Hindered magnetic dipole transition in the covariant light-front approach

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Hindered magnetic dipole transitions $\Upsilon(nS) \to \gamma \eta_b(n'S)$ are studied in the covariant light-front approach. Compared with the allowed magnetic dipole transitions, we find that results for hindered magnetic dipole transitions are sensitive to heavy quark mass and shape parameters of the light-front wave functions. It is possible to tune the parameters so that the predictions of branching fractions of $\Upsilon(2S, 3S) \to \gamma \eta_b$ are consistent with the recent experimental data, but the relevant decay constant of $\eta_b$ is much smaller than that of $\Upsilon(1S)$. We also generalize the investigation to the charmonium sector and find the same conclusion.

Since the photon energy in allowed magnetic dipole (M1) transitions is limited, hindered M1 transitions with changes of principal quantum numbers are believed to offer better chances to discover the pseudoscalar quarkoniums such as $\eta_b$. Several years ago, the CLEO collaboration set the upper bounds for this kind of decays:

$$B(\Upsilon(3S) \to \gamma \eta_b) < 4.3 \times 10^{-4},$$
$$B(\Upsilon(2S) \to \gamma \eta_b) < 5.1 \times 10^{-4}. \quad (1)$$

In 2008 the BaBar collaboration observed a peak in the photon energy spectrum at $E_\gamma = (921.2^{+2.1}_{-2.8} \pm 2.4)$ MeV in radiative $\Upsilon(3S)$ decay. This is viewed as the first observation of the $\eta_b$ meson which is the lowest bottomonium ground state. The branching ratio of this radiative decay is

$$B(\Upsilon(3S) \to \gamma \eta_b) = (4.8 \pm 0.5 \pm 1.2) \times 10^{-4}. \quad (2)$$

The measured mass

$$m(\eta_b) = (9388.9^{+3.1}_{-2.3} \pm 2.7)\text{MeV}$$

gives the mass split

$$\delta m_{\eta_b(1S)} = m(\Upsilon(1S)) - m(\eta_b) = (71.4^{+2.3}_{-3.1} \pm 2.7)\text{MeV}. \quad (3)$$

Subsequently the $\eta_b$ meson has also been observed in the radiative $\Upsilon(2S)$ decay:

$$B(\Upsilon(2S) \to \gamma \eta_b) = (3.9 \pm 1.1^{+1.1}_{-0.9}) \times 10^{-4}. \quad (4)$$

The ratio of the two branching fractions is measured as

$$R \equiv \frac{B(\Upsilon(2S) \to \gamma \eta_b)}{B(\Upsilon(3S) \to \gamma \eta_b)} = 0.82 \pm 0.24^{+0.20}_{-0.19}. \quad (5)$$

The recently updated results by the CLEO collaboration are consistent with measurements by the BaBar collaboration taking into account the uncertainties

$$B(\Upsilon(3S) \to \gamma \eta_b) = (7.1 \pm 1.8 \pm 1.1) \times 10^{-4}, \quad \text{[6]}$$
$$B(\Upsilon(2S) \to \gamma \eta_b) < 8.4 \times 10^{-4}. \quad \text{[7]}$$
Decay widths of M1 transitions can be expressed in a well-known formula

\[ \Gamma(n^3S_1 \rightarrow \gamma n'^1S_0) = \frac{4}{3} \alpha e_Q^2 \left( \frac{k^3}{m_Q} \right) \left| \int_0^\infty dr r^2 R_{n'0}^* (r) J_0 \left( \frac{kr}{2} \right) R_{n0}(r) \right|^2. \]  

(8)

\( \alpha = 1/137 \) is the fine-structure constant, \( e_Q \) denotes the charge in unit of \( |e| \) of the transition quark, and \( m_Q \) denotes the quark mass. \( k \) is the energy of the photon in the vector meson rest frame. For allowed transitions \( (n = n') \), the emitted photon is typically soft. One can expand the spherical Bessel function \( J_0(\frac{kr}{2}) \) by \( k \). Radiative corrections and nonrelativistic corrections can be systematically studied in the effective field theory [5]. In particular the predicted branching ratio of \( J/\psi \rightarrow \gamma \eta_c \) is well consistent with the experimental data.

