Nonabelian gauge field dynamics on matrix D-branes

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Abstract

We construct a calculational scheme for handling the matrix ordering problems connected with the appearance of D-brane positions taking values in the same Lie algebra as the nonabelian gauge field living on the D-brane. The formalism is based on the use of an one-dimensional auxiliary field living on the boundary of the string world sheet and taking care of the order of the matrix valued fields. The resulting system of equations of motion for both the gauge field and the D-brane position is derived in lowest order of the $\alpha'$-expansion.
1 Introduction

Dirichlet branes, i.e. hypersurfaces to which the endpoints of strings are confined, play a fundamental role in the recent developments of the string theory duality pattern [1, 2, 3] and the connected genesis of a unique underlying theory. Such D-branes arise from open strings with free ends by T-dualizing type I string theory or have to be added to type II theories in order to fulfill all the duality requirements. In $\sigma$-model language the boundary of the string world sheet couples to a gauge field $A_M(Y)$ living on the D-brane. Equations of motion for this field, the brane position $f^\mu(Y)$ as well as the remaining target space fields can be obtained from the conformal invariance condition of the 2D field theory. For abelian $A$ and $f$ this program has been performed in ref. [4]. The equations of motion for these two fields turn out to be equivalent to the stationarity condition of the Born-Infeld action constructed out of the field strength related to $A$ and the metric and antisymmetric tensor on the brane induced from the target space.  

For the generalization to the case of a nonabelian gauge symmetry it is natural to assign values out of the same Lie algebra to both the gauge field and the brane position. This is due to the following property of the T-duality image of open strings with free ends coupled to an abelian gauge field. The gauge field components in the isometry directions become just the fields describing the position of the D-brane in the dual theory. An extensive discussion of this point is contained in [5, 6, 7, 8], examples for applications are found in [9]. The formal extension of this rule to nonabelian gauge fields leads to the notion of a matrix valued brane and string endpoint position [10, 3], respectively. If one tries to apply this concept e.g. to the Yang-Mills equation on the D-brane:

$$h^{MN} \hat{D}_M F_{NL} = 0 ,$$  \hspace{1cm} (1)

one immediately runs into a serious ordering problem. $h^{MN}$ as the inverse induced metric on the D-brane depends on the matrix valued brane position and becomes a matrix with respect to the gauge indices. The ordering problem concerns the construction of this matrix per se, as well as its ordering with respect to $\hat{D} F$.

The main objective of this paper is to decide this ordering problem on the basis of a well defined calculational scheme and to demonstrate the possibility of explicit calculations within this scheme.

As a guiding principle for our formulation of the $\sigma$-model describing strings coupled to matrix D-branes we require T-duality equivalence to a theory with open strings having free endpoints for the special situation of target space fields independent of the coordinates orthogonal to the brane. The string couples in the bulk of its world surface to the target space metric $G$, the dilaton $\Phi$ and an antisymmetric tensor field $B$. After the formulation of the model we drop $\Phi$ in the following lowest order calculations, since at this level its effect is governed by classical considerations. Our model is designed to describe the NS-sector of the T-dual of type I strings ($B = 0$) or the string D-brane interaction in type II.  

2 On some subtleties concerning this statement we will comment in a forthcoming paper [24].

3 We look at lowest order in $\alpha'$. In spite of some guesses [11, 12] there exists no proven nonabelian generalization of the Born-Infeld action [13].
II theories \((B \neq 0)\). We will not discuss the extension to include RR bosonic fields along the line of [13].

To introduce notations and the use of the \(\zeta\)-auxiliary field formalism we summarize in section 2 the result of the path integral treatment of T-duality given in [3]. The next section is devoted to a definition of the matrix D-brane model in a covariant manner and for generic target space and brane fields. The following calculation of RG \(\beta\)-functions is done in analogy to [3]. The main new aspects concern the consequent bookkeeping of all the effects due to the presence of the auxiliary field and the handling of explicit boundary parameter dependent Dirichlet conditions in the intermediate steps of the calculation. In this sense section 4 delivers the necessary formulae for the expansion of the action around a classical configuration obeying both the equations of motion in the bulk as well as the boundary conditions. In section 5 we discuss the propagator of the quantum field describing the fluctuations of the string world sheet and the effects of the auxiliary field perturbation theory on the counter term evaluation. The conclusions will add some remarks concerning interpretation and work to be done.

