Joint User and Power Scheduling in Downlink NOMA over Fading Channels

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Abstract—Non-orthogonal multiple access (NOMA) has been considered one of the most promising radio access techniques for next-generation cellular networks. In this paper, we study a joint user and power scheduling problem in downlink NOMA over fading channels. Specifically, we focus on a stochastic optimization problem to maximize the average weighted sum rate while ensuring given minimum average data rates of users. To address this challenging problem, we first develop an opportunistic user and power scheduling algorithm (OUPS) based on the duality and stochastic optimization theory, which transforms our stochastic problem into a series of deterministic ones to maximize the instantaneous weighted sum rate for each time slot. As a good feature, OUPS is an online algorithm that can make decisions without requiring information on the underlying distribution of the fading channels, making it effectively applicable to a variety of practical applications. Afterward, we develop a simple heuristic algorithm with very low computational complexity, called user selection and power allocation algorithm (USPA), for the instantaneous weighted sum rate maximization problem that is embedded in OUPS. Through the numerical results, we demonstrate that USPA provides near-optimal performance despite very low computational complexity and OUPS ensures given minimum average data rates of users.

Index Terms—Non-orthogonal multiple access (NOMA), opportunistic scheduling, power allocation, user scheduling.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is promising as a multiple access technique for beyond 5G cellular networks due to its high spectral efficiency and user connectivity [1]. Although various NOMA techniques have been proposed thus far, this paper focuses on power-domain NOMA [2]. In power-domain NOMA, superposition coding (SPC) and successive interference cancellation (SIC) are employed at the transmitter and receiver sides, respectively. With SPC and SIC, power domain diversity gain can be achieved by appropriate user scheduling and power allocation. Hence, user scheduling and power allocation have become important research issues.

There are many studies on either user selection, power allocation, or both in NOMA systems [3]–[8]. To be specific, in [3], the authors have studied the sum rate maximization problem, and developed a heuristic algorithm for joint user selection and power allocation based on the water-filling algorithm. On the other hand, the weighted sum rate maximization problem has been studied in [4], [5]. In [4], the authors have developed a heuristic user selection and power allocation algorithm that iteratively decides whether to select one or both for every two consecutive users based on their weights and channel gains. In [5], the authors have developed a power allocation algorithm based on geometric programming (GP) and a user selection algorithm based on a method of alternately updating user selection and power allocation using a matching game. Although the (weighted) sum rate maximization has been studied in the above studies [3]–[5], quality of service (QoS) constraints have not been considered.

In [6]–[8], studies have been conducted taking into account minimum data rate requirements as QoS constraints. To be specific, in [6], the authors have proven that the power allocation problem for the sum rate maximization is a convex problem that can be solved by standard algorithms for convex optimization. In [7], the authors have developed an optimal user selection and power allocation algorithm for the sum rate maximization based on the exhaustive search and the branch-and-bound approach. However, due to its impractical computational complexity, they have proposed another heuristic algorithm with the aid of matching theory and successive convex approximation. In [8], the authors have proposed a NOMA-based mixture transceiver architecture that applies SPC and SIC to each group and opportunistically increases the multiplexing gain while providing full diversity order based on channel-adaptive user grouping and proper power allocation.

Despite extensive studies on either user selection, power allocation, or both in NOMA systems, there are still some limitations. First, most of the previous studies, including [3]–[8], have focused on optimization problems from a snapshot perspective. That is, user selection has been concentrated, rather than user scheduling, under fixed channel conditions while considering the instantaneous (weighted) sum rate objective and QoS constraints. Thereby, in practical systems over time-varying fading channels, a significant drop in overall performance can occur since such instantaneous QoS constraints should always be satisfied even for users with very low channel gains. However, such instantaneous QoS constraints are not essential in most applications. Second, although various problems for the (weighted) sum rate maximization have been studied in depth, including [3]–[8], the weighted sum rate maximization problem with considering QoS constraints, that is an NP-hard problem, has not been well studied.

Motivated by the above observations, in this paper, we develop a joint user and power scheduling algorithm with very low computational complexity in downlink NOMA over fading channels to maximize the average weighted sum rate while
ensuring given minimum average data rates of users. We first develop an opportunistic user and power scheduling algorithm (OUS) based on the duality and stochastic optimization theory. As a merit, OUS is an online algorithm that can make decisions with only the instantaneous channel status without requiring knowledge of the underlying distribution of the fading channels, making it effectively applicable to a variety of practical applications. OUS requires solving a user selection and power allocation problem to maximize the instantaneous weighted sum rate for each time slot, which is known as challenging nonconvex mixed-integer programming. Although the weighted sum rate maximization problem has been studied in [4], [5] as mentioned before, their algorithms are too complex to be executed in every time slot. Hence, we additionally develop a very simple heuristic algorithm, called user selection and power allocation algorithm (USPA), which has low computational complexity and provides near-optimal performance. Through the numerical results, we first verify that our USPA provides good performance comparable to the optimal one, and then show that OUS with USPA guarantees given minimum average data rates of all users.

The rest of the paper is organized as follows. In Section II we present the system model. In Section III we formulate a stochastic user and power scheduling problem and develop OUS to solve it. In Section IV we develop USPA with very low computational complexity, which is exploited in OUS. Numerical results are presented in Section V by following the conclusion in Section VI.

