Attractive vortex interaction and the intermediate-mixed state of superconductors

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The magnetic vortices in superconductors usually repel each other. Several cases are discussed when the vortex interaction has an attractive tail and thus a minimum, leading to vortex clusters and chains. Decoration pictures then typically look like in the intermediate state of type-I superconductors, showing lamellae or islands of Meissner state or surrounded by Meissner state, but with the normal regions filled with Abrikosov vortices that are typical for type-II superconductors in the mixed state. Such intermediate-mixed state was observed and investigated in detail in pure Nb, TaN and other materials 40 years ago; last year it was possibly also observed in MgB$_2$, where it was called “a totally new state” and ascribed to the existence of two superconducting electron bands, one of type-I and one of type-II. The complicated electronic structure of MgB$_2$ and its consequences for superconductivity and vortices are discussed. It is shown that for the real superconductor MgB$_2$ which possesses a single transition temperature, the assumption of two independent order parameters with separate penetration depths and separate coherence lengths is unphysical.

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1. VORTEX LATTICE FROM GL THEORY

The most successful phenomenological theory of superconductors was conceived in 1950 by Ginzburg and Landau (GL)\textsuperscript{[1]}. When written in reduced units (length unit $\lambda$, magnetic field unit $\sqrt{2}H_c$, energy density unit $\mu_0H_c^2$, where $\lambda$ is the magnetic penetration depth and $H_c$ the thermodynamic critical field) the GL theory contains only one parameter, the GL parameter $\kappa = \lambda/\xi$ ($\xi$ is the superconducting coherence length). The kind of solutions of GL theory is quite different when $\kappa < 1/\sqrt{2}$ or $\kappa > 1/\sqrt{2}$. Physically, this is due to the fact that the energy of the wall between a normal and a superconducting domain is positive when $\kappa < 0.71$ (type-I) and negative when $\kappa > 0.71$ (type-II). This implies that a type-II superconductor is unstable against the spontaneous formation of many small superconducting and normal domains. A thesis student of Lev Landau in Moscow, Alexei Abrikosov in 1953 discovered a periodic solution of the GL theory which describes the occurrence of a regular lattice of vortex lines (published in 1957\textsuperscript{[2]}). Each Abrikosov vortex (or fluxon, flux line) carries one quantum of magnetic flux $\Phi_0 = h/2e = 2.07 \cdot 10^{-15}$ Tm$^2$, and the supercurrent circulates around a singular line on which the complex GL function $\psi(x,y,z)$ (or superconducting order parameter $|\psi|^2$) is zero. For finding this vortex lattice solution, in which Landau initially did not believe\textsuperscript{[2]}, Abrikosov obtained the Nobel prize in physics 50 years later in 2003.

Within the original GL theory in bulk superconductors, with increasing applied magnetic field $B_a = \mu_0H_a$ the superconductor is first in the Meissner state which has induction $B = 0$. When $B_a$ reaches the lower critical field $B_{c1}$ Abrikosov vortices start to penetrate. With further increasing $B_a$ the vortex lines form a more and more dense triangular lattice with induction $B = (2/\sqrt{3})\Phi_0/\alpha^2$ ($\alpha =$ vortex spacing). When $B_a$ and $B$ reach the upper critical field $B_{c2} = \mu_0H_{c2} = \Phi_0/(2\pi\xi^2)$, the order parameter vanishes and the superconductor becomes normal conducting. Since $B(B_a)$ increases monotonically (see Fig. 1 (top)), one may say that the pressure of the vortex lattice is positive and the vortices repel each other, held together by the applied magnetic field $B_a$, which acts as an external pressure. From thermodynamics follows that in equilibrium (in absence of pinning and surface barriers) one has $B_a = \mu_0H_a = \mu_0\partial F/\partial B$ where $F(B,\kappa)$ is the GL free energy density. The numerical GL solution $B_a(B)$ of the vortex lattice\textsuperscript{[3]} for $1/\sqrt{2} \leq \kappa \leq 10$ may be fitted well by the expression (with $B = B/B_{c2}$, $h = B_a/B_{c2}$, $m = b - h = M/B_{c2}$, $h_{c1} = B_{c1}/B_{c2}$):

$$h(b) = h_{c1} + \frac{c_1b^3}{1 + c_2b + c_3b^2},$$

$$h_{c1} = \frac{\ln \kappa + \alpha(\kappa)}{2\kappa^2}, \quad \alpha(\kappa) = \frac{1}{2} + \frac{1 + \ln 2}{2\kappa - \sqrt{2} + 2},$$

