PAPER

Witnessing topological Weyl semimetal phase in a minimal circuit-QED lattice

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Abstract

We present an experimentally feasible protocol to mimic topological Weyl semimetal phase in a small one-dimensional circuit-QED lattice. By modulating the photon hopping rates and on-site photon frequencies in parametric spaces, we demonstrate that the momentum space of this one-dimensional lattice model can be artificially mapped to three dimensions accompanied by the emergence of topological Weyl semimetal phase. Furthermore, via a lattice-based cavity input-output process, we show that all the essential topological features of Weyl semimetal phase, including the topological charges associated with Weyl points and the photonic surface states connecting the Weyl points as open arcs, can be unambiguously detected in a circuit with four dissipative resonators by measuring the reflection spectra. These remarkable features may open up a new prospect for simulating topological phases with well-controlled small quantum artificial lattices.

1. Introduction

Circuit quantum electrodynamics (QED) has achieved great experimental progresses in the past decade, including demonstrating quantum error correction based on surface code [1, 2] and cat code [3]. This system now has become one of the leading platforms for studying quantum optics and scalable quantum computing [4, 5]. The recent experimental progresses achieved in circuit QED endow this system with high coherence superconducting qubits [6] and microwave resonators [7]. Besides, such system also possesses tunable system parameters [8–10] and straightforward connectivity [11]. These advantages push this system further toward quantum simulation [12–15]. In particular, circuit-QED lattices formed by coupling microwave resonators and superconducting qubits provide a natural platform to realise analogue quantum simulators. A wide range of many-body physics has been studied in such simulators in the past years [16–24]. Experimentally, digital quantum simulation of quantum spin models has recently been demonstrated in superconducting circuits [25, 26].

The recent advance of topological photonics has opened a door for studying topologically protected states with photons [27, 28]. In particular, photonic topological insulator states nowadays have been extensively studied in different photonic systems [29–34]. In the context of circuit QED, the methods to produce photonic synthetic gauge fields have been investigated in a two-dimensional (2D) coupled microwave resonator lattice [35–37]. Based on engineering photonic magnetic fields and photonic interactions, integer and fractional
photonic topological insulators states were also studied [38–41]. Another route to realise photonic topological phase is based on dimension reduction [42–46]. It is found that the topological features of 2D photonic topological insulators could be observed in a one-dimensional (1D) system [42, 43], where one parameter that can be tuned periodically has been introduced as an artificial dimension in addition to the real spatial dimension.

On the other hand, topological Weyl semimetal has now attracted significant attention because its low-energy excitations are Weyl fermions [47–50]. This gapless semimetal phase is a new topological state of matter, extending topological classification to go beyond gapped topological insulators and opening a new era for condensed matter physics. Specifically, the conduction and valence bands of topological Weyl semimetal touch nontrivially at pairs of Weyl points [47]. The experimental realisation of topological Weyl semimetal was first reported in TaAs materials, where both the Weyl cones and the Fermi arcs were observed [51, 52]. Weyl points have also been observed in photonic crystal system [53] and have been theoretically studied in cold atoms [54] and acoustic systems [55]. However, the question of directly measuring the topological charge associated with each Weyl point remains unsolved either in solid-state materials or in artificial lattice systems.

In this paper, we propose a feasible protocol to mimic topological Weyl semimetal phase with a 1D circuit-QED lattice. In this lattice, the photon hopping between nearest neighbour resonators and the frequency shift of the resonators can be tuned by externally control circuit with state-of-the-art circuit-QED technology, which allows us to parameterise these two parameters and employ them as two artificial momentum dimensions. Combining these two synthetic momentums with the momentum in the real space, we construct a synthetic three-dimensional (3D) momentum space, and we find that four Weyl points emerge in the first Brillouin zone. The resulted gapless phase is further demonstrated as a topological Weyl semimetal phase by numerically analysing the topological charges of the Weyl points and the Fermi arc surface states. More importantly, with a lattice-based cavity input-output process, we show that all the essential features of topological Weyl semimetal can be unambiguously measured from four-coupled resonators with dissipation. In particular, we design a simple strategy to measure the topological charge associated with each Weyl point, which has not been shown in the literature before. Such remarkable results render us a minimal and realistic platform to observing topological Weyl semimetal physics and make our theory attractive to the experimentalists in the circuit-QED community.

