Nonlinearity of the stress-strain dependence of quasi-isotropic CFRP laminate under re-static loading

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Abstract. As part of the task of studying the degradation of material properties under re-static loading of a quasi-isotropic CFRP laminate the nonlinearity of the stress-strain dependence in the areas of active deformation is proved. The optimal curve approximating the experimental data from the point of view of the problem is chosen. The change in the derivatives of this curve can be use $\sigma$ to assess the degradation of the material properties during re-loading.

1. Introduction
Studying the behavior of CFRP laminates in the field of applied forces from the standpoint of a system evolving to a critical state [1], it is necessary to determine several measurable physical quantities, the change of which will make it possible to assess the degradation of material properties with an increase in the number of cycles of re-static loading. The task is to use the nonlinear properties of CFRP laminate itself as an indicator of degradation, if the assumed nonlinearity and its subsequent development can be established. The number of measured physical quantities is minimal – "time", "stress" and "strain".

The report focuses on the first part of the problem posed - a functional description of the experimental stress-strain dependence when testing a specimen of a quasi-isotropic structural FCRP laminate in one of the cycles of re-static loading.

2. Materials and test methods
The specimen is cut from a quasi-isotropic FCRP laminate with $[0/\pm45_2/0_2/90_2/\pm45_2/0]_s$ staking in the 0 direction. The dimensions of specimen working zone are as follows: length 140 mm, cross section $50 \times 6.03$ mm$^2$, center hole $\varnothing 14$ mm.

The test was carried out in hydraulic grips of the INOVA IK-6033 servo-hydraulic testing machine. The system for collecting and processing experimental information was based on electronic equipment and software from National Instruments (USA).

Quasi-static uniaxial tension with displacement control $S$ in accordance with figure 1. The duration of odd sections is 125 seconds, and even sections - 50 seconds. The modulus constant velocity of grip displacement on odd sections $V_s = 4.4 \, \mu m \cdot s^{-1}$, on even sections $V_s = 0 \, \mu m \cdot s^{-1}$ (figure 1, 2). Figure 3 shows an asymmetric change in the nominal stress $\sigma$ (hereinafter referred to as "stress") in the cross section of a specimen $50 \times 6.03$ mm$^2$ depending on time. An extensometer with a 20 mm base was installed on the side plane of the specimen symmetrically to the central hole. The average strain $\varepsilon$ was measured on a 20mm base of the extensometer (hereinafter referred to as "strain").
3. Research results

The experimental points \((\varepsilon_i, \sigma_i)\) under loading of the specimen in accordance with figure 1 are shown in figure 4.

As a result of linear approximation of the entire array of points \((\varepsilon_i, \sigma_i)\) in the sections of the loading diagram from the 1-st to the 8-th inclusive, the averaged equation has been obtained:

\[
\sigma = 78724 \cdot \varepsilon - 0.3074 \tag{1}
\]

When considering the experimental points \((\varepsilon_i, \sigma_i)\) on each separate section of the specimen loading diagram (figure 1), it is established that the dependences between them are individual and cannot be described by equation (1). As an illustration, figures 5 and 6 in an enlarged scale show the experimental points \((\varepsilon_i, \sigma_i)\) at minimum and maximum strains, respectively. Sections 1, 3, 6, 8 of the loading diagram belong to active deformation \((\sigma \uparrow)\), and sections 2, 4, 5, 7 to passive deformation \((\sigma \downarrow)\) [2, 3].
To prove the nonlinearity of the experimental dependence $\sigma_i = f(\varepsilon_i)$ and its subsequent mathematical description, we select the points $(\varepsilon_i, \sigma_i)$ of the 1-st and 3-rd sections of the active ($\sigma^{\uparrow}$) deformation of the specimen, combining them into a single array of the conditional stress-strain diagram $\sigma_i = f^*(\varepsilon_i)$ (figure 7). As a result of linear approximation of the specified array of points, we obtain an averaged equation

$$\sigma = 78334 \cdot \varepsilon + 1.7454 \quad (2)$$

in which the coefficient at $\varepsilon$ (elastic modulus) is slightly less than the coefficient at $\varepsilon$ in equation (1), which is evident. The nonlinearity of the experimental dependence $\sigma_i = f^*(\varepsilon_i)$ is confirmed by the behavior of the array of points (figure 8), each of which is a tangent modulus $E(\varepsilon_i)$ calculated from three adjacent points of the conditional stress-strain diagram $\sigma_i = f^*(\varepsilon_i)$, with subsequent one point advance in the direction of increasing strain.

