Effect of Strength Anisotropy on the Stability of Natural Slopes

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Abstract. It is well known that low plasticity soft clays have large variations in undrained shear strength with the direction of loading. Laboratory experiments show typically a cross anisotropic behavior, where the undrained shear strength in compression is significantly larger than the undrained shear strength in extension. The total stress based NGI-ADP model, as available in PLAXIS, captures such shear strength anisotropy well, when applied to embankments on or excavations from a horizontal or almost horizontal terrain. However, for non-horizontal terrain the direction of the in-situ principal stresses are inclined, and the axis of anisotropy are hence expected to be somewhat inclined too. As for natural slopes, the effect on the calculated factor of safety due to this inclination is not yet well documented. Previous studies show that this effect might not be a negligible effect and can increase the factor of safety by about 10% for gentle natural slopes. In order to study this, a new ADP model has been implemented in PLAXIS. The formulation is inspired by results from DSS laboratory testing where samples were consolidated under inclined effective stresses before shearing in the same or the opposite direction of the initial shear stress. As expected, the model shows higher factors of safety when applied to a slope than a conventional analysis. The paper then discusses to what extent this represents a real safety margin that has previously been neglected.

1. Introduction

1.1. Background

It is well recognized that marine clays are anisotropic in nature. It is expected that deposition (sedimentation) direction has played an important role in forming this anisotropy in natural marine clays. Generally, one expects anisotropy in permeability, strength and electrical/thermal conductivities. As both particles contact orientation and heterogeneity tend to be influenced by the way the material is deposited. For the mechanical behavior of natural clays, one may distinguish between the effect of the inherent anisotropy and that of stress induced anisotropy [1]. For horizontal sedimentation it is expected that the material would be cross-anisotropic with the vertical direction being the axis of symmetry. And, for this case the inherent and the stress induced anisotropy cannot easily be distinguished as the sedimentation direction and direction of major principal effective stress coincide. From a practical point of view, engineers need tools capable of reproducing the anisotropy of clays at continuum level. Therefore, the total stress based model by Grimstad et al. [2] has become a quite popular model in Norway for taking the measured anisotropy in undrained shear strength into

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account in engineering problems. The model assumes cross anisotropic behavior by a shift in the classical Tresca criterion towards the triaxial compression side to reproduce measured values of active (compression), direct and passive (extension) strength, $s^c$, $s^d$ and $s^e$ respectively. This total stress model is pragmatic as an engineering tool as it can easily adopt to a design profile of undrained strengths. Whereas, an effective stress model would predict the level anisotropy and magnitude of the actual undrained strength using many indirect parameters. However, the effective stress models are built within a more fundamentally correct framework and can model evolution of anisotropy (fabric). Since the use of these more fundamentally correct models are more difficult to adopt to practical problems the simpler tools tend to be exploited beyond the intentions of the developers. And, even though the model by Grimstad et al. [2] is intended for horizontally deposited clays, engineers and some researchers have extended the use to calculation of stability of natural slopes (e.g. Liu et al. [3]). In natural slopes it is expected that the angle of deposition and stress condition under consolidation will play a role on the fabric and heterogeneity of the clay, and thereby the anisotropy in undrained strength. It could be that the principal direction of fabric no longer coincides neither with the vertical direction nor with the direction of the major principal stress. As an example, Andersen [4] did direct simple shear (DSS) tests on a Norwegian quick clay under different consolidation shear stresses ($\tau_{c0}$). He showed that the undrained shear strength increased with increasing consolidation shear stress, and that the effect on the strength and thereby the FOS could be as high as about 30%. However, since the mean effective stress could be increased as well, for higher and higher consolidation shear stress during the consolidation stage in a DSS device, these results are inconclusive with respect to directly quantify the effect of this stress induced anisotropy. The above-mentioned study shows that it is difficult to empirically quantify this effect on the calculated factor of safety. Therefore, this article attempts to quantify the magnitude in possible change in the calculated factor of safety for a slope, when the initial stress condition is considered, within certain justified limitations. This work was done as part of the MSc study by the first author.

1.2. Analytical results for an infinite slope

Consider an infinite, plane strain, clay slope with a predefined sliding plane parallel with the surface, Figure 1a. Then consider that the soil strength is described with a shifted Tresca yield surface, where the center of the failure surface is shifted $\eta^* \cdot \sigma_0'$. Here $\eta^*$ is a factor between 0 and 1, and $\sigma_0'$ is the initial effective stress tensor in $x'z'$ coordinates, as given in eq. (1). For a special case of $\sigma_0'$, when the Earth Pressure Coefficient at rest between $x'$ and $z'$, $K_0'$, is 1.0, the shift of the Tresca surface is only in direction of the shear stress $\tau_{x'z'}$ and equals $\eta \cdot \tau_{x'z'}$. In order to keep the shift “independent” of $K_0'$ eq. (2) defines the relation between $\eta$ and $\eta^*$. Figure 2 gives a graphical representation in stress space of an initial stress situation for a point in a slope.

