A study on wavelet selection in power signal denoising

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Abstract. In the wavelet threshold denoising of power signal, the selection of wavelet has an important influence on the denoising effect, and the wavelet generating function has diversity, if not selected properly, it will directly lead to the failure of denoising. Firstly, an operator is introduced to modify the threshold of each scale to better reflect the variation of wavelet coefficients of signal and noise with scale. Then a controllable threshold function is proposed to adapt to different soft and hard characteristics, and it is used to denoise the wavelet coefficients. Based on the study of the characteristics of wavelet, such as orthogonality, vanishing moment, support length and symmetry, four principles of wavelet selection in power signal denoising are proposed. The voltage sag and harmonics model are established, and db5, coif1 and sym2 wavelets are selected to decompose the signal to the fourth scale for denoising. The signal-to-noise ratio, mean square error and the detailed features of the reconstructed signal after denoising are compared. The experimental results show that the orthogonal wavelet db5 with high vanishing moment order and long support length has better denoising effect than coif1 and sym2, which proves the correctness of the wavelet selection principles proposed in this paper.

1. Introduction
With the development of distributed generation, a large number of non-linear power electronic equipment and impulsive and fluctuating loads are more and more connected to the power system, which brings a variety of power quality problems. Because of the electromagnetic interference of the external environment, the error of the monitoring equipment and other factors, the detected power quality signals are inevitably overlapped with noise. The existence of noise will worsen the effectiveness of disturbance detection and recognition methods, so denoising is an important prerequisite for power quality control [1-2].

Wavelet transform provides a powerful tool for denoising with its good time-frequency localization performance. Wavelet threshold denoising is an important application of wavelet transform in the field of signal processing [3-6]. Different from the unique determination of the generating function (sine and cosine function) in the Fourier transform, the selection of the wavelet generating function is diverse. At present, there is no generally accepted principle to choose the wavelet generating function, which is usually determined by qualitative analysis and experimental comparison. Different literatures select different wavelets [7-10], such as db4 wavelet in literature [7], sym and coif wavelets in
literature [8], and db2-6 wavelets in literature [9] for experiments. But these literatures have not explained the specific wavelet selection principle in detail, nor explored the influence of the different characteristics of these different wavelets on the denoising results. In power signal denoising, because of the different characteristics of each wavelet, the effect of denoising is very different by choosing different wavelet to decompose and reconstruct the signal. To solve this problem, on the basis of a new threshold function and modified threshold, the influence of wavelet orthogonality, order of vanishing moment, support length and symmetry on denoising effect is studied, and four principles of wavelet selection in power signal denoising are proposed.

2. Wavelet threshold denoising algorithm

The main basis of wavelet threshold denoising is the difference of energy distribution between noise and effective signal after wavelet transform. After wavelet transform, the wavelet coefficients of noise are small, and the wavelet coefficients of effective signal are large. According to this, a suitable threshold is selected to divide the signal and noise, so as to achieve the purpose of signal-to-noise separation. The specific steps of wavelet denoising are as follows: first, determine the wavelet generating function and the optimal decomposition scale, carry out wavelet transform on the noisy signal, and get the wavelet coefficients at each decomposition scale; second, calculate the threshold of each scale, use the threshold function to threshold the wavelet detail coefficients of each scale, and get the wavelet coefficients after denoising; finally, use the wavelet inverse transform to reconstruct the approximate coefficients after wavelet decomposition and the detail coefficients after thresholding denoising, so as to obtain the denoised signal [11]. The process of wavelet threshold denoising is shown in Figure 1.

The traditional hard threshold function is a discontinuous function, which will produce some discontinuities and is difficult to deal with mathematically. Soft threshold function is a continuous function, which overcomes the problem of discontinuity of hard threshold function. However, the shrinkage of wavelet coefficients whose absolute value is greater than the threshold will cause the loss of certain high-frequency information, and then the edge of the signal will be fuzzy [12]. In view of the shortcomings of traditional hard and soft threshold functions, a new adjustable threshold function is proposed, whose mathematical expression is:

$$y = \begin{cases} x - \frac{nt}{2} + \frac{nt}{14e^{3x}} & |x| \geq T \\ 0 & |x| < T \end{cases}$$

(1)

In formula (1), n is an adjustable parameter, and its value range is $n \in [0,2]$. When n=0, the new threshold function is equal to the hard threshold function; when n=2, the new threshold function is equal to the soft threshold function. By controlling parameter n, the new threshold function can have many different soft and hard characteristics.

