Measurement of the double-differential inclusive jet cross section in proton–proton collisions at \( \sqrt{s} = 13 \) TeV

CMS Collaboration

CERN, 1211 Geneva 23, Switzerland

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Abstract A measurement of the double-differential inclusive jet cross section as a function of jet transverse momentum \( p_T \) and absolute jet rapidity \( |y| \) is presented. The analysis is based on proton–proton collisions collected by the CMS experiment at the LHC at a centre-of-mass energy of 13 TeV. The data samples correspond to integrated luminosities of 71 and 44 pb\(^{-1}\) for \( |y| < 3 \) and \( 3.2 < |y| < 4.7 \), respectively. Jets are reconstructed with the anti-\( k_T \) clustering algorithm for two jet sizes, \( R \), of 0.7 and 0.4, in a phase space region covering jet \( p_T \) up to 2 TeV and jet rapidity up to \( |y| = 4.7 \). Predictions of perturbative quantum chromodynamics at next-to-leading order precision, complemented with electroweak and nonperturbative corrections, are used to compute the absolute scale and the shape of the inclusive jet cross section. The cross section difference in \( R \), when going to a smaller jet size of 0.4, is best described by Monte Carlo event generators with next-to-leading order predictions matched to parton showering, hadronisation, and multiparton interactions. In the phase space accessible with the new data, this measurement provides a first indication that jet physics is as well understood at \( \sqrt{s} = 13 \) TeV as at smaller centre-of-mass energies.

1 Introduction

Quantum chromodynamics (QCD) is the fundamental theory describing strong interactions among partons, i.e., quarks and gluons. Inclusive jet production (\( p + p \rightarrow \text{jet} + X \)) is a key process to test predictions of perturbative QCD (pQCD) over a wide region in phase space. To compare with measurements, the parton-level calculations must be complemented with corrections for nonperturbative (NP) effects that involve the modeling of hadronisation (HAD) and multiparton interactions (MPI). Previous measurements at the CERN LHC have been carried out by the ATLAS and CMS Collaborations at centre-of-mass energies \( \sqrt{s} = 2.76 \text{ TeV} \) [1, 2], 7 TeV [3–7], and at lower \( \sqrt{s} \) by experiments at other hadron colliders [8–12]. The measurements at 2.76 and 7 TeV centre-of-mass energies were found to be in agreement with calculations at next-to-leading order (NLO) in the strong coupling constant \( \alpha_s \) over a wide range of jet transverse momentum \( p_T \) and rapidity \( y \). With the latest data from the LHC Run 2, these tests of pQCD are extended to cover the new energy regime of \( \sqrt{s} = 13 \) TeV.

In this paper, a measurement of the double-differential inclusive jet cross section is presented as a function of the jet \( p_T \) and absolute jet rapidity \( |y| \). The jets are clustered with the anti-\( k_T \) jet algorithm [13] as implemented in the FASTJET library [14]. Two jet sizes \( R \) are used: the larger value \( R = 0.7 \) corresponds to the standard jet size chosen in most QCD jet analyses made by the CMS Collaboration because it favourably compares to fixed-order predictions [15]. A second, smaller value of \( R \) emphasizes different aspects of perturbative and nonperturbative QCD and permits complementary tests to be performed [16–18]. Moreover, the choice of \( R = 0.4 \) as a new CMS default jet size that replaces the previous one of 0.5 in LHC Run 1 analyses will allow direct comparisons between jet measurements made by ATLAS and CMS.

The proton–proton collision data were recorded by the CMS experiment at a centre-of-mass energy of 13 TeV in 2015. The data samples correspond to integrated luminosities of 71 and 44 pb\(^{-1}\) for ranges in rapidity of \( |y| < 3 \) and \( 3.2 < |y| < 4.7 \), respectively. The smaller amount of data for the forward rapidity range is explained by more difficult operating conditions at the very start of data taking, which reduced the event sample certified for physics analyses. The results are compared to fixed-order predictions at NLO precision, complemented with electroweak and nonperturbative corrections, and to predictions of various Monte Carlo (MC) event generators that combine leading-order (LO) or NLO pQCD with the modeling of parton showers (PS), HAD, and MPI.

*e-mail: cms-publication-committee-chair@cern.ch
2 The CMS detector

The central feature of the CMS apparatus is a superconducting solenoid of 6 m internal diameter, providing a magnetic field of 3.8 T. Within the solenoid volume are a silicon pixel and strip tracker, a lead tungstate crystal electromagnetic calorimeter (ECAL), and a brass and scintillator hadron calorimeter (HCAL), each composed of a barrel and two endcap sections. Forward calorimeters extend the pseudorapidity coverage provided by the barrel and endcap detectors to the region $3.0 < \eta < 5.2$. Muons are measured in gas-ionisation detectors embedded in the steel flux-return yoke outside the solenoid. In the region $|\eta| < 1.74$, the HCAL cells have widths of 0.087 in $\eta$ and 0.087 radians in azimuth ($\phi$). In the $\eta$-$\phi$ plane, and for $|\eta| < 1.48$, the HCAL cells map onto $5 \times 5$ ECAL crystals arrays to form calorimeter towers projecting radially outwards from close to the nominal interaction point. At larger values of $|\eta|$, the size in rapidity of the towers increases and the matching ECAL arrays contain fewer crystals. Within each tower, the energy deposits in ECAL and HCAL cells are summed to define the calorimeter tower energies, subsequently used to provide the energies and directions of hadronic jets. The particle-flow (PF) event algorithm [19,20] reconstructs and identifies each individual particle with an optimised combination of information from the various elements of the CMS detector. The energy of photons is directly obtained from the ECAL measurement. The energy of electrons is determined from a combination of the electron momentum at the primary interaction vertex as determined by the tracker, the energy of the corresponding ECAL cluster, and the energy sum of all bremsstrahlung photons spatially compatible with originating from the electron track. The momentum of muons is obtained from the curvature of the corresponding track. The energy of charged hadrons is determined from a combination of their momenta measured in the tracker and the matching ECAL and HCAL energy deposits, corrected for zero-suppression effects and for the response function of the calorimeters to hadronic showers. Finally, the energy of neutral hadrons is obtained from the corresponding ECAL and HCAL energy. When combining information from the entire detector, the jet energy resolution typically amounts to 15 % at 10 GeV, 8 % at 100 GeV, and 4 % at 1 TeV, to be compared to about 40, 12, and 5 % obtained when the ECAL and HCAL alone are used. A more detailed description of the CMS detector, together with a definition of the coordinate system used and the relevant kinematic variables, can be found in Ref. [21].

