Pion Model

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1 Introduction

Yukawa introduced the concept of Pions as a possible mediator for the strong forces, as then visualized. Later these were discovered. They have been considered to be typical elementary particles because of their role in strong interactions. Based on this Hayakawa [1, 2] suggested a cosmological model that also included weak interactions. So also the author put forward a completely different cosmology in this journal [3, 4], working with fluctuations and pions, thus correctly predicted the accelerated expansion of the universe, when the standard Big Bang model of that time stated the exact opposite. In later work [5] the author has proposed a model, which starts with the Quantum Vacuum pictured as a collection of zero point oscillators or alternatively Planck scale oscillators (Cf.ref.[6]). At this stage a Ginzburg-Landau phase transition leads to the next stage or phase of the universe with elementary particles [7]. This can be seen as follows.

Let us consider an array of \( N \) particles, spaced a distance \( \Delta x \) apart, which behave like oscillators that are connected by springs. We then have [8, 9, 10, 11]

\[
 r = \sqrt{N\Delta x^2} \tag{1}
\]

\[
 ka^2 \equiv k\Delta x^2 = \frac{1}{2}k_B T \tag{2}
\]

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where \( k_B \) is the Boltzmann constant, \( T \) the temperature, \( r \) the extent and \( k \) is the spring constant given by

\[
\omega_0^2 = \frac{k}{m} \tag{3}
\]

\[
\omega = \left( \frac{k}{m} a^2 \right)^{\frac{1}{2}} \frac{1}{r} = \omega_0 \frac{a}{r} \tag{4}
\]

We now identify the particles with Planck masses and set \( \Delta x \equiv a = l_P \), the Planck length. It may be immediately observed that use of (3) and (2) gives \( k_B T \sim m_P c^2 \), which of course agrees with the temperature of a black hole of Planck mass. Indeed, Rosen [12] had shown that a Planck mass particle at the Planck scale can be considered to be a Universe in itself with a Schwarzschild radius equalling the Planck length. Furthermore in the above characterization a typical elementary particle like the pion can be considered to be the result of \( n \sim 10^{40} \) Planck masses.

Using this in (1), we get \( r \sim l \), the pion Compton wavelength as required. Whence the pion mass is given by

\[
m = m_P / \sqrt{n} \tag{5}
\]

as indeed is the case. Further, in this latter case, using (1) and the fact that \( N = n \sim 10^{40} \), and (2), i.e. \( k_B T = k l^2 / N \) and (3) and (4), we get for a pion and (5),

\[
k_B T = \frac{m^3 c^4 l^2}{\hbar^2} = mc^2,
\]

which of course is the well known formula for the Hagedorn temperature for elementary particles like pions [13].

In this model we end up with the Hagedorn temperature [6]. Now the importance of this is that in the Hagedorn theory Hadrons condense at this temperature, and these Hadrons would now be pions. In other words from the Quantum Vacuum through a phase transition we are lead to the formation of pions [14].

## 2 A Mass Spectrum

From these considerations various attempts have been made by several authors for a simple or fundamental mass spectrum [15]. We note that all
of non-leptonic matter is made up of quarks. They interact via the interquark or QCD force. In this picture, pions are bound-states of a quark and an anti-quark. The QCD potential is given by

$$U(r) = -\frac{\alpha}{r} + \beta r$$ (6)

where in units \(\hbar = c = 1\), \(\alpha \sim 1\). The first term in (6) represents the Coulombic part while the second term represents the confining part of the potential. As is well known the potential in (6) explains two well known features viz., quark confinement and asymptotic freedom.

Let us consider the pion made up of two quarks along with a third quark, one at the centre and two at the ends of an interval of the order of the Compton wavelength, \(r\). Then the central particle experiences the force

$$\frac{\alpha}{(\frac{\pi}{2} + r)^2} - \frac{\alpha}{(\frac{\pi}{2} - r)^2} \approx -\frac{2\alpha x}{r^3}$$ (7)

where \(x\) is the small displacement from the mean position. Equation (7) gives rise to the Harmonic oscillator potential, and the whole configuration resembles the tri-atomic molecule.

