Symplectic structure of $\mathcal{N} = 1$ supergravity with anomalies and Chern–Simons terms

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Abstract

The general actions of matter-coupled $\mathcal{N} = 1$ supergravity have Peccei–Quinn terms that may violate gauge and supersymmetry invariance. In addition, $\mathcal{N} = 1$ supergravity with vector multiplets may also contain generalized Chern–Simons terms. These have often been neglected in the literature despite their importance for gauge and supersymmetry invariance. We clarify the interplay of Peccei–Quinn terms, generalized Chern–Simons terms and quantum anomalies in the context of $\mathcal{N} = 1$ supergravity and exhibit conditions that have to be satisfied for their mutual consistency. This extension of the previously known $\mathcal{N} = 1$ matter-coupled supergravity actions follows naturally from the embedding of the gauge group into the group of symplectic duality transformations. Our results regarding this extension provide the supersymmetric framework for studies of string compactifications with axionic shift symmetries, generalized Chern–Simons terms and quantum anomalies.

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1. Introduction

Matter couplings in low-energy effective actions of string compactifications generically depend on scalar fields, such as the moduli. An important example of such a scalar field dependence is provided by non-minimal kinetic terms for gauge fields$^3$ (here enumerated by an index $A$),

$$e^{-1} \mathcal{L}_1 = -\frac{1}{4} \text{Re} \ f_{AB} \mathcal{F}^{A}_{\mu\nu} \mathcal{F}^{B}_{\rho\sigma} + \frac{1}{4} \text{Im} \ f_{AB} \mathcal{F}^{A}_{\mu\nu} \mathcal{F}^{B*}_{\rho\sigma},$$

(1.1)

$^3$ We use the non-Abelian field strength $\mathcal{F}^A_{\mu\nu} = F^A_{\mu\nu} + W^B_{\mu} W^C_{\nu} f^A_{BC}$, where $F^A_{\mu\nu} = 2\delta_{[\mu} W^A_{\nu]}$ is the Abelian part. The tilde denotes the Hodge dual, as is further specified in the appendix.
where the gauge kinetic function $f_{AB}(z)$ is a non-trivial function of the scalar fields, $z^i$, which, in $\mathcal{N} = 1$ supersymmetry, has to be holomorphic. The second term in (1.1) is often referred to as the Peccei–Quinn term.

If, under a gauge transformation with gauge parameter $\Lambda^A(x)$, some of the $z^i$ transform non-trivially, this may induce a corresponding gauge transformation of $f_{AB}(z)$. If this transformation is of the form of a symmetric product of two adjoint representations of the gauge group,

$$\delta(\Lambda) f_{AB} = \Lambda^C \delta_C f_{AB},$$

$$\delta(C) f_{AB} = f_{CA}^D f_{DB} + f_{CB}^D f_{DA},$$

(1.2)

with $f_{CA}^B$ being the structure constants of the gauge group, the kinetic term (1.1) is obviously gauge invariant. This is what was assumed in the action of general matter-coupled supergravity in [1].

If one also takes into account other terms in the (quantum) effective action, however, a more general transformation rule for $f_{AB}(z)$ may be allowed:

$$\delta(C) f_{AB} = i C_{AB,C} f_{CA}^D f_{DB} + f_{CB}^D f_{AD}.$$

(1.3)

Here, $C_{AB,C}$ is a constant real tensor symmetric in the first two indices, which we will recognize as a natural generalization in the context of symplectic duality transformations.

If $C_{AB,C}$ is non-zero, this leads to a non-gauge invariance of the Peccei–Quinn term in $\mathcal{L}_1$:

$$\delta(C) e^{-1} \mathcal{L}_1 = \frac{i}{3} C_{AB,C} \Lambda^C \mathcal{T}^{\mu}_{\nu\rho\sigma} \mathcal{F}^\mu_{\nu\rho\sigma}.$$

(1.4)

For rigid parameters, $\Lambda^A = \text{constant}$, this is just a total derivative, but for local gauge parameters, $\Lambda^A(x)$, it is obviously not. If (1.1) is a part of a supersymmetric action, the gauge non-invariance (1.4) also induces a non-invariance of the action under supersymmetry, as we will recall in section 3.

In order to understand how this broken gauge and supersymmetry invariance can be restored, it is convenient to split the coefficients $C_{AB,C}$ into a sum,

$$C_{AB,C} = C_{AB,C}^{(s)} + C_{AB,C}^{(m)}; \hspace{1cm} C_{AB,C}^{(s)} = C_{(AB,C)}, \hspace{1cm} C_{AB,C}^{(m)} = 0,$$

(1.5)

where $C_{AB,C}^{(s)}$ is completely symmetric and $C_{AB,C}^{(m)}$ denotes the part of mixed symmetry. Terms of the form (1.4) may then in principle be cancelled by the following two mechanisms, or a combination thereof:

(i) As was first realized in a similar context in $\mathcal{N} = 2$ supergravity in [3] (see also the systematic analysis [4]), the gauge variation due to a non-vanishing mixed part, $C_{AB,C}^{(m)} \neq 0$, may be cancelled by adding a generalized Chern–Simons term (GCS term) that contains a cubic and a quartic part in the vector fields

$$\mathcal{L}_{CS} = \frac{i}{2} C_{AB,C}^{(CS)} \mathcal{T}^{\mu}_{\nu\rho\sigma} \left( \frac{1}{2} W^A_{\mu} F^B_{\nu\rho\sigma} + \frac{1}{2} A^A_{\mu} D^D_{\nu\rho\sigma} W^E_{\mu} W^C_{\rho} W^B_{\sigma} \right).$$

(1.6)

This term depends on a constant tensor $C_{AB,C}^{(CS)}$, which has also a mixed symmetry structure. The cancellation occurs provided the tensors $C_{AB,C}^{(m)}$ and $C_{AB,C}^{(CS)}$ are the same. It has been shown in [5] that such a term exists as well in rigid $\mathcal{N} = 1$ supersymmetry.

(ii) If the chiral fermion spectrum is anomalous under the gauge group, the anomalous triangle diagrams lead to a non-gauge invariance of the quantum effective action of the form

4 This construction of general matter couplings has been reviewed in [2]. There, the possibility (1.3) was already mentioned, but the extra terms necessary for its consistency were not considered.

5 This corresponds to the decomposition $\mathcal{T} = \mathcal{F} \cdot \mathcal{F} \cdot \mathcal{F} \cdot \mathcal{F}$. 
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\[ d_{ABC} \Lambda^C \frac{\mathcal{F}^{A\mu}_\nu}{\Lambda^\nu} \tilde{F}^{\mu\nu B} \] with a symmetric tensor $d_{ABC} \propto \text{Tr}(\{T_A, T_B\} T_C)$. If $C^{(s)}_{AB,C} = d_{ABC}$, this quantum anomaly cancels the symmetric part of (1.4). This is the Green–Schwarz mechanism.

As has recently been emphasized in [6], both the Green–Schwarz mechanism and the GCS terms are generically needed to cancel the anomalies in orientifold models with intersecting D-branes. Moreover, it is argued in [6] that non-vanishing GCS terms might have observable consequences for certain variants of $Z'$ bosons. On the other hand, as described in [5], GCS terms may also arise in certain flux and generalized Scherk–Schwarz compactifications. Finally, they also play a role in the manifestly symplectic formulation of gauged supergravity with electric and magnetic potentials and tensor fields introduced in [7].

In view of these applications, it is surprising that the full interplay between gauge invariance and (local) supersymmetry in the presence of GCS terms and quantum anomalies is still only partially understood. In fact, before the work of [5], supersymmetric GCS terms were only studied in the context of extended supersymmetry [3, 8–12]. We would like to point out, however, that there is an important qualitative difference between $\mathcal{N} = 1$ and $\mathcal{N} \geq 2$ supersymmetry. In extended supersymmetry, the $C^{AB,C}_{AB,C}$ of (1.3) have no symmetric part. This has already been pointed out in [3] for the vector multiplets in $\mathcal{N} = 2$ supergravity, at least in the presence of a prepotential. The equation $C^{(s)}_{AB,C} = 0$ is also the basis of the manifestly symplectic formulation [7], where it is motivated by constraints known from $\mathcal{N} = 8$ supergravity. In $\mathcal{N} = 1$ supergravity, by contrast, we find that the symmetric part of $C^{AB,C}$ may be present and could in principle cancel quantum anomalies. This is consistent with the above-mentioned results on extended supergravity theories, because only $\mathcal{N} = 1$ supergravity has the chiral fermions that could possibly produce these quantum anomalies.

It is the purpose of this paper to give a systematic discussion of the structure of general $\mathcal{N} = 1$ supersymmetry with anomaly cancellation and GCS terms. We will do this for a general gauge kinetic function and an arbitrary gauge group with quantum anomalies. We also consider the full coupling to supergravity and discuss its embedding into the framework of the symplectic duality transformations. This generalizes the work of [5], which was restricted to linear gauge kinetic functions of theories without quantum anomalies and to rigid supersymmetry. As far as supersymmetry is concerned, the quantum anomalies of the gauge symmetries are as important as a classical violation of gauge invariance, because the quantum anomalies of the gauge symmetries also lead to supersymmetry anomalies as a consequence of the supersymmetry algebra. The consistent gauge and supersymmetry anomalies have been found for supergravity in [13]. Our result for the non-invariance of the sum of the kinetic terms and GCS terms in the classical action matches with the results of [13].