3 Event selection and jet reconstruction

The measurement is based on data samples collected with single-jet high-level triggers (HLT) [22]. Eight single-jet HLT paths are considered, seeded by Level 1 triggers based on calorimetric information. They require, in the full rapidity coverage of the CMS detector, at least one jet in each event with $p_T > 60, 80, 140, 200, 260, 300, 400, \text{or } 450 \text{ GeV}$. All triggers, except the one with the highest threshold, are prescaled. The relative efficiency of each trigger is estimated using lower-$p_T$-threshold triggers, and found to exceed 99 % in the $p_T$ regions shown in Table 1. The absolute trigger efficiency is measured using a tag and probe method [23] based on events selected with a single-jet trigger threshold of 40 GeV, a back-to-back dijet system, and a probe jet matched to a HLT trigger object. This trigger has an efficiency greater than 99 % for selecting an event with a jet of $p_T > 80 \text{ GeV}$.

The main physics objects in this analysis are PF jets, reconstructed by clustering the Lorentz vectors of the PF candidates with the anti-$k_T$ (AK) clustering algorithm for the two jet sizes $R = 0.7$ and 0.4 that will be referred to as AK7 and AK4, respectively. In order to reduce the contribution to the reconstructed jets from additional proton–proton interactions within the same or neighbouring bunch crossings (pileup), the technique of charged hadron subtraction [24] is used. Pileup produces unwanted calorimetric energy depositions and additional tracks. The charged hadron subtraction reduces these effects by removing charged particles that originate from pileup vertices. The average number of pileup interactions observed in these data is $\approx 19$. During data collection the LHC operated with a 50 ns bunch spacing.

Reconstructed jets require small energy corrections to account for residual nonuniformities and nonlinearities in the detector response. Jet energy scale (JES) [23] corrections are obtained using simulated events, generated with PYTHIA8.204 [25] with tune CUETM1 [26] and processed through the CMS detector simulation, and in situ measurements with dijet, photon+jet, and Z+jet events. An offset correction is applied to account for the extra energy clustered into jets due to the contribution of neutral particles produced by additional pileup interactions within the same or neighbouring bunch crossings.

### Table 1

| HLT path | $p_T$ range (GeV) |
|----------|------------------|
| PFJet_60 | 114–133          |
| PFJet_80 | 133–220          |
| PFJet_140| 220–300          |
| PFJet_200| 300–430          |
| PFJet_260| 430–507          |
| PFJet_300| 507–638          |
| PFJet_400| 638–737          |
| PFJet_450| $>737$           |

For $\eta < 1$.

**Note:** The measurement is based on data samples collected with single-jet high-level triggers (HLT) [22]. Eight single-jet HLT paths are considered, seeded by Level 1 triggers based on calorimetric information. They require, in the full rapidity coverage of the CMS detector, at least one jet in each event with $p_T > 60, 80, 140, 200, 260, 300, 400, \text{or } 450 \text{ GeV}$. All triggers, except the one with the highest threshold, are prescaled. The relative efficiency of each trigger is estimated using lower-$p_T$-threshold triggers, and found to exceed 99 % in the $p_T$ regions shown in Table 1. The absolute trigger efficiency is measured using a tag and probe method [23] based on events selected with a single-jet trigger threshold of 40 GeV, a back-to-back dijet system, and a probe jet matched to a HLT trigger object. This trigger has an efficiency greater than 99 % for selecting an event with a jet of $p_T > 80 \text{ GeV}$.

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The JES correction, applied as a multiplicative factor to the jet four-momentum vector, depends on the jet $\eta$ and $p_T$ values. The typical correction is about 10% for a central jet with a $p_T$ of 100 GeV, and decreases with increasing $p_T$.

Events are required to have at least one primary vertex (PV). If more than one primary vertex is present, the vertex with the highest sum of the squared $p_T$ of the associated tracks is selected. This selected vertex is required to be reconstructed from at least five charged-particle tracks and must satisfy a set of quality requirements, including $|z_{PV}| < 24$ cm and $\rho_{PV} < 2$ cm, where $z_{PV}$ and $\rho_{PV}$ are the longitudinal and transverse distances of the primary vertex from the nominal interaction point in the CMS detector. Jets with $p_T > 114$ GeV are grouped in seven different $|y|$ bins. Additional selection criteria are applied to each event to remove spurious jet-like signatures originating from isolated noise patterns in certain HCAL regions. To suppress noise patterns, tight identification criteria are applied [27]: each jet should contain at least two particles, one of which is a charged hadron, and the jet energy fraction carried by neutral hadrons and photons should be less than 90%. These criteria have an efficiency greater than 99% for genuine jets.

