Absorption in the particle oscillations

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Abstract

Particle oscillations in absorbing matter are considered. The approach based on the optical potential is shown to be inapplicable in the strong absorption region. Models with Hermitian Hamiltonian are analyzed.

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1 Introduction

In particle oscillations in the medium absorption can play an important role, for example, in the $K^0\bar{K}^0$ [1-4] and $n\bar{n}$ [5-8] oscillations. In this paper we consider $n\bar{n}$ transitions in the medium followed by annihilation

$$n \rightarrow \bar{n} \rightarrow M.$$  \hspace{1cm} (1)

Here $M$ are the annihilation mesons. The reason for considering this process is that the absorption (annihilation) of $\bar{n}$ is extremely strong.

In the standard approach (later on referred to as a potential model) the $\bar{n}$-medium interaction is described by antineutron optical potential $U_{\bar{n}}$. We have objections to this model (Sect. 2). The alternative models with a Hermitian Hamiltonian are considered in Sect. 3.

For the model with a Hermitian Hamiltonian two possibilities exist: a model with bare and a model with dressed propagators. In the model with a bare propagator (Sect. 3.1) infrared singularity takes place. For solving the problem a field theoretical approach with a finite time interval has been proposed [8-10]. For the model with a dressed propagator (Sects. 3.2 and 3.3) a $S$-matrix approach is used. For the models with bare and dressed propagators we directly calculate the off-diagonal matrix element without using the optical potential.

The results of the potential model and the model with a Hermitian Hamiltonian are compared in Sect. 4. The main statement is given in the paragraph below Eq. (23): the potential model contains double counting. This has been proved in the standard $S$-matrix approach. This fact should be emphasized particularly.

In Sect. 5 the results are summarized. The problems of the model with a Hermitian Hamiltonian are pointed out as well. The restriction on the free-space $n\bar{n}$ oscillation time $\tau$ critically depends on the description of absorption. In this regard, the main goal of this paper is to consider the absorption model itself.

2 Potential model

We consider process (1). In the standard approach [5-7] the $n\bar{n}$ transitions in the medium are described by Schrödinger equations

$$(i\partial_t - H_0)n (x) = \epsilon_{n\bar{n}} \bar{n}(x),$$  \hspace{1cm} (i)  

$$(i\partial_t - H_0 - V)\bar{n}(x) = \epsilon_{n\bar{n}} n(x),$$  \hspace{1cm} (ii)

$$H_0 = -\nabla^2 / 2m + U_n,$$

$$V = U_{\bar{n}} - U_n = \text{Re}U_n + i\text{Im}U_n.$$  \hspace{1cm} (2)
Im\(U_{\bar{n}} = -\Gamma/2, \bar{n}(0,x) = 0\). Here \(U_n\) and \(U_{\bar{n}}\) are the potential of \(n\) and the optical potential of \(\bar{n}\), respectively; \(\epsilon_{n\bar{n}}\) is a small parameter with \(\epsilon_{n\bar{n}} = 1/\tau\), where \(\tau\) is the free-space \(n\bar{n}\) oscillation time, \(\Gamma\) being the annihilation width of \(\bar{n}\).

In the lowest order in \(\epsilon_{n\bar{n}}\) the process width is [5-7]

\[
\Gamma_{pot} = \frac{\epsilon_{n\bar{n}}^2}{(\text{Re}V)^2 + (\Gamma/2)^2} \Gamma. \tag{3}
\]

\(U_{\bar{n}}\) is the basic element of the model. In this connection the following problems arise:

1. The optical model was developed for the Schrodinger type equations. The physical meaning of \(\text{Im}U_{\bar{n}}\) follows from the corresponding continuity equation. Coupled Eqs. (2) give rise to the following equation:

\[
(\partial_t^2 + i\partial_t(V + 2H_0) - H_0^2 - H_0 V + \epsilon_{n\bar{n}}^2)n(x) = 0. \tag{4}
\]

The continuity equation cannot be derived from (4).

2. To get \(\Gamma_{pot}\), the optical theorem or condition of probability conservation are used. However, the S-matrix is essentially non-unitary.

3. The structure and \(\Gamma\)-dependence of (3) provoke some objections. Due to this an alternative model should be considered.

