On the Seesaw Scale in Supersymmetric SO(10) Models

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ABSTRACT: The seesaw mechanism, which is responsible for the description of neutrino masses and mixing, requires a scale lower than the unification scale. We propose a new model with spinor superfields playing important roles to generate this seesaw scale, with special attention paid on the Goldstone mode of the $U(1)_{B-L}$ symmetry breaking.

KEYWORDS: Unification, seesaw mechanism
1 Introduction

Supersymmetric (SUSY) Grand Unified Theories (GUTs) of SO(10)\cite{1,2} are very important in searching for the new physics beyond the Standard Model (SM). First, the running behaviors of the three gauge couplings suggest that in the Minimal SUSY SM (MSSM) they unify at a scale around $2 \times 10^{16}$ GeV\cite{3–8} which is called the GUT scale $\Lambda_{GUT}$. Second, every generation of quarks and leptons are contained in a spinor representation $16$ of SO(10) which also has the component of a right-handed neutrino. Since in SO(10) the extra $U(1)_{B-L}$ symmetry needs to be broken with the corresponding Vacuum Expectation Value (VEV) giving the Majorana masses to the right-handed neutrinos, the low energy data on neutrino masses and mixing can be beautifully described by the seesaw mechanism\cite{9–17}.

Detailed studies\cite{18–26} suggest that there are difficult problems to be solved in the SUSY SO(10) models. First, as in other SUSY GUT models, proton decays are mainly induced by the dimension-five operators mediated by the color-triplet Higgsinos whose masses of the order $\Lambda_{GUT}$ are not large enough to suppress proton decays\cite{27–30}. Second, in SO(10) there is a conflict between the seesaw scale and gauge coupling unification. The low energy neutrino data suggest that the right-handed neutrinos couple to field with a VEV of the order $\Lambda_{seesaw} \sim 10^{14}$ GeV, while gauge coupling unification disfavors any intermediate scale\cite{22}. Third, in the MSSM there is a pair of weak doublets whose masses are negligibly smaller compared to other masses in the same representations which are of the order $\Lambda_{GUT}$. The large Doublet-Triplet Splitting (DTS) cannot be simply realized without introducing delicate mechanisms.

In recent studies, by going beyond the minimal model, all these major difficulties have been solved in a somehow complicated renormalizable model\cite{31}. Usually in renormalizable models the $U(1)_{B-L}$ symmetry is broken by the tensor representations $126 + \overline{126}$. In [31]...
there are two pairs of Higgs in $126 + \overline{126}$ responsible to the $U(1)_{B-L}$ breaking following [32]. Only one $\overline{126}$ with a $\Lambda_{\text{seesaw}}$ VEV couples to the MSSM matter superfields, while the other VEVs responsible for the $U(1)_{B-L}$ breaking take $\Lambda_{\text{GUT}}$ values and SO(10) breaks directly into the SM gauge symmetry. Since $\Lambda_{\text{seesaw}}$ is not a real intermediate scale, the gauge coupling unification maintains. Furthermore, although all the color triplets in this model have $\Lambda_{\text{GUT}}$ masses, their effective triplet masses are found to be $\frac{\Lambda_{\text{GUT}}^2}{\Lambda_{\text{seesaw}}}$ which are used to suppress proton decays. Also, natural DTS is realized through the Dimopoulos-Wilczek (DW) Mechanism [33–37] of missing VEV. The model [31] has been examined numerically to be realistic in [38].

It can be seen that $\Lambda_{\text{seesaw}}$ plays a very important role in realistic models. In [31] it simply uses a global symmetry whose breaking scale is taken around $\Lambda_{\text{seesaw}}$. The seesaw mechanism of type-I explains the typical low energy neutrino masses as

$$m_\nu = -\frac{v_{EW}^2}{\Lambda_{\text{seesaw}}}$$

(1.1)

where $v_{EW}$ is the electroweak scale of the SM at which the Dirac masses for the neutrinos are taken. However, if we rewrite (1.1) as

$$\frac{m_\nu}{v_{EW}} = -\frac{v_{EW}}{\Lambda_{\text{seesaw}}}$$

(1.2)

it only means the replacement of a very small number $10^{-10}$ on the LHS by another one on the RHS. To be more realistic, we need to give an explanation on this value $\Lambda_{\text{seesaw}}$. This has been done in [39] where a third pair of $126 + \overline{126}$ are introduced which couple to a SO(10) singlet charged under an anomalous $U(1)_A$ whose VEV is naturally at the string or the reduced-Planck scale $\Lambda_{st} \sim 10^{18}$ GeV according to the Green-Schwarz mechanism[40–43]. Consequently, the seesaw scale is generated as $\Lambda_{\text{seesaw}} \sim \frac{\Lambda_{GUT}^2}{\Lambda_{st}}$.

The seesaw scale is so important that we need to know if there are other models which can generate it, especially through the Green-Schwarz mechanism. Usually in the renormalizable SO(10) models, only the tensors $126 + \overline{126}$ are responsible for the $U(1)_{B-L}$ breaking while the spinors $16 + \overline{16}$ cannot appear. However, this is not necessarily true if these spinor Higgs differ explicitly from the three generations of the matter superfields.

