Supersymmetry and Kaon physics

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Abstract. Kaon physics has played an essential role in testing the Standard Model and in searching for new physics with measurements of CP violation and rare decays. Current progress of lattice calculations enables us to predict kaon observables accurately, especially for the direct CP violation, $\epsilon' / \epsilon$, and there is a discrepancy from the experimental data at the $2.9\sigma$ level. On the experimental side, the rare kaon decays $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are ongoing to be measured at the SM accuracy by KOTO at J-PARC and NA62 at CERN. These kaon observables are good probes for new physics. We study supersymmetric effects; the chargino and gluino contributions to $Z$ penguin, in kaon observables.

1. Introduction
Kaon physics is one of the most powerful probes of physics beyond the standard model (SM), and sensitive to high scale new physics (NP). The most exciting topic in kaon physics is the anomaly in the direct CP violation, $\epsilon' / \epsilon$. Recently, the hadron matrix elements of the $K \rightarrow \pi \pi$ decay have been determined with lattice QCD by the RBC-UKQCD collaborations [3], and the SM prediction of the direct CP violation is obtained as

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\text{SM}} = (1.38 \pm 6.90) \times 10^{-4}. \quad [\text{RBC-UKQCD}]$$

(1)

The hadronic uncertainties are reduced by the use of CP-conserving data as [4]

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}, \quad [\text{Buras et al.}]$$

(2)

which has been confirmed [5]

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\text{SM}} = (0.96 \pm 4.96) \times 10^{-4}. \quad [\text{Kitahara et al.}]$$

(3)

All these results are below the experimental world average [6] from NA48 [7] and KTeV [8,9] collaborations,

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\exp} = (16.6 \pm 2.3) \times 10^{-4}. \quad (4)$$

In particular, Eqs. (2) and (3) disagree with the experimental data at the $2.9\sigma$ level. This may suggest a NP model providing enhancement of $\epsilon' / \epsilon$.

Rare kaon decays $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are also good probes for NP because they are theoretically clean modes and strongly suppressed in the SM. Especially, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a

$^1$ Based on the works with Motoi Endo, Satoshi Mishima, Daiki Ueda [1], and with Morimitsu Tanimoto [2].
CP violating mode and provides the direct measurement of the CP violating phase in the CKM matrix. The SM predictions for these processes are given as [10]

\[
\begin{align*}
\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} &= (3.4 \pm 0.6) \times 10^{-11}, \\
\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} &= (8.4 \pm 1.0) \times 10^{-11}.
\end{align*}
\]

On the experimental side, KOTO and NA62 are ongoing to measure these modes at the SM accuracy. The data of these branching ratios are given as follows [11,12]:

\[
\begin{align*}
\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{exp}} &< 2.6 \times 10^{-8} \text{ (90\%C.L.)}, & \text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} &= (1.73^{+1.15}_{-1.05}) \times 10^{-10}. \tag{7}
\end{align*}
\]

In this article, we discuss the effects of supersymmetry (SUSY) models on these kaon observables and the possibilities of solving the discrepancy in $\epsilon'/\epsilon$. We focus on chargino and gluino $Z$-penguin contributions in the scenario with large trilinear couplings, not proportional to the Yukawa couplings. They do not decouple even the SUSY breaking scale is high once relevant mass insertion (MI) parameters are fixed, and give large effects to kaon observables.

2. $Z$-penguins in supersymmetry

In this article, we discuss chargino [1] and gluino [2] $Z$-penguin contributions. Let us consider chargino contributions to the $Z$-penguin diagrams (Fig. 1). They are described by the flavor-violating $Z$-boson vertex,

\[
\mathcal{L}_{\text{eff}} = \frac{-g^3}{8\pi^2 \cos \theta_W} Z_{ds} s_L \gamma_\mu d_L Z^\mu + \text{h.c.}, \tag{8}
\]

where the coupling $Z_{ds}$ includes SM and SUSY contributions,

\[
Z_{ds} = Z_{ds}^{(\text{SM})} + Z_{ds}^{(\text{SUSY})}. \tag{9}
\]

