Inference of Short-Term Maximum Crack Width of RC Flexural Members Based on Interval Estimation Method

Jitao Yao¹, Hanbing Yan¹* and Mingdong Yan¹

¹School of Civil Engineering, Xi’an University of Architecture & Technology, Xi’an, Shaanxi, 710055, China

*Corresponding author’s e-mail: hanbing_yan@163.com

Abstract. The design method based on observation and experimental model only considers the design of ultimate limit state in related research, the application range is not wide. This paper takes the short-term maximum crack width of RC flexural members as an example, puts forward the application of this method in the design of serviceability limit state. Dimensionless method was used to collect 4665 pieces of RC flexural member crack width data, and the normality test was performed to obtain the type of probability distribution obeyed by the crack width. Considering the situation without parameter information and known coefficient of variation, a direct estimation formula for short-term maximum crack width of RC flexural members based on interval estimation method is proposed.

1. Introduction

Among the design method of approximate probability limit state, compared with the partial safety factor method and the design value method, the method supported by structure experiment will have better demonstration and pertinence. The ISO 2394:1998[1] first introduced structure experiment as an intermediate step into the design process, and put forward the design method based on observations and experimental models, which was used to infer resistance of member. That is, to conduct a number of targeted experiments, analyze the results directly, and obtain the resistance design value of the designed members. The method mainly includes interval estimation method, Bayesian method and inference method base on analysis model, the essence of all of which is small sample inference.

Interval estimation method and Bayesian method are so direct and simple that they are widely used. To avoid the doubt of choice, the ISO 2394:2015[2] as updated standards and the EN 1990:2002[3] adopt only Bayesian method. Paper 4 points out the deficiency of traditional Bayesian method and propose a new Bayesian method with confidence level, it gets the same result as interval estimation method. It can be seen that interval estimation method is more reasonable than traditional Bayesian method.

Up to now, the relevant research about design method based on observations and experimental models still remains in the design of ultimate limit state, however, in the process of experiment, deformation, crack resistance, crack spacing and width will appear at the same time, all of which can be applied to the design of serviceability limit state.

In this paper, the short-term maximum crack width of RC flexural members is taken as an example to expand the application of design method based on observations and experimental models in the design of serviceability limit state, and a set of small sample inference method based on interval estimation method is proposed.
2. Probability characteristics of crack width

Before inferring the maximum crack width of a member, its fundamental probabilistic property should be known first, mainly including probability distribution type and coefficient of variation.

2.1. Probability distribution type

The ISO 2394:2015 in the inference of member resistance, the results considered by normal distribution are relatively conservative. When the sample size is small, the guarantee rate and variability are large, the design value of resistance may even be negative. The EN 1990:2002 takes into account the situation that the resistance obeys lognormal distribution to avoid this phenomenon. Therefore, in the inference of the maximum crack width, the probability distribution type of the crack width should be determined first to ensure that the result is more reasonable.

In this paper, the set-valued statistic analysis is used to determine the probability distribution type of crack width. In the process, in order to eliminate the influence of experiment design inconsistencies on the statistical results, use \( \omega = \omega_i / \overline{\omega} \) as the statistic to nondimensionalize the samples, where \( \omega_i \) is a crack width, and \( \overline{\omega} \) is the average width of all cracks in the same member.

This paper reviews some experimental results of RC flexural members\(^5\)-\(^7\), and collect 4665 pieces of short-term crack width data. Test the normality of the collected data by SPSS, the results are shown in Figure 1.

![Histogram and Standard Q-Q figure](image1)

Figure 1. Original data analysis results

In order to test the degree of lognormal distribution of the data, convert all the data into their logarithms, and the analysis results of the logarithmic data are shown in figure 2.

![Histogram and Standard Q-Q figure](image2)

Figure 2. Logarithmic data analysis results

By comparing the histogram and the standard Q-Q graph between original data and logarithmic data, it can be seen that the normality of the original data is more obvious than that of the logarithmic data.
Therefore, we consider that the short-term crack width of RC flexural members obeys the normal distribution. This paper is going to study based on the probability characteristics of normal distribution.

2.2. Coefficient of variation

In statistics, the coefficient of variation is the quotient of the population standard deviation and the population mean, which affects the dispersion of the distribution. When the coefficient of variation is known, it is helpful to improve the accuracy of inference.

