Spatially quantifying the leadership effectiveness in collective behavior

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Abstract. Among natural biological flocks/swarms or mass social activities, when the collective behavior of the followers has been dominated by the direction or opinion of one leader group, it seems difficult for later-coming leaders to reverse the orientation of the mass followers, especially when they are in quantitative minority. This paper, however, reports a counter-intuitive phenomenon, i.e. Following the Later-coming Minority, provided that the later-comers obey a favorable distribution pattern that enables them to spread their influence to as many followers as possible within a given time and to be dense

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enough to govern these local followers they can influence directly from the beginning. We introduce a discriminant index to quantify the whole group’s orientation under competing leaderships, with which the eventual orientation of the mass followers can be predicted before launching the real dynamical procedure. From the application point of view, this leadership effectiveness index also helps us to design an economical way for the minority later-coming leaders to defeat the dominating majority leaders solely by optimizing their spatial distribution pattern provided that the premeditated goal is available. Our investigation provides insights into effective leadership in biological systems with meaningful implications for social and industrial applications.

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1. Introduction

Regarding biological flocks/swarms, their collective behavior always depends on interactions among group members. In many cases, only a few individuals have pertinent global information [1], such as knowledge about the location of a food source [2], an obstacle [3] or a migration route [4]. It is known that several species can disseminate specific signals that help guide uninformed individuals [1, 4, 5]. On the other hand, valuable leadership may be correlated with age, status or reputation, and it is very common, for many species, that experienced group members play an important role in helping the less experienced. Here, we define leadership as ‘the initiation of new directions of locomotion by individuals, which are then readily followed by other group members’ [6]. It was demonstrated that a small proportion of informed individuals or leaders are sufficient to guide the navigating behavior of the whole group, e.g. foraging fish schools and bee swarms heading for new nest sites [1, 7].

Nevertheless, the nature of bio-groups is not always that simple, since it often happens that leaders within a group may differ from each other in their preferred directions due to different experiences or motivations. This divergence also happens frequently in human society, e.g. different political parties can possess totally different beliefs, and different demographic groups are faithful to different brands. Consensus decisions are very important for biological groups as they allow groups to remain together despite difference in individual preference and consequently help prevent individuals from losing the benefits associated with being part of a large group. Therefore, this kind of mass decision phenomenon with diverging opinions has been extensively studied by scholars previously [1, 8]. However, it is still a pending yet challenging problem whether the group consensus decisions could be controlled or adjusted at a low cost. For instance, it is often encountered that, when the mass followers’ orientation is completely dominated by one leader group, another leader group, which is in quantitative
minority, enters aiming at reversing the followers’ orientation to its own. In biological groups, migrating or foraging creatures have developed an effective way, such as waggle-dance [4] or navigational skill [5] of the scouts in bee swarms, to deviate to a new promising direction at a low cost of additional leadership. Moreover, some bird flocks [9] have developed a hierarchical leading mechanism characterized by a delayed information transmission network in the group. From the biomedical point of view, in the metastasis of malignant tumors [10], some ‘trailblazing’ cells located at the leading edge act as the leaders guiding the cancer cells invasion process. From the aspect of social science, in elections, a new social party always desires to defeat its opponents with as few extra seats in the legislature as possible. In marketing competition, consumers who used to prefer some old brands would switch to new ones with a more favorable advertising strategy.

In recent years, the study of effective leadership has attracted significant attention [1, 7, 11, 12]. In general, many recent works show that the whole group is likely to follow the majority rather than the minority under the guidance of divergent leadership [13]. For instance, the cost to the group as a whole is considerably higher for a ‘despotic’ than for a ‘democratic decision’ or ‘following the majority’ manner [7]. The larger the group, the smaller the proportion of informed individuals needed to guide the group [1]. In social science, it is shown that the public is apt to follow the majority when subjected to divergent opinions from different social parties [14, 15]. Synchronization of clapping [16], escaping panics [17] and group direction selection under divergent preferences [1] strongly suggest the rule of ‘following the majority’ as well. All these bring up an important question: Is it possible for minority latercoming leaders to defeat the dominating majority ones and how? In a closely relevant work addressing this problem, Dyer et al [8] have explicitly claimed that the spatial position of leaders affects both the speed and accuracy with which they can guide an uninformed group to a target. Moreover, they designed an effective way of letting two leaders start from the center and the periphery of the group. However, the underlying quantitative mechanism is still unknown so far, and hence a unified index has not been found to measure the effectiveness of leadership or to predict the future alignment of the mass followers beforehand. This paper aims at quantitatively investigating the role of individuals’ spatial distribution in leadership effectiveness, and at originating a unified leadership index for distribution optimization.

