Manifestly $\text{SL}(2, \mathbb{R})$ Duality-Symmetric Forms in ModMax Theory

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Abstract: In this paper, we will investigate a manifestly $\text{SL}(2, \mathbb{R})$-invariant structure for the energy-momentum tensor of ModMax theory as a nonlinear modification of Maxwell electrodynamics which includes conformal invariance as well. In the context of this theory, we show that the energy-momentum tensor of the generalized Born-Infeld theory can also be written in the same invariant form. We will find manifestly duality-invariant parts of the actions corresponding to the invariant couplings $\lambda$ and $\gamma$ in these theories. It can be shown that the resultant actions correspond to the irrelevant and marginal $T\bar{T}$-like deformations, respectively.

Keywords: Duality in Gauge Field Theories, Effective Field Theories, Field Theories in Lower Dimensions

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1 Introduction

It has been found different classes of non-linear electrodynamic (NED) theories as candidates to the extension of Maxwell electrodynamics in the literature [1–4]. Also, one can find some recent developments in this area of research in refs. [5–8]. Recently, it has been discovered a nonlinear extension of Maxwell’s theory, which not only is invariant under electromagnetic duality transformations, but also is a conformally invariant theory. This electromagnetic modification is known as the ModMax theory [9] and is described by the following Lagrangian density

\[ \mathcal{L}_{MM} = \cosh(\gamma)S + \sinh(\gamma)\sqrt{S^2 + P^2}, \]  

(1.1)

where \( S = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \) and \( P = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} \) are two Lorentz invariant variables, which \( \tilde{F}^{\mu\nu} = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \) is the Hodge dual of the electromagnetic field strength \( F_{\mu\nu} \). As is obvious from (1.1), the ModMax theory comprises a positive constant \( \gamma \) so that for \( \gamma = 0 \), it reduces to the Maxwell Lagrangian [10]. The generalization of the ModMax theory in the context of conformal invariant p-form gauge theories was studied in ref. [11].

It has been shown [9, 11] that the ModMax and the Born-Infeld theories can be combined into a duality-symmetric generalized Born-Infeld (GBI) electrodynamics as follows

\[ \mathcal{L}_{BI\gamma} = \frac{1}{\lambda} \left[ 1 - \sqrt{1 - \lambda(2\mathcal{L}_{MM} + \lambda P^2)} \right], \]  

(1.2)

where in the weak field limit \( \lambda \to 0 \), the Lagrangian (1.2) reduces to the ModMax theory denoted by (1.1). More studies on the ModMax theory, such as the supersymmetric actions,
the nonlinear generalizations, and solutions of field equations can be found in refs. [12–19].

Also, an alternative form of the ModMax Lagrangian in terms of axion-dilaton-like auxiliary scalar fields coupling has been proposed in ref. [20]. ModMax theory has been more carefully studied and expanded in recent works [19, 21–24].

As well known, the Maxwell’s field equations are invariant under the electric-magnetic SO(2) duality transformations, however, the enhancement of this symmetry to the level of an action is another problematic task. This fact becomes even more complicated for theories featuring nonlinear interactions of the Maxwell fields. There are three approaches to construct theories of self-interacting Maxwell fields incorporating an electromagnetic SO(2) duality invariance; the Gaillard and Zumino [4, 25, 26] approach which has been further developed by Gibbons and Rasheed [27], the non-covariant first order Hamiltonian approach based on the work by Henneaux and Teitelboim [28] and further followed by Deser, Gomberoff, Henneaux and Teitelboim [29, 30], and the third one is the PST method proposed by Pasti, Sorokin and Tonin [31–34].

In the first systematic method to construct duality invariant theories, we have to define a doublet \( (\tilde{F}, G) \), where \( G_{\mu\nu} = -2 \frac{\partial L(S, P)}{\partial F_{\mu\nu}} \) is an antisymmetric tensor, to show that the equations of motion are invariant under SO(2) rotations. This requirement imposes a non-linear differential equation; \( G \tilde{G} - F \tilde{F} = 0 \), the so-called duality-invariant condition. In the second approach, SO(2) duality is an exact and explicit symmetry of the Lagrangian of the theory. Such a NED theory has a doublet of electric and magnetic fields \( (A_1, A_2) \) and the SO(2) duality is an ordinary 2d rotation on these doublets. Finally in the PST approach, if one introduces an auxiliary scalar field and a compensating gauge symmetry, it is possible to formulate a manifestly covariant action with the correct degrees of freedom.

We will follow the Gaillard-Zumino approach and present the duality-invariant actions for the ModMax and GBI (and axion-dilaton-GBI) theories which gives the correct equations of motion when one imposes the duality invariant condition as an extra constraint. As the main purpose of our study in this letter, we find a manifestly SL(2, R) invariant structure for the energy-momentum tensors of both the ModMax and the GBI theories. In this respect, although the corresponding Lagrangians are not invariant under SO(2) duality-symmetry, their derivatives with respect to the invariant parameters are. The invariant parameter could be a coupling constant or an external background field, such as the gravitational field, which does not change under duality rotations [25]. In this approach if a theory satisfies the duality-invariant condition, then the physical objects of the theory such as equations of motion [35], energy-momentum tensor [27, 36] and scattering amplitudes [37–39] are duality invariant. We are also interested in finding corresponding duality invariant theory in the form that are manifestly SL(2, R) invariant. Actually, the duality-invariant parts of the Lagrangian is given by the derivative of the original Lagrangian with respect to the constant parameters \( \gamma \) and \( \lambda \) [41].

The structure of this paper is as follows: in section 2, we enhance SO(2) duality in the ModMax as well as the GBI theories to the non-compact SL(2, R) group by coupling the electromagnetic field to the dilaton and axion fields. In section 3, we find the manifestly duality-invariant parts of the actions for a general non-linear GBI theories of electrodynamics. We show that the expansion of a duality-invariant Lagrangian in NED theories is compatible
with the deformed Lagrangian constructed by both irrelevant and marginal $T\bar{T}$ operators in NED theories. Finally, the section 4 is devoted to giving a brief summary of results and identifies some directions for future researches.

2 SL(2, R) symmetry in axion-dilaton ModMax Theory

The equations of motion of free Maxwell theory are $\partial_\nu F^{\mu\nu} = 0$ and $\partial_\nu \tilde{F}^{\mu\nu} = 0$. These equations obviously are transformed into each other under the rotation of $(F^{\mu\nu}, \tilde{F}^{\mu\nu}) \rightarrow (\tilde{F}^{\mu\nu}, -F^{\mu\nu})$. This duality, which is known as the electric-magnetic duality, is a special case of the SO(2) duality-symmetry. In this section we are going to study the SO(2) symmetry for the energy-momentum tensor of the ModMax theory and its extension to SL(2, R) symmetry by coupling the electromagnetic field to some axion and dilaton fields. Consequently, we find the SL(2, R) duality-symmetric structures for the energy-momentum tensor of the ModMax and GBI theories.

