Lattice-QCD simulations have shown that the transition of deconfinement in quantum chromodynamics (QCD) at vanishing baryon chemical potential \( \mu_B \) is crossover \([1]\). There has been much speculation that the crossover becomes a true first-order phase transition for larger values of \( \mu_B \). This suggests that the QCD phase diagram can exhibit a critical endpoint where the line of first order transition matches that of second order or analytical crossover \([2]\).

Chiral symmetry restoration is another QCD originated phase transition. It has been shown that the transition for \( \mu_B = 0 \) is crossover \([3]\). So there could also be a chiral critical endpoint in phase diagram. But it is unclear if the critical temperature of chiral symmetry restoration is above \([4]\), or equal to \([5]\), or below \([6]\) that of the deconfinement.

Locating the critical endpoints of QCD phase transitions by lattice calculation is still a formidable challenge. But if the critical endpoint is in the region accessible to current relativistic heavy ion collisions, it should be discovered experimentally.

Most of the current signatures for finding the critical point are focused on the anomalous, or non-monotonous, behavior of the observable at various incident energies \([7]\). The argument is that in infinite system, the correlation length \( \xi \) diverges when approaching the critical point. The contribution of this singularity to the observable is supposed to be proportional to \( \xi^\nu \). However, the data from RHIC and SPS in more than a decade accumulation show no sign of anomalous behavior as a function of \( \sqrt{s} \) \([8]\).

In relativistic heavy ion collisions, two nuclei move with relativistic velocity and collide as two contracted pancakes. More central collision makes overlapped area larger. It is just because the large number of strongly interacting nucleons in more central nuclear collisions make the transition between hadron and quark-gluon plasma possible. The centrality (or the system size) dependence of the observable is noticeable \([9]\).

Due to the finite size of system, no divergence can be practically observed at critical point. The physical quantities, which are divergent in infinite system, become finite and have a maximum, i.e., so called non-monotonous behavior. However, the position of the maximum changes with system size and deviates from the true critical point.

The appearance of non-monotonous behavior is not always associated with critical point. Taking one-dimensional Ising model as an example, there is no critical point in this model, but its specific heat in a finite system has non-monotonous behavior.

Moreover, the absence of non-monotonous behavior does not mean no critical point. The physical quantities like order parameter, which are finite in infinite system, have a monotonous behavior near critical point in a finite system \([10]\). Therefore, non-monotonous behavior is not a reliable criterion for the critical point of finite system.

An effective identification of critical point of finite system is the finite-size scaling, which was proposed from phenomenological \([11]\) and renormalization-group \([12]\) theories, and was approved by the Monte Carlo results of finite systems in different universal classes \([13]\).

In this letter, we first propose how to locate critical point by finite-size scaling. Then the data of \( p_t \) correlation at 6 centralities and 4 incident energies from RHIC/STAR are analyzed. The behavior of critical point is likely observed around \( \sqrt{s} = 62 \) and 200 GeV. Finally, we suggest how to confirm the findings and precisely locate the critical point in coming experimental study.

The main points of finite-size scaling can be described as the following. An observable \( Q \) of finite system is a function of temperature \( T \) and system size \( L \). When \( L \) is much larger than the microscopic length scale and \( T \) is in the vicinity of critical point \( T_c \), the observable \( Q(T, L) \) can be written in a finite-size scaling form \([11-13]\),

\[
Q(T, L) = L^{\lambda/\nu} F_Q(t L^{1/\nu}).
\]  

(1)

\( t = (T - T_c)/T_c \) is the reduced temperature and \( \lambda \) is the critical exponent of the observable. \( \nu \) is the critical exponent of the correlation length \( \xi = \xi_0 t^{-\nu} \).

Finite-size scaling not only characterizes the scaling behavior of thermodynamic quantities of finite system near critical point, but also provides criterion for locating
the critical point. At critical point $T = T_c$, the finite-size scaling function $F_Q$ in Eq. (1) becomes

$$F_Q(0) = Q(T_c, L)L^{-\lambda/\nu},$$

(2)

which is constant and independent of system size $L$. In the plot of $Q(T, L)L^{-\lambda/\nu}$ vs $T$, the critical point $[T_c, F_Q(0)]$ is a fixed point, where all curves of different system sizes converges to. Reversely, the appearance of fixed point indicates the existence of a critical point.

If the critical exponent $\lambda = 0$, like Binder cumulant ratio $[14]$, the fixed point can be obtained directly from the temperature dependence of this observable at different system sizes. This is why Binder cumulant ratio has been used very widely in determining critical point of finite-size system.

If the critical exponent $\lambda \neq 0$ and is unknown, the fixed point can be found by investigating the temperature dependence of $Q(T, L)L^{-\alpha}$ at different system sizes. When a fixed point is observed at a certain parameter $a_0$, it indicates the existence of a critical point and the parameter $a_0$ is related to the ratio of critical exponents, i.e., $\lambda/\nu = a_0$.

