Revisiting Frequency Reuse towards Supporting Ultra-Reliable Ubiquitous-Rate Communication

Jihong Park, Dong Min Kim, Petar Popovski, and Seong-Lyun Kim†

Abstract—One of the goals of 5G wireless systems stated by the NGMN alliance is to provide moderate rates (50+ Mbps) everywhere and with very high reliability. We term this service Ultra-Reliable Ubiquitous-Rate Communication (UR2C). This paper investigates the role of frequency reuse in supporting UR2C in the downlink. To this end, two frequency reuse schemes are considered: user-specific frequency reuse (FR_u) and BS-specific frequency reuse (FR_b). For a given unit frequency channel, FR_u reduces the number of serving user equipments (UEs), whereas FR_b directly decreases the number of interfering base stations (BSs). This increases the distance from the interfering BSs and the signal-to-interference ratio (SIR) attains ultra-reliability, e.g., 99% SIR coverage at a randomly picked UE. The ultra-reliability is, however, achieved at the cost of the reduced frequency allocation, which may degrade overall downlink rate. To fairly capture this reliability-rate tradeoff, we propose ubiquitous rate defined as the maximum downlink rate whose required SIR is above a certain threshold. We term this service Ultra-Reliable Ubiquitous-Rate Communication (UR2C), answering to the question how much data rate can stably be achieved everywhere. For example, the NGMN Alliance suggests that a rate of at least 50 Mbps should be supported practically everywhere.

I. MOTIVATION AND CONTRIBUTION

Two extremes of 5G wireless system designs are pursuing very high data rate, i.e., enhanced mobile broadband (eMBB), and enabling ultra-reliable low-latency communication (URLLC) with very low data rate [1]–[3]. This paper explores a missing piece between them, Ultra-Reliable Ubiquitous-Rate Communication (UR2C), answering to the question how much data rate can stably be achieved everywhere. For example, the NGMN Alliance suggests that a rate of at least 50 Mbps should be supported practically everywhere [1]. In this paper “practically” means that a randomly picked user equipment (UE) can ubiquitously attain a predefined data rate with 99% reliability, which is higher than the commonly considered 95% reliability of the wireless coverage [1], [3].

An important enabler of UR2C is ultra-dense base station (BS) deployment where BS/UE density ratio per communication resource exceeds 1 [4]. A larger density of BSs improves signal-to-interference ratio (SIR) since it increases the number of non-interfering BSs that have no serving UE within their

Fig. 1. Illustrations of FR_u and FR_b when the number of channels \( M = 3 \):
(a) In FR_u, a BS randomly selects a single channel for all associated UEs;
(b) In FR_b, a BS randomly assigns a single channel to each associated UE.
At a typical UE (filled blue), both FR_u and FR_b thereby increase interfering BS distance, compared to (c) the baseline’s when \( M = 1 \).

cells. This makes the desired received signal power grow faster than the interference increase when deploying more BSs.

The effect of ultra-densification can also be achieved by dividing the total bandwidth into frequency channels. It makes interfering BSs located farther away, yielding SIR improvement. This motivated us to reconsider the problem of frequency reuse. In a downlink scenario, we fine-tune frequency reuse to make it applicable for a randomly selected UE, i.e., a typical UE. We thereby aim at proposing an optimal frequency reuse scheme that achieves our target ultra-reliability, 99% SIR coverage. To this end, we suggest two frequency reuse techniques: BS-specific frequency reuse (FR_b) and user-specific frequency reuse (FR_u).

When the entire frequency bandwidth is divided into \( M \) number of channels, each BS in FR_u uses only a single channel that it selects randomly (see Fig. 1-a). In FR_u, a BS randomly assigns a single channel to each associated UE (Fig. 1-b). If all the assigned channels to UEs are not identical, its corresponding BS utilizes multiple number of channels. Nevertheless, average amount of FR_u’s channel use is still less than a baseline without frequency reuse, i.e., \( M = 1 \) (Fig. 1-c). As every scheme with \( M \) larger than 1, both FR_b and FR_u introduce loss in the data rate as they are only partially using the bandwidth.

To make the proposed schemes applicable not only for a cell-edge UE but also for a typical UE, it is therefore important to examine how much data rate degrades in return for reliability improvement. For this purpose, we propose ubiquitous rate \( R_u \) defined as the maximum ergodic capacity.

J. Park, D. M. Kim and P. Popovski are with Department of Electronic Systems, Aalborg University, Denmark (email: \{jihong, dmk, petarp\}@es.aau.dk).
†S.-L. Kim is with School of EEE, Yonsei University, Seoul, Korea (email: slkim@yonsei.ac.kr).