In the present work we propose a mechanism with the $16 + \overline{16}$ helping to generate the seesaw scale through the Green-Schwarz mechanism. Instead of using three pairs of $126 + \overline{126}$ as in [39], we find that the minimal setting needs two pairs of them and two pairs of $16 + \overline{16}$. In the meantime, the successful mechanism of suppressing proton decay is not lost. The DTS problem, however, will not be naturally realized which is beyond the present study.

In Section 2, we will analyze the simple SUSY SO(10) models with one pair of $126 + \overline{126}$ or $16 + \overline{16}$ to break $U(1)_{B-L}$. In Section 3, we will extend the minimal models to incorporate the mechanisms of the proton decay suppression and the seesaw scale generation. In Section 4 we will analyze the simplest model with both $126 + \overline{126}$ and $16 + \overline{16}$ existing together. We give the general form of the Goldstone mode for the $U(1)_{B-L}$ symmetry breaking whose proof is given in the Appendix. In Section 5 we will propose a new model of generating the seesaw scale. In Section 6 we will summarize.
2 The simple SUSY SO(10) Models

We start with analyzing the $U(1)_{B-L}$ breaking in the simple SO(10) models. The minimal SUSY SO(10) contains Higgs superfields in $H(10), \Delta(126) + \bar{\Delta}(126), \Phi(210)$\cite{20–23}. Here, $H(10)$ (and also $D(120)$ in some extended models\cite{25, 26}) is not responsible for the GUT symmetry breaking and is irrelevant. The $\Phi(210)$ can be replaced by $A(45) + E(54)$ as an alternative\cite{19}, and we will use the later as examples. The general superpotential relevant for the $U(1)_{B-L}$ breaking is

$$\Sigma(m_\Delta + \lambda A).$$

Labeling the VEVs using their representations under the $SU(4)_C \times SU(2)_L \times SU(2)_R$ subgroup of SO(10), they are

$$\hat{v}_R \equiv \langle \Delta(10, 1, 3) \rangle, \quad v_R \equiv \langle \Delta(\overline{10}, 1, 3) \rangle, \quad A_1 \equiv \langle A(1, 1, 3) \rangle, \quad A_2 \equiv \langle A(15, 1, 1) \rangle.$$  \hspace{1cm} (2.2)

Preserving SUSY at high energy requires the D-flatness condition

$$\hat{v}_R = v_R$$

and the F-flatness conditions

$$0 = F_{\hat{v}_R} = M_\Delta v_R, \quad 0 = F_{v_R} = \overline{v}_R M_\Delta,$$

where $M_\Delta = m_\Delta + \lambda A_0$ and $A_0 = -\frac{1}{8} A_1 - \frac{\sqrt{3}}{5} \overline{A}_2$. For nonzero $\hat{v}_R = v_R$,

$$M_\Delta = 0$$

which determines $A_0$. Full determinations of all the VEVs need to use the complete superpotential which is simple and is irrelevant here.

The singlets of the SM gauge group must contain a massless eigenstate which is the Goldstone mode for the $U(1)_{B-L}$ breaking. This mass matrix for the SM singlets is

$$\begin{pmatrix}
0 & M_\Delta & -\lambda \frac{1}{9} v_R & -\lambda \frac{\sqrt{3}}{5} \overline{v}_R \\
M_\Delta & 0 & -\lambda \frac{\sqrt{3}}{5} v_R & -\lambda \frac{\sqrt{3}}{5} \overline{v}_R \\
-\lambda \frac{\sqrt{3}}{5} v_R & -\lambda \frac{\sqrt{3}}{5} \overline{v}_R & 0 & * \\
-\lambda \frac{\sqrt{3}}{5} \overline{v}_R & -\lambda \frac{\sqrt{3}}{5} v_R & * & 0
\end{pmatrix},$$

where both the columns and the rows are ordering as $\Delta, \overline{\Delta}, A_1, A_2$. The “∗”s stand for irrelevant quantities. The massless eigenstate is easy to find to be a combination of SM singlets in $\Delta$ and $\overline{\Delta}$, since the upper two rows are not independent, neither are the left two columns, when (2.5) applies. In this simple situation, the existence of the Goldstone mode can be taken as an automatic result of the F-flatness conditions. In a full model $E(54)$ is also needed for the GUT symmetry breaking, but including $E$ in the mass matrix for the SM singlets will not change the above results since it does not couple with the SM singlets in $\Delta, \overline{\Delta}$. $E$ and the “∗”s in (2.6) do not enter into the eigenvalue equation of the Goldstone mode.
Now we use $\Psi(16) + \overline{\Psi}(1\overline{6})$ to break $U(1)_{B-L}$. The VEVs are denoted as

$$\psi = \langle \Psi(4,1,2) \rangle, \quad \overline{\psi} = \langle \overline{\Psi}(4,1,2) \rangle,$$

and the relevant superpotential is

$$\overline{\Psi}(m_\psi + \eta A)\Psi.$$

The D-flatness condition is

$$\overline{\psi} = \psi$$

and the F-flatness conditions are

$$0 = F\overline{\psi} = M_\psi \psi, \quad 0 = F_\psi = \overline{\psi} M_\psi,$$

where $M_\psi = m_\psi + \eta A'_0$ and $A'_0 = -2A_1 - \sqrt{6}A_2$. For nonzero $\overline{\psi} = \psi$,