The paper is organized as follows. In section 2 we explain how symplectic transformations act in $\mathcal{N} = 1$ supersymmetry, and how this leads to the generalized transformation (1.3) of the gauge kinetic function $f_{AB}$.

In the subsequent three sections, we first consider rigid supersymmetry. More concretely, in section 3 we explore the non-invariance of the kinetic terms of the vector multiplets under gauge and supersymmetry transformations caused by (1.3). In section 4, the GCS action and its role in the restoration of gauge and supersymmetry invariance are discussed. Third, in section 5, we consider the quantum anomaly as obtained in [13, 14]. Finally, we analyse the complete cancellation of the gauge and supersymmetry anomalies by using the results of the two previous sections.

More precisely, the anomalies have a scheme dependence. As reviewed in [6] one can choose a scheme in which the anomaly is proportional to $d_{ABC}$. Choosing a different scheme is equivalent to the choice of another GCS term (see item 1). We will always work with a renormalization scheme in which the quantum anomaly is indeed proportional to $d_{ABC}$. 
The generalization to supergravity is considered in section 6. It turns out that the GCS terms obtained before can just be added to the general actions of matter-coupled supergravity. To show how this works in practice, it is useful to look at a gauge group that is the product of an Abelian and a semisimple group. This setup was also considered in [6, 15] and [16, 17]. Our discussion in section 7 is close to the last reference, where it is mentioned that local counterterms turn the consistent mixed anomalies into a covariant mixed anomaly. This is the form of the anomaly that appears as variation of the vector multiplet kinetic terms. The GCS terms that we consider are precisely the counterterms that are mentioned in [17].

We finish with conclusions and remarks in section 8 and some notational issues are summarized in the appendix.

2. Symplectic transformations in $\mathcal{N} = 1$ supersymmetry

In this section, we derive the general form (1.3) of the gauge transformation of the gauge kinetic function from the viewpoint of symplectic duality transformations. We begin by recalling the essential elements of the duality transformations in four dimensions [18–21]. The general form of kinetic terms for vector fields can be written in several ways:

$$ e^{-\frac{1}{2} L_1} = -\frac{1}{2} \Re f_{AB} F^{\mu A}_{\mu
u} F_{\mu
u B} + \frac{1}{2} \Im f_{AB} F^{\mu A}_{\mu
u} F_{\mu
u B}, $$

$$ = -\frac{1}{2} \Re \left( f_{AB} F^{\mu A}_{\mu
u} F_{\mu
u B} \right) = -\frac{1}{2} \Im \left( F^{\mu A}_{\mu
u} G_{\mu\nu}^A \right), \quad (2.1) $$

where the dual field strength is defined as

$$ G_{\mu\nu}^A = -2i \frac{\partial e^{-\frac{1}{8} L_1}}{\partial F^{\mu A}_{\mu
u}} = i f_{AB} F^{\mu A}_{\mu
u}. \quad (2.2) $$

This shows that the Bianchi identities and field equations can be written as

$$ \partial^{\mu} \Im F^{A\nu}_{\mu
u} = 0 \quad \text{Bianchi identities}, $$

$$ \partial_{\mu} \Im G^{\mu\nu A}_{\nu} = 0 \quad \text{equations of motion}. \quad (2.3) $$

The set (2.3) is invariant under the duality transformations

$$ \begin{pmatrix} F'_{\mu\nu} \\ G'_{\mu\nu} \end{pmatrix} = S \begin{pmatrix} F_{\mu\nu} \\ G_{\mu\nu} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F_{\mu\nu} \\ G_{\mu\nu} \end{pmatrix}, \quad (2.4) $$

where the real matrices $A$, $B$, $C$ and $D$ satisfy

$$ A^T C - C^T A = 0, \quad B^T D - D^T B = 0, \quad A^T D - C^T B = \mathbb{1}. \quad (2.5) $$

This guarantees that $S$ is a symplectic matrix. In order to have $G'$ of the form (2.2), the kinetic matrix $f_{AB}$ is transformed into $f'_{AB}$, where

$$ i f' = (C + Di f)(A + Bi f)^{-1}. \quad (2.6) $$

Symmetries of the action (2.1) correspond to symplectic transformations with $B = 0$, for which the Lagrangian (2.1) transforms into itself plus a total derivative if $C \neq 0$:

$$ e^{-\frac{1}{2} L'_1} = -\frac{1}{2} \Im \left( F^{\mu A}_{\mu
u} G_{\mu\nu}^A \right) $$

$$ = -\frac{1}{2} \Im \left( F^{\mu A}_{\mu
u} G_{\mu\nu}^A + f_{AB} (C^T A)_{AB} F^{\mu A}_{\mu\nu} \right). \quad (2.7) $$

Not all of these rigid symmetries of the action can be promoted to gauge symmetries. For this to be possible, the field strengths $F^{A}_{\mu\nu}$ have to transform in the adjoint representation of the prospective gauge group. This determines the upper line of the transformation (2.4).

7 The duality transformations, and hence the formulae in the first part of this section, apply to the ungauged action.
We do not know a priori the transformation rule of $f_{AB}$ and hence of $G_{\mu
u A}$. The conditions (2.5), however, further restrict the corresponding symplectic matrices to a form, which, at the infinitesimal level, reads

$$S = \mathbb{1} - \Lambda^C S_C,$$

$$S_C = \begin{pmatrix} f_{CB}^A & 0 \\ C_{AB,C} & -f_{CA}^B \end{pmatrix},$$

(2.8)

where $C_{AB,C}$ is a real undetermined tensor, symmetric in its first two indices. According to (2.6), the kinetic matrix should then transform under the gauge transformations as

$$\delta(\Lambda) f_{AB} = \Lambda^C \delta_C f_{AB}, \quad \delta_C f_{AB} = i C_{AB,C} f_{CB} f_{AD} + f_{CA} f_{BD}.$$  

(2.9)

The last two terms state that $f_{AB}$ transforms in the symmetric product of two adjoint representations. The first term is the correction to this and corresponds to the possible generalization by axionic shift symmetries mentioned in the introduction. Note that the gauge kinetic function might now transform non-trivially also under Abelian symmetries.

The algebra of gauge transformations is

$$\left[\delta(\Lambda_1), \delta(\Lambda_2)\right] = \delta(\Lambda_3) = \Lambda_3^A \Lambda_1^B f_{AB}.$$  

(2.10)

In order that this algebra is realized by the symplectic transformations (2.8), the commutators of the matrices $S_A$ should be of the form

$$[S_A, S_B] = f_{AB}^C S_C.$$  

(2.11)

Written in full, this includes the equation

$$C_{AB,E} f_{CD}^E - 2 C_{AE,[C} f_{D]B}^E - 2 C_{BE,[C} f_{D]A}^E = 0,$$  

(2.12)

which is the consistency condition that can be obtained by acting with $\delta_D$ on (2.9) and antisymmetrizing in $[CD]$.

Whether or not the $C_{AB,C}$ can really be non-zero in a gauge theory, and to what extent this could be consistent with $\mathcal{N} = 1$ supersymmetry is the subject of the remainder of this paper.

We finally note that, in this section, we considered only the vector kinetic terms. The symplectic formulation also gives insight into other terms of the action, which has been explored in [22]. The additional terms to the action that we will discuss in this paper do not modify this analysis. This is due to the fact that these new terms do not involve the auxiliary fields $D$, while the analysis of [22] is essentially dependent on the terms that result from the elimination of these auxiliary fields.

3. Kinetic terms of the vector multiplet

Allowing for a nonvanishing shift $i C_{AB,C}$ in $\delta_C f_{AB}$ breaks both the gauge and supersymmetry invariance. In this section, we make this statement more precise and begin our discussion with some subtleties associated with the superspace formulation in the Wess–Zumino gauge.

3.1. The action

The vector multiplet in the $\mathcal{N} = 1$ superspace formulation is described by a real superfield. The latter has many more components than the physical fields describing an on-shell vector multiplet, which consists of one vector field and one fermion. The advantage of this redundancy is that one can easily construct manifestly supersymmetric actions as integrals over full or chiral superspace. As an example consider the expression

$$S_f = \int d^4 x \, d^2 \theta f_{AB}(X) W_\alpha^{A} W_\beta^{B} \epsilon^{\alpha \beta} + \text{c.c.}$$  

(3.1)
Here, $W^A = \frac{1}{2} D^2 D_a V^A$, or a generalization thereof for the non-Abelian case, where $V^A$ is the real superfield describing the vector multiplets labelled by an index $A$. The $f_{AB}$ are arbitrary holomorphic functions of a set of chiral superfields denoted by $X$.

The integrand of (3.1) is itself a chiral superfield. As we integrate over a chiral superspace, the Lagrangian transforms into a total derivative under supersymmetry. Formally, this conclusion holds independently of the gauge symmetry properties of the functions $f_{AB}(X)$.