4 Measurement of the double-differential inclusive jet cross section

The double-differential inclusive jet cross section is defined as

$$ \frac{d^2\sigma}{dp_T dy} = \frac{1}{\epsilon L} \frac{N_j}{\Delta p_T \Delta y}, \tag{1} $$

where $L$ is the integrated luminosity, $N_j$ is the number of jets in a bin of a width $\Delta p_T$ in transverse momentum and $\Delta y$ in rapidity, and $\epsilon$ is the product of the trigger and jet selection efficiencies, which is greater than 99%. The phase space in rapidity is subdivided into six bins from $y = 0$ to $|y| = 3$ with $|\Delta y| = 0.5$, and one bin from $|y| = 3.2$ to 4.7, the forward rapidity region. The bin width in $p_T$ is chosen in such a way that bin-to-bin migrations due to detector resolution are less than 50%. In each bin, the statistical uncertainty is derived through the formula $\sqrt{(4-3f)/(2-f)}\sqrt{N_{jets}}$, where $f$ corresponds to the fraction of events which contribute with exactly one jet in the bin [6]. This procedure corrects for possible multiple entries per event. The fraction $f$ is typically larger than 95% in the entire phase-space considered, thus the correction is small.

The double-differential inclusive jet cross section is corrected for the detector resolution and unfolded to the stable particle level [28]. In this way, a direct comparison of this measurement to results from other experiments and to QCD predictions is possible. Particles are considered stable if their mean path length $c\tau$ is greater than 10 mm.

The unfolding procedure is based on the iterative d’Agostini method [29], as implemented in the ROOUNFOLD software package [30], using a response matrix that maps the predicted distribution onto the measured one. The response matrix is derived from a simulation, that uses the theoretically predicted spectrum as input and introduces smearing effects by taking into account the jet $p_T$ resolution. The predicted spectrum is evaluated from fixed-order calculations based on the NLOJET++ v4.1.13 program [31,32] within the framework of the FASTNLO v2.3.1 package [33], using the CT14 [34] parton distribution functions (PDF). More details are presented in Sect. 5.1. The jet $p_T$ resolution is evaluated with the CMS detector simulation based on GEANT4 [35] using a QCD simulation from PYTHIA8 with tune CUETM1, after correcting for the residual differences between data and simulation [23]. The unfolded distributions differ from the distributions at detector level by 5–20%. The unfolding procedure can turn statistical fluctuations of the measured spectra into correlated patterns among the neighbouring bins. It has been verified that such effects are always within the statistical uncertainties of the unfolded distributions, which are larger than those of the detector-level distributions. The iterative unfolding procedure is regularized by limiting the number of iterations to four in each rapidity bin.

The main systematic uncertainties for the jet cross section measurements arise from the JES calibration and from the uncertainty in the integrated luminosity. The JES uncertainty, evaluated separately for AK7 and AK4 jets, is 1–3% in the central region ($|y| < 2$) and increases to 7–8% in the forward rapidity region ($3.2 < |y| < 4.7$) [23]. The JES uncertainty also includes the uncertainty carried by the charged hadron subtraction. The resulting uncertainties in the double-differential inclusive jet cross section range between 8% at central rapidities and low $p_T$ to 65% at forward rapidities and the highest $p_T$. The uncertainty in the integrated luminosity (2.7% [36]) propagates directly to the cross section.

The unfolding procedure is affected by uncertainties in the jet energy resolution (JER) parametrisation. Alternative response matrices are used to unfold the measured spectra. They are built by varying the JER parameters within their uncertainties [23]. The JER uncertainty introduces a 1–2% uncertainty in the measured cross section. The model dependence of the theoretical $p_T$ spectrum also affects the response matrix and thus the unfolding, but this uncertainty has negligible effects on the cross section measurement. The model dependence is checked using various PDF sets to calculate the theoretical $p_T$ spectrum.

Finally, an uncertainty of 1% is assigned to the cross section to account for residual effects of small inefficiencies from jet identification [15]. The total experimental systematic uncertainty of the measured cross section is obtained by summing in quadrature the individual contributions from JES, luminosity, JER, and jet identification uncertainties.
5 Theoretical predictions

5.1 Predictions from fixed-order calculations in pQCD

The theoretical predictions for the jet cross section are calculated at NLO accuracy in pQCD and are evaluated by using NLOJET++ within the framework of fastNLO. The cross sections are calculated at NLO for single inclusive jet production. The renormalisation and the factorisation scales ($\mu_r$ and $\mu_f$) are chosen to be equal to the jet $p_T$. Five quarks are assumed to be massless in the calculation, which is performed using four different PDF sets with NLO accuracy: CT14 [34], HERAPDF1.5 [37], MMHT2014 [38], and NNPDF3.0 [39], with the default values of the strong coupling $\alpha_s(M_Z) = 0.1180, 0.1176, 0.1200,$ and 0.1180, respectively.

The theoretical uncertainties are evaluated as the quadratic sum of the scale, PDF, $\alpha_s$, and NP uncertainties. The scale uncertainty is calculated by varying $\mu_r$ and $\mu_f$ in the following six combinations: $(\mu_r/p_T, \mu_f/p_T) = (1/2,1/2), (1,2), (1,1/2), (2,1)$ and $(2,2)$. The (asymmetric) scale uncertainty is determined through the maximal upwards and downwards deviations with respect to cross sections obtained with the default setting. The PDF and $\alpha_s$ uncertainties are calculated according to the prescription of CT14 at the 90% confidence level and scaled down to a 68.3% confidence level.

