Electron-positron annihilation into Dirac magnetic monopole and antimonopole: the string ambiguity and the discrete symmetries

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Abstract

We address the problem of string arbitrariness in the quantum field theory of Dirac magnetic monopoles. Different prescriptions are shown to yield different physical results. The constraints due to the discrete symmetries (C and P) are derived for the process of electron-positron annihilation into the monopole-antimonopole pair. In the case of the annihilation through the one-photon channel, the production of spin 0 monopoles is absolutely forbidden; spin 1/2 monopole and antimonopole should have the same helicities (or, equivalently, the monopole-antimonopole state should be p-wave \(^{1}P_{1}\)).
1. The experimental searches for Dirac magnetic monopoles produced (as real or virtual particles) in $e^+e^-$ annihilation have been conducted for several decades. For reviews on Dirac monopoles [1] see, e.g., [2–4]; recently, there have been discussions of the effects of virtual monopoles on the anomalous magnetic moment of the electron [3] and in Z-boson decay [4].

If we are to derive an upper bound on the monopole mass based on the negative results of these searches, we have to be able to calculate theoretically the amplitude of the monopole-antimonopole production in $e^+e^-$ annihilation for a given monopole mass. However, this calculation presents serious problems.

There are two sources of difficulties. The first is a well-known fact that the coupling constant (that is, the magnetic charge of the pole) should be very large if the Dirac quantization condition is to be true. That makes impossible the use of perturbation theory for practical calculations (although it can be used within an effective field theory approach). The second difficulty (which has not been as much popularized) is, perhaps, more fundamental. It has nothing to do with the magnitude of the coupling constant at all; rather, it is related to the existence of Dirac string – infinitely thin line of magnetic flux stretching from the magnetic pole to infinity. It has been often repeated in the literature that the Dirac quantization condition makes the string invisible. However, in reality the situation is far from being so simple and clear. This is especially true in the context of quantum field theory where monopoles are allowed to be created and annihilated (recall that the Dirac quantization condition was initially derived for a simple quantum mechanical system “electron plus monopole”). It is generally believed that the full quantum field theory does not depend on how we choose the position of the string which can be arbitrary. However, the peculiarity of the monopole theory is that the formulation of the theory cannot be made without recourse to the string concept in one or another form. In other words, the quantum theory of monopoles is not manifestly string-independent. Since the string fixes a specific direction, the theory is not manifestly Lorentz invariant either. Perhaps, it is a unique example of a physical theory possessing implicit Lorentz invariance which nevertheless cannot be formulated in a manifestly invariant way.
What are the practical implications of this fundamental theoretical feature? One consequence is this. Imagine that we forget for a moment about the large coupling constant and attempt to calculate some physical quantity in the first order of perturbation theory (such as the monopole-antimonopole production in $e^+e^-$ annihilation). The result will be discouraging because it will be ambiguous. More exactly, the result will depend explicitly on the string direction which is clearly unacceptable. Obviously, a serious question is how to deal with this type of situation. Consider, for example, the process of $e^+e^-$ annihilation into monopole-antimonopole pair (assumed to be fermions). It has a virtue of being physically interesting and simple enough at the same time. This process has been previously considered and a prescription has been given for elimination of string dependence which has been subsequently adopted. The resulting cross-section is not very different from the cross-section for the creation of a pair of usual fermion-antifermion.

However, we believe that the prescription is not entirely satisfactory. One reason for concern is that it only gives the value of the $squared modulus of the amplitude$, but not the $amplitude$ itself. Therefore, it would be difficult to generalize it for the cases when an $interference$ of two amplitudes is involved (for instance, if we want to calculate the interference between electromagnetic and $Z$-boson contributions to the monopole-antimonopole production in $e^+e^-$ annihilation).

One purpose of this paper is to consider an alternative procedure and see if the physical results would be the same. More specifically, we propose an alternative prescription based on the averaging of the amplitude over all possible directions of the string. This procedure has a clear physical meaning since the string is supposed to be unobservable. However, it leads to a drastically different answer: according to this prescription, the amplitude of $e^+e^-$ annihilation into the monopole-antimonopole pair should be zero to the lowest order of perturbation theory.

This result suggests that the task of extracting the physically meaningful results from the inherently ambiguous perturbative calculations should be considered as an open problem requiring further investigation. In this paper we try to circumvent this problem by using
only general principles of quantum field theory whose validity does not rely on the use of perturbation theory. It is natural to start with the consideration of the role of the discrete symmetries such as C and P transformations and to see what constraints are provided by these symmetries.