For the action (3.1) to be gauge invariant, we should have the condition
\[
\delta_C f_{AB} - f_{CA}^D f_{DB} - f_{AD} f_{CB}^D = 0, \quad (3.2)
\]
where $\delta_C$ denotes the gauge transformation under the gauge symmetry related to the vector multiplet denoted by the index $C$ as in (2.9).

Due to the large number of fields in the superspace formulation, the gauge parameters are not just real numbers, but are themselves full chiral superfields. To describe the physical theory, one wants to get rid of these extra gauge transformations and thereby also many spurious components of the vector superfields. This is done by going to the so-called Wess–Zumino gauge [23], in which these extra gauge transformations are fixed and the many spurious components of the real superfields are eliminated. Unfortunately, the Wess–Zumino gauge also breaks the manifest supersymmetry of the superspace formalism. However, a combination of this original ‘superspace supersymmetry’ and the gauge symmetries survives and becomes the preserved supersymmetry after the gauge fixing. The law that gives the preserved supersymmetry as a combination of these different symmetries is called the ‘decomposition law’, see e.g. equation (2.28) in [1]. Notice, however, that this preservation requires the gauge invariance of the original action (3.1). Thus, though (3.1) was invariant under the superspace supersymmetry for any choice of $f_{AB}$, we now need (3.2) for this action to be invariant under supersymmetry after the Wess–Zumino gauge.

This important consequence of the Wess–Zumino gauge can also be understood from the supersymmetry algebra. The superspace operator $Q_\alpha$ satisfies the anticommutation relation
\[
\{ Q_\alpha, Q_\beta^\dagger \} = \sigma^\mu_{\alpha \dot{\beta}} \delta_\mu. \quad (3.3)
\]
This equation shows no mixing between supersymmetry and gauge symmetries. However, after the Wess–Zumino gauge the right-hand side is changed to [24]
\[
\{ Q_\alpha, Q_\beta^\dagger \} = \sigma^\mu_{\alpha \dot{a}} D_\mu = \sigma^\mu_{\alpha \dot{a}} (\delta_\mu - W^A_{\mu} \delta_A), \quad (3.4)
\]
where $\delta_A$ denotes the gauge transformation. Equation (3.4) implies that if an action is invariant under supersymmetry, it should also be gauge invariant.

As mentioned before, the preservation of the Wess–Zumino gauges implies that the effective supersymmetry transformations are different from those in the original superspace formulation. It is shown in [24] that the resulting supersymmetry transformations of a chiral multiplet are
\[
\delta(\epsilon) z^i = \bar{\epsilon}_L \chi^i_L, \\
\delta(\epsilon) \chi^i_L = \frac{1}{2} \gamma^\mu \epsilon_R D_\mu z^i + \frac{1}{2} h' \epsilon_L, \\
\delta(\epsilon) h^i = \bar{\epsilon}_L \mathcal{D} \chi^i_L + \bar{\epsilon}_R \lambda^A_{\alpha} \delta_A z^i. \quad (3.5)
\]
where we have denoted the scalar fields of the chiral multiplets as $z^i$, the left-chiral components of the corresponding fermions as $\chi^i_L$, and the auxiliary fields as $h^i$, while $\lambda^A_{\alpha}$ is the gaugino of the vector multiplet $V^A$. These transformations are valid for any chiral multiplet, in particular, they can be applied to the full integrand of (3.1) itself. We will make use of this in section 3.2.
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Compared to the standard superspace transformations, there are two modifications in (3.5). The first modification is that the derivatives of $z^i$ and $\chi^i_L$ are covariantized with respect to gauge transformations. This covariant derivative acts on the chiral fermions $\chi^i_L$ as

$$D_\mu \chi^i_L = \partial_\mu \chi^i_L - W^A_\mu \delta_A \chi^i_L.$$  \hfill (3.6)

Here, the gauge variation of the chiral fermions, $\delta_A \chi^i_L$, can be expressed in terms of the gauge variation, $\delta_A z^i$, of the scalar fields, using the fact that supersymmetry and gauge transformations commute,

$$\delta(\epsilon) \delta_A \chi^i_L = \delta_A \delta(\epsilon) \chi^i_L = \bar{\epsilon}_L \delta_A \chi^i_L.$$  \hfill (3.7)

This leads to

$$\delta_A \chi^i = \frac{\partial \delta_A z^i}{\partial z^j} \chi^j.$$  \hfill (3.8)

The second modification is the additional last term in the transformation of the auxiliary fields $h^i$. The origin of this term lies in the contribution of the decomposition law for one of the gauge symmetries contained in the chiral superfield of transformations, after the Wess–Zumino gauge is fixed.

To avoid the above-mentioned subtleties associated with the Wess–Zumino gauge, we will use component field expressions in the remainder of this text. Therefore, we reconsider the action (3.1) and in particular its integrand. The components of this composite chiral multiplet are \[1\]

$$z(f W^2) = -\frac{1}{2} f_{AB} \bar{\lambda}^A \lambda^B,$$

$$\chi_L(f W^2) = \frac{1}{2} f_{AB} \left( \frac{1}{2} \gamma^\mu \gamma^\nu F^A_{\mu\nu} - i D^A \right) \lambda^B - \frac{1}{2} \partial_i f_{AB} \chi^i_L \bar{\lambda}^A \lambda^B,$$

$$h(f W^2) = f_{AB} \left( -\bar{\lambda}_L^A \gamma^\mu \gamma^\nu \tilde{F}^A_{\mu\nu} + \frac{1}{2} D^A \gamma^5 \gamma^5 \gamma^\mu \gamma^\nu \bar{\lambda}_L^A \right) + \partial_i f_{AB} \chi^i_L \left( -\frac{1}{2} \gamma^\mu \gamma^\nu F^A_{\mu\nu} + i D^A \right) \lambda^B,$$

(3.9)

where we used the notation $\partial_i = \partial/\partial z^i$. The superspace integral in (3.1) means that the real part of $h(f W^2)$ is (proportional to) the Lagrangian:

$$S_f = \int d^4 x \frac{1}{2} \text{Re} \left( f_{AB} \bar{\lambda}^A \lambda^B \right).$$

(3.10)

From (3.9) and (3.10), we read off the kinetic terms of $S_f$:

$$S_{f,\text{kin}} = \int d^4 x \left[ -\frac{1}{4} \text{Re} \left( f_{AB} \gamma^\mu \gamma^\nu F^A_{\mu\nu} \right) - \frac{1}{2} \text{Re} \left( f_{AB} \bar{\lambda}^A \gamma^\mu \gamma^\nu \tilde{F}^A_{\mu\nu} \right) + \frac{1}{4} \text{Im} \left( f_{AB} \bar{\lambda}^A \gamma^5 \gamma^\mu \gamma^\nu \lambda^B \right) + \frac{1}{4} \text{Im} \left( f_{AB} \bar{\lambda}^A \gamma^\mu \gamma^\nu \tilde{F}^A_{\mu\nu} \right) + \frac{1}{4} \left( D^A \gamma^5 \gamma^5 \gamma^\mu \gamma^\nu \bar{\lambda}_L^A \lambda^B \right) \right].$$

(3.11)

In comparison to [1], we have used a partial integration to shift the derivative from the gaugini to (Im $f_{AB}$) and rearranged the structure constants in the last term, so as to obtain a ‘covariant’ derivative acting on (Im $f_{AB}$). More precisely, we define

$$D_\mu f_{AB} = \partial_\mu f_{AB} - 2 W^C_{\mu} f_{C(A} \delta f_{B)D}.$$ 

(3.12)

In the case that the gauge kinetic matrix transforms without a shift, as in (3.2), the derivative defined in (3.12) is fully gauge covariant.

In section 2, we motivated a more general gauge transformation rule for $f_{AB}$, in which axionic shifts proportional to $C_{ABC}$ are allowed as in (2.9). Then (3.12) is no longer the full covariant derivative. The full covariant derivative instead has the new form

$$\hat{D}_\mu f_{AB} \equiv \partial_\mu f_{AB} - W^C_{\mu} C_{ABC} f_{AB} = D_\mu f_{AB} - i W^C_{\mu} C_{ABC}.$$ 

(3.13)

We should remark here that [5] restrict their work to the case in which $f_{AB}$ is at most linear in scalars, and these scalars undergo a shift. This is the most relevant way in which (2.9) can be realized.
The last term in (3.11) is therefore not gauge covariant for non-vanishing \( C_{AB,C} \). Hence, in the presence of the new term in the transformation of \( f_{AB} \) we replace the action \( S_f \) with \( \hat{S}_f \), in which we use the full covariant derivative, \( \hat{D}_\mu \), instead of \( D_\mu \). More precisely, we define

\[
\hat{S}_f = S_f + S_{\text{extra}}, \quad S_{\text{extra}} = \int d^4x \left( -\frac{1}{4} i W^C \epsilon^{ABC} \bar{\xi}^A \gamma^\mu \gamma^\nu \xi^B \right).
\] (3.14)

Note that we did not use any superspace expression to derive \( S_{\text{extra}} \) but simply added \( S_{\text{extra}} \) by hand in order to fully covariantize the last term of (3.11). As we will further discuss in the next section, \( S_{\text{extra}} \) can in fact only be partially understood from superspace expressions, which motivates our procedure to introduce it here by hand. We should also stress that the covariantization with \( S_{\text{extra}} \) does not yet mean that the entire action \( \hat{S}_f \) is now fully gauge invariant. The gauge and supersymmetry transformations of \( \hat{S}_f \) will be discussed in section 3.2.

