Acoustic fatigue life prediction of structure based on power spectral density method

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Abstract. In the field of aerospace engineering, a large amount of thin shell structure is used in order to reduce the weight. These structures will be damaged by acoustic fatigue at high levels of noise and vibration loads. Acoustic fatigue analysis of thin walled structure is a very important part of the aircraft structure design. Simulation and analysis of three different fixation methods for random acoustic loads of thin-walled structure of aluminium alloy were introduced in this article. A method of power spectral density (PSD) was calculated for sound pressure level respectively in the band limited white noise 145 dB, 148 dB, 151 dB combined with fatigue damage theory based on linear length of the sound. The length of the 7075 Aluminium alloy material is 300 mm, and the thickness is 1.5 mm. The results show that: in the case of different ways of holding, the maximum stress suffered by the aluminium alloy thin sheet appeared at the constraint location, and this position is dangerous position for structural fatigue where will first be destroyed by random loads. Under random acoustic loads, the strain cloud of aluminium alloy thin sheet structure is the same with the first vibration mode, so the random vibration response was most effected by the first modal. Acoustic fatigue of the different sound pressure level was calculated by using the method of PSD and Miner linear damage rule.

1. Introduction
Fatigue matter of aircraft consists of two parts, each of which is static fatigue and dynamic fatigue. Static fatigue refers fatigue failure of structure by alternating stress under cycling load. Dynamic fatigue refers fatigue cracking happened on the structure by disturbance of random load which could cause random dynamic response of structure. Dynamic fatigue consists of acoustic fatigue and oscillating fatigue.

Acoustic fatigue refers that alternating stress happening on the structure by excitation of acoustic load brings the phenomenon of fatigue failure of structure. Random noise existed in the aircraft during the flight; alternating stress which was aroused by noise could damage the structure, for example, looseness of rivet, flaw of skin and etc. These would influence safety of flight seriously and decrease life of operation.

Traditional life estimation of acoustic fatigue is experience by generalizing large numbers of experimental data, such as ESDU[2] and calculation chart in America[3] called nomogram. These methods almost need a relationship curve about RMS and numbers of cycles of life, the structure life could be found out from the curve by RMS of structure stress calculated by environment of load, material of structure and size of shape. But, since the difference of material and connection technology of structure, the types of nomogram are different, for example, with the serving environment of
aircraft becoming abominable, internal of skin surface was welded by stiffener rib or other safety methods, from which the new nomogram[4] came out continually. S-N curve of the typical structure part was accumulated by an amount of experiment abroad. Compared with S-N curve of regular fatigue, S-N curve of the typical structure part is not better on quality and quantity which could limit the application on project. With the accumulation of experience, scientists gradually found that despite getting the S-N curve of material, but since the randomness of acoustic load, the principal problem is how to make acoustic load be equal to the stress on the curve. Although the Monte-Carlo method which could change the content of frequency to simulate time could relieve this problem to calculate the random fatigue life of S-N curve, but it is not efficient, especially the random process of low amplitude and high frequency. Therefore, the key is how to analyse the fatigue failure and predict the fatigue life by using regular S-N curve and according to power spectral density function on frequency domain. So structure fatigue problem of random vibration load is changed into research how to estimate the life of structure fatigue failure in the effect of random load showed by power spectral density function. For example, Julien et al use the method of numerical simulation with finite element to get an influential parameter when bolt of air management system on the aircraft suffered random load by loading the random vibration on it. Next, the evolutionary process in the frequency domain was predicted by vector Gaussian process. At last, bolt reliabilities of vibration fatigue under different conditions were evaluated by approximating FORM method.

In this paper, the influence that thin-wall structure of aluminium suffered from load of different acoustic pressure level was researched by the method of power spectral density, and acoustic fatigue life of different acoustic pressure level was calculated. Simulation and analysis of random acoustic pressure was carried out with three kinds of methods to fix. Acoustic fatigue life of aluminium with length of a side 300mm and thickness of 1.5mm was calculated by power spectral density and damage theory of linear fatigue with Band-Limited White Noise of 145dB, 148dB and 151dB.

