Dual-tunable ferrite-ferroelectric dynamic magnonic crystal

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Abstract. A dynamic magnonic crystal based on a planar multiferroic structure consisting of a ferrite and a ferroelectric layers has been designed and theoretically investigated for the first time. Spatially periodic electric field applied to the ferroelectric layer turns the regular multiferroic waveguide into periodic one. An electrodynamic model of the dynamic magnonic crystal and theoretical analysis of its performance is presented.

Recent years an increased interest for investigation of spin waves propagating in spatially periodic magnetic film waveguide (the so-called magnonic crystals) takes place [1, 2]. A feature of the spin wave spectrum of such structures is the presence of band gaps which appear due to Bragg reflection. Dynamic magnonic crystals (DMC) cause a special interest [3, 4], because of the ability to "turn on" and "turn off" its periodic waveguiding properties. For example, a magnonic crystal made of a ferromagnetic film and a meander conductor located near its surface was described in [3]. The current flow caused a sine-like variation of the film’s internal magnetic field, which could be modulated by changing the amount of current. In the work [4] DMC was realized as a periodic structure with a variable width. The edges of the structure were screened by conductors with electric current.

Another topical trend is investigation of artificial materials which demonstrate multiferroic properties [5-9]. Among them are ferrite-ferroelectric layered structures, in which hybrid spin-electromagnetic waves (SEWs) may propagate [10]. Until now, the main attention was given to the investigation of spatially homogeneous ferrite-ferroelectric waveguiding structures, while spatially periodic structures received a small attention. For example, a propagation of SEWs in a multiferroic periodic structure fabricated with a one-dimensional magnonic crystal and a ferroelectric slab had been experimentally investigated for the first time in [11]. The dispersion and frequency-selective properties of periodic ferrite-ferroelectric structures had been theoretically studied in the works [12, 13].

This work is devoted to theoretical investigation of microwave properties of multiferroic dynamic magnonic crystal. Investigated structure is shown in figure 1. It consists of several layers and metal electrodes. The layers are a ferrite film (1), a dielectric substrate (2), and a ferroelectric slab (3). The ferroelectric slab is covered by thin metal electrodes. The electrode on the top side is a grid consisting of metal strips. Both electrodes are assumed to be transparent for the microwave fields in the structure due to small thickness which is much smaller than the skin depth [14]. Ferroelectric slab is assumed to be in a paraelectric phase with a high dielectric constant. The dielectric constant is controlled by the bias voltage applied to the top and bottom
electrodes. This assumption is correct for barium strontium titanate ceramic of the composition \( \text{Ba}_{0.5}\text{Sr}_{0.5}\text{TiO}_3 \) at the room temperature.

The ferrite-ferroelectric structure serves as a waveguide for the SEWs. The electrodes do not affect the propagation of SEWs in the structure. The electrodes produce periodic polarization of the ferroelectric layer, as is shown in figure 1. It is well known that the polarization of the ferroelectrics decreases their dielectric permittivity \( \varepsilon_d \). Therefore, application of the voltage to the electrodes produces periodic modulation of \( \varepsilon_d \) in the investigated structure. This periodic modulation can be "turned on" and "turned off" by the bias voltage. Thus, the described multiferroic structure is a dynamic magnonic crystal for the SEWs.

![Figure 1](image1.png)

**Figure 1.** Schematic representation of the dynamic magnonic crystal based on ferrite-ferroelectric structure.

Consider the development of the electrodynamic model of the structure. We assume that the SEWs propagate perpendicular to the grid electrode in the direction opposite to \( x \) axis in a tangentially magnetized (along \( z \) axis) ferrite layer surrounded by free space (Figure 1). In this configuration maximum of the SEW energy is located at the interface between the ferroelectric layer and the ferrite film. We do not take into account existence of the dielectric substrate (2) because it has a low dielectric constant (about 10).

