Study on Dynamic Model of Vibratory Roller - Soil System

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Abstract. In this paper, two-degree-of-freedom dynamics model of vibratory roller-soil vibration system is established. According to the theory of mechanical system dynamics, the solution of the model is solved, and the analytical solution of dynamic response and natural frequency of soil in vibration compaction process is derived and calculated. The influence of vibration frequency, amplitude and soil parameters on the vibration compaction effect is analyzed, which is of guiding significance to the selection of vibration parameters of vibratory roller.

1. Introduction
Vibratory roller by its own gravity and vibration wheel vibration compaction effect of various construction materials, widely used in road construction in the vibration of vibratory roller is a complicated stochastic process, in order to study the performance of vibratory roller, analyzing the dynamic changes of the basic parameters in the process of vibration compaction, must simplify the structure of the complex vibration system, and therefore the mathematical model of numerical analysis.

2. The establishment of dynamic model of vibration system
The principle of establishing dynamic model of vibratory roller-soil vibration system is as follows: first, the dynamic model should be consistent with the actual engineering situation; secondly, the model should be simplified, which is a simple mathematical calculation method.

Vibratory roller on the compaction work has a direct role is the former car part, mainly by the front car rack, shock absorber and vibration wheel. In the process of vibration compaction, the movement of the frame and the vibration wheel is close to that of the rigid body while the deformation of the vibration absorber and the compacted soil is obvious. Therefore, the law of motion of the frame and the vibration wheel can be described by the dynamic model. The front frame and the rear frame of the necklace can be simplified into a centralized quality, that is, the quality of getting on the vehicle, and the simplified vibration wheel can also be reduced to a concentrated quality, And spring and damping to represent the shock absorber.

The simplification of soil mass and the determination of parameters are the keys to establishing dynamic model. In a vibratory rollers - soil vibrations systems, he mass of the vibration is involved in the mass of a vibration wheel and a part of the soil that vibrates with it. Therefore, during the vibration compaction, the dynamic effect of the vibration wheel on the soil can be equivalently expressed by a mass-spring-damping system\textsuperscript{[6]}. The vibration exciter of vibrating roller is the eccentric mass of rotation, the exciting force is the direction of rotation of the centrifugal force of the banner, the direction of vibration of the wheel is a multi-directional spatial motion. However, the effect of vibration compaction, especially the compaction...
effect of soil below the surface layer, vertical vibration and static pressure play a major role. In order to simplify the problem, the model only studies the vertical direction of the movement.

Vibratory roller - soil vibration system dynamics model shown in Figure 1. Assuming that the vibration wheel is in contact with the soil at any moment, the origin of displacements $x_1$, $x_2$ and $x_3$ are the positions of their respective masses in the static equilibrium of the system. In order to simplify the dynamic equations and orient the descent, Learning equation\cite{2}:

\begin{align*}
m_1x''_1 + c_1(x'_1 - x'_1) + k_1(x_1 - x_2) &= 0 \\
m_2x''_2 + c_1(x'_2 - x'_2) + k_1(x_2 - x_1) + F_s &= F_0 \sin \omega t \\
m_3x''_3 + c_2x'_3 + k_2x_3 &= F_s
\end{align*}

$m_1$, $x_1$ — Equivalent mass and displacement of rack vertical vibration;  
$k_1$, $c_1$ — Shock absorber stiffness and damping;  
$m_2$, $x_2$ — Equivalent mass and displacement of vibration wheel;  
$m_3$, $x_3$, $k_2$, $c_2$ — Equivalent mass, displacement, stiffness and damping of soil;  
$F$, $\omega$ — exciting force and dynamic frequency;  
$F_s$ — Dynamic force of Vibration wheel and soil;

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Vibratory Roller - Soil Vibration System Dynamics Model}
\end{figure}

