Direct CP violation in three-body decays of the $B$ meson by resonance effects

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Abstract

We discuss direct CP violation in three-body decays of $B$ meson such as $B^\pm \to K^\pm \pi^+\pi^-$, which involves various intermediate resonance states, such as $B^\pm \to K^*_i\pi^\pm$ and $B^\pm \to \rho K^\pm$, where $K^*_i$ represents the $\pi K$ resonance. Due to the large final state interaction phases of the resonances, the CP asymmetry can be as large as the 25% level near the kinematical region where the $K^*_i$ and $\rho$ resonances overlap. By examining a Dalitz plot of this mode and combining a measurement of the branching ratio for $B^\pm \to \rho^\pm K$, it is possible to extract the weak phase, $\phi_3 = \text{arg}(-V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*))$, of the Kobayashi-Maskawa matrix.

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The origin of CP violation is one of the unresolved problems, although CP violation has been observed in the $K^0-\bar{K}^0$ mixing system for 30 years. CP violation occurs due to the complex phase of the Kobayashi-Maskawa (KM) matrix in the standard model (SM). The main purpose of B-factory experiments is to measure the phases of the KM matrix, defined by 

$$\phi_1=\arg(-V_{cd}V_{cb}^*/(V_{td}V_{tb}^*)),$$

$$\phi_2=\arg(-V_{ud}V_{ub}^*/(V_{td}V_{tb}^*))$$

and

$$\phi_3=\arg(-V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)),$$

and to check the unitarity nature of the KM matrix [2, 3]. It is known that $\phi_1$ can be measured in a so-called "gold plated" $B^0 \to J/\psi K_s$ mode [4]. This mode is expected to determine $\phi_1$ to good accuracy. On the other hand, measurements of $\phi_2$ and $\phi_3$ are considered to be rather difficult from the experimental point of view. Various methods to measure the weak phases are discussed in the literature [5]. In order to observe the CP-violating effects it is necessary to have different CP-conserving and CP-violating phases in an amplitude. The CP-violating phase is provided by the phase of the KM matrix. For the neutral $B$ meson the CP-conserving phase is provided by the phase of $B^0\to\bar{B}^0$ oscillation, whereas for the charged $B$ meson the CP-conserving phase is dominantly supplied by the final-state-interaction (FSI) phase. In general, it is difficult to reliably evaluate the FSI phase because of hadron dynamics. However, we can reliably calculate the strong phase in the case of resonances by using the Breit-Wigner form. In addition, the resonance provides a large CP-conserving phase, which may lead to a large direct CP-violation effect [6] and the resonance enhancement of CP violation is discussed in several decay modes [7, 8].

In this paper we propose another method to measure the angle $\phi_3$ using the direct CP violation in three-body decays of the $B$ meson, such as the $B^\pm \to K^{\pm}\pi^-\pi^+$ decay mode. Several resonances contribute to this decay mode, the effects which might enhance direct CP violation. There are two channels of two-body intermediate states in this decay mode. One is $B^\pm \to K^{\pm}\rho \to K^{\pm}\pi^-\pi^+$; the other is $B^\pm \to K_i^{\ast}\pi^{\pm} \to K^{\pm}\pi^-\pi^+$, where $K_i^{\ast}$ denotes the $\pi K$ resonances summarized in Table [2][3]. The $B^\pm \to K^{\pm}\rho$ process involves equally tree ($T$) and QCD penguin ($P$) diagrams, whereas $B^\pm \to K_i^{\ast}\pi^{\pm}$ has a dominant contribution from the QCD penguin ($P'$) diagram. The amplitudes can be written by

$$a_i \equiv A(B^- \to K_i^{\ast}\pi^-) = P'_i \ e^{i\delta'_i},$$

(1)

