Risk Assessment of Power Systems Considering CVaR

Jin Yu¹, Xingang Wang², Zhongping Yu¹, Dongsheng Xi³ and Mengdi Chen³*

¹ Economic and Technical Research Institute of Xinjiang Power Grid Co. Ltd of State Power Grid, Urumqi, Xinjiang, 830000, China
² Xinjiang Power Grid Co. Ltd of State Power Grid, Urumqi, Xinjiang, 830000, China
³ Beijing Smart China Energy Internet Research Institute, Beijing, 100000, China
*Corresponding author’s e-mail: chenmengdi@qdkj999.onexmail.com

Abstract: In view of the randomness of component fault and load, considering the deficiency of traditional single reliability index, this paper introduces the Conditional Value-at-risk method based on Monte Carlo simulation, and constructs the probability model of risk assessment considering the forced outage rate of unit and line. In order to quantify uncertainty caused by random factors, the value-at-risk and conditional value-at-risk of power system security was proposed and used as a grid security index in the model combined with reliability index, which can accurately describe secure operation level in power system under different operation parameters, and reflect the severity of the accident and weakness and the grid, for further research on safe operation of power system to offer reference and decision-making basis.

1. Introduction
With the deepening of electric power marketization, power system faced more and more uncertain factors such as the changeable and random rules of the electric power market and the frequent occurrence of bad weather, which brings great challenges to the system operation and planning. Therefore, a new problem is how to analyze and optimize the system operation and planning decisions from the viewpoint of risk [1].

For this reason, scholars have proposed a variety of deterministic and probabilistic model evaluation methods. The literature [2] compares probabilistic safety assessment with deterministic methods. A new evaluation method is needed to overcome the weakness of the deterministic method when the system is in a tight running state. In general, risk is defined as the product of probability and severity [3]. In the traditional evaluation methods, the line power flow is used as the standard to evaluate the line overload, and the result tends to be conservative [4-6]. The literature [7] introduces a new risk assessment method that can be used in power system planning, design, operation and maintenance. The literature [8-9] applies the risk theory andputs forward the risk assessment method for interlocking faults of power system. The literature [10] reliability assessment of power system based on the probability distribution of wind power output is considered, and the VaR and CVaR indexes generated by wind farm's connection to the grid are obtained, providing a basis for wind farm planning and operation. Although this method can get the overall reliability situation. However, there is still a lack of discussion on the mathematical properties of the index, and the randomness factor of the system is not fully considered.
This paper considers randomness and uncertainty of system component failure and load, and calculates the risk index according to CVaR method under a certain confidence interval. The risk index comprehensively and intuitively reflects the potential maximum risk faced by the system and the potential loss when the risk index exceeds the maximum risk under a given probability confidence level $\sigma$ in the instability. It can accurately describe the safe operation level of the power grid under different operating parameters, and can reflect the severity of accidents and the weakness of power grid. According to these, the overall measures of disaster prevention are given.

2. Model parameter estimation for Monte Carlo simulation

The Monte Carlo simulation method is used to find out the objective and universal law of the motion of things by grasping the geometric quantity and geometric characteristics of the motion of things and using mathematical and computer methods to simulate the motion of things; It is based on a probability model, and according to the process described in this model, the results of simulation experiments are used as the approximate solution of the problem [11].

2.1. Stochastic scenario modeling

Suppose the system is made up of $n$ components, $x_i$ represents the state of $i$ component, so $S = (x_1, x_2, ..., x_n)$. The system state is determined by a combination of states of system elements, and each element state can be determined by sampling the probability that the elements appear in that state.

The probabilistic characteristics of each element can be described by a uniform distribution between $[0,1]$, assuming that each element has two states of failure and operation, and component failures are independent of each other. Let $Q_i$ denotes the failure probability of $i$ components, a random number $R_i$ uniformly distributed between $[0,1]$ is generated for the $i$ element, so:

$$
\begin{align*}
    x_i &= \begin{cases} 
    0, R_i \geq Q_i \text{ (running status)} \\
    1, 0 \leq R_i \leq Q_i \text{ (fault states)}
    \end{cases}
\end{align*}
$$

Equation (1) can be used to determine a system state $S = (x_1, x_2, ..., x_n)$. The process is repeated $N$ times, and a set of $N$ system state samples is obtained $S = (S_1, S_2, ..., S_n)$.

In this paper, the probability of component failure is described by exponential distribution

$$
    P(E_i) = \lambda_i e^{-\lambda_i t}
$$

Where $\lambda_i$ is the failure rate of $i$ component. Reliability data provided by the power sector.

2.2. Abundance index calculation

Assuming that each system state has probability $P(S)$ and reliability index functions $F(S)$, then the expected value of the system index is:

$$
    E(F) = \sum_{S \in G} F(S)P(S)
$$

Where $G$ represents the system state set.