$$c_1 = \frac{(1 - h_{c1})^3/(h_{c1} - p)^2}{(1 - 3h_{c1}^2 + 2p)/(h_{c1} - p)},$$

$$c_2 = \frac{1 + (1 - h_{c1})(1 - 2h_{c1} + 2p)/(h_{c1} - p)^2}{(2\kappa^2 - 1)\beta_4 + 1} - 1,$$

$$p = \left((2\kappa^2 - 1)\beta_4 + 1\right)^{-1}$$

with $\beta_4 = 1.15960$ (1.18034) for the triangular (square) vortex lattice. At $\kappa = 1/\sqrt{2}$ the $B(B_a)$ is a vertical straight line at $B_a = B_{c1} = B_{c2}$ like in type-I superconductors. This means the vortices do not interact with each other and all vortex arrangements have the same free energy. When $\kappa = 1/\sqrt{2}$.

The vanishing vortex interaction for $\kappa = 1/\sqrt{2}$ can also be seen from Kramer’s\textsuperscript{[4]} asymptotic interaction between two vortices at large distance $r \gg \lambda$, $V(r) = d_1(\kappa)K_0(r/\lambda) - d_2(\kappa)K_0(\sqrt{2r}/\xi)$ where the constants are

\[ d_1(\kappa) = \frac{1}{\sqrt{2\pi}}, \quad d_2(\kappa) = \frac{1}{2\sqrt{\pi}}. \]
that this V(r) has a finite maximum V(0), approximately equal to half the self energy of a vortex with flux 2Φ₀, since the two terms diverging as ±ln r compensate.

For κ > 1/√2 the potential (2) is repulsive. At κ = 1/√2 it vanishes at all distances, which means that all vortex arrangements have the same energy. This is an exact result of the GL theory [7]. The particular case κ = 1/√2 and its surroundings (κ ≈ 0.71, T ≈ Tₕ) were investigated in detail in [8, 9].
2. TYPE-I SUPERCONDUCTORS

Superconductors with $\kappa < 1/\sqrt{2}$ are called as type I. For $B_a < B_c = \mu_0 H_c$ (thermodynamic critical field) these should exhibit ideal Meissner effect with $B = 0$ (except for a surface layer of thickness $\lambda$) and $B = B_a$ for $B_a > B_c$. But in samples of finite size the demagnetizing effects and the small $\kappa \approx 0.70$, round islands of Meissner phase are surrounded by a regular vortex lattice (“intermediate mixed state”).

As recently shown by Prozorov [12, 13], the equilibrium topology of the intermediate state are flux tubes (in absence of bulk pinning and geometric barrier). Interestingly, pin-free disks and strips of constant thickness show hysteretic magnetization curves due to edge barriers for penetrating flux tubes and exiting lamellae. This topological hysteresis of type-I superconductors was observed and calculated analytically already in 1974 [14], where it was shown that ellipsoidal samples exhibit no magnetic hysteresis. Magnetic hysteresis loops in pin-free type-I superconductor disk and strips look very similar as the irreversible magnetization loops in pin-free type-II superconductors with edge barrier [15]. In particular, they go through the origin ($M = B - B_a = 0$ at $B_a = 0$, no flux is trapped) and become reversible at large $B_a$ (at $B_a > B_{c1}/2$ for type-II).

The structure of the flux tubes or lamellae in type-I superconductors and their widening (mushrooming) and branching as they approach the surface were calculated, e.g., in [16, 17]. In thin films of type-I superconductors a vortex lattice occurs though $\kappa < 0.71$ [18]. In particular,
that in thin (and infinitely large) films in perpendicular positive curvature in this case. These GL results show thickness is the magnetic stray field dominates [20]. For the vortex 

$B_c \leq B \leq B_\alpha$ and normal state for $B_\alpha > B_c$. The dashed line indicates the enhancement of the penetration field when the wall energy of the normal and superconducting domains in the intermediate state is accounted for. Middle row: Type-II/1 superconductors with Ginzburg-Landau parameter $\kappa$ close to $1/\sqrt{2}$, for $N = 0$ exhibit a jump of height $B_0$ in their induction and magnetization at $B_\alpha = B_2$ due to an attractive interaction between flux lines. For $N > 0$, this jump is stretched over a finite range of $B_\alpha$, allowing one to observe an intermediate mixed state with domains of Meissner phase and vortex-lattice phase. Bottom row: Type-II/2 superconductors with $\kappa \gg 1/\sqrt{2}$. Meissner phase for $B_\alpha < (1 - N)B_2_1$, mixed state (vortex lattice) for $(1 - N)B_2 \leq B_\alpha \leq B_2$, and normal state for $B_\alpha > B_2$. For $\kappa = 0.5$ the shear modulus $c_{66}$ of this vortex lattice is positive at all inductions $0 < B < B_{\alpha 2}$ when the film thickness is $d \leq \xi$, see Figs. 9 and 13 in [19], and the magnetization curve $-m(h) = -m(b)$ (Fig. 4 in [19]) has positive curvature in this case. These GL results show that in thin (and infinitely large) films in perpendicular field $B_\alpha = B$ the vortices may repel each other even when $\kappa < 0.71$ since the long-range repulsive interaction via the magnetic stray field dominates [20]. For the vortex interaction in thin films of finite size see [21].

