Conditions for large non-Gaussianity in two-field slow-roll inflation

Christian T Byrnes¹, Ki-Young Choi²,³ and Lisa M H Hall⁴

¹ Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany
² Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
³ Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
⁴ Department of Applied Mathematics, University of Sheffield, Sheffield S3 7RH, UK
E-mail: C.Byrnes@thphys.uni-heidelberg.de, kiyoung.choi@uam.es and lisa.hall@sheffield.ac.uk

Received 16 July 2008
Accepted 8 September 2008
Published 8 October 2008

Online at stacks.iop.org/JCAP/2008/i=10/a=008
doi:10.1088/1475-7516/2008/10/008

Abstract. We study the level of primordial non-Gaussianity in slow-roll two-field inflation. Using an analytic formula for the non-linear parameter $f_{NL}$ in the case of a sum or product separable potential, we find that it is possible to generate significant non-Gaussianity even during slow-roll inflation with Gaussian perturbations at Hubble exit. In this paper we give the general conditions for obtaining large non-Gaussianity and calculate the level of fine-tuning required to obtain this. We present explicit models in which the non-Gaussianity at the end of inflation can exceed the current observational bound of $|f_{NL}| \lesssim 100$.

Keywords: cosmological perturbation theory, inflation

ArXiv ePrint: 0807.1101
1. Introduction

Standard slow-roll single-field inflation generates a quasi-scale invariant spectrum of Gaussian, adiabatic perturbations. While this is in agreement with current data, string theory and SUSY generically contain many scalar fields and it is important to test how these extra fields may change the predictions of the simplest models. In particular, the simplest models predict a level of non-Gaussianity which is too small to observe in the foreseeable future [1]–[3], so any detection would be of great significance.

In this paper we focus on the possibility of obtaining a large level of non-Gaussianity during slow-roll inflation and derive general conditions for when it may be large. We present simple, explicit two-field models which can saturate the observational bound. Although there are many models of inflation which generate a large non-Gaussianity [4]–[10], few of them do so during inflation except by breaking slow-roll, e.g. with a kink in the potential [11] or having a non-standard kinetic term, e.g. DBI inflation [12]. Other methods widely considered for generating a large non-Gaussianity include the curvaton scenario [13], modulated reheating [14] and an inhomogeneous end to inflation [15]–[19]. All these scenarios require an additional light scalar field that does not affect the dynamics during inflation but that becomes important either at the end of inflation or later.

Several authors have considered the possibility of generating non-Gaussianity during inflation for a separable potential. In particular Vernizzi and Wands calculated a general formula for the non-linearity parameter $f_{NL}$ which parameterizes the bispectrum in the case of a sum separable potential [20]; this was later extended to the case of a...
Conditions for large non-Gaussianity in two-field slow-roll inflation

product separable potential including non-canonical kinetic terms [21] and a sum separable potential for an arbitrary number of fields [22]. This has been further extended to the trispectrum (four-point function) [23]. For other approaches, see for example [24, 25].

Rather less work, however, has been done on analysing the formulae which have been calculated and are present in the literature. The results are rather long and depend on many parameters. In general they appear to give a slow-roll suppressed non-Gaussianity, subject to several simplifying assumptions that are additional to the slow-roll assumption and that are not always valid. Here we carefully consider several explicit models of two-field inflation and scan the parameter space more generally than has been done before. We show that it is possible to generate an extremely large non-Gaussianity during slow-roll inflation even when the field perturbations at Hubble exit are Gaussian. However this is only possible for specific values of the model parameters and initial conditions. We consider under which conditions a general model can generate a large non-Gaussianity and show that this always requires some fine-tuning of initial conditions.

Although previous papers have shown that it is possible to generate a narrow ‘spike’ of large non-Gaussianity while the inflaton trajectory turns a corner, the non-Gaussianity decays again very quickly after the corner [20, 24]. This spike of non-Gaussianity is associated with a temporary ‘jump’ in the slow-roll parameters [25]. In the models that we consider the non-Gaussianity is large over many e-foldings of inflation and all of the slow-roll parameters remain smaller than unity.

The paper is organized as follows. First we define relevant quantities and introduce some notation (section 2). Then, in the following two sections we derive the general conditions for generating a large non-Gaussianity during inflation, in section 3 for a product separable potential and in section 4 for a sum separable potential. In section 5 we give specific examples for product and sum potentials which can generate large non-Gaussianity. This includes a two-field model of hybrid inflation in section 5.3. In section 6 we extend the previous results to a generalized action with non-canonical kinetic terms. We conclude in section 7.

2. Background theory

In this paper, we consider an inflationary epoch driven by two scalar fields, whose dynamics are governed by the action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[ M_p^2 \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \chi \partial_{\nu} \chi - W(\varphi, \chi) \right].$$

(1)

Here $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass. We consider slow-roll inflation, during which all of the slow-roll parameters defined below are less than unity.

$$\epsilon_\varphi = \frac{M_p^2}{2} \left( \frac{W_\varphi}{W} \right)^2, \quad \epsilon_\chi = \frac{M_p^2}{2} \left( \frac{W_\chi}{W} \right)^2, \quad \epsilon = \epsilon_\varphi + \epsilon_\chi,$$

$$\eta_{\varphi\varphi} = M_p^2 \frac{W_{\varphi\varphi}}{W}, \quad \eta_{\varphi\chi} = M_p^2 \frac{W_{\varphi\chi}}{W}, \quad \eta_{\chi\chi} = M_p^2 \frac{W_{\chi\chi}}{W}.$$  

(2)

While it is physically interesting to consider the slow-roll parameters, it will be useful to use the definition of angles along or perpendicular to the background trajectory of the
Conditions for large non-Gaussianity in two-field slow-roll inflation

two inflationary fields [26]:
\[
\cos \theta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}, \quad \sin \theta = \frac{\dot{\chi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}},
\]
(3)
such that, in the slow-roll approximation,
\[
\frac{\epsilon_\phi}{\epsilon} = \cos^2 \theta, \quad \frac{\epsilon_\chi}{\epsilon} = \sin^2 \theta.
\]
(4)

Throughout this paper we use formulae given in [21] and we refer the reader to this paper and references therein for more details. In summary we define several observable quantities in terms of the primordial curvature perturbation \(\zeta\), which may be calculated using the \(\delta N\) formalism [27]–[31]. The number of e-foldings, \(N\), given by
\[
N = \int_{t_*}^{t_{\text{fin}}} H(t) \, dt,
\]
(5)
is evaluated from an initial flat hypersurface to a final uniform density hypersurface. The perturbation in the number of e-foldings, \(\delta N\), is the difference between the curvature perturbations on the initial and final hypersurfaces. In this paper we take the initial time to be the Hubble exit during inflation, denoted by \(t_*\). In this paper, we consider the final time to be a uniform density hypersurface just before the end of inflation, as in [20, 21]. We note that this differs from the hybrid scenario of [18], in which the final hypersurface is determined by a separate waterfall field. The curvature perturbation is given by [31]
\[
\zeta = \delta N = \sum_I N_{I} \delta \varphi_{I}* + \frac{1}{2} \sum_{IJ} N_{I,J} \delta \varphi_{I}* \delta \varphi_{J}* + \cdots,
\]
(6)
where \(N_{I} = \partial N/\partial (\varphi^*_I)\) and the index \(I\) runs over all of the fields. We will consider the power spectrum and bispectrum defined (in Fourier space) by
\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2) \frac{2\pi^2}{k_1^3} P_\zeta(k_1),
\]
(7)
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3).
\]
(8)

From this we can define three quantities of key observational interest, respectively the spectral index, the tensor-to-scalar ratio and the non-linearity parameter:
\[
n_\zeta - 1 = \frac{\partial \log P_\zeta}{\partial \log k},
\]
(9)
\[
r = \frac{P_T}{P_\zeta} = \frac{8 P_*}{M_p^2 P_\zeta},
\]
(10)
\[
f_{\text{NL}} = \frac{5}{6} \frac{k_1^3 k_2^3 k_3^3}{k_1^3 + k_2^3 + k_3^3} \frac{B_\zeta(k_1, k_2, k_3)}{4\pi^4 P_\zeta^2}.
\]
(11)

Here \(P_*\) is the power spectrum of the scalar field fluctuations and \(P_T = 8 P_* = 8H_*^2/(4\pi^2 M_p^2)\) is the power spectrum of the tensor fluctuations. Both the spectra are calculated at the end of inflation and we ignore any evolution after this time. For full
Conditions for large non-Gaussianity in two-field slow-roll inflation

