Spontaneous violation of chiral symmetry in QCD vacuum is the origin of baryon masses and determines baryon magnetic moments and their other static properties

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Abstract

A short review is presented of the spontaneous violation of chiral symmetry in QCD vacuum. It is demonstrated, that this phenomenon is the origin of baryon masses in QCD. The value of nucleon mass is calculated as well as the masses of hyperons and some baryonic resonances and expressed mainly through the values of quark condensates $\langle 0 | \bar{q}q | 0 \rangle$, $q = u, d, s$ – the vacuum expectation values (v.e.v.) of quark field. The concept of vacuum expectation values induced by external fields is introduced. It is demonstrated that such v.e.v. induced by static electromagnetic field results in quark condensate magnetic susceptibility, which plays the main role in determination of baryon magnetic moments. The magnetic moments of proton, neutron and hyperons are calculated. The results of calculation of baryon octet $\beta$-decay constants are also presented.

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This paper is dedicated to the 85-th anniversary of Spartak Timofeevich Belyaev. Among all of his striking qualities I would like to especially stress the following – his principality, the principality both in science, and in social life. Spartak (I tell him so, because of our old friendship) is not going to any compromise, leaves not a position, which he considers as principal, although he understands well, that this will result in serious troubles to him. For this quality (but not only for it!), I have a great respect to him and wish him to be such for long years.
1. Chiral symmetry of QCD and its spontaneous violation

As is well known [1,2] the masses of light $u, d, s$ quarks, which enter the QCD Lagrangian, especially the masses of $u$ and $d$ quarks from which the usual (nonstrange) hadrons are built, are very small as compared to the characteristic QCD mass scale $M_{\text{char}} \sim 1 \text{ GeV}$:

$$\frac{m_u}{M_{\text{char}}} < 0.01, \quad \frac{m_d}{M_{\text{char}}} < 0.01, \quad \frac{m_s}{M_{\text{char}}} \approx 0.15$$

(1)

In QCD the quark interaction is due to the exchange of vector gluonic field. Thus, if light quark masses are neglected, the QCD Lagrangian (its light quark part) becomes chirally invariant – the quark fields can be transformed as $q \rightarrow \gamma_5 q$ or $q \rightarrow e^{i\alpha\gamma_5} q$, $q = u, d, s$. That means that not only vector, but also axial currents are conserved. (With exception of singlet (in flavour) axial current, which is not conserved because of anomaly – see e.g. [3].) However, the chiral symmetry is not realized in the spectrum of hadrons and their low energy interactions. Particularly, it can be shown, that v.e.v. $\langle 0 | \bar{q} q | 0 \rangle$, which should be equal to zero, if chiral symmetry is fulfilled, is not zero in fact:

$$\langle 0 | \bar{q} q | 0 \rangle_{1 \text{GeV}} = - \frac{1}{2} \frac{m_u^2 f_\pi^2}{m_u + m_d} \approx - (254 \text{ MeV})^3, \quad q = u, d$$

(2)

– the Gell-Mann, Oakes, Renner theorem [4] (for the proof see [5]). Here $m_\pi$ is the pion mass, $f_\pi$ is the pion decay constant, $f_\pi = 130.7 \text{ MeV}$ [6]) $m_u, m_d$ – are $u$- and $d$- quark masses ($m_u \approx 3.0 \text{ MeV}, m_d \approx 7.0 \text{ MeV}$ at the normalization point $\mu = 1 \text{ GeV}$ [7].) In a chirally symmetric theory fermion states must be either massless or degenerate in parity. It is evident that baryons (particularly, the nucleon) do not possess such properties. This means that the chiral symmetry of the QCD Lagrangian is spontaneously broken. According to the Goldstone theorem the spontaneous breaking of symmetry leads to appearance of massless particles in the spectrum of physical states - Goldstone bosons. (The proof of the Goldstone theorem for the case of QCD is given in [5].) In QCD Goldstone bosons can be identified with the triplet of $\pi$-mesons in the limit $m_u, m_d \rightarrow 0, m_s \neq 0$ (SU(2) symmetry) or with the octet of pseudoscalar mesons ($\pi, K, \eta$) in the limit $m_u, m_d, m_s \rightarrow 0$ (SU(3) symmetry).

