Fast and Automatic Periacetabular Osteotomy Fragment Pose Estimation Using Intraoperatively Implanted Fiducials and Single-View Fluoroscopy

R B Grupp¹, R J Murphy², R A Hegeman³, C P Alexander⁴, M Unberath¹, Y Otake⁵, B A McArthur⁶,⁷, M Armand³,⁴,⁸ and R H Taylor¹

¹Department of Computer Science, Johns Hopkins University, Baltimore, MD, USA
²Auris Health, Inc., Redwood City, CA, USA
³Research and Exploratory Development Department, Johns Hopkins University Applied Physics Laboratory, Laurel, MD, USA
⁴Department of Orthopaedic Surgery, Johns Hopkins Medicine, Baltimore, MD, USA
⁵Graduate school of Information Science, Nara Institute of Science and Technology, Ikoma, Nara, Japan
⁶Department of Surgery and Perioperative Care, Dell Medical School, University of Texas, Austin, TX, USA
⁷Texas Orthopedics, Austin, TX, USA
⁸Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD, USA

E-mail: grupp@jhu.edu

September 2019

Abstract. Accurate and consistent mental interpretation of fluoroscopy to determine the position and orientation of acetabular bone fragments in 3D space is difficult. We propose a computer assisted approach that uses a single fluoroscopic view and quickly reports the pose of an acetabular fragment without any user input or initialization. Intraoperatively, but prior to any osteotomies, two constellations of metallic ball-bearings (BBs) are injected into the wing of a patient’s ilium and lateral superior pubic ramus. One constellation is located on the expected acetabular fragment, and the other is located on the remaining, larger, pelvis fragment. The 3D locations of each BB are reconstructed using three fluoroscopic views and 2D/3D registrations to a preoperative CT scan of the pelvis. The relative pose of the fragment is established by estimating the movement of the two BB constellations using a single fluoroscopic view taken after osteotomy and fragment relocation. BB detection and inter-view correspondences are automatically computed throughout the processing pipeline. The proposed method was evaluated on a multitude of fluoroscopic images collected from six cadaveric surgeries performed bilaterally on three specimens. Mean fragment rotation error was 2.4 ± 1.0 degrees, mean translation error was 2.1 ± 0.6 mm, and mean 3D lateral center edge angle error was 1.0 ± 0.5 degrees. The average runtime of the single-view pose estimation was 0.7 ± 0.2 seconds. The proposed method demonstrates accuracy similar to other state of the art systems which require optical tracking systems or multiple-view 2D/3D registrations with manual input. The errors reported on...
fragment poses and lateral center edge angles are within the margins required for accurate intraoperative evaluation of femoral head coverage.

**Keywords**: Periacetabular Osteotomy, 2D/3D Registration, Computer Assisted Surgery, X-ray Navigation, Orthopaedics

This submission includes a supplementary document. An additional video with detailed descriptions of the methods and example results is available online at: https://youtu.be/0E0U9G81q8g.

1. Introduction

Patients suffering from developmental dysplasia of the hip (DDH) typically have severe pain and reduced coverage of the femoral head, which can lead to joint osteoarthritis and subluxation of the femur (Gala et al., 2016). Joint-preserving pelvic osteotomies, such as the Periacetabular osteotomy (PAO), treat DDH by reorienting the hip joint for increased femoral head coverage (Ganz et al., 1988). Specifically for PAO, four osteotomies are performed about the acetabulum, fracturing it from the pelvis and allowing it to be adjusted to the desired pose (Ganz et al., 1988). In the conventional approach, PAO surgeons rely on 2D X-ray images, tactile feedback, experience and acumen to navigate the surgery (Ganz et al., 1988). Clinicians typically assess femoral head coverage intraoperatively using specific radiographic measurements, such as the lateral center edge (LCE) angle (Wiberg, 1939), derived from fluoroscopy (Troelsen, 2009). However, this approach does not indicate the full 3D alignment of the acetabular fragment, nor does it describe additional biomechanical parameters, which have the potential to improve surgical outcomes (Hipp et al., 1999; Armand et al., 2005; Armiger et al., 2009; Niknafs et al., 2013; Liu et al., 2016a). A 3D example of a relocated fragment and a corresponding 2D fluoroscopic view is shown in figure 1.

In this paper, we propose a processing pipeline that is capable of automatically reporting fragment poses from a single fluoroscopic view with mean runtimes below one second. The pipeline is inspired by Roentgen stereometric analysis (RSA) techniques, which use metallic ball-bearings (BBs) to track the movement of bones or surgical implants over time (Selvik, 1990).

Two constellations of BBs are injected into the patient’s pelvis prior to osteotomy: one co-located on the expected acetabular bone fragment and the other on the larger pelvis portion. The 3D locations of the BBs are reconstructed using three fluoroscopic views of the constellations. Once the acetabulum is relocated, the 3D orientation and position of the fragment is automatically calculated using a single fluoroscopic view.

Navigation systems using optical trackers impose significant storage constraints, require non-trivial administration, and are not yet commonly found across operating rooms. However, our method only requires a small BB injection device and fluoroscopy, which is already common throughout orthopaedic operating rooms for joint surgery.
Figure 1. Examples of an adjusted acetabular fragment visualized in 3D (a) and in a corresponding 2D fluoroscopic image (b). The fragment pose shown in (a) was estimated using the view shown in (b). A precise model of the acetabular fragment is not required by the proposed method; the 3D bone surfaces in (a) were constructed using a preoperative plan of the osteotomies. The anatomical axes of the anterior pelvic plane are also shown in (a): left/right (LR) as X-axis, inferior/superior (IS) as Y-axis, and anterior/posterior (AP) as Z-axis.

Therefore, we believe the proposed fluoroscopic method is more easily deployable than other approaches relying on optical tracking technology (Langlotz et al., 1997; Akiyama et al., 2010; Murphy et al., 2015; Liu et al., 2016b; Takao et al., 2017). Furthermore, the registration process with an optical tracker requires a certain amount of bone exposure and may become more challenging when using minimally invasive incisions (Troelsen et al., 2008). Compared to existing approaches which leverage fluoroscopy (Grupp et al., 2019), our method only requires a single fluoroscopic image per pose estimate, does not rely on any knowledge of the 3D fragment shape, and runs without user initialization in a fraction of the time.

After BB injection, the proposed method does not require any specialized equipment or additional workflow. Moreover, the pose estimation executes quickly and automatically between fluoroscopic captures. The primary clinical contribution of this paper is the ability to report 3D orientation and position of the acetabular fragment, while requiring minimal modification to an existing surgical workflow. In terms of technical contribution, this paper is the first method leveraging intraoperatively constructed fiducial constellations to automatically recover point correspondences and poses of multiple objects moving non-coherently in uncalibrated single-view fluoroscopy.

1.1. Related Work

Early navigation systems for PAO, and other pelvic osteotomies, relied on optical trackers (Langlotz et al., 1997, 1998; Mayman et al., 2002; Akiyama et al., 2010), or custom cutting guides (Radermacher et al., 1998; Otsuki et al., 2013). These systems only provided intraoperative assistance during performance of the acetabular osteotomies; pose estimates of the relocated fragment were not produced.

More recent systems have focused on reporting the pose of a relocated
fragment (Murphy et al., 2015; Liu et al., 2016b; Murphy et al., 2016; Takao et al., 2017; De Raedt et al., 2018; Grupp et al., 2019). Fragment pose updates may be provided in real-time by directly attaching an optically tracked rigid body to the fragment as demonstrated in (Liu et al., 2016b). However, attaching a large rigid body to the acetabular region is challenging, especially when using a minimally invasive technique specialized for PAO (Troelsen et al., 2008). In order to estimate fragment poses and avoid the attachment of an extra rigid body, (Murphy et al., 2015) and (Takao et al., 2017) digitize specific points on the fragment with an optically tracked pointer tool after each adjustment of the fragment. This digitization adds minor overhead to the operative time in (Murphy et al., 2015) and causes some ambiguity between rotation and translation in (Takao et al., 2017). In (Murphy et al., 2015), fragment pose errors ranged from 1.4° − 1.8° in rotation and 1.0 − 2.2 mm in translation.

Eliminating the need for optical tracking systems, (Grupp et al., 2019) used multiple fluoroscopic views to track the acetabular fragment, ipsilateral femur, and pelvis. A multiple-component intensity-based 2D/3D registration (Markelj et al., 2012) of patient anatomy was used, requiring no external objects and maintaining compatibility with any PAO approach. However, the method suffers from several limitations and constraints that interfere with a typical surgical workflow:

- An approximate AP fluoroscopic view, two additional views, and manual annotation of a single anatomical landmark are required to initialize the method
- Accuracy of the approach degrades as intraoperative fragment shapes differ from preoperatively planned shapes
- The computation time on state-of-the-art hardware is not real-time, approximately 25 seconds.

In order to overcome these limitations, the methods described in this paper leverage implanted BBs and extend RSA-related techniques to automatically track the migration of the acetabular fragment using a single view per adjustment.

