The inclusive $^{56}$Fe($\nu_e, e^-$)$^{56}$Co cross section

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We study the $^{56}$Fe($\nu_e, e^-$)$^{56}$Co cross section for the KARMEN neutrino spectrum. The Gamow-Teller contribution to the cross section is calculated within the shell model, while the forbidden transitions are evaluated within the continuum random phase approximation. We find a total cross section of $2.73 \times 10^{-49}$ cm$^2$, in agreement with the data.

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The search for neutrino oscillations is one of the most promising experimental pathways for possible physics beyond the standard model of electroweak interaction. KARMEN is one of the pioneering low-energy accelerator-based experiments searching for these modes. It consists of liquid scintillator housed in a 7000 ton shielding blockhouse. One of the experimental signals which the KARMEN experiment can observe is the energy distribution of the electron emitted in charged-current reactions. This gives the experiment also the capability to obtain nuclear structure informations, and in fact the KARMEN collaboration has measured several (inclusive and exclusive) neutrino-induced charged and neutral current cross sections on $^{12}$C, which is an important ingredient of the liquid scintillator. Recently the collaboration has also been able to determine inclusive ($\nu_e, e^-$) cross sections for $^{13}$C and $^{56}$Fe. The latter might be quite important for two reasons: First, it provides a benchmark to test the ability of nuclear models to describe neutrino-induced reactions on nuclei in the iron mass range as they are encountered in astrophysical scenarios like the supernova collapse. Second, future long baseline neutrino oscillation experiments like MINOS or OMNIS will use steel as detector material. Thus a rather reliable knowledge of the charged-current (and neutral-current) neutrino-induced cross section on $^{56}$Fe is desirable to support the potential ability to observe supernova neutrinos by this detector.

In this communication we present the first calculation (apart from a shell model estimate by Bugaev et al.) of the $^{56}$Fe($\nu_e, e^-$)$^{56}$Co cross section. The charged-current neutrino-induced cross section for a transition from a nuclear state with angular momentum $J$ and isospin quantum numbers $T, M_T$ to a final nuclear state ($J', T' M_T'$) is derived in Refs. $^8$. Its evaluation reduces to the calculation of the various vector and axial vector multipole operators between appropriate nuclear many-body states. To derive these states we have adopted the following strategy. In Ref. $^8$ it has been demonstrated that the inclusive cross section for forbidden transitions is rather well described within the continuum random phase approximation (RPA). This model is, however, less reliable for the allowed transitions which in the long wavelength approximation are mediated by the Gamow-Teller operator $\sigma T$. Here, the continuum RPA does not recover all nucleon-nucleon correlations necessary to reproduce the quenching of the Gamow-Teller strength. Such a task can be achieved in large-scale shell model calculations performed in the complete pf shell. On the other hand, at the higher neutrino energies present in the KARMEN $\nu_e$-spectrum a replacement of the $\lambda = 1^+$ multipole operator by $\sigma T$ is not quite appropriate. However, considering the momentum-transfer dependence of the operator within shell model approaches represents a very formidable computational effort, even on modern parallel computers. To reliably estimate the contribution of the $\lambda = 1^+$ operator to the cross section, we have therefore chosen a hybrid approach of shell model and RPA, as discussed below.

Our shell-model calculation of the Gamow-Teller part to the $^{56}$Fe($\nu_e, e^-$)$^{56}$Co cross section uses the Strassburg-Madrid codes ANTOINE $^9$ and NATHAN $^10$ developed by Caourier. The calculation has been performed within the complete pf shell adopting the recently developed modified KB3 interaction. In this force slight monopole deficiencies encountered in the original KB3 interaction have been corrected. Ref. $^2$ demonstrates that the modified KB3 version gives a fair account of nuclear spectra in the mass range $A = 41 - 65$ and reproduces the Gamow-Teller strength distributions in general very well, if the latter are scaled by the factor $(0.74)^2$ which accounts for the fact that complete $0\hbar\omega$ shell-model calculations overestimate the experimental GT strength by a universal factor $^3 \times 10^4$. We will adopt in the following the same universal scaling factor. With the code NATHAN it is now possible to calculate the total GT strength for $^{56}$Fe in the complete pf shell involving $7 413488, J^T = 0^+$ configurations (this corresponds to an
m-scheme dimension of 501 million). We obtain a total GT− strength (this is the direction in which a neutron is changed into a proton) of 8.85 units which agrees with the experimental value of 9.9 ± 2.4 [14]. However, a calculation of the GT− strength distribution \( S_{GT}(E) \), where \( E \) denotes the excitation energy in the final nucleus \(^{56}\text{Co}\), in the complete pf shell is still yet beyond present-day computer abilities. We have therefore truncated the model space in our calculation of the GT− strength distribution of \(^{56}\text{Fe}\) allowing a maximum of 5 particles to be excited from the \( f_{7/2} \) orbital to the rest of the pf shell in the final nucleus. Furthermore the Ikeda sum rule has been fulfilled within the model space. The m-scheme dimension of this calculation is 1.6 million for \(^{56}\text{Fe}\) and 19.8 million for \(^{56}\text{Co}\). In our truncated calculation we obtain a total GT− strength of 9.3 units, showing that the calculation at this level of truncation is nearly converged.

