Amplitudes of non-radial oscillations driven by turbulence

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Abstract. Turbulent motions in stellar convection zones generate acoustic energy, part of which is supplied to normal modes of the star. Amplitudes of these modes result from a balance between the efficiencies of excitation and damping processes in the convection zones. For radial oscillations, in the solar case, a good agreement between observations and theoretical modelling is reached (see for instance Belkacem et al. 2006b). We also compute the energy supplied by unit time by turbulent convection to the acoustic modes of α Cen A (HD 128620), a star which is similar but not identical to the Sun. In addition, we have developed a formalism that provides the excitation rates of non-radial modes excited by turbulent convection. It then allows us to investigate both p and g modes in the upper convection zones as well as in stellar convective cores. As a first application, we have estimated-and report here- the impact of non-radial effects on excitation rates and amplitudes of high-angular-degree solar p modes.

1. Introduction

Amplitudes of solar-like oscillations result from a balance between stochastic excitation and damping in the outermost layers of the convection zone. Accurate measurements of the rate at which acoustic energy is supplied to the solar p modes are available from ground-based observations (GONG, BiSON) as well as from spacecrafts (SOHO/GOLF and MDI). From those measurements and a comparison with theoretical models, it has been possible to demonstrate that excitation is due to eddy motions in the uppermost part of the convection zone and by advection of entropy fluctuations.

Thus, amplitudes of radial modes of the Sun have been extensively studied [1, 2, 3, 4] to infer some turbulent properties of the uppermost part of the convection zone. We show hereafter recent results obtained for those modes and point out remaining uncertainties. Radial modes of stars other than the Sun also promise to give us a better knowledge of the convective zones with different physical properties. Observational data for those stars just begin to provide sufficient constraints about turbulent layers. Then, we also present results concerning the modelling of amplitudes of α-Cen A compared to ground-base observations.

Further, we have developed a non-radial formalism [5] that give us access to high-angular-degree p modes as well as gravity modes. One can then investigate a wide variety of modes and stars. As a first application, we present the impact of non-radial effects on excitation rates and amplitudes of high-angular-degree solar p modes.

The paper is organized as follows: Sect. 2 introduces the general formalism. Section 3 presents the results obtained for radial modes in the Sun and α-Cen A. In Sect. 4, we show and discuss
results obtained for non-radial modes for the Sun. Finally, Section 6 is dedicated to discussions and conclusions.

2. General formulation

The estimation of the power supplied to the modes by turbulent convection is obtained from the non-radial generalization of the work as detailed in [4]. The resulting theoretical rates successfully reproduce the observed $p$ modes excitation rates [6]. The model takes two driving sources into account, the first one is related to the Reynolds stress tensor. The second one is caused by the advection of the turbulent fluctuations of entropy by the turbulent motions. The power fed into each radial mode, $P$, is given by:

$$\mathcal{P} = \frac{1}{8} I \left( C_R^2 + C_S^2 \right),$$

where $C_R^2$ and $C_S^2$ are the turbulent Reynolds stress and entropy contributions, respectively and $I = \int_0^M dm \vert \xi \vert^2$ is the mode inertia, $\xi$ is the mode displacement and $M$ is the mass of the star. The expressions for $C_R^2$ and $C_S^2$ are given by

$$C_R^2 = \frac{64 \pi^3}{15} \int \rho_0 \, d^3 x_0 \, R(r) \left( 1 + \frac{1}{3} S_w^2 \right) S_R(\omega_0),$$

$$C_S^2 = \frac{16 \pi^3}{3 \omega_0} \int \alpha_s \, d^3 x_0 \left( A_\ell + B_\ell \right) S_S(\omega_0),$$

where we have defined

$$S_R(\omega_0) = \int \frac{dk}{k^2} E(k) \, \int d\omega \chi_k(\omega + \omega_0) \chi_k(\omega)$$

$$S_S(\omega_0) = \int \frac{dk}{k^4} E(k) E_s(k) \, \int d\omega \chi_k(\omega + \omega_0) \chi_k(\omega)$$

where the functions $R(r), A_\ell$ and $B_\ell$ contain the eigenfunctions, their expression and derivation are detailed in [5]. $E(k)$ is the spatial turbulent kinetic energy spectrum, $\chi_k$ the time correlation function of the eddies, $\alpha_s = (\partial P/\partial s)_\rho$ where $s$ is the entropy, $P$ the pressure, $\rho$ the density, $\rho_0$ the equilibrium density profile, $\omega_0$ the eigenfrequency and $S_w$ the skewness [7].

