A Disturbance Observer-Based Current-Constrained Controller for Speed Regulation of PMSM Systems Subject to Unmatched Disturbances

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Abstract—The speed regulation problem of permanent magnet synchronous motor system is investigated under a noncascade structure. Despite its superiority in straightforward control design, simple parameter adjustment, and satisfying system dynamic performance, the structure brings two problems: the overcurrent protection and unmatched disturbance rejection. Under this structure, the current cannot any more be restricted by a reference value, and ought to be constrained within a certain range to ensure the circuit safety. Besides, unmatched disturbances, mainly caused by external load torques, may result in undesired interference and violate the constraint requirement, since they affect the current directly via the same channel. Toward that end, a novel current-constrained control algorithm is designed to tackle the current constraint and unmatched disturbances simultaneously. A disturbance observer (DOB) is utilized for unmatched disturbance estimation. A constraint coping mechanism is constructed to restrict the current. Then, a key nonlinear item is proposed by augmenting the disturbance estimation and the constrained current. Finally, a composite controller is proposed with concise structure and rigorous closed-loop stability analysis. Numerical and experimental tests further validate that the proposed control approach achieves promising speed tracking performance and guarantees the current constraint in the presence of unmatched disturbances.

Index Terms—Current constraint, disturbance observer (DOB), permanent magnet synchronous motor (PMSM), speed regulation, unmatched disturbance.

I. INTRODUCTION

PERMANENT magnet synchronous motors (PMSMs) have been paid abundant attention and broadly adopted in various practical applications, such as power generations, robotics, and electric vehicles [1]–[3], because of their compact structure, high efficiency, and power density [4].

With technical development, there is little difference between the control periods of the inner- and outer-loops. As an alternative, the noncascade structure is applied with straightforward control design, simple parameter adjustment, and satisfying system dynamic performance [5]–[9].

Despite its advantages, the noncascade structure inevitably brings about some problems

1) The overcurrent protection: unlike the cascade structure, the current cannot be restricted by a reference value any more. In such cases, the current is considered as a system state. Thus, the overcurrent protection is not negligible, since it would cause performance degradation or even system damage [10].

2) The unmatched disturbance rejection: under the noncascade structure, disturbances in the current loop are unmatched, which enter the system via a different channel from the control input. Mainly caused by external load torques, unmodeled dynamics and parameter perturbations, the unmatched disturbances have negative impact on system performance and stability [11]. It is of significance but challenge to reject them, since they influence the current directly via the same channel and thus may violate the constraint. And it is hard to eliminate their influence from the output [12].

Traditional control approaches (e.g., PID) do not explicitly consider the above problems in the control design. Commonly, they restrict the current by selecting relatively conservative control parameters and reject the disturbances in a robust way. Consequently, they cannot balance well among requirements of system dynamic performance, satisfaction of the current constraint, and unmatched disturbance rejection ability [13].

Nowadays, various elegant approaches have been investigated. One idea is to seek the invariant set (IS) in the domain of constraints [14], [15]. The states starting from the IS will always remain in it. Since the IS estimation depends highly on the choice of Lyapunov functions, it is challenging and requires considerable calculation to find the maximal positive IS [16]. Another effective way is the model predictive control, which
transforms control design issues into optimization problems [5], [6], [17]–[19]. By constructing a cost function including the constrained current, the current is restricted by control actions that minimize the given cost function. This kind of constraints can be regarded as soft ones as the constraints are realized via an indirect manner [13], [20]. Besides, the barrier Lyapunov function (BLF) is widely exerted, which is related to the constrained state and will increase to infinity when the constraint is approached [21]–[23]. The BLF-based backstepping control has had fruitful practical investigations, such as electrostatic microactuators [24], electro-hydraulic systems [25], and flexible crane systems [26]. Unfortunately, it is conservative to choose control parameters. Since in each recursive step, the virtual controller must be restricted in the constrained domain [23]. Moreover, few works have considered the control solutions to address unmatched disturbances, which have direct impact on the constrained state via the same channel.

Recently, the penalty term based state-constrained control has been researched, where a nonlinear function of the constrained state is used to regulate the control gain automatically [27], [28]. A current-constrained PID (CCPID) controller is implemented in PMSMs to restrict the current [28]. But the disturbance rejection issue is not explicitly taken into account in the control design. The unmatched disturbance is simply counteracted by an integral action. This may bring negative effect on control performance and parameter tuning due to performance couplings caused by integral actions [13].

