Mixed vortex states of light as information carriers

Zdeněk Bouchal and Radek Čelechovský
Department of Optics, Palacký University, 17 Listopadu 50, 772 07 Olomouc, Czech Republic
E-mail: Bouchal@optnw.upol.cz

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Abstract. An original method for encoding of information into the spatial structure of mixed vortex states of light is proposed. It is based on the creation of pseudo-nondiffracting vortex patterns representing a controllable superposition of optical vortices with different topological charges. Weight coefficients of the superposition serve as carriers of information. The different topological charges of the vortices are used as ‘markers’ enabling vortex spatial separation and information decoding. The proposed concept of information encoding is verified experimentally by means of a spatial light modulator.

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1. Introduction

Since the modern study of the orbital angular momentum (OAM) by Allen et al [1], great interest has been focused on optical beams with the helical wavefront characterized by a phase term \( \exp(i m \varphi) \) depending on the topological charge \( m \) and on the circular cylindrical coordinate \( \varphi \).
Such beams, usually called optical vortices (OVs), possess phase dislocations and their OAM has the discrete value of $m\hbar$ per photon. The OAM carried by optical beams is associated with a spiral flow of the electromagnetic energy and has important mechanical consequences [1]–[3]. They are manifested by trapping and rotation of microparticles and atoms [4] and can be useful in designing optical tweezers and spanners and light-powered micromechanical systems.

Unlike common light fields, the OVs possess additional degrees of freedom prospective for the transfer of information. Their application provides a possibility to increase the information density. It was shown in [5] that photons prepared in multidimensional states of the OAM allow the generation of quNits applicable in quantum information systems.

In this paper, encoding of information into the spatial structure of the mixed vortex field is proposed, analysed theoretically and verified experimentally. The concept of information decoding is also examined and demonstrated by numerical simulations. The present methods of the information encoding and decoding are based on physical principles explored in the framework of the nondiffracting propagation of light [6].

Nondiffracting beams possess unique properties useful for nonlinear optics, particle manipulation, electron acceleration, photolithography, metrology and potentially also for applications to free space communications. Although nondiffracting beams are not realizable in an exact form, their good approximation can be obtained experimentally. Such beams are known as pseudo-nondiffracting (P-N) beams. During free propagation, their transverse intensity profile remains nearly invariant in a range which can be changed by geometrical parameters of the experiment. Another important advantage of the P-N beams is their self-regeneration ability [7]. Owing to this property, the P-N beam disturbed by amplitude or phase obstacles revives and during free propagation gets its initial amplitude and phase profile (see the animation for an experimental demonstration of the effect).

Methods of generation of the P-N beams are very variable and enable creation of the beams with a predetermined intensity profile [8]. Recently, methods composed of OVs of the same topological charge have been adopted for generation of P-N vortex arrays [9, 10]. In this paper, the mixed vortex field, representing the superposition of OVs with different topological charges, is examined as a convenient carrier of information. Applying a spatial light modulator, the mixed vortex field of the form $U = \sum_{m=1}^{M} a_m u_m \exp(i m \varphi)$ can be generated directly by a single Gaussian beam. The number of superposed vortices $M$, their topological charges $m$, amplitudes $u_m$ and weight coefficients $a_m$ are encoded into the structure of the computer-generated hologram sent to the spatial light modulator. If the modulator is driven dynamically, the weight coefficients can represent bits of information encoded into the spatial structure of the mixed vortex field. After Fourier filtration performed behind the spatial light modulator, the vortex field propagates as the P-N field. At the receiver, the weight coefficients must be measured to decode information. In the proposed decoding scheme, it is performed by the spatial separation of the OVs. The method is realizable by means of amplitude and phase modulation and works in such a way that the weight coefficients of the OVs are identified as bright intensity spots localized at positions uniquely specified by their topological charges.

