Kinematic analysis and visual grasping algorithm of collaborative robot

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Abstract. This paper mainly studies the kinematics analysis and visual capture algorithm of collaborative robots. First, the mechanical structure of the cooperative robot was analyzed. Then, the kinematics equation of the cooperative robot was solved by numerical iterative method. The correctness and rapidity of the algorithm were verified by simulation. Finally, a visual compensation method based on image edge contour features is proposed. The multi-target area processing was used to realize the positioning of the workpiece's three-dimensional spatial information, and the collaborative robot was guided to capture the operation to ensure the positioning accuracy.

1. Introduction

Universal Robot is a six-joint collaborative robot. Each joint can be rotated 360 degrees, with force/torque control and feedback function, which can be used without protective fences, and its reliability and performance are superior to other industrial robots [1]. The research on Kinematic analysis and visual grasping algorithm of collaborative robot is the prerequisite for the practical use of collaborative robot. The literature [2] improved the algorithm of the data least squares (DLS) by the adjustment of the processing and damping factors of the singular position, and realized the real-time kinematics solution. The literature [3] has adopted the method of reducing the system dimension by eliminating the kinematics equation and transforming the inverse kinematics solution into a sixteen equation, and realized the real-time kinematics solution. Literature [4] indicates that the motion control and dynamics algorithm of mobile robot are studied deeply. The control law based on the lag switch is designed in the case of absolute position. Measure the distance and Angle of the obstacle. The formal properties of the closed loop system with shared control are established through the Lyapunov sample analysis. The simulation results and experimental results are presented to show the effectiveness of the algorithm.

2. Kinematic analysis of collaborative robot

2.1. D-H parameters of collaborative robot

The D-H parameters method was a common method used by Denavit and Hartenberg in 1955 to describe robots. Which consider the robot as a series of connecting links, connect the two adjacent connecting links with a rotate or translate joint, establish a coordinate system on the connecting links, and use the homogeneous transformation matrix to represent relative pose between two adjacent connecting links. The homogeneous transformation matrix of the end tool relative to the pedestal
coordinate system is obtained recursively, and the kinematic equation of the robot is obtained [5]. Craig proposed an improved D-H parameter that secures the coordinate system axis and origin of each link to the previous axis of the link in 1986 [6]. According to the chain rule of coordinate system transformation, the transformation matrix of the coordinate system \( \{i-1\} \) to coordinate system \( \{i\} \) is:

\[
{^iT} = \text{Rot}(x,a_i)\text{Trans}(a_i,0,0)\text{Rot}(z,\theta_i)\text{Trans}(0,0,d_i)
\]  

(1)

The transformation formula \( {^iT} \) obtained by multiplying the matrix is:

\[
{^iT} =
\begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & a_{i,1} \\
\sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -d \sin \alpha_{i-1} \\
\sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & d \cos \alpha_{i-1} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(2)

The relationship \( ^iT \) between the tool frame and base frame is:

\[
^iT = ^{T'}T'\cdots^{T_1'}T_1
\]  

(3)

2.2. Forward kinematics of collaborative robot

The kinematics model of the 6-degree-of-freedom collaborative robot is to establish the positional mapping relationship between the joints of the base coordinate system and the end effector, as shown in Figure 1. The link D-H parameters of collaborative robot are shown in Table 1. The kinematics model of the cooperative robot can be easily constructed according to the Modified D-H parameter.

![D-H coordinate system of collaborative robot](image)

**Figure 1.** D-H coordinate system of collaborative robot.

**Table 1.** Modified D-H parameters of collaborative robot.

| Link | \( \alpha_{i,1} \) | \( a_{i,1} \) | \( d_i \) | \( \theta_i \) |
|------|------------------|--------------|--------|--------|
| 1    | 0°               | 0            | \( d_i \) | 0°     |
| 2    | 90°              | 0            | 0      | 0°     |
| 3    | 0°               | \( a_2 \)    | 0      | 0°     |
| 4    | 0°               | \( a_4 \)    | \( d_4 \) | 0°     |
| 5    | 90°              | 0            | \( d_5 \) | 0°     |
| 6    | -90°             | 0            | \( d_6 \) | 0°     |

in which  \( a_2 = -612 \text{mm} \),  \( a_3 = -572.3 \text{mm} \),  \( d_4 = 127.3 \text{mm} \),  \( d_1 = 163.941 \text{mm} \),  \( d_5 = 115.7 \text{mm} \),  \( d_6 = 92.2 \text{mm} \).

