Statefinder Description in Generalized Holographic and Ricci Dark Energy Models

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We have considered the generalized holographic and generalized Ricci dark energy models for acceleration of the universe. If the universe filled with only GHDE/GRDE the corresponding deceleration parameter, EOS parameter and statefinder parameters have been calculated. Next we have considered that the mixture of GHDE/GRDE and dark matter in interacting and non-interacting situations. Also the mixture of GHDE/GRDE and generalized Chaplygin gas have been analyzed during evolution of the universe. The natures of above mentioned parameters have been investigated for interacting and non-interacting situations. Finally, it follows that the prescribed models derive the acceleration of the universe.

I. INTRODUCTION

Recent observations of the luminosity of type Ia supernovae indicate [1 - 3] an accelerated expansion of the universe and lead to the search for a new type of matter which violates the strong energy condition. The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as dark energy. The universe is spatially flat which consists of about 70% dark energy with negative pressure, 30% cold dark matters with baryons and some radiation. In the observational cosmology of dark energy, the equation-of-state parameter (EoS) \(w = \frac{p}{\rho}\) plays a central role, where \(p\) and \(\rho\) are pressure and energy density of the corresponding field respectively. For accelerating expansion of the universe, the EoS of dark energy must satisfy \(w < -1/3\). To explain the current accelerated expansion, many models have been presented, such as cosmological constant, quintessence [4,5], phantom [6], quintom [7] etc. The simplest candidate of the dark energy is a very small positive time independent cosmological constant \(\Lambda\), for which \(w = -1\). The cosmological constant remains unchanged while the energy densities of dust matter and radiation decrease rapidly with the expansion of our universe. Many dynamical dark energy models have been proposed as alternatives to the cosmological constant. We will discuss here two types of dark energy candidates i.e., holographic and Ricci dark energies.

The dark energy problem may be in essence an issue of quantum gravity [8]. However, by far, we have no any complete theory of quantum gravity, so it seems that we have to consider the effects of gravity in some effective quantum field theory in which some fundamental principles of quantum gravity should be taken into account. The holographic principle [9] is just a fundamental principle of quantum gravity, and based on the effective quantum field theory, it is pointed out that the quantum zero-point energy of a system with size \(L\) should not exceed the mass of a black hole with the same size [10], i.e. \(L^3 \Lambda^4 \leq L M_p^2\) where \(\Lambda\) is the ultraviolet (UV) cut-off of the effective quantum field theory, which is closely related to the quantum zero-point energy density, and \(M_p = 1/\sqrt{8\pi G}\) is the reduced Planck mass. This observation relates the UV cut-off of a system to its infrared (IR) cutoff. When we take the whole universe into account, the vacuum energy related to this holographic principle can be viewed as dark energy. The largest IR cutoff \(L\) is chosen by saturating the inequality, so that we get the holographic dark energy density [11] \(\rho_\Lambda = 3c^2 M_p^2 L^{-2}\) where \(c\) is a numerical constant characterizing all of the uncertainties of the theory, whose value can only be determined by observations. Here \(L\) is define by \(L = a \tau(t)\) where \(a\) is a scale factor and \(\tau(t)\) can be obtained from \(\int_0^{\tau(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^\infty \frac{dt}{a} = \frac{R_E}{a}\), where \(k = 0, \pm 1\) is the curvature index and \(R_E\) is the radius of event horizon [12] and we get \(\tau(t) = \frac{\sin(\sqrt{|k|} R_E/a)}{\sqrt{|k|}}\). On the basis of the holographic principle proposed by [13] several others have studied holographic model for dark energy [14]. Employment of Friedman equation [12] \(\rho = 3M_p^2 H^2\) where \(\rho\) is the total energy density and taking \(L = H^{-1}\) one can find \(\rho_m = 3(1 - c^2) M_p^2 H^2\). Thus either \(\rho_m\) or \(\rho_\Lambda\) behaves like \(H^2\). Thus, dark energy results as pressureless. If we take \(L\) as

\begin{align*}
\rho_\Lambda &= \frac{3c^2 M_p^2 L^{-2}}{a} \\
\rho_m &= \frac{3(1 - c^2) M_p^2 H^2}{a}
\end{align*}

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the size of the current universe, say, the Hubble radius \( \frac{1}{H} \), then the dark energy density will be close to the observational result. But, neither dark energy, nor dark matter has laboratory evidence for its existence directly.

