$O(\alpha_s)$ QCD Corrections to Spin Correlations in $e^-e^+ \rightarrow t\bar{t}$ process at the NLC

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Abstract

Using a Generic spin basis, we present a general formalism of one-loop radiative corrections to the spin correlations in the top quark pair production at the Next Linear Collider, and calculate the $O(\alpha_s)$ QCD corrections under the soft gluon approximation. We find that: (a) in Off-diagonal basis, the $O(\alpha_s)$ QCD corrections to $e_L^ue_L^+$ ($e_R^ue_R^+$) scattering process increase the differential cross sections of the dominant spin component $t_\uparrow\bar{t}_\downarrow$ ($t_\downarrow\bar{t}_\uparrow$) by $\sim 30\%$ and $\sim (0.1\% - 3\%)$ depending on the scattering angle for $\sqrt{s} = 400 GeV$ and $1 TeV$, respectively; (b) in Off-diagonal basis (Helicity basis), the dominant spin component makes up 99.8\% ($\sim 53\%$) of the total cross section at both tree and one-loop level for $\sqrt{s} = 400 GeV$, and the Off-diagonal basis therefore remains to be the optimal spin basis after the inclusion of $O(\alpha_s)$ QCD corrections.

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A special feature of the top quark is that due to its large mass, $m_t = 175 \pm 6\, GeV$, the lifetime of the top quark is very short. This has an important consequence that the top quark decays before it hadronizes, and the spin information is preserved from production to decay. Thus we can expect the spin orientation of the top quark to be observable experimentally. Since there are significant angular correlations between the decay products of the top quark and the spin of the top quark, the spin-spin correlations in the top quark pair production can be extracted by forming angular correlations among the decay products of the top quark and the top anti-quark. Thus the spin correlations in the top quark pair production can be used as a good observable for testing the standard model (SM).

In most papers on the spin correlations for top quark pair production at the Tevatron and the Next Linear Collider (NLC), the top quark spin is decomposed in the Helicity basis. Recently, Mahlon and Parke proposed the Generic spin basis and found that the "Off-diagonal" basis, a special case of the Generic spin basis, is a more optimized decomposition of the top quark spins for $e^+e^-$ colliders because the contribution from like-spin pairs of top quarks vanishes to leading order in perturbation theory.

The $O(\alpha_s)$ QCD corrections to the cross sections of $e^-e^+ \rightarrow t\bar{t}$ process were calculated in refs. In this letter, we first repeated the calculation of $O(\alpha_s)$ QCD corrections to this process. We then present, in the Generic spin basis, a general formalism of one-loop radiative corrections to the spin correlations in the top quark pair production at the NLC, and calculate the $O(\alpha_s)$ QCD corrections and give the numerical results.

In the SM, we consider the process

$$e^-e^+ \rightarrow t\bar{t}$$

at the NLC with $\sqrt{s} = (0.4, 1)$ TeV. The tree level $Ve^-e^+$ vertex can be written as

$$\Gamma_{vee}^\mu = ie\gamma^\mu (e_L^VP_- + e_R^VP_+)$$

where $P_{\pm} = (1 \pm \gamma_5)/2$, and the SM values for these coupling factors are $e_{L,R} = -1$ for $V = \gamma$, $e_L^Z = (2\sin^2\theta_W - 1)/(2\sin\theta_W\cos\theta_W)$ and $e_R^Z = \sin\theta_W/\cos\theta_W$ for $V = Z$, and the $\theta_W$ is the Weinberg angle.
The general $Vt\bar{t}$ ($V = \gamma, Z$) coupling can be written as

$$\Gamma_{Vt\bar{t}}^\mu = ie \left[ \gamma^\mu (A_V - B_V \gamma^5) + \frac{t_1^\mu - t_2^\mu}{2} (C_V - D_V \gamma^5) \right],$$

where $t_1^\mu$ ($t_2^\mu$) is the momentum of the outgoing top (anti-top) quark.

