Bulk plasmon-phonon polaritons in n-GaN

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Abstract. We studied theoretically and experimentally plasmon-phonon polaritons and
longitudinal plasmon-phonon oscillations in n-GaN epitaxial layers. The studies were carried
out on the epitaxial layers with various doping levels. Simulation of the reflectivity spectra
and dispersion relations of plasmon-phonon polaritons was performed in a wide frequency range.
Reflectivity spectra transformation associated with phonon damping and electron relaxation
processes has been revealed. Experimental studies of the reflectivity spectra have been
performed in the spectral range of 8–80 meV. The experimental spectra are well fitted by the
simulated ones. Results of the study can be used for contactless determination of the electron
concentration and mobility in GaN epitaxial layers.

1. Introduction
During recent years, bulk and surface polaritons are extensively researched in order to create mid-
infrared and terahertz selective sources [1–3]. Intense incoherent radiation of GaAs plasmon-phonon
polaritons was obtained experimentally in the range of 10–15 THz [1]. A plasmon-phonon polariton
(PPP) is a “hybrid” quasiparticle emerged in a polar semiconductor under interaction of a photon, a
polar phonon and a bulk plasmon. In the framework of the wave theory, the PPP is a transverse
electromagnetic wave propagating in a polar semiconductor.

Let us start with a general consideration of electromagnetic waves in an isotropic medium.
Propagation of a plane wave is described by the wave equation [4]:

\[ \mathbf{q} \cdot (\mathbf{q} \cdot \mathbf{E}) - q^2 \cdot \mathbf{E} = -\varepsilon(\omega) \frac{\partial^2}{c^2} \mathbf{E}, \]  

(1)

where \( \mathbf{q} \) is the wave vector, \( \mathbf{E} \) is the electric field strength, \( \varepsilon(\omega) \) is the
dielectric permittivity, \( \omega \) is the angular frequency of the wave.

In accordance with equation (1), two types of electromagnetic waves can propagate in a medium,
namely, longitudinal waves and transverse ones. Longitudinal waves (\( \mathbf{E} \parallel \mathbf{q} \))
can propagate only at frequency values which represent the roots of the following equation:

\[ \varepsilon(\omega) = 0. \]  

(2)

In the case of transverse waves (\( \mathbf{E} \perp \mathbf{q} \)), the dependence of the wave vector on the angular
frequency is described by the dispersion relation:
Let us consider propagation of electromagnetic waves in three particular objects: (i) an \( n \)-doped nonpolar semiconductor, (ii) a polar semiconductor without free charge carriers (a polar dielectric), and (iii) an \( n \)-doped polar semiconductor.

(i) The dielectric permittivity of an \( n \)-doped nonpolar semiconductor can be written as

\[
\epsilon(\omega) = \epsilon_{\infty} \left[ 1 - \frac{\omega_p^2}{\omega^2} \right],
\]

where \( \omega_p = \sqrt{\frac{4\pi e^2 N_e}{\epsilon m_e}} \) is the plasma frequency, \( N_e \) is the free electron concentration, \( \epsilon \) is the elementary charge, \( m_e \) is the electron effective mass, \( \epsilon_{\infty} \) is the high frequency permittivity \([5]\). Equation (4) is related to the simplest model of electron gas (so-called collisionless plasma) when the free electron relaxation time \( \tau = \infty \). Because equation (2) has only one root (\( \omega = \omega_p \)), the longitudinal electromagnetic waves can propagate at the frequency \( \omega_p \) only. These waves represent longitudinal collective oscillations of electron gas (plasmons). Within the framework of the considered approximation, the longitudinal waves have no dispersion, and their behaviour is illustrated by the blue line in figure 1 (a). Besides plasmons, specific transverse electromagnetic waves which are called “plasmon polaritons” can be excited in the considered medium as well. The plasmon polariton dispersion dependence calculated in accordance with equations (3) and (4) is shown in figure 1 (a) by the red line. In the long-wavelength limit (\( q = 0 \)), plasmons and plasmon polaritons have the same frequency: \( \omega = \omega_p \). For the region of rather small \( q \) when \( \omega \) is comparable with \( \omega_p \), it can be shown that the contribution of the electromagnetic field energy to the total plasmon polariton energy is comparable with the contribution of the mechanical energy of transverse electron oscillations. In other words, a plasmon polariton in this region is a proper hybrid of a photon (electromagnetic field) and a plasmon (electron oscillations). Under increase of the wave vector \( q \), the plasmon polariton dispersion \( \omega(q) \) asymptotically approaches the straight line \( \omega = q\epsilon\epsilon_{\infty}^{-1/2} \) (at frequencies \( \omega \gg \omega_p \)). This behavior indicates that the contribution of the electromagnetic field energy to the plasmon polariton energy is dominant, and the contribution of the mechanical energy of transverse electron oscillations is negligible. As it is known, the linear dispersion dependence \( \omega = q\epsilon\epsilon_{\infty}^{-1/2} \) describes the propagation of a “pure” photon in a medium with the dielectric permittivity \( \epsilon_{\infty} \). It should be emphasized that in the spectral interval \( 0 \leq \omega < \omega_p \) a plasmon polariton cannot be excited. In the reflection spectrum one can observe a band of total reflection (reflectivity \( r = 1 \)) in this frequency range.