2. Model formulation

For the generality of this work, we investigate leadership in two mainstream dynamic models of collective behavior, i.e. the Vicsek model [18] and Couzin’s three-circle (CTC) model [19]. In the former model, individuals align their motion to the average of their spatial neighbors to achieve global velocity synchronization. In the latter, each individual attracts or repels the agents that are far away from or close to itself, respectively (see the outer and inner layers of figure 1), and steers towards the average heading of its neighbors if they are located at a medium distance from itself (see the middle layer of figure 1). Although mainly originating from [20], the CTC model can be comprehended as inserting a Vicsek model layer into a standard attractive/repulsive model. On the one hand, the virtue of the Vicsek model lies in uncovering the emergence of synchronized velocity and the phase transition along increasing external noise intensity and particle density for self-driven particles. On the other hand, the merits of the CTC model lie in revealing the different formation mechanisms of three types of typical collective behavior, i.e. swarm, torus and migration, by changing the thickness of the
alignment layer (see the middle layer of figure 1). More precisely, a migration/torus/swarm occurs with a thick/thin/negligible alignment layer, respectively. Here, due to the objective of effective leadership investigation, we focus on the migration behavior, and hence the alignment layer should be thick enough. Importantly, despite the different dynamic features, we seek to understand whether defeating majority by minority is possible if the later-comers adopt a suitable distribution pattern. To quantify the effectiveness of the leadership, we propose an evaluation index that can predict reasonably the orientation of the mass followers under the guidance of diverged leadership solely based on the parameters of the distribution patterns.

The original Vicsek and CTC models are slightly modified to incorporate the influence of the earlier and later-coming leaders: two small proportions of the whole group of $N$ individuals are given two conflicting preferred directions of motion representing, for example, the directions to known food resources, migration targets, or the faithfulness for one political belief or commodity brand. More precisely, there are three types of individuals in the modified Vicsek and CTC models: (i) $N_r$ ($\ll N$) earlier-coming leaders moving rightwards whose dynamics are $\bar{x}_r(i)(t+1) = \bar{x}_r(i)(t) + \bar{v}_r$, $i = 1, \ldots, N_r$; (ii) a minority of $N_l$ ($< N_r$) later-coming leaders moving leftwards with dynamics $\bar{x}_l(i)(t+1) = \bar{x}_l(i)(t) + \bar{v}_l$, $i = 1, \ldots, N_l$; and (iii) $N_f$ uninformed individuals with dynamics $\bar{x}_f(i)(t+1) = \bar{x}_f(i)(t) + \bar{v}_f$, $i = 1, \ldots, N_f$ and $N_f = N - N_r - N_l$. Here, $\bar{x}_i$ denotes the position of individual $i$, $\bar{v}_r = v \cdot \angle 0^\circ$, $\bar{v}_l = v \cdot \angle \pi$, $\bar{v}_f(t) = v \cdot \angle \theta_f(t)$ with $v$ being a constant scalar speed, $\angle 0^\circ$, $\angle \pi$ and $\angle \theta_f(t)$ being the directions of the $N_r$ leaders, $N_l$ leaders and $N_f$ followers, respectively, with the definition $\angle \alpha = (\cos \alpha, \sin \alpha)^T$. Note that the leaders move without being affected by the others, and the followers cannot differentiate the leaders and the followers.

