A REVISED LIMIT OF THE LORENTZ FACTORS OF GAMMA-RAY BURSTS WITH TWO EMITTING REGIONS

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Abstract

Fermi observations of GeV emission from gamma-ray bursts (GRBs) have suggested that the Lorentz factor of some GRBs is around a thousand or even higher. At the same time, the same Fermi observations have shown an extended GeV emission indicating that this higher energy emission might be a part of the afterflow and it does not come from the same region as the lower energy prompt emission. If this interpretation is correct then we may have to reconsider the opacity limits on the Lorentz factor which are based on a one-zone model. We describe here a two-zone model in which the GeV photons are emitted in a larger radius than the MeV photons and we calculate the optical depth for pair creation of a GeV photon passing the lower energy photons shell. We find that, as expected, the new two-zone limits on the Lorentz factor are significantly lower. When applied to Fermi bursts the corresponding limits are lower by a factor of five compared to the one-zone model. It is possible that both the MeV and GeV regions have relatively modest Lorentz factors (∼200–400). This two-zone limit is significantly softer than the one-zone limit.

Key words: gamma-ray burst: general – radiation mechanisms: non-thermal – relativistic processes

1. Introduction

Highly relativistic motion, essential to overcome the compactness problem (Ruderman 1975), is a basic ingredient of all gamma-ray burst (GRB) models. The value of the bulk Lorentz factor, $\Gamma$, of the relativistic outflow is of utmost interest. It is essential for understanding the nature of the inner engine, the outflow and its acceleration and collimation mechanisms, the conditions at the emitting regions, and the radiation mechanism. So far the most robust method to estimate $\Gamma$ was using the compactness. The high energy photons set an upper limit on the optical depth for pair production (Fenimore et al. 1993; Piran 1995; Woods & Loeb 1995; Lithwick & Sari 2001). The observations of GeV photons from several bursts enabled the Fermi team to set very high ($\Gamma \gtrsim 1000$) lower limits on $\Gamma$ for those GRBs (Abdo et al. 2009a, 2009b; Ackermann et al. 2010). While other methods to estimate or limit $\Gamma$ depend on various assumptions (see, e.g., Zou & Piran (2010)), the compactness limit seems to be independent of any model assumptions and hence it is considered to be the most robust one.

However, recent Fermi observations have also shown that the onset of higher energy ($\sim 100$ MeV to a few GeV—denoted hereafter as GeV) emission lags after the onset of the lower energy ($\sim 100$ keV to a few MeV—denoted hereafter as MeV) prompt emission (Abdo et al. 2009a, 2009b; Ackermann et al. 2010; Ghisellini et al. 2010). Fermi confirmed earlier EGRET results that the GeV emission also lasts longer than the MeV emission (Hurley et al. 1994; Fishman & Meeegan 1995; González et al. 2003). These two facts suggest the possibility of a different origin for the MeV and the GeV emission. This would be the case, for example, if the GeV emission arises from an external shock afterglow (Kumar & Barniol Duran 2009, 2010; Ghisellini et al. 2010; Piran & Nakar 2010; Gao et al. 2009), from multi-zone internal shocks (Xue et al. 2008; Aoi et al. 2010; Zhao et al. 2010) or if the MeV emission is the quasi-thermal radiation of the baryonic outflow while the GeV emission is mainly from the subsequent internal shocks.\footnote{The synchrotron radiation of electrons accelerated by such internal shocks may peak at eV energies while the inverse Compton radiation can give rise to a significant GeV component. In such a scenario, the GeV emission and energetic ultraviolet/optical flare are tightly correlated.}

The compactness limits on the Lorentz factor are based, however, on an implicit assumption that the MeV and the GeV photon arise from the same region. We show here that the relaxation of this assumption reduces significantly the estimated lower limit on $\Gamma$ (see also Aoi et al. 2010; Li 2010; Zhao et al. 2010). The existence of two regimes leads to a rich variety of possibilities. We consider in the following the most natural configuration, which is also consistent with the temporal delay of the GeV emission; the MeV emission is produced at lower radii (say via internal shocks) and the GeV emission is produced at a larger radius (say via external shocks; see Figure 1). Other geometrical options that we do not consider are that the GeV emission is emitted at a lower radius than the MeV emission or that the MeV and the GeV emission are produced at different angular regions. Our estimates do not depend on the origin of the emission (e.g., internal or external shock) but just on the overall geometry of the system. We calculate the optical depth of a GeV photon passing through the MeV photons shell and we obtain new compactness limits on $\Gamma$.