2. The origin of baryon masses

As was already said, there are two facts clearly indicating on the spontaneous violation of chiral symmetry in QCD:
1) the existence of quark condensate and its typical hadronic scale;
2) large baryon (particularly, proton) masses.

The question arises: are these phenomena directly connected? At first sight it seems that it is not. For dimensional grounds we can write

$$m^3 = -c \langle 0 | \bar{q} q | 0 \rangle,$$

(3)

where $m$ the proton mass and $c$ some numerical constant. The substitution of the value of proton mass and of the numerical value $2$ in (3) gives: $c \approx 50$ – an unreasonable large number. In fact the relation of the form (3) takes place indeed and the approximate formula reads [8]

$$m^3 = -2(2\pi)^2 \langle 0 | \bar{q} q | 0 \rangle$$

(4)
The formula \( \Pi \) has no adjustable parameters and its accuracy is about 15%.

Let us now dwell on the derivation of a more precise expression for proton mass in QCD in terms of the v.e.v.’s of various operators. It is used the Operator Product Expansion (OPE) method [9], intensely exploited by Shifman, Vainshtein and Zakharov (SVZ) [10] in investigation of the properties of QCD and determination of the meson masses. Define the polarization operator:

\[
\Pi(p) = i \int d^4 x \epsilon^{i p x} \langle 0 | T\{\eta(x), \bar{\eta}(0)\} | 0 \rangle, \tag{5}
\]

where \( \eta(x) \) is three-quark current with proton quantum numbers built from \( u \) and \( d \) quark fields. The explicit form of the current \( \eta(x) \) is:

\[
\eta(x) = (u^a(x)C\gamma_\mu u^b(x))\gamma_5\gamma_\mu d^c(x)\epsilon^{abc}, \tag{6}
\]

where \( a, b, c \) are the colour indeces, \( \epsilon^{abc} \) is the unit antisymmetric tensor, \( C \) is charge conjugation matrix. (The argumentation, why this form of the current is the most suitable for determination of proton mass and other proton properties, was given in [8],[11].) The general form of \( \Pi(p) \) is

\[
\Pi(p) = p\Pi_1(p^2) + \Pi_2(p^2), \tag{7}
\]

where \( p = p_\mu \gamma_\mu \). The first term in \( \Pi \) conserves chirality, while the second violates chirality. The OPE is written separately for chirality conserving and chirality violating structures:

\[
\Pi(p^2) = \sum_n C_n^{(i)}(p^2)O_n^{(i)}(0), \quad i = 1, 2. \tag{8}
\]

In \( O_n^{(i)}(0) \) are v.e.v.’s of various operators, \( C_n^{(i)}(p^2) \) are the coefficient functions. OPE \( \Pi \) is valid at large negative \( p^2 \), more precisely at \( p^2 < 0, |p^2| R_c^2 \gg 1 \), where \( R_c \) is the confinement radius.

Consider first the chirality conserving structure \( \Pi_1(p^2) \), perform the OPE and classify the operators according to their dimensions \( d_n \). The operator of dimension 0 is the unit operator, \( O_1 = 1 \). Its contribution is proportional to \( p^4 \ln p^2 \). (The polynomical terms are disregarded, they are not contributing to the sum rules below.) The next term of OPE has the dimension \( d = 4 \) and is proportional to gluon condensate, \( O_4 = (\alpha_s/\pi)\langle 0 | G_{\mu\nu}G_{\mu\nu}^a | 0 \rangle \). (The quark masses are neglected.) The next in dimension is the \( d = 6 \) operator, proportional to the product of three gluonic fields has two loop integration and is strongly suppressed numerically. The corresponding diagrams (up to \( d = 8 \)) are shown in Fig.1. The construction of OPE for chirality violating structure \( \Pi_2(p^2) \) is similar. The lowest dimension operator is the quark condensate, \( O_3 = \langle 0 | \bar{q}q \ | 0 \rangle \), \( C_3^{(2)} \sim p^2 \ln p^2 \). The next in dimension operator has \( d = 5 \) and its v.e.v. has to the form:

