D-dimensional Ideal Quantum Gases in $Ar^n + Br^{-n}$ Potential

Ahmed Jellal$^a$ and Mohammed Daoud$^{a,b}$

$^a$ High Energy Physics Section
the Abdus Salam International Centre for Theoretical Physics
Strada Costiera 11, 34100 Trieste, Italy

$^b$ Department of Physics, Faculty of Sciences,
University Ibn Zohr, P.O.B. 28/S, Agadir, Morocco

Abstract

The paper is concerned with thermostatistics of both $D$-dimensional Bose and Fermi ideal gases in a confining potential of type $Ar^n + Br^{-n}$. The investigation is performed in the framework of the semiclassical approximation. Some physical quantities for such systems are derived, like density of states, density profiles and number of particles. Bose-Einstein condensation (BEC) is discussed in the high and low temperature regimes.

PACS: 05.30.-d, 05.30.Fk, 05.30.Jp

Keywords: Density of states, density profiles, number of particles, BEC

$^*$E-mail: jellal@ictp.trieste.it – jellal@youpy.co.uk

$^{†b}$ Permanent address
1 Introduction

In the recent past there has been increasing emphasis in quantum statistics of the ideal quantum gases in an arbitrary quantum potential since the recent experiment observations. The latter concerns the Bose-Einstein condensation [1] and the Fermi quantum degeneracy [2] for dilute alkali-metal atoms in magnetic or magneto-optical traps at very low temperature. These results have received renewed attention and much of this attention is paid to the study of ideal quantum gases confining in different external potentials. In particular, the Bose-Einstein condensation. This was examined in many occasions in the presence of typical external potentials, for example; harmonic potential [3–8], toroidal potential [9], double-well potential [10] and in the presence of an impurity [11]. More recently, an interesting results have been obtained by Salasnich. Ideal quantum gases in $D$-dimensional space and confining in power-law potential [12] have been studied. Among the results derived in this paper, we note the condition of the Bose-Einstein condensation (BEC). Indeed, it is shown that BEC can set up if and only if $D^2 + Dn > 1$.

In this paper we wish to extend the Salasnich analysis [12] to cover more general results. For this task, we investigate the ideal quantum gases trapped in the potential

$$U(r) = Ar^n + Br^{-n}$$

with $A, B$ strictly positives and $n$ is a non vanishing integer. For this type of potential, we give the thermostatistics properties which generalize the Salasnich ones. In the limit $B \to 0$, we recover the Salasnich results. We prove that BEC occurs in the high temperature regime.

This paper is organised as follows. In section 2, we give the quantum distribution functions in the framework of the semiclassical approximation for both bosons and fermions confining in a generic potential. Section 3 is devoted to the generalization of the Salasnich analysis when the particles, bosons and fermions, are trapped in the potential given by equation (1). We will see that in the limit $B \to 0$, one can recover the results obtained by Salasnich in [12]. Bosons and fermions are studied separately. For the fermionic gas evolving in the $U(r)$ potential, we give the corresponding Fermi functions, the finite
and zero temperature momentum distribution, and the number of particles. In a similar way, we define the Bose function corresponding to particles obeying the Bose-Einstein statistics, we compute the finite temperature non condensed momentum distribution and finally we give the relation satisfied by the Bose transition temperature below which the BEC takes place. Since, this relation can’t be solved in the general case, we restrict the BEC analysis in the high and low temperature regimes. The last section is devoted to the conclusions and perspectives of the present work.

2 Preliminaries

We shall briefly review the basic tools which will be useful to investigate the thermal properties of both Bose and Fermi ideal gases in a generic confining external potential.

