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Post-Cracking Response of Hybrid Recycled/Industrial Steel Fiber-Reinforced Concrete

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Synopsis: The present study aims at investigating the influence of Recycled Steel Fibers (RSFs) recovered from waste tires on the resulting post-cracking response of Fiber-Reinforced Concrete (FRC) mixtures, when they are employed for replacing conventional Industrial Steel Fibers (ISFs). It moves from the results of four-point bending tests carried out on a series of specimens made of Hybrid FRC, namely reinforced by both RFSs and ISFs. Then, the paper proposes a theoretical model based on a meso-mechanical formulation merged into a cracked-hinge approach. The model is capable of taking into account explicitly the diverse geometric and mechanical properties of RSFs and ISFs and, hence, it is employed for interpreting the results of the aforementioned bending tests. Some comparisons between the experimental results and the theoretical predictions are presented with the aim to corroborate the mechanical consistence of the proposed model.

Finally, it is worth highlighting that this study has been carried out at the STRuctural ENGineering Testing Hall (STR.ENG.T.H) of the University of Salerno, as part of the “SUPERCONCRETE” Project (H2020-MSCA-RISE-2014 – n. 645704).

Keywords: FRC, waste tires, hybrid FRC, modelling, cracked-hinge approach
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**INTRODUCTION**

In the last decades, several researches focused on the characterization of the mechanical response of cement-based composite reinforced with different kinds of short fibers randomly dispersed within the matrix [1]: for instance, metallic [2], synthetic [3], glass [4][5] and natural [6][7][8] fibers. On the other hand, a significant research effort focused on the suitability and efficiency of using various recycled materials and industrial by-products as sustainable concrete constituents [9][10] and, one of the most promising solution is to reuse recycled fibers obtained from waste tires. In fact, approximately 1.4 billion tons vehicle pneumatics are sold annually in the world and, consequently, many of them can be categorized as “end of life” tires [11][12]. From the technical point of view, the internal steel reinforcement of tires can be reused in partial to total replacement of industrial steel fibers commonly use in the so-called Fiber-Reinforced Concrete (FRC) [13].

In the present context, this study reports the results of an experimental campaign carried out at the STRuctural ENGineering Testing Hall (STR.ENG.T.H) of the University of Salerno (Italy) aimed at investigating the post-cracking behavior of concrete reinforced with both industrial and recycled fibers obtained from waste tires. Then, the paper proposes a theoretical model based on a meso-mechanical formulation merged into a cracked-hinge approach. The model is capable of taking into account explicitly the diverse geometric and mechanical properties of Recycled and Industrial Steel Fibers and, hence, it is employed for interpreting the results of the aforementioned bending tests.

**EXPERIMENTAL INVESTIGATION**

**Industrial and Recycled Steel fibers**

The Industrial Steel Fibers (ISFs) present a length equal to 33 mm (1.30 in.) whereas its diameter is equal to 0.55 mm (0.02 in.); they are characterized by an optimized shape presenting two hooked ends. Conversely, the Recycled Steel Fibers (RSFs) derived from waste tires present an average length and diameter equal to 25 mm (0.98 in.) and 0.25 mm (0.01 in.), respectively.

**Fiber-Reinforced Concrete**

The cement-based mixtures were produced by using Portland cement type CEM II/A-LL 42.5R in accordance with the EN 197-1 [14] and a water-to-cement ratio equal to 0.49. The fine aggregates (namely “sand”) were characterized by a maximum nominal diameter equal to 2 mm (0.08 in.) meanwhile the coarse ones were divided in two fractions: Class 1 with a nominal diameter ranging from 2 mm (0.08 in.) to 10 mm (0.39 in.) and Class 2
with from 10 mm (0.39 in.) to 20 mm (0.78 in.). In addition, an acrylic-based superplasticizer was added to the mixtures in order to achieve the necessary workability (i.e., for obtaining a slump class consistency S3 [15]).

Three FRC mixtures were produced by employing the aforementioned cement-based matrix and a variable amount and combinations of industrial and recycled steel fibers:

- one “Industrial -FRC” mixture made with 0.75 % in volume, i.e., about 60 kg/m$^3$ (3.74 lb/ft$^3$) of ISFs;
- one “Recycled -FRC” mixture prepared by replacing 100% of ISFs with RSFs;
- one “Hybrid-FRC” mixture prepared by using 30 kg/m$^3$ (1.87 lb/ft$^3$) of ISFs and 30 kg/m$^3$ (1.87 lb/ft$^3$) of RSFs.

