Modified Brans-Dicke theory with space-time anisotropic parameters

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\textbf{Abstract.} We consider the ADM formalism of the Brans-Dicke theory and propose a space-time anisotropic extension of the theory by introducing five free parameters. We find that the resulting theory reveals many interesting aspects which are not present in the original BD theory. We first discuss the ghost instability and strong coupling problems which are present in the gravity theory without the full diffeomorphism symmetry and show that they can be avoided in a region of the parameter space. We also perform the post-Newtonian approximation and show that the constraint of the Brans-Dicke parameter $\omega_{BD}$ being large to be consistent with the solar system observations could be evaded in the extended theory. We also discuss that accelerating Universe can be achieved without the need of the potential for the Brans-Dicke scalar.

\textbf{Keywords:} modified gravity, gravity

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1 Introduction

There are many alternative theories and extensions of the Einstein’s general relativity. Motivations to consider them are diverse. They range from a classical straightforward extension of general relativity like the scalar-tensor theory to quantum gravity such as string theory. A comprehensive review can be found in recent articles [1]. The classical modification is based on the observational rationale that the predictions of the general relativity has been successful only on the solar system scale and has never been tested on the cosmological scale. Recently, it has another input from cosmology, that is, to search for the theoretical foundation of the accelerating Universe [3, 4] has become one of the most fundamental problems in modern cosmology. There are two main avenues. The first one is to assume an unknown source of energy which is repulsive in nature referred to as the dark energy and various proposals for its origin have been put forward [5]. The other is to consider an alternative theory of the Einstein’s theory of general relativity and modify the gravity in the IR limit. We are still awaiting more accurate observational data to distinguish between them. Therefore, modification of general relativity is a very important subject and is attracting a great deal of interest with extensive research activities being reported (see [1] and references therein).

Among the many alternatives, scalar-tensor theory [6] is especially interesting. In this theory, the scalar field enters in a nontrivial manner, specifically through non-minimal coupling term and it is based on solid foundation of general relativity. Typical examples would be the Brans-Dicke (BD) theory [7] and the gravity with a dilaton field arising for instance in the string theory. The scalar field in this type of theory provides many distinctive theoretical aspects, for example like induced gravity [8–10] and explanation of the behavior of the universe in the early inflationary stage as well as the late stage [6] in the cosmological context.

Especially, we attempt to modify the BD theory which is known as the prototype of the scalar-tensor theory and is one of the simplest alternative to Einstein’s theory of general relativity (GR). BD theory was firstly designed to properly incorporate the Mach’s principle into General relativity by replacing the gravitational constant \(G\) with a scalar field \(\phi\), which can vary with space and time. It is well-known that the observational constraints on BD
theory is restricted by the astronomical tests in the solar system, i.e., the BD parameter $\omega_{\text{BD}} > 50000$, being obtained from the observable Post-Newtonian parameter $\gamma_{\text{obs}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [11, 12].

Modifications of Brans-Dicke theory have been actively pursued. Most notable is the introduction of the potential for the Brans-Dicke scalar. For example, in refs. [13, 14], BD theory with a suitable self-interacting potential for the BD scalar field was proposed to account for the accelerated expanding of the Universe. In another example, it can be shown that $f(R)$ gravity in the metric formalism is equivalent to BD theory with a potential and the parameter $\omega_{\text{BD}} = 0$ [15].

Another alternative is a space-time anisotropic extension. It is motivated by the Horava-Lifshitz (HL) gravity [16] which was proposed as a quantum gravity theory. It is based on anisotropic scaling of space and time as a fundamental symmetry, abandoning the Lorentz symmetry at short distance. In the IR limit, the theory reduces to GR when the Lorentz violating parameter $\lambda$ becomes 1. When $\lambda \neq 1$, the HL gravity becomes a Lorentz violating Einstein-Hilbert action and the extended HL gravity [17, 18] reduces to Lorentz violating scalar-tensor theory in IR. There has been considerable interest in the HL gravity theory and various aspects have been investigated [19]. Among others, a BD type of generalization of the Horava-Lifshitz gravity was shown to be possible within the detailed balance condition in ref. [20], and the resulting theory reduces in the IR limit to the usual BD theory with a negative cosmological constant. On the other hand, our main interest is to investigate the possibility of modifying the original BD theory in such a way that the resulting theory can accommodate space-time anisotropy which breaks Lorentz symmetry at low energy and we will test whether such a theory is phenomenologically viable. From the theoretical point of view, breaking of the full diffeomorphism invariance coming from the anisotropy puts strong constraint in the resulting theory. The breaking causes the theory to have an additional scalar degree of freedom, whose behavior could create serious problems, such as a ghost or classical instability or the strong coupling problem. On the experimental side, it should pass the solar system test which requires $\omega_{\text{BD}}$ has to be big in the original BD theory.

In this paper, we propose a more straightforward modification of BD theory starting from the ADM formalism and discuss the above issues. A close look at the ADM formalism reveals that the original BD theory can be extended by introducing four more free parameters, which is related with space-time anisotropy, thereby breaking Lorentz symmetry. Depending on the values of these parameters, the theory shows many interesting aspects which are not present in the original BD theory. For example, the large value of $\omega_{\text{BD}}$ in the original BD theory to be consistent with the solar system observations could be evaded in the modified theory. It can be also shown that the ghost instability and strong coupling problems which are present in the gravity theory without the full diffeomorphism symmetry can be avoided within some range of the free parameters. Another distinctive feature is that the accelerating Universe could be achieved without the help of some potential contrary to the original BD theory.