(ii) The dielectric permittivity of a polar semiconductor without free electrons can be described by the following expression \([4]\):

\[
\epsilon(\omega) = \epsilon_{\infty} \left[ 1 + \frac{\omega_{\text{LO}}^2 - \omega_{\text{TO}}^2}{\omega_{\text{TO}}^2 - \omega^2} \right],
\]

where \( \omega_{\text{LO}} \) is the longitudinal optical phonon frequency, \( \omega_{\text{TO}} \) is the transverse optical phonon frequency. In this medium, equation \( \epsilon(\omega) = 0 \) gives only one root: \( \omega = \omega_{\text{LO}} \). Subsequently, longitudinal electromagnetic waves can be excited at this frequency only. These waves are associated with longitudinal lattice vibrations (LO phonons). They have no dispersion as it is shown by the blue line in figure 1 (b). Besides, specific transverse electromagnetic waves (phonon polaritons) which represent coupled oscillations of the lattice and electromagnetic field can be excited in a polar
semiconductor without free electrons. These transverse oscillations cannot be separated because simultaneously electromagnetic waves excite the TO phonons and oscillating charges radiate electromagnetic waves. The phonon polariton dispersion dependence calculated in accordance with equations (3) and (5) is shown in figure 1 (b) by the red lines. It consists of two branches. The upper branch starts at the frequency \( \omega = \omega_{\text{LO}} \). It can be shown that at small \( q \) the contribution of the mechanical energy of the lattice oscillations to the total energy of phonon polariton is comparable with the energy of the electromagnetic field. At frequencies \( \omega >> \omega_{\text{LO}} \), the upper branch of the dispersion dependence of a phonon polariton asymptotically approaches the straight line \( \omega = q c \varepsilon_{\infty}^{-1/2} \). As it was mentioned above, this straight line corresponds to a “pure” photon in a medium with the dielectric permittivity \( \varepsilon_{\infty} \). In the high frequency limit, the contribution of the mechanical energy of the lattice oscillations to the total energy of phonon polariton becomes negligible. The lower branch of the phonon polariton dispersion dependence continuously covers the spectral interval from 0 to \( \omega_{\text{TO}} \). It
can be shown that in the limit of large \( q \) when \( \omega \rightarrow \omega_{\text{TO}} \) the contribution of lattice vibrations to the phonon polariton energy dominates the electromagnetic field contribution. In the spectral interval \( \omega_{\text{TO}} < \omega < \omega_{\text{LO}} \), transverse electromagnetic waves cannot be excited. Respectively, in the reflection spectrum one can observe total reflection \( (r = 1) \) in this frequency interval (the so-called Reststrahlen band).

(iii) Finally, let us consider an \( n \)-doped polar semiconductor which is the subject of the present work. In this case, the dielectric permittivity includes both the lattice and free electron contributions \([4, 5]\):

\[
\varepsilon(\omega) = \varepsilon_\infty \left[ 1 + \frac{\omega_{\text{LO}}^2 - \omega_0^2}{\omega_{\text{TO}}^2 - \omega^2} - \frac{\omega_e^2}{\omega^2} \right],
\]

