Analysis of MHD free convective stream past a vertical permeable plate and a warmth source through permeable material underneath oscillatory pull

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Abstract: The mass exchange with the warmth source on the free convective progression of a gooey incompressible electrically directing liquid past vertically permeable plate through a permeable medium in nearness of a cross over attractive field with time dependant penetrability and oscillatory pull is considered in this paper. The arrangement is acquired by the Perturbation strategy for speed field temperature and fixation dissemination. The impacts of different boundaries on the speed field, temperature, and fixation dissemination are examined through diagrams. Likewise, the skin contact and Sherwood number are determined.

Key words: Convective flow, Porous Medium, Oscillatory suction, MHD flow.

1. Introduction
The study of free convective MHD flow together with heat source and mass transfer through porous medium under oscillatory suction has attracted the attention of many researchers because of their applications in various branches of Science and Technology. The best example for heat transfer by convection is heating the rooms and buildings using radiators. Heat Transfer and mass transfer takes a wide role in manufacturing industries for the design of fins, nuclear power plants, gas tubines, steel rolling and various propulsion devices for aircraft, materials processing, combustion and furnace design, temperature measurements and energy utilization.

Ashraf A.Moneium and W.S.Hassanin [1] investigated, "The MHD flow result through a permeable medium past vertical plate under oscillatory suction". Mohammad Al Zubi [2] explained, "MHD of an oscillatory flow with heat and mass transfer in a porous medium with chemical reaction over a vertical permeable plate". V.G.Gupta et.al [3] have reported, "Convective effects on MHD flow of partially filled porous medium with heat transfer between vertical plates moving in opposite direction". Pradip Kumar Gaur and Abhay Kumar Jha [4] discussed about, "The hall effect of heat and mass transfer in viscous-elastic fluid through rotating porous medium". P. Mangathaia et.al [5]
analysed, "MHD free convective flow in the presence of radiation and heat generation past a vertical porous plate". Vidhya.M and Sundarammal Kesavan [6] studied, "Laminar convection flow between two vertical parallel plates through porous medium". P. Rama Krishna Reddy and M.C Raju [7] examined, "MHD free convective flow past a porous plate". V. Prabhakara Reddy et.al [8] considered, "Heat and mass transfer flow of chemically reactive and radiation absorption fluid in magnetic field". Govindarajan.A et.al [9] presented, "Chemical reaction effects on unsteady free convective MHD flow through porous medium in a rotating plate with heat and mass transfer". Ighoroje W.A. Okuyade and Tega Okor [10] described, "MHD convective chemically reacting flow with heat source, thermal radiation and oscillating wall temperature, concentration and suction effects". Yasir Ali and Arshad Alam Khan [11] investigated, "Exact solution of MHD flow over porous plate which is oscillating and translating". Vidhya.M. et.al [12] explained, "The discrete delay of polynomials determinant with blood cells transformation". Khem Chand and Nidhi Thakur [13] have reported, "Span-wise fluctuating hydro magnetic free convective flow past a hot vertical porous plate with thermal radiation and viscous dissipation". Gorakhnath Waghmode and S V Suneetha [14] examined, "Warmth and mass exchange on MHD turning stream of a thick-flexible liquid in a permeable medium". S. Sheeba Juliet et.al [15] have analysed, "Chemical reaction of MHD flow through vertical porous plates with heat and mass transfer and variable temperature". Farhad Ali et.al [16] studied, "Conjugate effects of heat and mass transfer on MHD free convective flow over an inclined plate in porous medium". Satya Sagar Saxena et.al [17] examined, "MHD free convective oscillatory fluid flow past a porous plate in porous medium in the presence of heat source with chemical reaction". Kumar V R et.al [18] considered, "Thermal diffusive free convective radiating flow over an impulsively started vertical porous plate". S.S. Das et.al [19] presented, "Mass transfer effects on unsteady hydro magnetic convective flow past a vertical porous plate with heat sources".

Considering the investigations revealed above, to the extent the concern of the creators information, the MHD streams past a vertical permeable plate through a permeable medium under the oscillatory attractions has gotten consideration. The fundamental goal of the current examination is to break down MHD free convective progression of a viscous incompressible liquid past a vertical permeable plate with warmth and mass exchange through permeable medium. Here the oscillatory suction is considering, the stream is free convective and uniform cross over magnetic field influenced perpendicular to the plate.

