Contact processes with competitive dynamics in bipartite lattices: effects of distinct interactions

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Abstract. The two-dimensional contact process (CP) with a competitive dynamics proposed by Martins et al (2011 Phys. Rev. E 84 011125) leads to the appearance of an unusual active-asymmetric phase, in which the system sublattices are unequally populated. It differs from the usual CP only by the fact that particles also interact with their next-nearest neighbor sites via a distinct strength creation rate, and for the inclusion of an inhibition effect, proportional to the local density. Aimed at investigating the robustness of such an asymmetric phase, in this paper we study the influence of distinct interactions for two bidimensional CPs. In the first model, the interaction between first neighbors requires a minimal neighborhood of adjacent particles for creating new offspring, whereas second neighbors interact as usual (e.g. at least one neighboring particle is required). The second model takes the opposite situation, in which the restrictive dynamics is in the interaction between next-nearest neighbor sites. Both models are investigated under mean field theory (MFT) and Monte Carlo simulations. In similarity with results by Martins et al, the inclusion of distinct sublattice interactions maintains the occurrence of an asymmetric active phase and re-entrant transition lines. In contrast, remarkable differences are presented, such as discontinuous phase transitions (even between the active phases), the appearance of tricritical points and the stabilization of active phases under larger values of control parameters. Finally, we have shown that the critical behaviors are not altered due to the change of interactions, in which the absorbing transitions...
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belong to the directed percolation (DP) universality class, whereas second-order active phase transitions belong to the Ising universality class.

**Keywords:** phase transitions into absorbing states (theory)

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### 1. Introduction

Nonequilibrium phase transitions into absorbing states have attracted considerable interest, not only for the description of several problems such as wetting phenomena, spreading of diseases, and chemical reactions [1, 2], but also for the search for experimental verifications [3]. In the most common cases, phase transitions are second-order and belong to the directed universality (DP) class [1]. However, the inclusion of distinct dynamics (such as diffusion, disorder, laws of conservation, noise and others) not only may drastically change the phase transition and critical behavior [2, 4], but also may exhibit new features such as Griffiths phases [5], formation of stable patterns [6], phase coexistence [7] and others [8]. Recently, Martins *et al.* [9] have introduced a two-dimensional contact process (CP) [10] with sublattice symmetry breaking, in which the dynamics is ruled by the competition between particle creation at nearest and next-nearest neighbor occupied sites, and the annihilation also depends on the local particle density. Particles interact with their first and second neighbors by means of a similar interaction rule, but the strengths of creation rates are different. In addition to the usual absorbing and active (symmetric) phases, mean field theory (MFT) and Monte Carlo (MC) analysis predict the appearance of an unusual active-asymmetric phase, in which, in contrast to the symmetric phase, the distinct sublattices are unequally populated. A phase transition between the symmetric and asymmetric phases is characterized by a spontaneous symmetry breaking. All absorbing phase transitions belong to the directed percolation (DP) [2] class, whereas the transitions between active phases belong to the Ising universality class. Inspired by recent studies [7, 11], in which the particle creation requiring a minimal neighborhood of occupied sites (instead of one particle as in the original CP) leads to the appearance of a discontinuous absorbing phase transition, here we take a further step in the work.
by Martins et al by including such a class of restrictive dynamics in order to raise three remarkable questions, as follows. First, does the competition between distinct sublattice interactions (instead of only distinct creation rates) change the topology of the phase diagram? Second, is the asymmetric phase maintained by changing the interaction rules? Third, are the classifications of phase transitions altered? To answer these questions, we analyze two distinct models taking into account a minimum neighborhood of adjacent particles. Models are analyzed via mean-field approximation and numerical simulations. Results have shown the asymmetric phase ‘survives’ by the change of interactions but pronounced changes in the phase diagram are found, such as discontinuous absorbing transitions, discontinuous transitions with spontaneous breaking symmetry (instead of continuous transitions, as typically observed), the appearance of tricritical points, critical end point and the extension of phases under larger values of control parameters.

This paper is organized as follows. In section 2 we describe the studied models and we show results under mean-field analysis. In section 3 we show numerical results and we compare them with those obtained in section 2. Conclusions are given in section 4.

