On the Gerasimov-Drell-Hearn sum rule for the deuteron

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Abstract

The Gerasimov-Drell-Hearn sum rule is evaluated for the deuteron by explicit integration up to a photon energy of 550 MeV including contributions from the photodisintegration channel and from coherent and incoherent single pion production as well. The photodisintegration channel converges fast enough in this energy range and gives a large negative contribution, essentially from the $^1S_0$ resonant state near threshold. Its absolute value is about the same size than the sum of proton and neutron GDH values. It is only partially cancelled by the single pion production contribution. But the incoherent channel has not reached convergence at 550 MeV.

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I. INTRODUCTION

The Gerasimov-Drell-Hearn (GDH) sum rule connects the anomalous magnetic moment of a particle with the energy weighted integral - henceforth denoted by \( I^{GDH} \) - from threshold up to infinity over the difference of the total photoabsorption cross sections for circularly polarized photons on a target with spin parallel and antiparallel to the spin of the photon. In detail it reads for a particle of mass \( M \), charge \( eQ \), anomalous magnetic moment \( \kappa \) and spin \( S \)

\[
I^{GDH} = \frac{4\pi^2 \kappa^2 e^2}{M^2} S = \int_0^\infty \frac{dk}{k} \left( \sigma^P(k) - \sigma^A(k) \right),
\]

(1)

where \( \sigma^{P/A}(k) \) denote the total absorption cross sections for circularly polarized photons on a target with spin parallel and antiparallel to the photon spin, respectively, and the anomalous magnetic moment is defined by the total magnetic moment operator of the particle

\[
\vec{M} = (Q + \kappa) \frac{e}{M} \vec{S}.
\]

(2)

This sum rule gives a very interesting relation between a magnetic ground state property of a particle and an integral property of its whole excitation spectrum. In other words, this sum rule shows that the existence of a nonvanishing anomalous magnetic moment points directly to an internal dynamic structure of the particle. Furthermore, because the lhs of (1) is positive, it tells us that the integrated, energy-weighted total absorption of a circularly polarized photon on a particle with its spin parallel to the photon spin is bigger than the one on a target with its spin antiparallel, if the particle possesses a nonvanishing anomalous magnetic moment.

The GDH sum rule has first been derived by Gerasimov [1] and, shortly afterwards, independently by Drell and Hearn [2]. It is based on two ingredients which follow from the general principles of Lorentz and gauge invariance, unitarity, crossing symmetry and causality of the Compton scattering amplitude of a particle. The first one is the low energy theorem of Low [3] and Gell-Mann and Goldberger [4] for a spin one-half particle which
later has been generalized to arbitrary spin \[3\]. The second ingredient is the assumption of an unsubtracted dispersion relation for the difference of the elastic forward scattering amplitudes for circularly polarized photons and a completely polarized target with spin parallel and antiparallel to the photon spin.

Since proton and neutron have large anomalous magnetic moments, one finds large GDH sume rule predictions for them, i.e., \(I^{GDH}_p = 204.8 \mu b\) for the proton and \(I^{GDH}_n = 233.2 \mu b\) for the neutron. Although this sum rule is known for more than 30 years, it has never been verified by a direct measurement. Early evaluation by Karliner \[8\], based on a multipole analysis of experimental data on pion photoproduction on the nucleon, did not give conclusive results due to the lack of data at higher energies, and even present day data do not allow a definite answer as to its validity (see e.g. \[9\]). The recent interest in this sum rule stems from the study of the spin dependent structure functions in deep inelastic scattering \[10\].

Applying the GDH sum rule to the deuteron, one finds a very interesting feature. On the one hand, the deuteron has isospin zero ruling out the contribution of the large nucleon isovector anomalous magnetic moments to its magnetic moment. Therefore, one expects a very small anomalous magnetic moment for the deuteron. In fact, the experimental value is \(\kappa_d = -0.143\) resulting in a GDH prediction of \(I^{GDH}_d = 0.65 \mu b\), which is more than two orders of magnitude smaller than the nucleon values. On the other hand, considering the possible absorption processes, we note first that the incoherent pion production on the deuteron is dominated by the quasifree production on the nucleons bound in the deuteron. Thus it is plausible to expect from these processes a contribution to the GDH value roughly given by the sum of the proton and neutron GDH values, i.e., 438 \(\mu b\). Additional contributions arise from the coherent \(\pi^0\) production channel. In order to obtain the small total deuteron GDH value one, therefore, needs a large negative contribution of about the same size for cancellation. Indeed, one has an additional channel not present for the nucleon, namely the photodisintegration channel which is the only photoabsorption process below the pion production threshold. A closer look shows in fact that at very low energies near threshold
a sizeable negative contribution arises from the $M1$-transition to the resonant $^1S_0$ state, because this state can only be reached if the spins of photon and deuteron are antiparallel, and is forbidden for the parallel situation.