\[
O_5 = -g\langle 0 | \bar{q}\sigma_{\mu\nu} \frac{\chi^\nu}{2} G_{\mu\nu}^a q | 0 \rangle \equiv m_0^2\langle 0 | \bar{q}q \ | 0 \rangle,
\]

\( C_5 \sim \ln p^2 \). The corresponding diagrams are presented in Fig.2 (up to dimension \( d = 9 \). In order to connect the polarization operator, calculated in terms of quark and gluon
condensates via OPE with the same polarization operator expressed in terms of hadronic variables $\Pi_{i}^{\text{phys}}(p^2)$ use the dispersion relation representation:

$$
\Pi_{i}^{\text{phys}}(p^2) = \frac{1}{\pi} \int_{0}^{\infty} \frac{\text{Im}\Pi_{i}^{\text{phys}}(s)}{s-p^2} ds + \text{subtractions}
$$

(10)

In the right hand side of (10) the model of hadronic spectrum is used:

$$
\text{Im}\Pi_{i}^{\text{phys}}(s) = \text{resonance} + \text{continuum}
$$

(11)

It is assumed, that the resonance is separated from continuum by the gap, the continuum is equal to the asymptotics of $\Pi_{i}(p^2)$ and starts at continuum threshold $s_0$. The equality

$$
\Pi_{i}(p^2) = \Pi_{i}^{\text{phys}}(p^2)
$$

(12)

is the desired QCD sum rule [10].

In case of proton the resonance contribution is equal:

$$
\text{Im}\Pi(p) = (p + m)\delta(p^2 - m^2)\lambda^2,
$$

(13)

where $\lambda$ is given by:

$$
\langle 0 \mid \eta \mid p \rangle = \lambda v_p
$$

(14)

and $v_p$ is the proton spinor. Eq.(12) is not well defined because of divergences at the left hand side and subtraction terms at the right hand side. In order to kill both and improve the convergence of OPE and dispersion relation representation SVZ suggested to apply the
Borel transformation to Eq. (12). Put $Q^2 = -p^2$. Borel transformation of the function $f(Q^2)$ is defined by:

$$Bf(Q^2) = \lim_{n \to \infty} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2}\right)^n f(Q^2).$$

(15)

If $f(Q^2)$ can be represent by dispersion relation –

$$f(Q^2) = \frac{1}{\pi} \int \frac{Imf(s)}{s + Q^2} ds,$$

(16)

then

$$B_{M^2}f(Q^2) = \frac{1}{\pi} \int e^{-s/M^2} Imf(s) ds.$$  

(17)

Borel transformation improves the convergence of OPE, since

$$B_{M^2} \frac{1}{(Q^2)^n} = \frac{1}{(M^2)^{n-1}(n-1)!}.$$  

(18)

Collecting all terms of OPE, represented by diagrams of Fig.'s 1 and 2 and applying the Borel transformation to (12) we get the sum rules for photon polarization operator $[8],[12],[13]$

$$M^6 E_2(M)L^{-4/9}c_0(M) + \frac{1}{4} b M^2 E_0(M)L^{-4/9} + \frac{4}{3} a_{qq}^2 c_1(M) - \frac{1}{3} a_{qq}^2 \frac{m_0^2}{M^2}$$

$$= \tilde{\lambda}_N^2 \exp\left(-\frac{m^2}{M^2}\right).$$

(19)
\[ 2a_{\bar{q}q}M^4E_1(M)c_2(M) + \frac{272\alpha_s(M)}{81\pi}a_{\bar{q}q}^3\frac{1}{M^2} - \frac{1}{12}a_{\bar{q}q}b = m\tilde{\lambda}^2_N\exp\left(-\frac{m^2}{M^2}\right). \quad (20) \]

