The Spin Tensor of Dark Matter and the Hubble Parameter Tension

Fernando Izaurieta, † Samuel Lepe, ‡ and Omar Valdivia §

1 Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile
2 Instituto de Física, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaíso, Chile
3 Facultad de Ciencias, Universidad Arturo Prat, Iquique, Chile

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Allowing for a nonvanishing spin tensor for cold dark matter (ω_DM = 0) has the consequence of giving rise to an effective FLRW dynamics with a small negative barotropic constant for an effective dark matter density (−1/3 < ω_eff ≤ 0). This turns out to solve the Hubble parameter tension in a straightforward way.

Keywords: Hubble tension, Spin tensor, Torsion

I. INTRODUCTION

The Hubble parameter tension has given rise to a considerable number of hypotheses to explain it (see Refs. [1–5]). They range from possible systematic measurement errors (See Ref. [6]), to modified dark energy [2, 7], to more exotic theories as nonminimal couplings, torsional topological invariants, or quadratic Poincaré Gauge Theory [1] (See Refs. [10–12]), among many others. An independent improvement in the measurement of H_0 can be expected in future using black hole mergers as dark standard sirens, see Ref. [13].

Among all this buzz of activity, Ref. [14] offered a particularly simple solution: dropping the hypothesis of coldness in dark matter and allowing for a small negative value for the barotropic constant (ω_DM = −0.0108) fixes the Hubble parameter tension. Some arguments may favor such a non-particle dark matter scheme (see Ref. [15]), but for many, it would seem like an exotic possibility.

This article offers an alternative: a nonvanishing spin tensor for cold dark matter (ω_DM = 0) may solve the problem along the same lines presented in Ref. [14]. The spin tensor of cold dark matter gives rise to an effective dynamics in the FLRW equations corresponding to a small and negative effective barotropic constant −1/3 < ω_eff ≤ 0, precisely as the one required in Ref. [14] to fix the Hubble tension.

Studying a nonvanishing spin tensor also opens the possibility of a nonvanishing torsion. In this context, this work presents a model as close to General Relativity (GR) as possible. We are aware that there are other, much more general nonvanishing torsion Lagrangians. However, here we are not following an “all-encompassing theory” perspective. Instead, we are looking for an approach as simple as possible.

II. THE SPIN TENSOR OF COLD, NON-INTERACTING DARK MATTER

Our knowledge of dark matter nature is scarce, and it has given rise to a vast number of models about its composition. This article does not add a new model to this enormous collection. Whatever it is, dark matter characterizes for its lack of interaction with standard model particles. Therefore, independently of its composition, it seems reasonable to expect extremely weak or absent decoherence effects for dark matter. The general picture we could expect from this is the one of delocalized and diffuse dark matter, with giant wave functions extending over vast distances in the Universe. It is in sharp contrast with the behavior of interacting baryonic matter: interactions give rise to decoherence and structure.

It also implies that some sound models we use for baryonic matter are no longer reasonable hypotheses for dark matter.

One of these cases is how we treat the spin tensor as a source of gravity. The contribution of the spin tensor to gravity appears when considering an “à la Palatini” approach, i.e., with the connection and the metric as independent degrees of freedom and without imposing the torsionless condition (Riemann-Cartan geometry) [16–24].

On this geometry, let us consider the usual Einstein-Hilbert and cosmological constant Lagrangian with minimally-coupled matter \( \mathcal{L} = \frac{1}{2k_4} \left[ R(\Gamma, g) - 2\Lambda \right] + \mathcal{L}_M \) (Einstein-Cartan gravity). The corresponding field equations read

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa_4 T_{\mu\nu}, \tag{1}
\]

\[
T^\lambda_{\mu\nu} - \delta^\lambda_\mu T^\gamma_{\gamma\nu} + \delta^\lambda_\nu T^\gamma_{\gamma\mu} = \kappa_4 \sigma^\lambda_{\mu\nu}. \tag{2}
\]

Here \( \kappa_4 = \frac{8\pi G}{3k_4} \), and \( R_{\mu\nu} \) and \( R \) correspond to the Ricci tensor and Ricci scalar constructed from the general connection \( \Gamma^\lambda_{\mu\nu} \) (and not the Christoffel). The \( \sigma^\lambda_{\mu\nu} \) stands for the spin tensor of \( \mathcal{L}_M \), \( \tau_{\mu\nu} \) for its stress-energy tensor, and \( T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \) for torsion.

