Stochastic Transmission of Sneezed and Coughed Droplets - Physicists’ Perspective

Santosh K. Das†
School of Physical Sciences, Indian Institute of Technology Goa, Ponda-403401, Goa, India

Jan-e Alam‡
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata- 700064, India and Homi Bhabha National Institute, Training School Complex, Mumbai - 400085, India

Salvatore Plumari§ and Vincenzo Greco§
Department of Physics and Astronomy, University of Catania, Via S. Sofia 64, I-95125 Catania, Italy and Laboratori Nazionali del Sud, INFN-LNS, Via S. Sofia 62, I-95123 Catania, Italy

Abstract

The motion of droplets ejected in the air during sneezing, coughing, breathing and speaking has been studied within the scope of Langevin stochastic differential equation. The effects of drag, diffusion and the gravity on the droplets during its motion in the air have been taken into account. We found that the distance traveled by a droplet released at an average human height by an infected individual strongly depends on the velocity of ejection and mass of the droplet. For example, a droplet of radius 100 µm and mass 4186 nano-gm released at a height of 1.7 meter will travel 2.35 meter distance horizontally before settling gravitationally. The same droplet with lower ejection velocity of 5 m/s will travel a horizontal distance of 0.55 meter approximately. We found that the gravitational force does not allow larger (and hence massive) droplets to remain suspended in the air for long. Because the gravitational force is superior to the the drag force exerted by the air molecules on larger droplets. However, for smaller droplets the drag force dominates over the gravitational force which allows it to be suspended in the air for longer time. As an instance a droplet of radius 2.5 µm may remain suspended in the air for more than half an hour. This suggests that a healthy individual should maintain temporal distance from an infected one to avoid virus suspended in the air. Therefore, as preventive strategies a healthy individual should not only maintain geometric distance from an infected one but also temporal distance to deter the floating smaller droplets by covering nose, mouth, etc. This approach will be useful for the prevention of current COVID19 pandemic.

PACS numbers: 12.38.Mh, 12.39.-x, 11.30.Rd, 11.30.Er

Keywords:

†Electronic address: jane@vecc.gov.in
‡Electronic address: salvatore.plumari@hotmail.it
§Electronic address: greco@lns.infn.it
I. INTRODUCTION

It was shown in 1938 [1] that the droplets released through coughing, sneezing, speaking or breathing contain microorganisms (bacteria, virus, fungi, etc) causing a large number of diseases. The understanding of the evolution of the droplets in space and time helps enormously in planning the preventive strategies for the diseases caused by the microorganisms. Therefore, we make an attempt in this work to study the evolution of the droplets by applying the laws of physics.

The motion of sufficiently small but macroscopic objects in fluid can be studied by solving Langevin equation [2, 3] with appropriate initial condition. The problem will not only be complex but unsolvable if one considers the interaction of the macroscopic object with the individual molecules of the fluid. Therefore, one considers the interaction of the macroscopic object with the thermal bath formed by the fluid molecules characterized by temperature and density. The interaction of the macroscopic object with the fluid (bath) is encoded in the drag and diffusion coefficients of the object in the fluid. The object may also be subjected to external forces (e.g. gravity or electromagnetic depending on the situation) which needs to be included in the equation.

For a physicist this set an appropriate stage to study the propagation of sneezed and coughed ejecta in the air within the scope of Langevin equation. The motion of the droplets (macroscopic object) discharged outside by any infected individual through sneezing and coughing in the air (fluid) can be studied by using the laws of statistical mechanics. In this context the Langevin equation can be applied to study the evolution of the droplets (post ejection) in the air where the droplets play the role of the macroscopic object and the air play the role of the fluid. The application of Langevin equation is justified here because the mass of the droplets are much higher than the mass of oxygen, nitrogen, etc molecules present in the air. Apart from its interaction with the air, however, the droplets are also subjected to the force of gravity because they have non-zero mass. Therefore, after the ejection the change in position of the droplets with time will be determined by: (1) the drag force exerted by the air on the droplet, (2) the diffusion of the droplet in the air and (3) the gravitational force acting on it. The velocity of the droplets will dissipate in the air in course of time. Droplets with larger mass will be subjected to less drag and diffusion but more gravitational force. Therefore, it will be interesting to check how these competitive forces influences the distance that the droplet traverse from the source (infected individual). This will indicate the distance (both geometric and temporal) that a healthy individual should maintain from an infected one to prevent virally transmitted diseases.

The present work is organized as follows. In the next section we discuss the Langevin equation along with the initial conditions used to solve it. Sections III and IV are devoted for presenting results and summary of the findings respectively.

