Nonclassical degree of states of single and bipartite systems

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Abstract

We consider experimental routes to determine the nonclassical degree of states of a field mode. We adopt a distance-type criterium based on the Hilbert-Schmidt metric to quantify the nonclassicality. The concept of nonclassical degree is extended for states of bipartite systems, allowing us to discuss a possible connection between nonclassicality and entanglement measures.

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There are several interesting states of the quantized electromagnetic field studied nowadays, either concerning with their properties or their creation in laboratories. They are considered as classical states when their Glauber-Sudarshan P-functions are regular, non-negative; otherwise, when one of such characteristics are not attained, they are said to be nonclassical [1]. Coherent states and mixed thermal states are representative examples of classical states. According to a theorem by Hillery [2], every pure field state which is not coherent, is nonclassical. This result leads us to an endless number of nonclassical states in quantum optics. These states exhibit quantum effects, the most traditional of them being: (i) antibunching [3], (ii) sub-Poissonian statistics [4] and (iii) squeezing [5]. There are even other examples of field states which do not exhibit these quantum effects, but their field quantization are required to explain some experimental results. One such situation occurs for the time evolution of atomic inversion when the atom interacts with a field inside a cavity: the collapse-revival effect [6] can be explained only when the field is quantized, even if it is in the (most classical) pure coherent state.

Traditional examples of nonclassical states of the radiation field are: (i) the well known number (Fock) state $|n\rangle$, exhibiting antibunching and maximum sub-Poissonian; (ii) the squeezed coherent state $|z,\alpha\rangle$, which may be sub-Poissonian or not, depending on the type of squeezing effect; (iii) the phase-state [7], which is nonclassical according to the Hillery’s theorem [2], but it does not exhibit any known quantum effect, a result which stimulated the investigation about the nonclassical depth of this state [8].

However, quantum properties do not occur simultaneously for all nonclassical states. For example, all squeezed-vacuum states are super-Poissonian while squeezed-coherent states may be sub-Poissonian. Also, all number states show maximum sub-Poissonian statistics but exhibit no squeezing and their antibunching effect diminishes when N increases [9].

The above considerations lead to the appropriated question about how much nonclassical a quantum state is. Various criteria have appeared in the literature to quantify the nonclassical character of a given state. One such trial was introduced by Mandel [10], defining the parameter $q = (\Delta \hat{n}^2 - \langle \hat{n} \rangle) / \langle \hat{n} \rangle$ to quantify the departure of the photon-number distribution of the state from the Poissonian statistics. However, this parameter is not able to contemplate other nonclassical properties. A proposal by Hillery [11] introduced a nonclassical distance of a state as the trace-norm of the difference between the density operator of the state and that of the nearest classical state. However, practical determination of
such distance is rather difficult. Following this trend, other more operational measures of nonclassicality were introduced: one by Dodonov et al. [12], via a Hilbert-Schmidt distance between density operators; another by Marian et al. [13], via the Bures-Uhlmann definition of distance between states. Measure of nonclassical properties [14] and observable criterion distinguishing nonclassical states were also considered recently [15].

A distinct route, based on the Cahill-Glauber representation [16], was early proposed by Lee [17], introducing the R-function as a real $\tau$-parametrized Gaussian convolution of the P-function. With this representation, Lee defined the nonclassical depth of a state as the minimum value of $\tau$ ($\tau_m$) yielding to a regular, non-negative, R-function acceptable as a classical distribution function. Later on, Lutkenhaus and Barnett [18] considered a similar phase-space measure of nonclassicality. Very recently [19], the phase-space and the distance-type measures of nonclassicality were compared; it was shown in [19] that the distance-type measure is sensitive to (non-Gaussian) superposition states (while $\tau_m$ is maximum for such states) and, also, it results equivalent to the phase-space measure introduced by Lee [17] for Gaussian pure states. So, we can consider the distance-type measure as an extension of that by Lee and in this report we will employ this criterion for the measure of the nonclassicality of a field state.