The impact of NP effects, i.e. MPI and HAD effects, is evaluated by using samples obtained from different MC event generators with a simulation of PS and MPI contributions. The following MC event generators are used to estimate the NP corrections: LO PYTHIA8 with tune CUETM1, LO HERWIG++ 2.7.0 [40] with tunes UE-EE-5C [41] and CUETS1 [26], and NLO POWHEG [42–44]. The matrix element calculation performed with POWHEG is interfaced to PYTHIA8 with three different tunes (CUETS1-CTEQ6L1, CUETS1-HERAPDF, and CUETM1) for the simulation of the underlying-event (UE) contributions. The cross section ratios between a nominal event generation interfaced to the simulation of UE contributions, and a sample without HAD and MPI effects are taken as correction separately in each considered rapidity range. In a compact formulation, the NP correction factors can be defined as

$$C^{\text{NP}} = \frac{d\sigma^{\text{PS}+\text{HAD}+\text{MPI}}/dp_T}{d\sigma^{\text{PS}}/dp_T},$$

where $\sigma^{\text{PS}+\text{HAD}+\text{MPI}}$ is the cross section obtained with an MC sample simulating the contribution of PS, HAD, and MPI, while $\sigma^{\text{PS}}$ includes only PS effects. Corrections obtained with various NLO and LO event generators are evaluated separately for the AK7 and AK4 jets. The average of the results from the NLO and LO event generators defines the central value of the NP corrections, which are fitted to a power-law function in jet $p_T$. The uncertainty in the NP corrections are evaluated by fitting the upper and lower values of the predictions of the different generators.

The combinations of PDF sets, matrix element calculations, and UE tunes used to evaluate the NP corrections are validated on UE, minimum bias and jet variables, and they are able to reproduce a wide set of observables [26]. The NP corrections are shown in Figs. 1 and 2, respectively, for AK7 and AK4 jets in a central ($0.5 < |y| < 1.0$) and a forward rapidity bin ($2.5 < |y| < 3.0$).

Fig. 1 Fits to the nonperturbative corrections obtained for inclusive AK7 jet cross sections as a function of jet $p_T$ for two rapidity bins: $0.5 < |y| < 1.0$ (left) and $2.5 < |y| < 3.0$ (right). The dotted lines represent the uncertainty bands, which are evaluated by fitting the envelopes of the predictions of the different generators used.
The NP corrections for the AK7 jets are ≈15 % (13 %) for $p_T \sim 114$ GeV in the region $0.5 < |y| < 1.0$ ($2.5 < |y| < 3.0$) and decrease rapidly for increasing $p_T$, flattening at values of ≈1 for $p_T \sim 200–300$ GeV, depending on the considered rapidity range. Because of the smaller cone size, AK4 jets are less affected by the MPI and HAD effects. In particular, the additional energy produced by MPI shrinks for decreasing radii $R$, while the out-of-cone losses due to HAD effects increase for smaller radii $R$. These two effects are responsible for NP corrections that fall below 1 for AK4 jets with $p_T > 200$ GeV at central rapidity. The NP corrections for AK4 jets are very close to unity in the phase space considered. For both cone sizes, the uncertainty assigned to the NP corrections is of the order of 1–2 %.

Electroweak effects, which arise from the virtual exchanges of massive gauge W and Z bosons, become sizable at high jet $p_T$ and central rapidity. Corrections to electroweak effects are shown in Fig. 3 for both AK7 and AK4 jets [45]. They range between 0.96 and 1.05, depending on the jet $p_T$ and rapidity, and are less than 3 % for $p_T < 1$ TeV and very similar between the two cone sizes. For jet measurements performed at a centre-of-mass energy of 7 TeV [46], electroweak corrections of 10–15 % are observed for jet $p_T > 1$ TeV in the $|y| < 1.0$ range, decreasing below 2 % for lower $p_T$, independent of the jet rapidity. Electroweak corrections are applied to the NLOJet++ predictions in a similar manner to the NP contributions.
5.2 Predictions from fixed-order calculations matched to parton shower simulations

The predictions from different MC event generators are compared to data. The HERWIG++ and the PYTHIA8 event generators are considered. Both of them are based on an LO 2 → 2 matrix element calculation. The PYTHIA8 event generator simulates parton showers ordered in $p_T$ and uses the Lund string model [47] for HAD, while HERWIG++ generates parton showers through angular-ordered emissions and uses a cluster fragmentation model [48] for HAD. The contribution of MPI is simulated in both PYTHIA8 and HERWIG++. In particular, PYTHIA8 applies a model [49] where MPI are interleaved with parton showering, while HERWIG++ models the overlap between the colliding protons through a Fourier transform of the electromagnetic form factor, which plays the role of an effective inverse proton radius. Depending on the amount of proton overlap, the contribution of generated MPI varies in the simulation. The MPI parameters of both generators are tuned to measurements in proton–proton collisions at the LHC [26], while the HAD parameters are determined from fits to LEP data. For PYTHIA8, the CUETM1 tune, which is based on NNPDF2.3LO [50, 51], is considered, while HERWIG++ uses the CUETS1 tune [26], based on the CTEQ6L1 PDF set [52].

