An interval type-2 FCM for color image segmentation

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Received: 14-September-2019; Revised: 20-December-2019; Accepted: 23-December-2019
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Abstract
In today’s digital life, the segmentation of images is a very important issue. Each image contains a big amount of data. The internal relation between data of one image is nonlinear and ambiguous. There is a big uncertainty to find all segments of an image. Therefore, there is a big necessity to find a segmentation method for handling high uncertainty. In this paper, some of the previous works that have done on image segmentation have been improved and extended. A novel fuzzy c-means (FCM) is applied for the segmentation of images that inherently have high uncertainty and vagueness. A new method is used based on interval type-2 fuzzy sets, and the idea of reducing higher-order sets to lower order to capture the uncertainty of the images based on the decisiveness method. The higher peak signal to noise ratio (PSNR) value and the Jaccard similarity value for colour images show better segmentation results for having better performance and better effects on segmenting real world images. The results show that the proposed algorithm handles the segmentation of colour images better than the previous type-1 and type-2 FCM.

Keywords
Fuzzy c-means algorithm, Image processing, Image segmentation, Uncertainty.

1. Introduction
In some image processing applications, segmenting the images is crucial and essential due to its usefulness for understanding images easily and extracting meaningful parts from them. The process is to divide images into different interesting sections. In this process numerous approaches have been introduced and studied in the literature, like histogram Thresholding [1, 2], Edge Detection [3, 4], Region Growing [5, 6], Watershed Transformation [7, 8]. In the most recent decade fuzzy techniques are introduced in several fields of image processing to process image data with vagueness and ambiguity characteristics. The fuzzy c-means (FCM) algorithm is a popular technique that is widely used in the field of segmentation [9–11]. But it is founded that with type-1 fuzzy sets, ambiguity might not be taken well in the images because of the complexity of the image visual structures. Besides, with ordinary type-1 fuzzy sets, satisfactory results might not be obtained, so using, the type-2 fuzzy logic would be necessary to manage uncertainty exists in real-world problems.

1.1 Type-1 and type-2 fuzzy sets
The fuzzy logic systems are based on the fuzzy sets. Fuzzy sets are sets with elements that can only participate with a certain degree of membership function. Therefore, a fuzzy set does not have a crisp boundary. When the universe, $D_x$, is continuous and infinite, a fuzzy set $X$ in $D_x$ can be defined type-1 and type-2 fuzzy logic as follows respectively:

$$X = \{(x, \mu_x(x)) | x \in D_x\}$$

$$\hat{X} = \{(x, u, \mu_A(x, u)) | x \in X, u \in J_x \subseteq [0,1]\}$$

T2FSs are the continuation of type-1 [12, 13], they extended to provide additional information in the secondary membership function, which they are not only depend on a variable, it also depends on variable $u$.

1.2 Type-1 FCM
Fuzzy c-means [14] is one of the clustering algorithms that group data objects based on similarity, in image processing, FCM divides pixels in images to different segments based on their contribution degrees to different segments. One of the popular and powerful techniques used in cluster...
analysis and medical imaging is FCM, the algorithm works in step by step procedure it forms a fuzzy partition matrix during its implementation and also requires a cluster center along with objective function. The Equation 3 defines the objective function of the algorithm, its value should be decreased in any step of its execution.

\[
J_m = \sum_{i=1}^{N} \sum_{j=1}^{C} \mu_{ij}^m \left\| x_i - c_j \right\|^2
\]

FCM objective function decreases subject to constraints of membership grades, we can calculate membership values \( \mu_{ij} \) and update cluster centres \( c_j \) as follows:

\[
\mu_{ij} = \frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m}} \quad \text{for } i \in \{1, \ldots, N\}, \ j \in \{1, \ldots, C\}
\]

\[
C_j = \frac{\sum_{i=1}^{N} \mu_{ij} x_i}{\sum_{i=1}^{N} \mu_{ij}}, \text{ for } j = 1 \ldots C
\]

The algorithm execution steps will stop when \( \|U(k+1) - U(k)\| < \varepsilon \). When \( \varepsilon \) is some criteria for the termination of the algorithm, its value is always between 0 and 1, and \( k \) represents the repetition steps.

