Heavy Quarkonium at zero and finite temperature: an effective field theory perspective

Nora Brambilla

Physik-Department, Technische Universität München, James-Franck-Str. 1, 85748 Garching, Germany

Abstract. I discuss quarkonium physics at zero and finite temperature in the framework of nonrelativistic effective field theories.

Keywords: Heavy Quarks, Effective Field Theories, NRQCD, pNRQCD
PACS: 12.38.-t, 11.10.Wx, 25.75.Nq

QUARKONIUM PHYSICS AND EFTS

Effective field theories for the description of quarkonium processes have been newly developed and are being developed and provide a unifying description as well as a solid and versatile tool giving well definite predictions [1, 2, 3]. They rely on one hand on high order perturbative calculations and on the other hand on lattice simulations, the recent progress in both fields having added a lot to the theory reach. Heavy quarkonium is a multiscale system and as such is on one hand particularly suitable to be treated in an effective field theory framework. On the other hand the existence of many scales in quarkonium makes it a unique system to study complex environments. Quarkonium probes all the regimes of QCD, from the high energy region, where an expansion in the the coupling constant is possible, to the low energy region, where nonperturbative effects dominate. It probes also the intermediate region between the two regimes. In particular for quarkonium system with a very small radius the interaction turns out to be purely perturbative while for system with a large radius with respect to the confinement scale the interaction turns out to be nonperturbative. In the complex environment of heavy ion collisions quarkonium suppression constitutes a unique probe of deconfinement and quark gluon plasma formation [4]. The different radius of the different quarkonia states induces the phenomenon of sequential suppression, allowing to use quarkonium as a kind of thermometer for the measurement of the temperature of the formed medium. On similar ground quarkonium may constitute a special probe to be used in the study of a nuclear medium. The large mass, the clean and known decays mode make quarkonium an ideal probe of new physics in some well defined window of parameters of physics beyond the Standard Model [2, 3].

The modern approach to quarkonium physics consists in taking advantage of the hierarchy of non-relativistic energy scales in the system by constructing a suitable hierarchy of effective field theories (EFTs) [1].

The energy scales are: the heavy-quark mass (hard scale), $m$, the typical momentum transfer (soft scale), $p \sim mv$, whose inverse sets the typical distance, $r$, between the
heavy quark and the antiquark, and the typical kinetic energy (ultrasoft scale), $E \sim mv^2$, whose inverse sets the typical time scale of the bound state. The heavy-quark bound-state velocity $v$ is a small quantity $v \ll 1$ ($v^2 \sim 0.1$ for $b\bar{b}$, $v^2 \sim 0.3$ for $c\bar{c}$, $v^2 \sim 0.01$ for $t\bar{t}$), the mass is a large quantity $m \gg \Lambda_{\text{QCD}}$, $\alpha_s(m) \ll 1$. For energy scales close to $\Lambda_{\text{QCD}}$, perturbation theory breaks down and one has to rely on nonperturbative methods. Regardless of this, the nonrelativistic hierarchy of scales: $m \gg p \sim 1/r \sim mv \gg E \sim mv^2$ also persists below the $\Lambda_{\text{QCD}}$ threshold. While the hard scale is always larger than $\Lambda_{\text{QCD}}$, different situations may arise for the other two scales depending on the considered quarkonium system. The soft scale, proportional to the inverse typical radius $r$, may be a perturbative ($\gg \Lambda_{\text{QCD}}$) or a nonperturbative scale ($\sim \Lambda_{\text{QCD}}$) depending on the physical system. The first case is likely to happen only for the lowest charmonium and bottomonium states. We do not have direct information on the radius of the quarkonia systems, and thus the attribution of some of the lowest bottomonia and charmonia states to the perturbative or the nonperturbative soft regime is at the moment still ambiguous. The ultrasoft scale may still be perturbative only in the case of $t\bar{t}$ threshold states. All quarkonium scales get entangled in a typical amplitude involving a quarkonium observable. In particular, quarkonium annihilation and production happen at the scale $m$, quarkonium binding happens at the scale $mv$, which is the typical momentum exchanged inside the bound state, while very low-energy gluons and light quarks (also called ultrasoft degrees of freedom) live long enough that a bound state has time to form and, therefore, are sensitive to the scale $mv^2$. Ultrasoft gluons are responsible for phenomena like the Lamb shift in QCD.