The velocity of the $f_i$th follower, i.e. $\bar{v}_f(i)(t)$, has a constant speed $v$ and a direction

$$\theta_f(t+1) = \langle \theta_f(t) \rangle_r + \Delta \theta_f,$$

where $\langle \theta_f(t) \rangle_r$ denotes the average direction of individuals within a circle of radius $r$ surrounding individual $i$ (including itself), which is given as $\arctan(\sin(\theta_f(t))/\cos(\theta_f(t)))$, if $\cos(\theta_f(t)) > 0$, or $\arctan(\sin(\theta_f(t))/\cos(\theta_f(t))) + \pi$ if $\cos(\theta_f(t)) < 0$, where $\langle \sin(\theta_f(t)) \rangle_r$ and $\langle \cos(\theta_f(t)) \rangle_r$ denote the average sine and cosine values. In equation (1), the external noise $\Delta \theta_f$ is a random number chosen with a uniform probability from $[-\xi/2, \xi/2]$. This kind of aligning mechanism to the neighbors can nicely mimic the local dynamics of *go with the stream* in both bio-groups and human society [18].

By contrast, in the CTC model as shown in figure 1, apart from the average direction of the neighbor(s), their attractive/repulsive interactions will also be considered in the updating rule.
merely the alignment one(s) in normalized steady-state alignment index $V_i$. To intensify the leadership mechanism. The global orientation of the whole group is defined as the as to guarantee the emergence of migration behavior, and the individual number is increased to which the alignment layer is set to be much thicker than the repulsion and attraction layers so which merges the second and third cases of equation (3). However, in the standard CTC model, due to the incorporation of conflicting leaders, it becomes harder for the group to achieve a stable synchronized status and hence the oscillation of the group synchronization performance will be remarkably intensified. Thereby, we adopt the slightly modified CTC model (2) and (3) instead to highlight the role of the leaders and to accelerate the propagation of leadership.

Although the velocities change with time, in both Vicsek and CTC models, the acceleration of moving angles is neglected and hence the direction $\theta_i$ is updated simultaneously at each running step, which is related to the ‘dissipative friction forces’ in isokinetic thermostats [18].

To launch our investigation, at the beginning, it is assumed that there are no leaders moving leftwards. Once the orientation of the $N_f$ followers has completely aligned with $N_r$, the $N_l$ leaders appear to compete with the $N_r$ leaders, namely trying to reverse the orientation of the $N_f$ followers. Without loss of generality, we set in both Vicsek and CTC models a square moving region with periodic boundary conditions, the scalar constant speed $v = 0.03$ and the external noise $\xi = 0.1\pi$. Specifically, in the Vicsek model, we set the vision scope radius $r = 1$ as in [18]. In the CTC model as shown in figure 1, we set $R_r = 0.05$, $R_m = 1.0$, $R_u = 1.2$, in which the alignment layer is set to be much thicker than the repulsion and attraction layers so as to guarantee the emergence of migration behavior, and the individual number is increased to intensify the leadership mechanism. The global orientation of the whole group is defined as the normalized steady-state alignment index $V_m = 1 - \theta_u/(\pi/2)$, where $\theta_u$ denotes the steady-state direction of the whole group; thus the values 1, −1 and 0 of $V_m$ mean that the whole group is
completely following the $N_r$ leaders, the $N_l$ leaders and no biased direction, respectively. With these settings, we are now ready to carry out numerical simulations for further investigation of the nature of effective leadership.

3. Numerical simulations and analysis

Now recall the key problem that this paper addresses: Is it possible for the $N_l$ leaders to defeat the $N_r$ leaders and how? Questions closely relevant to this issue have already kindled the interest of not only physicists and biologists but also social scientists and market economy researchers for many years [1, 8, 14, 21]. In most of the relevant studies, it is generally shown that the followers (or most of them at least) would be likely to follow the majority leaders since each individual aligns with the average direction of its neighbors. It can be easily obtained that, if there are two conflicting leader groups in the neighborhood of an individual, the individual definitely tends to follow the majority one. In other words, for a given local region around one individual, the majority always defeats the minority, which thus implies the first decisive parameter for effective leadership, i.e. $P_1$) Persuasive Intensity: leaders’ persuasive power of governing the followers they influence.