Many candidates such as cosmological constant, quintessence, phantom, holographic dark energy, etc. have been proposed to explain the acceleration. Ricci dark energy, which is a kind of holographic dark energy [15] taking the square root of the inverse Ricci scalar as its infrared cutoff and this model is also phenomenologically viable. Gao et al [16] proposed the dark energy density proportional to the Ricci scalar \( R \) i.e., \( \rho_X \propto R \) is called the Ricci dark energy. This model works fairly well in fitting the observational data, and it could also help to understand the coincidence problem. Moreover, in this model the presence of event horizon is not presumed, so the causality problem is avoided. Assuming the black hole is formed by gravitation collapsing of the perturbation in the universe, the maximal black hole can be formed is determined by the casual connection scale \( R_C \) [17], given by \( R_C = 1/\sqrt{\text{Max}(H+2H^2,-\dot{H})} \) for a flat universe, where \( H \) is the Hubble parameter, and if \( R_C = 1/\sqrt{(\dot{H}+2H^2)} \), it could be consistent with the current cosmological observations when the vacuum density appears as an independently conserved energy component. As we know, in flat FRW universe, the Ricci scalar is \( R = -6(\dot{H}+2H^2) \), which means the \( R_C \) is proportional to \( R \) and if one choices the casual connection scale \( R_C \) as the IR cutoff, the Ricci dark energy model is also obtained. There are several works on this Ricci dark energy model [18].

In this work, we have first defined the generalized holographic and generalized Ricci dark energy models in section II. The statefinder parameters in two fluid system has been presented in section III. The equation of state and statefinder parameters for generalized holographic and generalized Ricci dark energy models without dark matter and with dark matter in non-interacting and interacting scenarios have been calculated in sections IV - VI. In sections VII and VIII, the equation of state and statefinder parameters for generalized holographic and generalized Ricci dark energy models with generalized Chaplygin gas in non-interacting and interacting scenarios have been studied. Finally, some fruitful conclusions have been drawn.

II. TWO DARK ENERGY MODELS : DEFINITIONS

Xu et al [19] proposed two types of dark energy models, i.e., generalized holographic and generalized Ricci dark energy models as follows:

(i) Generalized Holographic dark energy (GHDE): The energy density of GHDE is given by,

\[
\rho_h = 3c^2 m_p^2 H^2 f(R/H^2)
\]

(1)

where \( c \) is a numerical constant and \( f(x) \) is a positive function defined as, \( f(x) = \alpha x + (1 - \alpha) \), \( \alpha \) is a constant. When \( \alpha = 0 \) then \( f(x) = 1 \) and when \( \alpha = 1 \) then \( f(x) = x \).

(ii) Generalized Ricci dark energy (GRDE): The energy density of GRDE is given by,

\[
\rho_r = 3c^2 m_p^2 R g(H^2/R)
\]

(2)

where \( g(y) \) is a positive function defined as, \( g(y) = \beta y + (1 - \beta) \), \( \beta \) is a constant. When \( \beta = 0 \) then \( g(y) = 1 \) and when \( \beta = 1 \) then \( g(y) = y \).

It is interesting that when \( f(x) = g(y) = 1 \), we recover the energy densities of original holographic and Ricci dark energies. Also when \( f(x) = x \) and \( g(y) = y \), we recover interchangeably the energy densities of original holographic and Ricci dark energies. Also when \( \alpha = 0 \) and \( \beta = 1 \) or when \( \alpha = 1 \) and \( \beta = 0 \) then \( \rho_h \) and \( \rho_r \) both are same. Now comparing (1) and (2), we see that when \( \beta = 1 - \alpha \) the generalized Ricci dark energy reduces to the generalized holographic dark energy and vice versa.

The Ricci scalar for non-flat FRW universe is given by, \( R = -6(\dot{H}+2H^2 + \frac{\kappa}{\rho_X}) \), where \( k = +1, 0 \) and \( -1 \) for closed, flat, and open geometries, respectively. We shall consider the flat universe \( (k = 0) \) where,
\[ R = -6(\dot{H} + 2H^2) \]  

In the next section, we shall prescribe the statefinder diagnostic pair and deceleration parameter in two fluid system.

### III. STATEFINDER DIAGNOSTICS FOR TWO FLUID SYSTEMS

The Einstein field equations for the mixture of dark matter and dark energy are,

\[ H^2 = \frac{1}{3}(\rho_m + \rho_X) \]  

and

\[ \dot{H} = -\frac{1}{2}(\rho_m + \rho_X + p_m + p_X) \]

Also the conservation equation is given by

\[ \dot{\rho}_m + \dot{\rho}_X + 3H(\rho_m + \rho_X + p_m + p_X) = 0 \]

where \( p_X \) and \( p_m \) denote pressures and \( \rho_X \) and \( \rho_m \) are the energy densities of dark matter and dark energy respectively. Thus here \( p_X = p_h \), \( \rho_X = \rho_h \) for generalized holographic dark energy and \( p_X = p_r \), \( \rho_X = \rho_r \) for generalized Ricci dark energy respectively.

The flat Friedmann model which is analyzed in terms of the statefinder parameters [20]. The trajectories in the \( \{ r, s \} \) plane of different cosmological models shows different behavior. The statefinder diagnostic of SNAP observations used to discriminate between different dark energy models. The statefinder diagnostic pair is constructed from the scale factor \( a(t) \). The statefinder diagnostic pair is denoted as \( \{ r, s \} \) and defined as,

\[ r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - \frac{1}{2})} \]

where \( q \) is the deceleration parameter given by, \( q = -\frac{\ddot{a}}{a} \).