## 2 Functional integral derivation of the dual \(\sigma\)-model

Our original \(\sigma\)-model describes an open string coupling in the bulk to the target space metric \(G_{\mu\nu}\), an antisymmetric tensor \(B_{\mu\nu}\) and the dilaton \(\Phi\) (collective notation by \(\Psi = (G, B, \Phi)\)). In addition it couples via its ends to a nonabelian gauge field \(A_\mu\) taking its values in the Lie algebra of a nonabelian gauge group \(\mathcal{G}\). We assume the existence of one Killing vector \(k^\mu(X)\) and the invariance of our model under the corresponding diffeomorphism. Choosing adapted coordinates all target space fields are independent of \(X^0\) (For \(A\) a gauge transformation may be necessary to reach this conclusion [6].)

\[
X^\mu = (X^0, X^M), \quad k^\mu = (1, 0), \quad \partial_0 \Psi = 0, \quad \partial_0 A_\mu = 0 .
\]  

The partition function is given by

\[
Z[\Psi, A] = \int DX^\mu e^{iS[\Psi, C = 0; X]} \text{tr} P e^{i \int_{\partial M} A_\mu dX^\mu}.
\]  

To streamline notation we have expressed \(Z\) in terms of an action \(S\) which below is allowed to depend on an abelian vector field \(C_\mu(X^M, s)\) with possibly explicit dependence on the parameter on \(\partial M\)

\[
S_M[\Psi; X] = -\frac{1}{4\pi\alpha'} \int_M d^2 z \sqrt{-g} \left( \partial_m X^\mu \partial_n X^\nu E_{\mu\nu}^{mn}(X(z)) + \alpha' R_2(X(z)) \right),
\]

\[
S[\Psi, C; X] = S_M[\Psi; X] + \int_{\partial M} \left( C_\mu(X^M(z(s)), s) \dot{X}^\mu - \frac{1}{2\pi} k(s) \Phi \right) ds,
\]

\[
E_{\mu\nu}^{mn}(X) = g^{mn} G_{\mu\nu}(X) + \frac{\epsilon^{mn}}{\sqrt{-g}} B_{\mu\nu}(X).
\]  

\(^4\)There is a notational inconsistency in section 3 of the hard copy of this paper, which has been corrected in the electronic version.
$R^{(2)}$ is the curvature scalar on the 2D manifold $M$, $k(s)$ the geodesic curvature on its boundary $\partial M$ parametrized by $z(s)$.

To disentangle the path ordering implied by the Wilson loop we introduce an one-dimensional auxiliary field $\zeta_a(s)$ living on the boundary $\partial M$ \cite{15, 16}. It has the propagator

$$\langle \bar{\zeta}_a(s_1)\zeta_b(s_2) \rangle_0 = \delta_{ab}\Theta(s_2 - s_1)$$

(5)

and couples to $X^\mu$ via the interaction term

$$i\bar{\zeta}_a A^{ab}_\mu (X(z(s)))\zeta_b(s)\partial_m X^\mu z^m(s).$$

(6)

Then we can write (choosing $0 \leq s \leq 1$)

$$Z = \int DX^\mu D\bar{\zeta} D\zeta \bar{\zeta}_a(0)\zeta_a(1)e^{iS_0[\bar{\zeta},\zeta]} \exp(iS[\Psi, \bar{\zeta} A\zeta; X]).$$

(7)

Under the $\zeta$-path integral we can repeat the dualization procedure for abelian $A$ \cite{16}. Due to the presence of $\zeta(s)$ in $C_0(X^M(z(s)), s) = \bar{\zeta}(s)A_0(X^M(z(s)))\zeta(s)$ the resulting Dirichlet condition depends on $s$ explicitly. With the help of the functional

$$\mathcal{F}[\Psi, C|f] = \int_{X^0(z(s))=f(X^M(z(s)), s)} DX^\mu \exp(iS[\Psi, C; X])$$

(8)

we can write the result for $Z$ as

$$Z = \int D\bar{\zeta} D\zeta \bar{\zeta}_b(0)\zeta_b(1)e^{iS_0[\bar{\zeta},\zeta]} \mathcal{F}[\Psi, \bar{\zeta} A\zeta] - 2\pi\alpha'\bar{\zeta} A_0\zeta].$$

(9)

The dual target space fields are given by

$$\tilde{A}_\mu = (0, A_M)$$

(10)

and the standard Buscher rules \cite{17} for $\tilde{\Psi} \leftrightarrow \Psi$.

For abelian $A$ the boundary condition for the dual model (note $X^M = \tilde{X}^M$) is \cite{3}

$$\tilde{X}^0(z(s)) = -2\pi\alpha' A_0(\tilde{X}^M(z(s)))$$

which constrains the end points of the string to the hypersurface $\tilde{X}^0 = -2\pi\alpha' A_0(\tilde{X}^M)$ with free movement inside this hypersurface (D-brane). In contrast a general $s$-dependent boundary condition as in \cite{8} has no D-brane interpretation. However, we have to keep in mind that in \cite{8} we need only boundary conditions of the type

$$\tilde{X}^0(z(s)) = -2\pi\alpha'\bar{\zeta}(s)A_0(\tilde{X}^M(z(s)))\zeta(s).$$

This type still allows a D-brane interpretation: The brane as a whole changes its position in the target space in dependence on $s$.

