Generalized Parton Distributions and Nucleon Form Factors

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Abstract

The Dirac and Pauli form factors of the proton and neutron are obtained in the framework of the generalized parton distributions (GPDs) with some simple momentum transfer dependence. It is shown that both sets of the existing experimental data on the form factors, obtained by the Rosenbluth and polarization transfer, can be described by changing only the slope of the GPDs $E$. The description of neutron form factors is substantially better when the proton data obtained by the studies of polarization transfer are used.

1 Introduction

The determination of the hadron structure is related with our understanding of the non-perturbative properties of the QCD. Generalized parton distributions (GPDs) [1] for $\xi = 0$ provide information about the distribution of the partons in impact parameter space [2]. It is correlated with $t$-dependence of GPDs. Now we cannot obtain this dependence from the first principles; instead, it may be obtained from the phenomenological description of the nucleon electromagnetic form-factors.

Following [3], we limit ourselves to the case of GPDs with $\xi = 0$ corresponding to the non-forward parton densities so that the form factors can be represented as

$$F_1^q(t) = \int_0^1 dx \, \mathcal{H}^q(x,t), \quad F_2^q(t) = \int_0^1 dx \, \mathcal{E}^q(x,t),$$

(1)

We assume the validity of Gaussian ansatz which was used in [3] to describe the form factors of proton. However, this ansatz leads to a faster decrease in $F_1$ at larger momentum transfer. Although this region is, strictly speaking, outside the domain of validity of QCD factorization involving GPDs, one may consider also the problem of $t$-dependence of GPDs at large $t$ [4]. It was shown that at large $x \to 1$ and momentum transfer the behavior of GPDs requires a (larger) power dependence on $(1-x)$ in the $t-$ dependent exponent:

$$\mathcal{H}^q(x,t) \sim \exp[a \, (1-x)^n \, t] \, q(x).$$

(2)

with $n \geq 2$. It was noted that $n = 2$ naturally gives rise to Drell-Yan-West duality between parton distributions at large $x$ and the form factors. Various more elaborated parameterizations were considered later, see e.g. [5].
2 Momentum transfer dependence of GPDs and proton form factors

Our proposal consists in the attempt to find a simple ansatz which will be good enough to describe the form factors of the proton and neutron taking into account a number of new data that have appeared in the last years. Let us keep the simple Gaussian ansatz but using some new conditions. To support the proposal [3] and [4] we chose the t-dependence of GPDs in the form

\[ H^q(x, t) = q(x) \exp\left[a_+ \frac{(1-x)^2}{x^b} t\right]; \quad E^q(x, t) = E^q(x) \exp\left[a_- \frac{(1-x)^2}{x^b} t\right], \]  

with the free parameters \( b = 0.4 \) (determined mostly by the power 2 of the factor \( 1-x \)), \( a_+ \) (\( a_+ \) - for \( H \) and \( a_- \) - for \( E \)). All these parameters were fixed by analyzing the data on the ratio of proton Pauli and Dirac form-factors. The function \( q(x) \) was taken in the same normalization point \( \mu^2 = 1 \text{ GeV}^2 \) as in [6], which is based on the MRST2002 global fit [7]. In all our calculations we restricted ourselves to the contributions of \( u \) and \( d \) quarks in \( H^q \) and \( E^q \) with \( E^u(x) = k_u/N_u(1-x)^{\kappa_1} u(x), \quad E^d(x) = k_d/N_d(1-x)^{\kappa_2} d(x) \), (where \( \kappa_1 = 1.53 \) and \( \kappa_2 = 0.31 \) [6]) According to the normalization of the Sachs form factors, we have \( k_u = 1.673, \quad k_d = 2.033, \quad N_u = 1.53, \quad N_d = 0.946 \). The parameters \( a_+ = 0.675 \) and \( a_- \) correspond to the two experimental methods of the determination of the ratio of the Pauli and Dirac form factors. Below we consider version (I - polarization transfer method) leading to \( a_- = 0.59 \) and version (II - Rosenbluth separation) leading to \( a_- = 0.7 \).

The proton Dirac form factor, calculated in this work and multiplied by \( t^2 \), is shown in Fig.1a in comparison with the other works ([6],[8]) and experimental data. One can see, that our calculations sufficiently well reproduces the behavior of experimental data not only at high \( t \) but also at low \( t \).

The ratio of the Pauli to the Dirac proton form factors multiplied by \( t \) is shown in Fig.1b. There are two different sets of experimental data. Firstly, one may extract the form factors of the proton from the unpolarised differential cross section by the Rosenbluth

![Figure 1a](image1a.png)  
**Figure 1a.** Proton Dirac form factor multiplied by \( t^2 \) (hard line - the present work, dot-dashed line - [3]; long-dashed line - [6]; the data for \( F^p \) are from [9].

![Figure 1b](image1b.png)  
**Figure 1b.** Ratio of the Pauli to Dirac proton form factors multiplied by \( t \) (hard and dot-dashed lines correspond to version (I) and (II)) of the present work, dotted line - [10]; long-dashed line - [6]); the data are from [11].
method. The other method uses the polarized differential cross section to obtain these form factors. In our model we can obtain the results of both methods by changing the slope of $E$. So we examined two versions differing by the slopes $a\_\_$. One can now use the information on the neutron form factors in order to choose the more realistic version.

3 Neutron form factors

Using the model developed for proton we can calculate the neutron form factors. For that the isotopic invariance can be used to relate the proton GPDs to the neutron ones, Hence, we do not change any parameters and preserve the same $t$-dependence of GPDs as in the case of proton.

Again, we take two values of the slope $a\_\_\_\_\_$ as in the case of the proton form factors with the same size, which correspond to version (I) and version (II) below.

Our calculation of the $G^n_E$ is shown in Fig. 2a. Evidently, the first version is in better agreement with experimental data. Therefore, neutron data support the results obtained by polarization transfer method.

This conclusion is supported by the calculations of $G^n_M$ shown in Fig.2b. In this case, it is clearly seen that our parameterization normalized using the proton form-factors ratio from the polarization experiments describes these neutron data quite well.

4 Conclusions

The proposed version of Gaussian $t$-dependence of GPDs reproduces the electromagnetic structure of the proton and neutron sufficiently well. We show that changing only the slope parameters $a\_\_\_\_$ of $E$ it is possible to obtain both the Rosenbluth and Polarization data on the ratio of Pauli and dirac electromagnetic proton form-factors. The description of neutron form-factors is essentially better with the slope parameter fitted to proton polarization transfer data. This is in accordance with the recent theoretical analysis [14].

Figure 2a. $G^n_E$ (hard and dot-dashed lines correspond to version (I) and (II)); experimental data from [12].

Figure 2b. $G^n_M$ (hard and dot-dashed lines correspond to version (I) and (II)); experimental data from [13].
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