Relaxation in stellar systems, and the shape and rotation of the inner dark halo

Scott Tremaine* and Jeremiah P. Ostriker
Princeton University Observatory, Peyton Hall, Princeton, NJ 08544-1001, USA

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ABSTRACT

Why do galactic bars rotate with high pattern speeds, when dynamical friction should rapidly couple the bar to the massive, slowly rotating dark halo? This long-standing paradox may be resolved by considering the dynamical interactions between the galactic disc and structures in the dark halo. Dynamical friction between small-scale halo structure and the disc spins up and flattens the inner halo, thereby quenching the dynamical friction exerted by the halo on the bar; at the same time the halo heats and thickens the inner disc, perhaps forming a rapidly rotating bulge. Two possible candidates for the required halo structures are massive black holes and tidal streamers from disrupted precursor haloes. More generally, gravitational scattering from phase-wrapped inhomogeneities represents a novel relaxation process in stellar systems, intermediate between violent relaxation and two-body relaxation, which can isotropize the distribution function at radii where two-body relaxation is not effective.

Key words: galaxies: evolution – galaxies: formation – galaxies: haloes – galaxies: kinematics and dynamics.

1 INTRODUCTION

Most of the mass in disc galaxies resides in three components: a thin, rapidly rotating disc of stars and gas, a centrally concentrated stellar bulge or spheroid, and an invisible dark halo that extends far beyond the outer edge of the stellar disc. The bulge rotates less rapidly than the disc, and the rotational properties of the halo are unknown. The halo is believed to form by hierarchical clustering of dark matter, which results from gravitational instability of small irregularities present at much earlier times; its structure depends on the cosmological model, but usually the halo is strongly triaxial (Dubinski & Carlberg 1991; Warren et al. 1992) and rotates only slowly (median spin parameter \( \lambda \sim 0.05 \); see, e.g., Steinmetz & Bartelmann 1995). The disc is believed to form from baryonic material that collects in the potential well of the dark halo. Numerical simulations (Katz & Gunn 1991; Dubinski 1994; Kepner & Bryan, in preparation) show that the growth of the disc modifies the structure of the halo at radii smaller than the outer edge of the disc: the halo becomes more nearly axisymmetric (\( b/a \approx 0.8 \)) and oblate (\( c/a = 0.6 \)), but remains slowly rotating.

In this paper we examine the long-term gravitational interactions between the inner parts of the disc and halo. In particular, we shall discuss the rate and consequences of angular momentum transfer from the disc to the halo, which tends to flatten and spin up the halo, while causing the disc material to thicken, heat, and drift towards the centre. Any irregularities in either the disc or halo tend to contribute to angular momentum transfer; we shall argue that the transfer is probably sufficiently strong to transform the inner halo from a slowly rotating oblate spheroid supported by pressure anisotropy into a rapidly rotating thickened disc.

This argument raises the terminological issue of whether a ‘halo’ requires a different name when it acquires the structure of a disc (and vice versa); we shall not address this issue here, and in effect we use the term ‘halo’ as shorthand for collisionless dark matter, and ‘disc’ as shorthand for baryonic luminous matter.

Rapid rotation of the inner halo may resolve a persistent puzzle about the structure of barred disc galaxies. A rotating bar embedded in a non-rotating dark halo loses angular momentum to the halo through dynamical friction, on a time-scale that is generally much less than a Hubble time. Thus bars should have small pattern speeds, corresponding to large values of the dimensionless ratio \( R = (\text{corotation radius})/(\text{radial extent of the bar}) \) – which is normally \( \approx 1 \) when a bar forms. This argument was first explicitly discussed by Weinberg (1985), although earlier numerical models of disc galaxies with live haloes by Sellwood (1980) exhibited rapid angular momentum transfer from the disc to the halo after a bar formed in the disc. Hernquist & Weinberg (1992) simulated systems containing a non-rotating halo and a rigid bar (with no disc), and concluded that the gravitational coupling was ‘sufficiently strong to remove all of the bar’s angular momentum on a time-scale much shorter than a Hubble time’. Little & Carlberg (1991a,b) followed the long-term evolution of barred disc galaxies with live haloes using a two-dimensional code, which greatly improves spatial resolution but requires a somewhat unrealistic flat halo. They found that \( R \) increased from 1 to 2 over \( \sim 30 \) initial bar rotation periods. The most recent and comprehensive simulations of this model have been reported by Kepner & Bryan (1995).
process are by Debattista & Sellwood (1998), who followed self-consistent N-body models of a barred disc and initially spherical halo. In the simulation with the most massive halo, they found that the bar pattern speed dropped by a factor of 5 (to $R = 2.3$) within 50 initial bar rotation periods. Only in models with low-density haloes did the bar pattern speed remain high.

