Hexahedral prism and IMU installation error calibration technique based on turntable transmission

Shuangshuang RUAN¹, Xiangxin GUO¹, Huichao SHAO¹, Jian LIU¹ and Zhen TU¹

¹. Leador Space Information Technology Co., Ltd.

Email: guoxiangxin@leador.com.cn

Abstract. A hexahedral prism and IMU installation error calibration technique based on turntable transmission is proposed. It effectively simplifies the calibration process of precise POS and SAR radar installation errors. Based on the zero position of the turntable, high-precision optical equipment was used to calibrate the hexahedral prism and the zero position of the turntable, then calibrated IMU and turntable zero position to realize the installation error of hexahedral prism and IMU through the error attitude matrix conversion. The calibration method is much simple and accurate. Test result shows that the heading angle and attitude angle calibration repeatability is better than 5°.

1. Introduction

Military requirements have facilitated the development of radar. In the field of airborne SAR (Synthetic Aperture Radar) imaging technology, To obtain the instantaneous position, attitude and velocity information of SAR, The combination of airborne INS and indexing mechanism is used in the traditional method. With the development of position and orientation technology, The high precision and small POS (Position and Orientation System) is becoming popular in airborne SAR system, because of the short distance and strong connection which can reduce the negative effects of deflection and time delays.

The establishment of SAR compensation model mainly depends on the accurate position and attitude information obtained by POS which installed on the SAR system. The antenna phase center position and attitude information which are used to simulate the desired trajectory can be obtained by coordinate transformation based on the POS measurement data. High quality SAR images are obtained by matching filtering and error compensation of the desired trajectory.

Because of the three axes of the IMU can't be materialized, the calibration of installation misalignment angle between SAR and IMU (Inertial Measurement Unit) hinders the application of POS in SAR system. The traditional calibration method of SAR is to measure the misalignment Angle between SAR and inertial navigation by optical equipment and target, on condition that the plane is leveled. The disadvantages of this method are long period and difficult to implement.

A method of materializing the three axes of the IMU can not only effectively simplify the calibration, but also general performance and stability improvements. In view of this, this paper proposes an installation error calibration method based on the transmission of three-axis turntable, which can calibrate the installation error Angle between IMU and hexagon prism, and then the three axes of IMU are shown through the hexagon prism.
2. Calibration Design

2.1. Calibration process

The calibration process based on the three turntable consists of four steps:

a. Making integrated components: Fixing the hexagon prism to the IMU through the fixture plate, Then fix the integrated components on the three-turnover table.
b. Calibrate the angle of Rotary tables and the hexagon prism.
c. Calibrate the angle of Rotary tables and IMU.
d. Calculate the angle of IMU and the hexagon prism.

The specific process is shown in the figure below:

2.2. Coordinate transformation
Coordinate system includes rotary table coordinate system, IMU coordinate system and prism coordinate system, as shown in the following figure:

**Figure 3. The Frame diagram**

The definition of the rotary table coordinate system \((O_1X_1Y_1Z_1)\): the axis \(O_1X_1\) is parallel to the rotating axis of the middle rotating frame of turntable, the axis \(O_1Y_1\) is parallel to the rotating axis of the inner rotating frame of turntable, the axis \(O_1Z_1\) is parallel to the rotating axis of the outer rotating frame of turntable. The attitude angle of the rotary table coordinate system relative to the geographic coordinate system is \(\theta_1, \gamma_1, \phi_1\). The direction cosine matrix is \(C_1^u\).

The definition of the IMU coordinate system \((O_2X_2Y_2Z_2)\): the axis \(O_2X_2\) points to the right of the IMU, the axis \(O_2Y_2\) points in front of the IMU, the axis \(O_2Z_2\) points above the IMU. The attitude Angle of the IMU coordinate system with respect to geographic coordinate system is \(\theta_2, \gamma_2, \phi_2\). The direction cosine matrix is \(C_2^u\).

The definition of the prism coordinate system \((O_3X_3Y_3Z_3)\): \(O_3X_3\) and \(O_3Y_3\) are normal directions of two orthogonal planes. The attitude Angle of the prism coordinate system with respect to geographic coordinate system is \(\theta_3, \gamma_3, \phi_3\). The direction cosine matrix is \(C_3^u\).

a. The transformation between the rotary table coordinate system and the IMU coordinate system

Because the INS cannot output its own attitude information under laboratory conditions, Therefore, the method of three-axis turntable calibration is adopted to realize the conversion of IMU coordinate system to the turntable coordinate system.

b. The transformation between the rotary table coordinate system and the prism coordinate system

\[
C_1^3 = (C_3^n)^T C_1^n \tag{1}
\]

