A Tale of Two Anomalies

Hooman Davoudiasl * and William J. Marciano †

Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA

The most recent determination of the fine structure constant $\alpha$ seems to point to a $\sim 2.4\sigma$ negative deviation in the measured electron anomalous magnetic moment $g_\mu - 2$. The corresponding experimental value for the muon, $g_\mu - 2$, has long had a $\sim 3.7\sigma$ deviation, in the positive direction. In this short letter, we point out that one real scalar, with a mass of $\sim 250 - 1000$ MeV, could explain the deviations in $g_\mu - 2$ and $g_e - 2$, through one- and two-loop processes, respectively. We briefly discuss potential implications of this simple scenario for low and high energy phenomena.

So far, neither the LHC experiments nor direct searches for dark matter have uncovered any signs of a “natural” Higgs sector nor weak scale dark matter states. However, there have been mild deviations from the Standard Model (SM) predictions over the years, some of which have disappeared with time. Of these, a long-standing one is the $\sim 3.7\sigma$ discrepancy between theory and experiment for the muon anomalous magnetic moment $g_\mu - 2$, which has withstood various theoretical refinements and is being currently remeasured at Fermilab with higher precision. While the final word on $g_\mu - 2$ remains to be decided by the new measurements and ongoing theoretical improvements of the SM prediction (see, for example, Refs. [3, 4]), the deviation has been a subject of intense phenomenological interest. As new physics at the TeV scale gets more constrained, the parameter space for weak scale models that could explain $g_\mu - 2$ starts to close.

Meanwhile, the search for new “dark” or “hidden” states at low mass scales $\lesssim 1$ GeV has recently been getting increasing attention [5, 6], partially spurred by astrophysical considerations related to DM models [7] and perhaps also by the dearth of indications for new high energy phenomena. In fact, $g_e - 2$ has emerged as an interesting target for dark sector searches, since light states with feeble couplings to the SM can in principle explain the anomaly. An early and motivated possibility was offered by the “dark photon” hypothesis, where a new vector boson that kinetically mixes with the photon [8] could have provided a solution [9]. This ideas and its simple extensions have now been essentially ruled out. However, other light states from a dark sector, for example a scalar that very weakly couples to muons could still furnish a potential solution [10].

A recent precise determination of the fine structure constant, $\alpha$, has introduced a new twist to this story. An improvement in the measured mass of atomic Cesium used in conjunction with other known mass ratios and the Rydberg constant leads to the new now most precise value [11]

$$\alpha^{-1}(\text{Cs}) = 137.035999046(27).$$

(For a detailed explanation of that prescription and its use in determining the SM prediction for the electron anomalous magnetic moment, $a_e = (g_e - 2)/2$, see the articles by G. Gabrielse in Ref. [12].) As a result, comparison of the theoretical prediction of $a_e^{\text{SM}}$ with the existing experimental measurement of $a_e^{\text{exp}}$ [13, 14] now leads to a discrepancy

$$\Delta a_e \equiv a_e^{\text{exp}} - a_e^{\text{SM}}$$

$$= [-87 \pm 28(\text{exp}) \pm 23(\alpha) \pm 2(\text{theory})] \times 10^{-14},$$

or when the uncertainties are added in quadrature

$$\Delta a_e = (-87 \pm 36) \times 10^{-14},$$

which represents a $2.4\sigma$ discrepancy that is opposite in sign from the long standing muon discrepancy previously mentioned. Note that the current discrepancy in Eqs. (2) and (3) results from an improvement in $\alpha^{-1}$ from 137.035999895(85) which previously [13] gave $\Delta a_e = -130(77) \times 10^{-14}$ and represented a $1.7\sigma$ effect. The central value has decreased in magnitude, but its significance has increased. The errors from the experimental determinations of $a_e$ and $\alpha$ are now the dominant sources of uncertainty.

Interestingly, simple dark photon models [9] and their extensions predict a positive deviation. Therefore, the negative $\sim 2.4\sigma$ deviation in $g_e - 2$ cannot be simultaneously explained together with the $\sim 3.7\sigma$ anomaly in $g_\mu - 2$ in those models, even if one could circumvent experimental constraints.

