THE YAGLOM LAW IN THE EXPANDING SOLAR WIND

G. GOGOBERIDZE$^{1,2}$, S. PERRI$^1$, AND V. CARBONE$^1$

$^1$ Dipartimento di Fisica, Università della Calabria, I-87036 Rende, Italy; g.gogoberidze@warwick.ac.uk
$^2$ Institute of Theoretical Physics, Ilia State University, 3/5 Cholokashvili Ave., 0162 Tbilisi, Georgia

Received 2013 March 1; accepted 2013 April 16; published 2013 May 13

ABSTRACT

We study the Yaglom law, which relates the mixed third-order structure function to the average dissipation rate of turbulence, in a uniformly expanding solar wind by using the two-scale expansion model of magnetohydrodynamic (MHD) turbulence. We show that due to the expansion of the solar wind, two new terms appear in the Yaglom law. The first term is related to the decay of the turbulent energy by nonlinear interactions, whereas the second term is related to the non-zero cross-correlation of the Elsässer fields. Using magnetic field and plasma data from WIND and Helios 2 spacecrafts, we show that at lower frequencies in the inertial range of MHD turbulence the new terms become comparable to Yaglom’s third-order mixed moment, and therefore they cannot be neglected in the evaluation of the energy cascade rate in the solar wind.

Key words: solar wind – turbulence

Online-only material: color figures

1. INTRODUCTION

The standard theory of the solar wind (Parker 1963) predicts the existence of a plasma flow, coming from the hot solar corona, that expands radially in the interplanetary medium without being heated further. In the case of a spherically symmetric adiabatic expansion, the proton temperature $T_p$ should vary as $T_p \sim R^{-4/3}$ with the distance from the sun $R$. However, observations from the Helios spacecraft from 0.3 to 1 AU showed a temperature radial profile $T_p \sim R^{-0.9}$ (Totten et al. 1995), thus implying in situ heating of the solar wind plasma during the expansion (see Marsch et al. 1982; Tu & Marsch 1995 and references therein). A first possible scenario for solar wind heating was proposed by Coleman (1968), who ascribed that phenomenon to the existence of a turbulent cascade, both in the magnetic and in the velocity fields, which is eventually dissipated producing internal energy. This has been considered an efficient mechanism at least for $R < 10$ AU. This idea has been supported by the fact that the proton temperature is positively correlated with the amplitude of magnetic field and velocity fluctuations (Belcher & Davis 1971). Kolmogorov’s theory for stationary, isotropic, and fully developed fluid turbulence (Kolmogorov 1941) predicts for the one-dimensional spectral energy density $E(k)$ in the inertial interval

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3},$$

where $C_k \approx 1.6$ is the Kolmogorov constant and $k$ is the wave number. A similar approach has also been extended to magnetized fluids as the solar wind (Iroshnikov 1963; Kraichnan 1965; Dobrowolny et al. 1980; Goldreich & Sridhar 1995). A recent study performed by MacBride et al. (2008) showed that the energy cascade rates derived by this method are not accurate enough and poorly agree with the proton heating rates.

Assuming a model for spherical symmetric expansion of the solar wind (Verma et al. 1995) and using proton temperature data, Vasquez et al. (2007) found the following expression for the proton heating rate per unit mass at 1 AU:

$$\varepsilon_p = 3.6 \times 10^{-5} T_p V_{sw} J \text{(kg s)}^{-1},$$

where $V_{sw}$ is the solar wind velocity in km s$^{-1}$ and the proton temperature $T_p$ is measured in K. Analysis of the solar wind data performed by Pilipp et al. (1990) and theoretical predictions by Leamon et al. (1999) suggested that the in situ electron heating rate is comparable to $\varepsilon_p$. Equation (2) implies that for fast warm streams in the solar wind the heating rate is of the order of $\varepsilon_p \sim 10^4$ J (kg s)$^{-1}$, whereas for relatively slow, cool streams a typical value of the heating rate is $\varepsilon_p \sim 10^2$ J (kg s)$^{-1}$.