The traditional universal threshold $\sigma_j \sqrt{2lnN}$ is fixed at every wavelet decomposition scale, which cannot well reflect the different propagation characteristics of noise and signal wavelet coefficients among different scales. Therefore, the operator $e^{(\frac{1}{2}-1)}$ is introduced to improve the threshold.

$$\lambda_j = \sigma_j \sqrt{2lnN} / e^{(\frac{1}{2}-1)}$$

(2)
In formula (2), $N$ is the total number of wavelet detail coefficients at the scale $j$; $j$ is the wavelet decomposition scale; $\sigma_j$ is the standard deviation of noise and the calculation formula is as follows:

$$\sigma_j = \frac{\text{median}(|cd_{j,k}|)}{0.6745}$$

(3)

In formula (3), $cd_{j,k}$ is the $k$-th wavelet detail coefficient at the $j$ scale after wavelet decomposition.

3. Characteristics of wavelet and selection principles of wavelet in denoising

3.1. Wavelet characteristics analysis

3.1.1. Orthogonality. Let $\Psi(t)$ satisfy the permissible condition. If the wavelet basis function obtained by binary expansion and translation, that is, $\Psi_{m,k}(t) = 2^{-\frac{m}{2}}\Psi(2^{-m} - k), m, k \in \mathbb{Z}$, constitutes the orthonormal basis of $L^2(\mathbb{R})$, then $\Psi(t)$ is called orthogonal wavelet. $\Psi_{m,k}(t)$ is called orthogonal wavelet basis function and the corresponding discrete wavelet transform is orthogonal wavelet transform [13]. Orthogonal wavelet transform can concentrate most of the signal energy in the low-frequency part, and leave a small proportion of the energy in the high-frequency part, which reflects the characteristics of energy concentration. The total energy before and after orthogonal wavelet transform is conserved, so the orthogonality reflects the anti-interference performance of wavelet function.

3.1.2. Order of vanishing moment. The intuitive understanding of N-order vanishing moment is to make the details of constant function and linear function become zero after N-1 wavelet transform. The magnitude of vanishing moment not only reflects the wavelet's ability to suppress polynomials, but also reflects the degree of energy concentration after wavelet transform [14]. When the vanishing moment is large to a certain extent, most of the values of the high-frequency part of the fine scale except for the singular points are small enough to be ignored. Therefore, the larger the vanishing moment is, the more concentrated the decomposed signal energy is, and the more advantageous it is to filter out the noise wavelet coefficients with small amplitude. However, as the order of vanishing moment increases, the corresponding calculation amount will increase, so it needs to be considered comprehensively in practice.

3.1.3. Support length. When time or frequency approaches infinity, the support length represents the convergence rate of wavelet function and scale function from finite value to zero. If the value of the wavelet function $\Psi(t)$ outside [a, b] is always zero, then the function $\Psi(t)$ is compactly supported on [a, b]. The corresponding wavelet is called compactly supported wavelet and the interval [a, b] is the length of the support set [15]. In order to provide a more practical finite impulse response filter with limited coefficients in the process of discrete wavelet decomposition, it is usually expected that the wavelet function will decay rapidly in the time domain and be tightly supported in the frequency domain.

3.1.4. Symmetry. The phase information of signal is mainly described by symmetry. For compactly supported wavelet function, its linear phase can be equivalent to the symmetry of wavelet, that is, the better the symmetry of wavelet, the better the linear phase.

3.2. Wavelet selection principles in power signal denoising

Based on the above analysis of wavelet characteristics, the principles of wavelet selection in power signal denoising are as follows.

Orthogonality. Orthogonality describes the degree of redundancy in wavelet representation of data. If wavelet can guarantee orthogonality, the coefficients in time-scale plane are uncorrelated, which can reduce the error, avoid the loss of signal energy and features, and help to reconstruct wavelet coefficients accurately.
Higher order of vanishing moment. The vanishing moment reflects the degree of energy concentration after wavelet transform, that is, the convergence rate of wavelet approximation smooth function. The higher the order of vanishing moment is, the stronger the detection ability for the signal mutation singular point is. The higher vanishing moment can make the calculated matrix sparser.

