Transmission Capacities for Overlaid Wireless Ad Hoc Networks with Outage Constraints

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Abstract—We study the transmission capacities of two coexisting wireless networks (a primary network vs. a secondary network) that operate in the same geographic region and share the same spectrum. We define transmission capacity as the product among the density of transmissions, the transmission rate, and the successful transmission probability (1 minus the outage probability). The primary (PR) network has a higher priority to access the spectrum without particular considerations for the secondary (SR) network, where the SR network limits its interference to the PR network by carefully controlling the density of its transmitters. Assuming that the nodes are distributed according to Poisson point processes and the two networks use different transmission ranges, we quantify the transmission capacities for both of these two networks and discuss their tradeoff based on asymptotic analyses. Our results show that if the PR network permits a small increase of its outage probability, the sum transmission capacity of the two networks (i.e., the overall spectrum efficiency per unit area) will be boosted significantly over that of a single network.

I. INTRODUCTION

Initiated by the seminal work of Gupta and Kumar [1], the studies for understanding the capacities of wireless ad hoc networks have made great progresses. Considering $n$ nodes that are randomly distributed in a unit area and grouped independently into one-to-one source-destination (S-D) pairs, Gupta and Kumar [1] showed that a typical time-slotted multi-hop architecture with a common transmission range and adjacent-neighbor communication can achieve a sum throughput that scales as $\Theta\left(\sqrt{n}/\log n\right)$. By using percolation theory, Franceschetti et al. [2] showed that the $\Theta\left(\sqrt{n}\right)$ sum throughput scaling is achievable. In [3], Grossglauser and Tse showed that by allowing the nodes to move independently and uniformly, a constant throughput scaling $\Theta(1)$ per S-D pair can be achieved. In [4], Baccelli et al. proposed a multi-hop spatial reuse ALOHA protocol. By optimizing the product between the number of simultaneous successful transmissions per unit area and the average transmission range, they showed that the transport capacity is proportional to the square root of the node density, which achieves the upper bound of Gupta and Kumar [1]. Weber et al. in [5] derived the upper and lower bounds on transmission capacity of spread-spectrum wireless ad hoc networks, where the transmission capacity is defined as the product between the maximum density of successful transmissions and the corresponding data rate, under a constraint on the outage probability.

All the above results focus on the capacity of a single ad hoc wireless network. In recent years, due to the scarcity and poor utilization of spectrum, the regulation bodies are beginning to consider the possibility of permitting secondary (SR) networks to coexist with licensed primary (PR) networks, which is the main driving force behind the cognitive radio technology [6]. In cognitive radio networks, the PR users have a higher priority to access the spectrum and the SR users need to operate conservatively such that their interference to the PR users is limited below an “acceptable level”. In this overlaid regime, the capacity or throughput scaling laws for both of the PR and SR networks are interesting problems. Recently, some preliminary works along this line appeared. In [7], Vu et al. considered the throughput scaling law for a single-hop cognitive radio network, where a linear scaling law is obtained for the SR network with an outage constraint for the PR network. In [8], Jeon et al. considered a multi-hop cognitive network with zero outage. In [9], Yin et al. assumed that the SR nodes know the location of each PR node. With an elegant transmission scheme, they showed that by defining a preservation region around each PR node, both networks can achieve the same throughput scaling law as a stand-alone wireless network, while the SR network may suffer from a finite outage probability. In [9], Yin et al. assumed that the SR nodes know the location of each PR node. With an elegant transmission scheme, they showed that by defining a preservation region around each PR node, both networks can achieve the same throughput scaling law as a stand-alone wireless network, while the SR network may suffer from a finite outage probability. In [9], Yin et al. assumed that the SR nodes know the location of each PR node. With an elegant transmission scheme, they showed that by defining a preservation region around each PR node, both networks can achieve the same throughput scaling law as a stand-alone wireless network, while the SR network may suffer from a finite outage probability.