2. Description of the Problem Formulation
Consider the unsteady free convective fluid flow of a viscous incompressible liquid which conducts power past a limitles vertical permeable plate through a permeable direct within the sight of a cross over attractive field with time-subordinate porosity and oscillatory attractions. Let (x, y) be the Cartesian arrange framework where the y-axis is along the infinite vertical permeable plate and the x-axis held opposite to the plate. Let u be the x-direction velocity.

Essential assumptions are made as follows:
- Every liquid extent are consistent.
- Expect that the plate and the liquid are in same temperature. Additionally the convergence of the species is raised or brought down.
- The liquid considered is electrically directing with exceptionally small Reynolds number.
- Consequently the attractive field created is irrelevant in contrast with the applied one.
- Porousness of the permeable medium is thought to be \( R^+(t^*)=K_p\left(1+\epsilon e^{\omega t^*}\right) \)
- The suction speed is thought to be \( v(t^*)=-v_0\left(1+\epsilon e^{\omega t^*}\right) \).
- Take pressure as constant.
- The plate is endless length, so all the factors are elements of y and t as it were.

Introduce following non-dimensional quantities to make the equations dimensionless:
\[
\frac{u}{v_0}, y = \frac{y}{v_0}, t = \frac{v_0^2 t^*}{4v}, S_c = \frac{v}{D}, \omega = \frac{4v_0\omega}{v_0^2}, P_r = \frac{\rho_cp_v}{K},
\]
\[ K_p = \frac{v^2 K^*}{v_0^2}, M^2 = \frac{\sigma B_0^2 v}{\rho v_0^2}, C = \frac{C_v - C_\infty}{C_v - C_\infty}, G_c = \frac{(C_w - C_\infty) g B v}{\nu^2} \]  

(1)

The governing equations in non-dimensional forms for Velocity, Temperature and Concentration are

\[ u_{yy} + f(t)u_y + C_1 u_y + T_1 G_y = \frac{u_{tt}}{4} + \left(M^2 + \frac{1}{f(t)}\right) u \]  

(2)

\[ T_{yy} + f(t) S_T T_y + Q_1 C_y = P_r \frac{T_{tt}}{4} + \frac{\beta c \rho}{k} \]  

(3)

\[ C_{yy} + f(t) S_C C_y = \frac{S e}{4} + \gamma C \]  

(4)

where \( f(t) = 1 + \epsilon e^{i \omega t} \) exposed to the accompanying limit conditions:

\[ u(0,t) = 0; \lim_{y \to \infty} T(y,t) = 0; \lim_{y \to \infty} C(y,t) = 0. \]  

(5)

3. Solution of the problem:

To research the solutions, we accept the arrangements in the accompanying structure

\[ u(y,t) = u_0(y) + \nu u_1(y) e^{i \omega t} \]  

(6)

\[ T(y,t) = T_0(y) + \nu T_1(y) e^{i \omega t} \]  

(7)

\[ C(y,t) = C_0(y) + \nu C_1(y) e^{i \omega t} \]  

(8)

where the amplitude \( \nu \ll 1 \) that means here the permeability variation is very little.

Apply (6) to (8) in equations (2) to (4) then equate the harmonic and non-harmonic terms.

Now we get the non-dimensional ODE as follows:

\[ u_0''(y) + (1 + A_1) u_0'(y) - A_2 u_0(y) = -C_0 G_c - T_0 G_T \]  

(9)

\[ u_1''(y) + (1 + A_1) u_1'(y) - \frac{i \omega}{4} A_2 u_1(y) = -C_1 G_0 A_1^{-1} - T_1 G_T A_1^{-1} \]  

(10)

\[ T_0''(y) + (1 + A_1) S_T T_0'(y) = \frac{\beta c \rho}{k} T_0(y) \]  

(11)

\[ T_1''(y) + (1 + A_1) S_T T_1'(y) - A_3 T_1(y) + \frac{Q_1}{A_1} C_1 = 0 \]  

(12)

\[ C_0'(y) + (1 + A_1) S_C C_0'(y) - \gamma C_0(y) = 0 \]  

(13)

\[ C_1'(y) + (1 + A_1) S_C C_1'(y) - \frac{S e}{4} \gamma C_1(y) = 0 \]  

(14)

where \( A_1 = M^2 + \frac{1}{k} \) and \( A_2 = A_1 + \frac{1}{4} i \omega \).

