Application of complex networks and indirect influences to non-forced migration

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Abstract. The migration phenomena of human populations is a well-known issue in social, economic, and sociophysics studies. A common effect of non-forced migration is the fact that important cities gain population over the first years and become overpopulated. Therefore, neighboring cities receive all migration and end up geographically merging with the important ones. Several studies have addressed the social and economic reasons behind this effect, but a mathematical model has been lacking. Here, we construct a migration complex network with population and migration dynamics and carry out an indirect influences analysis of those dynamics. Using this, we can measure the effect of migration on population growth across cities. The results show that the analysis of the indirect influences reveals interesting facts about the mentioned migration effect and address the measurement of this. Given this, urban planners and city administrations can make use of these findings to improve their migratory research.

1. Introduction

In many countries, migration represents a situation that raises economic, social, political, and environmental problems. Examples of this are the case of the United States of America with Mexican immigrants [1–3], the situation of people in the Middle East running escaping war [4,5], displaced Colombians migrating to capitals trying to survive [6–8] and the case of those who merely seek a better quality of life [9,11].

Academic researchers have dedicated investigations to this topic in several branches of science; there are applied mathematics articles using complex networks [12–15]. Other authors use a stochastic methodology [16]; in [17] the authors detail several migration models and many more research documents can be found [18–21]. The motivations to migrate can vary widely: violence, economy, social situations, and others. We now understand certain parts of the phenomena; however, it is still impossible to predict the next massive migration, or to control the effects, or to prevent it.

Addressing the phenomena with an urban, social, and economic interest, we can see the obvious problem in the city of Bogotá, Colombia. Being Colombia’s capital is the preferred destination for Colombians looking for better opportunities. Security, transportation, cultural and other problems are symptoms of its overpopulation. Neighboring towns to Bogotá, Colombia, such as Soacha, Fusagasugá, and Chía, have shown accelerated growth in the last years [22]. We call this effect as the neighbor effect, and some questions raise: it is accurate to
say that people that migrated have arrived at these towns instead of the capital? And, did the former capital inhabitants move to surrounding areas?

In Valle del Cauca, Colombia, the same symptoms are starting to occur. Therefore, we choose this department as a case study. It is possible, with available data, to make a simulation that predicts the behavior in the future. As such, we construct a dynamical network that models the migration phenomena, and then we solve it using numerical methods. Also, we apply indirect influences analysis to it to get an insight into migration phenomena and the particular effect described before. The definition of the dynamical network requires some new considerations about migration models, and the analysis of the indirect influences provides a way to measure the neighbor effect.

2. Materials and methods
We construct the migration complex network. Nodes are the different municipalities in Valle del Cauca, Colombia, and we associate the logistic growth with each population. Edges represent existing inter-municipal roads, which are the principal transportation means for migration. We then describe the migration dynamics and end up defining our network differential equation.

2.1. Nodes and population growth dynamics
For this case study, the forty-two towns in Valle del Cauca, Colombia, are nodes. We describe the population growth dynamics in each node by the logistic equation given in Equation (1), where \( \alpha_i \) represents the growth rate of node \( i \), \( x_i \) is the state variable that describes the number of individuals living in node \( i \) at time \( t \), and \( N_i \) is the loading capacity of node \( i \).

\[
G_i(x_i) = \alpha_i x_i \left( 1 - \frac{x_i}{N_i} \right). \tag{1}
\]

Now, in practice, the growth rate \( \alpha_i \) of a town is variable according to the year because of socio-economic factors. We consider these parameters as constants. In the same way, loading capacity \( N_i \) is also a variable depending on each node’s infrastructure, economic, sanitary, geographical, and other factors. We consider these parameters as constants equal to five times the initial population. This choice is arbitrary, but it allows us to see system evolution without excessive individuals in the simulation.

