Soft-Gluon Resummation: 
a Short Review

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Abstract

I briefly summarize some general features of soft-gluon contribution to inclusive cross-sections. The discussion includes the issue of soft-gluon singularities in infrared- and collinear-safe observables. All-order resummation and the ensuing varieties of QCD predictions are illustrated.

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1 Introduction

Sudakov resummation, that is, resummation of double-logarithmic perturbative contributions produced by soft-gluon radiation, has become a topic of broad interest in recent years. The reason is twofold.

On the theoretical side, our confidence in resummed calculations has highly increased: resummation techniques has been extended beyond double-logarithmic (DL) accuracy \[1\] and results at leading-logarithmic (LL) and next-to-leading logarithmic (NLL) order are available for many observables in different processes. In particular, using resummed calculations to NLL accuracy one can consistently introduce a meaningful definition (say $\overline{\text{MS}}$) of the QCD coupling $\alpha_s(\mu)$, investigate their theoretical accuracy by studying the renormalization-scale dependence and match the predictions with full next-to-leading order (NLO) calculations thus enlarging the applicability of perturbative QCD to wider kinematical regions \[2\]. The list of quantities evaluated to NLL order includes many $e^+e^-$ shape variables \[2\], $Q_L$-distributions in the Drell-Yan process \[3\] and cross sections for the production of high-mass systems via the Drell-Yan mechanism \[4\] and in deep-inelastic lepton-hadron scattering \[5\]. These quantities regard hard-scattering processes initiated by two coloured partons. Results at NLL order for multi-parton processes have begun to appear recently \[6\].

On the phenomenological side, many experimental results sensitive to soft-gluon resummation have become available and require detailed analyses. For instance, high-precision $e^+e^-$ data \[7\] from LEP and SLC strongly demand understanding of the Sudakov region. Last, but not least, renewed interest on soft-gluon corrections in hadronic collisions has been prompted by the high-$E_T$ tail of the one-jet inclusive cross-section measured at Tevatron \[8\].

Some features of soft-gluon radiation and resummation are briefly reviewed in the following Sections.

2 Soft-gluon effects in QCD cross sections

The finite energy resolution of any particle detector implies that physical cross sections are always inclusive over arbitrarily-soft particles produced in the final state. This inclusiveness is essential in QCD calculations. Higher-order perturbative contributions due to virtual gluons are infrared divergent and the divergences are exactly cancelled by radiation of undetected real gluons. Thus, perturbative cross sections are finite but, precisely speaking, the cancellation does not necessarily take place order by order in perturbation theory. In particular kinematic configurations real and virtual contributions can be highly unbalanced, spoiling the cancellation mechanism. As a result, soft-gluon contribution to QCD cross sections can still be

- either large
- or singular.

We comment on these points in turn.
The presence of large corrections that spoil the convergence of the perturbative expansion near the exclusive boundaries of the phase space of QCD observables is well known \[1\]. When the tagged final state is forced to carry most of the total energy available in the process (that is, one considers the quasi-elastic limit \(x \to 1\), where \(x\) generically denotes inelasticity variables), the radiative tail of real emission is strongly suppressed, producing the loss of balance with the virtual contribution. Then the cancellation of the infrared divergences bequeaths finite higher-order contributions of the type

\[
C_{nm} \alpha_s^n \ln^m (1 - x), \quad \text{with} \quad m \leq 2n, \tag{1}
\]

that can become large, \(\alpha_s \ln^2 (1 - x) \gtrsim 1\), even if the QCD coupling is in the perturbative regime \(\alpha_s \ll 1\). The logarithmically-enhanced terms in Eq. (1) are certainly relevant near the exclusive boundary \(x \to 1\). Moreover, their actual size in cross-section calculations depends on the coefficients \(C_{nm}\) and on the \(x\)-shape of parton densities. Thus, soft-gluon effects can be substantial also before reaching this extreme kinematic region. In these cases \[2\]-\[6\], the theoretical predictions can be improved by evaluating soft-gluon contributions to high orders and possibly resumming them to all orders in \(\alpha_s\).

Soft-gluon contribution can still be singular also for observables that fulfil the Sterman-Weinberg criteria \[9\] of infrared and collinear safety. This feature has recently been pointed out \[10\] in general terms. The singularities arise whenever the observable in question has a non-smooth behaviour in some order of perturbation theory at an accessible point \(x_0\), called critical point, inside the physical region of phase space. In this case, the lack of balance between real and virtual contributions is just produced by the sharpness of the distribution around the critical point \(x_0\) and, in particular, the following theorem applies \[10\]. If the distribution of the observable is discontinuous (e.g., it has a step) at that point in some order of perturbation theory, it will become infinite there to all (finite) higher orders. The infinite higher-order corrections have a double-logarithmic structure similar to that in Eqs. (1) after the replacement \(\ln(1 - x) \to \ln(x - x_0)\) and the singularities can appear either on one side or on both sides of the critical point.

One might think that infrared- and collinear-safe quantities, that at a certain perturbative order have a step-like behaviour inside the physical region, are quite abstract. In fact, this often can happen \[10\] \(i\) if the phase-space boundary for a certain number of partons lies inside that for a larger number, or \(ii\) if the observable itself is defined in a non-smooth way. The distribution of the \(C\)-parameter \[11\], a well-known event shape variable for \(e^+e^-\) annihilation final states, is an example of type \(i\). As shown in Fig. 1, this distribution has a stepwise (singular) behaviour at \(\mathcal{O}(\alpha_s)\) \((\mathcal{O}(\alpha_s^2))\) at the value \(C = C_0 = \frac{3}{4}\), which is the maximum value of the \(C\)-parameter for three final-state partons. Observables of type \(ii\) have typically double-sided singularities and are quite common in jet physics. Some examples are the jet shape, which describes the angular distribution of energy with respect to the jet axis \[12, 13\], and the momentum distribution of isolated photons, when the isolation criterion is defined in terms of energy-angle cutoffs \[14, 15\].