Tracing Eq. (2), the torsion reads

\[
T^\lambda_{\mu\nu} = \kappa_4 \left[ \sigma^\lambda_{\mu\nu} + \frac{1}{2} \left( \sigma^\gamma_{\mu\nu} \delta^\lambda_\gamma - \sigma^\gamma_{\gamma\nu} \delta^\lambda_\mu \right) \right]. \tag{3}
\]
The Eq. (3) implies that torsion does not propagate in a vacuum\(^2\), in sharp contrast with curvature. Even further, in the case of standard baryonic matter, the spin tensor arises as a small quantum effect. In almost any normal astrophysical situation, the torsion created through this mechanism is so negligible\(^3\) that it is possible to make light-hearted jokes about it (See the end of Chap. 8.4 of Ref. [31]).

Therefore, in the context of highly-localized interacting baryonic matter and the Einstein-Hilbert Lagrangian, torsion seems negligible. When considering a cosmological scale, it seems even more negligible: since torsion does not propagate in a vacuum, the effective spin tensor of a cosmological “gas of galaxies” is zero.

However, the same reasoning does not work for non-interacting cold dark matter. With no decoherence effects and intergalactic-wide dark matter wave functions, quantum effects as the spin tensor may be relevant. In this case, according to Eq. (3) the spin tensor can create a nonvanishing torsion at cosmic scales. This torsion cannot propagate in a vacuum, but it is not an obstacle to contribute to the cosmological evolution with a widely distributed spin tensor.

Nevertheless, lacking phenomenological knowledge of the dark matter, any hypothesis on the nature of its possible spin tensor would seem unsubstantiated. Again, some reasonable models for baryonic matter do not seem to apply to dark matter. The usual way to model the spin tensor of highly-interacting baryonic matter is as a Weyssenhof fluid\(^32,33\).

A feature of a Weyssenhof fluid is that its spin tensor does not obey isotropy of the Copernican principle\(^34\) in the short micro-scales of a highly interacting fluid. However, on a macro-scale, all the local anisotropies can be averaged, recovering the Copernican principle. The result is a theory like the ones of Refs. [35, 36], where the spin tensor of standard model fermions gives rise to a Big-Bounce model under the extreme high-densities of very early times.

The same reasoning does not seem to apply to non-interacting cold dark matter on broad cosmic scales, and a spin tensor obeying the Copernican principle seems more appropriate. Imposing the Copernican principle on the spin tensor \(\Sigma_{\lambda\mu\nu} = 0\), we get something of the form,

\[
\sigma_{\lambda\mu\nu} = \frac{2}{c^2 R_4} X(t) (g_{\lambda\mu} U_\nu - g_{\lambda\nu} U_\mu) + \frac{3}{c^2 R_4} Y(t) \sqrt{|g|} \epsilon_{\lambda\mu\nu\rho} U^\rho, \tag{4}
\]

where \(U^\rho\) is the co-moving 4-velocity, and \(X(t)\) and \(Y(t)\) are arbitrary functions of time. Eq. (4) is a well-known Ansatz; the second term sometimes receives the nickname of Cartan’s staircase See Ref. [28].

Since torsion depends algebraically on the spin tensor through Eq. (3), it is given by

\[
T_{\lambda\mu\nu} = \frac{1}{c^2} [X (g_{\lambda\rho} g_{\mu\nu} - g_{\lambda\nu} g_{\mu\rho}) - 2 Y \epsilon_{\lambda\mu\nu\rho} U^\rho]. \tag{5}
\]

Without a dark matter theory, we have no a priori information on \(X(t)\) or \(Y(t)\). However, it is possible to construct an “equation of state” for \(X(t)\) and \(Y(t)\) considering dimensional analysis. Putting all the torsional terms of \(R_{\mu\nu}\) and \(R\) at the right-hand side of Eq. (1), we get something of the form

\[
\dot{R}_{\mu\nu} - \frac{1}{7} g_{\mu\nu} \ddot{R} + \Lambda g_{\mu\nu} = \tau_{\mu\nu} + \tau^{\text{(T)}}_{\mu\nu}, \tag{6}
\]

where \(\dot{R}_{\mu\nu}\) and \(\ddot{R}\) are the standard torsionless Ricci tensor and scalar, and \(\tau^{\text{(T)}}_{\mu\nu}\) is the effective stress-energy tensor for torsion, given by

\[
\tau^{\text{(T)}}_{\mu\nu} = g_{\mu\nu} \left( \nabla_\rho K^{\alpha\rho\lambda} + \frac{1}{2} \left[ K^\alpha_{\lambda\rho} K^{\lambda\rho}_{\mu} - K^\alpha_{\lambda\rho} K^{\lambda\rho}_{\mu} \right] \right) + \frac{1}{2} \left( \nabla_\mu K^{\alpha}_{\lambda\nu} + \nabla_\nu K^{\alpha}_{\mu\lambda} + K^\alpha_{\lambda\nu} K^{\lambda}_{\mu\lambda} \right) \left( \nabla_\lambda + K^{\alpha}_{\lambda\alpha} \right) [K^{\lambda}_{\mu\nu} + K^{\lambda}_{\nu\mu}] \tag{7}
\]

where

\[
K_{\mu\nu\lambda} = \frac{1}{2} (T_{\mu\nu\lambda} - T_{\mu\lambda\nu} + T_{\lambda\mu\nu}) \tag{8}
\]

corresponds to what is called the contorsion (or contortion) tensor.

