Predicting the Isotropic Volumetric Compression Response of Hydrating Cemented Paste Backfill (CPB)

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Abstract Deep and high-stress mining results in stress transfers onto the previously placed backfill, and mines have recorded several MPa induced backfill stresses. Understanding the backfill-rock mass interaction is therefore critical. Previous work considered tabular ore bodies undergoing primarily one-dimensional compression and showed how the backfill reaction curves could be estimated from oedometer laboratory test results. This work considers massive orebodies and develops a similar approach based on isotropic compression curves. Isotropic compression tests exceeding 6 MPa are carried out on samples with 3.0 to 11.1% binder content, tested at 1-day cure time to 28-day cure time. The compression curve is characterized in three stages: initial elastic compression up to a yield point, followed by a transition stage to the start of a final stage with a linear post-yield compression line in $\varepsilon_v - \log(p')$ space. Because these isotropic compression tests are rare (the reported results are the first for Cemented Paste Backfill), attempts are made to relate the isotropic compression test parameters to parameters from the more commonly used Unconfined Compression Strength (UCS) tests. Unifying equations as functions of binder content and cure time are found to determine the initial yield stress and the peak strength from UCS tests. These are then related to the corresponding parameters in isotropic compression. Finally, the slope of the post-yield compression line is found as a function of UCS of similar CPB with the same binder content and cure time. Then the isotropic compression behavior of CPB is reconstructed as a function of binder content and curing time using UCS values of similar CPB. Although the calibrated parameters are specific to the studied mine’s materials, the framework is general and applicable to other mines’ CPBs.

Keywords Cemented paste backfill · Underground mining support · Isotropic consolidation · Isotropic hardening law · Laboratory tests

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List of Symbols

- $B_i$ Constant coefficients
- $CC$ Cement content
- $CT$ Curing time
- $\varepsilon$ Void ratio at the end of each isotropic consolidation step
- $\varepsilon_0$ Void ratio after curing
- $\varepsilon_C$ Total volumetric strain at the end of transitional compression segment
1 Introduction

The economic and environmental sustainability of bulk underground mining operations depends on timely backfilling of the large voids created by mining. This ensures regional stability of the surrounding rock mass, and when the mine’s own tailings are reused as backfill it also reduces the need for surface disposal of mine tailings and the attendant potentially adverse environmental impacts. Backfilling is particularly important in deep and high-stress mining for controlling progressive host rock closures resulting from ongoing mining, by partially transferring the load from the host rock onto the backfill. Cemented Paste Backfill (CPB), a mixture of mine tailings, water and binder, is the most rapid and efficient backfilling form that has gained increasing popularity in the mining industry over the past two decades.

An early attempt at measuring field stresses to understand the energy transfer from mining onto CPB was carried out by Hassani et al. (2001) at Chimo mine. They used stress measurements in the host rock and the backfill, coupled with displacement measurements in the host rock and quantified the energy absorbed by backfill during mining an adjacent stope. In their study, the induced stresses approached 1 MPa. Other subsequent studies have measured induced stresses in excess of 4 MPa and closure strains up to 15% (Thompson et al. 2009, 2011, 2012; Counter 2014; Raffaldi et al. 2019). While these induced stresses are smaller than the pre-existing field stresses, they are nonetheless much higher than confining stresses that are typically explored in mine backfill strength studies (such as consolidation and strength determination under different confining pressures and loading paths) conducted under controlled laboratory conditions.