$$M_\psi = 0$$

determining $A'_0$. The mass matrix for the SM singlets is

$$\begin{pmatrix}
0 & M_\psi & -\eta 2\overline{\psi} - \eta \sqrt{6}\psi \\
M_\psi & 0 & -\eta 2\psi - \eta \sqrt{6}\psi \\
-\eta 2\overline{\psi} - \eta \sqrt{6}\psi & -\eta 2\psi - \eta \sqrt{6}\psi & * & * \\
-\eta \sqrt{6}\psi & -\eta \sqrt{6}\psi & * & *
\end{pmatrix} (2.12),$$

where both the columns and the rows are ordering as $\Psi, \overline{\Psi}, A_1, A_2$. Again, the Goldstone mode exists following the F-flatness conditions. Also, we have not included explicitly $E$ which does not couple with the SM singlets in $\Psi, \overline{\Psi}$ and hence will not change the above results.

3 Generation of the seesaw scale in the realistic models

The simple models in Section 2 cannot be realistic. To suppress proton decay in the renormalizable models, the Higgs superfields which couple with the matter superfields in the MSSM need to be extended. The Yukawa sector is denoted as

$$W_{Yukawa} = \sum_{i,j=1,2,3} \Psi_i \Psi_j \left( Y_{10,ij} H_1(10) + Y_{120,ij} D_1(120) + Y_{126,ij} \Delta_1(126) \right), \quad (3.1)$$

where $\Psi_{1,2,3}$ are the MSSM matter superfields which do not contribute to the GUT symmetry breaking. Denoting those Higgs superfields which do not contribute to the Yukawa couplings as $H_2(10), D_2(120), \overline{\Delta}_2(126), \Delta_2(126)$, the mass matrix for the color-triplets is divided into $6 \times 6$ sub-matrices,

$$\begin{pmatrix}
0 & M_{12}^T \\
M_{12} & 0
\end{pmatrix} (3.2)$$

following the Higgs superpotential

$$m_{H_2 H_1} H_2 + m_{D_2 D_1} D_2 + m_{\Delta_2 \Delta_1} \overline{\Delta}_2 \Delta_2$$
+ A \left( H_1H_2 + H_1D_2 + H_2D_1 + D_1D_2 + D_1\overline{\Delta}_2 + D_1\Delta_2 + D_2\overline{\Delta}_1 + D_2\Delta_1 + \overline{\Delta}_1\Delta_2 + \overline{\Delta}_2\Delta_1 \right) \\
+ X \left( H_2^3 + D_2^3 + \overline{\Delta}_2^3 \right).

(3.3)

Here we have suppressed all dimensionless couplings for concise. Also, possible couplings

\[ EH_iH_j, \; ED_iD_j, \; E\Delta_i\Delta_j, \; E\overline{\Delta}_i\overline{\Delta}_j \]

(3.4)

need to be included if allowed.

For the SO(10) singlet X given a VEV \( \Lambda_{GUT} \) and all the other VEVs and mass parameters are at the GUT scale, all the triplets have GUT scale masses. In the mass matrix,

\[ \hat{M}_{12}^T \sim \Lambda_{GUT}, \; \hat{M}_{21}^T \sim \Lambda_{GUT}, \; \hat{M}_{22}^T \sim X. \]

(3.5)

The effective triplet mass matrix, which is got by integrating out those fields which do not appear in the Yukawa superpotential, has all entries \( \frac{\Lambda_{GUT}}{\sqrt{2}} \), thus the amplitudes for proton decay mediated by the color-triplet Higgsinos are suppressed for \( X \ll \Lambda_{GUT} \).

The superpotential (3.3) must be protected by extra symmetries, otherwise unwanted terms reappear and no suppression of proton decay can be assured. Charged under a global symmetry, in [32] the VEV of X is taken to be \( 10^{-2}\Lambda_{GUT} \). Consequently, the D-flatness condition is

\[ |v_{1R}|^2 + |v_{2R}|^2 = |v_{1R}|^2 + |v_{2R}|^2, \]

(3.6)

and F-flatness conditions are

\[ 0 = [v_{1R}, v_{2R}] \left[ \begin{array}{cc} 0 & M_{\Delta 12} \\ M_{\Delta 21} & X \end{array} \right], \quad 0 = \left[ \begin{array}{c} 0 \\ M_{\Delta 21} \end{array} \right] [v_{1R}, v_{2R}], \]

(3.7)

where \( M_{\Delta,ij} = m_{\Delta,ij} + \lambda_{ij}A_0 \).

