We present a study of event-by-event fluctuations of transverse momentum in π+p and K+p collisions at 250 GeV/c and the corresponding PYTHIA Monte Carlo results for π+p collisions. The dependence of Φ_{p_t} on event and particle variables are investigated in detail. We find that Φ_{p_t} are all negative for different average transverse momentum per event sample. The \langle p_t \rangle_N ∼ N correlation may not be the only origin of Φ_{p_t}.

1 Introduction

The investigation of single events has attracted a lot of attentions in heavy-ion collisions. In which, a method suggested by M. Gaździcki has been frequently used. It is defined by

$$\Phi_{p_t} = \sqrt{\langle Z^2 \rangle} - \sqrt{\langle z^2 \rangle}$$

where $z_i = p_{t_i} - \langle p_t \rangle$ is a single-particle transverse momentum variable, $Z = \sum_{i=1}^{N} z_i$ is an event variable with summation running over all $N$ final-state particles in an event, and $\langle \cdots \rangle$ denotes the average over all events in the sample. It was shown that the method removes the trivial fluctuations caused by the variation of impact parameter and by statistics, so that the dynamical event-by-event fluctuations of transverse momentum can be studied. The method is also supposed to be sensitive to the correlations between multiplicity and average transverse momentum. The larger this correlation, the stronger the $\Phi_{p_t}$. It is proposed that vanishing of this correlation can be used as a signal of “equilibration” of the system created in A-A collisions.

To understand the behavior of $\Phi_{p_t}$ in heavy-ion collisions, it is important to see how $\Phi_{p_t}$ depends on the correlation between average transverse momentum and multiplicity and how it relates to event and particle variables in hadron-hadron collisions.

We’ll show this behavior on π+p and K+p collisions at 250 GeV/c in NA22. Our analysis will be focused on the central rapidity range $-2 \leq y \leq 2$ and best momentum resolution range 0.001 GeV/c ≤ $p_t$ ≤ 10 GeV/c.
2 \( \Phi_{p_t} \) for the full sample

The negative correlation between charged \( n \) and \( \langle p_t \rangle_n \) (full triangles) in both the \( \pi^+p \) and \( K^+p \) collision data at 250 GeV/c is reproduced in Fig. 1, and compared to the expectation (open triangles) from PYTHIA which underestimates both \( \langle p_t \rangle_n \) at low \( n \) and the \( n \) dependence of \( \langle p_t \rangle_n \).

The \( \Phi_{p_t} \) of the full sample is a non-zero positive value of \( 29.06 \pm 1.68 \) MeV/c, while the PYTHIA for \( \pi^+p \) collisions is \( 18.95 \pm 0.95 \) MeV/c, thus underestimating the event-by-event fluctuation.

3 The dependence of \( \Phi_{p_t} \) on event variables

Now that \( \Phi_{p_t} \) describes the event fluctuations of transverse momentum, it is interesting to compare the event-by-event fluctuation strength in subsamples with different values of \( n \) and \( \bar{p}_t = \frac{1}{n} \sum_{i=1}^{n} p_t_i / n \).

First, we divide the full sample into two multiplicity subsamples a) \( n < \langle n \rangle \) and b) \( n \geq \langle n \rangle \). The \( \Phi_{p_t} \) values for these two subsamples are given in Table 1. Both of them are positive and the bigger multiplicity subsample has larger event-by-event fluctuation. For more detail, the dependence of \( \Phi_{p_t} \) on \( n \) is plotted in Fig. 2. \( \Phi_{p_t} \) is always positive and increases with increasing multiplicity \( n \). PYTHIA shows the same trend, but with lower \( \Phi_{p_t} \) values.

| Sample | Number of events | \( \langle p_t \rangle \) GeV/c | \( \langle n \rangle \) | \( \langle Z^2 \rangle \) MeV^2/c^2 | \( \langle \sigma^2 \rangle \) MeV^2/c^2 | \( \Phi_{p_t} \) MeV/c |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| a)     | 25844           | 0.396           | 3.99            | 77969.83        | 69680.73        | 15.26 \pm 1.92  |
| \pi^+p MC | 50669        | 0.336           | 4.04            | 47077.93        | 43113.39        | 9.87 \pm 1.08   |
| b)     | 17836           | 0.380           | 9.99            | 92164.61        | 73047.67        | 33.44 \pm 2.93  |
| \pi^+p MC | 45721        | 0.328           | 9.34            | 54901.86        | 44999.70        | 22.18 \pm 1.12  |

Then, the full sample is separated into two according to a) \( \bar{p}_t < \langle p_t \rangle \) and b) \( \bar{p}_t \geq \langle p_t \rangle \). Their corresponding \( \langle p_t \rangle_n \) vs. \( n \) correlation is plotted in Fig. 3.
We can see a positive correlation for subsample a) first increasing with multiplicity and then saturating. For subsample b), there is a negative correlation between \( \langle p_t \rangle_n \) and \( n \).

The values of \( \Phi_{p_t} \) for these two subsamples are given in Table 2. It is astonishing that they are both negative. The corresponding Monte Carlo results are provided in the same table and figure. Qualitatively, they show the same correlation tendencies as the experimental data, but quantitatively they underestimate the experimental ones. We wonder if \( \Phi_{p_t} \) can still present the event-by-event fluctuation in this case.