As defined above, \( f_{NL} \) is shape dependent, but it has been shown that the shape dependent part is much less than unity \([20,33]\). The ideal CMB experiment is only expected to reach a precision of \( f_{NL} \) around unity \([34]\), so we will calculate the shape independent part of \( f_{NL} \), denoted by \( f_{NL}^{(4)} \). Whenever the non-Gaussianity is large, \(|f_{NL}| > 1\), we can make the association \( f_{NL}^{(4)} \approx f_{NL} \). This \( (k \text{ independent}) \) part of \( f_{NL} \) and the spectral index can be calculated using the \( \delta N \) formalism,

\[
P_\zeta = \sum_I N_{II}^2 P_*,
\]

\[
n_\zeta - 1 = -2e^* + \frac{2}{H} \frac{\sum_{IJ} \dot{\phi} I N_{IJ}}{\sum_K N_{KK}^2},
\]

\[
f_{NL}^{(4)} = \frac{5}{6} \frac{\sum_{IJ} N_{IJ} N_{IJ} N_{IJ} N_{IJ}}{(\sum J N_{JJ}^2)^2}.
\]

There is no universal agreement over the sign of \( f_{NL} \) in the literature. We use the opposite sign convention to \([20,21]\), in order to be in agreement with the WMAP sign convention \([35]\). The latest observations from five years of WMAP data are

\[
n_\zeta = 0.96^{+0.014}_{-0.013}, \quad \text{(assuming } r = 0),
\]

\[
r < 0.2 \quad \text{(95\% CL)},
\]

\[
-9 < f_{NL}^{(\text{local})} < 111 \quad \text{(95\% CL)}.
\]

The ‘local’ bound on \( f_{NL} \) is based on the definition \( \zeta = \zeta_G + 3 f_{NL} \zeta_G^2 / 5 \), where \( \zeta_G \) is the Gaussian part of \( \zeta \) \([34,35]\). When more than one field direction during inflation contributes to \( \zeta \) in equation (6), then this definition of \( f_{NL} \) is not equivalent to the very commonly used definition equation (11), even though they have the same shape dependence (see e.g. \([36]\)). The bound on \( f_{NL}^{(\text{local})} \) may therefore not be applicable to the \( f_{NL} \) that we calculate. In this paper we will focus on the non-linearity parameter \( f_{NL} \). Constraints on this parameter are expected to improve by nearly an order of magnitude with the PLANCK satellite \([37]\).

3. Large non-Gaussianity in product potentials, \( W(\varphi, \chi) = U(\varphi)V(\chi) \)

We follow the terminology of \([20]\) and use the definitions

\[
u \equiv \frac{\epsilon^e}{\epsilon^e} = \cos^2 \theta^e, \quad v \equiv \frac{\epsilon^\chi}{\epsilon^\chi} = \sin^2 \theta^e.
\]

We will use a subscript or superscript ‘*’ to denote values evaluated at the time of Hubble exit and a subscripted ‘\( e \)’ will denote the time at \( t_e \). From equations (12) and (13), the power spectrum and spectral index for a product potential with canonical kinetic terms are

\[
P_\zeta = \frac{W_s}{24\pi^2 M_P^4} \left( \frac{u^2}{\epsilon_{\varphi}^e} + \frac{v^2}{\epsilon_{\chi}^e} \right),
\]

\[
n_\zeta - 1 = -2e^* - 4 \frac{\eta_{\varphi \varphi}^\epsilon / 2\epsilon_{\varphi}^e + \eta_{\chi \chi}^\epsilon / 2\epsilon_{\chi}^e}{u^2/\epsilon_{\varphi}^e + v^2/\epsilon_{\chi}^e}.
\]
The non-linear parameter \( f_{\text{NL}}^{(4)} \) becomes \[ f_{\text{NL}}^{(4)} = \frac{5}{6} \left( \frac{2}{u^2/\epsilon_\varphi + v^2/\epsilon_\chi} \right)^2 \left[ \frac{u^3}{\epsilon_\varphi^2} \left( 1 - \frac{\eta_{\varphi \varphi}}{2 \epsilon_\varphi^2} \right) + \frac{v^3}{\epsilon_\chi^2} \left( 1 - \frac{\eta_{\chi \chi}}{2 \epsilon_\chi^2} \right) - \left( \frac{u}{\epsilon_\varphi} - \frac{v}{\epsilon_\chi} \right)^2 \mathcal{A}_p \right], \]

where

\[
\hat{\eta} \equiv \frac{\epsilon_\chi \eta_{\varphi \varphi} + \epsilon_\varphi \eta_{\chi \chi}}{\epsilon},
\]

\[
\mathcal{A}_p \equiv -\frac{\epsilon_\varphi \epsilon_\chi}{(\epsilon_\varphi^2 \epsilon_\chi^2)} \left[ \hat{\eta}^2 - 4 \left( \frac{\epsilon_\varphi \epsilon_\chi}{\epsilon} \right)^2 \right].
\]

The slow-roll parameters, defined by equation (2), are

\[
\epsilon_\varphi = \frac{M_p^2}{2} \left( \frac{U_{\varphi}}{U} \right)^2 = \epsilon \cos^2 \theta, \quad \epsilon_\chi = \frac{M_p^2}{2} \left( \frac{V_\chi}{V} \right)^2 = \epsilon \sin^2 \theta,
\]

and

\[
\eta_{\varphi \varphi} = M_p^2 \frac{U_{\varphi \varphi}}{U}, \quad \eta_{\varphi \chi} = M_p^2 \frac{U_{\varphi \chi}}{W}, \quad \eta_{\chi \chi} = M_p^2 \frac{V_{\chi \chi}}{V}.
\]

Using a few auxiliary functions (involving only \( \theta^* \) and \( \theta^e \)), we may write equation (21) as

\[
f_{\text{NL}}^{(4)} = \frac{5}{6} \left( 2j_p(\theta^*, \theta^e) \epsilon^* - f_p(\theta^*, \theta^e) \eta_{\varphi \varphi}^* + g_p(\theta^*, \theta^e) \eta_{\chi \chi}^* + 2h_p(\theta^*, \theta^e, X)(\hat{\eta}^2 - 4 \sin^2 \theta^e \cos^2 \theta^e) \right],
\]

where the auxiliary functions are defined as

\[
f_p(\theta^*, \theta^e) \equiv \left( \frac{u^2 \sin^4 \theta^*}{u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*} \right)^2 = \tan^4 \theta^* - \frac{\tan^2 \theta^*}{(\tan^2 \theta^* + \tan^4 \theta^e)^2} \cos^2 \theta^e;
\]

\[
g_p(\theta^*, \theta^e) \equiv \left( \frac{v^3 \cos^4 \theta^e}{u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*} \right)^2 = \tan^8 \theta^e;
\]

\[
h_p(\theta^*, \theta^e) \equiv \left( \frac{u \sin^2 \theta^* - v \cos^2 \theta^e}{u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*} \right)^2 = \tan^2 \theta^e \left( \tan^2 \theta^* - \tan^2 \theta^e \right)^2 \sec^2 \theta^e;
\]

\[
j_p(\theta^*, \theta^e) \equiv \left( \frac{u^3 \sin^4 \theta^* \cos^2 \theta^* + v^3 \sin^2 \theta^* \cos^4 \theta^*}{u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*} \right)^2 = \frac{\sin^2 \theta^* \left( \tan^2 \theta^* + \tan^6 \theta^e \right)}{\cos^2 \theta^e \left( \tan^2 \theta^* + \tan^4 \theta^e \right)^2}.
\]

By analysing the functions \( j_p, f_p, g_p \) and \( h_p \) over the range of allowed values for \( \theta^* \) and \( \theta^e \), it is possible to locate regions of parameter space which give large \( f_{\text{NL}}^{(4)} \). We find that the function \( j_p(\theta^*, \theta^e) \) satisfies \( 0 \leq j_p \leq 1 \) and therefore the \( 'j' \)-term can never lead to significant values of \( f_{\text{NL}}^{(4)} \). We therefore ignore this term in the analysis that follows.