and equation (2) gives two different roots (we denote them as \( \omega_+ \) and \( \omega_- \)). Subsequently, longitudinal electromagnetic waves can propagate at these two frequencies. These oscillations are known as the coupled plasmon-phonon modes, they have no dispersion (see two blue lines in figure 1 (c)). Besides, specific transverse electromagnetic waves (plasmon-phonon polaritons) can be excited in a doped polar semiconductor. These waves represent coupled transverse oscillations of the electromagnetic field, lattice and electron gas which cannot be separated. The plasmon-phonon polariton dispersion dependence calculated in accordance with equations (3) and (6) is shown in figure 1 (c) by the red lines. It has two branches. The upper branch starts at the frequency \( \omega = \omega_+ \) and at frequencies \( \omega \gg \omega_+ \) asymptotically approaches the straight line \( \omega = q \epsilon \epsilon_\infty^{-1/2} \) which corresponds to a "pure" photon in a medium with the dielectric permittivity \( \epsilon_\infty \). It can be shown that for the upper branch in the limit \( q \rightarrow \infty \) the contribution of the electromagnetic field energy to the total plasmon-phonon polariton energy is dominant. In contrast to the cases (i) and (ii), here one can see two spectral intervals \( 0 < \omega < \omega_+ \) and \( \omega_{\text{TO}} < \omega < \omega_+ \) where the transverse electromagnetic waves cannot be excited. Respectively, in the reflection spectrum one can observe total reflection \( (r = 1) \) in both these frequency intervals.

It should be emphasized that in the above consideration we neglected the optical phonon damping and electron relaxation processes. However, as it will be shown below, the both factors are very important.

The goal of our work is to study theoretically and experimentally plasmon-phonon polaritons and longitudinal plasmon-phonon oscillations in \( n \)-GaN epitaxial layers. The studies are carried out on the epitaxial layers with various doping levels. In the paper, we present theoretical consideration taking into account the optical phonon damping and electron relaxation processes. Simulation of the reflectivity spectra and dispersion relations of plasmon-phonon polaritons was fulfilled in a wide frequency range. Experimental studies of the reflectivity spectra were performed in the spectral range of 8–80 meV. It has been shown that the considered theoretical model provides a good match with the experimental results.

2. Longitudinal plasmon-phonon modes in \( n \)-GaN

To take into account the optical phonon damping and electron relaxation processes, instead of equation (6) we used a more general expression for the dielectric permittivity of a polar semiconductor \([4, 5]\):

\[
\varepsilon(\omega) = \varepsilon_\infty \left[ 1 + \frac{\omega_{\text{LO}}^2 - \omega_0^2}{\omega_{\text{TO}}^2 - \omega^2} - \frac{\omega_e^2}{\omega^2 - i\omega\tau^{-1}} \right],
\]
where $\gamma$ is the damping constant for TO phonons, $\tau$ is the electron relaxation time determined by the mobility $\mu$: $\tau = \mu m_e/e$. It should be noted that equation (6) can be regarded as a particular case of equation (7) with $\gamma = 0$ and $\tau = \infty$.

We calculated frequencies of the longitudinal plasmon-phonon modes ($\omega_+$ and $\omega_-$) using equations (2), (7) and the Lyddane–Sachs–Teller relation: $\omega^2_{\text{LO}} / \omega^2_{\text{TO}} = \varepsilon_0 / \varepsilon_\infty$, where $\varepsilon_0$ is the static permittivity. The calculations were carried out for bulk GaN at various free electron concentrations $N_e$ in the range from $10^{17}$ to $10^{20}$ cm$^{-3}$. The following parameters of GaN were used [2]: $\varepsilon_\infty = 5.4$, $\varepsilon_0 = 9.5$, $\hbar \omega_{\text{TO}} = 69.3$ meV, $\gamma = 7.5 \times 10^{11}$ s$^{-1}$, $m_e = 0.2 m_0$. The dependence of $\tau$ on $N_e$ was derived from the experimental data on the electron concentrations and mobilities in $n$-GaN samples with various doping levels [6].

The resulting dependences of the frequencies $\omega_+$ and $\omega_-$ upon electron concentration $N_e$ are shown in figure 2 by the red lines. Besides, we calculated similar dependences ignoring the optical phonon damping and electron relaxation processes ($\gamma = 0$, $\tau = \infty$). The latter dependences are shown in figure 2 by the solid blue lines.

![Figure 2](image_url)

**Figure 2.** Angular frequencies of the longitudinal plasmon–phonon modes in $n$-GaN as a function of electron concentration. Red lines: calculation taking into account the optical phonon damping and electron relaxation processes. Solid blue lines: calculation ignoring these factors. The dashed blue line shows the concentration dependence of the plasma frequency $\omega_p$. Vertical lines denote electron concentrations in the samples under experimental study.