Thus a wide range of analytic arguments and numerical simulations concur that high-density haloes strongly decelerate bars. On the other hand, observations show that most bars are rapidly rotating. Bar pattern speeds can be estimated in several ways. (i) By comparing the shocks in models of gas flow with dust lanes in barred spiral galaxies. Using this approach Athanassoula (1992) estimated $R = 1.2 \pm 0.2$. (ii) From the rotation field of the old disc population and the equation of continuity. This method has been applied to the SB0 galaxy NGC 936 and yields $R = 1.4 \pm 0.3$ (Kent & Glaudell 1989; Merrifield & Kuijken 1995). (iii) By identifying photometric and kinematic properties of stars and gas with Lindblad or corotation resonances; this approach is probably less reliable, since some disc features may be transitory, and also the pattern speed of the spiral structure may differ from that of the bar (Sellwood & Sparke 1988). A recent review by Elmegreen (1996) concludes that $R = 1.2 \pm 0.2$ in early-type galaxies; bars in late-type galaxies may rotate more slowly, but so far there are very few observations.

Thus there is a significant and growing contradiction between theory ($R \gg 1$) and observations ($R \sim 1$). Debattista & Sellwood (1998) argue that the absence of slowly rotating bars can only be explained if the halo contributes less than $\sim 25$ per cent of the radial force at 2 disc scalelengths; this in turn requires that the disc provides almost 90 per cent of the total circular speed ($\gamma = v_{\text{disc}}/v_{\text{tot}} \sim 0.87$; see Olling 1995 for a more precise definition). The relative contribution of the disc and dark halo to the radial force in spiral galaxies is controversial and uncertain (Botttema 1976; Broeils & Courteau 1997; Courteau & Rix 1997; Sackett 1997; Dehnen & Binney 1998; Fuchs 1999; Sellwood 1999), but there is little direct evidence that the very high value of $\gamma$ required by Debattista & Sellwood is present in most disc galaxies. For example, careful discussions of mass models for the Galaxy are given by Olling & Merrifield (1998) and Dehnen & Binney (1998); the former paper describes nine models with $\gamma_0$ in the range 0.66–0.83, and the latter plots eight models with $\gamma$ in the range 0.57–0.81. Even the so-called ‘maximum disc’ models of external galaxies – which have the largest possible disc mass consistent with constant mass-to-light ratio in each component, the observed rotation curve and an isothermal dark halo – have a median $\gamma_0$ of only 0.85 (Sackett 1997), which is slightly lower than Debattista & Sellwood require.

A rapidly rotating inner halo could resolve this apparent contradiction, by reducing or even reversing the drag on a rotating bar.

2 DISC–HALO GRAVITATIONAL TORQUES

Disc–halo angular momentum transfer can result from structure in either the disc or halo. We first examine the effects of disc structure (Section 2.1), and find that no known disc features transfer angular momentum rapidly enough to spin up the inner halo. Halo structures (Section 2.2) are more efficient.

2.1 Structure in the disc

The disc properties of spiral galaxies exhibit a wide variety of structure, including molecular clouds, spiral arms, and central bars. All of these can transfer angular momentum to the halo.

