Energy transfer from space-time into matter and a bouncing inflation from Covariant Canonical Gauge theory of Gravity

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Cosmological solutions for covariant canonical gauge theories of gravity are presented. The underlying covariant canonical transformation framework invokes a dynamical space-time Hamiltonian consisting of the Einstein-Hilbert term plus a quadratic Riemann tensor invariant with a fundamental dimensionless coupling constant \(\gamma\). A typical time scale related to this constant, \(\tau = \sqrt{8\pi G\gamma}\), is characteristic for the type of cosmological solutions: for \(t \ll \tau\) the quadratic term is dominant, the energy momentum tensor of matter is not covariantly conserved, and we observe modified dynamics of matter and space-time. On the other hand, for \(t \gg \tau\), the Einstein term dominates and the solution converges to classical cosmology. This is analyzed for different types of matter and dark energy with a constant equation of state. While for a radiation dominated universe solution the cosmology does not change, we find for a dark energy universe the well known de-Sitter space. However, we also identify a special bouncing solution (for \(k = 0\)) which for large times approaches the de-Sitter space again. For a dust dominated universe (with no pressure) deviations are seen only in the early epoch. In late epoch the solution asymptotically behaves as the standard dust solution.

Keywords: field theory – gravitation – gauge field theory – Hamiltonian – Palatini formalism

INTRODUCTION

The Covariant Canonical Gauge Gravitation (CCGG) is a theory derived from the canonical transformation theory in the Hamiltonian picture [1]. It naturally identifies the affine connection as the fundamental gauge field, and does has to be symmetric (including torsion). In addition, it complements the Einstein Hilbert action by an additional quadratic Riemann invariant.

The additional quadratic term formally corresponds to a squared momentum field that equips space-time with kinetic energy. Moreover, by its presence the covariant conservation of the stress energy tensor is violated, as many modified theories of gravity in the Palatini approach do. For a symmetric connection, we got a relation between the covariant conservation of the energy momentum tensor and the metricity condition [2]. But in the case of non-symmetric connection, the covariant conservation of the energy momentum tensor is violated.

The objective of this paper is to investigate the impact of that quadratic term for elementary cosmological solutions in the Friedman model, and showing how the space-time could affect the dynamics of the universe and the behavior of matter.

We use natural units with \(\hbar = c = 1\). The signature of the metric is \(g_{\mu\nu} = \text{diag}(1, -1, -1, -1)\). Small Greek indices run from 0 to 3 and denote the number of space-time dimensions; small Latin indices run from 1 to 3 and denote spatial dimensions only.

COVARIANT CANONICAL GAUGE THEORY OF GRAVITY

A closed description of the coupled dynamics of fields and space-time geometry has been derived in [1], where the gauge formalism yields, on the basis of the theory amended covariant Hamiltonian:

\[
\mathcal{H}_\text{Dyn} = \frac{1}{4g_1^1} \bar{q}^{\alpha\beta} \eta_{\alpha} \eta_{\beta} \eta_{\gamma} \eta_{\delta} - g_{\alpha\beta} \bar{q}^{\alpha\beta} g_{\alpha\beta} + g_{4} \sqrt{g} \mathcal{H}_m, \tag{1}
\]

where the tensor densities \(\bar{q}^{\alpha\beta}\) and \(\bar{q}^{\alpha\beta}\) denote the canonical momenta of the metric \(g_{\alpha\beta}\) and of the connection coefficient \(\gamma^{\alpha\beta}_{\gamma}\) respectively. The \(\mathcal{H}_m\) is the Hamiltonian of the matter fields. The basic quantity that the formulation gives is that affine connection \(\gamma^{\alpha\beta}_{\gamma}\) in conjunction with their conjugates, \(q_{\alpha\beta\gamma}\) are internal dynamic variables of the system. [3] derives the matter energy momentum tensor, which is defined as:

\[
\Theta^{\alpha\beta}_{\gamma} = \frac{2}{\hbar} \partial_{\bar{q}^{\alpha\beta}} \mathcal{H}_m, \tag{2}
\]

takes the form:

