Investigation of gauge-fixed pure $U(1)$ theory at strong coupling

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We numerically investigate the phase diagram of pure $U(1)$ gauge theory with gauge fixing at strong gauge coupling. The FM-FMD phase transition, which proved useful in defining Abelian lattice chiral gauge theory, persists also at strong gauge coupling. However, there the transition seems no longer to be continuous. At large gauge couplings we find evidences for confinement.

1. INTRODUCTION

Recently it has been shown that nonperturbative gauge fixing can be applied successfully to decouple the longitudinal gauge degrees of freedom (dof) from Abelian lattice chiral gauge theories (LχGT) in the limit of zero gauge coupling. After one gauge transforms a gauge-noninvariant LχGT proposal like the Smit-Swift model or the domain wall waveguide model, one picks up the longitudinal gauge dof, the radially frozen scalars, explicitly in the action. The job of gauge fixing is to find a phase transition where the gauge symmetry would be restored with the scalars decoupled.

The gauge fixing proposal has also been successful in describing pure $U(1)$ gauge theory where pure QED is recovered at weak gauge coupling. In the present work we extend the study of gauge-fixed pure $U(1)$ gauge theory on lattice at strong gauge couplings. The major concern here is whether the FM-FMD transition, where the gauge symmetry is restored and was useful in defining LχGT, still exists at large $g$ and if so, what is the order of the transition. The gauge-fixed pure $U(1)$ theory can be looked upon as another lattice regularization of the strongly coupled $U(1)$ theory and its strong coupling properties like confinement can also be investigated.

2. GAUGE-FIXED PURE $U(1)$ THEORY

The gauge-fixed pure gauge action for compact $U(1)$, where the ghosts are free and decoupled, is:

\[ S_B(U) = S_G(U) + S_{Gf}(U) + S_{Ct}(U) \]  

where $S_G$ is the usual Wilson plaquette action, $S_{Gf}$ is the gauge fixing term and $S_{Ct}$ are appropriate counterterms given by,

\[ S_G = \frac{1}{g^2} \sum_{x<\mu<\nu} (1 - \text{Re} U_{\mu\nu x}) \]  

\[ S_{Gf} = \tilde{\kappa} \left( \sum_{x<\mu<\nu} \Box(U)_{\mu\nu x} - \sum_x B_x^2 \right) \]  

\[ S_{Ct} = -\kappa \sum_{\mu x} (U_{\mu x} + U_{\mu x}^\dagger) \]  

where $\Box(U)$ is the covariant lattice Laplacian with $\tilde{\kappa} = 1/(2\xi g^2)$ and

\[ B_x = \sum_{\mu} \left( \frac{A_{\mu x} - \mu + A_{\mu x}}{2} \right)^2 , \]  

where $A_{\mu x} = \text{Im} U_{\mu x}$. $S_{Gf}$ is not just a naive transcription of continuum covariant gauge fixing term it has in addition appropriate irrelevant terms. As a result, $S_{Gf}$ has a unique absolute minimum at $U_{\mu x} = 1$, validating weak coupling perturbation theory around $g = 0$ or

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$\kappa = \infty$ and in the naive continuum limit reduces to $1/2\xi \int d^4x (\partial_\mu A_\mu)^2$. Validity of weak coupling perturbation theory together with perturbative renormalizability helps determining the form of the counter terms to be present in $S_{CT}$. It turns out that the most important bosonic counterterm is the gauge field mass counterterm given by (4).

Our philosophy here has been to take a lattice theory given by (4) having the correct weak coupling limits, and then try and find out the strong coupling properties of the same theory. This is the best one can do.

### 2.1. Phase structure at weak coupling

One can have insight into the phase structure of the weak coupling theory from the leading order effective potential $V_{cl}$, obtained by perturbative expansion of $U_{\mu x} = \exp igA_{\mu x}$ around $U_{\mu x} = 1$ and then requiring the gauge potential $A_\mu$ to be constant,

$$V_{cl} = \kappa \left[ g^2 \sum_\mu A_\mu^2 + \cdots \right] + \frac{g^4}{2\kappa} \left[ \left( \sum_\mu A_\mu^2 \right) \left( \sum_\mu A_\mu^4 \right) + \cdots \right]. \quad (6)$$

$\kappa \equiv \kappa_c = 0$ signals a continuous phase transition (because the gauge boson mass obviously vanishes) between two broken phases: a ferromagnetic (FM) phase for $\kappa > 0$ and a directional ferromagnetic (FMD) phase for $\kappa < 0$. FMD phase is characterize by rotational noninvariance (minimum of $V_{cl}$ is at $A_\mu \neq 0$). At the FM $\sim$ FMD transition the longitudinal gauge dof do not scale and hence get decoupled.