2. Calculation of fatigue life based on power spectral density

The analysis method of power spectral density in frequency domain is a power spectral density function with stress response (PSD) by disturbance aroused by external noise and dynamic simulation or finite-element analysis to find out the hazardous position of the structure. Besides, the fatigue damage accumulation and fatigue life [7] of hazardous position could be calculated by stress range of probability density function from PSD.

The theoretical basis of PSD is a random process theory, which the experimental process of stress and time was assumed to be a sample of actual stress process. From this we could get the distribution about energy of different frequencies in random process or strength of random stress rather than random amplitude order of designing samples. The stress amplitude and average stress of random load is random and variable which make the calculation of random fatigue complicated. Let us assume that the signal which will be calculated is a steady process of narrow-band, the problem would be simplified. Under these circumstances, stress amplitude is random, the problem would be solved after obtaining the probability distribution of stress amplitude. For a steady random process, X(t) could be expressed by average in time domain, the Auto-correlation function is shown in formulas(1).

\[ R_x(\tau) = \mathbb{E}[x(t)x(t+\tau)] \] (1)

In the formula, \( \mathbb{E} \) stands for expectation, PSD for steady random process in frequency domain. Random process is shown in the form of bilateral power spectral density. Standing for an Auto-correlation function of a random process, and it is shown in formula (2).

\[ S_\omega(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau)e^{-j\omega \tau} d \tau \] (2)

Indeed, an Auto-correlation function usually be defined, and PSD is converted to single side shown in formula (3)
\[ G_\omega(\omega) = \begin{cases} 2S_{S_S}(\omega) & 0 < \omega < \infty \\ S_{S_S}(0) & \omega = 0 \end{cases} \quad (3) \]

PSD records a lot of statistical information about random process. The area under PSD curve stands for mean square of time. Moment of order \( n \) of PSD is defined in the formula (4).

\[ m_n = \int_{-\infty}^{\infty} \omega^n S_{S_S}(\omega)d\omega = 2\int_{0}^{\infty} \omega^2 S_{S_S}(\omega)d\omega = \int_{0}^{\infty} \omega^2 G_\omega(\omega)d\omega \quad (4) \]

Under certain conditions of nonzero mean, variance is shown in formula (5)

\[ \sigma^2 = \int_{0}^{\infty} G(f)df \quad (5) \]

In the formula, \( G(f) \) stands for power spectrum density function.

For an exemplar which the measured process about stress and time is an actual random process, \( S_{S_S} \) stands for power spectrum density function, therefore, the peak probability density function of random process is shown in formula (6)

\[ p(s) = \frac{1}{\sigma} \sqrt{\frac{1}{2\pi}} \exp \left[ -\frac{s^2}{2\sigma^2} \right] + \frac{\sqrt{\alpha}}{2\sigma^2} \left[ 1 + \text{erf} \left( \frac{s}{\sqrt{2}(1-\alpha)} \right) \right] \exp \left( -\frac{s^2}{2\sigma^2} \right) \quad (6) \]

In the formula, \( \text{erf} \) stands for probability integral (error function), \( \frac{\sqrt{\alpha}}{2\sigma^2} \) for an irregular factor of random load.

\[ \alpha = \sqrt{\frac{\int_{0}^{\infty} f^2 G(f)df}{\int_{0}^{\infty} f^4 G(f)df}} \quad (8) \]

In addition, a few parameters could be obtained from PSD. \( f_0 \) stands for average crossing zero of positive slope with one second, which also stand for average frequency of random load.

\[ f_0 = \sqrt{\frac{\int_{0}^{\infty} f^2 G(f)df}{\int_{0}^{\infty} G(f)df}} \quad (9) \]

\( n_0 \) stands for average of positive amplitude with cycle time.

\[ n_0 = \sqrt{\frac{\int_{0}^{\infty} f^4 G(f)df}{\int_{0}^{\infty} f^2 G(f)df}} \quad (10) \]

For a random signal of narrow band, the amplitude probability density could be calculated by peak probability density. In the formula (10), \( \alpha \) is ordered to 1, therefore, we could get the probability density function of stress amplitude in narrow band, which is shown in formula (11).

\[ P(S_a) = \frac{S_a}{2\sigma^2} \exp \left( -\frac{1}{2\sigma^2} \right) \quad (11) \]

It is obviously to observe that amplitude probability density function of random signal obeys Rayleigh distribution.