3. Risk Assessment Methods

3.1. Value at risk and conditional value at risk

The traditional reliability assessment is to calculate a total reliability index value (LOLP, EENS, etc.), which is a single reliability index value and hides a large number of information. As one of the digital features of random variables, the feedback of expected value index is limited, so the expected value
index alone cannot realize the complete and thorough cognition of the security risk level of power grid. Because the reliability index with the change of random variable is also a random variable, it will cause a lot of information loss.

At present, some scholars have introduced the ideas of value at risk (VaR) and conditional value at risk (CVaR) into the field of power system risk assessment and achieved certain results [12-13]. The risk value of power system VaR refers to the potential maximum loss faced by the system under the confidence level of a given confidence level in a certain period of time in the future, which can be expressed as:

\[ P(V > VaR_\sigma) = 1 - \sigma \]  \hspace{1cm} (4)

Where: \( \sigma \) is confidence; \( V \) is the system risk index of the study; \( VaR_\sigma \) is value at risk under confidence; \( P(V > VaR_\sigma) \) is the probability that system index is greater than value at risk.

VaR combines the potential risk of the system with the possibility of risk occurrence can reflect the scale of risk loss and its confidence level. However, the risk value index cannot accurately reflect the potential loss when the risk index exceeds the VaR, that is, it cannot reflect the shape of the distribution. For this reason, some scholars have further put forward the concept of conditional risk value, that is, at a certain confidence level, the system risk exceeds the VaR conditional mean value, which can be expressed as:

\[ CVaR_\sigma = \int_{VaR_\sigma}^{\infty} vp(v)dv \]  \hspace{1cm} (5)

Where: \( v \) is the value of system risk index under study \( V \); \( p(v) \) is probability density value of \( v \).

3.2. Risk measurement based on CVaR method

Here, the risk is expressed as system load loss in case of failure. \( V \) is used to represent the load loss of the system. Because component failures and loads are random variables, there may be multiple \( V \) satisfying the above problem description. Note that \( V \) units are MW, and associated with system standby, and therefore the risk value (VaR) can be defined as: Within a certain period of time in the future, the probability of less than \( \sigma \) is determined that the loss of load in a certain period of a system under random disturbance is \( V \) less than \( VaR_\sigma \), the maximum value of all these VaR is the risk value.

Let the probability density distribution of random variable \( y \), loss \( V \) be \( p(v) \), Then formula (5) the probability of not exceeding the threshold is:

\[ \sigma(y) = \int_{V_{VaR}}^{\infty} p(v)d(v) \]  \hspace{1cm} (6)

\[ VaR_\sigma = \max\{V \in R: \sigma(y) \leq \sigma\} \]  \hspace{1cm} (7)

It means the maximum risk value when the confidence level is satisfied. The value of \( VaR_\sigma \) can be positive or negative. When the signal level \( \sigma \) is set to 99%, when \( VaR_\sigma \) is positive, it means that there is a 99% probability that the maximum load loss of the system is \( VaR_\sigma \), or a 1% probability that the load loss is greater than \( VaR_\sigma \); When \( VaR_\sigma \) is negative, it refers to the reserve capacity. There is a 99% probability that the reserve capacity of the system is greater than \( VaR_\sigma \).

The concept of CVaR is introduced to describe the potential loss when the risk exceeds VaR and reflect the shape of the distribution. CVaR refers to the loss exceeding the conditional mean of VaR, and its mathematical expression is:

\[ CVaR = VaR + E(V - VaR|V > VaR) = E(V|V > VaR) \]  \hspace{1cm} (8)

Thus, the expression containing the probability density function is:
\[ CVaR = \frac{1}{1 - \sigma} \int_{v > VaR} p(v) \, dv \]  

(9)

The calculation of CVaR is simplified by introducing the following function [14].

\[ F(y) = VaR + \frac{1}{1 - \sigma} \int_{v < y} [v - VaR] + p(v) \, dy \]  

(10)

In addition, discrete points are used to approximately replace the integral in Equation (10). \( K \) is the sampling number, so Equation (10) can be reduced to:

\[ F(y) = VaR + \frac{1}{K(1 - \sigma)} \sum_{k=1}^{K} [v - VaR] \]  

(11)

It is proved by literature [14].

\[ CVaR = \min F(y) \]  

(12)

In this case, you can also get the corresponding VaR values.

Because the reliability index of power system actually reflects the risk of insufficient power supply capacity, the risk value is more suitable for the reliability index analysis of power network. The introduction of VaR and CVaR can provide a comprehensive and unified measurement index for power system reliability, and contain more decision information than a single reliability index, which can better guide decision-making.

