Long-range dynamics of magnetic impurities coupled to a two-dimensional Heisenberg antiferromagnet

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We consider a two-dimensional Heisenberg antiferromagnet on a square lattice with weakly coupled impurities, i.e. additional spins interacting with the host magnet by a small dimensionless coupling constant \( g \ll 1 \). Using linear spin-wave theory, we find that the magnetization disturbance at distance \( r \) from a single impurity behaves as \( \delta S^z \sim g/r \) for \( 1 \ll r \ll 1/g \) and as \( \delta S^z \sim 1/(gr^3) \) for \( r \gg 1/g \). Surprisingly the magnetization disturbance is inversely proportional to the coupling constant! The interaction between two impurities separated by a distance \( r \) is \( \delta e \propto g^2/r \) for \( 1 \ll r \ll 1/g \) and \( \delta e \propto 1/r^3 \) for \( r \gg 1/g \). For large distances, the interaction is therefore universal and independent of the coupling constant. We have also found that the frequency of Rabi oscillations between two impurities is logarithmically enhanced compared to the decay width \( \omega_{\text{Rabi}} \propto g^2 \ln(1/gr) \) at \( 1 \ll r \ll 1/g \). This leads to a logarithmic enhancement for NMR and EPR line broadening. All these astonishing results are due to the gapless spectrum of magnetic excitations in the quantum antiferromagnet.

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I. INTRODUCTION

The interplay of magnetic impurities with strongly correlated electron systems has attracted considerable attention over the past decade. The discovery of high-temperature superconductors stimulated studies of impurities in two-dimensional Mott-insulators with long-range antiferromagnetic order\(^2\), where the copper-oxide parent compounds are driven to superconductivity by doping with holes or electrons. At low doping, the dopants are localized and it is therefore insightful to study the limit of isolated static holes, which have been realized experimentally\(^3\) and extensively studied theoretically\(^4,5,6,7\). There is a considerable body of work devoted to impurity bonds and added spins\(^8,9,10,11,12,13,14\), a generalization of the static hole case, and an unexpected behavior of the impurity magnetic susceptibility at low temperature has been revealed quite recently\(^15,16,17,18,19,20,21,22\). There is also a separate and very interesting Kondo-like effect of an impurity in a magnetic system close to an O(3) quantum critical point\(^18,19,20,21,22\). However, in this work, we consider impurities (added spins) in a system with long-range antiferromagnetic order. The very unusual behavior we find, closely related to that observed for the magnetic susceptibility and other quantities in the papers\(^8,18,19,20,21,22,23\), is due to gapless Goldstone spin-wave excitations.

The rest of the paper is organized as follows. In section \( \ref{sec:II} \) we formulate the model, derive the spin-wave vertices and introduce an effective Hamiltonian describing the interaction between impurities, which is explicitly calculated in sections \( \ref{sec:III} \) and \( \ref{sec:IV} \) as a function of the distance between impurities. A similar problem has been considered earlier in Ref. \(^8\) but only the case of small separations has been addressed, here we focus on the long-range behavior. The magnetization disturbance in the host antiferromagnet induced by a single impurity is then calculated in section \( \ref{sec:V} \) and finally section \( \ref{sec:VI} \) presents our conclusions.

II. MODEL

We consider one and two spin \( \sigma = \frac{1}{2} \) impurities coupled to an isotropic Heisenberg antiferromagnet \( (J > 0) \) on a square lattice with lattice spacing \( a = 1 \). The impurities are connected to the origin and site \( \mathbf{r} \) of the antiferromagnet. The Hamiltonian of this system reads

\[
H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \left( \mathbf{\sigma}_1 \cdot \mathbf{S}_0 + \mathbf{\sigma}_2 \cdot \mathbf{S}_r \right). \tag{1}
\]

Here \( \langle i,j \rangle \) denote nearest-neighbors, \( S_i \) is a spin-\( S \) operator at site \( r_i \) and \( \sigma_j \) are spin-\( \frac{1}{2} \) operators describing the impurities, which are either both ferromagnetic \( (J' < 0) \) or antiferromagnetic \( (J' > 0) \). After integration over quantum fluctuations of the system, one obtains the effective Hamiltonian for the interaction between impurities

\[
H_{\text{eff}} = J' \left[ \langle S_0^z \rangle \sigma_1^z + \langle S_r^z \rangle \sigma_2^z \right] + \epsilon (\mathbf{r}) \sigma_1^z \sigma_2^z + \left( M (\mathbf{r}) \sigma_1^z \sigma_2^z + H.c. \right). \tag{2}
\]

