On the use of black hole binaries as probes of local dark energy properties

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Accretion of dark energy onto black holes will take place when dark energy is not a cosmological constant. It has been proposed that the time evolution of the mass of the black holes in binary systems due to dark energy accretion could be detectable by gravitational radiation. This would make it possible to use observations of black hole binaries to measure local dark energy properties, e.g., to determine the sign of $1 + w$ where $w$ is the dark energy equation of state. In this Letter we show that such measurements are unfeasible due to the low accretion rates.

1. INTRODUCTION

Experimental evidence shows that the universe currently undergoes an accelerated expansion driven by an unknown form of energy, dubbed dark energy. This dominant energy component can – at least phenomenologically – be described as a perfect fluid with density $\rho_{de}$ and a (possibly time dependent) equation of state $w \equiv p_{de} / (\rho_{de} c^2)$, where $p_{de}$ is the dark energy pressure. Current measurements are consistent with the dark energy density being approximately 70% of the critical density of the universe, $\rho_{de} \sim 10^{-29} \text{g cm}^{-3}$, and the equation of state being close to $w = -1$, i.e., the value corresponding to a cosmological constant. Since the density is very low and we do not expect dark energy to cluster to any large extent, all the information we have on dark energy is through its impact on the largest scales of the universe via the expansion rate of the universe. One of the foremost experimental and theoretical tasks in cosmology is to pin down the equation of state for dark energy, specifically to constrain or detect any deviations from the cosmological constant value of $w = -1$.

It has been proposed that dark energy can be detected also on smaller scales [1,2]. Any such local measurements would constitute a powerful confirmation of the existence of dark energy. In Mersini-Houghton and Kelleher [1], the possibility to measure local properties of dark energy using gravitational radiation from binary systems of supermassive black holes is investigated. The basic idea is that accretion of dark energy by black holes will lead to a mass change when dark energy is not a cosmological constant. The mass accretion would affect the dynamics of the system and in turn alter the waveform of the gravitational radiation produced by the binaries during the inspiraling phase. Measuring this waveform, through LIGO or LISA, would then make it possible to constrain the values of $w$ and $\rho_{de}$. Alternatively, the change in orbital period of the system could be measured using electromagnetic radiation, if present. In this Letter we claim that this effect is negligible and that this form of local measurement of $\rho_{de}$ and $w$ is not possible.

In Section 2 of the Letter, we compare the mass accretion due to the dark energy with that of the interstellar medium and dark matter. In Section 3 we look at the radial separation needed for dark energy accretion to dominate over the effect over gravitational radiation in a binary system of two supermassive black holes. In both cases we show that dark energy accretion cannot be considered a measurable effect, even for a binary system. In Section 4 we consider two concrete examples to validate this conclusion.

2. MASS ACCRETION

The formula for a spherically symmetric, adiabatic, steady-flow gas accretion onto a star at rest with respect to the gas was developed by Bondi in 1952 [3]. Since the properties of the mass accretion rate is determined by the gravitational field far away from the accreting object, it can be used to estimate the accretion rate for a black hole with respect to the interstellar medium. For a medium with mean free path length smaller than the accretion radius, this can be expressed as [4]

$$\dot{m} \cong 1.4 \times 10^{11} \left( \frac{m}{m_\odot} \right)^2 \times \left( \frac{\rho(\infty)}{10^{-24} \text{ g cm}^{-3}} \right) \left( \frac{c_s(\infty)}{10 \text{ km s}^{-1}} \right)^{-3} \text{ g s}^{-1} . \quad (1)$$

Here $\rho(\infty)$ is the density of the surrounding medium at infinity and $c_s(\infty)$ is the sound speed of the medium at infinity. For a supermassive black hole with mass $10^8 m_\odot$, this leads to an accretion rate of order $\dot{m} \sim 10^{27} \text{ g s}^{-1} \sim 20 m_\odot \text{ yr}^{-1}$ in the case of a constant gas supply.

In the case of an object moving with velocity $v$ with respect to the gas, which is at rest far away from the hole, the accretion rate becomes, for $v^2 \gg c_s^2$ (see [3])

$$\dot{m} \cong 5.2 \times 10^6 \left( \frac{m}{m_\odot} \right)^2 \times \left( \frac{\rho(\infty)}{10^{-24} \text{ g cm}^{-3}} \right) \left( \frac{v}{300 \text{ km s}^{-1}} \right)^{-3} \text{ g s}^{-1} . \quad (2)$$

The reference velocity $300 \text{ km s}^{-1}$ is established from the movement of a black hole in the gas of a merging galaxy.
(see [5]). The accretion rate is thus diminished by a factor $10^9$.

