Abstract

The viability of a possible cosmological scenario is investigated. The theoretical framework is the constrained next-to-minimal supersymmetric standard model (cNMSSM), with a gravitino playing the role of the lightest supersymmetric particle (LSP) and a neutralino acting as the next-to-lightest supersymmetric particle (NLSP). All the necessary constraints from colliders and cosmology have been taken into account. For gravitino we have considered the two usual production mechanisms, namely out-of-equilibrium decay from the NLSP, and scattering processes from the thermal bath. The maximum allowed reheating temperature after inflation, as well as the maximum allowed gravitino mass are determined.
1 Introduction

There is accumulated evidence both from astrophysics and cosmology that about 1/4 of the energy budget of the universe consists of so called dark matter, namely a component which is non-relativistic and neither feels the electromagnetic nor the strong interaction. For a review on dark matter see e.g. [1]. Although the list of possible dark matter candidates is long, it is fair to say that the most popular dark matter candidate is the lightest supersymmetric particle (LSP) in supersymmetric models with R-parity conservation [2]. The superpartners that have the right properties for playing the role of cold dark matter in the universe are the axino, the gravitino and the lightest neutralino. By far the most discussed case in the literature is the case of the neutralino (see the classic review [3]), probably because of the prospects of possible detection.

However, the gravitino is another very interesting candidate for cold dark matter, since its interactions are completely determined by the supergravity lagrangian [4], in contrast to what happens to the neutralino or the axino case, where the interactions depend on the chosen model. Unfortunately, gravitino belongs to the class of new exotic particles that can be potentially dangerous for cosmology, and it is therefore escorted by the so-called gravitino problem [5]. The mass of the gravitino strongly depends on the SUSY-breaking scheme, and can range from eV scale to scales beyond the TeV region [6, 7, 8]. In particular, in gauge-mediated SUSY-breaking schemes [6] the gravitino mass is typically less than 100 MeV, while in gravity-mediated schemes [7] it is expected to be in the GeV to TeV range. Finally, it must be noted that there are hybrid models of gauge- and gravity-mediation, in which gravity provides sub-dominant and yet non-negligible contributions [9]. Therefore, according to the precise mechanism for supersymmetry breaking, the gravitino can be either stable or unstable, with the corresponding gravitino cosmology. In general, the gravitino problem requires that the reheating temperature after inflation should be lower than $10^6 - 10^7$ GeV [10, 11], which poses serious difficulties to the thermal leptogenesis scenario [12].

In the present work we want to consider gravitino dark matter in the constrained next-to-minimal supersymmetric standard model (cNMSSM), assuming that the lightest neutralino is the next-to-lightest supersymmetric particle (NLSP), and taking into account the two usual gravitino production mechanisms to be discussed later on, namely the out-of-equilibrium decays of the NLSP, as well as scattering processes from the thermal bath.
This article is organized as follows. In the next section we present the theoretical framework. In section 3 we discuss all the relevant constraints from colliders and from cosmology, and we show our results. Finally, we conclude.

2 Theoretical framework

In the present article we work in the framework of the constrained next-to-minimal supersymmetric standard model (cNMSSM). We assume that the gravitino is the LSP, while the lightest neutralino is the NLSP. The gravitino is stable and plays the role of cold dark matter in the universe, while the neutralino is unstable and it decays to gravitino.

In what follows we review in short the particle physics model, namely the cNMSSM, as well as the gravitino production mechanisms.

2.1 Basics of cNMSSM

The most straightforward extension of standard model (SM) of particle physics based on SUSY is the minimal supersymmetric standard model (MSSM) [13]. It is a supersymmetric gauge theory based on the SM gauge group with the usual representations (singlets, doublets, triplets) and on $\mathcal{N} = 1$ SUSY. Excluding gravity, the massless representations of the SUSY algebra are a chiral and a vector supermultiplet. The gauge bosons and the gauginos are members of the vector supermultiplet, while the matter fields (quarks, leptons, Higgs) and their superpartners are members of the chiral supermultiplet.