The calculation of this effective Hamiltonian is the goal of the present work. The diagonal interaction term \( \epsilon (\mathbf{r}) \), as well as the off-diagonal term \( M (\mathbf{r}) \) are different, depending on whether the impurities are coupled to the same or different sublattices, see Fig. \( \ref{fig:II} \) (a) and (b) respectively. Without loss of generality, we place the origin on the up sublattice (sublattice “a”) and indicate the sublattice the second impurity is coupled to by the corresponding letter, see Figs. \( \ref{fig:II} \). In this way, “a” (“b”) refers to impurities coupled to the same (different) sublattice. We study the weak coupling regime \( |J'| \ll J \), so that the dimensionless
where excitations are described by operators creating spin-waves with $S^z = -1$ and $S^z = +1$ respectively, see Ref. 1 for a review. In this approximation, the Hamiltonian can be decomposed as $H = H_0 + V$, where

$$H_0 = E_0 + 4JS\sum_q \sigma_i^a \sigma_i^b + \beta^q_i \beta^q_j + J' S (\sigma_i^z \pm \sigma_j^z),$$

$$V = H_1^{(a)} + H_2^{(a, b)}, \text{ with}$$

$$H_1^{(a)} =$$

$$= - J' \sigma_i^z \frac{2}{N} \sum_{p, q} e^{i(p-q) \cdot r_i} (u_p \sigma_i^a + v_q \sigma_i^b) (u_q \sigma_i^a + v_q \sigma_i^b)$$

$$+ J' \sqrt{\frac{S}{N}} \left( \sigma_i^+ \sum_q e^{i q \cdot r_i} (u_q \sigma_i^a + v_q \sigma_i^b) + h.c. \right), \quad (5)$$

$$H_1^{(b)} =$$

$$= J' \sigma_i^z \frac{2}{N} \sum_{p, q} e^{i(p-q) \cdot r_i} (u_p \sigma_i^b + v_q \sigma_i^a) (u_q \sigma_i^b + v_q \sigma_i^a)$$

$$+ J' \sqrt{\frac{S}{N}} \left( \sigma_i^+ \sum_q e^{i q \cdot r_i} (u_q \sigma_i^b + v_q \sigma_i^a) + h.c. \right). \quad (6)$$

Here $E_0$ is the ground state energy of the antiferromagnetic host and $N$ the number of sites. The upper (lower) sign in the $\pm$ expression refers to the situation where the second impurity is coupled to a spin on sublattice “a” (sublattice “b”). This convention is used throughout the whole paper. The Bogoliubov parameters $u_q$ and $v_q$ are given by

$$u_q = \sqrt{\frac{1}{2\omega_q} + \frac{1}{2}}, \quad v_q = -\text{sgn}(q) \sqrt{\frac{1}{2\omega_q} - \frac{1}{2}},$$

with $\omega_q = \sqrt{1 - \gamma_q^2}$ and $\gamma_q = \frac{1}{2}(\cos q_x + \cos q_y)$, see Ref. 1. In this notation, the spin-wave dispersion is $\epsilon_q = 4JS\omega_q$. The interaction Hamiltonians and generate one and two spin-wave vertices summarized in Table I.

| Symbol | Operator | Factor for $r$ on “a” | or $r$ on “b” |
|--------|----------|------------------------|-----------------|
| ▲      | $\sigma^+ a_i^g a_i^h$ | $-\frac{2}{N} u_p b_q e^{i(p-q) r}$ | $\frac{2}{N} u_p b_q e^{-i(p-q) r}$ |
| △      | $\sigma^+ \beta^q_i \beta^q_j$ | $-\frac{2}{N} u_p b_q e^{i(p-q) r}$ | $\frac{2}{N} u_p b_q e^{-i(p-q) r}$ |
| ◆      | $\sigma^+ \beta^q_i \beta^q_j$ | $-\frac{2}{N} u_p b_q e^{i(p-q) r}$ | $\frac{2}{N} u_p b_q e^{-i(p-q) r}$ |
| ⧫      | $\sigma^+ \beta^q_i \beta^q_j$ | $-\frac{2}{N} u_p b_q e^{i(p-q) r}$ | $\frac{2}{N} u_p b_q e^{-i(p-q) r}$ |