Since this effective stress-energy tensor Eq. (7) is quadratic in torsion, it indicates that torsion scales as

\[
T_{\lambda\mu\nu} \sim \sqrt{\text{energy density}}. \tag{9}
\]

Therefore, it is reasonable to expect \(X(t)\) and \(Y(t)\) scaling in the same way. Even further, torsion has to vanish in a vacuum, and it seems natural to expect it to grow for higher dark matter densities. Since in the Lagrangian the only relevant energy densities are the cold dark matter density \(\rho_{\text{DM}}\) and \(\frac{\Lambda}{\kappa_c}\), it is natural to consider an Ansatz of the form

\[
X, Y \sim \left( \frac{K_{4\rho_{\text{DM}}}}{\Lambda} \right)^N \sqrt{\rho_{\text{DM}}}, \tag{10}
\]
where $N > -1/2$ is some number (in general, a different one for $X$ and $Y$). To assume there is no relation between torsion and $A$ amounts to the choice $N = 0$.

Let us remark that we are not making further hypotheses on the nature of dark matter. We are only using the Copernican symmetries and dimensional analysis to create the “equation of state” Ansatz (10). There are theories where fundamental fields (dark spinors) create spin tensors which satisfy the cosmological symmetries, see Ref. [37]. However, to keep the discussion general, we do not make any choice of a particular theory for dark matter. The Copernican symmetries and dimensional arguments suffice for our purpose.

III. THE FLRW EQUATIONS

Using the FLRW metric with flat spatial section and the Eq. (5), the field equation (1) takes the form

$$\frac{3}{c^2} \left[(H + X)^2 - Y^2\right] = \Lambda = \kappa_4 (\rho_b + \rho_{DM}) ,$$

and

$$\frac{1}{c^2} (\dot{H} + \dot{X}) + H (H + X) + \frac{1}{2} \left[(H + X)^2 - Y^2\right] = \frac{1}{2} \Lambda = -\kappa_4 \frac{1}{2} (p_b + p_{DM}) ,$$

where $\rho_b$ and $p_b$ are the density and pressure of baryons. Afterward, we impose the cold dark matter pressureless condition $p_{DM} = 0$, but for now, let us keep the term so we can follow its role in the equations.

Let us impose the condition that baryonic matter does not directly interchange energy with dark matter and neither with torsion. The former is an empirical fact, and the second is, to the best of our knowledge, an excellent approximation (except for very early times). This lack of interaction with baryonic matter implies the usual conservation law for it

$$\frac{d}{dt} \rho_b + 3H (\rho_b + p_b) = 0 .$$

Using this and some straightforward manipulation, it is possible to write the field equations as

$$\frac{3}{c^2} (H + X)^2 - \Lambda - \kappa_4 \left(\rho_b + \rho_{DM} + \frac{3Y^2}{\kappa_4 c^2}\right) = 0 ,$$

$$\frac{d}{dt} \rho_b + 3H (\rho_b + p_b) = 0 ,$$

$$\frac{d}{dt} \left(\rho_{DM} + \frac{3Y^2}{\kappa_4 c^2}\right) + 3H \left(\rho_{DM} + \frac{2Y^2}{\kappa_4 c^2} + p_{DM}\right) + X \left(\rho_b + \rho_{DM} + 3 [p_b + p_{DM}] - 2 \frac{\Lambda}{\kappa_4}\right) = 0 .$$

