Electromagnetic self-energies of vector mesons and electromagnetic mass anomaly of the massive Yang-Mills particles

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Abstract

A systematic method developed by the authors to evaluate the one-loop electromagnetic self-energies of the low-lying mesons is extended to the calculation of the vector sector including $\rho$, $\omega$, and $\phi$-mesons. The theoretical result of $\rho^0 - \rho^\pm$ electromagnetic mass difference is in agreement with the measurements. An interesting effect called as electromagnetic mass anomaly of the massive Yang-Mills particles is further discussed. There is no new parameter in this study.
1 Introduction

The investigation on the electromagnetic (EM) masses of the low-lying mesons has a long history, however, which is mainly on evaluating the EM masses of pseudoscalar $\pi$ and $K$-mesons (see Ref. [1] and references therein). In Ref. [1] (hereafter referred as paper I), we have presented a systematic investigation on the EM mass splittings of the low-lying mesons including $\pi$, $K$, $a_1$, $K_1$, and $K^*$ in the framework of the $U(3)_L \times U(3)_R$ chiral theory of mesons [2]. The present paper is the continuation of paper I. We are to calculate the EM self-energies of the vector $\rho$, $\omega$, and $\phi$ mesons to one-loop order and $O(\alpha_{\text{EM}})$.

Chiral quark model is originated by Weinberg [3], and developed by Manohar and Georgi [4]. In Ref. [5], the vector meson ($\omega$—meson) is firstly introduced into this model in order to study quark spin contents in chiral soliton model. The author of Ref. [2] further extended it to include the low-lying $1^-$ (vector) and $1^+$ (axial-vector) mesons (called as $U(3)_L \times U(3)_R$ chiral theory of mesons). The $U(3)_L \times U(3)_R$ chiral theory of mesons has been investigated extensively [1, 6, 7, 8] and its theoretical results agree well with the data.

Chiral perturbation theory (ChPT), which is expanded in powers of derivatives of the meson fields, is rigorous and phenomenologically successful in describing the physics of the pseudoscalar mesons at very low energies [4]. In Ref. [10], starting from the $U(3)_L \times U(3)_R$ chiral theory of mesons, and by using path integral method to integrate out the vector and axial-vector resonances, the authors have derived the chiral coupling constants of ChPT ($L_1$, $L_2$, $L_3$, $L_9$, and $L_{10}$). The results are in good agreement with the experimental values of the $L_i$ at $\mu = m_\rho$ in ChPT. Therefore, the QCD constraints discussed in Ref. [11] are met by this theory.

It has been pointed out in [3] that vector meson dominance (VMD) [12, 13] has been introduced into the $U(3)_L \times U(3)_R$ chiral theory of mesons. This means that the electromagnetic interaction of the mesons has been well established, which makes it possible to evaluate the EM self-energies of the low-lying mesons systematically.
In paper I, the logarithmic divergences coming from meson-loop diagrams which contribute to the EM mass splittings of $\pi$, $K$, $a_1$, $K_1$, and $K^*$ mesons have been factorized by using the intrinsic parameter $g$ of the theory (see paper I for details). However, when this method is extended to the cases of $\rho$, $\omega$, and $\phi$-mesons, the circumstances will be complicated. The Feynman diagrams contributing to the EM self-energies of $\rho$-mesons can be divided into three kinds, which have been shown in Fig. 1. Fig. 1.1 is tree diagram, and it contributes the finite result; Fig. 1.2 involves only logarithmic divergences, which could be evaluated by using the same method in paper I. In the framework of the $\text{U}(3)_L \times \text{U}(3)_R$ chiral theory of mesons, the explicit one-loop Feynman diagrams which belong to Fig. 1.2 will be drawn in Figs. 8, 9, and 10; It will meet some difficulties in computing Fig. 1.3 because the quadratic or higher order divergences will emerge in the Feynman integrations of these loop diagrams. Therefore, it is unsuitable to factorize these divergences by using $g$ in which only the logarithmic one is involved.

However, if watching these Feynman diagrams carefully, one will find that the contribution from Fig. 1.3 could be ignored. Considering one-loop correction to the $\rho - \gamma$ vertex (Fig. 2), and comparing Fig. 1.3 with Fig. 2, one will see that the contribution of Fig. 1.3 to EM self-energies of $\rho$-mesons should vanish if the one-loop renormalized $\rho - \gamma$ vertex is used in Fig. 1.1.

The expressions of VMD in the $\text{U}(3)_L \times \text{U}(3)_R$ chiral theory of mesons have been derived in Ref. [2], which can be read as

$$\frac{e}{f_{\rho}}\{-\frac{1}{2}F_{\rho \mu \nu}^\mu J_{\mu \nu}^\rho + A_{\mu}^\mu J_{\mu}^\rho\},$$

(1)

$$\frac{e}{f_{\omega}}\{-\frac{1}{2}F_{\omega \mu \nu}^\mu J_{\mu \nu}^\omega + A_{\mu}^\mu J_{\mu}^\omega\},$$

(2)

$$\frac{e}{f_{\phi}}\{-\frac{1}{2}F_{\phi \mu \nu}^\mu J_{\mu \nu}^\phi + A_{\mu}^\mu J_{\mu}^\phi\},$$

(3)
where
\[
\frac{1}{f_\rho} = \frac{1}{2} g, \quad \frac{1}{f_\omega} = \frac{1}{6} g, \quad \frac{1}{f_\phi} = -\frac{1}{3\sqrt{2}} g.
\]
(4)

\( J^\rho_\mu, J^\omega_\mu, \) and \( J^\rho_\mu \) are the corresponding hadronic currents for \( \rho^0, \omega, \) and \( \phi \) mesons respectively.

