Hysteretic Depinning and Dynamical Melting for Magnetically Interacting Vortices in Disordered Layered Superconductors

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We examine the depinning transitions and the temperature versus driving force phase diagram for magnetically interacting pancake vortices in layered superconductors. For strong disorder the initial depinning is plastic followed by a sharp hysteretic transition to a 3D ordered state for increasing driving force. Our results are in good agreement with theoretical predictions for driven anisotropic charge density wave systems. We also show that a temperature induced peak effect in the critical current occurs due to the onset of plasticity between the layers.

A vortex lattice subject to quenched disorder offers an ideal system for studying glassy dynamics governed by the competition between the ordering forces (the vortex-vortex repulsion) and disorder \[ \Phi \]. One of the key issues of glassy dynamics is the nature of the depinning transition separating a pinned state of the lattice at small external drives from the sliding regime that occurs when the external force exceeds a threshold depinning value. The early numerical simulations revealed that in the system with strong disorder plastic depinning occurs with the vortices tearing past one another and flowing in complex patterns \[ \Phi \]. The onset of plasticity at depinning was later confirmed in experiment \[ \Phi \] as well as subsequent numerical studies \[ \Phi \] and was demonstrated to play a crucial role in the pre-melting peak effect where the critical current exhibits a striking increase as a function of temperature or field \[ \Phi \] prior to vortex lattice melting and related phenomena. As the applied driving force is further increased, the effect of the quenched disorder is reduced and the vortex lattice can show a remarkable reordering transition at a driving force \[ F_c \] \[ \Phi \]. At this transition a considerable portion of the vortex lattice order is restored and the motion is highly coherent. The reordering force \[ F_c \] is expected to diverge as the temperature \( T \) approaches the vortex lattice melting temperature \( T_m \) \[ \Phi \]. Transport measurements provide strong evidence for this scenario \[ \Phi \]. Additional studies on the depinning and reordering transition and the nature of the coherently moving phases have been conducted theoretically \[ \Phi \], numerically \[ \Phi \], and experimentally \[ \Phi \] \[ \Phi \], showing both non-hysteretic continuous depinning transitions and abrupt hysteretic \( V(I) \) curves. The appearance of the hysteretic portion of the \( V(I) \) curves depends on how the vortex lattice was prepared: the initial current ramp shows a large hysteretic critical current but subsequent ramps produce only continuous non-hysteretic \( V(I) \) characteristics \[ \Phi \].

The underlying mechanisms of depinning and its interplay with the reordering transition remain an open question. The issues to be clarified include a determination of the conditions under which the depinning transition shows hysteretic or switching responses, as well as an understanding of the nature of these phenomena. The mechanism by which a three-dimensional lattice depins or reorders is also an open question, as well as the issue of whether the onset of plasticity coincides with a sudden jump in the critical current (the peak effect).

Some of these issues were addressed in \[ \Phi \] in a context of a driven three-dimensional anisotropic charge density wave system (CDW) with disorder. It was shown that if disorder is sufficiently strong \[ \Phi \], the dependence of the lattice velocity on the applied drive becomes two-valued or bistable: at the same applied force (close to the depinning force of the topologically disordered phase) the lattice can either remain at rest or slide depending on the history. This means that the depinning transition shows switching behavior and hysteresis. The details of depinning are determined by particulars of the system involved. For a layered anisotropic CDW considered in \[ \Phi \] the depinning occurs in two stages. The initial plastic depinning, where the decoupled 2D CDWs slide independently in each layer, is followed by a sharp transition or crossover at higher drives to a faster coherently moving 3D solid CDW phase. Reversing the process reveals hysteresis: pinning (immobilizing) of a coherent 3D CDW occurs at smaller drives. It was also shown that for weak disorder there is only a single non-hysteretic depinning transition directly to the 3D coherently moving state.

