Angular momentum transfer in primordial discs and the rotation of the first stars

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ABSTRACT
We investigate the rotation velocity of the first stars by modelling the angular momentum transfer in the primordial accretion disc. Assessing the impact of magnetic braking, we consider the transition in angular momentum transport mode at the Alfvén radius, from the dynamically dominated free-fall accretion to the magnetically dominated solid-body one. The accreting protostar at the centre of the primordial star-forming cloud rotates with close to breakup speed in the case without magnetic fields. Considering a physically-motivated model for small-scale turbulent dynamo amplification, we find that stellar rotation speed quickly declines if a large fraction of the initial turbulent energy is converted to magnetic energy \( \gtrsim 0.14 \). Alternatively, if the dynamo process were inefficient, for amplification due to flux-freezing, stars would become slow rotators if the pre-galactic magnetic field strength is above a critical value, \( \gtrsim 10^{-9.2} \) G, evaluated at a scale of \( n_{\text{H}} = 1 \text{ cm}^{-3} \), which is significantly higher than plausible cosmological seed values (\( \sim 10^{-15} \) G). Because of the rapid decline of the stellar rotational speed over a narrow range in model parameters, the first stars encounter a bimodal fate: rapid rotation at almost the breakup level, or the near absence of any rotation.

Key words: accretion, accretion discs – methods: numerical – stars: formation – stars: magnetic field – stars: Population III – stars: rotation.

1 INTRODUCTION
One of the outstanding challenges in modern cosmology is the formation process of the first generation of stars, the so-called Population III (Pop III). They influence all subsequent star and galaxy evolution in the early Universe, through their input of ionizing radiation and heavy chemical elements. The latter sensitively depends on the final fate of Pop III stars (Karlsson et al. 2013). There have been no direct observations yet, but the nature of the first stars has been elucidated by theoretical studies, in particular with numerical simulations of increasing physical realism (for recent reviews, see Bromm 2013; Greif 2015; Barkana 2016). Furthermore, there are a number of indirect empirical constraints, exhibiting the imprint of the first stars. Among them are extremely metal-poor (second-generation) stars, which retain the characteristic fingerprint of their parent Pop III stars in the chemical abundance pattern (e.g. Frebel et al. 2005; Beers & Christlieb 2005; Keller et al. 2014), metal-poor stellar systems in high-z galaxies, such as the enigmatic CR7 source (Sobral et al. 2015), and the gravitational wave signal emitted from the merger of massive black hole binaries (e.g. Abbott et al. 2016; Hartwig et al. 2016). There is need to further develop the theoretical model for the formation and evolution of the first stars to predict their observational signature, in light of the upcoming suite of next-generation telescopes, such as the James Webb Space Telescope (JWST).

The evolution and death of Pop III stars mainly depend on two parameters, their mass and rotation state. Specifically, stellar mass is the crucial parameter in determining their overall impact on the early Universe (e.g. Heger & Woosley 2002; Heger et al. 2003). According to theory, non-rotating zero-metallicity stars end their lives as core-collapse supernovae (CCSNe) in the mass range \( 15 < M_* \, M_\odot < 40 \) (on the main sequence), pair-instability supernovae (PISNe) in the range \( 140 < M_* \, M_\odot < 260 \), or directly collapse into black holes (BHs) for \( 40 < M_* \, M_\odot < 140 \) or \( 260 < M_* \, M_\odot \). Recent three-dimensional, gravito-radiation-hydrodynamics simulations examine in detail how massive the first stars can grow during the accretion phase (e.g. Hosokawa et al. 2011, 2016; Stacy et al. 2012, 2016; Susa 2013; Susa et al. 2014; Hirano et al. 2014).

As the second governing parameter, stellar rotation has been considered (e.g. Maeder & Meynet 2012). The dependence of stellar evolution on rotation is stronger for lower-metallicity (Brott et al. 2011), emphasizing the need to extend our understanding to Pop III. More rapid rotation enhances both mass loss and internal chemical mixing, which in turn greatly affect the path of stellar evolution, e.g. by establishing homogeneous chemical abundance distributions. Select consequences are the nucleosynthetic abundance pattern in massive Pop III stars (e.g. Chiappini et al. 2011), and the lowering of the minimum mass for a PISN to occur (e.g. Chatzopoulos & Wheeler 2012; Yoon et al. 2012). The latter is crucial to
correctly assess the prospects for future PISN surveys with JWST (e.g. Hummel et al. 2012). Furthermore, extremely rapid rotators with more than half the breakup speed encounter violent fates, such as gamma-ray bursts (GRBs), providing highly energetic probes of the early Universe (e.g. Bromm & Loeb 2006; Levan et al. 2016; Toma et al. 2016). As a final example, very massive (≥ 250 M⊙), rapidly rotating, low-metallicity stars might constitute single progenitors for a subset of the observed gravitational wave signals (D’Orazio & Loeb 2017).