2. The Model

Consider two emitting regions denoted by $M$, for MeV, and $G$, for GeV, respectively (see Figure 1). The MeV (GeV) emission region has a radius $R_M (R_G)$ (for simplicity we consider emission from thin shells) with $R_M \ll R_G$, a Lorentz factor $\Gamma_M (\Gamma_G)$ and an angular width $\theta_{M,J}$ ($\theta_{G,J}$). We assume that the MeV and GeV jets are aligned and both are pointing toward us. In this configuration, a GeV photon passes through a shell of MeV photons on its way to the observer (see Figure 2). Our goal is to estimate the optical depth for pair production of a GeV and an...
MeV photons shell. The width of the MeV photons shell is\(^5\)
\[ \Delta M \simeq c T_{90} / (1 + z), \]  
(1)
where \( T_{90} \) is the observed duration of the MeV pulse and \( z \) is the redshift of the burst. A GeV photon, emitted along the \( x \)-axis at \( R_G \) and \( \theta_G \) is immersed in MeV photons until it leaves the MeV shell at \( R_{\text{max}}(R_G, \theta_G) \) (see Figure 2)
\[ R_{\text{max}}(R_G, \theta_G) = \frac{R_G}{1 - 2 c T_{90} / [(1 + z)^2 R_G^2]} , \]  
(2)
for \( \theta_G^2 > 2 c T_{90} / [(1 + z) R_G] \), while \( R_{\text{max}}(R_G, \theta_G) \) → ∞ for \( \theta_G^2 \leq 2 c T_{90} / [(1 + z) R_G] \).

Assume, for simplicity, that the emitted flux of the MeV photons is constant over time then the number density of the MeV photons is
\[ n_{0,\text{MeV}}(R) \simeq \frac{L_M}{(1 + z)^2 E_p^2 4 \pi R^2 c} , \]  
(3)
where \( L_M \) is the isotropic equivalent MeV luminosity, \( E_p \) is the observed peak energy, and \( R_G < R < R_{\text{max}}(R_G, \theta_G) \) is the radius. The spectrum of the MeV photons is described by the Band function:
\[ n_{\text{MeV}}(E_M) \simeq n_{0,\text{MeV}}(R) \left\{ \begin{array}{ll}
(\frac{E_p}{E_M})^{-\alpha_M} & E < E_p, \\
(\frac{E_p}{E_M})^{-\beta_M} & E_p < E < E_{\text{max}},
\end{array} \right. \]  
(4)
where, in view of the two-region model, the MeV component has an upper limit to the energy: \( E_{\text{max}} \). Fortunately, \( \tau(E_G) \) is insensitive to the exact value of \( E_{\text{max}} \).

At \( R_G \), the MeV photons are beamed with an angular width \( 1/\Gamma_M \) along the radial direction outward. Since \( R_G > R_M \) the MeV photons travel almost radially outward. Relative to the GeV photons, that move along the \( x \)-axis, the angular width of the MeV photons is of order \( R_M / (R \Gamma_M) \) and it decreases with \( R \). This leads to two angular regimes: along the \( x \)-axis (for \( \theta_G < R_M / (R \Gamma_M) \)), the MeV photons have very small angles \( \lesssim R_M / (R \Gamma_M) \) relative to the GeV photons. This leads to a very small optical depth along the axis. For \( \theta_G > R_M / (R \Gamma_M) \), the angular spread of the MeV photons can be neglected and the typical angle between the MeV and the GeV photons is simply \( R_G / R \).

