Primordial Non-Gaussianities

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This contribution gives an overview on primordial non-Gaussianities from a theoretical perspective. After presenting a general formalism to describe nonlinear cosmological perturbations, several classes of models, illustrated with examples, are discussed: multi-field inflation with non-standard Lagrangians, modulaton fields, curvaton fields. In the latter case, a special emphasis is put on the isocurvature perturbations, which could leave a specific signature in non-Gaussianities.

§1. Introduction

A potentially promising probe of the early Universe, which has been studied very actively in the last few years, is the non-Gaussianity of the primordial perturbations. Whereas the simplest models of inflation, based on a single field with standard kinetic term produce undetectable levels of non-Gaussianity, a significant amount of non-Gaussianity can be produced in scenarios with i) non-standard kinetic terms; ii) multiple fields; iii) a non standard vacuum; iv) a breakdown of slow-roll evolution.

In this contribution, after summarizing the formalism to describe primordial non-Gaussianities, we review three categories of models that could produce detectable non-Gaussianities. First, we discuss inflationary models with generalized Lagrangians involving multiple scalar fields and non standard kinetic terms. Second, we consider non-Gaussianity generated by modulaton fields, or light scalar fields which are spectator fields during inflation but can affect some cosmological transition. Finally, we present some aspects of non-Gaussianity arising from curvaton models, in particular the issue of isocurvature non-Gaussianity.

§2. Nonlinear cosmological perturbations

In this section, we first present a geometrical description of the nonlinear cosmological perturbations. We then summarize the traditional statistical description of primordial perturbations, which are used to relate early Universe models with cosmological observations.

2.1. Covariant approach

Instead of the traditional metric-based approach, our discussion below is based on a more geometrical approach. Let us consider a spacetime with metric $g_{ab}$ and some perfect fluid characterized by its energy density $\rho$, its pressure $P$ and its four-velocity $u^{a}$. The corresponding energy momentum-tensor is given by

$$T_{ab} = \rho u_{a}u_{b} + P(g_{ab} + u_{a}u_{b}). \quad (2.1)$$
Let us also introduce the expansion along the fluid worldlines,
\[ \Theta \equiv \nabla_a u^a, \] (2.2)
and the integrated expansion
\[ N \equiv \frac{1}{3} \int d\tau \Theta, \] (2.3)
where \( \tau \) is the proper time defined along the fluid worldlines. By noting that \( \Theta/3 \) corresponds to the Hubble parameter \( H \) in a homogeneous and isotropic spacetime, one can interpret \( \Theta/3 \), in the general case, as a local Hubble parameter and \( a_{\text{loc}} \equiv e^N \) as a local scale factor, \( N \) being the local number of e-folds.

The conservation law for the energy-momentum tensor, \( \nabla_a T^a_b = 0 \), implies\(^3,4\) that the covector
\[ \zeta_a \equiv \nabla_a N - \frac{\dot{N}}{\dot{\rho}} \nabla_a \rho = \nabla_a N + \frac{\nabla_a \rho}{3(\rho + P)} \] (2.4)
satisfies the relation
\[ \dot{\zeta}_a \equiv \mathcal{L}_u \zeta_a = -\frac{\Theta}{3(\rho + P)} \left( \nabla_a P - \frac{\dot{P}}{\dot{\rho}} \nabla_a \rho \right), \] (2.5)
where a dot denotes a Lie derivative along \( u^a \), which is equivalent to an ordinary derivative for scalar quantities (e.g. \( \dot{\rho} \equiv u^a \nabla_a \rho \)).

If \( w \equiv P/\rho \) is constant, the above covector is a total gradient and can be written as
\[ \zeta_a = \nabla_a \left[ N + \frac{1}{3(1+w)} \ln \rho \right]. \] (2.6)

On scales larger than the Hubble radius, the above definition is equivalent to the non-linear curvature perturbation on uniform density hypersurfaces\(^5\)
\[ \zeta = \delta N - \int_{\bar{\rho}}^\rho H \frac{d\bar{\rho}}{\dot{\rho}} = \delta N + \frac{1}{3} \int_{\bar{\rho}}^\rho \frac{d\bar{\rho}}{(1+w)\dot{\rho}}. \] (2.7)
The above equation is simply the integrated version of (2.4).

The covector \( \zeta_a \), or the corresponding scalar quantity \( \zeta \), can be defined for the global cosmological fluid or for any of the individual cosmological fluids (this approach can also be extended to the case of interacting fluids\(^6\)).

2.2. Power spectrum and higher order correlation functions

We now discuss the statistical properties of the cosmological perturbation \( \zeta \). We first define the power spectrum
\[ \langle \zeta_{k_1} \zeta_{k_2} \rangle \equiv (2\pi)^3 \delta^{(3)}(k_1 + k_2) P_\zeta(k_1), \] (2.8)
where the Fourier modes are defined by
\[ \zeta_k = (2\pi)^3 \int d^3 x \ e^{-ik\cdot x} \zeta(x). \] (2.9)
Beyond the Gaussian case, the first quantity of interest is the three-point function, or its Fourier transform, called the bispectrum and defined by

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \equiv (2\pi)^3 \delta^{(3)} \left( \sum_i k_i \right) B_\zeta(k_1, k_2, k_3). \]  

(2.10)

Equivalently, one often uses the so-called $f_{\text{NL}}$ parameter, which can be defined in general by

\[ B_\zeta(k_1, k_2, k_3) \equiv \frac{6}{5} f_{\text{NL}}(k_1, k_2, k_3) [P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]. \]  

(2.11)

Similarly, the Fourier transform of the connected four-point function defines the trispectrum, according to

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle_c \equiv (2\pi)^3 \delta^{(3)} \left( \sum_i k_i \right) T_\zeta(k_1, k_2, k_3, k_4). \]  

(2.12)

The various correlation functions defined above can in principle be measured, or at least constrained, by observations of CMB fluctuations or large scale structure.

