GALAXY CLUSTERS, A NOVEL LOOK AT DIFFUSE BARYONS WITHSTANDING DARK MATTER GRAVITY

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INTRODUCTION

Diffuse baryons and dark matter (DM) constitute the major components of clusters and groups of galaxies, with the former energized and shining by continual struggle against the latter. The DM accounts for some 6/7 of the total masses $M \sim 10^{13} - 10^{15} M_\odot$ from poor groups to rich clusters, making for average densities $\rho \sim 10^{-26}$ g cm$^{-3}$ with its constituent ('collisionless') particles entertaining little or no interactions other than gravity. Thus the DM sets the overall gravitational ('collisionless') particles entertaining little or no interactions against gravity. As a result, the ICP at microscopic scales constitutes a very simple formalism constituting our ICP Supermodel. With a few parameters, this accurately represents the runs of density $n(r)$ and temperature $T(r)$ as required by up-to-date X-ray data on surface brightness and spectroscopy for both cool core (CC) and non cool core (NCC) clusters; the former are marked by a middle temperature peak, whose location is predicted from rich clusters to groups. The Supermodel inversely links the inner runs of $n(r)$ and $T(r)$, and highlights their central scaling with entropy $n_e \propto k_b^{-1}$ and $T_e \propto k_b^{-0.35}$, to yield radiative cooling times $t_c \approx 0.3 (k_b / 15$ keV cm$^{-3})^{1.2}$ Gyr. We discuss the stability of the central values so focused: against radiative erosion of $k_b$ in the cool dense conditions of CC clusters, that triggers recurrent AGN activities resetting it back; or against energy inputs from AGNs and mergers whose effects are saturated by the hot central conditions of NCC clusters. From the Supermodel we derive as limiting cases the classic polytropic $\beta$-models, and the 'mirror' model with $T(r) \propto \sigma^2(r)$ suitable for NCC and CC clusters, respectively; these limiting cases highlight how the ICP temperature $T(r)$ strives to mirror the DM velocity dispersion $\sigma^2(r)$ away from energy and entropy injections. Finally, we discuss how the Supermodel connects information derived from X-ray and gravitational lensing observations.

Subject headings: Dark matter — galaxies: clusters: general — gravitational lensing — methods: analytical — X-rays: galaxies: clusters

1. INTRODUCTION

Diffuse baryons and dark matter (DM) constitute the major components of clusters and groups of galaxies, with the former energized and shining by continual struggle against the latter. The DM accounts for some 6/7 of the total masses $M \sim 10^{13} - 10^{15} M_\odot$ from poor groups to rich clusters, making for average densities $\rho \sim 10^{-26}$ g cm$^{-3}$ with its constituent ('collisionless') particles entertaining little or no interactions other than gravity. Thus the DM sets the overall gravitational wells virialized within radii $R$ up to a few Mpc, where all bodies in dynamical equilibrium — from single particles to whole galaxies — possess or acquire an 1-D velocity dispersion $\sigma^2 \approx GM/5R \sim 10^3$ km s$^{-1}$. The bulk of the baryons, to a fraction again close to 6/7, is found in the diffuse form of a hot intracluster plasma (the ICP), mostly comprised of protons with the neutralizing electrons at number densities $n \sim 10^{-3}$ cm$^{-3}$, and equilibrium temperatures $k_B T \approx m_p \sigma^2/2 \sim 5$ keV ($k_B$ being the Boltzmann constant and $m_p$ the proton mass) well above most ionization potentials. This we know since Cavaliere et al. (1971) found in the first Uhuru data clear evidence of a new class of bright X-ray extragalactic sources associated with the deep, stable gravitational wells of the galaxy systems, emitting from the ICP they contain thermal bremsstrahlung powers $L_X \sim 2 \times 10^{-27} n^2 R^3 T^{1/2} \sim 10^{42} - 10^{45}$ erg s$^{-1}$. The notion has been nailed down beyond all doubts by observations of the extended nature of these sources (Gursky et al. 1972) and high excitation, coronal-like lines (Mitchell et al. 1976; Sarazin 1988) that also pointed toward definite, somewhat subsolar metallicities.

Such temperatures and densities make the ICP an extremely good plasma, in fact the best in the Universe ever, as its constituent particles in the DM gravitational wells acquire a large kinetic relative to electrostatic energy (at mean separations $d = n^{-1/3} \sim 10$ cm), their ratio being of order $k_B T / e^2 n^{1/3} \sim 10^{12}$; this astounding value is to be compared with its counterparts: $10^3$ in stellar interiors, or $3 \times 10^5$ in the pre-recombination Universe. This holds despite gravity being so exceedingly feeble at microscopic levels as to attain a mere $G m_p^2 / e^2 \sim 8 \times 10^{-37}$ of the strength that marks the electromagnetic interactions; it holds because the plasma condition

$$10^{12} \sim k_B T / e^2 n^{1/3} \equiv G m_p^2 / e^2 \times d / 10 R \times N$$

is dominated by the huge number $N \equiv M / m_p \sim 10^{73}$ expressing in proton units the total DM mass with its overwhelming gravity. As a result, the ICP at microscopic scales constitutes a very simple, nearly perfect gas of particles with 3
degrees of freedom and effective mass $\mu m_p$ with $\mu \approx 0.6$. At intermediate scales of some 10 kpc these sense mostly the electromagnetic interactions; by the latter the electrons absorb or emit radiation, while the protons share their energy over a mean free path $\lambda_{pp} \approx 10(k_B T/5\text{keV})^2 (n/10^{-3}\text{cm}^{-3})^{-1}$ kpc, with electrons following suit over some $40 \lambda_{pp}$ toward full thermal equilibrium.

By the same token, the macrophysics at cluster scales gets interestingly complex, as the ICP constitutes a faithful archive preserving memory of the different energy inputs occurring throughout sizes of Mpcs and over timescales of 10 Gyr. We shall see that the ICP physics is governed, in a nutshell, by the interplay of electromagnetic interactions transferring energy to the plasma, and of the DM bulk gravity proceeding to blend and readjust it over cluster scales.

To disentangle these processes, it will prove technically convenient to use two synthetic and formally analogous quantities. As for the ICP, the adiabat $k \equiv k_B T/n^{5/3}$ is straightforwardly related to the thermodynamic specific entropy $s = 3k_B/2 \times \log k + \text{const}$. Usually named ‘entropy’ for short, this quantity is endowed with time-honored, effective properties like: increasing (decreasing) by energy gains (losses) other than adiabatic compressions (expansions); and controlling the ICP settlement into gravitational wells.

As to the DM, the analogous quantity $K = \sigma^2/\rho^{5/3}$ is increasingly found to play – in spite of all the traditional objections leveled to defining ‘entropy’ in a collisionless medium dominated by long range self-gravity like the DM – similar roles to a true entropy; in fact, it increases during the fast collapse with associated major mergers that set up the halo bulk, and stays put during the subsequent slow mass accretion producing a quiet development of the outskirts from the inside out. These two stages have been recently recognized in sophisticated N-body simulations that follow the development of halos embedding galaxies or galaxy systems from initial cold DM perturbations (e.g., Zhao et al. 2003; Diemand et al. 2007). From simulations, during quiet stages the simple powerlaw run $K(r) \propto r^n$ is found to apply remarkably well with definite slopes close to $\alpha \approx 1.25$ throughout the structure’s main body, as first stressed by Taylor & Navarro (2001) and confirmed by many others (e.g., Dehnen & McLaughlin 2005; Hoffman et al. 2007; Ascasibar & Gottlöber 2008; Vass et al. 2009; Navarro et al. 2009).

We shall find these two entropies to be most useful in computing and relating the macroscopic, static equilibria of DM and baryons in the same gravitational wells as set by the former. This is perceived on just inspecting their related equilibrium conditions for the density runs $n(r)$ and $\rho(r)$, namely, the hydrostatic vs. Jeans equation in the form

$$\frac{1}{\mu m_p} \frac{d}{dr} (n^{5/3} k) = -\frac{GM(< r)}{r^2} = \frac{1}{\rho} \frac{d}{dr} (\rho^{5/3} K). \quad (2)$$

While the rich contents of the second equality have been discussed in detail by Lapi & Cavaliere (2009a [hereafter LC09], see also § 2 for a recap), in the present paper we will focus on the first equation. To this purpose, we make use of a physical model for $k(r)$ to derive the detailed pressure structure of the ICP.

