Crow Search based Multi-objective Optimization of Irreversible Air Refrigerators

Oguz Emrah TURGUT *

Abstract: This study proposes the optimum performance of the irreversible air refrigerators through recently developed metaheuristic algorithm called Crow search algorithm by means of finite time thermodynamics. Finite time thermodynamics is based on choosing the optimum pathways for any kind of thermodynamic system in order to reach the maximum efficiency of the thermodynamic cycle. Handful of objectives for assessing the performance of the irreversible air refrigerators such as coefficient of performance (COP), exergetic efficiency ($\eta_1$), ecological coefficient of performance (ECOP), thermos economic optimization (F), and thermos ecologic optimization functions (ECF) have been successfully applied on the system. Three optimization scenarios have been studied for the multi objective optimization of irreversible air refrigerators. First scenario evaluates the concurrent optimization of objectives including exergetic efficiency ($\eta_1$), coefficient of performance (COP), and ecological coefficient of performance (ECOP). In second scenario, coefficient of performance (COP), thermos economic parameter (F), and thermos ecological coefficient of performance (ECOP) have been simultaneously maximized to retain optimum working point of the cycle. Third case studies the simultaneous optimization of the imposed objectives such as second law efficiency ($\eta_2$), coefficient of performance (COP), and thermos ecological function (ECF). Widely known decision-making theorems of LINMAP, TOPSIS, and Shannon’s entropy theorem have been applied on the Pareto curve constructed by the non-dominated solutions to decide the most favorable solution on the frontier.

Keywords: Brayton refrigerators, Decision making, Crow Search Algorithm, Multi objective optimization, thermoeconomic optimization

1. Introduction

Environmental restrictions occurred by the protection of the Ozonosphere against the hazardous chlorofluorocarbon (CFC) gas has urged researchers to seek more environmental friendly alternatives as a refrigerant in refrigeration industry. Among of all options, the performance of air refrigeration cycles has drawn more attention than the other alternatives, as the working fluid of the cycle (air) is free, safe and non-toxic. In addition, air cycle equipment is conveniently reliable and prone to reduce the maintenance cost and system downtime. The performance of an ordinary air refrigeration cycle does not worsen as much as that of a vapour compression cycle unit when it is operated far away from its design point. Another advantage of an air refrigeration cycle is that air cycles can procure higher temperature difference as compared to vapour-compression cycles. This advantage can lead two important points: one is that cryogenic air can be produced through the utilization of this cycle, and the other is that, released heat to the ambient can produce at moderate and useful temperature ranges, which, if accompanied by cooling, can develop efficient and low-grade energy. As a result of these specific advantages, many researchers have devoted themselves to optimize the working parameters of a Brayton refrigerator in order to attain better system performance [1-4].

There has been many literature studies concerning with the performance optimization of non-regenerative Brayton cycles. Most of these studies consider the cooling load, coefficient of performance, exergetic efficiency, and thermoeconomic aspects as an optimization objective to be determined. As a preliminary study, Wu [5] investigated the heat exchanger effect on the power input of a gas refrigerator. A basic Brayton cycle coupled with two heat exchangers was analyzed and optimized by means of finite time thermodynamics approach. Chen et al. [6] mathematically analyzed the effect of the heat resistance on the performance of an air cycle refrigerator through finite time heat transfer approach. Interactions between cooling load, pressure ratio, and COP (coefficient of performance) values had been clearly identified and mathematical expressions had been derived. The results developed from mathematical models revealed that the cooling load has a parabolic dependence on COP rates. Zhou et al. [7] carried out numerical tests regarding to the performance analysis and optimization of an end or reversible air refrigerator by considering the cooling density, which is the ratio of cooling load to maximum specific volume in the cycle, as an optimization objective. Numerical experiments showed that pressure ratios and allocation of heat exchangers have considerable effect on the optimum system design. Luo et al. [8] examined the performance of an air refrigeration cycle by applying finite time thermodynamics theory as an evaluation criterion. Cycle characteristics were thoroughly studied with varying cycle parameters. Chen and Su [9] carried out numerical analysis related to exergetic efficiency optimization for an irreversible Brayton cycle. The objective function considered in the mentioned study, exergetic efficiency, is defined as the ratio of output exergy rate to the input exergy rate of the system. Analytically obtained solutions concerning with exergetic efficiencies were compared with those of determined by the traditional methods. Numerical calculations also took into account and evaluated the heat leak between hot and cold reservoirs and the effect of ratio of two reservoirs on the exergetic efficiencies. Tu et al.[10] used finite time thermodynamics theory to optimize a real air refrigerator with a simple irreversible variable-temperature heat