In this paper, we study the coexistence of two ad hoc networks with different transmission scales (power and/or transmission range) based on the transmission capacity defined in [5]. We extend the definition of transmission capacity from a single network to two overlaid networks. Different from the approaches in [7], [8], and [9], we resort to stochastic geometry tools to quantify the transmission capacities for both the PR and SR networks without defining any preservation regions. By considering the mutual interferences from the two networks, we discuss the tradeoff of the transmission capacities between them. The results show that if we permit a slight increase over the outage probability of the PR network, the sum transmission capacity of the overlaid networks will be boosted significantly over that of a single network.

The rest of the paper is organized as follows. The network
model and symbol notations are described in Section II. The transmission capacity for a single network case is analyzed in Section III. The transmission capacities for the PR and SR networks and their tradeoff are discussed in Section IV. The numerical results and observations are given in Section V. Finally, Section VI summarizes our conclusions.

II. NETWORK MODEL AND SYSTEM SETUP

Consider the scenario where a network of PR nodes and a network of SR nodes coexist in the same geographic region, and assume that the PR network is the legacy network, which has a higher priority to access the spectrum. The prerequisite condition for introducing a new SR network into the territory of the PR network is that the outage probability increment of the PR network is upper-limited by a target constraint $\Delta \epsilon$, where $\Delta \epsilon$ usually takes a very small value.

We assume that at a certain time instance the distribution of PR TXs follows a homogeneous Poisson point process (PPP) $\Pi_0$ of density $\lambda_0$, and the distribution of SR TXs follows another independent homogeneous PPP $\Pi_1$ of density $\lambda_1$. Our goal is to evaluate the outage probability of the PR network, $P^0$, and that of the SR network, $P^1$, which are functions of the TX node densities $\lambda_0$ and $\lambda_1$. The specific definitions of outage probability will be given in Section III and Section IV. Similar to that in [5], in order to evaluate the outage probabilities, we condition on a typical PR (or SR) RX at the origin, which yields the Palm distribution for (PR or SR) TXs. Following the Slivnyak’s theory in stochastic geometry [10], these conditional distributions also follow homogeneous PPPs with the corresponding densities (i.e., $\lambda_0$ and $\lambda_1$, respectively). Let $\{X_i \in \mathbb{R}^2, i \in \Pi_0\}$ and $\{Y_j \in \mathbb{R}^2, j \in \Pi_1\}$ denote the locations of the PR TXs and the SR TXs, respectively, $|X_i|$ and $|Y_j|$ denote the distances from PR TX $i$ and SR TX $j$ to the origin, respectively. An attempted transmission is successful if the received signal-to-interference-plus-noise ratio (SINR) at the reference RX is above a threshold, $\beta$; otherwise, the transmission fails, i.e., an outage occurs. We use $\beta_0$ and $\beta_1$ to represent the SINR thresholds for the PR network and the SR network, respectively.

For simplicity, we limit our discussion to single-hop transmissions, and assume that all PR TXs use the same transmission power $\rho_0$, and all PR transmissions are over the same distance $r_0$. Similarly, all SR TXs use the same transmission power $\rho_1$ over the same transmission distance $r_1$. For the wireless channel, we only consider the large-scale path-loss, and ignore the effects of shadowing and small-scale multipath fading. As such, the normalized channel power gain $g(d)$ is given as

$$g(d) = \frac{A}{d^\alpha},$$

where $A$ is a system-dependent constant, $d$ is the distance between the TX and the corresponding RX, and $\alpha > 2$ denotes the path-loss exponent. In the following discussion, we normalize $A$ to be unity for simplicity. The ambient noise is assumed to be additive white Gaussian noise (AWGN) with an average power $\eta$. We assume that all the PR TXs and the SR TXs use the same spectrum with bandwidth normalized to be unity.

As in [5], we define transmission capacity as follows.

**Definition 1:** Transmission capacity $C^\epsilon$ of a randomly-deployed wireless network is defined as the product among the maximum density $\lambda'$ of transmissions, the common transmission data rate $R$, and $(1 - \epsilon)$ with $\epsilon$ an asymptotically small outage probability. Therefore, we have

$$C^\epsilon = R\lambda'(1 - \epsilon).$$

As noted in [5], $C^\epsilon$ also represents the unit-area spectral efficiency of the successful transmissions.