Our approach develops the line extensively pursued in the literature at increasingly sophisticated levels. A simple isothermal or polytropic state has been adopted by, e.g., Cavaliere & Fusco-Femiano (1976, 1978), Balogh et al. (1999), Dos Santos & Doré (2002), and Ostriker et al. (2005). Re-fined models were based on a more articulated entropy run, consisting of an outer ramp produced by gravitational accretion shocks going on in the aftermath of cluster formation (see Tozzi & Norman 2001, Voit et al. 2003, Lapi et al. 2005 [hereafter LCM05]), plus a central entropy floor of nongravitational origin (e.g., Babul et al. 2002; Voit et al. 2002; Voit 2005). The latter may be contributed by a number of processes: cooling that selectively removes some low-entropy gas from the inner regions (see Bryan 2000; Voit & Bryan 2001; McCarthy et al. 2004); preheating of the ICP due to supernovae (SNe)/active galactic nuclei (AGNs) within the cluster progenitors (see Cavaliere et al. 1997; LCM05; McCarthy et al. 2008); the direct action within the cluster of central AGNs (Valageas & Silk 1999; Wu et al. 2000; Scannapieco & Oh 2004; LCM05). Additional gravitational events in the form of major mergers (Balogh et al. 2007; McCarthy et al. 2007) that now and then punctuate the equilibrium state of formed clusters (Cavaliere et al. 1999) may reach down to the center and enhance the entropy there.

Here we improve over these previous studies in the following respects. As to the DM potential well, we adopt the physical $\alpha$-profiles obtained from solving the Jeans equation rather than assuming the empirical NFW fit (see § 2). As to the ICP equilibrium, we present a novel analytical solution for the ICP density and temperature runs in terms of the ICP entropy distribution (see § 3). The distribution that we use comprises an outer ramp and a central floor, marked by two parameters physically assessed in terms of steady, self-similar accretion shocks plus additional energy inputs due to central AGNs and mergers (see § 3.1). We insert the entropy distribution in our equilibrium solution to obtain the ICP ‘Super-model’, that effectively represents with two ICP parameters the extended profiles of the X-ray surface brightness and temperature for either classes: the cool core (CC) and the non cool core (NCC) clusters, see § 4. We discuss the stability against cooling and feedback of the ensuing central conditions (see § 5). We recover classic models for the ICP distribution of density and temperature as limiting cases of the Supermodel valid in different radial ranges and for different central entropy levels (see § 6). We provide specific predictions for future X-ray observations of the ICP, and model-independent tests of the entropy runs underlying the ICP Supermodel (see § 7).

Throughout this work we adopt a standard, flat cosmology (see Spergel et al. 2007) with normalized matter density $\Omega_M = 0.27$, dark energy density $\Omega_\Lambda = 0.73$, and Hubble constant $H_0 = 72 \text{km s}^{-1} \text{Mpc}^{-1}$.

### 2. $\alpha$-PROFILES FOR THE DM HALOS

From LC09 we recall for use in the present paper a number of basic features in the equilibrium of the DM halos embedding the ICP, as described by the Jeans equation with entropy $K \propto r^n$ (see § 1); in compact form this reads

$$\gamma = \frac{3}{5} + \frac{3}{5} \frac{\alpha^2}{\sigma^2}.$$  \quad (3)

With $\alpha \equiv \log K/\log r$ set to a constant, this yields the changing density slope $\gamma(r) \equiv -\log \rho/\log r$ in terms of the increasing ratio to $\sigma^2(r)$ of the halo circular velocity squared $\nu_c^2(r) = GM(< r)/r$ that provides normalization and running estimates for the gravitational potential. The above equation provides the detailed cluster gravitational well, within which the ICP is to adjust.
The physically relevant values of $\alpha$ are pinned down from the structures’ cosmogonic development. Based on the straightforward scaling laws $\sigma^2 \propto M/r$ and $r \propto M^{1/3}$, it is seen that the density is to scale as $K \propto rM^{1/3}$, a run clearly implying a radial slope $\alpha \gtrsim 1$. The detailed time dependence of the accretion rate $\dot{M}/M$ and the related quantities may be derived semianalytically within the standard ΛCDM cosmogony; the result for the allowed range is $\alpha = 1.25 - 1.3$ from groups to massive clusters, narrowed down to

$$\alpha \approx 1.27 - 1.3$$  \hspace{0.5cm} (4)

for average masses $M \sim 10^{14} - 10^{15} M_\odot$ from poor to rich clusters of main interest here. In the structure’s development these values obtain at the transition epoch from the stage of fast collapse to that of slow accretion (relative to the running Hubble time), when the potential well attains its maximal depth marked by $\dot{M}t/M \sim 1$. Such a two-stage development turns out to be in tune with the intensive, detailed $N$-body simulations recalled in § 1. In addition, the simulations support a value of $\alpha$ in the range above, which stays put from the center throughout the halo bulk as the late quasi-equilibrium configuration develops from the inside out.

These values may be inserted into the Jeans equation, to find (as pioneered by Taylor & Navarro 2001 and Dehnen & McLaughlin 2005) all the viable solutions for $\rho(r)$, that we name ‘$\alpha$-profiles’ and illustrate in Fig. 1. These feature a monotonic radial run satisfying physical central and outer boundary conditions, i.e., zero gravitational force (corresponding to a flat minimum of the potential) and finite (hence definite) overall mass, respectively.

The ensuing behavior of the $\alpha$-profiles, basic to the ICP equilibrium, is illustrated in the top panel of Fig. 1, and highlighted by the analytic expressions of the slopes

$$\gamma_\alpha = \frac{3}{5} \alpha, \quad \gamma_0 = 6 - 3\alpha, \quad \gamma_\beta = \frac{3}{2} (1 + \alpha).$$ \hspace{0.5cm} (5)

These start from the central ($r \to 0$) value $\gamma_0 \approx 0.76 - 0.78$, progressively steepen to $\gamma_0 \approx 2.19 - 2.1$ at the point $r_0$ that marks the halo main body, and steepen further into the outskirts to the value $\gamma_\beta \approx 3.41 - 3.44$ at around the virial radius $R$ before going into a final cutoff. The $\alpha$-profiles are seen (see LC09) to correspond to a maximal value $\kappa_{\text{crit}}(\alpha) = v_\text{c}^2/\sigma^2 \approx 2.6 - 2.5$ for the relative gravitational pull at the point $r = r_p \geq r_0$ where $v_\text{c}^2(r)$ peaks (see also bottom panel of Fig. 1). After Eq. (3) this also implies at $r_p$ a maximal slope $\gamma_\beta = 3(\alpha + \kappa_{\text{crit}})/5 \approx 2.32 - 2.28$.

At variance with the empirical NFW formula (Navarro et al. 1997) that features angled central potential or pressure and diverging mass, for the $\alpha$-profiles the inner slope is considerably flatter, and the outer one is steeper as to result in a definite overall mass. The radial range $r > r_2$ where the density profile is steeper than a reference slope $\gamma = 2$ may be specified in terms of the usual concentration parameter $c \equiv R/r_2$, that may be viewed as a measure of central condensation and/or outskirts’ extension. In up-to-date numerical simulations (see Zhao et al. 2003; Diemand et al. 2007) this is found to take on value $c \approx 3.5$ at the transition redshift $z_t$, and to increase thereafter to current values $c \approx 3.5(1 + z_t)$ up to $c \approx 10$ for the fraction about 10% of rich clusters with early transition epoch $z_t \sim 1.5$. Instead, values $c = 4 - 5$ apply to the much more numerous clusters with recent transition epochs.

Another basic feature of the $\alpha$-profiles is provided by the peaked run of the gravitationally acquired dispersion

$$\sigma^2(r) \propto K(r)\rho^{2/3}(r),$$ \hspace{0.5cm} (6)

see bottom panel of Fig. 1. Towering above the central and the outer drops related to the cold nature of the DM, the peak results from the central rise of $K(r)$ and the outer steep falloff of $\rho(r)$, and will have a striking counterpart in the ICP. The characteristic values of $\sigma^2$ for the $\alpha$-profiles are set in terms of minimal values for $\sigma^2 = v_\text{c}^2/\kappa_{\text{crit}}$; this is related to limited randomization during accretion of the inflow kinetic energy gained by the initially cold DM particles.