Due to the energy dependence of the phase space, the cross section is sensitive to the Gamow-Teller strength distribution rather than only to the total strength. Our shell-model calculation reproduces the experimental GT− strength distribution quite well, as can be seen in Fig. 2 of Ref. [12].

For the Gamow-Teller transition we then have the total cross section

\[
\sigma(E_\nu) = \frac{G_F^2 \cos^2 \theta_c}{\pi} \int p_e E_e F(Z + 1, E_e) S_{GT}(E) dE_e ,
\]

(1)

where \( E_\nu \) is the neutrino energy, \( G_F \) is the Fermi constant, \( E_e \) and \( p_e \) are the energy and momentum of the outgoing electron, respectively, \( \theta_c \) the Cabibbo angle, \( E = E_\nu - E_e \) due to energy conservation, and \( F(Z+1, E_e) \) the Fermi function which accounts for Coulomb distortion of the outgoing electron wave function.

In the KARMEN experiment the neutrino beam has a Michel energy spectrum,

\[
n(E_\nu) = \frac{96E_\nu^2}{M_\mu} (M_\mu - 2E_\nu) ,
\]

(2)

where \( M_\mu \) is the muon mass. Folding Eq. (1) with the spectrum (2) yields a partial \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section of \( 139 \times 10^{-42} \text{ cm}^2 \) for the Gamow-Teller transitions. As mentioned above, our treatment of the \( \lambda = 1^+ \) multipole is only correct for small neutrino energies. To estimate the effect introduced by replacement of the \( \lambda = 1^+ \) operator by the momentum-transfer independent Gamow-Teller we have performed two calculations within the random phase approximation. In the first we adopted the complete \( \lambda = 1^+ \) multipole operator [6], in the second we replaced this operator by \( \sigma T \). We then find partial \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross sections of \( 341 \times 10^{-42} \text{ cm}^2 \) and \( 431 \times 10^{-42} \text{ cm}^2 \), respectively. From these calculations we conclude that the replacement of the full \( \lambda = 1^+ \) operator by the Gamow-Teller operator increases the cross section by about 20%. If we correct our shell model estimate by this factor, we derive at our final result for the partial \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section of \( 110 \times 10^{-42} \text{ cm}^2 \).

The contributions of the other transitions to the cross section have been calculated using the continuum RPA. For nuclei with closed-shell configurations this model is well documented in the literature [17]. We have recently generalized this formalism to allow also for partial occupancies in the parent ground state [3]. From our shell-model calculation we find the following occupation numbers for the pf shell orbitals: \( n_{7/2} = 5.30, n_{3/2} = 0.32, n_{1/2} = 0.07 \) and \( n_{5/2} = 0.30 \) for protons \( n_{7/2} = 7.29, n_{3/2} = 1.57, n_{1/2} = 0.44 \) and \( n_{5/2} = 0.70 \) for neutrons. We assumed a completely occupied \(^{40}\text{Ca} \) core. The single-particle energies were derived from an appropriate Woods-Saxon potential. We used the experimental values for the proton and neutron separation energies. As the residual interaction we adopted the finite-range G-matrix derived from the Bonn potential [13].

The other allowed transition is mediated by the \( 0^+ \) operator, which reduces to the Fermi transition in the limit of vanishing momentum transfer. The Fermi matrix element is easily obtained from isospin symmetry, yielding a total Fermi strength of \( S_F = (N-Z) \) which is concentrated in the isobaric analog state at 3.5 MeV in \(^{56}\text{Co} \). The Fermi contribution to the \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section is then readily calculated within the RPA approach. We find a partial cross section of \( 53 \times 10^{-42} \text{ cm}^2 \).

