Conceptual Study of Possibility for Droplets to Achieve Superheated in Edge Tokamak Plasmas

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Abstract. A fusion plasma is one of misty plasmas, because it always mixes with droplets produced from tokamak wall erosion and melting of dust by plasma heating. It is unavoidable for droplets to interact with edge transport barrier (ETB) during their flights. ETB is the transition zone between edge and core plasmas, where the ETB plasma is hot and dense. Thus, impurity dust and droplets should barely penetrate into core plasmas. It is clearly that vaporization is a mechanism responding to the circumstance. However, its detail is needed to be studied. The paper proposes the mechanisms to produce the heavy and localized impurity deposition in tokamaks through vaporization by superheating in ETB. The preliminary results suggests that droplets can achieve superheating when they travels in ETB towards core plasmas where de-pressurization exists along the path. In addition, there is a zero liquid pressure in ETB as a cutoff where liquid droplet cannot be existed. This leads to electrostatic breakups and strong vaporization, both of which ensure that dust and droplets hardly penetrate to core plasmas.

1. Introduction
Operating a tokamak in high confinement mode (H-mode), which is achieved by several auxiliary heating mechanisms, greatly improves ion and electron temperatures and number densities in core plasma. Meanwhile, they are drastically decreased in edge plasma, which mostly covers Scrape-Off-Layer (SOL). The steep gradients of those temperatures and densities are formed between core and edge plasmas with a very narrow width of a few centimetres. We name it as Edge Transport Barrier (ETB). For example, refs. [1]-[3] have reported of such plasma profiles. Superheating occurs at which an object temperature exceeds boiling temperature. Liquid rapidly becomes vapor. One way to initiate superheating is de-pressurization of surroundings. In addition, J. E. Shepherd and B. Sturtevant (1982) [4] reported of vapor explosion, induced by instability-driven surface oscillation.

In this work, we have been conducted a preliminary test on the assumption, proposed by N. Somboonkittichai and M. Coppins (2015) [5], that the steepness of edge transport barrier (ETB) should encourage superheating on impurity droplets as an additional tokamak dust and droplet destruction process, which produces localized impurity release next to tokamak core plasma.

2. Methodology
Our main aim is to investigate whether or not it is possible for superheating to be existed in tokamaks. In this section, we provide the background of methodology and physics we have used.
We call a plasma containing droplets as *misty plasma* [9]. Plasma particles, i.e. ions and electrons, directly heat and charge a droplet by deposition. They also press a droplet by physical momentum transfer. In other words, plasma exerts several pressures on the droplet. Liquid pressure \( P_{liq} \) as a net pressure on the droplet was determined by the use of Young-Laplace equation, i.e. a pressure balance equation at steady state. Boiling and evaporation take place where vapor pressure \( P_{vap} \) is equal to liquid pressure \( P_{liq} \), \( P_{liq} = P_{vap} \). Therefore, Clausius-Clapeyron equation was adopted to evaluate boiling temperature \( T_{boil} \) at a liquid pressure \( P_{liq} \). We also conducted an update on droplet temperature \( T_d \) by the use of simple plasma heating model combined with selected plasma parameters in near-SOL and near-core ETB and compared it with \( T_{boil} \). Simply, if \( T_d > T_{boil} \), superheating should happen. It needs to be noted that the material properties were taken from [6]-[8].

2.1. Modified Young-Laplace Equation
An ordinary Young-Laplace equation [10] explains a balance between inwards \( (P_{in}) \) and outwards \( (P_{out}) \) pressures on an equilibrium liquid surface, \( P_{in} = P_{out} \). In this study, \( P_{in} \) consists of \( P_{st} \) (pressure due to surface tension), \( \Gamma_i \) (ion pressure) and \( P_e \) (electron pressure), while \( P_{out} \) is \( P_{es} \) (electrostatic pressure). Hence, we obtained and adopted the modified Young-Laplace equation (MYLE) as \( P_{liq} = P_{st} + \Gamma_i + P_e - P_{es} \).

