Chaotic inflation on the brane and the Swampland Criteria

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(Dated: Draft October 4, 2018)

In this paper, we show that single-field chaotic inflation on the brane with the potential $V = a\phi^p$ is compatible with the Swampland criteria. The spectral index and the running spectral index are within experimental bounds for $0 < p \leq 2$. The tensor to scalar ratio is within observational bounds if $p \lesssim 0.35$.

\section{I. INTRODUCTION}

If we have a UV completed theory, it should be able to give some insight to our low-energy effective theory. Even if we do not have a UV completed theory yet, if the theory is believed to have some general properties, it could shed some light to our model building. Recently it was proposed that if an effective field theory can be embedded consistently in quantum gravity, it has to satisfy two criteria \cite{1,2}. These criteria provide such strong constraints, that they are incompatible to many inflation models \cite{3-5}. These two criteria are as follows:

- Scalar field excursion in reduced Planck units in field space are bounded from above \cite{6}
  \begin{equation}
  \frac{\Delta \phi}{M_P} < O(1).\tag{1}
  \end{equation}

- The slope of the scalar field potential satisfies a lower bound \cite{7}
  \begin{equation}
  M_P \left|\frac{V'}{V}\right| > c \sim O(1),\tag{2}
  \end{equation}
  whenever $V > 0$.

The number of e-folds of conventional single field slow roll inflation is given by

\begin{equation}
N = \frac{1}{M_P^2} \int \frac{V}{V'} d\phi \simeq \frac{\Delta \phi}{M_P \sqrt{V}},\tag{3}
\end{equation}

where the approximation is obtained by assuming $V'/V$ is independent of $\phi$. Therefore for single field inflation the first criterion divided by the second one roughly gives the number of e-folds during inflation. This suggests that it may not be possible to have a large enough number of e-folds. In addition, if the primordial density perturbation comes from inflaton fluctuation, the second constraint seems to be incompatible with current observation of tensor to scalar ratio.

One possibility to evade those criteria in single-field inflation is to consider a curvaton-like mechanism \cite{8}. Another possible way is given in \cite{9}, where it is pointed out that quintessential brane...
inflation is compatible with swampland criteria, however, the unacceptably high tensor to scalar ratio is predicted unless the initial state have a non-Bunch-Davies component. In addition, it is shown in [10] that warm inflation or k-inflation may also be compatible with the criteria.

In this paper, we consider chaotic inflation on the brane and show that it can satisfy both swampland criteria and the tensor to scalar ratio constraints.

II. INFLATION ON THE BRANE

In a braneworld scenario where our four-dimensional world is a 3-brane embedded in a higher-dimensional bulk, the Friedmann equation can be modified as [11–17]

\[ H^2 = \frac{1}{3M_P^2} \rho \left[ 1 + \frac{\rho}{2\Lambda} \right], \]  

(4)

where \( \Lambda \) provides a relation between the four-dimensional Planck scale \( M_4 \) and five-dimensional Planck scale \( M_5 \) through

\[ M_4 = \sqrt{\frac{3}{4\pi}} \left( \frac{M_5^2}{\sqrt{\Lambda}} \right) M_5, \]  

(5)

where \( M_P = M_4/\sqrt{8\pi} \simeq 2.4 \times 10^{18} \, \text{GeV} \) is the reduced Planck scale. We will set \( M_P = 1 \) in the following. The nucleosynthesis limit implies that \( \Lambda \gtrsim (1 \, \text{MeV})^4 \sim (10^{-21})^4 \). A more stringent constraint, \( M_5 \gtrsim 10^5 \, \text{TeV} \), can be obtained by requiring the theory to reduce to Newtonian gravity on scales larger than 1 mm, this corresponds to \( \Lambda \gtrsim 5.0 \times 10^{-53} \).

For inflation on the brane, the slow-roll parameters are modified into [17]

\[ \epsilon \equiv \frac{1}{2} \left( \frac{V'}{V} \right)^2 \frac{1}{\left( 1 + \frac{V}{2\Lambda} \right)^2} \left( 1 + \frac{V}{\Lambda} \right), \]  

(6)

\[ \eta \equiv \left( \frac{V''}{V} \right) \left( \frac{1}{1 + \frac{V}{2\Lambda}} \right). \]  

(7)

The number of e-folds is

\[ N = \int_{\phi_i}^{\phi_e} \left( \frac{V}{V'} \right) \left( 1 + \frac{V}{2\Lambda} \right) d\phi. \]  

(8)

The spectrum is

\[ P_R = \frac{1}{12\pi^2 V'^2} \left( 1 + \frac{V}{2\Lambda} \right)^3. \]  

(9)

The spectral index is

\[ n_s = 1 + 2\eta - 6\epsilon. \]  

(10)

The running spectral index is given by

\[ \alpha = -\frac{dn_s}{d\ln k} = -\frac{dn_s}{dN}. \]  

(11)
III. CHAOTIC INFLATION ON THE BRANE

Let us consider chaotic inflation on the brane with a potential of the form

\[ V = a\phi^p. \]  

(12)

From the potential we can obtain

\[ \frac{V'}{V} = \frac{p}{\phi}, \]  

(13)

\[ \frac{V''}{V} = \frac{p(p - 1)}{\phi^2}. \]  

(14)

A small \( p \) may be motivated from backreaction of the inflaton potential energy on heavy scalar fields [21] as well as flux flattening in axion monodromy inflation [22]. In the following, we will assume \( V/\Lambda \gg 1 \) so that the brane effect is significant. We will see that this imposes an upper bound to \( \Lambda \).

