Crumpling an ordinary thin sheet transforms it into a structure with unusual mechanical behaviors, such as logarithmic relaxation, emission of crackling noise, and memory retention. A central challenge lies in understanding how these behaviors emerge from the underlying complex geometry. Here we combine global force measurements and 3D imaging to correlate the global mechanical response and the underlying geometric transformations. We find that the response of crumpled sheets to cyclic strain is intermittent, hysteretic, and encodes a memory of the largest applied compression. Using 3D imaging we show that these behaviors emerge due to an interplay between localized and interacting geometric instabilities in the sheet. A simple model confirms that these minimal ingredients are sufficient to explain the observed behaviors. Finally we show that after training multiple memories can be encoded, a phenomenon known as return point memory. Our study lays the foundation for understanding the complex mechanics of crumpled sheets, and presents an experimental and theoretical framework for the study of memory formation in systems of interacting instabilities.

We begin by studying the global mechanical response of crumpled elasto-plastic sheets to cyclic strain. We find that the resulting force-displacement relations form hysteretic loops decorated with a myriad of abrupt force jumps. After a small number of cycles these converge to approximate limit cycles with repeating features. We show that the response is history dependent, encoding a memory of the largest applied compression. Interestingly, after training the sheets also exhibit return point memory with high accuracy.

Next, we correlate the global response to local transformations in the sheets’ geometry. We find that each force jump in the response corresponds to the snapping of a localized bistable degree of freedom (Hysteron). The global mechanical response arises from the collective dynamics of many such instabilities. These are coupled to each other, and encode the observed memories in their collective states and transitions. Simulations of a network of interacting hysterons reproduce the observed behaviors. Altogether, our picture forms a basis for studying the complex mechanics of coupled bistable elements.

Global response - memory and intermittency - Thin Mylar sheets are crumpled several times and then opened to an approximately flat configuration. In contrast to its floppy, uncrumpled ancestor, the crumpled-then-flattened sheet is stiffer and can easily carry its own weight if held at one end. When forcefully bent the sheet yields, but via a series of discrete jumps accompanied by audible popping sounds. If the force is removed, the crumpled sheet tends to retain its bent shape, and resists reversing the deformation.

To measure this response we load the sheets into two custom mechanical testers, shown in Fig. 1b. The sheets are held at two opposite edges, where one edge is fixed and the displacement of the other is controlled by a linear motorized stage. The force exerted back by the sheet is measured using a load cell connected to the fixed end (see Methods). We note, that in all experiments below, the measured stresses are much smaller than those applied during crumpling, therefore we expect negligible generation of new plastic deformations.

We begin by considering the mechanical response of a sheet that is strained periodically. We vary the stage displacement \( \Delta \) at a slow constant rate between \( \Delta_{\text{min}} \) and \( \Delta_{\text{max}} \). A typical example of the resulting force-displacement curve is presented in Fig. 1a. Two prominent features are observed. First, the mechanical response is hysteric – the force depends on the direction of change in the displacement. Secondly, the force curves are not smooth but rather decorated with a multitude of sudden force jumps. After a small number of cycles the hysteresis curves converge to an approximate
limit cycle, in which force jumps occur at nearly the same displacement values. Over many cycles we observe slow creep: a sluggish decrease in the measured force and a slow drift of the displacement values of the force jumps (see inset of Fig. 1b). Here we regard these creep processes as secondary effects and focus on the prominent, near-repeating features.

Next, we study the effect of changing the maximal displacement, after the system has reached a limit cycle in the interval $[\Delta_{\text{min}}^0, \Delta_{\text{max}}^0]$. We find two distinct behaviours, depending on whether the new upper limit $\Delta_{\text{max}}^1$ is larger or smaller than the previous, $\Delta_{\text{max}}^0$. For any $\Delta_{\text{max}}^1 < \Delta_{\text{max}}^0$ the system immediately falls into a new limit cycle, in which the force-displacement curve is fully enclosed by (and partially overlaps with) the previous curve, as shown for example in Fig. 1c. This transition is reversible: upon returning to the initial $\Delta_{\text{max}}^0$ the previous limit cycle is immediately recovered, identified by the same pattern of force jumps (see inset in Fig. 1c).

