An improved component modal synthesis-based nonproportional mistuning method (ICMS-NPMM) is proposed to investigate mistuned turbine blisks (MTBs) since the high-fidelity finite element models (HFEMs) involve large number of computations, which leads to low calculation efficiency. To reduce degrees of freedom and suppress the flutter of MTB, it is divided into mistuned blade structure and tuned disk structure, and the intentional mistuning is considered. Furthermore, the mistuned parameters, nonproportional mistuning, and complex loads are also considered. Firstly, the basic theory of ICMS-NPMM is investigated; secondly, the model of MTB is established via ICMS-NPMM; finally, the intentionally mistuned design of modal shape amplitudes (MSAs) is investigated via ICMS-NPMM.

Results indicate that the calculation efficiency is enhanced via ICMS-NPMM relative to that of via HFEM. In addition, the sensitivity and the flutter are decreased; meanwhile, the amplitude fluctuations of MSAs are distinctly decreased and become comparatively smooth. This investigation provides an important guidance for the vibration characteristic study of complex mechanical structures in engineering practice.

1. Introduction

The turbine blisks work in complex environment and bear complicated loads such as centrifugal force and thermal shock. Mistuning induces vibration and reduces the service life of turbine blisks. Besides, vibration causes the reduction of reliability and durability for turbine blisks. Moreover, the mistuning of turbine blisks is difficult to be identified and predicted. It is necessary to perform the design of mistuned turbine blisks (MTBs) to ensure the performance of aero-engine. The mistuned blisks have been researched such as vibration performance, reliability, and optimization. [1–3].

In general, the performance of mistuned blisks is affiliated to the frequency and output. For instance, the influence of mistuning on the blade-disk coupling was investigated [4]. The frequency, critical speeds, and Campbell diagram were studied [5]. Based on the investigation of the frequency, lots of scholars studied the response, and the forced and transient responses were conducted by establishing the sector finite element model (FEM) or lumped mass model, while these models were rarely applied to engineering [6–8].

As for the above studies, it is necessary to build the model of blisks firstly. Usually, the accuracy of blisks is related with the model. The lumped mass model was constructed to study the blisks [9], but the model is simplified severely and its precision is not high. Thus, the continuous parameter model is investigated and its precision is higher than the lumped mass model. The beams or shell was used to analyze the natural frequency or vibration response for a rotational shaft-disk-blade system or a compressor blade.
2. Improved Component Modal Synthesis and Reduced-Order Model

For the complex mechanical structure such as MTB, the computational efficiency is very low for HFEM. To deal with this problem, the ROM is adopted to investigate the modal shape amplitudes of MTB. Usually, blade stiffness mistuning which is a constant parameter was considered in classical component modal synthesis (CCMS). This method is named the CCMS-based proportional mistuning method (CCMS-PMM). However, the change of the modal and response is different proportion in practical engineering. Actually, mistuning parameters are constantly changing during the operation of blisks. This method is termed as the CCMS-based nonproportional mistuning method (CCMS-NPMM), which only considers the elastic modulus of blade as nonproportional mistuned parameter [39]. However, different mistuned blades have different modes and responses. Therefore, not only nonproportional stiffness mistuning but also nonproportional density and thickness mistuning are considered for different blades, which is named the improved component modal synthesis-based nonproportional mistuning method (ICMS-NPMM). And the modal shape amplitudes of MTB are analyzed by ICMS-NPMM.

Due to the influence of manufacturing error, material dispersion, installation error, and wear on MTB, the mistuning is inevitable. According to the working condition, the blade is taken as mistuned structure and disk is taken as tuned structure in this work.