2.1 Electric-magnetic duality in the ModMax theory

According to [42], a general electric-magnetic duality rotation on an angle $\alpha$ is given by

$$
\begin{align*}
G_{\mu\nu} &\rightarrow \cos \alpha \, G_{\mu\nu} + \sin \alpha \, \tilde{F} \\
\tilde{F}_{\mu\nu} &\rightarrow \cos \alpha \, \tilde{F}_{\mu\nu} - \sin \alpha \, G_{\mu\nu}.
\end{align*}
$$

(2.1)

This rotational symmetry is called the SO(2)-duality symmetry. Assume a finite SO(2)-duality rotation of the angle $\pi/2$, then we have the following relations for the fields and invariant parameters

$$
\left( G^{\mu\nu}, \tilde{F}^{\mu\nu} \right) \rightarrow \left( \tilde{F}^{\mu\nu}, -G^{\mu\nu} \right), \quad \left( g_{\mu\nu}, \gamma, \lambda \right) \rightarrow \left( g_{\mu\nu}, \gamma, \lambda \right).
$$

(2.2)

One can find the $G$-tensor for the ModMax theory (1.1) as follows

$$
G_{\mu\nu} = -2 \frac{\partial L_{MM}}{\partial F^{\mu\nu}} = \cosh(\gamma)F_{\mu\nu} + \sinh(\gamma) \frac{F_{\mu\nu}S + \tilde{F}_{\mu\nu}\mathcal{P}}{\sqrt{S^2 + \mathcal{P}^2}}.
$$

(2.3)

It has been found in ref. [27] that the energy-momentum tensor of a NED theory can be written as

$$
T_{\mu\nu} = g_{\mu\nu} \mathcal{L}(S, \mathcal{P}) + F^{\rho}_{\mu} G_{\nu\rho},
$$

(2.4)

therefore, by considering the above relation, the energy-momentum tensor of the ModMax theory (1.1) can be found as

$$
T_{\mu\nu} = T_{\mu\nu}^{\text{Max}} \left( \cosh(\gamma) + \frac{S}{\sqrt{S^2 + \mathcal{P}^2}} \sinh(\gamma) \right),
$$

(2.5)

where $T_{\mu\nu}^{\text{Max}} = F_{\rho\mu} F^{\rho}_{\nu} + g_{\mu\nu} S$ is the energy-momentum tensor of the Maxwell theory. Now, in order to study the behavior of the energy-momentum tensor (2.5) under the nonlinear
duality transformations (2.2), we define the following symmetric structure that is invariant under these transformations

\[ N_{\mu\nu} = G_{\mu}^{\alpha} G_{\alpha\nu} + \tilde{F}_{\mu}^{\alpha} \tilde{F}_{\alpha\nu}. \]  

(2.6)

Also substituting the ModMax $G$-tensor (2.3) in eq. (2.6), one can find the symmetric invariant structure corresponding to the ModMax theory

\[ N_{\mu\nu} = -2 \cosh(\gamma) \left( \cosh(\gamma) + \frac{S}{\sqrt{S^2 + P^2}} \sinh(\gamma) \right) T_{\mu\nu}^{\text{Max}} \]

\[ + 2 \sinh(\gamma) \left( \cosh(\gamma) \sqrt{S^2 + P^2} + \sinh(\gamma) S \right) g_{\mu\nu}. \]  

(2.7)

Considering the energy-momentum tensor (2.5) and the invariant structure (2.7), we obtain a manifestly invariant $T_{\mu\nu}$ of the form

\[ T_{\mu\nu} = -\frac{1}{2 \cosh(\gamma)} \left[ N_{\mu\nu} - \frac{1}{4} N_{\rho\rho} g_{\mu\nu} \right], \]  

(2.8)

where $N_{\rho\rho} = 8 \sinh(\gamma) \left( \cosh(\gamma) \sqrt{S^2 + P^2} + \sinh(\gamma) S \right)$ is the trace of symmetric structure $N_{\mu\nu}$. Note also that $N_{\rho\rho}$ vanishes at $\gamma = 0$ and we achieve the standard Maxwell energy-momentum tensor.

### 2.2 ModMax theory coupled to an axion-dilaton field

Consider the following nonlinear $\text{SL}(2,R)$ transformation

\[ \tau \rightarrow \frac{p\tau + q}{r\tau + s}, \quad \Lambda = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \in \text{SL}(2,R), \]  

(2.9)

where $\tau = C_0 + i e^{-\phi_0}$ is a complex axion-dilaton field. Now, for a Lagrangian in which the general NED theory is coupled with the axion field so that the equations of motion are $\text{SL}(2,R)$ invariant, one should add the term $\frac{1}{4} C_0 F_{\mu\nu} \tilde{F}^{\mu\nu}$ to the primary NED Lagrangian. On the other hand, the contribution of dilatonic field is given by the field redefinition $F_{\mu\nu} \rightarrow e^{-\frac{\phi_0}{2}} F_{\mu\nu}$ [26]. Thus, the final theory appears an axion-dilatonic NED theory denoted by Lagrangian density $\hat{\mathcal{L}}(\tau, F) = \hat{\mathcal{L}}(e^{-\frac{\phi_0}{2}} F) + \frac{1}{4} C_0 F_{\mu\nu} \tilde{F}^{\mu\nu}$. In this regard, the SO(2) duality group in eq. (2.2) enhances to the $\text{SL}(2,R)$ symmetry group. Due to this consideration, one can find the ModMax theory coupled to axion-dilaton field [13] as

\[ \hat{\mathcal{L}}_{\text{MM}}(\tau, F) = \hat{\mathcal{L}}_{\text{MM}} \left( e^{-\frac{\phi_0}{2}} F \right) - C_0 \mathcal{P} = \cosh(\gamma) e^{-\phi_0} S + \sinh(\gamma) \sqrt{e^{-2\phi_0} (S^2 + P^2)} - C_0 \mathcal{P}. \]  

(2.10)