In this short letter, we would like to point out that a minimal model based on a single light real scalar $\phi$, can in principle explain the inferred values of both $g_\mu - 2$ and $g_e - 2$, in a relatively economical fashion. We will show that a two-loop process Barr-Zee diagram [16, 17] can explain $g_e - 2$ while a one-loop contribution could be the primary origin of $g_\mu - 2$ [10, 18], mediated by the same scalar $\phi$. For a more detailed discussion of these loop processes and their contributions to the electron...
and muon anomalous magnetic moments see Ref. [19], where the authors discuss the relative contributions of one- and two-loop diagrams, but focus on the case of a pseudoscalar boson. Here, we focus on the effect of a light scalar where the Barr-Zee contribution represents an extension of earlier work in Ref. [10]. Work on the contribution of Barr-Zee type diagrams to $g_{\mu} - 2$ in the context of two Higgs doublet models and supersymmetry can be found in Ref. [20].

Let us consider the following Lagrangian for the real scalar $\phi$ of mass $m_\phi$

$$L_\phi = -\frac{1}{2} m_\phi^2 \phi^2 - \sum_f \lambda_f \phi \bar{f} f \frac{\kappa_\gamma}{4} \phi F^\mu_\nu F^\mu_\nu,$$  \hspace{1cm} (4)

where we only include an explicit coupling to a fermion $f$ with strength $\lambda_f$ and have omitted various kinetic terms and fermion masses. In this work, we allow $f$ to correspond to known quarks and leptons, as well as other potential more massive charged fermions. The $\lambda_f$ are constrained by phenomenology, as will be discussed later. We assume that the coupling to photons, through the field strength tensor $F^\mu_\nu$, is governed by the the constant $\kappa_\gamma$ which has mass dimension $-1$. The sum over couplings to $f$ will induce a contribution to $\kappa_\gamma$, but we do not specify all charged states that couple to $\phi$. Later, when we choose numerical values for our parameters we will discuss the range of values that may be expected on general grounds, consistent with various phenomenological constraints.

We will start with $g_{\mu} - 2$, assumed to be dominated by the one-loop diagram in Fig.1 which is given by [10, 21, 22]

$$\Delta a_\ell = \frac{\lambda_\ell^2}{8\pi^2} x^2 \left[ \sum_{\gamma} (1 + z)(1 - z)^2 \right] (1 - \frac{m_\phi^2}{m_\ell^2}) + \frac{\lambda_\ell^2}{8\pi^2} x^2 \left[ \sum_{\gamma} (1 + z)(1 - z)^2 \right]$$  \hspace{1cm} (5)

for lepton $\ell$ of mass $m_\ell$ and $x \equiv m_\ell/m_\phi$.

Current experimental constraints, as illustrated in Ref. [22], allow $0.25 \text{ GeV} \lesssim m_\phi \lesssim 1 \text{ GeV}$ and $\lambda_\mu \approx (1 - 3) \times 10^{-3}$, roughly corresponding to a range of parameters that can explain the $3.7\sigma$ deviation in $g_{\mu} - 2$, as above, by [3]

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (2.74 \pm 0.73) \times 10^{-9},$$  \hspace{1cm} (6)

which we will approximate as $\Delta a_\mu \approx 3 \times 10^{-9}$. Let us choose, for concreteness,

$$m_\phi = 250 \text{ MeV} \text{ and } \lambda_\mu = 10^{-3},$$  \hspace{1cm} (7)

which according to Eq. (5) yields $\Delta a_\mu \approx 3 \times 10^{-9}$. Since we will focus on $m_\phi$ values significantly larger than $m_\mu = 105.7 \text{ MeV}$ in this work, we roughly have $\Delta a_\mu \propto (\lambda_\mu/m_\phi)^2$ not far from our reference parameters in Eq. (7). However, for $x \ll 1$, one finds $\Delta a_\ell \propto -x^2 (\ln x^2 + 7/6)$ which deviates from the $1/m_\phi^2$ scaling.

Let us now address the mild deviation in Eq. (3). Here, we will concentrate on the “Barr-Zee” diagram in Fig.2, whose leading contribution for a lepton $\ell$ is given by [19, 25]

$$\Delta a_\ell \equiv \frac{\lambda_\ell}{4\pi^2} \frac{\kappa_\gamma}{m_\phi} \ln(\Lambda/m_\phi),$$  \hspace{1cm} (8)

where $\Lambda$ is an ultraviolet cutoff scale. We will later discuss the form of $\lambda_\ell$ obtained from integrating out heavy charged fermions of mass $m_f$ in the two-loop Barr-Zee diagram. In that case, one can show $\ln(\Lambda/m_\phi) = 13/12 + \ln(m_f/m_\phi)$ [24].

