Neutrino Rates in Color Flavor Locked Quark Matter

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Abstract

We study weak interaction rates involving Goldstone bosons in Color Flavor Locked (CFL) quark matter. Neutrino mean free path and the rate of energy loss due to neutrino emission in a thermal plasma of CFL pions and kaons is calculated. We find that in addition to neutrino scattering off thermal mesons, novel Cherenkov like processes wherein mesons are either emitted or absorbed contribute to the neutrino opacity. Lack of Lorentz invariance in the medium and loss of rotational invariance for processes involving mesons moving relative to the medium allow for novel processes such as $\pi^0 \to \nu \bar{\nu}$ and $e^- \pi^+ \to \nu_e$. We explore and comment on various astrophysical implications of our finding.
I. INTRODUCTION

QCD at high baryon density is expected to be a color superconductor. For three massless flavors, a symmetric ground state called the Color Flavor Locked (CFL) phase, in which BCS like pairing involves all nine quarks, is favored \cite{1}. This, color superconducting, phase is characterized by a gap in the quark excitation spectrum. Model calculations indicate that the gap $\Delta \sim 100$ MeV for a quark chemical potential $\mu \sim 500$ MeV \cite{2,3}. In this phase the $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B$ symmetry of QCD is broken down to the global diagonal $SU(3)$ symmetry due to pairing between quarks at the Fermi surface. Gluons become massive by the Higgs mechanism. The lightest excitations in this phase are the nonet of pseudo-Goldstone bosons transforming under the unbroken, global diagonal $SU(3)$ as an octet plus a singlet and a massless mode associated with the breaking of the global $U(1)_B$ symmetry (For a recent review see \cite{4}).

The lack of quark particle-hole excitations in dense quark matter at temperatures less than the critical temperature for color superconductivity ($T_c \sim 0.6\Delta$) will affect thermodynamic and transport properties of this phase. This can have important astrophysical implications if quark matter were to exist in the core of a neutron star. Several authors have recently explored some aspects of how color superconductivity might impact neutron star observables. These works have provided insight on: role of color superconductivity on the phase transition density; the nature of the interface between nuclear matter and CFL matter \cite{5}; its response to magnetic fields \cite{6}; and the thermal evolution of both young and old neutron stars \cite{7,8}. In this article we calculate weak interaction rates for neutrino production and scattering in the CFL phase and contrast it with earlier estimates of similar rates in normal(unpaired) quark matter.

Neutrinos play a central role in the early and late time thermal evolution of neutron stars. Weak interaction rates in the superconducting phase are therefore essential to make connections between color superconductivity and observable aspects associated with neutron star thermal evolution. Neutron stars are born in the aftermath of a core collapse supernova.
explosion. The inner core of a newly born neutron star is characterized by a temperature $T \sim 30$ MeV and a lepton fraction (lepton number/baryon number) $Y_L \sim 0.3$ implying $\mu_e \equiv \mu_{\nu_e} - \mu_Q \sim \mu_{\nu_e} \sim 200$ MeV. The high temperature and finite lepton chemical potentials are a consequence of neutrino trapping during gravitational collapse. The ensuing thermal evolution of the newly born neutron star, during which it emits neutrinos copiously, has generated much recent interest \cite{10,11}. Several aspects of this early evolution can be probed directly since neutrinos emitted during the first several tens of seconds can be detected in terrestrial detectors such as Super-Kamiokande and SNO. This study is motivated by the prospect that were Color-Flavor-Locked quark matter to exist in neutron stars at early times it would result in observable and discernible effects on the supernova neutrino emission.

We can expect significant differences in the weak interaction rates between the normal and the CFL phases of quark matter since the latter is characterized by a large gap in the quark excitation spectrum. Thus, unlike in the normal phase where quark excitations near the Fermi surface provide the dominant contribution to the weak interaction rates, in the CFL phase, it is the dynamics of the low energy collective states— the flavor pseudo-Goldstone bosons that are relevant. As a first step towards understanding the thermal and transport properties of this phase which are of relevance to core collapse supernova studies, we identify and calculate the weak interaction rates for neutrino production and scattering. The article is organized as follows: In §2 we briefly describe the effective theory describing Goldstone bosons in the CFL phase; In §3 we calculate the rate of neutrino reactions of interest; and in §4 we discuss how our findings might affect the early evolution of a newly born neutron star.

II. EFFECTIVE THEORY FOR GOLDSTONE BOSONS

There are several articles that describe in detail the effective theory for the Goldstone bosons in Color-Flavor-Locked quark matter \cite{12,13}. We briefly review some aspects of the effective theory in this section. It is possible to parameterize low energy excitations
about the $SU(3)$ symmetric CFL ground state in terms of the two fields $B = H/\sqrt{24f_H}$ and $\Sigma = e^{2i(\pi/f_H + \eta'/f_A)}$, representing the Goldstone bosons of broken baryon number $H$, and of broken chiral symmetry, the pseudo-scalar octet $\pi$, and the pseudo-Goldstone boson $\eta'$, arising from broken approximate $U(1)_A$ symmetry. Turning on nonzero quark masses, explicitly breaks chiral symmetry and induces a gap in the spectrum as the Goldstone bosons acquire a mass due to this explicit breaking of chiral symmetry. In addition, dissimilar quark masses induce new stresses on the system, acting in a manner analogous to an applied flavor chemical potential and favoring meson condensation. This was recently pointed out by Bedaque and Schafer [19]. In this work we include the stress induced by the strange quark mass, but we will assume that it is not strong enough to result in the meson condensation.

