Spontaneous formation of density waves in granular matter under swirling excitation

Cite as: Phys. Fluids 33, 081701 (2021); doi:10.1063/5.0056143
Submitted: 6 May 2021 · Accepted: 13 July 2021 ·
Published Online: 3 August 2021

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ABSTRACT

We study here the spontaneous clustering of a submonolayer of grains under horizontal circular shaking. The clustering of grains occurs when increasing the oscillation amplitude beyond a threshold. The dense area travels in a circular fashion at the driving frequency, which even exceeds the speed of driving. It turns out that the observed clustering is due to the formation of density waves. The analysis of a phenomenological model shows that the instability of the uniform density profile arises by increasing the oscillation amplitude and captures the non-monotonic dependence of the transition amplitude of the clustering on the global density of the system. Here, the key ingredient is that the velocity of individual grains increases with the local density. The interplay of the dissipative particle–particle interaction and frictional driving of the substrate results in this dependence, which is tested with discrete element method simulations.

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INTRODUCTION

Owing to its non-equilibrium nature, granular materials exhibit phenomena of self-organization across orders of magnitude of length scales, from gold panning1 to astrophysics.2,3 Those phenomena are largely represented by the clustering instability, non-uniform density distribution developing out of an initially homogeneous state. Clustering has been observed both in freely cooling granular gases4,5 and in driven systems.6 Such a collective behavior leads to pattern formation,7 segregation,8,9 phase separation,9 and shear banding.10 Though clustering of granular matter exhibits some generic features across various systems, to unravel the underlying mechanism, one may need to take peculiarities of any given experimental protocol, e.g., the type of energy input, into account.11 The discovery of new features challenges the existing concepts and theories.12 Unraveling the physics mechanism represents a crossroad of hydrodynamics, non-equilibrium statistical mechanics, and the phenomenological theory of pattern formation, which has attracted interest over decades. One may refer to Refs. 3 and 13 and references therein for an overview. In this article, we study a submonolayer of beads under horizontal agitations. Constant frictional driving of the substrate distinguishes it from vertically vibrated systems. Strip-like patterns were reported in such systems subjected to a one-dimensional oscillation.7 Under two-dimensionally vibrated systems, strip-like patterns were reported in such systems subjected to a one-dimensional oscillation.7 The discovery of new features may need to take peculiarities of any given experimental protocol, e.g., the type of energy input, into account.11 The discovery of new features challenges the existing concepts and theories.12 Unraveling the physics mechanism represents a crossroad of hydrodynamics, non-equilibrium statistical mechanics, and the phenomenological theory of pattern formation, which has attracted interest over decades. One may refer to Refs. 3 and 13 and references therein for an overview. In this article, we study a submonolayer of beads under horizontal agitations. Constant frictional driving of the substrate distinguishes it from vertically vibrated systems. Strip-like patterns were reported in such systems subjected to a one-dimensional oscillation.7 Under two-dimensionally vibrated systems, strip-like patterns were reported in such systems subjected to a one-dimensional oscillation.7

EXPERIMENTAL RESULTS

For the system studied here, there are three experimental parameters: the oscillation strength, the global packing density, and the ratio of the grain size to the container size. We first study a reference system specified, and later, we investigate the influence of various parameters to the system behavior. The submonolayer consists of \( N_{\text{tot}} = 6930 \) polydisperse Zirconium Oxide spheres of diameter \( [0.6...0.8] \) mm with uniform distribution and the mean \( d = 0.7 \) mm. The grains located on an acrylic plate are confined by a 3D-printed polylactic acid (PLA) circular sidewall of diameter \( D = 82 \) mm and height 5 mm. The global packing density is given by the area ratio

\[
\phi = \frac{N_{\text{tot}} \cdot \pi \cdot \langle d^2 \rangle}{A_{\text{tot}}} = \frac{N_{\text{tot}} \cdot \pi \cdot \langle d^2 \rangle}{6 \cdot \pi \cdot (D / 2)^2}
\]

where \( N_{\text{tot}} \) is the total number of grains, \( \langle d^2 \rangle \) is the mean grain diameter, and \( A_{\text{tot}} \) is the area of the circular sidewall. The global packing density is calculated as the area fraction occupied by the grains, namely, the ratio of the grain area to the total area.

