Approximation of the parallel robot working area using the method of nonuniform covering

E V Gaponenko¹, D I Malyshev¹, L Behera²

¹ Belgorod State Technological University named after V. G. Shukhov, 46, Kostyukov Str., Belgorod, 308012, Russia
² Indian Institute of Technology Kanpur, Kanpur 208016, India
E-mail: gaponenkobel@gmail.com, gaponenko.ev@bstu.ru

Abstract. The article describes the application of optimization algorithms for solving the problem of determining the working area of a parallel 6-DOF relative mechanism device, which includes a flat 3-RRR mechanism and a tripod mechanism. The method for approximation of the set of nonlinear inequalities system solutions describing constraints on the geometric parameters of the robot, based on the concept of nonuniform coverings, is considered. Based on the method, external and internal approximations are obtained, given as a set of parallelepipeds. The working area of the relative mechanism device has been obtained, taking into account the restrictions on the geometric parameters of the workpiece located in a moving coordinate system located in the center of a flat 3-RRR mechanism. The moving coordinate system is located in the center of the planar mechanism.

1. Introduction
Robots with parallel kinematics are increasingly used in industry, including mechanical engineering for performing various technological operations. Such robots have a number of positive properties, which include high performance, increased structural rigidity, increased accuracy of the operations performed. Questions of structural synthesis, methods for studying the working space, optimization of the trajectory of movement of such mechanisms are discussed in detail in [1-5]. An important characteristic of parallel robots is working area, within which there must be a working tool when performing technological operations. Finding the working area of such robots is much more difficult than for robots with a serial architecture. They are characterized by the ambiguity of the solution of the kinematics problem, that is, the different positions of the driving rods can correspond to one position of the output link.

Let us determine the working area with application of known geometric, numerical methods, methods of discretization. In [6-8], it is shown that interval analysis is an effective tool for approximating the working area. In [9–11], approximation of convex bodies with the help of polyhedra is considered. The approximation of the reachability region of dynamical systems was studied in [12, 13]. In [14], an approximation of the image of a compact set under mapping using the concept of an ε-effective shell was proposed and a method for constructing it was given. It is also worth noting the work [15] devoted to the construction of three-dimensional computational grids in regions of complex shape. The method of nonuniform covering for approximation of the set of solutions of a nonlinear inequalities system was considered in [16] and the application of this method to determine the working area of certain types of
planar robots was considered in [17-19]. The covering of the set P is called the set of n-dimensional parallelepipeds $P_i$, $i \in \overline{1,k}$, such that:

$$P \subseteq \bigcup_{i \in \overline{1,k}} P_i$$

and for each $P_i$, $i \in \overline{1,k}$, at least one of the three conditions is satisfied:

$$\max_{i \in \overline{1,k}} \max_{i \in P_i} g_j(x) < 0$$

$$\max_{i \in \overline{1,k}} \min_{x \in P_i} g_j(x) < 0$$

Inequalities (1) and (2) are not satisfied and $d(P_i) \leq \delta$, where $\delta$ is a given positive value that determines the accuracy of the approximation.

On the basis of the considered method, external and internal approximation sets are constructed. The inner approximation set is included in the set of solutions of the inequality system, and the outer one includes it. Both sets are combinations of n-dimensional parallelepipeds. This method allows one to approximate the set of solutions of systems of equalities or inequalities describing the working area of the robot. The mathematical transformation of the equations of communication of some robots makes it possible to reduce the dimensionality and, consequently, the computation time. The use of parallelepipeds of large dimension allows you to avoid significant mathematical transformations, and then project them on the coordinate axes necessary for visualization. For approximation of the working area of the mechanism of relative manipulation in the moving coordinate system, approximation sets of individual modules of the mechanism will be used.