The functions \( f_p, g_p \) and \( h_p \) (as functions of \( \theta^* \) and \( \theta^e \)) are plotted in figure 1. The main point to note is that these prefactors are large (and can be very large) in two regions:

A \( \cos^2 \theta^* \ll 1 \) and \( \cos^2 \theta^e \ll 1 \) (\( \epsilon_\chi \gg \epsilon_\varphi \)). In this region, \( f_p \gg g_p \) and \( h_p \sim f_p \sin^2 \theta^e \).

B \( \sin^2 \theta^* \ll 1 \) and \( \sin^2 \theta^e \ll 1 \) (\( \epsilon_\varphi \gg \epsilon_\chi \)). In this region, \( g_p \gg f_p \) and \( h_p \sim g_p \cos^2 \theta^e \).
Conditions for large non-Gaussianity in two-field slow-roll inflation

Figure 1. The contour plot of the functions $f_p$, $g_p$ and $h_p$, in the plane of $\theta^*$ and $\theta^e$. The bottom and left-hand axes show the angles $\theta^*$ and $\theta^e$, respectively. The top and right-hand axes show $\sin^2 \theta^*$ and $\sin^2 \theta^e$.

In both of these regions, these prefactors can account for a significant level of $f_{NL}^{(4)}$, despite the relevant terms in equation (26) having slow-roll factors. Each of these regions describes one of the fields dominating over the other in kinetic energy. Note, however, that single-field inflation (i.e. with one field static) will not lead to a large value of non-Gaussianity, since these fractions vanish exactly for $\dot{\chi}^e = 0$ or $\dot{\phi}^e = 0$ ($\theta^e = 0$ or $\pi/2$ respectively).

Due to the symmetry, we shall focus on region B and explicitly write down full conditions for both regions A and B in section 3.1. To find the condition to give large non-Gaussianity, we will concentrate on the terms including the functions of $g_p(\theta^*, \theta^e)$ and $h_p(\theta^*, \theta^e)$, since the other terms cannot give large non-Gaussianity as discussed above. The large $f_{NL}^{(4)}$ is given by

$$f_{NL}^{(4)} \approx \frac{5}{6} g_p(\theta^*, \theta^e) \left[ -\eta_{\chi\chi}^* + 2 \cos^2 \theta^e \left( \tilde{\eta}^e - 4 \sin^2 \theta^e \cos^2 \theta^e \epsilon e \right) \right],$$

$$\approx \frac{5}{6} \frac{\sin^6 \theta^e}{(\sin^2 \theta^* + \sin^2 \theta^e)^2} \left[ -\eta_{\chi\chi}^* + 2 \eta_{\chi\chi}^e \right]. \quad (28)$$

This bound is shown graphically in figure 2. Here we assumed that $|\eta_{\chi\chi}| \gg \sin^2 \theta^e |\eta_{\chi\chi}^e - 4 \epsilon e|$. For $|f_{NL}|$ to be larger than unity, the function $g_p(\theta^*, \theta^e)$ must be bigger than the inverse of the slow-roll parameters in the square bracket. In this way, from equation (28) we find the general way to obtain large non-Gaussianity, $|f_{NL}^{(4)}| \gtrsim 1$,

$$\sin^2 \theta^* \lesssim \sin^4 \theta^e \left( \frac{1}{\sqrt{\sin^2 \theta^e G_p}} - 1 \right), \quad G_p = \frac{6}{5} \left| -\eta_{\chi\chi}^* + 2 \eta_{\chi\chi}^e \right|^{-1}. \quad (29)$$
Figure 2. A blown-up graph of region B. The condition for \( g_p \) in equation (29) is plotted for \( G_p = 1000 \) with a white line. It can be seen that this condition encloses the contour for \( g_p \) larger than some constant, \( G_p \).

From this condition we can derive some corollaries. Firstly, from \( \partial g_p / \partial (\sin^2 \theta_e) = 0 \) and the definition of \( g_p \), the initial and final angle of the trajectory are constrained:

\[
\sin^2 \theta^* < \frac{1}{3} \left( \frac{3}{4} \right)^2 \frac{1}{G_p^2}, \quad \text{and} \quad \sin^2 \theta^c < \frac{1}{G_p},
\]

and secondly we see that \( \sin^2 \theta \) must typically grow by at least two orders of magnitude during inflation, since we require

\[
\frac{\sin^2 \theta^c}{\sin^2 \theta^*} > 4G_p.
\]

In this region of large non-Gaussianity, the power spectrum from equation (19), the spectral index from equation (20) and the tensor-to-scalar ratio in region B are

\[
\mathcal{P}_\zeta \simeq \frac{W_*}{24\pi^2 M_p^2 \epsilon^*} (1 + \tilde{r}),
\]

\[
n_\zeta - 1 \simeq -2\epsilon^* + 2 \frac{-2\epsilon^* + \eta^*_\varphi^* + \tilde{r} \eta^*_\chi^*}{1 + \tilde{r}},
\]

\[
r \simeq 16\epsilon^* (1 + \tilde{r})^{-1},
\]

where

\[
\tilde{r} = \frac{\sin^4 \theta^c}{\sin^2 \theta^*} > 0.
\]

The spectrum of curvature perturbations can be dominated by the fluctuations in the slow-rolling field \( \varphi \) or \( \chi \) depending on \( \tilde{r} \), which can be smaller or bigger than unity. We note, however, that the tensor-to-scalar ratio can only be suppressed in multiple-field
Conditions for large non-Gaussianity in two-field slow-roll inflation

inflation, since the tensor power spectrum is unchanged from the single-field value and $P_\zeta$ is enhanced. The spectral index is changed by the presence of $\tilde{r}$, but remains slow-roll suppressed.

3.1. Condition for large non-Gaussianity in product potentials

If we summarize, the general conditions for obtaining large $f_{\text{NL}}^{(4)}$ can be written simply. Ultimately, the two-field system must have one field which dominates the evolution throughout. This corresponds to being in region A or B (depending on which field dominates). Hence, one field must dominate the evolution almost fully at the beginning, while the other field gains a relatively large percentage of kinetic energy by the end of inflation. Specifically, the slow-roll parameters and the value of $f_p$ or $g_p$ must be such as to give $|f_{\text{NL}}^{(4)}| \gtrsim 1$ in equation (26).

For region A ($\cos^2 \theta \ll 1$), the condition is

$$\cos^2 \theta^* \lesssim \cos^4 \theta^c \left(\frac{1}{\sqrt{\cos^2 \theta^c F_p}} - 1\right), \quad F_p = \frac{6}{5} \left| -\eta_{\varphi \varphi}^* + 2\eta_{\varphi \varphi}^e \right|^{-1} \quad \text{(Region A)}, \quad (36)$$

where we have assumed that $|\eta_{\varphi \varphi}| \gg \cos^2 \theta^c |\eta_{\chi \chi} - 4\epsilon^e|$. For region B ($\sin^2 \theta \ll 1$), the condition is

$$\sin^2 \theta^* \lesssim \sin^4 \theta^c \left(\frac{1}{\sqrt{\sin^2 \theta^c G_p}} - 1\right), \quad G_p = \frac{6}{5} \left| -\eta_{\chi \chi} + 2\eta_{\chi \chi}^e \right|^{-1} \quad \text{(Region B)}, \quad (37)$$

where we have assumed that $|\eta_{\chi \chi}| \gg \sin^2 \theta^c |\eta_{\varphi \varphi} - 4\epsilon^e|$. This condition also leads to a large growth of the subdominant slow-roll parameter. For regions A and B respectively,

$$\frac{\cos^2 \theta^c}{\cos^2 \theta^*} = \frac{\epsilon^e_{\varphi} \epsilon^*_{\varphi}}{\epsilon^e_{\chi} \epsilon^*_{\chi}} \gtrsim 4F_p \quad \text{(Region A)}, \quad \frac{\sin^2 \theta^c}{\sin^2 \theta^*} = \frac{\epsilon^e_{\chi}}{\epsilon^e_{\varphi} \epsilon^*_{\chi}} \gtrsim 4G_p \quad \text{(Region B)} \quad (38)$$

3.2. Direct observation of $f_{\text{NL}}^{(4)}$ and fine-tuning

Equations (36) and (37) encode the level of fine-tuning required on the initial and final slow-roll parameters to obtain an observable level of non-Gaussianity. As an example, if observations find $f_{\text{NL}}^{(4)} \sim 10$, for the standard order of slow-roll parameters ($\eta \sim 0.01$), one requires $f_p$ or $g_p \sim 1000$. In the case of region B, this corresponds to

$$\frac{\epsilon^e_{\chi}}{\epsilon^e} = \sin^2 \theta^e \lesssim 10^{-7}, \quad \text{and} \quad \frac{\epsilon^e_{\varphi}}{\epsilon^e} = \sin^2 \theta^e \lesssim 10^{-3}. \quad (39)$$