The transition, therein, was realized by increasing the global packing density \( \phi_{\text{tot}} \), while keeping the oscillation amplitude constant. The transition packing density is reduced by larger oscillation amplitude, an example of “freezing by heating.”13 However, the mechanism of the spontaneous clustering is still unknown, and the motion within the clustering region is not investigated in detail. Here, we confirm that, for a given \( \phi_{\text{tot}} \), the clustering is achieved by increasing the oscillation amplitude. The transition is abrupt and sensitive to the amplitude (see the supplementary material). The mechanism is explored by analyzing the motion of individual particles, a phenomenological model, and discrete element method (DEM) simulations.

\[
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\text{ABSTRACT} & \\
\text{INTRODUCTION} & \\
\text{EXPERIMENTAL RESULTS} & \\
\end{align*}
\]
$\phi_{\text{tot}} = N_{\text{tot}}(d_{g}/D)^2$. The inclination of the bottom plate is smaller than 0.02 mm/m. The container is subjected to anti-clockwise circular oscillation in the horizontal plane of frequency $f = 5$ Hz. The oscillation amplitude is varied in the range $A = [5…13]$ mm. Note that here, the amplitude represents the diameter of the circular oscillation path [see Fig. 1(b)]. In this range of agitation and $\phi_{\text{tot}} = 0.505$, grains rarely jump over each other and, thus, the packing remains two-dimensional. The system is illuminated by a LED panel from the bottom, and the dynamics are captured by a high-speed camera (Mikrotron MC1362) at the top at a constant frame rate of 500 Hz. The camera is fixed in the laboratory frame of reference. The velocity of particles is obtained by multiplying the distance traveled between consecutive frames by the frame rate, corresponding to a time interval of 0.002 s. In the following, the analysis is done in this frame for reference. To avoid the potential influence of the boundary layers, we exclude particles closer than $15d_g$ to the sidewall from the analysis [see Fig. 1(a)].

Upon oscillation, grains roll and slide on the substrate and collide with each other and the sidewall. The density distribution changes with the oscillation amplitude. Figure 1 shows the average of images of the system during ten cycles at two oscillation amplitudes. For $A = 5$ mm, the system is homogeneous. For $A = 11$ mm, a high-density region appears close to the center. The observed clustering transition is very sensitive to the oscillation amplitude. The cluster disappears within ten cycles after decreasing $A$ from 11 to 10 mm. The reversibility of the transition highlights the uniqueness of the clustering in the current work with respect to that in a vertically vibrated monolayer, where hysteresis is observed. Furthermore, in our experiments, the dense area moves anti-clockwise, in the same direction as the oscillation (Multimedia view).

This reversible transition is so abrupt that it can be well recognized by naked eyes, which is further confirmed by quantitative measurements of the average and the variance of the local packing density $\phi_{\text{loc}}$ in the region of interest. The transition is accompanied by a jump of the average of $\phi_{\text{loc}}$, and the variance reaches a peak just below the transition, indicating the emergence of unstable small clusters (see an example in the supplementary material). The quantitative definitions of $\phi_{\text{loc}}$ will be given below.

In Fig. 1(b), there are dense areas at the periphery of the packing as well. Those dense boundary layers may be sustained over many cycles and move in a counter-intuitive way, opposite to the swirling motion. This motion mode will be investigated in another work. Nevertheless, the occurrence of a dense area near the periphery is not surprising. The frictional driving of the substrate introduces both linear and angular momentum of grains. The rotational degree of freedom of grains reduces the linear momentum transfer from the substrate. Therefore, the linear speed of grains is $2/7$ of that of oscillation, ignoring interactions between particles. On one hand, this velocity difference leads to compression on the periphery of the packing via collisions between the particles and the sidewall. On the other hand, it always leaves an empty area not containing grains near the sidewall, as if the packing only occupies a fraction of the total area (see Fig. 1, for example). In consequence, the packing density in the region of interest, $\phi_{\text{loc}}$, is typically larger than $\phi_{\text{tot}}$. As long as the packing remains two-dimensional, $\phi_{\text{loc}}$ can be estimated via $\phi_{\text{loc}}/\phi_{\text{tot}} = (1 - 2A^2/D^2)^2$. However, this effect could not explain the observed spontaneous clustering in the central area, for instance, the sensitivity of the clustering to $A$ (see the supplementary material). To understand the mechanism of clustering, we study the dynamics of individual particles.

