Anomalous magnetic moment of muon and L-violating Supersymmetric Models

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Abstract

We consider L-violating Supersymmetric Models to explain the recent muon $g_\mu - 2$ deviation from the Standard Model. The order of trilinear L-violating couplings which we require also generate neutrino mass which is somewhat higher than expected unless one considers highly suppressed $L - R$ mixing of sfermions. However, without such fine tuning for sfermions it is possible to get appropriate muon $g_\mu - 2$ deviation as well as neutrino mass if one considers some horizontal symmetry for the lepton doublet. Our studies show that $g_\mu - 2$ deviation may not imply upper bound of about 500 GeV on masses of supersymmetric particles like chargino or neutralino as proposed by other authors for $R$ parity conserving supersymmetric models. However, in our scenario sneutrino mass is expected to be light ($\sim 100$ GeV) and $e - \mu - \tau$ universality violation may be observed experimentally in near future.

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There is recent indication [1] that the anomalous magnetic moment of muon (AMMM) differs from its Standard Model value by $\Delta a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 42(16) \times 10^{-10}$. If it is further confirmed in future experiments this will be a strong evidence of new physics beyond the Standard Model apart from the other evidence coming from the neutrino oscillation experiments [2, 3, 4] indicating massive neutrinos. Both these evidences of new physics - anomalous magnetic moment of muon and neutrino mass, might be related in some models like in the leptonic Higgs doublet model considered in [3] or in $R$-parity violating Supersymmetric Models which we are considering here.

There are several explanations of this recent AMMM experiment [1] in the context of Supersymmetric Models [4, 5] and also in the context of non-Supersymmetric Models [1]. Recently, $R$ parity violation in Effective Supersymmetry has been considered [5] to explain the deviation in $a_\mu$. To avoid the constraint coming from muon neutrino mass they have considered particularly the semileptonic $L$ violating couplings $\lambda'_{231}$ and $\lambda'_{233}$ instead of $\lambda_{233}$. However the product of these couplings also contribute to muon neutrino mass. Unless one considers highly suppressed $L - R$ mixing of sfermions in general it is difficult to satisfy the general upper bound $^3$ on active neutrino mass $\sim 2.2$ eV.

In this letter we have shown that in $R$ parity violating Supersymmetric model considering upper bound on the neutrino masses and also considering the recent observation of $\Delta a_\mu$ [1] one gets constraint on soft susy breaking parameters due to the requirement of suppression of $L - R$ mixing of sfermions. However, this indicates certain amount of fine tuning in the SUSY parameters. On the other hand, it seems natural to consider some horizontal symmetry [11, 12, 13] for the lepton doublet superfields in the $R$ violating Supersymmetric Model for which it is possible to get simultaneously large $\Delta a_\mu$ as observed [1] as well as the light neutrino mass satisfying the present upper bound of about 2.2 eV. Furthermore, in this scenario only sneutrino mass is required to be light about 100 GeV whereas unlike $R$ conserving Supersymmetric Model there are no upper bounds on masses of other supersymmetric particles like chargino or neutralino [7].

We shall discuss first how in $R$ violating Supersymmetric Model [14] anomalous magnetic moment of muon get some contribution through $L$ violating couplings and how in such scenario majorana neutrino mass is also generated. In $R$ parity violating Supersymmetric Model as the three lepton supermultiplet $L_m$ $m = 1, 2, 3$ and down-type Higg supermultiplet transform identically we denote these four supermultiplets as $L_\alpha$ $\alpha = 0, 1, 2, 3$. Imposing $Z_3$ baryon triality one can write the most general renormalizable $L$ violating terms in the superpotential [14] as:

$$W = \epsilon_{ij} \left[ -\mu_\alpha L_\alpha^i H_U^j H_c + \frac{1}{2} \lambda_{\alpha\beta\gamma} L_\alpha^i L_\beta^j E_m + \lambda'_{\alpha\beta\gamma} L_\alpha^i Q_n^j \bar{D}_m - h_{nm} H_U^i Q_n^j U_m \right]$$

