Horizontal, Anomalous $U(1)$ Symmetry for the More Minimal
Supersymmetric Standard Model

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(February 1997)

Abstract

We construct explicit examples with a horizontal, “anomalous” $U(1)$ gauge
group, which, in a supersymmetric extension of the standard model, reproduce
qualitative features of the fermion spectrum and CKM matrix, and suppress
FCNC and proton decay rates without the imposition of global symmetries.
We review the motivation for such “more” minimal supersymmetric standard
models and their predictions for the sparticle spectrum. There is a mass hier-
archy in the scalar sector which is the inverse of the fermion mass hierarchy.
We show in detail why $\Delta S = 2$ FCNC are greatly suppressed when compared
with naive estimates for nondegenerate squarks.
The minimal supersymmetric standard model (MSSM) with stop-induced electroweak symmetry breaking naturally stabilizes the electroweak scale, but fails to exhibit the accidental global symmetries of the standard model (SM) which inhibit flavor-changing neutral currents (FCNC), lepton flavor violation (LFV), electric dipole moments (EDMs) and proton decay. To agree with experimental bounds, such processes must be suppressed by imposing approximate global symmetries to produce degeneracy or alignment in the squark and slepton mass matrices. Recently, Cohen, Kaplan, and Nelson [1] have proposed an alternative framework in which the gauginos, higgsinos, and third generation squarks are sufficiently light to naturally stabilize the electroweak scale, while the first two generations of squarks and sleptons are sufficiently heavy to suppress FCNC, LFV, and EDMs below experimental bounds. Such models are “more” minimally supersymmetric than the MSSM in the sense that they do not require the ad hoc supposition of degeneracy or alignment. The anomalous $U(1)_X$ used by Dvali and Pomarol [2], and Binetruy and Dudas [3] to break supersymmetry (hereafter DPBD mechanism) could induce the required mass hierarchy, provided third generation matter carries no $X$-charge. In this letter, we show that an anomalous $U(1)_X$ can also explain the order of magnitude of fermion masses and mixing angles, and suppress proton decay from dimension-5 operators. Our criteria for assigning $U(1)_X$ charges differ significantly from those considered previously in this context [3][4].

The DPBD mechanism of supersymmetry breaking requires a gauged $U(1)_X$ with positively and negatively $X$-charged matter superfields $P$ and $N$. If $X$-symmetry is anomalous (i.e. $\text{tr}(X) \neq 0$) below some scale $M$, the effective theory below this scale includes a Fayet-Iliopoulos term $\sim g_X^2 \text{tr}X_M^2$ [3]. For mathematical consistency, one can either imagine that $M$ is the Planck scale and the anomaly is canceled via the Green-Schwarz mechanism [3], or that $M$ is some other scale at which anomaly-canceling matter lives. If the superpotential also contains a term of the form $mPN$, introduced either explicitly or dynamically by nonperturbative physics, the various fields obtain vacuum expectation values

$$\phi_N = N_1 = \epsilon M$$

(1)
\[ F_P = P_{\theta \theta} = \epsilon m M \]
\[ D_X = (V_X)_{\theta \theta \theta} = m^2 / g^2 \]

where \( \epsilon \) is a computable number, typically somewhat smaller than unity\(^1\).

The D-term vev gives each scalar matter field \( \phi \) a mass squared \( X_\phi m^2 \) proportional to its \( X \)-charge; thus \( m \) is a mass scale for \( X \)-charged scalar matter. Our phenomenological philosophy, following ref. \(^4\), is to set \( m \) high relative to a standard model physics scale and move the first and second generation squarks and sleptons to the scale \( m \) by giving their corresponding superfields positive \( X \)-charge. A straightforward 1-loop calculation will verify that the electroweak scale can be reproduced with less than 10% cancelation between bare higgs masses squared and 1-loop radiative corrections only if all particles which couple strongly to the higgses weigh in at less than \( \sim 1 \) TeV, the so-called \(^\sim \)’t Hooft bound”. Since top and left-handed bottom squarks couple strongly to higgses, naturalness of the electroweak scale requires them to remain lighter than \( \lesssim 1 \) TeV. We accomplish this by assigning the corresponding superfields zero \( X \)-charge. Since we do not wish to break any of the standard model gauge symmetries at a scale \( \sim m \), all chiral superfields charged under SM gauge groups must have nonnegative \( X \)-charge.

