Basis of Diagrammatic Deformation Model for Core Reinforced Concrete Structures Calculating

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Abstract. The basis of the diagrammatic deformation model for core reinforced concrete structures calculating are physical relations. The construction of physical relations linking the moments and the normal force of the cross section of the core element in the general case of oblique bending and oblique eccentric compression with the curvature and relative deformation at the level of the selected longitudinal axis is given. The transformation of diagrams in relation to the deformation of concrete and rebar in the frozen state on the basis of experimental data [8, 9] is proposed. The general system of physical relations has been established for the calculation of reinforced concrete core structures using modern computational methods in the case of simultaneous action of stress in two planes and the variable in the cross-section thickness negative temperature of concrete and rebars freezing. Physical relations are designed to calculate reinforced concrete core structures that are operated under the simultaneous action of force loads and a significant variable in cross-section of negative temperature.

1. Introduction
Nonlinear properties of reinforced concrete in the calculation of core reinforced concrete structures are the most completely taken into account in the diagram deformation model, which is based on the use of real nonlinear diagrams of concrete deformation, as well as reinforcement bar with linear and nonlinear sections deformation.

A diagram model of core reinforced concrete structures at normal climatic conditions calculating was developed in [1]. This model was included in the Code SP 63.13330.2012 “Concrete and won concrete construction. Design requirements”, as well as in the Manual “Statically indeterminate reinforced concrete structures. Diagram methods of automated calculation and design” [2].

In the construction of diagrams [1, 2], the nonlinear concrete diagram proposed in [3] was used. In [4] this diagram was transformed to describe a nonlinear section of reinforcement deformation.

In the works [5-7] the transformation of diagrams in relation to the deformation of concrete and rebar in the frozen state on the basis of experimental data [8, 9] is proposed.

On the basis of these transformed diagrams, we consider the development of a nonlinear diagram model for the calculation of core reinforced concrete structures on the simultaneous action of loads and variables in the cross-sections of negative temperatures (variable cross-section freezing of concrete and reinforcement bar).
2. Materials and methods

2.1. Characteristics of the cross section of the rod element

![Figure 1. Rod element cross section calculation scheme.](image)

We consider core structures of arbitrary cross-section subjected to the action of moments $M_x$ and $M_y$ in the planes $Z0X$ and $Z0Y$ and the normal force $N_z$ applied at the cross-section point 0 along the $Z$ axis (Figure 1). Let's divide the cross-section of the beam into $i$ small concrete elements according to Figure 1 and select the reinforcing rods with numbers $j$ and $k$.

Let's denote: $A_{bi}$ – areas of small elements, $\sigma_{bi}$ – concrete stresses within small elements, $Z_{bix}, Z_{biy}$ – centers’ coordinates of gravity of $i$ elements in the $X$ and $Y$ axes. Elements reinforced with reinforcing bars, where $j$ – number of rods, $\sigma_{sj}$ – stress in reinforcing bars, $A_{sj}$ – area of the reinforcing bar, $Z_{sjx}, Z_{sjy}$ – the coordinates of the rebars in the $X$ and $Y$ axes. During loading, cracks may appear in the section. In cracks, concrete elements $i$ are disabled, and rebar numbers $j$ are assigned new indexes $k$, where $\sigma_{sk}$ – stresses of rebar in cracks, $A_{sk}$ – area of the rebar, $Z_{skx}, Z_{sky}$ – coordinates of rods intersected by cracks. The numbers of these rods are excluded from list $j$.

2.2. Determination of concrete and reinforcing bar relative deformations

The total relative deformations of concrete $\varepsilon_i$ of the $i$-th element will consist of deformations caused by stresses $\sigma_{bi}$ and the action of temperature $T_i$ at the level of element $i$,

$$\varepsilon_i = \varepsilon_{bi} + \varepsilon_{mi} = \frac{\sigma_{bi}}{E_{bi} \nu_{bi}} + \alpha_{bi} T_i,$$  \hspace{1cm} (1)

where $E_{bi}$ – modules of concrete elasticity at level $i$ in the frozen state, and $\nu_{bi}$ – coefficient of concrete secant modulus change determined according to the stress-strain diagram of concrete in the
frozen state at level \( i \), \( \alpha_{bi} \) – coefficient of concrete thermal strains at the level of the \( i \)-th element. Recommendations for determining \( E_{bTi}, \nu_{bTi}, \alpha_{bTi} \) are given in [6, 7].

