Induced decoherence and entanglement by interacting quantum spin baths

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The reduced dynamics of a single or two qubits coupled to an interacting quantum spin bath modeled by a XXZ spin chain is investigated. By using the method of time-dependent density matrix renormalization group (t-DMRG), we go beyond the uniform coupling central spin model and evaluate nonperturbatively the induced decoherence and entanglement. It is shown that both decoherence and entanglement strongly depend on the phase of the underlying spin bath. We show that in general, spin baths can induce entanglement for an initially disentangled pair of qubits. Furthermore, when the spin bath is in the ferromagnetic phase, because qubits directly couple to the order parameter, the reduced dynamics shows oscillatory type behavior. On the other hand, only for paramagnetic and antiferromagnetic phases, initially entangled states suffer from the entanglement sudden death. By calculating concurrence, the finite disentanglement time is mapped out for all phases in the phase diagram of the spin bath.

I. INTRODUCTION

Spin qubits are promising candidates for quantum information processing due to their long decoherence and relaxation time.\textsuperscript{1-5} Some schemes, such as solid state spin qubits, further enjoy the potential scalability via the integration with nanotechnology.\textsuperscript{6-9} However, spin qubits are not totally immune from the ubiquitous decoherence. To describe the bath that causes the decoherence of spin qubits, it is known that in some cases, the bath is better modeled by spins instead of delocalized oscillators, resulting in the so-called spin baths.\textsuperscript{10} It has been argued that the influence of spin baths may be qualitatively different from bosonic baths and non-Markovian dynamics can easily emerge.\textsuperscript{11-14} Due to the growing interest in spin baths, the decoherence behavior and the entanglement dynamics of few qubits coupled to spin baths have been studied extensively in recent years. Early works focused on the decoherence due to independent spins.\textsuperscript{15} Here although the proposed model formally resembles a spin boson model, non-Markovian already emerges even when bath modes are not interacting.\textsuperscript{16} In real baths, however, spins are not independent. It is therefore important to include effects due to interactions of spins in the bath. Nonetheless, the inclusion of the intra-spin interaction in the bath complicates the problem and only for some limited models with high symmetry, exact reduced dynamics can be identified.\textsuperscript{17} Beyond models with exact solutions, approximated dynamics was obtained by using mean-field or perturbative approaches\textsuperscript{18} to handle more generic models. The most common model employed in these works is the "central spin model", where the qubits are uniformly coupled to all spins of the bath. While analytical derivations are possible in these models, they are less realistic and are more difficult to be implemented experimentally. A non-perturbative approach that can capture the non-Markovian effects induced by interacting spin bath with generic coupling to qubits is hence highly desirable.

To overcome the difficulty associated with interacting spins, we utilize the method of time-dependent density matrix renormalization group (t-DMRG)\textsuperscript{19,20} to investigate the reduced dynamics of single or two qubits coupled to an interacting spin chain. Recently, t-DMRG has been used to study the single qubit pure dephasing induced by a XXZ anisotropic spin chain.\textsuperscript{21} The advantage of t-DMRG is its ability to calculate reduced dynamics even when the spin bath is not integrable and the coupling is not uniform. Due to the accumulation of errors, t-DMRG will eventually run away at large time\textsuperscript{22} but this does not impose serious limitation since for the study of quantum information we are mostly interested in some smaller time scale. In this work we apply the method of t-DMRG to investigate both pure dephasing and general decoherence model of qubits coupled to spin baths. Single qubit decoherence as well as two qubits (dis)-entanglement dynamics are investigated. It is shown that both decoherence and entanglement strongly depend on the phase of the spin bath. In general, we find that spin baths can induce entanglement for an initially disentangled pair of qubits. However, when the spin bath is in the ferromagnetic phase, because qubits directly couple to the order parameter, the reduced dynamics shows oscillatory type behavior. On the other hand, only for paramagnetic and antiferromagnetic phases, initially entangled states suffer from the entanglement sudden death.\textsuperscript{23,24}

To quantify the single spin decoherence, we evaluate the evolution of the Loschmidt echo.\textsuperscript{25} We analyze the relation between the short time Loschmidt echo decay parameter and the quantum phases of the spin bath, as it has been pointed out that these two are closely related, especially when a symmetry breaking occurs in the bath.\textsuperscript{26} We use the temporal evolution of concurrence\textsuperscript{16} to study the entanglement dynamics. One important issue of entanglement dynamics is the possibility of creating entanglement through a common bath for originally disentangled qubits. It has been shown that induced entanglement via a common bath is possible for bosonic and fermionic baths.\textsuperscript{17,18} For spin baths such a possibility has been explored for non-interacting spin bath\textsuperscript{27} as
well as interacting ones, but is restricted to uniform coupling models. It will be shown later in this paper that induced entanglement is possible for local coupling model considered in this work. We note that the induced entanglement is also closely related to recent proposals of quantum communication and teleportation via spin chain.

