Heat transport in ultra-thin dielectric membranes and bridges

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Phonon modes and their dispersion relations in ultrathin homogenous dielectric membranes are calculated using elasticity theory. The approach differs from the previous ones by a rigorous account of the effect of the film surfaces on the modes with different polarizations. We compute the heat capacity of membranes and the heat conductivity of narrow bridges cut out of such membranes, in a temperature range where the dimensions have a strong influence on the results. In the high temperature regime we recover the three-dimensional bulk results. However, in the low temperature limit the heat capacity, $C_V$, is proportional with $T$ (temperature), while the heat conductivity, $\kappa$, of narrow bridges is proportional to $T^{3/2}$, leading to a thermal cut-off frequency $f_c = \kappa/C_V \propto T^{1/2}$.

INTRODUCTION

More and more precise measurements are needed in science (e.g. in astrophysics) and therefore very sensitive detectors are required. However, even if all the technological difficulties are removed and the detector components would be perfect, the thermal fluctuations and the discreteness of particle and energy fluxes through the detector would still limit the precision of the measurements. To achieve the desired sensitivity, the detectors have to be a very convenient design (see for example Refs. [1, 4, 5, 6]). SiN bridges have been investigated experimentally by several authors (as examples, see [7, 8, 9]). The thickness of such a membrane is of the order of hundreds of nm, which is very small as compared to its other dimensions. The membrane may also be cut, so that the detector lies on its central and wider part, which is connected to the bulk material by narrow bridges (See Fig. 1).

Thermal characteristics of SiN membranes and bridges have been investigated experimentally by several authors (as examples, see [10, 11]). SiN is an amorphous dielectric and, if we neglect here the dynamic fluctuations (see review [9] and references therein), its low temperature thermal properties are determined only by phonons. At temperatures below a few degrees Kelvin, the bulk material (assumed to be isotropic) is well described by the Debye model, with typical values for the longitudinal and transversal sound velocities $c_l = 6200$ m/s and $c_t = 10300$ m/s, respectively [8]. Hence, the heat capacity can be expressed as $C_V^{3D} = K_1 T^3$, where $T$ is the temperature and $K_1$ is a constant. If the mean free path of the phonons, $\ell$, is assumed to vary as $\ell \propto \lambda^{-s}$, where $\lambda$ is the phonon wavelength, the heat conductivity has the expression $\kappa^{3D} = K_2 T^{-3-s}$, where again $K_2$ is a material dependent constant (see for example Ref. [9]).

Note that for the above given values of sound velocity, and at temperatures below 1 K, the bulk dominant phonon wavelength, $\lambda_{dom} \approx \hbar c/(1.6k_B T)$ (see [9], Chap. 8), is of several hundreds of nanometers. Thus it is comparable with the smallest dimensions of the structures measured in Refs. [10, 11]. Typically in these experiments, the measurements made at the lower end of the temperature interval showed qualitative differences from the measurements at the upper end of the interval, due to finite-size effects.

In this paper we will improve the model of Refs. [10, 11] by calculating rigorously the phonon modes and their dispersion relations in ultrathin membranes and narrow bridges. We shall show the thermodynamical consequences of the calculations. Such calculations appear to be well known to those working in the field of elasticity theory (see for example Refs. [12, 13]), but apparently are not so familiar to the condensed matter community. Therefore, to make the paper more read-
able, we shall give some details of our calculations. To describe these effects, a simplified model was recently used\[10]. Namely, it was assumed that the phonon modes are quantized along the direction perpendicular to the membrane surface. In this way, the three-dimensional (3D) (quasi)continuous phonon spectrum splits into what one could call two-dimensional (2D) branches\[10]. At low enough temperatures, only the lowest branches are populated by phonons and the membrane is described as a 2D isotropic phonon gas. In this situation, if the mean free path of the phonons varies as $l \propto \lambda^{-s}$, the heat capacity has the form $C^{2D}_V = K_q T^2$ and the thermal conductivity is $\kappa^{2D} = K_4 T^{2-s}$, where $K_3$ and $K_4$ are material dependent constants. This model describes the change of the exponent of the temperature dependence of heat conductivity, as observed in Refs.\[1, 9, 11], and could fit relatively well the experimental data\[10]. On the other hand, the observation that in narrow bridges the exponent of the temperature dependence of the heat conductivity apparently converges to 1.5 as the width of the bridge decreases\[1, 3], as well as the increase of the ratio $\kappa/C_V$ with temperature\[3], cannot be explained in this model.