From Eqs. (6) and (7), we have

\[ \epsilon = \frac{2p^2\Lambda}{a\phi^{p+2}}. \]  

(15)

\[ \eta = \frac{2\Lambda p(p - 1)}{a\phi^{p+2}}. \]  

(16)

\[ \frac{\eta}{\epsilon} = \frac{p - 1}{p}. \]  

(17)

Inflation ends at \( \phi = \phi_e \) when one of the slow-roll parameters becomes of order one, namely \( \max\{\epsilon, |\eta|\} = 1 \). We can see from Eq. (17) that if \( p = 0.5 \), \( \epsilon = |\eta| \). However, if \( p > 0.5 \), \( \epsilon > |\eta| \). In this case, we have

\[ \phi_e^{p+2} = \frac{2p^2\Lambda}{a}. \]  

(18)

On the other hand, if \( 0 < p < 0.5 \), \( \epsilon < |\eta| \), therefore

\[ \phi_e^{p+2} = \frac{2\Lambda p(1 - p)}{a}. \]  

(19)

From Eq. (8), for \( p > 0.5 \), we obtain

\[ N = \int_{\phi_e}^{\phi_e} \frac{a\phi^{p+1}}{2p\Lambda} = \frac{a\phi_e^{p+2}}{2p(p + 2)\Lambda} - \frac{p}{p + 2}. \]  

(20)

For \( 0 < p < 0.5 \), we obtain

\[ N = \int_{\phi_e}^{\phi_e} \frac{a\phi^{p+1}}{2p\Lambda} = \frac{a\phi_e^{p+2}}{2p(p + 2)\Lambda} - \frac{1 - p}{p + 2}. \]  

(21)

\[^1\text{Monomial potentials with fractional powers have been considered in string theory [18–20].}\]
In both cases, comparing with $N \sim 60$, we can neglect the contribution of $\phi_e$, therefore at horizon exit we have

$$\phi_e^{p+2} = \frac{120p(p + 2)\Lambda}{a}. \quad (22)$$

Substitute this result into Eq. (15), we obtain

$$\epsilon = \frac{p}{60(p + 2)}. \quad (23)$$

From the above equation and Eqs. (17) and (10), the spectral index is given by

$$n_s = 1 - \frac{2 + 4p}{N(p + 2)} = 1 - \frac{1 + 2p}{30(p + 2)}. \quad (24)$$

Interestingly, this result does not depend on the parameters $a$ nor $\Lambda$ as long as our assumption $V/\Lambda \gg 1$ is valid. The spectral index as a function of $p$ from Eq. (24) is plotted in Fig. 1. Depending on the value of $p$, the spectral index $n_s$ is more or less within the observational constraints $[23]$. The running spectral index is given by

$$\alpha = -\frac{2 + 4p}{N^2(p + 2)} = -\frac{2 + 4p}{3600(p + 2)}, \quad (25)$$

which is plotted in Fig. 2. As we can see from the plot, the running spectral index is in accordance with the Planck data $|\alpha| \lesssim 0.01$ [23]. Substituting Eq. (22) into Eq. (9), we obtain

$$P_R = \frac{1}{96\pi^2} \frac{4^{p+2}}{p^{p+2}} \frac{2^{p-3}}{p^{p-3}} (p + 2) \frac{4^{p+2}}{p^{p+2}} \Lambda^{p+2} a^{p+2} = (5 \times 10^{-5})^2, \quad (26)$$
where CMB normalization is imposed. This gives a relation between $a$ and $\Lambda$ for some fixed $p$. Hence from Eq. (22), we can obtain

$$\phi_i = 8.66 \times p^{\frac{p+2}{2p+4}} [120(p + 2)]^{\frac{2p+4}{2p+6}} \Lambda^{\frac{1}{6}}. \quad (27)$$

Since $V$ is decreasing during inflation, if our assumption $V/\Lambda \gg 1$ is satisfied at the end of inflation, say $V/\Lambda > 100$, it is satisfied during inflation. For $p > 0.5$, this imposes a condition

$$\Lambda^{\frac{1}{6}} < 1.15 \times 10^{-2} \times 2^{\frac{p}{2p+4}} \left[ \frac{p}{120(p + 2)} \right]^{\frac{2p+4}{2p+6}}. \quad (28)$$

