Flexural behavior of concrete beams with steel bar and FRP reinforcement
Seongeun Kim and Seunghun Kim
Department of Architecture Engineering, Hanbat National University, Daejeon, Republic of Korea

ABSTRACT
The purpose of this paper is to investigate the flexural behaviour of concrete beams with deformed steel bars and FRP bars for reinforcement. From experimental and analytical works, stiffness, crack, strength, and deflection behaviour was analysed and compared with ACI318 and ACI440. There were six specimens, which were tested by the variables of reinforcement combination (steel-steel, steel-GFRP, steel-CFRP) and concrete strength (40 MPa, 60 MPa). Test results showed that specimens with steel-FRP reinforcements had two times higher strength than the specimens with steel-steel reinforcements and that deflection of specimens was similar. Nominal flexural strengths of specimens by ACI318 and ACI440 were 113 ~ 142% of flexural strengths by the test. From finite element analysis, the reliability of the analysis was demonstrated by comparison between appropriate analysis model and test results.

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1. Introduction

Reinforced concrete structures, which are considered as products of industrial development, integrate concrete and steel bars in constructing building structures. The reinforced concrete has replaced traditional structural materials such as wood and steel because the steel reinforcing helps to resist external force (tensile stress). In addition, it has many advantages including durability, constructability, and economic feasibility. However, the neutralisation of the reinforced concrete, caused by ageing-related material deterioration, can result in corrosion of the steel bars. The corrosion naturally reduces the durability of the whole structure, causes structural problems, and accordingly raises the repair cost of the structure. In order to improve the performance of a reinforced concrete structure, many studies have focused on using various materials for repair and reinforcement. Within this context, some studies attempted to apply FRP (fiber reinforced plastic) bars to reinforce concrete structures. Japan (JSCE 1997) and Canada (CAN/CSA S806-02 2002), have established design procedures specifically for the use of FRP reinforcement for concrete structures.

The advantages of the FRP bars include high strength, low weight, low thermal conductivity, and excellent corrosion resistance. They can prevent the deterioration of a structure and improve its durability, which may enhance the economic feasibility of the structure by reducing its repair or reinforcement cost. However, as the elasticity modulus of FRP bar (35 ~ 50 GPa for GFRP bars and 120 ~ 150 GPa for...
CFRP bars) is lower than that of the steel bars (200 GPa), it usually causes large deflection and cracks. Moreover, unlike steel bars having inelastic behaviour after its yield strength, the FRP bar shows perfect elastic-brittle behaviour without the yield point. Such a great mechanical and physical difference demands to pay special care and attention when applying FRP bars to a reinforced concrete structure.

Some relevant regulations (including ACI440.1R) have been suggested to cover the use of FRP bars, but those specifications are limited to single placement of the FRP bar, while the combined application of FRP bars and steel bars has not been specified. Steel bars and FRP bars are expected to be complementary to each other to address the problems of corrosion and deflection when placing FRP bars in the bottom part of steel bars. Unfortunately, only a few related experimental and analytical data (El Refai et al. 2015) are available; therefore, no reliable criteria have been provided for evaluating the strength and deformation performance of the combined application of FRP bars and steel bars. This study began with reviewing the precedent studies and focused on the possibility of controlling the failure patterns of beams, their crack depth, crack width, and examining the results of flexural rigidity and deflection behaviour. Thereafter, an experiment was conducted by applying recently suggested criteria. Furthermore, the effect of the concrete strength was analysed by setting strength variables, which was a basic study to suggest some criteria for RC beams using the hybrid reinforcement of FRP and steel bars. The purpose of this study is to propose not only an analytical model of beams reinforced by FRP, but also to provide a reasonable analytical model to predict the flexural behaviour of beams using different types of reinforcing bars with different strengths.

2. Analysis of precedent studies

2.1. Studies on FRP

The provisions of ACI 440.1R-15 are limited to the single reinforcement of FRP bar, and its equation for predicting deflection is not applicable to the case where different types of reinforcing bars are used. This is because the reduction factor of each reinforcing bar should be applied. Although ACI 440.2R-08 covers the combined use of steel bar and FRP, it provides the case where FRP plate is added to or embedded in the surface of an existing structure as a standard, and no criterion is given for predicting deflection when using combinations of the two materials.

