Theoretical Review of K-Physics

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Abstract

We review several aspects of K-Physics: i) Main targets of the field, ii) The theoretical framework for K-decays, iii) Standard analysis of the unitarity triangle, iv) $\varepsilon'/\varepsilon$, v) Rare and CP violating K-decays, vi) Comparision of the potentials of $K \to \pi \nu \bar{\nu}$ and CP-B asymmetries. vii) Some aspects of the physics beyond the Standard Model.
1 Introduction

The theoretical review of K-physics presented below is based on the theoretical summary talk which I have given at the K-physics workshop held in Orsay this summer. The experimental summary has been presented by Bruce Winstein.

This workshop has shown very clearly a great potential of K-physics in testing the Standard Model, testing its possible extensions and searching for exotic phenomena such as lepton number violation, CPT and Quantum Mechanics violations. In view of space limitations not all topics can be presented here and I will frequently refer to other talks contained in these proceedings.

I have organized the material as follows:

Section 2 gives a ”Grand View” of the field, discussing its most important targets, recalling the CKM matrix and the unitarity triangle and presenting briefly the theoretical framework.

Section 3 discusses the by now standard analysis of the unitarity triangle (UT).

Section 4 summarizes the present status of $\varepsilon'/\varepsilon$.

Section 5 summarizes the present status of the four stars in the field of rare K-decays: $K_L \rightarrow \pi^0 e^+ e^-$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \mu \bar{\mu}$. This section ends with a classification of K- and B-decays from the point of view of theoretical cleanliness.

Section 6 compares the potentials of CP asymmetries in B-decays and of the very clean decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in determining the parameters of the CKM matrix. Here necessarily a short discussion of CP violation in B-decays will be given.

Section 7 offers a brief look beyond the Standard Model.

Section 8 gives a very short outlook.

Section 9 contains some remarks on this workshop.

2 Grand View

2.1 Main Targets of K-Physics

Let us list the main targets of K-Physics:

- The parameters of the CKM matrix. In particular: the parameters $\lambda$, $\eta$, $\rho$, the element $|V_{td}|$ and $\sin 2\beta$,

- CP violation and rare decays in the Standard Model,

- Low energy tests of QCD, tests and applications of non-perturbative methods such as: lattice, chiral perturbation theory, $1/N$ expansion, QCD sum rules, hadronic sum rules,

- Physics beyond the Standard Model such as supersymmetry, left-right symmetry, charged higgs scalars, leptoquarks, lepton number violations etc.,

- Truly exotic physics related to CPT violations and Quantum Mechanics violations.
All of these targets have been discussed at this workshop. In particular searches for CPT violations and tests of Quantum Mechanics for which DAΦNE is clearly an excellent machine, have been elaborated on by Cline, Di Domenico, Ellis, Tsai, Kostelecky and Huet. CPT tests outside the K system have been discussed by Gabrielse and Okun. Finally very interesting results on CP, T and CPT tests at CPLEAR and phase measurements at CERN and Fermilab have been presented by Le Gac and Pavlopoulos and vigorously discussed at the round table discussion on the phase measurements. Related issues have been presented by Khalfin. Since these aspects are already summarized by Bruce Winstein I will not include them in my talk. Let me then move to the CKM matrix which is central for this field.

2.2 The CKM Matrix and the Unitarity Triangle

An important target of particle physics is the determination of the unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa matrix [1, 2] which parametrizes the charged current interactions of quarks:

$$J^{cc}_\mu = (\bar{u}, \bar{c}, \bar{t}) L \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$ (1)

The CP violation in the standard model is supposed to arise from a single phase in this matrix. It is customary these days to express the CKM-matrix in terms of four Wolfenstein parameters [3] $(\lambda, A, \varrho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and $\eta$ representing the CP violating phase:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2 & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$ (2)

Because of the smallness of $\lambda$ and the fact that for each element the expansion parameter is actually $\lambda^2$, it is sufficient to keep only the first few terms in this expansion.

Following [4] one can define the parameters $(\lambda, A, \varrho, \eta)$ through

$$s_{12} \equiv \lambda \quad s_{23} \equiv A\lambda^2 \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta)$$ (3)

where $s_{ij}$ and $\delta$ enter the standard exact parametrization [5] of the CKM matrix. This specifies the higher orders terms in (2).

The definition of $(\lambda, A, \varrho, \eta)$ given in (3) is useful because it allows to improve the accuracy of the original Wolfenstein parametrization in an elegant manner. In particular

$$V_{us} = \lambda \quad V_{cb} = A\lambda^2$$
$$V_{ub} = A\lambda^3(\varrho - i\eta) \quad V_{td} = A\lambda^3(1 - \varrho - i\eta)$$ (4, 5)

where

$$\varrho = \varrho(1 - \frac{\lambda^2}{2}) \quad \eta = \eta(1 - \frac{\lambda^2}{2})$$ (6)
A useful geometrical representation of the CKM matrix is the unitarity triangle obtained by using the unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

(7)

rescaling it by $|V_{cd}V_{cb}| = A\lambda^3$ and depicting the result in the complex $(\bar{\rho}, \bar{\eta})$ plane as shown in fig. 1. The lengths CB, CA and BA are equal respectively to 1,

$$R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} |V_{ub}|$$

and

$$R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} |V_{td}|. \quad (8)$$

The triangle in fig. 1, $|V_{us}|$ and $|V_{cb}|$ give the full description of the CKM matrix. Looking at the expressions for $R_b$ and $R_t$ we observe that within the standard model the measurements of four CP conserving decays sensitive to $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$ can tell us whether CP violation ($\eta \neq 0$) is predicted in the standard model. This is a very remarkable property of the Kobayashi-Maskawa picture of CP violation: quark mixing and CP violation are closely related to each other.

There is of course the very important question whether the KM picture of CP violation is correct and more generally whether the standard model offers a correct description of weak decays of hadrons. In order to answer these important questions it is essential to calculate as many branching ratios as possible, measure them experimentally and check if they all can be described by the same set of the parameters $(\lambda, A, \bar{\rho}, \eta)$. In the language of the unitarity triangle this means that the various curves in the $(\bar{\rho}, \bar{\eta})$ plane extracted from different decays should cross each other at a single point as shown in fig. 2. Moreover the angles $(\alpha, \beta, \gamma)$ in the resulting triangle should agree with those extracted one day from CP-asymmetries in B-decays. More about this below.

Since the CKM matrix is only a parametrization of quark mixing and of CP violation and does not offer the explanation of these two very important phenomena, many physicists hope that a new physics while providing a dynamical origin of quark mixing and CP violation will also change the picture given in fig. 2. That is, the different curves based on standard model...
Figure 2: The ideal Unitarity Triangle. For artistic reasons the value of $\bar{\eta}$ has been chosen to be higher than the fitted central value $\bar{\eta} \approx 0.35$.

expressions, will not cross at a single point and the angles $(\alpha, \beta, \gamma)$ extracted one day from CP-asymmetries in B-decays will disagree with the ones determined from rare K and B decays.

Clearly the plot in fig. 2 is highly idealized because in order to extract such nice curves from various decays one needs perfect experiments and perfect theory. We will see in section 6 that for certain decays such a picture is not fully unrealistic. Generally however the task of extracting the unitarity triangle from the experimental data is not easy. In order to understand this we have to discuss the present theoretical framework.

2.3 Theoretical Framework

The basic problem in the calculation of branching ratios for K decays and other physical observables in the K system is related to strong interactions. Although due to the smallness of the effective QCD coupling at short distances, the gluonic contributions at scales $O(M_W, M_Z, m_t)$ can be calculated within the perturbative framework, the fact that K mesons are $q\bar{q}$ bound states forces us to consider QCD at long distances as well. Here we have to rely on existing non-perturbative methods, discussed briefly below, which are not yet very powerful at present.

The question then is whether we could somehow divide the problem into two parts: the short distance part, under control already today, and the long distance part which hopefully will be fully under control when our non-perturbative tools improve. One could even hope that in certain decays the non-perturbative contributions could be measured and subsequently used in other decays so that one could make predictions in some cases already today without the
necessity for non-perturbative calculations.

The Operator Product Expansion (OPE) combined with the renormalization group approach can be regarded as a mathematical formulation of the strategy outlined above. This framework brings in local operators \( Q_i \) which govern “effectively” the transitions in question and the amplitude for an exclusive decay \( M \rightarrow F \) is written as

\[
A(M \rightarrow F) = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i(\mu) \langle F | Q_i(\mu) | M \rangle
\]

where \( M \) stands for the decaying meson, \( F \) for a given final state and \( V_{\text{CKM}} \) denotes the relevant CKM factor. \( C_i(\mu) \) stand for the Wilson coefficient functions (c-numbers). The scale \( \mu \) separates the physics contributions in the “short distance” contributions (corresponding to scales higher than \( \mu \)) contained in \( C_i(\mu) \) and the “long distance” contributions (scales lower than \( \mu \)) contained in \( \langle F | Q_i(\mu) | M \rangle \). By evolving the scale from \( \mu = \mathcal{O}(M_W) \) down to lower values of \( \mu \) one transforms the physics information at scales higher than \( \mu \) from the hadronic matrix elements into \( C_i(\mu) \). Since no information is lost this way the full amplitude cannot depend on \( \mu \). This is the essence of the renormalization group equations which govern the evolution (\( \mu \)-dependence) of \( C_i(\mu) \). This \( \mu \)-dependence must be cancelled by the one present in \( \langle Q_i(\mu) \rangle \). Generally this cancellation involves many operators due to the operator mixing under renormalization.

