Influence of installation error on roundness error measurement

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Abstract. The installation tilt causes the actual measured circular contour to be an elliptical contour [1], and the installation eccentricity causes the sampled points uneven [2], which can both affect the measurement accuracy of the roundness error [3, 4]. The tilt and eccentricity error must be separate from the measured data when measuring the cylindrical workpiece roundness error with a cylindricity measuring instrument to obtain the true contour information of the cylindrical workpiece itself. In this paper, the mathematical models of the workpiece installation tilt and eccentricity are established based on the least square roundness error evaluation algorithm. The influence of the workpiece installation tilt and eccentricity on the roundness error measurement are simulated by MATLAB, and an actual measurement is analysed. The results show that after leveling and centering the workpiece, the influence of the tilt and eccentricity error of the workpiece installation on roundness error measurement can be neglected when the Taylor565H type cylindricity measuring instrument is used for micron order roundness error measurement.

Key words: Roundness error, installation tilt, installation eccentricity, simulation

1. Introduction

Since the precision rotary shaft system and high measurement accuracy, the cylindricity measuring instrument is an authoritative method for roundness error measurement [5]. The cylindricity measuring instruments are mainly divided into two categories according to the overall layout and the rotary mode, the rotary table type and the rotary probe type, which are suitable for measuring small and large workpieces respectively. The measured workpiece is placed on the measuring table, and the axis of the workpiece is adjusted to be coaxial with the rotating spindle of the cylindricity measuring instrument. The probe is in contact with the measured contour, and ideal circle is formed since the relative motion of the two. The actual contour is compared with the ideal circle to obtain the radius variation of the actual contour, and the roundness error value is calculated by data post-processing [6]. The installation
error of the workpiece on the instrument, the rotation error of the instrument spindle, and the installation error of the probe are the main factors affecting the measurement accuracy [7]. It is necessary to accurately level and center the workpiece to reduce the influence caused by workpiece installation during roundness error measurement before using the cylindricity measuring instrument to measure the rotary workpieces. This paper studies the relationship between installation error and roundness error measurement based on the least square roundness error evaluation method. The mathematical models of workpiece installation tilt and eccentricity are established. The influence of tilt and eccentricity on roundness error measurement are simulated by MATLAB, and an actual measurement is analyzed. The results show that after leveling and centering the workpiece, the influence of the tilt and eccentricity error of the workpiece installation on roundness error measurement can be neglected when the Taylor565H type cylindricity measuring instrument is used for micron order roundness error measurement.

2. Error modeling

2.1. Mathematical model of least square roundness error

As shown in Fig. 1, assuming that the measured circle contour is on plane XOY, where O denotes the origin of the coordinates, \((R \cos \theta_i, R \sin \theta_i)\)\((i = 0,1,2...m-1)\) are the values in cartesian coordinate system of the \(m\) measured points on plane XOY. The least square center \(O'(x_0, y_0)\) and the least radius of square circle \(R\) can be calculated by the following formula [8].

\[
\begin{align*}
  x_0 &= \frac{2}{m} \sum_{i=1}^{m} R_i \cos \theta_i \\
  y_0 &= \frac{2}{m} \sum_{i=1}^{m} R_i \sin \theta_i \\
  R &= \frac{1}{m} \sum_{i=1}^{m} R_i
\end{align*}
\]  

And the roundness error can be expressed as:

\[
e = \max\left(\sqrt{(R_i \cos \theta_i - x_0)^2 + (R_i \sin \theta_i - y_0)^2}\right) - \min\left(\sqrt{(R_i \cos \theta_i - x_0)^2 + (R_i \sin \theta_i - y_0)^2}\right)
\]  