The roles of the two spin tensor modes are extremely different. Let us sacrifice generality for simplicity, and let us impose $X = 0$ to focus our attention only on the behavior of the $Y$ component. Defining an effective dark matter density and pressure given by

$$\rho_{eff} = \rho_{DM} + \frac{3}{c^2 \kappa_4} Y^2 ,$$

$$p_{eff} = p_{DM} - \frac{1}{c^2 \kappa_4} Y^2 ,$$

in the $X = 0$ case the field equations adopt the canonical form

$$\frac{3}{c^2} H^2 = \Lambda + \kappa_4 (\rho_b + p_{eff}) ,$$

$$\frac{d}{dt} \rho_b + 3H (\rho_b + p_b) = 0 ,$$

$$\frac{d}{dt} \rho_{eff} + 3H (p_{eff} + p_{eff}) = 0 .$$

The only difference with the standard ΛCDM cosmology is the fact that for the case of cold dark matter $p_{DM} = 0$, the effective pressure is still negative $p_{eff} < 0$. Let us use the Ansatz (10) with $N = 0$ to write down

$$Y = \alpha_Y c \sqrt{\frac{\kappa_4}{3} \rho_{DM}} ,$$

where $\alpha_Y$ is a “torsiotropic” constant of proportionality. In the case of cold dark matter $p_{DM} = 0$ the effective barotropic constant is given by

$$\omega_{eff} = \frac{p_Y}{\rho_Y} = \frac{1}{3 (1 + 1/\alpha_Y^2)} ,$$

and therefore $-1/3 < \omega_{eff} \leq 0$. In other words, the spin tensor component has the effect of creating a non-particle FLRW effective behavior for otherwise standard cold dark matter. Let us emphasize that this small negative pressure $p_{eff}$ is only an effective artifact at the level of FLRW evolution, and it does not change the characteristic dark matter speed. Dark matter in our model is still standard $\omega_{DM} = 0$ pressureless cold dark matter. The only difference with the standard case is that we assume that one of the components of the spin tensor does not vanish.

On the other hand, in the standard model of particles, torsion interacts only with fermions in a extremely weak manner\(^4\). However, when passing all the torsional terms at the right-hand side of the field equations as in Eq. (6), it behaves as an extra source of standard torsionless Riemannian gravity. For this reason, some authors have suggested that the whole dark matter phenomenology could be due to torsion (See Ref. [38]). We do not go as far, but the point is that torsion behaves as an extra dark source for the torsionless piece of gravity.

\(^4\) Again, assuming a minimal coupling. There is no experimental reason to assume otherwise, but if there were nonminimal couplings, torsion would be easier to detect, see Ref. [30].
Therefore, in the current context, we have to distinguish between dressed and bare dark matter. When we measure torsionless Riemannian effects and attribute them to dark matter, what we are measuring is the altogether combined effect of “bare” dark matter and torsion, \( \rho_{\text{DM}} + \frac{3}{c^2 \kappa_4} \). In other words, torsion amplifies the effect of dark matter, creating the “dressed” version \( \rho_{\text{eff}} \) we observe as a dark source.

The solution to the Hubble parameter tension presented in Ref. [14], but rephrased in terms of “dressed” dark matter, would be the following. According to the PLANCK report Ref. [39], they measured in a model-independent way the combination \( \Omega_b h^2 \) for baryons and dark matter (where \( h \) is the “adimensional” Hubble parameter given by \( H_0 = 100 h \, \text{km} \, \text{Mpc}^{-1} \)) as

\[
\Omega_b^{(\text{PLANCK})} h^2 = 0.0224 \pm 0.0001 , \quad \Omega_{\text{DM}}^{(\text{PLANCK})} h^2 = 0.120 \pm 0.001 .
\]

The tabulated values of \( \Omega_x \) and \( H_0 \) at \( z = 0 \) are predictions assuming standard \( \Lambda \)CDM, not direct measurements.

Since the values of PLANCK assume a vanishing pressure for dark matter, this is equivalent to say that it is a model-independent measurement of dark matter density \( \rho_{\text{DM}}^{(\text{CMB})} \) at \( z = 1089 \). Using the value Eq. (25), it is simple to check (omitting errors) that

\[
\rho_{\text{DM}}^{(\text{CMB})} = \frac{3 H_0^2 \Omega_{\text{DM}}^{(\text{PLANCK})}}{\kappa_4 c^2} (z + 1)^3 \bigg|_{z=1089} = 0.262 \, \text{J} \, \text{m}^{-3} .
\]

In the context of our model, measurements of Riemannian geometry-dependent features (as CMB data) are sensitive only to the whole dressed effective dark matter density \( \rho_{\text{eff}} \), and not to the bare one \( \rho_{\text{DM}} \). Therefore, what is being measured by PLANCK data corresponds in our model to

\[
\rho_{\text{eff}} \bigg|_{z=1089} = \rho_{\text{DM}}^{(\text{CMB})} ,
\]

and not to the bare density \( \rho_{\text{DM}} \bigg|_{z=1089} \).