Another important issue is the disentanglement dynamics of an initially entangled state. It has attracted much attention in recent years since Yu and Eberly and Jakóbczyk and Jamróz predicted that two initially entangled state without interaction can become completely disentangled at finite time. This feature has been termed entanglement sudden death (ESD). ESD has been studied theoretically within various models and has been demonstrate experimentally. These models, however, are restricted to Markovian bosonic bath or classical noise. In this work we explore for the first time if ESD-like phenomena can occur for spin baths in non-Markovian regime. In particular, we will show that only when the spin bath are paramagnetic or antiferromagnetic, the phenomenon of ESD occurs, while when the spin bath is in the ferromagnetic phase, concurrence shows oscillatory behavior. As understanding the nature of the (dis)-entanglement dynamics constitutes an important step for quantum engineering these systems, our results are of practical usage for future quantum information processing.

The paper is organized as follows. In Sec. II we present our model Hamiltonian and briefly discuss how to apply t-DMRG to analyze the model Hamiltonian. In Sec. III we present our results of single qubit decoherence while in Sec. IV the results of (dis)-entanglement dynamics are presented. In Sec V we summarize and discuss implication of our results.

II. THEORETICAL FORMULATION

We consider a system-bath model which is described by the total Hamiltonian \( H = H_{sys} + H_{bath} + H_{int} \), where \( H_{sys} \) is the Hamiltonian of a single or two qubits system, \( H_{bath} \) is the Hamiltonian of a spin bath and \( H_{int} \) represents the interaction between qubits and the bath. We shall set \( H_{sys} = 0 \) but our method can be applied to a generic \( H_{sys} \). We shall assume that the spin bath is a spin chain characterized by the \( XXZ \) Heisenberg model

\[
H_{bath} = J \sum \left( S^x_i S^x_{i+1} + S^y_i S^y_{i+1} + \Delta S^z_i S^z_{i+1} \right),
\]

where \( J > 0 \). It is known that the \( XXZ \) Heisenberg model has a very rich structure. The system is ferromagnetic for \( \Delta < 1 \), antiferromagnetic (Ising-type) for \( \Delta > 1 \), and critical (XY-type) for \( -1 < \Delta < 1 \). It also encompasses the \( XY \) model where \( \Delta = 0 \). The most general linear coupling between qubit \( A ) \) and the bath can be expressed as

\[
H_{int} = \sum_{i,a} \epsilon_i \delta_{A(B)} S^a_i,
\]

where \( \alpha = x, y, z \) and \( i = 1, \ldots, N \). Here \( \epsilon_i \) characterizes the coupling of the spin qubit to the \( i \)th spin in the spin chain. For most situations, the more interesting cases are \( \epsilon < 0 \) and hence we shall concentrate on negative \( \epsilon \). Our numerical method, however, can be applied equally well to cases with positive \( \epsilon \). In our work, both the Ising coupling and isotropic Heisenberg coupling will be considered. The Ising coupling \( (\epsilon^x_i = \epsilon^y_i = 0) \) gives rise to a pure dephasing model while the isotropic Heisenberg coupling \( (\epsilon^x_i = \epsilon^y_i = \epsilon^z_i \neq 0) \) induces both dephasing and energy relaxation. The range of the coupling is crucial for characterizing the interaction of qubits to the spin bath. For uniform coupling \( \epsilon_i \) is independent of \( i \). This is unrealistic but for uniform coupling the Loschmidt echo and the entanglement dynamics can be calculated exactly by using Jordan-Wigner transformation when the spin bath is the \( XY \) model \( (\Delta = 0) \). However, the more realistic coupling model is the local coupling model in which only the coupling to the closest spin is nonvanishing. Nonetheless, there is no analytic solutions known for this model. The reduced dynamics is less studied but is more relevant to real experiments. In this case, if the spin bath is ferromagnetic, the qubit couples directly to the order parameter (the magnetization); while if the spin bath is antiferromagnetic or paramagnetic, the qubit does not couple to the order parameter. Hence the reduced dynamics exhibits completely different behavior in different phases. As the local coupling model is more relevant to real experiments, in the following we shall concentrate on the local coupling model.

