Modelling of two-and four-terminal thin-film silicon tandem solar cells

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Abstract. We have performed semi-empirical modelling of amorphous silicon/microcrystalline silicon thin-film solar cells in tandem optical configuration, under series (two-terminal) and independent (four-terminal) electrical connection. The four-terminal connection relaxes the constraint of current matching between the cells. Computer simulations indicate an increase in maximum initial efficiency from 10.0\% (two-terminal) to 10.8\% (four-terminal). Following degradation of the amorphous silicon top-cell the figures are 8.5\% and 9.8\% respectively, with an optimum top-cell absorber layer thickness in four-terminal connection of 150 nm. Changes in solar spectral quality have been simulated by applying weighted spectra, and improvements in efficiency favouring four-terminal connection occur under red- and blue-rich conditions. However, these typically convey only a small fraction of the total annual insolation, and gains of 3\% or less in annual electrical output are predicted. Possible increased optical and electrical losses due to additional contact layers in four-terminal connection are not taken into account.

1. Introduction

Tandem solar cells comprising amorphous silicon (a-Si:H) top cell/microcrystalline silicon (μc-Si:H) bottom cell enable higher efficiencies to be achieved in comparison with equivalent single-junction cells due primarily to reduced photocarrier thermalisation loss resulting from more effective spectral matching [1]. An additional benefit is that the a-Si:H top-cell absorber layer may be made thinner than in a single cell, resulting in improved resistance to light-induced degradation [2]. Given the compatibility of the deposition processes for each sub-cell, these advantages have led to sustained scientific and commercial interest in such ‘micromorph’ cells and modules over the past decade [3-5]. The great majority of tandem cells employ series electrical connection of sub-cells to give a two-terminal (2T) device, which results naturally from the deposition process. 2T connection imposes an important boundary condition - that the current in both sub-cells must remain equal. To achieve optimal power output under these conditions requires precise current matching, e.g. by careful adjustment of absorber-layer thickness. However, if the sub-cells are electrically decoupled in a four-terminal (4T) device then their operating points are independently controllable, ensuring maximum power transfer at all times.

A number of operational advantages may in principle be gained by 4T connection. Since the a-Si:H top-cell need no longer support the generation current of the μc-Si:H bottom-cell, it can be thinner and will be more stable as a result. The ‘intermediate tunnel junction’ between the sub-cells and its attendant resistive losses is obviated. Variations in service conditions such as changes in intensity and
spectral quality of the incident light will not unbalance the cell. For example, sunlight from a sun low in the sky contains a greater proportion of red light. Assuming the tandem cell has been optimized for AM1.5, this will pass through the top-cell and generate most current in the bottom-cell. Thus in 2T connection the power deliverable to an external load under these conditions would be strongly limited by the top-cell. Similarly, if the light contains a larger blue fraction then the tandem becomes bottom-cell limited. In 4T connection no such limitation exists, and provided the maximum-power point of each component cell is tracked independently the electrical matching remains optimal.

Despite these potential advantages, little practical interest has been shown until recently in the development of 4T micromorph cells. Madan [6] has described a range of experiments on prototype 4T structures, and outlined possible production pathways. Kondo et al [7] used a 4T structure to measure the component currents in a tandem cell thereby enabling a more detailed analysis. Dadouche et al [8] have conducted a computer simulation study of 4T cells. They used a numerical solution of the device equations for each cell, enabling parameters such as defect densities and top-cell layer thickness to be adjusted. Their simulations are carried out for the case of a polymorphous silicon top-cell. They conclude, in common with Madan [6], that a thin top-cell leads to optimal performance in a degraded 4T micromorph, i.e. one in which this sub-cell has a high defect density. A study of 4T connected modules was recently performed by Hudansky et al [9], who found significant improvements in initial efficiency over commercial micromorph modules, under both standard test conditions and real outdoor conditions. Curiously, similar stabilised efficiencies following degradation were obtained, contrary to expectations.

