Absence of physical walls in hot gauge theories

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This paper shows that there are no physical walls in the deconfined, high-temperature phase of $Z(2)$ lattice gauge theory. In a Hamiltonian formulation, the interface in the Wilson lines is not physical. The line interface and its energy are interpreted in terms of physical variables. They are associated with a difference between two partition functions. One includes only the configurations with even flux across the interface. The other is restricted to odd flux.

1. INTRODUCTION

This paper discusses some aspects of the global $Z(N)$ symmetry of finite-temperature gauge theory. It contributes to recent discussions of the physics of $Z(N)$ phases and interfaces. The example of $Z(2)$ gauge theory is treated here.

The Wilson line carries a nontrivial representation of the global $Z(N)$ symmetry. In the confining phase, $\langle L \rangle = 0$, and the ensemble is $Z(N)$ symmetric. In the high $T$ phase, $\langle L \rangle$ takes one of $N$ distinct values proportional to the $N$th roots of unity $z$ in $Z(N)$, and the $Z(N)$ symmetry is broken.

In the Hamiltonian description, the physical variables are the group elements on the links of the spatial lattice. In a Lagrangian formulation, there are also group elements on links in the inverse-temperature direction. These are unphysical, auxiliary variables introduced to enforce the Gauss law constraints. The Wilson line is constructed from the unphysical variables. It is a projection operator that forces the gauge field to be in a fundamental rather than a singlet state at the spatial position of the line. The global $Z(N)$ symmetry of the Lagrangian formulation is not physical; it acts as the identity on all physical states. There is a single physical, high-temperature phase, which is the same for all $z$.

If there were $N$, degenerate, physically-distinct, high-temperature phases, then there could be large regions of space in different phases and separated by interfaces. These walls could have a physically-significant interface tension. In the line variables, there are phase separations and interfaces. However, as noted above, the $N$ phases of the Wilson lines are not physically distinct. Thus one may question the physical relevance of the line interfaces.

It is shown below that there are no interfaces in the physical variables associated with the interfaces in the lines. The physical effect associated with the interfaces in the line variables is the “conservation” mod 2 of the total flux along the direction perpendicular to the line interface. The interface tension for the lines is related to a difference between two partition functions. One includes only the configurations with even flux across the interface. The other is restricted to odd flux.

Because there are no physically distinct high-temperature phases of the Yang-Mills field, there are also no metastable states in the presence of matter. Matter simply lowers the free energy. It is shown that this can be understood in terms of the physical variables as an increase in entropy.

2. NO PHYSICAL WALLS IN HOT YANG-MILLS THEORY

This section treats gauge theory without matter fields. The Hamiltonian formulation in the $A_0 = 0$ gauge is used. The physical degrees of freedom are gauge group elements on links of the spatial lattice. The flux basis specifies that the amplitude to be at different points in the group
is given by a matrix element from an irreducible representation. Physical states are configurations of physical variables with the additional condition that the Gauss law constraints are satisfied. In the case of $Z(2)$, the flux is either zero or one on each link. The constraints are that each site must have an even number of links with ones. To enforce these constraints, the unphysical variables are introduced in the Lagrangian formulation. These are the group elements on links in the fourth direction or the fourth component of the gauge field.

If the partition function is written as a sum over physical states, then the unphysical variables have no work to do and need not even appear. Clearly, interfaces in the unphysical variables are not physical per se. So the question is: do they reflect the existence of interfaces in the physical variables?