At this point, a pertinent question emerges: Is it possible to determine experimentally the nonclassical degree of a field state? To answer this question, we will use a distance-type criterion to characterize the nonclassicality of field states, and we shall restrict our analysis to pure states. Metrics can be introduced in the Hilbert space as functions of the fidelity, which corresponds to the quantum-mechanical transition probability between two pure states, $F(|\Phi\rangle, |\Psi\rangle) = |\langle \Phi | \Psi \rangle|^2$. As examples, we mention the Bures-Uhlmann and the Hilbert-Schmidt distances between two pure states ($|\Phi\rangle$ and $|\Psi\rangle$), which are given by

$$d^{BU}(|\Phi\rangle, |\Psi\rangle) = \left(2 - 2 \sqrt{F(|\Phi\rangle, |\Psi\rangle)}\right)^{1/2}, \quad (1)$$
$$d^{HS}(|\Phi\rangle, |\Psi\rangle) = \sqrt{2 - 2 F(|\Phi\rangle, |\Psi\rangle)} . \quad (2)$$

A distance-type measure of nonclassicality, for pure states, can be defined as the minimum of any monotonically increasing function of the distance between the state and an arbitrary coherent state of the field mode. On these grounds, following [19], we define the nonclassical degree of the pure state $|\Psi\rangle$ as the minimum value of one half of the squared Hilbert-Schmidt...
distance between $|\Psi\rangle$ and an arbitrary coherent state $|\beta\rangle$, that is,

$$D_{|\Psi\rangle} = \min_{\{\beta\}} \left[ 1 - |\langle \beta | \Psi \rangle|^2 \right] = 1 - \pi \max_{\{\beta \in \mathbb{C}\}} Q_{|\Psi\rangle} (\beta),$$

(3)

where $Q_{|\psi\rangle} (\beta)$ is the Husimi Q-function corresponding to the state $|\Psi\rangle$. In other words, the quantity $D_{|\Psi\rangle}$ defined in (3) will be used to determine the degree of nonclassicality of the pure state $|\Psi\rangle$, coherent states being taken as the most classical between the quantum states of the field mode.

This measure of nonclassicality is slightly distinct from those in [15,16]; besides being simpler, it also makes ease the comparison with the nonclassical depth $\tau_m$ used in [17], as shown in [13]. Note that $D_{|\alpha\rangle} = 0$ for coherent states, since $\max_{\{\beta \in \mathbb{C}\}} Q_{|\alpha\rangle} (\beta) = \pi^{-1}$, as it should. On the other hand, for number states one obtains $D_{|n\rangle} = 1 - n^ne^{-n}/n!$ showing distinct nonclassical degree for different number states, the upper bound ($D = 1$) being reached in the limit $n \to \infty$. This result differs from that emerging in the context of phase-space measure [17], where $\tau_m = 1$ for all number states $|n\rangle$, no matter the value of $n$. At first glance, it may seem strange that a number state with $n$ large is more nonclassical, in the sense of the distance measure $D_{|n\rangle}$, than a state with smaller $n$, say $|1\rangle$. However, for a number state $|n\rangle$, the expectation value of the electromagnetic field vanishes identically while its energy is proportional to $n$; clearly, such a state is more distant from coherent states as larger $n$ is. Another distinction between these two criteria comes from the fact that, while the nonclassical depth $\tau_m$ introduced in [17] arises from the minimum of the R-function, the nonclassical degree $D_{|\Psi\rangle}$ of [19], for pure states, arises from the maximum value of the Husimi Q-function.

According to the Eq.(3), the experimental determination of $D_{|\Psi\rangle}$ is obtained via the Husimi Q-function. Then the question posed above can be transposed to: “How determining experimentally the Q-function?” In a previous paper [20], we have proposed an experimental arrangement to measure the Q-function. The strategy follows the projection synthesis scheme proposed by Pegg-Barnett [21]. Accordingly, we can write $Q_{|\psi\rangle} (\beta) = \pi^{-1} \langle \beta | \hat{\rho} | \beta \rangle$, where $\hat{\rho} = |\Psi\rangle \langle \Psi|$ is the density operator describing the field whose nonclassical degree is to be determined from Eq.(3), and $|\beta\rangle$ stands for a coherent state. It was shown in [20] that $Q_{|\psi\rangle} (\beta) = \text{Tr}(\hat{\rho} \hat{\Pi})$, where $\hat{\Pi} = K |\beta\rangle \langle \beta|$, $K$ standing for a constant. When we choose $\hat{\Pi} = K |\theta\rangle \langle \theta|$, with $|\theta\rangle$ being the phase-state, the method in [21] allows one to determine the phase-distribution $P(\theta)$. Both cases require specific states used as auxiliary reference
fields, the \textit{reciprocal binomial state} in \cite{21} and the \textit{complementary-coherent state} in \cite{20}. Proposals for generation these two states in travelling modes were recently suggested \cite{22, 23}. So, by combining the experimental result for the Q-function with its connection with the nonclassical degree $D_{|\Psi\rangle}$ given in Eq. (3), one obtains the experimental value of the quantity $D_{|\Psi\rangle}$.