The key point is that the 2 \times 2 matrix in (3.7) has one but only one zero eigenvalue. Then, in the symmetric mass matrix of the SM singlets, ordering as \( \Delta_1, \Delta_2, \overline{\Delta}_1, \overline{\Delta}_2, A_1, A_2 \),

\[ \begin{pmatrix}
0 & M_{\Delta 21} & X & -\lambda_{21} \frac{v_{2R}}{\sqrt{2}} & -\frac{\sqrt{3}}{\sqrt{2}} v_{2R} \\
M_{\Delta 21} & X & -\lambda_{21} \frac{v_{1R}}{\sqrt{2}} & -\frac{\sqrt{3}}{\sqrt{2}} v_{1R} & -\lambda_{21} \frac{v_{1R}}{\sqrt{2}} \\
-\lambda_{12} \frac{v_{2R}}{\sqrt{3}} & -\lambda_{12} \frac{v_{1R}}{\sqrt{3}} & -\lambda_{12} \frac{v_{2R}}{\sqrt{3}} & -\lambda_{12} \frac{\sqrt{3}}{\sqrt{2}} v_{2R} & -\lambda_{21} \frac{v_{2R}}{\sqrt{3}} \\
-\lambda_{12} \frac{v_{2R}}{\sqrt{3}} & -\lambda_{12} \frac{v_{1R}}{\sqrt{3}} & -\lambda_{12} \frac{v_{2R}}{\sqrt{3}} & -\lambda_{12} \frac{\sqrt{3}}{\sqrt{2}} v_{2R} & -\lambda_{21} \frac{v_{2R}}{\sqrt{3}}
\end{pmatrix}\]

(3.8)

The upper-left 4 \times 4 sub-matrix has two zero eigenvalues following (3.7), while in the upper-right 4 \times 2 sub-matrix there is only one independent row and in the lower-left 2 \times 4 sub-matrix there is only one independent column. Consequently, there is only one massless eigenstate which is the Goldstone mode whose components are all from the \( B - L \) charged fields \( \Delta_{1,2} \) and \( \overline{\Delta}_{1,2} \).

Note that \( M_{\Delta 12} \) and \( M_{\Delta 21} \) cannot be zero simultaneously without fine-tuning parameters. We choose \( M_{\Delta 21} = 0 \) and solve the D- and F-flatness conditions, then

\[ v_{1R} = \frac{X}{M_{\Delta 12}}, \; v_{2R} \sim v_{1R} \sim \Lambda_{GUT}, \; v_{2R} = 0. \]

(3.9)
It is $v_{1R}$ which gives the masses to the right-handed neutrinos, and this seesaw scale is now generated through the VEV of the SO(10) singlet $X$.

In [32], although the link between the seesaw scale and the proton decay suppression has been setup, the VEV of the SO(10) singlet is simply put in by hand and is not a satisfactory. The global symmetry has been further replaced by an anomalous $U(1)_A$ symmetry in [39], where a third pair of $\Delta_3 + \Delta_3$ are introduced so that the last term (the term containing $X$) in (3.3) is replaced by

$$A'(H_2 + D_2)(\Delta_3 + \Delta_3) + A'(\Delta_2 \Delta_3 + A'(\Delta_3 \Delta_3 + S \Delta_3 \Delta_3),$$

(3.10)

where $A'$ is a new 45 and $S$ is a SO(10) singlet. Due to the Green-Schwarz mechanism, the VEV of the $S$ is taken to be the string scale $\Lambda_{\text{st}} \sim 10 \Lambda_{\text{GUT}}[44, 45]$. The D-flatness condition is

$$|v_{1R}|^2 + |v_{2R}|^2 + |v_{3R}|^2 = |v_{1R}|^2 + |v_{2R}|^2 + |v_{3R}|^2,$$

(3.11)

and the F-flatness conditions are

$$0 = [v_{1R}, v_{2R}, v_{3R}] \begin{bmatrix} 0 & M_{\Delta 12} & 0 \\ M_{\Delta 21} & 0 & M_{\Delta 23} \\ 0 & M_{\Delta 32} & S \end{bmatrix} v_{1R} = 0 = \begin{bmatrix} 0 & M_{\Delta 12} & 0 \\ M_{\Delta 21} & 0 & M_{\Delta 23} \\ 0 & M_{\Delta 32} & S \end{bmatrix} \begin{bmatrix} v_{1R} \\ v_{2R} \\ v_{3R} \end{bmatrix},$$

(3.12)

where $M_{\Delta 23} \sim A'_0$, $M_{\Delta 32} \sim A'_0$. Then, the $U(1)_{B-L}$ symmetry breaking requires the determinant of the $3 \times 3$ matrix in (3.12) is zero so that it has one massless eigenstate. Again we choose $M_{\Delta 21} = 0$ which gives

$$v_{1R} = \frac{M_{\Delta 23} M_{\Delta 32}}{S M_{\Delta 12}} v_{2R}, \quad v_{3R} = -\frac{M_{\Delta 23}}{S} v_{2R}, \quad v_{2R} \sim v_{1R} \sim \Lambda_{\text{GUT}}, \quad v_{2R} = v_{3R} = 0.$$  

(3.13)

Solving all the other F-flatness conditions shows that $A'_0 \sim \frac{1}{\sqrt{10}} \Lambda_{\text{GUT}}$, then the seesaw scale $v_{1R} \sim 10^{-2} \Lambda_{\text{GUT}}$ is generated. The mechanism of proton decay suppression keeps working, since in getting the effective triplet mass matrix, a first step of integrating out $\Delta_3 + \Delta_3$ is needed, which amounts to replacing $X$ in the simple model by $A'_0 \sim 10^{-2} \Lambda_{\text{GUT}}$.

In the models using only $\Psi(16) + \overline{\Psi}(16)$ to break the $U(1)_{B-L}$, the seesaw scale can be generated similarly. However, such kind of models are non-renormalizable. In the next Section we will study in a renormalizable model where both $126 + \overline{126}$ and $\Psi(16) + \overline{\Psi}(16)$ are present.