The partial \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross sections for selected forbidden transitions are listed in Table 1. From the forbidden transitions only the \( J^\pi = 1^- \) and \( 2^- \) dipole multipoles are important. We find a summed cross section of \( 111 \times 10^{-42} \text{ cm}^2 \) for the forbidden transitions, which is only slightly smaller than the contributions stemming from the allowed transitions.

Adding up the allowed and forbidden transitions, we find a total \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section of \( 2.73 \times 10^{-40} \text{ cm}^2 \), which agrees with the KARMEN result, \( 2.56 \pm 1.08(\text{stat.}) \pm 0.43(\text{syst.}) \times 10^{-40} \text{ cm}^2 \) [14]. Fig. 1 shows the differential \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section as function of excitation energy in \(^{56}\text{Co} \). To test the importance of the partial occupancy formalism for the forbidden transitions, we have repeated the continuum RPA calculation with a parent ground state whose occupation numbers have been taken from the independent-particle model. Thus, \( n_{7/2} = 6 \) for protons, and \( n_{7/2} = 8 \) and \( n_{3/2} = 2 \) for neutrons. The respective partial cross sections are also listed in Table 1. Confirming the results and arguments given in Ref. [14], we observe that an improved description of the parent ground state, i.e. by considering partial occupancies, does not change the results noticeably. We now find a total cross section of \( 2.62 \times 10^{-40} \text{ cm}^2 \).

In summary, we have calculated the \(^{56}\text{Fe}(\nu_\mu,e^-)^{56}\text{Co} \) cross section for a neutrino spectrum corresponding to the KARMEN experiment. While the Fermi contribution for the transition to the isobaric analog state is trivial,
we calculate the Gamow-Teller piece to the cross section within a shell model approach which has been proven to correctly describe the GT− strength distribution and total strength for $^{56}$Fe. For the forbidden transitions we employed the continuum RPA. Combining all relevant multipoles we find a total $^{56}$Fe($\nu_e$, $e^-$)$^{56}$Co cross section for the KARMEN neutrino spectrum of $2.73 \times 10^{-40}$ cm$^2$ which agrees with the experimental value obtained by the KARMEN collaboration, $256 \pm 108$(stat.) $\pm 43$(syst.) $\times 10^{-42}$. Despite the rather large experimental uncertainty, this agreement is promising and shows that with the currently chosen combination of nuclear models (shell model for allowed transitions and continuum RPA for forbidden transitions) one should be able to reliably evaluate neutrino-induced cross sections on nuclei in the iron mass range as they are of interest in supernova simulations. Such calculations are in progress.

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TABLE I. Partial cross sections $\sigma_j^\pi$ for the $^{56}$Fe($\nu_e$, $e^-$)$^{56}$Co reaction induced by muon-decay-at-rest neutrinos. The second and third columns give the (continuum RPA) results calculated without and with the consideration of partial occupations in the ground state. The Gamow-Teller result ($J^e = 1^+$) has been calculated within the shell model. The $J = 0^+$ multipole contribution reflects the Fermi transition to the isobaric analog state. The cross sections are in units of $10^{-42}$ cm$^2$.

| multipole | $\sigma_j^\pi$ | $\sigma_j^\pi$ |
|-----------|----------------|----------------|
| $1^+$     | 110.6          | 109.8          |
| $0^+$     | 45.7           | 52.7           |
| $0^-$     | 0.7            | 0.7            |
| $1^-$     | 48.2           | 48.9           |
| $2^+$     | 6.7            | 6.9            |
| $2^-$     | 43.6           | 47.2           |
| $3^+$     | 6.8            | 6.4            |
| $3^-$     | 0.4            | 0.5            |
| $\Sigma$  | 262.7          | 273.1          |
FIG. 1. Differential $^{56}$Fe($\nu_e$, $e^-$)$^{56}$Co cross section for the KARMEN neutrino spectrum as function of excitation energy in $^{56}$Co. The figure shows the allowed contributions, while the insert gives the contributions of the $1^-$ and $2^-$ multipolarities. The allowed contributions have been folded with a Gaussian of 0.5 MeV FHWM at energies below 5 MeV and with 1 MeV FHWM above 5 MeV.