Since its introduction in the 1970’s, RSA has been used for a variety of applications (Selvik, 1990), including the longitudinal analysis of orthopaedic implant migration (Valstar et al., 2002), bone growth (Kärholm et al., 1984), and even PAO stability (Mechlenburg et al., 2007). Recent work in the RSA community has incorporated 2D/3D registration technology to track the movement of bones and implants without relying on inserted BBs (De Bruin et al., 2008; Seehaus et al., 2012). Similar to (Grupp et al., 2019), these methods require manual input and multiple X-ray views.

The implantation of BBs for intraoperative fragment tracking during PAO was first introduced in (Murphy et al., 2013) and (Armand et al., 2018). However, this approach is not easily incorporated into a surgical workflow, since it requires: the manual identification of BB correspondences, multiple post-osteotomy views, and a calibrated CBCT C-Arm.

Several methods for automatic 3D BB reconstruction have been developed by the
CBCT community (Hamming et al., 2009; Yaniv, 2009; Dang et al., 2012; Choi et al., 2014). These methods require a calibrated C-Arm, leverage more than three projections, or rely on an orbital motion constraint to help establish correspondences. For intraoperative coronary artery reconstruction, epipolar constraints have been applied to automatically prune invalid point correspondences between two (Yang et al., 2009), and three (Blondel et al., 2006), fluoroscopic views. Structure-from-Motion pipelines operate in a similar fashion and use the dense correspondences found in large photographic collections to reconstruct rigid structures in 3D (Snavely et al., 2008; Westoby et al., 2012; Schonberger and Frahm, 2016).

Methods using a known 3D marker constellation, two 2D X-ray views, and varying levels of manual interaction have been developed for patient positioning and motion compensation in radiation therapy (Schweikard et al., 2000; Litzenberg et al., 2002; Aubry et al., 2004).

Single plane RSA was proposed in order to avoid the less-common, bi-planar, imaging devices used in RSA (Yuan et al., 2002). However, the method requires the use of a calibration cage, does not address the establishment of 2D/3D correspondences, and was only evaluated for single object pose recovery.

Automatic pose and correspondence estimation between a single rigid collection of BBs and one fluoroscopic view was described in (Tang et al., 2000) and (Kang et al., 2013). The poses of custom tailored fiducial objects using lines and ellipses may also be computed automatically in a single view (Jain et al., 2005; Steger et al., 2013). These poses are generally restricted to tracking the relative motion of the C-Arm, since the relationship between the patient’s anatomy and the intraoperatively inserted fiducial is typically unknown.

The method of (Tang et al., 2000) was incorporated into fluoroscopic systems for estimating the poses of multiple BB constellations required for knee kinematics (Tang et al., 2004; Ioppolo et al., 2007). However, the mechanism used to identify constellation membership and establish 2D/3D correspondences, was not described.

The pipeline proposed in this paper is able to accurately, quickly, and automatically provide pose estimates of a relocated bone fragment during PAO. No reliance on external tracking devices is required. Furthermore, the pose estimation method does not require: a calibrated C-Arm, multiple-views, a specific constellation pattern, accurate knowledge of the fragment shape, or any manual establishment of correspondence.

2. Materials and Methods

The method introduced in this paper requires some preoperative processing and two distinct phases during the surgery. CT scanning, segmentation of the anatomy, and anatomical landmark digitization make up the preoperative processing. The first intraoperative phase is performed only once and consists of BB injection and reconstruction. Pose estimation of the acetabular fragment from a single fluoroscopic view represents the second intraoperative addition. In order to achieve the desired
Figure 2. A summary of the surgical workflow proposed for this method, including the data required for each step. The key contributions of this work are the BB reconstruction and single-view pose estimation components, which are highlighted in gray. Full workflows for the reconstruction and pose estimation components are shown in figures 5 and 6, respectively.

amount of femoral head coverage, it is typical for a surgeon to iterate between collecting fluoroscopy and adjusting the fragment. Therefore, our processing combines intelligent pruning and GPU acceleration to avoid any significant delay to the workflow. Figure 2 shows the workflow of the proposed method at a high level. Full details of the preoperative processing, intraoperative BB reconstruction, and intraoperative fragment pose estimation are now provided.

2.1. Preoperative Processing

Preoperative processing proceeds identically to that in (Grupp et al., 2019), which we briefly describe here. A lower torso CT scan is obtained and resampled to have 1 mm isotropic voxel spacing. An automated method (Krčah et al., 2011) is used for an initial segmentation of the pelvis and femurs; any inconsistencies around the femoral head and acetabulum are cleaned up manually. Anatomical landmarks are manually annotated to define the anterior pelvic plane (APP) coordinate system (Nikou et al., 2000), and also for later use as initialization of pre-osteotomy pelvis registrations. The origin of the APP is set at the center of the ipsilateral femoral head, and the mapping from APP coordinates to the CT volume coordinates is denoted by $T_{APP}^V$. Six additional landmarks are manually annotated in order to create a planned fragment shape, which is only used to visualize the intraoperative movement of the fragment. Examples of the APP axes orientation and a planned fragment shape are shown in figure 1.
Figure 3. The Halifax bead injection device used in four of the cadaver surgeries is shown on the left. On the right, a pre-osteotomy fluoroscopic image is shown with automatic detections of injected beads highlighted by yellow circles; every injected BB was detected. The larger BBs were used to help establish the ground truth pose of the fragment and as such, are not used and not detected during intraoperative pose estimation.

2.2. Intraoperative BB Reconstruction

A Halifax Biomedical Inc. injection device is used to implant two, four-BB constellations onto the ipsilateral side of the patient’s pelvis, with one constellation lying on the area expected to lie on the acetabular fragment and the other on the larger pelvis fragment. The BBs are injected after performing soft-tissue dissection, but prior to osteotomy.

Three distinct fluoroscopic views are collected while the patient anatomy remains stationary. For each view, the radial symmetry algorithm (Loy and Zelinsky, 2003) is used to automatically locate each BB in 2D. The 3D locations of each BB are constructed by recovering the relative pose information of each view, establishing inter-view BB correspondences, and performing triangulation (Hartley and Zisserman, 2003).

Using the strategy laid out in (Grupp et al., 2019), relative poses between the three views are recovered by performing 2D/3D rigid registrations of the patient’s preoperative pelvis to each view. Since some of our pre-osteotomy views have excessive pelvic tilt and violate the approximate AP view assumption, we select more than the single landmark described in (Grupp et al., 2019) for initialization of the pipeline. Appendix A describes the parameters used for the intensity-based registrations.

BB correspondences are automatically established using a combination of anatomical information and the multiple-view geometry between the three C-Arm poses. Two of the views are selected to create a candidate set of two-view, single-BB, correspondences and triangulated 3D points. Although we have made no assumptions about the geometry of these views, one of the views was always an approximate AP orientation with a variable amount of pelvic tilt. The candidate correspondences are created by first considering all possible combinations of single-BB correspondences.
Figure 4. A visual example of the pre-osteotomy reconstruction process for a single BB. Three fluoroscopic views used for reconstruction are shown in (a), (b), and (c). The initial two-view triangulations are derived from (a) and (b), while (c) is used for re-projections of initial triangulations. Regions pertinent to this example are indicated by yellow boxes, and are magnified in the bottom row. 3D renderings of the patient’s ipsilateral hemi-pelvis and the relative location of the C-Arm detector for the first two views are shown in (d). The green circle in (a) indicates the location of a detected BB, whose 3D location is to be reconstructed. In (b), the green circle shows the detected BB location with true correspondence to BB in (a); the red square and yellow diamond show detected locations with incorrect correspondence. In (d), the three colored spheres are initial triangulations of the BB from (a) when matched with the BBs of varying colors in (b). The red sphere is not located on the pelvis and its candidate correspondence is pruned. However, the green and yellow spheres are located on the pelvis and must be checked using (c). Lines between the X-ray source and BB locations on the detector are colored consistently with (a), (b), and (c); note the intersection between the green lines. The green circle in (c) indicates the detected location of the BB in true correspondence with the green circles in (a) and (b). The green “X” is the re-projection of the green sphere from (d) and the yellow asterisk is the re-projection of the yellow sphere. Since the green sphere was triangulated using a correct correspondence, its re-projected distance to the BB detection in (c) is very small compared to the re-projected distance of the yellow sphere, which was triangulated using an incorrect correspondence.
between the two views, and pruning candidates that result in a triangulated point located more than 10 mm away from the pelvis surface. The red sphere shown in figure 4 is an example of a correspondence pruned in this way. Candidate three-view correspondences are constructed by pairing each of the remaining two-view correspondences with every 2D BB detection in the third view. For each candidate three-view correspondence, the two-view 3D triangulation is re-projected into the third view and the distance to the hypothesized 2D match is recorded. Intuitively, re-projection distances for valid correspondences should be smaller than distances from invalid matches, as shown with the green and yellow re-projections in figure 4. Correct three-view correspondences are established by greedily selecting the candidate correspondences with minimum re-projection distances in the third view. The final 3D reconstructions are triangulated using the correct three-view correspondences. In this way, the third view is used to enforce consistency and refine the 3D triangulation. A visual example is shown in figure 4 and a more formal description is located in Appendix B.

