Quantum field description of the finite-width effects

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Abstract

The model of unstable particles with random mass is suggested to describe the finite-width effects. The phenomenological manifestation of mass smearing is discussed in the framework of the model.

1. Introduction

Quantum field description of the unstable particles (UP) with a large width runs into some problems, which are under considerable discussions [1]. These problems have both the conceptual and technological status and arise due to UP lie somewhat outside the traditional formulation of quantum field theory [2]. We can not treat the UP with large width as asymptotic state and include it into the set of initial or final states. Moreover, perturbative approach is unfit in the resonance neighborhood. These conceptual problems are connected with methodological difficulties, such as an ambiguity in definition of mass and width. Therefore, the new quantum field approach [2] (Bohm et al), phenomenological models [3] and effective theories of UP [4] are actual now.

The convolution method [5] is convenient and clear phenomenological way to evaluate the instability or finite-width effects (FWE). This method describes FWE in the processes $\Phi \rightarrow \phi_1 \phi \rightarrow \phi_2 \phi_3 \ldots$, where $\phi$ is the UP with large width. The intermediate unstable state $\phi$ is simulated by the final state $\phi$ in the decay $\Phi \rightarrow \phi_1 \phi$ with invariant mass, described by Breit-Wigner-like (Lorentzian) distribution function. The phenomenological expression for
a decay rate has convolution form [5]:

\[ \Gamma(\Phi \to \phi_1 \phi) = \int_{q_1^2}^{q_2^2} \Gamma(\Phi \to \phi_1 \phi(q)) \rho(q) dq^2, \tag{1} \]

where \( \rho(q) = M \Gamma_\phi(q)/\pi |P(q)|^2 \). In Eq. (1) \( \rho(q) \) is probability density of invariant mass distribution, \( P(q) = q^2 - M^2 + i M \Gamma_\phi(q) \) (Altarelli et al), \( \Gamma(\Phi \to \phi_1 \phi(q)) \) and \( \Gamma_\phi(q) \) are partial width of \( \Phi \) and total width of \( \phi \) in the stable particle approximation, when \( m_\phi^2 = q^2 \).

The formula for a decay rate, which has a close analogy to the Eq.(1), was applied to the description of FWE in \( B \) and \( \Lambda \) decay channels with \( \rho(770) \) and \( a_1(1260) \) in the final states [6]. It was shown that the contribution of FWE to the decay rates of these channels are large (20-30 %) and the account of it significantly improves a conformity of experimental data and theoretical predictions. Analogous results were obtained in Ref. [3] for the dominant decay channels of \( \Phi(1020) \), \( \rho(770) \) and \( K^*(892) \). The decay rates of the near-threshold decay channels \( t \to WZb, cWW, cZZ \) were calculated with help of convolution formula (CF) in Ref. [5]. It was shown in these works, that the FWE play a significant role in the near-threshold processes.

The convolution formula (1) was derived in Ref. [7] by direct calculation with help of the decay-chain method. In this work the contribution of all decay-chain channels of UP is described by function \( \rho(q) = q \Gamma(q)/\pi |P(q)|^2 \). The essential elements of this derivation for vector and spinor UP are the expressions \( \eta_{mn} = -g_{mn} + q_m q_n / q^2 \) and \( \hat{\eta} = \hat{q} + q \) for numerators of vector and spinor propagators (\( \hat{q} = q_i \gamma^i \)). The convolution formula was derived for the decay chain \( t \to bW \to bfjfj \) in the limit of massless fermions \( f \) in Ref. [5] (Galderon and Lopez-Castro). Quantitative analysis of convolution and decay-chain calculations of the \( t \to WZb \) decay rate was fulfilled in Ref. [5] (Altarelli et al). The formula for a decay rate, which is similar to (1), was received in Ref. [3] for the case of the scalar UP within the framework of the ”random mass” model. The UP is described in this model by the quantum field with a ”smeared” (fuzzy) random mass in accordance with the uncertainty principle for energy and lifetime of unstable quantum system [8]. The FWE is connected with this fundamental principle, which gives the relation \( \delta m * \tau \approx 1 \), that is \( \delta m \approx \Gamma \) in the rest frame of reference (\( \delta E = \delta m, c = \hbar = 1 \)) [3]. So, the uncertainty principle leads to the interpretation of kinematic value \( q^2 \) in Eq.(1) as a random mass square. Thus, the intermediate states of UP, which are traditionally defined as virtual, in the neighborhood of \( q^2 = M^2 \) are not differ from real ones in accordance with the uncertainty principle. This interpretation is connected with a smearing of mass shell and with above mentioned definition of \( \eta_{mn} \) and \( \hat{\eta} \), which are proportional to the polarization matrix for the vector and spinor UP (see section 3). As it was noted in Ref. [7], this proportionality leads to the factorization of the expression for
width in the decay-chain method, and, as consequence, to the CF (1). Thus, the suggested model is theoretical framework of the convolution method, which takes into account the uncertainty principle.

In this paper we consider the generalization of the model [3], which includes vector and spinor fields (Section 2). Within the framework of this generalized model the CF is derived for UP of arbitrary type (Section 3). To determine the probability density $\rho(m)$, which is an analogue of $\rho(q)$ in Eq.(1), we put a connection between the model and effective theory of UP with modified propagators, used in Ref. [7] (Section 4). It was shown in the section 4, that this connection leads to Lorentzian probability density $\rho(m)$ and to the traditional description of UP in the intermediate state by dressed propagator, as a special case of suggested approach. The model is applicable to the decay processes of type $\Phi \rightarrow \phi_1\phi(q)$, that is describes UP in a final state, and leads to the convolution formula (1) for UP of arbitrary type. In the Section 5 we have considered some examples of FWE manifestations in a various regions of the particle physics. The contributions of FWE (or mass smearing) into the decay rates of $\phi(1020) \rightarrow K\bar{K}$, $B^0 \rightarrow D^+\rho^+$, $W \rightarrow f_1f_2$ decays and into the oscillations in the systems of neutral mesons $M^0 - \bar{M}^0$ are evaluated within the framework of the model.