II. THE LANGEVIN EQUATION

We write down the Langevin equation in the presence of gravitational field for the motion of the droplets of mass ($M_i$) in the air as follows [4, 5]:

$$\frac{dr_i}{dt} = v_i$$

$$M_i \frac{dv_i}{dt} = -\lambda v_i + \xi(t) + F_G$$

(1)

(2)
where \( dr_i \) and \( dv_i \) are the shift of the coordinate and velocity in each discrete time step \( dt \), \( i \) stands for the Cartesian components of the position and velocity vectors. \( \lambda \) in Eq. 2 is the drag coefficient. The first term of Eq. 2 represents the dissipative force and the second term represents the stochastic (diffusive) force, \( \xi \) regulated by the diffusion coefficient \( D \). Due to the stochastic nature of \( \xi(t) \), it is also called noise. We study the evolution with a white noise ansatz for \( \xi(t) \), i.e \( \langle \xi(t) \rangle = 0 \) and \( \langle \xi(t) \xi(t') \rangle = D \delta(t-t') \). White noise describes a fluctuating field without memory, whose correlations have an instantaneous decay called \( \delta \) correlation. The third term in Eq. 2, \( F_G \) represents the gravitational force \( = M \dot{g}, \dot{g} = 9.8 \text{ m/s}^2 \). The drag coefficient has been estimated by using Stokes’ Law. The diffusion coefficient is related to the drag coefficient through the Einstein fluctuation-dissipation relation [3].

We solve these equation by using Monte-Carlo techniques with the inputs discussed below. For the initial spatial coordinate we use, \( x = y = 0 \) and \( z = H_0 \), where \( H_0 \) is the height (1.7 meter) at which the droplet is released (nose/mouth). We distribute the initial velocity uniformly in the \( x-y \) plane, where \( v_z = 0 \). The gravitational force act on the \( z \) direction. We take spherical droplets of various radii (\( R \)) ranging from 5\( \mu \)m to 100\( \mu \)m [6]. The density of the droplet is taken as, \( d = 1000 \text{ Kg/m}^3 \). Then the mass is determined by the relation \( M = 4\pi R^3 d/3 \). The value of the drag coefficients, \( \lambda \) is determined by using Stokes’ law as \( \lambda = 6\pi \eta R \). For a droplet of radius 100\( \mu \)m, \( \lambda = 33912 \text{ Nmg/sec} \) where we have taken the viscous coefficient of air as \( \eta = 1.8 \times 10^{-5} \text{ Kg/(meter-sec)} \) [7]. By using Einstein relation we get the value of the diffusion coefficient, \( D = K_B T \lambda \) where \( K_B = 1.38 \times 10^{-23} \text{ J/}^\circ \text{K} \), is the Boltzmann constant and \( T \) is the temperature. We have taken \( T = 30^\circ \text{C} \) and obtained \( D = 0.1418 \times 10^{-3} \text{ (Nmg)}^2 \text{ m}^2 \text{sec}^{-3} \). For a droplet of radius 100\( \mu \)m we get \( M = 4186 \text{ Nbm} \). The initial velocity of the droplets has been varied from 5 to 21 m/s [8].

### III. RESULTS

The contamination depends on the mass and initial velocity of the droplets. However, the droplets will have different sizes (and hence masses) and initial velocities at the time of ejection through coughing and sneezing. Therefore, we provide results for a range of droplet masses and initial velocities.

We consider a droplet of radius 100 \( \mu \)m and mass 4186 nano-gm ejected at initial time \( t = 0 \) with velocity varying from from 5 to 21 m/s. The droplet is ejected at a (initial) height, \( H_0 = 1.7 \text{ meter} \) \((x = y = 0)\), that is the initial spatial coordinate of the droplet is \((x, y, z) = (0, 0, 1.7 \text{meter})\). The magnitude of the velocity is distributed uniformly in the \( x-y \) plane with \( v_z = 0 \). With these inputs we solve the Langevin equations, Eq. 2. We use \( H(t) \) as the height of the droplets which changes with time and \( L(t) = \sqrt{x^2 + y^2} \) is the horizontal distance traveled by the droplet which is also a time dependent quantity.

