Experimental Divertor
Similarity Database Parameters

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December 1995

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Abstract

A set of experimentally-determined dimensionless parameters is proposed for characterizing the regime of divertor operation. The objective is to be able to compare as unambiguously as possible the operation of different divertors and to understand what physical similarities and differences they represent. Examples from Alcator C-Mod are given.

Background

Recent theory has shown that exactly scaled similarity experiments on tokamak divertors, where atomic physics is important in addition to plasma physics processes, are not feasible because there are no free parameters. However, similarity comparisons can be obtained in principle if either some of the plasma parameters [1], or some of the geometric parameters [2], or both, are unimportant. Moreover, such comparisons can be made practical if only approximate similarity is required. Approaches to obtaining similarity with ITER divertor regimes have been explored under various assumptions about divertor transport scaling.

There are now several moderately similar divertor tokamaks, with increasing quantities of divertor data, that span a range of dimensionless regimes. In addition to the theoretical approaches to identifying similarity scaling, it seems most natural now to examine the experimental results to discover what the values of relevant dimensionless parameters actually are, and therefore to try to compare from experiment to experiment so as to discover the underlying physics that is similar or different in them. To do this requires a set of dimensionless parameters to be indentified and, as far as possible, measured on each experiment. With these stored in appropriate databases, it ought to be possible quickly to identify dimensionless regimes. Alternatively, the parameters can be evaluated from a less processed database of raw data once the form of the parameters are agreed, provided that the database contains the relevant raw data.

The purpose of this note is to propose parameters that determine the regimes of the relevant physics, and are experimentally measurable. Since judgement must be exercised in the choice of the exact parameters to include in the set, these choices should not be regarded either as definitive or as exhaustive. However, it is hoped that the set will form the basis for discussion, initial comparisons, and further refinement.
The Formal Parameters

As has been discussed elsewhere [1,2] the dimensionless parameters deriving from the intensive variables (i.e. density, temperature, magnetic field, etc.) of the plasma can be considered to be

- $\rho^*$: The ratio of Larmor radius to characteristic size.
- $\nu^*$: The collisionality – ratio of size to mean-free-path.
- $\lambda_D^*$: The ratio of Debye length to size.
- $\beta$: The ratio of plasma kinetic to magnetic pressure.
- $T^*$: The ratio of plasma temperature to characteristic atomic energy.
- $f_j$: The ion (or neutral) density of a species, $j$, as a fraction of electron density.

Of these, it is commonly supposed that $\lambda_D^*$ is unimportant. The parameters $f_j$ are undoubtedly important but often not explicitly considered.

In addition to these parameters there is a host of other dimensionless parameters that can be considered to be geometric. These include

- Relative shapes of the plasma and divertor cross-section, including such quantities as divertor flux expansion factors and relative depth.
- Ratios of poloidal to toroidal field, and hence field-line angle, safety factor, and so on.

Moreover, geometric considerations are essential for the intensive parameters too, because their values depend upon the choice of their exact definition and where they are measured. Therefore it is important to be explicit about how the intensive parameters are to be defined.

The parameters listed above are sufficient to define the similarity of equivalent systems. If two systems are found with all these parameters the same, then all relevant parameters are equal for the two systems. However, in comparing systems which are not exactly similar, there are other dimensionless parameters that describe important processes in the divertor. It may be of interest to document their values explicitly even if they are in principle determined by the parameters above. We here propose one such vital parameter: the ratio of the charge-exchange mean-free-path to the SOL thickness.

The following proposals for specific choices of parameters' definitions are based on judgement about what is important and what it is generally measured. They are therefore open to refinement. However, it is hoped that some degree of agreement on parameter definitions can be obtained along these lines.
The Specific Parameters

There are, roughly speaking, three poloidal positions in the SOL that may be considered characteristic. They are, (1) the main chamber, (2) the x-point, (3) the target. By the "main chamber", we mean the part of the SOL that is adjacent to the core plasma and sufficiently far from the x-point as not to be significantly influenced by it. The x-point is the transition between regions of the SOL adjacent to the main plasma and to the private flux region. This point is representative of the upstream divertor. The target is where the field lines intersect the solid surface.