### 3 Models with a Hermitian Hamiltonian

The interaction Hamiltonian of process (1) is

\[
\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + \mathcal{H},
\]

\[
\mathcal{H}_{n\bar{n}} = \epsilon_{n\bar{n}} \bar{\Psi}_n \Psi_{\bar{n}} + \text{H.c.}, \tag{5}
\]

where \(\mathcal{H}_{n\bar{n}}\) and \(\mathcal{H}\) are the Hamiltonians of \(n\bar{n}\) conversion [5] and the \(\bar{n}\)-medium interaction, respectively. The background neutron potential is included in the neutron wave function:

\[
n(x) = \Omega^{-1/2} \exp(-ix), \tag{6}
\]

\(p = (\epsilon, p), \epsilon = p^2/2m + U_n\).

#### 3.1 Model with a bare propagator

The \(n\bar{n}\) conversion comes from the exchange of Higgs bosons with \(m_H > 10^5\) GeV. The \(\bar{n}\) annihilates in a time \(\tau_a \sim 1/\Gamma\). We deal with a two-step process with a characteristic time \(\tau_a\).

The general definition of the antineutron annihilation amplitude \(M_a\) is

\[
<M0| T \exp(-i \int dx \mathcal{H}(x)) - 1 |0\bar{n}_p> = N(2\pi)^4 \delta^4(p_f - p_i)M_a. \tag{7}
\]
Here $|0n_p>$ is the state of the medium containing the $\bar{n}$ with the 4-momentum $p = (\epsilon, p)$; $<M|$ denotes the annihilation mesons, $N$ includes the normalization factors of the wave functions. The antineutron annihilation width $\Gamma$ is expressed through $M_a$:

$$\Gamma = N_1 \int d\Phi |M_a|^2,$$

where $N_1$ is the normalization factor.

The amplitude of process (1) $M_1$ is given by

$$<M_0|T\exp(-i\int dx(\mathcal{H}_{n\bar{n}}(x) + \mathcal{H}(x))) - 1|0n_p> = N(2\pi)^4\delta^4(p_f - p_i)M_1.$$  (9)

In the lowest order in $\mathcal{H}_{n\bar{n}}$ one obtains

$$M_1 = \epsilon_{n\bar{n}}G_0M_a,$$  (10)

$$G_0 = \frac{1}{\epsilon_\bar{n} - p_\bar{n}^2/2m - U_\bar{n} + i0},$$  (11)

where $G_0$ is the antineutron propagator. Since $p_\bar{n} = p$, $\epsilon_\bar{n} = \epsilon$, then $G_0 \sim 1/0$. $M_a$ contains all the $\bar{n}$-medium interactions followed by annihilation including antineutron rescattering in the initial state. So in this case the antineutron propagator is bare.

We deal with infrared singularity. For solving the problem a field theoretical approach with a finite time interval has been proposed [9]. The process (1) probability was found to be [10]

$$W(t) \approx W_f(t) = \epsilon_{nn}^2 t^2,$$  (12)

where $W_f$ is the free-space $n\bar{n}$ transition probability. Equation (12) leads to a very strong restriction on the free-space $n\bar{n}$ oscillation time: $\tau = 10^{16}$ yr.

### 3.2 Auxiliary process

Starting from (5) and (6) we have drawn the singular amplitude $M_1$. To gain a better understanding of the problem, we consider the $n\bar{n}$ transitions in the medium followed by $\beta^+$-decay:

$$n \rightarrow \bar{n} \rightarrow \bar{\bar{p}}e^+\nu.$$  (13)

The neutron wave function is given by (6). The interaction Hamiltonian is

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + \mathcal{H}_W + V\bar{\Psi}_\bar{n}\Psi_\bar{n},$$  (14)

where $V$ is defined by (2), $\mathcal{H}_W$ is the Hamiltonian of the decay $\bar{n} \rightarrow \bar{\bar{p}}e^+\nu$. In the lowest order in $\mathcal{H}_{n\bar{n}}$ the amplitude $M_2$ is

$$G = \frac{1}{\epsilon_\bar{n} - p_\bar{n}^2/2m - U_\bar{n} + i0} = \frac{1}{\epsilon - p^2/2m - (U_n + V) + i0} = \frac{1}{V^*},$$

$$M_2 = \epsilon_{n\bar{n}}GM_d.$$  (15)
where $M_d$ is the amplitude of the $\beta^+\text{-decay}$, $G$ is the antineutron propagator.