By employing the \( \rho - \gamma \) vertex in eq. (1), the decay width of \( \rho \to e^+e^- \) is
\[
\Gamma(\rho \to e^+e^-) = \frac{4\pi \alpha^2_{\text{EM}} m_\rho}{f_\rho^2} \frac{m_\rho}{3}.
\]
(5)

Here \( \alpha_{\text{EM}} = \frac{e^2}{4\pi} \). When \( g=0.39 \) \(^2\), the numerical result of \( \Gamma(\rho \to e^+e^-) \) is 6.53 keV, which is in good agreement with the experimental data 6.77±0.32 keV \(^4\). It is expected that one-loop corrections to the \( \rho - \gamma \) vertex are very small, and can be ignored.

It is enough to evaluate the contributions from Fig. 1.1 and 1.2 to EM self-energies of \( \rho \)-mesons. The effect of Fig. 1.3 is included by renormalizing the \( \rho - \gamma \) vertex. In fact, Fig. 1.3 is not the 1PI (one particle irreducible) diagram for the one-loop EM self-energies of \( \rho \)-meson. Thus, it is straightforward to use the method developed in paper I to factorize the logarithmic divergences from mesonic loop diagrams, then to get a finite result of \( \rho^0 - \rho^\pm \) EM mass difference. The difficulties in calculating the EM self-energies of \( \omega \) and \( \phi \)-mesons can be circumvented similarly.

It has been pointed out by Sakurai \(^{13}\) that there are two ways of writing down VMD. In eqs. (1)-(3) (called as VMD1), the photon-meson coupling have two approaches: the first one is the direct coupling of photon and mesons \( (A^\mu J^i_\mu, i=\rho, \omega, \) and \( \phi) \), the second one is the interaction through the neutral vector mesons including \( \rho^0, \omega, \) and \( \phi \).

The other representation of VMD (called as VMD2) \(^3\), in which the photon-mesons

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\(^2\)if setting \( g=0.39 \) instead of \( g=0.35 \), we will get much better fit for the data than that in Ref. \( \| \), and \( g=0.39 \) has been set in paper I and Refs. \( \|, \|, \| \).

\(^3\)In Ref. \( \| \), the two representations of VMD have been referred as VMD1 and VMD2.
coupling must be through the neutral vector mesons, is expressed as follows

\[-\frac{e m^2}{f_\rho} \rho^\mu A^\mu + \frac{1}{2} \left(\frac{e m_\rho}{f_\rho}\right)^2 A_\mu A^\mu, \tag{6}\]

\[-\frac{e m^2}{f_\omega} \omega^\mu A^\mu + \frac{1}{2} \left(\frac{e m_\omega}{f_\omega}\right)^2 A_\mu A^\mu, \tag{7}\]

\[-\frac{e m^2}{f_\phi} \phi^\mu A^\mu + \frac{1}{2} \left(\frac{e m_\phi}{f_\phi}\right)^2 A_\mu A^\mu \tag{8}\]

As proved by Kroll, Lee, and Zumino [12], and by Sakurai [13], VMD1 and VMD2 are equivalent in describing the electromagnetic interaction of the mesons. Fortunately, in the framework of the $U(3)_L \times U(3)_R$ chiral theory of mesons and employing VMD2, the loop diagram (Fig. 1.3) contributing to the EM self-energies of the vector mesons which contains quadratic or higher order divergences will disappear automatically, and only the logarithmic divergence is involved in the rest loop diagrams. Our calculations show that, without considering Fig. 1.3, contributions to EM self-energies of $\rho$-mesons by using VMD1 are entirely equivalent to ones by using VMD2 directly.

The purposes of the present paper are twofold:

1. The investigation of EM self-energies of the vector mesons has been concerned by particle physicists. Here, we are to extend the method used in paper I to calculate the one-loop EM self-energies of the low-lying mesons in the case of vector mesons including $\rho$, $\omega$, and $\phi$-mesons, which makes the investigation in paper I much more complete and systematic.

2. An interesting effect, EM masses anomaly of the massive Yang-Mills particles, has been revealed in Ref. [16], which states that the non-Abelian gauge structure of the massive Yang-Mills particles makes the EM mass of the neutral $K^*(892)$ larger than that of the charged one. This claim will be re-examined in the cases of $\rho$ meson and other massive Yang-Mills particles elaborately.

The paper is organized as follows. In Sec. 2, the equivalence of VMD1 and VMD2 is discussed briefly. As examples, we show that, in the framework of the $U(3)_L \times U(3)_R$ chiral
theory of mesons, VMD1 and VMD2 could reduce the same results of the pion EM form factor and the EM mass splitting of $\pi^\pm - \pi^0$, and the well-known result of $m_{\pi^\pm}^2 - m_{\pi^0}^2$ obtained by Das, Guralnik, Mathur, Low, and Young [18] is reproduced. Sec. 3, the EM self-energies of the vector mesons including $\rho$, $\omega$, and $\phi$ are evaluated, and the EM mass anomaly of the massive Yang-Mills particles is discussed. Sec. 4, we give the summary of the results.

