Exclusive production of hard dijets and magnetic susceptibility of QCD vacuum

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We argue that coherent production of hard dijets by linearly polarized real photons can provide direct evidence for chirality violation in hard processes, the first measurement of the magnetic susceptibility of the quark condensate and the photon distribution amplitude. It can also serve as a sensitive probe of the generalized gluon parton distribution. Numerical calculations are presented for HERA kinematics.

The QCD vacuum is a highly complex state. It has a nontrivial particle and energy density characterized by quark and gluon condensates and complicated dynamical properties that characterize its response to external probes. In particular, consider quarks and antiquarks in the QCD vacuum in a constant (electro)magnetic field \(|\mathbf{B}\rangle\). In the weak field, the induced magnetization of the vacuum is proportional to the applied field, the quark density \(|\bar{q}q\rangle\), the quark electric charge \(e_q\) and a constant \(\chi\) that is called magnetic susceptibility. In relativistic notation

\[
\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = e_q \chi \langle \bar{q}q\rangle F_{\alpha\beta} \tag{1}
\]

where \(F_{\alpha\beta}\) denotes the electromagnetic field strength. If the magnetic field is varying with a certain frequency, the magnetic susceptibility is replaced by the corresponding response function which is nothing else but the photon distribution function in the infinite momentum frame.

To be more precise, the wave function of a real photon contains both the perturbative chiral-even (CE) contribution of the quark-antiquark pair with opposite helicities, and the nonperturbative chiral-odd (CO) contribution with quarks having the same helicity which is due to the chiral symmetry breaking. The perturbative CE contribution is singular \(\sim 1/|\mathbf{r}|\) at small transverse distances \(r\) and well known:

\[
\langle 0|\bar{q}(0)\gamma_+^{1/2}\gamma_5 q(x)|\gamma^{(\lambda)}(q)\rangle =
\]

\[
= i\epsilon_{\alpha\beta}(x)\int_0^1 du e^{-iu(qx)} \left( (\epsilon_{\alpha\lambda} \cdot \mathbf{r})(2u - 1) \pm i\epsilon_{ik} \mathbf{r} \epsilon_{k}^{(\lambda)}(q) \right), \tag{2}
\]

where \(\epsilon_{ik} = \epsilon_{ik+}\) is the two-dimensional antisymmetric tensor in the transverse plane, \(\epsilon_u = (2/3)\sqrt{4\pi\alpha_{EM}}\), \(\epsilon_d = -(1/3)\sqrt{4\pi\alpha_{EM}}\), etc. The nonperturbative CO contribution is regular at small transverse separations (apart from the logarithms) and can be parametrized by the photon distribution amplitude \(\phi_\gamma(u, \mu)\):

\[
\langle 0|\bar{q}(0)\sigma_{\alpha\beta}q(x)|\gamma^{(\lambda)}(q)\rangle =
\]

\[
= i\epsilon_{\alpha\beta} \langle \bar{q}q \rangle (\epsilon_{\alpha\lambda} q_{\beta} - \epsilon_{\beta\lambda} q_{\alpha}) \int_0^1 du e^{-iu(qx)} \phi_\gamma(u, \mu). \tag{3}
\]

Here the normalization is chosen as \(\int du \phi_\gamma(u) = 1\), and \(u\) stands for the momentum fraction carried by the quark.

The distribution amplitude \(\phi_\gamma(u, \mu \geq 1 \text{ GeV})\) is believed to be not far from the asymptotic form

\[
\phi_\gamma(u) = 6u(1 - u). \tag{4}
\]

The magnetic susceptibility was estimated using the vector dominance approximation and QCD sum rules:

\[
\chi(\bar{q}q) \simeq 40 - 70 \text{ MeV} \quad (\text{at } \mu = 1 \text{ GeV}). \tag{5}
\]

However, any direct experimental evidence on both \(\chi\) and \(\phi_\gamma(u)\) is absent. The CO contribution in photoproduction was discussed only once in [6] for the vector meson production at large \(t\).

In this letter we argue that this structure can be studied in experiments for the exclusive hard dijet production off nucleons (and nuclei)

\[
\gamma + N \to (\bar{q}q) + N, \tag{6}
\]

similar to the recent studies of coherent dijet production by incident pions by the E791 collaboration [3]. In particular, we will show that perturbative (chirality conserving) and nonperturbative (chirality violating) contributions can be separated by the different dependence on the longitudinal momentum of the dijets and on the azimuthal angle.