4 Breaking $U(1)_{B-L}$ in models with both $126 + \overline{126}$ and $16 + \overline{16}$

Now we include both a pair of $126 + \overline{126}$ and a pair of $\Psi(16) + \overline{\Psi}(16)$ in the same model. We will have some general observation on the $U(1)_{B-L}$ breaking in this case.

The relevant superpotential is

$$\Delta M \Delta + \overline{\Psi} M \Psi + \overline{\Psi} \Psi + \Delta \overline{\Psi}.$$

(4.1)

The D-flatness condition is

$$2 |\overline{v}_{R}|^2 + |\overline{\psi}|^2 = 2 |v_{R}|^2 + |\psi|^2,$$

(4.2)
and the F-flatness conditions are

\[
0 = F_{v_R} = M_\Delta v_R + \psi \bar{\psi}, \\
0 = F_{\psi_R} = v_R M_\Delta + \bar{\psi} \psi, \\
0 = F_{\psi} = M_\psi \psi + 2 v_R \bar{\psi}, \\
0 = F_{\bar{\psi}} = \bar{\psi} M_\psi + 2 v_R \psi.
\]

Unlike in the models discussed in the previous Sections, the equations in (4.3) are nonlinear and no simple solutions can be directly read off.

Ordering the bases as \(\Delta, \bar{\Delta}, \Psi, \bar{\Psi}, A_1, A_2\), the symmetric mass matrix for the SM singlets is

\[
\begin{pmatrix}
0 & M_\Delta & 0 & 2 \bar{\psi} & -\frac{\lambda}{5} v_R & -\frac{\sqrt{2}}{5 \sqrt{2}} v_R \\
M_\Delta & 0 & 2 \psi & 0 & -\frac{\lambda}{5} v_R & -\frac{\sqrt{2}}{5 \sqrt{2}} v_R \\
0 & 2 \bar{\psi} & 2 v_R & M_\psi & -\eta 2 \bar{\psi} & -\eta \sqrt{6} \psi \\
2 \bar{\psi} & 0 & M_\psi & 2 v_R & -\eta 2 \bar{\psi} & -\eta \sqrt{6} \psi \\
-\frac{\lambda}{5} v_R & -\frac{\lambda}{5} v_R & -\eta 2 \bar{\psi} & -\eta 2 \bar{\psi} & * & * \\
-\lambda v_R & -\lambda \sqrt{2} v_R & -\eta \sqrt{6} \psi & -\eta \sqrt{6} \psi & * & *
\end{pmatrix}
\]

(4.4)

Following (4.3), the upper four rows in (4.4) can be combined into a row with all its entries being zeros as the Goldstone mode. Explicitly, the Goldstone mode is

\[
\frac{2 v_R}{N} \Delta - \frac{2 v_R}{N} \bar{\Delta} + \frac{\psi}{N} \bar{\psi} - \frac{\bar{\psi}}{N} \psi
\]

(4.5)

whose physical meaning is very obvious. The factors “2” and/or “-” correspond to the \(U(1)_{B-L}\) charges. This simply follows the F-flatness conditions (4.4). Here

\[
N = \left( |2 v_R|^2 + |2 v_R|^2 + |\psi|^2 + |\bar{\psi}|^2 \right)^{1/2}
\]

is the normalization factor. The Goldstone modes in the simple models of Section 2 are special cases of (4.5). Also, it can be found that in the models discussed in Section 3, those SM singlets with zero VEVs \((v_{2R} = 0 \text{ or } v_{3R} = 0)\) do not enter into the Goldstone constituents. We will give a simple proof for the formula of the Goldstone’s constituents in the Appendix.

5 The present model

In constructing models which successfully suppress proton decay and generate the seesaw scale through the Green-Schwarz mechanism, the mass matrix for the color-triplets needs to be the form of (3.2) so that two pairs of \(126/\overline{126}\) are needed, and the sub-matrix \(\tilde{M}_{22}^T\) need to be generated through the couplings of \(\Delta_{1,2}/\bar{\Delta}_{1,2}\) with several pairs of \(\Psi/\bar{\Psi}\).

The superpotential for the \(U(1)_{B-L}\) breaking is

\[
\widetilde{\Delta}_1 M_{\Delta 12} \Delta_2 + \widetilde{\Delta}_2 M_{\Delta 21} \Delta_1 + \overline{\Psi}_k M_{\psi k} \Psi_k + f_{ijk} \widetilde{\Delta}_i \Psi_j \Psi_k + g_{ijk} \Delta_j \Psi_k \overline{\Psi}_i,
\]

(5.1)
where $f$'s and $g$'s are dimensionless couplings. $M_{\Delta 12}$ and $M_{\Delta 21}$ need to be taken at the GUT scale for the particles in $\Delta_{1,2}/\overline{\Delta}_{1,2}$ to have masses of this scale so that we can take $M_{\Delta ij} = m_{\Delta ij} + \lambda_{ij} A_0$, while $M_{\Psi_{4i}}$'s must involve couplings with a large VEV of SO(10) singlet and their explicit forms have not been determined yet.