Figure 1. Least square circle

2.2. Mathematical model of installation tilt

As shown in Fig. 2, assuming that there exist no shape error on the cylindrical workpiece, the axis of the workpiece does not coincide with the spindle of the cylindricity measuring instrument just since the
workpiece installation tilt. The measured contour is an ellipse contour. Let the angle between the workpiece axis and the cylindricity measuring instrument spindle to be \( \alpha \), so the long axis of the elliptical contour can be expressed as \( a = r / \cos \alpha \), short axis \( b = r \), the eccentricity \( H \alpha = h \tan \alpha \). Due to the influence of tilt, the actual measured points are collected with O as the center of rotation. So the coordinate of each point is \( (a \cos \theta + h \tan \alpha, b \sin \theta) \). The distance from each point on the measured contour to the center of rotation is:

\[
r = \sqrt{(a \cos \theta + h \tan \alpha)^2 + (b \sin \theta)^2}
\]  

(3)

2.3. Mathematical model of installation eccentricity

When the center of the measured circular contour and the rotary spindle are coincided in point \( O \), the coordinates of the point on the circle of radius \( r \) can be expressed as \( (r \cos \theta, r \sin \theta) \). Assuming that there exist no shape error on the cylindrical workpiece, the center of the measured circular contour does not coincide with the center of the cylindricity measuring instrument just since the workpiece installation eccentricity as shown in Fig. 3. The center of the circular contour is \( O(a, b) \), and the coordinates of the point corresponding to the center of the spindle O should be \( (r \cos \theta + a, r \sin \theta + b) \)

3. Simulation

3.1. Tilt error

According to above mathematical model, the influence of the cylindrical workpiece with radius of 50mm and 100mm on roundness error measurement of the contour at height \( h = 180 \text{mm} \) when the tilt angle is gradually increased to 10 degrees are simulated by MATLAB. The relationship between roundness error and tilt angle are shown in Fig. 4. The simulation results show that the roundness error caused by
workpiece installation tilt is gradually increased with the increase of the angle, for the same tilt angle, the roundness error increases with the increase of the radius.

![Graph showing the relationship between roundness error and tilt](image1)

**Figure 4.** Relationship between roundness error and tilt

3.2. **Eccentricity error**

According to above mathematical model, assuming that the workpiece is installed with only eccentricity on the X-axis, the influence of the cylindrical workpiece with radius of 50mm and 100mm on the roundness error measurement of the circular contour when the installation eccentricity is gradually increased to 1mm is simulated by MATLAB. The relationship between roundness error and eccentric magnitude is shown in Fig. 5. The simulation results show that the roundness error caused by the eccentricity of the workpiece is gradually increased with the increase of eccentric magnitude. For the same installation eccentricity, the roundness error becomes smaller with the increase of the radius.

![Graph showing the relationship between roundness error and eccentricity](image2)

**Figure 5.** Relationship between roundness error and eccentricity

3.3. **Actual measurement analysis**

As shown in Fig. 6, the axial and radial rotation motion accuracy of the Taylor 565H type cylindricity measuring instrument are all about 30 nm, and the straightness of its column is less than 70 nm[8]. The instrument can perform automatic leveling and centering. The leveling accuracy of this cylindricity measuring instrument can reach 0.3um, and the centering accuracy is better than 0.8um.
During the measurement, the whole length of the workpiece is split into several equidistant sections, and the probe measures the sections from the bottom to the top, and each section is uniformly sampled to conclude the radial variation of the measured points in the polar coordinate system [9]. The least square axis of the measured workpiece can be expressed as 

\[ a = u_i + \alpha z_j, \quad b = u_z + \beta z_j, \]

The distance from each measured point to the least square axis is:

\[ R_y = \sqrt{(R_y \cos \theta_j - a)^2 + (R_y \sin \theta_j - b)^2} = \sqrt{(R_y \cos \theta_j - (u_i + \alpha z_j))^2 + (R_y \sin \theta_j - (u_z + \beta z_j))^2} \quad (4) \]

Where \( j \) represents different sampled sections, \( i \) represents different sampled points, and \( R_y \) represent the coordinate value variations of the measured points. For uniform sampling, the least square radius and the least square center can be obtained by the following formula, where \( (\mu_i, \mu_j, 0) \) is the coordinate of the intersection point of the axis and the XY plane, which characterizes the position of the axis. \( \alpha, \beta \) are the direction parameters of the axis, \( R \) is the least square radius.

