Heavy-light physics with NRQCD

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First results are obtained for B mesons using a heavy propagator calculated using NRQCD, and a light Wilson propagator. Results from 13 quenched configurations of size $16^3 \times 48$ at $\beta = 6.0$ give a value for $f_B$ of less than 200 MeV and a $B^* - B$ splitting of 32(8) MeV. Superior signal/noise behaviour is observed over static propagators on the same configurations. No extrapolation to the $b$ mass for the heavy quark is required.

1. INTRODUCTION

B physics is an exciting area of topical interest. Results for B decay constants and mixing parameters are urgently needed by the experimental community and can best be provided by calculations in lattice QCD. There has been some controversy, however, in interpreting the lattice results. The static approximation gives large values for $f_B$ with a large spread from group to group [1]. The poor signal/noise properties of this approximation presumably give rise to this variation. Calculations to date using Wilson propagators for the heavy quark have required an extrapolation in the heavy quark mass to the $b$. The use of NRQCD for the heavy quark gives a result with good signal/noise properties right at the $b$ mass and the method is presented here.

NRQCD is an effective field theory, appropriate to heavy quark physics, which has had great success to date in the simulation of heavy-heavy systems, both $b\bar{b}$ and $c\bar{c}$ [2,3].

In that case the NRQCD action is viewed as an expansion in powers of $v^2/c^2$ where $v$ is a typical velocity inside the heavy-heavy system. Simulations have been done recently by the NRQCD collaboration using leading and next-to-leading terms in this expansion [2,3]. The coefficients of the sub-leading terms can be taken at their tree-level values when the input gauge fields are transformed as

$$U_\mu \rightarrow \frac{U_\mu}{u_0} \quad (1)$$

$u_0$ is taken as the fourth root of the average plaquette $\bar{u}$. What this transformation does is to include in each term the radiative tadpole corrections to all orders in perturbation theory.

The success of the spectrum calculated [2,3] gives us confidence in the action and in the coefficients of each term. That the associated perturbation theory is well behaved and understood is shown by a study of the zero and finite momentum energies of the $\Upsilon$ as a function of bare quark mass [2]. The bare quark mass that should appear in the action can then be fixed by setting the kinetic mass of a known state, say the $\Upsilon$, to its experimental value. This gives a bare quark mass for the $b$, $M_{b0} = 1.7$ at $\beta = 6.0$ [2].

To study heavy-light systems we can use the same NRQCD action for the heavy quark as has been used in the heavy-heavy simulations [2,3]. There are then no parameters to be fixed at all and all masses are obtained as predictions.

For heavy-light systems the importance of different terms in the action changes. The momentum scale inside the mesons is set now by the dynamics of the light quark and, to leading order, is independent of $M$, the mass of the heavy quark. The importance of terms in the NRQCD action then depends only on their power of $1/M$. Terms at $1/M^2$ which were included at next-to-leading order in a heavy-heavy simulation are effectively suppressed. We can neglect them when working to $1/M$ for a heavy-light simulation [3].

The results described here use an NRQCD action for the heavy quark of the following form

$$\mathcal{L}_{NRQCD} = -\bar{\Psi} D_\mu \Psi + \frac{i}{2M} \frac{D_\mu \Psi^+}{2M}$$
The heavy quark propagators are calculated with the usual evolution equation \( \frac{g}{2M} \Psi^\dagger \sigma_B \Psi \) (2)

The heavy quark propagators are calculated with the usual evolution equation and with \( U_\mu \) fields transformed as in equation 1. The bare heavy quark mass was taken as 2.0 to simulate a b quark on lattices at \( \beta = 6.0 \). Subsequent work on the heavy-heavy spectrum as described above \( \beta \) shows this value for the mass to be 15% high. This initial study used \( 13 \times 16^3 \times 48 \) lattices fixed to Coulomb gauge.

The heavy quark propagators were combined with light Wilson propagators at two \( \kappa_l \) values, 0.154 and 0.155. \( \kappa_c \) is determined at this \( \beta \) value to be 0.157. The light quark propagators were calculated from a \( \delta \) function source and given a \( \sqrt{2\kappa_l} \) rescaling.