For broadband signal, amplitude probability density has relation with method of cycle counting which is used. If we used the method of amplitude average counting, the amplitude probability density could be expressed by formula (12)
Accordingly, amplitude probability density function of broadband signal is a combination of Rayleigh distribution and Gaussian distribution, especially, amplitude probability density function obeys Gaussian distribution while \( \alpha = 0 \). Three-zone method came up with Steinberg which is based on Gaussian distribution and Miner linear damage rule is combined and shown in Figure (1). The assumption of this method is the probability which is more than \( 100\% - 99.73\% = 0.27\% \) is 0. The formula used Miner method to calculate the total injury is shown in formula (9) and (10).

\[
D = \frac{n_{\sigma_1}}{N_{\sigma_1}} + \frac{n_{\sigma_2}}{N_{\sigma_2}} + \frac{n_{\sigma_3}}{N_{\sigma_3}}
\]  

(13)

In the formula, \( n_{\sigma} \) is equal or less than the actual circulation amount \((0.6831 \nu \cdot T)\) of \( \sigma \) level. \( n_{\sigma} \) is equal or less than the actual circulation amount \((0.271 \nu \cdot T)\) of \( 2\sigma \) level. \( n_{\sigma} \) is equal or less than the actual circulation amount \((0.0433 \nu \cdot T)\) of \( 3\sigma \) level. \( N_{\sigma_1}, N_{\sigma_2}, N_{\sigma_3} \) are respectively corresponding to the actual circulation amount by the fatigue curve of \( 1\sigma, 2\sigma, 3\sigma \) stress level. It is an effective process [11] to use \( 1\sigma, 2\sigma, 3\sigma \) stresses and equating average frequency to calculate random fatigue.

\[
P(S_a) = \frac{S_a}{2\alpha \sigma^2} e^{-\frac{1}{2\alpha}(\frac{S_a}{\sigma})^2}
\]  

(12)

3. The power density spectrum analysis of structure

3.1. Analysis of modality

Prior to analyse random load based on power spectrum density, the modality of structure needs to be analysed to get the intrinsic frequency and natural modal of vibration. The 7075 aluminium alloy thin sheet with sides of 300mm, thickness of 1.5mm was analysed by ANSYS software to get the modality. The mechanical property parameters of 7075 aluminium material are shown in table 1.

| Material Properties of Aluminium Alloy 7075 |
| E (GPa) | \( \nu \) | \( \rho \) (g/mm\(^3\)) | \( \sigma_b \) (MPa) |
|---------|--------|----------------|-------------------|
| 71      | 0.33   | 2.8            | 538               |

Since the length and wide are much bigger than thickness, the modal is analysed by shell element. It could not only get the accuracy result, but also decrease the time of calculation. The length of grid is set up to 0.005mm after finishing the modal. Finally, the structure of aluminium alloy thin sheet was divided into 3600 hexahedral units. The aluminium alloy thin sheet was constricted in three kinds of operating modes, operating mode 1: fixed constraints with four edges; operating mode 2: fixed constraints with two parallel edges; operating mode 3: fixed constraints with X model. The three kinds of constrains are shown in figure 2.
The range of intrinsic frequency is set up to 0–1000 Hz. Maximum frequency of aluminium alloy thin sheet structure is set up to 1000 Hz. Intrinsic frequency and vibration mode are shown in table 2. (Limitation of paper introduces three kinds of operating modes of the first four vibration modes)

Table 2. Natural frequency and modal shape under different constraint of aluminium alloy thin sheet

| Operating mode 1 | Operating mode 2 | Operating mode 3 |
|------------------|------------------|------------------|
| First order 146.94Hz | First order 90.3Hz | First order 235.69Hz (Symmetry) |
| Second order 299.63Hz (Symmetry) | Second order 106.95Hz | Second order 513.56Hz (Symmetry) |
| Third order 441.69Hz | Third order 177.66Hz | Third order 731.23Hz (Symmetry) |
| Forth order 536.99Hz | Forth order 249.16Hz | Forth order 937.82Hz (Symmetry) |
3.2. Analysis of random spectrum

After an analysis of modality, random spectrum was analysed by PSD method. PSD is a curve of relationship about power density and frequency, which could be displacement, speed, acceleration, force and stress. The analysed input of random spectrum is single point or multipoint of PSD actuation curve applied on node. At the same time, the output was responded of PSD worked on node (displacement, stress et al), and it could be used to predict the fatigue life.