We now briefly outline the procedure to evaluate the reduced dynamics of the qubits and other derived quantities. For a given set of parameters, we first employ static DMRG to find the ground state \( |G \rangle \) of the spin chain, where open boundary condition (OBC) is used. We assume that at \( t = 0 \) the initial total state is a product state of the form: \( |\Phi(0)\rangle = |\psi_{sys}(0)\rangle |G\rangle \), where \( |\psi_{sys}(0)\rangle \) is some particular system state that we are interested in. Formally the evolution of the reduced density matrix can be obtained by evolving first the total state

\[
|\Phi(t)\rangle = e^{-iHt} |\psi_{sys}(0)\rangle |G\rangle,
\]

then tracing off the spin bath

\[
\rho_{sys}(t) = \text{Tr}_{bath}[|\Phi(t)\rangle \langle \Phi(t)|].
\]

Loschmidt echo and concurrence then can be evaluated from \( \rho_{sys}(t) \). In general evolving such a state is a formidable task. t-DMRG, however, provides a way to efficiently evolve such a state with high accuracy for a quasi-one dimensional system. We note that the degrees of freedom of the qubits are kept exactly during the t-DMRG calculation by targeting an appropriate state. The dimension of the truncated Hilbert space is set to be \( D = 100 \). For short time decay simulation we set \( \delta t = 10^{-3} \) in the Trotter slicing while for entanglement dynamics we set \( \delta t = 0.1 - 0.5 \) to balance the Trotter error and truncation error.
III. SINGLE QUBIT DECOHERENCE

In this section, we present our results for the single qubit decoherence which is characterized by the Loschmidt echo. The Loschmidt echo has been used extensively to quantify the single qubit decoherence, especially its connection to the quantum criticality of the spin baths.\[13,34,35\] Loschmidt echo can be understood intuitively as follows: Consider an initially disentangled total state \((C_{+}|+⟩ + C_{−}|−⟩) \otimes |G⟩\), at some later time \(t\) it will evolve into an entangled state \(C_{+}(t)|+⟩ \otimes |Ψ_{+}(t)⟩ + C_{−}(t)|−⟩ \otimes |Ψ_{−}(t)⟩\). Loschmidt echo, defined as \(\mathcal{L}(t) \equiv |⟨Ψ_{+}(t)|Ψ_{−}(t)⟩|^2\), clearly measure the coherence between \(|+⟩\) and \(|−⟩\). When \(\mathcal{L} = 1\) the qubit is disentangled from the bath while when \(\mathcal{L} = 0\) the qubit is totally entangled with the bath.

We start by noting that for numerical calculation on finite length, all dynamics will show quasi-periodic behavior. The quasi-period is known as the revival time. Since in our numerical calculation, the spin bath is a chain of finite length, it is essential to identify the revival time for each length to avoid unphysical results due to revival. As a zeroth order approximation, the revival time for each length to avoid unphysical results due to revival.

FIG. 1: (a)(b)(c) \(\mathcal{L}(t)\) as a function of time for different lengths and \(\Delta\). (d)Revival time as a function of length. The coupling is of Heisenberg type with \(\epsilon = −0.3\).

To compute the single qubit decoherence, we couple the qubit to a single site of spin chain, which is taken to be the middle site of the chain to suppress boundary effects. We tune the spin bath to different quantum phases by changing the parameter \(\Delta\). In the ferromagnetic regime, a small uniform external field is introduced during the static-DMRG calculation but is turned off during time evolution. It is numerically checked that the numerical results reported below are insensitive to the magnitude of the applied external field.

