Economic mechanisms for improving the efficiency of the territorial distribution of production points and temporary storage facilities

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Abstract. This article discusses the algorithms for finding the optimal solution of problems related to the location of temporary storage of goods, warehouses, firms for processing raw materials and shops selling the final product in the transport network. An algorithm is also proposed for finding a compromise solution to the problem of maximizing profits for each agent.

1. Introduction
The warehouse placement problems are considered to be quite important regional issues. Usually there is no problem with single warehouse firms. However, large companies face warehouse placement difficulties when operating in international markets. There are two types of establishing warehouse network: centralized and decentralized. In the first case, one large warehouse is present, in the second case - several warehouses are scattered in different sales regions [1-5].

The capacity of flows of goods, their rational organization, demand in the sales market, the size of the sales region, the concentration of consumers in it, the location of suppliers and customers relative to each other are the main criteria by which the location and the number of warehouses are determined.

Thus, a problem of optimal warehouse placement arises. In order to build a new warehouse or buy an existing warehouse and operate it, a significant capital investment is required. On the other hand, there is also a reduction in distribution costs due to the maximum proximity of warehouses to customers. Similarly, a problem of optimal placement of temporary storage facilities for products and raw materials arises. It is also necessary to solve problem of the territorial location of factories processing raw materials and shops selling products. Thus, there are several optimization problems for which one can find a solution using the model described below.
2. Informal statement of the problem

Suppose there is a certain plane \( \pi \) on which some finite transport network \( C = (N, p, d) \) is set, where \( N = \{x_1, x_2, ..., x_k\} \) is a finite set of vertices, \( p \) is a function of transport costs, and \( d \) being limited traffic capacities. Denote by \( d = d(d_1, ..., d_{N_A}) \) and \( d' = d'(d'_1, ..., d'_{N_B}) \) traffic capacity for each type of raw material and finished products. \( N_A, N_B \) is the number of subtypes for each type of raw material respectively. Similarly, we denote the functions of transport costs: \( p = p(p_1, ..., p_{N_A}) \) and \( p' = p'(p'_1, ..., p'_{N_B}) \). Consider the following scheme: "the extraction of raw materials \( \rightarrow \) their storage location \( \rightarrow \) production \( \rightarrow \) temporary storage of products \( \rightarrow \) store". Thus, we do not consider cases when parameters \( d \) and \( d' \) or \( p \) and \( p' \) are simultaneously present on the edges. Furthermore, we can use simple notation \( p \) and \( d \), assuming either raw materials or ready made products depending on the context. We will also introduce such concepts as mining points \( A = \{A_1, ..., A_{N_A}\} \) and raw materials storage points \( S = \{S_1, ..., S_{k_1}\} \), raw material processing plants \( B = \{B_1, ..., B_{N_B}\} \) and stores that sell finished goods \( M = \{M_1, ..., M_{m}\} \).

Furthermore, we assume that our points of extraction of raw materials are fixed, i.e. they are located at strictly defined nodes. For each plant, a set of different types of raw materials is set, and one type of finished product is obtained from it. Every buyer wants to buy some amount of goods. Goods are delivered from the warehouse (to firms and shops), and each warehouse corresponds to a given type of raw material and goods. The concept of value of each final product is introduced. It represents the sum of the value of the goods at the point of extraction of raw material, transportation costs for the delivery first to the storage warehouse, and then to the firm, production costs, costs for delivering goods to the store and storage costs in warehouses [6-7].

Several agents are being considered. They own production facilities, transportation, road junctions and raw materials extraction points. It is also important to introduce the concept of stocks to help gauge agents’ interest. Each agent receives a certain percentage of the profit in each area. If stocks do not exist in an area, then there is zero income. Let us determine what costs are incurred by each type of agent. Those agents involved in road junctions and transportation spend money on road maintenance, as well as servicing or replacing vehicles and wages for workers. Those agents who own production facilities and points of extraction of raw materials, incur some fixed costs: for example, utility bills (gas, electricity, water), maintenance costs, rent and salary. Those agents who deal with warehouses have maintenance costs (insurance, fire safety, wages to workers, and maintenance of equipment in a warehouse). The size of variable cost is small, so it can be omitted [8].