2. Methods

One-dimensional circuit-QED lattice. We consider a 1D array of superconducting transmission line resonators and Xmon qubits alternatively connected to each other. The cross−shape Xmon qubit has recently attracted much attention, as it can be used to construct scalable quantum circuits [6, 11]. The Xmon qubit can be connected to different circuit elements simultaneously, including resonators or external qubit control lines. Such features allow us to build the circuit-QED lattice shown in figure 1. In this lattice, each unit cell contains two resonators, labelled $a_n$ and $b_n$ respectively. The two resonators $a_n$ and $b_n$ in the $n$th unit cell are both coupled to the Xmon qubit $Q_{na}$, and the two resonators $b_n$ and $a_{n+1}$ that belong to two adjacent unit cells are both coupled to the Xmon qubit $Q_{nb}$. Meanwhile, another Xmon qubit is embedded inside each resonator to provide additional control over the frequency shift of the resonator modes. Specifically, we assume the two resonators $a_n$ and $b_n$ in the same unit cell are coupled to the Xmon qubits $Q_{na}$ and $Q_{nb}$, respectively. The total Hamiltonian of this system can be written as

$$H_q = \sum_n \sum_{x=1,2,3,4} \frac{\omega_x}{2} \tau_{nx}^+ \tau_{nx}^- + \omega_c (a_n^\dagger a_n + b_n^\dagger b_n) + g_x \tau_n^+ (a_n + b_n) + g_x \tau_n^- (a_n + b_n) + g_x \tau_n^+ a_n + g_x \tau_n^- b_n + H.c.$$  

(1)

where $\omega_c$ and $\omega_x$ are the frequencies of the superconducting Xmon qubits (with $\tau_{nx}$ operators) and the transmission line resonators, respectively, $g_x$ is the coupling strength between the Xmon qubit $Q_{nx}$ and $Q_{nx+1}$.
the resonators $a_n$ and $b_n$ ($a_n$ and $a_{n+1}$), and $g_a (g_b)$ is the coupling strength between the resonator $a_n (b_n)$ and the Xmon qubit $Q_{an}$ ($Q_{bn}$).

The system is operated in the dispersive regime, where the qubit frequencies are far off resonance from that of the resonators [56]. In this regime, the qubit-resonator couplings result in effective photon hopping between adjacent resonator modes with the hopping rates controllable via adjusting the qubit frequencies. Such coupling also generates frequency shift of the resonator modes, which can also be manipulated by adjusting the qubit frequencies. Here we assume all Xmon qubits are prepared in their ground state so that the resonators and the Xmon qubits are effectively decoupled except for the qubit-mediated photon hopping and qubit-induced resonator frequency shift. As we will show later, external microwave driving on the resonators will be employed for the detection of the topological effects. For this purpose, we choose to discuss this system in the rotating frame with respect to the external driven frequency $\omega_{dr}$ and in the interaction picture with respect to the qubit frequencies $\omega_{q}$. The total effective Hamiltonian in the dispersive regime then has the form

$$H = \sum_n (J_1 a_n^\dagger b_n + J_2 a_{n+1}^\dagger b_{n-1} + H.c.) + \left(\Delta_c - \chi - \frac{g_a^2}{\Delta_a}\right) a_n a_n + \left(\Delta_c - \chi - \frac{g_b^2}{\Delta_b}\right) b_n b_n,$$

(2)

where the qubit-assisted photon hopping rates between resonators are $J_{1,2} = -\frac{g_{a,b}^2}{\Delta_{1,2}}$, with the cavity (qubit) detuning $\Delta_c = \omega_c - \omega_{dr}$, the resonator frequency shift induced by the inline Xmon qubits is $\chi = -\frac{g_a^2}{\Delta_a} - \frac{g_b^2}{\Delta_b}$, and as one can see, the photon hopping rates and the resonator frequency shift could be directly tuned by changing the qubit detuning $\Delta_c (x = 1, 2, a, b)$ via external control circuits to adjust the qubit frequency $\omega_q$. The Xmon-qubit-based circuits in figure 1 have already been studied experimentally and the control of the qubit frequency is a standard technology [1, 2, 11, 57]. Such circuits have also been studied in other parameter regimes, where quantum phase transitions for cavity polaritons or spin particles can be observed [19, 58]. We also want to mention that tunable photon hopping between resonators can also be generated with other circuit setups, such as a tunable SQUID loop [39] or a tunable capacitor [60].