To improve the disturbance rejection ability, several control strategies have combined the disturbance observer (DOB) technology [29]–[35] with the penalty term [36]–[38]. However, the above methods do not tackle the unmatched disturbance issue in essence. To be specific, in [36], only the matched disturbance is considered and handled in current-constrained dc–dc converter systems. In [37] and [38], unmatched disturbances in PMSMs are considered within the current-constrained control strategies, but lack of rigorous closed-loop stability and performance analysis. To conclude, for many motion control systems with second-order dynamics, the overcurrent protection together with unmatched disturbance problems are neither explicitly considered nor fundamentally solved. Furthermore, it is significant and meaningful to investigate a general control algorithm for state-constrained nonlinear systems subject to unmatched disturbances.

This article simultaneously tackles the overcurrent protection and unmatched disturbance rejection problems for speed regulation of PMSMs. To begin with, a DOB is designed to estimate the unmatched disturbance. Following the observer design, a key nonlinear item including a feed-forward compensation part and a constraint coping mechanism part, is proposed by augmenting the disturbance estimation and the constrained state. The former is introduced to attenuate unmatched disturbances, which is related to the system structure. The latter is utilized to restrict the current, which is with respect to the current and automatically increases when the current tends to its boundary. As such, a novel DOB-based current-constrained controller (DOB-CC) is obtained. Robust closed-loop stability analysis, together with numerical and experimental tests have verified its effectiveness. The main contributions and novelties of this article are as follows.

1) A general control algorithm is investigated for state-constrained nonlinear systems subject to unmatched disturbances, which affect the constrained state directly via the same channel. To the best of our knowledge, it is the first time in the literature to explicitly solve this problem with rigorous closed-loop stability analysis.

2) For speed regulation of PMSMs, a new current-constrained control algorithm is proposed to simultaneously fulfill the overcurrent protection task and compensate undesirable effects of unmatched load torque variations.

3) By virtue of a key nonlinear item via a DOB and a constraint coping mechanism, the proposed control algorithm not only exhibits promising control performance, but also guarantees the current constraint effectively, even in the presence of unmatched disturbances.

4) The proposed control algorithm can be easily extended to many motion control systems with second-order dynamics due to the concise control structure and the complete theoretic guarantees.

II. PROBLEM FORMULATION

A. Mathematical Model

The mathematical model of a surface-mounted PMSM in the d–q frame is formulated as [11], [28]

\[
\begin{align*}
\dot{\omega} &= \frac{1}{J} \left( n_p \psi_f i_q - B \omega - T_L \right), \\
\dot{i}_d &= \frac{1}{L_d} \left( -R_s i_d + n_p \omega L_i q + u_d \right), \\
\dot{i}_q &= \frac{1}{L_q} \left( -R_s i_q - n_p \omega L_i d - n_p \psi_f \omega + u_q \right)
\end{align*}
\]

where \( u_d, u_q, i_d, \) and \( i_q \) are the stator voltages and currents of the \( d \) and \( q \)-axes, respectively; \( \omega \) is the angular velocity; \( n_p \) is the number of pole pairs; \( R_s \) is the stator resistance; \( L \) is the stator inductance; \( \psi_f \) is the rotor flux linkage; \( T_L \) is the load torque; \( J \) is the rotor inertia, and \( B \) is the viscous frictional coefficient.

Here, the reference value of the \( d \)-axis current \( i_{q}^{\text{ref}} \) is set to zero to obtain the maximum torque-to-current ratio [39]. If the controller of \( i_d \) loop works well, one obtains \( i_d = i_d^* = 0 \). In such case, system (1) reduces to

\[
\begin{align*}
\dot{\omega} &= \frac{1}{J} \left( n_p \psi_f i_q - B \omega - T_L \right), \\
\dot{i}_q &= \frac{1}{L_q} \left( -R_s i_q - n_p \psi_f \omega + u_q \right)
\end{align*}
\]

Suppose that the current \( i_q \) is bounded by a constant \( c > 0 \), which is a safety threshold and often selected as 2~3 times of the rated current in industrial applications [10]. Since the load torque variations influence the current directly, a large load torque change may violate the current constraint. Thus, it is reasonable to assume that the load torque \( T_L \) changes within a range related to \( c \) to satisfy the constraint requirements.

Assumption 1: The load torque \( T_L \) has a positive boundary, which is defined as \( D = (n_p \psi_f I_N - B \omega_{\text{ref}})/2 \).

Remark 1: Since the rated electromagnetic torque \( n_p \psi_f I_N \) is obviously larger than the maximum viscous torque \( B \omega_{\text{ref}} \), where
$I_N$ is the rated current. With the selection of $c$, i.e., $c > I_N$, it yields that $n_p \psi_f c > B \omega_{\text{ref}}$, thus $\dot{D} > 0$.