An alternative method for the derivation of the energy cascade rate is to use the Yaglom law, which relates the third-order structure function to the mean energy cascade rate and represents one of the most fundamental relations in the theory of turbulence (Yaglom 1949; Chandrasekhar 1967; Frisch 1995). The generalization of the Yaglom law for magnetohydrodynamic (MHD) turbulence was derived by Politano & Pouquet (1995, 1998). Sorriso-Valvo et al. (2007), Carbone et al. (2009), and MacBride et al. (2008) used solar wind data from various satellites to derive the energy cascade rate by means of the Yaglom law for isotropic MHD turbulence. The latter showed that the energy cascade rates obtained through the Yaglom law are in qualitative agreement with the predictions of Equation (2). Recently, Osman et al. (2011) used multispacecraft data from the Cluster mission to study the energy cascade rate via the anisotropic form of the Yaglom law. It should also be noted that the relatively high level of measurement uncertainties in the plasma data has much less influence on Yaglom’s third-order moment than on the second-order moment of MHD turbulence (Gogoberidze et al. 2012).

In all those previous studies, homogeneity of turbulence has been assumed, while solar wind expansion has not been taken into account. In this paper, we study the influence of the solar wind expansion on the Yaglom law. Namely, using the two-scale expansion model developed by Zhou & Matthaeus (1990), where the “global” variable related to the solar wind expansion is considered as a “slow” variable, whereas nonlinear interactions are mainly determined by local, “fast” variables, we derive a modified expression for the Yaglom law. Other studies of the Yaglom law in the presence of various inhomogeneities include the influence of uniform large-scale velocity shear (Wan et al. 2009, 2010; Stewarz et al. 2011) and the expanding box effect (Hellinger et al. 2013). We show that two extra terms appear in the Yaglom law due to the solar wind expansion. The first extra term is related to non-WKB decay of the turbulent energy due to nonlinear interactions, whereas the second term is caused by the interaction between large-scale fields and the...
cross-correlation of the small-scale inward and outward propagating Alfvén waves. Using data from WIND and Helios 2 spacecrafts, we show that the novel terms in the expansion-modified Yaglom law become comparable to Yaglom’s third-order moment at larger timescales in the inertial range of solar wind turbulence and, therefore, make significant contribution to the estimate of the energy dissipation rate. 

The paper is organized as follows: the modified Yaglom law in the uniformly expanding environment is derived in Section 2, the solar wind data analysis is presented in Section 3, and the conclusions are given in Section 4.

2. THEORETICAL ANALYSIS

We consider incompressible MHD turbulence in the presence of a constant magnetic field \( \mathbf{B}_0 \). The Elsässer variables \( \mathbf{Z}^\pm = \mathbf{v} \pm \mathbf{b}/\sqrt{4\pi \rho} \), the eigenfunctions of counterpropagating Alfvén waves, are usually considered as the most fundamental variables to study MHD turbulence (Biskamp 2003). The dynamics of the Elsässer variables is governed by the incompressible MHD equations

\[
\left( \frac{\partial}{\partial t} + \mathbf{V}_A \cdot \nabla \right) \mathbf{Z}^\pm + (\mathbf{V}^\pm \cdot \nabla) \mathbf{Z}^\pm + \nabla \rho + \lambda^\pm \nabla^2 \mathbf{Z}^\pm + \lambda^\mp \nabla^2 \mathbf{Z}^\mp = 0.
\]  

(3)

Here \( \rho \) is the total (hydrodynamic plus magnetic) pressure, \( \mathbf{V}_A \equiv \mathbf{B}_0/\sqrt{4\pi \rho} \) is the Alfvén velocity related to the background magnetic field, \( \rho \) is the mass density, and \( \lambda^\pm = (\nu \pm \mu)/2 \) where \( \nu \) is the kinematic viscosity and \( \mu \) is the magnetic diffusivity. Although the solar wind plasma is weakly collisional and, consequently, turbulent fluctuations are mainly damped by kinetic mechanisms, in our further analysis we will include the collisional dissipation terms in Equation (3) similar to the other studies (Polito et al. 1998; Carbone et al. 2009).

Since Yaglom’s law is observed at relatively small scales in the inertial range of the solar wind turbulence, the precise form of the dissipation mechanism, acting at very small scales, seems to be unessential for the present study.