A hierarchy of EFTs may be constructed by systematically integrating out modes associated to high energy scales not relevant for quarkonium. Such integration is made in a matching procedure enforcing the equivalence between QCD and the EFT at a given order of the expansion in $v$. The EFT realizes a factorization at the Lagrangian level between the high energy contributions, encoded into the matching coefficients, and the low energy contributions, carried by the dynamical degrees of freedom. Poincaré symmetry remains intact in a nonlinear realization at the level of the NR EFT and imposes exact relations among the matching coefficients [8].

At the scale $m$ the suitable EFT is NRQCD [5], which follows from QCD by integrating out the scale $m$. As a consequence, the effective Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$. The field of quarkonium production has seen terrific progress in the last few years both in theory and in experiments, for a review see [2,3].

For what concerns decays, recently, substantial progress has been made in the evaluation of the factorization formula at order $v^7$ [9], in the lattice evaluation of the NRQCD matrix elements [10] and in the data of many hadronic and electromagnetic decays [2]. The data are clearly sensitive to NLO corrections in the Wilson coefficients and presumably also to relativistic corrections. Improved theory predictability would entail the lattice calculation or data extraction of the NRQCD matrix elements and perturbative resummation of large contribution in the NRQCD matching coefficients. The new data on hadronic transitions and hadronic decays pose interesting challenges to the theory.

Lattice NRQCD calculations have undergone a steady development in last few years see [2,3].
At the scales $mv$ and $mv^2$ the suitable EFT is potential NonRelativistic QCD (pNRQCD) [6, 7], which follows from NRQCD by integrating out the scale $mv$. As a consequence, the effective Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$, inherited from NRQCD, and an expansion in $r$

$$\mathcal{L}_{pNRQCD} = \int d^3r \sum_n \sum_k \frac{c_n(\alpha_s(M), \mu)}{M^n} V_{n,k}(r, \mu', \mu) r^k \times O_k(\mu', Mv^2, ...),$$

where $O_k$ are the operators of pNRQCD that live at the low-energy scale $mv^2$, $\mu$ is the NRQCD factorization scale, $\mu'$ is the pNRQCD factorization scale and $V_{n,k}$ are the Wilson coefficients of the EFT that encode the contributions from the scale $r$ and are non-analytic in $r$. Looking at the equations of motion of pNRQCD, we may identify $V_{n,0}$ with the $1/m^n$ potentials that enter the Schrödinger equation and $V_{n,k\neq0}$ with the couplings of the ultrasoft degrees of freedom, which provide corrections to the Schrödinger equation. This EFT is close to a Schrödinger-like description of the bound state. The bulk of the interaction is carried by potential-like terms $V_{n,0}$, but non-potential interactions $V_{n,k\neq0}$, associated with the propagation of low-energy degrees of freedom ($Q\bar{Q}$ colour singlets, $Q\bar{Q}$ colour octets and low energy gluons), are generally present. They start to contribute at NLO in the multipole expansion of the gluon fields and are typically related to nonperturbative effects [7].

In what follows we will focus on the EFT at the scale $mv$ and $mv^2$. Then, there are several cases for the physics at hand. In the case in which the EFT has been constructed [7, 16, 1], i.e. for states below threshold, the work is currently going in calculating higher order perturbative corrections in $v$ and $\alpha_s$ for processes of interest, resumming the logarithms in the ratio of the scales that may be sizeable, calculating or extracting nonperturbatively low energy correlators and extending the theory with the addictions of electromagnetic effects [21] and the consideration of $QQQ$ and $QQq$ systems [15]. The issue here is precision physics and the study of confinement. Close to threshold the EFT has not yet been constructed and the degrees of freedom have still to be identified [31, 3]. At finite temperature the EFT is being constructed and the existing results hint at a new physical picture with possible application at heavy ion collisions at LHC.