Before proceeding we can make a quick check on (7). We use the fact that the frequency is given by

$$\omega = \left(\frac{\alpha^2}{m_\pi r^3}\right)^{\frac{1}{2}} = \frac{\alpha}{(m_\pi r^3)^{\frac{1}{2}}}$$

whence the mass of the pion \(m_\pi\) is given by

$$(\hbar \omega \equiv \omega = m_\pi)$$ (8)

Remembering that \(r = 1/m_\pi\), use of (8) gives \(\alpha = 1\), which of course is correct.

To proceed, the energy levels of the Harmonic oscillator are now given by,

$$\left(n + \frac{1}{2}\right)m_\pi$$

If there are \(N\) such oscillators, then over the various modes the energy of the particle is given by

$$E = \sum_{r=1}^{3N} \left(n_r + \frac{1}{2}\right)\hbar \omega = l \left(n + \frac{1}{2}\right)\hbar \omega$$
m and n being positive integers. The mass of the particle $P$ is now given by

$$m_P = l \left( n + \frac{1}{2} \right) m_\pi$$ \hspace{1cm} (9)

It is remarkable that the above simple formula reproduces for the different integral values of $l$ and $n$ reproduces the masses of all the known Bosons with an error of less than 1% in most of the cases and exceptionally less than about 2%.

The above theory suggests that these particles are composed of (or can decompose into) pions. An immediate example is the $K_0$ Meson which can break into two or even three pions.

More generally in a collision of the above particle, at high energies we can expect the following

$$A + B \rightarrow C + D + \Delta$$ \hspace{1cm} (10)

where $A$, $B$, $C$, are particles from the above table, $D$ are pions if any and $\Delta$ is the energy (or conversely).

3 Remarks

1. Interestingly six decades ago Nambu had given the empirical formula $n$ or $(n + \frac{1}{2}) \times$ (the mass of the pion) for about half a dozen particles, remembering that at that time such a small number of particles were known [18]. However this formula was nothing more than a curiosity because it was completely ad hoc and devoid of any dynamics.

2. Just as the nucleus can be split into sub-constituents we could conceive of an elementary particle being split into its constituent quarks with the release of energy. This however is forbidden by the quark confining force in the QCD potential. The above considerations on the other hand suggest that the particles could be split or fused into other particles and pions with the release of energy.

3. We quickly observe that the above mass spectrum formula [9] works remarkably well for all known non-leptonic Fermions too [6]. This appears surprising because it must be borne in mind that this derivation is insensitive to quantum numbers such as spin and so on.
On the other hand, pions being Hadrons, it is expected that they contribute to the masses of the other Hadrons. In any case it is expected that reactions (10) hold good for Fermions as well, provided that there is a balance of other quantum numbers. Indeed, as is well known such pion production has been observed at the RHIC, over the years in proton-proton and other collisions [19] [20] [21].

In any case the very fact that all non leptonic particle masses are covered (with error limits), by a single formula in terms of the pion mass is in itself very suggestive and interesting. In other words (9) holds good for all known elementary particles. Further (10) maybe important in the context of very high energy collisions taking place at 14 TeV expected at LHC in 2013.