It is well known that smooth average properties of the solar wind vary on length scales of the order of the heliospheric distance \( R \). On the other hand, turbulent fluctuations have a correlation length that is much smaller than \( R \) (Matthaeus et al. 2005). This scale separation allows for formulation of the so-called WKB-like transport equations for MHD turbulence in the solar wind (Tu et al. 1984; Zhou & Matthaeus 1990). In the framework of this approach, the magnetic field and the velocity are considered as a sum of mean and fluctuating parts. The mean parts depend only on global, slowly varying coordinate (denoted by \( \mathbf{R} \) hereafter) and is related to the large-scale inhomogeneities in the solar wind plasma, whereas fluctuating (turbulent) fields depend on both the slow and a rapidly varying coordinate, denoted here by \( \mathbf{r} \). Large and small scales can be separated, assuming ergodicity (Matthaeus & Goldstein 1982), by introducing an ensemble averaging operation \( \langle \cdot \rangle \). Here we use the two-scale expansion model presented in Zhou & Matthaeus (1990): equations for fluctuations in the expanding environment are

\[
\begin{align*}
\frac{\partial \mathbf{z}^\pm}{\partial t} + (\mathbf{U} \mp \mathbf{V}_A) \cdot \nabla \mathbf{z}^\pm + \frac{\mathbf{z}^\pm - \mathbf{z}^\mp}{2} \cdot \nabla \cdot \left( \frac{\mathbf{U}}{2} \pm \mathbf{V}_A \right) \\
+ \mathbf{z}^\pm \cdot \nabla \mathbf{U} \pm \frac{1}{\sqrt{4\pi \rho}} \mathbf{B}_0 + (\mathbf{z}^\mp \cdot \nabla) \mathbf{z}^\pm \\
= -\frac{1}{\rho} \nabla \rho + \lambda^\pm \nabla^2 \mathbf{z}^\pm + \lambda^\mp \nabla^2 \mathbf{z}^\mp.
\end{align*}
\]  

(4)

Here \( \mathbf{z}^\pm \) denotes the turbulent fluctuations of the Elsässer variables and \( \mathbf{B}_0 \) is the large-scale mean magnetic field. This equation is derived assuming incompressibility with respect to the small-scale variable \( \mathbf{r} \), i.e., \( \nabla \cdot \mathbf{v} = \mathbf{V}_A \cdot \rho = 0 \).

In hydrodynamics, a mean or large-scale flow sweeps the small-scale eddies without affecting the energy transfer between length scales. In MHD, a mean (or large-scale) magnetic field \( \mathbf{B}_0 \) sweeps oppositely propagating fluctuations \( \mathbf{z}^\pm \) and \( \mathbf{z}^\mp \), which affects the energy transfer. Therefore, contrary to hydrodynamics, in the case of MHD turbulence both linear (sweeping) and nonlinear (straining) timescales are important for the dynamics of turbulence in the inertial interval (for a review, see Zhou et al. 2004 and Zhou 2010). The concept of scale separation introduced above implies that the energy cascade timescale (which in general case depends on both the sweeping and straining of turbulent fluctuations) is much less compared to the characteristic timescale of the solar wind expansion \( R/\mathbf{U} \).

For further simplifications of the analysis, we make several additional assumptions: we consider a uniform radial expansion, i.e., \( \mathbf{U} = (U, 0, 0) \) with \( U = \) constant, and a constant large-scale mean magnetic field \( \mathbf{B}_0 \); we also assume \( U \gg V_A \), which is reasonable in the case of the solar wind (being \( U \sim 400 \text{ km s}^{-1} \) and \( V_A \sim 40 \text{ km s}^{-1} \)), and finally we assume \( \lambda^+ = \lambda^- = \nu \).

For a uniform expansion, \( \mathbf{V}_0 = 2U/R \), so that Equation (4) reduces to

\[
\frac{\partial \mathbf{z}^\pm}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{z}^\mp + \mathbf{V}_A \cdot \nabla \mathbf{z}^\pm + (\mathbf{z}^\mp \cdot \nabla) \mathbf{z}^\pm + (\mathbf{z}^\pm - \mathbf{z}^\mp) \frac{U}{2R} \\
= -\frac{1}{\rho} \nabla \rho + \nu \nabla^2 \mathbf{z}^\pm + \nu \nabla^2 \mathbf{z}^\mp.
\]  

(5)

As we see, due to the expansion, two extra terms appear in Equation (5) compared to the standard incompressible MHD equation for the turbulent fluctuations (Bruno & Carbone 2005). The second term on the left-hand side of Equation (5) describes convective acceleration related to the solar wind expansion, whereas the fifth term describes interaction between large-scale fields and the cross-correlation of the Elsässer fields (Zhou & Matthaeus 1989).