We would finally like to emphasize that, in the context of \( N = 1 \) supersymmetry, there is a priori no further restriction on the symmetry of \( C_{AB,C} \) apart from its symmetry in the first two indices. This, however, is different in extended supersymmetry, as is most easily demonstrated for \( N = 2 \) supersymmetry, where the gauge kinetic matrix depends on the complex scalars \( X^A \) of the vector multiplets. These transform themselves in the adjoint representation, which implies

\[
\delta(\Lambda) f_{AB}(X) = X^E \Lambda^C f_{EC} \delta_D f_{AB}(X). \tag{3.15}
\]

Hence, this gives, from (2.9),

\[
iC_{AB,C} = X^E f_{EC} \delta_D f_{AB}(X) - f_{CA} \delta_D f_{BD} - f_{CB} \delta_D f_{AD}, \tag{3.16}
\]

which leads to \( C_{AB,C} X^A X^B X^C = 0 \). As the scalars \( X^A \) are independent in rigid supersymmetry, this implies that \( C_{(AB,C)} = 0 \).

### 3.2. Gauge and supersymmetry transformations

The action \( S_f \) is gauge invariant before the modification of the transformation of \( f_{AB} \). In the presence of the \( C_{AB,C} \) terms, the action \( \hat{S}_f \) is not gauge invariant. However, the non-invariance comes only from one term. Indeed, terms in \( \hat{S}_f \) that are proportional to derivatives of \( f_{AB} \) do not feel the constant shift \( \delta_C f_{AB} = i C_{AB,C} + \cdots \). They are therefore automatically gauge invariant. Also, the full covariant derivative (3.13) has no gauge transformation proportional to \( C_{AB,C} \), and also Re \( f_{AB} \) is invariant. Hence, the gauge non-invariance originates only from the third term in (3.11). We are thus left with

\[
\delta(\Lambda) \hat{S}_f = \frac{1}{4} iC_{AB,C} \int d^4x \Lambda^C f_{EC} \hat{\partial}^\mu \hat{\partial}^\nu \hat{f}^{AB}. \tag{3.17}
\]

This expression vanishes for constant \( \Lambda \), but it spoils the local gauge invariance.

We started to construct \( S_f \) as a superspace integral, and as such it would automatically be supersymmetric. However, we saw that when \( f_{AB} \) transforms with a shift as in (2.9), the gauge symmetry is broken, which is then communicated to the supersymmetry transformations by

\footnote{The same argument can be made for supergravity in the symplectic bases in which there is a prepotential. However, that is not the case in all symplectic bases. Bases that allow a prepotential are those where \( X^A \) can be considered as independent \cite{22, 25}. An analogous argument for other symplectic bases is missing. This is remarkable in view of the fact that spontaneous breaking to \( N = 1 \) needs a symplectic basis that allows no prepotential \cite{26}. Hence, for the \( N = 2 \) models that allow such a breaking to the \( N = 1 \) theories that we are considering in this paper, there is also no similar argument for the absence of a totally symmetric part in \( C_{AB,C} \), except that for \( N = 2 \) there are no anomalies that could cancel the corresponding gauge variation, due to the non-chiral nature of the interactions.}
the Wess–Zumino gauge fixing. The $C_{AB,C}$ tensors then express the non-invariance of $S_f$ under both gauge transformations and supersymmetry.

To determine these supersymmetry transformations, we consider the last line of (3.5) for $\{z^i, \chi^i, h^i\}$ replaced by $\{z(f W^2), \chi(f W^2), h(f W^2)\}$ and find

$$\delta (\epsilon) S_f = \int d^4x \text{Re} \left[ iC_{AB,C} \left[ -\bar{\epsilon} R \gamma^\mu W^C_\mu \left( \frac{1}{4} \gamma^{\rho \sigma} f_{A}^{\rho A} - \frac{1}{2} iD^A \right) \chi_L^B - \frac{1}{2} \bar{\epsilon} R^{\gamma^A \epsilon^B \gamma_L^C} \bar{\epsilon} R \bar{\chi} \right] \right].$$

(3.18)

The first term in the transformation of $h(f W^2)$ is the one that was already present in the superspace supersymmetry before going to Wess–Zumino gauge. It is a total derivative, as we would expect from the superspace rules. The other two terms are due to the mixing of supersymmetry with gauge symmetries. They vanish if $z(f W^2)$ is invariant under the gauge symmetry, as this implies by (3.7) that $\chi(f W^2)$ is also gauge invariant.

Using (3.9) and (2.9), however, one sees that $z(f W^2)$ is not gauge invariant, and (3.18) becomes, using also (3.8),

$$\delta (\epsilon) S_f = \int d^4x \text{Re} \left( \frac{1}{2} C_{AB,C} \epsilon^{\mu \nu \rho \sigma} W^C_\mu \left( -\frac{1}{2} C_{\rho A}^{\nu B} \gamma^B - \frac{3}{2} iC_{(AB,C)} \bar{\epsilon} R^{\gamma^A \epsilon^B \gamma_L^C} \right) \right).$$

(3.19)

Note that this expression contains only fields of the vector multiplets and none of the chiral multiplets.

It remains to determine the contribution of $S_{\text{extra}}$ to the supersymmetry variation, which turns out to be

$$\delta (\epsilon) S_{\text{extra}} = \int d^4x \text{Re} iC_{AB,C} \left[ -\frac{1}{2} W^C_\mu \gamma^\mu \gamma^A - \frac{1}{2} iD^A \right].$$

(3.20)

By combining this with (3.19), we obtain, after some reordering,

$$\delta (\epsilon) \hat{S}_f = \int d^4x \text{Re} \left( \frac{1}{2} C_{AB,C} \epsilon^{A B \rho \sigma} W^C_\rho \left( -\frac{1}{2} C_{\rho A}^{B} \gamma^B - \frac{3}{2} iC_{(AB,C)} \bar{\epsilon} R^{\gamma^A \epsilon^B \gamma_L^C} \right) \right).$$

(3.21)

In sections 4 and 5, we describe how the addition of GCS terms and quantum anomalies can cancel the left-over gauge and supersymmetry non-invariances of equations (3.17) and (3.21).

4. Chern–Simons action

4.1. The action

Due to the gauged shift symmetry of $f_{AB}$, terms proportional to $C_{AB,C}$ remain in the gauge and supersymmetry variation of the action $\hat{S}_f$. To re-establish the gauge symmetry and supersymmetry invariance, we need two ingredients: GCS terms and quantum anomalies. The former were in part already discussed in [3–5]. They are of the form

$$S_{\text{CS}} = \int d^4x \frac{1}{2} C_{AB,C}^{(\text{CS})} \epsilon^{\mu \nu \rho \sigma} \left( \frac{1}{3} W^C_\mu W^A_\nu F^B_\rho + \frac{1}{4} f_{DE}^A W^D_\mu W^E_\nu W^C_\rho \right).$$

(4.1)

The GCS terms are proportional to a tensor $C_{AB,C}^{(\text{CS})}$ that is symmetric in $(A, B)$. Note that a completely symmetric part in $C_{AB,C}^{(\text{CS})}$ would drop out of $S_{\text{CS}}$ and we can therefore restrict $C_{AB,C}^{(\text{CS})}$ to be a tensor of a mixed symmetry structure, i.e. with

$$C_{(AB,C)}^{(\text{CS})} = 0.$$  

(4.2)

A priori, the constants $C_{AB,C}^{(\text{CS})}$ need not be the same as the $C_{AB,C}$ introduced in the previous section. For $\mathcal{N} = 2$ supergravity [3] one needs them to be the same, but we will, for
$N = 1$, establish another relation between both, which follows from supersymmetry and gauge invariance requirements.

As was described in [5], the GCS terms can be obtained from a superfield expression:

$$S_{\text{CS}} = c_{AB,C}^{(\text{CS})} \int d^4x \, d^4\theta \left[ -\frac{2}{3} V^C \Omega^{AB}(V) + \left( f_{DE}^B V^C \, D^\mu V^A \, D^2(D_\alpha V^D) + \text{c.c.} \right) \right], \quad (4.3)$$

where $W$ is the restriction $W_{\mu
u} = Z_{AB} \epsilon_{\mu
u}$ into the superspace expression $S$ in (3).

The full non-Abelian superspace expression (4.3) is valid only in the Wess–Zumino gauge, where it reduces to the bosonic component expression (4.1) plus a fermionic term [5]:

$$S_{\text{CS}} = S_{\text{CS}} + (S_{\text{CS}})_{\text{term}}, \quad (S_{\text{CS}})_{\text{term}} = \int d^4x \left( -\frac{1}{4} i c_{AB,C}^{(\text{CS})} W_{\mu}^C \gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\nu \right). \quad (4.4)$$

where we used the restriction $c_{AB,C}^{(\text{CS})} = 0$ from (4.2).