**B. Control-Oriented Model**

The control objective is to make the rotating speed track a given reference signal, ensuring the current constraint in the presence of unmatched disturbances.

First, the rotating speed tracking error is defined as $e_1 = \omega_{\text{ref}} - \omega$, where $\omega_{\text{ref}}$ is the constant reference rotating speed. Then, defining $e_2 = -n_p \psi_f k_q + B \omega_{\text{ref}}$ as the system state, and the control-oriented model is written as

\[
\begin{align*}
\dot{e}_1 &= \frac{1}{J} (-a_1 e_1 + e_2 + d) \\
\dot{e}_2 &= \frac{1}{L} [f(e_1, e_2) + u] \\
y &= e_1
\end{align*}
\]

(3)

where $u = ba_q$ is the control input, and $y$ is the system output, with

$a_1 = B, b = -n_p \psi_f, d = T_L$

\[f(e_1, e_2) = -n_p^2 \psi_f^2 e_1 - R_s e_2 + (n_p^2 \psi_f^2 + R_s B) \omega_{\text{ref}}.\]

(4)

For simplicity of the theoretical analysis, denoting $M_1 = n_p \psi_f c + B \omega_{\text{ref}}$ and $M_2 = n_p \psi_f c - B \omega_{\text{ref}}$. From the previous analysis, we have $M_1 > 0$ and $M_2 > 0$. In this way, the current constraint $|i_q| < c$ is equivalent to the state constraint $e_2 \in (-M_2, M_1)$. And the control problem is transformed to stabilize $e_1$ in the presence of the state constraint $e_2 \in (-M_2, M_1)$ and the unmatched disturbance $d$.

**C. Motivation of Control Design**

To explain our motivation more illustratively, two kinds of the existing control methods are analyzed theoretically.

1) **CCPID Controller**

The CCPID controller is expressed as [28]

\[u = -k_p e_1 - k_I \int_0^t e_1 d\tau - f(e_1, e_2) - \left[ k_D + \frac{l}{(M_1 - e_2)(M_2 + e_2)} \right] e_2 = -k_p e_1 - k_I \int_0^t e_1 d\tau - k'_{D} e_2 - f(e_1, e_2)\]

(5)

where $k'_{D} = k_D + \frac{l}{(M_1 - e_2)(M_2 + e_2)}$, and $k_p, k_D, k_I, l$ are positive control parameters. Since the penalty term $\frac{l}{(M_1 - e_2)(M_2 + e_2)}$ increases to infinity when the current tends to its boundary, the control action can restrict the current.

Combining (3) and (5), it yields

\[
\dot{e}_2 = \frac{1}{L} \left( -k_p e_1 - k_I \int_0^t e_1 d\tau - k'_{D} e_2 \right)
\]

(6)

and it can be further obtained that

\[
\ddot{e}_1 + a_2 \dot{e}_1 + a_3 e_1 + a_4 e_2 = \frac{k_D}{LJ} d + \frac{1}{L} \dot{d}
\]

(7)

where $a_2 = \frac{k_p}{J}$, $a_3 = \frac{k_p}{LJ} + \frac{a_1 k_D}{LJ}$, and $a_4 = \frac{k_I}{J}$.

**Remark 2:** Although the current can be restricted, the controller can hardly react promptly to alleviate the influence of $\dot{d}$ and $\ddot{d}$ on the tracking error $e_1$. Moreover, the integral action may bring adverse effects, e.g., large OSs and long ST.

2) **Traditional DOBC**

The controller is expressed as [31]

\[u = -k_1 e_1 - k_2 \left( e_2 + \dot{d} \right) - f(e_1, e_2)\]

(8)

where $k_1, k_2$ are positive control parameters, and $\dot{d}$ is the disturbance estimation provided by a DOB.

Combining (3) and (8), it yields

\[
\begin{align*}
\dot{e}_1 &= \frac{1}{J} (-a_1 e_1 + e_2 + d) \\
\dot{e}_2 &= \frac{1}{L} \left[ -k_1 e_1 - k_2 \left( e_2 + \dot{d} \right) \right].
\end{align*}
\]

(9)

Denoting $e_d = d - \dot{d}$ as the estimation error, one obtains

\[
\dot{e}_1 + k'_1 e_1 + k'_2 e_2 = \frac{1}{J} \dot{d} + \frac{k_2}{LJ} e_d
\]

(10)

where $k'_1 = \frac{k_1}{J} + \frac{k_2}{L}$ and $k'_2 = \frac{k_1}{LJ} + \frac{a_1 k_2}{LJ}$.