The emitted photon in hindered M1 transitions is rather energetic and the convergence of the expansion in \( k \) becomes poor. On the other aspect, the decay amplitude is zero at the leading power in \( k \) since the different wave functions are orthogonal. Higher power terms contribute and they are sensitive to the treatments of the nonperturbative dynamics in transition form factors. For more than twenty years, hindered M1 transitions have received extensive interests but theoretical predictions vary over several orders [6]. The recent experimental data could offer an opportunity to investigate the dynamics in the quarkonia transition and in particular might be helpful to constrain the form of the wave functions.

In this work, we use the covariant light-front quark model (LFQM) to investigate the M1 transitions and examine whether the commonly-used light-front wave functions (LFWF) can simultaneously explain the experimental data for both allowed and hindered M1 transition. The light-front QCD may be the only potential candidate to reconcile the low energy quark model and the high energy parton model. The LFQM [7] can give a full treatment on spins of hadrons using the so-called Melosh transformation. Physical quantities are represented as the overlap of LFWF. These wave functions are expressed in terms of the internal variables of the quark and gluon degrees and thus are manifestly Lorentz invariant. To preserve the covariance and remove the dependence of physical quantities on the direction of the light-front, Jaus proposed the covariant LFQM in which the zero-mode contributions are systematically included [8]. The application of the covariant LFQM to the s-wave and p-wave decay constants and form factors is very successful [9–14]. (see also Ref. [17]).

Under this framework, the transition \( \Upsilon(1S) \rightarrow \gamma \eta_b(1S) \) has already been studied in Ref. [15, 16].

In the following we employ the light-front decomposition of the momentum \( P = (P^-, P^+, P^\perp) \), where \( P^\pm = P^0 \pm P^3 \). These momenta can be expressed in terms of the internal variables \( (x_i, p^\perp) \) as:

\[ p^+_1, p^+_{1,2} = x_{1,2} P^+, p_{1,2\perp} = x_{1,2} P^\perp \pm p^\perp, \]

(9)

with \( x_1 + x_2 = 1 \). With these internal variables, one can define some useful quantities

\[ M_0^2 = (e_1 + e_2)^2 = \frac{p^2}{x_1} + \frac{m_1^2}{x_2}, \]

\[ p_z = \frac{x_2 M_0}{2} - \frac{m_1^2 + p^2}{2x_2 M_0}, \]

(10)

\[ e_i = \sqrt{m_i^2 + p_i^2 + p_z^2}. \]

here \( e_i \) can be interpreted as the energy of the quark/antiquark and \( M_0 \) can be viewed as kinetic invariant mass of the meson system. The transition amplitude of \( V(P, \epsilon_V) \rightarrow P \gamma^*(q, \epsilon_\gamma) \) is usually
parametrized as

$$A = i e \epsilon_{\mu\nu\rho\sigma} \epsilon^\rho_V \epsilon^\nu_V q^\sigma P^\rho V(q^2), \quad (11)$$

where the photon is firstly taken as off-shell $q^2 \neq 0$ and the convention $\epsilon^{0123} = 1$ is adopted. Feynman diagrams for this process are given in Fig. 1, in which the photon is emitted from the quark shown in (a) or from the antiquark shown in (b). Due to the charge conjugation invariance, these two diagrams provide identical contributions. It is straightforward to evaluate these two diagrams:

$$A = -i e Q N_c \int \frac{d^4 p_1}{(2\pi)^4} \left\{ \frac{H_V H_P}{N_1 N_2 N_1'} s^a_{\mu\nu} + \frac{H_V H_P'}{N_1 N_2 N_2'} s^b_{\mu\nu} \right\} \epsilon^\gamma \epsilon^\nu_V, \quad (12)$$

where $N_i = p_i^2 - m_i^2$, $N'_i = p'_i - m_i^2$ and

$$s^a_{\mu\nu} = \text{Tr} \left[ \left( \gamma_\nu - \frac{p_1\nu - p_2\nu}{W_V} \right) (-\not{p}_2 + m_2)\gamma_5 (\not{p}_1' + m_1) \gamma_\mu (\not{p}_1 + m_1) \right], \quad (13)$$

$$s^b_{\mu\nu} = \text{Tr} \left[ \left( \gamma_\nu - \frac{p_1\nu - p_2\nu}{W_V} \right) (-\not{p}_2 + m_2)\gamma_\mu (-\not{p}_2' + m_2)\gamma_5 (\not{p}_1 + m_1) \right]. \quad (14)$$