The critical point can also be found directly from the system size dependence of the observable. Taking logarithm in the both sides of Eq. (1), it becomes

$$\ln Q(T, L) = \lambda/\nu \ln L + \ln F_Q(t L^{1/\nu}).$$

(3)

At critical point $t = 0$, the second term of Eq. (3) becomes a constant and $\ln Q(T_c, L)$ becomes a straight line with respect to $\ln L$. If system is away from the critical point, the second term of Eq. (3) is no longer a constant. It gives an additional size dependent contribution to the observable and makes $\ln Q(T, L)$ deviate from the straight line with respect to $\ln L$.

It is found recently that the finite-size scaling holds not only for thermodynamic quantities like order-parameter, susceptibility, and so on, but also for various cluster sizes [15] and their fluctuations [16]. Therefore, the finite-size scaling of various critical related observable could be used to identify critical point and its critical exponents.

In relativistic heavy ion collision, correlation and fluctuation of final state particles is regarded as critical related observable [17]. Although much attention have been drawn in measuring them, but influence of system size has been neglected. The available data for system size study is very few. The $p_t$ correlation at Au+Au collisions from RHIC/STAR [9] is the only data which can be used for the analysis, where the centrality dependence of $p_t$ correlation at 4 incident energies are well presented [9]. But the errors of the data at $\sqrt{s} = 20$ GeV are much larger than that at other collision energies. The $p_t$ correlation is defined as

$$P(\sqrt{s}, L) = \frac{1}{N_e} \sum_{k=1}^{N_e} \sum_{i=1}^{N_t} \left( \sum_{j=1, j \neq i}^{N_t} (p_{t,i} - \langle p_{t} \rangle)(p_{t,j} - \langle p_{t} \rangle) \right) N_k(N_k - 1),$$

(4)

$N_e$ is the number of event, $p_{t,i}$ is the transverse-momentum of the $i$th particle in each event, and $N_k$ is the number of particles in the $k$th event. $\langle \ldots \rangle$ is the average over event sample.

Collision energy is the controllable condition. Here we let it play the role of temperature in the analysis of finite-size scaling. The size of the formed matter is mainly limited by the size of overlapping transverse region, which is proportional to the number of participant nucleons and is quantified as centrality. So the initial mean size of the formed matter can be approximately estimated by the square root of participants, $\sqrt{N_{\text{part}}}$. We choose dimensionless (or relative) size,

$$L = \sqrt{N_{\text{part}}}/\sqrt{2N_A},$$

(5)

as scaled mean size of initial system, where $N_A$ is the number of nucleons of incident nucleus. The system size at transition should be a monotonically increasing function of $L$. The position of critical point is insensitive to the concrete form of this function, but only the critical exponents changes with it. As the first step, the system size at transition is assumed to be proportional to $L$.

In the case of a few critical points, the finite-size scaling of $p_t$ correlation in the vicinity of each critical collision energy $\sqrt{s}_{c,i}(i = 1, 2, \ldots)$ can be written as

$$P(\sqrt{s}, L) = L^{\lambda/\nu} F_{P_{t,i}}(e_i L^{1/\nu}).$$

(6)

e_i = (\sqrt{s} - \sqrt{s}_{c,i})/\sqrt{s}_{c,i}$ is the reduced collision energy at $i$th critical point, which is unknown in priori. $\lambda_i$ is the $i$th critical exponent of $p_t$ correlation. In the following, we demonstrate how to locate the critical point by the data of $p_t$ correlation from RHIC/STAR.

Firstly, we change the centrality dependence of $p_t$ correlation at different collision energies in Ref. [9] to the collision energy dependence at different sizes (or centralities). The results are shown in Fig. 1(a). Since in the most peripheral collisions, the size of the formed matter is too small to be inside the asymptotic region of finite-size scaling, we choose six centralities at mid-central and central collisions to do the analysis. The sizes corresponding to the 6 centralities are indicated in the legend of Fig. 1(a). It is clear that at a given collision energy, the correlation strength increases with the decrease of system size. The influence of finite size is obvious.

If critical collision energy of QCD phase transition is in the range of incident energy at RHIC, the behavior of fixed point should be observable. So we multiply $P(\sqrt{s}, L)$ by a size factor $L^{-\alpha}$ with different $\alpha$ to see how it changes with the system size $L$. Varying $-a$ from small to large, it is interesting to see that at collision energy $\sqrt{s} = 62$ GeV, all points of different sizes move firstly toward each other, then well converge at $-a_{0.1} = 2.09$, and finally move again apart from each other. The corresponding steps and typical $a$ values are presented in Fig. 1(b), (c), and (d) respectively, where the errors in each sub-figures come from the measure of $P(\sqrt{s}, L)$ only, and the errors of $N_{\text{part}}$ are not included.
The energy dependence of $p_t$ correlations at different sizes $L$ (or centralities). Data come from RHIC/STAR. (b), (c) and (d) are $p_t$ correlation multiplied by the factor, $L^{-a}$, with $-a = 1.0$, $2.09$ and $4$, respectively.