3. ATTRACTIVE VORTEX INTERACTION

The first successful decoration experiments evaporated an iron wire in a 1 Torr He atmosphere that produced “magnetic smoke” which settles on the surface of the superconductor and marks the ends of the flux tubes or flux lamellae (Fig. 2,3a) or of the vortices (Figs. 3c,4,5) [22]. In the type-II superconductors Vanadium and Niobium (see Fig. 3 of [11]) and PbIn alloys they showed more or less regular triangular vortex lattice with structural defects, e.g., dislocations and vacancies. These pictures were consistent with the prediction from GL theory that the vortices repel each other. However, later decoration of very pure Nb showed a domain structure similar to that observed in type-I superconductors but with the normal domains replaced by domains of vortex lattice. The Figs. 3a and 3b compare the laminar domains in the type-I superconductor Tantalum and in the type-II superconductor Nb. In an optical microscope these domains look very similar, but the electron microscope (Fig. 3c) reveals that in Nb the domains which carry the magnetic flux consist of vortex lattice with constant lattice spacing $a_0$.

In Fig. 4 (top) one sees islands of Meissner state embedded in a vortex lattice, while Fig. 4 (bottom) shows Meissner state with islands of vortex lattice. In Fig. 5 tongues of vortex lattice penetrate from the specimen edge into the Meissner state of high-purity polycrystalline Nb.

This observation of an upper limit $a_0$ of the vortex spacing in the intermediate state of low-$\kappa$ type-II superconductors, and its explanation by an attractive interaction, suggests that the magnetization curves in the ideal (pin-free) case should exhibit a jump of height $B_0 = (2/\sqrt{3})\Phi_0/a_0^2$ at the field of first vortex penetration. This means, as soon as it is energetically favorable for vortices to penetrate, they immediately jump to their equilibrium distance $a_0$, which for $N = 0$ means a uniform induction $B_0$, see Fig. 1 (bottom). Such jumps of $B(B_\alpha)$ from 0 to $B_0$ were indeed observed in clean Nb and in lead alloys with increasing content of Thallium (which changes $\kappa$), and excellent agreement of $B_\alpha$ with the directly measured $a_0$ was observed [22–24]. Jumps $B_\alpha$, and thus vortex attraction, were also observed in the TaN system with varying nitrogen content and various $\kappa = 0.35$ to $0.5$. See also the reviews [11,27].

The jump $B_0$ in the ideal magnetization curve $B(B_\alpha)$ was confirmed by computations [28] based on the microscopic BCS-Gor’kov theory that reduces to the GL theory in the limit $T \to T_c$. Earlier computations [29] confirming the vortex attraction were based on the extended GL theory of Neumann and Tewordt [30] that keeps all correction terms to lowest order in $1 - T/T_c$. The BCS-Gor’kov computations [28] for clean superconductors yield an S-shaped curve $B(B_\alpha)$ with an unstable part (dashed curve in Fig. 1 (bottom)) that has to be replaced by a vertical line of height $B_0$ that cuts the area under the dashed line in two equal parts (Maxwell construction). Useful interpolation formulae for this S-shape are given in [31].

The phase diagram of low-$\kappa$ clean superconductors in the $\kappa - T$ plane ($t = T/T_c$) was measured by Auer and Ullmaier [26] and computed from BCS-Gor’kov-Eilenberger
theory by Klein [32]; this is discussed in [33]. Near $\kappa = 0.71$ with increasing $t$, this phase diagram shows the transition from type-I to usual type-II (termed type-II/2) via a region where the vortices attract each other, termed type-II/1 superconductors. Figure 6 shows the magnetization curves $-M(B_n)$, $M = B-B_n$, for type-I, II/1, and II/2 superconductors shaped as cylinders ($N = 0$, left column) and spheres ($N = 1/3$, right column), schematic from [27]. Recently by neutron-scattering a kind of phase diagram of vortex lattices and normal and intermediate-mixed states has been obtained for pure Nb cylinders of 14 mm length and 4 mm diameter [34]. So, vortex attraction in pure Niobium is still an active field.

