Disentangling Nonlocality and Teleportation

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Abstract

Quantum entanglement can be used to demonstrate nonlocality and to teleport a quantum state from one place to another. The fact that entanglement can be used to do both these things has led people to believe that teleportation is a nonlocal effect. In this paper it is shown that teleportation is conceptually independent of nonlocality. This is done by constructing a toy local theory in which cloning is not possible (without a no-cloning theory teleportation makes limited sense) but teleportation is. Teleportation in this local theory is achieved in an analogous way to the way it is done with quantum theory. This work provides some insight into what type of process teleportation is.

1 Introduction

Entangled states in quantum theory can be used both to demonstrate that quantum theory is not a local hidden variable theory and to teleport quantum states from one place to another. The fact that entanglement can be used to do both these things has led people to assume that teleportation is a nonlocal effect. In this paper it is shown that teleportation is conceptually independent of nonlocality. This is done by constructing a toy local theory in which cloning is not possible (without a no-cloning theory teleportation makes limited sense) but teleportation is. Teleportation in this local theory is achieved in an analogous way to the way it is done with quantum theory. This work provides some insight into what type of process teleportation is.

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The question of whether teleportation is essentially nonlocal has been raised in two papers. Popescu [3] showed that certain mixed states called Werner states which have been shown by Werner not to violate any Bell inequalities can nevertheless be used to teleport quantum states with a greater fidelity than could be achieved without any entanglement. More recently, Braunstein [4] has shown how teleportation is possible with continuous variables. He points out that it is possible to consider only states having positive Wigner functions and that the measurements he considers can all be functions of $q$ and $p$. If we are restricted to such measurements on a state with a positive Wigner function then Bell [5] has showed that it is possible to construct a local hidden variable model. However, it is possible to consider different states and different measurements, and then there will be nonlocality (see [6]).

In general, it is very difficult to differentiate between two physical concepts within the same physical theory. If that physical theory has both of the corresponding properties then these properties are likely to be manifest at the same time making it difficult to disentangle them. However, by considering alternative physical theories, we can hope to show that two physical concepts are distinct.

It is the no-cloning theorem which makes quantum teleportation interesting. In standard classical physics it is possible in principle to make measurements on a system which establish a complete
description of the state without disturbing the system. This complete description of the state can then be used to build another system in the same state. Hence, a possible teleportation scenario would be that such a complete measurement is made on a classical system, the resulting information is sent to another location, and a new copy is created. The original copy could be destroyed and this would seem like teleportation. However, there is no reason to destroy the original. On the other hand, if there is a no-cloning theorem which says it is impossible to make a copy of a system when the state of the system is unknown then there can be no dilemma about whether to destroy the original. It must have been destroyed if there is now a copy somewhere else. Hence we will take the existence of a no-cloning theorem to be part and parcel of what we mean by teleportation. Since classical physics as it stands does not have a no-cloning theorem, we will invent an alternative physical theory which does and which is explicitly local. This theory is only intended to illustrate the central point of this paper (it does not correspond to any real physical system).

A number of real optical experiments have been performed to demonstrate quantum teleportation [7]. Since these are real experiments the detectors are less than 100% efficient. Taking advantage of this fact Risco-Delgado [8] has shown that it is possible to build a local hidden variable model which agrees with the results the first two of these experiments (he has not considered the third). However, this model does not support perfect teleportation and it is not clear whether a no-cloning theorem can be derived. Hence, they are not useful for our present purpose. Nevertheless, they played a role in motivating the approach taken here.

2 A toy local theory

We will now state the seven postulates of the toy local theory.

2.1 State description postulate

Postulate 1 A system consists of particles and each particle can be in one of four states labeled 0, 1, 2, 3.

A system of N particles has the state X = (x_1, x_2, ..., x_N) where x_n \in \{0, 1, 2, 3\} is the state of the nth particle. The number of possible states X is 4^N.

2.2 State measurement and preparation postulates

Postulate 2 There exist measurement apparatuses which can be used to extract information about the state of a system (though, due to postulates 4 and 5 below, only partial information can be extracted). A given measurement, A, will have R outcomes labeled r = 0, 1, 2, ..., R - 1. Associated with the rth outcome is the set of possible states A_r = \{X_{r1}, X_{r2}, ..., X_{rL}\}

A given X does not appear in more than one such set. I.e. the sets A_r are disjoint. Furthermore, each possible X must appear in one of the sets A_r.

