Diffraction mode selection in planar lasers with Bragg resonators

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Abstract. Using the coupled-wave approach supplemented by the quasi-optical approximation, we investigate the possibilities of diffraction mode selection with respect to the transverse index in planar distributed-feedback lasers. Quality factors and spatial structures of planar Bragg resonators with finite width and length of the corrugate area were found. Allowable values of the Fresnel parameter were determined at which the diffraction losses at the resonator edges lead to effective discrimination of modes with large numbers of transverse variations and thus provide conditions for the onset of a stationary single-mode laser generation regime.

1 Introduction

Bragg structures are widely used in quantum and classical electronics as a means of provision distributed feedback (DFB) \cite{1-3} for efficient mode selection with respect to the longitudinal (parallel to the Bragg lattice vector) index as a necessary condition of narrow-band generation. However, state-of-the-art technology capabilities allow for manufacturing active media with transverse (lateral) dimensions up to several hundred wavelengths \cite{4, 5}. For such media, the problem of transverse radiation coherence becomes of immediate importance.

In the present paper, we show that, in certain limits determined by the Fresnel parameter value, the described problem is solved by means of mode diffraction selection when modes with larger numbers of variations with respect to the transverse coordinate have larger diffraction losses so their decay is faster and they are excluded from generation. In fact, such method of mode selection is used in open quasi-optical resonators \cite{2} and can obviously be applied to planar Bragg structures and distributed feedback lasers based on such structures.

2 The model and basic equations of a planar DFB laser

We consider a planar dielectric plate with a section of shallow sine corrugation, $b(z) = b_0 + b_1 \cos(kz)$ with a length of $l_z$ and a width of $l_x$ (see Fig.1), where $b_0$ - is the...
average thickness of the layer, \( b_1 \) is the corrugation amplitude, \( d \) is its period, \( \bar{h} = 2\pi / d \).

Under conditions of Bragg resonance \( 2h \approx \bar{h} \), eigenmodes in such a structure are formed by two counter-propagating wave beams with electrical field that can be presented as

\[
E_\pm = \text{Re}[a_\pm(y)A_\pm(z,x,t)\exp(i(\omega_0 t \mp \bar{h}z))].
\]

Here \( \omega_0 \) is the Bragg frequency taken as the carrier one, \( A_\pm(z,x,t) \) are the slowly varying wavebeam amplitudes, \( a_\pm(y) \) are the transverse field structures of the modes of the regular waveguide (i.e. the dielectric plate in the optical wavelength range and the planar waveguide in the microwave range). The amplification of the waves by the active medium and their mutual scattering taking into account the transverse diffraction is described by the following parabolic-type equation system:

\[
\frac{\partial A_+}{\partial Z} + \frac{\partial A_-}{\partial \tau} + i \frac{\partial^2 A_+}{\partial X^2} + \frac{\partial^2 A_-}{\partial X^2} + i f(X) A_\pm = \frac{\nu}{2} \left( 1 - p \left( |A_+|^2 + 2|A_-|^2 \right) \right),
\]

\[
\frac{\partial A_-}{\partial Z} + \frac{\partial A_+}{\partial \tau} + i \frac{\partial^2 A_-}{\partial X^2} + \frac{\partial^2 A_+}{\partial X^2} + i f(X) A_\pm = \frac{\nu}{2} \left( 1 - p \left( |A_-|^2 + 2|A_+|^2 \right) \right). \tag{1}
\]

Here \( \tau = \alpha v_g t \), \( Z = \alpha z \) and \( X = \sqrt{2\alpha h}x \) the normalized time and coordinates, \( L_z = \alpha L_z \) and \( L_x = \sqrt{2\alpha h}L_x \) are the longitudinal and transverse dimensions of the corrugated area, \( v_{gr} \) is the group velocity of the partial waves, \( \alpha \) is the wave coupling coefficient [1].

Function \( f(X) \) defines the transverse profile of the corrugation (Fig.1). For the description of the amplification, we used the simplest and the most general instantaneous model of the active medium characterized by the amplification parameter \( \nu \) and by the saturation parameter \( \rho \).

