Staggered Chiral Perturbation Theory with Heavy-Light Mesons

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We merge heavy quark effective theory with staggered chiral perturbation theory to calculate heavy-light (B, D) meson quantities. We present results at NLO for the B(D) meson decay constant in the partially quenched and full QCD cases, and discuss the calculation of the form factors for B(D) → π(K)ℓν decays.

The lattice can make a major contribution to the understanding of flavor physics through the computation of the properties of heavy-light mesons (see Ref. [1] for reviews). A promising approach for these systems is to use staggered light quarks so that the simulations can be performed with a heavy quark, the latter gets so much energy by exchanging a gluon of momentum $\pi/a \approx 5\text{GeV}$, which is safely higher than physical or simulated pion masses. Heavy quarks with energy this high affect low-energy physics only through renormalization.

Technically, the criterion $\Delta E(\pi/a) \gg m_\pi$ allows us to neglect, from the effective Symanzik action of the lattice theory, “mixed” 4-quark operators (products of heavy and light bilinears) that violate taste. At $\mathcal{O}(a^2)$, then, all taste-violations in the Symanzik action are in the light quark sector, i.e., the same as in Refs. [2,3].

We stress that we are not taking into account discretization effects due to the heavy quarks, only those that arise specifically from light-quark taste violations. Ideally, with a highly improved heavy quark, the former effects would be negligible. With currently used heavy quark actions, these effects are not negligible and must be estimated and/or extrapolated away separately.

Once the Symanzik action is known, combining HQET and SxPT is a straightforward generalization of continuum heavy-light $\chi$PT [7]. The heavy-light mesons are combined into a single field and its conjugate

$$H_a = \frac{1}{2} [\gamma^\mu B_{\mu a} - \gamma_5 B_a] , \quad \overline{H}_a = \gamma_5 H_a^\dagger \gamma_0$$

while the light mesons are collected in $\Sigma = \sigma^2 = \exp(i\Phi/f)$. $\Phi$ is the $12 \times 12$ matrix (for 3 flavors of light quarks) that contains the pions

$$\Phi = \begin{pmatrix} U & \pi^+ & K^+ \\ \pi^- & D & K^0 \\ K^- & K^0 & S \end{pmatrix},$$

where $U = U_\alpha T_\alpha$, $K^+ = K^+_a T_a$, etc., and the $T_a$ are the 16 taste matrices [4,6]. Under chiral $SU(12)_L \times SU(12)_R$ we have the transformations

$$H_a \rightarrow H_b U^\dagger_{ba} , \quad \overline{H}_a \rightarrow U_{ab} \overline{H}_b$$

$$\sigma \rightarrow L\sigma R^\dagger , \quad \Sigma \rightarrow L \Sigma R^\dagger$$

with $L, R, U \in SU(12)$.

To leading order in $1/m_Q$, there are three expansion parameters: $m_\pi \sim \sqrt{m_q}$ ($m_q$ is a generic...
light quark mass), $a^2$, and $k$, the heavy-light residual momentum. We assume $k \sim m_s$ and $m_q \sim a^2$ for our power counting. We write the Lagrangian as $L = L_{1,2} + L_3$, where

$$
L_{1,2} = i \text{tr}_0 (\overline{H}_\nu \gamma^\mu (\delta_{ab} \partial_\mu + i \gamma^\nu \partial_\mu) \gamma^5 H_b)
+ g \text{tr}_0 (\overline{H}_a \gamma^5 \gamma_5 \gamma^5 A^a_\mu) + L_{\chi PT}. \tag{5}
$$

$L_{\chi PT}$ is the pion $S \chi PT$ Lagrangian found in Ref. [6]. $M$ is the light quark mass matrix, $\gamma_\mu \equiv (i/2) [\gamma^1 \partial_\mu + \sigma \partial_\mu \sigma^\dagger]$, and $A_\mu \equiv (i/2) [\gamma^5 \partial_\mu \sigma - \sigma \partial_\mu \sigma^\dagger]$. The terms in $L_{1,2}$ contribute to the NLO chiral logarithms, while the terms in $L_3$ are one order higher in the calculation, and thus can at most contribute to analytic terms at NLO. Part of $L_3$ is a new taste-breaking potential, $\gamma H$, containing both heavy and light mesons. Decay constants can be extracted from the matrix element $\langle 0 | A_{\nu,\alpha}^a | B_{x,\nu} | v \rangle = -i f_{B_x} m_{B_x} \nu^\nu \delta_{ab}$, where $A_{\nu,\alpha}^a$ is the axial current which destroys a $B_{x,a}$ meson, $x$ denotes the light quark flavor, and $\alpha$, its taste. The decay constant $f_{B_x}$ is independent of the light quark taste due to the symmetry $\sigma \rightarrow \xi_{\nu}^{(3)} \sigma \xi_{\nu}^{(3)}$, $H \rightarrow H_{\xi_{\nu}^{(3)}}$. The corresponding chiral operator is $A_{\nu,\alpha}^{T,a} = \frac{i \tau^a}{2} \text{tr}_0 [(1 - \gamma_5) \gamma^\nu \gamma_5 H_b] \sigma_{ba}$, where $\gamma^\nu \gamma_5 H_b$ projects the $x$ flavor block of $H_b$. We write the decay constant as $f_{B_x} \frac{\sqrt{m_{B_x}}}{\sqrt{m_{B_{\alpha}}}} = \kappa (1 + \delta f_{B_x}/(16\pi^2 f^2))$. There are two types of non-zero one-loop diagrams which contribute to the chiral logarithms, one coming from corrections to the current itself [Fig. 1(a)] and the other from wavefunction renormalization [Fig. 1(b)]. The crosses in Fig. 1 refer to one or more insertion of the two-point hairpin diagrams discussed in Ref. [6], for the singlet, axial, and vector taste flavor-neutral mesons. These correspond to disconnected quark level diagrams. We account for the transition from four to one tastes per flavor in the same way as in the calculations for light meson quantities [3].