However, the degree of stellar rotation in Pop III stars remains highly uncertain. One has to contend with the computational difficulty of following the protostellar accretion process for long times (~Myr), while also achieving the extremely high densities to resolve the protostellar surface. The most precise three-dimensional hydrodynamics simulation for first star growth could only follow the accretion for ten years after initial protostar formation (Greif et al. 2012). Other studies considered longer accretion times, but needed to evaluate the accreted angular momentum at the numerical resolution scale, which was much larger than the actual stellar surface (Stacy et al. 2011, 2013). The currently available results suggest a final rotation close to the breakup speed, but have not yet taken into account the entire angular momentum transfer over all relevant scales.

To develop a more realistic picture, the possible spin-down from such near-breakup rotation due to magnetic stresses should be considered. This question is related to the so-called ‘angular momentum problem’ in present-day (Population I) star formation, where the observed initial angular momentum of a cloud core must be efficiently redistributed or removed during collapse (e.g. McKee & Ostriker 2007; Larson 2010). As a possible solution, the angular momentum endowed to the protostar via the accreting gas may be limited by the magnetic braking on the accretion disc. Specifically, there are two main theoretical ideas to achieve this regulation (for a review, see Littlefair 2014): magnetic star-disc interaction (‘disc locking’: e.g. Matt & Pudritz 2005a; Matt et al. 2010) and stellar winds (e.g. Matt & Pudritz 2005b; Matt et al. 2012). In a recent synthesis, Rosen et al. (2012) develop an angular momentum evolution model for massive stars, considering both magnetic and gravitational torques.

In existing simulations of primordial accretion discs, however, magnetic effects are often neglected because of the tiny cosmological seed strength (Xu et al. 2008; Widrow et al. 2012). In some numerical studies, the fields are significantly amplified during the subsequent cloud collapse (Sur et al. 2010, 2012; Federrath et al. 2011; Turk et al. 2012). The resulting magnetic stresses could thus influence the long-term disc evolution, accretion and fragmentation process (Machida & Doi 2013). Later on, a magnetically-driven jet could affect the subsequent evolution (e.g. Latif & Schleicher 2016). However, the overall impact on stellar rotation remains uncertain. In this paper, we construct a simple, semi-analytical model to evaluate angular momentum transport, mediated by the primordial accretion disc. We derive a critical magnetic field strength above which the stellar rotation quickly declines via magnetic braking. This idealized estimate needs to be updated with future, fully self-consistent simulations, but is useful as an initial exploration.

The remainder of the paper is organized as follows. We begin by describing our semi-numerical methodology in Section 2. Section 3 presents the model results, in particular the rotational speed of the first stars depending on models of the magnetic field strength amplification. In Section 4, we provide our main conclusions and discuss their implications.

2 METHODOLOGY

The aim of this study is to evaluate the dependence of stellar rotation on magnetic field strength for Pop III stars. To circumvent the difficulty of rigorous magneto-hydrodynamical (MHD) numerical simulations, we construct an idealized semi-analytical model (see Fig. 1). We compute the transfer of angular momentum onto the protostar via accreting gas using the results of cosmological hydrodynamical simulations. Although these baseline simulations do not include any MHD effects, we still have a valid representation of the gas density and kinematics during the early stages of collapse, when magnetic fields are not yet dynamically important (see Fig. 2). We then analytically estimate the distribution of magnetic field strength (Section 2.1). The mode of angular momentum transfer undergoes a transition at the Alfvén radius, R_A, inside of which magnetic pressure overcomes the dynamical ram pressure (Section 2.2). Our model evaluates the resulting angular momentum imparted to the protostar surface (Section 2.3), and yields the resulting stellar rotational velocity (Section 2.4).

2.1 Field amplification during cloud collapse

The distribution of magnetic field strength throughout the star-forming cloud is a necessary ingredient for our modelling, but is not self-consistently calculated within the cosmological simulation. This is justified in the cosmological context due to the initial weakness of any seed field, such as those generated through astrophysical mechanisms, including the Biermann battery (e.g. Biermann 1950) and Weibel instability (e.g. Schlücker & Shukla 2003). During the subsequent cloud collapse, however, the B-field can be further amplified via gravitational compression (e.g. Xu et al. 2008), the small-scale turbulent dynamo process, whose amplification rate depends on numerical resolution (e.g. Sur et al. 2010, 2012; Federrath et al. 2011; Turk et al. 2012; Schober et al. 2012), and the magneto-rotational instability (MRI; e.g. Balbus & Hawley 1991; Hawley & Balbus 1991; Balbus & Hawley 1998; Silk & Langer 2006). During the later accretion phase, the tangled magnetic fields resulting from the small-scale dynamo could be realigned coherently via the α-Ω dynamo in differentially rotating discs (e.g. Pudritz & Silk 1989; Tan & Blackman 2004; Latif & Schleicher 2016). The magnetic field strength may finally reach the equipartition value, providing dynamically important magnetic stresses.
To date, there have been no MHD simulations which can investigate the entire amplification process of magnetic field strength, from the tiny cosmological seed to the final value inside a newborn star. Therefore, we adopt idealized amplification models during the collapse stage of the star-forming cloud and determine the magnetic field distribution at the beginning of the accretion phase.

