilde{e}_0, \quad e_i(\varepsilon) = \frac{1}{g(\varepsilon)} \tilde{e}_i, \]

with \( \tilde{e}_0 \) and \( \tilde{e}_i \) are related to the energy independent frame fields. It is notable that \( E \) cannot exceed \( E_P \), so, \( 0 < \varepsilon \leq 1 \). In other words, the gravity’s Rainbow produces a correction to the spacetime metric that becomes significant when the particle’s energy approaches the Planck energy. In the gravity’s rainbow context and by combining various gravities the black hole and cosmological solutions have been studied in some literatures. For example, \( F(R) \) gravity’s rainbow [50,51], Gauss-Bonnet gravity’s rainbow [52,53], dilatonic gravity’s rainbow [54,55], Galileon gravity’s rainbow [56] and Lovelock gravity’s rainbow [57]. Also, in Refs , showed that by considering a special limit on rainbow functions, we encounter with nonsingular universes in Einstein and Gauss-Bonnet gravity’s rainbow [58,59]. Remnant for all

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black objects and also absence of black holes at LHC due to gravity’s rainbow have been investigated in refs. \[60–62\]. Modified TOV in gravity’s rainbow and investigation the properties of neutron stars and dynamical stability conditions have been perused \[63\].

On the other hand and in order to have massive gravitons, GR must be modified, because the gravitons are massless particles in GR. Therefore, Fierz and Pauli were the first to study the theory describing the massive gravitons \[64,65\]. Later, Boulware and Deser has been found out that this theory of massive gravity suffers a ghost instability at the non-linear level \[66,67\].

Recently, there has been a great interest in modification of the GR on nonlinear level to include massive gravitons. Among the studies done in this regard, one can point out Refs. \[68–70\] in which a stable nonlinear massive gravity \[71,72\] was employed to conduct the investigations.

The first three dimensional black hole solutions in the presence of the cosmological constant were introduced by Bañados, Teitelboim, and Zanelli which are known as BTZ black holes \[73\]. Later, it was shown that these solutions have central roles in understanding several issues such as black hole thermodynamics \[74,76\], quantum gravity, string, gauge theory, the AdS/CFT conjecture \[77,78\] and investigation of gravitational interaction in low dimensional spacetime \[79\]. The charged BTZ black hole is the correspondence solution of adS-Maxwell gravity in three dimensions \[74,80,81\]. Recently, charged BTZ black holes with two generalizations of the massive gravity and gravity’s rainbow have been investigated \[82–84\].

Hawking radiation of black holes improved our knowledge toward quantum theory of gravity. Then Bekenstein showed that there is a lower bound for event horizon area of black hole as \[85\]

\[
(\Delta A)_{\text{min}} = 8\pi l_p^2, \tag{4}
\]

in which \(l_p\) is the Planck length. It is notable that this lower bound independ of parameters of black holes. On the other hand, Quasinormal mode (QNM) frequencies are known as the characteristic sound of black hole. Therefore we used of this viewpoint and consider an adiabatic invariant quantity as (or action of the oscillating horizon)

\[I_{\text{adiabatic}} = \int \frac{dE}{\Delta \omega(E)}, \tag{5}\]

where \(\Delta \omega = \omega_{n+1} - \omega_n\), \(E\) and \(\omega\) are the energy and frequency of QNM, respectively. Later Hod and Kunstatter calculated the area spectrum by considering the real part of QNM frequency. Next, Maggiore \[89\] refined Hod’s idea by proving that the physical frequency of QNM is determined by its real and imaginary parts. A new method proposed by Majhi and Vagenas in order to quantize the entropy without using QNM. They used the idea of related to an adiabatic invariant quantity to Hamiltonian of the black hole and extracted an equally spaced entropy spectrum with its quantum to be equal to the results obtained by Bekenstein \[90\]. In the tunneling picture, we can consider horizon of black hole as an oscillate periodically when the particle tunnels in or out. Therefore we used of this viewpoint and consider an adiabatic invariant quantity as (or action of the oscillating horizon)

\[I = \int p_i dq_i, \tag{6}\]

in which \(p_i\) is the corresponding conjugate momentum of the coordinate of \(q_i\) \((i = 0,1\) where \(q_0 = \tau\) and \(q_1 = r_h\), in which \(\tau\) and \(r_h\) are related to the Euclidean time and the horizon radius, respectively). By using the Hamilton’s equation \((\dot{q}_i = \frac{dH}{dp_i})\), one can rewrite the equation \[89\] as

\[I = \int \int_0^H dH d\tau + \int \int_0^H \frac{dH}{r_h r_h} dr_h = 2 \int \int_0^H \frac{dH}{r_h} dr_h, \tag{7}\]

where \(H\) is the Hamiltonian of system and \(r_h = \frac{dr}{d\tau}\). Now we want to calculate the above adiabatic invariant quantity, so we consider a static metric in gravity’s rainbow as

