Testing Modified Gravity Theories by Constraining the Theories Parameter Using Radius and Mass Observations of Brown Dwarfs

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Abstract. Modified Gravity (MOG) theories modify Poisson equation in the Newtonian limit. This MOG is also important as test-bed to analyze whether General Relativity (GR) is consistent with observations. We investigate the mass and radius of Brown Dwarfs predicted by modified Poisson equation from Beyond Horndeski (BH) and Eddington-Inspired-Born-Infeld (EiBI) theories. Brown Dwarfs are sub-stellar objects that are difficult to be detected because of their low luminosity. One of the most important properties of brown dwarfs which recently observed by Johnson and Montet[1] are radii and masses. In this note, we derive the equation of state of the stars analytically, wherein electrons part, we use exact non-relativistic degenerate ideal Fermi gas result. We also derive modified hydrostatic stellar equation predicted by MOG theories and solve them numerically to obtain their profile and mass-radius relation for each parameter on both MOG theories. We compare our theoretical results with the ones obtained Johnson and Montet [1].

1. Introduction
MOG which succeed to overcome singularities problem in general relativity and served as alternatives to the existence of dark matter, need to be subjected to solar system or compact objects tests. In this research we focus on sub-stellar object such as brown dwarfs predictions of MOG (see Ref. [2] for similar study but focus on white dwarfs). Brown dwarfs are difficult to be detected because of their low luminosity. To study their luminosity, we need to know the properties of the objects, including radii and masses. From observation point of view, radii of brown dwarfs can be determined only if brown dwarfs is in transit period or existed in eclipse system. However, many progress in observation are reported recently (see Ref. [3] and the references therein). The first brown dwarf that is detected by Kepler data is found in LHS 6343 system. Johnson calculated mass of brown dwarf is $62.7 \pm 2.4 M_{\text{Jup}}$ and the corresponding radius is $0.833 \pm 0.021 R_{\text{Jup}}$. Montet also found brown dwarf with mass of $62.1 \pm 1.2 M_{\text{Jup}}$ and radii $0.782 \pm 0.013 R_{\text{Jup}}$. These results[1] depend on calculated mass with precision of 1.9% and depend on calculated radius with precision of 1.4% [1, 3]. We will compare these observation data to our theoretical results which are obtained use BH and EiBI theories in non-relativistic limit or each in a form of modified Poisson equation.
2. Formalism

2.1. Modified Gravity Theories

The Vainshtein mechanism is only partially operational in BH family theories which has an incomplete suppression of the fifth-force in the interior of extended sources due to the time-dependence of the cosmological field [4]. BH family theories is defined in action as

\[ S = \int \sqrt{-g} M^2_{pl} \left( \frac{R}{2} + X + \frac{L_4}{\Lambda^4} \right) d^4x + S_M. \]  

(1)

\( M_{pl} = (8\pi G_N)^{-1} \) is planck mass, \( X = -\frac{1}{2} \partial_t \phi \partial^t \phi \) and \( L_4 = X[\Box \phi^2] - \phi_{ab} \phi^{ab} - (\phi^a \phi^b \phi^c \phi^d) \). \( F_5 = 2\alpha^2 F_N \) is a consistent solution of the equation and the force is unscreened. Through Vainshtein mechanism, few astrophysical signatures can be observed. One can see the detail derivation in Ref.[4], where the final modified Poisson equation of BH action is

\[ \nabla^2 \Phi(r) = 4\pi G_N \rho(r) + \frac{\gamma}{4} G_N \frac{d^3M(r)}{dr^3}. \]  

(2)

\( \gamma \) is a dimensionless parameter which characterizes the deviation of of BH theory to general relativity (GR). Note that if \( \gamma > 0 \) in the new term in Eq. (2) then more mass is concentrated in stellar core [5].

We also investigate EiBI theory. The full action for EiBI theory[7],

\[ S = \frac{1}{8\pi G_N} \int d^4x(\sqrt{|g_{ab} + \kappa R_{ab}(\Gamma)|} - \lambda \sqrt{-g}) + S_m(g_{ab}, \chi_m), \]  

(3)

where \( |.| \) represents the determinant. We will arrive at field equation after varying EiBI action,

\[ \sqrt{-q} q^{ab} = \lambda \sqrt{-g} g^{ab} - 8\pi G_N \kappa \sqrt{-g} T^{(m)ab}, \]  

(4)

where \( q^{ab} \) is the inverse of \( q_{ab} \) and \( T^{(m)}_{ab} \) is energy-momentum tensor. The lowest correction Einstein equation in Newtonian limit will give modified Poisson equation. The modified Poisson equation for EiBI is expressed by [6],

\[ \nabla^2 \Phi(r) = 4\pi G_N \rho(r) + \frac{\kappa}{4} \nabla^2 \rho(r), \]  

(5)

where the correction term can work as repulsive or attractive force depending on the sign of the parameter \( \kappa \) and it can be shown that this parameter corresponds to effective polytropic equation of state parameter \( K \) through \( K = \frac{\kappa}{8} \) relation [6].

