Capacity Analysis of UAV Communication System Based on FD-NOMA

Chunyu Niu\textsuperscript{1}, Xiangdong Jia\textsuperscript{1,2*}, Yaping Lv\textsuperscript{1} and Pengshan Ji\textsuperscript{1}

\textsuperscript{1}College of Computer Science and Engineering, Northwest Normal University, Lanzhou 730070, China
\textsuperscript{2}Wireless Communication Key Laboratory of Jiangsu Province, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
Email: 1640803172@qq.com; jiaxd@nwnu.edu.cn

Abstract. In order to improve the communication quality between unmanned aerial vehicle (UAV) and ground users. An UAV communication system model based on full-duplex and non-orthogonal multiple access (NOMA) technology is proposed, and the capacity of the system model in urban scenarios is analyzed. First, the accurate capacity expression of the system model is given; then, the calculation problem of the exponential integral function in the formula is solved by introducing the Q function, and the approximate closed-form expression of the exponential integral function is obtained, and then the approximate closed-form expression of the capacity is obtained; second, the coefficient factor is used to obtain a more accurate approximate closed-form expression; finally, simulation and numerical results show that increasing the number of UAV or NOMA power vector can achieve better capacity performance.

Keywords. UAV communication; full-duplex (FD); non-orthogonal multiple access (NOMA); capacity analysis.

1. Introduction

1.1. Background

Unmanned aerial vehicles (UAVs) have received extensive research attention in recent years due to their mobility and flexibility [1-2]. Compared with traditional fixed base stations (BS), using UAV as an air BS to provide communication services for ground users (GU) can greatly improve communication performance, such as higher data rates and wider coverage [3]. Compared with traditional air-to-ground wireless communication, UAV can quickly adjust and deploy according to needs, and has fully controllable mobility. Therefore, it is used in information countermeasures, air fire balance and strike, major disaster near-air detection, personnel search and rescue, and emergency airdrops. The field has broad application prospects [4-6]. In recent years, many scholars have conducted a lot of research on UAV communication. Many documents have shown that deploying one or more fixed UAVs can increase network capacity and coverage [7-9], and shorten the distance to GU by adjusting the location can improve communication performance.

1.2. Related Work and Contribution

Compared with half-duplex (HD), full-duplex (FD) is a technology that allows uplink and downlink transmission simultaneously, doubling system capacity. Non-orthogonal multiple access (NOMA) is an important technology for 5G wireless communication, which not only improved the frequency
utilization but also enabled more users or devices to access the network [10-14]. Unlike traditional orthogonal multiple access (OMA), NOMA is based on the idea that multiple users can share a single resource block at the same time, for example subcarrier or spread spectrum code [15]. Large amount of multiplexing in the power domain for these users need for successive interference removal demodulation on the receiving end. In the existing literature, reference [16] researched from the perspective of rate and proposed a communication system model of FD-BS and multi-HD users, which proved the superiority of FD-NOMA over HD-NOMA. It lays a theoretical foundation for the research of this paper. In reference to the problem of spectrum shortage in UAV communication, reference [17] proposed FD-NOMA and HD-OMA schemes, and gave their closed interrupt probability expressions, which proved that FD-NOMA UAV communication improves the feasibility of spectrum utilization. However, the capacity performance of such a system has not been analyzed in the above literatures.

In this paper, we propose a UAV communication system based on FD-NOMA. The specific work is as follows:

- The proposed FD-NOMA scheme can allow users with different user quality of service (QoSs) and transmission rate requirements to transmit and receive at the same time.
- Through the analysis of the ergodic capacity of the system, the exact expression of the capacity is derived, the complex calculation problem of exponential integral function is solved, and the approximate closed expression is obtained. The analysis results show that the approximate closed expression has lower computational complexity and less error.
- Numerical results show that better capacity performance can be obtained by increasing the number of UAV and NOMA power, and that FD self-interference and channel noise will affect the performance of the proposed scheme.