The theoretical analysis and experimental verification of the information encoding and numerical estimations of the information decoding demonstrate the potential of the present method. The information encoding is not restricted to application of the P-N Bessel Gauss modes; a base of the Laguerre Gauss modes can also be created by the proposed method. Interferometric methods [11] enabling a sorting of single photons on the basis of their OAM can be adopted to improve the information decoding.
2. Description of the mixed vortex field

Creation of mixed vortex fields is based on a variability of nondiffracting P-N fields. Applying an integral treatment, a general P-N monochromatic field with angular frequency $\omega$ can be expressed in the form

\[ U(r, t) = \exp(i\omega t) \int_0^{2\pi} A(\psi) g(r, \theta, \psi) \, d\psi. \]  

(1)

The integral describes the P-N field propagating along the $z$-axis as a superposition of inclined Gaussian beams $g(r, \theta, \psi)$ whose axes create a conical surface with the vertex angle $2\theta$. The $z$-direction is the cone axis. The axes of the separate Gaussian beams used in the superposition are specified by the azimuthal angle $\psi$ and their complex amplitudes are modified by $A(\psi)$. As is known [6], the complex amplitude (1) describes the P-N field even if the function $A(\psi)$ is arbitrarily changed. In this paper, this degree of freedom is used for the generation of the mixed vortex fields with a multi-helix phase. In that case the azimuthal modulation $A(\psi)$ is chosen as

\[ A(\psi) = \sum_{m=1}^{M} \sum_{n=1}^{N} a_m t_m(\psi) s_n(\Delta x_n, \Delta y_n, \psi), \]  

(2)

where $a_m$ denotes the weight coefficients and $t_m$ and $s_n$ are the modulating functions enabling formation of the phase topology of the $m$th vortex mode and the shift of the vortex centre to the position with coordinates $(\Delta x_n, \Delta y_n)$, respectively. They are given by

\[ t_m = \exp(im\psi), \]  

(3)

\[ s_n = \exp[i\alpha(\Delta x_n \cos \psi + \Delta y_n \sin \psi)], \]  

(4)

where $\alpha$ is the parameter influencing the spot size of the superposed vortex modes. The integration (1) performed with the given azimuthal modulation $A$ results in

\[ U(r, t) = u_0(z)G(r, t) \sum_{m=1}^{M} \sum_{n=1}^{N} a_m u_{m,n}(r_n) \exp(im\varphi_n), \]  

(5)

describing an array of the Bessel beams carried by a background Gaussian beam $G$ modified by a propagation function $u_0$. The Gaussian beam can be conveniently described by the confocal parameter $q_0$ or by the complex parameter $q = z + i q_0$. The function $u_0$ changes under a free propagation and depends on both $\alpha$ and $q$. It can be expressed as

\[ u_0(z) = 2\pi \exp \left[ i\frac{\alpha^2 z}{2k} \left( 1 - \frac{z}{q} \right) \right]. \]  

(6)

The complex amplitude $G$ has the known form

\[ G(r, t) = \frac{w_0}{w} \exp \left( -\frac{r^2}{w^2} - \frac{kr^2}{2R} + i\Phi \right) \exp(i\omega t - ikz), \]  

(7)

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where \( r^2 = x^2 + y^2 \), and \( \omega, k, w_0, w, R \) and \( \Phi \) denote the angular frequency, the wavenumber and the parameters of the common Gaussian beam, respectively. Since the axes of the Gaussian beams used in integral (1) are inclined by the angle \( \theta \) with respect to the \( z \)-axis, the beam parameters are transformed as

\[
w_0^2 = \frac{2q_0}{k},
\]

(8)

\[
w = w_0 \left[ 1 + \left( \frac{z}{q_0 \cos \theta} \right)^2 \right]^{1/2},
\]

(9)

\[
R = \frac{z}{\cos \theta} + \frac{q_0^2 \cos \theta}{z},
\]

(10)

\[
\Phi = \arctan \left( \frac{z}{q_0 \cos \theta} \right).
\]

(11)