Similarly, the total relative deformations of reinforcing bars \( j \) in the section without cracks are:

\[
\varepsilon_j = \varepsilon_{ij} + \varepsilon_{sTj} = \frac{\sigma_{ij}}{E_{ijTj}v_{ijTj}} + \alpha_{ijTj} T_j,
\]

in the section with cracks they are:

\[
\varepsilon_k = \varepsilon_{sk} + \varepsilon_{sTk} = \frac{\sigma_{sk}}{E_{sTk}v_{sTk}} + \alpha_{sTk} T_k,
\]

where \( E_{sTj}, E_{sTk} \) are the rebar deformation modules in the frozen state at levels \( j \) and \( k \), and \( \nu_{sTj}, \nu_{sTk} \) are the coefficients of the rebar secant modules change on nonlinear sections of the rebar deformation diagram (on linear sections \( v_{ijTj} = 1, v_{sTk} = 1 \)); \( \alpha_{sTj}, \alpha_{sTk} \) – coefficients of temperature deformations of rebars at levels \( j \) and \( k \); \( T_j, T_k \) – freezing temperature of rods \( j \) and \( k \), \( \Psi_{sTk} \) – V. I. Murashev’s coefficient [10], which takes into account the effect of partial adhesion of reinforcement and concrete in the areas between cracks on the average deformations of the reinforcement.

The \( \Psi_{sTk} \) coefficient can be determined from the dependencies given in [1], replacing the \( \varepsilon_{sk} \) with the \( \varepsilon_{sTk} \).

Negative temperature values should be included with a minus sign in all dependencies.

In the case of a constant cross-section temperature:

\[
\begin{align*}
E_{bTi} &= E_{bT}, E_{sTj} = E_{sT}, \\
v_{bTi} &= v_{bT}, v_{sTj} = v_{sT}, \\
T_i &= T_j = T_k = T.
\end{align*}
\]

3. Results

From formulas (1) – (3) follows:

\[
\begin{align*}
\sigma_{bi} &= (\varepsilon_i - \alpha_{bTi} T)E_{bTi} v_{bTi}, \\
\sigma_{sTj} &= (\varepsilon_{sTj} - \alpha_{sTk} T)E_{sTj} v_{sTj}, \\
\sigma_{sTk} &= (\varepsilon_{sTk} - \alpha_{sTk} T)E_{sTk} v_{sTk} / \Psi_{sTk}.
\end{align*}
\]

We assume that the change in the total relative deformations of concrete (\( \varepsilon_i \)) and rebars (\( \varepsilon_j, \varepsilon_k \)) corresponds to the hypothesis of flat sections. According for example to [10]

\[
\begin{align*}
\varepsilon_i &= \varepsilon_{iz} + r_x Z_{hx} + r_y Z_{hy}, \\
\varepsilon_j &= \varepsilon_{ij} + r_x Z_{gx} + r_y Z_{gy}, \\
\varepsilon_k &= \varepsilon_{ik} + r_x Z_{sk} + r_y Z_{sk}.
\end{align*}
\]

where \( \varepsilon_{iz} \) is the relative deformations of the element at the Z-axis level, \( r_x, r_y \) – the curvatures of the element in the planes XOZ and YOZ, which are determined by the second derivatives of the deflections \( Wx \) along the X-axis and \( Wy \) along the Y-axis:

\[
r_x = \frac{\partial^2 W}{\partial Z^2}, \quad r_y = -\frac{\partial^2 W}{\partial Z^2}.
\]

Substituting the value of \( \varepsilon_i, \varepsilon_j, \varepsilon_k \) from (6) into the dependence (5), we find
\[
\sigma_{ix} = (\varepsilon_{oZ} + r_zZ_{ax} + r_xZ_{ny})E_{ot}V_{ot} - \alpha_{ot}E_{ot}V_{ot}T_i, \\
\sigma_{iy} = (\varepsilon_{oZ} + r_zZ_{ax} + r_xZ_{ny})E_{ot}V_{ot} - \alpha_{ot}E_{ot}V_{ot}T_i, \\
\sigma_{ik} = (\varepsilon_{oZ} + r_zZ_{ax} + r_xZ_{ny})E_{ot}V_{ot} - \alpha_{ot}E_{ot}V_{ot}T_i.
\]