In the $t\bar{t}$ center of mass frame (CMS), the scattering plane is defined to be the X-Z plane where the electron is moving along the $+Z$ direction and $\theta_t$ is the scattering angle of top quark, and we also set $\phi_t = 0$. The Born helicity amplitudes for the process (1) are obtained by summing the contributions from both the $Z$ and $\gamma$:

$$M_0(h_{-e^-}, h_{+e^+}, h_t, h_{\bar{t}}) = \frac{2e^2}{\sqrt{s}} \left[ M_0(h_{-e^-}, h_{+e^+}, h_t, h_{\bar{t}})^\gamma + M_0(h_{-e^-}, h_{+e^+}, h_t, h_{\bar{t}})^Z R(s) \right].$$

where $s = 4E^2$ is the total energy in CMS, $E$ is the energy of the electron beam, and $R(s) = s/(s - M_Z^2) \frac{1}{\sqrt{2}}$.

In the *Generic spin basis*, as illustrated by Fig.1 in ref.\[3\], the top quark (anti-top quark) spin states are defined in the top quark (anti-top quark) rest-frame, where one decomposes the top (anti-top) spin along the direction $\hat{s}_t$ ($\hat{s}_{\bar{t}}$), which makes an angle $\xi$ with the anti-top (top) momentum in the clockwise direction. Thus, the state $t_\uparrow \bar{t}_\uparrow$ ($t_\downarrow \bar{t}_\downarrow$) refers to a top with spin in the $+\hat{s}_t$ ($-\hat{s}_\bar{t}$) direction in the top rest-frame, and an anti-top with spin $+\hat{s}_\bar{t}$ ($-\hat{s}_t$) in the anti-top rest-frame.

In the *Generic spin basis*, the amplitudes $M_0(h_{-e^-}, h_{+e^+}, \hat{s}_t, \hat{s}_{\bar{t}})$ for the process $e^-e^+ \rightarrow t\bar{t}$ can be generally written as

$$M_0(- + t_\uparrow \bar{t}_\uparrow \text{ or } t_\downarrow \bar{t}_\downarrow) = \mp \left[ m_tA_L \sin \theta \cos \xi - (E_A \cos \theta + K_B \sin \theta) \sin \xi \right],$$

$$M_0(- + t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) = -(E_A \cos \theta + K_B \sin \theta) \cos \xi - m_tA_L \sin \theta \sin \xi$$

$$\mp (E_A + KB \cos \theta),$$

$$M_0(+ - t_\uparrow \bar{t}_\uparrow \text{ or } t_\downarrow \bar{t}_\downarrow) = \mp \left[ m_tA_R \sin \theta \cos \xi - (E_A \cos \theta - K_B \sin \theta) \sin \xi \right],$$

$$M_0(+ - t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) = -(E_A \cos \theta - K_B \sin \theta) \cos \xi - m_tA_R \sin \theta \sin \xi$$

$$\pm (E_A - KB \cos \theta),$$

1At NLC with $\sqrt{s} \geq 400$GeV, the imaginary part of the $Z$ propagator can be neglected safely.
where $K = (E^2 - m^2)^{1/2}$. The amplitudes in Helicity basis can be obtained easily by setting $\cos \xi = \pm 1$ in Eqs.(5)-(8). The form factors $A_{L,R}$ and $B_{L,R}$ are defined as

$$
A_{L,R} = \frac{4e^2 E}{s} \left( e_{L,R}^\gamma A_\gamma + e_{L,R}^Z A_Z(s) \right),
$$

(9)

$$
B_{L,R} = \frac{4e^2 E}{s} \left( e_{L,R}^\gamma B_\gamma + e_{L,R}^Z B_Z(s) \right).
$$

(10)

At tree level, the form factors $(A_V, B_V)$ ($V = \gamma, Z$) appeared in Eqs.(3),(9) and (10) are

$$
A_\gamma^0 = \frac{2}{3}, \quad B_\gamma^0 = 0, \quad A_Z^0 = \frac{3 - 8 \sin^2 \theta_W}{12 \sin \theta_W \cos \theta_W}, \quad B_Z^0 = \frac{1}{4 \sin \theta_W \cos \theta_W}.
$$

(11)

When we make a special choice for the angle $\xi$ in the Generic spin basis,

$$
\tan \xi = \frac{m_t A_L^0 \sin \theta}{EA_L^0 \cos \theta + KB_L^0},
$$

(12)

the Generic spin basis turns into the so-called Off-diagonal basis\[5\] for $e^-_L e^+_R$ scattering\[3\], then we will have

$$
\frac{d\sigma_0}{d \cos \theta}(- + t_\uparrow \bar{t}_\uparrow \text{ or } t_\downarrow \bar{t}_\downarrow) = 0
$$

(13)

$$
\frac{d\sigma_0}{d \cos \theta}(- + t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) = \left( \frac{\beta}{32\pi s} \right) |M_0(- + t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow)|^2
$$

(14)

$$
\frac{d\sigma_0}{d \cos \theta}(+ - t_\uparrow \bar{t}_\uparrow \text{ or } t_\downarrow \bar{t}_\downarrow) \approx 0
$$

(15)

$$
\frac{d\sigma_0}{d \cos \theta}(+ - t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) = \left( \frac{\beta}{32\pi s} \right) |M_0(+ - t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow)|^2
$$

(16)

where the subscript 0 means the tree level physical quantities.