More generally, the angle between an MeV photon emitted radially outward at \( (R_G, \theta_M) \) and a GeV photon emitted parallel to the \( x \)-axis at \( (R_G, \theta_G) \) is:
\[ \theta = \arctan \frac{\sqrt{(R_M \sin \theta_M)^2 + (R_G \sin \theta_G)^2 - 2(R_M \sin \theta_M)(R_G \sin \theta_G) \cos \phi}}{R_G \cos \theta_G - R_M \cos \theta_M + \Delta} , \]  
(5)
where \( \Delta = R - R_G \) is the distance the GeV photon travels before it collides with the MeV photon and \( \phi \) is the angle of a given MeV photon relative to the axis parallel to the GeV photon axis.

A GeV photon, with an (observed) energy \( E_G \), and an MeV photon, with an (observed) energy \( E_M \), can produce a pair if \( E_M \geq E_{\text{min}}, \) where
\[ E_{\text{min}} = \frac{2(m_e c^2)^2}{(1 + z)^2 E_G (1 - \cos \theta)} \simeq \frac{4(m_e c^2)^2}{(1 + z)^2 \theta^2 E_G} . \]  
(6)
The cross section behaves like (e.g., Jauch & Rohlich 1980)
\[ \sigma = \frac{3}{16} \sigma_T (1 - \hat{\beta}^2) \left[ (3 - \hat{\beta}^2) \ln \frac{1 + \hat{\beta}}{1 - \hat{\beta}} - 2 \hat{\beta} (2 - \hat{\beta}^2) \right] , \]  
(7)
where
\[ \hat{\beta} = \sqrt{1 - \frac{2 m_e c^2}{(1 + z) E_M (1 + z) E_G} (1 - \cos \theta)}^{-1} , \]  
(8)
\( \hat{\gamma} = 1 / \sqrt{1 - \hat{\beta}^2} \) and \( \sigma_T \simeq 6.65 \times 10^{-25} \text{ cm}^2 \) is the Thompson cross section.

We can estimate now the overall optical depth for the GeV photon:
\[ \tau(\theta_G, E_G) = \int_{R_G}^{R_{\text{max}}(R_G, \theta_G)} d R \int_{0}^{\theta_M / \theta_G} d \theta_M \int_{0}^{2 \pi} d \phi \int_{E_{\text{min}}}^{E_{\text{max}}} d E_M \frac{d^2 n_{\text{MeV}}}{d E_M d \theta_M d \phi} \sigma (1 - \cos \theta) . \]  
(9)

The effective optical depth for an observed photon is averaged over all angles:
\[ \exp[-\tau(E_G)] = \int_{0}^{\theta_M / \theta_G} d \theta_G \int_{0}^{\theta_M / \theta_G} D(3 + \hat{\beta}_G) e^{-\tau(\theta_G, E_G)} \theta_G d \theta_G , \]  
(10)

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5 Zhao et al. (2010) consider erroneously an MeV shell width of \( R/2 \Gamma \) (see their Equation (10)).
where $D = 1/(\Gamma_G (1 - \beta_{bulk} \cos \theta_G))$ is the Doppler factor, $\beta_{bulk} = \sqrt{1 - 1/\Gamma_G^2}$ is the bulk velocity, and $\beta_G$ is the photon index of the GeV emission. It is interesting to note that Equation (9) is independent of $\Gamma_G$. The overall dependence of $\tau$ on $\Gamma_G$ arises from Equation (10) where $\Gamma_G$ determines (via $D$) the effective width of the integration.

3. A SIMPLIFIED MODEL

We can simplify the above model and obtain an almost analytic formula by making a few approximations. We show later that the full numerical results indeed agree with these formulæ. First, we approximate the MeV spectrum in the relevant energy range using a single power law: $N_{MeV}(E_M) \propto E_M^{-\alpha_M}$. Second, we approximate the cross section (Equation (7)) as $\sigma \approx \sigma_T/(3 \gamma^2)$ (This form allows us for an analytic integration. The numerical factor is chosen by comparison of the analytic approximate results with the full numerical solution). Third, we divide the analysis to two regimes: for $\theta_G > R_M/\langle R_T \rangle$, all the MeV photons move radially and the collision angle is simply $\theta \approx \theta_M$. For $\theta_G < R_M/\langle R_T \rangle$, the scatter in the directions of the MeV photons is important and we approximate the collision angle as $\theta \approx R_M/\langle R_T \rangle$.