The above attractive interaction originates from BCS-corrections to the GL theory (which, strictly spoken, is valid only at $T = T_c$) and at $0 < T < T_c$ occurs for $0.71 \leq \kappa \leq 1.5$ [28]. The London theory that follows from GL theory when $\kappa^2 \gg 1$ and $b \ll 1$, has purely repulsive vortex interaction, namely, the first term in Eq. (2). However, many corrections to the simple GL or London theories are expected to modify the monotonically decreasing interaction potential at large distances, $V(r) \propto \exp(-r/\lambda)$, such that $\lambda$ becomes complex. This, in principle, causes an oscillating potential, whose first minimum may occur at large distances where the amplitude of the potential is small.

Such corrections may have various origins, e.g., the spatially varying order parameter may lead to non-local electrodynamics [35]. Linear nonlocal electrodynamics relates the supercurrent density $j$ to the vector potential $A$ (in Fourier space and London gauge) by a kernel $Q(k)$, $j(k) = -Q(k)A(k)$. The simple “local” London theory has $Q(k) = Q(0) = \lambda^{-2}$. An elegant non-local extension of London theory uses the Pippard kernel $Q_P(k)$ [30], which introduces a Pippard length $\xi_P$. This Pippard theory leads to a vortex field $H(r)$ and vortex potential $V(r) = H(r)\Phi_0$ that oscillates at large distances (“field reversal” [37, 38]). The corresponding BCS kernel $Q_{BCS}(k)$ is an infinite sum of such Pippard kernels and its vortex field $H(r)$ and potential $V(r)$ exhibit similar oscillations [37], see also [38]. We note that some of the early theories quoted in [26] were flawed.

In anisotropic superconductors vortex attraction may even occur in the linear London theory, which now has different penetration depths $\lambda_a, \lambda_b$, and $\lambda_c$ for supercurrents flowing along the crystalline a, b, and c axes [27]. A field reversal and vortex attraction may then occur when the vortices are at an oblique angle [34, 41].

In layered superconductors an anisotropic pair potential between the pancake vortices is investigated by Feigelman et al. [42] and Clem [43]. This potential has an intra-layer logarithmic repulsive part and an inter-layer attractive part, both arising from the magnetic interaction. This potential is used by Jackson and Das [44, 45] for the study of freezing of vortex fluids by a density functional method, yielding a nearly first order transition from a fluid to a triangular Abrikosov vortex lattice. Brandt et al. [46] predicted long-ranged fluctuation-induced attraction of vortices to the surface in layered superconductors by a type of Casimir force. This work inspired Blatter and Geshkenbein [47] to study long-range van der Waals type inter-vortex attraction in the anisotropic layered materials, which arises from the thermal fluctuations of vortices. At low magnetic fields, the inter-vortex separation being very large, even weak attraction influences the phase diagram near the $H_{c1}$ boundary. While the above forces are of thermal origin, van der Waals attraction mediated by impurities is studied by Mukherji and Nattermann [48].

It is conceivable that also a two-component GL theory may lead to an oscillating vortex-vortex interaction with attractive tail and more or less pronounced minimum if its input parameters are chosen appropriately. Such a potential was calculated in [49, 50]. In the same work some of the decoration [49] and scanning SQUID microscopy [50] images taken during field cooling of MgB$_2$ platelets suggest that at the (unknown) temperature where the depicted vortex positions during field cooling were frozen, the vortex interaction had a minimum that determined the dominating vortex distance in the observed vortex chains and clusters.

While Refs. [49, 50] do not mention any of the above listed other cases of vortex attraction, the supplement [51] refers to previously observed and well understood vortex attraction in pure Nb. Note that possible observation of vortex attraction in MgB$_2$ does not mean that the input parameters and interaction potential of simulations that lead to similar vortex arrangements are true or unique. In the following the case of the 2-band superconductor MgB$_2$ will be discussed in some detail.

4. BEYOND THE STANDARD BCS MODEL

The BCS theory is the standard model for understanding various basic properties of conventional metallic superconductors. In this simple theory the Fermi surface is isotropic. The attractive electron-phonon interaction is a constant in a small shell around the FS. In 1958 Gorkov showed the equivalence of the BCS theory with the one-order parameter isotropic Ginzburg-Landau theory. The latter is a powerful phenomenological theory suitable for practical applications. On the other hand a more rigorous approach is the Eliashberg theory suitable for strong coupling of electron-phonon interaction.