The centres of the separate Bessel spots can be shifted to the positions defined by coordinates \((\Delta x_n, \Delta y_n)\) so that an array of predetermined form can be created. The Bessel spots are described by a complex amplitude \( u_{m,n} \) and a phase term \( \exp(im\varphi_n) \) depending on the complex variables \( r_n \) and \( \varphi_n \). They can be comprehended as generalized circular cylindrical coordinates

\[
r_n = (\bar{x}_n^2 + \bar{y}_n^2)^{1/2},
\]

(12)

\[
\varphi_n = \arctan \left( \frac{\bar{y}_n}{\bar{x}_n} \right).
\]

(13)

The off-axis shifts of the beam spot centres are expressed by the transformations

\[
\bar{x}_n = x - \frac{q}{z + q} \Delta x_n,
\]

(14)

\[
\bar{y}_n = y - \frac{q}{z + q} \Delta y_n.
\]

(15)

The amplitude \( u_{m,n} \) then can be written by means of the Bessel function of the first kind and the \( m \)th order \( J_m \),

\[
u_{m,n}(r_n) = i^{-m}J_m \left[ \alpha \left( 1 + \frac{z}{q} \right) r_n \right].
\]

(16)

The complex amplitude (5) describes the P-N mixed vortex field representing the superposition of OVs with different topological charges. The parameters of the mixed vortex field such as the number of superposed OVs, their weight coefficients, spot sizes, topological charges and spatial positions of the vortex centres depend on the choice of the function \( A(\psi) \) given by (2). This dependence is important as \( A(\psi) \) can be simply changed experimentally. In the limit case \( w_0 \to \infty \), the \( z \)-dependence of the complex amplitude \( U \), given by (5), appears only
Figure 1. An optical set-up demonstrating the principle of generation of the P-N mixed vortex fields. The complex mask is composed of amplitude and phase masks. It creates the mixed vortex composed of four OVs with the topological charges \( m = 1, 3, 5 \) and 7, respectively. Illumination of the mask by two inclined collimate beams results in creation of two spatially separated P-N mixed vortex fields.

Parameters of the mixed vortex field can be controlled based on the principle depicted in figure 1. As follows from (1), P-N beams can be comprehended as an interference field of the inclined Gaussian beams whose axes create a conical surface with the angle \( 2\theta \). Controlling the amplitudes and phases of the Gaussian beams, the intensity profile of the generated interference field representing the P-N beam can be changed to a required form. A simple experimental set-up can be designed to obtain a superposition of the inclined Gaussian beams (figure 1). In the simplest case, it consists of the amplitude mask transmitting light through a thin annular ring and the Fourier lens with Gaussian transparency. If the annular ring is placed at the focal plane of the lens and illuminated by a plane wave (a collimate laser beam) propagating along the \( z \)-axis, the separate points of the ring emit spherical waves which become Gaussian beams behind the lens. Their axes are inclined by the angle \( \theta \) with respect to the \( z \)-axis, and is given by \( \theta = \arctan(\rho_0/f) \), where \( \rho_0 \) is the radius of the ring and \( f \) denotes the focal length of the lens.