The Higgs sector in the MSSM is enhanced compared to the SM case. There are now two Higgs doublets, $H_u, H_d,$ (or $H_1, H_2$) for anomaly cancelation requirements and for giving masses to both up and down quarks. After electroweak symmetry breaking we are left with five physical Higgs bosons, two charged $H^\pm$ and three neutral $A, H, h$ ($h$ being the lightest). Since we have not seen any superpartners yet, SUSY has to be broken. In MSSM, SUSY is softly broken by adding to the Lagrangian terms of the form

- Mass terms for the gauginos $\tilde{g}_i, M_1, M_2, M_3$
  \[ M \tilde{g} \tilde{g} \] (1)

- Mass terms for sfermions $\tilde{f}$
  \[ m_f^2 \tilde{f} \tilde{f} \] (2)
• Masses and bilinear terms for the Higgs bosons $H_u, H_d$

$$m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + B\mu(H_u H_d + h.c.)$$

(3)

• Trilinear couplings between sfermions and Higgs bosons

$$AY \tilde f_1 H \tilde f_2$$

(4)

In the unconstrained MSSM there is a huge number of unknown parameters \cite{14} and thus little predictive power. However, motivated by the grand unification idea, the constrained MSSM (CMSSM) assumes that gaugino masses, scalar masses and trilinear couplings have (separately) a common, universal, value at the GUT scale, like the gauge coupling constants do. CMSSM is therefore a framework with a small controllable number of parameters, and thus with much more predictive power. In the CMSSM there are four parameters, $m_0, m_{1/2}, A_0, \tan\beta$, which are explained below, plus the sign of the $\mu$ parameter from the Higgs sector. The magnitude of $\mu$, as well as the B parameter mentioned above, are determined by the requirement for a proper electroweak symmetry breaking. However, the sign of $\mu$ remains undetermined. The other four parameters of the CMSSM are related to

• Universal gaugino masses

$$M_1(M_{GUT}) = M_2(M_{GUT}) = M_3(M_{GUT}) = m_{1/2}$$

(5)

• Universal scalar masses

$$m_{\tilde f_i}(M_{GUT}) = m_0$$

(6)

• Universal trilinear couplings

$$A_{ij}^u(M_{GUT}) = A_{ij}^d(M_{GUT}) = A_{ij}^l(M_{GUT}) = A_0\delta_{ij}$$

(7)

•

$$\tan\beta \equiv \frac{v_1}{v_2}$$

(8)

where $v_1, v_2$ are the vevs of the Higgs doublets and $M_{GUT} \sim 10^{16}$ GeV is the Grand Unification scale.
Unfortunately, the CMSSM suffers from the so-called $\mu$ problem \cite{[15]}. This problem is elegantly solved in the framework of the next-to-minimal supersymmetric standard model (NMSSM) \cite{[16]}. In addition to the MSSM Yukawa couplings for quarks and leptons, the NMSSM superpotential contains two additional terms involving the Higgs doublet superfields, $H_1$ and $H_2$, and the new superfield $S$, a singlet under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ \cite{[17]}

$$W = \epsilon_{ij} \left( Y_u H_2^i Q^j u + Y_d H_1^j Q^i d + Y_e H_1^i L^j e \right) - \epsilon_{ij} \lambda S H_1^i H_2^j + \frac{1}{3} \kappa S^3$$ \hspace{1cm} (9)

where we take $H_1^T = (H_1^0, H_1^-)$, $H_2^T = (H_2^+, H_2^0)$, $i, j$ are $SU(2)$ indices, and $\epsilon_{12} = 1$. In this model, the usual MSSM bilinear $\mu$ term is absent from the superpotential, and only dimensionless trilinear couplings are present in $W$. However, when the scalar component of $S$ acquires a VEV, an effective interaction $\mu H_1 H_2$ is generated, with $\mu \equiv \lambda(S)$.