1.3. Partial Symbol Description
The meaning of some symbols is shown in table 1.

| Symbol | Meaning |
|--------|---------|
| $\beta_0$ | The channel gain with a reference distance of 1m |
| $p$ | Transmission power |
| $\eta$ | The self-interference coefficient |
| $\mu$ | SNR |
| $\sigma^2$ | Channel noise power. |

2. System Model
In this paper, a FD-NOMA-based UAV communication system model is constructed, which is composed of $M$ UAVs and $N$ GUs. As shown in figure 1, the number of UAVs and GUs is represented by set $M=\{1,2,\cdots,M\}$ and set $N=\{1,2,\cdots,N\}$ respectively. UAV flies at an altitude of $H$ above the ground, and the communication between UAVs and GUs is completed by FD-NOMA. In the urban and crowded scenario, because there are a large number of reflection and refraction links between UAVs and GUs, this paper uses the Rayleigh fading channel model, that is, to establish a main line-of-sight path (LoS) between UAVs and GUs.

As shown in figure 2, in order to express clearly, we establish a three-dimensional coordinate system, the position of the $i$-th UAV is represented as $(x_i,y_i,H)$, the position of the $j$-th GU is represented as $(a_j,b_j,0)$, and the horizontal positions of the $i$-th UAV and $j$-th GU are $s_i=(x_i,y_i)$ and $k_j=(a_j,b_j)$, respectively. The channel gain between the $i$-th UAV and $j$-th GU can be expressed as:
where $D_{i,j}$ represents the square of the distance from the $i$-th UAV to $j$-th GU.

3. Capacity Analysis

In a fading channel which varies with time, it can be divided into fast fading channel and slow fading channel according to the speed of the channel changing with time. On the premise that the receiver knows the complete channel information and the transmitter only knows the information distribution, the channel capacity is usually divided into ergodic capacity and interrupt capacity. The ergodic capacity is the average instantaneous capacity of all states, while the outage capacity is used to describe the system performance in slowly changing channels. UAV communication is usually a time-varying channel, so this system uses ergodic capacity.

On the basis of Shannon’s theorem, the capacity expression of the $i$-th UAV to $j$-th GU in the air can be obtained:

$$ R_{i,j} = \log_2 \left( 1 + \frac{p_{i,j} g_{i,j}}{\sum_{l \neq j} p_{i,l} + \eta \hat{P}_{i,k} + \sigma^2} \right) $$

where $p_{i,j}$ represents the transmission power from the $i$-th UAV to the $j$-th GU, $\sum_{l \neq j} p_{i,l}$ represents the co-channel interference of adjacent users after SIC. $\eta \hat{P}_{i,k}$ means FD uplink self-interference. $\eta \in [0,1]$. The larger the $\eta$ is, the stronger the FD self-interference is, and the smaller the $\eta$ is, the weaker the FD self-interference is.

The capacity of the $i$-th UAV can be expressed as:

$$ R_i = \sum_{j=1}^N R_{i,j} $$

The total capacity of $M$ UAVs can be expressed as:

$$ R_{sum} = \sum_{i=1}^M R_i $$

That is:
\[ R_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{p_{i,j} g_{i,j}}{\sum_{i=j+1}^{N} p_{i,j} + \eta \bar{p}_{i,k} + \sigma^2} \right) \]  

(5)

The channel noise power value is normalized to obtain:

\[ R_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{\mu \bar{\lambda}_{i,j} g_{i,j}}{\sum_{i=j+1}^{N} \bar{\lambda}_{i,j} + \eta \bar{\lambda}_{i,k} + 1} \right) \]  

(6)

where \( \bar{\lambda}_{i,j}, \bar{\lambda}_{j,i}, \bar{\lambda}_{i,k} \) represents the allocated NOMA power coefficient that is consistent with the normalized channel noise power value during FD transmission.

The probability distribution function of the instantaneous signal-to-interference-to-noise ratio of each slot is:

\[ f(\delta_{i,j}) = \frac{1}{\delta_{i,j}} e^{-\frac{\delta_{i,j}}{\delta_{i,j}}} \]  

(7)

where \( \delta_{i,j} = \frac{\mu \bar{\lambda}_{i,j}}{\mu \left( \sum_{i=j+1}^{N} \bar{\lambda}_{i,j} + \eta \bar{\lambda}_{i,k} \right) + 1} \) is the average channel gain power of each GU.