III. ASYMPTOTIC ANALYSIS OF THE TRANSMISSION CAPACITY: SINGLE NETWORK CASE

In this section, we derive the asymptotic result (asymptotic over vanishingly-small outage probability values) for the transmission capacity of the PR network when the SR network is absent. As an example, we focus on the case when the path-loss exponent $\alpha = 4$, over which we build an asymptotic analysis framework that is useful for the future study over the cases of general $\alpha$ values.

When the SR network is absent, denote the target outage probability of the PR network over per-link SINR as $\epsilon_0$. Then we have

$$P^0 = \text{Prob} \left( \frac{\rho_0 r_0^{-\alpha}}{\eta + \sum_{i \in \Pi_0} \rho_0 |X_i|^{-\alpha}} \leq \beta_0 \right) = \epsilon_0. \quad (3)$$

Rewrite (3) as

$$\text{Prob} (X \geq T_0) = \epsilon_0, \quad (4)$$

where $X = \sum_{i \in \Pi_0} \rho_0 |X_i|^{-\alpha}$ and $T_0 = \frac{\rho_0 r_0^{-\alpha}}{\beta_0} - \eta$. The moment generating function (MGF) of $X$ is given by [11]

$$\Phi_X(s) = \exp \left[ -\pi \lambda_0 \frac{\beta_0^\frac{\alpha}{2}}{\sqrt{\beta_0}} \Gamma \left( 1 - \frac{\alpha}{2} \right) \right]. \quad (5)$$

When $\alpha = 4$, we have

$$\Phi_X(s) = \exp \left[ -\pi^2 \lambda_0 \beta_0^\frac{1}{2} s^2 \Gamma \left( 1 - \frac{\alpha}{4} \right) \right]. \quad (6)$$

Via the inverse Laplace transform, we obtain the probability density function (PDF) of $X$ as

$$f_X(x) = \frac{\pi}{2} \lambda_0 \sqrt{\rho_0} x^{-\frac{7}{2}} \exp \left( -\frac{\pi^3}{4x} \lambda_0 \rho_0 \right), \quad (7)$$

and the corresponding cumulative density function (CDF) of $X$ as

$$F_X(x) = 2Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0}}{\sqrt{2tx}} \right). \quad (8)$$

From (3), we have

$$\text{Prob} (X \geq T_0) = 1 - 2Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0}}{\sqrt{2T_0}} \right). \quad (9)$$
Combined (4) and (9), it is clear that the following condition has to be satisfied:

\[ Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0}}{\sqrt{2T_0}} \right) = \frac{1 - \epsilon_0}{2}. \tag{10} \]

When \( \epsilon_0 \to 0 \) such that \( \frac{\pi^2 \lambda_0 \sqrt{\rho_0}}{\sqrt{2T_0}} \to 0 \), with Taylor series expansion, we obtain the maximum allowable value (via the monotonicity of the Q function) of \( \lambda_0 \) asymptotically for \( \alpha = 4 \) as

\[ \lambda_0^{\epsilon_0} = \frac{\epsilon_0}{\pi} \left( \frac{T_0}{\rho_0} \right)^{\frac{3}{2}} = \frac{\epsilon_0}{\pi} \left( \frac{r_0^4 - \eta}{\beta_0} \right)^{\frac{3}{2}}. \tag{11} \]

As we can see from (11) that when the outage probability \( \epsilon_0 \) is very small, the density of TXs is a linear function of \( \epsilon_0 \). Therefore, the transmission capacity of the PR network is given by

\[ C_0^{\epsilon_0} = R_0 \lambda_0^{\epsilon_0} (1 - \epsilon_0), \tag{12} \]

where \( R_0 \) is the data rate when the transmission between the TX and its associated RX is successful, which is set to be same for all the links.

