Design of conformal composite absorber with non-uniform resistors via characteristic basis function method

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Abstract: A conformal composite absorber with non-uniform resistors was designed to achieve broadband absorption characteristics. An improved characteristic basis function method was proposed, which separates the load-resistors matrices in the inverse of impedance matrices by using the Woodbury matrix inverse lemma. The extraction of characteristic basis functions was accelerated, and the total time of solving the matrix equation was reduced obviously. Finally, a prototype was fabricated and measured with the optimized resistors. The measured results showed that proposed conformal composite absorber achieved good absorption in the 2.8–8.0 GHz for different polarized incident wave.

Keywords: broadband, conformal composite absorber, characteristic basis function method (CBFM), non-uniform resistors

Classification: Microwave and millimeter-wave devices, circuits, and modules

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Conformal absorbing materials can be mounted on various aircraft to reduce radar cross section (RCS) without disturbing their aerodynamic characteristics [1]. In recent years, electromagnetic metamaterials have attracted special attention, due to their characteristics of chiral, frequency selection and absorption characteristics [2, 3, 4]. Composite absorbers with planar periodic structure were extensively researched to achieve broadband absorption or selective absorption in [5, 6, 7, 8]. In addition, several conformal composite absorbers (CCA) were realized to achieve narrow-band absorption [9, 10]. However, they have rarely been reported as conformal radar absorbers with broadband absorption.
When the CCA is wrapped on a curved structure, the fundamental characteristics of planar periodic structures, namely the periodicity and the infinite extent, are lost [11, 12]. It makes the analysis and design of CCA more difficult. The equivalent absorption boundary method and full-wave analysis method are the main methods for analyzing such problems [13, 14, 15]. The full-wave analysis method can analyze the detail structure of absorbing materials, and provide accurate current results, but it needs large computation cost. The moment method, a kind of full-wave analysis method, has more advantages for solving scattering problems compared with other full-wave analysis method. The characteristic basis function method (CBFM) reduces the dimension of the matrix equation formed by moment method, via decomposing the large structure into blocks. It has higher computing efficiency [16, 17, 18, 19] when solving electromagnetic problems such as frequency selection surfaces and microstrip antenna arrays.

In this paper, a CCA was designed by using double hexagonal loops (DHL) with non-uniform resistors. An improved CBFM was proposed, which separated the load-resistors matrices in the inverse of impedance matrices by Woodbury matrix inverse lemma. Compared with the traditional CBFM, it accelerated extracting the characteristic basis functions, and reduced the total time of solving the matrix equation obviously. The values of non-uniform resistors were optimized by a genetic algorithm and verified by HFSS software. Finally, a prototype was fabricated and measured. The measured results showed that the proposed CCA overcame the effect of bending and achieved good broadband absorption.

2 Design and optimization of CCA

DHL was chosen because of its good polarization insensitivity and dense lattice distribution in planar structure [5]. An array of DHL with a period of $p$ is printed on a conductor-backed dielectric substrate, as shown in Fig. 1(a). The structure of DHL-CCA is showed as Fig. 1(b). The substrate has a relative permittivity of $e_r$ and a thickness of $t_1$. The thickness of the air layer is $t_2$. The length of each side of the outer loop is $d_1$, and that of the inner loops is $d_2$. The width of outer loop is $s_1$, and that of inner loops is $s_2$. The resistors are inserted into each side of hexagonal loops. The resistors in the outer loop and the inner loop of the $i$-th unit have the values of $R_{OUT}^i$ and $R_{IN}^i$, respectively.

![Fig. 1. DHL-CAA (a) array (b) structure](image-url)
With this type, a CAA with non-uniform resistors was proposed to overcome the effect of bending on the absorption properties. An improved CBFM was also proposed and applied in the optimization of non-uniform resistors. The electric field integral equation was established for solving the scattering problem of the CCA, and then it was converted into a matrix equation by the moment method of Rao-Wilton-Glisson (RWG) basis function. According to the outline of the unit (the dotted line as shown in Fig. 1), the CCA was partitioned into hexagonal sub-block regions with number of $M$. To ensure continuity of the current on the metal plate, the sub-block area was expanded outward [17]. The number of the RWG basis functions in each sub-block was $N_e$. Since the loaded resistors affected only the elements on the main diagonal [20], the CBFM-based matrix equation of the CCA was expressed as follows

$$
\begin{bmatrix}
Z_{11} + Z_{11}^R & Z_{12} & \cdots & Z_{1M} \\
Z_{21} & Z_{22} + Z_{22}^R & \cdots & Z_{2M} \\
\vdots & \ddots & \ddots & \vdots \\
Z_{M1} & Z_{M2} & \cdots & Z_{MM} + Z_{MM}^R
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_M
\end{bmatrix}
= 
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_M
\end{bmatrix}
$$

