Arc Motion in an Obliquely Imposed Alternating Magnetic Field

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Abstract. The arc motion is theoretically investigated under an alternating magnetic field imposed obliquely to the arc. The arc is known to oscillate on a 2-D plane when the alternating magnetic field is imposed perpendicularly to the arc. If the alternating magnetic field is imposed obliquely to the arc, then it is expected that the arc oscillates not on the 2-D plane but in a 3-D space. For this study, 3-D simulation was performed on the motion of the plasma gas under an alternating magnetic field crossing obliquely to the arc. It was also assumed that a streamline of the plasma gas represented the arc profile. The momentum equation for the plasma gas was solved together with the continuity equation. Governing parameters for the gas motion are \( \theta \) (crossing angle), \( v_0 \) (initial velocity of the plasma gas), and \( \lambda \). Parameter \( \lambda \) is defined as \( \lambda = (I_a B_0)/Q_0 \). Numerical results are reported under different operating conditions such as magnetic flux densities and the angles between the arc and the magnetic flux. If the crossing angle is larger than \( 4/\pi \), the arc might be extinguished because of the drastic increase of the arc length.

Keywords. Transferred arc, Alternating magnetic field, Momentum equation, 3-D simulation

1. Introduction

Arcs, which are widely used as high-intensity heat sources in various industrial fields, have power output that is easily controllable by adjusting the arc current. Various chemical atmospheres can be provided using different plasma gases. Among different modes of arc operation, a transferred arc is often used in many material processes such as welding, cutting, melting, and refining metals because of its high-efficiency heat transfer. Thermal and electromagnetic pinch effects stabilize the arc, which concentrates the heat flow into a small area. This property is desirable to obtain high temperatures in a narrow zone of the material, but it is not convenient for heating wide areas of materials.

Several trials were attempted to expand the arc heat flow [1–3]. The authors obtained a wide heat source by imposing the alternating magnetic field perpendicularly to a transferred arc [4–6]. As shown in Figure 1, the arc is driven by the electromagnetic force induced from the interaction between the magnetic field and arc current. The high-speed oscillating arc can be regarded as an expanded heat source with a width of the oscillation amplitude. In addition to conventional arc characteristics, a magnetically driven arc presents the following advantages: (1) the arc width can be varied easily by adjusting the imposed magnetic field, and (2) the density distribution of heat flux can be controlled by varying the waveform of the imposed magnetic field. For example, if the waveform of the magnetic field is rectangular, then the heat flow is concentrated at both ends of the oscillation.

In previous work on a magnetically driven arc, theoretical, and experimental investigation was restricted to the magnetic field perpendicular to the arc, where the arc moved two-dimensionally in a single plane. In the present work, the authors consider the arc motion under the magnetic field.
imposed obliquely at an arbitrary crossing angle to the arc. In such an arrangement, the arc is expected to move in a three-dimensional space.

![Figure 1.](image1.png) **Figure 1.** (a) Schematic illustration of the arrangement used to produce the magnetically driven arc, and (b) a typical expanded arc [5].

2. Modeling of an arc motion under the external magnetic field at an arbitrary crossing angle

2.1. Governing equation

A theoretical model is considered under the arrangement shown in Figure 2. A DC transferred arc is generated between cathode in a plasma torch and a metallic plate that serves as an anode. Plasma gas is supplied to the torch and is ejected from the torch orifice. The origin of the coordinate system used in the theoretical modeling is located at the torch orifice exit. The direction and strength of the imposed magnetic field change periodically with time, but they are uniform in space. Physical parameters and their nomenclature are presented in Table 1.

![Figure 2.](image2.png) **Figure 2.** Schematic illustration of the arrangement used to produce the magnetically driven arc. The magnetic field is imposed at the crossing angle of $\theta$.

| Symbol | Nomenclature |
|--------|--------------|
| $\rho$ | Density of plasma gas [kg/m³] |
| $P$    | Pressure of plasma gas [Pa]   |
| $\mu$  | Coefficient of viscosity [Pa/s] |
| $v$    | Velocity of plasma gas [m/s]  |
| $j$    | Arc current density [A/m²]    |
| $F$    | External force (Electromagnetic force) [N] |
The fundamental concept of the present modeling is based on the idea that the stream line of the plasma gas after the torch orifice represents the arc profile because the arc current is able to pass only through the conductive plasma gas. The relations governing the plasma gas motion are a continuity equation and a momentum equation. Their general forms are written respectively as shown below.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}
\]

\[
\rho \frac{dv}{dt} = F - \nabla p + \mu \Delta v + \frac{\mu}{3} \nabla (\nabla \cdot \mathbf{v}) \tag{2}
\]

To simplify the consideration, the following assumptions are used.