Where \(C_1^n\) is the cosine function of rotation Angle \(\theta_1, \gamma_1, \phi_1\) of the rotary table, \(C_3^n\) is the cosine function of prism attitude Angle \(\theta_3, \gamma_3, \phi_3\).

c. The transformation between the IMU coordinate system and the prism coordinate system

\[
C_2^3 = (C_3^n)^T C_1^n \tag{2}
\]

Where \(C_2^n\) is the cosine function of rotation Angle \(\theta_2, \gamma_2, \phi_2\) of the rotary table, The corresponding attitude Angle of \(C_2^n\) is \(\theta_2, \gamma_2, \phi_2\) which can be obtained by post-processing of dynamic test data.

3. Calibration of prism and turntable

3.1. Measurement of horizontal attitude Angle of prism
Because every face of a hexahedral prism is a plane, the reflected light will be reflected back in parallel, no matter where the light comes from.

The structure of a hexagon prism $ABCD-A_1B_1C_1D_1$ is shown in figure 4 below:

![Figure 4. The structure of a hexagon prism](image)

1) Where the plane $ABCD-A_1B_1C_1D_1$ is approximately perpendicular to the $O_3Y_3$ axes of the prism, let's define it as plane 1;
2) Where the plane $BB_1C_1C$ is approximately perpendicular to the $O_3X_3$ axes of the prism, let's define it as plane 1;
3) The verticality between adjacent faces of a prism is known to be less than $2"$;
4) The attitude angle of prism is defined as three deviation angles of geographic coordinate system and carrier coordinate system which is formed by the axes $A_1A, A_1B_1, A_1D_1$.

The yellow cross wire of the auto-collimation theodolite is seen to coincide with the black cross wire, when the beam $S_0S_1$ emitted by the theodolite is reflected vertically back, as shown in figure 5.

![Figure 5. The image shown by the eyepiece of the theodolite](image)

At this point, the optical axis is perpendicular to both the prism surface and the auto-collimation theodolite mirror, the vertical angle $\angle a_1$ rotated by the collimating theodolite is made up of the plane $BB_1C_1C$ and the geographic horizontal plane, it is also the angle between the normal line $AB$ of plane $BB_1C_1C$ and the geographic plane, the same principle is used for collimation measurements of other planes.

Through the measuring process of theodolite collimation, the angle between two mutually perpendicular planes of a prism and the geographic horizontal plane can be obtained, mark $\angle a_1$, $\angle a_2$, they are also the angle between the two horizontal axes of the prism coordinate system and the geographic coordinate system plane.

According to the relative rotation relation between the geography system and the load system, angle $\theta_3, \gamma_3, \phi_3$ is defined as the angle between the prism coordinate system and the geographic coordinate system:

$$
\frac{x_n}{y_n} = \Gamma_3^{\frac{x}{y}} \quad \frac{x_3}{y_3} = \Gamma_3^{x/y} \quad \frac{x_n}{y_n} = \Gamma_3^{x/y}
$$

(3)

Where,
According to the projection relation of axes $O_3X_3$ and $O_3Y_3$ under the geography system in formula (4), we can calculate the Angle between them and the geographic horizon, as:

$$
\tan \alpha_1 = \frac{-\sin r_3 \cos \theta_3}{\sqrt{(\cos r_3 \cos \varphi_3 - \sin r_3 \sin \theta_3 \sin \varphi_3)^2 + (\cos r_3 \sin \varphi_3 + \sin r_3 \sin \theta_3 \cos \varphi_3)^2}}
$$

$$
= \frac{-\sin r_3 \cos \theta_3}{\sqrt{(\cos r_3)^2 + (\sin r_3)^2(\sin \theta_3)^2}}
$$

$$
\tan \alpha_2 = \frac{\sin \theta_3}{\sqrt{(\cos \theta_3 \cos \varphi_3)^2 + (-\cos \theta_3 \sin \varphi_3)^2}} = \tan \theta_3
$$

$$
\tan r_3 = \frac{\tan \alpha_1}{\sqrt{(\cos \theta_3)^2 - (\tan \alpha_1)^2(\sin \theta_3)^2}}
$$

In formula (5), $\angle a_1$ and $\angle a_2$ are the horizontal attitude angles measured by the auto-collimation theodolite.

