Behavior of the Abrasive Medium in a Single-Axis Vibration System

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Abstract. The article deals with the physical basis of the behavior of free abrasive grains in a single-axis vibration system. A mathematical model is made that represents the layered division of the container into layers without mixing. The established operating mode can be considered as a quasi-static state, which allowed us to determine the parameters of the speed and density of the layers of abrasive material in the working area. The optimal loading height of the working environment and the operating modes of the installation were determined.

1. Introduction
Vibration treatment of parts with free abrasive is common as a scale removal operation [1], for deburring [2], and for forming the surface layer [3]. This method assumes that the parts to be processed and the abrasive medium to be processed are placed in a container that oscillates [4]. Under the influence of vibration, the processing medium acquires the property of fluidity [5] and is suspended in the container [6]. This makes it possible for abrasive particles to slide over the surfaces of the processed parts [7], as well as to penetrate into hard-to-reach parts, thereby ensuring a fairly uniform treatment of all its surfaces.

With increasing requirements for product quality, the technology of vibroabrasive processing of parts of complex shapes will soon be widely used in industrial enterprises [8]. The use of vibroabrasive technology for finishing surfaces is hindered by the lack of a mathematical model of processing with a given accuracy [9]. To do this, at the initial stage, it is necessary to determine the behavior of the abrasive medium in the container.

2. Methods
The influence of the position of the part body relative to the moving system on the volume and localization of material removal has been studied in sufficient detail [10], [11]. Both periodic [12] and continuous [10] unloading of the fixed part is considered. Due to the small 0.5-2 g/min [13] removal of the material, most of the work is aimed at intensifying the process using electrochemical processing [14], magnetic vibration [15], and vibrothermomechanical [16] processing of parts. The message of additional forced movement also leads to an intensification by 2-15 [17] times. For simple parts, a spindle [18] is used that performs reciprocating movements. Most of the works are experimental in nature [19] and do not lead to mathematical models of directed material removal [20].

Modern technology allows you to connect a robot manipulator that can perform a given spatial movement [21]. To determine the degree of removal of material in various layers and parts of the
installation, which uses the rotation of the abrasive material with simultaneous vibration, a mathematical model was compiled.

In this case, the container is divided by height into n infinitesimal layers, in the working area of which idealized conditions apply, allowing each layer to be considered a conditionally solid plate.

According to the model, the i-th plate is affected by the forces of viscosity (1), elasticity (2) and plasticity (3) on the side of the (i-1) and (i+1) plates.

\[
F_{ni} = c \cdot (y_i-y_{i-1})/(x_i-x_{i-1}) + c \cdot ((y_{i+1}-y_i)/(x_{i+1}-x_i)) \quad (1)
\]

\[
F_{yi} = q_i \cdot k \cdot (d-x_i+x_{i-1}) - q_{i+1} \cdot k \cdot (d-x_{i+1}+x_i) \quad (2)
\]

\[
F_{pi} = q_i \cdot b_i \cdot l \cdot (d-x_i+x_{i-1}) - q_{i+1} \cdot b_i \cdot l \cdot (d-x_{i+1}+x_i) \quad (3)
\]

where i = 2, n+1; c is a viscosity coefficient; \( y_i = dx_i/dt \) is the speed of the i-th plate; \( x_i \) is a coordinate of the i-th plate; \( q_i \) is the coefficient that takes into account the distance between adjacent plates or between the container and the plate; \( q_i = 0 \), at \( x_i-x_{i-1} \geq d \); \( q_i = 1 \), at \( x_i-x_{i-1} < d \); where i = 2, n+2; \( b_i \) is the coefficient that takes into account the relative speed of adjacent plates and the relative speed of the plates and the container; \( b_i = 0 \), at \( y_i-y_{i-1} \geq D \); \( b_i = 1 \), at \( y_i-y_{i-1} < D \); where i = 2, n+2; \( K \) is the plasticity coefficient; \( n+2 \) is the sticking coefficient; \( d = D/(n+1) \) is the distance between plates; \( D \) is the feeding height of the working medium in the container; \( n \) is the number of plates on which the feeding is based.

The interaction of the working medium with the container and the part being processed in the gravity field is described by the equations (4):

\[
dy/dt=(F_u+F_y+F_Ni-mg)/m; \quad (4)
\]

\[
dx/dt=y_i; \quad x_{ni}=A \cdot \sin(2 \pi \cdot \varphi \cdot t-\pi/2)+A; \quad (5)
\]

\[
x_{n+2}=x_1+H; \quad y_1=2 \pi \cdot \varphi \cdot A \cdot \cos(2 \pi \cdot \varphi \cdot t-\pi/2); \quad y_{n+2}=y_1;
\]

\[
F_{Ni}=-\xi \cdot \rho_p \cdot v^2 \cdot 2 \cdot S_k;
\]