It is clear that the above theory reduces to $\hat{\mathcal{L}}_{\text{Max}}(\tau, F)$ in the limit of $\gamma \rightarrow 0$. So, the energy-momentum tensor obtained from Lagrangian (2.10), only differs up to a dilaton field factor from the energy-momentum tensor of the pure ModMax theory. This means that $\hat{T}_{\mu\nu} = e^{-\phi_0} T_{\mu\nu}$ where the tensor $T_{\mu\nu}$ is given by eq. (2.5). We can also find the $G$-tensor corresponding to the axion-dilaton ModMax theory (2.10) from $\hat{G}_{\mu\nu} = -\frac{1}{2} \frac{\partial \hat{\mathcal{L}}_{\text{MM}}(\tau, F)}{\partial F^{\mu\nu}}$. Thus, we have the antisymmetric tensor $\hat{G}_{\mu\nu} = \hat{G}_{\mu\nu} - C_0 \tilde{F}_{\mu\nu}$ where $\tilde{G}_{\mu\nu} = -\frac{\partial \hat{\mathcal{L}}_{\text{MM}}}{\partial F^{\mu\nu}} = e^{-\phi_0} G_{\mu\nu}$ and $G_{\mu\nu}$ is given in eq. (2.3).
According to \cite{27}, for any NED theory one can extend the duality transformations (2.2) to the corresponding SL(2, R) duality group. In the case of ModMax theory, the corresponding SL(2, R) duality transformations is described by
\[ \mathcal{F}_{\mu\nu} \equiv \left( \tilde{G}_{\mu\nu} - C_0 \tilde{F}_{\mu\nu} \right) \rightarrow (\Lambda^{-1})^T \left( \tilde{G}_{\mu\nu} - C_0 \tilde{F}_{\mu\nu} \right), \] (2.11)
and\footnote{Note that the matrix \( \mathcal{M}_0 \) here is the inverse of the matrix \( \mathcal{M} \) in ref. \cite{27}.}
\[ \mathcal{M}_0 = e^{\phi_0} \begin{pmatrix} |\tau|^2 & C_0 \\ C_0 & 1 \end{pmatrix}, \quad \mathcal{M}_0 \rightarrow \Lambda \mathcal{M}_0 \Lambda^T. \] (2.12)
Using the transformations (2.11) and (2.12), we can find a symmetric structure in the form of \( \hat{N}_{\mu\nu} = (\mathcal{F}^T)^{\alpha\beta} \mathcal{M}_0 \mathcal{F}_{\alpha\beta} \) which is manifestly invariant under the SL(2, R) duality transformations. That is,
\[ \hat{N}_{\mu\nu} = e^{-\phi_0} \tilde{F}_{\mu\alpha} \tilde{F}_{\nu\alpha} + e^{\phi_0} \tilde{G}_{\mu\alpha} \tilde{G}_{\nu\alpha} = e^{-\phi_0} N_{\mu\nu}, \] (2.13)
where we have used \( \tilde{G}_{\mu\nu} = e^{-\phi_0} G_{\mu\nu} \) and eq. (2.6) to obtain the second line. On the other hand, the energy-momentum tensor corresponding to eq. (2.10) is given as follows
\[ \hat{T}_{\mu\nu} = -\frac{1}{2 \cosh(\gamma)} \left[ \hat{N}_{\mu\rho} - \frac{1}{4} \hat{N}_{\rho\rho} g_{\mu\nu} \right], \] (2.14)
which is a manifestly SL(2, R) invariant structure in the axion-dilaton ModMax theory.

### 2.3 SL(2, R) symmetry in the GBI theory

Following the discussion in the previous section, one can construct a GBI theory with SL(2, R) symmetry group by adding an axion-dilaton field to BI\( \gamma \) theory given by eq. (1.2), i.e., as \( \hat{L}_{BI\gamma}(\tau, F) = \hat{L}_{BI\gamma}(e^{-\phi_0} F) - C_0 P \). Due to this identification we obtain
\[ \hat{L}_{BI\gamma}(\tau, F) = \frac{1}{\lambda} \left[ 1 - \sqrt{1 - \lambda \left( 2 e^{-\phi_0} (\cosh(\gamma) S + \sinh(\gamma) \sqrt{(S^2 + P^2)}) + \lambda e^{-2\phi_0} P^2 \right)} - C_0 P \right]. \] (2.15)

Similar to what we have a priori done for the ModMax theory, we are interested in the SL(2, R) invariant structure for the energy-momentum of GBI theory in (2.15). Therefore, using the standard definition of the energy-momentum tensor and after a straightforward calculation, we obtain
\[ \hat{T}_{\mu\nu} = e^{-\phi_0} \frac{F_{\mu\alpha} F_{\nu}^{\alpha}(\cosh(\gamma) \sqrt{S^2 + P^2} + \sinh(\gamma) S)}{x \sqrt{S^2 + P^2}} \]
\[ + e^{-\phi_0} \frac{(2 \lambda \cosh(\gamma) S \sqrt{S^2 + P^2} + \lambda \sinh(\gamma) (P^2 + 2 S^2) + e^{\phi_0} (x - 1) \sqrt{S^2 + P^2}) g_{\mu\nu}}{\lambda x \sqrt{S^2 + P^2}}, \] (2.16)
where, for simplicity, we use the shorthand $x = \sqrt{1 - \lambda(2e^{-\phi_0} \left( \cosh(\gamma) S + \sinh(\gamma) \sqrt{(S^2 + P^2)} \right) + \lambda e^{-2\phi_0} P^2)}$.

The $\lambda$-expansion of the above energy momentum tensor up to order $\lambda$ is as follows

$$\tilde{T}_{\mu\nu} = e^{-\phi_0} \left( \cosh(\gamma) + \frac{\sinh(\gamma) S}{\sqrt{S^2 + P^2}} \right) T_{\mu\nu}^{\text{Max}} + e^{-2\phi_0} \lambda \left[ \left( \cosh(2\gamma) S + \frac{\sinh(2\gamma) (P^2 + 2S^2)}{2\sqrt{S^2 + P^2}} \right) T_{\mu\nu}^{\text{Max}} \right] - \frac{1}{2} \left( \cosh^2(\gamma) P^2 + S \left( \cosh(2\gamma) S - \sinh(2\gamma) \sqrt{S^2 + P^2} \right) \right) g_{\mu\nu} + \mathcal{O}(\lambda^2) + \ldots \quad (2.17)$$

The leading term in the expansion of (2.17) is the energy-momentum tensor of the axion-dilaton ModMax theory that we found in terms of $\text{SL}(2,R)$ invariant structure in eq. (2.14). In fact, if the eq. (2.17) is to be invariant under the $\text{SL}(2,R)$ transformations, one should find it invariant at any order of $\lambda$ as well.

In comparison to the axion-dilaton ModMax theory, the antisymmetric $G$-tensor of generalized theory in eq. (2.15) could be found from $\tilde{G}_{\mu\nu} = -2 \frac{\partial F_{\mu\nu}(\gamma,F)}{\partial \mu\nu}$. However, one can write the result as $\tilde{G}_{\mu\nu} = \tilde{G}_{\mu\nu} - C_0 \tilde{F}_{\mu\nu}$, where

$$\tilde{G}_{\mu\nu} = e^{-\phi_0} \frac{\sinh(\gamma)(F_{\mu\nu} S + \tilde{F}_{\mu\nu} P)}{\sqrt{S^2 + P^2}} + e^{-\phi_0} \frac{\cosh(\gamma) F_{\mu\nu} + e^{-2\phi_0} \lambda P \tilde{F}_{\mu\nu}}{x}. \quad (2.18)$$

The $\lambda$-expansion of the above tensor up to order $\lambda$ is as follows

$$\tilde{G}_{\mu\nu} = e^{-\phi_0} \left[ \cosh(\gamma) F_{\mu\nu} + \frac{\sinh(\gamma)(F_{\mu\nu} S + \tilde{F}_{\mu\nu} P)}{\sqrt{S^2 + P^2}} \right] + e^{-2\phi_0} \lambda \left[ \cosh^2(\gamma) \tilde{F}_{\mu\nu} P + \frac{\cosh(\gamma) \sinh(\gamma) \tilde{F}_{\mu\nu} PS}{\sqrt{S^2 + P^2}} + \frac{F_{\mu\nu}(2 \cosh(2\gamma) S \sqrt{S^2 + P^2} + \sinh(2\gamma) (P^2 + 2S^2))}{2\sqrt{S^2 + P^2}} \right] + \mathcal{O}(\lambda^2) + \ldots \quad (2.19)$$