This work has been supported in part by the Danish Ministry of Higher Education and Science (EliteForsk Award, Grant Nr. 5137-00073B), and partly by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (NRF-2014R1A2A1A11053234).
that guarantees a target $\eta$ SIR coverage probability for a given SIR threshold $t$. For instance, $R_{0.99}$ for unit resource allocation is $0.99 \cdot \log_2(1+t)$ bps, where $t$ is given by taking the inverse function of SIR coverage probability to the target reliability constraint equation $\Pr(\text{SIR} \geq t) = 0.99$.

In this respect, we utilize stochastic geometry, and provide the closed-form downlink ubiquitous rate at a typical UE. Its derivation follows in part from preceding works that provide closed-form downlink SIR coverage at a typical UE in a form of a hypergeometric function [3], [6]. However, inverting a hypergeometric function to solve the reliable constraint equation is not analytically viable. We instead propose novel closed-form SIR bounds by using a hypergeometric function transformation technique, and thereby derive the desired ubiquitous rate in a closed form. Based on this result, we optimize FR$_b$ and FR$_u$ with respect to $M$, and investigate how to achieve the UR2C’s target 50 Mbps ubiquitous rate with 99% reliability under different deployment density scenarios.

The contributions of this paper are summarized as follows.

- Optimized frequency reuse designs for UR2C are proposed (see Proposition 3 with Figs. 3 and 4).
- Closed-form ubiquitous rates and SIR reliabilities under the frequency reuse schemes are derived (Proposition 2).
- Closed-form SIR reliability approximation for UR2C as well as closed-form SIR reliability bounds are provided (Lemmas 2 and 3).

II. SYSTEM MODEL

In a downlink cellular network, BSs are uniformly distributed with density $\lambda_b$, resulting in a homogeneous Poisson point process (PPP). Independently, UEs are also uniformly distributed with density $\lambda$, leading to another homogeneous PPP with density $\lambda$. Each UE associates with the nearest BS, forming a Poisson-Voronoi tessellation [7].

The entire frequency bandwidth $W$ is divided into $M_i$ channels, where hereafter the subscript $i \in \{b, u, o\}$, respectively, denotes FR$_b$, FR$_u$, and a baseline model. In FR$_b$, each BS randomly selects a single frequency channel, and serves its associated UEs only via the selected channel. In FR$_u$, a BS randomly assigns a single channel to each of its associated UE, and serves them respectively through the UEs’ assigned channels. In the baseline, BSs neglect frequency reuse, and serve their associated UEs by using the entire bandwidth, i.e., $M_o = 1$. For all cases, multiple UEs using the same bandwidth are served by the BS through an equal TDMA allocation.

A BS occupies each channel when at least a single UE is served via the channel; otherwise, the channel remains void for that BS and the BS is considered inactive. For $\lambda$ density of UEs served by a BS via a single channel, the BS’s corresponding channel occupancy probability $p_c(\lambda)$ is given as $p_c(\lambda) = 1 - (1 + \lambda/3.5\lambda_0)^{-3.5}$ [8], which is a monotone increasing (or decreasing) function of $\lambda$ (or $\lambda_0$). In sparse networks $\lambda_0/\lambda \rightarrow 0$, $p_c(\lambda)$ becomes 1, while for ultra-dense networks $\lambda_0/\lambda \rightarrow \infty$, $p_c(\lambda)$ decreases towards 0 at the rate $\lambda/\lambda_0$ [4].

For each non-void channel, a BS allocates unity power, and transmits signals. The transmitted signals experience path-loss attenuation with the exponent $\alpha > 2$ and Rayleigh fading with unit mean. Noise power is assumed to be much smaller than interference, and is thus neglected as in [4]–[6].

III. PROBLEM FORMULATION; UBQUITOUS RATE MAXIMIZATION

At a typical UE, the reliability of SIR $\mathcal{P}_t(M_i)$ when there are $M_i$ channels is defined as:

$$\mathcal{P}_t(M_i) := \Pr(\text{SIR}(M_i) \geq t)$$

(1)

where $\text{SIR}(M_i)$ is the SIR with $M_i$, elaborated in Section IV. Note that $\mathcal{P}_t(M_i)$ is a monotone increasing function of $M_i$. As $M_i$ increases, interfering BSs become located farther away, which thereby improves $\text{SIR}(M_i)$ and so does $\mathcal{P}_t(M_i)$.

Increasing $M_i$, however, may decrease data rate since it reduces per-UE resource allocation at least by $M_i$. Precisely, the typical UE’s downlink rate $R_t(M_i)$ is given as

$$R_t(M_i) := \frac{W}{M_i} \cdot \frac{1}{N_i} \cdot \log_2(1+t)$$

(2)

where $N_i$ is the number of UEs served by the same BS via the same channel as the typical UE’s.