After pruning reconstructed BBs on the contralateral side, the process is completed by classifying the remaining BBs as fragment/non-fragment using a $K$-Means clustering ($K = 2$).

The workflow of the entire reconstruction process is shown in figure 5.

Annotation speed and computation time is not a critical factor at this point in the procedure, since the BB constellations are not required until the fragment has been relocated; osteotomies may be performed immediately after the three fluoroscopic views are obtained.

2.3. Intraoperative Pose Estimation

After ostotomies have been performed and the acetabular fragment has been relocated, a single fluoroscopic image may be used to recover the fragment’s pose with respect to the APP, $\Delta_{\text{APP}}$. Once the the poses of the ilium and fragment BB constellations with respect to the C-Arm, $T_{TIL}^C$ and $T_{TFR}^C$, are recovered, $\Delta_{\text{APP}}$ is computed as in (1).

$$\Delta_{\text{APP}} = T_{TAPP}^V T_{TFR}^C T_{TIL}^C$$  

(1)

Since the BB constellations are constructed in the original pelvis volume coordinate frame, the composition of $T_{TIL}^C T_{TFR}^C$ is valid and maps points on the preoperative fragment region to their adjusted locations. Using $\Delta_{\text{APP}}$, the current pose of the fragment may be visualized (figure 1), and pose parameters or biomechanical (e.g. LCE) angles may also be displayed. Figure 6 depicts the entire, end-to-end, pose estimation workflow.

As was the case for each fluoroscopic view used for pre-osteotomy BB reconstruction, the radial symmetry algorithm is used to detect each BB in the 2D fluoroscopic image automatically. Since the 3D/2D BB correspondences are not yet established, it is not feasible to directly apply classic PnP approaches (Hartley and Zisserman, 2003) for calculation of $T_{TIL}^C$ or $T_{TFR}^C$. Since manual identification is tedious and error-prone, an automatic method to establish correspondences is the appropriate intraoperative strategy. One possible, although naive, approach would be to enumerate
Figure 5. A workflow overview of the intraoperative BB reconstruction process. Three separate 2D/3D pelvis registrations of each fluoroscopic view are performed to recover the relative poses of the C-Arm. Triangulations from all possible single-BB correspondences in the first two views are computed, and pruned using the 3D pelvis segmentation. Any remaining, invalid, correspondences are eliminated by re-projecting into the third view and checking for consistency with 2D BB detections. Using the correct three-view correspondences, the BBs are re-triangulated, and K-Means is used to label each BB as belonging to the ilium or fragment constellation.

over all possible correspondences and their poses. DRRs and similarities with the fluoroscopic image would be computed for each candidate pose, with the actual pose implied by the best similarity score. There are 1,680 possible correspondences when all 8 BBs are detected in the view. Since screws and K-wires are used to fix the fragment, false BB detections are common and may result in 12 BB detections, yielding 11,880 possible correspondences. For bilateral cases, with BBs, screws, or K-wires on the contralateral side, 24 BB detections could be possible, yielding 255,024 possible correspondences. Examples of 2D BB detections in fluoroscopy images are shown in supplementary figures S-1, S-2, and S-3, with the number of BB detections per image varying between 7 and 21. The sheer number of possible poses precludes the brute-force strategy from working in an intraoperatively compatible timeframe. However, we shall describe a procedure for pruning the number of candidate poses by several orders of magnitude, enabling the required similarity scores to be intraoperatively computed through GPU acceleration.

2.3.1. General Pose Pruning Strategy For a given 4-BB constellation, the general pruning strategy enumerates over each 3-BB sub-constellation. Furthermore, the full
Figure 6. Complete workflow used for single-view relative pose estimation of the acetabular fragment. Gray-shaded boxes correspond to the ilium and fragment BB constellation pose estimate workflows described in figures 8 and 11, respectively. The relative pose of the bone fragment is calculated using the BB constellation poses.

set of possible 3-BB 3D/2D correspondences for each sub-constellation is examined. Potential solutions to the P3P problem (Fischler and Bolles, 1981) are considered for each set of correspondences. Since we are concerned with pose estimation using fluoroscopic imagery, our approach to the P3P problem assumes that the BB constellation lies between the X-ray source and detector. This assumption enables solutions to be ignored which: are impossible given the rigid structure of the constellation, or which place the BB constellation too close to the X-ray source. Many poses that would be produced from incorrect 3-BB correspondences are discarded in this way. In addition to the point sets and hypothesized correspondences, a set of source-to-detector ratios is also required as input to the P3P solver. The source-to-detector ratios are used to back-project one of the 2D BB detections to possible 3D locations, simplifying the pruning problem. Full details of this approach are described in Appendix C. Solutions reported by the P3P solver are further pruned according to anatomical constraints. The candidate source-to-detector ratios and anatomical constraints differ for the ilium and fragment BB constellations. An overview of the general pruning approach is shown in figure 7.

2.3.2. Ilium BB Pose Estimation The pose of the ilium BB constellation is recovered first and is then used to assist with establishing the pose of the fragment BB constellation. Figure 8 shows the high level data flow for pose estimation of the ilium constellation. A set of 129 uniformly spaced source-to-detector ratios is used for each ilium P3P invocation: \{0.6 + 0.003125k | k = 0, 1, \ldots, 128\}. Using the APP coordinate
Figure 7. The data flow of the general pruning strategy used during BB constellation pose estimations. Dashed boxes indicate input data and processing that will be specific for either ilium or fragment processing.

Figure 8. Overview of the ilium BB constellation pose estimation process. The workflow of the general pruning strategy (figure 7) is re-used here and highlighted in gray, with inputs specific to ilium pruning emphasized by dashed borders. Since the general pruning strategy returns multiple possible poses and BB correspondences, image intensities are used to select the best candidate pose. The pose is further refined by a 2D/3D intensity-based registration of the pre-osteotomy pelvis, with success criteria automatically verified by the number of ilium BBs matched through re-projection.
frame, a reference AP orientation of the pre-osteotomy pelvis, with respect to the C-Arm, is constructed and used for pruning anatomically implausible ilium poses. The AP orientation has the following properties: the patient is supine with the X-ray detector placed anteriorly, the AP axis is parallel to the C-Arm depth axis, the IS axis is parallel to the 2D image row axis with the top of the image more superior than the bottom, and the LR axis is parallel to the 2D image column axis. Each candidate P3P pose is examined to obtain the difference in orientation from the reference AP pose and an Euler decomposition is used to obtain rotation angles about each anatomical axis. Poses are pruned when the magnitude of any Euler angle is greater than 60°. Using such a large range of allowable angles permits all reasonable C-Arm orientations while eliminating highly unlikely poses, such as those that place the detector beneath, or nearly orthogonal with, the surface of the operating table. An example of a pose pruned using this logic is shown in the top row of figure 9. Using each remaining candidate ilium pose, the original fragment BB constellation is projected into the view; e.g. where the fragment BBs would be located in 2D had the fragment not been moved. Since the majority of fragment movement consists of rotation, the re-projected fragment BBs should lie nearby to 2D BB detections. Poses are pruned when less than 3 of the fragment BBs are projected inside the bounds of the 2D image. For each projected fragment BB, the distance to the nearest 2D detection is calculated, and the three BBs with the smallest nearest distances are recorded. When the mean distance associated with these three BBs is greater than 200 pixels the candidate ilium pose is pruned.

Once the general pruning strategy has been completed for the ilium BB constellation, the naïve brute-force, intensity-based, approach is used to select the best ilium pose from the remaining candidates. The bottom row of figure 9 shows several examples of image similarity calculated from poses derived from different correspondences. This pose is used as initialization for an intensity-based 2D/3D registration of the pre-osteotomy pelvis to the fluoroscopic image. Details of the intensity-based registration parameters are listed in Appendix A. Using the pose estimate computed during the intensity-based registration, final ilium BB correspondences are established by re-projecting the 3D ilium BBs into 2D. Correspondences are greedily assigned based on the minimum 2D distances between projected BB locations and detected 2D BB locations. However, no correspondence is established for projected BBs with minimum distances greater than 10.5 pixels. When less than two correspondences are established we consider the algorithm to have failed in establishing the ilium pose and no further processing is performed. The ilium pose is set to the intensity-based registration pose when exactly two correspondences are established. When three or four correspondences are established, the ilium pose is refined by optimizing over the corresponding ilium BB re-projection distances starting from the intensity-based pose as the initial guess.

The set of 2D BB detections is pruned down to exclude: BBs already matched to the ilium, and any BBs that are distant from the expected location of the fragment. A BB is considered distant if the closest, re-projected, fragment BB is greater than 200
Figure 9. The top row shows an ilium pose pruned for excessive difference from the reference AP orientation (137° about the AP axis). The green sphere indicates the X-ray source with a green line connecting to the principal point on the X-ray detector. For this example, the candidate correspondences were able to satisfy the constraints of the P3P solver. However, the implausibility of the pose reveals the incorrectness of the correspondences. The bottom row depicts several examples of ilium poses and correspondences used for initialization of the full-pelvis intensity-based, 2D/3D, registration. Green edges, derived from a specific pelvis pose, are overlaid over the intraoperative fluoroscopic image. Agreement between the overlaid edges and base image indicates agreement between the hypothesized pose and true pose. Image similarity scores are listed in the bottom right of each overlay. The scores are computed from DRRs, computed at each candidate pose, and the intraoperative fluoroscopic image. Lower scores indicate better similarity, with the bottom right example representing the most likely pose of the four. pixels away. This is a variation of the process previously used for pruning ilium poses by re-projection of 3D fragment BBs.

