Abstract—This article analyzes the global stability of synchronous reference frame phase-locked loops (SRF-PLLs) from a large signal viewpoint. First, a large-signal model of SRF-PLL is accurately established, without applying any linearization method. Then, according to the phase portrait and Lyapunov argument, the global performance of SRF-PLL is discussed in the nonlinear frame. Compared with the small-signal analysis methods, the proposed analysis, not relying on the small-signal model and linearization method, provides a global discussion of the SRF-PLL performance. The contributions of this article are as follows. First, it is found that SRF-PLL has infinite equilibrium points, including stable points and saddle points. Second, it provides a way to divide the global region of SRF-PLL into many small regions. In each small region, the SRF-PLL only has one stable equilibrium point. And for any initial states \((\tilde{\theta}(t_0), \tilde{\omega}(t_0))\) in a small region, all states \((\tilde{\theta}(t), \tilde{\omega}(t))\), \(t > t_0\) will remain in this small region, and SRF-PLL will converge to the unique stable equilibrium point of this small region. Third, by dividing the global region of SRF-PLL into many small regions, it is found that when the frequency of grids varies largely, the SRF-PLL will converge to a new equilibrium point that is far away from the original equilibrium point. It is the reason why the frequency convergence of SRF-PLL has many oscillations and SRF-PLL has a rather slow dynamic, when the frequency changes largely. The experimental results are provided to verify the proposed global stability analysis of SRF-PLL.

Index Terms—Global stability analysis, grid synchronization, large-signal model, phase-locked loop, synchronous reference frame.

I. INTRODUCTION

RECENTLY, literature [1], [2] provide many grid synchronization methods, such as second-order generalized integrator-based PLLs (SOGI-PLLs) [3]–[5], transfer delay phase-locked loops (TD-PLLs) [6], and the adaptive observers [7]. Synchronous reference frame phase-locked loop (SRF-PLL) is a classical grid-synchronization method and has been widely used in many kinds of applications. For example, with the help of SRF-PLL, many power electronic converters achieve the connections with utility grids, which are 50- or 60-Hz fixed-frequency grids [8], [9].

SRF-PLL estimates the phase and frequency of the grid voltage, and it provides the information to achieve grid synchronization of grid-tied converters. The structure of SRF-PLL is shown in Fig. 1, and it contains three parts, including the phase detector (PD), the loop filter (LF), and the voltage-controlled oscillator (VCO) [10]. In details, the output of the PD contains the phase error information. LF is a PI controller and it attenuates the high-frequency components from the PD output. VCO generates the estimated phase, which follows the actual phase of the grid voltage.

Recently, it is found that SRF-PLL causes an important effect on the stability of power grids, and several papers have discussed the relationship between SRF-PLL and grid stability [11]–[16]. Generally, most existing methods simplify SRF-PLL as a small-signal model [10] and discuss the SRF-PLL stability in the linear frame. However, the analysis of SRF-PLL is only a local result, and these methods are only available for the case where the difference between the initial estimated frequency \(\tilde{\omega}\) and grid frequency \(\omega\) is sufficiently small. In addition, the SRF-PLL is used for grid synchronization in some varying-frequency power systems, such as more electric aircraft (MEA) grids. In the varying-frequency power grids, the frequency changes largely. For example, the frequency range in MEA grids is from 360 to 800 Hz. Hence, large frequency jumps should be discussed for the SRF-PLL in these applications [17], [18]. Unfortunately, these methods fail to give an accurate analysis for SRF-PLL in the varying-frequency power grids.

Furthermore, some literature, such as [19] and [20], provide a large-signal analysis of the SRF-PLL. The arguments in these...
methods imply that SRF-PLL converges to the equilibrium point (0, 0), globally. However, when the frequency changes largely, the experiment results of SRF-PLL show that the SRF-PLL converges to a different equilibrium point and suffers a slow dynamic. This experimental phenomenon cannot be explained by these methods.

In this article, a global performance analysis of SRF-PLL is provided in the nonlinear frame. First, a large-signal model is established according to the structure of SRF-PLL, which does not rely on the linearization. Then, the phase portrait and Lyapunov argument are proposed to discuss the global performance of the SRF-PLL. Experimental results further confirm the proposed theoretical analysis. The main contributions of this article are as follows.

1) It is found that SRF-PLL has infinite equilibrium points that are located at \((n\pi, 0), \; n = 0, \pm 1, \pm 2, \ldots\). In them, \((2k\pi, 0)\) are stable points and \((2k + 1)\pi, 0)\) are saddle points, where \(k = 0, \pm 1, \pm 2, \ldots\). When the states are on stable points or saddle points, the states of the SRF-PLL will remain at this equilibrium points for all future time.