SM Yukawa couplings appear in the MSSM as terms in the superpotential of the form \( LRH \), where \( L \) and \( R \) are \( SU(2) \) doublet and singlet matter superfields, respectively, and \( H \) is the appropriate higgs superfield, either the up-type \( h_u \) or the down-type \( h_d \). Most such terms are forbidden by \( X \)-symmetry because \( LRH \) is not, in general, an \( X \)-singlet. However, nonrenormalizable operators, induced in the effective superpotential by physics at the scale \( M \), of the form
\[
\frac{(LRH)_{\theta \theta} (N^n)}{M^n} = \epsilon^n \Psi_L \Psi_R \phi_H
\]

\(^1\)If we consider the DPBD mechanism in the context of tree-level supergravity (SUGY) with \( M \) set equal to the Planck mass, the vevs obtained above are perturbed only slightly, \( \phi_P \) and consequently \( F_N \) get small vevs, and the gravitino mass \( m_{3/2} \sim \epsilon m \).
mimic SM Yukawa couplings because of the expectation value of $\phi_N$. Here $n$ is a sum of $X$-charges chosen so that $LRHN^n$ is an $X$-singlet. Assuming such operators are in fact induced with coefficients whose ratios are $O(1)$, we can, given a set of $X$-charge assignments for matter superfields, make order-of-magnitude estimates of SM mass matrix elements

$$\begin{align*}
\lambda^u_{ij} &\propto \epsilon^{q_i + \bar{u}_j + h_u} \tan \beta \\
\lambda^d_{ij} &\propto \epsilon^{q_i + \bar{d}_j + h_d} \\
\lambda^l_{ij} &\propto \epsilon^{\ell_i + \bar{e}_j + h_d}
\end{align*}$$

(5)

where we have used symbols for the fields to indicate their respective $X$-charges, and

$$\tan \beta \equiv \langle \phi_{h_u} \rangle / \langle \phi_{h_d} \rangle .$$

(6)

Perturbative diagonalization yields fermion masses and intergenerational CKM matrix elements

$$\begin{align*}
m^u_i &\propto \epsilon^{q_i + \bar{u}_i + h_u} \tan \beta \\
m^d_i &\propto \epsilon^{q_i + \bar{d}_i + h_d} \\
m^e_i &\propto \epsilon^{\ell_i + \bar{e}_i + h_d} \\
V_{ij} &\sim \epsilon^{|q_i - q_j|}.
\end{align*}$$

(7)

Such mass matrix models were introduced by Froggatt and Nielsen [7] and have since been explored by many authors [8]. The existence of such a $U(1)_X$ horizontal symmetry underlying SM mass matrices makes the prediction for intergenerational CKM matrix elements

$$V_{12}V_{23} \sim V_{13}.$$ 

(8)

Since $V_{12} \sim \epsilon$, $V_{23} \sim \epsilon^2$, and $V_{13} \sim \epsilon^3$, for $\epsilon \sim 1/5$, this relation is empirically confirmed. In order that the top quark Yukawa coupling be unsuppressed, we must have $q_3 = \bar{u}_3 = h_u = 0$. For $\epsilon \sim 1/5$, the CKM matrix elements are then best explained by $q_1 = 3$, $q_2 = 2$.

Squark mass matrix entries are also induced by nonrenormalizable operators generated by physics at the scale $M$. In particular, the operator

$$\frac{(A^* B)_1 (N^n)_1 (P^* P)_{\theta \bar{\theta} \theta \bar{\theta}}}{M^{2+n}} = \epsilon^{2+n} m^2 \phi_A^* \phi_B^*$$

(9)

with $n = X_B - X_A$ ( or $n = X_A - X_B$ with $N$ replaced by $N^*$ ), mimics a scalar mass matrix entry. Note that such operators give masses of order $\epsilon m$ even to scalars with vanishing $X$-charge. Since naturalness requires $\epsilon m \lesssim 1$ TeV, we obtain the constraint $m \lesssim 5$ TeV.
Note also that $SU(2)$-invariance requires that $A$ and $B$ in eq. (9) be either both left-handed doublet superfields or both right-handed singlet superfields; such operators induce no mixing between left- and right-handed squarks. Although left-right squark mixing exists in the models we consider, it is heavily suppressed. Neglecting left-right mixing, we write four uncoupled squark mass matrices.

\begin{align}
\lambda_{ij}^{uL,dL} &\propto \epsilon^{2+|q_i-q_j|} + X_{q_i} \delta_{ij} \\
\lambda_{ij}^{uR} &\propto \epsilon^{2+|\bar{u}_i-\bar{u}_j|} + X_{u_i} \delta_{ij} \\
\lambda_{ij}^{dR} &\propto \epsilon^{2+|\bar{d}_i-\bar{d}_j|} + X_{d_i} \delta_{ij}
\end{align} 

The second term in each expression accounts for the $D$-term contribution to masses, discussed above.

