Initial data for high-compactness black hole–neutron star binaries

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Abstract
For highly compact neutron stars, constructing numerical initial data for black hole–neutron star binary evolutions is very difficult. We describe improvements to an earlier method that enable it to handle these more challenging cases. These improvements were found by invoking a general relaxation principle that may be helpful in improving robustness in other initial data solvers. We examine the case of a 6:1 mass ratio system in inspiral close to merger, where the star is governed by a polytropic $\Gamma = 2$, an SLy, or an LS220 equation of state (EOS). In particular, we are able to obtain a solution with a realistic LS220 EOS for a star with compactness 0.26 and mass $1.98 M_\odot$, which is representative of the highest reliably determined neutron star masses. For the SLy EOS, we can obtain solutions with a comparable compactness of 0.25, while for a family of polytropic equations of state, we obtain solutions with compactness up to 0.21, the largest compactness that is stable in this family. These compactness values are significantly higher than any previously published results.

Keywords: neutron stars, initial value problem, black holes, relativity and gravitation, relativistic binaries

(Some figures may appear in colour only in the online journal)
1. Introduction

Among the prime candidate sources for ground-based gravitational wave detectors are binary systems containing inspiraling neutron stars: black hole–neutron star (BHNS) binaries, or systems containing two neutron stars. Besides being likely gravitational wave sources, such systems are the leading candidates to explain short gamma-ray bursts [1–5]. Radioactive decay of the neutron-rich material ejected by the merger may also power optical/infrared transients days after the merger [6–8], particularly for BHNS binaries with a large neutron star and a rapidly rotating black hole [9–13], and for binary neutron star mergers with compact neutron stars [14].

Gravitational waves from coalescing binaries are searched for and analyzed using the matched-filtering technique [15–18], which compares the detector output with a bank of templates that model the waves emitted by the source. Therefore accurate knowledge of the expected waveforms of incoming signals is required. While post-Newtonian templates are expected to be accurate when the binary is widely separated, they break down near merger. Fully relativistic numerical simulations of the last few orbits and the merger are needed to match onto the post-Newtonian waveforms Moreover, modeling of the subsequent electromagnetic and neutrino emission must also be done by a code that can deal with all the effects of strong-field gravity.

Numerical modeling of these systems is very challenging (see [19–22] for reviews). A key ingredient in such simulations is accurate initial data. Ideally, one would like a snapshot of the gravitational field and the matter distribution only a few orbits before merger but resulting from millions of years of slow inspiral. In general relativity, no exact way is known to do this because the nonlinear Einstein equations are too difficult to solve. So instead, various plausible approximations are made. The most common assumption is that the binary has had time to settle into a quasi-equilibrium state, the system being approximately time-independent in the corotating frame. Furthermore, as the viscous forces within the star are expected to be small, we do not expect much change in the spin of the star as the orbital radius decreases. For an initially nonspinning neutron star, this would lead to an irrotational velocity profile, another standard assumption. Because of gravitational wave emission, however, there is no exact equilibrium state. Accordingly, these conditions cannot all be perfectly satisfied simultaneously. Nevertheless, initial data incorporating these assumptions seems to work quite well in practice.

This paper will focus on initial data for BHNS binaries, and in particular systems where the neutron star has high compactness

\[ C = \frac{M_{NS}}{R}. \]  

Here \( M_{NS} \) is the ADM mass and \( R \) is the areal radius for an isolated star with the same equation of state (EOS) and baryon mass, and we use units with \( c = G = 1 \). The techniques introduced here should be equally applicable to neutron star–neutron star binaries.

Previous results on initial data for BHNS evolutions include the early work of Taniguchi et al [23] and Sopuerta et al [24], as well as more recent initial configurations generated by Taniguchi et al [25–27] and Grandclement [28]. Both Taniguchi and Grandclement use codes based on the LORENE package [29], and excise from the computational domain the region inside the apparent horizon of the black hole. An alternative method based on the puncture formalism, in which the constraints are solved both inside and outside the black hole horizon, has been proposed by Kyutoku et al [30]. The results from [30] were also obtained using
LORENE. A newer initial data code, COCAL, has been developed by Uryū and Tsokaros [31], but not yet applied to BHNS binaries.

Our own group has developed an independent code ([32], henceforth Paper I) that uses a multidomain spectral method to achieve high accuracy at a relatively low computational cost. The code is based on the spectral elliptic solver (Spells) developed by the Cornell-Caltech collaboration [33], and originally developed by Pfeiffer [34, 35] for the study of binary black hole initial data. While the mathematical formulation of the problem is very similar to [27] and [28], the numerical techniques are quite different. In particular, while all use multidomain spectral methods, the flexibility that Spells offers in choosing subdomain shapes and the form of the elliptic equations allows one to efficiently adapt the configuration of the numerical grid to the geometry of the system and obtain high-precision results at a very reasonable computational cost.

A drawback of the method described in Paper I is that it fails to converge to a solution when the compactness $C$ of the neutron star is too high. In fact, this seems to be a defect of all the published methods for solving the BHNS initial value problem. The maximum compactness that can be handled depends on the EOS. The easiest EOS for all methods is a polytrope because it is smooth inside the star, and the density goes linearly to zero near the surface. The method of Paper I can, for example, reliably produce binaries with an initial separation of $7M_0$ and a compactness up to 0.18. Here $M_0$ is defined via

$$M_0 = M_{BH} + M_{NS},$$

where $M_{BH}$ is the Christodoulou mass of the black hole and $M_{NS}$ is the neutron star mass as defined above. With some small modifications, described later, it can reach $C = 0.20$, very close to the maximum allowed value before the neutron star is unstable to gravitational collapse.

Treating realistic equations of state is more difficult. They tend to have nonsmooth behavior as the composition changes in various density regimes. They often have nonanalytic behavior or very steep slopes at the surface. And even smooth equations of state may be given in tabular form, which introduces its own nonsmoothness. For example, for the SLy EOS [36–38], the maximum compactness attainable by the method of Paper I is 0.16, corresponding to a neutron star mass of $1.27 M_⊙$. The other codes for producing BHNS initial data have reported a maximum compactness of $C = 0.196$ with a piecewise-polytropic EOS (for a $1.45 M_⊙$ neutron star) [39], while binary neutron star initial data has been obtained up to $C = 0.213$ (for a $1.6 M_⊙$ neutron star) [14]. Since a neutron star of mass $2 M_⊙$ is known to exist, this is clearly not adequate.