We show that the behaviour of the monopole-antimonopole system under discrete symmetries is rather different from that of standard fermion-antifermion or boson-antiboson system (standard means not carrying magnetic charge). In particular, there arise selection rules for the process of the monopole-antimonopole production through one-photon annihilation of an electron and positron. For spin 1/2 monopole the P and C symmetries require that the monopole and antimonopole have the same helicities. For spinless monopoles CP symmetry absolutely forbids the monopole-antimonopole production through the one-photon annihilation of an electron and positron.

2. The Feynman rules describing the interactions of photons and monopoles have the following form (Fig. 1):

\[ -ig\epsilon^{\mu\nu\lambda\rho}\gamma_{\nu}\gamma_{\lambda}q^{\rho} + i\epsilon. \]  

The photon and fermion propagators, as well as the photon-electron vertex, remain the same as in the standard QED. Note that in other formulations of the monopole quantum field theory the Feynman rules would be different (for details, see [4]). The most notable feature of these Feynman rules is the fact that they depend on the vector \( n \) which corresponds to the direction of the string. Thus, these Feynman rules are not manifestly invariant. However, it is believed that the full theory is nevertheless Lorentz-invariant, that is physical predictions should not depend on the specific direction of the vector \( n \).

Now, let us write down the amplitude of the process of the electron-positron annihilation into the monopole-antimonopole pair. The amplitude has the following form (Fig. 2):

\[ A = iegK^n\epsilon_{\mu\nu\lambda\beta}\gamma^{\mu}q_{n}^{\lambda}q_{n}^{\beta} \frac{1}{q^2} J^\mu. \]  

where
\[ J^\mu = \bar{v}_e(p_2)\gamma^\mu u_e(p_1), \quad K^\beta = \bar{u}_g(p_3)\gamma^\beta v_g(p_4). \]  

The dependence on \( n \) remains even after the squaring of the amplitude is made. An obvious question is how to make sense out of the \( n \)-dependent quantity. It has been suggested in Ref. 7 that one should drop the terms which have no pole in \( q^2 \) and thus to arrive at the following result:

\[ |A|^2 = \frac{e^2 g^2}{q^4} [(KJ)(JK^\dagger) - (JJ^\dagger)(KK^\dagger)]. \]  

However, the consistency of such a prescription can be questioned on the grounds that it gives the corrected value of the squared matrix element but not of the amplitude itself. Therefore, it would be difficult to generalize it for the cases when an interference of two amplitudes is involved (for instance, if we want to calculate the interference between electromagnetic and Z-boson contributions to the monopole-antimonopole production in \( e^+e^- \) annihilation). Another concern is whether Eq. (4) is positively definite or not. There exists a different approach to the problem of dealing with the \( n \) dependence. The idea is to average over all possible directions of \( n \). Since there are no physically preferred directions of \( n \), all the directions should be taken with the same weight. Because all these directions are physically indistinguishable, we have to perform averaging of the amplitude rather than of the squared matrix element. Therefore, we need to find the average value:

\[ \langle \frac{n}{qn} \gamma \rangle. \]  

By Lorentz invariance, it is sufficient to find this average value in a system where \( n^0 = 0 \) and, consequently, \( n^2 = 1 \):

\[ \langle \frac{n}{qn} \rangle = \frac{1}{4\pi} \int \frac{n}{qn} d\Omega. \]  

In evaluating this integral one should be careful about a possible singularity arising when the vector \( n \) becomes orthogonal to \( q \). Let us choose the \( z \) axis of the spherical coordinate system such as to be parallel to \( q \), and calculate the \( x, y, z \) components of the average:
\[
\frac{1}{4\pi} \int \frac{n_x}{-qn} d\Omega = -\frac{1}{4\pi|q|} \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt \int_0^{2\pi} \cos \phi d\phi,
\]

where \( t = \cos \theta \). Although the integral over \( \phi \) vanishes, we need to prove that the integral over \( t \) is not singular. For this purpose we have to invoke the \( qn+i\epsilon \) rule (or, in 3-dimensional terms, the \( qn-i\epsilon \) rule):

\[
\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt \rightarrow \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-i\epsilon} dt = \varphi \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt + i\pi \int_{-1}^{1} \delta(t) \sqrt{1-t^2} dt = i\pi
\]