When the spin is in the Ising antiferromagnetic regime or the XY critical regime, we find that in short time the behavior of the Loschmidt echo decay is Gaussian, \(\mathcal{L}(t) \sim e^{-αt^2}\), where \(α\) is the decay parameter.\[36\] In Fig. 2 we plot the decay parameter as a function of \(\Delta\) for the case of Ising coupling. In the ferromagnetic regime (\(\Delta < −1\)), the qubit decay is completely suppressed (\(α = 0\)). This is a consequence of the Ising coupling in which both \(|+⟩ \otimes |G⟩\) and \(|−⟩ \otimes |G⟩\) are eigenstates to the system and hence \(\mathcal{L}(t) = 1\).

Clearly, the decay parameter is largest in the critical regime (\(−1 < \Delta < 1\)) and it decreases gradually to zero as one moves into the antiferromagnetic regime (\(\Delta > 1\)). For single link scenario, the decay parameter is almost featureless within the critical regime. Our numerical results also show that if the qubit is coupled to multiple sites, decay parameter acquires a weak dependence on \(\Delta\) and the transition near \(\Delta = 1\) becomes less sharp (not plotted). Note that the decay parameter becomes sensitive to the magnitude of the small staggered field applied when the spin bath is close to the phase boundary (\(\Delta \sim 1\)). This is due to the fact that for finite \(N\) the barrier between two degenerate ground states is finite and approaches zero as \(\Delta\) approaches 1. The ground state
obtained by static DMRG includes a small mixture of the degenerate state, which is sensitive to the strength of the staggered field. For larger $\Delta$, the barrier between two degenerate ground states increases as one moves deep into the antiferromagnetic regime. As a result, the decay parameter becomes less sensitive to the strength of the stagger field for $\Delta \gg 1$.

We now turn to the case of Heisenberg coupling. We first note that in the ferromagnetic regime, the qubit couples to the order parameter. Therefore, the qubit is effectively in an average magnetic field $\langle \vec{S}_i \rangle$. As a consequence of the Heisenberg coupling, the qubit will precess about $\langle \vec{S}_i \rangle$. Since magnons are generated at the same time when the qubit evolves, $\langle \vec{S}_i \rangle$ starts to deviate from $1/2$ and results in oscillations in the reduced dynamics. Fig. 4 shows some typical oscillating behavior of $L(t)$ in this scenario. Clearly, the reduced dynamics is no longer Gaussian. Therefore, we shall not mark the ferromagnetic regime in the followings.

In Fig. 5 we plot the decay parameter as a function of $\Delta$ for the case of Heisenberg coupling. The overall behavior is very similar to the case of Ising coupling except that the decay parameter depends weakly on $\Delta$ in the critical regime. This is different from the Ising coupling case shown above but is similar to the multiple sites Ising coupling case. For both Ising and Heisenberg coupling we find a discontinuity in the behavior of $L(t)$ at $\Delta = -1$ and a first derivative discontinuity at $\Delta = 1$. These discontinuities coincides with the phase boundary of the underlying spin chain. Different behavior at the $\Delta = \pm 1$ can be traced back to the different nature of the ferromagnetic and antiferromagnetic transition and the close relation between the decoherence and the quantum criticality of the bath is clearly demonstrated.

IV. ENTANGLEMENT DYNAMICS

In this section, we investigate the entanglement dynamics of two qubits that couple to the spin bath. There are two central issues to be addressed. The first issue is the possibility of entanglement creation via the common spin bath for a pair of initially disentangled qubits without direct interaction. The second issue is the disentanglement dynamics of an initially entangled state. In particular, we would like to address the issue if qubits influenced by spin baths also suffer from entanglement sudden death and if the entanglement sudden death depends on the quantum phase that qubits couple to. To characterize the entanglement, we shall use concurrence as the measurement of entanglement. For a given reduced density matrix $\rho(t)$, the concurrence is defined as $C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$, where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots of the eigenvalues of the operator $\rho(\sigma^y \otimes \sigma^y)\rho^*(\sigma^y \otimes \sigma^y)$ and $\rho^*$ is the complex conjugation of $\rho$. 
FIG. 6: Entanglement dynamics for an initially disentangled pair of qubits for the case of Ising coupling. Here $\Delta = 0$ and $N = 80$.

FIG. 7: Entanglement dynamics for an initially disentangled pair of qubits for the case of Heisenberg coupling. Here $\Delta = 0$ and $N = 80$.

