The development of instability of Richtmyer–Meshkov in the interaction of a shock wave with a two-component medium consisting of light and heavy gas

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Abstract. On the basis of the previously formulated mathematical model of the mechanics of a two-speed two-temperature mixture of light and heavy gases, the evolution of a system of two cylinders of heavy gas after the passage of a shock wave through it is numerically investigated. The analysis of the obtained data on the dependence of the spatial expansion on time, as well as the kinematic characteristics of the system showed their satisfactory compliance with the experimental data for all initial separations of the cylinders. The influence of the initial separation of cylinders on the flow morphology is investigated.

1. Problem statement

The Richtmyer–Meshkov instability occurs at the interface of two media when a shock wave passes through it. The effect is interesting both from a fundamental and practical point of view. This instability takes place, for example, in supersonic combustion, laser thermonuclear fusion, in some astrophysical problems. A typical laboratory experimental setting for the study of this instability is the study of the evolution under the action of a plane shock wave of an expanding cylinder (or several cylinders) of a heavy gas flowing through a light gas under the action of gravity [1, 2]. This work is devoted to numerical modeling and study of the instability of Richtmeyer–Meshkov arising under the action of a shock wave in such and similar systems. In particular, the interaction of the shock wave with two columns of heavy gas in the atmosphere of the lung is considered.

The specificity of this problem is that the strongly nonequilibrium state of two gases with different densities is considered. The dynamics of such a system should be described in the framework of a two-fluid model [3]. The speed of the evolving system after passing through the shock wave is three orders of magnitude higher than the speed of the cylinder under the influence of gravity, which allows to solve the problem in a flat formulation, considering the interaction of the shock wave with a drop (drops) of heavy gas in the atmosphere of the lung. In Figure 1, the following notation is used: 1 refers to gas 1, 2 to the mixture of gases 1 and 2, 3 to the shock wave, and 4 to the transition region of mixing.
2. Method of solution

The parameters of the mixture of two different-density gases are described by the equations of the dynamics of mixtures:

\[
\frac{\partial U_i}{\partial t} + \frac{\partial F_i^{(1)}}{\partial x} + \frac{\partial F_i^{(2)}}{\partial y} = W_i,
\]

where

\[
U_i = \begin{bmatrix}
\rho_i \\
\rho_i u_i \\
\rho_i v_i \\
E_i
\end{bmatrix},
F_i^{(1)} = \begin{bmatrix}
\rho_i u_i \\
\rho_i u_i^2 + p_i \\
\rho_i u_i v_i \\
u_i E_i + p_i u_i
\end{bmatrix},
F_i^{(2)} = \begin{bmatrix}
\rho_i v_i \\
\rho_i v_i^2 + p_i \\
\rho_i v_i^2 + p_i \\
v_i E_i + p_i v_i
\end{bmatrix},
\]

\[
W_i = \begin{bmatrix}
0 \\
K(u_j - u_i) \\
K(v_j - v_i) \\
K[u_i(u_j - u_i) + v_i(v_j - v_i) + \beta_i (u_j - u_i)^2 + (v_j - v_i)^2] + q(T_j - T_i)
\end{bmatrix},
\]

\[
p_i = km_i T_i, E_i = \rho_i \left( e_i + \frac{u_i^2 + v_i^2}{2} \right), e_i = \frac{kT_i}{m_i(\gamma - 1)}, \rho_i = m_i n_i, i, j = 1, 2 (i \neq j).
\]

Here \( \rho_i \) is the partial density; \( u_i, v_i \) are the velocity components; \( e_i \) is the internal energy; \( p_i \) is the pressure; \( T_i \) is the temperature; \( m_i \) is the mass of a molecule of the \( i \)-th gas; \( n_i \) is the number density of molecules of the \( i \)-th gas; \( x, y \) are Cartesian coordinates, \( t \) is time; \( k \) is the Boltzmann constant; \( K = 16 \rho_1 \rho_2 \Omega_{12}^{(1)} / (3(m_1 + m_2)) \) is a parameter that characterizes the interaction of the gases; \( \Omega_{12}^{(1)} \) is the collision integral; \( \beta_i = m_i T_i / (m_1 T_1 + m_2 T_2) \); \( q = 3m_i K / (m_1 + m_2) \); \( E_i \) is the total energy of the \( i \)-th component; \( \gamma_i \) is the adiabatic index. The hard-sphere interaction potential is written as

\[
K = \frac{16}{3} \rho_1 \rho_2 \sigma_{12} \sqrt{\frac{k \pi}{2 m_1 T_1 + m_2 T_2}}, \quad \sigma_{12} = \frac{\sigma_1 + \sigma_2}{2}
\]
(σᵢ is the diameter of a molecule of the i-th gas).