2.3.3. **Fragment BB Pose Estimation** The fragment pose recovery is started by conducting the general pruning strategy over candidate fragment BB correspondences and poses. Since approximate depth of the BBs is known from the ilium pose recovery process, only 33 source-to-detector ratios are passed to the P3P solver. A reference source-to-detector ratio, \( \hat{r} \), is computed by mapping the centroid of the fragment 3D BB constellation into the C-Arm coordinate frame using the ilium pose. The source-to-detector ratios are then uniformly sampled about this reference: \( \{ \hat{r} \pm 0.003125k | k = 0, 1, \ldots, 16 \} \). Using each solution produced by the P3P solver, the relative pose of the fragment is computed using (1). Any relative pose with rotation magnitude greater than 60° or translation magnitude greater than 30 mm is pruned.
Figure 10. An example of an implausible fragment pose which was pruned due to a large rotation of 142°. Despite their incorrectness, the candidate correspondences used to compute this candidate fragment pose were able to satisfy the P3P solver constraints.

Figure 10 shows an example fragment pose that was pruned in this fashion.

Due to the difficult nature of the chiseling process, the true shape of the acetabular fragment usually differs from the preoperatively planned shape. For this reason, image similarities are not used to select the best candidate returned from the general pruning process. Instead, the best candidate is selected by choosing the pose yielding the largest number of matching BBs and the smallest mean re-projection distance. The match criterion used for ilium matches is reused here. When less than 3 BBs are matched, the method reports failure. However, the fragment pose is refined by an optimization over re-projection distances if at least 3 BBs are matched. The optimization is regularized by the translation magnitude of the fragment pose relative to the APP. This regularization is reasonable, since the fragment movement is believed to consist primarily of rotation and the approximate depth is known from the ilium pose. The specific workflow used for pose estimation of the fragment BB constellation is shown in figure 11.

It is important to note that the approach described here only requires correspondences to be established for three ilium BBs and three fragment BBs. Therefore, the proposed method provides some robustness to occlusion, since it is unlikely that more than one BB from a single constellation will be occluded for any given view. Likewise, it is still feasible to obtain fragment pose estimates when a single BB (per constellation) becomes dislodged from the bone.

2.4. Cadaver Experiments

Surgeries performed on three, non-dysplastic, cadaveric specimens were used to evaluate the proposed method. Specimens 1, 2, and 3 were aged 89, 87, 94 and were male, female, and male, respectively. Preoperative processing and planning was performed bilaterally for each specimen and six PAOs were performed by our surgeon co-author, B.A.M. The Halifax injector, using BBs of 1 mm diameter, was only used during surgeries for specimens 2 and 3. For specimen 1, bone burs were created on the surface of the pelvis,
and 1.5 mm diameter BBs were affixed with cyanoacrylate. The larger BBs were also inserted into specimens 2 and 3, but were only used for ground truth calculations. A comprehensive discussion of BB insertion and the fragment pose ground truth protocol is found in (Grupp et al., 2019). Ground truth poses for specimen 1 were calculated using the 2D/3D known BB constellation approach, whereas specimens 2 and 3 use the 3D/3D method.

Three fluoroscopic views were used to reconstruct the pre-osteotomy 3D BB constellations for each surgery. Pose estimation of the relocated fragment was conducted on 3 separate fluoroscopic images, each with different viewing geometries. All fluoroscopy was obtained using a Siemens CIOS Fusion C-Arm with 30 inch flat panel detector.

A video example of the full pipeline, from preoperative planning to intraoperative pose estimation is available online at: https://youtu.be/0E0U9G81q8g.

3. Results

3.1. Intraoperative BB Reconstruction

For pre-osteotomy BB reconstruction, there were no false BB detections or missed detections in the 2D images. Table 1 summarizes the reconstruction errors. The mean reconstruction error of the larger BBs implanted into specimen 1 was 2.6 mm. For specimens 2 and 3, the mean reconstruction error of the smaller, injected, BBs was 1.4
Table 1. A summary of BB reconstruction errors for each surgery, specified by the cadaver specimen number and operative side. The means and standard deviations of reconstruction errors are given for the separate ilium and fragment BB constellations and also the entire set of BBs. For each surgery, four BBs were reconstructed for each of the ilium and fragment constellations.

| Surgery | Reconstruction Errors (mm) |
|---------|-----------------------------|
|         | Ilium BBs | Fragment BBs | All BBs  |
| 1 Left  | 2.5 ± 0.4 | 3.2 ± 0.2 | 2.9 ± 0.5 |
| 1 Right | 2.1 ± 0.2 | 2.7 ± 0.5 | 2.4 ± 0.5 |
| 2 Left  | 1.6 ± 0.3 | 1.4 ± 0.2 | 1.5 ± 0.3 |
| 2 Right | 1.3 ± 0.5 | 1.1 ± 0.1 | 1.2 ± 0.3 |
| 3 Left  | 1.3 ± 0.3 | 1.1 ± 0.4 | 1.2 ± 0.3 |
| 3 Right | 1.4 ± 0.2 | 1.6 ± 0.2 | 1.5 ± 0.2 |

mm. The mean computation time for the entire reconstruction pipeline was 8.3 ± 0.4 seconds. When excluding the 2D/3D full pelvis registration time required for relative pose recovery of the C-Arm, the correspondence establishment and reconstruction took 0.7 ± 0.1 seconds. Timing measurements were conducted using a single NVIDIA P100 (PCIe) GPU and seven cores of an Intel Xeon E5-2680 v4 CPU.

3.2. Intraoperative Pose Estimation

A summary of the maximum number of ilium and fragment poses considered, and the actual poses considered due to pruning, is shown in Table 2. Due to the range of source-to-detector distances searched over, the maximum number of poses considered is greater than the maximum number of possible correspondences. On average, 99.6% of the maximum number of ilium poses are pruned by the P3P solver step. Using anatomical constraints, the remaining poses are pruned by an average of 95.9%. The maximum number of poses are pruned by an average of 97.3% for the fragment case, with anatomical constraints pruning 81.2% of the remaining poses on average.

Pose estimation was successfully performed on 18 total views (3 per surgery). Errors in rotation were below 3° for 12 of the 18 cases, with a mean of 2.4°. When the rotation errors were decomposed about anatomical axes, only rotation about the IS axis had errors greater than 3°. In terms of both mean and standard deviation, rotation measurements about the AP axis were the most accurate, followed by LR, and then IS. The maximum 3D LCE angle error was 1.8° and the mean was 1.0°. The mean translation error was 2.1 mm, and was less than 3 mm for 15 of the 18 estimates. Mean translation errors about the anatomical axes were all within 0.2 mm of each other, and the maximum difference between standard deviations was 0.3 mm. The entire listing of errors for each pose estimate is shown in Table 3.

Table 4 includes a full summary of the number of BB detections and matches in each image. All four ilium BBs were matched in 6 of the 18 cases and all four fragment BBs were matched in 16 of the 18 cases. The mean rotation, translation, and LCE angle
Table 2. A summary of the number of pose and correspondence combinations for the ilium and fragment BB constellations during the process of single-view fragment pose estimation for the three different views of each cadaver surgery. The maximum number of possible combinations are listed, along with the number after each pruning step. Each of the pose candidates after anatomical pruning for the ilium is used for initialization of the full-pelvis intensity-based 2D/3D registration. The maximum number of fragment poses and correspondences is lower than that of the ilium, since the ilium correspondences are established first and implausible fragment BB detections are pruned.