§In principle there may be a tree-diagram contribution to $B^\pm \to K_i^{\ast}\pi^{\pm}$ through a rescattering process, $B^\pm \to K^{\pm}\rho \to K_i^{\ast}\pi^{\pm}$ and this contribution will be discussed later.
\[ b_\rho \equiv A(B^- \to K^- \rho) = Te^{i\delta_T}e^{-i\phi_3} + Pe^{i\delta_P}, \]  

where \( \delta_T, \delta_P \) and \( \delta'_P \) represent the strong phases, and the tree-diagram contribution has the weak phase \( \phi_3 \). Concerning the corresponding amplitude for charge-conjugated modes \( \bar{a}_i \) and \( \bar{b}_\rho \), only the sign of \( \phi_3 \) should be changed. These amplitudes are normalized as

\[ \text{BR}(B^- \to K^+\pi^-) = |a_i|^2, \quad \text{BR}(B^- \to K^-\rho) = |b_\rho|^2. \]  

Although these modes have not been observed experimentally, the CLEO collaboration reported the branching ratios for the \( B \to \pi K \) modes as follows [10]:

\[ \text{BR}(B^+ \to K^0\pi^+) = (2.3^{+1.1}_{-1.0} \pm 0.2 \pm 0.2) \times 10^{-5}, \]  
\[ \text{BR}(B^0 \to K^+\pi^-) = (1.5^{+0.5}_{-0.4} \pm 0.1 \pm 0.1) \times 10^{-5}. \]  

This result indicates that the \( T, P, P' \) amplitudes for the \( B^+ \to K^0\pi^+ \) and \( B^0 \to K^+\pi^- \) processes are the same orders of magnitudes [11]: Since the \( B^0 \to \pi^+K^- \) and \( B^- \to \pi^-K \) transitions are essentially the same as the \( B^- \to K^-\rho \) and \( B^- \to K_i^*\pi^- \) transitions, respectively, these intermediate contributions can significantly interfere with each other in the \( B^\pm \to K^\pm\pi^+\pi^- \) decay modes. The intermediate resonances, which subsequently decay to two pseudo-scalars, have large FSI phases. We assume that these resonance contributions can be treated by the Breit-Wigner form:

\[ \Pi_{K_i^*}(s) = \frac{\sqrt{\Gamma_{K_i^*}/m_{K_i^*}}}{s - m_{K_i^*}^2 + im_{K_i^*}\Gamma_{K_i^*}}, \quad \Pi_{\rho}(s') = \frac{\sqrt{\Gamma_{\rho}/m_{\rho}}}{s' - m_{\rho}^2 + im_{\rho}\Gamma_{\rho}}, \]  

where \( s = (p_{K^-} + p_{\pi^+})^2, \quad s' = (p_{\pi^+} + p_{\pi^-})^2 \) and the \( m_i \)'s and \( \Gamma_i \)'s are the masses and widths of the resonances, respectively.

Assuming that the \( B^- \to K^-\pi^-\pi^+ \) decay amplitude is dominated by the above resonances, we can write the decay amplitude in a narrow-width approximation as follows:

\[ A(B^- \to K^-\pi^-\pi^+) = \sum_i \frac{a_i}{m_{K_i^*}} \sqrt{\frac{2B_{K_i^*}}{B_{K_i^*}} \Pi_{K_i^*}(s) \Theta(z)} + \frac{b_\rho}{m_\rho} \sqrt{\frac{2B_{\rho}}{B_{\rho}} \Pi_{\rho}(s') \Theta(z')}, \]
where \( B_{\rho}^{\pi\pi} \) and \( B_{K_1^*}^{K\pi} \) denote the branching ratios for \( \rho \rightarrow \pi\pi \) and \( K_1^* \rightarrow K\pi \), respectively, and \( \beta_{i}^{jk} \) is defined by

\[
\beta_{i}^{jk} = \sqrt{\frac{(1 - (m_j + m_k)^2/m_i^2)(1 - (m_j - m_k)^2/m_i^2)}{}}. \tag{8}
\]