The heavy quark propagators were calculated both from a \( \delta \) function source and from an exponentially smeared source \( \exp(-r/\rho_0) \), with \( \rho_0 = 3.0 \). There was one source per configuration. By combining either smeared or local heavy propagators with local light propagators we are able to study local-local, smeared-local and smeared-smeared correlation functions for the B meson. We also calculated heavy quark propagators in the static approximation (that is, using equation 2 with all \( 1/M \) terms switched off) from the same smeared sources. This enables us to directly compare the static and NRQCD signal/noise behaviour.

2. RESULTS

2.1. \( f_B \)

To make a B meson we used the naïve lowest order vertex operator, i.e. we combined the 2-component heavy antiquark propagator with the 2 upper spins of the light quark propagator. \( 1/M \) effects in the vertex operator were ignored at this stage. The zero-momentum B meson correlation function then has an exponential fall-off given by its energy in lattice units and an amplitude related to the decay constant, \( f_B \). For the NRQCD heavy quarks the signal/noise is such that it is possible to fit the local-local correlation function, see Figure 1. This has previously been shown not to be possible for static heavy quarks. Fitting to a single exponential from time slices 12-48 and using an svd algorithm with cut-off 1.0e-03 to invert the covariance matrix, we obtain at \( \kappa_l = 0.155: \)

\[
E = 0.48(1), \quad Z_L = 0.11(2), \quad \chi^2/\text{dof} = 1.7/4, \quad Q = 0.8. \quad (3)
\]

\( Z_L \) is the square root of the amplitude. \( \sqrt{2Z_L} \) is equal to \( f_B \sqrt{M_B} \) in the \( f_r = 132 \text{ MeV} \) convention, modulo a lattice-to-continuum renormalisation constant. This renormalisation constant has only been calculated for an NRQCD action with no \( \sigma_B \) term and with no \( u_0 \) transformation of the \( U_\mu \) \( \beta \). A fairly large \( O(g^2) \) contribution was found in that case, but a major component of it was the wave function renormalisation of the heavy quark. This is much reduced when tadpole effects are taken care of using eq. 1 \( \beta \). It seems likely that the renormalisation constant for the action used here is then much closer to 1.

Extrapolating linearly to \( \kappa_c \) for the light quark, we obtain 0.16(3)/\( Z_A \) for \( f_B \sqrt{M_B} \) in lattice units. This agrees with other UKQCD results in this region \( \beta \), for reasonable values of \( Z_A \).

The predicted value for \( f_B \) is then less than 200 MeV, using an inverse lattice spacing at \( \beta = 6.0 \) of 2.0 GeV. In the quenched approximation the value for the lattice spacing depends on the scale at which it is being determined; the \( a^{-1} \) value from \( b\bar{b} \) splittings being considerably larger than that from light hadrons \( \beta \). The scales appropriate to heavy-light mesons are presumably much more like those of light hadrons, so we use an \( a^{-1} \) from those calculations, as above.

The static calculation requires the calculation of both smeared-local and smeared-smeared correlation functions. The smeared-local correlation function is much noisier than the NRQCD smeared-local function (see Figure 2), but the smeared-smeared correlation function is only slightly noisier than its NRQCD counterpart.

The poor noise properties of the local static correlation functions are because of the absence of a mass in the heavy-heavy channel that appears in the noise \( \beta \). Smearing has the effect of producing such a mass by effectively introducing the heavy quark potential at non-zero \( R \) into the heavy-heavy cross-terms. This is a wholly
Figure 1. The effective mass for the local-local correlation function for the $B$ meson using an NRQCD propagator for the heavy quark.

desirable effect for the static smeared-local correlation function and the noise is much reduced. For NRQCD the non-locality of the source has the effect of increasing the noise over the local-local function, although an earlier plateau is seen. In both cases smearing at the sink introduces more noise than having a local sink.

We can fit a single exponential to the ratio of the square of the smeared-local to the smeared-smeared correlation functions for the static case. The amplitude is then the effective local-local amplitude. We obtain at $\kappa_l = 0.155$

$$E = 0.47(1), Z_L = 0.21(1),$$
$$\chi^2/dof = 6.7/13, Q = 0.9.$$  \hfill (4)

Of necessity there was a more restricted range in time (15 - 25) than for the NRQCD fit. We believe that this accounts for the fact that the errors are not significantly worse.