2 VMD1 and VMD2

VMD predates the standard model. It has not been derived from the standard model directly, but nevertheless enjoys phenomenological supports in describing hadronic electromagnetic interactions. In the $U(3)_L \times U(3)_R$ chiral theory of mesons, the basic lagrangian of the theory is (Hereafter we use the notations in paper I)

\[
\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + e_0 Q \gamma \cdot A + \gamma \cdot a_5 - m u(x))\psi(x)
\]

\[
+ \frac{1}{2} m_1^2 (\rho^\mu \rho_\mu + \omega^\mu \omega_\mu + a^\mu a_\mu + f^\mu f_\mu)
\]

\[
+ \frac{1}{2} m_2^2 (K^a_\mu K^{a\mu} + K_1^\mu K_1^\mu)
\]

\[
+ \frac{1}{2} m_3^2 (\phi^\mu \phi_\mu + f_1^\mu f_{s\mu}) + \mathcal{L}_{EM}
\] (9)

where $u(x) = \exp[i\gamma_5(\tau_i \pi_i + \lambda_a K^a + \eta + \eta')](i=1,2,3$ and $a=4,5,6,7)$, $a_\mu = \tau_i a^i_\mu + \lambda_a K^a_1 + (\frac{2}{3} + \frac{1}{\sqrt{3}} \lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}} \lambda_8)f_{s\mu}$, $v_\mu = \tau_i \rho^i_\mu + \lambda_a K^{a_1}_\mu + (\frac{2}{3} + \frac{1}{\sqrt{3}} \lambda_8) \omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}} \lambda_8) \phi_\mu$, $A_\mu$ is the photon field, $Q$ is the electric charge operator of $u$, $d$ and $s$ quarks, $\psi$ is quark-fields, $m$ is a parameter related to the quark condensate, and $\mathcal{L}_{EM}$ is the kinetic lagrangian of photon field. Thus, the use of path integral method to integrate out the quark fields will reduce the effective lagrangian of the mesons.

Since the photon field $A_\mu$ has been introduced into eq. (9), it is very natural to deduce the explicit expressions of VMD in the mesonic effective lagrangian, which are just the eqs.(1)-(3). The full lagrangian which describes the interaction of photon and the neutral vector
mesons could be expressed as follows,

\[ \mathcal{L}_{\text{VMD1}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{4} \rho^0_\mu \rho^{0 \mu} + \frac{1}{2} m^2 \rho^0_\mu \rho^{0 \mu} + \rho^{0 \mu} J^\rho_\mu + \frac{e}{f_{\rho}} \left( -\frac{1}{2} F^{\mu \nu} \rho^0_\mu + A^\mu J^\rho_\mu \right). \] (10)

For simplicity, here we only discuss the interactions between \( \rho^0 \) and the photon. VMD1 is manifestly gauge invariant for the neutral vector meson \( \rho^0 \) coupling with the conserved current in strong interaction. As pointed out by Sakurai [13], there is an equivalent way of writing down a gauge invariant expression for hadronic electromagnetic interaction, which is

\[ \mathcal{L}_{\text{VMD2}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} \rho^0_\mu \rho^{0 \mu} + \frac{1}{2} m^2 \rho^0_\mu \rho^{0 \mu} + \rho^{0 \mu} J^\rho_\mu - \frac{e m^2}{f_{\rho}} \rho^0_\mu A^\mu + \frac{1}{2} \left( \frac{e m^2}{f_{\rho}} \right)^2 A^\mu A^\mu. \] (11)

From VMD1 to VMD2, one can use the following transformation,

\[ \rho^0_\mu \rightarrow \rho^0_\mu - \frac{e}{f_{\rho}} A_\mu, \]
\[ A_\mu \rightarrow \sqrt{1 - \left( \frac{e}{f_{\rho}} \right)^2} A_\mu, \] (12)
\[ e \rightarrow e \sqrt{1 - \left( \frac{e}{f_{\rho}} \right)^2}. \]

It seems that the mass term of the photon field in eq. (11) violates gauge invariance of the electromagnetic interaction. However, the \( \rho \cdot A \) term in this equation contributes an imaginary mass term of the photon, which exactly cancels the contribution from mass term of the photon. To illustrate this, one could calculate the modification of the photon propagator using the new interaction lagrangian including the new mass term of the photon (Fig. 3).

From Fig. 3, one will obtain

\[ iD(k^2) = \frac{-i}{k^2} + \frac{-i e m^2}{f_{\rho}} \frac{1}{k^2 - m^2} \frac{k}{k^2} + \frac{-i e m^2}{f_{\rho}} \frac{k^2}{k^2} + \frac{-i e^2 m^2}{f_{\rho}} \frac{k}{k^2} + \ldots \]
\[ = \frac{-i}{k^2} + \frac{-i e m^2}{f_{\rho}} \frac{k^2}{k^2 - m^2} \frac{k}{k^2} + \frac{-i e^2 m^2}{f_{\rho}} \frac{k}{k^2} + \ldots \]
Using the operator identity
\[
\frac{1}{A - B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{B} + \frac{1}{A} B \frac{1}{A} B \frac{1}{A} + \ldots,
\]
we have
\[
iD(k^2) = \frac{-i}{k^2 - \frac{e^2 m^2_{\pi}}{f^2_{\rho}} k^2 - m^2_{\rho}} \rightarrow \frac{-i}{k^2} (1 - \frac{e^2}{f^2_{\rho}})
\]
(13)
for small \(k^2\). Thus, we are simply left with a change in EM coupling constant \(e\)
\[
e^2 \rightarrow e^2(1 - \frac{e^2}{f^2_{\rho}}).
\]

