Hybrid spin Hall nano-oscillators based on ferromagnetic metal/ferrimagnetic insulator heterostructures

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Abstract

Spin-Hall nano-oscillators (SHNOs) are promising spintronic devices to realize current controlled GHz frequency signals in nanoscale devices for neuromorphic computing and creating Ising systems. However, traditional SHNOs --- devices based on transition metals --- have high auto-oscillation threshold currents as well as low quality factors and output powers. Here we demonstrate a new type of hybrid SHNO based on a permalloy (Py) ferromagnetic-metal nanowire and low-damping ferrimagnetic insulator, in the form of epitaxial lithium aluminum ferrite (LAFO) thin films. The superior characteristics of such SHNOs are associated with the excitation of larger spin-precession angles and volumes. We further find that the presence of the ferrimagnetic insulator enhances the auto-oscillation amplitude of spin-wave edge modes, consistent with our micromagnetic modeling. This hybrid SHNO expands spintronic applications, including providing new means of coupling multiple SHNOs for neuromorphic computing and advancing magnonics.
Introduction.

High efficiency oscillators are essential to accelerate the application of spintronics for neuromorphic computing\(^1\)-\(^4\), Ising systems\(^5\) and magnonic devices\(^6\)-\(^8\) among other applications. Spin-Hall nano-oscillators (SHNOs) are one of the important approaches to achieve these applications due to their two-dimensional geometry, which permits coupling multiple SHNOs in a plane\(^9\)-\(^11\), as well as their ease of fabrication. Several geometries of SHNOs have been proposed in previous studies, such as nanodisk\(^12\)-\(^14\), nanowire\(^15\)-\(^17\), and nanoconstriction types\(^18\)-\(^21\). However, these SHNOs generally have high threshold currents, low emission powers and poor quality factors because of the nature of their constituent materials, specifically the large magnetic damping in transition metal ferromagnets. In recent years, attention has focused on ferrimagnetic insulators\(^23\)-\(^26\) due to their extremely low damping and consequently high magnon conductivity\(^27\), which is very favorable for spintronic applications. At the same time, this low damping characteristic facilitates the formation of spin-Hall effect induced auto-oscillations; indeed, ferrimagnetic insulator-based nano-oscillators have been demonstrated with yttrium iron garnet/Pt bilayers\(^25\)-\(^26\). Nevertheless, they suffer from low power emission due to their small inverse spin Hall effect signals\(^28\). Joule heating also limits their application at room temperature\(^28\). One way to overcome these drawbacks is by creating a new type of hybrid SHNOs based on ferromagnetic metal-ferrimagnetic insulator heterostructures. Interesting physics emerges when coupling thin layers of these two types of materials\(^29\)-\(^32\). When the two layers are weakly coupled, there are two distinct spin resonances, associated with acoustic and optical modes. However, when they are strongly coupled, the two layers act collectively, leading to magnetic properties inherited from both layers\(^33\), specifically a lower effective damping. Thus, SHNOs fabricated from such heterostructures can take the advantage of the low damping from the ferrimagnetic insulator layer and yet maintain a strong electrical signal from the ferromagnetic metal layer.

Theoretical studies have shown that a uniform spin current applied to an extended magnetic thin film does not support the formation of auto-oscillations due to the emergence of nonlinear damping from magnon-magnon interactions\(^34\). However, by concentrating spin current in a small region, linear spin-wave mode auto-oscillation states can be stabilized\(^35\). Later, it was shown that nonlinear localized modes\(^36\) can be also achieved due to the suppression of magnon-magnon interactions, which has been experimentally demonstrated in point-contact type and disk-type SHNOs\(^12\)-\(^14\),\(^37\). Meanwhile, if the device geometry is confined (e.g. a nanowire or nanoconstriction), auto-oscillations can still be excited in a localized region that leads to a potential well that limits spin-wave propagation\(^16\)-\(^20\). These self-localized modes can have much smaller threshold currents than linear modes due to lower radiative loss in an auto-oscillation state.