It is supposed that $d$ has a constant steady-state value and is accurately reconstructed, then it yields

\[
\dot{e}_1 + k'_1 e_1 + k'_2 e_2 = 0
\]

(11)

which implies that $e_1$ can converge to zero asymptotically by choosing the control parameters properly.

Given that $e_1 = 0$, we have

\[
\dot{e}_2 = -\frac{k_2}{LJ} (e_2 + \dot{d}).
\]

(12)

**Remark 3:** It implies that the DOBC cannot rigidly ensure the current constraint. Usually, it selects relatively conservative control parameters to avoid excessive current. But this inevitably sacrifices the dynamic performance to some extent.

**III. CONTROLLER DESIGN**

In this section, a DOB is first utilized for the unmatched disturbance estimation. Second, a key nonlinear item containing a feed-forward compensation part and a fast constraint coping mechanism part is proposed. Then, a composite controller is developed with rigorous closed-loop stability analysis.

**A. Design of the DOB**

To acquire the estimation value $\hat{d}$ of the unmatched disturbance $d$ in system (3), the DOB is designed as [31]

\[
\begin{align*}
\dot{z} &= -p z + \frac{\alpha}{L} (a_1 e_1 - e_2 - p e_1) \\
\dot{d} &= z + p e_1
\end{align*}
\]

(13)

where $z$ is the observer state, and $p > 0$ is the observer gain, respectively.

Assumption 2: The derivative of $d$ in system (3) has a positive boundary, which is defined as $d'_2 = \sup_{t \geq 0} |d(t)|$.

**Lemma 1** (see [29]): For system (3) under Assumptions 1 and 2, if $d$ satisfies that $\lim_{t \to \infty} \dot{d}(t) = 0$, then the estimation error $e_d = d - \hat{d}$ converges to zero asymptotically.
According to (13), one obtains
\[ \dot{e}_d = \dot{d} - \dot{d} = \dot{d} - \frac{p}{J} e_d. \] (14)

Under Assumption 2, it yields
\[ |e_d(t)| \leq e^{-\frac{5t}{2}} e_d(0) + \frac{J}{p} \left( 1 - e^{-\frac{5t}{2}} \right) d_d^2 \leq D \] (15)

where \( D = \sup_{t>0} |e_d(t)| \) is the upper bound of \( e_d(t) \) and it decreases as \( p \) is increasing.

**B. Design of the Composite Controller**

For system (3), the DOB-CC controller is designed as
\[ u = -k_1 e_1 - f(e_1, e_2) - \left[ k_2 + \frac{l}{(M_1 - e_2)(M_2 + e_2)} \right] (e_2 + \dot{d}) \] (16)

and the actual control law is expressed as
\[ u_q = \frac{k_1}{n_p \psi_f} (\omega_{ref} - \omega) + R_s i_q + n_p \psi_f i_\omega \\
+ \frac{1}{n_p \psi_f} \left( k_2 + \frac{l}{c^2 - \tilde{i}_q^2} \right) B \omega_{ref} - n_p \psi_f i_q + \dot{d} \] (17)

where \( k_1, k_2, \) and \( l \) are positive control parameters.

In (16), the constraint coping mechanism \( \frac{l}{(M_1 - e_2)(M_2 + e_2)} \) is a nonlinear continuous function when \( e_2 \in (-M_2, M_1) \). If \( i_q \) tends to the limitation \( \pm c \), i.e., the state \( e_2 \) tends to the barrier boundary \( M_1 \) or \( -M_2 \), the function will tend to infinity. In this manner, the controller restricts the current effectively by regulating the controller gain automatically.

Combining (14) and (16) with (3), the closed-loop system is written as
\[
\begin{aligned}
\dot{e}_1 &= -\frac{n_p}{L} e_1 + \frac{1}{L} (e_2 + \dot{d}) + \frac{1}{2} e_d \\
\dot{e}_2 &= -\frac{n_p}{L} e_1 - \frac{1}{L} \left[ k_2 + \frac{l}{(M_1 - e_2)(M_2 + e_2)} \right] (e_2 + \dot{d}) \\
\dot{e}_d &= \dot{d} - \frac{p}{J} e_d
\end{aligned}
\] (18)

where \( e = [e_1, e_2, e_d]^T \) is denoted as the state vector.

**Theorem 1**: Under Assumptions 1 and 2, if the initial state \( e_2(0) \in (-M_2, M_1), \) i.e., \( i_q(0) \in (-c, c), \) the state \( e \) of the closed-loop system (18) asymptotically converges to a bounded compact set \( \Omega_B = \{ e | \alpha V(e) \leq \frac{1}{2} d_d^2 \} \), where \( \alpha > 0 \) is specified later. Meanwhile, the condition \( e_2 \in (-M_2, M_1) \) holds. Namely, the rotating speed \( \omega \) asymptotically tracks the reference value \( \omega_{ref} \) to a bounded compact set, and the current constraint \( |i_q| < c \) is guaranteed.