2.1 Quantum distribution functions

We start with a confined quantum gases (identical bosons or fermions). The average number of occupation for the \( k \)-mode is

\[
N_k = \frac{1}{e^{\beta (\epsilon_k - \mu)} + 1},
\]

for the fermions and

\[
N_k = \frac{1}{e^{\beta (\epsilon_k - \mu)} - 1},
\]

for bosons. In equations (2) and (3), \( \mu \) is the chemical potential, \( \epsilon_k \) are the energy of particles, and \( \beta = \frac{1}{k_B T} \) the reciprocal temperature. Then, the total average occupation number \( N \) of the particles is given by

\[
N = \sum_k N_k,
\]

which leads to fix the parameter \( \mu \). For more details about the physical meaning of the chemical potential \( \mu \) see ref. [12].
In conclusion, equations (2) and (3) will be written in the semiclassical approximation in the next subsection and will be constituted a starting point of our study for the bosons and fermions, respectively.

### 2.2 Semiclassical approximation

In the semiclassical approximation (the thermodynamical limit), the energy spectrum may be considered as a continuum. This situation occurs when the number of particles is large and the energy level spacing is small. Thus, the quantum distribution functions can be replaced by the so-called phase-space distribution \[12\]

\[ n(r, p) = \frac{1}{e^{\beta(\epsilon(r, p) - \mu)} + 1}, \quad (5) \]

where \( r = (r_1, r_2, \ldots, r_D) \) and \( p = (p_1, p_2, \ldots, p_D) \) are the position and momentum of \( D \)-dimensional system under consideration. In equation (5), plus and minus refer to fermions and bosons respectively. To obtain the total distribution, we compute the integral over all \( 2D \)-dimensional phase-space. Then, we get

\[ N = \int \frac{d^D r d^D p}{(2\pi\hbar)^D} n(r, p), \quad (6) \]

which can be written as follows

\[ N = \int d^D r \ n(r) = \int d^D p \ n(p), \quad (7) \]

where the spacial and momentum distributions are given, respectively, by

\[ n(r) = \int \frac{d^D p}{(2\pi\hbar)^D} \ n(r, p), \quad (8) \]

and

\[ n(p) = \int \frac{d^D r}{(2\pi\hbar)^D} \ n(r, p). \quad (9) \]

Introducing the density of states

\[ \rho(\epsilon) = \int \frac{d^D r d^D p}{(2\pi\hbar)^D} \delta(\epsilon - \epsilon(r, p)), \quad (10) \]
with $\delta(x)$ is the usual Dirac function. The average total occupation number of particles can be written as

$$N = \int_0^\infty d\epsilon \rho(\epsilon) \frac{1}{e^{\beta(\epsilon-\mu)} + 1}. \quad (11)$$

The phase-distribution, in the case of fermions at zero temperature (i.e where the chemical potential $\mu$ coincides with Fermi energy $E_F$), takes the simplest form

$$n(r,p) = \Theta(E_F - \epsilon(r,p)), \quad (12)$$

where $\Theta(x)$ is the well-known Heaviside function. The case of bosons was discussed in great detail in [12] (see also the references therein). When, the external potential is taken into account, the classical single-particle energy is defined as

$$\epsilon(r,p) = \frac{p^2}{2m} + U(r), \quad (13)$$

from which one can prove that the semi-classical density function is given by

$$\rho(\epsilon) = \left(\frac{m}{2\pi\hbar^2}\right)^{\frac{D}{2}} \frac{1}{\Gamma\left(\frac{D}{2}\right)} \int d^Dr \left(\epsilon - U(r)\right)^{\frac{D-2}{2}}, \quad (14)$$

where $\Gamma(n)$ is the Gamma function. It is interesting to note that equation (14) is valid for all external potentials. We now close this section related to the main tools which will be used in the following sections investigating the thermostatistics of bosons and fermions trapped in the potential given by eq.(1).

### 3 Ideal gases in the $Ar^n + Br^{-n}$ potential

In many experiments with alkali-metals atoms, the external trap can be modelled by a harmonic potential [14]. The effects of adiabatic change in the trap can be represented by power-law potentials [12]. In general, to find the momentum distribution, the Fermi temperature and the Bose temperature, it is necessary to specify the external potential.

#### 3.1 Fermionic gas

To discuss the statistical properties of a fermionic gas embedded in the potential $U(r)$, let us start by defining the Fermi function. As we will see, this definition generalizes the one
given in [12] and allows the computation of physical quantities, of interest in the study of the system under consideration, as, for instance, the density of states and number of particles.