2. The model of unstable particles with a random mass

The effect of mass smearing is described by the wave packet with some weight function $\omega(\mu)$, where $\mu$ is random mass parameter [3]. The model field function, which simulates UP in the initial, final or intermediate states, is represented by the expression:

$$\Phi_\alpha(x) = \int \Phi_\alpha(x, \mu)\omega(\mu)d\mu .$$  

(2)

In Eq.(2) $\Phi_\alpha(x, \mu)$ are the components of field function, which are determined in the usual way when $m^2 = \mu$ is fixed (stable particle approximation). The limits of integration will be defined in the sections 3 and 4.

The model Lagrangian, which determines ”free” unstable field $\Phi(x)$, has the convolution form:

$$L(\Phi(x)) = \int L(\Phi(x, \mu))|\omega(\mu)|^2 d\mu .$$  

(3)

In Eq.(3) $L(\Phi(x, \mu))$ is standard Lagrangian, which describes model ”free” field $\Phi(x, \mu)$ in stable particle approximation ($m^2 = \mu$).

From Eq.(3) and prescription $\partial\Phi(x, \mu)/\partial\Phi(x, \mu') = \delta(\mu - \mu')$ it follows Klein-Gordon
equation for the spectral component:

$$ (\Box - \mu)\Phi_\alpha(x, \mu) = 0. $$  \hfill (4)

As a result we have standard momentum representation of field function for fixed mass parameter $\mu$:

$$ \Phi_\alpha(x, \mu) = \frac{1}{(2\pi)^{3/2}} \int \Phi_\alpha(k, \mu)\delta(k^2 - \mu)e^{ikx}dk. $$  \hfill (5)

All standard definitions, relations and frequency expansion take place for $\Phi_\alpha(k, \mu)$, but the relation $k^0_\mu = \sqrt{k^2 + \mu}$ defines smeared (fuzzy) mass-shell due to random $\mu$.

The expressions (2) and (3) define the model "free" unstable field, which really is some effective field. This field is formed by interaction of "bare" UP with decay products and includes nonperturbative self-energy contribution in the resonant region. Such an interaction leads to the spreading (smearing) of mass, that is to the transition from $\rho^{st}(\mu) = \delta(\mu - M^2)$ for the bare particles to some smooth density function $\rho(\mu) = |\omega(\mu)|^2$ with mean value $\mu_0 \approx M^2$ and $\sigma_\mu \approx \Gamma$. So, the UP is characterized in the discussed model by the weight function $\omega(\mu)$ or probability density $\rho(\mu)$ with parameters $M$ and $\Gamma$ (or real and imaginary parts of pole). A similar approach has been discussed by Matthews and Salam in Ref. [8].

The commutative relations for model operators have an additional $\delta$-function:

$$ [\dot{\Phi}_\alpha^-(\bar{k}, \mu), \Phi_\beta^+(\bar{q}, \mu')]_\pm = \delta(\mu - \mu')\delta(\bar{k} - \bar{q})\delta_{\alpha\beta}, $$  \hfill (6)

where subscripts $\pm$ correspond to the fermion and boson fields. The presence of $\delta(\mu - \mu')$ in Eq.(6) means an assumption - the acts of creations and annihilations of particles with various $\mu$ (random mass square) don’t interfere. So, the parameter $\mu$ has the status of physically distinguishable value as random $m^2$. This assumption directly follows from the interpretation of $q^2$ in Eq. (11) as random parameter $\mu$. By integrating both side of Eq.(6) with weights $\omega^*(\mu)\omega(\mu')$ one can get standard commutative relations

$$ [\dot{\Phi}_\alpha^-(\bar{k}), \Phi_\beta^+(\bar{q})]_\pm = \delta(\bar{k} - \bar{q})\delta_{\alpha\beta}, $$  \hfill (7)

where $\Phi_\alpha^\pm(\bar{k})$ is full operator field function in momentum representation:

$$ \Phi_\alpha^\pm(\bar{k}) = \int \Phi_\alpha^\pm(\bar{k}, \mu)\omega(\mu)d\mu. $$  \hfill (8)

It should be noted that Eq.(7) follows from Eq.(6) when $\int |\omega(\mu)|^2d\mu = 1$.

The expressions (2) and (5) are the principal elements of the discussed model. The weight function $\omega(\mu)$ in Eq.(2) (or $\rho(\mu)$) is full characteristic of UP and the relations (6) define the structure of the model amplitude and of the transition probability (section 3).
The probability density $\rho(\mu)$ will be defined in the fourth section by matching the model propagator to renormalized one.

With help of traditional method one can get from Eqs. (2), (4) and (6) the expression for the unstable scalar Green function [3]:

$$
\langle 0|T(\phi(x), \phi(y))|0 \rangle = D(x-y) = \int D(x-y, \mu) \rho(\mu) d\mu. \quad (9)
$$

In Eq. (9) $D(x, \mu)$ is standard scalar Green function with $m^2 = \mu$, which describes UP in an intermediate state:

$$
D(x, \mu) = \frac{i}{(2\pi)^4} \int \frac{e^{-ikx}}{k^2 - \mu + i\epsilon} dk. \quad (10)
$$

The right side of the Eq. (9) is Lehmann-like spectral (on $\mu$) representation of the scalar Green function, which describes the propagation of scalar UP. Taking into account the connection between scalar and vector Green functions, we can get the Green function of the vector unstable field:

$$
D_{mn}(x, \mu) = -(g_{mn} + \frac{1}{\mu} \frac{\partial^2}{\partial x^m \partial x^n}) D(x, \mu) = \frac{-i}{(2\pi)^4} \int \frac{g_{mn} - k_m k_n / \mu}{k^2 - \mu + i\epsilon} e^{-ikx} dk. \quad (11)
$$