The leading terms of the effective Lagrangian describing the octet Goldstone boson field $\pi$ is given by

$$L = \frac{1}{4}f_\pi^2 \left[ \text{Tr} \nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \text{Tr} \nabla \Sigma \cdot \nabla \Sigma^\dagger \right] + f_\pi^2 \left[ \frac{a}{2} \text{Tr} \tilde{M} \left( \Sigma + \Sigma^\dagger \right) + \frac{\chi}{2} \text{Tr} \left( \Sigma + \Sigma^\dagger \right) \right]$$

$$\nabla_0 \Sigma = \partial_0 \Sigma - i \left[ X_L \Sigma - X_R \Sigma \right].$$

1\ The decay constant $f_\pi$ has been computed previously [15] and is proportional to the quark chemical potential. $X_{L,R}$ are the Bedaque-Schafer terms: $X_L = -\frac{M M^\dagger}{2\mu}$, $X_R = -\frac{M M^\dagger}{2\mu}$, and $\tilde{M} = |M| M^{-1}$. A finite baryon chemical potential breaks Lorentz invariance of the effective theory. The temporal and spatial decay constants can thereby differ. This difference is encoded in the velocity factor $v$ being different from unity. An explicit calculation shows that $v = 1/\sqrt{3}$ and is common to all Goldstone bosons, including the massless $U(1)_B$ Goldstone boson [15].

1\ If $K^0$ condensation occurs, as discussed in Ref. [19, 20], the ground state is reorganized and the excitation spectrum is modified. The weak interaction rates in this phase is currently under investigation and will be reported elsewhere.
At asymptotic densities, where the instanton induced interactions are highly suppressed and the $U(1)_A$ symmetry is restored, the leading contributions to meson masses arise from the $\text{Tr} \tilde{M} \Sigma$ operator whose coefficient $a$ has been computed and is given by $a = \frac{3 \Delta^2}{\pi^2 f^2}$. 

At densities of relevance to neutron stars the instanton interaction may become relevant. In this case a $\bar{q}q$ condensate is induced and consequently the meson mass term can receive a contribution from the operator $\text{Tr} M \Sigma$. Its coefficient $\chi$ at low density is sensitive to the instanton size distribution and form factors. Current estimates indicate that the instanton contribution to the $K^0$ mass lies in the range $5 - 120$ MeV. The meson masses are explicitly given by

$$m_{\pi^\pm} = a(m_u + m_d)m_s + \chi(m_u + m_d)$$
$$m_{K^\pm} = a(m_u + m_s)m_d + \chi(m_u + m_s)$$
$$m_{K^0} = a(m_d + m_s)m_u + \chi(m_d + m_s).$$

In this article we will assume the instanton contribution to the $K^0$ mass is $\sim 50$ MeV (corresponding to $\chi \sim 15$ MeV) at $\mu = 400$ MeV and $\Delta = 100$ MeV. With this choice, the kaon mass is too large to allow for $K^0$ condensation.

To incorporate weak interactions, we gauge the Chiral Lagrangian in the usual way by replacing the covariant derivative by

$$\mathcal{D}_\mu \Sigma = \nabla_\mu \Sigma - \frac{ig}{\sqrt{2}}(W^\pm_\mu \tau^\pm + W^-_\mu \tau^-) \Sigma - \frac{ig}{\cos \theta_W} Z_\mu (\tau_3 \Sigma - \sin^2 \theta_W [Q, \Sigma]) - i \bar{e} \tilde{A} [Q, \Sigma]$$

The time component of $\nabla_\mu$ includes the Bedaque-Schafer term as described above. The fields $W^\pm_\mu, Z_\mu$ describe weak gauge bosons. The charge matrix is diagonal $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ as well as weak-isospin matrix $\tau_3 = \frac{1}{2} \text{diag}(1, -1, -1)$ whereas $\tau^+$ and $\tau^-$ are the isospin raising and lowering operators which incorporate Cabibbo mixing. The weak coupling constant is related to Fermi coupling constant via the standard relation $\sqrt{2} g^2 = 8 G_F M_W^2$ where $M_W$ is a mass of the $W$ gauge boson and the mass of $Z$ boson $M_Z \cos \theta_W = M_W$ where $\theta_W$ is Weinberg angle. The last term is the modified electromag-
netic coupling of mesons to the massless $\tilde{A}$ photon in the CFL phase, where the charge $\tilde{e} = e \cos \theta$ and $\theta$ is the mixing angle between the original photon and the eighth gluon [6].

For momenta small compared to $f_\pi$ we can expand the nonlinear chiral Lagrangian to classify diagrams as the first order (proportional to $f_\pi$) and the second order (independent of $f_\pi$). The first order diagrams, involving two leptons and a meson are shown in the Fig. 1. In Fig. 1(a) the charged current decay of charged pions and kaons is shown. Fig. 1(b) shows the neutral current decay of $\pi^0 \to \nu\bar{\nu}$. In vacuum, the latter process is forbidden by angular momentum conservation. However, as we will show later, it is allowed for finite momentum pions in the CFL medium. Diagrams in the Fig. 1(c) and 1(d) show processes involving two mesons. The process in Fig. 1(c) is the two body correction to the charged current decay of the decay of charged kaons and pions interacting with the neutral mesons. And Fig. 1(d) depicts neutrino pair emission from the annihilation of charged Goldstone bosons $K^\pm$ and $\pi^\pm$ and neutrino-meson scattering.