(1)

in which the coefficients $\lambda_{\alpha\beta\gamma}$ are antisymmetric under the interchange of the indices $\alpha$ and $\beta$. $H_U$ is the up-type Higgs supermultiplet, $Q$ are chiral superfields containing quark doublets and $U_m$ and $D_m$ are the singlet up and down type quark supermultiplets respectively and $E_m$ are the singlet charged lepton supermultiplet. Some soft supersymmetry breaking terms relevant for our discussion are

$$V_{\text{soft}} = \left( M_L^2 \right)_{\alpha\beta} \tilde{L}_\alpha^i \tilde{L}_\beta^i + \left( M_E^2 \right)_{mn} \tilde{E}_m^i \tilde{E}_n^i - \left( \epsilon_{ij} b_\alpha \tilde{L}_\alpha^i H_U^j + H.c. \right) + \epsilon_{ij} \left( \frac{1}{2} a_{\alpha\beta\gamma} \tilde{L}_\alpha^i \tilde{L}_\beta^i \tilde{E}_m \right)$$

---

$^3$This follows if one considers the bound from Tritium beta decay as well as the mass squared differences of different flavor of active neutrinos taking part in oscillation [3].
\( + \ a'_{\alpha m} \bar{\tilde{L}}_i \tilde{Q}_n \tilde{D}_m - (a_U)_{nm} H_l^i \tilde{Q}_n^i \tilde{U}_m + h.c. \) \\

in which the single \( B \) term of the \( R \) conserving minimal Supersymmetric standard Model (MSSM) has been extended to the 4 component vector \( b_\alpha \). Similarly \( A \) parameters of MSSM has been extended to \( a_{\alpha \beta m} \) and \( a'_{\alpha m} \). Here \( a_{\alpha \beta m} \) are antisymmetric in the first two indices \( \alpha \) and \( \beta \). In a convenient notation such \( A \) and \( B \) parameters have been written above as

\[
\begin{align*}
  a_{\alpha \beta m} &= \lambda_{\alpha \beta m} (A_E)_{\alpha \beta m} , \\
  a'_{\alpha m} &= \lambda_{\alpha m} (A_D)_{\alpha m} ,
\end{align*}
\]

\((a_U)_{nm} = h_{nm} (A_U)_{nm} \), \( b_\alpha = \mu_\alpha B_\alpha \)

We assume that only the neutral scalar fields acquire vacuum expectation values and we write \( \langle h_U \rangle = v_u / \sqrt{2} \) and \( \langle \tilde{\nu}_\alpha \rangle = \nu_\alpha \). If \( \mu_\alpha \) and \( \nu_\alpha \) are almost aligned at the low energy scale by some mechanism then only the \( \mu_i \) (where \( i = 1..3 \)) can be rotated away. However, if such alignment condition is achieved at some scale of supersymmetry breaking normally it breaks down at the low energy scale. Due to this misalignment parametrised as \( \xi_i = v_i / v_0 - \mu_i / \mu_0 \), particularly the mixing of neutralino/neutrino, chargino/charged lepton and slepton/Higgs mixing occur [15]. We shall neglect such mixing considering \( \mu_i \) to be very small and also we neglect the contribution to AMMM coming from the effective trilinear terms obtained from bilinear terms with suitable redefinition of \( L \) and \( H \) superfields. To get appreciable \( \Delta a_\mu \) we need large trilinear \( \lambda_{ijk} \) or \( \lambda'_{ijk} \) couplings as will be shown later.

As contribution to AMMM comes also due to mixing between \( L \) type and \( R \) type charged slepton/d-squarks in the Feynman diagrams we discuss this mixing in brief for any particular generation. The \( L - R \) unitary mixing matrix is given by

\[
U^I = \begin{pmatrix}
  \cos \phi & \sin \phi \\
  -\sin \phi & \cos \phi
\end{pmatrix}
\]

\[(3)\]

Corresponding to the \( 2 \times 2 \) \( LR \) block of the charged slepton squared mass matrix

\[
M^2(\tilde{l}) = \begin{pmatrix}
  L^2 + m_i^2 & \bar{A} m_l \\
  \bar{A} m_l & R^2 + m_i^2
\end{pmatrix}
\]

\[(4)\]

where \( L^2 = M^2(1)_l + (T_3 - e \sin^2 \theta_w) m_z^2 \cos 2\beta \), \( R^2 = M^2(2)_l + e \sin^2 \theta_w m_z^2 \cos 2\beta \) and \( A = A_{0l} - \mu_0 \tan \beta \) with \( T_3 = -1/2 \) and \( e = -1 \) for the down type charged sleptons. \( M^2(1)_l \), \( M^2(2)_l \) and \( A_{0l} \) are \( R \) parity conserving soft supersymmetry breaking parameters. The charged slepton mass eigenstates are given by

\[
\tilde{l}_i = U^I_{1i} \tilde{l}_L + U^I_{2i} \tilde{l}_R
\]

\[(5)\]