Here we present a device simulation procedure rather different to that of Dadouche et al [8]. It is based on a semi-empirical approach, and makes use of suitably-scaled experimental $J-V$ curves. The power delivered in each connection for a given top-cell thickness is evaluated, in both un-degraded and light-soaked states. This enables the optimum thickness in both states to be determined for 2T and 4T connections. We have also simulated the effects of varying solar spectral quality over typical outdoor conditions, by including a weighting function applied to the AM1.5 spectrum coupled with statistical data on the irradiance distribution vs. average photon energy. The potential benefits of the 4T connection are identified and discussed in the context of previous studies.

2. Modelling

The model assumes a series optical structure, in which the top cell, whose thickness may be varied, filters the light absorbed by the bottom cell, which remains constant in thickness throughout. This is shown schematically in figure 1. No account is taken of any optical or electrical losses arising from additional contact layer(s) required for 4T connection. In order to determine the short-circuit currents, the product of the quantum efficiency ($QE$) and the AM1.5 spectrum, filtered in the case of the bottom cell, is integrated over the entire wavelength range, separately for each cell. These photocurrents are

![Figure 1. Principle of four-terminal tandem solar cell](image)
then used to scale representative J-V curves of good-quality un-degraded a-Si:H and μc-Si:H solar cells along the current density axis, thereby providing an approximation to the characteristics of the component cells. It is accepted that this cannot provide an exact match to the true characteristic, as alongside a general increase in current density JSC, both fill-factor FF and open-circuit voltage VOC generally decrease with increasing cell thickness (and are also wavelength-dependent). However, over the i-layer thickness range of interest, the increase in JSC is the dominant factor. To obtain the 2T J-V curve, the scaled characteristics are solved simultaneously by graphical means for a range of equal currents, and from this, the maximum power under 2T connection may be calculated. For the 4T case, the unrestricted maximum powers for both cells are calculated, and then added together to obtain the total power.

The QE curves used for the top- and bottom-cells are shown in figures 2 and 3 respectively. In the case of the top-cell, the curves are generated by taking a thick (1 μm) un-degraded cell, and then multiplying by the optical absorption curve for the relevant thickness of a-Si:H. This artifice essentially assumes the short-wavelength QE is thickness-independent, and the long-wavelength QE is not limited by extraction, only by generation of carriers. These curves are then used to scale the J-V characteristics in figures 4 and 5, in the manner described above.

The effect of top-cell photo-degradation is included in our simulation in a simple way. The data of Klaver et al [2] and Platz et al [10] reveal that while all three solar cell parameters JSC, VOC and FF are reduced by photo-degradation, FF is most strongly affected. For an i-layer thickness of 600 nm or less, JSC falls by less than 10% on 1000 hours exposure to AM1.5, and VOC by less than 3%. However, FF falls roughly linearly with thickness, by up to 30%. The FF data from those sources are re-drawn in figure 6. In order to model the FF degradation, the output from the a-Si:H cell in 2T and 4T configurations is reduced in proportion to the ratio of the degraded to the un-degraded FF value at the appropriate thickness.

The influence of solar spectral variations is included in the simulations by means of weighted AM1.5 spectra. This broadly matches a range of insolation conditions, such as those reported by Krishnan et al [11]. The weight is set by a linear function, and the spectrum quantified in terms of its average photon energy (APE) calculated over the range 350 to 1050 nm. This range is commonly adopted in work on silicon solar cells, and in this context the APE of the AM1.5 spectrum is 1.88 eV. The total power between these limits was normalised to a constant value for all simulations, enabling direct calculation of the conversion efficiency. Typical spectra obtained are shown in figure 7. The integration procedure is then applied to obtain the short circuit currents, the J-V characteristics scaled, and then solved numerically for 2T connection as described above. In these simulations, for simplicity

**Figure 2.** QE vs. i-layer thickness d<i> for a-Si:H top cell. See text for how thickness variation is obtained.

**Figure 3.** Effective QE for μc-Si:H bottom cell of thickness 1.1 μm, including filtering by a-Si:H top-cell of thickness d<i>.
Figure 4. Construction used to obtain the 2T tandem-cell characteristic from top- and bottom-cells (T and B), evaluated at points 1, 2 and 3. The maximum-power points for the separate cells (4T connection) are marked.