Assuming that the charged states that contribute to $\kappa_\gamma$ could be of GeV scale masses (e.g. the $\tau$), we roughly have $\ln(\Lambda/m_\phi) \sim 2 - 3$. For $m_\phi = 250 \text{ MeV}$, as above, and $\lambda_\ell \kappa_\gamma \approx -3 \times 10^{-8} \text{ GeV}^{-1}$, Eq. (8) yields $\Delta a_\ell \approx -9 \times 10^{-13}$, roughly at the level of the apparent experimental anomaly in Eq. (3). The negative sign can be obtained from the choice of various same sign Yukawa couplings, including those that contribute to $\kappa_\gamma$. For concreteness, in the case of the electron, with $m_e = 0.511 \text{ MeV}$, we will take

$$\lambda_e = 3 \times 10^{-4} \text{ and } \kappa_\gamma = -10^{-4} \text{ GeV}^{-1}.$$  \hspace{1cm} (9)

Here, we have assumed that the sign of the $\kappa_\gamma$ is negative, which is a choice corresponding to positive fermion Yukawa couplings to $\phi$, as will be described later.

Note that the chosen value of $\lambda_e$ in Eq. (9) does not follow naive scaling with the lepton mass, $\lambda_e/\lambda_\mu = m_e/m_\mu$, in reference to that of $\lambda_\mu$ in Eq. (7). However, since $\phi$ is not assumed to control the masses of the leptons, this is not an inconsistent choice and can be easily obtained.
from a simple effective theory that does not have hierarchical charged lepton interactions. We would also like to mention that for values of $\ln(\Lambda/m_\phi)$ larger than those assumed in the preceding discussion, one could choose smaller values of $|\lambda_e,\mu|$ due to the enhanced contributions of the Barr-Zee diagrams to both smaller values of $\kappa$, and of the wrong sign to account for the anomaly in $\phi$ charged fermions. For example, for possible couplings of $\phi$ to $\tau$ and charm of $\lambda_f \sim 10^{-2}$, we expect to obtain the requisite photon coupling, to obtain the $a_e$ discrepancy for $\lambda_e = 3 \times 10^{-4}$. One can see this from the estimate (see, for example, Ref. [26])

$$k_{\gamma} \approx -\frac{2\lambda_f}{3\pi m_f} \frac{Q_f^2 N_f'}{m_{\phi}}, \quad (11)$$

where $Q_f$ is the charge of the fermion, $m_f \sim 1$ GeV is of the order $\tau$ or charm mass, and $N_f'$ is the number of colors of fermion $f$. The above formula for $k_{\gamma}$ is obtained in the limit that $m_f \gg m_\phi$. We note that, with our conventions, for $\lambda_f > 0$ one obtains $k_{\gamma} < 0$. Other couplings, such as the one to muons, can enhance the above estimate for $k_{\gamma}$, assuming constructive interference from same sign Yukawa couplings.

In summary, we have shown that a simple model, comprising a singlet scalar $\phi$ of mass $\gtrsim 250$ MeV and $\sim 10^{-3}, 10^{-4}$ couplings to the muon and the electron, respectively, can account for a $\sim 3.7\sigma$ discrepancy in the muon $g - 2$ and a $\sim 2.4\sigma$ discrepancy in the electron $g - 2$ of opposite sign. The former anomaly is mediated through a one-loop diagram, whereas the latter originates from a two-loop Barr-Zee diagram, using aphe-
nomenologically allowed coupling of the scalar to photons. The model could give rise to lepton pair signals in rare meson or tau decays, as well as those in electron and muon scattering processes. A simple effective theory that does not lead to naive scaling of the scalar-lepton couplings with the lepton mass can realize our scenario. The effective theory, in turn, could arise from TeV-scale charged vector-like fermions coupled to the SM Higgs and \( \phi \). In that case, the LHC could potentially discover those fermions, which would shed further light on the underlying physics manifested in the possible deviations of the electron and muon \( g - 2 \).

