Calibration of triaxial magnetometer with ellipsoid fitting method

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Abstract. The accuracy of the triaxial strapdown magnetometer is limited by the scaling factor, three-axis orthogonality, offset errors, hard iron and soft iron effects of the platform, a calibration method of the triaxial strapdown magnetometer is proposed in this paper. Firstly, the integrated error model of the triaxial magnetometer is established without simplification, then the error calibration parameters are calculated by the ellipsoid fitting method, and the output of triaxial strapdown magnetometer can be calibrated by the calibration parameters. At last, simulation experiments are built for verification of the proposed calibration method. The simulation results show that the proposed method can reduce 94.6% of magnetometer errors and magnetic inference. The proposed calibration method can effectively calibrate the triaxial strapdown magnetometer.

1. Introduction
At every location on the earth, the geomagnetic field vector has a direction, which can be used as a reference frame to aircraft, ships and so on[1,2]. The triaxial magnetometer is an important equipment for geomagnetic navigation, however the triaxial magnetometer onboard a maneuverable platform is subjected to the scaling factor, three-axis orthogonality, offset errors, hard iron and soft iron effects of the maneuverable platform, therefore the triaxial strapdown magnetometer must be calibrated before use.

The error calibration of triaxial magnetometer has already been researched for many years, there are many literatures about the error calibration, the calibration method of the triaxial magnetometer can be classified into two kinds: the vector calibration[3] and the scale calibration[4-8]. Vector calibration needs a 3D Helmholtz coil system and a high precision tri-axial non-magnetic platform, the calibration process is complicated and the equipment of vector calibration is very expensive. Compared to vector calibration, scale calibration is much easier to realize, scale calibration only needs a homogenous magnetic field and a scalar proton magnetometer, the error calibration parameters can be solved by rotating the triaxial magnetometer in a uniform magnetic field and presenting it in different attitudes. Pang[9] proposed an error calibration method of the triaxial magnetometer based on Gauss-Newton iteration algorithm, however the hard iron and soft iron effects of the platform are ignored in this method. Zhang[10] proposed a liner calibration method of the triaxial magnetometer based on liner error model, however higher(more than second) order terms are neglected in this liner calibration method.
In this paper, a calibration method of the triaxial strapdown magnetometer is proposed. Firstly, the integrated error model of the triaxial magnetometer is established without simplification, then the error calibration parameters is calculated by the ellipsoid fitting method, and the output of triaxial strapdown magnetometer can be calibrated by the calibration parameters. The proposed method is easy to realize, both the vector magnetometer errors (scaling factor, three-axis orthogonality, offset errors) and magnetic interference (hard iron, soft iron) can be calibrated in the proposed method. At last, simulations with a triaxial fluxgate magnetometer is carried out for verification of the proposed calibration method.

2. Integrated error model of the triaxial magnetometer

2.1 Vector magnetometer error model

The triaxial fluxgate magnetometer is subjected to scaling factor, axis orthogonality and offset errors. Axis orthogonality is that the three axes of the triaxial fluxgate magnetometer may not be perfectly orthogonal, the actual three axis may not coincide with the three-orthogonal-axis. As shown in figure 1, suppose that $O - X_0Y_0Z_0$ is an ideal sensor’s orthogonal coordinate system, and $O - XYZ$ is the actual coordinate system. Suppose that $OZ$ is completely aligned with axis $OZ_0$. The plane $YOZ$ is coplanar with the plane $Y_0OZ_0$. $\psi$ denotes angle between the axis $OY$ and $OY_0$. $\theta$ denotes angle between the axis $OX$ and the plane $X_0OY_0$. $\phi$ denotes angle between the axis $OX_0$ and the projection of $OX$ in the plane $X_0OY_0$.

![Figure 1. Schematic diagram of the model of non-orthogonal error](image)

Each axis of the magnetometer has different biases and sensitivities, so we suppose that $k_x, k_y, k_z$ are the scale factors and $O = [O_x, O_y, O_z]^T$ are the biases for $OX, OY, OZ$ axes. Then outputs of the magnetometer can be converted to the orthogonal coordinates $O - X_0Y_0Z_0$ from $O - XYZ$ by the mathematical model. The calibration model can be established as

$$B^m = SC_{NO}B^r + O$$

(1)

$B^m$ are actual outputs of magnetometer in sensor frame, and $B^r$ are calibrated output of magnetometer in the orthogonal coordinates $O - X_0Y_0Z_0$, the scale factor matrix $S = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$, and the non-orthogonal error matrix $C_{NO} = \begin{bmatrix}\cos\theta\cos\phi & \cos\theta\sin\phi & \sin\theta \\ 0 & \cos\psi & \sin\psi \\ 0 & 0 & 1 \end{bmatrix}$.