The approach of [3] is different as compared to to earlier studies of the dijet photoproduction [5] in that the exclusive dijet final state is identified by requiring that the jet transverse momenta are compensated to a high accuracy within the diffractive cone and making some additional cuts. This approach seems to work for the case of coherent dijet production from nuclei by incident
follows from twist counting. The different \( \phi \) in a state with orbital angular momentum torization is more adequate for HERA energies and, in

\[
q_1 = z p_1 - \frac{q_1^2}{z s} p_2, \quad q_2 = \bar{z} p_1 + \frac{q_2^2}{\bar{z} s} p_2 - q_{\perp}
\]

so that \( z \) is the longitudinal momentum fraction and \( q_{\perp} \) the transverse momentum of the quark jet. Hereafter we use the shorthand notation \( \bar{u} = (1 - u) \) for any longitudinal momentum fraction \( u \). Note that we consider the forward limit, when transverse momenta of the jets compensate each other. In this kinematics the mass of the produced \( q\bar{q} \) pair is equal to \( M^2 = q_{\perp}^2/z \bar{z} \), and the momentum of the outgoing nucleon \( p'_{\perp} = p_2(1 - \xi)/(1 + \xi) \), where \( \xi = M^2/(2s - M^2) \approx M^2/2s, s = (p_1 + p_2)^2 \).

Fig. 1 presents one example of the existing 31 LO contributions in the symmetric notation \[13\]. \( y + \xi \) and \( y - \xi \) are the t-channel gluon momentum fractions with respect to \( (p_2 + p'_{\perp})/2 \). The full calculation of the NLO contribution goes beyond the tasks of this letter. For the reasons explained below it is necessary, however, to include the leading NLO contribution at large energies (enhanced by \( \log \xi \) which corresponds to an additional gluon exchange between the t-channel gluons, see Fig. \[13\]). Including this contribution, the imaginary part of the amplitude equals

\[
\text{Im} \mathcal{J}_{CE} = \xi \mathcal{H}_g(\xi, \xi) + \frac{\alpha_s N_c}{\pi} \int_0^1 \frac{dy}{y + \xi} \mathcal{H}_g(y, \xi), \quad (10)
\]

where \( \mathcal{H}_g(y, \xi) = d\mathcal{H}_g(y, \xi)/dy \big|_{y=\xi} \). The real part is smaller and can be neglected in the first approximation.

For high energies alias \( y \to 0 \), \( \mathcal{H}_g(y, \xi) \sim y^{-\Delta} \), where in perturbation theory \( \Delta \sim \alpha_s \log s/q_{\perp}^2 \) has to be treated as a small parameter. Therefore, despite the fact that the two terms in \[14\] appear in different orders in the collinear expansion, they are of the same order as far as the counting of energy logarithms is concerned. This feature is specific for real photons and can be traced to the fact that the LO amplitude in \[9\] only contains a (rather unusual) double-pole, but no single poles (c.f. \[11\]). Since \( \mathcal{H}_g(y, \xi) \sim G(y) \) at \( y \gg \xi \), and as the factor \( \alpha_s N_c / (\pi y) \) is nothing but the low-y limit of the DGLAP gluon splitting function, the integral in Eq. \[10\] can be identified to logarithmic accuracy with the unintegrated gluon distribution \( f(\xi, q^2) = \partial G(\xi, q^2)/\partial \ln q^2 \). This contribution corresponds to the one considered in \[11\] in the \( k_{\perp} \) factorization approach. The first contribution in \[11\] is analogous to Eq. (42) in \[13\].

For the nonperturbative CO contribution the large momenta are not allowed in the photon vertex and the factorization formula contains a convolution with the photon distribution amplitude. In this case an additional hard gluon exchange is mandatory and the diagram in Fig. \[10\] presents one example of the existing 31 LO contributions, cf. Fig. 11 in Ref. \[14\]. The calculation of this contribution is similar to the case of pion diffraction dijet production considered in much detail in \[13\]. Here we only present the final result \( C_F = (N_c^2 - 1)/(2N_c) \):
Integration over the quark longitudinal momentum fraction contains a logarithmic divergence at the end points \( u \to 0, 1 \) which signals that the collinear factorization is violated. The divergent contribution is purely imaginary and reads

\[
\mathcal{J}_{\text{CO}}^{IR} = 2 i \mathcal{H}_g(\xi, \xi) \left( N_c z \frac{\xi^2 + z^2}{2 N_c} \right) \int_{u_{\text{min}}^l}^{1} \frac{du}{u^2} \phi_s(u),
\]

where we have introduced an infrared cutoff \( u_{\text{min}}^l = \mu_{\text{IR}}^2/q_\perp^2 \). In numerical calculations we use \( \mu_{\text{IR}} = 500 \text{ MeV} \). The origin of the factorization breaking is that both final and initial state interactions are present and lead to pinching of the integration contour in the so-called Glauber region, see [4] for a detailed discussion. In the present context violation of factorization is probably not surprising since the CO contribution is suppressed by a power of \( q_\perp^2 \) compared to the leading twist.

Another important integration region for the quark momentum fraction in [12] is \( \xi \ll |u - z| \ll 1 \) which produces a logarithmic enhancement in energy:

\[
\mathcal{J}_{\text{CO}}^{\text{DLA}} = 4 i N_c \phi_s(z) \int_{\xi}^{1} \frac{dy}{y + \xi} \mathcal{H}_g(y, \xi).
\]

Hence in the double-logarithmic approximation collinear factorization is valid for the CO contribution as well.

