Final-state Polarization in $B^0_s$ Decays

Alakabha Datta,1 David London,2 Joaquim Matias,3 Makiko Nagashima,2 and Alejandro Szynkman2

1Dept. of Physics and Astronomy, 108 Lewis Hall, University of Mississippi, Oxford, MS 38677-1848, USA
2Physique des Particules, Université de Montréal,C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7
3IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

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Certain $B^0_s \to V_1V_2$ decays ($V_i$ is a vector meson) can be related by flavor SU(3) symmetry to corresponding $B^0_d \to V_3V_4$ decays. In this paper, we show that the final-state polarization can be predicted in the $B^0_s$ decay, assuming polarization measurements of the $B^0_d$ decay. This can be done within the scenario of penguin annihilation (PA), which has been suggested as an explanation of the unexpectedly large transverse polarization in $B \to \phi K^*$. PA is used to estimate the breaking of flavor SU(3) symmetry in pairs of decays. Two of these for which PA makes a reasonably precise prediction of the size of SU(3) breaking are $(B^0_s \to \phi \phi, B^0_s \to \phi K^{*0})$ and $(B^0_s \to \phi K^{*0}, B^0_d \to \bar{K}^{*0}K^{*0})$. The polarization measurement in the $B^0_d$ decay can be used to predict the transverse polarization in the $B^0_s$ decay, and will allow a testing of PA.

INTRODUCTION

We consider $B \to V_1V_2$ decays ($V_i$ is a vector meson). Since the final-state particles are vector mesons, when the spin of these particles is taken into account, this decay is in fact three separate decays, one for each polarization (one longitudinal, two transverse). Naively, within the standard model (SM), the transverse amplitudes are suppressed by a factor of size $m_V/m_B$ ($V$ is one of the vector mesons) with respect to the longitudinal amplitude. Then one expects the fraction of transverse decays, $f_T$, to be much less than the fraction of longitudinal decays, $f_L$.

However, it was observed that these two fractions are roughly equal in the decay $B \to \phi K^*$: $f_T/f_L \simeq 1$. A similar effect was later seen in $B \to \rho K^*$ decays.

If one goes beyond the naive SM, there are two explanations which account for this “polarization puzzle.” The first is penguin annihilation (PA) $\bar{b}O_sq\bar{q}O_q$, where $q = u, d$ ($O$ are Lorentz structures, and color indices are suppressed). With a Fierz transformation, these operators can be written as $\bar{b}O'q\bar{q}O'$ and a gluon can now be emitted from one of the quarks in the operators, and can then produce a pair of $s, \bar{s}$ quarks. They then combine with the $\bar{s}, q$ quarks to form the final states $\phi K^{*0}$ ($q = u$) or $\phi K^{*+}$ ($q = d$). These are annihilation contributions. Normally such terms are expected to be small as they are higher order in the $1/m_b$ expansion, and thus ignored. However, within QCD factorization (QCDf) it is plausible that the coefficients of these terms are large. (Within perturbative QCD (PQCD), the penguin annihilation is calculable and can be large, though it is not large enough to explain the polarization data in $B \to \phi K^*$.

In QCDf, due to the appearance of endpoint divergences, PA is not calculable, but is modeled. These divergences are regulated with a cut-off, introducing several arbitrary parameters. There is therefore an enormous uncertainty in the size of the PA amplitude as one varies these unknown parameters within certain chosen limits.

It is also possible within QCDf that the transverse amplitudes receive significant contributions from perturbative rescattering from charm intermediate states. However, the transverse amplitudes could be purely dominated by PA. In this paper we explore the consequences of the scenario in which PA contributions are large and dominant to see what type of testable predictions result.

The second SM explanation is rescattering. The idea is that nonperturbative rescattering effects involving charm intermediate states, generated by the operator $\bar{b}O'q\bar{q}O'$, can produce large transverse polarization in $B \to \phi K^*$. A particular realization of this scenario is the following. Consider the decay $B^+ \to D^{*+}_{s} \bar{D}^{*0}$. Since the final-state vector mesons are heavy, the transverse polarization can be large. The state $D^{*+}_{s} \bar{D}^{*0}$ can now rescatter to $\phi K^{*+}$. If the transverse polarization $T$ is not reduced in the scattering process,
this will lead to $B^+ \to \phi K^{*+}$ with large $f_\pi/f_L$. (A similar rescattering effect can take place for $B_d^0 \to \phi K^{*0}$.)