According to the Formula (2) and the D-H parameters in Table 1, the transformation matrix of each link of the cooperative robot can be obtained as shown Formula (4).
\[ \mathbf{T} = \begin{bmatrix} c_{\theta_1} & -s_{\theta_1} & 0 & 0 \\ s_{\theta_1} & c_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

Multiplying the transformation matrices, the forward kinematics equation of the collaborative robot is:

\[ \mathbf{T} = \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} = \begin{bmatrix} -s_c s_{\theta_4} + c_c (s_{\theta_3} + c_\theta c_{\theta_5}) & -c_c s_{\theta_4} - s_c (s_{\theta_3} + c_\theta c_{\theta_5}) & s_{\theta_3} - c_\theta c_{\theta_5} & a_c c_{\theta_3} + a_\theta s_\theta + d_d c_{\theta_3} + d_d s_\theta s_{\theta_3} - d_d c_{\theta_3} c_{\theta_5} \\ -c_c s_{\theta_4} - s_c (s_{\theta_3} + c_\theta c_{\theta_5}) & -s_c s_{\theta_4} + c_c (s_{\theta_3} + c_\theta c_{\theta_5}) & -s_{\theta_3} - c_\theta s_{\theta_5} & a_\theta c_{\theta_3} - a_c s_\theta - d_d s_{\theta_3} - d_d c_\theta c_{\theta_3} \\ c_{\theta_4} - s_\theta c_{\theta_5} & s_{\theta_4} + c_\theta c_{\theta_5} & -c_{\theta_5} & d_d c_{\theta_3} + d_d s_\theta s_{\theta_3} + d_d c_{\theta_3} c_{\theta_5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

In which, \( c_i = \cos \theta_i, c_{\theta_i} = \cos(\theta_i + \theta_j + \theta_k), s_{\theta_i} = \sin(\theta_i + \theta_j + \theta_k) \), \( i, j = 1 \sim 6 \).

2.3. Inverse kinematics of collaborative robot

1) Solve \( \theta_i \)

The left and right sides of the Equation (3) simultaneously by the left multiplication the inverse transformation \( \mathbf{T}^{-1} \) of \( \mathbf{T} \) can be obtained:

\[ \mathbf{T}^{-1} \mathbf{T} = \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{T} \]

The elements (3, 3) and (3, 4) at the two ends in the matrix obtained by the Equation (6) are equal, so:

\[ c_i = a_i s_i - a_i c_i \\
\]

Simultaneous solution can get two solutions of \( \theta_i \) in the range \([-\pi, \pi]\)

\[ \theta_i = \arctan(2(d_d a_i - d_i a_i - p_i) - \arctan(2(d_d, \pm \sqrt{(d_d a_i - p_i)^2 + (d_d a_i - p_i)^2 - d_i^2})) \]

2) Solve \( \theta_5 \)

Substituting \( \theta_i \) to Equation (7), the solution of \( \theta_5 \) that can be obtained

\[ \theta_5 = \arctan(2(\sqrt{-(a_s \sin \theta_5 - a_c \cos \theta_5)^2, a_s \sin \theta_5 - a_c \cos \theta_5)}) \]

3) Solve \( \theta_6 \)

The elements (3, 1) and (3, 2) at both ends of the matrix obtained by the Equation (3) are equal, and obtained:

\[ c_5 s_6 = n_5 s_6 - n_5 c_6 \\
- s_5 s_6 = a_5 s_6 - a_5 c_6 \]

When \( \sin \theta_5 \neq 0 \), the solution of \( \theta_6 \) in the range \([-\pi, \pi]\) can be obtained

\[ \theta_6 = \arctan(2(\frac{a_c \sin \theta_6 + a_c \cos \theta_6}{a_c \sin \theta_6 - a_c \cos \theta_6})) \]

When \( \sin \theta_5 = 0 \), the robot was in a singular position, and the directions of the joint axes 4 and 6 were the same or opposite, and only the sum or difference of the \( \theta_4 \) and \( \theta_6 \) was solved. At this time, the value of \( \theta_6 \) can be arbitrarily selected, and then the corresponding value of \( \theta_5 \) is calculated.