Notation: Scalars, vectors, and sets are denoted by italic, boldface, and calligraphic letters, respectively. \( \mathbb{E}[\cdot] \) denotes a statistical expectation operator. For a set \( S \), \( (a_i)_{i \in S} \) denotes a vector that consists of elements in the set \( \{a_i : i \in S\} \). For a complex number \( a \), \(|a|\) denotes its absolute value. For a real-valued vector \( a \), \( |a|^+ \) is a vector each of whose elements is chosen as the maximum value among zero and itself.

II. SYSTEM MODEL

We consider downlink NOMA in a single-cell that consists of one single-antenna base station (BS), whose maximum transmission power is limited to \( P_{\text{max}} \), and a set of \( N \) single-antenna users denoted by \( N = \{1, 2, \ldots, N\} \). We assume a time-slotted system over block fading channels, in which the channel condition of each wireless link changes from one time slot to another, whereas it remains constant during a time slot. Let \( \{h_{it}, t = 1, 2, \ldots\} \) be the fading process associated with User \( i \), where \( h_{it} \) is a complex-valued continuous random variable representing the channel response from the BS to User \( i \) in time slot \( t \). We assume that \( \{h_{it}\} \) is a stationary and ergodic process. In succession, we denote a channel vector in time slot \( t \) by \( \mathbf{h}^t = (h_{it})_{i \in N} \) and its support by \( \mathcal{H} \). We assume that information on the underlying distribution of \( \mathbf{h}^t \) is unknown to the BS since it is practically difficult to obtain such a priori information. However, an instantaneous channel vector is known to the BS at the beginning of each time slot, so that user scheduling and power allocation can be carried out by the BS based on it.

According to the NOMA principle, multiple users can be simultaneously scheduled with different levels of power in the same time slot. Let \( x_i^t \) be the message with unit energy intended for User \( i \) in time slot \( t \), i.e., \( \mathbb{E}[|x_i^t|^2] = 1 \), \( p_i^t \) be power allocated to User \( i \) in time slot \( t \), and \( q_i^t \) be a user selection variable whose value is 1 if User \( i \) is selected in time slot \( t \) and 0 otherwise. The BS employs SPC and transmits the signal \( x_i^t = \sum_{i \in N} q_i^t \sqrt{p_i^t} x_i^t \) to users subject to the maximum transmission power constraint, i.e., \( \mathbb{E}[|x_i^t|^2] = \sum_{i \in N} q_i^t p_i^t \leq P_{\text{max}} \).

Then, the received signal at User \( i \) is given as

\[
y_i^t = h_i^t \sum_{i \in N} q_i^t \sqrt{p_i^t} x_i^t + n_i^t, (1)
\]

where \( n_i^t \) is the additive Gaussian noise of User \( i \) in time slot \( t \) with zero-mean and variance \( \sigma_i^2 \), i.e., \( n_i^t \sim \mathcal{CN}(0, \sigma_i^2) \). Each user performs SIC to decode its target message from its received signal. To be specific, User \( i \) first decodes the signals intended for any other User \( j \) whose normalized channel gain is not higher than itself, i.e., \( |h_i^t|^2/\sigma_i^2 \leq |h_j^t|^2/\sigma_j^2 \), and then subtracts them from the received signal \( y_i^t \). In succession, User \( i \) decodes its target message \( x_i^t \) by treating the signals intended for users with higher normalized channel gains than itself as interference signals. With a typical assumption that each user can successfully decode the signals intended for users with weaker channels, the maximum achievable rate of User \( i \) in time slot \( t \) can be calculated as

\[
R_i(p^t, q^t; h^t) = q_i^t \log_2 \left( 1 + \frac{|h_{i.t}|^2 p_{i.t}^t}{\sum_{j \in N: |h_{j.t}|^2 > |h_{i.t}|^2} p_{j.t}^t + \sigma_i^2} \right), (2)
\]

where \( p^t = (p_i^t)_{i \in N} \) and \( q^t = (q_i^t)_{i \in N} \).

III. OPPORTUNISTIC USER AND POWER SCHEDULING

We study a joint user and power scheduling problem whose objective is to find optimal user selection and power allocation for each time slot to maximize the average weighted sum rate, while satisfying the minimum average data rate of each user. The joint user and power scheduling problem is formulated as

\[
\begin{align*}
\text{maximize} \quad & \frac{1}{T} \sum_{t=1}^{T} \sum_{i \in N} w_i R_i(p^t, q^t; h^t) \\
\text{subject to} \quad & \sum_{t=1}^{T} R_i(p^t, q^t; h^t) \geq \bar{R}_i, \forall i \in N, \quad (3a) \\
& p^t \in \mathcal{P}, \quad q^t \in \mathcal{Q}, \quad \forall t, \quad (3c)
\end{align*}
\]

where \( w_i \) and \( \bar{R}_i \) are the weight and minimum average data rate requirement of User \( i \), respectively. \( \mathcal{P} = \{(v_i)_{i \in N} : \sum_{i \in N} v_i \leq P_{\text{max}}, v_i \geq 0, \forall i \in N\} \), and \( \mathcal{Q} = \{(v_i)_{i \in N} : v_i \in \{0, 1\}, \forall i \in N\} \). The constraints (3b) and (3c) guarantee the minimum average data rate requirement of each user and the range of power allocation and user selection vectors, respectively. Note that since the channel fading is assumed to be ergodic, the long-term time average converges almost surely
to the expectation for almost all realizations of the fading process. Thus, denoting a channel vector in a generic time slot by \(h\), we can transform the above problem into

\[
(P) \quad \text{maximize} \quad \mathbb{E}_h \left[ \sum_{i \in \mathcal{N}} w_i R_i (p^h, q^h, h) \right] \tag{4a}
\]

subject to \(\mathbb{E}_h [R_i (p^h, q^h, h)] \geq \bar{R}_i, \forall i \in \mathcal{N}, \tag{4b}\)

