Study of high-frequency oscillations and waves in Hall thruster plasma using two-dimensional full kinetic axial-azimuthal model (2D 3V Full PIC)

I Khmelevskoi¹², D Tomilin¹ and A Lovtsov¹²

¹Department of Electrophysics, Keldysh Research Centre, 8 Onezhskaya Street, Moscow 125438, Russia
²Department of Aerophysics and Space Research, Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow region 141700, Russia

*E-mail: khmelevskoi@kerc.msk.ru

Abstract. The paper presents the results of Hall thruster plasma simulation using a 2D 3V axial-azimuthal Full PIC model. Two types of solution are investigated in detail for two different magnitude of the magnetic field. In the calculations, different types of azimuthal waves were observed. By their characteristics these waves can be corresponded to the electron drift instabilities, gradient-drift waves and ion plasma resonance. The ion plasma resonance was observed in calculations with the magnetic field increase.

1. Introduction

Hall thrusters have long history of various purposes spacecrafts onboard application. Hall thruster (HT) is a coaxial plasma device that uses electrostatic and magnetic forces. Electrostatic forces are used to accelerate an ionized propellant. Magnetic forces are used both to limit the mobility of electrons and to provide an efficient ionization [1-3].

Collisions with neutral particles and interactions with the walls cannot explain the experimental value of electron current. Modelling the Hall thruster plasma allows to investigate the processes responsible for the electron conductivity. Axial-radial geometry is typical for modelling of hall thruster [4-6]. For models in this geometry, it is possible to take into account correctly the interactions with the wall and secondary electron emission. Nevertheless, these models do not give the experimental value of the electron current without artificial additive (Bohm conductivity) [7]. The dependence of Bohm conductivity is inversely proportional to the magnetic field value and the Bohm coefficient. This dependence is a generalization of a large amount of experimental data. Also, azimuthal oscillations and waves in a plasma can significantly affect the electron current in HT. However, this effect can be directly taken into account either in three-dimensional [8] or in 2D axial-azimuthal models [9-13]. Three-dimensional models of real HT devices require high computational resources. Axial-azimuthal models cannot precisely take into account the real configuration of the magnetic field in the HT and the plasma interactions with walls. It is difficult to obtain prediction of HT operating parameters using such axial-azimuthal models. Despite these disadvantages, axial-azimuthal models can be used to research the influence of azimuthal waves on the dynamics of electrons.

In Ref. [9] the authors developed a two-dimensional PIC model for studying the plasma stability in the electron drift direction. The authors suggested that the plasma instabilities could explain the
experimentally observed values of the electron current. In Ref. [10–13] the influence of electron drift instability (EDI) on the dynamics of electron component of plasma was studied using the two-dimensional axial-azimuthal PIC models. The influence of the propellant type (Xe, He, Ar, Kr) on the HT discharge in the axial-azimuthal plane was studied using the 2D PIC/MCC model [14]. The authors of 2D3V Full PIC axial-azimuthal model [15] took into account diamagnetic effects. Two types of azimuthal instabilities were observed in the result of modelling: gradient drift waves [16, 17] and electron drift instability. In addition, the influence of the azimuthal component on the dynamics of electrons was studied in the axial-azimuthal plane using a hybrid fluid-particle-in-cell model [18].

This paper presents the results of two-dimensional modelling of a Hall thruster plasma in the axial-azimuthal plane using 2D3V Full PIC model. This model does not require approximation of Bohm diffusion to reproduce correct level of electron transport. The aim of this research is to study the dependence of the solution from the magnitude of the magnetic field. The research is focused on the azimuthal waves and instabilities in the HT plasma.

This work was presented at 18th International Conference “Aviation and Cosmonautics” (Moscow, 18-22 November, 2019).

