Baryon, Charged Hadron, Drell-Yan and $J/\psi$ Production in High Energy Proton-Nucleus Collisions

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Abstract

We show that the distributions of outgoing protons and charged hadrons in high energy proton-nucleus collisions are described rather well by a linear extrapolation from proton-proton collisions. The only adjustable parameter required is the shift in rapidity of a produced charged meson when it encounters a target nucleon. Its fitted value is 0.16. Next, we apply this linear extrapolation to precisely measured Drell-Yan cross sections for 800 GeV protons incident on a variety of nuclear targets which exhibit a deviation from linear scaling in the atomic number $A$. We show that this deviation can be accounted for by energy degradation of the proton as it passes through the nucleus if account is taken of the time delay of particle production due to quantum coherence. We infer an average proper coherence time of $0.4\pm0.1$ fm/c, corresponding to a coherence path length of $8\pm2$ fm in the rest frame of the nucleus. Finally, we apply the linear extrapolation to measured $J/\psi$ production cross sections for 200 and 450 GeV/c protons incident on a variety of nuclear targets. Our analysis takes into account energy loss of the beam proton, the time delay of particle production due to quantum coherence, and absorption of the $J/\psi$ on nucleons. The best representation is obtained for a coherence time of $0.5$ fm/c, which is consistent with Drell-Yan production, and an absorption cross section of 3.6 mb, which is consistent with the value deduced from photoproduction of the $J/\psi$ on nuclear targets.
Introduction

Propagation of a high energy particle through a medium is of interest in many areas of physics. High energy proton-nucleus scattering has been studied for many decades by both the nuclear and particle physics communities \[1\]. Such studies are particularly relevant for the Relativistic Heavy Ion Collider (RHIC), which will collide beams of gold nuclei at an energy of 100 GeV per nucleon, and for the Large Hadron Collider (LHC), which will collide beams of lead nuclei at 1500 GeV per nucleon \[2\].

There are two extreme limits of a projectile scattering from a nucleus. When the cross section of the projectile with a nucleon is very small, as is the case for neutrinos, Glauber theory says that the cross section with a nuclear target of atomic number \(A\) grows linearly with \(A\). When the cross section with an individual nucleon is very large, as is the case for pions near the delta resonance peak, the nucleus appears black and the cross section grows like \(A^{2/3}\). A more interesting case is the production of lepton pairs with large invariant mass, often referred to as Drell-Yan, in proton-nucleus collisions. Both the elastic and inelastic cross sections for proton-nucleon scattering are relatively large, but the partial cross section to produce a high mass lepton pair, being electromagnetic in origin, is relatively small. Experiments have shown that the inclusive Drell-Yan cross section grows with \(A\) to a power very close to 1. The theoretical interpretation is that the hard particles, the high invariant mass lepton pairs, appear first and the soft particles, the typical mesons, appear later due to quantum-mechanical interference, essentially the uncertainty principle. These quantum coherence requirements also lead to the Landau-Pomeranchuk-Migdal effect \[3\]. Deviations from the power 1 by high precision Drell-Yan experiments \[4\] at Fermi National Accelerator Laboratory (FNAL) suggest that it may be possible to infer a finite numerical value for the coherence time. That is one of our goals.

Another of our goals is to take this coherence time effect into account when extracting an absorption cross section for \(J/\psi\) on nucleons for \(J/\psi\) particles produced in high energy proton-nucleus collisions. This cross section is of great interest in the ongoing analysis and debate over whether quark-gluon plasma is formed in high energy nucleus-nucleus collisions.

Before addressing these goals it is imperative to have a basic description of high energy proton-nucleus collisions which reproduces the essential data on outgoing baryons and mesons. We will use one particular theoretical approach, but it important to realize that any model or extrapolation which incorporates the same basic features will lead to the same conclusions we find here.

