Loss of superfluidity in a Bose-Einstein condensate via forced resonant oscillations

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Abstract

We predict the loss of superfluidity in a Bose-Einstein condensate in an axially symmetric harmonic trap alone during resonant collective oscillations via a classical dynamical transition. The forced resonant oscillation can be initiated by (a) a periodic modulation of the atomic scattering length with a frequency that equals twice the radial trapping frequency or multiples thereof, or by (b) a periodic modulation of the radial trapping potential with a frequency that equals the radial trapping frequency or multiples thereof. Suggestion for future experiment is made.

Key words: Bose-Einstein condensation, Superfluid-insulator transition
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The first experimental observation of quantum phase effects on a macroscopic scale such as interference of matter waves [1] was made in a Bose-Einstein condensate (BEC) in an axially-symmetric harmonic trap. Later on more controlled studies of interference of matter waves were performed with BEC loaded on a harmonic plus an one-dimensional optical lattice trap [2,3,4,5,6,7]. More recently, a three-dimensional optical lattice trap has been employed by Greiner et al. [8,9,10] in the study of the formation of an interference pattern. The formation of the interference pattern is a consequence of phase coherence in the BEC across the optical lattice sites.

In the experiments with optical lattice, phase coherence in the BEC is generated by quantum tunneling of atoms from one optical lattice site to another leading to a communication among various sites. As the strength of the optical lattice barrier is larger than the typical energy of the system by about two orders of magnitude, classical movement of uncondensed cold atoms is prohibited across the optical lattice barriers. It is the atoms of the BEC which

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experience this miraculous flow through the high barriers, which is a manifestation of superfluidity. More surprising is the observation that all superfluids are necessarily condensates, but all condensates are not superfluids. There have been both theoretical and experimental studies of breakdown of superfluidity in BEC via quantum [8,11] and classical [12,13,14,15] transitions.

As superfluidity is necessarily a quantum phenomenon the experimental consideration of Greiner et al. [8,9,11] on the loss of superfluidity in a BEC trapped on a three-dimensional optical lattice potential via a quantum phase transition is worth mentioning. The long-range phase coherence in the condensate along the entire optical lattice is a sign of communication among various sites which is necessary for developing superfluidity in the condensate. Equal phase at all points or a slowly (and orderly) varying phase are the ideal examples of coherent phase. On the other hand, a rapidly (and randomly) varying phase in space is generally incoherent. The phase on an optical lattice site and the number of atoms in that site play the roles of conjugate variables obeying the Heisenberg uncertainty principle of quantum mechanics [9]. In the superfluid state the coherent phase is considered to be known and consequently the number of atoms on each site is unknown thus allowing a free movement of atoms from one site to another [8]. The loss of phase coherence is associated with a Mott insulator state where the phase is entirely arbitrary across the optical lattice sites and the number of atoms at each site is fixed and consequently, their free passage from one site to another is stopped.

Greiner et al. [8,11] demonstrated that, as the strength of the optical lattice traps is increased, the quantum tunneling of atoms from one optical site to another as well as the communication among different optical lattice sites are stopped resulting in a superfluid to Mott insulator quantum phase transition in the BEC [8]. As the strength of the optical lattice traps in the Mott insulator state is reduced the superfluidity is restored in a short time [8,11]. This reversible quantum phase transition may occur at absolute zero (0 K) and is driven by Heisenberg’s uncertainty principle [8] and not by thermal fluctuations involving energy as in a classical phase transition. As the temperature approaches absolute zero all thermal fluctuations die out and at 0 K classical phase transitions are necessarily excluded.

Following a suggestion by Smerzi et al. [12], Cataliotti et al. [13] demonstrated in a novel experiment the loss of superfluidity in a BEC trapped in an one-dimensional optical-lattice and harmonic potentials when the center of the harmonic potential is suddenly displaced along the optical lattice through a distance larger than a critical value. Then a modulational instability of classical nature takes place in the BEC. Consequently, it cannot reorganize itself quickly enough via phase coherent states and the superfluidity of the BEC is lost. Later on, it has been suggested that superfluidity could be lost in a resonant collective oscillation of a BEC on an one-dimensional optical
lattice potential arising from a periodic modulation of the scattering length [14] or a periodic modulation of the radial trapping potential [15].

