Spinning deformations of the D1-D5 system
and a geometric resolution of Closed Timelike Curves

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Abstract

The $SO(4)$ isometry of the extreme Reissner-Nordstrom black hole of $\mathcal{N} = 1$, $D = 5$ supergravity can be partly broken, without breaking any supersymmetry, in two different ways. The “right” solution is a rotating black hole (BMPV); the “left” is interpreted as a black hole in a Gödel universe (GBH). In ten dimensions, both spacetimes are described by deformations of the D1-D5-pp-wave system with the property that the non-trivial Closed Timelike Curves (CTC’s) of the five dimensional manifold are absent in the universal covering space of the ten dimensional manifold. In the decoupling limit, the BMPV deformation is normalizable. It corresponds to the vev of an IR relevant operator of dimension $\Delta = 1$. The Gödel deformation is sub-leading in $\alpha'$ unless we take an infinite vorticity limit; in such case it is a non-normalizable perturbation. It corresponds to the insertion of a vector operator of dimension $\Delta = 5$. Thus we conclude that from the dual (1+1)-CFT viewpoint the $SO(4)$ R-symmetry is broken ‘spontaneously’ in the BMPV case and explicitly in the Gödel case.

1 Introduction

Since the work of Vafa and Strominger [1] and Callan and Maldacena [2], the D1-D5 system has been a particularly instructive D-brane configuration for the study of black holes in string theory (see [3] [4] [5] [6] for reviews). In this paper we give still another example of its richness. We present a simple, supersymmetry preserving, deformation of the D1-D5 system with angular momentum, interpreted in five dimensions as a black hole in Gödel’s universe (GBH).¹ The latter is a supersymmetric solution to $\mathcal{N} = 1$, $D = 5$ Supergravity recently found, albeit not interpreted as a black hole in a rotating universe, by Gauntlett et al. [14].

Gödel’s universe [15] has intrigued relativists for over 50 years. It is a solution to Einstein’s equations with an apparently harmless type of matter: a pressureless, positive energy density, perfect

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²Different supersymmetric deformations of the D1-D5 system with angular momentum have been considered in [11] [12] [13].
fluid and a negative cosmological constant. However, the solution is plagued with Closed Timelike Curves (CTC’s), which make the Cauchy problem ill-defined and seemingly violate causality. Still, attempts have been made to make sense of spacetimes with CTC’s at the classical level \[16\] and the discussion concerning their consistency shifted to the quantum realm \[17\].

String theory has already shed some light on this discussion using exactly the D1-D5 system. A five dimensional, supersymmetric, rotating, asymptotically flat black hole solution, dubbed BMPV black hole \[7\], has an entropy described in terms of states of the dual (1+1) dimensional CFT, by the AdS/CFT correspondence. As the rotation of the black hole is increased beyond a threshold, we will have ‘naked’ CTC’s all over the spacetime; at this point the CFT states one could associate to the spacetime violate the CFT unitarity bound \[8\]. Thus, quantum mechanics breaks down when the spacetime becomes a usable ‘Time machine’.

In an apparently unrelated development \[14\], a Gödel type solution was found in five dimensions which is maximally supersymmetric. Remarkably, as we shall show, a slight modification of the BMPV black hole describes a black hole in this universe, a solution that preserves one half of the supersymmetry of \(D = 5, N = 1\) Supergravity. It follows that the Gödel universe black hole (GBH) can also be seen as a deformation of the D1-D5-pp-wave system, and that we can use known machinery to analyze this solution from several viewpoints.

We start, in section 2, by reviewing the solutions of Gauntlett et al. \[14\] of interest to us.

In section 3 we will study comparatively the classical five dimensional geometry of the BMPV black hole versus the black hole in Gödel’s universe (GBH). They belong to the same class of solutions and differ by the choice of a right versus left invariant one-form for squashing a three-sphere. We will discuss the near horizon geometries -two types of ‘squashed’ \(AdS_2 \times S^3\), the zero mass limits -Gödel spacetime and a singular ‘repulson’- and the supersymmetry of the solutions. Concerning supersymmetry we note that whereas the supersymmetry of the Gödel black hole is enhanced in the zero mass limit, the one of the BMPV black hole is not. We point out that the entropy of the BMPV black hole depends both on mass and angular momentum, whereas the one of GBH depends solely on mass. From this viewpoint it is reasonable to say that the BMPV is a rotating black hole whereas the GBH is a static black hole in a rotating universe.

In section 4 we uplift the solutions to 10D as deformations of the D1-D5-pp-wave system. We observe that the non-trivial CTC’s of the five dimensional solution become topological and hence absent in the universal covering space of the ten dimensional solution. This had already been noticed for the BMPV black hole in \[8\] and subsequently arose in the supergravity description of supertubes \[9\]. But it is the first time a geometric ‘resolution’ of CTC’s in a Gödel-type spacetime is given. This resolution is quite a sensitive process. In particular, the non-trivial CTC’s are still present in the IIA or M-theory uplifting of the five dimensional Gödel solution \[14\]. So, the process does not survive T-duality. Moreover, as we shall see, considering a more general family of five dimensional solutions, the resolution in type IIB does not occur. But it is striking that it works for the two paradigms of spaces with CTC’s: a rotating black hole and a Gödel universe.

By taking the decoupling limit we observe that, in the dual Conformal Field Theory, the BMPV angular momentum is associated to a relevant operator (in the IR), whereas the Gödel angular momentum is associated to an irrelevant one. The latter, however, vanishes as \(\alpha’ \rightarrow 0\) unless we take a (double) scaling limit of infinite spacetime vorticity. The BMPV spacetime ‘perturbation’ is normalizable and thus it is not a deformation of the (1+1) dimensional CFT, it is a ‘deformation’ of
the states describing the static black hole, which acquire R-charge; thus the SO(4) R-symmetry is broken spontaneously. The Gödel universe black hole, in the infinite vorticity limit, corresponds to a non-normalizable perturbation, and therefore to a deformation of the (1+1) dimensional CFT by the insertion of the aforementioned irrelevant operator; the SO(4) R-symmetry is explicitly broken.

Five dimensional black holes have been under the spotlight recently for a different reason: a counterexample to black hole uniqueness theorems was found [18]. This counterexample pertains uncharged, rotating black holes. Uniqueness seems, however, to cover static [19] and supersymmetric [20] black holes. We believe the supersymmetric black hole in a rotating universe discussed herein is still another example of the surprises and richness of higher dimensional theories. Moreover, it shows how two different black hole spacetimes, both with angular momentum and within the same theory can be very clearly distinguished by the dual string theoretical description.

2 Supersymmetric solutions of $\mathcal{N} = 1$ Supergravity in D=5

The minimal supergravity theory in five spacetime dimensions was constructed in [23, 24]. We take the action to be

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - F^2 - \frac{2}{3\sqrt{3}} \varepsilon^{\alpha\beta\gamma\mu\nu} F_{\alpha\beta} F_{\gamma\mu} A_\nu \right),$$

(2.1)

where $F = dA$, $\varepsilon$ is the Levi-Civita tensor, related to the Levi-Civita tensor density by $\varepsilon^{\alpha\beta\gamma\delta\mu} = \varepsilon^{\alpha\beta\gamma\delta\mu}/\sqrt{-g}$ and we use a ‘mostly plus’ signature. The equations of motion are

$$R_{\mu\nu} = 2 \left( F_{\mu\alpha} F_{\nu}^{\alpha} - \frac{1}{6} g_{\mu\nu} F^2 \right), \quad D_{\mu} F^{\mu\nu} = \frac{1}{2\sqrt{3}} \varepsilon^{\alpha\beta\gamma\mu\nu} F_{\alpha\beta} F_{\gamma\mu}. \quad (2.2)$$

