Stochastic aggregation model for the multifractal distribution of matter

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1 Introduction

Two main features of the observable distribution of visible matter are the space correlations of galaxy positions and the mass function of galaxies. As discussed in Pietronero and Sylos Labini on this issue ([1], see also [2],[3]), the concept of multifractal (MF) distribution (including the masses) naturally unifies these two properties. Hence with the knowledge of the whole MF spectrum one obtains information on the correlations in space as well as on the mass function.

From a theoretical point of view one would like to identify the dynamical process that leads to such a MF distribution. In order to gain some insight into this complex problem we have developed a simple stochastic model that includes some of the basic properties of aggregation process and allows us to pose a variety of interesting question concerning the possible dynamical origin of the MF distribution. The dynamics is characterised by some parameters, that have a direct physical meaning in term of cosmological processes. In this way we can relate the input parameters of the dynamics to the properties of the final configuration and produce a sort of phase diagram.

2 The dynamical friction

We suppose that the structures are formed by the aggregation of smaller objects: the main point which we consider is that this process likely to be dependent on the environment in which it takes place. If two particles collide, in order to form a bound state they have to dissipate a certain amount of energy. The basic mechanism of energy dissipation via gravitational interaction is the dynamical friction, that is a systematic deceleration effect to which a test particle undergoes moving through a cloud of other particles. It is a consequence of the fluctuating force acting on the test particle due to the varying complexion of particles neighbours, so it is environment dependent.
Figure 1: The probability of making an irreversible aggregation $P_a$ during a collision is greater in denser regions (a) than in sparse ones (b) and it is more efficient in more dense regions. The dynamical friction, in addition to the local density, depends also from other quantities: the relative velocity and the relative mass of the test particle with respect to the velocity and mass of the background particles [4]. A complete study of this effects as well as the computation of the typical time scales of the dynamical friction in cosmological environment is still in progress. In our simulation when two particles collide there is a probability $P_a$ of irreversible aggregation and probability $1 - P_a$ to scatter. In such a manner the gravitational interaction is simulated only through the aggregation probability $P_a$ that is made dependent from some parameters, $P_a = P_a(\alpha, \beta, ..)$, that define the dynamics of the aggregation process. The environment dependence of the dynamical friction breaks the spatial symmetry of the aggregation process: this is one of the fundamental element that originates a fractal (and multifractal) distribution.

3 The simulations

We consider an aggregation process that growths via binary collisions between particles. The collision rate is uniform in space while the probability of forming an irreversible aggregate, can be a complex function of the local density. We consider a two dimensional grid with $N = 256^2$ sites and periodic boundary condition. At each time step a new particle of unitary mass is added to the system and has probability $P(k; i, j)$ of going in the site $(i, j)$:
Clearly in this model the total mass is not conserved. The total number of particles is \( N = 10^4 \). The probability is normalized by the condition:

\[
\sum_{i,j} P(k; i, j) = 1 \tag{1}
\]

To study the effect of the non linear dynamics we start from the trivial case in which the probability constant and equal in each site. In this case we obtain the binomial distribution as mass function: of course the spatial symmetry of the distribution is not broken and the homogeneity in space is conserved at all times.

The second step is to consider the aggregation probability dependent to the mass of then interacting particles \( P(k; i, j) = \frac{1}{A}(1 + m(k, i)^\alpha) \) where \( A \) is the normalization constant obtained from eq.(1). The parameter \( \alpha \) triggers the effect of the perturbation on the probability due to the mass present in the site. If \( \alpha < 1 \) the perturbation is small and the mass function continues to be bell-shaped. Otherwise if \( \alpha > 1 \) the mass function becomes a power law, with the the exponent that depends from the parameter \( \alpha \) followed by a the cut-off \( M^* \) that depends from the time \( k \), as the mass is not conserved. Even in this case there is not any spatial dependence of the merging process so that the distribution remains homogeneous at all times.

We now consider the environment effect on the merging phenomenon. A way to estimate the local density, and mimicking the dissipation effect in the simulation, is to assign an influence function to each particle: the influence function of the particle in the site \((i', j')\) with mass \( m(k; i', j') \) at the time \( k \) on the site \((i, j)\) is described by:

\[
f(k; i', j', i, j) = \exp \left( -\frac{d(i', j', i, j)}{m(k; i', j')^\beta} \right) \tag{2}
\]

\( d(i', j', i, j) \) is the distance between the site \((i, j)\) where the collision occurs and the generic site \((i', j')\). The multiparticle influence function is \( F(k; i, j) = \sum_{i', j'} f(k; i', j', i, j) \). We keep the aggregation probability to be \( P(k; i, j) = \frac{1}{A}(1 + F(k; i, j)^\alpha) \). We have introduced two free parameters \( \alpha \) and \( \beta \). For a larger value of \( \alpha \) the aggregation occurs in even denser regions, so that the clustering is stronger and the fractal dimension lower: this parameter describes the effect of many particles influence. The parameter \( \beta \) tunes the influence of the size of the mass of each particle: if \( \beta \) is enhanced the larger aggregate dominates the smaller.
To study the spatial properties of the simulation we compute the integral density-density correlation is:

\[ G(\vec{r}) = \int_0^R d\vec{r} \langle \rho(\vec{r}_0) \rho(\vec{r} + \vec{r}_0) \rangle \sim R^D \]

if \( D = d = 2 \) the system is homogeneous otherwise if \( D < d \) is fractal.

At early times the probability is constant for each site and the effect of the perturbation due to the mass distribution is small. For this reason the first sites are occupied in a random manner. Once a certain mass distribution has been developed, the energy dissipation mechanism becomes dominant for aggregation. We can see in the simulation a transient from an homogeneous distribution towards a fractal behaviour: there is a rapid increment of the aggregation probability. The fractal dimension of the final state does not depend from the initial condition but only from the parameters of the dynamics: \( D = D(\alpha, \beta) \). The mass function of the final distribution is a power law with an exponential tail. In a certain region of the phase-space of the dynamics there is the spatial symmetry breaking of the aggregation process and the non linear dynamics generates spontaneously the self-similar fluctuations of the asymptotic state that is an attractive fixed point: there is not any dependence from initial conditions: In the asymptotic time limit (infinite mass) all the sites will be occupied and the growth of the aggregates follows in a self-similar way leading to a MF distribution: the mass function is Press-Schechter like. The breaking of the spatial symmetry in the aggregation probability will lead asymptotically to a region that will never filled,
Figure 3: \( a \): The integrated density-density correlation function: the final states has dimension \( D = 1.6 \) (\( \alpha = 3 \) and \( \beta = 0.5 \)). \( b \): the mass function in the same case

the voids, because the density inside is lower, and with the time evolution always depressed, than the density in cluster.

4 Conclusion

We consider an aggregation process in which the formation of structure is a process that depends from the local environment. The energy dissipation mechanism through the gravitational interaction is the dynamical friction, that strongly depends from the local density. The aggregation is an environment dependent process and this breaks spontaneously the spatial symmetry with respect to the formation of strictures. We have shown that in this model the asymptotic distribution is not generated by an amplification of the initial density fluctuations but the non linear dynamics leads spontaneously the self-similar fluctuations of the asymptotic state, so that there is not any crucial dependence from initial conditions. The necessary ingredients for a dynamics in order to generate a fractal (multifractal) distribution are the breaking of the spatial symmetry, and the Self-Organized nature of the dynamical mechanism.
References

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