To estimate the transfer rate, we adopt a simple model in which the circular speed of the disc, $v_c$, is independent of radius, and a fraction $f_d$ of the gravitational force in the disc plane is due to a spherical halo; the remaining fraction $f_h = 1 - f_d$ arises from the self-gravity of the disc. In this model the halo density and disc surface density are (Binney & Tremaine 1987)

$$\rho_h(r) = f_h \frac{v_c^2}{4\pi Gr^2}, \quad \Sigma_d(r) = f_d \frac{v_c^2}{2\pi Gr^2}.$$  

(1)

If the torque per unit disc area exerted on the halo is $N$, then the disc material in the radius range $[r, r + dr]$ loses angular momentum at the rate $2\pi N dr$; if we assume that this angular momentum is gained by the halo material in the same radius range, with mass $4\pi r^2 \rho_h(r) dr$ and characteristic specific angular momentum $-r v_c$, then the halo is expected to flatten on a characteristic time

$$\tau = \frac{2 f_h \rho_h(r)^2 v_c}{N} = \frac{f_h v_c^3}{2\pi GM}.$$  

(2)

First, consider the effects of discrete masses in the disc. We suppose that a fraction $f_{d,i}$ of the disc mass is in objects of mass $M$ and radius $R_i$. Each such object loses specific angular momentum to the halo by dynamical friction, at a rate (Binney & Tremaine 1987, equation 7-24)

$$L = -0.43 f_{d,i} \frac{GM}{r} \ln \Lambda,$$  

(3)

where $\Lambda = r/R_i$. The torque per unit area on the halo is then

$$N = f_d \Sigma_d |L|,$$  

and the halo flattening time is

$$\tau = 2.3 \frac{f_{d,i} \rho_h(r)^2 v_c}{f_d GM \ln \Lambda}$$

$$= 1 \times 10^{12} \frac{v_c}{f_d GM} \left( \frac{0.1}{v_c} \right) \left( \frac{200 \text{km s}^{-1}}{3 \text{kpc}} \right)^2$$

$$\times \left( \frac{10^6 M_\odot}{M} \right) \left( \frac{10}{\ln \Lambda} \right).$$

The largest disc objects are giant molecular clouds (GMCs). These have a wide range of masses, but their contribution to dynamical friction depends on the weighted average $(M_i^2)/M_i$, which is dominated by the largest clouds at mass roughly $10^6 M_\odot$ (e.g. Combes 1991). The largest GMCs may not be bound, but unbound objects transfer angular momentum just as well. The fraction of the total mass in molecular gas ranges from 4 per cent for Sa galaxies to 25 per cent for Scd galaxies (Young & Scoville 1991), so $f_{d,i} = 0.1$ is a reasonable average. We then see from equation (4) that GMCs do not transfer significant angular momentum to the bulk of the inner halo (although individual GMCs at radii $\lesssim 1$ kpc do spiral inwards in less than a Hubble time; see Stark et al. 1991).

Next, we examine angular momentum transfer by spiral structure in the disc. A tightly wrapped spiral pattern with azimuthal wavenumber $m$, pattern speed $\Omega_p$, radial wavenumber $k$ and fractional surface-density amplitude $\Sigma_i/\Sigma_0$ exerts a torque per unit area on the halo (Mark 1976)

$$N = \frac{2 \pi^2 m^2 \Omega_p^2 \rho_h G \Sigma_i}{k^2 a^3} \left( \frac{\Sigma_i}{\Sigma_0} \right)^2,$$  

(5)

where $a$ is the one-dimensional velocity dispersion in the halo, and the halo distribution function is assumed to be Maxwellian. In our model, $a = v_c/\sqrt{2}$, and, using equations (1) and (2), the halo flattening time is

$$\tau = \frac{3 \pi^{1/2}}{4} \frac{m^2}{\Omega_p f_d} \left( \frac{\Sigma_i}{\Sigma_0} \right)^2.$$  

(6)
where \( i = \tan^{-1}(mkr) \) is the pitch angle of the pattern. An optimistic set of parameters is \( m = 2, \ i = 20^\circ \), \( f_d = 0.7 \), \( \Omega_p^{-1} = 5 \times 10^5 \) yr, and \( \Sigma/\Sigma_d = 0.2 \), which yields \( \tau = 10^{12} \) yr, too long to be of interest. Unusually strong and open spiral patterns could transfer angular momentum more rapidly, but these are normally transitory, induced for example by an encounter with a nearby galaxy, or else closely associated with a central bar.