\[
\frac{1}{8\pi G} G^{\alpha}_{\beta} \equiv g_1 Q^{\alpha}_{\beta} \equiv \Theta^{\alpha}_{\beta}, \tag{3}
\]

where \(G^{\alpha}_{\beta}\) is the Einstein tensor:

\[
G^{\alpha}_{\beta} \equiv R^{\alpha}_{\beta} - \frac{1}{2} g^{\alpha}_{\beta} R, \tag{4}
\]

and the tensor \(Q^{\alpha}_{\beta}\) is the contribution from the quadratic Riemann term:

\[
Q^{\alpha}_{\beta} \equiv R^{\alpha}_{\beta\gamma\delta} R_{\gamma\delta\xi} - \frac{1}{2} g^{\alpha}_{\beta} R^{\gamma\delta\xi} R_{\gamma\delta\xi}. \tag{5}
\]
From the basic formulation, the connection is being the Christoffel symbol: The coupling constants are related to the physical quantities:

$$g_1 g_2 = \frac{1}{16\pi G}, \quad 6g_3 g_3^2 + g_3 = \frac{\Lambda}{8\pi G}. \quad (6)$$

Then the CCGG equation, generalizing the Einstein equation, can be written as

$$\Theta^\nu_\mu + \theta^\nu_\mu = 0, \quad (7)$$

where $\theta^\nu_\mu$ is the energy-momentum tensor of matter (stress tensor) balancing the strain tensor. The non-quadratic terms in the Riemann tensor of Eq. (3), that we get when $g_1 \to 0$, represent exactly the Einstein equation with cosmological constant $\Lambda$.

**The stress energy tensor conservation**

As discussed in [5] the covariant divergence of the stress energy-momentum tensor does not vanish for modified theories of gravity beyond Lovelock theories in Palatini formalism. That means that theories beyond General Relativity and Gauss-Bonnet will not provide a covariantly conserved stress tensor:

$$\Theta^\nu_\mu = 0. \quad (8)$$

This can be seen as follows. While the covariant divergence of the Einstein tensor vanishes identically, $G^\mu_\nu = 0$, this is not the case for the quadratic term in the strain tensor. A direct calculation leads (see Appendix A) to

$$Q^\nu_\xi = R^{\xi \lambda \alpha \beta} R_{\gamma \beta \alpha \lambda} \neq 0. \quad (9)$$

Hence

$$- \Theta^\nu_\mu \equiv g_1 Q^\nu_\mu = \theta^\nu_\mu \neq 0. \quad (10)$$

This implies that we should expect energy and momentum transfer between space-time (described by the quadratic Riemann term in the strain tensor) and the energy-momentum density of matter, described by the stress tensor. We will come back to this conjecture below. In the following we illuminate using the Friedman model how the CCGG theory modifies cosmology.

**COSMOLOGICAL IMPLICATIONS**

**Generalized Friedman equation**

The (FLRW) Friedman-Lemaitre-Robertson-Walker ansatz is the standard model of cosmology dynamics based on the assumption of a homogeneous and isotropic universe at any point, commonly referred to as the cosmological principle. The symmetry considerations lead to the FLRW metric

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]. \quad (11)$$

Herein, $a(t)$ defines the dimensionless cosmological expansion (scale) factor, whereas $K$ denotes the positive, negative, or zero special curvature $K$ of the spatial slice. In the following, we determine the expansion factor dynamics $a(t)$ by means of our generalized field equation (1). To set up the source term, the universe is usually modeled as a perfect fluid. The appropriate energy-momentum tensor is then

$$\theta^\nu_\xi = \text{diag}(\rho, -p, -p, -p). \quad (12)$$

The density, $\rho$, and the pressure, $p$, refer to all types of matter present in the universe. Due to the symmetry properties they can only depend on the universal time $t$. The density $\rho(t)$ and the pressure $p(t)$ are not independent but related via an equation of state, which, for a perfect fluid, is characterized by a constant parameter $\omega$:

$$\omega = \frac{p}{\rho} \quad \text{or} \quad \omega = \text{const.} \quad (13)$$