This SUSY contribution $Z_{ds}^{(\text{SUSY})}$ is written in terms of MI parameters $(\delta^u_{LR})_{ij}$,

\[
(\delta^u_{LR})_{ij} = \frac{\sqrt{2}}{m^2} (T^u_{ij})_{ij}, \quad (\delta^u_{RL})_{ij} = \frac{\sqrt{2}}{m^2} (T^u_{ij})_{ji}, \tag{10}
\]

where $(T^u_{ij})_{ij}$ is the trilinear scalar couplings. The coupling $Z_{ds}^{(\text{SUSY})}$ is given by the up-type squark-chargino diagrams (Fig. 1). Focusing on the MI parameters, $(\delta^u_{LR})_{13}$ and $(\delta^u_{LR})_{23}$, one obtains

\[
Z_{ds}^{(\text{SUSY})} \simeq (\delta^u_{LR})_{13}(\delta^u_{LR})_{23} H_0(x_{\tilde{q}W}), \tag{11}
\]

where $H_0(x_{\tilde{q}W})$ is the loop function. The coupling $Z_{ds}^{(\text{SUSY})}$ does not decouple even if superparticles are heavy as long as the product of the mass insertion (MI) parameters $(\delta^u_{LR})_{13}(\delta^u_{LR})_{23}$ is fixed [13,14]. These features are guaranteed by the SU(2)$_L$ breaking, which is provided by $(\delta^u_{LR})_{13}$ and $(\delta^u_{LR})_{23}$ in Eq. (11). Although CP-violating FCNCs of Kaon are tightly constrained by the indirect CP violation of Kaon or electric dipole moments, SUSY contributions to them decouple in heavy SUSY scenarios. Thus, the discrepancy in $\epsilon'/\epsilon$ may be explained by the $Z$-penguin contributions. Gluino contributions to the $Z$-penguin also have the same structures. In our analysis, we calculate in the mass eigenstates basis. We study two scenarios with different setup:
Chargino contributions

We study the chargino contributions to the Z penguin especially with the vacuum stability constraint [1]. Since the MI parameters are proportional to scalar trilinear couplings, the chargino Z-penguin contributions are constrained by requiring the stability of the electroweak (EW) vacuum. This condition is not relaxed even if SUSY particles are heavy. In this work, charge-color breaking (CCB) vacua or potential directions unbounded from below (UFB) have been studied along with $\epsilon' = \epsilon [13, 14]$. However, their analyses follow the strategy of Ref. [15, 16], and the vacuum decay rate has not been examined. The vacuum decay is studied, and we discuss whether the current discrepancy of $\epsilon'/\epsilon$ is explained by the chargino Z-penguin contributions.

Gluino contributions

The gluino contributions to the Z penguin are also studied [2]. The interesting point of the gluino contribution is that it gives the equal left-handed and right-handed Z couplings. In the scenario with the large right-handed Z couplings, we can enhance $\epsilon'/\epsilon$ and the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$ decay simultaneously [17]. We discuss whether the gluino contributions can enhance $\text{BR}(K_L \to \pi^0\nu\bar{\nu})$ with satisfying the data of $\epsilon'/\epsilon$ at the high-scale SUSY model.

3. Chargino contribution

We study the chargino contributions to Z penguin especially with considering the vacuum decay rate seriously. According to Eq. (11), large $\epsilon'/\epsilon$ is achieved when $\tilde{u}_L$ and $\tilde{c}_L$ have a large mixing with $t_R$. The left-right mixing is proportional to the scalar trilinear coupling $(T_U)_{ij}$. Large flavor-violating trilinear couplings may generate instabilities of the EW vacuum [18]. Requiring that the lifetime of the EW vacuum is longer than the present age of the universe, the trilinear couplings, or equivalently $(\delta_{LR}^u)_{13}$ and $(\delta_{LR}^u)_{23}$, are constrained.

The vacuum decay rate per unit volume is expressed as

$$\Gamma/V = A \exp(-S_E),$$  \hspace{1cm} (12)
The gluino contributions to the $\bar{Z}$ penguin is discussed in this section. We consider the high-scale SUSY model at $\mathcal{O}(10)$ TeV. As a simple set-up for it, we take the gluino, wino and bino masses $M_i (i = 3, 2, 1)$ with $\mu$ and $\tan\beta$, and the masses of sbottom $b_1$ and $b_2$ as:

$$M_3 = 10 \text{ TeV}, \quad M_2 = 3.3 \text{ TeV}, \quad M_1 = 1.6 \text{ TeV}, \quad \mu = 10 \text{ TeV}, \quad \tan\beta = 3$$

and $m_{\tilde{b}_1} = 10 \text{ TeV}, \quad m_{\tilde{b}_2} = 15 \text{ TeV}$. 

