Application of Structural Mechanics Solver in Internal Force Calculation of Frame Structure

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Abstract. The internal force calculation of frame structure is a difficult problem in civil engineering graduation design. This article introduces the basic principles of bending moment distribution method and structural mechanics solver method. Taking a two-story, two-span frame structure as an example, a comparative study is carried out and the results show that there is certain error, and analyze the cause of the error. The research has certain guiding significance for the teaching of structural mechanics and civil engineering graduation design.

1. Introduction
The internal force calculation of frame structure is the focus and difficulty of concrete structure design and graduation design, and it is one of the most frequently used structural forms of multi-storey buildings[1-2]. The composition of the frame structure includes beams, slabs, columns and foundations. The joints of beam and column are generally rigid, while the joints of column and foundation are mostly rigid, with clear structural transmission force. The frame structure mainly bears dead load, live load, wind load, and seismic load. Dead load and live load are generally vertical, wind load and seismic load are horizontal. The internal force and lateral displacement of the multi-layer frame structure are calculated by hand, approximate methods are used in general. When calculating the internal force under the vertical load, there are the layered method, the bending moment distribution method, and the iterative method; and there are the inflection point method and the improved inflection point method (D value method) under the horizontal load. These methods adopt different assumptions and have different calculation results, but they can generally meet the requirements of engineering design accuracy. This paper mainly carries out the comparative study of internal force calculation between the bending moment distribution method and the structural mechanics solver under the action of vertical load [3].

2. Basic Principles

2.1. Bending Moment Distribution Method
Under the action of vertical load, the lateral displacement produced by the more regular frame is very small and can be ignored. The internal force of the frame is simplified by moment distribution method.
2.1.1. Basic Assumptions
(1) The influence of the lateral displacement of the frame structure on its internal force is not considered;
(2) The load on each layer of beams only affects the internal forces of the beams of this layer and its upper and lower columns, and the influence on the internal forces of other layers of beams and columns is negligible. (The internal force referred to in this assumption does not include the column axial force)

2.1.2. Calculation Steps
(1) Calculate the moment distribution coefficient of each node;
(2) Calculate the fixed-end bending moment of the frame beam;
(3) Calculate the unbalanced bending moments of each node, and perform the first allocation of the unbalanced bending moments of all nodes at the same time (no bending moment transmission is carried out in the meantime);
(4) Transfer the distributed bending moments of all rod ends to their distal ends at the same time (for rigid frames, the transfer coefficients are all 1/2);
(5) The new unbalanced bending moment generated by the transmission of the bending moment of each node is allocated for the second time, so that each node is in a balanced state;
(6) The final bending moment of each rod end is obtained by superimposing the fixed end bending moment, distributed bending moment and transmission bending moment.

2.2. Structural Mechanics Solver Method
The structural mechanics solver is a computer-aided analysis and calculation software for teachers, students and engineering and technical personnel. The solution includes the geometric composition, determinate structure, statically indeterminate structure, displacement, internal force, influence lines, free vibration, elastic stability, ultimate load, etc., and covers almost all the problems in classical structural mechanics. More importantly, all use accurate algorithms to give accurate answers [4].

The internal force and deformation algorithm of the structural mechanics solver for the plane frame is shown in the equation in Figure 1. For the plane frame structure, the internal force and displacement are calculated by the conventional matrix displacement method, and the stiffness equation of the structure $K\Delta = P$ is solved to obtain the node displacement, and then calculate the internal force and deformation of the rod [5].

\[
\begin{bmatrix}
\frac{E_A}{l} & 0 & 0 & -\frac{E_A}{l} & 0 & 0 \\
0 & \frac{12EI}{l^2} & 6EI/l & 0 & -\frac{12EI}{l^2} & 6EI/l \\
0 & 6EI/l & 4EI/l & 0 & -6EI/l & 2EI/l \\
-\frac{E_A}{l} & 0 & 0 & \frac{E_A}{l} & 0 & 0 \\
0 & -\frac{12EI}{l^2} & 6EI/l & 0 & \frac{12EI}{l^2} & -6EI/l \\
0 & 6EI/l & 2EI/l & 0 & -6EI/l & 4EI/l
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
w_4 \\
w_5 \\
w_6
\end{bmatrix}
= \begin{bmatrix}
F_x \\
F_y \\
M_x \\
M_y \\
M_z \\
M_z
\end{bmatrix}
\]

Figure 1. Stiffness Equation of Matrix Displacement Method.

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500mm and a width of 250mm. Assuming that the frame structure is only subjected to vertical loads, its sizes are 2.5kN/m and 4.0kN/m respectively, and the material elastic modulus is $3.0 \times 10^7$kN/m$^2$.

### 3.1. Bending Moment Distribution Method

Under the action of vertical load, the calculation of the second moment distribution method is assisted by EXCEL. The specific process is shown in Table 1 (Unit: kN·m).