One can see that the dependence $\omega_+ (N_e)$ is not affected by the electron relaxation time $\tau$. The dependence approaches the LO phonon frequency $\omega_{\text{LO}}$ at low free electron concentrations. At high free electron concentrations (when $\omega_p > \omega_{\text{LO}}$, i.e. at $N_e > 2 \times 10^{19}$ cm$^{-3}$) $\omega_+ (N_e) \approx \omega_p (N_e)$. Such a
behavior is associated with redistribution of lattice and electron contributions to the dielectric permittivity with doping level increase.

The dependence $\omega_\epsilon(N_e)$ at high free electron concentrations approaches the TO phonon frequency $\omega_{TO}$ (and also is not affected by the electron relaxation time). This is due to electron screening of longitudinal optical oscillations. On the contrary, at low free electron concentrations ($N_e < 10^{18}$ cm$^{-3}$) the dependence $\omega_\epsilon(N_e)$ is strongly affected by the electron relaxation time. In the hypothetical case of collisionless plasma ($\tau = \infty$), $\omega_\epsilon(N_e) = \omega_{p}(N_e)\sqrt{\frac{\epsilon_{\infty}}{\epsilon_0}}$. However, the results of calculation based on the experimental dependence $\tau(N_e)$ drastically differ (see figure 2). In particular, with decreasing electron concentration $\omega_\epsilon$ approaches zero much rapidly than $\omega_{p}\sqrt{\frac{\epsilon_{\infty}}{\epsilon_0}}$.

3. Dispersion of plasmon-phonon polaritons and simulated reflectivity spectra

Let us consider excitation of the transverse electromagnetic wave (plasmon-phonon polaritons) in a semi-infinite polar semiconductor under normal incidence of the plane electromagnetic wave with a frequency $\omega$ at the planar vacuum/semiconductor interface. Under these conditions, the wave excited inside a semiconductor is homogeneous and propagates in the direction perpendicular to the interface. The surfaces of constant phase and constant amplitude are parallel to the interface. If the dielectric permittivity of a semiconductor is characterized by equation (7), propagation of the wave inside a semiconductor can be described by the complex wave vector $q = \frac{\omega}{c}(n + ik)$, where $n$ is the refractive index and $k$ is the extinction coefficient which are determined by the dispersion relation (3). We carried out these calculations for plasmon-phonon polaritons in bulk $n$-GaN with different electron concentrations and plotted the dependencies of $\text{Re}(q) = \frac{\omega}{c}n$ and $\text{Im}(q) = \frac{\omega}{c}k$ upon photon energy $\hbar\omega$. These two dependencies completely describe the dispersion of plasmon-phonon polaritons. The results of simulation for the $n$-GaN crystals with electron concentrations of $2\times10^{18}$ and $2\times10^{19}$ cm$^{-3}$ are presented in figure 3 by the red lines. We assumed the same value of electron mobility (179 cm$^2$/V·s) for both samples. Other parameters of GaN are specified in section 2. For comparison, the results of dispersion simulation based on a simplified model ignoring dissipation processes ($\gamma = 0$, $\tau = \infty$) are presented in the same figure by the solid blue lines. The reflectivity spectra $r(\hbar\omega)$ calculated for the vacuum/semiconductor interface in the framework of both models are presented in figure 3 as well. The calculation of the reflectivity was performed using Fresnel’s formula for the case of normal incidence:

$$r = \frac{(n - 1)^2 + k^2}{(n + 1)^2 + k^2}.$$  \hspace{1cm} \text{(8)}$$

If dissipation processes are ignored, the simulation predicts existence of two spectral ranges where $\text{Re}(q)$ is equal to zero: $[0, \hbar\omega_{TO}]$ and $[\hbar\omega_{TO}, \hbar\omega_{p}]$, see the solid blue lines in figure 3 (a, d). Thus, outer illumination of a polar semiconductor in these spectral ranges cannot excite plasmon-phonon polaritons inside the crystal. Obviously, in both ranges the refractive index $n = 0$ and, in accordance with equation (8), the reflectivity is equal to 1 (see the solid blue lines in figure 3 (c, f)).
Figure 3. Dispersion dependencies of plasmon-phonon polaritons (a, b, d, e) and reflectivity spectra (c, f) for $n$-GaN. Simulation for two electron concentrations: $N_e = 2 \cdot 10^{18}$ cm$^{-3}$ (a, b, c) and $2 \cdot 10^{19}$ cm$^{-3}$ (d, e, f). Red lines: calculation taking into account the optical phonon damping and electron relaxation processes. Solid blue lines: calculation ignoring dissipation processes.