![Fig. 1. Scheme of DFB laser.](image)

With respect to the longitudinal coordinate, at the edges of the corrugated section, boundary conditions corresponding to the absence of waves incoming from outside are used:

\[
A_+(X,\tau)|_{Z=0} = 0, A_-(X,\tau)|_{Z=L_z} = 0. \tag{3}
\]

With respect to the transverse coordinate, at some distance from the Bragg structure \( X = \pm B / 2 \), the radiation boundary conditions are set [6], which correspond to free diffraction of the partial wavebeams:
The field distribution inside the structure does not depend on the location of the boundary. Based on the system (4), one can find the longitudinal and transverse power flows from the Bragg structure as

$$P^\pm_z (\tau) = \int_{-B/2}^{B/2} |A_x|^2 \, dX \bigg|_{z=0,t_z}$$  

(6)

$$P^\pm_x (\tau) = \text{Im} \left[ \int_0^{L_z} \frac{\partial A_x^*}{\partial X} A_x \, dZ \right] \bigg|_{X=\pm B/2}$$  

(7)

Ratio of these flows is denoted as

$$K = \left( P^+_x + P^-_x \right) / \left( P^+_z + P^-_z \right).$$  

(8)

### 3 Simulations results

#### 3.1 Eigenmodes of the finite-width planar Bragg reflector

In order to obtain the eigenmodes’ spatial distributions and decrements, the Bragg structure was excited by a short electromagnetic pulse incident from the left boundary. In the process of the field decay described by the Eqs (1)-(2) with zero right-hand part and boundary conditions (3)-(5), the extraction of the highest-Q mode took place.

![Fig. 2. Time dependence of the field amplitude in the process of excitation of Bragg resonator by external pulse and its subsequent decay (curve 1). The descending stage of above dependence in the logarithm scale (curve 2). Lower insert contains the field spectrum at the intermediate time interval: $\tau \in [20,45]$, and the upper one contains the spectrum at the final interval: $\tau \in [115,140]$, $L_z = 5$, $L_x = 8$, $N = 1$.](https://doi.org/10.1051/itmconf/2019306012)
At the initial stage in the field spectrum (inserts in Fig. 2)
\[ S(\Omega) = \int_{-\tau_1}^{\tau_2} A_+ (\tau) \exp(-i\Omega \tau) d\tau \]  
(9)
lower-frequency symmetric and anti-symmetric modes are present (\( \Omega \) is the normalized frequency shift from the Bragg frequency).

However, Q-factors of the lower-frequency modes are larger than those of the higher-frequency ones, and, as can be seen from the Fig.2, at the final stage only the lower-frequency symmetric mode remains. Thus the extraction of the highest-quality modes with set symmetry takes place. In the system under consideration, this approach allows to determine the spatial structures of the modes (Fig.3) together with their eigenfrequencies (Fig.4) and decrements (Fig.5).

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**Fig. 3.** Spatial structures of the symmetric (left) and the anti-symmetric (right) modes. \( L_z = 5 \), \( L_x = 8 \), \( N = 1 \).

**Fig. 4.** Dependence of the frequency shift of the higher-frequency (1), lower-frequency anti-symmetric (2) and symmetric (3) modes of the Bragg resonator on the Fresnel parameter \( N \). Dot-dashed line denotes the asymptotic values determined by relation (10). \( L_z = 5 \).
It is well-known [1, 2] that in neglection of the transverse diffraction of the field, two modes exist with equal decrements (Q-factors) at the frequencies given by (dot-dashed lines in Figs. 4 and 5)
\[
\Omega_n = \pm \left(1 + \frac{n^2 \pi^2}{2L_z^2} + i \frac{n^2 \pi^2}{L_z^2}\right),
\]
up- and down-shifted with respect to the Bragg frequency. Diffraction losses break this Q-factor degeneracy for Bragg resonators with transverse size corresponding to a Fresnel parameter
\[
N = L_x^2 / \lambda L_z = L_x^2 / (4\pi L_z),
\]
is of order of 1.