It is proposed that intensive dimensionless parameters be defined at each of the three positions. Generally, in present experiments there are measurements at the target and in the main chamber but perhaps not at the x-point, although efforts are currently devoted to obtaining this data.

The radial scale-length with which plasma characteristic lengths are to be compared is the width of the scrape-off layer, $\Delta$. It is proposed that this be defined as half the pressure e-folding length. This e-folding length is the distance from the separatrix to where the plasma electron pressure $(nT)$ is $\exp(-1)$ of its value at the separatrix, or, equivalently for an exponential profile, the ratio of the radial integral of the pressure to its separatrix value. The practical advantage of using the pressure is that, except for detached divertors, the pressure is approximately constant along the flux surface so the $\Delta$ can be evaluated either from main-chamber or from target measurements. (A potential disadvantage is that we need both $T$ and $n$ to be measured.) Half the exponential width is used so as to be typical of the near-separatrix region. Note, of course, that $\Delta$ varies with poloidal position.

The flux surface at distance $\Delta$ is proposed also as the radial position at which the plasma parameters such as $T_e$ and $n_e$ are to be evaluated. Their values there will be slightly smaller (by about 20%) than the separatrix values in the main chamber, which are most often quoted. However, the advantage of using the values at $\Delta$ is that they are more representative of the whole SOL. It is found that the divertor target values exactly on the separatrix are very often strongly influenced by the private flux region and by neutrals, rendering them even less typical of the divertor fan as a whole.

For the gyroscale parameter, $\rho^*$, it is proposed that this be defined for singly charged ions as

$$\rho^* \equiv \frac{(m_i T_e)^{1/2}}{eB_i} / \Delta.$$  

This uses the ion Larmor radius at the sound speed, ignoring the ion temperature. Corrections for finite $T_i$ can be incorporated as an auxiliary parameter $T_i/T_e$. It also uses the toroidal field whose variation as a function of position is weak, so that the $\rho^*$ variation from place to place is caused primarily by temperature differences. Physically, the poloidal Larmor radius may be key. It then requires the extra geometric parameter of the field-line
angle, $B_p/B_t$. The ratio of the local poloidal Larmor radius to the local scrape-off length is almost independent of poloidal position (because each has a $B_t/B_p$ factor and they cancel) and is equal to $\rho^*B_t/B_p$ using midplane values.

The Coulomb collisionality of greatest interest is that for parallel motions. It is proposed that the parallel connection length, $L$, for the magnetic field be documented for two regions: the main chamber and the divertor. We denote by $L_m$ the complete connection length from target to target and by $L_x$ the connection length from x-point to target. (The inner and outer divertor connection lengths may be different in single-null configurations, and there are two different total connection lengths for double-null configurations.) The connection length has a logarithmic singularity at the separatrix arising from the x-point region. Connection lengths are therefore measured on the flux surface that is a distance $\Delta$ outside the separatrix, where the plasma parameters are measured. It is proposed that the mean-free-path for Coulomb collisions be taken as that for electron-ion momentum transfer, sometimes called “slowing down”. Using the plasma formulary [3] expression for thermal electrons, $\nu_e = 2.9 \times 10^{-6}(n/cm^{-3})\ln \Lambda(T_e/eV)^{-3/2}$ and the characteristic speed, $(T_e/m_e)^{1/2}$ the electron mean-free-path is

$$\ell_e = 1.45 \times 10^{-4} \frac{(T_e/eV)^2}{(n/10^{20}m^{-3})} \text{ m},$$

taking the coulomb logarithm, $\ln \Lambda$, to be 10. We then define

$$\nu^* = L/\ell_e,$$

for the different connection lengths.