We define the Fermi function as follows

\[
f_n^{(B)}(z) = \frac{1}{\Gamma(n)} \int_0^\infty dy \ y^{n-1} \frac{ze^{-ay-by-1}}{1+ze^{-ay-by}}.
\] (15)

where \(a\) and \(b\) are two arbitrary positive constants. In the particular case \(b = 0\), we get after rescaling the variable

\[
f_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty dy \ y^{n-1} \frac{ze^{-y}}{1+ze^{-y}}.
\] (16)

where \(f_n(z)\) is the Fermi function definition [12] for a fermionic system in the \(U(r) = Ar^n\) potential. We note that, the Fermi function can be expanded, for \(|z| < 1\), as

\[
f_n^{(B)}(z) = \frac{2(b/a)^{n/2}}{\Gamma(n)} \sum_{j=0}^{\infty} (-1)^{j+1} z^j K_n(2\sqrt{ab}),
\] (17)

in terms of the modified Bessel function \(K_n(2\sqrt{ab})\) [13]

\[
K_n(2\sqrt{ab}) = \frac{1}{2} \left(\frac{a}{b}\right)^{n/2} \int_0^\infty dy \ y^{n-1} e^{-ay-by^{-1}}.
\] (18)

The density of states of a fermionic quantum gas in the potential eq.(1) can be calculated from equation (14). So, we have to compute the integral of type

\[
\rho^B(\epsilon) = \left(\frac{m}{2\pi \hbar}\right)^{D/2} \frac{1}{\Gamma(D/2)} \int d^D r \ (\epsilon - Ar^n - Br^{-n})^{D-2}.\] (19)

In this order, we put \(y = r^n\) and solve the following equation

\[
\epsilon - Ay - By^{-1} = 0.
\] (20)

The latter equation has two positive solutions

\[
u = \frac{\epsilon}{2A} + \sqrt{(\frac{\epsilon}{2A})^2 - \frac{B}{A}}, \quad \alpha = \frac{\epsilon}{2A} - \sqrt{(\frac{\epsilon}{2A})^2 - \frac{B}{A}}.
\] (21)
when $\epsilon \geq 2\sqrt{AB}$ is satisfied since the radius of the sphere $r$ should be a real positive.

Using the relation $d^D r = \frac{D\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2} + 1)}$ giving the $D$-dimensional unit sphere and the equations (21), the integral (19) can be written as

$$\rho_B(\epsilon) = \left(\frac{m}{2\pi\hbar}\right)^{\frac{D}{2}} \frac{D\pi^{\frac{D-2}{2}} A^{\frac{D-2}{2}}}{n\Gamma(\frac{D}{2})\Gamma(\frac{D}{2} + 1)} \int_0^u dy \, y^{\frac{D}{2} - \frac{D}{2}} (\alpha + y)\frac{\Gamma(\frac{D}{2} - 1)}{\Gamma(\frac{D}{2})^2} (u - y)^{\frac{D}{2} - 1},$$

which can be solved using the integral representations of the hypergeometric functions.

Finally, we obtain

$$\rho_B(\epsilon) = \left(\frac{m}{2\sqrt{\pi\hbar}}\right)^{\frac{D}{2}} \frac{D\pi^{\frac{D}{2}} A^{\frac{D}{2}}}{n\Gamma(\frac{D}{2})\Gamma(\frac{D}{2} + 1)} \frac{\Gamma(\frac{D}{2} - 1)}{\Gamma(\frac{D}{2})^2} 2F_1\left(1 - \frac{D}{2}, \frac{D}{2}; 1; \frac{u}{\alpha}\right).$$