2) It provides a way to divide the global region of the SRF-PLL into many small regions. In each small region, the SRF-PLL only has one stable equilibrium point. And for any initial states \((\theta(t_0), \omega(t_0))\) in a small region, all states \((\theta(t), \omega(t)), t > t_0\) will still remain in this small region, and SRF-PLL will converge to the unique stable equilibrium point of this small region.

3) By dividing the global region of SRF-PLL into many small regions, it is found that when the frequency of grids varies largely, the SRF-PLL will converge to a new equilibrium point that is far away from the original equilibrium point. It is the reason why the estimated frequency of SRF-PLL has many oscillations and a rather long transient process, when the frequency changes largely.

The rest of this article is organized as follows. Section II briefly reviews the small-signal analysis method for SRF-PLL. The large-signal model of SRF-PLL is established in Section III. Following, global performance analysis of SRF-PLL is given in Section IV. Experimental tests are, then, provided in Section V. Finally, Section VI concludes this article.

**II. REVIEW OF THE CLASSICAL SMALL SIGNAL METHOD**

In this section, the small-signal analysis method of SRF-PLL is briefly reviewed. For ideal power grids, the grid voltages \(v_a\), \(v_b\), and \(v_c\) are

\[
v_a = V \cos(\theta) = V \cos(\omega t + \phi)
\]

\[
v_b = V \cos\left(\theta - \frac{2\pi}{3}\right) = V \cos\left(\omega t + \phi - \frac{2\pi}{3}\right)
\]

\[
v_c = V \cos\left(\theta + \frac{2\pi}{3}\right) = V \cos\left(\omega t + \phi + \frac{2\pi}{3}\right)
\]

where \(V\), \(\omega\), and \(\theta\) are the amplitude, frequency, and phase angle, respectively. Applying the \(abc-dq\) transformation

\[
T = \frac{2}{3}
\begin{bmatrix}
\cos(\hat{\theta}) & \cos(\hat{\theta} - \frac{2\pi}{3}) & \cos(\hat{\theta} + \frac{2\pi}{3}) \\
\sin(\hat{\theta}) & -\sin(\hat{\theta} - \frac{2\pi}{3}) & -\sin(\hat{\theta} + \frac{2\pi}{3})
\end{bmatrix}
\]

where \(\hat{\theta}\) is the estimation of \(\theta\), and the grid voltages (1)–(3) are deduced as

\[
v_d = V \cos(\theta - \hat{\theta})
\]

\[
v_q = V \sin(\theta - \hat{\theta})
\]

(6) is strongly nonlinear. To simplify the model of SRF-PLL, a linearization assumption is provided as

(A1) The initial estimation error of the phase angle \(\theta - \hat{\theta}\) is sufficient small.

With this assumption, (6) is linearized as

\[
\sin(\theta - \hat{\theta}) \approx \theta - \hat{\theta}
\]

From (6) and (7), it is carried out

\[
v_q = V \sin(\theta - \hat{\theta}) \approx V(\theta - \hat{\theta})
\]

Hence, when satisfying the assumption (A1), the SRF-PLL in Fig. 1 is linearized, as shown in Fig. 2. And the small-signal model of the SRF-PLL is established as [10]

\[
E_\theta(s) = \frac{s^2}{s^2 + k_p s + k_i}
\]

where \(k_p\) and \(k_i\) are the parameters of PI.

According to the small-signal model, such as (9), the small-signal analysis discusses the SRF-PLL performance. However, it is limited by the linearization condition (A1), and the analysis of SRF-PLL is only a local result. Meanwhile, it is difficult to confirm whether a frequency/phase jump is suitable for the small-signal analysis from the rigorous theoretical viewpoint. And the small-signal analysis method fails to give an accurate analysis of SRF-PLL in the varying-frequency power grids.