These $\alpha$-profiles actually depend weakly on $\alpha$ in the named range (see Fig. 1), and though derived from the isotropic Jeans Eq. (3) prove to be stable against addition of reasonable anisotropies. The latter are described by the standard Binney (1978) parameter $\beta$, which numerical simulations suggest to increase outwards from central values $\beta(0) \gtrsim -0.1$ mean-
ing weakly tangential anisotropy, toward $\beta \leq 0.5$ that implies prevailing radial motions (see Dehnen & McLaughlin 2005; Hansen & Moore 2006; also Höst et al. 2009). Such density profiles turn out to be slightly flattened at the center and considerably steepened into the outskirts.

The $\alpha$-profiles are also stable against educated variations in the condition $\alpha = \text{const}$. Actually, the cosmogenic buildup given by our semianalytic computation yields a slow decrease of $\alpha(r)$ into the outskirts, as these develop from the inside out at late times after the transition. Again in keeping with the simulations, the outer development does not affect the inner gravitational potential and the equilibrium described by the Jeans equation, that (with its central boundary condition) also works from the inside out. In fact, for the present use we have computed the density profiles associated with $\alpha(r)$, and checked these to be very close to the parent $\alpha$-profiles but for being somewhat steeper into the outskirts, with densities lower by about 15% in the vicinity of the virial radius.

It is to be stressed that the $\alpha$-profiles for $M \sim 10^{14} - 10^{15}M_\odot$, especially with full $\alpha(r)$ and anisotropies, turn out to provide an optimal fit to the surface density runs as derived from gravitational lensing observations in around massive clusters that just require central slopes flatter and outer slopes steeper than the standard NFW (cf. Broadhurst at al. 2008). Such a fit for A1689 requires high values $c \sim 10$ and early transition epochs $z_t \approx 1.5$, related conditions that the two-stage development predicts to occur in about 10% of the rich clusters (Lapi & Cavaliere 2009b).

We will adopt these $\alpha$-profiles depending on the two DM key parameters $\alpha$ and $c$ to describe the gravitational potential containing the ICP, to which we now turn.

3. THE ICP EQUILIBRIUM

We will focus on clusters in equilibrium conditions, in between the punctuating violent mergers for which we refer the reader to the review by Markovich & Vikhlinin (2007). When the DM halo is close to equilibrium, even more so will be the pervading ICP; in fact, the sound crossing time $R/(5k_B T/3\mu m_p)^{1/2}$ is seen to be somewhat shorter than the dynamical time $R/c$ on recalling from § 1 that $k_B T \approx \mu m_p \sigma^2$ holds. In these conditions the ICP is governed by the hydrostatic, Jeans-like equation provided by the first and the second member of Eq. (2). In a compact form similar to Eq. (3) this writes

$$g \equiv \frac{3}{5} a + \frac{3}{5} b,$$

once the ICP entropy $k(r)$ is given and used to factor out the product $d(k n^{5/2})/dr$ in the original Eq. (2). In fact, the entropy slope $a \equiv d\log k/d\log r$ enters Eq. (7) to yield the density slope $g(r) \equiv -d\log n/d\log r$, concurring with the gravitational pull measured by the (squared) ratio $3 b(r) / 5 \equiv 3\mu m_p v^2(r) / 5 k_B T(r)$ of the DM circular velocity (discussed in § 2) to the sound speed.

The above constitutes just a first order differential equation, that transforms to linear in terms of the variable $n^{5/2}(r)$. By textbook recipes (cf. Dwight 1961) this is amenable to a simple solution in form of a quadrature, that is conveniently written as

$$T'(r) = \bar{k}(r) \bar{r}^{-2/3}(r) \equiv \bar{k}^{-1/5}(r) [1 + \frac{2}{5} b R \int_r^{1} \frac{d\bar{r}'}{\bar{r}'} v^2(r') \bar{k}^{-3/5}(r')] .$$

Here barred variables are normalized to their boundary value at $r = R$ where $b(r)$ takes on the value $b_R$, while $v^2(r)$ is taken from the $\alpha$-profiles with its weak dependence on $\alpha$.

In Fig. 2 (top panel) we plot the integral appearing in the above equation, along with the simple analytical fit presented in Appendix A. Note three circumstances. First, $2 b_R / 5 \approx 1$ holds (e.g., $2 b_R / 5 \approx 1.06 - 1.02$ on using $b_R = 2.65 - 2.55$, see Eq. [10]). Second, at the center the integral dominates in the square bracket; thus the memory of the boundary condition is swamped while the integral is numerically found to scale as $k^{-1/4}$ with the central entropy $k_c$, see Fig. 2 (bottom panel). Third, on approaching the center where the $r$-dependence of the bracket is already saturating, Eq. (8) implies $T(r) \propto n^{-1}(r) \propto k^{3/5}(r)$. The issue to stress is that, rather than a model, the above Eq. (8) has the standing of a theorem in hydrostatics valid for the ICP of clusters close to equilibrium.

3.1. ICP entropy

What actually needs physical modeling is the entropy run $k(r)$. We base upon the notions that entropy is erased by radiative cooling on the timescale $t_c \approx 65 (k_B T / 5\text{keV})^{1/2} (n/10^{-3} \text{cm}^{-3})^{-1}$ Gyr (Sarazin 1988), while substantial raising
requires shocks (see Appendix B) as are driven by supersonic outflows from center and set by inflows across the boundary.

3.1.1. Entropy deposited at the boundary

At the outer end, the mathematics of Eqs. (7) requires one boundary condition fixing \( b_R \). On the observational side, \( T(r) \) is found to decline slowly toward the virial radius (Molendi & Pizzolato 2001), and at such a rate it would take tens of Mpc’s to decline smoothly from keV values to those some \( 10^{-2} \) times lower as prevailing in the external medium. So at \( r \approx R \) a discontinuity is to occur and terminate the hot ICP, at variance with the smooth if steeper decline of the DM density. In fact, most of the transition takes place across a few mean free paths \( \lambda_{pp} \) at the accretion shocks produced when external gas supersonically falls into the DM potential.

Numerical simulations (e.g., Tormen et al. 2004; see their Fig. 8) show that accretion of the smooth gas and minor lumps making up a major fraction of the accreted mass drives a complex patchwork of shocks mostly within an outer layer with thickness of order \( 10^{-3} \) \( R \); across such a layer the standard conservation laws of mass, momentum and energy may be applied leading to the classic Rankine-Hugoniot jump conditions; to within 10\% the results are similar to a coherent shock, roughly spherical and located at \( r \sim R \) (see LCM05 for a complete treatment, and Ettori & Fabian 1998 for possibly delayed electron equilibrium). The impact of major lumps reaching down to the center will be dealt with in § 3.1.3.

The jump conditions take on a particularly simple form for cold inflow into rich clusters that cause strong shocks with Mach numbers squared \( \mathcal{M}_c^2 \) considerably exceeding unity (see Appendix B). In strong shocks lingering at \( r \approx R \) (and expanding along with the cluster’s development), maximal conversion of infall energy occurs over a radial range of order \( \lambda_{pp} \), to yield

\[
k_R T_R = \frac{2}{3} \frac{\mu m_p v_R^2}{\Delta \phi_R},
\]

where \( 2 v_R^2 \Delta \phi_R \) is the kinetic energy per unit mass of the gas freely falling across the potential drop down to \( r = R \) from the turning point where infall starts (see LCM05). Thus we find

\[
b_R = \frac{3}{2 \Delta \phi_R},
\]

with values 2.65–2.55 corresponding to \( \alpha = 1.27–1.3 \) and \( \Delta \phi_R \approx 0.57–0.59 \).

At the boundary the equilibrium Eq. (7) yields \( g = 3 (a + b_R)/5 \). Meanwhile, powerlaw approximations \( k \propto r^p \), \( n \propto r^{-q} \) apply in the vicinity of \( r = R \), so \( m(r) \propto r^{3-q} \) describes the ICP mass \( m \) in the outer layer, and correspondingly

\[
k \propto m^{(1-q)/3} \propto m^{5a/3(5-a-b)}
\]

obtains.