Following Gauntlett et al. [14], the supersymmetric solutions with a timelike Killing vector field of $\mathcal{N} = 1$ Supergravity in D=5 can be written as

$$ds^2 = -f^2[dt + \omega]^2 + f^{-1} h_{ij} dx^i dx^j;$$

$$F = \frac{\sqrt{3}}{2} d(f[dt + \omega]) - \frac{1}{\sqrt{3}} G^+. \quad (2.3)$$

The five dimensional manifold, $\mathcal{M}$, is a non-trivial line bundle, $\mathcal{M} \xrightarrow{\pi} \mathcal{B}$, with the following properties:

a) The base space, $\mathcal{B}$, is a four dimensional hyper-Kähler manifold with metric, $h_{ij};$

b) $f$ is a globally defined function on $\mathcal{B}$ and $\omega$ is a locally defined one-form on $\mathcal{B}$. The two form $fd\omega$ is split in its self-dual and anti-self-dual parts (with respect to the hyper-Kähler metric) as

$$fd\omega = G^- + G^+; \quad (2.4)$$

c) The Bianchi identity and equation of motion for the gauge field require

$$dG^+ = 0, \quad \Delta f^{-1} = \frac{2}{9} (G^+)^{ij} (G^+)_{ij}. \quad (2.5)$$

\[ \text{This solution assumes } f > 0; \text{ an analogous set of solutions exists with } f < 0. \]
Note that if we choose $G^+=0$, that is $d\omega$ is anti-self-dual, the equations of motion simply require $f^{-1}$ to be harmonic on $\mathcal{B}$. This includes all known supersymmetric solutions prior to [14], with a timelike Killing vector field, in particular the BMPV black hole [7] (which is a special case of the solutions in [25, 26]).

2.1 Solutions with flat base space

We take $\mathcal{B}$ to be $\mathbb{R}^4$, and write its metric in terms of the left or right invariant one forms (respectively) on $S^3 \simeq SU(2)$:

$$ds_{\mathbb{R}^4}^2 = dr^2 + \frac{r^2}{4} \left[ (\sigma_L^1)^2 + (\sigma_L^2)^2 + (\sigma_L^3)^2 \right] = dr^2 + \frac{r^2}{4} \left[ (\sigma_R^1)^2 + (\sigma_R^2)^2 + (\sigma_R^3)^2 \right].$$

(2.6)

The explicit form of $\sigma_{R,L}^i$ in terms of Euler angles can be found in [14]. Here, we note that this one-forms obey

$$d\sigma_R^i = \frac{1}{2} \epsilon_{ijk} \sigma_R^j \wedge \sigma_R^k, \quad d\sigma_L^i = -\frac{1}{2} \epsilon_{ijk} \sigma_L^j \wedge \sigma_L^k.$$ 

(2.7)

The difference in sign is important in the following. Such difference is associated to the different sign choice for the two $SU(2)$ algebras in $SO(4)$. In terms of the dual vector fields to $\sigma_{L,R}^i$, which we denote as $\xi_{L,R}^i$, we have

$$[\xi_R^i, \xi_R^j] = -\epsilon_{ijk} \xi_R^k, \quad [\xi_L^i, \xi_L^j] = \epsilon_{ijk} \xi_L^k.$$ 

(2.8)

The natural ansatz for the one-form $\omega$ is

$$\omega = g(r) \sigma_{L,R} ,$$

(2.9)

where $\sigma_{L,R}$ is any of the left-invariant or any of the right-invariant one-forms; we treat the two cases in parallel. It follows that

$$G^+ \equiv \frac{f}{2} (d\omega + *d\omega) = \frac{f}{2} \left( \dot{g} \pm \frac{2}{r} g \right) \left[ dr \wedge \sigma_{L,R} \pm \frac{r}{2} d\sigma_{L,R} \right].$$

(2.10)

The top (bottom) signs correspond to right (left) invariant one-forms. This convention will be used throughout the paper. Requiring $dG^+ = 0$, yields the condition

$$f (r \dot{g} \pm 2g) = \ell r^{\pm 2},$$

(2.11)

where $\ell$ is a dimensionful constant. Feeding this back in (2.10), we find,

$$G^+ = \pm \frac{\ell}{4} d\left( r^{\pm 2} \sigma_{L,R} \right), \quad \Rightarrow \quad (G^+)_{ij} (G^+)_{ij} = \frac{4\ell^2 r^{\pm 4}}{r^4}.$$ 

(2.12)

Solving the remaining equation of motion (2.5) with (2.12), we obtain [14]

$$f^{-1} = \begin{cases} \lambda + \frac{\mu}{r^2} + \frac{\ell^2}{9} r^2, \\ \lambda + \frac{\mu}{r^2} + \frac{\ell^2}{27} r^6 \end{cases}, \quad g(r) = \begin{cases} j \frac{r^2}{2} + \ell \left( \frac{\mu}{2} + \frac{\lambda r^2}{4} + \frac{\ell^2 r^4}{54} \right), \\ j r^2 - \ell \left( \frac{\lambda}{4 r^2} + \frac{\mu}{6 r^4} + \frac{\ell^2 r^8}{270} \right) \end{cases}.$$ 

(2.13)
\( \lambda, \mu, j \) are integration constants. Notice that the dimensions of \( j, \ell \) are \([ j_L, \ell_L ] = L^3, [ j_R, \ell_R ] = L^{-1} \). They are both vorticity (or ‘angular momentum’) parameters since they give non-trivial contributions to the \( dt dx^i \) terms in the metric and to the magnetic dipole term \( dx^i \wedge dx^j \) in the gauge field. But whereas ‘\( \ell \)’ contributes as a source for the function \( f \), ‘\( j \)’ decouples from it.

To write down the two solutions more explicitly we parameterize \( SU(2) \) by Euler angles \(( \theta, \phi, \psi )\).

The metric on the base space is then

\[
ds_{\mathbb{R}^4}^2 = dr^2 + \frac{r^2}{4} \left( d\theta^2 + d\psi^2 + d\phi^2 + 2 \cos \theta d\psi d\phi \right) .
\]

(2.14)

Pick up the following left and right invariant one-forms

\[
\sigma^3_L = d\phi + \cos \theta d\psi , \quad \sigma^3_R = d\psi + \cos \theta d\phi .
\]

(2.15)

The metric and gauge potential for the two solutions may then be written as

\[
ds^2 = -f^2 [dt + g \sigma^3_{L,R}]^2 + f^{-1} ds_{\mathbb{R}^4}^2 , \quad A = \frac{\sqrt{3}}{2} \left( f dt + \left[ fg - \frac{\ell}{6} r^{\pm 2} \right] \sigma^3_{L,R} \right) .
\]

(2.16)

3 Properties of the five dimensional spacetimes

In the remaining of this paper we will focus on the solutions (2.16) with \( \ell = 0 \), since these are sufficiently rich. These solutions are characterized by three parameters: \( \mu, j, \lambda \). The right solution is the BMPV black hole. We will interpret the left solution as a black hole in a Gödel type universe (GBH). For \( \lambda = 0 \) we will be looking at the near horizon geometries of the two types of black holes. For \( \mu = 0 \) we will be looking at their zero mass limits. Let us discuss the several cases in detail.