Predictions based on NLO pQCD are also considered using the POWHEG package matched to PYTHIA8 parton showers and including a simulation of MPI. The POWHEG sample uses the CT10nlo PDF set [53]. Various tunes in PYTHIA8 are used for the UE simulation, which differ in the choice of the

![Fig. 4 Double-differential inclusive jet cross section as function of jet $p_T$. On the left, data (points) and predictions from NLOJet++ based on the CT14 PDF set corrected for the NP and electroweak effects (line) are shown. On the right, data (points) and predictions from POWHEG (PH) + PYTHIA8 (P8) with tune CUETM1 (line) are shown. Jets are clustered with the anti-$k_t$ algorithm ($R = 0.7$).](image1)

![Fig. 5 Double-differential inclusive jet cross section as function of jet $p_T$. On the left, data (points) and predictions from NLOJet++ based on the CT14 PDF set corrected for the NP and electroweak effects (line) are shown. On the right, data (points) and predictions from POWHEG (PH) + PYTHIA8 (P8) with tune CUETM1 (line) are shown. Jets are clustered with the anti-$k_t$ algorithm ($R = 0.4$).](image2)
Fig. 6  Ratio of measured values to theoretical prediction from NLOJet++ using the CT14 PDF set and corrected for the NP and electroweak effects. Predictions employing three other PDF sets are also shown for comparison. Jets are clustered with the anti-$k_t$ algorithm with a distance parameter of 0.7. The error bars correspond to the statistical uncertainties of the data and the shaded bands to the total experimental systematic uncertainties.
Fig. 7 Ratio of measured values to theoretical prediction from NLO-Jet++ using the CT14 PDF set and corrected for the NP and electroweak effects. Predictions employing three other PDF sets are also shown for comparison. Jets are clustered with the anti-$k_t$ algorithm with a distance parameter of 0.4. The error bars correspond to the statistical uncertainties of the data and the shaded bands to the total experimental systematic uncertainties.
Fig. 8 Ratio of measured values to predictions from POWHEG (PH) + PYTHIA8 (P8) with tune CUETM1. Predictions employing four other MC generators are also shown for comparison, where PH, P8, and Hpp stands for POWHEG, PYTHIA8, and HERWIG++ (HPP), respectively. Jets are clustered with the anti-$k_t$ algorithm with a distance parameter of 0.7. The error bars correspond to the statistical uncertainties of the data and the shaded bands to the total experimental systematic uncertainties.
Fig. 9 Ratio of measured values to predictions from POWHEG (PH) + PYTHIA8 (P8) with tune CUETM1. Predictions employing four other MC generators are also shown for comparison, where PH, P8, and Hpp stands for POWHEG, PYTHIA8, and HERWIG++ (HPP), respectively. Jets are clustered with the anti-\(k_t\) algorithm with a distance parameter of 0.4. The error bars correspond to the statistical uncertainties of the data and the shaded bands to the total experimental systematic uncertainties.
PDF set and the HAD parameters: the CUETM1, and tunes CUETS1-CTEQ1L and CUETS1-HERAPDF, which use the CTEQ6L1 and the HERAPDF1.5LO [54] PDF sets, respectively. The HAD parameters for the CUETM1 tune are taken from the Monash tune [55], while the 4C tune provides these in both CUETS1 tunes. All these combinations of POWHEG matrix element and UE-simulation tunes reproduce with very high precision the UE and jet observables at various collision energies [26].

6 Comparison of theoretical predictions and data

Figures 4 and 5 show the double-differential inclusive jet cross section measurements, presented as a function of $p_T$ for seven $|y|$ ranges, after unfolding for detector effects, using the anti-$k_t$ algorithm with $R = 0.7$ and 0.4, respectively. The measurements are compared to the NLOJet++ predictions based on the CT14 PDF set, corrected for NP and electroweak effects (left), and to the predictions from POWHEG + PYTHIA8 with tune CUETM1 (right). The data are consistent with the predictions over a wide range of jet $p_T$ from 114 GeV up to 2 TeV.

The ratios of data over the NLOJet++ predictions using the CT14 PDF set are shown in Fig. 6 for the AK7 jets. The error bars on the points correspond to the statistical uncertainties, and the shaded bands correspond to the total experimental systematic uncertainties. For comparison, predictions employing three alternative PDF sets are also shown. Figure 7 shows the results for the AK4 jets. Overall, a good agreement within the uncertainties is observed between the data and predictions in the entire kinematic range studied, for both jet cone sizes. However, for $R = 0.4$, the cross sections are systematically overestimated by about 5–10 %, while a better description is provided for jets reconstructed with $R = 0.7$. The relatively poor agreement for $R = 0.4$ is due to PS and soft-gluon resummation contributions, which are missing in fixed-order calculations, and that are more relevant for smaller jet cone sizes because of out-of-cone effects.

The ratios of data over predictions from POWHEG + PYTHIA8 with tune CUETM1 are shown in Figs. 8 and 9 for the AK7(AK4) jets. The error bars on the points correspond to the statistical uncertainties and the shaded bands to the total experimental systematic uncertainties. For comparison, four other MC predictions are also shown. There is an overall good level of agreement within the uncertainties between data and predictions from POWHEG + PYTHIA8 with various tunes for both cone sizes, in the entire kinematic range studied. The agreement of data with PYTHIA8 and HERWIG++ is poor in absolute scale. The HERWIG++ event generator shows good agreement with the data in shape for all rapidity bins, while PYTHIA8 agrees well in shape with the data for only $|y| < 1.5$.

7 Summary

A measurement of the double-differential cross section as a function of jet $p_T$ and absolute rapidity $|y|$ is presented for two jet sizes $R = 0.4$ and 0.7 using data from proton–proton collisions at $\sqrt{s} = 13$ TeV collected with the CMS detector. Data samples corresponding to integrated luminosities of 71 and 44 pb$^{-1}$ are used for absolute rapidities $|y| < 3$ and for the forward region $3.2 < |y| < 4.7$, respectively.

