Creation of three-species $^{87}\text{Rb} - ^{40}\text{K} - ^6\text{Li}$ molecules: interfering for the best

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Abstract. An ultracold three-species Bose–Fermi–Fermi degenerate atomic mixture $^{87}\text{Rb} - ^{40}\text{K} - ^6\text{Li}$ was realized very recently (Tagliber M et al 2008 Phys. Rev. Lett. 100 010401). Here, we study the creation of heteronuclear triatomic molecules in this mixture, and show that a constructive triple-path interference can lead to an almost ideal conversion rate, in comparison with the single- or double-path cases. The important effect of the initial population imbalance on the atom–molecule dark state is also investigated.

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1. Introduction

The experimental realization of Bose–Einstein condensates (BEC) in dilute atomic gases has led to an explosion of research in nonlinear and quantum atom optics [1]–[6]. A remarkable recent development is the creation of homonuclear and heteronuclear molecules not only in purely bosonic samples [7] but also in degenerate Fermi–Fermi [8] or Bose–Fermi [9] mixtures, by using either optical photo-association (PA) [10] or magnetic Feshbach resonance (FR) techniques [11]–[14]. For example, in a recent experiment, Ospelkaus et al [15] created ultracold $^{40}$K$^{87}$Rb heteronuclear molecules with a transfer rate as high as 84%. In view of the anisotropic properties of the dipole–dipole interaction, heteronuclear molecules are of particular interest for a number of applications such as the study of new and exotic quantum phases, quantum information science and quantum computing [15], and the investigation of the crossover between a molecular BEC and a Bardeen–Cooper–Schrieffer-like gas of paired fermions [16, 17]. In a closely related development, we mention experimental work on atom–molecule dark states [18] and proposed frequency-chirped generalizations [19, 20] of the stimulated Raman adiabatic passage (STIRAP) technique [21]–[23] to maintain an approximate coherent population trapping (CPT) state by compensating for particle collisions.

Although most of the recent efforts have focused on diatomic molecules, the assembly of more complex ultracold objects also attracts much interest. Notable developments include the recent experimental evidence on the Efimov trimer state Cs$_3$ [24, 25] and on the molecular tetramer Cs$_4$ [26]. Based on the FR-assisted PA method, the creation of the heteronuclear trimer A$_2$B was also studied in a two-species Bose condensate, where the existence of two different reaction paths and a double-slit-like interference led to an enhanced reaction rate [27]. Hence it is important to identify the effects that can arise from the more complex interference effects associated with the creation of the heteronuclear trimer ABC from a three-species atomic mixture.

Very recently, Taglieber et al [28] realized experimentally for the first time a quantum-degenerate three-species Bose–Fermi–Fermi mixture, $^{87}$Rb–$^{40}$K–$^6$Li. This provides a promising new platform for the study of the possible creation of ABC trimers as well as the investigation of Fermi–Fermi or Bose–Fermi mixtures within one sample. As an interesting extension of this work, the intriguing effects of color superfluidity and quantum-phase transitions in three-species ultracold atomic fermions have also been discussed [29, 30]. We also note that, by considering the dissociation dynamics of the ABC trimer, Moore and Vardi [31] predicted the possibility of an almost complete selectivity between two dissociation channels of the ABC trimer, a result of the interplay between Bose enhancement and competition between modes for a finite number of initial molecules.

The purpose of the present paper is to show that, by starting from an $^{87}$Rb–$^{40}$K–$^6$Li mixture, one can exploit a constructive multi-path interference in the atom–molecule conversion. In particular, the triple-path case can be almost ideal and far more efficient than the single- or double-path cases. We also point out that, throughout the process of multi-path conversion, the choice of the external fields needs to account for the imbalance in the initial atomic populations in order to keep the dimer states unpopulated, that is, to guarantee that it acts as an (approximate) dark state. While the proposed scheme is experimentally challenging, recent progress in the manipulation of atom–dimer or even dimer–dimer resonances [26, 32, 33] indicates that it may become possible in the not too distant future.
2. The single-path scheme: role of population imbalance

As illustrated in figure 1, there are three different paths that lead to the creation of the heteronuclear trimer ABC in an $^{87}$Rb–$^{40}$K–$^6$Li mixture via the FR-assisted PA method. They involve the intermediate fermionic dimers AB or AC or the bosonic dimer BC, respectively. The basic idea is to couple the atoms to the molecular dimers via FR, the dimers being in turn photoassociated with the atoms to form bound trimers. The key to the method under investigation in this paper is to arrange the PA laser pulses in such a way that the dimer states are dark states. In that case the dimer population never becomes significant, thereby eliminating the collision losses that would otherwise significantly reduce the conversion process.