The $\zeta$-integration results in ordering the matrices sandwiched between $\bar{\zeta}(s)$ and $\zeta(s)$ with respect to increasing $s$. But after performing the functional integral over the world
surfaces $X^\mu(z)$ there is no longer any correlation between a given target space point and $s$. The situation is improved if one treats (8) and (9) within the background field method (bm). Then both in $\mathcal{F}_{bm}$ and $Z_{bm}$ all dependence on target space coordinates is realized via a classical string world sheet configuration $X_{cl}$. The result of the $\zeta$-integration is then

$$Z_{bm}[\Psi, A; X_{cl}] = \text{tr} \mathcal{F}_{bm}[\tilde{\Psi}, \tilde{A}; \tilde{X}_{cl}] - 2\pi \alpha' A_0] .$$

(11)

Path ordering now refers to the classical path $\tilde{X}_{cl}(z(s))$ and involves both the matrices appearing in the second argument of $\mathcal{F}_{bm}$ as well as $A_0$ entering via the argument specifying the boundary condition.

The insertion of a matrix as boundary condition is performed after $\mathcal{F}_{bm}$ has been calculated with scalar (not matrix valued) $s$-dependent boundary condition. In this formalism we can avoid wondering about the target space interpretation of matrix valued boundaries. The situation is similar to dimensional regularization, the change from integer $n$ to complex $\hat{n}$ is performed after $\int d^n x...$ has been calculated for integer $n$.

3 Covariant definition of the matrix D-brane model

In analogy to [4] we now generalize our consideration to the case of several abelian isometries, define the resulting dual model in a covariant way and extend it to generic target space fields $\Psi$ at the end. For this purpose we describe a matrix D-brane with a p-dimensional world hypersurface by matrix valued target space coordinates

$$f^\mu(Y^N), \quad N = 1, \ldots, p ,$$

taking their values in the Lie algebra of $G$. The open string under discussion couples in the bulk as usual to the target space fields $\Psi$. In addition there is a nonabelian gauge field $A_M(Y)$ living on the D-brane [3]. Under a gauge transformation with $\Omega(Y) \in G$ the field $A$ transforms as a standard gauge field while $f$ transforms homogeneously $f^\mu \to \Omega f^\mu \Omega^{-1}$. The Dirichlet boundary condition as well as the coupling of the string world sheet boundary to $A_M$ will be formulated with the help of the one-dimensional auxiliary field $\zeta$ of the previous section.

Let again $z(s)$ parametrize the string world sheet boundary in 2D parameter space. The Dirichlet boundary condition relates the target space image $X^\mu(z(s))$ of this $z$-space curve to a curve on the D-brane $Y^N(s)$

$$X^\mu(z(s)) = \bar{\zeta}(s) f^\mu(Y^N(s)) \zeta(s) .$$

(12)

The relevant action is ($S_M$ from [3])

$$S[\Psi, A; \bar{\zeta}, \zeta; X] = S_M[\Psi; X] + S_{\partial M} ,$$

$$S_{\partial M} = \int_{\partial M} \left( \bar{\zeta}(s) A_N(Y(s)) \zeta(s) \dot{Y}^N - \frac{1}{2\pi} k(s) \Phi(X(z(s))) \right) ds .$$

(13)

\footnote{This field corresponds to $\tilde{A}_N(X^N) = A_N(X^N)$ in the previous section.}
With the covariant version of (8)

$$F[\Psi, \overline{\zeta} A \zeta | \overline{\eta} f \eta] = \int_{(12)} DX^\mu \exp(iS[\Psi, A; \overline{\zeta}, \zeta; X])$$  \hspace{1cm} (14)$$

the partition function is finally given by

$$Z[\Psi, A] = \int D\overline{\zeta}(s) D\zeta(s) \overline{\zeta}_b(0) \zeta_b(1) e^{iS_0[\overline{\zeta}, \zeta]} F[\Psi, \overline{\zeta} A \zeta | \overline{\zeta} f \eta]$$

$$= \text{tr} PF[\Psi, A[f].$$  \hspace{1cm} (15)$$

Before turning in the next sections to the calculation of lowest order RG $\beta$-functions for the model defined above, we still provide the background expansion of the boundary condition (12). If $X$ and $Y$ are varied around a configuration satisfying (12) at fixed $\zeta$, $\overline{\zeta}$ we get a gauge non-covariant result. Therefore, we combine the variation of $Y$ with an adapted gauge transformation of $\zeta$, $\overline{\zeta}$ ($C$ denotes the straight line connecting $Y$ and $Y + \delta Y$)

$$\zeta \rightarrow \zeta + \delta \zeta = P \exp(i \int_C A_N dY^N) \zeta$$

$$= \zeta + i\delta Y^M A_M \zeta - \frac{1}{2} \delta Y^M \delta Y^N (A_M A_N - i\partial_M A_N) \zeta + O((\delta Y)^3),$$  \hspace{1cm} (16)$$

which leads to

$$\delta X^\mu = \delta Y^M \overline{\zeta} D_M f^\mu \zeta + \frac{1}{2} \delta Y^M \delta Y^N \overline{\zeta} D_M D_N f^\mu \zeta + O((\delta Y)^3).$$  \hspace{1cm} (17)$$

$D_M = \partial_M - i[A_M, \cdot]$ is the gauge covariant derivative.