Baryon and Charged Hadron Production

For a basic description of high energy proton-nucleus scattering we prefer to work with hadronic variables rather than partonic ones. We make a straightforward linear extrapolation from proton-proton scattering. This extrapolation, referred to as LEXUS, was detailed and applied to nucleus-nucleus collisions at beam energies of several hundred GeV per nucleon in ref. \[5\]. Briefly, the inclusive distribution in rapidity \(y\) of the beam proton in an elementary proton-nucleon collision is parameterized rather well by

\[
W_1(y) = \lambda \frac{\cosh y}{\sinh y_0} + (1 - \lambda) \delta(y_0 - y),
\]

where \(y_0\) is the beam rapidity in the lab frame. The parameter \(\lambda\) has the value 0.6 independent of beam energy, at least in the range in which it has been measured, which is 12 – 400 GeV. It may be
interpreted as the fraction of all collisions which are neither diffractive nor elastic. As the proton cascades through the nucleus its energy is degraded. Its rapidity distribution satisfies an evolution equation \( \frac{\cosh y}{\sinh y_0} \sum_{k=1}^{i} \left( \frac{i}{k} \right) \frac{\lambda^k (1-\lambda)^{i-k}}{(k-1)!} \left[ \ln \left( \frac{\sinh y_0}{\sinh y} \right) - (1-\lambda)^i \delta(y_0 - y) \right] \] \( i \)

This distribution then gets folded with impact parameter over the density distribution of the target nucleus as measured by electron scattering.

The net proton distributions are shown for 200 GeV protons incident on S and Au targets in Fig. 1. The data is from NA35 \[^8\]. The theoretical curves fall below the data when the rapidity is less than 1, and for good reason. The projectile deposits energy in the target which subsequently boils off low energy nucleons, but this effect is not included in the curves. The minor discrepancy near the beam rapidity may be due to an inadequate parametrization of the elementary pp distribution eq. (1), or to experimental cuts and acceptances in momentum space, or to both. Otherwise the agreement is very good and without free parameters.

\[ W_i(y) = \frac{\cosh y}{\sinh y_0} \sum_{k=1}^{i} \left( \frac{i}{k} \right) \frac{\lambda^k (1-\lambda)^{i-k}}{(k-1)!} \left[ \ln \left( \frac{\sinh y_0}{\sinh y} \right) - (1-\lambda)^i \delta(y_0 - y) \right] \] \( i \)

**Figure 1:** The rapidity distribution of net protons in p+S and p+Au collisions at a beam energy of 200 GeV. The data are from NA35 \[^8\]. The curves are calculated with LEXUS with no free parameters. The curves undershoot the data at low rapidity because target evaporation is not taken into account.
Every inelastic collision produces an average number of negatively charged hadrons given by the simple formula
\[
\langle h^- \rangle_{NN} = 0.784 \left( \frac{\sqrt{s} - 2m_N - m_\pi}{s^{1/8}} \right)^{3/4}.
\] (3)

The negatively charged hadrons are Gaussian distributed in rapidity with a width given by \(\sqrt{\ln(\sqrt{s}/2m_N)}\) in customary notation. The rapidity distribution of negatively charged hadrons, as computed in the way described, is nearly centered in the nucleon-nucleon center-of-momentum (c.m.) frame, whereas data taken for p+S and p+Au collisions at 200 GeV \[8\] are skewed towards the target rest frame. If one allows for a small rapidity shift of 0.16 whenever a produced hadron encounters a struck target nucleon, chosen with a sign corresponding to a slowing down of the hadron relative to the nucleon, one obtains the curves shown in Fig. 2. The paucity of computed hadrons compared to

Figure 2: The rapidity distribution of negatively charged hadrons in pp, p+S, and p+Au collisions at a beam energy of 200 GeV. The pp data are from \[9\] and the p+S and p+Au data are from NA35 \[8\]. The curves are calculated with LEXUS with a rapidity shift of 0.16 per struck nucleon.

data at small rapidity in p+Au collisions is undoubtedly due to a further cascading and particle production by struck nucleons in such a large nucleus. This physics could be incorporated by a more detailed cascade code followed by nuclear evaporation, but is not essential for our purposes in this paper. Apart from that, the description of the data is very good, including absolute normalization, especially considering that there is only one free parameter.
Figure 3: The transverse momentum distributions for the same p+S and p+Au collisions as in Fig. 2. The curves are calculated with LEXUS with a rapidity shift of 0.16 per struck nucleon.

As the proton cascades through the nucleus it undergoes a random walk in transverse velocity. This broadens the transverse momentum distribution of the produced hadrons relative to pp collisions in the way described in ref. [5]. The transverse momentum distributions, for various windows of rapidity, are shown in Fig. 3. There are no free parameters apart from the rapidity shift which was
already fitted to be 0.16.

