Population statistics of beamed sources. II: Evaluation of Doppler factor estimates.

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ABSTRACT

In a companion paper we presented a statistical model for the blazar population, consisting of distributions for the unbeamed radio luminosity function and the Lorentz factor distribution of each of the BL Lac and Flat Spectrum Radio Quasar (FSRQ) classes. Our model has been optimized so that it reproduces the MO JAVE distributions of apparent speeds and redshifts when the appropriate flux limit is applied and a uniform distribution of jet viewing angles is assumed for the population. Here we use this model to predict the Doppler factor distribution for various flux-limited samples for which Doppler factors have been estimated in a variety of ways (equipartition, variability + equipartition, inverse Compton dominance) on a blazar-by-blazar basis. By comparing the simulated and data-estimated Doppler factor distributions in each case, we evaluate the different methods of estimating blazar Doppler factors. We find that the variability Doppler factors assuming equipartition are the ones in the best agreement with our statistical model, whereas the inverse Compton Doppler factor method is only suitable for FSRQs.

Key words: galaxies: active – galaxies: blazars – jets – Doppler factors

1 INTRODUCTION

Blazar observations are shrouded in relativistic effects, due to their preferential alignment of their jets close to our line of sight (Blandford & Königl 1979). Decomposing relativistic effects from intrinsic properties would allow us to probe the processes important to jet astrophysics including the jet–black-hole connection, the structure and evolution of jet magnetic fields, the evolution of flaring events in the jet rest frame, and particle acceleration in jets.

The jet Doppler factor is a key quantity in any such effort. It is the Doppler factor that determines how much flux densities are boosted and timescales compressed in the observer frame. Additionally, the Doppler factor is a different function of the bulk Lorentz factor $\Gamma$ and the viewing angle $\theta$ than the one determining the apparent speeds of jet components. Measurement of both these quantities allows one to solve for both $\Gamma$ and $\theta$. For this reason, measurements of Doppler factors on a blazar-by-blazar basis have been actively pursued. Several methods have been proposed, including causality arguments, (Aharonian et al. 2007, Porstsch et al. 2003, Claussen-Brown et al. 2013), emission region geometry (Fan et al. 2013, 2014), assumed high-energy emission processes (Ghisellini et al. 1993), and the assumption of equipartition between jet magnetic fields and relativistic electron energy densities (Guo & Dalp 1993, Readhead 1994, Lähteenmäki & Valtaoja 1999, Hovatta et al. 2009).

However, it is not straightforward to evaluate the accuracy of such estimates. Often these estimates represent only lower limits to the true jet Doppler factors; in other cases, different methods produce different results for the same sources. Since each method uses several different assumptions that might not hold, it is impossible to determine which method provides the most accurate estimate of the Doppler factor of a source on a blazar by blazar basis.

Here, we take a statistical approach to evaluating the accuracy of various techniques for estimating the Doppler factor of a blazar jet. In a companion paper (Liodakis & Pavlidou 2014, hereafter Paper I), we presented a population model each of the BL Lac and Flat Spectrum Radio Quasar (FSRQ) classes of blazars. The model consists of distributions for the intrinsic unbeamed 15 GHz radio luminosity and the Lorentz factor of the jets. Its parameters were optimized using well-measured observable quantities: redshift, and apparent velocity ($\beta_{app}$) as measured by the

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MOJAVE program (Monitoring Of Jets in Active galactic nuclei with VLBA Experiments, Lister & Homan 2003). We can use this model in order to derive Doppler factor distributions for any flux-limited sample, independently of variability, flux, or equipartition brightness temperature.

Since our approach is independent of all assumptions entering methods of Doppler factor estimation in individual blazars, and is able to adequately describe blazars as a population, we are presented with a unique opportunity: we can use the derived Doppler factor distributions in order to compare them with those obtained using various single-blazar techniques, and evaluate which method of calculating Doppler factors for individual sources is most accurate. At the same time, we can evaluate whether the assumptions used in each of these methods hold.

This paper is organized as follows. In §2 we describe our model and the resulting Doppler factor distributions, and in §3 the various Doppler factor estimation techniques we test in the following sections. In §4 we compare our statistical results with known methods of determining Doppler factors and in §5 we will discuss the results of our comparison, how they relate to blazar physics, as well as the applicability of the derived Doppler factor distributions from our model on individual sources. We summarize our conclusions in §6.