**III. LARGE-SIGNAL MODEL FOR THE SRF-PLL**

A small-signal model is based on the linearization method, and hence, it only describes the performance of SRF-PLL in a local region. To analyze the global performance of SRF-PLL, a large-signal model is established as follows.
According to the LF and VCO parts in SRF-PLL, it is deduced that
\[
\dot{\theta} = \int \dot{\omega} \, ds
\]
\[
\dot{\omega} = k_p v_q + k_i \int v_q \, ds + \omega_c
\] (10)
where \(\omega_c\) is a constant value. It implies that
\[
\dot{\dot{\theta}} = \dot{\omega}
\]
\[
\dot{\dot{\omega}} = k_p \dot{v}_q + k_i v_q.
\] (11)

Defining \(\tilde{\theta} = \hat{\theta} - \theta\) and \(\tilde{\omega} = \hat{\omega} - \omega\), and noting
\[
\dot{\theta} = \omega, \quad \dot{\omega} = 0
\] (12)
it yields
\[
\dot{\tilde{\theta}} = \tilde{\omega} - \omega = \tilde{\omega}
\]
\[
\dot{\tilde{\omega}} = k_p \dot{v}_q + k_i v_q.
\] (13)

From (6), it is deduced
\[
\dot{v}_q = -V \cos \theta \tilde{\dot{\theta}} = -V \tilde{\omega} \cos \theta
\] (14)
Submitting (6) and (14) into (13) leads to
\[
\dot{\tilde{\theta}} = \tilde{\omega}
\]
\[
\dot{\tilde{\omega}} = -k_i V \sin \theta - k_p V \tilde{\omega} \cos \tilde{\theta}.
\] (15)
This is the large-signal model for SRF-PLL. Compared with the small-signal model, the proposed large-signal model, not relying on linearization and (A1), describes the performance of SRF-PLL, globally. Observing (15), the large-signal model of SRF-PLL is strongly nonlinear. In the next section, the global performance of SRF-PLL is analyzed based on the proposed model (15).

IV. LARGE-SIGNAL ANALYSIS

For nonlinear systems, the phase portrait is a powerful tool to analyze stability, globally. In this section, the global performance of SRF-PLL is discussed according to the phase-portrait analysis. Based on the large-signal model (15), the phase portrait of SRF-PLL is shown in Fig. 3, with \(k_p = 92\) and \(k_i = 4232\). And the step-by-step way is provided in the Appendix to draw the Fig. 3.

From Fig. 3, the following can be found.
1) SRF-PLL has infinite equilibrium points that are located at \((n\pi, 0), n = 0, \pm 1, \pm 2, \ldots\). In these, \((2k\pi, 0)\) are stable points and \(((2k + 1)\pi, 0)\) are saddle points, where \(k = 0, \pm 1, \pm 2, \ldots\).

2) In Fig. 3, the blue lines divide the global region of SRF-PLL into many small regions. The Appendix provides a way to draw these special lines. For each saddle point, there exist two special lines that converge to this saddle point, although the saddle points are unstable. And these special lines divide the global region of SRF-PLL into many small regions. In each small region, the SRF-PLL only has one stable equilibrium point.

When the states are on stable points or saddle points, the states of SRF-PLL remain at the equilibrium points for all future time.
SRF-PLL will converge to the unique stable equilibrium point of this small region.

3) When the frequency of grids varies largely, the states of SRF-PLL will belong to a new small region. Hence, SRF-PLL will converge to the unique stable equilibrium point of this new small region. This stable point is far away from the original equilibrium point, and the frequency convergence of SRF-PLL has many oscillations. In this case, the SRF-PLL has a rather long transient process.

In the following, a strict theoretical analysis is provided to discuss the global performance of SRF-PLL.

According to the definition of equilibrium points shown in the Appendix, the equilibrium points for the SRF-PLL (15) are the real roots of the equation

\[ 0 = \tilde{\omega} \]

\[ 0 = -k_i V \sin \tilde{\theta} - k_p V \omega \cos \tilde{\theta}. \]  

(16)

Solving the algebraic equations (16) implies that the SRF-PLL has infinite equilibrium points and these points are located at \((n\pi, 0), n = 0, \pm 1, \pm 2, \ldots\). It is defined that \(\tilde{\theta}_n = \tilde{\theta} - n\pi\) and \(\tilde{\omega}_n = \tilde{\omega} - 0\). From (15), it is obtained that

\[ \begin{pmatrix} \dot{\tilde{\theta}}_n \\ \dot{\tilde{\omega}}_n \end{pmatrix} = A_n \begin{pmatrix} \tilde{\theta}_n \\ \tilde{\omega}_n \end{pmatrix} \]  

(17)

where

\[ A_n := \begin{bmatrix} 0 & 1 \\ -k_i V \cos(n\pi) & -k_p V \cos(n\pi) \end{bmatrix}. \]

If \(n = 2k\pi, k = 0, \pm 1, \pm 2, \ldots\), it yields \(\cos(n\pi) = 1\). Thus, a direct calculation implies that the matrix \(A_n\) has two eigenvalues with negative real part. This means that the equilibrium points \((2k\pi, 0), k = 0, \pm 1, \pm 2, \ldots\) are stable, and they are called saddle points. In Fig. 3, points \((0, 0)\) and \((2\pi, 0)\) are stable points.

