Mesonic Superfluidity in Isospin Matter under Rotation

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We investigate the mesonic superfluidity in isospin matter under rotation. Using the two-flavor NJL effective model under the presence of global rotation, we demonstrate two important effects of the rotation on its phase structure: a rotational suppression of the scalar-channel condensates, in particular the pion superfluidity region; and a rotational enhancement of the rho superfluidity region with vector-channel condensate. A new phase diagram for isospin matter under rotation is mapped out on the $\omega - \mu_I$ plane where three distinctive phases, corresponding to $\sigma$, $\pi$, $\rho$ dominated regions respectively, are separated by a second-order line at low isospin chemical potential and a first-order line at high rotation which are further connected at a tri-critical point.

INTRODUCTION

Recently there have been rapidly growing interests in understanding the properties and phase structures of matter under global rotation. Examples of such physical systems come from a variety of different areas, such as the hot quark-gluon plasma in peripheral heavy ion collisions [1–9], dense nuclear matter in rapidly spinning neutron stars [10–12], lattice gauge theory in rotating frame [13], cold atomic gases [14–16] as well as certain condensed matter materials [17, 18].

Rotation provides an interesting new type of macroscopic control parameter, in addition to conventional ones such as temperature and density, for a many-body system. In particular it has nontrivial interplay with microscopic spin degrees of freedom through the rotational polarization effect and could often induce novel phenomena. For example, there are highly nontrivial anomalous transport effects such as the chiral vortical effect and chiral vortical wave in rotating fluid with chiral fermions [19–25]. Furthermore, if the underlying materials contain fermions that may form condensate via pairing, their phase structure can be significantly influenced by the presence of global rotation [26–37]. A generic effect is the rotational suppression of fermion pairing in the zero angular momentum states, which has been demonstrated for e.g. chiral phase transition and color superconductivity in the strong interaction system [26]. Given the suppression of scalar pairing states in these systems, it is natural to wonder what may happen to pairing states with nonzero angular momentum e.g. spin-1 condensate of fermionic pairs. In general, one would expect them to be enhanced by rotation which prefers states with finite angular momentum and tends to polarize the angular momentum along the rotational axis. It is of great interest to examine this in concrete physical systems.

In this Letter, we perform the first analysis on the influence of rotation on the phase structure of isospin matter — the Quantum Chromodynamics (QCD) matter at finite isospin density (or equivalently chemical potential) which implies an imbalance between the u-flavor and d-flavor of quarks in the system [38, 39]. Such isospin matter is relevant for understanding the properties of neutron star materials which have a significant mismatch between the number of neutrons and protons and thus also between u-quarks and d-quarks. Also the dense matter created in low energy heavy ion collisions bears significant isospin density arising from stopping of initial beam nuclei. It is also possible to simulate such isospin matter with two-component cold fermionic gases.

One particularly interesting phenomenon in isospin matter is the pion superfluidity [40–45]: while at low isospin density the system contains a chiral $\sigma$ condensate (from quark-anti-quark pairing in the scalar channel), it changes into a pion condensate (from quark-anti-quark pairing in the pseudo-scalar channel) at high isospin density. Since both are pairing states zero angular momentum, one would expect a rotational suppression effect on both. Furthermore, rotation may induce condensation in other mesonic channels arising from quark-anti-quark pairing in non-zero angular momentum states such as the $\rho$-channel. We will perform the first systematic study of all these possible mesonic superfluidity pairing states simultaneously in the isospin matter under global rotation. We will show that there are indeed suppression of scalar pairing and enhancement of vector pairing due to fluid rotation, with the emergence of rho superfluidity phase at high isospin density under rapid rotation. Such analysis will further allow us to envision and map out a new phase diagram on the rotation-isospin parameter plane with highly nontrivial phase structures.

FORMALISM

To investigate the mesonic superfluidity in isospin matter, we will adopt a widely-used effective model, namely the two-flavor Nambu-Jona-Lasinio (NJL) model with
four-fermion interactions in various channels at finite isospin chemical potential $\mu_I$:  

$$ L = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0 + \frac{\mu_I}{2} \gamma_0 \tau_3)\psi + L_4^I + L_5^I, $$  

$$ L_4^I = G_s \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5 \tau \psi)^2 \right], $$  

$$ L_5^I = -G_v (\bar{\psi}\gamma_\mu \tau \psi)^2. $$  

In the above, the $m_0 = 5$ MeV is the light quark mass parameter while $G_s = G_v = 5.03$ GeV$^{-2}$ are the scalar and vector channel coupling constants respectively. The NJL-type effective model also requires a momentum cut-off parameter $\Lambda = 650$ MeV. These choices are quite standard, leading to the correct pion mass and decay constant in the vacuum as well as a vacuum expectation value (VEV) of $\sigma$ field to be $\sigma_0 = 2 \times (250 \text{MeV})^3$.