The cosmology we have adopted throughout this work is $H_0 = 71 \text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_m = 0.27$ and $\Omega_L = 1 - \Omega_m$ (Komatsu et al. 2009). This choice was made so that our cosmological parameters agree with the MOJAVE analysis (Lister et al. 2009a).

2 DOPPLER FACTOR DISTRIBUTIONS

In this section we give a short description of our blazar population model. For a more detailed description see Paper I.

We optimized our model using data from the MOJAVE survey (Lister et al. 2009b). MOJAVE uses a statistically complete flux-limited sample (Arshakian, Ros & Zensus 2000). Samples determined only by strict statistical criteria are crucial for population studies such as ours. We removed outliers and any source that showed unusual behavior (jet bending, inward motions etc.) as indicated by Lister et al. (2000a, 2013), and separated the sample in two sub-populations: BL Lac objects, and Flat Spectrum Radio Quasars (FSRQs). A flux limit, at 1.5 Jy, serves as a constraint determining how beamed the set of the observed sources is.

We assumed single power law distributions for the Lorentz factor,

$$p(\Gamma) \propto \Gamma^{-\alpha}, \quad (1)$$

and the unbeamed luminosity function (Lister & Marscher 1997; Cara & Lister 2008; Chatterjee et al. 2008; Abdo et al. 2010b), and have adopted a pure luminosity evolution model described in (Padovani & Urry 1992; Padovani 1992).

$$n(L_\nu, z) \propto \left( \frac{L_\nu}{T(z)} \right)^{-A}, \quad (2)$$

where $T(z)$ is the lookback time. We assumed random viewing angles $\theta$ (i.e. $\cos \theta$ uniformly distributed between -1 and 1). Thus, our model parameters are the power law indices for the Lorentz factor distribution ($\alpha$) and the luminosity function ($A$), and the evolution parameter ($\tau$).

For every set of model parameters, we compared simulated and observed distributions for apparent speeds and redshifts of sources obeying the flux limit. We rejected any model for which the Kolmogorov-Smirnov test (K-S test) delivered a probability < 5% of consistency between observed and simulated distributions. Our optimal model is the one which minimized the product of the K-S probability values for these two distributions. The best fit parameters for the FSRQ population were $\alpha = 0.57 \pm 0.001; A = 2.6 \pm 0.01; \tau = 0.26 \pm 0.001$. For the BL Lac population we found $\alpha = 0.738 \pm 0.002; A = 2.251 \pm 0.02$. Note that the BL Lac parameters do not include the evolution parameter: we found that the BL Lac luminosity function is consistent with no evolution. The error represents scanning step and statistical variations in simulated distributions. We have also explored the threshold of the acceptability of our model, by keeping all the parameters but one to the best fit value, and
changing the other towards higher or lower values until the K-S test threshold of 5% is violated. The limits of the parameters are shown in Table 1. For each model (determined by a set of the parameters and each flux limit value, we can produce distributions of derived quantities, including Doppler factors, viewing angles, and timescale modulation factors. These results are presented in detail in Paper I along with a detailed discussion on the reasoning behind our assumptions and our optimization algorithm.

In Figs. 1 and 2 we review the Doppler factor distributions produced by our model for BL Lacs and FSRQs respectively for a 1.5 Jy - limited sample. The Doppler factor is given by

\[ \delta = \frac{1}{\Gamma(1 - \beta \cos \theta)}, \]  

where \( \beta \lesssim 1 \) is the speed of the jet in units of the speed of light, which is connected to the Lorentz factor through,

\[ \Gamma = \frac{1}{\sqrt{1 - \beta^2}}. \]

The distribution obtained from our optimal model is plotted, in each case, with the thick black solid line. To give a sense of the uncertainty in these distributions due to the uncertainty in our model parameters we plot, with other line types and colors, the resulting distributions when each model parameter is at the limit that still gives apparent jet speed and redshift distributions acceptable within our 5% threshold, while all other parameters are kept at their optimal value.

3 SINGLE-BLAZAR DOPPLER FACTOR ESTIMATES

In this section, we review various techniques that have been used in the literature to derive Doppler factor estimates for individual blazars.

3.1 Inverse Compton Doppler Factors

The inverse Compton Doppler Factor \( \delta_{IC} \) (Ghisellini et al. 1993) is derived based on the requirement that the Synchrotron self-Compton (SSC) flux density should not exceed the observed flux density at high frequencies. The SSC emission consists of photons produced by inverse-Compton upscattering of synchrotron photons by the same relativistic electrons that produce them, and, in that sense, is a guaranteed high-energy component in any region containing magnetic fields and relativistic electrons.