Moreover, if \(n = (2k + 1)\pi, k = 0, \pm 1, \pm 2, \ldots\), the matrix \(A_n\) has an eigenvalue with positive-real part, and the other with negative real part. Thus, these equilibrium points \(((2k + 1)\pi, 0), k = 0, \pm 1, \pm 2, \ldots\) are unstable, and they are called saddle points. In Fig. 3, points \((-\pi, 0)\), \((\pi, 0)\), and \((3\pi, 0)\) are saddle points.

According to the definition of equilibrium points, it concludes that when the initial states of SRF-PLL are on stable points or saddle points, the states of SRF-PLL will remain at this equilibrium points for all future time.

According to the abovementioned analysis of saddle points, the SRF-PLL in the small region of saddle points is unstable. It implies that the initial states in this region will go far away the saddle points, while there still exists and only exists two special lines around each saddle point. When the initial states are on the special lines, SRF-PLL will converge to the saddle point [21]. In Fig. 3, the blue lines are these special lines.

Moreover, these special lines divide the global region of SRF-PLL into infinite small regions. In Fig. 3, Regions I and II are highlighted as an example of different regions. When the initial states fall in different regions, SRF-PLL will converge to different stable points \((2k\pi, 0), k = 0, \pm 1, \pm 2, \ldots\). In each small region, a strict Lyapunov analysis is proposed for SRF-PLL to show the stability as follows.

It is defined as the coordination transformation

\[ \tilde{\theta}_{2k} = \tilde{\theta} - 2k\pi \text{and} \tilde{\omega}_{2k} = \tilde{\omega} - 0. \]  

(18)

From (15), it is expressed as

\[ \dot{\tilde{\theta}}_{2k} = \tilde{\omega}_{2k} \]

\[ \dot{\tilde{\omega}}_{2k} = -k_i V \sin \tilde{\theta}_{2k} - k_p V \tilde{\omega}_{2k} \cos \tilde{\theta}_{2k}. \]  

(19)

Consider the Lyapunov function

\[ U(\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) = \int_0^{\tilde{\theta}_{2k}} k_i V \sin(y) dy + \frac{1}{2} \tilde{\omega}_{2k}^2. \]  

(20)

It is easy to verify that \(U(\tilde{\theta}_{2k}, \tilde{\omega}_{2k})\) is positive in the compact set \(\{ (\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) \in \mathbb{R}^2 | U(\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) \leq k_i V \}\).

From (19) and (20), it is deduced that

\[ \dot{U}(\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) = k_i V \sin(\tilde{\theta}_{2k}) \tilde{\omega}_{2k} - k_i V \sin(\tilde{\theta}_{2k}) \tilde{\omega}_{2k} \]

\[ - k_p V \cos(\tilde{\theta}_{2k}) \tilde{\omega}_{2k}^2 \]

\[ = - k_p V \cos(\tilde{\theta}_{2k}) \tilde{\omega}_{2k}^2 \leq 0. \]  

(21)

Hence, (21) is negative semidefinite, and SRF-PLL in this region is stable.

In the following, the steady-state errors of SRF-PLL are analyzed by LaSalle’s theorem. Let \(S = \{ (\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) \in \mathbb{R}^2 | U(\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) = 0 \}\). Note that

\[ \dot{U} = 0 \Rightarrow k_p V \cos(\tilde{\theta}_{2k}) \tilde{\omega}_{2k}^2 \Rightarrow \tilde{\omega}_{2k} = 0. \]  

(22)

Hence, \(S = \{ (\tilde{\theta}_{2k}, \tilde{\omega}_{2k}) \in \mathbb{R}^2 | \tilde{\omega}_{2k} = 0 \}\). Let \((\tilde{\theta}_{2k}(t), \tilde{\omega}_{2k}(t))\) be a solution that belongs identically to \(S\)

\[ \tilde{\omega}_{2k} \equiv 0 \Rightarrow \dot{\tilde{\omega}}_{2k} \equiv 0 \Rightarrow k_i V \sin(\tilde{\theta}_{2k}) \equiv 0. \]  

(23)