As the trace of the quadratic Riemann tensor vanishes, it does not contribute to the trace equation. So independently of the dimensionless constant $g_1$, which is associated with the quadratic Riemann tensor terms, the metric (11) yields the following inhomogeneous second-order equation for the expansion factor:

$$-\frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 - 2M + \frac{K}{a^2} = 0, \quad (14)$$

with

$$M(t) = \frac{1}{3} \left[ 2\pi G ( \rho(t) - 3p(t) ) + \Lambda \right]. \quad (15)$$

On the basis of the metric (11), the non-contracted equation (14) yields two more differential equations for the expansion factor $a(t)$ for the indices $\alpha, \xi = 0$ and $a, \xi = 1$

$$-8\pi G g_1 \left[ \left( \frac{\dot{a}^2 + K}{a} \right)^2 - \ddot{a}^2 \right] + \dot{a}^2 + K - \frac{1}{3} \Lambda a^2 = \frac{8\pi G}{3} \rho a^2, \quad (16a)$$

$$8\pi G g_1 \left[ \left( \frac{\dot{a}^2 + K}{a} \right)^2 - \ddot{a}^2 \right] + 2\dot{a}\ddot{a} + \dot{a}^2 + K - \Lambda a^2 = -8\pi G p a^2. \quad (16b)$$

For those generalized Friedman equation, two extreme cases are obtained: One case is when $g_1 = 0$,

$$\left( \frac{\dot{a}^2}{a^2} + \frac{K}{a^2} - \frac{1}{3} \Lambda \right) = \frac{8\pi G}{3} \rho, \quad (17a)$$

$$2\dot{a}^2 + \left( \frac{\dot{a}}{a} \right)^2 + K - \Lambda = -8\pi G p, \quad (17b)$$
where the original Friedman equations are restored.

The second case is when \( g_1 \to \infty \), and the contribution of the \( g_1 \) terms dominates the Friedman equations such that

\[
-g_1 \left( \frac{a^2 + K}{a} \right)^2 - \ddot{a}^2 = \frac{1}{3} \rho_{g_1 \to \infty} a^2, \tag{18a}
\]

\[
-g_1 \left( \frac{\dot{a}^2 + K}{a} \right)^2 - \ddot{a}^2 = \rho_{g_1 \to \infty} a^2. \tag{18b}
\]

In this limit the equation of state, defined as

\[
\omega_{g_1 \to \infty} := \frac{p_{g_1 \to \infty}}{\rho_{g_1 \to \infty}} = \frac{1}{3}
\]

is that of radiation. This asymptotic behavior is not surprising because now the strain tensor is dominated by the quadratic term which is traceless as in the case of radiation. As Eqs. (14) and (16) must hold simultaneously, we can solve Eq. (14) for \( \ddot{a} \) and insert it into Eq. (16):

\[
32\pi G g_1 M \left( \dot{a}^2 + K - M \dot{a}^2 \right) + \dot{a}^2 + K - \frac{1}{4} \Lambda a^2 = \frac{8\pi G}{3} \rho a^2. \tag{20}
\]

The term proportional to \( g_1 \) thus gives rise to modified cosmic dynamics, as compared to the conventional Friedman equation, to which Eq. (20) reduces when we set \( g_1 = 0 \).

According to Eq. (10), the covariant divergence of the energy-momentum tensor (12) must be equal to the covariant derivative of the quadratic Riemann tensor terms. Considering a classical content of the universe, namely dust and radiation, and we can neglect torsion and assume metricity. For the zero component we then get

\[
T^0_{\alpha \beta} = -\dot{\rho} - \frac{\ddot{a}}{a} (\rho + p) = g_1 R^0_{\beta \lambda \alpha} K^\beta_{\rho \lambda \alpha}. \tag{21}
\]

For the metric (11) only the zero component of the vector of the covariant divergence of the quadratic Riemann tensor terms does not vanish, and with Eq. (13) we find

\[
\frac{1}{3} a \dot{\rho} + \dot{a} (\rho + p) = 2 g_1 \frac{\ddot{a}}{a^3} \left( -\dot{a}^2 \ddot{a} - a \dddot{a} + 2 \dot{a}^3 + 2 \dot{a} K \right). \tag{22}
\]