All the above studies on the destruction of superfluidity in a BEC were performed with an optical lattice trap and the loss of superfluidity in both classical and quantum cases has been traced to the fixing of a specific atom(s) at a definite lattice site with no mobility [8,9,12]. An interesting question is if, in case of the loss of superfluidity via the classical transitions above [14,15], the forced tunneling of the atoms through the optical lattice barriers plays a fundamental role. We find that the answer to this question is negative and demonstrate that the superfluidity can also be destroyed in a classical resonant oscillation of a BEC in a harmonic trap alone without any accompanying optical lattice trap. In view of comments in the literature [4,8,9] this is surprising. Effectively, the optical lattice is found to have no effect on the loss of mobility of the atoms associated with quantum fluctuations due to Heisenberg’s uncertainty principle.

In the present study the forced classical resonant oscillation of the BEC is initiated by a periodic modulation of the scattering length or the radial trapping potential. A periodic modulation of the scattering length [16] or the radial trapping potential [17,18] is known to lead to collective resonant oscillation in an axially-symmetric BEC. In previous studies we demonstrated that such forced oscillations in the presence of a joint harmonic and optical traps lead to a breakdown of superfluidity [14,15]. A similar breakdown of superfluidity of a BEC in a harmonic trap alone reveals that the optical lattice trap does not play a decisive role in the loss of superfluidity via collective resonant oscillation. The optical lattice trap is, however, fundamental in the formation of the interference pattern upon release from the traps, which plays a decisive role in the detection of superfluidity in these studies. The strong optical lattice potential essentially cuts the BEC into phase coherent pieces upon release from the joint traps which is fundamental in the formation of the interference pattern.

Another way of detection of phase coherence and superfluidity of a BEC is by cutting it into phase coherent pieces using a laser as in the experiment by Andrews et al. [1]. Upon free expansion these pieces form an interference pattern. If the BEC is cut into two pieces the interference pattern consists of a large number of dark (absence of matter) and bright (matter) patches [1]. However, we show that if the initial BEC is cut into many equal coherent pieces as in the experiment with optical lattice, only three prominent bright spots can be generated. In the present numerical simulation with an axially symmetric harmonic trap, the initial wave function $\psi$ of the BEC is cut into many equal pieces by setting $\psi(\rho, z = \pm jd) = 0$ with $j = 0, 1, 2, 3,$...etc. Here $d$ is the spacing in the axial $z$ direction and $\rho$ corresponds to the radial direction. Subsequently, for a sufficiently small $d (\sim 0.1 - 1\mu m)$, upon release
from the trap such a coherent fragmented BEC will form an interference pattern of three bright spots.

We consider here two ways of initiating the collective resonant oscillation of the BEC. First, it is initiated near a Feshbach resonance \[19\] by a periodic modulation of the repulsive atomic scattering length \( a > 0 \) via \( a \rightarrow a + \tilde{A} \sin(\Omega t) \) where \( t \) is time, \( \tilde{A} \) an amplitude, and \( \Omega \) the frequency of modulation. Such modulation of the scattering length can be realized experimentally near a Feshbach resonance by manipulating an external background magnetic field. When \( \Omega = 2\omega \) or multiples thereof, resonant collective oscillation can be generated in the BEC, where \( \omega \) is the radial trapping frequency \[16\]. This resonant oscillation destroys the superfluidity of the BEC provided that the condensate is allowed to experience this oscillation for a certain interval of time called hold time.

Next we generate the resonant collective oscillation by a periodic modulation of the radial trapping potential \( V_\rho \) in the axially symmetric BEC via \( V_\rho \rightarrow V_\rho[1 + \tilde{B} \sin(\Omega t)] \) with \( \tilde{B} \) the amplitude of modulation. When \( \Omega = \omega \) or multiples thereof, resonant collective oscillation can be generated in the BEC \[17\]. This phenomenon has also been explored experimentally \[20\]. We find that this resonant oscillation also destroys superfluidity of the BEC after some hold time.