**Four-point bending tests**

For each mixture, three cubes (150 mm x 150 mm x 150 mm) and three prismatic specimens (150 mm x 150 mm x 600 mm) were prepared: the cubic samples were tested in compression, while the prismatic ones were tested under four-point bending. All tests were performed after 28 days of curing and, before performing the four-point bending tests, the prismatic specimens were notched in the middle for about 45 mm in depth. Experimental tests were conducted according to the procedures described in UNI 11039 [16][17] and UNI EN 12390-4 [18], for bending and compression, respectively. Specifically, the four-point bending tests were performed in displacement control and dedicated transducers monitored the Crack Tip Opening Displacement (CTOD) in the two sides of the notch tip were employed in order to measure the average CTOD (i.e., CTOD$_m$).

**EXPERIMENTAL RESULTS**

The plain cement-based mixture presented an average compressive strength, at 28 days, of 25 MPa (3626 psi). The results highlighted that, as already documented in the literature, the presence of fibers only slightly influences the resulting compressive strength of FRCs (i.e., between 5% and 10% with respect to the corresponding plain mixture). More details regarding the influence of the Recycled Fibers on the resulting compressive strength are available in a study recently published by the authors [19].

Fig. 1 reports the experimental results obtained from the four-point bending tests performed on the notched samples with the scope of characterizing the post-cracking response of the mixtures under investigation. In each graph, the line represents the mean curve (based on three tested samples), whereas the light area indicates the range of variation of the resulting curves.

The “Industrial-FRC” mixture (see Fig. 1a) presents a post-cracking response characterized by a significant improvement of toughness in comparison with an unreinforced plain mixture: this is due to the well-known bridging action of the ISFs. Then, the analysis of the green curve reported in Fig. 1b also highlights the influence of the complete replacement of ISFs with the with an equal volume of recycled ones and, it is worth highlighting that the “Recycled-FRC” mix is characterized by a similar post-cracking behavior with that corresponding to “Industrial-FRC”. At first sight, this was a somehow “unexpected” result, since a significant decay in the post-cracking response was observed in a previous studies when ISFs were completely replaced with an equal amount of recycled ones [13]: a more in-depth analysis unveiled that the RSFs employed in the present research have an average aspect ratio significantly (almost twice) higher than the ones adopted in other studies, therefore, the RSFs have a more efficient behavior [19]. Similarly, also in the case of “Hybrid-FRC” (see Fig. 1c) the presence of RSFs in partial replacement of ISFs does not significantly affects the performance of FRC.

A more comprehensive analysis can be conducted by considering the representative parameters defined by the UNI-11039 part 1 [16] and 2 [17]: the first crack strength ($f_{fl}$), the work capacity indices and the ductility indices. The first crack strength ($f_{fl}$) is defined as follow:

$$f_{fl} = -\frac{P_{fl} \cdot l}{b \cdot (b - a_0)^2}$$

(1)

where $P_{fl}$ represents the first crack load (in N), $b$, $h$ and $l$ are the width (in mm), height (in mm) and span length (in mm) of the tested beam, respectively, while $a_0$ (in mm) represents the notch depth.

Then, the work capacity indices (i.e., $U_1$ and $U_2$, energy absorption values) represent the areas under the vertical load P-CTOD curve in a representative range for the Serviceability Limit State (i.e., considering a CTOD ranging between CTOD$_D$ and CTOD$_D + 0.6$ mm) and for the Ultimate State (i.e., considering CTOD ranges between CTOD$_D + 0.6$ mm and CTOD$_D + 3.0$ mm), respectively. Finally, the ductility indices ($D_0$ and $D_1$) can be determined with the following equations:

$$D_0 = \frac{f_{eq(0-0.6)}}{f_{fl}}$$

(2)

$$D_1 = \frac{f_{eq(0.6-3)}}{f_{eq(0-0.6)}}$$

(3)

where $f_{eq(0.6)}$ represents the first equivalent post-cracking strength, supposed to be significant for the Serviceability Limit State (evaluated as a function of the $U_1$ parameter) [17], whereas the second equivalent post-cracking strengths one (i.e., $f_{eq(0.6-3)}$) is rather relevant for the Ultimate State (evaluated as a function of the $U_2$.
If the ductility indices range between 0.5 and 0.9, the post-cracking response of the FRC mixtures is defined as “softening”, if \( D_0 \) and \( D_1 \) range between 0.9 and 1.1 the FRC is considered “plastic”, meanwhile it is defined as “hardening”, for \( D_0 \) and \( D_1 \) higher than 1.1.