Figure 1 Equilibrium considerations in tangential shear stress for an arbitrary slice of soil in an infinite slope along predefined slide plane parallel to the surface.
\[ \sigma_{\theta}' = \begin{bmatrix} \sigma_{x'z'} & \tau_{x'z'} \\ \tau_{x'z'} & \sigma_{x'z'} \end{bmatrix} = \begin{bmatrix} K_0' & \sigma_{x'z'} \\ \sigma_{x'z'} & \sigma_{x'z'} \end{bmatrix} \]  

(1)

\[ \eta^* = \frac{\eta}{\sqrt{1 + \left(1 - \frac{K_0'}{2} \cot \theta \right)^2}} \]  

(2)

**Figure 2.** Shifted failure surface in the plane strain deviatoric stress plane. Right figure shows the general concept in arbitrary Cartesian coordinates. Left figure shows the special case defined above in x’ z’ Cartesian coordinates.

Where \( \theta \) is the slope angle. Consider then a failure mechanism of the infinite slope involving sliding of the clay layer along the interface towards the firm layer below, and assume associated flow, then the kinematics decides the stress state for which the slope will fail for a given factor of safety, \( FOS \). It can be shown that eq. (3) represents the increase in \( FOS \) with increase in \( \eta \).

\[ FOS_{\text{ani}} = FOS_{\text{iso}} + \eta^* \]  

(3)

Where \( FOS_{\text{ani}} \) is the \( FOS \) considering anisotropic strength and \( FOS_{\text{iso}} \) for isotropic strength. Or expressed as the change, \( \Delta FOS \):

\[ \Delta FOS = \eta^* \]  

(4)

The recommended “ADP”-factors [5] would for a low plasticity clay with an assumed initial mobilization, for horizontal terrain, \( \tau_0/s_u^C \), of 0.70, results in a value of \( \eta = 0.46 \). Figure 3 shows the failure surface and initial stress situation for the NGI-ADP model for a cross-anisotropic situation, i.e. for an arbitrary stress point below the surface. For \( K_0' = 0.50 \) and a slope with inclination 1:5, then the calculated increase \( \Delta FOS = 0.29 \). This increase in calculated \( FOS \) is significant considering many natural slopes with initially calculated quite low \( FOS \) for long planar failure surfaces, but is in line with the 30% described by Andersen [4] for \( FOS_{\text{iso}} = 1.0 \). However, note that, the NGI-ADP, Tresca type, criterion assumes \( s_u^P = s_u^A \) (plane strain passive undrained strength) and \( s_u^C = s_u^A \) (plane strain active undrained strength), when neglecting that the corners are a bit rounded. Which implies that the calculated value is on the high side. Also note that the mechanism here does not involve any geometrical change to the slope, as would be the case for e.g. a road embankment construction.
1.3. Scope of this article
In Nordal et al. [6] it was concluded that for the particular case of the Vestfossen slide (an embankment placed in a slope caused the slide in 1984), the above-mentioned effect could be about 10%. Nordal et al. [6] used a model, ADPX, to quantify this effect. However, the study was limited to a case of Vestfossen and by the capability of the ADPX model to reproduce the in-situ strength anisotropy. Therefore, this article presents an improved material model, called ADPX3, and applies this in a Monte Carlo simulation to quantify the “error” in traditional FOS calculations. It should be noted that, in case of sensitive materials, [7] and Fornes and Jostad [8] demonstrated that the reduction in FOS could be as high as 20% due to progressive failure. Further, Jostad and Lacasse [9] demonstrated potential importance of 3D effects on the FOS (for insensitive clays), indicating an increase as high as 50% for very narrow geometrical limitations. However, the combination of these three, different effects, i.e. decrease due to strain-softening, increased due to 3D effect and increase due to anisotropy, is not a part of this study. It is important to note that it would not be wise to superpose these effects, as the calculated failure mechanism is altered as well.