\(p^h \in \mathcal{P}, q^h \in \mathcal{Q}, \forall h \in \mathcal{H}, \tag{4c}\)

where \(\bar{p} = (p^h)_{h \in \mathcal{H}}, \bar{q} = (q^h)_{h \in \mathcal{H}},\) and \(p^h = (p^h_i)_{i \in \mathcal{N}}\) and \(q^h = (q^h_i)_{i \in \mathcal{N}}\) are the power allocation and user selection vectors corresponding to a channel vector \(h\), respectively. Note that for each time slot with channel vector \(h\), we can select users and allocate power according to the solution for \(q^h\) and \(p^h\), respectively, obtained by solving Problem \((P)\).

There are two main challenges in solving Problem \((P)\). One is that information on the underlying distribution of \(h\) is practically unknown, and the other is that Problem \((P)\) is nonconvex mixed-integer stochastic programming. To resolve these challenges, we employ a dual approach and a stochastic subgradient method as in [9], [10]. To this end, we first define a Lagrangian function \(L\) associated with Problem \((P)\) as

\[
L(\bar{p}, \bar{q}, \lambda) = \mathbb{E}_h \left[ \sum_{i \in \mathcal{N}} w_i R_i (p^h, q^h, h) \right] + \sum_{i \in \mathcal{N}} \lambda_i \left[ \mathbb{E}_h [R_i (p^h, q^h, h)] - \bar{R}_i \right] = \mathbb{E}_h \left[ \sum_{i \in \mathcal{N}} (w_i + \lambda_i) R_i (p^h, q^h, h) \right] - \sum_{i \in \mathcal{N}} \lambda_i \bar{R}_i, \tag{5}\]

where \(\lambda = (\lambda_i)_{i \in \mathcal{N}}\) is a nonnegative Lagrangian multiplier vector corresponding to the constraint \((4b)\). Based on the above Lagrangian function, the dual problem associated with Problem \((P)\) is defined as

\[
(D) \quad \text{minimize} \quad \lambda \geq 0 \quad F(\lambda), \tag{6}\]

where \(\geq\) is the elementwise inequality, \(\mathbf{0}\) is a zero vector, and

\[
F(\lambda) = \text{maximize} \quad L(\bar{p}, \bar{q}, \lambda) \tag{6}
\]

subject to \(p^h \in \mathcal{P}, q^h \in \mathcal{Q}, \forall h \in \mathcal{H}.

Since Problem \((P)\) is nonconvex, even if its dual problem \((D)\) is optimally solved, the duality gap is generally nonzero. However, the duality gap vanishes in our problem, resulting in no loss of optimality. We prove this in the following theorem.

**Theorem 1.** The strong duality holds between Problem \((P)\) and its dual problem \((D)\).

**Proof.** See Appendix A. \(\blacksquare\)

By Theorem 1 we deduce that OUPS, an algorithm to be developed subsequently to solve the dual problem \((D)\), can be used to obtain the optimal solution to problem \((P)\).

Prior to discussing how to solve the dual problem \((D)\), we focus on finding its objective function, \(F(\lambda)\). That is, we first solve the maximization in \((6)\). We can observe in \((5)\) that the first term is separable in terms of each channel vector, and the second term is independent of the power allocation variable. Hence, for any given Lagrangian multiplier \(\lambda\), the maximization in \((6)\) can be solved by separately solving the following subproblem for each given channel vector \(h\):

\[
(D^h) \quad \text{maximize} \quad \sum_{i \in \mathcal{N}} (w_i + \lambda_i) R_i (p^h, q^h, h). \tag{7}\]

The expectation has disappeared in Problem \((D^h)\), so that it can be solved without knowledge of the underlying distribution of \(h\). Note that, for given \(\lambda\) and \(h\), Problem \((D^h)\) turns into a user selection and power allocation problem to maximize the instantaneous weighted sum rate with weight \(w_i + \lambda_i\). An algorithm to solve this problem, called USPA, will be developed in the next section.

We now focus back on solving the dual problem \((D)\). Even though optimal user selection and power allocation can be obtained for each \(h\) and \(\lambda\) by solving Problem \((D^h)\), the underlying distribution of \(h\) is still required to solve the dual problem \((D)\). However, such a priori knowledge is unknown in most practical applications. Nevertheless, thanks to the fact that the dual problem \((D)\) is a convex stochastic programming problem, we can solve it using the stochastic subgradient method, where the Lagrangian multiplier is iteratively updated as

\[
\lambda^t+1 = [\lambda^t - \eta^t v^t]^+, \tag{8}\]

where \(\lambda^t\) and \(\eta^t\) are the Lagrangian multiplier vector and the step size in time slot \(t\), respectively, and \(v^t\) is the stochastic subgradient of \(F(\lambda)\) with respect to \(\lambda\) at \(\lambda = \lambda^t\). By Danskin’s min-max theorem [12], the stochastic subgradient, \(v^t = (v^t_i)_{i \in \mathcal{N}},\) can be determined as

\[
v^t_i = R^t_i - \bar{R}_i, \forall i \in \mathcal{N}, \tag{9}\]

where \(R^t_i\) is the instantaneous data rate of User \(i\) in time slot \(t\), which is achieved by selecting users and allocating power according to the solution to Problem \((D^h)\) given \(h^t\) and \(\lambda^t\). When the Lagrangian multiplier vector follows this update process, it converges almost surely to the optimal solution, \(\lambda^*\), of the dual problem \((D)\) if the step size \(\eta^t\) satisfies

\[
\eta^t \geq 0, \sum_{t=1}^{\infty} \eta^t = \infty, \text{ and } \sum_{t=1}^{\infty} (\eta^t)^2 < \infty. \tag{10}\]

The proposed OUPS is outlined in Algorithm 1.