| Surgery | Proj. | # Ilium Pose/Correspondence Candidates | # Frag. Pose/Correspondence Candidates |
|---------|-------|---------------------------------------|---------------------------------------|
|         |       | Before Pruning | After P3P Pruning | After Anat. Pruning | Before Pruning | After P3P Pruning | After Anat. Pruning |
| 1 Left  | 1     | 885,456 | 2,567 | 155 | 7,920 | 139 | 34 |
| 1 Left  | 2     | 681,120 | 2,381 | 80 | 7,920 | 157 | 40 |
| 1 Left  | 3     | 4,117,680 | 6,230 | 168 | 15,840 | 198 | 22 |
| 1 Right | 1     | 510,840 | 843 | 24 | 15,840 | 178 | 33 |
| 1 Right | 2     | 260,064 | 645 | 17 | 7,920 | 97 | 18 |
| 1 Right | 3     | 371,520 | 472 | 14 | 7,920 | 153 | 52 |
| 2 Left  | 1     | 108,360 | 450 | 12 | 3,168 | 68 | 9 |
| 2 Left  | 2     | 510,840 | 590 | 10 | 3,168 | 89 | 6 |
| 2 Left  | 3     | 1,126,944 | 1,204 | 10 | 3,168 | 89 | 10 |
| 2 Right | 1     | 108,360 | 1,097 | 70 | 3,168 | 156 | 30 |
| 2 Right | 2     | 371,520 | 858 | 23 | 3,168 | 142 | 17 |
| 2 Right | 3     | 885,456 | 1,269 | 32 | 3,168 | 106 | 22 |
| 3 Left  | 1     | 371,520 | 783 | 35 | 3,168 | 122 | 12 |
| 3 Left  | 2     | 1,408,680 | 1,359 | 48 | 792 | 12 | 4 |
| 3 Left  | 3     | 173,376 | 958 | 50 | 3,168 | 100 | 12 |
| 3 Right | 1     | 173,376 | 1,639 | 129 | 3,168 | 106 | 22 |
| 3 Right | 2     | 108,360 | 1,139 | 85 | 3,168 | 108 | 17 |
| 3 Right | 3     | 260,064 | 698 | 58 | 792 | 26 | 5 |

errors for estimates with 4 ilium BBs matched were 1.7°, 2.1 mm, and 1.0°, respectively. With less than 4 ilium BBs matched, the mean errors were 2.8°, 2.0 mm, and 1.1°, respectively. With 4 fragment BBs matched, the mean rotation, translation, and LCE angle errors were 2.3°, 2.1 mm, and 1.0°, respectively. The mean errors were 3.1°, 1.5 mm, and 1.6°, when less than 4 fragment BBs matched.

In the third view for the left side of specimen 1, one ilium BB was outside the image bounds and not detected. On the left side of specimen 2, one of the ilium BBs was occluded by K-wire in each view and therefore not detected. Analysis of the postoperative CT revealed that this BB was actually dislodged by either: performance of the ilium osteotomy or insertion of the K-wire. The missed ilium detections in views 1 and 2 on the right side of specimen 2, were occluded by screws. Occlusion by K-wire also caused the missed ilium detection in view 2 on the right side of specimen 3. However, according to the postoperative CT this ilium BB was also displaced from the bone. The
missed fragment BB detections were caused by K-wire occlusion.

The mean computation time for the single-view pose estimation was $0.7 \pm 0.2$ seconds, and was measured using the same hardware used to record reconstruction times.

Thumbnails of each fluoroscopy image used for fragment pose estimation during the cadaver experiments are found in supplementary figures S-1, S-2, and S-3.

4. Discussion

Although one third of fragment pose estimates had rotation errors larger than $3^\circ$, LCE angle errors were well below the $3^\circ$ success criteria identified in (Grupp et al., 2019). This indicates that the proposed method is able to quantify the amount of femoral head coverage, resulting from an intraoperatively relocated acetabulum, within clinically acceptable error thresholds.

Given the automatic nature of the method and the relatively quick runtime, it should be feasible for clinicians to smoothly move between making pose adjustments to
Table 4. A summary of the number of BBs detected in each image and the number matched by the pose estimation process. The total number of 2D BB detections includes false alarms on screws and BB detections on the contralateral side. The number of ilium and fragment BB detections, indicate the number of BBs detected from the appropriate constellation; a number less than 4 implies missed-detections. The number of ilium and fragment BB matches is the number of final correspondences established per constellation for a given set of ilium and fragment poses.

| Surgery | Proj. | # Total Detected | Ilium BBs | Fragment BBs |
|---------|-------|------------------|-----------|--------------|
|         |       |                  | # Detected | # Matched | # Detected | # Matched |
| 1 Left  | 1     | 13               | 4         | 2          | 4           | 4          |
| 1 Left  | 2     | 12               | 4         | 4          | 4           | 4          |
| 1 Left  | 3     | 21               | 3         | 2          | 4           | 4          |
| 1 Right | 1     | 11               | 4         | 4          | 4           | 4          |
| 1 Right | 2     | 9                | 4         | 2          | 4           | 4          |
| 1 Right | 3     | 10               | 4         | 4          | 4           | 4          |
| 2 Left  | 1     | 7                | 3         | 3          | 4           | 4          |
| 2 Left  | 2     | 11               | 3         | 3          | 4           | 4          |
| 2 Left  | 3     | 14               | 3         | 3          | 4           | 4          |
| 2 Right | 1     | 7                | 3         | 3          | 4           | 4          |
| 2 Right | 2     | 10               | 3         | 3          | 4           | 4          |
| 2 Right | 3     | 13               | 4         | 3          | 4           | 4          |
| 3 Left  | 1     | 10               | 4         | 4          | 4           | 4          |
| 3 Left  | 2     | 15               | 4         | 4          | 3           | 3          |
| 3 Left  | 3     | 8                | 4         | 4          | 4           | 4          |
| 3 Right | 1     | 8                | 4         | 3          | 4           | 4          |
| 3 Right | 2     | 7                | 3         | 3          | 4           | 4          |
| 3 Right | 3     | 9                | 4         | 3          | 3           | 3          |

the fragment, taking fluoroscopic shots, and receiving feedback regarding the current pose estimate.

The mean rotation error when less than the full number of BBs were matched in either constellation, was 1.1° greater than the mean rotation error over the cases matching full BB constellations. However, mean translation and mean LCE angle error were less effected by unmatched ilium BBs. When only 3 fragment BBs were matched, the mean LCE angle error was 0.6° larger than the mean LCE angle error associated with all fragment BBs matched. Therefore, the number of matched BBs in each constellation may be used to convey confidences in the estimated poses. When less than 4 fragment BBs are matched, confidence in any rotation and LCE angle would be lowered. For cases when all 4 fragment BBs were matched, but less than 4 ilium BBs were matched, confidence in LCE angle would remain unaffected, however confidence in general rotation would be reduced.

Highlighting the robustness of the method, all LCE errors remained below the 3° error threshold, even when BBs were missing from the view or occluded. The method was also robust against BBs which were displaced from the pelvis, but remained in the field of view and detected. This was demonstrated on the right side of specimen 3,
where an ilium BB had been dislodged from the bone and was detected in views 1 and 2. Since the P3P solver does not return solutions for non-rigid transformations of the constellations, the displaced BB was not matched, despite the detection of all four ilium BBs in these views.

Average BB reconstruction errors for specimen 1 were greater than those of specimens 2 and 3. This was most likely caused by larger 2D BB localization errors for specimen 1. This is to be expected, since the BBs used for specimen 1 were larger than those used for specimens 2 and 3. Only one of the six LCE errors for specimen 1 was greater than 1°, indicating that the proposed method is not dependent on a single size of BBs.

The performance of the method compares favorably to the fiducial-free method (FFM) proposed in (Grupp et al., 2019). When a postoperatively segmented fragment shape was retrospectively used for pose estimation, the FFM was reported to have mean rotation error of 2.2°, mean translation error of 2.2 mm, and mean LCE error of 1.1°. The mean rotation error of the proposed method is only slightly larger than the FFM mean rotation error, while translation and LCE angle errors of the proposed method are slightly smaller. When the FFM uses an intraoperatively refined version of a preoperatively planned fragment shape, the mean rotation, translation, and LCE angle errors increase to 3.5°, 2.5 mm, and 1.8°, respectively. Considering that an accurate segmentation of the fragment shape is not available intraoperatively, the method proposed in this paper provides a more accurate assessment of fragment pose and femoral head coverage than the FFM.

In the current state of practice, preoperative CT data cannot be effectively used for intraoperative assessment of anatomical angles. As a result, contemporary preoperative imaging usually consists solely of standing radiographs. Although the proposed method requires a preoperative CT of the patient to be collected, the patient specific CT may be replaced with a statistical atlas of pelvis anatomy (Otake et al., 2015) in the future. In this approach, the patient’s anatomy would be reconstructed using a deformable 2D/3D registration between patient-specific 2D X-ray images and the atlas (Sadowsky et al., 2007; Hurvitz and Joskowicz, 2008; Zheng, 2010; Kang et al., 2016). A precise cartilage model is required for a comprehensive biomechanical analysis, including estimates of the joint contact pressure (Armiger et al., 2009). Since a statistical atlas may not be capable of satisfactorily reconstructing the cartilage model, a lower-dose, partial CT of the patient’s acetabulum may be used to augment the statistical model (Chintalapani et al., 2010; Grupp et al., 2016).

The only manual portion of the intraoperative pipeline is the annotation of anatomical landmarks during BB reconstruction. Using recent advances in fluoroscopic deep learning (Bier et al., 2019), we believe it may be possible for these landmarks to be localized automatically. This would result in a completely automatic intraoperative pipeline, further reducing the impact on existing surgical workflows.

Although the registration framework leveraged from (Grupp et al., 2019) is a highly optimized C++ library with OpenCL GPU acceleration, the pruning algorithms
described in this paper were implemented as serial C++ routines. We believe faster execution times should be possible, since the candidate poses and correspondences evaluated in each pruning phase are not dependent on one another, and the computations may be done in parallel.

Due to the possibility of BBs becoming dislodged from the bone, some clinicians may find the permanent insertion of BBs into patients unacceptable. We believe that this outcome may be avoided by using structures temporarily affixed to the bones during surgery, and then removed after the fragment is satisfactorily fixed in place. The development of such structures is the topic of future work, however two possibilities include: a deformable mesh of BBs that could be impressed on the bone surface, or percutaneously inserted wires which would pierce into the pelvis bone.