\[ds^2 = -\frac{\psi(r,\varepsilon)}{f^2(\varepsilon)} dt^2 + \frac{1}{g^2(\varepsilon)} \left[ \frac{dr^2}{\psi(r,\varepsilon)} + r^2 d\varphi^2 \right]. \tag{8}\]
It is notable that, we can obtain \( r_h \) by using \( \psi (r_h) = 0 \). Finding the oscillating velocity of black hole horizon, we can calculate the equation (7). In the tunneling picture, when a particle tunnels in or out, horizon of black hole will expand or shrink due to gain and loss the mass of black hole. Since the tunneling and oscillation happen simultaneously, therefore the tunneling velocity of particle is equal and opposite to the oscillating velocity of black hole horizon \( (\dot{r}_h = -\dot{r}) \). Also, we have to Euclideanize the introduced metric (8) by using the transformation \( t \rightarrow -i \tau \). So we have

\[
\text{d}s^2 = \frac{\psi (r, \varepsilon)}{f^2(\varepsilon)} \text{d}t^2 + \frac{1}{g^2(\varepsilon)} \left[ \frac{dr^2}{\psi (r, \varepsilon)} + r^2 d\varphi^2 \right].
\]

It is notable that, when a photon travels across the horizon of black hole, the radial null path (or radial null geodesic) is given by

\[
\text{d}s^2 = d\varphi^2 = 0 \rightarrow \dot{r} = \pm i \left( \frac{g(\varepsilon) \psi (r, \varepsilon)}{f(\varepsilon)} \right),
\]

in which the negative sign denotes the incoming radial null paths and also the positive sign represents the outgoing radial null paths. It is notable that, we consider the outgoing paths (the positive sign of Eq. (10)) in order to calculation of area spectrum, because these paths are more related to the quantum behaviors under consideration.

So, the shrinking velocity of the black hole horizon is as

\[
\dot{r}_h = -\dot{r} = -i \left( \frac{g(\varepsilon) \psi (r, \varepsilon)}{f(\varepsilon)} \right).
\]

Using the above equation and Eq. (7) we have

\[
I = 2 \int_0^H \frac{dH}{r_h}dr_h = -2i \left[ \int_0^H \frac{\text{d}H}{g(\varepsilon)\psi (r, \varepsilon)} \right] dr.
\]

In order to solve this adiabatic invariant quantity (Eq. (12)), we use definition of Hawking’s temperature with relation to the surface gravity on the outer horizon \( (r_+) \) as \( T_{bh} = \frac{\kappa}{2\pi} \), in which \( \kappa \) is surface gravity.

Inasmuch as the area spectrum and also the entropy spectrum spacing change with respect to the change in \( r_h \),

\[
\text{II. ENTROPY SPECTRUM OF BTZ BLACK HOLES IN MASSIVE GRAVITY’S RAINBOW}
\]

BTZ black holes in massive gravity’s rainbow by considering the metric of 3-dimensional spacetime with the following line element

\[
\text{d}s^2 = -\frac{\psi (r, \varepsilon)}{f(\varepsilon)^2} \text{d}t^2 + \frac{1}{g(\varepsilon)^2} \left( \frac{dr^2}{\psi (r, \varepsilon)} + r^2 d\varphi^2 \right),
\]

in which \( \psi (r, \varepsilon) \) is the metric function of our black holes and the functions \( f(\varepsilon) \) and \( g(\varepsilon) \) are rainbow functions have been obtained in ref. [101]. The metric function in this gravity is \[101\]

\[
\psi (r, \varepsilon) = -\frac{\Lambda(\varepsilon) r^2}{g(\varepsilon)^2} - m_0(\varepsilon) - 2G(\varepsilon) f(\varepsilon)^2 q(\varepsilon)^2 \ln \left( \frac{r}{l(\varepsilon)} \right) + \frac{m(\varepsilon)^2 c(\varepsilon)c_1(\varepsilon)r}{g(\varepsilon)^2},
\]

where

- \( \Lambda(\varepsilon) \) is the cosmological constant parameter, \( m_0(\varepsilon) \) is the mass parameter, \( 2G(\varepsilon) \) is the gravitational constant parameter, \( q(\varepsilon)^2 \) is the area spectrum parameter, \( l(\varepsilon) \) is the scale parameter, and \( c(\varepsilon) \) and \( c_1(\varepsilon) \) are rainbow functions parameter.
where $m_0(\varepsilon)$ is an energy dependant integration constant related to the total mass of the BTZ black holes. The electric potential ($U$) and the electric charge ($Q$) are calculated as follows \[101\]

\[
U(\varepsilon) = -q(\varepsilon) \ln \left( \frac{r_+}{l(\varepsilon)} \right).
\]

\[
Q(\varepsilon) = \frac{1}{2} f(\varepsilon) G(\varepsilon) q(\varepsilon).
\]

