Lubrication performance of conical spiral-groove bearings with rough surfaces

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Abstract. A mixed lubrication model of conical spiral-groove bearings is established by integrating partial fluid lubrication theory and G-W statistical theory. Considering the effect of bearing surface roughness, the equivalent average Reynolds equation is derived based on average flow model, which is solved by boundary fitted coordinate system and finite difference method. The G-W contact model is used to calculate the axial force and friction force produced by the contact of multi asperities. The coefficients of friction are calculated and measured under different rotational speed and the theoretical results are in accordance with the test results. The results show that comparing with bearings with deep groove, ones with shallower groove have lower transition speed and lower friction coefficient at low speeds.

1 Introduction
Conical spiral-groove bearings have been adopted for high-speed rotating machines, such as centrifuges, flywheels, machine spindles et al. because of their remarkable hydrodynamic effect, the capability of carrying both axial and radial loads, self-sealing performance, and good stability in the case of small eccentricities. Many studies on static and dynamic characteristics of this type of bearings are on the base of the assumption of full-film lubrication [1-4]. In fact, hydrodynamic bearings are usually in mixed lubrication, boundary lubrication or even dry friction during startup and shutdown transient operation. Analyzing the single lubricated state cannot fully reveal the evolution of different lubrication regime. There are, however, only a few reports that evaluate hydrodynamic performance of conical spiral-groove bearings throughout the entire range of operating conditions.

It needs to consider the combined effects of surface roughness and asperities contact when the spiral-groove bearings are in mixed lubrication at low speed. As for the influence of surface roughness on the hydrodynamic performance of spiral-groove bearings, a typical approach is to establish the average Reynolds equation by means of average flow model, which is popular in dealing with rough plain bearings [5-7]. In addition, several literature have considered other effects on lubrication performance of spiral-groove bearings, such as fluid centrifugal inertia force, sleeve elastic deformation, cavitation occurrence et al [8-10].

In general, there are three methods to calculate the load carried by asperities. G-W contact model is a classical statistical model for analyzing elastic or elastic-plastic contact between rough and smooth surfaces, the disadvantage of which is that the shape and height distribution of asperities are too
simple. On the base of G-W contact model, K-E contact model further improves the elastic-plastic contact. While M-B contact model uses fractal theory to establish the elastic-plastic contact model, which is on the base of fractal parameters and limited to the surface with fractal characteristics [11-16].

In this paper, combining with partial lubrication and asperities contact is proposed to analyze the lubrication performance of conical spiral-groove bearings. The average Reynold equation is used to calculate hydrodynamic pressure, while G-W contact model is adopted to compute the force of asperities contact. The calculated results of the friction coefficients are compared with the measured results to verify the validity of the proposed model. Then the effect of depth of groove on hydrodynamic performance is discussed.

2 Theoretical approach

2.1 Partial lubrication model

Figure 1 shows the geometry configuration of a conical spiral-groove bearings treated here. The physical coordinate system \((r, \theta, z)\) is fixed to the shaft with twelve spiral grooves, \(s\) is the meridian coordinate. In order to analyze the performance of spiral-groove bearings more accurately, the physical coordinate system \((r, \theta, z)\) could be transformed into the computational coordinate system \((r, \eta, z)\) by means of boundary fitting coordinate, \(\xi\) is the meridian coordinate in computational coordinate system, and the transformation relation between these two coordinate systems is:

\[
\begin{align*}
\xi &= s \\
\eta &= \theta - \frac{1}{\sin \alpha \tan \beta} \ln \left( \frac{s}{s_s} \right)
\end{align*}
\] (1)

Where, \(\alpha\) is the half angle of the cone, \(\beta\) is the angle of spiral groove, \(s_s\) is the base radius of the spiral line.

![Figure 1. The geometry configuration of a conical spiral-groove bearing.](image)