The derivation of the Yaglom law in MHD turbulence (Polito & Pouquet 1998) has already been presented in detail without the expansion effects by Carbone et al. (2009). Derivation of the Yaglom law implies the following steps: one should consider Equation (5) in two different points, \( \mathbf{x}' \) and \( \mathbf{x} \), thus allowing to define \( \mathbf{R} = (\mathbf{x}' + \mathbf{x})/2 \) and \( \mathbf{r} = \mathbf{x}' - \mathbf{x} \). Subtracting those equations in \( \mathbf{x}' \) and \( \mathbf{x} \), we can derive dynamical equations for the differences of the Elsässer fields, \( \Delta \mathbf{z}^\pm = \mathbf{z}^\pm(\mathbf{x}') - \mathbf{z}^\pm(\mathbf{x}) \). We then multiply the obtained equations by \( \Delta \mathbf{z}^\pm \), and ensemble average and take the trace. Under the assumption of local homogeneity, one finally ends up with the dynamical equation for the evolution of \( \langle |\Delta \mathbf{z}^\pm|^2 \rangle \) (Equation (12) in Carbone et al. 2009). It can be readily shown that the two extra terms related to the expansion evolve as follows: the third term on the left-hand side of Equation (5) gives \( \partial \langle |\Delta \mathbf{z}^\pm|^2 \rangle \), whereas the fourth term gives \( \mathbf{U} \langle |\Delta \mathbf{z}^\pm|^2 \rangle / R - \mathbf{U} \langle \mathbf{z}^\mp \cdot \Delta \mathbf{z}^\pm \rangle / R \). Therefore, the dynamical equation for \( \langle |\Delta \mathbf{z}^\pm|^2 \rangle \) takes the form

\[
\frac{\partial}{\partial t} \langle |\Delta \mathbf{z}^\pm|^2 \rangle + \nabla \cdot (\Delta \mathbf{z}^\mp \Delta \mathbf{z}^\pm) + \frac{U}{R} \frac{\partial}{\partial R} (R \langle |\Delta \mathbf{z}^\pm|^2 \rangle) \\
- \frac{U}{R} \langle \mathbf{z}^\mp \cdot \Delta \mathbf{z}^\pm \rangle = 2\nu \nabla^2 \langle |\Delta \mathbf{z}^\pm|^2 \rangle - \frac{4}{3} \nabla \cdot (\mathbf{e}^\pm \mathbf{r}).
\]  

(6)

Here \( \mathbf{e}^\pm = \nu (\partial \mathbf{z}^\pm / \partial \mathbf{r}) \) (where indices indicate the summation over the vector components) are the pseudo energy dissipation
rates of the corresponding Elsässer fields. Equation (6) is valid even for anisotropic MHD turbulence (Carbone et al. 2009). A further simplification implies an assumption of local isotropy. Although MHD turbulence is known to be anisotropic in the inertial range due to the influence of the mean magnetic field (see, e.g., Biskamp 2003), a study performed by Stewarz et al. (2009) showed that the analysis of solar wind data based on isotropic and anisotropic versions of the Yaglom law gives very similar results for the energy dissipation rate of MHD turbulence, therefore, the assumption of isotropy is reasonable for the purposes of the present study. In this framework, the gradient and the Laplacian operators become

\[ \nabla = \frac{2}{r} + \frac{\partial}{\partial r}, \quad \nabla^2 = \left( \frac{2}{r} + \frac{\partial}{\partial r} \right) \frac{\partial}{\partial r}. \]  

(7)

Considering stationary turbulence (thus dropping the first term on the left-hand side of Equation (6)), which could be a reasonable approximation in certain fast wind streams, the integration of Equation (6) in the inertial range (where the influence of the dissipation term on the RHS can be neglected) yields

\[ \langle \Delta z_+^2 \Delta z_-^2 \rangle + \frac{U}{R^2} \int_0^R \frac{y^2}{\Delta z} \left( \frac{\partial}{\partial R} (R \Delta z_+^2) \right) dy - \frac{U}{R^2} \int_0^R y^2 (\Delta z_+ \cdot \Delta z_-) dy = -\frac{4}{3} \varepsilon \tau. \]  