The ergodic capacity can be written as:

\[ R_{i,j} = \int_{0}^{\infty} \log_2 \left( 1 + \delta_{i,j} \right) \frac{1}{\delta_{i,j}} e^{-\frac{\delta_{i,j}}{\delta_{i,j}}} d\delta_{i,j} \]  

(8)

The exact ergodic capacity expression of FD-NOMA-based UAV communication system in urban scenario can be written as follows:

\[ R_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} e^{x} E\left[ \frac{1}{\delta_{i,j}} \right] \log_2 e \]  

(9)

where \( E(x) = \int_{s}^{x} \frac{e^{-t}}{t} dt \) is an exponential integral function. In order to solve the closed-form expression of \( E(x) \), \( E(x) \) can be decomposed into:

\[ E(x) = \int_{s}^{x} \left( \frac{e^{-t}}{\sqrt{t}} \right) \left( \frac{1}{\sqrt{t}} \right) dt \]  

(10)

By comparing the Gaussian \( Q \) function \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{s}^{x} e^{-\frac{t^2}{2}} dt \), it is found that the first term \( \frac{e^{-t}}{\sqrt{t}} \) in the disassembly formula of \( E(x) \) is related to the derivative of the Gaussian \( Q \) function, that is:

\[ \frac{e^{-t}}{\sqrt{t}} = -2\sqrt{\pi} \frac{d}{dt} \left( Q(\sqrt{2t}) \right) \approx 4\sqrt{\pi} \sum_{n=1}^{\infty} p_n e^{-2n^2t} \]  

(11)

We can quote the general approximation of \( Q(\sqrt{2t}) \) given by \( Q(\sqrt{2t}) \approx \sum_{n=1}^{\infty} p_n e^{-2n^2t} \), bring (11) into (10) and solve the integral. It is found that there is still a \( Q \) function in the formula. We use its exponential approximation to replace this function and obtain the approximate closed expression of \( E(x) \):
\[ E_i(x) \approx 4\pi p_n q_n \sum_{n=1}^{s+1} \sum_{m=1}^{s+1} p_m e^{-q_m^2 q_n^2} \]  
(12)

where \( p_n \) and \( q_n \) are constant coefficients, \( p_n = \frac{\theta_n - \theta_{n+1}}{\pi} \) and \( q_n = \left( \frac{\cot(\theta_{n+1}) - \cot(\theta_n)}{\theta_n - \theta_{n+1}} \right)^{\frac{1}{2}} \), respectively, and \( 0 \leq \theta_0 < \theta_1 < ... < \theta_n < ... < \theta_{s+1} = \frac{\pi}{2}, n \in [0, s+1] \); In the same way, it can be obtained
\[ Q_m = \frac{\cot(\theta_{m+1}) - \cot(\theta_m)}{\theta_m - \theta_{m+1}}, \quad p_m = \frac{\theta_m - \theta_{m+1}}{\pi}, \quad \text{and} \quad 0 \leq \theta_0 < \theta_1 < ... < \theta_m < ... < \theta_{t+1} = \frac{\pi}{2}, m \in [0, t+1]. \]  
\( s \) and \( t \) are positive integers that control the accuracy of the approximation.