Only the ground state $|0\rangle$ is the true stationary quantum state. The states $|1\rangle$, $|\uparrow\rangle$, and $|2\rangle$ decay to the ground state with emission of spin-waves. Using the Fermi golden rule and the decay matrix elements presented in Tab. II one finds the following widths of excited states with respect to the emission of magnons

$$\Gamma_1 = 2\gamma_1 J,$$

$$\Gamma_2 = 2\Gamma_1.$$ \quad (8)

The three-level system is well defined, since $E_1 - E_0 \gg \Gamma_1$. But the diagonal and off-diagonal interaction energies $\epsilon(r)$ and $M(r)$ in the effective Hamiltonian have limited meaning because of the finite lifetime. Only in the ground state $|0\rangle$, the diagonal interaction energy is well defined. In the spin flip states $|1\rangle$, $|\uparrow\rangle$, and $|2\rangle$ the diagonal interaction energy does not make much sense, because we will see that it is always much smaller than the corresponding decay width. However, there is a regime where the off-diagonal interaction $M(r)$ is larger than the decay width and hence leads to Rabi oscillations between states $|1\rangle$ and $|\uparrow\rangle$. 

FIG. 1: Schematic picture of the antiferromagnetic host with two antiferromagnetic impurities coupled to the origin and site $r$ on the same sublattice (a) and on different sublattices (b). 

TABLE I: Spin-wave vertices for an impurity coupled to site $r$ on sublattice “a” or “b”.

$\epsilon_q = 4JS\omega_q$. The interaction Hamiltonians and generate one and two spin-wave vertices summarized in Table I.

Let us return to the effective Hamiltonian $H_2$. The first two terms are obvious and do not require calculations. They simply generate three energy levels, $E_0 = -J'(S^z)$, $E_1 = E_T = 0$, and $E_2 = J'(S^z)$. To be specific, let us consider the case shown in Fig. I(a), with impurities coupled to the same sublattice, then

$$|0\rangle = |↓, ↓\rangle,$$

$$|1\rangle = |↓, ↑\rangle,$$

$$|\uparrow\rangle = |↑, ↓\rangle,$$

$$|2\rangle = |↑, ↑\rangle.$$ \quad (7)
III. DIAGONAL INTERACTION $\epsilon(r)$ BETWEEN IMPURITIES

In the leading order of the $\frac{1}{N}$-expansion, the interaction energy $\epsilon(r)$ arises in second, third and fourth orders of perturbation theory, describing the exchange of two spin-waves between impurities. The corresponding contributions in usual Rayleigh-Schrödinger perturbation theory are

\[
\delta \epsilon_2 = \sum_{n \neq 0} \frac{(0\langle Vn | n\langle V0)}{(\epsilon_0 - \epsilon_n)} ,
\]

\[
\delta \epsilon_3 = \sum_{n, m \neq 0} \frac{(0\langle Vn | n\langle V| m\langle m|V0)}{(\epsilon_0 - \epsilon_n)(\epsilon_0 - \epsilon_m)} ,
\]

\[
\delta \epsilon_4 = \sum_{n, m, k \neq 0} \frac{(0\langle Vn | n\langle V| m\langle m|V| k\langle k|V0)}{(\epsilon_0 - \epsilon_n)(\epsilon_0 - \epsilon_m)(\epsilon_0 - \epsilon_k)} ,
\]

where $V$ is the perturbation [1]. In general, the expressions for third and fourth order energy corrections are more complex than those in [9]. The complication is due to contributions similar to [9], but in which intermediate states $m, n, k$ coincide with the initial state $|0\rangle$. Fortunately, the perturbation [10] does not allow such intermediate states and therefore Eqs. [9] are valid. The matrix elements $\langle n|V|m\rangle$ are given in Table [11] and it is convenient to represent the corrections [10] by diagrams where each vertex corresponds to some particular matrix element $\langle n|V|m\rangle$. The diagrams describing the $\frac{1}{N}$-corrections are shown in Figs. 2 and 3 for antiferromagnetic impurities ($J' > 0$) coupled to the same and different sublattices respectively. Ferromagnetic impurities ($J' < 0$) generate similar diagrams with interchanged $\alpha$- and $\beta$-spin-waves. The first diagram in Fig. 2 for instance represents the expression

\[
-2J'^2N\frac{1}{S^2}q^2 \omega_p^2 \omega_q^2 e^{-i(p-q) \cdot r} N^2 (|J| S + 4JS\omega_p) 4JS(\omega_p + \omega_q) (|J'| S + 4JS\omega_p) ,
\]

with a factor 2 taking into account the similar process with exchanged impurities.