3.1 \( \lambda = 0 ; j, \mu \neq 0 \): Squashed \( AdS_2 \times S^3 \) as near horizon geometries

The solutions reads

\[
ds^2 = -\frac{r^4}{\mu^2} \left[ dt + jr^{2} \sigma^3_{L,R} \right]^2 + \frac{\mu}{r^2} dr^2 + \frac{\mu}{4} \left[ (\sigma^1_{L,R})^2 + (\sigma^2_{L,R})^2 + (\sigma^3_{L,R})^2 \right] \]

\[
A = \frac{\sqrt{3}}{2} \frac{\mu}{r^2} \left[ dt + jr^{2} \sigma^3_{L,R} \right] .
\]

(3.1)

For \( j = 0 \) this is \( AdS_2 \times S^3 \) which is the near horizon geometry of the five dimensional Reissner-Nordström black hole. Turning on \( j \) we deform the \( AdS_2 \times S^3 \) solutions in two different ways corresponding to the left and the right solutions. They are still homogeneous spaces. For the right case this was discussed in [21]. The isometry group for \( AdS_2 \times S^3 \) includes both left \(( \xi^L_i \) \) and right \(( \xi^R_i \) \) invariant vector fields on \( S^3 \). For \( j \neq 0 \), \( \xi^L_1, \xi^L_2 \) \( ( \xi^R_1, \xi^R_2 ) \) are not Killing vector fields of the left (right) solution [3.1] any longer, since they act as \(( \mathcal{L} \) denotes Lie derivative)

\[
\mathcal{L}_{\xi^L_i} \sigma^j_L = \epsilon^{ijk} \sigma^k_L , \quad \mathcal{L}_{\xi^R_i} \sigma^j_R = -\epsilon^{ijk} \sigma^k_R .
\]

(3.2)

However the action of the remaining vector fields is sufficient to ensure that the isometry group acts transitively on the manifold. In particular these spacetimes are completely non-singular.
Despite these similarities, the left and right solution are quite different. Denoting by $\Delta_{L,R}$ the coefficient of $(\sigma_{L,R}^3)^2$ in (3.1), we have

$$\Delta_{L,R} = \mu^4 - j^2/\mu^2 r^{4+4}.$$  \hspace{1cm} (3.3)

Whereas, for the left solution, this coefficient always becomes negative for sufficiently large $r$, in the right solution its ability to change sign depends crucially in the ratio $4j^2/\mu^3$. The behavior of the latter is associated to the over-rotating versus under-rotating regimes of the BMPV black hole (to be reviewed in section 3.3), and one should think about the right solution as the near horizon geometry of such black hole. The behavior of the left solution is analogous to the four dimensional Gödel spacetime. This is a homogeneous, non-isotropic, rotating, non-expanding universe. Since $\Delta_L$ becomes negative at large $r$, there are angular (hence closed) directions becoming time like, and hence closed timelike curves (CTC’s). Due to the absence of horizons these can be deformed to pass all over the spacetime leading to possible causality violations. Note that when $\Delta_{L,R} = 0$ the metric is not singular and this is not a null surface; also there is no paralleled propagated singularity when crossing this surface (curvature seen by a freely falling observer), i.e. the components of the Riemann tensor on a parallel propagated frame are well behaved.

It is not a surprise that this Gödel-type solutions appears as a deformation of the $AdS_2 \times S^3$ vacuum. In fact, the original Gödel solution (without the trivial flat direction) can be thought of as a deformation of $AdS_3$ [22]. More specifically, since $AdS_3 \equiv SL(2,\mathbb{R})$, we consider the family of geometries

$$ds^2 = -\alpha^2 (\sigma^0_L)^2 + (\sigma^2_L)^1 + (\sigma^3_L)^2,$$  \hspace{1cm} (3.4)

where $\sigma^\mu$ are left-invariant one-forms on $SL(2,\mathbb{R})$ and $\alpha$ the deformation parameter. For $\alpha^2 = 1$ we have $AdS_3$. For $\alpha^2 = 2$ we have the Gödel solution. Explicitly this family of metrics can be written as

$$ds^2 = -\alpha^2 \left[ dt + \sinh^2 \frac{r}{2} d\phi \right]^2 + \frac{1}{4} (dr^2 + \sinh^2 r d\phi^2).$$  \hspace{1cm} (3.5)

To describe Anti-de-Sitter space the time direction must be periodic, hence leading to CTC’s. However, these are trivial CTC’s (topological) which means they are resolved by going to the universal covering space, $\tilde{AdS}_3$, for which $-\infty < t < \infty$. If $\alpha^2 > 1$, we cannot resolve all CTC’s even if we go to the universal covering space, since the coefficient of $d\phi^2$ becomes

$$\sinh^2 r - 4\alpha^2 \sinh^4 (r/2),$$  \hspace{1cm} (3.6)

therefore negative for sufficiently large $r$. These are non-trivial CTC’s (geometric) since they are homotopic to a point. This is the Gödel case.

3.2 $\mu = 0$; $\lambda, j \neq 0$: Gödel-type universe and Singular Repulson

The solution is now

$$ds^2 = -\left[ dt + jr^{\mp 2} \sigma_{L,R}^3 \right]^2 + ds^2_{\mathbb{R}^4}, \quad A = \frac{\sqrt{3}}{2} jr^{\mp 2} \sigma_{L,R}^3.$$  \hspace{1cm} (3.7)

Replacing $jr^{\mp 2} \rightarrow jr^n$, the Ricci scalar is

$$R = 2j^2 (n^2 + 4) r^{2n-4}.$$  \hspace{1cm} (3.8)
This is constant everywhere for the left solution. On the other hand, the coefficient of \((\sigma^3_{L,R})^2\) is
\[
\Delta_{L,R} = \frac{r^2}{4} - \frac{j^2}{r^{\pm4}},
\]
and hence CTC’s will be present at large \(r\) for the left solution. Indeed, this is a homogeneous space of the Gödel type. Note that for \(n \neq 2\) the spacetime is not homogeneous and it is singular at either \(r = 0\) or infinity.

The right solution is asymptotically flat and has a timelike curvature singularity at \(r = 0\). But such a singularity is unattainable by freely falling observers. The radial geodesic equation for a point particle of mass \(m\), angular momentum \(\omega\) (all of it in one 2-plane, the same 2-plane as the background space’s \(j\)), energy \(E\), and affine parameter \(\tau\) is
\[
\left(\frac{dr}{d\tau}\right)^2 = \frac{4E^2}{r^2}\Delta_{L,R} - \left(m^2 + \frac{4\omega^2}{r^2}\right) - \frac{8jE\omega}{r^{\pm2+2}}.
\]
(3.10)

For small \(r\) the dominating term always becomes negative, hence prohibiting geodesics from entering such region. The geometry is a singular repulson. For the Gödel case the opposite happens: for large enough \(r\) a negative term dominates. In either case a freely falling observer, with \(jE\omega < 0\) can cross \(\Delta_{L,R}\) into the region where CTC’s form, but cannot penetrate ‘too much’ into this region. For the physical interpretation and further discussion of the geodesic motion see [28].

3.3 \(j, \mu, \lambda \neq 0\): Black Hole in Gödel universe and BMPV black hole

Without loss of generality let us take \(\lambda = 1\), which can always be achieved by coordinate transformations. It is convenient to replace the isotropic radial coordinate, \(r\), by a Schwarzschild-type radial coordinate, \(\rho\), related by \(\rho^2 = r^2 + \mu\). The solution is written explicitly as
\[
\begin{align*}
\frac{ds^2}{\Delta^2} &= \frac{4E^2}{\rho^2}\Delta_{L,R} - \left(m^2 + \frac{4\omega^2}{\rho^2}\right) - \frac{8jE\omega}{\rho^{\pm2+2}}.
\end{align*}
\]
(3.11)

We have denoted
\[
\Delta = 1 - \frac{\mu}{\rho^2}.
\]
(3.12)

For \(j = 0\) this is the five dimensional extreme Reissner-Nordström solution. There is a timelike curvature singularity at \(\rho = 0\) and a degenerate black hole event horizon at \(\rho_H = \sqrt{\mu}\). The spacetime is asymptotically flat with the typical Carter-Penrose diagram for extreme RN black holes. For \(j \neq 0\) we still have a curvature singularity at \(\rho = 0\), since the Ricci scalar is
\[
R = \frac{16j^2(\rho^2 - \mu)^{2+2} - 2\mu^2\rho^2}{\rho^8}.
\]
(3.13)

The coefficient of \((\sigma^3_{L,R})^2\) in the metric becomes
\[
\Delta_{L,R}(\rho) = \begin{cases} 
\frac{\rho^2}{4} - \frac{j^2}{\rho^4} & (R) \\
\frac{\rho^2}{4} - j^2\left(\rho - \frac{\mu}{\rho}\right)^4 & (L)
\end{cases}.
\]
(3.14)
The solution is invariant under $j, t \to -j, -t$. So we take $j$ positive. Define $\rho_C$ by $\Delta_{L,R}(\rho_C) = 0$, which is the boundary of the region where angular directions become timelike.