Denoting the atom–dimer coupling strength via FR as $\lambda_1'$, with detuning $\delta$, and denoting the Rabi frequency of the PA field as $\Omega_1'$, with detuning $\Delta$, the simplest energy density corresponding to the path-AB case is given in the Hartree approximation by (see [34])

$$E_{AB} = \sum_{i \neq j} \frac{1}{2} \chi_{ij}^2 |\psi_i|^2 |\psi_j|^2 + \delta |\psi_{d_1}|^2 + \lambda_1' \left[ |\psi_{a_1}|^2 |\psi_{a_2}|^2 + h.c. \right] + (\Delta + \delta) |\psi_g|^2$$

$$- \Omega_1' \left[ |\psi_{g_1}|^2 |\psi_{c_1}|^2 + h.c. \right] + \sum_{i=(a,g)} \frac{1}{2} \chi_i |\psi_i|^4 + \sum_{f=(b,c,d_1)} \frac{3}{2} A_f' |\psi_f|^{10/3}. \quad (1)$$

Here, $\psi_i$ represents the atomic or molecular probability amplitude, the indices $i, j = a, b, c, d_1, g$ stand for the atoms, dimers or trimers, respectively, and $A_f' = (6\pi^2)^{2/3}/2M_f$, with $M_f$ ($f = b, c, d_1$) denoting the mass of the fermionic components. The terms proportional to $\chi_{ij} = 2\pi a_{ij}/M_{ij}$ describe the s-wave collisions between the species $i$ and $j$, $a_{ij}$ is the s-wave scattering length, and $M_{ij} = M_iM_j/(M_i + M_j)$ is the reduced mass ($\chi_i' = \chi_{ii}'$). Note that there are no s-wave collisions between fermionic atoms of the same species, so that the kinetic energy dominates the intra-species interaction. Hence it is a good approximation to consider only the kinetic energies for same-species fermions, but to assume that the collisions are the dominating terms for the bosons and for inter-species fermion–fermion interactions (see [34] for a similar treatment of atom–dimer conversion).
The ordinary mean-field approach [35, 36] is suitable for treating systems with large numbers of condensed bosonic particles, but is of course inadequate for fermions. For the present system containing a large number of fermions, brute-force numerics becomes rapidly prohibitive. Here, we follow the approach of Lu and Li [34] by starting from the mean-field Lagrangian density [37, 38] and using the Euler–Lagrange equations to derive mean-field dynamic equations. We then consider a single evolution equation for each particle species, noting that due to the different nature of the self-interaction terms, there are important differences between the mean-field dynamic equations for fermions and those for bosons. With these considerations in mind, we introduce the mean-field Lagrange density

\[ \mathcal{L} = \frac{i}{2} \sum_i \left( \psi_i^* \frac{\partial \psi_i}{\partial t} - \psi_i \frac{\partial \psi_i^*}{\partial t} \right) - E_{AB} \]

for the system, and substitute it into the Euler–Lagrange equations

\[ \frac{\partial \mathcal{L}}{\partial \psi_i^*} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi_i^*)} \right) = 0. \]

This yields a set of coupled evolution equations for each species. For example the path AB is described by the following equations of motion, with \( \phi_i = \psi_i / \sqrt{n} \) and \( n \) the initial particle density:

\[
\begin{align*}
\frac{d\phi_a}{dt} &= -i \sum_j \chi_{aj} |\phi_j|^2 \phi_a - i\lambda_1 \phi_d \phi_b^*, \\
\frac{d\phi_b}{dt} &= -i \sum_{j \neq b} \chi_{bj} |\phi_j|^2 \phi_b - i\lambda_1 \phi_a \phi_d^* - iA_b |\phi_b|^{4/3}, \\
\frac{d\phi_c}{dt} &= -i \sum_{j \neq c} \chi_{cj} |\phi_j|^2 \phi_c + i\Omega_1 \phi_b \phi_d^* - iA_c |\phi_c|^{4/3}, \\
\frac{d\phi_d}{dt} &= -i \sum_{j \neq d} \chi_{dj} |\phi_j|^2 \phi_d - i\lambda_1 \phi_a \phi_b + i\Omega_1 \phi_b \phi_d^* \\
&- (\gamma + i\delta) \phi_d - iA_d |\phi_d|^{4/3}, \\
\frac{d\phi_g}{dt} &= -i \sum_j \chi_{gj} |\phi_j|^2 \phi_g - i(\Delta + \delta) \phi_g + i\Omega_1 \phi_d \phi_c.
\end{align*}
\]