It should be stressed that the use of the renormalization group is necessary in order to sum up large logarithms \( \log \frac{M_W}{\mu} \) which appear for \( \mu = \mathcal{O}(1 - 2 \text{ GeV}) \). In the so-called leading logarithmic approximation (LO) terms \( (\alpha_s \log \frac{M_W}{\mu})^n \) are summed. The next-to-leading logarithmic correction (NLO) to this result involves summation of terms \( (\alpha_s)^n(\log \frac{M_W}{\mu})^{n-1} \) and so on. This hierarchic structure gives the renormalization group improved perturbation theory.

I will not discuss here the technical details of the renormalization group and of the calculation of \( C_i(\mu) \). They can be found in a recent review and in the contributions of Martinelli and Nierste to these proceedings. Let me just list a few operators which play an important role in the phenomenology of K decays. These are \( (\alpha \text{ and } \beta \text{ are colour indices}) \):

**Current–Current:**

\[
Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \quad Q_2 = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A}
\]

**QCD–Penguin and Electroweak–Penguin:**

\[
Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A} \quad Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}
\]

\( \Delta S = 2 \text{ and } \Delta B = 2 \) Operators:

\[
Q(\Delta S = 2) = (\bar{s}d)_{V-A}(\bar{d}s)_{V-A} \quad Q(\Delta B = 2) = (\bar{b}d)_{V-A}(\bar{d}b)_{V-A}
\]

**Semi–Leptonic Operators:**

\[
Q_{9V} = (\bar{s}d)_{V-A}(\bar{e}e)_V \quad Q_{10A} = (\bar{s}d)_{V-A}(\bar{e}e)_A
\]
Figure 3: Typical Penguin and Box Diagrams.

\[ Q_{\bar{\nu}\nu} = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \quad Q_{\bar{\nu}\mu} = (\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A} \]  

The rather formal expression for the decay amplitudes given in (9) can always be cast in the form [7]:

\[ A(M \to F) = \sum_i B_i V_{CKM}^i \eta_{QCD}^i F_i(m_t, m_c) \]  

which is more useful for phenomenology. In writing (15) we have generalized [9] to include several CKM factors. \( F_i(m_t, m_c) \), the Inami-Lim functions [8], result from the evaluation of loop diagrams with internal top and charm exchanges (see fig. 3) and may also depend solely on \( m_t \) or \( m_c \). In the case of new physics they depend on masses of new particles such as charginos, stops, charged Higgs scalars etc. The factors \( \eta_{QCD}^i \) summarize the QCD corrections which can be calculated by formal methods discussed above. Finally \( B_i \) stand for nonperturbative factors related to the hadronic matrix elements of the contributing operators: the main theoretical uncertainty in the whole enterprise. In leptonic and semi-leptonic decays for which only the matrix elements of weak currents are needed, the non-perturbative \( B \)-factors can fortunately be determined from leading tree level decays reducing or removing the non-perturbative uncertainty. In non-leptonic decays this is generally not possible and we have to rely on existing non-perturbative methods. A well known example of a \( B \)-factor is the renormalization group invariant parameter \( B_K \) defined by

\[ B_K = B_K(\mu) \left[ \alpha_s(\mu) \right]^{-2/9} \quad \langle \bar{K}^0 | Q(\Delta S = 2) | K^0 \rangle = \frac{8}{3} B_K(\mu) F_K^2 m_K^2 \]  

For simplicity we did not show the NLO correction in \( B_K \). \( B_K \) plays an important role in the phenomenology of CP violation in \( K \to \pi\pi \).

In order to achieve sufficient precision the Wilson coefficients or equivalently the QCD factors \( \eta_{QCD}^i \equiv \eta_i \) have to include both the leading and the next-to-leading (NLO) corrections. These
corrections are known by now for the most important and interesting decays and are reviewed in [3]. We will discuss the impact of NLO calculations below.

Let us recall why NLO calculations are important for the phenomenology of weak decays:

- The NLO is first of all necessary to test the validity of the renormalization group improved perturbation theory.
- Without going to NLO the QCD scale $\Lambda_{\overline{MS}}$ extracted from various high energy processes cannot be used meaningfully in weak decays.
- Due to the renormalization group invariance the physical amplitudes do not depend on the scales $\mu$ present in $\alpha_s$ or in the running quark masses, in particular $m_t(\mu_t)$, $m_b(\mu_b)$ and $m_c(\mu_c)$. However in perturbation theory this property is broken through the truncation of the perturbative series. Consequently one finds sizable scale ambiguities in the leading order, which can be reduced considerably by going to NLO.
- In several cases the central issue of the top quark mass dependence is strictly a NLO effect.

Clearly in order to calculate the full amplitude in (15) or (9) also the $B_i$ factors or the matrix elements $\langle F | Q_i(\mu) | M \rangle$ have to be evaluated. Since they involve long distance contributions one is forced in this case to use non-perturbative methods. Several non-perturbative methods have been discussed at this workshop. Let me make only a few remarks here. The details can be found in the corresponding contributions to these proceedings.

The lattice calculations have been discussed by Martinelli and Kilcup. The progress in this field is rather slow but eventually this could be the most powerful method to evaluate hadronic matrix elements. Yet it seems that in the case of non-leptonic decays we have to wait 5 to 15 years before calculation at the level of 10% (statistic and systematic) could be achieved. Some lattice results will be given below.

Other methods have been reviewed by Donoghue in a very enjoyable talk and specifically discussed by Bijlens, Bertolini, Fabbrichesi, Fajfer, Pich, Singer and Soldan. In particular Bijlens stressed the great potential of DAΦNE in testing chiral perturbation theory. At this point I can only recommend to read his contribution and to study “The second DAΦNE Physics Handbook” which is a great piece of work. Yet since chiral perturbation theory is based mainly on symmetries it has its own limitations which are particularly seen in the important work of Kambor, Missimer and Wyler [9] where the the general form of $\Delta S = 1$ non-leptonic Lagrangian at $O(p^4)$ has been worked out. At this level there are too many unknown parameters (counter terms) which cannot be determined experimentally and one has to use some additional dynamical methods like $1/N$ [10, 11, 12] in order to have some predictions. Yet as stressed by Pich, reviewing his work with de Rafael, Ecker and other spanish masters (Bruno and Prades), in other decays such as $K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $K^+ \to \pi^+\gamma\gamma$ (seen for the first time by BNL-E787) the situation is very different and predictions can be made easier. See also a recent work on $K^+ \to \pi^+\gamma\gamma$ in [13]. One should hope that the status of non-leptonic decays in the framework of chiral perturbation theory will improve in the DAΦNE era.
Donoghue discussed also his interesting work with Gabbiani which uses dispersion relations. Bertolini and Fabbrichesi on the other hand presented their extensive calculations of $\Delta I = 1/2$ rule and of $\varepsilon'/\varepsilon$ in the framework of the Chiral Quark Model. This model has potential uncertainties related to the value of the gluon condensate, which plays an important role in enhancing the $\Delta I = 1/2$ transitions. This ”gluon condensate effect” has been already identified by Pich and de Rafael in their work of 1991 [14] and is another expression of the $\Delta I = 1/2$ enhancement through quadratic cut-off dependence in the $1/N$ approach of Bardeen, Gérard and myself [10] which we discussed almost ten years ago. Yet it is important to see such effects in different settings. Let us hope that one day this rule will be understood at a fully quantitative level and that the relation of the approaches mentioned above to the diquark approach of Stech and Neubert [15] will be clarified. Some insight here can be gained from the work of Jamin and Pich [10].

Clearly the list of non-perturbative methods has to include QCD sum rules [17] which play a substantial role in estimating non-perturbative parameters. Finally I would like to mention the interesting work on the extended Nambu-Jona-Lasinio model by Bijnens, Bruno and de Rafael [18]. Other non-perturbative strategies are reviewed in [19].

Needless to say all the non-perturbative methods listed above have at present considerable limitations and only time will show whether the situation can be considerably improved. Consequently the dominant theoretical uncertainties in the decay amplitudes reside in the matrix elements of $Q_i$ or the corresponding $B_i$ factors.

After these general remarks let us move to the presentation of some results obtained in this framework and its confrontation with the experimental data.

3 Standard Analysis

3.1 Basic Formulae

At present there is still a rather limited knowledge of the shape of the unitarity triangle. The standard analysis using the available experimental and theoretical information proceeds essentially in five steps:

Step 1:
From $b \to c$ transition in inclusive and exclusive B meson decays one finds $|V_{cb}|$ and consequently the scale of UT:

$$|V_{cb}| \Rightarrow \lambda |V_{cb}| = \lambda^3 A$$

(17)

Step 2:
From $b \to u$ transition in inclusive and exclusive B meson decays one finds $|V_{ub}/V_{cb}|$ and consequently the side $CA = R_b$ of UT:

$$|V_{ub}/V_{cb}| \Rightarrow R_b = 4.44 \cdot \frac{|V_{ub}|}{|V_{cb}|}$$

(18)
Step 3:
From the observed indirect CP violation in $K \rightarrow \pi\pi$ described experimentally by the parameter $\varepsilon_K$ and theoretically by the imaginary part of the relevant box diagram in fig. 3 one derives the constraint:

$$\bar{\eta} \left[ (1 - \bar{\rho}) A^2 \eta_2 S(x_t) + P_0(\varepsilon) \right] A^2 B_K = 0.226 \quad S(x_t) = 0.784 \cdot x_t^{0.76} \quad (19)$$

where

$$P_0(\varepsilon) = [\eta_3 S(x_c, x_t) - \eta_1 x_c] \frac{1}{\lambda^4} \quad x_t = \frac{m_t^2}{M_W^2} \quad (20)$$

Equation (19) specifies a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane. Here $B_K$ is the non-perturbative parameter defined in (10) and $\eta_2$ is the QCD factor in the box diagrams with two top quark exchanges. Finally $P_0(\varepsilon) = 0.31 \pm 0.02$ summarizes the contributions of box diagrams with two charm quark exchanges and the mixed charm-top exchanges. $P_0(\varepsilon)$ depends very weakly on $m_t$ and its range given above corresponds to $155 \, GeV \leq m_t \leq 185 \, GeV$. The NLO values of the QCD factors $\eta_1$, $\eta_2$ and $\eta_3$ are given as follows $[20, 21, 22]$:

$$\eta_1 = 1.38 \pm 0.20 \quad \eta_2 = 0.57 \pm 0.01 \quad \eta_3 = 0.47 \pm 0.04 \quad (21)$$

The quoted errors reflect the remaining theoretical uncertainties due to $\Lambda_{MS}$ and the quark masses. The references to the leading order calculations can be found in $[3]$. The factor $\eta_1$ plays only a minor role in the analysis of $\varepsilon_K$ but its enhanced value through NLO corrections $[21]$ is essential for the $K_L - K_S$ mass difference.