Even further, assuming that the correct value of \( H_0 \) is given by direct measurements as the ones from Refs. [40, 41], (e.g. \( H_{\text{dir}} = h_{\text{dir}} 100 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \) with \( h_{\text{dir}} = 0.742 \pm 0.018 \)), and allowing for a small negative effective barotropic constant \( -1/3 < \omega_{\text{eff}} \leq 0 \), it is clear that the values of \( \Omega_b \) and \( \Omega_{\text{DM}} \) slightly change in comparison to the tabulated \( \Omega_b^{(\text{PLANCK})} = 0.049 \) and \( \Omega_{\text{DM}}^{(\text{PLANCK})} = 0.265 \). For a start, the baryonic density parameter would correspond to

\[
\Omega_b = \frac{\left( \Omega_b^{(\text{PLANCK})} h^2 \right)_{\text{tabulated}}}{h_{\text{dir}}^2} = 4.07 \times 10^{-2} .
\]

In a similar way, the parameter \( \Omega_{\text{eff}} \) depends on \( \omega_{\text{eff}} \) through

\[
\Omega_{\text{eff}} = \frac{\kappa_4 c^2}{3 H_{\text{dir}}^2} (z + 1)^{3(1 + \omega_{\text{eff}})} \rho_{\text{DM}}^{(\text{CMB})} \bigg|_{z=1089} = \frac{\left( \Omega_{\text{DM}}^{(\text{PLANCK})} h^2 \right)_{\text{tabulated}}}{(z + 1)^{3\omega_{\text{eff}}}} \bigg|_{z=1089} ,
\]

\[
= 0.218 \, 1090^{3\omega_{\text{eff}}} .
\]

From the flat space spatial geometry condition \( \Omega_{\text{eff}} + \Omega_b + \Omega_{\Lambda} = 1 \), we have \( \Omega_{\text{eff}} = 0.273 \). Therefore, \( \omega_{\text{eff}} \) has the same small negative value already known from Ref. [14],

\[
\omega_{\text{eff}} = -1.08 \times 10^{-2} .
\]

Using Eq. (23) we can read back \( \alpha_Y \) as

\[
\alpha_Y = \sqrt{\frac{1}{1 + 1090^{3\omega_{\text{eff}}}}} = 0.183 .
\]

Therefore, from Eqs. (23) and (22) we can see that torsion amplifies the dark matter density for only a small factor,

\[
\rho_{\text{eff}} = \left( 1 + \alpha_Y^2 \right) \rho_{\text{DM}} = 1.03 \rho_{\text{DM}} ,
\]

and the bare torsion density parameter \( \Omega_{\text{DM}} \) corresponds finally to

\[
\Omega_{\text{DM}} = \frac{1}{1.03} \Omega_{\text{eff}} = 0.265 ,
\]

which coincides precisely with the one tabulated by PLANCK. Therefore, the departure from \( \Lambda \)CDM is tiny. The only density parameter changing a bit is the one associated with baryons, decreasing from \( \Omega_b^{(\Lambda \text{CDM})} = 0.049 \) to \( \Omega_b = 0.041 \). The difference is due to the energy density associated with \( \frac{3}{c^2 \kappa_4} \).

IV. CONCLUSIONS

We have proved that a nonvanishing spin tensor for cold dark matter mimics the effect of a small negative barotropic constant, \( \omega_{\text{eff}} = -1.08 \times 10^{-2} \). The small negative pressure created through this mechanism suffices to explain the Hubble parameter tension, starting with the CMB initial conditions measured by PLANCK. The work of Ref. [14] already proved that this small negative barotropic constant can solve the tension problem, but the novelty of the mechanism shown here is that it does
not require “exotic” physics. Instead, it only requires allowing one of the dark matter spin tensor components to have a non-zero value.

The departure from ΛCDM is tiny, and everything is in agreement with CMB’s initial conditions. The only density parameter that changes a bit in comparison to standard ΛCDM is the one of baryonic matter, lowering from the standard ΛCDM value 0.049 to 0.041. The difference is due to the small energy density $\frac{3}{2}Y^2$ associated with the spin. In the current model, the tiny negative pressure of this spin component solves the tension problem.

To use a negligible spin tensor for baryonic matter is an excellent approximation for practical purposes, due to its interactions and decoherence. In contrast, none of these conditions seem to be true for dark matter, and therefore to neglect its spin tensor seems unjustified. On the other hand, it is hard to imagine a direct measurement of the dark matter spin tensor in the foreseeable future. Since we have not detected dark matter particles, such measurement is still equivalent to measure torsion, a problem well known for its experimental difficulty [30]. However, a vanishing torsion background has effects on the amplitude and polarization of gravitational waves propagating on it (see Refs. [27, 29]). It may open a way to test these ideas experimentally and to check whether or not dark matter has spin and generates torsion.

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