We have already argued that the up-type higgs $h_u$ must not carry $X$-charge. If the down-type higgs $h_d$ carries charge $X$, a scalar $h_u h_d$ mass mixing or $\mu B$ term is induced by the operator

\[
\frac{(h_u h_d)_1 (N^X)_1 (P^* P)_{\theta\theta\bar{\theta}\bar{\theta}}}{M^{2+X}} = \epsilon^{2+X} m^2 \phi_u \phi_d .
\]

Furthermore, by eq. (9), the diagonal higgs’ mass squared $m_{h_u}^2 \sim \epsilon^2 m^2$ and $m_{h_d}^2 \sim (X + \epsilon^2) m^2$. These results can be used in conjunction with the approximate relation

\[
\tan \beta \sim \frac{m_{h_u}^2 + m_{h_d}^2}{\mu B}
\]

which is valid for $\tan \beta \gtrsim 1$, to show that

\[
\tan \beta \sim 1 \quad \text{for } X = 0 \quad \tan \beta \sim \frac{1}{\epsilon^{2+X}} \quad \text{for } X > 0 .
\]

When neither higgs carries $X$-charge, $\tan \beta \sim 1$, and the small values of $m_b/m_t$ and $m_\tau/m_\ell$ must be explained by assigning nonzero $X$-charge to $\bar{d}_3$ and to $\ell_3$ or $\bar{\ell}_3$. Although the corresponding scalars will then acquire masses of order $m$, a heavy right-handed bottom squark and heavy third generation sleptons do not upset the naturalness of the electroweak scale, because these particles couple only weakly to higgses. When $h_d$ carries $X$-charge, $\tan \beta$
is large and the low-energy effective theory below the scale \( m \) contains only a single higgs scalar. Since the couplings of down-type quarks to \( h_d \) are suppressed by at least \( X \) factors of \( \epsilon \), the bottom to top mass ratio at short distances is at most \( \epsilon^{2+2X} \) in such scenarios. Since \( m_b/m_t \sim \epsilon^3 \) is the smallest ratio consistent with experiment, we are forced to reject models with any \( X \)-charge in the higgs sector.

Gaugino mass terms also arise as nonrenormalizable operators. In particular, terms of the form

\[
\frac{(W_\mu W^\mu)_1 (N)_1 (P)_{\theta\bar{\theta}}}{M^2} = \epsilon^2 m \Psi_\lambda \Psi_\lambda \tag{16}
\]

induce gaugino masses of order \( \epsilon^2 m \sim 200 \text{ GeV} \). A higgsino mass term of the same order is induced by the operator

\[
\frac{(h_u h_d)_{\theta\bar{\theta}} (N^*)_1 (P^*)_\theta\bar{\theta}}{M^2} = \epsilon^2 m \Psi_{h_u} \Psi_{h_d}. \tag{17}
\]

Gauginos and higgsinos are the lightest sparticles in these models.

We should note that there are numerous superpotential terms allowed by all symmetry considerations which we nonetheless reject because they give \( M \)-scale masses to MSSM fields. Such “dangerous” terms include, for example, \( h_u h_d P N/M \), which gives \( \mu B \sim \epsilon^2 mM \). The possibility of dangerous superpotential terms is a familiar problem for supersymmetric

\[\text{\textsuperscript{2}}\]

The authors of \[\text{\textsuperscript{10}}\] argued that \( \tan \beta \) could never be naturally large in a model with only two higgs doublets. Our conclusions here do not contradict their result, because the presence of multiple scales in the soft supersymmetry breaking terms violates one of their assumptions.

\[\text{\textsuperscript{3}}\]

Supergravity gives a contribution to \( \mu B \sim \epsilon^2 m^2 \) when \( M \) is the Planck scale and \( X = 1 \). This implies \( \tan \beta \sim 1/\epsilon^2 \), allowing us to obtain \( m_b/m_t \sim m_\tau/m_t \sim \epsilon^3 \), provided no third generation superfield carries \( X \)-charge. Although such models are interesting because of this constraint and because the corresponding low-energy effective theories have only a single higgs, we have been unable to find a set of \( X \)-charge assignments of this type which suppresses FCNC sufficiently to be phenomenologically viable.
models containing fields with vevs at high scales. Here we merely reiterate that, although we
cannot forbid such terms on symmetry grounds, the nonrenormalization theorem guarantees
that they will not be generated if absent initially. Setting dangerous terms equal to zero is
thus at least technically natural. There are no dangerous Kähler potential terms, which is
fortunate, because the Kähler potential is not protected by any nonrenormalization theorem.