At tree level, the form factors $A_V$ and $B_V$ ($V = \gamma, Z$) are very simple, and $C_V = D_V = 0$. When we consider the one-loop corrections, the form factors may become much more complicated. In general, including the one-loop corrections, the form factors can be written as

$$
A_V = A_V^0 + \delta A_V, \quad B_V = B_V^0 + \delta B_V
$$

(17)

where $\delta A_V$ and $\delta B_V$ represent the one-loop corrections.

\[2\]As pointed in ref.[3], there are two Off-diagonal basis for $e^-_L e^+_R$ and $e^-_R e^+_L$ scattering respectively. But these two Off-diagonal bases are almost identical[3] and we also only use the Off-diagonal basis for $e^-_L e^+_R$ defined by Eq.(12) even when discussing the case of $e^-_R e^+_L$ scattering.
At one-loop level, in the *Generic spin basis*, the general differential cross sections for \(e^-e^+\) scattering are
\[
\frac{d\sigma}{d\cos\theta}(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_i) = \frac{\beta}{32\pi s} |M_0(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_i) + \delta M(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_i)|^2, \tag{18}
\]
where \(M_0\) is the tree level amplitudes given in Eqs.(5)-(8) and the \(\delta M\) represents the one-loop contributions, which are given by
\[
\delta M(- + t_\downarrow \bar{t}_\downarrow \bar{t}_\downarrow) = \mp \left[ (m_t \delta A_L - K^2 C_L) \sin \theta \cos \xi \right.
\]
\[- (K \delta B_L + E \delta A_L \cos \theta) \sin \xi \sin \theta \right] - EKD_L \sin \theta, \tag{19}
\]
\[
\delta M(- + t_\downarrow \bar{t}_\downarrow \bar{t}_\uparrow) = -(m_t \delta A_L - K^2 C_L) \sin \theta \sin \xi - (K \delta B_L + E \delta A_L \cos \theta) \cos \xi \cos \theta \cos \theta
\]
\[
\mp (E \delta A_L + K \delta B_L \cos \theta), \tag{20}
\]
\[
\delta M(+ - t_\uparrow \bar{t}_\downarrow \bar{t}_\uparrow) = \mp \left[ (m_t \delta A_R - K^2 C_R) \sin \theta \sin \xi \right.
\]
\[- (E \delta A_R \cos \theta - K \delta B_R) \sin \xi \cos \theta \right] - EKD_R \sin \theta, \tag{21}
\]
\[
\delta M(+ - t_\uparrow \bar{t}_\downarrow \bar{t}_\uparrow) = -(m_t \delta A_R - K^2 C_R) \sin \theta \sin \xi - (E \delta A_R \cos \theta - K \delta B_R) \cos \xi \cos \theta \cos \theta
\]
\[
\mp (K \delta B_R \cos \theta - E \delta A_R). \tag{22}
\]
with
\[
\delta A_{L,R} = \frac{4e^2E}{s} \left( e^{\gamma}_{L,R} \delta A_{\gamma} + e^{Z}_{L,R} \delta A_{Z} R(s) \right), \tag{23}
\]
\[
\delta B_{L,R} = \frac{4e^2E}{s} \left( e^{\gamma}_{L,R} \delta B_{\gamma} + e^{Z}_{L,R} \delta B_{Z} R(s) \right), \tag{24}
\]
\[
C_{L,R} = \frac{4e^2E}{s} \left( e^{\gamma}_{L,R} C_{\gamma} + e^{Z}_{L,R} C_{Z} R(s) \right), \tag{25}
\]
\[
D_{L,R} = \frac{4e^2E}{s} \left( e^{\gamma}_{L,R} D_{\gamma} + e^{Z}_{L,R} D_{Z} R(s) \right), \tag{26}
\]
The above formulae are valid for the general cases corresponding to the vertex couplings as given in Eq.(3).