**Fig. 2, Fig. 3 and Fig. 4** summarize the results obtained for the aforementioned parameters determined for the FRC mixtures under investigation. The results indicate that the presence of Recycled Steel Fibers slightly increases the first crack strength (see Fig. 2). In fact, the reference Industrial FRC presents a first crack strength equal to 3.26 MPa meanwhile for both Recycled and Hybrid FRCs higher values were registered. On the contrary, the results in terms of ductility indices (\( D_0 \) and \( D_1 \) in Fig. 4) highlight that the presence of Recycled fibers slightly reduce the resulting FRC toughness. In fact, the \( D_0 \) changes from 1.06 (for Industrial FRC) to 1.01 (for Recycled FRC) and to 1.02 (for Hybrid FRC).

Based on these results, it can be concluded that the post-cracking behavior of “Industrial-FRC”, “Recycled-FRC” and “Hybrid-FRC” for small crack openings can be defined as a plastic type, as in all cases \( D_0 \) ranges between 0.9 and 1.1. Conversely, when it comes to the ductility index \( D_1 \), the presence of RSFs moves the post-cracking behavior from hardening (\( D_1 \) equal to 1.24 for “Industrial-FRC”) to a plastic (with values ranging between 0.9 and 1.1 in all cases in which RSFs were used).

**THEORETICAL MODELLING**

This section presents a theoretical model and the related numerical procedure, based on the cracked hinge model [19], intended at simulating the cracking and post-cracking behavior of specimens of Industrial, Recycled and Hybrid FRCs subjected to bending tests.

The cracked hinge model [19] describes the crack as a local discontinuity (hinge), that is confined to a certain region of width “s”. Outside these rigid boundaries, the uncracked element can be modeled using classical beam theory or appropriate elastic theory.

**Stress-strain and stress-crack opening for concrete matrix**

To describe the concrete tensile behavior a stress-strain diagram (\( \sigma-\varepsilon \)) should be used for the uncracked concrete, and a stress-crack opening diagram (\( \sigma-w \)) should be taken into account for the cracked section.

For uncracked concrete, a bilinear stress-strain relation may be used, as given in the following equations:

\[
\sigma_{ct} = \begin{cases} 
E_{ct} \cdot \varepsilon_{ct} & \text{for } \sigma_{ct} \leq 0.9 f_{ctm} \\
 f_{ctm} \cdot \left( 1 - 0.1 \cdot \left( \frac{0.00015 - \varepsilon_{ct}}{0.00015} \right) \frac{f_{ctm}}{f_{ctm}} \right) & \text{for } 0.9 f_{ctm} \leq \sigma_{ct} \leq f_{ctm}
\end{cases}
\]

(4)

where, \( E_{ct} \) is the tangent modulus of elasticity (in MPa), \( \varepsilon_{ct} \) is the tensile strain, \( \sigma_{ct} \) is the tensile stress (in MPa) and \( f_{ctm} \) is the tensile strength (in MPa).

The first branch follows a law linear-elastic (that is poorly affected by the presence of the fibers and which is a function of the properties of the matrix), while the second branch is a linear approximation of the micro-cracking behavior. At tensile stresses of about 90% of the tensile strength micro-cracking starts to reduce the stiffness in a small damage zone. The micro-cracks grow and form a discrete crack at stresses around the tensile strength. For a cracked section a bilinear approach for the stress-crack opening relation can be estimated by the following equations:

\[
\sigma_{ct} = \begin{cases} 
f_{ctm} \cdot \left( 1.0 - 0.8 \cdot \frac{w}{w_1} \right) & \text{for } w \leq w_1 \\
f_{ctm} \cdot \left( 0.25 - 0.05 \cdot \frac{w}{w_1} \right) & \text{for } w_1 \leq w \leq w_\varepsilon
\end{cases}
\]

(5)

where, \( w \) is the crack opening (in mm), \( w_1 = G_f/f_{ctm} \) (in mm) when \( \sigma_{ct} = 0.2 f_{ctm} \), \( w_\varepsilon = 5G_f/f_{ctm} \) (mm) when \( \sigma_{ct} = 0 \) and \( G_f \) is the fracture energy (in N/mm). In the absence of experimental data, \( G_f \) for normal weight concrete may be estimated by [21]:

\[ G_f = 73f_{ctm}^{0.18} \]

(6)

where \( f_{ctm} \) is the mean compressive strength (in MPa).