The graph of the curve

$$E(\varepsilon) = -8.484 \cdot 10^{11} \cdot \varepsilon^3 + 5.782 \cdot 10^9 \cdot \varepsilon^2 - 1.168 \cdot 10^7 \cdot \varepsilon + 8.486 \cdot 10^4 \quad (3)$$

approximating the specified array of points is shown in figure 9. The tangent modulus $E(\varepsilon)$ changes during deformation, the experimental dependence $\sigma_i = f^*(\varepsilon_i)$ is nonlinear. By simple integration of equation (3), it is possible to obtain the equation of the conditional stress-strain diagram $\sigma = f^*(\varepsilon_i)$ in the form of a polynomial of the 4-th degree.

The equation of the conditional stress-strain diagram $\sigma = f^*(\varepsilon_i)$ in the form of a polynomial of the 4-th degree can also be obtained by approximating the original array of points $(\varepsilon_i, \sigma_i)$ (figure 7):

**Figure 5.** Specimen loading diagram (figure 1): experimental points $(\varepsilon_i, \sigma_i)$ at minimum strain.

**Figure 6.** Specimen loading diagram (figure 1): experimental points $(\varepsilon_i, \sigma_i)$ at maximum strain.

**Figure 7.** 1-st and 3-rd sections of the specimen loading diagram: experimental points $(\varepsilon_i, \sigma_i)$.

**Figure 8.** 1-st and 3-rd sections of the specimen loading diagram: an array of values of the tangent modulus $E(\varepsilon_i)$.

**Figure 9.** Approximation of the array of points $(\varepsilon_i, \sigma_i)$.
\[ \sigma = -1.535 \times 10^{11} \varepsilon^4 + 1.358 \times 10^9 \varepsilon^3 - 4.046 \times 10^6 \varepsilon^2 + 8.279 \times 10^4 \varepsilon + 5.360 \times 10^{-1} \quad (4) \]

In this case, the slope of the function graph \( \sigma = f^*(\varepsilon) \) to the positive direction of \( x \)-axis (figure 10) is determined by the first derivative of the function (4) at the strain \( \varepsilon \):

\[ \frac{d\sigma}{d\varepsilon} = -6.140 \times 10^{11} \varepsilon^3 + 4.074 \times 10^9 \varepsilon^2 - 8.092 \times 10^6 \varepsilon + 8.279 \times 10^4 \quad (5) \]

The second derivative of the function (4) at the strain \( \varepsilon \) (figure 11) characterizes the direction of the convexity (↑up, ↓down) of the function graph, and also allows us to determine the inflection points (ip1, ip2), in which the direction of the convexity changes:

\[ \frac{d^2\sigma}{d\varepsilon^2} = -1.842 \times 10^{12} \varepsilon^2 + 8.148 \times 10^9 \varepsilon - 8.092 \times 10^6 \quad (6) \]

Figure 12 compares the slope of the function graph \( \sigma = f^*(\varepsilon) \) to the positive direction of the strain axis equation (5) with the curve of the tangent modulus \( E(\varepsilon) \) equation (3) obtained by the methods described above. There are no qualitative differences and there is an insignificant relative shift in the strain of the second inflection points (ip2) of the graphs of the functions \( d\sigma/d\varepsilon \) and \( E(\varepsilon) \). The maximum (at \( \varepsilon = 0 \)) quantitative difference is about 2.5%, which indicates that it is acceptable to use both methods to identify the nonlinearity of the stress – strain dependence of a quasi-isotropic structural FCRP laminate.
4. Conclusions
The conducted studies of the experimental stress-strain dependence for a specimen of quasi-isotropic structural FCRP laminate in one of the cycles of re-static loading allowed to establish the nonlinearity of this dependence, to determine the type of curve approximating the experimental data, and to minimize the number of nonlinearity parameters of the approximating curve – the slope of the graph, the direction of the convexity and the inflection points.

In the future, it is intended to use the change of the nonlinear part of the stress – strain dependence as an indicator of the degradation of material properties under re-static loading (figures 13 and 14) [4].

References
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