The Path Integral of Feynman and "Information Modelling" of Processes and Systems

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ABSTRACT

The given article example of physical analogies to be entered information space-time. The opportunity of Poincare group use is shown for transition from one frame in another, for this purpose is entered invariant velocity of transition of the information. For calculation of information processes probability amplitudes is offered using path integral of Feynman.

1. INTRODUCTION

The prominent aspect of studying of complex systems is the opportunity of the account of all processes and the phenomena occurring in system. The present time most studied are physical processes. However even at studying physical processes there is a number of problems for the decision which it is necessary to use various models simultaneously. The examples of such problems are descriptions of systems where thermodynamic and electromagnetic processes simultaneously proceed. The one of ways of modelling of such systems is the description of these systems in "information language" i.e. when parameters of system, conditions of system are compared to units of measure of the information, with possible values of the information and with mechanisms of perception of the information. Such modelling allows to abstract from consideration of concrete processes, but it is correct to consider them in the general behaviour of studied system.

2. THE "INFORMATION MODELLING" OF PROCESSES AND SYSTEMS

The given section of article is considered the problem so-called "information modelling of system" for real systems, from example physical systems. The given method of without generality restriction can be applied for chemical, biological and social systems. The as a rule, the consideration and description of not isolated physical systems is reduced consideration set the physical system and the physical system environment. This is allows considering physical system as subsystem of environment. This approach is considering problem about interaction of two subsystems: physical systems on the one hand and environments on the other hand. Let's consider some dynamic physical system of not isolated from an environment. Let this subsystem is described by some parameters set:

\[ \alpha = \{ \alpha_1, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n \} , \] (1)

here following designations are entered: the \( \alpha \) – is designations of set parameters of consideration physical system, the \( \alpha_i \) – is certain parameters of set parameters, the \( n \) is general number of parameters for consideration physical system. For example, as such by parameter in Eq. of (1) can be chosen total moment of system, orbital moment of system, spin, electric charge, magnetic moment, etc. The parameters set are allowed unequivocally describing physical system. These parameters is possible solve for functions of physical system condition, for example, total energy of system, entropy of system, electric field intensity, magnetic field intensity, etc. Thus are receiving described set for the systems condition defined through set parameters of system:

\[ \varphi (\alpha) = \{ \varphi_1 (\alpha), \varphi_2 (\alpha), \ldots, \varphi_{m-1} (\alpha), \varphi_m (\alpha) \} , \] (2)

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here following designations are entered: the $\varphi$ – is designations of set system condition of consideration physical systems, the $\varphi_j$ – is certain system condition of set system condition, the $m$ is general number of system condition relation defined through set parameters of system. In case at the system are coursing some processes or not isolated system are influencing some actions which lead to change of parameters values and functions of condition (which are defined through parameters of system). These processes can be defined by some equations set, represented through parameters and conditions of system:

$$
\begin{align*}
\psi_1 [\varphi (\alpha), \alpha] &= 0 , \\
\psi_2 [\varphi (\alpha), \alpha] &= 0 , \\
& \vdots \\
\psi_{l-1} [\varphi (\alpha), \alpha] &= 0 , \\
\psi_l [\varphi (\alpha), \alpha] &= 0 .
\end{align*}
$$

(3)

Here $\psi_k$ – is equation defined processes for system and the $l$ is general number of processes equations. This is necessary to note, that in among equations of processes can containing differential equations from functions of condition and parameters. The given equations set are representing the dynamics description, system condition and system parameters change. The supposition is well-known processes for physical system. However at all parameters and possible system conditions account are necessary to face problem of their description because of their complexity the joint account. This especially important is considering similar of set properties and systems description methods.

"'Information modelling" physical system consisting that parameters, conditions and the processes describing system, are classified as follows:

1. in physical systems are allocated parameters, conditions of system, and processes which can be interpreted, as data carriers. The given characteristics are compared to some points of abstract information space.

2. the values of parameters and conditions of system, and also some processes proceeding in system, can be compared to value of the information being in the given point of information space.