It is important to test these explanations in order to determine whether new physics is or is not present. The polarization puzzle has been mainly seen in $b \to \bar{s}$ transitions. However, if PA or rescattering is the true explanation, one also expects to observe large $f_\pi/f_L$ in $b \to \bar{d}$ decays. In Ref. [5] such decays were discussed, and it was observed that the most promising transitions were those which are dominated by penguin amplitudes. The decays in the scenario in which PA dominates these amplitudes ($A_1$) are related by

$$\bar{b} \to s\bar{s}s \quad \text{and} \quad \bar{b} \to \bar{s}d\bar{d} : \quad B_d^0 \to \phi K^{0*},$$

$$B_d^0 \to \phi\phi, K^{0*}\bar{K}^{0*},$$

$$B \to \phi K^{*+}, \rho^* K^{*0}.\quad (1)$$

(Decays which also receive tree contributions are not included in the current analysis.)

Now, all of these decays are the same under flavor SU(3), which treats $d$, $s$ and $u$ quarks as equal. The idea is that, given a measurement of the polarization in one decay, one can predict the polarization in another decay using PA or rescattering. However, in relating the two decays, the effect of SU(3) breaking must be included. We can relate the transverse amplitudes of SU(3)-related decays in the scenario in which PA dominates these amplitudes. On the other hand, this relation is unknown in rescattering, which involves long-distance contributions. For this reason, in this paper we consider only PA.

We note that the transverse ($A_{||,\perp}$) and helicity amplitudes ($A_{\pm}$) are related by $A_{||,\perp} = (A_+ \pm A_-)/\sqrt{2}$. However, $A_-$ for $B$ decays ($A_+ \to B$ decays) has an extra spin-flip suppression with respect to $A_+$. Consequently, we neglect $A_-$ ($A_{||,\perp}$) and hence define $|A_{||}|^2 \equiv |A_{\perp}|^2 + |A_+|^2$ (i.e. $A_T$ includes both transverse amplitudes).

First, consider the pair of decays $B_s^0 \to \phi \phi$ and $B_d^0 \to \phi K^{0*}$. The main PA contributions to the transverse amplitudes are (the penguin-annihilation term arises only from penguin operators with an internal $\bar{t}$ quark)

$$A_T(B_s^0 \to \phi \phi) = |V_{tb}V_{ts}| f_\pi f_L^2 2(b_3^{(\phi\phi)} + b_4^{(\phi\phi)}),$$

$$A_T(B_d^0 \to \phi K^{0*}) = |V_{tb}V_{ts}| f_\pi f_L^2 f_0 f_{K^*} b_3^{(\phi K^{0*})}.\quad (2)$$

where $b_3^{(\phi\phi)}$ and $b_4^{(\phi\phi)}$ are the QCDF terms corresponding to PA [10]. Throughout the paper, we have dropped the overall factor of $G_F/\sqrt{2}$. (Absolute values are taken for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements because we are not interested in CP-violating observables here, but rather the rate.)

As noted earlier, QCDF also contains the spin flips. Consequently, it is reasonable to ignore $\alpha_4$, and we do so here.

The annihilation coefficient $b_3$ is given by

$$b_3^{(\phi\phi)} = \frac{C_F}{N_c} \left[ (C_5 + N_c C_6) A_3^i + C_5 A_3^j + C_3 A_3 \right],\quad (3)$$

where $N_c$ is the number of colors, the $C_i$ are Wilson coefficients, and the calculable infrared divergences are found in the $A_3^{f,f}$ (due to the endpoint singularity of the final-state distribution amplitudes). The superscripts ‘$i$’ and ‘$f$’ refer to gluon emission from the initial- and final-state quarks, respectively. The subscript ‘$i$’ refers to the Dirac structure ($V - A$) $\otimes$ ($V - A$), while ‘$f$’ refers to the Dirac structure $(-2)(S - P) \otimes (S + P)$.

