Phenomenological Lambda-Nuclear Interactions

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[ABSTRACT]

Variational Monte Carlo calculations for $^4\Lambda H$ (ground and excited states) and $^5\Lambda He$ are performed to decipher information on $\Lambda$-nuclear interactions. Appropriate operatorial nuclear and $\Lambda$-nuclear correlations have been incorporated to minimize the expectation values of the energies. We use the Argonne $\upsilon_{18}$ two-body $NN$ along with the Urbana IX three-body $NNN$ interactions. The study demonstrates that a large part of the splitting energy in $^4\Lambda H$ ($0^+ - 1^+$) is due to the three-body $\Lambda NN$ forces. $^{17}\Lambda O$ hypernucleus is analyzed using the $s$-shell results. $\Lambda$ binding to nuclear matter is calculated within the variational framework using the Fermi-Hypernetted-Chain technique. There is a need to correctly incorporate the three-body $\Lambda NN$ correlations for $\Lambda$ binding to nuclear matter.

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§1. Introduction

The study of the response of a many-body system to a hyperon gives insight into the structure of baryon-baryon interactions. The binding energy data of light $s$-shell hypernuclei provide a unique opportunity to know more about the $\Lambda$-nuclear interactions, particularly on their spin-dependence. In the past, basically two approaches have been followed. The first one involves Brueckner-Hartree calculations using Nijmegen $YN$ potential with and without higher order correction to single-particle energies[1,2]. This method uses the large $\Lambda N \rightarrow \Sigma N$ coupling which gives considerably lower binding energy for $^5\Lambda He$. Any attempt to correct this leads to poor agreement with the scattering data. The second method is primarily based on reliable variational techniques, mostly using simplified $NN$ interactions[3,4]. We follow this approach but use realistic Argonne $\upsilon_{18}$ $NN$ interaction[5] along with Urbana IX three-body $NNN$ interaction[6,7].

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ical approach we follow is consistent with meson-theoretic models as well as available low energy scattering data. The \( \Lambda N \rightarrow \Sigma N \) coupling is effectively taken care of by inclusion of the phenomenological \( \Lambda NN \) potential. The \( \Lambda NN \) potential consists of both the dispersive and two-pion-exchange (TPE) kind as employed in previous studies\cite{4}. The hypernuclei considered in this work are \( \frac{4}{3} \Lambda H \), \( \frac{4}{3} \Lambda H^* \) (\* on \( \frac{4}{3} \Lambda H \) refers to the excited \( 1^+ \) state), and \( \frac{5}{3} He \).

The interaction parameters which we find based on our \( s \)-shell results are later used to make estimates of the binding energy of \( ^{17}_{\Lambda}O \). We also study the \( \Lambda \) binding to nuclear matter by using the Fermi-Hypernetted-Chain (FHNC) technique\cite{4,8}. This study gives an indication of the implications of our \( s \)-shell results on heavier hypernuclear systems.

In section 2 we describe the Hamiltonian used in this work. Section 3 gives the wave function approach. In section 4 we discuss the results and finally in section 5 we give the conclusion and comments.

\section*{2. The Hamiltonian}

The complete hypernuclear Hamiltonian consists of the nuclear Hamiltonian \( H_{\Lambda^{-1}}^{A^{-1}} \) and the lambda Hamiltonian \( H_{\Lambda} \). The nuclear Hamiltonian \( H_{\Lambda^{-1}}^{A^{-1}} \) is given by

\[
H_{\Lambda^{-1}}^{A^{-1}} = -\sum_{i=1}^{A-1} \frac{\hbar^2}{2m_i} \nabla_i^2 + \sum_{i<j}^{A-1} V_{ij} + \sum_{i<j<k}^{A-1} V_{ijk}, \tag{2.1}
\]

where \( V_{ij} \) and \( V_{ijk} \) are the two-nucleon \( NN \) and three-nucleon \( NNN \) potentials, respectively, and \( m_i \) is the mass of the nucleon.

The two-body \( NN \) interaction employed here is the Argonne \( v_{18} \) interaction\cite{5}. The first fourteen operator components of this model are charge-independent and are an updated version of the Argonne \( v_{14} \) potential\cite{9}. Three additional charge-dependent and one charge-asymmetric operators are added along with a complete electromagnetic interaction, containing the Coulomb, Darwin-Foldy, vacuum polarization, and magnetic moment terms with finite-size effects. The potential has been fit directly to the Nijmegen \( pp \) and \( np \) scattering data base\cite{10,11}, low-energy \( nn \) scattering parameters, and deuteron binding energy. For the three-body \( NNN \) potential we use the Urbana model\cite{6,7} consisting of the two-pion-exchange (TPE) part of Fujita and Miyazawa\cite{12} and a repulsive phenomenological spin-isospin independent term. We have used the Urbana \( IX \) model\cite{7} of the interaction where the values of the strength parameters are used in conjunction with the Argonne \( v_{18} \) interaction.

The lambda Hamiltonian \( H_{\Lambda} \) is given by

\[
H_{\Lambda} = -\frac{\hbar^2}{2m_{\Lambda}} \nabla_{\Lambda}^2 + \sum_{i=1}^{A-1} V_{i\Lambda} + \sum_{i<j}^{A-1} V_{ij\Lambda}, \tag{2.2}
\]

where \( V_{i\Lambda} \) and \( V_{ij\Lambda} \) are the two-body \( \Lambda N \) and three-body \( \Lambda NN \) potentials, respectively, and \( m_{\Lambda} \) is the mass of the \( \Lambda \) particle. The first terms of Eqs.(2.1) and (2.2) pertain to the total kinetic energy of the nucleons and \( \Lambda \), respectively.

The two-body \( \Lambda N \) potential \( V_{\Lambda N} \) includes a central potential\cite{4} of the same form for the singlet and triplet spin states. These have a theoretically reasonable attractive tail due to the TPE in accord with Urbana type potentials \cite{13} with spin- and space-exchange terms.
\[ V_{AN} = \left[ (V_c(r) - \tilde{V}) (1 - \epsilon + \epsilon P_x) + \frac{1}{4} V_\sigma \bar{\sigma}_\Lambda \cdot \bar{\sigma}_N \right] T_\pi^2(r), \]  

where \( \tilde{V} \) and \( V_\sigma \) are the spin-average and spin-dependent strengths, respectively. \( P_x \) is the Majorana space-exchange operator, \( \epsilon \) is the corresponding exchange parameter, \( V_c \) is the Woods-Saxon repulsive core, and \( T_\pi \) is the one-pion-exchange (OPE) tensor potential shape modified with a cutoff. Further details can be found in Ref. [4].

In this study we consider potential parameters that are consistent with low energy \( \Lambda p \) scattering data that essentially determine the value of spin-average strength \( \tilde{V} = 6.15 \pm 0.05 \text{ MeV} \) [4].

For hypernuclei with zero-spin core nuclei, such as \( ^5_\Lambda \text{He} \), the major contribution arises from the spin-average strength \( \tilde{V} \) while the spin component contributes very little. The spin dependence \( V_\sigma \) is assumed to be positive, which is consistent with hypernuclear spins of mass 4 systems. We find that for the s-shell hypernuclei (\( A \leq 5 \)) the s-state interaction is dominant but the higher partial wave interactions, in particular, the p-state, also make a small but significant contribution contrary to earlier studies [14]. The importance of the p-state contribution becomes significant due to the \( \Lambda \)-nuclear correlations.

