On the peculiarities in the rotational frequency evolution of isolated neutron stars

Abstract The measurements of pulsar frequency second derivatives have shown that they are $10^2$–$10^6$ times larger than expected for standard pulsar spin-down law, and are even negative for about half of pulsars. We explain these paradoxical results on the basis of the statistical analysis of the rotational parameters $\nu$, $\dot{\nu}$ and $\ddot{\nu}$ of the subset of 295 pulsars taken mostly from the ATNF database. We have found a strong correlation between $\ddot{\nu}$ and $\nu$ for both $\dot{\nu} > 0$ and $\dot{\nu} < 0$, as well as between $\nu$ and $\dot{\nu}$. We interpret these dependencies as evolutionary ones due to $\nu$ being nearly proportional to the pulsars’ age. The derived statistical relations as well as “anomalous” values of $\ddot{\nu}$ are well described by assuming the long-time variations of the spin-down rate. The pulsar frequency evolution, therefore, consists of secular change of $\nu$, $\dot{\nu}$ and $\ddot{\nu}$ according to the power law with $n \approx 5$, the irregularities, observed within a timespan as a timing noise, and the variations on the timescale larger than that timespan – several tens of years.

Keywords methods: data analysis — methods: statistical — pulsars: general

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1 Introduction

The spin-down of radio pulsars is due to the conversion of their rotation energy into emission. According to the ”classical” approach, their rotational frequencies $\nu$ evolve obeying the spin-down law $\dot{\nu} = -K\nu^n$, where $K$ is a positive constant that depends on the magnetic dipole moment and the moment of inertia of the neutron star, and $n$ is the braking index. The latter can be determined observationally from measurements of $\nu$, $\dot{\nu}$ and $\ddot{\nu}$ as $n = \nu\dot{\nu}/\ddot{\nu}^2$. For a simple vacuum dipole model of pulsar magnetosphere $n = 3$; the pulsar wind decreases this value to $n = 1$; for multipole magnetic field $n \geq 5$ (Manchester & Taylor 1977). At the same time the measurements of pulsar frequency second derivatives $\ddot{\nu}$ have shown that their values are much larger than expected for standard spin-down law and are even negative for about half of all pulsars. The corresponding braking indices range from $-10^6$ to $10^6$ (D’Alessandro et al. 1993; Chukwude 1993; Hobbs et al. 2004).

It was found that the significant correlations between $|\ddot{\nu}|$ (|$\dot{P}$|) and $\dot{\nu}$ ($\langle \dot{P} \rangle$) demonstrate the increase of the absolute values of the $\nu$ and $P$ second derivatives for younger (with faster slow-down) pulsars (Cordes & Downs 1985; Arzoumanian et al. 1994; Lyne 1999). The anomalously high and negative values of $\ddot{\nu}$ and $n$ may be interpreted as a result of low-frequency terms of the “timing noise” – a complex variations of pulsars rotational phase within a timespan (D’Alessandro et al. 1993).

It is clear that the timespan of observations is by no means intrinsic to the pulsar physics. Indeed, the variations of rotational parameters may take place on larger timescales as well. However, the timescale of observations naturally divides it into two separate classes of manifestations – the residuals in respect to the best fit for the timing solution (the “timing noise”) and the systematic shift of the best fit coefficients (i.e. in the measured values of $\nu$, $\dot{\nu}$, $\ddot{\nu}$) relative to some mean or expected value from the model. The latter effect may be called the “large timescale timing noise”.

Up to date we know nearly 200 pulsars for which the timespan of observations is greater than 20 years, and the values of their $\ddot{\nu}$ still turn out to be anomalously large (Hobbs et al. 2004).

For example, for the PSR B1706-16 pulsar, variations of $\ddot{\nu}$ with an amplitude of $10^{-24}$ s$^{-3}$ have been detected on a several years timescale (see Fig.7 in (Hobbs et al. 2004)), with the value of $\ddot{\nu}$ depending on the time interval...
selected. However, the fit over the entire 25 year timespan gives a value of $\dot{\nu} = 3.8 \times 10^{-25}$ s$^{-3}$ with a few percent accuracy (which leads to a braking index $\approx 2.7 \cdot 10^5$).

In the current work we provide observational evidence of the nonmonotonic evolution of pulsars on timescales larger than the typical contemporary timespan of observations (tens of years), using the statistical analysis of measured $\nu$, $\dot{\nu}$ and $\ddot{\nu}$. We estimate the main parameters of such long timescale variations and discuss their possible relation to the low-frequency terms of the timing noise. We have also derived the parameters of pulsar secular spin-down.