1. Pressure is constant
2. Plasma gas is incompressible outside the torch
3. Viscous effects are negligible
4. External body force is an electromagnetic one written as \( \mathbf{F} = \mathbf{j} \times \mathbf{B} \)
5. The alternating magnetic field frequency is so low that the imposed magnetic field can be assumed as steady.

Consequently, eq. (2) takes the following form.

\[
\rho \frac{dv}{dt} = j \times B \tag{3}
\]

According to the fundamental concept by which the arc current passes along the channel of the plasma gas flow, the vector of plasma gas velocity and that of arc current are denoted as

\[
\mathbf{j} = -\alpha \cdot \mathbf{v}, \quad (4)
\]

where \( \alpha \) is a positive constant. A minus sign in eq. (4) results from the anti-parallel relation between the plasma gas motion and the arc current. Consequently, the following relation is obtained.

\[
\rho \frac{dv}{dt} = -\alpha (\mathbf{v} \times \mathbf{B}) \tag{5}
\]

The validity of the relation simplified in eq. (5) was confirmed in an earlier experiment conducted under the imposed magnetic field perpendicular to the arc [4]. In the present study, the magnetic flux density has not only an x-component but also a z-component, as shown in Figure 2.

\[
\mathbf{B} = (B_x, 0, B_z), \quad B_x^2 + B_z^2 = B_0^2
\]

\[
B_x = B_0 \sin \theta, \quad B_z = B_0 \cos \theta \tag{6}
\]

Then, eq. (5) expressed in vector form is decomposed into three scalar equations as shown below.

\[
\rho \frac{dv_x}{dt} = -\alpha v_z B_z \tag{7}
\]

\[
\rho \frac{dv_y}{dt} = -\alpha (v_z B_x - v_x B_z) \tag{8}
\]
\[\rho \frac{dv_x}{dt} = \alpha v_y B_x\]  

(9)

Differentiating eq. (8) with respect to \(t\) and substituting eqs. (7) and (9), following eq. (10) is obtained.

\[\frac{d^2 v_y}{dt^2} + \lambda^2 v_y = 0\]  

(10)

Therein, \(\lambda\) is defined as shown below.

\[\lambda = \frac{\alpha}{\rho} B_0\]  

(11)

Similarly, eq. (7) and eq. (9) can be transformed respectively into eq. (12) and eq. (13).

\[\frac{d^2 v_x}{dt^2} = -\frac{\alpha}{\rho} B_z \left\{ -\frac{\alpha}{\rho} \left(v_z B_x - v_x B_z\right) \right\}\]  

(12)

\[\frac{d^2 v_z}{dt^2} = \frac{\alpha}{\rho} B_x \left\{ -\frac{\alpha}{\rho} \left(v_z B_x - v_x B_z\right) \right\}\]  

(13)

Rearranging both eqs. (12) and (13), the following eqs. (14) and (15) are obtained, each of which corresponds respectively to the equation for \(v_x\) and that for \(v_z\).

\[\frac{d^2 \left( \frac{\rho}{\alpha} \right)^2 \frac{1}{B_x B_z} \frac{d^2 v_x}{dt^2}}{dt^2} + \frac{B_0^2}{B_x B_z} v_y = 0\]  

(14)

\[\frac{d^2 \left( \frac{\rho}{\alpha} \right)^2 \frac{1}{B_x B_z} \frac{d^2 v_z}{dt^2}}{dt^2} + \frac{B_0^2}{B_x B_z} v_y = 0\]  

(15)

Integrating the equations above with respect to \(t\),

\[\frac{d^3 v_x}{dt^3} + \lambda^2 \frac{dv_x}{dt} + C_1 = 0\]  