In this article, we demonstrate a new type of SHNO that combines a ferromagnetic transition metal Py with an epitaxial thin film ferrimagnetic insulator, lithium aluminum ferrite (LAFO). This hybrid SHNO expands spintronic applications, including providing new means of coupling multiple SHNOs for neuromorphic computing and can advance designs for magnonics. Furthermore, compared to conventional Py/Pt SHNOs, this hybrid SHNO is superior in all important characteristics having a reduced threshold current, stronger emission power and higher quality factor.

Results and Discussion.

Our heterostructures are composed of two different lithium aluminum ferrite compositions (Li\(_{0.5}\)Al\(_{1.5}\)Fe\(_{1.5}\)O\(_4\) (LAFO) or Li\(_{0.5}\)Al\(_{0.5}\)Fe\(_2\)O\(_4\) (LFO)) (x nm)/Py(5nm)/Pt(5nm) layers with varied LAFO or LFO thickness x (including x=0, i.e., just Pt/Py layers). The Py/Pt layers are patterned into 400 nm wide nanowires with a 400 nm gap between two Au contact pads as shown in Fig. 1a. Detailed deposition and fabrication conditions are in Methods. We fabricated devices with LAFO: LAFO4/Pt5, LAFO10/Pt5, LAFO20/Pt5, and LFO:
LFO15/Py5/Pt5 with the numbers being the layer thicknesses in nm. Lastly, a Py5/Pt5 reference device was deposited on a sapphire substrate.

![Figure 1](image)

**Figure 1.** (a) Schematic of the hybrid SHNO device and power spectral density (PSD) measurement setup. (b) FMR frequency versus resonance field for various unpatterned thin films and heterostructures, including for reference, Py5/Pt5 bilayers and LAFO and LFO thin films. (c) FMR linewidth as a function of frequency for the same samples.

To determine the magnetic properties in different Py/LAFO samples, ferromagnetic resonance spectroscopy (FMR) measurements were carried out on unpatterned thin films and heterostructures via a vector network analyzer (VNA) technique. Effective magnetization $M_{\text{eff}}$ and anisotropy field $H_a$ are obtained by fitting resonance peaks to the Kittel model $f = \mu_0^2 \gamma^2 / 2 \pi \sqrt{(H + H_a)(H + H_a + M_{\text{eff}})}$, where $H$ is the external magnetic field, $M_{\text{eff}}$ is the effective magnetization, $\gamma$ is the gyromagnetic ratio, and $\mu_0$ is the vacuum permeability. Both LAFO and LFO have magnetocrystalline anisotropy with an easy axis along $<110>$ and hard axis along $<100>$ directions that is characterized by an in-plane anisotropy field $H_a$. Gilbert damping constants $\alpha$ are obtained by measuring the FMR linewidth as a function of the frequency. The data and fits are shown in Fig. 1b&c and the fitting parameters are listed in Supplementary Table S1. There was always only one FMR absorption peak observable, indicating that the two magnetic layers in our Py/LAFO heterostructures are strongly coupled. Further, the $M_{\text{eff}}$ of Pt/Py/LAFO falls between the $M_{\text{eff}}$ of bare LAFO or LFO and Py layers, as expected for two ferromagnetically coupled magnetic layers. To analyze the change
of $M_{\text{eff}}$ and $\alpha$ in the heterostructures, a macrospin model based on Landau-Lifshitz-Gilbert (LLG) equation considering two strongly coupled magnetic layers is used (see Supplementary Note 7). When two magnetic layers are strongly ferromagnetically coupled the acoustic mode resonance condition will be set by the weighted mean of the magnetic properties of the two individual layers, $\bar{M}_{\text{eff}} = \frac{(t_{\text{Py}}M_{\text{Py},\text{eff}} + t_{\text{LAFO}}M_{\text{LAFO},\text{eff}})}{(t_{\text{Py}}M_{\text{Py}} + t_{\text{LAFO}}M_{\text{LAFO}})}$. The slow increase of the higher frequency mode of the two devices associated unpatterned structures we conducted spin–torque FMR (ST-FMR) on both 2$\mu$m wide stripe devices and 400nm wide nanowire devices. Figures 2a and b show the ST-FMR spectra of 400nm wide nanowire devices. In contrast to the FMR spectra, ST-FMR shows two dominant resonances, whose linewidth and peak amplitude are sensitive to bias current. In contrast, for 2$\mu$m width stripe devices only one ST-FMR peak is seen (Supplementary Fig. S2a).