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**Algorithm 1: Opportunistic user and power scheduling**

1. Initialize: \(\lambda^0 = \mathbf{0}\), and \(t = 1\)
2. for each time slot \(t\) do
   3. Obtain a solution to Problem \((D^h)\) using USPA.
   4. Transmit a signal based on the obtained solution.
   5. Update \(\lambda^t\) according to \((7)\) and \((8)\).
   6. \(t \leftarrow t + 1\).

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IV. USER SELECTION AND POWER ALLOCATION

In this section, we develop USPA to solve Problem (D^h). To alleviate the intractability of Problem (D^h) caused by the presence of integer (combinatorial) variables, i.e., user selection variables, we first transform it into

\[
\text{(Q_1)} \quad \max_{p \in P} \sum_{i \in N} \tilde{w}_i R_i(p^h; \mathbf{h}),
\]

where \( R_i(p^h; \mathbf{h}) \) is defined as \( R_i(p^h, q^h; \mathbf{h}) \) with \( q_i^h = 1 \), and \( \tilde{w}_i = w_i + 4_i \). Although the user selection variable, \( q^h \), is excluded from the variables, an optimal solution to Problem (D^h) can be easily obtained from that to Problem (Q_1).

**Theorem 2.** Let \((p^h)^{†}\) be an optimal solution to Problem (Q_1). Then, the optimal solution, \((p^h)^∗, (q^h)^∗\), to Problem (D^h) can be obtained as

\[
(p^h)^∗ = (p^h)^{†} \quad \text{and} \quad (q^h)^∗ = (q^h_i)_{i \in N},
\]

where for all \( i \in N \), \( q^h_i = 1 \) if \((p^h)^{†} > 0\) and \( 0 \) otherwise.

**Proof.** See Appendix B

Using Theorem 2 we focus on Problem (Q_1) rather than Problem (D^h). For notational simplicity, we omit \( \mathbf{h} \) in both \( p^h \) and \( p^h \) and assume, without loss of generality, that the normalized channel gains are ordered such that \( |h_i|^2 / \sigma_i^2 \leq |h_j|^2 / \sigma_j^2 \) if \( i < j \). Then, Problem (Q_1) can be equivalently recast as

\[
\text{(Q_2)} \quad \max_{p_1, p_2, \ldots, p_N} \sum_{i = 1}^N \tilde{w}_i \log \left( 1 + \frac{|h_i|^2 p_i}{|h_i|^2 \sum_{j > i} p_j + \sigma_i^2} \right)
\]

subject to \( \sum_{i = 1}^N p_i \leq P_{\max} \), \( p_i \geq 0, \quad i = 1, 2, \ldots, N \).

Note that since Problem (Q_2) is nonconvex, we cannot apply standard algorithms for convex optimization. Furthermore, as mentioned before, the problem should be solved with low computational complexity. To cope with these challenges, we first find candidate users to whom power may be allocated, and then address how to optimally allocate power to them.

**Definition 1.** A last SIC user refers to a user who does not experience any interference signal after the SIC process.

We begin with an assumption that User \( k \) is selected as the last SIC user. Thus, \( p_k \) will be given a certain value, and \( p_i \) for \( i > k \) will be zero so that User \( k \) does not experience any interference signal after the SIC process. Under this assumption, \( p_k, p_{k+1}, \ldots, p_N \) are no longer part of the optimization variables. Note that how to select the last SIC user and how much power to allocate to it will be discussed later. Then, Problem (Q_2) can be reformulated as

\[
\text{(Q_3)} \quad \max_{p_1, p_2, \ldots, p_{k-1}} \sum_{i = 1}^{k-1} \tilde{w}_i \log \left( 1 + \frac{|h_i|^2 p_i}{|h_i|^2 \sum_{j > i} p_j + \sigma_i^2} \right)
\]

subject to \( \sum_{i = 1}^{k-1} p_i + p_k \leq P_{\max} \), \( p_i \geq 0, \quad i = 1, 2, \ldots, k-1 \).

Note that Problem (Q_3) is a problem for finding candidate users under the assumption that User \( k \) has been selected as the last SIC user. Now, based on the fact that the interference power is usually much greater than the noise power, we assume that the noise signals of users who suffer from the interference signals are negligible, i.e., \( \sigma_i^2 \approx 0 \) for \( i < k \). Then, by letting \( \rho_i = \sum_{j = 1}^{k} p_j \), we can approximate Problem (Q_3) as

\[
\text{(Q_4)} \quad \max_{\rho_1, \rho_2, \ldots, \rho_{k-1}} \sum_{i = 1}^{k-1} \tilde{w}_i \log_2 \left( \frac{\rho_i}{\rho_{i+1}} \right)
\]

subject to \( \prod_{i = 1}^{k-1} \frac{\rho_i}{\rho_{i+1}} \leq P_{\max}, \quad \rho_i \geq 1, \quad i = 1, 2, \ldots, k-1, \)

where the constraints are equivalent to those in Problem (Q_3), derived using simple arithmetic operations. In succession, by letting \( r_i = \log_2 (p_i / \rho_{i+1}) \) for \( i < k \), where \( r_k = \log_2 (p_k) \), and taking the logarithm of the both sides of the constraints, we can restate Problem (Q_4) as

\[
\text{(Q_5)} \quad \max_{r_1, r_2, \ldots, r_{k-1}} \sum_{i = 1}^{k-1} \tilde{w}_i r_i
\]

subject to \( r_i + r_k \leq \log_2 (P_{\max}), \quad r_i \geq 0, \quad i = 1, 2, \ldots, k-1. \)