Assuming a power law energy distribution for the electrons, homogeneous magnetic fields, and the observation frequency (\( \nu_{\text{obs}} \)) to be the self-absorption frequency of the core component dominating at that frequency, the Doppler factor would be:

\[ \delta_{IC} = f(\alpha) F_m \left[ \frac{\ln(\nu_o/\nu_{\text{obs}})}{\ln(\nu_o/F_m^\beta \nu_{\text{obs}})} \right]^{1/(4+2\alpha)} (1+z), \]  

where \( F_m \) is the synchrotron flux density at \( \nu_o \) and \( F_{\chi} \) the X-ray flux density both in Jy, \( \theta_d \) the angular size of the core in milli arcseconds, \( \nu_{\chi} \) is in keV, \( \nu_m \) is in GHz and \( \nu_o \) the synchrotron high energy cutoff which is assumed to be 10^{14} Hz. The function \( f(\alpha) \) is given by (Ghisellini 1987) to be \( f(\alpha) \simeq 0.08\alpha + 0.14 \).

Equation (5) is applicable in the case of a discrete jet (\( p = 3 + \alpha \)). For the continuous jet case (\( p = 2 + \alpha \)) the Doppler factor is related to the one of (Eq5) by

\[ \delta_{2+\alpha} = \delta_{3+\alpha}^{(2+\alpha)/3(2+\alpha)}. \]

A more detailed description of the model can be found in Ghisellini et al. (1993); Guíjosa & Dalla (1996).

3.2 Equipartition Doppler Factors

Equipartition Doppler factors (Readhead 1994) use the assumption of equipartition between electrons and magnetic fields in a radio emission region to calculate an intrinsic brightness temperature and, from there, a Doppler factor through comparison to an actual observed brightness temperature. Different incarnations of this method differ in the way they calculate the angular size of the emission region (direct observation through VLBI or variability timescales and causality arguments.)

3.2.1 VLBI Equipartition Doppler Factors

The angular size of a uniform self-absorbed source in order to have equipartition of the radiating particles and the magnetic field, or else the equipartition angular size is (Scott & Readhead 1977).

\[ \theta_{eq} = 10^3 (2h)^{1/17} F(\alpha)(1 - (1 + z)^{-1/2})^{-1/17} \times S_p^{8/17} (1+z)^{(15-2\alpha)/34} (\nu_p \times 10^3)^{-(2\alpha+35)/34} \text{mas}, \]  

where \( h \) is the dimensionless Hubble parameter, \( S_p \) is in Jy and \( \nu_p \) in GHz. The function \( F(\alpha) \) is described in Scott & Readhead (1977). \( S_p \) and \( \nu_p \) have not been corrected for the beaming effect, thus the observed values are related to the intrinsic ones through \( S_p = \delta^{-2} S_{\text{obs}} \) and \( \nu_p = \delta^{-1} \nu_{\text{obs}} \). Assuming the observed angular size is \( \theta_d = \theta_{eq} \) the equipartition Doppler factor is,

\[ \delta_{eq} = \left\{ \left[ 10^4 F(\alpha)^{34} (1 - (1 + z)^{-1/2})/2h \right]^{-2} (1 + z)^{15-2\alpha} \times S_{\text{obs}}^{-10\alpha} \theta_{d}^{-34} (\nu_{\text{obs}} \times 10^3)^{-(2\alpha+35)} \right\}^{1/(13-2\alpha)}. \]  

The equipartition Doppler factor can also be expressed as the ratio of the observed brightness temperature over the maximum intrinsic brightness temperature (\( T_{b,\text{int}} \)). Since

| Table 1. Limits of the model parameters presented as deviations from the optimal value, for which the 5% K-S test requirement is still met. |
|----------------------------------|
| BL Lacs | FSRQs |
| \( a_{min} \) | 0.738-1.2 | 0.57-0.1 |
| \( a_{max} \) | 0.738+0.9 | 0.57+0.2 |
| \( A_{min} \) | 2.251-0.7 | 2.6+0.1 |
| \( A_{max} \) | 2.251+0.3 | 2.6+0.3 |
| \( \tau_{min} \) | - | 0.26-0.01 |
| \( \tau_{max} \) | - | 0.26+0.04 |
for powerful synchrotron radio sources $T_{b,int}$ is equal to the equipartition temperature ($T_{eq}$): 

$$\delta_{eq} = \frac{T_{b,obs}}{T_{eq}}.$$

A detailed description of the method can be found in Readhead (1994); Gujios & Daly (1990); Britzen et al. (2007).