4) Solve \( \theta_i \)

By multiplying \( \mathbf{T}^{-1} \mathbf{T}^{-1} \) both right and left sides of Equation (6), you can get:
The elements (1, 4) and (2, 4) at the two ends of the matrix obtained by Equation (12) are equal, which yields:

\[
\begin{align*}
&d_1(n_1c_1 + n_2s_1)s_2 + d_2(o_1c_1 + o_2s_1)c_2 - d_3(a_1c_1 + a_2s_1) + p_1c_1 + p_2s_1 = a_1c_2 + a_2s_2 \\
&n_1d_1c_1 + o_1d_2c_2 - a_1d_3 + p_1 - d_1 = a_1s_2 + a_2s_2
\end{align*}
\]

Simultaneous solution can get two solutions of \( \theta_3 \) in the range \([-\pi, \pi]\)

\[
\theta_3 = \arctan(\pm\sqrt{a_2^2 - (r_{31}^2 + r_{32}^2 - a_1^2)^2})
\]

5) Solve \( \theta_3 \)

Substituting \( \theta_3 \) into Equation (13), you can get:

\[
\theta_3 = \arctan(2(\pm\sqrt{a_2^2 - (r_{31}^2 + r_{32}^2 - a_1^2)^2} \cdot r_{43} - a_2^2 - a_1^2))
\]

6) Solve \( \theta_4 \)

The elements (1, 1) and (2, 1) at both ends of the matrix obtained by the Equation (12) are equal, you can get:

\[
\begin{align*}
&c_{31} = (n_1c_1 + n_2s_1)c_2c_3 - (o_1c_1 + o_2s_1)c_3s_2 - (a_1c_1 + a_2s_1)s_3 \\
&s_{31} = n_1c_1c_3 - a_1s_1s_3 - a_2s_3
\end{align*}
\]

Simultaneous solution can get two solutions of \( \theta_4 \) in the range \([-\pi, \pi]\)

\[
\theta_4 = \arctan(2(n_1c_1c_3 - o_1s_1c_3 - a_1s_1c_3 - (o_1c_1 + o_2s_1)c_3s_2 - (a_1c_1 + a_2s_1)s_3) - \theta_2 - \theta_3)
\]

2.4 Kinematics simulation of collaborative robot

After the kinematics of the robot is solved, it needs to be verified. The spatial sinusoid is taken as an example to verify. According to the inverse kinematics algorithm, the values of the joint angles are calculated and the corresponding motion sequences are solved, as shown in Figure 2. The solid line indicates the position generated by the trajectory planner, and the broken line indicates the position of the trajectory obtained by the robot through the inverse solution. It can be seen from the figure that the two curves overlap to meet the accuracy requirements.
3. Visual compensation algorithm

3.1. Camera calibration

Use the flat dot array calibration plate that comes with Halcon software for camera calibration and subsequent camera and robot hand-eye calibration. The calibration plate is $50 \text{mm} \times 50 \text{mm}$, with 49 dots, and the center point of adjacent dots is 12.5mm, its coordinate information can also be expressed in $7 \times 7$ matrix form. After the calibration plate is selected and the camera is installed, the calibration plate at different positions in the field of view is photographed by the camera and transmitted to the Halcon software, as shown in Figure 3, and the calibration template is processed separately by a program prepared in advance.

![Figure 3. Camera calibration.](image)

The picture finds the calibration area and the array dot, and calculate the coordinates of the center of the array dot and the approximate position between the calibration plate and the camera. After the software program detects the coordinates of the center of the mark in the calibration plate, according to the above calibration algorithm. The principle is to find the camera's internal parameters as well as external parameters. Some parameters such as Camera focal length $f = 7.83581$, Camera distortion coefficient $Kappa = -8428.59$, average error $e = 0.186955$, and Internal reference matrix $K$ solved by calculation are as follows:

$$
K = \begin{bmatrix}
640.594 & 0 & 325.506 \\
0 & 583.732 & 289.676 \\
0 & 0 & 1
\end{bmatrix}
$$

(18)

3.2. The targets match of feature recognition

The edges of the image contour as one of the most basic characteristics of digital image, is a kind of important information in the image, said its invariance various conditions are applicable, and it has good properties is not sensitive to noise [7-10]. Contour matching is based on the edge features of the workpiece in the image to identify the image. The contour matching method can be used to minimize the amount of data computation, to keep the structure information in the image as far as possible, to ignore the secondary information and to accurately identify the artifacts. The artifact feature extraction results, as shown Figure 4.
Figure 4. Feature recognition.