3.2. Measurement of Course Angle of prism

When the outer frame axis of the turntable points to scale 0, the normal of the mirror which fixed to the outer frame overlap due north. The horizontal Angle $\angle \Phi_1$ of the mirror is measured with a self-collimating theodolite, The horizontal Angle $\angle \Phi_2$ of the prism is measured with a self-collimating theodolite, The Angle formed by the horizontal component of the prism normal $O_3Y_3$ and the north direction of the geography system is:

$$
\varphi_0 = 180^\circ - \Phi_2 + \Phi_1
$$

Assume that the normal is parallel to the $y_{3h}$ axis of the prism as shown below:
The Angle formed by the horizontal component of the prism normal and the north direction of the geography system is:

$$\phi_n = \theta$$  \hspace{1cm} (7)

The attitude Angle of prism can be obtained according to formula (5) and formula (7). The cosine matrix $C_n^a$ between prism coordinate system and geographic coordinate system can be calculated.

3.3. Measurement of attitude Angle of turntable

The attitude Angle of the rotary table coordinate system is the output Angle of the three-axis rotary table, include the inner frame angle, middle frame angle, outer frame angle, Let's call it Angle $\theta_1, \gamma_1, \phi_1$. Thus the cosine matrix $C_1^a$ between the rotary table coordinate system and the geographic coordinate system can be calculated.

3.4. The transformation between a prism and a turntable coordinate system

According to the cosine matrix $C_3^a$ between the prism coordinate system and the geographic coordinate system, The cosine matrix $C_1^a$ between the rotary table coordinate system and the geographic coordinate system, The relative Angle relation between prism coordinate system and rotary table coordinate system can be calculated:

$$C_1^a = (C_1^a)^T C_3^a$$  \hspace{1cm} (8)

If the relative Angle between the prism coordinate system and the rotary table coordinate system is a small Angle, The corresponding transformation matrix can be approximated as a function of attitude Angle difference between two coordinate systems:

$$C_1^a = \begin{bmatrix}
1 & (\phi_1 - \phi_3) & -(\theta_1 - \theta_3) \\
-(\phi_1 - \phi_3) & 1 & (\gamma_1 - \gamma_3) \\
(\theta_1 - \theta_3) & -(\gamma_1 - \gamma_3) & 1
\end{bmatrix}$$  \hspace{1cm} (9)

4. IMU and Turntable Calibration Method

After IMU is calibrated at the factory, its output coordinate system is shell system $O_2X_2Y_2Z_2$. In actual turntable installation, IMU has a certain angle relationship with the turntable system $O_1X_1Y_1Z_1$. The IMU output coordinate system $O_2X_2Y_2Z_2$ can be converted to the turntable output axis $O_1X_1Y_1Z_1$ according to the multi-position test and rate test of turntable, and the calibration accuracy is angle-second.

The original output of the IMU is $\tilde{f}_{ib}^b$ and $\tilde{w}_{ib}^b$, and the cross-coupling error relative to gyro obtained by the indoor calibration IMU and turntable is:
The cross-coupling error relative to accelerometer is:

\[
E_a = \begin{bmatrix}
0 & E_{gxy} & E_{gxz} \\
E_{gyx} & 0 & E_{gyz} \\
E_{gxx} & E_{gy} & 0
\end{bmatrix}
\]  \hspace{1cm} (10)

The relative gyro zero bias is 
\[\epsilon = [\epsilon_x, \epsilon_y, \epsilon_z]^T,\]

\[\nabla = [\nabla_x, \nabla_y, \nabla_z]^T,\]

the relative gyro scale coefficient is 
\[K_g = [K_{gx}, K_{gy}, K_{gz}]^T,\]

the relative accelerometer zero bias is:

\[
E_a = \begin{bmatrix}
0 & E_{axy} & E_{axz} \\
E_{ayx} & 0 & E_{ayz} \\
E_{ax} & E_{ay} & 0
\end{bmatrix}
\]  \hspace{1cm} (11)

According to the calibration compensation formula, the corresponding device error is compensated to IMU output, so that the IMU output coordinate system is converted to the virtual IMU output coordinate system \(O_{1}X_1Y_1Z_1\) (i.e. turntable coordinate system).

In calibration process, it can reduce the impact between various parameters if the \(O_{1}X_1Y_1Z_1\) system coincides in parallel with the \(O_{2}X_2Y_2Z_2\) system.

5. Prism and IMU error angle calculation method
After calibrating the relative relationship of prism-turntable and IMU-turntable, the calculation of the prism and IMU error angle can be realized by using conversion matrix.