*Ege University, Department of Mechanical Engineering, Izmir, TURKEY
*Corresponding Author Email: octurgut@hotmail.com
reservoir. The allocation of heat exchanger inventory and ratio of the thermal capacities between the working fluid and heat reservoirs were optimized in order to obtain maximum cooling load and COP rates. The variable influences of pressure ratio, the efficiencies of expander and compressor, and the thermal capacities of the working fluids on the performance of the cycles were examined by numerical parametric studies. Recent years have witnessed the application of thermos economization optimization studies by various researchers. This new term was primarily benefited in the works of Yan and Lin [11], and Chen et al. [12]. In these studies, the mentioned term stated above which was introduced by Angulo-Brown [13] for power generation cycles was modified to evaluate the performance of the refrigeration cycles and defined as \( ECF = Q_L - \beta L T_o \sigma \) where \( \beta L = T_L / (T_H - T_L) \) is the COP value for reversible Carnot refrigerator, \( T_o \) is the ambient temperature, \( \sigma \) is the entropy generation rate, and \( Q_L \) is the cooling load. Using this modified criterion, Kalaiselvam et al. [14] investigated the optimal performance of two-stage refrigerator and analytically obtained the optimal conditions of cooling load, entropy generation rate, and coefficient of performance rates. Wouagfack and Tchinda [15] considered ecological coefficient of performance (ECOP) as an objective function to optimize the working parameters of irreversible refrigeration absorption system. Analytically derived performance parameters including optimal temperatures had been optimized with respect to maximum ECOP values and corresponding effects of heat leakage rates, internal irreversibility, and the source temperature ratios had been explicitly discussed. Ust and Sahin [16] carried out numerical experiments on the performance of an irreversible Carnot refrigerator model by taking ECOP as an optimization objective. This term was previously defined as a ratio of the cooling load to the entropy generation rate. Analytically derived optimal temperatures of working fluids, optimum coefficient of performance, and optimal cooling load was obtained through the maximization of ECOP values. Ust [17] examined the thermo-ecological performance analysis of an ordinary reversible Brayton refrigerator. The optimum design parameters along with their maximum ECOP rates were determined analytically. Following these studies, Ust[18] extended his earlier works [16,17] and investigated the ecological performance of irreversible regenerative Brayton refrigerator by considering the ECOP objective function. The influences of regeneration and heat source temperature ratios were examined. Interested readers can also find related papers about finite time thermoeconomic optimization in the literature[18 - 28]

Finite time thermoeconomic optimization evaluates the economic aspects of thermosyoptic systems using finite time thermodynamics. Sahin and Kodal [29] firstly presented thermoeconomic performance criteria, which is the cooling load per unit cost, in order to take into account of both investment and energy consumption rates. In their study, the optimum cycle parameters of end or reversible heat pumps and refrigerators those maximizing the objective function described above. Kodal et al.[30] investigated the effects of internal irreversibility and heat leakages on the refrigerators and heat pumps based on the thermoeconomic criterion with using finite time thermodynamic theory. Using finite time thermodynamic, thermoeconomic analysis of absorption irreversible refrigerators and heat pumps were performed by Kodal et al [31]. Analytically obtained optimal design parameters were determined at the maximum thermoeconomic objective function rates. This study is a pioneer work on multi objective optimization of irreversible air refrigeration cycles since, after a comprehensive literature survey, it was found that there is no records for the multi objective performance optimization of air cycle refrigerators. This paper considers three different optimization scenarios for optimum design of irreversible heat pump based on reversed Brayton cycle. Three decision-making theories of LINMAP, TOPSIS and Shannon’s entropy theory have been utilized to determine final best solution among the set of non-dominated solutions.

2. Thermodynamical modelling of irreversible air refrigeration cycle

Figure 1(a-b) gives the basic depiction of an irreversible Brayton cycle along with its corresponding T-S diagram.

The cycle operates between extreme temperatures of heat source at \( T_i \) (cold and refrigerated space) and heat sink temperature at \( T_o \) (warm ambient). As its nature, cycle at hand includes two isobaric processes (the processes those operating between 2-3 and 4-1) and two non – isentropic processes ( the compression process 1-2 and the expansion process 3-4). Heat transfer from heat source to the refrigerator ( \( \dot{Q}_L \) ) and the rate of heat exchange from the refrigerator to the heat sink ( \( \dot{Q}_H \) ) can be simply calculated as in Eq. (1) and Eq. (2)

\[
\dot{Q}_L = U_L A_L \left( LMTD \right)_L = \dot{C}_w (T_i - T_o) \tag{1}
\]

\[
\dot{Q}_H = U_H A_H \left( LMTD \right)_H = \dot{C}_w (T_2 - T_3) \tag{2}
\]