Unfortunately, the literature contains no data on the response of CPB to high confining stress. Uniaxial compressive strength (UCS) of CPB has been studied quite extensively (e.g., Kesimal et al. 2005; Klein and Simon 2006; Orejarena and Fall 2010; Yi et al. 2015; Gorakhki and Bareither 2017; and Xu et al. 2018)), in part because it is the least resource intensive, but also there are many limit equilibrium design methods that depend on UCS as a key input (e.g., Mitchell et al. (1982) and Potvin et al. (2005)). Although there have been progress regarding preparing the laboratory specimens to be more representative of field conditions (Benzaazoua et al. 2006) less attention has been paid to tests that study mechanical properties and effects of cemented bond breakage on the behavior of cured CPB under more complex loading conditions (Pierce et al. 1998; Yilmaz et al. 2010, 2015; Fang and Fall 2020; Jafari et al. 2020a, b, 2021). Therefore, new information about CPB’s response to high confining pressures is needed to have safe and economical backfill desing. Jafari et al. (2020a) showed how laboratory oedometer test results could be used to understand the backfill-rock mass interaction for tabular ore bodies undergoing predominantly one-dimensional compression. In this work a similar approach is used to understand how backfill would respond to predominantly isotropic loading. The volumetric hardening law is one of the main components of constitutive models for structured soils. This law employs the notion of a loading surface, whose evolution depends on a hardening parameter that is identified by either irreversible void ratio or volumetric strain. Therefore, in order to determine this parameter, material behavior under isotropic compression should be studied.

The majority of the studies on soil behavior under isotropic pressure have focused on uncemented sands (Lade et al. 1996; Jefferies and Been 2000; Consoli et al. 2005; Lade and Bopp 2005), with fewer studies on cemented sands and clays (summarized in Table S1 in the Supplementary Data). There are no results for cemented silts, which is the common material in Cemented Paste Backfill. Furthermore, in all

\begin{align*}
\varepsilon_{v, Tot}^v & \quad \text{Total volumetric strain} \\
\varepsilon_{v, YS}^v & \quad \text{Total volumetric strain at yield stress} \\
K & \quad \text{Bulk Modulus} \\
m & \quad \text{Intercept of post-yield compression line in } \varepsilon_{v, Tot}^v - \log (p') \text{ space} \\
P_{ISO} & \quad \text{Pressure corresponding to the intersection of post-yield compression line and pressure axis} \\
PYCLS & \quad \text{Post-yield compression line slope in } \varepsilon_{v, Tot}^v - \log (p') \text{ space} \\
p' & \quad \text{Effective mean isotropic stress} \\
S_{YS} & \quad \text{Slope at yield stress in } \varepsilon_{v, Tot}^v - \log (p') \text{ space} \\
\sigma_C & \quad \text{Stress at the end of transitional compression segment} \\
\sigma_{ISO, YS} & \quad \text{Yield stress in isotropic compression test} \\
\sigma_{UCS, YS} & \quad \text{Yield stress in UCS test}
\end{align*}
previous studies the experiments were carried out on “cured” samples (typically 28-day), however in mining the loading could occur much earlier due to ongoing proximate mining. Therefore, in the current work a range of binder contents and cure times is considered representative of practical mining conditions. Previous work (Jafari et al. 2020a) showed that the Unconfined Compressive Strength (UCS) can be determined as a function of binder content and cure time, and so it is also desired to relate the isotropic compression response to the UCS test results if possible, as the UCS is more conventional and readily carried out. Although the calibrated test results will be specific to the materials tested, it is desired that a framework be demonstrated by which other mine’s materials can be similarly tested in a more efficient way with fewer resource-intensive tests such as isotropic compression tests.

2 Experimental Program

2.1 Material Description

The tailings used in this study were collected from the Williams mine located east of Marathon, Ontario, Canada, and mainly contain silt-size crushed rock with 86% fine as shown in Fig. 1. Silicates were the main mineral constituent of the tailings, and small amounts of sulphur oxides were detected. It should be noted that the concentration of sulphur-bearing minerals (particularly pyrite and pyrrhotite) is low as compared to other hard rock mine tailings reported in the literature, as will be considered subsequently. Atterberg limits were determined in accordance with ASTM D4318-05. The liquid limit (LL) of the tailings is 23%, the plastic limit (PL) is 18%, and the plasticity index (PI) is 5%. The specific gravity of tailings is 2.768 ± 0.009 determined using Helium Stereopycnometer in accordance with ASTM D5550-14. 2014 and the specific surface of tailings is 0.344 m^2/g measured based on ASTM C204-18e1. The water collected from the mentioned mine’s paste plant and ordinary Portland cement were other constituents of this CPB. More information about the chemical compositions of tailings and cement, and concentrations of ions in the process water may be found in Jafari et al. (2020c).