### 3.2.2 Variability Doppler Factors

In this case, the time evolution of a radio flare is used to calculate the brightness temperature of the emission region. The Doppler factor is then obtained by setting that variability brightness temperature equal to the equipartition brightness temperature.

$$\delta_{var} = \left(\frac{T_{b,var}}{T_{b,int}}\right)^{1/3}.$$

where $\delta_{var}$ is the Doppler factor for each observed component.

#### 3.3 Single-component Causality Doppler Factors

The underlying assumption of this method is that the variability timescale of a resolved jet component is determined by the light travel time across the component, rather than loss processes (Jorstad et al. 2002, 2006). This method relies upon the observational determination of both the angular size of the component and the variability timescale, defined as:

$$\Delta t_{var} = \frac{dt}{\ln(S_{max}/S_{min})},$$

where $S_{max}$ and $S_{min}$ are the measured maximum and minimum flux densities, respectively, and $dt$ is the time between $S_{max}$ and $S_{min}$. The Doppler factor can be calculated from:

$$\delta_{var} = \frac{sd_L}{c\Delta t_{var}(1+z)}.$$

where $d_L$ is the luminosity distance, and $s$ is the angular size of the component, equal to 1.6a for a Gaussian, equal to the full width at half maximum, measured at the epoch of $S_{max}$. After calculating a Doppler factor for each observed component for a specific source, the weighted average of these values is assigned to a source, with the weights being inversely proportional to the uncertainty in apparent velocity of each component.

This method is resource-expensive in that it requires multiepoch VLBI monitoring for each source. For this reason, Doppler factors at this stage have been calculated by this method, to our knowledge, only for 5 BL Lac objects and 8 FSRQs. As a result, a statistical evaluation for this method is rendered impractical due to low statistics.

### 3.4 Gamma-ray Opacity Doppler factors

The calculation of $\gamma$-ray opacity Doppler factors is based on the requirement that the $\gamma$--ray emission region must be transparent to gamma rays. This calculation is based on the assumption that the gamma-rays are produced in the same region as gamma rays. Causality arguments are used to connect variability timescales with the emission region size. The Doppler factor is then given by:

$$\delta_{\gamma} \approx \left[1.54 \times 10^{-3}(1+z)^{4+2\alpha}\left(\frac{d_L}{\text{Mpc}}\right)^2\right]^{1/(4+2\alpha)}\times\left[\left(\frac{\Delta T}{\text{hours}}\right)^{-1}\left(\frac{F_{\gamma\text{eV}}}{\mu Jy}\right)\left(\frac{E_{\gamma}}{\text{GeV}}\right)\right]^{1/(4+2\alpha)},$$

where $\alpha$ is the X-ray spectral index, $F_{\gamma\text{eV}}$ the flux density at 1 keV in $\mu$Jy, $E_{\gamma}$ is the energy at which the $\gamma$-rays are detected in GeV, $d_L$ the luminosity distance as described in [3] and $\Delta T$ is the time scale in hours defined as:

$$\Delta T = \left(\frac{(1+z)R}{c\delta_{\gamma}}\right).$$

where $R$ is the size of the emission region. A variation of this method uses UV photons as the target photon field and similar arguments to derive a Doppler factor.

In this work, we do not statistically test this technique, because the calculation of a statistical Doppler factor distribution requires a well-defined, 15 GHz radio flux-limited sample. Because of the significant scatter in the radio/gamma-ray flux correlation (Pavlidou et al. 2012), it is not straightforward to calculate a single radio flux limit for a gamma-ray selected sample, even if the latter is flux-limited. At the same time, the fact that gamma-ray opacity arguments can only provide lower limits to the true Doppler factor complicates the statistical comparison of these Doppler factors to other datasets and models. However, such a comparison would be principle be very interesting, especially if it could confirm whether $\gamma$-ray and radio emitting regions have different outflow velocities (Georganopoulos & Kazanas 2003) and hence different Doppler factors.

