Exploring the spectrum of QCD using the lattice

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Abstract. The calculation of the spectrum of QCD is key to an understanding of the strong interactions, and vital if we are to capitalize on the experimental study of the spectrum. In this paper, we describe progress towards understanding the spectrum of resonances of both mesons and baryons from lattice QCD, focusing in particular on the resonances of the $I = 1/2$ nucleon states, and of charmonium mesons composed of the heavy charmed quarks.

1. Introduction

The strong interaction, one of the forces of the Standard Model of particle and nuclear physics, is responsible for binding both protons and neutrons into nuclei, and also the primordial gluons and the lightest quarks into pions, protons, neutrons and other so-called hadrons. The interaction arises from a quantum field theory known as Quantum Chromodynamics (QCD).

In order to really understand QCD and hence test whether it is the complete theory of the strong interaction, we must know the spectrum of mesons and baryons that it implies and test those spectra against high quality data. The complete combined analysis of available experimental data on the photoproduction of nucleon resonances is the 2009 milestone in Hadronic Physics (HP), and the measurement of the electromagnetic properties of the low-lying baryons is an HP 2012 milestone. The observed spectrum of QCD provides little direct evidence of the presence of the gluons. However, QCD admits the possibility of exotic mesonic states of matter in which the gluonic degrees of freedom are explicitly exhibited. The search for such states will be an important component of the upgraded JLab@12GeV.

Given the intense experimental efforts in hadron spectroscopy, the need to predict and understand the hadron spectrum from first principles calculations in QCD is clear. Hence, an important goal of the effort of the USQCD Collaboration is the study of the resonance spectrum of QCD. The remainder of this paper is laid out as follows. In the next section, we introduce lattice QCD, and outline the methodology for looking at resonances on the lattice. We then
describe recent progress at understanding the resonance spectrum, focusing first on nucleon resonances and then on the meson sector. We conclude with prospects for future calculations.

2. Lattice QCD

The interactions of the quarks and gluons at very short distances, such as those probed at the LHC, are weak, a property known as asymptotic freedom, for which Gross, Politzer, and Wilczek won the 2004 Nobel Prize. This enables the interactions to be expanded in terms of a small coupling constant. At longer distances, typical of the binding of the quarks and gluons into hadrons, the coupling becomes strong, and the theory highly non-linear. Here, the only means of solving, as opposed to modelling, QCD is through numerical calculations on the lattice.

Lattice gauge calculations solve QCD on a four-dimensional lattice, or grid, of points in Euclidean space. The quarks reside on the points of the grid, whilst the gluons are associated with the links joining those points. Lattice calculations proceed through a Monte Carlo method, in which ensembles of gauge configurations are generated with a probability distribution prescribed by the Euclidean QCD action. Lattice QCD calculations have always been at the leading edge of exploiting the most powerful supercomputing resources available, helped by the highly regular nature of the problem.

The calculation of the ground-state spectrum has been a benchmark calculation of lattice QCD since its inception. Figure 1 shows a summary of the low-lying light-hadron masses compared with their experimental values, measured on anisotropic clover lattices designed for the study of resonance spectroscopy[1]; details of the computational methodology are given in the poster of Balint Joo.

A comprehensive picture of resonances requires that we go beyond a knowledge of the ground state mass in each channel, and obtain the masses of the lowest few states of a given quantum
number. This we can accomplish through the use of the variational method[3, 4]; we calculate a matrix of correlator functions

\[ C_{ij}(t) = \sum_\vec{x} \langle O_i(\vec{x}, t) O_j^\dagger(\vec{0}, 0) \rangle, \]

where \( \{O_i; i = 1, \ldots, N\} \) are a basis of interpolating operators with given quantum numbers. We then solve the generalized eigenvalue equation

\[ C(t)u = \lambda(t, t_0) C(t_0)u \]

to obtain a set of real (ordered) eigenvalues \( \lambda_n(t, t_0) \), where \( \lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_{N-1} \). At large Euclidean times, these eigenvalues then delineate between the different masses

\[ \lambda_n(t, t_0) \longrightarrow e^{-M_n(t-t_0)} + O(e^{-\Delta M_n(t-t_0)}) \]

where \( \Delta M_n = \min\{| M_n - M_i |; i \neq n\} \). The eigenvectors \( u \) are orthogonal with metric \( C(t_0) \), and a knowledge of the eigenvectors can yield information about the partonic structure of the states. Crucial to the application of the variational method is the use of a basis of interpolating operators that have a good overlap with the low-lying states of interest. The cubic lattice employed in our calculations does not admit the full rotational symmetry of the continuum, but rather the more restricted symmetry of the octahedral group. Thus states at rest are classified according to the irreducible representation (irreps) of the cubic group, and for spectroscopy calculations, interpolating operators must be constructed that transform irreducibly under the cubic group; this task has been the prerequisite for our study both of baryons[5, 6], and of mesons.