Here

\[ a_{\bar{q}q} = -(2\pi)^2\langle 0 | \bar{q}q | 0 \rangle = 0.65 \text{ GeV}^3, \quad (21) \]

\[ b = (2\pi)^2\langle 0 | \frac{\alpha_s}{\pi}G_{\mu\nu}^2 | 0 \rangle, \quad (22) \]

\[ L = \frac{\alpha_s(\mu^2)}{\alpha_s(M^2)}, \quad (23) \]

\[ E_n(M) = \frac{1}{n!} \int_0^{s_0/M^2} z^n e^{-z}dz. \quad (24) \]

\[ \tilde{\lambda}^2 = 2(2\pi)^2\lambda^2, \quad (25) \]

\( \mu \) is the normalization point and \( c_0, c_1, c_2 \) are \( \alpha_s \) corrections [14],[15],[16],[17]. (In the derivation of Eq.’s (19),(20) the factorization hypothesis [10] for contributions of operators of higher dimensions (\( d \geq 6 \)) was assumed: the saturation of such contribution by vacuum intermediate state. This assumption is legitimate at large number of colours \( N_c \) and the corrections to factorized formulae are of order \( 1/N_c^2 \sim 10\% \).) As will be shown below, Eq.’s (19),(20) are valid at \( M^2 = 0.9 \sim 1.5 \text{ GeV}^2 \). Let us perform the rough approximation: neglect all higher order terms of OPE and continuum contribution (\( s_0 \rightarrow \infty, E_i = 1 \)), as well as \( \alpha_s \)-corrections (\( L = 1, c_k = 1 \)). Put \( M^2 = m^2 \) and divide (20) by (19). We get Eq.(4) presented above. The substitution of the numerical value of quark condensate (2) gives: \( m = 1.09 \text{ GeV} \) in comparison with the experimental value \( m_{\text{exp}} = 0.94 \text{ GeV} \). Go now to a more exact treatment of Eq.’s (19),(20). The values of \( m \), found as the ratio of (20) to (19) and \( \tilde{\lambda}^2 \) from (19) and (20) at \( m = m_{\text{exp}} = 0.94 \text{ GeV} \) are plotted in Fig.3 as functions of \( M^2 \). (For the values of parameters – see [7].)

**Fig. 3.** The proton mass sum rules, Eqs. (19) and (20). The dashed and dash–dotted curves give \( \tilde{\lambda}^2 \), determined from (19) and (20) respectively, using the experimental value of \( m \) (left scale). The solid line gives \( m \) as the ratio of (20) to (19).
Fig. 4. The diagrams corresponding to quark pair mixing, $\bar{q}q \rightarrow \bar{q}'q'$, and resulting in deviations from Eq. (30).

As is seen from Fig. 3 the $M^2$-dependence of $m$ and $\bar{\lambda}^2$ is very weak at $0.9 < M^2 < 1.5$ GeV and the values of $\bar{\lambda}^2$ found from (19) and (20) differ less than 5% at $M^2 \approx 1$ GeV$^2$. The final result of the proton mass calculation is [8],[12],[13] (see also [7],[18]):

$$m = 0.98 \pm 0.10 \text{ GeV} \quad (26)$$

$$\bar{\lambda}^2 = 3.2 \pm 0.6 \text{ GeV}^6 \quad (27)$$

The masses of hyperons and various baryon resonances were calculated in a similar way [8],[19] with a good coincidence with experiment. Particularly, the mass of $\Delta$-isobar was found to be equal to

$$m_\Delta = 1.30 \pm 0.18 \text{ GeV} \quad (m_\Delta \text{ exp} = 1.23 \text{ GeV}) \quad (28)$$

The conclusion from all the said above is: the appearance of baryon masses and their numerical values is caused by chiral symmetry violation in QCD vacuum.