2.2. Equation of State

Brown dwarf fail to have a stable hydrogen burning sequence and instead derive their stability from electron degeneracy pressure [8]. We integrate the Fermi-Dirac integral exactly using the polylogarithm functions \( \text{Li}_x(x) \). The electrons are mainly non-relativistic due to relatively low temperature and density. The expression for the pressure of a degenerate Fermi gas at finite temperature is

\[ P_F = \frac{2}{5} \alpha \mu \frac{1}{2} - \frac{1}{8} \alpha \mu \frac{3}{2} \log(1 + e^{-\beta \mu}) + \frac{3}{2} \alpha \beta^{-2} \mu \frac{1}{2} \frac{\pi^2}{6} \]
\[ + \frac{3}{4} \alpha \beta^{-2} \mu \frac{1}{2} \text{Li}_2(-e^{-\beta \mu}) - \frac{3}{4} \alpha \beta^{-3} \mu^{-1/2} \text{Li}_3(-e^{-\beta \mu}) \]
\[ + \left( \frac{\pi^2}{6} - \frac{9}{4} \right) \alpha \beta^{-1} \mu \frac{3}{2} + \left( \frac{9}{8} - \frac{\pi^2}{3} \right) \alpha \beta^{-2} \mu \frac{1}{2} + \left( \frac{\pi^2}{2} \frac{3}{8} \right) \alpha \beta^{-3} \mu^{-1/2} \]
\[ + \frac{1}{8} \alpha \mu \frac{5}{2}. \]  

(6)
where \( \alpha = (2/3)(4\pi(2m)^{3/2}/(2\pi\hbar)^3) \), \( \beta = (k_BT)^{-1} \). We introduce degeneracy parameter \( \psi \) as central temperature of brown dwarf. It has the same order of magnitude as the electron Fermi Temperature,

\[
\psi = \frac{k_B T}{\mu_F} = \frac{2m_e}{h^2} \left( \frac{m_H \mu_e}{3\pi^2 \rho} \right)^{2/3} k_BT,
\]

where \( \mu_F \) is the electron Fermi energy in the degenerate limit. \( \mu_e = 1.143 \) is a fixed parameter of number of baryons per electron [5].

The interior of brown dwarf is composed by electrons and ions. Thus, the total pressure comes from both contributions i.e., \( P = P_F + P_{ion} \). The pressure due to ions for an ionized hydrogen gas can be expressed by \( P_{ion} = \frac{k_B T}{\mu m_H} \). The total equation of state for brown dwarf pressure is then

\[
P = \frac{2}{5} \alpha A^{5/2} \left( \frac{\rho}{\mu_e} \right)^{5/3} \left\{ 1 - \frac{5}{16} \psi \log(1 + e^{-\frac{1}{\psi}}) + \frac{15}{8} \psi^2 \left( \frac{\pi^2}{3} + Li_2(-e^{-\frac{1}{\psi}}) \right) \right. \\
- \frac{15}{16} \psi^3 Li_3(-e^{-\frac{1}{\psi}}) + \frac{5}{2} \left( \frac{\pi^2}{6} - \frac{9}{4} \right) \psi \\
+ \left. \frac{5}{2} \left( \frac{9}{8} - \frac{\pi^2}{3} \right) \psi^2 + \frac{5}{2} \left( \frac{3\pi^2}{16} \right) \psi^3 + \frac{5}{16} + \alpha \psi \right\},
\]

(8)

where \( \alpha = \frac{5\mu_e}{2\mu} \).