The D-flatness condition is

$$2|v_{1R}|^2 + 2|v_{2R}|^2 + \sum_k |\overline{\psi}_k|^2 = 2|v_{1R}|^2 + 2|v_{2R}|^2 + \sum_k |\psi_k|^2,$$

(5.2)

while the F-flatness conditions are

$$0 = F_{v_{1R}} = M_{\Delta ij} v_{jR} + f_{ik}^j \overline{\psi}_k \psi_l,$$

$$0 = F_{v_{2R}} = \overline{v}_{1R} M_{\Delta ij} + g_{ik}^j \overline{\psi}_k \psi_l,$$

$$0 = F_{\overline{\psi}_k} = M_{\Psi_{4i}} \psi_l + g_{lk}^j v_{jR} \overline{\psi}_l,$$

$$0 = F_{\psi_l} = \overline{\psi}_k M_{\Psi_{4i}} + f_{ik}^j v_{jR} \overline{\psi}_k.$$  

(5.3)

Explicitly, the second equation in (5.3) gives

$$\overline{v}_{1R} = -g_{ik}^2 \overline{\psi}_k \overline{\psi}_l (M^{-1})_{21}, \quad \overline{v}_{2R} = -g_{ik}^1 \overline{\psi}_k \overline{\psi}_l (M^{-1})_{12},$$

then, to generate a VEV of the seesaw scale for $\overline{v}_{1R}$, at least one VEV $\overline{\psi}$ is lower than $\Lambda_{GUT}$. If there is only one $\overline{\psi}$, the second equation gives also a low $\overline{v}_{2R}$. Then the D-flatness condition (5.2) suggests that the $U(1)_{B-L}$ breaks at a scale lower than $\Lambda_{GUT}$, which violates gauge coupling unification. Thus we must have at least two pairs $\Psi_{4,5} + \overline{\Psi}_{4,5}$ in the spinor representations whose labels are different from the matter superfields $\Psi_{1,2,3}$. Furthermore, we can diagonalize the coupling $g_{ik}^2$ so that only $g_{44}^2 \neq 0$ without lost of generality. Also, the coupling $f_{ik}^j$ must be zero so that $\Psi_{4,5}$ are different from $\Psi_{1,2,3}$ which couple with $\overline{\Delta}_1$ through the Yukawa couplings.

We have tried using many different forms of $M_{\Psi_{4i}}$, for all F-flatness conditions fulfilled. We find the following successful model by introducing two SO(10) singlets $S, S'$ and one more $A'(45)$ in addition to the original $A(45) + E(54)$. Suppressing all the dimensionless couplings, the full superpotential for the GUT symmetry breaking is

$$W = \overline{\Delta}_1 M_{\Delta 12} \Delta_2 + \overline{\Delta}_2 M_{\Delta 21} \Delta_1 + S \overline{\Psi}_4 \psi_5 + m_{54} \overline{\Psi}_3 \psi_5 + A \overline{\Psi}_3 \psi_5 + \Delta_1 \overline{\Psi}_5 + \overline{\Delta}_2 \Psi_4^2 + \Delta_2 \overline{\Psi}_4 + S' A' A + S A^2 + \frac{1}{2} m_A A^2 + \frac{1}{2} m_E E^2 + A^2 E + E^3,$$

(5.4)

which is protected by an anomalous $U(1)_A$ under which the charges of the superfields are listed in Table 1. Here explicitly $M_{\Delta ij} = m_{\Delta ij} - \lambda_{ij} \frac{1}{2} A_1 - \lambda_{ij} \frac{\sqrt{2}}{\sqrt{5}} A_2$.

| charge  | $\Delta_1$ | $\Delta_2$ | $\Delta_3$ | $\Psi_4$ | $\Psi_5$ | $\Psi_3$ | $A, E$ | $S$ | $S'$ | $A'$ | $\Psi_{1,2,3}$ |
|---------|-------------|-------------|-------------|-----------|----------|----------|--------|-----|------|------|----------------|
| $-6b - 4c$ | $-2c$ | $2c$ | $6b + 4c$ | $-3b - 2c$ | $-c$ | $c$ | $b$ | $b + 2c$ | $b + c$ | $-b - c$ | $3b + 2c$ |

Table 1. SO(10) multiplets and their $U(1)_A$ charges.
In string models the anomalous $U(1)_A$ symmetry is only anomalous in the effective theory below the string scale. The D-term of such an $U(1)_A$ symmetry gets a non-zero Fayet-Iliopoulos term $\xi$ related to the string scale as\cite{40–43}

$$D_A = -\xi + \sum Q_i |\varphi_i|^2,$$

$$\xi = \frac{M^2}{192\pi^2} \text{Tr} Q, \quad (5.5)$$

where the sum includes all scalar fields $\varphi_i$ present in the theory with nonzero $U(1)_A$ charges $Q_i$. Then the SO(10) singlet gets a non-zero VEV\cite{44, 45}

$$S \sim 10^{-1} M_{st} \sim 10 M_G \quad (5.6)$$

from the anomalous D-term to preserve SUSY. Numerically a variation of order one in $S$ is reasonable in (5.6).