The first term of resource allocation $W/M_i$ in (2) indicates the maximum resource allocation per UE when a BS serves only a single UE, which is reduced by $1/M_i$. The actual resource allocation to a typical UE is $E[1/N_i]$ portion of $W/M_i$ since the maximum amount is equally allocated to the multiple UEs sharing the same channel and BS with the typical UE’s. According to [8], the reciprocal of $E[1/N_i]$ can be calculated by multiplying (i) the density of common UEs using the same channel as the typical UE’s and (ii) average cell size of the BSs serving such common UEs. In FR$_b$, each BS utilizes only a single channel, so the entire UEs with density $\lambda$ become the common UEs. Next, the density of the BSs serving the common UEs is $p_c(\lambda)\lambda_b$. The average size of cells having the common UEs then becomes $1/(p_c(\lambda)\lambda_b)$ because the average cell size of BSs is the reciprocal of the BS density [9]. As a result, $E[1/N_b] = p_c(\lambda)\lambda_b/\lambda$. In the baseline, common UE density is $\lambda$, so the same calculation applies, i.e., $E[1/N_b] = E[1/N_u]$. In FR$_u$, common UE density is

| Notation | Meaning |
|----------|---------|
| FR$_b$, FR$_u$ | BS/User-centric frequency reuse |
| $W$ | Total frequency bandwidth |
| $M_i$ | # Channels where $i \in \{b, u, o\}$ resp. for FR$_b$, FR$_u$ and baseline |
| $\text{SIR}(M_i)$ | SIR under $M_i$ number of channels |
| $N_i$ | # UEs sharing the same BS and channel with a typical UE |
| $\mathcal{P}_t(M_i)$ | SIR reliability with SIR threshold $t$ |
| $\eta$ | Target SIR reliability; e.g. 0.99 for 99% SIR reliability |
| $R_t(M_i)$ | Average rate, given as $W/M_i \cdot E[1/N_i] \cdot \log(1+t)$ |
| $E[1/N_i]$ | Ubiquitous rate, maximum average rate s.t. $\mathcal{P}_t(M_i) \geq \eta$ |
| $\lambda_b, \lambda$ | BS and UE densities |
| $p_c(\lambda)$ | Channel occupancy probability for serving UE density $\lambda$ |
TABLE II

| Scheme | Resource Allocation | Interferer Density |
|--------|---------------------|--------------------|
|        | \(W/M_i\)         | \(E[1/N]\)         | \(\lambda_i\)      |
| Baseline | \(W\)              | \(p_c(\lambda)\lambda_b/\lambda\) | \(p_c(\lambda)\lambda_b\) |
| FRW | \(W/M_b\)         | \(p_c(\lambda)\lambda_b/\lambda\) | \(p_c(\lambda)\lambda_b/M_b\) |
| FRA | \(W/M_a\)         | \(p_c(\lambda/M_a)\lambda_b/(\lambda/M_a)\) | \(p_c(\lambda/M_a)\lambda_b\) |

reduced by \(1/M_a\), and this enlarges the average cell size by replacing \(\lambda\) with \(\lambda/M_a\), as summarized in Table II.

Consequently, in return for resource allocation reduction as \(M_i\) increases, it improves SIR reliability in (1), and thereby increases spectral efficiency in (2) for a fixed target reliability. To achieve UR2C, we capture this reliability-rate tradeoff between (1) and (2), and formulate our optimization problem.

\[(P0) \text{ maximize } R_i(M_i) \]
\[\text{ s.t. } \mathcal{P}_t(M_i) \geq \eta \]

It is noted that \(R_i(M_i)\) and \(\mathcal{P}_t(M_i)\) respectively are monotone increasing and decreasing functions of the SIR threshold \(t\). Therefore \(R_i(M_i)\) is maximized when \(\mathcal{P}_t(M_i) = \eta\) holds, which is the reliability constraint equation. In this respect, we define ubiquitous rate \(R_\eta(M_i)\) as the maximum rate guaranteeing \(\mathcal{P}_t(M_i) \geq \eta\).

Our objective is to derive \(R_\eta(M_i)\) for each frequency reuse scenario. To this end, we first derive a closed-form \(\mathcal{P}_t(M_i)\) approximation (Lemma 3), and thereby easily solve the reliability constraint equation with respect to \(t\). For this given optimal \(t\), we tractably optimize \(R_\eta(M_i)\) with respect to \(M_i\) under an ultra-reliable regime, i.e. \(\eta \approx 1\).

IV. CLOSED-FORM RELIABILITY VIA NORMALIZED SIR

This section derives closed-form SIR reliability approximation for UR2C (Lemma 3). This enables closed-form ubiquitous rate expressions in the next section (Proposition 2). To this end, we consider the following two techniques.