3.3. Risk assessment Model

In the sequential simulation of power network reliability, power flow calculation will be carried out first after each system state is extracted to determine whether the operation constraints such as node voltage overrun and line overload are violated. If the operation constraints are violated, the optimal load reduction calculation is carried out. A risk assessment study is carried out on behalf of the ELC (expected of load curtailments) of load loss. When the components in the system fail, the output of each unit and the load reduction of each node are taken as the control variables, and the minimum total load reduction is taken as the objective function of the model. The optimal cut load model based on DC power flow can be described as follows:

\[ \begin{align*}
& \min \sum_{j=1}^{n} C_i \\
\text{s.t.} & P_L = B_L A B^{-1} (PG - PD + C) \\
& \sum_{i=1}^{n} P_j + \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} PD_i \\
& PG_{i_{\text{min}}} \leq PG_i \leq PG_{i_{\text{max}}} \\
& 0 \leq C_i \leq PD_i \\
& -P_{L_{\text{min}}} \leq P_{Li} \leq P_{L_{\text{max}}}
\end{align*} \]  

(13)

\( C \) is the vector of node load removal, \( P_L \) is the vector composed of branch active power; \( B_L \) is a square matrix, whose diagonal elements are each branch admittance, \( A \) is the branch - node correlation matrix, \( B \) is node admittance matrix, \( PG \) and \( PD \) are the generator output and load power vectors respectively.

Solution process of short-term power system risk assessment includes following three main steps:

a. Random sampling for \( Z \) times based on failure rate and load distribution characteristics;

b. According to \( Z \) sampling values, fault analysis and repeated optimal load cutting model, get \( Z \) annual power shortage;
c. Based on the above results, the optimization problem represented by formula (10) and (11) is solved, and the system risk measurement index is obtained.

4. Analysis of examples

4.1. Test result

Consider the IEEE RTS-79 system as an example. Select the annual maximum load (2850 MW) corresponding to the time of the system load, the power reference value is 100 MVA. According to Monte Carlo simulation, the value at risk satisfying a certain confidence interval is calculated $\sigma_{VaR}$ and $\sigma_{CVaR}$ shown in Table 1.

| confidence level $\sigma$ | $VaR$ (MW) | $CVaR$ (MW) |
|---------------------------|------------|-------------|
| 0.95                      | 142        | 273.7574    |
| 0.99                      | 412        | 517.9248    |

On the basis of the previous definition of risk value and conditional risk value, the confidence interval $\sigma$ is 0.95 and 0.99. The physical meaning is: under a random disturbance in the next year, 95% of the blackout loss load is not more than 142 MW; and the expected value of the loss load is 273.7574 MW, the coordinated represent the blackout risk level of the system more comprehensively.

Table 2. Load loss expectation index of bus nodes

| bus | ELC (MW) |
|-----|----------|
| 18  | 997.9148 |
| 15  | 969.5648 |
| 13  | 825.1652 |
| 10  | 647.4858 |

Table 2 is the result data analysis of four lines with large expected value of total load loss of bus (load point): because bus risk index can help to determine the weak link of the system.

From the bus node risk index, the four lines are in a high risk state (maximum loss of load) when the system fails, and from the load demand, the load demand of the four bus is large, and once the main network system fails, the structure of the system will be seriously unstable.

4.2. Analysis and discussion of VaR and CVaR

4.2.1. The influence of different levels of confidence

Suppose that the system load at the corresponding time of the annual maximum load (2850 MW) is fixed. Considering the peak load, different $VaR_\sigma$ and $CVaR_\sigma$ values can be obtained for different confidence levels $\sigma$, such as 85% and 99%. The curve is shown in figure 1. As can be seen from the figure, both curves of $VaR_\sigma$ and $CVaR_\sigma$ decrease monotonously with the confidence level, and the value of $CVaR_\sigma$ is always larger than that of $VaR_\sigma$, because according to the definition of $CVaR_\sigma$, it is equal to the average capacity of all volumes greater than or equal to $VaR_\sigma$. At $x=0.88$, $CVaR_\sigma$ is less than 0. It shows that when the confidence level is 88%, the system will not lose load due to random disturbance, and the smaller the probability, the larger the reserve capacity.
4.2.2. The influence of the power of line reconstruction
As system $VaR$ and $CVaR$ results are shown in Table 3. It can be seen from the table that increasing the intensity of line transformation can reduce the scale of power outage after the accident. However, it is found that the corresponding $VaR$ and $CVaR$ are not different under the condition of different line transformation, the main reason is that the line is broken due to accidental factors, resulting in a small part of the area load cannot be supplied, and the upgrading of the line has no effect on this situation; The main cause of large-scale system accidents is the chain failure after accidental factors, and the transformation of the line is obviously of great significance in dealing with the chain failure.

