Numerical Analysis and Study of the Dynamic Response of Structural Systems for Machinery Foundations

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Abstract: This work aims the deterministic dynamic analysis, in the time and frequency domain, of a reinforced concrete floor supported by a pre-cast pile foundation system, when subjected to the excitation produced by a large compressor installed in an industry for the production of air gases. The concrete slab presents a dimension of 12 m × 15 m, required to support a compressor-motor assembly weighting 1,900 kN and positioned at a height of 4m of the investigated concrete floor. In this investigation, two numerical models were developed and the difference between these models is characterized by the discretization of the support points (pre-cast concrete piles). The developed numerical model adopted the usual mesh refinement techniques present in finite element method simulations implemented in the CSi SAP2000 V.17.2.0 software. Based on the developed analysis methodology, the dynamic structural response of the foundation system is evaluated in terms of natural frequencies, vibration modes, displacements, velocities, and accelerations. The maximum values of the dynamic response of the system are compared with the limit values recommended by standards and project recommendations, aiming a careful evaluation, regarding the performance of the structure in terms of excessive vibrations and the economic aspects involved in the design of the foundation system. Finally, the obtained results of the two developed numerical models are compared, as to evaluate if there are benefits in refining the support points modelling.

Key words: Machinery foundation, structural dynamics, numerical analysis.

1. Introduction

The design of foundations systems for rotating machinery consists in a complex task for civil engineers, due to the several actions involved in the problem present different natures, such as static or dynamic forces. This design, if not well performed, may result in overestimated foundations (anti-economic design) or even, in the opposite, deficient foundations systems, causing damages to the equipment, to the foundation or even on near structures, generating production problems and bringing risks to safety and people’s health [1-4].

The design of machinery foundations involves, in a general way, the analysis of the rotating equipment and the dynamic forces existing during its operation, the evaluation of soil characteristics, the development of structural model and, certainly, the study of the dynamic structural response, verification of displacements, velocities and maximum acceleration and its tolerances, the design of isolators (only when necessary), the structural design and, also, the verification of maximum soil stresses [5-8].

Rotating machines provoke dynamic forces that are transmitted to foundation as vibrational movements, or in case of sensitive machines, that are subjected to the foundation vibrations [9-11]. The machines can be classified as: (a) those that produce impact forces such as hammers; (b) those that provoke periodic forces such as piston compressors or explosion motors; (c) machines of high-velocity, such as turbines and rotating compressors; (d) other machines. These equipments also can be classified based on their operating frequency values: (a) low to medium frequency, those with frequencies under 500 RPM; (b) medium to high frequency, presenting frequencies between 500 and 1,000 RPM; (c) high frequency, with frequencies over 1,000 RPM [12].
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Considering the study of the dynamic structural response of a foundation system, the main objective of the structural engineer is to analyse the maximum values of the displacements, velocities and accelerations, assuring that the equipment-foundation system works in a safe way, presenting if possible natural frequencies as far as possible from resonating frequencies, avoiding a resonance condition, in order to guarantee the adequate operation of the equipment and also attending human comfort criteria [13-17].

This way, in this investigation a deterministic dynamic analysis of a reinforced concrete slab supported by a pre-cast pile foundation system, subjected to the excitation produced by a large compressor was performed, in the time and frequency domain. In this analysis the natural frequencies, vibration modes, displacements, velocities and accelerations were evaluated, and after that the response maximum values were compared with the limits recommended by standards and design codes recommendations.

2. Investigated Structural Model

The investigated structural model is composed by a reinforced concrete floor supported by 20 pre-cast reinforced concrete piles of diameter equal to 0.50 m and length of 8.50 m. The floor dimensions are equal to 12 m × 15 m (in plan). The motor is located above a pedestal of dimensions equal to 5.55 m × 3.15 m (in plan) and 3.84 m height. The compressor is located above two pedestals, the first one presenting 2.40 m × 1.50 m (in plan) and the second 2.40 m × 0.60 m (in plan), both 2.66 m height. The concrete of the floor is C30 Class (ABNT NBR 6118) and the elasticity modulus is equal to 2.60 × 10⁴ MPa. The concrete of the pre-cast piles is C20 class and the elasticity modulus is equal to 2.13 × 10⁴ MPa, see Fig. 1.

