Reducing the Heterogeneity of Payoffs: an Effective Way to Promote Cooperation in Prisoner’s Dilemma Game

Luo-Luo Jiang,† Ming Zhao,‡ Han-Xin Yang,§ Joseph Wakeling,∥ Bing-Hong Wang,¶ and Tao Zhou

1Department of Modern Physics, University of Science and Technology of China, Hefei 230026 P. R. China
2Department of Physics, University of Fribourg, Chemin du Musée 3, CH-1700 Fribourg, Switzerland
3The Research Center for Complex System Science, University of Shanghai for Science and Technology and Shanghai Academy of System Science, Shanghai, 200093 P. R. China

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In this paper, the total payoff of each agent is regulated to reduce the heterogeneity of the distribution of the total payoffs. It is found there is an optimal regulation strength where the fraction of cooperation is prominently promoted, too weak or too strong of the strength will have little effects or result in the disappearance of the cooperators. It is also found that most of the cooperators are not distributed in isolation but form the cooperator clusters, and to promote the cooperation the only way is to enlarge the size of the cooperator clusters. Finally, we try to explain the emergence of larger clusters and prove the existence of the optimal regulation strength. Our works provide insight into the understanding of the relations between the distribution of payoffs and the cooperative behaviors.

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I. INTRODUCTION

Cooperation is a widespread and important phenomenon in natural and social systems, which is the foundation for the sustainable development of creatures. However, individuals are instinctively self-interested, which will drive them to cheat to obtain more benefits rather than to cooperate. Therefore, understanding the conditions for the emergence and promotion of cooperation is one of the fundamental and central problems in biological, social, and economic science [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Cooperative phenomenon can be well described by game theory. As one of the representative games, prisoner’s dilemma game (PDG) seizes the characteristics of the conflict between the selfish individuals and the collective interests. In PDG, when most of the individuals take the cooperation strategy, the collective interests is optimized, but as to an individual, if it cheats when its opponents cooperate, it will profit much greater than it cooperates and its opponents will profit little, even none. Thus, more and more individuals will cheat, as a result the cooperation will decrease. Ultimately, all the individuals will receive lower payoffs than they take the cooperation strategy.

There are many mechanisms that can promote the cooperation of PDG, such as repeated interaction [1], spatial extensions [12], reciprocity [13], and partly randomly contacts [14]. Very recently, payoffs had also been found playing an crucial role in promoting cooperation in PDG [15, 16, 17]. Particularly, Perc found that Gaussian-distributed payoff variations is more successful in promoting cooperation than Levy distribution of payoffs [15], indicating heterogeneity of payoffs will do harm to the cooperation amongst egoistic individuals. Then a natural question has arisen: what is the effective way to regulate the total payoff of each agent to optimize the cooperation? Here in this paper, we try to give an answer to this question.

As we all know, taxation is of fundamental importance for every country, it is not only a way to raise funds but also an effective way to regulate the incomes of individuals to maintain the social stability. In this paper, based on the idea borrowed from the tax policy, we regulate the total payoff of each agent to decrease the broadness of the distribution of payoffs. We will show that when the payoffs are regulated to some extent, the fraction of cooperation will be greatly promoted, however, if the regulation is too strong, the cooperation will be suppressed again. We also try hard to give a convincing explanation for the promotion of cooperation.

II. MODEL

In the spatial PDG, agents located on a square lattice follow two simple strategies: cooperation (C) or defect (D), described as the form of vector:

\[
\phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

When a cooperator meets a cooperator, both of them get reward 1, and when a defector meets a defector, they each get 0. And when a cooperator meets a defector, it gets 0, but the defector receive temptation \(b\), \(1 < b < 2\). The above rule can be expressed by a matrix:

\[
\psi = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}.
\]

which is called payoff matrix, and the parameter \(b\) characterizes the temptation to defect against cooperation. Each agent plays PDG with its four neighbors. Therefore, the total payoff of the player \(i\) is the sum of payoffs after \(i\) interacts...
with its 4 neighbors, which is written as:

\[ P_i = \sum_{j \in \Lambda_i} \phi_i^T \phi_j, \]  