Functions $H_V, H_P$, depending on the four momentum of the internal quarks, are the wave functions for vector and pseudoscalar mesons. In the absence of singularity in the $H_V, H_P$, integrating over the minus component of the internal momentum will pick up the pole in the propagators. Then the functions $H_V, H_P$ are replaced by $h_V, h_P$ which only depends on the plus component and the transverse momentum and $W_V$ is replaced by $w_V$

$$h_V = h_P = (M^2 - M_0^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2M_0}} \phi(x_2, p_\perp),$$

$$w_V = M_0 + m_1 + m_2, \quad (15)$$

where $\phi(x_2, p_\perp)$ denotes the momentum distribution inside the meson. In order to preserve the covariance, it is suggested that the zero mode contributions should be added in the covariant LFQM. The inclusion of the zero-mode contribution corresponds to a proper way to integrate out the minus component and after the integration expressions for the form factor $V(q^2)$ are given as follows [15, 16]

$$V(q^2) = \frac{e Q}{8\pi^3} \int dx_2 d^2 p_\perp \frac{2 \phi_V(x_2, p_\perp) \phi_P(x_2, p'_\perp)}{x_1 M_0 M'_0} \left( m_Q + \frac{2}{w_V} (p_\perp^2 + (p_\perp \cdot q_\perp)^2) \right), \quad (16)$$

and the decay width of $V \rightarrow P \gamma$ is then evaluated as

$$\Gamma(V \rightarrow P \gamma) = \frac{4\pi\alpha}{3} \frac{(m_V^2 - m_P^2)^3}{32\pi m_V^3} |V(0)|^2. \quad (17)$$

In the $m_Q \rightarrow \infty$ limit, the heavy quark inside the quarkonium moves nonrelativistically. Integrating out the hard off-shell degrees of freedom, one reaches an effective field theory known as nonrelativistic QCD [18] to deal with the low energy dynamics. In a heavy quarkonium, the square of the transverse momentum $p_\perp^2$ is of the order $A^2_{\chi QCD}$ and the momentum fraction of the heavy quark $x_2 \simeq 1/2$. The kinetic invariant mass $(M_0, M'_0)$ is roughly $2m_Q$. At the leading power in $1/m_Q$, the form factor $V(0)$ in Eq. (16) is reduced to

$$V(0) \simeq \frac{e Q}{8\pi^3} \int dx_2 d^2 p_\perp \frac{\phi_V(x_2, p_\perp) \phi_P(x_2, p'_\perp)}{m_Q}. \quad (18)$$
\[ V(p) \rightarrow P\gamma(p') \]

FIG. 1: Feynman diagrams for \( V \rightarrow P\gamma \).

This formula, with the LFWF discussed in the following, could formally reproduce the leading power behavior shown in Eq. (8) in the momentum space.

For hindered M1 transition, the transition amplitude is from the higher power corrections and one needs a specific approach to calculate them. As we can see from the above discussion, the form factors in the covariant LFQM are expressed as the overlap of the LFWF. It contains the nonperturbative coupling of the quark pair with a meson. Except for some limited cases, these quantities cannot be derived from the first principle. Practically one often adopts a phenomenological form. The commonly-used wave function for the \( S \)-wave mesons with lowest principle number is the Gaussian-type

\[
\phi(x_2, p_\perp) = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} e^{i \epsilon_1 \epsilon_2} \sqrt{\frac{dp_z}{dx_2}} \exp \left( -\frac{p_z^2 + p_\perp^2}{2\beta^2} \right),
\] (19)

\[
\frac{dp_z}{dx_2} = \frac{e_1 e_2}{x_1 x_2 M_0}.
\]

In the case of the wave functions of the \( 2S, 3S \) mesons, we adopt the solutions of the harmonic-oscillator problem

\[
\phi_{2S}(r) = \sqrt{\frac{1}{6}} \left( \frac{\beta^2}{\pi} \right)^{3/4} \left( \frac{3}{4} - \frac{\beta^2 r^2}{2} \right) \exp \left( -\frac{\beta^2 r^2}{2} \right),
\]

\[
\phi_{3S}(r) = \sqrt{\frac{2}{15}} \left( \frac{\beta^2}{\pi} \right)^{3/4} \exp \left( -\frac{\beta^2 r^2}{2} \right) \times \left( \frac{15}{4} - 5\beta^2 r^2 + \beta^4 r^4 \right).
\] (20)