(16)

and

\[\frac{d^3 v_z}{dt^3} + \lambda^2 \frac{dv_z}{dt} + C_2 = 0\]  

(17)

Further integration of eq. (16) and eq. (17) result in the following equations.

\[\frac{d^2 v_x}{dt^2} + \lambda^2 v_x + C_1 t + C_3 = 0\]  

(18)

\[\frac{d^2 v_z}{dt^2} + \lambda^2 v_z + C_2 t + C_4 = 0\]  

(19)

The constants of integration, \(C_1, C_2, C_3, C_4\) are determined respectively by considering the initial conditions for the plasma gas velocity as

\[ (v_x, v_y, v_z) = (0, 0, v_0), \text{ at } (x, y, z, t) = (0, 0, 0, 0) \]  

(20)

Using the relations of eqs. (7), (8), and (9), further initial conditions are obtained as eqs. (21), (22), and (23).

\[ \left( \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \right)_{t=0} = (0, -\lambda v_0 \sin \theta, 0), \text{ at } (x, y, z, t) = (0, 0, 0, 0) \]  

(21)
\[ \begin{align*}
\left( \frac{d^2 v_x}{dt^2}, \frac{d^2 v_y}{dt^2}, \frac{d^2 v_z}{dt^2} \right)_{x=0, y=0, z=0} &= \left( \lambda^2 v_0 \sin \theta \cos \theta, 0, -\lambda^2 v_0 \sin^2 \theta \right), \text{ at } (x, y, z, t) = (0, 0, 0) \\
\left( \frac{d^3 v_x}{dt^3}, \frac{d^3 v_y}{dt^3}, \frac{d^3 v_z}{dt^3} \right)_{t=0} &= \left( 0, -\lambda^3 v_0 \sin \theta, 0 \right), \text{ at } (x, y, z, t) = (0, 0, 0).
\end{align*} \]

Considering these initial conditions of eqs. (20), (21), (22), and (23), eq. (18) and eq. (19) are expressed as

\[ \frac{d^2 v_x}{dt^2} + \lambda^2 v_x + \lambda^2 v_0 \sin \theta \cos \theta = 0 \] (24)

and

\[ \frac{d^2 v_z}{dt^2} + \lambda^2 v_z - \lambda^2 v_0 \cos^2 \theta = 0 \] (25)

Then, substituting the following relations eq. (26) to eq. (10), eq. (24) and eq. (25),

\[ v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}, \]

the equations which govern the displacement of the plasma gas are finally rewritten in the following forms:

\[ \frac{d^3 x}{dt^3} + \lambda^2 \frac{dx}{dt} + \lambda^2 v_0 \sin \theta \cos \theta = 0 \] (27)

\[ \frac{d^3 y}{dt^3} + \lambda^2 \frac{dy}{dt} = 0 \] (28)

\[ \frac{d^3 z}{dt^3} + \lambda^2 \frac{dz}{dt} - \lambda^2 v_0 \cos^2 \theta = 0 \] (29)

Solving these differential equations, the position of the plasma gas with \( t, \{x(t), y(t), z(t)\} \) is obtained. Initial conditions at \( t = 0 \) for eqs. (27), (28), and (29) are as shown below:

\[ \begin{align*}
(x, y, z)_{t=0} &= (0, 0, 0) \\
\left( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)_{t=0} &= (0, v_0) \\
\left( \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2} \right)_{t=0} &= (0, \lambda v_0 \sin \theta, 0).
\end{align*} \] (30)

2.2. Conservation of kinetic energy for the plasma gas

The equation shown as eq. (5) induces the conservation of kinetic energy for the plasma gas easily. The scalar products of vector \( \mathbf{v} \) to both sides of eq. (5) results in

\[ \rho \left( \frac{dv}{dt} \right) \cdot \mathbf{v} = -\alpha \mathbf{v} \times \mathbf{B} \cdot \mathbf{v}. \] (31)

The left-hand-side of eq. (31) is written as

\[ \rho \left( \frac{dv}{dt} \right) \cdot \mathbf{v} = \frac{d}{dt} \left( \frac{1}{2} \rho \mathbf{v}^2 \right), \] (32)

and its right-hand-side is zero. Therefore,

\[ \frac{1}{2} \rho \mathbf{v}^2 = \text{constant}. \] (33)
This relation is that of the kinetic energy conservation for the plasma gas during the flight. The velocity of the plasma gas might change its direction, but $v^2$ must always equal $v_0^2$ where $v_0$ represents the initial velocity of the plasma gas. Because the plasma gas is ejected in the downward direction, the initial velocity $v_0$ is negative.