This requires very special initial values for the fields.
4. Large non-Gaussianity in sum potentials, \( W(\varphi, \chi) = U(\varphi) + V(\chi) \)

In this section we find the general condition for large non-Gaussianity with a sum separable potential, \( W(\varphi, \chi) = U(\varphi) + V(\chi) \). Defining

\[
u \equiv \frac{U^* + Z^e}{W^*}, \quad v \equiv \frac{V^* - Z^e}{W^*},
\]

with

\[
Z^e = \left( \frac{V^e \epsilon_\varphi^e - U^e \epsilon_\chi^e}{\epsilon_e} \right) = V^e \cos^2 \theta^e - U^e \sin^2 \theta^e,
\]

the power spectrum and spectral index are given by [20]

\[
P_{\zeta} = \frac{W_*}{24 \pi^2 M_*^4} \left( \frac{u^2}{\epsilon_\varphi^e} + \frac{v^2}{\epsilon_\chi^e} \right),
\]

\[
n_{\zeta} - 1 = -2 \epsilon^e - 4 \frac{u}{\epsilon_\varphi^e} (1 - (\eta^*_{\varphi\varphi}/2\epsilon_\varphi^e) u) + v (1 - (\eta^*_{\chi\chi}/2\epsilon_\chi^e) v) \frac{u^2}{\epsilon_\varphi^e} + \frac{v^2}{\epsilon_\chi^e}.
\]

The non-linear parameter \( f_{\text{NL}}^{(4)} \) is [20]

\[
f_{\text{NL}}^{(4)} = \frac{5}{6} \frac{2}{(u^2/\epsilon_\varphi^e + v^2/\epsilon_\chi^e)^2} \left[ \frac{u^2}{\epsilon_\varphi^e} \left( 1 - \frac{\eta^*_{\varphi\varphi}}{2\epsilon_\varphi^e} u \right) + \frac{v^2}{\epsilon_\chi^e} \left( 1 - \frac{\eta^*_{\chi\chi}}{2\epsilon_\chi^e} v \right) + \frac{u}{\epsilon_\varphi^e} - \frac{v}{\epsilon_\chi^e} \right]^2 A_S,
\]

where we define

\[
\hat{\eta} \equiv \frac{(\epsilon_\chi \eta_{\varphi\varphi} + \epsilon_\varphi \eta_{\chi\chi})}{\epsilon} = \eta_{\varphi\varphi} \sin^2 \theta + \eta_{\chi\chi} \cos^2 \theta,
\]

\[
A_S \equiv -\frac{W^2 \epsilon_\varphi^e \epsilon_\chi^e}{W_* (\epsilon_e)^2} [\epsilon^e - \hat{\eta}^e] = -\frac{W^2}{W_*} \cos^2 \theta^e \sin^2 \theta^e [\epsilon^e - \hat{\eta}^e].
\]

The slow-roll parameters, defined by equation (2), are

\[
\epsilon_\varphi = \frac{M^2}{2} \left( \frac{U_\varphi}{U + V} \right)^2 = \epsilon \cos^2 \theta, \quad \epsilon_\chi = \frac{M^2}{2} \left( \frac{V_\chi}{U + V} \right)^2 = \epsilon \sin^2 \theta,
\]

and

\[
\eta_{\varphi\varphi} = M^2 \frac{V_{\varphi\varphi}}{U + V}, \quad \eta_{\chi\chi} = 0, \quad \eta_{\varphi\chi} = M^2 \frac{V_{\varphi\chi}}{U + V}.
\]

Like in the analysis of a product potential, it is possible to rewrite equation (44):

\[
f_{\text{NL}}^{(4)} = \frac{5}{6} \left[ 2 j_s \epsilon^e - f_s \hat{\eta}_{\varphi\varphi}^e - g_s \eta_{\chi\chi}^e - 2 h_s \frac{W^2}{W_*} (\epsilon^e - \hat{\eta}^e) \right],
\]
where the following auxiliary functions have been used:

\[ f_s(u, \sin^2 \theta^*) = \frac{u^3 \sin^4 \theta^*}{(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*)^2}, \]

\[ g_s(u, \sin^2 \theta^*) = \frac{v^3 \cos^4 \theta^*}{(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*)^2}, \]

\[ h_s(u, \sin^2 \theta^*) = \frac{\sin^2 \theta^* \cos^2 \theta^* (u \sin^2 \theta^* - v \cos^2 \theta^*)^2}{(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*)^2}, \]

\[ j_s(u, \sin^2 \theta^*) = \frac{(u^2 \sin^4 \theta^* \cos^2 \theta^* + v^2 \cos^4 \theta^* \sin^2 \theta^*)}{(u^2 \sin^2 \theta^* + v^2 \cos^2 \theta^*)^2}. \] (50)

Note that \( u + v = 1 \) and hence these functions depend on just two variables, \( u \) and \( \theta^* \) (or \( v \) and \( \theta^* \)), and that \( 0 \leq u \leq 1 \) and \( 0 \leq v \leq 1 \). The first term of \( f_{\text{NL}}^{(4)} \) in equation (49) is always smaller than unity, since \( 0 \leq j_s \leq 1 \).

Once again there are two regions with potentially large non-Gaussianity:

C \( u \ll 1 \), and \( \cos^2 \theta^* \ll 1 \).

D \( v \ll 1 \), and \( \sin^2 \theta^* \ll 1 \).

We temporarily analyse this second region, region D, in which \( v \ll 1, \sin^2 \theta^* \ll 1 \) and \( g_s \) can be large. In this region, we can use the approximations

\[ g_s(v, \sin^2 \theta^*) \simeq \frac{v^3}{(\sin^2 \theta^* + v^2)^2}, \quad h_s(v, \sin^2 \theta^*) \simeq \frac{g_s(v, \sin^2 \theta^*) \sin^2 \theta^* \cos^2 \theta^*}{v}. \] (51)

In the special type of potential when \( V^e \simeq V^* \), such as hybrid inflation, \( v \sim \sin^2 \theta^* W^e/W^* \) and hence

\[ h_s(v, \sin^2 \theta^*) \simeq g_s(v, \sin^2 \theta^*) \cos^2 \theta^* \frac{W^*}{W^e}. \] (52)

The function \( g_s \) has exactly the same form as the function \( g_p \) for the product potential in the region where it is large; see equation (28), replacing \( \sin^2 \theta^* \) with \( v \). Hence it is large in exactly the same areas, and for \( g_s \gtrsim G_s \) we require

\[ \sin^2 \theta^* \lesssim v^2 \left( \frac{1}{\sqrt{v G_s}} - 1 \right), \quad G_s = \frac{6}{5} \left| -\eta_{\chi\chi^*}^* + 2\eta_{\chi\chi}^e \right|^{-1} \] (Region B). (53)

Finally, we can approximate \( f_{\text{NL}}^{(4)} \) in region D as

\[ f_{\text{NL}}^{(4)} \simeq \frac{5}{6} g_s \left[ -\eta_{\chi\chi^*}^* - \frac{2 W^2}{W^e} \sin^2 \theta^* \cos^2 \theta^* (e^e - \hat{\eta}^e) / v \right], \] (54)

where we have used \( u \simeq 1 \) and \( \cos^2 \theta^* \simeq 1 \). We may analyse this equation for the specific case when \( V^e \simeq V^* \), which can be true in many models because \( \sin^2 \theta^* \simeq (V^e/U^e)^2 \ll 1 \), so at least initially the potential \( V \) is extremely flat. In this scenario, \( v \sim \sin^2 \theta^* W^e/W^* \) and hence

\[ f_{\text{NL}}^{(4)} \simeq \frac{5}{6} g_s \left[ -\eta_{\chi\chi^*}^* - \frac{2 W^e}{W^*} \cos^2 \theta^* (e^e - \hat{\eta}^e) \right]. \] (55)

Indeed, the forms of the power spectrum and spectral index in this large non-Gaussianity region are the same as those for the product potential, equations (32) and (33).
4.1. Condition for large non-Gaussianity in sum potentials