2.1. Tuned Reduced-Order Model of Disk. For the substructure of tuned disks, the displacement is denoted as

\[
x = \begin{bmatrix} x_0 \\ x_\Pi \end{bmatrix} = \begin{bmatrix} \Phi_\Theta & \Psi_\Theta \\ \Phi_\Pi & \Psi_\Pi \end{bmatrix} \begin{bmatrix} p_\Phi \\ p_\Psi \end{bmatrix},
\]

where \( \Phi \) is the truncated set of normal modes; \( \Psi \) is the complete set of attachment modes; \( p_\Phi \) and \( p_\Psi \) are, respectively, the modal coordinates of \( \Phi \) and \( \Psi \); \( \Theta \) is the internal DOFs of disk; and \( \Pi \) is the interfacial DOFs between blades and disk.

By coordinate transformation, the mass and stiffness matrices of ROM for the disk are written as
2.2. Mistuned Reduced-Order Model of Blade. For the mistuned blade, the displacement is described as
\[ x' = I_p \psi = p'_\psi, \]
where \( I \) is the unit matrix and \( p'_\psi \) is the modal coordinates.

The mass and stiffness matrices are described as
\[
\mu' = I^T M I = M',
\]
\[
\kappa' = I^T K I = K',
\]
where \( M' \) and \( K' \) are, respectively, the mass and stiffness matrices in physical coordinates and \( \mu' \) and \( \kappa' \) are, respectively, the mass and stiffness matrices of mistuned blade ROM.

2.3. Substructure Improvement. Only the stiffness is considered as a constant mistuned parameter on the CCMS-PMM, i.e., the elastic modulus is expressed as
\[ E_n = E_0(1 + \delta_{nE}), \]
where \( E_0 \) is the nominal elastic modulus of the tuned turbine blisks; \( E_n \) is the \( n \)th blade elastic modulus of MTB; and \( \delta_{nE} \) is the mistuned parameter of the elastic modulus for the \( n \)th blade, which is a constant.

However, the \( \delta_{nE} \) is changing with the operation of MTB on the CCMS-NPMM. To cater the practical engineering, the nonproportional mistuning of blade thickness and mass is considered as well as that of the blade elastic modulus. And this methodology is termed as ICMS-NPMM.

When the mistuning of the \( i \)th blade thickness is written as
\[ \Delta d_i = d_0 \cdot A_{int} \cdot \sin \left( \frac{2\pi \cdot N_h}{N_b} (i - 1) \right), \]
\( d_0 \) is the blade thickness of disk, \( A_{int} \) is the mistuned amplitude, \( N_b \) is the number of blades, and \( N_h \) is the number of harmonics.

The \( i \)th blade thickness of MTB is defined as
\[ d_i = d_0 + \Delta d_i, \]
The mistuning of the \( i \)th blade stiffness is
\[ \Delta E_i = E_0 \cdot A_{int} \cdot \sin \left( \frac{2\pi \cdot N_h}{N_b} (i - 1) \right). \]
The \( i \)th blade elasticity modulus of MTB is defined as
\[ E_i = E_0 + \Delta E_i. \]

When the mistuning of the \( i \)th blade density is
\[ \Delta \rho_i = \rho_0 \cdot A_{int} \sin \left( \frac{2\pi \cdot N_h}{N_b} (i - 1) \right), \]
\( \rho_0 \) is the blade density of tuned blisks.

The \( i \)th blade density of MTB is defined as
\[ \rho_i = \rho_0 + \Delta \rho_i. \]

Three mistuned parameters are considered, which caters the real engineering.

2.4. Substructure Synthesis. To combine the disk and blades ROM, the coordinated condition is given, which is expressed as
\[ x_{11} = x', \]
\[ f_{11} = f \cdot I. \]

According to equations (1)–(3) and equation (12), the coordinated condition is rewritten as
\[ x_{11} = \Phi_1 p_\psi + \Psi_{11} p_{\psi} = p_\psi = x'. \]