It is the aim of the present paper to report on an evaluation of the GDH sum rule for the deuteron by explicit integration of the GDH integral up to a photon energy of 550 MeV including the photodisintegration channel as well as coherent and incoherent single pion photoproduction channels.

II. THE GDH SUM RULE FOR THE DEUTERON

For the deuteron, one can express the difference of the cross sections by the vector target asymmetry $\tau_{c10}$, i.e.,

$$\sigma^P(k) - \sigma^A(k) = \sqrt{6}\sigma^0\tau_{c10},$$  

where $\sigma^0$ denotes the unpolarized absorption cross section. Explicitly, $\sigma^0$ and the difference may be expressed in terms of the electric and magnetic multipole matrix elements by

$$\sigma^0 = \frac{4\pi}{9} \sum_{Lj\lambda} \left( |E_L(\lambda j)|^2 + |M_L(\lambda j)|^2 \right),$$  

$$\sigma^P(k) - \sigma^A(k) = 4\pi\sqrt{6} \sum_{LL'j\lambda} \left( -1 \right)^j \begin{pmatrix} L' & L & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} L' & L & 1 \\ 1 & 1 & j \end{pmatrix} \Re[ (E_{L'}^*(\lambda j) + M_{L'}^*(\lambda j))(E_L(\lambda j) + M_L(\lambda j))] ,$$

where $\lambda$ labels the possible final states of a given total angular momentum $j$. Note, that due to parity conservation one has in (4) and (5) either electric or magnetic contributions for a given multipolarity $L$ and state $\lambda j$.

We have evaluated explicitly the GDH sum rule for the deuteron by integrating the difference of the two total photoabsorption cross sections with photon and deuteron spins parallel and antiparallel up to a photon energy of 550 MeV. Three contributions have been included: (i) the photodisintegration channel $\gamma d \rightarrow np$, (ii) the coherent pion production $\gamma d \rightarrow \pi^0 d$, and (iii) the incoherent pion production $\gamma d \rightarrow \pi NN$. The upper integration limit
of 550 MeV has been chosen because on the one hand one finds sufficient convergence for the photodisintegration channel, while on the other hand, only single pion photoproduction has been considered, thus limiting the applicability of the present theoretical treatment to energies not too far above the two pion production threshold as long as significant contributions from multipion production cannot be expected. Indeed, the evaluation of $I^{GDH}$ for the nucleon by Karliner indicates that significant contributions from two-pion production start only above this energy. We will now discuss the three contributions separately.

A. Photodisintegration

In this case, the final states are the partial waves of $np$ scattering, and for a fixed $j$, one has four final partial waves which are labelled by $\lambda$ in the Blatt-Biedenharn convention. For the leading contributions $L = L' = 1$, one finds

$$\sigma^P(k) - \sigma^A(k) = -\frac{2\pi}{3} \sum_\lambda \left( 2|E^1(\lambda 0)|^2 + |E^1(\lambda 1)|^2 - |E^1(\lambda 2)|^2 
+ 2|M^1(\lambda 0)|^2 + |M^1(\lambda 1)|^2 - |M^1(\lambda 2)|^2 \right). \quad (6)$$

We first note, that $E1$-transitions lead to the $^1P_1$ and $^3P_j$ ($j = 0, 1, 2$) states. However, the isoscalar $^1P_1$ is largely suppressed while the triplet $^3P_j$ contributions almost cancel each other. The cancellation would be complete if spin-orbit and tensor forces could be neglected, because in this case the matrix elements are simply related by angular momentum recoupling coefficients. Thus, at low energies remain the $M1$-transitions, essentially to $^1S_0$ and $^3S_1$ states. Of these, the $^1S_0$ contributions is dominant because of the large isovector part of the $M1$-operator coming from the large isovector anomalous magnetic moment of the nucleon. It is particularly strong close to break-up threshold where the $^1S_0$ state is resonant. It can only be reached by the antiparallel spin combination resulting in a strong negative contribution to the GDH sum rule.

The photodisintegration channel is evaluated within the nonrelativistic framework as is described in detail in Ref. [11] but with inclusion of the most important relativistic contribu-


tions. Explicitly, all electric and magnetic multipoles up to the order \( L = 4 \) are considered which means inclusion of the final state interaction in all partial waves up to \( j = 5 \). For the calculation of the initial deuteron and the final n-p scattering wave functions we use the realistic Bonn potential (r-space version) \([12]\). In the current operator we distinguish the one-body currents with Siegert operators (N), explicit meson exchange contributions (MEC) beyond the Siegert operators, essentially from \( \pi \)- and \( \rho \)-exchange, contributions from isobar configurations of the wave functions (IC), calculated in the impulse approximation \([13]\), and leading order relativistic contributions (RC).