Where \( A_H \) and \( A_L \) are the corresponding total heat exchange areas, \( U_H \) and \( U_L \) are overall heat transfer coefficients for both hot and cold side heat exchangers, respectively. Assuming the ideal gas approximation with constant heat capacities \( \dot{C}_w \) is the heat capacitance rate of the working fluid. Logarithmic mean temperature differences for corresponding hot and cold sides are calculated by Eqs. (3-4)
The rate of exergy output (\( E_{\text{out}} \)) to the rate of exergy input (\( E_{\text{in}} \)) and formulated as Eq. (18)\[18]:

\[
\eta_E = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{\dot{Q}_{LT}}{\dot{Q}_{LT} + \dot{Q}_{HT} \left( \frac{T_0 - T_L}{T_0 - T_H} \right) - \dot{Q}_{LT} \left( \frac{T_0 - T_L}{T_0 - T_H} \right)}
\]

On the basis of Eqs. (5) and (6), \( T_3 \) and \( T_4 \) take the form of given equations Eqs. (19-20).

\[
T_3 = e_H T_H + T_2 \left( 1 - e_H \right)
\]

\[
T_4 = T_2 \left( 1 - e_L \right)
\]

The compression and expansion efficiencies in the irreversible Brayton refrigeration cycle are defined as Eqs. (21-22).

\[
\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}
\]

\[
\eta_E = \frac{T_3 - T_4}{T_3 - T_{4s}}
\]

Then, Eq.(21) and (22) yield to Eqs. (23-24)

\[
T_{2s} = \left( 1 - \eta_C \right) T_2 + \eta_C T_2
\]

\[
T_{2s} = \left( T_2 / \eta_E \right) + \left( 1 - \left( 1 / \eta_E \right) \right) T_1
\]

Using the ideal gas relation on the second law of thermodynamics yield the following formulation Eq. (25).

\[
\phi = \frac{T_{2s}}{T_1} = \frac{T_3}{T_4}
\]

Where \( \phi \) is the isentropic temperature ratio of the cycle. Eqs (23-25) transforms into the Eqs. (26-27)[17]

\[
T_2 = T_1 \left( \frac{\phi - 1 + \eta_C}{\eta_C} \right)
\]

\[
T_3 = T_1 \left( 1 - \eta_H \right) + \frac{\eta_C}{\eta_E} \left( 1 - \eta_C \right) T_1
\]

By substituting Eqs. (19), (20), (21), (24), (26), (27) into Eq (25), temperature \( T_1 \) becomes the function of the isentropic temperature ratio of the cycle and simplification parameters defined in Eqs (28-33)[17]

\[
T_1 = \frac{k_1 \phi + k_2}{k_1 \phi^2 + k_4 \phi + k_5}
\]

\[
k_1 = e_H T_H \left( 1 - \frac{1}{\eta_E} \right) - \frac{\epsilon_L T_L}{\eta_E \left( 1 - \epsilon_L \right)}
\]

\[
k_2 = e_H T_H
\]

\[
k_3 = \frac{1}{\eta_C} \left( 1 - \frac{1}{\eta_E} \right) \left( e_H - 1 \right)
\]

\[
k_4 = \frac{1}{\eta_C} \left( 1 - \frac{1}{\eta_E} \right) \left( e_H - 1 \right)
\]

\[
k_5 = \frac{1}{\eta_C} \left( 1 - \frac{1}{\eta_E} \right)
\]

Based on the construction of the formulation given in Eq (28),
The ecological coefficient of performance (ECOP) for an irreversible Brayton refrigeration cycle, which is the one of the objective functions to be simultaneously optimized in this study, is expressed by the ratio of the cooling load to entropy generation rate multiplied by the ambient temperature and formulated by the following equation, Eq. (37).

\[ ECOP = \frac{\dot{Q}_T}{T_0 \dot{S}} \]  

(37)

Another objective function that will be studied in this paper is thermo-economic optimization that was previously elaborated studied in [29-31]. Basic formulation of this optimization objective can be given as Eq. (38)

\[ F = \frac{\dot{Q}_T}{C_i + C_e + C_m} \]  

(38)

Where \( C_i, C_e, \) and \( C_m \) correspondingly represent the annual investment, energy consumption and the maintenance costs. Investment cost includes the annual expenditures of main components of the cycle including the heat exchanger, compressor and expansion devices along with their prime movers. For the heat exchangers, it is considered that total investment cost is proportional to the total heat exchange area. Moreover, investment costs related to the compressors and their drivers are considered proportional to the required input power. Therefore, the annual investment cost of the cycle becomes the function of the mentioned factors and can be accordingly defined as Eq. (39) [34]

\[ C_i = a (A_{H_i} + A_{L_i}) + b\dot{W}_m \]  

(39)

Where \( a \) is the capital recovery factor times investment cost per unit heat exchanger area with the dimensional representation of ncu/(year m\(^2\)). The coefficient for the investment costs of the compressors with their drivers, \( b \), is the capital recovery factor times investment cost per unit power with the dimensional representation of ncu/(year kW). The unit ncu represents the national currency unit. Annual energy consumption cost is the function of the power input and can be formulated as Eq. (40)[34]

\[ C_e = b_2\dot{W}_m \]  

(40)

Where the coefficient \( b_2 \) symbolizes the annual operation hours times the price per unit energy with dimensional representation of ncu/(year kW). Total maintenance cost of system is assumed to be proportional to the cooling rate and defined as Eq. (41)

\[ C_m = a_2\dot{Q}_T \]  

(41)

Where the coefficient \( a_2 \) stands for annual cost per unit energy input rate with the dimensional representation of ncu/(year kW). And finally, the last optimization objective considered for this study is thermo-ecological function (ECF) which is subtly described in introduction section.