For the 1+1+1 partially quenched chiral logs, we define the following sets of masses,

$$
\mu_{(3)} = \{ m_{u_0}^2, m_{D_0}^2, m_{S_0}^2 \}, \quad M_{j(3)} = \{ m_{x_0}^2, m_{x_0}^2, m_{x_0}^2 \}, \quad M_{V(4)} = \{ m_{x_0}^2, m_{x_0}^2, m_{v_0}^2, m_{v_0}^2 \}. \tag{6}
$$

Figure 1. One-loop diagrams contributing to the (a) current correction and (b) wavefunction renormalization.

where $t$ is a taste label. We then obtain

$$
\delta f_{B_x} = -\frac{1 + 3g_\pi^2}{2} \left\{ \frac{1}{16} \sum_{F,t} \ell(m_{\xi_{Ft}}^2) \right. \right.
+ \frac{1}{3} \sum_{j} \partial_{X_j} \left[ R_{j(3)}^{[3]}(M_{I(3)}^{(3)}; \mu_{I(3)}^{(3)}) \ell(m_{V}^{(3)}) \right]
+ a^2 \delta V \sum_{j} \partial_{X_j} \left[ R_{j(3)}^{[4]}(M_{V(4)}^{(4)}; \mu_{V(4)}^{(3)}) \ell(m_{V}^{(3)}) \right]
+ [V \rightarrow A] \right\}, \tag{7}
$$

where $\ell(m^2) = m^2 \ln m^2 + \text{finite volume corrections}$, $F$ labels sea-quark flavors, $t$ runs over the 16 tastes, and $j_I$ and $j_V$ run over the set of masses in the first argument of the $R_j$, the residues of the poles of the disconnected flavor-neutral propagators [3]. We define the derivative in Eq. (7) by $\partial_X \equiv \partial / \partial m_X^{(3)}$.

In the 2+1 ($m_u = m_d \neq m_s$) full QCD case, we have

$$
\delta f_B = -\frac{1 + 3g_\pi^2}{2} \left\{ \frac{1}{16} \sum_{t} \ell(m_{\xi_{Ft}}^2) \right. \right.
+ \frac{1}{2} \ell(m_{u_d}^2) + \frac{1}{6} \ell(m_{v_0}^2) \left. \right. \right.
+ a^2 \delta V \left[ \frac{m_{\pi_0}^2 - m_{\eta_0}^2}{m_{\pi_0}^2 - m_{\eta_0}^2} \ell(m_{\pi_0}^2) \right.
+ (\pi_0^0 \rightarrow \eta_0 \rightarrow \eta_0' \rightarrow \pi_0^0) \right\} + [V \rightarrow A]. \tag{8}
$$
where \( \pi_V^0 \to \eta_V \to \eta'_V \to \pi_V^0 \) represents two additional terms with cyclic replacements. There is a similar result for \( \delta f_{B_s} \).

To see the importance of the additional \( a^2 \) terms, we compare, in Fig. 2, the continuum version of the chiral logs as a function of valence quark mass with the complete lattice expression. The \( a^2 \) terms drastically change the behavior for lighter valence quark masses. We conclude that lattice data for heavy-light decay constants can be misleading; the data may look linear, but there can be large systematic errors if physical values are extracted by simple linear extrapolation.

Comparing our results for decay constants to the continuum expressions, we see that there is an easy way to generalize the continuum, partially quenched \( \chi \)PT expressions with \( N_{\text{sea}} \) degenerate quark flavors to SXPT. Terms \( \propto N_{\text{sea}} \), which arise from connected diagrams that at the quark level involve sea quark loops, become an average over tastes. Terms \( \propto 1/N_{\text{sea}} \) are disconnected; in SXPT we have \( I, V \), and \( A \) pieces. There can be additional minus signs in disconnected \( A \) and \( V \) terms compared to disconnected \( I \) terms, because of the anti-commutation relations for taste matrices. With this in mind, it is straightforward to write down the SXPT expressions for the form factors for \( B(D) \to \pi(K)\ell\nu \) decays using Ref. [9]. These expressions are however quite complicated and will be presented separately [8]. It will also be straightforward to extend these results to heavy-light B-parameters.

Fortunately, at this order the heavy-light SXPT chiral logarithms require no new parameters beyond what are already present in continuum heavy-light \( \chi \)PT and light meson SXPT. There will however be simple new analytic terms, proportional to \( a^2 \), in the complete expressions. Our results have already been used in analyzing lattice data for heavy-light form-factors and decay constants [3].

This work was supported by the U.S. DOE.

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