The Fourier transformations, with the Jacobi determinant \( \sqrt{\frac{dp_z}{dx_2}} \), are derived as

\[
\phi_{2S}(x_2, p_\perp) = 4 \sqrt{\frac{2}{3}} \left( \frac{\pi}{\beta^2} \right)^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp \left( -\frac{p_z^2 + p_\perp^2}{2\beta^2} \right) \times \left( \frac{p_z^2 + p_\perp^2}{\beta^2} - \frac{3}{2} \right),
\]

\[
\phi_{3S}(x_2, p_\perp) = 4 \sqrt{\frac{2}{15}} \left( \frac{\pi}{\beta^2} \right)^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp \left( -\frac{p_z^2 + p_\perp^2}{2\beta^2} \right) \left( \frac{(p_z^2 + p_\perp^2)^2}{\beta^4} - \frac{5(p_z^2 + p_\perp^2)}{\beta^2} + \frac{15}{4} \right). (21)
\]

This kind of LFWF for the \( 2S \) mesons has been used in Ref. [16] while its light-cone form is studied in Ref. [19]. It would be meaningful to use the allowed and hindered M1 transitions to constrain the forms of the LFWF and in particular to test the harmonic-oscillator wave functions given above, as most of the previous studies in the covariant LFQM mainly focus on mesons without any radial excitation.

Two kinds of inputs are required in the numerical analysis: masses (for hadrons and constituent quarks) and the shape parameters in the wave functions. For the heavy quarks, the difference between the current mass and the constituent mass is small. From the PDG [20], the current
masses are \( m_c = (1.27^{+0.07}_{-0.11}) \) GeV and \( m_b = (4.20^{+0.17}_{-0.07}) \) GeV in the \( \overline{\text{MS}} \) renormalization scheme. In this work, we will choose three different values for the constituent quark masses

\[
m_c = (1.1, 1.3, 1.5) \ \text{GeV}, \quad m_b = (4.0, 4.4, 4.8) \ \text{GeV}.
\]  

(22)

As for hadron masses, all of them used in the present analysis have been measured except the one of \( \eta_b(2S) \). We will use the mass split in the charm sector \( ^{(22)} \)

\[
\delta m_{\eta_c(1S)} = 116.5 \text{MeV}, \quad \delta m_{\eta_c(2S)} = 48.1 \text{MeV}
\]

to estimate the mass split for the \( \eta_b(2S) \)

\[
\delta m_{\eta_b(2S)} = \frac{\delta m_{\eta_b(1S)}}{\delta m_{\eta_c(1S)}} \delta m_{\eta_c(2S)} = 29.5 \text{MeV}.
\]  

(23)

Shape parameters of the LFWF are usually determined by decay constants of hadrons whose expressions are given in Ref. \( ^{(9)} \). Decay constants of \( \Upsilon(nS) \) are extracted from the partial decay width of the leptonic \( \Upsilon \to e^+ e^- \) decays

\[
\Gamma_{ee} \equiv \Gamma(\Upsilon \to e^+ e^-) = \frac{4\pi \alpha^2 e_b^2 f_\Upsilon^2}{3m_\Upsilon}.
\]  

(24)

The shape parameters (corresponding to \( m_b = 4.4 \pm 0.4 \) GeV and \( m_c = 1.3 \pm 0.2 \) GeV) are collected in table \( ^{11} \). In the case of \( \eta_b \) mesons, their decay constants are not measured at present. In the future, these decays may not be well constrained as the uncertainties in the ideal mode \( \eta_b \to 2\gamma \) would be typically large. In this work, we first try to tune the shape parameter so that the predictions of branching fractions of \( \Upsilon(2S, 3S) \to \gamma \eta_b \) decays are consistent with the data provided by the BaBar collaboration.