Below we will review new results focusing on the example of the calculation of the interquark potential at zero and at finite temperature in the different dynamical situations.

**QUARKONIUM POTENTIAL AT ZERO TEMPERATURE**

For states away from threshold we have a clear effective field description called pNRQCD, based on perturbative and lattice computations. This is nowadays the standard description.

The soft scale $mv$ may be larger or not than the confinement scale $\Lambda_{QCD}$ depending on the radius of the quarkonium system. When $mv^2 \sim \Lambda_{QCD}$, we speak about weakly-coupled pNRQCD because the soft scale is perturbative and the matching from NRQCD to pNRQCD may be performed in perturbation theory. When $mv \sim \Lambda_{QCD}$, we speak about strongly-coupled pNRQCD because the soft scale is nonperturbative and the matching from NRQCD to pNRQCD may not be performed in perturbation theory.
The potential is a Wilson coefficient of an EFT. In general undergoes renormalization, develops scale dependence and satisfies renormalization group equations which allow to resum large logarithms.

**weakly-coupled pNRQCD**

If the quarkonium system is small, the soft scale is perturbative and the potentials can be entirely calculated in perturbation theory [1].

Since the degrees of freedom that enter the Schrödinger description are in this case both $Q\bar{Q}$ color singlet and $Q\bar{Q}$ color octets, both singlet and octet potentials exist. The static singlet $Q\bar{Q}$ potential is pretty well known. The three-loop correction to the static potential is now completely known: the fermionic contributions to the three-loop coefficient [22] first became available, and more recently the remaining purely gluonic term has been obtained [23, 24].

The first log related to ultrasoft effects arises at three loops [13]. Such logarithm contribution at $N^3\text{LO}$ and the single logarithm contribution at $N^4\text{LO}$ may be extracted respectively from a one-loop and two-loop calculation in the EFT and have been calculated in [12].

The perturbative series of the static potential suffers from a renormalon ambiguity (i.e. large $\beta_0$ contributions) and from large logarithmic contributions. The singlet static energy, given by the sum of a constant, the static potential and the ultrasoft corrections, is free from ambiguities of the perturbative series. By resumming the large logs using the renormalization group equations and comparing it (at the NNLL) with lattice calculations of the static energy one sees that the QCD perturbative series converges very nicely to and agrees with the lattice result in the short range (up to 0.25 fm) and that no nonperturbative linear ("stringy") contribution to the static potential exist [14, 12].

In particular, the recently obtained theoretical expression [11] for the complete QCD static energy at NNNLL precision has been used to determine $r_0\Lambda_{\text{MS}}$ by comparison with available lattice data, where $r_0$ is the lattice scale and $\Lambda_{\text{MS}}$ is the QCD scale, obtaining $r_0\Lambda_{\text{MS}} = 0.622^{+0.019}_{-0.015}$ for the zero-flavor case. This extraction was previously performed at the NNLO level (including an estimate at NNNLO) in [25]. The same procedure can be used to obtain a precise evaluation of the unquenched $r_0\Lambda_{\text{MS}}$ value after short distance unquenched lattice data for the $Q\bar{Q}$ exist.

The static octet potential is known up to two loops [26]. Relativistic corrections to the static singlet potential have been calculated over the years and are summarized in [1].

In the case of $QQq$ baryons, the static potential has been determined up to NNLO in perturbation theory [15] and recently also on the lattice [29]. Terms suppressed by powers of $1/m$ and $r$ in the Lagrangian have been matched (mostly) at leading order and used to determine, for instance, the expected hyperfine splitting of the ground state of these systems.

