A dynamic study of a complex gear transmission

E Merticaru¹, V Merticaru jr²
¹Faculty of Mechanical Engineering, Gheorghe Asachi Technical University of Iaşi
²Faculty of Machine Manufacturing and Industrial Management, Gheorghe Asachi Technical University of Iaşi

E-mail: emertica@yahoo.com

Abstract. This paper presents a dynamic study of a complex gear transmission. The kinematics is presented. The parameters of dynamic model are presented. The differential equation of kinetic energy is written. Some discussions are made regarding the work of the mechanism. The aspects discussed in the paper are useful to those who design such mechanisms.

1. Introduction

Gear dynamics is still a topical field of study although it is an old engineering issue. Many authors have dealt with this problem along time. Meshing stiffness, bearing variable compliance, friction, unbalanced mass of the rotor, tooth wear were taken into account using various dynamic models [5-15]. However, the gear transmissions taken into account were very simple and not so complex as that one presented in this paper. The dynamic study presented in this paper takes into account a more complex gear transmission. The overall dynamic parameters of the gear transmission are taken into account. Elasticity of kinematic elements, meshing stiffness and friction are neglected.

2. Kinematics

The complex gear transmission which is taken into account for study it is presented in figure 1. This kind of gear is used, for example, for manufacturing cylindrical toothed wheels [1,2,4]. But this is not the only example of use of this kind of gear transmission.

The complex gear presented in figure 1 is driven by an electric motor which is acting the wheel $z_1$. Further, the flow of power is divided through gears $z_2$ and $z_4$. Then, the flow of power is divided, once again, through wheels $z'_3$ and $z_{10}$. At the wheel $z_9$ we get an output motion as a result of linear combination of angular speeds of wheels $z'_6$ and $z'_7$, and the angular speeds of wheels $z_9$ and $z_{11}$ are also correlated. The gear in figure 1 has 1 degree of mobility. In figure 1, $M_{m}$ is the motor torque and $M_{t1}$ and $M_{t2}$ are technological torques acting on wheels $z_9$ and, respectively $z_{11}$. Also, the $z_i$ represents the number of teeth of wheel "i", $\omega_i$ represents the angular speed of wheel "i". Also we suppose to know the inertia moments $J_i$ of kinematic elements "i" and the mass $m_8$ of satellite wheel $z_8$, and the radius of disposal $r_{67}$ of the satellite wheel $z_8$ (shown in figure 1). Some theoretical aspects regarding kinematics and dynamics of this kind of gear transmission are studied.

For the kinematic chain in figure 1, formed by wheels $z_1$, $z_2$, $z'_2$, $z_3$, $z_{10}$ and $z_{11}$, the transmission ratios are:

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$  \hspace{1cm} (1)

$$i_{23} = \frac{\omega_2}{\omega_3} = \frac{z_3}{z_{12}}$$  \hspace{1cm} (2)

$$i_{10-11} = \frac{\omega_3}{\omega_{11}} = \frac{z_{11}}{z_{10}}$$  \hspace{1cm} (3)
For the kinematic chain in figure 1, formed by wheels $z_1$, $z_2$, $z'_2$, $z_3$, $z'_3$ and $z'_7$, the transmission ratios are:

$$i_{37} = \frac{\omega_3}{\omega_7} = \frac{z_7}{z_3}$$

(4)

For the kinematic chain in figure 1, formed by wheels $z_1$, $z_4$, $z'_4$, $z_5$, $z'_5$ and $z'_6$, the transmission ratios are:

$$i_{14} = \frac{\omega_1}{\omega_4} = \frac{z_4}{z_1}$$

(5)

$$i_{45} = \frac{\omega_4}{\omega_5} = \frac{z_5}{z_4}$$

(6)

$$i_{56} = \frac{\omega_5}{\omega_6} = \frac{z_6}{z_5}$$

(7)

Hence, from relations (1)-(7), we get:

$$\omega_2 = \frac{\omega_1}{i_{12}}$$

(8)

$$\omega_3 = \omega_1 \cdot i_{13}$$

where $i_{13} = \frac{z_2}{z_1} \cdot \frac{z_3}{z_2}$

(9)

$$\omega_{11} = \omega_1 \cdot i_{11-11}$$

where $i_{11-11} = \frac{z_2}{z_1} \cdot \frac{z_3}{z_2} \cdot \frac{z_{11}}{z_{10}}$