Previous work on the high-temperature, homogeneous phases of the lines $[3][4]$ makes the existence of physical interfaces seem very unlikely. It was shown that the physical configurations asrewd to the Ising model results from using site variables to enforce the restriction on configurations, and then doing the $\theta$ sums. The site variables are the Ising spins. First, consider the sum of $\theta(l)$ over the $2d = 6$ $l$’s contained in the set $I(i)$ of links with endpoint $i$:

$$
\Sigma(i) = \sum_{l \in I(i)} \theta(l).
$$

To force this to be even at each site, introduce the site variables $s(i)$ that take the values $\pm 1$. The factor

$$
\frac{1}{2} \sum_{s(i) = \pm 1} s(i)^\Sigma(i)
$$

has the desired effect. A factor like this is introduced into the partition function sum for each site $i$. The spins $s(i)$ are the same as the Wilson lines of the $Z(2)$ gauge theory. This model is equivalent to the Ising model for the spins $s(i)$ with the Ising $\beta$ related to the gauge $T$ by $e^{-\sigma/T} = \tanh \beta$. Note that I never refer to the Ising model $1/\beta$ as temperature. Temperature always refers to the $T$ appearing in $[3][4]$. Large $T$ gives large $\beta$, and ordered Ising spins. It is convenient to introduce $\mu = \sigma/T$. In the high $T$ region,

$$
e^{-2\beta} \sim \mu/2.
$$

To discuss interfaces, it helps to introduce boundary conditions that force at least one interface in the lines to appear. Use solid-cylindrical geometry: space is finite in the transverse $x$ and $y$ directions and very long in the $z$ direction. There

The partition function is

$$
Z = Tr'[e^{-H/T}] = \sum_{C'} \langle \theta | e^{-H/T} | \theta \rangle
$$

$$
= \sum_{C'} e^{-(1/T)\Sigma_I \sigma \theta(l)}.
$$

In a gauge theory, the Hamiltonian comes from the transfer matrix. There is a factor of this form and a factor from the spatial plaquette term in the Lagrangian. It is sufficient to consider the high-temperature and strong-coupling limit. In that case, the plaquette term can be neglected in a first approximation. The equivalence of this flux model to the Ising model results from using site variables to enforce the restriction on configurations, and then doing the $\theta$ sums. The site variables are the Ising spins. First, consider the sum of $\theta(l)$ over the $2d = 6$ $l$’s contained in the set $I(i)$ of links with endpoint $i$:

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e^{-2\beta} \sim \mu/2.
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To discuss interfaces, it helps to introduce boundary conditions that force at least one interface in the lines to appear. Use solid-cylindrical geometry: space is finite in the transverse $x$ and $y$ directions and very long in the $z$ direction. There
are periodic boundary conditions in the $x$ and $y$ directions. The transverse area in dimensionless lattice units is $A$. Let $L$ be large and positive. Apply the boundary conditions $s = -1$ for $z = -L/2$ and $s = 1$ for $z = L/2$. The limit of interest is $L \to \infty$ followed by $A \to \infty$.

First review the picture in terms of the unphysical spin variables. The high-temperature region is the large-$\beta$, ordered phase for the Ising spins. The interface is approximately flat and has a Boltzmann weight $e^{-2\beta A}$. Thus, the energy per unit area $\alpha$ is related to $\beta$ by $\alpha/T \approx 2\beta$ as $\beta \to \infty$. The partition function with the indicated boundary conditions is $Z'$ and is a sum with an odd number of interfaces. The partition function with the same boundary conditions at each end and an even number of interfaces is $Z$. The ratio is

$$Z'/Z = e^{-(F' - F)/T} \cong 1 - (F' - F)/T.$$  (6)

In the large $L$ limit, the result of the sums is the excess free energy

$$F' - F = 2Te^{-2La} \quad \text{with} \quad \epsilon = e^{-2\beta A}. \quad (7)

This relates the activity for the wall $e^{-2\beta A}$ to $Z'/Z$ and $F' - F$. The approximations are valid for large $L$ and $Le^{-2\beta A} \gg 1$.

Now consider the same situation in terms of flux variables. In the high-temperature, deconfined phase with $\mu = \sigma/T$ small, there is dense, percolating flux. The flux is almost random except for the constraint that there be an even number of links with flux at each site.

To enforce the constraints, there is a factor given in (3) at each site. For a site on the $z = -L/2$ boundary, this becomes $(-1)^g$. This gives a factor of $-1$ for each link with flux coming into the system from the boundary.