From the previous section, it is clear that, to get large non-Gaussianity, we need a constraint on the function $g_s$; for region C,

$$\cos^2 \theta^* < u^2 \left( \frac{1}{\sqrt{u F_s}} - 1 \right), \quad F_s = \frac{6}{5} \left| -\eta_{\phi\phi}^* + 2\eta_{\phi\phi}^c \right|^{-1} \quad \text{(Region C).}$$

In the same way, for region D, we obtain

$$\sin^2 \theta^* < v^2 \left( \frac{1}{\sqrt{v G_s}} - 1 \right), \quad G_s = \frac{6}{5} \left| -\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^c \right|^{-1} \quad \text{(Region D).}$$

As in equation (30), we find exact upper limits on the parameters:

$$\cos^2 \theta^* < \frac{1}{3} \left( \frac{3}{4} \right)^4 \frac{1}{F_s^2}, \quad u < \frac{1}{F_s} \quad \text{and} \quad \frac{u}{\cos^2 \theta^*} > 4 F_p \quad \text{(Region C)},$$

$$\sin^2 \theta^* < \frac{1}{3} \left( \frac{3}{4} \right)^4 \frac{1}{G_s^2}, \quad v < \frac{1}{G_s} \quad \text{and} \quad \frac{v}{\sin^2 \theta^*} > 4 G_p \quad \text{(Region D).}$$

5. Examples

In this section we give specific examples, two of which can give large non-Gaussianity and one which cannot. We note that, in all the following examples, we have one parameter that acts as a normalization factor in $W_*$. This parameter can be fixed to normalize the amplitude of the perturbations.

5.1. Quadratic times exponential potential, $W(\varphi, \chi) = (1/2)e^{-\lambda \varphi^2/M_P^2}m^2\chi^2$

We consider a product potential, with $U(\varphi) = e^{-\lambda \varphi^2/M_P^2}$ and $V(\chi) = m^2\chi^2/2$, for which the slow-roll parameters are

$$\epsilon_\varphi = 2\lambda^2\frac{\varphi^2}{M_P^2}, \quad \eta_{\varphi\varphi} = -2\lambda + 4\lambda^2\frac{\varphi^2}{M_P^2}, \quad \epsilon_\chi = \eta_{\chi\chi} = \frac{2M_P^2}{\chi^2}, \quad \eta_{\varphi\chi} = -\frac{4\lambda \varphi}{\chi}. \quad (60)$$

We consider $\lambda > 0$ and slow-roll throughout inflation, so that the $\varphi$ field increases during inflation while the $\chi$ field decreases. In this slow-roll limit, the exponential function can be expanded so that the potential is dominated by the quadratic potential of the $\chi$ field, $\frac{1}{2}m^2\chi^2$, and corrected by the integration between the $\chi$ and $\varphi$ fields, $-\frac{1}{2}\lambda m^2\varphi^2\chi^2$, which might be important for preheating [38, 39]. Since inflation ends when the $\chi$ field rolls close to its minimum, i.e. when $\epsilon = 1 \approx \epsilon_\chi$, the only way to generate large non-Gaussianity is to start with small field of $\varphi$ (which corresponds to region A).

For this potential, the slow-roll solutions for $\varphi$ and $\chi$ lead to

$$\varphi = \varphi_* e^{2\lambda N}, \quad \chi^2 = \chi_*^2 - 4NM_P^2. \quad (61)$$
Conditions for large non-Gaussianity in two-field slow-roll inflation

Figure 3. The contour plot of $\log_{10}|f_{\text{NL}}^{(4)}|$ for example A, $W(\varphi, \chi) = \frac{1}{2} e^{-\lambda \varphi^2/M_P^2} m^2 \chi^2$. Here we used $\lambda = 0.04$ (left) and 0.05 (right). White regions indicate when $|f_{\text{NL}}^{(4)}| < 1$ and the dark centremost region indicates $|f_{\text{NL}}^{(4)}| > 100$.

Since $\lambda^2 \varphi^2/M_P^2 < 1$, we find $\eta_{\varphi\varphi} \sim \eta_{\chi\chi} \sim -2\lambda$. From equation (36), the constraints on the initial values of the field $\varphi$ can be obtained:

$$
\frac{M_P^2 \chi_e^4 e^{-12\lambda N_e}}{2\lambda^2 \chi_e^6} \lesssim \frac{\varphi_e^2}{M_P^2} \lesssim \frac{2M_P^2 e^{-4\lambda N_e}}{\lambda \chi_e^6},
$$

where $N_e \simeq (\chi_e^2 - \chi_e^2)/4M_P^2$ and $\chi_e^2 \simeq 2$. For the specific values of $\chi_e = 16M_P$ and $\lambda = \{0.03, 0.04, 0.05\}$, the constraints lead to

$$\{0.134, 2 \times 10^{-3}, 3 \times 10^{-5}\} \lesssim \frac{\varphi_e}{M_P} \lesssim \{0.128, 0.031, 0.0078\}.$$

In this range, the non-linearity parameter is well approximated as

$$
f_{\text{NL}}^{(4)} \simeq \frac{5}{6} \frac{\cos^6 \theta_e}{(\cos^2 \theta_e + \cos^4 \theta_e)^2} \eta_{\varphi\varphi}.
$$

The contour plots of $f_{\text{NL}}^{(4)}$ for two values of $\lambda$ ($\lambda = 0.04, 0.05$) are given in figure 3 using the full formula, equation (44), which is almost the same as equation (63). Here it can be seen that $f_{\text{NL}}^{(4)}$ can be very large when $\varphi_e \sim 10^{-3} M_P$ ($\lambda = 0.04$) or $\varphi_e \sim 10^{-4} M_P$ ($\lambda = 0.05$). We note that the analytic constraints differ slightly from the plots, due to the value of $\eta_{\varphi\varphi}$ used. In the analysis, we assume $\eta_{\varphi\varphi} \sim -2\lambda$, but use the exact form (equation (60)) in the plots.

The time evolution of $f_{\text{NL}}^{(4)}$ from the $\delta N$ formula is given in figure 4, where the final point is identified as some time during inflation. This method is also used in Vernizzi and Wands [20] for analytic evolution. We can see that $|f_{\text{NL}}^{(4)}|$ increases sharply around $N = 30$. This can be understood from the evolution of two fields: from this time, the velocity of $\varphi$ (or $\cos^2 \theta$) increases (as seen in the third and fourth plots of figure 4) to satisfy the condition equation (36) to obtain large non-Gaussianity. Then $f_{\text{NL}}^{(4)}$ begins to
Conditions for large non-Gaussianity in two-field slow-roll inflation

Figure 4. The analytic evolution of $f_{NL}^{(4)}$, $\epsilon_\chi$ and $\epsilon_\varphi$ for example A. The evolutions of both fields, $\varphi$ and $\chi$, are also shown. We used $\lambda = 0.05$ and $\varphi_* = 10^{-3} M_P$, $\chi_* = 16 M_P$. We numerically solved the full equations of motion until $\epsilon = 1$.

decrease around $N = 45$ as $\cos^4 \theta^e$ becomes bigger than $\cos^2 \theta^e$. Note that, around the region for large $|f_{NL}|$, the curvature perturbation of the $\varphi$ field becomes comparable to, and then dominates over, that of the $\chi$ field. Around $N = 58$, the $\chi$ field begins to roll so fast that $\cos^2 \theta$ decreases and $|f_{NL}^{(4)}|$ increases once more. However during this fast-roll stage, slow-roll breaks down and our analytic formula is not valid any longer. Thus we calculate final values when $\epsilon = 1$.

We note, for reference, that in this regime of large non-Gaussianity, the background trajectory must be at almost entirely in the one-field direction. We therefore expect reheating to take place in almost the same way as it would in normal chaotic inflation.

5.2. Quadratic product potential

In this section we show that a direct coupling between $\varphi$ and $\chi$, leading to the product potential

$$W = \frac{\lambda}{2} \varphi^2 \chi^2,$$  \hspace{1cm} (64)

cannot generate a large non-Gaussianity during inflation. From the equation $N = \int H \, dt$ it follows that the fields evolve according to

$$\varphi^2_e = \varphi^2_* - 4 N M_P^2,$$  \hspace{1cm} (65)

and similarly for $\chi$. Hence we have in region B, where $\sin^2 \theta = 1$ and $4 N M_P^2 < \varphi^2_* \ll \chi^2_*$,

$$\sin^2 \theta^e \simeq \frac{\epsilon_\varphi}{\epsilon_e} = \frac{\varphi^2_* - 4 N M_P^2}{\chi^2_* - 4 N M_P^2} < \frac{\varphi^2_*}{\chi^2_*} = \sin^2 \theta_*.$$

(66)

Hence $\sin^2 \theta$ decreases during inflation and this model cannot generate $|f_{NL}| > 1$.