where \( F_{Ni} \) is the force of interaction of the i-th plate with the workpiece; \( \xi \) is the coefficient of interaction; \( \rho_p=\rho_c \cdot \rho_p \); \( \rho_p \) is bulk density loaded into the container of the working medium; \( \rho_c \) is the density of the material working environment; \( \rho_p \) is relative density of packing of the working environment; \( H \) is the height of the container; \( v \) is the relative velocity of interaction of working medium with the workpiece; \( S_k \) is the ground area of the container; \( x_1 \) is the coordinate of the bottom of the container; \( x_{n+2} \) is the coordinate of the container cover; \( y_1, y_{n+2} \) is the speed of the bottom and lid of the container; \( m_i=\rho_c \cdot \rho_p \cdot S_k \cdot D/\pi \) is the mass of the i-th plate; \( g \) is the free fall acceleration; \( A \) is the amplitude of oscillation of the container; \( T \) is time.

The initial conditions at time \( t =0 \) are written (5): \( x_1=0; x_{n+2}=H; y_1=0; y_{n+2}=0 \).

\[
x_i=x_{i-1}+d-(\rho_c \cdot \rho_p \cdot S_k \cdot D/(\pi+2-i)) \cdot g/(K+l) \cdot n \quad (5)
\]

where \( i=2, n+1 \).

Thus, the problem of the motion of the working medium in an oscillating container is reduced to solving a system of (4) 2n first-order ordinary differential equations and four algebraic ones.

3. Results and discussions

To numerically solve the problem on a computer in order to optimize the process, it is necessary to select the values of the parameters included in the mathematical model, which include the time step of the Runge-Kutta method \( S_1 \) and the coefficients that set the properties of the working medium: elasticity coefficient - \( K \); plasticity coefficient – \( l \); viscosity coefficient - \( C \).

When selecting the values of the parameters of the elastic-viscoplastic step model of the \( S_1 \) method, several preliminary calculations were performed, during which the amplitude and frequency of vibrations of the container, its dimensions and loading of the working medium were selected as in the experimental setup. The initial values of \( K \) and \( L \) were chosen by analogy with the corresponding properties of structural steel.
The values of the initial data of the performed calculations and their results were analyzed. The model that most realistically describes the process of interaction of the working medium particles with each other, as well as with the bottom and lid of the container, is a plastic model, and the plasticity coefficient should be $l=10^7 \text{n/m}$.

![Figure 1](image)

**Figure 1.** Trajectory of the container and the working environment. $K=c=0; l=10^7 \text{n/m}; S_1=2\cdot10^{-5}c$ ; n=10.

The accuracy of calculations depends significantly on the time step of the Runge-Kutta method, which must correspond to $S_1\leq10^{-4}c$.

As the number of plates increases, the design characteristics of the process become more detailed, but in this case, a corresponding reduction in the time step is necessary to maintain the accuracy of the calculation.

The calculation time on a computer of one oscillation period with the number of plates $n = 4$ and the time step $t = 10^{-4}c$ is 0.2-0.7 minutes (exclusive or multi-user mode). As the number of plates increases or the time step decreases, the calculation time increases proportionally.

To determine the frequency or instability of fluctuations in the working environment, it is necessary to calculate for 4-5 periods.

The model was tested experimentally. Steel balls with a diameter of 3.0 mm were used as the working medium. During the experiments, the strain measurement of the force of the working medium on the workpiece, on the bottom and lid of the container was carried out, as well as photographing the position of the working medium particles at the characteristic moments of the working cycle (Fig. 2)
Figure 2. 1 and 2 – calculated graphs of the loading force on the part from above and below; 3 and 4 – experimental graphs of the loading force on the part from above and below. The number of layers of the working environment n=4.

4. Conclusions
When choosing the values of constants included in the calculation formulas ($K=c=0; \ell=10^7 \text{ m}^2/\text{m}; S_1=10^{-3} \text{ c}$), the proposed mathematical model describes the vibration treatment process with sufficient accuracy for practice (relative error 10~15%).

To calculate the amplitude and frequency of the working environment impact on the workpiece, the number of plates in the model can be significantly reduced, which leads to a proportional reduction in machine time.

It is established that at a frequency $f = 3 \text{ Hz}$ in the range of amplitudes $30 \leq A \leq 50 \text{ mm}$ at any values of $H$, $D$, $X_kg$ processing of the part does not occur.

The optimal loading height of the working medium in the studied range ($3 \leq f \leq 7 \text{ Hz}, 30 \leq A \leq 70 \text{ mm}$) was half the height of the container $D = 0.5 \cdot N$.

5. Summary
The mathematical model, the methodology and software can be used to calculate by computer the optimal values of the height of the container, load, working environment and the provisions of parts in the container and forces acting on the bottom and the lid of the container, upper and lower surfaces of the workpiece, trajectories and speed of movement of the working medium at any given value of the amplitude and frequency of oscillations, angle parts, any mechanical properties of the work environment.

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