In order to verify the performance of such combination using various loading conditions and variables, not only an experimental investigation but also an analytical approach is needed to ensure the validity of the study. Therefore, it is necessary to develop a reliable material model using FRP bars, which was previously mentioned as popular repair or reinforcement material in the introduction of this paper. Many studies have been performed to improve practicability and economic feasibility of using FRP reinforcing bars. Lee, Yang, and Yoon (2007) carried out a structural experiment by fabricating a total of 10 RC beam specimens with various types of reinforcing bars, hybrid arrangements, and fiber admixtures. The experimental results were used to compare the criterion and prediction equations proposed by the researchers in terms of crack, ultimate moment, and deflection. A new equation for predicting the deflection of beams reinforced by different types of reinforcing bars was also proposed.

Yang et al. (2011) examined the behaviours of beams reinforced by the combination of different types of bars and attempted to determine solutions to many problems of FRP reinforced beams. For these purposes, they conducted a total of 10 experiments and analysed the behaviours related to post cracking rigidity, crack patterns, ductility, and deflection. Based on the analysis, it was found that the combined placement could control large deflections, deep cracks, and reduce the cracks width.

Moon, Oh, and Ahn (2008) used a finite element analysis programme to investigate the effects of FRP reinforced beams design variables. The flexural behaviour of such beam was analysed. The suitability of the analysis model was verified by comparing its results to previous experimental results. Reinforcement ratio, elasticity modulus of FRP and compressive strength of concrete were the manipulated variables to compare their effects on the flexural rigidity of the members and the deflection-displacement relation to the theoretical results in ACI 440. The study results indicated that the behaviour was mostly affected by the reinforcement ratio, in addition to the increase in the compressive strength of concrete.

Bencardino, Condello, and Ombres (2015) conducted numerical prediction and an infinite element analysis for a total of 17 specimens including RC beam using FRP and steel bar on the same line. They proved the reliability of the FE model by comparing the numerical prediction and analytical results with the experimental results, thereby evaluating the effectiveness of the Italian guidelines. As explained above, most of the available data is limited to the experimental studies on FRP and the other types of reinforcing bars, as well as the theoretical equations. Even the analytical studies on FRP bar have focused only on single reinforcement or foreign codes. As a result, no sufficient data has been provided that can be applied...
2.2. Standard equation

Regarding the theoretical crack moment, Equation (1) given by ACI code was used, which covers the whole cross-section of the concrete member and considers its elastic behaviour.

$$M_{cr} = \frac{f_{rc} I}{Y}$$  (1)

Where $f_{rc}$ is the modulus of rupture of concrete, MPa. $I$ is a moment of inertia of gross concrete section about the centroidal axis, neglecting reinforcement, mm$^4$. And $Y$ is a distance from the centroidal axis of the cross section, neglecting reinforcement, to the tension face, mm.

The theoretical value of ultimate moment was calculated by Equation (2), which was proposed by the previous research based on ACI code.

$$M_n = A_{rf} \left( d_1 - \frac{a}{2} \right) + A_{sf} \left( d_2 - \frac{a}{2} \right) + A_s f_s \left( d' - \frac{a}{2} \right)$$  (2)

where $A_{rf}$ is area of FRP external reinforcement, mm$^2$. $A_s$ is area of inner flexural reinforcement, mm$^2$. $A_s'$ is section areas of compressive steel reinforcement, mm$^2$. $d_1$ is depths of outermost reinforcement, mm. $d_2$ = depths of inner reinforcement, mm. $d'$ is depths of compressive steel reinforcement, mm. $f_s$ is stress level in FRP reinforcement, MPa. $f_s$ is stress in inner flexural reinforcement, MPa. $f_s'$ is stress in compressive steel reinforcement, MPa. $a$ is depth of equivalent rectangular stress block, mm. ACI 440.1R-06 (2006) provides Equation (3) below for predicting the deflection of concrete beams. The equation has been derived by applying reduction factor $\beta_g$ to Branson equation in consideration of reduced tension stiffening effect due to the use of FRP bar.

$$I_e = \left( \frac{M_{cr}}{M_n} \right)^3 \beta_g I_g + \left[ 1 - \left( \frac{M_{cr}}{M_n} \right)^3 \right] I_{cr} \leq I_g$$  (3)

$$\beta_g = \frac{1}{5} \left( \frac{\rho_t}{\rho_{tb}} \right) \leq 1.0$$  (4)