The mass and stiffness matrices of MTB and their modal coordinates are expressed as
\[
\mu^{\text{syn}} = \mu + \begin{bmatrix} I^T \Phi_1^T M \Phi_1 & \Phi_1^T M' \Psi \\ \Psi_{11}^T M \Phi & \Psi_{11}^T M' \Psi \end{bmatrix},
\]
\[
\kappa^{\text{syn}} = \kappa + \begin{bmatrix} I + \Phi_1^T M \Phi_1 & \Phi_1^T M \Psi + \Phi_1^T M' \Psi \\ \Psi_{11}^T M \Phi + \Psi_{11}^T M \Psi & \Psi_{11}^T M' \Psi + \Psi_{11}^T M' \Psi \end{bmatrix},
\]
where \( p^{\text{syn}} = \begin{bmatrix} p_\psi \\ p_{\psi} \end{bmatrix} \)

The intrinsic frequencies of tuned blisks are still dense after a small mistuning (0%–5%), and the mistuned intrinsic frequencies can be described by a tuned intrinsic subset.
Thus, the $\Psi$ and $p_\phi$ in equations (14) and (15) can be ignored for the above ROM, and the ROM of MTB is rewritten as

$$\mu^{\text{sys}} = \begin{bmatrix} I + \Phi_{\Pi}^T M \Phi_{\Pi} \end{bmatrix},$$  \hfill (17)

$$\kappa^{\text{sys}} = \begin{bmatrix} \Lambda + \Phi_{\Pi}^T K \Phi_{\Pi} \end{bmatrix},$$  \hfill (18)

where $\Phi_{\Pi} = \mathbf{B} \text{diag}_{n=1,\ldots,N} [M_\iota^n]$; $K_\iota = \mathbf{B} \text{diag}_{n=1,\ldots,N} [K_\iota^n]$; $\mathbf{B} \text{diag} [\cdot]$ is block diagonal matrix; and $N$ is the number of blades.

If only the normal modal is used to describe blade vibration, the boundary displacement between blades and disk cannot be obtained; therefore, the additional boundary modal must be considered. Assume that the additional boundary modal is defined as

$$\begin{bmatrix} \Psi_{\text{CB}}^\iota \\ 1 \end{bmatrix},$$  \hfill (20)

where $\Psi_{\text{CB}}^\iota$ is the internal DOFs of blades.

The relationship of mass matrix $M_{\text{CB}}$ and stiffness matrix $K_{\text{CB}}$ are, respectively, described as

$$J_m(\Psi_{\text{CB}}^\iota) = \begin{bmatrix} \Psi_{\text{CB}}^\iota \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{\text{CB}}^\iota_m \\ M_{\text{CB}}^\iota_b \end{bmatrix} \begin{bmatrix} \Psi_{\text{CB}}^\iota \\ 1 \end{bmatrix},$$  \hfill (21)

$$J_k(\Psi_{\text{CB}}^\iota) = \begin{bmatrix} \Psi_{\text{CB}}^\iota \\ 1 \end{bmatrix}^T \begin{bmatrix} K_{\text{CB}}^\iota_m \\ K_{\text{CB}}^\iota_b \end{bmatrix} \begin{bmatrix} \Psi_{\text{CB}}^\iota \\ 1 \end{bmatrix},$$  \hfill (22)

where $i$ and $b$ are, respectively, the internal and boundary DOFs of blades.

The relationship of the minimum boundary modal $\Psi_{\text{CB},m}^\iota$ and $\Psi_{\text{CB},b}^\iota$ is, respectively, obtained via transforming of $M_{\text{CB}}$ and $K_{\text{CB}}$ based on the first-order differential of $J_m$ and $J_k$, which are expressed as

$$M_{\text{CB}}^\iota \Psi_{\text{CB},m}^\iota + M_{\text{CB}}^\iota_0 = 0,$$

$$K_{\text{CB}}^\iota \Psi_{\text{CB},m}^\iota + K_{\text{CB}}^\iota_0 = 0.$$  \hfill (23)