Thus, indeed, the \( t \)-integral is finite and, therefore,

\[
\frac{1}{4\pi} \int \frac{n_x}{-qn} d\Omega = 0.
\]

Furthermore, a similar argument shows that the \( y \)-component of the average value also vanishes:

\[
\frac{1}{4\pi} \int \frac{n_y}{-qn} d\Omega = 0.
\]

Now, the \( z \)-component is:

\[
\frac{1}{4\pi} \int \frac{n_z}{-qn} d\Omega = -\frac{1}{4\pi |q|} \int d\Omega = -\frac{1}{|q|}.
\]

Thus, finally, we obtain:

\[
\frac{1}{4\pi} \int \frac{n}{-qn} d\Omega = -\frac{q}{q^2}.
\]

Consequently,

\[
\langle \frac{n^\gamma}{qn} \rangle = \frac{q^\gamma}{q^2}.
\]

Now, if we insert this value into the \( e^+e^- \) annihilation amplitude, we obtain

\[
A = iegK^\beta \epsilon_{\mu\beta\gamma\delta} \frac{q^\gamma q^\delta}{q^4} J^\mu = 0.
\]

Thus, we arrive to the conclusion: \textit{if one uses the averaging procedure to eliminate the string dependence of the amplitude, than the one-photon amplitude of the \( e^+e^- \) annihilation...
into monopole-antimonopole pair turns out to be zero. To summarize, we have shown that two different prescriptions used to eliminate the string dependence of the amplitude lead to drastically different physical results. Therefore, we suggest to try to circumvent this problem by using only general principles of quantum field theory whose validity does not rely on the use of perturbation theory. It is natural to start with the consideration of the role of the discrete symmetries such as C, P and T transformations and to see what constraints are provided by these symmetries.

4. There exist several formulations of the quantum field theory with electric and magnetic charges. However, all of the formulations have been shown \cite{4} to be equivalent (except the formulation due to Cabibbo and Ferrari). Therefore, we will not need to specify exactly in which theoretical context are going to work. Rather, we will focus on the properties of the quantum field theory under the action of the discrete symmetries such as space reflection and charge conjugation. It can be shown \cite{8–10} that the quantum field theory of the electric and magnetic charges is invariant under the following discrete transformations (we use Majorana representation, and denote the magnetically charged fields by the subscript $g$):

\begin{align*}
C & : \mathbf{E}, \mathbf{H}, \psi, \psi_g \rightarrow -\mathbf{E}, -\mathbf{H}, \psi^\dagger, \psi_{g}^\dagger. \\
P & : \mathbf{E}(x), \mathbf{H}(x), \psi(x), \psi_g(x) \rightarrow -\mathbf{E}(-x), -\mathbf{H}(-x), \gamma^0\psi(-x), \gamma^0\psi_g^\dagger(-x) \\
T & : \mathbf{E}(t), \mathbf{H}(t), \psi(t), \psi_g(t) \rightarrow \mathbf{E}(-t), -\mathbf{H}(-t), \gamma^0\gamma^5\psi(-t), \gamma^0\gamma^5\psi_g^\dagger(-t).
\end{align*}

Here, a comment on terminology is in order. There is some confusion in the literature as to whether we should retain the names “P reflection” and “T inversion” for the above operations or we should call them “PM” and “TM” transformations, where M stands for the inversion of the magnetic charge. However, this difference is of semantical rather than of physical character; switching from one terminology to the other does not entail any physical consequences. In this paper we adopt the the first point of view, i.e. we keep the names parity and T inversion for the operations we have just introduced without making any further qualifications (the same view is adopted in \cite{3}).
Sometimes one can find in the literature the statements to the effect that the theory of monopoles is not invariant under P and T symmetries. These statements refer to the situation when the discrete symmetries are assumed to act on the magnetically charged particles in exactly the same way as they act on the electrically charged particles, that is their action on the magnetically charged states does not include the sign inversion of the magnetic charge. It is easy to see that if the discrete transformations are defined in that way, then the theory is indeed P and T non-invariant. However, the possibility to define P and T symmetries in such a way that they are conserved makes the “non-conserving” definition irrelevant.

Note also that we assume that there are no particles carrying simultaneously both the electric and magnetic charge; in other words, there are no dyons in the theory; in this case the conserving P and T operations do not exist [10].