2.1.1 Flux-freezing

The small magnetic seed field can be further amplified during cloud compression. If the $B$-field is tightly coupled to the collapsing gas through flux-freezing, the resulting amplification is described by a power law, $B \propto n_{H}^{\alpha}$, where $n_{H} = \rho m_{H}$ is the gas number density, normalized to the proton mass $m_{H}$. As basic model parameter, we specify the cosmological seed field, $B_{1}$, before the cloud collapse begins, at a gas number density of $n_{H} = 1 \text{ cm}^{-3}$. Figure 3(a) shows the resulting field amplification, employing the simulation data from Fig. 2.

2.1.2 Small-scale turbulent dynamo

The turbulent dynamo can lead to further amplification. To exemplify this, cosmological MHD simulations provide power-law fits to their results, to be compared to the flux-freezing expression, specifically $B \propto n_{H}^{0.83}$ (Federrath et al. 2011) and $B \propto n_{H}^{0.89}$ (Turk et al. 2012), shown in Figs 3(b) and (c). However, the amplification level via the turbulent small-scale dynamo depends on the numerical resolution, and no MHD simulation has reached convergence. Following a different route, Schober et al. (2015) present a physically-motivated model for the small-scale dynamo, indicating that the magnetic field can be highly amplified by energy conversion from the initial turbulent kinetic energy. Depending on the detailed properties of the turbulence, their study derives an upper limit of $\sim 30\%$ for the resulting saturation level.

Here, we apply the maximum saturation level of the magnetic field strength obtained from the physically-motivated model (Schober et al. 2015), $\alpha_{\text{sat}} = m_{\text{sat}} \epsilon_{0}$, where $m_{\text{sat}}$ is the magnetic energy density at saturation and $\epsilon_{0}$ the initial turbulent kinetic energy. The maximally amplified magnetic field strength, $B_{\text{sat}}$, then becomes

$$\frac{B_{\text{sat}} R_{*}^{2}}{8\pi} = \alpha_{\text{sat}} \frac{\rho v_{\text{turb}} R_{*}^{2}}{2},$$

where $v_{\text{turb}}$ is the turbulent velocity, depicted in Fig. 2(f). The saturation level, $\alpha_{\text{sat}}$, depends on the microphysical properties of the turbulence, and is determined by the magnetic Prandtl number, $Pm = \frac{Rm}{Re}$, where $Rm$ and $Re$ are the magnetic and hydrodynamical Reynolds numbers, respectively. Assuming Kolmogorov turbulence for the Pop III protostellar disc (Stacy et al. 2016), we show the resulting field amplification in Fig. 4, for two extreme cases: $\alpha_{\text{sat}} = 0.304$ ($Pm \gg 1$) and 0.0238 ($Pm \ll 1$).

2.1.3 Lower and upper limits

We also consider lower and upper limits for the $B$-field distribution. As minimum value, we assume a dipole field anchored to the central protostar. The perpendicular component in the equatorial plane at a distance $r$ from the star is

$$B_{\text{dipole}} = B_{0} \left( \frac{r}{R_{*}} \right)^{-3},$$

Figure 2. Initial conditions for semi-analytic $B$-field model. We show radial distributions of gas properties in the collapsed primordial cloud, as obtained in the underlying cosmological simulation: (a) gas number density, (b) enclosed gas mass, (c) rotation velocity, (d) radial infall velocity, (e) sound speed, and (f) turbulent velocity. The solid lines show analytical fits, provided in Appendix A, to the simulation results (dashed lines), evaluated at the beginning of the protostellar accretion phase.
Figure 3. Radial distributions of magnetic field strength, amplified during the collapse of a primordial star-forming cloud: (a) $B \propto \rho^{2/3}$, (b) $B \propto \rho^{0.83}$, and (c) $B \propto \rho^{0.89}$. The line colours represent the initial $B$-field strength when $n_H = 1 \text{ cm}^{-3}$, $B_1 = 10^{-14}$, $10^{-12}$, $10^{-10}$ and $10^{-8}$ G in panels (a - c), respectively. The two dashed lines are the maximum and minimum field strengths, $B_{\text{kin}}$ and $B_{\text{dipole}}$, respectively (see Section 2.1.3 for further discussion). The transition in angular momentum transfer mode occurs where the amplified field strength reaches the thick grey line, representing the condition $B = 4n_p\nu_{\text{rad}}^{-1}$.