A. Entanglement creation

It has been shown that entanglement can be created without direct interaction if two qubits interact with a common bosonic bath or a fermionic bath. Of particular interest to us is the onset time of the entanglement, the strength of the induced entanglement, and the time scale where the induced decoherence eventually takes over. These considerations are important in determining if such an induced entanglement is useful in real quantum computation. The issue is also closely related to the proposals of induced interaction via a common bath, where the effect of induced decoherence from the same bath is usually neglected during the derivation.

In Fig. 6 we plot the concurrence as a function of time using various coupling strength and inter qubit distance. The coupling between qubits and the spin bath is of Ising type, which gives rise to a pure dephasing model. We assume that the coupling strength is the same for two qubits ($c_1 = c_2$) and the initial state is taken to be $\frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$. We shall set $\Delta = 0$ but similar results can be obtained for $\Delta \neq 0$. Before we discuss our findings in more details we would like to comment that if Markovian approximation or uniform coupling assumption are taken then one can no longer discuss the inter-qubit distance dependence. The relation between entanglement dynamics and inter-qubit distance, however, is gaining interest since people begin to explore the non-Markovian effects of the bath. We terminate the simulation at half of the revival time, where usually Loschmidt echo reaches its minimum, to avoid the unphysical dynamics due to the revival. We also numerically check the finite size effect by comparing the entanglement dynamics from different chain lengths. We find that the results from different chain lengths agree with each other reasonably well. The length of the chain mainly set an upper bound for the simulation time.

We find that for this configuration it is possible to create entanglement via the spin bath. In particular, for weaker coupling strength the induced entanglement rises more slowly but can reach a higher value; while for stronger coupling the induced entanglement rises more rapidly. The maximal concurrence reached, however, is lower. This is because larger coupling strength also lead to stronger decoherence. It is also evident from the figure that the entanglement creating rate decreases as the inter qubit distance increases, which is typical for this kind of induced interaction. We find that for large enough $\epsilon$ and the smaller enough inter qubit distance the concurrence shows oscillatory behavior. In these cases, the coupling is strong enough to create concurrence oscillation but also weak enough to prevent the bath to totally disentangle the qubits. The delicate interplay between induced decoherence and induced entanglement indicates that using such an induced entanglement for quantum computation is a tricky task. One has to tune the coupling to be within the right window to balance the effect from each side.

In Fig. 7 we plot the concurrence as a function of time for the case of Heisenberg coupling, starting form the same initial condition. Qualitatively the behavior is similar to the case of Ising coupling. We find that the maximal entanglement that can be reached is smaller. This is because for Heisenberg coupling the Loschmidt echo always decays to zero regardless the coupling strength while for Ising coupling the minimal Loschmidt echo value is a decreasing function of the coupling strength and is not zero. We also find that the onset time is roughly proportional to the inter qubit distance. This is expected as the excitation of spin chain, which mediates the entanglement generation, travels with finite phase velocity. The time the excitation reaches the other qubit would be proportional to the inter qubit distance. However, the concurrence oscillation is absent, indicating that the induced interaction is weaker for Heisenberg coupling. We note that it is difficult to write down an exact form of the induced interaction unless Markovian approxima-
FIG. 8: Disentanglement dynamics for an initially disentangled pair of qubits when the bath is in the ferromagnetic phase. The coupling is of Heisenberg type, \( \epsilon = -0.3 \) and \( N = 80 \).

FIG. 9: Disentanglement dynamics for an initially disentangled pair of qubits when the bath is in the antiferromagnetic or the XY critical phase. The coupling is of Heisenberg type, \( \epsilon = -0.3 \), and \( N = 80 \).

B. Entanglement decay

Here we present our results for the disentanglement dynamics of an initially entangled state. To investigate the possibility of ESD in spin bath we start from an initial state of the form \( |\psi_{\text{sys}}(0)\rangle = \alpha |00\rangle + \beta |11\rangle \), with an initial concurrence \( C(0) = 2|\alpha\beta^*| \). Two qubits are set be 20 sites apart so that the decoherence of individual qubit are nearly independent and the coupling is of Heisenberg type. We first show a typical behavior of concurrence in the ferromagnetic regime in Fig. 8. Clearly, as for the reduced dynamics for a single qubit, the concurrence shows oscillatory behaviors. Hence qubits in the ferromagnetic regime do not suffer from ESD and the envelop of the entanglement decays exponentially.