1 Because for these sources the spectral index is very close to zero we have used $F_\nu \approx F_{15\text{GHz}}$ for nearby frequencies.
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4 COMPARISON WITH STATISTICAL DOPPLER FACTORS

In this section we compare our results on Doppler factor distributions from our blazar population models with data on Doppler factor estimates through different techniques found in the literature. Every data set we have worked upon is separated into two populations, one for the BL Lac objects and one for FSRQs. It was shown in Paper I that the model can adequately describe blazars as a population; however all derived distributions are sample-specific and flux-limit dependent. For this reason, if we want to compare any derived distribution with data, including the Doppler factor distributions, the flux-limit of the sample at hand must be taken into account. As a result, for each dataset and each object class, a distribution of statistical Doppler factors was derived by using our model and the flux limit of the corresponding sample of the data set we are comparing with. All the data we compare with are from flux-limited samples, either 0.35 Jy, 1 Jy or 2 Jy. Comparisons between statistical and single-blazar estimated Doppler factor distributions are made with the use of the Kolmogorov-Smirnov (K-S) test.

All the values presented in this work represent the probability value to be higher than 5%. For the case of the equipartition Doppler factor, the p-value for the K-S test for the BL Lac sample is extremely low, so no strong conclusions can be derived in this case.

We proceed to the data set analysed by Ghisellini et al. (1993) for the BL Lac objects and 108 FSRQs. The flux-limit for this sample is 0.35 Jy (Taylor et al. 1996).

The data set for the $\delta_{eq}$ features extremely low values, with the highest value for the BL Lac $\delta_{eq} \sim 1.11$ and for the FSRQs $\delta_{eq} \sim 4.72$. A K-S test confirmed that these distributions are not consistent with what is expected for these populations, with the p-value for the BL Lac $\sim 1.22 \times 10^{-7}\%$ and for the FSRQs $\sim 1.16 \times 10^{-76}\%$. For the case of the $\delta_{IC}$ we have excluded source 1732+389 from the FSRQ sample for being an outlier ($\delta_{IC} \approx 276$). The consistency between estimated and statistical Doppler factor distributions is rejected for FSRQs, with a K-S test returning a p-value of $\sim 2.15 \times 10^{-5}\%$. The value for the K-S test for the BL Lac sample is $\sim 6.7\%$. Note however that the statistics in the BL Lac sample is extremely low, so no strong conclusions can be derived in this case.

Since the K-S p-values for the equipartition Doppler factors are so low, plotting the cumulative distribution functions would not give any additional information. For this reason we only plot the probability density function (PDF) and the cumulative distribution function (CDF) for the $\delta_{IC}$ case, in Figs. 3 and 4.

We proceed to the data set analysed by Ghisellini et al. (1993) for the inverse Compton Doppler factor. The sample consists of 11 BL Lac objects and 108 FSRQs. The flux-limit for this sample is 1 Jy (Kuehr et al. 1993). We corrected for the continuous jet case (which we also assume in our
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We plot the PDF and CDF for the two classes in Figs. 5 and 6. We next test equipartition Doppler factors calculated by (Guijosa & Daly 1996). The sample consists of 32 BL Lac objects and 53 FSRQs. The flux-limit for this sample is 1 Jy (Ghisellini et al. 1993). The value of the KS-test for the BL Lac objects is \( \sim 1.5 \times 10^{-4}\% \) while for the FSRQs \( \sim 7.6 \times 10^{-3}\% \). Figures 7 and 8 show the probability density and cumulative distribution functions respectively of the statistical and estimated doppler factors.

Finally, we test variability Doppler factors (using the equipartition brightness temperature to derive a Doppler factor) using data from (Hovatta et al. 2009). They use a sample consisting of 22 BL Lac objects and 60 FSRQs. The flux-limit for this sample is 2 Jy (Valtaoja, Lahteenmaki & Terasranta 1992). For the BL Lac population, the value of the K-S test is \( \sim 54\% \) while for the FSRQ \( \sim 14\% \). The PDFs and CDFs for the comparison are shown in Figs. 9 and 10 respectively. This is the method that gives the best agreement with the statistical Doppler factors.

5 DISCUSSION

We have compared, in §4, results from our statistical, population model for blazars, with data provided in the literature for three distinct methods of calculating Doppler factors. The results of this comparison show that one of the methods, variability Doppler factors \( \delta_{\text{var}} \), is in agreement with statistical Doppler factors (K-S test BL Lac \( \sim 54\% \), FSRQs \( \sim 14\% \)) whereas the other two methods seem to
be drawn from completely different distributions (K-S test for both populations $\lesssim 10^{-3}\%$). This result is consistent with Lähteenmäki & Valtaoja (1999) arguing that variability Doppler factors using equipartition are a better and more accurate estimate of the Doppler factors of beamed sources than equipartition Doppler factors that rely on direct measurement of the angular size of the emission region, due to a weaker dependence on the observed brightness temperature (third root compared to first power).