2. Models

Let us consider a system of interacting particles placed on a square lattice of linear size \( L \) in which each site is empty or occupied by a particle. The dynamics is described as follows: particles in a given sublattice \( i \) (A or B) are created in empty sites with first- and second-neighbor transition rates \( \lambda_1 n_{1i}/q \) and \( \lambda_2 n_{2i}/q \) respectively, \( \lambda_1 \) and \( \lambda_2 \) being the strength of creation parameters, \( n_{1i} \) and \( n_{2i} \) denoting the number of particles in the first and second neighbors of the site \( i \) respectively, and \( q \) is the coordination number (reading 4 for a square lattice). In model 1, the interaction between first neighbors is taken into account only if \( n_{1i} \geq 2 \), in such a way that no contribution to the particle creation due to nearest-neighbor occupied sites occurs if \( n_{1i} \leq 1 \). In contrast, the interaction between second neighbors takes into account \( n_{2i} \geq 1 \) particles, as in the usual CP. Model 2 is the opposite case, in which the transition rate between first neighbors requires \( n_{1i} \geq 1 \) adjacent particles, but the interaction between second neighbors contributes only if \( n_{2i} \geq 2 \). In order to favor unequal sublattice populations, a term increasing with the number of nearest-neighbor particles, in the form \( \mu n_{1i}^2 \), is included in the annihilation rate [9]. If \( \mu = 0 \), one recovers the usual case in which a particle is spontaneously annihilated with rate 1.

To characterize the phase transitions, the sublattice particle densities \( \rho_i \) (\( i = A \) and B) are important quantities to measure. In the absorbing state both sublattices are empty, implying that \( \rho_A = \rho_B = 0 \). On the other hand, in an active-symmetric (as) phase \( \rho_A = \rho_B \neq 0 \), whereas in an active-asymmetric (aa) phase \( \rho_A \neq \rho_B \) and \( \rho = \frac{1}{2}(\rho_A + \rho_B) \neq 0 \). Hence, in contrast to the as phase, in the aa phase the sublattices are unequally populated and the phase transition is not ruled by the global density \( \rho \), but for the difference of sublattice densities given by \( \phi = \frac{1}{2}(\rho_A - \rho_B) \). Unlike the as phase, in which \( \phi = 0 \), in the aa phase it follows that \( \phi \neq 0 \).

2.1. Transition rates and mean-field analysis

From the above model definitions, we can write down the time evolution of sublattice densities \( \rho_A \) and \( \rho_B \), which correspond to one-site probabilities. Let the symbols • and □
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denote occupied sites belonging to the sublattices A and B, respectively. From the previous
dynamic rules, it follows that the time evolutions of $\rho_A$ and $\rho_B$ are given by

\[
\frac{d\rho_A}{dt} = \lambda_1 [2P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet) + 3P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet)] + \lambda_2 P(\bullet) - [1 + q^2 \mu P(\bullet)^2] P(\bullet),
\]

(1)

\[
\frac{d\rho_B}{dt} = \lambda_1 [2P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet) + 3P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet)] + \lambda_2 P(\bullet\bullet\bullet\bullet) - [1 + q^2 \mu P(\bullet)^2] P(\bullet).
\]

(2)

for the model 1 and

\[
\frac{d\rho_A}{dt} = \lambda_1 P(\bullet\bullet\bullet\bullet) + \lambda_2 [2P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet) + 3P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet)] - (1 + q^2 \mu P(\bullet)^2) P(\bullet)
\]

(3)

\[
\frac{d\rho_B}{dt} = \lambda_1 P(\bullet\bullet\bullet\bullet) + \lambda_2 [2P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet) + 3P(\bullet\bullet\bullet\bullet) + P(\bullet\bullet\bullet\bullet)] + P(\bullet\bullet\bullet\bullet\bullet) - (1 + q^2 \mu P(\bullet)^2) P(\bullet).
\]

(4)

for the model 2, where we are using the shorthand notations $\rho_A = P(\bullet)$ and $\rho_B = P(\bullet\bullet\bullet\bullet)$. Note that from the above model definitions it follows that equations (1) and (2) (model 1) and equations (3) and (4) (model 2) are symmetric under $A \Leftrightarrow B$. In terms of the order parameter $\phi$, the sublattice exchange implies that $\phi \Leftrightarrow -\phi$. Although the symmetric phase remains unchanged under $\rho_A \Leftrightarrow \rho_B$ (since $\phi = 0$), the above symmetry is broken in the asymmetric phase (corresponding to $\phi^* \Leftrightarrow -\phi^*$, where $\phi^*$ is the steady value). Thus a spontaneous symmetry breaking is expected to occur in the emergence of the aa phase.

