The Compressive and Rarefactive Dust Acoustic Solitary Waves with Two Different Temperatures for Both Electrons and Ions

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Abstract—The nonlinear propagation of dust acoustic solitary waves (DASWs) in an unmagnetized dusty plasma consisting of negatively charged dust fluid, Boltzmann distributed electrons with two different temperatures, Boltzmann distributed ions with two distinct temperatures are investigated. By employing the reductive perturbation technique that is valid for a small but finite amplitude limit, the Korteweg–de Vries (KdV) equation, have been derived. Our results reveal that the main quantities of DASWs (such as amplitude and width) are affected by electrons with two different temperatures (as well as ions), temperature ratios, and the number densities of two species of ions. It is shown that both positive and negative potential DASWs occur in this case.

Keywords: dust acoustic solitary waves, Korteweg–de Vries (KdV) equation, reductive perturbation technique, electrons with two different temperatures, Boltzmann distribution, unmagnetized dusty plasma

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1. INTRODUCTION

A dusty plasma which consisting of electrons, ions, charged dust particles is found in space environments, such as earth’s ionosphere, planetary rings, interstellar medium, Solar nebula, comet tails [1–7], in the laboratory devices such as dc and RF discharges machines, plasma processing reactors, fusion plasma devices, and in the industrial plasma [8–14]. In these devices, the particles are either formed as a result of agglomeration of reactive species or deliberately injected into the plasma [15, 16]. The presence of a dust component in the plasma can significantly modify the existing ordinary modes such as ion acoustic waves [17] and introduce new nonlinear waves at different dynamical scales. One of these modes is low-frequency dust acoustic (DA) wave (with a phase speed lying between the ion and dust thermal velocities), which was first predicted theoretically by Rao et al. [18]. In laboratory, the spectra of DA waves was experimentally observed by Barkan et al. [19]. The nonlinear DA waves have received a great deal of research interest in understanding the fundamental properties of localized electrostatic structures in space and laboratory dusty plasmas [20–28]. It is well known that electrons with two different temperatures are found to occur both in space and laboratory environment [29–31]. On the other hand, in laboratory and space plasmas, it is most evident that the ions are usually at different tempera-

1 The text was submitted by the authors in English.
KdV equation. They found that rarefactive and compressive potential DASWs can be formed in such dusty plasma.

Malik et al. [57] studied DASWs in a magnetized dusty plasma considering two different temperature ions and Boltzmann distributed electrons. In their investigations, they found that the effect of hot ions is significant when their number is very high as compared to the number of cold ions. Also they reported that the presence of more electrons in the dusty plasma increases the amplitude of the DASWs. Emanuddin et al. [58] theoretically investigated the propagation of DA waves in a magnetized dusty plasma consisting of two distinct temperature electrons, nonthermal ions and negatively charged mobile dust grains. Their analysis shows that the characteristics and the properties of the DASWs are significantly modified in the presence of external magnetic field. It was also shown that temperature ratio of ions, relative number densities of ions, and relative number densities of electrons are important parameters in the properties of the DASWs. Shamy et al. [59] studied DASWs for dusty plasmas consisting of hot dust fluid, nonisothermal ions and two different temperature electrons. They derived KdV equation and modified KdV equation at critical ion density. They have shown that the presence of a second component of electrons modifies the nature of DA solitary wave structures. The above-mentioned studies of DA solitary waves in a magnetized or an unmagnetized dusty plasma with two different temperature electrons or two different temperature ions show a good characteristic dependency of the properties of the DASWs. Shamy et al. [59] studied DASWs in a magnetized dusty plasma with two different temperature electrons and nonthermal ions and negatively charged mobile dust grains. Their analysis shows that the characteristics and the properties of the DASWs are significantly modified in the presence of external magnetic field. It was also shown that temperature ratio of ions, relative number densities of ions, and relative number densities of electrons are important parameters in the properties of the DASWs. Shamy et al. [59] studied DASWs for dusty plasmas consisting of hot dust fluid, nonisothermal ions and two different temperature electrons. They derived KdV equation and modified KdV equation at critical ion density. They have shown that the presence of a second component of electrons modifies the nature of DA solitary wave structures. The above-mentioned studies of DA solitary waves in a magnetized or an unmagnetized dusty plasma with two different temperature electrons or two different temperature ions show a good characteristic dependency of these nonlinear structures on the second temperature of electrons as well as ions in dusty plasma which motivate us to study DASWs in a dusty plasma with two different temperatures for both electrons and ions.

