Anisotropic scalar field emission from TeV scale black hole

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Abstract

Black holes are predicted to be copiously produced at the CERN Large Hadron Collider in the scenarios of TeV scale gravity. We report recent progress in studying decay of such a higher dimensional black hole in $D=4+n$ dimensions through the Hawking radiation into brane localized fields taking into account its angular momentum which is indispensable for realistic simulations. Presented is the greybody factors for a scalar field emission, which confirms our previous results in low energy approximation: (i) the existence of super-radiance modes and (ii) the non-trivial angular distribution of radiated scalar field. Phenomenological implications and future plans are discussed.

1 Introduction

Black hole is one of the most important key objects in theoretical physics. Its quantum behavior and thermodynamic property have played great roles in the path to understand yet unknown quantum theory of gravity but a direct experimental test has been believed almost impossible. Recently, the scenarios of large and warped extra dimension(s) have led to an amazing possibility of producing black holes at the CERN Large hadron Collider (LHC) with distinct signals [1, 2]. When the center-of-mass energy of a collision exceeds the Planck scale, which is of the order of TeV here, the cross section is dominated by a black hole production [3]. In this trans-Planckian energy domain, the larger the center-of-mass energy is, the larger the mass of the resulting black hole, and hence the better its decay is treated semi-classically via Hawking radiation. Main purpose of this talk is to discuss such decay signals.

In previous publications, we have pointed out that the production cross section of a black hole increases with its angular momentum, so that the produced black holes are highly rotating [4, 5]. (See also Ref. [6] for an earlier attempt.) The form factor for the production cross section, taking this rotation into account [4], is larger than unity and increases with the number of extra dimensions $n$. (The result is in good agreement with an independent numerical simulation of a classical gravitational collision of two massless point particles [7].) We note that this form factor is hardly interpretable without considering the angular momentum. It is essential to take into account the angular momentum of the black hole when we perform a realistic calculation of its production and evaporation.

Black holes radiate mainly into the standard model fields that are localized on the brane [8]. The master equation for brane field perturbation on the Myers-Perry higher dimensional rotating black hole background has been derived [11] for general fields with spin zero, one half and one (that is, for all the standard model fields). In the paper [4], we have shown analytic expressions for the greybody factor of $D=5$ (Randall-Sundrum) black hole by solving the master equation under the low energy approximation of radiating field. In this talk we present a generalized result to higher dimensional black hole in $D > 5$ for brane localized scalar field without relying on the low energy approximation and discuss its physical implications.

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2 Master equation for Brane field perturbation

The Teukolsky equation for spin \( s \) brane field in \( D = 4 + n \) dimensional Myers-Perry black hole background has been derived and the resultant equation is shown to be separable into the angular and the radial parts in our previous paper [4]. The angular function \( S \), the spin weighted spheroidal harmonics, is a regular function on \([0, \pi]\) which satisfies the equation

\[
0 = \frac{1}{\sin \vartheta \, d\vartheta} \left( \sin \vartheta \, dS \right) + \left[ (s - a \omega \cos \vartheta)^2 - \left( \frac{s}{\tan \vartheta} + \frac{m}{\sin \vartheta} \right)^2 + E - 2s^2 \right] S,
\]

with the eigenvalue \( E \) for a given angular mode \((l, m)\). This angular equation is exactly the same as the usual four dimensional Kerr black hole. The radial function \( R \) satisfies the equation

\[
0 = \Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{dR}{dr} \right) + \left[ \frac{K^2 - isK\Delta r}{\Delta} - \lambda_s + 4isr\omega + s(\Delta_{rr} - 2) \right] R,
\]

where \( \Delta = r^2 + a^2 - (r_H^2 + a^2)(r/r_H)^{-(n-1)} \), \( K = (r^2 + a^2)\omega - ma \) and \( \lambda_s = E - s(s + 1) - 2ma\omega + (a\omega)^2 \) with \( r_H \) and \( a \) being the horizon radius and the rescaled dimensionless angular momentum of the black hole. The \( \omega \) is the frequency of the radiated field that was assumed to be small in the derivation of the greybody factors in Ref. [4]. If the number of extra dimensions \( n \) is put to be zero, the radial equation reduces to the usual Teukolsky equation in four dimensions.