### 3. Crow Search Algorithm

In this study, the new emerged metaheuristic optimization algorithm called Crow Search will maintain multi objective optimization of the irreversible Brayton refrigeration cycle. Current trend in engineering optimization is to take advantage of the nature inspired metaheuristic algorithms in hard-to-solve design problems. Literature studies show that utilization of these strategies in engineering problems lead to the very efficient and effective results [35]. Various types of metaheuristic algorithms have been developed in recent years to solve variety of real world optimization problems. These algorithms are all nature inspired and simulate some principles of physics, biology and swarm intelligence [36]. The metaheuristic algorithms do not guarantee finding global optimal solution as they are based on stochastic search strategies. The most popular metaheuristic optimization algorithms those have been developed during two decades can be listed as follows. Genetic algorithm (GA) [37] is based on the Darwinian natural selection strategy and depends on the survival of the fittest. Differential evolution (DE) [38] adopts the perturbation schemes of the Genetic algorithm and tries to get the optimum results with a slight modification to the adopted schemes. Particle Swarm Optimization (PSO) [39] is a population based swarm intelligence algorithm, which mimics the social behavior of flocks of birds. Harmony Search (HS) [40] mimics the effort of the musician who aims to find the perfect state of the harmony determined by the aesthetic standards. Bat Algorithm (BAT) [41] is constructed on the echolocation of the micro bats. Firefly Algorithm (FA) [42] is based on the flashing light of the fireflies. Readers can easily observe that researchers have used only limited characteristics of nature and there is still room to develop more efficient metaheuristic algorithms. Based on this interpretation, Askarzadeh [43] proposed Crow Search Algorithm (CSA) which is built on the concept of subsistence behaviors of crows.

Crows are clever animals, as their nature. They have been considered as the most intelligent birds. They have the ability to remember faces when a strange looking one appear and warn each other with a sophisticated communication ways. [44 - 45] Crows are prone to watch other birds and observe where the watched bird hide their food, and they steal the food from when the owner leaves from the area. The crow who steals the food will take extra precautions in order to avoid being a future victim. As potential thieves, they use their own experiences to get rid of being pilfered, and can find safer courses to protect their catches from the pilferers [46].

According to the above-mentioned intelligent behaviors of the crows, CSA is developed obeying the principles below [43]

- Crows live in the form of flock
- Crows recalls their hiding space positions
- Crows observe other birds to do thievery
- Crows take care of their catches to avoid being pilfered by other birds

### 3.1. Implementation of Crow Search Algorithm

Table 1 gives the pseudo-code of the proposed CSA. This section provides the stepwise procedure for the implementation of CSA [43].
Table 1 Pseudo-code of Crow Search Algorithm

| Step | Description |
|------|-------------|
| 1.    | Initialize the algorithm parameters |
| 2.    | Optimization problem, the limits of the search space, and design variables are defined. After that, adjustable algorithm parameters including flock size (N), maximum number of iterations (maxiter), flight length (f_l), and awareness probability (AP) are valued. |
| 3.    | Initialize the position and memory of crows |
| 4.    | As the members of the flock, N crows are randomly positioned in D-dimensional space in Eq. (42). In this context, each crow symbolizes a sample solution and D is the number of design variables of the problem. |
| 5.    | Position update of the crows |
| 6.    | Evaluate the new position of the crows |
| 7.    | Evaluate the feasibility of the new position is checked. If the position is feasible then the crow updates its position, or else the crow stays in its current position. |
| 8.    | Check the boundary control and evaluate the feasibility of the each solution |

Step 1: Initialize the algorithm parameters
Optimization problem, the limits of the search space, and design variables are defined. After that, adjustable algorithm parameters including flock size (N), maximum number of iterations (maxiter), flight length (f_l), and awareness probability (AP) are valued.