In fact, the boundary modal is the smallest which contributes kinetic and potential energy to the blades. The relationship of normal modal and boundary modal corresponding to the mass and stiffness mistuning is, respectively, expressed as

$$\Phi_{\Pi,m}^\iota = \begin{bmatrix} \Phi_{\text{CB}}^\iota \\ \Phi_{\text{CB},m}^\iota \end{bmatrix} \begin{bmatrix} q_{\text{CB},m}^\iota \\ q_{\text{CB},r}^\iota \end{bmatrix},$$

$$\Phi_{\Pi,b}^\iota = \begin{bmatrix} \Phi_{\text{CB}}^\iota \\ \Phi_{\text{CB},b}^\iota \end{bmatrix} \begin{bmatrix} q_{\text{CB},b}^\iota \\ q_{\text{CB},r}^\iota \end{bmatrix},$$

where $q_{\text{CB},m}^\iota$, $q_{\text{CB},r}^\iota$, and $q_{\text{CB},b}^\iota$ are, respectively, the $n$th bladed normal and boundary modes.

Therefore, the relationship of all blades can be expressed as

$$\Phi_{\Pi,m}^n = (I \otimes \mathbf{U}_{\text{CB},m}^n) q_{\text{CB},m}^n,$$  \hfill (24)

$$\Phi_{\Pi,b}^n = (I \otimes \mathbf{U}_{\text{CB},b}^n) q_{\text{CB},b}^n,$$  \hfill (25)

where

$$\mathbf{U}_{\text{CB},m}^n = \begin{bmatrix} \Phi_{\text{CB}}^n \\ \Psi_{\text{CB},m}^n \end{bmatrix},$$

$$\mathbf{U}_{\text{CB},b}^n = \begin{bmatrix} \Phi_{\text{CB}}^n \\ \Psi_{\text{CB},b}^n \end{bmatrix},$$

$$q_{\text{CB},m}^n = \begin{bmatrix} q_{\text{CB},m}^n \\ q_{\text{CB},r}^n \end{bmatrix},$$

$$q_{\text{CB},b}^n = \begin{bmatrix} q_{\text{CB},b}^n \\ q_{\text{CB},r}^n \end{bmatrix},$$

where $\otimes$ is Kronecker product.

Equations (24) and (25) are substituted into equations (17) and (18), and the mass and stiffness matrices of ROM are, respectively, rewritten as

$$\mu^{\text{sys}} = I + (q_{\text{CB},m}^n)^T (I \otimes (\mathbf{U}_{\text{CB},m}^n)^T) M_{\text{CB}} (I \otimes (\mathbf{U}_{\text{CB},m}^n)^T) q_{\text{CB},m}^n,$$

$$\kappa^{\text{sys}} = I + \text{Bdiag}_{n=1,\ldots,N} [M_{\text{CB}}^n]^T M_{\text{CB}}^n (I \otimes (\mathbf{U}_{\text{CB},m}^n)^T) q_{\text{CB},m}^n,$$

$$\mu_{\Psi_{\Phi},r}^{\text{sys}} = (\Phi_{\text{CB}}^r, r)^T M_{\Phi_{\Psi},r} (\Phi_{\text{CB}}^r, r) + (\Phi_{\text{CB}}^r, r)^T M_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n + M_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n,$$

$$\mu_{\Psi_{\Phi},r}^{\text{sys}} = (\Psi_{\text{CB}}^r, r)^T M_{\Phi_{\Psi},r} (\Phi_{\text{CB}}^r, r) + (\Phi_{\text{CB}}^r, r)^T M_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n + M_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n,$$

$$\kappa_{\Psi_{\Phi},r}^{n} = (\Phi_{\text{CB}}^r, r)^T K_{\Phi_{\Psi},r} (\Phi_{\text{CB}}^r, r) + (\Phi_{\text{CB}}^r, r)^T K_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n + K_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n,$$