Note that the fermionic term in (4.4) is of a form similar to $S_{\text{extra}}$ in (3.14). More precisely, in (4.4) the fermions appear with the tensor $c_{AB,C}^{(\text{CS})}$, which has a mixed symmetry, (4.2). $S_{\text{extra}}$ in (3.14), on the other hand, is proportional to the tensor $c_{AB,C}^{(\text{CS})} + c_{AB,C}^{(\text{lo})}$. From this we see that if we identify $c_{AB,C}^{(\text{CS})} = c_{AB,C}^{(\text{CS})}$ and then do later, we can absorb the mixed part of $S_{\text{extra}}$ into the superspace expression $S_{\text{CS}}$. This is, however, not possible for the symmetric part of $S_{\text{extra}}$ proportional to $c_{AB,C}^{(\text{lo})}$, which cannot be obtained in any obvious way from a superspace expression. As we need this symmetric part later, it is more convenient to keep the full $S_{\text{extra}}$ as we did in section 3, as a part of $S_f$, and not include $(S_{\text{CS}})_{\text{term}}$ here. Thus, we will further work with the purely bosonic $S_{\text{CS}}$ and omit the fermionic term that is included in the superspace expression (4.3).

As an aside, we will show in the remainder of this subsection that for semisimple algebras the GCS terms do not bring anything new [4], at least in the classical theory. By this we mean they can be replaced by a redefinition of the kinetic matrix $f_{AB}$. This argument is not essential for the main result of this paper and the reader can thus skip this part. It shows, however, that the main application of GCS terms is for non-semisimple gauge algebras.

We start with the result [4] that if

$$c_{AB,C}^{(\text{CS})} = 2 f_{(A}^{CD} Z_{B)D}. \quad (4.5)$$

for a constant real symmetric matrix $Z_{AB}$, the action $S_{\text{CS}}$ can be reabsorbed in the original action $S_f$ using

$$f'_{AB} = f_{AB} + i Z_{AB}. \quad (4.6)$$

In fact, one easily checks that with the substitution of (4.5) into (2.9), the $C$-terms are absorbed by the redefinition (4.6). Equation (4.5) can be written as

$$C_{AB,C}^{(\text{CS})} = T_{C,AB}^{DE} Z_{DE}. \quad (4.7)$$

In the case that the algebra is semisimple, one can always construct a $Z_{AB}$ such that this equation is valid for any $C_{AB,C}^{(\text{CS})}$:

$$Z_{AB} = C_2(T)_{AB}^{CD} T_{E,CD}^{GH} g_{EF}^{(\text{CS})} C_{GH,F}^{(\text{CS})}, \quad (4.8)$$

where $g^{AB}$ and $C_2(T)^{-1}$ are the inverses of

$$g_{AB} = f_{AC}^D f_{BD}^C, \quad C_2(T)_{CD}^{EF} = g^{AB} T_{A,CD}^{GH} T_{B,GH}^{EF}. \quad (4.9)$$

These inverses exist for semisimple groups. To show that (4.8) leads to (4.7) one needs (2.12), which leads to

$$g^{HD} T_{H} \left( \frac{1}{2} C_{E}^{(\text{CS})} f_{DE}^C + T_{[D} \cdot C_{E]}^{(\text{CS})} \right) = 0, \quad (4.10)$$
where we have dropped doublet symmetric indices using the notation \( \cdot \) for contractions of such double indices. This further implies
\[
g^{AB}T_E \cdot T_B \cdot C_A^{(CS)} = C_\lambda(T) \cdot C_E^{(CS)},
\]
with which the mentioned conclusions can easily be obtained.

### 4.2. Gauge and supersymmetry transformations

The GCS term \( S_{CS} \) is not gauge invariant. Even the superspace expression \( S'_{CS} \) is not gauge invariant, not even in the Abelian case. So, just as for \( S_f \), we expect that \( S'_{CS} \) is not supersymmetric in the Wess–Zumino gauge, despite the fact that it is a superspace integral. This is highlighted, in particular, by the second term in (4.3), which involves the structure constants. Its component expression simply gives the non-Abelian \( W \wedge W \wedge W \wedge W \wedge W \) correction in (4.1), which, as a purely bosonic object, cannot be supersymmetric by itself.

For the gauge variation of \( S_{CS} \), one obtains
\[
\delta(\lambda)S_{CS} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^{(CS)} F^{\lambda}_{\mu\nu} + \frac{1}{8} \Lambda^C \left( 2C_{AB,D}^{(CS)} f_{CE}^B - C_{DA,B}^{(CS)} f_{CE}^B \right) 
+ C_{BE,D}^{(CS)} f_{CA}^B + C_{BC,D}^{(CS)} f_{AE}^B + C_{AB,C}^{(CS)} f_{DE}^B 
+ \frac{1}{2} \epsilon_{AC,B}^{(CS)} f_{DE}^B \right],
\]
where we used the Jacobi identity and the property \( C_{(CS)}^{(AB,C)} = 0 \).

A careful calculation finally shows that the supersymmetry variation of \( S_{CS} \) is
\[
\delta(\epsilon)S_{CS} = -\frac{1}{2} \int d^4x \epsilon^{\mu\nu\rho\sigma} \text{Re} \left[ C_{AB,C}^{(CS)} W_\mu W_\nu W_\rho W_\sigma + C_{A[B,C}^{(CS)} f_{DE]}^A W_\mu W_\nu W_\rho W_\sigma \right],
\]
where we used the Jacobi identity and the property \( C_{(AB,C)}^{(CS)} = 0 \).

### 5. Anomalies and their cancellation

In this section, we combine the classical non-invariances of \((\hat{S}_f + S_{CS})\) with the non-invariances induced by quantum anomalies.

#### 5.1. The consistent anomaly

The physical information of a quantum field theory is contained in Green’s functions, which in turn are encoded in an appropriate generating functional. Treating the Yang–Mills fields \( W_\mu \) as external fields, the generating functional (effective action) for proper vertices can be written as a path integral over the other matter fields,
\[
e^{-\Gamma[W_\mu]} = \int \mathcal{D}\phi \mathcal{D}\phi e^{-S(\phi, \phi)}. \]

The gauge invariance,
\[
\delta_\lambda \Gamma[W_\mu] = 0,
\]
of the effective action encodes the Ward identities and is crucial for the renormalizability of the theory. Even if the classical action, \( S \), is gauge invariant, a non-invariance of the path integral measure may occur and violate (5.2), leading to a quantum anomaly. Even though
the functional $\Gamma[W_\mu]$ is in general neither a local nor a polynomial functional of the $W_\mu$, the quantum anomaly,

$$\delta(\Lambda)\Gamma[W] = -\int d^4x \, \Lambda^A \left( D_\mu \frac{\delta \Gamma[W]}{\delta W_\mu} \right)_A = \int d^4x \, \Lambda^A A_A,$$

(5.3)
does have this property. More explicitly, for an arbitrary non-Abelian gauge group, the consistent form of the anomaly $A_A$ is given by

$$A_A \sim \epsilon^{\mu
u\rho\sigma} \text{Tr} \left( T_A \partial_\mu (W_\nu \partial_\rho W_\sigma + \frac{1}{2} W_\nu W_\rho W_\sigma) \right),$$

(5.4)

where $W_\mu = W_\mu^A T_A$, and $T_A$ denotes the generators in the representation space of the chiral fermions. Similarly there are supersymmetry anomalies, such that the final non-invariance of the one-loop effective action is

$$A = \delta(\Lambda)\Gamma[W] = \delta(\Lambda)\Gamma[W] + \delta(\epsilon)\Gamma[W] = \int d^4x (\Lambda^A A_A + \bar{\epsilon} \epsilon_A).$$

(5.5)

This anomaly should satisfy the Wess–Zumino consistency conditions [27], which are the statement that these variations should satisfy the symmetry algebra. For example, for the gauge anomalies these are

$$\delta(\Lambda_1) (\Lambda^A_2 A_A) - \delta(\Lambda_2) (\Lambda^A_1 A_A) = \Lambda^B_1 \Lambda^C_2 f_{BC} A_A,$$

(5.6)

If the effective action is non-invariant under gauge transformations, then also its supersymmetry transformation is non-vanishing. As we explained in section 3, this can for example be seen from the algebra (3.4).