For an example, it is possible to consider such important characteristic of quantum mechanical system, as spin projection of system. The spin projection as the system characteristic can be considered as some point in abstract information space, for example as one bit of the information. The value of spin projection of system can be compared to value of the information, belong in the given point of information space. For example, positive value of a spin projection is compared to one and negative value of spin projection is compared to zero.

For consideration of opportunity entering 4-dimensional information space-time and using group of Poincare transformations, it is necessary to find out criterion of existence or absence of invariant velocity of the information transitions. For example is considered the physical system in which the electromagnetic wave propagation with velocity of light in vacuum $-c$ and the wave length to equal $\lambda = 2 \ell_P$, where $\ell_P$ – is the length of Planck$^{*}$. That frequency of the propagation wave is also invariant value $\nu_p$, this is obvious:

$$
\nu_p = \frac{c}{2 \ell_P} ,
$$

(4)

Let by means of the electromagnetic wave is transferred some information, presented in the form of a binary code, for simplicity of reasoning and without restriction of the generality. In this frame, what transferred the information containing in one $\lambda$-bit is necessary transferred and accepted one semi-wave, as for recognition of one the information bit is enough accepted or transferred the semi-wave. In the given example by virtue of smallness the chosen the wave length conditionally is considered the point object — the material point is including in the quantity and the sense of the information. On the basis of above considered is possible made the following statement: physical transference of the information by means of

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$^{*}$The consideration of smaller length of a wave, for example, the value of the wave length equal to value of Planck’s length $\lambda = \ell_P$, does not represent interest if considered logic of the information bit perception. For perception of information bit necessary to apprehend the physical signal (wave) equal to the semi-wave length, minimally possible value of measured which is equal to length of Planck in physical system.$^{3}$
the electromagnetic wave answers displacement of the information some bits number in information space; otherwise this statement is made so: for moving one bit of the information is necessary and enough moving the semi-wave in physical space. Hence the information velocity is proportional to frequency half of Eq. of (4):

$$\nu_c = \frac{c \lambda_c}{\ell_P},$$  \hspace{1cm} (5)

If used formula for length Planck through fundamental physical constants: velocity of light in vacuum – c, Planck’s constant – ℏ the constant of gravitational interaction – G:

$$\ell_P = \sqrt{\frac{\hbar G_N}{c^3}},$$  \hspace{1cm} (6)

The having substitution the Eq. of (6) in Eq. of (5), gives the following equation for invariant value of information velocity:

$$\nu_c = \sqrt{\frac{\lambda_c^2 c^5 \hbar}{\hbar G_N}},$$  \hspace{1cm} (7)

In the considered example all parameters of a transferred electromagnetic wave were invariant, in any physical frame of reference. By virtue of it is approved: the length of a semi-wave is related with the information transferring in information space, transfer with invariant velocity Eq. of (7).

3. PROPERTY OF INFORMATION SPACE

Let’s note only, that position of the information should be described by means of covariant (or contravariant) the 4-vector of position – $x^\alpha$ and the corresponding 4-vector of velocity $v^\alpha$. The 4-vector should be transformed with usage of Poincare group realization in which as invariant velocity is used invariant value of information velocity – $\nu_c$. Eq. of (7), at transition from one frame of reference to other frame of reference:

$$x^\alpha = (x^0, x^1, x^2, x^3).$$  \hspace{1cm} (8)

The relativistic invariance consists in scalar product preservation at Poincare group transformations. At transition from one frame of reference in other frame of reference of the position 4-vector should be transformed on Poincare group representation, according has following obvious form for a 4-vector displacement transformation:

$$x'^\alpha = \Lambda^\alpha_\beta x^\beta + b^\alpha,$$  \hspace{1cm} (9)

where $\Lambda^\alpha_\beta$ — is Lorentz’s transformation matrix, $b^\alpha$ — is 4-vector of time-space displacement in information space.

