Enhancement of nonclassicality by frequency modulation in a nanomechanical system

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Abstract. The influence of the frequency modulation on the nonclassicality is explored in a system of nanomechanical resonator, which is a promising device in the fields of quantum information engineering. It is shown that the nonclassicality of the quantum state is dependent on the value of the modulation rate during the quantum evolution. A pronounced peak is observed in the curve of the nonclassicality versus the modulation rate. It indicates that the system’s nonclassicality can be enhanced by the modulation rate. The enhancement of the nonclassicality by the modulation rate is induced by the effects of the external driving source on the quantum interference between different energy levels. This is supported by the correspondences between the dependences of the nonclassicality indicator and the uncertainty of the Wigner distribution function on the modulation rate.

1. Introduction

Nonclassicality (or quantum effects) play key roles in quantum technologies and is one of the essential ingredients in quantum information [1]. The latter is a key resource in quantum information (e.g., quantum computer) [2-5]. In the fields with respect to quantum information, driven nonlinear systems are of great interests. They are important components in fields such as nano-mechanical resonators and superconductor circuits [6-13], which play key roles in the realization of quantum computer and quantum information processing [14-16]. The driving frequency of a driven system is modulated when it varies periodically with time. Frequency modulation is widely used in electrical communication [17]. It also has important applications in the quantum technologies (e.g., in quantum auto-resonance [18], quantum ladder [19] and quantum control [20]). However, the influences of frequency modulation on the nonclassicality has not been studied. This is the motivation for our work.

The driven system used here is a model of nanomechanical resonator. The latter is proposed as promising device in the fields related to quantum information and quantum computing [13-15]. The nonclassicality is measured by an indicator which is defined with the Wigner distribution function (WDF). Besides, the evolution of the driven system starts from a coherent state, whose value of the nonclassicality indicator is zero. It is demonstrated numerically that the growth of the nonclassicality indicator is significantly influenced by the modulation rate of the driving frequency (i.e., the rate at which the frequency is changed). In more detailed investigations, the curve of the nonclassicality versus the modulation rate is observed to have a remarkable peak. The latter means the nonclassicality can be greatly enhanced by varying the modulation rate of the driving frequency. Further comparisons show the correspondence between the dependences of the nonclassicality production and the quantum power spectrum on the rate of the frequency modulation. Such correspondence reveals that the
dependence of the nonclassicality growth on the modulation rate is induced by the effects of the modulated driving force on the energy-level interference during the evolution of the quantum state. This is supported by the comparisons between the curves of the nonclassicality indicator and the WDF dispersion versus the modulation rate.

2. Driven System and the Nonclassicality Indicator

The system used here is a nanomechanical resonator driven by periodical drive. In natural units, its Hamiltonian reads

\[ H = \Omega a^+ a - \frac{\alpha}{16}(a + a^+)^4 + f(a + a^+\cos[\phi_d(t)], \] (1)

where \( a \) and \( a^+ \) are the annihilation and creation operators, \( \Omega \) stands for the frequency of the fundamental mode of the system, \( \alpha \) is the nonlinear constant and \( f \) is the driving force. In Eq. (1), \( \phi_d(t) \) is the driving phase and varies periodically with a period \( T \). During a period, its rate versus time (i.e. the driving frequency) \( \omega_d = d\phi_d / dt = c \) and \( \phi_d(t) \) satisfies

\[ \phi_d(t) = ct^2/2, \] (2)

where \( c \) is the modulation rate and is also often called as chirp rate [21].

The quantum evolution of the system (1) is governed by the Schrödinger equation

\[ -i\hbar \frac{\partial}{\partial t} \langle \psi(t) \rangle = H(t) \langle \psi(t) \rangle. \] (3)

It can be numerically investigated by methods such as the spectral method [22].