Here, \( \chi_{ij} = \chi_{ij} n, \ A_f = A_f n^{2/3}, \lambda_j = \lambda_j \sqrt{n}, \ \Omega_l = \Omega_l \sqrt{n} \ (l = 1, 2, 3), \) and the decay rate \( \gamma \) is introduced phenomenologically to simulate the loss of dimers, based on the assumption that this decay dominates all other loss mechanisms, such as e.g. non-resonant scattering. As already mentioned we proceed to minimize this decay by exploiting the CPT dark state technique, which ideally permits the full transfer of the entrance-channel atomic state to the closed-channel molecular trimer state while keeping the intermediate dimer state unpopulated at all times. It is known that this method can be adapted to nonlinear situations as considered here by linearizing the nonlinear dynamics of the system around the intended CPT evolution [20].
The steady-state solution of equations (4) can be found with the help of the trial wavefunctions

\[ \phi_a = |\phi_a^0| e^{i\theta_a} e^{-i\mu_a t}, \quad \phi_b = |\phi_b^0| e^{i\theta_b} e^{-i\mu_b t}, \quad \phi_c = |\phi_c^0| e^{i\theta_c} e^{-i\mu_c t}, \]

\[ \phi_{d1} = |\phi_{d1}^0| e^{i(\theta_a + \theta_b)} e^{-i(\mu_a + \mu_b) t}, \]

\[ \phi_g = |\phi_g^0| e^{i(\theta_a + \theta_b + \theta_c)} e^{-i(\mu_a + \mu_b + \mu_c) t}, \] (5)

where \( \mu_a, \mu_b \) and \( \mu_c \) are the atomic chemical potentials. Inserting equations (5) into equations (4) and requiring that \( |\phi_g^0| = 0 \), one finds the CPT solution in the case of equal initial populations of the three atomic species,

\[ |\phi_g^0|^2 = \frac{1}{3[1 + (\Omega_1/\lambda_1)^2]}, \] (6)

under the generalized ‘two-photon’ resonance condition

\[ \Delta^{AB} = -\delta + \chi_1|\phi_a^0|^2 + \chi_2|\phi_b^0|^2 + \chi_3|\phi_c^0|^2 + \chi_4|\phi_{d1}^0|^2 + A_{ab}|\phi_b^0|^3 + A_{bc}|\phi_c^0|^3, \]

\[ \mu_A^a = \chi_{aa}|\phi_a^0|^2 + \chi_{ab}|\phi_b^0|^2 + \chi_{ac}|\phi_c^0|^2 + \chi_{ba}|\phi_{d1}^0|^2, \]

\[ \mu_B^b = \chi_{ab}|\phi_a^0|^2 + \chi_{bc}|\phi_c^0|^2 + \chi_{bg}|\phi_{d1}^0|^2 + A_{ab}|\phi_b^0|^3, \]

\[ \mu_C^c = \chi_{ac}|\phi_a^0|^2 + \chi_{cb}|\phi_b^0|^2 + \chi_{cg}|\phi_{d1}^0|^2 + A_{bc}|\phi_c^0|^3, \] (7)

where \( \chi_1 = \chi_{aa} + \chi_{ba} + \chi_{ca} - \chi_{gb} \), \( \chi_2 = \chi_{ab} + \chi_{cb} - \chi_{gb} \), \( \chi_3 = \chi_{ac} + \chi_{bc} - \chi_{gc} \), \( \chi_4 = \chi_{ag} + \chi_{bg} + \chi_{cg} - \chi_{gg} \), and the superscript ‘AB’ refers to the path AB. The general case with an initial population imbalance can be studied without any difficulty. We return to that point shortly.