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[1] G. W. Bennett et al. [Muon G-2 Collaboration], Phys. Rev. D 73, 072003 (2006) [hep-ex/0602035].
[2] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001
[3] T. Blum et al. [RBC and UKQCD Collaborations], [arXiv:1801.07224 [hep-lat]].
[4] A. Keshavarzi, D. Nomura and T. Teubner, [arXiv:1311.0029 [hep-ph]].
[5] J. D. Bjorken, R. Essig, P. Schuster and N. Toro, Phys. Rev. D 80, 075018 (2009) [arXiv:0906.0580 [hep-ph]].
[6] R. Essig, J. A. Jaros, W. Wester, P. H. Adrian, S. Andreas, T. Averett, O. Baker and B. Batell et al., [arXiv:1311.0029 [hep-ph]].
[7] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer and N. Weiner, Phys. Rev. D 79, 015014 (2009) [arXiv:0810.0713 [hep-ph]].
[8] B. Holdom, Phys. Lett. 166B, 196 (1986). doi:10.1016/0370-2693(86)91377-8.
[9] M. Pospelov, Phys. Rev. D 80, 095002 (2009) [arXiv:0811.1030 [hep-ph]].
[10] C. Y. Chen, H. Davoudiasl, W. J. Marciano and C. Zhang, Phys. Rev. D 93, no. 3, 035006 (2016) doi:10.1103/PhysRevD.93.035006 [arXiv:1511.04715 [hep-ph]].
[11] R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Mueller, Science 360, 191 (2018).
[12] G. Gabrielse in Lepton Dipole Moments, edited by B. Lee Roberts and William J. Marciano, World Scientific.
[13] T. Aoyama, T. Kinoshita and M. Nio, Phys. Rev. D 97, no. 3, 036001 (2018) doi:10.1103/PhysRevD.97.036001 [arXiv:1712.06060 [hep-ph]].
[14] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008) doi:10.1103/PhysRevLett.100.120801 [arXiv:0801.1134 [physics.atom-ph]].
[15] D. Hanneke, S. F. Hoogerheide and G. Gabrielse, Phys. Rev. A 83, 052122 (2011) doi:10.1103/PhysRevA.83.052122 [arXiv:1009.4831 [physics.atom-ph]].
[16] S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990) Erratum: [Phys. Rev. Lett. 65, 2920 (1990)]. doi:10.1103/PhysRevLett.65.2920, 10.1103/PhysRevLett.65.21
[17] J. D. Bjorken and S. Weinberg, Phys. Rev. Lett. 38, 622 (1977). doi:10.1103/PhysRevLett.38.622
[18] B. Batell, N. Lange, D. McKeen, M. Pospelov and A. Ritz, Phys. Rev. D 95, no. 7, 075003 (2017) doi:10.1103/PhysRevD.95.075003 [arXiv:1606.04943 [hep-ph]].
[19] G. F. Giudice, P. Paradisi and M. Passera, JHEP 1211, 113 (2012) doi:10.1007/JHEP11(2012)113 [arXiv:1208.6583 [hep-ph]]
[20] K. M. Cheung, C. H. Chou and O. C. W. Kong, Phys. Rev. D 64, 111301 (2001) doi:10.1103/PhysRevD.64.111301 [hep-ph/0103183].
[21] J. P. Leveille, Nucl. Phys. B 137, 63 (1978). doi:10.1016/0550-3213(78)90051-2
[22] D. Tucker-Smith and I. Yavin, Phys. Rev. D 83, 101702 (2011) doi:10.1103/PhysRevD.83.101702 [arXiv:1011.4922 [hep-ph]].
[23] B. Batell, A. Freitas, A. Ismail and D. McKeen, [arXiv:1712.10022 [hep-ph]].
[24] A. Czarnecki and W. J. Marciano, Phys. Rev. D 96, no. 11, 113001 (2017) Erratum: [Phys. Rev. D 97, no. 1, 019901 (2018)] doi:10.1103/PhysRevD.96.113001, 10.1103/PhysRevD.97.019901, 10.1103/PhysRevD.97.019901 [arXiv:1711.00550 [hep-ph]].
[25] W. J. Marciano, A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D 94, no. 11, 115033 (2016) doi:10.1103/PhysRevD.94.115033 [arXiv:1607.01022 [hep-ph]].
[26] M. Carena, I. Low and C. E. M. Wagner, JHEP 1208, 060 (2012) doi:10.1007/JHEP08(2012)060 [arXiv:1206.1082 [hep-ph]].