The function \( \Theta \) describes the angular component of the amplitude for the resonance decay \([8, 12]\). The explicit form depends on the spin of the resonance, and is given by

\[
\Theta(z) = \begin{cases} 
\sqrt{\frac{1}{2}}, & \text{spin } 0, \\
\sqrt{\frac{3}{2}z}, & \text{spin } 1, \\
\sqrt{\frac{5}{8}(3z^2 - 1)}, & \text{spin } 2,
\end{cases} \tag{9}
\]

where \( z(z') \) denotes \( \cos \theta (\cos \theta') \), and \( \theta (\theta') \) is the angle between the momentum of the \( B \) and \( K^- (\pi^-) \) mesons in the center-of-mass frame of the \( \pi^+ \pi^- (\pi^+ K^-) \) pairs. 

\( z(z') \) can be written as a function of \( s, s' \) as follows:

\[
z = \frac{2ss' + s^2 - s(m_B^2 + m_K^2 + 2m_\pi^2) + (m_B^2 - m_\pi^2)(m_B^2 - m_\pi^2)}{\sqrt{(s - (m_B + m_\pi)^2)(s - (m_B + m_\pi)^2)(s - (m_B - m_\pi)^2)(s - (m_B - m_\pi)^2)}}, \tag{10}
\]

\[
z' = \frac{\sqrt{s'(2s + s' - m_B^2 - m_K^2 - 2m_\pi^2)}}{\sqrt{(s' - 4m_\pi^2)(s' - (m_B + m_K)^2)(s' - (m_B - m_K)^2)}}. \tag{11}
\]

For simplicity, we hereafter consider the case that only the \( K_2^*(1430) \) and \( \rho \) resonances contribute to each \( \pi K \) and \( \pi\pi \) channel, respectively.\(^*\) We later discuss more general formula which includes all of the intermediate resonances. The differential decay width for \( B^- \rightarrow K^- \pi^- \pi^+ \) can be written as

\[
d\Gamma(B^- \rightarrow K^- \pi^- \pi^+)/dsds' \tag{12}\]

\[
= |a_{K_2^*}|^2 \text{Re} \left( f(K_2^*,K_2^*)(s,s') \right) + |b_{\rho}|^2 \text{Re} \left( g(\rho,\rho)(s,s') \right) \]

\[
+ \text{Re} \left( a_{K_2^*} b_{\rho}^* \right) \text{Re} \left( h(K_2^*,\rho)(s,s') \right) - \text{Im} \left( a_{K_2^*} b_{\rho}^* \right) \text{Im} \left( h(K_2^*,\rho)(s,s') \right). \]

The distribution on a Dalitz plot is described by the functions \( f, g \) and \( h \) in this formula, and are written as

\[
f(K_2^*,K_2^*)(s,s') = \frac{2}{m_{K_2^*}m_{K_2^*}} \sqrt{\frac{B_{K_2^*}^{K\pi}B_{K_2^*}^{K\pi}}{B_{K_1^*}^{\pi\pi}B_{K_2^*}^{K\pi}B_{K_1^*}^{K\pi}B_{K_2^*}^{K\pi}}} \left( \Pi_{K_1^*}(s)\Pi_{K_2^*}(s)^* \right) \Theta_{K_1^*}(z)\Theta_{K_2^*}(z), \tag{13}\]

\(^*\)The \( K_2^*(1430) \) resonance overlaps with the \( K_0^*(1430) \) resonance but we neglect it here. We will discuss this contribution later.
\[ g^{(f_i,f_j)}(s,s') = \frac{2}{m_f m_{f_j}} \sqrt{\frac{B_{f_i}^{f_j} B_{f_j}^{f_i}}{\beta_{f_j}^{f_i} \beta_{f_i}^{f_j} \beta_{f_i}^{f_j} \beta_{f_j}^{f_i}}} \{ \Pi_{f_i}(s') \Pi_{f_j}(s')^* \} \Theta_{f_i}^{f_j}(z') \Theta_{f_j}^{f_i}(z'), \]  