In Problem (Q_5), since \( r_1, r_2, \ldots, r_{k-1} \) are linearly combined in the objective function, it is obvious that all \( r_i \)'s except the one with the largest weight is zero. Also, by definition, we know that, for any \( i < k \), \( p_i \) is zero if \( r_i \) is zero. Hence, only the user with the largest weight is selected as a candidate user together with the last SIC user, i.e., User \( k \). We state this result in the following theorem.

**Theorem 3.** Under the assumption that the noise signals of users who suffer from the interference signals are neglected, by the solution to Problem (Q_2), at most two users are selected as candidate users to whom power may be allocated. To be specific, when User \( k \) is selected as the last SIC user, User \( \phi_k \) is accordingly selected as the other candidate user where

\[
\phi_k = \arg\max_{i < k} \{ \tilde{w}_i \}. \quad (11)
\]

By Theorem 3, we consider the following power allocation problem for the two-user case:

\[
\text{(P_2)} \quad \max_{p_{\phi_k}, p_k} \tilde{w}_{\phi_k} \log_2 \left( 1 + \frac{|h_{\phi_k}|^2 p_{\phi_k}}{|h_{\phi_k}|^2 p_k + \sigma_{\phi_k}^2} \right) + \tilde{w}_k \log_2 \left( 1 + \frac{|h_k|^2 p_k}{\sigma_k^2} \right)
\]

subject to \( p_{\phi_k} + p_k \leq P_{\max}, \quad p_{\phi_k} \geq 0, \quad p_k \geq 0. \)

This two-user power allocation problem can be optimally solved, and the solution comes in a closed form as a function of the channels and weights of the two users.
Theorem 4. The optimal solution, \( \{p^*_k, p^*_\phi \} \), to Problem (P2) is derived as

\[
p^*_k = \begin{cases} 
0, & \text{if } \hat{w}_k/\hat{w}_\phi < C_1, \\
\max_{p_k} \frac{\hat{w}_\phi \|h_{\phi_k}\|^2 \sigma^2_p - \hat{w}_k \|h_k\|^2 \sigma^2_p}{(\hat{w}_k - \hat{w}_\phi) \|h_{\phi_k}\|^2}, & \text{otherwise}, 
\end{cases}
\]

(12)

and

\[
p^*_\phi = P - p^*_k,
\]

(13)

where \( C_1 = \frac{\|h_{\phi_k}\|^2 \sigma^2_p}{\|h_k\|^2 \sigma^2_p} \) and \( C_2 = \frac{\|h_{\phi_k}\|^2 (\sigma^2_p + P \|h_k\|^2)}{\|h_k\|^2 (\sigma^2_p + P \|h_{\phi_k}\|^2)} \).

Proof. See Appendix C.

Using Theorem 4, the weighted sum rate, \( R^k_{\text{sum}} \), when User \( k \) is selected as the last SIC user can be given as

\[
R^k_{\text{sum}} = \hat{w}_\phi \log_2 \left( 1 + \frac{\|h_{\phi_k}\|^2 p^*_\phi}{\|h_{\phi_k}\|^2 p^*_k + \sigma^2_p} \right) + \hat{w}_k \log_2 \left( 1 + \frac{\|h_k\|^2 p^*_k}{\sigma^2_p} \right).
\]

(14)

In succession, the index, \( k^* \), of the optimal last SIC user can be obtained as

\[
k^* = \arg\max_{k \in \mathcal{N}} R^k_{\text{sum}}.
\]

(15)

The pseudocode of USPA is described in Algorithm 2.

Before we end this section, we discuss the computational complexity. Our USPA has linear computational complexity, i.e., \( \mathcal{O}(N) \), since \( R^k_{\text{sum}} \) can be calculated by the closed-form formula in (14). As a benchmark, we consider the GP-based algorithm proposed in [5], which provides an optimal solution to Problem (Q2) by solving its equivalent GP problem using interior point methods (IPM). Hence, we call it OPT. Note that the computational complexity of IPM for GP problems is known as \( \mathcal{O}((k + m)^{1/2}(m k^2 + k^3 + n^3)) \) in [13], where \( k, m, \) and \( n \) are the problem-dependent parameters. In our case, the parameters are given as \( k = 2N + 1, m = N + 1, \) and \( n = N \). Thus, OPT has nonlinear computational complexity of \( \mathcal{O}((3N + 2)^{1/2}(13N^3 + 20N^2 + 11N + 2) + L) \approx \mathcal{O}(13\sqrt{3}N^{7/2} + L) \), where \( L \) represents the computational complexity for converting from the solution to the GP problem back to that to Problem (Q2).

As a result, the computational complexity of OPT is much higher than that of USPA, and thus it is a heavy burden to run OPT at every time slot, compared to our USPA with low computational complexity. Comparison of computational complexity between the two methods will be shown experimentally in the next section.