### Table 2. \( \Phi_{p_t} \) and its related variables for the two different \( \bar{p}_t \) samples.

| Sample | Number of events | \( \langle p_t \rangle \) GeV/c | \( \langle n \rangle \) | \( \langle z^2 \rangle \) MeV^2/c^2 | \( \langle z \rangle \) MeV/c | \( \Phi_{p_t} \) MeV/c |
|--------|-----------------|-------------------------------|----------------|-----------------|-----------------|-----------------|
| a)     | 22844           | 0.309                         | 7.18           | 21511.77        | 38514.35        | -49.58±1.67     |
| \( \pi^+ p \) MC | 51960           | 0.270                         | 6.65           | 13410.97        | 26317.27        | -46.42±0.59     |
| b)     | 20836           | 0.478                         | 6.72           | 54062.54        | 98232.83        | -80.91±3.08     |
| \( \pi^+ p \) MC | 44430           | 0.404                         | 6.44           | 30923.91        | 56424.77        | -61.693±1.19    |

For more detail, \( \Phi_{p_t} \) for different \( \bar{p}_t \) subsamples is given in Fig. 4. We can see that \( \Phi_{p_t} \) is negative for any \( \bar{p}_t \) subsample and decreases monotonously with increasing \( \bar{p}_t \). PYTHIA gives the same trend, but with slightly less negative values.

## 4 The dependence of \( \Phi_{p_t} \) on the particle variable

Now we turn to discuss how \( \Phi_{p_t} \) behaves for different transverse momentum particles. Four samples are defined by the cut \( p_t > p_t^{cut} \) with \( p_t^{cut} = 0.1, 0.2, 0.3 \) and 0.4 GeV/c. It was shown in \( \text{Fig. 4} \) of \( \text{Fig. 4} \) of [1] that the lowest-\( p_t^{cut} \) subsample has the strongest negative correlation and the highest-\( p_t^{cut} \) sample has the strongest positive one.

In Table 3, the \( \Phi_{p_t} \) values are listed. Here we observe, the higher the \( p_t^{cut} \) and stronger the correlation, the smaller \( \Phi_{p_t} \). This is in contradiction with arguments that \( \Phi_{p_t} \) are mainly due to \( \langle p_t \rangle_n \) vs. \( n \) correlations (cf. Fig.4 of [1]).
Table 3. $\Phi_{p_t}$ and its related variables for different $p_t^{cut}$ subsamples.

| Sample      | Number of events | $\langle p_t \rangle$ GeV/c | $\langle N \rangle$ | $\langle z_2 \rangle$ MeV/$c^2$ | $\langle z_3 \rangle$ MeV$/c^2$ | $\Phi_{p_t}$ MeV$/c$ |
|-------------|------------------|-----------------------------|---------------------|-----------------------------|-----------------------------|---------------------|
| $p_t^{cut} > 0.1$GeV/$c$ | 43489             | 0.412                       | 6.45                | 82967.44                    | 68776.52                    | 25.79 ± 2.18        |
| $\pi^+ p$ MC | 95937             | 0.359                       | 5.96                | 47871.61                    | 40685.00                    | 17.09 ± 0.84         |
| $p_t^{cut} > 0.2$GeV/$c$ | 42912             | 0.474                       | 5.25                | 76051.82                    | 65047.70                    | 20.73 ± 2.14         |
| $\pi^+ p$ MC | 94390             | 0.420                       | 4.68                | 43234.06                    | 36107.48                    | 17.91 ± 0.87         |
| $p_t^{cut} > 0.3$GeV/$c$ | 41505             | 0.557                       | 3.95                | 71900.70                    | 63178.21                    | 16.79 ± 2.40         |
| $\pi^+ p$ MC | 90394             | 0.497                       | 3.36                | 39290.56                    | 32841.53                    | 17.00 ± 0.97         |
| $p_t^{cut} > 0.4$GeV/$c$ | 38583             | 0.649                       | 2.92                | 79183.20                    | 62918.54                    | 14.09 ± 2.85         |
| $\pi^+ p$ MC | 80820             | 0.583                       | 2.39                | 36320.72                    | 30920.49                    | 14.73 ± 1.14         |

5 The dependence of $\Phi_{p_t}$ on rapidity range

It is important whether the rapidity range used for the analysis influences the $\Phi_{p_t}$. So, $\Phi_{p_t}$ is given in Fig. 5 for different central rapidity ranges. When the rapidity range broadens from the center, $\Phi_{p_t}$ becomes larger and larger. Once the range spreads to wider than $-2 \leq y \leq 2$, $\Phi_{p_t}$ reaches its saturation value. So, this result confirms that the $\Phi_{p_t}$ we obtained from the central rapidity range $-2 \leq y \leq 2$, can represent the full one.

6 Summary

1. $\Phi_{p_t}$ is positive both for the full sample and for all samples of restricted multiplicity, and the high-multiplicity event sample has the strong event-by-event fluctuation. But it is negative for all samples of restricted $\bar{p}_t$.

2. The higher the $p_t$ in the sample, the stronger $\langle p_t \rangle_n$ vs. $n$ correlation, but the weaker $\Phi_{p_t}$. It turns out that this correlation may not be the only origin of event-by-event fluctuation.

3. Except for samples with particle transverse momentum larger than the average, the $|\Phi_{p_t}|$ values are underestimated by PYTHIA.

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