In Fig. 9 we plot the disentanglement dynamics of two states in the XY critical and antiferromagnetic regimes. All of the computations start from the same initial concurrence with two set of coefficients, \( \alpha/\beta = 1/\sqrt{3} \) or \( \beta/\alpha = 1/\sqrt{3} \), corresponding to two different initial states. We find that in the critical regime \((-1 < \Delta < +1)\) both states suffers from ESD. Furthermore, the entanglement dynamics of two states are identical in the critical regime, which is due to that the rotational symmetry is not broken in the critical regime and \( |00\rangle \) is equivalent to \( |11\rangle \) by \( Z_2 \) symmetry along the quantization axis. In both ferromagnetic and antiferromagnetic phase, where the rotational symmetry is broken, we find that the entanglement dynamics for these two states starts to deviate from each other. In most of the antiferromagnetic regime both states do not suffer from ESD. When \( \Delta \) is close to the phase boundary, however, both of two states that correspond to \( \alpha/\beta = 1/\sqrt{3} \) and \( \beta/\alpha = 1/\sqrt{3} \) suffer from ESD and have slightly different disentanglement time. In Fig. 10 we plot the inverse of disentanglement time, which is defined as the time when concurrence becomes zero, as a function of \( \Delta \). Starting from \( \Delta = -1 \), it shows monotonic decrease. Across the phase boundary, \( \Delta = 1 \), the inverse of disentanglement time develops a small bump and persists into the antiferromagnetic phase and finally decreases to zero around \( \Delta = 1.2 \). The existence of finite region with finite disentanglement time in the antiferromagnetic regime is due to that when \( \Delta \) approaches 1, the barrier between two degenerate ground states approaches zero. For finite \( N \) and finite value of
$\Delta - 1$, the ground state obtained by static DMRG includes a small mixture of the degenerate state so that it resembles the XY critical state and results in finite disentanglement time. In the thermodynamic limit ($N \to \infty$), the region with finite disentanglement time in the antiferromagnetic regime shrinks down, resulting discontinuity at the phase boundary $\Delta = 1$. The overall behavior found in the above is different from those reported in the early work in which two states investigated in the above possess different disentanglement dynamics and only one of them suffer from ESD. The difference is due to that the model adopted in Ref.\textsuperscript{22} includes the effect of spontaneous decay, breaking the symmetry between $|0\rangle$ and $|1\rangle$; while for spin bath in XY critical regime such symmetry breaking is absent. It is important to note that there exits a subregime, roughly around $-0.3 < \Delta < 0.6$, in which entanglement shows revival after some dark period. Note that the entanglement revival after some dark period was also reported in Ref.\textsuperscript{22} where a photonic multimode vacuum bath is assumed and the revival is attributed to the two photon decay. In our work, the origin of the revival is less clear. We believe that the existence of such subregime is due to the competition between disentanglement decay and Loschmidt echo decay. Within this subregime the Loschmidt echo decay more slowly, giving the system a chance to revive after the first ESD.

V. CONCLUSION

In summary, the decoherence and (dis)-entanglement dynamics induced by spin baths are investigated nonperturbatively by using t-DMRG. For both pure dephasing model (Ising coupling) and general decoherence model (Heisenberg coupling) we calculate the short time decay parameter of the Loschmidt echo. We find that in both cases the decay parameter is closely related to the phase of the underlying spin chain. In the ferromagnetic regime, the reduced dynamics shows oscillatory behavior while in the XY critical and antiferromagnetic regimes, the decay parameter shows a first derivative discontinuity at $\Delta = +1$. We evaluate the entanglement dynamics of a pair of initially disentangled qubits which are close to each other. We demonstrate that it is possible to induce entanglement via their common interaction with the spin bath. The competition between induced decoherence and entanglement can be easily seen in the coupling strength dependent behavior of the entanglement onset time, growth rate, and the maximal entanglement reached. Finally we investigate the disentanglement dynamics of a pair initially entangled qubits which are far from each other. For the two initial states we studied, we find that their disentanglement dynamics are identical and suffer from ESD in the critical regime. Their disentanglement dynamics begin to deviate from each other in both the ferromagnetic and the antiferromagnetic regime. They no longer suffer from ESD in the ferromagnetic regime but still suffer from ESD if the chain is near the antiferromagnetic transition. It is shown that the inverse of finite disentanglement time has a close relation to the phase of the spin bath and shows monotonic decrease behavior as one moves into the antiferromagnetic regime.

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