It results in \(\tilde{\theta}_{2k} \equiv 0\). Therefore, the only solution that can stay identically in \(S\) is the trivial solution \((\tilde{\theta}_{2k}(t), \tilde{\omega}_{2k}(t)) \equiv 0\). According to the LaSalle’s theorem [21], it concludes that for any initial states \((\tilde{\theta}_{2k}(t_0), \tilde{\omega}_{2k}(t_0)) \in \Omega\), the solutions \((\tilde{\theta}_{2k}(t), \tilde{\omega}_{2k}(t))\) approach 0 as \(t \to \infty\). Hence, the estimations of SRF-PLL converge to the stable points \((2k\pi, 0), k = 0, \pm 1, \pm 2, \ldots\), as

\[ \lim_{t \to \infty} \tilde{\theta}_{2k}(t) = 2k\pi \]  

(24)

\[ \lim_{t \to \infty} \tilde{\omega}_{2k}(t) = 0. \]  

(25)

From the abovementioned analysis, the global region of SRF-PLL is divided into infinite small stable regions. In each small region, the SRF-PLL has an unique stable equilibrium point \((2k\pi, 0), k = 0, \pm 1, \pm 2, \ldots\) and for any initial states \((\tilde{\theta}(t_0), \tilde{\omega}(t_0)) \in \text{such a small region}, \) all states \((\tilde{\theta}(t), \tilde{\omega}(t)), t > t_0\) will remain in this small region, and SRF-PLL will converge to the unique stable equilibrium point of this small region. Moreover, for any state \((\tilde{\theta}(t_0), \tilde{\omega}(t_0))\) of the SRF-PLL in a small region, the distance between this state point and the boundary of the small region can be applied to evaluate the SRF-PLL performance. When the frequency/phase jump is larger than
the distance, the states of SRF-PLL will jump to a new small region, and converge to a new equilibrium point. In particular, when the frequency jumps largely, the states of SRF-PLL belong to a new small region, and the equilibrium point of the new region is far away from the original one. In this case, although SRF-PLL converges to the new stable equilibrium point, the frequency convergence of SRF-PLL has many oscillations and suffers a long transient process. The experimental results provide the detail discussion of this phenomenon in the next section.

### V. EXPERIMENTAL RESULTS

In this section, SRF-PLL is implemented on a DSP28335-based platform, and the abovementioned analysis is verified by the following experimental results. The parameters of SRF-PLL are selected as $k_p = 92$ and $k_i = 4232$. The initial states of SRF-PLL are chosen as Cases A–G, as shown in Table I. The experiments are tested under four parts as follows.

#### A. Part 1: The Initial States are in Region I

In this part, SRF-PLL is implemented under Cases A and B, in which the initial states of SRF-PLL are in the Region I. Figs. 4 and 5 are, respectively, the experimental results and the phase portrait of SRF-PLL in Cases A and B.

In Cases A and B, the initial states $(\bar{\theta}(t_0), \bar{\omega}(t_0))$ are $(0, 20)$ and $(0.5\pi, -20)$, respectively, and these initial states are all in Region I. From Figs. 4 and 5, it is observed that all states $(\bar{\theta}(t), \bar{\omega}(t))$, $t > t_0$ in Cases A and B remain in the Region I, and SRF-PLL converges to $(0, 0)$, which is the unique stable equilibrium point of Region I. It confirms the conclusion in Section IV. Meanwhile, observing the phase portrait in Fig. 5, it is found that the frequency convergence has no oscillations, which implies that the dynamics in Cases A and B are fast according to the theoretical analysis in Section IV. It is verified by observing the transient time of Cases A and B, as shown in Fig. 4.

#### B. Part 2: The Initial States are in Region II

In this part, SRF-PLL is tested in Cases C and D, and the initial states of these cases are all in the Region II. Figs. 6 and