On the other hand, pursuing the scaling for \( K(r) \) recalled in § 2, LC09 show that the DM entropy behaves as \( K \propto M^{4/3}/M^{2/3} \propto M^{1/2} \) on considering that \( M \propto r^{4/3} \) holds in the standard \( \Lambda \)CDM cosmogony for \( z < 0.5 \) during the slow accretion stage. As external gas and DM are accreted in cosmic proportion, in the outer layer \( n \propto M \) applies, and it follows that the ICP entropy at the boundary can be expressed as

\[
K \propto m^{3/2}.
\]

Then on equating the exponents in Eqs. (11) and (12) one finds

\[
a_R = \frac{45 - 9 b_R}{19} \quad \text{and} \quad g_R = \frac{27 + 6 b_R}{19}.
\]

With the values of \( b_R \) discussed above, these yield the slopes \( a_R \approx 1.1 \) and \( g_R \approx 2.2 \), both considerably flatter than the corresponding DM values. Note from Eq. (13a) how the slope \( a_R \) at the boundary decreases if \( b_R \) is increased. We expect such a decrease to take place for clusters with high concentrations corresponding to shallow outer potentials (see Lapi & Cavaliere 2009b), that decrease the outer drops \( \Delta \phi_R \) entering \( b_R \) through Eq. (10). E.g., a value \( \alpha \approx 10 \) (holding for a cluster with an early transition, see § 2) in place of the usual \( \alpha \approx 4 \) (holding for clusters with a recent transition) implies \( \Delta \phi_R \) to lower to 0.47, \( b_R \) to grow to 3.2, and \( a_R \) to decrease to 0.85 as may be the case for A1689 (see Lemze et al. 2008; Lapi & Cavaliere 2009b); meanwhile, from Eq. (13b) \( g_R \) increases to 2.4. A flat entropy slope may be also produced when the boundary shock is weakened by substantial preheating of the infalling gas; such a condition is relevant for poor clusters and groups with comparatively low potentials (see LCM05), e.g., \( \alpha \approx 0.8 \) applies to systems with \( M \lesssim 10^{14} \) \( M_\odot \) with external preheating at levels around 1/2 keV per particle.

On the other hand, after Eqs. (13) high values of \( a \) with an upper bound at 45/19 \( \approx 2.4 \) (in keeping with the variance observed by Cavagnolo et al. 2009) correspond to low values of \( b_R \), hence to large \( \Delta \phi_R \); these imply flat densities and temperatures sustained to high values in the outskirts.

3.1.2. Entropy stratification

No other major sources or sinks of energy and entropy occur inward of the boundary at a few Mpc’s down to the central \( 10^2 \) kpc. So throughout the ICP bulk, the entropy slope is to stay at its boundary value \( a = a_R \), and in rich clusters with standard concentration

\[
k(r) \propto r^{1.1} \quad \text{(14)}
\]

is to hold, a result similar to Tozzi & Norman (2001), LCM05, and Voit (2005).

On the other hand, while the simple thermodynamics of the ICP maintains the run of \( k(r) \) in this powerlaw form, its more complex adiabatic readjustments within the gravitational well cause the density slope \( g(r) \) to flatten out inward; this is granted by Eq. (7), as the inequality \( b(r) < b_R \) (i.e., \( v_g^2(r)/k g T(r) < 1 \)) progressively strengthens away from the boundary. As a result, the ICP slope \( g(r) \) not only starts out but also stays generally flatter than DM’s \( \gamma(r) \) inside of the boundary, as given by

\[
g(r) = 3[a + b_r]/5 < 3[\alpha + \kappa(r)]/5 = \gamma(r).
\]

This implies the ICP density run \( n(r) \) at the boundary to parallel the DM’s \( \rho(r) \) at some other point well inside the cluster’s main body.

3.1.3. Entropy in the central region

Next we discuss the central value \( k_c \) of the entropy and its origins. Recall from § 1 that the central entropy may be produced by shocks driven by substantial merging events reaching the center (McCarthy et al. 2007; Balogh et al. 2007), and by AGNs residing in the central bulge-dominated galaxies (see Valageas & Silk 1999; Wu et al. 2000; Cavaliere et al. 2002; Scannapieco & Oh 2004; LCM05). In addition, a
basal entropy level may be left over by SNe and AGNs that preheated the gas in the volume due to collapse into, or to accrete onto the cluster (Cavaliere et al. 1997; LCM05, McCarthy et al. 2008). To begin with, consider preheating; this may be expressed in terms of entropy advected across the shock by the currently infalling gas (see Appendix B). However, such a process is known to be effective only within limits: if always strong, it would flatten the bulk entropy slope of most clusters to values α < 1 (see § 3.1.1 and Fig. 3), and produce really flat entropy profiles in most groups at variance with the observations (see Balogh et al. 1999; Pratt & Arnaud 2003; Rasmussen & Ponman 2004); unless tailored to cluster precollapse size, it would flatten the bulk entropy slope of most clusters to values α around 1.27 and an average concentration value c ≡ R/r_{25} = 5 (see § 2). In the panel representing T(r), we plot the DM velocity dispersion σ^2(r) as a thick dashed line, and the circular velocity v_c(r) as a thin dashed line; we also show average data for the CC (circles), NCC (squares), and UNC (triangles) classes from Leccardi & Molendi (2008). The hatched area outlines the central range hardly accessible to current resolutions.

Fig. 3.— Radial runs of the ICP density, mass, entropy, temperature, cooling time, and polytropic index given by the Supermodel. Solid lines are for different values of the central entropy k_c = 5 × 10^{-2} (red), 2.5 × 10^{-2} (cyan), and 0 (blue); dotted lines refer to the ‘mirror’ model discussed in § 6.1 with β = 0.75; dashed lines illustrate the underlying DM distribution taken from the α-profile with α = 1.27 and an average concentration value c ≡ R/r_{25} = 5 (see § 2). In the panel representing T(r), we plot the DM velocity dispersion σ^2(r) as a thick dashed line, and the circular velocity v_c(r) as a thin dashed line; we also show average data for the CC (circles), NCC (squares), and UNC (triangles) classes from Leccardi & Molendi (2008). The hatched area outlines the central range hardly accessible to current resolutions.

Values of k_c in the range 10 – 30 keV cm^2 as frequently observed despite fast radiative erasure point toward entropy inputs frequently refreshed; a promising path is provided by sizeable and recurrent energy injections by central AGNs, triggered into activity by renewed fueling in a low-entropy, dense environment of their underlying supermassive black holes (BHs). These are observed to undergo outbursts able to inject, nearly independent of cluster mass, energies ∆E up to 10^{62} erg or a few keV per particle, yielding k_c of several tens keV cm^2. Cooling on the scale of 10^{-1} Gyr may be offset as the several supermassive BHs inhabiting the many bulge-dominated galaxies in the central region of a rich cluster alternatively kindle up, and collectively recur over such timescales (see Nusser et al. 2006; Ciotti & Ostriker 2007; Conway & Ostriker 2008). In particular, outbursts lasting for a few 10^8 yr can continuously drive out to 300 kpc pressurized blastwaves terminating into a shock with Mach numbers sustained at M^2 ≥ 3, as computed in detail by LCM05 and observed by Forman et al. (2005), Nulsen et al. (2005), McNamara & Nulsen (2007), see also Appendix B.

All these processes contribute to raise the entropy over an extended region around the center, adding to the outer run given by Eq. (14); the combined entropy profile may be described by the simple parametric expression (see Voit 2005 and references therein)

\[
\dot{k}(r) = \dot{k}_c + (1 - \dot{k}_c) r^\alpha
\]  

represented in Fig. 3. Having computed the outer powerlaw slope a around 1.1 (see § 3.1.1) and having discussed central entropy levels k_c from a few tens to several hundreds keV cm^2, up to several 10^2 keV cm^2 (see Appendix B). With timescales for free radiative cooling of order

\[
t_c \approx 0.3 \left( \frac{k_c}{15 \text{keV cm}^2} \right)^{3/2} \left( \frac{k_B T}{5 \text{keV}} \right)^{-1} \text{Gyr},
\]  

(Voit & Donahue 2005), the above levels take several Gyrs to decrease substantially. At the other end, a much lower entropy state would rapidly run down to values lower still. So a key point is to secure intermediate but persisting entropy levels.