Define $x \equiv R/R_M$ and $x_{max} \equiv R_{max}/R_G = 1/[1 - 2\Delta M/(\theta_G R_G)]$. For large angles the collision angle between the MeV and GeV photons is $\theta \approx \theta_M G/R_\theta = \theta_G/x$. The minimal energy of the MeV photon for pair production is

$$E_{M,\text{min}} \sim \frac{4(m_em)^2}{(1+z)^2 E_M \theta_G^2}. \quad (11)$$

The number density of the MeV photons

$$N(E_M) \approx n_{0,MeV}(R) \frac{E_M}{E_p}^{-\alpha_M} \approx \frac{(1+z)^{2\alpha_M}}{4 \pi R_G^2 \Gamma_G^2 c(2 \pi m_e c^2)^{2\alpha_M}} \frac{E_M^{-\alpha_M}}{E_p^{-\alpha_M}} \theta_G^{-2\alpha_M} x^{-\gamma \alpha_M} \gamma^{-\alpha_M}. \quad (12)$$

where $\gamma \equiv E_M/E_{M,\text{min}} = \bar{\gamma}$. Collecting the above expressions:

$$\tau(\theta_G) = \int_{R_G}^{R_{max}(\theta_G,\theta_T)} dR \int_{E_{M,\text{min}}(\theta, R)}^{E_{M,max}} dE_M \cdot N(E_M) \times (1 - \cos \theta) \sigma_T/(3 \gamma^2) \approx \frac{(1+z)^{2\alpha_M - 3} L_M \epsilon_M^{-\alpha_M - 1} \epsilon_p^{-\alpha_M - 2} \sigma_T}{6 \times (2 \pi m_e c^2)^{2\alpha_M}} \times \frac{1}{2\alpha_M + 1} \left[ \left( 1 - x_{max}^{2\alpha_M - 1} \right) \left( 1 - \frac{1}{4} (1+z)^2 \epsilon_{max} \epsilon_G^2 \right)^{-\alpha_M} \right] \approx \frac{1}{2\alpha_M + 1} \left( 1 - \frac{1}{\epsilon_{max}} \right) \frac{1}{x_{max}^2}. \quad (13)$$

where $\epsilon_G \equiv E_G/m_e c^2$ and $\epsilon_{max} \equiv E_{M,\text{max}}/m_e c^2$. We use $R_G = \eta \sqrt{\tau_c T_G} / (1+z)$ to express $R_G$ in terms of $T_G$ and $\Gamma_G$, where $T_G$ is the time when the GeV photon is observed. Generally $\eta \lesssim 1$ with $\eta = 1$ for an external shock and $\eta < 1(\sim 0.01)$ for an internal shock. As we see later the result is not very sensitive to $\eta$. Realizing that the GeV photons are mainly coming from $\theta_G \approx 1/\Gamma_G$, we can now invert this equation to obtain a rough estimate of the minimal Lorentz factor. The solution will be consistent if $\theta_G < R_G (R_G \Gamma_G)$, namely if $\Gamma_G > R_G \Gamma_M/R_M$. To do so we assume that the factor in the square brackets of Equation (13) is of order unity. We obtain

$$\Gamma_{G,\text{min}} \approx \left[ \frac{(1+z)^{2\alpha_M - 3} L_M \epsilon_M^{-\alpha_M - 1} \epsilon_p^{-\alpha_M - 2} \sigma_T}{6 \times (2 \pi m_e c^2)^{2\alpha_M}} \right]^{1/2\alpha_M - 1} \approx 34 \left( \frac{30 \times 10^{\alpha_M}}{\alpha_M 34 \alpha_M^{-2} \epsilon_p^{-1} \epsilon_M^{-1} R_G^{-2} \theta_G^{2\alpha_M}} \right)^{1/2\alpha_M - 1} \times L_{M,\text{max}} \epsilon_{G,\text{max}}^{-\alpha_M - 1} \epsilon_{p,0}^{-\alpha_M - 2} \eta^{-1} T_G^{-2\alpha_M}, \quad (14)$$

where the notation $Q_x = Q/10^x$ is used and $\epsilon_p \equiv E_p/m_e c^2$. For $\alpha_M = 2$, the overall coefficient is 34. Note the weak (with a power $-1/(2\alpha_M + 2) \sim -1/6$ to $-1/2$) dependence of $\Gamma_{G,\text{min}}$ on $\eta$.