Algorithm 2: User Selection and Power Allocation

1. for each \( k \in \mathcal{N} \) do
2. Suppose Users \( k \) and \( \phi_k \) are selected using Theorem 5.
3. Find optimal power allocation using Theorem 4.
4. Calculate \( R^k_{\text{sum}} \) using (14).
5. Find \( k^* \) using (15).
6. Select Users \( k^* \) and \( \phi_k \) as the optimal candidate users.
7. Allocate power to them according to Theorem 3.

V. Numerical Results

We consider a single cell with one BS with a maximum transmission power of 43 dBm and 5 users. According to [15], we set the large-scale path loss to \( 128.1 + 37.6 \log_{10}(d_{km}) \) dB, where \( d_{km} \) is the distance in kilometers, and consider the log-normal shadow fading with a standard deviation of 8 dB and the small-scale fading with coefficients following independent and identical zero-mean unit-variance complex Gaussian distributions. The noise power for each user is set to \(-104\) dBm. The step size of the stochastic subgradient algorithm in (7) is set to \( \eta = 1/t \), which satisfies the conditions in (8), so that the convergence of the algorithm is guaranteed.

We first compare USPA and OPT in solving Problem (Q2). Fig. 1 shows the comparison results for 1000 independent trials. In each trial, the user weights are randomly set between 0 and 1 then normalized by their sum, and the distance between each user and the BS is randomly set between 20 m to 500 m. Fig. 1a shows the performance comparison results over 1000 trials in terms of the weighted sum rate. As shown in this figure, the performance of USPA is very close to that of OPT. On average, the performance difference between the two methods over 1000 trials is only 0.0412 bps/Hz (0.7%). Fig. 1b shows the frequency histogram of the number of users selected. Fig. 1c shows the average weighted data rate of each user, where the user indices are sorted in decreasing order.

\[^1\]Unlike our USPA where power allocation is achieved by the closed-form formula, there is a numerical precision errors in OPT because the power allocation is achieved through an iterative algorithm. For this reason, near zero but not zero power allocation is achieved although zero power allocation is optimal. Under this circumstance, for fair comparison, we have regarded only users with data rates above 0.1 bps/Hz as selected users.
order of the weighted data rate. According to Fig. 1(b) in OPT, the probability of selecting three or more users reaches about 25\% which is not small. However, according to Fig. 1(d) since the sum of the weighted data rates of the first two users occupies over 96\% of the weighted sum rate, the last three users do not significantly contribute to the performance. That is why the performance of USPA is very close to that of OPT.

In addition, it is worth noting that in obtaining the above numerical results, OPT have taken about 3900 times more execution time than USPA. These results verify that not only does USPA provide good performance close to the optimal one, but it also has very low computational complexity.

Now, we evaluate the performance of our OUPS by comparing the following three types of OUPS. One is OUPS-USPA in which user selection and power allocation are performed by USPA. Another is OUPS-OPT in which user selection and power allocation are performed by OPT. The other is OUPS-OMA in which only one user who can provide the highest instantaneous weighted data rate using full power is selected at each time slot. Under the scenario where 5 users with equal weights are located 20 m, 140 m, 260 m, 380 m, and 500 m away from the BS, we show the performance results over 10,000 time slots in Fig. 2. Fig. 2(a) shows the average sum rate when the minimum average data rate requirement of each user is 2 bps/Hz. From the results, not only does OUPS-USPA provide much higher average sum rates than OUPS-OMA, but also it provides a good performance close to that of OUPS-OPT. In Fig. 2(b) we show the average data rates of each user for OUPS-USPA for different minimum average data rate requirements. The first bar for each user represents its average data rate when there are no minimum average data rate requirements. In this case, only the user closest to the BS achieves a very high data rate, whereas the rest of the users achieve very low data rates of nearly 0 bps/Hz. On the other hand, the second, third, and fourth bars for each user represent its average data rates when the minimum average data rate requirements are given as 1 bps/Hz, 2 bps/Hz, and 3 bps/Hz, respectively. The results show that the QoS constraints for all users are well satisfied, and the user closest to the BS achieves the highest average data rate among all users so that the average sum rate is maximized. Numerical results demonstrate that our proposed OUPS-USPA provides near-optimal performance while ensuring the given QoS constraints.

VI. Conclusion

In this paper, we have studied the joint and power scheduling problem to maximize the average weighted sum rate while ensuring given QoS constraints in the downlink NOMA. We have first developed an opportunistic scheduling algorithm, OUPS, that fully exploits time-varying channels, and then developed an internal algorithm with very low computational complexity, USPA, that maximizes the instantaneous weighted sum rate. Numerical results validate that our proposed scheduling technique not only provides good performance comparable to the optimal one, but also ensures QoS constraints. This study will be the cornerstone of our future work on scheduling for multi-carrier NOMA systems.

APPENDIX A

Proof of Theorem 1

To prove that the strong duality holds between Problem (P) and its dual problem (D), we employ the time-sharing condition proposed in [36], which is defined as follows.