### Table I: Different Cases of Experimental Results

| Parts | Cases | Initial states (Phase and frequency jump) | Regions | Converged stable points |
|-------|-------|------------------------------------------|---------|------------------------|
| Part 1 | Case A | $\bar{\theta}(t_0) = 0$ rad, $\bar{\omega}(t_0) = 20$ Hz | Region I | $\bar{\theta}(\infty) = 0$ rad, $\bar{\omega}(\infty) = 0$ Hz |
|        | Case B | $\bar{\theta}(t_0) = 0.5\pi$ rad, $\bar{\omega}(t_0) = -20$ Hz | Region I | $\bar{\theta}(\infty) = 0$ rad, $\bar{\omega}(\infty) = 0$ Hz |
| Part 2 | Case C | $\bar{\theta}(t_0) = 2\pi$ rad, $\bar{\omega}(t_0) = 20$ Hz | Region II | $\bar{\theta}(\infty) = 2\pi$ rad, $\bar{\omega}(\infty) = 0$ Hz |
|        | Case D | $\bar{\theta}(t_0) = 2.5\pi$ rad, $\bar{\omega}(t_0) = -20$ Hz | Region II | $\bar{\theta}(\infty) = 2\pi$ rad, $\bar{\omega}(\infty) = 0$ Hz |
| Part 3 | Case E | $\bar{\theta}(t_0) = \pi - 0.01$ rad, $\bar{\omega}(t_0) = 0$ Hz | Region I | $\bar{\theta}(\infty) = 0$ rad, $\bar{\omega}(\infty) = 0$ Hz |
|        | Case F | $\bar{\theta}(t_0) = \pi + 0.01$ rad, $\bar{\omega}(t_0) = 0$ Hz | Region II | $\bar{\theta}(\infty) = 2\pi$ rad, $\bar{\omega}(\infty) = 0$ Hz |
| Part 4 | Case G | $\bar{\theta}(t_0) = 0$ rad, $\bar{\omega}(t_0) = -45$ Hz | Region VI | $\bar{\theta}(\infty) = -8\pi$ rad, $\bar{\omega}(\infty) = 0$ Hz |

Fig. 5. SRF-PLL phase portrait of Cases A and B.
In Cases C and D, the initial states \( \tilde{\theta}(t_0), \tilde{\omega}(t_0) \) are \((2\pi, 20)\) and \((2.5\pi, -20)\), respectively, and these initial states are in the Region II. From Figs. 6 and 7, it is found that all states \((\tilde{\theta}(t), \tilde{\omega}(t))\), \(t > t_0\) in Cases C and D are in the Region II, and SRF-PLL converges to \((2\pi, 0)\), which is the unique stable equilibrium point of the Region II. It further confirms the conclusion in Section IV. Meanwhile, from the phase portrait in Fig. 7, it is observed that the frequency convergence has no oscillations in this part, which implies that the dynamics in Cases C and D are fast according to the theoretical analysis in Section IV. It is further confirmed by Fig. 6.
C. Part 3: The Initial States are Closed but in Different Regions

In this part, SRF-PLL is implemented in Cases E and F. The initial states are closed but in different regions. Figs. 8 and 9 are the experimental results and the phase portrait of this part, respectively.

In Case E, the initial state \((\tilde{\theta}(t_0), \tilde{\omega}(t_0))\) is \((\pi - 0.01, 0)\), and it is in the Region I. In Case F, the initial state \((\tilde{\theta}(t_0), \tilde{\omega}(t_0))\) is \((\pi + 0.01, 0)\), and it is in the Region II. The two initial states of Cases E and F are closed but in the different small regions. From Fig. 8 and 9, it is observed that all states \((\tilde{\theta}(t), \tilde{\omega}(t))\), \(t > t_0\) in Case E belong to the Region I, whereas all states \((\tilde{\theta}(t), \tilde{\omega}(t))\), \(t > t_0\) in Case F remain in the Region II. SRF-PLL in Case E converges to the unique stable equilibrium point \((0, 0)\) in the Region I, whereas SRF-PLL in Case F converges to the unique stable equilibrium point \((2\pi, 0)\) in the Region II, although the initial states in Cases E and F are rather closed, and the distance between them is only 0.02 rad. The experimental results further confirm the theoretical analysis in Section IV. The phase portrait in Fig. 9 shows that the frequency convergence in Cases E and F has no oscillation. From the conclusion in the last section, the SRF-PLL dynamics in Cases E and F are fast, which is verified by the experimental results in Fig. 8.

D. Part 4: The Initial State is Far Away From the Unique Stable Equilibrium Point of the Small Region

In this part, SRF-PLL is tested in Case G. Figs. 10 and 11 are the phase portrait and the experimental results of this part, respectively.

In Case G, the initial state \((\tilde{\theta}(t_0), \tilde{\omega}(t_0))\) is \((0, -45)\). The initial state is in Region VI and far away from the unique stable equilibrium point \((-8\pi, 0)\) in Region VI. From Figs. 10 and 11, it is observed that all states \((\tilde{\theta}(t), \tilde{\omega}(t))\), \(t > t_0\) in Case G belong to Region VI, and SRF-PLL converges to the unique stable equilibrium point \((-8\pi, 0)\) in Region VI. It confirms the conclusion of Section IV. Meanwhile, from Fig. 10, it is found that the frequency convergence in Case G occurs oscillations. It will result in slow dynamics, according to the analysis in Section IV. Observing Fig. 11, the dynamic of the SRF-PLL is slow, which further confirms the conclusion in last section.