**Drell-Yan Production**

Having satisfied ourselves that we have a reasonable quantitative description of the soft hadronic physics, we now turn to a description of the Drell-Yan. Figure 4 is a schematic of two limits and an intermediate situation. One limit is full energy degradation of the proton as it traverses the nucleus. Produced hadrons appear immediately with zero coherence time, causing the proton to have less energy available to produce a Drell-Yan pair at the backside of the nucleus. The other limit is usually referred to as Glauber, although this is a bit of a misnomer. Produced hadrons, being soft on the average, do not appear until after the hardest particles, the Drell-Yan pair, have already appeared. This is the limit of a very large coherence time, and it allows the proton to produce the Drell-Yan pair anywhere along its path with the full incident beam energy. An intermediate case is one of finite, nonzero coherence time. By the time the proton wants to make a Drell-Yan pair on the backside of the nucleus, hadrons have already appeared from the first collision but not from the second. Therefore the proton has more energy available to produce the Drell-Yan pair than full zero coherence time but less energy than with infinite coherence time. This ought to result in an $A$ dependence less than 1, with the numerical value determined by the coherence time. It is instructive to contemplate the relative importance of energy loss and coherence time for an 800 GeV proton incident on a very large nucleus, such as a neutron star: Can one imagine the proton reaching the backside of a neutron star and producing a Drell-Yan pair without having suffered any energy loss?

Consistent with our philosophy to describe everything in terms of hadronic variables we should use a parametrization of measured Drell-Yan cross sections in pp and pn collisions. However, we need these over a very broad energy range because of the decreasing energy of the proton as it cascades through the nucleus, and such broad measurements have not been made. Therefore, we compute the Drell-Yan yields in individual pp and pn collisions using the parton model with the GRV structure functions [10] to leading order with a K factor. These structure functions distinguish between pp and pn collisions. We have compared the results to pp collisions at the same beam energy of 800 GeV [11] and found the agreement to be excellent for all values of $x_F$.

The experiment E772 [4] measured the ratio $\sigma_{pA}^{DY}/(\sigma_{pd}^{DY}/2)$. Were there no energy loss and all nuclei were charge symmetric this ratio would be equal to $A$. The experiment measured muon pairs with invariant mass $M$ between 4 and 9 GeV and greater than 11 GeV to eliminate the $J/\psi$ and $\Upsilon$ contributions. The data has been presented in 7 bins of Feynman $x_F$ from 0.05 to 0.65. (Recall that $x_F$ is the ratio of the muon pair longitudinal momentum to the incident beam momentum in the nucleon-nucleon c.m. frame.) Data for exemplary values of $x_F$ are shown in Figs. 5 to 7. The data should fall on the dashed line if the ratio of cross sections is $A$. There is a small but noticeable departure for tungsten and at the largest value of $x_F$. This is to be expected if energy loss plays a role as it must affect the largest target nucleus and the highest energy muon pairs the most [12].

We have computed the individual cross sections $\sigma_{pA}^{DY}$ with a variable time delay. The proton cascades through the nucleus as described earlier, but we assume that the energy available to produce a Drell-Yan pair is that which the proton has after $n$ previous collisions. Thus $n = 0$ is full energy loss and $n = \infty$ is zero energy loss. We have taken the resulting proton-nucleus Drell-Yan cross section, multiplied it by 2, divided it by the sum of the computed pp and pn cross sections and display the results in Figs. 5 to 7. The lower edge of the shaded regions in the figure corresponds to $n = 4$ and
Figure 4: Schematic of a high energy proton passing through a nucleus. The upper panel represents full energy loss: hadrons are produced immediately and the proton has less energy available for each subsequent collision. The middle panel represents partial energy loss: there is a finite time delay before hadrons are produced and so the proton has more energy available to create a high energy Drell-Yan pair. The bottom panel represents no energy loss: the usual Glauber picture.
Figure 5: The ratio of the pA Drell-Yan cross section to the proton-deuterium cross section divided by 2 for a beam energy of 800 GeV. The data are from E772. The dashed line assumes a scaling linear in the atomic number A. The shaded region represents our calculations with a coherence time ranging from 4 to 6 proton-nucleon collisions, both elastic and inelastic. We computed with target nuclei C, Ca, Fe, W and Pb and interpolate between with straight lines to guide the eye. The value of Feynman $x_F$ of the Drell-Yan pair is 0.65.