Assuming that the photon distribution amplitude \( \phi_s(z, \mu = q_\perp) \) is close to the asymptotic form [8], we obtain \( \mathcal{J}_{\text{CO}} \sim z(1 - z) \) for both integration regions, up to small corrections. The IR divergence in [13] does not have, therefore, any significant effect on the jet distribution but mainly influences the normalization.

In the numerical calculation presented below we have taken into account the full result in [10] and the imaginary part of the CO contribution in [14]. Calculations are done for HERA kinematics, \( s = 10000 \text{ GeV}^2 \) and the value \( \chi(\bar{q}q) = 50 \text{ MeV} \), cf. [8]. We use the parametrization of the generalized gluon distribution by Freund and McDermott [14] that is based on the MRST2001 leading-order forward distribution [17]. The transverse momentum dependence of the cross section integrated over \( \phi, z \) and \( t \) [18] is shown in Fig. 4.

\[
\mathcal{J}_{\text{CO}} = -\frac{1}{\pi} \int dy \int_{-1}^{1} du \mathcal{H}_g(y, \xi) \frac{\phi_s(u)}{u \bar{u}} \left\{ C_F \left( \frac{2 \xi}{y - \xi + i \epsilon} - \frac{1}{y - \xi} \right) + C_F \left( \frac{z \xi}{u \bar{u}} + 1 \right) + \frac{1}{2 N_c} \left( \frac{\xi}{u + \bar{u}} \right) \right\}.
\]

\[
\mathcal{J}_{\text{CO}} = \mathcal{H}_g(\xi, \xi) \left( N_c z \frac{\xi^2 + z^2}{2 N_c} \right) \int_{u_{\text{min}}^l}^{1} \frac{du}{u^2} \phi_s(u),
\]

\[
\mathcal{J}_{\text{CO}}^{\text{DLA}} = 4 i N_c \phi_s(z) \int_{\xi}^{1} \frac{dy}{y + \xi} \mathcal{H}_g(y, \xi).
\]

\[
\mathcal{J}_{\text{CO}}^{IR} = 2 i \mathcal{H}_g(\xi, \xi) \left( N_c z \frac{\xi^2 + z^2}{2 N_c} \right) \int_{u_{\text{min}}^l}^{1} \frac{du}{u^2} \phi_s(u),
\]

\[
\mathcal{J}_{\text{CO}}^{\text{DLA}} = 4 i N_c \phi_s(z) \int_{\xi}^{1} \frac{dy}{y + \xi} \mathcal{H}_g(y, \xi).
\]

\[
\mathcal{J}_{\text{CO}} = \mathcal{H}_g(\xi, \xi) \left( N_c z \frac{\xi^2 + z^2}{2 N_c} \right) \int_{u_{\text{min}}^l}^{1} \frac{du}{u^2} \phi_s(u),
\]

\[
\mathcal{J}_{\text{CO}}^{\text{DLA}} = 4 i N_c \phi_s(z) \int_{\xi}^{1} \frac{dy}{y + \xi} \mathcal{H}_g(y, \xi).
\]
The contribution is large in the region of intermediate distances. Our main result is that the nonperturbative important information on the photon structure at small separation of dijets with large transverse momenta can yield the perturbative contribution by a different way. On the other hand, the dijet cross section for large momentum $q_\perp \sim 5$ GeV. Identification of the curves is the same as in Fig. 2.

![Graph](image)

FIG. 3. The differential cross section $d\sigma_{\gamma+\rightarrow 2\text{jets}+}/dq_\perp^2 dz$ (a) and $d\sigma_{\gamma+\rightarrow 2\text{jets}+}/dq_\perp^2 d\phi$ (b) for jet transverse momentum $q_\perp = 5$ GeV. Identification of the curves is the same as in Fig. 2.

To conclude, we summarize our main points. In this letter we argue that studies of exclusive photoproduction of dijets with large transverse momenta can yield important information on the photon structure at small distances. Our main result is that the nonperturbative CO contribution is large in the region of intermediate $q_\perp \sim 2 - 4$ GeV and can be clearly separated from the perturbative contribution by a different $z$- and $\phi$-dependence. Observation of the CO contribution would be the first clear evidence for the chirality violation in hard processes and also provide the first direct measurement of the magnetic susceptibility of the quark condensate. On the other hand, the dijet cross section for large $q_\perp$ can serve to constrain the generalized gluon distribution.

On the theoretical side, we deviate from previous studies of the dijet production by consistently applying the collinear factorization in terms of generalized parton distributions. For the nonperturbative CO contributions the collinear factorization is, strictly speaking, broken. However, the sensitivity to the IR cutoff is relatively weak and can formally be eliminated by taking into account Sudakov-type corrections in the modified collinear factorization framework. We think that this technique is potentially more accurate and the results can be improved systematically by the calculations of higher-order corrections. In particular, the complete NLO calculation of the perturbative CE contribution would be very welcome because of cancellations that are discussed in the text.

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FIG. 4. Same as in Fig. 3 but for $q_\perp = 2$ GeV.

![Graph](image)