In this paper, we describe how to improve the algorithm of Paper I to allow high-compactness initial data to be calculated. For example, we show that for the SLy EOS, a solution with $C = 0.25$, corresponding to a neutron star mass of $1.86 M_⊙$, can be obtained. Furthermore, for the LS220 EOS [40], we can obtain a solution with $C = 0.26$, corresponding to a mass of $1.98 M_⊙$. These initial data can now be used in binary evolutions to study the effect of high compactness on the outcome of the merger.

The key principle underlying modification to the algorithm is quite simple and general. We expect it to be useful for other initial data codes as well. The idea is as follows: The algorithm consists of iteratively solving several coupled nonlinear elliptic and algebraic equations. During the iteration, the unknowns get updated. Many of the variables are updated by a relaxation procedure (equation (46) below), where only a fraction of the predicted update is applied. This relaxation improves the robustness of the algorithm. However, some variables are typically changed more abruptly. The general principle that allows the algorithm to converge even in difficult cases is all variables should be updated smoothly by relaxation.
In [41], it was observed that highly compact configurations may result in mathematical nonuniqueness of the solution, and a resolution of this problem was presented in the context of conformally flat formulations for the evolution equations. In contrast, the work reported here is not related to nonuniqueness. Rather, we are dealing with an iterative algorithm that displays poor convergence as the compactness increases. The changes to the algorithm restore good convergence.

In section 2 of this paper, we first review the solution procedure of Paper I and then describe the specific changes that allow high compactness models to converge. In section 3 we present high compactness results using polytropic, SLy, and LS220 EOS. Finally, in section 4 we offer closing remarks on these findings.

2. Methods

Our work is based on an implementation in the spectral Einstein code of the procedure described in Paper I, which was itself strongly inspired by Taniguchi et al [23]. We will begin with a review of the formalism used to solve the initial data problem for BHNS binaries, and the iterative procedure used to generate initial conditions in Paper I. We will then discuss the methods developed here to improve the robustness of the algorithm.

2.1. Formalism

The basic formalism used in this work is identical to that of Paper I. We begin with the $3+1$ decomposition of the space–time metric (see [42] for a review)

$$\mathrm{d}s^2 = -\alpha^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t).$$

Here $\alpha$ is the lapse function, $\beta^i$ is the shift vector and $\gamma_{ij}$ is the three-metric induced on a hypersurface $\Sigma(t)$ of constant coordinate time $t$. In this decomposition, the unit normal vector $n^\mu$ to $\Sigma(t)$ and the tangent vector $t^\mu$ to the coordinate line $t$ are related by

$$t^\mu = \alpha n^\mu + \beta^\mu,$$

with $n_\mu = (-\alpha, 0, 0, 0)$ and $\beta^\mu = (0, \beta^i)$. The extrinsic curvature of $\Sigma(t)$ is the symmetric tensor defined as

$$K_{\mu\nu} = -\nabla_\mu n_\nu - n_\nu \gamma^{\lambda}_{\mu \lambda} \nabla_\lambda (\ln \alpha) = -\frac{1}{2} \mathcal{L}_n \gamma_{\mu\nu},$$

where $\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ is the extension of the three-metric $\gamma_{ij}$ to the four-dimensional spacetime, and $g_{\mu\nu}$ is the four-metric of that spacetime. $\mathcal{L}_n$ is the Lie derivative along the normal $n^\mu$. By construction, $K^{\mu\nu}n_\mu = 0$ and we can restrict $K^{\mu\nu}$ to the three-dimensional tensor $K^{ij}$ defined on $\Sigma \times \Sigma$. The extrinsic curvature $K^{ij}$ is then divided into its trace $K$ and trace-free part $A^{ij}$:

$$K^{ij} = A^{ij} + \frac{1}{3} \gamma^{ij}K.$$

We treat the matter as a perfect fluid with stress–energy tensor

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + Pg_{\mu\nu},$$

where $\rho = \rho_0(1 + \epsilon)$ is the energy density, $\rho_0$ the baryon density, $\epsilon$ the specific internal energy, $P$ the pressure, and $u_\mu$ the fluid’s four-velocity. For the initial value problem, it is often convenient to consider the following projections of the stress tensor:
\[ E = \gamma^\mu \gamma_\mu n_\nu n_\nu, \] (8)
\[ S = \gamma^\mu \gamma_\mu \gamma^\nu T^\mu_\nu, \] (9)
\[ J^i = -\gamma^i \gamma^\nu n_\mu n_\nu. \] (10)

We then further decompose the metric according to the conformal transformation

\[ \gamma_{ij} = \Psi^4 \tilde{\gamma}_{ij}, \] (11)

Other quantities are assumed to have the following conformal transformations:

\[ E = \Psi^{-6} \tilde{E}, \] (12)
\[ S = \Psi^{-6} \tilde{S}, \] (13)
\[ J^i = \Psi^{-6} \tilde{J}^i, \] (14)
\[ A^{ij} = \Psi^{-10} \tilde{A}^{ij}, \] (15)
\[ \alpha = \Psi^{6} \tilde{\alpha}, \] (16)

\( \tilde{A}^{ij} \) is related to the shift and the time derivative of the conformal metric, \( \tilde{a}_{ij} = \partial_t \tilde{\gamma}_{ij} \) by

\[ \tilde{A}^{ij} = \frac{1}{2\tilde{\alpha}} [\mathcal{L}_\beta \tilde{\gamma}^{ij} - \tilde{a}^{ij}], \] (17)

where \( \mathcal{L} \) is the conformal longitudinal operator whose action on a vector \( V^i \) is

\[ (\mathcal{L} V)^i = \tilde{\nabla}^i V^i + \tilde{\nabla}^j V^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{\nabla}_k V^k, \] (18)

and \( \tilde{\nabla} \) is the covariant derivative defined with respect to the conformal three-metric \( \tilde{\gamma}_{ij} \).