The paper is organized as follows. In section 2, we consider space-time anisotropic BD action with five parameters which reduces to the ordinary (isotropic) BD action for particular choices of the four parameters. In section 3, we consider a quadratic action through the perturbation analysis of the space-time anisotropic BD gravity in order to investigate the pathological behaviors of scalar graviton mode. In section 4, we perform the perturbation analysis up to cubic order and check whether the strong coupling problem can be cured to this order. In section 5 we study the observational constraint on the space-time anisotropic
BD theory and compare with the experimental results. The conclusion and discussions are given in section 6.

2 Space-time anisotropic Brans-Dicke action

In order to construct a space-time anisotropic Brans-Dicke gravity with additional parameters, let us first consider an isotropic Brans-Dicke gravity in the ADM formalism, whose metric is parameterized by

\[ ds^2_{\text{ADM}} = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]  

(2.1)

where \( N \) is the lapse function, \( N_i \) is the shift function, and \( g_{ij} \) is the three dimensional metric.

For the ADM metric, Brans-Dicke action is given by

\[
S_{\text{BD}} = \int d^4x \sqrt{-g}(\phi R - \omega_{\text{BD}} \phi \nabla_\mu \phi \nabla^\mu \phi),
\]

(2.2)

where \( \phi \) and \( \omega_{\text{BD}} \) is the Brans-Dicke scalar and Brans-Dicke parameter respectively. \( g_{ij} \) and \( R \) are the three dimensional metric tensor and the Ricci scalar. The extrinsic curvature \( K_{ij} \) and \( \pi \) take the forms

\[
K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i),
\]

(2.3)

\[
\pi = \frac{1}{N}(\phi - \nabla_i \phi N^i),
\]

(2.4)

where the dot denotes differentiation with respect to \( t \). The first line of (2.2) is the kinetic (K) term and second line is the potential (V) term.

We first notice that the BD gravity (2.2) preserves the full diffeomorphism symmetry. We can extend it to space-time anisotropic BD (aBD) theory by introducing free parameters which explicitly break the diffeomorphism invariance and consider the most general action as follows\(^1\)

\[
S_{\text{aBD}} = S^K_{\text{aBD}} + S^V_{\text{aBD}},
\]

(2.5)

where

\[
S^K_{\text{aBD}} = \int dtd^3x N \sqrt{g} \left\{ \phi \left( K_{ij} K^{ij} - \lambda K^2 \right) - 2\eta_1 K \pi + \omega_1 \phi^{-1} \pi^2 \right\},
\]

(2.6)

\[
S^V_{\text{aBD}} = \int dtd^3x N \sqrt{g} \left( \phi R - 2\eta_2 \nabla_i \nabla^i \phi - \omega_2 \phi^{-1} \nabla_i \phi \nabla^i \phi \right).
\]

(2.7)

\(^1\)This extension is based on the fact that each term of \( 2K \pi \), \( 2g^{ij} \nabla_i \nabla_j \phi \), \( \phi^{-1} \pi^2 \), and \( \phi^{-1} g^{ij} \nabla_i \phi \nabla_j \phi \) in the action (2.2) is invariant under foliation-preserving diffeomorphism (2.8). We note that the action (2.5) is clearly different from a scalar tensor theory written in the unitary gauge \[21, 22\]. This is because the scalar tensor model obtained from applying the St"uckelberg trick does not produce our model.
In this action, the parameters $\lambda$, $\eta_{1,2}$, $\omega_{1,2}$ are dimensionless constants. Note that when $\lambda \neq 1$, $\eta_1 \neq 1$, $\eta_2 \neq 1$, and $\omega_1 \neq \omega_2$, the action (2.5) does not have the full diffeomorphism invariance, but is invariant under the foliation-preserving diffeomorphism:

$$\delta x^i = -\zeta^i(t,x), \quad \delta t = -f(t),$$

$$\delta g_{ij} = \delta_t \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \delta g_{ij},$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + f \dot{N}_i + \dot{f} N_i,$$

$$\delta N = \zeta^j \partial_j N + f \dot{N} + \dot{f} N,$$

$$\delta \phi = \zeta^k \partial_k \phi + f \dot{\phi}. \quad (2.8)$$

We point out that the parameter $\eta_1$ ($\eta_2$) is associated with deviation from the BD theory along the normal (spatial) direction of the leaves of the foliation. Moreover, we introduced two BD parameters $\omega_1$ and $\omega_2$ which need not to be the same in the anisotropic case. With the choice of the parameters of $\lambda = 1$, $\eta_1 = \eta_2 = 1$, and $\omega_1 = \omega_2 = \omega_{BD}$, the action (2.5) reduces to (isotropic) Brans-Dicke action (2.2). For a constant scalar field, it becomes Lorentz violating Einstein-Hilbert action with an anisotropic parameter $\lambda$. Note also that the space-time anisotropic BD action (2.5) is still invariant with respect to global (isotropic) conformal transformation

$$N \rightarrow \Omega N, \quad N_i \rightarrow \Omega^2 N_i, \quad g_{ij} \rightarrow \Omega^2 g_{ij}, \quad \phi \rightarrow \Omega^{-2} \phi, \quad (2.9)$$

where $\Omega$ is an arbitrary constant. We further remark that for $\Omega = \Omega(t,x)$, the conformal symmetry under the transformation (2.9) can be extended to local conformal symmetry if one chooses

$$\eta_1 = -\frac{1}{2}(1 - 3\lambda), \quad \omega_1 = \frac{3}{4}(1 - 3\lambda), \quad \eta_2 = 1, \quad \omega_2 = -\frac{3}{2}, \quad (2.10)$$

which implies that for $\lambda = 1$ case, i.e., $\eta_1 = \eta_2 = 1$, $\omega_1 = \omega_2 = -3/2$, the space-time anisotropic BD action (2.5) reduces to a conformally invariant isotropic action [23].