Considering the effect of surface roughness of bearings, the pressure flow factor and shear flow factor are introduced based on the Patir-Cheng average flow model [5,6]. It is assumed that the fluid is incompressible and in the condition of isothermal laminar flow. According to the relationship between these two coordinate systems, the average Reynold equation deduced in the computational coordinate system is given by
\[
s_{z} \frac{\partial}{\partial \eta} \left[ \phi_{h} \frac{h^{3}}{12 \mu J} \left( s_{z} \frac{\partial p}{\partial \eta} - s_{z} \frac{\partial p}{\partial \xi} \right) \right] - s_{z} \frac{\partial}{\partial \xi} \left[ \phi_{h} \frac{h^{3}}{12 \mu J} \left( s_{z} \frac{\partial p}{\partial \eta} - s_{z} \frac{\partial p}{\partial \xi} \right) \right] \\
+ r t_{\theta} \frac{\partial}{\partial \eta} \left[ \phi_{f} \frac{r h^{3}}{12 \mu J} \left( r \tilde{h}_{\eta} \frac{\partial p}{\partial \eta} - r \tilde{h}_{\eta} \frac{\partial p}{\partial \xi} \right) \right] - r t_{\theta} \frac{\partial}{\partial \eta} \left[ \phi_{f} \frac{r h^{3}}{12 \mu J} \left( r \tilde{h}_{\eta} \frac{\partial p}{\partial \eta} - r \tilde{h}_{\eta} \frac{\partial p}{\partial \xi} \right) \right]
\]
\[
= \frac{\omega r}{2} \left( s_{z} \frac{\partial \tilde{h}_{\eta}}{\partial \eta} - s_{z} \frac{\partial \tilde{h}_{\eta}}{\partial \xi} \right) + \frac{\omega r}{2} \sigma \left( s_{z} \frac{\partial \phi_{sh}}{\partial \eta} - s_{z} \frac{\partial \phi_{sh}}{\partial \xi} \right) + J \frac{\partial h}{\partial t_{\xi}}
\]
\]

Where,

\[
J = s_{z} (r \tilde{h}_{\eta} - s_{z} (r \tilde{h}_{\xi})
\]

Here, \( h \) is the nominal film thickness, \( \rho \) is the density of the fluid, \( \mu \) is the dynamic viscosity of the fluid, \( \sigma \) is the composite roughness of the surface profile of shaft and sleeve, \( \tilde{h}_{\eta} \) is the expected value of local film thickness. \( \phi_{h} \) and \( \phi \) denote the pressure flow factors in \( \theta - \) and \( \sigma - \) direction, respectively, \( \phi_{sh} \) denotes the shear flow factor in \( \theta - \) direction, which can be calculated according to reference [5,6].

After the local hydrodynamic pressure \( p \) is determined from average Reynolds equation which can be solved by means of finite volume and finite difference method [5,6], the axial load carried by hydrodynamic pressure is given by

\[
F_{ch} = \int_{A} J (p - p_{oc}) \sin \alpha d\eta d\xi + (p_{oc} - p_{a}) \cdot \pi R_{c}^{2}
\]

Where, \( p_{oc} \) is the static pressure in the closed cavity near the small end of bearings.

The hydrodynamic frictional torque is obtained from

\[
T_{h} = \int_{A} \left[ \frac{\mu r_{\theta}}{h} (\phi_{f} + \phi_{sh}) - \phi_{f} \frac{h}{2} \frac{\partial p}{\partial \theta} \right] r d\eta d\xi
\]

Where, \( \phi_{f} \) denotes the shear stress factor, \( \phi_{sh} \) and \( \phi_{f} \) denote shear stress correction factors defined on the basis of the average flow model [5,6].

2.2 Asperities contact model

It is assumed that only elastic deformation occurs on contact surfaces, and the contact is simplified as contact between elastic rough surfaces and flat surfaces. Here we adopt G-W contact model to build asperities contact model between rough surfaces of bearings [11]. The axial load carried by asperities contact can be given by

\[
F_{ca} = \sin \alpha \left[ \frac{4}{3} D_{sum} A_{e} E^{'2} \sigma^{3/2} F_{\eta}^{1/2} (h') \right]
\]

Where, \( D_{sum}, R_{s} \) and \( \sigma_{s} \) denote the density, the average radius and the standard deviation of asperities, \( E^{'} \) denotes composite elastic modulus, \( A_{e} \) is the nominal contact area, \( F_{\eta}^{1/2} (h') \) is the probability cylindrical function [11,17].

The friction torque produced by asperities contact is obtained from

\[
T_{a} = F_{ca} f_{a} \hat{r}
\]

Where \( f_{a} \) is the friction coefficient under boundary lubrication, which is measured by the test of flat journal bearings.
The total axial capacity of load is the sum of the hydrodynamic force $F_{z,h}$ and the contact force $F_{z,ct}$, and the total friction torque is the sum of the contact friction $T_n$ and the viscous resistance $T_\nu$.