In contrast, for any $\Delta_{\text{max}}^1 > \Delta_{\text{max}}^0$, the force-displacement curve changes irreversibly, as shown for example in Fig. 1d. After a short transient, the system converges to a new limit cycle, which generally has small overlap with the previous curve. Moreover, returning to $\Delta_{\text{max}}^0$, does not recover the original limit cycle. The new limit cycle is tilted with respect to the previous one, and exhibits a different pattern of force jumps (see inset of Fig. 1d). This indicates that the response is history dependent, and encodes a memory of the largest applied strain.

Return point memory - Having shown that when completing a cycle in the reversible regime the system recovers its initial state, we investigate whether more complex displacement cycles obey the same rule. The phenomenon where the system returns to its initial state irrespective of their displacement trajectory is known as return point memory (RPM) [16, 20, 21]. Systems exhibiting RPM can encode multiple memories [16].

The hallmark of RPM is a hierarchy of nested hysteresis loops. Each time the displacement returns to previ-
FIG. 2. **Return point memory** - A set of nested hysteresis loops indicating return point memory. The inset shows the displacement sequence.

ouisly visited value it recovers its previous state, closing a hysteresis loop. As a result, a displacement sequence that backtracks and closes smaller inner loops, results in a set of hysteresis loops that are confined within each other. The inset of Fig. 2 shows the displacement sequence that forms three loops (indicated in yellow, orange and purple). The resulting force-displacement curve yields three nested hysteresis loops in agreement with the occurrence of RPM. Finally, repeating a cycle in the parent interval (shown in blue) yields a hysteresis loop that overlaps the initial cycle (yellow) with a matching pattern of force-jumps. This amounts to strong evidence that return point memory is present.

**Mesoscopic excitations** - We now turn to investigate the mechanism underlying the mechanical and memory response reported above. Since the force jumps are a central feature of the mechanical response, we start by investigating their geometric origin. We characterise a single force jump by varying the motor displacement periodically in a small interval around one such event. The resulting force-displacement curve shows that each event is reversible (Fig. 3a). However, the threshold for the transition in the increasing strain direction is larger than the threshold required to reverse the transition. Thus, a single event defines a two-state hysteresis loop, a hysteron. Collecting the magnitudes of many force jumps in the increasing displacement phase of the cycle, we find their distribution is broad, spanning several decades in magnitude (see Fig. 3b).

The planar geometry of our crumpled sheets enables direct imaging of the geometric transformations occurring at these instabilities. Using a 3D scanner, we measure the topography of the sheet before and after a single force jump. Subtracting the two measurements reveals that each force jump is a result of a localized event, in which a small region of the sheet snaps suddenly, in a direction perpendicular to the plane of the sheet (Fig. 3c). For the most part, the bistable geometric features are tips of d-cones, or small flat facets that undergo buckling. We note that these instabilities are accompanied by audible acoustic emissions, and the resulting crackling noise was shown to exhibit a universal power-law distribution of intensities 3, 22.

To relate this mesoscopic picture to the macroscopic memory of largest strain, we cycle the displacement between $\Delta_0^{\text{min}}, \Delta_0^{\text{max}}$, and compare the topography of the sheet at the end of the cycle to the topography obtained
after varying the upper limit $\Delta_{1}^{\text{max}}$. For the reversible regime of Fig. 1, we find no discernible difference between the topographies (Fig. 3d). Namely, by the end of the protocol all hysterons return to their original state. This holds for any $\Delta_{1}^{\text{max}} < \Delta_{0}^{\text{max}}$, as well as for the nested cycles during RPM (see supplementary). In contrast, for the irreversible transition of Fig. 1, the topography changes significantly as well (see Fig. 3d). In particular, several hysterons have changed their state. This is generally true for any $\Delta_{1}^{\text{max}} > \Delta_{0}^{\text{max}}$. We therefore deduce that the memory of largest strain is encoded in the collective configuration of the hysterons.

**Interactions between hysterons** - To complete the mesoscopic description, we ask whether interactions between the bistable elements are present and affect the mechanical response. Interactions between hysterons are expected to be mediated through the sheet’s elasticity. Flipping a hysteron presumably modifies the local strain field, affecting the global transition thresholds of its neighbors [5, 19, 23, 24]. Interactions are known to play an important role in the occurrence of RPM, which is not entirely understood [16, 21, 25].

To probe for interaction we compare the measured behavior to the predictions of the Preisach model, which considers a collection of independent, non-interacting hysterons with fixed activation thresholds, driven by an external field [16, 19, 25, 27]. Here, to account for forces, one could envision a set independent bistable "springs", while the external field is the applied displacement (see supplementary information for detailed discussion). The essential assumption is that the activation thresholds of the individual hysterons are fixed, and are not affected by the state of the system, nor the history of applied displacements.