The parameters can be expressed as

\[ r = 1 + \frac{9}{2(\rho_X + \rho_m)} \left( \frac{\partial p_X}{\partial \rho_X} (\rho_X + p_X) + \frac{\partial p_m}{\partial \rho_m} (\rho_m + p_m) \right) \]

\[ s = \frac{1}{(\rho_X + p_m)} \left( \frac{\partial p_X}{\partial \rho_X} (\rho_X + p_X) + \frac{\partial p_m}{\partial \rho_m} (\rho_m + p_m) \right) \]

and

\[ q = \frac{1}{2} + \frac{3}{2} \left( \frac{\rho_X + p_m}{\rho_X + \rho_m} \right) \]
IV. GHDE AND GRDE MODELS WITHOUT DARK MATTER

We follow the work of Kim et al [21] in generalized holographic and generalized Ricci dark energy models. Here we consider that the universe is filled with only generalized holographic/Ricci dark energy. In this case $\rho_m = 0 = p_m$, so the first Friedmann equation (4) can be written as,

$$H^2 = \frac{1}{3}\rho_h$$ \hspace{1cm} (11)

Combining (1) and (3) we get the GHDE density as,

$$\rho_h = 3c^2[-6\alpha \dot{H} + (1 - 13\alpha)H^2]$$ \hspace{1cm} (12)

Combining this equation with (11) we get,

$$6\alpha \dot{H} = (1 - 13\alpha - \frac{1}{c^2})H^2$$ \hspace{1cm} (13)

Solving the above differential equation we get,

$$H^2 = H_0^2a^{\frac{1}{\alpha}(1-13\alpha-\frac{1}{c^2})}$$ \hspace{1cm} (14)

where $H_0$ is the integrating constant which is the present value of $H$. So from (11) and (14) we have,

$$\rho_h = 3H^2 = 3H_0^2a^{\frac{1}{\alpha}(1-13\alpha-\frac{1}{c^2})}$$ \hspace{1cm} (15)

Second Friedmann equation is given by (from (5)),

$$\dot{H} = -\frac{1}{2}(p_h + \rho_h)$$ \hspace{1cm} (16)

Using (13) - (15), the equation (16) becomes,

$$p_h = \frac{1}{3\alpha}(4\alpha + \frac{1}{c^2} - 1)H_0^2a^{\frac{1}{\alpha}(1-13\alpha-\frac{1}{c^2})}$$ \hspace{1cm} (17)

So the equation of state $w_h$ for generalized holographic dark energy model is defined as,

$$w_h = \frac{p_h}{\rho_h} = \frac{1}{9\alpha c^2}(4\alpha c^2 - c^2 + 1)$$ \hspace{1cm} (18)

This model generates dark energy if $w_h < -1/3$ i.e., if $\alpha < \frac{c^2 - 1}{c^2}$. The equation of state parameter $w_h$ has been drawn against $\alpha$ and $c$ in fig.1. From the figure, we have seen that $w_h$ decreases from positive to negative as $\alpha$ decreases and $c$ increases.

Now from (15) and (17) we get,

$$\frac{\partial p_h}{\partial \rho_h} = \frac{\partial p_h/\partial a}{\partial \rho_h/\partial a} = \frac{1}{9\alpha c^2}(4\alpha c^2 - c^2 + 1)$$ \hspace{1cm} (19)

So from (8), (9), (10), (18) and (19) we obtain $r$, $s$ and $q$ as

$$r = 1 + \frac{1}{18\alpha^2 c^4}(13\alpha c^2 - c^2)(4\alpha c^2 - c^2 + 1)$$ \hspace{1cm} (20)
FIG. 1: The variation of $w_h$ against $\alpha$ and $c$.

FIG. 2: The variation of $r$ against $s$.

FIG. 3: The variation of $q$ against $\alpha$ and $c$. 
\[ s = \frac{1}{9\alpha c^2} (13\alpha c^2 - c^2 + 1) \] (21)

and

\[ q = \frac{1}{6\alpha c^2} (7\alpha c^2 - c^2 + 1) \] (22)

The universe will be accelerating if \( q < 0 \) i.e., if \( \alpha < \frac{c^2 - 1}{c^2} \).

The variation of \( r \) against \( s \) has been drawn in fig.2. From the figure, we have seen that when \( r \) increases, \( s \) first decreases from positive to negative up to about \( r = 0.5 \) and then \( s \) increases from negative to positive.

The deceleration parameter \( q \) has been drawn against \( \alpha \) and \( c \) in fig.3. From the figure, we have seen that \( q \) decreases from positive to negative as \( \alpha \) decreases and \( c \) increases.

