Analysis of the Dynamic Behavior of a Vehicle Suspension when Passing over a Bump

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Abstract. Passing a vehicle over bumps generates sudden variations in acceleration with effects on passenger comfort. In this paper we aim to model the movement of a vehicle, considering only vertical movements, neglecting the movement of roll and pitch. Based on differential equations that govern dynamic behavior, a simulation model of motion is built in MATLAB, the Simulink\textsuperscript{TM} module. Suspensions of the vehicle will be considered as passive and semi-active. Passive and semi-active are still the most common suspensions, although active suspensions have been used lately, with mechanical parameters that characterize suspensions, stiffness and dampers being controlled. The paper analyzes the responses given by the suspensions to the passage over bumps, and how they can be mitigated.

1 Introduction

All suspensions in the automotive field are based on the classic arc-damper structure, also known as passive suspensions because they do not receive any outside energy input. These suspensions have inherent limitations as compared to suspensions, based on electror-rectal or magnetoreological fluids, which are called semi-active or active suspensions, as the outside energy is used only in the commissioning or is used to modify the mechanical characteristics [1]. Passive suspensions are based on a compromise in choosing a convenient ratio between elastic and damping characteristics to achieve acceptable performance across the range of working frequencies. In the theory of linear systems, the simplest arc-damping system, representing a system with a degree of freedom, has modeled motion based on a second order differential equation. If this system has a high damping then its performance is good in the area near the resonance frequency and has weaker performance outside of this area. If, however, that system has a weak damping, then its behavior is exactly the opposite [2]. The most well-known model for studies of the dynamic behavior of a passenger car is known as the model-quarter car. This model has been widely used to analyze the performance of passive, semi-active or active systems [3]. The modelling of the vehicle suspension, also on the model of the quarter car, but it has two degrees of freedom is given in Fig. 3. The model represents the vehicle system, reduced to

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the single wheel movement, i.e. the movement of the axle and the vehicle assumed to be only vertical and the perturbation to be identical on each wheel [4]. The main requirements for a suspension are: to isolate the car from road disturbances, ensuring passenger comfort; maintaining good manoeuvrability; maintaining a good road; supporting the static weight of the car. A simulation and analysis model (SAM) requires information on the mechanical parameters used [5]. Measurement of the mechanical characteristics of the suspension requires a static test facility. The simulation and analysis model (SAM) used in the work has 17 degrees of freedom, of which 6 degrees of freedom are required for the rigid body motion of the vehicle, and each wheel is considered to have two degrees of freedom, one for rotating the wheel the other for its vertical movement relative to the vehicle. Suspension modelling has been made so that they are pivoted relative to an instant centre. In this paper, the Simulink software is used to simulate possible laboratory or field tests. It demonstrates the ease with which they can reproduce the tests, regardless of their complexity, both under laboratory conditions and in field conditions.

2 Mathematical models

In Fig. 1 there is presented a very simplified model of a vehicle with one degree of freedom (1-DOF) that takes into consideration only the vertical movement, the pitch and roll movement are not taken into account. The vertical movement of the car is done at a height of 0.15 m. The car tire is considered to have 0.305 m, and the bump is modeled by a step input function. After a horizontal movement over 450 m, less 0.15 m from the zero level, it returns to this level over a distance of 1 m, as shown in Fig. 1

![Fig. 1. Simplified model of suspension](image)

To obtain the differential equations of the movement of a car, two cases are considered. In the first case, the mass of the tire and its damping and stiffness properties are negligible in relation to the mass of the vehicle and the elastic properties of the spring, and the damping properties of the damper. In this case, the simplified model is that of Fig. 2 [6]. In Simulink, the model considered with 1DOF, can be simulated starting from the expression of acceleration given in equation (1).

\[
\ddot{y} = \frac{c(z - y) + k(z - y)}{m},
\]  

(1)
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Fig. 2. Mathematical model for suspension system with 1-DOF

For the second case, both the mass of the wheel and the elastic and damping properties of the tire were taken into account. This gives a system with two degrees of freedom (2-DOF), shown simplified in the Fig 3 [7].

Fig. 3. Mathematical model for suspension system with 2-DOF

In order to realize the Simulink simulation scheme of the system with two degrees of freedom, the expressions of the two accelerations given in the equation (2) are explained from the differential equations of the movement.

\[
\begin{align*}
\ddot{y}_1 &= \frac{c(\dot{y}_2 - \dot{y}_1) + k(y_2 - y_1)}{m_1}, \\
\ddot{y}_2 &= \frac{c(\dot{y}_1 - \dot{y}_2) + k(y_1 - y_2) + c_i(\dot{z} - \dot{y}_2) + k_i(z - y_2)}{m_2}
\end{align*}
\]

(2)

To simulate the equations (2) in Simulink, it is necessary to give the initial conditions corresponding to the geometric form of the bump, which in this case are given by equations (3).

\[
z = \begin{cases} 
0 & t < h_{\text{start}} \\
-0.15m & h_{\text{start}} \leq t < h_{\text{end}} \\
0 & t \geq h_{\text{end}}
\end{cases}
\]

(3)
3 Simulink models

The input and output of the car wheels over a bump is modeled from the Simulink software point of view by two Step functions, which in fact constitute the initial conditions, conditions that mechanically generate disturbing forces. These forces according to equations (1) and (2) give the vertical accelerations of the two masses, and by integration their speeds and displacements are obtained. In first case it is considered the 1-DOF model, shown in Fig. 4 and in second case the 2-DOF model, shown in Fig. 5 [8-9].