The annihilation coefficient $b_4$ is

$$b_4^{(\phi\phi)} = \frac{C_F}{N_c} \left( C_4 A_4^i + C_6 A_4^j \right).\quad (4)$$

Here, $A_3^i \cong A_3^j$ has a suppression factor of $O(1/m_b^2)$ compared to the $A_3^{i,j}$ [10]. Thus, $b_3^{(\phi\phi)}$ and the third term in $b_3^{(\phi\phi)}$ can be neglected in Eq. (2).

Now, if we assume that the term containing $A_3^i$ in $b_3^{(\phi\phi)}$ dominates over the others (as we will see below, the error on this approximation is at the level of only a few percent), we find that

$$\frac{A_T(B_s^0 \to \phi \phi)}{2 A_T(B_d^0 \to \phi K^{0*})} = \frac{f_\pi f_L}{f_\pi f_{K^*}} \frac{A_3^{(\phi\phi)}}{A_3^{(\phi\phi)}}.\quad (5)$$

where $A_3^{(\phi\phi)}$ has the following integral form [10]:

$$A_3^{(\phi\phi)} = \frac{\pi \alpha_s}{\alpha} \int_0^1 dxdy \left\{ \frac{2m_1}{m_2} r_{\phi\phi} \phi_{\phi\phi}(x) \phi_{\phi\phi}(y) + \frac{2m_2}{m_1} r_{\phi\phi} \phi_{\phi\phi}(y) \phi_{\phi\phi}(x) \right\}.\quad (6)$$

Here $\phi_{\phi\phi}(x,y)$ is the twist-2 (twist-3) light-cone distribution amplitude, $x$ ($y$) stands for the momentum fraction carried by the quark in $V_1$ ($V_2$), and $r_{\phi\phi} = (2m_{\phi\phi}/m_b)f_{\phi\phi}^2/f_{\phi\phi}$. $r_{\phi\phi}$ is defined as [13]

$$\langle V(p,\epsilon^*) | \bar{q} \gamma_{\mu} q \gamma_{\nu} | 0 \rangle = f_{\phi\phi}^2 p_{\mu} \epsilon_\nu^* - p_{\nu} \epsilon_\mu^* .\quad (7)$$

It is useful to make a comment concerning the appropriate interpretation of Eq. (5): if a $K^*$ is one of the final particles, the argument of the distribution amplitudes (DAs) is the momentum fraction corresponding to the s quark, whereas for a $K^*$ meson, it is the momentum fraction of the d quark. That is, the DAs are defined as $\phi_{\phi\phi}(z) = \phi_{\phi\phi}(\bar{z})$, where $\bar{z} \equiv 1 - z$.

In order to estimate the ratio of the transverse amplitudes in Eq. (5), we need to know the amount of SU(3) breaking in the ratio of $A_3^{(\phi\phi)}$ and $A_3^{(\phi\phi)}$. To do this, further assumptions are necessary. In what follows, we adopt the same assumptions as those used by the authors of Ref. [10] to carry out their study.
1. the asymptotic form of the light-cone distribution (LCD) amplitudes,

2. a universal parametrization of the end-point singularities, i.e. independent of any particular decay mode,

3. a modeling of the singularities.

It is the first point about which there has been some debate. Certain references have calculated higher-order moments in the LCDs, suggesting that such “non-asymptotic LCDs” are important for some light mesons [14]. If so, then SU(3) breaking in these non-asymptotic pieces will also contribute to the ratio in Eq. (5). Unfortunately, this SU(3) breaking is not calculable, in which case our analysis below will not hold. Equally unfortunately, it is very difficult for experiment to determine moments in the LCDs, suggesting that such “non-asymmetries, these correspond to $B^+ \to \rho^+ K^{0*}$, $B^0_s \to \phi K^{0*}$, $B^+ \to \phi K^{*+}$, and $B^0_s \to \phi \phi$. The important point here is that the dependences on the momentum fractions in $A_i^f$ are the same for every decay belonging to this class. Therefore, in the comparison of any of these decays, the integrals containing the singularities cancel in the ratio of the transverse amplitudes, and this even before using a cutoff to regulate the end-point divergences.