Equation (22) can be considerably simplified by inserting the trace equation (14). With the definition of \( M \) from Eq. (15) we finally get (see appendix A)

\[
\frac{1}{3} \dot{\rho} + \dot{a} (\rho + p) = 8\pi G g_1 \dot{a} \left( \dot{p} - \frac{1}{3} \right). \tag{23}
\]

This equation provides a simple relation between the density and the pressure and the scale parameter. As expected, for \( g_1 = 0 \) we recover the covariant conservation of the stress energy tensor, as

\[
\frac{d\rho}{dt} + 3 \frac{\dot{a}}{a} (\rho + p) = 0 \tag{24}
\]

corresponds to \( T^0_{\alpha \beta} = 0 \). On the other hand, in the case \( g_1 \to \infty \), we obtain

\[
\dot{p} = \frac{1}{3} \dot{\rho}, \tag{25}
\]

which corresponds to the equation of state for radiation similar to Eq. (19).

Equipped now with the basic equation of cosmology (16) as modified by the CCGG theory, we shall in the following sections analyze the dynamics of the universe under the assumption of various but constant equations of state.

### Rescaling the equations

For solving the modified Friedman equations we express Eqs. (16) in terms of the Hubble parameter \( H = \dot{a}/a \) and its derivatives. For simplicity we prefer to assume the space to be flat, i.e. \( K = 0 \). The density and pressure equations are then rewritten as

\[
8\pi G \rho' = 3H^2 + 24\pi G g_1 \left( 2H^2 \dot{H} + H^2 \right) \tag{26a}
\]

\[
8\pi G p' = -3H^2 - 2\dot{H} + 8\pi G g_1 \left( 2H^2 \dot{H} + H^2 \right) \tag{26b}
\]

where the energy density and the pressure encompass dust, radiation and also include the cosmological constant interpreted as Dark Energy density and pressure, respectively:

\[
\rho' = \rho + \frac{\Lambda}{8\pi G} \tag{27a}
\]

\[
p' = p - \frac{\Lambda}{8\pi G} \tag{27b}
\]

This form reduces the modified Friedman equations from second order equations to first order equation. From the generalized Friedman equations, a typical time scale \( \tau \) emerges related to the basic constant \( g_1 \):

\[
\tau = \sqrt{8\pi G g_1} = \sqrt{g_1 / M_p} \tag{28}
\]

where \( M_p \) is the reduced Planck mass. This time scale is typical for the period of dominance of the quadratic term over the Einstein-Hilbert term in the full action. As we will see in the next section, for "short" times, \( t/\tau \ll 1 \), the quadratic term is dominant and violates the conservation of the stress tensor. For long times, \( t/\tau \gg 1 \), the Einstein term becomes more dominant and asymptotically the universe behaves as the familiar cosmological model \( \Lambda \)CDM where the covariant conservation of the stress tensor is assumed. The scale of \( g_1 \) can vary in the range \( 0 < g_1 < 10^{120} \) as explored in [3] which is equivalent to \( 0 < \tau < 600 \text{Gy} \).

The Friedman equations (16) can be simplified by rescaling the time coordinate to

\[
\tilde{t} := \frac{t}{\tau}, \quad \tilde{\rho} := (8\pi G)^2 g_1 \rho', \quad \tilde{H} := \tau H \tag{29}
\]

\[
\tilde{p} := (8\pi G)^2 g_1 p', \quad \tilde{H}^\prime := \frac{d\tilde{H}}{d\tilde{t}} \tag{30}
\]
depends only on the Hubble parameter: Eq. (31b) into Eq. (31a), we obtain a unified equation which generalized Friedman equations. By setting $(3\omega-1)$ the term $3\dot{H}^2$ be more dominant for the density equation, and for $\dot{t} \ll 1$ the term $3(2\dot{H}^2 \dot{H} + \ddot{H}^2)$ (which comes from the quadratic Riemann tensor) would be more dominant.

In the following we suppress the tilde above the rescaled time coordinate and the other quantities and replace prime by dot for time derivatives, unless needed for clarity.

