Holographic Signatures of Resolved Cosmological Singularities

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work with A. Schäfer, J. Schliemann, F. Mele, J. Münch
[arXiv:1612.06679 , arXiv:1804.01387]

Gauge/Gravity Duality 2018, Würzburg
**Scope of work:**
- Holography in non-string QG?
- Focus: loop quantum gravity
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Content:
- Short review of recent work
- Focus: Implications of bulk singularity resolution
Outline

1. Introduction and related work
2. Strategy
3. Example: Kasner-AdS
4. Conclusion
Quantisation of classical gravity in connection variables

Diffeomorphism-invariant extension of lattice gauge theory
Loop quantum gravity

- Quantisation of classical gravity in connection variables
- Diffeomorphism-invariant extension of lattice gauge theory

Main areas of progress

- 3+0 dimensions (topological), $\Lambda = 0$
  - [Ponzano, Regge '68; Turaev, Viro '92; Rovelli '93; Freidel, Louapre '04; Barrett, Naish-Guzman '08; ...]

- State counting / surface entropy
  - [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Krasnov '97-; Engle, Noui, Perez '07-; ...]

- Symmetry reduced quantisation $\rightarrow$ quantum cosmology
  - [Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; ...]
3+0 dimensions (topological), $\Lambda = 0$

- Partition function can be evaluated exactly
- Various dual statistical models for different boundary states

[Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15; Dittrich, Goeller, Livine, Riello '17]
### LQG and Holography (other work)

#### 3+0 dimensions (topological), $\Lambda = 0$
- Partition function can be evaluated exactly
- Various dual statistical models for different boundary states
  - [Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15; Dittrich, Goeller, Livine, Riello '17]

#### State counting
- State counting à la black hole entropy for general surfaces
- Augment discrete Ryu-Takayanagi formula for tensor networks by
  - [Hayden, Nezami, Qi, Thomas, Walter, Yang '16] to geometric formula [Han, Hung '16]
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Gravitational bulk singularities at least in classical limit
Classical limit and singularities

Gravitational bulk singularities at least in classical limit

Field theory picture:

- Non-perturbative string theory defined via AdS/CFT
- Quantum gravity from field theory
  
  [Hertog, Horowitz '04, '05; Das, Michelson, Narayan, Trivedi '06; Turok, Craps, Hertog '07; Barbón, Rabinovici '11; Smolkin, Turok '12;]
Classical limit and singularities

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Gravity picture:

- Quantum gravity resolves singularities!?
- Holographic dual of (resolved) singularities
Two-point correlators in Kasner

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]

Boundary: \( ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \quad p_i \in \mathbb{R} \)

Bulk: \( ds_5^2 = \frac{1}{z^2} \left( dz^2 + ds_4^2(t) \right) \)
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Geodesic approximation: (heavy scalar operators)

\[ \langle \mathcal{O}(x)\mathcal{O}(-x) \rangle \sim \exp(-\Delta L_{\text{ren}}) \]

\( \Delta \): conformal weight of \( \mathcal{O} \)
\( L_{\text{ren}} \): renormalised geodesic length
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Main result

Geodesic passing singularity \( \leftrightarrow \) finite distance pole in 2-point correlator
Effective bouncing metric

Strategy: modify 4d part, no large curvatures from z-direction

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Effective Bouncing Metric

Strategy: modify 4d part, no large curvatures from z-direction

\[ ds_5^2 = \frac{1}{z^2} \left( dz^2 + ds_4^2(t) \right) \]

- Quantum bounce interpolates between classical solutions [Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; . . . ]
- Transitions between different Kasner solutions [Gupt, Singh '12]

\[ \propto \left(-t\right)^{-1/6} \propto t^{5/6} \]

\[ x - \text{scale} \]
Improved 2-point correlators

\[ ds_5^2 = \frac{1}{z^2} \left( dz^2 + ds_4^2(t) \right) \]

Possible simplifications

- QG scale is 4d, no Kasner transitions \(\rightarrow\) analytic solution
- QG scale is 5d, no Kasner transitions \(\rightarrow\) numeric solution

5d scale + Kasner transitions not straightforward
(Ansatz too narrow, 5d QG theory required)
**IMPROVED 2-POINT CORRELATORS**

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All calculations give qualitatively similar results
Signatures of the resolved singularity

Dual of the resolved singularity

- Finite distance bump instead of pole
- Subdominant large distance contribution

Discussion

So far: prototype calculation
Goal: find system where independent field theory computation possible
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- So far: prototype calculation
- Goal: find system where independent field theory computation possible
Analytical result (above): no Kasner transition, 4d Planck scale [NB, Schäfer, Schliemann '16]

Numerics: 5d Planck scale + Kasner transitions qualitatively similar [NB, Mele, Münch '18]
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CONCLUSION

- Holographic aspects of LQG actively investigated
  - 3d gravity
  - Tensor networks
  - Singularity resolution

Thank you for your attention!
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Long distance behaviour

- Complex geodesics:
  \[ \langle \mathcal{O}(x)\mathcal{O}(-x) \rangle \xrightarrow{x \to \infty} \infty (\mathcal{L}_{\text{bdy}})^{-\frac{2\Delta}{1-p}} \]
  \[ \neq (\mathcal{L}_{\text{bdy}})^{-2\Delta} \] due to Kasner background breaking conformal symmetry

- Real singularity-free geodesic \( p < 0 \):
  \[ \langle \mathcal{O}(x)\mathcal{O}(-x) \rangle \xrightarrow{x \to \infty} \infty \lambda^{-2p\Delta} (\mathcal{L}_{\text{bdy}})^{-2\Delta} \]
  - Subdominant to complex contribution
  - Vanishes as \( \lambda \to 0 \)
General holography from QG

AdS/CFT relies on

- Asymptotic symmetry of AdS ↔ global CFT symmetry
- Geometry of AdS near boundary ↔ UV structure of CFT
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Derive dual theory directly from QG partition function!