**Proof**: Defining \( T \) as a positive constant or \( T = +\infty \). It is assumed that the condition \( e_2(t) \in (-M_2, M_1) \) is satisfied during \( t \in [0, T] \). There are three steps in the proof.

**Step 1**: To prove that the states of system (18) are bounded, when \( t \in [0, T] \).

The candidate Lyapunov function is chosen as
\[ V(e) = \frac{k_1 J}{2L} e_1^2 + \frac{1}{2} \left( e_2 + \frac{1}{2} \right) e_2^2 + \frac{1}{2} e_d^2 \] (19)

Taking the derivative of \( V(e) \) along system (18), one obtains
\[ \dot{V}(e) = \frac{k_1 J}{L} e_1 \dot{e}_1 + \left( e_2 + \dot{d} \right) \left( \dot{e}_2 + \dot{d} \right) + \alpha V(e) \]
\[ = -\frac{a_1 k_1}{L} e_1^2 - \frac{k_1}{L} e_1 e_d - \frac{k_2}{L} \left( e_2 + \dot{d} \right)^2 + \frac{p}{J} e_{ed} \left( e_2 + \dot{d} \right) \]
\[- \frac{p}{J} e_{ed} + e_d \dot{d} - \frac{L}{(M_1 - e_2)} \frac{e_{ed}^2}{2} \leq -\frac{a_1 k_1}{L} e_1^2 - \frac{k_2}{L} \frac{p}{J} \left( e_2 + \dot{d} \right)^2 - \frac{1}{2} \frac{(p - k_1^2)}{L} e_d^2 + \frac{1}{2} \frac{p}{J} e_{ed} \] (20)

If the control parameters are selected such that \( \frac{1}{2} k_1^2 < k_1 < \sqrt{\frac{p}{J} - 1} L \) and \( k_2 > \frac{p}{J} \), then we have
\[ \dot{V}(e) \leq -\alpha V(e) + \frac{1}{2} d_d^2 \] (21)

where \( \alpha = \min \{ \frac{a_1}{L}, \frac{k_1}{L}, \frac{k_2}{L}, \frac{p}{J}, \frac{k_1^2}{L} \} \) is positive. According to the Lyapunov theory, one can conclude that the states of system (18) are bounded, when \( t \in [0, T] \). Furthermore, we suppose that \( e_1 \) is bounded by a positive constant \( A_0 = \sup_{t>0} |e_1(t)|, t \in [0, T] \).

**Step 2**: To prove that the condition \( e_2 \in (-M_2, M_1) \) holds, i.e., the current constraint is ensured all the time:

For definition clarity, denoting \( G = (-\infty, +\infty) \times (-M_2, M_1) \times [-D, D] \) as the corresponding domain to the feasible zone of system (2) in Fig. 1(a). In Fig. 1(b), \( G_1 \) (blue, top) and \( G_2 \) (green, bottom) represent the corresponding values of \( e \) when \( i_q \) is close to its boundaries \( \pm c \), respectively; and \( G_3 \) (white, middle) is the rest part satisfying \( |i_q| < c \), which will be specified later.

Based on Step 1, denoting \( G_1 = [-A_0, A_0] \times [m_1, M_1] \times [-D, D], \) \( G_2 = [-A_0, A_0] \times (-M_2, -m_2] \times [-D, D], \) and \( G_3 = [-A_0, A_0] \times (-m_2, m_1) \times [-D, D], \) where \( m_1 \) and \( m_2 \) are chosen as
\[ m_1 = \max \{ |e_2(0)|, N_1, \bar{D} \} > 0 \]
\[ m_2 = \max \{ |e_2(0)|, N_2, \bar{D} \} > 0 \]  \hspace{1cm} (22)

with
\[ N_1 = \frac{M_1 \eta + \bar{D} l}{\eta + l}, \quad N_2 = \frac{M_2 \eta + \bar{D} l}{\eta + l} \]
\[ \eta = k_1 A_0 (M_1 + M_2), \quad \bar{D} = D + \bar{D}. \]  \hspace{1cm} (23)

To prove that the condition \( e_2(t) \in (-M_2, M_1) \) holds when \( t \in [0, T) \), the following three cases are discussed to prove that the states starting from \( G_1 \) and \( G_2 \) cannot cross the restricted boundaries when \( t \in [0, T) \).