The results are summarized in Fig. 1, where the cross section difference and the GDH integral is shown. The GDH values are listed in Tab. 1. One readily notes the huge negative contribution from the \( ^1S_0 \)-state at low energies (see the upper left panel of Fig. 1). Here, the effects from MEC are relatively strong resulting in an enhancement of the negative value by about 15 percent. It corresponds to the well-known 10 percent enhancement of the radiative capture of thermal neutrons on protons. Isobar effects are significant in the region of the \( \Delta \)-resonance, as expected. They give a positive contribution, but considerably smaller in absolute size than MEC. The largest positive contribution stems from RC in the energy region up to about 100 MeV (see the upper right panel of Fig. 1) reducing the GDH value in absolute size by more than 30 percent. This strong influence from relativistic effects is not surprising in view of the fact, that the correct form of the term linear in the photon momentum of the low energy expansion of the forward Compton scattering amplitude is only obtained if leading order relativistic contributions are included \([7]\). The total sum rule value from the photodisintegration channel then is \( I^{\text{GDH}}_{\gamma d \rightarrow np}(550 \text{ MeV}) = -413 \mu b \). Almost the same value is obtained for the Paris Potential \([14]\). Its absolute value almost equals within less than ten percent the sum of the free proton and neutron values. This may not be accidental since the large value is directly linked to the nucleon anomalous magnetic moment as is demonstrated by the fact that one finds indeed a very small but positive value \( I^{\text{GDH}}_{\gamma d \rightarrow np}(550 \text{ MeV}) = 7.3 \mu b \) if the nucleon anomalous magnetic moment is switched off in the e.m. one-body current operator (for further details see \([14]\)).
B. Coherent pion production

The theoretical model used to calculate the contribution of the coherent pion production channel is described in detail in Refs. [13] and [16]. The reaction is clearly dominated by the magnetic dipole excitation of the $\Delta$ resonance from which one expects a strong positive $I_{\gamma d \rightarrow d\pi^0}^{GDH}$ contribution. The reason for this is that the $\Delta$-excitation is favoured if photon and nucleon spins are parallel compared to the antiparallel situation. The model takes into account pion rescattering by solving a system of coupled equations for the $N\Delta$, $NN\pi$ and $NN$ channels. The most important rescattering mechanism is due the successive excitation and decay of the $\Delta$ resonance. The inclusion of the rescattering effects is important and leads in general to a significant reduction of the cross section in reasonable agreement with the differential cross section data available in the $\Delta$ region.

Fig. 2 shows the result of our calculation. One sees the strong positive contribution from the $\Delta$-excitation giving a value $I_{\gamma d \rightarrow d\pi^0}^{GDH}(550\text{MeV}) = 63\mu b$. The comparison with the unpolarized cross section, also plotted in Fig. 2, demonstrates the dominance of $\sigma^P$ over $\sigma^A$. Furthermore, one notes quite satisfactory convergence at the highest energy considered here.

C. Incoherent pion production

The calculation of the $\gamma d \rightarrow \pi NN$ contributions to the GDH integral is based on the spectator nucleon approach discussed in [17]. In this framework, the reaction proceeds via the pion production on one nucleon while the other nucleon acts merely as a spectator. Thus, the $\gamma d \rightarrow \pi NN$ operator is given as the sum of the elementary $\gamma N \rightarrow \pi N$ operators of the two nucleons. For this elementary operator, we have taken the standard pseudovector Born terms and the contribution of the $\Delta$ resonance, and a satisfactory description of pion photoproduction on the nucleon is achieved in the $\Delta$-resonance region [17]. Although the spectator model does not include any final state interaction except for the resonant $M_{1+}^{3/2}$
multipole, it gives quite a good description of available data on the total cross section demonstrating the dominance of the quasifree production process, for which the spectator model should work quite well.

The results are collected in Fig. 3. The upper part shows the individual contributions from the different charge states of the pion and their total sum to the cross section difference for pion photoproduction on both the deuteron and for comparison on the nucleon. One notes qualitatively a similar behaviour although the maxima and minima are smaller and also slightly shifted towards higher energies for the deuteron. In the lower part of Fig. 3 the corresponding GDH integrals are shown. A large positive contribution comes from $\pi^0$-production whereas the charged pions give a negative but - in absolute size - smaller contribution to the GDH value. Up to an energy of 550 MeV one finds for the total contribution of the incoherent pion production channels a value $I_{\gamma d \rightarrow NN\pi}^{GDH}(550 \text{ MeV}) = 167 \mu b$ which is remarkably close to the sum of the neutron and proton values for the given elementary model $I_n^{GDH}(550 \text{ MeV}) + I_p^{GDH}(550 \text{ MeV}) = 163 \mu b$. It underlines again that the total cross section is dominated by the quasifree process. However, as is evident from Fig. 3 convergence is certainly not reached at this energy. Furthermore, the elementary pion production operator had been constructed primarily to give a realistic description of the $\Delta$ resonance region. In fact, it underestimates the GDH integral up to 550 MeV by about a factor two compared to a corresponding evaluation based on a multipole analysis of experimental pion photoproduction data, as is discussed in the next section. For this reason we cannot expect that this model gives also a good description of experimental data above 400 MeV. But the important result is, that the total GDH contribution from the incoherent process is very close to the sum of the free proton and neutron GDH integrals which will remain valid for an improved elementary production operator.
III. SUMMARY AND CONCLUSIONS