2.2. Edges and migration condition
We choose network edges from a road map as the roads that connect two towns directly. For example, if when going from city \( A \) to \( B \), we must cross \( C \), then we only consider the roads from \( A \) to \( C \) and from \( C \) to \( B \). To analyze indirect influences, we define the weight of each edge as the number of persons migrating through it. We establish the migration condition \( c_i \) as 0.9 for \( i = 1, 2, \ldots, 42 \). These values obey the fact that individuals do not migrate from a node if conditions are desirable, i.e. loading capacity \( N_i \) is greater than the population. So, we define the migration function from the node \( i \) as described in Equation (2), where \( \kappa_i \) is the annual migration rate.

\[
f_i(x_i) = \begin{cases} 
0 & \text{if } x_i < 0.9 N_i \\
\kappa_i x_i & \text{if } x_i \geq 0.9 N_i 
\end{cases}. \tag{2}
\]

So, we configure a non-forced migration, as individuals only migrate when nodes are getting full. To compute \( \kappa_i \), we adapt the well-known gravitational migration model that computes the number of people traveling from region \( j \) to region \( i \), by the Equation (3), where \( k \) is a proportionality constant that depends on the study, and \( d_{ij} \) is the distance between regions.
\[ M_{ij} = k \frac{P_i P_j}{d_{ij}^2}. \] (3)

For simplicity, we consider \( k = 1 \) in this study, according to George Zipf’s considerations [24].

The most discussed problem with this model is its symmetry \( M_{ij} = M_{ji} \), which is something that real migrations don’t exhibit. To give asymmetry to the gravitational model, we define a population weighting as follows. First, if \( j \) is the starting node, we compute the relevance \( W_j(t) \) of the population in node \( j \) on time \( t \) among its neighbors using Equation (4), where \( A \) is the adjacency matrix of the network.

\[ W_j(t) = \frac{x_j(t)}{x_j(t) + \sum_{k=1}^{n} A_{kj}x_k(t)}. \] (4)

Second, for the adjacent nodes to \( j \) we define \( W_{i\{j\}}(t) \) as the relevance of the population in node \( i \) on time \( t \) among the neighbors of \( j \) by using Equation (5).

\[ W_{i\{j\}}(t) = \frac{A_{ij}x_i(t)}{x_j(t) + \sum_{k=1}^{n} A_{kj}x_k(t)}. \] (5)

Therefore, we describe our modified gravitational model by Equation (6). Notice that we add the time variable \( t \) to the basic gravitational migration model and replace \( P_i \) with \( W_{i\{j\}}(t)x_i(t) \) and \( P_j \) with \( W_j(t)x_j(t) \), so our model is now asymmetric, and takes into account the population in all the adjacent nodes of the migrational origin.

\[ M_{ij}(t) = W_{i\{j\}}(t)W_j(t)\frac{x_i(t)x_j(t)}{d_{ij}^2}. \] (6)

With this model, we define the migration rate from node \( i \) using Equation (7).

\[ \kappa_i = W_i(t) \sum_{j=1}^{n} W_{i\{j\}}(t)\frac{x_j(t)}{d_{ij}^2}. \] (7)

2.3. Migration dynamics

Now, provided the migration function \( f_i(x_i) \), individuals are moving between neighbor nodes but, we need to take destination preferences into account because not all nodes are equally attractive, thanks to the modified gravitational migration model. For that, we define the preference of moving from node \( j \) to node \( i \) by Equation (8).

\[ \gamma_{ij} = \frac{W_{i\{j\}}(t)\left(\frac{x_i(t)}{d_{ij}^2}\right)}{\sum_{k=1}^{n} W_{k\{j\}}(t)\left(\frac{x_k(t)}{d_{kj}^2}\right)}. \] (8)

Note that \( \sum_{i=1}^{n} \gamma_{ij} = 1 \) so the matrix \( \gamma = (\gamma_{ij}) \) has stochastic columns; to get the amount of people arriving from node \( j \) to node \( i \), we use the values of \( \gamma_{ij} \) and \( f_j(x_j) \) to obtain \( \gamma_{ij}f_j(x_j) \), which is how many people go from \( j \) to \( i \). Finally, with Equation (9) we obtain the total amount of people getting to node \( i \) from its neighbors at a certain time.

\[ g_i(t) = \sum_{j \in Ady(i)} \gamma_{ij}f_j(x_j). \] (9)
2.4. Network dynamics model