In Fig. 1, the results obtained with these inputs have been displayed. The (horizontal) distance, \( L \) traveled by the droplets from the source depends strongly on the initial velocity and mass. While a droplet of mass 4186 Nmg with small initial velocity, \( V_0 = 5 \text{ m/s} \) travels a distance, \( L \approx 0.55 \text{ meter} \), a droplet with larger initial velocity, \( V_0 = 21 \text{ m/s} \) travels 2.35 meter approximately. This droplet takes about 1.5 sec to fall in the ground due to gravity. Other droplets with intermediate values of \( V_0 = 15 \text{ m/s} \) and 10 m/s travel horizontal distances approximately 1.7 meter and 1.1 meter respectively. It may be mentioned here that a droplet of radius 200 \( \mu \)m takes about 0.73 sec to fall on the ground which may be compared with the value for free fall time \( t = \sqrt{(2H)/g} = 0.59 \text{sec} \) from a height 1.7 meter (please also see [9]). This indicates that free fall will be a reasonable approximation for droplets having

\[ M = \frac{4}{3} \pi \eta R^3 \]

\[ D = \frac{K_B T \lambda}{3} \]

\[ L(t) = \sqrt{x^2 + y^2} \]
FIG. 1: The change in height with time of the droplet of mass 4186 nano-gm with different initial velocities are depicted. The distance, $L$ traveled by the ejecta from the source of infection (at a height 1.7 meter) for different initial velocity ($V_0$) have been shown.

radii larger than 200 $\mu$m. The results discussed above can be viewed in a different way as follows. Fig. 2 shows the change in height of the droplets with horizontal distance at a time 2 sec after the ejection for different initial velocities ($V_0$). A droplet of mass 4186 Ngm with $V_0 = 21$ m/s (5 m/s) travel a horizontal distance, $L \approx 2.35$ meter (0.55 meter). The same droplet with intermediate $V_0$ values, 15 m/s (10 m/s) travels approximately 1.7 meter (1.1 meter). These results indicate that big (massive) droplets falls on the ground within a short time due to gravitational force but they travel larger distance due larger momentum as the drag force for such droplets is weaker than gravitational force. These results are consistent with the results displayed in Fig. 1.

How long a droplet takes to gravitationally settle on the ground or in other words how long it remains suspended in the air after it is ejected through sneezing or coughing? Of course that will depend on the mass of the droplet and the magnitude of the drag and diffusion coefficients. We assume that the droplet is ejected at a height 1.7 meter in the air with initial velocity 21 m/s. We find that a droplet of size 100 $\mu$m float in the air only for 1.5 sec approximately. For such large (massive) droplet the gravitational force dominates over drag and diffusion and consequently settle on the ground quickly. However, a droplet of smaller size (hence lighter too) of radius 5 $\mu$m and ejection velocity 21 m/s survives in the air for almost 520 sec i.e. $\approx 8.7$ minutes. We have also found that a droplet of radius 2.5 $\mu$m with same ejection velocity will remain suspended in the air for more than half an hour. Because for such lighter droplets drag and diffusion process play significant role and the effects of gravity is small. These results along with droplets with intermediate size are displayed in Fig. 3. It may be mentioned that if we set the values of drag and diffusion coefficients to zero then our numerical results are in excellent agreement with the results obtained by assuming free fall of the droplets. Results depicted in Fig 4 show that the droplet of size size 100 $\mu$m with initial velocity 21 m/s released from a height of 1.7 meter from the ground will
FIG. 2: Variation of $H$ with $L$ for a droplet of mass 4186 Ngm and radius 100 $\mu$m for different ejection velocities has been displayed.

FIG. 3: The variation of $H$ with time for droplets of different sizes (masses) and ejection velocity 21 m/s.

travel a distance slightly more than 2.35 meter. However, smaller size droplets with same initial velocity traverse a smaller distance because of their small momentum. For example, droplets of sizes 10, 25 and 50 $\mu$m travels distances of magnitude 0.024, 0.15, and 0.6 meters respectively. It should be clearly mentioned here that although the smaller droplets travel smaller distance but they survive in the air for longer time (Fig.3).

From practical point of view the question to be asked is - what is the geometric distance
FIG. 4: The change of $H$ with $L$ for droplets of different masses has been shown. The initial velocity is 21 m/s.

FIG. 5: The variation of the maximum horizontal distance ($L_{\text{max}}$) travelled by droplets as a function of radius for different ejection velocity.

$L_{\text{max}}$ that an healthy individual should keep from an infected one? The answer will depend on ejection velocity and mass of the droplets. In this context we study the variation of $L_{\text{max}}$ with the size of the droplets for $V_0 = 10$ m/s (average velocity of the droplets originating from coughing [8] and references therein) and a higher value of $V_0 = 21$ m/s (which is expected to give higher value of $L_{\text{max}}$). We observe that the maximum horizontal distance traveled by the droplets increases with its size (mass). For larger ejection velocity and radius we get
FIG. 6: The variation of the maximum time that the droplets remain suspended in the air with radius. The dependence of the maximum time on ejection velocity is found to be weak.

larger $L_{\text{max}}$. It is interesting to note that the value of $L_{\text{max}}$ for $V_0 = 10$ m/s (average velocity) and $R = 100 \mu$m $L_{\text{max}}$=1.1 meter. That is the distance to be maintained between an infected and a healthy person comes out to be about 1.1 meter if we consider the average ejection velocity. For $V_0 = 21$ m/s we get $L_{\text{max}} = 2.35$ meter.