Substituting $\tilde{G}_{\mu\nu}$ from (2.19) in eq. (2.13), we find the $\text{SL}(2,R)$ invariant structure corresponding to (2.15) up to order $\lambda$, i.e.,

$$\tilde{\mathcal{N}}_{\mu\nu} = -2e^{-\phi_0} \left[ \cosh(\gamma) \left( \cosh(\gamma) + \frac{S \sinh(\gamma)}{\sqrt{S^2 + P^2}} \right) T_{\mu\nu}^{\text{Max}} \right] - \frac{1}{2} \left( \cosh(\gamma) P^2 + \cosh(3\gamma) \left( P^2 + 2S^2 \right) + 2 \sinh(3\gamma) S \sqrt{S^2 + P^2} \right) g_{\mu\nu} + \mathcal{O}(\lambda^2) + \ldots \quad (2.20)$$

Now, by considering this invariant structure and after performing some lengthy calculations we are able to recast the energy-momentum tensor of the axion-dilaton $BI\gamma$ theory in
\[\hat{T}_{\mu\nu} = -\frac{1}{2 \cosh (\gamma)} \left( \hat{N}_{\mu\nu} - \frac{1}{4} \hat{N}_{\rho}^{\rho} g_{\mu\nu} \right) + \lambda \left[ a_0(\gamma) \hat{N}_{\rho}^{\rho} \hat{N}_{\mu\nu} + b_0(\gamma) \hat{N}_{\mu}^{\rho} \hat{N}_{\nu\rho} \right] + \ldots, \tag{2.21}\]

where the coefficients \(a_0(\gamma)\) and \(b_0(\gamma)\) are given by

\[a_0(\gamma) = -\frac{1}{16 \cosh^2 (\gamma)} + \frac{3}{16}, \quad b_0(\gamma) = \frac{1}{4 \cosh^2 (\gamma)} - \frac{3}{8}.\]

Note that in eq. (2.21) we have also used the identities \(F_{\mu\alpha} \tilde{F}_{\nu\alpha} = -g_{\mu\nu} P\) and \(F_{\mu\alpha} F_{\gamma\beta} F_{\gamma\beta} F_{\nu} = 2 F_{\mu}^{\alpha} F_{\alpha\nu} S + g_{\mu\nu} P^2\) that hold for any two-form \(F\). The explicit forms of the invariant structures that appear in eq. (2.21) have been mentioned in the appendix.

3 Manifestly duality-invariant actions

As mentioned before, the Lagrangian of the Maxwell and the ModMax theories are not invariant under the SL(2, \(R\)) transformation while their derivatives with respect to an invariant parameter are. The invariant parameter could be a coupling constant or an external background field, such as the gravitational field, which does not change under duality rotations. For instance, the energy-momentum tensor that is obtained from the variation of the action with respect to the gravitational field \(g_{\mu\nu}\) would be invariant under duality rotations (2.1) [25, 26]. In this paper we deal with two coupling constants \(\gamma\) and \(\lambda\) in the ModMax and its generalization GBI theories.

It has been shown in ref. [43] that the non-zero contribution of the duality-invariant action in electromagnetic theories comes from the variation of original theory with respect to the electromagnetic field strength. Also, in [41] it was shown that the derivative of the original Lagrangian \(L\) with respect to the coupling constant \(c\) of the form \(c \partial L / \partial c\) produces the corresponding invariant theory \(L_{\text{inv}}^c\). In other words, though the action is a dimensionless quantity in general field theories, its variation with respect to a dimensionful parameter is not necessarily. For example, in GBI theory since \(\lambda\) is a dimensionful parameter, the derivative of the action with respect to \(\lambda\) is dimensionful and one needs a coefficient of type \(\lambda\) so that it does not have a physical dimension. In contrast, for a theory with a typical dimensionless coupling constant such as the ModMax theory, the derivative itself can be an invariant quantity alone.

Due to this fact and according to (1.2), consider the invariant Lagrangians \(L_{\text{inv}}^\lambda\) and \(L_{\text{inv}}^\gamma\) which correspond to the coupling constants \(\lambda\) and \(\gamma\), respectively. Obviously, the Lagrangians of the Maxwell theory as well as the ModMax theory are independent of \(\lambda\), therefore there is no duality-invariant Lagrangian of type \(L_{\text{inv}}^\lambda\) for these theories. On the other hand, \(L_{\text{inv}}^\gamma\) has non-vanishing contribution only for the ModMax theory, while the derivative of the Lagrangian density of Maxwell theory with respect to \(\gamma\) vanishes. We ensue the following invariant action from the above discussion for the ModMax theory

\[\frac{\partial L_{\text{MM}}}{\partial \gamma} = L_{\text{inv}}^\gamma - L_{\text{inv-MM}} = \sinh (\gamma) S + \cosh (\gamma) \sqrt{S^2 + P^2}. \tag{3.1}\]
Comparing the duality-invariant Lagrangian (3.1) with eq. (2.7), we can find a manifestly SO(2) duality-symmetric form for the Lagrangian (3.1) as follows

\[ \mathcal{L}_{\text{inv-MM}}^\gamma = \frac{1}{8 \sinh(\gamma)} N^\rho_\rho. \]  

(3.2)

It is noticed that there is a similar consideration for the GBI theory (1.2) as well. In fact, variations of the GBI Lagrangian with respect to the couplings \( \lambda \) and \( \gamma \), which are respectively denoted by \( \mathcal{L}_{\text{inv-BI}\gamma}^\lambda \) and \( \mathcal{L}_{\text{inv-BI}\gamma}^\gamma \), are invariant under SO(2) duality group. In the rest of this section, we will derive the frameworks of these invariant theories in the context of SL(2, R) structures.

### 3.1 Duality-invariant Lagrangian: \( \mathcal{L}_{\text{inv-BI}\gamma}^\lambda \)

For the purpose of constructing the duality-invariant parts of the Lagrangian for GBI theory, let us briefly review the construction of this duality for an arbitrary electrodynamic theory \( \mathcal{L}(S, P) \) which satisfies the duality invariant condition. Now, one can define an invariant Lagrangian \( \mathcal{L}_{\text{inv}}^\lambda \) so that is duality-invariant as follows [5, 41, 43–45]

\[ -\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = \mathcal{L}_{\text{inv}}^\lambda = \mathcal{L}(S, P) + \frac{1}{4} F_{\mu\nu} G^{\mu\nu}. \]  

(3.3)

As we mentioned above, due to the lack of dependence on \( \lambda \), the invariant actions for both the Maxwell theory and the ModMax theory are trivial. For example, in the case of ModMax theory one can check this fact by inserting \( \mathcal{L}_{\text{MM}} \) from (1.1) and \( G_{\mu\nu} \) from (2.3) in eq. (3.3). Now, consider a family of NED theories that led to the Maxwell theory (or the ModMax theory) at the first order of coupling constant, i.e. \( \mathcal{O}(F^2) \). It seems that the nonzero duality-invariant contribution of these NED theories appears from the order of \( \mathcal{O}(F^4) \) [43], but we will show that this statement is not true for the ModMax theories. By comparing (2.4) and (3.3), one can find that \( \mathcal{L}_{\text{inv}}^\lambda = \frac{1}{4} T_\mu^\mu \). This interesting result asserts that the necessary condition for a NED theory to have nonzero contribution to duality-invariant Lagrangian (3.3) is that \( T_\mu^\mu \neq 0 \).