In the past three decades a number of novel superconductors have been discovered with many exotic properties beginning with the high-$T_c$ cuprates. Others are doped fullerides, ruthenides, nickel borocarbides, magnesium diborides and pnictides etc. Basic understanding of superconductors requires the correct information about its electronic and vibrational structures and electron-
phonon interaction. Fortunately an ab-initio theory like the density functional theory helps us to meet this type of requirement. By using a self-consistent electron potential arising out of ionic, Hartree, exchange and correlations, one computes the quasi-particle dispersion throughout the Brillouin zone and obtains the Fermi surface.

In a superconductor the formation of Cooper pairs depends heavily on the geometry of the Fermi surfaces. Then comes the electron-phonon interaction that mediates the pairing. In view of the complicated band structures and their Fermi surfaces, the novel superconductors are treated beyond the standard BCS model. We are reminded that single-sheeted Fermi surfaces only come from the simple metals like alkalis considered in the jellium model and those are not superconductors under ambient conditions. Therefore, if the multi-sheeted Fermi surfaces are connected in the entire Brillouin zone, then an anisotropic GL theory is useful. Otherwise for unconnected Fermi surfaces one can use either (i) a multi-band BCS theory [52] or (ii) a generalization of the GL theory invoking many (or many components of) order parameters [53]. This two-component order-parameter theory was used to explain successfully two specific-heat transitions in the high $T_c$ superconductor YBCO (see Choy et al. in [53]).

As said before, in reality the bands are formed due to ionic and electronic self-consistent potential. In principle we are dealing with a charged (Coulomb) system, hence the bands are not entirely independent, and further due to the electron-phonon interaction (whether small or large) the bands are coupled. For this reason multi-(inter- and intra-)band interaction terms explicitly appear in the microscopic Hamiltonian [52]. In the case of multi-order parameter GL theory, one has the superconducting free energy composed of individual parts of the GL free energy and the additional free energy due to coupling between the order parameters as mentioned above. This choice of coupling is non-unique. If $\psi_1$ and $\psi_2$ are two order parameters the coupling or interaction term can be one or more terms from the following list:

$$\gamma_1 |\psi_1|^2 |\psi_2|^2, \gamma_2 (\Pi \psi_1^+ \Pi \psi_2 + \hbar \omega), \gamma_3 (\psi_1^+ \psi_2 + \psi_1 \psi_2^+)$$

where $\Pi = \nabla + 2\pi i \mathbf{A}/\Phi_0$. The first term of this list is a systematic expansion of GL, the second term means coupling through the gradient of order parameter and vector potential $\mathbf{A}$, and the third term is the internal Josephson coupling term. This last term is supposed to provide a “minimal coupling” [54]. Instead of the phenomenological GL theory, we start with the microscopic Bogolyubov pairing Hamiltonian and use the mean-field approach; we derive the GL functional with a coupling term that will be Josephson-like. An important point here is that the coefficients $\alpha_i$ of $|\psi_i|^2$, and the coefficient $\gamma_3$ of the Josephson term, do depend both on the intra-band and inter-band pairing. Therefore, as a matter of principle the order parameters can never be independent.

There is a huge amount of literature on both microscopic and GL theory for the multi-band cases. We do not intend to review that here, see for example [55]. We shall return to the interacting bands in the next Sections.

In the earlier sections we have discussed the physics of vortex matter with particular emphasis on the value of $\kappa$ and the type of superconductivity (type-I or type-II). We also discussed the behavior of pure Nb at temperatures below the GL regime. In the conventional single gap superconductor within GL theory there are a well defined coherence length $\xi$ and penetration depth $\lambda$. In terms of them one defines $\kappa = \lambda/\xi$. As stated earlier its unique value, if less or bigger than 0.71, defines precisely type-I or type-II superconductors, in which the surface free energy between normal and Meissner domains is positive or negative, respectively. This definition tells about the equilibrium states of a superconductor. Here with this brief background we shall discuss the interesting physics of the novel superconductor MgB$_2$ and the vortex matter therein. Needless to say that it has become a news item in the past one year with the claim that a new form of matter is discovered [49, 50]. Detailed discussions follow in Sec. 6.