If the universe is filled with generalized Ricci dark energy rather than generalized holographic dark energy then all the above solutions are valid provided \( \alpha = 1 - \beta \). So the universe will be accelerating if \( \beta > \frac{8c^2 - 1}{7c^2} \).

\[ \text{V. GHDE AND GRDE MODELS WITH DARK MATTER : NON-INTERACTING SCENARIO} \]

Here we consider the universe is filled with the mixture of dark matter and GHDE and also there is no interaction between them. In this case the first Friedmann equation (4) can be written as,

\[ H^2 = \frac{1}{3} (\rho_h + \rho_m) \] (23)

Since there is no interaction, so dark matter and GHDE are separately conserved and hence the energy conservation equations for dark matter and dark energy are (from (6)),

\[ \dot{\rho}_m + 3H(\rho_m + p_m) = 0 \] (24)

and

\[ \dot{\rho}_h + 3H(\rho_h + p_h) = 0 \] (25)

Assume that the equation of state for dark matter is \( p_m = w_m \rho_m \). Putting it in the above equation and after solving the differential equation we get,

\[ \rho_m = \rho_{m0}a^{-3(1+w_m)} \] (26)

where \( \rho_{m0} \) is the integrating constant. Combining (12), (23) and (26) we have,

\[ 6\alpha c^2 \dot{H} + (13\alpha c^2 - c^2 + 1)H^2 = \frac{1}{3} \rho_{m0}a^{-3(1+w_m)} \] (27)

and after solving we get,

\[ H^2 = \frac{\rho_{m0}a^{-3(1+w_m)}}{3\{(4\alpha c^2 - c^2 + 1) - 9\alpha c^2 w_m\}} + H_1^2a^{-\frac{(13\alpha c^2 - c^2 + 1)}{3\alpha c^2}} \] (28)

where \( H_1 \) is the integrating constant. From (23), (25), (26) and (28) we have,
\[ \rho_h = 3H^2 - \rho_m = \left[ \frac{1}{\{(4\alpha c^2 - c^2 + 1) - 9\alpha c^2 w_m\}} - 1 \right] \rho_{m0}a^{-3(1+w_m)} + 3H_1^2a \left( \frac{13\alpha c^2 - 2c^2 + 1}{3\alpha c^2} \right) \] (29)

and

\[ p_h = \left[ \frac{1}{\{(4\alpha c^2 - c^2 + 1) - 9\alpha c^2 w_m\}} - 1 \right] w_m\rho_{m0}a^{-3(1+w_m)} + \left( \frac{4\alpha c^2 - c^2 + 1}{3\alpha c^2} \right) H_1^2a \left( \frac{13\alpha c^2 - 2c^2 + 1}{3\alpha c^2} \right) \] (30)

FIG. 4: The variation of \( w_h \) against \( a \) for \( w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2 \) for different values of \( \alpha = .1, .12, .15 \).

FIG. 5: The variation of \( w \) against \( a \) for \( w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2 \) for different values of \( \alpha = .1, .12, .15 \).

So the equation of state \( w_h \) for GHDE in this non-interacting scenario is obtained as,

\[ w_h = \frac{p_h}{\rho_h} = \left[ \frac{1}{\{(4\alpha c^2 - c^2 + 1) - 9\alpha c^2 w_m\}} - 1 \right] w_m\rho_{m0}a^{-3(1+w_m)} + \left( \frac{4\alpha c^2 - c^2 + 1}{3\alpha c^2} \right) H_1^2a \left( \frac{13\alpha c^2 - 2c^2 + 1}{3\alpha c^2} \right) \] (31)
FIG. 6: The variation of $r$ against $s$ for $w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2$ and $\alpha = .1$.

FIG. 7: The variation of $q$ against $a$ for $w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2$ and $\alpha = .1$.

Also the equation state for combined fluid is obtained as

$$w = \frac{p_m + p_h}{\rho_m + \rho_h} = \frac{w_m \rho_m a^{-3(1 + w_m)}}{(4\alpha c^2 - c^2 + 1) - 3\alpha c^2 w_m} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha c^2} \frac{H_1^2 a}{(13\alpha c^2 - c^2 + 1)} + 3H_1^2 a^{-\frac{(13\alpha c^2 - c^2 + 1)}{3\alpha c^2}}$$

(32)

The above expressions are very complicated, so the variation of $w_h$ against $a$ has been drawn in fig.4 for $w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2$ with different values of $\alpha = .1, .12, .15$. Fig.5 represents the variation of $w$ against $a$ for $w_m = .01, \rho_{m0} = 1, H_1 = 1, c = 2$ for different values of $\alpha = .1, .12, .15$. Also from (8), (9), (10), (26), (29) and (30) we get graphs of $s$ against $r$ and $q$ against $a$ in figures 6 and 7 respectively. We have seen that $s$ first increases from some positive value to $+\infty$ and after that $s$ also increases from $-\infty$ to some positive value as $r$ increases from negative to positive. We have seen that the EOS for GHDE $w_h$ keeps negative sign as evolution of the universe. When we have considered GHDE with dark matter without interaction, the EOS for combined fluid generates the negative sign. The deceleration parameter also gives us the negative sign, so the non-interacting model also generates dark energy. When $\alpha$ is replaced by $(1 - \beta)$, we get the similar results for GRDE in with dark matter.
VI. GHDE AND GRDE MODELS WITH DARK MATTER: INTERACTING SCENARIO