2.2 Magnetic interference
2.2.1 Hard iron
The hard magnetic material of the maneuverable platform will be magnetized in the geomagnetic field, because the hard magnetic material has a high coercive force and a high remanence value, so the magnetic field of the hard magnetic material can keep a long time. Due to the magnetic sensor and hard magnetic materials are fixed on the maneuverable platform, regardless of the platform attitude changes, magnetic field of hard magnetic material in triaxial magnetometer frame is constant. So the hard iron can be modeled as a constant vector \( h_b \),

\[
h_b = [h_{bx}, h_{by}, h_{bz}]^T
\]

2.2.2 Soft iron
The soft iron is generated by the interaction of the external magnetic field and the soft magnetic material, because the soft magnetic material has a low coercive force and a narrow hysteresis loop, so when the direction of the external magnetic field changes, the direction of the induced magnetic field will change. The magnitude and direction of the induced magnetic field is associated with the external magnetic field and coefficients of the soft magnetic material.

\[
S_{SI} = B_SI B^c
\]

Where \( S_{SI} \) is the soft iron effect, \( C_{SI} \) is the soft iron coefficients, \( B^c \) is the background field.

2.3 Integrated error model
Take all of the vector magnetometer errors and magnetic inference into consideration, an integrated error model is established,

\[
B^{m} = SC_{NO}(I + C_{SI})B^c + b_{O} + \varepsilon
\]

Where \( I \) is the unit matrix, \( \varepsilon \) is the measurement noise, however, measurement noise is really small compared to the other errors and magnetic inference, then equation (4) can be simplified to

\[
B^{m} = CB^c + b
\]

where \( C = SC_{NO}(I + C_{SI}) \), \( b = SC_{NO}b_{O} + \varepsilon \).

3. Ellipsoid fitting method
From equation (5), the relationship between the calibrated output and measured magnetic field can be written as

\[
B^c = C^{-1}(B^{m} - b)
\]

When the background magnetic field is constant, then the calibrated output of magnetometer should satisfy \((B^c)^T B^c = \text{const}\), Then equation (6) becomes

\[
(B^{m} - b)^T (C^{-1})^T C^{-1}(B^{m} - b) = \|B^b\|^2
\]

where \( B^b \) is the background field, suppose \( A = ((C^{-1})^T C^{-1})/\|B^b\|^2 \), then equation (7) becomes

\[
(B^{m} - b)^T A(B^{m} - b) = 1
\]

The matrix \( A \) is a symmetric matrix, suppose

\[
A = \frac{a_1}{2} \frac{a_4}{2} \frac{a_5}{2} \frac{a_2}{2} \frac{a_6}{2} \frac{a_3}{2} \]

(9)
Equation (8) is the equation of an ellipsoid, the equation implies that the measured magnetic field $B^m$ lie on the surface of an ellipsoid centered on $b$, so the ellipsoid fitting method is using the measured magnetic field data to estimate the parameters of the ellipsoid.

The quadric equation of an arbitrary ellipsoid is

$$F(\zeta, v) = a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 xy + a_5 yz + a_6 zx + a_7 x + a_8 y + a_9 z + a_{10} = 0$$

(10)

Where $\zeta = [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}]$ is the parameter matrix and $v = [x_1^2, y_1^2, z_1^2, \ldots, x_n^2, y_n^2, z_n^2]$ is the measurement data matrix respectively.

$F(\zeta, v)$ is the algebraic distance of the measurement data points to the ellipsoid surface. Then the ellipsoid fitting can be written as

$$\min \sum \|F(\zeta, v)\| = \min \zeta^T D^T D \zeta$$

s.t. $4a_1a_3 - a_2^2 > 0$ (11)

where $D = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1y_1 & x_1z_1 & x_1 & y_1 & z_1 & 1 \\ x_2^2 & y_2^2 & z_2^2 & x_2y_2 & x_2z_2 & x_2 & y_2 & z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & y_n^2 & z_n^2 & x_ny_n & x_nz_n & x_n & y_n & z_n & 1 \end{bmatrix}$

