Comparative study of impact of random environment on individual and combined Laguerre-Gauss modes

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Abstract. In this paper, the passage of individual and combined Laguerre-Gauss modes through the random medium with the given correlation function in the form of the Gaussian function is numerically investigated. The passage of the beams is described with the help of the extended Huygens-Fresnel principle, and the numerical calculation of the propagation operator is realized, using the fast Fourier transform. It is established, that the stability of the state of the orbital angular momentum of single modes, shown earlier is also observed for their superposition, while an increase in the degrees of freedom is provided, which can be used to encode the transmitted information.

1. Introduction
Communication systems, based on the work of space optics are becoming very competitive in the field of broadband access networks [1, 2]. As a consequence, it becomes necessary to study the influence of turbulent perturbations in the atmosphere on the intensity and other properties of laser beams [3].

The propagation of an optical signal in free space can be subject to distortions, associated with the turbulence of the medium [4]. Due to the properties of turbulence, random changes in the atmospheric refractive index may cause a distortion in the intensity of the laser radiation. Classical methods for describing the propagation of the wave through the turbulent atmosphere are based on applications of the Rytov method [5] and the method of parabolic equations [6].

With the help of these methods, the propagation of optical signals from a partially coherent source was investigated [7, 8], and also the features of the propagation of various laser beams in the turbulent medium, including high-order Gaussian beams [9], hollow beams [10], nondiffraction beams, beams of Bessel-Gauss and Airy type [11, 12], as well as cosine-shaped beams [13]. By this it was found, that higher-order Gaussian beams, including vortex beams [14-16], as well as various spatially structured beams, are broadened to a lesser degree by the influence of turbulence, than the fundamental Gaussian beam.

The Laguerre-Gauss modes are example of the beams with so-called screw phase dislocations or phase vortexes. As early as 1992 it was established, that the Laguerre-Gauss laser modes possess an orbital angular momentum [17]. Later it was shown, that the orbital angular momentum is a natural characteristic of all light beams with a vortex phase and can be produced in a conventional optical laboratory [18-21].

The beams, bearing an orbital angular momentum (vortex singular beams [22]) have an infinite number of possible states, which excites a huge theoretical and applied interest in these beams. In particular, they are used to seal the channels of information transfer in free space and in optical fibers
[23, 24]. As a rule, it should be noted, that the individual modes of laser radiation or individual beams of special type are considered. However, different superpositions of multi-mode beams can have a given orbital angular momentum [25, 26], including those, demonstrating other properties, for example, self-reproduction [27]. The stability of such beams to the inhomogeneities of the optical medium has not yet been investigated.

In this paper, the comparative simulation of the passage of individual and combined Laguerre-Gauss modes through a random medium with a given correlation function in the form of Gaussian function is considered. The passage of the beams is described with the help of the extended Huygens-Fresnel principle [28], and the numerical calculation of the propagation operator is realized using, the fast Fourier transform. The effect of random perturbations in the medium on the characteristics of the Laguerre-Gaussian modes, as well as on the superposition of modes with opposite indices of the vortex orders is investigated.

2. Theoretical foundations

The Laguerre-Gauss modes are described by the following expression [29]:

\[
GL_{n,m}(r, \phi) = \frac{1}{\sigma_0} \left[ \frac{2n!}{\pi(n+|m|)!} \right]^{1/2} \exp \left( -\frac{r^2}{\sigma_0^2} \right) \left( \frac{\sqrt{2r}}{\sigma_0} \right)^{|m|} L_{|m|}^{|m|} \left( \frac{2r^2}{\sigma_0^2} \right) \exp(i \phi),
\]

where \( L_{n,m}(x) \) is the generalized Laguerre polynomial; \( n \) and \( m \) are the orders of the modes, where \( m \) is whole number, and \( n \) is whole positive.