$$\kappa_{\Psi_{\Phi},r}^{n} = (\Psi_{\text{CB}}^r, r)^T K_{\Phi_{\Psi},r} (\Phi_{\text{CB}}^r, r) + (\Phi_{\text{CB}}^r, r)^T K_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n + K_{\Phi_{\Phi},r} \Psi_{\text{CB},m}^n.$$  \hfill (29)
Equations (27) and (28) can be used in various mistuned blisks, but mistuned value associated with boundary modal needs to be known; however, it is almost impossible to achieve. Therefore, assuming that the boundary displacement of the normal modal is smaller, or there is no mistuning of boundary unit, i.e., the kinetic and potential energy of boundary displacement is ignored. In this case, the mass and stiffness matrices of ROM for MTB can be expressed as

\[
\mu_{\text{syn}} \approx I + B \text{diag} \left( \begin{bmatrix} q_{\text{CB},m}^T \end{bmatrix} \mu_{\text{pp},r} q_{\text{CB},m} \right),
\]

\[
\kappa_{\text{syn}} \approx \Lambda + B \text{diag} \left( \begin{bmatrix} q_{\text{CB},m}^T \end{bmatrix} \kappa_{\text{pp},r} q_{\text{CB},m} \right).
\]

This ROM only has blade normal modal, and the kinetic and potential energy of boundary displacement is ignored. This model can be used to analyze the maximum modal shape amplitudes of MTB.

### 3. Modeling

MTB modeling is the foundation for investigating the DAs, SAs, and SEAs. Thus, the model parameters need to be given at first, and the main parameters of MTB are shown in Table 1. The variations of blade stiffness, density, and thickness are shown in Figures 1–3.

The interfaces between blades and disk as well as the interfaces between disk hole and disk are the key joints. Therefore, only the interface DOFs are calculated, which is a huge saving in terms of computational cost. The master DOFs of the MTB are shown in Figure 4.

Only the disk is regarded as superelement [40]; however, the reduced-order FEM of MTB is established via ICMS-NPMM in Figure 5.

To verify the DOFs are reduced by ISCMS, the elements are calculated as shown in Table 2.

It can be seen from Table 2 and Figure 5 that compared with classical SCMS and HFEM, ratios of decline are, respectively, 84.22% and 96.11%; thus, the DOFs of MTB are distinctly reduced so that the computational time is obviously shorter relative to CCMS and HFEM.

### 4. Maximum Modal Shape Amplitude Analysis

The MTB works in severe environment, and it is very sensitive to mistuning which can trigger vibration localization even high fatigue stress and fracture. Therefore, it is very necessary to investigate the modal shape under centrifugal force and thermal load. The difference from previous studies (localization factor as research object [40]) is that the modal shape amplitudes are regarded as the research object so as to make sure the variation tendency of most dangerous position and reasonably prevent MTB from destroying by mistuning. This method is simple and has high computational efficiency. To comprehensively study vibration characteristics of MTB, the DAs, SAs, and SEAs are investigated, respectively.

#### 4.1. Verification

The investigation indicates that the one vibration period of blisks is about 23-24-order frequencies, and the blisks are mainly affected by the low-order modes. Meanwhile, in order to improve the calculation efficiency, the first two vibration period modes are studied. Thus, the first 38 intrinsic frequencies of MTB with \( w = 1120 \text{ rad/s}, t = 1100^\circ \text{C} \) are discussed by ICMS-NPMM in Figure 6 to verify the effectiveness and rationality of ICMS-NPMM. The first 10, 20, 30, and 38 intrinsic frequencies are, respectively,
calculated by ICMS-NPMM, CCMS-NPMM, and HFEM in Figure 7 and Table 3.