The mixing angle \( \phi = \phi_k \) (where \( k \) correspond to the particular generation we are considering) for the charged slepton is given by

\[
\sin 2\phi_k = \frac{2 \bar{A} m_l}{\sqrt{(L^2 - R^2)^2 + 4 \bar{A}^2 m_i^2}}
\]

\[(6)\]

For \( L - R \) mixing of d-squark we replace the the slepton mixing angle \( \phi_k \) by \( \phi_d_k \) which is obtained similarly from the above equations after replacing \( m_l, e = -1, \ M^2(1)_l, M^2(2)_l \).
and $A_{0\ell l}$ with $m_{d}$, $e = -1/3$, $M^{2}(1)_{dd}, M^{2}(2)_{dd}$ and $A_{0dd}$ respectively. Similarly for $L - R$ mixing of $u$-squark we replace the the slepton mixing angle $\phi_{l_k}$ by $\phi_{u_k}$ which is obtained similarly from the above equations after replacing $m_t, T_3 = -1/2, e = -1, M^{2}(1)_{ll}, M^{2}(2)_{ll}$ with $m_u, T_3 = 1/2, e = 2/3, M^{2}(1)_{uu}, M^{2}(2)_{uu}$ respectively. Furthermore, in this case $A = A_{0uu} - \mu_0 \cot \beta$. For large $\mu_\alpha$ one should also include a term $-\mu_i \lambda_{ijk} \nu_\alpha/\sqrt{2}m_l$ for slepton and a term $-\mu_i \lambda'_{ijk} \nu_\alpha/(\sqrt{2}m_d)$ for $d$ squark [16].

Due to trilinear $\lambda_{ijk}$ couplings AMMM gets contribution from one loop diagram with (a) charged lepton and sneutrino (b) charged slepton and neutrino, in the internal lines. Photon line is attached with charged internal particle line. Due to $\lambda'_{ijk}$ couplings AMMM gets contribution from one loop diagram with (a) up type quark and down type squarks (b) down type quark and up type squarks, in the internal line. Photon line is attached with any one of the internal particle line. The chirality flip on the external muon line or the chirality flip in the internal sfermion line can be considered for both $\lambda$ and $\lambda'$ couplings. However, while considering the chirality flip on the internal line one requires $L - R$ mixing of sfermions. Then it will be difficult to get appreciable $\Delta a_\mu$ as well as small neutrino mass $\sim 2.2$ eV as $L - R$ mixing of sfermions is present in both $\Delta a_\mu$ and neutrino mass. This makes the contribution to $\Delta a_\mu$ from the chirality flip on the internal line lesser than that coming from the chirality flip on the external muon line and the contribution to $\Delta a_\mu$ coming from the chirality flip on the internal sfermion line may be ignored. If we consider the chirality flip on the external muon line [3, 7] then for $\lambda_{ijk}$ couplings

$$\Delta a_\mu \approx \sum_{i,k} \frac{m^2_\mu}{96\pi^2} \left[ 2 \left| \lambda_{i2k} \right|^2 \frac{m^2_{\tilde{\nu}_i}}{m^2_{\tilde{\nu}_i}} + 2 \left| \lambda_{i2k} \right|^2 \frac{m^2_{\tilde{\nu}_i}}{m^2_{\tilde{\nu}_i}} - \left| \lambda_{i2k} \right|^2 \frac{m^2_{\tilde{\nu}_i}}{m^2_{\tilde{\nu}_i}} - \left| \lambda_{i2k} \right|^2 \right]$$

(7)

and due to $\lambda'_{ijk}$ couplings

$$\Delta a_\mu \approx \sum_{i,j} \frac{m^2_\mu \left| \lambda'_{2jk} \right|^2}{32\pi^2 \left( m^2_{d_{kR}} - m^2_{d_j} \right)} \left[ 1 + \frac{2 r(u_j, d_{kR})}{1 - r(u_j, d_{kR})} \left\{ \frac{1}{2} + \frac{3}{1 - r(u_j, d_{kR})} \ln r(u_j, d_{kR}) \right\} \right]$$