Analogously Green function of the spinor unstable field:

$$
\hat{D}(x, \mu) = (i\hat{\partial} + \sqrt{\mu}) D(x, \mu) = \frac{i}{(2\pi)^4} \int \frac{\hat{k} + \sqrt{\mu}}{k^2 - \mu + i\epsilon} e^{-ikx} dk, \quad (12)
$$

where $\hat{k} = k_i \gamma^i$. These Green functions in momentum representation have a convolution structure:

$$
D_{mn}(k) = \int D_{mn}(k, \mu) \rho(\mu) d\mu, \quad \hat{D}(k) = \int \hat{D}(k, \mu) \rho(\mu) d\mu. \quad (13)
$$

3. The model amplitude and the convolution formula for a decay rate

In this section we consider the model amplitude for the simplest processes with UP in a final state and get the CF (1) as direct consequence of the model. The expression for a scalar operator field [3]:

$$
\phi^\pm(x) = \frac{1}{(2\pi)^{3/2}} \int \omega(\mu) d\mu \int \frac{a^\pm(\bar{q}, \mu)}{\sqrt{2q^0_\mu}} e^{\pm i q x} d\bar{q}, \quad (14)
$$

where $q^0_\mu = \sqrt{\bar{q}^2 + \mu}$ and $a^\pm(\bar{q}, \mu)$ are creation or annihilation operators of UP with momentum $q$ and mass square $m^2 = \mu$. Taking into account Eq. (6) we can get:

$$
[a^-(\bar{k}, \mu), \phi^+(x)]_-; [\phi^-(x), a^+(\bar{k}, \mu)]_- = \frac{\omega(\mu)}{(2\pi)^{3/2} \sqrt{2k^0_\mu}} e^{\pm ikx}, \quad k^0_\mu = \sqrt{k^2 + \mu}. \quad (15)
$$
The expressions (15) differ from standard ones by the factor $\omega(\mu)$ only. From this result it follows that, if $\hat{a}^+(k, \mu)|0\rangle$ and $\langle 0|\hat{a}^-(k, \mu)$ define UP with a mass $m^2 = \mu$ and a momentum $k$ in the initial and final states, then the amplitude for the decay of type $\Phi \to \phi\phi_1$ has the form:

$$A(k, \mu) = \omega(\mu)A^{st}(k, \mu),$$

(16)

where $A^{st}(k, \mu)$ is amplitude in a stable particle approximation when $m^2 = \mu$. This amplitude is calculated in a standard way and can include high corrections. Moreover, it can be effective amplitude for the processes with hadron participation [3, 5].

To define the transition probability of the process $\Phi \to \phi\phi_1$, where $\phi$ is UP with a large width, we should take into account the status of parameter $\mu$ as physically distinguishable value, which follows from Eq.(6). Thus, the amplitude at different $\mu$ don’t interfere and we have the convolution structure of differential (on $k$) probability:

$$d\Gamma(k) = \int d\Gamma^{st}(k, \mu)|\omega(\mu)|^2d\mu.$$  

(17)

In Eq.(17) the differential probability $d\Gamma^{st}(k, \mu)$ is defined in the standard way (stable particle approximation):

$$d\Gamma^{st}(k, \mu) = \frac{1}{2\pi}\delta(k_{\Phi} - k_{\phi} - k_1)|A^{st}(k, \mu)|^2d\bar{k}_{\phi}d\bar{k}_1,$$

(18)

where $k = (k_\Phi, k_\phi, k_1)$ denotes the momenta of particles. From Eqs.(17) and (18) it directly follows the known convolution formula for a decay rate

$$\Gamma(m_\Phi, m_1) = \int_{\mu_0}^{\mu_m} \Gamma^{st}(m_\Phi, m_1; \mu)|\rho(\mu)|d\mu,$$

(19)

where $\rho(\mu) = |\omega(\mu)|^2$ and $\mu_0, \mu_m$ are defined in Refs. [5, 7] as threshold and maximal invariant mass square of unstable $\phi$.

An account of high corrections in the amplitude (16) keeps the convolution form of Eq.(19). This form can be destroyed by accounting of the interaction between the products of UP ($\phi$) decay and initial $\Phi$ or final $\phi_1$ states. The calculation in this case can be fulfilled in a standard way, but UP in the intermediate state is described by the model propagator. However, a calculation within the framework of perturbative theory (PT) can not be applicable to the UP with large width, that is to the short-living particle. In any case, the applicability of the PT, the model approach or convolution method to the discussed decays should be justified by experiment. The correspondence of CM to the experimental data was demonstrated for some processes [3, 5, 6, 7], but this problem needs in more detailed investigation. In this connection we should note the analysis of higher-order corrections for
processes with UP [4]. The separation between factorizable and non-factorizable corrections make it possible to build the effective theory of UP [4].

When there are two UP with large widths in a final state $\Phi \to \phi_1\phi_2$, then in analogy with the previous case one can get double convolution formula:

$$\Gamma(m_\Phi) = \int \int \Gamma^{st}(m_\Phi; \mu_1, \mu_2)\rho_1(\mu_1)\rho_2(\mu_2)d\mu_1 d\mu_2.$$  \(20\)

The derivation of CF for the cases when there is vector or spinor UP in a final state can be done in analogy with the case of scalar UP. However, in Eqs. (14), (15) and (16) we have a polarization vector $e_m(q)$ or spinor $u_\nu^{\alpha\pm}(q)$, where q is on fuzzy mass-shell. As a result we get polarization matrix with $m^2 = \mu$. For the vector UP in a final state:

$$\sum e_m(q)e^*_n(q) = -g_{mn} + q_m q_n/\mu.$$  \(21\)

For the spinor UP in a final state:

$$\sum_{\nu} u_{\nu}^{\alpha\pm}(q)\bar{u}_{\beta}^{\nu\mp}(q) = \frac{1}{2q_\mu^0}(\hat{q} \pm \sqrt{\mu})_{\alpha\beta}.$$  \(22\)

In Eqs. (21) and (22) sum run over polarization and $q_\mu^0 = \sqrt{q^2 + \mu}$.