Finally, the soft SUSY breaking terms are given by \cite{[17]}

$$-\mathcal{L}_{\text{soft}} = m_{\tilde{Q}}^2 \tilde{Q}^* \tilde{Q} + m_{\tilde{U}}^2 \tilde{u}^* \tilde{u} + m_{\tilde{D}}^2 \tilde{d}^* \tilde{d} + m_{\tilde{L}}^2 \tilde{L}^* \tilde{L} + m_{\tilde{E}}^2 \tilde{e}^* \tilde{e} + m_{H_1}^2 H_1^* H_1 + m_{H_2}^2 H_2^* H_2 + m_S^2 S^* S + \epsilon_{ij} \left( A_u Y_u H_2^i \tilde{Q}^j \tilde{u} + A_d Y_d H_1^j \tilde{Q}^i \tilde{d} + A_e Y_e H_1^i \tilde{L}^j \tilde{e} + \text{H.c.} \right) + \left( -\epsilon_{ij} \lambda A_S H_1^j H_2^i + \frac{1}{3} \kappa A_\kappa S^3 + \text{H.c.} \right) - \frac{1}{2} \left( M_3 \lambda_3 \lambda_3 + M_2 \lambda_2 \lambda_2 + M_1 \lambda_1 \lambda_1 + \text{H.c.} \right)$$ \hspace{1cm} (10)

Clearly, the NMSSM is very similar to the MSSM. Despite the similarities between the two particle physics models, the Higgs sector as well as the neutralino mass matrix and mass eigenstates in the NMSSM are more complicated compared to the corresponding ones in the MSSM.

In particular, in the Higgs sector we have now two CP-odd neutral, and three CP-even neutral Higgses. We make the assumption that there is no CP-violation in the Higgs sector at tree level, and neglecting loop level effects the CP-even and CP-odd states do not mix. We are not interested in the CP-odd states, while the CP-even Higgs interaction and physical eigenstates are related by the transformation

$$h_a^0 = S_{ab} H_b^0$$ \hspace{1cm} (11)
where $S$ is the unitary matrix that diagonalises the CP-even symmetric mass matrix, $a, b = 1, 2, 3$, and the physical eigenstates are ordered as $m_{h_1^0} \lesssim m_{h_2^0} \lesssim m_{h_3^0}$.

In the neutralino sector the situation is again more involved, since the fermionic component of $S$ mixes with the neutral Higgsinos, giving rise to a fifth neutralino state. In the weak interaction basis defined by $\Psi_0^T \equiv \begin{pmatrix} \tilde{B}_0' = -i\lambda', \tilde{W}_3^0 = -i\lambda_3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S} \end{pmatrix}$, the neutralino mass terms in the Lagrangian are [17]

$$L_{\tilde{\chi}_0}^{\text{mass}} = -\frac{1}{2}(\Psi_0^T)^T M_{\tilde{\chi}_0} \Psi_0 + \text{H.c.},$$

with $M_{\tilde{\chi}_0}$ a $5 \times 5$ matrix,

$$M_{\tilde{\chi}_0} = \begin{pmatrix} M_1 & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta & 0 \\ 0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta & 0 \\ -M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\lambda s & -\lambda v_2 \\ M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\lambda s & 0 & -\lambda v_1 \\ 0 & 0 & -\lambda v_2 & -\lambda v_1 & 2\kappa s \end{pmatrix}$$

(13)

The above matrix can be diagonalised by means of a unitary matrix $N$

$$N^* M_{\tilde{\chi}_0} N^{-1} = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0})$$

(14)

where $m_{\tilde{\chi}_1^0}$ is the lightest neutralino mass. Under the above assumptions, the lightest neutralino can be expressed as the combination

$$\tilde{\chi}_1^0 = N_{11} \tilde{B}_0' + N_{12} \tilde{W}_3^0 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0 + N_{15} \tilde{S}$$

(15)

In the following, neutralinos with $N_{11}^2 > 0.9$, or $N_{15}^2 > 0.9$, will be referred to as bino- or singlino-like, respectively.

Similarly to the CMSSM, in the constrained next-to-minimal supersymmetric standard model the universality of $m_0, A_0, m_{1/2}$ at the GUT scale is again assumed, with the only parameters now being $\tan\beta, m_0, A_0, m_{1/2}, \lambda, A_k$ and the sign of the $\mu$ parameter can be chosen at will.

