Dispersion relation of the $\rho$ meson in hot and dense nuclear matter

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The dispersion relation of $\rho$ meson in both timelike and spacelike regimes in hot and dense nuclear medium is analyzed and compared with that of $\sigma$ meson based on the quantum hadrodynamics model. The contribution of free nucleons at $T=0$ is analyzed through one-loop NN interactions due to different Dirac- and Fermi-sea contributions at finite temperature and density.

Heavy-ion collision physics has stimulated intense investigations of the properties of strongly interacting, hot and dense nuclear matter\textsuperscript{1}. Among the proposed signals for detecting quark-hadron phase transition, dileptons and photons are considered to be the clearest ones because they can penetrate the medium almost undisturbed and reflect the property of the fireball formed in the initial stage of collisions\textsuperscript{2}. Furthermore, the dileptons from the decay of light vector mesons can be considered as possible signals of the partial chiral symmetry restoration. Especially, the property of $\rho$ in hot and dense environment has attracted much attention in the literature due to its relatively larger decay width compared with $\omega$ and $\phi\textsuperscript{3,4,5}$. It is interesting that the $\rho$ mass decreasing mechanism can be used to explain the low invariant mass dilepton enhancement in central $A-A$ collisions observed by CERES-NA45\textsuperscript{6,7,8}.

From the point of view of many-body theory, the collective effect of medium on a meson is reflected by its full propagator, which determines its dispersion relation as well as the response to the external source\textsuperscript{9,10,11}. Due to the broken Lorentz symmetry, the dispersion relations for longitudinal and transverse modes of vector mesons are different. However, the timelike and spacelike regimes of the pole positions of their in-medium propagators. The in-medium nucleon excitation.

We start from QHD-I to obtain the effective nucleon mass $M_N^*$ and effective chemical potential $\mu^*$ for discussing the in-medium meson property. In the relativistic Hartree approximation, the self-consistent equations for $M_N^*$ and $\mu^*$ can be written as\textsuperscript{10,17}

\begin{align}
M_N^* - M_N &= -\frac{g_\pi^2}{m_\pi^2} \frac{4}{(2\pi)^3} \int d^3p \frac{M_N^*}{\omega} [n_B + \bar{n}_B] \\
&\quad + \frac{g_\pi^2}{m_\pi^2} \frac{1}{\pi^2} \left[ M_N^4 \ln \left( \frac{M_N^*}{M_N} \right) - M_N^2 (M_N^* - M_N) \\
&\quad - \frac{5}{2} M_N^2 (M_N^* - M_N)^2 - \frac{11}{6} (M_N^* - M_N)^3 \right], \\
\mu^* - \mu &= -\frac{g_\rho^2}{m_\rho} \frac{\rho_B}{m_\rho^2},
\end{align}

where $\omega = \sqrt{M_N^*^2 + \rho^2}$ is the nucleon energy and the baryon density $\rho_B$ is defined by

$$\rho_B = \frac{4}{(2\pi)^3} \int d^3p [n_B - \bar{n}_B],$$

with $n_B(\bar{n}_B)$ being the Fermi-Dirac distribution functions for (anti-)baryons, respectively. The coupled equations can be solved numerically with the parameters determined by fitting the binding energy at normal nuclear density $\rho_0$ given in Table\textsuperscript{10}. The $M_N^*$ decreases with increasing density $\rho_B$ at fixed temperature(see Fig\textsuperscript{11}), analogously to the result of mean field theory neglecting the vacuum fluctuations\textsuperscript{10}. The effective chemical potential $\mu^*$ will affect the properties of mesons indirectly through the distribution functions.