**Three-dimensional Weyl semimetal phase.** By varying the the photon hopping rates and the shifts of the resonator frequency in parametric spaces, we can construct a series of lattice Hamiltonians. Specifically, we focus on lattice Hamiltonians confined to the following parameter conditions:

$$J_1 = J[1 - \cos(\theta_1)],$$

$$J_2 = J[1 + \cos(\theta_1)],$$

$$\frac{g_a^2}{\Delta_a} = J[1 - \cos(\theta_2)],$$

$$\frac{g_b^2}{\Delta_b} = J[1 + \cos(\theta_2)],$$

(3)

where the parameters $\theta_1$ and $\theta_2$ can be tuned from 0 to $2\pi$. By combining the lattice momentum $k_z$ associated with the 1D circuit, $\theta_1$, and $\theta_2$, we thus construct an effective 3D artificial momentum space with $k = (k_z, \theta_1, \theta_2)$. The conditions described in equation (3) can be realized by externally tuning the Xmon qubit frequencies: $\omega_c = \omega_{dr} - \frac{g_a^2}{|J|}[1 - \cos(\theta_1)]$, $\omega_a = \omega_{dr} - \frac{g_a^2}{|J|}[1 + \cos(\theta_1)]$, $\omega_b = \omega_{dr} + \frac{g_b^2}{|J|}(1 - \cos(\theta_2))$, and $\omega_b = \omega_{dr} + \frac{g_b^2}{|J|}(1 + \cos(\theta_2))$. Note that the cross-shaped Xmon qubits can be connected to multiple ports simultaneously. In addition to the ports that connect to the resonators, external current lines can be coupled to the qubits to generate flux bias and tune the qubit frequencies [1, 2, 11, 57]. Parameter deviations will inevitably occur during the tuning process. However, the topological features demonstrated below are robust against such deviations given the topological protection in this system. With a qubit-resonator coupling strength on the order of 300 MHz and a detuning on the order of 2 GHz, the photon hopping rates are in the range of 50 MHz, far exceeding the scale of the resonator bandwidth and the qubit decoherence rate.

Now we turn to study the underlying topological features emerged in our model in the synthetic first Brillouin zone: $k_z \in (0, \pi], \theta_1 \in (0, 2\pi], \theta_2 \in (0, 2\pi)]$. By substituting the parameters in equation (3) into the Hamiltonian (2) and applying the Fourier transformation in momentum basis, we obtain the Hamiltonian in the momentum space with

$$H = \sum_k \epsilon_{q}(k)q_{k},$$

where $\epsilon_{q}(k) = (a_k, b_k)\gamma$ is a vector composed of the momentum-space operators $a_k$ and $b_k$. The momentum-space matrix is

$$h(k) = \Delta_0 + h_x\sigma_x + h_y\sigma_y + h_z\sigma_z,$$

(4)

where $\Delta_0 = \Delta_c - 2J - I$, $h_x = 2J\cos(k_z)$, $h_y = 2J\cos(\theta_1)\sin(k_z)$, $h_z = J_1\cos(\theta_2)$, and $\sigma_{x,y,z}$ are the Pauli matrices spanned by the two momentum components $a_k$ and $b_k$. With this equation, we find that there are four band-touching points in the synthetic 3D first Brillouin zone with $k_y = (\frac{\pi}{7}, \frac{\pi}{7}, \frac{\pi}{7})$, respectively. The energy spectrum corresponding to equation (4) is plotted in figure 2(a). This result indicates that the low-energy spectrum around the band-touching points is gapless. This gapless phase has a resemblance to a 3D Weyl semimetal phase with $\theta_1$ and $\theta_2$ playing the role of the momenta $k_z$ and $k_x$. In the following, we will show that the four band-touching points are Weyl points.
For this purpose, we first expand the Hamiltonian \( h (k) \) around the four band-touching points \( k_w \). The corresponding low-energy Hamiltonians can be derived as