Studies on hypernuclei have shown that it is necessary to include a three-body \( \Lambda NN \) interaction in the Hamiltonian. We consider phenomenological \( \Lambda NN \) forces of the dispersive (spin-dependent and spin-independent) as well as the TPE kind, which arise from the suppression of \( \Sigma, \Delta, \ldots \) degrees of freedom by the medium, that is, the second nucleon.

The dispersive kind has a spin dependence that is given by

\[ V_{ANN}^{DS}(r_{ij\Lambda}) = W_\sigma T_\pi^2(r_{i\Lambda}) T_\pi^2(r_{j\Lambda}) \left[ 1 + \frac{1}{6} \bar{\sigma}_\Lambda \cdot (\bar{\sigma}_i + \bar{\sigma}_j) \right], \]  

The TPE part of the interaction is given by [15]

\[ W_p = -\frac{1}{6} C_p \{ \bar{\tau}_i \cdot \bar{\tau}_j \} \{ X_{k\Lambda} X_{j\Lambda} \} Y(r_{i\Lambda}) Y(r_{j\Lambda}), \]  

where \( X_{k\Lambda} \) is the OPE operator given by

\[ X_{k\Lambda} = (\bar{\sigma}_k \cdot \bar{\sigma}_\Lambda) + S_{k\Lambda}(r_{k\Lambda}) T_\pi(r_{k\Lambda}) \]  

with

\[ S_{k\Lambda}(r_{k\Lambda}) = \frac{3 (\bar{\sigma}_k \cdot r_{k\Lambda}) (\bar{\sigma}_\Lambda \cdot r_{k\Lambda})}{r_{k\Lambda}^2} - (\bar{\sigma}_k \cdot \bar{\sigma}_\Lambda). \]  

In Eq. (2.5) \( \{ \} \) represents the anticommutator term. \( Y_{k\Lambda}(r_{k\Lambda}) \) and \( T_{k\Lambda}(r_{k\Lambda}) \) are the usual Yukawa and tensor functions, respectively, with pion mass \( \mu = 0.7 \text{ fm}^{-1} \).

The \( \Lambda \)-nuclear interaction parameters, \( \tilde{V}, V_\sigma, C_p, \) and \( W_\sigma \) are considered as unknown. These are then fitted as a function of the \( B_\Lambda \) values that have been calculated using the s-shell results. Taking these values of \( \tilde{V}, V_\sigma, C_p, \) and \( W_\sigma \) we again perform variational calculations that give us the final results for \( ^4_\Lambda H, ^4_\Lambda H^*, \) and \( ^5_\Lambda \text{He} \). These results are later used to analyze \( ^{17}_\Lambda O \) and \( \Lambda \) binding to nuclear matter.

\section*{ §3. Wave Function and Approach}
The trial variational wave function we adopt is of the following form:

\[
| \Psi_v \rangle = \left[ 1 + \sum_{i<j<k} U_{ijk} + \sum_{i<j} U_{ij\Lambda} + \sum_{i<j} U_{ij}^{LS} + \sum_{i<j<k} U_{ijk}^{T}\right] \left[ \prod_{i<j<k} f_{ijk} \right] | \Psi_p \rangle .
\] (3.1)

The pair wave function \( | \Psi_p \rangle \) is a symmetrized product of two-body \((1 + U_{ij})\) and \((1 + U_{i\Lambda})\) correlation operators acting on a Jastrow trial function. This is written as

\[
| \Psi_p \rangle = \left[ S \prod_{i<j} (1 + U_{ij}) \right] \left[ S \prod_{i=1}^{A-1} (1 + U_{i\Lambda}) \right] | \Psi_J \rangle .
\] (3.2)

\( U_{ij} \) in Eq.(3.2) is defined as

\[
U_{ij} = \sum_{p=2,6} \left[ \prod_{k \neq i,j} f_{ijk}^{p}(r_{ik}, r_{jk}) \right] u_{p}(r_{ij}) O_{ij}^{p}
\] (3.3)

with \( O_{ij}^{p} = [1, \sigma_i \cdot \sigma_j, S_{ij}] \otimes [1, \tilde{\tau}_i, \cdot \tilde{\tau}_j] \). \( S_{ij} \) is the tensor operator. \( \sigma \) and \( \tilde{\tau} \) are the spin and isospin operators, respectively. The factor \( f_{ijk}^{p} \) suppresses spin-isospin correlations between two nucleons in the presence of a third one.

In principle, the \( U_{i\Lambda} \) correlation will consist of a spin and a Majorana space-exchange operator [16]:

\[
U_{i\Lambda} = \alpha_{\sigma} u_{\sigma}(r_{i\Lambda}) \tilde{\sigma}_i \cdot \tilde{\sigma}_j + \alpha_{px} u_{px}(r_{i\Lambda}) P_x ,
\] (3.4)

where \( \alpha_{\sigma} \) and \( \alpha_{px} \) are variational parameters and \( P_x \) is the space-exchange operator. In our calculations we have not included the space-exchange correlations since the calculations become complicated and time consuming. In any case the effect of these correlations is expected to be small for s-shell hypernuclei.

The spin dependent correlation \( u_{\sigma} \) given in Eq.(3.4) is defined as

\[
u_{\sigma} = \frac{(f_{s}^{\Lambda} - f_{t}^{\Lambda})}{f_{c}^{\Lambda}} ,
\] (3.5)

where \( f_{c}^{\Lambda} \) is the spin-average correlation function. \( f_{s}^{\Lambda} \) and \( f_{t}^{\Lambda} \) are the solutions of the quenched \( \Lambda N \) potential in singlet and triplet states, respectively, which are given by the following relation:

\[
\left[ -\frac{\hbar^2}{2\mu_{\Lambda N}} \nabla^2 + \tilde{V}_{s/t}(r_{\Lambda N}) + V_{\Lambda N}^{a} \right] f_{s/t}^{\Lambda} = 0 ,
\] (3.6)

where \( \tilde{V}_{s/t} \) is the quenched \( \Lambda N \) potential in singlet/triplet state. \( \mu_{\Lambda N} \) is the reduced mass of the \( \Lambda-N \) pair, while \( V_{\Lambda N}^{a} \) is an auxiliary potential.