2.2 Specimen Preparation and Testing Procedure

The mine tailings were shipped from the mine in separate buckets. The water content of each bucket was first determined by mixing the bucket for 15 min to ensure that the material was well blended. It should be noted that the water contents used in this work are defined as mass water/mass solid, as opposed to the definition commonly used in the mining industry, which is mass water/total mass. Based on what has been used in the CPB preparation in the mentioned mine, the desired initial water content (percent mass of water by total mass of the solid) of 38% was considered in specimen preparation, and consequently, tailings, mine process water, and cement values were determined and mixed to obtain the desired water content representative of mining conditions. A range of cement content between 3.0 to 11.1% (weight of cement to dry tailings) was considered in this study which is consistent with the range of cement content commonly used in practice (Landriault 1995; Potvin et al. 2005). The specimens were cast into split moulds in three layers. The moulds were then sealed with no drainage allowed, and the specimens were cured in a humid container with 96% relative humidity and temperature of 22 ± 1 °C for one day. Then, to prevent oxidization, after removing the top cap of the moulds, the specimens were submerged in the mine process water to cure until the desired testing time. The extracted specimens had 34.2 mm diameter, and the ratio of 2:1 between height and diameter was

![Fig. 1 Particle-size distribution of the mine tailings](image-url)
considered in specimen preparation. More information about the designed mould can be found in Jafari et al. (2017).

UCS tests were conducted under water with a constant strain rate of 0.03 percent per minute to ensure fully drained conditions during testing. Soaking the specimen during a low strain-rate test prevents the specimen from drying. Moreover, it prevents development of suction, which in turn ensures that the developed shear resistance is purely due to the bonding between soil grains. To conduct the isotropic compression test, a triaxial cell with capacity of 20 MPa was designed and fabricated (Fig. 2). In these tests, the specimens were always back-pressurized (to about 600 kPa) to ensure the fully saturated condition. The volumetric change was measured using a frictionless rolling diaphragm type volume change device (VCD) with 0.01 ml resolution. Using the pore pressure transducer installed in the top cap, the pore pressure in the specimen was monitored, and drainage was allowed from the bottom platen.

3 Laboratory Test Results and Discussion

This section provides and briefly discusses the UCS and isotropic compression test results carried out in this study. The following abbreviations are used in the tables and figures: UCS: uniaxial compressive strength test; ISO: isotropic compression test; CC: Cement Content; and CT: Curing Time. The number following each abbreviation indicates the value of the parameter presented by the abbreviation.

3.1 UCS Test Results

UCS tests were carried out on CPB specimens with four cement contents at five different curing times based on ASTM D2166/D2166M-16. Three tests were conducted for each combination of curing time and cement content, resulting in a total of 60 specimens being tested. The average and coefficient of variation of UCS measurements are presented in Table 1. It is noted that the coefficient of variation of UCS test results for each combination of cement content and curing time was within the acceptable range.
curing time is less than 7%, and generally lower for higher cement contents.

The UCS test results indicate that, as expected, UCS and yield stress in the UCS tests ($\sigma_{YS}$, see Sect. 3.1.1) generally increase with the increase of curing time at a certain cement content. Also, for all cement contents, the rate of increase in UCS seems much higher initially but declines over time (as can be seen in Table 1). This can be attributed to the process of cement hydration and the fact that over time, most of the cement particles have reacted with water, which would result in stiffening and progressive strength gaining of CPB. It should be noted that not all CPB material necessarily behave this way. For instance, the presence of sulphide minerals within the CPB has a deteriorative effect (chemical weathering) that leads to the reduction of UCS by curing time (Benzaazoua et al. 1999, 2002; Kesimal et al. 2004; Li et al. 2019). Also, Portland cement combined with fly ash, ground blast furnace slag, or other supplementary cementitious materials can have multiple reaction phases leading to more discontinuous strength gain with hydration relationships (e.g., Veenstra (2013)).