(14)

\[ h^{(K_2^*, f_j)}(s,s') = \frac{4}{m_{K_2^*} m_{f_j}} \sqrt{\frac{D_{K_2^*}^{f_j} D_{f_j}^{K_2^*}}{\beta_{K_2^*}^{f_j} \beta_{f_j}^{K_2^*} \beta_{K_2^*}^{f_j} \beta_{f_j}^{K_2^*}}} \{ \Pi_{K_2^*}(s) \Pi_{f_j}(s')^* \} \Theta_{K_2^*}^{f_j}(z) \Theta_{f_j}^{K_2^*}(z'). \]  

(15)

The above expressions are defined for general meson states, \( f_i = \rho, f_0 \), or \( f_2 \), which decay into the \( 2\pi \) state since we will later discuss the \( B^\pm \to f_i \pi^\pm \to \pi^+ \pi^- K^\pm \) mode in addition to the \( B^\pm \to \rho K^\pm \to \pi^+ \pi^- K^\pm \) mode. We can obtain the differential width for the charge-conjugated mode by simply changing the amplitude \( a_i(b_\rho) \) into \( \pi_i(\tilde{\rho}) \). In Fig. we show the kinematical functions \( \text{Re}(f^{(K_2^*, K_2^*)}) \), \( \text{Re}(g^{(\rho, \rho)}) \), \( \text{Re}(h^{(K_2^*, \rho)}) \) and \( \text{Im}(h^{(K_2^*, \rho)}) \) in the region \( R = \{ 0 < s, s' < 3\text{GeV}^2 \} \). It can be seen that the signs of \( \text{Re}(f^{(K_2^*, K_2^*)}) \) and \( \text{Re}(g^{(\rho, \rho)}) \) are always positive, whereas \( \text{Re}(h^{(K_2^*, \rho)}) \) and \( \text{Im}(h^{(K_2^*, \rho)}) \) change their signs around the \( K_2^*(1430) \) and \( \rho \) poles. Note that the \( K_2^*(1430) \) and \( \rho \) contributions can overlap around the region \( s \simeq m_{K_2^*}^2, s' \simeq m_{\rho}^2 \), and that there is a large interference in this region.

Let us discuss how we can determine the parameters in Eq. (12). Since one of the strong phases is irrelevant, we have six unknown parameters \( (T, P, P', \delta_T - \delta_P, \delta_P - \delta_P', \phi_3) \) in this formula. We can determine these parameters as follows. In the kinematical region \( s \simeq m_{K_2^*}^2 \) and \( s' \gg m_{\rho}^2 \), the \( K_2^* \) contribution dominates in the decay width. We can extract the amplitudes \( |a_{K_2^*}| \) and \( |\pi_{K_2^*}| \) by retaining only the first term in the Eq. (12) and the parameter \( P' \) can be determined as \( P' = |a_{K_2^*}| \). By examining the distribution on the Dalitz plot in the overlapping region \( s \simeq m_{K_2^*}^2 \) and \( s' \simeq m_{\rho}^2 \), we can extract \( \text{Re}(a_{K_2^*} b_{\rho}^*), \text{Im}(a_{K_2^*} b_{\rho}^*), \text{Re}(\pi_{K_2^*} \tilde{\rho}), \text{Im}(\pi_{K_2^*} \tilde{\rho}) \). Since we have already known \( |a_{K_2^*}| \) and \( |\pi_{K_2^*}| \), we can determine \( \text{Re}(b_{\rho} e^{i\delta_P}), \text{Im}(b_{\rho} e^{i\delta_P}), \text{Re}(\tilde{b}_{\rho} e^{i\delta_P}), \text{Im}(\tilde{b}_{\rho} e^{i\delta_P}) \). There are five unknown parameters in \( b \) and \( \tilde{b} \) but we have only four constraints. If we combine the branching ratio for \( B^- \to \rho^- K^- \), whose amplitude is written as

\[ A(B^- \to \rho^- K^-) = -\sqrt{2} P e^{i\delta_P}, \]  