We recall that the hypergeometric function $2F_1(\alpha, \beta; \gamma; z)$ is defined by

$$2F_1(\alpha, \beta, \gamma, z) = \frac{1}{B(\beta, \gamma - \beta)} \int_0^1 dy \, y^{\beta - 1}(1 - y)^{\gamma - 1}(1 - zy)^{-\alpha},$$

where $\text{Re}\gamma > 0, \text{Re}\beta > 0$ and

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} = B(\beta, \alpha),$$

is the so-called beta function or Euler's integral of the first kind, which admits the following integral representation

$$B(\alpha, \beta) = \int_0^1 dt \, t^{\alpha - 1}(1 - t)^{\beta - 1}.$$

It is interesting to remark that the following relation is satisfied

$$\lim_{B \to 0} (\alpha)^{\frac{3}{2}} 2F_1\left(1 - \frac{D}{2}, \frac{D}{2}; 1; \frac{u}{\alpha}\right) = \left(\frac{\epsilon}{A}\right)^{\frac{D}{2} - 1} \frac{\Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + 2)}{\Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + 2)}.$$

Using the latter relation, the density of states, in the limit $B \to 0$, is given by

$$\rho(\epsilon) = \left(\frac{m}{2\hbar^3}\right)^{\frac{D}{2}} \left(\frac{1}{A}\right)^{\frac{D}{2}} \frac{\Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + \frac{D}{2} + 1)}{\Gamma(\frac{D}{2} + 1) \Gamma(\frac{D}{2} + \frac{D}{2} + 1)} \epsilon^{\frac{D}{2} + 1}$$

which coincides with the result obtained in reference [12].

Using the above tools, other interesting properties of the present system (fermions) may be determined in a simple way:
i- The finite temperature momentum distribution

\[ n(p) = \frac{1}{(2\sqrt{\pi \hbar})^D} \frac{\Gamma(D/2 + 1)}{\Gamma(D/2 + 1)} f^{(B)}_{\frac{D}{n}}(e^{\beta(\mu - \frac{p^2}{2m})}). \] (29)

ii- The zero temperature momentum distribution

\[ n(p) = \frac{1}{(2\sqrt{\pi \hbar})^D} \frac{1}{\Gamma(D/2 + 1)} \Theta(E_F - \frac{p^2}{2m}) \left[ \frac{E_F - \frac{p^2}{2m}}{2A} + \left( \frac{E_F - \frac{p^2}{2m}}{2A} \right)^2 - \frac{B}{A} \right]^{\frac{D}{2}}, \] (30)

where we request that \( p \leq [2m(\epsilon - 2\sqrt{AB})]^\frac{1}{2} \) for the right term makes sense.

iii- Number of particles

The number of particles is given by the following integral

\[ N = \frac{1}{\Gamma(D/2 + 1)} \int \frac{d^Dp}{(2\pi \hbar)^D} \Theta(E_F - \frac{p^2}{2m}) \left[ \frac{E_F - \frac{p^2}{2m}}{2A} + \left( \frac{E_F - \frac{p^2}{2m}}{2A} \right)^2 - \frac{B}{A} \right]^{\frac{D}{2}} \] (31)

which can be reorganized as

\[ N = \frac{1}{(2\sqrt{\pi \hbar})^D} \frac{D\pi^{D/2}}{\Gamma(D/2 + 1)^2} \int_0^{[2m(\epsilon - 2\sqrt{AB})]^\frac{1}{2}} dp p^{D-1} \left[ \frac{E_F - \frac{p^2}{2m}}{2A} + \left( \frac{E_F - \frac{p^2}{2m}}{2A} \right)^2 - \frac{B}{A} \right]^{\frac{D}{2}}. \] (32)

A calculation more or less complicated, gives the following expression for the number of particles

\[ N = M f(\gamma) B(D/2, D/2) \left[ f_1(\gamma) \text{ _2F}_1(+1) - f_1^{-1}(\gamma) \text{ _2F}_1(-1) \right], \] (33)

where

\[ M = \frac{D(m^2)^{\frac{D}{2}}}{(AB)^{\frac{D}{2n} + \frac{D}{2}}} \frac{(\text{ _2F}_1)^{\frac{D}{2n} - \frac{D}{2}}}{A^{D/m}}, \]

\[ f(\gamma) = (\gamma^2 - 1)^{\frac{D-1}{2}} \frac{[\gamma - (\gamma^2 - 1)^{\frac{1}{2}}]^{\frac{D}{2n} - \frac{D}{2}}}{\gamma^2 - 1}, \]