Here we consider the universe is filled with the mixture of dark matter and GHDE and also there is an interaction between them. So dark matter and GHDE are not separately conserved and let the interaction term is defined by $3\delta H \rho_m$, where $\delta$ is the interaction parameter. From (6), we get the relations

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -3\delta H \rho_m$$

and

$$\dot{\rho}_h + 3H(\rho_h + p_h) = 3\delta H \rho_m$$

Assume that the equation of state for dark matter is $p_m = w_m \rho_m$. Putting it in the above equation and after solving the differential equation we get,

$$\rho_m = \rho_{m1} a^{-3(1+w_m+\delta)}$$

where $\rho_{m1}$ is the integrating constant. Combining (12), (23) and (35) we have,

$$6\alpha c^2 \dot{H} + (13\alpha^2 - c^2 + 1)H^2 = \frac{1}{3} \rho_{m1} a^{-3(1+w_m+\delta)}$$

and after solving we get,

$$H^2 = \frac{\rho_{m1} a^{-3(1+w_m+\delta)}}{3\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\} + H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}}$$

where $H_2$ is the integrating constant. From (23), (34), (35) and (37) we have,

$$\rho_h = \left[\frac{1}{\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\}} - 1\right] \rho_{m1} a^{-3(1+w_m+\delta)} + 3H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}$$

and

$$p_h = \left[\frac{w_m + \delta}{\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\}} - w_m\right] \rho_{m1} a^{-3(1+w_m+\delta)} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha c^2} H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}$$

So the equation of state for GHDE $w_h$ is obtained as,

$$w_h = \frac{\rho_{m1} a^{-3(1+w_m+\delta)} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha c^2} H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}}{\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\} - 1} \rho_{m1} a^{-3(1+w_m+\delta)} + 3H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}$$

Also the equation state for combined fluid is obtained as,

$$w = \frac{\rho_m + p_h}{\rho_m + p_h} = \frac{\frac{(w_m+\delta)\rho_{m1} a^{-3(1+w_m+\delta)}}{\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\}} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha c^2} H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}}{\frac{\rho_{m1} a^{-3(1+w_m+\delta)}}{\{4\alpha c^2 - c^2 + 1 - 9\alpha^2(w_m + \delta)\}} + 3H_2^2 a^{-\frac{(13\alpha^2-c^2+1)}{3\alpha c^2}}}$$

The above expressions are very completed, so the equation of state parameters $w_h$ and $w$ against $a$ have been drawn in figure 8 and 9 respectively for $w_m = .01, \rho_{m1} = 1, H_2 = 1, c = 2, \delta = .01$ with different values of $\alpha = .1, .12, .15$. Also from (8), (9), (10), (35), (38) and (39) we draw the graphs of $s$ against $t$ and $q$ against $a$ in figures 10 and 11 respectively for $w_m = .01, \rho_{m1} = 1, H_2 = 1, c = 2, \delta = .01, \alpha = .1$. We have seen that $s$ always decreases with positive sign as $r$ increases with negative level. We have seen that the EOS for GHDE $w_h$ keeps negative sign as evolution of the universe. When we have considered interacting GHDE with dark matter, the EOS for combined fluid generates the negative sign. The deceleration parameter also gives us the negative sign, so the interacting model also generates dark energy. When $\alpha$ is replaced by $(1 - \beta)$, we get the similar results for GRDE in interaction with dark matter.
VII. GHDE AND GRDE MODELS WITH GENERALIZED CHAPLYGIN GAS: NON-INTERACTING SCENARIO

Let the universe is filled with the mixture of generalized Chaplygin Gas and GHDE. This generalized Chaplygin Gas is considered a perfect fluid which follows the adiabatic equation of state. The equation of Generalized Chaplygin Gas [22] is given by,

\[ p_c = -\frac{B}{\rho_c^\gamma} \quad 0 \leq \gamma \leq 1, \quad B > 0. \]  

In this case we consider the universe is filled with the mixture of Chaplygin Gas and GHDE and also there is no interaction between them.

In this case the first Friedmann equation can be written as,

\[ H^2 = \frac{1}{3}(\rho_h + \rho_c) \]  

Since there is no interaction, so Chaplygin Gas and GHDE are separately conserved and hence the energy conservation equations for Chaplygin Gas and dark energy are,
FIG. 10: The variation of \( s \) against \( r \) for \( w_m = 0.01, \rho_{m1} = 1, H_2 = 1, c = 2, \delta = 0.01, \alpha = 0.1 \).