2.2. Pressures on Droplet
For a spherical droplet, \( P_{es} \), \( P_e \) and \( \Gamma_i \) are controlled by surface charges on a droplet surface. The equilibrium surface charges establish floating potential \( (\phi_{f,d}) \) on the droplet. To evaluate \( \phi_{f,d} \), we adopted the Orbital Motion Limited (OML) theory [11]. The OML floating potential can be approximated as \( \phi_{f,d} \approx -2.504 T_e [eV] \) for \( T_i = T_e \) or \( \beta = 1.0 \) [5], where \( \beta \) is the ratio of ion temperature \( (T_i) \) to electron temperature \( (T_e) \). We obtained and applied \( \phi_{f,d} \) from OML to evaluate \( P_{es} \), i.e. \( P_{es} = \frac{\epsilon t d e^2}{2 r_d^2} = \frac{3.135 \alpha r d^2 [eV]}{r_d^2} \) [12]. With regards to \( \Gamma_i \) and \( P_e \), M. Coppins (2010) [9] has provided

\[
\Gamma_i = \frac{1}{2} n_i e \beta T_e [eV] \left( 2 \left( \frac{2}{3} \Phi_{f,d} \right) \sqrt{\frac{\Phi_{f,d}}{\pi}} + \exp \left( \Phi_{f,d} \right) \operatorname{erfc} \left( \sqrt{\Phi_{f,d}} \right) \right),
\]

\[
P_e = \frac{1}{2} n_e e T_e [eV] \exp \left( -\Phi_{f,d} \right),
\]

where \( \Phi_{f,d} = \frac{\phi_{f,d}}{T_e [eV]} \) and \( \Phi_{f,d} = \frac{\phi_{f,d}}{T_i [eV]} \) is a net pressure on the droplet was determined by the use of Young-Laplace equation (MYLE) as \( P_{liq} = P_{st} + \Gamma_i + P_e - P_{es} \).

2.3. Simple Plasma Heating Model
In this study, the model included heating by incoming ion \( (\Delta_i) \) and electron \( (\Delta_e) \) fluxes on a droplet surface and cooling by thermal radiation flux, explained by Stefan-Boltzmann law, \( \Xi_{rad} = \epsilon \sigma' T_i^4 \), where \( \epsilon' \) is Stefan-Boltzmann constant and \( \epsilon \) is emissivity. Incoming ion \( (\Xi_i) \) and electron \( (\Xi_e) \) heat fluxes were determined by one-way Maxwellian heat flux [13], \( \Xi_i = 2 e T_i [eV] \Delta_i \) and \( \Xi_e = 2 e T_e [eV] \Delta_e \). We assumed a fusion plasma was thermally ionized, so that \( T_i \approx T_e \) and \( n_i = n_e = n_i' \). This is reasonable for a fusion plasma near separatrix, i.e. interface connecting between core and SOL plasmas. By the OML theory [11] and considering at steady state, \( \Delta_i = \Delta_e = n \sqrt{\frac{e T_i [eV]}{2 \pi m_e}} \exp \left( -\Phi_{f,d} \right) \). Finally, our heating model used to update \( T_d \) was

\[
T_{d,f} = T_{d,i} + \frac{3}{\rho c v_d} \left( 4 e T_e [eV] \Delta_e - \epsilon_d \sigma' T_{d,i}^4 + \epsilon_w \sigma' T_w^4 \right) \delta t,
\]

where \( T_{d,i} \) and \( T_{d,f} \) are initial and final droplet temperatures, \( \rho \) is a liquid density, \( c \) is a specific heat capacity, \( \delta t \) is a time-step for a droplet moving from initial to final position, \( T_w \) is an inner-wall temperature, i.e. \( T_w = 300 \) K, and \( \epsilon_d \) and \( \epsilon_w \) are emissivities of dust and wall materials.
2.4. Clausius-Clapeyron Equation