We end the discussion on the particle physics model here, by making a final remark regarding the differences between the CMSSM and the cNMSSM. In the CMSSM the lightest neutralino is mainly a bino in most of the parameter space, and low values of $m_0$ are disfavored because they lead to charged sleptons that are lighter than the neutralino $\chi_1^0$, while in the cNMSSM the lightest neutralino is mainly a singlino in large regions of the parameter space,
thanks to which the charged LSP problem can be avoided \[18\]. Furthermore, in the cNMSSM there are more mechanisms that contribute to the neutralino relic density \[18\].

2.2 Gravitino production

For the gravitino abundance we take the relevant production mechanisms into account and impose the cold dark matter constraint \[19\]

\[
0.1097 < \Omega_{cdm} h^2 = \Omega_{3/2} h^2 < 0.1165 \tag{16}
\]

At this point it is convenient to define the gravitino yield, \( Y_{3/2} \equiv n_{3/2}/s \), where \( n_{3/2} \) is the gravitino number density, \( s = h_{eff}(T) \frac{2\pi^2T^3}{45} \) is the entropy density for a relativistic thermal bath, and \( h_{eff} \) counts the relativistic degrees of freedom. The gravitino abundance \( \Omega_{3/2} \) in terms of the gravitino yield is given by

\[
\Omega_{3/2} h^2 = \frac{m_{\tilde{G}} s(T_0) Y_{3/2} h^2}{\rho_{cr}} = 2.75 \times 10^8 \left( \frac{m_{\tilde{G}}}{GeV} \right) Y_{3/2}(T_0) \tag{17}
\]

where we have used the values

\[
T_0 = 2.73 K = 2.35 \times 10^{-13} \text{ GeV} \tag{18}
\]

\[
h_{eff}(T_0) = 3.91 \tag{19}
\]

\[
\rho_{cr}/h^2 = 8.1 \times 10^{-47} \text{ GeV}^4 \tag{20}
\]

The total gravitino yield has two contributions, namely one from the thermal bath, and one from the out-of-equilibrium NLSP decay.

\[
Y_{3/2} = Y_{3/2}^{TP} + Y_{3/2}^{NLSP} \tag{21}
\]

The contribution from the thermal production has been computed in \[20\], \[21\], \[22\]. In \[20\] the gravitino production was computed in leading order in the gauge coupling \( g_3 \), in \[21\] the thermal rate was computed in leading order in all Standard Model gauge couplings \( g_Y, g_2, g_3 \), and in \[22\] new effects were taken into account, namely: a) gravitino production via gluon \( \rightarrow \) gluino + gravitino and other decays, apart from the previously considered \( 2 \rightarrow 2 \) gauge scatterings, b) the effect of the top Yukawa coupling, and c) a proper treatment of the reheating process. Here we shall use the result of \[20\] since
the corrections of \[21, 22\] do not alter our conclusions. Therefore the thermal gravitino production is given by

\[ Y_{3/2}^{TP} = 0.29 \times 10^{-12} \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( 1 + \frac{1}{3} \frac{m_{\tilde{g}}^2}{m_G^2} \right) \]  \hspace{1cm} (22)

or, approximately for a light gravitino, \( m_{\tilde{G}} \ll m_{\tilde{g}} \)

\[ Y_{3/2}^{TP} \simeq 10^{-13} \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{m_{\tilde{G}}} \right)^2 \]  \hspace{1cm} (23)

with \( m_{\tilde{G}} \) the gravitino mass and \( m_{\tilde{g}} \) the gluino mass. At this point it must be noted that all the relevant particles here (gluons, quarks, gluinos, squarks and gravitinos) are supposed to be in thermal equilibrium, and thus the reheating temperature after inflation should be at least 1 TeV. This is going to be important later on.

The second contribution to the gravitino abundance comes from the decay of the NLSP

\[ \Omega_{3/2}^{NLSP} h^2 = \frac{m_{\tilde{G}}}{m_{NLSP}} \Omega_{NLSP} h^2 \]  \hspace{1cm} (24)

with \( m_{NLSP} \) the mass of the NLSP, and \( \Omega_{NLSP} h^2 \) the abundance the NLSP would have, had it not decayed into the gravitino.