\[ f_1(\gamma) = \gamma - (\gamma^2 - 1)^{\frac{1}{2}}, \]

\[ f_1^{-1}(\gamma) = \frac{1}{f_1(\gamma)}, \]

\[ \gamma = \frac{E_F - \frac{p^2}{2m}}{2A}, \]

\[ \text{ _2F}_1(\pm 1) = \text{ _2F}_1\left(-\frac{D}{n} + \frac{D}{2} \pm 1; \frac{D}{2}; D, -\frac{2(\gamma^2 - 1)^{\frac{1}{2}}}{\gamma - (\gamma^2 - 1)^{\frac{1}{2}}} \right), \]

and \( B(D/2, D/2) \) is given by eqs.(25,26).
The number of particles is expressed in terms of the hypergeometric and beta functions. It is clear that this relation is complicated and we believe that a more detailed study of it can generate more interesting results. We will return to this matter in a subsequent paper.

To close this subsection, we note that the results derived in ref. [12] related to the fermionic gas, can be reproduced just by taking the limit $B \rightarrow 0$.

### 3.2 Bosonic gas

We now examine the condensation of a system of bosons confining in the potential $U(r)$ (eq.(1)). We start by defining the Bose function as follows

$$g^{(B)}_{n}(z) = \frac{1}{\Gamma(n)} \int_{0}^{\infty} dy \, y^{n-1} \, \frac{ze^{-ay-by-1}}{1-ze^{-ay-by-1}}.$$  \hspace{1cm} (34)

For $|z| < 1$, we have

$$g^{(B)}_{n}(z) = \frac{2(\frac{b}{a})^\frac{n}{2}}{\Gamma(n)} \sum_{j=1}^{\infty} z^j \, K_n(2j \sqrt{ab}),$$  \hspace{1cm} (35)

where $K_n(2\sqrt{ab})$ is given by eq.(18). For $B \rightarrow 0$, we show that

$$g_n(z) = \frac{1}{\Gamma(n)} \int_{0}^{\infty} dy \, y^{n-1} \, \frac{ze^{-y}}{1+ze^{-y}},$$  \hspace{1cm} (36)

which is nothing but the definition of the Bose function given in [12] corresponding to the bosons in the potential $U(r) = Ar^n$.

Using the definition given by eq.(34), one obtain the following results:

**i- The finite temperature non condensed momentum distribution**

$$n(p) = \frac{1}{(2\pi \hbar)^D} \frac{\Gamma(\frac{D}{2}+1)}{\Gamma(\frac{D}{2}+1)} \frac{\Gamma(\frac{D}{2}+1)}{\Gamma(\frac{D}{2}+1)} \, g^{(B)}_{n}(e^{\beta(\mu-\frac{p^2}{2m})}).$$  \hspace{1cm} (37)

**ii- The Bose transition temperature**

To obtain the Bose-temperature, we have to calculate the number of particles

$$N = \int d^D p \, n(p).$$  \hspace{1cm} (38)
As in the case of ordinary bosons (without interaction), we define the Bose-temperature by taking the chemical potential \( \mu = 0 \). By introducing \( \mu = 0 \) in eq.(37), we get

\[
N = \frac{D\pi^{\frac{D}{2}}}{(2\sqrt{\pi\hbar})^D} \frac{\Gamma(D/n + 1)}{\Gamma(D/2 + 1)^2} \int_0^\infty dp \, p^{D-1} g^{(B)}_D (e^{-\frac{\beta p^2}{2\hbar}}).
\] (39)

With the help of eq.(35), it is easy to show that

\[
N = 2\left(\frac{B}{A}\right)^\frac{D}{2n} \frac{2m}{\beta} \frac{D\pi^{\frac{D}{2}}}{(2\sqrt{\pi\hbar})^D} \frac{\Gamma(D/n + 1)}{\Gamma(D/2 + 1)} \frac{1}{\Gamma(D/2 + 1)} \sum_{j=1}^\infty \frac{K_n (2j\beta\sqrt{AB})}{j^{D/n}}.
\] (40)