In the case of $QQQ$ baryons, the static potential has been determined up to NNLO in perturbation theory [15] and also on the lattice [28]. The transition region from a Coulomb to a linearly raising potential is characterized in this case also by the emergence of a three-body potential apparently parameterized by only one length. It has been shown
that in perturbation theory a smooth genuine three-body potential shows up at two loops.

**strongly-coupled pNRQCD**

If the quarkonium system is large, the soft scale is nonperturbative and the potentials cannot be entirely calculated in perturbation theory \(^1\). Then the potential matching coefficients are obtained in the form of expectation values of gauge-invariant Wilson-loop operators. In this case, heavy-light meson pairs and heavy hybrids develop a mass gap of order \(\Lambda_{\text{QCD}}\) with respect to the energy of the \(Q\bar{Q}\) pair, the second circumstance being apparent from lattice simulations. Thus, away from threshold, the quarkonium singlet field \(S\) is the only low-energy dynamical degree of freedom in the pNRQCD Lagrangian \(^{16, 1, 18}\) (neglecting ultrasoft corrections coming from pions and other Goldstone bosons). The singlet potential \(V_S(r)\) can be expanded in powers of the inverse of the quark mass; static, \(1/m\) and \(1/m^2\) terms were calculated long ago \(^{16}\). They involve NRQCD matching coefficients (containing the contribution from the hard scale) and low-energy nonperturbative parts given in terms of static Wilson loops and field-strength insertions in the static Wilson loop (containing the contribution from the soft scale). Such expressions correct and generalize previous finding in the Wilson loop approach \(^{17}\) that were typically missing the high energy parts of the potentials, encoded into the NRQCD matching coefficients and containing the dependence on the logarithms of \(m\), and some of the low energy contributions. Poincaré invariance \(^8\) establishes exact relations between the potentials of the type of the Gromes relation between spin dependent and static potentials \(^{19}\).

In this regime of pNRQCD, we recover the quark potential singlet model. However, here the potentials are calculated in QCD by nonperturbative matching. Their evaluation requires calculations on the lattice or in QCD vacuum models \(^{32}\).

Recent progress includes new, precise lattice calculations of these potentials obtained using the Lüscher multi-level algorithm \(^{27}\).

The nonperturbative potentials for the \(QQ\) and \(QQq\) have been obtained in the second reference of \(^{15}\) and in \(^{30}\).

**QUARKONIUM CLOSE OR ABOVE THRESHOLD**

In the most interesting region, the region close to threshold where many new states, conceivably of an exotic nature have been recently discovered, no EFT description has yet been constructed nor the appropriate degrees of freedom clearly identified \(^{31, 3}\). An exception is constituted by the \(X(3872)\) that displays universal characteristics related to its being so close to threshold, reason for which a beautiful EFT description could be obtained \(^{20}\).

The threshold region remains troublesome also for the lattice, although several excited states calculations have been recently being pioneered.

Lattice results about the the crosstalk of the static potential with a pair of heavy-light mesons in the lattice have recently appeared \(^{33}\) but further investigations appear to be necessary.
QUARKONIUM POTENTIAL AT FINITE TEMPERATURE

The study of quarkonium in media has recently undergone crucial developments. Large datasets from heavy-ion collisions have recently become available at RHIC displaying new features related to the quark gluon plasma formation characteristics like the particular structure of jet quenching and the very low viscosity to entropy ratio. In particular the quark gluon plasma looks more like a liquid than a plasma and the use of perturbative expansion appears to be justified only at temperature bigger than the deconfinement one.

The suppression of quarkonium production in the hot medium remains one of the cleanest and most relevant probe of deconfined matter.

However, the use of quarkonium yields as a hot-medium diagnostic tool has turned out to be quite challenging for several reasons. Quarkonium production has already been found to be suppressed in proton-nucleus collisions by cold-nuclear-matter effects, which themselves require dedicated experimental and theoretical attention. Recombination effects may play an additional role and thus transport properties may become relevant to be considered. Finally, the heavy quark-antiquark interaction at finite temperature $T$ has to be obtained from QCD.