Note that Eq. (2) depends on the closure model that is used to express the fourth-order moments of velocity involved in the theory in terms of the second ones. The most commonly used closure model at the level of fourth-order moments is the quasi-normal approximation (QNA). The QNA is strictly valid for normally distributed fluctuating quantities with zero mean, leading to a vanishing skewness ($S_w$). However, the uppermost part of the convection zone is a turbulent convective system composed of essentially two flows and the associated probability distribution functions of the fluctuations of the vertical velocity and temperature do not follow gaussian laws [8]. Hence the use of the QNA is not valid. A more sophisticated closure model was then proposed by [7]. By taking into account both the effect of the skew introduced by the presence of two flows and the effect of turbulence onto each flow, [7] thus proposed a closure model that takes into account the presence of plumes, leading to a non-vanishing skewness ($S_w$) in Eq. (2).

3. Radial $p$ modes

3.1. The Sun

Figure 1 summarizes our comparisons between recent helioseismic constraints obtained by [9] and theoretical calculations obtained by [6]. It confirms the result of [10] that a Lorentzian eddy-time correlation function (see Eq. (4)) yields in a better agreement with the observations than
Figure 1. Rates (in linear scale) at which energy is injected into the solar $p$ modes. The filled circles correspond to the helioseismic constraints obtained by [9]. The lines correspond to different theoretical calculations: the solid line uses the Lorentzian function (LF) and the CMP, the dashed line is as the solid line with only the contribution of the Reynolds stress, the dot-dashed line uses the LF and the QNA closure model, the triple-dot-dashed line uses the Gaussian function (GF) and the CMP.

a Gaussian eddy-time correlation function (GF). Figure 1 also shows that the use of the QNA results in a significant underestimation of the contribution of Reynolds stress to mode excitation. On the other hand, the closure model with plumes (CMP) proposed by [7] significantly increases this contribution. The theoretical calculations limited to the Reynolds stress underestimates only by $\sim 15\%$ the maximum in the excitation rates $P$ derived by [9]. Then, when the entropy contribution is added, the theoretical calculations based on this improved closure model fit the helioseismic data.

Despite those improvements, some discrepancies remain at high frequency ($\nu > 4$ mHz) and at low frequency ($\nu < 3$ mHz). At high frequency they are attributed to the closure model as well as to the entropic contribution that are still not satisfactorily well enough modelled in the super-adiabatic region. We also point out that the calculation of the mode excitation rates requires the computation of the mode eigenfunctions. The latter are obtained from 1D stellar models. In such stellar models, the structure of the outer layers is very poorly modelled. As most of the excitation occurs near the photosphere, this poor description is likely to introduce some biases on the predicted excitation rates in particular at high frequency.

At low frequencies, a possibly overestimation of the Reynolds stress contribution can be attributed to the frequency dependent factor ($\chi_k$). In [6], calculations assuming a Gaussian and a Lorentzian for $\chi_k$ are presented. It shows that the frequency-dependent factor $\chi_k$ is likely between these two regimes. In the quasi-adiabatic convection zone, plumes are well-formed, and the convective system must be treated as composed of two flows. Hence, the upflows that are less turbulent can be modelled by a Gaussian, but downflows are turbulent creating a departure from a Gaussian.

3.2. $\alpha$ Cen A

We have computed the energy supplied by unit time by turbulent convection ($P$) to $\alpha$ Cen A acoustic modes [11]. Observational constraints on $P$ are derived from the amplitude spectrum obtained by [12] and from two different estimates of the averaged mode line widths, namely that obtained on one hand by [13] and on the other hand by [12].

We find that our theoretical estimations of $P$, which assume a Lorentzian eddy-time correlation function ($\chi_k$) and the Closure Model with Plumes (CMP) proposed by [7], lie in the observed domain. On the other hand, when a Gaussian function is chosen for $\chi_k$, $P$ is significantly underestimated. The comparison with the seismic data for $\alpha$ Cen A thus confirms the results obtained by [10] in the solar case, that-is $\chi_k$ significantly departs from a Gaussian.
As in [10], we attribute the departure of $\chi_k$ from a Gaussian to diving plumes (i.e. down flows), which are more turbulent than granules (i.e. the up flows).

The changes in $P$ induced by including the CMP rather than the QNA, by assuming the LF rather than the GF and by including or not the driving source due to the entropy fluctuations are found of the same orders than in the case of the Sun. This is not surprising since the surface layers of $\alpha$ Cen A are not significantly different from those of the Sun. Indeed $\alpha$ Cen A has an effective temperature $T_{\text{eff}} = 5810$ K and a gravity log $g = 4.305$, which are relatively close to that of the Sun [14].

Differences between the theoretical calculations that use the CMP and those based on the Quasi-Normal Approximation (QNA) (see Fig. 2) as well as differences between calculations including the driving by the entropy fluctuations and those that do not include it (not shown), are of the same orders than the observational uncertainties associated with the two available data sets. The present seismic constraints therefore are unable to discriminate between the two closure models investigated in this work (CMP and QNA) nor to validate or not the contribution of the entropy fluctuations. This emphasises the need for more accurate seismic data for $\alpha$ Cen A.