**Figure 3.** (left) $(\epsilon'/\epsilon)_{\text{SUSY}}$ is shown as a function of $m_{\tilde{q}}$. The Wino mass $m_{\tilde{W}}$ is 1, 2, 3 TeV for the blue solid, dashed and dotted lines, respectively, while it is equal to $m_{\tilde{q}}$ on the black line. On the red (orange) region, $\Delta (\epsilon'/\epsilon)$ is saturated at the 1σ (2σ) level. The SM value follows Ref. [4]. (right) Correlation between BR($K_L \to \pi^0 \nu\bar{\nu}$) and $(\epsilon'/\epsilon)_{\text{SUSY}}$ is shown.

where $S_E$ is estimated at the semi-classical level, which is called the bounce action [19] and calculated by CosmoTransition 2.0a2 [20] in this work. The prefactor $A$ is not determined at this level, and we adopt an order-of-estimation analysis: the typical energy scale $A \sim (100 \text{ GeV})^4$ or $(10 \text{ TeV})^4$. The lifetime of the EW vacuum is longer than the age of the universe if the bounce action satisfies

$$S_E \gtrsim 400.$$  \hspace{1cm} (13)\]

We discuss whether the current discrepancy of $\epsilon'/\epsilon$ is explained by the chargino $Z$-penguin contributions with satisfying the constraints especially from the vacuum stability condition. The vacuum decay rate is estimated to derive an upper bound on the size of $(T_U)_{i3}$ by requiring $S_E \gtrsim 400$. In Fig 2, the bound of $(\delta_{LR}^U)_{i3}$ is shown as a function of $m_{\tilde{q}} \equiv m_{\tilde{Q}_i} = m_{\tilde{C}_i}$. Due to the relation Eq. (10), the limit becomes severer as the SUSY scale increases. Therefore, the SUSY contributions to $\epsilon'/\epsilon$ decrease according to Eq. (11).

In the left plot of Fig. 3, the SUSY contributions to $\epsilon'/\epsilon$ are shown as a function of $m_{\tilde{q}}$. Here, $|\langle T_U \rangle_{i3}|$ is set at $S_E = 400$, and $|\langle T_U \rangle_{13}| = |\langle T_U \rangle_{23}|$ is assumed. The CP-violating phase is taken to be maximal. There is a degree of freedom in choosing $m_{\tilde{W}}$, and it is set to be 1, 2, 3 TeV and $m_{\tilde{q}}$ as reference cases. It is found that the current discrepancy of $\epsilon'/\epsilon$ can be explained; the SUSY scale can be as large as 4–6 TeV, depending on the choice of $m_{\tilde{q}}$. In the right plot of Fig. 3, correlation between BR($K_L \to \pi^0 \nu\bar{\nu}$) and $(\epsilon'/\epsilon)_{\text{SUSY}}$ is shown. BR($K_L \to \pi^0 \nu\bar{\nu}$) decreases as $\epsilon'/\epsilon$ increases unless $\epsilon'/\epsilon$ is very large. The current discrepancy implies that BR($K_L \to \pi^0 \nu\bar{\nu}$) is predicted to be less than 60% of the SM prediction.
On the other hand, we take the masses of the first and second family up-type and down-type squarks around 15 TeV within 5 \(- 15\% \) relevantly. The flavor mixing parameters \( s_{ij}^{dL} \) are set as \( s_{ij}^{dL} = 0, 0.3 \) ( \( i, j = 1, 2 \) ).

