Hydro-dynamic damping theory in flowing water

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Abstract. Fluid-structure interaction (FSI) has a major impact on the dynamic response of the structural components of hydroelectric turbines. On mid-head to high-head Francis runners, the rotor-stator interaction (RSI) phenomenon always has to be considered carefully during the design phase to avoid operational issues later on. The RSI dynamic response amplitudes are driven by three main factors: (1) pressure forcing amplitudes, (2) excitation frequencies in relation to natural frequencies and (3) damping. The prediction of the two first factors has been largely documented in the literature. However, the prediction of fluid damping has received less attention in spite of being critical when the runner is close to resonance. Experimental damping measurements in flowing water on hydrofoils were presented previously. Those results showed that the hydro-dynamic damping increased linearly with the flow. This paper presents development and validation of a mathematical model, based on momentum exchange, to predict damping due to fluid structure interaction in flowing water. The model is implemented as an analytical procedure for simple structures, such as cantilever beams, but is also implemented in more general ways using three different approaches for more complex structures such as runner blades: a finite element procedure, a CFD modal work based approach and a CFD 1DOF approach. The mathematical model and all these implementation approaches are shown to agree well with experimental results.

1. Introduction

Due to the current demands of the hydro-energy market, hydraulic turbine manufacturers are required to highly optimize all turbine components in order to maximize efficiency and minimize cost. For optimized hydroelectric turbines, dynamic phenomena due to Fluid Structure Interaction (FSI) are of particular concern. For mid-head to high-head Francis runners, one of the most damaging FSI phenomena can be the Rotor-Stator Interaction (RSI) which has led to catastrophic failure in the past [1] [2]. This phenomenon has been largely studied and is now well understood [3]. At Andritz Hydro, conservative analyses are performed for all new Francis runner designs in order to ensure low dynamic stresses due to RSI [4].

The RSI dynamic response calculation is based on three main factors: the hydraulic excitation predicted by Computational Fluid Dynamics (CFD) [3], the structural natural frequency prediction in water using Finite Element Analysis (FEA) [6] and the hydro-dynamic damping. While the first two factors are industry standard practice, hydrofoil fluid damping has received very little attention in the literature in spite of being critical when the runner is close to resonance. Experimental hydrofoil fluid damping measurements have been presented previously [7]-[9]. This paper presents the theory development of a mathematical model, based on momentum exchange, for the calculation of the
hydraulic damping. An analytical calculation is applied to a simple cantilever beam structure and three methods are presented for the application on more complex structures such as Francis runner blades: a finite element procedure, a CFD modal work based approach and a CFD 1 DOF approach. Validations of the theory and of the three implementation approaches are presented using comparisons with hydrofoil fluid damping measurements.

2. Theory development

2.1. Fluid forces

The approach is based on the momentum exchange between the structure and the flowing fluid [10]. Consider an infinitesimal fluid element with constant mass flowing on the surface of a vibrating structure as illustrated in figure 1.

\[ f = ma = m \frac{d^2z}{dt^2} \]  

As the particle normal position is implicitly a function of its location on the structure and the time, the full time derivative is found as the sum of its partial derivatives with respect to \( x \), \( y \) and \( t \).

\[ \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} + \frac{\partial z}{\partial t} \]  

Therefore, using the tangential velocities, \( v_x = \frac{\partial x}{\partial t} \) and \( v_y = \frac{\partial y}{\partial t} \), and supposing that the flow velocity is constant in time, \( \frac{\partial v_x}{\partial t} = \frac{\partial v_y}{\partial t} = 0 \), the force is found as follows with contributing terms of inertia \( f_m \), damping \( f_c \) and stiffness \( f_k \):

\[ f = f_m + f_c + f_k \]

\[ f_m = m \frac{\partial^2 z}{\partial t^2} \]

\[ f_c = m \left( 2v_x \frac{\partial^2 z}{\partial x \partial t} + 2v_y \frac{\partial^2 z}{\partial y \partial t} \right) \]

\[ f_k = m \left( v_x^2 \frac{\partial^2 z}{\partial x^2} + v_y^2 \frac{\partial^2 z}{\partial y^2} + 2v_x v_y \frac{\partial^2 z}{\partial x \partial y} + v_x \left( \frac{\partial v_x}{\partial x} \frac{\partial z}{\partial x} + \frac{\partial v_y}{\partial x} \frac{\partial z}{\partial y} \right) + v_y \left( \frac{\partial v_x}{\partial y} \frac{\partial z}{\partial x} + \frac{\partial v_y}{\partial y} \frac{\partial z}{\partial y} \right) \right) \]