### 3 Scalar graviton mode in the quadratic action

In order to investigate the behavior of graviton mode in the Minkowski background ($g_{ij} = \delta_{ij}$, $\phi = \phi_0$), we first consider the following scalar perturbations of the metric and scalar field for the Minkowski background up to the linear order

$$N = e^\alpha, \quad N_i = \partial_i \beta, \quad g_{ij} = e^{-2\psi} \delta_{ij}, \quad \phi = \phi_0 + \varphi. \quad (3.1)$$

In these perturbations, $\alpha$, $\beta$, $\psi$, and $\varphi$ are functions of space and time. Substituting the above perturbations into the action (2.5), one finds the quadratic action ($\Box \equiv \partial_t \partial^t$)

$$S^{(2)} = \int dt d^3x \left\{ 3\phi_0(1 - 3\lambda)\dot{\psi}^2 + 2\phi_0(1 - 3\lambda)\varphi \Box \beta + \phi_0(1 - \lambda)(\Box \beta)^2 + 2\eta_1(3\dot{\varphi} + \dot{\varphi} \Box \beta) \\
+ \omega_1 \phi_0^{-1} \dot{\varphi}^2 + 4\varphi \Box \psi + 4\phi_0 \alpha \Box \psi - 2\phi_0 \psi \Box \varphi - 2\eta_2 \alpha \Box \varphi - 2\phi_0^{-1}(\partial_t \varphi)^2 \right\}. \quad (3.2)$$

Variation of the fields $\alpha$ and $\beta$ of the quadratic action leads to the (local) Hamiltonian and momentum constraints

$$\varphi - \frac{2}{\eta_2} \phi_0 \psi = 0, \quad (3.3)$$

$$\Box \beta + \frac{1}{1 - \lambda} \left( 1 - 3\lambda + \frac{2\eta_1}{\eta_2} \right) \dot{\psi} = 0, \quad (3.4)$$

respectively.
From the constraints (3.3) and (3.4), we can replace the fields $\varphi$ and $\Box \beta$ with $\psi$. After taking the integration by parts, the quadratic action (3.2) can be rewritten as
\[
S^{(2)} = 2 \int dtd^3x \left\{ -\frac{1}{c^2_\psi} \dot{\psi}^2 + \left( \frac{2\omega_2}{\eta_2^2} + \frac{4}{\eta_2} - 1 \right) \phi_0 \Box \psi \right\}, \tag{3.5}
\]
where
\[
c^2_\psi = \frac{1 - \lambda}{\phi_0} \left\{ 3\lambda - 1 + 2\frac{\eta_1^2}{\eta_2^2} - \frac{4\eta_1}{\eta_2} + \frac{2(\lambda - 1)\omega_1}{\eta_2^2} \right\}^{-1}. \tag{3.6}
\]
We notice two things. First is that the perturbation $\varphi$ can be completely eliminated due to the Hamiltonian constraint (3.3) in favor of the scalar graviton mode. The other is that in the limit $\eta_2 \to \infty$, we find the action is exactly the same with the quadratic action in the HL gravity (for $\phi_0 = 1$)
\[
S^{(2)}_{\text{HL}} = 2 \int dtd^3x \left\{ -\frac{1}{c^2_{\psi\text{HL}}} \dot{\psi}^2 - \psi \Box \psi \right\}, \tag{3.7}
\]
with $c^2_{\psi\text{HL}} = (1 - \lambda)/(3\lambda - 1)$. It can be easily checked that in the quadratic action (3.7), the scalar graviton $\psi$ has serious problems, i.e., classical instability in the case of $c^2_{\psi\text{HL}} < 0$ or the presence of ghost when $c^2_{\psi\text{HL}} > 0$. Interestingly, in our framework such pathological behaviors do not show up with
\[
c^2_\psi < 0, \quad 0 < \frac{2\omega_2}{\eta_2^2} + \frac{4}{\eta_2} - 1, \tag{3.8}
\]
which can be satisfied with a wide range of the parameters region.

Before passing, we make the following remarks on the number of scalar degrees of freedom in our proposed model. There seem to be two scalar modes that are propagating at the perturbation level; one coming from the BD scalar and the other scalar from the gravitational sector which is present whenever $\lambda \neq 1$ [16]. However, as we see from the perturbation equation (3.3) the Hamiltonian constraint yields a direct connection between the two scalars. It is to be noticed that this kind of relation is not peculiar to space-time anisotropic theory. It is persistent even in the covariant theory as long as the scalar field is non minimally coupled like "$\phi R$", and this coupling is the origin of the connection between the two scalar modes. That is why there is only one scalar degree of freedom in the pure BD theory, for example, which corresponds to $\eta_2 = 1$ in eq. (3.3). In fact, this would be true to more generalized non minimal coupling like the scalar tensor theory of the type $f(\phi)R$. However, it can be checked that on a cosmological background with time dependent $\phi_0(t)$, $\alpha^2$ term is present in the quadratic action and subsequently, one is left with two independent

\[\text{This limit of } \eta_2 \to \infty \text{ is reminiscent of } \alpha \to \infty \text{ which is the coefficient of } (\nabla_i N/N)^2 \text{ term introduced in ref. [17] (see also ref. [24] for details).}\]
\[\text{It is well-known that a propagating scalar graviton mode does not exist in Einstein-Hilbert action with the full diffeomorphism invariance. More precisely, one can check from eq. (3.2) that in Einstein-Hilbert action (} \lambda = 1, \varphi = 0, \eta_{1,2} = \omega_{1,2} = 0) \text{ and conformally equivalent case (} \lambda = 1, \eta_{1,2} = 1, \omega_{1,2} = -3/2), \text{ the Hamiltonian and momentum constraints yield } S^{(2)}_{\psi} = 0, \text{ which implies that there is no the propagating scalar graviton mode (see appendix for details).}\]
\[\text{We would like to thank the anonymous referee for suggesting this point.}\]
degrees of freedom $\psi$ and $\varphi$. In the appendix we elaborate on these points by explicitly showing the details.