**SOLUTIONS WITH CONSTANT EQUATIONS OF STATE**

In this section we analyze three limiting cases of a radiation-dominated (Traceless stress tensor, $\omega = \frac{1}{3}$), dark energy dominated ($p = -\rho$, $\omega = -1$), and matter-dominated ($p = 0$, $\omega = 0$) universes.

A constant equation of state simplifies the rescaled generalized Friedman equations. By setting $p = \omega \rho$, and substituting Eq. (31b) into Eq. (31a), we obtain a unified equation which depends only on the Hubble parameter:

$$\ddot{H}^2 \left(3(\omega-1)\dot{H} + 2\right) + \dot{H}^2 \left(2(3\omega-1)\dot{H} + 3(\omega+1)\right) = 0$$

(32)

Now it becomes obvious that for radiation ($\omega = \frac{1}{3}$) but also for dark energy ($\omega = -1$) a number of terms cancel out.

**Radiation dominance**

For this case, there is no formal deviation from the trace equation at all, because the traces of both, the quadratic Riemann term (strain tensor) and the stress energy tensor vanish. Hence, the trace equation gives:

$$2\ddot{H} + \dot{H}' = 0 \quad \Rightarrow \quad a(t) \sim t^\frac{1}{3}$$

(33)

which corresponds to Eq. (32) for $\omega = \frac{1}{3}$ For this Hubble parameter, the new terms in Eq. (31a) and (31b) are identically zero,

$$2\ddot{H}^2 \dot{H}' + \dot{H}^2 = 2H^2 \dot{H}' + H^2 = 0$$

(34)

and both equations reduce to the original Friedman equation, and there are no deviations from the standard radiation solution as all terms from the (traceless) quadratic Riemann tensor are identically zero.

**Vacuum dominance**

Vacuum solution means a universe void of matter and radiation but with dark energy present. The equation of state is in this case $\omega = -1$ or $p = -\rho$. Substituting this identity into the density and the pressure equations yields the differential equation

$$\ddot{H}' \left(2\ddot{H}' + 4\ddot{H}^2 - 1\right) = 0$$

(35)

A trivial solution is $\ddot{H} = 0$ which is the standard inflation solution for which, as in the radiation case (34), the contribution from the quadratic Riemann tensor equals to zero. The energy density and the pressure are constant, hence we recover the standard Einstein Ansatz with cosmological constant. Notice, though, that by rescaling, the cosmological constant

$$\Lambda \rightarrow \Lambda \left(8\pi G\right)^2 g_1$$

(36)

grows with growing $g_1$, the clock ticks faster via $\dot{t} \sim t/\sqrt{g_1}$. The second solution results if the expression in parentheses vanishes, $2\ddot{H}' + 4\ddot{H}^2 - 1 = 0$. The solution for the scale factor is readily determined with two different solutions:

$$a(t) = a_0 \sqrt{\cosh \left(\frac{t-t_0}{\tau}\right)}$$

(37)

for $|\ddot{H}| < 1/2$ or:

$$a(t) = a_0 \sqrt{\sinh \left(\frac{t-t_0}{\tau}\right)}$$

(38)
for $|\tilde{H}| > 1/2$. Here $a_0$ and $t_0$ are integration constants. For $t = t_0$ the scale parameter in (37) assumes the minimal value $a_{\text{min}} = a_0$. This solution which is symmetric with respect to $t - t_0$ describes a bouncing universe. Starting from minus infinite time it decelerates to a full stop, at time $t = t_0$ and finite scale factor $a_0$, to rebound thereafter into an exponential inflation phase. For the second solution (38), the universe starts with $a = 0$ which describes a Big Bang with asymptotic exponential inflation.