For cold dark matter, the accretion rate is suppressed by a factor $[c_s(\infty)/c]^2$ since it is collisionless [6]. For dark matter densities in the galaxy core of $\rho \sim 10^{-25}$ g cm$^{-3}$ and sound speed $c_s = 100$ km s$^{-1}$ (see [7]), this yields an accretion rate of $\dot{m} \sim 10^{16}$ g s$^{-1} \sim 10^{-10} m_\odot$ yr$^{-1}$. In the case of a binary system of two supermassive black holes, we thus expect the mass accretion to be dominated by interstellar gas with a bulk flow similar to the velocities of the black holes.

We now turn to the question whether dark energy accretion is non-negligible. A relativistic formula for the accretion flow onto compact objects was developed by Michel [8]. The relativistic treatment gives corrections of order unity to the accretion flow onto a black hole [6]. A general treatment of black hole accretion of a perfect fluid was presented in Babichev et al. [9]. In particular, the dependence on the sign of the accretion of the equation of state of the fluid was investigated. For dark energy, taken to be in the form of a perfect fluid, the result is

$$\dot{m} = \frac{4\pi AG^2\rho_{de}(\infty) m^3 (1 + w)}{c^3}.$$  \hspace{1cm} (3)

The constant $A$ is determined by demanding continuity of the flow from infinity to the black hole horizon. It depends on the equation of state and is of order unity. In the rest of the Letter we use $A = 1$. $\rho_{de}(\infty)$ is the dark energy density at infinity. The equation of state determines the sign of $\dot{m}$, which lies at the heart of the proposal to do local measurements of $w$ using binary black hole systems. In passing, we note that Eq. (3) is also valid for an ultrarelativistic gas in which case $w = 1/3$. Since $\rho_{cmb} \leq 10^{-4} \rho_{de}$, accretion of cosmic microwave background photons will be negligible compared to dark energy accretion, unless the dark energy equation of state is extremely close to unity.

Putting Eq. (3) in a form similar to Eq. (1) yields

$$\dot{m} \approx 6.7 \times 10^{-8} (1 + w) \times \left(\frac{m}{m_\odot}\right)^2 \left(\frac{\rho_{de}(\infty)}{10^{-29} \text{ g cm}^{-3}}\right) \text{ g s}^{-1}. \hspace{1cm} (4)$$

For a supermassive black hole with mass $10^8 m_\odot$ and a dark energy density close to the critical energy density, the accretion rate is of the order $\dot{m} \sim 10^8$ g s$^{-1} \sim 10^{-18} m_\odot$ yr$^{-1}$. Comparing the accretion rate of dark energy with that from the interstellar medium and dark matter, we see that the possibility of using gravitational radiation observation to measure the sign of $\dot{m}$ due to dark energy accretion is effectively zero. We have not studied the possible numerical results of the investigations carried out in [2], which describes the effect of dark energy on the quasinormal modes associated with the ringdown phase of the black hole resulting from the coalescence. In principle, however, this effect is the result of the mass accretion given in Eq. (3), and as we have seen, the accretion rate is several orders of magnitude lower than that from the interstellar medium and dark matter. We therefore anticipate that the effect of dark energy accretion on the quasinormal modes will also be negligible.

Assuming that dark energy accretion is the dominating mechanism for black hole mass change, we can solve Eq. (3) to obtain

$$m(t) = \frac{m_0}{1 - \tau}, \hspace{1cm} (5)$$

where

$$\tau = 3 \times 10^{10} \left(\frac{\rho_{de}(\infty)}{10^{-29} \text{ g cm}^{-3}}\right)^{-1} \left(\frac{m_0}{m_\odot}\right)^{-1} \text{ s}. \hspace{1cm} (6)$$

For a $m = 10^8 m_\odot$ black hole, $|1 + w| \sim 0.1$ and a dark energy density close to the critical, this corresponds to $\tau = 10^{26}$ years or approximately $10^{16}$ times the age of the universe.

Although miniscule, we note that dark energy accretion will be larger than the mass loss due to Hawking radiation for black holes masses larger than $m \sim 10^{-9} m_\odot$, i.e., a tenth of the mass of the moon.