For $\theta_G \lesssim R_M/\Gamma_M$, the typical collision angle of the MeV photons is $\theta \lesssim \theta_M R_M/R_G$. The optical depth is similar to Equation (13), with $\tau \sim R_M/\Gamma_M$ replacing $\theta_G$ and $x_{max} \rightarrow \infty$:

$$\tau(\theta_G) \approx \tau(\theta_G = 0) = \frac{(1+z)^{2\alpha_M - 3} L_M \epsilon^{-\alpha_M - 1} \epsilon_p^{-\alpha_M - 2} \sigma_T \theta_G^{2\alpha_M}}{6 \times (2 \pi m_e c^2)^{2\alpha_M}} \times \frac{1}{2\alpha_M + 1} \left( 1 - \frac{1}{4} (1+z)^2 \epsilon_{max} \epsilon_G \left( \frac{R_G}{R_G \Gamma_M} \right)^{2\alpha_M} \right). \quad (15)$$

Interestingly, this formula depends on $\Gamma_M$, the Lorentz factor of the MeV emitting region, and it is independent of $\Gamma_G$. For typical values, $z = 1$, $\alpha_M = 2$, $E_{max} = E_G = 10$ GeV, $E_p = 1$ MeV, $L_M = 10^{33}$ erg, $R_M = 10^{15}$ cm, $R_G = 10^{16}$ cm, we obtain $\Gamma_{M,\text{min}} \approx 8$, which only provides a very weak constraint on the bulk Lorentz factor of the MeV region. If we set $R_G = R_M$, this expression reduces to the one-zone case, where the relevant collisions occur at $\theta = 1/\Gamma_M$ in the observer’s frame. Taking $R_M = R_G = 2T_G c \delta T$, where $\delta T = 0.1$ s is the typical duration of $\gamma$-ray pulse, $\Gamma_M \gtrsim 200$, consistent with Lithwick & Sari (2001).

4. RESULTS

Figure 3 depicts the dependence of $\tau$ on $\theta_G$ for a set of typical parameters: $z = 1$, $\delta M = 50$ s and $\Delta M = 7.5 \times 10^{11}$ cm, $E_G = E_{max} = 10$ GeV, $E_p = 1$ MeV, $L_M = 10^{33}$ erg, and $T_G = 100$ s. $\beta_G$ is always chosen to be 2. $\bar{\tau} \approx 1$ for these parameters (with $\alpha_M = 1$, $\beta_M = 2$) for $\Gamma_{G,\text{min}} = 40$. Note that numerical experiments reveal that to obtain the average effective optical depth using Equation (10) we have to integrate up to $\sim 4/\Gamma_G$. The results in this figure depict both the full (Equation (9)) and the simplified (Equation (13)) calculations. The later are depicted by the two thicker lines which are almost superposed on the corresponding numerical results, showing consistency of the analytical solution with the full numerical one. As the angle increases, the small-angle approximation $(\sin \theta \approx \theta)$ breaks down and the approximate solution slightly diverges from the numerical one. However, the deviation is small and it usually takes place in a regime that is not critical for the overall optical depth.
One can clearly see different segments of power-law dependence of the optical depth on $\theta_G$. For a given GeV photon $\theta_G$ determines the colliding angle and hence $E_{M,\text{min}}$. As most photons are in the lowest energy and the cross section is largest near $E_{M,\text{min}}$, the optical depth is dominated by the low-energy photons at $E_{M,\text{min}}$ and the photon index can be simply taken the one of the lines are encoded using a combination of colors and line types. $\alpha_M = 0, 1, 2$ are represented as dashed, solid, and dotted lines, respectively. The two thick lines are the analytical solution from Equation (13) for $\alpha_M = \beta_M = 1, 2$. A second transition takes place when $x_{\text{max}}$ approaches unity, namely the width of the interaction region is small compared to $R_G$. According to Equation (13), $\tau(\theta_G) \propto \theta_G^{2\alpha_M}$ for $x_{\text{max}} \gg 1$ and $\tau(\theta_G) \propto \theta_G^{-2\alpha_M}$ for $x_{\text{max}} \sim 1 + 2M/(\theta_G^2 R_G) \to 1$. The transition between the two takes place as $\theta_G \approx \sqrt{2\Delta M/R_G} \sim \sqrt{2cT_90/(2\Gamma_G^2 cT_G)} \sim \sqrt{T_90/(\Gamma_G^2 T_G)}$. For some parameters the two transitions may coincide with one and we have chosen $L_M$ so that the two transitions are clearly seen.