The first inspection of the phase diagrams can be achieved by performing one-site mean-field analysis. It consists of replacing a given $n$-site probability by a product of $n$ one-site probabilities, in such a way that equations (1) and (2) become

\[
\frac{d\rho_A}{dt} = \lambda_1 \rho_B^2 (1 - \rho_A) [3 - 3 \rho_B + \rho_B^2] + \lambda_2 (1 - \rho_A) \rho_A - (1 + q^2 \mu \rho_B^2) \rho_A
\]

(5)

\[
\frac{d\rho_B}{dt} = \lambda_1 \rho_A^2 (1 - \rho_B) [3 - 3 \rho_A + \rho_A^2] + \lambda_2 (1 - \rho_B) \rho_B - (1 + q^2 \mu \rho_A^2) \rho_B,
\]

(6)

for model 1 and equations (3) and (4) become

\[
\frac{d\rho_A}{dt} = \lambda_1 (1 - \rho_A) \rho_B + \lambda_2 \rho_A^2 (1 - \rho_A) [3 - 3 \rho_A + \rho_A^2] - (1 + q^2 \mu \rho_B^2) \rho_A
\]

(7)

\[
\frac{d\rho_B}{dt} = \lambda_1 (1 - \rho_B) \rho_A + \lambda_2 \rho_B^2 (1 - \rho_B) [3 - 3 \rho_B + \rho_B^2] - (1 + q^2 \mu \rho_A^2) \rho_B
\]

(8)

for model 2. The steady solutions are obtained by taking $d\rho_A/dt = d\rho_B/dt = 0$ in both cases and for a given set of parameters $\lambda_1, \lambda_2$ and $\mu$ we can obtain $\rho_A$ and $\rho_B$ by solving the system of two coupled equations, from which we have built the phase diagrams, as shown in figure 1. From now on, we are going to refer to $\phi$ only in terms of its absolute value, calculated by $\phi = \frac{1}{2} |\rho_A - \rho_B|$.  

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In conformity with results by Martins et al [9], in which the aa phase is not stable for $\mu = 0$, we have considered $\mu = 1$ in all cases. Such a value is lower than considered in [9], in order to exploit the role of distinct sublattice interactions in the aa phase. In particular, for both models the system is constrained in the ab phase for low $\lambda_1$ and $\lambda_2$, whereas for sufficiently large $\lambda_2$ and $\lambda_1$ both $\rho_A$ and $\rho_B$ are close to 1 and the system is in the as phase. The aa phase is located for intermediate values of control parameters and hence the phase diagrams are re-entrant. For model 1 the transition line between the ab and as phases starts at $(\lambda_1, \lambda_2) = (0, 1)$ and ends at $(4.1, 0)$. It is second-order for low $\lambda_1$ but becomes discontinuous by increasing $\lambda_1$ with a tricritical point located at $(0.33, 1)$. In addition, the phase transitions between active phases (the as–aa and aa–as transition lines) are second-order, starting at $(\lambda_1, \lambda_2) = (0, 1)$ and $(0, 20)$, respectively and ending at $(2.85, 7.70)$. In summary, MFT shows that the aa phase is similar to that studied in [9] for model 1 and the inclusion of a distinct nearest-neighbor interaction provokes qualitative changes only in the ab–as transition line.

In contrast, MFT predicts more substantial differences for model 2 than than the above mentioned results, as a result of a distinct interaction between next-nearest neighbors. No symmetric phase is presented for low $\lambda_1$, in such a way that the ab–as and as–aa transition lines (presented in model 1) give rise to the ab–aa coexistence line and meet the ab–as and as–aa lines in a triple point R located at $(0.71, 2.12)$. Also in contrast with above mentioned results the as–aa and aa–as transition lines are first-order for low $\lambda_1$ and become continuous in tricritical points located at $(1.50, 2.35)$ and $(2.43, 18.7)$, respectively, giving rise to corresponding critical lines. Both critical lines meet at $(4.47, 8.76)$. Finally, the aa phase extends for larger values of $\lambda_1$ and $\lambda_2$, but the ab appears for lower values of $\lambda_1$, as a result of non restrictive dynamics in the nearest-neighbor sites. A tricritical point located at $(0.983, 0.693)$ separates the coexistence from the critical ab–as transition lines.
3. Numerical results

Numerical simulations have been performed for square lattices of linear sizes \(L\) (ranging from \(L = 20\) to \(80\)) and periodic boundary conditions. For model 1, the actual MC dynamics is described as follows.