The inertial force \( f_m \) represents the added mass effect of the water that is known to modify substantially the natural frequencies of a structure from air to water [6]. As observed from experiments
[7]-[9], the damping force $f_c$ is proportional to the flow velocity. The stiffness force, which depends on
the curvature of the structure and the fluid velocity, would affect the natural frequency of the structure
in flowing water compared to still water. Experimental results did not show a clear variation of the
hydrofoil natural frequency under flowing water [7]-[9]. The stiffness force may therefore not be
important and will be assumed to be small ($f_k \rightarrow 0$).

The total fluid force on the structure is obtained from the surface integration. Assuming uniform
flow along the $x$ direction, $v = v_x$ and $v_y = 0$, the total force is given by:

$$F = t_w \rho_w \int \left( \frac{\partial^2 z}{\partial t^2} + 2 \nu \frac{\partial^2 z}{\partial x \partial t} \right) dx dy$$

(4)

where $\rho_w$ is the fluid density and $t_w$ the virtual thickness of the fluid that represents the zone of the
fluid near the structure that has an influence on its dynamic response.

2.2. Fluid work
Damping is the dissipated energy and can therefore be related to the work done by the fluid force over
one vibration cycle. Assuming harmonic motion, the deflection of the structure, $u(x,y,t) = z$, is
expressed in terms of its spatial and temporal components undergoing vibration at frequency $\Omega$.

$$u = u_0(x,y) \sin(\Omega t)$$

$$\frac{\partial u}{\partial t} = \Omega u_0(x,y) \cos(\Omega t)$$

$$\frac{\partial^2 u}{\partial t^2} = \Omega^2 u_0(x,y) \sin(\Omega t)$$

(5)

The work done by the fluid force is calculated from the integral of the force from equation (4) over
the motion of the structure, $u$. To obtain the work done over one cycle, the integration variable is
changed from $u$ to $t$.

$$W_F = t_w \rho_w \int \int \left( \frac{\partial^2 u}{\partial t^2} + 2 \nu \frac{\partial^2 u}{\partial x \partial t} \right) dx dy du$$

$$W_F = t_w \rho_w \int_0^{2\pi} \int_0^{\pi} \left( \frac{\partial^2 u}{\partial t^2} + 2 \nu \frac{\partial^2 u}{\partial x \partial t} \right) \Omega u_0(x,y) \cos(\Omega t) \, dx \, dy \, dt$$

$$W_F = t_w \rho_w \int_0^{2\pi} \int_0^{\pi} \left( \Omega^2 u_0^2 \sin(\Omega t) \cos(\Omega t) \right. + 2 \nu \Omega^2 \cos^2(\Omega t) u_0(x,y) \frac{\partial u_0(x,y)}{\partial x} \bigg) dx \, dy \, dt$$

$$W_F = t_w \rho_w 2\pi \nu \Omega \int_0^{\pi} u_0(x,y) \frac{\partial u_0(x,y)}{\partial x} \, dx \, dy$$

(6)

It can be seen that the inertial force does not contribute to the work done over one vibration cycle.
Only damping force dissipates energy.

2.3. Damping work
The work done by the damping of an under-damped vibrating system can also be obtained from the
total potential energy, $U$, and the non-dimensional damping ratio which is the ratio between the system
damping and the value of the critical damping $\zeta = c/c_{cr}$.

$$W_D = 4\pi \zeta U$$

(7)

The total potential energy of the system, including both structure and fluid, is given from the sum of
the kinetic energies of the structure alone and of the fluid due to the motion of the structure at the
natural frequency of the structure in the fluid $\Omega = \omega_n$. The mass per unit area of the structure is
expressed as $t_s \rho_s$. 

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This damping work $W_D$ is equal to the damping work done by the fluid forces from equation (6) and the ratio between water mass and total mass can be expressed by natural frequencies where $\omega_s$ is the natural frequency of the structure alone or in air and $\omega_{sw}$ is the natural frequency of the structure in the fluid. The damping ratio can therefore be expressed as follows:

$$\zeta = \frac{v}{\omega_{sw}} \left[ 1 - \left( \frac{\omega_{sw}}{\omega_s} \right)^2 \right] \frac{\iint u_0(x,y) \frac{\partial u_0(x,y)}{\partial x} dxdy}{\iint u_0^2(x,y) dx dy}$$

(9)

The damping ratio, close to resonance depends on the fluid velocity parallel to the plate $v$, the structures natural frequencies in vacuum and in the fluid $\omega_s$ and $\omega_{sw}$, and the mode shape of interest $u_0(x,y)$.

