INSTANTON INDUCED PHENOMENA IN THE EFFECTIVE
STANDARD MODEL

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It is shown that in the Effective Standard Model new phenomena can arise due
to the presence of small instanton configurations. The chief result is that under
certain conditions new hidden coupling constants could exist in the model. In the
Electroweak sector that might result in the possibility of observing $B + L$ violating
processes due to a highly non perturbative contribution to the *hol grail function*.
The same phenomenon might occur in the $QCD$ sector of the theory and could be
observed in the DIS experiments at HERA.

1 Introduction

According to the Principle of Renormalizability all the interaction terms whose
coupling constant has negative energy dimension should be discarded from
our fundamental Lagrangians because they give rise to non renormalizable
theories.

A better insight came after the work of Wilson who has pioneered a real
change in our way of conceiving renormalization. It is no more simply regarded
as a welcome technical device to get rid of the unwanted divergencies in our
physical quantities but rather as the necessary manifestation of the difference
in the description of Physical phenomena at different scales. The energy cutoff
energy also acquires a physical meaning, it represents the maximal scale at
which the theory itself makes sense.

Non renormalizability generally means that it is not possible in the framework of perturba-
tion theory to absorb all the infinities appearing in the computation of the physical quantities
simply adjusting a finite number of bare parameters. Recently this definition of renormal-
izability has been enlarged to include also theories with non renormalizable terms.

What is important for the purpose of our contribution is that even under this larger definition of
renormalizability the analysis is still performed within perturbation theory.
In this novel perspective the non renormalizable operators acquire a different ontological status. Consider a Langrangian containing such operators. It may be proven that at every order in perturbation theory the contributions to any physical quantity coming from those terms are always suppressed by negative powers of the cutoff. As far as the energy scale at which the process under consideration occurs is very far from the cutoff scale, these contributions are practically negligible. The technical notion of non renormalizable operator is replaced by the more physical one of irrelevant operator. The renormalizable operators similarly are called relevant operators.

The alert reader has certainly remarked that we have explicitly stressed that the classification of operators mentioned above has been done within the framework of perturbation theory. We mention now two situations where such a classification might suffer from serious changes.

i) In order to study the running of the coupling constants of our model we generally first identify a fixed point and then linearize the evolution equations. Consequently this classification is valid until we reach the edge of the linearization region. Nothing can be said about the role of the irrelevant operators outside of this region. In this volume an explicit example is given of such a violation of universality behaviour together with the possible physical consequences of such a phenomenon.

ii) Another possible source of violation of the universality behaviour is related to the presence of non trivial saddle points. In the path integral language the perturbative analysis mentioned above corresponds to a semiclassical approximation in the presence of trivial saddle points. If the action under investigation possess non trivial saddle points, due to this very non perturbative effect the above classification might fail. The purpose of this contribution is to give an explicit example of such a failure together with the possible interesting physical consequences associated with it.

In Sec. 2 we review and partially extend the results already presented on the analysis of an effective $SU(2)$ model. In Sec. 3 we discuss the instanton induced effects in the light of our results.

2 $SU(2)$ in the presence of irrelevant operators

We believe that the strongest argument which suggests that these kind of models really deserve our attention comes once again from the Wilsonian approach to field theory. Any model but the Theory of Everything is the low energy approximation of some more fundamental one. In the process of eliminating higher energy degrees of freedom, irrelevant terms appear in our effective

\footnote{To be precise the renormalizable operators are the relevant and the marginal ones.}
Lagrangians.

Let’s consider an SU(2) Lagrangian containing all sort of irrelevant terms as for instance higher power or higher derivative terms

\[ L_{\text{eff}} = -\frac{1}{4g^2}G_{\mu\nu}^a \left( 1 + \sum_n \frac{c_n}{\Lambda^{2n}} (D^2)^n + \sum_m \frac{d_m}{\Lambda^{2m}} (G_{\alpha\beta}^b g^{b\alpha\beta})^m + \cdots \right) G^{a\mu\nu}. \]  

(1)

\( \Lambda \) is the cutoff of the theory and \( D \) is the covariant derivative.

