The combat application of queuing theory model in formation ship to air missile air defense operations

Zhitong Zhong¹ and Lin Zhang¹
¹ Dalian Naval Academy, Dalian, Liaoning Province, 116018, China
* Corresponding author’s e-mail: 1136321823@QQ.com

Abstract. This paper is trying to analyze the dynamic process of air defense operations by using queuing theory synthetically, targeting at the firepower optimization and effectiveness evaluation in air defense operations of surface vessel formation. It also establishes the service model of intercepting air targets by single type and multi type ship to air missiles, and finally comes up with the optimization model of air defense firepower. Vaule for formation configuration, optimization of air defense firepower and effectiveness evaluation.

1. Preface
In the future sea battlefield, surface ships will face multiple groups, multiple directions, multiple targets, multiple waves, multiple types of dense and continuous attacks. The battle command of cooperative air defense of naval fleet has become a hot topic in the study of naval competition in various countries[1]. In order to counter the threat of air targets, the formation combat must use the formation ship to air missile weapon system efficiently and coordinately, and form a layered defense system according to the fire area and function characteristics of each ship to air missile, so as to effectively intercept the coming targets. In this paper, the queuing theory is used to describe the process of air defense and antimissile confrontation of ship to air missile dynamically, to analyze the operational state of the system, to calculate the interception probability. It also takes this queuing theory to analyze the number of firepower channels, the operational time of weapons, the probability of single damage and the penetration probability of air attack targets, so as to provide a strong theoretical basis for optimizing the distribution of fire power of air defense of formation ships and exerting the operational efficiency of weapons.

2. Basic assumptions
2.1 In the air defense and antimissile operations at sea, each ship to air missile system is taken as the service station, the air targets that come in succession are taken customers, the target flow is the simplest flow (Poisson flow), the air attack density is λ, that is to say, the arrival interval time of two adjacent targets follows the negative index distribution of λ;

2.2 Ship to air missile intercepts the incoming air targets one after another. The shooting process can be regarded as a random service process, and the shooting time follows a negative exponential distribution. The fire density of a single target channel of the ship to air missile system is respectively, 

μ = \frac{1}{t_{fw}}, \text{ and the average shooting time of the ship to air missile system} t_{fw} [2];
2.3 For the long-range ship to air missile, its firepower area is large and the target stays for a long time. Once it flies out of the firepower area, it will break through. Therefore, the ship to air missile weapon can be regarded as a random service system with limited waiting time. The average number of enemy targets that break through without being shot by the ship to air missile in unit time is \( n_v = \frac{1}{t_d} \), where \( t_d \) is the average staying (flying) time of the air target in the firepower area of the ship to air missile.

2.4 The first come first serve (shooting) rule is adopted by ship to air missile to intercept air target. Supposed that the fire channel of the air raid target is idle during the movement time in the launching area, the idle fire channel can be used to fire towards the first arrived target, that is to follow first come first serve rule. If the air attack target enters the air defense area and all the fire channels are occupied, and unable to intercept the coming targets, it is considered that the air attack target will break through the fire area without the influence of the fire unit.

2.5 The operational effectiveness of weapons is usually influenced by the detection probability, the shooting probability and the killing probability of a single shot. As Figure 1, Figure of flow chart that air target have been served.

3. Practical use

3.1 Fire effectiveness model

It is assumed that the surface warship formation consists of warships equipped with ship to air missiles and the size of the formation air defense area is divided into \( k \) sections. Assuming that all air targets penetrate in the same direction in turn, the probability of air targets penetrating from the first section of the fire area is \( p_k \), the formation has M-type ship to air missiles in total, among which the number of fire channels of the \( i \)th type ship to air missiles is \( n_{i,a} \), the number of deployed in the \( k \) section is \( n_{i,a,k} \), and meets the requirements of \( \sum x_j = N \). According to the queuing theory, the formation of each type of ship to air weapon system is taken as a waiter, the continuous target is regarded as a customer, and the fight process is regarded as a service process[3].

3.1.1 Single type ship to air missile fire effectiveness

According to the performance of the ship to air missile, the intercepting depth of the fire area is large, the air target has a long flight time in the fire area, and only one type of ship to air missile serves for the incoming target. Therefore, the air defense process can be regarded as a limited waiting system.
Suppose that the number of incoming air targets is $N$, and $\alpha = \frac{\lambda}{\mu}$, $\beta = \frac{\nu}{\mu}$, at this time, it is a queuing system of fire channels (service station) of a single type ship to air missile. According to queuing theory [4], the target intercepting probability of ship to air missile is:

$$ P_{e} = 1 - P_{k} = 1 - \frac{\beta}{\alpha} \sum_{k=0}^{\infty} \frac{s\alpha^{k}}{k!} \prod_{n=1}^{\infty} \left( n_{ka} + k\beta \right) $$

In the formula (1), $\beta = \frac{\nu}{\mu}$ is the average number of targets that have not been fired but have penetrated, and $S$ is the average number of targets that have not been served but still remain in the fire area when the fire channel is saturated.