To describe the propagation of the laser beam in the medium with random distortions, the extended Huygens-Fresnel principle is used [28]:

\[
E(u, v, z, t) = -\frac{i}{2\pi z} \exp(ikz) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_0(x, y) \exp \left( i\frac{k}{2z} \left( x-u \right)^2 + \left( y-v \right)^2 \right) + \Psi(x, y, u, v, z) e^{-i\omega t} \, dx \, dy,
\]

where \( E_0(x, y) \) is the field in the input plane (at \( z=0 \)); \( E(u, v, z, t) \) is the field at distance \( z \) from the input plane; \( \Psi(x, y, u, v, z) \) is random complex phase, associated with the turbulence of the atmosphere. Expression (2) corresponds to the Rytov method [6, 30, 31], and the function describes random deviations of the phase function of the spherical wave, propagating from the initial plane to the output plane.

Due to the fact, that the theoretical and experimental parameters of turbulent media are considered in statistical aspect, the extended Huygens-Fresnel principle is used only to analyze the average characteristics of light beams, such as the average intensity [7-13, 32]. However, in order to investigate the conservation of the state of the orbital angular momentum, in this paper we simulate the propagation of vortex beams and their superpositions through the random environment on the basis of individual statistical realizations.

To illustrate the approach used, it is needed to write down a special case of the extended Huygens-Fresnel principle (2) for the one-dimensional input function, as well as discarding the time dependence:

\[
E(u, z) = -\frac{i}{2\pi z} \exp(ikz) \int_{-\infty}^{\infty} E_0(x) \exp \left( i\frac{k}{2z} \left( x-u \right)^2 + \psi(x, u, z) \right) \, dx.
\]

Let’s assume, that the correlation function of the random field does not depend on the specific coordinates \( x_1 \) and \( x_2 \) but only on their difference. Moreover, it does not depend on coordinates in the output plane, but depends only on the distance \( z \) to it [30, 31]:

\[
R_u \left[ \exp[\psi(x_1, u, 0)], \exp[\psi(x_2, u, z)] \right] = \exp \left[ -\frac{(x_1 - x_2)^2}{\rho_0^2} \right],
\]
where $\rho_0(z) = (0.545C_n^2k^2z)^{-3/5}$ is the coherence length of the propagation of the spherical wave through the turbulent atmosphere, $C_n^2$ is the structural constant of the medium, $U$ is the random field.

To determine the state of the orbital angular momentum of the beam, the scalar product of the beam and optical vortexes are used:

$$c_p = \frac{2\pi}{R} \int_0^R \int_0^\varphi E(r, \varphi, z) \exp(-i\varphi) r \, dr \, d\varphi,$$

where $E(r, \varphi, z)$ is the analyzed beam; $\exp(i\varphi)$ is the optical vortex of the $p$-th order (angular harmonic); $c_p$ is the value of the weighting factor of the $p$-th order.

Perform operation (5) optically, and simultaneously for several orders of optical vortexes it is possible with the help of the multichannel (multi-order) diffractive optical element [34, 35]. The values of the coefficients of the expansion in the angular harmonics (5) make it possible to determine the state of the orbital angular momentum of the laser beam [36, 37]:

$$\mu = \left( \sum_p \left| c_p \right|^2 \right)^{-1} \left( \sum_p \left| c_p \right|^2 \right)^{-1}.$$  \hspace{1cm} (6)

3. Modeling results

The simulation of propagation of the individual beams and superpositions of the Laguerre-Gauss modes was performed using a fast Fourier transform. The width of the input area is 2x2 mm, the wavelength is 633 nm, $\sigma_0 = 0.2$ mm.

Figure 1 shows the propagation of the Laguerre-Gauss beam in the random medium at distances of 310 mm, 610 mm and 910 mm. Table 1 shows the weight content of angular momenta, depending on the range of propagation.

As can be seen from Figure 1 and Table 1, even when propagating in the random environment, the content of the corresponding optical vortex in the beam is remained. The greatest values of the weight coefficients are attained at $p = m$.