The results of \[ \Phi \] are very general, and the particular model used there can be straightforwardly generalized to other periodic media in quenched disorder. A detailed study of the character of the many-valued \( v - F \) dependence near depinning was recently carried out in \[ \Phi \] within the framework of a mean-field model for a visco-elastically driven vortex configuration, where a dynamic bistability and, accordingly, a hysteretic plastic depinning was found to occur for sufficiently strong disorder. Note that the layered CDW model of \[ \Phi \] can
be conveniently applied directly to highly anisotropic superconductors where the magnetic interactions between pancake vortices dominate \[1\]. In this system both a 2D regime, in which vortices in each layer move independently from the other layers, and a 3D regime, where vortices in different layers form coherently moving lines, should exist \[15,16\].

In this work we investigate the dynamics of magnetically interacting vortices in a three-dimensional layered superconductor to numerically test the predictions of Ref. \[1\]. Our model is relevant to anisotropic layered superconductors such as BSCCO as well as artificially layered superconductors in which the magnetic interactions dominate. Recent numerical simulations with this model have found that for increasing applied driving force, a transition from a decoupled plastically flowing state to a coupled 3D coherently moving lattice occurs \[15,16\]. A sharp static 2D to 3D disorder induced transition also appears as a function of field and disorder \[15,16\]. In this model, the dynamic phase diagram as a function of driving force and temperature, as well as hysteretic and switching behavior in the current-voltage characteristics, have not been studied until now. We find that for strong disorder, the initial depinning is 2D with the vortices flowing plastically and independently in any one layer. For increased drives we see a sharp transition to a coherently moving 3D vortex lattice which manifests itself in an abrupt increase in the vortex velocities. We observe strong hysteresis when the driving current is cycled. We have also investigated the driving versus temperature phase diagram.

We consider a three-dimensional layered superconductor in which the vortices are modeled as repulsive point particles confined to a layer with an equal number of vortices per layer. The overdamped equation of motion for vortex \(i\) is given by

\[ f_i = -\sum_{j=1}^{N_v} \nabla_i U(\rho_{i,j}, z_{i,j}) + f_i^{pp} + f_d + f^T = \eta v_i, \]

where \(N_v\) is the number of vortices, \(\rho_{i,j}\) and \(z_{i,j}\) are the distance between pancakes \(i\) and \(j\) in cylindrical coordinates, and \(\eta = 1\). The system has periodic boundaries in-plane and open boundaries in the \(z\) direction \[18\]. The magnetic energy between pancakes is

\[ U(\rho_{i,j}, 0) = 2d\epsilon_0 \left( 1 - \frac{d}{2\lambda} \right) \ln \frac{R}{\rho} + \frac{d}{2\lambda} E_1 \]

\[ U(\rho_{i,j}, z) = -s_m \frac{d^2\epsilon_0}{\lambda} \left( \exp\left(-z/\lambda\right) \ln \frac{R}{\rho} - E_2 \right) \]

where \(\epsilon_0 = \Phi_0^2/(4\pi\lambda)^2\), \(\lambda\) is the London penetration depth, \(R = 22.6\lambda\), the maximum radial distance, \(E_1 = \int_0^\infty d\rho' \exp\left(-\rho'/\lambda\right)/\rho', \quad E_2 = \int_0^\infty d\rho' \exp\left(-\sqrt{\rho^2 + \rho'^2}/\lambda\right)/\rho'\), \(d = 0.005\lambda\) is the inter-layer spacing, and we set \(s_m = 2.0\). The uncorrelated pins are modeled by parabolic traps that are randomly distributed in each layer. The vortex-pin interaction is given by

\[ f_i^{pp} = -\sum_{k=1}^{N_v} \frac{N_v}{k} \frac{f_p}{\rho} \frac{(r_i - r_k^p)}{\Theta((\xi_p - r_i - r_k^p)/\lambda)}, \]

where the pin radius \(\xi_p = 0.2\lambda\), the pinning force \(f_p = 0.02f_0^p\), and \(f_0 = \epsilon_0/\lambda\). We have simulated a 16\(\lambda\) x 16\(\lambda\) system with a vortex density of \(n_v = 0.35/\lambda^2\) and a pin density of \(n_p = 1.0/\lambda^2\) in each of \(L = 8\) layers. This corresponds to \(N_v = 80\) vortices and \(N_p = 256\) pins per layer, with a total of 640 pancake vortices. For our finite temperature simulations the temperature is modeled as a Langevin kick \(f^T\) where \(<f(t)^T> = 0\) and \(<f(t)^T f(t)^T> = 2k_B T \delta_{ij}\).