For observables only sensitive to gluons and light quarks, a very successfull EFT called Hard Thermal Loop (HTL) effective theory has been derived in the past [43] by integrating out the hardest momenta propotional to $T$ from the dynamics. However, considering also heavy quarkonium in the hot QCD medium, one has to consider in addition to the thermodynamical scales in $T$ also the scales of the nonrelativistic bound state and the situation becomes more complicate.

In the last few years, there has been a remarkable progress in constructing EFTs for quarkonium at finite temperature and in rigorously defining the quarkonium potential. In [34, 35], the static potential was calculated in the regime $T \gg 1/r \gtrsim m_D$, where $m_D$ is the Debye mass and $r$ the quark-antiquark distance, by performing an analytic continuation of the Euclidean Wilson loop to real time. The calculation was done in the weak-coupling resummed perturbation theory. The imaginary part of the gluon self energy gives an imaginary part to the static potential and hence a thermal width to the quark-antiquark bound state. In the same framework, the dilepton production rate for charmonium and bottomonium was calculated in [36, 37]. In [38], static particles in real-time formalism were considered and the potential for distances $1/r \sim m_D$ was derived for a hot QED plasma. The real part of the static potential was found to agree with the singlet free energy and the damping factor with the one found in [34]. In [39], a study of bound states in a hot QED plasma was performed in a non-relativistic EFT framework. In particular, the hydrogen atom was studied for temperatures ranging from $T \ll m\alpha^2$ to $T \sim m$, where the imaginary part of the potential becomes larger than the real part and the hydrogen ceases to exist. The same study has been extended to muonic hydrogen in [40], providing a method to estimate the effects of a finite charm quark mass on the dissociation temperature of bottomonium.

An EFT framework in real time and weak coupling for quarkonium at finite temperature was developed in [42] working in real time and in the regime of small coupling $g$, so that $gT \ll T$ and $v \sim \alpha_s$, which is expected to be valid for tightly bound states: $Y(13)$, $J/\psi$, ... .

Quarkonium in a medium is characterized by different energy and momentum scales;
there are the scales of the non-relativistic bound state that we have discussed at the beginning, and there are the thermodynamical scales: the temperature $T$, the inverse of the screening length of the chromoelectric interactions, i.e. the Debye mass $m_D$ and lower scales, which we will neglect in the following.

If these scales are hierarchically ordered, then we may expand physical observables in the ratio of such scales. If we separate explicitly the contributions from the different scales at the Lagrangian level this amounts to substituting QCD with a hierarchy of EFTs, which are equivalent to QCD order by order in the expansion parameters. As it has been described in the previous sections at zero temperature the EFTs that follow from QCD by integrating out the scales $m$ and $mv$ are called respectively Non-relativistic QCD (NRQCD) and potential NRQCD (pNRQCD). We assume that the temperature is high enough that $T \gg gT \sim m_D$ holds but also that it is low enough for $T \ll m$ and $1/r \sim mv \sim m_D$ to be satisfied, because for higher temperature the bound state ceases to exist. Under these conditions some possibilities are in order. If $T$ is the next relevant scale after $m$, then integrating out $T$ from NRQCD leads to an EFT that we may name NRQCD$_{HTL}$, because it contains the hard thermal loop (HTL) Lagrangian [43]. Subsequently integrating out the scale $mv$ from NRQCD$_{HTL}$ leads to a thermal version of pNRQCD that we may call pNRQCD$_{HTL}$. If the next relevant scale after $m$ is $mv$, then integrating out $mv$ from NRQCD leads to pNRQCD. If the temperature is larger than $mv^2$, then the temperature may be integrated out from pNRQCD leading to a new version of pNRQCD$_{HTL}$ [44]. Note that, as long as the temperature is smaller than the scale being integrated out, the matching leading to the EFT may be performed putting the temperature to zero.