Remark: According to theoretical part, the states of SRF-PLL will remain at the saddle points for all future time when the states first stand on its saddle point. Meanwhile, there exists two special lines for each saddle point, and the SRF will converge to the saddle points when its initial states are on these special lines. However, due to the truncation errors of the experiment platform, the initial states of SRF-PLL are difficult to exactly stand on saddle points or special lines, and its experiments are hard to be provided in practical cases.
VI. Conclusion

In this article, the global performance of SRF-PLL was analyzed in the nonlinear framework. The large-signal model of SRF-PLL was accurately established, and phase portrait and Lyapunov argument was proposed to analyze the global stability of SRF-PLL. It was found that SRF-PLL had infinite equilibrium points, including stable points and saddle points. Furthermore, a way was provided to divide the global region of SRF-PLL into many small regions. Each small region only had a stable equilibrium point. And for any states in a small region, the states of SRF-PLL still belonged to this small region for all future time, and SRF-PLL will converge to the unique stable point of this small region. In addition, it was found that when the frequency of grids varied largely, the SRF-PLL converged to a new equilibrium point that was far away from the original equilibrium point. In that case, there were many oscillations for frequency dynamic, and the SRF-PLL had a rather long transient process. Experimental tests were also provided to verify the validity of the theoretical discovery.

APPENDIX

In this section, the related nonlinear system theory is introduced, including the definitions of phase portraits, equilibrium points, stable points, and saddle points. Then, the step-by-step way is provided to draw the phase portraits of SRF-PLL.

Consider a second-order nonlinear system

\[ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = f_1(x_1, x_2) \]
\[ \dot{x}_2 = f_2(x_1, x_2). \quad (26) \]

Let \( x(t) = (x_1(t), x_2(t)) \) be the solution of (26) that starts at a certain initial state \( x(0) = (x_1(0), x_2(0)) \). The locus of the \( x_1 - x_2 \) plane of the solution \( x(t) \) for all \( t \geq 0 \) is a curve that passes through the point \( x(0) \). The \( x_1 - x_2 \) plane is usually called the phase plane. The family of all solution curves in \( x_1 - x_2 \) plane is called the phase portrait of (26).

A point \( x = x^* \) is said to be an equilibrium point of (26) if it has the property that whenever the state of the system starts at \( x^* \), it will remain at \( x^* \) for all future time. The equilibrium points of (26) are the real roots of the equation

\[ \begin{aligned}
&f_1(x_1, x_2) = 0 \\
&f_2(x_1, x_2) = 0.
\end{aligned} \quad (27) \]

Let \( p = (p_1, p_2) \) be an equilibrium point of (26). It is defined that \( y_1 = x_1 - p_1 \) and \( y_2 = x_2 - p_2 \). Under a sufficiently-small neighborhood the equilibrium point and the nonlinear system (26) is linearized as

\[ \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (28) \]

where

\[ A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \bigg|_{x = p}. \quad (29) \]

When all the eigenvalues of the matrix \( A \) have negative real part, the equilibrium point \( p \) is called the stable point. When the matrix \( A \) has an eigenvalue with positive real part, and an eigenvalue with negative real part, the equilibrium point \( p \) is called the saddle point.

In the following, a step-by-step way is shown how to obtain the phase portraits of SRF-PLL.

Step 1: For the large-signal model (15) of the SRF-PLL, find all equilibrium points and determine the type of isolated ones via linearization. From the abovementioned analysis, the equilibrium points \( (2k\pi, 0) \), \( k = 0, \pm 1, \pm 2, \ldots \), are stable points, and the equilibrium points \( (2k + 1)\pi, 0) \), \( k = 0, \pm 1, \pm 2, \ldots \) are saddle points.

Step 2: Select a bounding box in the state plane and select the initial points inside the bounding box. That is, choose different initial values \( (\hat{\theta}(0), \hat{\omega}(0))^T \) such that

\[ \hat{\theta}_{\text{min}} \leq \hat{\theta}(0) \leq \hat{\theta}_{\text{max}}, \quad \hat{\omega}_{\text{min}} \leq \hat{\omega}(0) \leq \hat{\omega}_{\text{max}}. \quad (30) \]