To find the maximum and minimum functions in parallelepipeds, you can use interval analysis methods that use the developed Snowgoose library in C++. However, in this case, due to the multiple occurrence of variables, errors may occur that affect the result. Therefore, the search for the minimum and maximum of a function in a parallelepiped is found using grid approximation. The method was developed and tested on models of robots with 2 and 3 degrees of freedom in the framework of the project of the Russian Science Foundation, agreement No. 16-19-00148.

2. Formulation of the problem
Let us consider a relative mechanism device (Fig. 1), which consists of two modules.

![Figure 1. 3D-model of the relative mechanism device.](image)

The first module is made on the basis of a tripod, the center of the movable platform of which can perform translational movement along the Z axis and rotational one around the X and Y axes, as well as additional displacements of the output link during its turns relative to the horizontal axes imposed by the kinematic chains of the tripod. The second module is made based on a flat 3-RRR mechanism,
performs translational movement along the $X$ and $Y$ axes, the rotational around the $Z$ axis. Thus, the relative mechanism device has 6 degrees of freedom. It can be used to perform part processing operations and other operations if the working tool 1 is mounted on the movable platform of tripod 2, and the workpiece is on the movable platform 3-RRR mechanism 3. An important characteristic of parallel robots is the size of the working area in which technological operations and this working area should be, whenever possible, maximum.

We set the task of determining the working area of the robot, taking into account the restrictions on the geometric parameters of the workpiece located on the mobile platform 3. The mechanism is shown in Fig. 2. The $Z_0$ and $Z_1$ axes lie on one straight line, perpendicular to the fixed base of the tripod $D_1D_2D_3$, and the platform of the flat 3-RRR mechanism and passing through the center of the circumscribed circle of triangle $A_1A_2A_3$. In the initial position, that is, when the center of the circumscribed circle $A_1A_2A_3$ coincides with the center of the moving plate of the flat 3-RRR mechanism, the moving coordinate system $X_2Y_2Z_2$ coincides with the coordinate system $X_0Y_0Z_0$.

3. Algorithm for constructing the working area of the robot.

To construct the working area of the relative mechanism device, taking into account the constraints on the geometric parameters of the workpiece, it is necessary to find a variety of positions $p$. $O'$ in the moving coordinate system $X_2Y_2Z_2$ located in the center of the flat 3-RRR mechanism. When determining the set of positions of $p$. $O'$ using combinations of points from the boxes that describe the working areas of the tripod and the 3-RRR mechanism, it becomes necessary to calculate the coordinates of $p$. $O'$ a significant number of times. For example, in the case of the separation of parallelepipeds with a uniform grid of $100 \times 100$ and the number of parallelepipeds in each of the lists $n = 1000$, the number of calculations and results to be saved for each of the coordinates is $10^{18}$ times. In this regard, it is necessary to determine the set of positions by the “reverse” method consisting in checking the points of a set, which includes the position of the coordinate $p$. $O'$ in the moving coordinate system $X_2Y_2Z_2$. For set points, their coordinates in the $X_1Y_1Z_1$ coordinate system are determined using transformation matrices including linear displacements and platform rotations derived from parallelepipeds describing the working area of the 3-RRR mechanism. Then their entry into the tripod working area is checked. Thus, the relation connecting the coordinates $O'$ in the systems $X_2Y_2Z_2$ and $X_1Y_1Z_1$ (Fig.2) has the form:

![Figure 2. The scheme of the relative mechanism device.](image-url)
\[ O'_1 = M_{0.1} \cdot M_{2.0} \cdot O'_2 \]

- the vector of the point \( O' \) of the working tool in the fixed coordinate system

\[ O'_2 = [x_2, y_2, z_2] \] - vector of the point \( O' \) coordinates in the moving coordinate system

\[ X_1Y_1Z_1 \] associated with the tripod.

\[ X_2Y_2Z_2 \] which must be checked for entry into the working area of the tripod.

\[ \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 & x_0 \\ \sin \varphi & \cos \varphi & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] - matrix that takes into account the transformation from the \( X_2Y_2Z_2 \) coordinate system to \( X_0Y_0Z_0 \), where \( \varphi \) is the angle of rotation of the coordinate system \( X_0Y_0Z_0 \) relative to the \( Z_0 \) axis; \( x_0, y_0 \) are linear displacements along the \( X_0 \) and \( Y_0 \) axes, respectively.

\[ \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] - matrix that takes into account the transition from the coordinate system \( X_0Y_0Z_0 \) to \( X_1Y_1Z_1 \), where \( h_1 \) is the distance from the center of the platform of the 3-RPP mechanism to the center of the fixed platform of the tripod.