In frame of if product of the 4-vector most on itself will turnout the following equality for its component, following of requirements of relativistic invariance is considered:

$$x^2 = x^\beta x_\beta = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = \nu_c^2 \tau^2,$$ \hspace{1cm} (10)

where $x^0 = \nu_c t$ — is lights bit; $t$ — is time in the laboratory system of reference, in conformity coincides in due course in physical system; $\nu_c \tau$ — is the 4-interval, invariant value; $\tau$ — is intrinsic time which, according to it coincides with intrinsic time in considered physical system. From the received obvious kind of a square of the 4-interval in this frame have relation between infinitesimal time intervals in intrinsic system of references and laboratory system of references:

$$\nu_c^2 (d \tau)^2 = \nu_c^2 (d t)^2 + (d x^1)^2 + (d x^2)^2 + (d x^3)^2.$$  \hspace{1cm} (11)

Let’s consider the problem on displacement in information space-time and calculation of its mean value. For this purpose we shall consider mean displacement between two points:

$$\langle \Delta x^\alpha \rangle = \langle (x^\alpha_b - x^\alpha_a) \rangle,$$ \hspace{1cm} (12)
Without restriction of a generality have considered displacement of the information along one axis, for example is axis of $x^1$. The considering 4-velocity vector component at obvious form define as the derivative of the information position 4-vector on intrinsic time:

$$v^\alpha = \frac{d x^\alpha}{d \tau} = \frac{1}{\sqrt{1 - (v/v_c)^2}} (v^0, v^1, v^2, v^3), \quad v = \sqrt{v^1^2 + v^2^2 + v^3^2},$$

and enter the following dimensionless 4-vector which components are the 4-velocity attitude invariant value of information velocity Eq. of (7):

$$\beta^\alpha = \frac{v^\alpha}{v_c} = \frac{1}{\sqrt{1 - \beta^2}} \left(1, \beta^1, \beta^2, \beta^3\right), \quad \beta^i = v^i/v_c.$$

The velocity of displacement of the information along this axis, at the relativistic description have designated for convenience $u^1$:

$$u^1 = v_c \frac{\beta^1}{\sqrt{1 - \beta^2}}.$$

The displacement from a point $x^1_a$ in the point $x^1_b$ should be expressed by following integral from velocity along the axis $x^1$:

$$\Delta x^1 = x^1_b - x^1_a = \int_{x^1_a}^{x^1_b} u^1 \, d \tau = v_c \int_{x^1_a}^{x^1_b} \frac{\beta^1}{\sqrt{1 - \beta^2}} \, d \tau.$$

The considering 4-velocities scalar product obvious form:

$$v^\alpha v_\alpha = \frac{1}{1 - (v/v_c)^2} \left[(v^0)^2 - (v^1)^2 - (v^2)^2 - (v^3)^2\right] \equiv v_c^2,$$

and 4-velocity vector component Eq. of (13) at statement of Eq. of (16) is received following integral:

$$\Delta x^1 = \int_{x^1_a}^{x^1_b} v^1 \, d t,$$

where $t$– is time in a laboratory frame of reference. The continuous change of component of the displacement vector along the axis $x^1$ should be casual parameter, therefore measured displacement should be mean displacement calculated on rules of calculation of a continuous random variable, not concretizing the form of the random variable probability density (to the question of calculation of density should be return later, considering considering path integrals of Feynman), should be calculate an mean displacement:

$$\langle \Delta x^1 \rangle = \langle \int_{x^1_a}^{x^1_b} v^1 \, d t \rangle.$$