Summing these contributions, we obtain the interaction energy in the leading $\frac{1}{N}$-order

\[
\delta \epsilon(r) = \frac{4J'^2}{SN^2} \sum_{p, q} \frac{A_{p, q} e^{i(p-q) \cdot r}}{B_{p, q}} ,
\]

with

\[
A_{p, q}^{(a)} = J'^2 q_p^2 \left( u_p^2 (\omega_p + \omega_q)^2 - 2v_p^2 \omega_q^2 \right) + 8J' q_p^2 \omega_p \omega_q (u_p^2 (\omega_p + \omega_q) - 2v_p^2 \omega_q) - 32 J'^2 q_p^2 \omega_p^2 \omega_q^2
\]

\[
A_{p, q}^{(b)} = -2u_p q_p v_p v_q \omega_p \omega_q
\]

\[
B_{p, q}^{(a)} = (\omega_p + \omega_q) (|J| + 4J\omega_p)^2 (|J'| + 4J\omega_q)^2
\]

\[
B_{p, q}^{(b)} = (\omega_p + \omega_q) (|J'| + 4J\omega_p) (|J'| + 4J\omega_q)
\]
The sum extends over momenta in the magnetic Brillouin zone. In the case of ferromagnetic impurities (\(J' < 0\)), the Bogoliubov parameters in (10) have to be interchanged.

For large distances, \(r \gg 1\), the interaction energy comes from small momenta, \(p,q \approx 1/r \ll 1\). One can therefore approximate the Bogoliubov parameters and the dispersion by

\[
\omega_q \approx \frac{q}{\sqrt{2}}, \quad u_q \approx \sqrt{\frac{1}{2q}} \quad \text{and} \quad v_q \approx -\sqrt{\frac{1}{2q}}, \quad (11)
\]

and hence simplify the interactions (10)

\[
\delta \epsilon^a(r) \approx \frac{Jg^2}{S} \frac{1}{(2\pi)^2} \int \frac{e^{i(p-q) \cdot r}}{(p+q)} d^2q, \quad \delta \epsilon^b(r) \approx -\frac{Jg^2}{S} \frac{1}{(2\pi)^2} \int \frac{e^{i(p-q) \cdot r}}{(p+q)} d^2q. \quad (12)
\]

Here \(g\) is the dimensionless coupling constant. Since both integrals are ultraviolet convergent, we extend the integration domain to infinity. There is no difference between ferro- and antiferromagnetic (\(J' = \pm |J'|\)) impurities in this approximation.

We first consider very large distances \(r \gg 1/g\). In this case, momenta in (12) are limited by \(p,q \ll g\) and the corrections to the ground state energy are equal to

\[
\delta \epsilon(r) \approx \pm \frac{J}{16\sqrt{2\pi}} \frac{1}{Sr^3} \quad (r \gg 1/g) \quad (13)
\]

Interestingly, in this limit, the interaction between impurities is independent of their coupling to the antiferromagnetic host \(J'\).

In the case of intermediate distances between impurities, \(1 \ll r \ll 1/g\), a similar calculation using

\[
\int_0^\infty \frac{J_{0(pq)}J_{0(qr)}dpdq}{p+q} = \frac{1}{\sqrt{2} \pi},
\]

yields

\[
\delta \epsilon(r) = -\frac{g^2}{2\sqrt{2\pi}} \frac{J}{Sr^3} \quad (1 \ll r \ll 1/g). \quad (14)
\]

If the separation between the impurities is small, \(r \approx 1\), the approximation (11) for the Bogoliubov parameters and the dispersion is no longer valid and the integrals in (10) have to be calculated numerically. In order to circumvent finite-size effects, we extrapolate the results obtained from lattices with up to \(N^2 = 100 \times 100\) sites by a polynomial \(\delta \epsilon(r) = a(r) + b(r)/N + c(r)/N^2\). Even for very small distances, the interaction between additional spins is surprisingly well fitted by the intermediate distance attractive behavior (13). The situation is analogous to the magnetization disturbance, see section V, for which Fig. 5 provides an illustration of qualitatively similar fits. The attractive interaction (14) found in the weak coupling regime is independent of the sublattice. In comparison, in the strong coupling limit \(|J'| \geq J\) an attractive interaction is found for nearest neighbors, but the interaction between impurities on next-nearest sites is repulsive.