Step 2: Initialize the position and memory of crows
As the members of the flock, N crows are randomly positioned in D-dimensional space in Eq. (42). In this context, each crow symbolizes a sample solution and D is the number of design variables of the problem.

\[
\text{Crows} = \begin{bmatrix}
0 \quad 1 \quad 1 \quad \cdots \quad 1 \\
0 \quad 2 \quad 2 \quad \cdots \quad 2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 \quad N \quad N \quad \cdots \quad N
\end{bmatrix}
\]  

Eq. (43) shows the initialization of the memory of each crow in the flock.

\[
\text{Memory} = \begin{bmatrix}
1 \quad 2 \quad \cdots \quad 1 \\
1 \quad \cdots \quad \cdots \quad 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 \quad \cdots \quad \cdots \quad \cdots \quad 1
\end{bmatrix}
\]  

Step 3: Evaluate the fitness of each crow in the flock Each crow in the flock is evaluated through its corresponding fitness value.

Step 4: Update position of the crows
Crows update their position in the search domain with the following procedure. Assume that crow \(i\) wants to move to a new position. Then, this crow selects one of the crows from the flock (for example crow \(j\)) and follows it to find the position of the hidden food concealed by this crow \(m\). Consequently, the new position of the crow \(i\) is determined by the following Eq. (44).

\[
x_{i, \text{iter}+1} = x_{i, \text{iter}} + f_l \times f_{\text{iter}} \times (m_{\text{iter}} - x_{i, \text{iter}}) \]  

Where \(r\) is the Gaussian random number generated between 0 and 1 and \(f_{\text{iter}}\) is the flight length of crow \(i\) at iteration \(\text{iter}\).

Step 5: Check the boundary control and evaluate the feasibility of the each solution
Position of each crow in the flock is checked. If the position of the crow extends the prescribed boundaries of the search space, then this crow is restricted into the predefined search domain. After that, the feasibility of the new position is checked. If the the position is feasible then the crow updates its position, or else the crow stays in its current position.

Step 6: Evaluate the fitness values of the new position of the crows
The new position of each crow in the flock is evaluated by its corresponding fitness function value.

Step 7: Memory update
Following procedure given in Eq. (45) shows the memory update mechanism of each crow in the flock.

\[
x_{i, \text{iter}+1} = \begin{cases}
 x_{i, \text{iter}} + f_l \times f_{\text{iter}} \times (m_{\text{iter}} - x_{i, \text{iter}}) & \text{if } f(x_{i, \text{iter}+1}) > f(m_{\text{iter}}) \\
 x_{i, \text{iter}} & \text{otherwise}
\end{cases}
\]  

Eq. (45) shows that if the objective function value of the updated position of a crow is better than the fitness value of the memorized position, then the crow updates its memory.

Step 8: Check termination criteria
Step 4 to Step 7 are repeated until predetermined maximum number of iteration value is reached. At the end of the iterations, the best memory position in the memory matrix with respect to objective function value is taken as the optimum solution of the problem along with its respective fitness value.

3.2. Multi objective optimization
Multi objective optimization is a real world problem solving strategy, which includes conflicting objectives with a number of equality and inequality constraints to be optimized simultaneously. This problem solving strategy has become popular amongst the researcher community as most of the design problem inherits more than one objective, which should be concurrently solved for reasserting the optimum design. Traditional optimization methods based on single optimization procedure yields one optimum solution for the problem at hand. Therefore, the use of multi objective optimization in system design or problem solution is inevitable to some degree since it comprises set of non-dominated solutions those satisfying the conflicting objectives at acceptable levels [47]. These solutions on the search space built up the Pareto frontier along with the ideal and nadir solutions. Pareto frontier is constructed by the set of solutions, which are trade-off solutions between the various objective functions. Best compromising solution on the Pareto curve can be selected through the decision-making theories such as LINMAP, TOPSIS, fuzzy, and Shannon’s entropy theory. Multi objective optimization problem can be mathematically defined as Eq. (46).

\[
\text{Find } \bar{x}(x_i) \quad \forall i = 1, 2, 3, \ldots, N_{\text{par}} \\
\text{Arg min or Arg max } f_i(\bar{x}) \quad \forall i = 1, 2, 3, \ldots, N_{\text{obj}} \\
g_j(\bar{x}) = 0 \quad \forall j = 1, 2, 3, \ldots, m \\
h_k(\bar{x}) \leq 0 \quad \forall k = 1, 2, 3, \ldots, n
\]  

Where \(\bar{x}\), \(N_{\text{par}}\), \(f_i(\bar{x})\), \(N_{\text{obj}}\), \(g_j(\bar{x})\), and \(h_k(\bar{x})\) correspondingly represent design variables vector, number of design variables, objective functions, number of objectives, equality and inequality constraints [48].
3.3. Decision making strategies in multi objective optimization

Pareto frontier comprises set of non-dominated solutions, which are tradeoff between conflicting objectives. Multi objective optimization uses various types of decision-making theories in order to select the most feasible solution on the Pareto curve. There are plenty of decision-making approaches in the literature [49]. This study utilizes three decision-making methods including TOPSIS, LINMAP, and Shannon’s entropy approach. Before utilizing these decision-making theories, all the objective functions must be unified and re-scaled. For this aim, objective functions should be non-dimensionalized with using Euclidian, linear, and fuzzy non-dimensioned methods given below.