Figure 6 indicates that the errors of inherent frequencies for MTB are 0.004%∼0.102% and 0.006%∼0.159% by ICMS-NPMM and CCMS-NPMM relative to that of HFEM. The inherent frequencies acquired by ICMS-NPMM and CCMS-NPMM are practical unanimity with that of HFEM. Thus, the accuracy of SCMS-NPMM and CCMS-NPMM is comparable to that of HFEM. Meanwhile, the computational time and saving ratio are given in Table 3 and Figure 7. The computational time of MTB is, respectively, reduced by 7.51%∼29.13% and 10.48%∼49.15% with CCMS-NPMM and ICMS-NPMM relative to HFEM. It is observed that the larger of the orders, the higher of the time saving ratio. Thus, the calculative time of ICMS-NPMM is lower than that of CCMS-NPMM, which indicates that the ICMS-NPMM is superior to CCMS-NPMM. Also, it can be seen from Table 3 that the time-saving rate is higher with the rise of the frequency order and the main reason is that the energy of the blade disk cannot be uniformly transmitted out due to the existence of mistuning. The accumulated energy increases with the improvement of order so as to the calculation time becomes slower and slower for the HFEM. However, ICMS-NPMM proposed solved the problem.

4.2. Modal Shape Amplitudes. The MTB is subjected to common action of centrifugal force and thermal load; therefore, it is necessary to investigate the intrinsic shape amplitudes. To study the common effect of centrifugal force and thermal load on the modal shape amplitudes, the DAs, SAs, and SEAs of the tuned and mistuned blisks are researched under interaction of centrifugal force and thermal load. The first 38 order DAs, SAs, and SEAs are respectively, calculated using ICMS-NPMM, as shown in Figures 8(a), 9(a), and 10(a); the deviations of DAs, SAs, and SEAs are shown in Figures 8(b), 9(b), and 10(b); the increments of DAs, SAs, and SEAs and their comparison are shown in Figures 8(c), 9(c), and 10(c). The explanation of Figures 8–10 is shown in Table 4.

As seen in Figures 8–10, the variation tendencies of DAs, SAs, and SEAs under the combined action with speeds and temperatures are analogical to those of only speeds or temperatures considered; however, there are some differences.

Figure 8 indicates that the DAs are closer to those of only speeds considered with the combined action of speeds and temperatures, and the research indicates that DAs are more effected by speed than temperature. Besides, the change trend of DAs is analogical to those of only speeds or temperatures taken into account and corresponding values are reduced. Also, the mistuning is more effected than the speed when the speed and temperature simultaneously act in MTB.

Figure 9 manifests that the instability of SAs is observed, the danger rises obviously, and the destroy of MTB increases from the 24th order when the temperatures and speeds are both considered. The main reason is the fluctuation obviously increases from the 24th order model. In addition, the variable quantities of SAs arise in low-order and high-order frequencies when the temperature is changeless and the speed raises, or the speed is changeless and temperature rises, which intensifies destruction of MTB.

Figure 10 shows that the SEAs are sensitive to the temperature in high-order frequency and the MTB is more likely failure from the 24th order when the temperature enhances to 1100°C. In addition, the deviations of SEAs increase sharply in the first vibration period. Distinctly, the destructive of MTB increases when the rotational speed is invariant and temperature is enhanced to a certain degree. Meanwhile, the SEAs in the 9th order frequency occur larger fluctuation when the temperature is changeless but speed is varied. Therefore, the SEAs are impacted by speed in low-order frequency. This investigation indicates that the effect of temperature on SEAs in high-order frequency is larger. The increment of SEAs is a bit more complicated, and the fluctuation is observed in low-order frequency.
Figure 4: Master DOFs of the MTB. (a) Interfaces of master DOFs. (b) Master DOFs.

Figure 5: Reduced-order FEM of MTB. (a) Disk cohesion and blade substructure. (b) Blade cohesion and disk substructure.

Table 2: The elements of MTB.

| Element nos. | HFEM | Classical SCMS | Ratio of decline (%) | ISCMS | Ratio of decline (%) |
|--------------|------|----------------|----------------------|-------|----------------------|
| 1102519      |      | 173911         | 84.22                | 42998 | 96.11                |

Figure 6: Intrinsic frequency analysis.