The formulae (19) and (20) describe FWE in full analogy with the phenomenological convolution method [5] and with some cases of the decay-chain method [5, 7]. Thus, we consider the quantum field basis for CM, which takes into account the fundamental uncertainty principle and is in a good agreement with the experimental data on some decays. To evaluate FWE for the case, when UP is in an initial state, we must account the process of UP generation. When UP is in an intermediate state, then the description of FWE is equivalent to the traditional one, but the model propagators are determined by Eqs. (9) - (13).

4. Determination of random mass distribution function

The possibility of $\rho(\mu)$-determination directly follows from the connection of the decay-chain method (DCM) and convolution method [7]. As was shown in Ref. [7], this connection leads to the convolution formula (11), where in accordance with uncertainty principle we interpret the value $q^2$ as random mass square parameter $\mu$, which distribution is described by the expression:

$$\rho(\mu) = \frac{1}{\pi} \frac{\sqrt{\pi} \Gamma(\mu)}{|P(\mu)|^2}.$$  \(23\)
In Eq. (23) $\Gamma(\mu)$ is $\mu$-dependent full width and $P(\mu)^{-1}$ is propagator’s denominator. It should be noted, that the convolution structure of Eq. (1) and universal structure of Eq. (23) don’t depend on the definition of $P(\mu)$. In general $P(\mu)$ has a complex pole structure $\mu - \mu_R$ and can be approximated by the Breit-Wigner $\mu - M^2 + iM\Gamma(\mu)$ [5] or another phenomenological formulae. The expression (23) is very simple and convenient in practical calculations of decay rate, where the error of approximation is small.

Here we’ll consider the definition of $\rho(\mu)$ from the matching model propagators to standard dressed ones [3]. This consideration is rather methodological than practical and demonstrates the connection between model and traditional descriptions. Let us associate the model propagator of scalar unstable field (9) with standard one:

$$\int \frac{\rho(\mu)d\mu}{k^2 - \mu + i\epsilon} \leftrightarrow \frac{1}{k^2 - m^2(k^2) + i\epsilon \Pi(k^2)},$$

(24)

where $\Pi(k^2)$ is conventional self-energy of scalar field. With help of an analytical continuation of the expressions (24) on complex plane $k^2 \rightarrow k^2 \pm i\epsilon$ and prescription [9]:

$$\Pi(k^2 \pm i\epsilon) = \text{Re}\Pi(k^2) \pm i\text{Im}\Pi(k^2)$$

(25)

the conformity (24) can be represented by the equality

$$\int_0^\infty \frac{\rho(\mu) d\mu}{k^2 - \mu + i\epsilon} = \frac{1}{k^2 - m^2(k^2) \pm i\epsilon \Pi(k^2)},$$

(26)

where $m^2(k^2) = m_0^2 + \text{Re}\Pi(k^2)$. With account of round pole rules and $d\mu = d(\mu \mp i\epsilon)$, $\rho(\mu \mp i\epsilon) = \rho(\mu) \pm O(i\epsilon)$ two Eqs. (26) can be combined into the equality ($\mu \pm i\epsilon \rightarrow z$):

$$\oint \frac{\rho(z) dz}{z - k^2} = \frac{1}{k^2 - m^2(k^2) - i\epsilon \Pi(k^2)} - \frac{1}{k^2 - m^2(k^2) + i\epsilon \Pi(k^2)}.$$  

(27)

The left side of Eq. (27) is Cauchy integral, which equal to $2\pi i \rho(k^2)$ and after a change $k^2 \rightarrow \mu$ in the final expression for $\rho$ we have:

$$\rho(\mu) = \frac{1}{\pi} \frac{\text{Im}\Pi(\mu)}{[\mu - m^2(\mu)]^2 + [\text{Im}\Pi(\mu)]^2}.$$  

(28)

The expression (28) for $\rho(k^2)$ in Breit-Wigner approximation is usually exploited within the framework of convolution method. From Eq. (28) and definition $\rho(\mu) = |\omega(\mu)|^2$ it follows:

$$\omega(\mu) = \sqrt{\frac{\text{Im}\Pi(\mu)}{\mu - m^2(\mu) \pm i\epsilon \Pi(\mu)}}.$$  

(29)

The ambiguity of sign in (29) is not essential because the expression $|\omega(\mu)|^2$ only enters into the physical values. In the parametrization $\text{Im}\Pi(\mu) = \sqrt{\pi} \Gamma(\mu)$ we have relativistic Breit-Wigner $\omega(\mu)$ and Lorentzian $\rho(\mu)$, which coincides with the expression (23) for renormalized...
Inserting the expression (28) into the left side of Eq.(24) one can check with help of Cauchy method the self-consistency of Eqs.(24) and (28).