Figure 5. Tandem cell (2T connection) J-V obtained from construction illustrated in figure 4.

we are using the un-degraded J-V characteristic of the top-cell and maintain a constant top-cell thickness of 300 nm.

The power delivered under 2T and 4T connections may be estimated by asserting that the fractional changes in open-circuit voltage and fill-factor are substantially smaller than those in the current, and may thus be neglected. The $V_{OC}$ values used in our simulations were 0.90 V and 0.52 V for a-Si:H and µc-Si:H sub-cells respectively, and 1.42 V for the tandem cell. $FF$ was assumed constant at 70% throughout.

3. Results and discussion

3.1. Effect of top-cell thickness on 2T and 4T performance

The power produced by each cell under 4T conditions vs. top-cell thickness is shown in figure 8. In the un-degraded state, the top cell power increases with increasing thickness. The curve gradually levels off, whereas in practice the output reduces above 400 nm due to recombination loss exceeding the gain.
from increased optical absorption. This feature is illustrated in the simulations of Dadouche et al [8]. In the degraded state, the inclusion of the fill-factor term results in a peak in output from the top-cell for a thickness of 250-300 nm. The bottom-cell output falls rapidly as the top-cell thickness increases up to 100 nm, reflecting strong top-cell absorption in the blue/green portion of the spectrum. Above 300 nm, the bottom-cell output continues to fall as progressively more of the visible spectrum is absorbed in the top-cell, but does so more gradually as the deep red/IR spectral content lies below the a-Si:H band gap.

The efficiency of the 2T and 4T connected tandems, in initial and degraded states, is shown in figure 9, as a function of top-cell thickness. In the initial state, the efficiencies of both 2T and 4T connections continue to rise up to 600 nm top-cell thickness. The 2T efficiency falls as the thickness is further increased and the system becomes bottom-cell limited (not shown on this scale). The 4T efficiency tends towards a plateau. If there is no additional recombination loss in the top-cell with increasing thickness, then the highest 4T tandem efficiency will be achieved with a thick top-cell, because carriers created close to the band edges, by band-gap energy photons, undergo the minimum thermalisation loss, and will thus generate more electrical energy when absorbed in the top-cell than the bottom cell. In practice, recombination increases as the internal electric field falls, which reduces $FF$ [2], and the 4T curve will reach a maximum dependent on the intrinsic defect density in the undegraded top-cell. We note that the 4T connection (10.8%) yields a higher efficiency than the 2T (10.0%) connection since in the former case both cells operate at maximum power transfer.

![Figure 7. Weighted solar spectra generated using linear filter function. Spectra are normalised to contain equal power between 350 and 1050 nm.](image)

![Figure 8. Power delivered by top- and bottom-cells in independent connection (4T). It is assumed that the bottom-cell parameters do not degrade.](image)
For i-layer thicknesses > 80 nm, degradation reduces the power produced by a given top-cell, in either configuration. The 2T optimized efficiency of 8.5% is reached for a top-cell thickness around 350 nm. However, the 4T peak efficiency of 9.8% occurs at 100-150 nm. There is a higher carrier thermalisation loss than in the former case, since more higher-energy photons will pass through the thinner top-cell to the lower bandgap bottom-cell, but this is more than recouped by reduced Joule heating in the top-cell by the bottom cell due to the current mismatch. Under these conditions, the model predicts that the top-cell delivers some 45% of the total tandem power in 4T, compared with 65% in 2T configuration.