Fig. 4.— ICP projected distributions of emission-weighted temperature, spectroscopic-like temperature, X-ray surface brightness, and f.o.s. SZ effect. Lines styles as in Fig. 3.
4. THE SUPERMODEL FOR CC AND NCC CLUSTERS

The run of \( k(r) \) in the handy form Eq. (17) may be inserted into the hydrostatic equilibrium Eqs. (8) to yield the Supermodel for the ICP disposition. Away from the boundary this links \( T(r) \) and \( n(r) \) inversely; such an inverse link is particularly clear at the very center, as is seen from the dependencies \( T_c(k_c) \) and \( n_c(k_c) \) obtained from Eqs. (8) on considering the explicit scalings \( T_c \propto k_c^{3/5} \) and \( n_c \propto k_c^{-3/5} \) with the further factor supplied by the dominant integral that scales as \( k_0^{0.25} \) (see Appendix A). Thus the overall scaling laws read

\[
T_c \propto k_c^{0.35}, \quad n_c \propto k_c^{-1}; \quad (18)
\]

these yield \( T_c \propto n_c^{-0.35} \), from which we see clearly how low/high \( T_c \) correspond to high/low \( n_c \). In addition, it it is easily perceived, and is seen from Fig. 3, that the central runs of \( n(r) \) are angled or core-like, depending on \( k_c \) being low or high.

There is much more. Fig. 3 show that the Supermodel provides simultaneous, accurate descriptions to both the full profiles of \( T(r) \) as directly given by Eq. (8), and of the surface brightness in X rays provided on integrating along the l.o.s. the volume emissivity for optically thin thermal bremsstrahlung; this reads \( 2.4 \times 10^{-27} L_X \) erg s\(^{-1} \) cm\(^{-3} \) with \( L_X \propto n^2 T^{1/2} \) (emission lines add for \( k_B T \lesssim 2 \) keV, see Sarazin 1988). After Eq. (8) one has

\[
L_X(\bar{r}) \propto \bar{k}^{-9/10}(\bar{r})\left[1 + 2/5 b_R \int_1^{1/\bar{r}} \bar{k}^{-3/5}(\bar{r}') \bar{v}_c^2(\bar{r}')/\bar{r}'^{7/2}\right]^{7/2}, \quad (19)
\]

with the central scaling given by \( L_X(0) \propto k_c^{-1.8} \). These profiles depend: strongly on the value of \( k_c \) that primarily governs the
central pressure and hence the central density run (see Fig. 3); weakly on $a$ (and $b_k$) that governs the middle run; mildly on the DM concentration $c$ (see § 2) that governs the outer decline toward the boundary values.

We illustrate in Figs. 3 and 4 how straightforwardly the Supermodel describes various observables for both classes of Cool Core (CC) and Non Cool Core clusters (NCC) as identified by Leccardi & Molendi (2008), and also for their intermediate class of UNC clusters. In Figs. 5, 6, and 7 we focus on the specific cases of the clusters A2218, A2204, and A1413 for which both high-resolution XMM-Newton and preliminary Suzaku data are available.

From these results is clearly seen how CC clusters are marked out by the presence of a peak of $T(r)$ at $r \approx 0.1 - 0.2 R$ (or equivalently by $T_c < T_R$). The condition for the peak to occur is highlighted on recalling from Eq. (8) that

$$T(r) \propto k_r^{2/3}(r)$$  

(20)

applies, as the ICP counterpart of Eq. (6) for the DM. Given that $n(r)$ rises monotonically inward, $T(r)$ will peak and then decline toward the center when $k_{3/5}(r)$ decreases strongly toward a low value of $k_c$, as is the case with the CC clusters. On the other hand, $T(r)$ will rise to a central, roughly isothermal plateau for sufficiently high values of $k_c$.

From the condition for a maximum to occur in the functional form of $T(r)$ as given by Eq. (8), on using $v_\alpha^2(r)$ from the $\alpha$-profiles with $\alpha = 1.27 - 1.3$ the threshold value for the peak reads

$$k_c \approx 2.5 \times 10^{-2} ;$$  

(21)

with boundary values $k_R \approx 1500 - 2000$ keV cm$^2$, these correspond to $k_c \approx 40 - 50$ keV cm$^2$. In closer detail, the peak looms out (the case of UNC clusters) for $k_c \approx 2.5 \times 10^{-2}$, and stands out (the case of CC clusters) for $k_c \lesssim 10^{-2}$ corresponding to $k_c \approx 15 - 20$ keV cm$^2$.

We stress that a finite $T_c \neq 0$ constitutes a natural feature of the equilibrium for CC clusters rather than some peculiarity of cooling flows (see discussion by Peterson & Fabian 2006). This holds with realistically small values of $k_c$; but even if $k_c$ were formally null, $T_c$ would decline toward the very center with a somewhat flatter powerlaw $r^{3/5}$ compared to $\sigma^2 \propto r^{3/5}$. This behavior is consistent with the nonradiative runs of the hydrodynamical simulations by Borgani (2007, see his Fig. 1).

A feature typical of the Supermodel is the peak of $T(r)$ closely following the maximum of the DM velocity dispersion $\sigma^2(r)$, not that of the circular velocity $v_c^2(r)$, see Fig. 3 (middle right panel). As the former moves considerably downward with masses ranging from $10^{15}$ to $10^{13} M_\odot$ (see Fig. 1 and LC09), we predict the peak of $T(r)$ should also move to progressively lower radii in going from rich to poor clusters and groups; preliminary data by Nagai et al. (2007) support the prediction, but robust data require secure Chandra calibrations.

Finally, the Comptonization parameter for the SZ effect (Sunyaev & Zel’dovich 1972) obtains from integrating along a l.o.s. the volume quantity $\mathcal{Y} \propto p$ in terms of the (thermalized) electron pressure $p = nk_b T$; after Eq. (8) this means

$$\mathcal{Y}(\vec{r}) \propto [1 + 2/5 b_R \int_0^{1} d\bar{r} \bar{k}_{3/5}^{2/3}(\bar{r}) v_\alpha^2(\bar{r}) / \bar{r}^{1/2}] ,$$  

(22)

to the result plotted in Fig. 4. In the central region this scales as $\mathcal{Y}(0) \propto k_c^{-0.65}$.

5. STABILITY OF THE CENTRAL CONDITIONS

Having shown how the Supermodel focuses the central conditions, here we pursue the discussion ending § 3, and argue that they are robust against energy losses or additions.

5.1. From Cool Coses to cooling cores, and back

Plainly, the CC state produced by the Supermodel differs not only from a cooling flow but also from a freely cooling core; in fact, cooling is not included in Eqs. (7) as they stand. However, the Supermodel focuses the conditions for enhanced radiation and fast cooling to set in, namely, low finite $T_c \propto k_c^{0.35}$ linked to high $n_c$ so as to imply

$$t_c \approx 0.3 \left( \frac{k_c}{15 \text{keV cm}^2} \right)^{1.2} \text{Gyr} .$$  

(23)

Fig. 3 (bottom left panel) illustrates the run $t_c(r)$ of the cooling time throughout a CC cluster.
So far as it goes for the Supermodel proper. The sequel of the story is long accepted in general terms, to imply that such an enhanced radiation will lead to entropy loss, that in turn will further lower $T_c$ and increase $n_e$, so shortening $t_c$ and opening the way for a classic cooling catastrophe to set in (White & Rees 1978; Blanchard et al. 1992). A possible happy end to the cooling story in clusters has been widely proposed and discussed (see Binney & Tabor 1995; Cavaliere et al. 2002; Voit & Donahue 2005; Ciotti & Ostriker 2001, Tucker et al. 2007), to the effect that, before the conditions run away into a full catastrophe, the ICP condensing around central massive galaxies and mixing with their ISM is very likely to kindle up AGN activities by renewing mass accretion onto their powerhouses, the supermassive BHs lurking and starving at most galactic centers. Thus recurrent loops are conceivably started out by cooling that rekindles AGNs, that in turn feed back energy into the surrounding medium to eventually quench the BH accretion flow and ultimately yield quasi-steady, widespread conditions with $k_c$ reset over timescales of some $10^8$ yr to values around 15 keV cm$^{-2}$.

5.2. Saturation in Non Cool Cores

On the other hand, NCC conditions prevail above the divide at $k_c \approx 40 - 50$ keV cm$^{-2}$. We understand the fewer NCC relative to the CC clusters primarily on the basis of strong input inputs, in the tail of the AGN luminosity function $N(L)$ (e.g., Richards et al. 2006); this implies for the output statistics $\Delta E N(\Delta E) \propto L N(L)$ a sharp decline above $\Delta E \approx 5 \times 10^{61}$ erg. We also note that hotter cluster centers tend to impair or prevent the supersonic condition necessary for driving strong pressurized blastwaves (see Appendix B), i.e., $M_c^2 \approx 1 - \Delta E/E \gtrsim 3$ corresponding to $\Delta E/2k_bT_c \geq 1$; meanwhile the entropy deposited reads $k_c \approx \Delta E/(1 - \Delta E/2E)^{3/2}$. This will saturate the effects of multiple inputs, if any, occurring within a cooling time.