The right solution is the BMPV spacetime: a rotating, asymptotically flat black hole. We have CTC’s in the region $\rho^3 < 2j \equiv \rho_C^3$. If $\rho_C < \rho_H$ the CTC’s are hidden behind the horizon. This is the under-rotating regime. For $\rho_C > \rho_H$, the CTC’s are naked, and can be deformed to pass through any spacetime point. The surface $\rho = \rho_H$ is now timelike; it is highly repulsive, and no causal geodesic can go beyond it, becoming an effective boundary for the spacetime \[28\]. This is the over-rotating regime, which is similar to the repulson described in the last subsection with the crucial difference that the repulsive surface is now $\rho = \rho_H \neq 0$, hence non-singular. Since the spacetime is asymptotically flat one can define ADM mass and angular momentum vector (corresponding to the two independent two-planes) of the BMPV black hole. One obtains \[27\]

$$\begin{align*}
M &= \frac{3\pi \mu}{4G_5}, \\
\vec{J} &= \left(0, j\frac{\pi}{2G_5}\right).
\end{align*}$$  

The entropy of this black hole is

$$S_{BMPV} = \frac{\pi^2}{2G_5} \sqrt{\mu^3 - 4j^2},$$

which becomes ill defined in the over-rotating case. A very curious property of the BMPV spacetime is that the angular velocity of the horizon is zero \[27\]. This is a necessary condition for supersymmetry, since susy is incompatible with an ergo-region.

The left solution has CTC’s both for large and for small radial coordinate, since the second term in \[3.14\] dominates in both regimes. But $\Delta_L$ is not always negative. At $\rho = \sqrt{\mu}$ it is positive. Can this surface be interpreted as a horizon? It is certainly a null surface, as can be seen by a standard analysis. First introduce regular coordinates on the horizon. This is achieved by choosing a retarded time and a new angular coordinate,

$$d\tilde{v} = dt + a(\rho)d\rho, \quad \begin{cases} 
\frac{d\tilde{\psi}}{d\rho} = \frac{d\psi + b(\rho)d\rho}{\sqrt{\Delta_{L,R}(\rho)}} \\
\frac{d\tilde{\phi}}{d\rho} = \frac{d\phi + b(\rho)d\rho}{\sqrt{\Delta_{L,R}(\rho)}} \Rightarrow \tilde{\sigma}_{L,R}^3 = \frac{d\tilde{\psi} + \cos \theta d\phi}{d\tilde{\phi} + \cos \theta d\psi} = \frac{\sigma_{L,R}^3 + b(\rho)d\rho}{\sigma_{L,R}^3 + \frac{2j(\rho^2 - \mu)^{\frac{1}{2}}}{\rho \sqrt{\Delta_{L,R}}}}.
\end{cases}$$

(Top/bottom refer to Right/Left solutions). The functions $a(\rho), b(\rho)$ are chosen as to eliminate the $d\rho^2$ and $d\rho \tilde{\sigma}_{L,R}^3$ terms in the new metric. This requirement yields

$$a(\rho) = \frac{2\rho^3 \sqrt{\Delta_{L,R}}}{(\rho^2 - \mu)^2}, \quad b(\rho) = \frac{2j(\rho^2 - \mu)^{\frac{3}{2}}}{\rho \sqrt{\Delta_{L,R}}}.$$  

The metric becomes

$$d\tilde{s}^2 = d\tilde{v} \left[-\Delta^2 d\tilde{v} + \frac{\rho}{\sqrt{\Delta_{L,R}}} d\rho - 2j\Delta^{\frac{1}{2}} \rho^{\frac{1}{2}} \tilde{\sigma}_{L,R}^3 \right] + \frac{\rho^2}{4} \left[ (\sigma_{L,R}^1)^2 + (\sigma_{L,R}^2)^2 \right] + \Delta_{L,R}(\tilde{\sigma}_{L,R}^3)^2.$$  

We want the new coordinates to cover the surface $\rho = \sqrt{\mu}$. This requires $\Delta_{L,R}(\sqrt{\mu}) > 0$. Whereas for the right solution this is only true for the under-rotating case, for the left solution this is true for any value of $j$. Consider now the family of surfaces $S(\rho) = 0$. The normal vector to the surface has norm

$$\partial_{\mu} S \partial_{\nu} S g^{\mu\nu} = g^{\rho\rho} = \Delta^2.$$  

8
The surface at \( \rho = \sqrt{\mu} \) is therefore null. Hence the solution is rightly interpreted as a black hole in Gödel’s Universe. Its entropy, computed from the horizon area, is the same as that of a static black hole

\[
S_{\text{Gödel BH}} = \frac{\pi^2}{2G_5} \sqrt{\mu^3}.
\]  

Both entropies will be analyzed from the viewpoint of the D1-D5 system in section 4.

### 3.4 Supersymmetry

The solution (2.16) with \( \ell = 0 \) takes the form

\[
ds^2 = -(e^0)^2 + \delta_{ij} e^i e^j, \quad F = \frac{\sqrt{3}}{2} d e^0,
\]

with the frames

\[
e^0 = f (dt + \omega), \quad e^i = f^{-1/2} dx^i.
\]

The Killing spinor equation for (2.1) yields

\[
0 = d \epsilon + \frac{i}{4} \omega_{ab} \Gamma^{ab} \epsilon + \frac{i}{4\sqrt{3}} (e^a \Gamma^{bc} a - 4e^b \Gamma^c) F_{bc} \epsilon
\]

\[
= e^0 \left( \frac{\partial_\epsilon}{f} + \frac{f^2 a_{ij} \Gamma^{ij}}{8} + \frac{i \partial_k f \Gamma^i}{2 f^{1/2}} \right) (1 - i \Gamma^0) \epsilon + e^k \left( f^{1/2} (\partial_k - \omega_k \partial_t) \epsilon - \frac{\partial_k f}{2 f^{1/2}} i \Gamma^0 \epsilon + \frac{\partial_i f \Gamma^i_k}{4 f^{1/2}} + \frac{f a_{ki} \Gamma^0}{4} \right) (1 - i \Gamma^0) \epsilon - \frac{f^2 \Gamma^0}{4} (a_{ki} + \ast a_{ki}) \epsilon.
\]

We have denoted by \( a_{ij} \) the components of the two form \( a = d\omega \) and by \( \ast a \) the Hodge dual of \( a \) on \( \mathbb{R}^4 \). If we take \( a \) to be anti-self-dual on \( \mathbb{R}^4 \) the last term vanishes and we find the obvious set of Killing spinors

\[
\epsilon = f^\frac{1}{2} \epsilon_0, \quad \epsilon_0 = i \Gamma^0 \epsilon_0,
\]

corresponding to four independent Killing spinors (one half of the vacuum supersymmetry). This is the supersymmetry preserved by the solutions in section 3.3: the black hole in Gödel’s universe and the BMPV black hole.