Of course, the size of the SU(3) breaking will depend on the pairs of decays considered [see Eq. (8)]. For example, we expect the pair $B^0_s \to \phi \phi$ and $B^+ \to \phi K^{*+}$ to be as good as $B^0_s \to \phi \phi$ and $B^0_s \to K^{0*}$. However, the SU(3) breaking in $B^+ \to \rho^+ K^{0*}$ and $B^+ \to \phi K^{*+}$ also turns out to be about the same size. In all cases, the SU(3) breaking can be worked out as we have done above.

Another class in which the final states have the same dependence on $x$ and $y$ is given by the decays $b(B^0_s) \to \bar{d}d$ and $b(B^0_s) \to \bar{s}s$. Even if one of the most promising $B^0_s$ decay modes to be measured in the near future, $B^0_s \to K^{0*} K^{0*}$, belongs to this class, we cannot use it to make predictions because its decay-class partner, $B^0_s \to \rho^0 K^{*0}$, typically receives tree contributions. Therefore, this class is not very useful since it only contains one pure penguin decay.

The third class is defined by the transition $b \to \bar{d}s\bar{s}$. This includes the decays $B^0_s \to \phi K^{0*}$ and $B^0_s \to K^{0*} K^{0*}$, and this pair is particularly promising. The prediction for the ratio of the transverse amplitudes is

$$\frac{A(T(B^0_s \to \phi \phi))}{A(T(B^0_s \to \phi K^{0*}))} = \frac{f_{\phi}^0}{f_{\phi}^0 f_{K^{0*}}} \left(\frac{2v^0_{\phi}}{m_{\phi}^2 + \frac{m_{\phi}^2 r^0_{\phi}}{m_{K^{0*}}}} \right),$$

where $f_{\phi}^0/f_{\phi}^0 = 1.22 \pm 0.03$ [13], $f_{\phi} = 221 \pm 3$ MeV [13], and $f_{K^{0*}} = 218 \pm 4$ MeV [13]. The values of $f_{\phi}^0$ in $r^0_{\phi}$ are theoretically estimated. We have 13 $f_{\phi}^0 = f_{\phi}^0 = 175 \pm 25$ MeV. Since the decay constants and the meson masses are known, the SU(3) breaking in Eq. (8) is well-controlled for this pair.

What this says is that the transverse polarization amplitude in $B^0_s \to \phi \phi$ is predicted by PA to be related to that in $B^0_s \to \phi K^{0*}$ through Eq. (8). Thus, once one makes the polarization measurement in the $B^0_s$ decay, one can test PA by making the equivalent measurement in the $B^0_s$ decay.

The key ingredient in the above analysis is to take two decays in which the final states have the same dependence on the momentum fractions $x$ and $y$. However, although the pair of decays considered above is the most promising for the analysis, it is not unique. In fact, all decays in a special class have the same dependence on $x$ and $y$. This class contains $b(B^0_s, B^+) \to \bar{d}d\bar{s}d$ (the parenthesis indicates that this transition includes only $B^0_s$ or $B^+$, and not $B^0_s$ decays) and $b \to \bar{s}s\bar{s}$ penguin decays. In addition, only decays to ground-state spin-1 mesons are included (excited mesons have different DAs in general). Thus, excluding those decays which also receive tree contributions, these correspond to $B^+ \to \rho^+ K^{0*}$, $B^0_s \to \phi K^{0*}$,
In passing, we note the following. Previously, we mentioned that we use the same assumptions as those in Ref. [10]. Although we work within the same restricted theoretical framework as this reference, and although it is true that the size of the error in our predictions could be affected by large uncertainties related to the choice of this particular scenario, we emphasize that our results go beyond the analysis made in Ref. [10]. There, due to the parametrization of the infinities, the uncertainties in the individual transverse amplitudes turn out to be at the level of one hundred percent for most of the decay channels [19]. Instead, we show here that it is possible to obtain more accurate predictions with the same theoretical inputs used in the treatment of the divergent integrals when specific decays are compared.