A full cohomological analysis of anomalies in supergravity was made by Brandt in [13, 14]. His result (see especially (9.2) in [14]) is that the total anomaly should be of the form

$$A = \sum_\mu \left[ d_{ABC} F_\mu^{B} + \left( d_{ABD} f_{DE}^{B} + \frac{1}{2} d_{ABC} f_{DE}^{B} \right) W_\mu^D W_\nu^E \right] \tilde{F}^{\mu\nu},$$

(5.7)

$$\bar{\epsilon} \epsilon_A = \text{Re} \left[ \frac{i}{2} d_{ABC} \bar{\epsilon}_R \epsilon_L \Lambda^A_{R} \Lambda^B_L + i d_{ABC} W^C \tilde{F}^{\mu\nu} \epsilon_L \gamma_\mu \Lambda^B_R + \frac{1}{2} d_{ABC} f_{DE}^{A} \epsilon_L \gamma_\nu \Lambda^{C}_{DE} W_\mu^D W_\sigma^E \epsilon_L \gamma_\rho \Lambda^B_R \right].$$

(5.8)

The coefficients $d_{ABC}$ form a totally symmetric tensor that is not fixed by the consistency conditions. Comparison with (5.4) implies that they are of the form

$$d_{ABC} \sim \text{Tr} \left( [T_A, T_B] T_C \right).$$

(5.9)

5.2. The cancellation

Since the anomaly $A$ is a local polynomial in $W_\mu$, one might envisage a cancellation of the quantum anomaly by the classically non-gauge invariant terms in the action in the spirit of the Green–Schwarz mechanism.

The sum of the variations of the kinetic terms, (3.17) and (3.21), and of the variations of the GCS term, (4.12) and (4.13), simplifies if we set

$$C_{ABC}^{(CS)} = C_{ABC}^{(m)} = C_{ABC}^{(C)} - C_{ABC}^{(e)},$$

(5.10)

10This result is true up to local counterterms. The latter are equivalent to a redefinition of the $C_{ABC}^{(CS)}$. This is the same as the scheme dependence mentioned in [6], which is also equivalent to a modification of these GCS terms.
and then use the consistency condition (2.12) for the tensor $C_{ABC}$. The result is

$$\delta(\Lambda_1)(\hat{S}_f + S_{CS}) = \frac{1}{4i} \int d^4 x \Lambda^C \left[ C_{ABC}^{\text{(s)}} F_{\mu \nu}^B + \frac{3}{2} C_{ABC}^{\text{(s)}} f_{DE}^E W_\mu^D W_\nu^E \right] F^{\mu \nu A},$$

$$\delta(\epsilon)(\hat{S}_f + S_{CS}) = \int d^4 x \text{Re} \left[ -\frac{3}{8} C_{ABC}^{\text{(s)}} f_{DE}^E \bar{\epsilon}_R^B \epsilon_L^A \left( \bar{W}_\mu W_\nu^C \bar{\epsilon}_{LY} \epsilon_R^B \right) \right].$$

The integrand of these expressions cancel the gauge anomaly (5.7) and supersymmetry anomaly (5.8) if we set

$$C_{ABC}^{\text{(s)}} = d_{ABC}. \quad (5.12)$$

Thus, if $C_{ABC}^{\text{(m)}} = C_{ABC}^{\text{(CS)}}$ and $C_{ABC}^{\text{(s)}} = d_{ABC}$, both gauge and supersymmetry are unbroken, in particular anomaly-free. Note that this does not mean that any anomaly proportional to some $d_{ABC}$ can be cancelled by a $C_{ABC}^{\text{(s)}}$. A gauge kinetic function with an appropriate gauge transformation induced by gauge transformations of scalar fields such that (5.12) holds may simply not exist. Our analysis only shows that if (5.12) holds, and $C_{ABC}^{\text{(m)}} = C_{ABC}^{\text{(CS)}}$, the theory is gauge and supersymmetry invariant.

6. Supergravity corrections

In this section, we generalize our treatment to the full $\mathcal{N} = 1, d = 4$ supergravity theory. We check supersymmetry and gauge invariance of the supergravity action and show that no extra GCS terms (besides those already added in the rigid theory) have to be included to obtain supersymmetry or gauge invariance.

The simplest way to go from rigid supersymmetry to supergravity makes use of the superconformal tensor calculus [28–31]. A summary in this context is given in [2]. Compared to the rigid theory, the additional fields reside in a Weyl multiplet, i.e. the gauge multiplet of the superconformal algebra, and a compensating multiplet. The Weyl multiplet contains the vierbein, the gravitino $\psi_\mu$, and an auxiliary vector, which will not be important for us. The compensating multiplet enlarges the set of chiral multiplets in the theory by one. The full set of fields in the chiral multiplets is now $(X^I, \Omega^I, H^I)$, which denote complex scalars, fermions and complex auxiliary fields, respectively. The physical chiral multiplets $(z_i, \chi_i, h_i)$ form a subset of these such that $I$ runs over one more value than $i$. As our final results depend only on the vector multiplet, this addition will not be very important for us, and we do not have to discuss how the physical ones are embedded in the full set of chiral multiplets.

When going from rigid supersymmetry to supergravity, extra terms appear in the action (3.10); they are proportional to the gravitino $\psi_\mu$. The integrand of (3.10) is replaced by the so-called density formula, which is rather simple due to the use of the superconformal calculus [32]:

$$S_f = \int d^4 x e \text{Re} \left[ h(f W^2) + \bar{\psi}_\mu R^\mu \chi_L(f W^2) + \frac{1}{2} \psi_\mu R^\mu \psi_{\nu R}^\nu (f W^2) \right], \quad (6.1)$$

where $e$ is the determinant of the vierbein. For completeness, we give the component expression of (6.1). It can be found by plugging in the relations (3.9), where we replace the fields of the chiral multiplets with an index $i$ by the larger set indexed by $I$, into the density formula (6.1). The result is
\[
\hat{S}_f = \int d^4x e \left[ \text{Re} \ f_{AB}(X) \left( -\frac{1}{4} \mathcal{F}^A_{\mu\nu} \mathcal{F}^{AB} - \frac{1}{2} \hat{\lambda}^A \gamma^\mu \hat{\lambda}^B + \frac{1}{2} D^A D^B 
+ \frac{1}{8} \bar{\psi}_\mu \gamma^\nu (\mathcal{F}^A_{\nu\rho} + \hat{\mathcal{F}}^A_{\nu\rho}) \gamma^\rho \lambda^B 
+ \frac{1}{4} \text{Im} \ f_{AB}(X) \mathcal{F}^A_{\mu\nu} \hat{\mathcal{F}}^{AB} 
+ \frac{1}{4} \left( \hat{D}_\mu \text{Im} \ f_{AB}(X) \right) \hat{\lambda}^A \gamma^\rho \lambda^B 
+ \left\{ \frac{1}{2} \partial_I f_{AB}(X) \right\} \left[ \hat{\Omega}^I_L \left( -\frac{1}{2} \gamma^{\mu\nu} \hat{\mathcal{F}}^A_{\mu\nu} + i D^A \right) \lambda^B_L = \frac{1}{2} \left( H^I + \bar{\psi}_\mu R^\mu \Omega^I_L \right) \hat{\lambda}^A \lambda^B_L \right\} 
+ \frac{1}{4} \partial_I \partial_J f_{AB}(X) \bar{\Omega}^I_L \bar{\Omega}^J_L \hat{\lambda}^A \lambda^B_L + h.c. \right] \right].
\]

where the hat denotes full covariantization with respect to gauge and local supersymmetry, e.g.
\[
\hat{\mathcal{F}}^A_{\mu\nu} = \mathcal{F}^A_{\mu\nu} + \bar{\psi}_\mu \gamma_{\lambda} \lambda^A.
\]

Note that we use already the derivative \( \hat{D}_\mu \text{Im} \ f_{AB}(X) \), covariant with respect to the shift symmetries, as explained around (3.13). Therefore, we denote this action as \( \hat{S}_f \) as we did for rigid supersymmetry.

The kinetic matrix \( f_{AB} \) is now a function of the scalars \( X^I \). We thus have in the superconformal formulation
\[
\delta_C f_{AB} = \partial_I f_{AB} \delta_C X^I = i C_{AB,C} + \cdots.
\]

Let us first consider the supersymmetry variation of (6.2). Compared with (3.21), the supersymmetry variation of (6.2) can only get extra contributions that are proportional to the \( C \)-tensor. These extra contributions come from the variation of \( H^I \) and \( \Omega^I_L \) in covariant objects that are now also covariantized with respect to the supersymmetry transformations and from the variation of \( e \) and \( \lambda^A \) in the gauge covariantization of the \( \left( \hat{D}_\mu \text{Im} \ f_{AB}(X) \right) \) term. Let us list in more detail the parts of the action that give these extra contributions.