In order to support our analysis we have solved numerically equations (4) assuming that initially the atomic populations are the same and that there are no molecules present. Typical results are shown in figure 2. In that specific example, the external fields are \( \lambda_1 = 9.07 \times 10^5 \text{s}^{-1} \) \( (l = 1 \text{ for the path AB}) \) and

\[ \Omega_1(t) = \Omega_{1,0} \text{sech}(t/\tau), \] (8)

with \( \Omega_{1,0}/\lambda_1 = \lambda_1 \tau = 20 \). The collision parameters are chosen as \[34\] \( \chi_{aa} = 0.0056\lambda_1, \chi_{ab} = 0.23\lambda_1 \) and \( \chi_{ac} = \chi_{bc} = 0.0098\lambda_1 \), and \( A_{ab} = 0.3\lambda_1, A_c = 0.5762\lambda_1, A_{ad_1} = 0.09\lambda_1 \) and \( A_{d_2} = 0.1\lambda_1 \). All other collision values are taken as zero due to the lack of good estimates. We have carried out a number of additional simulations for a large set of plausible collisional parameters and found results similar to those of figure 2.

Figure 2(a) shows the creation of trimers ABC for \( \delta = \pm 1 \) when considering only the single-path AB. We also carried out simulations for \( \delta = \pm 2 \) and \( \delta = \pm 3 \), and the final conversion rate \( |\phi_g(t = \infty)|^2 \) was found to always exceed 0.22 in those cases for negative detunings (the maximum theoretical value is of course 0.33). In contrast, for \( \delta = 1 \) (or other positive detunings, such as \( \delta = 2 \) or \( \delta = 3 \)), the trimer populations initially followed CPT solutions, but remarkable unstable features appeared at about \( t = 86/\lambda_1 \), leading to final conversion rates less than 0.1. The reason for a smaller trimer population than predicted from CPT is that we cannot maintain the two-photon resonance condition \( \Delta^{AB} \) for the whole duration of the \text{sech} pulse.

Figure 2(b) is a plot of the conversion rate as a function of \( \delta \) for various ratios of the initial particle numbers, i.e. \( N_a, N_b, N_c = 1:1:1, 1:2:1, \) or \( 2:1:2 \), respectively, illustrating
the significant effect of any initial population imbalance. We conclude the single-path discussion by remarking that we have found very similar results for the other paths (paths AC and BC), with the same CPT state as in equation (6) and the same two-photon resonance condition as in equations (7). Note also that in the short-time limit it is possible to recover the mean-field dynamical behavior from a full quantum treatment [39].

3. The double-path case: interfering for the better

We now show that any two of the three single paths can be combined to achieve a significantly enhanced conversion rate, a result of constructive quantum interferences between these paths. As a concrete example, we consider the two paths AC and BC, for which the energy density is (see [34])

\[ E_{AC+BC} = \sum_{i \neq j} \frac{1}{2} \chi^2_{ij} |\psi_i|^2 |\psi_j|^2 + \delta (|\psi_{di}|^2 + |\psi_{dc}|^2) + (\Delta + \delta) |\psi_d|^2 \]

\[ + \lambda_2 [\psi_{d2}^* \psi_a \psi_c + \text{h.c.}] + \lambda_3 [\psi_{d3}^* \psi_b \psi_c + \text{h.c.}] - \Omega_2 [\psi_{g2}^* \psi_d \psi_b + \text{h.c.}] \]

\[ - \Omega_3 [\psi_{g3}^* \psi_{d3} \psi_a + \text{h.c.}] + \sum_{i=(a,d_3,g)} \frac{1}{2} \chi^2_{i} |\psi_i|^4 + \sum_{f=(b,c,d_2)} \frac{1}{2} A_{f}^* |\psi_f|^{10/3}. \]  (9)

It is straightforward to show that, under the same two-photon resonance condition as in the single-path cases, the steady-state ansatz leads to $|\phi^{0}_{d2}|^2 = |\phi^{0}_{d3}|^2 = 0$ and

\[ \lambda_2 |\phi^0_a| |\phi^0_c| = \Omega_2 |\phi^0_g| |\phi^0_b|, \quad \lambda_3 |\phi^0_b| |\phi^0_c| = \Omega_3 |\phi^0_g| |\phi^0_a|, \]  (10)

from which we can calculate the CPT value of the trimer population as

\[ |\phi^0_g|^2 = \frac{(\lambda_3 / \Omega_3) (\lambda_2 / \Omega_2)^2}{\lambda_3 / \Omega_3 + 2\lambda_2 / \Omega_2 + 3(\lambda_3 / \Omega_3)(\lambda_2 / \Omega_2)^2}. \]  (11)
Figure 3. Interfering for the better in atom–trimer conversion. Panel (a) shows the population of the trimers and of atoms A and C for $\delta = \pm 1$. Here $A_d = 0.1$ and all other parameters are as in the single-path case. Panel (b) shows the final trimer population $|\phi_g(t = \infty)|^2$ as a function of $R_1$ for $\delta = -1$.