(i) A particle \(i\) is randomly selected from a list of currently \(N\) occupied sites.

(ii) The particle \(i\) (for instance belonging to the sublattice A) is annihilated with probability \(p_a = (1 + \mu n_B^2)/(1 + \mu n_B^2 + \lambda_1 + \lambda_2)\), \((n_B\) being its number of nearest-neighbor particles) and with complementary probability \(p_c = 1 - p_a\) the creation process is selected.

(iii) If the particle creation is performed, with probabilities \(\lambda_1/(\lambda_1 + \lambda_2)\) and \(\lambda_2/(\lambda_1 + \lambda_2)\) the first- and second-neighbor particle interactions will be chosen, respectively.

(iv) If the first (second) neighbor interaction is chosen, one of its first (second) neighbors \(j\) is randomly selected and a particle will be created, provided \(j\) is empty and if at least two (one) of its first (second) neighbors are occupied.

For model 2 the last rule is replaced in such a way that the particle is created in the site \(j\) provided it is empty and at least one (two) of its first (second) neighbors are occupied.

Numerical simulations have been improved by employing the quasi-stationary method \([12]\). Briefly the method consists of storing a list of \(M\) active configurations (typically one stores \(M = 2000\) configurations) and whenever the system falls into the absorbing state a configuration is randomly extracted from the list. The ensemble of stored configurations is continuously updated, where in practice for each MC step a configuration belonging to the list is replaced with probability \(\tilde{p}\) (typically one takes \(\tilde{p} = 0.01\)) by the actual system configuration, provided it is not absorbing.

Numerical simulations exhibit distinct behaviors in the case of continuous and discontinuous transitions and hence distinct analyses are performed to characterize them. In the former case, relevant thermodynamic quantities present algebraic behavior close to the critical point. In particular, the order parameter \(\phi\) and its variance \(\chi = \langle \phi^2 \rangle - \langle \phi \rangle^2\) behaves as \(\phi \sim (\lambda - \lambda_c)^{\beta}\) and \(\chi \sim (\lambda - \lambda_c)^{\gamma}\) respectively where \(\beta\) and \(\gamma\) are associated critical exponents. Besides, at the critical point \(\lambda_c\), \(\phi\) and \(\chi\) also exhibit power-law behaviors when simulated for finite system sizes. According to the finite size scaling theory \([1]\), they behave as \(\phi \sim L^{-\beta/\nu_\perp}\) and \(\chi \sim L^{\gamma/\nu_\perp}\) respectively where \(\nu_\perp\) is the critical exponent associated with the spacial length correlation. For the DP universality class in two dimensions, \(\beta\), \(\nu_\perp\) and \(\beta/\nu_\perp\) read 0.5834(30), 0.7333(75) and 0.796(9) respectively, whereas for the Ising universality class \(\beta\), \(\gamma\) and \(\nu_\perp\) read 1/8, 7/4 and 1 respectively.

For locating the critical points, we study the crossing among ‘cumulants’ curves. In particular, a cumulant appropriate for absorbing transitions (\(\rho\) being the order parameter) is the moment ratio given by \(U_2 = \langle \rho^2 \rangle / \langle \rho \rangle^2\). For DP transitions in two dimensions, it assumes the universal value \(U_{2c} = 1.3257(5)\) at the critical point. In contrast, for the
transition between active phases, a proper quantity to be studied is the fourth-order Binder cumulant [13]

\[ U_4 = 1 - \frac{\langle \phi^4 \rangle}{\langle \phi^2 \rangle^2}, \tag{9} \]

where in the present case \( \phi \) is the difference between sublattice densities, defined previously. The study of the above quantity is understood by recalling that the aa and as phases are similar to the ferromagnetic and paramagnetic ones found in the Ising model, respectively. At the critical point, for systems belonging to the Ising universality class, \( U_4 \) assumes the universal value \( U_{4c} = 0.61069 \ldots \) and hence the crossing among distinct \( L \)s will provide us an estimation of the critical point.