(3)

where \( \Lambda_i \) denotes four neighbors of individual \( i \). In classical PDG, an agent updates its strategy according to the following rule: the agent \( i \) plays PDG with its neighbors, then randomly selects a neighbor \( j \), and adopts its strategy with probability

\[
G_{i \rightarrow j} = \frac{1}{1 + \exp[(P_i - P_j)/T]},
\]

(4)

where \( T \) characterizes the stochastic noise. For \( T = 0 \), the individual always adopts the best strategy determinately, while irrational changes are allowed for \( T > 0 \). In numerical simulation, noise level is often set as \( T = 0.1 \) because a few irrational behavior is common in real economic systems.

In our regulation scheme, we regulate the total payoffs:

\[ W_i = P_i^\alpha, \]

(5)

where \( \alpha > 0 \) is the regulation parameter. It is notable that \( \alpha = 0 \) is forbidden because it is meaningless for \( P_i \). In the case \( \alpha = 1 \), the present model restores to classical PDG. When \( \alpha \) decreases from 1 to 0, the heterogeneity of the payoffs distribution is also depressed, and when \( \alpha \) goes to 0, the differences of total payoffs for different agents disappear. In this paper, we replaced the total payoffs \( P_i \) and \( P_j \) in equation (4) by the regulated payoffs \( W_i \) and \( W_j \), and investigate how the cooperation will be affected by this regulation.

### III. Simulation and Analysis

In order to describe the evolution process of the game, we employ the fraction of cooperations as an order parameter

\[ \rho_C = \frac{1}{L^2} \sum_{i=1}^{L^2} \phi_i^T \left( \begin{array}{c} 1 \\ 0 \end{array} \right). \]

(6)

Based on a periodic boundary lattice with size of 100 x 100, an extensive Monte Carlo numerical simulation is performed with random initial states. After the system reaches dynamic equilibrium, \( \rho_C \) is calculated and the final results are obtained after the averaging of 10000 times.

Figure I(a) shows the cooperation fraction \( \rho_C \) as a function of \( b \) at different values of \( \alpha \). It displays that \( \rho_C \) decreases monotonically with the increasing of \( b \), no matter what \( \alpha \) is. Most interestingly, the cooperation are greatly affected by the parameter \( \alpha \) for fixed \( b \): in a large region of \( \alpha \), \( \rho_C \) will be increased, indicating the reduction of heterogeneity of payoffs will improve the cooperation. It worth noting that there is at least one optimal value of \( \alpha \) where \( \rho_C \) takes its maximum, larger or smaller \( \alpha \) will cause the decreasing of \( \rho_C \). Thus, to quantify the effects of \( \alpha \) on the promotion of cooperation for different \( b \), we present the dependence of \( \rho_C \) on \( \alpha \) in Fig. I(b). Clearly, in the \( \alpha \) region (0.37, 1.0), \( \rho_C \) is larger than the case of classical PDG (\( \alpha = 1.0 \)), and at the point \( \alpha = 0.5 \), \( \rho_C \) reaches its maximum, where the cooperation is promoted prominently. But when \( \alpha < 0.2 \) or \( \alpha > 1.4 \), there will be no cooperator, which means that both higher and lower value of \( \alpha \) will jeopardize cooperation. It is especially worth noting that our regulation scheme is more powerful for larger temptation \( b \), for example, at \( b = 1.005 \), for the classical PDG (\( \alpha = 1.0 \)), the fraction of cooperation \( \rho_C \) is 0.3855 and for the best case of our regulation scheme (\( \alpha = 0.5 \)), \( \rho_C = 0.5060 \), the increment is \( \Delta \rho_C = 0.1205 \); but for a larger temptation \( b = 1.020 \), the fraction of cooperation \( \rho_C \) will increase from 0.0 to 0.3736, with the increment \( \Delta \rho_C = 0.3736 \).