The results show that the cement content and curing time considerably affect CPB strength. Here, a power function and a factor of the form $CT^{m}.CC$ are proposed to capture both effect of cement content and curing time on UCS and $\sigma_{YS}^{UCS}$ (Fig. 3). Similar power function and factor are used by Consoli et al. (2012) to capture the effect of variation of cement content and void ratio in cemented sands on the UCS of cemented sand. The best-fit power functions are shown in Fig. 3. This fitting approach works in part because the strength development with time is a relatively monotonic, smooth and continuous function. It should be noted that this is not always the case, as mentioned above. Therefore, the proposed fitting function is specific to the studied backfill in terms of both its mathematical form as well as the calibrated parameters, and other mine’s materials will require similar laboratory testing programs to determine if the mathematical formulations require adaptation or the material parameters require recalibration.

### Table 1 Summary of UCS tests

| Test case  | Cement content [%] | Curing time [days] | Void ratio ($e_0$) | Yield stress [kPa] | UCS [kPa] | UCS coefficient of variation [%] |
|------------|-------------------|-------------------|-------------------|-------------------|-----------|---------------------------------|
| UCS-CC3.0-CT01 | 3.0               | 1                 | 1.03              | 12.15             | 27.88     | 5.89                            |
| UCS-CC3.0-CT03 | 3                 | 1                 | 1.03              | 23.92             | 49.24     | 2.42                            |
| UCS-CC3.0-CT07 | 7                 | 1                 | 1.01              | 31.56             | 62.81     | 1.83                            |
| UCS-CC3.0-CT14 | 14                | 1                 | 1.02              | 45.96             | 89.09     | 3.35                            |
| UCS-CC3.0-CT28 | 28                | 1                 | 1.02              | 59.05             | 109.07    | 3.25                            |
| UCS-CC5.3-CT01 | 5.3               | 1                 | 1.02              | 28.59             | 61.83     | 7.00                            |
| UCS-CC5.3-CT03 | 3                 | 1                 | 1.01              | 56.77             | 111.60    | 5.11                            |
| UCS-CC5.3-CT07 | 7                 | 1                 | 1.00              | 80.16             | 150.54    | 2.39                            |
| UCS-CC5.3-CT14 | 14                | 0.99              | 114.21            | 203.71            | 4.86      |
| UCS-CC5.3-CT28 | 28                | 1                 | 134.37            | 235.29            | 4.66      |
| UCS-CC7.5-CT01 | 7.5               | 1                 | 1.01              | 65.63             | 135.38    | 3.35                            |
| UCS-CC7.5-CT03 | 3                 | 1                 | 1.00              | 123.64            | 223.85    | 2.63                            |
| UCS-CC7.5-CT07 | 7                 | 0.99              | 178.90            | 295.16            | 4.11      |
| UCS-CC7.5-CT14 | 14                | 0.99              | 230.00            | 366.17            | 2.51      |
| UCS-CC7.5-CT28 | 28                | 0.99              | 279.33            | 446.30            | 3.24      |
| UCS-CC11.1-CT01 | 11.1              | 1                 | 1.00              | 133.31            | 242.76    | 2.48                            |
| UCS-CC11.1-CT03 | 3                 | 0.98              | 311.24            | 539.64            | 1.30      |
| UCS-CC11.1-CT07 | 7                 | 0.97              | 421.47            | 677.50            | 1.49      |
| UCS-CC11.1-CT14 | 14                | 0.96              | 520.93            | 788.33            | 1.77      |
| UCS-CC11.1-CT28 | 28                | 0.96              | 579.05            | 873.34            | 3.65      |
3.1.1 Yield Stress in UCS Tests (σ\textsubscript{YS})

Yield is the process in which a material changes from a state of predominantly elastic behavior to one of predominantly plastic behavior (Bieniawski 1967). It is generally accepted that the cemented bonds control the yield behavior of cemented soils (Malandraki and Toll 2000). Determining this value is sometimes difficult due to the gradual onset of cementation bond breakage (Rotta et al. 2003). This may be even more complicated because yield in cemented or generally structured soils is known to depend on stress path, initial confining pressure, and also drainage conditions (Smith et al. 1992; Coop and Atkinson 1993). For the tested CPB specimens however, determining the yield stress was relatively straightforward.