Using the standard definition of the Hawking temperature ($T = \frac{\hbar \kappa}{2\pi}$), the surface gravity is obtained by considering the metric (14) as \[101\]

\[
\kappa = \frac{1}{2\pi} \sqrt{-\frac{1}{2} (\nabla_{\mu} \chi^\nu) (\nabla^\mu \chi^\nu) - \frac{1}{2} \left( \frac{g(\varepsilon) \psi'(r, \varepsilon)}{f(\varepsilon)} \right)}.
\]

Therefore, the Hawking’s temperature and the electric potential of these black holes are \[101\]

\[
T = \frac{\hbar \kappa}{2\pi} = \frac{\hbar}{4\pi} \left( \frac{g(\varepsilon) \psi'(r, \varepsilon)}{f(\varepsilon)} \right),
\]

\[
U(\varepsilon) = -\frac{2Q(\varepsilon)}{f(\varepsilon) G(\varepsilon)} \ln \left( \frac{r_+}{l(\varepsilon)} \right).
\]

The entropy of black holes can be obtained by employing the area law as \[101\]

\[
S = \frac{\pi}{2g(\varepsilon)} r_+.
\]

The total mass of these solutions obtained as \[101\]

\[
M = \frac{m_0(\varepsilon)}{8 f(\varepsilon)}.
\]

Here, we want to quantize the entropy of this black hole by using the adiabatic invariant quantity and Bohr-Sommerfeld quantization rule. Considering the equations (12) and (13) we have

\[
I = \oint p_i dq_i = -4i \int_{r_{in}}^{r_{out}} \int_0^H \frac{dH}{\psi(r)} \frac{dH}{g(\varepsilon)} \times \frac{f(\varepsilon)}{g(\varepsilon)}.
\]

In order to solve the above equation, we use of the near horizon approximation, so $\psi(r)$ can be Taylor expanded in the following form

\[
\psi(r, \varepsilon) = \psi(r, \varepsilon)_{r=r_+} + (r - r_+) \psi'(r, \varepsilon)_{r=r_+} + \ldots.
\]

The first term is zero ($\psi(r, \varepsilon)_{r=r_+} = 0$). Also, by using the Cauchy integral theorem, the equation (23) and Eq. (19) reduce to

\[
I = \oint p_i dq_i = 4\pi \int_0^H \frac{dH}{\kappa} = 2\hbar \int_0^H \frac{dH}{T}.
\]

The Smarr-formula for BTZ black hole in massive gravity’s rainbow is

\[
dM = dH = TdS - UdQ.
\]

Therefore, the equation (25) become

\[
\oint p_i dq_i = 2\hbar S \left[ 1 + \frac{U(\varepsilon) f(\varepsilon) G(\varepsilon)}{2Q(\varepsilon)} \ln \left( G(\varepsilon) \left[ 2\Lambda(\varepsilon) r - m^2(\varepsilon) c(\varepsilon) c_1(\varepsilon) \right] r + 8Q^2(\varepsilon) g^2(\varepsilon) \right) \right].
\]

On the other hand, the Bohr-Sommerfeld quantization rule is given by

\[
\oint p_i dq_i = 2\pi n\hbar, \quad n = 1, 2, 3, \ldots.
\]
Comparing the equations (27) and (28), one can obtain the entropy spectrum as

\[
S = \pi n \frac{U(\varepsilon)}{2\Omega(\varepsilon)} \ln \left\{ G(\varepsilon) \left[ 2\Lambda(\varepsilon) r - m^2(\varepsilon) c(\varepsilon) c_1(\varepsilon) \right] r + 8Q^2(\varepsilon) g^2(\varepsilon) \right\}.
\]

(29)

So, the BTZ black hole in massive gravity’s rainbow is quantized. As one can see, the entropy spectrum depend on the cosmological constant, the electrical charge, the massive parameter and the rainbow functions. In other words, the entropy spectrum depends on the value of BTZ black hole in massive gravity’s rainbow.

III. CONCLUSIONS

In this paper, we have considered the BTZ black holes in the presence of massive gravity’s rainbow. We have studied the quantization of the entropy of these black hole by using an adiabatic invariant integral method put forward by Majhi and Vagenas with modification from proposed by Jiang and Han, and also the Bohr-Sommerfeld quantization rule. Next, we have showed that the entropy of BTZ black hole in massive gravity’s rainbow is quantized. In addition, we observed that, the entropy spectrum of this black hole depends on the parameter of massive gravity, rainbow functions, the cosmological constant, the Newton’s gravitational constant and also the electrical charge. In other words, the existence massive and gravity’s rainbow have been affect on the entropy spectrum of BTZ black hole in this gravity.

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