Let us focus on the special cases $\eta_1 = \eta_2$ (otherwise noticed) which have rather special consequences. For $\eta_1 = \eta_2 = \eta$, the quadratic action (3.5) becomes

$$S^{(2)} = 2\phi_0 \int dt d^3x \left\{ \left( 3 + \frac{2\omega_2}{\eta^2} \right) \dot{\psi}^2 + \left( \frac{2\omega_2}{\eta^2} + \frac{4}{\eta} - 1 \right) \psi \Box \psi \right\},$$

and it shows that the dependency on the parameter $\lambda$ for the scalar graviton completely disappears. This also implies the quadratic action (3.5) with (3.6) is devoid of the singularity associated with the limit $\lambda \to 1$. The condition (3.8) of avoiding the classical instability and ghost mode in this case is given by

$$\omega_1 > -\frac{3}{2} \eta^2, \quad \omega_2 > -2 + \frac{1}{2} (\eta - 2)^2.$$

Note that $\omega_2$ always has to be greater than $-2$ for any value of $\eta$. For $\eta = 1$, $\omega_1 > -\frac{3}{2}$ and $\omega_2 > -\frac{3}{2}$, which coincides the ghost free case of the isotropic BD theory [25], $\eta_1 = \eta_2 = 1$, $\omega_1 = \omega_2 = \omega_{BD} > -\frac{3}{2}$. Another case of interest is when $\eta = 2$. In this case, we have $\omega_1 > -6$, $\omega_2 > -2$. We will see that for this particular value, $\omega_2$ does not have to be a large value in order to pass the solar test as in the original BD theory [see eq. (5.7)]. As $\eta$ becomes larger, $\omega_1$ can be more negative, whereas $\omega_2$ shifts away from the value $-2$ in the positive direction.

4 Strong coupling in the cubic action

In the previous section, we investigated the behavior of graviton mode in the quadratic action and showed that the pathological behavior can be cured in a wide range of parameters with the condition of eq. (3.8) being satisfied. Another important task is to examine if the theory gets strongly coupled at the cubic action level in the limit of $\lambda \to 1$ [26, 27].

In order to study with the cubic order interaction terms [28], we first recall the non-linear scalar perturbations (3.1) around the Minkowski background and substitute this into the action (2.5). After some tedious manipulations one can find that the cubic-order action is given by

$$S^{(3)} = \int dt d^3x \left\{ -2\varphi (\partial \psi)^2 + 2\phi_0 \psi (\partial \psi)^2 - 2\alpha \phi_0 (\partial \psi)^2 - 4\varphi \psi \Box \psi + 4\alpha \varphi \Box \psi + 2\phi_0^2 \Box \psi + 2\phi_0 \psi \Box \psi - 4\phi_0 \psi \Box \psi - \eta_2 \alpha \Box \varphi - \eta_2 \psi \Box \varphi + 2\eta_2 \alpha \Box \varphi - 2\eta_2 \psi \Box \varphi - 2\eta_2 \Box \varphi \right\}$$

$$+ 2\eta_2 \alpha \varphi \Box \varphi - 2(1 - 3\lambda) \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \phi_0 \psi \Box \beta$$

$$+ 4\phi_0 \psi \Box \beta \dot{\psi} - \lambda \phi_0 \psi \Box \beta - 3(1 - 3\lambda) \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \phi_0 \psi \Box \beta$$

$$+ 2\alpha (\partial \beta)^2 + \varphi \alpha \Box \beta - 2(1 - 3\lambda) \varphi \Box \beta + \varphi \Box \beta$$

$$+ 2(1 - 3\lambda) \psi \Box \beta - \lambda \phi_0 \psi \Box \beta - 2(1 - 3\lambda) \psi \Box \beta - 2(1 - 3\lambda) \psi \Box \beta$$

$$+ 2(1 - 3\lambda) \psi \Box \beta - \lambda \psi \Box \beta - 2(1 - 3\lambda) \psi \Box \beta - 2(1 - 3\lambda) \psi \Box \beta$$

$$+ \alpha \psi \Box \beta - 2\eta_1 \left[ 9\psi \varphi \varphi - 3\alpha \psi \varphi + 3\psi \varphi \varphi \dot{\beta} + \varphi \dot{\beta} + \varphi \varphi \dot{\beta} + \varphi \dot{\beta} \right]$$

$$+ \alpha \psi \Box \beta - \omega_1 \left( \phi_0^{-1} \alpha \varphi^2 + 3\phi_0^{-1} \psi \varphi^2 + \varphi \varphi^2 + 2\phi_0^{-1} \dot{\varphi} \dot{\varphi} \right) \right\}.$$
Using the first-order Hamiltonian and momentum constraints (3.3), (3.4) obtained in the previous section, the above action (4.1) reduces to

\[ S^{(3)} = \phi_0 \int dt d^3x \left\{ A_1 \psi (\partial_i \psi)^2 + A_2 \dot{\psi} \partial_i \psi \partial_i \left( \frac{\dot{\psi}}{\delta} \right) + A_3 \dot{\psi} \left( \partial_i \partial_i \frac{\dot{\psi}}{\delta} \right) + A_4 \dot{\psi} \psi{^2} \right\} , \quad (4.2) \]