The Jastrow wave function \( | \Psi_J \rangle \) is given by

\[
| \Psi_J \rangle = \left[ \prod_{i=1}^{A-1} f_{c}^{\Lambda}(r_{i\Lambda}) \prod_{i<j} f_{c}(r_{ij}) \right] | \Phi \rangle .
\] (3.7)

Here \( | \Phi \rangle \) is an antisymmetric product of single particle wave function with the desired \((J, T)\). The initial uncorrelated state \( \Phi \) has no coordinate dependence and is real. For example, consider the following \( \Phi \) states for \( ^4_{\Lambda}H \) and \( ^4_{\Lambda}H^* \) expressed in the spin-isospin basis with the appropriate \((J, T)\) states,
\[ \frac{4}{3} H \left( J=0, T=\frac{1}{2} \right) = C \left( \frac{1}{2}, \frac{1}{2}, 0; \frac{1}{2}, -\frac{1}{2}, 0 \right) 3H_{J=\frac{1}{2}}^{s} \Lambda_{-\frac{3}{2}}^{s} + C \left( \frac{1}{2}, \frac{1}{2}, 0; \frac{1}{2}, \frac{1}{2}, 0 \right) 3H_{J=\frac{1}{2}}^{s} \Lambda_{\frac{3}{2}}^{s} \]

and

\[ \frac{4}{3} H^* \left( J=1, T=\frac{1}{2} \right) = C \left( \frac{1}{2}, \frac{1}{2}, 1; \frac{1}{2}, \frac{1}{2}, 1 \right) 3H_{J=\frac{1}{2}}^{s} \Lambda_{\frac{3}{2}}^{s} , \] (3.8)

where \( C \) represents the Clebsch Gordon coefficients[17]. \( 3H_{J=\frac{1}{2}}^{s} \) and \( 3H_{J=\frac{1}{2}}^{s} \) are the uncorrelated \( \Phi s \) for triton, whereas \( \Lambda_{-\frac{3}{2}}^{s} \) and \( \Lambda_{\frac{3}{2}}^{s} \) are the spin-down and spin-up states, respectively of the \( \Lambda \) particle.

The spin-orbit correlation \( U^{LS}_{ij} \) is given by

\[ U^{LS}_{ij} = [u_{i\sigma}(r_{ij}) + u_{i\sigma'}(r_{ij}) \vec{p}_{i} \cdot \vec{p}_{j}] (\mathbf{L} \cdot \mathbf{S})_{ij} . \] (3.10)

The eight radial functions, \( f_{c}(r_{ij}) \), \( u_{p=2,6}(r_{ij}) \), \( u_{i\sigma}(r_{ij}) \), and \( u_{i\sigma'}(r_{ij}) \) are obtained from approximate two-body Euler-Lagrange equations with variational parameters[18].

\( U^{TNN}_{ijk} \) is a three-body correlation induced by the three-nucleon interaction, \( V_{ijk} \). The other correlations incorporated in the wave function are a spatial three-body NNN correlation \( f_{ijk}^{c} \), along with \( U_{ijk} \) that consists of a spin-orbit and an isospin three-body correlation. Further details can be found in Ref. [19].

The three-body \( \Lambda NN \) correlation \( U_{ij\Lambda} \) has the following form:

\[ U_{ij\Lambda} = \tilde{V}_{ij\Lambda}(\delta_1, \delta_2) , \] (3.11)

where \( \tilde{V}_{ij\Lambda} \) differs from \( V_{ij\Lambda} \) through the cutoff factor \( c \) of the usual Yukawa and tensor functions. \( \delta_1 \) and \( \delta_2 \) are variational parameters which multiply the \( \Lambda NN \) interaction parameters, \( C_{p} \) and \( W_{o} \), respectively.

No attempt has been made to vary the two-body \( NN \) and three-body \( NNN \) correlation parameters[19] as their effect has been found to be small[20] and only the variational parameters of the wave function pertaining to \( \Lambda \) have been varied to obtain a minimum in the energy. The optimum values of these parameters which are used in our final calculations are given in Tables[1] and [2].

We calculate energy expectation values using Monte Carlo (MC) integration[21,22]. The expectation values are sampled both in configuration space and in the order of operators in the wave function by following a Metropolis random walk[23]. The mathematical expressions used to evaluate the energy expectation values are given below.

The energy expectation value for the pure nucleus is given by

\[ \langle E^A_{N} \rangle = \frac{\langle \Psi^A_{N-1} | H^A_{N-1} | \Psi^A_{N-1} \rangle}{\langle \Psi^A_{N-1} | \Psi^A_{N-1} \rangle} , \] (3.12)

where \( \Psi^A_{N-1} \) is the wave function of the mass (A-1) nucleus and \( H^A_{N} \) is the nuclear Hamiltonian.

The energy expectation value for the hypernucleus is given by

\[ \langle E^A_{H} \rangle = \frac{\langle \Psi^A_{H} | H^A_{H} | \Psi^A_{H} \rangle}{\langle \Psi^A_{H} | \Psi^A_{H} \rangle} , \] (3.13)
where $\Psi^A_{\Lambda H}$ is the wave function of the mass ‘A’ hypernucleus and $H^A_{\Lambda H}$ is the hypernuclear Hamiltonian.

Therefore, binding energy of $\Lambda$ to the hypernucleus is given by

$$-B_{\Lambda} = \langle E^A_{\Lambda H} \rangle - \langle E^{A-1}_N \rangle .$$

(3.14)

The nuclear and hypernuclear wave functions, $\Psi^{A-1}_N$ and $\Psi^A_{\Lambda H}$ are optimized with respect to the variational parameters to obtain the minimum in the energies.

The $B_{\Lambda}$ value for each hypernuclei is calculated from the variational results using Eq.(3.14). The $B_{\Lambda}$ value is thus written as a function of the adjustable parameters in the $\Lambda$ Hamiltonian $H_{\Lambda}$, and is used to determine the set of parameters which are consistent with the experimental $B_{\Lambda}$ values[26,27].

§4. Results and Discussion

Table[3] gives the variational results for the nuclei, namely, $^4He$ and $^3H$ calculated using the two-body $NN$ Argonne $v_{18}$ interaction[5] and three-body $NNN$ Urbana $IX$ interaction[6,7] with relevant correlations. The numbers appearing in parentheses in all the tables in this work indicate the statistical error in the last digit. These calculations have been performed on similar lines as those by Wiringa et al.[5,19] and the results conform to theirs. These results also check very well with the recent calculations of Forest et al.[24], who use a truncated version of the Argonne $v_{18}$ interaction.

Next we calculate the energy expectation values for the $s$-shell hypernuclei, namely $^\Lambda_4H$, $^\Lambda_5H^*$, and $^\Lambda_5He$ using the two-body $NN$ Argonne $v_{18}$ and three-body $NNN$ Urbana $IX$ interactions along with the two-body $\Lambda N$ and three-body $\Lambda NN$ interactions with appropriate correlations incorporated in the wave function. Variational calculations have been performed for different values of spin-average potential strength, $\bar{V}$ (6.10, 6.15, and 6.20 MeV). The different values of the space-exchange parameter $\epsilon$ used in this work are 0.24 ($\bar{V}$=6.20 MeV), 0.19 ($\bar{V}$=6.15 MeV), and 0.14 ($\bar{V}$=6.10 MeV) [25].

The bulk calculations consist of the energy expectation values for each $\bar{V}$ as a function of the interaction parameters, $V_\sigma$, $C_p$, and $W_o$. In Table[4] we illustrate one such set of results for the ground state of $^\Lambda_4H$. Our results demonstrate that the $B_{\Lambda}$ values for $^\Lambda_5He$, $^\Lambda_5H$, and $^\Lambda_5H^*$ show similar trends with the spin-average potential strength $\bar{V}$ and $\Lambda NN$ interaction parameters, $C_p$ and $W_o$. As expected, $B_{\Lambda}$ increases with $\bar{V}$. $B_{\Lambda}$ also increases significantly with the increase in $C_p$, while it decreases with $W_o$. As expected, the dependence on $\bar{V}$, $C_p$, and $W_o$ is more pronounced for $A = 5$ than for $A = 4$ systems. This result is in accord with the earlier calculations where only simplified $NN$ interactions have been used[4].