**Linear non-dimensionalization approach**

Objective functions can be nondimensionalized by the linearization strategy described in Eqs. (47-48).

\[ F^n_y = \frac{F_y}{\max(F_y)} \quad (\text{For maximization problem}) \]  

\[ F^n_y = \frac{F_y}{\max(1/F_y)} \quad (\text{For minimization problem}) \]  

**Euclidian approach**

Objective functions can be non-dimensionalized through the Euclidian approach as following equation Eq. (49).

\[ F^n_y = \frac{F_y}{\sqrt{\sum_{i=1}^{m} (F_y)^2}} \]  

**Fuzzy approach**

Objective functions of the optimization problem can be non-dimensionalized via the fuzzy approach as defined in Eqs. (50-51).

\[ F^n_y = \frac{F_y - \min(F_y)}{\max(F_y) - \min(F_y)} \quad (\text{For maximization problem}) \]  

\[ F^n_y = \frac{\max(F_y) - F_y}{\max(F_y) - \min(F_y)} \quad (\text{For minimization problem}) \]  

In Eqs (47 - 51), \( F_y \) symbolizes the matrix of objectives at some of the solutions on the Pareto frontier and \( i \) stands for the index of the non-dominated solutions on the Pareto curve and \( j \) symbolizes the index of each objective of the optimization problem. The fuzzy Bellman–Zadeh decision making theories utilizes the fuzzy non-dimensionalization procedure whereas the LINMAP, TOPSIS and Shannon’s entropy theory methods use Euclidian non-dimensionalization procedure.

3.3.1. LINMAP decision making approach

Ideal point on the Pareto curve is the solution, which is the optimization result of the each objective irrespective of satisfaction to the other objectives. In multi objective optimization, it is obvious that optimal solution of each solution can not be obtained as it is acquired in single objective optimization. Therefore, ideal solution is not located on the Pareto frontier. In LINMAP decision making approach, after making Euclidian non-dimensionalization of all objectives in the objective space, the distance between the each solution on the Paretor frontier and ideal solution is calculated by the following equation Eq. (52).

\[ d_i = \sqrt{\sum_{j=1}^{n} \left(F_{ij} - F_{ij}^{ideal}\right)^2} \]  

Where \( n \) represents the number of objectives, \( i \) stands for index of the solution on the Pareto curve \((i=1,2,3,...,m)\), and \( F_{ij}^{ideal} \) is the ideal value of the \( j \)th objective obtained through the single objective optimization. LINMAP method selects the solution with minimum distance from the ideal point as a final optimum solution, which can be shown as Eq. (53)[51]:

\[ i_{final} = i \in \min\left(d_i\right); \quad i = 1, 2, 3, ..., m \]  

3.3.2. TOPSIS decision making approach

This method utilizes the nadir solution (non-ideal point) instead of the ideal point. The nadir point in the objective space is the solution in which each objective function has its worst value. Therefore, this procedure calculates the Euclidian distance of the each point on the Pareto curve from the nadir point with the given equation in Eq. (54).

\[ d_i = \sum_{j=1}^{n} \left(F_{ij} - F_{ij}^{nadir}\right)^2 \]  

In this method, final form of the Euclidian distance, \( d_i \), is calculated as Eq. (55)[52]:

\[ d_i = \frac{d_i}{d_{i+}+d_{i-}} \]  

Solution with a maximum \( d_i \) is selected as a final desired output, and respectively \( i_{nadir} \) is the index of the final selected solution given as Eq. (56).

\[ i_{final} = i \in \max\left(d_i\right); \quad i = 1, 2, 3, ..., m \]  

3.3.3. Shannon’s entropy approach

This method takes into account of the weight of the alternatives based on the \( L_{ij} \) in non-dimensional \( F_{ij} \) matrix with \( n \) number of solutions on the curve and \( m \) objective functions. The elements of \( L_{ij} \) is calculated as Eq. (57)[53]:

\[ L_{ij} = \frac{F_{ij}}{\sum_{i=1}^{n} F_{ij}}, \quad i = 1, 2, 3, ..., m \quad j = 1, 2, 3, ..., m \]  

Shannon’s entropy value can be obtained by the following equation Eq. (58).

\[ SE_j = -M \sum_{i=1}^{n} L_{ij} \ln L_{ij}, \quad M = 1/\ln(n) \]  

Deviation degree (\( D_j \)) is computed by Eq. (59).

\[ D_j = 1 - SE_j \]  

The weight of \( j \)th objective is calculated by the Eq. (60).

\[ W_j = \frac{D_j}{\sum_{j=1}^{m} D_j} \]  

Finally, this yields to Eq. (61).

\[ Y_j = L_{ij} W_j \]  