Overall, we predict, in the degraded state, a significant benefit of the 4T connection over the 2T connection – an improvement in absolute photovoltaic efficiency of some 1.3%, and a more stable output. However, the percentage figures quoted here should be taken as indicative, rather than absolute, since they depend on the quality of the ‘initial’ J-V characteristics and the approximations inherent in this model.

We now compare our results with those of Dadouche et al [8] in more detail, beginning with the component cells (4T connection, figure 8). While the initial and degraded top-cell powers show very similar dependence on top-cell thickness, the bottom-cell power contribution is rather smaller in their case, even though the bottom cell in their model is slightly thicker (1.5 µm against 1.1 µm). At a top-cell thickness of 100 nm, our model predicts a bottom-cell contribution of 5.5 mW/cm², compared with 3 mW/cm². The figures are 4 mW/cm² and 2.5 mW/cm² at 400 nm thickness. Comparison with experimental data [12] supports the view that our bottom-cell currents are too large. This is probably because we do not take account of reflections, scattering, absorption in extra TCO layers etc.

Comparing the total output in 2T and 4T connections (figure 9) with the equivalent figures in Dadouche et al [8], there is reasonable overall agreement between the curves, although some differences in detail exist. The main discrepancies are: (i) In 2T connection our model predicts little degradation for top-cell thicknesses below 150 nm, rising to a 25% relative power loss at 600 nm. In their model, the relative loss is around 15%, and is independent of top-cell thickness between 50 and 500 nm. (ii) In 4T connection, for a degraded top-cell, we observe a clear maximum in efficiency around 150 nm, whereas in their study it is almost independent of top-cell thickness. (iii) For a degraded cell, our model predicts an increase in optimized efficiency from 8.5% (2T) to 9.8% (4T), a 1.3% improvement, against a prediction of only 0.2% in their model. At present we have no clear explanation for these discrepancies.

3.2. Effect of solar spectral distribution on 2T and 4T performance
The simulated efficiency for a tandem cell comprising an un-degraded top-cell of thickness 300 nm coupled with a 1.1 µm bottom-cell, under 2T and 4T connection over a range of spectral conditions is
presented in figure 10. Under 2T connection, the tandem is limited by the top-cell current under red-rich conditions, and by the bottom-cell current under blue-rich conditions. The optimum current-match for this simulation is obtained with a slightly blue-rich spectrum relative to AM1.5 – at an $APE$ of 1.91 eV as against 1.88 eV. Under 4T connection, current matching is no longer an issue, and a progressive improvement in efficiency is obtained with increasing blue content. This is because an increasing proportion of photons have an energy above the a-Si:H bandgap, and the electron-hole pair thus generated by these photons is subject to a smaller thermalisation loss than in the µc-Si:H cell.

However, figure 10 does not convey the true picture, since it does not take into account the prevalence of red-rich or blue-rich conditions over a period of time. In order to predict the likely improvement obtainable outdoors, some statistical information is required. Figure 11 shows a histogram of irradiance distribution (vs. $APE$) over a three-year cycle after Minemoto et al [13], re-scaled so that the peak irradiance lies at zero $APE$. By means of this distribution we have estimated the annual energy output, by summing the electrical energy produced by each histogram element, and then calculating an ‘annual average’ efficiency by dividing by the total irradiance. The result is shown in figure 12. This plot shows the effect of shifting the mean value of the distribution shown in figure 11 relative to the matched condition in 2T connection (at $APE = 1.91$ eV in this case). There is little change in the 4T response, since the underlying efficiency rises monotonically with $APE$. However in comparison to figure 10, the 2T response shows a lower peak in annual efficiency, some 0.3% below the corresponding 2T value. This is because the ‘wings’ of the distribution are converted at a lower efficiency than the centre. The 2T response falls either side of the peak because the cell is no longer matched to the $APE$ at the centre of the distribution. This indicates that a 2T module

![Figure 10. Simulation of photovoltaic efficiency for 2T and 4T connected tandem cells, vs. average photon energy $APE$.](image10)

![Figure 11. Histogram showing solar irradiance distribution over three annual cycles [13]. 90% lies within ±0.05 eV of the mean $APE$.](image11)
matched to AM1.5, but not to the local outdoor conditions, may incur losses in annual efficiency of around 0.5%. Further work is required to determine the prevalence of local variations in APE but a preliminary investigation suggests a range of 1.86 – 1.92 eV.