As expected for LO predictions, the MC event generators PYTHIA8 and HERWIG++ exhibit significant discrepancies in absolute scale with respect to data, which are somewhat more pronounced for the case of HERWIG++. In contrast, the shape of the inclusive jet $p_T$ distribution is well described by HERWIG++ in all rapidity bins. Predictions from PYTHIA8 start deviating from the observed shape as $|y|$ increases.

In the comparison between data and predictions at NLO in perturbative QCD including corrections for nonperturbative and electroweak effects, it is observed that jet cross sections for the larger jet size of $R = 0.7$ are accurately described, while for $R = 0.4$ theory overestimates the cross section by 5–10 % almost globally. In contrast, NLO predictions matched to parton showers as performed with POWHEG + PYTHIA8 for two different tunes, perform equally well for both jet sizes. This result is consistent with the previous measurement performed at $\sqrt{s} = 7$ TeV [15], where it was observed that POWHEG + PYTHIA8 correctly describes the $R$ dependence of the inclusive jet cross section, while fixed-order predictions at NLO were insufficient in that respect.

This measurement is a first indication that jet physics is as well understood at $\sqrt{s} = 13$ TeV as at smaller centre-of-mass energies in the phase space accessible with the new data.

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University of Florida, Gainesville, USA
D. Acosta, P. Avery, P. Bortignon, D. Bourilkov, A. Brinkerhoff, A. Carnes, M. Carver, D. Curry, S. Das, R. D. Field, I. K. Furic, J. Konigsberg, A. Korytov, P. Ma, K. Matchev, H. Mei, P. Milenovic65, G. Mitselmakher, D. Rank, L. Shchutska, D. Sperka, L. Thomas, J. Wang, S. Wang, J. Yelton

Florida International University, Miami, USA
S. Linn, P. Markowitz, G. Martinez, J. L. Rodriguez

Florida State University, Tallahassee, USA
A. Ackert, J. R. Adams, T. Adams, A. Askew, S. Bein, B. Diamond, S. Hagopian, V. Hagopian, K. F. Johnson, A. Khatiwada, H. Prosper, A. Santra, M. Weinberg

Florida Institute of Technology, Melbourne, USA
M. M. Baarmand, V. Bhopatkar, S. Colafranceschi66, M. Hohlmann, D. Noonan, T. Roy, F. Yumiceva

University of Illinois at Chicago (UIC), Chicago, USA
M. R. Adams, L. Apanasevich, D. Berry, R. R. Betts, I. Bucinskaite, R. Cavanaugh, O. Evdokimov, L. Gauthier, C. E. Gerber, D. J. Hofman, P. Kurt, C. O’Brien, I. D. Sandoval Gonzalez, P. Turner, N. Varelas, Z. Wu, M. Zakaria, J. Zhang

The University of Iowa, Iowa City, USA
B. Bilki67, W. Clarida, K. Dilsiz, S. Durgut, R. P. Gandrajula, M. Haytmyradov, V. Khristenko, J.-P. Merlo, H. Mermerkaya68, A. Mestvirishvili, A. Moeller, J. Nachtman, H. Ogul, Y. Onel, F. Ozok69, A. Penzo, C. Snyder, E. Tiras, J. Wetzel, K. Yi

Johns Hopkins University, Baltimore, USA
I. Anderson, B. Blumenfeld, A. Cocoros, N. Eminizer, D. Fehling, L. Feng, A. V. Gritsan, P. Maksimovic, M. Osherson, J. Roskes, U. Sarica, M. Swartz, M. Xiao, Y. Xin, C. You

The University of Kansas, Lawrence, USA
A. Al-bataineh, P. Baringer, A. Bean, J. Bowen, C. Bruner, J. Castle, R. P. KennyIII, A. Kropivnitskaya, D. Majumder, W. Mcbrayer, M. Murray, S. Sanders, R. Stringer, J. D. Tapia Takaki, Q. Wang

Kansas State University, Manhattan, USA
A. Ivanov, K. Kaadze, S. Khalil, M. Makouski, Y. Maravin, A. Mohammadi, L. K. Saini, N. Skhirtladze, S. Toda

Lawrence Livermore National Laboratory, Livermore, USA
D. Lange, F. Rebbasoo, D. Wright

University of Maryland, College Park, USA
C. Anelli, A. Baden, O. Baron, A. Belloni, B. Calvert, S. C. Eno, C. Ferraioli, J. A. Gomez, N. J. Hadley, S. Jabeen, R. G. Kellogg, T. Kolberg, J. Kunkle, Y. Lu, A. C. Mignerey, Y. H. Shin, A. Skuja, M. B. Tonjes, S. C. Tonwar

Massachusetts Institute of Technology, Cambridge, USA
A. Apyan, R. Barbieri, A. Baty, R. Bi, K. Bierwagen, S. Brandt, W. Busza, I. A. Cali, Z. Demiragli, L. Di Matteo, G. Gomez Ceballos, M. Goncharov, D. Hsu, Y. Iiyama, G. M. Innocenti, M. Klute, D. Kovalskyi, K. Krajczar, Y. S. Lai, Y.-J. Lee, A. Levin, P. D. Luckey, A. C. Marini, C. Meginn, C. Mironov, S. Narayan, X. Niu, C. Paus, C. Roland, G. Roland, J. Saffeld-Nebgen, G. S. F. Stephens, K. Sumorok, K. Tatar, M. Varma, D. Velicanu, J. Veverka, J. Wang, T. W. Wang, B. Wyslouch, M. Yang, V. Zhukova