The created P-N beam has the transverse intensity profile given by the product of the zero-order Bessel function and the Gaussian function. It is called the Bessel–Gauss beam and does not represent a vortex beam. Its central spot is bright and remains nearly unchanged.
in the region whose length can be approximated by \( L \approx \frac{w_0}{\theta} \), where \( w_0 \) is the waist radius of the Gaussian function describing transparency of the Fourier lens. The size of the central spot of the created beam also depends on the angle \( \theta \) and its radius can be approximated by \( r \approx \frac{1}{\alpha} \), where \( \alpha = k \sin \theta \). The bright beam centred on the axis is obtained if the used annular ring does not change the amplitude and phase of the collinearly impinging collimate beam. If the spiral phase mask \( t_m = \exp(i m \psi) \) is placed near the annular ring mask, the bright P-N beam becomes a dark vortex beam with the topological charge \( m \). Its spot size and length \( L \) exhibit the same dependence on the parameter \( \theta \) as in the case of the bright P-N beam. If the spiral mask is replaced by the complex mask with transparency \( \sum_{m=1}^{M} \exp(i m \psi) \), the mixed vortex field is created. It is composed of \( M \) vortices with different topological charges. If the mask is illuminated by a collimate beam impinging along the \( z \)-axis, the mixed vortex field is centred on the \( z \)-axis. The situation changes if the mask is illuminated by an inclined collimate beam. It can be approximated by a plane wave whose phase is given by \( \exp(-i k_n \cdot r) \). In that case, the created mixed vortex field still propagates along the \( z \)-axis but its centre is shifted to the off-axis position. At the points of the annular ring, the phase of the plane wave can be expressed as \( \exp[-i \rho_0 (k_{xn} \cos \psi + k_{yn} \sin \psi) - i k_{zn} z] \). If we compare the phase part depending on the transverse coordinates with the phase modulation causing a transverse shift of the vortex centre (4), we can write

\[
\begin{align*}
    k_{xn} &= -\frac{\alpha}{\rho_0} \Delta x_n, \\
    k_{yn} &= -\frac{\alpha}{\rho_0} \Delta y_n.
\end{align*}
\]

The transverse components of the wavevector of the inclined plane wave \( k_{xn} \) and \( k_{yn} \) are specified by the required coordinates of the centre of the created vortex field \( \Delta x_n \) and \( \Delta y_n \). If the complex mask \( \sum_{m=1}^{M} \exp(i m \psi) \) is illuminated by \( N \) plane waves (collimate beams) impinging from different directions \( \sum_{n=1}^{N} \exp(-i k_n \cdot r) \), the variable vortex patterns can be generated. They are composed of \( N \) mixed vortex fields centred at predetermined positions of the transverse plane. Each mixed vortex field is composed of \( M \) vortices with different topological charges \( m \). The complex mask illustrated in figure 1 creates superposition of the OVs with topological charges \( m = 1, 3, 5 \) and \( 7 \), respectively. The complex mask is composed of the amplitude and phase mask (phase values are demonstrated by grey levels in figure 1). Illumination of the mask by two collimate beams impinging from different directions results in two spatially separated P-N mixed vortex fields illustrated in figure 1. In experiments, the geometrical parameters of the vortex patterns can be controlled and adjusted in a very wide range. They significantly depend on parameters \( \theta \) and \( w_0 \). The total dimensions of the generated pattern are restricted by the waist radius \( w_0 \), the transverse size of the separate mixed vortex fields can be approximated by \( 1/\alpha \) and the positions of their centres are given by the inclinations of the impinging collimate beams (figure 1). The created pattern is damped by a Gaussian envelope with the waist radius \( w_0 \) so that the coordinates of the vortex centres must fulfil the condition \( |\Delta x_n| < w_0 \) and \( |\Delta y_n| < w_0 \). The length of the range where the mixed vortex field appears is still approximated by \( L \approx \frac{w_0}{\theta} \).

In comparison with common light beams, the mixed vortex field (5) provides additional degrees of freedom applicable to a new way of information encoding into the spatial structure of light. A simple set-up illustrated in figure 1 demonstrates the basic principle of the vortex field creation. Although it is realizable (the amplitude and phase masks can be prepared

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photolithographically), its applications are restricted. The reason is that \( N \) input beams must be available to create \( N \) spatially separated mixed vortex patterns. Furthermore, the created superposition of vortices can be changed only by the change of the complex mask. Both disadvantages are removed by an experimental set-up based on an application of a spatial light modulator. In that case, the general mixed vortex fields (5) can be created dynamically from the single Gaussian beam. The following analysis and experimental verification are focused on the information encoding performed by the spatial light modulation.