IV. ASYMPTOTIC ANALYSIS OF THE TRANSMISSION CAPACITY: OVERLAIRED NETWORK CASE

A. Transmission Capacity of the PR Network

When the SR network is present, it introduces interference to the PR network and the outage probability of the PR network will be increased. If we set the target outage probability increment of the PR network as \( \Delta \epsilon \), we have

\[ p^0 = \operatorname{Prob} \left( \frac{\rho_0 r_0^{-\alpha}}{\eta + \sum_{i \in \Pi_0} \rho_0 |X_i|^{-\alpha} + \sum_{j \in \Pi_1} \rho_1 |Y_j|^{-\alpha}} \leq \beta_0 \right) = \epsilon_0 + \Delta \epsilon. \tag{13} \]

With \( Y = \sum_{j \in \Pi_1} \rho_1 |Y_j|^{-\alpha} \), (13) can be rewritten as

\[ \operatorname{Prob}(X + Y \geq T_0) = \epsilon_0 + \Delta \epsilon. \tag{14} \]

The MGF of \( Y \) is given by

\[ \Phi_Y(s) = \exp \left[ -\pi \lambda_1 \rho_1 ^{-\alpha} s \frac{r_1^{-\alpha}}{\beta_1} \left( 1 - \frac{2}{\alpha} \right) \right]. \tag{15} \]

Define \( Z = X + Y \) such that the MGF of \( Z \) is given by

\[ \Phi_Z(s) = \Phi_X(s) \Phi_Y(s) = \exp \left[ -\pi s \frac{r_1^{-\alpha}}{\beta_1} \left( 1 - \frac{2}{\alpha} \right) \left( \lambda_0 \rho_0 ^{-\alpha} + \lambda_1 \rho_1 ^{-\alpha} \right) \right]. \]

For \( \alpha = 4 \), we have

\[ \Phi_Z(s) = \exp \left[ -\pi s \frac{r_1^{-\alpha}}{\beta_1} \left( \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right) \right]. \tag{16} \]

and the PDF of \( Z \) is given by

\[ f_Z(z) = \frac{\pi}{2} \left( \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right)^{-\frac{3}{2}} \times \exp \left[ -\pi z \frac{3}{4} \left( \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right)^2 \right]. \tag{17} \]

Applying (17) in (14), we have

\[ 1 - 2Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1}}{\sqrt{2T_1}} \right) = \epsilon_0 + \Delta \epsilon, \tag{18} \]

i.e.,

\[ Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1}}{\sqrt{2T_0}} \right) = \frac{1 - \epsilon_0 - \Delta \epsilon}{2}. \tag{19} \]

When \( \epsilon_0 \to 0 \) and \( \Delta \epsilon \to 0 \), with bivariate Taylor series expansion, we obtain

\[ 1 - \frac{\pi \lambda_0 \sqrt{\rho_0}}{2 \sqrt{T_0}} - \frac{\pi \lambda_1 \sqrt{\rho_1}}{2 \sqrt{T_1}} = 1 - \epsilon_0 - \Delta \epsilon. \tag{20} \]

If we choose \( \lambda_0 = \lambda_0^{\epsilon_0} \) as in (11), the maximum allowable value of \( \lambda_1 \) corresponding to a target outage probability increment \( \Delta \epsilon \) is given by

\[ \lambda_1^{\Delta \epsilon} = \frac{1}{\pi} \left( \frac{T_0}{\rho_1} \right)^{\frac{3}{2}} \Delta \epsilon = \frac{1}{\pi} \left( \frac{\rho_0}{\rho_1} \right) \left( \frac{r_0^4 - \eta}{\beta_0} \right)^{\frac{3}{2}} \Delta \epsilon, \tag{21} \]

and the transmission capacity of the PR network is given by

\[ C_0 = R_0 \lambda_0^{\epsilon_0} (1 - \epsilon_0 - \Delta \epsilon). \tag{22} \]

As shown in (20), when the SR network is presented, the outage probability of the PR network can be approximated by an affine function of \( \lambda_0 \) and \( \lambda_1 \) over asymptotically small \( \epsilon_0 \)'s and \( \Delta \epsilon_0 \)'s.