Postulate 3 If the state of the system is X then the outcome of a measurement of A will be r where X \in A_r. Since X only appears in one of the sets A_r, there can only be one outcome for a given X.

Postulate 4 After a measurement of A having outcome r the state of the system will be X_r with probability \frac{1}{L} (i.e. the state is selected randomly from the set A_r).

From postulates 3 and 4 we see that if a measurement is repeated on a system the outcome will be the same. Postulate 4 has the consequence that a measurement will, in general, disturb the state of a system. This limits the amount of information that can be extracted about the state of a system by repeated measurements. The set A_r can be represented...
by the matrix
\[ A_r = \begin{pmatrix}
  x_1^1 & x_2^1 & \cdots & x_N^1 \\
  x_1^2 & x_2^2 & \cdots & x_N^2 \\
  \vdots & \vdots & \ddots & \vdots \\
  x_1^L & x_2^L & \cdots & x_N^L
\end{pmatrix} \]

The entries in the \( l \)th row are the elements of \( X_{rl} \) and the \( n \)th column pertains to the \( n \)th particle. For \( L = 5 \) a typical column might be
\[ \begin{array}{c}
  2 \\
  0 \\
  0 \\
  2 \\
  2
\end{array} \quad (1) \]

**Postulate 5** The sets \( A_r \) can be chosen in any way which is consistent with the constraints imposed in the postulate 2 and the following additional constraints. The sets \( A_r \) must be such that (i) in each column there are at least two different values of \( x \) and (ii) each value which occurs must occur in the column for at least 25% of the entries.

To illustrate postulate 5 we see that the column in (1) is ok because (i) there are at least two different values, in this case 0 and 2, and (ii) because each value which occurs does occur for at least \( \frac{1}{4} \) of the entries, in this case the ratios are \( \frac{2}{5} \) and \( \frac{3}{5} \). Postulate 5, like postulate 4, limits the amount of information that can be extracted by measurement. If all the entries in a particular column were 0 then, if that outcome were recorded, we could be certain that the state of the corresponding particle is 0 both before and after the measurement. However, this is not possible. Postulate 5 implies that there must be at least a 25% chance that the state is something else.

### 2.3 State Manipulation postulates

It is possible to manipulate the particles which make up a system though only in accordance with the following rules.

**Postulate 6** The particle can be moved around in space (though at sub-luminal speeds). A joint measurement can only be made on two or more particles if they in the same place.

This postulate is necessary since we are constructing a local theory.

**Postulate 7** There exist apparatuses which make it possible to manipulate each individual particle without knowing its state in such a way that if the particle is in the state \( x \) it will go to the state \( U(x) \) where \( U \) is a one to one function.

These manipulations are taken to be analogous to unitary transformations in quantum theory. We will be particularly interested in the operation \( U_k \) where
\[ U_k(x) = (x + k) \mod 4 \quad (2) \]

We will call such operations rotations.

### 3 Examples of measurements

#### 3.1 One particle

Consider making measurements on a single particle. In this case \( N = 1 \) so there will only be one column in the matrix representation of the sets \( A_r \). One possible measurement is defined by the matrices
\[ A_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \]

We see that both \( A_0 \) and \( A_1 \) are consistent with postulates 2 and 5. Imagine that Alice is given a single particle which has \( x = 0 \) but that she does not know what its state is. She could perform the measurement described by these matrices. Since \( x = 0 \) she will get outcome 0 corresponding to \( A_1 \) by postulate 3. She will now know that the original state was either 0 or 1 but she will not know which. After this measurement the new state of the particle could be either 0 or 1 (with even probabilities) by postulate 4. Alice will not know which. She could repeat the measurement many times and each time she would get the same outcome. The state \( x \), is in some sense, a hidden variable since a particle cannot be prepared with a known \( x \). The preparable states of the system

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are combinations such as [50% of 0 and 50% of 1]. An alternative measurement is defined by the matrices

\[ B_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad B_1 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \]

If Alice now gives the particle to Bob and he measures \( B \) there is a 50% chance he will get outcome 0 and a 50% chance he will get outcome 1. If he gets outcome 0 then since the state Alice gave him was known to be [50% of 0 and 50% of 1] they can conclude that the state of the particle Alice gave him was \( x = 1 \). However, the state of the system after Bob’s measurement will be [50% of 1 and 50% of 2] corresponding to \( B_0 \). Furthermore they do not know whether \( x = 1 \) was the state of the system that was originally given to Alice (in our example it was not).