According to the experiment data of 1429 cracks in more than 40 beams, Chinese standard GB 50010-2012[8] obtains an approximate normal distribution \( N(1.0, 0.4^2) \) for \( \bar{\omega} \), from which, the coefficient of variation is about 0.4. In this paper, the coefficient of variation of 4665 cracks is 0.377, it’s close to that of the former.

3. The inference of short-term maximum crack width

It is assumed that the short-term crack width \( \omega \) presented in the experiment is subject to normal distribution \( N(\mu, \sigma^2) \), \( \mu \) is the population mean of the crack width \( \omega \), and \( \sigma \) is the population standard deviation. The inferential value of the maximum short-term crack width of the member is \( \omega_{\text{max,s}} \), which is the upper \( 1-p \) quantile of \( \omega \). \( p \) is the guarantee rate of \( \omega_{\text{max,s}} \), which satisfies

\[
P\{\omega \leq \omega_{\text{max,s}}\} = p
\]

(1)

\[
\omega_{\text{max,s}} = \mu + z_{1-p} \sigma
\]

(2)

Where: \( z_{1-p} \) is the upper \( 1-p \) quantile of the standard normal distribution.

3.1. Inference without parameter information

When there is no parameter information, that is, when the coefficient of variation is unknown, the standard deviation in the inference adopts the sample standard deviation \( S \), which can be used to construct the pivot

\[
\frac{\omega_{\text{max,s}} - \bar{\omega}}{S/\sqrt{n}} \sim t(n-1, z_{1-p} \sqrt{n})
\]

about \( \omega_{\text{max,s}} \), where \( n \) is the sample size, and \( \bar{\omega} \) is the sample mean of \( \omega \).

According to the interval estimation method, the maximum crack width should be estimated at the upper limit, order

\[
P\left\{ \frac{\omega_{\text{max,s}} - \bar{\omega}}{S/\sqrt{n}} \leq t(n-1, z_{1-p} \sqrt{n}) \right\} = P\left\{ \omega + \frac{t(n-1, z_{1-p} \sqrt{n}) \sigma}{\sqrt{n}} \geq \omega_{\text{max,s}} \right\} = C
\]

(3)

In the formula, \( t(n-1, z_{1-p} \sqrt{n}) \) is the upper \( 1-\alpha \) quantile of non-central t-distribution with \( n-1 \) as the degree of freedom and \( z_{1-p} \sqrt{n} \) as the non-central parameter. \( \alpha \) is the significance level; \( C \) is the confidence level, and \( C = 1 - \alpha \). From this, we can get the upper limit estimate, that is, the short-term maximum crack width is inferred as

\[
\omega_{\text{max,s}} = m + \frac{t(n-1, z_{1-p} \sqrt{n}) \sigma}{\sqrt{n}}
\]

(4)

Where: \( m \) and \( s \) are respectively the realization values of \( \bar{\omega} \) and \( S \).

3.2. Inference when coefficient of variation is known

When the coefficient of variation is known, the population standard deviation corresponding to the sample mean can be constructed through the sample mean, \( \sigma = m \cdot \delta \). In this case, construct the pivot

\[
\frac{\omega_{\text{max,s}} - z_{1-p} \sigma - \bar{\omega}}{\sigma/\sqrt{n}} \sim N(0,1)
\]

about \( \sigma \), order
\[ P \left( \frac{\omega_{\text{max},s} - z_{1-p} \sigma - \bar{\omega}}{\sigma/\sqrt{n}} \leq z_a \right) = P \left( \frac{\bar{\omega} + (z_{1-p} + z_{1-p}) \sigma}{\sqrt{n}} \geq \omega_{\text{max},s} \right) = C \]  

(5)

Where: \( z_a \) is the upper \( \alpha \) quantile of the standard normal distribution. Therefore, when the coefficient of variation is known, the short-term maximum crack width of the member is inferred as

\[ \omega_{\text{max},s} = m + \left( \frac{z_{1-C}}{\sqrt{n}} \right) \sigma \]  

(6)

Where: \( z_{1-C} \) is the upper \( 1-C \) quantile of the standard normal distribution.