The Clausius-Clapeyron equation [14] represented the relationship between $P_{\text{vap}}$ and its associated $T_{\text{boil}}$. Kelvin correction term [15] modified the equation because a droplet surface is curved. Consequently, $T_{\text{boil}}$ was evaluated from the Clausius-Clapeyron-Kelvin (CCK) equation as 

$$ T_{\text{boil}} = \left( \frac{A+B}{T_{\text{ref}}-\ln\left( \frac{P_{\text{vap}}}{P_{\text{ref}}} \right)} \right), $$

where $A = \frac{2\sigma m}{k_BT_{\text{ref}}r_d^e}$, $B = \frac{H_{\text{vap}}}{R}$, $R = 8.314\text{JK}^{-1}\text{mol}^{-1}$, $H_{\text{vap}}$ is a latent heat of vaporization, $m$ is an atomic mass of a dust material, and $P_{\text{ref}}$ and $T_{\text{ref}}$ are reference $P_{\text{vap}}$ and $T_{\text{boil}}$, respectively.

3. Results and Discussions

As reported by N. Endstrasser et al. (2011) [16], the most population of collected spherical W dust in ASDEX-U is around 1 $\mu$m in size. In this study, we assumed 1 $\mu$m for the tungsten (W) droplet radius ($r_d$). To update droplet temperature ($T_d$) between near-SOL and near-core in ETB, we used eq. 3 with electron number density ($n_e$) and electron temperature ($T_e$) from the H-mode plasma profiles of JET [1] and ASDEX-U [2] in the study. For simplicity, we approximated that $n_e$ and $T_e$ linearly reduced from near-core to near-SOL in ETB along minor (radial) position in the poloidal cross-section of a tokamak. We assumed that a droplet was initially occupied melting temperature ($T_{\text{melt}} = 3687\text{K}$ for tungsten (W) [6]). Emissivities for tungsten (W) and beryllium (Be) are 0.35 [7] and 0.61 [8], respectively. W was for the test droplets and the ASDEX-U wall and Be was for the JET wall. It has been reported that a micron-size macroparticle can occupy the velocity of 100-m/s if it reaches separatrix [17]. The timescale for a droplet moving in ETB of tokamaks, e.g. JET and ASDEX-U, between near-core and near-SOL was approximately in an order of a few $10^{-4}$ s. In figures 1 and 2, the minor position of "0" refers to near-SOL ETB at the bottom side of the ETB profiles.

Figure 1 shows $P_{\text{liq}}$ (by MYLE), $P_{\text{es}}$, $P_{\text{st}}$, $\Gamma_i$ and $P_e$ on the W droplets in both JET (see figure 3 in ref. [1]) and ASDEX-U (see figure 4 in ref. [2]) H-mode profiles. $P_{\text{liq}}$ decreases when the droplet travels towards core plasma. Higher $n_e$ and $T_e$ in core plasma increase ion and electron fluxes, so that $P_{\text{es}}$ overcomes $P_{\text{st}} + \Gamma_i + P_e$. Although, in there $\Gamma_i$ and $P_e$ also increase, they are still several orders less than $P_{\text{st}}$ and $P_{\text{es}}$ on the 1-$\mu$m droplet. $P_{\text{st}}$ dominates in SOL but not in core plasma. As a result, $P_{\text{liq}}$ on the droplet greatly decreases in near-core ETB. By considering MYLE, $P_{\text{liq}}$ tends to be zero when $P_{\text{es}}$ dominates $P_{\text{st}}$, i.e. $P_{\text{es}} \geq P_{\text{st}}$ approximately. This happens on the way in ETB at the approximated distances of 0.013 m for JET and 0.022 m for ASDEX-U with respect to near-SOL, while the widths of ETB between near-SOL and near-core of JET and ASDEX-U are around 0.05 m [1] and 0.03 m [2]. We also anticipated that the droplets electrostatically breakup and vaporize, which was suggested as the process for localized dust destruction near core plasma [5, 9], at the distances beyond the position of zero-$P_{\text{liq}}$. $T_{\text{boil}}$ of the W droplets in the ETB of JET and ASDEX-U have been shown in figure 2. $T_{\text{boil}}$ decreases from SOL to core plasma. This corresponds to $P_{\text{liq}}$ decreasing from SOL to core plasma due to the reduction of $n_e$ and $T_e$, as shown in figure 1. This implies that evaporation should be enhanced in near-core ETB due to the lower $T_{\text{boil}}$. In other words, impurity should be difficult to survive in liquid phase because temperature range between melting and boiling is shorten in near-core ETB.