THE MODEL

In Refs.\[11, 13] it was considered simply that the longitudinal and transversal polarized phonon modes are independently quantized by Neumann boundary conditions imposed at the membrane surfaces. Nevertheless, in a rigorous analysis it must be taken into account that the modes with different polarization couple at a free surface\[12] and because of this, the phonon modes in thin membranes are “distorted” and show features that go beyond a simple treatment of the quasi 2D phonon gas.

For concreteness, we perform calculations for the practically important case of SiNx membranes with thickness $100 - 200\text{nm}$ having parallel surfaces. Other dimensions of the membranes are usually of the order of $100\text{\mu m}$. Consequently, the membranes can be considered as infinitely long and wide. It is further assumed that the material is isotropic. The eigenmodes of these kind of systems are well known from acoustics and are called Lamb waves\[12].

Similar to electricity theory, the physical acoustic fields can be expressed by a scalar and a vector potential, $\Phi$ and $\vec{\Psi}$. The velocity fields of the longitudinal $l$ and transversal $t$ phonon modes are then defined as $\vec{v}_l = \nabla \cdot \Phi$ and $\vec{v}_t = \nabla \times \vec{\Psi}$. Simple wave equations can be derived for the potentials, $\Delta \Phi = c_l^{-2} \partial^2_{\tau^2} \Phi$ and $\Delta \vec{\Psi} = c_t^{-2} \partial^2_{\tau^2} \vec{\Psi}$. Here $c_l$ and $c_t (\leq c_q)$ are the longitudinal and transversal sound velocities, respectively. In an infinite three-dimensional space this yields one longitudinal and two transversal plane wave solutions.

When however the system is restricted to finite size, the boundaries of the system lead to a coupling of longitudinal and transversal modes. This coupling is due to the boundary condition at a free surface: the total stress should vanish. If the wave incident on the surface is polarized along the $y$ direction (see Fig. 2a), the free boundary condition is satisfied if the reflected wave has the same polarization. Such a wave is called a horizontal shear wave ($h$-wave). The $h$-waves do not mix at the boundaries with waves of different polarizations and they form the typical “box eigenmodes”, with the dispersion relation

$$\frac{\omega^2_{h,n}}{c_t^2} = \left(\frac{n\pi}{b}\right)^2 + k^2_{||}. \quad (1)$$

Here $k_{||}$ is the component of the wave vector parallel to the membrane surface and $b$ is the thickness of the membrane. On the other hand, if the polarization of the incident wave is either longitudinal or transversal, but in the plane $xz$, then the reflected wave will always be a superposition of longitudinally and transversally polarized waves. These two waves have different propagation velocities, and the condition to get eigenmodes is that the plane waves reconstruct each other after two reflections. The longitudinal and transversal components of the eigenmode have the same wave vector component $k_{||}$ along the membrane surfaces, but different components perpendicular to the surfaces, which we shall call $k_{\perp}$ and $k'_{\perp}$, respectively. The frequency of the eigenmode is then $\omega^2 = c_l^2 (k^2) = c_l^2 (k')^2$. These equations give a relation between the components $k_{\perp}$ and $k'_{\perp}$. The eigenmodes obtained in this way fall into two classes: symmetric (s-wave) and antisymmetric (a-wave), according to the symmetry of the velocity field of the wave with respect to the plane $z = 0$. The dispersion relation of the $s$- and $a$-waves are given by the equations

$$\tan\left(\frac{\pi k_{\perp}}{2}ight) = -\frac{4k_l^4 k_{\perp}^2 k'^2_{\perp}}{[(k_{\perp})^2 - k_{||}^2]^2} \quad (2)$$