In many models, the final perturbation $\zeta$ depends on the fluctuations of one or several scalar fields generated during inflation and the so-called $\delta N$-formalism\(^7,8\) provides a powerful method to evaluate, at least formally, the primordial non-Gaussianity generated on large scales.\(^9\) The underlying idea is to describe, on scales larger than the Hubble radius, the non-linear evolution of perturbations generated during inflation in terms of the perturbed expansion from an initial hypersurface (usually taken at Hubble crossing during inflation) up to a final uniform-density hypersurface (usually during the radiation-dominated era). Using the Taylor expansion of the number of e-folds given as a function of the initial values of the scalar fields,

\[ \zeta \simeq \sum_I N,I \delta \varphi^I_* + \frac{1}{2} \sum_{IJ} N,IJ \delta \varphi^I_* \delta \varphi^J_*, \]  

(2.13)

one finds,\(^9,10\) in Fourier space,

\[
\begin{align*}
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle &= \sum_{IJK} N,IJN,K \langle \delta \varphi^I_{k_1} \delta \varphi^J_{k_2} \delta \varphi^K_{k_3} \rangle \\
&+ \frac{1}{2} \sum_{IJKL} N,IJN,KL \langle \delta \varphi^I_{k_1} \delta \varphi^J_{k_2} (\delta \varphi^K_* \ast \delta \varphi^L_*)_{k_3} \rangle + \text{perms.}
\end{align*}
\]  

(2.14)

From this expression, one can distinguish two types of non-Gaussianity, depending on whether the first line or the second line dominates.

In the first case, non-Gaussianities arise from the three-point function of the scalar field(s), as models with non-standard kinetic terms.\(^11\)–\(^13\) This leads to a specific shape of non-Gaussianity, called equilateral because the dominant contribution comes from configurations where the three wavevectors have similar length $k_1 \sim k_2 \sim k_3$. 
In the second case, non-Gaussianities arise from the nonlinear dependence of $N$ of the scalar field(s). Assuming quasi-Gaussian scalar field fluctuations, one finds that (2.14) leads to a bispectrum of the form (2.11) with

$$6 \frac{f_{NL}}{5} = \frac{N_I N_J N^{IJ}}{(N_K N^K)^2}. \tag{2.15}$$

This corresponds to another shape of non-Gaussianity, usually called local or squeezed, for which the dominant contribution comes from configurations where the three wavevectors form a squeezed triangle.

The present observational constraints\(^{14}\) on these two main types of non-Gaussianity are

$$-10 < f_{NL}^{(local)} < 74 \quad (95\% \text{ CL}), \quad -214 < f_{NL}^{(equil)} < 266 \quad (95\% \text{ CL}). \tag{2.16}$$

Note that other types on non-Gaussianity can be produced, such as the folded shape (which peaks at $k_3 \sim k_1 + k_2$) arising from a non-standard vacuum.\(^{13}\)

Extending the Taylor expansion (2.13) up to third order, one can compute in a similar way the trispectrum.\(^{15}\) For local non-Gaussianity, the trispectrum can be written in the form\(^{16}\)

$$T_\zeta(k_1, k_2, k_3, k_4) = \tau_{NL} [P(k_{13})P(k_3)P(k_4) + 11 \text{ perms}]$$

$$+ \frac{54}{25} g_{NL} [P(k_2)P(k_3)P(k_4) + 3 \text{ perms}], \tag{2.17}$$

with

$$\tau_{NL} = \frac{N_{IJ} N^K N^{IJ} N^K}{(N_L N^L)^3}, \quad g_{NL} = \frac{25}{54} \frac{N_{IJ} N^K N^{IJ} N^K}{(N_L N^L)^3} \tag{2.18}$$

and where $k_{13} \equiv |k_1 + k_3|$. The present constraints on these parameters, assuming the data do not contain any isocurvature contribution, are\(^{17}\)

$$-7.4 \times 10^{-5} g_{NL} < 8.2 \quad (95\% \text{ CL}), \quad -0.6 < 10^{-4} \tau_{NL} < 3.3 \quad (95\% \text{ CL}).$$

The next section will be devoted to models with several inflatons described by a very general Lagrangian, with multi-field DBI inflation as a concrete illustration. Later, we will consider in turn two types of scenarios leading to local type non-Gaussianity: modulatons and curvatons.

§3. Generalized multi-field inflation

Although it is obviously simpler to deal with a single inflaton, high energy physics models usually predict the existence of many scalar fields. Multi-field inflation should thus be considered seriously. Moreover, many models involve kinetic terms that are not standard. It is therefore useful to develop a formalism that is as general as possible and try to tackle multi-field models described by an action of the form

$$S = \int d^4 x \sqrt{-g} \left[ \frac{R}{16\pi G} + P(X^{IJ}, \phi^K) \right], \tag{3.1}$$
where $P$ is an arbitrary function of $N$ scalar fields and their associated $N(N+1)/2$
kinetic terms

$$X^{IJ} = -\frac{1}{2} \nabla_\mu \phi^I \nabla^\mu \phi^J.$$  (3.2)

This includes multi-field models of the form

$$P(X, \phi^I) = X - V(\phi^I), \quad X \equiv G_{IJ} X^{IJ},$$  (3.3)

where $G_{IJ}$ is an arbitrary metric in field space, as well as k-inflation models, which depend on very general single-field Lagrangians.