The nonclassicality of a quantum system can be evaluated by the negativity of the WDF. The WDF of \( \langle \psi(t) \rangle \) in natural units reads

\[ W(x, p; t) = \pi^{-1/2} \int d\kappa e^{2i\kappa p} \langle x - \kappa | \psi(t) \rangle \langle \psi(t) | x + \kappa \rangle. \] (4)

Based on the negativity of the WDF, the nonclassicality of the quantum state can be measured by

\[ \nu = 2 \int \int W^-(x, p; t) dq dp / \int \int W(x, p; t) dq dp, \] (5)

where \( W^-(x, p; t) \) denotes the negative values of the WDF and \( \int \int W^-(x, p; t) dq dp \) is the volume of the integrated negative part of the WDF [23,24]. It increases from 0 to 1, as the volume of the negative part of the WDF increases from 0 to its maximum. That is, the increase of \( \nu \) indicates the increase of the nonclassicality of the quantum state.
3. Dependence of the Nonclassicality on the Frequency Modulation

Following the equations shown above, the nonclassicality of the system in the driven system (1) are explored. The quantum evolution of the system starts from a coherent state, i.e.

$$|\psi(0)\rangle = e^{-|\gamma|^2/2} \sum_{k=0}^{\infty} (\gamma^k / \sqrt{k!}) |k\rangle,$$

where $\gamma$ is complex [25]. Coherent states play important roles in quantum information [25-27]. The values of their nonclassicality indicators equal to zero. Besides, the parameter values are $\Omega = 1$, $\alpha = 1/10$, $f = 2$ and $\gamma = 1 + i$ in natural units. The period of the modulated frequency is 10.

As mentioned above, the value of the nonclassicality indicator is zero at $t = 0$. It grows with the time during the quantum evolution, due to the nonlinearity of the system. Furthermore, the rates of the nonclassicality growth for different modulation rates are obviously different, as is displayed by Fig. 1(a). Figure 1(a) illustrates the growth of the nonclassicality indicator with the time $t$ for three different values of the modulation rate. The values of $c$ in Fig. 1(a) are 0.5, 1.4 and 3. It is easy to see that the growth of the nonclassicality indicator for $c = 0.5$ is faster than those for $c = 1.4$ and $c = 3$. This indicates that the influences of the frequency modulation rate on the growth of the nonclassicality.

Further simulations are performed with the modulation rate varying from 0 to 3.5. The results are exhibited in Fig. 1(b). In Fig. 1(b), the growth of the nonclassicality $\nu$ with the time is indicated by $\nu$.

4. Spectrum and the Dispersion of the WDF

As shown above, the increase of $\nu$ arises from the increase of the negativity of the WDF. It is known that the negativity of the WDF arises from the interference fringes, which is induced the interference between quantum energy levels [28]. In a driven nonlinear system, the driving force and thus the modulation rate of the driving frequency can affect the interference of the energy levels by affecting the energy-level transition. As a result, the nonclassicality shows dependence on the modulation rate of the driving frequency during the system’s evolution.

According to the studies on the Liouville dynamics[29], the energy levels of the system’s evolution are able to be evaluated via the Fourier spectral analysis of $\langle x(t) \rangle$ with $\langle x(t) \rangle = \int x\omega(x, p; t) dx dp$. In Fig. 2, we present the Fourier spectra of $\langle x(t) \rangle$ for $c = 0.5$, $c = 1.4$ and $c = 3$. It is shown by Fig. 2 that the Fourier spectrum significantly depends on the modulation rate of the driving frequency. Comparing Figs. 1(a) and 2, one can find that the nonclassicality grows with the density and width of the Fourier spectrum. This confirms the above argument that the modulation rate influences the nonclassicality production by means of affecting the energy levels.
The enhancement of the energy-level interference can induce the growth of the WDF dispersion. Thus, the former can be indicated by the growth of the dispersion of the WDF in the phase space. The latter can be measured by the uncertainty of the WDF in the phase space and is indicated by $\Delta$ here. In other words, the increase of $\Delta$ reveals the increase of the energy-level interference. In Fig. 3(a), we present the time evolution of the dispersion of the WDF for $c=0.5$, $c=1.4$ and $c=3$. They correspond to the curves in Fig. 1(a). It is obvious that the increase of the nonclassicality production corresponds to the increase of the dispersion of the WDF.

For more detailed comparison, we calculate the average value of $\Delta(t)$, i.e., $\overline{\Delta} = \tau^{-1} \int_{0}^{\tau} \Delta(t)dt$. The dependence of $\overline{\Delta}$ on the frequency modulation can be seen from Fig. 3(b). Comparisons between Fig. 1(b) and Fig. 3(b) show that clear correspondence exists between the curve of $\overline{\Delta}$ versus the frequency modulation rate $c$ and that of $\overline{\Delta}$ versus $c$. This further supports the argument presented above.