FIG. 2: Reflection of elastic waves at the free surfaces of the membrane of thickness $b$. A horizontal shear wave reflects also as a horizontal shear wave (a), while a wave longitudinally or transversally polarized into the $xz$ plane reflects as a superposition of longitudinally and transversally polarized waves (b).
and
\[
\frac{\tan(\frac{b}{2}k_{⊥}^4)}{\tan(\frac{b}{2}k_{∥}^4)} = -\frac{4k_{∥}^4 k_{⊥}^4 k_{∥}^2}{((k_{∥}^4)^2 - k_{∥}^2)^2},
\]
respectively. The equations (2) and (3), together with \(\omega^2 = c_s^2(k^4)^2 = c_t^2(k^4)^2\) form a set of transcendental equations, which can be solved only numerically. The branches of each dispersion relation are shown in Fig. 3. As can be seen, the \(h\)-modes are just usual “box modes”. Without mixing at the membrane surfaces, all the modes would be of this type and the results would be identical to the ones of Refs. 10, 11. However, due to the coupling of the longitudinal and transversal modes, the dispersion relations of the \(a\)- and \(s\)-modes show some interesting properties.

At \(k_{∥} = 0\), all the excited branches satisfy the relation \(\partial \omega / \partial k_{∥} = 0\), but for the \(s\)- and \(a\)-modes, unlike for the \(h\)-modes, \(\partial^2 \omega / \partial k_{∥}^2\) may be either positive or negative. If \(\partial^2 \omega / \partial k_{∥}^2 < 0\), the dispersion curve has a minimum at some value \(k_{∥} > 0\). The lowest branch of the \(h\)-modes is a straight line,
\[
\omega_{h,0} = c_t k_{∥}
\]
but not for the \(s\)- and \(a\)-modes. For the \(s\)-modes, \(\partial \omega_{s,0} / \partial k_{∥} > 0\), so the group velocity of long wavelength \(s\)-phonons is different from zero. On the other hand, for the \(a\)-modes \(\partial \omega_{a,0} / \partial k_{∥} = 0\) and \(\partial^2 \omega_{a,0} / \partial k_{∥}^2 > 0\). Therefore the group velocity of the \(a\)-modes is zero at long wavelength and from this point of view the \(a\)-phonons are similar to massive particles.

Since the low temperature thermal properties of the membranes are determined by the dispersion relations of the lowest branches at long wavelengths, we shall take a closer look at these.

**Lowest branch of the symmetric modes:** If \((b/2)k_{∥}\) converges to zero, the solutions \((b/2)k_{∥}\) and \((b/2)k_{∥}\) of Eq. (4), and satisfying \(c_s^2(k^4)^2 = c_t^2(k^4)^2\) approach also zero, in such a way that \(k_{∥}^4\) is real, while \(k_{⊥}^4 \equiv i\tilde{k}_{⊥}^4\) is imaginary. Then Eq. (2) reduces to
\[
\tan(\frac{b}{2}k_{⊥}^4) \approx \frac{k_{∥}^4 k_{∥}^4 k_{∥}^2}{((k_{∥}^4)^2 - k_{∥}^2)^2}. \tag{5}
\]
The solution \(k_{∥}^4 = 0\) is unphysical (does not satisfy the boundary conditions if plugged into the stress formulae), so the only solution is \((k_{∥}^4)^2 - k_{∥}^2 = 4(\tilde{k}_{⊥}^4)^2 k_{∥}^2\), which yields the dispersion relation
\[
\omega_{s,0} = 2c_s \sqrt{c_t^2 - c_s^2} k_{∥} \equiv c_s k_{∥}. \tag{6}
\]
So the dispersion relation is linear in the long wavelength limit. The group velocity of the \(s\)-modes is \(c_s\), which, since \(0.5 \leq 1 - c_s^2/c_t^2 < 1\) [14], takes values between \(\sqrt{2}c_t\) and \(2c_t\).

Lowest branch of the antisymmetric modes: In the case of antisymmetric modes, if \((b/2)k_{∥}\) converges to zero, then both \((b/2)k_{∥}\) and \((b/2)k_{∥}\) converge to zero, but taking imaginary values: \(k_{∥} \equiv ik_{∥}\) and \(k_{∥} \equiv ik_{∥}\), leads to a valid solution [14]. Expanding Eq. (3) and using the equation \(\omega^2 = c_s^2(k^4)^2 = c_t^2(k^4)^2\), we get the quadratic dispersion relation
\[
\omega_{a,0} = \frac{\hbar}{2m^*} k_{∥}^2, \tag{7}
\]
as for massive particles of “effective mass”
\[
m^* = \hbar \left[ 2c_t \sqrt{(c_t^2 - c_s^2)/3c_t^2} \right]^{-1}. \tag{8}
\]
A plot of the lowest branches of the \(h\)-, \(s\)- and \(a\)-modes in the long wavelength limit is shown in Fig. 4.