In the following, as an example, we show the equivalence of VMD1 and VMD2 by calculating pion EM form factor. In Ref. [2], the interaction \(\mathcal{L}_{\rho\pi\pi}\) has been derived
\[
\mathcal{L}_{\rho\pi\pi} = 2 \epsilon_{ijk} \rho_{\mu}^{i} \pi^{j} \partial^{\mu} \pi^{k} - \frac{2}{\pi^2 f^2_{\pi} g} [(1 - \frac{2c}{g})^2 - 4\pi^2 c^2] \epsilon_{ijk} \rho_{\mu}^{i} \partial_{\nu} \pi^{j} \partial^{\mu} \pi^{k},
\]
(14)
where \(c = \frac{f^2_{\pi}}{2gm^2_{\rho}}\), and \(f_{\pi}\) is the decay constant of pion with value 0.186 GeV. Thus, the pion EM form factor \(F_{\pi}(k^2)\) can be defined through the amplitude of process \(\gamma \rightarrow \pi^+ \pi^-\)
\[
\mathcal{M}_{\gamma \rightarrow \pi^+ \pi^-} = -e(q^+ - q^-)^{\mu} F_{\pi}(k^2),
\]
(15)
here \(q\) and \(k\) are momentum of pion and the virtual photon respectively. From VMD1 (as shown in Fig. 4.1), we have
\[
F_{\pi}(k^2) = (1 + \frac{k^2}{m^2_{\rho} - k^2}) \frac{f_{\rho\pi\pi}(k^2)}{f_{\rho}}
\]
(16)
where
\[
f_{\rho\pi\pi}(k^2) = \frac{2}{g} \left\{ 1 + \frac{k^2}{2\pi^2 f^2_{\pi}} [(1 - \frac{2c}{g})^2 - 4\pi^2 c^2] \right\}.
\]
Note that the second term in eq. (16) due to the direct interaction of \( \rho \) and \( \gamma \) vanishes at \( k^2 = 0 \), and it is therefore very important to take into account the effect of the contact \( J_\mu^\rho A^\mu \) interaction.

From VMD2 (as shown in Fig. 4.2), one obtains

\[
F_\pi(k^2) = \frac{m_\rho^2}{m_\rho^2 - k^2} \frac{f_\rho\pi(k^2)}{f_\rho}. \tag{17}
\]

It is obvious that the two approaches lead to the identical result from eq. (16) and eq. (17).

Now, we show that VMD1 and VMD2 are equivalent in evaluating the EM mass splitting of pions. In paper I, the EM mass difference between \( \pi^\pm \) and \( \pi^0 \) in the chiral limit has been obtained in terms of VMD1. Here we will re-examine this calculation by employing VMD2.

The Feynman diagrams contributing to \( (m_\pi^\pm - m_\pi^0)_\text{EM} \) have been shown in Fig. 5 and Fig. 6, which correspond to ones from VMD1 and VMD2 respectively. The diagrams which receive contributions from the interaction \( \mathcal{L}_{\rho\pi\pi} \) have been neglected because their contributions vanish in the chiral limit. This point has been demonstrated in paper I. Since there are no the direct vertices of \( \pi \) and \( \gamma \), the number of the Feynman diagrams contributing to EM self-energies of pions in terms of VMD2 has been much more decreased.

Comparing Fig. 5 and Fig. 6, one will find that if the effective propagators in Fig. 7.1 and Fig. 7.2 are equivalent, the contributions of Fig. 5 and Fig. 6 must be the same to the EM self-energies of pion-mesons. Note that in Fig. 7.1, the \( \rho - \gamma \) vertex comes from VMD1, which is momentum dependent; in Fig. 7.2, the \( \rho - \gamma \) vertex is from VMD2. It is easy to prove that Fig. 7.1 and Fig. 7.2 both contribute

\[
-\frac{i \epsilon^2}{k^2 f_\rho^2} \left[ \frac{m_\rho^4}{(k^2 - m_\rho^2)^2} (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) + \alpha \frac{k_\mu k_\nu}{k^2} \right], \tag{18}
\]

where \( \alpha \) is the gauge parameter. Because there is always a factor \( \frac{\epsilon}{f_\rho} \) in the coupling of the photon with the current \( J_\mu^\rho \) in VMD1, this factor has been considered in calculating Fig. 7.1. Eq. (18) has been obtained by Lee and Nieh in Ref. [17]. Therefore, it is not surprising
that the result of EM self-energies of pions in terms of VMD1 is the same as one in terms of VMD2.

The straightforward calculation of Fig. 6 leads to

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}} = \frac{i e^2}{f_\pi^2} \int \frac{d^4k}{(2\pi)^4} (D - 1) m_{\rho}^4 \left( \frac{F^2 + \frac{k^2}{4\pi^2}}{k^2(m_{\rho}^2 - m_{\rho}^2)^2} \right) \left[ 1 + \frac{\gamma^2 F^2 + \frac{k^2}{4\pi^2}}{g^2 k^2 - m_a^2} \right].
\]

This is just eq. (34) in paper I, which is derived in terms of VMD1. The gauge independence of \((m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}}\) can be proven in the same way as that in paper I.