2. BASIC EQUATION

We consider a five-component plasma, which consists of negatively charged dust grains, two types of isothermal electrons with temperatures \( T_{el} \) and \( T_{eh} \), two types of isothermal ions with temperatures \( T_{il} \) and \( T_{ih} \). The charge neutrality at equilibrium requires that

\[
n_{il0} + n_{il0} = n_{el0} + n_{eh0} + n_{il0} Z_d, \tag{1}
\]

where \( n_{il0} \) is the unperturbed dust number density, \( n_{el0} \) and \( n_{eh0} \) are equilibrium electron number densities at low \( (T_{el}) \) and high \( (T_{eh}) \) temperature electrons, respectively; \( n_{il0}, n_{ih0} \) are the unperturbed ion number densities at low \( (T_{il}) \) and high \( (T_{ih}) \) temperature ions, respectively, \( Z_d \) is the number of charge on the dust grains. From Eq. (1), we have

\[
\alpha = \frac{n_{il0} Z_d}{n_{il0}} = 1 - \delta_1 - \delta_2 + \delta_3, \tag{2}
\]

where \( \delta_1 = n_{il0}/n_{il0} \), \( \delta_2 = n_{eh0}/n_{ih0} \), and \( \delta_3 = n_{eh0}/n_{il0} \).

According to one-dimensional propagation, the dynamics of dust particles are governed by normalized equations of fluid equations:

\[
\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \tag{3}
\]

\[
\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{4}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = n_d + \delta_1 e^{-\theta_0} + \delta_2 e^{-\theta_1} - \frac{1}{\alpha} e^{-\theta_0} - \delta_3 e^{-\theta_1}, \tag{5}
\]

where \( \theta_i = T_{il}/T_{el}, \theta_2 = T_{il}/T_{eh}, \theta_3 = T_{il}/T_{el}, \) \( n_d \) is the dust particle number density normalized to \( n_{il0} \), \( u_d \) is the dust fluid velocity in \( x \) direction normalized by \( C_d = \sqrt{Z_d T_{il}/m_d} \), \( m_d \) is dust mass, \( \phi \) is the electrostatic potential normalized by \( T_{il}/e \) with \( e \) being the magnitude of the electron charge. The time \( t \) and space \( x \) variables are normalized by the dust plasma period \( \omega_d^{-1} = \frac{m_d}{4\pi n_{il0} Z_d^2 e^2} \) and the Debye length \( \lambda_d = \frac{T_{il}/4\pi n_{il0} Z_d e^2} {\sqrt{\rho_d}} \), respectively. In order to derive the KdV equation, we employ the reductive perturbation technique [60] and introduce stretched coordinates as follows:

\[
\xi = \epsilon^{1/2}(x - v_0 t), \quad \tau = \epsilon^{1/2} t, \tag{6}
\]

where \( \epsilon \) is a small parameter characterizing the strength of nonlinearity \( (0 < \epsilon < 1) \) and \( v_0 \) is the nonlinear wave phase velocity which normalized by \( C_d \).

The variables \( n_d, u_d, \phi \) can be expanded in a power series of \( \epsilon \):

\[
n_d = 1 + \epsilon n_{d1} + \epsilon^2 n_{d2} + \ldots, \tag{7}
\]

\[
u_d = \epsilon u_{d1} + \epsilon^2 u_{d2} + \ldots, \tag{8}
\]

\[
\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + \ldots. \tag{9}
\]

Substituting Eqs. (7)–(9) along with the stretching coordinates (7) into Eqs. (3)–(5) and writing them with the lowest order of \( \epsilon \), we obtain the following relations:

\[
n_{d1} = \frac{1}{v_0} u_{d1}, \quad u_{d1} = -\frac{1}{v_0} \phi_1, \tag{10}
\]

\[
n_{d2} = \frac{1 + \delta_1 \theta_1 + \delta_2 \theta_2 + \delta_3 \theta_3}{\alpha} \phi_1, \tag{11}
\]

\[
v_0 = \sqrt{\frac{\alpha}{1 + \delta_1 \theta_1 + \delta_2 \theta_2 + \delta_3 \theta_3}}. \tag{12}
\]