where $B_*$ is the field strength at the stellar surface $R_*$. We adopt $B_* = 1 \text{ kG}$ as fiducial value, based on the surface field strength observed for present-day, pre-main-sequence stars (e.g. Johns-Krull 2007). Massive Pop III protostars may well behave differently, but we defer such a more complete exploration to future work. For the radius of the accreting protostar, for simplicity we adopt a constant value $R_* = 30R_\odot$, consistent with the radii employed in our protostellar evolution model, discussed below (see Fig. 5c).

We estimate the upper limit for the $B$-field by assuming that the magnetic pressure, or energy density, must be less than the local dynamical pressure to continue cloud collapse and gas accretion, $B^2 \leq 4\pi n_p c_s^2$, where $c_s$ is the sound speed. The resulting maximum $B$-field is

$$B_{\text{kin}} = 0.14 G \left( \frac{n_H}{10^{10} \text{ cm}^{-3}} \right)^{12} \left( \frac{c_s}{3 \text{ km s}^{-1}} \right)^4. \tag{3}$$

Our results are largely consistent with recent studies of the dynamo-amplified field strength in primordial discs, arguing that fields can reach close to the saturation, or energy equipartition, level (Latif & Schleicher 2016).\(^1\) We finally assume that a significant fraction of the magnetic stress can be organized into a large-scale, coherent topology, e.g. by following the small-scale turbulent dynamo with an $\alpha-\Omega$ dynamo, or by imposing the stellar dipole field on to the surrounding disc. As a consequence, the inner disc, where magnetic stresses may dominate, could be forced into solid-body rotation (see the discussion below).

2.2 Angular momentum transport modes

Our model considers the magnetic impact on angular momentum transfer in the infalling gaseous material. We assume that the rotational velocity evolution undergoes a transition at the Alfvén radius, $R_A$ (see Fig. 1). At $r > R_A$, where magnetic pressure is negligible, the gas accretion proceeds dynamically. Initially, the gas experiences free-fall collapse with constant angular momentum, such that $v_{\text{rot}} = \text{const}$, where $v_{\text{rot}}$ is the rotational velocity. After the formation of a centrifugally supported disc, the gas eventually joins the disc and migrates through the accretion disc with the Keplerian rotation speed, $v_{\text{Kepl}} = \sqrt{GM_\ast/r}$, where $M_\ast \geq M_\odot$ is the mass enclosed within radius $r$, which is in turn close to the mass of the central protostar, $M_\ast$. At $r < R_A$, on the other hand, magnetic stresses dominate the accretion process. We further assume that, as a consequence of the dominant magnetic stress, the disc is locked into solid-body rotation, such that $v_{\text{rot}} = \text{const}$.

\(^1\) Note that our estimate for the maximally amplified field, $B_{\text{kin}}$, is consistent with the upper limit for the saturation level achievable for the small-scale dynamo (Schober et al. 2015).
The angular momentum evolution of accreting gas thus depends on the initial distance from the star, as follows. For a gas element accreted from an initial radius, \( r_{\text{init}} \), the rotational velocity at the stellar surface, \( R_\star \), is
\[
v_{\text{rot}} R_\star \equiv \frac{R_\star}{r_{\text{init}}} v_{\text{rot}} r_{\text{init}},
\]
if \( r_{\text{init}} \leq R_\star \), and
\[
v_{\text{rot}} R_\star = \left( \frac{R_\star}{R_\star} \right) v_{\text{rot}} R_\star = \left( \frac{R_\star}{R_\star} \right) \min \left[ \left( \frac{r_{\text{init}}}{R_\star} \right) v_{\text{rot}} r_{\text{init}}, v_{\text{Kep}} \right],
\]
if \( r_{\text{init}} > R_\star \), respectively. In the latter expression, we evaluate the Keplerian speed at the Alfvén radius, \( v_{\text{Kep}} = v_{\text{Kep}} R_\star \). The character of dynamical infall switches from free-fall collapse to disc accretion when the rotational velocity reaches \( v_{\text{Kep}} \).