From the local Gauss law constraints, it follows that the total flux $\sum_{x,y} \theta_z(x, y, z) \mod 2$ on a transverse slice of longitudinal links is independent of $z$. This will be referred to as flux “conservation”. Thus each configuration can be described as even or odd. The effect of the boundary conditions is to weight the odd configurations with an extra factor of $-1$. The partition function sum

$$Z = \sum_{\text{even}} e^{-H/T} + \sum_{\text{odd}} e^{-H/T} \quad (8)$$

is replaced by

$$Z' = \sum_{\text{even}} e^{-H/T} - \sum_{\text{odd}} e^{-H/T}. \quad (9)$$

This gives

$$Z'/Z = \frac{(Ze - Zo)}{(Ze + Zo)} \quad (10)$$

$$= 1 - 2e^{-(F_o - F_e)/T}. \quad (11)$$

The last step is correct for large $(F_o - F_e)$. This free energy difference is proportional to the length $L$, so that (11) is consistent with (8). Thus, the free energy difference per unit length of odd flux verses even flux is the quantity of interest.

The next part of the discussion shows the direct connection between the spin interfaces and the conservation of flux. Consider the structure of the partition function. The role of the spins is to enforce the local constraints on flux. In the calculation leading to (4), the spins are constant on transverse planes. Thus, they are enforcing the weaker constraint of flux conservation. The calculation begins from (2), inserts the factors (4), does the flux sum, and then does the spin sum. It is manifest in the calculation that the interfaces enforce the flux conservation. The result is

$$(F_o - F_e)/T = 2L(\frac{\mu}{2})^A. \quad (12)$$

Since $\mu$ and $\beta$ are related by (3) this is equivalent to (8).

This shows that the interfaces in the unphysical spin variables are associated with flux conservation i.e. the total flux $\mod 2$ on a transverse slice of links is independent of longitudinal position.

With that established, one can redo the calculation of $Z'$ by a much easier method using flux variables only (4). The partition function $Z'$ is the difference of two contributions. It is the sum of all the even configurations minus the sum of all the odd configurations as in (8). There is no reference to interfaces. The result is the same as that from the spin calculation, but the associated physical picture is completely different. In flux, it is the Gauss law constraints rather than interfaces that are important.
3. EFFECTS OF MATTER

Matter fields in the fundamental representation explicitly break the global $Z(N)$ symmetry. The $N$ line states are no longer degenerate. Above the critical temperature, there can be metastable states of the line variables separated by walls [3].

The important effect of the matter is that it breaks the global symmetry. This can be studied in a simpler situation where the matter is represented by an external field $h$ linearly coupled to the Wilson lines: $h^* L + h L^*$. In a general situation with $Z(2)$ global symmetry, the constraint effective potential with $h \neq 0$ is a double well with one side lower than the other. As $h \to 0$, this becomes the symmetric double well with degeneracy. Thus the existence of the metastable state is linked to the existence of degeneracy.

However, without matter, there is a single, physical, high-temperature phase. Thus, there is no degeneracy that could be connected to multiple states, some metastable, when symmetry breaking is present. I conclude that in the physical flux picture, there are no metastable states.

Nevertheless, matter does affect the free energy. It is interesting to describe this effect entirely in the flux picture. When the partition function is expressed in terms of flux variables, the effect of $h$ is to allow odd sites with a density controlled by $h$. This increases the number of allowed configurations (i.e. increases the entropy), increases $Z$, and decreases $F$.

4. CONCLUSION

While calculations in terms of the physical flux variables and in terms of the unphysical lines lead to the same result, the associated interpretations are completely different. Interfaces in the line variables may be a convenient device for making approximate calculations of physical quantities in terms of unphysical variables. However, one should be cautious with heuristic arguments that rely upon the physical reality of the interfaces. In terms of physical variables, there are no interfaces or metastable states.

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