The expansion up to second order in the linear perturbations of the action (3.1) is useful to obtain the classical equations of motion for the perturbations and to calculate the spectra of the primordial perturbations generated during inflation. Working for convenience with the scalar field perturbations $Q^I$ defined in the spatially flat gauge, the second order action can be written in the compact form

$$S_{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[ (P_{IJ}) + 2P_{(M,J),(IK)}X^{MK} \dot{Q}^I \dot{Q}^J - P_{IJ} h^{ij} \partial_i Q^I \partial_j Q^J - M_{KL} Q^K Q^L + 2 \Omega_{KI} Q^K \dot{Q}^I \right],$$  (3.4)

where the mass matrix $M_{KL}$ and the mixing matrix is $\Omega_{KI}$ can be expressed in terms of the $X^{IJ}$, of $P$ and its derivatives.

An interesting example, which combines non-standard kinetic terms with a possibly multi-dimensional inflaton space, is multi-field DBI (Dirac-Born-Infeld) inflation. In these models, motivated by string theory, inflation is due to the motion of a $D3$-brane in an internal six-dimensional compact space. The dynamics of the brane is governed by the Dirac-Born-Infeld Lagrangian, hence the name of these models.

After some appropriate reparametrizations, the Lagrangian can be written in the form

$$P = -\frac{1}{f(\phi^I)} \left( \sqrt{\det(\delta^\mu_\nu + f G_{IJ} \partial^\mu \phi^I \partial^\nu \phi^J)} - 1 \right) - V(\phi^I),$$  (3.5)

where the potential arises from the brane’s interactions with bulk fields or other branes. This Lagrangian can be written explicitly in the form (3.1) upon using

$$\det(\delta^\mu_\nu + f G_{IJ} \partial^\mu \phi^I \partial^\nu \phi^J) = 1 - 2fG_{IJ}X^{IJ} + 4f^2X^{[I}_J X^{J]} + 8f^3X^{[I}_J X^{J} X^K + 16f^4X^{[I}_J X^{J} X^K X^L},$$  (3.6)

where the field indices are lowered by the field metric $G_{IJ}$, which corresponds to the metric of the internal compact space, and the brackets denote antisymmetrization over the indices. Note that the dilaton and the various form fields are ignored in the Lagrangian (3.5), but they can also be included in the analysis of the cosmological perturbations generated by these models.

As in other multi-field inflationary models, it can be convenient to decompose the perturbations into a so-called (instantaneous) adiabatic mode, along the...
background velocity in field space, and (instantaneous) entropic modes, which are orthogonal to the adiabatic direction. Focussing, for simplicity, on the two-field case, where there is a single entropic degree of freedom, one can decompose the scalar field perturbations as

$$Q^I = Q_\sigma e^I_\sigma + Q_s e^I_s,$$

(3.7)

where the adiabatic vector $e^I_\sigma$ and entropic vector $e^I_s$ are normalized (via the field space metric $G_{IJ}$). The perturbations generated during inflation can then be determined by using the standard techniques, which gives

$$P_{Q_\sigma \ast} \simeq \frac{H^2}{4\pi^2}, \quad P_{Q_s \ast} \simeq \frac{H^2}{4\pi^2 c_s^2},$$

(3.8)

(the subscript * here indicates that the corresponding quantity is evaluated at sound horizon crossing $k c_s = a H$). For small $c_s$, the entropic modes are thus amplified with respect to the adiabatic modes.

Since we are in a multi-field scenario, the curvature perturbation can evolve after sound horizon crossing, and the spectrum of the final curvature perturbation, which is probed by cosmological observations, can be formally written as

$$P_R = (1 + T^2_{RS}) P_{R_*} = (1 + T^2_{RS}) \frac{H^4}{4\pi^2 \epsilon c_s},$$

(3.9)

where $T_{RS}$ quantifies the transfer from the entropic into the adiabatic modes.

Let us now discuss non-Gaussianities in multi-field DBI inflation. The three-point correlation functions of the scalar fields can be computed from the third order action, which is given, in the small sound speed limit, by

$$S_{(3)} = \int dt \, d^3x \left\{ \frac{a^3}{2c_s^2} \left[ (\dot{Q}_\sigma)^3 + c_s^2 \dot{Q}_\sigma (\dot{Q}_s)^2 \right] \right.$$  
$$- \frac{a}{2c_s^2} \left[ \dot{Q}_\sigma (\partial Q_\sigma)^2 - c_s^2 \dot{Q}_\sigma (\partial Q_s)^2 + 2c_s^2 \dot{Q}_s \partial Q_\sigma \partial Q_s \right] \right\}.$$  

(3.10)

The contribution from the scalar field three-point functions to the coefficient $f_{NL}$ is found to be given by

$$f_{NL}^{(3)} = -\frac{35}{108 c_s^2 (1 + T^2_{RS})},$$

(3.11)

which is similar to the single-field DBI result, but with a suppression due to the transfer between the entropic and adiabatic modes.

Interestingly, multi-field DBI inflation could also produce a local non-Gaussianity in addition to the equilateral one. Finally, let us mention that the trispectrum in multi-field DBI inflation has also been computed.

§4. Modulations

Significant non-Gaussianity can arise when a cosmological transition in the history of the Universe depends on some light scalar field, which has previously acquired
some fluctuations during the inflationary phase. Consequently, in different regions of the Universe where the value of the scalar field is slightly different, the cosmological transition and the subsequent cosmological evolution will differ. In this way, the fluctuations of the scalar field, which we will call a modulation, are converted into curvature fluctuations.