**Fiber-matrix interface**

The post-peak behavior of the material is governed by the tensile strength of the fibers along the failure plane. Optimal behavior post-peak period is guaranteed by the presence of a large number of fibers through the failure plane: more fibers cross this plane in the region of the post-peak, greater is the pull-out strength, denoting an
improved toughness. Then, in order to consider the interaction between the matrix and the fiber, a pull-out test is considered. In fact, considering a fiber, of a length “dx”, immersed in the matrix and subjected to a tensile force, a simple equilibrium equation can be written:

\[
\frac{\pi \cdot d_f^2}{4} ds = \tau \cdot \pi \cdot d_f \cdot dx
\]  

(7)

from which it is obtained:

\[
\frac{ds}{dx} = \frac{4}{\tau} d_f
\]  

(8)

Introducing the bond stress-strain linear (\(\sigma = E \cdot \varepsilon\)) and substituting in equation (8), it is obtained:

\[
\frac{d\varepsilon_s}{dx} = \frac{4}{E \cdot d_f \cdot \tau}
\]  

(9)

Considering that the fiber undergoes an elongation equal to “\(ds\)”, it is gotten for the congruence:

\[
\varepsilon_f = \frac{ds}{dx}
\]  

(10)

Replacing this expression in equation (9), it is obtained:

\[
\frac{d^2 s}{dx^2} = \frac{4}{E_i \cdot d_f} \cdot \tau
\]  

(11)

where \(\tau\) represents the shear stress acting along the perimeter of the fiber and \(s\) is the slip. The simplifying assumption considers a relationship between these two parameters, shown in Fig. 5. In this graph the dashed curves represent the relationships between these two parameters in “reality”.

The area under the curve represents the fracture energy of the fiber \((G_{fibres})\) and \(s_o\) is the last slip corresponding to \(\tau_o\). A possible solution of the Equation (8) is the following:

\[
s(x) = \frac{2\tau}{E_i d_f} x^2 + Ax + B
\]  

(12)

Introducing the boundary conditions:

\[
\begin{align*}
  s'(0) &= 0 \\
  s(l) &= s_o
\end{align*}
\]  

(13)

it is obtained \(A=0\) and \(B = \frac{2\tau}{E_i d_f} l^2\).

Then, the trend of the slips are described by the following equation:

\[
s(x) = s_o - (1^2 - x^2) \frac{2\tau}{E_i d_f}
\]  

(14)

Imposing the limiting case \(s_o = s_{th}\) it is derived the maximum value of the anchorage length \(l_{b,\text{max}}\):

\[
s_o - l^2 \frac{2\tau s}{E_i d_f} = 0 \Rightarrow l_{b,\text{max}} = \sqrt{\frac{E_i d_f s_o}{2\tau s}}
\]  

(15)

It is possible have two cases:

\[
F_{\text{FB}} = \begin{cases} 
\pi \cdot d_f \cdot l_b \cdot \left( \frac{G_{fibres}}{s_o} \right) & \Rightarrow l_b \leq l_{b,\text{max}} \\
\pi \cdot d_f \cdot l_{b,\text{max}} \cdot \left( \frac{G_{fibres}}{s_o} \right) & \Rightarrow l_b \geq l_{b,\text{max}} 
\end{cases}
\]  

(16)

where \(F_{\text{FB}}\) is the force which the fibers transmit to the matrix (Fibers Force Bridging).

**Numerical implementation**

A numerical code was realized for implementing and solving the theoretical model formulated in the previous subsection: it is capable to reproduce, in terms of load-CMOD and load-CTOD curves, the behavior of specimens subjected to three or four-point bending tests. The operating procedure of this code is described below.

First of all, it has been defined the geometry of the specimen, the depth of the notch, the length of the cracked hinge and type of bending test (i.e., three- or four-point). Then, it has been included the geometrical and mechanical characteristics of the materials constituting the specimen (concrete and fibers).

The section of the specimen, less of the notch, is then divided into many layers and it has been chosen the reference system. At this point, note the geometry and the volume percent of fibers (\(\rho_f\), the code sets, knowing the type of
fiber, uniquely determined by the parameters $E_f$ (elastic modulus) and $f_y$ (yield strength or ultimate strength), how many are the fibers present in the specimen, through the following relation:

$$n_f = \frac{V_f}{V_i} = \frac{\rho_f V_i}{V_c}$$  

(17)

where, $n_f$ is the number of the fibers, $V_f$ is the volume of the fibers, $V_i$ is the volume of the single fiber and $V_c$ is the volume of the cement-based matrix.