The amplitudes for the leading order processes are given by

$$A_{\pi^0 \to \nu\bar{\nu}} = \frac{G_F}{\sqrt{2}} f_\pi \tilde{p}_\mu j_Z^\mu$$

$$A_{\pi^\pm \to e\nu} = G_F f_\pi \cos \theta_C \tilde{p}_\mu j_W^\mu$$

$$A_{K^\pm \to e\nu} = G_F f_\pi \sin \theta_C p_\mu j_W^\mu$$

FIG. 1. Feynman graphs showing the coupling of the neutral and charged leptonic current to one and two meson states.
where $\tilde{p}_\mu = (E, v^2 \tilde{p})$ is the modified four-momentum of Goldstone boson and $j_W^\mu$ and $j_Z^\mu$ describe the charged leptonic current and neutral leptonic currents. $\theta_C$ is the Cabbibo mixing angle. Note that the meson "four momenta" that appear in the matrix element do not correspond to the on shell four momenta of the mesons. This is because the covariant derivatives contains the in medium velocity and for the case of kaons, the energy shift arising from the Bedaque-Schafer term.

At next to leading order, the amplitude is independent of $f_\pi$. The amplitude for the charged current process is given by

$$A_{\Phi^\pm \Phi^0 \to e\nu} = -iC \frac{G_F}{2} (\tilde{p}_1 - \tilde{p}_2)_\mu j_W^\mu$$  \hspace{1cm} (5)$$

where the coupling coefficient $C = \sin \theta_C$ for $\pi^0K^-, \pi^0K^+$; $C = \sqrt{2} \sin \theta_C$ for $\pi^-K^0, \pi^+K^0$; $C = \sqrt{2} \cos \theta_C$ for $K^-K^0, K^+\bar{K}^0$ and $C = 2 \cos \theta_C$ for $\pi^-\pi^0, \pi^+\pi^0$. Neutrinos couple to the charged mesons via the neutral current. This leads to processes such as the annihilation of $\pi^+\pi^- \to \nu\bar{\nu}$ and $K^+K^- \to \nu\bar{\nu}$, whose amplitude is given by

$$A_{\Phi^+ \Phi^- \to \nu\bar{\nu}} = -i \frac{G_F}{\sqrt{2}} (1 - 2 \sin^2 \theta_W) (\tilde{p}_1 - \tilde{p}_2)_\mu j_Z^\mu$$  \hspace{1cm} (6)$$

where $p_1, p_2$ are momenta of Goldstone bosons. It also gives rise to neutral current neutrino-meson scattering given by the amplitude

$$A_{\nu \Phi^\pm \to \nu \Phi^\pm} = -i \frac{G_F}{\sqrt{2}} (1 - 2 \sin^2 \theta_W) (\tilde{p}_1 + \tilde{p}_2)_\mu j_Z^\mu .$$  \hspace{1cm} (7)$$

In addition to the flavor octet of Goldstone bosons, the massless Goldstone boson associated with spontaneous breaking of $U(1)_B$ couples to the weak neutral current. This is because the weak isospin current contains a flavor singlet component. Amplitude for processes involving the $U(1)_B$ Goldstone boson $H$ and the neutrino neutral current is given by

$$A_{H\nu\bar{\nu}} = \frac{4}{\sqrt{3}} G_F f_H \tilde{p}_\mu j_Z^\mu ,$$  \hspace{1cm} (8)$$

where $\tilde{p}_\mu = (E, v^2 \tilde{p})$ is the modified four momentum of the Goldstone boson. The decay constant for the $U(1)_B$ Goldstone boson has also been computed in earlier work and is given
by $f_H^2 = 3\mu^2/(8\pi^2)$ \cite{15}. Further, we note that a $HHZ^0$ vertex is absent in the lowest order Lagrangian ($H$ boson does not carry baryon number, isospin or hypercharge).

III. THERMODYNAMICS AND NEUTRINO RATES

The dispersion relations for Goldstone modes in the CFL phase are unusual. They are easily computed by expanding the Lagrangian to second order in meson fields and are given by

$$E_{K^\pm} = \pm \frac{m_s^2}{2\mu} + \sqrt{v^2p^2 + m_{K^\pm}^2}, \quad E_{\pi^\pm} = \sqrt{v^2p^2 + m_{\pi^\pm}^2},$$
$$E_{K^0} = -\frac{m_s^2}{2\mu} + \sqrt{v^2p^2 + m_{K^0}^2}, \quad E_{\bar{K}^0} = \frac{m_s^2}{2\mu} + \sqrt{v^2p^2 + m_{K^0}^2}.$$  \hspace{1cm}  (9)

They violate Lorentz invariance and the induced effective chemical potential arising from the analysis of Bedaque and Schafer \cite{19} breaks the energy degeneracy of the positive and negative charged kaons. This making the $K^+$ lighter than the $K^-$ and naturally results in an excess positive charge in the meson sector at finite temperature. Electric charge neutrality of this phase demands electrons and consequently an electric charge chemical potential is induced. This novel phenomena, akin to the thermoelectric effect, modifies the number densities of the individual mesons in the plasma.