A strong constraint on the values of $X$-charges can be obtained by noting \[11\] that
integrating out matter above the scale $m$ to produce a low-energy effective theory introduces
an effective Fayet-Iliopoulos term for hypercharge

$$
\frac{g^2 m^2}{(4\pi)^2} \left[ \text{tr}(XY) \ln \left( \frac{M^2}{m^2} \right) - \text{tr}(XY \ln X) \right]
$$

which can lead to disastrous color and electric charge breaking minima of the scalar potential. Because of the large log in the first term\[^4\] we are forced to require

$$
\text{tr}(XY) \sim 0 .
$$

Since this trace requirement involves all matter superfields in the theory, it prevents us from
considering quarks and leptons separately. It can be accommodated nicely if we assign $X$-
charges to $SU(5)$ multiplets, i.e. $\ell_i = \bar{d}_i$ and $q_i = \bar{u}_i = \bar{e}_i$. Unfortunately, such an assignment predicts

$$
\frac{m_e}{m_\mu} \sim \frac{m_d}{m_s} \quad \frac{m_\mu}{m_\tau} \sim \frac{m_s}{m_b}
$$

the first of which is off by an order of magnitude. This problem, which plagues all $SU(5)$-
respecting models, has been addressed by Georgi and Jarlskog \[12\], who showed how $SU(5)$
group theory factors can give the successful prediction

$$
\frac{m_e}{m_\mu} \sim \frac{m_d}{9m_s}.
$$

\[^4\]The calculation of the first term in this expression can and should be improved by using the
renormalization group, however its magnitude and our conclusions are not affected.
If the mixed standard model–$U(1)_X$ anomalies are cancelled entirely by the Green–Schwarz mechanism, and if $\sin^2 \theta_W = 3/8$ at short distances, then the $X$-charges must satisfy the constraints (hereafter referred to as the GS constraints)

$$\text{tr} (X T_a T_b) \propto \text{tr} (T_a T_b) \quad \text{tr} (X^2 Y) = 0 \quad (22)$$

where $T_a$ are generators of SM gauge group transformations. These are automatically satisfied if $X$-charge assignments respect $SU(5)$ symmetry, and we do not give any examples which satisfy the GS constraints which are not consistent with $SU(5)$. Since additional, superheavy matter fields (which might, for example, get mass from the vev of $\phi_N$) could also contribute to the mixed anomalies, we will not impose the GS constraints.

In table 5, we give the best examples of $X$-charge assignments which approximately reproduce all CKM matrix elements and known fermion mass ratios, and which satisfy the hypercharge trace constraint eq. (19). We also give an example which is consistent with $SU(5)$ symmetry, but in which several fermion mass ratios are off by factors of up to ten. For all models, we have assumed $\epsilon \sim 1/5$, $h_u = h_d = 0$, and $\tan \beta \sim 1$. We mark the model consistent with $SU(5)$ symmetry with an asterix.

We will now proceed to derive the implications of our charge assignments for FCNC and proton decay rates.

Given the quark and squark mass matrices derived above, we can determine the intergenerational mixing matrix elements which appear at quark-squark-gluino vertices; these are relevant to the computation of the supersymmetric contributions to FCNC amplitudes. If the similarity transformation $V^\dagger_L \lambda^{qL} V_R$ diagonalizes a quark mass matrix, then the left- and right-handed squark mass matrices in the basis of quark mass diagonalizing superfields are

$$\bar{\lambda}^{\tilde{q}L} = V^\dagger_L \bar{\lambda}^{\tilde{q}L} V^\dagger_L \quad \bar{\lambda}^{\tilde{q}R} = V^\dagger_R \bar{\lambda}^{\tilde{q}R} V^\dagger_R \quad (23)$$

A short algebraic exercise will show that, provided the $i$th and $j$th generation squarks are degenerate, i.e. carry the same $X$-charge, the order-of-magnitude of the off-diagonal element $\lambda^{\tilde{q}_i \tilde{q}_j}$ is unaffected by this transformation. If, on the other hand, the squarks are nondegenerate, the the order of magnitude of an off-diagonal element is
\[ \bar{\lambda}^i_j = \max \left( \lambda^q_i, V_{ij} \right). \]

The quark-squark-gluino mixing matrices are just the matrices \( Z \) which diagonalize \( Z^\dagger \bar{\lambda}^j_i Z \).

The \( \epsilon \)-dependence of their entries can be determined by perturbative diagonalization.