2 Statistical analysis of the ensemble of pulsars

Our statistical analysis is based on the assumption that numerous measurements of the pulsar frequency second derivatives reflect their evolution on the timescale larger than the duration of observations, and uses the parameters of 295 pulsars.

From 389 objects of ATNF catalogue (Manchester et al., 2003) with known $\dot{\nu}$ we have compiled a list of "ordinary" radio pulsars with $P > 20$ ms and $\dot{P} > 10^{-17}$ s s$^{-1}$, excluding recycled, anomalous and binary pulsars, and with relative accuracy of second derivative measurements better than 75%. It has been appended with 26 pulsars from other sources (D’Alessandro et al., 1993; Chukwude, 1993). The parameters of all pulsars have been plotted on the $\ddot{\nu} - \dot{\nu}$ diagram (Fig. I).

The basic result of the statistical analysis of this data is a significant correlation of $\dot{\nu}$ and $\ddot{\nu}$, both for 168 objects with $\ddot{\nu} > 0$ (correlation coefficient $r \approx 0.90$) and for 127 objects with $\ddot{\nu} < 0$ ($r \approx 0.85$). Both groups follow nearly linear laws, however they seem to be not exactly symmetric relative to $\ddot{\nu} = 0$. We divided both branches into 6 intervals of $\ddot{\nu}$, computed mean values and its standard deviations of $\ddot{\nu}_\pm$ in each and rejected the hypothesis of their symmetry with a 0.04 significance level. The absolute values of $\ddot{\nu}_+$ are systematically larger than corresponding $\ddot{\nu}_-$ (the difference is positive in 5 intervals of 6). Also, the difference of analytical fits to branches is positive over $-10^{-11} \div -10^{-15}$ s$^{-3}$ interval of $\ddot{\nu}$. These are the arguments in favour of a small positive asymmetry of branches.

We found an obvious correlation of $\dot{\nu}$ with the characteristic age $\tau_{ch} = -\frac{1}{2} \frac{\dot{\nu}}{\nu}$ ($r = 0.96$, $\dot{\nu} \sim \tau_{ch}^{-1.16\pm0.02}$). These parameters are nearly proportional, which leads to a significant correlation of $\tau_{ch}$ both with $\ddot{\nu}$ ($r = 0.85$ for the positive branch and $r = 0.75$ for the negative one) and with $\eta$ ($r = 0.75$ and $r = 0.76$ correspondingly).

The correlations found are fully consistent with the results published in Cordes & Downs (1985), Arzoumanian et al. (1994); Lyne (1999), as well as in Urama et al. (2006). However, the branches with $\ddot{\nu} > 0$ and $\ddot{\nu} < 0$ in those works were not analyzed separately from each other (not as $|\dot{\nu}|$).

Young pulsars confidently associated with supernova remnants are systematically shifted to the left in Fig. 1 (open symbols). The order of their physical ages roughly corresponds to that of their characteristic ages. This means that any dependence on $\dot{\nu}$ or $\tau_{ch}$ reflects the dependence on pulsar age.

The $\ddot{\nu} - \dot{\nu}$ diagram (Fig. I) may be interpreted as an evolutionary one. In other words, each pulsar during its evolution moves along the branches of this diagram while increasing the value of its $\dot{\nu}$ (which corresponds to the increase of its characteristic age). However, there is an obvious contradiction: for the negative branch, $\dot{\nu}$, being negative, may only decrease with time (since $\ddot{\nu}$ is formally the derivative of $\dot{\nu}$), and the motion along the negative branch may only be backward! This contradiction is easily solved by assuming non-monotonic behaviour of $\ddot{\nu}(t)$, which has an irregular component ($\delta \ddot{\nu}$) along with the monotonic one ($\ddot{\nu}_{ev}$), where the subscript “ev” marks the evolutionary value. In such interpretation the value of $\nu_{ev}$ must be positive, which will lead to a positive asymmetry of the branches. Moreover, the evolutionary increasing of $\dot{\nu}$ implies a positive evolutionary value of $\ddot{\nu}$.