Figure 4 depicts $\Gamma_{G,\text{min}}$ as a function of the MeV luminosity $L_M$. As one can expect (see Equation (14)) $\Gamma_{G,\text{min}}$ increases with $L_M$. The optical depth for $T_G = 1$ s (thin solid line, $\eta = 1$) is much larger than the other ones for which $T_G = 100$ s, again in agreement with Equation (14). If $\alpha_M = \beta_M$, $\Gamma_{G,\text{min}}$ increases with $L_M$ as a single power law. This is consistent with Equation (14). For $\alpha_M \neq \beta_M$, the relationship breaks into two power-law segments that are dominated by different parts of Band spectrum of the MeV photons. As the luminosity increases the Lorentz factor increases, and the dominant contribution to the optical depth arises from $\theta_G \approx 1/\Gamma_G$, with a lower effective $\theta_G$. $E_{M,\text{min}}$ increases and hence for large values of $L_M$ the opacity is dominated by the high energy spectral slope, $\beta_M$. Conversely, for low values of $L_M$ the opacity is dominated by $\alpha_M$. The transition takes place at $E_{M,\text{min}} \approx E_p$. The two-zone limits, $\Gamma_{G,\text{min}}$, for four Fermi bursts are shown in Table 1 together with the single-zone limits, $\Gamma_{\text{min}}$, and some parameters of the bursts. While the single-zone limits, $\Gamma_{\text{min}}$, are of order 1000 and even larger, the two-zone limits are around 200 (400), for $\eta = 1(0.01)$. It should be stressed that in two of these bursts, GRB 080916c and GRB 090510, the highest energy GeV photon used to determine the single-zone limit is coincident with a large MeV flux, while an 11 GeV photon is contemporaneous in GRB 090902b with an MeV spike. Still in both GRB 080916c and GRB 090926a, MeV photons are observed after the end of the prompt MeV ($T_G > T_90$). Thus, it is not clear whether the single zone or the two-zone limit should be used. If the GeV photons are from an external shock, another direct constraint on $\Gamma_G$ arises from the dynamics of

### Table 1

| GRB     | $z$ | $T_90$ (s) | $\alpha_M$ | $\beta_M$ | $E_R$ (MeV) | $L_{\text{iso}}$ (erg s$^{-1}$) | $E_G$ (GeV) | $T_G$ | $\Gamma_{G,\text{min}}$ | $\Gamma_{\text{min}}$ | ref |
|---------|-----|------------|------------|-----------|-------------|-------------------------------|------------|-------|------------------------|----------------------|-----|
| 080916C | 4.35| 66         | 1.02       | 2.21      | 1.17        | $7 \times 10^{53}$             | 13.22      | 40    | 193 (414)              | 880                  | 1   |
| 090510  | 0.903| 0.5        | 0.48       | 3.09      | 5.1         | $4.6 \times 10^{53}$            | 3.4        | 0.5   | 150 (277)              | 1200                 | 2   |
| 090926A | 1.822| 30         | 0.61       | 3.87      | 0.8         | $3.2 \times 10^{53}$            | 33.4       | 82    | 120 (218)              | 1000                 | 3.4 |
| 090926A | 2.1062| 20        | 0.693      | 2.34      | 0.27        | $4.2 \times 10^{53}$            | 19.6       | 26    | 150 (318)              | 1200                 | 5   |

**Notes.** Shown are the low-energy spectral parameters $\alpha_M, \beta_M$, and $E_R$ as well as the luminosity, $L_{\text{iso}}$ and the energy, $E_G$; and time $T_G$ of the highest energy GeV photon as well as the two zone, $\Gamma_{G,\text{min}}$, and the single zone, $\Gamma_{\text{min}}$, limits.