In the 3 + 1 formalism, the Einstein equations are decomposed into a set of evolution equations for the metric variables as a function of \( t \), and a set of constraint equations on each hypersurface \( \Sigma(t) \). The initial data problem consists in providing quantities \( g_{\mu\nu}(t_0) \) and \( K_{\nu\rho}(t_0) \) that satisfy the constraints on \( \Sigma(t_0) \) and represent initial conditions with the desired physical properties (e.g. masses and spins of the objects, initial orbital frequency, eccentricity, and so forth). We solve the constraint equations using the extended conformal thin sandwich (XCTS) formalism [43], in which the constraints take the form of five nonlinear coupled elliptic equations. The XCTS equations can be written as

\[ 2\tilde{\alpha} \left[ \tilde{\gamma}^{ij} \left( \frac{1}{2\tilde{\alpha}} (\mathcal{L}_\beta \tilde{\gamma})^{ij} \right) - \tilde{\gamma}^{ij} \left( \frac{1}{2\tilde{\alpha}} \tilde{a}^{ij} \right) - \frac{2}{3} \Psi^6 \tilde{\nabla}^k K - 8\pi \Psi^4 \tilde{J} \right] = 0, \] (19)

\[ \tilde{\nabla}^2 \Psi - \frac{1}{8} \Psi \tilde{R} - \frac{1}{12} \Psi^5 K^2 + \frac{1}{8} \Psi \gamma \tilde{A}^{ij} \tilde{A}_{ij} + 2\pi \Psi^{-1} \tilde{E} = 0, \] (20)

\[ \tilde{\nabla}^2 (\tilde{a} \Psi) - (\tilde{a} \Psi) \left[ \frac{1}{8} \tilde{R} + \frac{5}{12} \Psi^4 K^2 + \frac{7}{8} \Psi \gamma \tilde{A}^{ij} \tilde{A}_{ij} + 2\pi \Psi^{-2} (\tilde{E} + \tilde{S}) \right] = -\Psi^2 (\partial_t K - \beta^k \partial_k K). \] (21)

We solve these equations for the conformal factor \( \Psi \), the densitized lapse \( \tilde{a} \Psi \) and the shift \( \beta^i \). In the equations, \( \tilde{E}, \tilde{S} \) and \( \tilde{J}^i \) determine the matter content of the slice. The variables \( \tilde{\gamma}_{ij}, \tilde{a}_{ij} = \partial_t \tilde{\gamma}_{ij}, K \) and \( \partial_t K \) are freely chosen.
If we work in a coordinate system corotating with the binary, $\tilde{u}_{ij} = 0$ and $\partial_t K = 0$ are natural choices for a quasi-equilibrium configuration. $K$ and $\gamma_{ij}$ are chosen to match analytical superpositions of solutions for boosted black holes and neutron stars (see Paper I for details).

In addition to solving these equations for the metric variables, we must impose some restrictions on the matter. In particular, the stars should be in a state of hydrostatic equilibrium in the comoving frame. This involves solving the Euler equation and the continuity equation, under the assumption that the spacetime has a helicoidal Killing vector $\xi$ given by

$$\xi = \partial_t + \Omega_0 \partial_\phi,$$

for some constant $\Omega_0$ associated with the orbital angular velocity of the binary with respect to a distant inertial observer (see e.g. [44]).

For an irrotational binary, the first integral of the Euler equation then leads to the condition

$$h\gamma = C,$$

where $C$ is a constant, the enthalpy $h$ is defined as

$$h = 1 + e + \frac{P}{\rho_0}$$

and we have introduced

$$\gamma = \gamma_\alpha \gamma_0 (1 - \gamma_{ij} U^i U^j_0),$$

$$\gamma_0 = (1 - \gamma_{ij} U^i_0 U^j_0)^{-1/2},$$

$$\gamma_\alpha = (1 - \gamma_{ij} U^i U^j)^{-1/2},$$

$$U^i_0 = \frac{\beta^i}{\alpha}.$$

The three-velocity $U^i$ is defined by

$$u^u = \gamma_{\alpha} (n^\mu + U^\mu),$$

$$U^\mu n_\mu = 0.$$

The choice of $U^i$, which is unconstrained in this formalism, is an important component in determining the initial conditions in the neutron star. For irrotational binaries (nonspinning neutron stars), there exists a potential $\phi$ such that

$$U^i = \frac{\gamma^{-4} \gamma^u}{h \gamma_{\alpha}} \partial_j \phi.$$

The continuity equation can then be written as a second-order elliptic equation for $\phi$:

$$\frac{\rho_0}{h} \nabla^\mu \nabla_\mu \phi + (\nabla^\mu \phi) \nabla_\mu \frac{\rho_0}{h} = 0.$$
More explicitly written, this equation becomes

\[
\rho_0 \left\{ -\gamma^{ij} \partial_i \partial_j \phi + \frac{h_j \beta \Psi^4}{\alpha} \partial_i \gamma_n + hK_{\gamma n} \Psi^4 + \left( \gamma^{ij} \frac{h_j}{\alpha \Psi^2} \right) \partial_i \partial_j \phi \right\}
\]

\[
= \gamma^{ij} \partial_i \phi \partial_j \rho_0 - \frac{\gamma^{ij} \alpha \partial_j \rho_0}{\alpha} - \left( \ln \frac{h}{\alpha \Psi^2} \right) \partial_i \phi.
\]

(33)

Once we have obtained \( h \) from the metric and \( U^i \) via equation (23), the other hydrodynamical variables can be recovered if we close the system by the choice of an EOS for cold neutron star matter in \( \beta \)-equilibrium, \( P = P(\rho_0) \) and \( \epsilon = \epsilon(\rho_0) \).

The outer boundary conditions of our system of equations are quite simple. We require the metric to be asymptotically flat and in Minkowski coordinates in the inertial frame. We solve the initial data problem in a frame that is rotating at constant angular velocity \( \Omega_0 = (0, 0, \Omega_0) \), and can be infalling with a radial velocity \( a_0 r \), and boosted with a velocity \( v_{\text{boost}} \). This leads to the boundary conditions

\[
\beta = \Omega_0 \times r + a_0 r + v_{\text{boost}},
\]

(34)

(35)

(36)

In the standard quasi-equilibrium formalism (e.g. [23, 44]), \( \Omega_0 \) is a free parameter which is set so that the center of the neutron star experiences no net radial forces (see below), while \( a_0 = v_{\text{boost}} = 0 \). It is, however, possible to instead choose \( \Omega_0 \) and \( a_0 \) to reduce the initial eccentricity of the binary. The quasi-equilibrium formalism typically produces binary initial data with eccentricity \( e \lesssim 0.01 \), while choosing \( \Omega_0 \) and \( a_0 \) according to the eccentricity reduction algorithm derived by Pfeiffer et al [45] for binary black holes can reduce the eccentricity of a BHNS system by more than an order of magnitude (see Paper I). Changing \( v_{\text{boost}} \) can be a convenient method to obtain initial data with no net linear momentum (see [9, 46] and discussion below). Typically, we place the outer boundary of our computational domain at \( r = 10^9 M-10^{10} M \). The multidomain spectral method is very convenient for being able to reach such large outer boundaries.