The methods that define the yield stress can generally be classified into two main groups based on (1) a clear change in stress–strain behavior (Anagnostopoulos et al. 1991; Coop and Atkinson 1993; Kavvadas et al. 1994; Rotta et al. 2003); and (2) a clear change in stiffness-strain behavior on the logarithmic or semilogarithmic scale (Malandraki and Toll 1996, 2000, 2001). In this study, both methods were used to determine the yield stress and essentially identical results were obtained; values from the second method are reported.

Figure 4 shows the ratio of yield stress to the strength of the material in the UCS test which is an indication of the domain of elastic behavior of the material based on applied stress. This ratio increases by curing time in the specimens with identical cement content; for example, for the specimens with 11.1% cement content, the ratio increases from 0.55 for the specimens with one-day curing time to 0.66 for the specimens with 28 days curing time, i.e. an approximately 20% increase. Also, the specimens with higher cement content show higher yield stress to strength ratios at the same curing time.

![Graph showing the variation of yield stress to strength ratio with curing time for different cement contents](image-url)
3.2 Isotropic Compression Test Results

The variation of total volumetric strain ($\varepsilon_{\text{Tot}}^v$) with respect to effective isotropic pressure ($p'$) are shown in Fig. 5 for specimens with 3.0, 5.3, 7.5, and 11.1% cement contents. The summary of individual specimen test results is presented in Table 2. These results show that the typical isotropic compressive behavior of CPB is similar to other cemented soils. This means that CPB shows low volumetric strain change under low effective isotropic pressure up to the yield stress. Here, the pressure at which the material behavior begins to deviate from linearity is defined as the isotropic yield stress ($\sigma_{\text{YS}}^{\text{ISO}}$). Beyond $\sigma_{\text{YS}}^{\text{ISO}}$, the rate of volumetric change against the increase of isotropic pressure increases followed by linear behavior at higher pressures. Figure 5 further shows that $\sigma_{\text{YS}}^{\text{ISO}}$ increases with increasing cement content and curing time, and that the corresponding curves expand and become steeper.

![Fig. 5](image-url)  
**Fig. 5** Effective isotropic compression response of specimens with: a 3.0%; b 5.3%; c 7.5%; and (d) 11.1% cement content at different curing times.
4 General Equation of Volumetric Strain Variation Versus Isotropic Pressure

A careful study of all isotropic compression tests shows that for all cement content and curing time combinations, CPB response under isotropic pressure can be defined by three stages of behavior, namely, elastic, transitional compression, and post-yield compression. These three stages have also been observed in cemented sands and clays under isotropic pressure (Table S1). To illustrate, Fig. 6a highlights these as segments AB, BC, and CD for the ISO-CC7.5-CT28 specimen. Segment AB shows the pre-yield compression where CPB exhibits elastic behavior up to the yield stress (σ_{YS}), which can be defined by the bulk modulus of the material (shown as PrYCS). Beyond the yield stress, cement bond breakage progresses, and the CPB response deviates from linear elastic behavior. Once bond breakage is started, the impact on the volumetric response is considerable, causing a concave response labelled as TC_S (segment BC). Finally, in segment CD, as the isotropic pressure increases, more cement bonds break and the volumetric response of CPB relies more on the movement and rearrangement of silt-size crushed rock and broken hydration products. This is called the post-yield compression segment and is labelled as PYCS. Figure 6b shows the same interpretation as above except that the elastic volumetric strain corresponding to yield stress (ε_{YS}^p) is subtracted from the data; this alternative presentation prevents visual clutter when curves from multiple curing times are superimposed (Fig. 7). Note the subsequent change of coordinates of points B, C, D, and particularly PISO which will be used in presenting the proposed equations in the following sections.