Maximum \( Y_j \) is selected by Shannon’s entropy approach to decide the final optimum solution with using the index of maximum point in \( Y_j \). That is, Eq. (62).

\[ i_{final} = i \in \max(Y) \]  

4. Results and Discussion

Three case studies are taking into account in this paper to optimize the pathways between the state point temperatures of irreversible
air refrigerators (Brayton refrigerators). First case explains the multi-objective optimization of exergetic efficiency ($\eta_{II}$), coefficient of performance (COP), and ecological coefficient of performance (ECOP). Second case deals with the concurrent optimization of the problem objects of coefficient of performance (COP), thermo-economic parameter ($F$), and thermo-ecological coefficient of performance (ECOP). Third case simulates the multi-objective optimization of three problem objectives such as second law efficiency ($\eta_{II}$), coefficient of performance (COP), and thermo-ecological function (ECF). These problem parameters are iteratively optimized through Crow search algorithm until the termination criteria of the optimization method is satisfied. Six design variables including the effectiveness of the heat exchanger at hot side ($\varepsilon_H$), the effectiveness of the heat exchanger at the cold side ($\varepsilon_L$), compression efficiency ($\eta_C$), expansion efficiency ($\eta_E$), internal conductance ($\xi$), and isentropic temperature ratio ($\phi$) have been considered to be optimized in order for attaining the efficient design of the irreversible air refrigerators in terms of finite time thermodynamics. The whole system is designed under the following operating conditions: $C_w = 1050.0$ (W/m²K), $T_L = 260.0$ (K), $T_0 = 290.0$ (K), $U_H = 500.0$ (W/m²K), $U_L = 500.0$ (W/m²K), $a = a_2 = 0.15$, $b = b_2 = 0.1$. Apart from these definitions, there exists a conceptual parameter in the context of multi-objective optimization called “the deviation index.” This expression is used to select the best final answer obtained from the different decision making theorems at hand. Calculation of this term is explicitly detailed in Ahmadi et al. [54]. As the deviation index rate is close to zero value, it means that its corresponding solution tends to reach optimum point while leaving away from the non-ideal point.

4.1. Results of the case study 1

This case maintains the multi-objective optimization of exergetic efficiency ($\eta_{II}$), coefficient of performance (COP), and ecological coefficient of performance (ECOP) of an irreversible air refrigerator. Figure 2 depicts the pareto frontier built on the basis of the set of the non-dominated solutions of these aforementioned design objectives. As seen from Figure 2, final solution obtained by LINMAP method is superior to the remaining decision making theorems. Table 2 reports the optimum solutions retained through three different decision-making theorem along with their corresponding design variables and state point temperatures. Outcomes of the LINMAP optimizer are selected as the best final answer due to its corresponding deviation index value, which is closer to zero than the others.

| Design variables | TOPSIS | LINMAP | Shannon’s entropy theory |
|------------------|--------|--------|--------------------------|
| $\varepsilon_H$  | 0.8985 | 0.8945 | 0.8669                   |
| $\varepsilon_L$ | 0.8907 | 0.8993 | 0.8983                   |
| $\eta_C$         | 0.9494 | 0.9484 | 0.9496                   |
| $\eta_E$         | 0.9499 | 0.9488 | 0.9497                   |
| $\xi$            | 0.0115 | 0.0106 | 0.0101                   |
| $\phi$           | 1.5221 | 1.4894 | 1.4678                   |

| Temperatures |
|--------------|
| $T_1$        | 255.735  |
| $T_2$        | 256.498  |
| $T_3$        | 256.851  |
| $T_4$        | 257.258  |
| $T_5$        | 257.324  |
| $T_6$        | 257.324  |
| $T_7$        | 257.324  |
| $T_8$        | 257.324  |
| $T_9$        | 257.324  |
| $T_{10}$     | 257.324  |

| Objective functions |
|---------------------|
| $\eta_{II}$         | 0.3044  |
| COP                 | 1.0076  |
| ECOP                | 1.4488  |

| Deviation index |
|-----------------|
| 0.1287          |
| 0.0931          |
| 0.2451          |

Fig. 2 Pareto frontier for case study 1
4.2. Results of the case study 2

This case deals with the simultaneous optimization of coefficient of performance (COP), thermoeconomic parameter (F), and thermoeccological coefficient of performance (ECOP) to scrutinize the optimum state point temperatures of the irreversible air refrigerators through Crow Search Algorithm. Figure 3 plots the scatter of non-dominated solutions over the objective function domain, along with the final best solutions obtained by three decision-making theorems. Table 3 reports the best results acquired by three different decision makers accompanied with optimum design variables and state point temperatures. Results retrieved from Table 3 indicates that all decision makes finds the same optimal output which reassuring the idea that the obtained result is the best final solution with a deviation index of 0.018.