Angular momentum transfer by dynamical friction on a central bar has been discussed in the Introduction. Although the frictional forces are strong enough to despin the bar, it is much harder to spin up the halo, because the reservoir of angular momentum in the bar is too small. Bars typically contain \( \approx 30 \) per cent of the disc luminosity and extend only to \( \approx 30 \) per cent of the disc radius, so their moment of inertia is only a few per cent of the disc’s; thus, transferring all of the bar’s angular momentum to the halo — as noted, a rapid process — would still impart negligible halo rotation.

We conclude that the known disc structures cannot spin up the inner halo.

### 2.2 Structure in the halo

The dark halo is believed to form by gravitational instability of small density fluctuations in the early Universe. In standard hierarchical models, dense, low-mass haloes form first, and then merge to build up successively more massive but less dense objects. The process terminates at a redshift \( z \), given roughly by \( 1 + z = \Omega_m^{-1} \), where \( \Omega_m \) is the density parameter. When a small halo merges with a larger one, it loses energy by dynamical friction and spirals towards the centre until it is tidally stripped or completely disrupted, at a radius \( r \sim \sqrt{M/M_d}^{1/3} \), where \( r \) and \( M \) are the radius and mass of the small halo and \( M_d \) is the mass of the larger halo interior to \( r \). The disrupted halo is stretched into a tidal streamer; the length of the streamer after a time \( t \) is roughly \( \Delta r = (\pi P/(r_d) t) \sim (\pi P)(M/M_d)^{1/3} \), where \( P \) is the orbital period (Tremaine 1993; Johnston 1998). Such streamers can survive as distinct — though unbound — structures in phase-space for a Hubble time or longer; since the halo material is collisionless, the streamers never overlap in phase-space, but simply become longer as phase mixing proceeds, forming a kind of phase-space spaghetti. Structure resulting from incomplete phase-mixing is already well-known in several other contexts: moving groups in local disc stars (Eggen 1965; Dehnen 1998), shells in elliptical galaxies (Hernquist & Quinn 1988; Malin 1999), clumps in kinematic surveys of the metal-weak halo (Majewski, Munn & Hawley 1996), and the disrupted Sagittarius dwarf galaxy (Ibata et al. 1997).

A rotating disc exerts dynamical friction on a tidal streamer. To estimate the frictional force, consider the idealized case in which a single streamer of length \( L \approx r \) pierces the disc at right angles. At distances \( \ll L \), we can approximate the streamer as an infinite, straight wire, and we can approximate the wire as stationary, since the velocity vector of its material is approximately aligned with the streamer. We can approximate the disk as a uniform, infinite, zero-thickness sheet moving past the streamer at velocity \( v_c \). If the linear density in the streamer is \( \lambda = M/L \), the frictional force exerted on it from disc material with impact parameter \( b \ll L \) is

\[
F_{\text{drag}} = \frac{4\pi^2 G^2 \lambda^2 \Sigma_d b_1}{v_c^3},
\]

where \( b_1 \sim L \) is the maximum impact parameter considered. This formula involves the same approximations as the standard Chandrasekhar formula for dynamical friction on a point mass (Binney & Tremaine 1987), but is somewhat less reliable because the drag is linear (rather than logarithmic) in the poorly determined maximum impact parameter.

This drag force is exerted only while the streamer intersects the disc. If the streamer follows a circular orbit of radius \( r \), then it intersects the disc for a fraction \( Li/(2\pi r) \) of its orbit, so the average drag force is

\[
\langle F_{\text{drag}} \rangle = \frac{2\pi G^2 \lambda^2 \Sigma_d L b_1}{v_c^3}. \tag{8}
\]

Disc material with impact parameter \( b \gg L \) sees the streamer as a point mass, and the time-average of the frictional force component from this material is

\[
\langle F_{\text{drag}} \rangle = \frac{2^{1/2} G^2 \lambda^2 \Sigma_d L^2 \ln \Lambda}{v_c^3}, \tag{9}
\]

where \( \Lambda = b^2/b_1 \), and \( b_1 \) and \( b_1 \) are the maximum and minimum impact parameters considered. Equations (8) and (9) are similar except for the Coulomb logarithm in the latter, but the ratio \( b/b_1 \) is not usually large, so material with \( b \ll L \) and \( b \gg L \) contributes comparable drag forces.