The transition we consider is classical in nature and can be treated \[12\] by the mean-field nonlinear Gross-Pitaevskii (GP) equation \[21\]. The time-dependent axially symmetric BEC wave function \( \Psi(r; t) \) at position \( r \) and time \( t \) is described by the following GP equation

\[
\left[ -i \hbar \frac{\partial}{\partial t} - \frac{\hbar^2 \nabla^2}{2m} + V(r) + gN|\Psi(r; t)|^2 \right] \Psi(r; t) = 0, \tag{1}
\]

where \( m \) is the mass and \( N \) the number of atoms in the condensate, \( g = 4\pi\hbar^2a/m \) the strength of interatomic interaction, with \( a \) the atomic scattering length, and \( V(r) = \frac{1}{2}m\omega^2(\rho^2 + \nu^2z^2) \) the trapping potential where \( \omega \) is the angular frequency of the harmonic trap in the radial direction \( \rho \), \( \nu \omega \) that in the axial direction \( z \), with \( \nu \) the aspect ratio. The normalization condition is \( \int d\mathbf{r} |\Psi(r; t)|^2 = 1 \).

In the axially-symmetric configuration, the wave function can be written as \( \Psi(r, t) = \psi(\rho, z, t) \). Now transforming to dimensionless variables \( \hat{\rho} = \sqrt{2}\rho/l \), \( \hat{z} = \sqrt{2}z/l \), \( \tau = t\omega \), \( l = \sqrt{\hbar/(m\omega)} \), and \( \varphi(\hat{\rho}, \hat{z}; \tau) \equiv \hat{\rho}^{1/3}/\sqrt{8}\psi(\rho, z; t) \), Eq. (1) becomes

\[
\left[ -i \frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial \hat{\rho}^2} + \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} - \frac{\partial^2}{\partial \hat{z}^2} + \frac{1}{4} \left( \hat{\rho}^2 + \nu^2 \hat{z}^2 \right) \right] \varphi(\hat{\rho}, \hat{z}; \tau) = 0.
\]
\[
+ \frac{V_{\text{opt}}}{\hbar \omega} - \frac{1}{\rho^2} + 8\sqrt{2\pi n} \left| \frac{\varphi(\hat{\rho}, \hat{z}; \tau)}{\rho} \right|^2 \varphi(\hat{\rho}, \hat{z}; \tau) = 0,
\]

where nonlinearity \( n = Na/l \). In terms of the one-dimensional probability \( P(z, t) \equiv 2\pi \int_0^\infty d\hat{\rho}|\varphi(\hat{\rho}, \hat{z}, \tau)|^2/\hat{\rho} \), the normalization of the wave function is given by \( \int_{-\infty}^\infty d\hat{z}P(z, t) = 1 \). The probability \( P(z, t) \) is useful in the study of the present problem under the action of the optical lattice, specially in the investigation of the formation and evolution of the interference pattern after the removal of the trapping potentials.

We use the parameters of the experiment of Cataliotti et al. [4] with repulsive \(^{87}\text{Rb}\) atoms where the radial trap frequency was \( \omega = 2\pi \times 92 \text{ Hz} \). For the mass \( m = 1.441 \times 10^{-25} \text{ kg} \) of \(^{87}\text{Rb}\) the harmonic oscillator length \( l = \sqrt{\hbar/(m\omega)} = 1.126 \text{ \(\mu\)m} \) and and the dimensionless time unit \( \omega^{-1} = 1/(2\pi \times 92) \text{ s} = 1.73 \text{ ms} \). We solve Eq. (2) numerically using a split-step time-iteration method with the Crank-Nicholson discretization scheme described recently [22]. The time iteration is started with the known harmonic oscillator solution of Eq. (2) with \( n = 0: \varphi(\hat{\rho}, \hat{z}) = [\nu/(8\pi^3)]^{1/4} \hat{\rho} e^{-(\hat{\rho}^2 + \hat{z}^2)/4} [22] \). First we calculate the ground-state wave function of the system for \( n = 5 \) and \( \nu = 0.5 \) which we use in numerical simulation. To investigate the superfluidity of the BEC we set \( \varphi(\hat{\rho}, \hat{z}) = 0 \) for \( \hat{z} = \pm 0.3j \), \( j = 0, 1, 2, 3, \ldots \). This cuts the initial BEC in slices of width 0.3 in the axial direction. The free expansion of a phase coherent initial BEC sliced in this fashion will lead to a prominent interference pattern revealing the superfluidity. Essentially, similar slices result in a BEC trapped in an one-dimensional optical lattice as in the experiment by Cataliotti et al [4]. The atom cloud released from one piece expand, and overlap and interfere with atom clouds from neighboring piece to form the robust interference pattern due to phase coherence. The pattern consists of a central peak and two symmetrically spaced peaks moving in opposite directions [4,5,7]. Although we employ the dimensionless space units \( \hat{\rho} \) and \( \hat{z} \) and time unit \( \tau \) in numerical calculation, the results are reported in actual units: \( r \text{ \(\mu\)m} \), \( \rho \text{ \(\mu\)m} \), \( z \text{ \(\mu\)m} \) and \( t \text{ ms} \). In the conversion we used the parameters of the experiment of Cataliotti et al. [4], e.g., \( \rho = 0.8\hat{\rho} \text{ \(\mu\)m} \), \( z = 0.8\hat{z} \text{ \(\mu\)m} \), and \( t = 1.73\tau \text{ ms} \).