First there is a coupling of \( \bar{\Omega}^I_L \) with a gravitino and gaugini, coming from
\[
S_1 = \int d^4x e \left[ -\frac{1}{4} \partial_I f_{AB} \bar{\Omega}^I_L \gamma^{\mu\nu} \hat{\mathcal{F}}^A_{\mu\nu} \lambda^B_L \right] + h.c.
\]
\[
\to \delta(\epsilon) S_1 = \int d^4x e \left[ -\frac{1}{8} i C_{AB,C} W^C_{\rho} \lambda^A_L \gamma^\rho \lambda^B_L \right] + h.c.
\]

We used the expression (6.3) for \( \hat{\mathcal{F}}^A_{\mu\nu} \) and (3.5) where \( D_\mu X^I \) is now also covariantized with respect to the supersymmetry transformations, i.e. \( \hat{D}_\mu F^I \). There is another coupling between \( \Omega^I_L \), a gravitino and gaugini, that we will treat separately:
\[
S_2 = \int d^4x e \left[ \frac{1}{4} \partial_I f_{AB} \bar{\Omega}^I_L \gamma^\mu \psi_\mu R \hat{\lambda}^A_L \lambda^B_L \right] + h.c.
\]
\[
\to \delta(\epsilon) S_2 = \int d^4x e \left[ \frac{1}{8} i C_{AB,C} W^C_{\rho} \bar{\epsilon}_R \gamma^\rho \psi_\mu R \hat{\lambda}^A_L \lambda^B_L + \cdots + h.c. \right].
\]

A third contribution comes from the variation of the auxiliary field \( H^I \) in \( S_3 \), where
\[
S_3 = \int d^4x e \left[ -\frac{1}{4} \partial_I f_{AB} H^I \bar{\lambda}^A_L \lambda^B_L \right] + h.c.
\]

The variation is of the form
\[
\delta_C H^I = \bar{\epsilon}_R \gamma^\rho D_\rho \Omega^I_L + \cdots = -\frac{1}{2} \bar{\epsilon}_R \gamma^\rho \gamma^\mu \hat{D}_\mu \bar{\Omega}^I_L \psi_\mu R + \cdots = \frac{1}{2} \delta_C X^I W^C_\nu \bar{\epsilon}_R \gamma^\nu \gamma^\rho \psi_\mu R + \cdots.
\]
Therefore we obtain
\[
S_3 = \int d^4x e \left[ -\frac{1}{4} \partial_I f_{AB} H^I \bar{\lambda}_a \lambda_a^B + \text{h.c.} \right]
\]
\[
\rightarrow \delta(\epsilon) S_3 = \int d^4x e \left[ -\frac{1}{8} i C_{AB,C} W^C_{\mu} \bar{\psi}_{\mu R} \gamma^\mu \psi_{\mu L} \bar{\lambda}_a \lambda_a^B + \ldots + \text{h.c.} \right].
\]
(6.9)

Finally, we need to consider the variation of the vierbein \( e \) and the gaugini in a part of the covariant derivative on \( \text{Im} f_{AB} \):
\[
S_4 = \int d^4x e \left[ \frac{1}{4} i C_{AB,C} W^C_{\mu} \bar{\psi}_{\mu R} \gamma^\mu \gamma^5 \lambda_a \lambda_a^B \right]
\]
\[
\rightarrow \delta(\epsilon) S_4 = \int d^4x e \left[ -\frac{1}{4} i C_{AB,C} W^C_{\mu} \bar{\psi}_{\mu R} \gamma^\mu \gamma^5 \psi_{\mu L} \bar{\lambda}_a \lambda_a^B + \frac{1}{4} \bar{\epsilon} R Y^\rho \gamma^\rho \psi_{\mu L} \bar{\lambda}_a \lambda_a^B \right]
\]
\[
+ \frac{1}{4} \bar{\epsilon} R Y^\rho \gamma^\rho \psi_{\mu R} \epsilon R \bar{\lambda}_a \lambda_a^B \right] + \frac{1}{4} i C_{AB,C} W^C_{\mu} \bar{\psi}_{\mu R} \epsilon R \bar{\lambda}_a \lambda_a^B + \ldots + \text{h.c.} \right].
\]
(6.10)

It requires some careful manipulations to obtain the given result for \( \delta(\epsilon) S_4 \). One needs the variation of the determinant of the vierbein, gamma matrix identities and Fierz relations.

In the end, we find that \( \delta(\epsilon) (S_1 + S_2 + S_3 + S_4) = 0 \). This means that all extra contributions that were not present in the supersymmetry variation of the original supergravity action vanish without the need of extra terms (e.g., generalizations of the GCS terms). We should also remark here that the variation of the GCS terms themselves is not influenced by the transition from rigid supersymmetry to supergravity because it depends only on the vectors \( W^A \), whose supersymmetry transformations have no gravitino corrections in \( \mathcal{N} = 1 \).

Let us check now the gauge invariance of terms proportional to the gravitino. Neither terms involving the real part of the gauge kinetic function, \( \text{Re} f_{AB} \), nor its derivatives violate the gauge invariance of \( \hat{S}_f \). The only contributions to gauge non-invariance come from the pure imaginary parts, \( \text{Im} f_{AB} \), of the gauge kinetic function. On the other hand, no extra \( \text{Im} f_{AB} \) terms appear when one goes from rigid supersymmetry to supergravity and, hence, the gauge variation of \( \hat{S}_f \) does not contain any gravitini. This is consistent with our earlier result that neither \( \delta(\epsilon) \hat{S}_f \) nor \( S_{\text{CS}} \) contain gravitini.

Consequently, the general \( \mathcal{N} = 1 \) action contains just the extra terms (4.1), and we can add them to the original action in [1].

7. Specializing to Abelian \( \times \) semisimple gauge groups

We mentioned at the end of section 4.1 that simple gauge groups do not lead to non-trivial GCS terms. Therefore we consider now a relevant case: the product of a (one-dimensional) Abelian factor and a semisimple gauge group. This will allow us to clarify the relation between our results and previous work, in particular [16, 17]. In these papers, the authors study the structure of quantum consistency conditions of \( \mathcal{N} = 1 \) supergravity. More precisely, they clarify the anomaly cancellation conditions (required by the quantum consistency) for a U(1) \( \times \) \( G \) gauge group, where \( G \) is semisimple. We introduce the notations \( F_{\mu \nu} \) and \( G^a_{\mu \nu} \) for the Abelian and semisimple field strengths, respectively.

In this case, one can look at ‘mixed’ anomalies, which are those proportional to \( \text{Tr} (Q T_a T_b) \), where \( Q \) is the U(1) charge operator and \( T_a \) are the generators of the semisimple algebra. Following ([17], section 2.2), one can add counterterms such that the mixed anomalies...
proportional to $\Lambda^a$ cancel and one remains with those that are of the form $\Lambda^0 \text{Tr} \left( Q G_{\mu \nu} G^{\mu \nu} \right)$, where $\Lambda^0$ is the Abelian gauge parameter. Schematically, it looks like

$$\delta(\Lambda) L_{ct} : \quad - \Lambda^a A^a_{\text{mixed con}} + \Lambda^0 A^0_{\text{mixed con}}$$

(7.1)

where the subscripts ‘con’ and ‘cov’ denote the consistent and covariant anomalies, respectively. The counterterms $L_{ct}$ have the following form:

$$L_{ct} = \frac{1}{3} Z \varepsilon^{\mu \nu \rho \sigma} C_{\mu} \text{Tr} \left[ Q \left( W_\nu \partial_\rho W_\sigma + \frac{3}{4} W_\nu W_\rho W_\sigma \right) \right], \quad Z = \frac{1}{4\pi^2},$$

(7.2)

where $C_{\mu}$ and $W_\mu$ are the gauge fields for the Abelian and semisimple gauge groups respectively. The expressions for the anomalies are:

$$A^a_{\text{mixed con}} = \frac{1}{4} Z \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left[ T^a Q \partial_\mu (C_\nu \partial_\rho W_\sigma + \frac{1}{4} C_\nu W_\rho W_\sigma) \right],$$

$$A^0_{\text{mixed con}} = \frac{1}{4} Z \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left[ Q \partial_\mu (W_\nu \partial_\rho W_\sigma + \frac{1}{4} W_\nu W_\rho W_\sigma) \right],$$

$$A^0_{\text{mixed cov}} = \frac{1}{8} \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left[ Q G_{\mu \nu} G_{\rho \sigma} \right].$$

(7.3)

The remaining anomaly $A^0_{\text{mixed con}}$ is typically cancelled by the Green–Schwarz mechanism.

We will compare this now with our results for general non-Abelian gauge groups, which we reduce to the case Abelian $\times$ semisimple. The index $A$ is split into 0 for the U(1) and $a$ for the semisimple group generators. We expect the GCS terms (4.1) to be equivalent to the counterterms in [17] and the role of the Green–Schwarz mechanism is played by a U(1) variation of the kinetic terms $f_{ab}$, hence by a $C$-tensor with non-trivial components $C_{ab,0}$.

It follows from the consistency condition (2.12) that

$$C_{0a,0} = C_{00,a} = 0$$

(7.4)

and the $C_{ab,0}$'s are proportional to the Cartan–Killing metric in each simple factor. We write here

$$C_{ab,0} = Z \text{Tr}(QT_a T_b),$$

(7.5)

where $Z$ could be arbitrary, but our results will match the results of [17] for the value of $Z$ in (7.2).

We will not allow for off-diagonal elements of the gauge kinetic function $f_{AB}$:

$$f_{0a} = 0 \quad \Rightarrow \quad C_{0a,b} = 0.$$  

(7.6)

There may be non-zero components $C_{00,0}$ and $C_{ab,c}$, but we shall be concerned here only with the mixed ones, i.e. we have only (7.5) different from zero.