Step 3: For each initial values, calculate the trajectories using numerical algorithm. To find the trajectory passing through a point \( (\hat{\theta}(0), \hat{\omega}(0))^T \), solve the equation

\[ \begin{aligned}
\dot{\theta} &= \dot{\omega} \\
\dot{\omega} &= -k_1 V \sin \hat{\theta} - k_p V \omega \cos \hat{\theta}
\end{aligned} \quad (31) \]

in the forward time (with positive \( t \)) and in reverse time (with negative \( t \)). Solution in reverse time is equivalent to solution in forward time of the equation

\[ \begin{aligned}
\dot{\theta} &= -\dot{\omega} \\
\dot{\omega} &= k_1 V \sin \hat{\theta} + k_p V \omega \cos \hat{\theta}.
\end{aligned} \quad (32) \]

Step 4: For saddle points \( ((2k + 1)\pi, 0) \), \( k = 0, \pm 1, \pm 2, \ldots \), use linearization to generate the stable and unstable trajectories. Specifically, let the eigenvalues of the matrix \( A_{2k+1} \) be \( \lambda_1 > 0 > \lambda_2 \) and the corresponding eigenvectors be \( v_1 \) and \( v_2 \). The unstable trajectories are drawn by solving the equation in forward time

\[ \begin{aligned}
\dot{\hat{\theta}} &= \hat{\omega} \\
\dot{\hat{\omega}} &= -k_1 V \sin \hat{\theta} - k_p V \omega \cos \hat{\theta}
\end{aligned} \quad (33) \]

where \( \alpha \) is a small positive number. The stable trajectories are drawn by solving the equation in backward time

\[ \begin{aligned}
\dot{\hat{\theta}} &= \hat{\omega} \\
\dot{\hat{\omega}} &= -k_1 V \sin \hat{\theta} - k_p V \omega \cos \hat{\theta}
\end{aligned} \quad (34) \]

That is, solve the equation in forward time

\[ \begin{aligned}
\dot{\hat{\theta}} &= -\hat{\omega} \\
\dot{\hat{\omega}} &= k_1 V \sin \hat{\theta} + k_p V \omega \cos \hat{\theta}
\end{aligned} \quad (35) \]
The lines by solving (34) and (35) previously converge to saddle points and divide the global region of SRF-PLL into many small regions. In such a small region, the SRF-PLL only has one stable equilibrium point. For any initial states within these small regions, all states $(\hat{\theta}(t), \hat{\omega}(t))$ in a small region, all states $(\hat{\theta}(t), \hat{\omega}(t)), t > t_0$ in this case will remain in this small region, and SRF-PLL will converge to the unique stable equilibrium point of this small region, which is proven in Section IV. In addition, (34) and (35) are related to the SRF-PLL’s parameters $k_p$ and $k_i$. Hence, the shapes and the sizes of the small regions are related with the parameters of SRF-PLL.

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Jin Huang received the Ph.D. degree in mechanical engineering from Xidian University, Xian, China, in 1999.

From 2001 to 2002, he was a Visiting Researcher with the Department of Mechanical Engineering, The University of British Columbia, Vancouver, BC, Canada. He is currently a Professor and the Dean of the School of Electro-Mechanical Engineering, Xidian University, and the Director of the Key Laboratory of Electronic Equipment Design, Ministry of Education. He has authored or coauthored more than 100 papers in various peer-reviewed journals and conference proceedings, and he holds more than 50 patents. His research interests include flexible electronics, mechatronics, and 3-D printing.

Dr. Huang is the Fellow of the Chinese Institute of Electronics and was the Deputy Secretary General of the Electro-Mechanical Engineer Society of China.

Yong Yang (Senior Member, IEEE) received the B.S. degree in automation from Xiangtan University, Xiangtan, China, in 2003, the M.S. degree in electrical engineering from Guizhou University, Guiyang, China, in 2006, and the Ph.D. degree in electrical engineering from Shanghai University, Shanghai, China, in 2010.

He is currently an Associate Professor with the School of Rail Transportation, Soochow University, Suzhou, China. From 2017 to 2018, he was a Visiting Scholar with the Center for High Performance Power Electronics, The Ohio State University, Columbus, OH, USA. He has coauthored more than 70 journal and conference papers. His research interests include model-predictive control in power electronic converters, distributed energy resource interfacing, and high-performance motor drive control.

Wei Hang received the B.S. degree in mechatronics from Nanjing Normal University, Nanjing, China, in 2006, and the Ph.D. degree in industrial science from Ibaraki University, Mito, Japan, in 2014.

He is currently a Lecturer with the School of Mechanical Engineering, Zhejiang University of Technology, Hangzhou, China. His research interests include power electronics, ultra-precision machining, and smart manufacturing.