It should be noted that the ion-ion mean-free-path is essentially the same as the electron, at the same temperature. The ion-electron mean-free-path is $(m_i/m_e)^{1/2}$ longer. It should also be noted that the mean free path for epithermal electrons ($v \approx 3.7\nu_e$) responsible for heat transfer is much longer than the thermal value, by a factor approximately 17 accounting for both electron and ion collisions and using the fast electron expressions. This latter collisionality determines whether the SOL is in the sheath- or conduction-limited regimes.

For the dimensionless temperature, relative to characteristic atomic physics energy scale, there is no obvious atomic energy that ought to be preferred; therefore the temperature in some convenient units is the natural choice of definition. Electron volts seems the natural choice. More importantly, the electron and ion temperatures may well be different. For the atomic processes the electron temperature is undoubtedly more important. The ion temperature is not completely insignificant, especially for such processes as charge exchange, but its importance as a dimensionless parameter relative to atomic energies is much less.
The key geometric parameters are associated with the plasma shape and the field line angle. From the point of view of the scrape-off layer, the significant shape parameters are presumably (1) the ratio of the connection length in the divertor to the connection length in the main chamber and (2) the ratio of the flux surface spacing in the divertor to the spacing in the main chamber (i.e. the flux expansion factor), which we will choose to define at a distance $\Delta$ outside the separatrix so as to be well defined at the x-point poloidal position. These together also determine the ratio of the divertor poloidal depth to the plasma radius. Moreover, the flux expansion factor is equal to the ratio of the field line angles in the divertor and main chamber. However, one value of field line angle is still required to define the field geometry, it can be taken simply as $B_p/B_t$ at the midplane.

A parameter that may also be important for the details of recycling and related processes at the target is the angle in the poloidal plane of intersection between the separatrix and the divertor target. We will denote this $\alpha_p$. A “horizontal” target plate generally has $\alpha_p < \pi/2$, while a “vertical” plate has $\alpha_p > \pi/2$. The total three-dimensional angle of intersection of the field-line and the target ($\alpha_t$ say) is then given by $\sin \alpha_t = \sin \alpha_p / (1 + B_t^2/B_p^2)^{1/2}$, evaluated at the target.

An auxiliary parameter of great importance is the collisionality of neutral flow across the divertor. For symmetric charge-exchange of hydrogen, the reaction rate is $<\sigma v> \approx 10^{-14} T_i^{0.33} \text{m}^{-3} \text{s}^{-1}$ for temperatures of interest. Therefore, the mean-free-path can be taken as $\ell_{cx} = (2\mathcal{E}/m)^{1/2}/n<\sigma v> \text{m}$, where $\mathcal{E}$ is the neutral energy. We can consider specified energy (e.g. cold or Franck-Condon) neutrals, or else hot thermal neutrals with $\mathcal{E} = T_i/2$. We define representative cases as the cold expression for 3 eV atoms:

$$\ell_{cx,\text{cold}} \approx 2.4 \times 10^{-2}[(T_i/eV)/\mu]^{-0.33}(n_i/10^{20} \text{m}^{-3})^{-1} \text{ m}$$

and the hot expression for thermalized atoms:

$$\ell_{cx,\text{hot}} \approx 1.4 \times 10^{-2}[(T_i/eV)/\mu]^{0.17}(n_i/10^{20} \text{m}^{-3})^{-1} \text{ m}.$$ 

Electron excitation and ionization processes are also of great importance. However, the plethora of different rates makes the task of defining explicit excitation lengths overwhelming. In order of magnitude, the ionization rate above about 10 eV temperature is about a factor of two less than the charge-exchange rate. So that the ionization length is similar to the charge-exchange length. However, as the electron temperature falls below 10 eV, ionization rapidly becomes negligible.