Technique #1. SIR normalization. At a typical UE, let \(S(\lambda_b)\) denote its received desired signal power when associating with the nearest BS out of all BSs with density \(\lambda_b\). Similarly, \(I_c(\lambda_i)\) denotes aggregate interference where \(\lambda_i\) is interferer density and \(r\) the typical UE’s association distance. In the baseline utilizing the entire bandwidth, all the BSs that have at least a single UE perform downlink transmissions interfering with the typical UE. Therefore, \(\lambda_b = p_c(\lambda)\lambda_b\). In FRW, \(1/M_b\) portion of such non-void BSs become interferers, and thus \(\lambda_b = p_c(\lambda)\lambda_b/M_b\). In FRA, only the BSs having at least a single UE that is served through the same channel as the typical UE’s with probability \(1/M_a\) become interferers. It leads to \(\lambda = p_c(\lambda/M_a)\lambda_b\), as summarized in Table II.

Next, we utilize a transformation between BS density \(\lambda_b\) and received signal power \(S(\lambda_b)\) (or \(\lambda_i\) and \(I_c(\lambda_i)\)). According to mapping theorem [7], \(P\) times transmission power increase for all BSs is almost surely (a.s.) identical to \(P^{\frac{1}{2}}\) times BS densification without their power increase, from the typical UE’s received signal power point of view. By exploiting this relationship, we can transform any SIR(M_i) = \(S(\lambda_b)/I_c(\lambda_i)\) into its normalized value SIR = \(S(1)/I_c(1)\) specified in the following lemma.

Lemma 1. (SIR normalization, Corollary 2.35 in [7]) SIR(M_i) and its normalized SIR satisfy the following relationship.

\[\text{SIR}(M_i) = \left(\frac{\lambda_b}{\lambda_i^{\frac{1}{2}}}\right) \text{SIR} \text{ a.s.} \quad (3)\]

This allows us to focus only on \(\lambda_i\) when examining the proposed scheme’s impact on SIR.

Technique #2. Hypergeometric function transformation. When \(\lambda_i = \lambda_b\), it has been known that the baseline model’s downlink SIR coverage at a typical UE is the reciprocal of a Gauss-hypergeometric function [3], [6], given as

\[P_t(M_i) = \frac{1}{\text{F}_2(1, -\frac{1}{\alpha}; 1 - \frac{1}{\alpha}; -t)^{-1}} \quad (4)\]

where a Gauss hypergeometric function \(\text{F}_2(a, b; c; z) := \sum_{n=0}^{\infty} \frac{\alpha(a)_n(b)_n}{z^n} \) and Pochhammer symbol \((x)_n\) is defined as \(\Gamma(x + n)/\Gamma(x)\) for the Gamma function \(\Gamma(x + 1) := x!\).

Applying Lemma 1 generalizes this result for an arbitrary \(\lambda_i > 0\) by using Lemma 1.

\[P_t(M_i) = \Pr \left(\text{SIR} \geq \left[\frac{\lambda_b}{\lambda_i^{\frac{1}{2}}}\right] \right) \quad (5)\]

\[= \frac{1}{\text{F}_2(1, -\frac{1}{\alpha}; 1 - \frac{1}{\alpha}; -\left[\frac{\lambda_i}{\lambda_b}\right]^{\frac{1}{2}} t)^{-1}} \quad (6)\]

The last step follows from (4), which is also applicable for SIR. Setting \(\lambda_i = \lambda_b = 1\) while replacing \(t\) with \((\lambda_i/\lambda_b)^{\frac{1}{2}} t\) yield such a result.

The result (6) is, however, still not applicable for analytically solving the reliability constraint equation \(P_t(M_i) = \eta\) since the inverse function of (6) is unknown. We detour this problem by utilizing Pfaff’s Gauss-hypergeometric function transformation [10], specified in the following lemma.

Lemma 2. (Reliability Bounds) SIR reliability \(P_t(M_i)\) is upper and lower bounded as follows.

\[\frac{\alpha}{2\pi \text{csc} \left(\frac{\pi}{2\alpha}\right)} \left(1 + \left[\frac{\lambda_i}{\lambda_b^{\frac{1}{2}}}\right] t\right)^{-\frac{\pi}{2}} \leq \mathcal{P}_t(M_i) \leq \left(1 + \left[\frac{\lambda_i}{\lambda_b^{\frac{1}{2}}}\right] t\right)^{-\frac{\pi}{2}} \quad (7)\]

Proof: Consider Pfaff’s transformation [10] given as follows.

\[\text{F}_1(c - a, b; c; 1/1 - x^{-1}) = (1 - x)^{-c} \cdot \text{F}_1(a, b; c; z) \quad (8)\]

Setting \(x = (\lambda_i/\lambda_b)^{\frac{1}{2}}\) and applying \(a = b = -\frac{1}{\alpha}, c = 1 - \frac{1}{\alpha}\), and \(z = (1 + 1/x)^{-1}\) yields \(\mathcal{P}_t(M_i) = c_0(1 + x)^{-\frac{1}{2}}\) where \(c_0 := \text{F}_1(-\frac{1}{2}, -\frac{1}{2}; 1 - \frac{1}{2}; 1/1 + x)\) is a monotonically decreasing function of \(x\), having maximum \(\frac{\pi}{4\alpha} \text{csc} \left(\frac{\pi}{4\alpha}\right)\) and minimum 1 respectively when \(x\) approaches 0 and \(\infty\). Applying these inequalities provides the desired result.