In the case of first-order transitions, the probability order-parameter distribution is an important quantity to characterize them, since in contrast to second-order transitions it presents a bimodal shape at the phase coexistence. Hence, a two peak probability distribution for larger system sizes will be used as the indicator of a phase coexistence.

After presenting the methodology, let us show numerical results and compare them with the MFT ones. In figure 2 we show the phase diagram obtained from numerical simulations for model 1. The topology of the phase diagram is similar to that obtained from MFT, including the existence of absorbing, symmetric and asymmetric phases and the following ab–as, as–aa and aa–as transition lines. Also in similarity to MFT, the ab–as line is continuous for low \( \lambda_1 \) and becomes discontinuous by increasing such a nearest-neighbor creation parameter, whereas the phase transitions between active phases are second-order. However, some differences from the MFT are observed. In particular, in similarity with the results by Martins et al in [9], the aa phase is placed for lower values...
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Figure 3. Moment ratio $U_2$ versus $\lambda_2$ for distinct $L$s and $\lambda_1 = 0.01$.

Figure 4. On the left, log–log plot of $\rho$ versus $L$ for $\lambda_1 = 0.01$ and $\lambda_{2c} = 1.6515$. On the right, log–log plot of $\rho$ versus $y = \lambda_2 - \lambda_{2c}$ for $\lambda_1 = 0.01$ and $L = 80$. The left and right curves have slopes $\beta/\nu_\perp = 0.796(9)$ and $\beta = 0.5834(30)$ respectively.

of control parameters than those obtained from the MFT. In contrast, the ab–as transition line extends for relatively larger $\lambda_1$s.

After describing the main features of the phase diagram, let us show some explicit results for distinct points of the phase diagram. Starting from the ab–as transition line, in figure 3 we plot the moment ratio $U_2$ for distinct system sizes and $\lambda_1 = 0.01$. Note that all curves cross at $\lambda_{2c} = 1.6515(5)$ with $U_2 = 1.34(2)$, which is close to the universal DP value $1.3257(5)$. In fact, as shown in figure 4, for $\lambda_{2c} = 1.6515$ we find the critical exponents $\beta/\nu_\perp = 0.794(2)$ and $\beta = 0.584(2)$, which are compatible with the DP values (solid lines). Results for other critical points (not shown) confirm that the second-order transitions between the ab and as phases belong to the DP universality class.

In figure 5 we show results for $\lambda_1 = 18$. For $L = 80$ and $\lambda_2 = 0.3618$, the probability distribution $P_\rho$ presents two equal peaks at distinct densities ($\rho = 0.0002$ and 0.501) and
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Figure 5. For model 1, the quasi-stationary probability distribution $P_\rho$ for $\lambda_1 = 18$, $\lambda_2 = 0.3618$ and $L = 80$. In the inset, $P_\phi$ for the difference of sublattice densities $\phi$.

Figure 6. Reduced fourth-order cumulant $U_4$ versus $\lambda_2$ for distinct $L$s and $\lambda_1 = 0.05$. In the inset, we plot the order parameter (continuous lines) and the system density $\rho$ (dotted) versus $\lambda_2$ for $L = 80$.

together with a single peak centered at $\phi \sim 0$ for $P_\phi$, the phase transition between the ab and as phases for $\lambda_1 = 18$ is first-order.

Next, we study the phase transition between active phases, whose results are exemplified for $\lambda_1 = 0.05$ and shown in figure 6. Note that for $2.5 < \lambda_2 < 5.0$, $\phi$ has a sharp increase followed by a less pronounced change of $\rho \neq 0$ (inset of figure 6), signaling the emergence of the as–aa phase transition. Results for $U_4$ show that all curves (for distinct system sizes) cross at $\lambda_{2c} = 4.55(5)$ with $U_4 = 0.605(5)$, which is very close to the universal value $U_4 = 0.61069$ and highlights that such second-order phase transition belongs to the Ising universality class [9]. By increasing further $\lambda_2$, $\phi$ reaches a maximum and starts decreasing until vanishing. Such sharp behavior, accompanied by a smooth variation of $\rho$, are consistent with the aa–as phase transition. We see that all cumulant
curves cross at $\lambda_{2c} = 11.36(5)$ with $U_{4c} = 0.60(1)$, which is also consistent with the Ising value. Note that in the aa phase $U_4 \rightarrow 2/3$ by increasing the system $L$, signaling that the spontaneous symmetry breaking is similar to that found in the Ising model. To confirm the above expectations, we analyze the order-parameter variance $\chi$ for finite system sizes, whose results are shown in figure 7. At the above critical points, we find the exponents $\gamma/\nu = 1.75(1)$ and $1.75(1)$, which are in good accordance with the value $7/4$ and hence confirm the above expectations.