2.4. Cantilever beam example

Consider a cantilever beam, as illustrated in figure 2, with fluid flowing down its longitudinal axis. The natural frequency of the beams’ first bending mode, without any effects from the fluid, is known from elementary vibration theory. The beam thickness is given by $t_s$, $L$ is the beam length $E$ is the Young modulus and $\rho_s$ is the beam density.

$$\omega_s = 1.017 \frac{t_s}{L} \sqrt{\frac{E}{\rho_s}}$$

(10)

The cantilever beams’ natural frequency in water can be obtained by finite element analysis with acoustic elements or analytically from the following equation based on the Rayleigh quotient. Both methods give very similar results.

$$\omega_{sw} = 1.017 \frac{1}{L^2} \sqrt{\frac{t_s^2 E}{t_s \rho_s + t_w \rho_w}}$$

(11)

\(\rho_w\) is the fluid density and $t_w$ is the fluid virtual thickness that can be obtained from Brennans’ suggestion for fluid added mass around a rectangular cross-section of width $b$ [11].

$$t_w = \frac{\pi b}{4}$$

(12)
The cantilever beams’ first bending mode shape can be approximated through a polynomial expression as shown in the plot of figure 2. Equation (9) can then be solved for the cantilever beam. The damping is plotted in figure 3 as a function of the flow velocity, clearly showing that the damping is directly proportional to the fluid velocity.

3. Implementation for complex structures
In order to apply the damping theory developed in the previous section to more complex structures such as Francis runners, a more practical approach needs to be developed. In this section, three possible approaches are presented: a finite element approach and two computational fluid dynamics (CFD) approaches, one based on the modal work and one using a 1DOF model.

3.1. Finite element implementation
Using the FE approach, the displacement function, $u(x, y, z)$, is represented as a set of discretized displacements, $\mathbf{u}$, and shape functions, $N$.

$$u(x, y, z) = N\mathbf{u} \tag{13}$$

The standard FE structural stiffness, $\mathbf{K}_s$, and mass, $\mathbf{M}_s$, matrices are formulated as follows:

$$\mathbf{K}_s = \iiint B^T DB\,dxdydz \tag{14}$$

$$\mathbf{M}_s = \iiint \rho_s N^T N\,dxdydz \tag{14}$$

where $N$ are the shape functions, $B$ contains partial derivatives of the shape functions that represent the strain, and $D$ is the material elastic constitutive matrix. The equations of motion are then:

$$\mathbf{M}_s \ddot{\mathbf{u}} + \mathbf{K}_s \mathbf{u} = \mathbf{F}_E - \mathbf{F}_w \tag{15}$$

where $\mathbf{u}$ is a vector of the nodal displacements, an overdot indicates differentiation with respect to time, $\mathbf{F}_E$ are the external excitation forces, including the fluid generated excitation forces, and $\mathbf{F}_w$ contains the damping forces due to the fluid. The quantity $\mathbf{F}_w$ is calculated by assuming the damping forces act as a pressure, $P$, on the structure and integrating over the surface area $A$. Using the expression for the force due to FSI from equation (4), $\mathbf{F}_w$ is given by:

$$\mathbf{F}_w = \iiint \left[2\rho_w t_w v_x N^T \frac{\partial N}{\partial x}\,d\mathbf{u} + 2\rho_w t_w v_y N^T \frac{\partial N}{\partial y}\,d\mathbf{u} + \rho_w t_w N^T \mathbf{N} \mathbf{u} \right] dA \tag{16}$$

$\mathbf{F}_w$ is then simplified as follows where $\mathbf{C}_w$ is the damping matrix due to the motion of the fluid and $\mathbf{M}_w$ is the mass matrix due to the inertial load from the presence of the fluid.

$$\mathbf{F}_w = \mathbf{C}_w \mathbf{\ddot{u}} + \mathbf{M}_w \mathbf{\dot{u}} \tag{17}$$

Substituting back $\mathbf{F}_w$ into the equation of motion (15), the equation of motion of the whole system of structure and fluid is obtained:

$$[\mathbf{M}_s + \mathbf{M}_w] \ddot{\mathbf{u}} + \mathbf{C}_w \mathbf{\dot{u}} + \mathbf{K}_s \mathbf{u} = \mathbf{F}_E \tag{18}$$

