Analysing periodic amplitude changes in RR Lyrae and other types of variable stars

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Abstract. Various types of pulsating stars show periodic amplitude changes. The Fourier spectra of these stars typically exhibit a structure of either two or three close frequencies. But the existence of a close peak in the spectrum does not necessarily mean that a second pulsation mode causes the variation in amplitude. Therefore, a test concerning the phasing behaviour at different stages of the beat cycle was applied to determine whether the cyclic behavior is caused by beating or by other effects. On this basis a comparison is drawn between several different types of pulsators. Nonlinear effects in RR Lyrae stars pose an additional challenge to the method. For all examined stars we could show that two separate modes are present and that the amplitude changes are likely caused by beating.

1. Motivation

In many RR Lyrae stars, amplitude variability - the so-called Blazhko effect - is observed. In the Fourier spectrum this behaviour manifests itself in two or three closely spaced frequencies. Many authors take it for granted that two separate excited modes are present and their beating with each other causes the observed amplitude variations. But peaks in the Fourier spectrum are not necessarily independent modes, and the Blazhko effect is still an unsolved problem, i.e., the cause for the amplitude variability is not clear. A variation in amplitude inevitably leads to a multiplet structure in the Fourier diagram, also when caused by other effects than beating (see Figure 1). Therefore it is necessary to prove that beating is present before assuming that both peaks which can be found in the power spectrum are real modes.

\textbf{Figure 1.} Why multiple peaks in a Fourier diagram are no positive proof for beating: Just one frequency was used to produce this set of artificial data, but a variation in amplitude leads to the observed triplet structure in the Fourier diagram.
Figure 2. Stellar data compared to the simulations. Left panel: RR Lyrae, right panel: SS Fornacis. Dots show the real data, the green line indicates a simulation of beating between two modes, the brown line is a simulation of amplitude change that is not caused by beating. The difference between the two simulations can be seen in the phase (lower panel) where beating leads to a clear variation while amplitude variation of a single mode yields a constant phase. For each star amplitudes of the frequencies in the simulation were chosen as found in the Fourier transform of the data, i.e., an amplitude ratio of 5.2 for RR Lyrae and 6.9 for SS For respectively.

2. The Method
In their paper Close frequency pairs in Delta Scuti stars [1] the authors used the phasing behaviour to differentiate between amplitude variability of a single mode and beating between two excited modes. This method was successfully applied to the Delta Scuti star BI CMi. The variable phase technique relies on the fact that the two scenarios of a) a single mode with variable amplitude and b) two modes beating with each other lead to very different phasing behavior. In the case of equal amplitudes of the two modes, a 180 degree phase jump occurs at the time of minimum amplitude, while with a single frequency the phase remains constant. In the case of unequal amplitudes a variation of the phase with its steepest slope at the time of minimum amplitude is expected to occur instead of the phase jump (see lower panel of Figure 2).

3. RR Lyrae stars
For our investigations of the Blazhko effect we applied the test to two Blazhko RR Lyrae stars SS For and RR Lyr which were observed in the framework of the Blazhko project (for details about the data sets see Kolenberg et al. 2006 [2] and Kolenberg et al. 2007 [4]). The data sets were divided into phase bins of the modulation period. A Fourier analysis was performed and
for each phase bin a fit was calculated using the following formula:

\[ m(t) = \sum_{i=1}^{n} A_i \sin(2\pi f t + \varphi_i) \]

\( A \) and \( \varphi \) are the amplitudes and phases respectively, \( f \) is the frequency and \( n \) is the number of frequencies. The amplitude \( A_1 \) and phase \( \varphi_1 \) of the fundamental frequency were plotted against modulation phase. The result was then compared to simulations of a beating on the one hand and a variation of amplitude on the other hand. The light curves of RR Lyrae stars are very non-sinusoidal. Therefore, 7 harmonics were included in the simulations (\( n=8 \)). Amplitudes of the modes and their harmonics were chosen as found in the stellar data. It can be seen that the behavior of the RR Lyrae stars fits the model of beating between two modes very well (see Figure 2). In the case of beating, the third component of the observed triplet structure can be explained as a combination frequency following Breger & Kolenberg [3].

Errors were estimated using the standard formulae

\[ \sigma(A) = \sqrt{\frac{2}{N}} \sigma(m) \]

\[ \sigma(\varphi) = \frac{\sigma(A) \ 1}{A \ 2\pi} \]

multiplied by a factor of two as suggested by Handler et al. 2000 [5]. Some additional errors might be introduced by the data sampling. At some Blazhko phases the light curve changes very quickly, resulting in differing light curves within the related phase bin. Here, data sampling properties might lead to slightly deviating amplitudes and phases.

![Figure 3](image)

**Figure 3.** Comparison of different treatment of harmonics. •: Change of \( A_1 \) and \( \varphi_1 \) during the Blazhko cycle while amplitudes and phases of harmonics were fixed, ○: Change of \( A_1 \) and \( \varphi_1 \) during the Blazhko cycle while all amplitudes and phases of harmonics were allowed to change during the Blazhko cycle.
3.1. **Influence of nonlinearities**
The method of investigation of the phase behaviour was so far only applied to stars with sinusoidal light variations. Therefore, its application to RRab stars with their very nonlinear light curves was treated with caution. Moreover, it is well known that the light curve shape of RR Lyrae stars changes during the Blazhko cycle. Also, secondary features like the so-called bump are known to change during the modulation [6] resulting in changing amplitudes of the harmonics which describe the shape of the light curve. As mentioned earlier, 7 harmonics were included into the fit. In the first iteration, the amplitudes and phases of the harmonics were fixed to the values that best fit the complete data set, while only the amplitude and phase of the fundamental frequency was allowed to change from bin to bin. The result of this analysis was chosen for Figure 2. Then, the same analysis was also performed, allowing all amplitudes and phases to vary. The results turned out to be very similar. Figure 3 compares the solutions of the two different methods.

4. **The sdB star PG 1605+72**
The pulsating hot subdwarf PG 1605+72 was observed extensively by Kilkenny et al. (1999) [7]. Photometric results show a strong variation of the amplitude on a timescale of about a week which corresponds to a frequency pair at 181.7251 and 181.5872 c/d. When applying the phase jump method to this star the results turn out to be a textbook example: a clear discontinuity of phase can be seen exactly at the time of minimum amplitude (Figure 3). This explicitly favors the two-mode hypothesis (beating).

![Figure 4. Analysis of PG 1605+72. Top panel: amplitude behaviour during a modulation cycle, bottom panel: phase behaviour. Dots are the results obtained from stellar data, solid lines indicate the simulation of a beating between two modes with nearly equal amplitudes as it is the case for PG 1605+72. Error bars are smaller than the symbols.](image)

5. **Conclusions**
The variable phase technique was used to investigate the two well-observed Blazhko RR Lyrae stars SS For and RR Lyr. For both stars, the results agree with the model of two separate modes beating with each other and causing the amplitude variations. Changing light curve shapes do
not cause problems, both methods of treating the harmonics yield comparable results. For the sdB star PG 1605+72 it could be shown that the amplitude changes are caused by beating.

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