Table 3. Standard test system results data

| Strength of line reconstruction now 1.005 1.050 |  |
|---|---|---|
| VaR (Times per year) | 215.0000 | 195.0000 | 187.0000 |
| CVaR (MW·h per year) | 334.0315 | 336.2758 | 324.2088 |

4.2.3. Influence of load distribution
Figure 2 shows that the other parameters are the same (all initial operating parameters), the total load $L$ of the system is different $VaR$ and $CVaR$.

Figure 2. Change curves of $VaR$ and $CVaR$ under different total system loads
The confidence level is 99%, $VaR$ and $CVaR$, the sum decreases with the increase of load, which accords with the actual physical meaning of the larger the system load and the smaller the reserve. Therefore, when the load peak period occurs, the reasonable balance of power generation and load can reduce the occurrence of power outages and reduce the system risk. Knowing the risk level of the system under different load levels can guide the short-term maintenance plan to avoid the high risk load level.

5. Conclusion
In this paper, two short-term adequacy evaluation indexes of risk value and conditional risk value under system failure are cited, and a risk assessment method with conditional risk value is proposed.
This method establishes a quantitative relationship between component failure, system adequacy and load randomness. The research results provide a new method for the adequacy decision-making and dispatching operation of modern power system with a large number of random fluctuation power supply and load. The following conclusions were drawn from the study:

1. The randomness test of abundance index sequence is carried out to verify that the power system risk measurement has the premise of using the VaR method.

2. By using adequacy index and VaR method, the reliability of the system is measured. This paper holds that Monte Carlo simulation method can calculate VaR and CVaR values with good early warning accuracy and high accuracy. Because Monte Carlo simulation method is based on setting the failure rate from the random process, and through a large number of historical data to obtain the corresponding parameters of the sample data to be simulated, a new set of data in accordance with the law of historical data is obtained to reflect the variation law of adequacy index. In this way, a large number of scenarios can be produced, but the adequacy index has no possibility and probability, such as large power outages and other extreme cases. These cases are understood as the data showing the peak and thick tail characteristics in the samples that conform to the law of non-strict normal distribution.

References

[1] Chen, W.H., Luo, L. (2008) Risk assessment and prevention control of power system line overload. East China Electric Power, 07: 42-45.
[2] McCalley, J., Asgarpoor, S., Bertling, L., Billinion, R., Chao, H., Chen, J., Endrenyi, J., Fletcher, R., Ford, A., Grigg, C., Hamoud, G., Logan, D., Meliopoulos, A. P., Ni, M., Rau, N., Salvadori, L., Schilling, M., Schlumberger, Y., Schneider, A. and Singh, C., (2004) Probabilistic security assessment for power system operations. In: IEEE Power Engineering Society General Meeting, 2004. 212-220.
[3] Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S. S. and Zhang, P., (2011) Risk assessment of cascading outages: Part I — Overview of methodologies. In: 2011 IEEE Power and Energy Society General Meeting. 1-10.
[4] (1989) Safety and Economic Operation of Power System -- Model and Method.
[5] Guo, Y.J. (1985) Power System Reliability Principle and Application (Volume 1).
[6] Guo, Y.J. (1986) Power System Reliability Principle and Application (Volume 2).
[7] Zhou, J.Q., Zhao, X. (2006) Study on Risk Assessment Method and Application Case of Power System. China Power, 08: 77-81.
[8] Li, R.R., Zhang, Y., Jiang, Q.Y. (2006) Risk Assessment of Complex Power System Chain Fault. Power grid technology, 10: 18-23.
[9] Zhang, S., Liu, Y.M., Mei, X.L., et al. (2011) Risk Assessment of Chain Fault of Power System Based on Real-time Operating Conditions. Huazhong Electric Power, 02: 40-44.
[10] Huang, H.Y., Yu, W.J. (2013) Reliability Evaluation of Power System Considering Wind Power Output Probability Distribution. Power grid technology, 09: 2585-2591.
[11] Xie, F., Chen, L.J., Qin, J.C. (2011) Comparative Study on Measuring Exchange Rate Risk between Monte Carlo Simulation Method and Historical Simulation Method. Economist, 10: 21-23.
[12] Mei, S.W., He, F., Zhang, X.M., Wu, S.Y. and Wang, G. (2009) An Improved OPA Model and Blackout Risk Assessment. IEEE Transactions on Power Systems, 24: 814-823.
[13] Xue, Z.Y., Zhou, M., Li, G.Y. (2014) Short-term Abundance Decision of Power System Based on Conditional Value at Risk. Proceedings of the CSEE, 01: 96-104.
[14] Uryasev, S., (2000) Conditional value-at-risk: optimization algorithms and applications. In: IEEE/IAFE/INFORMS Conference on Computational Intelligence for Financial Engineering. New York. 49-57.