Oration for Andrew Wiles

Fanfare

We honour Andrew Wiles for his supreme contribution to number theory, a contribution that has made him the world’s most famous mathematician and a beacon of inspiration for students of math; while solving Fermat’s Last Theorem, for 350 years the most celebrated open problem in mathematics, Wiles’s work has also dramatically opened up whole new areas of research in number theory.

A love of mathematics

The bulk of this eulogy is mathematical, for which I make no apology. I want to stress here that, in addition to calculations in which each line is correctly deduced from the preceding lines, mathematics is above all passion and drama, obsession with solving the unsolvable. In a modest way, many of us at Warwick share Andrew Wiles’ overriding passion for mathematics and its unsolved problems.

Three short obligatory pieces

Biography  Oxford, Cambridge, Royal Society Professor at Oxford from 1988, Professor at Princeton since 1982 (lamentably for maths in Britain). Very many honours in the last 5 years, including the Wolf prize, Royal Society gold medal, the King Faisal prize, many, many others.

Human interest story  The joy and pain of Wiles’s work on Fermat are beautifully documented in John Lynch’s BBC Horizon documentary; I particularly like the bit where Andrew takes time off from unravelling the riddle that has baffled the world’s best minds for 350 years to tell bed-time stories to little Clare, Kate and Olivia.
Predictable barbed comment on Research Assessment  It goes without saying that an individual with a total of only 14 publications to his credit who spends 7 years sulking in his attic would be a strong candidate for early retirement at an aggressive British research department.

Fermat–Wiles in three minutes

Fermat’s Last Theorem: A perfect cube cannot be written as the sum of two perfect cubes, a perfect fourth power cannot be written as the sum of two perfect fourth powers, and likewise, a perfect nth power cannot be written as the sum of two perfect nth powers. In other words, for any \( n > 2 \), the equation

\[
a^n + b^n = c^n
\]

does not have any integer solutions with \( a, b, c \neq 0 \).

Over the 350 years since Fermat’s celebrated margin, any number of mathematicians have tried their hands at this, from 10 year olds in public libraries through to the most distinguished professors. A popular approach is to argue by contradiction: if \( a, b, c \) are nonzero integers satisfying Fermat’s equation \((*)\), you try to argue that \( a, b, c \) are very special, in fact eventually so special that they can’t exist. Any prime dividing the right-hand side of Fermat’s equation \((*)\) divides it \( n \) times, and you could try to argue that it can’t also divide the left-hand side \( n \) times. About 150 years ago, a number of people noted that the left-hand side splits as a product of \( n \) factors in the ring of cyclotomic integers, these factors being more-or-less coprime, and thought that they could see a way through from this to a contradiction; in the course of explaining why this approach fails, Kummer invented algebraic number theory and the class group of an algebraic number field, and paved the way for class field theory.

A key twist on the argument by contradiction was invented in the early 1980s by the German mathematician Gerhard Frey: if \( a, b, c \) are nonzero integers satisfying Fermat’s equation \((*)\), consider the equation

\[
y^2 = x(x + a^n)(x + c^n),
\]

where \( a, b, c \) are considered fixed. This equation in \( x, y \) is called an elliptic curve: it is the curve obtained as the graph of the function square root of \( x(x + a^n)(x + c^n) \). (The name “elliptic” comes from the fact that equations of this form arise in Euler’s integral formula for the arc length of an ellipse.)
Just as before, the aim is to argue that Frey’s curve (**) is very special, in fact eventually so special that it can’t exist. (The special thing is that the discriminant of the cubic polynomial on r-h.s. of (**) is \(a^n b^p c^p\), which has many repeated prime factors.) Frey’s idea was immediately taken up by a number of mathematicians, who hoped to exploit the encyclopaedia of results on elliptic curves accumulated since the time of Fermat and Euler.

The deepest fact about elliptic curves, and the essential achievement of Wiles’ work from the mid 1980s, is the Taniyama–Shimura conjecture: every elliptic curve over the integers is “modular”, that is, parametrised by modular forms. A modular form is a function having very strong symmetry with respect to an arithmetic group – it is thus an object of complex analysis, hyperbolic geometry, representation theory, arithmetic and algebraic geometry. It was known from the late 1980s that the Taniyama–Shimura conjecture would imply that the Frey curves do not exist, hence prove Fermat’s last theorem. The received wisdom from the 1970s was that Taniyama–Shimura was most likely to be true, but unlikely ever to be proved. So it might well have remained without Wiles’ 7 year odyssey in his attic. For the details of the proof, I borrow a phrase from Fermat: *Hoc elogium exiguitas non caperet.*

In one sense, all this talk of Fermat and what’s happened over the last 350 years is extremely misleading, because the real impact of Wiles’ work lies in the future. Possibly the single biggest issue in the mathematics of the next century is a vast generalisation of class field theory and of the Taniyama–Shimura conjecture called the *Langlands program:*

> modular forms parametrise the representations of the Galois group of the rational number field.

While it certainly sees off Fermat’s last theorem, Wiles’s work, and its current development at his hands and those of his students and successors, is a searchlight illuminating this maze.

**Presentation**

Mr Chancellor, in the name of the council, I present to you for admission to the degree of Doctor of Science, *honoris causa,*

Andrew Wiles

Miles Reid, Jul 1998