New Light on an Old Idea:
The Starobinsky Model from Superconformal D-term Inflation

Valerie Domcke
DESY, Hamburg

in collaboration with
W. Buchmüller, K. Kamada, K. Schmitz

arxiv 1210.4105, 1306.3471
Inflation in a Nutshell

- slowly rolling homogeneous scalar field
  → exponential expansion of the universe.

- 'stretched' quantum fluctuations
  → CMB inhomogeneities: $A_s$, $n_s$, $r$

- hybrid inflation:
  → end of inflation =
    GUT-scale phase transition
  → in SUSY: F- and D-term

BSM physics in the sky?
Inflation after Planck

Planck '13

Valerie Domcke — DESY — 26.09.2013 — Page 3
⇒ new interest in $R^2$-inflation (Starobinsky model)
The Starobinsky model of $R^2$ Inflation

An old idea [Starobinsky '80]

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_R = \frac{1}{2} \left( R + \frac{1}{6M^2} R^2 \right)
\]

rewritten as a scalar-tensor theory [Whitt '84]

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_R \simeq \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{3}{4} M^2 \left( 1 - \exp \left(-\sqrt{\frac{2}{3}} \sigma \right) \right)^2.
\]

..in new light: 1305.1247, 1306.3417, 1306.5220, 1307.1137, 1307.3537, 1307.4343, 1309.2015,...
The Starobinsky model of $R^2$ Inflation

An old idea [Starobinsky '80]

$$\frac{1}{\sqrt{-g}} \mathcal{L}_R = \frac{1}{2} \left( R + \frac{1}{6M^2} R^2 \right)$$

rewritten as a scalar-tensor theory [Whitt '84]

$$\frac{1}{\sqrt{-g}} \mathcal{L}_R \simeq \frac{1}{2} R - \frac{1}{2} g^\mu{}^\nu \partial_\mu \sigma \partial_\nu \sigma - \frac{3}{4} M^2 \left( 1 - \exp \left( -\sqrt{\frac{2}{3 \sigma}} \right) \right)^2.$$
The Starobinsky model of $R^2$ Inflation

An old idea [Starobinsky '80]

$$\frac{1}{\sqrt{-g}} \mathcal{L}_R = \frac{1}{2} \left( R + \frac{1}{6M^2} R^2 \right)$$

rewritten as a scalar-tensor theory [Whitt '84]

$$\frac{1}{\sqrt{-g}} \mathcal{L}_R \simeq \frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{3}{4} M^2 \left( 1 - \exp \left( -\sqrt{\frac{2}{3}} \sigma \right) \right)^2.$$
Superconformal D-term inflation
GUT scale inflation $\rightarrow$ symmetry principle $\rightarrow$ Superconformal D-term inflation $\rightarrow$ Supergravity.

Valerie Domcke — DESY — 26.09.2013 — Page 5
superconformal Models of Inflation: a Toy Model

- supergravity + local conformal symm. $\rightarrow$ superconformal symmetry in particular local conformal transformations:
  \[ g_{\mu\nu} \rightarrow e^{-2\alpha(x)} g_{\mu\nu}, \quad z \rightarrow e^{\alpha(x)} z, \quad \bar{z} \rightarrow e^{\alpha(x)} \bar{z} \]

- conformally invariant Lagrangian:
  \[ \frac{\mathcal{L}_{\text{toy}}}{\sqrt{-g}} = + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{12} \phi^2 R(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - \frac{1}{12} \sigma^2 R(g) - \frac{\lambda}{4} \sigma^4 \]
Superconformal Models of Inflation: a Toy Model

- Supergravity + local conformal symm. $\rightarrow$ superconformal symmetry in particular local conformal transformations:

$$
g_{\mu\nu} \mapsto e^{-2\alpha(x)} g_{\mu\nu}, \quad z \mapsto e^{\alpha(x)} z, \quad \bar{z} \mapsto e^{\alpha(x)} \bar{z}
$$

- Conformally invariant Lagrangian:

$$
\mathcal{L}_{\text{toy}} \sqrt{-g} = \frac{1}{2} g^{\mu\nu} (\partial_{\mu} \phi)(\partial_{\nu} \phi) + \frac{1}{12} \phi^2 R(g) - \frac{1}{2} g^{\mu\nu} (\partial_{\mu} \sigma)(\partial_{\nu} \sigma) - \frac{1}{12} \sigma^2 R(g) - \frac{\lambda}{4} \sigma^4
$$
Superconformal Models of Inflation: a Toy Model

- supergravity + local conformal symm. $\rightarrow$ superconformal symmetry
- in particular local conformal transformations:
  \[ g_{\mu\nu} \mapsto e^{-2\alpha(x)} g_{\mu\nu}, \quad z \mapsto e^{\alpha(x)} z, \quad \bar{z} \mapsto e^{\alpha(x)} \bar{z} \]