We will now present expressions for the dimensional and dimensionless parameters in table form. We use slightly abbreviated notation for the dimensional parameters as noted in Table 1. Also $\mu$ denotes the bulk ion mass in units of the proton mass.
Table 1. Dimensional Parameters.

| Parameter | Units       | Definition                                      |
|-----------|-------------|-------------------------------------------------|
| $n_{20}$  | $10^{20} m^{-3}$ | Electron density at Δ-surface                   |
| $T$       | eV          | Electron temperature at Δ-surface               |
| $B_t$     | T           | Toroidal magnetic field                         |
| $L$       | m           | Parallel connection length                      |
| $\Delta$  | m           | Half radial midplane distance to 1/e pressure point |

Qualified by position subscripts: $m$ main chamber, $x$ x-point, and $t$ target.

Table 2. Dimensionless parameters in practical units (eV, m, T).

| Parameter | Formula | Description                                      |
|-----------|---------|-------------------------------------------------|
| $L_x / L_m$ |         | Relative parallel length of divertor.            |
| $B_{pt} / B_{pm}$ |   | Flux expansion factor at target.                |
| $B_{px} / B_{pm}$ |   | Flux expansion factor at x-point.               |
| $B_{pm} / B_{tm}$ |   | Field line angle in main chamber (midplane).    |
| $\alpha_p$  |         | Poloidal angle of intersection with target.     |
| $\rho^*$    | $1.02 \times 10^{-4} \left( \frac{\mu T}{B_t \Delta} \right)^{1/2}$ | Inverse of gyrowidth of SOL.                   |
| $\nu^*$     | $6.9 \times 10^3 \frac{L_{px}^{1/2}}{T_{px}}$ | Parallel Coulomb Collisionality.               |
| $\beta$     | $8.0 \times 10^{-5} \frac{n_{20} T}{B_t^2}$ | SOL toroidal beta.                             |
| $T$         |         | Temperature                                      |
| $T_i / T_e$  |         | Relative ion temperature.                        |
| $f_j$       | $n_j / n_e$ | Relative species density.                       |
| $\Delta / \ell_{cx,\text{cold}}$ | $42 \Delta n_{20} (T_i / \mu)^{0.33}$ | Transverse cold neutral collisionality.        |
| $\Delta / \ell_{cx,\text{hot}}$ | $71 \Delta n_{20} (T_i / \mu)^{-0.17}$ | Transverse thermalized neutral collisionality. |

Non-geometric quantities qualified by position subscripts: $m$ main chamber, $x$ x-point, and $t$ target.

A spreadsheet [4] has been set up whereby the dimensionless parameters are calculated once the dimensional parameters of Table 1 and additional independent parameters (mostly geometric) of Table 2 are input. An illustrative example of this spreadsheet is given in Table 3.
Alcator C-Mod Examples

The parameters defined above have been included in the Alcator C-Mod edge database and are calculated automatically, based primarily on probe data (embedded and fast-scanning probes). We illustrate the range of parameters on C-Mod by several figures based on operation during March through April 1995. Fig 1 shows a plot of poloidal field against separatrix density; the current grouping shows that the field depends primarily on plasma current, as expected. (This data is all at toroidal field of 5.3T.) Fig 2 plots the dimensionless parameters $\rho^*$ versus $\nu^*$. The value of $\nu^*$ ranges from about 10 to over 400, corresponding to the observed range of regimes from sheath-limited to conduction-limited. Fig 3 plots $\beta^*$ versus $\Delta$. The betas are low; the widths a few millimeters. All these data are main-chamber parameters (denoted ‘mid’ in the plots).

In Fig 4 we show the total connection length (target to target) as a function of the ratio of $\Delta$ to the poloidal Larmor radius (i.e. $(B_p/B_t)l/p^*$). Connection length varies primarily with current. The SOL width is typically only a few times the poloidal gyrolength, thus emphasizing the importance of cross-field drifts, as has been experimentally observed. Fig 5 illustrates the values of the ‘cold’ neutral collisionality at the outer divertor (x-axis) and the main chamber (y-axis). The neutral collisionality is generally less than about 0.4 in the main chamber, implying that a significant fraction of neutrals can cross the SOL. At the divertor, however, the collisionality is much higher (particularly at high densities) and the divertor leg is therefore ‘thick’ to neutrals.