Thus, we have put the correspondence between the model [2] - [6] and some effective theory of UP with renormalized propagator of scalar UP. To establish such a correspondence for the vector UP we insert \( \rho(\mu) \) into the model propagator (13) with \( D_{mn}(k, \mu) \), defined by (11) for vector unstable field:

\[
\int_0^{\infty} -\frac{g_{mn} + k_m k_n / \mu}{k^2 - \mu + i\epsilon} \frac{1}{\pi} \frac{Im\Pi(\mu)}{[\mu - m^2(\mu)]^2 + [Im\Pi(\mu)]^2} d\mu =
\]

(30)

With help of Eq.(25) and above used method we can represent the second part of Eq.(30) in the form (\( \mu \rightarrow z = \mu \pm i\epsilon \)):

\[
\frac{1}{2i\pi} \int_0^{\infty} -\frac{g_{mn} + k_m k_n / \mu}{k^2 - \mu + i\epsilon} \frac{1}{\mu - m^2(\mu) - iIm\Pi(\mu)} - \frac{1}{\mu - m^2(\mu) + iIm\Pi(\mu)} d\mu.
\]

(31)

The right side of Eq.(31) is similar to the expression for propagator of vector UP, which leads to the convolution formula (1) in the decay-chain method [7]. The numerator of this effective propagator coincides with \( \eta_{mn}(k) \), which was used in [7]. In Eqs.(30) and (31) the value \( \Pi(k^2) \) is defined for vector field as transverse part of polarization matrix [1]. The calculations of \( \Pi(k^2) \) in effective theory (unstable hadrons) or in gauge theory (Z,W-bosons) can run into some difficulties. In the first case loop calculation can be ambiguous and we should use traditional Breit-Wigner approximation \( m^2(\mu) \approx M^2 \) and \( Im\Pi(\mu) \approx \mu \Gamma(\mu) \). To escape the gauge-dependence in the second case we can use pole definitions of mass and width [1].

The description of \( \rho(\mu) \) by the universal function (28) for scalar and vector fields can be justified by the general structure of parametrization for bosons:

\[
m^2(q^2) = m_0^2 + Re\Pi(q^2), Im\Pi(q^2) = q\Gamma(q^2).
\]

(32)

In the case of unstable fermion we have another parametrization scheme:

\[
m(q^2) = m_0 + Re\Sigma(q^2), Im\Sigma(q^2) = \Gamma(q^2).
\]

(33)

So, the definition of the fermion function \( \rho(\mu) \) demands an additional analysis. If we choose for fermion UP the universal density function (23), which follows from convolution method [7], then we must do exchange \( Im\Pi(\mu) \rightarrow \sqrt{\pi}Im\Sigma(\mu) \) in the Eq.(28). Inserting the result into Eq.(13) with \( \hat{D}(x, \mu) \), defined by Eq.(12), we can get the correspondence between the model propagator of fermion unstable field and the effective theory one:

\[
\int \frac{\hat{k} + \sqrt{\mu}}{k^2 - \mu + i\epsilon} \rho(\mu)d\mu \rightarrow \frac{\hat{k} + k}{k^2 - m^2(k^2) - i\epsilon\Sigma(k^2)}.
\]

(34)
where \( k = \sqrt{(kk)} \). The numerator of the right side of Eq. (34) coincides with the expression \( \hat{\eta} \) in Ref. [7].

The transitions (24), (31) and (34) establish the correspondence between the discussed model and some effective theory of UP in the framework of traditional QFT approach. These transitions follow from the determination of \( \rho(\mu) \), that is from the accounting of interaction, which forms the wave packet (2) and mass smearing. Above mentioned effective theory has a close analogy with the traditional description of UP in the intermediate state as a special case of discussed approach. The most important feature of the effective theory, chosen in such a way, is the possibility to connect the decay-chain method and convolution method within the framework of this theory [7]. So, we have some self-consistency of the discussed model, effective theory, convolution and decay-chain method. However, due to some difficulties, which arise in traditional approach, the search of alternative \( \rho(\mu) \) - definition is actual now.

5. Phenomenological consequences of the model

The phenomena of mass smearing take place on the various hierarchical levels due to fundamental character of uncertainty principle. The value of FWE in the particle physics depends on the relations \( \Gamma/M \) and \( \Gamma/(M - M') \), where \( M \) and \( M' \) are the masses of UP and total masses of decay products. So, FWE is large in the decay or generation of UP with a large decay width \( \Gamma \) or in the near-threshold processes. There are many examples of the particles with a large value of \( \Gamma/M \) in the hadron physics, for instance \( \Gamma_{\rho}/M_{\rho} \approx 0.2 \), and we can observe a large effect in these cases. The fundamental UP have, as a rule, negligible widths, except \( Z, W \) bosons and \( t \) quark, which have \( \Gamma/M \sim 10^{-2} \). So, FWE can be discovered by means of the precision measurements of decay characteristics or in the near-threshold processes. In this section we offer a short review of mass-smearing phenomena and consider some examples of the processes, where FWE play a significant role.

One of the most pure FWE in the hadron physics takes place in the near-threshold decays \( \phi(1020) \to K^+K^-, K_L^0K_S^0 \). The ratio of branchings does not depend on hadron factors in a good approximation [10] and is equal to the ratio of phase space:

\[
R = \frac{B(\phi \to K^+K^-)}{B(\phi \to K_L^0K_S^0)} = \left( \frac{1 - 4m_+^2/m_\phi^2}{1 - 4m_0^2/m_\phi^2} \right)^{3/2},
\]

where \( m_+ = m(K^\pm) \) and \( m_0 = m(K^0) \). Inserting the values of masses into Eq. (35) we get the discrepancy between theoretical and experimental \( R \), which was discussed in Ref. [10]:

\[
R^{th} = 1.53; \quad R^{exp} = 1.45 \pm 0.03.
\]

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The various corrections to $R^{th}$ have been calculated in [10] (Bramon et al), but the discrepancy remains (Fermi’s ”Golden Rule” puzzle). Suggested model doesn’t directly describe FWE in the processes with fixed energy, for instance in the process $e^+e^- \rightarrow \phi(1020) \rightarrow \bar{K}K$. Here we consider model description of FWE in the process of type $X \rightarrow Y \phi(1020) \rightarrow Y \bar{K}K$.