(8)

This equation represents a generalization of the Yaglom law for uniformly expanding solar wind. The second term on the left-hand side of Equation (8) is related to the nonlinear interactions of the fluctuations. Indeed, as was shown by Marsch & Tu (1990), in the absence of nonlinear interactions and cross-correlations, the energy densities of counterpropagating Alfvén waves would have a WKB dependence on heliospheric distance \( E^\pm(R) \rho(R)^{-1/2}(U^2 \mp V_\perp^2) = \text{constant} \) (Marsch & Tu 1989). In the considered case of the uniform expansion \( U = \text{constant} \), and for \( U \gg V_A \), it is easy to show that for fluctuations following WKB scaling, namely, \( E^\pm(R) \sim 1/R \), the second term on the left-hand side in Equation (8) vanishes. Therefore, this term is directly related to the decay of the turbulent energy by means of nonlinear interactions during radial expansion. Note that a similar term also appears in the modified Yaglom law for decaying hydrodynamic turbulence (Danaila et al. 1999), which is known to dominate over Yaglom’s third-order term for relatively large separation \( \tau \).

The second extra term related to the solar wind expansion (the third term on the left-hand side of Equation (8)) is due to the cross-correlation of the Elsässer fields. The identity \( \langle \Delta z_+ \cdot \Delta z_- \rangle = \langle \Delta z^2 \rangle - \langle \Delta b^2 \rangle/4\pi\rho \) shows that the cross-correlation is proportional to the so-called residual energy, i.e., the difference between kinetic and magnetic energies. Taking into account that both solar wind data and numerical simulations of MHD turbulence show an excess of magnetic energy with respect to the energy contained in the velocity fluctuations at all scales in the inertial range (see, e.g., Tu & Marsch 1991; Bavassano et al. 1998; Gogoberidze et al. 2012), we conclude that the third term on the left-hand side of Equation (8) is positive.

Finally, it is worth noting that a dimensional analysis of Equation (8) yields for a ratio between the Yaglom third-order term and the terms related to the expansion to be of the order of \( (v_{rms}/L)/(U/R) \), where \( v_{rms} \) is the rms value of the velocity fluctuations. At 1 AU, this ratio has the order of \( \geq 10 \) and one can conclude that expansion effects are small (or even negligible) in the inertial range of the solar wind turbulence. As we show below, this is not the case and at least for larger timescales in the inertial range these terms are of the same order of magnitude. The reason is that the Yaglom law contains an ensemble average of a third-order quantity that does not have a specific sign, and consequently, the result obtained after averaging is significantly reduced compared to the dimensional estimate \( \langle \Delta z_+^2 \Delta z_-^2 \rangle \sim v_{rms}^2 \).

To study the relative importance of the third-order and of the solar wind expansion terms, we performed analysis of the data from the WIND and Helios 2 satellites. The decay term (second term on the left-hand side) contains derivative with respect to the heliospheric distance, so that it requires data of turbulent fluctuations at least at two different heliospheric distances. For this reason, first we use 3 s resolution plasma and magnetic field data from the WIND satellite to compare the third-order Yaglom and the cross-correlation terms, then we use 81 s resolution data from Helios 2 collected from the same corotating plasma stream at different heliospheric distances (Marsch & Tu 1990; Tu & Marsch 1995) to assess the possible importance of the decay term.