An important goal of the present study is to learn about the role of $V_\sigma$, $C_p$, and $W_o$ through the $0^+ - 1^+$ energy splitting in $^\Lambda_4H$ and $^\Lambda_5H^*$. We place limits on the values of these parameters, consistent with the following experimental $B_{\Lambda}$ values:

$$B_{\Lambda}(^\Lambda_4H) = 2.22 \pm 0.04 MeV,$$
\[
B_\Lambda(\frac{3}{4}H^*) = 1.12 \pm 0.06 \text{MeV},
\]
\[
B_\Lambda(\frac{5}{4}He) = 3.12 \pm 0.02 \text{MeV}.
\]

Values of \(B_\Lambda(\frac{3}{4}H)\) and \(B_\Lambda(\frac{3}{4}H^*)\) are averages for those of \(\frac{3}{4}H\) and \(\frac{3}{4}He\). Limits on the parameters \(V_o\), \(C_p\), and \(W_o\) are determined by the uncertainties in the experimental \(B_\Lambda\) values.

For a given value of \(\bar{V}\) we did a \(\chi^2\) fit for the calculated energy expectation values according to the relation

\[
B_\Lambda(V_o, W_o, C_p) = y_1 V_o + y_2 W_o + y_3 C_p + y_4 W_o^2 + y_5 C_p^2 + y_6 W_o C_p + B_\Lambda^o,
\]

where \(B_\Lambda^o\) is the corresponding value of \(B_\Lambda\) for \(V_o = C_p = W_o = 0\) for each hypernuclear species. The coefficients \(y_1 \cdots y_6\) are varied to give a minimum in the \(\chi^2\) that is defined as

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{B_\Lambda(V_o, C_p, W_o) - B_\Lambda}{\Delta B_\Lambda} \right)^2.
\]

Here \(N\) is the total number of energy calculations for a particular hypernucleus with different values of \(V_o\), \(C_p\), and \(W_o\). \(B_\Lambda\) is the calculated value of the \(\Lambda\) separation energy and \(\Delta B_\Lambda\) is the corresponding Monte Carlo statistical error. The values of the coefficients \(y_1 \cdots y_6\), as determined from this procedure, are displayed in Table[5]. In all the cases considered the \(\chi^2\) values are \(\leq 1\) which demonstrates the goodness of the fit. There shall be correlated error bars on the coefficients \(y_1 \cdots y_6\) which would be reflected in the uncertainties in determining \(V_o\), \(C_p\), and \(W_o\). We find it more convenient to consider the uncertainties in the experimental values while placing limits on \(V_o\), \(C_p\), and \(W_o\). We hope to compensate some of the uncertainties associated with the \(y\)'s by giving a generous allowance to the experimental \(\Delta B_\Lambda\) values as well as by taking into account the Monte Carlo statistical errors in the calculation of the energies.

We use the coefficients \(y_1 \cdots y_6\) of Table[5] to obtain a fit with respect to the experimental \(B_\Lambda\) values, treating \(V_o\), \(C_p\), and \(W_o\) as parameters to determine the best fit. We again construct a \(\chi^2\) fit using Eq.(4.3) but now “\(N\)” refers to the factor “3” for the three hypernuclear species and \(B_\Lambda\) refers to the experimental \(B_\Lambda\) values. The \(\chi^2\) fit is minimized with respect to \(C_p\) and \(W_o\) for a given value of \(V_o\). In Fig. 1 we plot \(\chi^2\), \(C_p\), and \(W_o\) as a function of \(V_o\) for \(\bar{V} = 6.15\text{MeV}\). It is seen that both \(C_p\) and \(W_o\) decrease with increase in \(V_o\), the effect being more pronounced for \(C_p\).

Fig. 2 displays the calculated values of \(B_\Lambda\) as a function of \(V_o\) for \(\frac{3}{4}H\), \(\frac{3}{4}H^*\), and \(\frac{5}{4}He\). Within the accuracy of the graphs the \(B_\Lambda\) values for \(\frac{3}{4}H\) and \(\frac{5}{4}He\) do not show any dependence on \(V_o\) in the range 0.09–0.26 MeV. As one may expect, the \(B_\Lambda\) values for \(\frac{3}{4}H^*\) depend sensitively on \(V_o\), and thus in turn on the spin dependence of \(C_p\) and \(W_o\).

The \(\chi^2\) stays very close to zero (which corresponds to almost an exact fit to \(B_\Lambda\) (exp) values) for \(V_o = 0.176 \pm 0.015\text{MeV}, C_p = 1.64 \pm 0.03\text{MeV}\) and \(W_o = 0.026 \pm 0.001\text{MeV}\). Most of the deviation from zero of the \(\chi^2\) values in Fig. 1 arise from \(\frac{3}{4}H^*\). The dotted horizontal lines of Fig. 2 display the limits on the experimental \(B_\Lambda\) value of \(\frac{3}{4}H^*\), consistent with the experimental error bar of \(\pm 0.06\text{MeV}\). This places limits on the values of \(V_o\), thus, in turn on \(C_p\) and \(W_o\). However, the actual error bars on these parameters would be larger due to Monte Carlo statistical errors. To take this into account, we made a number of energy calculations for \(V_o\) in the range 0.10 to 0.24 MeV. The corresponding values of \(C_p\) and \(W_o\) have been taken from the calculations of Fig. 1. We could
obtain acceptable fits to the energies for $V_\sigma$ in the range 0.12–0.23 MeV. This gives for $\bar{V}=6.15$ MeV:

$$V_\sigma = 0.176 \pm 0.05, C_p = 1.64 \pm 0.15, W_o = 0.026 \mp 0.003. \quad (4.4)$$

A similar study for $\bar{V}=6.20$ MeV gives

$$V_\sigma = 0.125 \pm 0.05, C_p = 1.52 \pm 0.15, W_o = 0.025 \mp 0.003. \quad (4.5)$$

We have been able to obtain good fits only for $\bar{V} = 6.15$ and 6.20 MeV. The value of $\bar{V} = 6.10$ MeV does not reproduce the correct binding energies of $s$-shell hypernuclei. Therefore, we have not carried out any error analysis for $\bar{V}=6.10$ MeV. For the sake of completeness we mention the best parameter values for $\bar{V}=6.10$ MeV:

$$V_\sigma = 0.193, C_p = 1.84, W_o = 0.027. \quad (4.6)$$

We can note from Table[6(A)] that the $\Lambda N$ spin potential has a non zero contribution even in a closed-shell system such as $^4\Lambda$He. This arises because of the $\Lambda N$ spin-spin correlations incorporated in the wave function.

Comparing the results for the core nuclei (Table[3]) with the results for the hypernuclei (Tables[6(A-C)]) we note, in general, a shrinking of the core nuclei by about 20% in all the hypernuclei due to the presence of the $\Lambda$ particle. This decrease in radii of the core nuclei would imply that the lambda wave functions are closer for larger $A$. This also contributes to the fact that the dependence of $B_A$ on $\bar{V}$, $C_p$, and $W_o$ is more pronounced for the mass 5 than for the mass 4 hypernuclei. The change in the $d$-state probability is found to be small in all cases.