For example, in all our models, \( \lambda^{dl}_d \) and \( V^d_L \) have the form

\[
\lambda^{dl}_d = \begin{pmatrix}
3 & \epsilon^3 & \epsilon^5 \\
\epsilon^3 & 2 & \epsilon^4 \\
\epsilon^5 & \epsilon^4 & \epsilon^2
\end{pmatrix}, \quad V^d_L = \begin{pmatrix}
1 & \epsilon & \epsilon^3 \\
\epsilon & 1 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & 1
\end{pmatrix}.
\] (25)

This implies

\[
\bar{\lambda}^{dl}_d = (V^d_L)^\dagger \lambda^{dl}_d V^d_L = \begin{pmatrix}
3 & \epsilon & \epsilon^3 \\
\epsilon & 2 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon^2
\end{pmatrix}.
\] (26)

which is diagonalized by the left-handed down-type quark-squark-gluino mixing matrix

\[
Z^{dl}_L = \begin{pmatrix}
1 & \epsilon & \epsilon^3 \\
\epsilon & 1 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & 1
\end{pmatrix}.
\] (27)

Although equation (27) only indicates the leading order scaling with \( \epsilon \) of each matrix element, we can actually derive additional constraints on the matrix. For instance, since unitarity requires the first and second columns to be orthogonal and \( Z^*_{31} Z_{32} \) contributes to the inner product of the columns only at \( \mathcal{O}(\epsilon^5) \), we have

\[
Z^{dl*}_{21} = -Z^{dl}_{12} + \mathcal{O}(\epsilon^3)
\] (28)

Such constraints from unitarity will prove crucial in our calculations of squark contributions to FCNC, to which we now proceed.

Several groups \([13,14]\) have calculated the squark-glùino box contributions to FCNC for arbitrary squark masses and quark-squark-gluino mixing matrices. In particular, their contribution to the \( K_L - K_S \) mass difference \( \Delta m_K \), whose measured value places stringent limits on any FCNC beyond the SM, is
\[ \frac{\Delta m_K}{m_K} = \alpha_S^2(m) f_K \left[ \frac{11}{54} \frac{Z_{i1}^d Z_{i2}^d Z_{i1}^{dR} Z_{i2}^{dR}}{m_{Li}^2 - m_{Lj}^2} \ln \left( \frac{m_{Li}^2}{m_{Lj}^2} \right) + \frac{11}{54} \frac{Z_{i1}^{dR} Z_{i2}^{dR} Z_{i1}^{dR} Z_{i2}^{dR}}{m_{Ri}^2 - m_{Rj}^2} \ln \left( \frac{m_{Ri}^2}{m_{Rj}^2} \right) + \left( \frac{1}{9} - \frac{2}{27} R \right) \frac{Z_{i1}^{dL} Z_{i2}^{dL} Z_{i1}^{dR} Z_{i2}^{dR}}{m_{Li}^2 - m_{Rj}^2} \ln \left( \frac{m_{Li}^2}{m_{Rj}^2} \right) \right] \]

where

\[ R = \left( \frac{m_K}{m_s + m_d} \right)^2 \sim 10. \] (30)

This expression relies on vacuum insertion and PCAC to obtain matrix elements of quark operators between \( K^0 \) and \( \bar{K}^0 \), and the approximation that \( m_{\tilde{g}} \ll m_{\tilde{q}} \). Corrections to this expression from nonzero gaugino masses are less than 10%. Although in principle left-right squark mixing exists and contributes to FCNC, not only are these mixing angles themselves smaller than those amongst left- and right-handed squarks separately, but the contributions of these mixings to \( \Delta m_K \) are also down relative to those included above by \((m_{\tilde{g}}/m_{\tilde{q}})^2\). The full expression for \( \Delta m_K \) with arbitrary gluino mass and squark mixing is relegated to an appendix. For each set of charge assignments, we have computed the minimum value of \( m \) which will just suppress \( \Delta m_K \) to its observed size, and listed the result in table 5.

We illustrate the calculation of \( m_{\text{min}} \) by computing the contribution of the first and second generation left-handed down squarks to expression (29). Using the unitarity constraint relating \( Z_{12}^{dL} \) to \( Z_{21}^{dL} \) derived earlier, we obtain a result to leading order in \( \epsilon \) proportional to

\[ (Z_{21}^{dL})^2 \left[ 1 \frac{X_1}{X_1} + 1 \frac{X_2}{X_2} - \frac{2}{X_1 - X_2} \ln \left( \frac{X_1}{X_2} \right) \right] \] (31)

where \( X_1 \) and \( X_2 \) are the \( X \)-charges of the first and second generation left-handed down-squarks. For \( X_1 = X_2 \), the factor in brackets vanishes, illustrating the squark degeneracy mechanism of FCNC suppression. For our nondegenerate \( X \)-charge assignments, the expression reduces to

\[ \epsilon^2 \left[ 1 \frac{3}{3} + 1 \frac{3}{2} - 2 \ln \left( \frac{3}{2} \right) \right] \sim 0.022 \epsilon^2. \] (32)
This is much smaller than the $\sim \epsilon^2$ value one might have expected for “generically” nondegenerate squark masses.