![Figure 2](image2.png)

**Figure 2.** The spectrum of SEWs in the ferrite-ferroelectric structure.
The theoretical model was developed in two stages. In the first stage we derive the dispersion equation and calculate the dispersion characteristics of the SEWs in the homogeneous ferrite-ferroelectric waveguiding structure. In order to derive the dispersion equation, we use a method described in [15]. In this method four Helmholtz equations for the variable component of the electric field $E_z$ of TE wave are obtained from Maxwell’s equations. The first and fourth of them correspond to free half-spaces above and below the structure, the second equation describes the propagation of electromagnetic waves (EMWs) in the ferroelectric layer, and the third one describes the propagation of the SEWs in the ferrite film taking into account the microwave magnetic permeability tensor. The solutions of the Helmholtz equations give expressions for the components $E_z$ in each layer. Further $H_z$ components of magnetic field are obtained from $E_z$ via the Maxwell’s equations. Substitution of $E_z$ and $H_z$ into the electrodynamic boundary conditions at the layers’ interfaces gives us a system of six linear homogeneous algebraic equations with respect to six variables. The vanishing of the system’s determinant results in the following dispersion relation:

$$
\tan(\kappa_d L_d) \left[ \tan(\kappa_f L_f) A + (\kappa_0^2 - \kappa_d^2) \mu_{\perp} \right] + \kappa_d \left[ \tan(\kappa_f L_f) B + 2\kappa_0 \mu_{\perp} \right] = 0, \quad (1)
$$

where $A = (\kappa_d^2 \mu_{\perp} - \frac{\kappa_0 \kappa}{{\mu}}) \left( \frac{\kappa \mu}{{\kappa_f \mu}} - \frac{\kappa_0 \mu_{\perp}}{\kappa_f} \right) + 2\kappa_0 \kappa_f$, $B = \frac{\kappa_0^2 \mu_{\perp}^2}{\kappa_f} - \frac{k^2}{\kappa_f \mu_{\perp}} - \kappa_f$, $k$ is a SEW longitudinal wave number, $L_d$ and $L_f$ are the thicknesses of the ferroelectric and ferrite layers, respectively, $\kappa_0 = \sqrt{k^2 - \omega^2 \mu_0 \varepsilon_0}$, $\kappa_d = \sqrt{\omega^2 \mu_0 \varepsilon_0 \varepsilon_d - k^2}$, and $\kappa_f = \sqrt{\omega^2 \mu_0 \mu_{\perp} \varepsilon_0 \varepsilon_f - k^2}$ are transverse wave numbers in free space, in the ferroelectric layer and in ferrite layer, respectively, $\mu_0 = 4\pi \times 10^{-7}$ H·m$^{-1}$, $\varepsilon_0 = 8.85 \times 10^{-12}$ F·m$^{-1}$, $\mu_{\perp} = \mu - \frac{\alpha^2}{\mu}$, $\mu = \frac{\omega M (\omega H + \omega M)}{(\omega H^2 - \omega^2)}$, $\alpha = \frac{\omega M}{(\omega H^2 - \omega^2)}$, $\omega H = 2\pi \gamma H$, $\omega M = 2\pi \gamma \cdot 4\pi M_s$, $\gamma = 2.8$ MHz/Oe, $H$ is the magnetic field, $M_s$ is the saturation magnetization.

As a second step we calculate the dispersion relation of the SEWs in periodically polarized ferrite-ferroelectric structure. We use following formula obtained by the method of coupled waves [16]:

$$
cos KA = \cos k_1 l_1 \cos k_2 l_2 - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin k_1 l_1 \sin k_2 l_2, \quad (2)
$$

where $K$ is a Bloch wave vector, $A$ is a period of the structure, $k_1$ and $k_2$ are wave numbers of SEWs propagating in polarized and unpolarized sections of the structure with $\varepsilon_{d1}$ and $\varepsilon_{d2}$, respectively, $l_1$ and $l_2$ are the distances passed by the wave in the polarized and unpolarized sections of the structure (see

Figure 3. Amplitude-frequency characteristics of unpolarized (grey line) and periodically polarized (black line) ferrite-ferroelectric structure.

