Three-body hadronic structure of low-lying \(1/2^+\) Σ and \(Λ\) resonances

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Abstract. We discuss the dynamical generation of some low-lying \(1/2^+\) Σ’s and \(Λ\)’s in two-meson one-baryon systems. These systems have been constructed by adding a pion in the \(S\)-wave to the \(KN\) pair and its coupled channels, where the \(1/2^-\) \(\Lambda(1405)-\)resonance gets dynamically generated. We solve Faddeev equations in the coupled-channel approach to calculate the \(T\)-matrix for these systems as a function of the total energy and the invariant mass of one of the meson-baryon pairs. This squared \(T\)-matrix shows peaks at the energies very close to the masses of the strangeness \(-1\), \(1/2^+\) resonances listed in the particle data book.

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1 Introduction

Several claims of excitation of a narrow resonance at \(\sim 1540\) MeV were made in the \(pK^0\) final state in different experiments [1], which were performed to confirm the existence of the pentaquark state, \(\Theta^+\) [2]. The \(pK^0\) system could possess a total strangeness +1 or −1 and, hence, these states could very well correspond to, for example, a \(\Sigma^+\) [3,4]. Actually, the existing \(\Sigma\) and \(\Lambda\) resonances [5], in this energy region, are not well understood. The poor status of these low-lying, \(S = −1\) states is evident from the following facts: a) The spin-parity assignment for many of these states is unknown, e.g., for \(\Sigma(1480), \Sigma(1560), \) etc., b) the partial-wave analyses and production experiments have been often kept separately in the particle data group listings, e.g., for \(\Sigma(1620), \Sigma(1670), \) etc. other times, e.g., in case of \(\Lambda(1600), \) it is stated that existence of two resonances, in this energy region, is quite possible [5].

There are hints for some of them, like the \(\Lambda(1600)\) and the \(\Sigma(1660), \) to decay to three-body final states like \(\pi^0\pi^0\Lambda\) and \(\pi^0\pi^0\Sigma, \) etc. [6,7]. This indicates that the information of similar three-body contributions to the wave functions of the states, in this energy region, could be important and that this, so far unknown, information could lead to a better understanding of the low-lying \(\Sigma\) and \(\Lambda\) resonances. The \(\pi-\Sigma\) interaction has been studied extensively in chiral unitary dynamics, where the \(\Lambda(1405)\) gets dynamically generated (with a two-pole structure [8]). Is it possible that adding another pion to this strongly attractive system could give rise to a three-body bound state? We have carried out a study of such systems for total spin-parity \(J^P = 1/2^+\) and, indeed, find evidence for four \(\Sigma\) and two \(\Lambda\) known resonances [5] to have a three body, i.e., two-meson one-baryon, structure, which we discuss in the following text. The chiral dynamics has been used earlier in the context of the three-nucleon problems, e.g., in [9]. This is the first time when the chiral dynamics is applied to solve the Faddeev equations for two-meson one-baryon systems.

2 Formalism

We solve Faddeev equations in the coupled-channel approach. These coupled systems have been constructed by pairing up all the possible pseudoscalar mesons and \(1/2^+\) baryons, which couple to strangeness \(=−1\) and by adding a pion to the pair finally. We end up with twenty-two coupled channels for a fixed total charge [10]. In order to generate dynamically the \(\Lambda(1405) \) \(S_{01} \) \((J^P = 1/2^-)\) in the meson-baryon sub-systems, all the interactions have been written in the \(S\)-wave, which implies that the total \(J^P\) of the three-body system is \(1/2^+\).