3 Constraints and results

- Spectrum and collider constraints: We have used NMSSMTools \[23\], a computer software that computes the masses of the Higgses and the superpartners, the couplings, and the relic density of the neutralino, for a given set of the free parameters. We have performed a random scan in the whole parameter space (with fixed \( \mu > 0 \) motivated by the muon anomalous magnetic moment), and we have selected only those points that satisfy i) theoretical requirements, such as neutralino LSP, correct electroweak symmetry breaking, absence of tachyonic masses etc., and ii) LEP bounds on the Higgs mass, collider bounds on SUSY particle masses, and experimental data from B-physics \[24, 25\]. For all these "good" points the lightest neutralino is either a bino or a singlino, and contrary to the case where neutralino is the dark matter particle, here we do not require that the neutralino relic density falls within the allowed WMAP range mentioned before.
As we have already mentioned, the total gravitino abundance, and not the neutralino one, should satisfy the cold dark matter constraint \[ \Omega_{cdm} 10^{97} < \Omega_\text{cdm}h^2 < 0.1165 \] (25) that relates the reheating temperature after inflation to the gravitino mass as follows

\[ 0.11 = A(m_{\tilde{G}}, m_{\tilde{g}})T_R + \frac{m_{3/2}}{m_{NLSP}}\Omega_{NLSP}h^2 \] (26)

For a given point in the cNMSSM parameter space, the complete spectrum and couplings have been computed, and we are left with two more free parameters, namely the gravitino mass and the reheating temperature after inflation. The gravitino mass is directly related to the SUSY-breaking scheme, while the precise range of values of the reheating temperature is crucial for the baryon asymmetry generation mechanism. The thermal production contribution cannot be larger than the total gravitino abundance, and for this we can already obtain an upper bound on the reheating temperature

\[ T_R \leq 4.1 \times 10^9 \left( \frac{m_{\tilde{G}}}{100 \, \text{GeV}} \right) \left( \frac{TeV}{m_{\tilde{g}}} \right)^2 \, \text{GeV} \] (27)

Assuming a gluino mass \( m_{\tilde{g}} \sim 1 \, \text{TeV} \), we can see that for a heavy gravitino, \( m_{\tilde{G}} \sim 100 \, \text{GeV} \), it is possible to obtain a reheating temperature large enough for thermal leptogenesis. However, for a light gravitino, \( m_{\tilde{G}} \sim 1 \, \text{GeV} \), we cannot obtain a reheating temperature larger than \( T_R \sim 10^7 \, \text{GeV} \).

- In scenarios in which gravitino is assumed to be the LSP, the NSLP is unstable with a lifetime that is typically larger than BBN time \( t_{BBN} \sim 1 \, \text{sec} \). Energetic particles produced by the NSLP decay may dissociate the background nuclei and significantly affect the primordial abundances of light elements. If such processes occur with sizable rates, the predictions of the standard BBN scenario would be altered and the success of the primordial nucleosynthesis would be spoiled. BBN constraints on cosmological scenarios with exotic long-lived particles predicted by physics beyond the Standard Model have been studied [10, 26]. Previous investigations have shown that the neutralino NLSP scenario with a gravitino mass \( m_{3/2} \geq 100 \, \text{MeV} \) and a neutralino lifetime in the range \( (10^4 - 10^8) \, \text{sec} \) is already disfavored [27], while the stau NLSP is still a viable scenario. The neutralino NLSP scenario can still be rescued if we avoid the stringent BBN constraints, namely if the neutralino lifetime becomes either larger than the age of the universe or...
lower than the BBN time. The first possibility is realized in the degenerate gravitino scenario \[28\], where the neutralino is extremely long-lived, and the only constraint comes from the cold dark matter bound.