(1)

where $Z_{ij}$ was the mutual impedance matrix of the $j$-th sub-block against the $i$-th sub-block, which was only generated by metal and dielectric. In the $i$-th sub-block ($i, j \in 1, 2, \ldots, M$), the current solution was expressed as $I_i$, the source was $V_i$, and the load-resistors matrix was $Z_{i}^R$.

Then we extracted the primary characteristic basis functions. The number of resistors in each unit was $N_R$. When the resistors were loaded in the units, they can be represented as only $N_R$ values adding onto the main diagonal of $Z_{ii}$, so each load-resistors matrix was a sparse diagonal array of rank $N_R$, and it has $N_R < N_e$. The primary characteristic basis function $I_i^P$ in $i$-th sub-block can be expressed as

$$
I_i^P = (Z_{ii} + Z_{ii}^R)^{-1}V_i
$$

(2)

In the way similar to the singular value decomposition, we selected two appropriate matrices $U$ and $\Psi$ (whose sizes were $N_R \times N_e$) to decompose $Z_{ii}^R$, so we got $Z_{ii}^R = U\Lambda\Psi^H$, where $\Lambda$ was the $N_R$ order diagonal matrix, the superscript of $\Psi^H$ meant conjugate transpose of $\Psi$. Using the Woodbury matrix inverse lemma, we decomposed the inverse of the sum of $Z_{ii}$ and $Z_{ii}^R$ in Eq. (2) as follows

$$
(Z_{ii} + Z_{ii}^R)^{-1} = Z_{ii}^{-1} - Z_{ii}^{-1}U(E + \Lambda\Psi^HZ_{ii}^{-1}U)^{-1}\Lambda\Psi^HZ_{ii}^{-1}
$$

(3)

where $E$ was an $N_R$ order unit matrix. Since each unit had the same metal and dielectric structure, the self-coupled impedance matrix $Z_{ii}$ of each sub-block was the same. However, the load-resistors matrix $Z_{ii}^R$ of each sub-block was different. By decomposing the inverse of the sum of the matrices in Eq. (3), the inversion of $N_e$ order matrices for $M$-times was simplified to the inversion of $N_R$ order matrices for once and the inversion of $N_R$ order matrices for $M$-times. The amount of computation can be reduced in the extraction of characteristic basis functions.

Based on the Foldy-Lax equations in which mutual-coupling effects among all scatterers can be included, the secondary basis functions $I_i^{S1}$ and $I_i^{S2}$ were extracted as follows, where the inverse matrices for solving the secondary basis functions have been solved from above in Eq. (3).
\[ I_i^{S1} = -(Z_{ii} + Z_{ii}^R)^{-1} \sum_{j=1(j \neq i)}^{M} Z_{ij}I_j^p \]
\[ I_i^{S2} = -(Z_{ii} + Z_{ii}^R)^{-1} \sum_{j=1(j \neq i)}^{M} Z_{ij}I_j^{S1} \]

The second-order secondary basis functions were adopted to ensure calculation accuracy. Using the characteristic basis functions to reduce Eq. (1), a 3M × 3M matrix equation was obtained, where the \(ij(i \neq j)\) region in the matrix equation can be expressed as follows, and in the \(ij(i = j)\) region, the \(Z_{ij}\) element in Eq. (5) should be replaced with the \((Z_{ii} + Z_{ii}^R)\).