In the most general case, we consider three possible mesonic superfluidity scenarios: condensation of $\sigma$, $\pi$ or $\rho$ fields respectively. Following the standard mean-field method, we introduce the corresponding condensates:

$$ \sigma = \langle \bar{\psi}\psi \rangle, \quad \pi = \langle \bar{\psi}i\gamma_5 \tau \psi \rangle, \quad \rho = \langle \bar{\psi}i\gamma_0 \tau_3 \psi \rangle. $$

Furthermore, we are considering such a system under global rotation around $\hat{z}$-axis with angular velocity $\bar{\omega} = \omega \hat{z}$. To do this, one can study the system in the rotating frame and rewrite the spinor theory with the curved metric associated with the rotating frame [26]. In such a description, the main new effect is a global polarization term in the Lagrangian density:

$$ L_R = \bar{\psi}^{\dagger} \left[ (\bar{\omega} \times \bar{x}) \cdot (-i\bar{\partial}) + \omega \cdot \vec{S}_{4\times4} \right] \psi $$

where $\vec{S}_{4\times4} = \frac{1}{2} \text{Diag} (\bar{\sigma}, \bar{\sigma})$ is the spin operator with $\vec{\sigma}$ the $2 \times 2$ Pauli matrices. Physically, this term polarizes both the orbital and spin angular momenta to be aligned with global rotation axis and its effect is identical for particles or antiparticles.

Putting all these together, one obtains the following thermodynamic potential for isospin matter under rotation following standard thermal field theory calculations:

$$ \Omega = G_s (\sigma^2 + \pi^2) - G_v \rho^2 - \frac{N_c N_f}{16\pi^2} \sum_n \int d^3k \int dk_z [J_{n+1}(kr)^2 + J_n(kr)^2] $$

$$ \times T \left[ \ln \left(1 + \exp\left(\frac{\omega^+ - (n + \frac{1}{2})\omega}{T}\right)\right) + \ln \left(1 + \exp\left(\frac{\omega^+ - (n + \frac{3}{2})\omega}{T}\right)\right) + \ln \left(1 + \exp\left(\frac{-\omega^+ - (n + \frac{1}{2})\omega}{T}\right)\right) + \ln \left(1 + \exp\left(\frac{-\omega^+ - (n + \frac{3}{2})\omega}{T}\right)\right) \right] $$

with quasiparticle dispersion relations given by

$$ \omega^\pm = \sqrt{4G_s^2 \pi^2 + (\sqrt{(m_0 - 2G_s \sigma)^2 + k^2 + k_z^2} \pm \tilde{\mu}_I)^2} $$

where $\tilde{\mu}_I = \frac{\mu_I}{T} + G_v \rho$. At given temperature $T$ and isospin chemical potential $\mu_I$, one then determines the mean-field condensates by solving the gap equations:

$$ \frac{\partial \Omega}{\partial \sigma} = \frac{\partial \Omega}{\partial \pi} = \frac{\partial \Omega}{\partial \rho} = 0. $$

These equations can be numerically solved. For situations with multiple solutions, the true physical state should be determined from the absolute minimum of the thermodynamic potential. Note also under rotation, the system is no longer homogeneous with thermodynamic quantities varying with radial coordinate $r$. For the numerical results to be presented later, we use a value $r = 0.1$ GeV$^{-1}$. This is a rather modest value that ensures $\omega r \ll 1$ in all our calculations and that renders finite boundary effect to be negligible. The qualitative features of our findings do not depend on this particular choice and generally the rotational effect increases with larger value of $r$. For more discussions see e.g. [27].

**ROTATIONAL SUPPRESSION OF PION SUPERFLUIDITY**

We first demonstrate the rotational suppression of scalar pairing channels, i.e. mesonic condensates arising from quark-anti-quark pairing in the zero total angular momentum $J = 0$ states. While such rotational suppression was previously proposed as a generic phenomenon in fermion pairing transitions and demonstrated for e.g. chiral condensate or color superconductivity [26], it has never been examined for the mesonic superfluidity in isospin matter. In our case, the scalar pairing channels include the condensates of both $\sigma$ (scalar) and $\pi$ (pseudo-scalar) fields. To show this effect clearly, we will temporarily “turn off” the vector channel in the present section by putting $G_v$ and $\rho$ to zero in Eq.(6).

In the case without rotation, this system has been very well studied. At low isospin density the system is vacuum-like with only a nonzero $\sigma$ condensate which would decrease with increasing density. At certain high enough isospin density, the $\pi$ condensate starts to form via a second-order phase transition — a phenomenon called pion superfluidity [40–44]. Here we focus on the influence of the rotation on this phenomenon.

In Fig. 1, we show the sigma and pion condensates $\sigma$ and $\pi$ (scaled by $\sigma_0$) as a function of $\omega$ at $T = 20$ MeV (upper) and $T = 100$ MeV (lower) for several different values of $\mu_I$. As one can see, there is indeed a generic suppression on both the $\sigma$ and $\pi$ condensates, due the fact that the presence of global rotation always “prefers” states with nonzero angular momentum and thus disfavors these $J = 0$ mesonic pairing channels. What’s most interesting is the case at high isospin density, where the system is in a pion superfluid phase with nonzero pion
condensate without rotation. But with increasing rotation, this condensate eventually approaches zero via either a first-order (at low T) or second-order (at high T) transition. Thus the spontaneously broken isospin symmetry in the pion superfluidity phase can be restored again under rapid rotation, which is a new effect.

