Research of Small-Scale Turbulence’s Parameters Distribution

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Abstract. The article analyzes the influence of seasonal and temporal factors, as well the water areas geographic coordinates on the distribution parameters value of turbulized interlayers in depth. As the initial data for these studies, vertical profiles of the temperature and velocity of water masses were used.

1. The topicality of the problem
Most types of water movements in the ocean have a turbulent character, that is, all hydrodynamic characteristics (velocity, pressure, temperature, etc.) are changing randomly, chaotically in space and time under practically unchanged external conditions. The presence of oscillations, which have different periods and amplitudes, indicates a complex, multiscale internal structure of turbulent flows. And namely a random character, multiscale structure and increased ability to transfer the amount of motion, heat and passive impurities are the main features of turbulent flows, distinguishing them from laminar ones. Turbulent regime arises from the loss of hydrodynamic stability with respect to small perturbations evolved in the laminar flow. Turbulent flows, as a phenomenon, have a very strong effect on objects that fall into the turbulence region, since intensive mixing is observed in these areas, large gradients and pulsations of the particle velocities, which significantly affect the destruction of materials.

Today the problem of protection and rational use of water resources has acquired a global character. In the mathematical modeling of processes that affect the water quality, a joint research of the hydrophysical and chemical-biological parameters of aquatic environments is necessary. Hydrophysical processes to a large extent form the habitat of hydrobionts, determine the transfer and sedimentation of substances, the processes intensity of pollution and self-purification of water bodies. Since the turbulence of the aquatic environment is characterized by intermittency, i.e. the vertical layer has sections with laminar flow and with pronounced turbulence, then obtaining a picture of this intermittency is an important task, both for researchers of this phenomenon, and for seafarers, fishing, etc.

2. Analysis of existing solutions
In [1], the parameters of turbulent interlayers are analyzed and mathematical models for their determination are given. The main characteristics of intermittency are the number of turbulized
interlayers \( n \) in a given vertical section, the distance between the interlayers \( r \) and their thickness \( h \). Since the mechanisms for the formation of turbulence are very diverse (the instability of surface wind waves, the shear instability of the flow velocity, the temperature stratification, the instability of internal waves and their overturning), the parameters characterizing the arrangement's vertical picture of such layers (\( \tau \)-layers) are random variables and for their description functions of probability density are used:

\[
p_1(n) = \frac{1}{n!} (\mu H)^n \exp(-\mu H),
\]

where

\( p_1(n) \) - the probability that in a layer of thickness \( H \) there are \( n \) \( \tau \)-layers;

\( \mu \) - distribution parameter;

the parameter estimation \( \mu \) is the average number of \( \tau \)-layers per 1m.

\[
F_1(h_i) = 1 - \exp(-\alpha h_i)
\]

(2)

where

\( \alpha \) - distribution parameter;

\( h_i \) - distance between \( \tau \)-layers.

\[
F_2(h_r) = 1 - \exp(-\beta h_r)
\]

(3)

\( \beta \) - distribution parameter;

\( h_r \) - thickness of the \( \tau \)-layer.

The last two parameters are sufficient to obtain a location picture of the interlayers in depth. The probability density functions parameters of these characteristics are random variables and depend on the depth of the interlayer. In order to research these relationships, a large amount of experimental data is needed, therefore, in [1] an empirical formula is proposed for computing the parameters \( \alpha \) and \( \beta \), which includes the data of all preliminary sounding in the investigated water area region:

\[
g_k(\mu/k, \sum_{i=1}^{k} h_i) = \frac{1}{(k-1)!} \mu^{k-1} \exp(-\mu \sum_{i=1}^{k} h_i)
\]

(4)

where

\( k \) - the number of measurements;

\( \sum_{i=1}^{k} h_i \) - total thickness of interlayers.

The values of \( k \) and \( \sum_{i=1}^{k} h_i \) are given in the tables [1]. Calculations of these parameters are based on the large volume data of preliminary soundings throughout the entire ocean studies time.

3. Results of the research

The purpose of this paper is to investigate factors affecting the form and parameters of the distances distribution between turbulized interlayers and their thicknesses, as well determining the functional dependence of the distribution parameters \( \alpha \) and \( \beta \) in depth. For research use of vertical velocity and temperature profiles. It is proposed to take the geographical coordinates of the water area in which the
study is conducted, the day time and the season of the experiment as factors affecting the view and distribution parameters of turbulent layers. In this regard, all the experimental data were broken down into blocks in accordance with the indicated factors, namely:

- to research the influence of the water areas the Black Sea, Middle and Southern Caspian regions were chosen;
- spring (April), summer (August) and autumn (November) were chosen for the influence research of the season;
- the morning (8:00) and the evening (17:00) were chosen to research the influence of day time.

It was taken into account that the other factors are identical inside each block.

Figures 1 to 3 show the probability density functions of the distances (a) between the turbulized layers and their thickness (b) for each block separately. Analysis of the results shows that the probability density functions of distances and thicknesses have exponential law, the parameters value of which are influenced by the geographical coordinates of the water area, day time and season.

**Figure 1.** Density distribution functions of distances between turbulent layers (a) and layer thickness (b) in August. △ - Northern Caspian; □ - South Caspian; x - Black Sea.