In Minkowski space, the polarization tensor $\Pi^{\mu\nu}(k)$ of $\rho$ can be divided into two parts with the standard projection tensors $P_L^\mu$ and $P_T^\mu$ according to

$$\Pi^{\mu\nu}(k) = \Pi_L(k)P_L^\mu + \Pi_T(k)P_T^\mu$$

with

$$\Pi_L(k) = \frac{k^2}{k^2}\Pi^{00}(k), \quad \Pi_T(k) = \frac{1}{2} P_T^{ij}\Pi_{ij}(k).$$

With the effective Lagrangian for $\rho NN$ interactions\textsuperscript{18}

$$\mathcal{L}_{\rho NN} = g_{\rho NN} \left[ \bar{\Psi}_a \gamma^\mu \tau^a \Psi V_\mu^a - \frac{\kappa_\rho}{2M_N} \bar{\Psi} \gamma^\mu \tau^a \sigma_{\mu\nu} \partial^\nu V_\mu^a \right],$$
where $V^\mu_\rho$ is the $\rho$ meson field and $\Psi$ the nucleon field, the polarization tensor is given in random phase approximation (RPA) by

$$\Pi^{\mu\nu}(k) = 2g^2_\rho NN T \sum_{p_0} \int \frac{d^3P}{(2\pi)^3} \text{Tr} \left[ \Gamma^\mu(k) \frac{1}{p-M_N} \Gamma^\nu(-k) \frac{1}{(p-k)-M_N} \right],$$

with $\Gamma^\mu = \gamma^\mu + (ik_\rho/2M_N)\sigma^{\mu\nu}k_\nu$. The temperature and effective chemical potential are hidden in the zero-component of nucleon momentum via $p_0 = (2n+1)\pi T i + \mu^*$. With the residue theorem, one can separate the polarization tensor into two parts

$$\Pi^{\mu\nu}(k) = \Pi_{F}^{\mu\nu}(k) + \Pi_{D}^{\mu\nu}(k),$$

where $\Pi_{F}^{\mu\nu}(k)$ corresponds to the particle-antiparticle contribution of the Dirac sea at $T=0$ and $\Pi_{D}^{\mu\nu}(k)$ arises from the particle-hole contribution\[14\ [22] [23]. The various components of $\Pi_{D}^{\mu\nu}(k)$ are listed in the appendix and $\Pi_{F}^{\mu\nu}(k)$ can be found in Ref.[16].

For vector meson excitation in the medium, the dispersion relations determined by the pole positions of the full propagator $D^{\mu\nu}$ for longitudinal and transverse branches are different, while the pole masses determined by taking the limit $\Pi_{L,T}(k_0,|k| \to 0)$ for $L$ and $T$ modes are consistent\[16]. As pointed out in Ref.\[13\,], the screening mass determined by $\Pi_L(0,|k| \to 0)$ can be related to the isospin fluctuations. In general case including the Dirac sea contribution and the vacuum mass $m_\rho$, the screening masses are defined by the pole positions of full propagators related with the finite momentum self-energy $\Pi(0, k)$\[12\ [13\]

$$k^2 + m_\rho^2 + \Pi_{L,T}(0, k) = 0.$$  

The screening (Debye) masses $M_D = -i|k|$ are the inverse Debye screening lengths and reflect the collective effects of medium, i.e., the damping characteristic $e^{-|k|x}$ of the excitations with purely imaginary wave numbers. The self-consistent numerical results for $M_D$ determined by\[8\] are indicated in Fig[11] where the effective nucleon mass $M_N^*$ and the effective pole mass of $r$ are also shown for comparison.