(8)

where $r(u_j, d_{kR}) = \left( m_{u_j}/m_{d_{kR}} \right)^2$. Particularly for $j = 3$ and $k = 1$ AMMM gets contribution through $\lambda'_{231}$ coupling. For $m_{d_3} \sim 1$ TeV one may consider $\lambda'_{231} \sim 1$. For $m_{d_1} \sim 200$ GeV one gets $\Delta a_\mu \sim 10^{-9}$. Due to the stringent constraint on other $\lambda'_{2jk}$ couplings [18] it is difficult to get significant contribution to $\Delta a_\mu$. As for example $\lambda'_{231}$ coupling (which has been considered in ref. [3]) may be considered about 1 but in that case $b$ squark has to be considered in the TeV range to satisfy the present experimental bound [13] but as its’ mass also appear in the denominator it is not possible to get appreciable contribution to $\Delta a_\mu$. For $\lambda_{i2k}$ or $\lambda_{i2k}$ couplings it is possible to get appreciable contribution to $\Delta a_\mu$ for various values of $i$ and $k$. If one considers the mass of charged slepton in the TeV range then most of these couplings can be about 1 satisfying the present experimental constraint [13]. In this case if one considers the mass of sneutrino somewhat light (about 100 GeV say) then $\Delta a_\mu \sim 10^{-9}$ and also it is interesting to note that the negative contributions are very small due to high charged slepton mass.

We like to mention here why the diagrams contributing to AMMM with chirality flip on the internal line are small. With chirality flip in $u$ squark or $d$ squark internal particle line


the one loop diagram gives

\[
\Delta a_\mu = \sum_{j,k} \frac{N_c m^2_{\mu}}{8\pi^2 m^2_d} \lambda'_{2jk} \lambda'_{2kj} \left[ m_{d_j} \sin 2\phi_{u_k} \left\{ \frac{2}{3} F_1(r_{d_j}) + \frac{1}{3} F_2(r_{d_j}) \right\} + m_{u_j} \sin 2\phi_{d_k} \left\{ \frac{1}{3} F_1(r_{u_j}) + \frac{2}{3} F_2(r_{u_j}) \right\} \right]
\]

where

\[
F_1(r_{d_j,u_j}) = \frac{1}{2(1-r)^2} \left[ 1 + \frac{2 r}{1-r} \ln r \right]; \quad F_2(r_{u_j}) = \frac{1}{2(1-r)^2} \left[ 3 - \frac{2 r}{1-r} \ln r \right]
\]

and \(r_{d_j,u_j} = (m_{a_j,d_j}/m_\tilde{q})^2\) and \(m_\tilde{q}\) is the scalar squark mass in the loop. However, after diagonalising the \(L - R\) mixing matrix there are two squarks \(\tilde{q}_1\) and \(\tilde{q}_2\) in the internal line. So there will be further suppression in the above \(\Delta a_\mu\) by a factor \((m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}) / (m^2_{\tilde{q}_1} m^2_{\tilde{q}_2})\).

It is possible to get appreciable \(\Delta a_\mu\) due to top mass which comes for \(j = 3\). However, in that case \(\Delta a_\mu\) is also proportional to \(L - R\) mixing of \(d\) squark. But this mixing will also be present in the neutrino mass matrix. So it will not be possible to get small neutrino mass as well as large \(\Delta a_\mu\). For \(\lambda_{ijk}\) couplings with chirality flip on the internal line there will be diagram contributing to \(\Delta a_\mu\) with charged slepton and neutrino in the internal line. However, as \(\Delta a_\mu\) as well as neutrino mass - both are proportional to \(L - R\) mixing of charged lepton, such diagrams cannot give appreciable \(\Delta a_\mu\). So we conclude that the contributions to \(\Delta a_\mu\) coming from the chirality flip on the internal line can be ignored.

We next like to show what happens to neutrino mass if one considers such higher values of \(\lambda_{ijk}\) or \(\lambda'_{ijk}\) couplings of about 1. As we have ignored the mixing of neutrinos with neutralinos, neutrino does not get mass at the tree level. However, at one loop level all the neutralinos will acquire significant mass as the trilinear couplings are required to be large for AMMM. From Figure 1, one loop contribution to the neutrino mass matrix \((m_{\nu})_{ij}\) due to \(\lambda'_{ijk}\) couplings is given by

\[
(m_{\nu})_{ij} = \frac{3}{32\pi^2} \sum_{n,k} \lambda'_{i kn} \lambda'_{j nk} m_{d_k} \sin 2\phi_{d_n} \ln \frac{m_1}{m_2}
\]

where \(m_{1,2}\) are the non-degenerate squark masses obtained after diagonalising the the squark mass matrix inducing \(L - R\) mixing. Similarly, for \(\lambda_{ijk}\) couplings the one loop contribution to the neutrino mass matrix is given by

\[
(m_{\nu})_{ij} = \frac{1}{32\pi^2} \sum_{n,k} \lambda_{i kn} \lambda_{j nk} m_{l_k} \sin 2\phi_{l_n} \ln \frac{m_1}{m_2}
\]