With above parameters, results for the branching ratios in \( \Upsilon \) decays are collected in table \( ^{11} \) and table \( ^{13} \), in which we have used three sets inputs for the shape parameters and three different values for the quark mass. Several remarks are give in order. Firstly, from these tables one can see that the decay constant of the \( \eta_b \) is not very sensitive to the shape parameter and the quark mass \( m_b \). Secondly the allowed channels \( \Upsilon(nS) \to \eta_b(nS)(n = 1, 2) \) are not sensitive to these inputs either. On the contrary, results for hindered channels are strongly dependent on the input parameters, which is not beyond expectation. If we adopt the \( m_b = 4.4 \) GeV and \( \beta_{\eta_b} = 0.93 \) GeV, theoretical predictions of branching fractions of \( \Upsilon(2S, 3S) \to \gamma \eta_b \) are consistent with the experimental data given in Ref. \( ^{2, 3} \). The ratio of branching fractions is predicted as 0.66.
TABLE II: Branching ratios (in units of $10^{-4}$) of M1 transitions: $\Upsilon(nS) \rightarrow \gamma \eta_b(n'S)$. The $b$ quark mass is used as $m_b = 4.4$ GeV.

| $\beta_{\eta_b}$ | 0.90 | 0.93 | 0.96 |
|-------------------|------|------|------|
| $f_{\eta_b}$      | 389  | 406  | 424  |
| $\mathcal{B}(\Upsilon(1S) \rightarrow \eta_b)$ | 2.7  | 2.8  | 2.9  |
| $\mathcal{B}(\Upsilon(2S) \rightarrow \eta_b)$ | 21.3 | 5.0  | 0.003|
| $\mathcal{B}(\Upsilon(3S) \rightarrow \eta_b)$ | 2.3  | 7.7  | 18.6 |
| $R$               | 9.4  | 0.66 | $1.5 \times 10^{-4}$ |
| $\beta_{\eta_b(2S)}$ | 0.75 | 0.8  | 0.85 |
| $f_{\eta_b(2S)}$  | 343  | 372  | 400  |
| $\mathcal{B}(\Upsilon(2S) \rightarrow \eta_b(2S))$ | 0.3  | 0.3  | 0.4  |
| $\mathcal{B}(\Upsilon(3S) \rightarrow \eta_b(2S))$ | 40.3 | 3.1  | 6.7  |

TABLE III: Branching ratios (in units of $10^{-4}$) of M1 transitions: $\Upsilon(nS) \rightarrow \gamma \eta_b(n'S)$. The shape parameters of $\eta_b$ and $\eta_b(2S)$ are used as $\beta_{\eta_b} = 0.93$ GeV; $\beta_{\eta_b(2S)} = 0.80$ GeV.

| $m_b$   | 4.0  | 4.4  | 4.8  |
|---------|------|------|------|
| $f_{\eta_b}$ | 419  | 406  | 394  |
| $\mathcal{B}(\Upsilon(1S) \rightarrow \eta_b)$ | 3.3  | 2.8  | 2.5  |
| $\mathcal{B}(\Upsilon(2S) \rightarrow \eta_b)$ | 24.8 | 5.0  | 0.003|
| $\mathcal{B}(\Upsilon(3S) \rightarrow \eta_b)$ | 3.3  | 7.7  | 14.4 |
| $R$     | 7.5  | 0.66 | 0.0002|
| $m_b$   | 4.0  | 4.4  | 4.8  |
| $f_{\eta_b(2S)}$ | 379  | 372  | 364  |
| $\mathcal{B}(\Upsilon(2S) \rightarrow \eta_b(2S))$ | 0.4  | 0.3  | 0.3  |
| $\mathcal{B}(\Upsilon(3S) \rightarrow \eta_b(2S))$ | 18.2 | 3.1  | 0.055|

In the heavy quark limit, pseudoscalar mesons and vector mesons belong to the same supermultiplet and their decay constants are almost the same. If we adopt the same decay constant for $\eta_b$ with its vector partner, the shape parameter is determined as $\beta_{\eta_b} = 1.451$ GeV and theoretical predictions of branching ratios are given as

\[
\mathcal{B}(\Upsilon(1S) \rightarrow \gamma \eta_b) = 3.2 \times 10^{-4},
\]
\[
\mathcal{B}(\Upsilon(2S) \rightarrow \gamma \eta_b) = 5.8\%,
\]
\[
\mathcal{B}(\Upsilon(3S) \rightarrow \gamma \eta_b) = 13.7\%.
\]  

The latter two results are much larger than the experimental data given earlier. These inconsistencies between theoretical results and the data may imply either the inappropriate form of the LFWF used in this analysis, or a small decay constant for $\eta_b$ compared with that for $\Upsilon$.