In deriving eq. (19) or eq. (34) in paper I, the calculations on EM-masses are up to the fourth order covariant derivatives in effective lagrangians [2]. In the remainder of this section, we find that the well-known result of \((m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}}\) given by Das et al [18] can be reproduced if the EM self-energies of pions receives the contributions only from the second order derivative terms. In this case, the interaction lagrangians \(L_{\rho\rho\pi\pi}\) and \(L_{\rho\pi\alpha}\) [eqs.(18)(19) in paper I] will be simplified as follows

\[
L_{\rho\rho\pi\pi} = \frac{2F^2}{g^2 f_\pi^2} \epsilon^{ij} \epsilon^{\rho\mu} \left( \pi^2 \delta_{ij} - \pi_i \pi_j \right), \quad L_{\rho\pi\alpha} = - \frac{2F^2}{f_\pi g^2} \epsilon^{ij} \epsilon^{\rho\mu} \pi^k a_i \pi^j.
\]

Thus, in the chiral limit, the EM mass difference of pions is

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}} = \frac{i 3\alpha^2}{f_\pi^2} \int \frac{d^4k}{(2\pi)^4} m_{\rho}^4 \left( \frac{F^2}{k^2(m_{\rho}^2 - m_{\rho}^2)^2} \right) \left( 1 + \frac{F^2}{g^2(k^2 - m_a^2)} \right). \tag{20}
\]

The Feynman integration in eq.(20) is finite. So it is straightforward to get the result of \((m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}}\) after performing this integration, which is

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}} = \frac{3\alpha_{\text{EM}} m_{\rho}^4}{8\pi f_\pi^2} \left\{ \frac{2F^2}{m_{\rho}^2} - \frac{2F^4}{g^2(m_{\alpha}^2 - m_{\rho}^2)(m_{\rho}^2 - m_{\rho}^2) + \frac{1}{m_{\rho}^2 - m_{\rho}^2} \log m_{\rho}^2} \right\}. \tag{21}
\]

Because we only consider the second order derivative terms in the lagrangian, the relation between \(m_{\alpha}\) and \(m_{\rho}\) is \(m_{\alpha}^2 = \frac{F^2}{g^2} + m_{\rho}^2\) instead of eq.(4) in paper I. Thus, we can get

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}} = \frac{3\alpha_{\text{EM}} m_{\rho}^2 m_{\alpha}^2}{4\pi} \frac{m_{\rho}^2}{m_{\alpha}^2 - m_{\rho}^2} \log \frac{m_{\rho}^2}{m_{\alpha}^2}. \tag{22}
\]
When substituting the relation \( m_a^2 = 2m_\rho^2 \), which can be derived from the Weinberg sum rules \[^{13}\,\] , into eq.(21), we have

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{\text{EM}} = \frac{3\log 2}{2\pi} \alpha_{\text{EM}} m_\rho^2,
\]

which is exactly the result obtained by Das et al. \[^{18}\,\] , and serves as the leading term of eq.(19).

### 3 EM self-energies of the vector mesons and EM mass anomaly of the massive Yang-Mills particles

The subject of EM contributions of vector mesons is in fact very old. The early attempts are in \[^{17,20,21,}\,\] , and the recent are in \[^{22,23}\,\] . In Ref. \[^{1,16}\,\] , the EM masses of \( K^*(892) \) have been evaluated in the framework of the \( U(3)_L \times U(3)_R \) chiral theory of mesons. In this section, we will deal with the EM self-energies of \( \rho \), \( \omega \), and \( \phi \)-mesons to one-loop order and \( O(\alpha_{\text{EM}}) \).

The one-loop Feynman diagrams contributing to the EM self-energies of \( \rho \)-meson are only from the massive Yang-Mills (MYM)self-interactions and Gauging Wess-Zumino-Witten (GWZW) anomaly interactions in the effective lagrangian of the \( U(3)_L \times U(3)_R \) chiral theory of mesons \[^{2}\,\] . This case is the same as that of \( K^*(892) \)-meson \[^{16}\,\] .

From Ref. \[^{2}\,\] , MYM lagrangian related to \( \rho \)-meson is

\[
\mathcal{L}_{\text{MYM}} = -\frac{1}{8} Tr(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu - \frac{i}{g}[\rho_\mu, \rho_\nu] - \frac{i}{g}[a_\mu, a_\nu])^2 + \text{mass terms of } \rho.
\]

Thus, the three-point and four-point MYM interaction vertices which contribute to EM self-energies of \( \rho \)-meson can be read from eq. (24)

\[
\mathcal{L}_{\text{MYM}}^{(3)} = \frac{1}{g} \epsilon_{ijk} \partial_\mu \rho_i^j \rho_j^i \rho_k^\nu,
\]

\[
\mathcal{L}_{\text{MYM}}^{(4)} = \frac{1}{g^2} (\rho_i^j \rho_i^j \rho_i^j \rho_i^j - \rho_i^j \rho^j \rho^j \rho^j)
\]
The GWZW anomaly lagrangian in the present theory contributing to EM self-energies of ρ-meson is [2]

$$\mathcal{L}_{GWZW}^{\omega\rho\pi} = -\frac{3}{\pi^2 g^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \rho^\alpha_i \partial_\beta \pi^i.$$  \hspace{1cm} (27)

Now, combining VMD1 [eqs. (1), (2)] or VMD2 [eqs. (6), (7)], one can ev aluate the one-loop EM self-energies of ρ-meson in the standard way presented in paper I. The corresponding diagrams are shown in Figs. 8, 9, and 10. Note that all the diagrams will contribute to EM self-energies of ρ-meson if we use VMD1 while only Figs. 8.3, 9.3 and 10.3 give contributions if using VMD2. In the following, we calculate these diagrams separately.