\[
H_w (q) = -2Jq_x \sigma_x + 2Jq_y \sigma_y + Jq_z \sigma_z,
\]

where \( q = (q_x, q_y, q_z) = k - k_w \) and \( k \) is the synthetic momentum near the touching points. One can find that the above Hamiltonians are exactly the Weyl Hamiltonians and the band-touching points are the Weyl points. In general, the Weyl Hamiltonian can be expressed as \( H_w = \sum_{ij} v_{ij} q_i \sigma_j \), which describes a massless Dirac fermion with a fixed chirality defined as \( Ch = \text{sign}(\text{Det} [v_{ij}]) \). For our system, the chirality for the two Weyl points \( W_1 = (\pm \pi, \pi, 0) \) and \( W_2 = (\pi, \pm \pi, 0) \) is 1; and the chirality becomes \(-1\) for the two Weyl points \( W_3 = (\pi, \pi, \pi) \) and \( W_4 = (\pi, -\pi, \pi) \). The essential topological features of the Weyl semimetal phase are analysed below.

**Magnetic monopoles at the Weyl points.** States at the Weyl points can be viewed as topological defects in the momentum space and are extremely stable. This is a key proof that connects the Weyl semimetal to a topological phase. For a translationally invariant system, infinitesimal transformation of the Weyl Hamiltonian only shifts the Weyl points in energy or momentum, but does not remove them from the energy spectrum. In this sense, the Weyl points possess absolute stability, which hints that the Weyl points have topological protection and can be related to a topological index. In fact, it is found that the topological charges of the Weyl points are associated with quantised source of Berry curvature [47]. If we assume the ground state of the Weyl Hamiltonian as \( \psi (q) \), the Berry curvature for this Weyl node is expressed as \( \mathcal{F}_{ijkl} = i \langle [\nabla_i \psi (q)] \times [\nabla_j \psi (q)] \rangle \). Then the topological charge of the Weyl point can be characterised by the first Chern number,

\[
C = \frac{1}{2\pi} \oint S \mathcal{F} \cdot dS,
\]

where the integration is perform on a closed surface \( S \) in the momentum space surrounding the Weyl point. In our system, with the Weyl Hamiltonians (5), the Berry curvatures for the Weyl points \( W_{1,2} \) and \( W_{3,4} \) are calculated as \( \mathcal{F}_{1,4} = q/2|q|^3 \) and \( \mathcal{F}_{2,3} = -q/2|q|^3 \), respectively. In figure 2(b), we plot the Berry curvature around the four Weyl points in our model. As one can see, a Weyl point can be viewed as a magnetic monopole. The Berry curvature and the Berry flux number are associated with the magnetic field and the quantised magnetic charge of such monopole. From figure 2(b), one can easily find that the Chern numbers (monopole charges) for the Weyl points \( W_{1,4} \) and \( W_{2,3} \) are \( C = 1 \) and \( C = -1 \), which are exactly equal to the chiralities of the four Weyl points and give the physical meaning of the chiralities.

**Topologically protected surface states connecting the Weyl points.** Another distinguishing topological feature for the Weyl semimetal phase in electronic systems is the existence of topologically protected Fermi arc surface states [47]. Such surface states at the Fermi energy in a Weyl semimetal form a Fermi arc, contrary to the usual Fermi surface that forms a closed loop. In contrast, our system is a bosonic photon lattice, the analogue of the Fermi energy and Fermi arcs are the zero-energy and the photonic surface states connecting the Weyl points. For simplicity, we call the analogue of Fermi arcs in our photonics system as zero-energy-surface-states (ZESS) arcs. In figure 3, we calculate the edge states of our model when the circuit-QED lattice is under an open boundary condition. The result shows that there are two sheets of edge states around zero energy. They are localised in the left-most and the right-most resonators, respectively, corresponding to the surface states localised on the two surfaces in the underlying 3D artificial lattice. The ZESS arcs appear as the intersection (dashed lines in figure 3) of the two sheets of the edge states connecting the Weyl points. To be more precise, there are two ZESS arcs in our system, which appear as two lines that connect the projections of
the Weyl points \( W_1 (W_2) \) and \( W_4 (W_3) \) on the surface of the Brillouin zone. More specifically,

\[
\begin{align*}
\text{First ZESS arc: } & \quad (\theta_1, \theta_2) = \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \leftrightarrow \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \\
\text{Second ZESS arc: } & \quad (\theta_1, \theta_2) = \left( -\frac{\pi}{2}, -\frac{\pi}{2} \right) \leftrightarrow \left( \frac{\pi}{2}, \frac{\pi}{2} \right)
\end{align*}
\]