At the surface of the neutron star, the boundary condition on \( \phi \) can be easily inferred from the \( \rho_0 = 0 \) limit of equation (33):

\[
\gamma^{ij} \partial_i \phi \partial_j \rho_0 = \frac{h_j \beta \Psi^4}{\alpha} \partial_i \rho_0.
\]

(37)

If the boundary condition is applied exactly on the surface, \( \partial_i \rho_0 \) is proportional to the unit normal to the boundary. However, in practice, we apply this boundary condition on our current best estimate of the surface (see below), and there may thus be slight differences between \( \partial_i \rho \) and the normal to the boundary.

We also excise a spherical region surrounding the center of the black hole. On the excision surface, we impose boundary conditions derived in [47, 48], which ensure that the excision surface is an apparent horizon (see also Paper I for details). This boundary condition comes with a freely specifiable vector \( \Omega_{\text{BH}} \), which is effectively used to set the spin of the black hole. The free choice of the radius of the excised region sets the mass of the black hole.

Finally, we discuss how a first guess for the orbital angular velocity \( \Omega_0 \) can be obtained.

We define the center of the neutron star as in [44], by requiring that it correspond to a maximum of the enthalpy \( h \), i.e. that the force-balance equation
\( \nabla \ln h = 0. \)  

(38)

Is satisfied at the desired location of the neutron star center.

Neglecting any infall velocity, this condition guarantees that the binary is in a circular orbit. Of course, this is only an approximation as there is really some infall velocity, but this still leads to low eccentricity binaries with \( e \approx 0.01 \). From the integrated Euler equation (23), we can write this condition as

\[
\nabla \ln h = \nabla \left( \ln \frac{\gamma_0}{\alpha \gamma} \right) = 0,
\]

(39)

or, by using the definitions of \( \gamma_0 \) and \( \gamma \),

\[
\nabla \ln (\alpha^2 - \gamma_i \beta^i) = -2\nabla \ln \gamma.
\]

(40)

If we decompose the shift \( \beta \) into its inertial component \( \beta_0 \) and its comoving component according to

\[
\beta = \beta_0 + \Omega_0 \times r + \dot{a}_0 r,
\]

(41)

we obtain a quadratic equation for the orbital angular velocity \( \Omega_0 \) (neglecting the dependence of \( \gamma \) on the orbital angular velocity \( \Omega_0 \)). In practice, we solve for \( \Omega_0 \) by projecting equation (40) along the line connecting the center of the two compact objects\(^6\). Once a quasi-equilibrium solution has been obtained, lower eccentricity systems can be generated by modifying \( \Omega_0 \) and \( \dot{a}_0 \), following the methods developed by Pfeiffer et al [45].

Diagnostic quantities that are useful in describing the properties of the system include the ADM energy and total angular momentum. These quantities are typically defined as integrals on \( S_\infty \), the sphere at infinity, which is not convenient for computations. Integrating by parts, we can transform these expressions into integrals on any sphere \( S \) enclosing all matter and singularities and, when needed, a volume integral on \( V \), the region of the initial slice lying outside \( S \). Assuming, on \( S \) and within \( V \) (but not necessarily inside \( S \)), conformal flatness, \( K = 0 \), and no constraint violations, this gives:

\[
M_{\text{ADM}} = -\frac{1}{2\pi} \oint_{S_\infty} \delta^i \partial_j \phi \, dS_i
= -\frac{1}{2\pi} \left( \oint_{S} \delta^i \partial_j \phi \, dS_i - \frac{1}{8} \int_{\mathcal{V}} \phi^2 K_{ij} K^{ij} dV \right).
\]

(42)

\[
J_{\text{ADM}}^i = \frac{1}{8\pi} \oint_{S_\infty} (\alpha K^{ij} - y K^{ij}) dS_i
= \frac{1}{8\pi} \oint_{S} (\alpha K_{ij} - y K_{ij}) \delta^i \phi^2 dS_i.
\]

(43)

The decomposition into surface and volume integrals is not unique, but we found these expressions convenient, as the contribution of the volume terms decreases at least as \( 1/r \) away from the center of mass, reducing the sensitivity to small numerical errors at spatial infinity.

In our initial data, we also require that the total ADM linear momentum vanishes (see section 2.4). It is computed in a very similar way:

\(^6\) Along the other directions, the enthalpy \( h \) is corrected so that force balance is enforced at the center of the star, according to the method described in Paper I.
\[ p_{\text{ADM}}^j = \frac{1}{8\pi} \int_{\mathcal{S}_{\text{in}}} \delta^i_j K^{ij} dS_j \]
\[ = \frac{1}{8\pi} \int_{\mathcal{S}_3} \delta^{ij} \delta^{k\ell} K_{ij} \phi^2 dS_j, \]  
\[ (44) \]

Finally, the baryon mass of the neutron star is
\[ M_{\text{NS}}^b = \int_{\text{NS}} \rho_0 \Psi^6 \sqrt{\frac{\tilde{g}}{1 - \gamma_j U^j U^j}} dV, \]
\[ (45) \]

with \( \tilde{g} \) the determinant of the three-metric \( \tilde{g}_{ij} \). \( M_{\text{NS}}^b \) is the quantity that is used during the iterative solve step to fix the mass of the neutron star (see section 2.2).