Nevertheless, it is observed that, apart from the group size, the interactions between individuals are strongly dependent on their spatial distribution. Admittedly, if the majority and the minority share the same spatial distribution pattern, the majority has either larger influence area or higher particle density, which intensifies their leadership. However, if the leader group sizes are identical, the spatial starting position of leaders affects both the speed and accuracy with which they can guide an uninformed group to a target [8]. Thereby, in order to quantitatively investigate the influence of the spatial distribution of the leadership, it is quite necessary and natural to seek assistance from both social and natural biological collective behavior studies. For example, Dyer et al [8] and Aube and Shield [21] have found that, for human groups in mass circular motion or panic evacuation, having a mixture of leaders positioned in the center and the periphery increases the speed and accuracy of group synchronization. The reason is that this kind of distribution diversity can ‘stretch’ the leadership to influence as many followers as possible. Impressively, this kind of leadership ‘stretching’ mechanism can also find its counterpart in biological flocks/swarms. In these natural groups, effective navigation generally benefits from the informed individuals spreading out through the group. Beckman et al [22] reported that informed scout bees guide uninformed swarms to a new nest by flying through the whole swarm to indicate the direction of travel. In migrating bird flocks or fish schools, accurate navigation can be achieved if the experienced individuals are spread out through the group [8]. All these biological experimental findings bring up another decisive parameter for effective leadership, i.e. $P_2$) Effective Range: the initial influential range of the leader group. We will demonstrate later that distinct dynamics will emerge with different $P_2$.

Now it is clear that the leader group size is not the sole key factor for leadership effectiveness, and hence it can be hypothesized that the minority leaders may defeat the majority ones provided that they have better $P_1$, $P_2$ or both. Next we will verify such a hypothesis. One may note that $P_1$ and $P_2$ are actually competing since $P_1$ favors congregated leader clusters, while $P_2$ tends to spread the cluster out to traverse the whole group, and hence such a contradiction constitutes the main challenge of the problem.
To address this question, as shown in figure 2, we quantify $P2$ by the length of the leaders’ distribution region perpendicular to the movement direction of the leader, namely the normal length $\eta$, and characterize the persuasive intensity $P1$ by the reciprocal of the average spatial distance among the leaders, namely the clustering factor $\sigma = 1/(\frac{1}{2N} \sum_{i,j \in \tilde{N}, j \neq i, \| \vec{x}_i - \vec{x}_j \|_2 \leq \tilde{d}_{2\tilde{N}} \| \vec{x}_i - \vec{x}_j \|_2)}$, where $\tilde{d}_{2\tilde{N}}$ denotes the geographical distance between the $2\tilde{N}$th closest pair of the leader group $\tilde{N}$. With these parameter settings, we implemented simulations with the $N_l$ leaders initially randomly distributed in the $L \times L$ square (figure 2(b)), as considered in most previous works. Afterwards, we examine the effectiveness of $N_l$ leader groups on some typical distribution patterns, such as the $L \times L$ square, $L \times L$ diagonal, $L \times 1$ vertical, $1 \times L$ horizontal and $1 \times 1$ dotted regions as shown in figures 2(b)–(f), respectively. According to the two factors $P1$ and $P2$ of effective leadership, one can expect that the figure 2(d) vertical pattern outperforms the figure 2(b) square pattern because they share the identical $\eta$ but the former has larger $\sigma$. Anologically, the figure 2(d) vertical pattern can also be expected to be superior to the figure 2(e) horizontal pattern since they have the same $\sigma$ but the former have larger $\eta$. However, it is difficult to compare figure 2(d) vertical and figure 2(f) dotted patterns, since the former has larger $\eta$. 