Target space distances along curves subject to (12), (16) are measured with the induced metric

$$h_{MN}(Y(s), s) = f^\mu_{i,M}(Y(s), s) \cdot f^\nu_{i,N} \cdot G_{\mu \nu}(\overline{\zeta}(s)f(Y(s))\zeta(s)),$$  \hspace{1cm} (18)$$

where we defined

$$f^\mu_{i,M}(Y(s), s) = \overline{\zeta}(s) D_M f^\mu(Y(s)) \zeta(s).$$  \hspace{1cm} (19)$$

In these formulae we have indicated the functional dependences in full detail, since it will be important for later use to know at which places matrices appear after performing the $\zeta$-integration. Now we want to develop a calculus of covariant derivation with respect to this induced metric, which is covariant with respect to the gauge group $G$, too. Motivated by (19) one has to replace the derivative $\partial_N$ by

$$\partial_N - i\overline{\zeta} A_N \frac{\partial}{\partial \zeta} + i\zeta_b A^{ab}_N \frac{\partial}{\partial \zeta_a}.$$

\footnote{We avoid to introduce a new symbol for it.}
As far as $Y$-dependence appears via quantities sandwiched between $\zeta$ and $\bar{\zeta}$, the final recipe is to replace $\partial_N$ by $D_N$ applied to the quantity under consideration sandwiched between $\bar{\zeta}, \zeta$. Then the Levi-Civita connection related to (18) is

$$\gamma_{MNL} = \frac{1}{2} [(MN, L) + (ML, N) + (NM, L) + (NL, M) - (LM, N) - (LN, M)] + f^\mu_M \cdot f^\nu_N \cdot f^\lambda_L \cdot \Gamma_{\mu\nu\lambda}(\bar{\zeta}f\zeta) ,$$

$$(MN, L) = \bar{\zeta}(s) D_M D_N f^\nu(Y(s))\zeta(s) \cdot f^\lambda_L \cdot G_{\nu\lambda}(\bar{\zeta}f\zeta) ,$$

with $\Gamma_{\mu\nu\lambda}$ denoting the target space connection coefficients.

Introducing Riemann normal coordinates $\xi^\mu$ and $\eta^N$ for the target space and the brane, respectively, we get finally

$$\xi^\mu = \eta^M f^\mu_M + \frac{1}{2} \eta^M \eta^N K^\mu_{MN} + O(\eta^3) ,$$

with

$$K^\mu_{MN} = \bar{\zeta} D_M D_N f^\nu(\bar{\zeta}f\zeta) \cdot f^\mu_L \cdot G_{\alpha\beta}(\bar{\zeta}f\zeta) - \gamma^\lambda_{MN} f^\mu_A .$$

## 4 Background field expansion of the action

We expand around a classical string configuration satisfying the stationarity condition both in the bulk of the string world surface $M$ (equation of motion) and on the boundary $\partial M$ (generalized Neumann boundary condition). The equation of motion for $X$ (We drop the index $cl$ used in section 2.) is taken into account by dropping linear terms in $\xi$ in the bulk. The generalized Neumann boundary condition has to be written down explicitly, since it will need some discussion below (t and n denote tangential and normal components, respectively.)

$$\partial_n X^\alpha G_{\alpha\mu} \bar{\zeta} D_M f^\mu\zeta - \partial_t X^\alpha B_{\alpha\mu} \bar{\zeta} D_M f^\mu\zeta - 2\pi\alpha' \partial_t Y^\lambda \bar{\zeta} F_{AM}\zeta = 0 .$$