Of all other Doppler factor estimation techniques we tested, there were only two cases where our statistical analysis indicated agreement between our statistical model and the data: the BL Lac sample in Britzen et al. (2007) and the FSRQ sample in Ghisellini et al. (1993).

In the first case, even though the value of the K-S test is marginally acceptable ($\sim 6\%$), it is clear from the low value of the Doppler factors (maximum value $\sim 7$) and from Figs. 4 and 5 that the agreement is far from good; however, the low number of sources in the BL Lac sample (11 sources) is preventing a strong conclusion either way through the K-S test.

In the second case, the test gives a probability of $\sim 10\%$ for consistency, which is above the limit set in §2. Since in these cases there is an excess of low values for the Doppler factors, we have compared them with distributions resulting from the model using $a_{\text{max}}$ and the model using $\lambda_{\text{min}}$ since these give a higher fraction of low Doppler factor values that our optimal population model.

Hand, many BL Lacs are intermediate synchrotron peaked (ISP) and high synchrotron peaked (HSP), so part of their X-ray flux can be produced by synchrotron emission, which would result in the inverse Compton Doppler factors underestimating the true Doppler factor of their jets. Indeed, it is clear from Fig. 4 that the Doppler factors of the BL Lacs are underestimated compared to the expectations from our optimal population model.

It is troubling that the consistency of $\delta_{IC}$ with the expectations from our population model for FSRQs is so different between the (Ghisellini et al. 1993) sample and the (Britzen et al. 2007) sample. While the first is acceptable, the second is rejected with a probability of $\sim 2.15 \times 10^{-5}\%$. Six orders of magnitude in difference might indicate poor sample selection or/and large errors in measurements. This difference might also be due to the time difference between radio and X-ray flux measurements. The main assumption of the inverse Compton Doppler factor method is that synchrotron-emitting electrons up-scatter synchrotron photons to X-rays. The X-ray and radio flux densities must therefore be measured at the same time. Any time difference in observations might result in non-corresponding flux densities. A better evaluation of this technique could be achieved by systematically pursuing simultaneous radio and X-ray observations for a radio-selected flux-limited sample of low-synchrotron peaked sources.

For those Doppler techniques and samples that the hypothesis of being drawn from our optimal statistical model is rejected by the K-S test, we have also performed comparisons with distributions drawn from a statistical model with parameters other than the optimal, but still within the limits of acceptability discussed §2. Since in these cases there is an excess of low values for the Doppler factors, we have compared them with distributions resulting from the model using $a_{\text{max}}$ and the model using $\lambda_{\text{min}}$ since these give a higher fraction of low Doppler factor values that our optimal model (see Figs. 11 and 12) for both populations. A K-S test indicated that agreement with these distributions is also rejected.

Figure 11. Variability Doppler factors plotted against inverse Compton Doppler factors for the FSRQs that are common between the Hovatta et al. (2009) and Ghisellini et al. (1993) samples.

Figure 10. Cumulative distribution function of statistical Doppler factors for the BL Lacs (solid line) and FSRQs (dashed line) overplotted with variability Doppler factors using equipartition from Hovatta et al. (2009) (X for BL Lacs, + for FSRQs).
Although the variability Doppler factors using equipartition result in the best agreement with our statistical model (and, by extension, with MOJAVE results on jet speeds), the agreement is not perfect. First, there is a pileup of sources between Doppler factors of 10 and 15 in both classes of sources. This is a result of the monitoring cadence limit: high Doppler factors result in very compressed timescales, and the fastest flare in a source may not be resolvable at a given cadence. Higher time resolution in the flux density curves will provide more accurate results. For the FSRQ sample (Hovatta et al. 2009) we also see a deficit of low Doppler factor values (see Fig. 10). This effect can be attributed to the fact that low-boosting sources also feature smaller timescale compression; longer time scales can in turn make identifying the peak of the flare difficult.