3. Encoding, transmission and decoding of information

The application of the mixed vortex field (5) to the information encoding is enabled by the fact that it can be obtained by the direct transformation of the single Gaussian beam. This is the fundamental difference from the previous experiments on vortex patterns obtained by the interference of separately generated vortex beams. The generated mixed vortex field possesses degrees of freedom which are not available by the common light field. As additional parameters, the magnitudes and the signs of the topological charges and the weight coefficients of the OVs can be applied to the information encoding. While the weight coefficients are used as carriers of bits of information, the topological charges serve as ‘markers’ enabling information decoding. Although the position of the vortex centres can also be used as additional degrees of freedom, they are not applied in the paper. We operate with the coaxial OVs whose centres coincide. This special case is given by (5), used with \( N = 1 \) and \( \Delta x_1 = \Delta y_1 = 0 \). The main goal of the paper is not optimizing the encoding scheme but only demonstrating its original principle utilizing the spatial structure of the mixed vortex field.

In the common encoding process, a sequence of \( M \) pulses is required to transmit \( M \) bits of information. Applying the mixed vortex field, \( M \) bits of information can be encoded into the spatial structure of the single input carrying \( M \) coaxial OVs. It can be done simply by modulating the function \( A(\psi) \). Its application enables a generation of \( M \) OVs with topological charges \( m \) and the weight coefficients \( a_m \) whose values are 0 or 1. In that case, \( M \) bits of information \( a_1, a_2, \ldots, a_M \) are represented by the mixed vortex field with the complex amplitude

\[
U_{a_1, \ldots, a_M} = u_0 \sum_{m=1}^{M} a_m u_m \exp(im\varphi).
\] (19)

It propagates as a P-N field and is applicable to the free-space communications. The length of the region \( L \), where the field is available without significant changes, depends on the parameters \( w_0 \) and \( \alpha \) used in (5). It can be approximated by \( L \approx kw_0/\alpha \) and controlled in the experiment. It represents distance of the information transmission and its restriction is given by the technical parameters of the optical set-up used.

To decode information at the receiver, the weight coefficients \( a_m \) must be identified by a simple measurement. This is possible by a method utilizing different topological charges, of the OVs used in the encoded mixed vortex state. Assuming that \( M \) bits of information are encoded into the field composed of \( M \) coaxial OVs with different topological charges, its complex amplitude is given by (19). In that case, the general relation (5) has been used with \( N = 1, \Delta x_1 = \Delta y_1 = 0, r_1 = r \) and \( \varphi_1 = \varphi \). The amplitude and phase of the vortex field carrying information are conveniently modulated at the receiver. The modulation is performed by the
complex transparency $T$ given by

$$
T = \sum_{m=1}^{M} u_m^* \exp[-i(m\varphi + \Phi_m)], \tag{20}
$$

where $\Phi_m = k_{xm}x + k_{ym}y$ is the phase of the plane wave enabling a spatial separation of the OVs by means of a Fourier transform. In this way, the $m$th OV is localized at the position given by the components $k_{xm}$ and $k_{ym}$ of the wave vector. After the modulation, the vortex field is in the state enabling decoding of information. Its complex amplitude $U'_{a_1,\ldots,a_M} = TU_{a_1,\ldots,a_M}$ can be written in the form

$$
U'_{a_1,\ldots,a_M} = u_0 \sum_{m=1}^{M} a_m |u_m|^2 \exp(-i\Phi_m) + u_0 \sum_{m,n=1, m \neq n}^{M} a_m u_n u_n^* \exp[i(m-n)\varphi - i\Phi_n]. \tag{21}
$$