For neutralino NLSPs, the dominant decay mode is \( \chi \rightarrow \gamma \tilde{G} \), for which the decay width is \[27, 29\]

\[
\Gamma(\chi \rightarrow \gamma \tilde{G}) = \frac{|N_{11} \cos \theta_W + N_{12} \sin \theta_W|^2}{48\pi M_\ast^2} \frac{m_\chi^5}{m_\tilde{G}^2} \left[ 1 - \frac{m_\tilde{G}^2}{m_\chi^2} \right]^3 \left[ 1 + 3 \frac{m_\tilde{G}^2}{m_\chi^2} \right]
\]  

(28)

where \( M_\ast \) is the Planck mass, \( m_\chi \) is the neutralino mass, and \( \theta_W \) is the weak angle. This decay contributes only to EM energy. If kinematically allowed, the neutralino will also decay to gravitino and Z boson, or gravitino and light standard model Higgs boson \( h \). The leading contribution to hadronic energy is from \( \chi \rightarrow Z \tilde{G}, h \tilde{G} \). These decays produce EM energy for all possible \( Z \) and \( h \) decay modes (except \( Z \rightarrow \nu \bar{\nu} \)), but they may also produce hadronic energy when followed by \( Z, h \rightarrow q \bar{q} \). The decay width to \( Z \) bosons is \[27, 29\]

\[
\Gamma(\chi \rightarrow Z \tilde{G}) = \frac{|-N_{11} \sin \theta_W + N_{12} \cos \theta_W|^2}{48\pi M_\ast^2} \frac{m_\chi^5}{m_\tilde{G}^2} F(m_\chi, m_\tilde{G}, m_Z) \times \left[ \left( 1 - \frac{m_\tilde{G}^2}{m_\chi^2} \right)^2 \left( 1 + 3 \frac{m_\tilde{G}^2}{m_\chi^2} \right) - \frac{m_\tilde{G}^2}{m_\chi^2} G(m_\chi, m_\tilde{G}, m_Z) \right]
\]  

(29)

where

\[
F(m_\chi, m_\tilde{G}, m_Z) = \left[ \left( 1 - \left( \frac{m_\tilde{G} + m_Z}{m_\chi} \right)^2 \right) \left( 1 - \left( \frac{m_\tilde{G} - m_Z}{m_\chi} \right)^2 \right) \right]^{1/2}
\]  

(30)

\[
G(m_\chi, m_\tilde{G}, m_Z) = 3 + \frac{m_\chi^3}{m_\tilde{G}^3} \left( -12 + \frac{m_\tilde{G}}{m_\chi} \right) + \frac{m_\chi^4}{m_\tilde{G}^4} \left( 3 - \frac{m_\tilde{G}^2}{m_\chi^2} \right)
\]  

(31)

with \( m_Z \approx 91 \text{ GeV} \) the mass of the Z boson.

The decay width to the Higgs boson is \[27, 29\]

\[
\Gamma(\chi \rightarrow h \tilde{G}) = \frac{|N_{13}S_{11} + N_{14}S_{12} + N_{15}S_{13}|^2}{48\pi M_\ast^2} \frac{m_\chi^5}{m_\tilde{G}^2} F(m_\chi, m_\tilde{G}, m_h) \times \left[ \left( 1 - \frac{m_\tilde{G}}{m_\chi} \right)^2 \left( 1 + \frac{m_\tilde{G}}{m_\chi} \right)^4 - \frac{m_h^2}{m_\chi^2} H(m_\chi, m_\tilde{G}, m_h) \right]
\]  

(32)
where \( S_{ij} \) are three of the components of the mixing matrix in the Higgs sector for the CP-even mass eigenstates, \( F \) is as given in (30), and

\[
H(m_\chi, m_\tilde{G}, m_h) = 3 + 4 \frac{m_\tilde{G}}{m_\chi} + 2 \frac{m^2_\tilde{G}}{m^2_\chi} + 4 \frac{m^3_\tilde{G}}{m^3_\chi} + 3 \frac{m^4_\tilde{G}}{m^4_\chi} + \frac{m^4_h}{m^4_\chi}
\]

\[
- \frac{m^2_h}{m^2_\chi} \left( 3 + 2 \frac{m_\tilde{G}}{m_\chi} + 3 \frac{m^2_\tilde{G}}{m^2_\chi} \right)
\]

with \( m_h \) the mass of the Higgs boson. Therefore, the neutralino lifetime is given by

\[
\tau = \frac{1}{\Gamma}, \quad \Gamma = \Gamma(\chi \rightarrow \gamma \tilde{G}) + \Gamma(\chi \rightarrow Z \tilde{G}) + \Gamma(\chi \rightarrow h \tilde{G})
\]