In Fig. 4, we show the predicted region for BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) versus BR(\( K^+ \rightarrow \pi^+\bar{\nu}\nu \)) without imposing \( \epsilon_K \). The green line corresponds to the Grossman-Nir bound [21]. The dashed red lines denote the \( 1\sigma \) experimental bounds for BR(\( K^+ \rightarrow \pi^+\bar{\nu}\nu \)).

\[ \text{Figure 4. The predicted region for BR}(K_L \rightarrow \pi^0\bar{\nu}\nu) \text{ versus BR}(K^+ \rightarrow \pi^+\bar{\nu}\nu) \text{ without imposing } \epsilon_K. \text{ The green line corresponds to the Grossman-Nir bound [21]. The dashed red lines denote the } 1\sigma \text{ experimental bounds for BR}(K^+ \rightarrow \pi^+\bar{\nu}\nu). \]

In this article, we have discussed the SUSY effects on kaon observables \( \epsilon'/\epsilon \) and \( K \rightarrow \pi\nu\bar{\nu} \). The predicted BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) versus \( \epsilon'/\epsilon \), where the \( Z \) coupling satisfies the condition of eq.(15). The vertical solid red line denotes the central value of the experimental data, and the dashed ones denote the experimental bounds with 3\( \sigma \) for \( \epsilon'/\epsilon \). The pink denotes the SM prediction.

\[ \text{Figure 5. The predicted BR}(K_L \rightarrow \pi^0\bar{\nu}\nu) \text{ versus } \epsilon'/\epsilon, \text{ where the } Z \text{ coupling satisfies the condition of eq.(15). The vertical solid red line denotes the central value of the experimental data, and the dashed ones denote the experimental bounds with } 3\sigma \text{ for } \epsilon'/\epsilon. \text{ The pink denotes the SM prediction.} \]

On the other hand, we take the masses of the first and second family up-type and down-type squarks around 15 TeV within 5 \(- 15\% \) relevantly. The flavor mixing parameters \( s_{ij}^{dL} \) are set as \( s_{ij}^{dL} = 0, 0.3 \) ( \( i, j = 1, 2 \) ).

In Fig. 4, we show the predicted region for BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) versus BR(\( K^+ \rightarrow \pi^+\bar{\nu}\nu \)), with imposing \( \epsilon_K \). The left-right (LR) mixing angle \( \theta_{LR}^0 \), which is correspond to the trilinear couplings, is fixed to 0.3. It is found that these two observables are enhanced significantly due to the large LR mixing angle. The direction of the enhancement of BR(\( K \rightarrow \pi\nu\bar{\nu} \)) corresponds to the specific phase regions of the flavor mixing parameters, which are constrained by \( \epsilon_K \).

Let us discuss the correlation between BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) and \( \epsilon'/\epsilon \). The Z-penguin contributions to them are studied [17], and it is pointed out that a simultaneous enhancement of them is realized on the following condition for the flavor violating Z couplings \( \Delta_{LR}^d(Z) \) :

\[ |\text{Im}\Delta_{LR}^d(Z)| < |\text{Im}\Delta_{LR}^d(Z)| < 3.3|\text{Im}\Delta_{LR}^d(Z)|. \]  

Since the gluino contributions give the equal left-handed and right-handed Z couplings, they can satisfy the above condition. In Fig. 5, we show the correlation between BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) and \( \epsilon'/\epsilon \), where Z coupling satisfies the above condition. The constraint from \( \epsilon_K \) is also imposed.

It is remarkable that the Z-penguin mediated by the gluino can enhance BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) with satisfying the data of \( \epsilon'/\epsilon \). While the estimated \( \epsilon'/\epsilon \) fits the observed value, the branching ratio of \( K_L \rightarrow \pi^0\bar{\nu}\nu \) increases up to \( 1.0 \times 10^{-10} \). The correlations with \( K_L \rightarrow \mu^+\mu^- \), and the B meson rare decays, \( B^0 \rightarrow \mu^+\mu^- \) and \( B_s \rightarrow \mu^+\mu^- \) are also discussed in [2].

5. Conclusion

In this article, we have discussed the SUSY effects on kaon observables \( \epsilon'/\epsilon \) and \( K \rightarrow \pi\nu\bar{\nu} \). The recent analyses of the SM prediction of \( \epsilon'/\epsilon \) have reported a discrepancy from the experimental value, and we discuss the possibilities that the chargino and gluino Z-penguin contribution explain the discrepancy.