(10)

$$\omega_7 = \omega_1 \cdot i_{17}$$

where $i_{17} = \frac{z_2}{z_1} \cdot \frac{z_3}{z_2} \cdot \frac{z_7}{z_3}$

(11)

$$\omega_4 = \frac{\omega_1}{i_{14}}$$

(12)

$$\omega_5 = \frac{\omega_1}{i_{15}}$$

where $i_{15} = \frac{z_2}{z_1} \cdot \frac{z_5}{z_4}$

(13)

$$\omega_6 = \frac{\omega_1}{i_{16}}$$

where $i_{16} = \frac{z_2}{z_1} \cdot \frac{z_5}{z_4} \cdot \frac{z_6}{z_5}$

(14)

For the differential gear and $z_p \rightarrow z_9$, the transmission ratios are:

$$i_{p6} = \frac{\omega_p - \omega_6}{\omega_9 - \omega_p} = \frac{z_9}{z_6}$$

(15)
From relations (15) and (16) we get the Willis relation:

\[ i_{g7}^p = \frac{\omega_6 - \omega_p}{\omega_7 - \omega_p} = \frac{z_7}{z_6} \tag{16} \]

From relations (15) and (16) we get the Willis relation:

\[ i_{g7}^p = \frac{\omega_6 - \omega_p}{\omega_7 - \omega_p} = -\frac{z_5}{z_6} \text{ where } z_7 = z_6 \tag{17} \]

and

\[ i_{g7}^p = -1 \tag{18} \]

From relations (11), (14) and (15)-(18) we get:

\[ \omega_p = \frac{\omega_6 + \omega_7}{2} \tag{19} \]

that is:

\[ \omega_p = \omega_1 \cdot i_{1p} \tag{20} \]

where

\[ i_{1p} = \frac{l_{1p} + l_{16}}{2l_{16}} \tag{21} \]

From relations (11), (14), (15)-(18) we get:

\[ \omega_8 = \omega_1 \cdot i_{18} \tag{22} \]

where

\[ i_{18} = \frac{1}{l_{16}} + (l_{12} + l_{16}) (l_{88}^{-1}) \frac{1}{2l_{16}l_{177}l_{88}} \tag{23} \]

From relations (20) and (24) we get:

\[ \omega_9 = \omega_1 \cdot i_{19} \tag{26} \]

where

\[ i_{19} = \frac{i_{1p} + i_{19}}{i_{1p} i_{19}} \tag{27} \]

From relations (19) and (25) there can be seen that \( \omega_p \) and \( \omega_9 \) are linear combination of \( \omega_6 \) and \( \omega_7 \).

Also, from relations (10) and (26) it can be seen that \( \omega_{11} \) and \( \omega_9 \) follow the relation:

\[ \omega_{11} = \frac{\omega_6}{i_{19} i_{11}} \tag{28} \]

3. Dynamics

Let's modelate the mechanism in figure 1 as a dynamic model of rotation (reducing element model) \[3\]. We take as reducing element the kinematic element 1. The reduced moment of inertia of the dynamic model is:

\[ I_{red} = I_1 \cdot \left(\frac{\omega_1}{\omega_1}\right)^2 + I_2 \cdot \left(\frac{\omega_2}{\omega_2}\right)^2 + I_3 \cdot \left(\frac{\omega_3}{\omega_3}\right)^2 + I_{11} \cdot \left(\frac{\omega_{11}}{\omega_1}\right)^2 + I_4 \cdot \left(\frac{\omega_4}{\omega_4}\right)^2 + I_5 \cdot \left(\frac{\omega_5}{\omega_5}\right)^2 + \]

\[ + I_6 \cdot \left(\frac{\omega_6}{\omega_6}\right)^2 + I_7 \cdot \left(\frac{\omega_7}{\omega_7}\right)^2 + I_p \cdot \left(\frac{\omega_p}{\omega_p}\right)^2 + I_8 \cdot \left(\frac{\omega_8}{\omega_8}\right)^2 + m_8 \cdot \left(\frac{\omega_p \cdot \omega_6}{\omega_1}\right)^2 + I_9 \cdot \left(\frac{\omega_9}{\omega_9}\right)^2 \tag{29} \]

After performing mathematical calculations on relation (29), there results:

\[ I_{red} = I_1 + I_2 \cdot \left(\frac{1}{i_{12}}\right)^2 + I_3 \cdot \left(\frac{1}{i_{13}}\right)^2 + I_{11} \cdot \left(\frac{1}{i_{11-11}}\right)^2 + I_4 \cdot \left(\frac{1}{i_{14}}\right)^2 + I_5 \cdot \left(\frac{1}{i_{15}}\right)^2 + \]

3
$$+J_{6} \cdot \left( \frac{1}{i_{16}} \right)^{2} + J_{7} \cdot \left( \frac{1}{i_{17}} \right)^{2} + J_{p} \cdot \left( i_{1p} \right)^{2} + \beta_{8} \cdot \left( i_{1p} \cdot r_{67} \right)^{2} + J_{9} \cdot \left( i_{19} \right)^{2}$$  \hspace{1cm} (30)

It can be observed that $J_{\text{red}}=\text{constant}$.