The mode transition occurs where the magnetic pressure balances the ram pressure of spherical (free-fall) accretion, \( \dot{B}^2 = 4\pi \rho v_{\text{rad}}^2 \), where \( v_{\text{rad}} \) is the radial infall velocity. Here, the dynamical and magnetic cloud properties are given by their respective radial distributions (Figs. 2, 3 and 4). This criterion thus implicitly defines the Alfvén radius. It is convenient to work in terms of a local function, \( R_{A,\star} \), to evaluate the transition criterion, as follows. Substituting the shell-crossing mass accretion rate, \( \rho = M4\pi R^2 v_{\text{rad}} \), we arrive at the expression
\[
R_{A,\star} \approx 9 \times 10^6 R_\odot \left( \frac{B}{0.1 \text{G}} \right)^{-1} \left( \frac{M}{0.04 M_\odot \text{ yr}^{-1}} \right)^{12} \left( \frac{v_{\text{rad}}}{3 \text{ km s}^{-1}} \right)^{12}.
\]
(6)

The Alfvén radius is then given by the condition \( R_{A,\star} \approx r \). During the accretion process, each gas element migrates inwards toward the central protostar, possibly entering a region where magnetic pressure becomes dominant. Because the radial distribution of the infall velocity is very similar to that of the sound speed (see Figs. 2d and 3), the crossing point between the \( B \)-field distribution and the upper limit, \( B_{\text{lim}} \), represent the transition scales, depending on the seed field strength (Fig. 3) and the saturation level (Fig. 4).

### 2.3 Protostellar radial evolution

We model the radial evolution of the accreting protostar as in Stacy et al. (2011), and refer the reader to their section 3.2.2 for details. A newborn protostellar core gradually expands during the initial adiabatic accretion phase, growing as
\[
R_\star \equiv 50 \left( \frac{M_\star}{1 M_\odot} \right)^{13} \left( \frac{M_\star}{M_{\text{crit}}} \right)^{13} R_\odot,
\]
(7)
where \( M_{\text{crit}} = 4.4 \times 10^{-3} M_\odot \text{ yr}^{-1} \) is the critical accretion rate above which stellar radiative feedback, opposing continued gas accretion, becomes efficient (Omukai & Palla 2003). The stellar radius shrinks during the following Kelvin-Helmholtz (KH) contraction phase as
\[
R_\star = 140 \left( \frac{M_\star}{10 M_\odot} \right)^{-2} \left( \frac{M_\star}{M_{\text{crit}}} \right)^{13} R_\odot.
\]
(8)

The phase shift occurs when \( R_\star \) becomes smaller than \( R_1 \). If the stellar rotational speed reaches the breakup value during the KH contraction, we assume that the star rotates at the breakup speed by slowing the radial contraction to
\[
R_3 = \exp \left[ \ln R_{\text{pre}} - \frac{d}{dt} \ln M_\star \right] R_\odot,
\]
(9)
where \( R_{\text{pre}} \) denotes the stellar radius at the previous step. Finally, the contracting star reaches the zero-age main sequence (ZAMS) at a radius
\[
R_4 = 4.65 \left( \frac{M_\star}{100 M_\odot} \right)^{0.61} R_\odot.
\]
(10)

Recently, this model has been updated (Stacy et al. 2016), but we adopt the earlier version for simplicity.

### 2.4 Protostellar angular momentum

Employing the protostellar radial evolution model above, we proceed to evaluate the angular momentum history of the accreting protostar as follows. The time-dependent mass growth, \( M_\star(t) \), is obtained from the simulation data, represented by an analytical fit \( M_\star(t) \) (Fig. 5a and Equation A7). When the stellar mass grows from \( M_\star - dt \) to \( M_\star \), the newly accreted gas mass is \( \Delta M_\star = M_\star - M_\star - dt \). We assume that the gas element migrates from a distance \( r \) where \( M_\star \), arriving at the protostar with a rotational velocity given by Equations 4 or 5. Then, the total angular momentum acquired by the star is \( J_\star = r_\star M_\star v_{\text{Kep}} \), where \( J_{\text{acc}} = r_\star M_\star v_{\text{Kep}} \). The resulting angular momentum of the star with mass \( M_\star \) and radius \( R_\star \), modelled as a solidly rotating sphere for simplicity, is \( J_\star = \Omega I_\star \), where \( \Omega \) is the angular velocity and \( I_\star = 2M_\star R_\star^2/5 \) the moment of inertia. Finally, the equatorial rotational velocity of the accreting star is
\[
V_r = R_\star \Omega = \frac{5J_\star}{2R_\star M_\star}.
\]
(11)

We note that we ignore the possible inclination of the accretion flow with respect to the rotational axis (e.g. Bate et al. 2010; Fielding et al. 2015), and assume accretion through an equatorial disc.

### 2.5 Time evolution of accreting protostar

Figure 5 summarizes the evolution of the Pop III protostar, resulting from our idealized modelling. The analytical fitting functions well reproduce the simulation results (Figs. 5a and b). Although the fitting formulae smooth over some detailed features, such as the episodic accretion, which can act to quickly expand the stellar radius (Hosokawa et al. 2016; Sakurai et al. 2016), the overall evolution is properly represented. We note that the extrapolation to times beyond \( \sim 10^5 \) yr is very uncertain.