Then we consider the assumption (5) described in the previous section. For the plasma gas, the travelling time from the torch orifice to the anode, $\tau_{\text{travel}}$, is estimated as

$$\tau_{\text{travel}} = \frac{L}{|v_0|},$$  \hspace{1cm} (34)

whereas the characteristic value $\tau_{\text{field}}$ which expresses the time to change the magnetic field is evaluated as

$$\tau_{\text{field}} = \frac{1}{f}.$$

Therein, $f$ represents the frequency of the imposed alternating magnetic field. If the following condition is satisfied, it is reasonable that the imposed magnetic field is assumed to be steady during the flight of the plasma gas, as

$$\tau_{\text{field}} \gg \tau_{\text{travel}},$$  \hspace{1cm} (36)

or

$$f \ll \frac{|v_0|}{L}.$$  \hspace{1cm} (37)

We must bear in mind that our consideration is restricted to the condition written in eq. (37).

2.3. Determination of $\lambda$.

We consider the relation between $\lambda$ and the operating parameters such as the arc current, the imposed magnetic flux density, and the plasma gas flow rate. The governing equations (27), (28), (29) under the initial conditions (30) show that the displacement of the plasma gas with time (or the arc profile) is determined by "$\lambda$", "$v_0$" and "$\theta$". Using the relation of eq. (4), eq. (11) is rewritten as

$$\lambda = -\frac{j}{\rho_0^2}B_0 = \frac{j}{\rho_0 v_0}B_0.$$  \hspace{1cm} (38)

If the cross sectional area of the plasma gas stream tube is represented as $S$, then the flow rate of the plasma gas $Q_0$ is expressed as $\rho_0 S$, whereas the arc current $I_a$ is equal to $jS$ according to the assumption that the arc current passes along the stream line of the plasma gas. Consequently, the following relation is obtained.

$$\lambda = \frac{I_a B_0}{Q_0}.$$  \hspace{1cm} (39)

3. Numerical Results and Discussion

Numerical calculations were performed. Commercial software (Mathematica; )[9] was used to solve the differential equations shown as (27), (28), and (29) under the initial condition of eq. (30). The values of parameters used in the present calculation are presented in Table 2. Except for crossing angle, numerical values shown in Table 2 are the same as those used in the previous work [4]. The numerical value of $\lambda$ was determined easily by operating parameters such as arc current $I_a$, imposed magnetic flux density $B_0$, and mass flow rate of plasma gas $Q_0$, according to the relation of eq. (39). Because the imposed magnetic field is alternating, the magnetic flux density, $B_0$ varies between $B_{\text{max}}$ to $-B_{\text{max}}$.

| Parameters                          | Symbol | Value               |
|------------------------------------|--------|---------------------|
| Arc current [A]                    | $I_a$  | 90                  |
| Flow rate of plasma gas [kg/s]     | $Q_0$  | $3.0 \times 10^{-4}$|
| Initial velocity of plasma gas [m/s]|[ $v_0$ | -80                 |
| Magnetic flux density [T]          | $B_0$  | $20 \times 10^{-3}$ |
Crossing angle [rad] \( \theta \) \( 0 - \pi/2 \)
Standoff distance between torch and anode [m] \( L \) \( 0.05 \)
Frequency of the alternating magnetic field [Hz] \( f \) \( 50 \)

Under the conditions presented above, \( \tau_{\text{travel}} \) was estimated at \( 0.6 \times 10^{-3} \) [s] and \( \tau_{\text{field}} \) was \( 20.0 \times 10^{-3} \) [s]. The condition expressed in eq. (36) was satisfied in the present calculation.