The contributions from all three channels and their sum are listed in Tab. I. A very interesting and important result is the large negative contribution from the photodisintegration channel near and not too far above the break-up threshold with surprisingly large relativistic effects below 100 MeV. Hopefully, this low energy feature of the GDH sum rule could be checked experimentally in the near future.

For the total GDH value from explicit integration up to 550 MeV, we find a negative value $I_{d}^{GDH}(550 \text{ MeV}) = -183 \mu b$. However, as we have mentioned above, some uncertainty lies in the contribution of the incoherent pion production channel because of shortcomings of the model of the elementary production amplitude above the $\Delta$ resonance. If we use instead of the model value $I_{\gamma d \rightarrow NN\pi}^{GDH}(550 \text{ MeV}) = 167 \mu b$ (cf. previous section) the sum of the GDH values of neutron and proton by integrating the cross section difference obtained from a multipole analysis of experimental data (fit SM95 from [18]), giving $I_{n}^{GDH}(550 \text{ MeV}) + I_{p}^{GDH}(550 \text{ MeV}) = 331 \mu b$, we find for the deuteron $I_{d}^{GDH}(550 \text{ MeV}) = -19 \mu b$, which we consider a more realistic estimate. Since this value is still negative, a positive contribution of about the same size should come from contributions at higher energies in order to fulfil the small GDH sum rule for the deuteron, provided that the sum rule is valid. These contributions should come from the incoherent single pion production above 550 MeV because here convergence had not been reached in contrast to the other two channels, photodisintegration and coherent pion production, and in addition, from multipion production.

It remains as a task for future research to improve the elementary pion photoproduction operator above the $\Delta$ resonance. But for this also precise data on $\sigma^P - \sigma^A$ from a direct measurement is urgently needed. Furthermore, for the reaction on the deuteron, the influence of final state interaction has to be investigated, too. Because the large cancellation between the various contributions requires quite a high degree of precision for the theoretical description. For this reason, also at least two-pion production contributions have to be
considered in order to obtain more reliable predictions at higher energies.
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TABLE I. Various contributions of the photodisintegration channel to the GDH integral for the deuteron integrated up to 550 MeV in $\mu$b.

|     | N   | N+MEC | N+MEC+IC | N+MEC+IC+RC |
|-----|-----|-------|----------|-------------|
| N   | $-619$ | $-689$ | $-665$   | $-413$      |

TABLE II. Contributions of the different absorption channels to the GDH integral for the deuteron integrated up to 550 MeV in $\mu$b.

| Channel                 | $\gamma d \rightarrow np$ | $\gamma d \rightarrow d\pi^0$ | $\gamma d \rightarrow np\pi^0$ | $\gamma d \rightarrow nn\pi^+$ | $\gamma d \rightarrow pp\pi^-$ | Total       |
|------------------------|----------------------------|-------------------------------|--------------------------------|-------------------------------|-------------------------------|-------------|
|                        | $-413$                     | $63$                          | $288$                          | $-35$                         | $-86$                         | $-183$      |
FIG. 1. Contribution of the photodisintegration channel to the GDH sum rule for the deuteron.

Two upper and lower left panels: difference of the cross sections in various energy regions; lower right panel: $I_{\gamma d\rightarrow np}^{GDH}$ as function of the upper integration energy. Dashed curves: N, dash-dot: N+MEC, dotted: N+MEC+IC, and full curves N+MEC+IC+RC.
FIG. 2. Contribution of the coherent $\pi^0$ production to the GDH sum rule for the deuteron. Left panel: difference of the cross sections (full curve), the dashed curve shows the unpolarized cross section; right panel: $I_{\gamma d \rightarrow d\pi^0}^{GDH}$ as function of the upper integration energy.

FIG. 3. Contribution of the incoherent $\pi$ production to the GDH sum rule for the deuteron and the nucleon. Upper part: difference of the cross sections; lower part: $I_{\gamma d \rightarrow NN\pi}^{GDH}$ as function of the upper integration energy. Full curves for the deuteron, dotted curves for the nucleon. In the case of $\pi^0$ production, the dotted curve shows the summed proton and neutron contributions.