Number counts and redshift distribution of gravitational arcs are computed in the field of massive clusters of galaxies to probe the universe at high redshift. Using an accurate modelling for the cluster mass distribution and a model for the spectrophotometric evolution of galaxies, the redshift distribution of gravitational arclets is computed in the field of cluster Abell 2218 and in the Hubble Deep Field where a cluster is artificially located. Counts are very well reproduced in the B band but an important population appears at high redshift which is not seen in deep spectroscopic surveys. Unfortunately, the very high sensitivity of the counts with respect to the model for galaxy evolution and to the mass distribution prevents from estimating the cosmological parameters with arcs statistics. Future works have to concentrate on high redshift clusters and take advantage of objects with smaller distortions.

1 Number counts and redshift distribution of gravitational arclets

A powerful way to investigate the population of high redshift galaxies is to use the magnification by the gravitational potential of massive clusters of galaxies. The spectrophotometric evolution of galaxies can then be probed by counts and redshift distribution of gravitational arcs.

The number counts of gravitational arclets in the field of a massive cluster of galaxies are a competition between, first, the magnification of the luminosity by the cluster potential that makes more objects visible and, second, the surface dilution that decreases the surface density of arclets by the same factor as for the magnification. If the surface density of galaxies up to magnitude \( m \) is \( n(<m) = n_010^{\alpha m} \), the ratio of the density of arcs over the density of field galaxies is \( \frac{n_{\text{arc}}(<m)}{n(<m)} = M^{2.5\alpha-1} \) where \( M \) is the magnification. For observations at faint level in the blue band, the counts remain roughly unchanged while from the \( R \) to \( K \) wavebands (\( \alpha < 0.4 \)) a depletion takes place with respect to an empty field (Broadhurst 1995, Fort et al. 1996).

The number of arcs brighter than magnitude \( m \) with an axis ratio greater than \( q_{\text{min}} \) and a surface brightness brighter than \( \mu_0 \) within the field of a cluster of galaxies is:

\[
N(m, q_{\text{min}}, \mu_0) = \sum_i \int_{z_l}^{z_{\text{max}}} \int_{q_{\text{min}}}^{\infty} \int_{L_{\text{min}}}^{L_{\text{max}}} \frac{dV}{dz} dz dq dL dq \Phi_i(L, z)
\]

The sum is over the different morphological types \( i \). \( z_l \) is the lens redshift and \( z_{\text{max}}(\mu_0, i) \) is the redshift cutoff corresponding to the limit in central surface brightness \( \mu_0 \). \( S(q, z, H_0, \Omega) \) is the angular area in the source plane (at redshift \( z \)) that gives arcs with an axis ratio between \( q \) and \( q + dq \). The luminosity \( L \) is given by the model of Bruzual and Charlot (1993, GISSEL96) using the results of Pozzetti et al. (1996).

It was required as a preliminary step that counts and redshift distribution in empty field and in various wavebands are correctly reproduced. We used the model for galaxy evolution of...
Figure 1: Left: Number counts of arclets in A2218 with the F702W filter, per bin of one magnitude \((a/b > 2, \mu_{F702W} \leq 25.5)\). \(\circ\): observed counts. Predicted counts are displayed for three mass distributions: SIS (dotted line), bimodal mass distribution of Kneib et al. (1995) (dashed line) and a mass distribution including substructures from Kneib et al. (1996) (solid line). Error bars correspond to statistical fluctuations. Right: Redshift distribution of arclets in A2218 per bin of 0.05 in \(z\) \((a/b > 2, R_{F702W} \leq 23.5 \text{ and } \mu_R \leq 24)\). Same notations as for the left figure. The histogram corresponds to the redshifts compiled by Ebbels et al. (1997), ordinate is in arbitrary units.

Bruzual and Charlot (1993) with the prescriptions of Pozzetti et al. (1996) (see Bézecourt et al. 1998 for details). In order to check the sensitivity to the cluster mass distribution, three different potentials are used for cluster A2218. The simpler one is a singular isothermal sphere (SIS) whose velocity dispersion is determined by the location of a giant arc at \(z = 0.702\) (\(\sigma = 1031 \text{ km s}^{-1}\)). The second one is a bimodal mass distribution centered on the two main galaxies of the cluster (Kneib et al. 1995). The more complex potential used was obtained by Kneib et al. (1996) and includes galaxy scale components to account for the numerous substructures in the cluster. Unless specified otherwise, \(H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}\), \(q_0 = 0.5\) and \(\Omega_\Lambda = 0\).

## 2 Results

### 2.1 Absolute number counts in cluster A2218

The detection of elongated objects in A2218 was performed in the frame of the WFPC2 HST image (6200 sec.) in filter F702W (Kneib et al. 1996).