**Figure 2.** Density distribution functions of distances between turbulent layers (a) and layer thicknesses (b) (Middle Caspian). x - April; □ - August; △ - November.
It follows from Fig. 1 that for the Black Sea the average values of the distances between the turbulent interlayers and their thickness are of the order of 4-5 m, while they exceed the same parameters for the Caspian region. The Northern Caspian region is characterized by the smallest distances between the layers (2.5-3.5 m) and their sizes (2 - 3 m). Such data are consistent with theoretical provisions on the influence of water salinity on the occurrence of turbulent bursts [2].

Figure 2 shows an insignificant difference in the size and position of the interlayers, depending on the season for the same area of sea. Nevertheless, it follows from the figure that the largest average values of the interlayer parameters are characteristic for the summer season and less in spring and autumn. In this case, these data are explained by a decrease in temperature in the spring and autumn periods compared to the summer, which is also one of the mechanisms for the formation of turbulence [2].

According to Fig. 3 in the evening, a decrease in the characteristic distances between the layers and their thickness is observed, which can also be explained by a decrease in the air temperature.

From previous researches it follows that the distribution parameters of the characteristic distances between the layers and their thicknesses $α$ and $β$ are random variables. Statistical analysis of these parameters was carried out in two directions [3]:

- distributions of parameters $α$ and $β$ for random values of distances between layers and their vertical sizes, obtained for the whole depth of sounding in the Middle Caspian region, in August are analyzed;

- values $α$ and $β$ of probability density functions of distances distributions and thicknesses of turbulent layers were obtained from fixed depth horizons, for example, 20 meters, and their statistical analysis was performed.

Fig. 4 shows the histograms of the distribution of the parameter $a$ of the distance density function between the layers (a) and the parameter $β$ of the density function of the thickness of the $τ$-layer (b) obtained as a result of the first direction of investigations for the Black Sea in August. Fig. 5 gives the curves of the probability density functions for the parameters $α$ (a) and $β$ (b), the depths distributed horizontally, obtained for the Middle Caspian during the summer period.

Based on the type of histograms, we put forward a hypothesis about the normal law of distribution of the parameters $α$ and $β$. From here we obtained point estimates of the mathematical expectation and standard deviation for the characteristic distances and sizes of the $τ$-layers in the Black Sea in August:

$$Mα = 5,12; \, \sigmaα = 0,77;$$
$$Mβ = 4,78; \, \sigmaβ = 1,02.$$
In Fig. 4 also shows the density function of the parameters $\alpha$ and $\beta$ on the normal distribution law. The obtained results do not contradict the results of studies [1], where the formation of $\tau$-layers with distances between them commensurate with their thickness at medium thicknesses of layers of 4-6 m scale is observed for the Black Sea water area in summer.

As follows from the results shown in Fig. 5 estimates of mathematical expectation and standard deviation over fixed depth horizons vary in depth. To analyze the trends of their change in Fig. 6 shows the vertical profiles of the temperature typical for the given water area in August, and Fig. 7 shows the dependences graphs of the mathematical expectation’s estimates and variance for the parameters $\alpha$ and $\beta$ on the depth. A comparative study of these figures shows that the distance between the $\tau$-layers decreases in the thermocline region (10-110 m) and sharply increases upon transition to a quasihomogeneous layer. The change in the thickness of the interlayers is also observed in the region of transition to a quasihomogeneous layer. The same observation concerns the mean square deviations of the parameters under study. The obtained data are interesting in that they confirm the influence of temperature changes on the mechanisms of formation of turbulent perturbations.

Using the approximation methods, we determine the coefficients of the polynomial describing the dependence of the mathematical expectation’s estimates of the parameter $\alpha$ by the depth. For a polynomial of degree four $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, describing the function $M(\alpha)$, the following values of the coefficients were obtained: $a_4 = 1.57E-07, a_3 = -5.8E-05, a_2 = 0.005, a_1 = 0.038, a_0 = 1.115$. In Fig. 6 for the mathematical expectation of the parameter $\alpha$ is a graph constructed from the obtained polynomial. The integral error is 1.23%.
Figure 5. The probability density functions of the parameters $\alpha$ (a) and $\beta$ (b) are distributed in depth.

4. Conclusions

The results of the performed analysis confirm the theoretical assumptions about the normal distribution law of the random parameters $\alpha$ and $\beta$, and also that these parameters are not only random, but also depend on the depth. According to the research results the following results were obtained:

1. It has been revealed that the parameters $\alpha$ and $\beta$ of the distances distribution between the $\tau$-layers and their thickness depend on the time factors and geographical coordinates of the investigated water area. This influence is determined by changes in the temperature regime, salinity and density of water.
2. The distribution of the parameters $\alpha$ and $\beta$ along the depth horizons has been analyzed and it is established that these parameters tend to increase in the thermocline region and sharply decrease upon transition to the quasihomogeneous layer. These parameters are inversely proportional to the distance between the $\tau$-layers and the thickness.

3. The coefficients of the polynomial, approximating the experimental estimates of the mathematical expectation and the standard deviation for parameters $\alpha$ and $\beta$ of the $\tau$-layer distribution are determined. The integral error does not exceed 1.5%.

The results of the research can be used to develop mathematical models for constructing a pattern of intermittency of turbulence in depth [4-6].
Figure 7. Graphs of the mathematical expectation and variance estimates for the parameter $\alpha$ (a) and $\beta$ (b) from the depth.

5. References
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