Under which conditions can we have extra Killing spinors? Take the remaining set of spinors

\[
\epsilon = \chi(x^\mu) \epsilon_0, \quad \epsilon_0 = -i \Gamma^0 \epsilon_0,
\]

where \( \chi(x^\mu) \) is some matrix, to find the condition

\[
0 = e^0 \left( \frac{\partial_\epsilon}{f} + \frac{i \chi \partial_i f \Gamma^i}{f^{1/2}} \right) \epsilon_0 + e^k \left( f^{1/2} (\partial_k - \omega_k \partial_t) \chi + \frac{\partial_k f}{2 f^{1/2}} \chi + \frac{\partial i f \Gamma^i_k}{2 f^{1/2}} - i f^2 a_{ki} \Gamma^i \right) \epsilon_0.
\]

If we do not allow the spinor to be time dependent, \( \chi = \chi(x^i) \), we find that \( f \) must be a constant, which we take to be unity, and the condition

\[
\partial_k \chi = i a_{ki} \Gamma^i \chi.
\]

---

3We have taken \( \Gamma_{01234} = i \).
A constant \( f \) corresponds to the solutions with \( \mu = 0 \) in section 3.2, thus we now ask if we can solve (3.28) for such solutions. First notice that cartesian coordinates on \( \mathbb{R}^4 \) are related to the ones of the form (2.14) by

\[
x^1 + ix^2 = r \cos (\theta/2)e^{i(\phi+\psi)/2}, \quad x^3 + ix^4 = r \sin (\theta/2)e^{i(\psi-\phi)/2}.
\]

(3.29)

Thus, in cartesian coordinates

\[
\sigma^3_{L,R} = \frac{2}{r^2} \left[ x^1 dx^2 - x^2 dx^1 \pm (x^3 dx^4 - x^4 dx^3) \right] \equiv \frac{2}{r^2} (I^3_{L,R})_{ij} x^i dx^j,
\]

(3.30)

which defines the constant antisymmetric matrices \((I^3_{L,R})_{ij}\) and

\[
\omega = 2jr^{\mp 2}-(I^3_{L,R})_{ij} x^i dx^j \Rightarrow a = d\omega = \left\{ \begin{array}{l}
\frac{2j}{r^4}(I^3_{R})_{ij} \left\{ \delta^i_k - 4\frac{x_k x^i}{r^2} \right\} dx^k \wedge dx^j \\
2j(I^3_{L})_{ij} dx^i \wedge dx^j
\end{array} \right.. \tag{3.31}
\]

It is straightforward to check that \( a = -\star a \) for both cases.

For the left case \( a \) is a constant matrix and (3.28) becomes

\[
\partial_k \chi = i4j(I^3_{L})_{ki} \Gamma^i \chi \Rightarrow \chi = \exp (i4j(I^3_{L})_{ki} \Gamma^i x^k).
\]

(3.32)

This shows the Gödel type universe of section 3.2 is maximally supersymmetric [14].

For the right case one can check that the integrability condition for (3.28)

\[
\partial_l a_{kj} \Gamma^j \chi = 0,
\]

(3.33)

is not obeyed. Hence, the singular repulsion of section 2.3 preserves only half of the vacuum supersymmetry.

Although we have not checked, it is natural that supersymmetry is also going to be enhanced in the near horizon geometry for both solutions. For the right solution this has been checked in [29, 27]. For the left solution this would yield another maximally supersymmetric solution of the Gödel type, (3.1).

### 4 Ten dimensional interpretation in IIB String Theory

Consider type IIB supergravity with only the graviton, dilaton and Ramond-Ramond two form potential being excited. The equations of motion are

\[
D_M \partial^M \phi = \frac{e^\phi}{12} F_{MNP} F^{FNP}, \quad D_M (e^\phi F^{MNP}) = 0,
\]

(4.1)

\[
R_{MN} = \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{e^\phi}{4} F_{MPQ} F^{NPQ} - \frac{1}{48} g_{MN} e^\phi F_{PQR} F^{PQR},
\]

which can be derived from the action (which is therefore a consistent truncation of IIB Sugra)

\[
S^{(10)} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left( R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{2 \cdot 3!} e^\phi F_{MNP} F^{MNP} \right).
\]

(4.2)
As usual $F = dC$. Split the ten-dimensional coordinates as $x^M = (x^\mu, y^i)$, where $\mu = 0 \ldots 4$, $i = 1 \ldots 5$. The vectors $\partial/\partial y^i$ are assumed to be Killing vector fields. Then, perform a Kaluza-Klein reduction with the ansatz,

$$\begin{align*}
ds^2 &= e^{\alpha(\chi)} \left[ e^{\beta(x)} \hat{g}_{\mu\nu} dx^\mu dx^\nu + e^{-4\alpha(\chi)} \left( dy^1 + \hat{A}_\mu^{(1)} dx^\mu \right)^2 \right] + e^{-3\beta(x)/4} ds^2(\mathbb{T}^4) , \\
C &= \frac{1}{2} \hat{B}_{\mu\nu} dx^\mu \wedge dx^\nu + \hat{A}_\mu^{(2)} dx^\mu \wedge dy^1 , \\
\phi &= \hat{\phi}(x) .
\end{align*}$$

(4.3)

Hatted quantities are five dimensional, $ds^2(\mathbb{T}^4)$ is the metric on a flat four-torus and we have singled out one of the extra-dimensions, $y^1$, which can have off diagonal terms in the metric and non trivial legs in the Ramond-Ramond field, hence giving rise to two extra gauge fields. The powers of the several exponential factors in the metric ansatz are chosen as to obtain the five dimensional Einstein frame, keeping two constants, $a, b$, which can be chosen as to yield canonical normalization for the kinetic terms of the scalar fields. Such choice gives $117b^2 = 8, 6a^2 = 1$. It is also convenient to define the new scalar $\chi(x) = b\psi(x)/a - \lambda(x)$. We obtain the following five dimensional action

$$S^{(5)} = \frac{1}{2\kappa^2} \int d^5x \sqrt{-\hat{g}} \left( \hat{R} - \frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{1}{2} \partial_\mu \hat{\chi} \partial^\mu \hat{\chi} - \frac{1}{2} \partial_\mu \hat{\psi} \partial^\mu \hat{\psi} - \frac{1}{2 \cdot 3!} e^{2\hat{\phi}+2\hat{\chi}-4\hat{\psi}} \hat{H}_{\mu\nu\alpha} \hat{H}^{\mu\nu\alpha} \\
- \frac{1}{4} e^{4\hat{\chi}-5\hat{\psi}} \hat{F}_{(1)}^{(1)} \hat{F}_{(1)}^{(1)} - \frac{1}{4} e^{\hat{\phi}-2\hat{\alpha}+\hat{\psi}} \hat{F}_{\mu\nu}^{(2)} \hat{F}_{\mu\nu}^{(2)} \right) ,$$

(4.4)

The field strengths are defined as $\hat{F}^{(i)} = \hat{A}^{(i)}$ and $\hat{H} = d\hat{B} - \hat{A}^{(1)} \wedge d\hat{A}^{(2)}$, where the latter combination naturally arises in the reduction procedure. Moreover, denoting the volume of $\mathbb{T}^4$ by $(2\pi)^4 V$ and the radius of the remaining circle of compactification by $R$ we have $(2\pi)^3 V R \kappa^2 = \kappa^2$.