In what follows, we will calculate the \(O(\alpha_s)\) QCD corrections to the top quark pair production as an interesting example. The relevant Feynman diagrams include the QCD virtual corrections as well as the real bremsstrahlung graphs.

We will use dimensional regularization to regulate all the ultraviolet divergences in the virtual one-loop corrections. To regulate the infrared divergences, we introduce a gluon
mass parameter $\lambda$ in the gluon propagator, which is justified since the non-Abelian nature of QCD does not enter at the order in $\alpha_s$. We also adopt the on-shell mass renormalization scheme.

For $O(\alpha_s)$ QCD virtual corrections, the form factor $D_V (V = \gamma, Z)$ is still zero, while the non-zero factors $\delta A_V, \delta B_V$ and $C_V$ are

\[
\begin{align*}
\delta A_\gamma &= \frac{\alpha_s}{3\pi} A^0_\gamma U(\beta), \\
\delta A_Z &= \frac{\alpha_s}{3\pi} A^0_Z U(\beta), \\
\delta B_\gamma &= 0, \\
\delta B_Z &= \frac{\alpha_s}{3\pi} B^0_Z [U(\beta) - 2V(\beta)], \\
C_\gamma &= -\frac{\alpha_s}{3\pi m_t} C^0_\gamma V(\beta), \\
C_Z &= -\frac{\alpha_s}{3\pi m_t} A^0_Z V(\beta),
\end{align*}
\]

(27)

(28)

(29)

with

\[
U(\beta) = 1 + \frac{\beta^2}{\beta} \left[ Sp\left(\frac{2\beta}{\beta - 1}\right) - Sp\left(\frac{2\beta}{1 + \beta}\right) + \pi^2\right] - 3\beta \ln[X_t] - 4
+ \left[ \frac{1 + \beta^2}{\beta} \ln[X_t] + 2 \right] \ln\left[ \frac{m^2_t}{\lambda^2} \right],
\]

(30)

where $\beta = \sqrt{1 - 4m^2_t/s}$, $V(\beta) = (1 - \beta^2) \ln[X_t]/\beta$, $X_t = (1 - \beta)/(1 + \beta)$, and $Sp(z) = - \int_0^z \ln(1 - t)/t \, dt$ is the Spence function. Since we have neglected the width of $Z$ gauge boson, only the real parts of $\delta A_V, \delta B_V$ and $C_V$ will contribute to the corrections, and therefore we show only the real parts of them in Eqs. (27)-(29). In fact, the contributions from the imaginary parts of $\delta A_V, \delta B_V$ and $C_V$ are negligibly small even if we keep the imaginary parts of the $Z$ boson propagator, by our numerical calculations.

Although ultraviolet divergences have canceled in differential cross sections in Eq.(18), the infrared-divergent part is still present.

We regulate the infrared divergences associated with the soft real-gluon emission using the same gluon mass parameter $\lambda$. Under the soft gluon approximation (SGA), the amplitude induced by the soft real gluon emissions can be written in a factorized form proportional to the tree level amplitude. The corresponding differential cross sections therefore are the form of

\[
\frac{d\sigma_{\text{soft}}}{d\cos\theta}(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_t) = \eta_{\text{soft}} \frac{d\sigma_0}{d\cos\theta}(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_t)
\]

(31)

The factor $\eta_{\text{soft}}$ can be written as

\[
\eta_{\text{soft}} = G_{1R} + G_{\delta} + G_{\text{fin}}
\]

(32)
Here $G_{IR}$ represents the infrared divergent part, $G_\delta$ is the truncation part, and $G_{fin}$ is the finite part, and we have
\[
G_{IR} = \frac{4\alpha_s}{3\pi} \ln[\frac{\chi^2}{s}] \left(1 + \frac{s - 2m_t^2}{s\beta} \ln[X_i]\right), \quad (33)
\]
\[
G_\delta = -\frac{4\alpha_s}{3\pi} \ln[\frac{4\omega_\beta}{s}] \left(1 + \frac{s - 2m_t^2}{s\beta} \ln[X_i]\right), \quad (34)
\]
\[
G_{fin} = -\frac{4\alpha_s}{3\pi} \left\{ \frac{1}{\beta} \ln[X_i] + \frac{s - 2m_t^2}{s\beta} \left[2Sp(1 - X_i) + \frac{1}{2}\ln^2[X_i]\right]\right\} \quad (35)
\]
where $\omega_\beta$ is the fixed maximum gluon energy.

