Extremal \( t \) processes: Elliptical domain of attraction and a spectral representation

T. Opitz

ISM (CC51), Université Montpellier 2
Place Eugène Bataillon
34095 Montpellier cedex 5

Abstract

The extremal \( t \) process was proposed in the literature for modeling spatial extremes within a copula framework based on the extreme value limit of elliptical \( t \) distributions (Davison, Padoan and Ribatet (2012)). A major drawback of this max-stable model was the lack of a spectral representation such that for instance direct simulation was infeasible. The main contribution of this note is to propose such a spectral construction for the extremal \( t \) process. Interestingly, the extremal Gaussian process introduced by Schlather (2002) appears as a special case. We further highlight the role of the extremal \( t \) process as the maximum attractor for processes with finite-dimensional elliptical distributions. All results naturally also hold within the multivariate domain.

Keywords: elliptical distribution, extremal \( t \) process, max-stable process, spectral construction

1. Introduction

Davison et al. (2012) survey the statistical modeling of spatial extremes and provide a global view on available models and their interconnections. Among these models, the extremal \( t \) process represents a max-stable process that generalizes the \( t \) extreme value copula to infinite dimension. It is well defined, yet no direct construction was known back then which lead the authors to class it among copula models characterized by their motivation from multivariate considerations. An application to Swiss rainfall data in that paper bears witness of its versatility for extremal dependence modeling. In the following, we show that the extremal \( t \) process provides a natural connection between two prominent max-stable model classes, namely Schlather’s extremal Gaussian process (Schlather (2002)) and the Brown-Resnick process defined in Brown and Resnick (1977) and revisited in a more general context in Kabluchko et al. (2009). The connection to the Brown-Resnick process was detailed for the multivariate context in Nikoloulopoulos et al. (2009) and is related to the study of elliptical triangular arrays with the Hüsler-Reiss distribution (Hüsler and Reiss (1989), Falk et al. (2011)) as the maximum attractor (Hashorva (2005)). It was then interpreted for the spatial context in Davison et al. (2012). The extremal \( t \) dependence structure is further proposed for semi-parametric inference in a multivariate context by Klüppelberg et al. (2007) and Klüppelberg et al. (2008). We conceive a spectral representation for the extremal \( t \) process that generalizes
the one of the extremal Gaussian process. It renders direct simulation possible for moderately large general degrees of freedom.

The remainder of the paper is organized as follows: Section 2 gives some background in extreme value theory and reviews results for elliptical distributions. Spectral constructions of multivariate extremal t distributions and extremal t processes are presented in Section 3, along with a statement on the domain of attraction for processes with finite-dimensional elliptical distributions. We conclude with a discussion and potential future developments in Section 4.

The following notational conventions shall apply in the remainder of the paper: If not stated otherwise, operations on vectorial arguments like maxima or arithmetic operations must be interpreted componentwise. Vectors are typeset in bold face, in particular the vector constants \( \mathbf{0} = (0, \ldots, 0)^T \) and \( \mathbf{1} = (1, \ldots, 1)^T \). Rectangular bounded or unbounded sets are given according to notations like \([u, v] = [u_1, v_1] \times \cdots \times [u_d, v_d]\) or \((0, \infty) = (0, \infty) \times \cdots \times (0, \infty)\). The complementary set of a set \( B \) in \( \mathbb{R}^d \) is written \( B^c \). The truncation operator \( x^+ = \max(x, 0) \) maps negative values to 0. The indicator function of a set \( B \) is denoted by \( \chi_B(\cdot) \).

2. Extreme value theory

For a more detailed account of max-stability and extreme value theory in general we refer the reader to the textbooks of Beirlant et al. (2004) and de Haan and Ferreira (2006).