It is worth emphasizing that the above method concerns with states of travelling modes of the quantized light field. What about experimental method concerning with states of trapped fields inside a high-Q cavity? In this case, there is an experimental arrangement proposed by Lutterbach and Davidovich \cite{24} to determine the Wigner W-function describing a stationary field inside a cavity. However, Q- and W-functions, which are Gaussian convolutions of the Glauber-Sudarshan P-function, are related by

$$Q(\beta) = \frac{2}{\pi} \int d^2 \alpha W(\alpha) e^{-2|\alpha-\beta|^2},$$

or, conversely,

$$W(\alpha) = \exp\left(-\frac{1}{2} \frac{\partial}{\partial \alpha} \frac{\partial}{\partial \alpha^*}\right) Q(\alpha).$$

This equation, therefore, allows one to get the Q-function via a Gaussian convolution of the Wigner function. Alternatively, one can directly reconstruct the Q-function of an initial state in a lossy cavity, as proposed in \cite{25}.

We should mention that other experimental proposals, also furnishing the Wigner function, can be found in the literature for stationary \cite{26} and travelling \cite{27} fields.

So far we have restricted our discussion of the degree of nonclassicality to a single system, a mode of the electromagnetic field. A natural question is then what is the nonclassical degree of states of composite systems, for example two independent modes ($a$ and $b$) of the field. This will permit us to search for a possible correspondence between the degrees of nonclassicality and entanglement of field states.

To extend the distance-type measure of nonclassicality to bipartite systems one has to choose a set of states as reference, such states being considered as the most classical ones. A possibility is to take the set of product states \{\ket{\alpha, \beta} = \ket{\alpha}_a \otimes \ket{\beta}_b\}, \ket{\alpha}_a and \ket{\beta}_b being coherent states of modes $a$ and $b$ respectively, as the set of the most classical among pure states of the bipartite system. In this way the definition (3) can be generalized to

$$D_{|\Psi\rangle_{ab}} = \min_{\{\alpha, \beta\}} \left[1 - |\langle \alpha, \beta | \Psi\rangle_{ab}|^2\right]$$

$$= 1 - \pi^2 \max_{\{\alpha, \beta \in C\}} Q_{|\Psi\rangle_{ab}}(\alpha, \beta),$$

where $Q_{|\Psi\rangle_{ab}}(\alpha, \beta) = \pi^{-2} \langle \alpha, \beta | \hat{\rho}_{ab} | \alpha, \beta \rangle$ stands for the Husimi function of the pure state $|\Psi\rangle_{ab}$. As naturally expected, $D_{|\alpha\rangle_a \otimes |\beta\rangle_b} = 0$. Notice that if $|\Psi\rangle_{ab}$ is a product state, that
is $|\Psi\rangle_{ab} = |\phi_1\rangle_a \otimes |\phi_2\rangle_b$, the Q-function is equal to the product of the Husimi functions corresponding to the states of the parts separately, $Q_{|\phi_1,\phi_2\rangle}(\alpha, \beta) = Q_{|\phi_1\rangle}(\alpha) Q_{|\phi_2\rangle}(\beta)$. Thus, for product states, the distance-type degree of nonclassicality can be expressed in terms of the nonclassical degrees of the factor states, that is

$$D_{|\phi_1\rangle_a \otimes |\phi_2\rangle_b} = D_{|\phi_1\rangle_a} + D_{|\phi_2\rangle_b} - D_{|\phi_1\rangle_a} D_{|\phi_2\rangle_b}.$$  \hfill(6)

In particular, one finds $D_{|0\rangle_a \otimes |n\rangle_b} = D_{|n\rangle_a \otimes |0\rangle_b} = D_{|n\rangle} = 1 - n^2 e^{-n}/(n!)$ and $D_{|n\rangle_a \otimes |n\rangle_b} = 1 - n^2 e^{-2n}/(n!)^2$.

Let us now consider the nonclassical degree for entangled states of a bipartite system. Entanglement [28] is widely believed to be the fundamental trace distinguishing quantum mechanics from classical mechanics, and it is crucial for aspects of quantum information [29], such as quantum teleportation, quantum cryptography and quantum computation. One should then expect to exist, somehow, a correspondence between entanglement and nonclassicality of states.

To address this question consider, for simplicity, the families of normalized states

$$|\Psi^{(\pm)}\rangle_{ab} = \sqrt{\xi} |0,1\rangle \pm \sqrt{1-\xi} |1,0\rangle$$ \hfill(7)

where $0 \leq \xi \leq 1$; these entangled states interpolate between the one-photon product states $|1,0\rangle$ and $|0,1\rangle$ of a bipartite system, which are the limits $\xi = 0$ and $\xi = 1$ respectively.