After the introduction of basic concepts and notation, we define the problem. The problem is to minimize costs for each agent. It can be seen is a multi-agent interaction problem. Note that the territorial location of production points and food storage points is also a part of our task. By introducing common profit parameter and subtracting considered costs from it, we arrive at the following problem – a problem of each agent’s profit maximization. The task can be divided into the following steps [9-12]:

- Compiling a mathematical description for the given conditions.
- Introduction of cost functions (for mining, transport, production).
- Description of algorithms and their application in determining the optimal solutions and equilibrium positions for a given task.

3. Formalization, construction and analysis of the multi-agent interaction model
Let a finite transport network $C = (N, p, d)$ be given on a plane, where $N$ is a finite set of nodes, $p$ are transport cost functions ($p : (N, N) \to R^1$), set on network edges, $d$ - limited traffic capacity functions ($d : (N, N) \to R^1$). The edge of the network is an ordered pair of nodes $(x_i, x_j)$. The network is represented in figure 1 [13-15].

$A_i$ is a number of points of extraction of raw materials, $A_1, ..., A_{N_A}$ are the points of extraction of raw materials, $C_i^A = C_i^A (C_1^A, ..., C_{N_A}^A)$ is the cost of the $i$-th set of raw materials, $S_i (S_1, ..., S_{N_S})$ is cost function of the $i$-th set of raw materials. For each edge $(x_i, x_j)$ of the network between points of extraction of raw materials ($A_1, ..., A_{N_A}$) and points of temporary storage of products ($S_1, ..., S_{N_S}$), the transportation cost functions of moving the $i$-th unit of raw materials ($p_i^A (A_j, S_k)$, where $j = 1...N_B$ - number of types of raw materials, $k = 1..., K_A$ - number of temporary storage of raw materials), are set. A non-negative number $p_i^A (A_j, S_k)$ is matched to each type of raw material. Thus, it is possible to create a matrix $P = (p_{ij}), i = 1...N_B, j = 1...N_A$.

![Transport network](image)

**Figure 1.** Transport network.

The network has free nodes, in which $N_B$ firms for processing of raw materials into final products $B_1...B_{N_B}$ are located. In order to produce one unit of finished products, a certain set of raw materials is required $f_i = (V_1, ..., V_{N_V}), i = 1..., N_B$ is the number of types of final products. Cost to produce one unit of goods is $C_i$, hence cost of production of the $i$-th set of finished products is represented by function $C_{i_n} = C_{i_n} (C_{i_n} ... C_{B_{n_B}})$. Similarly, we denote the functions of transportation costs: $p_{i_k}^A (S_k, B_j)$, where $j = 1...N_B$ is the number of production points, $k = 1..., K_A$ is the number of temporary storages of raw materials. This function assigns a nonnegative number $p_{i_k}^A (S_k, B_j)$ to each edge $(x_i, x_j)$ for each type of raw material.

Furthermore, the following notation is introduced: $p_{i_k}^A (S_k, B_j)$ is the function of transport costs between the temporary storages of raw materials and plants for each edge $(x_i, x_j)$; $p_{ij}^A (S_k, B_j)$ -
function values for each type of raw material, where \( j = 1,...,N_B \) is the number of production points, \( k = 1,...,K_A \) is the number of temporary storages of raw materials; \( B_1...B_{NB} \) - finished products; \( S'_j = S'_j(S_1,...,S_{K'_A}) \) – the cost function of storing the \( i \)-th set (of certain type) of the final product; \( p^B_j(B_j,S'_j) \) - functions of transportation costs for moving a single \( i \)-th product, where \( j = 1...N_B \) is the number of production points, \( k = 1...K_B \) is the number of temporary storage facilities of finished products; \( p^B_j(B_j,S'_j) \) - a function that associates transport costs of a certain type of finished product with each edge; \( M_1,...,M_m \) - network nodes in which \( M \) stores are located and demand is defined for each store; \( m_i = (W_1,...,W_{N_p}) \) - a set of finished products that are purchased for the \( i \)-th store; \( p^B_j(S'_k,M) \) - functions of transportation costs for moving products between temporary storage facilities of final products and stores; \( p^M_j(x_i,x_j),...,p^M_j(x_i,x_j) \) - the value of the function for each type of finished product [16-18].