Due to the topology of the configuration space the path integral defining the theory has saddle points which become the self dual instantons for \( c_n = d_m = \cdots = 0 \). We now introduce the collective symbol \( c \) to indicate the whole set of irrelevant coupling constants and denote such a distorted instanton, \( A_{\mu}(c,\rho)(x) \). \( \rho \) is the scale parameter of the instanton configuration, which is introduced in a somehow arbitrary fashion by requiring that the distorted instanton be a self dual instanton with size \( \rho \) for \( c=0 \).

The tree level action for this configuration is of the form

\[ S_{\text{inst}}(\rho) = \frac{8\pi^2}{g^2} \left( 1 - f(c, \rho \Lambda) \right) \]  

(2)

where \( \frac{8\pi^2}{g^2} \) is the scale independent usual instanton action. The dimensionless function \( f \) is a complicated function of the whole set of irrelevant coupling constants \( c \) as well as of the dimensionless product \( \rho \Lambda \). Depending on the actual value of the coupling constants \( c \) the function \( S_{\text{inst}}(\rho) \) as a function of \( \rho \) may or may not have a minimum for a value \( \bar{\rho} \) of \( \rho \) which, for dimensional reasons is \( \bar{\rho} \sim \frac{1}{\Lambda} \). When there is no minimum no surprises occur in the model. Let’s then analyze the case when such a minimum exists. In particular the value of \( S_{\text{inst}}(\bar{\rho}) \) at the minimum can be positive or negative. When \( S(\bar{\rho}) \) is negative new interesting phenomena occur. Here we limit our analysis to the case \( S_{\text{inst}}(\bar{\rho}) > 0 \).

Let’s call \( h \) the value of \( f \) at the minimum. We have

\[ S_{\text{inst}}(\bar{\rho}) = \frac{8\pi^2}{g^2} \left( 1 - h \right) \]  

(3)

Computing now the contribution to the partition function coming from the instanton sector, \( Z_1 \), normalized as usual with the contribution from the zero winding number sector, \( Z_0 \), we get

\[ \left( \frac{Z_1}{Z_0} \right) = C \frac{V}{g^8} \int_0^\infty \frac{\rho^4}{\rho^8} e^{-\frac{\rho^4}{8\pi^2} \left( 1 - f(c,g,\rho \Lambda) \right)} D(\rho \Lambda) \]  

(4)
where \( D \) stands for the contribution of the fluctuation determinant, \( V \) is the quantization volume and \( C \) is a numerical constant whose value is of no importance for us. The infrared catastrophe is ignored by the introduction of the infrared cut-off, \( \mu \) which may be regarded as the \( \Lambda \) parameter of the theory (say \( \Lambda_{QCD} \) in \( QCD \)).

The contribution of instantons as the function of the size parameter has a well pronounced peak at \( \rho \approx \bar{\rho} \). Another important region is in the infrared where the loop corrections increase. In order to separate the contribution of the large, i.e. cut-off independent instantons from that of the stable saddle point in the vicinity of the cut-off we split the scale integration into two parts by the help of a scale parameter \( m \), \( \Lambda_{QCD} \ll m < \Lambda \),

\[
\int_0^{\mu_0} d\rho \cdots = \int_0^{\mu_0} d\rho \cdots + \int_{\mu_0}^{\mu_1} d\rho \cdots .
\]  

The first and the second integral will be referred as the contribution of the mini and the large instantons and denoted as \( (Z_1/Z_0)_L \) and \( (Z_1/Z_0)_M \) respectively. The final result is

\[
\left( \frac{Z_1}{Z_0} \right)_L = C \frac{V}{g^4} \int_0^{\mu_0} d\rho \frac{\rho^5 e^{-\frac{8\pi^2}{g^2}(1-f)}}{\rho^5 \rho^{\Lambda}} \sim \frac{3}{10} C \frac{V A^4}{g^4} e^{-\frac{8\pi^2}{g^2}} \left( \frac{\Lambda}{\mu} \right)^{\frac{10}{3}}
\]

\[
\left( \frac{Z_1}{Z_0} \right)_M = C \frac{V}{g^4} \int_0^{\mu_0} d\rho \frac{\rho^5 e^{-\frac{8\pi^2}{g^2}(1-f)}}{\rho^5 \rho^{\Lambda}} \sim C \frac{V A^4}{g^4} e^{-\frac{8\pi^2}{g^2}(1-h)} \rho^{\bar{\rho} \Lambda} \sim \frac{C V}{g^4} e^{-\frac{8\pi^2}{g^2} \Lambda \mu} \left( \frac{\Lambda}{\mu} \right)^{\frac{10}{3}} \left( \frac{11}{5} h \right)
\]  