The reduced torque of the dynamic model is:

$$M_{\text{red}} = M_{m} \cdot \frac{\omega_{1}}{\omega_{1}} - M_{t1} \cdot \frac{\omega_{6}}{\omega_{6}} - M_{t2} \cdot \frac{\omega_{11}}{\omega_{11}}$$  \hspace{1cm} (31)

After performing the mathematical calculation on relation (31), there results:

$$M_{\text{red}} = M_{m} - M_{t1} \cdot i_{19} - M_{t2} \cdot \frac{1}{i_{1-11}}$$  \hspace{1cm} (32)

The differential equation of kinetic energy is:

$$dE = dL$$  \hspace{1cm} (33)

where:

$$E = \frac{1}{2} \cdot J_{\text{red}} \cdot \omega_{1}^{2}$$  \hspace{1cm} (34)

and

$$dL = M_{\text{red}} \cdot d\varphi_{1}$$  \hspace{1cm} (35)

where $d\varphi_{1}$ is the elementary angle of rotation of element 1.

From relations (33), (34) and (35) there results:

$$J_{\text{red}} \cdot \omega_{1} \cdot \frac{d\omega_{1}}{d\varphi_{1}} = M_{m} - M_{t1} \cdot i_{19} - M_{t2} \cdot \frac{1}{i_{1-11}} = M_{\text{red}}$$  \hspace{1cm} (36)

or

$$J_{\text{red}} \cdot \frac{d\omega_{1}}{dt} = M_{m} - M_{t1} \cdot i_{19} - M_{t2} \cdot \frac{1}{i_{1-11}} = M_{\text{red}}$$  \hspace{1cm} (37)

In general, for a tool-machine, $M_{m}, M_{t1}$ and $M_{t2}$ are depending on $\omega_{1}$, that is:

$$M_{m} = M_{m}(\omega_{1})$$  \hspace{1cm} (38)

$$M_{t1} = M_{t1}(\omega_{1})$$  \hspace{1cm} (39)

$$M_{t2} = M_{t2}(\omega_{1})$$  \hspace{1cm} (40)

and

$$M_{\text{red}} = M_{\text{red}}(\omega_{1})$$  \hspace{1cm} (41)

From relation (37) there results:

$$\frac{d\omega_{1}}{dt} = \frac{M_{\text{red}}(\omega_{1})}{J_{\text{red}}}$$  \hspace{1cm} (42)

By integrating differential equation (42), there can be obtained the relation of $\omega_{1}$ function of time:

$$\omega_{1} = \omega_{1}(t)$$  \hspace{1cm} (43)

In function of mathematical expression of $M_{\text{red}}(\omega_{1})$, the relation (42) can be integrated by mathematical calculus or by numerical means. After obtaining the $\omega_{1} = \omega_{1}(t)$, the angular velocities $\omega_{11}$ and $\omega_{9}$ can be calculated using relations (10) and (26).

4. Discussions

The angular speed $\omega_{9}$ is a linear combination of speeds $\omega_{6}$ and $\omega_{7}$. The angular speeds $\omega_{9}$ and $\omega_{11}$ are correlated by relation (28). Sometimes, the kinematic chain $z_{1}-z_{4}-z'_{4}-z_{5}-z'_{5}-z'_{6}$ can be disabled by introducing in the design of the mechanism, a clutch. In this case, depending of the location of the clutch, some of the angular speeds $\omega_{6}$, $\omega_{5}$, $\omega_{6}$ will be equal to zero. In this case, it is necessary to provide a mean to fix (to block) the solar wheel $z_{6}$ so that the differential gear to be transformed into a planetary gear. In this case, $\omega_{9}$ will be a function only of $\omega_{7}$. When $\omega_{6}=0$, $\omega_{5}=0$, $\omega_{6}=0$ the expression
of $J'_{\text{red}}$ can be obtained by making the terms in $\omega_4$, $\omega_5$, $\omega_6$ equal to zero. Of course that $\omega_p$ will be modified by making $\omega_6=0$ in relation (19). In this case, it can be observed that the new value of $J'_{\text{red}}$:

$$J'_{\text{red}} \leq J_{\text{red}}$$ (44)

This fact leads to decreasing of the reduced moment of inertia and to increasing of the nonuniformity of the speed of working. This is why it is better to place the clutch closer to wheel $z_6$, so that the wheels $z_4$, $z_4'$, $z_5$ and $z_5'$ to remain in motion, acting like a flywheel.