### 3.1.2 Cooperative air defense effectiveness of multi type ship to air missile

When the formation of various types of ship to air missiles cooperate in air defense operations, limited by the formation of air defense fire area, usually each type of ship to air missile only fires at the target once, and the shooting service time is small. According to the assumption of the fire effectiveness model, the M-type ship-to-air missile constitutes a combined random service system [5-6], which can be regarded as a multi fire channel vanishing system. The interception probability of each type of ship-to-air missile against the incoming air target is:

$$ P_{e} = 1 - \frac{\alpha^{m}}{m!} \cdot \sum_{i=1}^{m} \frac{(\lambda t_{i})^{m}}{i!} = 1 - \frac{\alpha^{m}}{m!} \cdot \sum_{i=1}^{m} \frac{(\lambda t_{i})^{m}}{i!} $$

According to the tactical and technical performance of the ship to air missile, the damage probability of each type of ship to air missile to the target is $Q_{d1}$, and the damage probability of the incoming target to a certain type of ship to air missile is $Q_{d1} = N(1 - P_{e})Q_{d1}$.

Then a combined random service system is set up by the M-type ship to air missiles, and of which the damage probability to the incoming air target is:

$$ Q_{e} = \frac{\sum_{i=1}^{k} n_{i}(1 - P_{e})Q_{d1}}{N} $$

### 3.2 Optimization model of formation air defense firepower

The purpose of air defense operation of Naval Formation is to achieve the maximum comprehensive defense efficiency of air defense weapon system [7-9], that is to intercept the maximum number of enemy’s air targets, that is to say, the air target penetration probability is the minimum, and to try utmost to protect the safety of the ship or the adjacent ships of the formation as much as possible. In the practical operation, it is impossible to know the enemy’s attack direction in advance, and one may only rely on the information system to detect and find the hostile incoming air targets. It can be seen that there is a great uncertainty in the enemy’s attack direction in the air defense operation. The possible attacking direction of the enemy can be predicted according to the practical battlefield situation and air attack weapon performance and other related data. The air defense tasks are distributed as per the balance principle, under which the Air defense fire distribution shall be carried out. Therefore, the air defense fire optimization of formation ship to air missile should consider all
kinds of possible attacking situations of air targets, analyze the battlefield information comprehensively, strengthen the air defense firepower in the key threat direction, and take other operational directions into account for the ship to air missile with formation cooperative operation ability, so as to achieve the best comprehensive defense effect of formation ship to air missile system. If the comprehensive defense effectiveness of the formation ship to air missile system is assumed to be \( W \), the force deployment should be based on the principle of the maximum comprehensive defense effectiveness of the system, that is, the minimum penetration probability of the enemy. According to the probability of the enemy entering each defense area and the ship to air missile intercepting probability which is computed by the built-in queuing theory as well as the full probability formula, the target intercepting probability of the formation ship to air missile system in the combat area can be obtained[10]. Thus the fire optimization model is as follows:

\[
\max W = \sum_{i=1}^{k} n_i (1 - P_{si}) Q_{di} = 1 - \sum_{i=1}^{k} \left[ (1 - P_{si}) Q_{di} \right]
\]

4. Conclusion

In this paper, it is meant to use queuing theory, model simulation and analysis method, to analyze and evaluate the operational effectiveness of naval air missile system, as for the changeable formation configuration of air defense threat, it still has strong practicability to use the model flexibly to evaluate the operational effectiveness of air defense. At the same time, in order to facilitate the analysis, the use of fire and the process of confrontation in air defense operations are simplified, which may not be consistent with the facts of air defense operations in surface warship formation. The complex problems such as multi-platform cooperation, firepower optimization distribution, firepower channel fire conversion will be the focus of the next research.

Reference

[1] Qiu,Z.M.(2010) Future maritime cooperative warfare technology. Command control and simulation, 32(3):118-120

[2] Zhao, G.T.Yu,Y.M.L, Q.Wang, C.(2014) Research on Combat Effectiveness of Antiaircraft Missile Weapon System Based on Queuing Theory. Modern Defence Technology, 42(1):19-23

[3] Zhang, J.Q. Li,T.B. Ren,X.W.(2009) Modeling and simulation technology of warship combat system [M]. Naval Engineering University, Wuhan.

[4] Xing,C.F .Li M.Y. W,L. (2008)Effectiveness analysis of Shipborne Weapon System. National Defence Industry Press, Beijing.

[5] Przemieniecki JS, (2000)Mathematical Methods in Defense Analyses. AIAA Education Series,

[6] Liu,Z.C. Yan,J.G. (2012) The Application of the Queuing Theory on the Defense System. College Mathematics, 28(5):119-123

[7] Min,H.Q. Li,X, Zhu, C.W. (2006)The application of queuing theory in the shooting effectiveness of missile defense system. Command control and simulation, 28(3):88-90.

[8] Xing,Q.H. Liu,F.X. (2006)Regional air defense deployment optimization system modeling. System engineering and electronic technology, 28(5)712-715.

[9] Zhang,S. Wang, Y.L. CaoZ.Y. (2005)Evaluation model of ground air defense deployment scheme. Fire and command control, 30(5):15-18.
[10] Zhao, P. J., Li, J. G. (2017) An Optimization Model for Air Defense Troops Deployment Based on Queuing Theory. Fire Control & Command Control, 42(11):38-42.