Kaon, pion and the total electric charge density of the meson gas, normalized to the photon number are shown in Fig. 2. The results are shown as a function of temperature for a quark chemical potential of 400 MeV (corresponding to $f_\pi = 83$ MeV) at which we have chosen the pairing gap $\Delta = 100$ MeV. The quark masses are set at $m_u = 3.75$ MeV $m_d = 7.5$ MeV and $m_s = 150$ MeV and instanton induced coefficient of the $\text{Tr}M\Sigma$ operator $\chi = 16$ MeV as mentioned earlier. With these assumptions, at zero temperature the rest energy of mesons are given by: $E_{\pi^\pm} \simeq 30$ MeV; $E_{K^+} \simeq 26$ MeV; $E_{K^-} \simeq 82$ MeV; $E_{K^0} \simeq 24$ MeV and $E_{\bar{K}^0} \simeq 80$ MeV. Consequently, the kaon number is dominated by the $K^0$ and $K^+$ mesons and a finite electron chemical potential favors $\pi^-$ mesons over the $\pi^+$. It is surprising that, with increasing temperature, the plasma has more mesons than photons. Despite
FIG. 2. Number densities of the mesons, the charge density and the number of $e^+e^-$ pairs, normalized by the photon number density. The electron chemical potential required to maintain electric charge neutrality at finite temperature is also shown.

being massive, the meson number is enhanced for two reasons. First, the velocity factor $v = 1/\sqrt{3}$ results in a soft dispersion relation resulting in reduced Boltzmann suppression of the high momentum modes. The number density is enhanced by a factor $1/v^3$. For the same reason, the density of massless $H$ bosons (not shown in the figure) is also larger than the photon density by the factor $1/(2v^3)$. Secondly, the Bedaque-Schafer term which acts like an effective chemical potential favoring strange quarks, and the induced negative electric charge chemical potential required to maintain charge neutrality enhances the number of mesons with strange quarks and negative charge.

Prior to discussing neutrino rates in the finite temperature plasma we briefly comment on the temperature range over which our analysis is expected to be valid. The effective theory description of the finite temperature phase is valid only if the typical meson energy is small compared to $2\Delta$. For larger energies, meson propagation is strongly attenuated since
they can decay into quark quasi-particle excitations. Further, with increasing temperature, we should expect coefficients of the effective theory such as $f_\pi$, $v$ and the meson masses to change. On general grounds, we expect these changes to become relevant as $T$ approaches $T_c$. In our analysis, we ignore these changes and restrict ourselves to temperatures $T$ small compared to $T_c$. At $\mu = 400$ MeV and $\Delta = 100$ MeV corresponding to $T_c \sim 60$ MeV we expect our analysis to provide a fair description of the plasma for $T \lesssim 30$ MeV.

A. Neutrino Opacity

Novel processes such as the $\nu \to H\nu$, $\nu \to \pi^0\nu$ and $\nu_e \to \pi^+e^-$ are allowed in this phase owing to the fact that mesons can have a space like dispersion relation. These processes can be thought of as arising due to Cherenkov radiation of mesons. Dispersion relations for the pions and kaons indicate that they are space like four momenta when their 3-momentum satisfies:

$$p_\pi > \frac{m_\pi}{\sqrt{1-v^2}} \quad ; \quad p_K > \frac{\sqrt{m_K^2(1-v^2)+X^2-X}}{1-v^2}, \quad (10)$$

where $X = m_\pi^2/(2\mu)$ for $(K^+, K^0)$ and $X = -m_\pi^2/(2\mu)$ for $(K^-, \bar{K}^0)$. Consequently, only high energy neutrinos, with energy greater than $E_{TH} = m_\pi/\sqrt{1-v^2}$, can emit pions as Cherenkov radiation. In contrast, the massless $U(1)_B$ Goldstone boson has a space like dispersion relation for all momenta. Thus, neutrinos of all energies can Cherenkov radiate $H$ bosons as they traverse the dense CFL medium.

We can define the neutrino mean free path for these processes as the neutrino velocity times the rate of emission of Cherenkov mesons. This is given by

$$\frac{1}{\lambda_{\nu\to\phi l}(E_\nu)} = \frac{1}{2E_\nu} \int \frac{d^3p_\phi}{(2\pi)^32E_\phi} \int \frac{d^3p_l}{(2\pi)^32E_l}(2\pi)^4 \delta^4(P_\nu - P_\phi - P_l)|A_{\nu\to\phi l}|^2 \quad (11)$$

where $E_\nu$ is the initial neutrino energy, $P_\phi$ is the meson four momentum and $P_l$ is the final state lepton four momentum. We label the final state lepton as $l$ to account for the fact that