The characteristic timescale $T$ of such variations must be much shorter than the pulsar life time and at the same time much larger than the timescale of the observations. As it evolves, a pulsar repeatedly changes sign of $\ddot{\nu}$, in a spiral-like motion from branch to branch, and spends roughly half its lifetime on each one. The asymmetry of the branches reflects the positive sign of $\nu_{ev}(t)$, and therefore, secular increase of $\nu_{ev}(t)$ (i.e. all pulsars in their secular evolution move to the right on the $\ddot{\nu} - \dot{\nu}$ diagram). Systematic decrease of branches separation reflects the decrease of the variations amplitude and/or the increase of its characteristic timescale.
Any well known non-monotonic variations of $\dot{\nu}(t)$, like glitches, microglitches, timing noise or precession, will manifest themselves in a similar way on the $\nu - \dot{\nu}$ diagram and lead to extremely high values of $\dot{\nu}$ (Shemar & Lyne 1996; Stairs et al. 2000). However, their characteristic timescales vary from weeks to years, and they are detected immediately. But here the variations on much larger timescales are discussed, and their study is possible only statistically, assuming the ergodic behaviour of the ensemble of pulsars.

3 Non-monotonic variations of pulsar spin-down rate on large timescales

Variations of the pulsar rotational frequency may be complicated – periodic, quasi-periodic, or completely stochastic. Generally, it may be described as a superposition

$$\nu(t) = \nu_{ev}(t) + \delta\nu(t),$$

where $\nu_{ev}(t)$ describes the secular evolution of pulsar parameters and $\delta\nu(t)$ corresponds to irregular variations. Similar expressions describe the evolution of $\dot{\nu}$ and $\ddot{\nu}$ after a differentiation. The $\delta\nu(t)$ satisfies the obvious condition of zero mean value $\langle \delta\nu(t) \rangle \approx 0$ over the timespans larger than characteristic timescale of the variations. The amplitude of the observed variations of $\dot{\nu}$ is related to the dispersion of this process as $\sigma_{\delta\nu} = A_{\nu} = \sqrt{\langle (\delta\nu)^2 \rangle}$. The second derivative $\ddot{\nu}$ is the only parameter significantly influenced by the timing variations (see Section 4). Thus one can assume that the measured values of $\nu$ and $\dot{\nu}$ may be considered to be evolutionary ones, $\nu_{ev}$ and $\dot{\nu}_{ev}$ (since $\delta\nu$ and $\delta\ddot{\nu}$ are small).

Using relations described above, the secular behavior $\nu(t)$ (or $\nu(\dot{\nu})$) may be found by plotting the studied pulsar group onto the $\nu - \dot{\nu}$ diagram (Fig. 2). The objects with $\dot{\nu} > 0$ and $\dot{\nu} < 0$ are marked as filled and open circles, correspondingly. It is easily seen that the behavior of these two sub-groups is the same, which is in agreement with the smallness of the pulsar frequency variations in respect to the intrinsic scatter of $\nu(\dot{\nu})$. However, a strong correlation between $\nu$ and $\dot{\nu}$ ($r \approx 0.7$) is seen, and

$$\dot{\nu} = -C\nu^n,$$

where $C = 10^{-15.26 \pm 1.38}$ and $n = 5.13 \pm 0.34$. So, the secular evolution of the “average” pulsar is according to the “standard” spin-down law with $n \approx 5!$ This result is very interesting on its own, especially since the $\nu$ and $\dot{\nu}$ are always measured as independent values. The braking index $n \approx 5$ may suggest the importance of multipole components of pulsar magnetic field, or the deviation of the angle between pulsar rotational and dipole axes from $\pi/2$ (Manchester & Taylor 1977).

Note here, that the width of the fit on Fig. 2 is quite small – only about 1.5 orders of magnitude. If the spin-down is even approximately close to being described by the vacuum dipole model, then $C$ should scale as $\left( B_0 \sin\alpha \right)^2$, where $B$ is the polar field and $\alpha$ is the magnetic inclination angle. But the range of $\left( B_0 \sin\alpha \right)^2$ over the pulsar population is expected to be of many orders of magnitude. Therefore, Fig. 2 once again shows that simple vacuum dipole model is not adequate to observations. The braking index $n \approx 5$ allows the interpretation of $C$ in terms of the quadrupole spin-down. It could be explained also in the frame of electric current mechanism of pulsar spin-down (Gurevich et al. 1993).

As has been stated above, the dependency between $\nu$ and $\tau_{ch}$ is consistent with a power law with a slope of $-1.16 \pm 0.02$. This value is significantly different from $-1.0$ which would be the case when the $\nu - \dot{\nu}$ correlation is absent and roughly consists with a slope of 5.13 in the $\nu(\dot{\nu})$ dependency (the values are different on a 2.5$\sigma$ level). But $n$ is strongly dependent on the $\nu - \tau$ slope value, so $-1.16$ seems to be more or less consistent with the measured value of $n$.