### 3. RADIAL CHANGE

In principle, mass accretion in a binary black hole system would give an imprint on the gravitational waves produced in the system. We make the assumption that the dark energy does not carry any angular momentum, and that the angular momentum transfer during dark energy accretion therefore is zero. The reason for this is twofold. First, when dark energy is not a cosmological constant, it does admit spatial and temporal variations. These variations are, however, on such large scales that it is hard to see how they would effectively transfer angular momentum to a binary system. Unless the dark energy in the vicinity of the binary system has an angular velocity orders of magnitude larger than that of the binary system, the order of magnitude of the numerical results of this Letter will not be affected. Secondly, we want to isolate the effect of the mass accretion on the radial separation, which is done under the assumption of zero angular momentum transfer. To affect the waveform, the radial change induced by the mass accretion under conservation of angular momentum would have to be of the same order of magnitude as the radial change induced from the back-reaction of gravitational radiation. In the following we will see that the radial separations needed for the two effects, dark energy accretion and gravitational radiation, to be of the same order of magnitude is on the scale of parsecs. At these separations however, both effects are negligible. In the following we take
which gives
\[ \rho_{de} \sim 10^{-29} \text{g cm}^{-3} \]
when investigating the effects of dark energy accretion. Since we are only interested in order of magnitude estimates of the effect, it does not matter if \(1 + w\) is positive or negative.

We will consider a binary system with two black holes of equal mass \(m\) during the inspiraling phase in a semi-circular orbit. For separations much larger than the Schwarzschild radii of the black holes, \(r \gg r_s = 2Gm/c^2\), we can describe a binary black hole system using Keplerian dynamics. Such a system will produce gravitational radiation that carry energy away from the system at a rate
\[ P_{gw} = \frac{64 G^4 m^5}{5 c^5 r^5}. \tag{7} \]
Equating the orbital energy loss with the power output, \(dE_{\text{orbital}}/dt = P_{gw}\), gives the rate of change of the radial separation \(r\) of the two masses:
\[ \dot{r} = -\frac{128 G^3 m^3}{5 c^5 r^3}. \tag{8} \]
To see the effect of dark energy accretion on the radial separation, we use the fact that angular momentum must be conserved in the process. Putting \(\dot{J} = 0\) and using Kepler’s law gives
\[ \dot{r} = -\frac{2\dot{m}r}{m}. \tag{9} \]
The change in orbital energy due to dark energy accretion is
\[ P_{\text{acc}} = -\frac{2G\dot{m}m}{r}. \tag{10} \]
To consider the joint effect of gravitational radiation and dark energy accretion we put
\[ \frac{dE_{\text{orbital}}}{dt} = P_{gw} + P_{\text{acc}}, \tag{11} \]
which gives
\[ \dot{r} = -\frac{128 G^3 m^3}{5 c^5 r^3} - \frac{2\dot{m}r}{m}, \tag{12} \]
that is, the total rate of change is equal to the sum of the individual contributions. Eq. (12) is valid as long as \(\dot{m} \ll m\omega\), where \(\omega\) is the angular frequency of the system given by,
\[ \omega = \left(\frac{Gm_{\text{tot}}}{r^3}\right)^{1/2}, \tag{13} \]
where \(m_{\text{tot}}\) is the total mass of the binary system. If the mass accretion rate is of the same order as \(m\omega\), we would have to change the derivation of the gravitational waveform (since the waveform depends on the second time derivative of the quadrupole moment) and thus in turn alter the formula for the energy radiated away from the system. Eq. (12) is different from the formula for radial change derived in Mersini-Houghton and Kelleher [1], the reason being that they include the rest energy in the orbital energy and does not include \(P_{\text{acc}}\) in the time derivative of the orbital energy in Eq. (11). Although we do not agree on the formula for the time derivatives of the orbital energy and the separation, these differences do not affect the main conclusions of this Letter.

Combining Eqs. (3) and (12), we see that the dark energy accretion term is proportional to \(m\) whereas the gravitational radiation term is proportional to \(m^3\). The effect of dark energy accretion would therefore have a larger impact for black holes with small masses. On the other hand, the frequency of the gravitational wave is proportional to the square root of the mass during the inspiraling phase, and in order for the gravitational radiation to be detectable with LISA or LIGO, we need a system of supermassive black holes. LISA [16] will be sensitive in the frequency range 0.03 mHz – 0.1 Hz whereas LIGO has its maximum sensitivity around 100 Hz [10].

In Fig. (1), we plot the radial change due to both terms of Eq. (12) with respect to the radial separation in the region where they overlap. The system consists of two black holes with equal masses \(m = 10^8 M_\odot\). We see that the effects are of the same order of magnitude at separations \(r \sim 10^{19} \text{m}\) with \(|\dot{r}| \sim 10^{-14} \text{m s}^{-1}\). In general, for a system of two black holes with equal masses \(m\), the separation needs to be at least \(r \sim 10^{15} (m/M_\odot)^{1/2} \text{m}\) in order for dark energy to dominate over gravitational radiation, corresponding to \(|\dot{r}| \sim 10^{-26} (m/M_\odot)^{3/2} \text{m s}^{-1}\) or \(|\dot{r}/r| = 10^{-41} (m/M_\odot) \text{s}^{-1}\). The gravitational radiation angular frequency is given by $10^{-12} (m/M_\odot)^{-1/4} \text{Hz}$; beyond the detection level of LISA or LIGO for masses $m \gtrsim 10 \text{ kg}$.