In Fig. 1(a) we show the average vortex velocities in the direction of drive, \(V_x = <v_x>_\rho\), as the applied driving force \(f_d\) is increased and then decreased, and in Fig. 1(b) we show the corresponding \(c\)-axis correlation function, \(C_z = 1 - \langle\langle(r_i - \rho - r_{i+1} - \rho + \rho_0)(0)/2\rangle\rangle\Theta(\rho_0/2 - (r_i - \rho - r_{i+1} - \rho_0))\), where \(\rho_0\) is the vortex lattice constant. At \(f_d = 0\) the vortices are pinned and uncorrelated in the \(z\)-direction as is indicated by the fact that \(C_z\) has a low value. The initial depinning, at \(f_{dp} = 0.01f_0^p\), is 2D with the vortices remaining uncorrelated in the \(z\)-direction. Near \(f_d = 0.013\) the vortex velocities sharply increase, coinciding with a sharp transition to a coupled moving 3D state as seen in the jump up in \(C_z\). At higher drives the vortex velocity linearly increases with increasing \(f_d\). Upon reducing \(f_d\) from this linear regime the system remains in the 3D state for driving forces well below \(f_d = 0.013\). At \(f_d = 0.007\) there is a sudden transition to a 2D pinned state as seen in the drop in \(V_x\) and \(C_z\). If the applied driving force is again increased the velocities and \(C_z\) will follow the same hysteresis loop. In Fig. 1(c,d) we show \(V_x\) and \(C_z\) for the same system as in Fig. 1(a,b) but with a lower disorder strength of \(f_p = 0.01f_0^p\). Here the depinning transition is non-hysteretic with a single pinning threshold to the 3D coherently moving state as seen by the fact that \(C_z = 1.0\) at all times. For a given coupling strength value \(s_m\), there is a critical pinning force above which there is a static 3D-2D transition \[18\]. Hysteric \(V(I)\) curves can only be observed above, but close to, this transition. The width of the hysteresis in the \(V(I)\) curves is largest for disorder strengths just above the static 2D-3D transition and gradually narrows for increasing disorder strength.

The behavior we observe is in excellent agreement with the theoretical predictions of \[1\]. We note that unlike the anisotropic charge-density wave system, where only inter-plane plasticity or slipping can occur, in the vortex system in-plane plasticity is possible within each layer. The 2D-3D transition in the case of the vortices occurs by simultaneous recoupling of the vortices between layers and the recoiling of the vortices in plane. To illustrate this, in Fig. 2(a) we show a top view of the vortices in
the moving 2D regime, indicating that the vortices are uncorrelated from layer to layer and are disordered in plane. In contrast, Fig. 2(b) shows that in the moving 3D regime, the vortices are aligned between layers and possess a triangular ordering.

In Fig. 3 we show the driving force versus temperature phase diagram for a system with \( f_p = 0.01 f_0 \). The temperature is plotted in units of \( T_m = 0.00045 \), the temperature at which the clean system undergoes melting in the form of a single sharp transition from a 3D lattice to a 2D pancake gas. As shown in Fig. 3, at \( T/T_m < 0.005 \) for low drives there is a 3D pinned phase where the vortices remain aligned. At these temperatures, for \( f_d/f_0 = 0.001 \) there is a non-hysteretic depinning transition directly to the coherently moving 3D state.