When accounting for dissipation processes ($\gamma \neq 0$, $\tau \neq \infty$), the quantity $\text{Re}(q)$ becomes non-zero at all photon energies (see the red lines in figure 3 (a, d)). Consequently, the reflectivity becomes...
less than 1 in the whole spectrum (see the red lines in figure 3 (c, f). It means that dissipation allows exciting transverse electromagnetic waves (PPP) at any photon energy, in particular in the ranges $[0, h\omega_\perp]$ and $[h\omega_{t0}, h\omega_\parallel]$. It should be emphasized that in the spectral range $[h\omega_{t0}, h\omega_\parallel]$ a significant decrease of the reflectivity due to dissipation processes occurs at the high-frequency edge only. However, at the low-frequency edge of this range, the decrease of the reflectivity is rather small and one can see a sharp peak with high reflectivity at $h\omega \approx h\omega_{t0}$. This peak results from the sharp peak in the spectrum of $\text{Im}(q)$ (and, consequently, the spectrum of $k$), see the red lines in figure 3 (b, e).

Under electron concentration increase from $2 \cdot 10^{18}$ cm$^{-3}$ (figure 3 (a–c)) to $2 \cdot 10^{19}$ cm$^{-3}$ (figure 3 (d–f)), the low-frequency dip in the reflectivity spectrum (around $h\omega_\perp$) becomes significantly narrower but the high-frequency one (around $h\omega_\parallel$) broadens. These peculiarities are associated with redistribution of the phonon and plasmon contributions to the coupled plasmon-phonon modes with doping level increase. Namely, at a lower doping level the mode $h\omega_\perp$ is mainly formed by the plasmon contribution ($h\omega_\perp \approx h\omega_p$), and the mode $h\omega_\parallel$ is mainly formed by the phonon contribution ($h\omega_\parallel \approx h\omega_k$). On the contrary, at a higher doping level the mode $h\omega_\perp$ is mainly formed by the phonon contribution ($h\omega_\perp \approx h\omega_{t0}$) and the mode $h\omega_\parallel$ is mainly formed by the plasmon contribution ($h\omega_\parallel \approx h\omega_p$).

In the considered case of low-mobility electrons, dissipation of the PPP energy due to electron relaxation processes is more intensive in comparison with dissipation caused by phonon damping. This is a key issue for understanding the reflectivity spectrum change under transition from the simplified model (blue curves in figure 3 (c, f)) to general one taking into account dissipation processes (red curves in figure 3 (c, f)). At a higher doping level (figure 3 (f)), corresponding decrease of the reflectivity at photon energy close to $h\omega_\perp$ is more significant than nearby $h\omega_\parallel$. This is associated with plasmon domination in plasmon-phonon polariton at photon energies close to $h\omega_\parallel$ and phonon domination at photon energies close to $h\omega_\perp$. At a lower doping level, there is a reciprocal situation (figure 3 (c)).

4. Experimental study of $n$-GaN epilayer reflection

Epitaxial layers of gallium nitride doped with Si were grown by MOVPE on a sapphire substrate. Free electron concentration and mobility in sample 1 were $5 \cdot 10^{18}$ cm$^{-3}$ and 189 cm$^2$/V·s, respectively (the thickness of the epilayer was 10 μm). The parameters of sample 2 were $3.6 \cdot 10^{19}$ cm$^{-3}$, 122 cm$^2$/V·s and 6.2 μm, respectively. Sample characterization was carried out by means of the Hall effect and conductivity measurements at room temperature.

The reflectivity spectra were measured by a Fourier spectrometer in the range of 8–80 meV at room temperature. A globar and a pyroelectric detector were used as a source of radiation and a detector, respectively. The incidence angle was about 11°. The experimental spectra are shown in figure 4.