Even if future experiments could reach such an ultralow frequency bandwith, the effect of both gravitational radiation and dark energy accretion is negligible, thus also excluding electromagnetic detection of changes in the period of the system.

If dark energy accretion is the dominating effect, we can solve for radius and the angular frequency as a function of time,
\[ r(t) = r_0 \left(1 - \frac{t}{\tau}\right)^2, \tag{14}\]
and
\[ \omega(t) = \omega_0 \left(1 - \frac{t}{\tau}\right)^{-7/2}, \tag{15}\]
where again \(\tau\) is given in Eq. (6), corresponding to \(\tau \sim 10^{26}\) years for a typical system of supermassive black hole binaries.

The time to coalescence due to gravitational radiation for equal masses \(m = 10^8 M_\odot\) with separation \(r = 10^{19} \text{m}\)
is $r \sim 10^{24}$ years, also far beyond the Hubble time. Other effects, such as dynamical friction with surrounding stars and gas will undoubtedly dominate, and are needed to bring the system to a separation where gravitational radiation becomes important (roughly $10^3$ Schwarzschild radii). To bring the system down to separations of a few hundreds of a parsec using external mechanisms is known as the "final parsec problem", and dark energy accretion is clearly not a driving force in these circumstances.

4. EXAMPLES

Two concrete examples where dark energy accretion is claimed to be of importance are given in Mersini-Houghton and Kelleher [1] which we review and comment on here. The first one is the radio galaxy 0402+379 [13]. It is believed to have two supermassive black holes in its center with a separation of 7.3 parsecs ($\sim 2.2 \times 10^{17}$ m) and total mass $1.5 \times 10^8 m_{\odot}$. The orbital period is 150 000 years. The radial change due to gravitational radiation is $|\dot{r}_{gw}| \sim 10^{-9}$ m s$^{-1}$ and the change due to dark energy accretion is $|\dot{r}_{de}| \sim 10^{-16}$ m s$^{-1}$. The merging time due to gravitational radiation is $\tau \sim 10^{18}$ years. This is an example of a system that will spend the majority of its lifetime in the separation region $0.1 - 10$ parsec unless an external mechanism, such as dynamical friction, carries angular momentum from the system. In any case, unlike Mersini-Houghton and Kelleher [1], we find that dark energy accretion will be completely negligible in this system, as is evident from the smallness of $\dot{r}_{de}$.

The next example is the quasar OJ287. This object has been observed since 1891 and shows a 12 year periodic optical outburst. This is believed to be the result of the passing of a black hole of mass $m_1 \sim 10^8 m_{\odot}$ through the accretion disk which belongs to a black hole of mass $m_2 \sim 10^{10} m_{\odot}$ [14]. The system is believed to coalesce in $\tau \sim 10^5$ years. To model the system correctly, post-Newtonian approximation schemes must be applied. This means that the system can be used for precise testing of general relativity and the effect of gravitational radiation [15]. The system is described by a post-Newtonian Keplerian orbit with eccentricity $e = 0.66$. Thus, we cannot use Eq. (12). To obtain an order of magnitude comparison between the effect of gravitational radiation and dark energy accretion, we idealize the system as being in a circular orbit with equal masses $10^9 m_{\odot}$ (as is done in Mersini-Houghton and Kelleher [1]). If we take the semi-major axis as the radius of the orbiting black hole, we have $r \sim 10^{15}$ m. This result in a radial change of $|\dot{r}_{gw}| \sim 10$ m s$^{-1}$ and $|\dot{r}_{de}| \sim 10^{-18}$ m s$^{-1}$. Again, we find that the effect from gravitational radiation is the dominating effect, contrary to the claims in Mersini-Houghton and Kelleher [1].

5. CONCLUSIONS

We have investigated the effect of dark energy accretion in binary supermassive black hole systems. When comparing the mass change due to accretion from the interstellar medium, dark matter and dark energy, we find that the effect of dark energy accretion for all practical purposes is negligible. We also compare the effects from dark energy accretion and the loss of energy through gravitational wave emission. At the separations between the binary constituents needed for dark energy accretion to dominate over gravitational radiation, and under the assumption that the rotation of dark energy in the vicinity of the binary system is not greatly exceeding that of the binary system, we find the radial change induced by the accretion to be too small to be observable. Thus, even in an idealized setting with no gas accretion, the effect of dark energy accretion during the inspiraling phase does not have a measurable impact on the dynamics of the system. Doing local measurements of the equation of state of dark energy, as proposed in Mersini-Houghton and Kelleher [1], can therefore unfortunately not be considered possible.

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