1.3 Interval type-2 FCM

Hwang and Rhee [15] introduced the interval type-2 fuzzy FCM method as enhanced method for capturing and managing high uncertainty, they have worked on managing uncertainty in the field of pattern recognition focusing on FCM algorithm. They extended a pattern set in to an interval type 2 fuzzy sets (IT2FS) by involving two different values of fuzzifier \( m \) (\( m_1 \) and \( m_2 \)) which creates a foot of uncertainty (FOU) for the fuzzifier and modified the procedure of FCM. IT2 FCM has two membership functions, upper and lower memberships and it has two fuzzifier values \( m_1 \) and \( m_2 \), representing different fuzzy degrees and providing different objective functions as listed in the Equation 6 and Equation 7:

\[
J_{m1} = \sum_{i=1}^{N} \sum_{k=1}^{C} \mu_{ij}^{m_1} \left\| x_i - c_j \right\|^2
\]

\[
J_{m2} = \sum_{i=1}^{N} \sum_{k=1}^{C} \mu_{ij}^{m_2} \left\| x_i - c_j \right\|^2
\]

Both upper and lower interval type-2 fuzzy membership functions can be calculated using the Equation 8 and Equation 9.

Upper membership function:

\[
\bar{\mu}_{f}(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_1}}, & \text{if } \frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_1}} < \frac{1}{\varepsilon} \\
\frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_2}}, & \text{Otherwise}
\end{cases}
\]

Lower membership function

\[
\underline{\mu}_{f}(x_i) = \begin{cases} 
\frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_1}}, & \text{if } \frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_1}} \geq \frac{1}{\varepsilon} \\
\frac{1}{\sum_{k=1}^{C} \left( \frac{d_{ik}}{d_{kj}} \right)^{2/m_2}}, & \text{Otherwise}
\end{cases}
\]

The foot of uncertainty of fuzzifier \( m \) is obtained by the two fuzzifiers \( m_1 \) and \( m_2 \).

2. Our proposed method

In our proposed method for image segmentation, we used IT2FCM for colour image segmentation with some modification, we employed a filtering step into the process of segmentation because FCM and IT2FCM are both sensitive to noises, median filter [16] which is a none-linear type filter is used as a first step for eliminating noises from images especially impulse noises, none-linear filters are good due to their robustness for removing noise pixels without blurring edges. In this paper, the interval type-2 fuzzy sets are used as basic membership functions in fuzzy clustering. The type reduction which is applied to interval type-2 fuzzy membership functions is decisiveness method which is proposed in [17] and [18]. In decisiveness type reduction method, the interval type-2 is reduced to type one using the below equation for calculating \( \mu_d \) as reduced type membership degree:

\[
\mu_d = \mu_p - \left( \frac{1}{2} \right) u^2
\]

In which: \( \mu_p = \frac{\mu_u(x_i) + \mu_l(x_i)}{2} \) and \( u = \frac{\mu_u(x_i) - \mu_l(x_i)}{2} \)

One of the advantages of this determiner is that the more the uncertainty bandwidth increases, the determined value of a point decreases in a linear manner. For instance, if \( u=1 \), then \( \mu_{du}=0 \) and if \( u=0 \), then \( \mu_{du}=1 \) and its fluctuations between these two points is in accordance with \( u \). After choosing a decisiveness type reduction method which has a powerful effect on uncertainty handling, the applied algorithm for image segmentation is proposed as below:
Steps of our IT2FCM method for colour image segmentation

Step 1) Apply median filtering image to input images.

Step 2) Set the number of the cluster prototypes \((2 \leq c \leq N)\) and fuzzifiers \(m, m_1\) and \(m_2\). Set \(\epsilon > 0\) to a very small value and initialize the cluster center \(V\) randomly.

Step 3) Estimating FCM membership \(\mu_{ij}\) using \(m\) and \(m_2\) in the following orders:
- Compute the distance between input patterns and cluster centers \(d_{ik}^2 = \|x_k - v_i\|\) (where \(\|x_k - v_i\|\) is the euclidean norm).
- \[\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{1}{d_{ik}^{m_2}}\right)^{m_1}}; \quad i \in \{1, \ldots, N\}, \quad j \in \{1, \ldots, c\}\]

Step 4) Calculating The upper and lower membership functions \(\tilde{\mu}^{(1)}_{ij}, \tilde{\mu}^{(2)}_{ij}\) using Equations 8 and 9.

Step 5) Updating cluster centers \(V^{t+1}\) using Equation 10.
- \[v_j = \frac{\sum_{i=1}^{N} \mu_{ij} x_i}{\sum_{i=1}^{N} \mu_{ij}} \quad \text{(11)}\]

Step 6) Applying decisiveness type-reduction as follows in Equation 12:
- \[\mu_j(x_i) = \frac{\mu^{R}_{j}(x_i) + \mu^{L}_{j}(x_i)}{2} - \left(\frac{\mu^{R}_{j}(x_i) - \mu^{L}_{j}(x_i)}{2}\right)^2, \quad j = 1, \ldots, c \quad \text{(12)}\]

3. Experimental results

I have tested FCM and our proposed algorithm on randomly chosen images from the publicly-available ground-truth segmentation databases, like BSDS500 from the Berkeley Segmentation Dataset and Benchmark as showed some image examples from the dataset in Figure 1. Both algorithms are simulated using MATLAB 2016b. A 3 x 3 kernel window is used for median filtering. And the fuzzifier parameters are set like: \(m = 3, m_1 = 2, m_2 = 7\).