4.1. Modulated reheating

A typical example is the modulated reheating scenario\(^\text{(27), (28)}\) where the decay rate of the inflaton, \(\Gamma\), depends on a modulation \(\sigma\).

A simple way to compute the curvature perturbation is to calculate the number of e-folds between some initial time \(t_i\) during inflation, when the scale of interest crossed out the Hubble radius, and some final time \(t_f\). For simplicity, let us assume that, just after the end of inflation at time \(t_e\), the inflaton behaves like pressureless matter (as is the case for a quadratic potential) until it decays instantaneously at the time \(t_d\) characterized by \(H_d = \Gamma\). At the decay, the energy density is thus \(\rho_d = \rho_e \exp[-3(N_d - N_e)]\) and is transferred into radiation, so that, at a subsequent time \(t_f\), one gets

\[
\rho_f = \rho_e \exp[-4(N_f - N_d)] = \rho_e \exp[-3(N_f - N_e) - (N_f - N_d)].
\]

(4.1)

Using the relation \(\Gamma = H_d = H_f \exp[2(N_f - N_d)]\) to eliminate \((N_f - N_d)\) in (4.1), we finally obtain

\[
N_f = N_e - \frac{1}{3} \ln \frac{\rho_f}{\rho_e} - \frac{1}{6} \ln \frac{\Gamma}{H_f}.
\]

(4.2)

This implies that the nonlinear curvature perturbation can be simply expressed as

\[
\zeta = \zeta_{\text{inf}} - \frac{1}{6} \ln \left( \frac{\Gamma(\sigma)}{\Gamma} \right),
\]

(4.3)

where \(\zeta_{\text{inf}}\) represents the contribution to the curvature perturbation from the inflaton fluctuations. At linear level, this leads to the curvature power spectrum

\[
P_\zeta = P_{\zeta_{\text{inf}}} + \frac{1}{36} \left( \frac{\Gamma(\sigma)}{\Gamma} \right)^2 P_{\delta \sigma^2} = P_{\zeta_{\text{inf}}} + \frac{1}{36} \left( \frac{\Gamma(\sigma)}{\Gamma} \right)^2 \left( \frac{H_*}{2\pi} \right)^2.
\]

(4.4)

By expanding (4.3) up to second and third orders in \(\delta \sigma^2\), one can easily determine the bispectrum and trispectrum for the curvature perturbation. Using the expressions (2.15) and (2.18), one finds that the associated nonlinear parameters are given by

\[
f_{NL} = 5 \left( 1 - \frac{\Gamma''}{\Gamma^3} \right) \Xi^2,
\]

(4.5)

and

\[
\tau_{NL} = \frac{36}{25} f_{NL} \Xi, \quad g_{NL} = \frac{50}{3} \left( 2 - 3 \frac{\Gamma''}{\Gamma^3} + \frac{\Gamma^2 \Gamma''}{\Gamma^3} \right) \Xi^3,
\]

(4.6)

where \(\Xi = 1 - (P_{\zeta_{\text{inf}}}/P_\zeta)\) represents the fraction of the curvature power spectrum due to the modulation.
4.2. Modulated trapping

One can also envisage the possibility that a modulaton field affects the cosmological evolution during inflation. This is the case in the modulated trapping scenario,\(^{29}\) which relies on the resonant production of particles during inflation.\(^{30}\) In this model, the inflaton \(\phi\) is coupled to other fields, for example to some fermions \(\psi\) via with the interaction Lagrangian
\[
L_{\text{int}} = \lambda \phi \bar{\psi} \psi.
\]

If, during inflaton, the effective mass of \(\psi\), \(m_{\text{eff}} = m - \lambda \phi\) becomes zero, this triggers a burst of production of these particles. Even if these particles are quickly diluted by the expansion, their backreation will affect the evolution of the inflaton, governed by the equation of motion
\[
\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = \lambda n_s \left( \frac{a}{a_*} \right)^{-3} \Theta(t - t_*).
\]
Indeed, the term on the right hand side induces a temporary slow-down of the inflaton, which leads to a slightly longer phase of inflation. This brief trapping of the inflaton thus manifests itself as an increment of the number of e-folds until the end of inflation:
\[
N = N_{\text{std}}(\phi) + \Delta N_{\text{trapping}}. \quad (4.8)
\]

Let now assume that this trapping depends on some modulaton, for example via the coupling between the inflaton and the particles, and occurs well after the fluctuations of the modulaton (on observable cosmological scales) have been generated. This is in contrast with other scenarios\(^{30} - 33\) where the trapping occurs approximately when cosmological scales exit the Hubble radius, which leads to special features in the CMB spectrum as well as specific non-Gaussianity.\(^{34}\)

Using the Taylor expansion of (4.8), where only the second term \(\Delta N\) depends on the modulaton \(\sigma\):
\[
\zeta = \delta N = \frac{dN_{\text{slow-roll}}}{d\phi} \delta \phi + \cdots + \Delta N_{,\sigma} \delta \sigma + \frac{1}{2} \Delta N_{,\sigma\sigma} \delta \sigma^2 + \frac{1}{6} \Delta N_{,\sigma\sigma\sigma} \delta \sigma^3 \quad (4.9)
\]
(highest order derivatives with respect to the inflaton are ignored, because they give negligible non-Gaussianities), one can compute the power spectrum and non-Gaussianity of the curvature perturbation, generated by the modulated trapping scenario.\(^{29}\) According to (2.15), the corresponding non-linearity parameter for the bispectrum is given by
\[
\frac{6}{5} f_{NL} = \left( \frac{\Delta N_{,\sigma}^2 \Delta N_{,\sigma}}{N_{,\phi}^2 + (\Delta N_{,\phi})^2} \right)^2 \left( \frac{P_{\zeta}^{\text{trapping}}}{P_{\zeta}} \right) = \frac{N_{,\phi}^2}{(\Delta N_{,\phi})^2} = \frac{36}{25} f_{NL}^2 \frac{\Delta N_{,\sigma}}{(\Delta N_{,\phi})^2}. \quad (4.10)
\]