- conformally invariant Lagrangian:
  \[ \frac{\mathcal{L}_{\text{toy}}}{\sqrt{-g}} = + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{12} \phi^2 R(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - \frac{1}{12} \sigma^2 R(g) - \frac{\lambda}{4} \sigma^4 \]
superconformal Models of Inflation: a Toy Model

- supergravity + local conformal symm. \( \rightarrow \) superconformal symmetry
  in particular local conformal transformations:

\[
g_{\mu\nu} \mapsto e^{-2\alpha(x)} g_{\mu\nu}, \quad z \mapsto e^{\alpha(x)} z, \quad \bar{z} \mapsto e^{\alpha(x)} \bar{z}
\]

- conformally invariant Lagrangian:

\[
\mathcal{L}_{\text{toy}} = \sqrt{-g} \left( + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{12} \phi^2 R(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - \frac{1}{12} \sigma^2 R(g) - \frac{\lambda}{4} \sigma^4 \right)
\]
superconformal Models of Inflation: a Toy Model

supergravity + local conformal symm. \(\rightarrow\) superconformal symmetry
in particular local conformal transformations:

\[ g_{\mu\nu} \mapsto e^{-2\alpha(x)} g_{\mu\nu}, \quad z \mapsto e^{\alpha(x)} z, \quad \bar{z} \mapsto e^{\alpha(x)} \bar{z} \]

conformally invariant Lagrangian:

\[ \mathcal{L}_{\text{toy}} = \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{12} \phi^2 R(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - \frac{1}{12} \sigma^2 R(g) - \frac{\lambda}{4} \sigma^4 \]

gauge-fixing \(\phi = \sqrt{6}M_P\):

\[ \mathcal{L}_{\phi=\sqrt{6}M_P} = \frac{M_P^2}{2} R(g) - \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) + \frac{1}{12} \sigma^2 R(g) + \frac{\lambda}{4} \sigma^4 \right] \]

\[ \begin{aligned} \text{Einstein-Hilbert term} \quad & \text{conformally invariant, non-minimally coupled to } R \\ \Rightarrow \text{Simple structure in Jordan frame, complicated in Einstein frame} \end{aligned} \]
Frame function & Kähler potential ($|\bar{z}|^2 \equiv \delta_{\alpha\bar{\beta}} \bar{z}^\alpha \bar{z}^{\bar{\beta}}$):

\[
\Xi = -|\bar{z}^0|^2 + |\bar{z}|^2 \quad z^0 = \sqrt{3} M_P \quad \Phi = -3 M_P^2 + |\bar{z}|^2
\]

\[
K = -3 M_P^2 \ln \left( -\frac{1}{3 M_P^2} \Phi(z, \bar{z}) \right)
\]

\Rightarrow \text{Canonical superconformal supergravity (CSS) models} \quad \text{[Ferrara et al '11]}

\[
\frac{1}{\sqrt{-g_J}} \mathcal{L}_J = \frac{1}{2} M_P^2 (R(g_J) + 6 A_\mu^2) - \frac{1}{6} R(g_J)|\bar{z}|^2 - \delta_{\alpha\bar{\beta}} g_{J}^{\mu\nu} (\tilde{\nabla}_\mu \bar{z}^\alpha) (\tilde{\nabla}_\nu \bar{z}^{\bar{\beta}}) - V_J
\]

- canonical kinetic terms in Jordan frame
- conformal coupling to $R$
- $V_J \sim$ global SUSY scalar potential
\[ W = \lambda S \phi_+ \phi_-, \quad \Phi = -3M_P^2 + |S|^2 + |\phi_+|^2 + |\phi_-|^2 + \frac{\chi}{2} (S^2 + \bar{S}^2) \]

Approximate superconformal symmetry, broken by
- Planck mass \( M_P \)
- Fayet-Illiopoulos term \( \sqrt{\xi} \sim M_{\text{GUT}} \)
- holomorphic \( \chi \)-term in frame function

Features
- non-minimal coupling to gravity or non-canonical kinetic terms
- supergravity corrections under control
- two-field inflation model \( S = (\sigma + i\tau)/\sqrt{2} \)
- low spectral index, large cosmic string contribution
The Starobinsky model from superconformal D-term inflation
The Large Field Limit

- canonical normalization of the inflaton $\sigma$ in the Einstein frame

$$\frac{d\sigma}{d\hat{\sigma}} = \frac{1}{\sqrt{K_{SSS}}} \approx \sqrt{\frac{2}{3}} \left(1 - \frac{\chi}{6} \sigma^2\right) - \frac{\chi}{3} \sigma \rightarrow \sigma^2 = \frac{6}{\chi} \left(1 - C \exp \left(\sqrt{\frac{2}{3}} \hat{\sigma}\right)\right)$$