Definition A.1. Let \( \{ \tilde{p}_z, \tilde{q}_z \} \) and \( \{ \tilde{p}_y, \tilde{q}_y \} \) be optimal solutions to Problem (P) with \( \bar{R} = \bar{R}_z \) and \( \bar{R} = \bar{R}_y \), respectively, where \( \bar{R} = (\bar{R}_i)_{i \in N} \). Then, Problem (P) is said to satisfy the time-sharing condition if for any \( \bar{R}_z, \bar{R}_y \) and for any \( 0 \leq \theta \leq 1 \), there always exists a feasible solution \( \{ \tilde{p}_z, \tilde{q}_z \} \) such that

\[
\mathbb{E}_h \left[ q_{z,i}^h R_{i}(\tilde{p}_z^h, h) \right] \geq \theta \bar{R}_{i,z} + (1 - \theta) \bar{R}_{i,y}, \quad \forall i \in N, \quad (A.1)
\]

\[
\mathbb{E}_h \left[ \sum_{i \in N} q_{z,i}^h w_i R_{i}(\tilde{p}_z^h, h) \right] \geq \theta \mathbb{E}_h \left[ \sum_{i \in N} q_{z,i}^h w_i R_{i}(\tilde{p}_z^h, h) \right] + (1 - \theta) \mathbb{E}_h \left[ \sum_{i \in N} q_{y,i}^h w_i R_{i}(\tilde{p}_y^h, h) \right]. \quad (A.2)
\]

In addition to the definition of the time-sharing condition in [36], it has been proven that if an optimization problem satisfies the time-sharing condition, the strong duality holds regardless of the convexity of the problem. Hence, we prove...
Theorem 1 by showing that Problem (P) satisfies the time-sharing condition. It can be easily verified by setting
\[
\{p^*_i, q^*_i\} = \begin{cases} 
\{p^i, q^i\} & t \leq \theta T, \\
\{p^*_i, q^*_i\} & t \geq \theta T,
\end{cases} 
\]  
(A.3)

for \(0 \leq \theta \leq 1\). With a feasible solution defined in (A.3), we can derive, for all \(i \in N^c\),
\[
\mathbb{E}_h [q^i R_i (p^*_i; h)] = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} q^i_{t,x} R_i (p^*_i; h^t) 
= \lim_{T \to \infty} \frac{1}{T} \left( [\theta T] \sum_{t=1}^{\lfloor \theta T \rfloor} q^i_{t,x} R_i (p^*_i; h^t) + \sum_{t=\lfloor \theta T \rfloor + 1}^{T} q^i_{t,x} R_i (p^*_i; h^t) \right)
= \theta [\sum_{t=1}^{\lfloor \theta T \rfloor} q^i_{t,x} R_i (p^*_i; h^t) + \sum_{t=\lfloor \theta T \rfloor + 1}^{T} q^i_{t,x} R_i (p^*_i; h^t)] + (1 - \theta) \mathbb{E}_h [q^i_{\lfloor \theta T \rfloor} R_i (p^*_i; h)]
\]
\(\geq \theta R_{i,x} + (1 - \theta) R_{i,y}\),

(A.4)

where \(\lfloor \cdot \rfloor\) is the floor function that gives the largest integer not exceeding its argument. Similarly, we can also derive
\[
\mathbb{E}_h \left[ \sum_{i \in N} q^i R_i (p^*_i; h) \right] = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i \in N} q^i_{t,x} R_i (p^*_i; h^t)
= \lim_{T \to \infty} \frac{1}{T} \left( [\theta T] \sum_{t=1}^{\lfloor \theta T \rfloor} q^i_{t,x} R_i (p^*_i; h^t) + \sum_{t=\lfloor \theta T \rfloor + 1}^{T} q^i_{t,x} R_i (p^*_i; h^t) \right)
= \theta \mathbb{E}_h \left[ \sum_{i \in N} q^i_{\lfloor \theta T \rfloor} R_i (p^*_i; h) \right] + (1 - \theta) \mathbb{E}_h \left[ \sum_{i \in N} q^i_{\lfloor \theta T \rfloor} R_i (p^*_i; h) \right].
\]  
\(\text{(A.5)}\)

Hence, the time-sharing condition holds for Problem (P). 

\[\blacksquare\]

**Appendix B**

**Proof of Theorem 2**

Since \(q^h\) is either 0 or 1, the objective value of Problem (D^h) is less than or equal to the optimal value of Problem (Q), i.e., \(\sum_{i \in N} w_i R_i (p^h, q^h; h) \leq \sum_{i \in N} \tilde{w}_i R_i (p^h, q^h; h)\), for any \(p^h \in \mathcal{P}\) and \(q^h \in \mathcal{Q}\). Hence, by letting \((p^h)^\dagger\) be an optimal solution to Problem (Q), we have
\[
\sum_{i \in N} \tilde{w}_i R_i (p^h, q^h; h) \leq \sum_{i \in N} \tilde{w}_i R_i ((p^h)^\dagger; h),
\]  
(B.1)

for all \(p^h \in \mathcal{P}\) and \(q^h \in \mathcal{Q}\). Consider \((p^h)^\ast = (p^h)^\dagger\) and \((q^h)^\ast = (q^h)^\dagger\) such that \(q^i_1 = 1\) if \((p^h)^\dagger\) \(> 0\) and \(q^i_0 = 0\) otherwise. By simple arithmetic operations, it can be easily verified that
\[
\sum_{i \in N} \tilde{w}_i R_i ((p^h)^\ast, (q^h)^\ast; h) = \sum_{i \in N} \tilde{w}_i R_i ((p^h)^\dagger; h).
\]  
(B.2)

Hence, an optimal solution to Problem (D^h) is given as \(\{(p^h)^\ast, (q^h)^\ast\}\), which is obtained from the optimal solution, \((p^h)^\dagger\), to Problem (Q1).