The phase diagram for model 2 is shown in figure 8. In similarity with model 1 and results by Martins et al, the inclusion of restrictive interaction between next-
For $L = 80$ and $\lambda_1 = 0.5$, in (a) the order parameter $\phi$ (continuous lines) and the system density $\rho$ (dashed lines) versus $\lambda_2$. In (b) and (c) probability distributions $P_\phi$ and $P_\rho$ (inset) for $\lambda_2 = 6.39$ and $\lambda_2 = 50.96$ respectively.

Figure 9. For $L = 80$ and $\lambda_1 = 0.5$, in (a) the order parameter $\phi$ (continuous lines) and the system density $\rho$ (dashed lines) versus $\lambda_2$. In (b) and (c) probability distributions $P_\phi$ and $P_\rho$ (inset) for $\lambda_2 = 6.39$ and $\lambda_2 = 50.96$ respectively.

nearest neighbor sites also maintains the aa phase for intermediate values of $\lambda_2$. However, confirming some MFT expectations, there are more pronounced differences with respect to the above mentioned results. More specifically, the phase as exists solely at larger values of $\lambda_1$, in such a way that no ab–as transition line is presented for low $\lambda_1$. Besides this, the aa phase is constrained by transition lines that are first-order and become critical by increasing $\lambda_1$. Hence, in contrast to the above mentioned results, the symmetry breaking occurs through a discontinuous phase transition for low $\lambda_1$. Also unlike previous cases, tricritical points separate the as–aa and aa–as coexistence lines from the respective critical curves. As a result of restrictive interaction between next-nearest neighbor particles, the aa phase extends for much larger values of control parameters than model 1 and those from [9]. Also confirming the MFT expectations, the phase transition between ab and as phases is critical and become discontinuous by lowering $\lambda_1$. Despite the above similarities, remarkable differences from MFT results are presented. There is no triple point in which ab, as and aa phases coexist. Instead, the critical as–aa line meets the coexistence line ab–aa in a critical end point (e) (located at $[\lambda_1, \lambda_2] = [0.58(1), 7.37(1)]$), giving rise to the ab–as phase coexistence. Furthermore, the aa phase extends for much larger $\lambda_2$ and lower $\lambda_1$ than those obtained from MFT, but the critical line ab–as extends for larger values of $\lambda_1$ than MFT predictions.

In order to exemplify all of the above features of the phase diagram, now we show explicit results for distinct points of the phase diagram. Starting from the ab–aa and aa–as coexisting phases, in figure 9 we show explicit results for $\lambda_1 = 0.5$. For low $\lambda_2$ the system is constrained in the ab phase and at a threshold value ($\lambda_2 \sim 6.39(1)$ for $\lambda_1 = 0.5$), both $\rho$ and $\phi$ change abruptly, signaling the ab–aa phase coexistence. As for model 1, in the aa phase $\rho$ presents a smooth variation, implying that the change of $\phi$ as $\lambda_2$ increases comes mainly from the spatial redistribution of particles in sublattices. In addition, the
aa phase extends for notably larger values of $\lambda_2$. Probability distributions in figure 9(b) reinforce the ab–aa phase transition being first-order, with two peaks (centered at $\phi \sim 0$ and $\phi \sim 0.25$ for $P_\phi$ and $\rho \sim 0$ and $\rho \sim 0.4$ for $P_\rho$), in consistency with the observed jumps. At a second threshold value ($\lambda_2 = 50.96(1)$ for $\lambda_1 = 0.5$) $\phi$ vanishes abruptly (with $\rho$ presenting a certain increase), signaling the aa–as phase transition. Once again, probability distributions in figure 9(c) confirm such a transition to be first-order, with two peaks centered at $\phi \sim 0$ and $\phi \sim 0.45$ ($\rho \sim 0.52$ and $\rho \sim 0.73$) for $P_\phi$ ($P_\rho$).