As shown in Fig. 2, the lower bound in Lemma 2 is only accurate for low SIR reliability when \(t \to \infty\), so is not
applicable for UR2C design. On the contrary, the upper bound has potential for the use in an ultra-reliable regime. This motivates us to further improve the upper bound.

With this end, we modify the upper bound so that it can asymptotically converge to the exact curve in the ultra-reliable regime, i.e. $t \to 0$. This only requires replacing the outermost exponent $2/\alpha$ with $2/(\alpha - 2)$, leading to another lower bound.

**Lemma 3.** (Closed-Form Ultra-Reliability) SIR reliability $\mathcal{P}_t(M_t)$ is lower bounded as follows.

$$\mathcal{P}_t(M_t) \geq \left(1 + \left[\frac{\lambda_i}{\lambda_b}\right]^{\frac{2}{\alpha}} t\right)^{-\frac{1}{2/\alpha}} \tag{9}$$

where the equality holds when $(\lambda_i/\lambda_b)^{\frac{2}{\alpha}} t \to 0$

**Proof:** See Appendix.

As Fig. 2 shows, Lemma 3 lower bound is accurate in an ultra-reliable regime. In the following we therefore utilize this as an approximation, i.e. $\mathcal{P}_t(M_t) \approx \left(1 + [\lambda_i/\lambda_b]^{\frac{2}{\alpha}} t\right)^{-\frac{1}{2/\alpha}}$.

### V. Ubiquitous Rate with Frequency Reuse

In the first subsection, closed-form ubiquitous rates for $FR_b$ and $FR_u$ are derived. Based on this result, $FR_b$ and $FR_u$ are optimized in the second subsection.

#### A. Closed-Form Ubiquitous Rate

Applying Lemma 3 with $\lambda_i$ summarized in Table II yields the closed-form reliabilities for $FR_b$, $FR_u$, and the baseline.

**Proposition 1.** (Reliability) For $\eta \approx 1$, the reliabilities of $FR_b$, $FR_u$, and the baseline for SIR threshold $t$ are given as follows.

$$FR_b: \quad \mathcal{P}_t(M_b) \approx \left[1 + \left(\frac{p_c(\lambda)}{M_b}\right)^\frac{2}{\alpha} t\right]^{-\frac{1}{2/\alpha}} \tag{10}$$

$$FR_u: \quad \mathcal{P}_t(M_u) \approx \left[1 + p_c\left(\frac{\lambda}{M_u}\right)^\frac{2}{\alpha} t\right]^{-\frac{1}{2/\alpha}} \tag{11}$$

$$\text{Baseline:} \quad \mathcal{P}_t(1) \approx \left[1 + p_c(\lambda)^{\frac{2}{\alpha}} t\right]^{-\frac{1}{2/\alpha}} \tag{12}$$

The closed-form representations makes it possible to invert the reliability constraint equation $\mathcal{P}_t(M_t) = \eta$ with respect to $t$. Applying this $t$ to the rate $R_t(M_t)$ in (2) leads to the following closed-form ubiquitous rates.

**Proposition 2.** (Ubiquitous Rate) For $\eta \approx 1$, ubiquitous rates of $FR_b$, $FR_u$, and the baseline are given as follows.

$$FR_b: \quad R_{\eta}(M_b) \approx \frac{\eta W p_c(\lambda) \lambda_b}{\lambda_b^2} \log_2 \left(1 + \frac{c_\eta M_b}{p_c(\lambda)^{\frac{2}{\alpha}}}\right) \tag{13}$$

$$FR_u: \quad R_{\eta}(M_u) \approx \frac{\eta W p_c\left(\frac{1}{M_u}\right) \lambda_b}{\lambda} \log_2 \left(1 + \frac{c_\eta}{p_c\left(\frac{1}{M_u}\right)^{\frac{2}{\alpha}}}\right) \tag{14}$$

$$\text{Baseline:} \quad R_{\eta}(1) \approx \frac{\eta W p_c(\lambda) \lambda_b}{\lambda} \log_2 \left(1 + \frac{c_\eta}{p_c(\lambda)^{\frac{2}{\alpha}}}\right) \tag{15}$$

where $c_\eta := \left(\frac{\alpha}{2} - 1\right)\left(1 - \eta\right)$

**Proof:** See Appendix.

This result specifies the impacts of frequency reuse and ultra-densification on ubiquitous rate as below.

**Remark 1.** (Impact of $M_t$) A larger $M_t$ decreases resource allocation, and yet increases spectral efficiency. Therefore, optimizing $M_t$ is required.