This is because the edge states are ill-defined in the projection of the Weyl points, where the gap is closed and there are bulk states. In figure 3, in order to verify this, we plot the density distributions of two selected edge states, one of which is close to the centre of the ZESS arc and the other is near the projection of the Weyl point. It turns out that the edge states near the projection of the Weyl point penetrate more into the bulk of the lattice; whereas the state near the centre of the ZESS arc is mainly located at the edge of the lattice. This result explains why the ZESS arcs end at the projections of the Weyl points. Although our system is bosonic by nature, one can find that the spectrum of the surface state is the same as that of a fermionic system and the zero energy in the spectrum is analogue to the Fermi energy.

**Lattice-based cavity input-output process.** The essential topological features of Weyl semimetal phase can be directly measured using a lattice-based cavity input-output process. Different from fermionic systems, our model employs a bosonic system where multiple photons can occupy one eigenstate at the same time. Inspired by this fact, one can excite the circuit-QED lattice to occupy a particular energy eigenstate by applying appropriate external microwave field driving at designated driving frequency to change \( \Delta_0 \) and matching to the eigenenergy of the whole lattice, and also by applying the driving on proper resonator so that it has the maximal wave function overlap with the eigenstate of the lattice. With this method, one can drive the lattice to occupy the photonic edge states or bulk states, depending on the choice of the driving frequency and the driven lattice sites. The signatures of topological features in Weyl semimetal phase is extracted by detecting the steady-state cavity output in the presence of finite resonator dissipation.

The above lattice-based method and the final steady state are concretely formalised as follows. Each resonator in the lattice is firstly assumed to be driven by an external microwave pulse. In the rotating frame with respect to the driving frequency \( \omega_d \), the driving Hamiltonian is

\[
H_d = \sum_n \left( \Omega_{na} a_n^+ + \Omega_{nb} b_n^+ \right) + H_c,
\]

where \( \Omega_{na}, \Omega_{nb} \) are the driving amplitudes on the two resonators in the \( nth \) unit cell. When the resonator dissipation is taken into account, the final steady state of the resonator lattice can be solved using a standard Lindblad master equation for this system. In particular, the expectation value of the resonator field \( a_j \) in the steady state can be expressed as

\[
\langle a_j \rangle = -i \langle [a_j, H + H_d] \rangle + \kappa \sum_n \langle L(a_n) a_j \rangle,
\]

where the Lindblad term \( L(a_n) a_j = a_n a_j a_n^+ - \frac{1}{2} \{ a_n^+ a_n, a_j \} \) and \( \kappa \) is the resonator decay rate. To obtain a simple formula for the expectation values of all the resonators in the lattice, we choose to work in the new bases.
\( \vec{a} = (\{a_i\}, \{b_i\}, \ldots, \{a_n\}, \{b_n\})^T \) and \( \vec{\Omega} = (\Omega_{ia}, \Omega_{ib}, \ldots, \Omega_{na}, \Omega_{nb})^T \). Based on the condition of the steady-state solution

\[
\langle a_i \rangle = 0,
\]

we can get the expectation value of all the resonator fields in the steady state and further write them in a compact form,

\[
\vec{a} = -\left( \Delta_0 + T - i \frac{\kappa}{2} \right)^{-1} \vec{\Omega},
\]

where the elements of matrix \( T \) are defined by \( T_{na, nb} = T_{nb, na} = J_1, T_{na, (n-1)b} = T_{(n-1)b, na} = J_2, T_{na(b), na(b)} = \pm J_1 \cos(\theta_2) \).