### 3.2 Two particles

The state of two particles is represented by \( X = (x_1, x_2) \). We will be interested in only one possible measurement that can be made on two particles. This is described by the matrices

\[ B_0 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0 & 1 \\ 3 & 0 \\ 2 & 1 \\ 3 & 2 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 & 2 \\ 1 & 3 \\ 2 & 0 \\ 3 & 1 \end{pmatrix} \quad B_3 = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \\ 3 & 0 \end{pmatrix} \]

These matrices satisfy the rules in postulates 2 and 5. This measurement on two particles will play the same role the Bell measurement plays in quantum teleportation. Furthermore, it can be used to prepare the analogue of the four Bell states. Imagine the two particles are initially in some unknown state. Now a \( B \) measurement is performed. If the outcome is 0 corresponding to \( B_0 \) then the state will be given by one of the rows in the \( B_0 \) matrix. Looking at this matrix we see that the two particles will become correlated and their state will be \((y, y)\) where \( y \) is unknown. Similar remarks apply to the other outcomes. For the outcome \( r \) corresponding to \( B_r \) the state will be \((y, y - r \mod 4)\). Any of these states can be changed into any other by applying the appropriate rotation to one particle using the operation defined in (3).

### 4 A no-cloning theorem

As stressed earlier, a no-cloning theorem is essential to give meaning to teleportation. We will now prove that within our toy local theory it is not possible to produce clones of particles in unknown states. We will take an operational approach to cloning. Thus, imagine that Peter prepares a particle in some state by making a measurement on it. As a result of his measurement he might know that the state is something like [50% of 0 and 50% of 1]. He now gives this particle to Alice. Alice does not know what measurement Peter used to prepare the particles. Alice is challenged to give two particles back to Peter which are, so far as Peter can establish, in the same state as the original. One of these two particles is to be regarded as the original and the other as the clone. Peter will then repeat the same measurement he used to prepare the particles to test how well Alice has done. If he gets the same outcome as he did originally but now for both particles then Alice has passed the test. This test is repeated a large number of times. Alice must pass each time to convince Peter that she can produce clones. We will now prove that this is impossible – Alice must fail this test sometimes.

Assume Peter prepares the state by measuring \( P \) where

\[ P_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad P_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \]

He could, of course, have chosen any other measurement. Assume further that he gets outcome 0. This means that the state is, from his point of view, [50% of 0 and 50% of 1]. Let’s assume that actually the state of the particle is 1 (though Peter will not know this). Peter passes the particle on to Alice. Alice receives a particle in state 1 (though she does not know its state is 1) but no information about how it was prepared. Imagine now that on another occasion
that Peter prepares the state by measuring \( P' \) where

\[
P'_0 = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad P'_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}
\]

and he gets outcome 1. This means that the prepared state, from Peter’s point of view, is [50% of 1 and 50% of 2]. We can assume that the state of the particle is actually 1 (again Peter will not know this). Peter passes the particle on to Alice. Again, Alice receives a particle in state 1 (though she does not know this) and no information about how it was prepared. From Alice’s point of view these two cases are identical. The only way she can hope to pass the test in all such cases is to prepare two particles in the state 1.

To do this let us imagine that she has \( N \) – 1 particles. When she receives the particle from Peter she has \( N \) particles. She will perform some operations on the particle and then give two particles to Peter who will perform his test. There are only two types of operation she can perform on the particles. She can perform measurements on all the particles as described in postulates 2–5 and she can perform manipulations on individual particles as described by postulate 7. It is clear that the manipulations described in postulate 7 cannot help since they only act on individual particles and effectively do nothing more than relabel the states. Hence, assume Alice makes a measurement \( C \). This measurement will have a certain number of outcomes. In the particular case we are discussing we want to produce two particles in the state 1. Hence, for at least one outcome, \( r_1 \) say, the matrix \( C_{r_1} \) must have at least two 1’s in at least one row (say the first row) and the positions of these 1’s in this row must be known by Alice (since she must know which particles to give to Peter). Assume that the 1’s are in the first two columns (so that, in the case of obtaining this outcome, Alice will give Peter particles 1 and 2). Then the matrix will have the form

\[
C_{r_1} = \begin{pmatrix} 1 & 1 & \cdots \\ u & v & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}
\]

In the first column at least 25% of the entries must be different to 1 by postulate 5. Assume that \( u \) is one such entry (\( u \neq 1 \)). If the outcome is \( r_1 \) then after the measurement the state will randomly selected from one of the rows in \( A_{r_1} \). However, at least 25% of these are of the form \((u, v, \ldots)\) where \( u \neq 1 \). Hence, when Alice gives the first two particle to Peter, they will sometimes fail the test because they will not always be in the state \((1, 1)\). This proves that Alice cannot successfully clone 100% of the time.