Figure 5(d) shows the stellar rotational velocity evolution without magnetic fields present. This is the baseline case where we can compare to results from previous simulations of Pop III star formation. Specifically, we consider a set of simulations with different spatial resolutions (Greif et al. 2012; Stacy et al. 2011; Hirano & Bromm 2017, see also Appendix B). Each simulation provides estimates for the mass (\( M_\star \)), radius (\( R_\star \)) and angular momentum (\( j_\star \)) of the central core, at the resolved scales (see Fig. B1). The lowest resolution simulation covers the entire history of the protostellar accretion phase (\( \sim 10^5 \) yr), whereas at highest-resolution, only the initial \( \sim 10^5 \) yr are covered. For simplicity, we assume that the mass accretion histories at the protostellar surface and the resolved scale are the same, such that \( M_\star = M_\star \). Applying our idealized model (see above) to this input data, we derive estimates for the resulting stellar rotation velocity (coloured lines in Fig. 5).

These cases represent an intermediate approach between fully consistent, ab-initio simulations of protostellar core formation, and our idealized, semi-analytic formalism here. Encouragingly, the extrapolated rotational velocities at different radial and time scales from the simulations are well reproduced by our model. The future
goal, however, is to achieve realistic simulations, capable of resolving the protostellar core scale, while at the same time being able to continue to the end of the accretion phase. As argued in this paper, it will be crucial to also include MHD effects in such next-generation simulations.

3 STELLAR ROTATION HISTORIES

We investigate how the stellar rotation speed changes due to the magnetic braking by considering the switch in angular momentum transfer mode at $R_A$ (Equation 6). In so doing, we explore different amplification models for the magnetic field in the primordial star-forming cloud.

In the absence of any magnetic field, $B_1 = 0$, the stellar rotational velocity gradually grows to $\sim 100\text{km s}^{-1}$ during the first expansion phase, and further increases in the KH contraction stage (see Fig. 5d). In this case, the rotation speed continues to increase up to $\geq 1,000\text{km s}^{-1}$ due to the angular momentum introduced via the accreting gas, and finally reaches an asymptotic value at the ZAMS transition, $\sim 10^7 \text{yr}$ after initial protostar formation. Due to the final spin-up, the rotational velocity almost reaches the breakup speed, $V_{\text{crit}}$, towards the end of the stellar mass growth (dashed line in Fig. 6). Without any effects capable of removing angular momentum, massive Pop III stars would therefore typically become rapid rotators, as has previously been established (e.g. Stacy et al. 2011, 2013).

The coloured lines in Fig. 6 represent stellar rotation histories, normalized to the breakup speed, $V_{\text{crit}}$, for different amplification models. Magnetic braking suppresses the stellar spin-up within $R_A$, such that the rotational degree decreases during the accretion of material within $R_A$, endowed with a reduced rotational velocity of $v_{\text{crit}}R_{\text{crit}} < 1$ (Equation 4). The subsequent evolution sensitively depends on the efficiency of the small-scale dynamo process, to be discussed next.

3.1 Efficient dynamo

In Figure 6(a), we show results for the physically-motivated model of the small-scale turbulent dynamo. The stellar rotation histories are markedly different depending on the assumed saturation levels, measured by $\alpha_{\text{sat}}$. In the maximally amplified case with $\alpha_{\text{sat}} = 0.304$ ($\text{Pm} \gg 1$), the transition at the Alfvén radius occurs at larger disc radius (Fig. 4), such that the rotational degree continues to decrease for a prolonged time. On the other hand, when $\alpha_{\text{sat}} = 0.0238$ ($\text{Pm} \ll 1$), the transition occurs close to the stellar surface, such that there is no effective magnetic braking. When the saturation level falls below $\alpha_{\text{sat}} \geq 0.14$, the stellar rotation history abruptly changes from the non-rotating case to the one at almost breakup speed. Therefore, if the saturation level of the small-scale dynamo amplification exceeds this critical value, Pop III stars become slow rotators, independent of the initial magnetic field.

3.2 Inefficient dynamo

In the case of an inefficient small-scale turbulent dynamo ($\alpha_{\text{sat}} < 0.14$), some degree of effective magnetic braking could still occur, if compressional flux-freezing were to sufficiently amplify the magnetic seed field. Any suppression of the stellar spin-up would then depend on the pre-galactic magnetic field strength, $B_1$ (Fig. 6b). The resulting rotation histories are now more complex. After the initial braking phase, the rotation velocity increases again, when the
gas outside $R_A$ migrates to the stellar surface, with rotational velocity depending on the initial radius as $v_{\text{rot}} = R_A R_\text{d}$. During the KH contraction phase, the rotation speed also decreases because of the dependence of accreting angular momentum, $\dot{J}_\text{acc} = v_{\text{rot}} R_\text{d} M_\ast = R_\ast R_A^3 v_{\text{rot}} r_\text{d} M_\ast$, on the contracting stellar radius $R_\ast$. The final rise at $t \sim 10^5$ yr occurs after the ZAMS is reached.