Figure 2c shows the ST-FMR frequency–resonance field spectra of a 400nm nanowire and a 2$\mu$m stripe LAFO20/Py5/Pt5 device. We find that the dispersion of the higher frequency mode of the two devices overlap, with a fit to the Kittel model giving $\mu_0 M_{\text{eff}}=0.86$ T, close to that found from the FMR spectra of the associated unpatterned film. We thus attribute this feature to a bulk mode (BM), a spin excitation that is uniform across the width of the device. The lower frequency mode only appears in the 400nm wide device and is associated with a much lower $\mu_0 M_{\text{eff}}=0.65$ T. We attribute this lower frequency mode to an edge mode (EM), as indicated in Figs. 2a&b, and this conclusion is supported by micromagnetic simulations as discussed below. Two modes of this type have been reported in previous studies\textsuperscript{15–17,20}.

Hybrid SHNO devices show the onset of auto-oscillations at a threshold current. Figures 2d–g shows the power spectral density (PSD) as a function of bias current at fixed field $H=0.0817$ T for $\phi=70^\circ$. In all the devices, the auto-oscillation frequency redshifts with increasing bias current, a characteristic of localized modes in nanowire SHNOS\textsuperscript{16,17}. This is not a pure heating effect (see Supplementary Fig. S4). Interestingly, the threshold current $I_{\text{th}}$ drops dramatically between the reference sample, Py5/Pt5, and the sample with LAFO, LAFO4/Py5/Pt5, and then the $I_{\text{th}}$ slowly increases with the thickness of LAFO. The slow increase of $I_{\text{th}}$ with the thickness of LAFO layer agrees well with the expectations of the macrospin model that predicts, $I_{\text{th}} \propto \left( \alpha_{\text{Py}} t_{\text{Py}} M_{\text{Py},\text{eff}} + \alpha_{\text{LAFO}} t_{\text{LAFO}} M_{\text{LAFO},\text{eff}} \right) \bar{M}_{\text{eff}}$, consistent with ST-FMR results obtained from the 2$\mu$m width stripe and 400nm width nanowire samples (Supplementary Fig. S2b,c). However, the drop of $I_{\text{th}}$ from Py5/Pt5 to LAFO4/Py5/Pt5 cannot be explained by this model. We note this decrease in $I_{\text{th}}$ is observed in ST-FMR studies conducted on both 400nm nanowire and 2$\mu$m width strip samples. So it does not depend sensitively on sample geometry. It is thus possible that the spin current generated from the Py layer itself acts on the LAFO to increase the spin torques and reduce $I_{\text{th}}$.\textsuperscript{39–42} Previous studies have experimentally shown spin-orbit torque can be generated from a single magnetic layer\textsuperscript{19,43}. In addition, the edge mode can be dominant in hybrid devices and cause a mode-related change of nonlinear damping\textsuperscript{44,45}, which would reduce radiative loss and thus $I_{\text{th}}$. In addition, in hybrid devices the dominant EM will cause mode-related nonlinear damping change, which would reduce radiative loss and thus $I_{\text{th}}$. Nevertheless, further study is required to explain this drop of $I_{\text{th}}$.  

\begin{figure}
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\caption{Example figure caption.}
\end{figure}
Notice that compared to the Py5/Pt5 sample, the slopes of the redshift increase in all LAFO samples. This is likely due to larger spin precession angles and the emergence of a nonlinear self-localized mode\textsuperscript{18}. Interestingly, the relative magnitude of EM and BM measured from ST-FMRs and PSDs both follow the same trend: the dominant mode transitions from a BM in Py5/Pt5 to an EM in LAFO20/Py5/Pt5, which simultaneously increases the performance of the oscillators. This transition will be discussed in detail in the next section. Threshold currents and auto-oscillation currents for different devices are listed in Supplementary Table S2.