Where $M_n$ is applied maximum moment, N-mm. $I_{cr}$ is moment of inertia of transformed cracked section, mm$^4$. $\rho_t$ is FRP reinforcement ratio, and $\rho_{tb}$ is FRP reinforcement ratio producing balanced strain condition. Equation (3) proposed by ACI 440.1R-06 cannot be applied directly to the case where different types of reinforcing bars are used, since it requires the application of different reduction factors for different reinforcing bars. Accordingly, the equation can be used by dividing the cross section according to the type of reinforcing bar. A virtual cross section should be equated with the depth of neutral axis of the original cross section, and the sum of beam widths of the virtual cross section should be equal to the beam width of the original cross section. In addition, the sum of the second moment of cracked section of virtual cross section should be equal to that of the original cross section. Applying Equation (3) to the virtual cross section thus obtained, each effective moment of inertia is obtained and then the effective moment of inertia of the original cross section can be obtained from the sum of all the effective moments of inertia.

ACI 440.1R-06 is based on the idea of a Bischoff and Scanlon equation. Thus, it does not apply any reduction factor to the new equation for effective moment of inertia, which is different from the Branson equation.

$$I_e = \frac{I_{cr}}{1 - \frac{M_{cr}}{M_n} \left( \frac{M_{cr}}{M_n} \right)^3 \left[ 1 - \frac{I_{cr}}{I_n} \right]} \leq I_g$$  (5)

Where $I_{cr}$ is moment of inertia of cracked section transformed to concrete, $I_g$ is gross moment of inertia, $M_n$ is maximum service load moment in member. Equation (5) does not need any correction factor. Thus, it can be applied to any types of reinforcing bars. Consequently, the equation is directly used to predict the deflection of specimens with hybrid reinforcement.

Recently, Equation (6), which was proposed by ACI 440.1R-15, has been modified by incorporating the additional factor $\gamma$, which explains the change of rigidity along the longitudinal direction of members, into the original equation proposed by Bischoff (Bischoff and Gross 2011). Factor $\gamma$ depends on loading and boundary conditions. It expresses the length of the non-crack area and the change of rigidity of the members’ cracked area. Branson’s original equation shows an average weight of cracks and the rigidity of cracked member, while Bischoff’s equation proposes a method of expressing an average weight of ductility. Applying the average weight of ductility is a better way of showing the flexural reaction of each crack according to the length of members.

$$I_e = \frac{I_{cr}}{1 - \gamma \left( \frac{M_{cr}}{M_n} \right)^2 \left[ 1 - \frac{I_{cr}}{I_n} \right]} \leq I_g \text{ where } M_{cr} \leq M_n$$  (6)
The elastic deflection by 2-point loading was calculated by the following Equation (8).

$$\Delta = \frac{P l^3}{12E I} \left( \frac{3a_s}{4l} - \left( \frac{a_s}{l} \right)^4 \right)$$

Where $P$ is indirect load carrying two concentrated load effects, $N$. $l$ is span length of member, mm. $a_s$ is distance from the point of support to the loading point, mm.

3. Experiment

3.1. Experimental design and method

Figure 1. shows detailed views of the specimens, while Table 1 presents the specimens’ specifications. At the bottom of the cross section, the specimens have double reinforcement using steel bar and FRP bar (or steel bar) as flexural tension members for the upper and lower parts, respectively. For the purpose of this experiment, a total of six specimens were used with variables related to the types of bottom reinforcing bar and concrete strength, etc. All the specimens were designed and fabricated with a length of 2,400 mm (span 2,200 mm), a cross sectional area of 150 × 250 mm, the depth of outer reinforcing bar of 215 mm and the depth of inner reinforcing bar of 165 mm. All the specimens were over-reinforced and were fabricated to be destroyed by concrete crushing. D10 deformed bar was used as a compression bar and inner reinforcing bar. For outer reinforcing bar, two 10 mm deformed bars, two 13 mm GFRP, and two 9 mm CFRP were used.

In this experiment, a UTM (Universal Testing Machine) was used to apply force. The experiment continued until every specimen was destroyed. The strain and deflection of each reinforcing bar were automatically measured from the moment of loading. Furthermore, the crack pattern and width were also measured for each displacement. A gauge was attached to the centre of every reinforcing bar to measure strain, and an LVDT (linear voltage differential transformer) was installed at the centre of the beam to measure the mid span deflection.