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Finally, the amplitude-frequency characteristics (AFC) of the multiferroic periodic structure are calculated by the following formula:

\[ H = 20 \log e^{-\alpha x}, \]  

where \( \alpha = \left| \frac{d k(\omega)}{d H} \right| \left( \Delta H + \varepsilon d \frac{d k(\omega)}{d \varepsilon d} \right) \tan \delta \) is a spatial damping decrement of the SEWs [17], \( \Delta H \) is the half-width of the curve of the ferromagnetic resonance, \( \tan \delta \) is a tangent of dielectric losses of the ferroelectric layer.

The dispersion characteristics of the SEWs propagating in the homogeneous ferrite-ferroelectric structure were calculated with the use of the dispersion equation (1). An example of the calculation results is shown in figure 2. The calculation was carried out for the ferrite layer having parameters corresponding to single-crystal yttrium iron garnet (YIG) film: \( 4\pi M_s = 1750 \) G, \( L_f = 10 \) μm, \( H = 1000 \) Oe, \( \varepsilon_{f} = 14 \), \( \Delta H = 0.5 \) Oe. The calculation was also carried out for the ferroelectric layer having parameters corresponding to the ceramic solid-state solution of Ba\(_{0.5}\)Sr\(_{0.5}\)TiO\(_3\): \( \varepsilon_d = 1500 - 2500 \), that depends on the applied electric field, \( L_d = 500 \) μm, \( \tan \delta = 10^{-2} \). As can be seen from the figure, a region of a strong dispersion of the SEWs is located at the wave number region from 25 rad/cm to 50 rad/cm. This area corresponds to the maximum hybridization between the spin waves in the ferrite film and the EMWs in the ferroelectric slab. The most significant modulation of the dispersion of the SEWs under the application of the electric field to the ferroelectric slab occurs in this area.

**Figure 4.** Width of the first band gap (squares) and its centre frequency (triangles) as a function of period for \( l_1=100 \) μm and \( l_2=300 \) μm (a, b) and for \( l_1=300 \) μm and \( l_2=100 \) μm (c, d) as well as for \( L_f=5 \) μm (left graphs) and \( L_f=10 \) μm (right graphs).
The dispersion and transmission characteristics of the SEWs in the DMC for different $\varepsilon_{d}$ are also calculated. Application of the voltage leads to the appearance of band gaps in the SEWs spectrum and, consequently, to the appearance of characteristic dips in the AFC of the waveguiding structure (see figure 3). The first band gap is relatively wide while the other are relatively narrow. This is due to the fact that the first band gap is located in the area of the SEWs strong dispersion. The centre frequency of the first band gap, as well as its width increases with increase of the bias voltage.

The calculations were made for a variety of the structure parameters. Figure 4 shows the dependences of the centre frequency of the first band gap and its width on the period of the structure with $l_{1}$≠$l_{2}$. The analysis of the results shows that the increase of the period $A$ of the structure increases the width of the first band gap. This is due to the reduction of the Bloch wave vector $K_{0}$=π/$A$ with an increase in $A$. Therefore the frequency of the first band gap shifts, and it is getting closer to the area of strong dispersion. According to the above discussion, the application of voltage leads to a significant change of the dispersion in that area. It leads to the broadening of the band gap.

Similar calculations were made for the structure with $l_{1}$=$l_{2}$. For example, the calculations for $\varepsilon_{d1}$ = 2000 and $\varepsilon_{d2}$ = 2500 and the ferrite film thickness $L_{f}$=5 μm show that the increase of the structure’s period leads to increase of the first band gap, which is equal to 21 MHz, and the decrease of its centre frequency, which is equal to 70 MHz. As well as for the film thickness $L_{f}$=10 μm the increase of the first band gap is equal to 37 MHz and the decrease of its centre frequency is equal to 130 MHz. Thus, it is seen from the results that it is more preferably to use relatively thick ferrite films to increase the electric tunability of the ferrite-ferroelectric DMC.

In conclusion, it is easy to achieve an efficient electrical tuning of one or more band gaps that are located at the frequencies of the strong dispersion of SEWs by choosing the geometry of the DMC and physical parameters of the multiferroic layered structure. Thus, dynamic magnonic crystals based on the planar multiferroic structure consisting of ferrite and ferroelectric layers can find a variety of applications.

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