3. Baryon magnetic moments

The vacuum in QCD can be considered as continuous medium. Under the influence of external electromagnetic field $F_{\mu \nu}$ quark pairs in the vacuum are polarized: it appears to be induced by the field vacuum expectation value [20],[13]

$$\langle 0 | \bar{q}\sigma_{\mu \nu}q | 0 \rangle_F = \chi_q \langle 0 | \bar{q}q | 0 \rangle F_{\mu \nu}, \quad q = u, d, s. \quad (29)$$

We restrict ourselves by consideration of the constant electromagnetic field, the electric charge $e = \sqrt{4\pi \alpha_{em}}$ is included in $F_{\mu \nu}$. The factor $\langle 0 | \bar{q}q | 0 \rangle$ is separated in (29), since $\langle 0|\bar{q}\sigma_{\mu \nu}q | 0 \rangle$ is violating chirality. This phenomenon is similar to paramagnetism or diamagnetism in matter. It can be shown [20],[13] that in a good approximation

$$\chi_q = e_q \chi \quad (30)$$

where $e_q$ is the $q$-quark charge. The arguments are the following. The appearance of the charge of some other quark $q'$, not coinciding with $q$, $q' \neq q$ is caused by the diagram of Fig.4 However, the diagram of Fig.4 is zero in any order of perturbation theory because of chirality conservation. Chirality violation might appear due to instantons, but for massless quarks the loop in Fig.4 vanishes – its $G$-parity in colour space is negative, $G_{\text{colour}} = -1$ [21]. The amplitude of Fig.4 has some resemblance to $\varphi - \omega$ mixing. Therefore the experimental smallness of $\varphi - \omega$ mixing is also an argument in favour of (20). The universal constant $\chi$ is called the quark condensate magnetic susceptibility. For consideration of the problem add to QCD Lagrangian the term, corresponding to interaction with electromagnetic field

$$L' = \int d^4x j_{\mu}^{el} A_\mu^{el} = \int d^4x j_{\mu}^{el} \frac{1}{2} x_\nu F_{\nu \mu}. \quad (31)$$
The schematical representation of $h \rightarrow h^*(h^* \rightarrow h)$ transitions in the external field.

The Fock-Schwinger gauge for e.m. field: $x_\mu A_\mu(x) = 0$ is used. The polarization operator in the linear approximation in $F_{\mu\nu}$ has the form:

$$\Pi(p) = i \int d^4xe^{ipx} \langle 0 | T\{\eta(x), \bar{\eta}(x)\} | 0 \rangle_F = \Pi^{(0)}(p) + \Pi^{(1)}_{\mu\nu}(p)F_{\mu\nu}.$$  \hspace{1cm} (32)

We are interested in $\Pi^{(1)}_{\mu\nu}(p)$. Perform OPE and classify the v.e.v. of operators according to their dimensions. The operator of the lowest dimension with $d = 2$ is $F_{\mu\nu}$ itself. The next, with $d = 3$, and for this reason the most important, is $\langle 0 | \bar{q}\sigma_{\mu\nu}q | 0 \rangle_F$. There are two v.e.v.'s operators of dimension 5

$$g\langle 0 | \bar{q} \gamma_5 \lambda^n G_{\mu\nu} q | 0 \rangle_F = \kappa_q F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle$$ \hspace{1cm} (33)

$$-ig\varepsilon_{\mu\nu\lambda\sigma} \langle 0 | \bar{q} \gamma_5 \lambda^n G_{\lambda\sigma} q | 0 \rangle_F = \xi_q F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle.$$ \hspace{1cm} (34)