2. Theory for general case

2.1. Model ADPX3
In the previous mentioned study by Hicher et al. [1], strain contour diagrams in the π-plane was constructed for triaxial tests with varying Lode angle. Figure 4 presents the drawn contours of equal strain for Kaolinite clay, where five drained shear tests were conducted on initial anisotropic consolidated samples (right) and four tests on initial isotropic consolidated samples (left). Even though these tests were drained tests, a shift, in the contours for 3% and 5% strain, towards the initial compression direction is observed. This is even more clear at lower strain levels, but this effect vanishes for large strains, which indicates high degree of stress/strain induced anisotropy and very limited amount of inherent anisotropy in this material.
Figure 4. Contours of equal strain in the π-plane for drained shearing on (initial) cross-anisotropic and isotropic consolidated samples of Kaolinite, adapted from Hicher et al. [1]. Note that the graph is flipped around the $\sigma_z$ axis, and therefore $\sigma_z$ becomes $\sigma_y$ on the other respective sides. The dotted lines represent the stress paths followed in the drained triaxial tests.

Kirkgard and Lade [10] presented undrained true triaxial data on natural San Francisco bay mud, data in Figure 5. The samples were isotropic reconsolidated, which might have some effect on the undrained response under shearing, but anisotropy in undrained shear strength was nevertheless observed. When the failure criterion of the NGI-ADP model is applied (to the right in the figure) it underestimates the passive undrained shear strength for this case. This possible defect of the NGI-ADP model is also been pointed out by e.g. Krabbenhoft et al. [11] as the NGI-ADP deliberately does not distinguish between true anisotropy/fabric and the inherent Lode angle dependency (also isotropic soils will give $s^{\text{L}}_u < s^{\text{C}}_u$). Since it is not common to have data for the passive strength, the NGI-ADP criterion would in most cases give conservative undrained shear strength estimates. However, for natural slopes the special case of cross-anisotropy is no longer valid. Instead, the failure surface shifts towards the initial stress state, $\sigma_0^i$, rather than towards the 'strength center', $(s^{\text{A}}_u - s^{\text{P}}_u)$. Therefore, a more general criterion, able to predict level of stress induced anisotropy, must be used instead. Following a similar procedure as Grimstad et al. [2] a shifted deviatoric stress vector, $\hat{s}$, is written as:

$$\hat{s} = s - \eta$$

(5)

Where $s$ is the deviatoric stress vector (in Voigt notation) and the shift vector $\eta$ is defined as:

$$\eta = \frac{\eta_i}{\eta_j} \left( \sigma_{i,j} - \sigma_{i,j}^0 \right) + \frac{\xi_i}{\xi_j} \left( \sigma_{i,j} - \sigma_{i,j}^0 \right)$$

$$\eta = \begin{bmatrix} \frac{\eta}{3} (\sigma_{xy} - \sigma_{xy}^0) + \frac{\xi}{3} (\sigma_{xy} - \sigma_{xy}^0) \\ \frac{\eta}{3} (\sigma_{yz} - \sigma_{yz}^0) + \frac{\xi}{3} (\sigma_{yz} - \sigma_{yz}^0) \\ \frac{\eta}{3} (\sigma_{zx} - \sigma_{zx}^0) + \frac{\xi}{3} (\sigma_{zx} - \sigma_{zx}^0) \\ \frac{\eta + 2\xi}{3} \sigma_{y0} \\ \zeta \sigma_{y0} \\ \zeta \sigma_{z0} \end{bmatrix}$$

(6)

Here $\eta$, $\xi$ and $\zeta$ are introduced as separate parameters. Which allow for distinguishing the gravitational direction, $z$, from the others. However, following the previous statement regarding stress induced versus inherent anisotropy, $\eta = \xi = \zeta$ is used in the rest of this study. Indicating only stress induced
anisotropy, with a shift towards the current in-situ stress condition. The Matsouka-Nakai criterion is used as the underlying criterion, following the formulation of Grimstad et al. [12]. Which reduces the necessary input parameters to \( \tilde{s}_u, \eta \) and \( K_0' \) when describing a normally consolidated cross-anisotropic material, without any attraction, \( a \), term. To the left in Figure 5 the ADPX3 model is compared to the data by Kirkgard and Lade [10], using \( K_0' = 0.6, \eta = 0.15 \) and \( \tilde{s}_u/\sigma_u' = 0.34 \). The final failure criterion is given in eq. (7). Associated flow (in the \( \pi \)-plane) is assumed and isotropic elasticity is used, which, completes the elastoplastic formulation. It should be noted that there is limited evidence on validity of the assumption of associated flow. As an example Krabbenhøft et al. [11] argued for a non-associated flow for their AUS (Anisotropic Undrained Strength) model.