For $FR_u$, the foregoing remark comes from the fact that $p_c(\lambda/M_u)$ is a monotone decreasing function of $M_u$.

**Remark 2.** (Impact of Densification) BS densification improves both spectral efficiency and resource allocation.

By the definition $p_c(\lambda)$ decreases with $\lambda$, and asymptotically converges toward 0 at the rate of $\lambda/\lambda_b$. This makes the density of the BSs occupying a single channel is upper bounded by the entire UE density, i.e. $p_c(\lambda)\lambda_b \leq \lambda$. Until reaching this bound, BS densification at least provides a marginal gain in resource allocation. Deploying more BSs also increases spectral efficiency by improving SIR reliability as specified in Lemma 3, leading to the above remark.

**Remark 3.** (Indistinguishable Condition) The ubiquitous rates of $FR_b$ and $FR_u$ are indistinguishable if $M_b$ and $M_u$ satisfy $p_c(\lambda/M_u) = p_c(\lambda)/M_b$, which is not always ensured since $M_u$ and $M_b$ are integers no smaller than 1.

It straightforwardly follows from comparing (13) and (14). This remark predicts that the optimal ubiquitous rates of $FR_b$ and $FR_u$ are not always identical yet very close to each other, to be verified by simulation in the next section.

**Remark 4.** (Identically Optimal Condition) For the optimal $M^*_t$ maximizing its ubiquitous rate, the optimal ubiquitous rates $R^*_\eta(M^*_t)$ and $R^*_\eta(M^*_u)$ become identical when both $M^*_b, M^*_u \to 1$ or $\lambda_b \to \infty$.

The case $M^*_t \to 1$ makes $FR_b$ and $FR_u$ converge to the baseline having $M_b = 1$. For $\lambda_b \to \infty$, indistinguishable condition in Remark 3 is always satisfied since $p_c(\lambda)$ and $p_c(\lambda/M_u)$ converge to 0.
B. Optimal Frequency Reuse Design

The optimal $M^*_i$ maximizing the ubiquitous rate is found as follows.

**Proposition 3.** (Optimal $M^*_i$) For $\eta \approx 1$, the optimal $M^*_i$ maximizing $R_\eta(M_i)$ is given as below:

$$M^*_b \approx \arg \min_{M_b} \left( 1 + \frac{p_c (\lambda M_u^* \eta^2)}{c_p M_b^* \eta^2} \right) \log_2 \left( 1 + \frac{c_p M_u^* \eta^2}{p_c (\lambda M_u^* \eta^2)} \right) - \frac{\alpha}{2}$$

$$M^*_u \approx \arg \min_{M_u} \left[ c_u + p_c \left( \frac{\lambda}{M_u} \right)^2 \right] \log_2 \left( 1 + \frac{c_u}{p_c (\lambda M_u^* \eta^2)} \right) - \frac{\alpha c_u}{2}$$

**Proof:** See Appendix.

For $\lambda_b \to 0$ or $\infty$, the optimal $M^*_i$’s converge as follows.

**Corollary 1.** (Asymptotically Optimal $M^*_i$) For $\eta \approx 1$, the asymptotic behavior of the optimal $M^*_i$ with respect to BS density $\lambda_b$ is given as follows.

$$M^*_b = M^*_u = \begin{cases} \infty & \text{if } (1 - \eta) \lambda_b \eta^2 \to 0 \\ 1 & \text{if } (1 - \eta) \lambda_b \eta^2 \to \infty \end{cases}$$

In the first diverging $M^*_i$, we can notice that the increase in $M^*_i$ comes from a higher reliability requirement $\eta$ and/or lower BS density. This implies exploiting frequency reuse becomes more effective for the UR2C design in sparse networks. The second converging $M^*_i$ to unity follows from the fact that BS ultra-densification can solely provide the target ultra-reliability while no longer relying on frequency reuse.

VI. NUMERICAL EVALUATION

Default system parameters in this section are given as follows: $\eta = 0.99$, $\alpha = 4$, and $W = 100$ MHz [2]. Applying Proposition 3 to Proposition 2 via numerical simulation yields the following frequency reuse design guideline for UR2C.

$\text{FR}_0$ and $\text{FR}_u$ increases ubiquitous rate, further improved by higher $\eta$ and $\alpha$. Fig. 3 shows the maximized ubiquitous rates with $\text{FR}_0$ and $\text{FR}_u$ based on Proposition 3 (see yellow dotted and blue solid curves) outperform the baseline result without frequency reuse (black dash-dotted). The gain in ubiquitous rate, $R_\eta(M^*_i)/R_\eta(1)$, increases for higher $\eta$ as it makes frequency reuse more effective to cope with the target reliability requirement. Compared to the baseline, the maximum gain with $\eta = 0.999$ is 3.15 times larger than the gain with $\eta = 0.99$. For higher $\alpha$, the gain also increases since SIR grows along with $\alpha$ under an interference-limited regime [4]–[6], amplifying the effect of frequency reuse in ubiquitous rate improvement. When $\alpha$ becomes 6 from 4, it increases the gain by 2.03 times. For a given target ubiquitous rate 50 Mbps, both $\text{FR}_b$ and $\text{FR}_u$ improve ubiquitous rate for all considered scenarios, while not converging to the baseline. This leads to reducing the required BS densification by up to 46%.