Concerning the parameter $B_K$, the most recent analyses using the lattice methods summarized by Kilcup here and recently by Flynn $[23]$ give $B_K = 0.90 \pm 0.06$. The 1/N approach of $[24]$ gives $B_K = 0.70 \pm 0.10$. A recent confirmation of this result in a somewhat modified framework has been presented by Bijnens and Prades $[25]$ who gave plausible arguments for the difference between this result for $B_K$ and the lower values obtained by using the QCD Hadronic Duality approach $[20]$ ($B_K = 0.39 \pm 0.10$) or using the $SU(3)$ symmetry and PCAC ($B_K = 1/3$) $[27]$. For $|V_{cb}| = 0.040$ and $|V_{ub}/V_{cb}| = 0.08$ such low values for $B_K$ require $m_t > 200$ GeV in order to explain the experimental value of $\varepsilon_K$ $[28, 1, 21]$. The QCD sum rule results are in the ballpark of $B_K = 0.60$ $[29]$. In our numerical analysis presented below we will use $B_K = 0.75 \pm 0.15$ (see table 1).

Step 4:
From the observed $B_d^0 - \bar{B}_d^0$ mixing described experimentally by the mass difference $(\Delta M)_d$ or by the mixing parameter $x_d = \Delta M/\Gamma_B$ and theoretically by the relevant box diagram of fig. 3 the side $BA = R_t$ of the UT can be determined:

$$R_t = 1.0 \cdot \left[ \frac{|V_{td}|}{8.7 \cdot 10^{-3}} \right] \left[ \frac{0.040}{|V_{cb}|} \right] \quad (22)$$

with

$$|V_{td}| = 8.7 \cdot 10^{-3} \left[ \frac{200 \, MeV}{\sqrt{B_{B_d} F_{B_d}}} \right] \left[ \frac{170 \, GeV}{m_t(m_t)} \right]^{0.76} \left[ \frac{(\Delta M)_d}{0.45/ps} \right]^{0.5} \sqrt{\frac{0.55}{\eta_B}} \quad (23)$$
Here $\eta_B$ is the QCD factor analogous to $\eta_2$ and given by $\eta_B = 0.55 \pm 0.01$ [21]. Next $F_{B_d}$ is the B-meson decay constant and $B_{B_d}$ denotes a non-perturbative parameter analogous to $B_K$.

There is a vast literature on the lattice calculations of $F_{B_d}$ and $B_{B_d}$. The most recent world averages are given by Flynn [23]:

$$F_{B_d}\sqrt{B_{B_d}} = 175 \pm 25 \text{ MeV} \quad B_{B_d} = 1.31 \pm 0.03$$

(24)

This result for $F_{B_d}$ is compatible with the results obtained using QCD sum rules [20]. An interesting upper bound $F_{B_d} < 195 \text{ MeV}$ using QCD dispersion relations can be found in [21]. In our numerical analysis we will use $F_{B_d}\sqrt{B_{B_d}} = 200 \pm 40 \text{ MeV}$. The experimental situation on $(\Delta M)_d$ has been recently summarized by Gibbons [32] and is given in table 1. For $\tau(B_d) = 1.55 \text{ ps}$ one has then $x_d = 0.72 \pm 0.03$.

**Step 5:**

The measurement of $B^0 - \bar{B}^0$ mixing parametrized by $(\Delta M)_s$ together with $(\Delta M)_d$ allows to determine $R_t$ in a different way. Setting $(\Delta M)_d^{\max} = 0.482 / \text{ps}$ and $|V_{ts}/V_{cb}|^{\max} = 0.993$ (see table 1) I find a useful formula:

$$(R_t)_{\text{max}} = 1.0 \cdot \xi \sqrt{\frac{10.2 / \text{ps}}{(\Delta M)_s^{\max}} \quad \xi = \frac{F_{B_s}\sqrt{B_{B_s}}}{F_{B_d}\sqrt{B_{B_d}}}$$

(25)

where $\xi = 1$ in the SU(3)–flavour limit. Note that $m_t$ and $|V_{cb}|$ dependences have been eliminated this way and that $\xi$ should in principle contain much smaller theoretical uncertainties than the hadronic matrix elements in $(\Delta M)_d$ and $(\Delta M)_s$ separately.

The most recent values relevant for (25) are:

$$(\Delta M)_s > 9.2 / \text{ps} \quad \xi = 1.15 \pm 0.05$$

(26)

The first number is the improved lower bound quoted in [32] based in particular on ALEPH and DELPHI results. The second number comes from quenched lattice calculations summarized by Flynn in [23]. A similar result has been obtained using QCD sum rules [33]. On the other hand another recent quenched lattice calculation [34] not included in (26) finds $\xi \approx 1.3$. Moreover one expects that unquenching will increase the value of $\xi$ in (26) by roughly 10% so that values as high as $\xi = 1.25 - 1.30$ are certainly possible even from Flynn’s point of view. For such high values of $\xi$ the lower bound on $(\Delta M)_s$ in (26) implies $R_t \leq 1.37$ which as we will see is similar to the bound obtained on the basis of the first four steps alone. On the other hand for $\xi = 1.15$ one finds $R_t \leq 1.21$ which puts an additional constraint on the unitarity triangle cutting lower values of $\tilde{g}$ and higher values of $|V_{td}|$. In view of remaining large uncertainties in $\xi$ we will not use the constraint from $(\Delta M)_s$ below.

### 3.2 Numerical Results

#### 3.2.1 Input Parameters

The input parameters needed to perform the standard analysis are given in table 1. The details on the chosen ranges of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ can be found in [32]. Clearly during the last two years
there has been a considerable progress done by experimentalists and theorists in the extraction of $|V_{cb}|$ from exclusive and inclusive decays. In particular I would like to mention important papers by Shifman, Uraltsev and Vainshtein \cite{35}, Neubert \cite{36} and Ball, Benecke and Braun \cite{37} on the basis of which one is entitled to use the value given in table 1. In the case of $|V_{ub}/V_{cb}|$ the situation is much worse but progress in the next few years is to be expected in particular due to new information coming from exclusive decays \cite{38, 32} and the inclusive semileptonic $b \to u$ rate \cite{35, 37, 39}.

Next it is important to stress that the discovery of the top quark by CDF and D0 and its impressive mass measurement summarized recently by Tipton \cite{40} had an important impact on the field of rare decays and CP violation reducing considerably one potential uncertainty. At this point it should be recalled that the parameter $m_t$, the top quark mass, used in weak decays is not equal to the one used in the electroweak precision studies at LEP, SLD and FNAL. In the latter investigations the so-called pole mass is used, whereas in all the NLO calculations $m_t$ refers to the running current top quark mass normalized at $\mu = m_t$: $\overline{m}(m_t)$. For $m_t = \mathcal{O}(170$ GeV), $\overline{m}(m_t)$ is typically by 8 GeV smaller than $m_t^{Pole}$. This difference matters already because the most recent pole mass value has a very small error, 175 ± 6 GeV \cite{40}, implying 167 ± 6 GeV for $\overline{m}(m_t)$. In this review we will often denote this mass by $m_t$.

| Quantity                       | Central | Error     |
|--------------------------------|---------|-----------|
| $|V_{cb}|$                       | 0.040   | ±0.003    |
| $|V_{ub}/V_{cb}|$               | 0.080   | ±0.020    |
| $B_K$                          | 0.75    | ±0.15     |
| $\sqrt{B_d F_{B_d}}$           | 200 MeV | ±40 MeV   |
| $\sqrt{B_s F_{B_s}}$           | 240 MeV | ±40 MeV   |
| $m_t$                          | 167 GeV | ±6 GeV    |
| $(\Delta M_d)^{(4)}$           | 0.464 ps$^{-1}$ | ±0.018 ps$^{-1}$ |
| $\Lambda^{(4)}_{MS}$           | 325 MeV | ±80 MeV   |

Table 1: Collection of input parameters.