2.2. Iterative construction of initial data

In the previous section, we have reduced the initial data problem to a set of five nonlinear elliptic equations for the metric quantities \( \Psi, \tilde{h}, \tilde{\Psi} \) and \( \beta' \) evaluated for a given matter content (equations (19)–(21)), one elliptic equation for the velocity potential \( \phi \) evaluated for a given background spacetime (equation (33)), and a set of algebraic equations allowing us to compute the hydrodynamics variables from the choice of a constant \( C \) in the first integral of Euler’s equations (equation (23)) and an initial orbital angular velocity \( \Omega_0 \), initial infall velocity \( \dot{a}_0 r \) of the binary, and possible boost velocity \( v_{\text{boost}} \). We also implicitly required the knowledge of the location of the surface of the neutron star, so that equation (33) can be solved with the boundary condition (37), the location of the center of each compact object, the radius of the excised region around the black hole, and the vector \( \Omega_{\text{BH}}^j \) setting the spin of the black hole. Given the coupled nature of those equations, we need a procedure to robustly produce self-consistent initial configurations. This is done through an iterative method in which we

- Solve the XCTS equations for the metric at fixed matter content, and use the solution to get an updated guess for the metric.
- Evaluate the position of the surface of the star from the condition \( h = h(\rho_{\text{in}} = \rho_{\text{min}}) \), and use this to update the boundary of the computational domain on which we solve equation (33).
- Compute the ADM linear momentum \( P \), and if necessary move the center of the black hole to drive \( P \) towards zero (or, alternatively, modify \( v_{\text{boost}} \) to obtain the same result, see section 2.4).
- Compute an updated guess for the angular velocity \( \Omega_0 \) from equation (40).
- Modify the radius of the excision region and the vector \( \Omega_{\text{BH}}^j \) to drive the mass and spin of the black hole towards their desired values.
- Compute an updated guess for the constant \( C \) in equation (23) to drive the mass of the neutron star to its desired value.
- Correct the enthalpy to ensure force balance at the center of the neutron star. (Choosing \( \Omega_0 \) according to equation (40) guarantees force-balance along the axis separating the two neutron stars, but minor corrections to \( h \) are needed to satisfy that condition in other directions, as described in Paper I.)
- Solve equation (33) for the velocity potential \( \phi \), and compute an updated guess for \( \phi \) (and all the derived hydrodynamical variables).
- If the procedure has converged to the requested accuracy, stop. Otherwise, go back to the first step.
This algorithm requires $\sim 100$--150 iterations to converge at the lowest grid resolution used to compute the initial data. We then increase the resolution of the grid following the adaptive technique described in [46]. The higher resolution solve steps, which start from a much better initial guess, only require 20--50 iterations each to converge.

In Paper I, the new guesses for the different free variables were computed through a variety of methods. In some cases we used a relaxation procedure: a variable $u$ with value $u_{\text{old}}$ that gave a solution $u^*$ was updated according to

\begin{equation}
    u_{\text{new}} = \lambda u^* + (1 - \lambda)u_{\text{old}}
\end{equation}

with $\lambda$ a relaxation parameter chosen to be in the range 0.3--0.5. Relaxation of this kind is well known in numerical analysis to often allow convergence of iterative procedures when the full update $\lambda = 1$ leads to divergence. Relaxation was done in Paper I for the solution of the XCTS equations, as well as for $\phi, \Omega_0, C,$ and $\Omega_{\text{BH}}$. On the other hand, the center of each object and the surface of the neutron star were modified through discontinuous jumps towards the expected solution $u^*$, once $u^*$ was deemed to have converged to a stable value. In this paper, we show that the robustness of the algorithm requires a more consistent approach in which all variables are relaxed towards the self-consistent solutions. Following this principle, two ingredients in the solution procedure of Paper I need to be modified: the algorithms determining the location of the surface of the neutron star and of its center. These modifications are described in the following subsections.

2.3. Neutron star surface adjustment

An important aspect of the solver is the numerical domain that is used to solve the equations. We employ the domain described in Paper I section III.A, and shown here in figure 1. The physical domain is covered by a number of overlapping spectral subdomains; choosing the location and resolution of the subdomains allows the domain to be well-matched to the problem. A particularly important detail (we have found) is that since the neutron star surface is a physical discontinuity, it should be placed at a subdomain boundary to avoid Gibbs oscillations in the numerical solution. This requirement was also found in Paper I, although we used a different technique to enforce it. We employ a modification of the method described in Paper I in section III.A.2 and in item 3 of the iterative procedure of section III.C.

We define the neutron star surface as the location where the enthalpy (24) reaches some target value. Note that in our code we assume beta-equilibrium and zero temperature, and so this choice is equivalent to choosing the surface to lie at constant pressure. However, we do not use the value $P = 0$. The reason is that tabulated equations of state have many discontinuous features at low density, and an extended low-density atmosphere. Trying to exactly capture those features in the same spectral subdomain as the core of the star significantly degrades the accuracy of the solution in high density regions. Instead we fix the surface at a constant nonzero pressure which is sufficiently low that only a small amount of matter is outside that surface, and sufficiently high that many of the kinks in the low-density EOS are in another subdomain. This choice defines the subdomain boundary and the location where we enforce the boundary condition on the velocity potential. The region outside this boundary is still permitted to contain matter, but the velocity flow is not irrotational. The solutions we obtain are not sensitive to the exact value of the pressure chosen to define the surface as long as only a small amount of mass is outside this value.

In practice, the enthalpy is computed from the metric and the fluid velocity by assuming hydrostatic equilibrium, equation (23). Since the neutron star surface may deviate slightly from the subdomain boundary during the solution, the equation for $h$ is solved outside the star
as well as inside. In principle, the enthalpy should take on a constant value everywhere in the vacuum region, but numerically $h$ is allowed to take values below $h(\rho = 0)$, which ensures better behavior in the surface-finding algorithm. Similarly, the baryon density $\rho$ is allowed to take unphysical values $\rho < 0$ in regions in which $h < h(\rho = 0)$ to guarantee that $\nabla \rho$ is continuous—a desirable property when using spectral methods.

The location of the surface can be found with a simple one-dimensional root finding algorithm along the collocation directions, which lets us find the stellar radius $R(\theta, \phi)$ as a function of angle. This is then used to define the stellar surface coefficients via an expansion in scalar spherical harmonics:

$$R(\theta, \phi) = \sum_{l} \sum_{m=-l}^{l} R_{lm} Y_{lm}^{m}(\theta, \phi) . \quad (47)$$

Note that for $m \neq 0$, the coefficients $R_{lm}$ will be complex-valued in general, but because $R(\theta, \phi)$ is real-valued, $R_{l,-m} = (-1)^{m}(R_{lm})^{*}$ and so we can define

$$S_{lm} = \begin{cases} (-1)^{m} \sqrt{\frac{2}{\pi}} \text{Re}(R_{lm}) & m \geq 0 \\ (-1)^{m} \sqrt{\frac{2}{\pi}} \text{Im}(R_{lm}) & m < 0 \end{cases} \quad (48)$$

to store all independent components of the $R_{lm}$ coefficients.
The coefficients in (48) are used to update the domain boundary for the neutron star subdomains used by the solver. However, since the enthalpy may have non-negligible errors early in the solve, the computed surface may not be in the right place and may have large jumps between iterations. This is similar to the difficulties that are encountered in the elliptic solve. Accordingly, we follow a relaxation scheme as in (46) in updating the \( S_{xy} \) coefficients defined in (48), which are computed in each step and used to define the mapping shown in (77) and (78) in Paper I. We have found that choosing the same value for this relaxation parameter as for the one controlling the metric and matter relaxation gives good results.