To visualize the motion of the dense area, we select a circular path in the lab frame and calculate the local density profile along this path. The path is concentric with the moving line of the center of the bottom plate but has a larger diameter of 22 mm. The local density $\phi_{\text{loc}}$ is defined for individual grains in a circular neighborhood with a diameter of $5d_g$. The upper bound of $\phi_{\text{loc}}$ is $\phi^* \approx 0.9$ corresponding to the hexagonal packing, $\phi_{\text{loc}}$ along this circular path is plotted vs time in Fig. 2(a). At a given time, there is a jump of $\phi_{\text{loc}}$ in space. The maximum of $\phi_{\text{loc}}$ travels along the selected path at the same frequency as the driving (5 Hz). For lower $A$, the density pattern disappears

![Figure 1](https://doi.org/10.1063/5.0056143.1)

**FIG. 1.** The average of the experimental images during 10 cycles in the reference frame of the moving container for two oscillation amplitudes: (a) $A = 5$ mm and (b) $A = 11$ mm. Within the cylindrical container, the background is bright. The darker a region looks, the denser it is. In (a) the dashed circle indicates the region of interest. In (b), dynamic clustering can be observed near the center. The red circle indicates a stationary path of a 22 mm diameter in the lab frame of reference, along which the local density profile in Fig. 2(a) is computed. See the main text for more information. The green solid circle indicates the typical region where individual grains would access during one oscillation cycle. The white circle represents the anti-clockwise oscillation path in scale, whose diameter is 11 mm. The position of both displayed images in the oscillation cycle is denoted as ⋆. Multimedia view: https://doi.org/10.1063/5.0056143.1

![Figure 2](image-url)

**FIG. 2.** (a) Local density profile along a circular path vs time for $A = 11$ mm and $\phi_{\text{tot}} = 0.505$, where clustering close to the center is observed. The circular path used here is highlighted in red in Fig. 1(b). The exact position of the path is given in the main text. For the same experiment, the local density $\phi_{\text{loc}}$ and the velocity of a grain $v_g$ on the circular path are plotted vs time in (b) and (c), respectively. See the main text for the definition of $\phi_{\text{loc}}$. **Letters**
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STABILITY ANALYSIS

The dependence of $v_s$ on $\phi_{loc}$ reveals the mechanism leading to the clustering. For a given $A$, consider the continuity equation

$$\frac{\partial \phi_{loc}}{\partial t} + \nabla \cdot (\phi_{loc} \mathbf{v}) = 0. \quad (1)$$

If the velocity of particles tends to relax toward the local presumed velocity $\mathbf{V}(\phi_{loc})$, it is readily to show that an increasing function of $\mathbf{V}$ on $\phi_{loc}$ would promote the formation of shock waves of $\phi_{loc}$.\(^{18,19}\)

However, the density wave is only observed for $A \geq 11$ mm, which suggests that the collisions between grains introduce an equivalent term of pressure sustaining the homogeneous state. Therefore, the equation for the velocity field can be written as

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{V} = \frac{\mathbf{V}(\phi_{loc}) - \mathbf{v}}{\tau} - \frac{1}{\phi_{loc}} \nabla p(\phi_{loc}).$$

(2)

The first term at the right-hand side describes the tendency of the local velocity to match the presumed velocity, $\mathbf{V}_{loc}$, where $\tau$ is the time scale of the relaxation of $\mathbf{v}$ toward $\mathbf{V}_{loc}$. As the interactions between particles are via contacts/collisions, $\tau$ is defined by the collision time scale.