There is a drastic decline in final rotation velocity, $V / V_{\ast, \text{crit}} \simeq 0.2$ at $t = 10^5$ yr, for $B_1 \simeq 10^{-2}$ G. If the pre-galactic seed field is less than this threshold value, the rotation recovers the high spin of the zero $B$-field case. Because this decline occurs over such a narrow range in $B_1$, the first stars encounter a bimodal fate, where they either rapidly rotate, with close to the breakup speed, or exhibit virtually no rotation at all. Simulated cosmological seed fields, $\sim 10^{-15}$ G at $n_H = 1$ cm$^{-3}$, are less than this critical value, such that the first stars are predicted to typically be rapid rotators unless there are additional amplification processes present.

4 SUMMARY AND CONCLUSIONS

Overall, our model suggests that the spin-down of the first stars requires efficient small-scale dynamo activity, or flux-freezing amplification from relatively high initial magnetic field strengths. Specifically, our analysis indicates that there is a bifurcation in the final rotational state of a Pop III star, exhibiting rotation either at near-zero or close to breakup speed. We may speculate that such bimodality is also reflected in observables, such as the nucleosynthetic patterns in extremely metal-poor stars (e.g. Chiappini et al. 2011). One key parameter in deciding which rotational fate is more common, or typical, is the magnetic Prandtl number in the disc. Existing studies suggest that this number varies over many orders of magnitude during the collapse (e.g. Schober et al. 2012), and guidance from present-day star formation is inconclusive (e.g. Oishi & Mac Low 2011). Therefore, future MHD simulations are needed to self-consistently study the properties of discs in primordial star formation.

The final degree of rotation is crucially important for the subsequent stellar evolution of Pop III stars. A key example is rotationally induced mixing, reducing the chemical abundance gradient throughout the star (e.g. Maeder & Meynet 2012). Specifically, massive stars with close to breakup rotation, $V / V_{\ast, \text{crit}} \gtrsim 0.5$, can undergo chemically homogeneous evolution, and enter a Wolf-Rayet phase (e.g. Yoon & Langer 2005; Woosley & Heger 2006, for the primordial case). The final fate and death of the first stars also sensitively depend on the rotation velocity (e.g. Yoon et al. 2012, see their fig. 12). Rapid stellar rotation will decrease the mass threshold for triggering energetic events, such as PISNe and hypernovae (e.g. Nomoto et al. 2003; Chatzopoulos & Wheeler 2012), or facilitate the occurrence of collapsar-driven GRBs at high redshifts (e.g. Wang et al. 2012). As an important implication for Pop III feedback, rapidly rotating stars are less puffed up and thus bluer, resulting in the increased emission of ionizing photons. The surrounding HII region would then be more extended, thereby affecting the impact of subsequent SN feedback.

The detailed differences in stellar evolution sensitively depend on the stellar model employed (e.g. Ekström et al. 2008). In addition, magnetic fields may also affect stellar evolution. An example is the Spruit-Taylor dynamo which can transport angular momentum from the core to the envelope (Spruit 2002). There are a number of rotationally-induced dynamical effects, such as a bar-like instability, which enables angular momentum transfer via gravitational torques, acting to slow down the core (Lin et al. 2011). Recently, Lee & Yoon (2016) report that rapid rotation can control the mass growth of massive Pop III stars, through the so-called $\Omega^2$ limit, where centrifugal forces boost the effectiveness of radiation pressure. As another example, Haemmerlé et al. (2018) present an evolution model of a supermassive Pop III star which implies that the surface rotation velocity is maintained at less than 10–20% of the Keplerian value to satisfy the $\Omega^2$ limit.
The magnetic braking can also change the morphology of the accreting gas envelope and disc. Machida & Doi (2013) perform magneto-hydrodynamic simulations to investigate disc formation and fragmentation in the magnetized primordial cloud, whose magnetic field is amplified according to $B \propto \rho^{3/2}$. They conclude that the star formation mode changes from binary or multiple fragmentation to single protostar formation, if the ambient star formation mode changes from binary or multiple fragmentation to single protostar formation, if the ambient star formation mode changes from binary or multiple fragmentation to single protostar formation, if the ambient star formation mode changes from binary or multiple fragmentation to single protostar formation.