Similarly, for the other case where we have a Doppler factor estimation method that is in statistical agreement with our optimal model (the inverse Compton Doppler factors of Ghisellini et al. 1993 for FSRQs), the agreement is also not perfect. There is a pileup of sources at low Doppler factor values compared to the expectation from our statistical model. This is expected if in the sample under study there is a considerable number of sources where the SSC emission is not the sole source of X-rays; or, if there are errors entering due to the non-simultaneity of X-ray and radio measurements.

In Fig. 11 we plot variability Doppler factors using equipartition from Hovatta et al. (2009) and inverse Compton Doppler factors from Ghisellini et al. (1993) for the 31 common FSRQ sources in their samples. We have indicated the regions of pileup of sources discussed above for each method in grey. Although a correlation can be seen for the remaining sources, the statistics are very low for a strong statement to be made. We note however that such pileups in the distribution of Doppler factors in a flux-limited sample where none are expected from a population model can be a good indicator regarding the sources for which the results of a particular technique should be used with caution.

Finally, we have evaluated whether the theoretically derived Doppler factor distributions can be combined with measurements of the flux density and apparent jet speed of a source to yield an estimate of the Doppler factor in individual sources. We created two sub-samples from the common sources in the MOJAVE sample and (Hovatta et al. 2009), for which we also have flux density measurements (Lister et al. 2009a). These consist of 12 BL Lacs and 39 FSRQs respectively. For each of these sources, we generated a distribution of Doppler factors as follows: starting from our blazar population model for the relevant class of sources (BL Lacs or FSRQs), we randomly drew luminosities, Lorentz factors, and viewing angles according to their respective distributions. However, instead of keeping only sources that obeyed a specific flux limit, we only kept sources with flux density within 10% of the mean flux density of the source at hand.

We show these distributions for three of these sources in Fig. 12. These three sources have the additional property that their variability and inverse Compton Doppler factor estimates are very close to each other. The values of their variability Doppler factors are 18.4, 16.1, and 10.7, for 0106+013, 0234+285, and 1730-130 respectively. We can see that all three distributions have a very significant spread. The mean and standard deviation of these distributions are 25.36 ± 4.96 for 0106+013, 19.00 ± 11.57 for 0234+285, and 25.86 ± 2.88 for 1730-130.

Figures 13 and 14 show the statistical Doppler factors (mean and standard deviation of the Doppler factor distribution produced for each source as described above) plotted against the variability Doppler factors for the BL Lacs and the FSRQs respectively. We can see that while there is agreement within 2σ for most sources, the uncertainties of the statistical Doppler factors are so large that they erase any correlation between the two quantities on a source-by-source basis. We conclude that our population model is not constraining enough when applied to individual sources, and thus we strongly advise against using it to derive information about single objects.
Figure 14. Statistical versus variability (using equipartition) Doppler factors for individual FSRQs. The error bars represent 1σ of the statistical Doppler factor distribution for each source.

6 CONCLUSIONS

We have used our population models for BL Lacs and FSRQs to evaluate different techniques of calculating Doppler factors in individual sources. Our conclusions can be summarized as follows.

- Variability Doppler factors using equipartition, when calculated for a flux-limited sample, result in a distribution that is in good agreement for both samples (FSRQs & BL Lacs) with the distribution produced by our population model when the same flux limit is applied.

Since the only observables entering the optimization of our population model are apparent jet speeds and source redshifts, our model contains no assumption regarding variability, causality, or equipartition. The agreement between our model distributions and the distributions of variability Doppler factors points to a self-consistent picture in radio between jet speeds and Doppler factors. Additionally, this agreement can only be achieved if equipartition as discussed by Scott & Readhead (1977), Readhead (1994) indeed holds in blazars as a population.

- Inverse Compton Doppler factors can adequately describe Flat Spectrum Radio Quasars as a population, while they are unable to describe BL Lacs, likely because some of the latter produce a significant fraction of their X-ray flux through synchrotron radiation. This conclusion is also in support of the self-synchrotron Compton model, a main assumption of the inverse Compton Doppler factor method, being responsible for most of the X-ray flux in a large fraction of FSRQs.

- The main limitation of the variability Doppler factor method assuming equipartition appears to be monitoring cadence. For this reason, long-term high-cadence blazar monitoring (such as the OVRO 15GHz monitoring program, Richards et al. 2011) can be an invaluable tool in deriving accurate Doppler factor estimates on a blazar-by-blazar basis.

- Population models such as the one described here are unable to yield reliable and useful estimates for Doppler factors of individual blazars, and for this reason they should not be used in this fashion.

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