In order to improve the accuracy of equation (12), this paper introduces a coefficient factor \( C \), which can be further written as follows:
\[ E_2(x) \approx C 4\pi p_n q_n \sum_{n=1}^{s+1} \sum_{m=1}^{s+1} p_m e^{-q_m^2 q_n^2} \]  
(13)

Through a large number of simulation experiments, it is found that the closed-form expression \( E_2(x) \) is in good agreement with the exact expression \( E(x) \) when \( C = \frac{1}{4} \), and the error is less than 0.00001. Therefore, a more accurate approximate closed expression can be obtained:
\[ E_3(x) \approx \pi p_n q_n \sum_{n=1}^{s+1} \sum_{m=1}^{s+1} p_m e^{-q_m^2 q_n^2} \]  
(14)

The approximate closed-form expression of capacity can be obtained by bringing (14) into (9):
\[ R_{\text{max}} \approx \pi \log_2 \left( \sum_{m=1}^{N} \sum_{n=1}^{s+1} \sum_{m=1}^{s+1} e^{-q_m^2 q_n^2} p_m p_n e^{-\theta_m \delta_{11}} \right) \]  
(15)

From the above formula, it can be seen that the system capacity is determined by \( M, N, \delta_{11} \), and the system capacity increases with the increase of \( M \) and \( N \). In addition, the accuracy of the expression is affected by \( s \) and \( t \). The correctness of the expression will be verified by the simulation results.

4. Simulation and Numerical Analysis

In this section, the validity of the approximate closed-form expression of the exponential integral function is verified by simulation experiments, and the effects of different parameters on the system capacity, as well as the effects of the number of devices and NOMA power vectors on the system capacity are compared. All the simulation experiments are carried out on the Matlab platform.

First of all, we verify the results of the exponential integral function \( E(x) \), the approximate closed-form expression \( E_1(x) \), and the approximate closed expression \( E_2(x) \) when taking the coefficient factor \( C = \frac{1}{4} \). As shown in figure 3, from the simulation results in the figure, we can see that \( E(x) \), \( E_1(x) \) and \( E_2(x) \) have similar curvature, and there is a large error between \( E_1(x) \) and \( E(x) \), while the error between the improved \( E_2(x) \) and the exact expression \( E(x) \) is only 0.00001, which shows the correctness of the approximate expression \( E_2(x) \).
After ensuring the correctness of $E(x)$, the approximate closed-form expression of capacity is further derived. Figure 4 verifies the influence of the number of UAV devices $M$ and NOMA power vector $a_i = [\lambda_1, \cdots, \lambda_N]$ on the system capacity, and compares the simulation results of different NOMA power vectors ($a_1 = [0.5, 1, 1.5], a_2 = [1, 2, 3], a_3 = [2, 4, 6]$) at $M = 1, a_1 = [0.5, 1, 1.5]$ and $M = 2$ where $2 \leftrightarrow 3, a_i, a_j$ means that under FD-NOMA, two UAV devices transmit information to three GU, whose NOMA power vectors are $a_i$ and $a_j$, respectively. From the simulation results, it can be seen that with the increase of $M$, the system capacity will increase, and the larger capacity can be obtained by increasing the NOMA power vector.

Finally, we compare the throughput of the system under different NOMA power coefficient $\lambda_{i,k}$ in FD-NOMA scheme. In order to demonstrate the effectiveness of the simulation, we only set one variable $\lambda_{i,k}$, and the other parameters are set to a fixed value. We can see from figure 5 that the smaller the NOMA power coefficient is, the greater the system throughput is, which is due to the increase of FD self-interference. When $\lambda_{i,k} = 10$, SNR has little effect on system throughput. When $\lambda_{i,k} = 1, 0 < \text{SNR} < 10\text{dB}$, the system capacity increases with the increase of SNR. However, when SNR $> 10\text{dB}$, it has little effect on the system throughput. When $\lambda_{i,k} = 0.1$, SNR becomes the main factor affecting system throughput.
5. Conclusion
This paper proposes a UAV communication system model based on FD-NOMA, deduces the exact expression of the system ergodic capacity in the urban scene, solves the calculation problem of the exponential integral function in the formula, and further deduces the approximate closed-form expression of the capacity. In order to verify the correctness of the expression, a series of simulation experiments are done in this paper. The simulation results show that better system capacity can be obtained by adding UAV equipment and increasing NOMA power vector. Finally, the effects of NOMA power coefficient and SNR on the reachable throughput of the system are compared. The smaller the NOMA power coefficient is, the greater the system reachable throughput is. Future work includes optimizing the layout of UAV to maximize the communication capability of UAV.

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