By analogy with (20) $\kappa_q$ and $\xi_q$ are proportional to quark charge:

$$\kappa_q = e_q \kappa_1, \hspace{0.5cm} \xi_q = e_q \xi$$ \hspace{1cm} (35)

Among 6-dimensional operator the vacuum average

$$\langle 0 | \bar{q} q | 0 \rangle \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F$$ \hspace{1cm} (36)

is accounted for (within the framework of the factorization hypothesis). In order to obtain the sum rules for polarization operator in the external electromagnetic field we need its dispersion relation representation. $\Pi_{\mu\nu}(p)_F$ corresponds to three point function. So, generally, we shall start from consideration of $\Pi_{\mu\nu}(q;p_1,p_2)$, where $q$ - is the momentum carried by electromagnetic field. The spectral representation of any structure function $\Gamma(p_1^2,p_2^2,q^2)$ of $\Pi_{\mu\nu}(q;p_1,p_2)$ is given by the double dispersion relation in variables $p_1^2,p_2^2$ at fixed $q^2 \leq 0$:

$$\Gamma(p_1^2,p_2^2,q^2) = \int_0^\infty \int_0^\infty \frac{\rho(s_1,s_2,q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2 + P(p_1^2)f(p_2^2,q^2) + P(p_2^2)f(p_1^2,q^2).$$ \hspace{1cm} (37)

In the final result the limit $q^2 \rightarrow 0, p_1^2 \rightarrow p_2^2 = p^2$ will be performed. The proton contribution corresponds to the term in $\rho(s_1,s_2,q^2)$, proportional to $\delta(s_1 - m^2)\delta(s_2 - m^2)$ and the double pole in $\Gamma$:

$$\Gamma(p^2) \sim \frac{1}{(p^2 - m^2)^2}.$$ \hspace{1cm} (38)

In Eq. (37) the inelastic transitions, represented by Fig.5, contribute to subtraction terms. Their contribution is proportional to:
\[
\frac{A}{(p^2 - m^2)(p^2 - m'^2)},
\]  
(39)
i.e. has a single pole at \(p^2 = m^2\). There are three tensor structures of \(\Pi_{\mu\nu}(p)\):

\[
\not{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}, \quad i(p_{\mu}\gamma_\nu - p_\nu\gamma_\mu) \not{p}, \quad \sigma_{\mu\nu}
\]  
(40)
The first structure conserves chirality, two last are violating chirality. The structure function at the last structure is not suitable for obtaining the sum rules, because of a bad convergence of OPE and large contribution of instantons \([22]\). For the first two structures Eq. (37) in the limit \(q^2 \to 0, p_1^2 \to p_2^2 \to p^2\) reduces to

\[
\Gamma(p^2) = \int \frac{\rho(s)}{(s - p^2)} + \frac{A}{s - p^2}.
\]  
(41)
(The treatment of a more general case, which arises, e.g. if \(\alpha_s\) corrections are taken into account, can be performed using the method of \([23]\).) I will not present here the sum rules for the proton and neutron separately. (They are presented in \([13]\)). It is possible to obtain the combination of the sum rules for proton and neutron, where all unknown susceptibilities \(-\chi, \kappa\) and \(\xi\) are excluded. These sum rules can be represented in the form:

\[
\mu_p e_d - \mu_n e_u + M^2(A_p e_d - A_n e_u) = \frac{4a_{qq}^2 e^{m^2/M^2}(e_u^2 - e_d^2)}{3\lambda^2} L^{4/9},
\]  
(42)
\[
\mu_p^a e_u - \mu_n e_d + M^2(B_p e_u - B_n e_d) = \frac{4a_{qq}^2 m M^2}{\lambda^2} e^{m^2/M^2} (e_u^2 - e_d^2).
\]  
(43)
Here the constants \(A\) and \(B\) represent the contributions of inelastic transitions for the first and second structures correspondingly, the indexes \(p\) and \(n\) mean proton and neutron, \(\mu\) are the magnetic moments (in nuclear magnetons) \(\mu^\mu_p\) is the proton anomalous magnetic moment. In order to eliminate the unknown single-pole contributions still remaining on the l.h.s. of Eqs. (42), (43) we apply the differential operator \(1 - M^2\partial/\partial M^2\) to these equations and obtain