In bold face is the value for $\eta = 1$ and brackets is the value for $\eta = 0.01$. (1) Abdo et al. 2009a; (2) Ackermann et al. 2010; (3) Abdo et al. 2009b; (4) de Palma et al. 2009; (5) Swenson et al. 2010.

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**Figure 3.** Optical depth for a photon with $E_G = 10$ GeV as a function of $\theta_G$. Different lines correspond to different spectral indices. The parameters of the lines are encoded using a combination of colors and line types. $\alpha_M = 0, 1, 2$ are represented as blue, black, and red, respectively. $\beta_M = 1, 2, 3$ are represented as dashed, solid, and dotted lines, respectively. The two thick lines are the analytical solution from Equation (13) for $\alpha_M = \beta_M = 1, 2$.

**Figure 4.** Minimal Lorentz factor $\Gamma_{G,\text{min}}$ as a function of the MeV luminosity $L_M$. Different curves correspond to different photon indices. The parameters of the lines are encoded using a combination of colors and line types. $\alpha_M = 0, 1, 2$ are represented as blue, black, and red, respectively. $\beta_M = 1, 2, 3$ are represented as dashed, solid, and dotted lines respectively. The thick gray line has $T_90 = T_G = 1$ s, corresponding to a short GRB.
external shock: \( \Gamma_G \approx 240 E_{\gamma,52}^{1/8} n_0^{-1/8} (1 + z)^{3/8} T_{G,P,1}^{-3/8} \) (Sari & Piran 1999). With observed GeV peak emission time \( T_{G,P} \approx 10 \) s (Ghisellini et al. 2010), \( E_k \sim E_{\gamma,iso} \sim 10^{55} \) erg and a circumburst density \( n \approx 1 \) cm\(^{-3} \), we obtain \( \Gamma_G \approx 740 \). This value is larger than the lower limits obtained from the two-zone compactness estimate. However, it is quite uncertain, in view of the uncertainty in the determination of \( T_{G,P} \).

5. CONCLUSIONS

Following various indications that the high energy (GeV) emission in GRBs is produced in a different region than the lower energy (MeV) emission, we derived here revised compactness limits on the Lorentz factor of GRB outflows within a two-zone model. We considered the “natural” model in which the GeV emission is produced at larger radii than the MeV emission. This would arise, for example, if the MeV emission is produced by internal shocks and the GeV emission by the afterglow, as has been suggested recently by several authors (Kumar & Barniol Duran 2009, 2010; Ghisellini et al. 2010). We calculated the optical depth for pair production by a GeV photon passing through the MeV photons shell. Our results reduce to the one-zone model when the emission region of the MeV and GeV coincide.

Collisions between the GeV and MeV photons occur in the two-zone model at larger radii than the prompt emission radius, the density of the MeV emission is smaller (than in the prompt emission regime) and the MeV photons are more collimated along the line of sight. Consequently, the optical depth is smaller compared to the one-zone case and the compactness constraint on the Lorentz factor becomes weaker. The new constraint that we find is only for the Lorentz factor of the GeV region. The constraint on the Lorentz factor of the MeV-emitting region, arising from the optical depth of the GeV photon, is rather weak. This limit does not contradict neutrino-driven jets (Aloy et al. 2005) or the magnetic-driven jets (Komissarov et al. 2009; Tchekhovskoy et al. 2009).

For a canonical set of parameters, like \( z = 1 \), \( \delta t_M = 50 \) s, \( E_G = E_{\max} = 10 \) GeV, \( E_P = 1 \) MeV, \( L_M = 10^{53} \) erg, and \( T_G = 100 \) s, the constraint on the GeV region is \( \Gamma_G \gtrsim 100 \). When we apply the two-zone constraint to four Fermi bursts, we find minimal Lorentz factors of about 200–400, about one fifth to one half (depending on \( \eta \)) of the one-zone limit which is in the order of 1000. We conclude that one should proceed with care when applying the one-zone limits to the Fermi data and unless we can verify that the GeV emission is indeed produced in the same region as the lower energy prompt emission we should consider the more relaxed two-zone limits.

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