B. Transmission Capacity of the SR Network

Denote the outage probability of the SR network as \( \epsilon_1 \), the outage probability of the SR network is given by

\[ p^1 = \operatorname{Prob} \left( \frac{\rho_1 r_1^{-\alpha}}{\eta + \sum_{i \in \Pi_0} \rho_0 |X_i|^{-\alpha} + \sum_{j \in \Pi_1} \rho_1 |Y_j|^{-\alpha}} \leq \beta_1 \right) = \epsilon_1. \tag{23} \]

Rewrite (23) as

\[ \operatorname{Prob} \left( Z \geq \rho_1 \frac{r_1^{-\alpha}}{\beta_1} - \eta \right) = \epsilon_1. \tag{24} \]

Define \( T_1 = \rho_1 \frac{r_1^{-\alpha}}{\beta_1} - \eta \), and we have

\[ \operatorname{Prob}(Z \geq T_1) = \epsilon_1. \tag{25} \]

Similar to (19), we obtain

\[ Q \left( \frac{\pi^2 \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1}}{\sqrt{2T_1}} \right) = 1 - \epsilon_1. \tag{26} \]

When \( \epsilon_1 \to 0 \), with bivariate Taylor series expansion, we have

\[ 1 - \frac{\pi \lambda_0 \sqrt{\rho_0}}{2 \sqrt{T_1}} - \frac{\pi \lambda_1 \sqrt{\rho_1}}{2 \sqrt{T_1}} = 1 - \epsilon_1. \tag{27} \]

Therefore, the outage probability of the SR network is given by

\[ \epsilon_1 = \frac{\pi}{\sqrt{T_1}} \left( \lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right). \tag{28} \]
and the transmission capacity of the SR network is given by

\[ C_1^r = R_1 \lambda_1^r (1 - \epsilon_1), \]  

(29)

where \( R_1 \) is the data rate adopted by successful SR links.

On the other hand, if we set the target outage probability of the PR network to be \( \epsilon_0 + \Delta \epsilon \), and set the target outage probability of the SR network to be \( \epsilon_1 \) simultaneously, we could choose the value of \( \lambda_1^r \) in (29) as follows

\[ \lambda_1^r = \min (\lambda_1^{\epsilon_{\text{SR}}}, \lambda_1^{\epsilon_{\text{PR}}}), \]

(30)

where \( \lambda_1^{\epsilon_{\text{SR}}} \) is given by (via (23))

\[ \lambda_1^{\epsilon_{\text{SR}}} = \frac{\epsilon_1}{\pi} \left( \frac{r_1^\alpha - \eta}{\rho_1} \right)^{\frac{1}{2}} - \lambda_0^c \sqrt{\frac{\rho_0}{\rho_1}}. \]

(31)

C. Sum Transmission Capacity of the Overlaid Network

When the SR network is present, based on the above analyses, the sum transmission capacity of the overlaid networks is given by

\[
C_s^\epsilon = C_0^\epsilon + C_1^\epsilon = R_0 \lambda_0^c (1 - \epsilon_0 - \Delta \epsilon) + R_1 \lambda_1^r (1 - \epsilon_1) = R_0 \lambda_0^c \left( 1 - \frac{\pi}{\sqrt{T_0}} (\lambda_0^c \sqrt{\rho_0} + \lambda_1^r \sqrt{\rho_1}) \right) + R_1 \lambda_1^r \left( 1 - \frac{\pi}{\sqrt{T_1}} (\lambda_0^c \sqrt{\rho_0} + \lambda_1^r \sqrt{\rho_1}) \right) = (R_0 \lambda_0^c + R_1 \lambda_1^r) - \frac{\pi}{\sqrt{T_0}} (\lambda_0^c \sqrt{\rho_0} + \lambda_1^r \sqrt{\rho_1}) \times \left( \frac{R_0}{\sqrt{T_0}} + \frac{R_1}{\sqrt{T_1}} \right). \]

(32)

Compared to the single network case, the gain of the transmission capacity (i.e., the overall spectrum efficiency) of the overlaid networks over that of a single network is given by

\[ K_0 = \frac{C_s^\epsilon}{C_0^\epsilon} \approx 1 + \frac{C_1^\epsilon}{C_0^\epsilon}. \]

(33)

D. Tradeoff of the Transmission Capabilities

Here we consider two setups to study the tradeoff between the transmission capacities of the PR network and the SR network. The first setup is that we change the value of \( \Delta \epsilon \) only, and fix other parameters (\( \rho_0, \rho_1, r_0, r_1, \beta_0, \beta_1, \eta, \) and \( \epsilon_0 \)). The second setup is that we change the value of \( \rho_1 \), and let other parameters (\( \rho_0, r_0, r_1, \beta_0, \beta_1, \eta, \epsilon_0 \), and \( \lambda_1 \)) be fixed.