Similar arguments, combining relations among matrix elements imposed by unitarity with the explicit formula (29), can be applied to the remaining terms. We find that the X-charge assignments chosen to reproduce the observed quark and lepton mass ratios provide significantly more suppression of FCNC than naive estimates for generically nondegenerate squarks would suggest. For charge sets A and C, the dominant supersymmetric contribution to FCNC, and thus the strongest constraint on $m_{\text{min}}$, comes from box graphs containing a left-handed first or second generation squark and a right-handed third generation squark. For sets B and D, graphs with both left and right-handed first and second generation squarks dominate.

In all cases $m_{\text{min}}$ is low enough that the predicted masses of sparticles coupled strongly to the higgs satisfy the ’t Hooft bound ($\epsilon m_{\text{min}} \lesssim 1$ TeV) arising from the assumption of 1-loop naturalness of the electroweak scale.

CP violation could still produce a complex phase for $\Delta m_K$ which would give unacceptably large $\epsilon_K$. For generic phases of $O(\pi/2)$, suppressing the contribution to $\epsilon_K$ to the observed level would require $m_{\text{min}}$ values roughly an order of magnitude larger. Such a large values would give third generation sparticle masses which imply a fine-tuning of the electroweak scale to better than one part in $10^3$, in gross violation of the ’t Hooft bound. Instead, we will simply assume imaginary contributions to $\Delta m_K$ to be small. Although we have no explanation for the near-reality of the supersymmetry breaking terms within the context of our model, a more detailed model incorporating spontaneous CP violation could surely be found which would suppress $\epsilon_K$ (see, for example, ref. [15]). Imposition of CP symmetry at short distances is theoretically attractive, since CP is a gauge symmetry in certain theories with extra dimensions at the Planck scale, such as string theory [16].

Another potential problem arises when we consider the requirement of electroweak scale naturalness at two loops, where there is a contribution to the higgs mass squared proportional to $m^2$ which is enhanced by a large logarithmic factor, $\ln(M^2/m^2)$. Dimopoulos and
Giudice computed the effects of heavy squark and slepton masses on the stability of the electroweak scale using the 2-loop renormalization group equations, and concluded that requiring that contributions to the higgs mass squared should not have to cancel to better than 10% implies that all squarks and sleptons are lighter than $\sim 2$–5 TeV (DG bound). All of our models, with $m$ set to $m_{\text{min}}$, are in mild violation of this requirement. Note, however, that, were we to allow the same 10% fine tuning amongst the squark contributions to $\Delta m_K$ that we have already allowed in the higgs sector, each $m_{\text{min}}$ would be about a factor of three lower and all the models would satisfy the 2-loop constraint. Alternatively, the DG bound could be relaxed by lowering the scale $M$ of $U(1)_X$ breaking, which would reduce, in turn, the size of the large logarithms which enhance loop corrections.

The possible appearance in the effective superpotential of dimension-5 operators of the form $qqql$ and $\bar{u}\bar{d}\bar{e}$, which generically induce proton decay at a rate far above the experimental limits even when suppressed by the Planck scale, has long posed a problem for supersymmetric extensions of the standard model. The existence of $X$-charge suppresses such operators by allowing only higher-dimension operators of the form

$$\frac{N^n(qqq\ell, \bar{u}\bar{d}\bar{e})}{M^{n+1}} \sim \frac{e^n}{M}(qqql, \bar{u}\bar{d}\bar{e})$$

where $n$ is chosen to form a $U(1)_X$ gauge-invariant operator. The most severe constraint on our models comes from the operator $q_1q_1q_2\ell_3$ which causes a proton to decay into a kaon and a tau neutrino. A simple one-loop calculation for $m_p \ll m_{\tilde{g}} \ll m_{\tilde{q}}$ and naive dimensional analysis show that the rate is

$$\Gamma_p \sim \frac{\alpha_5^2(m_{\tilde{g}}) m_{\tilde{g}}^2 m_p^2 e^{2n}}{4 (4\pi)^5 m_{\tilde{g}}^4 M^2} \sim \alpha_5^2(m) \frac{\epsilon^{4+2n}}{4096\pi^5} \frac{m_p^5}{M^2 m_{\tilde{q}}^4}.$$  