The method may also be used for cadaveric PAO training. Pose estimates provided by the system could act as feedback for the mental estimates of the surgeon. In this way, the system may improve surgeons’ association of tactile sensing and fluoroscopic interpretation with a fragment’s true pose.

Future work will also apply the techniques developed here for fluoroscopic osteotome tracking during PAO. The osteotome could be augmented with a pattern of BBs about it’s shaft to enable processing similar to what is done on the acetabular fragment in this paper.

5. Conclusion

This paper has proposed a new method for pose estimation of acetabular fragments using flouroscopy and two constellations of intraoperatively implanted BBs. Cadaveric studies have shown that the method is able to provide clinically accurate estimates of the LCE angle, a well-established indicator of femoral head coverage. Once the BB constellations have been reconstructed in 3D, all fragment poses are calculated automatically using a single-view, and in sub-second runtime. No other surgical equipment beyond a flat panel C-Arm and BB injector is required. The C-Arm does not need to be calibrated, encoded, or motorized. Unlike other fluoroscopic approaches, accurate knowledge of the bone fragment’s shape is not necessary. For these reasons, the proposed method provides minimal deviation from the standard surgical workflow, and should be easily mastered by clinicians already performing RSA.

Acknowledgments

We are grateful for the assistance of Mr. Demetries Boston during the cadaveric surgeries. This research was supported by NIH/NIBIB grants R01EB006839, R21EB020113, MEXT/JSPS KAKENHI 26108004, Johns Hopkins University Internal Funds, and a Johns Hopkins University Applied Physics Laboratory Graduate Student Fellowship. Part of this research project was conducted using computational resources at the Maryland Advanced Research Computing Center (MARCC).
Appendix A. Intensity-Based 2D/3D Registration Parameters

The pelvis-as-fiducial, intensity-based, registration parameters described in (Grupp et al., 2019) are exactly those used for the pre-osteotomy BB reconstruction phase. Each registration in the reconstruction phase runs two resolutions, $8\times$ and $4\times$ downsampling in 2D. In order to overcome large initialization offsets from ground truth, a computationally expensive, evolutionary optimization strategy is used at the $8\times$ level. The less computation-intensive, BOBYQA strategy (Powell, 2009), is used for optimization at the second level.

In order to avoid delays in the surgical workflow, a small execution time is desired during the post-osteotomy pose estimation phase. Therefore, the BOBYQA strategy is used at a single resolution level of $8\times$ downsampling in 2D. We believe a local optimization strategy is sufficient, since solutions reported by the P3P solver, using correct BB correspondences, should lie within some convex ball of the ground truth pose. Box constraints on the $se(3)$ parameter space are specified as in (A.1), where the X and Y axes are roughly aligned with image columns and rows, respectively, and the Z axis is aligned with the source-to-detector axis.

$$\{\pm15^\circ, \pm15^\circ, \pm30^\circ, \pm50, \pm50, \pm100\}$$

(A.1)

All other registration parameters remain identical to those of the reconstruction phase.

Appendix B. Pre-Osteotomy BB Reconstruction

Denote the sets of 2D detected BB locations as $P_v \subset \mathbb{R}^2$ for each view $v = 1, 2, 3$. Let $\mathcal{T} : \wp(\mathbb{R}^2) \rightarrow \mathbb{R}^3$ denote the triangulation operator used to reconstruct a 3D point from a collection of 2D points; $\wp$ indicates the power set operator. Let $\mathcal{D} : \mathbb{R}^3 \rightarrow \mathbb{R}$ denote the minimum distance of between a 3D point and the pelvis surface. Let $\mathcal{P}_v : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ denote the projection operator, applying a perspective projection of 3D points into the imaging plane of view $v$. As shown in (B.1), an initial set of correspondences and 3D triangulations, $A$, are computed for each of the candidate correspondences and any points lying further than $T$ mm away from the pelvis surface are pruned.

$$A = \{(p, q, \mathcal{T}(p, q)) | p \in P_1, q \in P_2, \mathcal{D}(\mathcal{T}(p, q)) < T\}$$

(B.1)

The remaining triangulated points are re-projected into the third view and the 2D distances to each BB detection are recorded, shown in (B.2).

$$B = \{(p, q, r, d) | (p, q, x) \in A, r \in P_3, d = \|\mathcal{P}_3(x) - r\|_2\}$$

(B.2)

The following book-keeping sets are initialized: the final set of 3D reconstructed points $C = \{}$, and sets indicating whether a 2D BB detection has been used for 3D reconstruction, $R_v = \{}$ for $v = 1, 2, 3$. We now iterate through $B$ in increasing order according to the re-projection component, $d$. For each $(p, q, r, d) \in B$, if $p \notin R_1$, $q \notin R_2$, and $r \notin R_3$, let $y = \mathcal{T}(p, q, r)$. If $\mathcal{D}(y) < T$, then the point is a suitable reconstruction; update the book-keeping sets: $C = C \cup \{y\}$, $R_1 = R_1 \cup \{p\}$, $R_2 = R_2 \cup \{q\}$, and
\( R_3 = R_3 \cup \{ r \} \). Once iteration over \( B \) is complete \( C \) represents the final set of 3D BB reconstructions. Iteration may be terminated early when any \( R_v \) is equal to \( P_v \) for \( v = 1, 2, 3 \).

Appendix C. Obtaining P3P Solutions

We wish to find plausible transformations that rigidly map three 3D model points into a C-Arm coordinate frame, so that their projected locations in 2D match a set of corresponding 2D points. Let \( B_1, B_2, B_3 \) denote the 3D model points and let \( b_1, b_2, b_3 \) be their (hypothesized) corresponding 2D points in the fluoroscopic image. The problem is simplified by assuming that the approximate depth, or proportion along the source-to-detector line, of \( B_1 \) in the C-Arm frame is known. Given this information, we know the location of \( B_1 \) with respect to the C-Arm, denoted as \( \tilde{B}_1 \). For \( j = 2, 3 \), let \( \tilde{B}_j(t) = s + t(\hat{b}_j - s) \) denote the lines which \( B_2 \) and \( B_3 \), with respect to the C-Arm, may possibly lie on. The X-ray source position is denoted by \( s \) and the 3D location on the X-ray detector, corresponding to \( b_j \), is denoted by \( \hat{b}_j \).

For specific values of \( t_2 \) and \( t_3 \), a potential pose is given by solving the 3D/3D corresponding point set registration (Horn, 1987) between \( \{ \tilde{B}_1, \tilde{B}_2(t_2), \tilde{B}_3(t_3) \} \) and \( \{ B_1, B_2, B_3 \} \). We find the four possible combinations of \( t_2 \) and \( t_3 \) and use the known shape of the 3D model to prune implausible poses.

Let \( l_{ij} = \| B_i - B_j \|_2 \) denote an inter-BB distance of the 3D model; the \( l_{ij} \) are known quantities. Let \( \tilde{l}_{ij}(t) = \| \tilde{B}_i - \tilde{B}_j(t) \|_2 \); the \( \tilde{l}_{ij}(t) \) are unknown quantities. Using \( l_{12} \) and \( l_{13} \), we find plausible values of \( t \) for \( \tilde{B}_2 \) and \( \tilde{B}_3 \) by solving (C.1) for \( j = 2, 3 \).

\[
\min_t \left( \tilde{l}_{ij}^2 - \tilde{l}_{ij}(t)^2 \right) \quad \text{(C.1)}
\]

Using MATLAB 2019a, derivatives and formulas for the possible minimizers of (C.1) were symbolically calculated; a maximum of 2 minimizers are possible. Let \( t_j^{(1)} \) and \( t_j^{(2)} \) denote the two solutions of (C.1) for \( j = 2, 3 \). Poses are pruned when (C.2), (C.3) and (C.4) are not satisfied for the combinations of \( j = 2, 3, k = 1, 2 \), and \( m = 1, 2 \).

\[
0.6 \leq t_j^{(k)} \leq 1.0 \quad \text{(C.2)}
\]

\[
1 - \epsilon \leq \frac{\tilde{l}_{1j}(t_j^{(k)})}{l_{ij}} \leq 1 + \epsilon \quad \text{(C.3)}
\]

\[
1 - \epsilon \leq \frac{\| \tilde{B}_2(t_j^{(k)}) - \tilde{B}_3(t_j^{(m)}) \|_2}{l_{23}} \leq 1 + \epsilon \quad \text{(C.4)}
\]

Pruning using (C.2) constrains objects to lie closer to the X-ray detector than the X-ray source. Pruning using (C.3) and (C.4) constrains \( \{ \tilde{B}_1, \tilde{B}_2(t_2), \tilde{B}_3(t_3) \} \) to have the same shape as \( \{ B_1, B_2, B_3 \} \). The pruning should be conducted in a greedy fashion in order to avoid unnecessary computation. A toy example depicting the geometries described here is shown in figure C1. For all experiments in this paper, \( \epsilon = 0.01 \).
Figure C1. A toy example of the P3P problem showing the four possible solutions when mapping the BB constellation into the C-Arm coordinate frame. See the text of Appendix C for a full explanation of notation. This drawing represents a specific source-to-detector distance used to estimate $\hat{B}_1$. For each of $B_2$ and $B_3$, two possible locations with respect to the C-Arm are shown. The inter-BB length to $B_1$ is preserved for all 4 solutions. However, visual comparisons of the dashed purple line in the volume coordinate frame with the corresponding lines in the C-Arm coordinate frame reveal that none of the candidate lengths between $B_2$ and $B_3$ are valid. Therefore, no solutions would be reported for this source-to-detector distance.