FIG. 11: The variation of \( q \) against \( a \) for \( w_m = 0.01, \rho_{m1} = 1, H_2 = 1, c = 2, \delta = 0.01, \alpha = 0.1 \).

\[
\dot{\rho}_c + 3H(\rho_c + p_c) = 0 \quad (44)
\]

and

\[
\dot{\rho}_h + 3H(\rho_h + p_h) = 0 \quad (45)
\]

from which we get,

\[
\rho_c = [B + \rho_{c0}a^{-3(1+\gamma)}]^{\frac{1}{1+\gamma}} \quad (46)
\]

where \( \rho_{c0} \) is the integrating constant. Hence we get,

\[
p_c = -[B + \rho_{c0}a^{-3(1+\gamma)}]^{\frac{1}{1+\gamma}} + \rho_{c0}a^{-3(1+\gamma)}[B + \rho_{c0}a^{-3(1+\gamma)}]^{\frac{1}{1+\gamma}} \quad (47)
\]

Putting the value of \( p_c \) and \( \rho_c \) we get,

\[
6\alpha c^2 \dot{H} + (13\alpha c^2 - c^2 + 1)H^2 = \frac{1}{3}[B + \rho_{c0}a^{-3(1+\gamma)}]^{\frac{3}{1+\gamma}} \quad (48)
\]
and after solving we get,

\[ H^2 = \frac{1}{9\alpha c^2} e^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} \int \left[ B + \rho_c e^{-3(1+\gamma)x} \right] \frac{1}{1+\gamma} e^{\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} dx + H_{c0}^2 a^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} \]  

(49)

where \( x = \ln a \) and \( H_{c0} \) is the integrating constant. Hence we have,

\[ \rho_h = \frac{1}{3\alpha c^2} e^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} I(x) - \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} e^{\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} + 3H_{c0}^2 a^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} \]  

(50)

where,

\[ I(x) = \int \left[ B + \rho_c e^{-3(1+\gamma)x} \right] \frac{1}{1+\gamma} e^{\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} dx \]  

(51)

Now, from energy conservation equation we get,

\[ p_h = -\rho_h - \frac{1}{3} \frac{\partial \rho_h}{\partial x} \]  

(52)

which gives,

\[ p_h = \frac{(4\alpha c^2 - c^2 + 1)}{27\alpha^2 c^4} e^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} \int \left[ B + \rho_c e^{-3(1+\gamma)x} \right] \frac{1}{1+\gamma} e^{\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} dx \]

\[ - \frac{1}{9\alpha c^2} \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} + B \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha^2} H_{c0}^2 a^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} \]  

(53)

So the equation of state for GHDE \( w_h \) is obtained as,

\[ w_h = \frac{p_h}{\rho_h} = \frac{(4\alpha c^2 - c^2 + 1)}{27\alpha^2 c^4} y(x) I(x) - \frac{1}{3\alpha^2 c^2} \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} + B \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} + \frac{(4\alpha c^2 - c^2 + 1)}{3\alpha^2} H_{c0}^2 y(x) \]

\[ \frac{1}{3\alpha^2} y(x) I(x) - \left[ B + \rho_c a^{-3(1+\gamma)} \right] \frac{1}{1+\gamma} + 3H_{c0}^2 y(x) \]  

(54)

where, \( y(x) = e^{-\frac{(13\alpha^2 - 2c^2 + 1)}{3\alpha^2}} x \).

Also the equation state for combined fluid is obtained as,
FIG. 13: The variation of $w$ against $a$ for $\rho_c = 1, B = 1, \gamma = .1, H_{c0} = 1, c = 2, \alpha = .1$.

FIG. 14: The variation of $s$ against $r$ for $\rho_c = 1, H_{c0} = 1, c = 2, \alpha = .1, B = 1, \gamma = .1$.

FIG. 15: The variation of $q$ against $a$ for $\rho_c = 1, H_{c0} = 1, c = 2, \alpha = .1, B = 1, \gamma = .1$. 
\[ w = \frac{p_c + p_h}{\rho_c + \rho_h} = \frac{p_c + \left(\frac{4\alpha c^2 - c^2 + 1}{2\gamma\alpha c^2}\right) y(x) I(x) - \frac{1}{9\alpha c^2} \left[ B + \rho_{c0}a^{-3(1+\gamma)} \right] \left( \frac{3}{3\alpha c^2} \right)}{\rho_c + \frac{1}{3\alpha c^2} y(x) I(x) - \left[ B + \rho_{c0}a^{-3(1+\gamma)} \right] \left( \frac{3}{3\alpha c^2} \right) H_{c0}^2 y(x)} H_c^2 \]  