Note that this expression for $\Gamma_p$ is depressed from the result for the MSSM with a single SUSY scale not only by several powers of $\epsilon$, but also by the factor $(m_{\tilde{g}}/m_{\tilde{q}})^2$. This additional suppression arises from a gluino mass insertion in the relevant diagram. Experimental limits on the proton lifetime require $n > 5$ for $m \sim 5$ TeV and $M$ set equal to the Planck mass. All $X$-charge assignments under consideration give a proton lifetime of greater than $10^{40}$ years.
with \( m = m_{\text{min}} \) and a Planck-scale \( M \), and are consistent with proton lifetime limits for any \( M \gtrsim 10^{15} \) GeV. Dimension-6 proton-decay inducing operators will be similarly suppressed, but these are not a phenomenological problem for \( M \) set to the Planck scale. Finally, we should note that the \( U(1)_X \) model does not alleviate the problem of \( B \) and \( L \) violating dimension-4 operators, which must still be forbidden by imposing a symmetry such as \( R \)-parity. Our philosophy has been to avoid the imposition of global symmetries; \( R \)-parity, however, could easily arise automatically as a consequence of a spontaneously broken \( B-L \) gauge symmetry \([18]\).}

The horizontal, anomalous \( U(1) \) gauge group models presented here have many unusual and attractive features. The charge assignments considered reproduce the observed fermion mass hierarchy and CKM mixing matrix elements. These same charge assignments predict a most unusual pattern of superpartner masses. Squarks and sleptons are highly nondegenerate, with a mass hierarchy that is the mirror-image of the fermion mass hierarchy: generically light third generation and progressively heavier second and first generation sparticles. In particular, top and left-handed bottom squarks are predicted to weigh in between 500 GeV – 1 TeV. For some charge assignments, some other third generation sparticles are also found at this scale. In other cases, they are found at the higher mass scale (2–10 TeV) of the first and second generation superpartners. Gauginos and higgsinos are predicted to be the lightest (100–200 GeV) superpartners. We have shown that, despite their nondegeneracy, the sparticles make acceptably small contributions to FCNC. Also, dimension-5 proton decay amplitudes are suppressed sufficiently to satisfy experimental bounds. The low masses of third generation sparticles which couple strongly to the higgs allow the electroweak symmetry-breaking scale to be reproduced with only mild fine-tuning. On the downside, our models do not explain the smallness of observed CP violation or the absence of certain dangerous terms generically allowed in the superpotential.

Aside from these few but noteworthy unresolved mysteries, we have successfully used a horizontal, anomalous, broken \( U(1) \) gauge symmetry to construct several phenomenologically acceptable “more” minimally supersymmetric extensions of the standard model. Such
models offer the tantalizing prospect of allowing us to one day determine Froggatt-Nielsen charges experimentally by simply measuring squark and slepton mass ratios. From a theoretical perspective, perhaps the most attractive feature of these models is their unification of the supersymmetry-breaking and flavor physics sectors into a single sector with an uncomplicated gauge group and small number of additional matter superfields.

Acknowledgements

After this work was completed, a complementary study, which considered the case of a $U(1) \times U(1)$ horizontal gauge group [19], appeared. One of the authors (D.W.) would like to thank Francois Lepeintre for helpful discussions. This work was supported by the U.S. Department of Energy, grant DE-FG03-96ER40956.

I. APPENDIX

Previous studies have given formulae for SUSY contributions to FCNC amplitudes only in the limit of squark near-degeneracy [14] or without explicitly evaluating various integrals which appear in the general case [13]. We therefore present, for future reference, a formula for the squark-gluino box contribution to the $K^0 - \bar{K}^0$ mixing amplitude valid for arbitrary masses and mixing angles.

$$
\langle K^0 | H | \bar{K}^0 \rangle = \alpha_s^2 m_K f_K^2 \times
$$

$$
\left\{ M_1 \left( \frac{11}{36} A_{i,j} + \frac{1}{9} B_{i,j} \right) Z_{1L,i}^* Z_{2L,i} Z_{1L,j}^* Z_{2L,j} + \right.
$$

$$
\left. M_1 \left( \frac{11}{36} A_{i,j} + \frac{1}{9} B_{i,j} \right) Z_{1R,i}^* Z_{2R,i} Z_{1R,j}^* Z_{2R,j} \right\}.
$$