Now the F-flatness conditions are

\begin{align*}
0 &= F_{\psi_1 R} = M_{12} v_{2R}, \\
0 &= F_{\psi_2 R} = M_{21} \bar{v}_{1R} + \bar{\psi}_4, \\
0 &= F_{\bar{\psi}_2 R} = M_{21} v_{1R} + \psi_4^2, \\
0 &= F_{\psi_1 R} = M_{21} v_{2R} + \psi_5^2, \\
0 &= F_{\psi_4} = S \bar{\psi}_5 + 2 \bar{\psi}_4 v_{2R}, \\
0 &= F_{\bar{\psi}_4} = m_{S \bar{\psi}_5} + 2 \bar{\psi}_4 v_{2R}, \\
0 &= F_{\bar{\psi}_5} = m_{S \bar{\psi}_5} - 2 A_1' \psi_5 - \sqrt{6} A_2' \psi_5 - 2 v_{1R} \bar{\psi}_5, \\
0 &= F_{S} = \bar{\psi}_4 \psi_5 + A_1^2 + A_2^2, \\
0 &= F_{S'} = A_1' A_1 + A_2' A_2, \\
0 &= F_{A_1'} = -2 A_1' \bar{\psi}_5 + S' A_1 + 2 S A_1', \\
0 &= F_{A_2'} = -\sqrt{6} \bar{\psi}_5 \psi_5 + S' A_2 + 2 S A_2', \\
0 &= F_{A_1} = m_{AA} + \sqrt{3} A_1 E + S' A_1' - \frac{1}{5} v_{1R} v_{2R} - \frac{1}{5} \bar{v}_{2R} v_{1R}, \\
0 &= F_{A_2} = m_{AA} - \frac{2}{\sqrt{15}} A_2 E + S' A_2' - \sqrt{3} \bar{v}_{1R} v_{2R} - \frac{\sqrt{3}}{5 \sqrt{2}} \bar{v}_{2R} v_{1R}, \\
0 &= F_{E} = m_{EE} + \frac{\sqrt{3}}{2 \sqrt{5}} E^2 + \frac{\sqrt{3}}{2 \sqrt{5}} A_1^2 - \frac{1}{\sqrt{15}} A_2^2. \quad (5.7)
\end{align*}

Solving all these equations gives one set of the solutions which require $v_2 = 0$ and $\psi_5 = 0$ and give the relations

$$\bar{v}_{1R} = -\frac{\bar{\psi}_4}{M_{12}}, \quad \bar{\psi}_4 = \frac{2 A_1' + \sqrt{6} A_2'}{S} \psi_5, \quad A_{1,2}' = -\frac{S'}{S} A_{1,2}.$$

Taking $S = 10 M_G$, the other VEVs are naturally

\begin{align*}
\bar{v}_{1R} &\sim 10^{-3} M_G, \quad \bar{v}_{2R} \sim M_G, \quad v_{1R} \sim M_G, \quad v_{2R} = 0, \\
\bar{\psi}_4 &\sim 10^{-2} M_G, \quad \bar{\psi}_5 \sim M_G, \quad \psi_4 \sim M_G, \quad \psi_5 = 0, \quad (5.8) \\
E &\sim M_G, \quad A_{1,2} \sim M_G, \quad A_{1,2}' \sim 10^{-2} M_G, \quad S' \sim 10^2 M_G.
\end{align*}
Now a VEV $\varphi_1 \sim 10^{13}$ GeV is generated as the seesaw scale, a factor of 10 smaller than that got in [39]. However, if in (5.6) $S$ is taken a smaller value, then this seesaw scale can be $\sim 10^{14}$ GeV now, comparable to that in [39].

It can be also checked that the Goldstone mode for the $B - L$ breaking has components only from $\overline{\Delta}_{1,2}, \Delta_1, \overline{\Psi}_{4,5}, \Psi_4$,

$$\frac{2v_1R}{N} \Delta_1 - \frac{2v_1R}{N} \overline{\Delta}_1 - \frac{2v_2R}{N} \Delta_2 + \frac{\psi_4}{N} \Psi_4 - \frac{\overline{\psi}_4}{N} \overline{\Psi}_4 - \frac{\psi_5}{N} \Psi_5,$$

by the F-flatness conditions. This agrees with the observation made in Section 4 that those fields with null $B - L$ breaking VEVs cannot enter into the Goldstone mode.

In the full model, $H_1(10)$ and $D_1(120)$ are introduced to have the same $U(1)_A$ charges as $\Delta_1$'s. We introduce $H_2$ and $D_2$ with the same charges as $\Delta_2$'s. We also impose a $Z_2$ symmetry, under which only the matter superfields $\Psi_{1,2,3}$ are odd, to suppress unwanted couplings. The full Higgs superpotential consistent with the $U(1)_A \times Z_2$ symmetry is

$$m_{H12}H_1H_2 + m_{D12}D_1D_2 + m_{\Delta12}\overline{\Delta}_1\Delta_2 + m_{\Delta21}\overline{\Delta}_2\Delta_1 + E H_1H_2 + E D_1D_2 + A (H_1H_2 + H_1D_2 + H_2D_1 + D_1\overline{\Delta}_2 + D_1\Delta_2 + D_2\overline{\Delta}_1 + D_2\Delta_1 + \overline{\Delta}_1\Delta_2 + \overline{\Delta}_2\Delta_1) + S\overline{\Psi}_4\Psi_5 + m_{54}\overline{\Psi}_4\Psi_4 + A\overline{\Psi}_5\Psi_5 + \Delta_1\overline{\Psi}_5^2 + \overline{\Delta}_2\Psi_5^2 + (H_2 + D_2 + \Delta_2)\overline{\Psi}_4^2 + S^A A + SA^2 + \frac{1}{2}m_A A^2 + \frac{1}{2}m_E E^2 + A^2 E + E^3.$$