**Figure 2.** (a) Illustration of effective leadership factors $P1$ and $P2$ by clustering factor $\sigma = 1/(\frac{1}{2N} \sum_{i,j \in \tilde{N}, j \neq i, \| \vec{x}_i - \vec{x}_j \|_2 \leq \tilde{d}_{2\tilde{N}} \| \vec{x}_i - \vec{x}_j \|_2)$ and normal length $\eta$ (or the spread length perpendicular to the moving direction), respectively, where $\tilde{N}$ can be $N_l$ or $N_\hat{r}$, and $\tilde{d}_{2\tilde{N}}$ denotes the geographical distance between the $2\tilde{N}$th closest pair of the leader group $\tilde{N}$. (b)–(f) Five typical different distribution patterns of the leaders. Here, group size $N = 500$, $L = 10$, the dashed and solid lines denote the moving area boundary and leaders’ initial distribution region boundary, respectively, and each $\sigma$ is an average over 200 independent initial distributions of $N_l = N_\hat{r} = 10$ leaders on the Vicsek model.
Figure 3. Orientation reversion of the mass followers under the later-coming $N_l$ leaders obeying five typical different distribution patterns given in figures 2(b)–(f), respectively. In (a), the Vicsek model and (b) the CTC model, $N = 1000$, $N_r = N_l = 30$, $L = 10$; in (c), the Vicsek model and (d) the CTC model, $N = 10000$, $N_r = N_l = 300$, $L = 100$. Panels (e) and (f) demonstrate $\eta$ for the Vicsek and CTC models with $L = 10$, respectively. In all the panels, initially, at $t = 0$, all of the follower group $N_f$ has already completely aligned with $N_r$, and then the $N_l$ leaders join. Here, the subscripts ‘$b, c, d, e, f$’ in (e) and (f) represent the distribution patterns in figures 2(b)–(f), respectively. Each point is an average over 200 independent runs and each curve in (a) and (b) (respectively (c) and (d)) takes 6.11 h (respectively 388.9 h) CPU time on a VC++ 6.0, Intel Xeon E5520 dual-core 64 bit 2.26 GHz CPU and 16G RAM computational unit. Note that the simulations are run on a Legend ShenTeng GPU Server Grid system composed of 100 such computational units, and the simulation threads can be calculated in parallel; thus the computational consumption is acceptable.
Figure 4. Leadership propagation in the Vicsek model. (a) $L \times L$ square distribution (figure 2(b)) versus $1 \times 1$ dotted distribution (figure 2(f)); (b) $L \times L$ square distribution (figure 2(b)) versus $L \times 1$ vertical distribution (figure 2(d)). Here, $N = 500$, $N_l = N_r = 10$, $L = 10$, $\xi = 0$ (see equation (1)); the red, blue and green particles denote the followers moving rightwards ($|\theta_i| \in [0, \frac{\pi}{20})$), leftwards ($|\theta_i| \in [\frac{19}{20}\pi, \pi)$) and along unbiased directions ($|\theta_i| \in [\frac{\pi}{20}, \frac{19}{20}\pi)$). For conciseness, we omit the leaders $N_l$ and $N_r$ to focus on the orientations of the followers $N_f$.

From simulations of the temporal evolution of $V_m$ as shown in figures 3(a)–(d) on the Vicsek and CTC models, respectively, five interesting and concrete phenomena are observed:

(i) Figures 2(d) and (f) defeat the earlier-coming figure 2(a) and reverse the orientation of the followers, while figures 2(b), (c) and (e) do not.

(ii) Figures 2(d) and (e) are the most and least effective patterns, respectively.

(iii) Compared to the Vicsek model, it is harder and takes longer for $N_l$ leaders to reverse the $N_f$ followers’ direction in the CTC model.

(iv) Figure 2(e) is less effective in the CTS model than in the Vicsek model.

(v) The global tendency of $V_m$ almost remains the same for different sized particle groups.

First, let us explain the difference between the Vicsek and CTC models, as shown by phenomena (iii) and (iv). This difference is rooted in the attractive/repulsive interactions of the CTC model in figure 1, which have remarkably compressed the spatial distribution region of the followers $N_f$ before they completely follow the earlier-coming leaders $N_l$. By mathematical analysis [3], if the interactive proximity net of the CTC model is jointly connected, then the whole group will eventually converge to a compact lattice formation where the distance between
Figure 5. Later-coming \(N_l\) leaders obeying \(L \times 1\) vertical pattern (see figure 2(d)) versus earlier-coming \(N_r\) leaders obeying \(L \times L\) square pattern (see figure 2(b)). (a) \(V_m\) from the model simulations on the Vicsek model; (b) \(V_{m1}\) approximated by equation (4) in the Vicsek model; (c) \(V_m\) from the model simulations on the CTC model; (d) \(V_{m1}\) approximated by equation (4) in the CTC model. Here, \(N = 1000, L = 10\); the regions above the white dashed lines represent the steady-state values of \(V_m\) and \(V_{m1}\) with \(N_l < N_r\) after \(M = 15\,000\) running steps. The warm and cold color regions represent the orientation of following the majority \(N_r\) and minority \(N_l\), respectively.

