Analysing Block Stability under Varying Resultant Force for Arbitrary Shapes

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Abstract. Block theory is a basis theory in Rock mechanism taking advantages of analysing finiteness, removability, and mechanical stability of convex blocks under different engineering conditions. However, classic block theory does not give solutions for non-convex blocks which are very common in civil project, such as underground edges, corners, and portals. To enhance availability of block theory, a general algorithm which can analyse stability of blocks with arbitrary shapes is proposed in this paper. In the proposed algorithm, the joint pyramid for block of arbitrary shapes can be computed, sliding mode of block under varying resultant force can be seen immediately from 3D view by means of an interactive visualization mechanism, and parallel or nonadjacent sliding faces of blocks are determined immediately for given resultant force. With this algorithm, blocks of arbitrary shapes can be analysed, and users do not need to interpret graphs of block theory to take advantage of its accuracy and effectiveness. The validation of the proposed algorithm is verified, and it was used to investigate the stability of the left bank rock slope of a dam. The results show that the algorithm is correct, effective, and feasible for use in rock engineering project.

1. Introduction

Block theory [1] has been applied in rock engineering for more than 30 years and got good application effect in some famous civil engineering projects [2, 3]. Block theory has already become fundamental analysis methods in computational rock mechanics. With classic block theory, finiteness, removability, and stability of convex blocks can be analyzed by joint planes and space planes that faces of convex block locate. And mechanical stability of blocks is computed according to removability, frictional angles of joint planes and resultant force applied on the block. Then, suggestions based on analysis result are proposed for design and construction of rock engineering. Joint pyramid (JP) is the core concept of block theory, it is convex pyramid intersected by inward half spaces that faces of blocks stay on. Evacuation pyramid (EP) is like JP that is intersection of evacuation half spaces. Block pyramid (BP) is intersection of joint pyramid (JP) and evacuation pyramid (EP). Blocks generated by tunnel boundary and joints are shown in Figure 1, and key blocks are labelled with joint pyramid code; the label “01” means above an orange joint plane and below a green joint plane. As we can see, removability of blocks can be judged directly by joint pyramid of convex blocks. Figure 2 gives an example of stereographic projection graph of 3 joint set, each joint set is represented by a great circle, and joint pyramids are represented by close areas labeled with joint pyramid codes.
The equilibrium region graph [1] is used to analyze stability of given joint pyramid under different resultant force, e.g., water pressure. Figure 3 gives an example, the regions labelled with single number represents single face sliding mode; the regions labelled with two number represents double face sliding mode; the region labelled with $S$ represents stable mode; the region labelled with $0$ represents lifting mode, i.e., block move without sliding faces. The dashed loops in graph are friction contours. The closer the contour is to the region 0, the higher friction angle is required to keep block stable.

There are two ways to analyze blocks with block theory: (1) get occurrences of joint disks by field survey, find dangerous joint pyramids, and estimate maximum key blocks. Subsequently, volumes, weight and shapes of the biggest dangerous key blocks can be obtained. (2) generate blocks by algorithms based on computational geometry [4], get orientation of joint planes that faces of blocks stay on, and judge stability of these blocks. Because the occurrences of joints are complex and difficult to obtain, stochastic approaches are usually adopted to generate joints [5]. These results of block theory are important references for design and support in rock engineering project.

Although block theory is a powerful method, it has become a sophisticated analysis tool that few researchers could take advantage of. To make block theory more applicable, researchers focused on nonconvex blocks [6, 7] and more complex application scenarios [8, 9]. To make block theory easier to use [10] and understand, three-dimensional visualization for stereographic projection [11] and equilibrium region graph [12] have been implemented. But graphs of block theory still need to be interpreted by engineers to understand stability of 3D blocks, which needs long time of practicing, complex shapes of blocks exacerbate this situation, thus there is still a gap between method and application.