The latter equation gives a relation between the number of particles (or density) and Bose-temperature below which we obtain a Bose-Einstein condensation. Due to the complicated sum appearing in (40), the Bose temperature can’t be calculated, in general, in a simple way. However, in the some particular situations, the solutions of this equation can be found. Indeed, in the high and low temperature regimes (which are two interesting situations from an experimental point of view) some information about BEC can be derived:

**High temperature**: This case corresponds to \( \beta \to 0 \). In this limit the functions \( K_n (2j\beta\sqrt{AB}) \) take the form \[ K_n (2j\beta\sqrt{AB}) \approx \frac{1}{2} \frac{\Gamma(D/n)}{(2j\beta\sqrt{AB})} \frac{\zeta(D/2 + D/n)}{\beta^{D/n}}. \] (41)

Therefore, eq.(40) implies

\[
N = \left(\frac{m}{\hbar^2}\right)^\frac{D}{2n} \frac{\Gamma(D/n + 1)}{\Gamma(D/2 + 1)} \frac{\zeta(D/2 + D/n)}{\beta^{D/n}}.
\] (42)

This result is similar to the one found in ref.[12]. From this relation we conclude that just restricting ourselves to high temperature limit, we find the Salasnich analysis related to the BEC. Indeed, it is shown that the latter takes place when the condition \( D/2 + D/n > 1 \) is satisfied. Strictly speaking, we obtain the condensed fraction as follows

\[
\frac{N_0}{N} = 1 - \left(\frac{T}{T_B}\right)^{D/n}.
\] (43)

where \( N_0 \) is the number of particles occupying the single-particle ground-state of the system when the temperature is below \( T_B \).

**Low temperature**: Equivalently to \( \beta \to \infty \), then we have \[ K_n (2j\beta\sqrt{AB}) \approx 0. \] (44)
In that case, we get a number of particles \( N \to 0 \) corresponding to a temperature \( T \to 0 \). This result is compatible with the literature \[13\].

## 4 Conclusion

In this work, we concentrated on the influence of confining potential \( U(r) = Ar^n + Br^{-n} \) on the main statistical properties of the ideal quantum gases (bosons or fermions) in \( D \)-dimensions. We discussed the derivation of the density of states, spacial and momentum distributions in the thermodynamical limit. For fermions, we have calculated the Fermi energy and for bosons, the phenomenon of Bose-Einstein condensation is discussed in terms of the reciprocal temperature \( (\beta) \). Two situations were considered \((\beta \to 0)\) and \((\beta \to \infty)\) corresponding to high and low temperature, respectively. In the limit \( B \to 0 \), our results reproduce the Salasnich ones ref.\[12\] concerning the confining power-law potential.

An important result of the present work concerns the Bose-Einstein condensation, in the high temperature domain, which occurs when \( \frac{D}{2} + \frac{D}{n} > 1 \), where \( D \) is the space dimension and \( n \) the exponent appearing in the expression of \( U(r) \). Another result concerns to the low temperature case where the number of particles vanishes.

To finalize this paper, it should be noted that the present investigation of the ideal quantum gases embedded in the potential \( U(r) \) (eq.(1)) in \( D \)-dimensions, constitutes now a very interesting topic to learn more about Bose-Einstein condensation. In fact, as we have indicated above, the BEC should be investigated in more detail in general cases solving equation (40). We believe that more important informations can be obtained in this case. We will return on this subject in a forthcoming work \[17\].

## Acknowledgment

A. Jellal is grateful to Prof. S. Randjbar-Daemi for the kind invitation to visit the High Energy Section of the Abdus Salam International Centre for Theoretical Physics (AS-
ICTP). M. Daoud acknowledges hospitality of (AS-ICTP) and would like to thank Prof. Yu Lu. The authors are thankful to Prof. Salasnich for the discussions about his work reference [12] and for his kind comment concerning this paper. The authors are greatly indebted to Prof. G. Thompson for reading the manuscript.

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