the upper edge to $n = 6$. Overall the best representation of the data lies in this range. This collision number shift is easily converted to a coherence time. Let $\tau_0$ be the coherence time in the c.m. frame of the colliding nucleons. This is essentially the same as the formation time of a pion since most pions are produced with rapidities near zero in that frame. The first proton-nucleon collision is the most important, so boosting this time into the rest frame of the target nucleus and converting it to a path length (proton moves essentially at the speed of light) gives $\gamma_{\text{cm}} c \tau_0 \approx \sqrt{\gamma_{\text{lab}}/2 c \tau_0}$. This path length may then be equated with $n$ times the mean free path $l = 1/\sigma_{\text{NN}}^{\text{tot}} \rho$. Using a total cross section of 40 mb and a nuclear matter density of 0.155 nucleons/fm$^3$ we obtain a path length of 8±2 fm and a proper coherence time of 0.4±0.1 fm/c corresponding to $n = 5 \pm 1$.

This value of the proper coherence time is just about what should have been expected a priori. In the c.m. frame of the colliding nucleons at the energies of interest a typical pion is produced with an energy of $E_\pi \approx 500$ MeV. By the uncertainty principle this takes a time of order $\hbar c/E_\pi \approx 0.4$ fm/c.
A process related to Drell-Yan is the production of $J/\psi$ which we shall address now. This is also a relatively hard process and so both energy loss of the beam proton and the Landau-Pomeranchuk-.
Migdal effect must be taken into account. However, there is an additional effect which plays a role, and that is the occasional absorption or breakup of the \( J/\psi \) in encounters with target nucleons. (The inelastic interaction of one of the leptons in Drell-Yan production with target nucleons is ignorably small.) The absorption cross section, \( \sigma_{\text{abs}} \), has been estimated in a straightforward Glauber analysis without energy loss and with an infinite coherence/formation time to be about 6-7 mb \[13\]. This has formed the basis for many analyses of \( J/\psi \) suppression in heavy ion collisions. Any anomalous suppression may be an indication of the formation of quark-gluon plasma \[2, 14\], hence the importance of obtaining the most accurate value of \( \sigma_{\text{abs}} \) possible. This cross section has also been inferred from photoproduction experiments of \( J/\psi \) on nuclei from which a value much less than that has been obtained \[15\]. This has been a puzzle. One attempt to resolve this apparent discrepancy consists of modeling the produced \( J/\Psi \) state as a pre-resonant color dipole state with two octet charges \[16\]; however, the results are only semi-quantitative.

![Figure 8](image)

**Figure 8:** Branching ratio into muons times cross section to produce \( J/\psi \) with \( x_F \geq 0 \) in proton-nucleus collisions at 200 GeV/c. The data is from NA38 \[17, 19\]. The dashed line is \( A \) times the nucleon-nucleon production cross section. The solid curve represents full energy loss with zero coherence/formation time, while the banded region represents partial energy loss with a coherence/formation time within the limits set by Drell-Yan production. (Computations were done for C, Al, Cu, W and U and the points connected by straight lines to guide the eye.)

In order to compute the production cross section of \( J/\psi \) in proton-nucleus collisions we need a parametrization of it in the more elementary nucleon-nucleon collisions. For this we call upon the
parametrization of a compilation of data by Lourenço [17].

\[ B\sigma_{NN\rightarrow J/\psi}(x_F > 0) = 37 \left(1 - m_{J/\psi}/\sqrt{s}\right)^{12} \text{nb} \]  

(4)

Here \( B \) is the branching ratio into dimuons and \( x_F \) is the ratio of the momentum carried by \( J/\psi \) to the beam momentum in the center of mass frame \((-1 < x_F < 1)\). Due to the degradation in momentum of the proton as it traverses the nucleus it is important to know the \( x_F \) dependence of the production. The Fermilab experiment E789 has measured this dependence at 800 GeV/c [18] to be proportional to \((1 - |x_F|)^5\). Assuming that this holds at lower energy too we use the joint \( \sqrt{s} \) and \( x_F \) functional dependence and magnitude:

\[ \frac{d\sigma_{NN\rightarrow J/\psi}}{dx_F} = 6\sigma_{NN\rightarrow J/\psi}(x_F > 0)(1 - |x_F|)^5. \]  

(5)

The cross section in proton-nucleus collisions can now be computed in LEXUS with no ambiguity.