2.1. Max-stability

Let \( Z, Z_1, Z_2, \ldots \) be a sequence of independent and identically distributed (iid) random vectors in \( \mathbb{R}^d \) \((d \geq 1)\) with nondegenerate univariate marginal distributions. We say that \( Z \) follows a max-stable distribution \( G \) if sequences of normalizing vectors \( a_n > \mathbf{0} \) and \( b_n \) \((n = 1, 2, \ldots)\) exist such that the equality in distribution

\[
\max_{i=1, \ldots, n} a_n^{-1}(Z_i - b_n) \overset{d}{=} Z \sim G
\]

holds for the componentwise maximum. A full characterization of multivariate max-stable distributions leads to rather technical expressions. For our purposes, it is convenient to focus on common \( \alpha \)-Fréchet marginal distributions \( G_j(z_j) = \Phi_a(z_j) = \exp(-z_j^{-\alpha})\chi_{(0,\infty)}(z_j) \) \((j = 1, \ldots, d)\) for some tail index \( \alpha > 0 \). Monotone and parametric marginal transformations allow reconstructing all admissible univariate max-stable marginal scales in \( \mathbb{1} \) from this particular marginal scale. More precisely, the class of univariate max-stable distributions is partitioned into the class of \( \alpha \)-Fréchet distribution under strictly increasing linear transformations and further the so-called Gumbel and Weibull classes.

With \( \alpha \)-Fréchet marginal distributions, the standard exponent measure \( \mathbb{M} \) can be defined on \([0, \infty) \setminus \{0\}\) by \( \mathbb{M}((0, z]) = -\log \mathbb{P}(Z^a \leq z) \) with the convention \( -\log 0 = \infty \) and characterizes the dependence structure in \( G \) on a standardized scale; it is uniquely defined by the dependence function \( M(z) = \mathbb{M}((0, z]) \) which takes the value \( \infty \) whenever \( \min z_j = 0 \) such that \( z \notin (0, \infty) \). The extremal coefficient \( M(1) \in [1, d] \) can serve as an indicator of the strength of extremal dependence, ranging from full dependence associated with the value 1 to independence associated with the value 0 (cf. Schlather and Tawn (2003)).

In the infinite-dimensional domain, we call a stochastic process \( Z = \{Z(s), s \in S \subset \mathbb{R}^p\} \) \((p \geq 1)\) with a non-empty Borel set \( S \) max-stable if its finite-dimensional distributions are max-stable. If \( Z_1, Z_2, \ldots \) are iid copies of \( Z \), then sequences of functions \( a_n(s) > 0 \) and \( b_n(s) \) \((n \geq 1)\) exist such that \( \{\max_{i=1, \ldots, n} a_n(s)^{-1}(Z_i(s) - b_n(s))\} \overset{d}{=} \{Z(s)\} \).
2.2. Domain of attraction

Let \(X, X_1, X_2, \ldots\) be a sequence of iid random vectors in \(\mathbb{R}^d\) with distribution function \(F\). For suitably chosen normalizing sequences, relation (1) can hold asymptotically in the sense of distributional convergence with nondegenerate marginal distributions in the limit \(Z\):

\[
\max_{i=1, \ldots, n} a_n^{-1}(X_i - b_n) \xrightarrow{d} Z \quad (n \to \infty).
\]

We say that the distribution \(F\) of \(X\) is in the max-domain of attraction (MDA) of the max-stable distribution \(G\) of \(Z\), or simply that \(X\) is in the MDA of \(Z\). Normalizing sequences are not unique and the limit distribution \(G\) is unique up to a linear transformation. If normalizing constants can be chosen such that all the univariate marginal distributions \(G_j\) are of the same \(\alpha\)-Fréchet type, then the particular choice of \(b_n = 0\) is admissible. In this case, the convergence in distribution (2) is equivalent to

\[
n \mathbb{P}(a_n^{-1}X \leq z) \to M(z^n) \quad \text{for all } z \in (0, \infty).
\]

For \(d = 1\), we have \(n \mathbb{P}(a_n^{-1}X \geq z) \to z^{-\alpha} (z > 0)\), and then \(X\) is said to be regularly varying at \(\infty\) with index \(\alpha > 0\) or just regularly varying in the remainder of this paper, denoted as \(X \in RV_\alpha\). The normalizing sequence can be chosen as \(a_n = \inf\{x : P(X \geq x) \leq n^{-1}\}\).

For stochastic processes, the notion of MDA is defined in the sense of the convergence of all finite-dimensional distributions according to (2).