As another check, we will extend the above discussion into the charmonium sector: $\psi(nS) \rightarrow \gamma \eta_c(n'S)$. The relevant experimental data is given as

\[
\mathcal{B}(J/\psi \rightarrow \gamma \eta_c) = (1.7 \pm 0.4)\%,
\]
\[
\mathcal{B}(\psi(2S) \rightarrow \gamma \eta_c) = (3.4 \pm 0.5) \times 10^{-3},
\]
\[
\mathcal{B}(\psi(2S) \rightarrow \gamma \eta_c(2S)) < 7.4 \times 10^{-4}.
\]
TABLE IV: Results for the branching ratios of M1 transitions involving \( \eta_c \) in the covariant LFQM (in units of \( 10^{-2} \) for \( J/\psi \to \eta_c \) and \( 10^{-3} \) for \( \psi(2S) \to \eta_c \)). In the left sector, the mass is used as \( m_c = 1.3 \) GeV; while in the right sector, the shape parameter is \( \beta_{\eta_c} = 0.55 \) GeV.

| \( \beta_{\eta_c} \) | 0.5 0.55 0.6 | \( m_c \) | 1.1 1.3 1.5 |
|-------------------|-----------|--------|-------------|
| \( f_{\eta_c} \) | 253 281 308 | \( f_{\eta_c} \) | 283 281 276 |
| \( B(J/\psi \to \eta_c) \) | 3.0 3.1 3.1 | \( B(J/\psi \to \eta_c) \) | 3.8 3.1 2.5 |
| \( B(\psi(2S) \to \eta_c) \) | 5.5 4.2 33.4 | \( B(\psi(2S) \to \eta_c) \) | 0.6 4.2 15.3 |

Again with proper inputs the results in table IV could be consistent with these data, but the relevant decay constant of \( \eta_c \) is much smaller than that of its vector partner \( J/\psi \). Our result for \( B(\psi(2S) \to \eta_c(2S)) \)

\[
B(\psi(2S) \to \gamma\eta_c(2S)) = (6.3^{+0.2+1.2}_{-1.2-1.0}) \times 10^{-4} \tag{29}
\]

is approaching the experimental upper bound. The first uncertainties are from \( m_c = (1.3 \pm 0.2) \) GeV and the second uncertainties are from \( \beta_{\eta_c(2S)} = (0.45 \pm 0.05) \) GeV. In Ref. [16], the inputs \( m_c = 1.56 \) GeV, \( \beta_{\eta_c} = 0.820 \) GeV and \( \beta_{J/\psi} = 0.613 \) GeV are adopted and prediction of \( B(J/\psi(1S) \to \gamma\eta_c) \) is consistent with the data reported by the CLEO collaboration [22]. If we adopt these parameters, the branching ratio of \( \psi(2S) \to \gamma\eta_c \) is larger than the experimental data by two orders

\[
B(\psi(2S) \to \gamma\eta_c) = 18.2\% \tag{30}
\]

Inspired by these inconsistencies, one may get the same conclusion as in the bottomonium sector.

In conclusion, we studied the hindered M1 transitions \( \psi(2S) \to \gamma\eta_c \) and the \( \Upsilon(nS) \to \gamma\eta_b(1S, 2S) \) in the covariant light-front quark model. Compared with the allowed M1 transitions, theoretical results for hindered channels are found to strongly depend on quark masses and the shape of the light-front wave functions. Because of this sensitivity, these transitions offer good laboratories to study the dynamics between the transitions and constrain the forms of the light-front wave functions. Using the harmonic-oscillator wave functions and the experimental data, our results show that the decay constants of the pseudoscalar mesons are required to be much smaller than the decay constants of their vector partners. With the precise decay constants measured in various other channels in the future, different forms of the wave functions could be directly tested in the channels studied here. A more practical way in the future is to incorporate more radiative transitions between heavy quarkonium states as in Ref. [23].

Note added: During the long-time revision of this work, a similar paper [24] appears. One of the important differences is: we have shown that hindered M1 transitions are sensitive to the shape of the wave functions as they should.

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