3. Solve Dynamics Equations

3.1 Solve the displacement $x_1$, $x_2$  
When the vibration wheel is in the grounded state, $x_2 = x_3$, so the vibration roller - soil vibration system dynamics equation (1.1) can be expressed as the following matrix form:
\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2 + m_3
\end{bmatrix}
\begin{bmatrix}
    x_1^s \\
    x_2^s
\end{bmatrix}
+ \begin{bmatrix}
    c_1 & -c_1 \\
    -c_1 & c_1 + c_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
+ \begin{bmatrix}
    k_1 & -k_1 \\
    -k_1 & k_1 + k_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    F_0 \sin \omega t
\end{bmatrix}
\] (2.1)

Because the model is a linear time-invariant system, the excitation is a harmonic force, so the steady state response is the resonance displacement. To solve the dynamic equation (2.1), the excitation force is expressed by \( F_0 e^{j\omega t} \) and suppose:
\[
x_1 = A_1 e^{j(\omega - \alpha_1)} \quad x_2 = A_2 e^{j(\omega - \alpha_2)} \quad (2.2)
\]

\( A_1, A_2 \) — the amplitude of \( x_1, x_2 \);
\( \alpha_1, \alpha_2 \) — the phase angle of \( x_1, x_2 \)

Formula (2.2) substituted into formula (2.1):
\[
(k_1 - m_1 \omega^2 + ic_1 \omega) A_1 e^{j(\omega - \alpha_1)} - (k_1 + ic_1 \omega) A_2 e^{j(\omega - \alpha_2)} = 0
\] (2.3)

\[
[k_1 + k_2 - m_2 \omega^2 + ic_1 + c_2 \omega] A_2 e^{j(\omega - \alpha_2)} - (k_1 + ic_1 \omega) A_2 e^{j(\omega - \alpha_2)} = F_0 e^{j\omega t}
\] (2.4)

Formula (2.3) and formula (2.4) can be found:
\[
A_1 = F_0 \sqrt{\frac{k_1 + c_1^2 \omega^2}{(k_1 - m_1 \omega^2)(k_1 - m_2 \omega^2) - \omega^2 (c_1 c_2 + k_1 m_2)}}
\]
\[
\alpha_1 = \tan^{-1}\frac{\omega (c_1 (k_1 - m_1 \omega^2 - m_2 \omega^2) + c_2 (k_1 - m_1 \omega^2))}{(k_1 - m_1 \omega^2)(k_1 - m_2 \omega^2) - \omega^2 (c_1 c_2 + k_1 m_2)} - \tan^{-1}\frac{c_1 \omega}{k_1}
\]
\[
A_2 = F_0 \sqrt{\frac{(k_1 - m_1 \omega^2)^2 + c_1^2 \omega^2}{(k_1 - m_1 \omega^2)(k_1 - m_2 \omega^2) - \omega^2 (c_1 c_2 + k_1 m_2)}}
\]
\[
\alpha_2 = \tan^{-1}\frac{\omega (c_1 (k_1 - m_1 \omega^2 - m_2 \omega^2) + c_2 (k_1 - m_1 \omega^2))}{(k_1 - m_1 \omega^2)(k_1 - m_2 \omega^2) - \omega^2 (c_1 c_2 + k_1 m_2)} - \tan^{-1}\frac{c_1 \omega}{k_1 - m_1 \omega^2}
\]

3.2 Solve the dynamic frequency \( \omega_1, \omega_2 \)

In the working condition without damping, \( c_1 = c_2 = 0 \) vibration roller - soil vibration system dynamics equation is:
\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2 + m_3
\end{bmatrix}
\begin{bmatrix}
    x_1^s \\
    x_2^s
\end{bmatrix}
+ \begin{bmatrix}
    k_1 & -k_1 \\
    -k_1 & k_1 + k_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    F_0 \sin \omega t
\end{bmatrix}
\] (2.5)

From formula (2.2) and formula (2.5) derived:
\[
\begin{bmatrix}
    k_1 - m_1 \omega^2 & -k_1 \\
    -k_1 & k_1 + k_2 - (m_2 + m_3) \omega^2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    F_0 \sin \omega t
\end{bmatrix}
\] (2.6)