We can express the relation in Eq. (8) in a polytropic equation of state relation as \( P = K \rho^{5/3} \) which \( 5/3 \) is related to polytropic index \( n = 3/2 \), if we define

\[
K = \frac{(3\pi^2)^{2/3} h^2}{5m_e m_H \mu_e^{5/3}} (1 + \gamma + \alpha \psi),
\]

(9)

where \( \alpha = (2/3)(4\pi(2m)^{3/2}/(2\pi\hbar)^3) \), \( \psi \) is a degeneracy parameter and correction term \( \gamma \) is expressed by,

\[
\gamma = -\frac{5}{16} \psi \log(1 + e^{-1/\psi}) + \frac{15}{8} \psi^2 \left( \frac{\pi^2}{3} + Li_2(-e^{-1/\psi}) \right) - \frac{15}{16} \psi^3 Li_3(-e^{-1/\psi}) + \frac{5}{2} \left( \frac{\pi^2}{6} - \frac{9}{4} \right) \psi \\
+ \frac{5}{2} \left( \frac{9}{8} - \frac{\pi^2}{3} \right) \psi^2 + \frac{5}{2} \left( \frac{3\pi^2}{2^2} \right) \psi^3 + \frac{5}{16}.
\]

(10)

This constant depends on central density of stellar interior which rules the distribution of density and temperature in all points of brown dwarf.

2.3. Newtonian Limit of Hydrostatic Equilibrium Stellar Equation

Eq. (11) of stellar structure can be derived by considering the cylindrical slab in the star with two forces, one is from inside to outside which is known as electron degeneracy and the other is the gravitational force from the outside to inside. Both is in equilibrium. While Eq. (12) is derived from mass continuity. The explicit form of both equations are

\[
\frac{dP(r)}{dr} = -\frac{d\rho(r)}{dr} \rho(r)
\]

(11)

\[
\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)
\]

(12)
Figure 1. Beyond Horndeski Gravity on Mass and Radius Relation

We can use this equation to solve the stellar equation of Brown Dwarfs. We can obtain the modified Poisson equation of two modified gravity theories from subsection (2.3) from Eqs. (11) and (12). The modified Poisson equation for BH theory is

$$\frac{d\rho}{dr} = \frac{1}{K \Gamma \rho(r)^{\Gamma-1}} - \frac{1}{K \Gamma \rho(r)^{\gamma} G_N \pi r^2 \rho(r)} \left\{ \frac{G_N M(r)}{r^2} \right\}, \quad (13)$$

where $\gamma$ is a dimensionless parameter of BH theory. We also can obtain the stellar equation for EiBI theory as

$$\frac{d\rho}{dr} = -\frac{1}{K \Gamma \rho(r)^{\Gamma-1}} + \frac{2}{3} \frac{\kappa}{\rho(r)} \left\{ \frac{G_N M(r)}{r^2} \right\}, \quad (14)$$

where $\kappa$ is a dimensionless parameter of EiBI theory.

3. Result and Discussion

We solve differential equation from subsection (2.3) numerically using fouth order Runge-Kutta (RK4) dimensionless parameter with initial boundary condition $M(0) = 0$ and $\rho(0) = \rho_c$ where $\rho_c$ is central density of brown dwarf ($10^4 - 10^6$ kg/m$^3$) [9] to obtain mass-radius relation for brown dwarfs.

For BH Gravity theory, $\gamma$ parameter can be positive or negative. For $\gamma > 0$, the correction term in hydrostatic equation works as repulsive force, while for $\gamma < 0$, it works as attractive force. As the result in modified mass and radius relation, the radius is increased by increasing $\gamma$ value.

This situation is quite similar to those of EiBI gravity theory. The similarity can be seen obviously by comparing Fig. (1) of BH and Fig. (2) of EiBI. Parameter $\kappa$ in EiBI can also work as attractive or repulsive force depending on $\kappa$ sign.
Interestingly, Both theories predict the minimum mass of brown dwarf. Note that the minimum mass of Hydrogen Burning (MMHB) has been studied quite detail in Ref.[5]. Furthermore, we obtain the allowed parameter range between $-0.6 \leq \gamma \leq 0.4$ using observational constraints of Johnson et. al and thighter constraints for $\gamma = -0.2$ using Montet et. al data (LHS6343C). Note that in BH theory, Sakstein obtain the constraint limit for $\gamma < -2/3$ in order brown dwarfs have a stabilized stellar configuration [5]. In the case of EiBI theory, We can obtain by using constraint from Johnson et. al the parameter range between $-2.5 \leq \kappa \rho_A \leq 3$. Here $\rho_A = \rho_\odot \times 10^9$. We also obtain tighter constraint for $\kappa \rho_A = -1$ by using Montet et al data (LHS6343C).

4. Conclusion
The free parameter of two modified gravity theories gives significant effects to masses and radii relation of brown dwarfs. These are the consequences of correction term in modified Poisson equations that can work as repulsive or attractive force term so that it can strength or weaken gravitational field in stellar objects in general. We can obtain the limit of free parameter of both theories from the brown dwarf observational masses and radii. The corresponding masses and radii constraints by Johnson et. al and Montet et al which are shown in Figs. (1) and (2) can tightly constraint The free parameter of both modified gravity theories.

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