Figure 2. (a) ST-FMR measurements of 400nm nanowire device for (a) Py5/Pt5 and (b) LAFO20/Py5/Pt5 at 7 GHz with the applied field at an angle $\phi=70^\circ$ to the wire. (c) Kittel model fitting curves of LAFO20/Py5/Pt5 for 2µm stripe (blue circles) device and 400nm nanowire device. Green diamond shows fit for the peaks from bulk mode and red square for the peaks from edge mode. Maps of PSDs as a function of frequency and dc bias at a fixed field $H=0.0817$ T for $\phi=70^\circ$ for nanowire devices consisting of (d) Py5/Pt5, (e) LAFO4/Py5/Pt5, (f) LAFO10/Py5/Pt5, and (g) LAFO20/Py5/Pt5. The output power increases significantly for the thickest LAFO sample studied as indicated by the colorscales above each PSD map.

To investigate the spin-wave modes of Py and Py/LAFO SHNO heterostructures, micromagnetic simulations were carried out using MuMax3 (see Methods\textsuperscript{46}). Spin currents were applied solely to the 400 nm Py nanowire’s center region to mimic the device’s current distribution. The simulation is run until a steady state response is observed. The time evolution of magnetization was then converted to the frequency domain by Fast Fourier transform (FFT). Figure 3a shows the spatial-averaged FFT amplitude in the center region of Py for different samples, confirming the experimental observation of two dominant auto-oscillation modes. From these simulations, we can find that the auto-oscillation frequencies and their trends are in excellent agreement with our experimental results: (i) the resonance frequency is almost not changed from the Py5/Pt5 device to the LAFO4/Py5/Pt5 device, then increases with the thickness of LAFO. This is mainly due to
variations in the net $\overline{M}_{\text{eff}}$, as the resonance frequency depends strongly on $\overline{M}_{\text{eff}}$; and we find that the resonance frequency changes closely follow the trends in $\overline{M}_{\text{eff}}$ determined by FMR and ST-FMR, shown in Supplementary Table S1; (ii) the high-frequency mode has a higher amplitude in Py5/Pt5, while the low-frequency mode gradually becomes dominant with increasing LAFO thickness; (iii) the peak amplitude and quality factor for LAFO20/Py5/Pt5 is significantly higher than that of Py5/Pt5. To identify the reason behind this transition, pixel-wise spatial FFTs were conducted on Py5/Pt5 and LAFO20/Py5/Pt5 simulations. Spatial FFT images of the Py layer from Py5/Pt5 at the auto-oscillation frequencies, obtained from Fig. 3a, are shown in Fig. 3b. We find that the low-frequency peak at $f=6.21$ GHz is concentrated on the edge of the Py stripe, while the peak at $f=7.17$ GHz is dominant in the middle. This observation is consistent with our assignment of the modes in the previous section and earlier studies$^{15,16}$.