Now, we are ready to define the differential equation that describes the population dynamics in the node \( i \). Combining the internal population dynamics at node \( i \), the migration function from \( i \), and the number of people getting to \( i \), we get the Equation (10).

\[
\frac{dx_i}{dt} = \alpha_i x_i \left(1 - \frac{x_i}{N_i}\right) - f_i(x_i) + \sum_{j \in \text{Ady}(i)} \gamma_{ij} f_j(x_j). \tag{10}
\]

To simplify, we give a vector equation for the entire network. Let \( \gamma = (\gamma_{ij}) \) be the matrix of movement preferences, let \( x = (x_1, x_2, \ldots, x_{42}) \) be the populations vector, let \( N = (N_1, N_2, \ldots, N_{42}) \) be the loading capacities vector, let \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{42}) \) be the growth rates vector, and let \( f(x) = (f_1(x_1), f_2(x_2), \ldots, f_{42}(x_{42})) \) be the migration function. Therefore, the dynamical equation of the network is described by the Equation (11), where \( G(x) = (G_1(x_1), G_2(x_2), \ldots, G_{42}(x_{42})) \) and \( I \) is the 42 × 42 identity matrix. Equation (11) is a Filippov system \([25, 26]\) and its numerical integration is straightforward.

\[
\frac{dx}{dt} = G(x) + (\gamma - I)f(x). \tag{11}
\]

3. Results

Given the network dynamical equation, we solve it numerically, and also we compute the indirect influences and dependencies during each integration iteration. Then we analyze and discuss the results.

3.1. Numerical solution and interpretation

Given the size of the network, we select some nodes to show their population evolutions and indirect influences and dependencies. To do this, we compute the average of centralities and select the twelve most representative. We consider the following centrality measures: degree, closeness, betweenness, and communicability. We provide the plots of centralities and average of centralities as annexes. For the numerical integration, we use the RK4 Runge-Kutta method \([27, 28]\), and a 0.1 time step. As initial values for population we use the Departamento Administrativo Nacional de Estadística (DANE) of Colombia projections of population \([29]\).

As we can see in Figure 1 and Figure 2, the population growth in Cali, Colombia, is faster than in the other towns because its initial population is the biggest, and so is its migration preference. In the other municipalities, the population growth is moderate, and all population levels reach an equilibrium.

There is an unexpected behavior near 50-time units, a non-smooth peak in Cali and Palmira, Colombia, graphics. At that moment, the simulation reached Cali’s loading capacity, and many individuals move from Cali to Palmira, Colombia. Although Buenaventura and Palmira, Colombia, have similar population levels, Buenaventura, Colombia, didn’t get the migration attention. Starting there, the population in Cali, Colombia, doesn’t present any growth and settle below its loading capacity. Palmira’s, Colombia, case is different, the simulation exceeded this town loading capacity, and the population converges above that level. To explain this behavior, we make an analysis of indirect influences.

3.2. Indirect influences analysis

To explain the behavior that the network exhibits we use indirect influences and dependencies via the PWP method. We use the matrices \( D(t) = \gamma(t) \text{diag}(f(x(t))) \) that show the number of individuals traveling between nodes at each time. To perform the calculations, we escalate the
numbers in these matrices, so we compute $D^\star(t) = \frac{D}{\sum D}$ each moment, where $\sum D$ represent the sum of all $D$ entries.

As we can see in Figure 3 and Figure 4, during the first iterations, there are no indirect dependencies in the network. When the first towns reach 90% of their loading capacity near 25 five-time units, Cali, Colombia, raises as the most dependent town for migration because its population is growing. Meanwhile, other towns exhibit disordered behavior. When the principal town reaches its loading capacity, near 50-time units, migration now focuses on the next relevant city that turns out to be Palmira, Colombia. Cali’s, Colombia, dependency falls, and Palmira’s, Colombia, becomes higher than the former’s. From that moment, Palmira, Colombia, is the center of the migration, followed by Cali, Colombia.

![Figure 1](image1.png)  
**Figure 1.** Population evolution for selected Colombia municipalities, Cali, El Cerrito, Buga, Tulía, San Pedro, and Cartago.