Table[7] gives the breakup of the $0^+ - 1^+$ splitting contributions in $^4\Lambda H$ and $^4\Lambda H^*$ arising from the $\Lambda N$ spin-dependent strength $V_\sigma$ and the three-body $\Lambda NN$ interaction $V_{\Lambda NN}$ for different values of spin-average strength $\bar{V}$. It can be noted from this table that the energy difference between $^4\Lambda H$ and $^4\Lambda H^*$ is consistent with the total contribution from $V_\sigma$ and $V_{\Lambda NN}$ within the error bars of the MC calculations. It can be seen that a large part ($\sim \frac{2}{3}$) of the splitting comes from the three-body $\Lambda NN$ potential. The two-body contribution arising from $V_\sigma$ is around $\sim \frac{1}{3}$ of the total splitting. This is in contrast to the earlier studies [4,28,29] wherein the $0^+ - 1^+$ splitting has been thought to have arisen mainly from the spin dependence of the two-body $\Lambda N$ potential. The present study clearly demonstrates that $V_{\Lambda NN}$ plays a significant role in explaining the splitting. This also results in a reduced $V_\sigma$ as compared to the value of 0.23$\pm$0.02 found in Ref. [4], though in our case the error bar on $V_\sigma$ is much larger due to reasons discussed earlier. In the present study for $V_\sigma=0.23$, half of the splitting arises because of $V_\sigma$ and the remaining half from the three-body $\Lambda NN$ forces. For the extreme case, in particular, for $V_\sigma=0.12$, the three-body forces contribute nearly $\frac{2}{3}$ of the total splitting. It would be desirable to have an independent fix on $V_\sigma$, for example, from a more refined $\Lambda p$ scattering data. This can enlighten us further on the three-body $\Lambda NN$ forces.

The Majorana space-exchange contribution for the various hypernuclei have been found to be small but significant in all $s$-shell hypernuclei. It is in the range 0.1 – 0.7 MeV for $\epsilon$ in the range 0.14 – 0.24 and as expected has a linear dependence with the Majorana exchange parameter $\epsilon$.

A few variational calculations have been carried out with only the $\Lambda NN$ potentials and no $\Lambda NN$ correlations. We find that without the $\Lambda NN$ correlations $^4\Lambda He$ is not bound. We also notice that the
contributions from $V^{2s}_{\Lambda NN}$ and $V^{DS}_{\Lambda NN}$ become more repulsive without the correlations and the total contribution from the three-body $\Lambda NN$ potentials become positive thereby decreasing $B_{\Lambda}$. We have performed a few energy calculations for $\frac{1}{2}H$, $\frac{1}{2}H'$, and $\frac{1}{2}H''$ but with the three-body $\Lambda NN$ part of the Hamiltonian completely switched off. We find in this case that $\frac{1}{2}H$ is overbound by about 2.34 MeV for $\tilde{V} = 6.20$ MeV and by about 1.39 MeV for $\tilde{V} = 6.15$ MeV. In general, the results for $\frac{1}{2}H$ show that it is underbound while $\frac{1}{2}H''$ is overbound without the three-body $\Lambda NN$ interactions for all values of $\tilde{V}$. Both these studies show the importance of the three-body $\Lambda NN$ potentials and correlations in obtaining a consistent fit to the $B_{\Lambda}$ values for all the $s$-shell hypernuclei considered here. In particular, we notice the importance of the three-body $\Lambda NN$ potentials and correlations in obtaining a consistent fit to the $B_{\Lambda}$ values for all the $s$-shell hypernuclei considered here. In particular, we notice the importance of the three-body $\Lambda NN$ potentials and correlations in obtaining a consistent fit to the $B_{\Lambda}$ values for all the $s$-shell hypernuclei considered here.

Implications of $s$-shell results on $^{17}_A O$: We now examine the $^{17}_A O$ hypernucleus in relation to the two- and three-body $\Lambda$-nuclear potential parameters that we find from our analysis of $s$-shell hypernuclei as described earlier. Usmani, Pieper, and Usmani\textsuperscript{[16]} (referred to as UPU) have carried out MC calculations for the $^{17}_A O$ hypernucleus using the $v_6$ part of the older Argonne $v_{14}$ potential \textcite{9}. For the three-nucleon potential they use the same form (Urbana model) as in the present study but with different strength parameters ($A_o = -0.0333$ and $U_o = 0.0038$). Their trial wave function consists of pair and triplet operators acting on a single-particle determinant. In many respects, these calculations are similar to our present calculations. The difference lies in the treatment of non-central correlations for which they use the Cluster Monte Carlo method with up to four-baryon clusters. The central correlations are treated exactly. We carry out this study in the hope of analyzing further our estimated $s$-shell $\Lambda$-nuclear parameters. UPU have given the following empirical relation for $C_p$ in the range 0-1 MeV and $W_o$ in the range 0-0.02 MeV:

$$B_{\Lambda} = 27.3 - 8.9 C_p + 11.2 C_p^2 + 870.0 W_o.$$  \hspace{1cm} (4.7)

This equation relates the $B_{\Lambda}$ of $^{17}_A O$ with $C_p$ and $W_o$. In order to test the consistency of our results with $B_{\Lambda}(^{17}_A O)$, we assume that relation (4.7) holds for our values of $C_p$ and $W_o$. UPU have done calculations for spin-average strength $\tilde{V} = 6.16$ MeV with space-exchange parameter $\epsilon = 0.3$. Our values of $\tilde{V} = 6.15$ MeV with $\epsilon = 0.17$ are closest to theirs. We thus need to modify (4.7) for our values of $\tilde{V}$ and $\epsilon$. Unfortunately, there is no simple method to scale relation (4.7) for $\tilde{V} = 6.15$ MeV, as the scaling can be considerably non-linear. However, since the two values of $\tilde{V}$ are very close, we assume that this will not affect the results much. The correction for $\epsilon$ is simple, since in the absence of space-exchange correlation the space-exchange energies are expected to be linear with $\epsilon$. Thus, relation (4.7) can be modified as:

$$B_{\Lambda} = 27.3 + (\epsilon - \epsilon_o)(1 - P_x) V_{\Lambda N} - 8.9 C_p + 11.2 C_p^2 + 870.0 W_o ,$$ \hspace{1cm} (4.8)

where $\epsilon_o = 0.3$ as taken by UPU, $P_x$ is the space-exchange operator and $\langle V_{\Lambda N} \rangle$ is the energy expectation value of the $\Lambda N$ potential.

Using the entries for $v_o(r)(1-\epsilon)$ and $v_o(r)\epsilon P_x$ from Table II of Ref. \textcite{16} and using our values of $C_p = 1.6407$ and $W_o = 0.0255$ for $\tilde{V} = 6.15$ MeV we obtain

$$B_{\Lambda} = 23.3 \pm 1.6 \text{ MeV}.$$
This is considerably larger than the empirical estimate\textsuperscript{16} $\sim 13.0 \pm 0.4$ MeV. Thus the results of the present study are incompatible with those of Ref. \textsuperscript{16}. The reason for this incompatibility may largely lie in the use of $\nu_6$ part of the $\nu_{18}$ Hamiltonian for $^{17}_\Lambda O$. Another reason can be attributed to the use of relations (4.7) and (4.8) for large values of $C_p$ and $W_o$ for which these relations may not be adequate. This discrepancy can probably be resolved by carrying out calculations for $^{17}_\Lambda O$ with Argonne $\nu_{18}$ Hamiltonian, which, at the moment is an extremely challenging task.