(16)

by using the isospin symmetry \( [13] \), we can therefore fix all the parameters, including the weak phase, \( \phi_3 \). Note that we assume that there is no tree-diagram contribution to the \( B^- \to K^- \pi^- \) amplitude in Eq. (11). In general the tree-diagram contribution can be induced through the rescattering process such as \( B^- \to \rho K^- \to K_2^* \pi^- \). If we
take into account the tree-diagram contribution, which can be written as $T \cdot e^{i \delta_T} e^{-i \phi_3}$, we have two unknown parameters, $\delta_T'$ and $T'$, in addition to the above six unknown parameters. We cannot determine $\phi_3$ since there are only seven independent constraints even if we combine the branching ratio for $B^\pm \rightarrow \rho^\pm \bar{K}$. Although it is hard to calculate the corrections from the rescattering effects quantitatively, it is estimated to be of order 10% for the $B \rightarrow \pi K$ modes [14]. Unless this rescattering effect dominates in the amplitudes, we can expect to determine $\phi_3$ with reasonable accuracy.

The CP-violating quantity $A_{CP}$ is defined as

$$A_{CP} \equiv \frac{d\Gamma(B^- \rightarrow K^- \pi^+ \pi^-)/dsds' - d\Gamma(B^+ \rightarrow K^+ \pi^+ \pi^-)/dsds'}{d\Gamma(B^- \rightarrow K^- \pi^+ \pi^-)/dsds' + d\Gamma(B^+ \rightarrow K^+ \pi^+ \pi^-)/dsds'}, \quad (17)$$

where we can write the denominator and numerator as follows:

$$d\Gamma(B^- \rightarrow K^- \pi^+ \pi^-)/dsds' + d\Gamma(B^+ \rightarrow K^+ \pi^+ \pi^-)/dsds' = 2P' \left[ T^2 + P^2 + 2TP \cos \delta_T' \cos \phi_3 \Re \left( f(K_2^-,K_2^+) \right) \right] \Re \left( g^{(\rho,\rho)} \right) + 2 \left( T^2 + P^2 + 2TP \cos \delta_T' \cos \phi_3 \Re \left( h^{(K_2^+,\rho)} \right) \right), \quad (18)$$

$$d\Gamma(B^- \rightarrow K^- \pi^+ \pi^-)/dsds' - d\Gamma(B^+ \rightarrow K^+ \pi^+ \pi^-)/dsds' = -2T \left[ 2P \sin \delta_T' \Re \left( g^{(\rho,\rho)} \right) + P' \sin \delta_T' \Re \left( h^{(K_2^+,\rho)} \right) \right] \sin \phi_3. \quad (19)$$

The CP asymmetry appears as an asymmetry of the distribution on the Dalitz plot for $B^+$ and $B^-$ decays. Let us consider here the integrated asymmetry in the region $R$. When we integrate the functions $\Re(h^{(K_2^+,\rho)})$ and $\Im(h^{(K_2^+,\rho)})$ in this region, they almost vanish because the functions change their signs around the resonance poles,

$$\int_R \left( d\Gamma(B^- \rightarrow K^- \pi^+ \pi^-)/dsds' + d\Gamma(B^+ \rightarrow K^+ \pi^+ \pi^-)/dsds' \right) \approx \int_R \left[ 2P^2 \Re \left( f(K_2^-,K_2^+) \right) + 2 \left( T^2 + P^2 + 2TP \cos \delta_T' \cos \phi_3 \Re \left( g^{(\rho,\rho)} \right) \right) \right]. \quad (20)$$
In order to see how the CP asymmetry can be large, it is necessary to evaluate the hadronic matrix elements, such as $T$, $P$ and $P'$, which we cannot calculate reliably. Instead of evaluating the hadronic matrix elements based on a specific hadronic model, let us make a crude estimation assuming the relation $T = P = P'$. We can obtain the maximal CP asymmetry by choosing the integration of Eq.(19). If we define

$$A_R \equiv \frac{\int_R dsds' \left| \text{Re} \left( h(K_2^+,\rho)(s,s') \right) \right|}{\int_R dsds' \left| \text{Re} \left( f(K_1^+,K_2^+)(s,s') + 2\text{Re} \left( g(\rho,\rho)(s,s') \right) \right) \right|},$$