The derived potential $V$ describes the real-time evolution of a quarkonium state in a thermal medium. At leading order, the evolution is governed by a Schrödinger equation. In an EFT framework, the potential follows naturally from integrating out all contributions coming from modes with energy and momentum larger than the binding energy. For $T < V$ the potential is simply the Coulomb potential. Thermal corrections affect the energy and induce a thermal width to the quarkonium state; these may be relevant to describe the in medium modifications of quarkonium at low temperatures. For $T > V$ the potential gets thermal contributions, which are both real and imaginary.

General findings in this picture are:

- The thermal part of the potential has a real and an imaginary part. The imaginary part of the potential smears out the bound state peaks of the quarkonium spectral function, leading to their dissolution prior to the onset of Debye screening in the real part of the potential (see, e.g. the discussion in [41]). So quarkonium dissociation appears to be a consequence of the appearance of a thermal decay width rather than being due to the color screening of the real part of the potential; this follows from the observation that the thermal decay width becomes as large as the binding energy at a temperature at which color screening may not yet have set in.
- Two mechanisms contribute to the thermal decay width: the imaginary part of the gluon self energy induced by the Landau-damping phenomenon (existing also in QED) [34] and the quark-antiquark color singlet to color octet thermal break up (a new effect, specific of QCD) [46]. Parametrically, the first mechanism dominates.
for temperatures such that the Debye mass $m_D$ is larger than the binding energy, while the latter dominates for temperatures such that $m_D$ is smaller than the binding energy.

- The obtained singlet thermal potential, $V$, is neither the color-singlet quark-antiquark free energy nor the internal energy. It has an imaginary part and may contain divergences that eventually cancel in physical observables [46].
- Temperature effects can be other than screening, typically they may appear as power law corrections or a logarithmic dependence [46, 39].
- The dissociation temperature goes parametrically as $\pi T_{\text{melting}} \sim m_g^{4/3}$ [39, 41].

The EFT provides a clear definition of the potential and a coherent and systematical setup to calculate masses and widths of quarkonium at finite temperature. In [45] heavy quarkonium energy levels and decay widths in a quark-gluon plasma, below the melting temperature at a temperature $T$ and screening mass $m_D$ satisfying the hierarchy $m_\alpha_s \gg \pi T \gg m_\alpha_s^2 \gg m_\alpha_s^5$. This situation is relevant for bottomonium 1S states ($\Upsilon(1S), \eta_b$) at the LHC. It has been found [45] that: at leading order the quarkonium masses increase quadratically with $T$ which in turn implies the same functional increase in the energy of the dileptons produced in the electromagnetic decays; a thermal correction proportional to $T^2$ appears in the electromagnetic quarkonium decay rates; at leading order a decay width linear with the temperature is developed which implies a tendency to dissolve by decaying to the continuum of the colour-octet states.

In [46, 47] the Polyakov loop and the correlator of two Polyakov loops at finite temperature has been calculated at next-to-next-to-leading order in the weak coupling regime and at quark-antiquark distances shorter than the inverse of the temperature and for Debye mass larger than the Coulomb potential. The calculation has been performed also in the EFT framework [46] and a relation between the Polyakov loop correlator and the singlet and octet quark-antiquark correlator has been established in this setup [42].

First attempts to generalize this new picture to the nonperturbative regime have been undertaken in [48].

ACKNOWLEDGMENTS

We acknowledge financial support from the DFG cluster of excellence “Origin and structure of the universe” (http://www.universe-cluster.de).

REFERENCES

1. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005)
2. N. Brambilla et al. [Quarkonium Working Group], arXiv:hep-ph/0412158
3. N. Brambilla et al. [Quarkonium Working Group], CLNS 10/2066, TUM-EFT 11/10 (2010).
4. T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
5. W. E. Caswell and G. P. Lepage, Phys. Lett. B 167 (1986) 437; G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51 (1995) 1125 [Erratum-ibid. D 55 (1997) 5853].
6. A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. 64 (1998) 428.
7. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. B 566 (2000) 275.