All the infrared divergence will cancel after adding the real corrections in Eq.(31) to the virtual corrections in Eq.(18), as they must. It is easy to see that the \textit{Off-diagonal basis} defined at tree level is still valid after the inclusion of QCD virtual corrections and soft real gluon emission contributions. With the same value of $\xi$ as given in Eq.(12), the $O(\alpha_s)$ QCD corrected differential cross sections in the \textit{Off-diagonal basis} are
\[
\frac{d\sigma}{d\cos\theta}(- + t_\uparrow t_\uparrow) = \frac{d\sigma}{d\cos\theta}(- + t_\downarrow t_\downarrow) = 0 \quad (36)
\]
\[
\frac{d\sigma}{d\cos\theta}(- + t_\uparrow t_\downarrow) = \frac{d\sigma_0}{d\cos\theta}(- + t_\uparrow t_\downarrow) \left\{ 1 + \frac{\alpha_s}{3\pi} \left[ X(\beta) + 2 \left( \frac{KV(\beta)}{M_0(- + t_\uparrow t_\downarrow)} \right) \right] \right\}, \quad (37)
\]
\[
\frac{d\sigma}{d\cos\theta}(+ - t_\uparrow t_\downarrow) = \frac{d\sigma_0}{d\cos\theta}(+ - t_\uparrow t_\downarrow) \left\{ 1 + \frac{\alpha_s}{3\pi} \left[ X(\beta) + 2 \left( \frac{KV(\beta)}{M_0(+ - t_\uparrow t_\downarrow)} \right) \right] \right\}, \quad (38)
\]
\[
\frac{d\sigma}{d\cos\theta}(+ - t_\downarrow t_\downarrow) = \frac{d\sigma_0}{d\cos\theta}(+ - t_\downarrow t_\downarrow) \approx 0 \quad (39)
\]
\[
\frac{d\sigma}{d\cos\theta}(+ - t_\uparrow t_\downarrow) = \frac{d\sigma_0}{d\cos\theta}(+ - t_\uparrow t_\downarrow) \left\{ 1 + \frac{\alpha_s}{3\pi} \left[ X(\beta) + 2 \left( \frac{KV(\beta)}{M_0(+ - t_\uparrow t_\downarrow)} \right) \right] \right\}, \quad (40)
\]
\[
\frac{d\sigma}{d\cos\theta}(+ - t_\downarrow t_\downarrow) = \frac{d\sigma_0}{d\cos\theta}(+ - t_\downarrow t_\downarrow) \left\{ 1 + \frac{\alpha_s}{3\pi} \left[ X(\beta) + 2 \left( \frac{KV(\beta)}{M_0(+ - t_\uparrow t_\downarrow)} \right) \right] \right\}, \quad (41)
\]
with
\[
X(\beta) = \frac{1 + \beta^2}{\beta} \left[ Sp\left( \frac{2\beta}{\beta - 1} \right) - Sp\left( \frac{2\beta}{1 + \beta} + \pi^2 \right) + 2 \left[ \frac{1 + \beta^2}{\beta} \ln[X_t] + 2 \right] \ln \frac{m_t^2}{4\omega_g^2} 
- \frac{4(1 + \beta^2)}{\beta} Sp(1 - X_t) - \frac{4 + 3\beta^2}{\beta} \ln[X_t] - \frac{1 + \beta^2}{\beta} \ln^2[X_t] - 4 \right].
\] (42)

From above analytical expressions, we can see that the one-loop QCD corrections do not change the spin configuration of top quark pairs. For $e_L^- e_R^+$ scattering, the contribution from the up-up (UU) and down-down (DD) top quark pairs vanish at the $O(\alpha_s)$ order in Off-diagonal spin basis. For the $e_R^- e_L^+$ scattering, the contributions from the UU and DD pairs of top quarks are also very close to zero. The cross sections in Helicity basis can be obtained easily by setting $\cos \xi = 1$ in the above formulae.

In the numerical calculation, we use the following parameters as standard input\[7\],
\[
\alpha_s(M_Z) = 0.118, \quad m_Z = 91.187\,\text{GeV}, \quad \sin^2 \theta_W = 0.2315,
\]
\[
\alpha = \frac{1}{128}, \quad m_t = 175\,\text{GeV}, \quad \Gamma_Z = 2.491\,\text{GeV}.
\] (43)

For the running of $\alpha_s$, we use $\sqrt{s}$ as the renormalization scale. For the infra-red cutoff $\omega_g$ we take $\omega_g = (\sqrt{s} - 2m_t)/5$ under the soft gluon approximation.