Note that no additional degrees of freedom have been added to the original equations of motion. The only additional computational effort required is the integration of the new $\mathbf{C}_w$ and $\mathbf{M}_w$ matrices that should be minimal compared to the solution time. It is assumed that the natural frequencies and mode shapes of the undamped structure for the $i$th mode, with no fluid effects, ($\omega_{i0}$), and ($\rho_i$) are known, and also the undamped natural frequencies in static fluid ($\omega_{ho}$), is known. These quantities could be
determined from an analogous FE model using standard structural and acoustic elements. It is also assumed that the mode shapes with and without fluid effects are identical. At this point, the only remaining problem is that the virtual fluid thickness, \( t_w \), needed to determine \( C_w \) and \( M_w \), is not known. It turns out that \( t_w \) can be determined from a few simple matrix manipulations. First, the unit damping, \( C_w' \) and unit mass, \( M_w' \), matrices are assembled without prior knowledge of \( t_w \).

\[
C_w' = \int \int 2\rho_w v_x N^T \frac{\partial N}{\partial x} + 2\rho_w v_y N^T \frac{\partial N}{\partial y} \, dA
\]

\[
M_w' = \int \int \rho N^T N \, dA
\]

Next, the mode shapes are normalized to the stiffness matrix using the natural frequencies that include the fluid effects. Note that any stiffness effects due to the flowing water are neglected and the stiffness matrix includes only the structural stiffness.

\[
(\mathbf{p})^T \mathbf{K}_s (\mathbf{p})_l = (\omega_{sv})_l^2
\]

\[
(\mathbf{p})^T [\mathbf{M}_s + t_w \mathbf{M}_w'] (\mathbf{p})_l = 1
\]

It is then possible to solve for \( t_w \).

\[
t_w = \frac{1 - (\mathbf{p})^T \mathbf{M}_w (\mathbf{p})_l}{(\mathbf{p})^T \mathbf{M}_w' (\mathbf{p})_l}
\]

Once the virtual thickness is known, \( C_w \) and \( M_w \) can be found from a simple scalar multiplication of \( t_w \) with \( C_w' \) and \( M_w' \). The damping ratio, for the \( i \)th mode, is then found as

\[
\zeta_i = \frac{(\mathbf{p})^T C_w (\mathbf{p})_l}{2(\omega_{ki})_l}
\]

In practice, elements to form \( C_w \) and \( M_w \) are surface elements that cover the wetted surface of the discretized solid structure as shown in figure 4. No additional degrees of freedom are needed since the fluid domain is not discretized. Determination of the virtual thickness, \( t_w \), and damping ratio, \( \zeta_i \), requires little computational effort based on simple matrix manipulations and from information that may already be available. The effective velocity needs to be determined by a separate flow analysis. The damping model was implemented using the finite element approach as a MATLAB [14] computer code called DAMPEL.

Damping measurements in flowing water have been performed on three hydrofoils [7]-[9]. Results from this damping test are used for validation of the damping model as implemented in DAMPEL. A schematic of the tested hydrofoils is shown in figure 6, and their mode shape deflexion for which damping was measured is shown in figure 7.

**Figure 4.** Mesh of solid structural elements with surface damping elements.

**Figure 5.** DAMPEL results compared to test data for H0 hydrofoil.
Uniform flow velocities corresponding to the test conditions were specified into DAMPEL and the experimentally determined natural frequency in static fluid was used to determine \( t_n \). The results for one of the hydrofoils from the test, H0, are presented in figure 5 and compared to DAMPEL predictions. Good agreement is obtained between DAMPEL and the experimental data.

![Schematic of tested hydrofoils.](image)

**Figure 6.** Schematic of tested hydrofoils.

**Figure 7.** Fixed hydrofoil first mode shape deflexion

### 3.2. CFD approaches

Hydrodynamic damping can be determined on a mode by mode basis from CFD analysis using two different approaches: the modal work approach and the 1DOF approach. For both of these approaches to be valid, it must be assumed that the influence of the flow on the mode shape relative to still water is negligible. These methods have also been developed for aerodynamic damping [12] and implementations in commercial codes such as Ansys CFX [13] have been shown in recent code previews.