To explain why the cooperation are so prominent at \( \alpha = 0.5 \), we give the snapshots of the distribution of cooperation when \( \alpha \) takes three typical values: 0.3, 0.5 and 1.0, which are shown in Fig II. Clearly, most of the cooperators are not distributed in isolation but form some clusters. When the payoffs are not regulated (\( \alpha = 1.0 \)) or regulated too much (\( \alpha = 0.3 \)), there are only a few cooperator clusters in the system, but when \( \alpha = 0.5 \), there will emerge so many cooperator clusters that the cooperation are remarkably promoted.

We also count the number of cooperator clusters, which has no notable changing for a large region of \( \alpha \), but the maximal and average size of the cooperator clusters will be greatly increased in some region of \( \alpha \), which is shown in Fig III(a) and the inset. Moreover, the distribution of cooperator clusters at the three values of \( \alpha \) are also plotted in Fig III(b). It is clear, when \( \alpha = 0.5 \), there are much more large clusters (\( S_C > 86 \)) than when \( \alpha = 0.3 \) or 1.0, and the small size clusters (\( 4 < S_C < 80 \)) are less than the two cases.

From the above simulation results we can assert, when \( \alpha \) increases from 0.0, there is no cooperator in the system until
α reaches some threshold, then the cooperators emerge, with α’s increasing, more and more agents becomes cooperators and the cooperator clusters become larger and larger, when α reaches 0.5, the number of cooperators, the maximal and average size of cooperator clusters all reach their maximums, and further increase α, where the payoffs are not regulated much, the cooperations are repressed again, more and more agents prefer to cheat, the cooperation decrease again, till disappear. It seems that the emergence of larger cooperator clusters cause the promotion of cooperation since when the fraction of cooperation is promoted the number of cooperator clusters will not change but the size of them will be greatly enlarged, as shown in insets of Fig 3(a). Our assertion is consistent with the previous researches that cooperators survive by forming compact clusters. And the cooperative agents along boundary resisting against defectors can be enhanced by, for example, heterogeneous structure [21], attractiveness of the neighbors [19,20], and stochastic interactions [21].

Here we try to explain how the cooperator clusters are formed and why the average cluster size reaches maximum at α = 0.5. In our simulation, the initial state, cheat or cooperate, of each agent is assigned randomly, from the statistical theory we know that at the very beginning there are already some cooperator clusters, to enlarge the already exist clusters, the defectors along the boundary of the cooperator clusters should be more inclined to change their strategies to cooperate, and the probability is decided by two aspects, one is the number of cooperators of their 4 neighbors and the other is the probability $G_{i\rightarrow j} = 1/(1 + \exp[(W_i - W_j)/T])$. Along the boundary of the cooperator clusters, the defector must have neighbors who are cooperators, so their total payoffs $P_D$ are not less than b, and for the cooperators, they must have at least one neighbor who is also cooperator, which makes their total payoffs $P_C$ take values 1, 2 or 3 but 0, as a result, $P_D$ may larger or smaller than $P_C$. When $P_D > P_C$, with α’s decreasing from 1, $P_D - P_C > 0$ becomes smaller and smaller, correspondingly, $G_{D\rightarrow C}$ becomes larger and larger. But when $P_D < P_C$, with α’s decreasing from 1, $P_D - P_C < 0$ becomes larger and larger, correspondingly, $G_{D\rightarrow C}$ becomes smaller and smaller. As a consequence, when $P_D > P_C$, smaller α is better for the formation of cooperator clusters but when $P_D < P_C$, larger α is better, thus, there must be a right value of α, which is optimal for the formation of the cooperator clusters, that is the reason why the fraction of cooperator is optimized at α = 0.5 in our simulation.

From the above analysis we can also concluded that the more cooperators of a defector’s neighbors are, the easier for it to become a cooperator. The reverse situation can also be proved that if most of the neighbors are defectors for a cooperator, the easier for it to becomes a defector. By far, we can give a picture of the evolution of the cooperator clusters: there must be broad boundary where the defectors around are changed to cooperators and acute boundary where the cooperators become defectors, which can be seen in Fig 4 A larger cluster can be divided into some smaller clusters when it is cut by defectors (Fig 4 from (b) to (c)), that is the reason why the clusters could not keep growing. Certainly, several clusters can also form a large one (Fig 4 from (c) to (d)). So the clusters evolve endlessly in a system.