First we study the destruction of superfluidity in the condensate after the application of a periodic modulation of the scattering length resulting in a similar modulation of nonlinearity \( n \) in Eq. (2) via

\[
n \rightarrow n + A \sin(\Omega \tau),
\]

where \( A \) is an amplitude. In the present study we employ amplitude \( A = 3 \). For present \( n = 5 \), this will restrict the modulated nonlinearity to have always positive values corresponding to atomic repulsion. Negative values of nonlinearity corresponding to atomic attraction may lead to collapse and instability [21,23] and will not be considered here. The BEC has been found to
Fig. 1. One-dimensional probability $P(z,t)$ vs. $z$ and $t$ for the BEC under the action of modulation (3) of the nonlinearity with $n = 5$, $\Omega = 2\omega$, and $A = 3$ and upon the removal of the traps after hold times (a) 0, (b) 17 ms, (c) 35 ms, and (d) 69 ms.

execute resonant collective oscillation when the modulation frequency $\Omega$ is an even multiple of the radial trap frequency $\omega$ [16]. In the presence of an one-
We explicitly study the destruction of superfluidity in the condensate upon the application of modulation (3) of the scattering length leading to a resonant oscillation at $\Omega = 2\omega$. The loss of superfluidity only takes place if the BEC is allowed to experience the resonant oscillation for a certain hold time. For numerical simulation we allow the BEC to evolve on a lattice with $\rho \leq 20 \mu m$ and $20 \mu m \geq z \geq -20 \mu m$ after the modulation (3) is applied and study the system after different hold times. The one-dimensional probability $P(z,t)$ is
plotted in Figs. 1 (a), (b), (c) and (d) for hold times 0, 17 ms, 35 ms and 69 ms, respectively. For hold time 17 ms, prominent interference pattern is formed upon free expansion. In Fig. 1 (a) three separate piece in the interference pattern corresponding to three distinct trails can be identified. The interference pattern is slowly destroyed at increased hold times as we can see in Figs. 1 (b), (c) and (d). As the hold time increases the maxima of the interference pattern mix up upon free expansion and finally for the hold time of 69 ms the interference pattern is completely destroyed as we find in Fig. 1 (d). As the BEC is allowed to evolve for a substantial interval of time after the application of the periodic modulation in the scattering length, a dynamical instability of classical nature sets in which destroys the superfluidity [12,13].

Next we investigate how the phase $\delta$ over the condensate behaves as the hold time is increased and what happens to the modulus $|\psi(\rho, z)|$ of the wave function. For this purpose we plot the phase and the modulus of the wave function of the BEC’s of Figs. 1 (a), (b), (c), and (d), respectively, in Figs. 2 (a), (b), (c), and (d) after hold times 0, 17 ms, 35 ms, and 69 ms. For hold time 0 the wave function is smooth and the phase is slowly varying over the full condensate corresponding to perfect phase coherence. But as the hold time is increased the phase varies rapidly over the condensate. The total variation of phase over the BEC for hold times 0, 17 ms, 35 ms, and 69 ms are 1.5 rad, 15 rad, 20 rad, and 45 rad, respectively. For larger hold times the modulus $|\psi(\rho, z)|$ of the wave function also develops a nonsmooth irregular behavior via the dynamical instability. The rapid variation of $\delta$ and the nonsmooth irregular wave function over the BEC for larger hold times are responsible for the destruction of superfluidity. Hence the superfluidity in the BEC could be destroyed in the absence of an optical trap due a classical collective resonant oscillation in the condensate initiated by a periodic modulation of the scattering length with a frequency $\Omega = 2\omega$. Such resonances appear for $\Omega = 2\omega j$ [16], where $j = 1, 2, 3, ...$ and we have verified that similar breakdown of superfluidity also occurs for $\Omega = 4\omega$. The resonance becomes more vigorous as $A$ is increased and so is the destruction of superfluidity.