Figure 7: Computational time of different order frequencies.
Table 3: Computational time and time saving ratio of three methods.

| First n orders | HFEM $t$ (s) | CCMS-NPMM $t$ (s) | CCMS-NPMM $\gamma$ (%) | ICMS-NPMM $t$ (s) | ICMS-NPMM $\gamma$ (%) |
|----------------|--------------|-------------------|-------------------------|------------------|-------------------------|
| 10             | 11.45        | 10.59             | 7.51                    | 10.25            | 10.48                   |
| 20             | 18.67        | 15.89             | 14.89                   | 14.47            | 22.49                   |
| 30             | 28.79        | 21.75             | 24.45                   | 18.39            | 36.12                   |
| 38             | 56.78        | 40.24             | 29.13                   | 28.87            | 49.15                   |

Note: $t$ is the computational time of different order frequencies; $\gamma$ is the time saving ratio for CCMS-NPMM and ICMS-NPMM relative to HFEM.

Figure 8: Displacement amplitudes with centrifugal force and thermal load. (a) Das. (b) Deviation of Das. (c) Variable quantity of DAs.
4.3. Intentionally Mistuned Design. Section 4.2 manifests the localization of the modal shape amplitude caused by mistuning remarkably increases and exacerbates MTB compared with the TTB. This can induce flutter which can cause catastrophic damage to MTB, especially, to blades. In fact, the aeroengine will fluctuate or vibrate seriously due to the disturbance of air flow during the flight of aircraft, and the installation error or wear will result passive mistuning, which will induce cracks failure or fatigue fracture for blades. The intentionally mistuned design can reduce passive mistuning. To suppress the flutter, the intentionally mistuned design is investigated for MTB, this design can improve the safety and prolong its service life.

To decline the flutter of mistuned blisks, it is vital to find the most influential factors on the structural characteristics and the best combination. The mainly influence factors are shown in Figure 11 by sensitivity analysis [41]. However, the severe mistuning can cause intensive vibration of blades. The experimental study indicates that the density and elastic modulus of each three blades are the same, which can reduce

Figure 9: Stress amplitudes with centrifugal force and thermal load. (a) SAs. (b) Deviation of SAs. (c) Variable quantity of SAs.
the passive mistuning. Thus, the elastic modulus and density of blades can be reasonably designed to improve the robustness and reduce the flutter of MTB as well as decline its fluctuation, which are shown in Figure 12.

Then, the DAs, SAs, and SEAs of intentional MTB are investigated via ICMS-NPMM, and the results are drawn in Figures 13～15. Figures 13(a)～15(a) manifest that the fluctuations of DAs, SAs, and SEAs become smooth and steady in the first vibration period and distinctly decline in the second vibration period for intentionally MTB compared with passive MTB, which means the stability and safety enhances of blisks; meanwhile, it is observed that their variation tendencies of intentional mistuning are similar to those of tuning and passive mistuning, but the values are less than those of passive mistuning, which are significantly reduced. Figures 13(b)～15(b) indicate that the DAs, SAs, and SEAs are greatly affected by rotational speed in low-order
### Table 4: The explanation of Figures 8–10.