3.2. Experiment of materials

The surface of the FRP bar used for this experiment had a spiral lug to improve adhesion with concrete. The material properties of the steel bar and FRP bar were identified by the experiment of materials according to KS B 0801. The experiment was conducted three times for each material and the average value was reported as the experiment result. Table 2 shows the results of the materials experiment.

The specified design strength of concrete applied for forming the specimens was 40 MPa and 60 MPa. The test specimens were formed when the experiment of compressive strength was conducted. The measurements of compressive strength were 40.91 MPa and 61.69 MPa. Table 3 presents the concrete proportion.

![Figure 1. Detailed view of the beam and specimen cross-section.](image)

| Specimen |
|------------------|
| $f_{ck}$[MPa] |
| STEEL-40S 40 |
| GFRP-40S 40 |
| STEEL-60S 60 |
| GFRP-60S 60 |
| CFRP-40S 40 |
| CFRP-60S 60 |

| Bottom bar1 | Type-number-diameter | Area, $A_s$[mm$^2$] |
|------------------|------------------|-----------------|
| STEEL-40S 40 | SD400-2–10 mm | 142.6 |
| GFRP-40S 40 | GFRP-2–13 mm | 253.4 |
| STEEL-60S 60 | SD400-2–10 mm | 142.6 |
| GFRP-60S 60 | GFRP-2–13 mm | 253.4 |
| CFRP-40S 40 | CFRP-2–9 mm | 142.6 |
| CFRP-60S 60 | CFRP-2–9 mm | 142.6 |

| Bottom bar2 | Type-number-diameter | Area, $A_s$[mm$^2$] |
|------------------|------------------|-----------------|
| STEEL-40S 40 | SD400-2–10 mm | 142.6 |
| GFRP-40S 40 | GFRP-2–13 mm | 253.4 |
| STEEL-60S 60 | SD400-2–10 mm | 142.6 |
| GFRP-60S 60 | GFRP-2–13 mm | 253.4 |
| CFRP-40S 40 | CFRP-2–9 mm | 142.6 |
| CFRP-60S 60 | CFRP-2–9 mm | 142.6 |
4. Experimental results and discussion

4.1. Load-deflection relation and crack patterns

The graphs shown in Figure 3 illustrate the relationship between load and mid-span deflection. The two graphs were distinguished according to concrete strength. For the same concrete strength, in the first flexural crack, there was not much difference between the specimens with steel reinforcement and the specimens with hybrid FRP-steel reinforcement. However, the first cracking load of the specimens with steel reinforcement was slightly lower than that of specimens with hybrid FRP-steel reinforcement. This seems to be due to the fact that the steel bar restrains concrete more than the FRP reinforcing bar, thereby generating larger tensile stress.

After the first crack occurred, the rigidity of every specimen decreased and each specimen began to have a different behaviour. The specimens with steel reinforcement showed larger post-cracking rigidity than specimens with FRP reinforcement. The rigidity of STEEL-40S and STEEL-60S decreased drastically after the steel yielded, whereas GFRP-40S, GFRP-60S, CFRP-40S, and CFRP-60S showed less decreased rigidity; however, they were not much affected by the yield of tension reinforcement placed on the upper part. This may be because FRP complementarily resists the increasing load that cannot be resisted by the steel bar after it yields.

The members with FRP reinforcement showed some brittle behaviour depending on the properties of materials. The maximum strength of these members was more than twice that of the members with steel reinforcement, while the difference significantly increased with greater concrete strength. The displacement at the maximum strength was assessed to be more than 3.5 times the yield displacement of specimens with steel reinforcement. In addition, the larger the concrete strength became, the better the strength and deformation performance turned out to be.

Figure 2. presents schematic diagrams of the observed crack patterns of all the specimens, when they were destroyed. Every specimen was destroyed by concrete crushing. The first crack, which occurred at the mid-span, moved deep into compression edge. At the beginning, the increase of

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| Material     | d, [mm] | A, [mm²] | E, [GPa] | f_y, [MPa] | f_u, [MPa] |
|--------------|---------|----------|----------|------------|------------|
| STEEL-D10    | 9.5     | 71.3     | 173      | 446        | 577        |
| GFRP-D13     | 12.7    | 126.7    | 48       | -          | 1118       |
| CFRP-D9      | 9.5     | 71.3     | 103      | -          | 1655       |

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Table 3. Concrete proportion.