However, resonant oscillation arising from modulation (3) is not the only classical dynamical mechanism for the destruction of superfluidity. It can also be destroyed via a periodic modulation of the radial trapping potential [17]. To illustrate this we consider the following modulation of the radial trapping potential in (2) via

$$\hat{\rho}^2/4 \rightarrow (\hat{\rho}^2/4)[1 + B \sin(\Omega\tau)], \quad (4)$$

while the axial trapping potential is left unchanged. In this case parametric resonance appear when the modulation frequency $\Omega$ is a multiple of the trap frequency $\omega$ [17]. These resonances appear even for a small value of the amplitude $B$. However, they appear for a band of frequency around the multiples
Fig. 3. One-dimensional probability $P(z,t)$ vs. $z$ and $t$ for the BEC under the action of modulation (4) of the radial trapping potential with $n = 5$, $\Omega = \omega$, and $B = 0.5$ and upon the removal of the traps after hold times (a) 35 ms, (b) 69 ms, and (c) 104 ms.

We find that the superfluidity is destroyed if the modulation (4) of the radial trapping potential is continued for a certain hold time with $\Omega$ fixed at a resonant frequency. To illustrate the loss of superfluidity in this case we plot the
Fig. 4. One-dimensional probability $P(z,t)$ vs. $z$ and $t$ for the BEC under the action of modulation (4) of the radial trapping potential with $n = 5$, $\Omega = 1.2\omega$, and $B = 0.5$ and upon the removal of the traps after hold time 104 ms.

The one-dimensional probability distribution $P(z,t)$ vs. $z$ and $t$ in Figs. 3 (a), (b), and (c) for hold times 35 ms, 69 ms, and 104 ms, respectively, for $\Omega = \omega$ and $B = 0.5$. In Fig. 3 (a) we find that the superfluidity is maintained for a hold time of 35 ms and three prominent peaks are formed in this case. However, as the hold time is increased the clean interference pattern is gradually destroyed as can be seen from Figs. 3 (b) and (c). The same phenomenon is found to occur at a larger resonant frequency of modulation, where $\Omega$ is a larger multiple of $\omega$. The destruction of superfluidity is favored for a larger amplitude of modulation $B$.

If the frequencies $\Omega$ are off their resonance values and the amplitudes $A$ and $B$ in Eqs. (3) and (4), respectively, have moderate values, there is no loss of superfluidity after large hold times for modulations of scattering length or radial trapping potential. To illustrate this for modulation (4) with $B = 0.5$, $\Omega = 1.2\omega$ and a hold time of 104 ms we plot the density $P(z,t)$ vs. $z$ and $t$ in Fig. 4, where the clean interference pattern reappears with a small change in the modulation frequency $\Omega$ from the resonance value $\omega$ to a nonresonant value $1.2\omega$. The maintenance of superfluidity in Fig. 4 off the resonance should be contrasted with its loss in Fig. 3 (c) at resonance for the same hold time.

In conclusion, we studied the destruction of superfluidity in a cigar-shaped trapped BEC upon the application of a periodic modulation of the scattering length or of the radial trapping potential leading to a resonant collective excitation. In the absence of modulation, the formation of the interference pattern upon the removal of the trap clearly demonstrates the phase coherence. At the resonance frequencies a dynamical instability in the BEC leads to the destruction of superfluidity provided that the BEC is kept in the modulated trap for a certain hold time. Consequently, after release from the trap no interference pattern is formed. The superfluidity in the BEC is not destroyed...
when the frequency of modulation is changed to a nonresonant value. It is possible to study this novel way of the destruction of superfluidity experimentally and a comparison of those results with mean-field models will enhance our understanding of matter wave BEC.

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