The above expressions are very completed, so the equation of state parameters \( w_h \) and \( w \) against \( a \) have been drawn in figure 12 and 13 respectively for \( \rho_{c0} = 1, H_{c0} = 1, c = 2, \alpha = .1, B = 1, \gamma = .1 \). Also from (8), (9), (10), (46), (50) and (53) we draw the graphs of \( s \) against \( r \) and \( q \) against \( a \) in figures 14 and 15 respectively for \( \rho_{c0} = 1, H_{c0} = 1, c = 2, \alpha = .1, B = 1, \gamma = .1 \). We have seen that \( s \) first increases and then decreases as \( r \) increases and we see that \( s \) is always positive but \( r \) keeps negative sign. We have seen that the EOS for GHDE \( w_h \) keeps negative sign as evolution of the universe. When we have considered GHDE and GCG without interaction, the EOS for combined fluid generates the negative sign. The deceleration parameter also gives us the negative sign, so the non-interacting model also generates dark energy. When \( (1 - \alpha) \) is replaced by \( \beta \), we get the similar results for GRDE and GCG without interaction.

### VIII. GHDE AND GRDE MODELS WITH GCG: INTERACTING SCENARIO

Here we consider the universe is filled with the mixture of GCG and GHDE and also there is an interaction between them. So GCG and GHDE are not separately conserved and let the interaction term is defined by \( 3\delta H\rho_c \), where \( \delta \) is the interaction parameter. So we get the relations,

\[ \dot{\rho}_c + 3H(\rho_c + p_c) = -3\delta H\rho_c \]  
(56)

and

\[ \dot{\rho}_h + 3H(\rho_h + p_h) = 3\delta H\rho_c \]  
(57)

Assume that the equation of state for GCG is \( p_c = -B/\rho_c^\gamma \). Putting it in the above equation and after solving the differential equation we get,

\[ \rho_c = \left[ \frac{B}{1 + \delta} + \rho_{c1}a^{-3(1+\gamma)(1+\delta)} \right] \frac{1}{H_c^2} \text{ and } p_c = -B \left[ \frac{B}{1 + \delta} + \rho_{c1}a^{-3(1+\gamma)(1+\delta)} \right] \frac{1}{H_c^2} \]  
(58)

where \( \rho_{c1} \) is the integrating constant. So Thus we get,

\[ 6\alpha c^2 \dot{H} + (13\alpha c^2 - c^2 + 1)H^2 = \frac{1}{3} B \left[ \frac{1 + \delta}{1 + \delta} + \rho_{c1}a^{-3(1+\gamma)(1+\delta)} \right] \frac{1}{H_c^2} \]  
(59)

and after solving we get,

\[ H^2 = \frac{1}{9\alpha c^2} e^{(\frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2})x} \int \left[ \frac{B}{1 + \delta} + \rho_{c1}e^{-3(1+\gamma)(1+\delta)x} \right] \frac{1}{H_c^2} e^{(\frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2})x} dx + H_{c1}^2 a \left( \frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2} \right) \]  
(60)

where \( x = lna \) and \( H_{c1} \) is the integrating constant. Hence we have,

\[ \rho_h = \frac{1}{3\alpha c^2} e^{(\frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2})x} J(x) - \frac{B}{1 + \delta} \rho_{c1}a^{-3(1+\gamma)(1+\delta)} \frac{1}{H_c^2} + 3H_{c1}^2 a \left( \frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2} \right) \]  
(61)

where,

\[ J(x) = \int \left[ \frac{B}{1 + \delta} + \rho_{c1}e^{-3(1+\gamma)(1+\delta)x} \right] \frac{1}{H_c^2} e^{(\frac{13\alpha c^2 - c^2 + 1}{3\alpha c^2})x} dx \]  
(62)
ates dark energy. When \( \alpha \) the negative sign. The deceleration parameter also gives us the negative sign, so the interacting model also gener-
the universe. When we have considered GHDE and GCG with interactio
increases and keeps positive signs. We have seen that the EOS for GHDE
\( H \)
s\( (46), (50) \) and \( (53) \), we also draw the graphs of
\( \gamma = 0.1 \) for \( H_{c1} = 1, c = 2, \delta = 0.01, B = 1, \alpha = 0.1 \).

Now, from energy conservation equation we get,
\[
p_h = \delta \rho_c - \rho_h - \frac{1}{3} \frac{\partial \rho_h}{\partial x}
\]
which gives,
\[
p_h = \left( \frac{4 \alpha c^2 - c^2 + 1}{27 \alpha^2 c^4} \right) y(x) J(x) + \left( \delta - \frac{1}{9 \alpha c^2} \right) \left[ \frac{B}{1 + \delta} \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right]^{1+\gamma}
\]
\[
+ \left[ \frac{B}{1 + \delta} - \delta \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right] \left[ \frac{B}{1 + \delta} + \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right]^{1+\gamma} + \left( \frac{4 \alpha c^2 - c^2 + 1}{3 \alpha c^2} \right) H_{c1}^2 y(x)
\]
So the equation of state for GHDE \( w_h \) is obtained as,
\[
w_h = \frac{p_h}{\rho_h} = \left( \frac{4 \alpha c^2 - c^2 + 1}{27 \alpha^2 c^4} \right) y(x) J(x) \left[ H_{c1}^2 + (\delta - \frac{1}{9 \alpha c^2}) + \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right]^{1+\gamma}
\]
\[
+ \left[ \frac{B}{1 + \delta} - \delta \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right] \left[ \frac{B}{1 + \delta} + \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right]^{1+\gamma} + \left( \frac{4 \alpha c^2 - c^2 + 1}{3 \alpha c^2} \right) H_{c1}^2 y(x)
\]
where, \( y(x) \approx e^{\frac{1}{3} \frac{13 \alpha c^2 - c^2 + 1}{3 \alpha c^2} x} \).