As usual, for low (or nonzero) concentrations of the j-th gas, the Euler equations for the pure i-th gas are used, and the parameters of the second gas are determined from the relations

\[
\frac{\partial n_j}{\partial t} + \frac{\partial n_j u_j}{\partial x} + \frac{\partial n_j v_j}{\partial y} = 0, \quad u_j = u_i, \quad v_j = v_i, \quad T_j = T_i
\]

The transition from one system of equations to the other is performed provided that the maximum value of the molar and mass fractions of the i-th gas does not exceed 0.1%:

\[
\max(x_i, \alpha_i) \leq 0.1%,
\]

\[
x_i = n_i / (n_1 + n_2), \quad \alpha_i = \rho_i / (\rho_1 + \rho_2).
\]

The intensity of the shock wave is characterized by the Mach number. The problem is solved in the framework of two fluid inviscid equations of hydrodynamics. Beforehand, to take into account the diffusion taking place in the real system for a drop in a light gas, the diffusion equation is solved, so that at each time the density distribution is smooth. It is shown that the resulting solution is equivalent to the solution of the original problem in a viscous formulation, but such a two-stage solution is much more economical in terms of counting time. Figure 2 shows a comparison of the radial profiles of the molar (as well as mass) SF₆ concentration at \(t d = 10\) ms obtained by these two methods for a solitary droplet. The deviation between the profiles according to the norms \(C\) and \(L₂\), does not exceed 2%.

![Figure 2. Comparison of radial concentration profiles. — — not blurred rectangular profile \((D = 3.1\) mm), O — two-liquid inviscid dynamics, X — diffusion equation.](image)

The computation scheme for the spatial approximation of system (1) is constructed using the flux vector splitting method [5, 6]. In order for the solution to remain monotonic in regions of large gradients, the order of approximation is reduced by a continuous limiter, which provides better agreement with experimental data than the minmod limiter [5]. Furthermore, the implicit approximation of the right-hand sides of the system (1) proposed in [7] is used, which does not require an additional restriction on the time step imposed by the Courant condition.

\[
\frac{\bar{U}_i^{n+1} - U_i^n}{\tau} = -\left(\frac{\partial F^{(1)}(U_i^n)}{\partial x}\right) - \left(\frac{\partial F^{(2)}(U_i^n)}{\partial y}\right) + W(\bar{U}_i^{n+1})
\]

\[
\frac{U_i^{n+1} - U_i^n}{\tau} = \frac{1}{2} \left[\frac{\partial F^{(1)}(U_i^n)}{\partial x} + \frac{\partial F^{(2)}(U_i^n)}{\partial y}\right] + \left(\frac{\partial F^{(1)}(U_i^n)}{\partial x}\right) + \left(\frac{\partial F^{(2)}(U_i^n)}{\partial y}\right) - W(U_i^{n+1}) - W(\bar{U}_i^{n+1})
\]
\[ F^{(1)}(U_i) = F_i^{(1)} = F_i^{(1)+} + F_i^{(1)^-}; \]
\[ F_i^{(1)+} = F_i^{(1)}, \quad F_i^{(1)^-} = 0, \quad \text{if } M_{ix} \geq 1, \]
\[ F_i^{(1)+} = 0, \quad F_i^{(1)^-} = F_i^{(1)}, \quad \text{if } M_{ix} \leq 1, \]

\[ F_i^{(1)^+} = \begin{cases} 
& f_i^\pm \left( (\gamma_i - 1)u_i \pm 2a_i \right) / \gamma_i \\
& f_i^\pm v \\
& f_i^\pm \left( (\gamma_i - 1)u_i \pm 2a_i \right)^2 / \left[ 2(\gamma_i^2 - 1) + v_i^2 / 2 \right] 
\end{cases}, \]

\[ f^\pm = \pm \rho_i a_i \left[ (M_{ix} \pm 1)/2 \right], \quad \text{if } |M_{ix}| \leq 1, \]

where \( M_{ix} = u_i / a_i \), \( a_i \) is the sound speed of the \( i \)-th gas. The expressions for \( F^{(2)}(U_i) \) are obtained similarly in terms of the Mach number \( M_{iy} = v_i / a_i \).

\[ \left( \frac{\partial F^\pm}{\partial x} \right)_{l,m} = \frac{F^\pm_{l+1/2,m} - F^\pm_{l-1/2,m}}{h_x}, \]

\[ F^+_{l+1/2,m} = F^+_{l+1,m} - \frac{s}{4} \left[ (1 - s \cdot \xi) \Delta^+ + (1 + s \cdot \xi) \Delta^- \right] F^+_{l+1,m}, \]

\[ F^-_{l+1/2,m} = F^-_{l,m} + \frac{s}{4} \left[ (1 - s \cdot \xi) \Delta^- + (1 + s \cdot \xi) \Delta^+ \right] F^-_{l,m}, \]

\[ \Delta^+ F_{l,m} = \frac{F_{l+1,m} - F_{l,m}}{h_x}, \quad \Delta^- F_{l,m} = \frac{F_{l,m} - F_{l-1,m}}{h_x} \]

\[ \xi = 1/3, \quad s = \frac{2\Delta^- \Delta^+ + \varepsilon}{(\Delta^-)^2 + (\Delta^+)^2 + \varepsilon}, \quad \varepsilon = 10^{-6}. \]

This scheme ensures second-order accuracy with respect to time and third-order accuracy with respect to spatial variables in the region of smooth flows.