In the Off-diagonal spin basis, only one spin component is appreciably non-zero: the up-down ($t_\uparrow \bar{t}_\downarrow$) component for $e_L^- e^+$ scattering and the down-up ($t_\downarrow \bar{t}_\uparrow$) component for $e_R^- e^+$ scattering, as illustrated in Fig.1. In the Off-diagonal spin basis, we have
\[
\frac{d\sigma}{d\cos \theta}(e_L^- e^+ \rightarrow t_\uparrow \bar{t}_\downarrow) = \begin{cases} 
(0.23 \sim 0.70) \text{ pb, at tree level}, \\
(0.33 \sim 0.89) \text{ pb, with QCD correction}, \end{cases}
\] (44)

and
\[
\frac{d\sigma}{d\cos \theta}(e_R^- e^+ \rightarrow t_\downarrow \bar{t}_\uparrow) = \begin{cases} 
(0.09 \sim 0.33) \text{ pb, at tree level}, \\
(0.16 \sim 0.43) \text{ pb, with QCD correction}, \end{cases}
\] (45)

for $-1 \leq \cos \theta \leq 1$ and $\sqrt{s} = 400\,\text{GeV}$. Above results show that, the QCD corrections make the differential cross sections of dominant spin components larger by $\sim 30\%$ compared to the tree level ones for both $e_L^- e^+$ and $e_R^- e^+$ modes. For $\sqrt{s} = 1000\,\text{GeV}$, however, the increase of the corresponding differential cross sections induced by the inclusion of QCD corrections is less than 3\% as shown in Fig.2.
Fig. 3 shows the fractions of the total cross sections for the dominant spin components, defined in both the Helicity basis and the optimal *Off-diagonal basis* for $e^-_L e^+e^-$ scattering, as a function of the total energy $\sqrt{s}$. The fractions were defined as

$$R_{UD} = \frac{\sigma_{UD}}{\sigma_{UD} + \sigma_{DU}}, \quad (46)$$

$$R_{LR} = \frac{\sigma_{tL\bar{t}_R}}{\sigma_{tL\bar{t}_L} + \sigma_{tL\bar{t}_R} + \sigma_{tR\bar{t}_L} + \sigma_{tR\bar{t}_R}}. \quad (47)$$

In the *Off-diagonal spin basis*, the dominant spin component $t_L\bar{t}_L$ makes up the 99.8% (96.4%) of the total cross section for $\sqrt{s} = 400\,\text{GeV} (1\,\text{TeV})$ after including the $O(\alpha_s)$ QCD corrections. In the *Helicity basis*, however, the fraction for the dominant spin component $t_L\bar{t}_R$ is only 53% at $\sqrt{s} = 400\,\text{GeV}$, as shown by the lower two curves in Fig. 4. Although this fraction for $t_L\bar{t}_R$ will increase to $\sim 83\%$ at $\sqrt{s} = 1\,\text{TeV}$, it is still less than the corresponding ratio $R_{tL\bar{t}_L} = 96.4\%$ at $\sqrt{s} = 1\,\text{TeV}$. For the $e^-_R e^+e^-$ scattering we have similar results.

When defined in the optimal *Off-diagonal basis*, the spins of the $t$ and $\bar{t}$ pairs produced from polarized $e^-_L e^+$ and $e^-_R e^+$ scattering are uniquely determined, and this specific feature will not be changed after including the $O(\alpha_s)$ QCD corrections under the soft-gluon approximation. Because the contribution to spin correlations in the top quark pair production from the hard-gluon emissions is numerically very small, we here present only the results under the soft-gluon approximation.

For the sake of cross check, we also present the numerical results in the ordinary *Helicity basis*. Fig. 4 shows the differential cross sections for the process $e^-_L e^+ \to t_L, R \bar{t}$ and $e^-_R e^+ \to t_L, R \bar{t}$ in the *Helicity basis* with $\sqrt{s} = 400\,\text{GeV}$, and the corresponding differential cross sections are clearly well consistent with those shown in Fig. 8 of ref. [8].