The process width $\Gamma_2$ is

$$\Gamma_2 = \frac{\epsilon_{n\bar{n}}^2}{|V|^2} \Gamma_d.$$  \hfill (16)

The propagator is dressed due to the additional field $V$.

### 3.3 Model with a dressed propagator

We return to process (1). Let us try to compose a model with a dressed propagator. By analogy with (14) in the Hamiltonian $\mathcal{H}$ (see (5)) we separate out the scalar field $V_1$:

$$\mathcal{H} = V_1 \bar{\Psi} \bar{n} \Psi n + \mathcal{H}_a,$$  \hfill (17)

where $\mathcal{H}_a$ is the annihilation Hamiltonian. Now the antineutron annihilation amplitude $M_{an}$ is defined through $\mathcal{H}_a$:

$$<0\bar{n} | T \exp(-i \int dx \mathcal{H}_a(x)) - 1 | 0\bar{n}_p >= N (2\pi)^4 \delta^4(p_f - p_i) M_{an}.$$

The interaction Hamiltonian is given by

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + V_1 \bar{\Psi} \bar{n} \Psi n + \mathcal{H}_a.$$  \hfill (19)

In the lowest order in $\mathcal{H}_{n\bar{n}}$ the amplitude of process (1) is

$$G_d = G_0 + G_0 V_1 G_0 + ... = \frac{1}{(1/G_0) - V_1 + i0} = -\frac{1}{V_1}.$$  \hfill (20)

The antineutron propagator $G_d$ is dressed. $V_1$ plays the role of antineutron self-energy $\Sigma$. $M_3$ corresponds to the first order in $\mathcal{H}_{n\bar{n}}$ and all the orders in $V_1$ and $\mathcal{H}_a$. Compared to (7), $M_{an}$ is calculated through the reduced Hamiltonian $\mathcal{H}_a$ instead of $\mathcal{H}$.

The process width $\Gamma_3$ is

$$\Gamma_3 = N_1 \int d\Phi |M_3|^2 = \frac{\epsilon_{n\bar{n}}^2}{|V_1|^2} \Gamma_{an},$$

$$\Gamma_{an} = N_1 \int d\Phi |M_{an}|^2.$$  \hfill (21)

The amplitude $M_3$ is non-singular because the propagator is dressed. The antineutron self-energy $\Sigma = V_1$ appears due to separation of the field $V_1$. This procedure seems to be artificial and unjustified. There are no similar problems for process (13) since the self-energy and decay of $\bar{n}$ are generated by different fields $\mathcal{H}_W$ and $V_1$. This point should be given particular emphasis.

In any case $\Gamma_{an} \sim \Gamma$, and so

$$\Gamma_3 \sim \Gamma_{an} \sim \Gamma.$$  \hfill (22)
4 Discussion

First of all we compare the potential model with the model with a dressed propagator. In (21) we have to take the same parameters as in the potential model: \( V_1 = V \) and \( \Gamma_{an} = \Gamma \). Then we get

\[
\Gamma_3 = \frac{e^2_{\bar{n}n}}{(\text{Re}V)^2} \frac{1}{(\Gamma/2)^2} \Gamma.
\] (23)

Equation (23) coincides with (3). By means of the model with a dressed propagator we have obtained \( \Gamma_{pot} \). The antineutron annihilation width \( \Gamma \) is involved in the propagator (see (20), where \( V_1 = V \)) as well as vertex function which means double counting.

The same conclusion has been drawn in [8]. It was shown that double counting leads to full cancellation of the leading terms. However, in [8] the model with a bare propagator has been considered. The approach with a finite time interval was used, but it can provoke additional questions. The above-given consideration is transparent.

If we want to remove double counting, we have to make direct calculations of the off-diagonal matrix element (see Sect. 3).

Let us compare the \( \Gamma \)-dependence of the results. In (21) one should use realistic parameters. We take \( V_1 = \text{Re}V \), then we obtain

\[
\Gamma_3 = \frac{e^2_{\bar{n}n}}{(\text{Re}V)^2} \Gamma_{an} \approx \frac{e^2_{\bar{n}n}}{(\text{Re}V)^2} \Gamma.
\] (24)

Therefore, \( \Gamma_3 \sim \Gamma \). For the \( K^0\bar{K}^0 \) transitions in the medium followed by decay and regeneration of the \( K^0_S \)-component an identical \( \Gamma \)-dependence takes plays [11,12].