Given these two-body decay widths, the resulting values for the energy release parameters are

\[
B^\chi_{\text{EM}} \approx \frac{1}{2m_\chi}, \quad \epsilon^\chi_{\text{EM}} = \frac{m^2_\chi - m^2_\tilde{G}}{2m_\chi}
\]

\[
B^\chi_{\text{had}} \approx \frac{\Gamma(\chi \rightarrow Z \tilde{G})B^Z_{\text{had}} + \Gamma(\chi \rightarrow h \tilde{G})B^h_{\text{had}} + \Gamma(\chi \rightarrow q\bar{q} \tilde{G})}{\Gamma(\chi \rightarrow \gamma \tilde{G}) + \Gamma(\chi \rightarrow Z \tilde{G}) + \Gamma(\chi \rightarrow h \tilde{G})}
\]

\[
\epsilon^\chi_{\text{had}} \approx \frac{m^2_\chi - m^2_\tilde{G} + m^2_{Z,h}}{2m_\chi},
\]

where \( B^h_{\text{had}} \approx 0.9, B^Z_{\text{had}} \approx 0.7 \), and the three-body decay \( \Gamma(\chi \rightarrow q\bar{q} \tilde{G}) \sim 10^{-3} \Gamma \) [27].

- Finally, it must be noted that for long neutralino lifetimes, \( \tau_{NLS} \geq 10^7 \) sec, in addition to BBN constraints there are strong bounds from the shape of the cosmic microwave background (CMB) black-body spectrum [28, 30]. However, in our investigation we have found that the neutralino lifetime is always \( \tau_{NLS} \leq 10^2 \) sec, for gravitino masses \( m_\tilde{G} \leq 1 \) GeV, and therefore we do not need to worry about these bounds from the CMB shape.

Our main results are summarized in the figures below. Before starting to discuss the figures, let us first make a few comments. The precise neutralino composition depends on the values of the coefficients \( N_{ij} \) in (15), which in
turn depend on the values of the free parameters of the model. Roughly, for large coupling $\lambda = 0.1 - 0.5$ the neutralino is mainly a bino, while for small coupling $\lambda \ll 1$, the neutralino is mostly a singlino. We refer the interested reader to e.g. [18] for the relevant discussion. Furthermore, the neutralino lifetime is determined by the three decay channels to gravitino plus photon or Z boson or Higgs boson. These partial decay rates depend on the available phase space (masses) as well as the couplings (composition coefficients $N_{1i}$). Thus in the bino case, in which $N_{11} \simeq 1$ and the rest of the coefficients are very small, the decay rate to gravitino and Higgs boson is negligible, while in the singlino case, in which $N_{15} \simeq 1$ and the rest of the coefficients are very small, the decay rate to gravitino and Z boson is negligible. The decay channel to gravitino and photon gives the main contribution, while the decay rate to gravitino and Z boson (in the bino case) or to gravitino and Higgs boson (in the singlino case) modify the neutralino lifetime by a factor of twenty or fifty per cent respectively. Finally, for a given point in the cNMSSM parameter space, the neutralino lifetime is a function of the gravitino mass only. Imposing the BBN constraints we find the maximum allowed gravitino mass, and from the cold dark matter bound we can determine the maximum allowed reheating temperature.