The information decoding is performed by applying the Fourier transform. The obtained angular spectrum $\mathcal{U}'_{a_1,\ldots,a_M} \equiv F^i\{U'_{a_1,\ldots,a_M}\}$ can then be written as

$$
\mathcal{U}'_{a_1,\ldots,a_M} = \sum_{m=1}^{M} a_m \mathcal{U}'_m \left(v_x - \frac{k_{xm}}{2\pi}, v_y - \frac{k_{ym}}{2\pi}\right) + \sum_{m,n=1, m \neq n}^{M} a_m \mathcal{U}'_{m,n} \left(v_x - \frac{k_{xm}}{2\pi}, v_y - \frac{k_{yn}}{2\pi}\right), \tag{22}
$$

where

$$
\mathcal{U}'_m = F^i\{|u_0|u_m|^2\}, \tag{23}
$$

$$
\mathcal{U}'_{m,n} = F^i\{u_0 u_m u_n^* \exp[i(m-n)\varphi]\}, \tag{24}
$$

and $v_x$ and $v_y$ denote the spatial frequencies. The terms $\mathcal{U}'_m$ of the spatial spectrum are obtained as the Fourier transform of the complex amplitude $u_0|u_m|^2$ whose helical phase was eliminated by the complex conjugate phase of the transparency $T$. It results in the bright intensity spots centred at the positions $(k_{xm}/2\pi, k_{yn}/2\pi)$ of the Fourier plane. The terms $\mathcal{U}'_{m,n}$ follow from the Fourier transform of the complex amplitude $u_0 u_m u_n^* \exp[i(m-n)\varphi]$, which contains the helical phase for $m \neq n$. As is known, the helical phase changes the Fourier spectrum to the form of the annular ring. Its dark centre is localized at the position given by $(k_{xm}/2\pi, k_{yn}/2\pi)$. As is obvious, the transparency $T$ serves as the complete base for the projection of the OVs used in the superposition. The bright spots appear at positions related to the OVs with the topological charges $m$ only if their coefficients $a_m$ are equal to 1. The spots appearing at positions related to OVs with the topological charges $m$ and coefficients $a_m = 0$ remain dark. In this way, the weight coefficients of the superposition can be obtained by the intensity measurement performed at predetermined positions of the Fourier plane. As an example, the mixed vortex field composed of four OVs with the topological charges 1, 3, 5 and 7 is presented. Its complex amplitude is given as $U_{a_1a_3a_5a_7} = \sum_{m=1}^{4} a_{2m-1} u_{2m-1} \exp[i(2m-1)\varphi]$. The weight coefficients represent information cells carrying binary information. If the OV with the topological charge $2m-1$ is used in the superposition, the weight coefficient $a_{2m-1}$ is regarded as ‘1’ and in the other case as ‘0’. In this way, $2^4$ information codes can be created by means of the examined mixed vortex field.
Figure 2. A scheme of the channel enabling information transmission by means of the mixed vortex fields. Four bits of information are encoded into the P-N field $U_{1,3,5,7}$ representing the base of four coaxial vortices with topological charges 1, 3, 5 and 7. By encoding, the weight coefficients of the separate vortices can be controlled and represent the information cells. After the complex modulation $T$ and the Fourier transform $FT$ performed at the receiver, the information code 1100 is identified by two bright spots at predetermined positions of the detector $D$.

The channel enabling the transmission of four bits of information by means of the mixed vortex field is schematically illustrated in figure 2. The base of the information encoding represents a superposition of four coaxial OVs with the topological charges $m = 1, 3, 5$ and 7, respectively. The bits of information are encoded into the spatial structure of the field by the choice of the weight coefficients of the superposition. In the illustrated example, the coefficients of the OVs with the topological charges $m = 1$ and 3 are equal to 1 while the OVs with $m = 5$ and 7 are used with zero weight coefficients. The information code can then be written as 1100. The intensity profile of the mixed vortex field is P-N and remains unchanged in the region whose length can be controlled in the experiment. At the receiver, the mixed vortex field is transmitted through the amplitude and phase masks with the complex transparency $T$. After the Fourier transform, the detected field is represented by the bright intensity spots representing the OVs used in the superposition. The positions of the bright intensity spots are predetermined by complex transparency $T$ and identify the presence of vortices with $m = 1$ and 3 in the superposition. The positions of the separate OVs of the used base are denoted by arrows in figure 2. In this way, the information code 1100 is decoded by a simple intensity measurement. The intensity patterns of the mixed vortex fields representing information codes 1000 ($a_1 = 1, a_3 = a_5 = a_7 = 0$), 1100, 1110 and 1111 are illustrated in figure 3 (left panel). They were obtained by (5) for the propagation distance $z = L/2$ (the size of the panels is $40 \times 40$ mm$^2$, and the vortex pattern is available in the region of the length $L \approx 30$ m). The decoding of information was performed by the method described above. The bright intensity spots of the Fourier plane, illustrated on the right panel of figure 3, represent the information cells with the value of ‘1’. Their positions were defined by the phase functions $\Phi_{2m-1}$ of the transparency $T$. The information cells 1000, 0100, 0010 and 0001 are
Figure 3. A numerical simulation of intensity profiles of the mixed vortex fields $U_{1,3,5,7}$ representing the superposition of four coaxial OVs with the topological charges 1, 3, 5 and 7. The symbols ‘1’ and ‘0’ used in the demonstrated information codes 1000, 1100, 1110 and 1111 represent the weight coefficients of the separate OVs used in the superposition. In the proposed decoding, the used OVs are identified as bright intensity spots at predetermined positions.