(8)

The common moments \(M_x, M_y\), and the normal force \(N_z\) are expressed in terms of forces in concrete and rebar in terms of moments of these forces relative to the X and Y axes:

\[
M_x = \sum \sigma_{nx}A_{nx}Z_{ax} + \sum \sigma_{ny}A_{ny}Z_{ny} + \sum \sigma_{nk}A_{nk}Z_{nk}, \\
M_y = \sum \sigma_{nx}A_{nx}Z_{nx} + \sum \sigma_{ny}A_{ny}Z_{ny} + \sum \sigma_{nk}A_{nk}Z_{nk}, \\
N_z = \sum \sigma_{zz}A_{zz} + \sum \sigma_{zy}A_{zy} + \sum \sigma_{zk}A_{zk}.
\]

(9)

Substituting in (9) the stress values in concrete and rebar from the dependencies (8), we come to the final system of physical relations:

\[
\begin{bmatrix}
M_x \\
M_y \\
N_z
\end{bmatrix} + \begin{bmatrix}
M_{fx} \\
M_{fy} \\
N_{fz}
\end{bmatrix} = \begin{bmatrix}
D_{xy} & D_{yx} & D_{xz} \\
D_{xy} & D_{yx} & D_{yz} \\
D_{xz} & D_{yz} & D_{zz}
\end{bmatrix} \begin{bmatrix}
r_x \\
r_y \\
e_{iz}
\end{bmatrix},
\]

(10)

where the coefficients of the stiffness matrix and the conditional temperature moments \(M_{fx}, M_{fy}\), and the force \(N_{fz}\) are equal:

\[
D_{xx} = \sum E_{ot}V_{ot}A_{xx}Z_{xx}^2 + \sum E_{ot}V_{ot}A_{xy}Z_{yx}^2 + \sum E_{ot}V_{ot}A_{xk}Z_{xk}^2, \\
D_{yy} = \sum E_{ot}V_{ot}A_{xx}Z_{xx}^2 + \sum E_{ot}V_{ot}A_{yy}Z_{yy}^2 + \sum E_{ot}V_{ot}A_{yk}Z_{yk}^2, \\
D_{zz} = \sum E_{ot}V_{ot}A_{xx}Z_{xx}^2 + \sum E_{ot}V_{ot}A_{zy}Z_{zy}^2 + \sum E_{ot}V_{ot}A_{zk}Z_{zk}^2, \\
D_{xy} = \sum E_{ot}V_{ot}A_{xx}Z_{yx}^2 + \sum E_{ot}V_{ot}A_{yy}Z_{xy}^2 + \sum E_{ot}V_{ot}A_{yk}Z_{yk}^2, \\
D_{xz} = \sum E_{ot}V_{ot}A_{xx}Z_{xk}^2 + \sum E_{ot}V_{ot}A_{yy}Z_{y}Z_{xk} + \sum E_{ot}V_{ot}A_{zk}Z_{z}, \\
D_{yz} = \sum E_{ot}V_{ot}A_{xx}Z_{zx}^2 + \sum E_{ot}V_{ot}A_{yy}Z_{yz}^2 + \sum E_{ot}V_{ot}A_{zk}Z_{xk}^2, \\
D_{xz} = \sum E_{ot}V_{ot}A_{xx}Z_{xk}^2 + \sum E_{ot}V_{ot}A_{yy}Z_{xy}^2 + \sum E_{ot}V_{ot}A_{zk}Z_{z},
\]

(11)

\[
M_{fx} = \sum \sigma_{ot}T_EV_{ot}A_{ot}Z_{ax}, \\
M_{fy} = \sum \sigma_{ot}T_EV_{ot}A_{ot}Z_{ay}, \\
N_{fz} = \sum \sigma_{ot}T_EV_{ot}A_{ot}Z_{zk}.
\]

(12)

Thus, a general system of physical relations (10) has been established for the calculation of reinforced concrete core structures using modern computational methods in the case of simultaneous action of stress in two planes and the variable in the cross-section thickness negative temperature of concrete and rebar freezing.
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