The model prediction with account of FWE gives the ratio in the form:

$$R_M = \frac{\int_a^b \Gamma^+(m)\rho(m)dm^2}{\int_{a_2}^{b_2} \Gamma^0(m)\rho(m)dm^2},$$

(37)

where $a_1 = 4m_+^2$, $a_2 = 4m_0^2$, $b = E_{max}^2$, $\Gamma^+(m) \sim m(1 - 4m_0^2/m^2)^{3/2}$, $m_a = m(K^a)$ and $a = 0, \pm$. In the Breit-Wigner (BW) approximation, which is applicable to narrow resonances ($\Gamma_\phi/m_\phi \sim 10^{-3}$), the function $\rho(m)$ is defined by the expression:

$$\rho(m) = \frac{1}{\pi} \frac{m_\phi \Gamma_\phi}{(m^2 - m_\phi^2)^2 + m^2 \Gamma_\phi^2}$$

(38)

According to Eq.(37) the value $R_M$ depends on upper limit of integration $E_{max}$, which we have took in the interval of two-particle generation:

$$R_M = 1.43 - 1.41; \quad E_{max} = (1.5 - 2.0) \text{ Gev.}$$

(39)

Thus, the model account of FWE in the process under consideration gives the result, which is similar to experimental one (36). In the paper [10] (Fischbach et al.) close result was obtained with help of correction, caused by energy dependence of matrix element. This approach has some analogy with discussed treatment. With help of Eqs.(37) and (38) one can evaluate the contribution of FWE into the value $R$ for the decay channels $\phi(1020) \rightarrow \rho\gamma$, $f_0(980) \rightarrow \bar{K}K$ and other.

Hadron decays of type $H \rightarrow H_1H_2$ are direct objects of suggested model, when $H_1$ and (or) $H_2$ are the hadrons with a large width. The contribution of FWE into decay rates of the decays $B^0 \rightarrow D^-\rho^+$, $B^0 \rightarrow D^-a_1^+$ and $\Lambda_b^0 \rightarrow \Lambda_c^+\rho^-$, $\Lambda_b^0 \rightarrow \Lambda_c^+a_1^-$ were evaluated [6] in the approach, which is similar to CM. The result of calculations reveals that the contributions of FWE are large (from 20 to 40 percent) and its account improves the conformity of the experimental data and theoretical predictions. Here we consider the decay $B^0 \rightarrow D^-\rho^+$ and evaluate FWE according to discussed approach, which is in close analogy with one used in the Ref.[6] but gives other result. The two-body nonleptonic decays of B mesons have been studied by Bauer et al. [11] and reanalysed with account of FWE in Ref.[6]. Decay rate is given by

$$\Gamma(B^0 \rightarrow D^-\rho^+) = \frac{|A(m_\rho)|^2}{8\pi m_\rho^2} \kappa^3,$$

(40)
where $k$ is absolute value of final three-momentum in the rest frame of the $B^0$ meson and
\[ A(m_\rho) = \sqrt{2} G_F a_1 V_{ub} V_{ud}^* m_\rho F_1(m_\rho^2). \] (41)

In the Eq.(41) $a_1$ is Wilson coefficient, $f_\rho$ is decay constant and $F_1(m_\rho^2)$ is form factor at $q^2 = m_\rho^2$, which was approximated by a simple pole formula $F_1(q^2) = F(0)/(1 - q^2/m_{bc}^2)$. The expression (41) does not include FWE and gives a marked difference between theoretical and experimental values of branchings \[ B^{th} = 10.5 \cdot 10^{-3} \quad \text{and} \quad B^{exp} = (7.5 \pm 1.2) \cdot 10^{-3}. \] (42)

The contribution of FWE into $B^{th}(B^0 \to D^- \rho^+)$ was calculatrd in the Ref.\[ B^{th} = 5.78 \cdot 10^{-3}, \quad R = 0.55. \] (43)

where $\bar{B}^{th}$ is branchings with account of FWE and $R = \bar{B}^{th}/B^{th}$. We recalculate the ratio $R$ taking into consideration m-dependence of $f_\rho(m)$, $F_1(m)$ and $\Gamma_\rho(m)$ (in analogy with approach \[ R = \int_a^b B^{th}(m) \rho(m) \, dm^2 / B^{th}(m_\rho), \] (44)

where
\[ \rho(m) = \frac{1}{\pi} \frac{m \Gamma_\rho(m)}{(m^2 - m_\rho^2)^2 + m^2 \Gamma_\rho^2(m)}, \] (45)
\[ \Gamma_\rho(m) = \left(\frac{g_\rho^2}{48\pi}\right) m (1 - 4m_\pi^2/m^2)^{3/2}, \] (46)
\[ a = (2m_\pi)^2, \quad b = (m_B - m_D)^2. \] (47)

The expressions $f_\rho(m)$ and $F_1(m)$ are taken from Ref.\[ R = 0.82, \quad \bar{B}^{th} = 8.64 \cdot 10^{-3}. \] (48)

Thus, our approach leads to more realistic evaluation of FWE, that is $R$, which improves the conformity of the experimental data and theoretical predictions. There are many processes in the hadron physics with participation of the hadrons with a large total width, for instance $f_0(600)$, $\rho(770)$, $f_0(980)$, $a_0(980)$ etc. In these cases we must take into account FWE, particularly in the near-threshold processes. Another feature of the mass-smearing phenomenon can manifests itself through the mass dependence of the hadron factors, such
as decay constant and form factor, which have been taken into consideration in the discussed decay $B^0 \rightarrow D^- \rho^+$. 