3. Nucleon Resonance Spectrum

Baryons, containing three quarks, are emblematic of the non-Abelian nature of QCD, and of the three colors of the theory. An important goal in exploring baryons is attempting to discern the effective degrees of freedom that describe the spectrum; the search for so-called “missing resonances” focuses on whether the spectrum can be well described by a quark model, or whether an effective theory with fewer degrees of freedom, such as a quark-diquark picture, provides a more faithful description of the baryon spectrum.

There are three double-valued irreducible representations of the cubic group, denoted \( G_{1u/g}(2) \), \( H_{u/g}(4) \) and \( G_{2u/g}(2) \), where \( g \) and \( u \) refer to positive and negative parity, respectively, and the brackets contain the dimension of the irrep. \( G_1 \) contains continuum spins \( 1/2, 3/2, \ldots \), \( H_g \) spins \( 3/2, 5/2, \ldots \) and \( G_2 \) spins \( 5/2, 7/2, \ldots \). Thus, at any fixed lattice spacing \( a \), a state corresponding to spin-5/2 has four degrees of freedom in \( H \), and two in \( G_2 \), with degeneracies between the energies in the two irreps emerging in the continuum limit. Figure 2 shows the experimental nucleon spectrum as seen on the lattice; the limited number of irreps requires that for each channel we be able to isolate as many energy levels as possible.

The nucleon spectrum has been analysed in a calculation with two flavors of light Wilson fermions, at two values of the pion mass[7], building on an earlier calculation in the quenched approximation to QCD[8]. The data are shown in Figure 3. For the first time in a lattice calculation, we can identify a spin-5/2 state, but the multi-hadron states that should be seen in the spectrum appear elusive; the calculation of correlation functions that are expected to be sensitive to these multi-hadron contributions is an important goal for the collaboration.

4. Meson Resonances

The new Hall D of the JLab 12GeV upgrade centers on the study of meson states produced in photoproduction reactions in the GlueX detector. Photoproduction has been proposed, within
Figure 3. The left- and right-hand panels show the spectrum of $I = 1/2$ baryon resonance, indicated by the solid boxes, obtained on $N_f = 2$ Wilson fermion lattices at $m_\pi = 578$ and 416 MeV respectively[7]; the errors are indicated by the vertical width of the box. The open boxes show the expected thresholds for multiparticle states.

QCD-motivated models, as a favorable method for the production of “exotic” hybrid mesons, those mesons having $J^{PC}$ outside the set allowed to a fermion-antifermion pair. The hybrid hypothesis is that an excited gluonic field in addition to a quark-antiquark pair can give rise to these quantum numbers.

As a theater for developing our methodology, we have studied charmonium, composed of the heavier charm quark $c$ and its antiquark; the system is particularly attractive, in that it is both computationally far less demanding than systems composed of the light $(u, d, s)$ quarks, and because there is a wealth of high-precision experimental data. For the study of the spectrum, performed in the quenched approximation to QCD, we used the known continuum behavior of our operators to enable the spins of the states in the different lattice irreps to be identified[9], illustrated for the $J^{PC} = J^{--}$ and $J^{++}$ channels in Figure 4. Lattice QCD can further enable the electromagnetic properties of the states to be investigated. Thus the radiative transitions between some low-lying, non-exotic states were studied[10], and recently this was extended to include the radiative transition form factors for some of the higher-lying states in the spectrum, including those with exotic quantum numbers[11]. Most importantly, it was shown that radiative transitions to states with exotic quantum numbers were calculable, and furthermore the partial width for the exotic decay $\Gamma(\eta_c \rightarrow J/\psi \gamma)$ was shown to be comparable to the non-exotic width. The extension of this calculation to mesons composed of light quarks, using the $N_f = 2 \oplus 1$ anisotropic clover lattices[1], is in progress.

5. Conclusions
The calculation of the masses and properties of resonances is essential to fully capitalize on the investment in experimental facilities, and to provide reliable calculations to confront the experimental results. Thus the generation and analysis of lattices designed for spectroscopy is a central goal of the USQCD Collaboration’s program. The calculations outlined above are but the first stage in this program, with quark masses considerably higher than those realized in nature. An immediate challenge arises as we enter the regime where resonances are unstable under the
Figure 4. The left- and right-hand panels show the masses calculated in the $J^{PC} = J^{-+}$ and $J^{++}$ channels, respectively, categorized according to the lattice irreps, with the color coding indicating their estimated continuum spin assignments where known: Black ($J = 0$), Red ($J = 1$), Green ($J = 2$), Blue ($J = 3$), and Orange (unidentified).

strong interactions, and multi-hadron energies in the spectrum are expected to be important; a novel means of efficiently computing correlation functions in this regime has recently been outlined[12]. The advent of petascale computing is enabling calculations to be performed at progressively lighter values of the light-quark masses, and calculations with all masses fixed to their physical values are now, finally, within reach.

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