Among these fibers, generated randomly, only those that cut the crack are considered, therefore the program provides as output a table that indicates, for each of these fibers, the position with respect to the compression side of the specimen ($y_c$, hinge), the vertical and horizontal angle ($\phi_v$ and $\phi_h$), the diameter ($d_i$), the elastic modulus ($E_d$), the yield strength ($f_y$) and the smallest anchorage length ($l_a$), since, if a micro-crack is develops in correspondence of a fiber, it loses its grip initially in the shorter side still embedded in the concrete. At this point, the code calculates the position of the neutral axis $y_c$, through a simple equilibrium of forces to the translation and then derives the bending moment $M$ of the specimen. Thus, it is possible to know how much the load $F$ and the corresponding values of CMOD is or CTOD, through the following reports:

$$F = \begin{cases} 
4M/L & \text{for three–point bending test} \\
6M/L & \text{for four–point bending test} 
\end{cases}$$  

(18)

$$\text{CMOD} = \varphi(h - y_c)$$  

(19)

$$\text{CTOD} = \varphi(h - y_c - \text{notch})$$  

(20)

$$\varphi = \frac{M_{ct}}{(E_s I) \text{ layers}} \frac{bh^2 f_{ct}}{6} \frac{s}{\left( \frac{bh^3}{12} \right) \text{ layers}}$$  

(21)

Finally, the code run the simulation and provide the graphical representation of load-CMOD, load-CTOD and load-deflection curves.

**Comparison between experimental and numerical results**

Considering the experimental results obtained in this study, a numerical simulation was carried out. The “extreme” cases of samples with only one type of fiber were modelled (i.e., Industrial and Recycled FRCs), then the cases of “Hybrid-FRC” was obtained by simply varying the volume percent of the fibers ($\rho_f$). The $G_{F_{\text{fibers}}}$ and $s_u$ values, characterizing the bond behavior of the fibers, were calibrated in order to obtain the best fit between the numerical and the experimental curves. Specifically, for Industrial Steel fibers the $G_{F_{\text{fibers}}}$ equal to 23 N/mm (131 pound/in.) was obtained and an $s_u$ value equal to 7 mm (0.27 in.) was used; for the Recycled Steel fibers the $G_{F_{\text{fibers}}}$ and $s_u$ were assumed equal to 12 N/mm (68 pound/in.) and 6 mm (0.23 in.), respectively. A comparison between experimental and numerical results, for the case of “Hybrid-FRC”, is reported in **Fig. 6**. The results of this comparison highlight that the proposed model partially reproduces the load-CTOD curves: in fact, the first branch (i.e., up to the maximum load of the ascending branch) is well simulated and, conversely the post-cracking phase is underestimated.

**CONCLUSIONS**

The main objective of this work was to characterizes the mechanical response of Hybrid FRC, obtained by the combining of two types of steel fibers (i.e., industrial and recycled ones). More specifically, it described the key results obtained in an experimental campaign and proposes a theoretical model intended at analyzing this material at more in-depth observation scale. Based on the results proposed herein, the following observations can be highlighted:

- FRCs including Recycled Steel Fibers were characterized by a significant post-cracking toughness almost comparable with the one obtained for mixtures with only industrial ones;
- however, both partial and total replacement of Industrial Steel Fibers with an equal amount of Recycled ones slightly reduce toughness and ductility indices;
- the proposed model only partially reproduces the load-CTOD (or load-CMOD) curves (i.e., up to the peak stress), underestimating the post-cracking branch. Therefore, a further development of the present study could be performed by considering the possible improvements: a more representative the local bond slip law bond for the fibers; introducing a parameter taking into account of the fibers’ shape for the Industrial steel fibers and investigate the influence of dowel effect; considering a statistical distribution of mechanical and geometrical properties of the recycled steel fibers.
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Fig. 1 - Four-point bending tests - Experimental results for (a) Industrial, (b) Recycled and (c) Hybrid FRCs [1 in.=25.40 mm; 1 lb = 0.004448 kN]
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Fig. 2 - Crack strengths for FRCs [1000 psi = 6.895 MPa]

Fig. 3 - Experimental-Numerical simulation for (a) Industrial, (b) Recycled and (c) Hybrid FRCs [1000 psi = 6.895 MPa]

Fig. 4 - Ductility Indices [1000 psi = 6.895 MPa]
Fig. 5 - Bond $\tau$-s in the simplifying assumption

Fig. 6 - Experimental-Numerical simulation for Hybrid FRCs [1 in.$=25.40$ mm; 1 lb $= 0.004448$ kN]