\[
\begin{pmatrix}
    \vdots & \vdots & \vdots \\
    [I_i^p]^H Z_{ij}I_j^p & [I_i^{S1}]^H Z_{ij}I_j^{S1} & [I_i^{S2}]^H Z_{ij}I_j^{S2} \\
    \vdots & \vdots & \vdots \\
    [I_i^p]^H Z_{ij}I_j^p & [I_i^{S1}]^H Z_{ij}I_j^{S1} & [I_i^{S2}]^H Z_{ij}I_j^{S2} \\
    \vdots & \vdots & \vdots \\
    \vdots & \vdots & \vdots \\
\end{pmatrix}
\begin{pmatrix}
    a_i \\
    b_i \\
    c_i \\
\end{pmatrix}
= \begin{pmatrix}
    \vdots \\
    \vdots \\
    \vdots \\
\end{pmatrix}
\]

In order to verify the efficiency, the proposed method was compared with direct solution and traditional CBFM. There were two matrix equations for different size of CCA, one had the array of \(M = 11 \times 11\) with \(N_e = 112\) for each sub-block, and the other had the array of \(M = 13 \times 11\) with \(N_e = 300\). The number of resistors in each unit was \(N_R = 12\). The total time of solving these matrix equations was calculated at a single frequency point (3 GHz).

### Table I. Time comparison of solving matrix equation with different methods

| \(M \times N_e\)          | Direct solution | traditional CBFM | The improved CBFM |
|---------------------------|-----------------|-------------------|-------------------|
| \(M = 11 \times 11\)      | 108.5 s         | 4.2 s             | 2.6 s             |
| \(N_e = 112\)             |                 |                   |                   |
| \(M = 13 \times 11\)      | 7050.6          | 70.89 s           | 40.89 s           |
| \(N_e = 300\)             |                 |                   |                   |

As shown in Table I, compared with the traditional CBFM, the solving time of two matrix equations was reduced by 38.1% and 42.3%, respectively. As the ratio of \(N_e\) to \(N_R\) increased, the reduction of the solving time was more significant. In particular, during the process of optimizing, the metal and dielectric structure was unchanged, and the \(Z_{ij}\) of each sub-block just needed to be calculated once. Only the load-resistors matrix \(Z_{ii}^R\) needed to be updated with the step of optimization. Using the improved CBFM which separated load-resistors matrices in the inverse of the impedance matrices, the optimization efficiency can be improved further.

The current solution of each sub-block was calculated by
\[
I_i = a_iI_i^p + b_iI_i^{S1} + c_iI_i^{S2}
\]
where the undetermined coefficient \(a_i, b_i, c_i\) was obtained by solving Eq. (5). Then the RCS of the CCA can be obtained. The absorption properties of CCA were
represented by the reduction ratio of RCS in normal incidence [21]. It was expressed as \( \Delta_{RCS} \) and calculated as follows,

\[
\Delta_{RCS} = \frac{\delta_C}{\delta_M}
\]  

(7)

where \( \delta_C \) was the RCS of CCA, \( \delta_M \) was the RCS of cylindrical conduction backboard with same size.

Finally, the relationship between the absorption properties of \( \Delta_{RCS} \) and all values of the non-uniform resistors was established. An objective function was set up to make \( \Delta_{RCS} < -10 \) dB in the desired absorption band. The values of resistors had \( 2M \) variables, and they had a complicated function relationship with \( \Delta_{RCS} \), so the genetic algorithm was used to optimize this problem. The optimization process was not described in detail here.

3 Simulation and measured results

In order to verify the accuracy of the proposed method, a DHL-CCA was designed and analyzed as an example. It was proposed with: \( \varepsilon_r = 2.2, \ p = 25.98 \) mm, \( d_1 = 13.5 \) mm, \( d_2 = 7 \) mm, \( s_1 = s_2 = 0.5 \) mm, \( t_1 = 0.5 \) mm, \( t_2 = 13 \) mm. When using the uniform resistors in the planar composite absorbers or CCA, all units had the same resistors with \( R^{\text{OUT}} = 180 \) \( \Omega \) and \( R^{\text{IN}} = 100 \) \( \Omega \). When using the non-uniform resistors in the CCA, the values of non-uniform resistors were optimized by the genetic algorithm.

For the convenience of simulation and fabrication, the CCA was designed as a finite periodic structure with arrays of \( M = 13 \times 11 \), whose size was 300 \( \times \) 300 mm. As shown in Fig. 2(a), there were 13 units staggered in horizontal (x-axis) and 11 units distributed in vertical (y-axis). Then it was warped on the cylindrical carrier, whose radius of curvature was \( r = 120 \) mm, as shown in Fig. 2(b). Since the cylindrical CCA was only bended horizontally, the horizontal units were different with non-uniform resistors, and the vertical units were same. The values of resistors were optimized and rounded as \( R^{\text{OUT}} = [167, 394, 313, 362, 234, 378, 258, 378, 234, 362, 313, 394, 167], \ R^{\text{IN}} = [241, 200, 72, 65, 68, 246, 108, 246, 68, 65, 72, 200, 241], \) with the unit of \( \Omega \).