Similar arguments also apply to the possibly larger if generally rarer energy outputs associated to substantial mergers; these may be up to $\Delta E \lesssim 10^{60}$ ergs (see § 3.1.3), but by the same token the energies effectively transferred to the ICP in NCC conditions are especially prone to saturate after the first such event.

5.3. Concluding on central conditions

Taking up from § 3.1.3, we recall that the cooling-fueling-feedback machinery calls for a correlation between CC clusters and current central AGNs (the so called dichotomy, see Voit 2005) that finds support from a considerable body of observations (e.g., Mittal et al. 2009). This picture is currently under scrutiny in study cases such as provided by the poor cluster AWMM4; there the considerable value $k_c \approx 60$ keV cm$^{-2}$ calls for a fueling process unrelated to cooling, with energetics higher than implied by the current radio activity. In addition, no signatures or fossil imprints have yet been found of large energy inputs caused over the past $10^{-1}$ Gyr by either major AGN outbursts or mergers (Gastaldello et al. 2008, Gricantucci et al. 2008). This may constitute as of today one instance of an exceptional preheating level, standing out as a main component to $k_c$.

To complete the picture, AGNs as widespread agencies for rising the central entropies lead to understand the steep decline of the local $L_X - T$ correlation from clusters to groups, or the equivalent saturation in groups of the $k - T$ correlation (see Ponman et al. 2003; Pratt et al. 2009), in terms of comparable single outbursts in differently sized galaxy systems (Cavaliere et al. 2002, LCM05). Finally, AGN kinetic plus radiative outputs also lead to expect for clusters a non monotonic rise and fall of the relation $L_X - z$, consistent with the current data (Cavaliere & Lapi 2008).

In sum, the two main modes for energy injections, namely, central AGNs and major mergers, provide different levels of entropy input; whence we expect a bimodal distribution for the observed number of clusters as a function of the central entropy level $k_c$. In fact, the two peaks should be remolded to an actual distance considerably smaller than the factor $10$ separating the two maximal input levels; this is because the statistics will be eroded at low $k_c$ by fast cooling, while limited at high $k_c$ by the small number of strong input events. The expected outcome will be not unlike the findings recently presented by Voit (2008) and Cavagnolo et al. (2009).

Back to our main course, we conclude that both the CC and the NCC central conditions envisaged by the Supermodel are made robust by processes additional, but naturally geared to it.

6. LIMITING MODELS

While the Supermodel can yield accurate representations of the ICP state with a moderate amount of formalism, it is nevertheless worthwhile to have at hands simple limiting models amenable to prompt analytical computations.

6.1. Mirror dispersions

We take up the point made in § 4 as to $T(r)$ following $\sigma^2(r)$ for CC clusters, and show in Figs. 3 (middle right) and 6 (bottom panel) the good performance around the $T(r)$ peak and shortward (but for the very central range where $T(r)$ deviates upward to its value $T_c \approx k_b^2 T_c^2$ provided by the simple model with the ICP mirroring the DM dispersion

$$T = \sigma^2 / \beta .$$

The normalizations are included in the constant parameter $\beta \equiv \mu m_p \sigma^2 / k_b T_c$ that in Fig. 6 is fixed at 0.75, just the natural value it takes when evaluated at $R$.

This is similar to the approximation discussed by Cavaliere & Fusco-Femiano (1981), and similarly yields the density in the explicit form

$$n(r) = \bar{n}^2(r) \delta^2(\beta - 1)(r) .$$

Here the DM density $n(r)$ is provided by the $\alpha$-profiles of § 2, to imply

$$n(r) \propto \rho^2(\beta - 1) \rho^{5/3 - 3/2}(r) ,$$

which goes into the simple form $n(r) \propto \rho(r)$ toward the center in the range where $\rho(r) \to r^{3/8}$ holds but still Eq. (25) applies.

6.2. Polytropic $\beta$-models

For NCC clusters, instead, we make contact with the classic $\beta$-models discussed by Cavaliere & Fusco-Femiano (1978).

In the central range the contact obtains noting these clusters to be marked by high values of $k_c$ that cause a nearly flat central run of the entropy after Eq. (17). It is now matter of straightforward algebra to see that on taking $k(r) \approx k_c$ to a zeroth approximation, this may be extracted from the integral in Eqs. (8), to yield directly

$$T / T_c = (n/n_c)^{2/3} = 1 + 2 k_c / \Delta \phi_{c,s} / 5$$

(27)
Corresponding to a polytropic approximation with macroscopic index $\Gamma \equiv 5/3 + \log k/\log n = 5/3$. To a next approximation $\Gamma$ may be obtained by carrying further the expansion of the integral, to obtain

$$\Gamma \approx \frac{5}{3} \left[ 1 - \frac{2}{5} \frac{1 - \frac{c_r}{k_c}}{\Delta \phi_{cR}} \int_{r_c}^{R} \frac{d\phi}{dr} \frac{1}{r^3 a} \right],$$  

(28)

to be used in the expression

$$\frac{T}{T_m} = (n/n_c)^{1-1} = 1 + (\Gamma - 1) \beta_c \Delta \phi_{cR}/\Gamma.$$  

(29)

In the outer regions a similar expansion may be pursued both for CC and NCC clusters, to yield an effective index

$$\Gamma \approx 1 + \frac{2}{5} \frac{1}{\Delta \phi_{cR}} \int_{r_c}^{R} \frac{d\phi}{dr} \frac{1}{r^3 a^{3/5}}.$$  

(30)

In fact, Fig. 3 last panel shows that values roughly constant in the range $\Gamma = 1.1 - 1.2$ apply to the outer regions of all cluster categories.\(^5\)

Another approximation of the polytropic type is seen to apply for $r > r_m$, i.e., to the right of the peak of $\sigma^2$ (see Fig. 1), and may be formally based as follows. Consider that wherever density and temperature follow (piecewise) powerlaw runs, the elimination of $r$ provides a link of the polytropic form $nT \propto r^n$; a similar consideration applies to the DM, leading to define a corresponding index $\Lambda$. Thus we may write the first and the third sides of Eq. (2) in these terms, and equate them directly; then simple algebra provides the explicit relation

$$\frac{T}{T_m} \approx \frac{\beta_m}{\Gamma} \left( \frac{\sigma}{\sigma_m} \right) \frac{1}{\Lambda} + \frac{T_R}{T_m}.$$  

(31)

This, complementarily to Eq. (24), shows that the temperature run $T(r)$ tends to follow the DM dispersion $\sigma^2(r)$ except for the vicinity of the virial boundary where the shock condition sustains it at the value $T_R$, and for the very center where a finite if small $k_c$ matters.

Summarizing the trend highlighted by the limiting models, the passive mirror behavior of the ICP with $T \propto \sigma^2$ prevails unless is offset by energy inputs, as in fact occurs at the boundary for all clusters, and in the central region for the NCC clusters.

7. DISCUSSION AND CONCLUSIONS

This paper introduces a novel look to the Astrophysics of galaxy clusters, in terms of both the $\alpha$-profiles for the initially cold DM and of the Supermodel for the hot ICP.

As for the DM halos, we have taken up from LC09 the physical $\alpha$-profiles. These are based on the Jeans equilibrium between self-gravity and pressure modulated by the DM `entropy' run $K(r) \propto r^\alpha$. The latter is found from many recent $N$-body simulations (recalled in § 1) to apply with $\alpha$ closely constant within the halo body; we have semianalytically computed the halo two-stage development and obtained the narrow range $\alpha \approx 1.27 - 1.3$ from poor to rich clusters (see § 2 for details). The ensuing $\alpha$-profiles, depending on the two key parameters $\alpha$ and $c_c$, provide density runs $\rho(r)$ that satisfy physical central and outer boundary conditions at variance with the empirical NFW formula, and also yield better fits to detailed data from gravitational lensing in and around massive clusters (see Lapi & Cavaliere 2009b).