Longer support length. The compactly supported wavelet avoids the artificial truncation in the process of filtering, thus avoiding the error caused by truncation, improving the accuracy of application. The longer the support length is, the more suitable it is for the local analysis of frequency domain signals.

Low requirements for symmetry. Symmetry can ensure that the filtering characteristics of wavelet have linear phase, and there is no high requirement for linear phase in denoising.

3.3. Wavelet characteristics of db5, coif1 and sym2

Among many wavelets available at present, gaus, dmev, rbio, cgau, cmor, fbsp, shan wavelets do not have compactly supported orthogonality, biorthogonal, morlet, mexican hat wavelets do not have orthogonality, meyer wavelet does not have compactly supported, so these wavelets are not considered. Db, coiflet and symlet are compactly supported orthogonal wavelets. Limited to the space, three representative wavelets db5, coif1 and sym2 are selected here for denoising experiment, and their basic characteristics are shown in Table 1.

| Wavelet | Orthogonality | Order of vanishing moment | Support length | Symmetry         |
|---------|---------------|---------------------------|----------------|------------------|
| db5     | yes           | 5                         | 9              | approximate symmetry |
| coif1   | yes           | 2                         | 5              | approximate symmetry |
| sym2    | yes           | 2                         | 3              | approximate symmetry |

Mallat algorithm is a fast algorithm of orthogonal wavelet, and wavelet threshold denoising is based on Mallat algorithm. Mallat fast algorithm is the process of filtering the original signal through a filter bank \( \{h(n), g(n)\} \) composed of a low pass and a high pass filter. After passing through the filter bank, the signal is decomposed into low frequency component and high frequency component. The decomposition results reflect the approximate and detailed characteristics of the signal respectively. In order to make more precise observation of the signal, the low frequency components are decomposed according to the same method until the required decomposition scale [16]. In Mallat algorithm, the scale function \( \phi(t) \) corresponds to a low-pass filter \( h(n) \) and the wavelet function \( \psi(t) \) corresponds to a high-pass filter \( g(n) \). The scale function and wavelet function waveforms of db5, coif1 and sym2 are shown in Figure 2.
4. Comparative experiments

4.1. Contrast denoising effect from detail figure

According to IEEE1159 power quality monitoring standard, the disturbance signal is established, and the reference frequency is 50Hz. The sampling frequency selected in this paper is 12.8kHz which is commonly used in practice [17]. The simulation time is 0.2s and there are 2560 sampling points in total.

Limited to space, two kinds of power quality signals of sag and harmonic are taken as examples. Three kinds of wavelets, db5, coif1 and sym2, are selected to decompose signal to the fourth scale. The threshold value and threshold function follow the method in Section 2 and denoise according to the flow chart in Figure 1.

It can be seen from Figure 3 that after denoising with db5, coif1 and sym2 wavelets, Figure 3 (b), (c) and (d) are much smoother than Figure 3 (a), while Figure 3 (c) and (d) have some small burrs compared with Figure 3 (b). In order to observe the denoising effect of the three kinds of wavelets more clearly, the waveform of the dotted box in Figure 3 is amplified locally, and the corresponding details are enlarged as shown in Figure 4. It can be clearly seen from the detail enlarged Figure 4 that the waveforms after denoising with coif1 and sym2 wavelets are far from the original signal waveform, which deviate from the original signal irregularly and substantially. The waveform after denoising with db5 wavelet almost coincides with the original waveform, achieving the best denoising effect.

Figure 5 is the denoising result of harmonic. After denoising with three kinds of wavelets, Figure 5 (b), (c) and (d) are much smoother than Figure 5 (a). The corresponding detail enlarged Figure 6 also clearly shows that the waveform after db5 wavelet denoising is the closest to the original waveform, while the waveforms after coif1 and sym2 wavelets denoising deviate greatly. Therefore, the denoising effect of coif1 and sym2 wavelets is far worse than that of db5 wavelet.

Figure 3. (a) Voltage sag noisy signal;(b) db5 denoised signal; (c) coif1 denoised signal;(d) sym2 denoised signal.

Figure 4. Detailed enlarged view of dotted box in Figure 3.

Figure 5. (a) Harmonic noisy signal;(b) db5 denoised signal; (c) coif1 denoised signal; (d) sym2 denoised signal.