Now, in terms of creation (or annihilation) operators (rather than in terms of local fields) the transformation law for a magnetically charged fermion takes the form:

\[ Pa_g(p, s)P^{-1} = b_g(-p, s), \quad Pb^\dagger_g(p, s)P^{-1} = -a^\dagger_g(-p, s). \] (18)

For a magnetically uncharged fermion \( \psi \), the transformation law is:

\[ Pa(p, s)P^{-1} = a(-p, s), \quad Pb(p, s)P^{-1} = -b(-p, s). \] (19)

The law of C transformation of a magnetically charged fermion has the same form as for a fermion without magnetic charge:

\[ Ca_g(p, s)C^{-1} = b_g(p, s), \quad Cb_g(p, s)C^{-1} = a_g(p, s). \] (20)

Similarly, one can obtain the formulas for the T reversal but we will not need to use them in the present paper. Hence, we see a clear difference between the behavior of the states with the electric charge and the magnetically charged states. The parity and time inversion acting on the electrically charged states do not change the electric charge of these states, that is under P transformation the electron is carried into an electron with opposite momentum and, likewise, positron is transformed into positron state with the opposite momentum.
Similarly, under time inversion the electron state is transformed into the electron state with opposite momentum and spin; the positron is turned into the positron with opposite momentum and spin. So, the P and T transformation do not change the electric charge at all. On the contrary, for magnetically charged particles the situation is exactly opposite: the P and T reflections necessarily include the change of sign of the magnetic charge. For instance, P transformation acting on the magnetic monopole takes it into antimonopole with the opposite momentum; likewise, under P parity the antimonopole is transformed into monopole with the opposite momentum. The same is true for T reversal: the T transformation changes the monopole into antimonopole with opposite momentum and spin; the antimonopole is changed into monopole with inverse momentum and spin.

5. Now we are in a position to apply the discrete symmetries to consideration of specific physical processes in order to establish whether any selection rules can be obtained or not. Since the monopoles are expected to be relativistic, let us use the helicity basis for their consideration. In this basis the pair of monopole-antimonopole is described by a wave function \( \psi_{J M \lambda_1 \lambda_2} \) where \( J \) is the total angular momentum of the pair, \( M \) is the projection of \( J \) and \( \lambda_1 \) and \( \lambda_2 \) are the helicities of the monopole and antimonopole. The action of discrete symmetries is given by:

\[
P \psi_{J M \lambda_1 \lambda_2} = \psi_{J M - \lambda_2 - \lambda_1},
\]

\[
C \psi_{J M \lambda_1 \lambda_2} = (-1)^J \psi_{J M \lambda_2 \lambda_1}.
\]

Using these rules, we can construct the wave function that has the photon quantum numbers, i.e. \( J = 1 \), \( P = -1 \) and \( C = -1 \):

\[
\psi_{1 M} = \frac{1}{\sqrt{2}}(\psi_{1 M \frac{1}{2} \frac{1}{2}} - \psi_{1 M - \frac{1}{2} - \frac{1}{2}}).
\]

Thus we see that in order to couple to the photon, the monopole and antimonopole should have the same helicities. To further understand the physical meaning of this condition, let us consider the non-relativistic limit, in which the monopole -antimonopole pair is described
by the wave function $\psi_{JLSM}$ where $L$ and $S$ are the total orbital momentum and spin, respectively. The connection between the wave functions $\psi_{JLSM}$ and $\psi_{JMA_1A_2}$ is given by [3]:

$$
\psi_{JLSM} = \sum_{\lambda_1\lambda_2} \psi_{JMA_1A_2} \langle JM\lambda_1\lambda_2|JLSM \rangle,
$$

where the coefficients are expressed through the $3j$ symbols as follows:

$$
\langle JM\lambda_1\lambda_2|JLSM \rangle = (-i)^L (-1)^S \sqrt{(2L + 1)(2S + 1)} \left( \begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} & S \\
\lambda_1 & -\lambda_2 & -\Lambda \\
0 & \Lambda & -\Lambda
\end{array} \right).
$$

It can be shown that the wave function Eq. (23) corresponds to the state $S = 0, L = 1$ in the non-relativistic limit, i.e.:

$$
\psi_{110M} = \frac{1}{\sqrt{2}} \left( \psi_{1M\frac{1}{2} - \frac{1}{2}} - \psi_{1M\frac{1}{2} + \frac{1}{2}} \right),
$$

$$
\Lambda = \lambda_1 - \lambda_2
$$

In other words, the $J^{PC} = 1^{--}$ of the monopole-antimonopole pair corresponds to the $^1P_1$ state in the non-relativistic limit. This should be contrasted with the case of the standard fermion-antifermion pair (such as positronium or quarkonium) for which the $1^{--}$ state is $^3S_1$ (or $^3D_1$).