Now the standard expansion of $S_M[\Psi; X + \delta X]$ gives (e.g. [18, 19, 4])

$$S_M = S_M[\Psi; X] + \frac{1}{4\pi\alpha'} \int_M d^2 z \left( G_{\alpha\beta} \nabla_m \xi^\alpha \nabla^m \xi^\beta + \xi^\alpha \xi^\beta R_{\mu\alpha\beta\nu} \partial_m X^\mu \partial^n X^\nu \right. \left. - \frac{1}{2} \xi^\alpha \xi^\beta \nabla_\alpha H_{\mu\beta\nu} \epsilon^{mn} \partial_m X^\mu \partial_n X^\nu \right)$$

$$+ \frac{1}{2\pi\alpha'} \int_{\partial M} \left( \partial_n X^\alpha G_{\alpha\beta} \bar{\zeta} \xi^\beta - \partial_t X^\alpha B_{\alpha\beta} \bar{\zeta} \xi^\beta \right)$$

$$+ \frac{1}{2} \xi^\alpha \xi^\beta \nabla_\alpha B_{\beta\mu} \partial_\mu X^\nu + \frac{1}{2} \xi^\alpha \nabla_\nu \xi^\beta B_{\alpha\beta}) ds + O(\xi^3) .$$

\footnote{It is a consequence of free movement of the string endpoints inside the D-brane and is compatible with the Dirichlet condition which forbids movement orthogonal to the D-brane.}

\footnote{Since we are performing lowest order calculations only, we will consider flat 2D metric and drop the dilaton field $\Phi$ in the following.}
\[\nabla\] denotes the target space covariant derivative. In the expansion of the the gauge field dependent part \(S_{\partial M}\) in (13) use has to be made of the \(\bar{\zeta}, \zeta\) quantum equation of motion including all contact terms. To shorten the treatment we use instead the standard variation formulae for the Wilson loop (see e.g. [16] and refs. therein) and re-express them afterwards in the auxiliary field language. For 
\[
\text{tr} P U[Y] = \text{tr} P \exp \left( i \int_{\partial M} A_M dY^M \right)
\]
one has up to order \(O((\delta Y)^3)\)
\[
\text{tr} P U[Y + \delta Y] = \text{tr} P \left( U[Y] \exp \left( i \int ds (F_{MN} \dot{Y}^N \delta Y^M + \frac{1}{2} D_M F_{NK} \dot{Y}^K \delta Y^M \delta Y^N + \frac{1}{2} F_{MN} \delta Y^M \delta \dot{Y}^N) ) \right) \right) .
\]
The translation into \(\zeta\)-language is
\[
\text{tr} P U[Y + \delta Y] = \int D\bar{\zeta} D\zeta \ \bar{\zeta}_b(0) \zeta_b(1) e^{iS_0 + iS_{\partial M}[Y]}
\cdot \exp \left( i \int ds \ \bar{\zeta} (F_{MN} \dot{Y}^N \delta Y^M + \frac{1}{2} D_M F_{NK} \dot{Y}^K \delta Y^M \delta Y^N + \frac{1}{2} F_{MN} \delta Y^M \delta \dot{Y}^N) \right) .
\]
On the other side by comparing 
\[
\text{tr} P U[Y + \delta Y] = \int D\bar{\zeta} D\zeta \ \bar{\zeta}_b(0) \zeta_b(1) e^{iS_0 + iS_{\partial M}[Y + \delta Y]}
\]
with (26) we get the wanted expansion for \(S_{\partial M}\). In writing its final version we still express \(\delta Y\) in terms of the Riemann normal coordinates on the brane \(\eta\) and denote the covariant derivative with respect to both the gauge group \(G\) and the induced metric by \(\hat{D}_M\)
\[
S_{\partial M}[A; Y + \delta Y] = S_{\partial M}[A; Y] + \int ds \ \bar{\zeta} (F_{MN} \dot{Y}^N \eta^M + \frac{1}{2} \hat{D}_M F_{NK} \dot{Y}^K \eta^M \eta^N + \frac{1}{2} F_{MN} \eta^M \hat{D}_t \eta^N) \zeta .
\]
The sum of (24) and (27) yields the expansion of \(S\). For further simplification we imply the boundary condition (23) for the classical background configuration \(X, Y\), eliminate on \(\partial M \xi\) by (21) in favour of \(\eta\) and introduce in analogy to (18)
\[
b_{MN}(Y(s), s) = f_{\mu}^A(Y(s), s) \cdot f_{\nu}^\alpha \cdot B_{\mu\nu}(\bar{\zeta}(s) f(Y(s)) \zeta(s)) .
\]
Then, using
\[
\partial_t X^\alpha = \dot{Y}^A f_{\alpha}^A,
\nabla_t \xi^\beta = \hat{D}_t \eta^M f_{\nu}^\beta + \eta^M \dot{Y}^A \bar{\zeta} \hat{D}_A \hat{D}_M f_{\beta}^A \zeta + \eta^M \dot{Y}^A \Gamma_{\alpha\mu}^\beta f_{\alpha}^A f_{\nu}^\mu
\]
and
\[
\hat{D}_M b_{NA} = \bar{\zeta} \hat{D}_M D_N f_{\nu}^\alpha \zeta f_{\epsilon}^A B_{\nu\alpha} + f_{\nu}^\alpha \zeta \hat{D}_M D_A f_{\beta}^A \zeta B_{\nu\alpha} + f_{\nu}^\alpha f_{\epsilon}^A f_{\mu}^\mu \partial_\mu B_{\nu\alpha} ,
\]
we arrive up to \( O(\xi^3, \eta^3) \) at