**Appendix C**

**Proof of Theorem 4**

Since a larger transmission power results in a higher weighted sum rate, the first constraint in Problem (P2) can be replaced with \(p_{\phi_k} + p_k = P_{\max}\). Then, by substituting \(p_{\phi_k}\) into \(P_{\max} - p_k\). Problem (P2) can be equivalently transformed into a one-variable optimization problem as
\[
(P_2') \quad \text{maximize}_{0 \leq p_k \leq P_{\max}} \ g(p_k),
\]  
\(\text{(C.1)}\)

where
\[
g(p_k) = \tilde{w}_{\phi_k} \log_2 \left( \frac{|h_{\phi_k}|^2 P_{\max} + \sigma_{\phi_k}^2}{|h_{\phi_k}|^2 p_k + \sigma_{\phi_k}^2} \right) + \tilde{w}_k \log_2 \left( \frac{|h_k|^2 p_k + \sigma_k^2}{\sigma_k^2} \right).
\]  
\(\text{(C.2)}\)

The derivative of \(g(p_k)\) with respect to \(p_k\) is given as
\[
g'(p_k) = \frac{1}{\ln 2} \left[ \frac{-\tilde{w}_{\phi_k} |h_{\phi_k}|^2}{|h_{\phi_k}|^2 p_k + \sigma_{\phi_k}^2} + \frac{\tilde{w}_k |h_k|^2}{|h_k|^2 p_k + \sigma_k^2} \right] = \frac{(\tilde{w}_k - \tilde{w}_{\phi_k}) |h_{\phi_k}|^2 |h_k|^2 p_k + \tilde{w}_k |h_k|^2 \sigma_k^2 - \tilde{w}_{\phi_k} |h_{\phi_k}|^2 \sigma_k^2}{\ln 2 (|h_{\phi_k}|^2 p_k + \sigma_{\phi_k}^2) (|h_k|^2 p_k + \sigma_k^2)}.  
\]  
\(\text{(C.3)}\)

From the above equation, we have \(g'(p_k) = 0\) if and only if
\[
p_k = \frac{\tilde{w}_{\phi_k} |h_{\phi_k}|^2 \sigma_k^2 - \tilde{w}_k |h_k|^2 \sigma_k^2}{(\tilde{w}_k - \tilde{w}_{\phi_k}) |h_{\phi_k}|^2 |h_k|^2}.
\]  
\(\text{(C.4)}\)

With (C.3) and (C.4), the optimal solution, \(p_k^*\), to Problem (P2') can be derived by separately considering the following three cases:

1. Suppose \(\tilde{w}_k / \tilde{w}_{\phi_k} > 1\). Then, \(g'(p_k) < 0\) if \(p_k < \tilde{p}_k\) and \(g'(p_k) \geq 0\) otherwise, and \(p_k < 0\). It implies that \(g\) is an increasing function on \([0, P_{\max}]\). Thus, \(p_k^* = P_{\max}\).
2. Suppose \(\tilde{w}_k / \tilde{w}_{\phi_k} = 1\). Then, \(g'(p_k) \geq 0\) for any \(p_k \in [0, P_{\max}]\). It implies that \(g\) is an increasing function on \([0, P_{\max}]\). Thus, \(p_k^* = P_{\max}\).
3. Suppose \(\tilde{w}_k / \tilde{w}_{\phi_k} < 1\). Then, \(g'(p_k) < 0\) if \(p_k < \tilde{p}_k\) and \(g'(p_k) \leq 0\) otherwise. It implies that \(g\) is an increasing function on \([0, \tilde{p}_k]\) and a decreasing function on \([\tilde{p}_k, P_{\max}]\). Thus, \(p_k^* = 0\) if \(\tilde{p}_k < 0\), \(p_k^* = P_{\max}\) if \(\tilde{p}_k \geq P_{\max}\), and \(p_k^* = \tilde{p}_k\) otherwise. From (C.4), we can derive
\[
\tilde{p}_k < 0 \iff \frac{\tilde{w}_k}{\tilde{w}_{\phi_k}} \leq C_1, \quad \text{(C.5)}
\]
\[
\tilde{p}_k \geq P_{\max} \iff \frac{\tilde{w}_k}{\tilde{w}_{\phi_k}} \geq C_2, \quad \text{(C.6)}
\]
where \( C_1 = \frac{|h_{k_0}|^2 \sigma_k^2}{|h_k|^2 \sigma_k^2} \) and \( C_2 = \frac{|h_{k_0}|^2 (\sigma_k^2 + P_{\text{max}} |h_k|^2)}{|h_k|^2 (\sigma_k^2 + P_{\text{max}} |h_{\phi_k}|^2)} \).

Hence, \( p_k^* \) can be recast as

\[
p_k^* = \begin{cases} 0, & \text{if } \tilde{w}_k / \tilde{w}_{\phi_k} < C_1, \\ P_{\text{max}}, & \text{if } \tilde{w}_k / \tilde{w}_{\phi_k} \geq C_2, \\ \hat{p}_k, & \text{otherwise}. \end{cases}
\]

(C.7)

From the fact that \( |h_{\phi_k}|^2 / \sigma_{\phi_k}^2 \leq |h_k|^2 / \sigma_k^2 \), we can easily derive that \( C_1 \leq C_2 \leq 1 \). Then, by combining the results of the above three cases, we can conclude that \( p_k^* = 0 \) if \( \tilde{w}_k / \tilde{w}_{\phi_k} < C_1 \), \( p_k^* = P_{\text{max}} \) if \( \tilde{w}_k / \tilde{w}_{\phi_k} \geq C_2 \), and \( p_k^* = \hat{p}_k \) otherwise. Since \( p_{\phi_k} + p_k = P_{\text{max}} \), \( p_{\phi_k} = P_{\text{max}} - p_k^* \).

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