5. VORTICES IN MgB$_2$

In 2001 by a surprising discovery Nagamatsu et al. [57] reported $T_c$ of an intermetallic MgB$_2$ around 39 K. Soon after (see [58] database for the first three years) it was realized that MgB$_2$ offers a host of novel properties that has been explored during the past 9 years. Magnesium diboride has a crystallographic structure consisting of honeycomb boron layers separated by magnesium layers, which have also honeycomb structure. Magnesium atoms are ionized in this structure donating their $s$-electrons to the conduction band. There are two 3-dimensional $\pi$ bands arising from the boron $p_z$ orbitals. One of them is electron-like, whereas the other is hole-like. From the $p_x, p_y$ orbitals two $\sigma$ bands occur. They are 2-dimensional confined to the boron planes. Bonds within the boron layers are strongly covalent, whereas between the layers they are metallic. From this a complicated Fermi surfaces geometry emerges. Two cylinders around the $\Gamma$-A-$\Gamma$ lines are two $\sigma$ bands, and two webbed tunnels are attributed to $\pi$ bands, see Fig. 7 (from Mazin and Antropov [59]). This overall picture is consistent with all the electronic structure calculations [59].

It is further found [60] that the optical bond-stretching $E_{2g}$ phonons couple strongly to the holes at the top of $\sigma$ bands, whereas the 3-dimensional $\pi$ electrons are weakly coupled to the phonons. For this reason of different coupling strengths of $\sigma$ and $\pi$ bands, a two-gap superconductor merges. The $E_{2g}$ phonons strongly coupled with $\sigma$ band, produce high $T_c$ in MgB$_2$. A further coupling with the $\pi$ bands enhances
FIG. 7: Band structure and Fermi surface of MgB$_2$ from Mazin and Antropov [59]. Green and blue cylinders (hole-like) are from the $\sigma$ bands, and the blue (hole-like) and red (electron-like) tubular networks are from the $\pi$ bands.

In the past nine years a variety of experiments have shown that MgB$_2$ is a two band superconductor. The main experiments are specific heat [Choi et al. in [59, 62], point contact and tunnelling spectroscopy [63, 64], Raman spectroscopy [65], penetration depth measurements [66], angle-resolved photo-emission spectroscopy [67], small-angle neutron-scattering [68], and muon-spin relaxation studies [69] etc. The value of the gap on the $\sigma$ bands $\Delta_{\sigma}$ ranges from 6.4 to 7.2 meV, while on the $\pi$ bands $\Delta_{\pi}$ varies from 1.2 to 3.7 meV. Almost all parameters such as coherence lengths ($\xi_\sigma$ and $\xi_\pi$) [66], upper critical fields ($H_{c2}(\sigma)$ and $H_{c2}(\pi)$) [70, 71], and penetration depths ($\lambda_\sigma$ and $\lambda_\pi$) [66] are anisotropic. Besides this the anisotropic factor is also strongly temperature dependent. These experiments convincingly suggest that MgB$_2$ is a two band superconductor, where phonons mediate pairing in the respective bands. Further, it is investigated that inter-band coupling helps to increase $T_c$. In the absence of inter-band coupling, it may give two isolated superconductors with two lower $T_c$.

6. A NEW PHASE OF MgB$_2$!

In a recent paper Babaev and Speight [72] studied phenomenologically a two-band superconductor with two order parameters, starting with a free energy that includes a Josephson-like interaction $\gamma_3$ between the two bands. Choosing $\gamma_3 \rightarrow 0$ in the decoupled band limit, they considered $\kappa_i$ ($i = 1, 2$) in two different regimes to produce type-I ($\kappa_1 < 0.71$) and type-II ($\kappa_1 < 0$) materials. Their prediction leads to, what they call a “semi-Meissner state”, particularly in the case of very low magnetic fields. Babaev and Speight claim that this state is thermodynamically stable. Instead of homogeneous distribution, the vortices form aperiodic clusters or vortexless Meissner domains, arising out of short range repulsion and long range attraction. Below we shall argue that in the particular case of MgB$_2$, this is an unrealistic construction.

In what is claimed as a remarkable manifestation of a new type of superconductivity in MgB$_2$, Moshchalkov and co-workers [49, 50] reported last year a novel “type-1.5 superconductor”, contrary to type-I and type-II superconductors. It appears these authors are influenced by the idea of “semi-Meissner state” of Babaev and Speight. Apparently both names carry similar meaning. Let us analyse carefully the results of Moshchalkov et al.