Similar to the method of Appendix A.3 in (Fischler and Bolles, 1981), we perform this process over a range of source-to-detector distances in order to achieve robustness in depth for $\hat{B}_1$. A minimum of zero, and a maximum of four, poses are identified for each depth.

References

Akiyama, H., Goto, K., So, K. and Nakamura, T. (2010). Computed tomography-based navigation for curved periacetabular osteotomy, *J Orthop Sci* 15(6): 829–833.

Armand, M., Grupp, R., Murphy, R., Hegman, R., Armiger, R., Taylor, R., McArthur, B. and Lepisto, J. (2018). Biomechanical guidance system for periacetabular osteotomy, *Intelligent Orthopaedics*, Springer, pp. 169–179.

Armand, M., Lepistö, J., Tallroth, K., Elias, J. and Chao, E. (2005). Outcome of periacetabular osteotomy: joint contact pressure calculation using standing AP radiographs, 12 patients followed for average 2 years, *Acta Orthop* 76(3): 303–313.

Armiger, R. S., Armand, M., Tallroth, K., Lepistö, J. and Mears, S. C. (2009). Three-dimensional mechanical evaluation of joint contact pressure in 12 periacetabular osteotomy patients with 10-year follow-up, *Acta Orthop* 80(2): 155–161.

Aubry, J.-F., Beaulieu, L., Girouard, L.-M., Aubin, S., Tremblay, D., Laverdière, J. and Vigneault, E. (2004). Measurements of intrafraction motion and interfraction and intrafraction rotation of prostate by three-dimensional analysis of daily portal imaging with radiopaque markers, *Int. J. Radiat. Oncol. Biol. Phys.* 60(1): 30–39.
Bier, B., Goldmann, F., Zaech, J.-N., Fotouhi, J., Hegeman, R., Grupp, R., Armand, M., Osgood, G., Navab, N., Maier, A. and Unberath, M. (2019). Learning to detect anatomical landmarks of the pelvis in X-rays from arbitrary views, *Int J Comput Assist Radiol Surg* pp. 1–11.

Blondel, C., Malandain, G., Vaillant, R. and Ayache, N. (2006). Reconstruction of coronary arteries from a single rotational X-ray projection sequence, *IEEE Trans. Med. Imag.* 25(5): 653–663.

Chintalapani, G., Murphy, R., Armiger, R. S., Lepistö, J., Otake, Y., Sugano, N., Taylor, R. H. and Armand, M. (2010). Statistical atlas based extrapolation of CT data, *Proc. SPIE* pp. 762539–762539.

Choi, J.-H., Maier, A., Keil, A., Pal, S., McWalter, E. J., Beaupré, G. S., Gold, G. E. and Fahrig, R. (2014). Fiducial marker-based correction for involuntary motion in weight-bearing C-arm CT scanning of knees. II. Experiment, *Med Phys* 41(6Part1): 061902.

Dang, H., Otake, Y., Schafer, S., Stayman, J. W., Kleinszig, G. and Siewerdsen, J. (2012). Robust methods for automatic image-to-world registration in cone-beam CT interventional guidance, *Med Phys* 39(10): 6484–6498.

De Bruin, P., Kaptein, B., Stoel, B., Reiber, J., Rozing, P. and Valstar, E. (2008). Image-based RSA: Roentgen stereophotogrammetric analysis based on 2D–3D image registration, *J Biomech* 41(1): 155–164.

De Raedt, S., Mechelenburg, I., Stilling, M., Römer, L., Murphy, R. J., Armand, M., Lepistö, J., de Bruin, M. and Søballe, K. (2018). Reliability of computer-assisted periacetabular osteotomy using a minimally invasive approach, *Int J Comput Assist Radiol Surg* 13(12): 2021–2028.

Fischler, M. A. and Bolles, R. C. (1981). Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography, *Commun. ACM* 24(6): 381–395.

Gala, L., Clohisy, J. C. and Beaulé, P. E. (2016). Hip dysplasia in the young adult, *J. Bone Joint Surg.-Am Vol.* 98(1): 63–73.

Ganz, R., Klaue, K., Vinh, T. S. and Mast, J. W. (1988). A new periacetabular osteotomy for the treatment of hip dysplasias technique and preliminary results., *Clin. Orthop. Relat. Res.* 232: 26–36.

Grupp, R. B., Hegeman, R., Murphy, R., Alexander, C., Otake, Y., McArthur, B., Armand, M. and Taylor, R. H. (2019). Pose estimation of periacetabular osteotomy fragments with intraoperative X-ray navigation, *IEEE Trans. Biomed. Eng.*

Grupp, R., Otake, Y., Murphy, R., Parvizi, J., Armand, M. and Taylor, R. (2016). Pelvis surface estimation from partial CT for computer-aided pelvic osteotomies, *Orthop. Proc.*, Vol. 98, The British Editorial Society of Bone & Joint Surgery, pp. 55–55.

Hamming, N., Daly, M., Irish, J. and Siewerdsen, J. (2009). Automatic image-to-world registration based on X-ray projections in cone-beam CT-guided interventions, *Med Phys* 36(5): 1800–1812.

Hartley, R. and Zisserman, A. (2003). *Multiple view geometry in computer vision*, Cambridge university press.

Hipp, J. A., Sugano, N., Millis, M. B. and Murphy, S. B. (1999). Planning acetabular redirection osteotomies based on joint contact pressures., *Clin. Orthop. Relat. Res.* 364: 134–143.

Horn, B. K. (1987). Closed-form solution of absolute orientation using unit quaternions, *J Opt Soc Am A* 4(4): 629–642.

Hurvitz, A. and Jokowicz, L. (2008). Registration of a CT-like atlas to fluoroscopic X-ray images using intensity correspondences, *Int J Comput Assist Radiol Surg* 3(6): 493.

Ioppolo, J., Börlin, N., Bragdon, C., Li, M., Price, R., Wood, D., Malchau, H. and Nivbrant, B. (2007). Validation of a low-dose hybrid RSA and fluoroscopy technique: Determination of accuracy, bias and precision, *J Biomechan* 40(3): 686–692.

Jain, A. K., Mustafa, T., Zhou, Y., Burdette, C., Chirikjian, G. S. and Fichtinger, G. (2005). Ftraca robust fluoroscope tracking fiducial, *Med Phys* 32(10): 3185–3198.
Kang, X., Armand, M., Otake, Y., Yau, W.-P., Cheung, P. Y., Hu, Y. and Taylor, R. H. (2013). Robustness and accuracy of feature-based single image 2-D–3-D registration without correspondences for image-guided intervention, *IEEE Trans. Biomed. Eng.* **61**(1): 149–161.

Kang, X., Yau, W.-P. and Taylor, R. H. (2016). Simultaneous pose estimation and patient-specific model reconstruction from single image using maximum penalized likelihood estimation (MPLE), *Pattern Recognit.* **57**: 61–69.

Kårrholm, J., Hansson, L. I. and Selvik, G. (1984). Changes in tibiofibular relationships due to growth disturbances after ankle fractures in children., *J. Bone Joint Surg.-Am Vol.* **66**(8): 1198–1210.

Krčah, M., Székely, G. and Blanc, R. (2011). Fully automatic and fast segmentation of the femur bone from 3D-CT images with no shape prior, *Proc. IEEE Intl. Symp. Biomed. Imag.* pp. 2087–2090.

Langlotz, F., Bächler, R., Berlemann, U., Nolte, L.-P. and Ganz, R. (1998). Computer assistance for pelvic osteotomies., *Clin. Orthop. Relat. Res.* **354**: 92–102.

Langlotz, F., Stucki, M., Bächler, R., Scheer, C., Ganz, R., Berlemann, U. and Nolte, L.-P. (1997). The first twelve cases of computer assisted periacetabular osteotomy, *Comput. Aided Surg.* **2**(6): 317–326.

Litzenberg, D., Dawson, L. A., Sandler, H., Sanda, M. G., McShan, D. L., Ten Haken, R. K., Lam, K. L., Brock, K. K. and Balter, J. M. (2002). Daily prostate targeting using implanted radiopaque markers, *Int. J. Radiat. Oncol. Biol. Phys.* **52**(3): 699–703.

Liu, L., Ecker, T. M., Schumann, S., Siebenrock, K.-A. and Zheng, G. (2016a). Evaluation of constant thickness cartilage models vs. patient specific cartilage models for an optimized computer-assisted planning of periacetabular osteotomy, *PLoS ONE* **11**(1): e0146452.