For the next higher order, we obtain

\[
v_0^2 \frac{\partial n_{d2}}{\partial \xi} = \frac{\partial n_{d1}}{\partial t} + v_0 \frac{\partial (n_d u_{d1})}{\partial \xi} + v_0 \frac{\partial u_{d2}}{\partial \xi} \tag{13},
\]

\[
\frac{\partial u_{d1}}{\partial t} = \frac{\partial u_{d1}}{\partial \xi} + u_{d1} \frac{\partial \phi_2}{\partial \xi} - \frac{\partial \phi_1}{\partial \xi}. \tag{14}
\]
3. RESULTS AND DISCUSSIONS

Equation (19) clearly indicates that both positive and negative solitons exist. The solution (20) also stands for \( n_{d1} \) if we replace \( \phi_{ml} \) with \( -(1/v_0^2)\phi_{ml} \). The coefficient of the nonlinear term \( A \), which determines the polarity of solitary waves, depends on equilibrium density ratio parameters \( \delta_1, \delta_2, \delta_3 \) and temperature ratio parameters \( \theta_1, \theta_2, \theta_3 \). To preserve the physical meanings, the possible values of these parameters are chosen as \( T_{el} < T_{eh} < T_{ih} \) and \( n_{el0}, n_{eh0} < n_{ih0} \). As \( u_0 > 0 \), for \( A > 0 \), positive potential or rarefactive solitary waves (DASWs with density hump) are formed, and for \( A < 0 \), negative potential or compressive solitary waves (DASWs with density dip) are formed. It is clear that an increase in \( u_0 \) increases the amplitude of the soliton wave and decreases its width. For \( \delta_1, \delta_2 = 0 \), i.e., electron depleted plasma, and considering ions at one temperature (\( \theta_3 = 1 \)), the values of nonlinear coefficient \( A \) and dispersion coefficient \( B \) are in good agreement with the findings of [61]. Furthermore, by considering electrons at one temperature (\( \theta_1 = \theta_2 \)), the nonlinear coefficient \( A \) and dispersion coefficient \( B \) are similar to that obtained in [41, 49].

Figures 1 and 2 show the variation in the amplitude of the DASWs versus coordinate \( \xi \) and time \( \tau \). Figure 1 shows that small-amplitude solitary waves with negative potential (compressive solitary wave) are generated, while Fig. 2 shows that small amplitude solitons with positive potential (rarefactive solitary wave) are formed. The results in these figures show that the amplitude of the soliton is constant with increasing time \( \tau \).

Figure 3 indicates that compressive (Fig. 3a) and rarefactive (Fig. 3b) solitary waves are formed at different values of \( u_0 \). Obviously, in this figure, one can see that the amplitude (width) of both compressive
and rarefactive soliton increases (decreases) as $u_0$ increases.

Figure 4 which plotted $\phi_1$ versus $\chi$ depicts that there is a range of values of $\delta_3$ (the ratio of density of high-temperature ions to density of low-temperature ions), in which either compressive or rarefactive solitary waves coexist. It is clear that if $\delta_3 = 6$, compressive solitary waves will be generated in the dusty plasma, and for $\delta_3 = 7$, we will have rarefactive solitary wave in the dusty plasma.

Figure 5 depicts the soliton profile for two values of $\theta_3$ (the ratio of low-temperature ions to high-temperature ions). It is evident that the polarity of DA solitons (compressive or rarefactive) depends strongly on $\theta_3$. We see that compressive DA soliton can exist for $\theta_3 = 0.2$ ($T_{ch} = 2T_{cl} = 20T_{ih} = 100T_q$), while $\theta_3 = 0.05$ ($T_{ch} = 2T_{cl} = 5T_{ih} = 100T_q$) leads to appearance of rarefactive DA soliton.

Figures 6 and 7 show the variation in the amplitudes of the DASWs with the temperature ratios $\theta_1$, $\theta_2$, and $\theta_3$ for some specified values of ratio densities $\delta_1$, $\delta_2$, $\delta_3$ and constant speed of DASWs. These figures show that compressive and rarefactive DASWs may exist in our two-temperature electrons and two-temperature ions dusty plasma model.