Similar to Babaev and Speight, Moshchalkov et al. use a two-band GL functional with an inter-band coupling term. They constructed the GL parameters $\kappa_i = \lambda_i/\xi_i$ ($i$ stands for $\sigma$ and $\pi$ bands of MgB$_2$) from the measured values of energy gaps, Fermi velocities and plasma frequencies of respective bands. The values are so chosen that $\kappa_\sigma > 0.71$ (=3.68) and $\kappa_\pi < 0.71$ (= 0.66).

| $\lambda_{\sigma\sigma}$ | $\lambda_{\pi\pi}$ | $\lambda_{\sigma\pi}$ | $\lambda_{\pi\sigma}$ |
|-------------------------|------------------|----------------------|----------------------|
| 1.017                   | 0.448            | 0.213                | 0.155                | Gohubov et al. [60] |
| 0.96                    | 0.29             | 0.23                 | 0.17                 | Liu et al. [60]    |
| 0.78                    | 0.21             | 0.15                 | 0.11                 | Choi et al. [59]   |

Table 1. Matrix elements of electron-phonon interactions
Thus, MgB$_2$ has properties of both type-I and type-II simultaneously, unlike in the case of Nb where $\kappa$ is close to 0.71 as discussed above. This construction is in absence of the coupling term, so the bands are fully independent/noninteracting. We have stated earlier that energy gaps $\Delta_\sigma$ range from 6.4 to 7.2 meV, while the $\Delta_\pi$ varies from 1.2 to 3.7 meV. Moshchalkov et al. chose the value of $\Delta_\sigma = 7.1$ meV and $\Delta_\pi = 2.2$ meV from some experiments and computed $\xi_i$ from the one-band BCS relation $\xi_i = h\nu_F/(2\pi^2\Delta_i)$. Then they used calculated plasma frequencies of respective bands, from which they obtained the London penetration depths $\lambda_\sigma(0) = 48.8$ nm and $\lambda_\pi(0) = 33.6$ nm and finally $\kappa_i = \lambda_i/\xi_i$.

Here we note a few points.  
A. (1) The numerical estimates for $\xi$ are obtained by using the one-band BCS formula. (2) One band calculated plasma frequency is used to compute $\lambda$. One does not know how these are relevant to interacting two-band MgB$_2$.

B. For critical fields Moshchalkov et al. gave their measured values for $H_{c1}(0) = 241$ mT, $H_{c2}(0) = 5.1$ T for the $\sigma$ band. From these data they also obtained $\lambda_\sigma$ and $\xi_\sigma$ from the one band formula. The value of $H_{c1}(0) = 241$ mT measured for their sample is much larger than the values reported by others, which are in the range of 25 - 48 mT, see Fig. 25 of Ref. [71]. Another point of concern is the reported value of thermodynamic critical field $H_\sigma(0) = 230$ mT [73], whereas Moshchalkov et al. have $H_{c1}(0) = 241$ mT higher than $H_\sigma(0)$.

C. The $\lambda$ that Moshchalkov et al. estimated are smaller by a large factor as can be verified from the Table 1 of Golubov et al. [74]. This Table lists the values of $\lambda(0)$ from a variety of experiments. Smaller $\lambda$ can make the $\kappa$ smaller. This possibility has made $\kappa_\sigma < 0.71$. Another important point is that using two $\lambda$ for a single flux line is questionable. One has to solve a single equation for the magnetic field (in two-order parameter GL/BCS theory) to obtain a single $\lambda$, even in the decoupled band limit. One has $\lambda \propto (\sum_i |\psi_i|^2/m_i)^{-1/2}$, containing a sum of two superfluid densities.

An often heard argument is that whether there is one $\kappa$ or two $\kappa$, there will be only one surface free energy between Meissner state and normal state, which would be either positive, zero or negative. Thus, superconductors are either of type-I or type-II. However, this argument tacitly assumes that boundaries occur only between Meissner and normal states. But as figures of the intermediate-mixed state show (see Figs. 3 to 5), there the boundary occurs between Meissner and vortex states. In these experiments the wall energy between Meissner state and normal state is negative (leading to a vortex state) but the wall energy between Meissner state and the vortex state is positive, causing the observed domain structure. So, there are indeed experimental situations that exhibit features of both type-I and type-II behavior. As stated in Sec. 3, one condition for this intermediate-mixed state to occur is that the vortex interaction has an attractive tail, or that the theoretical magnetization curve is S-shaped, see Fig. 1 (bottom). But in addition, the demagnetization factor of the decorated specimen has to be $N > 0$, else only Meissner state or a uniform vortex state will occur; the detailed appearance further depends on pinning, on the edge barrier, and on the magnetic history. This intermediate mixed state has been known for long time from experiments [11, 22, 26, 94, 73] and computations [28, 29, 31–33] for pure Nb, TaN, PbIn, and PbTi. In this respect, the decoration and scanning SQUID results of [10, 50] are not so new.

As discussed above, several origins of vortex attraction are conceivable, depending on the material, its purity, magnetic history, and temperature. Computer simulations using such a vortex interaction (short-range repulsive, long-range attractive) yield vortex arrangements that are similar to the observed ones [49, 50]. The details of the resulting clusters or chains also depend on the type and strength of vortex pinning that is always present and should be included in such simulations, in particular at very low induction $B$.