**THERMODYNAMICS**

**Heat capacity**

The qualitative difference between the dispersion relations of the \(a\)-modes, on one hand, and the \(h\)- and \(s\)-modes, on the other hand (see Eqs. 4, 6 and 7), have important consequences on the thermodynamic properties of the membrane. To show this, let us first calculate the heat capacity of the membrane. Phonons obey Bose statistics, so the average population of each phonon mode is \(n(\omega) = 1/\exp(\beta \hbar \omega) - 1\). If we integrate over \(k_{∥}\) and sum-up the contributions of all the branches, we arrive at the expression
\[
C_V = \frac{A}{kB^2 2\pi} \sum_\sigma \sum_{m=0}^{\infty} \int_{k_{∥}^-}^{k_{∥}^+} dk_{∥} \frac{k_{∥}^4 (\hbar \omega_{m,\sigma})^2 \exp(\beta \hbar \omega_{m,\sigma})}{(\exp(\beta \hbar \omega_{m,\sigma}) - 1)^2}, \tag{9}
\]
where \(\sigma\) represents the \(h\)-, \(s\)-, and \(a\)-modes, while \(\sum_m\) is the summation over the branches. The frequencies \(\omega_{m,\sigma}\) depend also on \(k_{∥}\); \(A\) is the area of the membrane.

For uncoupled longitudinal and transversal modes, the dispersion laws have the form \(\omega = c_s \sqrt{(\pi m^2/\hbar^2) + k_{∥}^2}\). In that case the 3D-to-2D crossover in the phonon gas would manifest itself through a relatively rapid change of the temperature dependence from \(T^3\) to \(T^2\) at the temperature \(T_c \equiv \hbar c_t/2k_B\), as seen in Fig. 4(a) [10]. The corresponding asymptotic temperature dependences of the heat capacity, following from Eq. (9), are
\[
C_V \approx \begin{cases} \eta_1 T^3, & T \gg T_c, \\ \eta_2 T^2, & T \ll T_c. \end{cases}
\]
where
\[
\eta_1 = \frac{4\pi V k_{∥}^4 \Gamma(5) \zeta(4)}{(2\pi c_3 h)^{-3}} \quad \text{and} \quad \eta_2 = \frac{\pi A k_{∥}^4 \zeta(3)}{(2\pi c_2 h)^2}.
\]
progress.) In the high temperature limit, the expected
for such thin films; more detailed calculations are in
dynamical defects to the specific heat are unimportant
of the
shown in the limit of long wavelengths. The linear behaviour
of the phonon eigenmodes of a free standing thin membrane
FIG. 3: The branches of the dispersion relations of the phonon eigenmodes of a free standing thin membrane. (a) h-modes, (b) s-modes and (c) a-modes
complicated, converging finally to 1, as \( T \to 0 \) (see Fig. b).

Heat conductivity

A common way to increase thermal insulation of the
detector mounted on the membrane is to cut the mem-
brane so that the central part is connected to the bulk
material only by narrow bridges (see Fig. 4). Therefore,
another quantity of interest is the thermal conductivity
(\( \kappa \)) along the membrane and bridges. Reported widths
of the bridges of interest for us are roughly from 4 \( \mu \text{m} \)
upwards \([1, 3]\) (We do not discuss here the “dielectric
quantum wires”, like in Ref. \([15]\)). As the width of
the bridge becomes smaller than the phonon mean free
path in the uncut membrane, the interaction of phonons
with the bridge edges should become the main scatter-
ning mechanism. It is very difficult to solve this problem
either analytically or numerically for the most general
case. Instead, we shall try to extract the relevant physi-
ological results for our problem, using a realistic model. The
cutting process leaves the bridge edges very rough, so we
shall assume that the phonons scatter diffusively at the
gates, i.e. scattered phonons are uniformly distributed
over the angles and branches corresponding to the same
frequency, \( \omega \).