From (2) we may easily determine the relation between $\nu_{ev}$ and $\dot{\nu}$ as $\nu_{ev} = nC^{\frac{1}{n}}(-\dot{\nu})^{2-\frac{1}{n}}$, which is shown in Fig. 3 as a thick dashed line. The same relation may be also estimated directly by using the asymmetry of the branches seen on Fig. 3 as $\ddot{\nu}(\nu) = \frac{1}{2}(\dddot{\nu} + \ddot{\nu}_-)$, where $\dddot{\nu}_\pm$ are defined in Fig. 1. Such estimation, while being very noisy, is positive in the $-10^{-11} \div -10^{-15} \nu^{-2}$ range and agrees quantitatively with the previous one.

The amplitude of the $\ddot{\nu}$ oscillations, $A_{\ddot{\nu}}$, may be easily computed in a similar way, by using $\dddot{\nu}_+$ and $\dddot{\nu}_-$, as $A_{\ddot{\nu}} = \frac{1}{2}(\dddot{\nu}_+ - \dddot{\nu}_-)$. 

![Fig. 2](image_url) The $\nu - \dot{\nu}$ diagram for the pulsars with the measured second derivative. The filled symbols are objects with positive $\dot{\nu}$, the open ones – with negative. The behavior of both subsets is the same. The solid line represents the best fit, corresponding to the $n \approx 5$ braking index, the dotted ones – $1-\sigma$ range.
The physical reasons of the discussed non-monotonic variations of the pulsar spin-down rate may be similar to the ones of the timing noise on a short timescale. Several processes had been proposed for their explanation (Cordes & Greenstein [1981]) – from the collective effects in the neutron star superfluid core to the electric current fluctuations in the pulsar magnetosphere. Whether these processes are able to produce long timescale variations is yet to be analyzed. On a short timescale, the pulsars show different timing behaviour. But on the long timescale their behavior seems to be alike.

The argument in favor of the similarity between the discussed variations and the timing noise is the coincidence of the timing noise $\dot{\nu}$ amplitude extrapolated according to its power spectrum slope (Baykal et al. 1999) to the timescale of hundreds of years, with the $A_\nu$ derived from our analysis for the same $\dot{\nu}$, i.e. the same ages (see Fig. 3).

At the same time, there are several low noise pulsars with large or negative $\dot{\nu}$. For 19 of 45 pulsars studied in (D’Alessandro et al. 1995) the timing noise is nearly absent (RMS $< 1 \times 10^{-3} P$). Six of them have anomalous $\dot{\nu}$ measured in (Hobbs et al. 2004), which are well consistent with the $\dot{\nu}$ - $\dot{\nu}$ correlation (Cordes & Downs 1983; Arzoumanian et al. 1994) and have a wide range of $\dot{\nu}$. This shows a possible difference between timing noise and the long timescale variations described above.

In any case, the principal point is that all the pulsars evolve with long-term variations, and the timescale of such variations significantly exceeds several tens of years. That explains the anomalous values of the observed $\dot{\nu}$ and braking indices and gives reasonable values of the underlying secular spin-down parameters.

4 Discussion and conclusions

In general, it is impossible to estimate the amplitudes of the frequency and its first derivative variations $\dot{\nu}_f$ and $A_{\dot{\nu}}$ from the amplitude of the second derivative only (the knowledge of its complete power density spectrum is needed). However, if the spectral density is relatively localized and some characteristic timescale $T$ of the variations exists, it is possible to set some limits on it. A rough estimation is $A_{\dot{\nu}} \sim A_\nu T$, $A_{\ddot{\nu}} \sim A_\nu T$ and $A_{\nu/\dot{\nu}} \sim A_\nu T^2$. On a large timescale the variations can not lead to pulsar spin-up, so the variations of frequency first derivatives are much smaller than the secular ones, and $A_{\dot{\nu}} \sim A_\nu T \ll \dot{\nu}$, so $T \ll \dot{\nu}/A_{\dot{\nu}}$. So, for the $10^{-12} < \dot{\nu} < 10^{-15}$ $s^{-2}$ range and corresponding values of $A_{\dot{\nu}}$ from $10^{-23}$ $s^{-3}$ to $10^{-26}$ $s^{-3}$, the characteristic timescale $T_{\nu/\dot{\nu}} \sim 10^{11}$ s. Also, this characteristic timescale is obviously larger than the timespan of observations, so $50 < T < 3 \times 10^3$ years. Assuming the constancy of $T$ during the pulsar evolution and therefore the change of $A_{\dot{\nu}}$ with time, we get $A_{\dot{\nu}} \sim 10^{-3} \div 10^{-7}$ Hz. For such a model the pulsar frequency varies with the characteristic time of several hundred years and the amplitude from $10^{-3}$ Hz for young objects to $10^{-7}$ Hz for older ones.

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