Assuming $\Delta(T) = \Delta_0 \sqrt{1-(T/T_c)^2}$, at $T = 30$ MeV the gap is reduced by only 15 percent.
it could be either a neutrino or an electron. The amplitude for this process was calculated earlier and is proportional to $\mu$. Since these processes do not have mesons in the initial state they can occur at zero temperature. The rate for the process depends on the neutrino energy and is independent of the temperature in so far as the meson masses and $f_\pi$ is independent of temperature. Kinematics and 1 relatively strong $H\nu\bar{\nu}$ coupling (approximately thrice as large as the $\pi^0\nu\bar{\nu}$ coupling) makes Cherenkov radiation of the massless $H$ bosons the dominant reaction of this type. Neutrino mean free path due to the reaction $\nu \rightarrow H\nu$ can be calculated analytically using Eq. (11) and is given by

$$\lambda_{\nu \rightarrow H\nu}(E_\nu) = \frac{1}{2E_\nu} \int \frac{d^3p_\phi}{(2\pi)^3E_\phi^2} f(E_\phi) \int \frac{d^3p_l}{(2\pi)^3E_l^2} \delta^4(P_\nu + P_\phi - P_l)|A_{\nu\phi \rightarrow l}|^2$$ (13)

where $f(E_\phi)$ is the Bose distribution function for the initial state mesons. Reactions involving the $H$ boson dominate over other Cherenkov absorption processes due to their larger population and stronger coupling to the neutral current. For this case, we find the neutrino mean free paths is given by

$$\lambda_{\nu H \rightarrow \nu\nu}(E_\nu) = \frac{128}{3\pi} \left[ \frac{v(1-v)^2}{(1-v)} \right] g_2(\gamma) + \frac{2v}{(1-v)^2} g_3(\gamma) - \frac{(1+v)}{(1-v)^2} g_4(\gamma) \right] G_f^2 f_H^2 E_\nu^3$$ (14)

where $\gamma = 2vE_\nu/(1-v)T$ and the integrals $g_n(\gamma)$ are defined by the relation

$$g_n(\gamma) = \int_0^1 dx \frac{x^n}{\exp(\gamma x) - 1}.$$ (15)

In the limiting cases of high ($E_\nu \gg T$) and low ($E_\nu \ll T$) neutrino energy the above integral can be performed analytically. For $E_\nu \gg T$ we find that
$$\frac{1}{\lambda_{\nu H \rightarrow \nu}(E_\nu)} = \frac{32\zeta(3)}{3\pi} \left[ \frac{(1-v^2)^2}{v^2} \right] \left[ 1 + \frac{\pi^4}{30\zeta(3)} \frac{T}{E_\nu} - \frac{3\zeta(5)}{\zeta(3)} \frac{(1-v^2)}{v^2} \frac{T^2}{E_\nu^2} \right] G_F^2 \ f_H^2 \ T^3, \ (16)$$

and for the case when $E_\nu \ll T$ we find

$$\frac{1}{\lambda_{\nu H \rightarrow \nu}(E_\nu)} = \frac{16}{3\pi} \frac{(1+v)^2(1-\frac{v}{3})}{(1-v)} \frac{T^2}{E_\nu^2}. \ (17)$$

In contrast to processes involving the emission or absorption of mesons by neutrinos, the usual scattering process involves the coupling of the neutrino current to two mesons. As noted earlier, the amplitude for these processes vanishes for the $H$ meson and is suppressed by the factor $p/f_\pi$ where $p$ is the meson momentum for the flavor octet mesons. In the later case, despite the suppression, the reaction could be important because the kinematics is not restricted to space like mesons. Neutrino mean free path for the scattering process is given by

$$\frac{1}{\lambda_{\nu \phi^\pm \rightarrow \nu \phi^\pm}(E_\nu)} = \frac{1}{2E_\nu} \int \frac{d^3p_\phi}{(2\pi)^3 2E_\phi} f(E_\phi) \int \frac{d^3p'_\nu}{(2\pi)^3 2E'_\nu} \int \frac{d^3p'_\phi}{(2\pi)^3 2E'_\phi} \times (2\pi)^4 \delta^4(P_\nu + P_\phi - P'_\phi - P'_\nu) |A_{\nu \phi^\pm \rightarrow \nu \phi^\pm}|^2 \ (18)$$

The resulting neutrino mean free path for all three classes of processes discussed above is shown in Fig. 3. The results are presented as a function of the ambient temperature. We assume that the typical neutrino energy is determined by the local temperature and set $E_\nu = \pi T$ since this corresponds to mean energy of neutrinos in thermal equilibrium. Note, the mean free path due to Cherenkov radiation of mesons has no intrinsic temperature dependence. Temperature dependence of the result shown in Fig. 3 arises because we have set $E_\nu = \pi T$. At low temperature, we expect Cherenkov reactions involving the massless Goldstone boson to dominate. Interestingly, however, we find that this is the case even for $T \sim 30$ MeV. The total contribution to opacity from novel Cherenkov processes is also shown in the figure as a solid curve labeled $\lambda_{3B}$. The Cherenkov absorption reaction, $\nu H \rightarrow \nu$, makes the largest contribution to the neutrino opacity over the temperature range $T = 1 - 30$ MeV. Reactions involving kaons and pions become comparable only for $T \gtrsim 15$ MeV. Further, charged current reactions involving the absorption or emission of kaons are
FIG. 3. Neutrino mean free path in a CFL meson plasma as a function of temperature. The neutrino energy $E_\nu = \pi T$ and is characteristic of a thermal neutrino.

Cabbibo suppressed and contribute to less than a few percent of the total rate. Scattering reactions are found to be negligible for $T \lesssim 10$ MeV and make a 20% contribution to the total opacity at $T \sim 30$ MeV.