Estimating the frictional force is more delicate for long streamers (\( \Delta \phi \gg 2\pi \)). A long streamer intersects the disc several (\( N \approx \Delta \phi/2\pi \)) times. If the locations of these intersections are uncorrelated, then the total drag force would be given by the drag per streamer (equation 7 with \( b_1 \sim r \)) times \( N \), or

\[
\langle F_{\text{drag}} \rangle = \frac{2\pi G^2 \lambda^2 \Sigma_d L b_1}{v_c^3}. \tag{10}
\]

If, on the other hand, the intersections are localized in the disc — as they would be if the streamer had an orbit of moderate inclination and eccentricity and differential precession were not too fast — then the \( N \) streamers of linear density \( \lambda \) act like a single streamer of density \( \lambda N \). Replacing \( \lambda \) by \( \lambda N \) and \( b_1 \) by \( r \) in equation (7), we find

\[
\langle F_{\text{drag}} \rangle = \frac{G^2 \lambda^2 \Sigma_d L}{v_c^3}. \tag{11}
\]

We regard equation (11) as more reliable than equation (10) in most cases. In either case the drag force will eventually be shut off when differential precession smears the streamer into an axisymmetric structure.

Equations (8) or (9) for short streamers, and equation (11) for long streamers, can be summarized by

\[
\langle F_{\text{drag}} \rangle = g^2 G^2 \lambda^2 \Sigma_d \frac{L}{v_c^3}, \tag{12}
\]

where \( g \) is a dimensionless constant of order unity. The frictional torque flattens the streamer orbit into the disc plane in a time

\[
\tau = \frac{M_\star v_c}{F_{\text{drag}}} = \frac{v_c^3 r}{g^2 G^2 \Sigma_d M_\star}. \tag{13}
\]

For a numerical estimate of the flattening time, let \( f_s \) be the ratio of the mass of a streamer to the halo mass inside radius \( r \), \( f_s = GM/r^2 v_c^2 r \), and use equation (1) to eliminate \( \Sigma_d \). Then equation (13) yields

\[
\tau = \frac{2\pi r}{g f_s f_d v_c} \left( \frac{3 \times 10^{10} \, \text{yr}}{0.3} \right) \left( \frac{0.01}{f_s} \right) \left( \frac{3 \, \text{kpc}}{200 \, \text{km s}^{-1}} \right). \tag{14}
\]

At radii \( \lesssim 3 \) kpc, streamers containing even a few per cent of the halo mass can be dragged into flattened orbits within a Hubble time.
At this radius the precession time is of order $10^9$ yr for moderately flattened galaxies; assuming that the drag is quenched after a few precession times, then streamers containing $\approx 10$ per cent of the halo mass will be significantly flattened.

The principal effect of the frictional torque from the disc is to increase the $z$-component of the angular momentum of the streamer. The orbital energy of the streamer changes more slowly: since the gravitational field of the streamer is approximately stationary, the energy of both the streamer and disc are conserved. The total angular momentum of the streamer also changes relatively slowly: for example, the torque on an axisymmetric streamer is always perpendicular to the streamer plane and hence perpendicular to the angular momentum vector, so the magnitude of the angular momentum is unaffected by the friction. This process of increasing the halo mass will be significant.

To close this section, we examine the rate of angular momentum transfer if the halo is composed of compact objects of mass $M$ (e.g., black holes). Then the torque on the halo per unit disc area is (Ostriker 1983)

$$N = 2^{5/2} I_1 \ln \Lambda \frac{G^2 M \rho_h r}{\sigma} \nu_c,$$

where $I_1 = 0.474$. Using equations (1) and (2), the characteristic flattening time for the halo is then (cf. equation 4)

$$\tau = \frac{\pi \nu_c r^2}{2^{5/2} I_1 \ln \Lambda} \frac{GM}{\nu_c} \frac{200 \text{ km s}^{-1}}{3 \text{ kpc}}$$

$$= 1.4 \times 10^{11} \text{ yr} \left( \frac{0.7}{f_d} \right) \left( \frac{\nu_c}{200 \text{ km s}^{-1}} \right) \left( \frac{r}{3 \text{ kpc}} \right) \left( \frac{10^6 M_\odot}{M} \right) \left( \frac{10}{\ln \Lambda} \right).$$