Consider the following four parameters to obtain storage, transportation costs and costs for extraction of raw materials [19-20]:

- \( C^A f_i \sum_{j=1}^{N_A} C^A_i V_j \) - cost of production of a set of raw materials.
- \( S' f_i \sum_{j=1}^{N_A} S'_j V_j \) - storage costs.
- \( p^A(A_i,S'_j) \sum_{j=1}^{N_A} p^A(A_i,S'_j) V_j \) - transportation costs for moving raw materials from the extraction point to the storage facilities.
- \( p^A(S'_j,B_i) f_i \sum_{j=1}^{N_A} p^A(S'_j,B_i) V_j \) - transportation costs for transporting a set of raw materials from the storage warehouse to the firm.

Thus, the cost of purchasing a set of raw materials \( f_i = (V_1...V_{N_A}) \) is:

\[
C^*_k(f_i) = C^A f_i + p^A(A_i,B_j) f_i + Sf_i ,
\]

where \( k = 1,...,N_B \) is the number of plants processing raw materials into finished products or

\[
C^*_k(f_i) = \sum_{j=1}^{N_A} C^A_j v_j + \sum_{j=1}^{N_A} p^A_j(A_i,B_m) v_j + \sum_{j=1}^{N_A} S'_j v_j
\]

where \( k = 1,...,N_B \) is the number of plants processing raw materials into finished products.

Similarly, we calculate purchase price for the finished product [21-25]:

- The cost of producing a specific set of finished product \( C^B_{m_i} \sum_{j=1}^{N_A} C^B_j V_j \).
- Transportation costs for transporting specific set of finished product

\[
p^B(B_j,S'_j)m_i \sum_{j=1}^{N_A} p^B(B_j,S'_j) W_j .
\]

- The cost of storing a specific set of finished product \( S' m_i \sum_{j=1}^{N_A} S'_j W_j \).
- Transportation costs for transporting specific set of finished product
\[ p_i^B(B_j, S_j)m_i \text{or} \sum_{i=1}^{\text{NA}} p_i^B(B_i, S_i)w_i . \]

Thus, the costs to obtain a finished product for a particular store become:
\[ C_k^B(f_i) = C^B m_i + p^B(B_i, M)w_i + S^B m_i \]

where \( k = 1\ldots M \) is the number of stores selling final products.

The total cost of the finished product is represented as \( C = (C_1^B, \ldots, C_M^B) \). Next, we need to determine the distance between objects. For this, the concept of the Euclidean norm is introduced [26-27]:
\[ r(x_j, y_j) = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}, \]

where \((x_j, y_j)\) - coordinates of point \( x_j \).

Let \( R_i \) be the admissible distance between objects, \( i = 1,\ldots, N^2, N^2 \) - the possible number of edges. Based on all of the above, we can set the task of minimizing the total costs for the function: \( \eta = CM \rightarrow \min \).

It is necessary that the following conditions are met [28-30]:

- \( r_i \leq R_i \) - distance between objects (warehouse and firm, for example);
- \( f_i \leq d_i \) - edge bandwidth;
- \( p \geq 0, i = 1,\ldots, N^2 \), where \( N^2 \) - possible number of network edges - shipping costs;
- \( v_i \geq 0, i = 1,\ldots, N_A \) - raw material;
- \( \sum_{i=1}^{n} v_i \leq V, i = 1,\ldots, N_A \) - total raw material;
- \( W_i \geq 0, i = 1,\ldots, N_B \) - finished product;
- \( \sum_{i=1}^{n} W_i \leq W, i = 1,\ldots, N_B \) - total finished product.

4. Formalization of the algorithm for solving the problem

4.1. Definition of the shortest path matrix between all pairs of points

To start, we build the shortest-path matrix from any node \((x_i)\) of the network graph to any other node [31-32]:

\[
\begin{array}{cccc}
  & x_1 & x_2 & \ldots & x_n \\
 x_1 & 0 & a_{x_1x_2} & \ldots & a_{x_1x_n} \\
x_2 & a_{x_2x_1} & 0 & \ldots & a_{x_2x_n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_n & a_{x_nx_2} & a_{x_nx_2} & \ldots & 0 \\
\end{array}
\]

where \( a_{x_1x_j} \) is the path length between nodes \( X_i \) and \( X_j \). It is clear that the distance between same nodes is zero.