Requiring all scalars to be constant and $\hat{A}^{(1)} = \hat{A}^{(2)} \equiv \hat{A}$ (denote $\hat{F} = d\hat{A}$), the equations of motion of (4.1) yield the following conditions

$$\begin{align*}
\hat{H}_{\mu\nu\alpha} \hat{H}^{\mu\nu\alpha} &= -3 \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} , \\
D_\alpha F^{\alpha\beta} &= -\frac{1}{2} \hat{H}^{\beta\nu} \hat{F}_{\mu\nu} , \\
D_\mu \hat{H}^{\mu\alpha\beta} &= 0 , \\
\hat{H} = -\hat{\star}d\hat{A} , \\
\Rightarrow d\hat{B} &= \hat{A} \wedge d\hat{A} - \hat{\star}d\hat{A} ,
\end{align*}$$

(4.5)

Further imposing

$$\hat{H} = -\hat{\star}d\hat{A} ,$$

(4.6)

and performing the rescaling $\hat{A} \rightarrow 2\hat{A}/\sqrt{3}$ we recover the equations of motion of $\mathcal{N} = 1, D = 5$ supergravity. $\hat{\star}$ denotes Hodge duality with respect to $\hat{g}$. Hence, a solution, $(\hat{g}, \hat{A})$ of the five dimensional supergravity theory uplifts as the solution

$$\begin{align*}
ds^2 &= \hat{g}_{\mu\nu} dx^\mu dx^\nu + \left( dy^1 + \frac{2}{\sqrt{3}} \hat{A}_\mu dx^\mu \right)^2 + ds^2(\mathbb{T}^4) , \\
H &= dC = \frac{2}{\sqrt{3}} d\hat{A} \wedge \left( dy^1 + \frac{2}{\sqrt{3}} \hat{A} \right) - \frac{2}{\sqrt{3}} \hat{\star}d\hat{A} ,
\end{align*}$$

(4.7)

of type IIB (or IIA or I, since the dilaton decouples) supergravity. We apply this result to (2.3), taking the gauge potential to be

$$\hat{A} = \frac{\sqrt{3}}{2} \left( (f - 1) dt + f \omega - \frac{2}{3} h^+ \right) ,$$

(4.8)
where \( h^+ \) is the potential for the closed form \( G^+ \), \( dh^+ = G^+ \). We obtain the ten dimensional solution

\[
ds^2 = f \left[ -dt^2 + (dy^1)^2 - (1 - f^{-1})(dt - dy^1)^2 + 2\omega \left( dy^1 - dt - \frac{2}{3} h^+ \right) \right. \\
\left. - \frac{4}{3} h^+ f^{-1} \left( dy^1 - (1 - f) dt - \frac{1}{3} h^+ \right) \right] + f^{-1} h_{ij} dx^i dx^j + ds^2(\mathbb{T}^4), \tag{4.9}\]

\[
H = d \left[ f(dt + \omega) - \frac{2}{3} h^+ \right] \wedge \left( dy^1 - dt + f(dt + \omega) - \frac{2}{3} h^+ \right) - \mathcal{K} \left[ f(dt + \omega) - \frac{2}{3} h^+ \right].
\]

### 4.1 Higher dimensional interpretation of BMPV and Gödel black hole

Start by specializing \([4.9]\) to \([2.16]\) with \( \ell = 0 \). We find

\[
ds^2 = f \left[ -dt^2 + (dy^1)^2 - (1 - f^{-1})(dt - dy^1)^2 + \frac{2j}{r^{\pm 2}} (dy^1 - dt) \sigma^3_{L,R} \right] + f^{-1} ds^2_{\mathbb{R}^4} + ds^2(\mathbb{T}^4) \\
C = f \left( dt + \frac{j}{r^{\pm 2}} \sigma^3_{L,R} \right) \wedge (dy^1 - dt) + \frac{\mu}{4} \cos \theta d\phi \wedge d\psi. \tag{4.10}\]

The right solution is a special case of a more general solution with three different charges found in \([8]\). This suggests the following solution of type IIB supergravity

\[
ds^2_E = f_5^{-\frac{1}{2}} f_1^{-\frac{3}{4}} \left[ -dt^2 + (dy^1)^2 + f_K(dt - dy^1)^2 + \frac{2j}{r^{\pm 2}} (dy^1 - dt) \sigma^3_{L,R} \right] + f_5^\frac{3}{4} f_1^\frac{1}{2} ds^2_{\mathbb{R}^4} + \left( \frac{f_1}{f_5} \right)^{\frac{1}{4}} ds^2(\mathbb{T}^4), \\
C_{RR} = f_1^{-1} \left( dt + \frac{j}{r^{\pm 2}} \sigma^3_{L,R} \right) \wedge (dy^1 - dt) + \frac{P}{4} \cos \theta d\phi \wedge d\psi, \quad e^{-2(\phi - \phi_\infty)} = \frac{f_5}{f_1}. \tag{4.11}\]

The three functions \( f_1, f_5, f_K \) are given by:

\[
f_5 = 1 + \frac{P}{r^2}, \quad f_1 = 1 + \frac{Q}{r^2}, \quad f_K = \frac{Q_{KK}}{r^2}. \tag{4.12}\]

It is straightforward to verify this indeed solves \([4.11]\) for both left and right cases. Setting \( j = 0 \) we recover the standard D1-D5-pp wave system \([2]\). Setting \( P = Q_{KK} = Q \equiv \mu \), and denoting \( f_1 = f_5 = 1 + f_K \equiv f^{-1} \) we recover \([4.10]\).

For the ‘right’ solution \([4.11]\) can be interpreted as a D1-brane inside a D5 with a Brinkmann wave propagating along the string. For the ‘left’ solution one should think of \([4.11]\) as the D1-D5-pp-wave system in a rotating background, which is not asymptotically flat.

### 4.2 A resolution of causality violations

It was shown in \([8]\) that the non-trivial CTC’s of the BMPV black hole become trivial in its ten dimensional description, by virtue of the Kaluza-Klein ‘oxidation’. This is also true for the supersymmetric Gödel solution. Examining \([4.10]\) there are no obvious CTC’s; one does not identify a periodic direction that becomes timelike. However, the vector

\[
k = \alpha \partial / \partial y^1 + \beta \xi^{L,R}_3, \tag{4.13}\]
which is a linear combination of two spacelike vectors, has norm
\[ |\mathbf{k}|^2 = \frac{r^2}{4j} \alpha^2 + \frac{2jf}{r^2} \alpha \beta + \beta^2. \tag{4.14} \]

For \( j = 0 \) this is non-negative for any choice of \( \alpha, \beta \), at any spacetime point. For \( j \neq 0 \), asking when \( \mathbf{k} \) becomes null yields a quadratic equation in \( \beta/\alpha \) with discriminant binomial \(-\Delta_{L,R}\) (given by (3.14)). Hence, there will be real solutions for \( \Delta_{L,R} \leq 0 \); for \( \Delta_{L,R} < 0 \) we can have a timelike \( \mathbf{k} \).

Thus we can find timelike curves, with tangent vector \( \mathbf{k} \), when \( \Delta_{L,R} < 0 \). However these curves can not be closed until we make the \( y^1 \) direction compact. These are the CTC’s seen in five dimensions, with the crucial difference that they are not homotopic to a point any longer; they are trivial, since they are resolved in the universal covering space of the manifold. In [8] a pictorial description of the resolution process is given.

If we apply T-duality along the \( y^1 \) direction to (4.11) we obtain the IIA (string frame) geometry
\[ ds^2 = -\left(\frac{f_1 f_5}{1 + f_K}\right)^{-1/2} \left[ dt + \frac{j}{r^2} \sigma^3_{L,R} \right]^2 + (f_1 f_5)^{1/2} \left( \frac{(dy^1)^2}{1 + f_K} + ds^2_{\mathbb{R}^4} \right) \left( \frac{f_1}{f_5} \right)^{1/2} ds^2(\mathbb{T}^4), \tag{4.15} \]
which has non-trivial CTC’s. The T-duality has exactly canceled the effect of the Kaluza-Klein ‘oxidation’ and that is the reason why the M-theory uplifting of the Gödel solution studied in [14] still exhibits the same type of Closed Timelike Curves as in five dimensions.

Solution (4.15) is a Gödel type universe in the limit \( f_1 = f_5 = 1, f_K = 0 \), and it is a delocalized (in the \( y^1 \) direction) \( D_0/D_4 \) system in the limit \( j = f_K = 0 \). Thus it seems that the solution is rightly interpreted (for \( f_K = 0 \)) as a \( D_0/D_4 \) system in a Gödel type universe. Given that we showed the existence of supersymmetric solutions describing black holes in Gödel type universes it is not suprising that supersymmetric D-brane systems in Gödel type universes exist.