Similarly, the nonlinear coefficients of the trispectrum are
\[
\tau_{NL} = \left( \frac{\Delta N_{,\sigma\sigma}}{(\Delta N_{,\phi})^3} \right)^2 \Xi^3 = \frac{36}{25 \Xi} f_{NL}^2, \quad \gamma_{NL} = \frac{25 \Delta N_{,\sigma\sigma}}{54 (\Delta N_{,\phi})^3} \Xi^3. \quad (4.11)
\]
The most interesting situation occurs when the coupling $\lambda$ depends directly on the modulation, which leads to the nonlinear parameters

$$f_{\text{NL}} = \frac{1}{2e\beta} \left( 3 + 2 \frac{\lambda''}{\lambda'^2} \right) \Xi^2, \quad g_{\text{NL}} = \frac{1}{2e^2 \beta^2} \left[ 1 + 6 \frac{\lambda''}{\lambda'^2} + 4 \frac{\lambda'^3}{3 \lambda'^2} \right] \Xi^3,$$

where $\beta \equiv \text{Max}(\Delta \dot{\phi})/|\dot{\phi}_*|$ cannot exceed 1.

If $\lambda$ depends only linearly on $\sigma$, the first expression reduces to $f_{\text{NL}} = \frac{3\Xi^2}{(2e\beta)}$, which shows that it is quite easy to obtain a detectable level of non-Gaussianity in this scenario: for example, one gets $f_{\text{NL}} \simeq 55$ with $\beta = 0.01$ and $\Xi = 1$. Moreover, there is a specific relation between $\tau_{\text{NL}}$ and $g_{\text{NL}}$ which could be confronted with observations if these quantities can be measured, and thus distinguish this scenario from other scenarios leading to different relations between the nonlinear coefficients.\(^{35}\)

If the future cosmological data point to the existence of a significant amount of local non-Gaussianity, the modulated trapping scenario would thus represent a viable model, together with the modulated reheating or the curvaton scenario which we examine in the next section.

§5. Curvatons and isocurvature perturbations

The last example that we consider in this contribution is the curvaton scenario,\(^{36}\) or more precisely in the mixed curvaton and inflaton version\(^{37}\) where the inflaton fluctuations are also taken into account. The curvaton is a weakly coupled scalar field, $\sigma$, which is light relative to the Hubble rate during inflation, and hence acquires Gaussian fluctuations with an almost scale-invariant spectrum. After inflation the Hubble rate drops and eventually the curvaton becomes non-relativistic so that its energy density grows with respect to that of radiation, until it decays.

Many aspects of the curvaton scenario have been studied in the literature. Here, we wish to focus our attention on isocurvature perturbations that can be generated in this type of scenario, and their non-Gaussianity. Isocurvature non-Gaussianity, which has been investigated recently in several works,\(^{38)}\)–\(^{44)}\) could indeed be distinguished from the usual adiabatic non-Gaussianity and thus open a new window on the early Universe, if ever detected.

As a preliminary step we present a general formalism that computes systematically the evolution of the nonlinear perturbations of various fluids through a decay transition. We then apply this formalism to a scenario with a curvaton fluid, radiation and cold dark matter CDM and compute the adiabatic and isocurvature perturbations, up to third order.

5.1. Evolution of the perturbations due to the decay of some species

We now consider a very general setting where several cosmological fluids coexist, each of them characterized by the nonlinear curvature perturbation

$$\zeta_A = \delta N + \frac{1}{3(1+w_A)} \ln \frac{\rho_A}{\bar{\rho}_A},$$

where $\rho_A$ are the energy densities and $w_A$ are the equation of state parameters of the $A$ fluid. This is a generalization of the usual radiation fluid and is valid for any type of fluid, including dark matter and inflation fields.

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as follows from the definition (2.6). We wish to compute the curvature perturbations after the decay of one of these fluids, denoted \( \sigma \), which will later correspond to the curvaton.

In the sudden decay approximation, which we adopt here, the decay takes place on the hypersurface characterized by \( H_d = \Gamma_\sigma \), where \( H_d \) is the Hubble parameter at the decay and \( \Gamma_\sigma \) is the decay rate of \( \sigma \). Since \( H \) depends only on the total energy density, the decay hypersurface is a hypersurface of uniform total energy density, with \( \delta N_d = \zeta \), where \( \zeta \) is the global curvature perturbation. The equality between the sum of all energy densities, before the decay and after the decay, thus reads

\[
\sum_A \bar{\rho}_A e^{3(1+w_A)}(\zeta_A - \zeta) = \bar{\rho}_{\text{decay}} = \sum_B \bar{\rho}_B e^{3(1+w_B)}(\zeta_B - \zeta),
\]

where the subscripts \(-\) and \(+\) denote quantities defined, respectively, before and after the transition. In the above formula, we have used the non-linear energy densities of the individual fluids, which can be expressed in terms of their curvature perturbation \( \zeta_A \) by inverting the expression (5.1).