3 Greybody factor for Brane scalar field emission

Evolution of the mass and the angular momentum of a black hole can be determined by emissions via Hawking radiation up to the final Planck phase where a semi-classical description breaks down

\[
-\frac{d}{dt} \left( \begin{array}{c} M \\ J_r \end{array} \right) = \frac{1}{2\pi} \sum_{s,l,m} g_s \int d\omega \frac{\Gamma_{s,l,m}(\omega)}{e^{\omega - m\Omega/T} + 1} \left( \begin{array}{c} \omega \\ m \end{array} \right),
\]

where \( g_s \) is the effective degrees of freedom for spin \( s \) field, \( \Omega \) is the surface angular velocity of the rotating hole, \( T \) is Hawking Temperature and \( \Gamma_{s,l,m} \) is the greybody factor for a given angular mode \((l, m)\). The analytic expression for \( \Gamma_{s,l,m} \) for five dimensional black hole has been obtained in Ref. [4].

One immediate observation is that the Hawking radiation of \( m > 0 \) mode dominates over that of \( m < 0 \) mode when \( \Omega \to \infty \), i.e. when the hole is highly rotating. In this limit, the modes with \( m > 0 \) show super-radiance with negative greybody factor \( \Gamma_{s,l,m} < 0 \), namely Hawking absorption. This can in principle result in the initial increase of the area and entropy for a highly rotating black hole [10].

The numerical results of the greybody factors for the \( D = 5 \) black hole is in good agreement with the analytic expression in [4]. As an example for \( D > 5 \) case, we present a result of \( l = 1, m = 1 \) mode, where the effects of rotation and the existence of super-radiance (negative greybody factor) can be clearly seen. Complete results for \( D \geq 4 \) will be shown in [4]. We plot for a \( D = 10 \) non-rotating hole \( a = 0 \) (the thin line) and for rotating one \( a = 1 \) (the thick line) in Figure 1. A super-radiance does not exist in negative mode \( m = -1 \) which is not presented here. When black hole is highly rotating, i.e. having large \( a \), the difference between \( m = 1 \) and \( m = -1 \) modes leads to the non-trivial angular distribution of Hawking radiation even for a scalar field emission. This is a clear signature of the rotating black hole. According to the analytic expression for the \( D = 5 \) dimensional black hole [4], the criteria of super-radiance in \( l = 1 \) mode is: \( \omega - m\Omega < 0 \) since \( \Gamma \propto \omega - m\Omega \). The numerical result is consistent with this: for \( a = 1.0 \) which is denoted as the thick line in the figure, we obtain the super-radiance mode when \( \omega \lesssim \Omega \). Further results of the general \( D = 4 + n \) dimensional black holes with varying angular momentum for the general angular modes \((l, m)\) will be presented elsewhere [9].

4 Discussion

Here we have explained the importance of the angular momentum when one considers TeV scale black hole production and evaporation. New numerical results are reported: greybody factor for the brane scalar
emission from a general $D = 4 + n$ dimensional rotating black hole without relying on the low frequency expansions. We observe super-radiance in $m > 0$ modes for $\Omega > 0$. The existence of super-radiance mode is crucially important new feature of the rotating black hole. Nontrivial angular dependence of the emission is observed even for the scalar field, which will be a signature of rotating black hole.

To understand the actual evolution of a black hole and to predict the collider signature, we need further investigations. It is important to determine the greybody factors for spinor and vector fields \[4, 11\]. The evolution of the angular momentum and the mass is determined by integrating Eq. (3), once all the greybody factors are determined. In particular, the spin-down phase, which has been simply neglected so far, can be precisely described. One can in principle determine the angular momentum of the produced black hole from the nontrivial angular distribution of the signals. The super-radiance mode is a unique feature of rotating black hole and its phenomenological implications should be clarified. Detailed simulations of black hole evaporation at LHC are to be done. Some of these points will be presented soon \[9\].

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