More interesting magnetic behavior occurs in magnetic heterostructures. Spatial FFT images of Py and LAFO layers from the LAFO20/Py5/Pt5 simulation are shown in Fig. 3c. Compared to Py5/Pt5, the EM and BM of the Py layer excited in LAFO20/Py5/Pt5 shows a much larger oscillation amplitude. The magnetization of the LAFO layer oscillates coherently with that of the Py layer. The increased auto-oscillation amplitude is mainly due to a larger precession cone angle in the Py layer, caused by the lower effective damping constant. Meanwhile, the area of the EM expands in the bilayer system, which leads to a larger excited volume of moments, consistent with a higher quality factor. The out-of-plane expansion of both BM and EM is caused by the strong ferromagnetic coupling between the two layers, while the in-plane expansion of EM can be understood in this way: the exchange field generated by the LAFO layer tends to align the moments at the edge of Py nanowires against the demagnetization field, which decreases the effective field inhomogeneity, thus increasing the EM coherent oscillation volume. This is confirmed by plotting the transverse magnetization profile of the devices at the equilibrium (Supplementary Fig. S3a), where one can observe a smoother transition near the edge of Py layer in the LAFO20/Py5/Pt5 device. This expands the area of the localized mode, especially the EM, greatly enhancing the coherence of each mode and thus increasing the maximum power and quality factor of signals emitted from the oscillators.

Figure 3. (a) FFT amplitude spectrum as a function of frequency from micromagnetic simulations of different devices. The spectrum is acquired by doing FFT on the time revolution of spatial-averaged magnetization
\(\bar{m}_z(t)\) in the center region of nanowire excited by a spin current. (b) Top views of spatial FFT images on the Py layer of Py5/Pt5 obtained at EM and BM resonance frequencies. (c) Top view of spatial FFT images of the Py layer (top) and LAFO layer (bottom) of a LAFO20/Py5/Pt5 device. The image size for the Py5 layer is 1500x400 nm\(^2\) and for the LAFO20 layer is 1500x1500 nm\(^2\). The logarithmic colorscale is on the right where the color represents the FFT amplitude of \(m_z(x, y)\).

To better understand the properties of the auto-oscillation modes, maps of the PSD as a function of magnetic field at fixed bias current (1.15 times \(I_{th}\)) are plotted in Figs. 4a-d. Similar to the current dependent PSD map (Figs. 2d to g), BM and EM are observed. By fitting the two PSD peaks to Lorentzian functions, we can obtain the auto-oscillation frequencies and dispersion curves for the BM and EM as shown in Figs. 4e-h. This is shown in comparison to the FMR and ST-FMR results. In the Py5/Pt5 sample (Fig. 4a), only the BM is detectable, and its dispersion curve is slightly redshifted compared to that of FMR and ST-FMR data. However, contrary to the commonly seen self-localized mode\(^{47,48}\) in magnetic thin films, in a magnetic wire the propagation of spin waves is restricted in the transverse direction but allowed along the wire direction, preventing mode localization. This BM has a similar \(I_{th}\) compared to the uniform mode\(^{35}\), which is confirmed by the ST-FMR on 2\(\mu\)m stripes (Supplementary Fig. S2(b)). Instead, the EM is localized due to a self-induced potential well, leading to a localized quasi-linear auto-oscillation mode.

In LAFO/Py bilayers (Fig. 4b-d), due to the strong coupling between two magnetic layers, the center region of LAFO will precess coherently with Py. The exchange fields generated from the LAFO layer change the auto-oscillations frequencies in LAFO/Py/Pt samples. The increased difference between the dispersion curves of auto-oscillation and those obtained from FMR and ST-FMR supports the idea that the spin-wave modes are more localized in the LAFO containing devices. As shown in Figs. 4e-h, compared to the Py5/Pt5 sample, the dispersion curve of the BM of the LAFO4/Py5/Pt5 sample from PSD measurements is much lower than the FMR mode. This is one of the key characteristics of a mode that is more strongly localized.

**Figure 4.** PSD maps as a function of external magnetic field and frequency for (a) Py5/Pt5, (b) LAFO4/Py5/Pt5, (c) LAFO10/Py5/Pt5, and (d) LAFO20/Py5/Pt5 SHNOs. Resonance frequency as a function of external magnetic field for the device (e) Py5/Pt5, (f) LAFO4/Py5/Pt5, (g) LAFO10/Py5/Pt5, and (h) LAFO20/Py5/Pt5 obtained by FMR (brown dash dot), ST-FMR (green dash), and PSD (blue and red solid) measurements.
Dominant resonance modes are highlighted by a wider line. Notice the resonance frequency obtained from PSD has two distinctive peaks, which are associated with bulk mode and edge modes.