Both UCS and isotropic test results show that CPB behaves linearly in the elastic domain and therefore, this segment of isotropic compression response can be simply predicted by elastic properties of CPB. However, predicting CPB behavior after the yield stress is not as straightforward. Examining the results for all cement content and curing time combinations suggests that the nonlinear transitional segment (segment BC in Fig. 6a) can be captured by a third-degree
This set of equations must result in a continuous and differentiable function over the three segments mentioned which is preferable in any constitutive equations to prevent issues related to singular corner regions. To fulfill these conditions, the value and derivative (slope) at B (yield stress) and C (end of the transitional compression segment) should be equal; so:

\[
\begin{align*}
    a_1 \cdot \log (p')^3 + a_2 \cdot \log (p')^2 + a_3 \cdot \log (p') + a_4 &= \epsilon_{\text{Tot}}^{v'} (\sigma_{\text{YS}}^{\text{ISO}} \leq p' \leq \sigma_C) \\
    PYCLS \cdot \log(p') + m &= \epsilon_{\text{Tot}}^{v'} (\sigma_C \leq p')
\end{align*}
\]

where \( \sigma_{\text{YS}}^{\text{ISO}} \): yield stress in isotropic compression test, \( \sigma_C \): stress at the end of transitional compression segment, \( \epsilon_{\text{YS}}^{v'} \): total volumetric strain at yield stress, \( \epsilon_{\text{Tot}}^{v'} \): total volumetric strain at the end of transitional compression segment, \( m \): intercept of post-yield compression line in \( \epsilon_{\text{Tot}}^{v'} - \log(p') \) space.

Solving the above system of equations results in the following \( a_i \) coefficients:

\[
\begin{align*}
    a_1 \cdot \log (\sigma_{\text{YS}}^{\text{SO}})^3 + a_2 \cdot \log (\sigma_{\text{YS}}^{\text{SO}})^2 + a_3 \cdot \log (\sigma_{\text{YS}}^{\text{SO}}) + a_4 &= \epsilon_{\text{YS}}^{v'} \\
    a_1 \cdot \log(\sigma_C)^3 + a_2 \cdot \log(\sigma_C)^2 + a_3 \cdot \log(\sigma_C) + a_4 &= \epsilon_C^{v'} \\
    PYCLS \cdot \log(\sigma_C) + m &= \epsilon_C^{v'} \\
    3a_1 \cdot \log (\sigma_{\text{YS}}^{\text{SO}})^2 + 2a_2 \cdot \log (\sigma_{\text{YS}}^{\text{SO}}) + a_3 &= S_{\text{YS}} \\
    3a_1 \cdot \log(\sigma_C)^2 + 2a_2 \cdot \log(\sigma_C) + a_3 &= PYCLS
\end{align*}
\]

\[
a_1 = -\left(2m - 2\epsilon_{\text{YS}}^{v'} + PYCLS \cdot \log(\sigma_C) + PYCLS \cdot \log (\sigma_{\text{YS}}^{\text{SO}}) - S_{\text{YS}} \cdot \log(\sigma_C) + S_{\text{YS}} \cdot \log (\sigma_{\text{YS}}^{\text{SO}}) / (\log(\sigma_C) - \log (\sigma_{\text{YS}}^{\text{SO}}))^3 \right)
\]
\[ a_2 = (3m \cdot \log(\sigma_C) - 3\log(\sigma_{YS}^{ISO}) \cdot \epsilon_{YS}^v - 3\log(\sigma_C) \cdot \epsilon_{YS}^v + 3m \cdot \log(\sigma_{YS}^{ISO})) \\
+ 2PYCLS \cdot \log(\sigma_C)^2 + 2PYCLS \cdot \log(\sigma_{YS}^{ISO})^2 - 2S_{YS} \cdot \log(\sigma_C)^2 \\
+S_{YS} \cdot \log(\sigma_{YS}^{ISO})^2 + 2PYCLS \cdot \log(\sigma_C) \cdot \log(\sigma_{YS}^{ISO}) + S_{YS} \cdot \log(\sigma_C) \cdot \log(\sigma_{YS}^{ISO})) \\
/ \left( \log(\sigma_C) - \log(\sigma_{YS}^{ISO}) \right)^3 \]