In the above process, the external drive can be referred to as the process of injecting photons to the resonators, and thereby the above scheme is naturally related to the cavity input-output process. In our work, as we will demonstrate below, the topological features of the photonic Weyl semimetal can be observed by driving the resonator lattice to occupy the edge states and measuring the resulted steady-state response. In particular, we choose to excite the edge states localised in the left edge of resonator lattice with the driving frequency tuned to match the corresponding eigenenergy and the driving pulse applied only to the left-most resonator, as the left edge mode has the maximal probability of populating the left-most resonator. In this case, the driving pulses have a form \( \vec{\Omega} = (\Omega_{ia}, 0, \ldots, 0, 0)^T \), where \( \Omega_{ia} \) is the external driving amplitude. According to the cavity input-output formula, the output photon field \( a_{i \text{out}} \) from the resonator at the left edge is related to the input photon through \[61\]

\[
a_{i \text{out}} = a_{i \text{in}} + \sqrt{\kappa} a_1,
\]

where the input field \( a_{i \text{in}} \) is related to the external driving by \[62\]

\[
\sqrt{\kappa} a_{i \text{in}} = i \Omega_{ia}.
\]

Combining this equation with equation (10), the reflection coefficient from the left edge can be derived as

\[
r_1 = \frac{\langle a_{i \text{out}} \rangle}{\langle a_{i \text{in}} \rangle} = 1 + i \kappa \left( \Delta_0 + T - i \frac{\kappa}{2} \right)^{-1} J_1.
\]

In the following, we will demonstrate that this reflection coefficient can be used to measure all the essential features of the photonic Weyl semimetal in our system.

### 3. Results

**Measuring monopole charges of the Weyl points.** As shown before, the monopole charge or the chirality of a Weyl point is quantified by the first Chern number of the ground state on a momentum sphere surrounding the Weyl point. Meanwhile, we know that 2D topological insulators with no symmetry are characterised by the first Chern numbers defined in a 2D momentum space. In this sense, we can represent the 3D momentum sphere surrounding the Weyl point with two periodic quasimomentums and further use them to construct an artificial 2D momentum space. In such a space, the Hamiltonian around the Weyl point becomes gapped and can have a topological insulator state. In this way, the topological feature of the Weyl point is mapped into that of a topological insulator. The resulting merit is that the measurement of monopole charge of a Weyl point is transferred to the measurement of the topological invariant of a photonic topological insulator state, which avoids the experimental challenges in directly measuring the monopole charge.

To map the 3D momentum sphere \( S \) around the Weyl point \((k_{xw}, \theta_{1w}, \theta_{2w})\) into an effective 2D momentum space, we first employ a circle to represent the 2D momentum surface around the Weyl point in the \((\theta_1, \theta_2)\) plane, with the Weyl point as the centre of this circle. In this case, the momenta around the Weyl point in the \((\theta_1, \theta_2)\) plane can be written in the following simple form:

\[
\theta_1 = \theta_{1w} + \theta \cos(\theta),
\]

\[
\theta_2 = \theta_{2w} + \theta \sin(\theta),
\]

where \( \theta \) is the radius of the circle and the parametric angle \( \theta \) is within the range \( 0 \) and \( 2\pi \). If the radius \( \theta \) is smaller than half of the separation between two Weyl points, one can find that the 3D momentum sphere around each Weyl point can be safely simplified to a 2D momentum space formed by \((k_x, \theta)\), where \( \theta \) plays the role of an artificial momentum dimension. Under this mapping, the Hamiltonian \( h(k) \) around each Weyl point becomes \( h(k_x, \theta) \), and the monopole charge of each Weyl point has a standard form of a Chern number.
where $\mathcal{F}(k_x, \theta)$ is the Berry curvature of ground band defined in the 2D momentum space $\{ k_x \in [0, \pi], \theta \in [0, 2\pi] \}$. In fact, such Chern number is nothing but the topological index characterising the topological phase implied in the mapped Hamiltonian $h(k_x, \theta)$. Numerically, we calculated the above Chern numbers associated with the four Weyl points in our system. The result shows that their values agree well with their monopole charges (chiralities). So the energy spectrum corresponding to each Hamiltonian $h(k_x, \theta)$ will become gapped and supports a topological insulator state, with its topological invariant exactly as the monopole charge of the Weyl point.