### 3. NONPERTURBATIVE PHASE DIAGRAM

To obtain the phase diagram of the gauge-fixed pure $U(1)$ theory, given by the action (4), in $(\kappa, \kappa_c)$-plane for fixed values of gauge coupling, we defined the following observables:

$$E_\kappa = \frac{1}{4L^4} \left\langle \sum_{x,\mu} \text{Re} U_{\mu x} \right\rangle \quad (8)$$

$$V = \left\langle \left( \frac{1}{L^2} \sum_x \text{Im} U_{\mu x} \right)^2 \right\rangle. \quad (9)$$

$E_\kappa$ and $E_\kappa$ are not order parameters but they signal phase transitions by sharp changes. We expect $E_\kappa \neq 0$ in the broken symmetric phase FM and FMD and $E_\kappa \sim 0$ in the symmetric (PM) phase. Besides, $E_\kappa$ is expected to have a large slope at $2^{nd}$ order phase transition (infinite slope in the infinite volume limit) and a discrete jump at $1^{st}$ order transitions. The true order parameter is $V$ which allows us to distinguish the FMD phase, where $V \neq 0$, from the other phases where $V \sim 0$.

We also probed the pure gauge system by quenched staggered fermions by measuring the chiral condensate,

$$\langle \bar{\chi} \chi \rangle_{m_0} = \frac{L^4}{4} \sum_x \langle M_{xx}^{-1} \rangle \quad (10)$$

where $M$ is the fermion matrix and $m_0$ is the bare mass.

### 3.1. Numerical results

![Figure 1. Phase diagram for: (a) $g = 0.8$ (○), (b) $g = 1.0$ (□), (c) $g = 1.1$ (△), (d) $g = 1.2$ (◇), (e) $g = 1.3$ (▽) (filled symbols for $10^4$ and empty symbols for $4^4$).](image)

The Monte Carlo simulations were done with 4-hit Metropolis algorithm on $4^4$ and $10^4$ lattices.
We explored the phase diagram in \((\kappa, \bar{\kappa})\)-plane at gauge couplings \(g = 0.6, 0.8, 1.0, 1.1, 1.2, 1.3\) and \(1.4\) over a range of \(0.30\) to \(-2.30\) for \(\kappa\) and 0.00 to 1.00 for \(\bar{\kappa}\). Each data point consisted of 45,000 Metropolis sweeps for \(4^4\) lattice and 6,500 sweeps for \(10^4\) lattice. The autocorrelation length was less than 10 for both lattices.

Figure 2 collectively shows the phase diagram in \((\kappa, \bar{\kappa})\)-plane for the different gauge couplings. The diagram looks qualitatively the same for all gauge couplings. At weaker gauge couplings \((g = 0.6, 0.8, 1.0)\) the FM-FMD transition (the dotted lines roughly parallel to the \(g\)-axis) appears to be continuous, as found previously in reduced model [6] and as also in the weak coupling \((g = 0.6)\) investigation [4], done previously.

\[
\begin{align*}
\text{Figure 2. Discontinuity in } E_\kappa \text{ across FM-FMD transition, shown in dotted line.}
\end{align*}
\]

At strong couplings \((g = 1.2, 1.3, 1.4)\) the FM-FMD transition still exists, albeit at larger negative \(\kappa\) resulting in a slimming of the FMD phase. A closer look, as shown in fig. 3, at the nature of change of \(E_\kappa\) across FM-FMD at these couplings indicates a discrete jump, possibly implying a 1st order transition.

Figure 3 shows quenched chiral condensates near the FM-FMD transition (remaining in the FM phase) for different gauge couplings as a function of staggered fermion bare mass \(m_0\). The chiral condensates were computed with the Gaussian noise estimator method. Anti-periodic boundary condition in one direction was employed. Figure 3 clearly indicates that for weaker gauge couplings \((g < 1.1)\) the chiral condensates in the chiral limit (obtained by linear extrapolation) vanish. However, for stronger gauge couplings \((g > 1.1)\) the chiral condensates are not zero in the chiral limit. The above observation may signal confinement on the lattice for the strongly coupled gauge-fixed pure \(U(1)\) theory.

The gauge-fixed pure \(U(1)\) theory with a dimensionless mass counterterm has an expected weak coupling behavior persisting up to \(g = 1.0\). At stronger couplings there is indication for confinement. However, the confinement seems to be a lattice artifact because the FM-FMD transition probably is of 1st order.

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