3.2.2 $|V_{ub}/V_{cb}|$, $|V_{cb}|$ and $\varepsilon_K$

The values for $|V_{ub}/V_{cb}|$ and $|V_{cb}|$ in table 1 are not correlated with each other. On the other hand such a correlation is present in the analysis of the CP violating parameter $\varepsilon_K$ which is roughly proportional to the fourth power of $|V_{cb}|$ and linear in $|V_{ub}/V_{cb}|$. It follows that not all values in table 1 are simultaneously consistent with the observed value of $\varepsilon_K$. This is indirectly seen in \cite{28} and has been more explicitly emphasized last year by Herrlich and Nierste \cite{22} and in \cite{3}. Updating and rewriting the analytic lower bound on $m_t$ from $\varepsilon_K$ \cite{28} one finds \cite{3}

$$\frac{|V_{ub}|}{V_{cb}}_{min} = \frac{0.225}{B_K A^2 (2m_t^{0.76}A^2 + 1.4)}$$  \hspace{1cm} (27)
Figure 4: Lower bound on $|V_{ub}/V_{cb}|$ from $\varepsilon_K$.

This bound is shown as a function of $|V_{cb}|$ for different values of $B_K$ and $m_t = 173$ GeV in fig.4. We observe that simultaneously small values of $|V_{ub}/V_{cb}|$ and $|V_{cb}|$ although still consistent with the ones given in table 1, are not allowed by the size of the indirect CP violation observed in $K \rightarrow \pi\pi$.

3.2.3 Output of a Standard Analysis

The output of the standard analysis depends to some extent on the error analysis. This should be always remembered in view of the fact that different authors use different procedures. In order to illustrate this I show in table 2 the results for various quantities of interest using two types of the error analyses:

- Scanning: Both the experimentally measured numbers and the theoretical input parameters are scanned independently within the errors given in table 1.
- Gaussian: The experimentally measured numbers and the theoretical input parameters are used with Gaussian errors.

Clearly the ”scanning” method is a bit conservative. On the other hand using Gaussian distributions for theoretical input parameters can certainly be questioned. One could instead use flat distributions (with a width of $2\sigma$) for the theoretical input parameters as done in [II]. The latter method gives however similar results to the ”Gaussian method”. Personally I think that at present the conservative ”scanning” method should be preferred. In the future however when data and theory improve, it would be useful to find a less conservative estimate which
| Quantity | Scanning | Gaussian |
|----------|----------|----------|
| $|V_{td}|/10^{-3}$ | 6.9 – 11.3 | 8.6 ± 1.1 |
| $|V_{ts}/V_{cb}|$ | 0.959 – 0.993 | 0.976 ± 0.010 |
| $|V_{td}/V_{ts}|$ | 0.16 – 0.31 | 0.213 ± 0.034 |
| $\sin(2\beta)$ | 0.36 – 0.80 | 0.66 ± 0.13 |
| $\sin(2\alpha)$ | −0.76 – 1.0 | 0.11 ± 0.55 |
| $\sin(\gamma)$ | 0.66 – 1.0 | 0.88 ± 0.10 |
| Im$\lambda_t/10^{-4}$ | 0.86 – 1.71 | 1.29 ± 0.22 |
| $(\Delta M)_s ps$ | 8.0 – 25.4 | 15.2 ± 5.5 |

Table 2: Output of the Standard Analysis. $\lambda_t = V_{ts}^*V_{td}$.

most probably will give errors somewhere inbetween these two error estimates. The analysis discussed here has been done in collaboration with Matthias Jamin and Markus Lautenbacher. More details and more results can be found in [42].

In fig. 5 we show the range for the upper corner A of the UT. The solid thin lines correspond to $R_t^{max}$ from (25) using $\xi = 1.20$ and $(\Delta M)_s = 10/ps$, 15/ps and 25/ps, respectively. The allowed region has a typical "banana" shape which can be found in many other analyses [4, 11, 22, 13, 14, 15]. The size of the banana and its position depends on the assumed input parameters and on the error analysis which varies from paper to paper. The results in fig. 5 correspond to a simple independent scanning of all parameters within one standard deviation. Effectively such an approach is more conservative than using Gaussian distributions as done in some papers quoted above. We show also the impact of the experimental bound $(\Delta M)_s > 9.2/ps$ with $\xi = 1.20$ and the corresponding bound for $\xi = 1.30$. In view of the remaining uncertainty in $\xi$, in particular due to quenching, this bound has not been used in obtaining the results in table 2. It is evident however that $B_s^0 - B_s^0$ mixing will have a considerable impact on the unitarity triangle when the value of $\xi$ will be better known and the data improves. This is very desirable because as seen in fig. 5 our knowledge of the unitarity triangle is still rather poor. Similarly the uncertainty in the predicted value of $(\Delta M)_s$ using $\sqrt{F_s}F_{Bs}$ of table 1 is large with central values around 15/ps.

4 \[ \varepsilon'/\varepsilon \]

The measurement of $\varepsilon'/\varepsilon$ at the $10^{-4}$ level remains as one of the important targets of contemporary particle physics. A non-vanishing value of this ratio would give the first signal for the direct CP violation ruling out the superweak models. The experimental situation on Re$(\varepsilon'/\varepsilon)$ is unclear at present. While the result of NA31 collaboration at CERN with Re$(\varepsilon'/\varepsilon) = (23 \pm 7) \cdot 10^{-4}$ [46] clearly indicates direct CP violation, the value of E731 at Fermilab, Re$(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \cdot 10^{-4}$ [47], is compatible with superweak theories [48] in which $\varepsilon'/\varepsilon = 0$. Hopefully, in about two years the experimental situation concerning $\varepsilon'/\varepsilon$ will be
clarified through the improved measurements by the two collaborations at the $10^{-4}$ level and by the KLOE experiment at DAΦNE.

There is no question about that the direct CP violation is present in the standard model. Yet accidentally it could turn out that it will be difficult to see it in $K \to \pi \pi$ decays. Indeed in the standard model $\varepsilon'/\varepsilon$ is governed by QCD penguins and electroweak (EW) penguins. In spite of being suppressed by $\alpha/\alpha_s$ relative to QCD penguin contributions, the electroweak penguin contributions have to be included because of the additional enhancement factor $\text{Re}A_0/\text{Re}A_2 = 22$ relative to QCD penguins. With increasing $m_t$ the EW penguins become increasingly important [49, 50], and entering $\varepsilon'/\varepsilon$ with the opposite sign to QCD penguins suppress this ratio for large $m_t$. For $m_t \approx 200$ GeV the ratio can even be zero [70]. Because of this strong cancellation between two dominant contributions and due to uncertainties related to hadronic matrix elements of the relevant local operators, a precise prediction of $\varepsilon'/\varepsilon$ is not possible at present.

In spite of all these difficulties, a considerable progress has been made in this decade to calculate $\varepsilon'/\varepsilon$. First of all the complete next-to-leading order (NLO) effective hamiltonians for $\Delta S = 1$ [51, 52, 53], $\Delta S = 2$ [21, 20, 22] and $\Delta B = 2$ [21] are now available so that a complete NLO analysis of $\varepsilon'/\varepsilon$ including constraints from the observed indirect CP violation ($\varepsilon_K$) and the $B_d^0 - \bar{B}_d^0$ mixing ($\langle \Delta M \rangle_d$) is possible. The improved determination of the $V_{ub}$ and $V_{cb}$ elements of the CKM matrix [32], and in particular the determination of the top quark mass $m_t$ [40] had
of course also an important impact on $\varepsilon'/\varepsilon$. The main remaining theoretical uncertainties in this ratio are then the poorly known hadronic matrix elements of the relevant QCD penguin and electroweak penguin operators represented by two important B-factors ($B_6$ = the dominant QCD penguin $Q_6$ and $B_8$ = the dominant electroweak penguin $Q_8$), the values of the $V_{CKM}$ factors and as stressed in [52] the value of $m_s$ and $\Lambda_{\overline{MS}}$.

An analytic formula for $\varepsilon'/\varepsilon$ which exhibits all these uncertainties can be found in [54, 55]. A very simplified version of this formula is given as follows

$$
\frac{\varepsilon'}{\varepsilon} = 11 \cdot 10^{-4} \left[ \frac{\eta \lambda^5 A^2}{1.3 \cdot 10^{-4}} \right] \left[ \frac{140 \text{ MeV}}{m_s(2 \text{ GeV})} \right]^2 \left[ \frac{\Lambda_{\overline{MS}}^{(4)}}{300 \text{ MeV}} \right]^{0.8} \left[ B_6 - Z(x_t)B_8 \right]
$$

(28)

where $Z(x_t) \approx 0.18(m_t/M_W)^{1.86}$ and equals unity for $m_t \approx 200 \text{ GeV}$. This simplified formula should not be used for any serious numerical analysis.

Concerning the values of $B_6$ and $B_8$ one has $B_6 = B_8 = 1$ in the vacuum insertion estimate of the hadronic matrix elements in question. The same result is found in the large $N$ limit [66, 67]. Also lattice calculations give similar results: $B_6 = 1.0 \pm 0.2$ [58, 59, 60, 61] and $B_8 = 1.0 \pm 0.2$ [68, 60, 61, 61], $B_8 = 0.81(1)$ [24]. These are the values used in [22, 41, 6, 55]. In the chiral quark model one finds [62]: $B_6 = 1.0 \pm 0.4$, $B_8 = 2.2 \pm 1.5$ and generally $B_8 > B_6$. On the other hand the Dortmund group [24, 62] advocates $B_6 > B_8$. From [62] $B_6 = 1.3$ and $B_8 = 0.7$ can be extracted. As discussed by Soldan at this workshop, a new Dortmund calculation is in progress. Other technical details can be found in the talks of Bertolini and Martinelli.