| f_c, [MPa] | f_cm, [MPa] | W | C | B | S | G | AE |
|------------|-------------|---|---|---|---|---|----|
| 40         | 40.9        | 167| 328| 141| 850| 870| 5.16|
| 60         | 61.6        | 169| 487| 163| 733| 813|     |

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Table 2. Material properties of reinforcements.

| Material | d, [mm] | A, [mm²] | E_r, [GPa] | f_y, [MPa] | f_u, [MPa] |
|----------|---------|----------|------------|------------|------------|
| STEEL-D10| 9.5     | 71.3     | 173        | 446        | 577        |
| GFRP-D13 | 12.7    | 126.7    | 48         | -          | 1118       |
| CFRP-D9  | 9.5     | 71.3     | 103        | -          | 1655       |

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Figure 2. Crack pattern of specimens.
load and the increase in the number of cracks were taking place simultaneously. However, from a certain point, only crack width increased without increase in the number of cracks. More flexural cracks occurred more quickly to specimens with FRP reinforcement (GFRP-40S, GFRP-60S, CFRP-40S, CFRP-60S) than those with steel reinforcement (STEEL-40S, STEEL-60S). At failure, specimens with FRP reinforcement had more cracks but smaller crack width than specimens with steel reinforcement. This might be because the FRP had low elasticity modulus, thus generated larger deflection, which resulted in more cracks.

4.2. Comparison between theoretical and experimental strength

Table 4 presents theoretical and experimental values of crack and ultimate moments. Every member showed smaller experimental value than theoretical value of crack moment. The crack moment expressed by Equation (1) uses the flexural strength of the plain concrete and produces greater value than the crack moment of the reinforced concrete member. This is because the steel (or FRP) reinforcement within concrete restrains concrete near the tension zone of the beam and generates tensile stress. In every specimen, the experimental value was greater than the theoretical value by 20% on average. This difference was larger in the specimens with FRP reinforcement and increased along with the increase of strength. Every specimen was designed to undergo compression failure by concrete crushing and the ultimate compressive strain of concrete was assumed to be 0.003. However, the ultimate compressive strain of real concrete is more than 0.003. For this reason, the experimental value was greater than the theoretical value.

4.3. Comparison of deflection

By applying effective moment of inertia, which was obtained by using Equation (3) provided by ACI440.1R-06, Equation (5) of Bischoff and Scanlon, and Equation (6) given by ACI440.1R-15, to Equation (8), the deflection was calculated and is presented along with the experimental values in Table 5. The service load was assumed to be 0.33, which was equal to that of reference. (STEEL-40S = 8.54 kN·m, GFRP-40S = 13.71 kN·m, CFRP-

![Figure 3. Loads-deflection relationship of specimens.](image_url)
40S = 15.19 kN·m, STEEL-60S = 9.50 kN·m, GFRP-60S = 17.15 kN·m, CFRP-60S = 20.99 kN·m).

5. Analysis

5.1. Finite element model

For finite element analysis, VecTor2 was used. It is an online finite element analysis programme based on the modified compression field theory of reinforced concrete members. In the finite element model (shown in Figure 4), the external constraint condition was formed by a simply supported beam. Load was used as vertical concentrated load applied to a corresponding node.

5.1.1. Concrete model

The non-linear material model of concrete, which was used for this analysis, used a stress-strain curve (see Equation (9)) with normal concrete strength, which was proposed by Popovics. This model reflects the fact that the linearity of the rising section of the graph increases along with the increase of strength, and the increase in the maximum compressive stress is accompanied by the decrease of concrete ductility.

\[
f_{ci} = -\left(\frac{\epsilon_{ci}}{\epsilon_p}\right) f_{ck} \frac{n}{n-1} \left(\frac{\epsilon_{ci}}{\epsilon_p}\right) \text{ for } \epsilon_{ci} < 0 \tag{9}
\]

Where \( f_{ci} \) = corresponding to the peak compressive stress, MPa. \( \epsilon_{ci} \) = less compressive than the strain, \( n \) = The curve fitting parameter.

5.1.2. Reinforcing bar model

As shown in Figure 5, the model for the reinforcing was a stress-strain curve that consists of three sections.