| Figures 8(a)/9(a)/10(a) | Figures 8(b)/9(b)/10(b) | Figures 8(c)/9(c)/10(c) |
|--------------------------|--------------------------|--------------------------|
| Tuned blisks             | Mistuned blisks          | Deviations of mistuned blisks relative to tuned blisks |
| y1: \( w = 1120 \text{ rad.s}^{-1} \) | y4: \( w = 1120 \text{ rad.s}^{-1} \) | y1: \( w = 1120 \text{ rad.s}^{-1}, t = 780°C \) relative to \( w = 1120 \text{ rad.s}^{-1} \) |
| y2: \( t = 780°C \)     | y5: \( t = 780°C \)     | y2: \( w = 1250 \text{ rad.s}^{-1}, t = 780°C \) relative to \( w = 1120 \text{ rad.s}^{-1} \) |
| y3: \( w = 1120 \text{ rad.s}^{-1}, t = 780°C \) | y6: \( w = 1120 \text{ rad.s}^{-1}, t = 780°C \) | y3: \( t = 780°C \) |
| y7: \( t = 1100°C \)    | y9: \( t = 1100°C \)    | y4: \( t = 1100°C \) relative to \( w = 1120 \text{ rad.s}^{-1}, t = 780°C \) |
| y8: \( w = 1120 \text{ rad.s}^{-1}, t = 1100°C \) | y10: \( w = 1120 \text{ rad.s}^{-1}, t = 1100°C \) | y5: \( t = 1100°C \) relative to \( t = 780°C \) |
| y11: \( w = 1250 \text{ rad.s}^{-1}, t = 1100°C \) | y13: \( w = 1250 \text{ rad.s}^{-1}, t = 1100°C \) | y6: \( w = 1120 \text{ rad.s}^{-1}, t = 780°C \) relative to \( t = 780°C \) |
| y12: \( w = 1250 \text{ rad.s}^{-1}, t = 780°C \) & y14: \( w = 1250 \text{ rad.s}^{-1}, t = 780°C \) | y7: \( w = 1250 \text{ rad.s}^{-1}, t = 1100°C \) | y7: \( t = 1120 \text{ rad.s}^{-1}, t = 780°C \) relative to \( t = 780°C \) |

![Figures 8(a)/9(a)/10(a)](image1)

![Figures 8(b)/9(b)/10(b)](image2)

![Figures 8(c)/9(c)/10(c)](image3)

**Figure 11:** Sensitivity of the modal shape amplitudes for MTB. (a) Sensitivity of DAs. (b) Sensitivity of SAs. (c) Sensitivity of SEAs.

![Blade nos. vs Density vs Elasticity modulus](image4)

**Figure 12:** Intentionally mistuned design. (a) Density of blades. (b) Elastic modulus of blades.
frequency while by temperature in high-order frequency. Although the values significantly increase, they are stable dominated by blade vibration, which means the reliability of bladed disk enhances. Figures 13(c) ∼ 15(c) show that deviations of DAs, SAs, and SEAs in intentional mistuning relative to those of in passive mistuning in centrifugal force and thermal load are closer to those of only centrifugal force considered in the first vibration period and closer to those of only thermal load considered in the second vibration period. Figures 13(d) ∼ 15(d) reveal that the increments of DAs SAs and SEAs in intentional mistuning relative to in passive mistuning have changed much in the second vibration period than in the first vibration period, the main reason is that the change in passive mistuning is larger than in intentional mistuning.

It is seen from Figures 13–15 that the fluctuation of modal shape amplitudes is reduced and becomes smooth and steady of intentionally MTB; that is, the response is less sensitive to the variables and the robustness is improved.
Figure 14: Stress amplitudes of intentionally MTB. (a) SAs of tuning, passive, and intentional mistuning. (b) SAs of intentional mistuning. (c) Deviation of SAs. (d) Increment of SAs.
5. Conclusions

An improved component modal synthesis-based non-proportional mistuning method (ICMS-NPMM) is proposed, which considers elasticity modulus, thickness, and density of blades, as well as centrifugal force and thermal load. Based on this theory, the reduced-order FEM of turbine blisks is established and its DOFs are obviously reduced. The investigation can obtain the following conclusions:

1. The intentional mistuning is adopted, and MTB is decomposed into mistuned blade and tuned disk. Meanwhile, the substructures are improved and synthesized. The reduced-order FEM is established by ICMS-NPMM, which can be used to analyze small or large mistuning of turbine blisks.

2. The DAs, SAs, and SEAs of first 38 order frequencies of the passive MTB are investigated using ICMS-NPMM and analyzed the influence of mistuning, centrifugal force, and thermal load on them. This
Data Availability

The data used to support the findings of this study are currently under embargo while the research findings are commercialized. Requests for data, (6/12 months) after publication of this article, will be considered by the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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