The theoretical calculation of the reflectivity for the samples under experimental study was based on the model considering transmission and reflection of radiation in a three-layer system [7], vacuum/GaN/sapphire in our case. The experimental reflectivity spectra were fitted by the simulated dependences. The free electron concentration and mobility were used as fitting parameters. Good agreement between calculation and experiment (see figure 4) shows that the considered model of plasmon-phonon polaritons adequately describes the $n$-GaN reflection spectra in a wide range of electron concentrations. For sample 1, the best fit is given by the electron concentration $N_e = 2.8 \cdot 10^{18}$ cm$^{-3}$ and mobility $\mu = 180$ cm$^2$/V·s. For sample 2, the fitting results are the following: $N_e = 1.9 \cdot 10^{19}$ cm$^{-3}$ and $\mu = 179$ cm$^2$/V·s. One can see that these values slightly differ from the values determined by means of Hall and conductivity measurements. This difference can be explained in the following way.
The values of concentration and mobility obtained from Hall measurements characterize the epitaxial layer as a whole, and parameters determined from the reflectivity spectra are mainly related to the subsurface region of the epitaxial layer.

The reflectivity spectra demonstrate a series of peculiarities. In every spectrum, one can see a dip close to the position of the longitudinal plasmon-phonon mode \( \hbar \omega_p \). The reflectivity spectrum of the low-doped sample 1 demonstrates a broad dip nearby \( \hbar \omega_p \) while the heavily-doped sample 2 shows a narrow one at a corresponding spectral position. As mentioned above, such a change in the reflectivity spectrum with doping level increase is associated with redistribution of the phonon and plasmon contributions to the coupled plasmon-phonon mode \( \hbar \omega_p \). At a lower doping level, the mode \( \hbar \omega_p \) is mainly formed by the plasmon contribution (\( \hbar \omega_p \approx \hbar \omega_\text{p} \)), and at a higher doping level the mode \( \hbar \omega_p \) is mainly formed by the phonon contribution (\( \hbar \omega_p \approx \hbar \omega_\text{TO} \)). Besides, at a lower doping level the reflectivity dip at \( \hbar \omega_e \) is additionally greatly broadened due to rather fast electron relaxation processes. At higher doping level the corresponding reflectivity dip is only slightly extra-broadened due to rather weak phonon damping.

The experimental spectrum for the low-doped sample 1 (and the simulated spectrum as well) demonstrates the interference pattern in the spectral range of 50–70 meV. In this spectral range, the GaN epilayer is rather transparent that results in multiple reflection of radiation from the GaN/sapphire and GaN/vacuum interfaces. The spectra for sample 2 do not demonstrate any interference pattern because the heavily doped GaN layer in this sample is totally nontransparent in the operating spectral range.

5. Conclusion
To summarize, we studied theoretically and experimentally the bulk plasmon-phonon polaritons in \( n \)-GaN with various doping levels. The longitudinal plasmon-phonon modes in \( n \)-GaN were considered as well. The theoretical consideration was carried out taking into account the optical phonon damping and electron relaxation processes. The simulated dispersion dependencies of plasmon-phonon polaritons and reflectivity spectra for semi-infinite \( n \)-GaN are presented. The reflectivity spectra transformation associated with phonon damping and electron relaxation processes has been revealed. The experimental investigation was performed for the \( n \)-GaN epitaxial layers grown on a sapphire substrate. The reflectivity spectra were measured in the spectral range of
8–80 meV for $n$-GaN with two electron concentrations: $\sim 3 \times 10^{18}$ and $2 \times 10^{19}$ cm$^{-3}$. The theoretical calculation of the reflectivity for the samples under experimental study was based on the model considering transmission and reflection of radiation in a three-layer system (vacuum/GaN/sapphire). The experimental reflectivity spectra were fitted by the simulated ones. Good agreement between calculation and experiment shows that the considered model of plasmon-phonon polaritons adequately describes the $n$-GaN reflection spectra in a wide range of electron concentrations. It is shown, that reflectivity spectrum analysis can be used for contactless determination of the electron concentration and mobility in GaN epitaxal layers.

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References
[1] Širmulis E, Šilėnas A, Požela K, Požela J and Jucienė V 2014 Appl. Phys. A 115 199
[2] Melentev G A et al 2016 J. Appl. Phys. 119 093104
[3] Melentev G A, Yaichnikov D Yu, Shalygin V A, Vinnichenko M Ya, Vorobjev L E, Firsov D A and Suihkonen S 2016 J. Phys.: Conf. Series 690 012005
[4] Yu P Y and Cardona M 2010 Fundamentals of Semiconductors 4th ed (Berlin–Heidelberg: Springer)
[5] Ukhanov Yu I 1977 Optical Properties of Semiconductors, in Russian (Moscow: Nauka)
[6] Agekyan V F et al 2015 Phys. Solid State 57 787
[7] Born M and Wolf E 2003 Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light, 7th edition (Cambridge: Cambridge University Press) pp 735–789