\[
F = \sqrt{3} J_2 \frac{\sin \phi_p (I_{10} + 3a)}{\sqrt{3} \hat{c}_\theta + \hat{s}_\theta}
\]

Where:

\[
\sin \phi_p = \frac{2 \sin \phi_c}{\sqrt{3 + \sin^2 \phi_c}}
\]

\[
\sin \phi_c = \frac{3}{2} \frac{I_{10} + 3a}{\tilde{s}_u} \left( \sqrt{1 + 4 \left( \frac{\tilde{s}_u}{I_{10} + 3a} \right)^2} - 1 \right)
\]

\[
\hat{c}_\theta = \cos \left( \frac{1}{3} \arcsin \left( \sin \phi_c \cdot \frac{3 - \sin^2 \phi_c}{2} \cdot \sin 3\theta \right) \right)
\]

\[
\hat{s}_\theta = \sin \left( \frac{1}{3} \arcsin \left( \sin \phi_c \cdot \frac{3 - \sin^2 \phi_c}{2} \cdot \sin 3\theta \right) \right)
\]
2.2. Verification
To verify the model implementation and the procedure for calculation of FOS (i.e. the strength reduction procedure) radial loading paths to failure were compared to strength (phi-c) reduction for several different initial conditions. Figure 6 show the results of one series of analysis done on a single stress point, for a case when the material is initialized anisotropically towards a state with Lode angle of 0°. It demonstrates that the model and phi-c reduction procedure work as intended.

2.3. Parameter restrictions
The “unknown” $\eta$ parameter is expected to vary depending on the soil and hence a general recommendation is not given as part of this study. However, in order to give some estimate on acceptable levels of $\eta$, a restriction is set, i.e. $0 < \eta < \eta_{\text{max}}$. Where $\eta_{\text{max}}$ is found from eq. (12), that introduces three requirements. The first requirement is obvious, while the second requirement is put there to restrict the shear strength from exceeding the strength obtained with the critical state friction angle, $\phi_{cs}$. The last requirement is introduced for restricting the maximum ratio of the undrained shear strengths, $k_1 = s_u/c_{u}\text{E}$, here $k_1 = 3$ is used. Which is obtained from calibration to triaxial tests on cross-anisotropic low plasticity clay.

$$\eta_{\text{max}} = \min \left\{ \frac{1}{M_c \cdot \frac{p_0^{\gamma} + a_c}{p_0^{\gamma} + a} - M_p^{\gamma}} \frac{p_0^{\gamma} + a}{q_0}, \frac{k_1 \cdot M_p^{\gamma} - M_p^{\gamma}}{1 + k_1} \frac{p_0^{\gamma} + a}{q_0} \right\}$$

(12)

Where $p_0^{\gamma}$ and $q_0$ is the initial mean and deviatoric stress, and $M_c = 6\sin\phi_{cs}/(3-\sin\phi_{cs})$, $M_p^{\gamma} = 6\sin\phi_{p}/(3-\sin\phi_{p})$ and $M_p^{\gamma} = 6\sin\phi_{p}/(3+\sin\phi_{p})$.

3. FOS calculations of randomized finite slopes
The object is to investigate the difference between FOS with and without considering the effect of stress induced anisotropy for slope stability calculations using the ADPX3 model. To achieve this, Monte Carlo simulations were carried out, where $\Delta FOS$, was evaluated among the population, by comparing $FOS_{\text{iso}}$ and $FOS_{\text{ani}}$ for the same randomly generated slope stability problem.
3.1. Geometry and parameter variations

Slopes with the following attributes were analyzed in PLAXIS 2D (www.plaxis.nl). Slope ratio \( b^{-1} = H/W \), was uniformly distributed, \( b^{-1} \in [0,0.60] \), where \( W \) is the width of the slope and \( H \) is the height. A constant \( W \) of 10 m is used throughout. The parameter \( \phi_p \) was generated assuming a normal distribution, \( \phi_p \sim N(\mu = 25^\circ, \sigma = 5^\circ) \), where \( \mu \) is the mean value of the population and \( \sigma \) is the standard deviation. Also the two attractions values were randomly generated from a normal distribution \( \alpha_a = a \sim N(\mu = 10 \text{ kPa}, \sigma = 2.5 \text{ kPa}) \) (possible neg. values of \( a \) and \( \phi \) were not included). In this study the critical state friction angle, the drained Poisson’s ratio (note: \( K_0' = v/(1-v) \) for hor. terrain under elastic response) and the unit weight of the clay were not varied, \( \phi_{cs} = 35^\circ \), \( v = 0.3 \) and \( \gamma = 20 \text{ kN/m}^3 \). A load was added on top of the slopes and adjusted such that the isotropic safety factor was \( FOS_{iso} < 1.40 \). Finally, \( \eta \) was set equal to \( \eta_{max} \) (with \( k_i = 3 \)) in half of the analyses and equal to 1.0 in the remaining.