This is plotted in Fig. 3. By using Eq. (27) and the upper bound of $\Lambda^{\frac{1}{6}}$, we can obtain an upper bound of $\phi_i$, which is plotted in Fig. 4. We can see that for $0.5 \leq p \leq 2$ the first Swampland criterion is satisfied even for the upper bound of $\Lambda^{\frac{1}{6}}$. In order to check the second Swampland criterion, we plot the lower bound of $V'/V$ at $\phi_i$ in Fig. 5. Note that since $\phi$ decreases during inflation, we can see from Eq. (13) that $V'/V$ increases during inflation. This implies that if the second Swampland criterion is satisfied at $\phi_i$, it is satisfied during the whole period of inflation.

For $p < 0.5$, the corresponding condition for $\Lambda^{\frac{1}{6}}$ is

$$\Lambda^{\frac{1}{6}} < 1.15 \times 10^{-2} \times 2^{\frac{p}{2p+4}} \times p^{\frac{p}{2p+4}} [120(p + 2)]^{-\frac{2p+1}{2p+6}}. \quad (29)$$

This is plotted in Fig. 6 and the upper bound of $\phi_i$ is plotted in Fig. 7. The lower bound of $V'/V$ is plotted at $\phi_i$ in Fig. 8. As we can see in Figs. 4 and 6, we have $\Lambda^{\frac{1}{6}} \lesssim 10^{-3}$ in the range $0.0001 \leq p \leq 2$. Combined with the lower bound of $\Lambda$ given in Section II, we obtain

$$1.92 \times 10^{-9} \lesssim \Lambda^{\frac{1}{6}} \lesssim 10^{-3}. \quad (30)$$
This means that $\Lambda^{1/6}$ can be many orders smaller than the upper bound. Each time when we lower $\Lambda^{1/6}$ by one order from the upper bound, $\phi_i$ is reduced by one order, and $V'/V$ is increased by one order. Thus from Figs. 4, 5, 7, and 8 we can see that the Swampland criteria can be satisfied by judiciously choosing an appropriate $\Lambda^{1/6}$ within the allowed range.
FIG. 5: The lower bound of $\frac{V'}{V}$ as a function of $p$ when $0.5 \leq p \leq 2$ from the condition $V/\Lambda > 100$.

FIG. 6: The upper bound of $\Lambda^{1/6}$ as a function of $p$ when $0.0001 \leq p \leq 0.5$ from the condition $V/\Lambda > 100$. 
FIG. 7: The upper bound of $\phi_i$ as a function of $p$ when $0.0001 \leq p \leq 0.5$ from the condition $V/\Lambda > 100$.

FIG. 8: The lower bound of $V'/V$ as a function of $p$ when $0.0001 \leq p \leq 0.5$ from the condition $V/\Lambda > 100$. 
The tensor to scalar ratio can be obtained from Eq. (23) as \[ r = 24\epsilon = \frac{2p}{5(p + 2)}. \] (31)

which is plotted in Fig. 9. For \( p = 2/3 \), we have \( n_s = 0.971 \) and \( r = 0.1 \). For \( p = 0.35 \), we have \( n_s = 0.976 \) and \( r = 0.06 \). Therefore when we have \( p \lesssim 0.35 \), the tensor to scalar ratio \( r \) is within the Planck constraints combining with the BICEP2/Keck Array BK14 data, namely \( r < 0.064 \) [23].

IV. CONCLUSION AND DISCUSSION

In this paper, we have calculated the field value, the spectral index, the running spectral index the tensor to scalar ratio for chaotic inflation on the brane with a potential of the form \( V = a\phi^p \). In general, the Swampland criteria can be satisfied even for a very small \( p \). The value of the spectral index and its running is of an acceptable value in the range \( 0 < p \leq 2 \). The tensor to scalar ratio is \( r \lesssim 0.06 \) if \( p \lesssim 0.35 \). Thus we conclude that single-field chaotic inflation on the brane where the primordial density perturbation is from inflaton fluctuation with the Bunch-Davies initial state is compatible with the Swampland criteria.

\[ \text{For conventional slow-roll inflation, or } \frac{V}{\Lambda} \ll 1, \text{ the tensor to scalar ratio is given by } r = 16\epsilon. \text{ Therefore for the same } \epsilon, \text{ the tensor to scalar ratio } r \text{ would be bigger for } \frac{V}{\Lambda} \gg 1. \]
Acknowledgement

This work is supported by the Ministry of Science and Technology (MOST) of Taiwan under grant numbers MOST 106-2112-M-167-001 (C. M. L.), MOST 107-2119-M-001-030 (K. W. N.), MOST-105-2112-M-007-028-MY3 (K. C.) and MOST-107-2112-M-007-029-MY3 (K. C.).

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