Optimization Problems of the Characteristics of Vibroprotective Systems of a New Type

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Abstract. For protection of technical and biological objects against vibration influence in the low frequencies range and also in protecting of objects against the vibration ranges changing in time the automated active vibroprotective systems have recently found application. Modern approach to the analysis of vibration processes in the amortized equipment, in engineering installations and constructions of difficult configurations and the considerable spatial sizes is based on accounting of all set of communications, interactions and constructive complexity of real objects in which oscillatory processes have wave character. In article is offered the choice technique for the vibroprotective systems of a new type of their optimum rigidity at casual kinematic influences. It is offered to present initial information for calculation in the form of spectral density of casual processes. Using the operation of smoothing of a trajectory of initial casual kinematic influences considered in article calculation of the main probabilistic characteristics for calculation of accelerations is the possible.

1. Introduction
Now the automated vibroprotective systems widely are applied to protection of various technical objects against low-frequency vibration influence or the vibration ranges changing in time [1]. However use of traditional vibroinsulator s often doesn't solve a vibration insulation problem as there is a problem of the choice of an elastic element of optimum rigidity for vibroprotective system. On the one hand, the small rigidity of an elastic element is necessary for reduction of absolute acceleration of a subject to vibroprotection, on the other hand, to reduce his deformation – high. It is obviously possible to resolve this problem, using the newest systems of vibration insulation [2].

2. Problem statement and its solution
We will consider work of passive and active system of vibroprotection [3] which schematic diagram is submitted in the figure 1.

We will analyze the casual kinematic impact of h(t) on system in which subject to vibroprotection has the mass of m, and the system of vibration insulation has an elastic element of rigidity With and a damper with parameter b.
Figure 1. The schematic diagram of vibroprotective system.

Then it is possible to write down the system of the differential equations for definition of movement of a subject to vibroprotection of $y(t)$, acceleration and deformation of an elastic element $u(t) = h(t) - y(t)$ [4]:

$$
\begin{align*}
\dddot{y} & + 2n\ddot{y} + \omega_0^2 y = 2n\dot{h} + \omega_0^2 h \\
\dddot{u} & + 2n\ddot{u} + \omega_0^2 u = -\ddot{h} = a_0 \\
\dot{a} & = 2n\dot{u} + \omega_0^2 u 
\end{align*}
$$

(1)

Where $\omega_0^2 = \frac{C}{m}$ - square of frequency of own fluctuations, $2n = \frac{b}{m}$ - damping coefficient.

Let’s say that external casual kinematic influence of $h(t)$ is set by spectral density. Then transfer functions according to (1) can be defined as

$$
H_u i\omega = \frac{1}{\omega_0^2 - 2\omega^2 + 2ni\omega}
$$

(2)

$$
H_a i\omega = \frac{2ni\omega + \omega_0^2}{\omega_0^2 - 2\omega^2 + 2ni\omega}
$$

We will receive formulas for definition of dispersions of processes of deformation of an elastic element $u(t)$ and acceleration $a(t)$:

$$
S_u = \int_{-\infty}^{+\infty} |H_u i\omega|^2 S_{a_0} \omega d\omega \approx \frac{\pi}{2n\omega_0^2} S_{a_0} \omega_0
$$

(3)

$$
S_a = \int_{-\infty}^{+\infty} |H_a i\omega|^2 S_{a_0} \omega d\omega \approx \frac{\pi}{2n} \omega_0^2 + 4n^2 S_{a_0} \omega_0
$$

(4)

Where $S_{a_0} \omega_0$ - spectral density of process $a_0(t)$.

We will represent schedules of dependences of the received expressions for dispersions of processes from frequency $\omega_0$ that is from rigidity of system.

In the figure 2 the discrepancy of requirements for rigidity to the system of vibroprotection is traced: at increase in rigidity acceleration of a subject to vibroprotection increases, and deformation of an elastic element decreases. On the other hand, at reduction of rigidity deformation of an elastic element increases, and acceleration of a subject to vibroprotection decreases.
Figure 2. Schedules of dependences of dispersions of processes on frequency.

For determination of optimum rigidity of system it is necessary to consider still that requirements to the system of vibroprotection are formulated in various units of measure: on movements - in terms of length and on accelerations – in terms of acceleration [4].

If to determine optimum frequency $\omega_0^*$ from an acceleration dispersion restriction condition, we will receive concrete value $S_{a_0}^2$. Similarly, if to determine optimum frequency $\omega_0^{**}$ from a condition of restriction of dispersion of deformation of an elastic element $S_{u}^2 \leq S_{a}^2$, we will receive concrete value $S_{a}^2$. We will accept value of optimum frequency as intermediate value $\omega_0^{**}$, defined as $\omega_0 = \alpha \omega_0^* + \beta \omega_0^{**}$, where $\alpha$ – the importance (weight) of the first condition, $\beta$ – the importance (weight) of the second condition, $\alpha + \beta = 1$.