Now the conditions are reduced as follows:

\[ u_0(0) = u_1(0) = T_0(0) = T_1(0) = C_0(0) = C_1(0) = 1 \quad \text{and} \quad u_0 = u_1 = T_0 = T_1 = C_0 = C_1 \to 0 \quad \text{as} \quad y \to \infty \]  

(15)

By solving equations (9) to (14) under boundary conditions (15), we obtain

\[ u(y,t) = e^{m_3 y} + m_15 e^{m_13 y} - m_3 \nu + m_16 e^{m_13 y} + m_2 e^{m_2 y} + m_2 e^{m_2 y} + m_4 e^{m_4 y} + m_5 e^{m_5 y} + m_6 e^{m_6 y} + m_7 e^{m_7 y} + m_8 e^{m_8 y} \]  

(16)

\[ T(y,t) = e^{m_3 y} + A_1 \left[ e^{m_3 y} + \frac{Q_1}{A_1 m_{11}} (e^{m_{1 y}} - e^{m_{3 y}}) + \frac{Q_1 m_3}{m_{12}} (e^{m_{3 y}} - e^{m_{3 y}}) \right] \]  

(17)

\[ C(y,t) = e^{m_3 y} + A_1 e^{m_3 y} \]  

(18)

The Skin Friction can be calculated as follows:

\[ \left( \frac{\partial u}{\partial y} \right)_{y=0} = \tau = m_{13} + m_{15} (m_{13} - m_{11}) + m_{16} (m_{13} - m_5) + A_1 (1 - m_{22} - m_{23} - m_{24} - m_{25} - m_{26} - m_{27} + m_{3} m_{22} + m_{3} m_{23} + m_{7} m_{24} + m_{3} m_{25} + m_{7} m_{26} + m_{3} m_{27}) \]  

(19)

The Sherwood Number can be calculated as follows:

\[ - \left( \frac{\partial C}{\partial y} \right)_{y=0} = S_n = -(m_1 + A_1 m_3) \]  

(20)

The constants \( Q_1, A_1, B_1, B_9 \) and \( m_1 - m_{27} \) are defined in appendix section.
4. Result and Discussion
We have considered the free convective and mass exchange stream of an electrically directing viscous incompressible liquid through a permeable medium past an unbounded vertical permeable plate with time subordinate permeability and oscillatory draw inside seeing an appealing field which is uniform traverse. Arrangement got utilizing typical bother strategy for minimal steady permeability $\varepsilon$. The courses of action deduced deliberately for the speed field, temperature differentiation and obsession flow. The conduct of the distinctive stream boundaries like Schmidt number $Sc$, Porosity boundary $k_p$, Grashof number of mass transfer $G_c$, Grashof number of heat transfer $G_T$, Magnetic parameter $M$, Prandtl number $Pr$, Frequency of oscillation $\omega$ are discussed and depicted through graphs of Fig 1-6.

Fig 1. Effect of Schmidt number ($Sc$) on velocity profile with $G_c = 10, G_T = 2, \varepsilon = 0.002, \omega = 5$ and $\omega t = \frac{\pi}{2}$.

Fig 2. Effect of $Pr$ on velocity profile with $G_c = 10, G_T = 2, Sc = 0.20, \varepsilon = 0.002, M = 0.5, \omega = 5, and \omega t = \frac{\pi}{2}$.
Fig 3. The non-dimension frequency of oscillation($\omega$) effect on temperature profile with $G_c = 10, G_T = 2, P_r = 10, S_c = 0.22, \varepsilon = 0.002, M = 0$ and $\omega t = \frac{\pi}{2}$.

Fig 4. The Schmidt number ($S_c$) effect on temperature profile with $G_c = 10, G_T = 2, \varepsilon = 0.002, \omega = 0.5, M = 0.5$ and $\omega t = \frac{\pi}{2}$. 
Fig 5. The chemical reaction parameter ($\gamma$) effect on concentration profile with $G_c = 10, \varepsilon = 0.002, \omega = 5, M = 0.5$ and $\omega t = \frac{\pi}{2}$.

Fig 6. The Schmidt number ($S_c$) effect on concentration profile with $G_c = 10, \varepsilon = 0.002, \omega = 5, M = 0.5$ and $\omega t = \frac{\pi}{2}$.

The effect of Schmidt Number $S_c$ on the speed profile of the stream field keeping various limits consistent is showed up in Fig 1. The Schmidt number is taking in increasing order and it is found that the velocity of the stream field likewise increasing. Fig 2 delineate the impact of Prandtl number $P_r$ on velocity profile of the stream field. It shows that the speed of the fluid extended while heat moving boundary of the liquid $P_r$ is expanding. The impact of non-dimension frequency of oscillation $\omega$ on temperature shows in Fig 3. It clarifies the temperature of the liquid expanded when the frequency of oscillation of the fluid increases.