To study proton decay we need to know the full color-triplet mass matrix. The columns are ordered as $H_1, D_1, D_1', \overline{\Delta}_1, \overline{\Delta}_1', \Delta_1; H_2, D_2, D_2', \Delta_2, \overline{\Delta}_2, \Psi_4, \overline{\Psi}_5$, and the rows are ordered as the conjugations,

$$M_T = \begin{pmatrix}
0_{(6 \times 6)} & B_{12}(6 \times 6) & B_{13}(6 \times 2) \\
B_{21}(6 \times 6) & 0_{(6 \times 6)} & B_{23}(6 \times 2) \\
0_{(2 \times 6)} & B_{32}(2 \times 6) & B_{33}(2 \times 2)
\end{pmatrix}$$

where

$$B_{12} = \begin{pmatrix}
m_H - \frac{E}{\sqrt{15}} & \frac{\sqrt{2}}{3} A_2 & -\frac{i\sqrt{2}}{3} A_2 & -\frac{i\sqrt{2}}{3} A_1 \\
\frac{i\sqrt{2}}{3} A_2 & -\frac{E}{2\sqrt{15}} & 0 & \frac{iA_0}{\sqrt{30}} \\
-\frac{iA_0}{\sqrt{30}} & \frac{i\sqrt{2}}{3} A_2 & -\frac{E}{2\sqrt{5}} & m_{12} - \frac{iA_0}{5\sqrt{6}} \\
0 & \frac{i\sqrt{2}}{3} A_2 & \frac{i\sqrt{2}}{3} A_2 & m_{21} - \frac{iA_0}{5\sqrt{6}} \\
0 & 0 & 0 & m_{21} - \frac{iA_0}{5\sqrt{6}}
\end{pmatrix},$$

$$B_{13} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -i\sqrt{3}\psi_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{6}\psi_5 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$
The proton decay rates depend only on the effective triplet mass matrix corresponding to the superfields which appear in the Yukawa couplings (3.1). Integrating out those superfields which are absent in (3.1), we find that that all entries in this effective triplet mass matrix are infinities so that there is no color higgsino mediated proton decay and proton decays are all from gauge mediation in the present model. This is quite different from the model in [31, 32, 39] where although the dimension-five operators are safe from the data, but in general they dominate over the mechanism of gauge mediation[38]. However, since we have not dealt with the DTS problem in the present model, this conclusion need to be taken carefully in future studies.

6 Summary

In this paper, we have proposed an alternative model to generate the seesaw scale. Proton decays through dimension-five operators are absent which are very different from the models studied before. However, further work needs to be done to solve the DTS problem in the present model.
Appendix

A  The Goldstone mode for $U(1)$ symmetry breaking

Only those superfields $X$’s which contain the SM singlets may contribute to the GUT and especially to the $U(1)_{B-L}$ symmetry breaking. For a general consideration, the superpotential with $m$ different $X$’s is

$$W = \sum_{n_1 \cdots n_m} f_{n_1 \cdots n_m} X_1^{n_1} \cdots X_m^{n_m}, \quad (A.1)$$

with the conservation of the $U(1)$ charges

$$0 = f_{n_1 \cdots n_m} \sum_{a=1}^m n_a q_a. \quad (A.2)$$

Hereon $X_a$ represents the VEV and its F-flatness condition is

$$0 = F_{X_a} = \sum_{n_1 \cdots n_m} \frac{n_a}{X_a} f_{n_1 \cdots n_m} X_1^{n_1} \cdots X_m^{n_m}, \quad (A.3)$$

and the mass matrix elements of the SM singlets are

$$M_{ab} = \frac{\partial^2 W}{\partial X_a \partial X_b} = \sum_{n_1 \cdots n_m} \frac{n_a n_b}{X_a X_b} (1 - \frac{\delta_{ab}}{n_b}) f_{n_1 \cdots n_m} X_1^{n_1} \cdots X_m^{n_m}. \quad (A.4)$$

Acting the mass matrix on the column vector

$$\hat{G} = (q_1 X_1, \cdots, q_b X_b, \cdots, q_m X_m)^T \quad (A.5)$$

gives a column vector whose component is

$$\sum_{b=1}^m M_{ab} G_b = \sum_{b=1}^m \sum_{n_1 \cdots n_m} \frac{n_a n_b}{X_a} (1 - \delta_{ab}) f_{n_1 \cdots n_m} X_1^{n_1} \cdots X_m^{n_m} \quad (A.6)$$

$$= \sum_{n_1 \cdots n_m} X_a \left[ f_{n_1 \cdots n_m} \sum_{b=1}^m n_b q_b X_1^{n_1} \cdots X_m^{n_m} - q_a \sum_{n_1 \cdots n_m} \frac{n_a}{X_a} f_{n_1 \cdots n_m} X_1^{n_1} \cdots X_m^{n_m} \right]$$

$$= 0$$

following (A.2) and (A.3). Then $\hat{G}$ in (A.5) is the Goldstone mode of the $U(1)$ symmetry breaking.

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