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APPENDIX
| Particle and mass | Mass From Formula | Error % | $(l, n)$ |
|-------------------|-------------------|---------|---------|
| $\pi^\pm(139.57018) \pm 0.00035$ | 137 | $-1.43885$ | (2, 0) |
| $\pi^0(134.9766) \pm 0.0006$ | 137 | 1.481481 | (2, 0) |
| $K^\pm 4064 \pm 0.016$ | 496 | 1.9 | (1, 3) |
| $\eta(547.51) \pm 0.18$ | 548 | 0.182815 | (8, 0) |
| $f_0(600)(400 - 1200)$ | 616.5 | (2.75)0 | (1, 4) |
| $\rho(775.5) \pm 0.4$ | 753.5 | $-2.14286$ | (1.5) |
| $\omega(782) \pm 0.12$ | 753.5 | $-3.6445$ | (1.5) |
| $\eta'(958) \pm 0.14$ | 959 | 0.104384 | (2, 3) |
| $f_0(980) \pm 10$ | 959 | $-2.14286$ | (2, 3) |
| $a_0(980) \pm 1.2$ | 959 | $-2.14286$ | (2, 3) |
| $\phi(1020) \pm 0.020$ | 1027.5 | 0.735294 | (1.7) |
| $h_1(1170) \pm 20$ | 1164.5 | $(-0.47009)0$ | (1.8) |
| $b_1(1235) \pm 3.2$ | 1233 | $(-0.16194)0$ | (2.4) |
| $a_1(1260)1230 \pm 40$ | 1233 | $(-2.14286)0$ | (2.4) |
| $f_2(1270)1275.4 \pm 1.1$ | 1233 | $-2.91339$ | (2.4) |
| $f_1(1285)1281.8 \pm 0.6$ | 1301.5 | 1.284047 | (1.9) |
| $\eta(1295)1294 \pm 4$ | 1301.5 | 0.501931 | (1.9) |
| $\pi(1300) \pm 100$ | 1301.5 | 0.115385 | (1.9) |
| $a_2(1320)1318.3 \pm 0.6$ | 1301.5 | $-1.40152$ | (1.9) |
| $f_0(1370)(1200 - 1500)$ | 1370 | 0 | (4.2) |
| $h_1(1380)$ | 1370 | 0.72464 | (4.2) |
| $\pi_1(1400)1376 \pm 17$ | 1370 | $-2.14286$ | (4.2) |
| $f_1(1420)1426.3 \pm 0.9$ | 1438.5 | 1.302817 | (1, 10) |
| $\omega(1420)1400 - 1450$ | 1438.5 | $(1.302817)0$ | (1, 10) |
| $f_2(1430)$ | 1438.5 | 0.594406 | (1, 10) |
| $\eta(1440)(1400 - 1470)$ | 1438.5 | $-0.10417$ | (1, 10) |
| $a_0(1450)1474 \pm 19$ | 1438.5 | $-0.7931$ | (1, 10) |
| $\rho(1450)1459 \pm 11$ | 1438.5 | $-0.7931$ | (1, 10) |
| $f_0(1500)1507 \pm 5$ | 1507 | $(0.466667)0$ | (2, 5) |
| $f_1(1510)$ | 1507 | $-0.19868$ | (2, 5) |
| $f'_2(1525) \pm 5$ | 1507 | $-1.18033$ | (2, 5) |
| $f_2(1565)$ | 1575.5 | 0.670927 | (1, 11) |
| Particle and mass | Mass From Formula | Error % | $(l, n)$ |
|-------------------|------------------|---------|----------|
| $h_1(1595)$       | 1575.5           | −1.22257| (1, 11)  |
| $\pi_1(1600) \pm 8$ | 1575.5           | −1.53125| (1, 11)  |
| $\chi(1600)$     | 1575.5           | −1.53125| (1, 11)  |
| $a_1(1640)$       | 1644             | 0.243902| (8, 1)   |
| $f_2(1640)$       | 1644             | 0.243902| (8, 1)   |
| $\eta_2(1645) \pm 5$ | 1644             | (0.06079) | (8, 1)   |
| $\omega(1670)$   | 1644             | (1.55688) | (8, 1)   |
| $\omega_3(1670) \pm 4$ | 1644             | −1.55689 | (8, 1)   |
| $\pi_2(1670) \pm 3.2$ | 1644             | −1.55689 | (8, 1)   |
| $\phi(1680) \pm 20$ | 1712.5           | 1.934524| (1, 12)  |
| $\rho(1690) \pm 2.1$ | 1712.5           | 1.331361| (1, 12)  |
| $\rho(1700) \pm 20$ | 1712.5           | (0.735294) | (1, 12)  |
| $a_2(1700)$       | 1712.5           | 0.735294| (1, 12)  |
| $f_0(1710) \pm 6$ | 1712.5           | (0.146199) | (1, 12)  |
| $\eta(1760)$     | 1781             | 1.193182| (2, 6)   |