### Table 1. The weight content of the optical vortexes in the single Laguerre-Gauss beams after passing through the random medium.

| $z$  | $p = 0$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 4$ | $p = 5$ | $p = 6$ | $p = 7$ | $p = 8$ | $p = 9$ | $p = 10$ |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $n = 2$ | $m = 3$ | 0.0004 | 0.0394 | 0.0227 | 0.7585 | 0.0706 | 0.0436 | 0.0465 | 0.0023 | 0.0068 | 0.0003 | 0.0000 |
| $n = 3$ | $m = 5$ | 0.0017 | 0.0232 | 0.1258 | 0.7254 | 0.1098 | 0.0015 | 0.0033 | 0.0031 | 0.0041 | 0.0002 | 0.0011 |
| $n = 4$ | $m = 7$ | 0.0004 | 0.0014 | 0.0284 | 0.9295 | 0.0282 | 0.003 | 0.0019 | 0.004 | 0.0015 | 0.0004 | 0.0008 |
| $n = 5$ | $m = 9$ | 0.011 | 0.0005 | 0.0117 | 0.0053 | 0.0146 | 0.8909 | 0.0035 | 0.0084 | 0.0087 | 0.0215 | 0.008 |
| $n = 6$ | $m = 11$ | 0.0004 | 0.0005 | 0.0046 | 0.0108 | 0.0477 | 0.8377 | 0.0409 | 0.0392 | 0.0158 | 0.0008 | 0.0009 |
| $n = 7$ | $m = 13$ | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0028 | 0.0105 |

Figure 2 shows the propagation of superposition of the Laguerre-Gauss beams with opposite orders of the optical vortexes in opposite directions. Table 2, by analogy with Table 1, shows the content of the optical vortexes after passing through the superposition in the random environment.
Figure 1. Propagation of the single Laguerre-Gauss beams in the random environment.

Table 2. The weight content of the optical vortexes of superposition of the Laguerre-Gauss beams after passing through the random medium (n = 3).

|   |   | z = 310 | z = 610 | z = 910 |
|---|---|---------|---------|---------|
|   |   | p = -5  | p = -4  | p = -3  | p = -2  | p = -1  | p = 0   | p = 1   | p = 2   | p = 3   | p = 4   | p = 5   |
| m_1 = 5 | m_2 = 5 | 310 | 0.4542 | 0.0187 | 0.003  | 0.0008 | 0.0006 | 0.0022 | 0.0015 | 0.0011 | 0.0013 | 0.0172 | 0.4507 |
|   |   | 610 | 0.3599 | 0.0339 | 0.0028 | 0.0002 | 0.0003 | 0.0008 | 0.0009 | 0.0027 | 0.0081 | 0.0473 | 0.3627 |
|   |   | 910 | 0.4673 | 0.0077 | 0.0069 | 0.0069 | 0.0002 | 0.0013 | 0.0011 | 0.0071 | 0.4682 |       |       |
| m_1 = 3 | m_2 = 3 | 310 | 0.0014 | 0.0113 | 0.4592 | 0.0032 | 0.0021 | 0.0023 | 0.0075 | 0.0255 | 0.4596 | 0.0007 | 0.0012 |
|   |   | 610 | 0.0161 | 0.0512 | 0.3963 | 0.0047 | 0.0035 | 0.0083 | 0.0129 | 0.0387 | 0.4013 | 0.0084 | 0.0228 |
|   |   | 910 | 0.0003 | 0.0165 | 0.4195 | 0.0453 | 0.0066 | 0.0002 | 0.0166 | 0.4193 | 0.0387 | 0.0184 |
Figure 2. Propagation of the superposition of the Laguerre-Gauss beams in the random environment.

4. Conclusion
In this paper, the propagation of both single Laguerre-Gaussian modes and their combinations were simulated. A comparative analysis of the stability of the characteristics of the propagated laser beams to random perturbations in the medium was made. It is established, that the stability of the state of the orbital angular momentum of single modes, shown earlier is also observed for their superposition, while an increase in the degrees of freedom is provided, which can be used to encode the transmitted information.

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