To illustrate the procedure used in the estimation of errors, we focus on the first pair, $B^0_s \rightarrow \phi \phi$ and $B^0_d \rightarrow \phi K^{0*}$ (a similar reasoning holds for $B^0_s \rightarrow \phi K^{0*}$ and $B^0_d \rightarrow K^{0*} K^{0*}$, as well as several other decay pairs). We first write the $A_{1,2}^{f}$ after the parametrization of the infrared divergences is applied [10]

$$A_{1,2}^{f} \approx 18\pi s_{f} \frac{m_{I} m_{2}}{m_{I}^{2}} \left( \frac{2}{3} X_{L} + \frac{5}{2} - \frac{\pi^{2}}{3} \right),$$

$$A_{3}^{f} \approx 18\pi s_{f} \frac{m_{I}^{2} m_{2}}{m_{I}^{2}} \left( X_{L}^{2} - 2X_{A} + 2 \right),$$

$$A_{4}^{f} \approx 18\pi s_{f} \frac{m_{I}^{2} m_{2}^{2}}{m_{I}^{2}} \left( 2X_{A}^{2} - 5X_{A} + 3 \right).$$

We see that $A_{1,2}^{f}$ and $A_{3}^{f}$ are symmetric in the interchange of $V_{i} \leftrightarrow V_{2}$, while $A_{4}^{f}$ is antisymmetric. $X_{A}$ and $X_{L}$ contain the same input parameters, but have different end-point-divergence behavior:

$$X_{A} = \left( 1 + \rho_{A} e^{i\phi_{A}} \right) \frac{m_{I} m_{2}}{\Lambda_{h}},$$

$$X_{L} = \left( 1 + \rho_{L} e^{i\phi_{L}} \right) \frac{m_{I} m_{2}}{\Lambda_{h}}.$$

Here, $\Lambda_{h}$ is an input parameter ($\Lambda_{h} = 0.5 \text{ GeV}$ [10]), and $\phi_{A}$ is an arbitrary phase. We have taken $\Lambda_{h}$, $\phi_{A}$, and $\rho_{A}$ to be the same for every decay mode.

To study the relative significance of $b_{4}$ and the neglected terms in $b_{3}$, we evaluate the following ratios [Eqs. (3) and (11)]:

$$r_{b_{4}}^{(V_{i}V_{2})} = \frac{C_{4} + C_{6}}{C_{5} + N_{c} C_{6}} \frac{A_{1,2}^{(V_{i}V_{2})}}{A_{3}^{(V_{i}V_{2})}},$$

$$r_{b_{3}}^{(V_{i}V_{2})} = \frac{C_{3}}{C_{5} + N_{c} C_{6}} \frac{A_{3}^{(V_{i}V_{2})}}{A_{4}^{(V_{i}V_{2})}},$$

$$R_{b_{3}}^{(\phi K^{0*})} = \frac{C_{5}}{C_{5} + N_{c} C_{6}} \frac{A_{3}^{(\phi K^{0*})}}{A_{4}^{(\phi K^{0*})}}.$$

where $V_{i}V_{2} = \phi \phi, \phi K^{0*}$ (note that $R_{b_{3}}^{(\phi \phi)}$ is zero).

Although the values of these ratios are quite uncertain, in large part due to the (arbitrary) value of $\Lambda_{h}$, in virtually all cases it is found that $|r_{b_{4}}^{(V_{i}V_{2})}| \lesssim O(10^{-2})$ and $|r_{b_{4}}^{(V_{i}V_{2})}|, |R_{b_{3}}^{(\phi K^{0*})}| \lesssim O(10^{-3})$. One can see this as follows. First, the ratios of the relevant Wilson coefficients in Eq. (12) at $\mu = m_{b}/2$ are as follows [7]:

$$\frac{C_{4} + C_{6}}{C_{5} + N_{c} C_{6}} \approx 0.63,$$

$$\frac{C_{3}}{C_{5} + N_{c} C_{6}} \approx -0.11, \quad \frac{C_{5}}{C_{5} + N_{c} C_{6}} \approx -0.05.$$  (13)

Second, we have $\left( A_{1}^{(V_{i}V_{2})}/A_{3}^{(V_{i}V_{2})} \right) \sim O(m_{1} m_{2}/m_{n}^{2})$, and $\left( A_{3}^{(\phi K^{0*})}/A_{3}^{(\phi K^{0*})} \right) \sim \left( (a-b)/(a+b) \right) \approx 0.07$ (where $a$ denotes $(m_{K^{0*}}/m_{b})e^{\phi}$ and $b$ is given by $K^{0*} \leftrightarrow \phi$).