It can be seen in Fig. 2(c) that $\tau$ is much smaller than the oscillation period.\(^{20}\)

The second term represents the gradient of the pressure, $p$. This term prevents particles from staying in the denser region, where they bounce away from each other via frequent collisions.\(^{21}\) Note that both $\mathbf{V}_{loc}$ and $p$ are functions of $\phi_{loc}$, and $A$. The presumed velocity, $\mathbf{V}_{loc}$, largely follows the direction of the oscillation. Therefore, we only consider the flow in the oscillation direction, and Eq. (2) is reduced to a scalar equation. The one-dimensional equation corresponding to Eq. (2) was derived in the context of vehicular traffic models in Ref. 22.

The model embodied in Eqs. (1) and (2) admits a steady-state solution representing the uniform flow [$\phi_{loc} = \phi_0$ and $\mathbf{v} = \mathbf{V}_{loc}($$\phi_0)$]. Note that we use $\phi_0$ instead of $\phi_{tot}$ for $\phi_{loc}$. Such a homogeneous flow is stable against density perturbations, which provided\(^{22}\)

$$(-v_s^s)^2 < p'. \quad (3)$$

$V_s^s$ and $p'$ are the derivatives of $V_s$ and $p$ with respect to $\phi_{loc}$, for a given $A$. If condition 3 is violated, the destiny disturbance grows and travels at a higher velocity than $V_s(\phi_0)$ corresponding to homogeneous flow. The error function is fitted on the measured $V_s(\phi_{loc}, A)$ (see Fig. 3), and it is used as a substitute for $V_s$. Thus, the derivative, $V_s^s$, has Gaussian-like peaks. The pressure from collisions is estimated by

$$p = \frac{c_0 \sigma_0 v_s^2}{2} \frac{d^2}{d(d - d^*^*)} = c_0 \sigma_0 v_s^2 f(\phi_{loc}),$$

(4)

with the dimensionless parameter $c_0 \approx 2$ (see the supplementary material), whose value is later determined by comparing with the experimental observation. For individual grains, the velocity fluctuation $\delta v_s$ in its neighborhood is extracted. Similar to $v_s$, $\sigma_0 v_s^2(\phi_{loc}, A)$ is the average of $\sigma_0 v_s^2$ over the region of interest and time. $\delta = d_s/\sqrt{\phi_{loc}}$ represents the average distance between grains for a given $\phi_{loc}$, and $d^* = d_s/\sqrt{\sigma_0}$ corresponds to the hexagonal packing. The fraction on the right-hand side of Eq. (4) is purely geometrical and is referred to as $f(\phi_{loc})$ in the following. $f(\phi_{loc})$ increases with $\phi_{loc}$ and diverges when approaching $\phi_{tot}$.

In contrast, though increasing with $A$, $\sigma_0 v_s^2$ is largely constant in the range of $\phi_{loc} \in [0.3, 0.8]$ for a given $A$ (see the supplementary material). Therefore, the variant of $p$ is dominated by $f(\phi_{loc})$, and its derivative is approximated by $p' \approx c_0 \sigma_0 v_s^2 f(\phi_{loc})$.

Figure 4 illustrates the condition in Eq. (3). The packing density in the region of interest for $A < 11$ mm is indicated by a gray bar ($\phi_0 \in [0.61, 0.62]$). $c_0 = 1.8$ is chosen such that Eq. (3) is just violated for $A = 11$ mm at $\phi_0$, but not for smaller oscillation amplitude. Though both $p'$ (or $\sigma_0 v_s^2$) and $V_s^s$ increase with $A$, the relative increase in the latter is more significant.\(^{17}\) In consequence, the instability is triggered by increasing $A$ beyond the transition amplitude, $A_t$. This corresponds to the observed freezing by heating.

FIG. 3. The presumed value of the velocity of grains, $v_s$, increases with the local density, $\phi_{loc}$, for various oscillation amplitude $A$. $A = 11$ mm triggers the clustering/density wave close to the center of the packing. The dashed lines are fits of the error function to the data. The global packing density here is $\phi_{tot} = 0.505$.
In this case, the whole packing rotates clockwise, opposite to the swirl-motion. This distinctive motion mode will be further investigated in a following work.

