Study on Stationarity of Random Load Spectrum Based on the Special Road

Huawen Yan\textsuperscript{1}, Weigong Zhang\textsuperscript{1,2} and Dong Wang\textsuperscript{1}

\textsuperscript{1}School of Instrument Science and Engineering, Southeast University, Nanjing 210096, China
\textsuperscript{2}zhangwg@seu.edu.cn

Abstract. In the special road quality assessment method, there is a method using a wheel force sensor, the essence of this method is collecting the load spectrum of the car to reflect the quality of road. According to the definition of stochastic process, it is easy to find that the load spectrum is a stochastic process. However, the analysis method and application range of different random processes are very different, especially in engineering practice, which will directly affect the design and development of the experiment. Therefore, determining the type of a random process has important practical significance. Based on the analysis of the digital characteristics of road load spectrum, this paper determines that the road load spectrum in this experiment belongs to a stationary stochastic process, paving the way for the follow-up modeling and feature extraction of the special road.

1. Introduction
In probability theory and related fields, a stochastic or random process is a mathematical object usually defined as a collection of random variables. Stochastic processes are widely used as mathematical models of systems and phenomena that appear to vary in a random manner. They have applications in many technology and engineering fields such as image processing, signal processing, information theory and computer science. Historically, the random variables were associated with or indexed by a set of numbers, usually viewed as points in time, giving the interpretation of a stochastic process representing numerical values of some system randomly changing over time, such as growth of a bacterial population, but the system studied this paper is a stochastic process representing numerical values of road loads randomly changing over space. On a certain length of road, the loads applied to the car by the road are random variables based on space, and these loads in each replicate experiment are not the same, so that it is not one or a few random variables, but a family of infinite number of random variables\cite{1}. Therefore, it can be determined that the load spectrum is a typical stochastic process according to the definition of a stochastic process. Based on their properties, stochastic processes can be divided into various categories, which include Markov processes, Wiener processes, Poisson processes, renewal processes and stationary processes. The study of stochastic processes requires mathematical knowledge and techniques from probability, calculus, linear algebra, and topology as well as branches of mathematical analysis such as real analysis, and Fourier analysis, while different stochastic process requires different mathematical knowledge and techniques, so in order to determine the kind of road load spectrum, this paper was divided into three steps. In short, first, put forward the framework of theoretical analysis; second, design an experiment according to the theory; third, make a validation of the theory using the data collected by the experiment.
2. Theoretical analysis

Despite limited distribution function family of stochastic processes can characterize the statistical properties of stochastic processes. However, it is often difficult and even impossible to determine the distribution function family of the stochastic process in practical application, so it is necessary to introduce the basic digital features of the random process as the numerical feature of the random variable is introduced. These digital features not only can characterize the important characteristics of the stochastic process, but also can facilitate the operation and the actual measurement. Therefore, four commonly used numerical features, which are mean function, variance function, correlation function, covariance function, are introduced to study the road load spectrum in this paper.

2.1 Mean Function and Variance Function

\[ \mu(t) = E[X(t)] = \int_{-\infty}^{+\infty} x f(x; t)dx \]  
\[ \sigma^2 = D[X(t)] = E[(X(t) - \mu(t))^2] = \int_{-\infty}^{+\infty} [x - \mu(t)]^2 f(x; t)dx \]

2.2 Correlation Function and Covariance Function

The mean and variance are digital features that characterize the statistical properties of the process at each isolated time, and the numerical features that can reflect the statistical characteristics of the process at different times is required, since new digital features are required to describe the stochastic process [2].

\[ \rho(t_1, t_2) = E[X(t_1)X(t_2)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1x_2 f(x_1, x_2; t_1, t_2)dx_1dx_2 \]  
\[ \gamma(t_1, t_2) = \text{Cov}(X(t_1), X(t_2)) = E[(X(t_1) - \mu(t_1))(X(t_2) - \mu(t_2))] \]

From the statistical theory we can see that the covariance function and the variance can be expressed by the mean and the autocorrelation function, so the mean and autocorrelation functions are more important digital features. While, when the time parameter t is discrete, X(t) is the time series, and the load data studied in this paper belong to discrete time series.

2.3 Stationary Condition

The process \{X_t, t \in Z\} is said to be stationary if it satisfies the following three properties [3]:

1) \( E|X_t|^2 < \infty \) for all \( t \in Z \),
2) \( EX_t = m \) for all \( t \in Z \),
3) \( \gamma_X(t_1, t_2) = \gamma_X(t_1 + \tau, t_2 + \tau) \) for all \( t_1, t_2, \tau \in Z \).