The ICP, on the other hand, settles to equilibrium within the gravitational wells associated with the $\alpha$-profiles, under control from the thermodynamic entropy produced by boundary and central shocks driven by AGNs or major mergers, plus a possible preheating basal level (see § 3). These physical effects may be compounded in the two-parameter form $k(r) = k_c + (1-k_c) r^p$ with $p$ ranging from 0.8 to 1.1. The resulting equilibrium may be concisely rendered as a trend for the ICP to follow the DM in the passive behavior $T(r) \propto \sigma^2(r)$, in the radial range free from the energy inputs that steadily produce the outer boundary slope and intermittently refresh the central level $k_c$.

In detail, our Supermodel of § 4 provides accurate and extended representations for the runs of ICP temperature and density and of the related ICP observables (see Fig. 5, 6, and 7 for examples). These validate the assumption of hydrostatic equilibrium, and closely constrain not only the value of the concentration $c$ for the DM $\alpha$-profile but also the two ICP parameters $a$ and $k_c$. In fact, such representations hold for either Non Cool Core and Cool Core clusters, marked out, respectively, by a monotonic outer decline of $T(r)$ from a central plateau at $T_c \gtrsim T_R$, or by a middle peak. In the Supermodel these morphologies are produced by the central entropy $k_c$, being higher or lower than a threshold value $k_c \approx 20 - 50$ keV cm$^2$; correspondingly, the X-ray brightness features a flat corelike or a steep central run, based upon the same mildly cusped DM $\alpha$-profile. In a forthcoming paper, we will present a detailed analysis of an extended sample of NCC and UNC clusters, with the aims of disentangling the origin of the central energy inputs, and of evaluating the variance in the cluster ages through their outer DM concentrations (see § 2).

We stress that the Supermodel links $T(r)$ and $n(r)$ inversely from the bulk toward the central region (see § 4); in particular, it yields for the very central values the scaling $T_c \propto k_c^{1.35}$ and $n_c \propto k_c^{-1}$, implying $t_c \propto k_c^{1.2}$ for the cooling time. The stability of such values is argued in § 5. High $T_c$ combines with flat $n_c$ to produce in NCC clusters central conditions conducive to saturation of $k_c$ toward values around $10^5$ keV cm$^2$. In CC clusters, instead, low though finite $T_c$ combines with high $n_c$ into a condition paving the way to fast cooling; this condition is conducive to triggering intermittent, recurrent loops going through the stages: cooling, massive BH fueling, AGN energy feedback, that halts further fueling and activity; these loops make possible in the long term a quasi-steady state. In the center of NCC clusters, instead, hot conditions suppress AGN reactivations owing to lack of dense cool ICP crowded around the central cluster galaxies; in addition, they tend to saturate the effective energy coupling from the most powerful AGNs or mergers by preventing or impairing supersonic conditions conducive to strong shocks.

Concerning central conditions, we emphasize two points. First, in CC clusters the Supermodel predicts a finite (non-zero) central $T_c$ with no need for any twist in cooling flow theories; this rather constitutes a natural condition set by their equilibria at low $k_c$ and stabilized against cooling by recurrent AGN activity. Second, in NCC clusters we expect saturation to enforce stability of the higher $k_c$ levels set by the inputs from powerful AGNs and from the stronger if rarer mergers.

Moving into the middle radial range where energy sources may be neglected, the `mirror' behavior of Eqs. (24) and (31) prevails with $T(r)$ passively following $\sigma^2(r)$; a novel feature

\(^5\) For a quick evaluation of the index, consider that $\Gamma \equiv 5/3 + d \log k/\log n = 5/3 - a_k/g_k = b_R/g_R$.\]
emerging from the Supermodel is the very close location of the two respective peaks. This is because such two homologous quantities arise from a parallel response to the requirement of withstanding the common gravity for equilibrium (see § 6, also Fig. 3). This gravity-induced behavior is at the root of the remarkable effectiveness of the simple model $T(r) \propto \sigma^2(r)$, which for the CC clusters holds well down toward the center (see Fig. 6). As a consequence, the Supermodel predicts the peak of $T(r)$ to move toward progressively smaller radii in going from rich clusters to groups.

On approaching the boundary, the run of $T(r)$ again deviates upward from this passive trend (see § 3.1 and Appendix B), with a boundary value $T_b$ sustained by the energy input associated to infall. Here the passive ICP behavior is again expected to be broken by the energy transfer due to electromagnetic interactions and localized to a range $\Delta r \sim \lambda_{pp} \ll r$ (whilst bubbles or shocks starting from the center smear their energy out to some $10^2$ kpc, implying an effective $\Delta r/r \sim 1$). However, pinning down the outer deviations requires high sensitivity, currently achievable only with full use of the Suzaku capabilities.

Next we highlight an unexpected connection specifically emerging from the Supermodel. X-ray observations of clusters yield information concerning the concentration $c$ through the values of the outer entropy slope $\alpha$ and more directly from detailed fits to the surface brightness data; in fact, we expect $\alpha$ to be lower and the density profile to be steeper for early clusters with higher concentrations $c$ (see § 3, in particular below Eqs. [13]). This specific prediction may be tested through extended simulations covering high-$c$ halos and nonadiabatic processes as discussed, e.g., by Borgani (2007). In parallel, high concentrations are keenly pinpointed by gravitational lensing observations (Lapi & Cavaliere 2009b and references therein). This opens the way to the use of existing X-ray data as convenient pointers to targets for time-expensive gravitational lensing observations.

To conclude, we turn to contrast the ICP and the DM behaviors. Note that the two density runs $\rho(r) \propto \left(\sigma^2(r)/K(r)\right)^{1/2}$ and $n(r) \propto \left(k_b T(r)/k(r)\right)^{1/2}$ will differ even where $T \propto \sigma^2$ applies, to the extent that the DM and ICP entropy runs differ. This brings us to directly compare these two governing entropies.

In a nutshell, their common features stem from smooth, slow gravitational mass infall onto the outskirts, while their detailed runs both in the outer and in the central range reflect their different sensitivity to other energy inputs. Quantitatively, both the underlying key parameters $b_R$ and $n_{crit}$ are amenable to conversion of infall kinetic energy. They take on very close values $b_R \approx 2.65-2.55$ (see § 3) and $n_{crit} \approx 2.6-2.5$ (see § 2) at their corresponding fiducial points $r \approx R$ or $r \approx r_p$; for increasing $\alpha$ or concentration $c$, they decrease together since both depend on $1/\Delta \phi$ in terms of the relevant potential drops from the turning point to $R$ or $r_p$ (see Eq. [10] for the ICP and Eq. [13] in LC09 for the DM).

On the other hand, the differing features stem from local vs. non-local character of the energy conversion. The collisionless DM particles fall from the cluster surroundings well into the body, where their kinetic energy is non-locally and progressively randomized, and spreads out entropy by orbit superposition and stratification with widely distributed apocentrers (see LC09 and references therein). Correspondingly, in the DM halos $K(r) \equiv \sigma^2/\rho^{2/3}$ starts out in the outskirts with uniform values $\alpha \sim 1.1$ in all clusters, to steepen toward the body to universal values $1.27 - 1.3$ in a gently convex shape.

Meanwhile, in the ICP $k(r) \equiv k_b T/n^{2/3}$ starts out at the boundary $r \approx R$ from somewhat lower average values $\alpha \approx 1.1 - 1.2$, but with a considerable variance when large preheating and high DM concentrations are included, see § 3.1.1 and evidence referred to therein. The slope, if anything, flattens out toward an effective value $\alpha \approx 0$ in the presence of central energy inputs.

Beyond details, this concave (vs. convex) shape of $a$ toward the center, together with the sensitivity of its boundary values to outer potential and preheating constitute features specific to the ICP. Thus we conclude that basically similar gravitational processes in DM and ICP (randomization of bulk kinetic energy, but on different scales) with the addition of the ICP collisional sensitivity to other energy inputs, concur to produce dissimilar shapes for $K(r)$ and $k(r)$.