Each pair of adjacent particles remains constant. That is why the attractive/repulsive interactions compress the flock formation, and hence when the followers have completely aligned with the \(N_r\) earlier-coming leaders, the distribution has been substantially compressed, which decreases the influential area of the \(N_l\) later-coming leaders and hence \(\eta_{e,f} < 1\). Thereby, as shown in figures 3(e) and (f), the value of \(\eta\) has been greatly reduced, while \(\sigma\) remains unchanged, which nicely explains phenomena (iii) and (iv). Of course, by our numerical simulations, as the attractive and repulsive layers in the CTC model approach zero, the compression effect of \(\eta\) will shrink and thus the difference between the Vicsek and CTC models will gradually disappear.

With our analysis above, all these observations are self-consistent with the aforementioned hypothesis on the leadership factors \(\eta\) and \(\sigma\) and hence help us understand more deeply the roles of \(P1\) and \(P2\). More precisely, \(P1\) favors governing the followers locally and then propagating the influence to the proximity population, while \(P2\) has been realized by sufficiently spreading the \(N_l\) leaders out perpendicularly to their movement direction. Note that moderate noise will
not substantially influence the tendencies of the temporal evolution curves but just induce some mild oscillations.

From the temporal evolution of the spatial distribution patterns of the three follower subgroups moving leftwards, rightwards and along unbiased directions, one can see more clearly the leadership propagation procedure in figure 4. As shown in figure 4(a), initially all the followers are likely to follow the leaders \( N_f \); then as the \( N_f \) leaders appear from the top-right \( 1 \times 1 \) unit, most of the followers at the top-right corner have been governed by \( N_f \), which effectively deviate their adjacent followers after a while. After about 200 running steps, these re-directed followers begin to invade the internal regions of the mass followers following \( N_r \) and finally reverse the orientation of the mass followers. Analogously, as in figure 4(b), when \( N_l \) leaders obeying \( L \times 1 \) vertical pattern appear from the right side of the square cell, they are locally stronger than \( N_r \) around the right edge. Thereafter, the leadership of \( N_l \) begins to invade the red mass followers moving rightwards and finally reverse their directions about 360 steps later. Thereby, these leadership propagation procedures nicely explain the observations in figure 3, i.e. the spatial distribution pattern favoring larger \( P1(\sigma) \) and \( P2(\eta) \) will have stronger leadership. Still worth mentioning is that it takes considerable running steps for figure 2(f) to propagate its influence to remote followers, so that the converging time is much longer than those for figures 2(b)–(e).

Based on the aforementioned simulations and analysis, one can be delighted to infer that even if the \( N_f \) followers have been completely dominated by the \( N_l \) leaders, it is highly possible for the minority \( N_l \) leaders to reverse the followers’ opinion by their better distribution pattern. As shown in figures 5(a) and (c) for the Vicsek and CTC models, \( N_l \) and \( N_r \) are initially randomly distributed in a favorable \( L \times 1 \) vertical (see figure 2(d)) and \( L \times L \) square (see figure 2(b)) regions, respectively, with \( N_l < N_r \) as shown in the area above the white dashed line. It is easy to see that \( V_m \) will be a positive (respectively negative) value if \( N_f \) followers follow the majority \( N_r \) (respectively minority \( N_l \)) leaders. Indeed, it is observed in figures 5(a)–(c) that \( N_f \) would be even more likely to follow the minority \( N_l \), since the region corresponding to \( V_m < 0 \) (i.e. moving leftwards) is larger than that of \( V_m > 0 \) (i.e. moving rightwards). In order to show this appealing counter-intuitive phenomenon more vividly, we have highlighted the ‘minority-following’ and ‘majority-following’ regions, respectively, by cold (blue, purple, dark green and grey) and warm (yellow, rose, orange and red) colors in the space spanned by \( N_r \) and \( N_l \). For both the Vicsek and CTC models, the cold color region is surprisingly much larger than the warm one, implying that it is highly possible for the later-coming minority \( N_l \) leaders to completely reverse the orientations of all the \( N_f \) followers from \( V_m = 1 \) to \( V_m = −1 \) owing to their superior distribution pattern. Remarkably, due to the attractive and repulsive interactions, the ‘following minority region’ (the cold color region above the white dashed line) in the CTS model shrinks compared to the Vicsek model. On the other hand, the bottom-right triangle region under the dashed lines, where \( N_l > N_r \), certainly belongs to ‘following the majority’ region since the \( N_l \) has both numerical and distribution advantages, but we still exhibit it for completeness.