where

\[ A_1 = \frac{8 \omega_2}{\eta_2^2} + \frac{4 \omega_2}{\eta_2} + \frac{12}{\eta_2} - 2, \]

\[ A_2 = -\frac{8 \eta_1 + 4(1 - 3 \lambda) \eta_2}{(-1 + \lambda)^2 \eta_2^2} \left\{ 2 \eta_1^2 - 4 \eta_1 \eta_2 + (3 \lambda - 1) \eta_2^2 + 2(\lambda - 1) \omega_1 \right\}, \]

\[ A_3 = -\frac{3 \eta_2 - 2}{(-1 + \lambda)^2 \eta_2^2} \left\{ 2 \eta_1 + (1 - 3 \lambda) \eta_2 \right\}^2, \]

\[ A_4 = \frac{1}{(-1 + \lambda)^2 \eta_2^2} \left\{ (1 + \lambda)(3 \lambda - 1)(3 \eta_2 - 2) \eta_2^2 - 4 \eta_1^2 (2 \lambda + 3 \eta_2 (\lambda - 2)) - 4 \eta_1 \eta_2 (2 - 6 \lambda + 3 \eta_2 (1 + \lambda)) - 4 \omega_1 (\lambda - 1)^2 (3 \eta_2 + 2) \right\} . \]

Note that when \( \eta_2 \to \infty \) the above action (4.2) once again reduces to the cubic action in the HL gravity as

\[ S^{(3)}_{HL} = 2 \int dt d^3x \left\{ -\psi (\partial_i \psi)^2 + \frac{2}{c_{HL}} \dot{\psi} \partial_i \psi \partial_i \left( \frac{\dot{\psi}}{\delta} \right) + \frac{3}{2} \left[ -\frac{1}{c_{HL}} \psi \left( \partial_i \partial_i \frac{\dot{\psi}}{\delta} \right) + \frac{2 c_{HL}^2 + 1}{2 c_{HL}} \dot{\psi} \psi{^2} \right] \right\} . \quad (4.3) \]

It is well-known that the above action is confronted with the strong coupling problem in the limit \( \lambda \to 1 \) [26, 27]. On the other hand, in the case of \( \eta_1 = \eta_2 \equiv \eta \) of our main focus, the above coefficients \( A_2, A_3, A_4 \) considerably simplify to become

\[ A_2 = 12 \left( 3 + \frac{2 \omega_1}{\eta^2} \right), \quad A_3 = -9 \left( 3 - \frac{2}{\eta} \right), \quad A_4 = 9 - \frac{6}{\eta} - \frac{4 \omega_1 (2 + 3 \eta)}{\eta^2}, \]

which again shows that there is no dependency on the parameter \( \lambda \) for the scalar graviton up to the cubic order. To see this more closely, we introduce the canonical variable \( \hat{\psi} = \sqrt{2} \psi / |c_{\psi}| \) with \( |c_{\psi}|^{-2} = \phi_0 (3 + 2 \omega_1 / \eta^2) \), in terms of which the quadratic action (3.9) becomes

\[ S^{(2)} = \int dt d^3x \left\{ \hat{\psi}^2 + \left( \frac{2 \omega_2}{\eta^2} + \frac{4}{\eta} - 1 \right) \phi_0 |c_{\psi}|^2 \hat{\psi} \partial_i \hat{\psi} \partial_i \right\} . \quad (4.4) \]

By using the canonical variable \( \hat{\psi} \), the cubic action (4.2) can be written as

\[ S^{(3)} = \frac{1}{2 \sqrt{2}} \int dt d^3x \left\{ \left( \frac{8 \omega_2}{\eta^2} + \frac{4 \omega_2}{\eta^2} + \frac{12}{\eta} - 2 \right) \phi_0 |c_{\psi}|^3 \hat{\psi} (\partial_i \hat{\psi})^2 + 12 |c_{\psi}|^2 \hat{\psi} \partial_i \hat{\psi} \partial_i \left( \frac{\hat{\psi}}{\delta} \right) \right. \]

\[ -9 \left( 3 - \frac{2}{\eta} \right) \phi_0 |c_{\psi}|^3 \hat{\psi} \left( \partial_i \partial_i \frac{\hat{\psi}}{\delta} \right)^2 + \left( \left( 27 + \frac{6}{\eta} \right) \phi_0 |c_{\psi}|^3 - 2 \left( 3 + \frac{2}{\eta} \right) |c_{\psi}|^2 \right) \hat{\psi}^2 \left. \right\} . \quad (4.5) \]
This action clearly shows non dependency on the parameter $\lambda$ and that we do not need to consider any fine-tuning for the interactions to be regular.

Before closing this section, we comment on the case with local conformal invariance. It turns out that with the choice of the parameters as in (2.10), the time dependent cubic terms of $\psi$ in the action (4.2) vanish and we get

$$S^{(3)} = \phi_0 \int dt d^3x \left\{ -8\psi(\partial_i \psi)^2 \right\}.$$ \hspace{1cm} (4.6)

This shows that the strong coupling problem does not show up at the cubic-order perturbation of the action with the local conformal invariance.