To obtain this matrix, we use the Floyd algorithm. This algorithm can be used in our case, since we are dealing with a large number of edge pairs between pairs of vertices.
Next step is to build total costs matrix for each set of products at each point of sale: where \( x_i \) is the \( i \)-th shop, \( k_i \) is a set of goods (for a specific type), \( k_i(x_j) \) are total costs, \( i = 1...n, j = 1...m \).

### 4.2. Determination of payoff functions for players

The payoff functions for each player are needed in order to find a compromise solution in the placement problem. Recall that in our task, the players are certain agents. Each of them has production capacity (agent \( A_1 \)), transportation, transport junctions (agent \( A_2 \)) and raw materials extraction points (agent \( A_3 \)). We formulate an algorithm for finding the payoff functions [33-34]:

**Step 1. Calculate total demand**

This step allows one to determine the required amount of extracted resources and the amount of production of finished products.

**Step 2. Preliminary estimate of revenues and costs at each network node**

Let the first agent \( A_1 \) own the points of temporary storage facilities of raw materials, points of storage of final products, and transport. Let the second agent \( A_2 \) own production points. Let the third agent \( A_3 \) own shops located in the points of consumption.

Then the profit of the first agent (\( P_1 \)) is calculated by the formula:

\[
P_1 = I_1 - C_1,
\]

where \( I_1 \) is the income from providing storage services, \( C_1 \) is the cost of transporting and storing raw materials and finished products.

**Profit of the second agent (\( P_2 \))**:

\[
P_2 = I_2 - C_2,
\]

where \( I_2 \) is the income from the cost of manufactured products, \( C_2 \) is the cost of purchasing processed raw materials.

The output is calculated using the Cobb-Douglas function:

\[
Q = J K^\alpha L^\beta,
\]

where \( J \) is a coefficient of manufactured products, \( K^\alpha, L^\beta \) are the costs of raw materials of the form \( \alpha \) and \( \beta \), respectively.

**The profit of the third agent (\( P_3 \))** is calculated as follows:

\[
P_3 = I_3 - C_3,
\]

where \( I_3 \) is the income from the sale of final products, \( C_3 \) is the cost of purchasing goods from the manufacturer.

**Step 3. Consideration of possible options for the location of production points.**

At this step, the net profit of each agent is calculated separately from each other.

### 4.3. Algorithm of finding a compromise solution

Now, having the payoff functions for the players, we can find a compromise solution. We use the following algorithm:
Step 1. Construction of the payoff matrix ($\Gamma$)

$$\Gamma = (\alpha_i, m),$$

where $l$ is the number of players, $m$ is the number of situations in the game.

Step 2. Compilation of a vector called “ideal” ($M$)

$$M = \begin{pmatrix} M_1 \\ \vdots \\ M_l \end{pmatrix}, \quad M_i^\prime = \max_m \alpha_{i,m}.$$  

It consists of the maximum income that producers receive.

Step 3. Calculation of residuals

Residuals are deviations of the income of each manufacturer from the maximum income:

$$\Gamma_m = (M - \alpha_{i,m}) = (\beta_{i,m}).$$

Step 4. Arrange income Values

In each situation, we arrange the income values in ascending order so that the first line contains the smallest values, and the last line contains the largest values:

$$\max_l (\beta_{m,l}) = \max_l (M - \alpha_{i,m}).$$

Step 5. The principle of ”minmax”

Among the obtained maximum residuals choose the minimum value:

$$\min_m \max_l (M - \alpha_{i,m}).$$

It is possible to have several situations in the last line with the same minimum. If so, one needs to go to the line above and look for the minimum value there. The resulting situations are the desired compromise set (solution).

5. Conclusion

This article has solved a problem of territorial distribution of production points and temporary storage facilities via Mathematical Modeling. Multi-agent interaction model was formalized, constructed and analyzed. Constructed algorithm of finding a compromise solution for the problem of territorial distribution of production points and temporary storage facilities was introduced.

Acknowledgment

The work is partially supported by the RFBR grant # 18-01-00796.

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