The most of elementary (or fundamental) particles are unstable, however, the large width have $W, Z$ bosons and $t$ quark only. The ratio $\Gamma/M \sim 10^{-2}$ for these particles, that is the value of FWE can be measured in the precision experiments or in the near-threshold processes. The decay rates of the near-threshold decays $t \rightarrow WZ\bar{b}, cWW$ and $cZZ$ were calculated with account of FWE within the framework of CM and DCM in the Refs.\[5]. The contributions of FWE lead to substantial enhancement of decay rates, in particular of $B(t \rightarrow WZ\bar{b})$ and $B(t \rightarrow cZZ)$. For instance, the branchings without ($B$) and with ($\bar{B}$) account of FWE in the first case differ by an order of magnitude [5] (Altarelli et al.): $B(t \rightarrow WZ\bar{b}) \sim 10^{-7}$, $\bar{B}(t \rightarrow WZ\bar{b}) \sim 10^{-6}$. The description of these decays by suggested model does not differ from the one in the Ref.[5]. Here we consider the contribution of FWE into decay rates of the decay channels $W \rightarrow f_1\bar{f}_2$ and $Z \rightarrow f\bar{f}$. In the approximation of massless fermion the partial width $\Gamma \sim M^3$ and the ratio $R = \bar{B}/B$ is defined by the simple expression:

$$ R = \int_a^b m^3\rho(m) \, dm^2/M^3. \quad (49) $$

In the case of process $e^+e^- \rightarrow Z \rightarrow W^+W^-$ near the threshold ($\sqrt{s} \approx 2M_W$) $a \approx m_W^2$, $b \approx s$. In the Breit-Wigner approximation for $\rho(m)$ from the Eq.[49] we get $R_W \approx 1.04$. The same result takes place for the $Z$-pair generation near threshold. It should be noted, that the values of the limits of integration in the Eq.[49] crucially depends on the process of $W$ or $Z$ generation. In the processes of type $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ mass parameter is fixed $m = \sqrt{s}$ and we have the decay properties of $Z$ as function of $\sqrt{s}$. Thus, the contributions of FWE into decay properties of $Z$, $W$ bosons and $t$ quark must be taken into consideration in the precision measurements. The evaluation of these contributions can be fulfilled in the framework of the convolution method in a simple way. It should be noted that considered effects don’t influence on the precision measurements of $Z$ properties at fixed energy $\sqrt{s} \approx M_Z$.

The effect of the mass smearing can plays a significant role in the mixing, oscillation and CP violation in the systems of neutral mesons $M^0 - \bar{M}^0$. Large contribution of FWE into mixing is due to the width of the short-lived state is comparable with the splitting of mass $\Gamma_S \sim \Delta m = |m_S - m_L|$. So, the levels of the short-lived ($M_S$) and long-lived ($M_L$) states strongly overlap due to mass smearing and this effect can influence on the mixing. To illustrate this phenomenon we compare the values $\Gamma$ and $\Delta m$ in the case of $K^0 - \bar{K}^0$ and
In the case of $B^0 - \bar{B}^0$ systems \[11\]:

$$
\Gamma(K_S^0) = 7.30 \cdot 10^{-6} eV, \quad \Delta m_K = 3.48 \cdot 10^{-6} eV,
$$
$$
\Gamma(B_d^0) = 4.24 \cdot 10^{-4} eV, \quad \Delta m_{B_d} = 3.34 \cdot 10^{-4} eV,
$$
$$
\Gamma(B_s^0) = 4.43 \cdot 10^{-4} eV, \quad \Delta m_{B_s} \geq 94.8 \cdot 10^{-4} eV
$$

(50)

In the case of $B^0 - \bar{B}^0$ systems short- and long-lived components have near the same widths and the division is usually marked by $M_H$ (heavy) and $M_L$ (light) states. Now we consider the phenomenological consequences of mass smearing in the systems of neutral mesons.

The theoretical evaluations of mass splitting, which are based on short distance FCNC transition (box diagrams), do not account FWE. So, the contradictions can take place between the theoretical $\Delta m^{th}$ and experimental values $\Delta m^{exp}$, which follow from the oscillation experiments. Here we demonstrate this on the examples of $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems.

The experimental value of mass splitting $\Delta m^{exp}$ follows from the observable characteristic of oscillation $\chi$:

$$
\chi = \frac{x^2}{2(1 + x^2)}, \quad x = \frac{\Delta m^{exp}}{\Delta m^{th}} \quad (when \ y = \frac{\Delta \Gamma}{\Gamma} \ll x).
$$

(51)

When $\Gamma_S \sim \Delta m^{exp}$, then the measured $\chi$ has some effective value, averaged by the mass distribution:

$$
\bar{\chi} = \int \chi(m) \rho(m) \, dm^2,
$$

(52)

where according to Eq.(51) $\chi(m) = \Delta m^2(m)/2(\Delta m^2(m) + \Gamma^2)$ and $\Delta m(m) = |m - M_L|$. The value $\bar{\chi}$ can substantially differs from $\chi_0 = \Delta m_0^2/2(\Delta m_0^2 + \Gamma^2)$, where $\Delta m_0 = |m_L - \bar{m}_S|$ and $m_L$ is constant. As a result the mass splitting, which follows from the oscillation experiment, can significantly differs from the theoretical value $\Delta m_0$, which follows from the FCNC transitions. Here we consider this effect for the $K^0 - \bar{K}^0$ system in more detail. The theoretical value $\Delta m^{th}_K$ is defined by the expression \[12\]:

$$
\Delta m^{th}_K = \frac{G_f^2}{6\pi^2} m_W^2 m_K B_K F_K^2 (\eta_1 |U_{cs}^* U_{cd}|^2 s_0(x_c) \\
+ \eta_2 |U_{ts}^* U_{td}|^2 s_0(x_t) + 2\eta_3 |U_{cs}^* U_{cd} U_{ts}^* U_{td}| s_0(x_c, x_t)).
$$

(53)

In the Eq.(53):

$$
\begin{align*}
s_0(x_c) &= x_c, \quad s_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3x_t^2 \ln x_t}{2(1 - x_t)^3}, \quad x_c = \frac{m_c^2}{m_W^2}, \\
x_t = \frac{m_t^2}{m_W^2}, \quad s_0(x_s, x_t) &= x_c (\ln \frac{x_t}{x_c} - \frac{3x_t}{4(1 - x_t)} - \frac{3x_t^2 \ln x_t}{4(1 - x_t)^2}).
\end{align*}
$$