localized at the positions given by the spatial frequency $v_0$ as $(-v_0, -v_0)$, $(v_0, -v_0)$, $(v_0, v_0)$ and $(-v_0, v_0)$, respectively. In equations (19) and (20), demonstrating the decoding process, the dependence of amplitudes $u_m$ on the propagation coordinate $z$ is neglected; however this dependence was taken into account in the results demonstrated in figure 3. If the encoding is performed at the plane $z = z_0$, the complex amplitudes $u_m(x, y, z_0)$ are used in (19). During the

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4. Experimental verification of information encoding

The encoding of information into the spatial structure of the mixed vortex field is based on an application of field (5). It represents the required superposition of the OVs with different topological charges and can be generated by the transmission of the collimate laser beam through the spatial light modulator (SLM). It can be realized by means of the setup illustrated in figure 4. The light field (5), which we want to generate, is coherently superposed with the reference wave. The calculated intensity pattern representing the computer-generated hologram is sent to the SLM placed at the front focal plane of the first Fourier lens. If the SLM is illuminated by the laser beam, the spatial spectrum of the transmitted field appears at the back focal plane of the Fourier lens. After a convenient spatial filtering a very good approximation of the field (5) is obtained behind the second Fourier lens. The process can be described mathematically.
It is assumed that the hologram is created at the plane $z = 0$ by an interference of the field $U$ given by (5) with the inclined reference plane wave $U_R = u_R \exp[i\omega t - ik(x \sin \gamma + z \cos \gamma)]$ ($\gamma$ denotes the inclination angle of the transverse component of the wave vector with respect to the z-axis). The collimate laser beam illuminating the SLM is approximated by the plane wave $U_r = u_r \exp(i\omega t - ikz)$. If the complex amplitude of the transmitted field is denoted by $U_T = U_r|U + U_R|^2$, its angular spectrum $\overline{U_T}$ can be obtained by performing the two-dimensional Fourier transform $\overline{U_T}(\nu_x, \nu_y) = F^2\{U_T(x, y)\}$. It can be expressed in the form

$$\overline{U_T} = \overline{U}_{T_0} + \overline{U}_{T_1} + \overline{U}_{T_{-1}},$$

where $\overline{U}_{T_0}$, $\overline{U}_{T_1}$, and $\overline{U}_{T_{-1}}$ represent the diffraction orders 0, +1 and −1, respectively. They are given by