Now with these factors \( \eta_{r,f} \) and \( \sigma_{l,f} \) (normal lengths and clustering factors of the \( N_l \) and \( N_r \) leader clusters, respectively), it is possible and necessary to design a suitable index to make a concrete comparison between different leaderships. To this end, we further define an influencing region ratio \( R_\eta = \eta_l/\eta_r \) and a clustering intensity ratio \( R_\sigma = \sigma_l/\sigma_r \), by which we find that the effectiveness of leadership can be reasonably predicted. Let the \( N_r \) leaders randomly distribute in an unbiased \( L \times L \) square (figure 2(b) with \( L = 10 \)), and the \( N_l \) leaders (with \( N_l = N_r = 30 \)) randomly distribute in \( L \times L, L \times (L − 1), \ldots, L \times 1 \) rectangular regions,
Figure 6. Roles of the clustering factor ratio $R_\sigma$ and the normal length ratio $R_\eta$ in the orientation $V_m$ of the followers. (a) Effect of $R_\sigma$ on the Vicsek model; here $R_\eta = 1$. (b) Combined effects of both $R_\sigma$ and $R_\eta$ on the Vicsek model. (c) Effect of $R_\sigma$ on the CTC model; here $R_\eta = 0.5$. (d) Combined effects of both $R_\sigma$ and $R_\eta$ on the CTC model. Here, $N = 1000$, $N_l = N_r = 30$, $L = 10$; the approximate index $V_{m1}$ in equation (4) is compared to the actual $V_m$ value resulting from simulations to highlight its effectiveness.

respectively, and hence $R_\eta$ remains constant ($R_\eta = 1$ for the Vicsek model and $R_\eta = 0.5$ for the CTC model, as shown in figures 3(e) and (f), respectively), while $\sigma_l$ increases monotonically. As shown in figures 6(a) and (c) for both the Vicsek and CTC models, $V_m$ drops with increasing $R_\sigma$ until asymptotically approaching a saturation value of $-1$, indicating dominant leadership of the $N_l$ leaders. To investigate the role of $R_\eta$ and $R_\sigma$ simultaneously, let the $N_l$ leaders (with $N_l = N_r = 30$) randomly distribute in $L \times L$, $(L - 1) \times (L - 1), \ldots, 1 \times 1$ square regions, respectively; then $\eta_l$ is falling while $\sigma_l$ is rising along this distribution sequence. Figures 6(b) and (d) show that $V_{m1}$ can satisfactorily approximate $V_m$, which reduces with increasing $R_\eta$ and $R_\sigma$. As a consequence, one can confidently conclude that the minority later-coming leaders do have the potential to reverse the followers only if they have larger values of $\eta$, $\sigma$ or both.

The observation from these extensive simulations suggests that we could predict the leadership effectiveness for two competing leader groups solely based on a suitable combination of the geometrical parameters $\eta$ and $\sigma$. In fact, taking into consideration the maximal and minimal saturation values 1 and $-1$ of $V_m$ and $V_m(1, 1) = 0$, we hereby propose a discriminant
index $V_{m1}(R_\eta, R_\sigma)$ as

$$V_{m1}(R_\eta, R_\sigma) = w_1 \tanh(\gamma(1 - R_\sigma)) + w_2 \tanh(1 - R_\eta).$$  \hspace{1cm} (4)