3.2. Results and discussions

Figure 7 presents the results of the Monte Carlo simulations together with the analytical solutions. The analyzes of the randomly generated slopes confirmed that the possible increase in the FOS due to anisotropic shear strength is closely connected to the slope angle. When \( \eta \) is restricted, the effect of anisotropy is also strongly related to the difference between the two input friction angles and the parameter \( k_i \). For very steep slopes the increase \( \Delta FOS \) approaches the analytical solutions for infinite slopes when \( \eta = 1.0 \). This is because these situations only need a small load in order to obtain \( FOS_{iso} < 1.40 \) and hence the problem is therefore in more resemblance to the analytical problem of infinite slope. For the restricted cases (the squares in the figure) and for moderate slope angles, \( b^{-1} < 0.2 \), there is limited beneficial effect of the stress induced anisotropy in the calculated FOS (i.e. the extreme value for \( b^{-1} = 0.2 \) is \( \Delta FOS_{ext} = 0.09 \)).

Figure 7. Increase in Factor of Safety due to stress induced anisotropy with varying slope angles, \( \Delta FOS \) vs. \( b^{-1} \).
For the cases with no restriction on \( \eta \) all the resulting failure mechanisms were categorized in three groups. 1st toe failure for both anisotropic and isotropic case; 2nd cases when the mechanism changes from toe failure in isotropic case to a bearing capacity type of failure (i.e. behind toe failure or slope failure) for the anisotropic case; and finally, 3rd where both types of analyses gave bearing capacity type of failure. The results are shown in Figure 8. As one can observe, steep slopes give toe failure for both cases, while for more gentle slopes \((b^{-1} < 0.3)\) the anisotropic material will give bearing capacity type of failure while the isotropic still gives toe failure. While, for the isotropic material \( b^{-1} < 0.1 \) is necessary to give bearing capacity type of failure. It is believed that these results are influenced by the slope width, compared to the value of the attraction, and that for increasing width, the isotropic material will give bearing capacity type of failure for steeper slopes than \( b^{-1} = 0.1 \) as well (for the same value of \( \alpha \)).

![Figure 8. Increase in Factor of Safety due to stress induced anisotropy with varying slope angles, \( \Delta FOS \) vs. \( b^{-1} \). Results grouped after type of failure mechanism.](image-url)

### 4. Conclusions and recommendations

This study demonstrates that strength anisotropy induced by the consolidation stress, may have a significant influence on the factor of safety in undrained slope stability analyses. The effect is found to be positively correlated to slope angle. This conclusion is in accordance with the statement made by Andersen [4], “If this strength increase is neglected, the stability of a slope under additional undrained loading may be significantly underestimated”. However, for natural slopes with moderate inclination \((b^{-1} < 0.2)\) the increase found in the finite element analyses with ADPX3 model is significantly lower than the analytical solutions for infinite slopes. The main reason for this is that unlike the infinite slope the finite slopes where loaded at the top of the slope. This is done to reproduce a more realistic design scenario. However, this results in a bearing capacity type of problem that differs significantly from a planar surface of slope failure. For steep slopes \((b^{-1} > 0.5)\) the applied loads are limited (as the factor...
of safety already is marginal for the natural case) and the results, for unbounded $\eta$, are hence in these cases very much in line with the analytical solution for infinite slopes. As a failure mode consisting of a toe circle resembles more the planar surface used in the analytical equation.

In this study, the initialization of the stress situation was quite simplified using a constant $\nu = 0.3$ for all cases ($\rightarrow K_0 = 0.43$ for horizontal terrain). More analysis should be conducted with a distribution for $\nu$, or by using a more advanced anisotropic, effective stress based, soil model, where a parameter like $K_0^{\text{NC}}$ could be varied. Also, analysis with different values for $\eta$, $\zeta$ and $\xi$ should be done to cover other types of slope generation, e.g. slopes that are eroded to their current configuration. In such cases shear strength anisotropy could be more in line with the gravitational direction and not in line with current stress situation, i.e. $\eta > \zeta = \xi$. Analyses where the slope width is increased should also be done. It is expected that by increasing the slope width the failure mechanism for the isotropic material will be of bearing capacity type (slope) for even steeper slopes than $b^{1/3} = 0.1$.

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