The planar composite absorber, the CCA with uniform resistors and the CCA with non-uniform resistors were simulated by the improved CBFM, and they were also simulated by HFSS software as a comparison. The absorbers were illuminated...
by normal incident wave. The electric field of the vertically polarized incidence was along the axis of the cylinder, and that of the horizontally polarized was along the direction of bending. The absorption of horizontally and vertically polarized incidence was showed in Fig. 3(a) and (b), respectively. It was seen that the results simulated by the improved CBFM were in good agreement with the results given by HFSS, which verified the accuracy of the proposed method.

Then we analyzed the absorption of three kinds of absorbers. The planar composite absorber with the parameters above realized good absorption for both vertically and horizontally polarized incidence in the 2.1–8.9 GHz with $\Delta RCS < -10$ dB. When the CCA with uniform resistors was covered on the cylindrical carrier, the absorption bandwidth for horizontally polarized incidence was reduced to 2.7–8.2 GHz, and there was a peak of $-6.37$ dB at 4.3 GHz. The absorption bandwidth for vertically polarized incidence was reduced to 2.2–7.2 GHz. It was because the bending and the truncation make the amplitude and phase change non-uniformly. Since the incident angle on the units varied from one to another, the multiple scattering spectrums were excited. To overcome this phenomenon, the CCA with non-uniform resistors was optimized. Its absorption bandwidth for horizontally polarized incidence was simulated as 2.6–8.2 GHz, and that for vertically polarized incidence was 2.4–7.5 GHz. Although the improvement of bandwidth was small, it compensated the poor absorption properties in 3.5–4.5 GHz.

The non-uniform resistors CCA was fabricated and measured. The DHL with the array of $M = 13 \times 11$ were printed on a FR4 dielectric substrate with the dielectric constant of $\varepsilon_r = 2.2$. The measured state was showed in Fig. 4. The
incident wave direction was $k$, and $E_x$ and $E_y$ were the electric field directions of the horizontally and vertically polarized incidence, respectively. The resistors were chosen as the 0403 type chip resistors with a precision of 1%. Under the constraint of available value of resistors, the value of non-uniform resistors were $R_{OUT} = [165, 392, 300, 365, 232, 374, 261, 374, 232, 365, 300, 392, 165]$, $R_{IN} = [240, 200, 75, 62, 68, 240, 107, 240, 68, 72, 75, 200, 240]$. The measurement results were shown in Fig. 5. They were in good agreement with the simulation results in Fig. 3. The absorption properties were improved in the 3–4.5 GHz band, and the effect of bending and truncation on broadband absorption was reduced. The absorption bandwidth with $\Delta_{RCS} < -10$ dB for the horizontally polarized incidence was 2.8–8.8 GHz, and that for the vertically polarized incidence was 2.3–8.0 GHz. It realized good broadband absorption characteristics was 2.8–8.0 GHz for different polarized incident wave.

![Fig. 5. Comparison of measured results (a) horizontally polarized incidence, (b) vertically polarized incidence](image)

4 Conclusions

A conformal DHL-CAA with non-uniform resistors was designed. An improved CBFM was proposed to accelerate extracting of the characteristic basis functions. Compared with the traditional CBFM, it reduced the total time of solving the matrix equation obviously. The values of non-uniform resistors were optimized by the genetic algorithm. The results simulated by the proposed method were in agreement with that given by HFSS, and the accuracy was verified. Finally, a prototype was fabricated and measured. The measured results showed that the proposed CCA overcame the effect of bending on broadband absorption. It achieved good broadband absorption for different polarized incident waves. The absorption bandwidth with $\Delta_{RCS} < -10$ dB for the horizontally polarized incident was 2.8–8.8 GHz, and that for the vertically polarized incident was 2.3–8.0 GHz. The proposed design and optimization method have reference significance for reducing the stealth design of single-station RCS of aircraft targets.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under the contract number 61601492.