(21)

$$A_I \equiv \frac{\int_R dsds' \left| \text{Im} \left( h(K_2^+,\rho)(s,s') \right) \right|}{\int_R dsds' \left| \text{Re} \left( f(K_1^+,K_2^+)(s,s') + 2\text{Re} \left( g(\rho,\rho)(s,s') \right) \right) \right|},$$

(22)

$A_R$ and $A_I$ correspond to the optimal CP asymmetry in the case of $\delta_P = 0$, $\delta'_P = \phi_3 = \pi/2$ and $\delta_P = \delta'_P = 0$, $\phi_3 = \pi/2$, respectively. We also calculate CP asymmetry using other $\pi\pi$ resonances, such as $f_0$ and $f_2$ summarized in Table 2. Although the isospin relation cannot be used for $f_0$, $f_2$ and therefore we cannot determine $\phi_3$ from these modes, a large direct CP asymmetry may be observable. In Table 3 we present the values of $A_R$ and $A_I$ not only in case of $K_2^+$ and $\rho$, but also in case of other resonances. Although the actual CP asymmetry indeed depends on the poorly known value such as $\phi_3$, $\delta_P$ and $\delta'_P$, it is reasonable to expect that the CP asymmetry can be as large as 25% level at somewhere around the overlapping region.

It is straightforward to write a general formula which includes all of the intermediate resonances as follows:

$$d\Gamma(B^- \rightarrow K^-\pi^-\pi^+)/dsds'$$

$$= \sum_i |a_i|^2 \text{Re} \left( f(K_i^+,K_i^+)(s,s') \right) + \sum_i |b_i|^2 \text{Re} \left( g^{f_i,f_i}(s,s') \right)$$

$$+ 2 \sum_{i<j} \left\{ \text{Re} \left( a_i a_j^* \right) \text{Re} \left( f(K_i^+,K_j^+)(s,s') \right) - \text{Im} \left( a_i a_j^* \right) \text{Im} \left( f(K_i^+,K_j^+)(s,s') \right) \right\}$$

$$+ 2 \sum_{i<j} \left\{ \text{Re} \left( b_i b_j^* \right) \text{Re} \left( g^{f_i,f_j}(s,s') \right) - \text{Im} \left( b_i b_j^* \right) \text{Im} \left( g^{f_i,f_j}(s,s') \right) \right\}$$

If $\delta_P$ is large enough, it is possible to have a large direct CP violation in $B^\pm \rightarrow K^\pm\rho$. Hence we may observe the direct CP violation even in the region $s \gg m_{K_2^+}^2$, $s' \simeq m_\rho^2$. 

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\[ + \sum_{i,j} \left\{ \Re \left( a_i b_j^* \right) \Re \left( h^{(K_i^*f_j)}(s,s') \right) - \Im \left( a_i b_j^* \right) \Im \left( h^{(K_i^*f_j)}(s,s') \right) \right\}. \]

Since the mass of \( K_0^*(1430) \) is as same as that of \( K_2^*(1430) \), these resonances coincide at the kinematical region \( \sqrt{s} \simeq 1430 \text{ MeV} \). Even if the masses are degenerate, we can determine \(|a_{K_0^*}|, |a_{K_2^*}|, \Re(a_{K_0^*}a_{K_2^*})\) and \(\Im(a_{K_0^*}a_{K_2^*})\) by analyzing the Dalitz plot in the region \( s \simeq m_{K_0^*}^2 \) and \( s' \gg m_\rho^2 \) since the \( K_0^* \) and \( K_2^* \) contributions depend differently on \( s' \). Once \( a_{K_0^*} \) and \( a_{K_2^*} \) are known up to an overall phase, we can extract \( \Re(b_j), \Im(b_j), \Re(\bar{b}_j) \) and \( \Im(\bar{b}_j) \) similarly as in the case of \( K_2^* \) alone by examining the Dalitz plot in the overlapping region. In case of the \( \rho \) resonance we can again obtain another constraint from \( B^- \to \rho^- K \) with the isospin symmetry. We can therefore determine all the parameters including \( \phi_3 \). In case of \( f_0 \) and \( f_2 \) resonances we cannot determine \( \phi_3 \) although a large direct CP violation may be observable.