By virtue the measured invariant mean displacement value is the value $\lambda_c = 1$ bit, therefore any mean displacement along any space axis in considered information space should be proportional to the integer of bits. If to assume is received contradiction, that $\lambda_c$ is not invariant value that contradicts. Therefore is received, that any mean displacement is proportional to the integers of bits:

$$\langle \Delta x^1 \rangle = N_1 \lambda_c.$$

The analogously is possible to prove validity of the statement about proportionality of components to the integer of bits for the remained space components of the mean displacement vector. Thus consider, that equalities for all space coordinates are already proved by us. So, should be calculate mean value from time component of a 4-vector of displacement:

$$\Delta x^0 = x^0_b - x^0_a = v_c \int_{x^0_a}^{x^0_b} \frac{1}{\sqrt{1 - \beta^2}} \, d \tau. \quad \Delta x^0 = v_c [t_b - t_a] = v_c \Delta t.$$
The mean time interval should be received as random value:

\[ \langle \Delta x^0 \rangle = \nu_c \langle \Delta t \rangle. \quad (22) \]

For calculation of mean values in the left part uses relate between time intervals in laboratory and intrinsic frame of references (this frames of references is considering inertial frame of references):\(^3\)

\[ \Delta \tau = \Delta t \sqrt{1 - \beta^2}, \quad (23) \]

where \( \beta \) is expressed in velocity of movement of inertial frame of references readout concerning laboratory, as the result of averaging the given equation and substitution of result:

\[ \langle \Delta t \rangle = \frac{\lambda_c}{\sqrt{\langle v^2 \rangle}} \sqrt{N_1^2 + N_2^2 + N_3^2}. \quad (24) \]

The Eq. of (24) coincides with equation for time component. Thus, following parities turn out for a component of 4-dimensional displacement in information space:

\[ N_1 = \frac{\nu_c}{\lambda_c} \langle \Delta x^1 \rangle, \quad N_2 = \frac{\nu_c}{\lambda_c} \langle \Delta x^2 \rangle, \quad N_3 = \frac{\nu_c}{\lambda_c} \langle \Delta x^3 \rangle, \quad \langle \Delta t \rangle = \frac{\lambda_c^2}{\sqrt{\langle v^2 \rangle}} \sqrt{N_1^2 + N_2^2 + N_3^2}. \quad (25) \]

4. THE PATH INTEGRAL OF FEYNMAN

The probably to consider the question on information transference. The information transference has chance quantity, i.e. the information position in the given point \( x_a^\alpha \) can be found with some probability of initial point \( x_b^\alpha \) transition:

\[ \Omega (x_b^\alpha, x_a^\alpha) = |K (x_b^\alpha, x_a^\alpha)|^2 = K (x_b^\alpha, x_a^\alpha) K^* (x_b^\alpha, x_a^\alpha), \quad (26) \]

where \( K (b, a) \) – is generally complex amplitude of transition from a point \( x_a^\alpha \) to a point \( x_b^\alpha \), \( K^* (b, a) \) – is complexly interfaced amplitude. For transition amplitude calculation has used method path integral.\(^4\) Thus, action for transference information is defined through a 4-vector of momentum:

\[ S_0 (x_b^\alpha, x_a^\alpha) = \int_{x_a^\alpha}^{x_b^\alpha} L_0 d\tau = - \int p^\alpha dx^\alpha. \quad (27) \]

The transition amplitude between points \( x_a^\alpha \) and \( x_b^\alpha \) was possible to calculate as integral on all possible paths from action function, where as a constant has been used the certain value of information constant \( \hbar_c \):\(^3\)

\[ K (x_b^\alpha, x_a^\alpha) = \frac{1}{N_\infty} \int_\Gamma \exp \left[ -i S_0 (x_b^\alpha, x_a^\alpha) \right] \prod_{x_i^\alpha} \left( \frac{d^4 p (x_i^\alpha) d^4 x (x_i^\alpha)}{8 \pi^4} \right), \quad (28) \]

where \( \Gamma \) – is a of phase space volume in which there is the information transference on all possible paths; \( x_i^\alpha \) – is carried out \( i \)-partition number of information position 4-vector in information space, partition trajectory which moving the information from initial point \( x_a^\alpha \) in finite point \( x_b^\alpha \), it essentially depends on partition paths; \( d^4 p (x_i^\alpha) \) and \( d^4 x (x_i^\alpha) \) – is corresponding \( i \)-partition number to that splitting of phase 4-volume paths elements in momentum and coordinate spaces; \( N_\infty \) – is normalizing constant which can be defined from normalizing condition on unit of probability in all information space:

\[ \left| \frac{1}{N_\infty} K (\Gamma_\infty) \right|^2 = \frac{1}{N_\infty N_\infty} K (\Gamma_\infty) K^* (\Gamma_\infty) = 1, \quad (29) \]

where \( \Gamma_\infty \) – is all 4-volume in information space of the given information extends on all possible paths. The obvious form of paths integral have calculated between two points \( x_a^\alpha \) and \( x_b^\alpha \) Eq. of (28) which defines amplitude of information transference, and then obvious form of normalizing constant have calculate for free information transference frame of Eq. of (29).
The calculation of paths integral is spent by partition each path into such sites, where \( p^\alpha (x_i^\alpha) \) – is constant everyone \( i \)-partition number, \( x^\alpha (x_i^\alpha) \) – is varies linearly, the \( M \) parts number for each of paths will depend on away partition, nevertheless, in a limit at \( M \to \infty \) the following obvious forms is received for path integral:

\[
K(b, a) = \frac{1}{N^\infty} \int_\Gamma \exp \left[-i p^\alpha (x_b^\alpha - x_a^\alpha)\right] \frac{d^4p}{8\pi^2}.
\]  
(30)

The integral of Eq. of (30) will be possible calculate, knowing an obvious form of 4-vector of momentum \( p^\alpha \). For calculation of mean value is necessary know in obvious form 4-density of the information transition probability from initial point in final point. For this purpose have considered probability of such transition Eq. of (26) and have find density of probability in coordinate space:

\[
\rho(x_b^\alpha, x_a^\alpha) = \frac{d\Omega(x_b^\alpha, x_a^\alpha)}{d^4x} = \frac{dK(x_b^\alpha, x_a^\alpha)}{d^4x}K^*(x_b^\alpha, x_a^\alpha) + K(x_b^\alpha, x_a^\alpha) \frac{dK^*(x_b^\alpha, x_a^\alpha)}{d^4x}.
\]  
(31)

The received definition of probability density can be used at calculation of mean from observable values: 4-vectors of information displacement, a 4-vector of momentum, etc. Also the given equation Eq. of (31) can be used for updating model parameters in requirements, i.e. at calculation of mean displacement in information space:

\[
\langle (x_b^\alpha - x_a^\alpha) \rangle = \int_\Gamma (x_b^\alpha - x_a^\alpha) \rho(x_b^\alpha, x_a^\alpha) d^4x.
\]  
(32)

5. CONCLUSIONS

The considered direction of modelling of information space can be applied without the generality limitations to the description of any real systems for which studying information making system is important and thus the account of processes of the various nature, for example, physical and sociological processes is required. The given direction of information space modelling allows to consider, on the one hand, objective parameters of the information, on the other hand, allows to consider within the limits of the approach subjective features of perception of the information. The important question at modelling information space is a correct comparison of the phenomena and processes proceeding in physical (chemical, biological, social, etc.) system with three-dimensional coordinates of information space. As a whole, apparently to the author, the given relativistic direction of the modelling description of information space and information systems has greater prospects from the point of view of the description of information processes in one language and without dependence from type of considered system or especially their combinations.

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REFERENCES

[1] F. Baluška, D. Volkmann, and P. Barlow, “Eukaryotic cells and their cell bodies: Cell theory revisited,” Ann Bot 94, pp. 9–32, 2004.
[2] R. Boyd and P. J. Richerson, The origin and evolution of cultures (Evolution and Cognition), Oxford University Press, 2005.
[3] O. I. Shro, “The description of information in 4-dimensional pseudo-euclidean information space,” arXiv.org: 0709.0993, p. 40, 2007.
[4] R. P. Feynman and F. L. Vernon, “The theory of a general quantum system interacting with a linear dissipative system,” Ann. Phys. 281, pp. 547–607, 2000.