### 4. Suggestions for relevant parameters

In addition to the mean and standard deviation, the final inference formula also contains some unknown quantities, such as the sample size \( n \), the confidence level \( C \), and the guarantee rate \( p \). Now we give some suggestions for value selection, and the designer can also make flexible adjustments according to his own needs.

#### 4.1. Sample size

In order to avoid the influence of statistical uncertainty, the minimum number of samples should be set in the inference. Paper 9 points out that the number of samples should not be less than 5, otherwise the relative error of the inference results will increase rapidly.

#### 4.2. Confidence level

Considering the variability of random variables and setting the confidence level \( C \) in inference, we can reasonably and quantitatively consider the influence of statistical uncertainty related to the variability of random variables and sample size according to different situations, so that the inference results have clear engineering significance.

When the variability of random variables is strong, if we select a larger \( C \), it often leads to an overly conservative result, so we should select a relatively smaller \( C \); on the contrary, when the variability of random variables is weak, we should select a relatively larger \( C \) to avoid too rash inference results.

The coefficient of variation of concrete material strength is 0.16-0.23, and the confidence level is 0.7 when inferring the standard value of concrete material strength in accordance with the GB 50292-2015. There are similar suggestions in international standard, the ISO 2394:2015 suggests that the confidence level should be lessen than 0.75.

For the inference of the maximum crack width of the member, no recommended confidence level is proposed in the domestic and foreign standards. It is known that the coefficient of variation of crack width of RC flexural member is about 0.4, according to the above principles, we should select a relatively small \( C \) value. However, considering that this method is a small sample inference, in order to avoid the impact of statistical uncertainty, the confidence level should not be too small, and it is recommended to obtain a guarantee rate of 0.7. However, considering that this method is a small sample inference, in order to avoid the influence of statistical uncertainty, the confidence level should not be too small, now, we recommend that the guarantee rate is 0.6.

#### 4.3. Guarantee rate

When the GB 50010-2012 calculates the short-term maximum crack width \( \omega_{\text{max},s} \), there is

\[ \omega_{\text{max},s} = \tau_s \cdot \bar{\omega} \]  

(7)

In the formula, \( \tau_s \) is the short-term crack width expansion coefficient, 1.66 is taken for flexural members, and the guarantee rate is about 95%. In this paper, we also set the guarantee rate at 0.95, when the confidence level is 0.6, and the coefficient of variation is 0.4, the short-term crack width expansion coefficient corresponding to different sample size were shown in Table 1.
Table 1. Short-term crack width expansion coefficient

| n  | τ  | n  | τ  | n  | τ  |
|----|----|----|----|----|----|
| 5  | 1.703262 | 14 | 1.685025 | 23 | 1.679072 | 32 | 1.675856 |
| 6  | 1.699313 | 15 | 1.684107 | 24 | 1.678627 | 33 | 1.675582 |
| 7  | 1.696244 | 16 | 1.683276 | 25 | 1.678209 | 34 | 1.675321 |
| 8  | 1.69377 | 17 | 1.68252 | 26 | 1.677816 | 35 | 1.675071 |
| 9  | 1.691721 | 18 | 1.681827 | 27 | 1.677444 | 36 | 1.674831 |
| 10 | 1.689988 | 19 | 1.68119 | 28 | 1.677093 | 37 | 1.674601 |
| 11 | 1.688496 | 20 | 1.680602 | 29 | 1.676760 | 38 | 1.674381 |
| 12 | 1.687195 | 21 | 1.680055 | 30 | 1.676443 | 39 | 1.674169 |
| 13 | 1.686048 | 22 | 1.679547 | 31 | 1.676142 | 40 | 1.673965 |

It can be seen that when the coefficient of variation is known, the short-term crack width expansion coefficient here is slightly greater than 1.66, but with the increase of the sample size, the gap gradually reduces, and finally approaches 1.658, which is more in line with 1.66.

4.4. Value table

For the convenience of application, when \( p = 0.95 \), \( C = 0.6 \), the value table of \( \frac{t_{(n-1, 0.95)} z_1}{\sqrt{n}} \) and \( \frac{z_{0.95} - z_{1-p}}{\sqrt{n}} \) are given here, which is shown in Table 2.