**IV. CONCLUSION**

In conclusion, we regulate the total payoffs of each agent to narrow down the differences between agents in the spatial prisoner’s dilemma game, and find that there is an optimal regulation strength where the cooperation is greatly promoted, especially for larger temptation. Too strong of the regulation will depress the cooperation, even cause the disappearance of the cooperators. We reassure that it is the larger size not the
FIG. 4: Four successive evolutions of the cooperator clusters. From (b) to (c), a cluster is divided into two clusters and from (c) to (d), two clusters form a larger new one. Here, the area of $10 \times 10$ is the part of $100 \times 100$ in the system, and the light gray sites present cooperative agents.

The number of cooperator clusters that promote the cooperation. We also prove the existence of the optimal regulation strength and explain the formation of larger cooperator clusters. Our results provide quantitative analysis of payoffs' effects on cooperation in spatial prisoner's dilemma game. Compared with other factors of maintaining cooperation such as spatial extensions, reciprocity, and punishment, reducing the heterogeneity of payoffs is more realistic for it seizes individual fitness by game payoffs. Regarding economic process, our findings suggest that moderate tax policy can play an important role in maintaining cooperation among social individuals.

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[1] R. Axelrod, The Evolution of Cooperation (Basic books, New York, 1984).
[2] J. Hofbauer and K. Sigmund, Evolutionary Games and Population Dynamics (Cambridge University Press, Cambridge, UK, 1998).
[3] M. A. Nowak, Evolutionary Dynamics: Exploring the Equations of Life (Harvard University Press, Harvard, USA, 2006).
[4] F. C. Santos, J. M. Pacheco, and T. Lenaerts, Proc. Natl. Acad. Sci. U.S.A. 103, 3490 (2006).
[5] Z.-X. Wu, X.-J. Xu, Z.-G. Huang, S.-J. Wang, and Y.-H. Wang, Phys. Rev. E 74, 021107 (2006).
[6] G. Szabó and G. Fáth, Phys. Rep. 446, 97 (2007).
[7] R. Boyd, and S. Mathew, Science 316, 1858 (2007).
[8] F. C. Santos, M. D. Santos, and J. M. Pacheco, Nature (London) 454, 213 (2008)
[9] W.-X. Wang, J.-H. Lü, G.-R. Chen, and P. M. Hui, Phys. Rev. E 77, 046109 (2008).
[10] M. Perc, and A. Szolnoki, Phys. Rev. E 77, 011904 (2008).
[11] G. Szabó and A. Szolnoki, Phys. Rev. E 79, 016106 (2009).
[12] M. A. Nowak and R. M. May, Nature (London) 359, 826 (1992).
[13] M. A. Nowak and K. Sigmund, Nature (London) 437, 1291 (2005).
[14] J. Ren, W.-X. Wang, and F. Qi, Phys. Rev. E 75, 045101 (2007).
[15] M. Perc, Phys. Rev. E 75, 022101 (2007).
[16] J. Tanimoto, Phys. Rev. E 76, 041130 (2007).
[17] X.-J. Chen, and L. Wang, Phys. Rev. E 77, 017103 (2008).
[18] J. Gómez-Gardeñes, M. Campillo, L. M. Floría, and Y. Moreno, Phys. Rev. Lett. 98, 108103 (2007).
[19] J.-Y. Guan, Z.-X. Wu, Z.-G. Huang, X.-J. Xu and Y.-H. Wang, Europhys. Lett. 76, 1214 (2006).
[20] P. Langer, M. A. Nowak, and C. Hauert, J. Theor. Biol. 250, 634 (2007).
[21] X.-J. Chen, F. Fu, and L. Wang, Phys. Rev. E 78, 051120 (2008).