B. Neutrino Emissivity

Charged current decays of pions and kaons, and the novel neutral current decay of $\pi^0$ are the leading one body process contributing to neutrino emission. In vacuum, the amplitude for similar processes are proportional to the lepton mass due to angular momentum conservation. However, as discussed earlier, the dispersion relations for Goldstone modes violate Lorentz invariance. Consequently we find that the decay of finite momentum pions and kaons is not suppressed by the electron mass (note that decay into muons is highly suppressed because the meson mass and the temperature are less than the mass of the muon). This can be understood by noting that Goldstone modes at finite density are collective exci-
tations associated with deformations of the Fermi surface. In the rest frame of a meson that has finite momentum relative to the medium the Fermi Surface is not spherically symmetric. This breaks rotational invariance and angular momentum is no longer a good quantum number to describe the meson state. As a result, decay of pions and kaons into massless lepton pairs is allowed.

The rate of energy loss due to one body decays is computed using the standard formula

$$\dot{\epsilon}_{1B} = \int \frac{d^3p}{(2\pi)^32E} \int \frac{d^3k_1}{(2\pi)^32E_1} \int \frac{d^3k_2}{(2\pi)^32E_2} |A|^2(2\pi)^4\delta^4(P - k_1 - k_2)$$

(19)

where $P$, $k_1$ and $k_2$ are the four momenta of the meson, the neutrino and the charged lepton respectively. We neglect the electron mass in the describing the kinematics of these reaction because the typical electron momentum is large. Further, since the electron chemical potential we find is also small (compared to $T$) we are justified in neglecting the final state electron Pauli-blocking. This allows us to perform the integration over the final state lepton momenta using the identity

$$\int \frac{d^3k_1}{2E_1} \int \frac{d^3k_2}{2E_2} \delta^4(P - k_1 - k_2) k_1^\mu k_2^\nu = \frac{\pi}{24}(P^\mu P^\nu + g^\mu\nu P^2)$$

(20)

to obtain

$$\dot{\epsilon}_{1B} = G_F^2 \frac{f_\pi^2 C^2}{12\pi} \int \frac{d^3p}{(2\pi)^3} f(E_p) \left((\vec{P} \cdot P)^2 - \vec{P}^2 P^2\right).$$

(21)

where $P = (E_p, \vec{p})$ is the four momentum of the Goldstone boson and $C = \sin \theta_C$ for kaon decays. $C = \cos \theta_C$ for charged pion decays and $C = 1$ for the neutral current decay of the $\pi^0$. Decay processes can occur only when the meson four momentum is time like. Thus, the 1-body decay of $H$ boson is forbidden and only low momentum pions and kaons (See Eq. (10)) can participate in the decay process. This accounts for the saturation of the one body decay contribution to the emissivity with increasing temperature seen in Fig. 4.

Higher momentum, space like, states can participate in processes such as $e^\pm \phi^\mp \rightarrow \nu$. We find that these lepton absorption processes dominate the emissivity for $T \gtrsim 10 MeV$. Processes involving two mesons in the initial state such as $\phi^\pm \phi^0 \rightarrow e^\pm \nu$ and $\phi^+ \phi^- \rightarrow \nu\bar{\nu}$
FIG. 4. Rate of energy loss due to neutrino emitting reactions. Contributions due to one body decay (solid curve), electron absorption on mesons (dot-dashed curve) and process involving the annihilation of two mesons is shown. The purely leptonic process $e^+e^- \rightarrow \nu\bar{\nu}$ is also shown (double dot-dashed curve).

can be expected to become important only at high temperature. This is because Goldstone bosons are weakly interacting, the two particle amplitude is reduced by the factor $p/f_{\pi}$ compared to the one body amplitude, where $p$ is the relative momenta of the Goldstone bosons. Despite this suppression we consider the two meson annihilation process because the pion and kaon densities exceeds the lepton density at high temperature $^3$.

In Fig. 4, the rate of energy loss due to meson decay, lepton assisted decays ($e\phi \rightarrow \nu$) and two meson reactions are shown. For reasons discussed earlier, meson decay is the dominant

$^3$It is interesting to note that in vacuum, with normal dispersion relations, the two body processes would dominate over one body decay at modest temperatures due to angular momentum restrictions on decay process.
reaction for $T \lesssim 10$ MeV. Reactions involving two particles in the initial state are also shown and become important at higher temperature. Charged current absorption of electrons and positrons on charged mesons producing neutrinos dominates the emissivity for $T \gtrsim 10$ MeV. The two meson annihilation reactions contribute to only 10% of the total rate even at $T \sim 30$ MeV. The purely leptonic process $e^+e^- \rightarrow \nu\bar{\nu}$ is also shown in the figure. Its contribution is always smaller than the contribution arising from the mesons.

Other processes involving electrons in the initial state, such as $e^-\phi^+ \rightarrow \nu\phi^0$ could become important at low temperature where the electron density exceeds the meson density. We have ignored their contribution since this corresponds to $T \lesssim 5$ MeV where the one-body decay dominates. We also note when the temperature becomes comparable to the gap $\Delta$ neutrino emission reactions involving quark quasi-particles can become important. In particular, quark Cooper pair breaking has been shown to be relevant when $T \sim \Delta$ [23]. We expect the rate of this process to small at $T \sim 30$ MeV since it is suppressed by the factor $\exp(-2\Delta/T)$.