**Table 1. Calculation Process of Bending Moment Secondary Distribution Method.**

| Description | Upper Column | Lower Column | Right Beam | Left Beam | Upper Column | Lower Column | Right Beam | Left Beam | Upper Column | Lower Column |
|-------------|--------------|--------------|------------|-----------|--------------|--------------|------------|-----------|--------------|--------------|
| A           | 0.621        | 0.379        | 0.261      | 0.426     | 0.313        | 0.423        | 0.577      |
| B           | -17.28       | 17.28        | -12        | 12        |
| C           | 10.73        | 6.55         | -1.38      | -2.25     | -1.65        | -5.08        | -6.92      |
| D           | 2.24         | -0.69        | 3.27       | -0.52     | -2.54        | -0.83        | -1.48      |
| E           | -0.96        | -0.59        | -0.06      | -0.09     | -0.07        | 0.98         | 1.33       |
| F           | 12.01        | -12.01       | 19.12      | -2.86     | -16.26       | 7.07         | -7.07      |

Description: A-Distribution Coefficient; B-Fixed End bending Moment; C-First Distribution of Bending Moment; D-Transmission Bending Moment; E-Second Distribution of Bending Moment; F-Final Rod End Bending Moment.

### 3.2. Structural Mechanics Solver Method

#### 3.2.1. Calculation Parameters

**Table 2. Calculation Parameters.**

| Element | h (mm) | b (mm) | l (m) | E (kN/m$^2$) | A (m$^2$) | EA(kN) | I (mm$^4$) | EI(kN·m$^2$) |
|---------|--------|--------|-------|--------------|-----------|--------|-----------|-------------|
| 1       | 400    | 400    | 4.5   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 2       | 400    | 400    | 4.5   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 3       | 400    | 400    | 4.5   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 4       | 400    | 400    | 3.6   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 5       | 400    | 400    | 3.6   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 6       | 400    | 400    | 3.6   | 30000000.00 | 0.16      | 4800000| 2133333333 | 64000       |
| 7       | 500    | 250    | 7.2   | 30000000.00 | 0.125     | 3750000| 2604166667 | 78125       |
| 8       | 500    | 250    | 6.0   | 30000000.00 | 0.125     | 3750000| 2604166667 | 78125       |
| 9       | 500    | 250    | 7.2   | 30000000.00 | 0.125     | 3750000| 2604166667 | 78125       |
| 10      | 500    | 250    | 6.0   | 30000000.00 | 0.125     | 3750000| 2604166667 | 78125       |
3.2.2. Parametric Modeling

Node,1,0,0
Node,2,7.2,0
Node,3,13.2,0
Node,4,0,4.5
Node,5,7.2,4.5
Node,6,13.2,4.5
Node,7,0,8.1
Node,8,7.2,8.1
Node,9,13.2,8.1
Element,1,4,1,1,1,1,1
Element,2,5,1,1,1,1,1
Element,3,6,1,1,1,1,1
Element,4,7,1,1,1,1,1
Element,5,8,1,1,1,1,1
Element,6,9,1,1,1,1,1
Element,7,5,1,1,1,1,1
Element,1,6,1,1,1,1,1
Element load,7,3,2.5,0,1,90
Element load,8,3,2.5,0,1,90
Element load,9,3,4,0,1,90
Element load,10,3,4,0,1,90
Element material properties,1,6,4800000,64000,0,0,-1
Element material properties,7,10,3750000,78125,0,0,-1

3.2.3. Calculation Results

![Figure 3. Bending Moment Value of Structural Mechanics Solver (Unit: kN·m).](image-url)
3.3. Comparative Analysis

Table 3. Comparative Analysis Result

| Element Code | Rod End 1 (Machine value-Hand value)/Machine value | Rod End 2 (Machine value-Hand value)/Machine value |
|--------------|-----------------------------------------------|-----------------------------------------------|
| Machine value | Hand value | Machine value | Hand value |
| 1 | 1.10 | 0.96 | 13.1% | 2.26 | 1.91 | 15.4% |
| 2 | -0.39 | -0.31 | 21.0% | -0.58 | -0.63 | 9.3% |
| 3 | -0.90 | -0.58 | 35.3% | -1.50 | -1.15 | 23.2% |
| 4 | 7.12 | 7.75 | 8.8% | 11.37 | 12.01 | 5.6% |
| 5 | -2.58 | -1.91 | 25.9% | -3.55 | -2.86 | 19.5% |
| 6 | -5.08 | -4.91 | 3.3% | -7.29 | -7.07 | 3.0% |
| 7 | -9.38 | -9.66 | 3.0% | 11.20 | 11.69 | 4.4% |
| 8 | -8.05 | -9.16 | 13.9% | 6.57 | 6.06 | 7.8% |
| 9 | -11.37 | -12.01 | 5.6% | 18.98 | 19.12 | 0.8% |
| 10 | -15.42 | -16.26 | 5.4% | 7.29 | 7.07 | 3.0% |

Taking the calculation result of the structural mechanics solver as a standard, comparing the calculation results of the bending moment distribution method, it is found that there is a certain error. Bending moment secondary distribution method is a progressive method based on the displacement method, which can approximate the exact solution, but requires more iteration. If only two distributions are performed, generally only an approximate solution can be obtained. From the above results, it can be seen that the partial error value of the second distribution method of bending moment exceeds 15%. The author believes that for actual engineering calculations, designers should use caution.

4. Conclusion

The powerful solution function of the structural solver provides a computer numerical experiment platform for structural analysis, and is an auxiliary means for teachers to carry out structural mechanics teaching. Students can be liberated from the complicated calculation work by hand and deepen their understanding of theoretical knowledge. It is a beneficial tool for students to learn mechanics knowledge independently. Compared with the second distribution method of bending moment, it can be found that the structural mechanics solver has simple calculation operation and better calculation results. More importantly, the structural mechanics solver provides an effective way for the reform of the teaching model of graduate design with the main computer calculation and hand calculation supplemented.

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