The axion-dilaton \( BI\gamma \) theory satisfies both duality invariant condition \( G\tilde{G} - F\tilde{F} = 0 \) and the energy-momentum traceless condition \( T_\mu^\mu \neq 0 \). If we set \( \phi_0 = 0 \) and \( C_0 = 0 \) in (2.15) then we will find the corresponding duality invariant Lagrangian as follows

\[ \mathcal{L}_{\text{inv-BI}\gamma}^\lambda = \frac{1}{\lambda} \left[ 1 - \frac{1}{\sqrt{1 - \lambda(\mathcal{L}_{\text{MM}} + \lambda P^2)}} \right] + \lambda \frac{\mathcal{L}_{\text{MM}}}{\sqrt{1 - \lambda(2\mathcal{L}_{\text{MM}} + \lambda P^2)}}. \]  

(3.4)

In other words, a comparison between eq. (2.17) for \( \phi_0 = 0 \) and eq. (3.4) shows that the identity \( \mathcal{L}_{\text{inv}}^\lambda = \frac{1}{4} T_\mu^\mu \) works truly. Also, it is worth mentioning that the \( \lambda \)-expansion of Lagrangian (3.4) starts from the order of \( \lambda \), or equivalently \( \mathcal{O}(F^4) \).

Considering the duality invariant structure (2.6) in which \( G_{\mu\nu} \) is substituted from (2.19) with \( \phi_0 = 0 \), we find the \( \lambda \)-expansion of action (3.4) in the form that is manifestly duality invariant

\[ \mathcal{L}_{\text{inv-BI}\gamma}^\lambda = -\frac{\lambda}{2^7 \sinh^3(\gamma)} N_{\mu}^\mu N_{\nu}^\nu + \frac{\lambda^2 \cosh(\gamma)}{2^9 \sinh^4(\gamma)} N_{\mu}^\mu N_{\nu}^\nu N_{\alpha}^\alpha + \ldots, \]  

(3.5)
in which we have used the following identity

\[ N_{\mu\nu} N^{\mu\nu} = \frac{\cosh(2\gamma)}{4 \sinh^2(\gamma)} N_{\mu}^{\mu} N_{\nu}^{\nu}. \]  

As alluded before, the SO(2)-invariant action \( L_{\text{inv}}^{\lambda - BI\gamma} \) can be related to the action (1.2) by a differential flow equation

\[ -\lambda \frac{\partial L_{\text{inv}}^{\lambda - BI\gamma}}{\partial \lambda} = L_{\text{inv}}^{\lambda}. \]

Therefore, one can find the relationship of the duality invariant actions (3.4) to the irrelevant \( T\bar{T} \)-like deformation of the GBI theory [46–48] as well as Born-Infeld theory [49–51] (the GBI theory at the limit of \( \gamma \to 0 \)), that is \( L_{\text{inv}}^{\lambda} = -\frac{1}{8} \lambda O_{T2}^{\lambda} \) where \( O_{T2}^{\lambda} = T_{\mu\nu} T^{\mu\nu} - \frac{1}{2} T_{\mu}^{\mu} T_{\nu}^{\nu} \) is an irrelevant \( T\bar{T} \) operator.

### 3.2 Duality-invariant Lagrangian: \( L_{\text{inv}}^{\gamma - BI\gamma} \)

In this subsection, we consider the second duality-invariant parts of the action coming from the derivative of GBI Lagrangian (1.2) with respect to the parameter \( \gamma \). One can find that

\[ L_{\text{inv}}^{\gamma - BI\gamma} = \frac{\partial L_{\text{BI}\gamma}}{\partial \gamma} = \frac{\sinh(\gamma) S + \cosh(\gamma) \sqrt{S^2 + P^2}}{\sqrt{1 - \lambda (2L_{MM} + \lambda P^2)}}, \]

therefore, expanding the invariant action (3.7) up to order \( \lambda \) yields that

\[ L_{\text{inv}}^{\gamma - BI\gamma} = \sinh(\gamma) S + \cosh(\gamma) \sqrt{S^2 + P^2} \]

\[ + \lambda \left( \cosh(2\gamma) S \sqrt{S^2 + P^2} + \cosh(\gamma) \sinh(\gamma) \left( P^2 + 2S^2 \right) \right) + O(\lambda^2) + \ldots. \]

As the same steps that we did in the previous subsection, by considering the invariant structure (2.6) and \( \phi_0 = 0 \), we find the above action in the form that is manifestly duality invariant as follows

\[ L_{\text{inv}}^{\gamma - BI\gamma} = \frac{1}{8 \sinh(\gamma)} N_{\mu}^{\mu} - \frac{1}{64 \sinh^3(\gamma)} N_{\mu}^{\mu} N_{\nu}^{\nu} + O(\lambda^2) + \ldots. \]

Note that, the duality-invariant Lagrangian \( L_{\text{inv}}^{\gamma - BI\gamma} \) starts from the order of \( F^4 \), but the duality-invariant Lagrangian \( L_{\text{inv}}^{\lambda - BI\gamma} \) starts from the order of \( F^2 \). Actually, the nonzero contribution of the invariant action \( L_{\text{inv}}^{\gamma - BI\gamma} \) at the Maxwell order can be interested because the other known NED theories have no contribution for their corresponding invariant actions at this order.

In general, by replacing \( N \) with \( \hat{N} \), the results that we have found in this section extend to the case of \( \phi_0 \neq 0 \) and \( C_0 \neq 0 \). Doing so, we extend the SO(2) duality invariant actions (3.5) and (3.9) to corresponding SL(2, \( R \))-invariant cases. It would be also of interest to notice that the duality-invariant Lagrangian (3.7) can be related to a marginal \( T\bar{T} \)-like deformation of the ModMax theory [46]. One can show that \( L_{\text{inv}}^{\gamma - BI\gamma} = \frac{1}{2} O_{T2}^{\gamma} \), where \( O_{T2}^{\gamma} = \sqrt{T_{\mu\nu} T^{\mu\nu} - \frac{1}{4} T_{\mu}^{\mu} T_{\nu}^{\nu}} \) is a marginal operator. It has been discussed in refs. [48, 52, 53] that a dimensional reduction of the ModMax theory to two spacetime dimensions corresponds to the continuous marginal root-\( T\bar{T} \) deformation of free bosons.
4 Discussions

In this paper, we studied the duality-invariant structures of the conformal nonlinear modification of the Maxwell electrodynamics (ModMax theory) and its generalization. We found an SL(2, R) invariant form for the energy-momentum tensor of the ModMax theory in terms of some SL(2, R) duality-symmetric structures. We have also obtained the manifestly duality invariant structures for the energy-momentum tensor of the GBI theory.