If we reduce (3.17) using (7.4) and (7.5) we get

$$[\delta(\Lambda) \delta f]_{\text{mixed}} = \int d^4x \left[ \frac{1}{8} Z \Lambda^0 \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left( Q G_{\mu \nu} G_{\rho \sigma} \right) \right].$$

(7.7)

Splitting (7.5) into a totally symmetric and mixed symmetry part gives

$$C_{ab,0}^{(s)} = C_{0a,b}^{(s)} = \frac{1}{2} C_{ab,0} = \frac{1}{4} Z \text{Tr}(QT_a T_b),$$

$$C_{ab,0}^{(m)} = \frac{1}{2} C_{ab,0} = \frac{1}{2} Z \text{Tr}(QT_a T_b), \quad C_{0a,b}^{(m)} = -\frac{1}{4} C_{ab,0} = -\frac{1}{4} Z \text{Tr}(QT_a T_b).$$

(7.8)

We learned in section 5.2 that for a final gauge and supersymmetry invariant theory we have to take $C^{(s)} = C^{(m)}$, and hence the mixed part of the GCS action (4.1) reads in this case:

$$[S_{CS}]_{\text{mixed}} = \int d^4x \left[ \frac{1}{3} Z C_{\mu \nu} \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left( Q \left( W_\nu \partial_\rho W_\sigma + \frac{3}{4} W_\nu W_\rho W_\sigma \right) \right) \right].$$

(7.9)
Finally, we reduce the consistent anomaly (5.7) using $d_{ABC} = C_{ABC}^{(5)}$. We find

$$A_0 = -\frac{1}{6} Z \varepsilon_{\mu
u\rho\sigma} \text{Tr} \left[ Q \partial_\mu \left( W_\nu \partial_\rho W_\sigma + \frac{1}{2} W_\nu W_\rho W_\sigma \right) \right],$$

$$A_a = -\frac{1}{3} Z \Lambda^a \varepsilon_{\mu
u\rho\sigma} \text{Tr} \left[ T_a Q \partial_\mu \left( C_\nu \partial_\rho W_\sigma + \frac{1}{4} C_\nu W_\rho W_\sigma \right) \right],$$

where $G_{\mu\nu}$ is the Abelian part of the gauge field $G_{\mu\nu}$.

We can make the following observations:

(i) The mixed part of the GCS action (7.9) is indeed equal to the counterterms (7.2), introduced in [17].

(ii) The consistent anomalies (7.10), for which we based our formula on [13, 14], match those in the first two lines of (7.3). As we mentioned above, the counterterm has modified the resulting anomaly to the covariant form in the last line of (7.3).

(iii) We see that the variation of the kinetic term for the vector fields (7.7) is able to cancel this mixed covariant anomaly (this is the Green–Schwarz mechanism).

Combining these remarks, our cancellation procedure can schematically be presented as follows:

\[
\begin{align*}
\text{Anomalies:} & \quad \Lambda^a A_a^{\text{mixed con}} + \Lambda^0 A_0^{\text{mixed con}} \\
\delta(\Lambda) \mathcal{L}_{(CS)} : & \quad -\Lambda^a A_a^{\text{mixed con}} - \Lambda^0 A_0^{\text{mixed con}} + \Lambda^0 A_0^{\text{mixed cov}} \\
\delta(\Lambda) S_f : & \quad 0 \\
\text{sum:} & \quad + 0
\end{align*}
\]

(7.11)

8. Conclusions

In this paper, we have studied the consistency conditions that ensure the gauge and supersymmetry invariance of matter coupled $\mathcal{N} = 1$ supergravity theories with Peccei–Quinn terms, generalized Chern–Simons terms and quantum anomalies. Each of these three ingredients defines a constant three index tensor:

(1) The gauge non-invariance of the Peccei–Quinn terms is proportional to a constant imaginary shift of the gauge kinetic function parameterized by a tensor $C_{ABC}$. This tensor in general splits into a completely symmetric part and a part of mixed symmetry, $C_{ABC}^{(c)} + C_{ABC}^{(m)}$.

(2) Generalized Chern–Simons terms are defined by a tensor, $C_{ABC}^{(CS)}$, of mixed symmetry.

(3) Quantum gauge anomalies of chiral fermions are proportional to a tensor $d_{ABC}$, which, in the appropriate regularization scheme, can be chosen to be completely symmetric, $d_{ABC} \propto \text{Tr}(T_A T_B T_C)$.

We find the full quantum effective action to be gauge invariant and supersymmetric if

$$C_{ABC} = C_{ABC}^{(CS)} + d_{ABC}.$$  (8.1)

The inclusion of the quantum anomalies encoded in a non-trivial tensor $d_{ABC}$ is the key feature that distinguishes $\mathcal{N} = 1$ theories from theories with extended supersymmetry. Because of their possible presence, the Peccei–Quinn shift tensor $C_{ABC}^{(c)}$ can now have a non-trivial symmetric part, $C_{ABC}^{(c)}$. In the context of $\mathcal{N} = 2$ supergravity, the absence of such a completely symmetric part can be directly proven for theories for which there exists a prepotential [3].

We performed our analysis first in rigid supersymmetry. Using superconformal techniques, we could then show that only one cancellation had to be checked to extend the results to supergravity. It turns out that the Chern–Simons term does not need any gravitino
corrections and can thus be added as such to the matter-coupled supergravity actions. Our paper thus provides an extension to the general framework of coupled chiral and vector multiplets in $\mathcal{N} = 1$ supergravity.$^{11}$

Our results are interesting for a number of rather different applications. For example, in [7], a general setup for treating gauged supergravities in a manifestly symplectic framework was proposed. In that work the completely symmetric part of what we call $C_{AB,C}$ was assumed to be zero, following the guideline of extended supergravity theories. As we emphasized in this paper, $\mathcal{N} = 1$ supergravity theories might allow for a non-vanishing $C_{(s)AB,C}$, and hence a possible extension of the setup of [7] in the presence of quantum anomalies. It might be interesting to see whether such an extension really exists.

In [6], orientifold compactifications with anomalous fermion spectra were studied, in which the chiral anomalies are cancelled by a mixture of the Green–Schwarz mechanism and generalized Chern–Simons terms. The analysis in [6] was mainly concerned with the gauge invariance of the bosonic part of the action and revealed the generic presence of a completely symmetric and a mixed part in $C_{AB,C}$ and the generic necessity of generalized Chern–Simons terms. Our results show how such theories can be embedded into the framework of $\mathcal{N} = 1$ supergravity and supplements the phenomenological discussions of [6] by the fermionic couplings in a supersymmetric setting.

The work of [6] raises the general question of the possible higher-dimensional origins of GCS terms. In [5], certain flux and generalized Scherk–Schwarz compactifications [33, 34] are identified as another means to generate such terms. In [35], it was also shown that $\mathcal{N} = 2$ supergravity theories with GCS terms can be obtained by ordinary dimensional reduction of certain 5$D$, $\mathcal{N} = 2$ supergravity theories with tensor multiplets [36, 37]. It would be interesting to obtain a more complete picture of the possible origins of GCS-terms in string theory and supergravity theories.

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Appendix. Notation

We closely follow the notation of [2], but use real $\varepsilon_{0123} = 1$. The (anti)self-dual field strengths are the same as there, i.e.

$^{11}$We should emphasize that we only considered anomalies of gauge symmetries that are gauged by elementary vector fields. The interplay with Kähler anomalies in supergravity theories can be an involved subject [16, 17], which we did not study.
Symplectic structure of $\mathcal{N} = 1$ supergravity with anomalies and Chern–Simons terms

$$ F_{\mu\nu}^\pm = \frac{i}{2} (F_{\mu\nu} \pm \tilde{F}_{\mu\nu}), \quad \tilde{F}^{\mu\nu} = -\frac{i}{2} e^{-\frac{1}{2}i\varepsilon^{\nu\lambda\rho\sigma}} F_{\rho\sigma}. \quad (A.1) $$

One difference is that we use indices $A, B, \ldots$ for gauge indices, such that $\alpha, \beta$ can be used for 2-component spinors in superspace expressions. Square brackets around indices like $[AB]$ denote the antisymmetrization with total weight one, thus for two indices it includes a factor 1/2 for each combination.

For comparison with Wess and Bagger notations, the $\gamma^m$ are

$$ \gamma^m = \begin{pmatrix} 0 & i\sigma^m \\ i\bar{\sigma}^m & 0 \end{pmatrix}. \quad (A.2) $$

where these sigma matrices are $\sigma^m_{\alpha\beta}$ or $\sigma^{\dot{\alpha}\dot{\beta}}$. Spinors that we use are in their 2-component notation

$$ \chi = \begin{pmatrix} \chi^\alpha \\ \bar{\chi}^\dot{\alpha} \end{pmatrix}, \quad \bar{\chi} = (\chi^\alpha \bar{\chi}^\dot{\alpha}). \quad (A.3) $$

where $\bar{\chi} = \chi^T C = i\chi^{\gamma 0}$. Further translation is obtained by replacing in Wess–Bagger

$$ S \to S, \quad \chi \to \sqrt{2}\chi, \quad F \to F, \quad \lambda \to -\lambda, \quad D \to -D, \quad W_\mu \to W_\mu, \quad (A.4) $$

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