We focus on finite blocks in this paper, a general algorithm to analyze stability of block is proposed, as described in section 2. It computes joint pyramid for blocks of arbitrary shapes based on the classical block theory, by which removability of blocks can be computed. Besides, an interactive block stability analysis mechanism is achieved that faces of blocks are mapped to regions of graph, with which we can take advantages of the equilibrium region graph but saves time of practicing. Validation and conclusion will be given in section 3, section 4, and section 5. By means of the method proposed in this paper, dangerous key blocks of arbitrary shapes, such as those at the edges, corners, and portals of underground chambers can be identified by engineers quickly.

2. Algorithm implementation
To make block theory suitable for blocks of any shapes, this paper proposed a general algorithm. It simplifies faces of blocks of arbitrary shapes into joint planes, compute joint pyramid for these joint planes, and show sliding mode of blocks in 3D block view and equilibrium region graph at same time. The algorithm turns block theory from artworks to industrial products that everyone can take advantages of. As shown in Figure 4, it includes 5 steps:

1. group and tag faces of block by normal vectors and tag each face with an id according to its normal vector, see section 2.1.
2. compute joint pyramid based on normal vectors from first step, this step builds up relation between faces of block and joint pyramid, see section 2.2.
3. draw equilibrium region graph [1].
4. map regions in equilibrium region graph to joint pyramid, then relationship between regions and faces of blocks are built up mediately, see section 2.3.
5. compute resultant force according to interactive cursor position, and judge which region should be picked. When section is picked, region related faces of blocks will be highlighted too, see section 2.4.

2.1. Grouping and tagging block faces
To analyse stability of blocks, engineers must simplify faces of blocks into joint planes first, which is tedious and complex work. To make block theory easier to use, the proposed algorithm groups and tags faces of block by normal vectors and generates joint planes. It has following steps:
1. prepare a normal array \( N \) and normal id array \( NI \).
2. iterate faces of block and check whether this normal has already exists in \( N \).
3. if the normal does not exist, we identify current face normal with size of \( N \) and add new normal into \( N \), otherwise just use id of the existing normal.

The key steps are shown in Figure 5.

![Figure 4. Algorithm process.](image)

![Figure 5. Grouping and tagging faces of block.](image)

2.2. Computing the joint pyramid
According to classic block theory, a convex block is removable if and only if its joint pyramid is not empty. But blocks have various shapes in practice, we must find a general way to compute joint pyramid. From the work [13], to compute joint pyramid of blocks of arbitrary shapes, we only need to compute intersection of joint pyramids of all convex sub-blocks. i.e., the joint pyramid of non-convex block is the minimum pyramid that belongs to joint pyramids of any other removable convex sub-blocks. There are four steps to compute the joint pyramid for any number of joint planes even with concave edges, as shown in Figure 6:
1. get all inward normal of joint planes.
2. compute cross-product for every two normal vectors and save cross-product vectors if the dot-product between itself and any other normal vectors are all positive, then do the same procedure for the reverse vector of the cross-product vector.
3. reserve face pairs at same time for all cross-product vectors saved in step 2.
4. iterate all face pairs, sort them into end-to-end order, and reverse any face pairs if needed.
2.3. Tagging regions in graph to faces of block
As described above, the equilibrium region graph needs long time of practicing to read, and the better way is to show sliding mode directly with 3D block. Thus, the algorithm map faces of block to regions in graph via joint pyramid. Then, we get sliding faces according to tagged face normal vector id. It not only shows the result intuitively, but also makes full use of the characteristics of the graph. Regions in graph have 4 kinds: lifting region, stable region, single face sliding mode regions and double faces sliding mode regions. To unify process, we store regions in order: lift region $R_0$, stable region $R_S$, $R_{12}$, $R_2$, $R_{23}$, $R_3$, ..., $R_n$, $R_1$, where $R$ means Region, foot index is index of joint planes. After regions are ordered, whenever any region is picked, the corresponding joint planes of joint pyramid can be determined at same time.