2.3. A spectral representation for max-stable processes

The commonly used models for max-stable processes are generated with so-called spectral constructions whose first appearance dates back to the seminal paper of de Haan (1984). Schlather (2002) proposes to use a Poisson process \(\{V_i\} \sim \text{PRM}(v^{-2}dv)\) on \((0, \infty)\) and iid replicates \(Q_i\) of an integrable random process \(Q\), independent of \(\{V_i\}\) and with \(\mathbb{E}Q_i(s) = 1 (s \in S)\), in order to construct the max-stable process

\[
Z = \{Z(s)\} = \left\{ \max_{i=1, 2, \ldots} V_i Q_i(s) \right\} \quad (s \in S)
\]

with univariate marginal distributions of type \(\Phi_1\). It is possible to replace \(Q_i(s)\) by the zero-truncated value \(Q_i^+(s)\) in this construction. Subsequently, without loss of generality we assume that the points \(V_i\) are in descending order such that \(V_1 \geq V_2 \geq \ldots\) and \(V_1 \sim \Phi_1\). We obtain extremal Gaussian processes by choosing a centered and appropriately scaled Gaussian process \(W\) for \(Q\). Other choices of \(Q\) were considered (cf. Davison et al. (2012)), leading for instance to so-called Brown-Resnick processes (Brown and Resnick (1977), Kabluchko et al. (2009)). The dependence function of \(Z\) for a finite number of points \(s_1, \ldots, s_d \in S\) is

\[
M_{s_1, \ldots, s_d}(z) = \mathbb{E} \max_{j=1, \ldots, d} \left( z_{j}^{-1}Q^+(s_j) \right).
\]

2.4. Multivariate t distributions and extremal dependence

2.4.1. Elliptical distributions

**Definition 2.1** (Elliptically distributed random vectors). A random vector \(X\) in \(\mathbb{R}^d\) is said to follow a (non-singular) elliptical distribution if it allows for a stochastic representation \(X \overset{d}{=} \mu + RdA U\) with a deterministic location vector \(\mu\), an invertible \(d \times d\) matrix \(A\) that defines the dispersion matrix \(\Sigma = AA^T = (\sigma_{j_1, j_2})_{1 \leq j_1, j_2 \leq d}\) and a nondegenerate random variable \(R\) independent from a random vector \(U\) uniformly distributed on the Euclidean unit sphere \(\{x \in \mathbb{R}^d \mid x^T x = 1\}\). We call \(R\) the radial variable.
Here we prefer to remain within the framework of quadratic and non-singular $A$ to avoid an overly technical presentation for the more general cases (cf. Anderson and Fang (1990) for the general representation). Random vectors following an elliptical multivariate $t$ distribution are an important example: We say that an elliptically distributed random vector $X$ in $\mathbb{R}^d$ follows the multivariate $t$ distribution with $\nu > 0$ (general) degrees of freedom if $d^{-1}R_d^2 \sim F_{d,\nu}$, where $F_{d,\nu}$ is the $F$-distribution with degrees of freedom $d$ and $\nu$. We write $X \sim_{\nu}(\mu, \Sigma)$ and $\mathbb{P}(X \leq x) = t_{\nu}(x \mid \mu, \Sigma)$. The multivariate $t$ distribution can be constructed as a variance mixture of the multivariate normal distribution: With $\nu Y^{-1} \sim \text{Gamma}(0.5\nu, 2)$ ($\nu > 0$) and a multivariate normal random vector $W \sim N(0, \Sigma)$ that is independent of $Y$, we obtain $\mu + \sqrt{\nu}W \sim t_{\nu}(\mu, \Sigma)$, see for instance Demarta and McNeil (2005). This construction is readily generalized to the infinite-dimensional setting on a domain $S$: If $W$ is a centered Gaussian process with domain $S$ and covariance function $\text{Cov}$ and $\nu Y^{-1} \sim \text{Gamma}(0.5\nu, 2)$ independent of $W$, then we call the random process $\sqrt{\nu}W$ a (centered) $t$ random process on $S$ which is characterized by the degree of freedom $\nu$ and the dispersion function $\text{Cov}$ (cf. Reislien and Omre (2006)).