**Implications of s-shell results on $\Lambda$ binding to nuclear matter:** The presence of a $\Lambda$ particle inside nuclear matter can reveal information on the $\Lambda$-nuclear interactions. The well depth $D$ is identified with the separation energy for a $\Lambda$ in nuclear matter. It is an important parameter which can help to distinguish between different $\Lambda N$ potentials and also throw light on the $\Lambda NN$ interaction. $\Lambda$ binding to nuclear matter can put further constraints on the potential parameters, namely $\bar{V}$, $V_\sigma$, $W_o$, and $C_p$. With this aim in mind we have performed calculations for $D$ using the Fermi-Hypernetted-Chain (FHNC) technique\textsuperscript{4} to calculate the energy expectation values.

We have calculated the well depth $D$ variationally using the same underlying principle as for our s-shell hypernuclei. Our discussion on $D$ is based on the results given in Table\textsuperscript{8}. The empirical value of $D$ is now fairly well established at $29 \pm 1$ MeV \textsuperscript{30,31} at the normal nuclear matter density of $\rho_o \approx 0.16 \text{ fm}^{-3}$. These results clearly indicate that $\Lambda$ is underbound at the normal nuclear matter density, $\rho_o \approx 0.16 \text{ fm}^{-3}$. This indeed is a disturbing feature. Bodmer and Usmani\textsuperscript{4} have found that it is possible to obtain a consistent phenomenology with hypernuclear interactions which include the s-shell and the medium and heavy hypernuclei as well as $\Lambda$ binding to nuclear matter. Our results indicate that with the present available techniques of treating the nuclear matter this is not possible. The resolution of this paradox perhaps lies in the proper handling of the three-body correlations, particularly the $\Lambda NN$ correlations for nuclear matter. It may be noted that the contribution from the TPE $\Lambda NN$ forces $V_{2\pi}^{\Lambda NN}$ for nuclear matter is always positive\textsuperscript{4}. On the other hand, $V_{2\pi}^{\Lambda NN}$ for s-shell hypernuclei and $^{17}_\Lambda O$ is always negative and substantial. The three-body $\Lambda NN$ correlations that are taken in nuclear matter calculations always pertain to the shell\textsuperscript{4}. The reason for adopting this correlation lies in its simplicity. At present the techniques for incorporating the realistic three-body $\Lambda NN$ correlations for nuclear matter are not sufficiently developed as compared to those for the s-shell hypernuclei incorporated in this work. These affect the contribution from $V_{2\pi}^{\Lambda NN}$ quite substantially, even to the extent of reversing its sign in the presence of the $\Lambda NN$ correlations as can be seen in the present as well as in the $^{17}_\Lambda O$ studies. The correct incorporation of the three-body correlations in nuclear matter may affect the results to quite an extent, particularly those at high densities\textsuperscript{32}. The incorporation of these correlations is indeed a challenging task and is very much needed for the present work as well as for other related studies.

§5. **Conclusion and Comments**

From the results discussed in the previous section we note that the values of the spin-average strength $\bar{V} = 6.20$ and 6.15 MeV give a reasonable description of the s-shell hypernuclei. We
have been able to provide a consistent account of the $B_\Lambda$ values of $^4\Lambda H$, $^4\Lambda H^*$, and $^5\Lambda He$ using
the realistic Argonne $\upsilon_{18}$ NN interaction and Urbana IX NNN interaction alongwith $\Lambda$-nuclear
interactions with appropriate correlations. Our results for $B_\Lambda$ show very similar trends with the
spin-average strength $\bar{V}$ of the $\Lambda N$ interaction, and the $\Lambda NN$ interaction parameters, $C_p$ and $W_o$
for all the $s$-shell hypernuclei considered.

An important conclusion of our study is that about 25% to 50% of the $0^- - 1^+$ splitting
energy between the $^4\Lambda H$ and $^4\Lambda H^*$ comes from the $\Lambda N$ spin-dependent strength $V_\sigma$. The earlier
studies [4,28,29] attribute a larger part of the splitting to the spin dependence of the two-body $\Lambda N$
interactions. In contrast, our study indicates that the major part ($\sim$ 50% to 75%) of the splitting
is generated by the three-body $\Lambda NN$ forces.

Our study on $\Lambda$-binding to nuclear matter shows that $\Lambda$ is underbound. This indicates the
fact that there is a need to include the three-body correlations while treating nuclear matter.
This would require a different technique altogether and is a challenging problem in itself. Our
analysis on $^{17}\Lambda O$ also indicate the importance of the non-central correlations. It is possible that
the inconsistency between the results of Ref. [16] and our $s$-shell results is due to their neglecting
terms with higher than four-baryon clusters and thereby neglecting the contributions from
the non-central clusters. Moreover, our values of the $\Lambda NN$ interaction parameters, $C_p$ and $W_o$
are higher than those of Ref. [16] and which, in turn, would induce stronger $\Lambda NN$ correlations.

Contrary to the findings by Bando et al. [33] and Shinmura et al. [34] regarding the effect of
tensor forces on the overbinding problem, we find that the tensor forces do not play a significant
role. Further, separate studies by Carlson[28] and Hiyama et al. [29] and on four- and five-body
hypernuclei have shown that the binding energies and the splitting energies are not reproduced
correctly using the Nijmegen interactions which have strong tensor terms. However, the small
suppression effects expected from $\Lambda N$ tensor forces are already implicitly included in our phe-
nomenological dispersive $\Lambda NN$ force. Moreover, the Argonne $\upsilon_{18}$ potential used in our study has
a weak tensor part and this in fact, provides a much better binding to nuclei[5].

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### TABLE 1. \( \Lambda N \) Correlations Parameters

| \( \Lambda Z \) | \( V \) | \( \kappa_{AN} \) | \( a_{AN} \) | \( C_{AN} \) | \( R_{AN} \) | \( \alpha_s \) | \( \alpha_s^\dagger \) |
|----------------|-------|---------------|-------------|-------------|-------------|-------------|-------------|
| \( ^5\Lambda He \) | 6.20  | 0.117         | 0.50        | 2.0         | 1.0         | 0.965       | 0.80        |
|                | 6.15  | 0.110         | 0.50        | 2.0         | 1.0         | 0.970       | 0.95        |
|                | 6.10  | 0.095         | 0.50        | 2.0         | 1.0         | 0.940       | 0.95        |
| \( ^4\Lambda H \) | 6.20  | 0.12          | 0.70        | 2.0         | 1.0         | 0.95        | 0.70        |
|                | 6.15  | 0.10          | 0.70        | 2.0         | 1.0         | 0.95        | 1.20        |
|                | 6.10  | 0.08          | 0.70        | 2.0         | 1.0         | 0.95        | 0.70        |
| \( ^4\Lambda H^* \) | 6.20  | 0.095         | 0.70        | 2.0         | 1.0         | 1.0         | 0.70        |
|                | 6.15  | 0.065         | 0.70        | 2.0         | 1.0         | 1.0         | 0.70        |
|                | 6.10  | 0.050         | 0.70        | 2.0         | 1.0         | 1.0         | 0.70        |