Some of the present authors have recently designed a strategy to measure the photonic topological invariant based on a cavity input-output process \cite{43}. The method shows that when the in-gap edge mode of the photonic topological insulator system is driven by external input photons, the photonic topological invariant is just the winding number of the reflection coefficient phase varying with $\theta$, which can be tracked by the cavity input-output process. It is straightforward to apply this method here to measure the photonic topological invariant of the ground state of the mapped Hamiltonian $h(k_x, \theta)$. To this end, we use the lattice-based cavity input-output method presented above to drive the in-gap left edge states. The reflection coefficients from the left-most resonator $r_L(\theta)$ formulated in equation (13) are numerically calculated and the results are plotted in figure 4, corresponding to the mapped Hamiltonians around the Weyl points $W_1$ and $W_4$. In the plots, we also present the winding directions of the phase of reflection coefficients when $\theta$ is tuned from 0 to $2\pi$. The results in figure 4(a)–(b) is with respect to the case when the lattice size is 12. The results show that the winding numbers corresponding to the Weyl points $W_1$ and $W_4$ are 1 and $-1$, which give the measurement outcomes of the monopole charge of these two Weyl points. Furthermore, we also calculate the case in figure 4(c)–(d) when the lattice size is reduced to 4. Remarkably, the winding numbers remain the same as the case with a larger lattice. In all cases, we also take into account the influence of the cavity dissipation. It is found that the winding number is clearly observable if the cavity dissipation rate is smaller than the energy gap of $J^2$. The reason is that the frequency of input photons (analogue of Fermi level) will remain in the energy gap. Then the corresponding topological invariant will remain the same. In this sense, the above measurement is very robust to the fluctuation of the frequency of input photons. The reflection coefficient phase can be measured using the standard homodyne detection technology. Therefore, the monopole charges of Weyl points could be clearly measured even in a circuit with only four resonators.

\[ C = \frac{1}{2\pi} \int dk_x d\theta \mathcal{F}(k_x, \theta), \]  

Figure 4. Winding of the reflection coefficient phase as a function of $\theta$ for the Weyl point (a) $W_1$ and (b) $W_4$ with a lattice size $N = 12$. The two cases are also shown in (c) and (d), respectively, for a lattice size $N = 4$. The detuning is $\Delta_0 = -0.1J$, and the resonator dissipation rate is $\kappa = 0.1J$ (solid line), $\kappa = 0.7J$ (dashed line), and $\kappa = 1.5J$ (dash-dotted line). The arrows $\uparrow$ and $\downarrow$ signify the winding directions as anticlockwise and clockwise (defined in right-hand coordinate system), which stand for the positive and negative signs of the corresponding winding numbers.
Measuring the photonic surface states connecting the Weyl points. In the last section, we have shown that ZESS arcs break off at the projections of the Weyl points on the surface Brillouin zone. In particular, we study in detail how to measure the first Fermi arc line that ends at the projection of the Weyl points $W_1$ and $W_4$, labelled as $\theta_1 = \pm 0.2\pi$, $\theta_1 = 0.5\pi$). Such ZESS arc can be measured by probing the position of zero-energy left or right edge modes. That is because ZESS arcs are the intersection lines of the left and right edge states at zero energy. In fact, we only need to probe the position of zero-energy edge modes along $q_1$ axis when the systematic parameter is tuned to $\theta_1 = 0.5\pi$. With the lattice-based cavity input-output process, the position of zero-energy edge modes in the $q_1$ direction can be measured based on externally driving the edge resonators and then scanning the driven frequency around zero energy to extract the reflection spectra. Each zero-energy peak in such reflection spectra would give the position of zero-energy edge mode. The reason is because only zero-energy edge mode is a resonant eigenmode of the photonic lattice in the above external driven. Such a mode has the maximal probability to populate the lattice, while the other non-resonant mode would quickly decay into the vacuum state.