**Case 1:** If \( e(t) \in G_1 \), where \( e_2(t) \in [m_1, M_1], t \in [0, T) \). Taking the derivative of \( e_2(t) \), it yields
\[ \dot{e}_2 = \frac{-k_1 e_1}{L} - \frac{e_2 + \bar{d}}{L} \left[ k_2 + \frac{l}{(M_1 - e_2)(M_2 + e_2)} \right] \leq \frac{-k_1 A_0}{L} - \frac{l (e_2 - \bar{D})}{(M_1 + M_2)(M_1 - e_2)} < 0 \]  \hspace{1cm} (24)

and we have \( e_2(t) \dot{e}_2(t) < 0, t \in [0, T) \). It implies that the condition \( e_2(t) < M_1, t \in [0, T) \) cannot be violated and the state \( e \) will enter \( G_3 \) according to Step 1. In Fig. 1(b), the trajectory of Point A is drawn for instance.

**Case 2:** If \( e(t) \in G_2 \), where \( e_2(t) \in (-M_2, -m_2), t \in [0, T) \). Taking the derivative of \( e_2(t) \), it yields
\[ \dot{e}_2 = \frac{-k_1 e_1}{L} - \frac{e_2 + \bar{d}}{L} \left[ k_2 + \frac{l}{(M_1 - e_2)(M_2 + e_2)} \right] \geq \frac{-k_1 A_0}{L} - \frac{l (e_2 + \bar{D})}{(M_1 + M_2)(M_2 + e_2)} > 0 \]  \hspace{1cm} (25)

and we have \( e_2(t) \dot{e}_2(t) < 0, t \in [0, T) \). One obtains that the condition \( e_2(t) > -M_2, t \in [0, T) \) cannot be violated and the state \( e \) will enter \( G_3 \) likewise.

**Case 3:** If \( e(t) \in G_3 \), where \( e_2(t) \in (-m_2, m_1), t \in [0, T) \), then the condition \( e_2(t) \in (-M_2, M_1) \) is originally satisfied. Considering Cases 1 and 2, if \( e \) enters \( G_1 \) or \( G_2 \), it will finally move into \( G_3 \). And (21) reveals that infinite switches between Case 3 and Case 1/Case 2 cannot happen. In Fig. 1(b), the trajectory of Point B is drawn for instance.

Next, we will prove that \( T = +\infty \), which indicates the satisfaction of the condition \( e_2 \in (-M_2, M_1) \) all the time.

It is assumed that for any \( e(0) \in G \), \( e(t) \) is the solution of system (18) on \( G_3 = (-A_0 - 1, A_0 + 1) \times (-M_2, M_1) \times [-D, \bar{D}] \subset G, t \in [0, T) \). Then, we have \( e(t) \in G_3 \subset G_3, t \in [0, T) \). Since it is defined that \( T > 0 \) or \( T = +\infty \), on the basis of the continuation theorem of solution, it is obtained that \( T = +\infty \). Thus, \( e_2 \in (-M_2, M_1) \) always holds, i.e., the current constraint \( |i_q| < c \) is satisfied all the time.

**Step 3:** To prove that the state \( e \) of system (18) converges to a bounded compact set asymptotically.

From the above analysis, it is obtained that (21) holds when \( t \in [0, +\infty) \). Then, denoting a bounded compact set as \( \Omega_B = \{ \epsilon |aV(e) \leq \frac{1}{2} \theta_d^2 \} \), one obtains that the state \( e \) of system (18) converges to \( \Omega_B \) asymptotically. As such, the rotating speed \( \omega \) tracks the reference signal \( \omega_{ref} \) to a bounded compact set asymptotically. This completes the proof.

**Corollary I:** Under Assumptions 1 and 2, if the condition \( \lim_{t \to +\infty} \dot{d}(t) = 0 \) holds, and the initial state \( e_2(0) \in (-M_2, M_1) \), i.e., \( i_q(0) \in (-c, c) \), then the origin of the closed-loop system (18) is asymptotically stable and the condition \( e_2 \in (-M_2, M_1) \) always holds. Namely, the rotating speed \( \omega \) tracks the reference value \( \omega_{ref} \) asymptotically, while the current constraint \( |i_q| < c \) is guaranteed all the time.

**Proof:** The proof is analogous to that of Theorem 1, and thus is omitted here for space.

The control block diagram is presented in Fig. 2.

**Remark 4:** The proposed controller mainly focuses on handling the current constraint and unmatched disturbances for the speed regulation of PMSMs. When further considering the control input saturation, a safe distance away from the constraint boundary should be determined for the current, which helps provide a more flexible current constraint range for practical applications. Theoretical analysis will be investigated in the future work.