Observational upper limits to the mass of compact halo objects are reviewed by Carr (1994). Masses $M > 3 \times 10^6 M_\odot$ are strongly disfavoured because they cause excessive heating of the local disc (Lacey & Ostriker 1985; Lacey 1991), excessive distortion of gravitationally lensed radio jets (Wambsganss & Paczynski 1992; Garrett et al. 1994), unobserved gravitational lensing of gamma-ray bursts (Marani et al. 1999), and excessive growth of the central black hole in the Galaxy (Xu & Ostriker 1994). Some authors have argued for much more stringent upper bounds on the mass of compact halo objects, but we find these arguments unconvincing. (i) Rix & Lake (1993) estimate that halo objects with $M > 10^5 M_\odot$ would cause excessive heating of the stars in the dwarf irregular galaxy GR8. This is a very low-luminosity galaxy ($M_B = -10.6$, corresponding to $2.5 \times 10^5 L_\odot$ or $2 \times 10^4 L_\odot$), so even a dark halo containing $30$ times the mass in visible stars will comprise only $N \sim 10$ halo objects of $3 \times 10^6 M_\odot$. The crossing time in the halo is $t_c \sim 10^7$ yr, and in $N t_c \sim 10^7$ yr the halo will evaporate, leaving a single or binary halo object at the centre, which of course would not heat the stellar distribution. This argument predicts that the dark mass in GR8 may be concentrated at its centre, producing a quasi-Keplerian circular-speed curve, but testing this prediction is difficult: in fact, GR8 is one of the few galaxies with a declining $H_\alpha$ rotation curve, but there is a large and uncertain correction for pressure support (Carigian, Beaulieu & Freeman 1990). (ii) Compact halo objects can disrupt globular clusters, so the present population of clusters constrains the mass and density of halo objects (Wielen 1988; Moore 1993; Klessen & Burkert 1996; Arras & Wasserman 1999). Klessen & Burkert set an upper limit $M < 5 \times 10^4 M_\odot$, and Moore’s limit is even smaller. Their results are based on the assumption that the disruption time $t_d$ for a particular subset of globular clusters (central density between $10^3$ and $10^4 M_\odot$ pc$^{-3}$) must be long enough that more than $1$ per cent of them survive; for a Poisson process with destruction probability per unit time $t_d^{-1}$ this requires that $t_d > 0.22 t_H$, where $t_H$ is the age of the cluster population. However, if the present clusters represent the survivors of a much larger original population — as indicated by the efficiency of the destruction mechanisms associated with known components of the Galaxy (Gnedin & Ostriker 1997; Murali & Weinberg 1997) — then we expect a wide range of survival times in any snapshot of the population. In particular, if the distribution of lifetimes at formation is a power law, then we expect the fraction of clusters with lifetimes less than $t_d \ll t_H$ to be $\sim t_d/t_H$, so $\sim 20$ per cent of clusters would be expected to violate the Klessen & Burkert criterion.

We conclude from equation (16) that if the halo is composed of $10^6 M_\odot$ black holes, well below the allowed upper limit of $3 \times 10^7 M_\odot$, then we expect substantial halo flattening and rotation out to $\sim 1$ kpc, which may be sufficient to suppress halo dynamical friction on rapidly rotating bars.

### 3 CONSTRAINTS ON HALO SHAPES

The arguments in the preceding sections suggest that the inner parts of dark haloes may be thick, rapidly rotating discs. As noted earlier, bars are strongly coupled to the inner halo and thus should also rotate rapidly, as observed. We now ask whether other observations constrain the shapes of inner haloes. Although haloes formed by hierarchical clustering are generally triaxial, the subsequent growth of the disc tends to make the inner halo more nearly axisymmetric (cf. Section 1), and this expectation is confirmed by the upper limits to the ellipticities of most disc galaxies induced by the halo potential — typically $\leq 0.1$ (see Franx & de Zeeuw 1992, Kuijken & Tremaine 1994 and Rix 1996). Thus we assume that the inner halo is axisymmetric, and restrict our attention to the flattening of axisymmetric haloes.