- scalar potential

$$V(\hat{\sigma}) \approx \frac{g^2}{2} \xi^2 \left(1 + \frac{g^2 q^2 \xi^2}{8 \pi^2} (1 + \ln x)\right), \quad x = \frac{\Phi(\sigma_c) \sigma^2}{\Phi(\sigma) \sigma_c^2}$$

$$\approx V_0 \left(1 - 2 \exp \left(-\sqrt{\frac{2}{3}} \hat{\sigma}\right)\right)$$

- predictions

$$n_s \approx 1 - \frac{2}{N_*}, \quad r \approx \frac{12}{N_*^2}, \quad \frac{dn_s}{d \ln k} \approx -\frac{2}{N_*^2}$$
The Large Field Limit

- canonical normalization of the inflaton $\sigma$ in the Einstein frame
  \[ \frac{d\sigma}{d\hat{\sigma}} = \frac{1}{\sqrt{K_{SS}}} \simeq \sqrt{\frac{2}{3}} \left( 1 - \frac{x}{6} \sigma^2 \right) \rightarrow \sigma^2 = \frac{6}{\chi} \left( 1 - C \exp \left( \sqrt{\frac{2}{3}} \hat{\sigma} \right) \right) \]

- scalar potential matches Starobinsky model
  \[ V(\hat{\sigma}) \simeq \frac{g^2}{2} \xi^2 \left( 1 + \frac{g^2 q^2 \xi^2}{8\pi^2} (1 + \ln x) \right), \quad x = \frac{\Phi(\sigma_c) \sigma^2}{\Phi(\sigma) \sigma_c^2} \]
  \[ \simeq V_0 \left( 1 - 2 \exp \left( -\sqrt{\frac{2}{3}} \hat{\sigma} \right) \right) \]

- predictions match Starobinsky model
  \[ n_s \simeq 1 - \frac{2}{N_*}, \quad r \simeq \frac{12}{N_*^2}, \quad \frac{d n_s}{d \ln k} \simeq -\frac{2}{N_*^2} \]
The Large Field Limit

- canonical normalization of the inflaton $\sigma$ in the Einstein frame

$$\frac{d\sigma}{d\hat{\sigma}} = \frac{1}{\sqrt{K_{SS}} \sqrt{\hat{K} \bar{K}}} \approx \sqrt{\frac{2}{3}} \left(1 - \frac{\chi}{6} \sigma^2\right) \quad \rightarrow \quad \sigma^2 = \frac{6}{\chi} \left(1 - C \exp\left(\sqrt{\frac{2}{3}} \hat{\sigma}\right)\right)$$

- scalar potential matches Starobinsky model

$$V(\hat{\sigma}) \approx \frac{g^2}{2} \xi^2 \left(1 + \frac{g^2 q^2 \xi^2}{8\pi^2} \left(1 + \ln x\right)\right) \quad , \quad x = \frac{\Phi(\sigma_c) \sigma_c^2}{\Phi(\sigma) \sigma^2}$$

$$\simeq V_0 \left(1 - 2 \exp\left(-\sqrt{\frac{2}{3}} \hat{\sigma}\right)\right)$$

- predictions match Starobinsky model

$$n_s \simeq 1 - \frac{2}{N_*} \quad , \quad r \simeq \frac{12}{N_*^2} \quad , \quad \frac{dn_s}{d\ln k} \simeq -\frac{2}{N_*^2}$$
Starobinsky model
Superconformal D-term inflation
non-canonical kinetic terms in the Einstein frame, determined by superconformal structure of $K$

canonical normalization introduces exponential flattening of potential

reproduces Starobinsky model, including the numerical factor in the exponential
What’s Going on?

- Higgs inflation with non-minimal coupling to gravity
  → same scale-invariant Lagrangian in the large field limit
  → no coupling $\mathcal{O}(10^5)$ necessary here

- $SU(2, 1)/SU(2) \times U(1)$ no-scale supergravity
  → structure of $K$ → structure of kinetic terms
  → different superpotentials possible

- Conformal structure with $\chi = 0$, $V_J \propto (\phi^2 - h^2)^2$
  → ‘stretching’ of different potentials for large fields
  → ‘universality class in conformal inflation’

- Look-here effect?
Planck data favours the Starobinsky model from 1980

The Starobinsky model has recently been rediscovered in several ‘modern’ inflation models