Table 2. Value table

| n  | \( \frac{t_{(n-1, 0.95)} z_1}{\sqrt{n}} \) | \( z_{0.95} + z_{1-p} \) | n  | \( \frac{t_{(n-1, 0.95)} z_1}{\sqrt{n}} \) | \( z_{0.95} + z_{1-p} \) | n  | \( \frac{t_{(n-1, 0.95)} z_1}{\sqrt{n}} \) | \( z_{0.95} + z_{1-p} \) | n  | \( \frac{t_{(n-1, 0.95)} z_1}{\sqrt{n}} \) | \( z_{0.95} + z_{1-p} \) |
|----|---------------------------------|------------------|----|---------------------------------|------------------|----|---------------------------------|------------------|----|---------------------------------|------------------|
| 5  | 2.065781                        | 1.703262         | 17 | 1.751860                        | 1.682520         | 29 | 1.722623                        | 1.676760         |
| 6  | 1.919009                        | 1.699313         | 18 | 1.748077                        | 1.681827         | 30 | 1.721140                        | 1.676443         |
| 7  | 1.865255                        | 1.696244         | 19 | 1.744672                        | 1.681190         | 31 | 1.719738                        | 1.676142         |
| 8  | 1.835165                        | 1.693770         | 20 | 1.741585                        | 1.680602         | 32 | 1.718410                        | 1.675856         |
| 9  | 1.815173                        | 1.691721         | 21 | 1.738770                        | 1.680055         | 33 | 1.717151                        | 1.675582         |
| 10 | 1.800595                        | 1.689988         | 22 | 1.736188                        | 1.679547         | 34 | 1.715954                        | 1.675321         |
| 11 | 1.793232                        | 1.688496         | 23 | 1.733809                        | 1.679072         | 35 | 1.714814                        | 1.675071         |
| 12 | 1.780253                        | 1.687195         | 24 | 1.731607                        | 1.678627         | 36 | 1.713728                        | 1.674831         |
| 13 | 1.772739                        | 1.686048         | 25 | 1.729561                        | 1.678209         | 37 | 1.712690                        | 1.674601         |
| 14 | 1.766375                        | 1.685025         | 26 | 1.727654                        | 1.677816         | 38 | 1.711698                        | 1.674381         |
| 15 | 1.760890                        | 1.684107         | 27 | 1.725870                        | 1.677444         | 39 | 1.710748                        | 1.674169         |
| 16 | 1.756097                        | 1.683276         | 28 | 1.724197                        | 1.677093         | 40 | 1.709838                        | 1.673965         |

5. Conclusion

(1) As the most basic probability characteristic, the probability distribution type has a very important influence on the inferred results. However, the statistical characteristics of fracture widths in the world have not been clearly studied. There are two methods to consider it, normal distribution and lognormal distribution. In this paper, 4665 pieces of short-term crack width data of RC flexural members are studied, and it is considered that they are more reasonable to obey the normal distribution.

(2) The design method based on observations and experimental models can be fully applied to the design of serviceability limit state. This paper proposes the inference of short-term maximum crack width of RC flexural members based on interval estimation method, and gives the value proposal and
value table of relevant parameters which can better meet the actual needs.

References
[1] International Organization for Standardization. (1998) ISO 2394:1998 General principles on reliability for structures. Geneva, Switzerland.
[2] International Organization for Standardization. (2015) ISO 2394:2015 General principles on reliability for structures. Geneva, Switzerland.
[3] BSI. (2002) EN 1990:2002 Basic of structure design. London, UK.
[4] Yao, J.T., Cheng, K.K. (2015) Review and improvement on methods for inferring structural resistance design value in international standard and European code. Journal of Building Structures, 36(01): 111-115.
[5] Lu, C.H. (2011) Flexural and durability performance of RC members with cracks. Zhejiang University, Hangzhou, China.
[6] Guan, J.F. (2010) Research on theory and application of simulation model test for reinforced concrete structure. Dalian University of Technology, Dalian, China.
[7] Xi, Y.L., Xu, J.S. (1989) Crack width calculation and probability analysis of weakly reinforced concrete members. Industrial Construction, 1989(10): 33-37+26.
[8] GB 50010-2012 Code for design of concrete structures. Beijing, China.
[9] Yao, J.T., Cheng, K.K., Liu, W. (2016) Statistical uncertainty and its impact on the establishment of structural performance model by testing. Journal of Xi’an University of Architecture & Technology(Natural Science Edition), 2016,48(5): 639-642.