$$\overline{U}_{T_0} = F^2\{U_r(|U|^2 + |U_R|^2)\},$$

$$\overline{U}_{T_1} = u_R^* u_r \overline{U}(\nu_x + \nu_s, \nu_y),$$

$$\overline{U}_{T_{-1}} = u_R u_r \overline{U}^*(\nu_x - \nu_s, \nu_y),$$

where $\nu_s = \sin \gamma/\lambda$ and $\lambda$ denotes the wavelength. The spatial frequency $\nu_s$ enables separation and filtration of the undesirable diffraction orders. The Fourier transform is optically performed by the first Fourier lens with focal length $f$. Plane wave components of the angular spectrum related to the angular frequencies $\nu_x$ and $\nu_y$ are then localized at the points $x = f\lambda \nu_x$, $y = f\lambda \nu_y$ of the back focal plane of the lens. The angular spectrum of the required field multiplied by the constant term is given by the +1th diffraction order. The undesirable 0th and −1th diffraction orders can be removed by the spatial filter placed at the back focal plane of the first lens. Its application requires a sufficient separation of the diffraction orders. It can be changed by the inclination angle of the reference plane wave $\gamma$. After the inverse Fourier transform is performed by the second lens, the P-N mixed vortex field with complex amplitude $U_T$ approximating the required profile $U$ is obtained. The examined vortex superposition was experimentally realized by applying the set-up in figure 4. The spatially filtered and collimate beam generated by the He–Ne laser illuminates the SLM (CRL OPTO 1024 × 768 pixels), where its amplitude is modulated by the computer-generated hologram. After spatial filtering in the 4f system, the P-N beam representing the superposition of four coaxial vortices with the topological charges $m = 1, 3, 5, 7$ is obtained. The forms of the central parts of the holograms sent to the SLM are shown in the left panel of figure 5. They result in generation of the mixed vortex fields representing coaxial superposition of four OVs. The transverse intensity patterns representing the information codes 1000, 1100, 1110 and 1111 are shown in figure 5 (right panel). The results of the experiment are in very good agreement with the numerical simulation presented in figure 3 (left panel). A very useful property of the present mixed vortex fields follows from the fact that they represent P-N beams possessing a self-reconstruction property. If such beams are perturbed by amplitude or phase obstacles they are able to regenerate during a free-space propagation and to reconstruct their initial spatial distribution of the complex amplitude [7]. Results of the experimental demonstration of the effect are shown in the animation. The OV carrying information code 1000 is blocked by a rectangular nontransparent obstacle. During the free-space propagation its intensity profile is reconstructed to the initial form. A nearly full reconstruction is achieved at the distance approximated by $a/\theta$, where $a$ is a transverse dimension of the obstacle and $\theta$ denotes the beam angular parameter.
Figure 5. Experimentally realized information encoding. The central parts of the holograms representing information codes 1000, 1100, 1110 and 1111 (left panel) and intensity profiles of the generated mixed vortex fields (right panel).
5. Conclusions

In the paper, mixed vortex fields and their properties are described and examined. It is shown that the vortex fields possess additional degrees of freedom applicable to the information encoding. An experiment of information encoding into the spatial structure of light is proposed and realized by means of a spatial light modulator. A simple method enabling information decoding is also proposed and verified by numerical simulations. It is based on the spatial separation of the vortex beams used in the encoding process. The different topological charges of the OVs are used as ‘markers’ enabling decoding. The mixed vortex fields carrying information belong to the class of P-N fields possessing the self-healing property. In this paper, the self-reconstruction is demonstrated experimentally for the single vortex beam.

Furthermore, a basic principle of the spatial information encoding based on application of the mixed vortex fields is presented. The proposed channel for transferring information should be comprehended as a simple demonstration of the potential of the present method. Optimization and further development of the method are possible. For example, a recently published method [15] stimulates research focused on transformation of the free-space P-N modes to the fibre modes. Furthermore, quantum reconstruction methods can be adopted to improve the proposed information decoding. In that case, the complete characterization of the mixed vortex fields is possible from several transverse scans of the field intensity.

The coaxial mixed vortex fields described and realized in the paper represent nonresolvable superposition of the OVs with different topological charges and orbital angular momenta. Such fields are potentially applicable to quantum optics experiments [16].

The vortex arrays composed of OVs with a predetermined spatial separation are also applicable to particle manipulation and sorting and to design of micro-optomechanical pumps [17].

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