The acoustic pressure levels of White Gaussian Noise used in this paper are set up to 145dB, 148dB, 151dB to simulate by random noise fatigue. White Gaussian Noise is a random noise, in which every frequency has the identical power density, in other words, the power density spectrum distributes uniformly in whole frequency. The power density spectrum of White Gaussian Noise with limited bandwidth could be shown in formula (14).

\[ G_p(f) = 4 / \Delta f \times 10^{\left(\frac{SPL}{10}\right)} \] (14)

In the formula, \( SPL \) stands for the acoustic pressure level of bandwidth, \( \Delta f \) for width of frequency band. In this paper, width of frequency band is set up to 1 Hz~1000 Hz, the power density spectrum of White Gaussian Noise in limited bandwidth was calculated with acoustic pressure level of 145dB, 148dB, 151dB. The data is shown in table 3.

| Table 3. Limited bandwidth of the white noise power spectral density |
|------------------|---|---|---|
| **Acoustic pressure level (dB)** | 145 | 148 | 151 |
| **power density spectrum (\( \text{Pa}^2/\text{Hz} \))** | 126.6 | 252.6 | 504.1 |

As could be clearly seen from the above chart, power density spectrum converted from acoustic pressure level is a pressure spectrum. Therefore, in the ANSYS software, pressure spectrum was selected as input. Since the resistance of aircraft structure is minor (The damping ratio is between 0.005~0.05) and different to determine, so it was ignored in this paper. Analysis and calculation were processed after modality combination. On different operating modes, the stress and strain of aluminium alloy thin sheet with different acoustic pressure level are shown in table 4 and table 6. As could be seen from the stress image, maximum stress that aluminium alloy thin sheet suffered appeared in the constraint position in different constraint methods by random load, this position is also a hazardous location of structure fatigue, and it would be damaged at first by random load. Comparison of different constraints and identical random acoustic load actuation shows that fixed constraints with X model is suffered a minimum stress. Therefore, constraint with X model could decrease the fatigue damage of random acoustic load to the structure of aluminium alloy thin sheet. As could be seen from the strain image, the strain image based on random acoustic vibration is basically identical with vibration modality of first order of modality analysis (first order modality of operating mode 3 has four kinds of vibration modality, the shape of combination with four kinds of vibration modality is identical with strain image ). It could be determined that first order modality has a maximum influence on respond of structure vibration.
Table 4. Stress and strain cloud by random spectral analysis in condition one

| Stress (dB) | Stress Cloud | Strain Cloud |
|------------|--------------|--------------|
| 145 dB     | ![Stress Cloud Image] | ![Strain Cloud Image] |
| 148 dB     | ![Stress Cloud Image] | ![Strain Cloud Image] |
| 151 dB     | ![Stress Cloud Image] | ![Strain Cloud Image] |
Table 5. Stress and strain cloud by random spectral analysis in condition two

| Stress | Strain |
|--------|--------|
| 145dB  |        |
| 148 dB |        |
| 151 dB |        |

Stress

Strain
Table 6. Stress and strain cloud by random spectral analysis in condition three

| Stress | Strain |
|--------|--------|
| 145dB  |        |
| 148dB  |        |
| 151dB  |        |

4. Prediction of fatigue life
Experimental data about 7075 aluminium alloy material is fitted by the form of power function formula. The experimental data about material fatigue is shown in table 7.