For the chargino contributions to the Z penguin, we have studied the vacuum stability constraint. The chargino contributions are constrained by the vacuum stability condition, and it is found that the SUSY contributions can bridge the current discrepancy of \( \epsilon'/\epsilon \) if the SUSY masses are smaller than 4 \(- 6\) TeV. The discrepancy implies that BR(\( K_L \rightarrow \pi^0\bar{\nu}\nu \)) is about less than 60\% of the SM prediction.
The gluino contributions to the $Z$ penguin have also been studied. Even the SUSY breaking scale is high, around 10 TeV, BR($K_L \to \pi^0 \nu \bar{\nu}$) can be enhanced with imposing the constraint from $\epsilon_K$. It is remarkable that the gluino contributions can enhance BR($K_L \to \pi^0 \nu \bar{\nu}$) up to $1.0 \times 10^{-10}$ with satisfying the data of $\epsilon'/\epsilon$.

The SM predictions of $\epsilon'/\epsilon$ are expected to be improved in the near future. If the discrepancy would be confirmed, the chargino and gluino contributions could provide an attractive solution.

Besides, the predicted correlations with the branching ratios of $K \to \pi \nu \bar{\nu}$ will be tested by KOTO and NA62 in future. KOTO is ongoing to measure BR($K_L \to \pi^0 \nu \bar{\nu}$) at the 10% level of the SM value [22], and NA62 will measure BR($K^+ \to \pi^+ \nu \bar{\nu}$) with ~10% precision by 2018 [23]. Kaon physics gives an exciting impact on NP in the coming years.

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References

[1] M. Endo, S. Mishima, D. Ueda and K. Yamamoto, Phys. Lett. B 762 (2016) 493 [arXiv:1608.01444 [hep-ph]].
[2] M. Tanimoto and K. Yamamoto, PTEP 2016 (in press) arXiv:1603.07960 [hep-ph].
[3] T. Blum et al., Phys. Rev. D 91, no. 7, 074502 (2015) [arXiv:1502.00263 [hep-lat]].
[4] A. J. Buras, M. Gorbahn, S. Jäger and M. Jamin, JHEP 1511, 202 (2015) [arXiv:1507.06345 [hep-ph]].
[5] T. Kitahara, U. Nierste and P. Tremper, arXiv:1607.06727 [hep-ph].
[6] K. A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update.
[7] J. R. Batley et al. [NA48 Collaboration], Phys. Lett. B 544, 97 (2002) [hep-ex/0208009].
[8] A. Alavi-Harati et al. [KTeV Collaboration], Phys. Rev. D 67, 012005 (2003) Erratum: [Phys. Rev. D 70, 079904 (2004)] [hep-ex/0208007].
[9] E. T. Worcester [KTeV Collaboration], arXiv:0909.2555 [hep-ex].
[10] A. J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, JHEP 1511 (2015) 033 [arXiv:1503.02693 [hep-ph]].
[11] J. K. Ahn et al. [E391a Collaboration], Phys. Rev. D 81 (2010) 072004 [arXiv:0911.4789 [hep-ex]].
[12] A. V. Artamonov et al. [BNL-E949 Collaboration], Phys. Rev. D 79 (2009) 092004 [arXiv:0903.0030 [hep-ex]].
[13] G. Colangelo and G. Isidori, JHEP 9809, 009 (1998) [hep-ph/9808487].
[14] A. J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, Nucl. Phys. B 566, 3 (2000) [hep-ph/9908371].
[15] J. A. Casas and S. Dimopoulos, Phys. Lett. B 387, 107 (1996) [hep-ph/9606237].
[16] J. A. Casas, Adv. Ser. Direct. High Energy Phys. 21, 469 (2010) [hep-ph/9707475].
[17] A. J. Buras, JHEP 1604 (2016) 071 [arXiv:1601.00005 [hep-ph]].
[18] J. h. Park, Phys. Rev. D 83, 055015 (2011) [arXiv:1011.4939 [hep-ph]].
[19] S. R. Coleman, Phys. Rev. D 15, 2929 (1977) Erratum: [Phys. Rev. D 16, 1248 (1977)].
[20] C. L. Wainwright, Comput. Phys. Commun. 183, 2006 (2012) [arXiv:1109.4189 [hep-ph]].
[21] Y. Grossman and Y. Nir, Phys. Lett. B 398 (1997) 163 [hep-ph/9701313].
[22] J. K. Ahn et al., arXiv:1609.03637 [hep-ex].
[23] M. Mouhson [NA62 Collaboration], arXiv:1611.04979 [hep-ex].