Similar behaviors are verified for other values of $\lambda_2$. Numerical results show that aa–as transition lines become critical at ($\lambda_1, \lambda_2$) = (0.65(3), 44(1)). In figure 10 we show results for $\lambda_1 = 0.9$, in order to exemplify the second-order transition between active phases. In the interval $10 < \lambda_2 < 15$ $\phi$ increases substantially followed by small variation of $\rho$ $\neq 0$. The first crossing curves for $U_4$ occur at 13.65(5) with $U_4 = 0.61(1)$, which is consistent with a second-order Ising phase transition. The maximum value of $\phi$ (for $\lambda_1 = 0.9$) yields at $\lambda_2 \sim 25$, from which $\phi$ starts decreasing until vanishing and no pronounced changes of $\rho$ signal the aa–as phase transition. For such a transition, all reduced cumulant $U_4$ curves cross in the interval 34.5(2) with $U_4 = 0.59(2)$—also consistent with previous phase transitions. By measuring the critical exponents, we obtain in both cases values consistent with the value $7/4$, in similarity with previous results.

In the last analysis, we examine the transition between the absorbing and active (symmetric) phases, whose results are exemplified in figure 11 for $\lambda_1 = 1.5$. The probability distribution $P_\rho$ has equal height peaks centered at densities $\rho \sim 0$ and $\rho \sim 0.317$, and together with the single peak of $P_\phi$ centered at $\phi \sim 0$, such a result confirms the ab–as phase coexistence for $\lambda_1 = 1.5$.

Finally, we plot results for $\lambda_1 = 3$, in order to exemplify the critical ab–as transition, whose results are summarized in figure 12. In similarity with results for the model 1, all
Figure 11. Quasi-stationary order-parameter probability distribution $P_\rho$ for $\lambda_1 = 1.5$ and $L = 80$. In the inset, the $P_\phi$ for the order parameter $\phi$.

Figure 12. On the left, moment ratio $U_2$ versus $\lambda_2$ for $\lambda_1 = 3$ and distinct system sizes. On the right, log–log plot of $\rho$ versus $L$ at the transition point $\lambda_{2c} = 0.602$. The straight line has slope $\beta/\nu_\perp = 0.796(9)$.

$U_2$ curves cross at the point $\lambda_{2c} = 0.602(2)$ with $U_2 = 1.34(1)$, which is close to the DP value $1.3257(5)$. At the above crossing point, $\rho$ behaves algebraically with an exponent consistent with the DP value $\beta/\nu_\perp = 0.796(9)$, illustrating that the critical ab–as line belongs to the DP universality class.

4. Conclusions

The original two-dimensional contact process with creation at both nearest and next-nearest neighbors and particle suppression exhibits a novel phase structure presenting...
a continuous phase transition with spontaneous broken-symmetry phase and sublattice ordering [9]. Aimed at exploiting the robustness of such an asymmetric phase and the possibility of distinct phase transitions, in this paper we studied the effect of distinct sublattice interactions (instead of only distinct creation rates as in the original model). Two distinct models were considered. In both cases, results confirm that the competition between first- and second-neighbor creation rates and particle suppression are fundamental requirements for the presence of an asymmetric active phase. In addition, the inclusion of distinct competing interactions leads to novel phase structures, summarized as follows. A restrictive interaction between nearest-neighbor sites (model 1) changes the absorbing phase transition (in contrast to the original model), but not the asymmetric phase. More pronounced changes are found by taking the restrictive interaction between second-neighbor particles (model 2). It not only prolongs greatly the asymmetric phase under larger values of control parameters but also shifts the phase transitions, from continuous to discontinuous, even between the active phases. This latter result is particularly interesting since it reinforces the role of restrictive interactions as a minimal mechanism for the appearance of first-order phase transitions [7]. Initially studied for absorbing phase transitions, our results reveal that this ingredient is more general, changing the nature of distinct phase structures. Although predicted by the mean field theory, it is worth mentioning that discontinuous absorbing transitions under the studied restrictive interactions do not occur in one-dimensional systems [15]. The resemblance between as–aa and ferromagnetic–paramagnetic Ising model transition also provides a reason why such transitions can not occur in one-dimension. In fact, results obtained by Martins et al for the original version confirm this. As a final remark, we note that possible extensions of the present work include exploiting the influence of distinct dynamics (such as diffusion and annihilation rules) in the asymmetric phase. This should be addressed in an ongoing work.

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