6. Tests and Results

6.1. Measurement of horizontal attitude Angle of prism
In view of hexahedral prism and IMU installation error calibration technology based on turntable transmission, our team conducted the relevant tests in Mobile Measurement Technology Laboratory (Hubei Province Engineering Center) to test the hexahedral prism and the IMU of A28 model.
Table 1. Test equipment

| Serial number | Device Name      | Device Model | Description              |
|---------------|-----------------|--------------|--------------------------|
| 1             | POS IMU         | IMU-A28      | tested equipment, number: 1804001 |
| 2             | hexahedral prism| -            | tested Equipment         |
| 3             | three-axis rate | 3KT-660      | -                        |
|               | turntable       |              |                          |
| 4             | total station   | NET          | -                        |
| 5             | test computer   | Lenovo       | for data calculations    |
| 6             | tripod          | -            | for fixing total station  |
| 7             | centring base   | -            | for fixing total station  |

6.2. Test Results

According to the high-precise total station measurement results, results of attitude calculation between turntable and prism are shown in Table 2.

Table 2. Turntable and Prism Calculations

| Attitude | Turntable | Prism | Turntable-Prism |
|----------|-----------|-------|-----------------|
| pitch    | \(-0.13768337\) | \(-0.19010139\) | \(0.05241802\) |
| roll     | \(-0.05109609\) | \(-0.07552331\) | \(0.02442722\) |
| yaw      | \(179.8108478\) | \(179.4925746\) | \(0.31827311\) |

According to testing results of high-precise three-axis turntable, results of angle calculation between IMU and turntable are shown in Table 3.

Table 3. IMU and Prism Attitude Calculations

| Pitch (") | Roll (") | Yaw (") |
|-----------|----------|---------|
| 1         | -180.5   | 75.2    | 50      |
| 2         | -181.6   | 76.5    | 48.9    |
| 3         | -182.1   | 76.6    | 48.9    |
| average   | -181.4   | 76.1    | 49.3    |
| repeatability | 0.8      | 0.8     | 0.6     |

According to Table 2 and Table 3, results of angle calculation between IMU and prism are shown in Table 4.

Table 4. IMU and Prism Calculations

| Attitude | IMU-Turntable | Prism- Turntable | IMU-Prism |
|----------|---------------|------------------|-----------|
| pitch    | \(-181.4\)    | \(-188.7\)       | 7.3       |
| roll     | \(76.1\)      | \(-87.9\)        | 164       |
| yaw      | \(49.3\)      | \(-1145.8\)      | 1195.1    |
| repeatability | 0.8      | 0.9    | 0.9     |

7. Introduction

The hexahedral prism and IMU installation error calibration technology based on turntable transmission was proposed, which made full use of high-precise total station and indoor turntable. Through turntable zero transmission, the hexahedral prism and IMU installation error was quickly calibrated in indoor. Moreover, the angle relationship between hexahedral prism mirror face line and
IMU output axis system was measured to facilitate lately the calibration between IMU, SAR radar and optical loads. This method is simple and high-accurate, and the repeatability of calibration results is better than 5". This technology has been serviced by CLP's SAR radar project and has achieved good effect.

References
[1] Pan J, Zhang C, Niu Y, et al. Accurate calibration for drift of fiber optic gyroscope in multi-position north-seeking phase[J]. Optik-International Journal for Light and Electron Optics, 2014, 125(24): 7244-7246.
[2] Zhu F, Tan J B, Cui J W. Beam splitting target reflector based compensation for angular drift of laser beam in laser auto collimation of measuring small angle deviations[J]. Review of Scientific Instruments, 2013, 84(6):124-131.
[3] Bao Wei-Min, Shen Gong-Xun, Li Hua-Bin. Investigation on inertial platform multi-position rolling self-calibration[J]. Journal of Beijing University of Aeronautics and Astronautics, 2011, 37(4):462-465.
[4] Zhang Hua-Qiang, Zhao Yan, Chen Yu. New system calibration calibration method for strapdown[J]. Journal of Beijing University of Aeronautics and Astronautics, 2012, 38(4):459-463.
[5] Dong Chun-Mei, Chen Xi-Jun, Ren Shun-Qing. Systematic calibration method for strapdown INS[J]. Navigation Positioning & Timing, 2016, 3(4): 74-80.
[6] Bekkeng J K. Calibration of a novel MEMS inertial reference unit[J]. instrumentation and Measurement, IEEE Transactions on, 2009, 58(6):1967-1974.
[7] Si Gao-li, Ma Bu-chuan, Zheng Tao, et al. Study on automatic calibration method of the installation error of the prism assembly[J]. Navigation and Control, 2016, 5: 100-103.
[8] Dai Chen-Xi, Research on the key technology of disassembly-free calibration method for missile-borne SINS[D]. Southeast University, 2017.

Funding:
This research received the funding of National Key Research and Development Program of China, the project name is "Remote Sensing Observation Technology for Highfrequency Subsequent Unmaruned Aircraft Regional Networking", sub-topic name is "high-precise light small POS system", and sub-topic number is "2017YFB0503001-2".