University of Michigan, Ann Arbor, USA
A. C. Benvenuti, R. M. Chatterjee, A. Evans, A. Finkel, A. Gade, P. Hansen, S. Kalafut, S. C. Kao, Y. Kubota, Z. Lesko, J. Mans, S. Nourbakhsh, N. Ruckstuhl, R. Rusack, N. Tambe, J. Turkewitz

University of Mississippi, Oxford, USA
J. G. Acosta, S. Oliveros

University of Nebraska-Lincoln, Lincoln, USA
E. Avdeeva, R. Bartek, K. Bloom, S. Bose, D. R. Claes, A. Dominguez, C. Fangmeier, R. Gonzalez Suarez, R. Kamalieddin, D. Knowlton, I. Kravchenko, A. Malta Rodrigues, F. Meier, J. Monroy, J. E. Siado, G. R. Snow, B. Stieger
State University of New York at Buffalo, Buffalo, USA
M. Alyari, J. Dolen, J. George, A. Godshalk, C. Harrington, I. Iashvili, J. Kaisen, A. Kharchilava, A. Kumar, A. Parker, S. Rappoccio, B. Roozbahani

Northeastern University, Boston, USA
G. Alverson, E. Barberis, D. Baumgartel, M. Chasco, A. Hortiangtham, A. Massironi, D. M. Morse, D. Nash, T. Orimoto, R. Teixeira De Lima, D. Trocino, R.-J. Wang, D. Wood

Northwestern University, Evanston, USA
S. Bhattacharya, K. A. Hahn, A. Kubik, J. F. Low, N. Mucia, N. Odell, B. Pollack, M. H. Schmitt, K. Sung, M. Trovato, M. Velasco

University of Notre Dame, Notre Dame, USA
N. Dev, M. Hildreth, K. Hurtado Anampa, C. Jessop, D. J. Karmgard, N. Kellams, K. Lannon, N. Marinelli, F. Meng, C. Mueller, Y. Musienko, M. Planer, A. Reinsvold, R. Ruchti, G. Smith, S. Taroni, N. Valls, M. Wayne, M. Wolf, A. Woodard

The Ohio State University, Columbus, USA
J. Alimena, L. Antonelli, J. Brinson, B. Bylsma, L. S. Durkin, S. Flowers, B. Francis, A. Hart, C. Hill, R. Hughes, W. Ji, B. Liu, W. Luo, D. Puigh, B. L. Winer, H. W. Wulsin

Princeton University, Princeton, USA
S. Cooperstein, O. Driga, P. Elmer, J. Hardenbrook, P. Hebda, J. Luo, D. Marlow, T. Medvedeva, M. Mooney, J. Olsen, C. Palmer, P. Piouë, D. Stickland, C. Tully, A. Zuranski

University of Puerto Rico, Mayagüez, USA
S. Malik

Purdue University, West Lafayette, USA
A. Barker, V. E. Barnes, D. Benedetti, S. Folgueras, L. Gutay, M. K. Jha, M. Jones, A. W. Jung, K. Jung, D. H. Miller, N. Neumeister, B. C. Radburn-Smith, X. Shi, J. Sun, A. Svyatkovskiy, F. Wang, W. Xie, L. Xu

Purdue University Calumet, Hammond, USA
N. Parashar, J. Stupak

Rice University, Houston, USA
A. Adair, B. Akgun, Z. Chen, K. M. Ecklund, F. J. M. Geurts, M. Guilbaud, W. Li, B. Michlin, M. Northup, B. P. Padley, R. Redjimi, J. Roberts, J. Rorie, Z. Tu, J. Zabel

University of Rochester, Rochester, USA
B. Betchart, A. Bodek, P. de Barbaro, R. Demina, Y. t. Duh, T. Ferbel, M. Galanti, A. Garcia-Bellido, J. Han, O. Hindrichs, A. Khukhunaishvili, K. H. Lo, P. Tan, M. Verzetti

Rutgers, The State University of New Jersey, Piscataway, USA
J. P. Chou, E. Contreras-Campana, Y. Gershtein, T. A. Gómez Espinosa, E. Halkiadakis, M. Heindl, D. Hidas, E. Hughes, S. Kaplan, R. Kunnawalkam Elayavalli, S. Kyriacou, A. Lath, K. Nash, H. Saka, S. Salur, S. Schnetzer, D. Sheffield, S. Somalwar, R. Stone, S. Thomas, P. Thomassen, M. Walker

University of Tennessee, Knoxville, USA
M. Foerster, J. Heideman, G. Riley, K. Rose, S. Spanier, K. Thapa

Texas A&M University, College Station, USA
O. Bouhali, A. Castaneda Hernandez, A. Celik, M. Dalchenko, M. De Mattia, A. Delgado, S. Dildick, R. Eusebi, J. Gilmore, T. Huang, E. Juska, T. Kamon, V. Krutelyov, R. Mueller, Y. Pakhotin, R. Patel, A. Perloff, L. Perniè, D. Rathjens, A. Rose, A. Safonov, A. Tatarinov, K. A. Ulmer

Texas Tech University, Lubbock, USA
N. Akchurin, C. Cowden, J. Damgov, C. Dragoiu, P. R. Dudero, J. Faulkner, S. Kunori, K. Lamicchane, S. W. Lee, T. Libeiro, S. Undleeb, I. Volobouev, Z. Wang
Vanderbilt University, Nashville, USA
A. G. Delannoy, S. Greene, A. Gurrola, R. Janjam, W. Johns, C. Maguire, A. Melo, H. Ni, P. Sheldon, S. Tuo, J. Velkovska, Q. Xu