Table 7. The material fatigue test data of 7075 aluminium alloy

| Stress | Life | Stress | Life |
|--------|------|--------|------|
| 540    | 1.2×10^3 | 500    | 2×10^3 |
| 460    | 3×10^3    | 425    | 7×10^3 |
| 400    | 1.05×10^4 | 370    | 3×10^4 |
| 350    | 3.5×10^4 | 320    | 10^5  |
| 305    | 1.1×10^5 | 267    | 5×10^6 |
| 235    | 7×10^5    | 228    | 10^7   |
The data above was fitted with power function by software called SPSS. After the fitting stage, S-N curve formula is \[ N = 3.268 \times 10^{31} \cdot \sigma^{-10.526} \]. Fitted correlation parameter is 0.976. Fitted S-N curve is shown in figure 3. (The image is the connection of fitted point)

Figure 3 The S-N curve of 7075 aluminium alloy material

As stress images in table 5-4, 5-5, 5-6 show, the maximum stress that aluminium alloy thin sheet suffered by random noise in different acoustic pressure level is equal with 1\( \sigma \). It shows the present that the stress aluminium alloy thin sheet suffered is more than \( \frac{1}{2} \sigma \) is equal to 68%. Acoustic fatigue life of structure could be calculated by formula (13) and S-N curve.

In the case of 145dB White Noise, which is constrict \( 3\sigma = 116.4 \text{ MPa} \), and brought in form by operating mode 1, the stresses were separately determined to \( 1\sigma = 38.8 \text{ MPa} \), \( 2\sigma = 77.6 \text{ MPa} \), formula of S-N curve, we could obtain that \( N_{1\sigma} = 6.17 \times 10^{14} \), \( N_{2\sigma} = 4.18 \times 10^{11} \), \( N_{3\sigma} = 5.86 \times 10^{9} \)

\[ D = \frac{0.683n}{6.17 \times 10^{14}} + \frac{0.271n}{4.18 \times 10^{11}} + \frac{0.0433n}{5.86 \times 10^{5}} \]

When D was equal to 1, that meant the structure of aluminium alloy thin sheet was damaged, damaged position is the position that stress was maximum in table 3. Taking D to the equation would obtain the random fatigue life of aluminium alloy thin sheet structure is \( n = 1.244 \times 10^{11} \).

Table 8 shows that random fatigue life caused by three kinds of acoustic pressure level in different calculation methods and operating modes.

Table 8. The fatigue life of aluminium alloy thin plate structures under different sound pressure level

| Acoustic pressure level | Operating mode 1 | Operating mode 2 | Operating mode 3 |
|-------------------------|------------------|------------------|------------------|
| 145dB                   | \( 1.24 \times 10^{11} \) | \( 1.68 \times 10^{9} \) | \( 3.32 \times 10^{12} \) |
| 148dB                   | \( 5.36 \times 10^{8} \) | \( 7.23 \times 10^{6} \) | \( 1.42 \times 10^{10} \) |
| 151dB                   | \( 2.54 \times 10^{6} \) | \( 2.95 \times 10^{4} \) | \( 6.13 \times 10^{7} \) |

5. Conclusions
This paper introduces traditional methods about fatigue calculation and fatigue damage theory. The PSD method is used to calculate the life of random acoustic fatigue for 7075 aluminium alloy thin sheet which is excited by different acoustic levels of white noise. The result shows:

(1) The stress that aluminium alloy thin sheet suffered appeared in constrain position in the different methods of constrain. This position is the hazardous position which will be damaged at first when suffering load. Comparison of different constraints and identical random acoustic load actuation
shows that fixed constraints with X model is suffered a minimum stress. Therefore, constraint with X model could decrease the fatigue damage of random acoustic load to the structure of aluminium alloy thin sheet.

(2) With the excitation of random acoustic load, the strain image based on random acoustic vibration is basically identical with vibration modality of first order of modality analysis. It could be determined that first order modality has a maximum influence on respond of structure vibration.

(3) The life of random fatigue under different acoustic pressure level was calculated by power spectral density function and Miner linear damage rule. With the excitation of white noise at 145dB, 148dB and 151dB, we could get the acoustic fatigue life of aluminium alloy thin sheet. It could provide a reference and a method to predict acoustic fatigue life of structure on engineering.

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