![Figure 2](image2.png)  
**Figure 2.** Population evolution for selected Colombia municipalities, Palmira, Ansermanuevo, Andalucía, Zarzal, Yotoco, and Ginebra.

![Figure 3](image3.png)  
**Figure 3.** Indirect dependencies for selected Colombia municipalities, Cali, El Cerrito, Buga, Tulía, San Pedro, and Cartago.

![Figure 4](image4.png)  
**Figure 4.** Indirect dependencies for selected Colombia municipalities, Palmira, Ansermanuevo, Andalucía, Zarzal, Yotoco, and Ginebra.
Indirect influences are pretty different. As we can see in Figure 5 and Figure 6, Palmira, Colombia, exhibits a high peak of influence on the network near 50-time units. The big group of people leaving Cali, Colombia, affects the entire network, and Palmira’s, Colombia, influence drops down with an influence coefficient under 0.4. From that moment, Cali, Colombia, becomes the most influential town with an influence coefficient over 0.5, followed by Palmira, Colombia, with 0.4 and others with less than 0.05.

Figure 5. Indirect influences for selected Colombia municipalities Cali, El Cerrito, Buga, Tulúa, San Pedro, and Cartago.

Figure 6. Indirect influences for selected Colombia municipalities, Palmira, Ansermanuevo, Andalucía, Zarzal, Yotoco, and Ginebra.

According to the above, we can see in the simulation the behavior of indirect influences and dependencies; when a town has a bigger loading capacity and a larger population than others, the migration trend is towards it. However, when the principal city gets full, the migration trend is redirected to a neighboring municipality, even people migrate from the former relevant town. Moreover, although Palmira’s, Colombia, loading capacity gets surpassed, we can’t see another discontinuous segment in the population graphs. So, the two municipalities reinforce each other, as we can see thanks to influences and dependencies graphs.

The sociological concept of social capital justifies a preference for towns with more population [30–33]. This concept measures the capacity of a society to work using collaboration and collective action; therefore, a city with more population has more social capital, understood as more social, economic, political, and even recreational opportunities. The network behavior change happens when the town with all the benefits cannot hold more inhabitants. Common factors among overpopulated cities include insecurity, insalubrity, transportation chaos, and alike; these factors influence the migration decision being or not in the city. Preference is now towards neighboring towns with high social capital, without losing migration interest to the former.

4. Conclusion
We presented an application of a complex network based on a Colombian department to non-forced migration, and we also proposed an asymmetric gravitational model adapted for this use case. We found the numerical solution of the population in each node of the network. We showed that the study of the indirect influences allows us to measure the neighbor effect of migrational
trends in capitals and neighboring towns. So, we were able to explain the social capital effect seen empirically across capital cities around the globe.

Because of the model’s nature, we didn’t cover all migratory features. For example, we can consider a migration resistance, called tradition or attachment. Also, we can take into account any other factor, like a natural disaster, violence, government policies to generate migration preference, which modifies the parameters of several nodes. Another relevant factor to take into account is that towns near the department’s border can prefer municipalities in neighboring departments, and we can augment the model with those to get a more realistic simulation.