In Fig. 6 the maximum time ($t_{\text{max}}$) of suspension of the droplet in the air is plotted as a function of $R$ for $V_0 = 21$ m/s. The results clearly indicates that the $t_{\text{max}}$ decreases with increase in $R$ i.e. the smaller droplets remain suspended in the air for a longer time. This result may be used as a guideline to determine the temporal distance that an healthy individual should maintain from an infected one. The dependence of $t_{\text{max}}$ on $V_0$ is found to be mild.

The results displayed above suggest that droplets of larger mass travels larger distance within a small interval of time. For larger droplets gravity dominates over drag and diffusion. But the drag force exerted by the air molecules on the smaller droplets dominates over the gravitational force and hence they remain suspended in the air for longer time. Therefore, to formulate preventive strategies both of these factors (distance and time) should be kept in mind.

IV. SUMMARY

We have studied the motion of the sneezed and coughed droplets (containing microorganism) of different masses ejected with different velocities by an infected individual at a height 1.7 meter. We find that in one hand the larger droplets are quickly settled gravitationally but within a small interval of time they travel larger distance. A droplet of mass 4186 Ngm with ejection velocity 21 m/s travel a distance of 2.35 meter horizontally in a time less than 1.5 sec. However, the same droplet with ejection velocity 5 m/s will travel about 0.55 meter horizontal distance. Therefore, the ejection velocity of large droplets play an important role
to determine the distance that to be maintained between healthy and infected individuals. A droplet of radius 100 µm with maximum possible ejection velocity \([8]\) of magnitude 28.8 m/s travels a distance of 3.2 meter. But it is important to mention here that the abundance of such high velocity droplets will be small. We found free fall is a reasonable approximation for droplets having radius larger than 200 µm. On the other hand lighter droplets travel smaller distance but have the ability to float in the air for longer time because the drag and diffusion processes play significant role and the effect of gravity is small. For example a droplet of size 5 µm with ejection velocity 21 m/s will remain suspended in the air for more than 8 minutes although it will travel a distance of less than 20 cm. It is found that a droplet of radius 2.5 µm and \(V_0 = 21\) m/s remains suspended in the air for more than half an hour. Hence, smaller droplets determines the temporal interval that a healthy individual should maintain from an infected one. Therefore, as preventive strategies a healthy individual should not only maintain geometric distance but also deter the floating droplets by covering nose, mouth, etc.

It may be mentioned here that the smaller droplets may be created by the process of fragmentation of the larger droplets \([10]\) depending on the environment. Droplets may decay through evaporation process \([11]\) and the products of the evaporation may remain suspended in the air with the potential of causing infection. The evaporation process will strongly depend on the air temperature and relative humidity. The present computation has been done in still air. The diffusion of droplets in flowing air with the possibility of evaporation will be addressed future.

**Acknowledgement**

SKD would like to acknowledge IIT Goa for internal funding (No. 2020/IP/SKD/005) and Professor Barada Kanta Mishra for encouragement. SKD acknowledges useful discussions with Raja Mitra, Ashish Bhatheja, Thaseem Thajudeen, Sabyasachi Ghosh, Haridas Pai and Rudra Narayan Roy. JA would like to thank Dr Achintya Bhattacharyya for useful discussions.

[1] W. F. Wells and M. W. Wells, Am. J. Public Health Nations Health, 28, 343 (1938).
[2] F. Reif, Fundamentals of Statistical and Thermal Physics, Mcgraw-Hill International Editions, Singapore, 1985.
[3] R. K. Pathria, Statistical Mechanics, Butterworth-Heinemann, Oxford, 1996.
[4] S. K. Das, F. Scardina, S. Plumari and V. Greco, Phys. Rev. C 90, 044901 (2014).
[5] S. K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina and V. Greco, Phys. Lett. B 768, 260-264 (2017).
[6] Z. Y. Han, W. G. Weng and Q. Y. Huang, J R Soc Interface 10, 20130560 (2013).
[7] J. Einarsson and M. Mehlig, Phys. Rev. Fluids, 2, 063301 (2017).
[8] M. Vansciver, S. Miller, J. Hertzberg, Aerosol Sci. Technol. 45 (3) (2011) 415-422.
[9] D. A. Stariolo, arXiv:2004.05699 [physics.class-ph]
[10] B. E. Scharfman, A. H. Techet, J. W. M. Bush and L. Bourouiba, Exp. Fluids, 57, 24 (2016).
[11] L. A. Anchordoqui and E. M. Chudnovsky, arXiv:2003.13689