2. Model description
In Keldysh Research Centre a 2D3V Full PIC model of the Hall thruster plasma has been developed. A rectangle-shaped geometry with periodic boundary conditions in the azimuthal direction and free boundary conditions in the other direction (axial direction) was chosen. The domain scheme is shown in figure 1.

![Figure 1. The calculation region scheme.](image)

The region which called “Layer” in the figure 1 is the discharge region, which contains the most of the electric field. This geometry describes a sector of a fixed radius cylinder in the HT discharge channel. The channel height of the HT is considered constant for the entire computational domain. The radial magnetic field is perpendicular to the modelling plane. It corresponds to the \( B_z \) component in the model and is considered static. The distribution of the electric field potential is determined at each time step by solving the Poisson equation with periodic boundary conditions along the X axis and the Dirichlet
boundary conditions along the Y axis using the fast Fourier transform. At the cathode boundary the electron flux is maintained at the level necessary to support plasma quasi-neutrality in the near-cathode region.

Interactions between particles are modelled by the Monte Carlo method. The following types of interactions are taken into account: ionization collisions [19], elastic collisions of electrons with neutral particles [20], excitation collisions [20], Coulomb collisions of electrons with ions [21], electron interactions with walls. Collisions with the walls are modelled with the semi-empirical method. The method implies three steps. First, the probability of electron collision with the walls is calculated. Second, the Debye sheath is calculated with the condition that the ion flux to the wall is equal to the electron flux. Third, electrons with the energy larger than the Debye sheath potential reach the wall.

3. Simulation results
In present research the computational domain is a cylindrical sector with a length of 35 mm and a width of 64 mm. The height of the channel is equal to 14 mm. The calculation grid is \(141 \times 256\), the grid step is 0.25 mm. The time step is equal to \(4 \times 10^{-11}\) s. The potential difference between the anode and cathode boundary is equal to 500 V. The xenon consumption is 1 mg/s. To obtain a stable solution, the grid step must be smaller than the Debye radius in the calculation [22]. There is common technique to fulfil this condition [23]. The vacuum permittivity value was artificially increased by a factor \(\gamma = 150\). The time step is chosen to resolve plasma oscillations and cyclotron motion of electrons [22]. There are about \(10^6\) particles of each component in simulation domain. The calculations were carried out on a single CPU.

The computational domain size in the azimuthal direction is approximately equal to 0.5 \(\cdot \pi\) radian of the circumference of the HT laboratory model with the mean diameter equal to 77 mm. This laboratory model was developed and tested at Keldysh Research Centre [24]. The magnetic field (figure 2a) correspond to the magnetic field values of the HT laboratory model. The magnetic field values were obtained using the Magnet2D software [25]. The line 1 in figure 2a is 30% lower than the line 2. The first calculation was carried out with breathing mode oscillations taken into account. However, the main focus of the paper is the high-frequency oscillations which timescale is much smaller than breathing mode oscillations, consequently, it is possible to use permanent ionization function instead of self-consistent simulation of the ionisation process. Therefore the ionization function is chosen to be equal to the average value during the period of ionization oscillations (breathing mode) for all the further calculations.

The initial distributions of plasma components corresponds to the chosen ionisation function. First, the neutral particles are placed in simulation region in accordance with the permanent ionization function. Figure 2b shows the neutral gas concentration and the normalized ionization function. Second, the plasma components are placed in computational domain. The neutral particles do not move during this phase of the simulation. Particle flows get into the equilibrium when the integral of the ionization function is approximately equal to the difference of the particle fluxes across the \(Y\) direction boundaries. All the data presented further were obtained after the equilibrium was reached. Hereinafter, the solution with the lower magnetic field (red curve in figure 2a) is called solution 1, and the solution with higher magnetic field (blue line in figure 2a) is called solution 2.