For \(\lambda_{ijk} \sim 1\) or \(\lambda'_{ijk} \sim 1\) considering the upper bound on \(m_\nu\) as 2.2 eV one obtains the following bound on the \(L - R\) mixing of sfermions:

\[
\sin 2\phi \ln \frac{m_1}{m_2} \leq 6.3 \times 10^{-4} / m_{l,j,k}
\]

where \(m_{l,j,k}\) is the mass of the charged lepton of generation \(j\) or \(k\). Particularly for \(j = 3\) or \(k = 3\) for \(\lambda_{ijk}\) or \(\lambda'_{ijk}\) couplings this bound is highly stringent (about \(3.7 \times 10^{-7}\)).
means that the $A$ parameter should be very small compared to $\sqrt{L^2 - R^2}$ and that implies particularly for large $\tan \beta >> 1$ (which is theoretically preferred particularly for bottom-tau unification) the $\mu_0$ parameter should be very small. So certain amount of fine tuning is necessary as this parameter is present in unbroken supersymmetry. If one considers the $R$ parity violating contribution to $A$ parameter as mentioned earlier then it indicates other $\mu_i$ also should be small.

Next we shall show that we do not require such significant suppression of $L - R$ mixing to get appreciable $\Delta a_\mu$ as well as small neutrino mass if we consider horizontal symmetry for the lepton doublet. For this we consider the $R$ parity violating Supersymmetric Model in such a way that $l_e - l_\mu$ lepton number is unbroken and there is discrete horizontal symmetry $D$ between the first two generations of lepton. Under symmetry $D$ the following chiral superfields transform as

$$\begin{pmatrix} L_e \\ L_\mu \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} L_e \\ L_\mu \end{pmatrix}. \quad (14)$$

$$\begin{pmatrix} e^c \\ \mu^c \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} e^c \\ \mu^c \end{pmatrix}. \quad (15)$$

The superfields $L_\tau, \tau^c, Q_i, u^c_i$ and $d^c_i$ do not transform under $D$. Under $l_e - l_\mu$ and $D$ the most general superpotential in Eqn. (1) now takes the form

$$W = \epsilon_{ij} \left[ -\mu_{\alpha} L_\alpha = 0.3 H_U^i + \frac{1}{2} \left( \lambda_{131} L_e^i L_t^j e^c + \lambda_{232} L_\mu^i L_\tau^j \mu^c + \lambda_{123} L_e^i L_\mu^j \tau^c + \lambda_{033} L_\tau^i H_d^j \tau^c \right) + \lambda_{011} L_e^i H_d^j e^c + \lambda_{022} L_\mu^i H_d^j \mu^c \right] + \lambda_{nm} H_U^i Q_n^j D_m + \lambda_{3nm} L_\tau^i Q_n^j D_m - h_{nm} H_U^i Q_n^j U_m \right]$$

It is important to note here that there are only a few trilinear $\lambda$ and $\lambda'$ couplings and there will be no contribution to AMMM from $\lambda'$ coupling. Now $l_e$ and $H_d$ have identical transformation properties and apart from two Higgs vev there is tau sneutrino vev $v_3$. One can choose the appropriate basis so that $v_3 = 0$. $v_1$ and $v_2$ do not exist because of $l_e - l_\mu$ conservation. If $D$ symmetry is considered for the soft breaking terms then instead of Eqn. (2) one can write down similar terms like above for $V_{soft}$. Particularly $D$ symmetry will imply $(M_L^2)_{11} = (M_L^2)_{22}$ and $(M_E^2)_{11} = (M_E^2)_{22}$. After breaking of gauge symmetry $D$ symmetry can be kept unbroken. Although in the $D$ symmetric limit the neutrino mass for the 1-st two generations are zero, the electron and muon mass will also be degenerate. However, as has been discussed in ref. [12], by adding dimension two soft breaking terms for muon superfields breaking $D$ symmetry softly it is possible to get appropriate mass differences for electron and muon without affecting the neutrino mass spectrum. It is because instead of lepton mass entering the neutrino mass diagram it is the gaugino mass (having much larger magnitude) that enters into the charged lepton mass diagram.