- Finite region QG
- Boundary state / condition ↔ dual theory

\[ \langle \cdots \rangle_{\text{Dual theory}(\phi^i_b)} := Z_{\text{QG}} \left[ \phi^i_b \right] \]

→ Euclidean 3d gravity best understood / solvable

[cf. neg. cos. constant: Castro, Gaberdiel, Hartman, Maloney, Volpato '11]
$\mathbf{3+0 \ LQG, \ \Lambda = 0}$

3-dim. gravity is topological:

$$S = \int_M e_i \wedge F^i(A), \quad \delta e_i S = F^i(A) = 0$$
$3+0$ LQG, $\Lambda = 0$

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Path integral:

$$Z(M) = \int De D\mathcal{A} e^{i \int_M e_i \wedge F_i(A)} \rightarrow \int D\mathcal{A} \delta \left( F_i(A) \right)$$
**3+0 LQG, \( \Lambda = 0 \)**

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\]

Discretize on fixed simplicial decomposition:

\[
Z_{PR}(M) = \left( \prod_{\text{links } l} \int_{\text{SU}(2)} dg_l \right) \prod_{\text{faces } f} \delta \left( \prod_{l \in f} g^\epsilon_{l,f} \right)
\]

Needs regularization: Gauge fixing / quantum group
Holography from partition functions

Dual 2d Ising model [Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15]

- Tri-valent boundary graph $\Gamma$ on 2-sphere

$$\left(Z^{\text{Ising}}(\Gamma)\right)^2 Z^{\text{LQG}}(\Gamma) = \left(\prod_{\text{edges } e} \cosh(y_e)\right)^2 2^{2\#\text{vertices}}$$

- Ising couplings $y_e \leftrightarrow$ QG coherent state parameters

Dual "twisted" 6-vertex model [Dittrich, Goeller, Livine, Riello '17]

Four-valent boundary graph $\Gamma$ on twisted 2-torus

Only spin $1/2$ rep., "fuzzy parallelograms"

Torus twist + monodromy integration in 6-vertex model:

$$Z^{\text{LQG}}(\Gamma) = Z^{\text{6 vertex twisted}}(\Gamma)$$

Intertwiners $\leftrightarrow$ vertex parameters
## Holography from Partition Functions

### Dual 2d Ising Model

- Tri-valent boundary graph $\Gamma$ on 2-sphere

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### Dual “Twisted” 6-Vertex Model

- Four-valent boundary graph $\Gamma$ on twisted 2-torus
- Only spin $1/2$ rep., “fuzzy parallelograms”
- Torus twist + monodromy integration in 6-vertex model:

\[
Z^{\text{LQG}}(\Gamma) = Z^{6 \text{ vertex}}_{\text{twisted}}(\Gamma)
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- Intertwiners $\leftrightarrow$ vertex parameters
Random tensor networks

- Approximate ground states of interacting many-body Hamiltonians
- Different types, here MERA (gapless systems) [figures from Orús, arXiv:1407.6552]

\[ S_{EE}(L) \sim \min. \ # \ crossed \ legs \]

[Swingle '09; ...; Hayden, Nezami, Qi, Thomas, Walter, Yang '16; ...]
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Compared to
- Ryu-Takayanagi formula
- Tensor network ↔ real space renormalization ↔ AdS geometry
**Random tensor networks**

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![Diagram of tensor networks](image)

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Compares to

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→ Model for discrete holography

- How to relate to continuum geometry / continuum RT-formula?
Figure 3. The spatial region with boundary and its semi-classical geometry are built by a large number of polyhedra p ... glu-
ing tetrahedrap's. It effectively connects the spin-network states from each p, and consistently produces the

Deriving RT from random tensor networks

[Hayden, Nezami, Qi, Thomas, Walter, Yang ’16]

- Average over random tensors ↔ Ising model ↔ RT-surface as domain wall
- Discrete RT formula for constant large bond dimension D:
  \[ S_{EE}(L) = \log D \times \min. \# \text{crossed legs} \]
- Missing input: \( \log D \leftrightarrow \text{geometry} \)
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  $$S_{EE}(L) = \log D \times \min \text{ \# crossed legs}$$
- Missing input: $\log D$ ↔ geometry

LQG interpretation [Han, Hung '16, figure from Han, Hung: arXiv:1610.02134]
Geometric RT from LQG

- Codim. 2 area from bond dimension $\leftrightarrow$ surface (black hole) entropy
- State counting: [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Krasnov '97-; ...]

$$D \sim \exp(A)$$

- Generic codim. 2 surfaces and dimensions [Husain '98; NB '13, '14]
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Geometric RT from LQG

- Repeat computation for generic large bond dimensions $D \sim \exp(A)$
  - $\rightarrow$ discrete Nambu-Goto path integral
  - $\rightarrow$ minimal surface [Han, Hung '16]
- Correct entanglement spectrum from Wheeler-de Witt wave function in 3d [Han, Huang '17]
Test hypothesis of singularity resolution for consistency with holography
[c.f. Engelhardt, Horowitz '16]

Work with effective bouncing metric in simple models

Independent of underlying QG approach, e.g.
- String cosmology
- Loop quantum cosmology
- Modified gravity
- ...

Compute 2-point boundary correlators in geodesic approximation
(Neglect possible corrections to geodesic equation)

Check for consistency with CFT description