\[
S = S[\Psi, A; \bar{\zeta}, \zeta; X] + \frac{1}{4\pi \alpha'} \int_M d^2 z \left( G_{\alpha\beta} \nabla_m \xi^\alpha \nabla^m \xi^\beta + \xi^\alpha \xi^\beta R_{\mu\alpha\beta\nu} \partial_m X^\mu \partial^n X^\nu - \frac{1}{2} \xi^\alpha \xi^\beta \nabla_n H_{\alpha\beta\nu} \epsilon^{mn} \partial_m X^\mu \partial^n X^\nu + \xi^\alpha \nabla_m \xi^\beta H_{\alpha\beta\mu} \partial_n X^\mu \epsilon^{mn} \right) \]

\[+ \frac{1}{4\pi \alpha'} \int_{\partial M} \left( \eta^M \eta^N \nabla_n X^\alpha G_{\alpha\beta} K_{MN}^\beta + \eta^M \eta^N \hat{Y}^A (\hat{D}_M b_{NA} + 2\pi \alpha' \zeta \hat{D}_M F_{NA} \zeta) \right.

\[\left. - i \eta^M \eta^N \hat{Y}^A (F_{BM} f^a_{MN} B_{\nu} + \eta^M \hat{D}_\nu \eta^N (b_{MN} + 2\pi \alpha' \bar{\zeta} F_{MN} \zeta)) \right) ds .\]

Comparing this result with the abelian case \([4]\) we find just one additional structure, the \( [F, f] \) commutator term. All other modifications refer only to the appearance of the auxiliary fields. Note that in (31) besides the explicit \( \zeta \)'s there is more \( \zeta \)-dependence in \( G, B \), since the arguments of these fields on \( \partial M \) are given by (12), and in \( b, \hat{D}, f^\nu_{MN} \) and \( K \) via (28, 20, 19, 22).

## 5 Lowest order calculation of RG \( \beta \)-functions

To begin with we need the propagator for the quantum corrections \( \xi \) and their manifestation on the brane \( \eta \). The relation between \( \xi \) and \( \eta \) is a consequence of the Dirichlet condition implied as an external constraint on the integrand of the functional integral in (14). The choice of any further boundary condition, to make the propagator \( \langle \xi \xi \rangle_0 \) well defined, is a matter of technical convenience only. It has implications on the question concerning the set of boundary vertices contributing in the perturbative evaluation. For instance, the use of a propagator obeying the Neumann condition including the Lorentz force has allowed in \([21, 22]\) to sum in one graph all orders of \( \alpha' \) for the case of constant background fields. Unfortunately, we cannot repeat this trick for our case since the price we paid for handling the nonabelian structures is the explicit boundary parameter dependence. We did not succeed in constructing the corresponding propagator explicitly. However, to calculate counter-terms in lowest order it is necessary to know the short distance behaviour only:

\[
\langle \xi^\mu (z_1) \xi^\nu (z_2) \rangle_0 = D^{\mu\nu} (z_1, z_2) + N^{\mu\nu} (z_1, z_2)
\]

\[= -\alpha' (G^{\mu\nu} (X (z_2)) + O(z_1 - z_2)) \cdot \log |z_1 - z_2| .\] (32)

The last line is valid inside \( M \). The boundary behaviour is controlled by

\[
D^{\mu\nu} (z_1, z_2) = 0, \quad \text{if } z_1 \text{ or } z_2 \in \partial M ,
\]

\[
\nabla_n N^{\mu\nu} = 0 \quad \text{on } \partial M ,
\]

\[
N^{\mu\nu} (z(s_1), z(s_2)) = -2\alpha' \left( f^\mu_{MN} f^\nu_{MN} + O(s_1 - s_2) \right) \cdot \log |s_1 - s_2| .\] (33)
The arguments of $f^\mu_M$ and $h^{MN}$ are $Y(s_2), s_2$. Eqs. (33) and (21) also imply

$$\langle \eta^A(s_1)\eta^B(s_2) \rangle_0 = -2\alpha' \left( h^{AB} + O(s_1 - s_2) \right) \cdot \log |s_1 - s_2| .$$  \hspace{1cm} (34)

The existence of such a propagator is guaranteed at least in the vicinity of constant background fields and no explicit $s$-dependence [4]. We assume that there are no global obstructions. The simple Neumann condition guarantees that the boundary vertex $\xi \nabla_n \xi G$ arising from the partial integration of the kinetic term in (31) does not contribute.