The general expression for the heat current along the
rectangular bridge of total length \( l \) is

\[
\dot{Q} = -\frac{1}{\ell} \sum_{\sigma,m,k} \hbar \omega(k) \tau(k) c_{\sigma}^2(k) \frac{\partial n(\omega)}{\partial T} \frac{\partial T}{\partial x}
\]  

(11)

where \( \tau(k) \) is the (average) scattering time of a phonon
having wavevector \( k \) parallel to the membrane surface
and belonging to the branch \( (\sigma,m) \). To simplify the writ-
ing, the dependence on \( \sigma \) and \( m \) of the quantities in Eq. \( \textbf{[11]} \) was made implicit. The bridge lies along the \( x \) direction,
the \( z \) direction is perpendicular to the membrane. We
denoted by \( c_{\sigma,n}(k) \equiv \partial \omega_{\sigma,n}/\partial k \) the group velocity
of the phonons. We also assumed that \( \partial T/\partial x \) is not
too large, so that the linear approximation, \( \dot{Q} \propto \partial T/\partial x \),
holds. Let us now denote the scattering time of the

Here, \( 3/c_3^3 = 2/c_1^3 + 1/c_1^3 \), \( 3/c_2^3 = 2/c_2^3 + 1/c_2^3 \), and
\( V \) is the volume of the membrane. The exponent of
the temperature dependence of the heat capacity, \( p_C \equiv \partial \ln C_v/\partial \ln T \), reflects the dimensionality of the phonon
gas distribution: \( p_C(T \gg T_c) = 3 \) and \( p_C(T \ll T_c) = 2 \)
\([10, 11]\).

In the more rigorous case, i.e. when the dispersion
relations are given by Eqs. \([11]\), \([9]\), and \([7]\), due to the
quadratic dispersion relation of the lowest \( a \)-mode, we get
a different temperature behaviour for \( T \ll T_c \). Summing
only over the three lowest branches, we get

\[
C_v \approx \alpha k_B \left( \alpha T^2 + \beta T \right)
\]

\[
\alpha = \frac{3 \zeta(3) k_B^2}{\pi \hbar^2} \left( \frac{1}{c_1^2} + \frac{1}{c_2^2} \right), \quad \beta = \frac{\zeta(2) k_B m^*}{\pi \hbar^2}.
\]  

(10)

(Preliminary estimates show that the contribution of
dynamical defects to the specific heat are unimportant
for such thin films; more detailed calculations are in
progress.) In the high temperature limit, the expected
\( T^3 \) behaviour is obtained, but at temperatures around
\( T_c \), the temperature dependence of \( p_C \) is somewhat more

FIG. 4: The three lowest branches of the dispersion relations of the phonon eigenmodes of a free standing thin membrane. (a) h-modes, (b) s-modes and (c) a-modes
FIG. 5: The temperature exponents \( p_f \equiv \partial \ln f / \partial \ln T \) of the heat capacity \((f \equiv C)\) and heat conductivity \((f \equiv \kappa)\) in narrow bridges of thin films. The insets show the behaviour of the curves for small values of \( T/T_C \). (a) The temperature exponent of \( C_V \), as it would be if the modes were not coupled. The dimensionality-crossover around \( T_C = (\hbar \Delta / 2k_B) \cdot 6^{-1} \) can be seen here quite nicely. For a 100nm thin membrane the critical temperature of SiN\(_x\) is 237 mK. (b) The temperature exponent of the heat capacity of a thin membrane or a narrow bridge. (c) The temperature exponent of the heat conductivity along a narrow bridge.

phonons in the uncut membrane by \( \tau_{M,\sigma,m}(k_\|) \), while the scattering time at the bridge edges is \( \tau_{E,\sigma,m}(k_\|) \). The effective scattering time, \( \tau_{\sigma,m}(k_\|) \), is then

\[
\tau_{\sigma,m}(k_\|) = \tau_{M,\sigma,m}(k_\|) + \tau_{E,\sigma,m}(k_\|). 
\]