It is interesting to notice that this resolution of causality violations in higher dimensions does not survive to turning on the self-dual part of \( fd\omega \): the case \( \ell \neq 0 \). The ten dimensional metric describing this solution is
\[ ds^2 = f \left[ -dt^2 + (dy^1)^2 - (1 - f^{-1}) (dt - dy^1)^2 + \left\{ 2g \mp \frac{\ell}{3} r^2 f^{-1} \right\} (dy^1 - dt) \right. \]
\[ \left. \mp \frac{\ell}{3} r^2 dt \mp \frac{\ell}{6} r^2 \left( 2g \mp \frac{\ell}{6} r^2 f^{-1} \right) \sigma_{L,R} \right\} \sigma_{L,R} + f^{-1} ds^2_{\mathbb{R}^4} + ds^2(\mathbb{T}^4), \tag{4.16} \]
which can still admit non-trivial CTC’s since the coefficient of \( (\sigma_{L,R})^2 \) can be negative.

4.3 Decoupling limit

The decoupling limit [30] is taken in the string frame, \( ds^2 = \exp(\phi/2) ds^2_{\mathbb{R}^2} \). First we have to know the quantization of the metric parameters in terms of the string coupling, \( g \), the inverse string tension \( \alpha' \) and the volume of the compactification \( T^4 \), \( V \), and radius of the circle, \( R \). The usual U-duality arguments and quantization of Kaluza-Klein momentum yield
\[ Q = \frac{Q_{1g}(\alpha')^3}{V}, \quad P = Q_5 g \alpha', \quad Q_{KK} = \frac{g^2 (\alpha')^4 N_P}{R^2 V}, \tag{4.17} \]
where \( Q_1, Q_5, N_P \in \mathbb{N} \). Now we look at the solution (4.11), in the limit where

\[
\alpha', r, V \to 0, \quad U \equiv \frac{r}{\alpha'} = \text{fixed}, \quad v \equiv \frac{V}{(\alpha')^2} = \text{fixed}.
\]

In this limit, the resulting solution is

\[
\begin{align*}
\alpha' & \left\{ \frac{U^2}{\sqrt{Q_1 Q_5}} \left[ -dt^2 + (dy^1)^2 + \frac{g^2 N_P}{R^2 U^2} (dt - dy^1)^2 + \frac{2j}{(\alpha')^{1+2} U^{1+2}} (dy^1 - dt) \sigma^3_{L,R} \right] \\
& \quad + \sqrt{Q_1 Q_5} \frac{dU^2}{U^2} + d\Omega^2_3 \right\},
\end{align*}
\]

\[\alpha' v U \left( \frac{dU^2}{U^2} + d\Omega^2_3 \right) \wedge (dy^1 - dt) + \frac{\alpha' g Q_5}{4} \cos \theta d\phi_1 \wedge d\phi_2,
\]

where \( \tilde{g} = g/\sqrt{v} \) and the dilaton is constant. We still have to address the \( j \) terms in the solution. But before let us do some general considerations on AdS/CFT.

Consider a scalar, vector and tensor perturbation of the \( \text{AdS}_3 \times S^3 \) solution as follows:

\[
ds^2 = U^2[-dt^2 + (dy^1)^2] + \frac{dU^2}{U^2} + d\Omega^2 + \frac{T}{U^{n_t}} (dt - dy^1)^2 + \frac{V}{U^{n_v}} (dt - dy^1) d\theta + \frac{S}{U^{n_s}} d\theta^2.
\]

The mass dimensions of the couplings are

\[
|T| = n_t + 2, \quad |V| = n_v + 1, \quad |S| = n_s.
\]

If these couplings act as sources of operators in the dual CFT, we would have the CFT Lagrangian deformed by the terms, respectively,

\[
\int d^2 x T O_t, \quad \int d^2 x V O_v, \quad \int d^2 x S O_s.
\]

Thus, they would be associated to operators of mass dimension

\[
\Delta[O_t] = -n_t, \quad \Delta[O_v] = 1 - n_v, \quad \Delta[O_s] = 2 - n_s.
\]

But in AdS/CFT, not all perturbations of AdS are associated to deformations of the dual CFT. A given operator in the CFT is associated to two different perturbations in AdS. To see this in more detail consider the unperturbed solution (4.20). These coordinates cover a Poincaré patch of \( \text{AdS}_3 \). The coordinate transformation \( U = 1/z \) gives \( \text{AdS}_3 \) in Poincaré coordinates \((t, y^1, z)\), where the conformal flatness becomes explicit. The timelike conformal boundary is at \( U \to \infty \), and \( U = 0 \) is a Cauchy horizon.

If we consider a scalar perturbation of the unperturbed geometry by taking a massive scalar field on \( \text{AdS}_3 \), \( \Box \phi = m^2 \phi \), it behaves at large U as

\[
\phi \sim \frac{a}{U^{1-\sqrt{1+m^2}}} + \frac{b}{U^{1+\sqrt{1+m^2}}}
\]

The first perturbation is non-normalizable (the norm, \( \int \sqrt{-g} |\phi|^2 \), diverges as \( U \to \infty \)); the second one is normalizable.
From the viewpoint of the field theory living on the timelike boundary of $AdS_3$, which is conjectured to be a dual description of the physics in Anti-de-Sitter space [30], $U$ is the energy scale. The two scalar perturbations above correspond to an operator, $\mathcal{O}$, in the CFT. A non-normalizable perturbation in AdS corresponds to deforming the dual field theory, whose Hamiltonian becomes

$$H = H_{CFT} + a\mathcal{O} ,$$

whereas a normalizable perturbation corresponds to giving a vev to the operator

$$<0|\mathcal{O}|0> = b .$$

Thus we identify the non-normalizable perturbation as being associated to the deformation and hence we find

$$\Delta[\mathcal{O}_s] = 1 + \sqrt{1 + m^2} .$$

In this scalar case, the operator corresponding to any massive perturbation is an irrelevant (in the IR, important in the UV) operator in the CFT since $\Delta > 2$. Note that since the CFT is two dimensional marginal operators have dimension $\Delta = 2$.

If we consider a massive vector perturbation of the $AdS_3$ geometry, $D_\mu F^{\mu\nu} = m^2 A^\nu$, they behave at large $U$ as (for $m^2 \neq 0)$ [31] [32]

$$A_U \sim \frac{c}{U^{3-\sqrt{m^2}}} + \frac{d}{U^{3+\sqrt{m^2}}} , \quad A_i \sim \frac{c_i}{U^{-\sqrt{m^2}}} + \frac{d_i}{U^{+\sqrt{m^2}}} ,$$

where $x^i = (t, y^1)$ [31]. We focus on the $A_i$ perturbations. The first one is non-normalizable. The second one is normalizable ($\int \sqrt{-g} g^{ij} A_i A_j$ converges on the boundary). Again, identifying the non-normalizable perturbation with the deformation we find

$$\Delta[\mathcal{O}_v] = 1 + \sqrt{m^2} .$$

Thus, a massive, non-normalizable, vector perturbation in $AdS_3$ corresponds to a deformation by a relevant ($0 < m^2 < 1$), marginal ($m^2 = 1$) or irrelevant ($m^2 > 1$) operator. The case with $m^2 = 0$ is more subtle. In Poincaré coordinates, the solution for vector perturbations in $AdS_2$ is actually

$$A_i \sim K_\nu(z) + I_\nu(z) ,$$

where these are hyperbolic Bessel functions, and $\nu = \sqrt{m^2}$. For $\nu \neq 0$ these have small $z$ (large U) behavior given by [428]. For $\nu = 0$ these behave at small $z$ as

$$A_i \sim -\log z + 1 .$$

Thus, a constant behavior is the zero mass limit of the normalizable perturbation. This is the BMPV angular momentum perturbation. Naively, the norm of such constant perturbation seems to diverge logarithmically. However, studying the massless perturbation by introducing a small mass cut-off when computing the norm, and taking the mass to zero at the end, one finds it is indeed a normalizable perturbation. This is in agreement with the known description of the BMPV in terms of CFT states given below.
Finally, if we consider a (massless) tensor perturbation. The linearized equation of motion can be found in \[33\]. It behaves as we approach the boundary as (in the radiation gauge)

\[
h_{ij} \sim c_{ij}U^2 + d_{ij}. \tag{4.32}
\]

The first is a non-normalizable mode whereas the second is normalizable. They both correspond to an operator of dimension \(\Delta[O_i] = 2\), i.e. marginal, which is of course the energy momentum tensor in the CFT.