**Fig. 7** Prediction of \( \varepsilon_{Tot}^v - \varepsilon_{YS}^v \) variation versus \( \log(p') \) using coefficients calculated by Eq. 1 for: (a) 5.3%; (b) 7.5%; and (c) 11.1% cement content specimens under different curing times.
Solid lines in Fig. 7 show the predictions by Eq. 1 for CPBs with the tested cement contents and curing times, and both the transitional and post-yield compression segments are predicted reasonably well. It should be noted that no prediction of isotropic compression behavior is made for specimens with 3.0% cement content and 5.3% cement content at one-day curing time due to lack of at least three data points before the yield stress.

It is worth mentioning that the reported isotropic test results for varieties of cemented soils (Table S1) and other cohesive porous materials such as concrete and rock (Fossum and Brannon 2004; Warren et al. 2004; Foster et al. 2005) suggest similar three-stage behavior. Therefore, the proposed equation is not necessarily limited to CPB and may be considered generally applicable to other cohesive porous materials.

5 Estimating Parameters for Isotropic Compression Behavior

An interesting feature of Eq. 1 is that the coefficients \( a_i \) are functions of parameters with identifiable physical meaning (strain and stress at the yield and end of the transitional compression segment, slope at the yield point, intercept and slope of the post-yield compression line) and not solely determined by curve fitting/regression. Furthermore, it is possible to establish empirical relationships to predict these parameters based on cement content and curing time, and therefore UCS. This approach is particularly useful because compared to isotropic compression tests under high confining pressure, the UCS test is simpler, cheaper, and is widely used in many geological and mining projects. In addition, the physical
meaning behind the parameters used to calculate the $a_i$ coefficients makes it possible to study and potentially quantify the effect of hydration on the isotropic compressive response of CPB under different combinations of cement content and curing time. This is examined in the following sections.

5.1 5.1. Estimating $\sigma_{ISO}^{YS}$, $\sigma_C$, and $m$ Using UCS Test Results

Figure 8 shows that isotropic yield stress $\sigma_{ISO}^{YS}$ can be predicted as a linear function of UCS yield stress $\sigma_{UCS}^{YS}$ for all combinations of cement content and curing time. The pressure at which the volumetric strain of CPB deviates from linear behavior is on average 7% higher than that of the UCS yield stress. The isotropic pressure at the end of transitional compression segment ($\sigma_C$) was determined for each test. Each $\sigma_C$ is plotted against the corresponding average UCS in Fig. 9. This figure shows that it is reasonable to assume a linear relationship between the value of UCS and pressure at the end of transitional compression segment. This means that $\sigma_C$ can also be predicted based on UCS test results for the range of cement content and curing time studied here. It is worth mentioned that same linear relationship between yield stress in one dimensional test and UCS has been reported by Horpibulsuk et al. (2004) for cemented clay as well.

The intercept of post-yield compression line ($m$, see Fig. 6a) is another parameter required to predict behavior of CPB under isotropic pressure. Rearranging the post-yield compression line equation, the intersection of this line with the effective isotropic pressure axis can be defined as $P_{ISO}$ which is equal to $10^{-m/PYCLS}$ (Fig. 6a). This means that by having $P_{ISO}$ and $PYCLS$ the value of $m$ can be predicted. Figure 10 shows that $P_{ISO}$ also varies linearly with the corresponding average UCS for each combination of cement content and curing time.
5.2 Estimating $S_{YS}$, $\varepsilon_{YS}^v$, and $\varepsilon_{C}^v$

The isotropic compressive behavior represented by the pre-yield compression section is elastic and controlled by the cement bonds, therefore the bulk modulus can be used to determine the slope at yield stress ($S_{YS}$). The bulk modulus can be calculated either directly from isotropic compression tests or indirectly from other elastic properties considering linear elastic behavior observed for CPB. Figure 11 presents bulk modulus calculated from isotropic compression tests conducted in this study. As expected, by the increase of cement content and curing time the bulk modulus increases as well. Also, considering linear elastic behavior of CPB in the pre-yield compression section, using bulk modulus and yield stress values, the value of volumetric strain at yield stress ($\varepsilon_{YS}^v$) can be calculated.