\[
\mu_p e_d - \mu_n e_u = \frac{4a_{qq}^2}{3\lambda_N^2} (e_u^2 - e_d^2) \left(1 - M^2 \frac{\partial}{\partial M^2}\right) e^{m^2/M^2} L^{4/9},
\]  
(44)
\[
\mu_p e_u - \mu_n e_d = e_u + \frac{4a_{qq}^2 m}{\lambda_N^2} (e_u^2 - e_d^2) \left(1 - M^2 \frac{\partial}{\partial M^2}\right) M^2 e^{m^2/M^2}.
\]  
(45)
The magnetic moments \(\mu_p\) and \(\mu_n\) can be approximately determined by setting \(M = m\), disregarding anomalous dimensions and substituting for the residue \(\lambda_N^2\) the value

\[
\bar{\lambda}_N^2 = \frac{2a_{qq} M^4}{m} e^{m^2/M^2} \bigg|_{M^2 = m^2},
\]  
(46)
which follows from the mass sum rules \([20]\) neglecting both anomalous dimensions and continuum contributions. Solving in this approximation Eqs. (44), (45) we arrive at the elegant results:

\[
\mu_p = \frac{8}{3} \left(1 + \frac{1}{6} \frac{a_{qq}}{m^3}\right),
\]  
(47)
\[
\mu_n = -\frac{4}{3} \left(1 + \frac{2}{3} \frac{a_{qq}}{m^3}\right).
\]  
(48)
Numerically, at $a_{qg} = 0.65 \text{GeV}^3$ we get from (17), (18) $\mu_p = 3.01, \mu_n = -2.03$ in comparison with the experimental values $\mu_p = 2.79, \mu_n = -1.91$. In a more rigorous treatment the study of the $M^2$-dependence of Eqs. (14), (15) in the confidence interval $0.9 < M^2 < 1.3 \text{GeV}^2$ gives as the best fit the values

$$
\mu_p = 2.7, \quad \mu_n = -1.7
$$

(49)

with an estimated error of about 10%. The proton and neutron magnetic moment were also calculated in [24],[25]. In [24] the sum rule for the chirality conserving structure was only used and the susceptibility $\chi$ was estimated basing on the vector dominance model. In [25] it was shown, that gluon condensate contribution, neglected in the above consideration is very small indeed (less than 1%). The hyperon magnetic moments were calculated in a similar way [26]. The results are presented in the Table.

### Table

**Magnetic moments of the baryon octet.**

|       | $p$   | $n$   | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^-$ | $\Xi^0$  | $\Xi^-$  | $\Lambda$ | $\Sigma\Lambda$ |
|-------|-------|-------|-------------|-------------|-------------|-----------|-----------|------------|-----------------|
| sum rules | 2.70  | -1.70 | 2.70        | 0.79        | -1.12       | -1.65     | -1.05     | -0.72      | 1.54           |
| quark model | 2.79$^{a)}$ | -1.91$^{a)}$ | 2.67        | 0.78        | -1.09       | -1.44     | -0.49     | -0.61$^{a)}$ | 1.63           |
| experiment | 2.79  | -1.91 | 2.46        | -           | -1.16       | -1.25     | -0.65     | -0.61      | 1.61           |

a) Input data;
b) Approximate value, calculated on the basis of $SU(3)$ relations.