2.4.2. The maximum attractor

The multivariate $t$ distribution fulfills the MDA condition (2). For normalizing constants $b_n = 0$ and $a_n = n^{1/\nu}(\sigma_j^{2, 1/\nu})^j = 1, \ldots, d$ with $c_o = \Gamma(0.5(\nu + 1))^{-1}\nu^{-0.5\nu+1}\sqrt{\nu}\Gamma(0.5\nu)$ (cf. Table 2.1 on page 59 in Beirlant et al. (2004)), we obtain $\nu$-Fréchet marginal distributions in the max-stable limit distribution $G$ of the multivariate $t$ distribution $t_{\nu}(\mu, \Sigma)$. The dependence function of $G$ was derived by Nikoloulopoulos et al. (2009). Denote by $\Sigma^* = (\sigma_{j,j}^*)_{j=1,\ldots, d}$ the correlation matrix that corresponds to the dispersion matrix $\Sigma$, and by $\Sigma^*_{-j,-j} = (\sigma_{j,j}^*)_{j \neq j, j \neq j}$ or $\Sigma^*_{-j,-j} = (\Sigma_{-j,-j})^T = (\sigma_{j,j}^*)_{j \neq j, j \neq j}$ submatrices obtained by removing some of the rows or columns. Similarly for vectors, we write $z_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_d)^T$. Then

$$M_{\nu, \Sigma^*}(z) = \sum_{j=1}^{d} z_j^{-1} t_{\nu+1} \left( (z_{-j}/z_j)^{-1 \nu^{-1}} | \Sigma^*_{-j,-j}, (\nu + 1)^{-1} (\Sigma^*_{-j,-j} - \Sigma^*_{-j,-j} \Sigma^*_{-j,-j}^T) \right).$$

We refer to the max-stable limit as extremal $t$ distribution, and we call extremal $t$ process the max-stable limit of a $t$ random process which is a generalization of the extremal $t$ distribution. Its dependence structure is characterized by the set of dependence functions for all finite-dimensional distributions, hence by a general degree of freedom $\nu > 0$ and the correlation function

$$\text{Cov}^*(s_{j_1}, s_{j_2}) = [\text{Cov}(s_{j_1}, s_{j_1}) \text{Cov}(s_{j_2}, s_{j_2})]^{-0.5} \text{Cov}(s_{j_1}, s_{j_2})$$

which corresponds to the dispersion function $\text{Cov}$ and determines the matrices $\Sigma^*$ for all finite-dimensional distributions. Similar to the copula approach in the multivariate domain, it is convenient to call extremal $t$ process any max-stable process whose set of dependence functions is the same.

In general, a regularly varying radial variable $R_d \in \text{RV}_\alpha$ ensures the MDA condition for an elliptical distribution and is a necessary and sufficient condition for the presence of asymptotic dependence, see Theorem 4.3 in Hult and Lindskog (2002). Moreover, Theorem 3.1 of Hashorva (2006) establishes the equivalence of regular variation of $R_d$ to regular variation of any component $X_j$ of $X$. The index of regular variation $\alpha$ is the same across components and the radial variable, and the dependence function $M$ depends only on $\alpha$ and the correlation matrix $\Sigma^*$. For elliptical distributions in the Gumbel or Weibull MDA, one always obtains independence in the max-stable limit $G$. Since the multivariate $t$ distribution covers
the full range of indices \( \alpha > 0 \) (equal to the general degree of freedom \( \nu \)) and correlation matrices \( \Sigma^* \), the extremal \( t \) dependence structure is exhaustive within the class of asymptotically dependent elliptical distributions.

3. Main results

The following theorem establishes the extremal \( t \) process as the maximum attractor for processes with finite-dimensional elliptical distributions and asymptotic dependence.

**Theorem 3.1 (Elliptical domain of attraction).** Let \( X = \{ X(s), s \in S \} \) be a random process that has finite-dimensional elliptical distributions according to the dispersion function \( \text{Cov} \). Suppose that \( |\text{Cov}^*(s_{j1}, s_{j2})| < 1 \) for all \( s_{j1} \neq s_{j2} \), where \( \text{Cov}^* \) is the correlation function that corresponds to \( \text{Cov} \). Assume that one of the two following conditions is fulfilled for \( X \):

- At least one of the finite-dimensional distributions for \( d \geq 2 \) is in a multivariate MDA with asymptotic dependence.
- At least one of the univariate marginal distributions of \( X \) is regularly varying.