3. SOLAR WIND DATA ANALYSIS

In order to estimate Yaglom’s third-order term and the cross-correlation term, we use magnetic field data from the Magnetic Field Investigation instrument on board WIND at 3 s resolution (Lepping et al. 1995). Density and velocity data are provided by the three-dimensional plasma instrument (Lin et al. 1995). We use observations made during a quiet fast stream. The start time of the interval is 10:00 of 2008 February 2 and the end time is 00:00 of 2008 February 4. During this interval, the solar wind speed remained above 550 km s\(^{-1}\). The energy of compressive fluctuations was an order of magnitude lower than that of incompressible fluctuations and, consequently, magnetic and velocity fluctuations, being mainly Alfvénic, were dominated by the components perpendicular to the local mean field. We made use of Taylor’s hypothesis (Taylor 1938) \( r = \pm U \tau \) to relate temporal changes \( \tau \) in the observational data to spatial variations \( r \) of the turbulent fields by using the solar wind speed \( U \) as a transformation parameter. In the last expression, the plus sign corresponds to the case when the radial axis is directed toward the sun, for instance in geocentric solar ecliptic (GSE) coordinates (where the \( x \)-axis points toward the sun and the \( z \)-axis is perpendicular to the plane of Earth’s orbit around the sun), the minus sign corresponds to the case when the radial axis is directed away from the sun, as in the radial–tangential–normal (RTN) coordinate system. Using Taylor’s hypothesis, Equation (8) can be rewritten as

\[ aY^\pm + D^\pm + M = -\varepsilon, \]  

(9)

where \( Y^\pm = 3\langle \Delta z_+^2 \Delta z_-^2 \rangle \) is the Yaglom term, \( D^\pm = 3U \int_0^R y^2 \left( \partial/\partial R \right) (R \Delta z_+^2) dy/4R^3 \) is the decay term, \( M = -3U \int_0^R y^2 (\Delta z_+ \cdot \Delta z_-) dy/4R^3 \) is the cross-correlation term, and \( a \) is +1 if \( r = U \tau \) and −1 for \( r = -U \tau \), depending on the reference frame, as discussed above.

The energy cascade rate of MHD turbulence \( \varepsilon \) is defined as \( \varepsilon = (\varepsilon^+ + \varepsilon^-)/2 \), so that

\[ aY + D + M = -\varepsilon. \]  

(10)
Here $Y = (Y^+ + Y^-)/2$ is the mean Yaglom term and $D = (D^+ + D^-)/2$ is the mean decay term. The data from the WIND satellite are provided in GSE, so that $\alpha = 1$. The absolute value of the mean Yaglom term (solid line) and the cross-correlation term (dashed line) for the solar wind interval studied here are presented in Figure 1. The mean energy dissipation rate derived from the Yaglom relation $\varepsilon = -Y$ is $\varepsilon \approx 5000 \text{ J kg}^{-1} \text{s}^{-1}$ in qualitative agreement with previous studies (Sorriso-Valvo et al. 2007; MacBride et al. 2008; Osman et al. 2011). The mean proton heating rate for the studied interval, derived by means of Equation (2), $\varepsilon_p = 6050 \text{ J kg}^{-1} \text{s}^{-1}$ is indicated by the red horizontal line in Figure 1. It must be noted that the Yaglom’s term is derived by averaging the third-order mixed term that does not have a fixed sign, so that it requires much more data points for stable convergence with respect to the second-order moment with fixed sign. Indeed, it has been shown that the stable convergence requires up to $10^5$ data points (Podesta et al. 2009; Stewart et al. 2009). If this condition is not fulfilled, then the Yaglom law is observed only in certain solar wind streams (Sorriso-Valvo et al. 2007). Our interval contains about $5 \times 10^3$ points, thus giving quite stable results; they are found to be in agreement with other studies.

As is well known, the residual energy in the solar wind usually follows power-law scaling in the inertial interval $(\Delta z^* \cdot \Delta z^*) \sim \tau^\gamma$ with $\gamma \approx 0.7$ (see, e.g., Gogoberidze et al. 2012 and references therein). Thus, it is expected that in the inertial range the cross-correlation term in Equations (8) and (9) should behave as $M \sim \tau^\gamma$. On the other hand, at large scales, correlations between fluctuations at different points are weakened, i.e., $(\Delta z^* \cdot \Delta z^*) \approx 2(z^* \cdot \z^*)$, therefore, for very large time separations $M \rightarrow \text{constant}$. This behavior is clearly seen in Figure 1.