Figure 6. Detailed enlarged view of dotted box in Figure 5.
Results and discussions: Combined with the characteristics of each wavelet to further analyze the deep-seated reasons. The order of vanishing moment of db5, coif1 and sym2 are 5, 2 and 2 respectively. The experimental results prove the correctness of the selection principle (2) proposed in Section 3.2, that is, higher order of vanishing moment is more conducive to the denoising analysis in frequency domain. The support length of db5, coif1 and sym2 are 9, 5 and 3 respectively. The above waveform analysis verifies the correctness of the selection principle (3) proposed in Section 3.2, that is, the longer support length of wavelet has better denoising effect. The deviation of the denoised waveform from the original waveform is very small by using approximately symmetric db5 wavelet. It can be seen that symmetry has little effect on the denoising effect, which verifies the selection principle (4) proposed in Section 3.2, that is, there is no special requirement for symmetry in denoising. In conclusion, the denoising effect of db5 wavelet is much better than that of coif1 and sym2 wavelets.

4.2. Contrast denoising effect from SNR and MSE

SNR (signal to noise ratio) and MSE (mean square error) are usually used to evaluate the denoising effect, and the definition formulas are [18]:

\[
\text{SNR} = 10 \log \left( \frac{\sum_{t=1}^{T} s^2(t)}{\sum_{t=1}^{T} [s'(t) - s(t)]^2} \right) \quad \text{(4)}
\]

\[
\text{MSE} = \frac{1}{T} \sum_{t=1}^{T} [s'(t) - s(t)]^2 \quad \text{(5)}
\]

In formulas (4) and (5), \(s(t)\) is the original signal and \(s'(t)\) is the denoised signal.

For sag and harmonic signals, SNR comparison charts after denoising with db5, coif1 and sym2 wavelets are shown in Figure 7 and Figure 8, and corresponding MSE data are shown in Table 2. It can be seen from Figure 7 and Figure 8 that no matter what the SNR of the input signal is, the SNR after denoising with db5 wavelet is much higher than that of coif1 and sym2 wavelets. It can be seen from Table 2 that MSE after denoising with db5 wavelet is lower than that of coif1 and sym2 wavelets. The larger the SNR and the smaller the MSE, the better the denoising effect. The experimental results show that the db5 wavelet which meets the wavelet selection principles proposed in Section 3.2 has the best denoising effect. Then compare the coif1 and sym2 wavelets. Figure 7 and Figure 8 show that the SNR of coif1 wavelet denoising is slightly higher than that of sym2. Coif 1 and sym2 wavelets have the same order of vanishing moment, both of which are 2, while the support length of coif 1 wavelet is 5, which is higher than that of sym2 wavelet. The experimental results show that the selection principle (3) proposed in Section 3.2 is correct. Compared with the denoising effect of db5 wavelet, it is obvious that the difference of the order of vanishing moment has a greater impact on the denoising effect than the support length, so the order of vanishing moment is the key factor.

**Figure 7. Voltage sag SNR comparison chart.**

**Figure 8. Harmonic SNR comparison chart.**
### Table 2. Comparison of MSE after three kinds of wavelets denoising.

| Disturbance type | SNR of input signal (dB) | MSE data after denoising | Disturbance type | SNR of input signal (dB) | MSE data after denoising |
|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|
| Sag              |                          |                          | Harmonic         |                          |                          |
| 12               | 0.00171475               | 0.00189802               | 0.001951         | 12                       | 0.00250137               | 0.00462842               | 0.004851               |
| 16               | 0.00064525               | 0.00079642               | 0.000841         | 16                       | 0.001025                 | 0.00299866               | 0.003102               |
| 20               | 0.00030186               | 0.00040991               | 0.000421         | 20                       | 0.00043607               | 0.00122944               | 0.001347               |
| 24               | 0.00013037               | 0.00022516               | 0.000234         | 24                       | 0.00021847               | 0.00068014               | 0.000688               |
| 28               | 0.000044772              | 0.00014429               | 0.000151         | 28                       | 0.00011751               | 0.00041114               | 0.000426               |

### 5. Conclusions
In this paper, the characteristics of wavelet are studied, and then the principles of wavelet selection in power quality signal denoising are proposed, that is, orthogonal wavelet with higher order of vanishing moment and longer support length should be selected, but there is no special requirement for the symmetry of wavelet. For voltage sag and harmonic disturbance, db5, coif1 and sym2 wavelets are selected to denoise. The experimental results show that the db5 wavelet denoising waveform almost coincides with the original waveform, and the SNR is the largest and MSE is the smallest after denoising, and the denoising effect of db5 is far better than that of coif1 and sym2 wavelets, which proves the correctness and reliability of the wavelet selection principles proposed in this paper.