**DISCUSSIONS**

We have elaborated the mechanism of clustering by the instability analysis of the uniform flow in a phenomenological model [Eqs. (1) and (2)]. The key ingredient promoting the clustering is the dependence of the presumed velocity of grains on the local density. Why does such a dependence exist? We believe that it is a consequence of the interplay between the friction between grains and that between grains and the substrate. $F_{\text{gs}}$ and $F_{\text{gg}}$ denote the friction coefficients of the former and the latter, respectively, and $F_{\text{gs}}$ and $F_{\text{gg}}$ are the corresponding friction forces. Upon agitation, $F_{\text{gs}}$ accelerates not only the linear momentum but also the angular momentum of grains. As discussed above, without interactions, individual grains would reach a linear speed of $2/7$ of the oscillation speed, at the state of rolling without sliding. It also implies that a grain would reach a larger linear speed, if its rolling speed is reduced. Consider now the collision of two spherical grains rolling in the same direction (inset of Fig. 5). $F_{\text{gs}}$ counteracts the rolling of grains. The frustration of rotation effectively enhances the action of $F_{\text{gs}}$ on linear momentum transfer. $F_{\text{gs}}$ is proportional to kinetic pressure $p$, and $p$ increases with $\phi_{\text{loc}}$ and $A$ [Eq. (4)]. Therefore, grains tend to move faster in dense areas and/or under stronger oscillations. These arguments lead to the features of the function $v_j(\phi_{\text{loc}}, A)$ as shown in Fig. 3, where $v_j$ increases with both $\phi_{\text{loc}}$ and $A$. In order to further support this line of arguments, discrete element method simulations are performed using LIGGGHTS. For given $\gamma$ and $A$, by decreasing $\mu_{\text{gs}}$, the dependence of $v_j$ on $\phi_{\text{loc}}$ and the clustering are both suppressed. Meanwhile, as observed in experiments, reducing $A$ has a similar effect on clustering, which is confirmed for a few combinations of oscillation frequency and amplitude. The transition amplitude $A_c$ decreases with $f$, and a constant critical oscillation strength $A_c f^2$ can be identified for relatively low frequencies. However, at high frequency, $A_c$ deviates from this trend (see the supplementary material). Simulations quantitatively consistent with experiments will be appreciated to reveal the relative significance of parameters, where a careful examination of the force model, such as the coupling between rolling and sliding, is required.

The clustering phenomenon reported here could be reproduced in experiments with polydisperse grains of diameter of 0.8–1 mm,
various surface types (smooth and rough glass beads), aspheric grains (e.g., millet seeds), and a square container (see the supplementary material). Therefore, the principle of clustering is robust for the explored parameter range. For an even larger aspect ratio of $d_g/D$, a solid-like cluster and its reptation motion mode were reported,\textsuperscript{25,26} which is different from the phenomenology here in both the global density distribution [Fig. 1(b)] and the motion of the cluster [Fig. 2(a)]. Moreover, the liquid-solid transition was reported to be independent of the oscillation frequency.\textsuperscript{1} Therefore, further exploration of the parameter space is necessary to reach a complete quantitative understanding of the clustering in swirling granular matter, its growth/coarsening, and the potential relation to segregation.\textsuperscript{8} In particular, the abrupt increase in $v_g$ across $A_i$ (cf. Figs. 3 and S3) and its dependence on $\phi_{tot}$ indicate a certain non-local effect, which increases with $\phi_{tot}$ and helps in reducing $A_i$ for $\phi_{tot}$ between 0.50 and 0.64. Appropriate hydrodynamic treatment of this term is anticipated in future research.

SUPPLEMENTARY MATERIAL

See the supplementary material for details of DEM simulations and snapshots of clustering in other experimental configurations.

ACKNOWLEDGMENTS

Funding of this research by the Deutsche Forschungsgemeinschaft (German Research Foundation), Grant No. PO472/40, is gratefully acknowledged. The work was supported by the Interdisciplinary Center for Nanostructured Films (IZNF), the Central Institute for Scientific Computing (ZISC) and the Interdisciplinary Center for Functional Particle Systems (FPS) at Friedrich-Alexander University Erlangen–Nürnberg.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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\textsuperscript{20} Otherwise, $v_g$ would display an apparent delay relative to $\phi_{loc}$, e.g., reaching its maximum at a later time.

\textsuperscript{21} Unlike in the free-cooling granular gas, the particles here are subjected to constant driving from the substrate.

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