5. Conclusion

The influence of frequency modulation on of the nonclassicality is investigated in a system of nanomechanical device. It is shown numerically that the value of the nonclassicality indicator during the quantum evolution is obviously dependent on the frequency modulation rate. Further investigations show that the dependence of the nonclassicality on the modulation rate is relevant to the effects of the modulated driving force on the quantum interference. Since nonclassicality plays key role in quantum technologies, the results presented in this work provides the means for enhancing the nonclassicality via the frequency modulation. This may have potential applications in the fields of quantum technologies, which is closely tied to the nonclassicality.

Acknowledgements

The authors would like to thank the Central University’s Fundamental Research Funds (Project No. 2015MS78 and Project No. 2017MS165).

References

[1] P. Euibyung, Noncalssicality, Entanglement of optical fields, Quantum Cryptography: Quantum Infomation and Optics, VDM Verlag Dr. Müller, Riga, 2009, pp.1-28.
[2] V. V. Dodonov, Nonclassical states in quantum optics: a squeezed review of the first 75 years, J. Opt. B 4 (2002) R1-R33.
[3] V. Veitch, C. Ferrie, D. Gross, and J. Emerson, Negative quasi-probability as a resource for quantum computation, New J. Phys. 14 (2012) 113011.
[4] G. Vacanti et al, Nonclassicality of optomechanical devices in experimentally realistic operating regimes, Phys. Rev. A 88 (2013) 013851.
[5] M. Howard, J. Wallman, V. Veitch, and J. Emerson, Contextuality supplies the 'magic' for quantum computation, Nature 510 (2014) 351-355.
[6] S. Rips et al, Steady-state negative Wigner functions of nonlinear nanomechanical oscillators,
New J. Phys. 14 (2012) 023042.

[7] H. G. Craighead, Nanoelectromechanical systems, Science 290 (2000) 1532-1535.

[8] V. Peano and M. Thorwart, Macroscopic quantum effects in a strongly driven nanomechanical resonator, Phys. Rev. B 70 (2004) 235401.

[9] R. Kaltenbaek, et al, Chirp amplification of entropy growth in an open chirped-driven anharmonic oscillator, arXiv preprint arXiv:1503 (2015) 02640.

[10] M. Imboden, O. A. Williams and P. Mohanty, Nonlinear dissipation in diamond nanoelectromechanical resonators, Appl. Phys. Lett. 102 (2013) 103502.

[11] R. Almog, S. Zaitsev, O. Shtempluck and E. Buks, Noise squeezing in a nanomechanical duffing resonator, Phys. Rev. Lett. 98 (2007) 078103.

[12] S. Rips, I. Wilson-Rae and M. J. Hartmann, Nonlinear nanomechanical resonators for quantum optoelectromechanics, Phys. Rev. A 89 (2014) 013854.

[13] A. Lupascu et al, Quantum non-demolition measurement of a superconducting two-level system, Nat. Phys. 1074 (2006) 26-31.

[14] X. B. Zou, W. Mathis, Quantum information processing and entanglement with Josephson charge qubits coupled through nanomechanical resonator, Phys. Lett. A 324 (2004) 484-488.

[15] M. Pechal, P. Arrangoiz-Arrbiola and A. H Safavi-Naeini, Superconducting circuit quantum computing with nanomechanical resonators as storage, Quan. Sci. Tech. 4 (2018) 015006.

[16] E. N. Cleland and M. R. Geller, Superconducting Qubit Storage and Entanglement with Nanomechanical Resonators, Physical Review Letters 93 (2004) 070501.

[17] T. G. Thomas, S. C. Sekhar, Communication Theory, Tata-McGraw Hill, New Delhi, 2005, pp. 136-137.

[18] B. Amstrup, et al, Optimal control of quantum systems by chirped pulses, Phys. Rev. A 48 (1993) 3830.

[19] W. Balling, D. J. Maas and L. D. Noordam, Interference in climbing a quantum ladder system with frequency-chirped laser pulses, Phys. Rev. A 50 (1994) 4276.

[20] K. W. Murch, et al, Quantum fluctuations in the chirped pendulum, Nat. Phys. 7 (2011) 105-108.

[21] P. Symons, Digital waveform generation, Cambridge University Press, New York, 2014, pp. 242-246.

[22] A. N. Dumont and P. Brumer, Characteristics of power spectra for regular and chaotic systems, J. Chem. Phys. 88 (1988) 1481-1496.