On the Possibility of Abnormally Intense Radiation Due to the Rotation of Electron Around a Dielectric Sphere

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Abstract
The abnormally intense radiation due to the uniform rotation of electron around the equatorial plane of a dielectric sphere is obtained. It takes place when the sphere surface is at a specific distance from the electron orbit and when the Cherenkov condition for electron and the matter of the sphere is satisfied.

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1 Introduction
A number of important electromagnetic processes is conditioned by the matter: the Vavilov-Cherenkov radiation, the X-ray transition radiation, the radiation of channeled particles [1] - [9]. In this connection it is of interest to study an influence of the matter on the radiation of the relativistic charge rotating along a circle in a permanent magnetic field (synchrotron radiation [10, 11]).

The synchrotron radiation in an infinite uniform medium was studied in [12] and further in [2, 13]. The radiation of a nonrelativistic particle rotating uniformly around a dielectric sphere, and the radiation of the particle rotating in close proximity to the ideally conducting sphere were considered in the [14]. In [15, 16] the expressions were obtained for the spectral and spectral-angular distribution of the radiation intensity without restrictions on the orbit radius and velocity of a particle rotating around a sphere with an arbitrary dielectric permittivity.

In the present paper an analysis of the numerical calculations by the formulae obtained in [15, 16] is carried out. The peculiarities of the radiation conditioned by the matter of a sphere and by its size, are revealed.

2 Basic formulae
We present the basic formulae describing the radiation of a particle with the charge \( q \) and velocity \( v = \omega_e r_e \) uniformly rotating around a sphere in its equatorial plane (\( r_e \) is the radius of orbit). The magnetic permeability of the sphere we take equal to unity and consider its dielectric permittivity \( \varepsilon_0 \) as an arbitrary real quantity (we do not take into account the effects connected with the radiation absorption), the sphere radius \( r_o < r_e \). The radiation
intensity at the frequency $\omega = k\omega_c$ (after an averaging over the rotation period $2\pi/\omega_c$) is determined by the expression

$$I_k = 2 \frac{q^2 \omega^2}{c \sqrt{\varepsilon_1}} \sum_{s=0}^{\infty} \left( |a_{kE}(s)|^2 + |a_{kH}(s)|^2 \right),$$

(1)

where $\varepsilon_1$ is the dielectric permittivity of a medium surrounded the sphere,

$$a_{kE} = b_{l}(E) P^k_l(0) \sqrt{\frac{(l-k)!}{l(l+1)(2l+1)(l+k)!}}, \quad l = k + 2s,$$

$$a_{kH} = b_{l}(H) \sqrt{\frac{(2l+1)(l-k)!}{l(l+1)(l+k)!}} \cdot \frac{dP^k_l(y)}{dy}, \quad y = 0, \quad l = k + 2s + 1$$

(2)

are the dimensionless amplitudes describing the contributions of multipole of the electric and magnetic kinds, respectively. In Eq. (2) $P^k_l(y)$ are the associated Legendre polynomials, and $b_l$ is a factor depending on $k$, $x = r_0/r$, $\varepsilon_0$ and $\varepsilon_1$:

$$b_l(H) = i u_1 \left[ j_l(u_1) - h_l(u_1) \frac{\{j_l(xu_0), j_l(xu_1)\}}{j_l(xu_0)h_l(xu_1)} \right], \quad u_i = k \sqrt{\frac{\nu}{c}}.$$

$$b_l(E) = (l + 1) b_{l-1}(H) - l b_{l+1}(H) + \frac{1}{x^2} \left( \frac{1}{\varepsilon_0} - \frac{1}{\varepsilon_1} \right) \times$$

$$\times \left[ j_{l-1}(xu_0) + j_{l+1}(xu_0) \right] \frac{h_{l-1}(u_1) + h_{l+1}(u_1)}{l(1 + l) u_0 j_l(xu_0)} - \frac{1}{l} \frac{dP^k_l(y)}{dy},$$

(3)

where $h_l(y) = j_l(y) + in_l(y)$; $j_l$ and $n_l$ are the spherical Bessel and Neumann functions, respectively. In Eq. (3) the following notations are introduced:

$$\{a(xu_1), b(xu_2)\} = a \cdot \frac{\partial b}{\partial x} - \frac{\partial a}{\partial x} \cdot b, \quad f_l(y) = \frac{f_l(y)}{\{j_l(xu_0), h_l(xu_1)\}},$$

$$\varepsilon^L_{\nu} = \frac{u_1 j_{\nu}(xu_0) h_{l}(xu_1) / \varepsilon_1 - u_0 j_{\nu}(xu_0) h_{\nu}(xu_1) / \varepsilon_0}{u_1 j_{\nu}(xu_0) h_{l}(xu_1) - u_0 j_{\nu}(xu_0) h_{\nu}(xu_1)},$$

(4)

The derivation of Eq. (1) is given in [13, 14].