### Acknowledgements
Supported by “China Scholarship Council (201809960015)” and “The Fundamental Research Funds for Beijing University of Civil Engineering and Architecture(X20075)”.

### References
[1] Ibrahim Ahmad, Ghaeth Fandi, Zdenek Muller, et al. 2019 Voltage quality and power factor improvement in smart grids using controlled DG units[J] Energies 12(18) 3433-3451
[2] Jianwen Li, Gang Qin, Yonggang Li, et al. 2019 Research on power quality disturbance identification and classification technology in high noise background[J] IET Generation, Transmission & Distribution 13(9) 1661-1671
[3] Thirumala,K., Shantanu,T.J., Umaikar,A.C. 2017 Visualizing time-varying power quality indices using generalized empirical wavelet transform[J] Electr. Power Syst. Res 143 99–109
[4] Srivastava M, Georgieva E R, Freed J H. 2017 A new wavelet denoising method for experimental time-domain signals: pulsed dipolar electron spin resonance[J] The Journal of Physical Chemistry A 121 2452-24636
[5] Lan, Sheng, Hu, Yue-Qun, Kuo, Cheng-Chien. 2019 Partial discharge location of power cables based on an improved phase difference method[J] IEEE Transactions on Dielectrics and Electrical Insulation 26(5) 1612-1619
[6] Wang Weibo, Dong Ruiying, Zeng Wenru, et al. 2019 A wavelet de-noising method for power quality based on an improved threshold and threshold function[J] Transactions of China Electrotechnical Society 34(2) 409-418
[7] Sharif MI, Li JP, Sharif A. 2019 A noise reduction based wavelet denoising system for partial discharge signal[J] Wireless Personal Communications 108(3) 1329-1343
[8] Tang SF, Tong MM, He XM. 2014 The optimum wavelet base of wavelet analysis in coal rock microseismic signals[J] Advances in Mechanical Engineering 1-6
[9] Ding WS, Li ZG. 2019 Research on adaptive modulus maxima selection of wavelet modulus maxima denoising[J] Journal of Engineering-Joe 13 175-180
[10] Ismael Urbina-Salas, Jose R. Razo-Hernandez, David Granados-Lieberman, et al. 2017 Instantaneous power quality indices based on single side band modulation and wavelet Packet–Hilbert transform[J] IEEE Trans. Instrum. Meas 66(5) 1021-1031
[11] Liu Siyi, JIN Tao, Liu Dui. 2017 Power system low-frequency oscillation mode identification base on improved wavelet threshold de-noising and RCRSV-MP algorithm[J] Electric
[12] Kadri O, Baarir ZE, Schaefer G. 2019 Power shrinkage--curvelet domain image denoising using a new scale-dependent shrinkage function[J] Signal Image and Video Processing 13(7) 1347-1355

[13] Ingrid Daubechies. 2017 Ten Lectures Wavelets[M] Beijing: Posts and Telecom Press

[14] Tang Xianghong, Li Qiliang. Time frequency analysis and wavelet transform[M] Beijing: Science Press, 2020 195-197

[15] Li Dengfeng. 2019 The mathematical theory of wavelet analysis[M] Beijing: Science Press

[16] Sun Yankui, 2018 Wavelet transform and image and graph processing technology[M] Beijing: Tsinghua University Press

[17] Yan Wang, Qunzhan Li, Fulin Zhou, et al. 2019 A new method with hilbert transform and slip-SVD-based noise-suppression algorithm for noisy power quality monitoring[J] IEEE Transactions on Instrumentation and Measurement 68(4) 987-1001

[18] Yaping Deng, Lu Wang, Hao Jia, et al. 2019 A sequence-to-sequence deep learning architecture based on bidirectional gru for type recognition and time location of combined power quality disturbance[J] IEEE Transactions on Industrial Informatics 15(8) 4481-4493