| Design variables | TOPSIS | LINMAP | Shannon’s entropy theory |
|------------------|--------|--------|--------------------------|
| ε_H              | 0.8999 | 0.8999 | 0.8999                   |
| ε_L              | 0.8884 | 0.8884 | 0.8884                   |
| η_e              | 0.9497 | 0.9497 | 0.9497                   |
| η_c              | 0.9497 | 0.9497 | 0.9497                   |
| η_i              | 0.0139 | 0.0139 | 0.0139                   |
| φ                | 1.4604 | 1.4604 | 1.4604                   |

| Temperatures     |        |        |                          |
|------------------|--------|--------|--------------------------|
| T_1              | 256.477| 256.477| 256.477                  |
| T_2              | 374.581| 374.581| 374.581                  |
| T_3              | 380.834| 380.834| 380.834                  |
| T_4              | 326.086| 326.086| 326.086                  |
| T_5              | 228.434| 228.434| 228.434                  |
| T_6              | 223.272| 223.272| 223.272                  |

| Objective functions |        |        |                          |
|---------------------|--------|--------|--------------------------|
| COP                 | 1.0188 | 1.0188 | 1.0188                   |
| ECOP                | 1.4699 | 1.4699 | 1.4699                   |
| F                   | 1.6794 | 1.6794 | 1.6794                   |

| Deviation index     | 0.018  | 0.018  | 0.018                    |

**Fig. 3** Non-dominated solutions for case study 2

**Fig. 4** Non dominated solutions for case study 3
4.3. Results of the case study 3

This case simulates the multi-objective optimization of three problem objectives of second law efficiency ($\eta_\text{f}$), coefficient of performance (COP), and thermoeconomical function (ECF) by means of Crow Search Algorithm. Figure 4 shows the Pareto optimum solutions of three different problem objectives along with the final optimum results obtained through three decision-making theorems. Table 4 gives the optimum solution outputs and their corresponding state point temperatures and decision variables. Table 4 shows that the best solution is found by Shannon’s entropy with a deviation index value of 0.1953 that is closer to the zero. Other decision makers find the same results with a deviation index value of 0.2397.

5. Conclusion

This comprehensive study is mainly concerned with the multi-objective optimization of irreversible air refrigerators through Crow Search Algorithm by means of finite time thermodynamics. Three case studies have been considered to investigate the thermal behavior of the air Brayton cycle through the problem objectives of coefficient of performance (COP), Ecological coefficient of performance (ECOP), second law efficiency ($\eta_\text{f}$), thermoeconomical function (ECF), and thermoeconomic function criteria (F). Pareto optimal solutions between triple objectives have been selected by three decision-making theorems including TOPSIS, LINMAP, and Shannon’s entropy theory. The optimal values of design variables including the effectiveness of the heat exchanger at hot side ($\xi_\text{p}$), the effectiveness of the heat exchanger at the cold side ($\xi_\text{c}$), compression efficiency ($\eta_\text{c}$), expansion efficiency ($\eta_\text{f}$), internal conductance ($\zeta$), and isentropic temperature ratio ($\phi$) have been decided according to the deviation index rates which explains the feasibility of a particular decision making theory for a specific optimization problem. Apart from those, Crow Search Algorithm proves its efficiency and robustness on multi-objective optimization problems with paving the way for future improvements on thermodynamic cycle design.

Table 4 Optimum solutions obtained from three different decision makers for case study 3

| Design variables | TOPSIS | LINMAP | Shannon’s entropy theory |
|------------------|--------|--------|--------------------------|
| $\xi_\text{p}$   | 0.8799 | 0.8799 | 0.8444                   |
| $\xi_\text{c}$   | 0.7304 | 0.7304 | 0.8691                   |
| $\eta_\text{c}$  | 0.9848 | 0.9484 | 0.9437                   |
| $\eta_\text{f}$  | 0.9494 | 0.9494 | 0.9483                   |
| $\zeta$          | 0.0238 | 0.0238 | 0.0117                   |
| $\phi$           | 1.2945 | 1.2945 | 1.4441                   |

| Temperatures    |        |        |                          |
|-----------------|--------|--------|--------------------------|
| $T_1$           | 257.999| 257.999| 256.247                  |
| $T_2$           | 333.991| 333.991| 370.045                  |
| $T_3$           | 338.120| 338.120| 376.822                  |
| $T_4$           | 322.175| 322.175| 326.565                  |
| $T_5$           | 252.577| 252.577| 231.326                  |
| $T_6$           | 248.872| 248.872| 226.138                  |

| Objective functions |        |        |                          |
|---------------------|--------|--------|--------------------------|
| $\eta_\text{f}$     | 0.1731 | 0.1731 | 0.2936                   |
| COP                 | 0.3794 | 0.3794 | 0.9551                   |
| ECF                 | -35.397| -35.397| -56.005                  |
| Deviation index     | 0.2397 | 0.2397 | 0.1953                   |

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