### 2.4. ADM momentum control

To uniquely specify the initial conditions that we are attempting to produce, we need to fix the location of the centers of the black hole and neutron star, \( \mathbf{c}_{\text{BH}} \) and \( \mathbf{c}_{\text{NS}} \), the angular velocity \( \Omega_0 \), the radial velocity \( \dot{v}_r = a_0 r \), and the boost \( \mathbf{v}_{\text{boost}} \). We try to make these choices so that

- The linear ADM momentum of the system satisfies \( \mathbf{P}_{\text{ADM}} = 0 \).
- The objects are following circular orbits with the desired initial separation \( d \).
- The center of mass of the system is approximately at the origin of the chosen coordinate system.

The initial data solver finds constraint-satisfying configurations by following the iterative procedure described in section 2.2. The center of the neutron star is fixed at \( \mathbf{c}_{\text{NS}} = (-dM_{\text{BH}}/M_0, 0, 0) \). The quantities \( \Omega_0 \) and \( a_0 \) are chosen in order to minimize the orbital eccentricity of the system, following the iterative procedure developed for black hole–black hole binaries [45]. Alternatively, we can solve for \( \Omega_0 \) by requiring force balance at the center of the neutron star as described in section 2.1. Combined with the choice \( a_0 = 0 \), this leads to eccentricities of a few percent, and provides a good initial guess for the eccentricity reduction algorithm. This leaves us with the choices of \( \mathbf{c}_{\text{BH}} \) and \( \mathbf{v}_{\text{boost}} \), which are both made iteratively. The location of the black hole center can be modified at each step of the iteration, after we solve the constraint equations and evaluate the position of the neutron star surface (section 2.3). The choice of \( \mathbf{v}_{\text{boost}} \) comes as an outer boundary condition in the constraint equation for the shift. We have developed two different methods to choose \( \mathbf{c}_{\text{BH}} \) and \( \mathbf{v}_{\text{boost}} \).

The first (hereafter “position control”) is largely similar to the algorithm described in Paper I for spin-aligned binaries, and updated in [49] for black hole spins misaligned with the orbital angular momentum of the binary. In this method, \( \mathbf{c}_{\text{BH}} \) is initialized to \( \mathbf{c}_{\text{BH}}^0 = (dM_{\text{NS}}/M, 0, 0) \). At each step \( n \) of the iterative procedure, we measure the linear ADM momentum \( \mathbf{P}_n \), and the relative changes in each component \( \delta P^i = |P^i_n - P^i_{n-1}|/|P^i_0| \). If \( \delta P^i < \alpha_P \) for the largest component \( P^i_n \) of the ADM momentum and a freely specifiable parameter \( \alpha_P \), then we reset the components of \( \mathbf{c}_{\text{BH}} \) in the orbital plane of the binary (the \( x,y \) plane here) using the relaxation formula

\[
\mathbf{c}_{x,y}^{\text{new}} = \lambda_P \mathbf{c}_{x,y}^* + (1 - \lambda_P) \mathbf{c}_{x,y}^{\text{old}}
\]  

(49)

analogously to (46), with \( \mathbf{c}_{x,y}^* \) computed using the values of \( P_{x,y}^i \) and \( \mathbf{c}_{x,y} \) at the two latest steps \( i \) and \( j \) at which the location of the black hole center was modified:

\[
\mathbf{c}_{x,y}^* = \frac{c_{x,y}^i P_{x,y}^i - c_{x,y}^j P_{x,y}^j}{P_{x,y}^i - P_{x,y}^j}.
\]

(50)

For larger neutron stars, we have been using \( \alpha_P = 0.1, \lambda_P = 1 \). For compact stars, we find that changing the location of the black hole center more often, but by smaller increments,
works better. Accordingly, we use $\alpha_P = 0.4$, and $\lambda_P$ is chosen to equal the relaxation parameter in the elliptic solve. Equation (50) is inspired by the fact that, in Newtonian physics, a change $\delta c$ in $c_{BH}$ induces a change $\delta P = -\delta c \times \Omega$ in $P$. A similar updating algorithm is used for the vertical location of the black hole center, except that instead of trying to cancel the linear momentum of the system, we require vertical force balance $\langle \nabla h \rangle_z = 0$ at the center of the neutron star. Thus we use

$$c_i^* = \frac{c_i^0 (\nabla h)^0_i - c_i^0 (\nabla h)^i_z}{(\nabla h)^0_i - (\nabla h)^i_z}.$$  \hspace{1cm} (51)

The vertical component of the linear momentum, which cannot be easily controlled by moving the location of the compact objects, is instead canceled by a ‘boost’ given to the entire system through the chosen value $v_{\text{boost}}$ of the shift on the outer boundary. We set $v_{\text{boost}}^z = 0$ and update $v_{\text{boost}}^i$ using the same method as for the black hole center, but with

$$v_{i}^* = \frac{v_{i}^0 P_{j}^z - v_{j}^0 P_{i}^z}{P_{j}^z - P_{i}^z}.$$  \hspace{1cm} (52)

In the second method (hereafter ‘boost control’), the location of the center of the black hole is fixed to its expected value for a Newtonian binary orbiting around the origin of our coordinate system, $c_{BH} = (d M_{\text{NS}} / M, 0, 0)$. The constraint $P_{\text{ADM}} = 0$ is then satisfied by controlling all components of the linear momentum through the outer boundary condition on the shift. That is, we use

$$v_{i}^* = v_{i}^0 \frac{P_{j}^{k,x,y,z} - v_{j}^0 P_{i}^{k,x,y,z}}{P_{j}^{k,x,y,z} - P_{i}^{k,x,y,z}}.$$  \hspace{1cm} (53)

to reset the components of $v_{\text{boost}}$ whenever $\delta P^i < \alpha_P$.