It is interesting to discuss the in-medium property of scalar meson $\sigma$ with QHD and compare it with the vector mesons. At zero temperature, the property of $\sigma$ has been discussed in Refs.\[22\ [23\]. As pointed out in the introduction, the screening mass of $\sigma$ in spacelike limit has been discussed at zero temperature by considering the one-loop $NN^{-1}$ excitation with free nucleon gas\[14\]. At finite temperature, the $\sigma$ meson self-energy with RPA is

$$\Pi_\sigma(k) = 2g^2_\sigma T \sum_{p_0} \int \frac{d^3P}{(2\pi)^3} \text{Tr} \left[ \frac{1}{p-M_N} \frac{1}{(p-k)-M_N} \right],$$

which can be reduced to

$$\Pi_\sigma(k) = \frac{3g^2_\sigma}{2\pi^2} \left[ 3M_N^* - 4M_N^{-2} M_N + M_N^2 \right]$$

$$\left( (M_N^2 - M_N^{-2}) \int_0^1 \ln \frac{M_N^2 - x(1-x)k^2}{M_N^2} dx \right)$$

$$- \left( \int_0^1 (M_N^2 - x(1-x)k^2) \ln \frac{M_N^2 - x(1-x)k^2}{M_N^2} dx \right)$$

$$+ \frac{g^2_\sigma}{\pi^2} \int \frac{p_0 dp_0}{\omega} \left( \rho_B + \bar{\rho}_B \right) \left[ 2 + \frac{k^2 - 4M_N^*}{4p_0 |k|} (a + b) \right],$$

where

$$a = \ln \frac{k_0^2 - 2p|k| + 2k_0\omega}{k_0^2 + 2p|k| - 2k_0\omega}, \quad b = \ln \frac{k^2 - 2p|k| + 2k_0\omega}{k^2 + 2p|k| + 2k_0\omega}$$

with $k^2 = k^2 - k^2$. It is necessary to note again that here we discuss the full propagator $D_\sigma$ with the in-medium nucleons. The effective masses of $\sigma$ meson defined analogously to those of $r$ are also displayed in Fig[11].

The pole masses $m_\rho^*$, $M_N^*$, and $m_\sigma^*$ versus temperature are indicated in Fig[11]. Due to the tensor(magnetic) coupling and the relatively smaller coupling constant $g_{\rho NN}$ compared with $\omega$ meson, the invariant in-medium mass $M_\rho = \sqrt{k_0^2 - k^2}$ is almost a constant in the timelike region. The dispersion relation curves for the longitudinal

![FIG. 1: Effective masses as functions of scaled density at temperature $T = 100$ MeV. The solid line represents the effective nucleon mass(labeled as $N$). The dotted lines are for pole masses($\rho$ and $\sigma$, respectively), dot-dashed for transverse screening mass ($T$). The dashed lines indicate the longitudinal screening mass($L$) of $\rho$ and screening mass($\sigma^*$) of $\sigma$, respectively.](image-url)
and transverse branches almost coincide and can only be separated from each other in the spacelike screening region. For comparison, the spacelike tensors \( \Pi(0, k) \) of \( \rho \) and \( \sigma \) are shown in the lower panel of Fig. 2. For \( \rho \), in the limit \( |k| \to 0 \) the Dirac-sea and the tensor coupling contributions vanish, and the Fermi sea contributes only to the longitudinal mode. Therefore, the screening mass determined by the spacelike limit of \( \Pi(0, k) \) will be very different from what we showed in Fig. 1. The difference between the \( L \) and \( T \) modes dominated by the Fermi-sea contribution increases with increasing momentum \( K = |k| \). For \( \sigma \), \( \Pi(0, k) \) contains both Dirac- and Fermi-sea contributions in the spacelike limit. It is the Fermi-sea contribution which leads to a negative \( \Pi(0, k) \) in the low \( |k| \) region.

In summary, we have analyzed the dispersion relations of \( \rho \) and \( \sigma \) in hot and dense nuclear environment in the framework of QHD in timelike and spacelike regimes. The pole and screening masses of \( \rho \) are found to decrease with increasing density. Although the pole mass of \( \sigma \) decreases in the low density region, the screening mass behaves very differently from those of \( \rho \) at finite temperature and density. This difference is attributed to the corresponding Dirac- and Fermi-sea contributions.