3. Finite Element Modelling of the Structural System

The developed numerical models adopted the usual mesh refinement techniques present in finite element method simulations implemented in the CSi SAP2000 V.17.2.0 [18]. In this analysis, two different modelling strategies were investigated [1]. The first one has considered that the pre-cast reinforced concrete piles were simulated by translational spring elements calculated based on the axial stiffness of the piles. On the other hand, it must be emphasized that three-dimensional frame elements with tension, compression, torsion and bending capabilities were used to simulate the concrete piles in the second numerical modelling strategy, see Figs. 2 and 3 [1].

In both developed finite element models (Models I and II, see Figs. 2 and 3), the hexaedral solid elements were used to represent the concrete floor. The hexaedral solid element presents three translational degrees of freedom per node. The three-dimensional frame element presents six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the x, y, and z-axes [1]. It must be emphasized that in this investigation the soil behaviour was simulated considering a group of independent springs governed by a linear-elastic model.

The foundation applies a reaction in the column normal direction that is proportional to the column deflection. Considering the subsoil geotechnical profile and using the analysis procedure based on the Winkler model, the horizontal reaction coefficients on the piles...
were determined as a function of the type of the soil [19-22]. Based on the values of the horizontal reaction coefficients, the foundation stiffness parameters values were calculated. This way, the foundation stiffness parameters values were used to determine the spring’s stiffness used in the finite element model to simulate the soil behavior [19-22]. The spring elements that simulate the soil were discretized based on a range of length equal to 1m and placed at the transversal direction of the concrete pile section axis.

4. Eigenvalues and Eigenvectors Analysis

Initially, the modal analysis was performed aiming to obtain the natural frequencies values (eigenvalues) and the vibration modes (eigenvectors) of the developed numerical models (Models I and II), making possible to verify the dynamic structural behavior of the investigated system [23, 24]. This way, Table 1 presents the values of the natural frequencies of the system and Figs. 4-7 show the main mode shapes of the structure, when the numerical models I and II are considered in the investigation.

5. Dynamic Structural Response

In this study, the developed numerical models were analysed in the time and frequency domain for the evaluation of the foundation performance based on a forced vibration analysis. To do this, the investigated structural system (Figs. 1-3) was subjected to the dynamic excitations (harmonic dynamic loads) due to

Table 1  Natural frequencies: Models I and II.

| Vibration mode | Frequency (Hz) | Vibration mode | Frequency (Hz) |
|----------------|---------------|----------------|---------------|
| f_{01}         | 3.21          | f_{01}         | 1.99          |
| f_{02}         | 3.24          | f_{02}         | 2.01          |
| f_{03}         | 4.19          | f_{03}         | 2.58          |
| f_{04}         | 13.73         | f_{04}         | 13.58         |
| f_{05}         | 15.14         | f_{05}         | 14.86         |
| f_{06}         | 15.51         | f_{06}         | 15.26         |
| f_{07}         | 15.86         | f_{07}         | 15.67         |
| f_{08}         | 20.06         | f_{08}         | 19.92         |
| f_{09}         | 29.66         | f_{09}         | 29.39         |
| f_{10}         | 34.93         | f_{10}         | 34.13         |
| f_{11}         | 40.77         | f_{11}         | 39.46         |
| f_{12}         | 46.34         | f_{12}         | 44.99         |

Fig. 4  Mode shape #01 (Model II): horizontal translation along y-axis. (Model I frequency in blue; Model II in green).
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Fig. 5 Mode shape #02 (Model II): horizontal translation along x-axis. (Model I frequency in blue; Model II in green).