We will consider a case when the spectral density of process of external casual kinematic influence of $h(t)$ is set as

$$S_h(\omega) = \frac{1}{2} S_h^2 \delta |\omega| - \omega_h$$

Where $\delta |\omega| - \omega_h$ - Dirac's delta function, $\omega_h$ - the frequency of process of $h(t)$.

Spectral density for acceleration $a_0$ will be determined by a formula:

$$S_{a_0}(\omega) = S_h(\omega) = \frac{1}{2} S_{a_0}^2 \delta |\omega| - \omega_h$$

Where $S_{a_0}^2 = \omega_h^4 S_h^2$.

We will define dispersions, having substituted (6) in formulas (3) and (4), we will receive:
If \( \omega_h = \omega_0 \) we will receive:

\[
S_u^2 = \frac{S_{u_0}^2}{4n^2 \omega_0^2} \quad (9)
\]

\[
S_u^2 = S_{u_0}^2 \left( 1 + \frac{\omega_0^2}{4n^2} \right) \quad (10)
\]

Follows from the received ratios (7-10) that at increase in rigidity of vibroprotective system deformation of her elastic element decreases, and acceleration of a subject to vibroprotection increases.

The spectral density of kinematic influences in most cases has an appearance [6]:

\[
S_h^2 \omega = \frac{\alpha}{\pi} S_h^2 \frac{\omega^2 + \alpha^2 + \beta^2}{\omega^2 - \alpha^2 - \beta^2 + 4\alpha \omega^2} \quad (11)
\]

Where \( \alpha; \beta; S_h \) - some parameters.

But it isn't possible to use this formula as for calculations spectral density is necessary for acceleration \( S_{a_0}^2 \omega = S_h^2 \omega \) and also in view of the fact that casual kinematic influence of \( h(t) \) has no derivatives.

It is possible to eliminate this difficulty by means of application of operation of statistical smoothing to trajectories of casual processes [7].

If to apply this operation to a ratio (11), then it is possible to present the spectral density of the first derivative of process of \( h(t) \) thus:

\[
S_{h^*} \omega = \frac{\alpha}{\pi} S_h F_{h^*} \omega \quad (12)
\]

Where \( F_{h^*} \omega ; F_{h^*}^* \omega \) - the quasi-spectrums determined by formulas:

\[
F_{h^*} \omega = \frac{\omega^2 + i\omega \sqrt{\alpha^2 + \beta^2}}{\omega^2 - \alpha^2 - \beta^2 + 2\alpha \omega} \quad (13)
\]

\[
F_{h^*}^* \omega = \frac{\omega^2 - i\omega \sqrt{\alpha^2 + \beta^2}}{\omega^2 - \alpha^2 - \beta^2 - 2\alpha \omega} \quad (14)
\]

Here designation of an asterisk from above means transition to the complex interfaced functions.

We will determine the spectral density of process \( \dot{h} \) thus:

\[
\dot{S}_h \omega = \frac{\alpha}{\pi} F_{h^*} \omega - \lim_{\omega \to 0} F_h \omega \quad (15)
\]

Dispersion and average frequency of process will be defined thus:

\[
S_h = \alpha^2 + \beta^2 \quad (16)
\]

\[
\omega_h = \sqrt{\alpha^2 + \beta^2} \quad (17)
\]
We will define a smoothed second derivative of process thus:

$$\tilde{S}_{\omega} = S_{\omega} - \frac{\alpha^2 + \beta^2}{\omega^2} S_{\omega}$$  \hspace{1cm} (18)

Then according to the given formulas (3) and (4) it is possible to receive:

$$S_u^2 = \alpha^2 + \beta^2 \int_{-\infty}^{+\infty} H_u(i\omega) \int S_{\omega} \omega \ d\omega \approx \frac{\pi}{2n\omega_0^2} S_{\omega} \omega_0$$  \hspace{1cm} (19)

$$S_u^2 = \alpha^2 + \beta^2 \int_{-\infty}^{+\infty} H_u(i\omega) \int S_{\omega} \omega \ d\omega \approx \frac{\pi}{2n} \frac{\alpha^2 + \beta^2}{\omega_0^2} + \frac{4n^2}{\omega_0^2} S_{\omega} \omega_0$$  \hspace{1cm} (20)

Therefore, again we receive that increase in rigidity of vibroprotective system leads to reduction of deformation of her elastic element and increase of acceleration of a subject to vibroprotection. The received formulas (19) and (20) are also offered to be used in calculations as formulas (7-10).

Thus, the offered technique of the choice of rigidity of mechanical systems is recommended for calculation of optimum vibroprotective systems of various objects.

3. Conclusions

Despite theoretical difficulties of the choice of an elastic element of optimum rigidity for vibroprotective system, the effective technique for calculation of optimum vibroprotective systems of various objects is offered.

It is expedient to present information for calculation in the form of spectral density of not differentiable casual processes.

When using operation of statistical smoothing of trajectories of initial casual kinematic influences, determining probabilistic characteristics for accelerations is obviously possible.

4. References

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