7. SUMMARY AND CONCLUSIONS

Type-I superconductors with demagnetization factor $N > 0$ in the field range $(1 - N)B_c < B_a < B_a$ are in the intermediate state showing normal conducting lamellae or tubes inside which $B = B_a$, embedded in Meissner state with $B = 0$. Type-II superconductors in the field range $(1 - N)B_{c1} < B_a < B_{c2}$ contain vortices which repel each other (mixed state). Under several conditions the vortex interaction may have an attractive tail, leading to observation of vortex clusters or lamellae similar as in type-I superconductors, but filled with vortices, and to vortex chains. This “intermediate-mixed state” has been known since forty years in pure Nb, TaN [26], PbIn, and PbTi [73] at temperatures sufficiently low such that GL theory does not apply. The recent possible observation of vortex attraction in the two-band superconductor MgB$_2$, if one agrees with this interpretation of the decoration [49] and scanning SQUID [50] images, may be due to the complexity of the required microscopic theory. But we question if these images hint at “a totally new state” that “behaves in an extremely unusual way” [40]. It should also be considered that at the very low inductions of these images even very weak pinning will influence the vortex positions and may cause vortex clustering [76].

MgB$_2$ is a ten year old new inter-metallic superconductor which has many fascinating properties, particularly a high $T_c \approx 39$ K. We briefly discussed its electronic and geometrical structure in the pure case. For understanding the occurrence of superconductivity we showed in Fig. 7 its two sets of $\sigma$ and $\pi$ bands. The electrons in these bands are interacting via Coulomb and phonon
interactions. There are unambiguous calculations and experiments which suggest the high strengths of inter-band and comparable intra-band interactions in this two-band material. It has been a consensus that MgB$_2$ is a phonon-mediated superconductor. The higher $T_c$ is a result of coupling between the two Fermi surfaces (see e.g. Geerk et al. in [60]). This information provides a compelling reason to adopt a two-band model, either by microscopic BCS-Eliashberg or GL theory (see e.g. [52, 53], Zhitomirsky and Dao in [54, 77], and the review [78]). Thus an argument of two independent bands is never tenable for MgB$_2$ that exhibits one single $T_c$. Hence no separate two superconducting regimes can occur in MgB$_2$.

A single vortex in a two-band superconductor will have a complicated vortex core that results from solving two coupled order parameter equations. Contrary to the remark by Babaev and Speight [60], the core size and shape depends nontrivially on the coupling of two order parameters and also on temperature. The next issue is about the penetration depth $\lambda$. This quantity comes out of the GL equation for the vector potential. In the presence of electromagnetic field coupling with the order parameter $\gamma_2(\Pi \psi^\dagger\Pi \psi) + hc$ as mentioned earlier in Sec. 4, $\lambda$ depends on the superfluid densities in a nontrivial fashion, see Eq. 9 and Fig. 3 of [79]. In brief, mixing two components as visualized by Babaev et al. and Moshchalkov et al., will not produce a new kind of superconductivity, because in the mixture $\lambda$ is determined self-consistently. Table 1 of Golubov et al. [74] lists 12 values of the zero-temperature London penetration depths $\lambda_L(0)$ from various experiments with MgB$_2$; these values exceed the reported values of the coherence lengths $\xi$, and thus $\kappa \gg 0.71$. Therefore, type-I condition will be hardly satisfied. For this reason it will be instructive to assess the “semi-Meissner” or “type-1.5” conditions in relation to Moshchalkov’s experiments [49, 50].

Another question is: Can one have two different penetration depths $\lambda$ in the same material and for the same direction of the supercurrent density $j$? Usually $\lambda$ is defined via the general linear relation (valid for small $j$ and small $B$ in isotropic superconductors, see Sec. 3) $j(k) = -Q(k)A(k)$ by $Q(0) = \lambda^{-2}$ or $\lambda = Q(0)^{-1/2}$. In general one has $j = \delta F/\delta A$ where $F(j, \mathbf{A})$ is the free-energy functional of a given theory and $\mathbf{A}$ the vector potential. This definition allows only for one single $\lambda$.

In conclusion, from our careful analysis we argue that the hypothesis of “type-1.5 superconductivity” as put forward in [42, 50] and in the abstract of Moshchalkov’s talk at this conference, is unrealistic and unfounded for MgB$_2$ that has one single transition temperature $T_c$. Anyway, it cannot be precluded that the vortex-vortex interaction, if calculated microscopically (e.g., by the quasiclassical Eilenberger method [31, 52, 50, 51]) may have an attractive tail due to the complexity of the problem.

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