5.3. Hybrid inflation

We consider a model of two-field hybrid inflation,

$$W(\varphi, \chi) = W_0 \left( 1 + \alpha \frac{\varphi^2}{M_P^2} + \beta \frac{\chi^2}{M_P^2} \right),$$  \hspace{1cm} (67)

Journal of Cosmology and Astroparticle Physics 10 (2008) 008 (stacks.iop.org/JCAP/2008/i=10/a=008)
Conditions for large non-Gaussianity in two-field slow-roll inflation

which is vacuum dominated, i.e. which satisfies $|\alpha \varphi^2| \ll M_P^2$ and $|\beta \chi^2| \ll M_P^2$. Here we assume that inflation ends abruptly with another waterfall field which we do not write down in the potential above. We note that $f_{\text{NL}}$ may change depending on the details of how the waterfall field is coupled to the two inflaton fields [15, 16, 18], but in general this is unlikely to generate a large contribution to $f_{\text{NL}}$ without fine-tuning [17].

In this regime the slow-roll solutions are [5] (we can identify $\eta_\phi = 2\alpha$ and $\eta_\sigma = 2\beta$ in the notation of that paper)

$$\varphi = \varphi_* e^{-2\alpha N}, \quad \chi = \chi_* e^{-2\beta N},$$ (68)

and the slow-roll parameters are

$$\eta_{\varphi\varphi} = 2\alpha, \quad \eta_{\chi\chi} = 2\beta, \quad \eta_{\varphi\chi} = 0,$$

$$\epsilon_\varphi = 2\alpha^2 \frac{\varphi^2}{M_P^2} \ll |\eta_{\varphi\varphi}|, \quad \epsilon_\chi = 2\beta^2 \frac{\chi^2}{M_P^2} \ll |\eta_{\chi\chi}|.$$ (69)

We note that the dominant slow-roll parameters $\eta_{\varphi\varphi}$ and $\eta_{\chi\chi}$ are constant in this model.

For this case, since $W^e \simeq W^i$, from equation (54) (or equivalently equation (55)) in region D, where $\sin^2 \theta^* \ll 1$ and $v \simeq \sin^2 \theta^e \ll 1$, it follows that

$$f_{NL}^{(4)} \simeq \frac{5}{6} g_s [\eta_{\chi\chi} - 2(e^* - \dot{\eta}^*)] \simeq \frac{5}{6} \frac{\sin^6 \theta^e}{(\sin^2 \theta^* + \sin^4 \theta^e)^2} \eta_{\chi\chi}. \quad (70)$$

In the above we have used equation (51) for $g_s$ in region D. We hence require the condition, equation (57),

$$\sin^2 \theta^* \lesssim v^2 \left( \frac{1}{\sqrt{v g_s}} - 1 \right), \quad G_s = \frac{5}{8} |\eta_{\chi\chi}|^{-1},$$ (71)

and find the exact upper limits on the parameters:

$$\sin^2 \theta^* < \frac{1}{4} \left( \frac{\beta}{\alpha} \right)^2 \left( \frac{\lambda}{\lambda^*} \right)^4 |\eta_{\chi\chi}|^2, \quad \text{and} \quad v = \sin^2 \theta^e < \frac{5}{8} |\eta_{\chi\chi}|.$$ (72)

Noting that in region D, $\sin \theta = \beta \chi/(\alpha \varphi)$, from equation (68) we require $N(\alpha - \beta) > 1$ so that $\sin^2 \theta$ grows significantly during inflation.

In table 1, we give some explicit examples of values of $\alpha$, $\beta$, $\varphi_*$ and $\chi_*$ which lead to a large non-Gaussianity. Using equation (43) we also calculate the spectral index. The contour of $f_{NL}^{(4)}$ for this sum potential for a specific choice of $\alpha$ and $\beta$ is given in figure 5. The first example in table 1 shows that it is possible to have $f_{NL} \simeq 50$ and a scale invariant spectrum. The tensor-to-scalar ratio is also small in this example.

Our formula for $f_{NL}^{(4)}$ is also consistent with the calculation of [5] but our analysis of the parameter space is more extensive and in particular it allows for very small initial field values. We therefore find a region with much larger non-Gaussianity than was found in [5].

As a further check on the algebra, we note that it is also possible to analyse this model using the formalism of a product potential with $W(\varphi, \chi) = W_0 \exp(\alpha \varphi^2/M_P^2) \exp(\beta \chi^2/M_P^2)$. This is equivalent to the sum potential equation (67) in the limit of vacuum domination ($|\alpha| \varphi^2/M_P^2 \ll 1$ and $|\beta| \chi^2/M_P^2 \ll 1$). We have checked that this gives the same results. We note from table 1 that we require a very small value of
Conditions for large non-Gaussianity in two-field slow-roll inflation

Figure 5. The contour plot of $\log_{10}|f^{(4)}_{NL}|$ with the sum potential, $W(\varphi, \chi) = V_0(1 + \alpha \varphi^2 + \beta \chi^2)$ (example C). The parameters match the examples in table 1 and the contours for $\log_{10}|f^{(4)}_{NL}|$ are shown. White regions indicate when $|f^{(4)}_{NL}| < 1$.

Table 1. Showing some initial conditions for the hybrid inflation model that lead to large levels of non-Gaussianity. The spectral index, calculated from equation (43), and the tensor-to-scalar ratio are also shown.

| $\alpha$ | $\beta$ | $\varphi_*$ | $\chi_*$ | $f^{(4)}_{NL}$ | $n_{\xi} - 1$ | $r$ |
|---|---|---|---|---|---|---|
| 0.018 | -0.018 | 1 | 0.00018 | -42 | 0 | 0.006 |
| 0.04 | 0.005 | 1 | 0.0018 | 9.27 | 0.09 | 0.10 |
| 0.01 | -0.02 | 1 | 0.00037 | -11.1 | -0.02 | 0.026 |

$\chi_*$ in region D. If the value becomes too small then the motion of the $\chi$ field will become dominated by quantum fluctuations rather than the classical drift down the potential, $3H\dot{\chi} \simeq -W_\chi = -W_0\beta \chi$, which we have assumed. In order that we can neglect the effect of the quantum fluctuations, the condition we require on the background trajectory is $|\dot{\chi}|/H^2 > \sqrt{3/2}$ [40]. Using $P_{\zeta} \simeq 10^{-10}$, $H^2 \simeq W_0$ and equation (42), the condition that the classical trajectory is valid can be rewritten as $(1 + \tilde{r})\beta^2 \chi^2_*/(\alpha^2 \varphi^2_*) > 6P_{\zeta}$, where $\tilde{r} = \sin^4 \theta^*/\sin^2 \theta^*$. We have checked that this is satisfied for all of the examples given in table 1, but it does provide a significant constraint on the total parameter space.

Very recently Cogollo et al have calculated the effect of the loop correction to the primordial power spectrum and bispectrum [41]. This loop correction arises from taking into account the contribution to the power spectrum and bispectrum arising from terms in the $\delta N$ expansion which are non-leading in the expansion of the field perturbation $\delta \varphi$; see e.g. [42]. However they can still be significant if the coefficient of the term given by the derivative of $N$ is extremely large; e.g. [43]. These ‘higher order’ terms are usually neglected, but [41] has shown the first explicit example of an inflation model where they cannot be neglected. They consider a two-field hybrid model with the same potential as equation (67) in the special case of an unstable straight trajectory along one of the axes,
with $\alpha$ and $\beta$ both negative. In this case, they find that (for certain initial values) the loop correction is dominant and can generate an observable $f_{\text{NL}}$. We have checked that for the explicit values given in the table the loop correction does not dominate (under the assumption that the loop correction dominant here is the same as the one in [41]). However this does provide a further restriction on the allowed parameter space of the ‘tree level’ calculation of the model, which is assumed in equation (70). We plan to return to this issue, and make a more thorough investigation of the hybrid model in a future publication.