The target position of the workpiece is located, and the center of the target artifact is usually used as the comprehensive representation of image information. The \((p + q)\) order moment of the image is defined, as shown below:

\[
M_{pq} = \sum_{i=i_{\text{max}}}^{i_{\text{min}}} \sum_{j=j_{\text{min}}}^{j_{\text{max}}} i^p j^q f(i,j)
\]

(19)

In which \(f(i,j)\) is pixel quality; \(p = 0, 1, 2, q = 0, 1, 2\), \(M_{pq}\) is moment of image. The position of the center of mass can be solved by the central moment, the zero moment \(M_{00}\), \(i\) axis inertia moment \(M_{01}\) and the \(j\) axis inertia moment \(M_{10}\).

\[
\begin{align*}
M_{00} &= \sum_{i=i_{\text{max}}}^{i_{\text{min}}} \sum_{j=j_{\text{min}}}^{j_{\text{max}}} f(i,j) \\
M_{01} &= \sum_{i=i_{\text{max}}}^{i_{\text{min}}} \sum_{j=j_{\text{min}}}^{j_{\text{max}}} j f(i,j) \\
M_{10} &= \sum_{i=i_{\text{max}}}^{i_{\text{min}}} \sum_{j=j_{\text{min}}}^{j_{\text{max}}} i f(i,j)
\end{align*}
\]

(20)

The centroid coordinates are: \((M_{10}/M_{00}, M_{01}/M_{00})\)

4. Experimental results

The cylindrical, triangular, and rectangular shapes to be grasped for the experiment are randomly placed in the field of view of the camera. Three kinds of workpieces are respectively grasped by creating image templates of different shapes. Take the experiment. The position coordinates of the target workpiece in the world coordinate system and the coordinate pairs under the teach pendant are identified as shown in Table 2.

| Workpiece Type | Pose Type | \(X/(\text{mm})\) | \(Y/(\text{mm})\) | \(Z/(\text{mm})\) | \(R_x/(\text{rad})\) | \(R_y/(\text{rad})\) | \(R_z/(\text{rad})\) |
|---------------|-----------|----------------|----------------|----------------|------------------|------------------|------------------|
| cylindrical   | Image     | 588.478        | -210.373       | -147.290       | -1.65792         | 2.66187          | 0.05975          |
|               | pendant   | 588.308        | -210.343       | -147.290       | -1.65792         | 2.66187          | 0.05371          |
| triangular    | Image     | 537.934        | -252.901       | -147.043       | 0.38119          | 3.09001          | 0.04118          |
|               | pendant   | 537.824        | -252.751       | -147.290       | -1.13500         | 2.91595          | 0.04115          |
| rectangular   | Image     | 652.091        | -253.022       | -147.203       | -1.13500         | 2.91595          | 0.04713          |
|               | pendant   | 651.891        | -253.222       | -147.200       | -1.13500         | 2.91595          | 0.05710          |
The analysis of the above three types of workpiece grabbing experimental data shows that although there is a certain position and rotation angle deviation in the process of grabbing, the range of the error value satisfies the relevant requirements of the grabbing experiment, and the robot pair can be realized. Dynamic crawling of target artifacts.

5. Conclusions
This paper mainly studies the kinematics analysis and visual capture algorithm of collaborative robots. First, the mechanical structure of the cooperative robot was analyzed. Then, the kinematics equation of the cooperative robot was solved by numerical iterative method. The correctness and rapidity of the algorithm were verified by simulation. Finally, a visual compensation method based on image edge contour features is proposed. The multi-target area processing was used to realize the positioning of the workpiece's three-dimensional spatial information, and the collaborative robot was guided to capture the operation to ensure the positioning accuracy.

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