The solutions have significant differences. The graphs in figure 3 and 4 is obtained by averaging the parameters over the time of \(8 \cdot 10^{-7}\) s. Figures 3 a-d shows two-dimensional distributions of the electron concentration (figure 3a and figure 3b) and plasma potential (figure 3c and figure 3d). One-dimensional distributions is obtained by averaging the parameters along the azimuthal component (figure 3e and figure 3f). Graphs in the figure 3a, 3c, 3e correspond to solution 1, and graphs in figure 3b, 3d, 3f correspond to the solution 2. For the solution 2, the main potential drop is located near the maximum of the ionization function. A characteristic property of the solution 2 is the local maximum of concentration along the Y axis near the magnetic field maximum. Figure 3a and 3b show the short-wave perturbations (electron concentration). Similar oscillations were obtained in [10–13] and represent as electron drift instability. These waves are characterized by a wavelength in the range from the Debye
radius to the electron cyclotron radius. The waves propagate in the azimuthal direction at near ion-sound velocity.

Figure 2. (a) Relative value of the magnetic field \( B \) for solution 1 and 2 (correspond to the number in legend); (b) Relative value of the ionization function \( S \) and neutral concentration profile \( N_{n_0} \).

Figure 3. Electron concentration \( n_{e_1}, 1/m^3 \) for solutions 1 (a) and 2 (b), plasma potential \( \phi, V \) for solutions 1 (c) and 2 (d), one-dimensional electron and ion \( n_i, 1/m^3 \) concentration and plasma potential for solutions 1 (e) and 2 (f).
Figure 4 shows two-dimensional and one-dimensional distributions of the electron temperature and the drift velocity for two solutions. The electron temperature is shown in figure 4a-b and the drift velocity is shown in figure 4c-d. Figure 4e-f contains the one-dimensional distributions of the electron temperature and the drift velocities. The solution 1 (figure 4a, 4c, 4e) is characterized by a maximum of the electron temperature and drift velocities in the region of the magnetic field decrease (right part of computational domain). The maximum values of the parameters in the solution 2 (figure 4b, 4d, 4f) is shifted to the ionization region. The electron temperature and the drift velocity in the ionization region (according to figure 1) are significantly higher for the solution 2 than for the solution 1. Also, the solution 2 is characterized by the presence of a large azimuthal heterogeneity in the region of the magnetic field decrease. This heterogeneity occurs in the distributions of all plasma parameters: concentrations, electron temperature, electron drift velocity and plasma potential.
Next, the dispersion function of azimuthal waves in the region of magnetic field decrease was investigated. Figure 5a-b shows the two-dimensional Fourier transform in time and X coordinate for the fixed $Y = 17.5$ mm. For the solution 1 (figure 5a), the dispersion function is approximately linear. Figure 5c shows the electron azimuthal velocity distribution and the spectral density of waves’ phase velocity for the solution 1. The spectral density of waves’ phase velocity and the azimuthal velocity distribution are overlapped. It means that the $\omega / k \approx V_d$ condition is satisfied for several frequencies. Therefore, there is an interaction between the azimuthal waves and the motion of the electrons, which leads to an additional transport of electrons across the magnetic field. According to the characteristics of these oscillations, the observed azimuthal waves correspond to gradient drift ones [15-17, 26-28]: the wavelengths exceed the cyclotron radius of the electrons, and the frequencies are in the range from 1 to 200 MHz.

![Figure 5](image)

**Figure 5.** Dispersion functions of the azimuthal waves for solutions 1 (a) and 2 (b); (c) – electron drift velocity distribution and the spectral density of waves’ phase velocity ($\rho$); (d) – ratio of the electron concentration to the magnetic field for solutions 1 and 2 (correspond to the number in legend). Notifications: $\omega$ is frequency, $k$ is wave vector in the azimuthal direction, $\Omega_i$ is ion plasma frequency, $V_x$ – electron azimuthal velocity distribution function.

For the solution 2 the spectrum (figure 5b) significantly differs from the corresponding spectrum of the solution 1 (figure 5a). The main maximum is found at frequency equal to ion plasma frequency ($\Omega_i$), which matches to ion-plasma resonance. This instability represents a large azimuthal heterogeneity with a wavelength approximately equal to the size of the calculation domain.