6. Non-Gaussianity with non-canonical kinetic terms (product potentials)

Generally, the scalar fields need not only couple through the potential; they may also couple kinetically [44]–[47]. In this section, therefore, we consider inflation governed by the following generalized action:

$$S = \int d^4x \sqrt{-g} \left[ M_P^2 \frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} e^{2b(\varphi)} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - W(\varphi, \chi) \right].$$  (73)

When the kinetic energies are non-canonical, the local non-Gaussianity is altered from the previous sections. In the case of the sum potential, the modifications are non-trivial and are difficult to analyse. When the potential is of product form, however, we may at least partially use the previous analysis. We therefore concentrate only on a product potential, but our findings relate also to sum potentials.

With a product potential, the local non-Gaussianity is given by [21]

$$\frac{6}{5} f_{\text{NL}}^{(4)} = 2e^{-2b_\varphi + 2b_\chi} \left[ \frac{w^3}{e^\varphi} \left( 1 - \frac{\eta_{\varphi\varphi}}{2e^\varphi} \right) + \frac{v^3}{e^\chi} \left( 1 - \frac{\eta_{\chi\chi}}{2e^\chi} \right) + \frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{U} \right) \frac{vu^2\alpha}{(e^\chi)^2} \sqrt{\epsilon^\chi e^\varphi} - \left( \frac{u\alpha}{e^\varphi} - \frac{v}{e^\chi} \right)^2 e^{2b_\varphi - 2b_\chi} A_P \right],$$  (74)

where

$$\alpha = e^{-2b_\varphi + 2b_\chi} \left[ 1 + \frac{\epsilon^\varphi}{\epsilon^\chi} \left( 1 - e^{2b_\varphi - 2b_\chi} \right) \right]$$  (75)

$$= e^{-X} \left[ 1 + \tan^2 \theta^e \left( 1 - e^X \right) \right],$$  (76)

$$A_P = -\frac{e^\varphi e^\chi}{(e^\chi)^2} \left[ \eta_{\varphi\varphi} - \frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{U} \right) \frac{(\epsilon^\chi)^2}{\epsilon^\varphi} \sqrt{\epsilon^\chi} - 4 \frac{\epsilon^\varphi e^\chi}{e^\varphi} \right].$$  (77)

Using new auxiliary functions, equation (74) can be rewritten as

$$\frac{6}{5} f_{\text{NL}}^{(4)} = \left[ 2J(\theta^\star, \theta^e, X)e^e - F(\theta^\star, \theta^e, X)\eta_{\varphi\varphi}^s - G(\theta^\star, \theta^e, X)\eta_{\chi\chi}^s \right.$$

$$+ \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{U} \right) K(\theta^\star, \theta^e, X) \sqrt{\epsilon^\varphi e^\chi} - 2H(\theta^\star, \theta^e, X)$$

$$\times \left( \hat{n}^e - \frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{U} \right) \frac{\sin^4 \theta^e}{\cos \theta^e} e^\varphi \sqrt{\epsilon^\varphi} - 4 \sin^2 \theta^e \cos^2 \theta^e e^e \right) \right].$$  (78)
Figure 6. The function $F$ as defined in equation (79) is plotted for three values of $X$: (a) $X = -0.001$ (left), (b) $X = 0$ (middle) and (c) $X = 0.001$ (right). The central graph corresponds to $f_p$ since $X = 0$.

where

\begin{align}
F_p(\theta^*, \theta^e, X) &\equiv \frac{e^{-X} \alpha^3 \tan^4 \theta^*}{(\alpha^2 \tan^2 \theta^* + \tan^4 \theta^e)^2 \cos^2 \theta^e}, \\
G_p(\theta^*, \theta^e, X) &\equiv \frac{e^{-X} \tan^8 \theta^e}{(\alpha^2 \tan^2 \theta^* + \tan^4 \theta^e)^2 \sin^2 \theta^e}, \\
H_p(\theta^*, \theta^e, X) &\equiv \tan^2 \theta^e \frac{(\alpha \tan^2 \theta^* - \tan^2 \theta^e)^2}{(\alpha^2 \tan^2 \theta^* + \tan^4 \theta^e)^2}, \\
J_p(\theta^*, \theta^e, X) &\equiv e^{-X} \sin^2 \theta^* \frac{(\alpha^3 \tan^2 \theta^* + \tan^6 \theta^e)}{\cos^2 \theta^e (\alpha^2 \tan^2 \theta^* + \tan^4 \theta^e)^2}, \\
K_p(\theta^*, \theta^e, X) &\equiv e^{-X} \alpha^2 \cos^4 \theta^e \sin^2 \theta^e \frac{\sin^2 \theta^e}{\sin^2 \theta^*}.
\end{align}

If $X = 0$, then $\alpha = 1$ and $F_p = f_p$, $G_p = g_p$, $H_p = h_p$ and $J_p = j_p$. The function $F_p$ is plotted for various values of $X$ in figure 6. From this definition, it is clear that allowing $\alpha \neq 0$, the range of $\theta^e$ for which we can obtain large $f_{NL}^{(4)}$ opens up. We also note that the symmetry between two fields is broken, due to $b(\varphi)$, which is apparent in figure 6. Note, though, that we require $\theta^* \sim 0, \pi/2$ as before. Furthermore, when $X > 0$ (or $b_e > b_e$), we no longer require $\cos^2 \theta^e \ll 1$.

Expressions for the power spectra and spectral index with a non-canonical kinetic term are given in [21, 47].

7. Conclusion

We have made an in-depth investigation into the level of non-Gaussianity during two-field slow-roll inflation. We have shown that it is possible to generate a large level of
non-Gaussianity during inflation without violating slow-roll and when the inflaton field perturbations are Gaussian at Hubble exit.

The general conditions for generating a large non-Gaussianity show that the inflaton potential must have a specific shape so that the angle of the background trajectory can grow by about two orders of magnitude or more during inflation. In the case of an inflation potential made of a product of two quadratic potentials, this condition is not possible, so we conclude that this model cannot generate a large non-Gaussianity during slow-roll inflation. When the angle of the background trajectory grows sufficiently (in relative terms), we still need one of the fields to dominate throughout inflation, yet the remaining field cannot remain fully insignificant. For typical values of the slow-roll parameters, we require that initially the subdominant field, say $\varphi$, must satisfy $\cos^2 \theta^* = \epsilon^* / \epsilon^* \lesssim 10^{-7}$. This means that the field trajectory is almost exactly parallel to the $\chi$ axes initially and typically this requires a finely tuned initial condition. In the case of a product separable potential we then also require that the final value of the angle of the background trajectory lies in a narrow range much greater than the initial value (in relative terms) but still nearly parallel to the $\chi$ axis. The analysis for a sum separable potential is very similar but more complicated to analyse. The initial background trajectory must be similarly fine-tuned to be nearly parallel to one of the axes of the inflaton fields, but the tuning at the end of inflation is not written so simply. We have also shown how the constraints on generating a large non-Gaussianity may be eased if we generalize the inflaton model to allow a non-canonical kinetic term.

We have presented two explicit models where a large non-Gaussianity can be generated during slow-roll inflation. One is a product potential driven by a field with a quadratic potential, which ends inflation when it approaches the minimum of the potential. If the second field with an exponential potential has a sufficiently small initial value, then for certain values of the model parameter, $\lambda$, this field grows by the right amount to generate a large non-Gaussianity, $f_{NL} \sim 100$. We also consider the sum separable model of hybrid inflation. We provide a few specific choices of the model parameters, which are effectively $\eta_{\varphi \varphi}$ and $\eta_{\chi \chi}$; for a suitably large ratio of the initial field values this model generates a large non-Gaussianity. We find that this is possible if the $\eta$'s have the opposite sign (so that inflation takes place near a saddle point) and when both of the $\eta$'s have the same sign. However we have also found that if one of the field values is too small, then quantum fluctuations may perturb the background trajectory to the extent that the classical trajectory in field space is no longer valid. There is also the possibility that the large scale loop correction which we have not generally considered in this paper may not be negligible. We intend to return to these issues and make a more thorough investigation of the hybrid model in a future publication.