The solution of the Faddeev equations,

\[ T = T^1 + T^2 + T^3; \]
where the $T^i$ partitions are written in terms of two-body $t$-matrices ($t^i$) and three-body propagators ($g = [E - H]^{-1}$) as

$$T^i = t^i + t^i g [T^j + T^k] \quad (i \neq j, i, j \neq k = 1, 2, 3),$$  

(2)

requires off-shell two-body $t$-matrices as an input. However, we found that the off-shell contributions of the two-body $t$-matrices give rise to three-body forces which, when summed up for different diagrams, cancel the one arising due to the three-body forces. The three-body forces correspond to the integral, which could simplify the calculations. The $t^i g [T^j + T^k]$ term can be now extracted out of the loop integral since it depends on the on-shell variables. The off-shell dependence of $t^i$ and $g^{ij}$ has been included in the terms enclosed in the curly brackets, which we define as $F^{213}$ and name as off-shell factor (which should not be confused with the off-shell “form-factor”, often used in the literature. The off-shell factor, here, is just the nomenclature for a mathematical expression which retains the off-shellness of the terms which are extracted out of the integral). The integral is now left with two propagators and the off-shell factor $F^{213}$, which together we re-express and define as a loop function, $G^{213}$,

$$G^{213} = \int \frac{dk}{(2\pi)^3} \frac{1}{2E_1(k')} \frac{1}{2E_2(k'' + k_3)} \frac{2M_3}{2E_3(k'' + k'_2)} \times \left[ \frac{1}{\sqrt{s - E_1(k'')}} \delta - \frac{1}{E_2(k'' + k_3)} + \delta - \frac{1}{E_{13}(k'_2 + k'_3)} + i\epsilon \right] \left[ \frac{1}{\sqrt{s - E_3(k''')}} \delta - \frac{1}{E_3(k'_2 + k'_3)} + \delta - \frac{1}{E_{13}(k'' + k'_3)} + i\epsilon \right],$$

(3)

The diagram in fig. 2 can, hence, be re-written as $t^i g^{ij} t^j$. The formalism has been developed following the above procedure, i.e., by replacing $g$ by $G$, every time a new interaction is added. This leads to another form of the Faddeev partitions (eq. (2)), which, after removing the terms corresponding to the disconnected diagrams and by denoting the rest of the equation as $T_R$, can be re-written as

$$T_R^{ij} = t^i g^{ij} t^j + t^i G^{ijk} T_R^{jk} + t^i G^{ijkl} T_R^{jl} \quad i \neq j, \quad j \neq k = 1, 2, 3,$$

(4)

where $s_{13} = (P - K')^2$, $s_{12} = (P - K_3)^2$, $s_{23} = (k'')^2$, $s_{123} = (P - K'')^2$ with $P, K_3, k'_2$ and $k''$ representing the four momenta corresponding to the $P = 0, k_3, k'_2$ and $k''$, respectively. Our aim is to extract $t^i g^{ij} t^j$ out of the integral, which could simplify the calculations. The $t^2(s_{13})$ and $t^i(s_{12})$, in the equation above, depend on on-shell variables and can be factorized out of the loop integral but not $t^i$ and $g^{ij}$ [10]. This can be done if we re-arrange the loop integral as

$$t^2(s_{13}) \int \frac{dk''}{(2\pi)^3} \frac{1}{2E_1(k'')} \frac{1}{2E_3(k'' + k'_2)} \times \left[ \frac{1}{\sqrt{s - E_1(k'')}} \delta - \frac{1}{E_2(k'' + k_3)} - \frac{1}{E_{13}(k'_2 + k'_3)} + i\epsilon \right] \left[ \frac{1}{\sqrt{s - E_3(k''')}} \delta - \frac{1}{E_3(k'_2 + k'_3)} + \delta - \frac{1}{E_{13}(k'' + k'_3)} + i\epsilon \right],$$

(5)

The $T^j_R$ can be related to the Faddeev partitions (eq. (2)) as $T^i = t^i + T^j_R + T^j_R$, hence, giving six coupled equations instead of three (eq. (2)). These $T^j_R$ partitions correspond to the sum of all the possible diagrams with the last two interactions written in terms of $t^i$ and $t^j$. This re-grouping of diagrams is done for the sake of convenience due to the different forms of the $G^{ijkl}$-functions. We define $T_R$ as

$$T_R = \sum_{i \neq j = 1}^{3} T^j_R,$$