5 Teleportation in the toy local theory

To perform teleportation two particles, 2 and 3, are prepared in a Bell-like state as described in section 3.2. This state is of the form \((y, y)\) where \( y \) is unknown. The two particles separate from the place of preparation. Particle 2 goes towards Alice, and particle 3 goes towards Bob. Alice is given a particle in an unknown state \( x \). She takes this particle which we will call particle 1 together with particle 2 and performs the Bell-like measurement \( B \) defined in \((3,4)\). She will get some outcome \( r \) for this measurement. If the outcome is \( r = 0 \) we see from the matrix \( B_0 \) in equation \((3)\) that \( x \) for particle 1 is equal to \( y \) for particle 2. In general, if she gets outcome \( r \) then we see from the matrices \( B_r \) that \( y = x - r \mod 4 \). Particle 3 is in the state \( y \). Alice sends the information \( r \) to Bob and he can perform the manipulation \( U_r \) defined in \((2)\) on particle 3. After this its state will be \( x \). This means that, without knowing the state of particle 1, we have successfully transferred its state onto particle 3. This is teleportation. The state of particle 1 is completely randomised after the Bell-like measurement and so the analogue with quantum teleportation is very strong.

6 Discussion

We have shown that it is possible to have a local theory which has a no-cloning theorem and supports teleportation. The particular model here provides some insight into what is essential for teleportation. What happens in this model is the following. The state of particle 1 is probed at a ‘microscopic’ level by particle 2 in a way that no ‘macroscopic’ measur-
ing apparatus could probe it. It is this that makes it possible to extract full information about the state of particle 1 without running into the no-cloning theorem. This information is divided between the measurement result \( r \) which Alice and Bob can know and the state of particle 3 which they cannot know and which was originally classically correlated with the ‘probe’ particle 2. The original state can then be reconstructed in an analogous way to the way it is reconstructed in quantum teleportation. Nonlocality plays no role in this process.

Given this model, we must not assume that nonlocality is playing an essential role in quantum teleportation, though it may be the case that it is. There are two reasons for believing that nonlocality may play an important role in quantum teleportation: (1) As has been pointed out by Bennett [1], the amount of information needed to specify a general qubit is much greater than the two bits of information which is classically communicated during quantum teleportation. One might speculate that, when a qubit is teleported, the extra information is being carried by the nonlocal properties of the entangled state. On the other hand, it is not possible to extract more classical information from a qubit than the two classical bits communicated during teleportation and so there must remain questions about the reality of the quantum information apparently transmitted during teleportation. (2) If one qubit of an entangled pair is teleported it is possible to obtain a violation of Bell’s inequalities between the second qubit of this pair and the teleported qubit. Hence, the teleportation machine must be able to convey what ever quality is necessary for this nonlocal correlation. However, we cannot necessarily assert on the basis of this fact that nonlocality plays a role in quantum teleportation. It is possible that the extra information which establishes the nonlocal correlations is only transmitted in the process of measuring the quantities in Bell’s inequalities, and is not transmitted in the teleportation process.

Even if it is eventually concluded that quantum teleportation is a nonlocal process, examples like that of Popescu [3] suggest that there may be a regime in which imperfect teleportation is happening, but no nonlocality is in involved. In such a regime teleportation would still circumvent the no-cloning theorem by probing the system to be teleported at a microscopic level in much the same way as the toy local theory considered in this paper. The Werner states considered by Popescu have since been shown to be nonlocal if more complicated measurements are considered than those originally considered by Werner (Popescu [10]). If two-particle quantum states were found which are local and yet useful in teleportation then we could conclude that the type of process discussed in this paper also happens in quantum theory. States having bound entanglement [11] are obvious candidates to consider. However, while it seems likely, it has not yet been shown that such states will always satisfy Bell type inequalities. Furthermore, it is not known whether such states could be useful for teleportation although results derived so far are negative [12].

Whatever conclusions may eventually be drawn about nonlocality in quantum teleportation, it has been established in this paper that teleportation in theories which have a no-cloning theorem can be an entirely local phenomenon.

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