| $\pi(1800) \pm 14$ | 1781             | −1.05556| (2, 6)   |
| $f_2(1810)$       | 1781             | −1.60221| (2, 6)   |
| $\phi_3(1850) \pm 7$ | 1849.5           | (−0.02703) | (1, 13)  |
| $\eta_2(1870)$   | 1849.5           | −1.09626| (1, 13)  |
| $\rho(1900)$     | 1918             | 0.947368| (4, 3)   |
| $f_2(1910)$       | 1918             | 0.418848| (4, 3)   |
| $f_2(1950) \pm 12$ | 1918             | −1.64103| (4, 3)   |
| $\rho_3(1990)$   | 1986.5           | −0.17588| (1, 14)  |
| $X(2000)$        | 1986.5           | −0.675  | (1, 14)  |
| $f_2(2010) \pm 10$ | 1986.5           | (−1.16915) | (1, 14)  |
| $f_0(2020)$      | 1986.5           | 1.65842 | (1, 14)  |
| $a_4(2040) \pm 10$ | 2055             | 0.735294| (2, 7)   |
| $f_4(2050) \pm 10$ | 2055             | 0.243902| (2, 7)   |
| $\pi_2(2100)$    | 2123.5           | 1.119048| (1, 15)  |
| $f_2(2100)$      | 2123.5           | 1.119048| (1, 15)  |
| $f_2(2150)$      | 2123.5           | −1.23256| (1, 15)  |
| Particle and mass | Mass From Formula | Error %   | $(l, n)$ |
|-------------------|-------------------|-----------|----------|
| $\rho_2(2150)$    | 2123.5            | −1.23256  | (1, 15)  |
| $f_0(2200)$       | 2260.5            | 2.75      | (1, 16)  |
| $f_J(2220)$       | 2260.5            | 1.824324  | (1, 16)  |
| $\eta(2225)$     | 2360              | 1.595506  | (1, 16)  |
| $\rho_3(2250)$    | 2260              | 0.46667   | (1, 16)  |
| $f_2(2300)$       | 2297 ± 28         | 1.26087   | (2, 8)   |
| $f_4(2300)$       | 2329              | 1.26087   | (2, 8)   |
| $D_s(2317)$       | 2329              | 0.5       | (2, 8)   |
| $f_0(2330)$       | 2329              | −0.04292  | (2, 8)   |
| $f_2(2340)$       | 2339 ± 60         | −0.47009  | (2, 8)   |
| $\rho_5(2350)$    | 2329              | −0.89362  | (2, 8)   |
| $a_6(2450)$       | 2466              | −0.89362  | (4, 4)   |
| $f_6(2510)$       | 2534.5            | 0.976096  | (1, 18)  |
| $K^*(892) ± 0.26$ | 890.5             | −0.16816  | (1, 6)   |
| $K_1(1270)$       | 1233              | 2.91338   | (2, 4)   |
| $K_1(1400)$       | 1370              | −2.14286  | (4, 2)   |
| $K^*(1410)$       | 1438.5            | 2.021277  | (1, 10)  |
| $K_0^*(1430)$     | 1438.5            | 0.594406  | (1, 10)  |
| $K_2^*(1430)$     | 1438.5            | 0.594406  | (1, 10)  |
| $K(1460)$         | 1438.5            | −1.4726   | (1, 10)  |
| $Pentaquark(1.5GeV)$ | 1.5               | 0         | (2, 5)   |
| $K_2(1580)$       | 1575.5            | −0.28481  | (1, 11)  |
| $K(1630)$         | 1644              | 0.858896  | (8, 1)   |
| $K_1(1650)$       | 1644              | −0.36364  | (8, 1)   |
| $K^*(1680)$       | 1717 ± 27         | (1.934524)0 | (1, 12) |
| $K_2(1770)$       | 1781              | (0.621469)0 | (2, 6)   |
| Particle and mass | Mass From Formula | Error % | (l, n) |
|-------------------|-------------------|---------|--------|
| $K_3^0(1780) \pm 7$ | 1781 | (0.05618)0 | (2, 6) |