The energy density and pressure in this case can be investigated by substituting the scale parameter into Eq. (31a) and (31b). The surprising solution is also a constant energy density and pressure. Therefore from the quadratic Riemann term a new inflationary solution emerges, and the special vacuum solution still leads to a constant density as the cosmological constant. For investigating the physical behavior for a different point of view, we have restored the potential of the scale factor by demanding that the “total energy” would be zero, with the form:

$$\dot{a}^2 + V(a) = 0, \quad V(a) = -\frac{a^2}{4} \pm \frac{a_0^4}{4a^2}$$

where the sign “+” refers to the first solution (37) and the “−” sign refers to the second solution (38). The graph for those potentials shown in Fig. (1). Clearly from this potential we can see a bouncing solution bounces for $a_0$ in the left case, and see a singularity for the right case. The asymptotic values of the potential gives $V(a) \sim -a^2$ and from Eq. (39) the asymptotic solution will be $\dot{a}/a = H = \text{const}$ which is the standard inflationary solution. As we saw there are no big deviations for vacuum or radiation solutions, however for a matter dominant solution the picture looks much different.

**Matter dominance**

In the matter-dominated case the second Friedman equation, (31b) yields the differential equation for the Hubble parameter:

$$3\dot{H}^2 + 2\dot{H} = 2\ddot{H}^2 + \dot{H}^2,$$

where again the time here refers to the rescaled coordinate $\tilde{t}$.

Numerical solutions for different values of the initial condition $H_0 \equiv H(t = 0)$ are presented in Fig. (2). The physical evolution of the scale parameter changes for different values of $H_0$.

Only from the limiting cases of $g_1$ we obtained in the previous chapter (26a, 26b) we can deduce that the Hubble constant should be driven from the standard matter dominant solution (where $g_1 = 0$) to the radiation dominant solution (where $g_1 \to \infty$). For both cases $H \sim t^{-1}$. So the figures of the scale parameter do not shows a significantly different behavior of the solutions, which can be shown at this form of graphs. To see the physics in between different values of $g_1$, we define the quantity:

$$\rho \sim \frac{1}{a^6} \iff n = -\frac{1}{H \frac{dt}{d\tilde{t}} \ln \rho}$$

which gives the power dependence of the density versus the power of the scale factor for a power law universes, which for matter dominant is $n = 3$ and $n = 4$ is for radiation.

By this redefinition, the numerical solutions in Fig. (3) show the difference between physical evolution for different values of $H_0$. The larger the value of $h_0$, further the solution

![FIG. 2. The scale parameter and the energy density for matter dominant universe.](image-url)

![FIG. 3. The Power of the scale factor which gives the density vs. the rescaled time.](image-url)
starts away from $n = 3$ which is the standard dust solution. However all solutions go asymptotically to the power $n = 3$.

The energy momentum tensor is asymptotically covariant conserved for $t \gg \tau$, because at those times the Riemann square term from the action decays and we left with the standard Einstein term. For illuminating this fact in the case of dust universe, we plotted in Fig. (4) the covariant derivative of the stress energy tensor, numerically calculated in Eq. (21), versus the re-scaled time $\tilde{t}$.

As the values of the time scale $\tau$ increase, the time where the covariant derivative approaches zero is delayed. On the other hand when $\tau$ goes to infinity we expect also that the covariant derivative of the stress energy tensor will be conserved as the case of $\tau = 0$, because of the proof we have seen that asymptotically, for $\tau \to \infty$, the matter content of the universe assumes a radiation equation of states.

For illustrating energy transition between the quadratic term and the matter density term, we analyze the quantity $\Omega_m^{(0)} = \tilde{\rho}_m a^3$. In standard cosmology this quantity is a constant, but in our case it will be constant only asymptotically, for $\tilde{t} \gg 1$, as we can see from Fig. (5). The normalized energy density $\Omega_m$ converges to 1 in the "late epoch". In earlier times energy density is gradually transferred from space-time (especially the quadratic Riemann term) to matter up to the value we encounter in the late epoch.

**CONCLUSIONS**

In this paper we have investigated the impact of the covariant canonical gauge theory of gravity on simple cosmological scenarios. Because this formulation is based on the first order formalism, the energy-momentum is not covariantly conserved – as well known also for other theories beyond Lovelock. We interpret this as energy transfer from space-time to matter.