Figure 12 presents total volumetric strain at the end of the transitional compression segment ($\varepsilon_{C}^v$) for different cement contents and curing times. Examination of the data shows that the value of $\varepsilon_{C}^v$ is not affected by the studied range of cement content and curing time (which, as mentioned, are representative of current engineering practice) and can be taken as independent from cement content and curing time. Interestingly, the $a_l$ coefficients were found to be independent of $\varepsilon_{C}^v$.

5.3 Estimating Post-Yield Compression Line Slope ($PYCLS$)

Jafari et al. (2020b) studied the slope of the post-yield compression line ($PYCLS$) in the normalized void ratio-isotropic compression stress space. They noted that this slope increases with cement content and curing time, and proposed the following equation:
in which CT and CC are curing time (days) and cement content (percentage) of the specimen, respectively. The prediction of Eq. 3 for the volumetric strain of CPB is presented in Fig. 13 with the values of coefficients $B_i$ noted in Fig. 13a. Alternatively, there is the possibility of predicting $PYCLS$ using (logarithm of) UCS since the value of $PYCLS$ increases with curing time and cement content. This is shown in Fig. 14.

Having established relationships between the equation’s parameters and UCS based on cement content and curing time, it is now possible to interpolate to intermediate cement content and curing time combinations, determine the corresponding UCS, and therefore the expected isotropic compression response.

6 Conclusions

A series of uniaxial compression tests and isotropic compression tests were utilized to study the impact of different combinations of cement content and curing time on CPB’s isotropic compression behavior. The test results provide a better understanding of the effect of cementation on the isotropic compression response of this cemented silt-size crushed rock. Based on the test results a straightforward equation is proposed that can predict the isotropic compression response of CPB based on parameters with physical meaning — rather than solely curve-fitting — in the isotropic stress-volumetric strain space. In addition, other key experimental findings specific to the studied CPB are:

- UCS of this CPB with different cement content and curing time can be predicted using an exponential fitting function based on a $CT^B(CC$ factor (Fig. 3).
- The isotropic behavior of CPB can be predicted using three segments: the pre-yield compression, transitional compression, and the post-yield compression.
- In both uniaxial and isotropic compression tests CPB shows linear stress–strain behavior before reaching a yield stress. The yield stress in isotropic compression is on average 7% higher than in uniaxial compression.
- The slope of the pre-yield compression line can be determined using the bulk modulus, which can be predicted using a logarithmic fitting function based on curing time, with coefficients calibrated to the cement content (refer to Fig. 11).
- The slope of the post-yield compression line can be predicted using a combined logarithmic-exponential fitting function based on cement content and curing time (Fig. 13). Also, there is a logarithmic relationship between this slope and the corresponding UCS (Fig. 14).

The findings in this paper provide a framework for predicting the isotropic compression behavior of the material studied, within the stated ranges of test parameters. The calibrated fitting functions are specific to the material tested, but the testing procedures and analysis framework can be used as a basis to determine the isotropic compression behavior of similar backfill materials for other mines. Also, to better mimic the field performance of CPB, the specimens can be prepared under the loading conditions that are representative of effective stress paths recorded in field monitoring programs during mine backfilling.

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Author Contributions MJ developed the experiment strategy, conducted the experiment, interpreted the data, and wrote the manuscript. MG was edited the manuscript. The senior responsible author (SRA) of the publication is MG.

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Data Availability All data, models, and code generated or used during the study appear in the submitted article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.
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