**Fig. 5.** Dependence of the decrements of the higher-frequency (1), lower-frequency anti-symmetric (2) and symmetric (3) modes of the Bragg resonator on the Fresnel parameter \(N\). Dot-dashed line denotes the asymptotic values determined by relation (10). \(L_z = 5\).

**Fig. 6.** Dependence of the resonator’s quality of the Fresnel parameter \(N\) and K coefficient for lower-frequency symmetric (black curves, 1, 3) and anti-symmetric (gray curves, 2, 4) modes of the Bragg resonator obtained in CST simulations. For comparison, K coefficient obtained using Eqs. (1)-(2) is shown double ticks lines for symmetric (5) and anti-symmetric (6) modes. \(L_z = 5\).
In order to verify the quasi-optical model, we undertook a full 3D simulation based on CST Microwave Studio software package. The results presented in Fig.6 show good agreement with the results of analytical theory.

### 3.2 Settlement of a single-mode oscillation regime in a laser with a finite-width 1D planar Bragg resonator

Obviously, the selectivity of a Bragg resonator at relatively low values of the Fresnel parameter should provide a possibility of realization of stationary single-mode oscillation regime in the process of modes excitation by an active medium. This is confirmed by Fig.7 where the temporal dependence of the energy flow from the right edge of the laser is shown in cases of both symmetric and anti-symmetric initial conditions. One can see that due to the fact that at the chosen parameters the quality of the symmetric lower-frequency mode is much larger than those of other modes (see Fig. 6), stationary regime of oscillations should correspond to excitation of this mode at any initial conditions. Note however that at larger values of the Fresnel parameter when Q-factors of the symmetric and of the non-symmetric modes become closer, regimes of simultaneous excitation and beatings between these modes would be realized.

![Fig. 7. Onset of the stationary oscillation regime. Temporal dependence of the energy flow from the right edge of the laser is shown in cases of the symmetric (1) and anti-symmetric (2) initial conditions. In the insert, the field spectra at the initial stage are shown in case of anti-symmetric initial conditions (3) and settled single-mode oscillation (4).](image)

4. Conclusions

Thus, the undertaken analysis shows the efficiency of diffraction mode selection in planar Bragg structures at relatively small values of the Fresnel parameter, i.e. at the width $l_x \leq \sqrt{\lambda_z}$. Alongside that, active media with lateral sizes of several dozens wavelength (semiconductor heterostructures [4]) and sheet electron beams with similar oversize factors [7] for generation of microwave radiation already exist, which demand different approaches for provision of spatially coherent oscillation, for instance, the 2D distributed feedback [5].
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References

1. H. Kogelnik, *Theory of dielectric waveguides, in: Integrated Optics*, (Berlin/Heidelberg: Springer, v. 7. 1979)
2. A. Yariv, P. Yeh, *Optical Waves and Crystals, Propagation and Control of Laser Radiation* (Wiley-Interscience, 2002)
3. N.S. Ginzburg, V.Y. Zaslavskii, I.V. Zotova, A.M. Malkin, N.Y. Peskov, A.S. Sergeev, JETP Lett., 91(6), 266 (2010)
4. N.S. Ginzburg, V.R. Baryshev, A.S. Sergeev, A.M. Malkin, , Phys. Rev. A. 91, 053806 (2015)
5. G.S. Sokolovskii, V.V. Dudelev, I.M. Gadzhiev, S.N. Losev, A.G. Deryagin, V.I. Kuchinskii, E.U. Rafailov, W. Sibbett, Tech. Phys. Lett. 31, 824 (2005)
6. N.S. Ginzburg, I.V. Zotova, A.S. Sergeev, Radiophys. and Quant. Electron., 52(8), 568 (2009)
7. A.V. Arzhannikov, V.S. Nikolaev, S.L. Sinitsky, M.V. Yushkov, J. Appl. Phys. 72(4), 1657 (1992)