From Fig. 8, the EM mass square difference corresponding to the contributions of three-point MYM interactions can be calculated directly, which is

$$m_{EM}^2(\rho^\pm)_{MYM}^{(3)} = i e^2 \int \frac{d^4k}{(2\pi)^4} \frac{m_\rho^4}{k^2 - 2p \cdot k - m_\rho^2} \times [3k^2 + 4m_\rho^2, \frac{(k^2)^2}{m_\rho^2} - \frac{4(k \cdot p)^2}{k^2} + 2k \cdot p + \frac{\langle (k \cdot \rho_\mu)(k \cdot \rho_\mu) \rangle}{\langle \rho_\mu \rho_\mu \rangle}] (9 + \frac{k^2}{m_\rho^2} - \frac{2k \cdot p}{k^2}),$$  \hspace{1cm} (28)

where $p$ is 4-momentum of ρ-meson, and $p^2 = m_\rho^2$; $\langle \rho_\mu \rho_\mu \rangle = \int d^4x \langle \rho[\rho_\mu(x)\rho_\nu(x)]\rho \rangle$, and $i = 1, 2$. After performing the Feynman integrations in eq. (28), we have

$$m_{EM}^2(\rho^\pm)_{MYM}^{(3)} = -e^2 m_\rho^2 [\frac{3}{4}\chi_\rho + \frac{1}{16\pi^2}(\frac{11\pi}{4\sqrt{3}} - \frac{37}{12})].$$  \hspace{1cm} (29)

The logarithmic divergence is involved in $\chi_\rho$, which has been factorized by using the intrinsic parameter $g$ of the theory in paper I,

$$\chi_\rho = \frac{1}{g^2} + \frac{1}{32\pi^2} + \frac{1}{16\pi^2} \log \frac{f_\pi^2}{6(g^2 m_\rho^2 - f_\pi^2)},$$  \hspace{1cm} (30)

g = 0.39 has been fixed in paper I. Substituting the experimental values of $f_\pi$ and $m_\rho$ into eq. (29), we obtain the numerical result

$$m_{EM}^2(\rho^\pm)_{MYM}^{(3)} = -3.36 \times 10^{-4} \text{ GeV}^2.$$  \hspace{1cm} (31)
Similarly, the contribution to EM self-energies of $\rho$-meson (Fig. 9) from four-point MYM interaction is

$$m_{\text{EM}}^{(4)}(\rho^\pm)_{\text{MYM}} = -ie^2 \frac{9m_\rho^4}{4} \int \frac{d^4k}{(2\pi)^4 \frac{k^2}{(k^2 - m_\rho^2)^2}}.$$  

(32)

It is remarkable that eq. (32) is free of the divergence and independent of $g$, so it is straightforward to get

$$m_{\text{EM}}^{(4)}(\rho^\pm)_{\text{MYM}} = -\frac{9e^2}{64\pi^2m_\rho^2}.$$  

(33)

Numerically

$$m_{\text{EM}}^{(4)}(\rho^\pm)_{\text{MYM}} = -7.75 \times 10^{-4} \text{ GeV}^2.$$  

(34)

The contribution to the EM self-energies of $\rho$-mesons from GWZW anomaly can be evaluated by using eq. (27) [Fig. 10]. However, it is easy to see that the contribution of Fig. 10 leads to the same shift of the charged and neutral $\rho$-meson masses, so it has no contribution to the EM mass difference of $\rho^\pm - \rho^0$. The EM self-energies of $\rho$-meson in this case is easily obtained

$$m_{\text{EM}}^{(4)}(\rho^0)_{\text{GWZW}} = m_{\text{EM}}^{(4)}(\rho^\pm)_{\text{GWZW}} = \frac{e^2m_\rho^4}{128\pi^6g_\pi^2f_\pi^2} = 9.96 \times 10^{-5} \text{ GeV}^2.$$  

(35)

There is also no divergences involved in Fig. 10. We take $m_\rho^2 = m_\rho^2$ in deriving eq. (35), and its numerical result is small.