We find a static transition to the 2D state at \( T/T_m = 0.005 \), which we label \( T_{dc} \). For \( T > T_{dc} \) the depinning transition is 2D, and as the driving force is further increased a dynamically induced reordering of the vortices occurs. As \( T \) is further increased above \( T_{dc} \) the size of the 2D pinned phase is reduced, while the drive at which the 2D plastic flow to 3D coherent flow transition occurs diverges as \( T \) approaches \( T_m \), in agreement with the dynamical freezing model of Koshelev and Vinokur.

The thermally induced decoupling seen in Fig. 3 also coincides with a sharp increase in the critical current or a “peak effect.” There has been considerable work on dimensional transitions in the context of the peak effect in layered superconductors. The static 3D-2D transition and critical current increase in layered superconductors have been previously studied as a function of interlayer coupling and magnetic field \([23, 24]\). Koshelev and Kes have proposed theoretically that magnetically coupled vortices can show a first order transition from a 3D to a 2D system under certain conditions \([22]\). The sharpness in the 3D-2D transition we observe in our simulation suggests that it is first order in nature \([15, 17]\).

The 3D-2D transition can be seen as a temperature induced peak effect in which the combination of temperature and quenched disorder bring about a decoupling transition where the individual pancakes can adjust to the optimal pinning configurations. The phase diagram in Fig. 3 is very similar to the one proposed by Battacharya and Higgins \([8]\) in which there is an elastic depinning regime for a certain temperature range, above which the peak effect and the onset of plasticity simultaneously appear. In our model the peak effect is caused not by the onset of plasticity in the \( a-b \) plane but by the onset of plasticity in the \( c \)-axis \([7]\) with the plasticity only occurring for \( T > T_{dc} \). We find that the temperature \( T_{dc} \) moves further below \( T_m \) as the disorder in the sample is increased. Thus for large enough disorder the 2D phase extends all the way to \( T = 0 \), and the dynamic phase diagram becomes the same as the one proposed in Ref. \([9]\).

In Fig. 4 we show the \( V_x \) versus \( f_d \) curves, which correspond to experimental \( V(I) \) curves, for increasing \( T \). For \( T < T_{dc} \) there is a sharp elastic depinning transition at which all the vortices start moving at once in the 3D state. For \( T > T_{dc} \) the depinning response is more continuous since the 2D depinning occurs inhomogeneously, with certain regions moving while other regions remain pinned. As \( T \) is further increased in the 2D regime the depinning transition falls at driving forces lower than that at which the 3D depinning transition occurred. Although for these higher temperatures the 2D pancakes depin at a lower applied drive than the 3D state, the overall velocity above depinning is still less than that of a coherently
moving 3D system at the same drive so that a crossing of the IV curves occurs.

To summarize, we have numerically investigated the dynamics of magnetically coupled vortices in layered superconductors interacting with quenched disorder. For sufficiently strong disorder the depinning is 2D. Here the vortices flow plastically in each layer and are uncorrelated from layer to layer. For higher drives there is a sharp hysteretic transition to a coherently moving ordered 3D vortex lattice. For weak disorder there is a non-hysteretic depinning transition from a pinned 3D to a moving 3D state. The two stage 2D-3D hysteretic depinning transition is in good agreement with the theoretical predictions of Ref. [11]. We have also mapped out the dynamic phase diagram as a function of driving force and temperature. At low temperatures we observe non-hysteretic 3D pinned to 3D moving transitions. For increased temperature there is a static 3D-2D transition which coincides with a peak in the critical current as well as the onset of plasticity in the c-axis. The drive at which a dynamically induced 2D-3D dynamic transition diverges as \( T_m \) is approached. Our dynamic phase diagram is very similar to that proposed in Ref. [12].

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\[ T/T_m = 0.00027 \text{ and } 0.0013. \]

\[ V_T \text{ versus } f_d \text{ responses for varied } T. \text{ Circles: temperatures below the decoupling temperature } T_{dc}: T/T_m = 0.0005, 0.014, 0.05, 0.11, 0.22, 0.5, 0.9. \]