Improving meson two-point functions by low-mode averaging

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Some meson correlation functions have a large contribution from the low lying eigenmodes of the Dirac operator. The contribution of these eigenmodes can be averaged over all positions of the source. This can improve the signal in these channels significantly. We test the method for meson two-point functions.

1. Introduction

Simulations with light quarks face the problem of increasing noise in meson two-point functions with decreasing quark masses. Some of this noise is due to using only a single source for the meson instead of averaging over all possible positions of this source. We present a method to improve such meson correlation functions. More details can be found in \[1\]; for related work see \[2\] and \[3\]. The idea is to average the contribution of the low-lying eigenmodes of the Dirac operator \(D\) over all positions of the source on the lattice. The contribution of the high modes is calculated using the standard method, inverting the Dirac operator on a single or a few quark sources. Ref. \[3\] has introduced the name low mode averaging (LMA) for this method. LMA can have a considerable impact on the noise in the two-point function if it is dominated by the low-lying modes. We expect it to be particularly effective at low quark masses and for Dirac operators with good chiral symmetry like the overlap operator \(\mathbb{H}\) which we use in our test.

An example for the dominance of the low modes is the pseudo-scalar scalar difference shown in Fig. 1. The contribution from the 20 lowest eigenmodes is shown.

2. Method

We are considering zero momentum two-point functions of the form

\[
\langle C(t) \rangle = \left\langle \frac{1}{L^3T} \sum_{x',x'',t'} \text{tr} \Gamma_1 G(x',t'' + t;x'',t') \times \Gamma_2 G(x'',t'';x',t'' + t) \right\rangle,
\]

with \(G\) the propagator matrix,

\[
(D + m)G(x',t';x,t) = \delta_{x,x'} \delta_{t,t'},
\]

modest are used to precondition the inversion of the Dirac operator. The associated gain in speed of the inversion alone justifies their computation.
and $\Gamma$, the Dirac matrix corresponding to the meson in question. In a standard simulation, however, one does not compute the full propagator but only a row, by inverting the Dirac operator on a localized source at, e.g., $(x', t') = (0, 0)$. On a given set of $N$ configurations one thereby obtains an estimator $\langle C_1(t) \rangle$ for $\langle C(t) \rangle$.

$$\langle C_1(t) \rangle = \langle \frac{1}{L^4 T} \sum_x \text{tr} \Gamma_1 G(0, 0; x, t) \times \Gamma_2 G(x, t; 0, 0) \rangle . \quad (1)$$

However, using eigenmodes of the Dirac operator, one can compute $C(t)$ itself. As this is in general not possible for all eigenmodes, we split the propagator in a contribution from the low modes and one from the high modes $G = G_L + G_H$ with $G_L$ the propagator in spectral representation

$$G_L(x, t; x', t') = \sum_{j=1}^n \frac{\langle x, t | j \rangle \langle j | x', t' \rangle}{i\lambda_j + m} . \quad (2)$$

The sum is over the $n$ lowest eigenmodes of the Dirac operator $|j\rangle$ with eigenvalues $i\lambda_j$.

The two-point function is thereby separated into four different parts, one from the low modes alone, one from the high modes and two interference terms

$$C(t) = C_{LL}(t) + C_{HL}(t) + C_{LH}(t) + C_{HH}(t) . \quad (3)$$

The first contribution $C_{LL}$ can be expressed by the low-lying eigenmodes alone and can thus be averaged over all positions of the source. The other contributions are restricted to the usual one or a few quark sources.

3. Test of the method

To test the method we use 80 quenched gauge configurations of size $12^3 \times 36$ generated with the Wilson gauge action at $\beta = 5.9$. We use the overlap Dirac operator constructed from HYP smeared gauge links. The low mode averaging is done using the 20 lowest eigenmodes of the Dirac operator which are also used to precondition the computation of the propagator. The bare quark masses are between 0.015 and 0.035 which correspond to pseudo-scalar to vector meson mass ratios $m_{PS}/m_V$ ranging between about 0.4 to 0.64, see [5] for further details. On each of the configurations the inversion of the Dirac operator was done on two Gaussian sources with radius 3a, one located on time-slice $t = 0$, the other on $t = 16$. We average over these two positions. The $\Gamma$ matrices depend on the meson in question and we shall use the abbreviations given in Table 1.

![Table 1](image)

Table 1

| Label | $A_\mu$ | $V_\mu$ | $B_{\mu\nu}$ |
|-------|---------|---------|-------------|
| $P$   | $\gamma_5$ | $\gamma_5 \gamma_\mu$ | $\gamma_\mu$ $\gamma_\mu$ $\gamma_\nu$ |

Figure 2. The effective mass plots for the $PP-SS$ correlator at $m_q = 0.025$. The open circles are the effective mass without low mode averaging, the the signal given by the filled squares is averaged using 20 eigenmodes. To give an impression of the improvements in the various channels, the ratio of the uncertainty

![Figure 2](image)
Figure 3. Ratio of the error bars for $n = 20$ compared to $n = 0$ at time-slice $t = 5$ for the different meson correlators. Data uses two source points for the high eigenmode part of the correlator.

Figure 4. Dependence of the uncertainty in the extracted $PP - SS$ mass on the number of eigenmodes included at $am_q = 0.025$.

Figure 5. Same as Fig. 4 for the $B_{0i}$ mass.

At time-slice $t = 5$ with LMA over 20 eigenmodes to the uncertainty without LMA is shown in Fig. 3. In the $\gamma_5, \gamma_5\gamma_i, \gamma_0\gamma_i$ and $\gamma_i$ channels, we find an improvement of roughly 30%, which corresponds to a factor of 2 in statistics. The $\gamma_5\gamma_0$ and $\gamma_i\gamma_j$ cannot profit from the LMA. It is known that low modes make a small contribution to these correlators.

The ultimate goal of this method is to improve the errors of the extracted meson masses. In Figs. 4 and 5 we show the dependence of the error of the masses on the number of eigenmodes used in the LMA. We find that the improvement seems to saturate at 12 to 16 modes included and reaches about 30% for the PP-SS mass and about 40% for the vector meson from the $B_{0i}$.

4. Conclusion

We tested a method to improve meson two-point functions in lattice QCD. We found an improvement in the uncertainty of some masses corresponding to a factor of 2 in statistics. This improvement comes at virtually no additional cost because the cost of computing the low eigenmodes is justified by the acceleration of the inversion of the Dirac operator alone.

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