Fig. 6 Mode shape #03 (Model II): rotation around z-axis. (Model I frequency in blue; Model II in green).

Fig. 7 Mode shapes #09 (Model I) and #49 (Model II): bending about y-axis. (Model I frequency in blue; Model II in green).

Table 2  Motor and compressor design specifications.

| Equipment | Weight (kN) | Frequency (Hz) | \( F_0 \) (kN) |
|-----------|-------------|----------------|---------------|
| Motor     | 390         | 30             | 8.5           |
| Compressor| 630         | 30             | 6.3           |

It must be emphasized that the masses (inertial forces) and the dynamic excitations were considered acting on the centre of gravity of the equipment, motor and compressor, respectively. In sequence of the investigation, Table 3 presents the maximum displacements and velocities values of the equipment anchoring points (structural sections A to P; Fig. 8).

The maximum values of the dynamic structural response of the structural system (displacements and velocities) were obtained through CSi SAP2000 [18]. These maximum values were compared with the maximum values proposed by standards and design recommendations [25]. It is important to remember that usually the admissible amplitudes are informed by the equipment manufacturer. In any situation that these data are not provided, it is recommended to adopt the values of ISO 1940-1 [25]. In this paper, it was adopted the value of 60 \( \mu \)m for maximum displacements and 2.8 mm/s for maximum velocities.

Considering Table 3 results, obtained at the steady state response of the Models I and II, it can be concluded that the vertical displacements maximum
Table 3 Dynamic structural response of the system: displacements and velocities.

| Structural section | Model I (μm) | Model II (μm) | Model I (mm/s) | Model II (mm/s) |
|--------------------|--------------|--------------|---------------|---------------|
| A                  | 3.294        | 3.229        | 0.46          | 0.39          |
| B                  | 3.386        | 3.518        | 0.48          | 0.44          |
| C                  | 3.477        | 3.923        | 0.50          | 0.57          |
| D                  | 3.567        | 4.443        | 0.52          | 0.71          |
| E                  | 3.654        | 5.145        | 0.54          | 0.85          |
| F                  | 4.544        | 4.786        | 0.66          | 0.72          |
| G                  | 4.578        | 4.818        | 0.66          | 0.72          |
| H                  | 4.612        | 4.850        | 0.65          | 0.72          |
| I                  | 4.646        | 4.881        | 0.65          | 0.73          |
| J                  | 4.678        | 5.194        | 0.64          | 0.83          |
| K                  | 2.579        | 4.989        | 0.45          | 0.88          |
| L                  | 2.387        | 2.803        | 0.40          | 0.42          |
| M                  | 1.956        | 4.191        | 0.38          | 0.77          |
| N                  | 1.742        | 1.860        | 0.32          | 0.28          |
| O                  | 4.354        | 6.208        | 0.54          | 0.84          |
| P                  | 3.710        | 5.957        | 0.45          | 0.72          |

Limit amplitude (ISO 1940-1, 2003): 60 μm
Limit velocity (ISO 1940-1, 2003): 2.8 mm/s

In sequence, Figs. 9 and 10 show the dynamic structural response of the system (displacements and velocities), in time domain, based on the node K response. The structural section K (node K) generically represents the dynamic response of the structure.

Considering Table 3 results (accelerations), obtained at the steady state response of the Models I and II, it can be concluded that the vertical accelerations maximum values are too far of the recommended limit \([A_Z = 0.55 \text{ m/s}^2 < A_{Z\text{lim}} = 1.0 \text{ m/s}^2]\) [8] and the human comfort criteria was satisfied as well.

Fig. 8 Investigated structural sections: motor and compressor analysis.

values are very low \([U_Z = 6.21 \mu \text{m} < U_{Z\text{lim}} = 60 \mu\text{m}]\) [25]. Analysing the velocities maximum values, the authors can confirm that these values are also below the admissible recommended limit \([V_Z = 0.88 \text{ mm/s} < V_{Z\text{lim}} = 2.8 \text{ mm/s}]\) [25]. These values of displacements and velocities can corroborate the affirmation that the structure can be optimized.