According to the discussion in the previous sections, we determined a few critical properties of the LAFO layer which can guide us to design a better hybrid SHNOs: (i) low $\alpha$ ferrimagnetic insulator to have lower $\alpha$, (ii) higher $M_{\text{eff}}$ to make the auto-oscillation EM more localized and (iii) appropriate thickness to not increase the threshold current. To meet these criteria, LFO (15nm), which possess these properties (properties listed in Supplementary Table S1), was used in place of LAFO as the ferrimagnetic insulator in our device. As shown in Figs. 5a&b, strong auto-oscillation signals up to 30 dB over the noise floor (NF) are detected only associated with EMs. Compared to the LAFO samples, the auto-oscillations occur at higher frequency due to the larger $M_{\text{eff}}$ of the LFO layer. Figure 3c is the dispersion curves obtained from FMR, ST-FMR, and PSD measurements for LFO15/Py5/Pt5 sample. Since FMR spectra are measured at $\phi = 0^\circ$ (magnetic hard axis) and the ST-FMR spectra are measured at $\phi = 70^\circ$ (closer to the magnetic easy axis direction), a significant difference between these results occurs due to the large in-plane crystalline anisotropy of the LFO layer. The strong anisotropy also causes a crossing between the maximum PSD signal and ST-FMR curves at low fields in Fig. 5c. At this low field region, the sample is not magnetically saturated as we are not measuring the PSD with the field along the easy axis of LFO. This leads to multidomain states at low field and a resonance frequency that does not dependent monotonically on applied field. However, at higher field, the PSD dispersion curve is closer to what we obtained from ST-FMR. The auto-oscillation dispersion in this field range is redshifted relative to ST-FMR dispersion, which again indicates the formation of localized auto-oscillation modes. To systematically compare the performance of different samples, PSDs at a fixed field $H=0.045$ T obtained from the field dependent PSD maps are plotted in Fig. 5d. Clearly, with the optimization of the LAFO layer, the maximum power in LAFO/Py/Pt samples can be at least a 1000 times larger than that of the Py5/Pt5. We note that the anisotropic magnetoresistance (AMR) does not vary significantly in the different samples (see Supplementary Fig. S1) and is thus not an important factor in the change in device output power. By fitting the dominant peak of the PSDs with a Lorentzian function, we obtained both the maximum power and maximum quality factor from each device as shown in Fig. 5e. From all these results, compared to the conventional Py/Pt SHNOs, we can obtain orders of magnitude higher emission power and quality factor in hybrid low damping ferrimagnetic insulator (LAFO) ferromagnetic metal (Py) heterostructures, which provides a new platform for SHNOs.
In summary, our work presents a new hybrid type of SHNOs, which shows superior performance compared to conventional Py/Pt spin oscillators. In hybrid SHNOs much higher power emission and quality factor can be obtained relative to conventional Py/Pt SHNOs. To understand the mechanism behind the improved performance, ferromagnetic resonance measurements and micromagnetic simulations were carried out on both conventional Py/Pt SHNO and hybrid SHNOs. Results show that the two layers precess coherently in bulk mode and edge modes. Meanwhile, the localization of auto-oscillations reduces the threshold current and makes the edge mode the dominant power emission source rather than the bulk mode. Further, by designing the composition and thickness of the ferrimagnetic insulator layer, we successfully fabricated hybrid SHNOs with better performance by replacing LAFO with LFO. Our work expands the possibility of SHNOs for many types of spintronic applications, such as synchronizing electrically isolated SHNOs, for neuromorphic computing and for magnonic logic circuits.
Methods