We now compare the contributions coming from these two different regions by computing the ratio

\[
R = \frac{(Z_1/Z_0)_L}{(Z_1/Z_0)_M} \sim \left( \frac{\Lambda}{\mu} \right)^{\frac{10}{3} \left( 5 - 11h \right)}
\]  

The result here is that for \( h > \frac{5}{11} \) the mini-instantons dominate the path integral, which is otherwise dominated by the large instantons.

What are the consequences of such a result?

### 3 Coupling constants flow, a new relevant parameter

Suppose that we are in the case when the large instantons dominate the path integral (i.e. \( h < \frac{5}{11} \)) in order to extract the flow of the coupling constant \( g \) we demand as usual the cutoff independence of \( (Z_1/Z_0)_L \), i.e. we require
that \( \Lambda \frac{d}{d\Lambda}(Z_1/Z_0)_{L} = 0 \). We immediately obtain

\[
\Lambda \frac{d}{d\Lambda} \left( \Lambda^{2g} e^{-\frac{4\pi^2 g^2}{\pi}} \right) = 0 \quad \rightarrow \quad \beta(g) = -\frac{11}{24\pi^2} g^3 \quad (8)
\]

i.e. we get the same result of the perturbative analysis in the zero winding number sector. This is the well known t’Hooft result. It is by no means surprising that we get the same result as in the perturbative constant background, since the ultraviolet structure is the same on the flat or the large instanton background. By the way the large instantons are the only ones considered in the standard analysis.

If we now apply the same strategy to extract the beta function for \( g \) in the case when the mini-instantons dominate the path integral, i.e. we require that \( \Lambda \frac{d}{d\Lambda}(Z_1/Z_0)_{M} = 0 \), we find

\[
\Lambda \frac{d}{d\Lambda} \left( \Lambda^4 e^{-\frac{8\pi^2}{\pi^2(1-h)}} \right) = 0 \quad \rightarrow \quad \beta(g) = -\frac{1}{4\pi^2(1-h)} g^3 \quad (9)
\]

This is a very surprising and disturbing result. It seems we have run into a paradox. In fact in a theory as SU(2) we should distinguish between two different classes of observables. Those with a topological origin, as the topological susceptibility, mass of the \( \eta' \), certain massless Green functions, and observables with non topological origin, as for instance the Wilson loop or the cross section for an ordinary process. The renormalization of those observables insensitive to the topology, whatever is the value of \( h \), requires the beta function for \( g \) dictated by the perturbative analysis. On the other hand our result shows that the observables with topological origin are sensitive to the actual value of \( h \). For \( h > 5/11 \) it seems that their renormalization requires a different flow for \( g \), given by eq. (9). Is there any possibility to reconcile these results?

The very origin of this apparent contradiction lies in our conservative attitude to require a fine tuning only for the parameter \( g \) which we consider to be the only relevant parameter of the theory. For the parameter \( h \), which is the result of an intricate combination of all the irrelevant parameters of our effective theory, we do not require any fine tuning, according with the statement that it is irrelevant. We shouldn’t forget however that its appearance is related to the presence of non trivial saddle points in our model. As we have already mentioned in the Introduction there is no guarantee that the perturbative classification of the operators should still be valid in this case. Actually we may easily convince ourselves that this is indeed not the case. Due to the presence of such irrelevant operators, with dimensionful coupling constants
containing negative cutoff powers, the theory requires a sort of renormalization at the tree level, a phenomenon which cannot be accounted for by the power counting theorem, which is based on the analysis of the loop corrections only.