An important consequence of property 3) is that the variance of the tth component \( X_t \) of the stationary process \{X_t\} is a constant independent of t because \( \gamma_X(t_1, t_2) = \gamma_X(t_1 - t_2, 0) \). Accordingly, defining covariance function and correlation function of stationary process to be a unary function for convenience

\[ \rho(h) = \frac{\gamma_X(h)}{\gamma_X(0)} \quad |h| \leq n, \text{ where } n \text{ represents the number of samples} \]

3. Arrange the experiment

While a vehicle is on the road, because of the effect of the ground, some forces are applied to the wheel, namely the longitudinal force \( F_x \), the lateral force \( F_y \), the vertical force \( F_z \), as shown in figure 1A. It is generally known that the role of force is mutual, so the wheel force is equivalent to the road load. The load can be collected by the WFT system, and the block diagram is shown in figure 1B. The hardware components of the system include elastomer, collection-module, encoder, transmission-module, central collection box, GPS module and laptop.
The WFT system is a vehicle-to-road coupling system, so the measured road load is related to multiple factors, which include road type, vehicle type, speed, tire pressure and so on. Road load of different road is not the same, which is the reason for setting various special roads. Different vehicles have different suspension types and weights, and their loads on the road are naturally different. In addition, the impact of speed and tire pressure on road loads is also evident. The vehicle used in this experiment is the military active off-road vehicle, which total weight is equal to curb weight plus weight of testers and equipment for the 3.7t, and the tire pressure was adjusted to 2.7Kg/cm². The sensors in the WFT system are mounted on the left and right wheels of the front axle. The speed of the vehicle is maintained at 40km/h, and GPS module is responsible for vehicle speed compensation. The special road used in the experiment was the cobblestone road, which is shown in figure. 2A, and a total of 3000 groups sample loads data were collected. One group of the original loads data collected by the left WFT are shown in figure. 2B to figure. 2D. Owning to the right WFT is similar to the left one, so the data curve of the left WFT is drawn alone for a brief description.
4. Experimental verification

It can be seen from the figure 2 that each group of load data contains 1000 acquisition points. The range of $t$ can be obtained by combining the experiment and the stationary conditions, which is $1 \leq t \leq 1000$. The first item of the stationary conditions is $E[X_t]^2 < \infty$, therefore, if the inequality $\sum_{t=1}^{1000} E[X_t]^2 < \infty$ is established, $E[X_t]^2 < \infty$ naturally holds. $|X_t|^2$ of $F_x$ is shown in figure 3, and it is easy to calculate $E[X_t]^2 = 1.205 \times 10^{10}$, although it is large, but it is still a limited number. Therefore, the $\{X(t)\}$ of $F_x$ satisfy the first item of the stationary conditions, Similarly available $F_y$ and $F_z$ also satisfy the first item.

The second item of the stationary conditions is $E[X] = \mu$, which means that the mean value of each acquisition point must be constant. Similarly, $F_x$ was taken as an example to explain. $E[X] = [EX_1, EX_2, \ldots, EX_{100}]$ is shown in figure 4, and the fluctuation range of $E[X]$ is equal to its maximum value of $Ex$ minus its minimum value, which is 213.9N and is within the precision range of WFT System. It should be noted that the accuracy of the vertical force of the WFT system is 1% of its full scale [5], and the full scale of this WFT system is 40KN, so the error within 400N is acceptable and $E[X]$, can be regarded as a constant.
Fig 4. EX_t of Fx

The third condition of the stochastic process implies that the covariance function between any two moments of the stochastic process is independent of the time points and is only related to the distance between them. The sample autocorrelation function of \(X_t = (X_1, X_2, \ldots, X_{1000})\) is defined as follow.

\[
\gamma(h) = n^{-1} \sum_{j=1}^{n-h} (x_{j+h} - x') (x_j - x'), \quad 0 \leq h \leq n, \quad n = 1000.
\]

Fig 5. Sample correlation (coefficient) function \(\rho\), \(0 \leq h \leq 200\)

According to the above formula is very convenient to calculate the sample autocorrelation coefficient (\(\rho\)) which is shown in figure 5 with \(h\) from 1 to 200. It is obviously to know that the value of \(\rho\) is less than 0.2 except when \(h\) equals 0 from the curve in figure 5, which means it is enough to get a conclusion that any two moments of \(X_t\) has no obviously relationship.

5. Summary

Although only making longitudinal force \(F_x\) as an example for a brief description, while it also applies to the lateral force \(F_y\) and the vertical force \(F_z\). In this paper, the theoretical analysis and experimental verification show that the measured road load spectrum belongs to the stationary random sequence. So the road feature extraction and road type identification has a valid theoretical basis.

References
[1] Peter J. Brockwell, Richard A. Davis. Time series: Theory and Methods. Springer. 2011, p8 – p23.

[2] Duan Huming, et al. Feature parameter extraction and statistical analysis of road surface measurement data. Journal of Vibration and Shock, 2013, 32(1): p30-34.

[3] Yong-chen, W.G, The methods of power spectrum estimation based on measured pavement. 2011, IEEE. p. 2787 - 2790.

[4] Hong Yongfu. Automobile development engineering. Machinery Industry Press. 2014, p115-160.

[5] Lin, G., et al., A self-decoupled three-axis force sensor for measuring the wheel force. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 2014, 228(3): p. 319-334.