From formula (2.6) derived:
\[
\omega_1^2 = \frac{(k_1 + k_2) m_1 + k_1 (m_2 + m_3) - \sqrt{(k_1 + k_2) m_1 + k_1 (m_2 + m_3)^2 - 4 m_1 (m_2 + m_3) k_1 k_2}}{2 m_1 (m_2 + m_3)}
\]
\[ \omega_1 = \frac{(k_1 + k_2)m_1 + k_1(m_2 + m_3) + \sqrt{(k_1 + k_2)m_1 + k_1(m_2 + m_3)}^2 - 4m_1(m_2 + m_3)k_1k_2}{2m_1(m_2 + m_3)} \]

\[ \omega_1 \] is the first-order natural frequency of the vibratory roller-soil vibration system, which mainly reflects the response characteristics between the frame and the shock absorber, and has a great influence on the vibration-damping performance of the vibratory roller; \( \omega_2 \) is the second-order vibration frequency, mainly reflects the dynamic response characteristics between the vibration wheel and soil, soil compaction has a greater impact.

4. Analysis of the relationship between compaction degree and model parameters

As the vibration wheel by the simple harmonic excitation force, so the vertical acceleration of the vibration wheel is:

\[ \begin{align*}
\mathbf{x}_2^0 &= \omega^2 \mathbf{x}_2 = \omega^2 A_0 e^{(i\omega t - \omega_2 t)} \\
\end{align*} \]

In the second chapter has been solved A, B, can be seen from the above equation, in the exciting force, the vertical acceleration of the vibration wheel and soil stiffness, damping and dynamic frequency. Vibratory roller operation, the stiffness \( k_2 \) and damping of soil \( c_2 \) are:

\[ k_2 = 0.253 \cdot \frac{(2.5 - e)^2}{(1 + e)(1 - \nu^2)} \sin \beta \cdot \sqrt{RL} \cdot (\sigma_0)^{1/3} \cdot E^{-1} \]

\[ c_2 = 0.265 - 2.33 \cdot 10^{-3} \cdot \frac{m_2}{\rho R^3} \]

It can be concluded that with the decreases of porosity ratio \( e \), the soil stiffness increases. The porosity ratio and degree of compaction trend of the opposite, so the soil stiffness increases, the compaction also increases; soil stiffness decreases, the compaction also decreases.

Since the density of soil increases with the degree of compaction when the vibratory roller is working, it can be concluded from formula (3-3) that as the degree of compaction increases, the damping of the soil decreases.

In the vibratory roller work, the choice of a reasonable vibration frequency for improving the rolling efficiency and to improve the compaction of the density of the material is very important. According to the principle of resonance, the compaction effect of the vibratory roller reaches the best only when the excitation frequency is close to or equal to the natural frequency of the soil. However, the reliability of the whole machine is reduced due to the resonance and the work comfort is poor. Vibratory roller usually working frequency are concentrated in the 25HZ to 50HZ between, and based on experience, a reasonable excitation frequency should be slightly higher than the vibration system of the second natural frequency B, the empirical formula is:

\[ \omega = (\sqrt{2} \sim 2)\omega_2 \]

5. Conclusion

By solving the dynamic model of the vibratory roller - soil system and analyzing the influence of parameters on compaction degree, the following conclusions are drawn:

1) the vertical acceleration of the vibration wheel is positively correlated with the soil stiffness and is negatively correlated with the damping of soil. So the dynamics of vibration wheel parameters closely related to the changes and the change of the density of the ground material, the vibration of wheel vertical acceleration and the interaction between material compaction degree has positive correlation, in actual engineering, will detect wheel vibration acceleration value as the main parameter of calculating vibratory roller compaction is feasible.
(2) the second order natural frequency of vibration system with the increase of soil compactness improved, therefore in the process of vibration compaction, the soil has hardened, the best vibration frequency should be also changed from small to big, and slightly higher than the second order natural frequency.

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