5 Cosmological constraint on space-time anisotropic BD theory

In this section we investigate the cosmological tests, which can provide the observational constraints for alternative theories of gravitation. For this purpose, we first consider the solar system test. Taking into account the static part of the perturbations in section 3, the Lagrangian (3.2) with a static point-like source term of mass $M_s$ can be written by [29, 30]

$$L_{\text{static}} = \left\{ 4\varphi \Box \psi + 4\phi_0 \alpha \Box \psi - 2\eta_2 \varphi \Box \psi - \omega_2 \phi_0^{-1}(\partial_i \varphi)^2 \right\} - M_s \alpha \delta^3(x).$$ \hspace{1cm} (5.1)

From varying for the fields $\psi$, $\alpha$, and $\varphi$, we obtain the following equations:

$$\Box \varphi + \phi_0 \varphi - \phi_0 \Box \psi = 0,$$

$$4\phi_0 \Box \psi - 2\eta_2 \Box \varphi - \omega_2 \phi_0^{-1}(\partial_i \varphi)^2 - M_s \delta^3(x) = 0,$$

$$2 \Box \psi - \eta_2 \Box \alpha + \omega_2 \phi_0^{-1} \Box \varphi = 0.$$ \hspace{1cm} (5.2)

The corresponding solutions to eq. (5.2) are

$$\psi = \frac{\eta_2 + \omega_2}{2 + \omega_2} \alpha, \quad \varphi = \frac{(\eta_2 - 2)\phi_0}{\omega_2 + 2} \alpha,$$ \hspace{1cm} (5.3)

where $\alpha$ is

$$\alpha = -\frac{(\omega_2 + 2)M_s}{8\pi\phi_0(2\omega_2 - \eta_2^2 + 4\eta_2)|x|}.$$ \hspace{1cm} (5.4)

It is well-known that the linear expansion of the metric (3.1) can be expressed by the Newtonian potential $u$ and the Post-Newtonian parameter $\gamma$ as follows:

$$g_{00} = -N^2 = -1 - 2\alpha = -1 + 2u,$$

$$g_{ij} = (1 - 2\psi)\delta_{ij} = (1 + 2\gamma u)\delta_{ij},$$ \hspace{1cm} (5.5)

which implies that for the solution (5.3) $u = -\alpha$ and the parameter $\gamma$ can be written by

$$\gamma = \frac{\psi}{\alpha} = \frac{\eta_2 + \omega_2}{2 + \omega_2} = 1 + \frac{\eta_2 - 2}{\omega_2 + 2}.$$ \hspace{1cm} (5.6)

Let us concentrate on the case $\eta_1 = \eta_2 = \eta$ again. The solar system tests currently constrain $\gamma$ as [11]

$$|\gamma - 1| < 2 \times 10^{-5}.$$ \hspace{1cm} (5.7)
For the case of the BD theory ($\eta = 1$, $\omega_2 = \omega_{BD}$), we see that the above condition (5.8) together with (5.7) restricts the region of $\omega_{BD}$ to

$$
\omega_{BD} > 50000. \tag{5.9}
$$

Note, however, that unlike the BD theory, this large value constraint on $\omega_{BD}$ (5.9) can be circumvented in the space-time anisotropic BD case when $\eta$ is very close to 2 as can be seen from

$$
|\eta - 2| < 2 \times 10^{-5}(\omega_2 + 2) \tag{5.10}
$$

which is obtained from eq. (5.7). Especially, when $\eta = 2$, any $\omega_2 > -2$ is allowed which is also consistent with the stability and no-ghost condition of eq. (3.10). In addition, for the solution (5.4) of $\alpha$ one can find the effective Newton’s constant $G_{\text{eff}}$ as

$$
G_{\text{eff}} = \frac{\omega_2 + 2}{8\pi\phi_0(2\omega_2 - \eta^2 + 4\eta \omega_2)}, \tag{5.11}
$$

being obtained from the Newtonian potential, i.e., $u = G_{\text{eff}}M_s/|x|$. In the General Relativity limit ($\omega_2 = \omega_{BD} \to \infty$) we have $G_{\text{eff}} \to G_N$ for the substitution of $\phi_0 \leftrightarrow 1/(16\pi G_N)$ with the Newton’s constant $G_N$. It is interesting to notice that in the space-time anisotropic BD theory the reduction $G_{\text{eff}} \to G_N$ can be achieved alternatively with appropriate choices of the free parameters different from the BD theory. Especially, when choosing the $\eta = 2$ once again, $G_{\text{eff}}$ reduces to the Newton’s constant $G_N$ irrespective of the value of $\omega_2$.

On the other hand, we can also obtain the cosmological constraint from a ratio factor of $G_c/G_{\text{eff}}$ ($G_c$ being the effective cosmological gravitational constant defined in eq. (5.12)) which is related to the primordial helium abundance [31]. Substituting the FRW metric of ansatz and $\phi = \phi(t)$ into the action (2.5) with matter term included and varying the action with respect to the lapse function $N(t)$, we find the standard Friedmann equation can be written as

$$
H^2 = \frac{8\pi G_c}{3}\rho, \tag{5.12}
$$

where $H$ is the Hubble parameter, $\rho$ is total matter density of the Universe, and the effective cosmological gravitational constant $G_c$ is given by

$$
G_c = \frac{1}{8\pi\phi(3\lambda - 1)}. \tag{5.13}
$$

It is pointed out that $G_c$ is equivalent to $G_N$ for the substitution of $\phi = \phi_0 \leftrightarrow 1/(16\pi G_N)$ in the limit of $\lambda \to 1$. In our case for $\phi = \phi_0$, we obtain the cosmological constraint from the observational bound [31] of $G_c/G_{\text{eff}}$ as

$$
\left| \frac{G_c}{G_{\text{eff}}} - 1 \right| = \frac{3(\lambda - 1)}{3\lambda - 1} + \frac{(\eta - 2)^2}{(3\lambda - 1)(\omega_2 + 2)} < 0.125. \tag{5.14}
$$

Comparing (5.10) with the bound (5.14) by using (3.10), we find that the allowed range of the parameter $\lambda$ is

$$
-\frac{2}{3} < \lambda - 1 < 0.095 \approx 10^{-1}, \tag{5.15}
$$

which imposes a rather loose constraint around $\lambda = 1$ [17].