In the $D$ symmetric limit neutrino mass terms for the 1-st two generations are forbidden. Only $\lambda_{131} = -\lambda_{311}$ and $\lambda_{232} = -\lambda_{322}$ will give mass to $\tau$ neutrino. These $\lambda$ couplings will contribute to AMMM, however those contributions may be very small. On the other hand, $\lambda_{123}$ coupling will not contribute to neutrino mass but will contribute to AMMM as shown...
in Figure 2. This will give

\[
\Delta a_\mu = \frac{m_\mu^2 |\lambda_{123}|^2}{48\pi^2} \left[ \frac{1}{m_{\tilde{\nu}_e}^2} - \frac{1}{2m_{\tilde{\tau}}^2} \right]
\]

(17)

from the diagram with (a) $\tau$ lepton and $\tilde{\nu}_e$ in the internal line (shown in Figure 2) and (b) $\tau$ slepton and $\nu_e$ in the internal line (not shown in figures). Photon is attached with $\tau$ lepton for (a) and $\tau$ slepton for (b) and there is chirality flip on the external muon line. In $R$-violating Supersymmetry the lower bound on sneutrino mass is expected to be somewhat lower than 100 GeV due to sneutrino pair production and its subsequent decays to charged leptons at LEP \[20\]. For $m_{\tilde{\tau}}$ about a few TeV it is possible to consider $\lambda_{123} \sim 1$ \[18\]. In that case the negative contribution to AMMM is very small and considering the conservative bound $m_{\tilde{\nu}_e} \sim 100$ GeV one gets $\Delta a_\mu \sim 10^{-9}$.

In this scenario due to $l_e - l_\mu$ symmetry the rare processes like $\mu \rightarrow 3e$, $\mu \rightarrow e\gamma$, $\tau \rightarrow 3e$, $\tau \rightarrow \mu e e$ etc. are forbidden. However, due to $\lambda_{123}$ coupling $\mu$ will decay to $e$, $\tilde{\nu}_e$, $\nu_\mu$ through $\tau$-slepton exchange at the tree level apart from the Standard Model $W$ exchange diagram. So there will be $e - \mu - \tau$ universality violation. Our studies indicate that this violation might be observed in near future due to our requirement of higher value of $\lambda_{123}$ coupling (for $\tau$-slepton mass about a few TeV) to explain the presently observed $\Delta a_\mu$ with $l_e - l_\mu$ and $D$ symmetry in $R$ parity violating Supersymmetric Model.

There are some uncertainties in the calculation of the hadronic contribution to AMMM \[21, 22\] particularly in the dispersion integral approach to hadronic vacuum polarization effects and as such the uncertainties also exist in finding the amount of what should be the new physics contribution to AMMM. To satisfy the experimental data \[1\] the hadronic and the new physics contribution together \[21\] to AMMM should be $7350(153) \times 10^{-11}$. Total hadronic contribution as shown by Davier et al is $6294(62) \times 10^{-11}$ whereas the same as shown by Jegerlehner is $6974(105) \times 10^{-11}$. If the later is correct then it is easily possible to explain $\Delta a_\mu$ through $R$ parity violating interactions. However, if the hadronic contribution is really less then one have to carefully examine the possibility of the explanation of $\Delta a_\mu$ in terms of $R$ parity violating interactions.

Lastly we like to comment on the AMMM from $R$ parity conserving Supersymmetric model versus $R$ parity violating Supersymmetric Model. In the $R$ parity conserving case there will be diagrams a) with smuon and neutralino b) sneutrino and chargino, in the internal lines. If all the supersymmetric particles present in the loop are considered almost degenerate then the diagram with (b) will dominate otherwise for large $L - R$ mixing of smuons in some cases diagram with (a) may dominate depending on the right-handed smuon mass. However, in both the cases large $\tan \beta$ is preferred as that gives larger contribution to AMMM. For theoretically preferred value of $\tan \beta \sim 35$ (necessary for the bottom-tau unification) the maximum mass allowed is around 360 Gev for the chargino and/or neutralino and 420 Gev for the lightest sneutrino in the loop \[7\]. However, in the $R$-parity violating case as there is lepton number violation, Majorana neutrinos can be massive. To satisfy the upper bound on neutrino masses it seems natural to consider $l_e - l_\mu$ and $D$ symmetry. Then we require light $\tilde{\nu}_e$ mass $\sim 100$GeV but other slepton masses are rather heavy in the TeV range. Here, it is interesting to note that theoretically sneutrino could be the lightest supersymmetric particle.
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Figure 1: One loop diagram involving $L$-violating couplings generating neutrino mass.

Figure 2: One loop diagram involving $L$-violating couplings generating anomalous magnetic moment of muon mass.