In section 3, we have proposed two actions which are invariant under SL(2, R) symmetry, and consistent with two irrelevant and marginal T\overline{T}-like deformations. The first duality-invariant action (3.4) is related to the standard form of the expansion of the GBI Lagrangian with respect to \( \lambda \), and the second one (3.7) concerns with the expansion of the GBI Lagrangian with respect to \( \gamma \), proposed in ref. [11], which are manifestly SL(2, R)-duality invariant and include axion-dilaton couplings.

As custom, the correlation functions are fundamental observables in QFTs. For instance, it has been shown in refs. [54–57] that the deformed Lagrangian and stress tensor could obtained order by order from a \( T\overline{T} \)-product operator in two dimensional CFTs, which can be used to expand the partition function up to the second-order. It would be of interest to study the correlation functions in the context of the ModMax theory and check out the consistency of two duality-invariant actions and \( T\overline{T} \)-like deformations.

The extension of supersymmetric theories to NED theories are also of great interest [58, 59]. For example, the supersymmetric extension of the Born-Infeld Lagrangian has been studied in ref. [60] and in the case of ModMax theory this extension was discussed in refs. [12, 13]. Recently, I. Bandos et al. [12] (see also [13]) have shown that the Born-Infeld-like extension of superModMax theory is described by the following superfield Lagrangian density

\[
\mathcal{L}_{\text{susy–BI}} = \cosh(\gamma) \left\{ \int d^2\theta W^2 + \int d^2\bar{\theta} \bar{W}^2 + \int d^2\theta d^2\bar{\theta} W^2 \bar{W}^2 K(S, P) \right\}, \tag{4.1}
\]

where \( K(S, P) \) is given by

\[
K(S, P) = \frac{1}{\cosh(\gamma)} \frac{1}{(S^2 + P^2)} \left( \cosh(\gamma) S + \sinh(\gamma) \sqrt{\cosh^2(\gamma) S^2 + \cosh^2(\gamma) P^2} - \lambda^2 P^2 \right) - \cosh(\gamma) S. \tag{4.2}
\]

Here, the superfields \( S \) and \( P \) are

\[
S = -\frac{1}{16} \left( D^2 W^2 + \bar{D}^2 \bar{W}^2 \right), \quad P = \frac{i}{16} \left( D^2 W^2 - \bar{D}^2 \bar{W}^2 \right), \tag{4.3}
\]

where \( W^2 = W^\alpha W_\alpha \). \( W_\alpha \) and its conjugate \( \bar{W}_\dot{\alpha} = (W_\alpha)^* \), are anticommuting Weyl spinor chiral superfields satisfying in the conditions

\[
\bar{D}_\dot{\alpha} W_\alpha = 0, \quad D^\alpha W_\alpha = \bar{D}_\alpha \bar{W}^\dot{\alpha}. \tag{4.4}
\]

In the limit \( \gamma = 0 \), the Lagrangian (4.1) reduces to the Bagger-Galperin Lagrangian [60]. The duality-invariant condition for electromagnetic duality invariance of generic nonlinear
electrodynamic theories in refs. [25, 26], were generalized to superfield formulations of 
N = 1 supersymmetric theories in refs. [61, 62]. For generic N = 1 theories described by a 
Lagrangian L[\bar{W}, W] of the ModMax theory can be investigated as a future work.

The Born-Infeld-like extension of the supersymmetric ModMax theory in (4.1) satisfies the 
duality-invariance condition (4.5). Therefore, as shown in section 3, we expect to have two 
supersymmetric duality-invariant actions for the N = 1 supersymmetric Born-Infeld-like 
theory. These two actions are denoted by 

\[ L_{\text{susy-inv-BI}}^\lambda = -\lambda \frac{\partial L_{\text{susy-BI}}} {\partial \lambda}; \quad L_{\text{susy-inv-BI}}^\gamma = \frac{\partial L_{\text{susy-BI}}} {\partial \gamma}. \tag{4.6} \]

In [51] we have, a priori, studied the compatibility deformation of BI theory with N = 2 supersymmetric duality-invariant Lagrangian. The TT-deformation of the N = 1 supersymmetric ModMax theory with operator O_{\lambda}^3, was initially considered in ref. [47]. As final remark, extending the N = 1 and N = 2 supersymmetric duality-invariant Lagrangian of the ModMax theory can be investigated as a future work.

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A Expansion of invariant structures

In this appendix we write the explicit forms of invariant structures \( \hat{N}_\nu^\rho \) and \( \hat{N}_\mu^\rho \hat{N}_\nu^\rho \) that appear in eq. (2.21).

\[
\hat{N}_\nu^\rho = 8 e^{-\phi_0} \sinh(\gamma) \left( \cosh(\gamma) \sqrt{S^2 + \bar{P}^2} + \sinh(\gamma) S \right) \tag{A.1}
\]

and

\[
\hat{N}_\mu^\rho \hat{N}_\nu^\rho = -e^{-2\phi_0} \frac{2 \sinh(2\gamma) \left( P^2 + \cosh(2\gamma) (P^2 + 2S^2) + 2 \sinh(2\gamma) S \sqrt{S^2 + \bar{P}^2} \right)}{\sqrt{S^2 + \bar{P}^2}} T_{\mu\nu}^{\text{Max}}
\]

\[
+ 2 e^{-2\phi_0} \cosh(2\gamma) \left( P^2 + \cosh(2\gamma) (P^2 + 2S^2) + 2 \sinh(2\gamma) S \sqrt{S^2 + \bar{P}^2} \right) g_{\mu\nu}
\]

\[
+ \lambda e^{-3\phi_0} \left( -2 \cosh(\gamma) P^2 - 4 \cosh(3\gamma) P^2 - 2 \cosh(5\gamma) (P^2 + 4S^2) \right)
\]

\[
+ \frac{32 \cosh^2(\gamma) \sinh(\gamma) P^2 S}{\sqrt{S^2 + \bar{P}^2}} - \frac{24 \cosh^2(\gamma) \sinh(3\gamma) P^2 S}{\sqrt{S^2 + \bar{P}^2}} - \frac{8 \sinh(5\gamma) S^3}{\sqrt{S^2 + \bar{P}^2}} \right) T_{\mu\nu}^{\text{Max}}
\]

\[
+ \left( -2 \cosh(\gamma) P^2 S + 4 \cosh(3\gamma) P^2 S + \cosh(5\gamma) (6P^2 S + 8S^3) + \sinh(\gamma) 2P^2 \sqrt{S^2 + \bar{P}^2}
\]

\[
+ (4P^2 \sinh(3\gamma) + 2 \sinh(5\gamma) (P^2 + 4S^2)) \right) g_{\mu\nu} \sqrt{S^2 + \bar{P}^2} + O(\lambda^2) + \ldots \tag{A.2}
\]
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References

[1] J. Plebanski, *Lectures on nonlinear electrodynamics*, Cycle of lectures, The Niels Bohr Institute and NORDITA, Copenhagen, October 1968.