Sample deposition and fabrication. Epitaxial Li$_{0.5}$Al$_{1.5}$Fe$_{1.5}$O$_4$ and Li$_{0.5}$Al$_{0.5}$Fe$_2$O$_4$ films are grown on (001) MgAl$_2$O$_4$ (MAO) substrates at 400°C in 15mTorr O$_2$ at a laser fluence of 1.9J/cm$^2$ by pulsed laser deposition. The deposition of epitaxial LAFO with different compositions follows the previous study$^{59,50}$. After the growth of the ferrite thin films of varying thickness and composition, Py(5nm)/Pt(5nm) bilayers are deposited via a Kurt Lester magnetron sputtering system at room temperature. The reference sample Py5/Pt5 is deposited on a c-sapphire (0001) substrate. The as-deposited samples are then spin-coated with PMMA 493 4A and exposed by an Elionix 50 keV E-beam lithography system for the nanowires patterning. After developing, the samples are transferred to the Kurt Lester system for Ar plasma dry etching. After cleaning the residual resists, waveguides with 400 nm gaps between two contact pads are patterned again by E-beam lithography. Finally, Cr(5nm)/Au(50nm) contacts are deposited.

Experimental techniques. VNA-FMR is used for detecting the thin film samples ferromagnetic resonance. For pure LAFO samples, a field-modulated technique is used to achieve detection of the low linewidth resonance peaks. The samples are always mounted with the dc magnetic field along the in-plane [100] hard axis ($\varphi=0^\circ$) of the LAFO thin film. ST-FMR measurements are carried out in a probe station with the external field always applied at $\varphi=70^\circ$ with respect to the current direction. The field is modulated with a coil and signal is detected by a lock-in. The DC is applied via a Keithley 2400. PSDs are measured via Keysight N9030B spectrum analyzer with a noise floor extension option. Input signals are amplified by an internal 29 dB low-noise amplifier. During the measurement, the resolution bandwidth is always kept at 1MHz. A Keithley 2400 is used for applying a DC into the SHNOs. The noise floor in this setup is -125 dBm. To exclude the sample-to-sample variations of resistances, ST-FMRs, and PSD maps, additional samples with the same geometry and composition are measured and shown in Supplementary Note 6.

Micromagnetic simulations. Micromagnetic simulations are run by Mumax3 micromagnetic simulator$^{46}$. The mesh size is set to $300 \times 300 \times 5$, and each cell size is $5 \times 5 \times 5$ nm$^3$. This length is smaller than the exchange length of Py and LAFO. The top Py layer is designed as a stripe in the center with dimension $400 \times 1500 \times 5$ nm$^3$, while the bottom LAFO layer is extended to the boundaries and varied in thickness. Periodic boundary condition along the long axis of the Py nanowire are used to eliminate the demagnetization field from the end of Py stripe. The exchange constant between the Py and LAFO layers is taken to be half of the harmonic mean of two layers. To reduce the spin-wave reflection at the boundary, we set an exponentially increased damping region near the boundaries of the simulated region. The spin current applied is restricted to the center region of the Py nanowire with dimension $500 \times 400 \times 5$ nm$^3$, since most of the spin current is concentrated between two Au contacts. Threshold currents are found by running simulations over 200 ns and slowly increasing the applied current until $m_z$ starts to converge to a stable auto-oscillation state. In order to determine the auto-oscillation spectrum in the frequency domain, we set the current to be 1.2 times the threshold current found above and run the simulation for 500 ns. And then we use FFT algorithms to convert the magnetization evolution in the time-domain to the frequency-domain. This method can be used to generate the auto-oscillation spectrum of the full device using the spatial averaged $\overline{m}_z(t)$ or to generate the spatial profile of each auto-oscillation modes from $m_z(r, t)$. Simulation details for different samples are summarized in Supplementary Note 4 and used parameters are listed in Supplementary Table S3.

Data Availability.