In figure 5d the ratio of the electron concentration to the magnetic field is shown. The graph shows that the electron concentration is aligned with the magnetic field in the right part of computational domain. The magnetic field reduces in this part of the domain.
Figure 6. Electron current ($J_e$), ion current ($J_i$), total discharge current ($J_{\text{sum}}$) and the integral of the ionization function ($S_{\text{sum}}$) for solutions 1 (a) and 2 (b).

The electron, ion, total discharge currents and the integral of the ionization function along Y axis are shown in figure 6 for the solution 1 (figure 6a) and the solution 2 (figure 6b). The ion current at the exit plane of the thruster almost reaches the value of the integral of the ionization function. The electron current in the solution 2 is about two times higher than in the solution 1. The total discharge current is constant along the Y direction, so the electron current and ion current adjust to each other.

4. Discussion
Table 1 contains the integral parameters of the discharge per length in the azimuthal direction for the laboratory model [29] (experimental value) and for two solutions in the model. The ion current in the experiment and in the calculations is approximately at the same level for the same gas flow rate per length. The discharge current for solution 1 is 20% higher than one observed in the experiment. Such disagreement could be explained by the simplicity of the model of plasma-wall interaction: the channel height is assumed constant, when the discharge chamber has complex shape in the experiment. In addition, the model does not take into account the real 3D configuration of the magnetic field. Thus, only a qualitative agreement between the experimental and model results could be observed.

Table 1. Comparison between model solutions and experimental data

| Parameter of operation | Laboratory model of HT | solution 1 | solution 2 |
|------------------------|------------------------|------------|------------|
| Gas flow rate per length, mg/s/m | 15.625 | 15.625 | 15.625 |
| Discharge current per length, A/m | 13.7 | 16.7 | 21.8 |
| Ion current per length, A/m | 11.9 | 10.7 | 10.8 |

Different operational HT modes are observed in the experiments [24, 30-38]. In the experimental works, two of them were called the “jet” and “bell” mode [24, 30]. It was shown that the increase of the magnetic field magnitude could lead to the transition from the “jet” mode to the “bell” mode. The enlarged values of the discharge current are observed in the “bell” mode due to the increase of the electron current component. Also, the waves with azimuthal component in the frequency range 5–150 MHz were found in these two modes [30]. The dispersion function of these waves is close to the linear one. In the “bell” mode, the amplitude of the detected waves significantly exceeds the amplitude in the “jet” mode.

The solutions in the model depends on the magnitude of the magnetic field. In this case, our model could be a promising tool for researching the plasma processes during the transition between different operational modes.
5. Conclusion
A 2D3V axial-azimuthal Full PIC model of the Hall thruster plasma has been developed. Two different solutions depending on the magnetic field magnitude are investigated using this model. Simulation results provide to research the electron concentration, plasma potential, electron temperature, electron drift velocity and another. The dispersion functions of waves in the azimuthal direction were studied. The solutions show waves, which can be attributed by their characteristic to the electron-drift instability, gradient-drift waves and ion-plasma resonance. The ion current obtained in the model corresponds to the experimental values with good agreement. It is noted that the developed model is a promising tool for researching the processes of the HT transition between different operational modes.