(54)

The most detailed evaluation of hadron factors, which enter to Eq.(53), was fulfilled in Ref.\[12\], including the corrections beyond leading logarithm:

$$
B_K = 0.84, \quad \eta_1 = 1.38, \quad \eta_2 = 0.574, \quad \eta_3 = 0.47, \quad F_K = 160 MeV.
$$

(55)
Using (55) and other data from [11] with help of (53) we get $\Delta m_{K}^{th}$, which is substantially less than $\Delta m_{K}^{exp}$:

$$\Delta m_{K}^{th} = 2.34 \cdot 10^{-6} \text{eV}, \quad \Delta m_{K}^{exp} = 3.48 \cdot 10^{-6} \text{eV}. \quad (56)$$

The characteristics of oscillation, which correspond to these mass splitting, are defined by Eq.(51):

$$\chi_{K}^{th} = 0.047, \quad \chi_{K}^{exp} = 0.093 \quad (57)$$

This large discrepancy can be eliminated by accounting for the mass smearing effect. In the BW approximation Eq.(52) can be rewritten in the form:

$$\bar{\chi} = \frac{\Gamma}{\pi} \int_{a}^{b} \frac{(x + \Delta m)^2 \, dx}{((x + \Delta m)^2 + \Gamma^2)(4x^2 + \Gamma^2)}, \quad (58)$$

where $x = m - m_S$, $\Gamma = \Gamma_S$ and the limits of integration $(a, b)$ are defined by the interval of $m$ variation, which, however, is not correctly determined. To evaluate $\bar{\chi}$ we fix the interval $m_L \leq m \leq m_S + n\Delta m$, where $n = 5, \ldots, 100$. Inserting $\Delta m = \Delta m$ into Eq.(58) we get:

$$\bar{\chi}_{K} = 0.075 - 0.117 \quad (59)$$

Thus, the account of mass smearing in the $K^0 - \bar{K}^0$ mixing can significantly change the value $\chi$ and make it possible to establish the accordance between $\chi_{K}^{exp} = 0.093$ and $\Delta m_{K}^{th} = 2.34 \cdot 10^{-6} \text{eV}$. The value of the mass splitting in the $B_d^0 - \bar{B}_d^0$ system follows from the expression [12]:

$$\Delta m_{B} = \frac{G_F^2}{6\pi^2} m_W^2 M_B f_B^2 s_0(m_t^2/m_W^2)(V_{td}V_{tb}^*)^2, \quad (60)$$

where:

$$s_0(x_t) = 0.784x_t^{0.76}, \quad f_B\sqrt{B} = 220\text{MeV}, \quad \eta_B = 0.551 \quad (61)$$

Using the rest data from [11], we get the central value of $\Delta m_{B}^{th}$, which is slightly larger than the experimental one:

$$\Delta m_{B}^{th} = 3.51 \cdot 10^{-4} \text{eV}, \quad \Delta m_{B}^{exp} = 3.34 \cdot 10^{-4} \text{eV}. \quad (62)$$

Inserting the value $\Delta m_{B}^{th}$ into Eq.(58) we get for the same interval of $m$ variation:

$$\bar{\chi}_{B}^{th} = 0.18 - 0.20, \quad \chi_{B}^{exp} = 0.188 \pm 0.003 \quad (63)$$

Thus, we get the realistic result, but for the correct evaluation of FWE contribution in this case we need more precise measurement of CKM element $V_{td}$. The effect of mass smearing in $B^0_s - \bar{B}^0_s$ system is small due to inequality $\Delta m_{B} \gg \Gamma_B$. In this case from Eq.(51) we have $\chi \approx 0.5 \approx \bar{\chi}$. It should be noted, that investigation of the mass-smearing effect (or FWE) in the $M^0 - \bar{M}^0$ systems demands more detailed and self-consistent analysis, which must
include the fit of CKM mixing matrix. The above considered examples are the illustrations, which can stimulate further investigation of the discussed effect. The same conclusion takes place for the indirect CP violation, which caused by the mixing.

In this section we have considered some examples of processes in the various fields of particle physics, were FWE give large contributions. We fulfilled rough evaluations of these contributions to illustrate the important role of FWE in some specific cases. The conclusion follows from this short analysis, that mass-smearing effect should be taken into consideration in the hadron and particle physics.

6. Conclusions

The finite-width effects in the processes with participation of UP can be described by renormalized propagator, decay-chain method, convolution method and effective theory of UP. The convolution formula is convenient instrument for calculations of decay rate and gives the results in accordance with experiment. In this paper we have considered the model of UP with a random mass and derived the convolution formula as a direct consequence of the model. The model operator function and Lagrangian have a convolution structure and describe the mass-smearing effects in accordance with the uncertainty principle.

The principal element of suggested model is probability density function $\rho(\mu)$, which describes the main properties of UP. Traditional description of UP in the intermediate state by resonance line with complex pole (or by dressed propagator with mass and width as parameters) corresponds to the model description of UP in arbitrary state by function $\rho(\mu)$ with the same parameters. We have considered the determination of $\rho(\mu)$ from DCM and by matching the model propagator to renormalized one. The first approach is equivalent to the convolution method or truncated decay-chain method. The second one has some restrictions, caused by propagator renormalization peculiarities. It should be noted, that the mass-smearing effect follows from the fundamental uncertainty principle, then the search of $\rho(\mu)$ from the first principles is reasonable.

We have considered some examples of FWE manifestations in a various regions of the particle physics. The fulfilled analysis and evaluations of the FWE contributions to observable characteristics lead to the conclusion: these contributions can be large, an account of it improves the conformation of experimental data and theoretical predictions, convolution method gives a simple and convenient tool for evaluation of the effect.
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