\( \dagger \) Eq.(3.8); For all other correlation parameters refer to 16)

### TABLE 2. \( \Lambda NN \) Correlation Parameters for \( ^5\Lambda He \), \( ^4\Lambda H \), and \( ^4\Lambda H^* \)

| Parameters | \( V=6.20 \) MeV | \( V=6.15 \) MeV | \( V=6.10 \) MeV |
|------------|------------------|------------------|------------------|
| \( \hat{\delta}_1^\dagger \) | 0.364104         | 0.311733         | 0.257894         |
| \( \hat{\delta}_2^\dagger \) | 0.006096         | 0.004845         | 0.003766         |

\( \dagger \) Eq.(3.11)

### TABLE 3. Variational results for \( ^3H \) and \( ^4He \)

| Components | \( ^3H \)     | \( ^4He \)     |
|------------|---------------|---------------|
| Kinetic energy | 50.76(4)     | 106.85(6)     |
| NN Potential energy | -57.95(4)   | -129.30(6)    |
| NNN Potential energy | -1.13(3)     | -5.27(7)      |
| Total energy   | -8.32(2)     | -27.71(6)     |
| r.m.s.(proton) | 1.585(3)      | 1.478(2)      |
| r.m.s.(neutron) | 1.731(4)     | 1.478(2)      |
| d-state probability | 0.0933(1) | 0.1512(2) |

All Energies are in MeV and Radii in fm
TABLE 4. Energy expectation values (calculated and fitted) for $^{14}H$(Ground State) with $\bar{V}=6.20$ MeV

| $V_\sigma$ | $W_\sigma$ | $C_\rho$ | $<E_{cal}>$ | $<E_{fit}>$ |
|------------|------------|---------|------------|------------|
| 0.17       | 0          | 0       | 10.58(05)  | 10.60      |
| 0.17       | 0          | 1.0     | 11.61(07)  | 11.63      |
| 0.17       | 0.01       | 0       | 9.97(05)   | 9.97       |
| 0.17       | 0.01       | 1.0     | 10.96(05)  | 10.90      |
| 0.17       | 0.02       | 0       | 9.25(04)   | 9.31       |
| 0.17       | 0.02       | 1.0     | 10.21(05)  | 10.13      |
| 0.17       | 0.02       | 2.0     | 12.28(07)  | 12.31      |
| 0.17       | 0.005      | 0       | 10.23(05)  | 10.29      |
| 0.17       | 0.005      | 1.0     | 11.25(05)  | 11.27      |
| 0.17       | 0.005      | 2.0     | 13.58(08)  | 13.60      |
| 0.17       | 0.015      | 1.0     | 10.50(05)  | 10.52      |
| 0.22       | 0          | 0       | 10.82(05)  | 10.71      |
| 0.22       | 0          | 1.0     | 11.66(05)  | 11.74      |
| 0.22       | 0.01       | 0       | 10.02(05)  | 10.09      |
| 0.22       | 0.01       | 1.0     | 10.90(05)  | 11.01      |
| 0.22       | 0.01       | 2.0     | 13.36(08)  | 13.30      |
| 0.22       | 0.02       | 0       | 9.43(05)   | 9.43       |
| 0.22       | 0.02       | 1.0     | 10.21(05)  | 10.25      |
| 0.22       | 0.02       | 2.0     | 12.35(06)  | 12.43      |
| 0.22       | 0.005      | 1.0     | 11.47(05)  | 11.38      |
| 0.22       | 0.015      | 1.0     | 10.68(05)  | 10.63      |
| 0.27       | 0          | 0       | 10.79(05)  | 10.83      |
| 0.27       | 0          | 1.0     | 11.79(06)  | 11.86      |
| 0.27       | 0          | 2.0     | 14.32(08)  | 14.24      |
| 0.27       | 0.01       | 0       | 10.26(05)  | 10.20      |
| 0.27       | 0.01       | 1.0     | 11.13(05)  | 11.12      |
| 0.27       | 0.01       | 2.0     | 13.41(08)  | 13.41      |
| 0.27       | 0.02       | 0       | 9.60(05)   | 9.55       |
| 0.27       | 0.02       | 1.0     | 10.39(05)  | 10.36      |
| 0.27       | 0.02       | 2.0     | 12.56(06)  | 12.54      |
| 0.27       | 0.005      | 0       | 10.50(05)  | 10.52      |
| 0.27       | 0.005      | 1.0     | 11.46(05)  | 11.49      |
| 0.27       | 0.015      | 1.0     | 10.81(05)  | 10.75      |

$\epsilon=0.24$, $\kappa_{AN}=0.12$, $\alpha_s=0.95$, $\alpha_\sigma=0.7$
TABLE 5. $B_\Lambda$ as a function of coefficients $y^*_1$–6

| $\Lambda^A Z$ | $V$   | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |
|--------------|-------|-------|-------|-------|-------|-------|-------|
| $^5\Lambda He$ | 6.20  | -0.92 | -285.51 | 1.28 | 1701.78 | 1.50 | -37.62 |
|              | 6.15  | 1.19  | -280.03 | 0.99 | 1134.41 | 1.35 | -7.44  |
|              | 6.10  | -0.20 | -210.02 | 0.88 | 2997.79 | 1.19 | -44.03 |
| $^4\Lambda H$ | 6.20  | 2.27  | -61.23  | 0.35 | -141.55 | 0.68 | -10.53 |
|              | 6.15  | 2.21  | -59.03  | 0.32 | 453.47  | 0.52 | -8.14  |
|              | 6.10  | 1.55  | -43.48  | 0.23 | 298.23  | 0.37 | -4.96  |
| $^4\Lambda H^*$ | 6.20 | -0.89 | -112.62 | 0.37 | 639.67  | 0.68 | -8.51  |
|              | 6.15 | -0.98 | -65.63  | 0.27 | 208.58  | 0.45 | -6.68  |
|              | 6.10 | -0.81 | -53.21  | 0.12 | 334.70  | 0.36 | -5.69  |