**Acknowledgments**

The authors thank Carsten van de Bruck, Filippo Vernizzi and David Wands for useful comments. CB acknowledges financial support from the Deutsche Forschungsgemeinschaft. K-YC is supported by the Ministerio de Educacion y Ciencia of Spain under Proyecto Nacional FPA2006-05423 and by the Comunidad de Madrid under Proyecto HEPHACOS, Ayudas de I+D S-0505/ESP-0346. LMHH acknowledges support from STFC.
References

[1] Maldacena J M, 2003 J. High Energy Phys. JHEP05(2003)013 [SPIRES] [arXiv:astro-ph/0210603]
[2] Bartolo N, Komatsu E, Matarrese S and Riotto A, 2004 Phys. Rep. 402 103 [SPIRES] [arXiv:astro-ph/0406398]
[3] Seery D and Lidsey J E, 2005 J. Cosmol. Astropart. Phys. JCAP09(2005)011 [SPIRES] [arXiv:astro-ph/0506056]
[4] Bartolo N, Matarrese S and Riotto A, 2002 Phys. Rev. D 65 103505 [SPIRES] [arXiv:hep-ph/0112261]
[5] Alabidi L, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)015 [SPIRES] [arXiv:astro-ph/0604611]
[6] Enqvist K, Jokinen A, Mazumdar A, Multamaki T and Vaihkonen A, 2005 Phys. Rev. Lett. 94 161301 [SPIRES] [arXiv:astro-ph/0411394]
[7] Jokinen A and Mazumdar A, 2006 J. Cosmol. Astropart. Phys. JCAP04(2006)003 [SPIRES] [arXiv:astro-ph/0512368]
[8] Chen X, Huang M X, Kachru S and Shiu G, 2007 J. Cosmol. Astropart. Phys. JCAP01(2007)002 [SPIRES] [arXiv:hep-th/0605045]
[9] Barnaby N and Cline J M, 2006 Phys. Rev. D 73 106012 [SPIRES] [arXiv:astro-ph/0601481]
[10] Misra A and Shukla P, 2008 arXiv:0807.0996 [hep-th]
[11] Chen X, Easther R and Lim E A, 2007 J. Cosmol. Astropart. Phys. JCAP04(2006)003 [SPIRES] [arXiv:astro-ph/0611645]
[12] Silverstein E and Tong D, 2004 Phys. Rev. D 70 103505 [SPIRES] [arXiv:hep-th/0310221]
[13] Linde A D and Mukhanov V F, 1997 Phys. Rev. D 56 535 [SPIRES] [arXiv:astro-ph/9610219]
[14] Enqvist K and Sloth M S, 2002 Nucl. Phys. B 626 395 [SPIRES] [arXiv:hep-ph/0109214]
[15] Lyth D H and Wands D, 2002 Phys. Lett. B 524 5 [SPIRES] [arXiv:hep-ph/0110002]
[16] Moroi T and Takahashi T, 2001 Phys. Lett. B 522 215 [SPIRES] [arXiv:hep-ph/01010096]
[17] Enqvist K and Nurmi S, 2005 J. Cosmol. Astropart. Phys. JCAP10(2005)013 [SPIRES] [arXiv:astro-ph/0508573]
[18] Linde A D and Mukhanov V F, 2006 J. Cosmol. Astropart. Phys. JCAP04(2006)009 [SPIRES] [arXiv:astro-ph/0511736]
[19] Malik K A and Lyth D H, 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)008 [SPIRES] [arXiv:astro-ph/0604387]
[20] Sasaki M, Valiviita J and Wands D, 2006 Phys. Rev. D 74 103003 [SPIRES] [arXiv:astro-ph/0607627]
[21] Dutta B, Leblond L and Kumar J, 2008 arXiv:0807.0180 [hep-th]
[22] Vernizzi F and Wands D, 2006 J. Cosmol. Astropart. Phys. JCAP11(2006)006 [SPIRES] [arXiv:astro-ph/0603799]
[23] Choi K Y, Hall L M H and van de Bruck C, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)029 [SPIRES] [arXiv:astro-ph/0603799]
[24] Battledi T and Easther R, 2007 J. Cosmol. Astropart. Phys. JCAP03(2007)020 [SPIRES] [arXiv:astro-ph/0601026]
[25] Seery D and Lidsey J E, 2007 J. Cosmol. Astropart. Phys. JCAP01(2007)008 [SPIRES] [arXiv:astro-ph/0611034]
[26] Rigopoulos G I, Shellard E P S and van Tent B J W, 2007 Phys. Rev. D 76 083512 [SPIRES] [arXiv:astro-ph/0511034]
Conditions for large non-Gaussianity in two-field slow-roll inflation

Yokoyama S, Suyama T and Tanaka T, 2008 Phys. Rev. D 77 083511 [SPIRES] arXiv:0711.2920 [astro-ph]
[26] Gordon C, Wands D, Bassett B A and Maartens R, 2001 Phys. Rev. D 63 023506 [SPIRES]
[arXiv:astro-ph/0009131]
[27] Starobinsky A A, 1985 Pis. Zh. Eksp. Teor. Fiz. 42 124
Starobinsky A A, 1985 JETP Lett. 42 152 (translation)
[28] Sasaki M and Stewart E D, 1996 Prog. Theor. Phys. 95 71 [SPIRES] [arXiv:astro-ph/9507001]
[29] Sasaki M and Tanaka T, 1998 Prog. Theor. Phys. 99 763 [SPIRES] [arXiv:gr-qc/9801017]
[30] Lyth D H, Malik K A and Sasaki M, 2005 J. Cosmol. Astropart. Phys. JCAP05(2005)004 [SPIRES]
[arXiv:astro-ph/0411220]
[31] Lyth D H and Rodriguez Y, 2005 Phys. Rev. Lett. 95 121302 [SPIRES] [arXiv:astro-ph/0504045]
[32] Wands D, Bartolo N, Matarrese S and Riotto A, 2002 Phys. Rev. D 66 043520 [SPIRES]
[arXiv:astro-ph/0205253]
[33] Lyth D H and Zaballa I, 2005 J. Cosmol. Astropart. Phys. JCAP10(2005)005 [SPIRES]
[arXiv:astro-ph/0507608]
[34] Komatsu E and Spergel D N, 2001 Phys. Rev. D 63 063002 [SPIRES] [arXiv:astro-ph/0005036]
[35] Komatsu E et al (WMAP Collaboration), 2008 arXiv:0803.0547 [astro-ph]
[36] Byrnes C T, Sasaki M and Wands D, 2006 Phys. Rev. D 74 123519 [SPIRES] [arXiv:astro-ph/0611075]
[37] http://www.rssd.esa.int/index.php?project=Planck
[38] Kofman L, Linde A D and Starobinsky A A, 1994 Phys. Rev. Lett. 73 3195 [SPIRES]
[arXiv:hep-th/9405187]
[39] Kofman L, Linde A D and Starobinsky A A, 1997 Phys. Rev. D 56 3258 [SPIRES] [arXiv:hep-th/9704452]
[40] Creminelli P, Dubovsky S, Nicolas A, Senatore L and Zaldarriaga M, 2008 J. High Energy Phys.
JHEP09(2008)036 [SPIRES] arXiv:0802.1067 [hep-th]
[41] Cogollo H R S, Rodriguez Y and Valenzuela-Toledo C A, 2008 J. Cosmol. Astropart. Phys.
JCAP08(2008)029 [SPIRES] arXiv:0806.1546 [astro-ph]
[42] Byrnes C T, Koyama K, Sasaki M and Wands D, 2007 J. Cosmol. Astropart. Phys. JCAP11(2007)027
[SPIRES] arXiv:0705.4096 [hep-th]
[43] Boubekeur L and Lyth D H, 2006 Phys. Rev. D 73 021301 [SPIRES] [arXiv:astro-ph/0504046]
[44] Garcia-Bellido J and Wands D, 1995 Phys. Rev. D 52 6739 [SPIRES] [arXiv:gr-qc/9506056]
[45] Garcia-Bellido J and Wands D, 1996 Phys. Rev. D 53 5437 [SPIRES] [arXiv:astro-ph/9511029]
[46] Starobinsky A A, Tsujikawa S and Yokoyama J, 2001 Nucl. Phys. B 610 383 [SPIRES]
[arXiv:astro-ph/0107555]
[47] Di Marco F, Finelli F and Brandenberger R, 2003 Phys. Rev. D 67 063512 [SPIRES]
[arXiv:astro-ph/0211276]
Di Marco F and Finelli F, 2005 Phys. Rev. D 71 123502 [SPIRES] [arXiv:astro-ph/0505198]