The bookkeeping of divergent diagrams in the perturbative evaluation of the partition function $Z$ is the same as in the case of a string with free ends [18, 19, 20, 21]. There are only modifications in the classical factors multiplying the quantum fields in the vertices. Then at one-loop level we have to consider the tadpole graph constructed with the $\eta \eta$-vertex (corresponding to the first three terms in the boundary integral of (31)) and the bulk-boundary interference graph [19, 21, 4] constructed with the $\eta \hat{D} \eta$-vertex (last term in the boundary integral) and the $\xi \nabla \xi$-vertex (last term in the bulk integral). In addition all diagrams constructed out of these two basic diagrams by multiple insertion of the $\eta \hat{D} \eta$-vertex contribute. However, since the propagator is not known explicitly, we skip the otherwise possible summation [20, 4] and restrict ourselves to the above mentioned two basic diagrams. Altogether, then our final result will represent the beginning of an expansion in $\alpha'$ and $B_{\mu \nu}$.

In this sense the tadpole contribution to the counter-term action is ($\Lambda$ denotes the short distance cutoff.)

$$\frac{1}{2\pi} \log \Lambda \cdot h^{MN} \left( K_{MN}^\alpha G_{\alpha \beta \mu} \partial_n X^\beta \right) + \hat{Y}^A \left( \hat{D}_M b_{NA} + 2\pi \alpha' \hat{\zeta} \hat{D}_M F_{NA} \hat{\zeta} \right)$$

$$- \bar{\zeta} \left[ F_{AM}, f^\alpha \right] \xi f^\mu_N B_{\nu \alpha} \right) .$$

The bulk-boundary interference diagram yields

$$- \frac{1}{2\pi} \log \Lambda \cdot \frac{1}{2} \left( b^{MN} + 2\pi \alpha' \hat{\zeta} F^{MN} \hat{\zeta} \right) f^\mu_M f^\nu_N H_{\mu \nu \alpha} \partial_n X^\alpha .$$

In total this gives for the boundary part of the counter-term action up to higher orders in $\alpha'$ and $B$

$$\Delta S = \frac{\log \Lambda}{2\pi} \int_{\partial M} \left( (h^{MN} K_{MN}^\mu \mu + \frac{1}{2} (b^{MN} + 2\pi \alpha' \hat{\zeta} F^{MN} \hat{\zeta} ) f^\mu_M f^\nu_N H_{\mu \nu \alpha} \right) \cdot \partial_n X^\nu$$

$$+ h^{MN} \left( \hat{D}_M b_{NA} + 2\pi \alpha' \hat{\zeta} \hat{D}_M F_{NA} \hat{\zeta} - i \hat{\zeta} \left[ F_{AM}, f^\alpha \right] \xi f^\mu_N B_{\nu \alpha} \right) ds .$$  \hspace{1cm} (35)

Now the auxiliary field formalism has to do its last job. $\bar{\zeta}$ and $\zeta$ appear in (32) at many places, compare the last remark in section 4. As a general rule they always sandwich some

\[\text{The factor 2 of the boundary singularity relative to the bulk singularity has been discussed at length in [22].}\]
Fig.1  Gauge index “fine structure” of a local vertex of the type indicated in (36) for $N = 4$. The crosses denote a matrix.

Since $\Delta S$ is local in $s$ we need a procedure to perform $\zeta$-integrals of the type

$$I_{ab} = \int D\tilde{\zeta} D\zeta \exp \left( iS_0 + i \int \tilde{\zeta} A_N \zeta dY^N \right) \tilde{\zeta}_a(0)\zeta_b(1) \prod_{j=1}^{N} \tilde{\zeta}M_j\zeta(s).$$  (36)

The generalization to the necessary multiple insertions of such vertices is straightforward.

Due to the $\Theta$-function in the $\zeta$-propagator there are no $\zeta$-loops going forward and backward in one-dimensional $s$-space. Only $\zeta$-loops consisting out of propagators at coinciding points i.e. $\zeta$-tadpoles are allowed. However, due to the ambiguity of $\Theta(0)$ they are subject to renormalization ambiguities. If the Wilson loop under the functional integral in (3) is replaced by the full two point function $\langle \tilde{\zeta}_b(0)\zeta_b(1) \rangle$ (compare (4)) we decide to specify the definition of the $\zeta$ quantum theory by restriction to the sector of diagrams in which the gauge index flux is completely described by only one continuous line connecting $s = 0$ and $s = 1$. This concept is illustrated by fig.1 and fig.2, it is based on the fact that all vertices appearing in the calculations have a gauge index structure as in (36). Via inserted matrices it connects legs in a pairwise manner. Just this restriction we imply on the covariant definition of our model in section 3, too.

10Thinking e.g. in terms of Taylor expanded background fields.