For spin aligned binaries, position control generally results in a much smaller coordinate velocity for the center of mass of the system than boost control. The effect of the boost control methods on the evolution of the center of mass was observed for misaligned BHNS binaries in [49]. Because a large drift of the center of mass might introduce undesirable coordinate effects in the methods used to extrapolate the gravitational wave signal to infinity, or complicate the work of the control system used to evolve the binary in the comoving frame, we have generally preferred position control. However, each change of the location of the center of the black hole in the initial data solver introduces significant constraint violations in our solution. We find that, for very compact stars with $C \lesssim 0.2$, these changes can prevent convergence of the iterative procedure used to generate initial data, and so we avoid position control in these cases.

We note that in general relativity, neither of the methods presented here can provide us with initial data in which the center of mass of the binary is truly fixed. This is a relatively unimportant effect for the short simulations typically performed for BHNS systems. Indeed, for such systems, we are mainly concerned with obtaining a convergent, uniquely defined solution to the initial data problem with an acceptably low residual motion for the center of mass. A small residual drift of the center of mass can, however, be a much more serious issue when attempting to evolve a binary for hundreds of gravitational wave cycles, as for example in recent simulations of black hole binaries [50]. A more satisfactory solution to this problem was proposed and applied to binary black hole systems in Ossokine et al [46]. There, the centers of both compact objects are modified to satisfy the constraint $C_{\text{COM}} = 0$, with the center of mass of the system defined by
and $n$ the normal to the outer boundary of the computational domain, $S_\infty$. The separation vector between the two centers is also kept constant, and the condition $p_{\text{ADM}} = 0$ is imposed through boost control. Why not use the same method here? One of the main problems encountered when attempting to get convergent initial data for BHNS binaries is that changing the location of the black hole center introduces significant constraint violations in the solution. Changing the location of the center of the neutron star is, in our experience, even worse (and not done in our algorithm even for more forgiving systems with less compact neutron stars). The method from Ossokine et al [46] requires changes of the location of the center of both objects, and may thus be even more disruptive to the convergence of the iterative procedure used for BHNS binaries than the method controlling the position of the black hole center. Accordingly, we currently prefer the less rigorous but more convergent methods described in this paper. We note that using the condition $p_{\text{ADM}} = 0$ as an approximate condition to minimize the motion of the center of mass of the binary is the method currently used by all initial data solvers used for BHNS systems ([26, 30], Paper I).

3. Results

Using the methods described above, we were able to obtain solutions for binaries with various neutron star compactness for a variety of different equations of state. In all cases, we have chosen the black hole to be nonspinning, since the aim of the study was to develop methods for handling high neutron star compactness.

3.1. Polytropic $\Gamma = 2$ EOS

This case is one of the easiest for initial data solvers because the EOS is smooth and the density goes to zero linearly near the surface. Using the methods of Paper I and tweaking only the elliptic solve relaxation parameter $\lambda$, solutions for polytropic $\Gamma = 2$ equations of state with compactness up to 0.18 can be obtained. If we additionally incorporate an initial guess based on a lower-compactness binary instead of starting with an isolated neutron star configuration, a solution with $\tilde{C} = 0.20$ can be found. Finally, using the techniques described in section 2, we are able to obtain solutions with compactness up to $\tilde{C} = 0.21$ and mass of $1.4M_\odot$. For this EOS, there is a dynamic instability that sets in for compactness slightly higher than 0.21, and so this is roughly the highest compactness that we expect to be physically meaningful. In addition, other work [41] has found problems related to mathematical non-uniqueness when seeking solutions for stars that are dynamically unstable, so for both these reasons we avoid investigating such solutions. For polytropes, we can thus nearly reach the maximum stable compactness without modifications of the procedure in Paper I. Only the highest stable compactness $\tilde{C} = 0.21$ require the techniques introduced above. By contrast, for the SLy and LS220 equations of state, the methods of Paper I fail at much lower compactness.

We show in table 1 the results for applying this method to $\Gamma = 2$ polytropes of different compactness. The stellar mass is held constant at $1.4M_\odot$, and so the polytropic parameter $\kappa$ varies across these solutions and is included in the table. (Note that for polytropes, the mass can be rescaled by changing $\kappa$ without affecting $\tilde{C}$.) The mass ratio is 6:1 for all cases. The binding energy $E_b$ is computed by subtracting the sum of the ADM masses of the black hole and neutron star in isolation from the ADM energy of the system. The surface coefficients
shown are defined in (48). The residuals obtained as a function of resolution are shown in figure 2. In the figure, the residual $R_{\text{mat}}$ is the $L_2$ norm of the amount by which the metric equations (19)–(21) are nonzero at the grid points. The residual $R_{\text{rel}}$ is the $L_2$ norm of the difference between the left-hand and right-hand sides of equation (33). In the residuals we can see some aberrant behavior and jumps, but nevertheless the exponential convergence with resolution that characterizes spectral methods is apparent, especially if one focuses on the higher resolutions. The convergence with resolution also does not seem to vary among the different cases, so that compactness does not seem to be an issue here. One other thing to note here is that the initial distance $d$ between the black hole and neutron star is chosen to be quite close in all but the highest compactness cases, as compared with the values in tables 2, 5, and 6 in Paper I. (Convergence of the method deteriorates the closer together the black hole and the neutron star are.)

Table 1. Solved quantities for a family of BHNS binary configurations. For each configuration, the equation of state is a $\Gamma = 2$ polytrope chosen to yield $M_{\text{NS}} = 1.4 M_\odot$. The multiplier below two of the columns applies to the column above it. The columns show the compactness $\tilde{C}$, the polytropic parameter $\kappa$, the relaxation parameter $\lambda$, the initial separation $d$, the orbital angular velocity $\Omega$, the binding energy $E_b$, the ADM angular momentum $J$, and the ratio $S_{20}/S_{00}$ of surface coefficients defined in (48). The quantity $M_0$ is defined in (2). In all cases the black hole has no spin and the mass ratio is 6:1.