Expanding the first equality in (5.2) up to third order, one finds

\[
\zeta = \sum_A \lambda_A \left[ \zeta_A^- + \frac{\beta_A A}{2} (\zeta_A - \zeta)^2 + \frac{\beta_A^2 A}{6} (\zeta_A - \zeta)^3 \right],
\]

with the coefficients

\[
\beta_A \equiv 3(1 + w_A), \quad \lambda_A \equiv \frac{\tilde{\Omega}_A}{\tilde{\Omega}}, \quad \tilde{\Omega}_A \equiv (1 + w_A) \Omega_A, \quad \tilde{\Omega} \equiv \sum_A \tilde{\Omega}_A,
\]

where the abundance parameters are defined just before the decay: \( \Omega_A \equiv \bar{\rho}_A^- / \bar{\rho}_{\text{decay}} \). Note that, although the global perturbation \( \zeta \) appears on both sides of (5.3), this relation can be used iteratively in order to determine, order by order, the expression of \( \zeta \) in terms of all the \( \zeta_A^- \), up to third order.

Just after the decay, the energy density of the fluid \( \sigma \) is transferred into the one or several of the remaining fluids. Introducing the relative branching ratios \( \gamma_{A\sigma} \), this means that the energy density for any species \( A \), just after the decay of \( \sigma \), is simply given by

\[
\rho_{A+} = \rho_{A-} + \gamma_{A\sigma} \rho_{\sigma}.
\]

This relation, which is fully non-linear, can be reexpressed, upon using (5.1), in the form

\[
e^{\beta_A (\zeta_A+ - \zeta)} = (1 - f_A) e^{\beta_A (\zeta_A- - \zeta)} + f_A e^{\beta_{\sigma} (\zeta_{\sigma} - \zeta)},
\]

where the parameter

\[
f_A \equiv \frac{\gamma_{A\sigma} \Omega_\sigma}{\Omega_A + \gamma_{A\sigma} \Omega_\sigma}
\]

represents the fraction of the fluid \( A \) that has been created by the decay.

Expanding (5.6) up to third order, and using (5.3), one gets

\[
\zeta_{A+} = \sum_B T_A^B \left[ \zeta_B^- + \frac{\beta_B B}{2} (\zeta_B - \zeta)^2 + \frac{\beta_B^2 B}{6} (\zeta_B - \zeta)^3 \right]
\]
\[-\frac{\beta_A}{2} (\zeta_{A+} - \zeta)^2 - \frac{\beta_A^2}{6} (\zeta_{A+} - \zeta)^3 \]  

(5.8)

with the coefficients

\[ T_A^A = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_A + (1 - f_A), \]  

(5.9)

\[ T_A^\sigma = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_\sigma + f_A \frac{\beta_\sigma}{\beta_A}, \]  

(5.10)

\[ T_A^C = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_C, \quad C \neq A, \sigma. \]  

(5.11)

Finally, after using (5.3) again, one finds that (5.8) yields the full expression, up to third order, of all the post-decay curvature perturbations in terms of the pre-decay curvature perturbations:

\[ \zeta_{A+} = \sum_B T_B^A \zeta_{B-} + \sum_{B,C} U^{BC}_A \zeta_{B-} \zeta_{C-} + \sum_{B,C,D} V^{BCD}_A \zeta_{B-} \zeta_{C-} \zeta_{D-}, \]  

(5.12)

with

\[ U^{BC}_A \equiv \frac{1}{2} \left[ \sum_E \beta_E T_E^A (\delta_{EB} - \lambda_B)(\delta_{EC} - \lambda_C) - \beta_A (T_{AB} - \lambda_B)(T_{AC} - \lambda_C) \right], \]

and

\[ V^{BCD}_A \equiv -\frac{1}{2} \sum_{E,F} \beta_E T_{AE} (\delta_{EB} - \lambda_B) \lambda_F (\delta_{FC} - \lambda_C)(\delta_{FD} - \lambda_D) \]

\[ + \frac{1}{6} \sum_E \beta_E^2 T_{AE} (\delta_{EB} - \lambda_B)(\delta_{EC} - \lambda_C)(\delta_{ED} - \lambda_D) \]

\[ - \beta_A (T_{AB} - \lambda_B) \left[ U^{CD}_A - \frac{1}{2} \sum_E \beta_E \lambda_E (\delta_{EC} - \lambda_C)(\delta_{ED} - \lambda_D) \right] \]

\[ - \frac{1}{6} \beta_A^2 (T_{AB} - \lambda_B)(T_{AC} - \lambda_C)(T_{AD} - \lambda_D). \]

The above expression thus provides a systematic computation of the post-decay curvature perturbations for all fluids in a very general setting. For scenarios with several decay transitions, the perturbations can be obtained by combining the various expressions of the type (5.12) for each transition.

5.2. Mixed curvaton and inflaton scenario

We now apply the general formalism presented above to a scenario involving a curvaton \( \sigma \), behaving as a pressureless fluid, in addition to radiation \( (r) \) and CDM \( (c) \), which can lead to isocurvature perturbations. The formula (5.12) allows us to compute, in terms of the pre-decay perturbations, the perturbations \( \zeta_r \) and \( \zeta_c \) after the decay, or equivalently the adiabatic perturbation, which coincides with \( \zeta_r \) deep in the radiation era, and the CDM isocurvature perturbation

\[ S_c = 3(\zeta_c - \zeta_r). \]  

(5.13)
For simplicity, we restrict our analysis to the situation where
\[ \zeta_c = \zeta_r = \zeta_{\text{inf}}, \] (5.14)
by assuming that the CDM and radiation perturbations, before the curvaton decay, depend only on the inflaton fluctuations.