The reinforcing bars show linear behaviour before it yields. The phase before rupture shows either linear or non-linear behaviour, depending on parameter of hardening phenomenon. The tension and compressive reinforced stress \( f_s \) were determined by referring to Equation (10).

\[
f_s = \begin{cases} 
E_s \epsilon_s & \text{for } \epsilon_s < \epsilon_y \\
 f_y & \text{for } \epsilon_y < \epsilon_s \leq \epsilon_{sh} \\
 f_u + (f_y - f_u) \left(\frac{\epsilon_u - \epsilon_{sh}}{\epsilon_u - \epsilon_y}\right)^p & \text{for } \epsilon_{sh} < \epsilon_s \leq \epsilon_u \\
0 & \text{for } \epsilon_u < \epsilon_s
\end{cases}
\]

Where \( \epsilon_s \) is the reinforcement strain, \( \epsilon_y \) is the yield strain, \( \epsilon_{sh} \) is the strain at the onset of the strain hardening, \( \epsilon_u \) is the ultimate strain, \( P \) is the strain-hardening parameter.

5.1.3. FRP reinforcement model

With regard to the independent behaviour, FRP is one of the typically brittle materials, which shows linear behaviour before being crushed and undergoes sudden rupture. For this reason, the material behaviour of FRP reinforcement was modelled to be linear behaviour until the point of failure.

![Figure 4. FEA modeling.](image)

![Figure 5. Stress-strain response model of steel bars.](image)
5.1.4. Bond model

Full adhesion was assumed for the modelling adhesive materials, while large values of rigidity and strength were designated in order to prevent the deformation of the combined elements.

5.2. Finite element model

5.2.1. Load-deformation curve

In Figure 6, the load-deformation curves produced by the finite element analysis were compared with those obtained from experiments for a total of six specimens. Among the specimens, the predicted analytical results of the specimen with steel reinforcement (including its predicted initial elastic range, time of crack, and yield strength) were quite accurate, and its analysis graph and experiment graph showed very similar behaviour. In the cases of the remaining four specimens (GFRP, CFRP outer reinforcement), the analysis graphs showed a little greater strength than the experiment graphs, but the difference was not significant. From the experimental results, it can be speculated that such a difference was due to the brittle behaviour caused by the concrete crushing occurring before FRP reinforcing bar yielded.

Table 6 lists the experimental and analytical values of the crack moment and maximum moment. As is clearly shown by the table, the crack moment values obtained from the analysis are smaller than the experimental values, which seem to be due to the fact that the flexural reinforcement restrained the contraction of the concrete. On the other hand, when the maximum moment and the experimental value were compared by the finite element analysis, the ratio was 1.03 and the standard deviation was 0.06, and the maximum error rate was estimated to be within 11%.

This accordingly indicates that the analytical results produced by the finite element analysis programme used in this study, adequately predicted the experimental results.

6. Conclusion

The following conclusions have been drawn from the flexural experiment for the concrete beam with hybrid FRP-steel reinforcement, where the type of flexural reinforcement and concrete strength were used as the experimental variables.
(1) Every specimen showed a similar initial rigidity, but after cracks occurred, the rigidity of the specimens with FRP reinforcement became gradually lower than that of the specimens with steel reinforcement. It is speculated that the FRP generated more cracks that led to reducing its rigidity more than the steel.

(2) The specimens with FRP bar and reinforcing bars used as tensile reinforcements increased the load of the specimens continuously to the maximum strength as the FRP bar resisted the additional stresses even after the yielding of the rebar. The displacement at maximum strength for all specimens was 86 ~ 116% based on specimens with steel reinforcing bars only.

(3) Although the specimen with FRP reinforcement had a low elasticity modulus, its displacement at the maximum strength was over 3.5 times that of the specimen with steel reinforcement at its yield strength. Furthermore, the specimen with FRP reinforcement generated more cracks. However, the displacement difference between steel and FRP at the maximum strength showed an error of less than 15%, and the error decreased along with the increase in the concrete strength.

(4) A finite element analysis model was implemented to obtain the strain for each load and the values of load-deflection curves for all specimens. When the maximum moment and the experimental value were compared by the finite element analysis, the ratio was 1.03 and the standard deviation was 0.06. In sum, the strains showed a large error at the initial stage, but the error decreased along with the increase of load. The load-deflection curves had similar behaviour in both cases. Therefore, the reliability of the analysis model has been verified and will be utilised as database for future analysis of variables.

Notes on contributors
Seongeun Kim is a master’s student in the Department of Architectural Engineering at Hanbat National University, Republic of the Korea.

Seunghun Kim is a master's student in the Department of Architectural Engineering at Hanbat National University, Republic of the Korea.

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