Fig.2  Allowed and forbidden contributions. Use is made of the vertex notation introduced in fig.1
After this specification we see that in the evaluation of $I$ the sum of all permuted products of the matrices $M_j$ appears

$$I_{ab} = N! \left( \Theta(0) \right)^{N-1} \left( P(U \text{ Sym}(\prod_{j=1}^{N} M_j)) \right)_{ab}.$$ 

At this point we still have a renormalization ambiguity. We fix the ambiguity in $\Theta(0)$ by requiring that the whole formalism reproduces in the abelian case the then well known results. This means to fix depending the number of factors $N$: $(\Theta(0))^{N-1} = (N!)^{-1}$.

Altogether we found for our local vertices the identification

$$\prod_{j=1}^{N} (\bar{\zeta} M_j \zeta) \simeq \bar{\zeta} \text{ Sym} \left( \prod_{j=1}^{N} M_j \right) \zeta.$$ 

(37)

We define an operation $Q$ by

$$Q\{\prod_{j=1}^{N} (\bar{\zeta} M_j \zeta)\} = \text{ Sym} \left( \prod_{j=1}^{N} M_j \right).$$ 

(38)

and extend it linearly to sums of products of sandwiched matrices.

The RG $\beta$-functions for the gauge field $A$ and the brane position $f$ can be read off from (35). Using the just defined operation $Q$ the boundary part of the conformal invariance condition then becomes

$$Q\{h^{MN} (\hat{D}_M b_{NA} + 2\pi \alpha' \bar{\zeta} \hat{D}_M F_{NA} \zeta - i\bar{\zeta} [F_{AM}, f^\alpha] \zeta f^\nu_{N} B_{\nu \alpha})\} = 0$$

$$Q\{h^{MN} K^{\mu}_{MN} G_{\mu \nu} + \frac{1}{2} (b^{MN} + 2\pi \alpha' \bar{\zeta} F^{MN} \zeta) f^\alpha_{;M} f^\beta_{;N} H_{\alpha \beta \nu}\} = 0.$$ 

(39)

After the application of $Q$ there is present no longer any $\bar{\zeta}$ or $\zeta$. The manner how matrices were sandwiched between the auxiliary fields determines which matrices have to be handled as basic entities under the symmetrization procedure. E.g. $\hat{D}_M F_{NA}$ and $[F_{AN}, f^\alpha]$ appear as such basic matrices. Therefore, the commutators due to the nonabelian gauge structure will not be removed by the symmetrization.

We interpret our main result (39) as the system of equations of motion for the gauge field living on the brane and the brane position. At this stage we leave open the question whether e.g. the first equation is the gauge field equation or whether as in [4] a certain linear combination of the two equations plays this role. Such a linear combination arises if one uses the boundary condition (23) to transform the projection onto the brane of the $\partial_n X^\nu$ term in (35) into a $\dot{Y}^A$ term. For more discussion on this point we refer to a paper in preparation [24].

The structure $[F, f]B$ is a genuine nonabelian effect. It leads for $B \neq 0$ to a direct interaction between the gauge field and the brane position. For gauge groups whose Lie algebras contain multiples of the identity the whole system is translation invariant in target space, otherwise this invariance is broken.

\footnote{We assume that as usual in lowest order there is no difference between the RG $\beta$-functions and the Weyl anomaly coefficients [23].}
6 Conclusions

The use of the one-dimensional auxiliary field formalism allowed us to express the partition function for an open string with free ends coupling to a nonabelian gauge field after a T-duality transformation as certain functional integral over the auxiliary field (10). The integrand contains the functional $F$ which corresponds to the partition function of a theory which obeys explicit boundary parameter dependent Dirichlet conditions. As discussed at the end of section 2, the replacement of the function specifying the boundary condition by a matrix valued object (enforced by the auxiliary field integration) gives meaning to the notion of matrix valued D-brane position. Motivated by this consequence of T-duality we gave, again using the auxiliary field, a definition of a model describing a string bound with its ends to a matrix D-brane and coupling to a nonabelian gauge field on the D-brane as well as to generic target space background fields. We were able to demonstrate the possibility of concrete calculations by deriving the lowest order system of equations of motion for both the nonabelian gauge field on the D-brane and the matrix valued brane position.

With this calculation we simultaneously solved the ordering problem for all the matrix quantities involved. Of course the formalism can be extended to higher orders, too. This requires an analysis of the renormalization of the system of coupled quantum fields $\xi$ and $\bar{\xi}$, $\zeta$. At the present stage it seems to be open whether the overall symmetrization is the correct solution of the ordering problem in higher orders, too. This is due to the nested structure of renormalization.

Certainly the most interesting aspect of further work concerns the search for nontrivial dynamical effects implied by the equations of motion derived in this paper. We would also like to understand possible relations to recent work on D-branes in the context of noncommutative geometry [25, 26].

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