Figures 8 and 9 show the results of our calculation in comparison to data taken by NA38 [19] and NA51 [20], respectively. The dashed curves are \( A \) times the nucleon-nucleon production cross section; they obviously overestimate the data. The solid curves show the result of LEXUS with full energy degradation of the beam proton without account taken of the Landau-Pomeranchuk-Migdal effect; they obviously underestimate the data. The hatched regions represent the inclusion of the latter effect with a proper formation/coherence time \( \tau_0 \) in the range of 0.3 to 0.5 fm/c consistent with

Figure 9: Same as figure 8 but for a beam momentum of 450 GeV/c. Data is from NA51 [17, 20].
Drell-Yan production. The time delay is implemented as described previously: The energy available for the production of $J/\psi$ is that which the proton had $n$ collisions prior; that is, the previous $n$ collisions are ignored for the purpose of determining the proton’s energy. This is an approximate treatment of the Landau-Pomeranchuk-Migdal effect. The $n$ is related to the beam energy and to the coherence time $\tau_0$ in the center of mass frame of the colliding nucleons. Using, as before, a total cross section of 40 mb, a nuclear matter density of 0.155 nucleons/fm$^3$, and $0.3 < \tau_0 < 0.5$ fm/c we obtain $2 < n < 3$ at 200 GeV/c and $3 < n < 5$ at 450 GeV/c. As may be seen from the figures, the data is overestimated, indicating the necessity for nuclear absorption.

We now introduce a $J/\psi$ absorption cross section on nucleons and compute its effect within LEXUS in the canonical way \[13\]. When the $J/\psi$ is created there will in general be a nonzero number of nucleons blocking its exit from the nucleus. Knowing where the $J/\psi$ is created allows one to calculate how many nucleons lie in its path, and hence, to compute the probability that it will be dissociated into open charm. We choose a value of $\tau_0$ allowed by Drell-Yan measurements, mentioned above, and then vary $\sigma_{\text{abs}}$, assuming that it is energy independent. The lowest value of chi-squared for the 200 and 450 GeV/c data set taken together is obtained with $\tau_0 = 0.5$ fm/c and $\sigma_{\text{abs}} = 3.6$ mb. The results are shown in figures 10 and 11. The fitted values all lie within

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{Same data as in figure 8. The solid curve is the best fit of the model which includes beam energy loss with a coherence time of 0.5 fm/c ($n=3$ at this energy) and a $J/\psi$ absorption cross section of 3.6 mb.}
\end{figure}

one standard deviation of the data points. This is quite a satisfactory representation of the data. It means that both Drell-Yan and $J/\psi$ production in high energy proton-nucleus collisions can be
understood in terms of a conventional hadronic analysis when account is taken of the energy loss of the beam proton, the Landau-Pomeranchuk-Migdal effect, and nuclear absorption of the $J/\psi$ in the final state. It also means that the absorption cross section for $J/\psi$ inferred from high energy proton-nucleus collisions is consistent with the value inferred from photoproduction experiments on nuclei.

**Conclusion**

The analysis performed here can and should be improved upon. What we have done is a rough approximation to adding the quantum mechanical amplitudes for a proton scattering from individual nucleons within a nucleus. A more sophisticated treatment would undoubtedly lead to even better agreement with experiment, but the inferred value of the proper coherence time is unlikely to be much different than obtained with this first estimate. It will be very instructive to repeat this analysis in the language of partonic variables. Actually, the analysis with parton energy loss alone was reported by Gavin and Milana [21] with satisfactory results obtained for Drell-Yan if the quarks/antiquarks lose about 1.5 GeV/fm. Nuclear shadowing [22] needs to be taken into account too. The relationship among all these effects is not well-understood, nor is the relationship between these effects in partonic and hadronic variables. Finally, the implications for nucleus-nucleus collisions [23] will undoubtedly
be important; they are under investigation.

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