These illustrative examples document the parameter ranges spanned by this set of experiments. It is intended that inter-machine comparisons be carried out using the definitions described here.

References

[1] K. Lackner, Comments on Plasma Physics and Controlled Fusion, 15, 359 (1994).
[2] I.H. Hutchinson and G.C. Vlases, to be published in Nuclear Fusion.
[3] D.L. Book, Plasma Formulary, NRL publication 0084-4040 (1986).
[4] This can be obtained in by anonymous FTP from //pfc.mit.edu/hutch/divsim.zip

Figure Captions

Figure 1 Plot of similarity database parameters (SIM$_c$) poloidal field versus separatrix density (at midplane).
Figure 2 Plot of $\rho^*$ versus $\nu^*$ (at midplane).
Figure 3 Plot of $\beta^*$ versus $\Delta$.
Figure 4 Plot of total connection length versus $\Delta/\rho_p$ (at midplane).
Figure 5 Plot of cold neutral transverse collisionality ($\Delta/l_{c\perp,cold}$) at the midplane versus at the outer divertor.
Table 3. Spreadsheet for Calculation of Experimental Divertor Similarity Parameters

| Parameter         | Units | Value      |
|-------------------|-------|------------|
| B_t              | T     | 4.00E+00   |
| L_m              | m     | 2.20E+01   |
| L_x/L_m          |       | 1.60E-01   |
| B_pt/B_pm        |       | 3.70E-01   |
| B_px/B_pm        |       | 1.40E-01   |
| B_pm/B_tm        |       | 1.70E-01   |
| alpha_p          |       | 5.00E-01   |
| m_i/m_proton     |       | 2.00E+00   |

**Main chamber**

| n_{20}          | 10^{-20} | 8.40E-01 |
| T               | eV       | 6.00E+01 |
| T_i/T_e         |         | 1.00E+00 |
| Delta_x         | m       | 1.14E-02 |
| rho*            |         | 2.03E-02 |
| nu*             |         | 2.13E+01 |
| beta            |         | 2.80E-04 |
| D/l_cold        |         | 2.27E+00 |
| D/l_hot         |         | 6.07E-01 |
| D/rho_p         |         | 8.35E+00 |

**X-point**

| n_{20}          | 10^{-20} | 1.40E+00 |
| T               | eV       | 4.00E+01 |
| T_i/T_e         |         | 1.00E+00 |
| Delta_x         | m       | 1.14E-02 |
| rho*            |         | 2.03E-02 |
| nu*             |         | 2.13E+01 |
| beta            |         | 2.80E-04 |
| D/l_cold        |         | 2.27E+00 |
| D/l_hot         |         | 6.07E-01 |
| D/rho_p         |         | 8.35E+00 |

**Target**

| n_{20}          | 10^{-20} | 2.44E+00 |
| T               | eV       | 5.20E+00 |
| T_i/T_e         |         | 1.00E+00 |
| Delta_t         | m       | 4.32E-03 |
| rho*            |         | 1.94E-02 |
| nu*             |         | 2.19E+03 |
| beta            |         | 6.34E-05 |
| D/l_cold        |         | 7.64E-01 |
| D/l_hot         |         | 5.66E-01 |
| D/rho_p         |         | 8.77E+00 |
| alpha_t         |         | 3.01E-02 |

Denotes input value *
Figure 1. Plot of similarity database parameters (SIM.) poloidal field versus separatrix density (at midplane).
Figure 2: Plot of \( \rho \) versus \( \lambda \) at the midplane.
Figure 3. Plot of $\beta^*$ versus $\Delta$. 
Figure 4. Plot of total connection length versus $\Delta/\rho_p$ (at midplane).
Figure 5. Plot of cold neutral transverse collisionality (Δ/Δt) at the midplane versus at the outer divertor.