As observed in Fig 4, the dimensionless Schmidt number $S_c$ of ratio of momentum diffusity and mass diffusity increases, the fluid temperature is seen to be increasing. Fig 5 shows the effect of chemical reaction parameter on fluid concentration. The concentration of the fluid increases if the
chemical reaction parameter $\gamma$ increases. The effect of Schmidt Number Sc on the concentration profile is showed in Fig 6. It is observed that the concentration of the fluid decreases when the Schmidt number Sc, ratio of momentum diffusity and mass diffusity increases.

5. Conclusion
The following conclusions are found:
- The velocity of the flow increases with expanding Schmidt Number Sc and Prandtl Number $\Pr$.
- The temperature of the liquid increases with increasing the frequency of oscillation of the fluid $\omega$ and Schmidt Number Sc.
- The fluid concentration increases when the parameter $\gamma$ of chemical reaction increases. Also it has reversal behaviour when the Schmidt Number Sc increases.

6. References
[1] Ashraf A Moniem, W S Hassanian (2013) Applied Mathematics (4) 694-702
[2] Mohammad Al Zubi (2018) Modern Mechanical Engineering 8 179-191
[3] V G Gupta Ajay Jain, Abhay Kumar Jha (2016) Journal of Applied Mathematics and Physics (4) 341-358
[4] Pradip Kumar Gaur, Abhay Kumar Jha (2016) Open Journal of Fluid Dynamics (6) 11-19
[5] P Mangathaia, G V Ramana Reddy, B Rami Reddy (2016) int j chem sci 14(3) 1577-1597
[6] Vidhya M, Sundarammal Kesavan (2010) Proceedings of the International Conference on Frontiers in Automobile and Mechanical Engineering (Scopus Indexed), IEEE Explorer 248-251
[7] P Rama Krishna Reddy, M C Raju(2018) International journal of Pure and applied Mathematics 118(5) 507-529
[8] V Prabakara Reddy, R V M S S Kiran Kumar, G Viswanatha Reddy, P Durga Prasad, S V K Varma (2015) International Conference on Computational Heat and Mass Transfer, Elsevier Procedia Engineering (127) 575-582
[9] Govindarajan A, Ali Chamilka, Sundarammal Kesavan, Vidhya M (2014) Thermal Science supplement 2(Web of science) (18) 515-524
[10] Igboroje W A Okuyade, Tega Okor (2019) American Journal of Fluid Dynamics 9(2) 35-43
[11] Yasir Ali, Arshad Alam Khan (2018) Discrete and continuous Dynamical systems series 11(4)
[12] Vidhya, S Balamuralitharan, A Govindarajan (2015) International Journal of Pure and Applied Mathematics (Scopus Indexed) 98(3) 399-411
[13] Khem Chand, Nidhi Thakur(2016) Journal of Rajasthan Academy of Physical Sciences 15(4) 267-281
[14] Gorakhnath Waghmode, S V Suneetha (2019) International Journal of Applied Engineering Research(14)9 2107-2120
[15] S Sheeba Juliet, M Vidhya, E P Siva and A Govindarajan (2019) AIP Conference proceedings 2112, 020159
[16] Farhad Ali, Ilyas Khan, Samiulhaq, Sharidan Shafee (2013) Plos one(8)6 e65223
[17] Satya Sagar Saxena, Dhiresh Kumar Pathak, Sandeep Kumar(2014) Pelagia Research Library Available online 5(4) 30-45
[18] Kumar V R, Raju M C, Raju G S S,Varma S V K, Kumar (2016) J Phys Math Research Article Open Access 7(1) 1000156
[19] S S Das, S R Biswal, U K Tripathy, P Das(2011) JAFM 4(4) 91-100
7. Nomenclatures

C* - Species concentration
ε - small positive constant \( \ll 1 \)
C - Non-dimension species concentration
\( \rho \) - Density of fluid
\( G_c \) - Grashof number for concentration
\( \omega \) - Non-dimension frequency of oscillation
\( G_T \) - Grashof number of heat transfer
\( \omega^* \) - Oscillation frequency
D - Molecular diffusivity
v - Kinematic coefficient of viscosity
\( g \) - gravity
\( \sigma \) - Electrical conductivity
\( K' \) - Permeability of material
K - Quantity of thermal conductivity
\( K_p \) - Porosity quantity
\( \gamma \) - Chemical reaction parameter
M - Magnetic parameter
u - x-axis non-dimension velocity component