It can be noted in Figure 1 that at tens of minutes scales in the inertial range the Yaglom and the cross-correlation terms are of the same order of magnitude. The ratio of these two terms on the scale of 30 minutes $|Y|/M \approx 3.5$. We studied several tens of other intervals of fast solar wind streams and the mean value of this ratio at 30 minute scales was $|Y|/M \approx 2.5$. Usually, the mean energy cascade rate in the solar wind is determined as the average value of the dissipation rate derived via the relation $\varepsilon = -Y$ for time separations from 1 minute to 2 hr (MacBride et al. 2008). Since the cross-correlation term is always positive, neglecting $M$ can lead to an overestimation of the real energy dissipation rate by tens of percent. We also analyzed quasi-stationary intervals of slow solar wind streams and the analysis showed that the cross-correlation term in Equation (10) is much less important for slow streams. The typical value of the ratio is $|Y|/M \approx 10–20$. These findings can explain some significant differences found between the energy dissipation rates obtained using Equation (2) and the estimation of the Yaglom law in fast solar wind streams observed by Stewart et al. (2009). These authors found very good agreement between the proton heating rate and energy cascade rate for relatively cold, slow streams of the solar wind, whereas for hot, fast streams the Yaglom law provided significant overestimate of the cascade rate compared to the value predicted by Equation (2).

We also studied data from Helios 2 with 81 s cadence collected from the same corotating stream at different heliospheric distances (Bavassano et al. 1982; Marsch & Tu 1990). We used the data studied previously by Zhou & Matthaeus (1990). The first interval was on 1976 day 76 when the distance from the Sun was 0.65 AU. The start time of the second interval was 00:00 on 1976 day 50 and the end time was 22:04 on 1976 day 51 when the distance from the Sun was 0.87 AU (Marsch & Tu 1990). The data are provided in the RTN reference frame so that $\alpha = -1$ in Equation (10). The mean Yaglom (solid line) and the cross-correlation (dashed line) terms at 0.87 AU are presented in Figure 2. Their behavior is similar to the one found at 1 AU in the WIND data set, although the Yaglom term in Figure 2 is less stable. This is due to the fact that the data set contains only about 3000 points. The absolute value of the mean decay term $D$ (which is actually negative due to decay of the turbulent energy during evolution) is given by the dash-dotted line in Figure 2. We used a linear approximation for the estimation of the derivative with respect to the heliospheric distance in the $D$ term of Equation (9). In particular, for any variable $Q$ we assumed $\partial_R(Q(R)) \approx [Q(0.87 \text{ AU}) - Q(0.65 \text{ AU})]/[0.87 \text{ AU} - 0.65 \text{ AU}]$. As can be seen, the Yaglom and the decay terms are pretty much comparable within all the range of timescales considered. In this context, one could conclude that the use of the Yaglom law without the decay term cannot give a reliable estimation of the cascade rate. However, it is worth stressing that using two intervals relevant to the same corotating stream, gives, in fact,
an upper estimate of the decay term. Indeed, although the data belong to the same corotating stream, the time gap between the data sets is of several weeks, so that the assumption of stationary turbulence is questionable (indeed if the total turbulence energy changes not only because of nonlinear decay but also because of non-stationarity, both of those effects would influence the decay term); further, the rough two-point approximation of the derivative with respect to the heliospheric distance $R$ can also contribute to the overestimation of the real value of $D$.

4. CONCLUSIONS

In this paper, we studied the Yaglom law for MHD turbulence in the expanding solar wind using the two-scale expansion model by Zhou & Matthaeus (1990). We derived the Yaglom in the expanding solar wind using the two-scale expansion effect. One of them is related to the energy decay by nonlinear interactions in MHD turbulence, while the second is related to the non-zero cross-correlation of the Els¨asser fields. Using magnetic field and plasma data from WIND and Helios 2 spacecrafts, we show that for fast solar wind streams, at large timescales in the inertial range of solar wind turbulence, both the decay and the cross-correlation terms are comparable to the Yaglom’s third-order mixed moment, and, therefore, they can give a significant contribution in the assessment of the energy cascade rate. Thus, the disagreement between the proton heating rate estimate, obtained by using Equation (2), and the Yaglom law observed for fast streams in the solar wind can be ascribed to the fact that the extra terms in Equation (8) related to the expansion are usually neglected.

S.P.’s research is supported by “Borsa Post-doc POR Calabria FSE 2007/2013 Asse IV Capitale UmanoObiettivo Operativo M.2.” S.P. and V.C. acknowledge the Marie Curie Project FP7 PIRSES-2010-269297 “Turboplasmas.”

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