Within the limits of the expected theoretical errors (10–15%) the results of the sum rule calculations are in agreement with the data. The exceptional case is the $\Xi$ hyperon, where the difference between theory and experiment is larger. The latter can be addressed to a significant $M^2$ dependence of the sum rules for the $\Xi$-mass and magnetic moments, which in turn may be related to a larger role of $m_s$-corrections. In conclusion, it must be emphasized, that no new parameters, besides those found in the calculations of baryon masses, enter the above used sum rules for baryon magnetic moments. The quark condensate magnetic susceptibility was found with the help of the special sum rule [27],[28],[29]. The most precise result is [29]:

$$
\chi_1 \text{ GeV} = -(3.15 \pm 0.3) \text{ GeV}^{-2}.
$$

(50)

### 4. The nucleon coupling constants with axial current

The nucleon coupling constant with the isovector axial current $g_A$ determines the $\beta$-decay rate of neutron, the nucleon coupling constant $g_A^{(8)}$ with the 8-component of octet axial current determines (together with $g_A$) the $\beta$-decays of hyperon. Therefore their knowledge is very important. I will not dwell on the theoretical formulae for these quantities – their derivation is based on the method, similar to that used in the calculation of baryon magnetic moments – and presents here only the final results:

$$
g_A^{\text{theor}} = 1.24 \pm 0.05 \ [30],[31],[32] \ (\exp 1.269 \pm 0.003 \ [6])
$$

(51)

$$
g_A^{(8)\text{theor}} = 0.45 \pm 0.15 \ [33],[32] \ (\exp 0.59 \pm 0.02 \ [33])
$$

(52)
When calculating $g_A^{\text{theor}}$ and $g_A^{(8)\text{theor}}$ the sum rule for nucleon mass, as well as the numerical value of nucleon coupling constant $\lambda^2$ with quark current $\eta$ was exploited. The agreement with experiment is good, especially in the case of $g_A$. (The large error in $g_A^{(8)\text{theor}}$ arises from strong compensation of the main term in OPE – see [32].) In the limit of exact SU(3)-symmetry $g_A$ and $g_A^{(8)}$ are related with $\beta$-decay axial coupling constants in the baryon octet $F$ and $D$:

$$g_A = F + D$$
$$g_A^{(8)} = 3F - D$$

From (52),(53) we have

$$F^{\text{theor}} = 0.42 \pm 0.04; \quad D^{\text{theor}} = 0.82 \pm 0.08$$

Of a special interest is the nucleon coupling constant with singlet axial current

$$j_{\mu5}^{(0)} = \bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d + \bar{s}\gamma_\mu\gamma_5s$$

Due to the Bjorken sum rule this coupling constant is connected with the part of proton spin $\Sigma$, carried by quarks. (See [34] for review and references). The singlet axial current is not conserved because of anomaly. In order to find $\Sigma$ it was considered the sum rule for the topological current

$$Q_5 = \frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}_{\mu\nu}, \quad \tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}G_{\lambda\sigma}$$

The result is [32]

$$\Sigma = 0.30 \pm 0.18$$

The value of $\Sigma$ (57) agrees with the data (although with large errors, the review on proton spin structure is given in the book [35], see also [36]). This value agrees well with the vector based dominance model [37],[38],[39] establishing the smooth connection of the integrals of polarized structure function

$$\int g_1(x, Q^2)dx$$

at $Q^2 = 0$ and high $Q^2$. (The former is given by Gerasimov-Drell-Hearn sum rule [40],[41] and expressed through nucleon magnetic moments.)

5. Conclusion

1. It was demonstrated that the origin of baryon masses is the spontaneous violation of chiral symmetry in QCD vacuum – the existence of quark condensate.

2. Basing on this statement the proton mass was calculated with accuracy $\sim 10\%$, as well as hyperon masses and masses of various baryon resonances.

3. It was shown, that the same phenomenon – the existence of quark condensate and its polarizability in external electromagnetic field is responsible for the magnitudes of the proton, neutron and hyperons magnetic moments.

4. The consequence of this is, that the properties of baryons are determined by the properties of QCD vacuum and are weakly related to the structure of theory at small distances (existence of Higgs boson, its interaction with quark etc.)
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