8. Appendix

\[
Q_1 = \frac{\sqrt{(c_w-c_\infty)}}{\rho C_p v_0^4 (T_w-T_\infty)}
\]
\[
A_1 = \varepsilon e^{i \omega t}
\]
\[
A_2 = M^2 + \frac{1}{1 + A_1}
\]
\[
A_3 = \frac{\rho c_p}{4} i \omega + \frac{\rho C_p}{k}
\]
\[
m_1 = \frac{-\sqrt{(1+A_1)S_c^+} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_2 = \frac{-\sqrt{(1+A_1)S_c^-} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_3 = \frac{-\sqrt{(1+A_1)S_c^+} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_4 = \frac{-\sqrt{(1+A_1)S_c^-} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_5 = \frac{-\sqrt{(1+A_1)S_c^+} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_6 = \frac{-\sqrt{(1+A_1)S_c^-} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_7 = \frac{-\sqrt{(1+A_1)S_c^+} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_8 = \frac{-\sqrt{(1+A_1)S_c^-} \sqrt{S_c^2(1+A_1)^2 + 4\gamma}}{2}
\]
\[
m_9 = (1 + A_1)S_c
\]
\[
m_{10} = \frac{\rho c_p}{4} i \omega + \frac{\rho C_p}{k}
\]
\[
m_{11} = m_1 + m_9 - m_{10}
\]
\[
m_{12} = m_3^2 + m_3 m_9 - m_{10}
\]
\[
C_p - Specific heat at constant pressure
\]
\[
S_c - Schmidt number
\]
\[
T_1 - Thermal expansion coefficient
\]
\[
t^* - Time
\]
\[
Q_1 - Absorption of radiation parameter
\]
\[
t - Non-dimension time
\]
\[
\tau - Skin friction
\]
\[
u^* - Velocity component along x-axis
\]
\[
S_h - Sherwood number
\]
\[
\nu_t^* - Velocity of Suction
\]
\[
v_0 - Velocity of suction which is positive constant
\]
\[
y^* - y-axis distance
\]
\[
y - y-axis non-dimensional distance
\]
\[
\beta - Species concentration with coefficient of volume expansion.
\]
\[
\infty - Subscript condition far away from the plate
\]

\[
A_7 = 1 + \frac{Q_1}{A_1 m_{11}} + \frac{Q_1 m_3}{m_{12}}
\]
\[
m_{13} = \frac{-1(1+A_1)^2}{A_1 + \frac{\sqrt{(1+A_1)^2 + 4A_2}}{2}}
\]
\[
m_{14} = \frac{-1(1+A_1)^2}{A_1 - \frac{\sqrt{(1+A_1)^2 + 4A_2}}{2}}
\]
\[
m_{15} = \frac{G_c}{m_1^2 + (1+A_1)m_1 - A_2}
\]
\[
m_{16} = \frac{m_3^2 + (1+A_1)m_3 - A_2}{m_3}
\]
\[
A_8 = 1 + m_{15} + m_{16}
\]
\[
m_{17} = \frac{-1(1+A_1)^2}{A_1 - \frac{\sqrt{(1+A_1)^2 + 4\omega A_2}}{2}}
\]
\[
m_{18} = \frac{-1(1+A_1)^2}{A_1 + \frac{\sqrt{(1+A_1)^2 + 4\omega A_2}}{2}}
\]
\[
m_{19} = m_3^2 + (1 + A_1)m_3 - \frac{\omega_4}{4} A_2
\]
\[
m_{20} = m_3^2 + (1 + A_1)m_3 - \frac{\omega_4}{4} A_2
\]
\[
m_{21} = m_1^2 + (1 + A_1)m_1 - \frac{\omega_4}{4} A_2
\]
\[
m_{22} = \frac{-G_c}{A_1 m_{19}}
\]
\[
m_{23} = \frac{-G_c}{G_T a_{18} Q_1 b_{19}}
\]
\[
m_{24} = \frac{-G_c}{A_1 m_{10} m_{20}}
\]
\[
m_{25} = \frac{-A_1}{G_T b_{13} m_3}
\]
\[
m_{26} = \frac{-A_1 m_{12} m_{20}}{G_T Q_1 m_3}
\]
\[
m_{27} = \frac{-A_1 m_{12} m_{19}}{G_T Q_1 m_3}
\]
\[
A_9 = 1 - m_{22} - m_{23} - m_{24} - m_{25} - m_{26} - m_{27}
\]