Rix (1996) and Sackett (1996, 1999) review the tools that have been used to constrain the shape and mass of the dark halo in our Galaxy (Monet, Richstone & Schechter 1981). Unfortunately, the best available results are model-dependent and have large error bars: $0.3 < c/a < 0.6$ from Binney, May & Ostriker (1987), and $c/a > 0.34$ from van der Marel (1991).

The ellipticity of the mass distribution in edge-on disc galaxies can be determined from the shape of their X-ray isophotes. This method yields $c/a = 0.60 \pm 0.13$ for the $80$ galaxy NGC 1332 (Buote & Canizares 1996); this is greater than expected for disc-like dark matter, but the uncertainties are large and the result is dominated by halo radii well beyond the outer edge of the disc, where the flattening mechanism discussed here would not operate. Steinman-Cameron, Kormendy & Durisen (1992) fit a precessing-disc model to the dust lanes in the $80$ galaxy NGC 4753 and derive $0.84 < c/a < 0.9$; however, this result depends on an assumed age for the event that warped the galaxy disc.

The axis ratios of the dark haloes in polar-ring galaxies have been estimated by several investigators. Whitmore, McElroy & Schweizer (1987) examined three $50$ galaxies with polar rings and...
found axis ratios for the potentials of 0.86 to 1.05 (±0.2), corresponding to axis ratios for the density of roughly 0.58 to 1.15 (±0.6); however, two of these galaxies were re-examined by Sackett et al. (1994) and Sackett & Pogge (1995), who found much flatter haloes with axis ratios 0.3–0.5. In any case, polar rings measure the halo flattening at radii beyond the outer edge of the disc, where the flattening mechanism discussed here would not operate.

High-resolution H I data can be used to determine the thickness of the H I layer in highly inclined disc galaxies; at and beyond the edge of the optical disc this thickness is sensitive to the shape of the dark halo. The most thorough analysis of this kind, for the Scd galaxy NGC 4244 (Olling 1996), yields $c/a = 0.2^{+0.1}_{-0.0}$. Olling & Merrifield (1997) derive $c/a = 0.75 \pm 0.25$ for the Galaxy using both the thickness of the H I layer and local stellar kinematics.

Gravitational lensing by disc galaxies can constrain the halo mass and shape. Koopmans, de Bruyn & Jackson (1998) find $c/a > 0.5$ for the edge-on lensing galaxy B1600+434. Flattening the halo does not significantly affect the overall lensing cross-section of a galaxy, but it does enhance the fraction of images that exhibit characteristic geometries associated with disc lensing (Keeton & Kochanek 1998). Unfortunately, disc galaxies are expected to comprise only 10–20 per cent of gravitational lenses, so a large sample will be difficult to obtain.

Evidently the present data allow a wide range of halo shapes, with some indication that moderately flattened haloes ($c/a \sim 0.5$) are more common than spherical or disc-shaped haloes. The data provide no strong evidence for or against rotating thickened discs of dark matter such as those proposed in this paper, especially since the flattening is likely to be confined to the innermost few kpc of the halo.

4 DISCUSSION

Structure in galactic discs and dark haloes couples these two components together, leading to angular momentum transfer from disc to halo, spin-up and flattening of the halo, and heating and thickening of the disc. The time-scale and radial extent of this process are highly uncertain. Known structures in the disc are probably not able to flatten and spin up the inner halo significantly in a Hubble time, but halo structure is likely to be more efficient. In particular, we have identified two types of halo structure that may be able to spin up the halo and heat the disc, at least in the innermost few kpc of the galaxy: massive black holes, and tidal streamers resulting from hierarchical halo formation.