Here, $w_1$ and $w_2$ are the weights of the effects of $R_\sigma$ and $R_\eta$, respectively, and $\gamma$ is used to adjust the origin-traversing slope of the tanh(·) function, which endows $V_{m1}$ an essential degree of freedom. Parameters $w_1$, $w_2$ and $\gamma$ are to be determined. According to our extensive numerical simulations in figures 6(a)–(d), the parameter $\gamma \in [1.2, 2.6]$ yields satisfactory approximation performance. Thereby, without loss of generality, we set $\gamma = 2.4$ and then apply least-square estimation [23] to identify $w_1 = 1.0$, $w_2 = 0.4$ commonly for all the results in figures 6(a)–(d). Note that this index is self-consistent in the sense that (i) the maximal and minimal saturation values remain at 1 and −1 for the feasible ranges of $R_\sigma$ and $R_\eta$ and (ii) if either the value of $R_\sigma$ or $R_\eta$ remains constant, then $V_{m1}$ will be determined only by the other one. More importantly, the index holds for both the Vicsek and CTC models with different particle numbers, showing the generality of this investigation into leadership effectiveness. We still note that the difference between the approximating performances of $V_{m1}$ on the Vicsek and CTC models lies in the attractive and repulsive interactions of the latter model (see figure 1), which have compressed the distribution region of followers as shown in figure 3(f).

It is important to note that the definition of the clustering factor $\sigma$ naturally takes the effect of the leaders’ numbers $N_l$ and $N_r$ into account. Remarkably, equation (4) with the same parameters $\gamma = 2.4$, $w_1 = 1.0$ and $w_2 = 0.4$ can account for the effective leadership for fixed patterns, but varying numbers $N_l$ and $N_r$ (figure 5(b)). Comparison of figures 5(a)–(d) shows clearly that $V_{m1}$ based on the spatial distribution patterns of the leaders can simulate the orientation $V_m$ of the mass followers under divergent leaderships irrespective of the detailed alignment dynamics of self-propelling particle models. Thereby, with the index $V_{m1}(R_\eta, R_\sigma)$, one can predict the orientation of the mass followers under conflicting leaderships. By this means, an economical way can also be designed for the later-coming leaders to defeat the majority earlier-coming ones. Note that the followers’ density and distribution also play a significant role in the eventual orientation [8]. However, in order to focus on the leaders’ spatial distribution, we let the followers be initially evenly distributed in the entire $L \times L$ region without loss of generality and the density will remain constant for this entire region since no follower(s) can enter or escape.

Once again, in order to intensify their leadership and thereby defeat the dominating majority earlier-coming leaders, the later-coming leaders should spread out in the direction perpendicular to its movement direction (see figure 2(d)), which is analogous to the scanning manner of a ‘scanner’ (see figure 4(b)), so as to influence as many followers as possible within a given period. Meanwhile, they should congregate closely to accumulate enough power to ‘persuade’ the followers they can influence. With an appropriate pattern of spatial distribution, the later-coming ‘minority’ can successfully deviate the mass followers that have been dominated by the earlier-coming ‘majority’. In this sense, counter-intuitively, the uninformed followers will not follow the quantitative ‘majority’ but the better pattern of ‘leadership spatial distribution’.

4. Conclusion

In summary, uncovering the nature of effective leadership of collective behavior is of great theoretical and practical significance. We have shown that later-coming leaders, even in quantitative minority, have the potential to defeat the earlier-coming dominating ones, if only
the former obeys a better spatial distribution pattern. This kind of superior pattern favors a large influential region and a high clustering factor. In order to quantify these effects, we have proposed an index merely based on the geometrical parameters of the distribution patterns of the leaders, which provides a reasonably good prediction of the leadership effectiveness. With this index, one can also design an economical way for the later-coming leaders to defeat the majority earlier-coming ones provided that the premeditated target is available. Extensive simulations on two mainstream self-propelling particle models, i.e. the Vicsek and CTC models, strongly support the generality of our result on the effective leadership mechanism.

Our investigation has launched a new exploration on the essential rules that govern leadership potentials. In natural science, the results have the potential for explaining how biological groups, such as migrating birds, foraging insects or even metastasizing cancer cells, optimize their leaders’ distribution to improve leadership effectiveness. In escaping panics, the rules revealed here can facilitate effective distribution of security staff so as to quickly evacuate an escaping crowd. From the engineering point of view, industrial multi-agent systems (such as multi-robot groups and unmanned aerial vehicle formations) can also be expected to benefit from this work to improve their adaptability to new environments. Thus, our investigation sheds some light on the leadership effectiveness optimization among biological systems, with meaningful implications for social and industrial applications.

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