Then a max-stable limit process \( Z \) exists for \( X \) and is an extremal \( t \) process. Its dependence functions for finite-dimensional distributions are given by (6).

**Proof.** We play on the equivalence of the regular variation condition for the radial variable or for one of the components. It is clear from Theorem 4.2 in [Hult and Lindskog 2002](#) (in the following referred to as HL) and the equivalence that our first condition entails the second one. Now assume \( X(s_0) \) is regularly varying. For any bivariate vector \( (X(s_0), X(s))^T \) with \( s \neq s_0 \), the equivalence dictates that the radial variable \( R_2 \) associated to the bivariate random vector is regularly varying, hence there is asymptotic dependence due to HL. Since \( X(s) \) is also regularly varying, we can iterate this argument for all bivariate vectors \( (X(s_1), X(s_2)) \) with \( s_1 \neq s_2 \) to prove that there is bivariate asymptotic dependence. Applying anew HL for any collection of distinct sites \( s_1, ..., s_d \) yields regular variation for the associated radial variable \( R_d \). Consequently, all finite-dimensional distributions of \( X \) possess max-stable limit distributions of the extremal \( t \) type which then constitute the extremal \( t \) limit process \( Z \).

We now provide a multivariate spectral construction for the extremal \( t \) distribution based on elliptical distributions.

**Theorem 3.2 (Multivariate spectral construction).** Suppose the following items are given:

- a tail index \( \alpha > 0 \) and a correlation matrix \( \Sigma^* \),
- iid replications \( X_i \) of an elliptically distributed random vector \( X = (X_1, ..., X_d) \) with dispersion matrix \( \Sigma = \Sigma^* \) and location vector \( \mu = 0 \) such that the expectation \( m_\alpha = \mathbb{E}[(X_1^\top)^\alpha] \) is non-null and finite and
- a Poisson process \( \{ V_i \} \sim \text{PRM}(\alpha \nu^{-(\alpha+1)} dv) \) on \((0, \infty)\).

Define the componentwise maximum

\[
Z = m_\alpha^{-1} \max_{i=1,2,...} V_i X_i.
\]

Then \( Z \) follows the extremal \( t \) distribution with \( \alpha \)-Fréchet marginal distributions and dependence function \( M_{\alpha, \Sigma^*} \).

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Proof. Due to infinite number of points in the Poisson process \( \{ V_i \} \), we can replace \( X \) by \( X^+ \) in the construction \([7]\). By taking \( Z \) to the power of \( \alpha \), i.e. \( m_\alpha^{-1} \max_{i=1,2,\ldots} V_i = (X^+)^\alpha \), we obtain a special case of the construction \([9]\) which proves the max-stability and the \( \alpha \)-Fréchet marginal distributions of \( Z \). Since \( V_i \sim \Phi_\alpha \), the radial variable in the elliptical random vector \( V_i X_i \) is regularly varying with index \( \alpha \) due to the variant of Breiman’s theorem from Lemma 2.3 in Davis and Mikosch (2008); thus \( V_i X_i \) is in the MDA of the extremal \( t \) distribution. At the same time, \( V_i X_i \) is in the MDA of \( Z \) according to Lemma 3.1 in Segers (2012). We conclude that \( Z \) follows the extremal \( t \) distribution with dependence function \([36]\).

As a direct application of Theorem \([32,39]\) we are now able to present one possible spectral representation of extremal \( t \) processes via the corresponding Gaussian process.

**Corollary 3.1 (Spectral representation of extremal \( t \) processes).** Suppose the following items are given:

- a tail index \( \alpha > 0 \) and a correlation function \( \text{Cov}^* \),
- iid replications \( W_i \) of a standard Gaussian random field \( W \) on \( S \subset \mathbb{R}^p \) with dispersion function \( \text{Cov} = \text{Cov}^* \) and
- a Poisson process \( \{ V_i \} \sim \text{PRM}(\alpha_\nu^{-1}) \) on \( (0, \infty) \).