$\text{FR}_0$ mostly performs better while $\text{FR}_u$ sometimes outperforms for large BS density. Overall, the maximized ubiquitous rates of $\text{FR}_0$ and $\text{FR}_u$ keep crossing as BS density increases, but have no significant gap. As predicted in Remark 3, the reason follows from the fact that $M^*_i$’s are integers no smaller than 1. These coarse unit changes cannot effectively minimize the objective argument functions in Proposition 3, so constantly lead to peaky crossings as well as the gap between the results from Proposition 3 and a full search algorithm in Fig. 3, until $\text{FR}_0$ and $\text{FR}_u$ converge to the baseline. While having similar ubiquitous rates, $\text{FR}_0$ requires less number of channels as shown in Fig. 4, so is more preferable than $\text{FR}_u$, especially for small BS density where $M^*_u$ is unrealistically large. For moderate BS density, $\text{FR}_u$ sometimes provides up to 28% higher ubiquitous rate than $\text{FR}_0$, and it is thus important to compare them in this regime for UR2C design. For large BS density, frequency reuse gain in ubiquitous rate disappears as also anticipated in Remark 4 and Corollary 1.
Optimal FR₀ and FRᵤ provides similar per-UE resource allocations and spectral efficiencies. Fig. 5 illustrates that FR₀’s maximum per-UE allocation 𝑊/𝑀₀ is larger but its actual allocation fraction 𝐸[1/𝑁₀] within the maximum value due to multiple access is smaller. These two factors cancelled out each other, resulting in almost the same resource allocation due to multiple access is smaller. These two factors cancelled out each other, resulting in almost the same resource allocation as FRᵤ’s. Fig. 6 visualizes FR₀’s maximized spectral efficiency keeps crossing but becomes clearly larger under a small BS/UE density environment. This originates from the fact that FR₀ more aggressively reduces interference by forcing to use only a single channel (see Fig. 1-a). It is effective under small BS/UE density where SIR is low, yielding FR₀’s smaller interferer density normalized by BS density 𝜆ᵣ/λ₀. Note also that non-smooth FR₀ and FRᵤ curves in Figs. 5 and 6 are because 𝑀₀ and 𝑀ᵤ are integers as elaborated in Remark 3.

VII. Conclusion

In this paper we derive closed-form ubiquitous rate, and investigate two frequency reuse schemes FR₀ and FRᵤ so that they can ubiquitously achieve the UR2C’s target 50 Mbps data rate with 99% SIR reliability. Both of their optimal designs provide higher ubiquitous rates compared to the baseline without frequency reuse. This result is different from traditional network design guidelines that prefer not to use frequency reuse for only maximizing data rate without guaranteeing reliability. It would thus be worth revisiting more advanced frequency reuse techniques that additionally utilize spatial and power differences. To enable UR2C, it would also be an interesting topic to incorporate other reliability achieving schemes such as double associations for attaining diversity gain [11].

APPENDIX

A. Proof of Lemma 3

Let 𝑓(𝑥) = 2𝐹₁(1, − 2/α; 1 − 2/α; −𝑥)^−1. By the Gauss-hypergeometric function definition,

\[ f(𝑥) = \left[ \sum_{n=0}^{∞} \frac{(1)_{n}(-\frac{2}{α})_{n}(-x)^{n}}{(1-\frac{2}{α})_{n}} \right]^{-1} = 1 - \frac{2x}{α - 2} + O(x^2) \] (19)

The last step follows from Taylor’s expansion for 𝑥 → 0. This indicates \( \lim_{x \to 0} f'(x) = -2/(α - 2) \).

Consider \( g(x) := (1 + x)^{-\frac{2}{αx}} \), having the same slope as \( f'(x) \) as \( x \to 0 \). To verify \( g(x) \leq f(x) \), we consider the first and second derivatives of \( f(x) \) and \( g(x) \) for \( x > 0 \) as follows.

\[ f'(x) = -2\left[f(x) - (1 + x)^{-1} f(x)^2\right] \] (20)
\[ g'(x) = -2(1 + x)^{-\left(1 + \frac{2}{αx}\right)} \frac{αx}{α - 2} \] (21)
\[ f''(x) = \frac{2(2 + α) f(x) + (1 + x)^{-2} \left[8 - 2(α + 6 + 6x + 2αx) f(x)^2 \right]}{(αx)^2} \] (22)
\[ g''(x) = \frac{2α(1 + x)^{-2\left(1 + \frac{2}{αx}\right)}}{(α - 2)^2} \] (23)