Model independently, we propose two observational tests addressed at directly probing in clusters the two underlying entropies. The run $K(r)$ of the DM entropy can be derived from probing the $\alpha$-profile by gravitational lensing observations as recalled in § 2. The run $k(r)$ of the ICP entropy can be reconstructed as proposed by Cavaliere et al. (2005) and illustrated in Fig. 8 starting from the relation

$$k(r) = \gamma^{14/9}(r) \mathcal{L}_{\chi}^{10/9}(r),$$

that joins devolved observations of X-ray brightness and SZ effect irrespective of any modeling or assumption on hydrostatic equilibrium and of redshift information (Cavaliere et al. 1977). Such studies may be particularly useful in the ongoing search for early clusters (see Andreon et al. 2009).
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APPENDIX

THE INTEGRAL IN EQ. (8)

Here we provide an effective approximation to the integral $I(\tilde{r})$ appearing on the right-hand side of Eq. (8), in terms of the following analytical expression

$$I(\tilde{r}) \equiv \int_{F}^{1} \frac{d\tilde{r}'}{\tilde{r}'} \tilde{v}_{c}(\tilde{r}') \tilde{k}^{-3/5}(\tilde{r}') \approx A_{0} \exp(-A_{1} \tilde{r}^{A_{2}});$$

the fitting parameters $A_{0}, A_{1},$ and $A_{2}$ depend weakly on $\tilde{k}$, as specified in Table A1. The dotted lines in Fig. 2 (top panel) illustrate the effectiveness of such an approximation for our three standard values of $\tilde{k}$. Note that on approaching the center the integral is numerically found to scale with central entropy $k_{c}$ like $I_{c} \propto k_{c}^{-1/4}$, see Fig. 2 (bottom panel). This result is used in § 4 of the main text.

We plan to provide elsewhere a set of fitting formulae for $\rho(r)$ and $\tilde{v}_{c}^{2}(r)$ leading to an analytic expression for $I(r)$, as tools enabling direct data analysis and extensive precision fits.

ENTROPY PRODUCTION IN SHOCKS

While cooling may condense out the colder fractions of the ICP and indirectly raise the average entropy of the rest, it is generally agreed (see Cavaliere et al. 2002; Voit 2005) that substantial entropy production requires shockwaves. Here we recall from LCM05 the temperature, density and entropy jumps produced across a shock transitional layer.

Conservation of mass, energy and total stress across the latter lead to the classic Rankine-Hugoniot temperature jump

$$\frac{T_{2}}{T_{1}} = \frac{5}{16} \mathcal{M}^{2} + \frac{7}{8} - \frac{3}{16} \frac{1}{\mathcal{M}^{2}}.$$  

The subscripts 1 and 2 denote the pre- and post-shock quantities, and $\mathcal{M} \equiv (3 \mu m_{p} \tilde{v}_{1}^{2}/5 k_{B} T_{1})^{1/2}$ the shock Mach number; the quantities with tildes refer to the shock reference frame, which is convenient in the case of internal shocks driven, e.g., by AGNs.

On the other hand, in the case of accretion shocks it is more convenient to work in terms of the infall velocity $v_{1}$ and of the related Mach number $\mathcal{M} \equiv (3 \mu m_{p} v_{1}^{2}/5 k_{B} T_{1})^{1/2}$ measured in the center of mass frame. Assuming the downstream kinetic energy to be small, one finds the temperature jump in the form

$$\frac{T_{2}}{T_{1}} = 1 + \frac{4}{9} \mathcal{M}^{2} \left[ 1 + \sqrt{\frac{9}{4} \frac{1}{\mathcal{M}^{2}}} \right].$$

In either reference frame the density jump in terms of the pre- and post-shock temperatures reads

$$\frac{n_{2}}{n_{1}} = 2 \left( 1 - \frac{T_{1}}{T_{2}} \right) + \sqrt{4 \left( 1 - \frac{T_{1}}{T_{2}} \right)^{2} + \frac{T_{1}}{T_{2}}}.$$  

The entropy jump $K_{2}/K_{1} = (T_{2}/T_{1}/n_{2}/n_{1})^{2/3}$ may be easily composed from the relations above.

| $\tilde{k}$ | $\epsilon = 3.5$ | $\epsilon = 4.5$ | $\epsilon = 5.5$ |
|-----------|-----------------|-----------------|-----------------|
|           | $A_{0}$ | $A_{1}$ | $A_{2}$ | $A_{0}$ | $A_{1}$ | $A_{2}$ | $A_{0}$ | $A_{1}$ | $A_{2}$ |
| 0         | 17.77  | 2.86   | 0.49   | 17.82  | 2.72   | 0.47   | 27.59  | 2.64   | 0.47   |
| $10^{-3}$ | 17.47  | 2.86   | 0.49   | 22.33  | 2.71   | 0.48   | 26.88  | 2.63   | 0.48   |
| $10^{-2}$ | 15.93  | 2.83   | 0.54   | 19.99  | 2.66   | 0.54   | 23.72  | 2.56   | 0.53   |
| $5 \times 10^{-3}$ | 13.42  | 2.73   | 0.62   | 16.53  | 2.51   | 0.61   | 19.34  | 2.38   | 0.61   |
| $10^{-2}$ | 12.04  | 2.63   | 0.66   | 14.70  | 2.39   | 0.66   | 17.09  | 2.25   | 0.65   |
| $2 \times 10^{-2}$ | 10.06  | 2.44   | 0.72   | 12.14  | 2.18   | 0.71   | 13.99  | 2.02   | 0.71   |
| $5 \times 10^{-2}$ | 8.53   | 2.26   | 0.77   | 10.19  | 1.99   | 0.75   | 11.66  | 1.83   | 0.74   |
| $7.5 \times 10^{-2}$ | 7.64   | 2.14   | 0.79   | 9.08   | 1.87   | 0.77   | 10.34  | 1.71   | 0.76   |
| $10^{-1}$ | 7.03   | 2.05   | 0.81   | 8.31   | 1.78   | 0.79   | 9.44   | 1.62   | 0.77   |

Note. — In computing the integral Eq. (8) we have used the DM $\alpha$-profiles with $\alpha = 1.27$. The above fitting coefficients are given for three values of the concentration parameter; other values may be derived by standard interpolation techniques.
Handy expressions apply to strong shocks, when the above expressions reduce to

\[
T_2 \approx \frac{3 \mu M_p v_1^2}{16 k_b T_1} + \frac{7}{8} \approx \frac{1}{3 k_b T_1} + \frac{3}{2}, \quad \frac{n_2}{n_1} \approx \frac{4}{1} - \frac{15 T_1}{16 T_2};
\]

(B4)

these approximations actually apply to high/intermediate \( \mathcal{M}^2 \gtrsim 3 \); the corresponding entropy jumps read

\[
\frac{K_2}{K_1} \approx \frac{3}{16} 4^{2/3} \mu M_p v_1^2 \left( \frac{1}{2} \right) + \frac{1}{3} 4^{2/3} \mu M_p v_1^2 \left( \frac{1}{8} \right) \approx \frac{17}{1} 4^{2/3} \mu M_p v_1^2 \left( \frac{1}{8} \right),
\]

(B5)

In last relation, the second term on the r.h.s. expresses the contributions of the advected external entropy \( 0.84 K_1 \), and is relevant when \( k_b T_1 \gtrsim 0.16 \mu M_p v_1^2 \), that is, when relatively strong preheating affects the gas infalling into a poor cluster or a group.

When a central energy pulse is discharged into the equilibrium ICP with a density gradient, e.g. \( n(r) \propto r^{-2} \), a blast wave is sent out, that is, a non-linear perturbed flow that terminates into a leading shock and comprises bulk kinetic energy up to matching the thermal one (see Sedov 1959). When the pulse is short compared with the transit time and the effects of gravity and upstream pressure are neglected, the Mach number decreases radially as \( \mathcal{M}(r) \propto r^{-1/2} \). When the pulse is sustained during the transit time, also \( \mathcal{M}(r) \) will be. Even with gravity and upstream pressure considered, this time is held in the simple case of a pulse constant over the transit time through the region where \( n(r) \propto r^{-2} \) applies, and to hold also for other combinations of equilibrium gradients and pulse shapes (LCM05). For longer times, \( \mathcal{M}(r) \) declines and the blast dissipates its kinetic energy into the ICP.

In such cases the Mach number depends on the energy input \( \Delta E \) relative to the ICP total energy \( E \) in the affected volume, simply as \( \mathcal{M}^2 \approx 1 + \Delta E/E \); the condition for a strong shock \( \mathcal{M}^2 \gtrsim 3 \) clearly translates into \( \Delta E/2m_k T > 1 \). It turns out that the ICP may be (partly) evacuated from the central region to a residual average density \( n(1-\Delta n/n) \propto 1 - \Delta E/E \), with an associated entropy \( k \propto \Delta E/(1 - \Delta E/E)^{3/2} \).