We conducted several trials by assuming that the droplets started to move from 0, 0.005 and 0.01 m towards core plasma, and from 0.013, 0.008 and 0.004 m towards SOL in JET and from 0, 0.01 and 0.015 m towards core plasma and from 0.022, 0.018 and 0.01 m towards SOL in ASDEX-U. Figure 2 illustrates that $T_d$ rapidly increases to $T_{\text{boil}}$. This should result from strong heating by the ETB plasmas. The plasma heating timescale ($t_{\text{ph}}$), estimated by eq. 3 for $r_d = 1 \mu$m and $\Delta T_d = T_{d,f} - T_{d,i} = 1\text{K}$, was approximately in an order between $10^{-6}$ s in near-SOL and $10^{-9}$ s in near-core. The timescale of droplet motion in ETB ($t_{\text{dm}}$) was approximately in an order of $10^{-5}$-$10^{-2}$ s for the droplet velocities of 10-1000 m/s and the ETB width of 0.01-0.1
Figure 1. Various pressures on a 1-μm tungsten (W) droplet. ($P_i$ refers to $\Gamma_i$ in this paper.)

Figure 2. Boiling temperature in SOL ($T_{bSOL}$), melting temperature ($T_m = T_{melt}$), and boiling temperature along minor radius ($T_b = T_{boil}$) of tungsten (W) and $T_d$ of the test droplets inwardly injected from near-SOL towards near-core ($T_{din1}$, $T_{din2}$ and $T_{din3}$) and outwardly injected from near-core towards near-SOL ($T_{dout1}$, $T_{dout2}$ and $T_{dout3}$) of JET and ASDEX-U ETB.

Thus, $t_{ph} \ll t_{dm}$ suggests that $T_d$ greatly rises with the short distance. When $T_d = T_{boil}$, the droplets are more or less in liquid-vapor equilibrium at local $P_{liq}$ for some time and undergo normal evaporation. The superheating response timescale ($t_{sh}$) was estimated by eq. 21 in A. J. Robinson et al. (2004) [18]. It was approximately in an order between $10^{-6}$ s in near-SOL and $10^{-9}$ s in near-core for $r_d = 1$ μm, and $t_{sh} \ll t_{dm}$. This implies that the droplets rapidly respond to $T_d > T_{boil}$, when the locations of the droplets change plasma parameters and $T_{boil}$. This should lead to strong evaporation by a series of droplet superheating if a droplet with $T_d \rightarrow T_{boil}$ travels towards core plasma, where $T_{boil}$ and $P_{liq}$ significantly decrease. The circumstance results in a rapid and intensive impurity deposition in a fusion plasma. If a droplet goes into the zero-$P_{liq}$ region, it is expected to completely breakup and vaporize by an electrostatic effect. However, a droplet with $T_d \rightarrow T_{boil}$ tends to adapt itself to the liquid-vapor equilibrium (along the CCK $T_{boil}$ curves in figure 2) if the droplet travels towards SOL, where $T_{boil}$ and $P_{liq}$ increase.

4. Conclusions

The study suggests that it is possible for the tokamak molten droplets to be superheated in edge transport barrier (ETB), where plasma temperature and number density greatly and steeply increase from those of SOL. Droplet liquid pressure reduces along minor radial position towards core plasma. Zero droplet liquid pressure is found and believed to produce electrostatic breakups and then strong vaporization. If the droplets travel towards core plasma and avoid the zero liquid pressure region, it is possible for them to vaporize through de-pressurized superheating in ETB. Consequently, heavy and fast evaporation should be unavoidable. Both mechanisms represent intensive and localized impurity productions in the edge of a tokamak plasma.
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