![Simulink model for 1-DOF suspension system](image1)

![Simulink model for 2-DOF suspension system](image2)

To enter the simulation parameters of the system with 1-DOF and for running in MATLAB promt, the following commands are given:

```matlab
%Define parameters of the simulation
m=1000; %Car mass (kg)
k1=500; %Spring stiffness (N/m)
c1=100; %Damping coefficient (Ns/m)
V=20; %Car speed (m/s)
holedepth=0.15; %Hole depth (m)
hole_width=1; %Hole width (m)
holestart=450;
hstart=holestart/V; hend=(holestart+hole_width)/V;
%Run Simulink
```

To enter the simulation parameters of the system with 2-DOF and for running in MATLAB promt, the following commands are given:

```matlab
%Define parameters of the simulation
m1=850; %Sprung mass (kg)
```
3. Simulink models

The input and output of the car wheels over a bump is modeled from the Simulink software point of view by two Step functions, which in fact constitute the initial conditions, conditions that mechanically generate disturbing forces. These forces according to equations (1) and (2) give the vertical accelerations of the two masses, and by integration their speeds and displacements are obtained. In first case it is considered the 1-DOF model, shown in Fig. 4 and in second case the 2-DOF model, shown in Fig. 5 [8-9].

Fig. 4. Simulink model for 1-DOF suspension system

Fig. 5. Simulink model for 2-DOF suspension system

To enter the simulation parameters of the system with 1-DOF and for running in MATLAB prompt, the following commands are given:

% Define parameters of the simulation
m=1000; % Car mass (kg)
k1=500; % Spring stiffness (N/m)
c1=100; % Suspension damping constant (Ns/m)
V=20; % Car speed (m/s)
holedepth=0.15; % Hole depth (m)
hole_width=1; % Hole width (m)
holestart=450; hstart=holestart/V; hend=(holestart+hole_width)/V;
% Run Simulink

To enter the simulation parameters of the system with 2-DOF and for running in MATLAB prompt, the following commands are given:

% Define parameters of the simulation
m1=850; % Sprung mass (kg)
m2=150; % Unsprung mass (kg)
k=100; % Spring stiffness (N/m)
kt=400; % Tire stiffness (N/m)
c1=90; % Suspension damping constant (Ns/m)
c2=10; % Tire damping constant (Ns/m)
V=20; % Car speed (m/s)
holedepth=0.15; % Hole depth (m)
hole_width=1; % Hole width (m)
holestart=450; hstart=holestart/V; hend=(holestart+hole_width)/V;
% Run Simulink

4. RESULTS

The results obtained from the simulation are given in the graphic form registered by Scope. For each of the two car models with 1-DOF and 2-DOF, three regimes were analyzed according to different values of the damping coefficient.

Fig. 6. Optimal damping response for 1-DOF system

Fig. 7. Optimal damping response for 2-DOF system

For an optimal regime, a damping coefficient $c_1 = 100 \text{Ns} / \text{m}$ was obtained for the
system considered. All the other characteristics of the two mechanical suspensions of a car
are given in "Define parameters of the simulation".

For second, regime with damping close to the critical damping of the mass-spring-
damper system, the following damping coefficients were considered: \( c_1 = 1000 \text{Ns/m} \) for
1-DOF system, and for 2-DOF system consider \( c = 600 \text{Ns/m} \) and \( c_t = 500 \text{Ns/m} \).

![Fig. 8. The response of the 1-DOF system with supractical damping](image)

And in the third case with high stiffness it consider \( k_1 = 5000 \text{N/m} \) for 1-DOF and
\( k = 1000 \text{N/m} \) and \( k_1 = 4000 \text{N/m} \) for 2-DOF system.

![Fig. 9. The response of the 2-DOF system with supractical damping](image)
For second, regime with damping close to the critical damping of the mass-spring-damper system, the following damping coefficients were considered:

\[ c = 1000 \text{Ns/m} \]

for 1-DOF system, and for 2-DOF system consider

\[ c = 600 \text{Ns/m} \]
\[ c = 500 \text{Ns/m} \]

And in the third case with high stiffness it consider

\[ k = 5000 \text{N/m} \]
\[ k = 1000 \text{N/m} \]
\[ k = 4000 \text{N/m} \]

5. CONCLUSIONS

The results obtained from the simulation lead to the conclusion that in the analysis and design of a suspension one can go from modeling the car as a system with a degree of freedom, although the system with two degrees of freedom has a more complete response and more closer to reality. It is found that in the system response with two degrees of freedom two frequencies are present.

For small dampers or lack of damping in the system, an uncomfortable ride is obtained. Improved comfort is obtained around optimum damping. For some damping values, reverberations are produced. High damping diminishes the effect of reverberation. Simulink software can be used to simulate different situations in the real world.

In order to achieve optimum damping regimes, different semi-active dampers with electrorheological or magnetorheological fluids can be used. Using Simulink to simulate the behavior of these dampers can be useful in identifying rheological parameters of fluids.
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