Let us consider the first setup. When \( \epsilon_0 \) is fixed, \( \lambda_0 \) is also fixed, see (11). From (22), we can see that \( C_0^\epsilon \) is a linear function of \( \Delta \epsilon \). As such, when \( \Delta \epsilon \) is increased, \( C_0^\epsilon \) is reduced. Rewrite (29) as

\[
C_1^\epsilon = \frac{R_1}{\pi \sqrt{\rho_1}} \Delta \epsilon \left( 1 - \frac{T_0}{T_1} \epsilon_0 - \frac{T_0}{T_1} \Delta \epsilon \right). \]

(34)

From (34), we can easily verify that when \( \sqrt{T_1/T_0} > \epsilon_0 \), \( C_1^\epsilon \) is a convex function of \( \Delta \epsilon \), and when \( \Delta \epsilon < \frac{1}{2}(\sqrt{T_1/T_0} - \epsilon_0) \), \( C_1^\epsilon \) increases monotonically over \( \Delta \epsilon \).
In Fig. 2, we show the normalized asymptotic transmission capacity \( C_{\epsilon_0} / R_0 \) as a function of the outage probability \( \epsilon_0 \), and the upper and lower bounds of the transmission capacity based on the results derived in [5], which verifies the tightness of the upper bound.

\[ \text{Upper bound} \]

\[ \text{Lower bound} \]

\[ \text{Asymptotic result} \]

Figure 2. Normalized transmission capacity vs. outage probability for the PR network when SR network is absent.

B. Overlaid Network Case

The normalized transmission capacity of the PR network \( C_{\epsilon_0} / R_0 \) vs. the increment of the outage probability \( \Delta \epsilon \) of the PR network is shown in Fig. 3. As expected, \( C_{\epsilon_0} / R_0 \) is inversely proportional to \( \Delta \epsilon \). On the other hand, since \( C_{\epsilon_0} \) is a convex function of \( \epsilon_0 \); and when \( \epsilon_0 < 1 - \Delta \epsilon \), \( C_{\epsilon_0} \) increases over \( \epsilon_0 \) monotonically for a fixed \( \Delta \epsilon \).

\[ \text{Normalized transmission capacity of the PR network vs. increment of the outage probability of the PR network.} \]

Figure 3. Normalized transmission capacity of the PR network vs. increment of the outage probability of the PR network.

In Fig. 4, we show the normalized transmission capacity of the SR network \( C_{\epsilon_1} / R_1 \) as a function of \( \Delta \epsilon \), see (29). As shown in the figure, we see that \( C_{\epsilon_1} \) increases monotonically over \( \Delta \epsilon \), since the larger the values of \( \lambda_{\epsilon_1}' \) and \( \epsilon_1 \) are, but the effect of \( \lambda_{\epsilon_1}' \) on \( C_{\epsilon_1} \) is dominant when \( \epsilon_1 \) is small.

\[ \text{Normalized transmission capacity of the SR network vs. increment of the outage probability of the PR network.} \]

Figure 4. Normalized transmission capacity of the SR network vs. increment of the outage probability of the PR network.

In this paper, we extended the concept of transmission capacity defined for the single network case to overlaid network case. By considering the mutual interference effect across two overlaid networks, i.e., the PR network vs. the SR network, we derived the transmission capacities for these two networks and studied their tradeoffs. Different from the previous approach for the single network case, we resorted to obtain the asymptotic solutions for these capacities. The results

VI. CONCLUSIONS

In this paper, we show the tradeoff between the normalized transmission capacity of the PR network \( C_{\epsilon_1} / R_0 \) and that of the SR network \( C_{\epsilon_1} / R_1 \) when \( \Delta \epsilon \) changes as an intermediate variable. We see that \( C_{\epsilon_1} \) decreases over \( \epsilon_1 \), which verifies the result in Section IV.
showed that by letting a SR network coexist with a legacy PR network, the spectrum efficiency per unit area could be increased significantly. Although we focused on a simple pathloss channel model with single-hop transmissions, the results are meaningful and motivating us to study more complex cases in the future work.

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