The degree of entanglement, defined as the von Neumann entropy $S_N(\hat{\rho}) = -\text{Tr}(\hat{\rho} \ln \hat{\rho})$ of either the reduced density matrix $\hat{\rho}_a = \text{Tr}_b(\hat{\rho}_{ab})$ or $\hat{\rho}_b$, for such states depend on $\xi$ and is given by

$$E(\xi) = -[\xi \ln \xi + (1-\xi) \ln (1-\xi)]$$ \hfill(8)

irrespective of the sign taken in the superposition $|\Psi^{(\pm)}\rangle_{ab}$. For $\xi = 1$ or $\xi = 0$, corresponding to the non-entangled states $|0,1\rangle$ and $|1,0\rangle$ respectively, $E$ naturally vanishes while the maximum value of $E(\xi)$ (namely $\ln 2$) is reached for $\xi = 1/2$, in which case these entangled states look like the singlet or to one of the triplet elements of the Bell’s basis of the subspace of the bipartite system $(\mathcal{H}_a \otimes \mathcal{H}_b)$ spanned by $\{|0\rangle_a |0\rangle_b , |0\rangle_a |1\rangle_b , |1\rangle_a |0\rangle_b , |1\rangle_a |1\rangle_b \}$. For the states $|\Psi^{(\pm)}\rangle_{ab}$, the Husimi Q-function is given by

$$Q_{|\Psi^{(\pm)}\rangle_{ab}}(\alpha, \beta; \xi) = \frac{1}{\pi} \exp \left[ -(|\alpha|^2 + |\beta|^2) \right] \times \left| \sqrt{\xi \beta} \pm \sqrt{1 - \xi \alpha} \right|^2,$$ \hfill(9)

irrespective of the sign taken in the superposition $|\Psi^{(\pm)}\rangle_{ab}$. For $\xi = 1$ or $\xi = 0$, corresponding to the non-entangled states $|0,1\rangle$ and $|1,0\rangle$ respectively, $E$ naturally vanishes while the maximum value of $E(\xi)$ (namely $\ln 2$) is reached for $\xi = 1/2$, in which case these entangled states look like the singlet or to one of the triplet elements of the Bell’s basis of the subspace of the bipartite system $(\mathcal{H}_a \otimes \mathcal{H}_b)$ spanned by $\{|0\rangle_a |0\rangle_b , |0\rangle_a |1\rangle_b , |1\rangle_a |0\rangle_b , |1\rangle_a |1\rangle_b \}$. For the states $|\Psi^{(\pm)}\rangle_{ab}$, the Husimi Q-function is given by

$$Q_{|\Psi^{(\pm)}\rangle_{ab}}(\alpha, \beta; \xi) = \frac{1}{\pi} \exp \left[ -(|\alpha|^2 + |\beta|^2) \right] \times \left| \sqrt{\xi \beta} \pm \sqrt{1 - \xi \alpha} \right|^2,$$ \hfill(9)
and the nonclassical degree calculated by formula (5) is given by

$$D_{(\Psi)_{ab}}^{(\pm)}(\xi) = 1 - e^{-1}, \quad 0 \leq \xi \leq 1;$$  \hspace{1cm} (10)

that is, all members of both families of states (7) have degree of nonclassicality equal to the nonclassical degree of the states \(|0, 1\rangle\) and \(|1, 0\rangle\), irrespective of the weights in the superpositions. Therefore, the nonclassical degree of the states (7) is insensitive to their degrees of entanglement.

We now consider the families of states

$$|\Phi\rangle_{ab}^{(\pm)} = \sqrt{\xi}|0, 0\rangle \pm \sqrt{1 - \xi}|1, 1\rangle,$$  \hspace{1cm} (11)

with \(0 \leq \xi \leq 1\), which interpolate between the zero-photon state and a two-photon state (one in each mode) of the bipartite system. The degree of entanglement of states (11) is also given by Eq. (8) and, again, the maximum value of \(E\) occurs when \(\xi = 1/2\) for which states (11) become the other Bell’s states. For the states (11), one finds that the measure of nonclassicality (5) leads to the same results for both \((+)-\) and \((-)-\)superpositions, namely,