A well motivated example is superconformal D-term inflation

Symmetry structure of non-minimal coupling to $R$ (Jordan frame) or of non-canonical kinetic terms (Einstein frame) seem to play a crucial role
backup slides
The Starobinsky model in the Jordan frame

\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} (R + \alpha R^2) \]

equivalent to (check by doing Euler-Lagrange for \( \phi \)):

\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ (1 + 2\alpha \phi) R - \alpha \phi^2 \right] \]

Weyl rescaling \( \tilde{g}_{\mu\nu} = (1 + 2\alpha \phi) g_{\mu\nu} \) to Einstein frame:

\[ S = \frac{1}{2} \int d^4 x \sqrt{-\tilde{g}} \left[ \tilde{R} + \frac{6\alpha^2}{(1 + 2\alpha \phi)^2} \partial^\mu \phi \partial_\mu \phi - \frac{\alpha \phi^2}{(1 + 2\alpha \phi)^2} \right] \]

Canonical normalization \( \hat{\phi} = \sqrt{3/2} \ln[1 + \phi/(3M^2)] \) with \( \alpha = 1/6M^2 \)

\[ S = \frac{1}{2} \int d^4 x \sqrt{-\tilde{g}} \left[ \tilde{R} + (\partial_\mu \hat{\phi})^2 - \frac{3}{2} M^2 (1 - \exp(-\sqrt{2/3} \hat{\phi}))^2 \right] \]
Einstein vs Jordan frame

Einstein frame:

\[
\mathcal{L}_E^{\text{grav}} = \sqrt{-g_E} \frac{1}{2} R(g_E) M_P^2 ,
\]

\[
\mathcal{L}_E^{\text{scal}} = \sqrt{-g_E} \left[ g_E^{\mu \nu} K_{\alpha \beta}(\partial_\mu z^\alpha)(\partial_\nu \tilde{z}^\beta) - (V_E^F + V_E^D) \right] .
\]

Jordan frame:

\[
\mathcal{L}_J^{\text{grav}} = -\sqrt{-g_J} \frac{1}{6} \Phi(z, \bar{z}) R(g_J),
\]

\[
\mathcal{L}_J^{\text{scal}} = \sqrt{-g_J} \left[ \Phi A_\mu^2(z, \bar{z}) + \left( \frac{\Phi K_{\alpha \beta}}{3M_P^2} - \frac{(\partial_\alpha \Phi)(\partial_{\bar{\beta}} \Phi)}{\Phi} \right) g_J^{\mu \nu}(\partial_\mu z^\alpha)(\partial_\nu \tilde{z}^\beta) - V_J \right] .
\]

with

\[
V_J = \frac{\Phi^2}{9M_P^4} (V_E^F + V_E^D), \quad A_\mu = -\frac{i}{2\Phi} \left[ (\partial_\mu z^\alpha)\partial_\alpha \Phi - (\partial_\mu \bar{z})\partial_{\bar{\alpha}} \Phi \right]
\]
Here, consider scalar sector in SUGRA framework:

- **local Weyl transformations:**
  
  \[
  g_{\mu\nu} \rightarrow e^{-2\alpha(x)} g_{\mu\nu}, \quad z \rightarrow e^{\alpha(x)} z, \quad \bar{z} \rightarrow e^{\alpha(x)} \bar{z}
  \]
  
  \[
  R \rightarrow e^{2\alpha(x)} \left( R - 6 e^{\alpha(x)} \frac{1}{\sqrt{-g}} \partial_{\mu} \left[ g^{\mu\nu} \sqrt{-g} \partial_{\nu} e^{-\alpha(x)} \right] \right)
  \]
Scalar F-term potential:

\[ V_F^J = \left( \delta^{\alpha\bar{\beta}} W_\alpha \bar{W}_\beta + \frac{1}{\Delta_K} |\delta^{\alpha\bar{\beta}} W_\alpha \partial_{\bar{\beta}} \Phi - 3W|^2 \right) \]

F-term hybrid inflation: \( W = \lambda \phi (S_+ S_- - v^2) \)
Along \( S_\pm = 0 \):

\[ V_E^F = \Omega^4 \lambda^2 v^4 - \frac{\Omega^4 \lambda^2 v^4 |2\phi - \chi \bar{\phi}|^2}{3M_P^2 + \frac{\chi}{2} (\phi^2 + \bar{\phi}^2) + \chi^2 |\phi|^2} \]

→ large tachyonic mass for \( \phi \) due to linear term in \( W \) and \( W \neq 0 \) during inflation
A two-field inflation model

- $\chi = 0$: restoring the symmetry of the frame function
  → dynamics independent of phase of inflaton field
  → single-field inflation model
- $\chi \neq 0$: two-field inflation model, $S = \frac{1}{\sqrt{2}}(\sigma + i\tau)$