The curvaton fluid isocurvature perturbation before the decay, \( S_\sigma \), can be easily related to the curvaton field fluctuations in the case of a quadratic potential. Indeed, writing the (non-linear) energy density of the oscillating curvaton defined on the spatially flat hypersurfaces, characterized by \( \delta N = \zeta_r \) when the curvaton is still subdominant:
\[ \rho_\sigma = m^2 \sigma^2 = m^2 (\bar{\sigma} + \delta \sigma)^2 = \bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)} = \bar{\rho}_\sigma e^{S_\sigma}, \] (5.15)
leads to the relation
\[ e^{S_\sigma} = \left(1 + \frac{\delta \sigma}{\bar{\sigma}}\right)^2. \] (5.16)
Expanding this expression up to third order, and using the conservation of \( \delta \sigma / \sigma \) in a quadratic potential, we obtain
\[ S_\sigma = \hat{S} - \frac{1}{4} \hat{S}^2 + \frac{1}{12} \hat{S}^3, \] (5.17)
where the quantity
\[ \hat{S} \equiv 2 \frac{\delta \sigma}{\bar{\sigma}} \] (5.18)
is Gaussian.

Using the general expressions (5.12), one finds that the primordial curvature perturbation is given by
\[ \zeta_r = \zeta_{\text{inf}} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \frac{1}{6} z_3 \hat{S}^3, \] (5.19)
with
\[ z_1 = \frac{r}{3}, \quad z_2 = \frac{r}{18} \left(3 - 8r + \frac{4r}{\xi} - \frac{2r^2}{\xi^2}\right), \] (5.20)
\[ z_3 = \frac{r^2}{54} \left(\frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} + \frac{15r^2}{\xi^2} + 64r + \frac{18}{\xi} - 36\right), \] (5.21)
where the parameter
\[ \xi \equiv \frac{\gamma_{r\sigma}}{1 - (1 - \gamma_{r\sigma}) \Omega_\sigma} \] (5.22)
can be interpreted as the efficiency of the energy transfer from the curvaton into radiation (\( \xi = 1 \) if the curvaton decays only into radiation, i.e. \( \gamma_{r\sigma} = 1 \)), and
\[ r \equiv \xi \tilde{r}, \quad \tilde{r} = \frac{3 \Omega_\sigma}{4 - \Omega_\sigma}. \] (5.23)
For the primordial isocurvature perturbation, one obtains
\[ S_c = s_1 \dot{S} + \frac{1}{2} s_2 \dot{S}^2 + \frac{1}{6} s_3 \dot{S}^3, \] (5.24)
with
\[ s_1 = f_c - r, \quad s_2 = \frac{1}{6} \left( 3 f_c (1 - 2 f_c) + \frac{2 r^3}{\xi^2} - \frac{4 r^2}{\xi} + 8 r^2 - 3 r \right), \] (5.25)
\[ s_3 = -\frac{1}{2} f_c^2 (3 - 4 f_c) - \frac{r^2}{18} \left( \frac{6 r^3}{\xi^4} + \frac{24 r^2}{\xi^2} - \frac{4 r^2}{\xi} - \frac{48 r}{\xi} - \frac{15 r}{\xi^2} + 64 r + \frac{18}{\xi} - 36 \right). \] (5.26)

From these expressions, one can determine the power spectrum, the bispectrum and the trispectrum, which can be probed by observations.

The power spectrum for the total curvature perturbation follows from the linear part and is given by
\[ P_{\zeta r} = P_{\zeta \text{inf}} + \frac{r^2}{9} P_{\dot{S}} = \Xi^{-1} \frac{r^2}{9} P_{\dot{S}}, \] (5.27)
where \( \Xi \) represents the fraction of the power spectrum due to the curvaton contribution. The power spectrum for the isocurvature fluctuations is, according to Eq. (5.24),
\[ P_{S_c} = (f_c - r)^2 P_{\dot{S}}. \] (5.28)

Both curvature and isocurvature perturbations depend on the curvaton fluctuations and are therefore correlated, with the correlation coefficient:
\[ C = \frac{P_{S_c, \zeta r}}{\sqrt{P_{S_c} P_{\zeta r}}} = \varepsilon f \sqrt{\Xi}, \quad \varepsilon f \equiv \text{sgn}(f_c - r). \] (5.29)

In the pure curvaton limit (\( \Xi \approx 1 \)), adiabatic and isocurvature perturbations are either fully correlated, if \( \varepsilon f > 0 \), or fully anti-correlated, if \( \varepsilon f < 0 \). In the opposite limit (\( \Xi \ll 1 \)), the correlation vanishes. For intermediate values of \( \Xi \), the correlation is only partial, as can also be obtained in multifield inflation.\(^{47}\)

The isocurvature-to-adiabatic ratio
\[ \alpha = \frac{P_{S_c}}{P_{\zeta r}} = 9 \left( 1 - \frac{f_c}{r} \right)^2 \Xi, \] (5.30)
is strongly constrained by cosmological observations, the precise limits depending on the assumed level of correlation between the isocurvature and adiabatic perturbations (since the impact of isocurvature perturbations on the observable power spectrum depends crucially on this correlation\(^{48}\)). In terms of the parameter \( a \equiv \alpha/(1 + \alpha) \), the limits (based on WMAP+BAO+SN data) given in 14) are
\[ a_{|\Xi=0} < 0.064 \quad (95\% \text{ CL}), \quad a_{|\Xi=1} < 0.0037 \quad (95\% \text{ CL}), \] (5.31)
respectively for the uncorrelated case (\( \Xi = 0 \)) and for the fully correlated case (\( \Xi = 1 \)). According to (5.30), the observational constraint \( \alpha \ll 1 \) can be satisfied if \( |f_c - r| \ll r \) (which includes the case \( f_c = 1 \) with \( r \approx 1 \)) or if \( \Xi \ll 1 \), i.e. the curvaton contribution to the observed power spectrum is very small.
5.3. Adiabatic and isocurvature non-Gaussianities