For different crossing angles of \( \theta \), variations of arc profiles with magnetic flux density are depicted in Figure 3. As shown in Figure 3(a) in which \( \theta = \pi/2 \), the curvature of the profiles increases with increased \( B_0 \). Furthermore, the arc root changes its position from A, B, C, and D along the y axis. Beyond a critical strength of the magnetic flux density corresponding to point D, arc has no arc root on the anode plate, which means that the arc is not able to exist beyond a certain magnetic flux density. Figures 3(b)–3(d) show variations of arc profiles with \( \lambda \), respectively, for \( \theta = \pi/3 \), \( \theta = \pi/4 \), \( \theta = \pi/6 \).

Figure 3(a). Arc profile variation with the increase of the magnetic field at \( \theta = \pi/2 \).

In Figure 3(b), arc root movements are shown between the positive magnetic flux density (or the positive \( \lambda \)) to clarify the movement of the arc root on the anode. Because the arc movement is symmetric with respect to the x-z plane for the negative magnetic flux density, the total movement of the arc root can be imagined in the whole range of \(-B_{\text{max}}\) to \(B_{\text{max}}\).

Figure 3(b). Arc profile variation with the increase of the magnetic field at \( \theta = \pi/3 \).
Figure 3(c). Arc profile variation with the increase of the magnetic field at $\theta = \pi/4$.

Figure 3(d). Variation of arc profiles with increased magnetic field at $\theta = \pi/6$.

As shown in Figure 3(b), the arc root shifts with the increase of the magnetic flux density from points A to B, C, and D; then it jumps to D' and with the further increase of the magnetic flux density, it moves to the points E, F, and G. Under the numerical condition presented in Table 2, a discrete jump occurs at $B_0 = 6.4 \times 10^{-3}$ [T]. In Figure 3(c), the arc root moves with the increase of the magnetic field from A to B, C, D and E. At point E, the arc is tangential to the anode plane. Then it goes to F, G, and H. No discrete jump appears at $\theta = \pi/6$, as shown in Figure 3(d).

The arc root movements on the anode for different values of $\theta$ are depicted in Figure 4. Further detailed analyses for various crossing angles reveal that the discrete jump occurs in the case of $\theta > \pi/4$. The arc length, $H$, is evaluated as

$$ H = \int v \cdot dt = v_0 \tau_{\text{travel}}. $$

(40)

Its flight time of the plasma gas from the torch orifice to the anode, $\tau_{\text{travel}}$, satisfies the following relation:

$$ z(t = \tau_{\text{travel}}) = L. $$

(41)

Therein, $z(t)$ is obtained by solving eq. (29). The arc length variation with $\lambda$ is shown for various crossing angles in Figure 5, where the ratio of $H/L$, as the ordinate, is shown with $\lambda$ as the abscissa.
For $2 \pi \theta = \pi$, the maximum arc length is $2 \pi$ times longer than the stand-off distance at $\lambda = 1600$. When the crossing angle is between $2 \pi/3 < \theta < \pi$, a jump in the arc length occurs. When $\theta = \pi/3$, the arc elongates to five times larger than the stand-off distance. Such an arc length increase results in the drastic increase of the arc voltage. Theoretically, it is possible for the arc to exist, but the arc will practically disappear. If $\theta = \pi/4$, then no jump occurs. The maximum elongation $H/L$ is 2.54 at $\theta = \pi/4$. It becomes 1 when $\theta$ approaches 0.

4. Conclusions
Theoretical consideration was performed to investigate the arc motion under the alternating magnetic field imposed obliquely to the arc. The fundamental concept of the modeling was based on the idea that the stream line of the plasma gas represents the arc profile.

- Theoretical analyses reveal the following results:
  (1) Displacement of the plasma gas with time is determined solving a set of three third-order linear ordinary differential equations.
  (2) Governing parameters for the gas motion are $\theta$ (crossing angle), $v_0$ (initial velocity of the plasma gas), and $\lambda$.
  (3) Parameter $\lambda$ is defined as $\lambda = (I_u B_0)/Q_0$.
- Numerical calculations reveal the following results:
The arc under the obliquely imposed alternating magnetic field moves three-dimensionally in space, different from the two-dimensional movement found under the perpendicular magnetic field.

If a crossing angle is larger than $\pi/4$, then the arc root on the anode moves discretely at a certain value of $\lambda$. At this critical $\lambda$, the arc might be extinguished because of the drastic increase of the arc length.

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