#### 3.2.1. Modal Work approach.

Given that modern CFD codes can handle defined mesh motion during transient calculations, it is possible to combine modal motion with CFD flow calculations. The assumption must be made that the modes are orthogonal and independent from each other, and that the modal motion is of harmonic nature. A harmonic modal motion is then applied to a relevant structural surface at a given forcing frequency \( \Omega \). The energy exchange (modal work) \( W_{mod} \) between the structure and the flow is extracted. This modal work is the work done by the pressure and shear forces per cycle on the total area of the structure \( A \), due to the structural displacement \( u \). The shear forces will only contribute at free edges such as trailing edges of blades. Their effect is found to be two orders of magnitude smaller than the pressure effect and is therefore neglected. The modal work per cycle extracted from CFD analysis is then equal to the work per cycle from fluid forces in equation (6).

The structural modal energy of deformation in water \( W_{def} \) can be obtained from a modal analysis with acoustic and solid elements. If the structure vibrates at its natural frequency, the damping ratio can be obtained from the CFD fluid work \( W_F = W_{mod} \) and the rearranged equation (7):

\[
\zeta = \frac{W_{mod}}{4\pi W_{def}} \tag{23}
\]

The tested hydrofoil H0 from figure 6 and previous publications [7]-[9] is used as a validation case. Figure 8 shows the mid-trailing edge displacement, the instantaneous modal work and the calculated...
modal work per cycle for the H0 hydrofoil modal vibration under flowing water at 6 m/s. Equation (23) can then be used to calculate the damping ratio.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Damping plate H0, 6 m/s, 1.0 mm amplitude modal vibration: monitoring of max modal displacement (magenta), instantaneous modal work on profile (red) and evolution of integral modal work over full period (green).}
\end{figure}

3.2.2. 1DOF approach. As an alternative to the modal work approach, the dynamic behaviour of the structure can be represented by its modal parameters and a single degree of freedom (1DOF) mechanical oscillator model. This 1DOF model can be coupled with the CFD calculation using again the mode shape in still water from a prior FE analysis. Then added damping can be extracted from a free oscillation after some initial impulse-like excitation by means of logarithmic decrement. Tested hydrofoil H0 [7]-[9] is again used as a validation case. Figure 9 shows the H0 mid-trailing edge displacement after an impulse excitation under a water flow of 6 m/s.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{Damping plate H0: CFD with 1DOF oscillator model, evolution of time signal after impulse excitation.}
\end{figure}

3.2.3. CFD results. In figure 10 the results from the two CFD approaches are compared with the measurements. One can see that the results from both prediction methods compare exceptionally well with the experimental results for which data are available in a velocity range up to 17m/s. Within that velocity range the two approaches also compare well with one another. Above 17m/s the two CFD
approaches show slightly different behaviour. The reason for this is not fully clear up until now. It is possible that this is simply an artefact of post processing. As the damping gets stronger, fewer full oscillation cycles are present for post-processing before being damped out, resulting in a less accurate determination of the damping ratio. This will be investigated in more detail in the future.

Figure 10. Damping plate H0: Comparison of damping ratio from modal work and 1DOF approaches with experiment.

4. Discussions and conclusion
A theory of the forces due to a flowing fluid on a vibrating structure has been developed. The theory may be applied to simple structures, such as cantilever beams, to analytically calculate the damping added by flowing fluid. However, the damping calculation for more complex structures, such as runner blades, is geometrically too complicated for an analytical integration of the equations. Instead, three methods were presented to implement the damping calculation for complex structures: FEA method, CFD modal work method and CFD 1DOF method. Calculated damping values from all three methods were compared to damping measurements on hydrofoil H0 and all prediction methods show good agreement with the measurements.

The finite element approach requires the input of the flow velocity. A mean value can be easily obtained for a straight test section as for the damping plate validation case. However, the choice of such a velocity can become more problematic in a realistic Francis runner flow channel. The two CFD approaches include much more of the fluid physics and do not require such an external input. Also, the FEA approach supposes that the mode shape is not affected by the presence of the fluid. This condition may not be fulfilled by all types of structures. In the CFD approaches the mode shapes calculate with FEA in stationary water can be used. These are considered sufficiently close to the mode shapes in flowing water. The 1DOF approach is quite powerful because it requires only one simplifying assumption (mode shape unchanged by flow) and reveals the dynamic behaviour of the system in a straight forward manner. However, one disadvantage of the 1DOF approach lies in the difficulty of extracting modal parameters at high flow velocities when high damping is present and numerical stability issues may arise.

All three approaches were shown to be reliable for damping prediction in the case of the damping plate measurements. According to the limitations of each approach, they may all be of interest in the characterisation of different components and phenomena inside the entire hydraulic turbine designs.
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