$$D_{|\Phi\rangle_{ab}}^{(\pm)}(\xi) = \begin{cases} 1 - (1 - \xi) \exp \left[ -2 \left(1 - \sqrt{\frac{\xi}{1 - \xi}}\right) \right], & \xi \leq \frac{1}{2} \\ 1 - \xi, & \xi \geq \frac{1}{2} \end{cases}$$  \hspace{1cm} (12)

for \(\xi\) within the interval \([0, 1]\). One sees that \(D_{|\Phi\rangle_{ab}}^{(\pm)}(\xi)\) decreases from \(1 - e^{-2}\) for \(\xi = 0\) (which corresponds to the state \(|1, 1\rangle\)) to 0 (the nonclassical degree of \(|0, 0\rangle\)) when \(\xi = 1\), thus interpolating monotonically between the nonclassical degrees of the constituting states of the superpositions (11); again, no correlation between the degrees of nonclassicality and entanglement is found. The degrees of nonclassicality and entanglement, for the families of states (7) and (11), are plotted in Fig. 1 as a function of \(\xi\).

The preceding analysis shows that entanglement is a quantum property of states of bipartite systems which, as occur for others nonclassical properties, does not alone determines the degree of nonclassicality of a given state. However, for the families of states considered, the distance-measure of nonclassicality introduced is correlated with the nature of the photon statistics, as indicated by the Mandel factor defined by

$$q_{\psi} = \frac{\langle \psi|\hat{n}^2|\psi\rangle}{\langle \psi|\hat{n}|\psi\rangle} - \langle \psi|\hat{n}|\psi\rangle - 1.$$  \hspace{1cm} (13)

In fact, in \(\mathcal{H}_a \otimes \mathcal{H}_b\), one defines \(\hat{n} = \hat{n}_a \otimes 1_b + 1_a \otimes \hat{n}_b\), and using the closure relation \(1_{ab} = \sum |i\rangle_a |j\rangle_b \langle j|_b \langle i|_a\), one can easily calculate \(q_{\psi}\) for products of number states, \(|n\rangle_a \otimes |m\rangle_b\),
FIG. 1: Degrees of nonclassicality and entanglement, plotted as a function of the parameter $\xi$, for the families of states $|\Phi\rangle^\pm$ and $|\Psi\rangle^\pm$ respectively; the dotted line stands for the (common) degree of entanglement of these states. Finding always $q = -1$; such states are among the most sub-Poissonian states of a bipartite system, like the number state $|n\rangle$ for a single mode. The $q$-factor for superpositions of number-product states can also naturally be evaluated and one finds, for the families $|\Phi\rangle^\pm$ and $|\Psi\rangle^\pm$ respectively,

$$ q_{|\Psi\rangle_{ab}}^\pm (\xi) = -1, \quad (14) $$

$$ q_{|\Phi\rangle_{ab}}^\pm (\xi) = 2\xi - 1; \quad (15) $$

one sees that $q_{|\Psi\rangle_{ab}}^\pm (\xi)$ take the minimum value allowed, independently of $\xi$, while $q_{|\Phi\rangle_{ab}}^\pm (\xi)$ vary from $-1$, for $\xi = 0$, to $1$, when $\xi = 1$. Notice, in addition, that the Bell states $|\Phi\rangle_{ab}^{(\pm)} (\xi = 1/2)$ are Poissonian. One concludes that the $q$-factor correlates well with the nonclassical degree for the families of states $|\Phi\rangle^\pm$ and $|\Psi\rangle^\pm$.

In resume, we have discussed experimental routes for measuring the degree of nonclassicality of field states describing a single system. The extension of the notion of nonclassical degree (using the distance-type criterium) for states of bipartite systems allowed us to investigate possible connections between the degree of nonclassicality and the degree of entanglement. As illustrated in Fig. 1, such alluded correspondence does not exist for the families of states $|\Psi\rangle^\pm$ and $|\Phi\rangle^\pm$ considered here; $D_{|\Psi\rangle_{ab}}^{(\pm)}$ does not change by varying $\xi$ and $D_{|\Phi\rangle_{ab}}^{(\pm)}$ is a decreasing function of $\xi$ in the whole interval $0 \leq \xi \leq 1$, while $E(\xi)$ increases for $0 \leq \xi \leq 1/2$ and decreases for $1/2 \leq \xi \leq 1$. On the other hand, a correlation was found between the nonclassical degree and the Mandel factor for these families of states. Many other states of a bipartite system can be analyzed along these lines. Finally, we remark
that the notion of nonclassical degree we have introduced for bipartite states can be easily extended to states of multi-partite systems, allowing a comparison with the more general measure of entanglement recently presented in Ref. [30]. Such study is left for future work.

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