Then the parameter matrix can be calculated by the least square method, and $A$ can be calculated from the parameters of the ellipsoid. Since $A$ is a symmetric positive-definite matrix, then the error calibration matrix $C^{-1}$ can be calculated from the cholesky factorization of $A$.

$$A = ((C^{-1})^T C^{-1})/\|B^m\|^2 = G^T G$$

(12)

where $G$ is an upper triangular matrix, then $C^{-1}$ can be written as

$$C^{-1} = \|B^m\| G$$

(13)

And $b$ can be written as

$$b = -A^{-1} \begin{bmatrix} a_7 \\ a_8 \\ a_9 \end{bmatrix}$$

(14)

Once the calibration matrixes $C^{-1}$ and $b$ are get, the triaxial magnetometer can be calibrated from equation (6).

4. Simulation

In order to verify the effectiveness of the proposed method, numerical simulation with a triaxial fluxgate magnetometer is performed in this paper.

| Table 1. Error parameters of triaxial magnetometer |
|--------------------------------------------------|
| non-orthogonal error (rad) | [0.0028, 0.0026, 0.0035] |
| scale factors | [1.013, 0.994, 1.007] |
| biases (nT) | [92, 56, -84] |
| Hard iron (nT) | [632, -516, 943] |
| Soft iron | \[
\begin{bmatrix} 0.34 & -0.53 & 0.26 \\ 0.82 & 0.36 & -0.12 \\ -0.16 & 0.34 & 0.43 \end{bmatrix}\] |
The magnitude of the geomagnetic field is 53000nT, the declination angle is $-5^\circ$, and the inclination angle is $60^\circ$. In the proposed method, the triaxial magnetometer will be rotated in the uniform geomagnetic field, and the magnetic field measurement data of different attitudes will be recorded. The measurement noise of each measurement axis is independent Gaussian noise with mean 0nT and variance $25\text{nT}^2$. The error parameters are set as table 1.

We use 600 attitudes to solve the error parameters. And the parameter matrixes can be calculated from equation (13) and equation (14). The estimated parameter matrixes are shown as follows:

$$
C^{-1} = \begin{bmatrix}
0.6819 & -0.0977 & 0.0119 \\
0 & 0.6332 & -0.0474 \\
0 & 0 & 0.6716 \\
\end{bmatrix}, \quad b = \begin{bmatrix}
734.6502 \\
-454.5243 \\
866.2233 \\
\end{bmatrix}.
$$

then the measurement data of the triaxial magnetometer can be calibrated from equation (6). The calibrated results are shown in figure 2 and figure 3. Figure 2 is the spatial distribution of the raw data and the calibrated data. Figure 3 is comparison of the total magnetic intensity before and after calibration.

![Figure 2. Spatial distribution of the raw data and the calibrated data](image)

From figure 2, we can see that the uncalibrated data is on the surface of an ellipsoid, after calibration, the calibrated data is on the surface of a sphere. The proposed calibration method can effectively calibrate the triaxial magnetometer.

![Figure 3. Comparison of total magnetic intensity before and after calibration](image)
From figure 3, we can see that the fluctuations of the total magnetic intensity are almost 20000nT, after calibration, fluctuations are reduced to 22nT. According to the error statistical equation (15), the proposed method can reduce 94.6% of errors and magnetic inference.

$$\varepsilon = \frac{1}{n} \sum_{i=1}^{n} \left( \left\| B^i \right\| - \left\| \tilde{B}^i \right\| \right)$$

where \(\left\| B^r \right\|\) is the theoretical total magnetic intensity of the triaxial magnetometer, \(\left\| B^m \right\|\) and \(\left\| B^c \right\|\) are the measured total magnetic intensity and the calibrated total magnetic intensity respectively.

5. Conclusion
For the scaling factor, three-axis orthogonality, offset errors, hard iron and soft iron effects of the triaxial strapdown magnetometer, a calibration method of the triaxial strapdown magnetometer is proposed in this paper. Firstly, the integrated error model of the triaxial magnetometer is established without simplification, then the error calibration parameters is calculated by the ellipsoid fitting method. The proposed method is easy to realize, both the vector magnetometer errors (scaling factor, three-axis orthogonality, offset errors) and magnetic interference (hard iron, soft iron) can be calibrated in the proposed method. Simulation results show that the proposed method can reduce 94.6% of errors and magnetic inference. We can come to the conclusion that the proposed method can effectively calibrate the triaxial strapdown magnetometer.

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