To see the influence of rotation on the phase structure, one can compare compare the $T - \mu_I$ phase diagram of isospin matter with and without rotation. As shown in Fig. 2, the region of pion superfluidity phase is significantly reduced by the rotation. In particular due to the rotation, a new first-order transition line emerges at high-$\mu_I$ side which connects to the second-order line at low-$\mu_I$ side via a new tri-critical point (TCP).

**ENHANCED RHO SUPERFLUIDITY UNDER ROTATION**

Suppression of the scalar pairing implies opportunity for enhanced pairing of states with nonzero angular momentum, such as the $\rho$ condensate. Indeed, the $\rho$ state has $J = 1$ and should be favored by the presence of global rotation. While the emergence of $\rho$ condensate at high isospin density has been previously studied [45], the interplay between the rho condensate and rotation and the implication for phase structure of isospin matter is discussed for the first time here. To do this, we now consider the full thermodynamic potential in Eq.(6) and consistently solve the coupled gap equations of all three possible condensates in Eq.(8). In Fig.3, we compare the results for the sigma, pi and rho condensates $\sigma$, $\pi$, $\rho$ (scaled by the vacuum chiral condensate $\sigma_0$) as a function of isospin chemical potential, for $\omega = 0$ (upper), $\omega = 500\text{MeV}$ (middle) and $\omega = 600\text{MeV}$, respectively.

In the case without rotation (Fig.3 upper panel), the chiral condensate decreases with increasing $\mu_I$ while both pion and rho condensates start to grow for $\mu_I$ greater than the critical value at about 140MeV for a second order phase transition. The pion condensate dominates the system at large isospin chemical potential.

In the case with strong rotation, $\omega = 500\text{MeV}$ (Fig.3 middle panel), the situation becomes different. Both pion and rho condensates still start to grow for $\mu_I$ greater than the critical value. But at even higher isospin density, a new first-order transition occurs and the pion condensate drops to zero. In this new region, the rho condensate becomes dominant.

For even stronger rotation, $\omega = 600\text{MeV}$ (Fig.3 lower panel), the pion condensate disappears all together. With increasing isospin chemical potential $\mu_I$, there is a smooth crossover from a $\sigma$-dominated phase at low isospin density to a $\rho$-dominated phase at very high isospin density.

These results clearly demonstrate the influence of rotation on the mesonic superfluidity in isospin matter and envision a new phase diagram on the $\omega - \mu_I$ plane, as shown in Fig. 4. This new phase structure is characterized by three distinctive regions: a vacuum-like, sigma-dominated phase in the low isospin density and slow rotation region; a pion-superfluidity phase in the midto-high isospin density with moderate rotation; and a rho-superfluidity phase in the high isospin and rapid rotation region. A second-order transition line separates...
FIG. 3: (color online) The sigma, pi and rho condensates \( \sigma, \pi, \rho \) (scaled by the vacuum chiral condensate \( \sigma_0 \)) as a function of isospin chemical potential, for \( \omega = 0 \) (upper), \( \omega = 500 \text{MeV} \) (middle) and \( \omega = 600 \text{MeV} \), respectively.

The novel phase structure found here may be useful and relevant for understanding properties and phenomena in isospin-asymmetric nuclear matter in various physical systems. For peripheral heavy ion collisions at relatively low beam energies, such as experiments at the RHIC Beam Energy Scan or at the future FAIR and NICA facilities, the created matter has significant rotation as well as high isospin density due to stopping effect and the asymmetry between protons and neutrons in the initial nuclei. Further more, by including finite

FIG. 4: (color online) A new phase diagram on the \( \omega - \mu_I \) plane for mesonic superfluidity in isospin matter under rotation. Solid line stands for first-order phase transition and dashed line for second-order transition, while dotted line for crossover, with the star symbol denoting a tri-critical point (TCP) at \( (\mu_I = 165 \text{MeV}, \omega^c = 548 \text{MeV}) \).

de dense. A new phase diagram for isospin matter under rotation has been mapped out on the \( \omega - \mu_I \) plane where three distinctive phases, corresponding to \( \sigma, \pi, \rho \) dominated regions respectively, are separated by a second-order line at low isospin chemical potential and a first-order line at high rotation which are joint by a tri-critical point. While quantitatively the locations and nature of these phase boundaries may depend on model details, we expect such a three-region structure to be generic.

In the present study, we have not considered the finite size effect on the phase structure [27]. The finite size correction may further reduce the region of pion superfluidity, but the qualitative feature of the phase structure is expected to remain. Our calculation of the thermodynamic potential with rotation is under mean field approximation, and the inclusion of fluctuations beyond such approximation may change the precise locations of the tri-critical point or phase transition lines. But the influence of rotation on the mesonic superfluidity, revealed in this Letter, shall remain the same.

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baryon chemical potential at low temperature, one could study the phase structure of asymmetric dense cold nuclear matter with rotation, which is useful for study the structure and properties of rotating compact star. A detailed investigation of these systems will be carried out in the future and reported elsewhere.

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