To summarize, we have calculated the $O(\alpha_s)$ QCD corrections to the spin correlations in the top quark pair production at NLC in the SM. We start from the general forms of $Vt\bar{t}$ vertex ($V = \gamma, Z$), derive out the amplitudes in the *Generic spin basis*, and give the general formalism of including one-loop corrections to the differential cross section in top quark pair production at the NLC, and finally calculate the $O(\alpha_s)$ QCD corrections. We found that:
(a) In Off-diagonal basis, the $O(\alpha_s)$ QCD corrections to $e^+_Le^- (e^+_Re^-)$ scattering process increase the differential cross sections of the dominant spin component $t_\uparrow \bar{t}_\downarrow (t_\downarrow \bar{t}_\uparrow)$ by $\sim 30\%$ and $\sim (0.1\% - 3\%)$ depending on the scattering angle for $\sqrt{s} = 400\text{GeV}$ and $1\text{TeV}$, respectively.

(b) The Off-diagonal basis remains the optimal spin basis even after the inclusion of $O(\alpha_s)$ QCD corrections. At $\sqrt{s} = 400\text{GeV}$, the dominant spin components in both $e^-L e^+$ and $e^-R e^+$ scattering make up more than $99\%$ of the total cross section at both tree and one-loop level, but such fraction is only $\sim 53\%$ in the Helicity basis.

Note added. While preparing this manuscript the paper of J. Kodaira, T. Nasuno and S. Parke appeared where the QCD corrections to spin correlations in top quark production at $e^+e^-$ colliders is also calculated. Their numerical results under the soft gluon approximation are in very good agreement with ours.

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Figure Captions

Fig.1: The differential cross sections in the *Off-diagonal basis* for the $e^-_Le^+_R \rightarrow t\bar{t}$ processes UD ($t^\uparrow \bar{t}^\downarrow$), DU ($t^\downarrow \bar{t}^\uparrow$) and UU + DD ($t^\uparrow \bar{t}^\uparrow + t^\downarrow \bar{t}^\downarrow$), assuming $\sqrt{s} = 400\text{GeV}$ and $\omega_g = (\sqrt{s} - 2m_t)/5 = 10\text{GeV}$. The non-dominant $t^\downarrow \bar{t}^\uparrow$ ($t^\uparrow \bar{t}^\downarrow$) component for the $e^-_Le^+_R$ scattering is multiplied by a factor of 100.

Fig.2: The same as Fig.1 but for $\sqrt{s} = 1\text{TeV}$ and $\omega_g = (\sqrt{s} - 2m_t)/5 = 130\text{GeV}$. The non-dominant $t^\uparrow \bar{t}^\uparrow + t^\downarrow \bar{t}^\downarrow$ component for the $e^-_Re^+_R$ scattering is multiplied by a factor of 10.

Fig.3: The $\sqrt{s}$ dependence of fractions of total cross sections for dominant $t^\uparrow \bar{t}^\downarrow$ ($t^L\bar{t}^R$) component for $e^-_Le^+_L$ scattering. The two upper curves show the fractions in the *Off-diagonal basis*, while the two lower curves correspond to the fractions in the *Helicity basis*.

Fig.4: The differential cross sections in the *Helicity basis* for the $e^-_Le^+_L \rightarrow tL,R\bar{t}$ and $e^-_Re^+_R \rightarrow tL,R\bar{t}$ processes at $\sqrt{s} = 400\text{GeV}$, and assuming $\omega_g = (\sqrt{s} - 2m_t)/5 = 10\text{GeV}$. 


Off-Diagonal Basis

\[ \text{FIG. 1} \]
Off-Diagonal Basis

FIG. 2

\[ \frac{d\sigma}{d\cos\theta} (\text{pb}) \]

- - - - UD(TREE)
- - - - UD(QCD)
- - - - DU(TREE)
- - - - DU(QCD)
- - - - DD+UU(TREE)
- - - - DD+UU(QCD)
- - - - (UU+DD)*10(TREE)
- - - - (UU+DD)*10(QCD)
FIG. 3

Fraction of Cross Section

$S^{1/2}$ (GeV)
Helicity Basis

![Graph showing the dependence of \( \frac{d\sigma}{d\cos\theta} \) on \( \cos\theta \) for different helicity configurations. The graph compares the predictions from R(TREE), R(QCD), L(TREE), and L(QCD). The left and right sides of the graph correspond to different helicity configurations: \( e^-_L e^+ \) and \( e^-_R e^+ \).]