University of Virginia, Charlottesville, USA
M. W. Arenton, P. Barria, B. Cox, J. Goodell, R. Hirosky, A. Ledovskoy, H. Li, C. Neu, T. Sinthuprasith, X. Sun, Y. Wang, E. Wolfe, F. Xia

Wayne State University, Detroit, USA
C. Clarke, R. Harr, P. E. Karchin, P. Lamichhane, J. Sturdy

University of Wisconsin-Madison, Madison, WI, USA
D. A. Belknap, S. Dasu, L. Dodd, S. Duric, B. Gomber, M. Grothe, M. Herndon, A. Hervé, P. Klabbers, A. Lanaro, A. Levine, K. Long, R. Loveless, I. Ojalvo, T. Perry, G. A. Pierro, G. Polese, T. Ruggles, A. Savin, A. Sharma, N. Smith, W. H. Smith, D. Taylor, P. Verwilligen, N. Woods

† Deceased

1: Also at Vienna University of Technology, Vienna, Austria
2: Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China
3: Also at Institut Pluridisciplinaire Hubert Curien, Université de Strasbourg, Université de Haute Alsace Mulhouse, CNRS/IN2P3, Strasbourg, France
4: Also at Universidade Estadual de Campinas, Campinas, Brazil
5: Also at Centre National de la Recherche Scientifique (CNRS) - IN2P3, Paris, France
6: Also at Université Libre de Bruxelles, Bruxelles, Belgium
7: Also at Deutsches Elektronen-Synchrotron, Hamburg, Germany
8: Also at Joint Institute for Nuclear Research, Dubna, Russia
9: Now at British University in Egypt, Cairo, Egypt
10: Also at Zewail City of Science and Technology, Zewail, Egypt
11: Now at Fayoum University, El-Fayoum, Egypt
12: Now at Ain Shams University, Cairo, Egypt
13: Also at Université de Haute Alsace, Mulhouse, France
14: Also at CERN, European Organization for Nuclear Research, Geneva, Switzerland
15: Also at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
16: Also at RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany
17: Also at University of Hamburg, Hamburg, Germany
18: Also at Brandenburg University of Technology, Cottbus, Germany
19: Also at Institute of Nuclear Research ATOMKI, Debrecen, Hungary
20: Also at MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary
21: Also at University of Debrecen, Debrecen, Hungary
22: Also at Indian Institute of Science Education and Research, Bhopal, India
23: Also at Institute of Physics, Bhubaneswar, India
24: Also at University of Visva-Bharati, Santiniketan, India
25: Also at University of Ruhuna, Matara, Sri Lanka
26: Also at Isfahan University of Technology, Isfahan, Iran
27: Also at University of Tehran, Department of Engineering Science, Tehran, Iran
28: Also at Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, Iran
29: Also at Università degli Studi di Siena, Siena, Italy
30: Also at Purdue University, West Lafayette, USA
31: Also at International Islamic University of Malaysia, Kuala Lumpur, Malaysia
32: Also at Malaysian Nuclear Agency, MOSTI, Kajang, Malaysia
33: Also at Consejo Nacional de Ciencia y Tecnología, Mexico City, Mexico
34: Also at Warsaw University of Technology, Institute of Electronic Systems, Warsaw, Poland
35: Also at Institute for Nuclear Research, Moscow, Russia
36: Now at National Research Nuclear University ‘Moscow Engineering Physics Institute’ (MEPhI), Moscow, Russia
37: Also at St. Petersburg State Polytechnical University, St. Petersburg, Russia
38: Also at University of Florida, Gainesville, USA
39: Also at P.N. Lebedev Physical Institute, Moscow, Russia
40: Also at California Institute of Technology, Pasadena, USA
41: Also at Faculty of Physics, University of Belgrade, Belgrade, Serbia
42: Also at INFN Sezione di Roma Università di Roma, Rome, Italy
43: Also at National Technical University of Athens, Athens, Greece
44: Also at Scuola Normale e Sezione dell’INFN, Pisa, Italy
45: Also at National and Kapodistrian University of Athens, Athens, Greece
46: Also at Riga Technical University, Riga, Latvia
47: Also at Institute for Theoretical and Experimental Physics, Moscow, Russia
48: Also at Albert Einstein Center for Fundamental Physics, Bern, Switzerland
49: Also at Adiyaman University, Adiyaman, Turkey
50: Also at Mersin University, Mersin, Turkey
51: Also at Cag University, Mersin, Turkey
52: Also at Piri Reis University, Istanbul, Turkey
53: Also at Gaziosmanpasa University, Tokat, Turkey
54: Also at Ozyegin University, Istanbul, Turkey
55: Also at Izmir Institute of Technology, Izmir, Turkey
56: Also at Marmara University, Istanbul, Turkey
57: Also at Kafkas University, Kars, Turkey
58: Also at Istanbul Bilgi University, Istanbul, Turkey
59: Also at Yildiz Technical University, Istanbul, Turkey
60: Also at Hacettepe University, Ankara, Turkey
61: Also at Rutherford Appleton Laboratory, Didcot, UK
62: Also at School of Physics and Astronomy, University of Southampton, Southampton, UK
63: Also at Instituto de Astrofísica de Canarias, La Laguna, Spain
64: Also at Utah Valley University, Orem, USA
65: Also at University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia
66: Also at Facoltà Ingegneria, Università di Roma, Rome, Italy
67: Also at Argonne National Laboratory, Argonne, USA
68: Also at Erzincan University, Erzincan, Turkey
69: Also at Mimar Sinan University, Istanbul, Istanbul, Turkey
70: Also at Texas A&M University at Qatar, Doha, Qatar
71: Also at Kyungpook National University, Taegu, Korea