Different from the case of $K^*(892)$-meson, the direct $\rho^0$-photon coupling which comes from VMD can bring the tree diagram contributing to EM masses of $\rho^0$, which has been shown in Fig. 1.1. It is easy to evaluate its contribution

$$m_{\text{EM}}^{2}(\rho^0)_{\text{Tree}} = -\frac{e^2g^2}{4}m_\rho^2.$$  

(36)

Numerically

$$m_{\text{EM}}^{2}(\rho^0)_{\text{Tree}} = -2.06 \times 10^{-3} \text{ GeV}^2.$$  

(37)
Totally, $\rho^0 - \rho^\pm$ EM mass difference is

$$ (m_{\rho^0}^2 - m_{\rho^\pm}^2)_{\text{EM}} = -9.49 \times 10^{-4} \text{ GeV}^2, \quad (38) $$

$$ (m_{\rho^0} - m_{\rho^\pm})_{\text{EM}} = -0.62 \text{ MeV}. \quad (39) $$

The light quark mass terms of the QCD lagrangian are

$$ \mathcal{L}_m = -m_u \bar{u}u - m_d \bar{d}d - m_s \bar{s}s - ... $$

It is important to note that the isospin violating piece of $\mathcal{L}_m$, $-\frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u)$, transforms as $\Delta I = 1$ under the isospin subgroup. The operator which produces the $\pi^\pm - \pi^0$ (or $\rho^\pm - \rho^0$) mass difference must have $\Delta I = 2$, therefore $(m_{\pi^\pm} - m_{\pi^0})_{\text{qm}} = (m_{\rho^\pm} - m_{\rho^0})_{\text{qm}} = 0$ at the leading order in quark mass expansion (The subscript qm denotes the contribution from quark mass). This has been verified in the case of pions, as is well known that the $\pi^\pm - \pi^0$ mass difference is almost entirely electromagnetic in origin.

The experimental value of $m_{\rho^0} - m_{\rho^\pm}$ [14] has a large error bar

$$ m_{\rho^0} - m_{\rho^\pm} = 0.1 \pm 0.9 \text{ MeV}. \quad (40) $$

Recent result obtained by ALEPH Collaboration [24] is

$$ m_{\rho^0} - m_{\rho^\pm} = 0.0 \pm 1.0 \text{ MeV}. \quad (41) $$

Comparing with eqs. (40) and (41), one will find that our prediction [eq. (39)] is in agreement with the measurements. This means that EM mass correction to the mass difference of $\rho$-meson is important, although its value is much smaller than the corresponding one of pions.

The EM self-energies of $\omega$-meson can also be calculated. The tree level contribution from

$$ \omega \rightarrow \gamma \rightarrow \omega, \text{ which is} $$

$$ m_{\omega}^2 \text{EM (\omega)}_{\text{Tree}} = -\frac{e^2 g^2 m_{\omega}^2}{36} = -2.37 \times 10^{-4} \text{ GeV}^2. \quad (42) $$
Besides the contribution from tree diagram, eq. (27) also contributes one-loop diagrams to the EM self-energies of $\omega$-meson (as shown in Fig. 11), and the loop diagrams give the finite contributions,

$$m_{\text{EM}}^2(\omega)_{\text{1-loop}} = \frac{9e^2m^2_{\mu}m^2_{\omega}}{128\pi^6g^2f^2_\pi} = 4.62 \times 10^{-4} \text{ GeV}^2.$$  (43)

Thus the numerical result of the total EM self-energies of $\omega$-meson to one-loop order and $O(\alpha_{\text{EM}})$ is

$$m_{\text{EM}}^2(\omega)_{\text{total}} = 2.25 \times 10^{-4} \text{ GeV}^2,$$

$$(m_\omega)_{\text{EM}} = 0.14 \text{ MeV}. \quad (44)$$

In the framework of the $U(3)_L \times U(3)_R$ chiral theory of mesons, the vector mesons are written as

$$v_\mu = \tau_\alpha \rho^\alpha + \lambda_a K^a + \left(\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8\right)\omega_\mu + \left(\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8\right)\phi_\mu,$$

which means that the $\omega$-meson is free of $s\bar{s}$, $\phi$-meson is pure $s\bar{s}$, and the mixing of $\omega$ and $\phi$-meson is ideal. This is implied by the Okubo-Zweig-Iizuka (OZI) rule \[27\]. In the large $N_C$ limit, the calculation of the EM self-energies of $\phi$-meson will be much simpler, only the tree level contribution from $\phi \rightarrow \gamma \rightarrow \phi$. Therefore we have

$$m_{\text{EM}}^2(\phi) = -\frac{e^2g^2m^2_{\phi}}{18} = -8.06 \times 10^{-4} \text{ GeV}^2,$$

$$(m_\phi)_{\text{EM}} = -0.40 \text{ MeV}. \quad (46)$$

Empirically, the $U(3)_L \times U(3)_R$ symmetry is broken due to the strong U(1) anomaly. Therefore the corrections from the next leading order corrections of large $N_C$ expansion to the processes which are related to the $\omega$ and $\phi$ mixing are non-trivial. However, this is beyond the scope of the present paper.