The experiments reported above show a clear indication of interactions. This is most apparent in Fig. 1, when comparing the two limit cycles before and after surpassing the maximal displacement (yellow and blue curves respectively). Even though the displacement range is identical for the two cycles, the pattern of instabilities changes considerably. Particularly, the displacements where the instabilities occur are at different values, as identified from the spikes in the derivative of the force, shown in the inset of Fig. 1. As noted above, this is inconsistent with the non-interacting case, where the instabilities occur at fixed displacements set only by the individual thresholds.

Another intriguing effect seen in the new limit cycle is a tilt in the force-displacement curve compared to the initial curve. This again contrasts the predictions of the Preisach model, as we discuss in the supplementary material. This suggests that interactions allow the system to explore lower energy states (i.e., with smaller forces), thereby reducing its effective elastic modulus.

**A coupled hysterons model** - Our experiments have shown that the effective degrees of freedom of crumpled sheets are localized, coupled, bistable elements. To formulate a model we make further simplifying assumptions. We assume that the planar sheet can be approximated by a two dimensional system embedded with bistable elements. Though the majority of the motion in experiments occurs in the lateral dimension, we assume it yields an in-plane strain. Lastly, we incorporate interactions by assuming that the strain results in forces between the bistable elements.

A simple realization of such a model is a two dimensional disordered bonded network, where each bond is a bistable spring [28, 29] (see Fig. 4). Namely, each bond
is governed by a potential with two minima, given by:

$$U_i = \frac{c_i}{4} (\delta r_i)^4 - \frac{a_i}{2} (\delta r_i)^2. \quad (1)$$

Here, $\delta r_i$ denotes the distance from the local maximum. Each bond is defined by the two parameters $a_i$ and $c_i$, which set the locations of the two minima, $\delta r_{\text{min}} = \pm \sqrt{a_i/c_i}$. The overall length of the bond is taken to be significantly larger than $\delta r_{\text{min}}$. Additional details are presented in the methods section and supplementary information. In the simulations, the network is strained isotropically and quasi-statically; at each step the energy is minimized to reach force balance.

We measure the stress-strain relation and find smooth elastic intervals separated by sharp stress-drops. These instabilities originate from bonds transitioning between their two minima, as illustrated in the supplementary video. Under periodic drive the system may converge to a precise limit cycle, depending on the parameters of the model. Even when it does not converge, often consecutive cycles yield very similar stress-strain curves, and appear as approximate limit cycles.

Following the same protocols as in experiments, we find that the model recovers all essential experimental findings: memory of the largest strain, a tilt in the stress-strain curves and change to the activation pattern when increasing the strain amplitude, and, after training, the nested loops which characterize return point memory (Fig. 4b,c). Further characterization is provided in the supplementary information.

We observe that bonds generally deviate from their minima, due to a mismatch between their rest lengths. As a result, interactions are generally frustrated. This is demonstrated in the supplementary video, where residual stresses within the network are visible even when the global stress vanishes.

Conclusions & Outlook - We have studied the mechanical response of a crumpled sheets to cyclic strain. We found that the force displacement curves are intermittent, hysteretic and encode a memory of the largest strain. All these effects can be traced back to the collective response of multiple bistable snap-through instabilities spread across the sheet. These mesoscopic, localized and interacting degrees of freedom form the basis for understanding the mechanics of crumpled sheets. A model of a coupled hysterons network reproduces the observed behaviors and sheds light on the role of interactions. Altogether, our work offers an experimental and theoretical framework for the study of memory formation in systems composed of interacting instabilities.

The description of crumpled sheets as a disordered network of coupled bistable degrees of freedom suggests that frustration may play an important role in the mechanics of these systems. This suggests that crumpled sheets can be viewed as a mechanical spin glass with a complex energy landscape. Similar suggestions have been made in the context of origami [24, 30, 31]. This begs the question of whether the glass-like behaviors of crumpled sheets, such as logarithmic aging and Kovacs-like memory retention [11, 2], could also be understood using this framework, or perhaps additional ingredients are needed, such as material aging [32].

Remarkably, our system exhibits return point memory with high precision, allowing us to encode multiple memories through a series of nested hysteresis loops. The occurrence of RPM is surprising, given that the system traverses a complex sequence of displacements, a large number of instabilities and significant interactions. Thus, a sheet that is crumpled mindlessly and with little effort is transformed into a programmable material, without the careful design required in engineered mechanical metamaterials [23, 33, 34].