The datasets generated during and/or analyzed during the current study are available in Supplementary Materials and also available from the corresponding authors on reasonable request.
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Author Contributions Statement.
H.R., Y.S. and A.D.K. conceived the experiment, X.Y.Z., S.C., and D.A.O synthesized the LAFO and LFO thin films and performed part of the FMR characterization, while H.R. deposited the Py and Pt thin films. H.R. fabricated the SHNOs, performed the transport experiments and analyzed the data, including FMR, ST-FMR and PSD data. G.W. and H.R. performed the micromagnetic simulations. The manuscript was prepared by H.R. and A.D.K. in consultation with all other authors. All authors read and commented on the manuscript.

Competing Interests Statement.
The authors declare no competing interests.
Figures’ captions.

Figure 1. (a) Schematic of the hybrid SHNO device and power spectral density (PSD) measurement setup. (b) FMR frequency versus resonance field for various unpatterned thin films and heterostructures, including for reference, Py5/Pt5 bilayers and LAFO and LFO thin films. (c) FMR linewidth as a function of frequency for the same samples.

Figure 2. (a) ST-FMR measurements of 400nm nanowire device for (a) Py5/Pt5 and (b) LAFO20/Py5/Pt5 at 7 GHz with the applied field at an angle $\phi=70^\circ$ to the wire. (c) Kittel model fitting curves of LAFO20/Py5/Pt5 for 2μm stripe (blue circles) device and 400nm nanowire device. Green diamond shows fit for the peaks from bulk mode and red square for the peaks from edge mode. Maps of PSDs as a function of frequency and dc bias at a fixed field $H=0.0817$ T for $\phi=70^\circ$ for nanowire devices consisting of (d) Py5/Pt5, (e) LAFO4/Py5/Pt5, (f) LAFO10/Py5/Pt5, and (g) LAFO20/Py5/Pt5. The output power increases significantly for the thickest LAFO sample studied as indicated by the colorscales above each PSD map.

Figure 3. (a) FFT amplitude spectrum as a function of frequency from micromagnetic simulations of different devices. The spectrum is acquired by doing FFT on the time revolution of spatial-averaged magnetization $\bar{m}_z(t)$ in the center region of nanowire excited by a spin current. (b) Top views of spatial FFT images on the Py layer of Py5/Pt5 obtained at EM and BM resonance frequencies. (c) Top view of spatial FFT images of the Py layer (top) and LAFO layer (bottom) of a LAFO20/Py5/Pt5 device. The image size for the Py5 layer is 1500×400 nm$^2$ and for the LAFO20 layer is 1500×1500 nm$^2$. The logarithmic colorscale is on the right where the color represents the FFT amplitude of $m_z(x,y)$.

Figure 4. PSD maps as a function of external magnetic field and frequency for (a) Py5/Pt5, (b) LAFO4/Py5/Pt5, (c) LAFO10/Py5/Pt5, and (d) LAFO20/Py5/Pt5 SHNOs. Resonance frequency as a function of external magnetic field for the device (e) Py5/Pt5, (f) LAFO4/Py5/Pt5, (g) LAFO10/Py5/Pt5, and (h) LAFO20/Py5/Pt5 obtained by FMR (brown dash dot), ST-FMR (green dash), and PSD (blue and red solid) measurements. Dominant resonance modes are highlighted by a wider line. Notice the resonance frequency obtained from PSD has two distinctive peaks, which are associated with bulk mode and edge modes.

Figure 5. PSD maps for LFO15/Py5/Pt5 as a function of (a) bias current and (b) magnetic field. (c) Dispersion curves of LFO15/Py5/Pt5 obtained from FMR, ST-FMR, and PSDs measurements. (d) PSD spectrums of different samples at fixed $H=0.045$ T. Lines are shifted upward 15 dB for each spectrum (e) Max signal (dB over NF) and max Q factor for different devices obtained from the PSDs in (a) and Fig. 4a-d.