We can now turn to the figures where we discuss the two cases (bino or singlino) separately. The first three figures correspond to the bino case, while the last two figures correspond to the singlino case. For the bino case, it is important to notice that the neutralino relic density can take values larger than the usual ones by two orders of magnitude. The reheating temperature decreases with the neutralino relic density, and takes larger values for very low neutralino relic density. Figure 3 shows the maximum allowed reheating temperature after inflation versus the maximum allowed gravitino mass, both in GeV. Although it cannot be seen directly from the figures, the maximum possible gravitino mass in the bino case is $m_{\tilde{G}} \simeq 1$ GeV, and the corresponding reheating temperature is $T_R \sim 10^7$ GeV. Therefore, we see that a) the gravitino in this scenario must be much lighter than the rest of superpartners, and b) the reheating temperature after inflation is not large enough for thermal leptogenesis. We remark in passing that gravitino masses of the order of 1 GeV can be obtained in hybrid supersymmetric models of gauge- and gravity-mediation, in which gravity provides sub-dominant and yet non-negligible contributions [9].

For the singlino case, we show in figure 4, the gravitino mass in GeV versus neutralino relic density, and in figure 5 the maximum allowed reheating
temperature versus gravitino mass, both in GeV. This time the neutralino relic density is even larger than before, and gravitino now must be extremely light. This is due to the smallness of the coefficients $N_{11}, N_{12}$ in the decay rate to gravitino and photon. For the same lifetime as before, the gravitino mass must be several orders of magnitude lower than in the bino case. The last figure shows that in the singlino case the reheating temperature cannot be larger than about 200 GeV. However, this value is much lower than the minimum value required for the computation of the gravitino thermal production, and therefore we conclude that this scenario must be excluded.

We can understand these features as follows. First, recall that the WMAP bound for the cold dark matter abundance relates the gravitino mass to the reheating temperature, and we can obtain an upper bound on the reheating temperature for a given gravitino mass. From equation (27), and for a gluino mass $m_{\tilde{g}} \sim 1$ TeV, we see that when the gravitino is extremely light, $m_{\tilde{G}} \sim 10^{-6}$ GeV, the upper bound on the reheating temperature becomes $T_R \simeq 41$ GeV. We then need to understand why the gravitino becomes so light in the singlino case. Let us assume that we have in the parameter space a point that corresponds to the bino case, another point that corresponds to the singlino case, and that the Higgs mass, superpartner masses, as well as the neutralino abundance are the same for the two points. The only thing that is different is the composition coefficients for the neutralino. In the bino case the first coefficient is practically unity and the rest tiny, while in the singlino case the last coefficient is almost unity and the rest negligible. The BBN constraints determine the maximum possible gravitino lifetime, which is given essentially by the photon decay channel. We thus have for the neutralino lifetime

$$\tau \sim \frac{m_{\tilde{G}}^2}{|N_{11}|^2} \quad (40)$$

Therefore if in the singlino case $|N_{11}| \simeq 10^{-6}$, the gravitino mass becomes as low as $m_{\tilde{G}} \simeq 10^{-6}$ GeV.

## 4 Conclusions

In the framework of the cNMSSM, we have considered a possible cosmological scenario with the gravitino LSP and the neutralino NLSP. The gravitino is stable and plays the role of cold dark matter in the universe, while the neutralino is unstable and decays to gravitino. We have taken into account
the relevant gravitino production mechanisms, which are i) the NLSP decay, and ii) scattering processes from the thermal bath. Our results can be seen in the figures. We have found that i) the gravitino is necessarily very light, and ii) the reheating temperature after inflation is two orders of magnitude lower than the temperature required for thermal leptogenesis. The singlino scenario must be excluded, while in the bino case it is possible to have a gravitino in the gravity-mediated SUSY-breaking scheme.

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J. L. Feng, A. Rajaraman and F. Takayama, Phys. Rev. D 68 (2003) 063504 [arXiv:hep-ph/0306024].
Figure 1: Gravitino non-thermal production versus neutralino relic density for the bino case.

Figure 2: Reheating temperature (in GeV) versus neutralino relic density for the bino case.
Figure 3: Reheating temperature versus gravitino mass (both in GeV) for the bino case.

Figure 4: Gravitino mass (in GeV) versus neutralino relic density for the singlino case.
Figure 5: Reheating temperature versus gravitino mass (both in GeV) for the singlino case.