The effect of the temperature of two-electron species on the amplitude of DASWs is presented in Fig. 6. In Fig. 6a, the amplitude of compressive DASWs decreases with increasing both $\theta_1$ and $\theta_2$. While in Fig. 6b, polarity changes (amplitude is positive), and rarefactive solitons are formed. It can be seen that the amplitude of a soliton increases with the increase in $\theta_1$ and $\theta_2$. It should be noted that the amplitude change

![Figure 3](image3.png)

*Fig. 3.* (Color online) Variation in $\phi_1$ versus spatial coordinate $\chi$ for $u_0 = 0.1$ (dotted curve), 0.15 (dashed curve), 0.2 (solid curve): (a) negative potential for $\theta_1 = 0.2$, $\theta_2 = 0.1$ and (b) positive potential for $\theta_1 = 0.03$, $\theta_2 = 0.01$, and $\theta_3 = 0.05$. Other parameters are $\delta_1 = 0.2$, $\delta_2 = 0.3$, $\delta_3 = 12$.

![Figure 4](image4.png)

*Fig. 4.* Variation in $\phi_1$ versus spatial coordinate $\chi$ for density ratios $\delta_3 = 6$ (dashed curve) and 7 (solid curve). Other parameters are $\delta_1 = 0.2$, $\delta_2 = 0.6$, $\theta_1 = 0.02$, $\theta_2 = 0.01$, $\theta_3 = 0.08$, and $u_0 = 0.01$ ($T_{ch} = 2T_{cl} = 8T_{ih} = 100T_q$).

![Figure 5](image5.png)

*Fig. 5.* (Color online) Variation in $\phi_1$ versus spatial coordinate $\chi$ for $\theta_3 = 0.2$ (dashed curve) and 0.05 (solid curve). The other parameters are $\delta_1 = 0.4$, $\delta_2 = 0.7$, $\delta_3 = 12$, $\theta_1 = 0.02$, $\theta_2 = 0.01$, and $u_0 = 0.1$. 


by \( \theta_1 \) (due to the lower-temperature electron) is greater than the amplitude change by \( \theta_2 \) (due to the higher-temperature electron), which completely agrees with the result of Mamun et al. [35]. To see the effects of ratio ion temperature on amplitude of DASWs, electrostatic potential \( \phi_1 \) is plotted against \( \chi \) for different values of \( \theta_3 \) (the ratio of low-temperature ion to high-temperature ion). In Fig. 7a, the amplitude of the compressive DA solitary waves decreases with the increase in \( \theta_1 \). While in Fig. 7b, rarefactive solitons are appeared as \( \theta_1 \) increases, the amplitude of the DA solitary waves also increases. Therefore, the large difference between the values of low and high ion temperature (leads to positive potential) appears in an increase in the DA solitary amplitude and the slight difference between the values of low and high ion temperature (leads to negative) causes the decrease in the amplitude of solitons.

4. CONCLUSIONS

We have investigated the propagation of DASWs in an unmagnetized dusty plasma, consisting of Boltzmann distributed electrons with two distinct temperatures, negatively charged dust grains, and Boltzmann distributed ions with two different temperatures. Using the reductive perturbation technique, the KdV equation is obtained. The results show that the present model supports both rarefactive and compressive DASWs structure. The presence of two different types of isothermal electrons and two different types of isothermal ions, which appear in space plasma and labo-
ratory environment, modify notably the basic features (polarity, amplitude, etc.) of solitary waves. It was concluded that in compressive DASWs polarity, the amplitude of the solitons increases with the increase in both $\theta_1$ (ratio of low-temperature ion to low-temperature electron) and $\theta_2$ (ratio of low-temperature ion to high-temperature electron), and in rarefactive DASWs polarity, as $\theta_1$ and $\theta_2$ increase, the amplitude of solitary waves increases. We also found that the difference between the values of low and high ion temperature may be caused by the change in the DA soliton polarity. The positive potential solitary waves exist for lower values of the temperature ratio $\theta_1$ (ratio of low-temperature to high-temperature ions), while negative potentials appear for larger values of $\theta_2$. Furthermore, our results reveal that density ratio $\delta$ (ratio of the high-temperature to low-temperature ion densities) has effect on solitary wave structure and can change the polarity of DASWs.

CONFLICT OF INTEREST
The authors declare that they have no conflicts of interest.

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