**TABLE I: Parameters of parabola fits.**

| $\sqrt{s}$ (GeV) | 20  | 62  | 130 | 200 |
|-------------------|-----|-----|-----|-----|
| $c_2$             | 1.86± 0.93 | 0.6 ± 0.09 | 1.56± 0.41 | 0.77± 0.1 |
| $c_1$             | 3.9± 0.89 | 2.59± 0.09 | 3.43± 0.41 | 2.74± 0.1 |

At $\sqrt{s} = 200$ GeV, the points of different sizes show the same behavior and best converge at $-a_{0.2} = 2.08$. While in the whole process, the points of different sizes at energies $\sqrt{s} = 20$ (or 130) GeV never move close to each other as those at $\sqrt{s} = 62$ (or 200) GeV do. So there are likely two fixed points around $\sqrt{s} = 62$ and 200 GeV.

In order to confirm the position of fixed points, we study the $\ln L$ dependence of $\ln P(\sqrt{s}, L)$ for four incident energies, respectively. A parabola fit, $c_2(\ln L)^2 + c_1 \ln L + c_0$, is used at each collision energy. The better straight-line behavior results in smaller $|c_2|$, and larger ratio of $|c_1/c_2|$. The fit parameters, $c_2$ and $c_1$, for 4 collision energies are listed in Tab. 1. It shows that the better straight-line behavior happen to be at $\sqrt{s} = 62$ and 200 GeV, which are the same collision energies of fixed points found above. The data at these two energies can be well fitted, respectively, by the straight lines with slopes $a_{0.1}$ and $a_{0.2}$, obtained above by the fixed points. The results are shown in Fig. 2(a). While, the data at $\sqrt{s} = 20$ and 130 GeV are better fitted by parabola as shown in Fig. 2(b).

The same analysis has also been applied to the $p_t$ correlation normalized by the average $p_t$ over the whole sample. The analysis for normalized $p_t$ correlation at $\sqrt{s} = 62$ and 200 GeV show exactly the same behavior of fixed points and straight lines as what $p_t$ correlation demonstrates above. The critical exponents of normalized $p_t$ correlation are smaller than that of $p_t$ correlation.

So the critical collision energies are most probably around $\sqrt{s} = 62$ and 200 GeV, rather than near $\sqrt{s} = 20$ and 130 GeV. The same analysis for other critical related observable, such as the fluctuations of mean $p_t$ per event, the moments of multiplicity, the ratio of $K$ to $\pi$, and so on, will be greatly helpful in confirming the observed results. Therefore, the incident energy and centrality dependence of those observable are called for.

If there were additional collisions around $\sqrt{s} = 62$ and 200 GeV, we could determine the finite-size scaling function defined in Eq. (9). This is impossible at present since there are only two collision energies in addition to the critical ones, and they could be outside of the asymptotic region where finite-size scaling holds.

The findings of the two critical points may imply that deconfinement and chiral symmetry restoration occur at different temperatures. Which one is at the lower or higher temperature (energy) has to be confirmed finally from theoretical calculation. Two critical collision energies, $\sqrt{s} = 62$ and 200 GeV, are both within the range estimated by lattice calculation [18].

The similar ratios of critical exponents at two critical points is consistent with current theoretical estimation, which shows that all critical exponents of the deconfinement transition, in the same university as the 3-dimensional Ising model [19], are very close to that of
chiral symmetry restoration, in the same university as the 3-dimensional O(4) model with spin symmetry [20].

To the summary, we argue in this letter that finite-size effects of the formed matter in relativistic heavy ion collisions is not negligible. The finite-size scaling, rather than non-monotonous behavior of observable is a reliable criterion of the existence of critical point. Then we propose how to locate critical point by finite-size scaling. As an application, we analyze the data of $p_t$ correlation and its normalized one at 6 centralities and 4 incident energies from RHIC/STAR. Two fixed points, and therefore two critical points, are likely observed around $\sqrt{s} = 62$ and 200 GeV. They could be, respectively, related to the transition of deconfinement and chiral symmetry restoration predicted by lattice-QCD. The ratios of critical exponents at these two critical points are similar, in consistence with current theoretical estimation.

The confirmation of this observation requires the efforts from both theoretical and experimental sides. From experimental side, it is proposed to get more and better data on other critical related observable at current collision energies, and a few additional collisions around $\sqrt{s} = 62$ and 200 GeV. Then we can more precisely determine the critical endpoints and critical exponents.

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