† Includes contribution due to $\Lambda$-nuclear correlations

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TABLE 6(A). Variational Results for $^5\Lambda He$

| Components                  | $V = 6.20$       | $V = 6.15$       | $V = 6.10$       |
|-----------------------------|------------------|------------------|------------------|
| Nuclear Kinetic energy†     | 128.38(74)       | 128.16(71)       | 125.35(69)       |
| $NN$ Potential energy†      | -139.97(73)      | -140.27(69)      | -139.31(67)      |
| $NNN$ Potential energy      | -5.97(8)         | -5.91(8)         | -5.73(8)         |
| $\Lambda$ Kinetic energy    | 11.64(15)        | 11.11(15)        | 9.13(13)         |
| $\Lambda N$ P.E (central)  | -23.65(32)       | -21.69(30)       | -17.55(27)       |
| $\Lambda N$ P.E (spin)     | -0.0138(1)       | -0.0311(3)       | -0.0281(3)       |
| $\Lambda N$ Space exch. contribution | 0.763(13)     | 0.578(10)        | 0.395(7)         |
| $\Lambda NN$ P.E (Total)   | -1.91(9)         | -2.67(10)        | -3.53(10)        |
| $\Lambda NN$ P.E (T.P.E.)  | -8.43(16)        | -8.77(17)        | -8.20(17)        |
| $\Lambda NN$ P.E (dispersive) | 6.52(11)       | 6.10(11)         | 4.67(10)         |
| Total Energy               | **-30.75(14)**  | **-30.76(18)**  | **-31.27(12)**  |
| $B_\Lambda$                | **3.03(15)**    | **3.05(19)**    | **3.56(13)**    |
| r.m.s. radius (proton)     | 1.376(2)         | 1.379(2)         | 1.389(2)         |
| r.m.s. radius (neutron)    | 1.377(2)         | 1.379(2)         | 1.389(2)         |
| d-state probability        | 0.1568(2)        | 0.1568(2)        | 0.1557(2)        |

† Includes contribution due to $\Lambda$-nuclear correlations
| Components                  | $V = 6.20$     | $V = 6.15$     | $V = 6.10$     |
|-----------------------------|----------------|----------------|----------------|
| Nuclear Kinetic energy†     | 65.76(47)      | 62.82(47)      | 60.34(46)      |
| $NN$ Potential energy†      | -66.91(47)     | -65.10(46)     | -63.60(46)     |
| $NNN$ Potential energy      | -1.33(3)       | -1.25(3)       | -1.19(3)       |
| $\Lambda$ Kinetic energy   | 7.17(11)       | 5.99(9)        | 4.77(9)        |
| $\Lambda N$ P.E (central)  | -14.00(22)     | -11.55(20)     | -8.84(18)      |
| $\Lambda N$ P.E (spin)     | -0.291(3)      | -0.363(4)      | -0.300(4)      |
| $\Lambda N$ Space exch. contribution | 0.333(9) | 0.221(6)       | 0.143(4)       |
| $\Lambda NN$ P.E (Total)   | -1.33(6)       | -1.36(6)       | -1.48(6)       |
| $\Lambda NN$ P.E (T.P.E.)  | -2.87(8)       | -2.63(9)       | -2.47(9)       |
| $\Lambda NN$ P.E (dispersive) | 1.54(4) | 1.27(4)        | 0.99(3)        |
| **Total Energy**            | **-10.61(6)**  | **-10.59(5)**  | **-10.16(5)**  |
| $B_{\Lambda}$               | **2.28(6)**    | **2.27(6)**    | **1.83(6)**    |
| r.m.s. radius (proton)      | 1.408(2)       | 1.435(3)       | 1.460(3)       |
| r.m.s. radius (neutron)     | 1.516(3)       | 1.547(3)       | 1.577(3)       |
| d-state probability         | 0.0984(1)      | 0.0960(1)      | 0.0962(1)      |

† Includes contribution due to $\Lambda$-nuclear correlations

| Components                  | $V = 6.20$     | $V = 6.15$     | $V = 6.10$     |
|-----------------------------|----------------|----------------|----------------|
| Nuclear Kinetic energy†     | 63.78(49)      | 60.58(47)      | 57.96(46)      |
| $NN$ Potential energy†      | -65.71(48)     | -63.64(46)     | -62.36(45)     |
| $NNN$ Potential energy      | -1.34(3)       | -1.27(3)       | -1.20(3)       |
| $\Lambda$ Kinetic energy   | 6.40(12)       | 4.78(9)        | 3.4516(8)      |
| $\Lambda N$ P.E (central)  | -12.35(25)     | -9.42(19)      | -6.56(17)      |
| $\Lambda N$ P.E (spin)     | 0.086(1)       | 0.083(1)       | 0.065(1)       |
| $\Lambda N$ Space exch. contribution | 0.311(8) | 0.202(6)       | 0.110(4)       |
| $\Lambda NN$ P.E (Total)   | -0.57(6)       | -0.62(5)       | -0.73(5)       |
| $\Lambda NN$ P.E (T.P.E.)  | -3.11(9)       | -2.32(8)       | -1.98(8)       |
| $\Lambda NN$ P.E (dispersive) | 2.54(7) | 1.70(5)        | 1.24(5)        |
| **Total Energy**            | **-9.40(7)**   | **-9.31(5)**   | **-9.26(5)**   |
| $B_{\Lambda}$               | **1.08(7)**    | **0.99(5)**    | **0.94(5)**    |
| r.m.s. radius (proton)      | 1.431(3)       | 1.467(3)       | 1.494(3)       |
| r.m.s. radius (neutron)     | 1.542(3)       | 1.585(3)       | 1.618(3)       |
| d-state probability         | 0.0995(1)      | 0.0980(1)      | 0.0969(1)      |

† Includes contribution due to $\Lambda$-nuclear correlations
TABLE 7. Breakup of the $0^+ - 1^+$ splitting contributions

| Contribution | $V = 6.20$ | $V = 6.15$ | $V = 6.10$ |
|--------------|-----------|-----------|-----------|
| $V_\sigma$   | 0.377(3)  | 0.446(4)  | 0.365(4)  |
| $V_{\Lambda NN}$ | 0.76(8)  | 0.74(8)  | 0.75(8)  |
| Total        | 1.137(80)| 1.186(80)| 1.115(89)|
| Energy differences | 1.21(9)  | 1.28(7)  | 0.90(7)  |

The first row gives the contribution to splitting from $V_\sigma$. The second row gives the contribution arising from $V_{\Lambda NN}$. The third row gives the total of $V_\sigma$ and $V_{\Lambda NN}$. The last row gives the actual calculated energy difference between $^4\Lambda H$ and $^4\Lambda H^\ast$.

TABLE 8. Results for Nuclear Matter calculations

| $V$  | $\rho_o$ | $-D$    | $<T_\Lambda + V_{\Lambda N}>$ | Spc. exch. | $V_{\Lambda NN}$ |
|------|----------|---------|-------------------------------|------------|-----------------|
| 6.20 | 0.162    | -21.525 | -77.617                       | 8.561      | 47.530          |
| 6.15 | 0.162    | -17.819 | -72.628                       | 6.644      | 48.164          |
| 6.10 | 0.162    | -11.727 | -67.729                       | 4.798      | 51.205          |

All energies are in MeV. $\rho_o$ is the normal nuclear matter density in fm$^{-3}$. The third column gives the well depth $D$. The fourth column gives the value of $D$ without the $\Lambda NN$ forces and the space-exchange contribution, i.e. for $\epsilon = 0$. The fifth column gives the reduction in the contribution to $D$ due to the space-exchange part (Spc. exch.) of the $\Lambda N$ potential. The last column gives the contribution due to the three-body $\Lambda NN$ forces.
Figure 1. $\chi^2$, $C_{\rho}$, and $W_\rho$ as a function of $V_\rho$. 

$\frac{M_0 V}{\chi^2}$ vs $V_\rho$ (MeV)
Figure 2. $B_{\Lambda}$ as a function of $V_{\sigma}$

$\Lambda^3 He$

$\Lambda^4 H$

$\Lambda^4 H^*$