Here, we discuss the experimental feasibility to detect this CP violation. The detector and accelerator are assumed to be similar to the Belle detector in the KEK B-factory [2]. The decays of \( B \to K_2^*(1430) \pi \) and \( \rho K \) are used. Both branching fractions, i.e., \( \text{BR}(B \to K^*_2 \pi^\pm) \cdot \text{BR}(K_2^* \to K^+\pi^-) \) and \( \text{BR}(B \to \rho K) \cdot \text{BR}(\rho \to \pi^+\pi^-) \), are set to \( 10^{-5} \). The kinematical region of \( s < 3 \text{GeV}^2 \) and \( s' < 3 \text{GeV}^2 \) is about 12\% of the total phase space, considering the experimental acceptance and decay angular distributions. The total experimental acceptance is about 3\%, including the geometrical factor and other cuts, such as to suppress continuum background \((e^+e^- \to (\gamma) \to q\bar{q})\). The signal-to-noise ratio is estimated to be \( \sim 2 \). In the case of having a 25\% direct CP asymmetry, \( \sim 300 \text{ fb}^{-1} \) of the data on \( \Upsilon(4s) \) is necessary to obtain a \( 3\sigma \) asymmetry. A significant improvement can be expected, if the kinematical region is reduced to only around the resonance overlapping region.

In this paper we consider the direct CP violation in three-body decays, such as \( B^{\pm} \to K^{\pm}\pi^+\pi^- \). The \( B \) meson can decay to the three-body final state through two different channels, \( K_i^*\pi^\pm \) and \( \rho K^{\pm} \), and these amplitudes may interfere with the same orders of magnitude near the kinematical region where the \( K_i^*\pi^\pm \) and \( \rho K^{\pm} \) contributions overlap. The resonances \( K_i^* \) and \( \rho \) provide a large final-state-interaction phase, which is reliably calculated using the Breit-Wigner form. This yields a large CP asymmetry around the overlapping region, and we can expect the CP asymmetry to be as large as the 25\% level in this region. By measuring the distribution on the Dalitz plot and combining the measurement of branching ratio...
for $B^\pm \rightarrow \rho^\pm K$, it is possible to extract the weak phase, $\phi_3$.

This work was supported in part by the Grant-in-Aid of the Ministry of Education, Science, Sports and Culture, Government of Japan.
References

[1] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

[2] Belle collaboration, “A Study of CP Violation in B Meson Decays”, The technical design report, KEK Report 95-1, unpublished.

[3] BaBar collaboration, “Technical Design Report”, SLAC-R-95-457, March, 1995.

[4] A.B. Carter and A.I. Sanda, *Phys. Rev. Lett.* **45**, 952 (1980); *Phys. Rev.* **D23**, 1567 (1981); 
I.I. Bigi and A.I. Sanda, *Nucl. Phys.* **B193**, 85 (1981).

[5] For a review see, for example, Y. Nir and H.R. Quinn, *Ann. Rev. Nucl. Part. Sci.* **42**, 211 (1992); 
A.J. Buras and R. Fleischer, [hep-ph/9704370](http://arxiv.org/abs/hep-ph/9704370).

[6] D. Atwood, and A. Soni, *Z. Phys.* **C 64**, 241 (1994); *Phys. Rev. Lett.* **74**, 220 (1995).