IV. NUMERICAL TESTS RESULTS

A. Test Design

In this section, numerical tests are performed using MATLAB/Simulink. The specification is given in Table I. The rotating speed reference is 3000 r/min, the current constraint is 10 A, and the voltage saturation limit is 220 V.
In the simulation, the proposed method (16), i.e., the DOB-CC controller, is compared with the conventional DOBC and the CCPID controller. The parameter tuning criterion should comprehensively take into account system stability, dynamic performance, current constraint satisfaction, and robustness against unmatched disturbances. For fair comparisons, the control parameters of the DOB-CC, high-gain DOBC, and CCPID controllers are tuned such that the speed tracking profiles have similar nominal performance specifications. While the low-gain DOBC is tuned to guarantee the current constraints in the whole process. The control parameters of the simulations are presented as follows:

- For the high-gain DOBC: $k_1 = 8000$, $k_2 = 200$, $p = 200$
- For the low-gain DOBC: $k_1 = 6000$, $k_2 = 200$, $p = 200$
- For the CCPID controller: $k_P = 8000$, $k_D = 200$, $k_I = 85000$, $l = 50$
- For the DOB-CC controller: $k_1 = 8000$, $k_2 = 200$, $l = 50$, $p = 200$

**B. Numerical Results**

- **Test 1: At the start-up phase**

  Fig. 3(a)–(c) are system responses of $e_1$, $i_q$, and $u_q$, respectively. To evaluate the control results qualitatively, the performance specification including overshoot (OS), settling time (ST), and peak current (PC) are utilized for validation. The performance indices are tabulated in Table II.

  As shown in Fig. 3(a) and Table II, the speed response under the DOB-CC controller has comparable dynamic performance with the high-gain DOBC and CCPID controller, while the

**Table I**

| Symbol | Quantity | Nominal Values |
|--------|----------|----------------|
| $n_p$  | Poles    | 4              |
| $P_N$  | Rated power | 0.75kW         |
| $U_N$  | Rated voltage | 220V          |
| $I_N$  | Rated current | 4.7A          |
| $\omega_N$ | Rated rotating speed | 3000rpm       |
| $T_N$  | Rated load torque | 2.387N⋅m       |
| $\psi_f$ | Flux linkage | 0.081Wb        |
| $J$    | Rotor inertia | $2.35 \times 10^{-4}$kg⋅m$^2$ |
| $B$    | Viscous damping | $7.4 \times 10^{-4}$N⋅m⋅s/rad   |
| $R_s$  | Stator resistance | 0.8Ω          |
| $L_s$  | Stator inductance | 2.9mH        |

**Table II**

| Performance | OS(rpm) | ST(s) | PC(A) | RT(s) | RMSE(rpm) |
|-------------|---------|-------|-------|-------|-----------|
| DOBC(high)  | 0.206   | 0.072 | 10.268| 0.048 | 34.904    |
| DOBC(low)   | 0.202   | 0.116 | 9.956 | 0.090 | 40.772    |
| CCPID       | 0.204   | 0.083 | 9.951 | 0.112 | 78.692    |
| DOB-CC      | 0.203   | 0.079 | 9.953 | 0.055 | 35.606    |

**Table III**

| Performance | OS(rpm) | ST(s) | PC(A) | RT(s) | RMSE(rpm) |
|-------------|---------|-------|-------|-------|-----------|
| DOBC(high)  | 2.985   | 0.117 | 11.309| 0.113 | 35.758    |
| DOBC(low)   | 2.263   | 0.160 | 9.612 | 0.137 | 41.363    |
| CCPID       | 2.932   | 0.123 | 9.609 | 0.242 | 80.453    |
| DOB-CC      | 2.763   | 0.120 | 9.615 | 0.116 | 36.161    |
low-gain DOBC has longer ST. Regarding the current constraint problem, Fig. 3(b) and Table II illustrate that the DOB-CC and CCPID controllers keep a good balance between the speed regulation ability and satisfaction of the current constraint. While the PC of the high-gain DOBC exceeds constraint 10 A. It indicates that the DOBC ensures the current constraint by selecting more conservative parameters. But this sacrifices, the dynamic performance to some extent.

- **Test 2: Subject to different load torque disturbances**
  In what follows, we consider the current satisfaction and robustness under three controllers in the presence of the following three kinds of load torque disturbances.

  1) *Step form:* The load torque \( T_L \) increases from 1 N-m to 3 N-m at \( t = 0.4 \) s and decreases to 2.8 N-m at \( t = 0.7 \) s.

  2) *Sawtooth form:* The load torque \( T_L \) repeatedly and monotonically increases from 2.8 N-m to 3.2 N-m and decreases to 2.8 N-m, when \( 0.9 \leq t < 1.5 \) s.