The first three terms of this formula agree with the analogous terms of formula (II.1) of ref. [13], provided the coefficient $\frac{1}{36}$ in their formula is changed to $\frac{11}{36}$, a replacement also necessary for consistency with formulas appearing later in their article. The remaining three terms are in substantial disagreement with ref. [13]. All terms agree, in the appropriate limit, with the calculation of ref. [14], and are supported by an independent calculation [20].
\[
\left( \frac{5}{9}M_5 - \frac{1}{3}M_4 \right) A_{i,j} + \left( \frac{7}{3}M_4 + \frac{1}{9}M_5 \right) B_{i,j} \right] Z_{1L,i}^* Z_{2L,i}^* Z_{1R,j}^* Z_{2R,j}^* + \\
\left( \frac{17}{18}M_2 - \frac{1}{6}M_3 \right) B_{i,j} Z_{1L,i}^* Z_{2R,i}^* Z_{1L,j}^* Z_{2R,j}^* + \\
\left( \frac{17}{18}M_2 - \frac{1}{6}M_3 \right) B_{i,j} Z_{1R,i}^* Z_{2L,i}^* Z_{1R,j}^* Z_{2L,j}^* - \\
\left( \frac{11}{18}M_4 + \frac{5}{6}M_5 \right) A_{i,j} Z_{1L,i}^* Z_{2R,i}^* Z_{1R,j}^* Z_{2L,j}^* \right) \\
\] (36)

Here \( Z \) is the down-type quark-squark-gluino mixing matrix; the first and second indices run over left- and right-handed down-type quarks and squarks, respectively. The \( M \)'s are matrix elements of four-quark operators between \( K^0 \) and \( \bar{K}^0 \) states; their values from PCAC and vacuum insertion are listed for reference.

\[
M_1 = \langle K^0 | \bar{d}_L^\gamma \gamma \mu s_L^\alpha \bar{d}_L^\gamma \mu s_L^\beta | \bar{K}^0 \rangle = \frac{2}{3} \\
M_2 = \langle K^0 | \bar{d}_R^\alpha \gamma s_L^\alpha \bar{d}_R^\beta \gamma s_L^\beta | \bar{K}^0 \rangle = -\frac{5}{24} R \\
M_3 = \langle K^0 | \bar{d}_R^\alpha \gamma s_L^\alpha \bar{d}_R^\beta \gamma s_R^\beta | \bar{K}^0 \rangle = \frac{1}{12} R \\
M_4 = \langle K^0 | \bar{d}_R^\alpha \gamma s_L^\alpha \bar{d}_L^\beta \gamma s_R^\beta | \bar{K}^0 \rangle = \frac{1}{12} + \frac{1}{2} R \\
M_5 = \langle K^0 | \bar{d}_R^\alpha \gamma s_L^\alpha \bar{d}_L^\beta \gamma s_L^\beta | \bar{K}^0 \rangle = \frac{1}{4} + \frac{1}{6} R \\
\] (37) (38) (39) (40) (41)

Expressions for \( \tilde{M} \)'s are obtained from those for the corresponding \( M \)'s by the exchange \( L \leftrightarrow R \). The ratio \( R \) is defined by eq. [30]. The functions \( A_{i,j} \) and \( B_{i,j} \) depend on the masses of the \( i \)th and \( j \)th down-type squark and the gluino mass and have the explicit form

\[
A_{i,j} = \frac{m_g^2}{(m_i^2 - m_g^2)(m_j^2 - m_g^2)} + \frac{m_i^4}{(m_i^2 - m_j^2)(m_i^2 - m_j^2)} \ln \left( \frac{m_i^2}{m_j^2} \right) + \\
\frac{m_i^4}{(m_j^2 - m_i^2)(m_j^2 - m_i^2)} \ln \left( \frac{m_j^2}{m_i^2} \right) \\
B_{i,j} = \frac{m_g^2}{(m_i^2 - m_j^2)(m_j^2 - m_i^2)} + \frac{m_i^4 m_g^2}{(m_i^2 - m_j^2)(m_i^2 - m_j^2)} \ln \left( \frac{m_i^2}{m_j^2} \right) + \\
\frac{m_i^2 m_g^2}{(m_j^2 - m_i^2)(m_j^2 - m_i^2)} \ln \left( \frac{m_i^2}{m_j^2} \right) \\
\] (42) (43)

Equation [29] is obtained by ignoring mixing between left- and right-handed squarks, assuming PCAC and vacuum insertion values for the relevant matrix elements, and evaluating \( A \) and
$B$ in the $m_3 \to 0$ limit. Formula (4) of ref. [14] for SUSY contributions to FCNC in the limit of nearly degenerate squarks may be obtained by setting $m_i^2 = m_j^2 + \delta m_{ij}^2$ and expanding $A$ and $B$ up to terms quadratic in $\delta m_{ij}^2$. 
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TABLE I. X-charge assignments for left chiral superfields. The column $m_{\text{min}}$ gives the minimum value for $m$ consistent with FCNC. All models are also consistent with limits on proton decay. Set D is consistent with SU(5) symmetry and with the GS constraints.