In the potential model \( \Gamma_{pot} \sim \Gamma \) only at light absorption. Indeed, if \( \Gamma/2 \ll |\text{Re}V| \), then

\[
\Gamma_{pot} = \frac{e^2_{\bar{n}n}}{(\text{Re}V)^2} \Gamma \left[ 1 - \left( \frac{\Gamma}{2\text{Re}V} \right)^2 \right].
\] (25)

In the first approximation (25) coincides with (24). This agreement was expected since the dominant role was played by \( \text{Re}U_{\bar{n}} \).

If \( \Gamma/2 \gg |\text{Re}V| \),

\[
\Gamma_{pot} = \frac{4e^2_{\bar{n}n}}{\Gamma}.
\] (26)

\( \Gamma_{pot} \sim 1/\Gamma \), whereas \( \Gamma_3 \sim \Gamma \).

We consider the difference in the results in the region of strong absorption. It is seen from the ratio

\[
r = \frac{\Gamma_3}{\Gamma_{pot}} = \left( \frac{\Gamma}{2\text{Re}V} \right)^2.
\] (27)

For nuclear matter we take \( \Gamma = 100 \) MeV. If \( |\text{Re}V| = 50 \) MeV, then \( r = 1 \). If \( |\text{Re}V| = 10 \) MeV, then \( r = 25 \). When \( |\text{Re}V| \) decreases, \( \Gamma_3 \) and \( r \) increase.
In the oscillation of other particles (for example, $K^0\bar{K}^0$) the difference between $\Gamma_3$ and $\Gamma_{pot}$ is less, however this difference can be essential for the problem under study.

For the realistic parameters $\Gamma = 100$ MeV and $|\text{Re} V| = 10$ MeV, the lower limit on the free-space $n\bar{n}$ oscillations time is $\tau = 1.2 \cdot 10^9$ s. When $V_1 = 0$, the model with a dressed propagator converts to the model with a bare propagator. It gives $\tau = 10^{16}$ yr. On the basis of this one can accept that the lower limit on the free-space $n\bar{n}$ oscillations time is in the range $10^{16}$ yr $> \tau > 1.2 \cdot 10^9$ s.

Thus we conclude the following: (1) The smaller $|\text{Re} V|$ (antineutron self-energy), the greater the difference in the results (see (27)). It is a maximum for the model with a bare propagator. (2) In the strong absorption region $\Gamma_{pot} \sim 1/\Gamma$, whereas $\Gamma_3 \sim \Gamma$. (3) The potential model contains double counting.

Finally, in the strong absorption region the model with an optical potential is inapplicable. In the models with a Hermitian Hamiltonian the optical potential is not used. The conclusions made above do not depend on the specific models of the blocks $M_a$ and $M_{an}$.

5 Conclusion

The potential model is applicable only in the case of slight absorption.

If absorption is strong, the potential model is inapplicable: (1) It contains double counting. This is a main statement of this paper. (2) The $\Gamma$-dependence of the result is inverse: $\Gamma_{pot} \sim 1/\Gamma$, whereas $\Gamma_3 \sim \Gamma$. (3) The physical meaning of $\text{Im} U_n$ is uncertain. (4) The using of the optical theorem or condition of probability conservation contradicts the fact that the $S$-matrix is essentially non-unitary.

The model with a Hermitian Hamiltonian should be used. Two variants of the model have been considered: the model with bare and dressed propagators. If the scalar field $V_1 \to 0$ (the antineutron self-energy $\Sigma \to 0$), the model with a dressed propagator converts to the model with a bare propagator. In both variants the optical potential is not used. The amplitudes of annihilation $M_a$ and $M_{an}$ are defined through Hermitian Hamiltonians.

The chief drawback in the model with a dressed propagator is that the procedure of separation of $V_1$ is artificial and unjustified. In our opinion the model with a bare propagator is preferable. There are a lot of arguments in favor of the model with a bare propagator [10]. The only objection to this model is that it gives the result which essentially differs from the result of the potential model. The potential model has been considered above.

Since the problem is of a great nicety, further investigations in the framework of the approach with a Hermitian Hamiltonian are needed.
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