In our case, we employ a four-coupled resonator lattice, and the first ZESS arc line is numerically calculated in the down panel of figure 5(a). It is found that the ZESS arc ends at the points with $\theta_1 = \pm 0.2\pi$ instead of $\pm 0.5\pi$. When $\theta_1$ is bigger than $\theta_1$, an energy gap appears and the intersection ZESS arc line disappears. The reason for this mismatch is due to finite lattice size effect. To measure this ZESS arc, we choose to drive the left-most resonator to excite the left edge state. In particular, we have calculated the reflection spectra around zero energy for five particular $q_1$ values in the upper panel of figure 5(a), including three inside and two outside the ZESS arc. The results show that the reflection spectra for $q_1$ values inside the ZESS arc has a peak at the zero energy, which means that this mode is the left ZESS arc edge state. In this way, the whole ZESS arc shape is mapped out and

![Reflection spectra](image_url)
then the ZESS arc end points are measured with $\theta_{1\zeta} = \pm 0.2\pi$. Similarly, we also calculate the Fermi arc line and the reflection spectra for a lattice size of 12 in figure 5(b). It turns out that the ZESS arc ends at the points with $\theta_{1\zeta} = \pm 0.4\pi$. The measured ZESS arc ending points derived from the reflection spectra also agree well with this point. In both case, we also plot the reflection spectra for $\theta_1$ values outside the ZESS arc. It turns out that two symmetric peaks appear around zero energy, which agrees with the fact that an energy gap will appear when $\theta_1$ is outside the ZESS arc. Moreover, one also can find that the heights of the peaks in the reflection spectra become very low when $\theta_1$ is closer to $\pm 0.5\pi$. As shown in section IV.B, this is because the edge states near the projection of the Weyl point penetrate more into the bulk and the probability occupying the left-most or right-most resonator will be very small.

We have summarised the finite lattice size effect on the measured ZESS arc ending points in table 1. The results show that the measured ZESS arc ending points $\theta_{1\zeta}$ would be closer to the projections of Weyl points $\theta_1 = \pm 0.5\pi$ when the lattice size is increased to 20. However, the cost of getting closer is very high and it requires a larger lattice size. This is because the ZESS arc edge state is ill-defined at the projection of the Weyl point. Therefore, it is not necessary to observe the behaviour of ZESS arc approaching the projection of Weyl point. In this sense, we conclude that the ZESS arcs can be measured in a circuit with only four dissipative resonators, which thus provides us a minimal platform to observe the topological protected ZESS arcs.

### 4. Discussion

Intuitively, we may think that the introduction of resonator dissipation could destroy the topological states. However, our results shown above demonstrate that dissipation is, in fact, beneficial for measuring the topological features in Weyl semimetals. Especially, we find that the monopole charges of the Weyl points and the photonic surface states connecting the Weyl points as open arcs can be unambiguously measured in a circuit with only four dissipative resonators. In contrast to conventional methods simulating topological insulator states in photonic systems [29–41], which typically require engineering synthetic gauge field and a large size lattice, our finding greatly relaxes the experimental requirement and thus provides a minimal system to study the photonic topological insulator and semimetal phase. In addition to the edge-state spectrum, our method with lattice-based input–output process can be extended for realising photonic lattice Hamiltonian tomography in near feature, which allows us to extract the bulk linear energy spectrum with respect to the Weyl cones.

### 5. Conclusion

In summary, we present a minimal circuit-QED lattice model to mimic the topological features of a Weyl semimetal. Moreover, we show that the essential topological features of Weyl semimetal phase are clearly measured in a small dissipative superconducting circuit network. Considering recent advances in the technology for controlling arrays of superconducting qubits and resonators [1–3], our method and scheme can be realised in practical systems with finite dissipation. Hence this work can stimulate further experimental and theoretical interests on mimicking and studying topological phases with small artificial quantum lattice systems.

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| Lattice size $N$ | Parameters $\theta_{1\zeta}$ |
|------------------|-----------------------------|
| 4                | $\pm 0.2\pi$                |
| 6                | $\pm 0.3\pi$                |
| 8                | $\pm 0.35\pi$               |
| 12               | $\pm 0.4\pi$                |
| 20               | $\pm 0.45\pi$               |
| 36               | $\pm 0.48\pi$               |

Table 1. Finite lattice size effect on the measured ZESS arc ending points.
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