**TABLE I:** Parameters of QHD-I determined at normal nuclear density \( n_0 = 0.1484 \text{fm}^{-3} \). The masses are in (MeV).

| \( g_{\sigma}^2 \) | \( g_{\rho N N}^2 \) | \( m_\sigma \) | \( m_\omega \) | \( g_{\rho NN} \) | \( k_\rho \) | \( m_\rho \) | \( M_N \) |
|----------------|----------------|---------|---------|-------------|--------|---------|--------|
| 54.289         | 102.770        | 458     | 783     | 2.63        | 6.1    | 770     | 939    |

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**APPENDIX A**

The ingredients of \( \Pi_{\mu \nu}^D \) with the similar expressions of \( a \) and \( b \) in (3) are

\[
\Pi_{ij}^D(k) = \Pi_{1D}^0(k) + \Pi_{2D}^0(k) + \Pi_{3D}^0(k),
\]

\[
\Pi_{1D}^0(k) = 2 \left( \frac{g_{\rho NN}}{2\pi} \right)^2 \left( \frac{k_\rho}{2M_N} \right)^2 \int \frac{p^2 dp}{\omega} \frac{\omega(n_B + \bar{n}_B)}{2p|k|} [4 + \frac{k^2 - 4\omega^2 + 4\omega^2}{2p|k|}] a + (\omega \to -\omega),
\]

\[
\Pi_{2D}^0(k) = \frac{k^2|k|^2}{k^2} \Pi_{1D}^0(k);
\]

\[
\Pi_{ij}^D(k) = (A_1 + A_2 + A_3) \delta^{ij} + (B_1 + B_2 + B_3) \frac{k_i k_j}{k^2},
\]

\[
A_1 = \left( \frac{g_{\rho NN}}{2\pi} \right)^2 \int \frac{p^2 dp}{\omega} \frac{\omega(n_B + \bar{n}_B)}{2p|k|} \frac{4(k^2 + k_0^2)}{|k|^2}
\]

\[
- \frac{k^4 - 4\omega^2 + 4\omega^2}{2p|k|^2} a - (\omega \to -\omega),
\]

\[
A_2 = \frac{k^2}{k^2} \Pi_{2D}^0,
\]

\[
A_3 = -k^2 \left( \frac{g_{\rho NN}}{2\pi} \right)^2 \left( \frac{k_\rho}{2M_N} \right)^2 \int \frac{p^2 dp}{\omega} \frac{\omega(n_B + \bar{n}_B)}{2p|k|^2} \left[ \frac{4k^2}{k^2} + \frac{4k^2 \omega^2}{2p|k|^2} (\omega - k_0) + \frac{4(k^2 - 2k_0^2)}{2p|k|^2} a + (\omega \to -\omega) \right],
\]

\[
B_1 = \left( \frac{g_{\rho NN}}{2\pi} \right)^2 \int \frac{p^2 dp}{\omega} \frac{\omega(n_B + \bar{n}_B)}{2p|k|^2} \left[ -\frac{4(k^2 + 3k_0^2)}{k^2}
\]

\[
- \frac{k^4 - 3k_0^4 (\omega - k_0) + 2k^2 (k_0^2 + 2p^2 - 2k_0^2)}{2p|k|^2} a + (\omega \to -\omega) \right],
\]

\[
B_2 = \Pi_{2D}^0(k).
\]
\[ B_3 = \left( \frac{g_{\rho NN}}{2\pi} \right)^2 \left( \frac{k_\rho}{2 M_N} \right)^2 \int \frac{p^2 dp}{\omega} \left( n_B + \bar{n}_B \right) \left[ \frac{4(k_0^4 + k^4)}{k^2} \right. \\
+ \left. \frac{k^2(2k_0^4 + k^4)}{2} + 4k_0 k^2(k_0^2 + k^2) \omega - 4(2k^2 + k^2)k^2 p^2 + 4(2k_0^4 - k_0^2 k^2 + k^4) \omega^2}{2p|k|^3} \right] a + (\omega \to -\omega) \].

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