Liu, L., Zheng, G., Bastian, J. D., Keel, M. J. B., Nolte, L. P., Siebenrock, K. A. and Ecker, T. M. (2016b). Periacetabular osteotomy through the pararectus approach: technical feasibility and control of fragment mobility by a validated surgical navigation system in a cadaver experiment, *Int Orthop* **40**(7): 1389–1396.

Loy, G. and Zelinsky, A. (2003). Fast radial symmetry for detecting points of interest, *IEEE Trans. Pattern Anal. Mach. Intell.* (8): 959–973.

Markelj, P., Tomažević, D., Likar, B. and Permuš, F. (2012). A review of 3D/2D registration methods for image-guided interventions, *Med Image Anal* **16**(3): 642–661.

Mayman, D. J., Rudan, J., Yach, J. and Ellis, R. (2002). The kingston periacetabular osteotomy utilizing computer enhancement: a new technique, *Comput Aided Surg* **7**(3): 179–186.

Mechlenburg, I., Kold, S., Romer, L. and Soballe, K. (2007). Safe fixation with two acetabular screws after Ganz periacetabular osteotomy, *Acta Orthop* **78**(3): 344–349.

Murphy, R. J., Armiger, R. S., Lepistö, J. and Armand, M. (2016). Clinical evaluation of a biomechanical guidance system for periacetabular osteotomy, *J Orthop Surg Res* **11**(1): 1.

Murphy, R. J., Armiger, R. S., Lepistö, J., Mears, S. C., Taylor, R. H. and Armand, M. (2015). Development of a biomechanical guidance system for periacetabular osteotomy, *Int J Comput Assist Radiol Surg* **10**(4): 497–508.

Murphy, R., Otake, Y., Lepistö, J. and Armand, M. (2013). Computer-assisted X-ray image-based navigation of periacetabular osteotomy with fiducial based 3D acetabular fragment tracking, *Orthop. Proc.*, Vol. 95, The British Editorial Society of Bone & Joint Surgery, pp. 84–84.

Niknafs, N., Murphy, R. J., Armiger, R. S., Lepistö, J. and Armand, M. (2013). Biomechanical factors in planning of periacetabular osteotomy, *Front Bioeng Biotechnol* **1**.

Nikou, C., Jaramaz, B., DiGioia, A. M. and Levison, T. J. (2000). Description of anatomic coordinate systems and rationale for use in an image-guided total hip replacement system, *Proc. Med. Image Comput. Comput.-Assist. Interv.*, pp. 1188–1194.

Otake, Y., Murphy, R., Grupp, R., Sato, Y., Taylor, R. and Armand, M. (2015). Comparison of optimization strategy and similarity metric in atlas-to-subject registration using statistical deformation model, *Proc. SPIE*, International Society for Optics and Photonics, pp. 94150Q–94150Q.
Otsuki, B., Takemoto, M., Kawanabe, K., Awa, Y., Akiyama, H., Fujibayashi, S., Nakamura, T. and Matsuda, S. (2013). Developing a novel custom cutting guide for curved peri-acetabular osteotomy, *Int Orthop* **37**(6): 1033–1038.

Powell, M. J. (2009). The BOBYQA algorithm for bound constrained optimization without derivatives, *Cambridge NA Report NA2009/06, University of Cambridge, Cambridge*.

Radermacher, K., Portheine, F., Anton, M., Zimolong, A., Kaspers, G., Rau, G. and Staudte, H.-W. (1998). Computer assisted orthopaedic surgery with image based individual templates., *Clin. Orthop. Relat. Res.* **354**: 28–38.

Sadowsky, O., Chintalapani, G. and Taylor, R. (2007). Deformable 2D-3D registration of the pelvis with a limited field of view, using shape statistics, *Proc. Med. Image Comput. Comput.-Assist. Interv*, pp. 519–526.

Schonberger, J. L. and Frahm, J.-M. (2016). Structure-from-motion revisited, *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, pp. 4104–4113.

Schweikard, A., Glosser, G., Boddueluri, M., Murphy, M. J. and Adler, J. R. (2000). Robotic motion compensation for respiratory movement during radiosurgery, *Comput. Aided Surg.* **5**(4): 263–277.

Seehaus, F., Olender, G. D., Kaptein, B. L., Ostermeier, S. and Hurschler, C. (2012). Markerless roentgen stereophotogrammetric analysis for in vivo implant migration measurement using three dimensional surface models to represent bone, *J Biomechan* **45**(8): 1540–1545.

Selvik, G. (1990). Roentgen stereophotogrammetric analysis, *Acta Radiol* **31**(2): 113–126.

Snavely, N., Seitz, S. M. and Szeliski, R. (2008). Modeling the world from internet photo collections, *Int. J. Comput. Vis.* **80**(2): 189–210.

Steger, T., Hoßbach, M. and Wesarg, S. (2013). Marker detection evaluation by phantom and cadaver experiments for C-arm pose estimation pattern, *Proc. SPIE*, Vol. 8671, International Society for Optics and Photonics, p. 86711V.

Takao, M., Nishii, T., Sakai, T. and Sugano, N. (2017). Comparison of rotational acetabular osteotomy performed with navigation by surgeons with different levels of experience of osteotomies, *Int J Comput Assist Radiol Surg* **12**(5): 841–853.

Tang, T. S., Ellis, R. E. and Fichtinger, G. (2000). Fiducial registration from a single x-ray image: a new technique for fluoroscopic guidance and radiotherapy, *Proc. Med. Image Comput. Comput.-Assist. Interv*, Springer, pp. 502–511.

Tang, T. S., MacIntyre, N., Gill, H., Fellows, R., Hill, N., Wilson, D. and Ellis, R. E. (2004). Accurate assessment of patellar tracking using fiducial and intensity-based fluoroscopic techniques, *Med Image Anal* **8**(3): 343–351.

Troelsen, A. (2009). Surgical advances in periacetabular osteotomy for treatment of hip dysplasia in adults, *Acta Orthop* **80**(sup332): 1–33.

Troelsen, A., Elmengaard, B. and Søballe, K. (2008). A new minimally invasive transsartorial approach for periacetabular osteotomy, *J. Bone Joint Surg.-Am Vol.* **90**(3): 493–498.

Valstar, E. R., Nelissen, R. G., Reiber, J. H. and Rozing, P. M. (2002). The use of roentgen stereophotogrammetry to study micromotion of orthopaedic implants, *ISPRS J Photogramm* **56**(5-6): 376–389.

Westoby, M. J., Brasington, J., Glasser, N. F., Hambrey, M. J. and Reynolds, J. (2012). structure-from-motionphotogrammetry: A low-cost, effective tool for geoscience applications, *Geomorphology* **179**: 300–314.

Wiberg, G. (1939). Studies on dysplastic acetabulum and congenital subluxation of the hip joint with special reference to the complications of osteoarthritis, *Acta Chir Scand* **83**(58).

Yang, J., Wang, Y., Liu, Y., Tang, S. and Chen, W. (2009). Novel approach for 3-d reconstruction of coronary arteries from two uncalibrated angiographic images, *IEEE Trans. Image Process.* **18**(7): 1563–1572.
Yaniv, Z. (2009). Localizing spherical fiducials in C-arm based cone-beam CT, *Med Phys* **36**(11): 4957–4966.

Yuan, X., Ryd, L., Tanner, K. and Lidgren, L. (2002). Roentgen single-plane photogrammetric analysis (RSPA) a new approach to the study of musculoskeletal movement, *J. Bone Joint Surg.-Br* Vol. **84**(6): 908–914.

Zheng, G. (2010). Statistical shape model-based reconstruction of a scaled, patient-specific surface model of the pelvis from a single standard AP X-ray radiograph, *Med Phys* **37**(4): 1424–1439.

**Supplementary: Fluoroscopy Used For Pose Estimation**

The fluoroscopy views used for pose estimation in the cadaver experiments are shown in figures S-1, S-2, and S-3. Table 4 lists the total number of BB detections in each image. Only the smaller injected BBs are detected for the views of specimens 2 and 3; the larger BBs were used for establishing ground truth and not intraoperative pose estimation.

![Fluoroscopic images](image)

**Figure S-1.** The fluoroscopic images used for pose estimation in the surgeries for cadaver specimen 1. The detected BBs are overlaid as yellow circles. For this specimen, the larger radius parameter passed to the radial symmetry method caused several false detections on the screws. Projection 3 on the left side shows an example of an excessive number of detections (21), with 7 detections corresponding to BBs on the contralateral side, 6 false alarms triggered by screws, and the remaining 8 detections corresponding to the desired ipsilateral BBs.
**Figure S-2.** The fluoroscopic images used for pose estimation in the surgeries for cadaver specimen 2. The detected BBs are overlaid as yellow circles. A smaller radius parameter was passed to the radial symmetry method and resulted in no false detections; only injected BBs were detected.

**Figure S-3.** The fluoroscopic images used for pose estimation in the surgeries for cadaver specimen 3. The detected BBs are overlaid as yellow circles. A smaller radius parameter was passed to the radial symmetry method and resulted in no false detections; only injected BBs were detected.