Also the equation state for combined fluid is obtained as,
\[
w = \frac{p_c + p_h}{\rho_c + \rho_h} = \left( \frac{4 \alpha c^2 - c^2 + 1}{3 \alpha c^2 c^2} \right) y(x) \left( \frac{J(x)}{3 \alpha c^2} + H_{c1}^2 \right) - \frac{1}{9 \alpha c^2} \left[ \frac{B}{1 + \delta} + \rho_{c1} a^{-3(1+\gamma)(1+\delta)} \right]^{1+\gamma}
\]
\[
\frac{\left( J(x) \right)^2 + 3 H_{c1}^2}{y(x)}
\]
The above expressions are very completed, so the equation of state parameters \( w_h \) and \( w \) against \( a \) have been drawn in figure 16 and 17 respectively for \( H_{c1} = 1, c = 2, \gamma = 0.1, B = 1, \alpha = 0.01, \delta = 0.1 \). From (8), (9), (10), (46), (50) and (53), we also draw the graphs of \( s \) against \( r \) and \( q \) against \( a \) in figures 18 and 19 respectively for \( H_{c1} = 1, c = 2, \gamma = 0.1, B = 1, \alpha = 0.01, \delta = 0.1 \). We have seen that \( s \) always decreases from negative level as \( r \) increases and keeps positive signs. We have seen that the EOS for GHDE \( w_h \) keeps negative sign as evolution of the universe. When we have considered GHDE and GCG with interaction, the EOS for combined fluid generates the negative sign. The deceleration parameter also gives us the negative sign, so the interacting model also generates dark energy. When \( \alpha \) is replaced by \( (1 - \beta) \), we get the similar results for GRDE and CGC with interaction.
FIG. 17: The variation of $w$ against $a$ for $H_{c1} = 1, c = 2, \gamma = .1, B = 1, \alpha = .1, \delta = .01$.

FIG. 18: The variation of $s$ against $r$ for $H_{c1} = 1, c = 2, \gamma = .1, B = 1, \alpha = .1, \delta = .01$.

FIG. 19: The variation of $q$ against $a$ for $H_{c1} = 1, c = 2, \gamma = .1, B = 1, \alpha = .1, \delta = .01$. 
IX. DISCUSSIONS

In this work, we have considered the generalized holographic and generalized Ricci dark energy models for acceleration of the universe. If the universe filled with only GHDE and GRDE the corresponding deceleration parameter, EOS parameter and statefinder parameters have been calculated. The universe will be accelerating if the parameter $\alpha$ satisfies $\frac{\alpha}{c^2 - 1} < 0$ for GHDE model. If the universe is filled with GRDE rather than GHDE then all the above solutions are valid provided $\alpha = 1 - \beta$. So the universe will be accelerating if $\beta > \frac{8c^2 - 1}{c^2 - 1}$. If the universe is filled with the mixture of dark matter and GHDE/GRDE, the equation of state parameters and deceleration obey the negative sign in both interacting and non-interacting scenarios, which derive the acceleration of the universe. When we consider the mixture of generalized Chaplygin gas and GHDE/GRDE, the equation of state parameters and the deceleration parameter also generate negative sign which shows that the combined fluids derive the acceleration of the universe. In all the cases, we have verified the results in drawing the figures of $w_h$, $w$ and $q$ by choosing (in particular) the values $w_m = 0.01, c = 2, \alpha = 0.1$ and $\delta = 0.01$. The statefinder parameters have different natures in our interacting and non-interacting cases. If the universe filled with only GHDE/GRDE, then from the figure 2, we have seen that when $r$ increases, $s$ first decreases from positive to negative up to about $r = 0.5$ and then $s$ increases from negative to positive. For non-interacting models of GHDE, we have seen from figure 6 that, $s$ first increases from some positive value to $+\infty$ and after that $s$ also increases from $-\infty$ to some positive value as $r$ increases from negative to positive and from figure 13, we have also seen that $s$ first increases and then decreases as $r$ increases and we see that $s$ is always positive but $r$ keeps negative sign. But for interacting models of GHDE, we have seen from figure 10 that, $s$ always decreases with positive sign as $r$ increases with negative level and from figure 18 that, $s$ always decreases from negative level as $r$ increases and keeps positive signs.

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