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**METHODS**

Experimental

The experiments are performed on Mylar sheets, 25 and 90 microns thick and approximately 20cm by 20cm across. These were crumpled manually several times and then loaded the mechanical testers. The difference between the two setups is in the geometry of the testers’ edge constraints. The first has straight constraints, keeping the sheet approximately flat and allowing direct imaging of its topography. However, in this setup the response is limited, as the sheet tends to bulge along the axis of strain, leaving the transverse dimension flat. The second cylindrical configuration avoids these localized deformations, as the transverse curvature frustrates the bending along the strain axis. In all the results displayed above, topographic 3D imaging is done with the flat setup using opaque 90 $\mu$m thick Mylar, and a HP David 5 3D scanner. Force-displacement curves are measured using the cylindrical setup and transparent 25 $\mu$m thick Mylar. We note that the flat setup exhibits the same characteristic features in it’s response, including the memory of the largest strain, the signature of interactions, and return point memory.

Numerical model

The picture suggested by the experiments is that a crumpled sheet can be modeled as a disordered collection of coupled bistable mechanical degrees of freedom.
To test if these ingredients are sufficient to explain the observed behaviors, we study a simple model. We assume that a planar sheet can be approximated by a two dimensional elastic system embedded with bistable elements. A simple realization of this is a disordered bonded network where each bond is bistable, implemented through a potential with two minima. The length of the bond is denoted by \( r = r_0 + \delta r \), where \( \delta r \) is smaller than \( r_0 \), which signifies the average between the two bistable states. For simplicity the potential is chosen to be symmetric:

\[
U_i = \frac{C}{4} (\delta r)^4 - \frac{a_i}{2} (\delta r)^2
\]

Here, \( a_i > 0 \) and the two minima are at \( \delta r = \pm \sqrt{a_i/C} \). For simplicity, the constant \( c \) is chosen to be the same for all bonds. We consider two cases: (1) \( a_i = a_0 \) is the same for all bonds. (2) \( a_i \) is uniformly distributed between \([0, a_0]\). To avoid boundary effects we employ periodic boundary conditions.

We simulate quasi-static dynamics, by straining the system between 0 and \( \epsilon_{\text{max}} \). The strain is discretized into small steps, below \( 10^{-4} \). Each step of the dynamics consists of a small change to the strain, and then an energy minimization step to reach force balance, using the FIRE algorithm \[37\]. The qualitative behavior appears to be independent of the particular deformation, and throughout the paper we strain both axes equally. In Fig. 4 the number of nodes are \( N = 2048 \) , \( a_i \) are uniformly distributed between \([0, 0.04]\), \( C = 1 \), and the average \( <r_0> \approx 1 \).

**Disordered network preparation**

We prepare our two-dimensional network from amorphous packings of spheres at zero temperature \[38\]. This choice is made for convenience and we believe has little impact on the results. A pair of spheres interact when the inter-particle distance, \( r \), is below the sum of the radii \( R_i + R_j \), via the potential:

\[
V_{ij}(r) = \begin{cases} 
V_0 \left( 1 - \frac{r}{R_i + R_j} \right)^2 & r \leq R_i + R_j \\
0 & r > R_i + R_j
\end{cases}
\]

Force balance configurations are reached by minimizing the energy using FIRE algorithm \[37\]. The properties of packings depends on the distance from the jamming transition, which can be tuned through the pressure exerted on the box. In the limit of zero applied pressure the system has critical like behavior, characterized by anomalous elasticity and diverging length scales. To avoid these atypical behaviors we focus on the regime that is far from the jamming transition. The distance from the isostatic transition is often characterized by the excess coordination number \( \Delta Z = Z - Z_c \). Here, \( Z = 2N_b/N \) where \( N_b \) are the number of bonds, \( N \) is the number of nodes and \( Z_c \approx 2d \) \[39-41\]. All our simulation are at \( \Delta Z \approx 1.5 \), which far from the isostatic point \( \Delta Z = 0 \).

The packings are the converted to a bonded network, by identifying the nodes with the centers of spheres and connecting pairs of overlapping particles with a bond. The length of the bond \( r_0 \) is set by the distance between the particle pair. Initial we set \( \delta r = 0 \), however, this is the local unstable maximum and the system relaxes into a stable basin of attraction.

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