[2] G. Boillat, *Nonlinear electrodynamics-Lagrangians and equations of motion*, J. Math. Phys. 11 (1970) 941 [arXiv:0807.4039] [nSPIRE].

[3] Z. Białynicka-Birula and I. Białynicki-Birula, *Nonlinear effects in quantum electrodynamics. photon propagation and photon splitting in an external field*, Physical Review D 2 (1970) 2341.

[4] I. Białynicki-Birula, *Nonlinear Electrodynamics: Variations on a theme by Born and Infeld*, in Quantum Theory of Particles and Fields: birthday volume dedicated to Jan Lopuszański, B. Jancewicz and J. Lukierski eds, World Scientific (1983), pp 31.

[5] P. Aschieri, S. Ferrara and B. Zumino, *Duality Rotations in Nonlinear Electrodynamics and in Extended Supergravity*, Riv. Nuovo Cim. 31 (2008) 625 [arXiv:2112.03757] [nSPIRE].

[6] A. Delgani, M.R. Setare and S. Zarepour, *Self-energy problem, vacuum polarization, and dual symmetry in Born-Infeld-type U(1) gauge theories*, Eur. Phys. J. Plus 137 (2022) 859 [arXiv:2112.12118] [nSPIRE].

[7] D.P. Sorokin, *Introductory Notes on Non-linear Electrodynamics and its Applications*, Fortsch. Phys. 70 (2022) 2200092 [arXiv:2112.12118] [nSPIRE].

[8] J.M.A. Paixão, L.P.R. Ospedal, M.J. Neves and J.A. Helayël-Neto, *The axion-photon mixing in non-linear electrodynamic scenarios*, JHEP 10 (2022) 160 [arXiv:2205.05442] [nSPIRE].

[9] I. Bandos, K. Lechner, D. Sorokin and P.K. Townsend, *A non-linear duality-invariant conformal extension of Maxwell’s equations*, Phys. Rev. D 102 (2020) 121703 [arXiv:2007.09092] [nSPIRE].

[10] B.P. Kosyakov, *Nonlinear electrodynamics with the maximum allowable symmetries*, Phys. Lett. B 810 (2020) 135840 [arXiv:2106.07547] [nSPIRE].

[11] I. Bandos, K. Lechner, D. Sorokin and P.K. Townsend, *On p-form gauge theories and their conformal limits*, JHEP 03 (2021) 022 [arXiv:2012.09286] [nSPIRE].

[12] I. Bandos, K. Lechner, D. Sorokin and P.K. Townsend, *ModMax meets Susy*, JHEP 10 (2021) 031 [arXiv:2106.07547] [nSPIRE].

[13] S.M. Kuzenko, *Superconformal duality-invariant models and N = 4 SYM effective action*, JHEP 09 (2021) 180 [arXiv:2106.07173] [nSPIRE].

[14] Z. Avetisyan, O. Evnin and K. Mkrtchyan, *Democratic Lagrangians for Nonlinear Electrodynamics*, Phys. Rev. Lett. 127 (2021) 271601 [arXiv:2108.01103] [nSPIRE].

[15] S.I. Kruglov, *On generalized ModMax model of nonlinear electrodynamics*, Phys. Lett. B 822 (2021) 136633 [arXiv:2108.08250] [nSPIRE].

[16] A. Ballon Bordo, D. Kubizňák and T.R. Perche, *Taub-NUT solutions in conformal electrodynamics*, Phys. Lett. B 817 (2021) 136312 [arXiv:2011.13398] [nSPIRE].
[17] H. Nastase, *Coupling ModMax theory precursor with scalars, and BIon-type solutions*, arXiv:2112.01234 [nSPIRE].

[18] K. Mkrtchyan and M. Svazas, *Solutions in Nonlinear Electrodynamics and their double copy regular black holes*, JHEP 09 (2022) 012 [arXiv:2205.14187] [nSPIRE].

[19] J. Barrientos, A. Cisterna, D. Kubiznak and J. Oliva, *Accelerated black holes beyond Maxwell’s electrodynamics*, Phys. Lett. B 834 (2022) 137447 [arXiv:2206.15777] [nSPIRE].

[20] K. Lechner, P. Marchetti, A. Sainaghi and D.P. Sorokin, *Maximally symmetric nonlinear extension of electrodynamics and charged particles*, Phys. Rev. D 106 (2022) 016009 [arXiv:2206.04657] [nSPIRE].

[21] A. Bokulić, I. Smolić and T. Jurić, *Constraints on singularity resolution by nonlinear electrodynamics*, Phys. Rev. D 106 (2022) 064020 [arXiv:2206.07064] [nSPIRE].

[22] A. Banerjee and A. Mehra, *Maximally symmetric nonlinear extension of electrodynamics with Galilean conformal symmetries*, Phys. Rev. D 106 (2022) 085005 [arXiv:2206.11696] [nSPIRE].

[23] R.C. Pantig, L. Mastrototaro, G. Lambiase and A. Övgün, *Shadow, lensing and neutrino propagation by dyonic ModMax black holes*, arXiv:2208.06664 [nSPIRE].

[24] M.J. Neves, P. Gaete, L.P.R. Ospedal and J.A. Helayël-Neto, *Considerations on the ModMax electrodynamics in the presence of an electric and magnetic background*, arXiv:2209.09361 [nSPIRE].

[25] M.K. Gaillard and B. Zumino, *Duality Rotations for Interacting Fields*, Nucl. Phys. B 193 (1981) 221 [nSPIRE].

[26] M.K. Gaillard and B. Zumino, *Nonlinear electromagnetic selfduality and Legendre transformations*, in *A Newton Institute Euroconference on Duality and Supersymmetric Theories*, Cambridge U.K., April 7–18 1997, pp. 33–48 [hep-th/9712103] [nSPIRE].

[27] G.W. Gibbons and D.A. Rasheed, *SL(2,R) invariance of nonlinear electrodynamics coupled to an axion and a dilaton*, Phys. Lett. B 365 (1996) 46 [hep-th/9509141] [nSPIRE].

[28] M. Henneaux and C. Teitelboim, *Dynamics of chiral (self-dual) p-forms*, Physics Letters B 206 (1988) 650.

[29] S. Deser, A. Gomberoff, M. Henneaux and C. Teitelboim, *Duality, selfduality, sources and charge quantization in Abelian N form theories*, Phys. Lett. B 400 (1997) 80 [hep-th/9702184] [nSPIRE].

[30] S. Deser, A. Gomberoff, M. Henneaux and C. Teitelboim, *P-brane dyons and electric magnetic duality*, Nucl. Phys. B 520 (1998) 179 [hep-th/9712189] [nSPIRE].

[31] P. Pasti, D.P. Sorokin and M. Tonin, *Duality symmetric actions with manifest space-time symmetries*, Phys. Rev. D 52 (1995) R4277 [hep-th/9506109] [nSPIRE].

[32] P. Pasti, D.P. Sorokin and M. Tonin, *Space-time symmetries in duality symmetric models*, in *Workshop on Gauge Theories, Applied Supersymmetry, and Quantum Gravity*, Leuven Belgium, July 10–14, 1995, pp. 167–176 [hep-th/9509052] [nSPIRE].