In our experiments, we have used two clustering validation methods partition coefficient (PC) and partition entropy (PE) as formulated in Equations 13 and 14 respectively [19]. We also compared the quality of segmentation results of both FCM and IT2FCM by using Jaccard similarity coefficient and peak signal to noise ratio (PSNR) methods [20]. The Jaccard similarity coefficient that is also called intersection over union is defined in Equation 15, and PSNR is defined in Equation 16.

\[PC = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{c} \mu_{ij}^2 \quad \text{(13)}\]

\[PE = -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{c} \mu_{ij} \log_{10} \mu_{ij} \quad \text{(14)}\]

\[IoU = \frac{\text{GroundTruth}(i) \cap \text{segmented}(i)}{\text{GroundTruth}(i) \cup \text{segmented}(i)} \quad \text{(15)}\]

\[PSNR = 10 \log_{10} \left(\frac{255^2}{\text{MAE}}\right) \quad \text{(16)}\]

Where MAE is the mean absolute error of the segmented image computed as in Equation 17.

\[\text{MAE} = \frac{\sum_{i \in I_1(m,n) \cup I_2(m,n)} |I_1(m,n) - I_2(m,n)|^2}{m \times N} \quad \text{(17)}\]

Where \(I_1(m,n)\) is the segmented image and \(I_2(m,n)\) is the original image.

4. Discussion

For the validation clustering methods, the clustering algorithm gets the best result when the Partition coefficient is maximum and the partition entropy is the minimum. As we compared both methods in Table 1, it indicates that IT2FCM has a better performance in dealing with the uncertainties and obtained better cluster results.

For measuring quality results, the higher the PSNR value means better segmentation results and the Jaccard similarity value should be higher and close to 1 as possible. The Jaccard index, also known as the intersection over union and the Jaccard similarity coefficient, is a statistic used for gauging the similarity and diversity of images. Table 2 shows the segmentation performance between the two algorithms. It indicates that our proposed method has a better performance than the conventional type-1 FCM.

**Table 1** Numerical comparisons using validation methods
We also provide some examples of the segmentation results using IT2FCM in Figure 2.

The approach of measuring the similarity between two images is using PSNR and Jaccard parameters: An image is selected, to be more correct, a region-of-interest in each image then the pixel features are identified for comparing (e.g., red, blue, green and gradient orientation). Finally, similarity formula is set up. The results which are shown in Table 2 shows higher PSNR and Jaccard compare to their type-1 and traditional type-2 methods. Figure 2 confirms this optimization.

![Figure 2 Segmentation results using IT2FCM, left original images, and right are the segmentation results](image)

### Table 2 Performance measures of FCM and IT2FCM

| Images | FCM | IT2FCM |
|--------|-----|--------|
|        | PC  | PE     | PC  | PE     |
| 1      | 0.7370 | 0.4745 | 0.8562 | 0.2516 |
| 2      | 0.7466 | 0.4546 | 0.8125 | 0.3327 |
| 3      | 0.7429 | 0.4572 | 0.7804 | 0.3999 |
| 4      | 0.7373 | 0.4769 | 0.7934 | 0.3730 |
| 5      | 0.8124 | 0.3324 | 0.8456 | 0.2948 |
| 6      | 0.6848 | 0.5633 | 0.6956 | 0.5471 |
| 7      | 0.8294 | 0.3366 | 0.8651 | 0.2789 |
| 8      | 0.8254 | 0.3349 | 0.8282 | 0.3101 |
| 9      | 0.7382 | 0.4844 | 0.9284 | 0.1395 |
| 10     | 0.9192 | 0.1579 | 0.9455 | 0.1007 |

5. **Conclusion**

In this paper, an image segmentation technique based on FCM is enhanced and improved in term of handling uncertainty. Interval type-2 fuzzy sets and decisiveness type reduction methods are used in the proposed technique. Type-1 FCM algorithm and proposed algorithm are tested on different kinds of color images. The performance of the proposed algorithm for segmentation has been compared with previous type-1 FCM algorithm. The objective evaluation metrics like partition coefficient (PC) and partition entropy (PE) are used in their comparisons. Other performance measures (Jaccard similarity coefficient, and Peak signal to noise ratio) are also used to test the algorithms. Results show that our interval type-2 algorithm are better than the type-1 FCM algorithm and applying decisiveness method for type reduction of interval type-2 provides an active method for handling uncertainty in image segmentation.

**Acknowledgment**

None.

**Conflicts of interest**

The authors have no conflicts of interest to declare.
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