  3) *Sinusoidal form:* The load torque \( T_L \) is imposed as \( T_L = 0.3 \sin(20\pi t) + 0.2 \sin(10\pi t) + 2.8 \) N-m when \( 1.5 \leq t \leq 2 \) s.

The system responses are shown in Fig. 3. Corresponding with the numerical tests, the following load torque disturbances are taken into account in the experiments.

As shown in Fig. 3(a) and Table II, the speed responses of the proposed DOB-CC controller and the high-gain DOBC are similar. While the low-gain DOBC and the CCPID controller have longer RT and generate larger RMSE. Fig. 3(b) and Table II reveal that the DOB-CC controller ensures the current constraint 10 A, while the high-gain DOBC violates the constraint. Moreover, in Fig. 3(c), the control efforts of the four controllers are kept at the same level for fair comparisons.

As shown in Tests 1 and 2, the proposed DOB-CC controller attains the best performance among three methods. It balances well between speed regulation performance and robustness, while satisfying the current constraint in the whole process.

V. EXPERIMENTAL TESTS RESULTS

A. Test Design

For further validation, the experimental setup has been built and its configuration is shown in Fig. 4. The control and drive circuits are integrated in the platform TMDSHVMTRPFCKIT explored by Texas Instrument [40]. The current constraint is 10 A, the voltage saturation limit is 220 V, and the sampling frequency is 10 kHz. The control parameters are given as: \( k_1 = 7500, k_2 = 180, p = 200 \) for the high-gain DOBC; \( k_1 = 5500, k_2 = 180, p = 200 \) for the low-gain DOBC; \( k_p = 7500, k_D = 180, k_I = 80000, l = 50 \) for the CCPID controller; and \( k_1 = 7500, k_3 = 180, l = 50, p = 200 \) for the DOB-CC controller.

B. Experimental Results

- **Test 1: At the start-up phase**

  Fig. 5(a)–(c) are system responses of \( e_1, i_q, \) and \( u_q \), respectively. The corresponding performance indices are presented in Table III. Consistent with the numerical results, it is observed in Fig. 5 and Table III that the proposed DOB-CC controller achieves comparable dynamic performance with CCPID controller, i.e., similar OS and ST. Both PCs of them are lower than 10 A, which indicates that the proposed DOB-CC and CCPID controllers ensure the current constraint effectively. While the DOBC cannot guarantee the current constraint under the same dynamic performance requirement.

- **Test 2: Subject to different load torque disturbances**

  Corresponding with the numerical tests, the following load torque disturbances are taken into account in the experiments.

  1) *Step form:* The load torque \( T_L \) increases from 1 N-m to 3 N-m at \( t = 2.2 \) s and decreases to 2.8 N-m at \( t = 2.6 \) s.

  2) *Sawtooth form:* The load torque \( T_L \) monotonically increases from 2.8 to 3.2N-m and decreases to 2.8 N-m when \( 3.2 \leq t \leq 3.8 \) s.

  3) *Sinusoidal form:* The load torque \( T_L = 0.3 \sin(20\pi t) + 0.2 \sin(10\pi t) + 2.8 \) N-m imposed when \( 4 \leq t \leq 5 \) s.

  The system responses are shown in Figs. 6–8, and Table III presents specific speed regulation performance indices.

As shown in Figs. 6(a), 7(a), and 8(a) and Table III, the RT and RMSE of the DOB-CC controller and the high-gain DOBC are comparable, and smaller than those of the low-gain DOBC and CCPID controller. Table III and Figs. 6(b), 7(b), and 8(b) illustrate that the DOB-CC controller ensures the current constraint 10 A, while the high-gain DOBC fails to do so. Furthermore, Figs. 6(c), 7(c), and 8(c) indicate that each control effort is kept at the same level.

Hence, the experimental results further verify the effectiveness of the proposed DOB-CC method, which can simultaneously realize the current constraint task and compensate the

![Fig. 5. System responses at the start-up phase: (a) speed tracking error \( e_1 \), (b) \( q \)-axis current \( i_q \), (c) \( q \)-axis voltage \( u_q \).](image)
undesirable effects of unmatched load torque variations, while
maintaining satisfying speed tracking performance.

VI. CONCLUSION

The overcurrent protection and unmatched disturbance rejection problems was investigated simultaneously for speed regulation of PMSM systems. In this article, a DOB-CC controller was developed via a DOB and a constraint coping mechanism. Comprehensive theoretical analysis, together with numerical and experimental validation, had demonstrated its effectiveness and feasibility. A promising future research direction is to extend the proposed method to a higher order system with varying state constraints and control input saturations subject to matched/unmatched disturbances.

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