At this point it seems appropriate to summarize the present status of the value of the strange quark mass. The most recent results of QCD sum rule (QCDSR) calculations [63, 64, 65] obtained at $\mu = 1 \text{ GeV}$ correspond to $m_s(2 \text{ GeV}) = 145 \pm 20 \text{ MeV}$. The lattice calculation of [63] finds $m_s(2 \text{ GeV}) = 128 \pm 18 \text{ MeV}$, in rather good agreement with the QCDSR result. This summer a new lattice result has been presented by Gupta and Bhattacharya [70]. They find $m_s(2 \text{ GeV}) = 90 \pm 20 \text{ MeV}$ which is on the low side of all strange mass determinations. Moreover they find that unquenching lowers further the values of $m_s(2 \text{ GeV})$ to $70 \pm 15 \text{ MeV}$. Similar results are found by the FNAL group [71]. The situation with the strange quark mass is therefore unclear at present and hopefully will be clarified soon.

It should also be remarked that the decomposition of the relevant hadronic matrix elements of penguin operators into a product of $B_i$ factors times $1/m_s^2$ although useful in the $1/N$ approach is unnecessary in a brute force method like the lattice approach. It is to be expected that future lattice calculations will directly give the relevant hadronic matrix elements and the issue of $m_s$ in connection with $\varepsilon'/\varepsilon$ will disappear.

The most recent analysis of [55] using input parameters of table 1, $B_6 = 1.0 \pm 0.2$, $B_8 = 1.0 \pm 0.2$ and $m_s(2 \text{ GeV}) = 129 \pm 17 \text{ MeV}$ finds

$$
-1.2 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 16.0 \cdot 10^{-4}
$$

(29)

and

$$
\varepsilon'/\varepsilon = (3.6 \pm 3.4) \cdot 10^{-4}
$$

(30)

for the ”scanning” method and the ”gaussian” method respectively.
The result in (30) agrees rather well with the 1995 analysis of the Rome group [41] which finds \( \varepsilon'/\varepsilon = (3.1 \pm 2.5) \cdot 10^{-4} \). On the other hand the range in (29) shows that for particular choices of the input parameters, values for \( \varepsilon'/\varepsilon \) as high as \( 16 \cdot 10^{-4} \) cannot be excluded at present. Such high values are found if simultaneously \( |V_{ub}/V_{cb}| = 0.10 \), \( B_6 = 1.2 \), \( B_8 = 0.8 \), \( B_K = 0.6 \), \( m_s(2 \text{ GeV}) = 110 \text{ MeV} \), \( \Lambda_{\overline{\text{MS}}}^{(4)} = 405 \text{ MeV} \) and low values of \( m_t \) still consistent with the \( \varepsilon_K \) and the observed \( B_0^d - \bar{B}_0^d \) mixing are chosen. It is however evident from the comparison of (29) and (30) that such high values of \( \varepsilon'/\varepsilon \) and generally values above \( 10^{-3} \) are very improbable.

The authors of [53] calculating the \( B_i \) factors in the chiral quark model find using the scanning method a rather large range \(-50 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 14 \cdot 10^{-4} \). In particular they find in contrast to [52, 41, 6, 55] that negative values for \( \varepsilon'/\varepsilon \) as large as \(-5 \cdot 10^{-3} \) are possible. The Dortmund group [64] advocating on the other hand \( B_6 > B_8 \) finds \( \varepsilon'/\varepsilon = (9.9 \pm 4.1) \cdot 10^{-4} \) for \( m_s(2 \text{ GeV}) = 130 \text{ MeV} \) [53]. From the point of view of the analyses in [11, 55] such high values of \( \varepsilon'/\varepsilon \) for \( m_s(2 \text{ GeV}) = O(130 \text{ MeV}) \) are rather improbable within the standard model.

The situation with \( \varepsilon'/\varepsilon \) in the standard model may however change if the value for \( m_s \) is as low as found in [54]. Using \( m_s(2 \text{ GeV}) = 85 \pm 17 \text{ MeV} \) one finds [55]

\[
0 \leq \varepsilon'/\varepsilon \leq 43.0 \cdot 10^{-4}
\]

and

\[
\varepsilon'/\varepsilon = (10.4 \pm 8.3) \cdot 10^{-4}
\]

for the ”scanning” method and the ”gaussian” method respectively. We observe that the ”gaussian” result agrees well with the E731 value and as stressed in [53] the decrease of \( m_s \) with \( m_s(2 \text{ GeV}) \geq 85 \text{ MeV} \) alone is insufficient to bring the standard model to agree with the NA31 result. However for \( B_6 > B_8 \), sufficiently large values of \( |V_{ub}/V_{cb}| \) and \( \Lambda_{\overline{\text{MS}}} \) and small values of \( m_s \), the values of \( \varepsilon'/\varepsilon \) in the standard model can be as large as \((2-4) \cdot 10^{-3}\) and consistent with the NA31 result.

Let us hope that the future experimental and theoretical results will be sufficiently accurate to be able to see whether \( \varepsilon'/\varepsilon \neq 0 \) and whether the standard model agrees with the data. In any case the coming years should be very exciting.

5 Rare K Decays

5.1 The Decay \( K_L \to \pi^0 e^+ e^- \)

Let us next move on to discuss the rare decay \( K_L \to \pi^0 e^+ e^- \). Whereas in \( K \to \pi \pi \) decays the CP violating contribution is only a tiny part of the full amplitude and the direct CP violation as we have just seen is expected to be at least by three orders of magnitude smaller than the indirect CP violation, the corresponding hierarchies are very different for \( K_L \to \pi^0 e^+ e^- \).

At lowest order in electroweak interactions (one-loop photon penguin, \( Z^0 \)-penguin and W-box diagrams), this decay takes place only if CP symmetry is violated. The CP conserving contribution to the amplitude comes from a two photon exchange, which although of higher order in \( \alpha \) could in principle be sizable. The CP violating part can again be divided into a
direct and an indirect one. The latter is given by the \( K_S \rightarrow \pi^0 e^+ e^- \) amplitude times the CP violating parameter \( \varepsilon_K \).

Now as reviewed by Pich at this workshop, out of these three contributions only the directly CP violating contribution can be calculated reliably. The other two contributions are unfortunately very uncertain at present and the following ranges can be found in the literature:

\[
Br(K_L \rightarrow \pi^0 e^+ e^-)_{\text{cons}} \approx \begin{cases} 
(0.3 - 1.8) \cdot 10^{-12} \quad [72] \\
4.0 \cdot 10^{-12} \\
(5 \pm 5) \cdot 10^{-12} \quad [74] 
\end{cases}
\]

and \([73, 70, 73, 74]\)

\[
Br(K_L \rightarrow \pi^0 e^+ e^-)_{\text{indir}} = (1 - 5.) \cdot 10^{-12} \quad (34)
\]

In what follows, we will concentrate on the directly CP violating contribution. There are practically no theoretical uncertainties here because the relevant matrix element \( \langle \pi^0 | (\bar{s}d)_{V-A} | K_L \rangle \) can be extracted using isospin symmetry from the well measured decay \( K^+ \rightarrow \pi^0 e^+ \nu \). Calculating the relevant box and electroweak penguin diagrams and including LO \([77]\) and NLO QCD \([78]\) corrections one finds approximately:

\[
Br(K_L \rightarrow \pi^0 e^+ e^-)_{\text{dir}} = 4.4 \cdot 10^{-12} \left[ \frac{\eta}{0.37} \right]^2 \left[ \frac{|V_{cb}|}{0.040} \right]^4 \left[ \frac{\overline{m}_t(m_t)}{170 \text{GeV}} \right]^2 \quad (35)
\]

Scanning the input parameters of table 1 one finds \([42]\)

\[
Br(K_L \rightarrow \pi^0 e^+ e^-)_{\text{dir}} = (4.5 \pm 2.6) \cdot 10^{-12} \quad (36)
\]

where the error comes dominantly from the uncertainties in the CKM parameters. Thus the directly CP violating contribution is comparable to the other two contributions. It is however possible that the direct CP violation dominates in this decay which is of course very exciting. In order to see whether this is indeed the case improved estimates of the other two contributions are necessary.

A much better assessment of the importance of the indirect CP violation in \( K_L \rightarrow \pi^0 e^+ e^- \) will become possible after a measurement of \( Br(K_S \rightarrow \pi^0 e^+ e^-) \). Bounding the latter branching ratio below \( 1 \cdot 10^{-9} \) or \( 1 \cdot 10^{-10} \) would bound the indirect CP contribution below \( 3 \cdot 10^{-12} \) and \( 3 \cdot 10^{-13} \) respectively. The present bounds: \( 1.1 \cdot 10^{-6} \) (NA31) and \( 3.9 \cdot 10^{-7} \) (E621) are still too weak. On the other hand KLOE at DAΦNE could make an important contribution here.