We now introduce the ratio $R_1 = \eta_2 / \eta_3$ (with $\eta_l = \Omega_l / \lambda_l$) of the external fields of the two paths, and observe that the freedom of choice of $R_1$ provides additional flexibility in attempting to approach the ideal CPT conditions via the constructive interference between these paths.

Figure 3(a) shows an example of a numerical simulation of the mean-field dynamics for $R_1 = 1$ and $\delta = \pm 1$. It illustrates the significant enhancement in trimer production as compared with the single-path situation (for $\delta = 1$, the final conversion rates are 0.078 or 0.234 for the single- or double-path cases). We found a similar enhanced atom–trimer conversion for $\delta = \pm 2$ and $\delta = \pm 3$ when choosing $R_1 = 1$. Note that $R_1 = 1$ is the optimal choice for the conditions of these simulations, as illustrated in figure 3(b), which shows the conversion rate as a function of $R_1$ with a clear maximum at $R_1 = 1$. Note that this value of $R_1$ is strongly dependent on the initial atomic populations, which are taken to be equal in this example. We return to this point in the next section.

4. The triple-path case: interfering for the best

We finally turn to the triple-path formation of trimers. A point of particular importance is to determine the quantum interference of all three channels with multi-dark states (AB, AC and BC), so that an almost ideal conversion rate can be approached even for positive detunings. By using the steady-state ansatz with $\mu_a$, $\mu_b$ and $\mu_c$ as in equations (5), it is straightforward to find the CPT steady-state solution

$$|\phi_g^0|^2 = \frac{(\lambda_1 / \Omega_1)(\lambda_2 / \Omega_2)(\lambda_3 / \Omega_3)}{\lambda_1 / \Omega_1 + \lambda_2 / \Omega_2 + \lambda_3 / \Omega_3 + 3(\lambda_1 / \Omega_1)(\lambda_2 / \Omega_2)(\lambda_3 / \Omega_3)},$$

$$|\phi_b^0|^2 / |\phi_c^0|^2 = (\lambda_2 / \Omega_1) / (\lambda_1 / \Omega_2),$$

$$|\phi_b^0|^2 / |\phi_a^0|^2 = (\lambda_2 / \Omega_2) / (\lambda_3 / \Omega_2),$$

$$|\phi_a^0|^2 / |\phi_c^0|^2 = (\Omega_1 / \lambda_1) / (\lambda_2 / \Omega_2).$$

(12)
and $|\phi_{d_1}^0|^2 = |\phi_{d_2}^0|^2 = |\phi_{d_3}^0|^2 = 0$, under the same ‘two-photon’ resonance condition as equations (7).

Figure 4(a) plots the result of typical numerical simulations for $R_1 = R_2 = 1$, where

$$R_1 = \eta_2/\eta_3, \quad R_2 = \eta_1/\eta_2,$$

and $\eta_l/\lambda_l (l = 1, 2, 3)$. We find that stable trimer formation is always possible. Most importantly, and in contrast with the single- or double-path cases, the triple-path interference can lead to an almost ideal conversion rate, especially for $\delta = 1$. In fact, we can almost not distinguish the cases $\delta = 1$ and $\delta = -1$ in figure 4(a), both of which reach a final population of 0.2836. This indicates a significant improvement even over the double-path situation.

Figure 4(b) shows the final conversion rate $|\phi_0(t = \infty)|^2$ as a function of $R_1$ and $R_2$ for $\delta = -1$. We also carried out our simulations for a number of other cases. For example, the final atom–trimer conversion rate $|\phi_0(t = \infty)|^2$ is about 0.294 for $\delta = 0$.