\[
\begin{align*}
R &= \sum_{i=1}^{m} \sum_{j=1}^{n} R_y / mn \\
\mu_i &= 2 \sum_{i=1}^{m} \sum_{j=1}^{n} R_y \cos \theta_j / mn \\
\mu_j &= 2 \sum_{i=1}^{m} \sum_{j=1}^{n} R_y \sin \theta_j / mn \\
\alpha &= 2 \sum_{i=1}^{m} \sum_{j=1}^{n} R_y z_j \cos \theta_j / m \sum_{j=1}^{n} z_j^2 \\
\beta &= 2 \sum_{i=1}^{m} \sum_{j=1}^{n} R_y z_j \sin \theta_j / m \sum_{j=1}^{n} z_j^2
\end{align*}
\]

The roundness error of a cylindrical workpiece with radius \( R = 50 \text{mm} \) and length of 180mm as shown in Fig. 6 is measured. Before measurement, the upper end surface is taken as the leveling plane, and a circular contour at the middle position of the workpiece is taken as the centering circle. When leveling is performed, the flatness of a plane and the perpendicularity with respect to the spindle axis will be calculated. The aligning table is biased by a corresponding amount of compensation such that a line perpendicular to the measured plane is parallel to the spindle of the instrument. When centering is performed, a roundness of the workpiece is measured. From the measurement data, the cylindricity measuring instrument calculates the eccentric magnitude of the workpiece center corresponding to the
center of the spindle. The centering table moves the corresponding compensation amount so that the center of the workpiece is concentric with the spindle axis. After calculation, the axis equation of the actual workpiece can be expressed as 
\[ a = 0.0065 + (7.0 \times 10^{-5}) z, \quad b = -0.0017 + (1.2 \times 10^{-5}) z \]
the intersection point of the workpiece axis and the turntable is \((0.0065, -0.0017, 0)\), and the tilt angle of the workpiece axis to the Z axis is \(7.1 \times 10^{-5}\). According to the above simulation, the roundness error caused by the installation tilt is 6.4 nm, and the roundness error caused by the installation eccentricity is 5.34 nm. The roundness error of the whole workpiece are all around \(3 \, \mu m\), so the roundness error caused by installation error can be neglected in this actual measurement.

4. Conclusion
The mathematical model of workpiece installation tilt and eccentricity is established based on the least square roundness error evaluation method in this paper. The relationships between roundness error and installation error are simulated by MATLAB, and the roundness error caused by installation error for a certain actual measurement is analyzed. The results show that after leveling and centering the workpiece, the influence of the tilt and eccentricity error of the workpiece installation on roundness error measurement can be neglected when the Taylor565H type cylindricity measuring instrument is used for micron order roundness error measurement.

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References
[1] D. G. CHETWYND, Roundness measurement using limacons. Precision Engineering, 1979,1(3): 137-141.
[2] Q. Zhao, Modeling and identification of contour features and its application. Dissertation for Doctor degree of HeFei University of Technology, 2004:41.
[3] Y. Li, L. Wei, J. Deng, et al. Study on automatic leveling technology. Piezoelectrics & Acoustooptics, 2010, 32(6): 949-952.
[4] W. Jiang, Y. Gao, D. Feng, et al. Automatic leveling system for base-plane of large-size photoelectric equipment. Opt. Precision Eng. 2009, 17(5): 1039-1045.
[5] W. Gao, Satoshi Kiyono, Takamitu Sugawara, High-accuracy roundness measurement by a new error separation method. Precision Engineering, 1997, 21: 123–133.
[6] Y. Zhang C. Zuo Y. Liu C. Li, Present State and Expectation of Roundness Measurement Method Tool Engineering 2008.
[7] W. Sui, D. Zhang, “Four Methods for Roundness Evaluation,” Physics Procedia, 24, 2159-2164 (2012).
[8] taylor-hobson, “Talyrond 565H Range,” https://www.taylor-hobson.com/products/roundness-form/high-precision-roundness/talyrond-565h-range.
[9] D. J. W. Dawson, “Cylindricity and its measurement,” Int J Mach Tool Manu. 32(1–2), 247-253 (1992).