For a cosmological solution in a homogeneous and isotropic universe, the Friedman equations are modified. For $g_1 = 0$ we recover the original Friedman equations, and for $g_1 \to \infty$ the modification becomes dominant. The equation of state for this limit is similar to the equation of state of radiation. The coupling constant $g_1$ driving the strength of the quadratic term gives rise to a time scale $\tau = \sqrt{8 \pi G g_1}$. For $t \ll \tau$ the quadratic Riemann term becomes dominant, whereas for $t \gg \tau$ the Einstein term is dominant. Therefore, a deviation from the standard cosmology emerges only in the very early universe. Of course "early" refers to the time scale driven by the coupling constant $g_1$.

We have analyzed various scenarios of the Friedman universe filled with perfect fluids with constant equations of state. For pure radiation we find no deviation from the standard cosmological solution, as the new quadratic term is traceless alike the radiation stress tensor. For a dark energy solution the original inflationary solution is predicted, but in addition we get a bouncing and a Big Bang solution, both with asymptotic inflation. The dust solution is the most interesting one. The deviation from the standard Friedman equation arises only for $t \ll \tau$ where the covariant conservation of the stress energy tensor is not zero. In fact, because the Einstein tensor in the stress tensor is covariantly conserved, the quadratic Riemann and the matter density terms must be conserved together. This implies the interpretation that energy-momentum is transferred from space-time to matter. Asymptotically the solution settles at the standard dust solution.

In summary, the effect of the quadratic Riemann term in the CCGG equation leads to a modified dynamics of space-time and matter. Derived via the rigorous mathematical framework of canonical transformations from first principles, we encounter "Dark Energy" like effects that have its roots in the dynamics of the geometry of the universe. How a coherent geometrical theory of Dark Energy emerges in the CCGG theory, is subject of ongoing study.
APPENDIX - COVARIANT DIVERGENCE OF THE QUADRATIC TERM

We first set up directly the covariant derivative of the quadratic Riemann tensor expression from Eq. (5)

\[ Q^{\alpha}_{\xi,\alpha} = \left( R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} - \frac{1}{4} \delta_{\xi}^{\alpha} R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} \right)_{,\alpha} \]

\[ = R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} + R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} - \frac{1}{2} R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} \]

\[ = R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} (R^{\beta\xi\eta}_{\lambda} - \frac{1}{2} R^{\beta\xi\eta}_{\lambda}) + R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda}. \quad (42) \]

We now make use of the Bianchi identity for the covariant derivative of the Riemann tensor in spaces without torsion

\[ R^{\eta\beta\lambda\alpha}_{\eta\beta\lambda\alpha} + R^{\eta\beta\alpha\xi}_{\eta\lambda\alpha\xi} + R^{\eta\beta\xi\lambda}_{\eta\beta\xi\lambda} = 0, \]

which we insert into Eq. (42)

\[ Q^{\alpha}_{\xi,\alpha} = R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} \left( -R^{\beta\xi\eta}_{\lambda} + \frac{1}{2} R^{\beta\xi\eta}_{\lambda} + \frac{1}{2} R^{\beta\xi\eta}_{\lambda} \right) + R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda}, \]

and equivalently rewritten as

\[ Q^{\alpha}_{\xi,\alpha} = -\frac{1}{2} R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} \left( R^{\beta\xi\eta}_{\lambda} + R^{\beta\xi\eta}_{\lambda} \right) + R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda}. \]

The first term on the right-hand side vanishes as the sum of the derivatives of the Riemann tensor is symmetric in \( \alpha \) and \( \lambda \) while \( R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} \) is skew-symmetric in these indices. The final result is now

\[ R^{\xi\beta\lambda\alpha}_{\beta\eta\lambda} R^{\beta\xi\eta}_{\lambda} = \frac{6}{a^4} \left( a^2 \ddot{a} + aa\dot{a} - 2\dot{a}^2 - 2\dot{a}K \right). \quad (43) \]

Hence

\[ \frac{1}{3} a^2 \ddot{\rho} + a^2 \dot{a} (\dot{\rho} + p) = 2 g_1 \dot{a} a \frac{d}{dt}(a^2 - 2Ma^2 + K) \]

\[ + 4g_1 \dot{a}(2Ma^2 - aa) = -4g_1 a^3 \frac{d}{dt}M \quad (44) \]