5. Conclusions

In the present paper, a dynamic study of a complex gear transmission. The kinematics is presented. The parameters of dynamic model are presented. The differential equation of kinetic energy is written. Some discussions are made regarding the work of the mechanism. The study in the paper might be useful for the designers of such kind of gear transmissions. The study in this paper will be continued with some researches on this theme, taking into account a more complex dynamic model with friction and numerical values and concrete situations of integrating differential equation (42).

6. References

[1] Frumusanu G 2008 Utilaje si echipamente pentru prelucrari mecanice-I- Galati
[2] Korka Z 2014 Masini-unelte. Constructie si cinematica Editura Eftime Murgu Resita ISBN 978-606-631-042-0
[3] Merticaru V 1991 Probleme dinamice ale functionarii mecanismelor Editura Junimea Iasi ISBN 973-37-0088-6
[4] Predinean N 2015 Cinematica masinilor-unelte Editura A.G.I.R. ISBN 978-973-720-589-6
[5] Merticaru V, Merticaru E V, Merticaru V V jr. 2009 Caracteristicile cinematico-dinamice ale mecanismelor partea 1 Editura PIM Iasi Romania ISBN 978-606-520-508-6, ISBN 978-606-520-509-3
[6] Merticaru E V, Merticaru V, Merticaru V V jr. 2014 Caracteristicile cinematico-dinamice ale mecanismelor partea 2 Editura Performantica Iasi Romania ISBN 978-606-520-508-6, ISBN 978-606-685-121-3
[7] Hedrih (Stevanovic) K, Nikolic-Stanojevic V 2010 A Model of Gear Transmission: Fractional Order System Dynamics Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2010, Article ID 972873, 23 pages doi:10.1155/2010/972873
[8] Deur J, Asgari J, Hrovat D, Kovač P J, 2006 Modeling and Analysis of Automatic Transmission Engagement Dynamics-Linear Case Dyn. Sys. Meas. Control. Jun 2006 128(2): 263-277 (15 pages), https://doi.org/10.1115/1.2192827
[9] Fernandez-del-Rincon A, Garcia P, Diez-Ibarbia A, De-Juan A, Iglesias M, Viadero F 2017 Enhanced model of gear transmission dynamics for condition monitoring applications: Effects of torque, friction and bearing clearance Mechanical Systems and Signal Processing Volume 85, 15 February 2017, pp 445-467
[10] Fernandez-del-Rincon A, Iglesias M, De-Juan A, Diez-Ibarbia A, Garcia P, Viadero F 2016 Gear transmission dynamics: Effects of index and run out errors Applied Acoustics Volume 108, July, 2016, pp 63-83
[11] Nevzat Özugüven H, Houser DR 1988 Mathematical models used in gear dynamics—A review Journal of Sound and Vibration Volume 121, Issue 3, 22 March 1988, pp 383-411
[12] Kahraman A 1994 Planetary Gear Train Dynamics J. Mech. Des. Sep 1994, 116(3): 713-720 (8 pages), https://doi.org/10.1115/1.2919441
[13] Choy F K, Tu Y K, Zakrajsek J, Townsend D P 1990 Dynamics of multistage gear transmission with effects of gearbox vibrations https://www.researchgate.net/publication/24297980_Dynamics_of_multistage_gear_transmission_with_effects_of_gearbox_vibrations
[14] Atanasiu V, Doroftei I, Iacob M R, Leohchi D 2011 Nonlinear Dynamics of Steel/Plastic Gears of Servomechanism Structures MATERIALE PLASTICE 48, Nr. 1, 2011, www.revmaterialeplastice.ro
[15] Atanasiu V, Oprișan C, Leohchi D 2014 The Effect of Tooth Wear on the Dynamic Transmission Error of Helical Gears with Smaller Number of Pinion Teeth Applied