The present experimental bounds

\[
Br(K_L \rightarrow \pi^0 e^+ e^-) \leq \begin{cases} 
4.3 \cdot 10^{-9} \quad [79] \\
5.5 \cdot 10^{-9} \quad [80] 
\end{cases}
\]

are still by three orders of magnitude away from the theoretical expectations in the Standard Model. Yet the prospects of getting the required sensitivity of order \( 10^{-11} - 10^{-12} \) by 1999 are encouraging \([81]\). More details on this interesting decay can be found in the original papers and in the talk by Pich.
5.2 $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are the theoretically cleanest decays in the field of rare K-decays. $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is dominated by short distance loop diagrams (Z-penguins and box diagrams) involving the top quark. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ receives additional sizable contributions from internal charm exchanges. The great virtue of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is that it proceeds almost exclusively through direct CP violation \[82\] and as such is the cleanest decay to measure this important phenomenon. It also offers a clean determination of the Wolfenstein parameter $\eta$ and in particular as we will stress in section 6 offers the clearest measurement of $\text{Im} \lambda_t = \text{Im} V_{ts}^* V_{td}$ which governs all CP violating K-decays. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is CP conserving and offers a clean determination of $|V_{td}|$. Due to the presence of the charm contribution and the related $m_c$ dependence it has a small scale uncertainty absent in $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

The next-to-leading QCD corrections to both decays have been calculated in a series of papers by Buchalla and myself \[83\]. These calculations considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expressions \[84\], in particular in the charm contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Since the relevant hadronic matrix elements of the weak currents entering $K \rightarrow \pi \nu \bar{\nu}$ can be related using isospin symmetry to the leading decay $K^+ \rightarrow \pi^0 e^+ \nu$, the resulting theoretical expressions for $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are only functions of the CKM parameters, the QCD scale $\Lambda_{\text{MS}}$ and the quark masses $m_t$ and $m_c$. The isospin braking corrections calculated in \[83\] reduce the $K^+$ and $K_L$ branching ratios by 10% and 5.6% respectively. The long distance contributions to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ have been considered in \[86\] and found to be very small: a few percent of the charm contribution to the amplitude at most, which is safely negligible. The long distance contributions to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are negligible as well.

The explicit expressions for $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can be found in \[3\]. Here we give approximate expressions in order to exhibit various dependences:

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 0.7 \cdot 10^{-10} \left[ \left| \frac{V_{td}}{0.010} \right|^2 \left| \frac{V_{cb}}{0.040} \right|^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{2.3} + cc + tc \right]$$  (38)

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.8 \cdot 10^{-11} \left[ \frac{\eta}{0.37} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{2.3} \left| \frac{V_{cb}}{0.040} \right|^4$$  (39)

where in (38) we have shown explicitly only the pure top contribution.

The impact of NLO calculations is best illustrated by giving the scale uncertainties in the leading order and after the inclusion of the next-to-leading corrections:

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.00 \pm 0.22) \cdot 10^{-10} \Rightarrow (1.00 \pm 0.07) \cdot 10^{-10}$$  (40)

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.00 \pm 0.30) \cdot 10^{-11} \Rightarrow (3.00 \pm 0.04) \cdot 10^{-11}$$  (41)

The reduction of the scale uncertainties is truly impressive. The reduction of the scale uncertainty in $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ corresponds to the reduction in the uncertainty in the determination of $|V_{td}|$ from ±14% to ±4%.
Scanning the input parameters of table 1 one finds [12]:

\[
Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (9.1 \pm 3.2) \cdot 10^{-11}, \quad Br(K_L \rightarrow \pi^0\nu\bar{\nu}) = (2.8 \pm 1.7) \cdot 10^{-11}
\]  

where the errors come dominantly from the uncertainties in the CKM parameters.

The present experimental bound on \( Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) \) is 2.4 \cdot 10^{-9} [87]. A new bound 2 \cdot 10^{-10} for this decay is expected from E787 at AGS in Brookhaven in 1997. In view of the clean character of this decay a measurement of its branching ratio at this level would signal the presence of physics beyond the standard model. Further experimental improvements for this branching ratio are discussed by Littenberg in these proceedings and in [88]. The present upper bound on \( Br(K_L \rightarrow \pi^0\nu\bar{\nu}) \) from FNAL experiment E731 [89] is 5.8 \cdot 10^{-5}. FNAL-E799 expects to reach the accuracy \( O(10^{-8}) \) and a very interesting new proposal AGS2000 [90] expects to reach the single event sensitivity 2 \cdot 10^{-12} allowing a 10% measurement of the expected branching ratio. It is hoped that also JNAF(CEBAF), KAMI and KEK will make efforts to measure this gold-plated decay. Such measurements will also put constraints on the physics beyond the standard model [91]. We will return to both decays in section 6.

5.3 \( K_L \rightarrow \mu\bar{\mu} \)

The rare decay \( K \rightarrow \mu\bar{\mu} \) is CP conserving and in addition to its short-distance part, given by Z-penguins and box diagrams, receives important contributions from the two-photon intermediate state, which are difficult to calculate reliably [12, 13, 14, 15, 16] as discussed by Eeg at this workshop.

This latter fact is rather unfortunate because the short-distance part is, similarly to \( K \rightarrow \pi\nu\bar{\nu} \), free of hadronic uncertainties and if extracted from the existing data would give a useful determination of the Wolfenstein parameter \( \rho \). The separation of the short-distance piece from the long-distance piece in the measured rate is very difficult however.

The analysis of the short distance part proceeds in essentially the same manner as for \( K \rightarrow \pi\nu\bar{\nu} \). The only difference enters through the lepton line in the box contribution which makes the \( m_t \) dependence stronger and \( m_c \) contribution smaller. The next-to-leading QCD corrections to this decay have been calculated in [83]. This calculation reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expressions from ±24% to ±10%. The remaining scale uncertainty which is larger than in \( K^+ \rightarrow \pi^+\nu\bar{\nu} \) is related to a particular feature of the perturbative expansion in this decay [83]. An approximate expression for the short distance part is given as follows:

\[
Br(K_L \rightarrow \mu\bar{\mu})_{SD} = 0.9 \cdot 10^{-9} (1.2 - \bar{\theta})^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{3.1} \left[ \frac{|V_{cb}|}{0.040} \right]^4
\]

In the absence of charm contribution, "1.2" in the first parenthesis would be replaced by "1.0".

Scanning the input parameters of table 1 we find:

\[
Br(K_L \rightarrow \mu\bar{\mu})_{SD} = (1.3 \pm 0.6) \cdot 10^{-9}
\]

where the error comes dominantly from the uncertainties in the CKM parameters.
Now the full branching ratio can be written generally as follows:

\[ Br(K_L \to \mu \bar{\mu}) = |\text{Re} A|^2 + |\text{Im} A|^2 \quad \text{Re} A = A_{SD} + A_{LD} \]  

(45)

with \text{Re} A and \text{Im} A denoting the dispersive and absorptive contributions respectively. The absorptive contribution can be calculated using the data for \( K_L \to \gamma\gamma \) and is known under the name of the unitarity bound [97]. One finds \((6.81 \pm 0.32) \cdot 10^{-9}\) which is very close to the experimental measurements

\[ Br(K_L \to \mu \bar{\mu}) = \begin{cases} \quad (6.86 \pm 0.37) \cdot 10^{-9} \quad \text{(BNL791)} \quad [98] \\ \quad (7.9 \pm 0.6 \pm 0.3) \cdot 10^{-9} \quad \text{(KEK137)} \quad [99] \end{cases} \]

(46)

which give the world average:

\[ Br(K_L \to \mu \bar{\mu}) = (7.1 \pm 0.3) \cdot 10^{-9} \]  

(47)

The accuracy of this result is impressive (±4%). It will be reduced to (±1%) at BNL in the next years.

The BNL791 group using their data and the unitarity bound extracts \(|\text{Re} A|^2 \leq 0.6 \cdot 10^{-9}\) at 90% C.L. This is a bit lower than the short distance prediction in (14). Unfortunately in order to use this result for the determination of \( \rho \) the long distance dispersive part \( A_{LD} \) resulting from the intermediate off-shell two photon states should be known. The present estimates of \( A_{LD} \) are too uncertain to obtain a useful information on \( \rho \). It is believed that the measurement of \( Br(K_L \to e \bar{e} \mu \bar{\mu}) \) should help in estimating this part. The present result \((2.9 + 6.7 - 2.4) \cdot 10^{-9}\) from E799 should therefore be improved.

More details on this decay can be found in [98, 83, 81, 95] and in the talk by Eeg at this workshop. More promising from theoretical point of view is the parity-violating asymmetry in \( K^+ \to \pi^+ \mu^+ \mu^- \) [100, 101]. Finally as stressed by Pich at this workshop, the longitudinal polarization in this decay is rather sensitive to contributions beyond the standard model [102].

### 5.4 Classification

It is probably a good idea to end this section by grouping various decays and quantities into four distinct classes with respect to theoretical uncertainties. I include in this classification also B-decays and in particular CP asymmetries in B decays which I will briefly discuss in the following section.

#### 5.4.1 Gold-Plated Class

These are the decays with essentially no theoretical uncertainties:

- CP asymmetries in \( B_d \to \psi K_S \) and \( B_s \to \psi \phi \) which measure the angle \( \beta \) and the parameter \( \eta \) respectively,

- The ratio \( Br(B \to X_d \nu \bar{\nu})/Br(B \to X_s \nu \bar{\nu}) \) which offers the cleanest direct determination of the ratio \(|V_{td}/V_{ts}|\),
- Rare K-decays $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ which offer very clean determinations of $\text{Im} \lambda_t(\eta)$ and $|V_{td}|$ respectively.

5.4.2 Class 1

- CP asymmetry in $B^0 \to \pi^+ \pi^-$ relevant for the angle $\alpha$ and the CP asymmetries in $B^\pm \to D_{CP} K^\pm$, $B_s \to D_s K$ and $B^0 \to \bar{D}^0 K^*$ all relevant for the angle $\gamma$. These CP asymmetries require additional strategies in order to determine these angles without hadronic uncertainties.

- Ratios $\text{Br}(B_d \to l\bar{l})/\text{Br}(B_s \to l\bar{l})$ and $(\Delta M)_d/(\Delta M)_s$ which give good measurements of $|V_{td}/V_{ts}|$ provided the SU(3) breaking effects in the ratios $F_{B_d}/F_{B_s}$ and $\sqrt{B_d}F_{B_d}/\sqrt{B_s}F_{B_s}$ can be brought under control.