References
[1] Kim V 1998 Main Physical Features and Processes Determining the Performance of Stationary Plasma Thrusters. Journal of Propul. Power 14 (5) 736
[2] Morozov A I, Savelyev V V 2000 Fundamentals of Stationary Plasma Thruster Theory. Reviews of Plasma Physic 21 203 doi: 10.1007/978-1-4615-4309-1_2
[3] Morozov A I, Kislov A Ya, and Zubkov I P 1968 High-current Hall-current plasma accelerator. JETP Lett. 7 (172)
[4] Irishkov S V, Gorshkov O A, and Shagayda A A 2005 Fully kinetic modeling of low power Hall thrusters. International Electric Propulsion Conf. (Princeton University) pp 1-10
[5] Parra F I, Ahedo E, Fife J M and Martinez-Sanchez M 2006 A two dimensional hybrid model of the Hall thruster discharge. J. Appl. Phys. 100 023304
[6] Mikellides I G and Katz I 2012 Numerical simulations of Hall-effect plasma accelerators on a magnetic-field-aligned mesh. Physical Rev. E86 046703 doi: 10.1103/PhysRevE.86.046703
[7] Boeuf J P 2017 Tutorial: Physics and modeling of Hall thrusters. J. Appl. Phys. 121 011101 doi: 10.1063/1.4972269
[8] Taccogna F and Minelli P 2018 Three-dimensional particle-in-cell model of Hall thruster: The discharge channel. Physics of Plasmas 25 061208 doi: 10.1063/1.5023482
[9] Adam J C, Heron A and Laval G 2004 Study of stationary plasma thrusters using two-dimensional fully kinetic simulations. Phys. Plasmas 11(295) doi: 10.1063/1.1632904
[10] Coche P and Garrigues L 2014 A two-dimensional (azimuthal-azimuthal) particle-in-cell model of a Hall thrusters. Physics of Plasmas 21 023503 doi: 10.1063/1.4864625
[11] Boeuf J P and Garrigues L 2018 E × B electron drift instability in Hall thrusters: Particle-in-cell simulations vs. theory. Physics of Plasmas 25 061204 doi: 10.1063/1.5017033
[12] Lafleur T and Chabert P 2018 The role of instability-enhanced friction on 'anomalous' electron and ion transport in Hall-effect thrusters. Plasma Sources Sci. Technol. 27 015003
[13] Lafleur T, Martorelli R, Chabert P, Bourdon A, 2018, Anomalous electron transport in Hall-effect thrusters: Comparison between quasi-linear kinetic theory and particle-in-cell simulations. Physics of Plasmas 25 061202 doi: 10.1063/1.5017626
[14] Croes V, Tavant A, Lucken R, Martorelli R, Lafleur T, Bourdon A and Chabert P 2018 The effect of alternative propellants on the electron drift instability in Hall-effect thrusters: Insight from 2D particle-in-cell simulations. Physics of Plasmas, 25 063522 doi: 10.1063/1.5033492
[15] Chernenyshov T, Son E, Gorshkov O 2019 2D3V kinetic simulation of Hall effect thruster, including azimuthal waves and diamagnetic effect. Journal of Physics D: Applied Physics 52(44) doi: 10.1088/1361-6463/ab35cb
[16] Nikitin V, Tomilin D, Lovtsov A and Tarasov A 2017 Gradient-drift and resistive mechanisms of the anomalous electron transport in Hall effect thrusters. EPL 117 45001 doi: 10.1209/0295-5075/117/45001
[17] Tomilin D 2013 Gradient instabilities of electromagnetic waves in Hall thruster plasma. Phys. Plasmas 20 042103
[18] Lam C M, Fernandez E and Cappelli M A 2015 A 2-D Hybrid Hall Thruster Simulation That Resolves the E×B Electron Drift Direction. IEEE Transactions on Plasma Science 43(1) DOI:
References

[19] R C Wetzel, F A Baiocchi, T R Hayes and R S Freund 1987 Absolute cross sections for electron-impact ionization of the rare-gas atoms by the fast-neutral-beam method. Phys. Rev. 35(2) 559

[20] Hashimoto T and Nakamura Y 1990 Electron swarm parameters in xenon and its momentum transfer cross section, Papers of Gas Discharge Technical Committee ED-90-61 (Japan: IEE)

[21] Artsimovich I. A. and Sagdeev R Z 1979 Plasma Physics for Physicists (Atomizdat Publishers, Moscow)