Ulbrich et al [14] recently demonstrated, by applying blue or IR bias light from an LED in addition to the AM1.5 spectrum, that the current-matching condition for a two-terminal tandem cell coincides with a minimum in the fill factor, and that maximum power is obtained at a point where the top-cell short-circuit current is around 5% greater than that of the bottom cell. Taking this into account modifies the efficiency curve for the 2T tandem cell (figure 10), making the reduction in efficiency more gradual. As a consequence, the 2T annual average efficiency is improved, and the relative benefit of 4T connection is less pronounced. We are currently investigating the potentially important consequences of this, and will report our findings in a subsequent publication.

4. Conclusions

The performance of two-terminal (series connected) and four-terminal (independent) amorphous silicon/microcrystalline silicon tandem solar cells has been modelled using a semi-empirical approach, in which the short-circuit current obtained by integration of the product of the quantum efficiency curve and the solar spectrum is used to scale the $J-V$ curves. The current-voltage curve for the series-connected cells is then obtained by graphical solution. In order to calculate efficiencies it is assumed that (i) the open-circuit voltage and fill-factor for all cells are insensitive to spectral effects and thus remain constant throughout, (ii) in the case of the two-terminal cell the circuit current is the lesser of the two sub-cell photo-generated currents.

The model predicts improved overall efficiency in four-terminal connection, for both as-deposited and light-degraded cells, by 0.8% and 1.3% respectively, under AM1.5 illumination. Relaxation of the current-matching boundary condition enables a thinner top-cell than is optimal in two-terminal connection to be used, which reduces both the degree of degradation and the fraction of the total power delivered by the top-cell. An optimal top-cell absorber layer of around 150 nm (with a 1.1 μm bottom-cell) is indicated.

A filter function is introduced which enables the spectrum to be varied in a broadly realistic manner over the range typically encountered outdoors (average photon energy between 1.8 and 2.0 eV). The two-terminal tandem cell shows a peak in efficiency of 11.2% at $APE = 1.90$ eV, where its top- and bottom-cell currents are matched, but falls steeply to 9.6% and 10.4% at 1.8 and 2.0 eV respectively. In contrast, the four-terminal connection, in which both cells are operated at the maximum power point, shows a monotonic increase in efficiency, from 10.6% to 11.7%.

Figure 12. Annual-averaged efficiencies of 2T and 4T connected tandem cells vs. position of irradiance peak (figure 11) relative to 2T current-matched APE.
A more realistic long-term (annual-average) efficiency is calculated based on a typical annual irradiance vs. APE distribution. From this we estimate that four-terminal connection provides an improvement in absolute efficiency of at least 0.3%, but could exceed 0.5% (or an increase of 5% in annual energy production) under a realistic degree of mismatch.

Our model does not take into account coupling and losses between the sub-cells. In two-terminal connection ohmic loss can result from the intermediate tunnel junction, whereas in the four-terminal case there may be increased optical loss due to the need for thicker TCO layer(s) at the intermediate contact to support the current extracted.

Future plans include development of a model based on the ASA program suite [15-17], which fully takes into account the electrical and optical device properties, and may be calibrated against a series of top- and bottom-cells. This offers greater flexibility to examine the effect of varying the full range of design parameters, including additional TCO contacts, and allows refinements such as an intermediate reflector to be considered.

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