\footnote{In synchronous time $t$, the cosmological ADM metric (2.1) is given by $N = 1$, $N_i = 0$, and $g_{ij} = a(t)^2 \eta_{ij}$, where $a(t)$ is the scale factor [17, 32, 33], which corresponds to the FRW metric.}
Conclusion and discussion

In this paper, we constructed a space-time anisotropic Brans-Dicke gravity, which includes five free parameters, i.e., $\lambda$, $\eta_1$, $\eta_2$, $\omega_1$, and $\omega_2$. In the case of $\lambda = \eta_1 = \eta_2 = 1$ and $\omega_1 = \omega_2 = \omega_{BD}$, the gravity reduces to the ordinary Brans-Dicke action with a BD parameter $\omega_{BD}$. When fixing the scalar field $\phi$ to be a constant value, it becomes Lorentz-violating Einstein-Hilbert action with an anisotropic parameter $\lambda$.

We found that in the perturbation around the Minkowski background and constant scalar field the scalar graviton at the quadratic as well as cubic order in the space-time anisotropic BD action does not show any pathological behaviors within some parameter range. This suggests that the space-time anisotropic BD gravity can be a viable theoretical model. Especially, the case $\eta_1 = \eta_2$ reveals intriguing property of $\lambda$ independence of both the quadratic and cubic actions. Also in the context of cosmological model, we have checked that unlike the case of the BD theory which is imposed by $\omega_{BD} > 50000$ to be consistent with the experimental observations, this large value can be evaded in the space-time anisotropic BD theory with the special value of $\eta = 2$ which is one of the novel feature of the space-time anisotropic BD theory. But the origin of these special properties associated with $\eta = 2$ is a puzzling aspect which needs to be investigated further.

We conclude with a couple of comments on the issues related to the BD theory. First, it is well-known that in the standard BD model without potential, there is no accelerated expansion, so one has to consider a potential term \[13, 14, 25\]. However, it is found in the space-time anisotropic BD model including matter contribution that we have a de Sitter solution ($H = \text{const.}$) for the FRW metric, given by

\[ a = a_0 e^{Ht}, \quad \phi = \phi_0 e^{-3(1+\omega_m)Ht}, \quad \rho_m = \rho_0 a^{-3(1+\omega_m)} \]  

(6.1)

with constants $a_0$, $\phi_0$, $\rho_0$ and $P_m = \omega_m \rho_m$. In particular, $\lambda$ and $\rho_0$ satisfy the following relation:

\[ \lambda = 2\eta + (1 - \omega_m^2)\omega_1 + 1/3, \quad \rho_0 = -9\omega_m \phi_0 a_0^{3(1+\omega_m)} H^2(\eta + \omega_1(1 + \omega_m)). \]

(6.2)

For $\rho_m = 0$ (vacuum) case, the above relation yields

\[ \omega_1 = -2\eta + \lambda - 1/3. \]

(6.3)

It should be pointed out that in the BD limit, i.e., $\eta \to 1$, $\lambda \to 1$ the parameter $\omega_1 = \omega_{BD}$ becomes a negative value, $\omega_{BD} = -4/3$ which conflicts with the lower bound $\omega_{BD} > 50000$, even if it satisfies the ghost-free condition of $\omega_{BD} > -3/2$ [25]. However, in the space-time anisotropic BD case this problem can be circumvented by choosing $\eta$ to be $\eta > 4/3$ or $\eta < 0$ for the bound (5.15) which is obtained from substituting (6.3) into the ghost-free condition of eq. (3.10). It is worth noticing that the allowed range is not in conflict with the special case of $\eta = 2$.

The second one is a speculation about quantum gravity. In refs. [34–36], the one-loop effective action in the pure BD theory is calculated and it is shown that the BD theory is not renormalizable. However, it becomes a renormalizable theory if curvature squared terms and a scale invariant self-interaction are included [9]. Likewise, we suspect that self-interacting scale invariant anisotropic BD with curvature squared terms [16–18, 37] might constitute a UV completion of the theory. The details are beyond the scope of the present paper, but the subject deserves further investigations.
A The number of scalar modes in gravity models

We consider the space-time anisotropic BD action (2.5) and its perturbative action (3.2), which produce various limits:

(i) GR ($\phi = \phi_0$, $\lambda = 1$, $\eta_{1,2} = 0$, $\omega_{1,2} = 0$)

For this case, the quadratic equation (3.2) reduces simply to

$$S_{GR}^{(2)} = 2\phi_0 \int dt d^3x \left\{ -3\dot{\psi}^2 - 2\dot{\psi} \Box \beta + 2\alpha \Box \psi - \psi \Box \psi \right\},$$

which leads to the Hamiltonian and momentum constraint, $\Box \psi = 0$ and $\dot{\psi} = 0$, respectively. Therefore the number of scalar modes is zero because of $S_{GR}^{(2)} = 0$.