In the case of homogeneous medium ($\varepsilon_0 = \varepsilon_1 = \varepsilon$)

$$b_l(H) = i u j_l(u), \quad \nu = k \sqrt{\frac{\nu}{c}},$$

$$b_l(E) = i u (2l + 1) \left[ j_l(u) + \frac{1}{u} j_l(u) \right],$$

(5)

and therefore Eq. (1), naturally, does not depend on $x$. One can also be convinced that Eq. (1) is transformed into the known formula [2, 10, 12, 13, 17]

$$I_k = kvq^2 \omega^2 \sqrt{\varepsilon} \left[ 2 J_{2k}(2k\beta\sqrt{\varepsilon}) + (1 - \frac{1}{\varepsilon^2}) \int_0^{2k\beta\sqrt{\varepsilon}} J_{2k}(y)dy \right],$$

(6)

where $\beta = v/c$, $J_{k}(y)$ is the integer-order Bessel function, and $\varphi'(y) = d\varphi/dy$. 

3 Results of numerical calculations

In Fig.1 along the axis of ordinates we plotted an average number of electromagnetic field quanta

\[ n_k = \frac{2\pi I_k}{\hbar \omega_r^2}, \]  

(7)

radiated per one period of rotation of electron with the energy 2 MeV (the logarithmic scale), and along the axis of abscissa an order of radiated harmonic in the range \( 1 \leq k \leq 50 \) is plotted. The function \( n_k \) is presented for the four values of \( x \). The curves \( a, b, c, d \) are the polygonal lines connecting the points with different \( k \) and the same \( x_a, x_b, x_c, x_d \), respectively. The line \( a \) describes a rotation in vacuum \( (x_a = 0) \), and the line \( b \) describes a rotation in the continuous medium \( (x_b = \infty) \) with the dielectric permittivity \( \varepsilon = 3 \) (the Cherenkov condition is satisfied). The calculations were carried out by the formula (6). For simplicity the dependence of \( \varepsilon \) on \( k \) (the dispersion) is not taken into account. It followed from the plots that in a continuous media

\[ n_k(\infty) \leq \frac{ve^2}{\hbar c^2} \left( 1 - \frac{1}{\varepsilon \beta^2} \right) < \frac{e^2}{\hbar c} \approx 0.05 \]  

(8)

is larger than the analogous quantity \( n_k(0) \) in the empty space. A difference between \( n_k(\infty) \) and \( n_k(0) \) is conditioned by the contribution of the Cherenkov’s quanta. Along with this, the specific oscillations are revealed on the curve \( b \). They results from the interference of waves in the conditions when the velocity of the electromagnetic waves propagation is lower than the velocity of the source motion \( c/\sqrt{\varepsilon} < v \).

A similar pattern should be observed also in the case when a medium has finite sizes. In the section 2 we considered the case of a sphere with the radius \( r_o \), around of which electron rotates at the distance \( r_e - r_o \). The polygonal lines \( c \) and \( d \) represent the results of calculations by the formula (8) for the two fixed values \( r_o/r_e = 0.974733692 = x_c \) and \( 0.980861592 = x_d \), respectively. The dielectric permittivity of the sphere \( \varepsilon_0 = 3 \). Outside the sphere there is a vacuum \( (\varepsilon_1 = 1) \). The electron energy \( E_e = 2MeV \). As it is seen, the specific oscillations are observed also in this case. However, there are also the peaks, and on the corresponding harmonics \( (k = 26 \) for the case \( c \) and \( k = 40 \) for the case \( d \) the radiation is abnormally intensive:

\[ n_{26}(x_c) = 4300 \quad \text{for the curve } c, \]
\[ n_{40}(x_d) = 94 \quad \text{for the curve } d. \]  

(9)

At the same time on the neighbouring harmonics \( n_k(x) \) is of the order \( n_k(\infty) \).

In the empty space the radiation intensity \( I_k \) reaches a maximum on the harmonic with \( k_{max} = 26: I_{26}(0) = 0.96e^2/\omega_r^2/c. \) On this harmonic an influence of the sphere with the radius \( r_o = 0.974733692 r_e \) is the most intensive: \( I_{26}(x_c)/I_{26}(0) \approx 2.53 \cdot 10^6 \) (just this value of \( r_o \) is chosen in the case of the curve \( c \)). An analogous situation is possible also on other harmonics. For example, on the harmonic with \( k = 40 \) an influence of the sphere is maximal at \( r_o = 0.980861592 r_e \) (the curve \( d \)). In this case \( I_{40}(x_d)/I_{26}(0) \approx 55700. \)

Figs.2 and 3 show the dependence of \( n_k(x) \) on \( x \) for the harmonics with \( k = 26 \) and \( k = 40 \), respectively. In this plots also \( \varepsilon_0 = 3, \varepsilon_1 = 1 \) and \( E_e = 2MeV \). Against a
background of the oscillations of the function $n_k(x)$, the extremely narrow and very high peaks are observed (on the right-hand part the function $n_k(x)$ is shown in the vicinity of the maximal peak). Already at a small deviation (along the axis of abscissa) from the centre of any of these peaks $n_k$ rapidly decreases. Therefore the value $x = r_o/r_e$ must be fixed with a high accuracy (for example, by an external electric field sustaining a uniform rotation of a particle). The energy radiated per one period of the electron rotation, is equal to

$$\frac{2\pi}{\omega_e} I_k = k\hbar \omega_e n_k.$$  \hspace{2cm} (10)