[22] Hockney R W, Eastwood J W 1987 Computer Simulation Using Particles (McGraw-Hill Book Company)

[23] Liu H, Wu B, Yu D, Cao Y and Duan P 2010 Particle-in-cell simulation of a Hall thrusters. Journal of Physics D: Applied Physics 43(16) 165202 doi: 10.1063/1.5054009

[24] Khmelevskoi I A, Tomilin D A 2019 Parametric Study of Two Stable Forms of Discharge Burning in a Hall-Effect Thruster. Techn. Phys. 64(9) 1283 doi: 10.1134/S1063784219090068

[25] Shagayda A A 2007 Certificate of Software Application Registration No. 2007614083 p 24 [in Russian]

[26] Lazurenko A, Krasnoselskikh V and Bouchoule A 2008 Experimental Insights Into High-Frequency Instabilities and Related Anomalous Electron Transport in Hall Thrusters. IEEE Transactions on Plasma Science 36(5) 1977 doi: 10.1109/TPS.2008.2000972

[27] Tilinin G N 1977 Experimental study of high-frequency plasma oscillations in the Hall-current plasma accelerator Sov. Physics-Tech. Journal 47(8) 1684

[28] Esipchuk Y B and Tilinin G N 1976 Drift instability in a Hall-current plasma accelerator Sov. Physics-Tech. Journal 46(4) 718

[29] Tomilin D A, Lovtsov A S, Khmelevskoi I A 2019 Parametric studies of 1.5 and 2.5 kW Hall Thrusters with external layer. The 36th International Electric Propulsion Conf. (Austria, Vienna) IEPC-2019-342

[30] Khmelevskoi I A, Lovtsov A S, Tomilin D A 2019 Study of two Different Discharge Modes in Hall Thruster. The 36th International Electric Propulsion Conf. (Austria, Vienna) IEPC-2019-387

[31] Lovtsov A S, Tomilin D A and Shashkov A S 2014 Distribution of local plasma parameters in a channel of the Hall thruster at two different types of discharge burning. Tech. Phys. Letters 40(9) 754

[32] Azziz Y 2007 Experimental and Theoretical Characterization of a Hall Thruster Plume, Dissertation, Massachusetts Institute of Technology

[33] Conversano R W, Goebel D M, Mikellides I G, Hofer R R, Matlock T S, Wirz R E 2014 Magnetically Shielded Miniature Hall Thruster: Performance Assessment and Status Update. 50th AIAA/ASME/SAE/ASEE Joint Propulsion Conf. (USA, San Diego) DOI: 10.2514/6.2014-3896

[34] Kostin A N, Lovtsov A S, Vasin A I, Vorontsov V V 2013 Development and qualification of Hall thruster KM-60 and the flow control unit. The 33st International Electric Propulsion Conference (USA, Washington) IEPC-2013-055

[35] Hara K, Sekerak M J, Boyd I D and Gallimore A D 2014 Mode transition of a Hall thruster discharge plasma. J. Appl. Phys. 115 203304 doi: 10.1063/1.4879896

[36] Ding Y, Boyang J, Sun H, Wei L, Peng W, Li P, Yu D 2017 Effect of matching between the magnetic field and channel length on the performance of low sputtering Hall thrusters. Adv. Space Res. 63(3) 837 doi: 10.1016/j.asr.2017.11.003

[37] Ding Y, Sun H, Li P, Wei L, Xu Y, Peng W, Su H, Yu D 2017 Influence of hollow anode position on the performance of a hall-effect thruster with double-peak magnetic field. Vacuum 143 251 doi: 10.1016/j.vacuum.2017.06.030

[38] Han L, Wei L, Yu D 2016 Mode transition induced by the magnetic field gradient in Hall thrusters. J. Phys. D: Appl. Phys. 49 375203 doi: 10.1088/0022-3727/49/37/375203