References
[1] Lara J 2015 International migration and human capital in mexico: Networks or parental absence? *International Journal of Educational Development* 41 131
[2] Hamilton E R, Choi K H 2015 The mixed effects of migration: Community-level migration and birthweight in mexico *Social Science & Medicine* 132 278
[3] Valsecchi M 2014 Land property rights and international migration: Evidence from mexico *Journal of Development Economics* 110 276
[4] Staněk M 2017 The humanitarian crisis and civil war in syria: Its impact and influence on the migration crisis in europe *Kontakt* 19(4) e270 special Issues of Migration from the Health and Social Perspective
[5] Tuccio M W J 2018 Return migration and the transfer of gender norms: Evidence from the middle east *Journal of Comparative Economics* 46(4) 1006
[6] Balcells L and Steele A 2016 Warfare, political identities, and displacement in spain and colombia *Political Geography* 51 15
[7] Oslander U 2016 The banality of displacement: Discourse and thoughtlessness in the internal refugee crisis in colombia *Political Geography* 50 10
[8] S M J 2018 The impact of internal displacement on destination communities: Evidence from the colombian conflict *Journal of Development Economics* 131 132
[9] Chiara F, Franco D, Alessandro O 2018 Climate change, agriculture and migration: a survey *Sustainability* 10(5) 1405:1
[10] Grecequet M, DeWaard J, Hellmann J J and Abel G J 2017 Climate Vulnerability and Human Migration in Global Perspective *Sustainability* 9(5) 720:1
[11] Nica E 2015 Labor Market Determinants of Migration Flows in Europe *Sustainability* 7(1) 634
[12] Davis K, D’Odorico P, Laio F, Ridolfi L 2013 Global spatio-temporal patterns in human migration: a complex network perspective *PLOS ONE* 8(1) e53723
[13] Dong S, Yinxiu P and Wang Y 2013 A research on complex network of chinese interprovincial migration based on the fifth population census *21st International Conference on Geoinformatics* (Kaifeng: IEEE)
[14] Fagiolo G, Mastrorillo M 2013 Migration and trade: a complex-network approach *Social Science Research Network* 1
[15] Tan S, Liu J 2014 Characterizing the effect of population heterogeneity on evolutionary dynamics on complex networks *Scientific Reports* 4 1
[16] Cruceru A 2010 Statistical analysis of the migration phenomenon *Romanian Statistical Review* 11 1
[17] Aleshkovski I, Joutsev V 2006 Mathematical models of migration *System Analysis and Modeling of Integrated World Systems* vol 2 (Oxford: Eolss Publisher Co.) p 185
[18] Filho H, de Lima Neto F, Fusco W 2011 Migration and social networks - an explanatory multi-evolutionary agent-based model *IEEE Symposium on Intelligent Agent (IA)* (IEEE)
[19] Massey D, Arango D, Hugo G, Kouaouci A, Pellegrino A, Edward Taylor J 2002 *Worlds in Motion: Understanding International Migration at the End of the Millennium* (Oxford University Press)
[20] Massey D, España F 1987 The social process of international migration *Science* 237(4816) 773
[21] Tanasie A, Dracea R and Ladaru G R 2017 A Chaos Theory Perspective on International Migration *Sustainability* 9(12) 2355:1
[22] Guzman L A, Oviedo D and Bocarejo J P 2017 City profile: The bogotá metropolitan area that never was *Cities* 60 202
[23] Verhulst P F 1845 Recherches mathematiques sur la loi d’accroissement de la population *Nouveaux memoires de l’Academie Royale des Sciences et Belles-Lettres de Bruxelles* 18 1
[24] Zipf G 1946 The p1p2/d hypothesis: On the intercity movement of persons *American Sociological Review* 2 677
[25] Guardia M, Seara T M, Teixeira M A 2009 Generic bifurcations of low codimension of planar filippov systems *Journal of Differential Equations* 250(4) 1967
[26] Kuznetsov Y U, Rinaldi S, Gragnati A 2003 One-parameter bifurcations in planar filippov systems
International Journal of Bifurcations and Chaos 13 2157
[27] Runge C 1895 Ueber die numerische auflösung von differentialgleichungen Mathematische Annalen 46(2)
167–178
[28] Kutta W 1901 Beitrag zur näherungsweisen integration totaler differentialgleichungen Zeit. Math. Phys. 46
435
[29] Departamento Administrativo Nacional de Estadística (DANE) 2005 Censo General 2005 Demografía y
Población (Colombia: Departamento Administrativo Nacional de Estadística)
[30] Bourdieu P 1977 Outline of a theory of practice (Cambridge: Cambridge University Press)
[31] Portes A 1998 Social capital: its origins and applications in modern sociology Annual Review of Sociology
24 1
[32] Massey D, Aysa-Lastra M 2010 Social capital and international migration from latin america International
Journal of Population Research 2011 834145:1
[33] Putnam R 2001 Social capital: measurement and consequences Canadian Journal of Policy Research 2 41