Population imbalance often plays an important role in the physics of ultracold matter waves. For example, by starting from a two-spin-state mixture of ultracold fermionic atoms, the population imbalance can result in a superfluid to normal phase transition [40]. In the following we show that an initial populations imbalance can also significantly affect the dynamics of atom–molecule conversion. To this end we introduce parameters $P$ and $Q$ to characterize the initial atomic population imbalance as

$$P = \frac{n_a - n_b}{n_a + n_b + n_c}, \quad Q = \frac{n_a - n_c}{n_a + n_b + n_c},$$

where $n_a, n_b, n_c$ are the atomic densities. Since the total particle number is conserved we have that

$$|\phi_a^0|^2 + |\phi_b^0|^2 + |\phi_c^0|^2 + |\phi_0^0|^2 = 1 + \frac{P + Q}{3},$$

$$|\phi_a^0|^2 + |\phi_b^0|^2 + |\phi_c^0|^2 + |\phi_0^0|^2 = 1 - \frac{2P + Q}{3}.$$
\begin{align}
|\phi_e^0|^2 + |\phi_{a1}^0|^2 + |\phi_{a3}^0|^2 + |\phi_{b2}^0|^2 &= \frac{1 + P - 2 Q}{3},
\end{align}

and from the steady-state conditions we obtain
\begin{align}
|\phi_{a1}^0|^2 &= \eta_1 \eta_2 |\phi_{b2}^0|^2, \quad |\phi_{a3}^0|^2 &= \eta_1 \eta_3 |\phi_{b2}^0|^2, \quad |\phi_{b2}^0|^2 &= \eta_2 \eta_3 |\phi_{b2}^0|^2.
\end{align}

The CPT solution for the general case is then
\begin{align}
|\phi_{b2}^0|^2 &= \frac{(1 + P + Q)/3}{1 + \eta_1 \eta_2} = \frac{(1 - 2 P + Q)/3}{1 + \eta_1 \eta_3} = \frac{(1 + P - 2 Q)/3}{1 + \eta_2 \eta_3},
\end{align}

which can be re-expressed as
\begin{align}
P &= \frac{\eta_1 \eta_2 - \eta_3 \eta_1}{3 + \eta_1 \eta_2 + \eta_2 \eta_3 + \eta_3 \eta_1}, \quad Q = \frac{\eta_1 \eta_2 - \eta_2 \eta_3}{3 + \eta_1 \eta_2 + \eta_2 \eta_3 + \eta_3 \eta_1}.
\end{align}

Obviously, \( P = Q = 0 \) for \( \eta_1 = \eta_2 = \eta_3 \), a property closely related to the fact that in that case the trimer conversion rate is dramatically improved in the double-path case for \( R_1 = \eta_2 / \eta_3 = 1 \) (see figure 3(b)) and in the triple-path case for \( R_1 = \eta_2 / \eta_3 = 1, \ R_2 = \eta_1 / \eta_2 = 1 \).

In our earlier work on the creation of the trimer \( \text{A}_2\text{B} \) [27], we found that \( R = 2 \) (see [27], for the definition of \( R \)) leads to an optimal conversion rate. The present discussion clarifies the fact that this specific value is a result of the fact that our simulations assumed an initial ratio of 2 between atoms A and B. This value, quite different from the value of \( R = 1 \) found for equal initial populations, illustrates clearly the crucial role of the initial atomic population imbalance in any multi-dark state scheme of atom–molecule conversion, a point that was not discussed in [27].

Finally, we note that the timescale for conversion of the atoms into trimers can be much faster than the characteristic time associated with loss mechanisms. Maximizing trimer conversion efficiency for a specific pulse duration \( T \) and effective Rabi frequencies \( \Omega_l (l = 1, 2, 3) \) can be thought of as an optimization problem [41] whose solution involves selecting the optimal route to enhance the conversion rate while minimizing non-ideal factors.

5. Conclusion

In summary we have shown that the FR-assisted PA or STIRAP technique can be applied, at least in principle, to the creation of trimers with the recently realized \(^{87}\text{Rb}–^{40}\text{K}–^{6}\text{Li}\) three-species atomic mixture [28]. The coexistence of three reaction paths can lead to a significant enhancement of the atom–heteronuclear trimer conversion by interfering for the best. Our simulations show that stable atom–trimer conversion is always possible for certain parameters by keeping the intermediate dimer state unpopulated. The important effect of the initial atomic population imbalance on the multi-dark state atom–molecule conversion was also clearly demonstrated.

Our future work will consider the effect of quantum noise in the early stages of trimer production. We also plan to study the quantum properties of quantum ultracold gases with three-component fermionic species in optical lattices and to investigate the phase transition between atomic state, trion state and color superfluidity by using FR to change the atomic interaction [29]. In view of the rapid advances in achieving ultracold three-species atomic mixtures [28], the fascinating physics of coherent assembly and quantum control of
multi-species heteronuclear molecules [29] holds much promise of showing exciting new developments.

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