In the above calculations, we assumed that the impurities are in the ground state, see Eq. (7). To check the kinematic structure of the diagonal interaction in the effective Hamiltonian (2) one has to perform similar calculations for states (1), (1), and (2), see (7). In the case where both impurities are flipped with respect to their ground state configuration, the interaction is the same as in the ground state, since the situation corresponds to ferromagnetic coupling with \(J' > 0\). For an excited state with only one flipped impurity, a calculation analogous to (10) shows that the interaction has opposite sign and hence justifies the kinematic structure of the \(\sigma \cdot \sigma\) term in (2). We emphasize (see also end of section III), that because of the finite lifetime, the diagonal interaction in the spin-flipped states has limited meaning.

**IV. OFF-DIAGONAL INTERACTION M(r) AND RABI OSCILLATIONS BETWEEN IMPURITIES**

If a magnetic impurity is flipped with respect to the ground state configuration, e.g. in a nuclear magnetic resonance (NMR) or electron paramagnetic resonance (EPR) experiment, then the excited state has a finite lifetime due to emission of magnons, see first of Eqs. (8). However, there is another mechanism of spin relaxation, due to presence of distant similar magnetic impurities: a spin-wave exchange leads to Rabi oscillations between two impurities. In the effective Hamiltonian (2) this process is described by the off-diagonal term \(M(r)\). Diagrams for \(M(r)\) are shown in Fig. 4. Rabi oscillations can only be observed between impurities coupled to spins on the same sublattice, so

\[
M^{(a)}(r) \neq 0, \quad M^{(b)}(r) = 0.
\]

The oscillation frequency of this two-level system is proportional to the real part of the mixing matrix element

\[
M = \langle \uparrow, \downarrow | H | \downarrow, \uparrow \rangle,
\]

since the probability \(P(t)\) to find the system in a state with flipped impurities after the time \(t\) is equal to \(P(t) = \sin^2(2\Re M t)\). Using Tab. 4 to evaluate the diagrams shown in Fig. 4 we find

\[
M^{(a)}(r) = \frac{J^2}{N} \sum_q e^{iq \cdot r} \left( \frac{u_q^2}{J' - 4J\omega_q + i\delta} - \frac{u_q^2}{J' + 4J\omega_q} \right).
\]
described by the staggered magnetization. Let us con-

For large distances $r \gg 1$ we use the approximate Bo-
goliubov parameters \( \text{(15)} \). The mixing element becomes

$$M^{(a)}(r) = -Jg^2 \frac{2}{\pi} \int J_0(gr) \left( \frac{2d}{q^2 - g^2} + i\pi \delta(q-g) \right) dq$$

$$= Jg^2 \left( Y_0(gr) - iJ_0(gr) \right),$$

where \( Y_0 \) is the Neumann function. For very large dis-
tances between impurities, $r \gg 1/g$, the real part of the mixing matrix element is comparable to the imaginary one and both are much smaller than the width \( \text{(5)} \). In this case, there are no Rabi oscillations. However, in the intermediate regime, $1 \ll r \ll 1/g$, the real part

$$\text{Re} \ M^{(a)}(r) = Jg^2 \frac{2}{\pi} \ln gr$$

is logarithmically enhanced compared to the imaginary part and compared to the spin-wave width \( \text{(5)} \). Thus, in this regime, Rabi oscillations between impurities are well pronounced and this mechanism gives the main contribution to the effective width of the magnetic resonance line

$$\Gamma_{eff} \approx J \frac{4g^2}{\pi^2} |\ln gr|.$$  \hspace{1cm} \text{(16)}$$