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APPENDIX A: SIMULATION DATA AND FITTING FUNCTIONS

The numerical data used in our modelling (dashed lines in Figs. 2 and 5) are obtained from a three-dimensional cosmological simulation (Hirano et al. 2014). The simulation set-up and methodology are briefly summarized in section 3.1 in Hirano & Bromm (2017), where the same cloud is adopted as the initial condition. We represent the simulation data with the following fitting functions, spherically averaged around the new-born protostar, to be used in our model. The gas number density is (Fig. 2a)

\[ n_{\text{H}_1} = 10^{10} r^{-6.06} \times 10^4 R_{\odot}^{-2.2} \text{ cm}^{-3}, \]  

(A1)

the enclosed gas mass (Fig. 2b)

\[ M_{\text{enc}}(r) = \int_0^r 4\pi r'^2 \rho dr = 100 r \times 10^6 R_{\odot}^{-1} M_{\odot}, \]  

(A2)

the rotation velocity, as a function of enclosed mass (Fig. 2c),

\[ v_{\text{rot}}(r) = \begin{cases} 
10.0, \\
3.5 M_{\text{enc}} 10^4 M_{\odot}^{-0.3}, \\
3.5, \\
2.0 M_{\text{enc}} 10^4 M_{\odot}^{-0.3}, \\
2.0 \text{ km s}^{-1}, 
\end{cases} \]  

(A3)

the radial velocity (Fig. 2d)

\[ v_{\text{rad}}(r) = \begin{cases} 
7.0, \\
7.0 M_{\text{enc}} 0.106 M_{\odot}^{-0.15}, \\
1.0 M_{\text{enc}} 481.4 M_{\odot}^{-0.27}, \\
1.0 \text{ km s}^{-1}, 
\end{cases} \]  

(A4)

the sound speed (Fig. 2e)

\[ c_s(r) = \begin{cases} 
7.0, \\
7.0 M_{\text{enc}} 0.02 M_{\odot}^{-0.15}, \\
2.0 \text{ km s}^{-1}, 
\end{cases} \]  

(A5)

and the turbulent velocity (Fig. 2f)

\[ v_{\text{turb}}(r) = \begin{cases} 
9.0, \\
9.0 M_{\text{enc}} 0.04 M_{\odot}^{-0.55}, \\
1.0 M_{\text{enc}} 2 M_{\odot}^{-0.15}, \\
1.5 M_{\text{enc}} 40 M_{\odot}^{-0.1}, \\
1.3 M_{\text{enc}} 200 M_{\odot}^{-0.8}, \\
0.5 M_{\text{enc}} 10^3 M_{\odot}^{-0.1}, \\
0.8 M_{\text{enc}} 8 \times 10^3 M_{\odot}^{-2}, \\
1.5 M_{\text{enc}} 10^3 M_{\odot}^{-0.3}, \\
4.0 M_{\text{enc}} 5 \times 10^4 M_{\odot}^{-1} \text{ km s}^{-1}. 
\end{cases} \]  

(A6)

The mass accretion history of the protostar (Fig. 5a) is also fitted as

\[ M_* t = \begin{cases} 
30.92 r^{-1250} \text{ yr}^{1.50}, \\
30.92 r^{-1250} \text{ yr}^{0.65} M_{\odot}, 
\end{cases} \]  

(A7)

Figure B1. Summary of previous simulation data, with different numerical resolution and time coverage: (a) evolution of the hydrostatic core mass, (b) core radius, and (c) specific angular momentum. We show data from three studies: G12 (Greif et al. 2012; Stacy et al. 2013, dotted lines), HB17 (Hirano & Bromm 2017, dashed), and S11 (Stacy et al. 2011, dot-dashed). None of these simulations includes magnetic fields.

APPENDIX B: SUMMARY OF PREVIOUS SIMULATIONS

We consider previous simulation data of Pop III star formation, extracting estimates for the resulting stellar rotation, for comparison with our modelling (Fig. B1). Greif et al. (2012) is the only simulation to date which applied no sub-grid model, and directly resolved the optically thick protostellar core. However, their time coverage is limited to only \( \sim 10 \text{ yr} \) after initial protostellar core formation. Specifically, we use results for four different haloes (MH1 to MH4). Hirano & Bromm (2017) perform a series of simulations, resulting in hydrostatic cores with threshold densities \( n_H = 10^{15}, 10^{12} \), and \( 10^{10} \text{ cm}^{-3} \), above which gas collapse is suppressed by artificially enhanced opacity. These simulations were initiated from the same large-scale star forming site used in our baseline model (Hirano et al. 2014). We analyse the primary hydrostatic core in these three cases. Stacy et al. (2011) compute gas cloud collapse up to \( n_H = 10^{12} \text{ cm}^{-3} \) by adopting the sink particle method with sink radius 50 au \( \approx 10^7 M_{\odot} \). We consider the growth of two cases (sinks A and B) in that study.
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