The presence of singularities in the fixed-order expansion of infrared- and collinear-safe observables arises questions on the reliability of perturbative predictions, on the validity (interpretation) of the Sterman-Weinberg criteria in perturbation theory and on the effect of non-perturbative contributions. As discussed in Ref. \[10\] and recalled in the following Section, these problems have a satisfactory solution entirely within the context of perturbation theory. The solution is based on the resummation of the singular soft-gluon
contributions to all orders.

3 The varieties of Sudakov resummation

Soft-gluon exponentiation

The physical bases for all-order summation of soft-gluon contributions to QCD cross sections are dynamics and kinematics factorizations [16]. The first factorization follows from gauge invariance and unitarity: in the soft limit multi-gluon amplitudes fulfill generalized factorization formulae given in terms of a single-gluon emission probability that is universal (process independent). The second factorization regards kinematics and strongly depends on the observable to be computed. If, in the appropriate soft limit, the phase-space for this observable can be written in a factorized way, resummation is feasible in form of generalized exponentiation [4] of the single-gluon emission probability. Then, exponentiation allows one to define and carry out an improved perturbative expansion that systematically resums LL terms, NLL terms and so on.

Note that phase space depends in a non-trivial way on multi-gluon configurations and, in general, is not factorizable in single-particle contributions (soft-gluon exponentiation can be violated [17]). Moreover, even when phase-space factorization is achievable, it does not always occur in the space where the physical observable \( x \) is defined. Usually, it is necessary to introduce a conjugate space to overcome phase-space constraints. A typical example [4, 18] is the energy-conservation constraint that can be factorized by working in \( N \)-moment space, \( N \) being the variable conjugate to the energy \( x \) via a Mellin (or Laplace) transformation.

Large or singular soft-gluon contributions can have different origins (cf. Sect. 2) and resummation takes different exponentiation forms depending on kinematics. This leads to varieties of Sudakov effects.
Figure 2: Contribution of gluon resummation at order $\alpha_S^4$ and higher, relative to the truncated $O(\alpha_S^3)$ result, for the invariant-mass distribution of jet pairs at the Tevatron. The subscripts $ij$ refer to the various partonic channels.

a) Sudakov suppression

Soft-gluon resummation produces suppression of cross sections near the exclusive phase-space boundary. Typical examples are $e^+e^-$ event shapes in the two-jet limit [2]. Resummed predictions for these observables have been successfully compared with data from $e^+e^-$ annihilation at energies below [13], at [7] and above [20] the $Z^0$ peak. In particular, the predictions reduce the renormalization-scale dependence of pure NLO calculations, leading to measurements of $\alpha_S(M_Z)$ with smaller theoretical uncertainty.

b) Sudakov enhancement

In hadronic collisions the exclusive phase-space boundary can be approached in the production of systems of high mass near threshold. Cross sections are again suppressed in this regime. However, perturbative QCD does not apply to absolute cross sections. The perturbatively computable component is what is left after factorization of long-distance physics into universal parton distributions. Thus, perturbative calculations regard ratios of cross sections. In the factorization schemes ($\overline{\text{MS}}$ and DIS) that are most commonly used, it turns out that soft-gluon resummation enhances the predictions for these ratios. Moreover, parton density factorization requires exact implementation of energy conservation. It follows that resummation must be carried out in $N$-moment space and the inversion to the physical space has to be performed with care [18] to avoid the introduction of unjustified divergences [21, 22].

The effect of LL resummation on the production cross-sections of heavy-quarks and jets was evaluated in Ref. [18]. Figure 2 shows the result for the invariant-mass ($M_{jj}$) distribution of a jet pair at Tevatron. Here $\sigma^{(\text{res})}$ and $\sigma^{(3)}$ respectively denote the resummed cross-section and its truncation at order $\alpha_S^3$. One can see that resummation enhances the cross section by (at most) $10 \div 15\%$ when the inelasticity variable $\tau = M_{jj}^2/S$ is as large as $\tau \sim 0.5$.

c) Sudakov shoulder

The case of perturbative quantities with soft-gluon singularities inside the physical phase
space has, so far, received less attention in the literature. Much work on resummation remains to be done in order to reach a level of understanding comparable to that of soft-gluon contribution near the exclusive phase-space boundary.

We expect [10] on general grounds that finiteness will be restored at perturbative level by all-order summation. After resummation, one obtains a characteristic structure which is not only finite but smooth (infinitely differentiable) at the critical point. We called this structure a \textit{Sudakov shoulder} [10]. The existence of a Sudakov shoulder implies the restoration of validity of the Sterman-Weinberg criteria at infinite order in perturbation theory.

The dot-dashed curve in Fig. 1 illustrates the Sudakov shoulder obtained by performing DL resummation in the case of the $C$-parameter distribution [10]. The resummed prediction for $C > C_0 = \frac{3}{4}$ joins smoothly to the $\mathcal{O}(\alpha_s^3)$-calculation at (and below) the critical point. Note that the shoulder is nevertheless still quite sharp on the scale shown in Fig. 1.

The issue of soft-gluon singularities inside the physical phase space is of some importance for QCD phenomenology. Before using any “safe” observable to test the theory or to measure $\alpha_s$, one needs to identify the critical points of that observable and the expected behaviour in whatever order of perturbation theory is to be used. These points will need to be avoided in comparisons between fixed-order predictions and experiment. On the other hand, if resummed predictions can be obtained to single-logarithmic precision, then the behaviour at critical points represents an interesting new class of QCD predictions.

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