5.4.3 Class 2

Here I group quantities or decays with presently moderate or substantial theoretical uncertainties which should be considerably reduced in the next five years. In particular I assume that the uncertainties in $B_K$ and $\sqrt{B} F_B$ will be reduced below 10%.

- $B \to X_{s,d}\gamma$, $B \to X_{s,d} e^+e^-$, $B \to K^*(\rho) e^+e^-$

- $(\Delta M)_d$, $(\Delta M)_s$, $|V_{cb}|_{\text{incl}}$, $|V_{tb}|_{\text{incl}}$, $|V_{ub}/V_{cb}|_{\text{incl}}$

- Some CP asymmetries in B-decays reviewed for instance in [103]

- $\varepsilon_K$ and $K_L \to \pi^0 e^+e^-$

5.4.4 Class 3

Here we have a list of important decays with large theoretical uncertainties which can only be removed by a dramatic progress in non-perturbative techniques:

- CP asymmetries in most $B^\pm$-decays

- $B_d \to K^*\gamma$, Non-leptonic B-decays, $|V_{ub}/V_{cb}|_{\text{excl}}$

- $\varepsilon'/\varepsilon$, $K \to \pi\pi$, $\Delta M(K_L - K_s)$, $K_L \to \mu\bar{\mu}$, hyperon decays and so on.

It should be stressed that even in the presence of theoretical uncertainties a measurement of a non-vanishing ratio $\varepsilon'/\varepsilon$ or a non-vanishing CP asymmetry in charged B-decays would signal direct CP violation excluding superweak scenarios [108]. This is not guaranteed by several clean decays of the gold-plated class or class 1 [104] except for $B^\pm \to D_{CP} K^\pm$. 

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6 CP-B Asymmetries versus $K \to \pi \nu \bar{\nu}$

6.1 CP-Asymmetries in B-Decays

CP violation in B decays is certainly one of the most important targets of B factories and of dedicated B experiments at hadron facilities. It is well known that CP violating effects are expected to occur in a large number of channels at a level attainable at forthcoming experiments. Moreover there exist channels which offer the determination of CKM phases essentially without any hadronic uncertainties. Since CP violation in B decays has been already reviewed in two special talks by Nir and Nakada at this workshop and since in addition extensive reviews can be found in the literature [103, 104, 106], let me concentrate only on the most important points.

The CP-asymmetry in the decay $B_0^d \to \psi K_S$ allows in the standard model a direct measurement of the angle $\beta$ in the unitarity triangle without any theoretical uncertainties [107]. Similarly the CP asymmetry in the decay $B_0^d \to \pi^+ \pi^-$ gives the angle $\alpha$, although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [108].

We have then for the time-dependent asymmetries

$$A_{CP}(\psi K_S, t) = -\sin((\Delta M)_d t) \sin(2\beta) \quad A_{CP}(\pi^+ \pi^-, t) = -\sin((\Delta M)_d t) \sin(2\alpha + \theta_P) \quad (48)$$

where $\theta_P$ represents the "QCD penguin pollution" which has to be taken care of in order to extract $\alpha$. The most popular strategy is the isospin analysis of Gronau and London [108]. It requires however the measurement of $Br(B^0 \to \pi^0 \pi^0)$ which is expected to be below $10^{-6}$: a very difficult experimental task. For this reason other strategies avoiding this channel or estimating the size of the penguin contribution have been proposed. Since this is the K-physics workshop I will not review them here and refer to recent reviews [103, 106] where various alternative strategies for the determination of $\alpha$ and the issues of the determination of the angle $\gamma$ in the decays of class 1 [110, 111, 112] are discussed.

In what follows let us assume that the problems with the determination of $\alpha$ will be solved somehow. Since in the usual unitarity triangle one side is known, it suffices to measure two angles to determine the triangle completely. This means that the measurements of $\sin 2\alpha$ and $\sin 2\beta$ can determine the parameters $\rho$ and $\eta$. As the standard analysis of the unitarity triangle of section 3 shows, $\sin(2\beta)$ is expected to be large: $\sin(2\beta) = 0.58 \pm 0.22$ implying the integrated asymmetry $A_{CP}(\psi K_S)$ as high as $(30 \pm 10)\%$. The prediction for $\sin(2\alpha)$ is very uncertain on the other hand $(0.1 \pm 0.9)$ and even a rough measurement of $\alpha$ would have a considerable impact on our knowledge of the unitarity triangle as stressed in [4] and recently in [113].

6.2 UT from CP-B and $K \to \pi \nu \bar{\nu}$

Let us then compare the potentials of the CP asymmetries in determining the parameters of the standard model with those of the cleanest rare K-decays: $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ [113]. Measuring $\sin 2\alpha$ and $\sin 2\beta$ from CP asymmetries in $B$ decays allows, in principle, to
fix the parameters $\bar{\eta}$ and $\bar{\rho}$, which can be expressed as [114]

\[
\bar{\eta} = \frac{r_-(\sin 2\alpha) + r_+(\sin 2\beta)}{1 + r_+^2(\sin 2\beta)} \quad \bar{\rho} = 1 - \bar{\eta}r_+(\sin 2\beta)
\]

where $r_{\pm}(z) = (1 \pm \sqrt{1 - z^2})/z$. In general the calculation of $\bar{\rho}$ and $\bar{\eta}$ from $\sin 2\alpha$ and $\sin 2\beta$ involves discrete ambiguities. As described in [114] they can be resolved by using further information, e.g. bounds on $|V_{ub}/V_{cb}|$, so that eventually the solution (49) is singled out.

Alternatively, $\bar{\rho}$ and $\bar{\eta}$ may also be determined from $K_L \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ alone [115, 116]. An interesting feature of this possibility is in particular that the extraction of $\sin 2\beta$ from these two modes is essentially independent of $m_t$ and $V_{cb}$ [116]. This fact enables a rather accurate determination of $\sin 2\beta$ from $K \to \pi \nu \bar{\nu}$.

A comparison of both strategies is displayed in Table 3, where the following input has been used

\[
|V_{cb}| = 0.040 \pm 0.002 \quad m_t = (170 \pm 3) GeV
\]

\[
B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11} \quad B(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}
\]

The measurements of CP asymmetries in $B_d \to \pi \pi$ and $B_d \to J/\psi K_S$, expressed in terms of $\sin 2\alpha$ and $\sin 2\beta$, are taken to be

\[
\sin 2\alpha = 0.40 \pm 0.10 \quad \sin 2\beta = 0.70 \pm 0.06 \quad \text{(scenario I)}
\]

\[
\sin 2\alpha = 0.40 \pm 0.04 \quad \sin 2\beta = 0.70 \pm 0.02 \quad \text{(scenario II)}
\]

Scenario I corresponds to the accuracy being aimed for at $B$-factories and HERA-B prior to the LHC era. An improved precision can be anticipated from LHC experiments, which we illustrate with our choice of scenario II.

As can be seen in Table 3, the CKM determination using $K \to \pi \nu \bar{\nu}$ is competitive with the one based on CP violation in $B$ decays, except for $\bar{\rho}$ which is less constrained by the rare kaon processes. On the other hand Im$\lambda_t$ is better determined in the kaon scenario. It can be obtained from $K_L \to \pi^0 \nu \bar{\nu}$ alone and does not require knowledge of $V_{cb}$, which enters Im$\lambda_t$ when derived from $\sin 2\alpha$ and $\sin 2\beta$. This analysis suggests that $K_L \to \pi^0 \nu \bar{\nu}$ should eventually yield the most accurate value of Im$\lambda_t$. This would be an important result since Im$\lambda_t$ plays a central role in the phenomenology of CP violation in $K$ decays and is furthermore equivalent to the Jarlskog parameter $J_{CP}$ [117], the invariant measure of CP violation in the Standard Model,

\[
J_{CP} = \lambda(1 - \lambda^2/2)\text{Im}\lambda_t.
\]

There is another virtue of the comparison of the determinations of various parameters using CP-B asymmetries with the determinations in very clean decays $K \to \pi \nu \bar{\nu}$. Any substantial deviations from these two determinations would signal new physics beyond the standard model.