A rotating flattened halo may be needed to explain the high pattern speed of bars (Section 1). Since flattening the inner halo also thickens the inner disc, this process may form some galactic bulges, which, like discs, are metal-rich and rapidly rotating (Wyse, Gilmore & Franx 1997). Another way to make bulges from discs is through a buckling instability of bar structures in the discs (Combes & Sanders 1981; Raha et al. 1991). The hypothesis that some bulges may be produced from discs is supported by the inner cut-offs observed in disc surface-brightness profiles as they enter the bulge (Kormendy 1977), and the similar scalelengths of inner discs and bulges (Courteau, de Jong & Broeils 1996). Kormendy (1993) has argued persuasively that some bulges are really discs in terms of their dynamics and origin. On the other hand, the centres of bulges have much higher phase-space density than discs, and so cannot arise from discs through a collisionless process (Carlberg 1986); moreover, the strong similarity between bulges and elliptical galaxies of similar luminosity suggests that most bulges are formed in a manner similar to the family of equivalent ellipticals.

Alternatively, detailed simulations may show that tidal streamers and other unbound halo structure thicken the disc excessively in standard cosmological models, in which case we have an interesting new constraint on such models (effectively an extension of the arguments of Tóth & Ostriker 1992 from bound to unbound structures).

Dynamical friction on tidal streamers provides a natural mechanism to produce flattened dark haloes of non-baryonic material, and thus offers a counter-example to the usual belief that if the dark halo is dislike it must be baryonic. If this process is important, then we expect the inner haloes of spiral galaxies to be substantially flatter than haloes of ellipticals.

Relaxation from tidal streamers can also be important in elliptical galaxies. Gravitational scattering of stars by streamers can enhance the relaxation rate compared to the usual Chandrasekhar formulae for two-body relaxation (Binney & Tremaine 1987) and hence isotropize the distribution function at radii where two-body relaxation is ineffective (energy relaxation is less effective than isofropization, because the potential from a long streamer varies only slowly in time).

We may contrast relaxation from tidal streamers with two other relaxation processes in stellar systems: two-body and violent relaxation. The two-body relaxation rate is determined by assuming that the stellar system is as smooth as possible, so the only potential fluctuations arise from Poisson noise due to individual stars. Violent relaxation (Lynden-Bell 1967) is modelled by assuming that the potential fluctuations in the stellar system are as large and rapid as possible (spatial and time scales of order the system size and dynamical time), an assumption that is only valid during the initial collapse of the system. Relaxation from tidal streamers is intermediate – stronger than two-body relaxation, and weaker but longer lasting than violent relaxation – and arises because a stellar system only phase mixes gradually over many dynamical times.

One important unresolved question is how and when the streamers are finally mixed together. Small-scale irregularities in the disc and halo (GMCs, globular clusters, etc.) can mix the streamers, but relaxation arising from interactions between streamers may be more effective. This issue is also important for experiments that hope to detect the phase-space structure of dark matter particles (Sikivie & Ipser 1992).

The proposals made in this paper can be tested in a variety of ways. The physical processes can be studied by appropriate numerical simulations. Conventional $N$-body simulations of galaxy formation offer limited insight into phase mixing, because of numerical noise (Hernquist & Barnes 1990; Hernquist & Ostriker 1992; Steinmetz & White 1997). However, specially designed simulations are more powerful. A reasonable approximation is to turn off the gravitational forces among the disc particles and among the streamer particles, to suppress large-scale instabilities in the disc and numerical relaxation within the halo. The only remaining forces would be between disc and streamer particles, and between both sets of particles and a fixed galactic potential. A more complete treatment would include the mutual gravitational interactions between streamers. As an illustration of the importance of such interactions, consider a single streamer consisting of an axisymmetric ring of material in an inclined orbit. Eventually this streamer would be dragged precisely into the disc midplane. However, if there are several axisymmetric streamers with the same radius and different mid-planes, they cannot all be dragged into the disc midplane since this would lead to an increase in the phase-space density of the halo material, which violates the collisionless Boltzmann equation.
The efficiency of disc–halo coupling due to tidal streamers depends on the strength and properties of structure in the dark halo. Recent high-resolution numerical simulations (Klypin et al. 1999) based on a variant of the cold dark matter cosmological model support the picture presented in this paper of a rich hierarchy of small-scale substructure in the halo.

There are also direct observational tests. (i) Massive black holes scatter a fraction of the stars in the bulge and inner disc out to much larger radii; these should appear as a distinct population of small-scale substructure in the halo.

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