| $K_2(1820) \pm 13$ | 1849.5 | 1.620879 | (1, 13) |
| $K(1830)$ | 1849.5 | 1.065574 | (1, 13) |
| $K_0^0(1950)$ | 1918 | -1.64103 | (4, 2) |
| $K^0(1980)$ | 1986.5 | 0.328283 | (1, 14) |
| $K^+_1(2045) \pm 9$ | 2055 | (0.488998)0 | (2, 7) |
| $K_2(2250)$ | 2260.5 | 0.466667 | (1, 16) |
| $K_3(2320)$ | 2329 | 0.387931 | (2, 8) |
| $K^*_2(2380)$ | 2397.5 | 0.735294 | (1, 17) |
| $K_4(2500)$ | 2466 | -1.36 | (4, 4) |
| $K(3100)$ | 3082.5 | -0.56452 | (1, 22) |
| $D^\pm(1869.3)$ | 1849.5 | -1.05922 | (1, 13) |
| $D_0^+(1968.5) \pm 0.6$ | 1986.5 | 0.914402 | (1, 14) |
| $D^*_0(2007)2006.7 \pm 0.4$ | 1986.5 | -1.02143 | (1, 14) |
| $D^\pm_1(2010) \pm 0.4$ | 1986.5 | -1.16915 | (1, 14) |
| $D_s(2317)2317.3 \pm 0.6$ | 2329 | 0.51791 | (2, 8) |
| $D_{1}(2420)2422.3 \pm 1.3$ | 2397.5 | -0.92975 | (1, 17) |
| $D^{*+}_1(2420)$ | 2397.5 | -0.97067 | (1, 17) |
| $D_2^+(2460)2461.1 \pm 1.6$ | 2466 | 0.243902 | (4, 4) |
| $D^*_2(2460)2459 \pm 4$ | 2466 | 0.243902 | (4, 4) |
| $D^{*+}_{S1}(2536)2535.35 \pm 0.34$ | 2534.5 | -0.07885 | (1, 18) |
| $D_{S1}(2573) \pm 1.5$ | 2534.5 | -1.09631 | (1, 18) |
| $B^\pm(5278)2579 \pm 0.5$ | 5274.5 | -0.05824 | (1, 38) |
| $B^0(5279.4) \pm 0.5$ | 5274.5 | -0.09281 | (1, 38) |
| $B_3(5732)$ | 5754 | -0.47009 | (4, 10) |
| $B^0_s(5369.6)5367.5 \pm 1.8$ | 5343 | -0.49538 | (2, 19) |
| $B^{*+}_s(5850)$ | 5822.5 | -0.47009 | (1, 42) |
| $B^0_c(6400)6286 \pm 5$ | 6370.5 | 0.4609 | (3, 15) |
| $\eta c(1S)(2979)2980.4 \pm 1.2$ | 2945.5 | -1.12454 | (1, 21) |
| $J/\psi(1S)(3096)3096.916 \pm 0.011$ | 3082.5 | -0.46402 | (1, 22) |
| $\chi c_0(1P)(3415.1)3414.76 \pm 0.35$ | 3425 | 0.289889 | (2, 12) |
| $\chi c_1(1P)(3510.5) \pm 0.07$ | 3493.5 | -0.48426 | (1, 25) |
| $\chi c_2(1P)(3556)3556.20 \pm 0.09$ | 3562 | 0.168729 | (4, 6) |
| Particle and mass | Mass From Formula | Error % | $(l, n)$ |
|-------------------|-------------------|---------|----------|
| $\psi(2S)(3685.9)$ | 3699              | 0.355408| (2, 13)  |
| $\psi(3770)3771.1 \pm 2.4$ | 3767.5 | $(-0.06631)0$ | (1, 27) |
| $\psi(3836)$        | 3876              | 0.13    | (3, 9)   |
| $\chi(3872)3871.2 \pm 0.5$ | 3944 | 0.38    | (2, 14)  |
| $\chi_{c2}(28)3929 \pm 5$ | 4041.5 | (0.037129)0 | (1, 29) |
| $\psi(4040)4039 \pm 1$ | 4178.5 | (0.444712)0 | (1, 30) |
| $\psi(4160)4153 \pm 3$ | 4452.5 | 0.84937 | (1, 32)  |
| $\psi(4415)4421 \pm 4$ | 9453 | $-0.07716$ | (2, 34) |
| $\gamma(1S)(9460.3) \pm 0.26$ | 9864 | 0.041583 | (16, 4) |
| $\chi_{b0}(1P)(9859.9)$ | 9864 | $-0.29011$ | (16, 4) |
| $\chi_{b1}(1P)(9892.7) \pm 0.6$ | 9864 | $-0.49029$ | (16, 4) |
| $\gamma(2S)(10023) \pm 0.00031$ | 10001 | 0.21949 | (2, 36) |
| $\chi_{b0}(2P)(10232) \pm 0.0006$ | 10275 | 0.42026 | (2, 37) |
| $\chi_{b1}(2P)(10255) \pm 0.0005$ | 10275 | 0.1945027 | (2, 37) |
| $\gamma(3S)(10355) \pm 0.0005$ | 10343.5 | 0.11105 | (1, 75) |
| $\gamma(4S)(10580)10579.4 \pm 1.2$ | 10549 | $-0.29301$ | (2, 38) |
| $\gamma(10860)10865 \pm 8$ | 10891.5 | 0.290055 | (3, 26) |
| $\gamma(11020)11019 \pm 8$ | 11028.5 | 0.077132 | (1, 80) |