On the other hand unprecedented precision for all basic CKM parameters could be achieved by combining the cleanest K and B decays [114]. While $\lambda$ is obtained as usual from $K \to \pi e \nu$, $\bar{\rho}$ and $\bar{\eta}$ could be determined from $\sin 2\alpha$ and $\sin 2\beta$ as measured in CP violating asymmetries in $B$ decays. Given $\eta$, one could take advantage of the very clean nature of $K_L \to \pi^0 \nu \bar{\nu}$ to extract
Table 3: Illustrative example of the determination of CKM parameters from $K \to \pi \nu \bar{\nu}$ and from CP violating asymmetries in $B$ decays. The relevant input is as described in the text. Shown in brackets are the errors one obtains using $V_{cb} = 0.040 \pm 0.001$ instead of $V_{cb} = 0.040 \pm 0.002$.

|               | $K \to \pi \nu \bar{\nu}$ | $B \to \pi \pi, J/\psi K_S$ (I) | $B \to \pi \pi, J/\psi K_S$ (II) |
|---------------|--------------------------|-------------------------------|-------------------------------|
| $|V_{ud}|/10^{-3}$ | $10.3 \pm 1.1(\pm 0.9)$  | $8.8 \pm 0.5(\pm 0.3)$     | $8.8 \pm 0.5(\pm 0.2)$     |
| $|V_{ub}/V_{cb}|$ | $0.089 \pm 0.017(\pm 0.011)$ | $0.087 \pm 0.009(\pm 0.009)$ | $0.087 \pm 0.003(\pm 0.003)$ |
| $\bar{\theta}$ | $-0.10 \pm 0.16(\pm 0.12)$ | $0.07 \pm 0.03(\pm 0.03)$   | $0.07 \pm 0.01(\pm 0.01)$   |
| $\bar{\eta}$   | $0.38 \pm 0.04(\pm 0.03)$  | $0.38 \pm 0.04(\pm 0.04)$    | $0.38 \pm 0.01(\pm 0.01)$    |
| $\sin 2\beta$  | $0.62 \pm 0.05(\pm 0.05)$  | $0.70 \pm 0.06(\pm 0.06)$    | $0.70 \pm 0.02(\pm 0.02)$    |
| $\text{Im} \lambda_t/10^{-4}$ | $1.37 \pm 0.07(\pm 0.07)$ | $1.37 \pm 0.19(\pm 0.15)$ | $1.37 \pm 0.14(\pm 0.08)$ |

A or, equivalently $|V_{cb}|$. This determination benefits further from the very weak dependence of $|V_{cb}|$ on the $K_L \to \pi^0 \nu \bar{\nu}$ branching ratio, which is only with a power of 0.25. Moderate accuracy in $B(K_L \to \pi^0 \nu \bar{\nu})$ would thus still give a high precision in $|V_{cb}|$. As an example we take $\sin 2\alpha = 0.40 \pm 0.04$, $\sin 2\beta = 0.70 \pm 0.02$ and $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$, $m_t = (170 \pm 3) GeV$. This yields

$$\bar{\theta} = 0.07 \pm 0.01 \quad \bar{\eta} = 0.38 \pm 0.01 \quad |V_{cb}| = 0.0400 \pm 0.0013 \quad (54)$$

which would be a truly remarkable result. Again the comparison of this determination of $|V_{cb}|$ with the usual one in tree level B-decays would offer an excellent test of the standard model and in the case of discrepancy would signal physics beyond the standard model.

7 A Look beyond the Standard Model

In this review we have concentrated on rare decays and CP violation in the standard model. The structure of rare decays and of CP violation in extensions of the standard model may deviate from this picture. Consequently the situation in this field could turn out to be very different from the one presented here. It is appropriate then to end this review with a few remarks on the physics beyond the standard model. Much more elaborate discussion can be found in the talk of Nir presented at this workshop and in [118, 119].

7.1 Impact of New Physics

There is essentially no impact on $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ determined in tree level decays. This is certainly the case for the first two elements. In view of the smallness of $|V_{ub}|$ a small impact from the loop contributions (sensitive to new physics) to leading decays could in principle be present. However in view of many theoretical uncertainties in the determination of this element such contributions can be safely neglected at present.
There is in principle a substantial impact of new physics on the determination of \( \varrho, \eta, |V_{td}|, \text{Im}\lambda_t \) and generally on the unitarity triangle through the loop induced decays which can receive new contributions from internal chargino, charged Higgs, stops, gluinos and other exotic exchanges. If the quark mixing matrix has the CKM structure, the element \( |V_{ts}| \) on the other hand will be only slightly affected by these new contributions. Indeed from the unitarity of the CKM matrix \( |V_{ts}|/|V_{tb}| = 1 - \mathcal{O}(\lambda^2) \) and the new contributions could only affect the size of the \( \mathcal{O}(\lambda^2) \) terms which amounts to a few percent at most. This situation makes the study of new physics in rare B decays governed by \( |V_{ts}| \) somewhat easier than in rare K-decays and B decays which are governed by \( |V_{td}| \). Indeed in the latter decays the impact of new physics is felt both in the CKM couplings and in the \( m_t \) dependent functions, which one has to disentangle, whereas in the former decays mainly the impact of new physics on the \( m_t \) dependent functions is felt.

Similarly if no new phases in the quark mixing are present, the formulae for CP asymmetries in B-decays remain unchanged and these asymmetries measure again the phases of the CKM matrix as in the standard model. Thus even if there is some new physics in the loop diagrams we will not see it in the clean asymmetries directly if there are no new phases in the quark mixing matrix. In order to search for new physics the comparision of the values of CKM phases determined from CP asymmetries and from loop induced decays is then mandatory.

The situation becomes more involved if the quark mixing involves more angles and new phases and in addition there are new parameters in the Higgs, SUSY and generally new physics sector. For instance in such a case the "gold-plated" asymmetry in \( B \to \psi K_S \) would take the form [118]:

\[
A_{CP}(\psi K_S, t) = -\sin((\Delta M)_d t) \sin(2\beta + \theta_{NEW})
\] (55)

implying that not \( 2\beta \) but \( 2\beta + \theta_{NEW} \) is measured by the asymmetry.

This short discussion makes it clear that in order to search effectively for new physics it is essential to measure and calculate as many processes and compare the resulting CKM parameters with each other. Graphically this corresponds simply to figure 2. In this enterprise the crucial role will be played by very clean decays of the "gold-plated" class and of classes 1 and possibly 2 in which the new physics will not be hidden by theoretical uncertainties present in the decays of class 3.

### 7.2 Signals of New Physics

New Physics will be signaled in principle in various ways. Here are some obvious examples:

- Standard model predictions for various branching ratios and CP asymmetries will disagree with data,
- \((\varrho, \eta)\) determined in K-physics will disagree with \((\varrho, \eta)\) determined in B-physics,
- \((\varrho, \eta)\) determined in loop induced decays will disagree with \((\varrho, \eta)\) determined through CP asymmetries,
Forbidden and very rare decays will occur at unexpected level: $K_L \to \mu e$, $K \to \pi \mu e$, $d_N$, $d_e$, $D^0 - \bar{D}^0$ mixing, CP violation in D-decays \cite{120} etc.,

Unitarity Triangle will not close.

7.3 General Messages

Let us end this discussion with some general messages on New Physics which have been stressed in particular by Nir at this workshop.

As discussed by Gavela at this workshop, baryogenesis suggests that there is CP violation outside the Standard Model. The single CKM phase simply does not give enough CP Violation for the required baryon asymmetry \cite{121}. It is however not unlikely that large new sources of CP violation necessary for baryogenesis could be present at the electroweak scale \cite{122}. They are present for instance in general SUSY models and in multi-Higgs models.

It should be stressed that baryogenesis and the required additional CP violation being flavour diagonal may have direct impact on the electric dipole moments but have no direct impact on FCNC processes. However new physics required for baryon asymmetry could bring new phases relevant for FCNC.

Concentrating on SUSY for a moment, more general and natural SUSY models give typically very large CP violating effects \cite{123} and FCNC transitions \cite{124} which are inconsistent with the experimental values of $\varepsilon_K$, $K_L - K_S$ mass difference and the bound on the electric dipole moment of the neutron. In order to avoid such problems, special forms of squark mass matrices \cite{125} and fine tuning of phases are necessary. In addition one frequently assumes that CP violation and FCNC are absent at tree level. In the limiting case one ends with a special version of the MSSM in which to a good approximation CP violation and FCNC processes are governed by the CKM matrix and the new effects are dominantly described by loop diagrams with internal stop, charginos and charged higgs exchanges \cite{126}. It is then not surprising that in the quark sector new effects in MSSM compared with SM predictions for FCNC transitions are rather moderate, although for a particular choice of parameters and certain quantities still enhancements (or suppressions) by factors 2-3 cannot be excluded \cite{127}. Larger effects are expected in the lepton sector and in electric dipole moments. Similar comments about the size of new effects apply to multi-higgs models and left-right symmetric models.

Large effects are still possible in models with tree level FCNC transitions, leptoquarks, models with horizontal gauge symmetries, technicolour and top-colour models \cite{118,119}. Unfortunately these models contain many free parameters and at present the only thing one can do is to bound numerous new couplings and draw numerous curves which from my point of view is not very exciting.

On the other hand it is to be expected that clearest signals of new physics may come precisely from very exotic physics which would cause the decays $K_L \to \mu e$, $K \to \pi \mu e$, T-violating $\mu$-polarization in $K^+ \to \pi^0 \mu^+ \nu$ to occur. Also sizable values of $d_N$, $d_e$, $D^0 - \bar{D}^0$ mixing and of CP violation in D-decays and top decays are very interesting in this respect.

It should however be stressed once more that theoretically cleanest decays belonging to the
top classes of section 5 will certainly play important roles in the search for new physics and possibly will offer its first signals.

8 Outlook

There is clearly an exciting time ahead of us!

9 Final Remarks

This was a very enjoyable workshop brilliantly organized by the team lead by Lydia and Louis Fayard. In particular the interactions between theorists and experimentalists were very fruitful. Even if the first and the last word have been given to experimentalists (Rene Turlay, Bruce Winstein), the first and the last chairmanship have been given to theorists (Eduardo de Rafael, Fred Gilman). I do hope very much that these fruitful interactions between experimentalists and theorists will continue so that in the year 1999, when hopefully Lydia and Louis will organize another K-physics workshop in Orsay, we will all agree that $\varepsilon'/\varepsilon \neq 0$ and that the main K-physics target for the next decade should be the measurement of $K_L \to \pi^0\nu\bar{\nu}$.

Acknowledgements: I would like to thank Markus Lautenbacher for help in producing the figures and him, Gerhard Buchalla and Matthias Jamin for wonderful collaboration. This work has been supported by the German Bundesministerium für Bildung and Forschung under contract 06 TM 743 and DFG Project Li 519/2-1.

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