V. MAGNETIZATION CLOUD AROUND AN IMPURITY

An interesting question is how an additional spin influences the magnetic order in the host antiferromagnet, described by the staggered magnetization. Let us con-
sider an impurity $\sigma_1$ at the origin and a local magnetic field $h$ on site $r$. The Hamiltonian reads

$$H = J \sum_{(i,j)} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sigma_1 \cdot \mathbf{S}_0 + h S_z^r,$$

and the variation of the expectation value of the local magnetization $\delta S_z^r$ in the ground state is given by

$$\delta S_z^r = \frac{\partial \delta E}{\partial h},$$  \hspace{1cm} \text{(18)}$$

where $\delta E$ is the part of the energy dependent on $J'$. Now we can consider the same framework as for the calculation of the interaction between impurities. Using Eq. \textbf{15} together with the Rayleigh-Schrödinger perturbation theory, one can see that

$$\delta S_z^r = \mp 2\delta \epsilon(r),$$

where $\delta \epsilon$ is the energy correction described by those diagrams in Figs. \textbf{2} and \textbf{3} without spin flip of the second impurity. These diagrams give the following explicit expressions for the magnetization variation

$$\delta S_z^r = \frac{2J'}{SN^2} \sum_{p,q} \frac{C_{p,q} e^{i(p-q) \cdot r}}{D_{p,q}},$$  \hspace{1cm} \text{(19)}$$

with

$$C_{p,q}^{(a)} = 2J'u_q^2 v_p^2 \omega_q - J' u_q^2 v_p^2 (\omega_p + \omega_q) + 8Ju_q^2 v_p^2 \omega_p \omega_q,$$

$$C_{p,q}^{(b)} = -8Ju_q^2 u_p v_q v_p \omega_q,$$

$$D_{p,q} = (\omega_p + \omega_q) (|J'| + 4J \omega_p) (|J'| + 4J \omega_q)$$

assuming $J' > 0$. The expressions for the ferromagnetic case ($J' < 0$) are obtained by a change of the global sign and interchanged Bogoliubov parameters in \textbf{15}. Using the approximations \textbf{11} for $r \gg 1$, we find that the magnetization disturbance is equal to

$$\delta S_z^r = \pm \frac{1}{8\sqrt{2\pi}} \frac{J}{|J'|} \frac{1}{S^3} \left( r \gg 1/g \right),$$

in the very large distance limit, and

$$\delta S_z^r = \pm \frac{1}{8\sqrt{2\pi}} \frac{J}{|J'|} \frac{1}{S^r} \left( 1 \ll r \ll 1/g \right),$$

for the intermediate region. In the case of small separa-
tion from the impurity, one has to calculate the integrals in \textbf{15} numerically. Using the same finite-size extrapo-
lation scheme as in section \textbf{11} we find that the variation of the magnetization in the vicinity of the impurity is described by Eq. \textbf{21} down to $r = 1$. Fig. \textbf{4} displays the variation of the magnetization calculated numerically at $J' = -0.01$. The results are very close to the analytical expression \textbf{21}. In agreement with Ref. \textbf{3} an added spin always enhances the Néel order in the host magnet, independent of the sign of the exchange coupling. In contrast to a vacancy, this enhancement is not limited to nearest neighbor sites\textbf{12}, but extends over the whole magnet. It is also interesting to compare our result for an added spin to an in-plane impurity considered in Ref. \textbf{14}. If the impurity is placed inside the host, it weakens the Néel order of the surrounding spins, but the magnetization disturbance also decreases as $1/r^3$ for $r \gg 14$.

VI. CONCLUSION

To conclude, we have studied the long-range dynamics of one and two spin-$\frac{1}{2}$ impurities in a two-dimensional
Heisenberg antiferromagnet with on site spin-$S$ treated in the linear spin-wave approximation. The impurities are assumed to be weakly coupled to the host magnet by a small dimensionless coupling constant $g$. A systematical treatment of the corrections contributing to the leading order of the $1/r^3$ expansion leads to non-trivial long-range dynamics. The interaction between two impurities can be separated into two regimes: For very large separations ($r \gg 1/g$) it is universal (independent of $J'$) and decreases as $1/r^3$. The interaction is repulsive (attractive) for impurities coupled to the same (different) sublattices. In an intermediate region $1 \ll r \ll 1/g$, the interaction decreases only as $1/r$ and is attractive, independent of the sign of the exchange couplings. It is shown that Rabi oscillations between impurities coupled to spins on the same sublattice are possible and well pronounced in the intermediate regime. The effective Hamiltonian describing the interaction in terms of the impurity spins is derived. It exhibits an xyz anisotropy which leads to NMR and EPR line broadening. The magnetization disturbance in the host magnet induced by a single impurity is analyzed in the same framework. It is shown that the disturbance exhibits behaviors similar to the interaction energy, always enhancing the magnetic order in the antiferromagnetic host. Numerical results indicate that the intermediate regimes can be extended down to $r \approx 1$.