### 3 Experiment

#### 3.1 Apparatus and specimens

The experiments are carried out on the MM-W1A friction and wear test machine. A schematic diagram and a photograph of specimen fixture we designed are shown in figure 2. The upper specimen is attached to the spindle of friction test machine by the coupling, so the upper specimen could rotates about the axis. The lower specimen is put in the inside holder laid in the outside holder which is placed on the platform of the test machine and could be moved up and down along the vertical direction, the gap between inside and outside holders is filled with rubber rings.

![Figure 2](image.png)

**Figure 2.** The specimen fixture, (a) schematic diagram, (b) photograph.

As shown in Figure 3, the upper specimens are made of GCr15, the cone surfaces with hardness 750HV are lapped to a fine finish. The spiral grooves are ablated by laser marking machine. There are three types of upper specimens, including conical flat specimen, conical specimen with shallower spiral-groove(10um) and conical specimen with deep spiral-groove(50um). The lower specimen is made of aluminum bronze with surface hardness 250HV.

![Figure 3](image.png)

**Figure 3.** The photograph of specimens, (a) the upper specimens (b) the lower specimen.

#### 3.2 Procedure

The lubricant is SAE 100 oil without additives, and oil-bath lubrication is used in the test. In order to avoid excessive rise of oil temperature, after applying load, set upper specimen rotates at 200rpm for 2 minutes and then stop the machine until the oil is fully cooled. The next sequence increases 200rpm. To minimize the friction, each sequence is repeated several times, and the average value is taken as the final test results.

### 4 Results and discussion
Table 1 presents the input parameters for the numerical calculation.

| Items        | Values (unit) | Items        | Values (unit) |
|--------------|---------------|--------------|---------------|
| $\alpha$     | 30(°)         | $\sigma_1$   | 2.68(μm)      |
| $\beta$      | 20(°)         | $C^*$        | 0.870($\gamma=9$), 1.48($\gamma=1/9$) |
| $R_0$        | 6(mm)         | $r^*$        | 1.5 ($\gamma=9$), 0.42($\gamma=1/9$) |
| $R_c$        | 3(mm)         | $D_{mm}$     | $2.21 \times 10^9$(m$^{-2}$) |
| $R_i$        | 2(mm)         | $R_i$        | 7.5(μm)       |
| $h_g$        | 10,50(μm)     | $\sigma_s$   | 2.41(μm)      |
| $\mu$        | 0.385(Pa·s)   | $E(z_0^*)$   | 2.77(μm)      |

Figure 4(a) compares the results of numerical calculation and experiment for a specimen with shallower groove (10μm). The curves demonstrate the agreement between the numerical calculation and experiment. The friction coefficient drops rapidly with the rotational speed at first and then slowly increases when the speed exceeds a specific value, which indicates the transition from mixed lubrication regime to hydrodynamic regime. The rotational speed responding to the minimum friction coefficient value is the transition speed. The transition speed predicted theoretically (about 600 rpm) is slightly smaller than experiment result (about 800 rpm). The difference between calculation and experiment below the transition speed is a little smaller than that above the transition speed.

Figure 4(b) is for a specimen with deep groove (50μm). Comparing the results of figure 4(a) and 4(b), it implies that at low speeds, the shallower groove bearing has a lower friction coefficient than one with deep groove, but at high speed, it is opposite. Moreover, the bearing with shallower groove has lower transition speed.

Figure 5 (a) and (b) present variations in axial load and friction torque with speed, respectively. With the increase of speed, load carried by hydrodynamic pressure increases gradually until all of the external load, while load carried by contact force decreases gradually until zero, indicating that with increasing speed the hydrodynamic effect becomes stronger and the number of direct contact of asperities gets rarer. Meanwhile, with increasing speed the friction torque due to fluid viscous resistance increases gradually, while one due to asperities contact decreases. In general, friction coefficient of asperities contact is larger than that of fluid viscous resistance, so as shown in figure 5 (b), the friction torque due asperities contact under low speed is much larger than that due to fluid viscous resistance.
Figure 5. Calculation result of capacity of load, friction torque and rotational speed relationship for bearing with shallower groove, (a) capacity of load, (b) friction torque.

5 Conclusions

By means of experiments, the hydrodynamic effect produced by spiral-groove is obviously observed, and the transition speed from mixed lubrication regime to hydrodynamic regime can be detected.

The friction coefficient predicted by numerical calculation on the base of combining partial lubrication and asperities contact agrees well with the results of experiment.

At low speeds bearings with shallower groove has greater hydrodynamic effect, lower transition speed and friction. However, at high speeds bearings with deep groove has lower friction due to larger average film thickness.

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