**IV. CONCLUSIONS**

We have shown that novel processes in which mesons are either emitted or absorbed from neutrinos occur in the CFL plasma and contribute to neutrino opacity. Absorption of, thermal, massless $H$ bosons is the dominant reaction contributing to the neutrino opacity in the CFL phase for temperatures in the range $T = 1 - 30$ MeV. Cherenkov radiation of these mesons is the second most important process and is likely to be the dominant process for $T \lesssim 1$ MeV. With increasing temperature and the exponential growth of pion and kaon number densities in the CFL plasma additional reactions such as $\nu + \pi^0 \rightarrow \nu$ and $\nu_e + \pi^- \rightarrow e^-$ contribute to the neutrino opacity. We find that the mean free path for thermal neutrinos at $T = 10$ MeV is of the order of 10 meters and at $T = 5$ MeV it is similar to 100 meters. In the table below we compare the neutrino mean free path in CFL matter with those in nuclear matter and unpaired quark matter under similar conditions. To make these comparisons we
employ earlier estimates of neutrino mean free path in nuclear matter obtained by Reddy, Prakash and Lattimer [24] and in unpaired quark matter obtained by Iwamoto [25]. The quark chemical potential $\mu = 400 \text{ MeV}$ corresponds to a baryon density $n_B \sim 5n_0$ in quark matter, where $n_0 = 0.16 \text{ fm}^{-3}$ is the nuclear saturation density. In the table below, we compare mean free path of thermal neutrinos ($E_\nu = \pi T$) in these different phases at $n_B = 5n_0$.

| phase   | process              | $\lambda(T=5 \text{ MeV})$ | $\lambda(T=30 \text{ MeV})$ |
|---------|----------------------|-----------------------------|-----------------------------|
| Nuclear | $\nu n \rightarrow \nu n$ | 200 m                       | 1 cm                        |
| Matter  | $\nu_e n \rightarrow e^- p$ | 2 m                         | 4 cm                        |
| Unpaired| $\nu q \rightarrow \nu q$ | 350 m                       | 1.6 m                       |
| Quarks  | $\nu d \rightarrow e^- u$ | 120 m                       | 4 m                         |
| CFL     | $\lambda_{3B}$       | 100 m                       | 70 cm                       |
|         | $\nu \phi \rightarrow \nu \phi$ | $>10 \text{ km}$ | 4 m                         |

The findings presented in the table above indicate that neutrino mean free path in CFL matter is similar to or shorter than that in unpaired quark matter in the temperature range $T = 1 - 30 \text{ MeV}$. This, surprising, result arises solely due to the novel processes involving Cherenkov absorption and radiation of CFL Goldstone bosons.

Novel neutrino emitting processes such as $\pi^0$ decay to neutrino-anti neutrino pairs occur in the CFL phase. Charged current leptonic decay of mesons is also enhanced in the medium. They occur because, in the rest frame of the meson moving relative to the medium, the ground state breaks rotational invariance. With increasing temperature, this enhancement saturates since only low momentum mesons have a time like dispersion relation. At higher temperatures, reactions involving two particles overcome this kinematic constraint and dominate the emissivity. In the neutron star context, the high temperature emissivity is unlikely to play an important role in neutron star dynamics because neutrinos are effectively trapped and are described by local thermal distributions. We will restrict ourselves to $T \lesssim 10 \text{ MeV}$ to make the comparisons between the emissivity of the CFL phase with that of unpaired quark matter. In unpaired quark matter, Iwamoto finds that the emissivity due
beta decay of light quarks at $n_B = 5n_0$ to be $\dot{\epsilon}_{q\beta} \sim 2 \times 10^{36}$ erg cm$^{-3}$ s$^{-1}$ at $T = 5$ MeV and $\dot{\epsilon}_{q\beta} \sim 2 \times 10^{38}$ erg cm$^{-3}$ s$^{-1}$ at $T = 10$ MeV [25]. This is to be compared with our finding that $\dot{\epsilon}_{CFL} \sim 5 \times 10^{33}$ erg cm$^{-3}$ s$^{-1}$ at $T = 5$ MeV and $\dot{\epsilon}_{CFL} \sim 2 \times 10^{35}$ erg cm$^{-3}$ s$^{-1}$ at $T = 10$ MeV. The emissivity is roughly reduced by three orders of magnitude.

At lower temperature, emissivity in the CFL phase is exponentially suppressed, due to the paucity of thermal mesons, by the factor $\exp \left( -\frac{m}{T} \right)$, where $m$ is the mass of the lightest octet meson.

For a detailed discussion of neutrino emission in the CFL phase at sub MeV temperature, we refer the reader to an article by Jaikumar, Prakash and Schafer which was posted on the archive concurrently with this work [26].