The radiative losses are negligible if the cyclic frequency

$$\omega_e \ll \frac{E_e}{k\hbar n_k} \sim 10^{13} \frac{E_e}{MeV} \frac{10^8}{kn_k} Hz.$$  \hspace{2cm} (11)

An analogous pattern takes place for other $1 < \varepsilon_0 \leq 5$ and $E_e \leq 5MeV$, when the Cherenkov condition is satisfied (see Table 1). Moreover, in certain cases (see the 2-4th rows of Table 1) one can observe a superintensive radiation with

$$n_k > \frac{2\pi r_e}{\lambda_k} = k\frac{v}{c}.$$  \hspace{2cm} (12)

Table 1: The average number $n_k$ of electromagnetic field quanta emitted per revolution of electron.

| $k$ | $E_e$ MeV | Rotation in a continuous medium | Rotation around a sphere in a vacuum |
|-----|----------|-------------------------------|--------------------------------------|
|     |          | $\varepsilon = 1$ | $\varepsilon = 3$ | $\varepsilon = 5$ | $\varepsilon = 3$ | $\varepsilon = 5$ |
|     |          | $n_k$ | $n_k$ | $n_k$ | $n_k(\mu)$ | $\mu$ | $n_k(\mu)$ |
| 20  | 1        | $3.07 \cdot 10^{-4}$ | $2.37 \cdot 10^{-2}$ | $3.18 \cdot 10^{-2}$ | 6.6433228 | 4.13 | 5.2992 | 1.76 |
|     | 3        | $2.72 \cdot 10^{-3}$ | $3.32 \cdot 10^{-2}$ | $3.33 \cdot 10^{-2}$ | 0.5432354 | 201 | 3.482 | 0.34 |
|     | 5        | $3.00 \cdot 10^{-3}$ | $3.42 \cdot 10^{-2}$ | $3.63 \cdot 10^{-2}$ | 1.480803 | 133 | 2.596109 | 133 |
| 40  | 1        | $2.39 \cdot 10^{-6}$ | $1.93 \cdot 10^{-2}$ | $3.11 \cdot 10^{-2}$ | 0.82132 | 9.64 | 1.13910742 | 2260 |
|     | 3        | $1.57 \cdot 10^{-3}$ | $2.90 \cdot 10^{-2}$ | $3.77 \cdot 10^{-2}$ | 1.2224 | 0.65 | 0.9986 | 0.65 |
|     | 5        | $1.85 \cdot 10^{-3}$ | $3.22 \cdot 10^{-2}$ | $3.47 \cdot 10^{-2}$ | 4.801 | 0.16 | 1.50036 | 1.45 |

Note: $\varepsilon$ is the dielectric permittivity of the matter. In the case of a sphere for every three values of $k, E_e$ and $\varepsilon$ we chose and presented one value of the ratio of the sphere radius to the radius of the electron orbit $r_o/r_e = 1 - 0.01\mu$, for which $n_k(\mu)$ is considerably larger than $e^2/hc$.

The formulae (3) are not valid for electron rotating inside a spherical cavity in an infinite medium, and therefore we did not carry out the corresponding calculations.

The numerical calculations were duplicated by two independent programs. One of them, a more simple, was made with the help of the Mathematica, and an another, more fast-acting, on the Pascal language.
4 Conclusions

We calculated the intensity of radiation for electron with an energy of several MeV uniformly rotating around a sphere in its equatorial plane. The matter of the sphere is regarded as transparent, and its dielectric permittivity $1 < \varepsilon \leq 5$. It is obtained that on the average the $n > k$ quanta of the electromagnetic field may be radiated per revolution of electron, where $k$ is the number of the radiated harmonic ($k \leq 50$). In the absence of a sphere or at the rotation of electron in an infinite medium with the same $\varepsilon$, the analogous quantity $n_k < 0.05 \approx e^2/hc$. Such an intense radiation takes place when the sphere surface is at a specific distance from the electron orbit and when the Cherenkov condition for electron and the matter of the sphere is satisfied.

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Figure captions:

Fig.1: Average number $n_k(x)$ of electromagnetic field quanta emitted per revolution of electron, as a function of the radiated harmonic’s number $k$. The polygonal lines $a, b, c$ and $d$ differ by the value of $x$ (the ratio of the sphere radius to the radius of the electron orbit): $x_a = 0$ (vacuum), $x_b = \infty$ (infinite medium), $x_c \approx 0.9747337$, $x_d \approx 0.9808616$. The dielectric permittivity of the matter $\varepsilon = 3$, the electron energy $E_e = 2MeV$.

Fig.2: The same quantity, as in Fig.1, depending on $x$. A number of the radiated harmonic is fixed: $k = 26$. Here also $\varepsilon = 3$ and $E_e = 2MeV$. On the right-hand side the function $n_k(x)$ is plotted in the vicinity of the maximal peak.

Fig.3: The same dependence, as in Fig.2, in the case $k = 40$. 

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