Under the assumption of diffusive scattering at the bridge edges, \( \tau_{E,\sigma,m}(k_\|) \) can be estimated as \( \tau_{E,\sigma,m}(k_\|) = w/(\nu_{\sigma,n}(k_\|) \sin \vartheta) \), where \( \vartheta \) is the angle between \( k_\| \) and the \( x \)-direction, and \( w \) is the bridge width. If we transform the summation over \( k_\| \) in Eq. (11) into an integral and write \( \dot{Q} = -\kappa \cdot \partial T / \partial x \), we get

\[
\kappa = \frac{w}{2\pi} \sum_{\sigma,m} \int_0^{2\pi} \int_0^{\infty} dk_\| k_\| \hbar \omega \frac{u^2 \cos^2(\vartheta)}{u \sin(\vartheta)} \frac{\partial n}{\partial T} + \frac{1}{\tau_M} \partial T \\
= \frac{w^2}{2\pi} \sum_{\sigma,m} \int_0^{2\pi} dk_\| k_\| \hbar \omega \frac{\partial n}{\partial T} \int_0^{\infty} \frac{\cos^2(\vartheta)}{\sin(\vartheta)} + \frac{1}{\tau_M} \partial T 
\]

so in the limit of low temperatures \( \kappa \propto T^{3/2} \). The numerical results are shown in Fig. 5 (c).

If a membrane, which is connected to the bulk material by narrow bridges, is heated by an AC current, its thermal cut-off frequency has the expression \( f_c = G/C_V \equiv k/(\nu_{\sigma,m} \nu_{\sigma,n}) \), where \( \kappa \) is the thermal conductivity of the bridge and \( C_V \) is the heat capacity of the membrane. If all the modes are of the form \( \nu_{\sigma,m} = \nu_{\sigma,n} \cdot \tau_{M,\sigma,m}(k_\|) \) is the mean free path corresponding to \( \tau_{M,\sigma,m}(k_\|) \). Denoting \( a = w/\ell_M \) we can write the integral over \( \vartheta \) as

\[
C(a) \equiv \int_0^{2\pi} \frac{\cos^2(\vartheta)}{\sin(\vartheta)} + a \frac{\partial n}{\partial T} + a \frac{1}{\tau_M} \partial T \\
= 4 \left[ \frac{a\pi}{2} - 1 + \sqrt{1 - a^2} \log \frac{1 + \sqrt{1 - a^2}}{a} \right] 
\]

If edge-scattering dominates, i.e. \( w \ll \ell_M \), the integral only depends logarithmically on \( a \)

\[
C(k_\|) \approx 4 \log \left( \frac{2}{\omega} \right). 
\]

CONCLUSIONS

In summary, we used elasticity theory to calculate the phonon modes in ultrathin membranes made of homogeneous, isotropic silicon nitride (SiN\(_x\)). Using the dispersion relations thus obtained, we calculated the heat capacity and heat conductivity of the membrane and of bridges, cut out of such membranes. In the low temperature limit the phonon gas becomes two-dimensional
FIG. 6: The cut-off frequency of an AC-heated membrane, which is connected to the bulk material by narrow bridges. Here we normalized the function to its maximum value, so that \( f'_c = f_c/f_{c,0} \propto \kappa/(TC) \). For small values of \( T/TC \) \( f'_c \) becomes linear, which is shown in the inset.

and populates three branches of the dispersion relations, namely the lowest \( k_\parallel \), \( s \)-, and \( a \)-branches (see Fig. 3). At low temperatures, the dispersion relation corresponding to the lowest \( a \)-branch is quadratic in \( k_\parallel \), while the other two are linear, so at low temperatures the \( a \)-branch gives the dominant contribution. Quite surprisingly, this implies that the universal behavior of heat capacity in two-dimensional systems is obeyed also by the phonon gas at low temperatures, where \( C_V \propto T \) (see Ref. [16] and citations therein).

In the calculation of thermal conductivity along bridges, we assumed diffusive scattering of phonons at the bridge edges. This is justified by the fact that the cutting process leaves these edges very rough. If the width of the bridge decreases below a certain value (depending on the mean free path of the phonons in the uncut membrane) the interaction with the edges becomes the main scattering process for the phonons, see Eq. (12). In such a case, at low temperatures it was found that \( \kappa \propto T^{3/2} \) (Eq. 13).

If a membrane, connected to the bulk material by narrow bridges, is heated by an AC current, the amplitude of the temperature oscillations in the membrane has a cut-off around the frequency \( f_c = G/C_V \). In the low temperature limit, this cut-off frequency shows an increase with the temperature, as \( T^{1/2} \). Preliminary experimental results show an increase of \( f_c \) with \( T \), but on a temperature range much wider than the one in Fig. 6. This seems to suggest that at higher temperatures other processes have to be taken into account in the calculation of thermal characteristics of SiN\(_x\) membranes.

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