| $\tilde{C}$ | $\kappa$ | $\lambda$ | $d/M_0$ | $\Omega M_0$ | $E_b/M_0$ | $J/M_0^2$ | $S_{20}/S_{00}$ |
|------------|----------|-----------|---------|-------------|-----------|------------|----------------|
| 0.15       | 96.7     | 0.3       | 6.86    | 0.04648     | −6.69     | 0.422      | 4.261         |
| 0.16       | 89.7     | 0.3       | 6.86    | 0.04649     | −6.70     | 0.421      | 4.439         |
| 0.17       | 84.1     | 0.2       | 6.86    | 0.04650     | −6.71     | 0.421      | 4.474         |
| 0.18       | 79.8     | 0.2       | 6.86    | 0.04650     | −6.71     | 0.421      | 4.412         |
| 0.19       | 76.6     | 0.2       | 7.71    | 0.03967     | −6.19     | 0.434      | 3.574         |
| 0.20       | 74.4     | 0.2       | 7.71    | 0.03968     | −6.19     | 0.434      | 3.419         |
| 0.21       | 73.2     | 0.15      | 14.0    | 0.01740     | −3.85     | 0.526      | 1.441         |

$\times 10^{-3}$ $\times 10^{-3}$

3.2. SLy EOS

Using the methods of this paper, we can obtain solutions for an SLy EOS with compactness up to $\tilde{C} = 0.25$, which corresponds to a star with a mass of 1.86 $M_\odot$. For comparison, the maximum stable compactness for this EOS is about $\tilde{C} = 0.31$, corresponding to a mass of 2.04 $M_\odot$. Solutions were found for a variety of different distances, although as the compactness becomes greater, it becomes more difficult to obtain solutions for very close binaries. At the highest compactness, the closest binaries we attempted did not have a convergent solution, and so an evolution of such a system would need to simulate more than 20 orbits before a merger.

These results are shown in table 2. The mass ratio $q$ is close to 6, with slight variations between solutions. The lapse function for a representative case is shown in figure 3. The residuals obtained in the solve are shown in figure 4, plotted against $N^{1/3}$, the cube root of the number of points in the domain. Note that for the $\tilde{C} = 0.23$ and $\tilde{C} = 0.25$ cases, we have only plotted the convergence in the case of $d = 14 M_0$. Even more so than in the polytropic case, there are some jumps in the residuals, particularly in the velocity potential. Furthermore, jumps are present at all compactnesses instead of just low ones. However, once again, the
convergence does appear smoothly exponential at high resolution. The same value for the relaxation parameter is used for all of the various quantities that are updated via a relaxation scheme.

3.3. LS220 EOS

We obtain solutions for an LS220 tabulated EOS with compactness up to $\mathcal{C} = 0.26$, which corresponds to a star with a mass of $1.98 M_\odot$. For comparison, the largest known reliable neutron star masses are $1.97 \pm 0.04 M_\odot$ [51] and $2.01 \pm 0.04 M_\odot$ [52]. The LS220 EOS is the most realistic of the ones considered here, and so this is a very relevant result. The largest compactness for which this EOS yields a stable solution is $\mathcal{C} = 0.29$, corresponding to a mass of $2.04 M_\odot$.

These results are shown in table 3. In all cases, the mass ratio is 6:1. The residuals obtained in the solve are shown in figure 5, plotted against $N^{1/3}$, the cube root of the number of points in the domain. As before, we find exponential convergence, and obtain a final residual of $10^{-5}$ or better.
It is worth briefly discussing the differences in final residuals among the various solutions we obtain. Comparing the various cases, we find a solution for the metric quantities and velocity potential whose residual is worse for the most compact solutions than the least compact ones by about a factor of 10. Generally speaking, the metric residual reaches $10^{-5}$ in the worst cases and the velocity potential reaches $10^{-4}$. We consider this to be a suitable result for the purpose of evolving these systems through merger. We performed a very short examination of the behavior of our most compact LS220 solution in an evolution, and found that the convergence of the constraints and their values are quite acceptable, similar to what is
Figure 4. Spectral convergence for a family of BHNS binary configurations. For each configuration, the equation of state is SLy. Each panel shows the final residual in the metric solution ($R_{\text{met}}$) and in the velocity potential solution ($R_{\text{mat}}$) as a function of the cube root of the number of grid points in the domain. The vertical scale is logarithmic, and the exponential convergence with resolution is apparent.

Table 3. Solved quantities for a family of BHNS binary configurations. For each configuration, the equation of state is the LS220 tabulated equation of state. The multiplier below two of the columns applies to the column above it. The quantities shown are all as for table 1, with $\kappa$ replaced by the neutron star ADM mass $M_{\text{NS}}$ in units of $M_\odot$.

| $C$  | $M_{\text{NS}}$ | $\lambda$ | $d/M_\odot$ | $\Omega M_\odot$ | $E_{\nu}/M_\odot$ | $J/M_\odot^2$ | $S_{20}/S_{0}$ | $\times 10^{-3}$ | $\times 10^{-3}$ |
|------|----------------|-----------|-------------|-----------------|------------------|--------------|---------------|----------------|----------------|
| 0.17 | 1.40           | 0.2       | 14.0        | 0.01739         | $-3.84$          | 0.526        | 1.885         |                |                |
| 0.23 | 1.83           | 0.1       | 14.0        | 0.01739         | $-3.83$          | 0.526        | 1.347         |                |                |
| 0.25 | 1.94           | 0.08      | 16.0        | 0.01439         | $-3.18$          | 0.552        | 0.985         |                |                |
| 0.26 | 1.98           | 0.07      | 14.0        | 0.01739         | $-3.73$          | 0.525        | 1.075         |                |                |
seen with a much less compact star. In terms of matter, we see an initial anomaly in the density larger than in the low compactness case, but this anomaly decreases with resolution.

4. Conclusions

The problem of solving for initial data for a BHNS system is a complex one involving many interacting solution steps. In addition to the elliptic equations governing the system, multiple nonlinear constraints must be imposed simultaneously. High compactness stars cause difficulty with the various solvers that currently exist.

We have invoked as a general principle that all variables should be updated by a smooth relaxation procedure during the iteration. This has led us to focus on two of the algebraic constraints in particular that were not treated in this way in our previous work: aligning the surface with a subdomain boundary and enforcing zero linear momentum on the binary system.

We have found that by modifying the method of Paper I [32] to enforce these constraints with additional relaxation steps, we can obtain solutions for a wide variety of systems with...
quite high compactness stars, including stars with polytropic, SLy, and LS220 equations of state. This includes a solution for a binary with a physically realistic LS220 EOS star having a mass of $M = 1.98 M_{\odot}$. While systems whose components are very close together are still difficult to solve, we expect that these improvements to the algorithm should allow solutions to the BHNS initial data problem to be found for essentially all physically important cases.

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