Our solution in the decoupling limit \[4.19\] has two ‘perturbation’ of \(AdS_3 \times S^3\). The momentum \((N_P)\) ‘perturbation’ is a tensor perturbation, corresponding to the normalizable mode in \(4.32\). It corresponds to giving a vev to the components of the CFT energy momentum tensor \(T_{++}\) or \(T_{--}\). The angular momentum ‘perturbations’ \((j)\) are vector perturbations, which we now discuss in more detail.

### 4.3.1 The BMPV case

For the BMPV case, one quantizes the physical angular momentum of the spacetime \[8-15\]. Using \(G_5 = \pi(\alpha')^4g^2/(4VR)\), we find

\[
j = \frac{N_J(\alpha')^4g^2}{2VR} \Rightarrow \frac{j}{(\alpha')^2} = \frac{N_Jg^2}{2R}, \tag{4.33}
\]

where \(N_J \in \mathbb{N}\). This angular momentum term is the normalizable mode corresponding to a CFT operator of dimension \(\Delta = 1\). This is a relevant operator which is the reason why the BMPV entropy is sensitive to the spacetime angular momentum. Moreover, this is a normalizable perturbation, as discussed above. Hence, it corresponds to considering a different set of states (from the \(j = 0\) case) in the \(D1-D5\) CFT \[4\].

Such states were identified in \[7\]. The space transverse to the D1-D5 system is four dimensional and has an \(SO(4)\) isometry, which translates as the \(SO(4)\) R-symmetry of the CFT. Hence the spacetime angular momentum is translated as CFT charge. The two independent angular momentum parameters \(\vec{J} = (J_L, J_R)\) translate as the quantized charges, \((F_L, F_R)\), of the \(U(1)_L \times U(1)_R\) Cartan sub-algebra of the \(SO(4)\) R-symmetry.

The quantum states that describe the static black hole are states with CFT energy, i.e. \((L_0, \tilde{L}_0)\) eigenvalues, \((0, N_P)\). The states that describe the BMPV black hole are ‘deformed’ by carrying charge \((F_L, F_R) = (0, N_J)\) as well. It turns out that the operators that charge the states must carry by unitarity (which in this context means absence of negative norm states) a conformal weight of \(3N_J^2/(2c)\), where \(c\) is the central charge of the D1-D5 CFT. Thus, by unitarity \[8\]

\[
N_P > 3N_J^2/(2c), \tag{4.34}
\]

which in terms of the five dimensional black hole means that \(\mu^2 > 4j^2\) and thus we are restricted to the under-rotating case, where all CTC’s are hidden behind the event horizon of the black hole.

### 4.3.2 The Gödel black hole case

The angular momentum term in the decoupling limit metric is, in this case,

\[
\alpha' \frac{U^2}{g\sqrt{Q_1Q_5}} \left[2j(\alpha')^2U^2(dy^1 - dt)d\sigma_{L,R}^2\right]. \tag{4.35}
\]
Since there is no known way to define ‘physical angular momentum’ in Gödel’s universe, we cannot quantize \( j \), as we did for the BMPV case. This is because Gödel-type solutions are homogeneous spaces and thus, there is no asymptotic region in which the vorticity can be faced as a perturbation, much in the same way that a cosmological constant, however how small, cannot be faced as a perturbation of flat space. Not quantizing \( j \) is consistent with interpreting this term as a deformation of the theory as opposed to an additional quantum number. But this term is sub-leading in \( \alpha' \) in the decoupling limit, for finite \( j \). Thus, the decoupling limit of the Gödel black hole solution is the same as the one of the Reissner-Nordstrom black hole, i.e. (4.19) with \( j = 0 \). This explains why the black hole entropy is not seeing the spacetime angular momentum at all. Moreover, in the dual CFT, the description of the entropy is exactly the one of the preceding subsection with \( N_J = 0 \).

In order to make the angular momentum term non-vanishing in the decoupling limit, we must take a ‘double scaling limit’

\[
j \to \infty , \quad \text{keeping} \quad j(\alpha')^2 \equiv J = \text{fixed}
\]

and thus the duality can only say something about Gödel in the ‘infinite vorticity limit’. The angular momentum term is the non-normalizable perturbation associated to the insertion of a vector operator of dimension \( \Delta = 5 \) in the CFT. This operator is irrelevant in the IR, and thus it will not alter the black hole entropy. Notice that such insertion breaks the \( \text{SO}(4) \) R-symmetry explicitly in the CFT Lagrangian, which is in sharp contrast with the BMPV case.

## 5 Conclusions and Discussion

It has recently been emphasized by Townsend how angular momentum in supersymmetric systems leads to interesting and sometimes surprising configurations [34]. In this paper we have shown that a simple, supersymmetric deformation of the D1-D5 system with angular momentum is interpreted in five dimensions as a black hole in a Gödel-type universe. The sole existence of a black hole in Gödel’s universe is a novel result and it would be interesting to see if it depends on supersymmetry or not. The entropy of this black hole is ‘blind’ to the spacetime angular momentum and hence this black hole has as many quantum states as the Reissner-Nordstrom black hole of minimal supergravity in five dimensions. This entropy differs, therefore, from the BMPV black hole entropy. We have emphasized all over the text the parallelism between the BMPV and the Gödel black hole solution; they differ by a sign choice in the two \( \text{SO}(3) \) algebras in \( \text{SO}(4) \).

We have shown that in ten dimensions the ‘geometric’ and non-trivial CTC’s of the five dimensional solution become ‘topological’ and trivial. One might therefore ask why should the AdS/CFT duality care about the CTC’s if they can be resolved in ten dimensions. Note that the dual CFT description of the five dimensional spacetime requires the wave propagation direction to be compact, in order for the Kaluza-Klein momentum to be quantized, which corresponds to the quantized CFT energy.

Very recently [35] the enhançon mechanism was studied in the ten dimensional description of the BMPV black hole and argued that it helps to prevent the appearance of CTC’s (in situations where the enhançon is relevant, like \( K3 \) compactifications). From the present paper it is logical to ask what a similar analysis can say about the Gödel-type universe case. Concerning the ‘CTC’s resolving mechanism’ presented herein we would like to add that it does not work for all supersymmetric
spinning black holes in five dimensions. A counterexample was given in [10] for asymptotically AdS black holes. For completeness let us mention that recent work on strings on time dependent backgrounds involves timelike identifications (hence trivial CTC’s) in certain parts of flat space (see eg. [36, 37]). After Kaluza-Klein reduction non-trivial CTC’s develop, which is exactly the converse of our mechanism. However these are excluded from the lower dimensional manifold since the spacetime develops a curvature singularity before reaching the region with CTC’s.

Taking the decoupling limit of the solutions we find two very distinct situations. For the well known BMPV case, the angular momentum term corresponds to a normalizable vector perturbation in $AdS_3$. In the CFT it translates as a dimension $\Delta = 1$ operator getting a vacuum expectation value. This operator corresponds to states carrying R-charge, which break spontaneously the R-symmetry. For the Gödel case, the angular-momentum term is a non-normalizable perturbation. Hence we need not quantize its coefficient since it does not correspond to a quantum number. This term vanishes as we take $\alpha' \to 0$ unless we take a scaling limit for the spacetime vorticity. This explains why the CFT will be ‘blind’ to the finite spacetime vorticity. It would be interesting to understand if the infinite vorticity deformation of the CFT can be ruled out by unitarity.

Finally, to understand the properties of the solutions (2.16) with $\ell \neq 0$ both in 5 and 10 dimensions is an open question.

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