In Ref. \[16\], the EM masses of $K^*(892)$-meson has been calculated, especially, one effect called as EM mass anomaly of massive Yang-Mills particles has been revealed. In the case
of $\rho$-meson, it is easy to find that this conclusion also hold from eqs. (31) and (34) because the three-point and four-point MYM interactions contribute the negative values to the EM masses of $\rho^\pm$. Actually, this effect can also be seen in evaluating the EM self-energies of the axial-vector mesons $a_1$ and $K_1$ for these mesons are also introduced into the $U(3)_L \times U(3)_R$ chiral theory of mesons as the massive Yang-Mills particles. In paper I, EM masses of $a_1$ and $K_1$ have been calculated. Here we extract the contributions coming from the interaction lagrangians related to the three-point and four-point MYM vertices,

$$m_{EM}^2(a_{\pm})_{MYM}^{(3)} = -0.002688 \text{ GeV}^2,$$

$$m_{EM}^2(a_{\pm})_{MYM}^{(4)} = -0.000648 \text{ GeV}^2,$$

for $a_1$-meson, and

$$[m_{EM}^2(K_{1}^\pm) - m_{EM}^2(K_{1}^0)]_{MYM}^{(3)} = -0.003474 \text{ GeV}^2,$$

$$[m_{EM}^2(K_{1}^\pm) - m_{EM}^2(K_{1}^0)]_{MYM}^{(4)} = -0.000781 \text{ GeV}^2,$$

for $K_1$ meson. Here, it is obvious that the contributions from the MYM self-interaction make the EM masses of the neutral mesons larger than that of the charged one for the massive Yang-Mills particles.

It is expected that this is a non-trivial and unusual effect for the massive Yang-Mills particles because it is contrary to the common knowledge of the hadron’s EM-masses, such as $m(\text{neutron})_{EM} < m(\text{proton})_{EM}$, $m(\pi^0)_{EM} < m(\pi^+)_{EM}$, and $m(K^0)_{EM} < m(K^+)_{EM}$ [25]. The studies in Refs. [16, 26] show that the experiment favors to support this EM masses anomaly effect. However, the experimental information for the EM masses of the vector and axial-vector mesons is rather poor, so this effect needs to be further investigated.
4 Summary

In terms of two equivalent representations of VMD (VMD1 and VMD2), the method developed in paper I to calculate the one-loop EM self-energies of the low-lying mesons has been naturally extended to the case of the vector sector including $\rho$, $\omega$, and $\phi$-mesons. Because all the parameters of the theory have been fixed previously, this is a parameter free study in the present paper.

The theoretical result of the EM mass difference of $\rho$ meson is in agreement with the measurements, which means that the EM contribution is important to the $\rho^0 - \rho^\pm$ mass difference. The EM self-energies of $\omega$ and $\phi$-mesons which make the shifts for the mass of the mesons are also evaluated.

It has been shown that the effect of EM mass anomaly of the massive Yang-Mills particles, which has been revealed in Ref. [16], holds in the case of the $\rho$-meson. An elaborate analysis of the cases of $a_1$ and $K_1$ also supports this conclusion. This interesting effect should be further investigated, and also be tested by the experiments in the future.

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**Caption**

**Fig. 1** The diagrams contributing to electromagnetic self-energies of $\rho$–meson up to one-loop order.

**Fig. 2** The $\rho - \gamma$ vertex and its one-loop corrections.

**Fig. 3** The modification of the photon propagator by using VMD2. The $\times$ denotes the contribution from the mass term of the photon in eq. (11).
**Fig. 4** The pion electromagnetic form factor in terms of VMD1 and VMD2. Fig. 4.1 is from VMD1, and Fig. 4.2 is from VMD2.

**Fig. 5** The one-loop diagrams contributing to electromagnetic mass difference between $\pi^\pm$ and $\pi^0$ by using VMD1.

**Fig. 6** The one-loop diagrams contributing to electromagnetic mass difference between $\pi^\pm$ and $\pi^0$ by using VMD2.

**Fig. 7** The effective propagators coming from VMD. Fig. 7.1 is in terms of VMD1, Fig. 7.2 is in terms of VMD2.

**Fig. 8** The one-loop diagrams contributing to electromagnetic self-energies of $\rho$–meson from three-point MYM interactions. All three diagrams give contributions if using VMD1 while only Fig. 8.3 does if using VMD2.

**Fig. 9** The one-loop diagrams contributing to electromagnetic self-energies of $\rho$–meson from four-point MYM interactions. All three diagrams give contributions if using VMD1 while only Fig. 9.3 does if using VMD2.

**Fig. 10** The one-loop diagrams contributing to electromagnetic self-energies of $\rho$–meson from GWZW anomaly lagrangians. All three diagrams give contributions if using VMD1 while only Fig. 10.3 does if using VMD2.

**Fig. 11** The tree and one-loop diagrams contributing to electromagnetic self-energies of $\omega$–meson.
Fig. 1
\[\begin{align*}
\rho & \quad \gamma \\
\hline
& \quad \\
\end{align*} \] (2.1)

\[\begin{align*}
\rho & \quad \gamma \\
\hline
& \quad \\
\end{align*} \] (2.2)

Fig. 2
Fig. 3
\begin{align}
\begin{array}{c}
\pi \quad \gamma \quad \pi \\
\hline
\end{array} & + & \\
\begin{array}{c}
\pi \quad \rho \quad \pi \\
\hline
\end{array}
\end{align}

(4.1)

\begin{align}
\begin{array}{c}
\pi \quad \rho \quad \pi \\
\hline
\end{array} & \quad \gamma \\
\end{align}

(4.2)

Fig. 4
Fig. 5
Fig. 6
\[
\gamma \quad + \quad \rho \quad \gamma \quad + \quad \rho \quad \gamma \quad \rho \\
(7.1)
\]

\[
\rho \quad \gamma \quad \rho \\
(7.2)
\]

Fig. 7
Fig. 8
(9.1) \hspace{2cm} (9.2) \hspace{2cm} (9.3)

Fig. 9
Fig. 10
Fig. 11