From a theoretical point of view some work is still to be performed. We note that $\alpha$ Cen A has an iron to hydrogen abundance of $[\text{Fe/H}]=0.2$ [14]. The effect of a lower metallicity on the excitation rates remains to be investigated; this requires the computation of a 3D simulation with a non-solar abundance representative of the surface layers of the star (in progress).

4. High-angular degree solar $p$ modes

Figure 3 displays the rate ($P$) at which energy is supplied to the modes normalized to the excitation rates of radial modes ($P_{\text{rad}}$) in function of frequency. First, it is seen that the higher the angular degree ($\ell$) the more energy is supplied to the mode. This is due to the mode inertia ($I$) that decreases with $\ell$ since high-angular degree modes lie in the uppermost part of the outer layers.

In Fig. 3, one can distinguish between two types of modes, namely low-$n$ ($\leq 3$) and high-$n$ ($> 3$) modes (see Fig. 3).

- For high-$n$ modes, non-radial effects play a minor role in the excitation source terms. The dominant effect (see Fig. 3) is due to the mode inertia as discussed above.
- For lower values of $n$, there is a contribution to the excitation rates due to the horizontal terms in $R(r)$ defined in Eq. (1).

Figure 2. Rates $P$ at which energy is injected into the $p$ modes of $\alpha$ Cen A. The grey area represents the observations defined by merging the uncertainties associated with the two independent available observational data sets [11]. The lines correspond to different theoretical calculations: the solid line uses the Lorentzian function (LF) and the CMP, the dashed line uses the LF and the QNA closure model, the dot-dashed line uses the Gaussian function (GF) and the CMP.
Figure 3. The rate \( (P) \) at which energy is supplied to each \( \ell, n \) mode for \( \ell = 50, 100, 300 \) is divided by the excitation rate \( (P_{\text{radial}}) \) obtained for the \( \ell = 0, n \) mode. Computation of the theoretical excitation rates are performed as explained in [5].

For low-\( n \) modes, the horizontal contributions to the excitation become dominant whereas it is negligible for high-\( n \) modes in front of the radial contribution. This is due to the mode compressibility that becomes small because the radial contribution to the excitation rates is proportional to the mode compressibility. For those modes, the dominant contributions are horizontal and related to the shear of the mode.

Note that the present formalism is valid for \( \ell < 500 \), since it has been assumed that the spatial variation of the eigenfunctions is large compared to the typical length scale of turbulence, leading to what we call the separation of scales. For \( \ell > 500 \) the characteristic length of the mode becomes smaller than the characteristic length \( L_c \) of the energy bearing eddies. Those modes will then be excited by turbulent eddies with length-scale smaller than \( L_c \), i.e. lying in the turbulent cascade. These eddies inject less energy into the mode than the energy bearing eddies do, since they have less kinetic energy. We can then expect that – at fixed frequency – they received less energy from the turbulent eddies than the low-degree modes. A theoretical development is currently underway to properly treat the case of very high \( \ell \) modes.

5. Conclusion

For the Sun, the present excitation model gives the theoretical slope of the power at intermediate \( (\nu \in [2.2 \text{mHz} : 3.7 \text{mHz}]) \) frequencies which is in agreement with the observed data (see Fig. 1). We also find that including the CMP causes a global increase of the injected power. This brings the power computed with the Reynolds stress contribution alone closer to the observations. Furthermore, the power obtained by including both the Reynolds stress and the entropy fluctuation contributions reproduces the slope at low and intermediate \( (\nu < 3.7 \text{mHz}) \) frequencies although it slightly over-estimates the excitation rates (note however that Fig. 1 shows the 1 \( \sigma \) error bars).

For \( \alpha \) Cen A, theoretical calculations assuming an LF fit best the maximum oscillation amplitudes. It confirms the result obtained for the Sun. To go further one needs more accurate seismic data for \( \alpha \) Cen A. The CoRoT satellite (see the recent review by [15]) which was launched on December 27\(^{th} \) 2006 will soon provide us accurate measurements of mode amplitudes and line-widths for a larger set of stars. The quality of the data is expected to be significantly higher than current observations.

We have extended the [4] formalism in order to predict the amount of energy that is supplied to non-radial modes. We have demonstrated that non-radial effects are due to two contributions, namely the effect of inertia that prevails for high-order modes \( (n > 3) \) and non-
radial contributions in the Reynolds source term in $C_R^2$ (see Eq. (2)) that dominate over the radial one for low-order modes ($n < 3$). The present work focuses on $p$ modes, but the formalism is valid for both $p$ and $g$ modes. We will address the case of gravity modes in a forthcoming paper.

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