Fig. 9 Vertical displacement \(U_Z\) of node K: Models I and II.

Fig. 10 Vertical velocity \(V_Z\) of node K: Models I and II.
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Table 4  Dynamic structural response: accelerations.

| Structural section | Model I (m/s²) | Model II (m/s²) |
|--------------------|---------------|----------------|
| Q                  | 0.22          | 0.28           |
| R                  | 0.16          | 0.27           |
| S                  | 0.15          | 0.29           |
| T                  | 0.44          | 0.55           |
| U                  | 0.35          | 0.46           |
| V                  | 0.26          | 0.38           |
| W                  | 0.13          | 0.20           |
| X                  | 0.06          | 0.13           |
| Y                  | 0.13          | 0.25           |

Limit acceleration (ISO 1940-1, 2003): 1.0 m/s²

Having in mind a human comfort analysis, Table 4 presents the maximum accelerations values related to the structural sections Q through Y, as illustrated in Fig. 11, and in sequence, Fig. 12 shows the dynamic response of the structure (accelerations), in time domain, based on the node K response. Considering a frequency domain dynamic analysis, Fig. 13 presents the vertical displacements of node F. It is possible to see clearly the tendency of the structure presenting peaks of displacements related to the resonance situations, when the natural frequencies values of the both numerical models I and II are coincident with the excitations frequencies, associated to harmonic loadings acting on the structural system (motor and compressor). It must be emphasized that the Model II presented more strong and defined energy transfer peaks, with some oscillations on subharmonics. The

most parcel of energy transfer of the investigated structural system is associated to the resonance condition and occurs when the Model II is considered in the analysis.

6. Conclusions

This paper aims the deterministic dynamic analysis, in the time and frequency domain, of a reinforced concrete floor supported by a pre-cast pile foundation system, when subjected to the excitation produced by a large compressor installed in an industry for the production of air gases.
Based on the natural frequency values and vibration modes configurations and the maximum values of displacement, velocity and acceleration, calculated along this investigation, it was possible to properly evaluate the dynamic structural behavior of the structural model, considering two different strategies of numerical modelling. The first one has considered that the pre-cast reinforced concrete piles were simulated by translational spring elements and the second one considered three-dimensional frame elements with tension, compression, torsion and bending capabilities used to simulate the concrete piles.

The results obtained indicated that the foundation system analysed in this study fully attends the standards requirements \([U_Z = 6.21 \mu m < U_{Z\text{lim}} = 60 \mu m; V_Z = 0.88 \text{ mm/s} < V_{Z\text{lim}} = 2.8 \text{ mm/s} \text{ and } A_Z = 0.55 \text{ m/s}^2 < A_{Z\text{lim}} = 1.0 \text{ m/s}^2]\) [25]. It must be emphasized that the investigated structural system clearly can be optimized; the final concrete volume and also the final costs of the system can be significantly smaller.

Regarding the different types of strategies of modelling, it could be noted that in Model I, the steady state response begins approximately in 1s \((t = 1s)\), and when the Model II is considered in the analysis this time is three times larger \((t = 3s)\). In the frequency domain analysis, it can be observed that the energy transfer peaks are different between the two numerical models I and II, and the intensity is higher in the Model II, even more on the frequency of interest \((f = 30 \text{ Hz: excitation frequency of the motor-compressor})\).

This way, considering the results presented in this investigation, the authors would like to emphasize the importance of the consideration of the effects of interaction soil-structure, based on the modelling of the piles, as considered in the numerical model II (Model II), due to the fact that relevant differences can occur when the dynamic structural response is investigated.

Finally, this research work will proceed, based on a structural optimization of the foundation system, evaluating aspects related to the geometry of the system, but keeping in mind that all economic evaluation (optimization process) must take into account the aspects related to the design criteria, code recommendations and safety.

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