Now, let us consider spin 0 monopoles for we do not have any evidence concerning the possible value of the monopole spin. From the similar considerations as the above, it can be shown that the spinless monopole-antimonopole system has the following quantum numbers:

$$
P = 1, \quad C = (-1)^J,
$$

where $J$ is the total angular momentum of the system. Thus, the spinless monopole-antimonopole production through the one-photon $e^+e^-$ annihilation is absolutely forbidden. Next, it follows from Eq. (28) that in the state with the total angular momentum $J = 1$ the monopole-antimonopole pair has always $CP = -1$. Therefore, CP symmetry absolutely
forbids the $1^-$ and $1^{++}$ states of the monopole-antimonopole system. Note that this conclusion holds true even if P and C parities do not conserve separately, but CP does. This means that the decay of Z-boson into spin 0 monopole-antimonopole pair would be absolutely forbidden in a CP invariant theory.

Thus we have shown that C and P invariance imposes exact selection rules on the monopole-antimonopole state produced through the one-photon channel of $e^+e^-$ annihilation.

It remains to be investigated whether these selection rules can help us to understand why the monopoles have not been observed experimentally.

Recently the contribution of virtual monopoles to various physical processes has been examined in several papers. One of them was the contribution of virtual monopole-antimonopole pairs to the anomalous magnetic moment of the electron [3] (see Fig.3). Another process is the monopole loop contribution to the decay of Z boson into 3 photons [6]. By necessity, the calculation of all these diagrams involves the use of some prescription to eliminate the string dependence of physical results. As we have seen, the validity of the conclusions drawn with the help of such prescriptions remains uncertain and needs to be further investigated.

6. To summarize, we have examined several issues related to the processes of Dirac monopole-antimonopole production in high-energy collisions such as $e^+e^-$ annihilation. Perturbative calculations for such processes are known to be inherently ambiguous due to the arbitrariness of direction of the monopole string; this requires use of some prescription to obtain physical results. We argue that different prescriptions lead to drastically different physical results which suggests that at present we do not have an entirely satisfactory procedure for the elimination of string arbitrariness (this problem is quite separate from the problems caused by the large coupling constant). We then analyze the consequences of discrete symmetries (P and C) for the monopole production processes and for the monopole-antimonopole states. The P and C selection rules for the monopole-antimonopole states turn out to be different from those for the ordinary fermion-antifermion or boson-antiboson
systems. In particular, the spin $1/2$ monopole and antimonopole should have the same helicities if they are produced through one-photon annihilation of an electron and positron. In the case of spinless monopoles CP symmetry absolutely forbids the monopole-antimonopole production through the one-photon annihilation of an electron and positron. This work was supported in part by the Australian Research Council.
REFERENCES

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[1] P.A.M.Dirac, Proc. Roy. Soc. (London) A133, 60 (1931).

[2] V.I.Strazhev and L.M.Tomilchik, Fiz. El. Chast. Atom. Yad., 4, 187 (1973).

[3] S.Coleman, in Proc. International School on Subnuclear Physics "Ettore Majorana", Erice 1982.

[4] M.Blagojevic and P.Senjanovic, Phys. Reports 157, 233 (1988).

[5] S.Graf, A.Schafer and W.Greiner, Phys. Lett. 262, 463 (1991).

[6] A. De Rujula, Nucl. Phys. B435, 257 (1995).

[7] W.Deans, Nucl. Phys. B197, 307 (1982).

[8] N.F.Ramsey, Phys. Rev. 109, 225 (1959).

[9] S.Weinberg, Phys. Rev. 138, B988 (1965).

[10] D.Zwanziger, Phys. Rev. 176, 1489 (1968).

[11] J.D.Bjorken and S.D.Drell, Relativistic Quantum Fields McGraw-Hill, 1964.

[12] T.T.Wu and C.N.Yang, Nucl. Phys. B107 (1976) 365.

[13] V.B.Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, Relativistic Quantum Theory, 2nd ed., Oxford ; New York : Pergamon Press, 1971-74.
FIGURES

FIG. 1. Feynman rules for the photon-monopole interaction

FIG. 2. Electron-positron annihilation into monopole-antimonopole

FIG. 3. Contribution of virtual monopole-antimonopole pair to the anomalous magnetic moment of the electron
Fig. 3