Then the process defined by

\[
Z = \{ Z(s) \} = \left\{ m_\alpha^{-\alpha^{-1}} \max_{i=1,2,\ldots} V_i W_i(s) \right\} \quad (s \in S),
\]

with \( m_\alpha = \sqrt{\pi}^{-1} 2^{0.5(\alpha-2)} \Gamma(0.5(\alpha + 1)) \) and \( \Gamma(\cdot) \) the Gamma function, is an extremal \( t \) process with \( \alpha \)-Fréchet marginal distributions. Its dependence structure is characterized by a general degrees of freedom and the correlation function \( \text{Cov}^* \).

Proof. It remains to verify the value of \( m_\alpha \). Using the variable transformation \( y = 0.5x^2 \) yields

\[
m_\alpha = \int_0^\infty x^{\alpha}(2\pi)^{-0.5} \exp(-0.5x^2) dx = \int_0^\infty (2y)^{0.5\alpha}(2\pi)^{-0.5} \exp(-y)(2y)^{-0.5} dy,
\]

and gathering the involved constants leads to the desired representation of \( m_\alpha \).

4. Discussion

The \( \alpha \)-power \( \{ Z^{\alpha}(s) \} \) of \([3]\) establishes unit Fréchet marginal distributions as in the construction \([4]\) with \( Q = (W^+)^\alpha \). Clearly, we identify the extremal Gaussian process (Schlather (2002)) for \( \alpha = 1 \). When \( d = 2 \) and \( \sigma_{12}^* = 0 \) in Theorem \([32,39]\), the range of the extremal coefficient covers the open interval \((1.5, 2)\): For \( \alpha = 1 \) corresponding to the extremal Gaussian process, the value is known to be \( 1 + 0.5\sqrt{2} \). As the degree of freedom \( \nu = \alpha \) tends to infinity in \([6]\), the univariate \( t \)-distribution converges towards the normal distribution and its variance tends to 0 such that \( M(1) = 2 \lim_{\nu \rightarrow \infty} t_{\nu+1}(1 \mid 0, (1 + \nu)^{-1}) = 2 \) in \([6]\). As \( \nu \) tends to 0, we observe \( M(1) = 2 \lim_{\nu \rightarrow 0} t_{\nu+1}(1 \mid 0, (1 + \nu)^{-1}) = 2t_1(1 \mid 0, 1) = 2(\pi^{-1}(\text{arctan}(1) + 0.5) = 1.5 \). This helps understand the long-range dependence structure in models for extremal \( t \) processes since the applied correlation functions are usually non-negative and approach 0 as the distance between two points increases to infinity. In particular, extremal \( t \) processes can be considered more flexible than extremal...
Gaussian processes or Brown-Resnick processes which are both special cases; see Davison et al. (2012) for the case of the Brown-Resnick process which arises for some $\alpha$-dependent correlation structures as $\alpha$ tends to infinity. Moreover, the formulation of the dependence function (6) for the extremal Gaussian process ($\nu = \alpha = 1$) is more general than the bivariate expressions obtained by Schlather (2002) and lends itself more easily to interpretation.

Theorem 3.2 and Corollary 3.1 allow us to simulate extremal $t$ distributions and processes with the method devised in Theorem 4 of Schlather (2002), thus completing the range of max-stable models available for direct simulation. When the degree of freedom is large, the computational complexity may become very restrictive, making it difficult to assure a good quality of simulation. However, in this case the Hüsler-Reiss distribution (Hüsler and Reiss (1989); Falk et al. (2011)) in the multivariate case and the Brown-Resnick process in the infinite-dimensional case could be adequate proxies for some correlation structures. Future research should explore in more detail up to which degree of freedom $\alpha$ the simulation procedure is numerically feasible, and if the Brown-Resnick process provides an adequate substitute around and beyond the "critical" value of $\alpha$. The spectral construction (8) further opens the way for tackling conditional simulation in the theoretical framework developed by Dombry and Éyi-Minko (2011) and applied in Dombry et al. (2011) and Dombry and Ribatet (2012). Test procedures based on an estimate of $\alpha$ could be devised to check for the nested submodels of the extremal Gaussian or Brown-Resnick type.

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