The calculations were carried out in a rectangular region \([x_n, x_k; 0, y_k]\). A grid with a variable cell size was used. In region 5, a small cell size of 0.02 mm was used, and outside this region, the cell size was increased exponentially at a rate of 1.05 to 1.1, which eliminated the reflection effects at the boundaries. The length of these zones was up to 100 cells. To keep the droplet within region 5, the calculations were made in a coordinate system moving at a certain speed \( U_0 \) in the \( x \) direction.

Testing of the modeling technique is carried out on comparison of calculation data with experimental [1, 2]. As a light gas, air was considered, and the drop consisted of SF6. The shock wave Mach number was 1.2. The thermodynamic parameters before the shock wave corresponded to normal conditions.
3. Discussion of results

When a shock wave interacts with two drops, a complex structure consisting of four interacting vortices is generally formed. However, its character depends significantly on the distance $S$ between the centers of the droplets. A decrease in $S$ leads to a decrease in the density gradient in the region between the droplets and, as a consequence, to a decrease in the corresponding vorticity field. The changes in the forming vortex structures with increasing distance between the drops are illustrated in Figure 3. Density isolines are also shown here, in all cases the time of development of structures is the same and is equal to 750 $\mu$s.

![Figure 3. Images of structures at $t = 750 \mu s$. $S/D$=1.2, 1.5, 2.0, 3.0, 4.0.](image)

As can be seen, the structure of the four vortices develops only when $S/D$ is greater than a certain value. Indeed, at $S/D \leq 1.2$, the droplets are so close to each other that in the process of diffusion they are combined into one continuous region and the evolution of the system occurs as well as in the solitary droplet. On the other hand, it is clear that with the increasing distance between the vortices, a situation is eventually realized when the instability on each drop will develop almost independently. However, the vortex interaction long-range and mutual influence of vortices is noticeable at sufficiently large distances between the droplets. Several critical $S/D$ values can be distinguished. First, $S/D = 1.4$, starting from this distance, the droplets have internal vortices (although of very low intensity), resulting in separation of the droplets from each other (see Fig. 3). With a further increase in the $S/D$ parameter, the intensity of the internal vortices increases and aligns with the external ones when the diffusion regions cease to overlap, approximately at $S/D=3.0$. As a result, the upper drop rotates clockwise and the lower one in the opposite direction.

At $S/D = 2.5$, the direction of rotation of each droplet is inverted (second critical distance). This is due to the fact that the intensity of the internal vortices becomes comparable in magnitude with the external ones. But since they are at close range, it is their interaction that prevails. A further increase in the distance between the droplets weakens the interaction of internal vortices, which leads to a decrease in their angular velocity (Fig. 4). It was shown that the decrease in the rotation frequency of vortex structures is described by the ratio: $n = A \cdot (S/D)^{-b}$, where $A = 3160$ Hz and $b = 3.15$. So at $S/D = 24$ the rotational speed will be approximately equal to 0.25 Hz (the third critical distance is conditional). In this case, for the considered time $t = 1000 \mu s$, the rotation angle will not exceed 0.1 degrees.
Figure 4. Dependence of the rotation angle of the upper drop on time for different values of the S/D parameter: O – 5, □ – 6, ∆ – 8, X – 12.

Summary

The development of Richtmeyer–Meshkov instability in the interaction of a plane shock wave with an isolated column and two columns of heavy gas in a light gas atmosphere is numerically studied. To solve the problem of interaction of a passing shock wave with an infinite cylinder of heavy gas – sulfur hexafluoride located in the air, a two-speed continuum model and a developed high-precision numerical technology are used. The use of mathematical modeling technology allows firstly to detail the flow pattern, and secondly to investigate the phenomenon in a wider range of parameters.

Different distances between the centers of the cylinders were considered. We have identified three values (critical distance), in which the nature of the interaction between cylinder changes qualitatively. When the droplets are close together, a slight change in distance can significantly change the nature of the interaction. However, starting from a certain distance value (of the order $S/D = 4.0$) there is a smooth attenuation of the interaction intensity.

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