The significant new finding of this work is that the neutrino opacity of the CFL phase is similar or greater than that of unpaired quark matter. Relative to nuclear matter the CFL phase is only marginally less opaque. At lower temperature, it is roughly comparable to the opacity nuclear matter given that the latter is known only to within factor of a few at the densities or relevance to neutron stars. Despite the energy gap in the quark quasi-particle excitations spectrum the opacity remains large and there is no exponential suppression of neutrino cross sections even when $T \ll \Delta$. This is in sharp contrast to earlier findings of exponentially suppressed neutrino cross sections in the two flavor superconducting phase of quark matter which is devoid of Goldstone bosons in its excitation spectrum [1]. The astrophysical implication of this finding is that temporal aspects of neutrino diffusion inside the newly born neutron star could be similar to that found in earlier studies of neutrino transport in unpaired quark matter [11]. However, since we expect the specific heat of the CFL phase $C_V \sim T^3$ to be small compared to that of unpaired quark matter where $C_V \sim \mu^2 T$ the cooling rates could still differ and needs to be investigated. If the rate of energy loss via neutrino diffusion depends solely on neutrino mean free path, we can expect the smaller heat capacity to result in accelerated cooling due to a relatively more rapid rate of change of the ambient temperature. Nonetheless, we emphasize that the early evolution of a newly born neutron star is a complex process which depends on several microscopic ingredients and macroscopic conditions. In order to gauge how color superconductivity in
the neutron star core will impact observable aspects of early neutron star evolution the rates computed in this work, and in addition, the thermodynamic properties of the CFL phase need to be included in detailed numerical simulations of core collapse supernova. This is the only reliable means to bridge the gap between the exciting theoretical expectation of color superconductivity at high density and observable aspects of core collapse supernova – the neutrino count rate, the neutrino spectrum and the explosion itself.

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REFERENCES

[1] M. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999) \texttt{hep-ph/9804403}.

[2] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B 422, 247 (1998) \texttt{arXiv:hep-ph/9711395}.

[3] R. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998) \texttt{arXiv:hep-ph/9711396}.

[4] K. Rajagopal and F. Wilczek, \texttt{arXiv:hep-ph/0011333}.

[5] M. G. Alford, K. Rajagopal, S. Reddy and F. Wilczek, Phys. Rev. D 64, 074017 (2001) \texttt{arXiv:hep-ph/0105009}.

[6] M. G. Alford, J. Berges and K. Rajagopal, Nucl. Phys. B 571, 269 (2000) \texttt{arXiv:hep-ph/9910254}.

[7] D. Blaschke, T. Klahn and D. N. Voskresensky, Astrophys. J. 533, 406 (2000) \texttt{arXiv:astro-ph/9908334}.

[8] D. Page, M. Prakash, J. M. Lattimer and A. Steiner, Phys. Rev. Lett. 85, 2048 (2000) \texttt{arXiv:hep-ph/0005094}.

[9] G. W. Carter and S. Reddy, Phys. Rev. D 62, 103002 (2000) \texttt{arXiv:hep-ph/0005228}.

[10] A. Burrows and J. M. Lattimer, Astrophys. J. 307, 178 (1986). W. Keil and H. T. Janka, Astron. Astrophys. 296, 145 (1995). J. A. Pons, S. Reddy, M. Prakash, J. M. Lattimer and J. A. Miralles, Astrophys. J. 513, 780 (1999) \texttt{astro-ph/9807040}.

[11] J. A. Pons, A. W. Steiner, M. Prakash and J. M. Lattimer, Phys. Rev. Lett. 86, 5223 (2001) \texttt{arXiv:astro-ph/0102015}.

[12] D. K. Hong, M. Rho and I. Zahed, Phys. Lett. B 468, 261 (1999) \texttt{arXiv:hep-ph/9906551}.
[13] R. Casalbuoni and R. Gatto, Phys. Lett. B 464, 111 (1999) [arXiv:hep-ph/9908227].
[14] M. Rho, A. Wirzba and I. Zahed, Phys. Lett. B 473, 126 (2000) [arXiv:hep-ph/9910550].
[15] D. T. Son and M. A. Stephanov, Phys. Rev. D 61, 074012 (2000) [hep-ph/9910491];
    erratum, ibid. D 62, 059902 (2000) [hep-ph/0004095].
[16] C. Manuel and M. H. Tytgat, Phys. Lett. B 479, 190 (2000) [arXiv:hep-ph/0001095].
[17] S. R. Beane, P. F. Bedaque and M. J. Savage, Phys. Lett. B 483, 131 (2000) [arXiv:hep-ph/0002209].
[18] T. Schafer, [arXiv:hep-ph/0109052].
[19] P. F. Bedaque and T. Schafer, Nucl. Phys. A 697, 802 (2002) [arXiv:hep-ph/0105150].
[20] D. B. Kaplan and S. Reddy, Phys. Rev. D 65, 054042 (2002) [arXiv:hep-ph/0107265].
[21] T. Schafer, [arXiv:hep-ph/0201189].
[22] R. Casalbuoni, Z. Duan and F. Sannino, Phys. Rev. D 63, 114026 (2001) [arXiv:hep-ph/0011394].
[23] P. Jaikumar and M. Prakash, Phys. Lett. B 516, 345 (2001) [arXiv:astro-ph/0105225].
[24] S. Reddy, M. Prakash and J. M. Lattimer, Phys. Rev. D 58, 013009 (1998) [arXiv:astro-ph/9710113].
[25] N. Iwamoto, Ann. Phys. 141, 1 (1982)
[26] P. Jaikumar, M. Prakash and T. Schaefer, [arXiv:astro-ph/0203088].