2.4. Picking region on graph for given force
Due to block stability varies with different resultant forces, the most intuitive stability visualization way is showing resultant force and block sliding mode together in 3D view. Thus, after regions established, the algorithm gives the solution to pick the region for current cursor position. As we see, regions in equilibrium region graph are close areas consist of arcs. To find out whether a point is in given region, we just need to check if the resultant force is in the convex pyramid of given region. More specifically, (1) we get edge vectors of the convex pyramid corresponding to the given region. (2) compute normal of planes of the convex pyramid by cross every two adjacent vectors. (3) check if the unit vector corresponding to the cursor position is below any planes, if it does, the vector is out of this convex pyramid, otherwise the vector is in the convex pyramid. The key steps of these judgements are shown in Figure 7.

3. Validation
Nonconvex blocks are common in engineering, we test algorithm by concave block with one concave edge in this section. Faces of block are tagged with $f_1, f_2, f_3, f_4, f_5, f_6$, and $f_7$, and normal ids of faces are 1, 2, 3, 4, 5, 6 and 6. We choose face $f_5$ as space plane, and all other faces are used as joint planes, as shown in Table 1. The block is at double-faces sliding mode along face $f_1$ and $f_3$ under gravity, and the region $R_{13}$ is highlighted, as shown as Figure 8.
### Table 1. Occurrences of faces of concave block

| Face | Dip (◦) | Dip dir. (◦) | Normal id | Joint plane | Half space |
|------|---------|--------------|-----------|-------------|------------|
| f₁   | 58      | 140          | 1         | yes         | Below      |
| f₂   | 57      | 100          | 2         | yes         | Below      |
| f₃   | 90      | 330          | 3         | yes         | Below      |
| f₄   | 90      | 30           | 4         | yes         | Below      |
| f₅   | 68      | 240          | 5         | no          | Below      |
| f₆   | 0       | 0            | 6         | yes         | Above      |
| f₇   | 0       | 0            | 6         | yes         | Above      |

![Figure 8](image-url) Displaying sliding model both in equilibrium region graph and 3D view.

### 4. Engineering applications

The proposed algorithm is used to study the stability of the left bank of a dam in China. The left bank slope is excavated on a large scale, the average occurrence of the strata is 268° 39’ (dip direction ∠ dip), with some interlayer shear zones developed. There are three main sets of joints developed in the rock mass, as shown in Table 2. The joints are straight and closed, with a calcite film attached.

According to the qualitative analysis of engineering geology, the slope is a near-forward slope, and the blocks exhibit a single-face sliding mode along the strata. Based on the analysis of the block theory, when slope outer shear surface follows NWW group and slope inner shear surface follows NEE group, the biggest key block is formed as shown in Figure 9. The volume of this block is 45808 m³, and it is at single-face sliding mode under gravity along the interlayer shear zone, as shown in Figure 10.

### Table 2. Physical and mechanical properties of the structural surfaces

|                  | Interlayer shear zone | NWW    | NEE    | NNE    |
|------------------|-----------------------|--------|--------|--------|
| Average orientation | 268° 39’             | 192° 75’ | 350° 70’ | 97° 57’ |
| Coefficient of friction | 0.18–0.25             | 0.35   | 0.35   | 0.35   |

![Figure 9](image-url) Possible biggest key block.  
![Figure 10](image-url) Sliding mode of biggest key block under gravity.

### 5. Conclusion

In this paper, we propose a general block stability analysis algorithm. It has 3 advantages: firstly, faces of blocks are grouped by normal that parallel or nonadjacent sliding faces can be found immediately; secondly, joint pyramid for block of arbitrary shapes can be computed which greatly enhance availability.
of block theory; thirdly, regions of equilibrium region graph are mapped to faces of block via joint pyramid that sliding mode judged by graph can be intuitively shown in 3D block view. With this interactive method, engineers could analyse block stability and find key blocks of arbitrary shapes at any positions without costing of learning graphs of block theory, which could greatly promote the development and application of block theory.

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