Thus we can obtain the following:

\[ O'_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 & x_0 \\ \sin \varphi & \cos \varphi & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} \]

(3)

The algorithm for the approximation of the working area of the robot in the moving coordinate system is developed in the framework of the project of the Russian Science Foundation, the agreement number is 17-79-10512. The algorithm works with four lists of three-dimensional parallelepipeds: the two previously obtained lists \( P_{1f} \) and \( P_{2f} \), which will not change during the execution of the algorithm, and two changeable lists \( P \) and \( P_I \).

The algorithm (Fig.3) works as follows:

1. At the first step of the algorithm, the list of internal approximation \( P_I \) is empty, and the list \( P \) consists of only one parallelepiped \( Q \), which is guaranteed to include the working area.

2. We divide each of the parallelepipeds of the list of the lower mechanism \( P_{2f} \), and the dimension of the grid depends on the size of the parallelepiped.

3. We extract the parallelepiped \( Q_j, j \in 1, n, n \) from the list \( P \) and divide it with a uniform grid along each of the axes.

4. If for each of the points \( A_l \), of the parallelepiped \( Q \) \( j \) with coordinates \( (x_{2,l}, y_{2,l}, z_{2,l}) \) there is such point \( A_m \) of the parallelepipeds of the list \( P_{2f} \) with coordinates \( (x_{0,m}, y_{0,m}, z_{0,m}) \) that, when calculating the value of the position of coordinate \( p \), \( O' \) in the coordinate system \( X_1Y_1Z_1 \) of the tripod by the formula (11), the resulting point \( A_k \) belongs to the parallelepiped from the list \( P_{1f} \). Then the parallelepiped \( Q_j, j \in 1, n \) fully satisfies the conditions and is added to the list \( P_I \).

5. If for each of the points \( A_l \) there is no such point \( A_m \) that when calculating the value of the point of the position of the coordinate \( p \), \( O' \) in the coordinate system \( X_1Y_1Z_1 \) of the tripod by the formula (3), the resulting point belongs to the parallelepiped from the list \( P_{1f} \). Therefore, the parallelepiped does not meet the requirements and is excluded from further consideration.

6. In other cases, the parallelepiped is divided into two equal parallelepipeds along the edge with the greatest length. These parallelepipeds are entered at the end of the list \( P \).

7. If the next parallelepiped to be considered in the list \( P \) is less than the given approximation accuracy \( \delta \), then the algorithm completes its work.

8.
Figure 3. An algorithm for the working area of the relative mechanism device.
The simulation results with the projection on the plane are shown in Fig. 4.

Figure 4. The simulation results: a - in the projection on the $X_2Z_2$ plane, b - on the $Y_2Z_2$ plane, c - on the $X_2Y_2$ plane.

4. Conclusions

The developed algorithm showed its effectiveness. The computation time for the approximation accuracy $\delta = 6$ mm and the grid dimension 64x64x64 for functions calculating on a PC were 2 hours and 45 minutes. Approximation accuracy ($\delta = 0.006-0.06$ mm) of a given complex robot with 6 degrees of freedom compared to flat 3-RRR mechanisms with three degrees of freedom ($\delta = 0.006-0.06$ mm) decreases 100-1000 times. This is due to an increase in the dimension of the problem, and significant computational resources needed to improve accuracy. As a result, the working area has fuzzy boundaries, which may be due to insufficient computing power and the presence of singularity zones.

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