The observed and predicted number counts of arclets are in very good agreement in the \(B\) band considering the mass distribution of Kneib et al. (1996). Counts in the \(R\) band are shown in Figure 1. The predicted total number of arcs \((R_{F702W} \leq 23.5 \text{ and } a/b \geq 2)\) with the best mass model is lower than the observed number by a factor of 1.8. The bimodal model of Kneib et al. (1995) and the SIS model underpredict the counts by factors of respectively 1.5 and 2 with respect to the model of Kneib et al. (1996) including galaxy scale potentials. This is a clear justification for the use of accurate mass distributions in statistics on gravitational arcs.

### 2.2 Redshift distribution

The redshift distribution of arclets selected in the F702W filter is presented in Figure 1 and can be partially compared to the results of a spectroscopic survey of arclets (Ebbels et al. 1997). However, this survey may be biased in the arclets selection criterion and in the ability to determine a redshift which mostly depends on the existence of emission lines in the optical part of the spectrum. This observed distribution peaks at a value of \(z \simeq 0.6\), with only two objects at \(z > 1\). Figure 1 shows the redshift distribution of arclets expected with the same selection conditions than those adopted by Ebbels et al. (1997). A peak also appears at a similar redshift \((z \simeq 0.8\), solid curve\). The slight difference between the observed and the predicted peak is
due to the hypothesis of circularity for the sources which prevents from obtaining arcs at low redshift just behind the cluster because of a less efficient lensing power. However, another population of objects is expected at \( z > 2 \) which is not seen in the data. This high redshift tail is mainly produced by elliptical-type galaxies and shows the limit of the evolution model for young galaxies and early epochs.

3 Cosmological parameters

Figure 2 shows the predicted counts in A2218 for two different values of \( q_0 \), 0 and 0.5. The difference between the two curves is of the same order of magnitude as the uncertainty due to the mass distribution between mass modellings of Kneib et al. (1995) and (1996). This means that \( q_0 \) cannot be inferred from counts of gravitational arcs. A low \( q_0 \) doesn’t help to reconcile with the data, hence one could argue that it implies a high value for \( \Omega_L \) as it would increase the counts. However, this is unlikely as the counts of arclets in the \( B \) band are very well reproduced and a similar analysis performed in cluster A370 (Bézecourt et al. in preparation) doesn’t require a high value of \( \Omega_L \). Moreover, uncertainties in galaxy evolution are too high to allow for a determination of \( \Omega_0 \) and \( \Omega_L \) with arcs counts.

4 The Hubble Deep Field

The results presented above can suffer from a privileged line of sight in the direction of A2218 and a representative view of the distant universe is then needed. This sample of distant galaxies is given by the images of the Hubble Deep Field (Williams et al. 1996). The model developed in the previous sections can be pushed to fainter limits by locating artificially the mass distribution of cluster A2218 (Kneib et al. 1996) in front of the HDF. Miralles and Pelló (1998) have determined photometric redshifts for a sample of 1400 galaxies in the HDF enabling to lens these objects by a cluster like A2218.

The predicted counts of arcs in the field of the HDF (F450W filter) with an axis ratio greater than 2 are in excellent agreement with the counts of simulated arcs (Figure 3). The redshift distribution resulting from the combination of the mass model with the evolution model for objects selected in the \( B \) band shows a deficit at \( z < 0.5 \) due to the circularity of sources. On the contrary, an excess of sources at high-\( z \) appears as in Figure 3. A different approach is then needed for galaxy evolution at early epochs, particularly in the hierarchical scenario.
Figure 3: Left: Counts of arclets in the HDF lensed by A2218 artificially located in front of it (arclets with an axis ratio $a/b > 2$). $\circ$: simulated “observed” counts. Solid line: predicted counts for the lens model by Kneib et al. (1996) and $q_0 = 0.5$. Right: Redshift distribution of arclets in the HDF per bin of 0.4 in $z$ for $a/b > 2$ and $B_{F450W} < 27.5$ for the simulated “observed” arclets (histogram) and the predicted distribution (solid line).

5 Conclusions

It is now obvious that arcs statistics can only be performed with mass distributions including substructures, too simple potentials should be avoided. Number counts of arclets are in good agreement with observations in the $B$ band and the redshift distribution of arclets at $z \leq 1.0$ is correctly predicted by the model. The important population of arclets expected at $z \geq 1.0$, which is not observed in spectroscopic surveys, is highly dependent on the modelling of high redshift ellipticals and the role of dust absorption in the rest frame UV.

Unfortunately, the geometry of the universe cannot be determined with arcs statistics because of too important uncertainties in clusters mass distributions and in the model for galaxy evolution. Geometrical effects have now to be investigated with clusters at redshift greater than 0.5 considering also the weak lensing regime. High-$z$ lens efficiency relates more directly to cosmology and counts of all the background objects in a cluster field enable accurate measurement of the magnification bias that probes the high redshift population.

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