[7] G. Eilam, M. Gronau, R. R. Mendel, *Phys. Rev. Lett.* **74**, 4984 (1995); 
R. Enomoto and M. Tanabashi, *Phys. Lett.* **B386**, 413 (1996); 
S. Gardner, H.B. O’Connell and A.W. Thomas, [hep-ph/9705453](http://arxiv.org/abs/hep-ph/9705453).

[8] D. Atwood, G. Eilam, M. Gronau and A. Soni, *Phys. Lett.* **B341**, 372 (1995).

[9] Particle Data Group, R.M. Barnett *et al., Phys. Rev.* **D54**, 1 (1996).

[10] F. Würthwein, [hep-ex/9706010](http://arxiv.org/abs/hep-ex/9706010).

[11] R. Fleischer and T. Mannel, [hep-ph/9704423](http://arxiv.org/abs/hep-ph/9704423), [hep-ph/9706261](http://arxiv.org/abs/hep-ph/9706261).

[12] H.J. Lipkin, Y. Nir, H.R. Quinn and A.E. Snyder, *Phys. Rev.* **D44**, 1454 (1991); A.E. Snyder and H.R. Quinn, *Phys. Rev.* **D48**, 2139 (1993).

[13] A.S. Dighe, M. Gronau and J.L. Rosner, [hep-ph/9709223](http://arxiv.org/abs/hep-ph/9709223).

[14] A.F. Falk, A.L. Kagan, Y. Nir and A.A. Petrov, [hep-ph/9712223](http://arxiv.org/abs/hep-ph/9712223). 
M. Neubert, [hep-ph/9712224](http://arxiv.org/abs/hep-ph/9712224).
D. Atwood and A. Soni, hep-ph/9712287.
see also A. Buras, R. Fleisher and T. Mannel, hep-ph/9711262.

[15] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, 103 (1987).
| $K^*_i$     | spin | mass (MeV) | width (MeV) | $B(K^*_i \rightarrow K^-\pi^+)$ |
|------------|------|------------|-------------|---------------------------------|
| $K^*(892)$ | 1    | 896        | 51          | 0.67                            |
| $K^*_0(1430)$ | 0    | 1429       | 287         | 0.62                            |
| $K^*_2(1430)$ | 2    | 1432       | 109         | 0.33                            |

Table 1: Summary of $K\pi$ resonances.

| $f_i$   | spin | mass (MeV) | width (MeV) | $B(f_i \rightarrow \pi^+\pi^-)$ |
|---------|------|------------|-------------|---------------------------------|
| $\rho(770)$ | 1    | 769        | 151         | 1                               |
| $f_0(980)$  | 0    | 980        | 40-100      | 0.52                            |
| $f_2(1270)$ | 2    | 1275       | 185         | 0.57                            |

Table 2: Summary of $\pi\pi$ resonances.

| $K_i$ | $f_i$ | $A_R$ | $A_I$ |
|-------|-------|-------|-------|
| $K^*$ | $\rho$ | 0.13  | 0.087 |
| $K^*$ | $f_0$  | 0.15  | 0.10  |
| $K^*$ | $f_2$  | 0.26  | 0.25  |
| $K_0$ | $\rho$ | 0.20  | 0.22  |
| $K_0$ | $f_0$  | 0.26  | 0.28  |
| $K_0$ | $f_2$  | 0.26  | 0.27  |
| $K_2$ | $\rho$ | 0.26  | 0.25  |
| $K_2$ | $f_0$  | 0.29  | 0.26  |
| $K_2$ | $f_2$  | 0.26  | 0.25  |

Table 3: CP asymmetries defined in Eq.(21)-(22). Each row corresponds to the asymmetries, taking into account only one resonance for each $\pi\pi$ and $\pi K$ channel. Here the width for $f_0$ is assumed to be 70 MeV.
FIG. 1: Kinematical functions defined in Eq.(13). We take into account the $\rho$ and $K^*_2(1430)$ resonances for the $\pi\pi$ and $K\pi$ channels, respectively. $s = (p_K^- + p_{\pi^+})^2$ and $s' = (p_{\pi^-} + p_{\pi^+})^2$. 