The resolution of this apparent paradox is very simple. In eq. (9) we have actually another parameter, \( h \), which can be fine tuned. We can now keep the ratio \((Z_1/Z_0)_M\) finite and maintain the usual flow for \( g \), fine tuning appropriately the coupling constant \( h \). In other words, the two conditions

\[
\Lambda \frac{d}{d\Lambda} \left( \Lambda^4 e^{-\frac{8\pi^2}{g^2(1-h)}} \right) = 0 \quad \text{and} \quad \frac{8\pi^2}{g^2} = \frac{22}{3} \ln \frac{\Lambda}{\Lambda_0},
\]

where \( \Lambda_0 \) is the \( \Lambda \)-parameter of the theory, give

\[
h(\Lambda) = \frac{5}{11} + \frac{\kappa}{\ln(\Lambda/\Lambda_0)}. \tag{11}
\]

\( h \) is actually a relevant parameter of the theory. This is the very crucial and new result of our analysis. The theory posses two different phases. In the mini instantons dominated phase there is a new hidden relevant coupling constant and its flow is given by eq. (11). Note that the actual value of this new coupling constant at a given energy scale is a new external input which can be determined only via experiments, precisely the same way as the electron mass and charge in QED are determined by the experiments and are the external input of the theory.

We may now rewrite the expression for \((Z_1/Z_0)_M\) by the help of a physical finite scale \( \mu \) in the following manner

\[
\Lambda^4 e^{-\frac{8\pi^2}{g^2(1-h)}(1-h(\Lambda))] = \mu^4 e^{-\frac{8\pi^2}{g^2(\mu)}(1-h(\mu))} = \mu^4 e^{-\frac{8\pi^2}{g^2(\mu)}(1-\frac{5}{11} + \frac{\kappa}{\ln(\mu/\Lambda_0)})}. \tag{12}
\]

This last expression will be helpful to understand the physical implications of our result.

4 Instanton induced effects

Since the pioneering work of t’Hooft, it is well known that \( B+L \) is not strictly conserved in the Standard Model. Certain massless Green functions which vanish in perturbation theory, get a non perturbative contribution from the instanton sector. This effect is due to the anomalously non conserved current, but the small instanton suppression factor \( \exp\left(-\frac{4\pi}{\alpha_w}\right) \) makes it unobservable. At the beginning of the nineties there was a certain excitement due to the discovery of an enhancement factor for the \( B + L \) violating cross sections. It
was found that for those $B + L$ violating processes with high multiplicity of $W, Z$, and/or $H$ particles in the final state, the cross section get enhanced w.r.t. the t’Hooft suppression factor, growing with the center of mass energy, $s$, as

$$\sigma \sim \exp \left( -\frac{4\pi}{\alpha_w} \left( 1 - \text{const} \times \left( \frac{s}{M_{sph}^2} \right)^\frac{2}{3} \right) \right).$$

(13)

$M_s$ is the sphaleron mass. More detailed studies have subsequently shown that the approximations involved in those original computations lose their validity at energies far below the scale $M_s$ where the whole argument of the exponential, the so called holy-grail function, would become small. The final agreement was that at best the suppression exponent could reach a value of one half the original t’Hooft factor and then the effect would be still far beyond any possibility of observation.

Our result gives a novel mechanism which allows the holy-grail function to receive an O(1) contribution (see the last expression of eq. (12)) with an opposite sign with respect to the t’Hooft suppression factor, independently of any enhancement due to the high multiplicity of the final state. It opens again the door for the possible observation of $B + L$ violating processes within the framework of the Standard Model at accessible energies.

There is also another interesting possible application of the results presented above. During the last years, even though the scientific community has somehow given up the hope to observe instanton induced $B + L$ violating processes in the Standard Model, there has been an intense theoretical activity concerning the possible impact of instanton induced processes in QCD, particularly in DIS processes. The basic mechanism is actually the same, the only difference being that it has to be applied to $SU(3)$ rather then to $SU(2)$. This field is actually under active investigation and a dedicated working group exists in connection with the HERA experiments. The hope is to finally observe the impact of the presence of instantons in the cross sections for these processes. These experiments may turn out to be very important from the point of view of our result. In addition to the usual well studied contribution to the holy grail function our result suggests that an additional contribution might come from the presence of an it hidden coupling constant in QCD.

The comparison of the experimental cross sections with the theoretical predictions might eventually allow to measure the new scale introduced in the theory through the presence of the constant $\kappa$ in eq. (11).

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