Firstly, \( f'(x), g'(x) < 0, f''(x) > 0, \) and \( g''(x) \geq 0 \) for all \( x > 0 \) where the equality holds when \( x \to \infty \) and/or \( α \to \infty \). Secondly, \( \lim_{x \to 0} f'(x)/g'(x) = 1 \) and \( \lim_{x \to \infty} f'(x)/g'(x) \leq 1 \) where the equality holds when \( α \to \infty \). Lastly, \( f(0)/g(0) = 1 \) and \( \lim_{x \to \infty} f(x)/g(x) = 1 \) due to their original coverage probability definitions. As \( x \) increases, these conclude that \( f(x) \) and \( g(x) \) are monotonically decreasing, starting identically from unity while the decreasing slope of \( g(x) \) is steeper than that of \( f(x) \) due to \( f'(x)/g'(x) \leq 1 \) for all \( x > 0 \). This concludes the proof of \( f(x) \geq g(x) \) for all \( x \).
B. Proof of Proposition 2

Let \( a_i \) denote \( p_c(\lambda_b)/M_b, p_c(\lambda/M_u) \), and \( p_c(\lambda) \) respectively for FR_b, FR_u and the baseline. By applying Proposition 1, the reliability constraint equation \( P_t(M_i) = \eta \) is rephrased with respect to \( t \)

\[
t = \eta^{-(\alpha^2 - 1)} - 1 \quad \text{a}_i^{\alpha^2} \tag{24}
\]

\[
= \left[ 1 - (1 - \eta) \right]^{-(\alpha^2 - 1)} - 1 \quad \text{a}_i^{\alpha^2} \tag{25}
\]

In (25), applying Taylor expansion for \( \eta \approx 1 \) makes \( [1 - (1 - \eta)]^{-(\alpha^2 - 1)} \) become \( 1+c_\eta \). Applying this with Table II and a typical UE’s reliability \( \eta \) to (2) provides the desired result. ■

C. Proof of Proposition 3

According to Karush-Kuhn-Tucker (KKT) first and second order conditions, the local maximum of \( R_\eta(M_b) \) in (13) is the global maximum. Likewise, the local maximum of \( R_\eta(M_u) \) in (14) becomes its global maximum. Therefore, \( M_b \) and \( M_u \) can maximize \( R_\eta(M_b) \) and \( R_\eta(M_u) \) when respectively satisfying the first order conditions \( dR_\eta(M_b)/dM_b = 0 \) and \( dR_\eta(M_u)/dM_u = 0 \). Now that \( M_b \) and \( M_u \) are integers no smaller than 1, the values minimizing the LHS of the first order conditions provide the optimal numbers of channels, finalizing the proof. ■

REFERENCES

[1] N. Alliance, “Recommendations for NGMN KPIs and Requirements for 5G,” Jun. 2016.
[2] Qualcomm, “5G Vision Presentation,” Feb. 2016.
[3] P. Popovski, “Ultra-Reliable Communication in 5G Wireless Systems,” in Proc. 1st Int. Conf. on 5G for Ubiquitous Connectivity (5GU), Levi, Finland, Nov. 2014.
[4] J. Park, S.-L. Kim, and J. Zander, “Tractable Resource Management with Uplink Decoupled Millimeter-Wave Overlay in Ultra-Dense Cellular Networks,” IEEE Trans. Wireless Commun., vol. 15, pp. 4362–4379, Jun. 2016.
[5] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A Tractable Approach to Coverage and Rate in Cellular Networks,” IEEE Trans. Commun., vol. 59, pp. 3122–3134, Nov. 2011.
[6] X. Zhang and M. Haenggi, “A Stochastic Geometry Analysis of Inter-Cell Interference Coordination and Intra-Cell Diversity,” IEEE Trans. Wireless Commun., vol. 13, pp. 6655–6669, Dec. 2014.
[7] M. Haenggi, Stochastic Geometry for Wireless Networks. Cambridge Univ. Press, 2013.
[8] S. M. Yu and S.-L. Kim, “Downlink Capacity and Base Station Density in Cellular Networks,” Proc. IEEE WiOpt Wksp. on Spatial Stochastic Models for Wireless Networks (SpaSWIN), May 2013.
[9] D. Stoyan, K. W. S., and J. Mecke, Stochastic Gemoetry and its Applications. Wiley, 2nd ed., 1995.
[10] G. E. Andrews, R. Askey, and R. Roy, Special Functions. Cambridge Univ. Press, 1999.
[11] D. M. Kim and P. Popovski, “Reliable Uplink Communication through Double Association in Wireless Heterogeneous Networks,” IEEE Wireless Commun. Lett., vol. 5, pp. 312–315, Jun. 2016.