We evaluate the three ratios in Eq. (12) by considering many different values in the ranges $0 \leq \rho_{A} \leq 2$ and $0 \leq \phi_{A} \leq 2\pi$. We find that $|r_{b_{4}}^{(V_{i}V_{2})}|, |r_{b_{3}}^{(V_{i}V_{2})}|, |R_{b_{3}}^{(\phi K^{0*})}| \ll 1$ always, except for a singular behavior at $\phi_{A} = 0, 2\pi$. The largest contribution to the error arises from $r_{b_{3}}^{(V_{i}V_{2})}$ but it remains at the level of a few percent within the scanned region of the parameter space. Thus, we have covered a wide set of models of the infrared singularities. The point here is, although the precise values of the ratios are very uncertain, they are always small.

We therefore conclude that the PA dominance hypothesis leads to a clean prediction for the ratio of transverse amplitudes in the pair $B^0_s \rightarrow \phi \phi$ and $B^0_d \rightarrow \phi K^{0*}$ [Eq. (5)], although this result is a direct consequence of the particular modeling of the suppressed terms (e.g. asymptotic LCDs). We have also analyzed the pair $B^0_s \rightarrow \phi K^{0*}$ and $B^0_d \rightarrow K^{*0} K^{0*}$, as well as the pairs containing the charged modes ($B^+ \rightarrow \rho^{+} K^{0*}, B^+ \rightarrow \phi K^{*+}$), under the same set of assumptions, and we have obtained similar conclusions.

We have therefore seen that there are a number of decay pairs within a given class whose transverse polarizations are related. Some of these pairs involve a $B^0_s$ decay. Now, the B-factories BaBar and Belle have made many measurements of $B^0_s$ and $B^+$ mesons. But it is only relatively recently, at hadron colliders, that $B^0_s$ mesons have started to be studied. This will increase when the LHCb turns on. It should be possible to make measurements of the transverse polarization in some $B^0_s$ decays in the near future, and to test the PA/QCD hypothesis.

Above, we have presented the ratio of $A_{4}^{f}$’s for two pairs of decays. However, it is perhaps better to present a ratio of $f_{r}$’s since this is what will actually be measured. $A_{r}$ and $f_{r}$ are related by including information about the branching ratio (BR): $f_{r} = |A_{r}|^2/(\Gamma BR/PS)$, where $\Gamma$
PA makes a reasonably precise estimate of the SU(3) and explore its consequences. Which is the source of the large transverse polarization be estimated. We therefore assume that it is PA alone for the transverse amplitudes, and for special classes of decay pairs, that this SU(3) breaking can account. However, it is only within a specific scenario of PA dominance for the transverse amplitudes, and/or $\bar{b} \rightarrow d$ transitions.

The numbers have been obtained by taking values of masses and lifetimes from the Particle Data Group without errors, along with the theoretical estimates of the decay constants given above. The predictions given in Eq. (14) will yield a test of PA. If there are discrepancies in the measurements, this may indicate the presence of new physics, in $\bar{b} \rightarrow s$ and/or $\bar{b} \rightarrow d$ transitions.

The decays $B_d^0 \rightarrow K^{0*} K^{0*}$ and $B_d^0 \rightarrow \phi K^{0*}$ have both been measured, so that this information can be included in Eq. (14):

$$\frac{f_T(B_d^0 \rightarrow \phi K^{0*})}{f_T(B_d^0 \rightarrow K^{0*} K^{0*})} = 3.22 \pm 0.72$$

$$\frac{f_T(B_s^0 \rightarrow \phi K^{0*})}{f_T(B_s^0 \rightarrow \phi K^{0*})} = 0.62 \pm 0.12$$

The explicit testing of PA, probably in the near future at the LHCb.

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See, for example, P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005); P. Ball and A. N. Talbot, JHEP 0506, 063 (2005), and references therein.

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The authors in Ref. [10] overcome this difficulty by taking into account experimental data to fit the entire divergent transverse amplitude (i.e., $\hat{\alpha}_{4}$ in their notation).