The value for $f_\sqrt{M}$ is 0.27(2)/$Z_A$ after linear extrapolation to the chiral limit in $\kappa_l$. This is in agreement with other UKQCD static results at $\beta = 6.0$ \cite{10}, obtained using the improved clover action for the light quarks, although the $Z_A$ values will be slightly different (by 10%) in the two cases. The value for $f_B$ is around 300 MeV, much larger than for NRQCD heavy quarks.

2.2. $M_B$

Because of the form of the NRQCD action, the energy measured for a zero-momentum correlation function must be adjusted to give the mass of the meson. This adjustment by terms which are perturbatively calculable works very well for heavy-heavy mesons \cite{2}. For the B meson we have

$$M_B = Z_M M_b - E_0 + E.$$  \hfill (5)

All quantities are in lattice units. $Z_M$ is the heavy quark mass renormalisation and $E_0$ is the energy shift. Using $E_0 = 0.22$ and $Z_M = 1.14$ \cite{2} gives $M_B = 2.46(5)$ in lattice units after extrapolation to the light quark chiral limit. Multiplication by $a^{-1}$ is the major source of error, giving a result of 4.9(4) GeV, certainly consistent with experiment (5.3GeV).

Another determination of $M_B$ comes from the finite momentum propagators. A kinetic mass can be extracted from the ratio of the energies of lowest non-zero momentum to the zero-momentum. With only 13 configurations the results are too noisy to give a value from this method.

2.3. $B^*-B$

From the ratio of the local-local NRQCD correlators for the $B^*$ and the $B$ we can extract a value for the splitting between $B^*$ and $B$. A single exponential fit at $\kappa_l = 0.155$ to the ratio from time slices 5-20 gives

$$E = 0.016(3), A = 1.02(2),$$
$$\chi^2/dof = 1.9/3, Q = 0.6.$$  \hfill (6)

Both values of $\kappa_l$ give the same splitting so no chiral extrapolation can be done. Taking the value above and $a^{-1} = 2.0(1)$ GeV gives a splitting of 32(8) MeV, rather lower than the experimental value of 46 MeV. This may be a reflection of the rather low quark mass that we have used. If the splitting is proportional to $1/M_b$, then a quark mass of 1.7 rather than 2 would give a splitting of 38(8) MeV, consistent with experiment. Values obtained using heavy Wilson fermions have been much lower than this \cite{12}.  


2.4. $\Lambda_b$

We have calculated the NRQCD smeared(h)-local(l) correlation functions for the $\Lambda_b$. The effective mass plots are rather noisy, and the mass depends quite strongly on $\kappa_l$. Chiral extrapolation gives a splitting between the $\Lambda_b$ and the $B$ of 0.28(8) in lattice units. Converting to physical units gives the rather imprecise value at present of 60(20) MeV. The experimental result is 36(4) MeV.

3. CONCLUSIONS

The use of NRQCD for heavy quark propagators can give useful results for heavy-light mesons. Once the terms in the action are fixed from the very precise heavy-heavy calculations that are possible, there are no free parameters left. Correlation functions for the $B$ meson can be calculated directly at the $b$ mass (fixed from $\Upsilon$ spectroscopy) and they have good signal/noise properties.

This preliminary calculation on a small number of configurations at $\beta = 6.0$ finds a value for $f_B$ consistent with others using heavy Wilson propagators. This confirms from another source the existent of relatively large $1/M$ terms in $f \sqrt{M}$ at the the $B$. The usefulness of the approach here is that a direct comparison with static results calculated using the same smearing functions on the same configurations is possible.

A value for $M_B$ can be calculated since all parameters in the action are already fixed. This agrees with experiment. In addition the value for the $B^* - B$ splitting looks promising. It is reasonably close to experiment unlike values obtained up to now using heavy Wilson fermions.

Acknowledgements

This work was carried out on a Thinking Machines CM-200 supported by SERC, Scottish Enterprise and the Information Systems Committee of the UFC. I am grateful to my colleagues in the NRQCD collaboration for useful discussions.

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