Since we now deal with two observable quantities, namely adiabatic and isocurvature perturbations, the definition of the bispectrum can be extended to include both types of perturbations. In our particular case, where there is only one degree of freedom, $\hat{S}$, at the nonlinear level, one can show that the generalized bispectra (with indices $I = \{\zeta, S\}$) are of the form\(^{43}\)

$$B^{IJK}(k_1, k_2, k_3) = b^{IJK}_NL P_S(k_2)P_S(k_3) + b^{IKJ}_NL P_S(k_1)P_S(k_3) + b^{KIJ}_NL P_S(k_1)P_S(k_2)$$

(5.32)

with

$$b^{IJK}_NL \equiv N^{I(2)}_2 N^{J(1)}_1 N^{K(1)}_1,$$

(5.33)

where $N^{\zeta (2)}_2 = z_2$, $N^{S (2)}_2 = s_2$, $N^{\zeta (1)}_1 = z_1$, $N^{S (1)}_1 = s_1$, respectively.

Recalling that the usual, purely adiabatic, $f_{NL}$ is proportional to the bispectrum of $\zeta$ divided by the square of the power spectrum, one defines the analogs of $f_{NL}$ by dividing the coefficients $b^{IJK}_NL$ by the square of the ratio $P_{\zeta}/P_S = z_1^2/\Xi$, i.e.

$$\bar{f}^{IJK}_NL \equiv f^{IJK}_NL \equiv \frac{z_2^2}{z_1^2} b^{IJK}_NL.$$  

(5.34)

Taking into account the fact that the last two indices can be permuted, this leads to six different coefficients, explicitly given by the expressions

$$\bar{f}^{\zeta\zeta\zeta}_NL = \frac{z_2^2}{z_1^2} \Xi^2, \quad \bar{f}^{\zeta\zeta S}_NL = \frac{s_1^2z_2^2}{z_1^2} \Xi^2, \quad \bar{f}^{\zeta S\zeta}_NL = \frac{s_2^2}{z_1^2} \Xi^2,$$

(5.35)

$$\bar{f}^{\zeta SS}_NL = \frac{s_1^2z_2^2}{z_1^2} \Xi^2, \quad \bar{f}^{S\zeta\zeta}_NL = \frac{s_1s_2}{z_1^2} \Xi^2, \quad \bar{f}^{S SS}_NL = \frac{s_1^2s_2}{z_1^2} \Xi^2.$$  

(5.36)

Remarkably, these six different types of local non-Gaussianities can in principle be discriminated in the CMB data, independently of any specific primordial scenario, because they lead to six distinct angular bispectra. It would thus be extremely interesting to look for these new types of non-Gaussianities in the future CMB data in order to measure or constrain the six parameters $\bar{f}^{IJK}_NL$ that generalize the usual adiabatic parameter\(^{49}\).

The same analysis applies to the trispectra that combine adiabatic and isocurvature perturbations, leading to the generalized parameters\(^{44}\)

$$\tau^{IJKL}_NL \equiv \frac{N^{I(2)}_2 N^{J(1)}_1 N^{K(1)}_1 N^{L(1)}_1}{z_1^6} \Xi^3, \quad \gamma^{IJKL}_NL \equiv \frac{54}{25} g^{IJKL}_NL \equiv \frac{N^{I(3)}_3 N^{J(1)}_1 N^{K(1)}_1 N^{L(1)}_1}{z_1^6} \Xi^3,$$

(5.37)

where $N^{\zeta (3)}_3 = z_3$ and $N^{S (3)}_3 = s_3$. Taking into account the symmetries under permutations of the indices, one finds, for two observables ($I = \{\zeta, S\}$), 9 different parameters $\tau^{IJKL}_NL$ and 8 parameters $\gamma^{IJKL}_NL$.

An interesting question is whether one can find significant non-Gaussianities, while satisfying the bound on the isocurvature spectrum. As mentioned earlier, this isocurvature constraint can be satisfied with $\Xi \simeq 1$ if $f_c$ and $r$ are sufficiently
close. In this case, one finds that the purely adiabatic non-Gaussianity dominates. But in the alternative situation where \( \Xi \ll 1 \), one finds that, with respect to the purely adiabatic non-Gaussianity, the purely and mixed isocurvature ones are either enhanced by constant factors, if \( f_c \ll r \ll 1 \), or much more strongly enhanced with powers of \( (3f_c/r) \), if \( r \ll f_c \ll 1 \).

§6. Conclusions

As this contribution has tried to illustrate with a few explicit examples, the detection of primordial non-Gaussianities would have dramatic consequences.

First, since the simplest inflationary models, based on a single field in slow roll, predict a negligible amount of primordial non-Gaussianities, these models would have to be replaced with more elaborate models, involving several scalar fields, non standard kinetic terms or other features.

Second, since the measurement of non-Gaussianities contains potentially a lot of information, in particular concerning their shape, one could hope to discriminate between different categories of models, which would otherwise appear degenerate in their predictions of the power spectrum.

Even the simplest shape of non-Gaussianity, the local shape, can hide surprisingly rich variations if perturbations are generated by several scalar fields and if isocurvature perturbations survive. In such situation, purely adiabatic, purely isocurvature and mixed non-Gaussianities could coexist and the hierarchy between their amplitudes would provide invaluable information. Since isocurvature fluctuations are usually associated with the generation of dark matter and baryon asymmetry in the Universe, non-Gaussianity from isocurvature fluctuations, if detected in the future, would give us a lot of insight into the nature of dark matter, the mechanism of baryogenesis, and therefore into high energy physics.

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