(ii) HL gravity ($\phi = \phi_0$, $\eta_{1,2} = 0$, $\omega_{1,2} = 0$)

The quadratic equation (3.2) in this case reduces to

$$S_{HL}^{(2)} = \left\{ \int dt d^3x \left( 3\phi_0 (1 - 3\lambda) \dot{\psi}^2 + 2\phi_0 (1 - 3\lambda) \dot{\psi} \Box \beta + \phi_0 (1 - \lambda) (\Box \beta)^2 - 2\phi_0 \psi \Box \psi \right) \right. + 4\phi_0 \int dt \alpha(t) \int d^3x \Box \psi \right\}. \quad (A.1)$$

The second line in eq. (A.1) is due to the projectability\footnote{See refs. [17, 24] for the non-projectability case in the HL gravity.} condition of the original HL gravity, which implies that $\alpha$ is a function of $t$ only, $\alpha = \alpha(t)$. Variation of the fields $\alpha$ and $\beta$ of the quadratic action leads to the Hamiltonian and momentum constraints

$$\int d^3x \Box \psi = 0 \quad \text{and} \quad \Box \beta + \frac{1 - 3\lambda}{1 - \lambda} \dot{\psi} = 0.$$

Finally the quadratic action becomes

$$S_{HL}^{(2)} = 2\phi_0 \int dt d^3x \left\{ -\frac{1}{c_{HL}^2} \dot{\psi}^2 - \psi \Box \psi \right\},$$

with $c_{HL}^2 = (1 - \lambda)/(3\lambda - 1)$, which is the same as (3.7). Thus in the HL gravity the number of scalar modes is one.

(iii) BD gravity ($\lambda = 1$, $\eta_{1,2} = 1$, $\omega_{1,2} = \omega_{BD}$)

In this case, the quadratic equation (3.2) is given by

$$S_{BD}^{(2)} = \int dt d^3x \left\{ -6\phi_0 \dot{\psi}^2 - 4\phi_0 \dot{\psi} \Box \beta + 2(3\dot{\phi} \dot{\psi} + \dot{\phi} \Box \beta) + \omega_{BD} \phi_0^{-1} \dot{\phi}^2 + 4\phi \Box \psi + 4\phi_0 \alpha \Box \psi - 2\phi_0 \psi \Box \psi - 2\alpha \Box \phi - \omega_{BD} \phi_0^{-1} \partial_i \phi^2 \right\}.$$

The corresponding Hamiltonian and momentum constraints are

$$\Box \phi - 2\phi_0 \Box \psi = 0 \quad \text{and} \quad 2\phi_0 \dot{\psi} - \dot{\phi} = 0,$$
respectively. As a result, the quadratic action can be written as

$$S^{(2)}_{BD} = 2\phi_0 \int dt d^3x \left\{ (3 + 2\omega_{BD}) \dot{\psi}^2 + (3 + 2\omega_{BD}) \psi \Box \psi \right\}.$$  

Therefore the number of scalar modes is one. Furthermore, this aspect remains even for scalar tensor gravity of the type $f(\phi)R$ with $f$ an arbitrary function of $\phi$, where the Hamiltonian constraint would yield $\Box \varphi - 2f(\phi_0) \Box \psi / f'(\phi_0) = 0$. In this case, also a scalar and gravitational scalar modes are interrelated and the propagating degree of freedom for scalar modes is one.

However, the above property for the flat Minkowski background does not persist on a FRW universe. This is simply because in time dependent background, the presence of $\alpha^2$ term prevents a direct connection between the two scalar modes as in eq. (3.3). A straightforward calculation in the FRW background\textsuperscript{7} yields (terms containing only $\alpha$)

$$S^{(2)}_\alpha = a^3 \int dt d^3x \left\{ \alpha \left( 6\phi_0 H(1-3\lambda) - 6\eta_1 \dot{\phi}_0 \right) \dot{\psi} + \left( 6\eta_1 H - 2\omega_1 \phi_0^{-2} \dot{\phi}_0 \right) \varphi ight. 
+ \left( 9\phi_0 H^2(1-3\lambda) - 18\eta_1 H \dot{\phi}_0 + 3\omega_1 \phi_0^{-1} \dot{\phi}_0^2 \right) \psi - \left( 3H^2(1-3\lambda) - \omega_1 \phi_0^{-2} \phi_0^2 \right) \varphi 
+ \left( 2\phi_0 H(1-3\lambda) - 2\eta_1 \phi_0 \Box \beta + 4a^{-2} \phi_0 \Box \psi - 2a^{-2} \eta_2 \Box \varphi \right) \right. 
+ \alpha^2 \left[ \frac{3}{2} \phi_0 H^2(1-3\lambda) - 3\eta_1 H \phi_0 + \frac{\omega_1}{2} \phi_0^{-1} \phi_0^2 \right] \right\}, \quad (A.2)$$

which leads to the Hamiltonian constraint as follows:

$$\alpha F_1 + F_2(\psi, \varphi) = 0. \quad (A.3)$$

Here $F_1$ does not contain any of the two scalar modes $\psi, \varphi$ and $F_2(\psi, \varphi)$ is a function of the two fields. Note that in the Minkowski limit ($a \to 1, H \to 0$, and $\phi_0 = \text{const.}$), $F_1$ vanishes and $F_2$ yields just eq. (3.3). The Hamiltonian constraint (A.3) shows that two scalar modes are no longer dependent upon each other. Substitution of $\alpha$ from (A.3) into the full action containing all the other terms will give a perturbed quadratic action with two independent degrees of freedom $\psi$ and $\varphi$.

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\textsuperscript{7}The scalar perturbations of the metric and scalar field for the FRW background are given by

$$N = e^{\alpha}, \quad N_i = a^2 \partial_i \beta, \quad g_{ij} = a^2 e^{-2\phi} \delta_{ij}, \quad \phi = \phi_0 + \varphi,$$

where the scale factor $a$ and $\phi_0$ are function of $t$. 

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