[33] P. Pasti, D.P. Sorokin and M. Tonin, *On Lorentz invariant actions for chiral p forms*, Phys. Rev. D 55 (1997) 6292 [hep-th/9611100] [nSPIRE].

[34] P. Pasti, D.P. Sorokin and M. Tonin, *Covariant action for a D = 11 five-brane with the chiral field*, Phys. Lett. B 398 (1997) 41 [hep-th/9701037] [nSPIRE].

– 13 –
[35] M.B. Green and M. Gutperle, Comments on three-branes, *Phys. Lett. B* 377 (1996) 28 [hep-th/9602077] [SPIRE].

[36] K. Babaei Velni and H. Babaei-Aghbolagh, On SL(2, R) symmetry in nonlinear electrodynamics theories, *Nucl. Phys. B* 913 (2016) 987 [arXiv:1610.07790] [SPIRE].

[37] K. Babaei Velni and H. Babaei-Aghbolagh, S-dual amplitude and D3-brane couplings, *Phys. Rev. D* 99 (2019) 066007 [arXiv:1901.00198] [SPIRE].

[38] M.R. Garousi, Duality constraints on effective actions, *Phys. Rept.* 702 (2017) 1 [arXiv:1702.00191] [SPIRE].

[39] H. Babaei-Aghbolagh and M.R. Garousi, S-duality of tree-level S-matrix elements in D3-brane effective action, *Phys. Rev. D* 88 (2013) 026008 [arXiv:1304.2938] [SPIRE].

[40] N. H. Pavao, Effective Observables for Electromagnetic Duality from Novel Amplitude Decomposition, arXiv:2210.12800.

[41] P. Aschieri and S. Ferrara, Constitutive relations and Schroedinger’s formulation of nonlinear electrodynamics theories, *JHEP* 05 (2013) 087 [arXiv:1302.4737] [SPIRE].

[42] G.W. Gibbons and D.A. Rasheed, Electric-magnetic duality rotations in nonlinear electrodynamics, *Nucl. Phys. B* 454 (1995) 185 [hep-th/9506035] [SPIRE].

[43] J.J.M. Carrasco, R. Kallosh and R. Roiban, Covariant procedures for perturbative non-linear deformation of duality-invariant theories, *Phys. Rev. D* 85 (2012) 025007 [arXiv:1108.4390] [SPIRE].

[44] W.A. Chemissany, J. de Jong and M. de Roo, Selfduality of non-linear electrodynamic theories with derivative corrections, *JHEP* 11 (2006) 086 [hep-th/0610060] [SPIRE].

[45] W. Chemissany, R. Kallosh and T. Ortin, Born-Infeld with Higher Derivatives, *Phys. Rev. D* 85 (2012) 046002 [arXiv:1112.0332] [SPIRE].

[46] H. Babaei-Aghbolagh, K.B. Velni, D.M. Yekta and H. Mohammadzadeh, Emergence of non-linear electrodynamic theories from TT\_\_\_\_\_\_\_-like deformations, *Phys. Lett. B* 829 (2022) 137079 [arXiv:2202.11156] [SPIRE].

[47] C. Ferko, L. Smith and G. Tartaglino-Mazzucchelli, On Current-Squared Flows and ModMax Theories, *SciPost Phys.* 13 (2022) 012 [arXiv:2203.01085] [SPIRE].

[48] R. Conti, J. Romano and R. Tateo, Metric approach to a TT\_\_\_\_\_\_-like deformation in arbitrary dimensions, *JHEP* 09 (2022) 085 [arXiv:2206.03415] [SPIRE].

[49] R. Conti, L. Iannella, S. Negro and R. Tateo, Generalised Born-Infeld models, Lax operators and the TT perturbation, *JHEP* 11 (2018) 007 [arXiv:1806.1115] [SPIRE].

[50] C. Ferko, H. Jiang, S. Sethi and G. Tartaglino-Mazzucchelli, Non-linear supersymmetry and TT\_\_\_\_\_\_-like flows, *JHEP* 02 (2020) 016 [arXiv:1910.01599] [SPIRE].

[51] H. Babaei-Aghbolagh, K. Babaei Velni, D.M. Yekta and H. Mohammadzadeh, T\_\_\_\_\_\_-like flows in non-linear electrodynamic theories and S-duality, *JHEP* 04 (2021) 187 [arXiv:2012.13636] [SPIRE].

[52] H. Babaei-Aghbolagh, K. Babaei Velni, D. Mahdavian Yekta and H. Mohammadzadeh, Marginal TT\_\_\_\_\_\_-like deformation and modified Maxwell theories in two dimensions, *Phys. Rev. D* 106 (2022) 086022 [arXiv:2206.12677] [SPIRE].
[53] C. Ferko, A. Sfondrini, L. Smith and G. Tartaglino-Mazzucchelli, \textit{Root-\overline{T}T Deformations in Two-Dimensional Quantum Field Theories}, \textit{Phys. Rev. Lett.} \textbf{129} (2022) 201604 [arXiv:2206.10515] [inSPIRE].

[54] S. He and Y. Sun, \textit{Correlation functions of CFTs on a torus with a \overline{T}T deformation}, \textit{Phys. Rev. D} \textbf{102} (2020) 026023 [arXiv:2004.07486] [inSPIRE].

[55] S. He, Y. Sun and Y.-X. Zhang, \textit{\overline{T}T-flow effects on torus partition functions}, \textit{JHEP} \textbf{09} (2021) 061 [arXiv:2011.02902] [inSPIRE].

[56] M. He, S. He and Y.-h. Gao, \textit{Surface charges in Chern-Simons gravity with \overline{T}T deformation}, \textit{JHEP} \textbf{03} (2022) 044 [arXiv:2109.12885] [inSPIRE].

[57] S. He and Y.-Z. Li, \textit{Higher Genus Correlation Functions in CFTs with \overline{T}T Deformation}, \textit{arXiv:2202.04810} [inSPIRE].

[58] S. Deser and R. Puzalowski, \textit{Supersymmetric Nonpolynomial Vector Multiplets and Causal Propagation}, \textit{J. Phys. A} \textbf{13} (1980) 2501 [inSPIRE].

[59] S. Cecotti and S. Ferrara, \textit{Supersymmetric Born-Infeld Lagrangians}, \textit{Phys. Lett. B} \textbf{187} (1987) 335 [inSPIRE].

[60] J. Bagger and A. Galperin, \textit{A New Goldstone multiplet for partially broken supersymmetry}, \textit{Phys. Rev. D} \textbf{55} (1997) 1091 [hep-th/9608177] [inSPIRE].

[61] S.M. Kuzenko and S. Theisen, \textit{Supersymmetric duality rotations}, \textit{JHEP} \textbf{03} (2000) 034 [hep-th/0001068] [inSPIRE].

[62] S.M. Kuzenko and S. Theisen, \textit{Nonlinear selfduality and supersymmetry}, \textit{Fortsch. Phys.} \textbf{49} (2001) 273 [hep-th/0007231] [inSPIRE].