Analysis of the Influence of Tilt Angle on Star Identification of Digital Zenith Camera

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Abstract. Considering that the tilt angle of the digital zenith camera in the fine state affects the position of the star image on the CCD sensor, this paper established the CCD image coordinate calculation model in the horizontal state according to the principle of quaternion coordinate transformation, and analyzed the influence of the tilt angle on the star map identification combined with the experimental data. The experimental results show that the tilt angle has a significant impact on the position of star imaging, but does not affect the results of star map identification.

1. Introduction
Star map identification improves the initial solution data for astronomical positioning of digital zenith camera. During the process of star map identification, it is necessary to adjust the output value of the two sensitive axes of the inclinometer mounted on the instrument to about $10^\circ$, so that the instrument is in the fine state. It can be seen that even in the fine state, the digital zenith camera also has a small tilt angle. The tilt of the instrument causes the optical axis to tilt, which causes the position of the star image to change on the CCD sensor. So based on the quaternion coordinate transformation principle, the CCD image coordinates in the horizontal state are obtained by using the reading of the inclinometer and the CCD image coordinates under the fine state in this paper. Then on this basis, the influence of the tilt angle on the star map identification is analyzed by calculating the distance between the image points.

2. Star map identification principle
The theoretical image coordinates $(X, Y)$ of the star can be calculated from reference (Hirt C, Bürki B, et al, 2010) So the distance between the theoretical points of the stars can be calculated as follows:

$$d_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$$

(1)

Star image on the CCD sensor of the digital zenith camera, the position of the image point is represented by pixels. Assuming that the position of the image point on the CCD sensor is $(x, y)$, so the distance between the image points can be calculated as follows:

$$d_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}$$

(2)
When the distance between the theoretical image points of the three stars and the distance between the image points satisfy the following formula, the star map is successfully matched.

\[
\begin{align*}
|d_{ij} - d_{mn}| &\leq \varepsilon \\
|d_{jk} - d_{nh}| &\leq \varepsilon \\
|d_{ik} - d_{mh}| &\leq \varepsilon 
\end{align*}
\]

(3)

In the formula3, the value of \( \varepsilon \) is generally 1~2 pixels.

3. Calculation model of horizontal state image coordinates

A two-axis inclinometer mounted on the lens barrel can measure the tilt component of the rotating shaft of digital zenith camera relative to the horizontal plane (Zhaofa ZHOU, Xianyi LIU, et al 2015). Through the output values \( m, n \) of the two sensitive axes of the inclinometer, the angle \( \theta_1, \theta_2 \) between the two coordinate axes of the CCD image coordinate system and the horizontal plane are(Xianyi LIU,Zhao-fa ZHOU, et al, 2016):

\[
\begin{align*}
\theta_1 &= k_1 m \cos \beta - \sin \beta (k_1 n - k_1 m \cos \varepsilon) \sec \varepsilon \\
\theta_2 &= k_2 m \sin \beta + \cos \beta (k_2 n - k_2 m \cos \varepsilon) \sec \varepsilon 
\end{align*}
\]

(4)

In the formula 4, \( \beta \) is the angle between the inclinometer and the CCD image coordinate system. \( \varepsilon \) is the shear angle between the two sensitive axes of the inclinometer. \( k_1, k_2 \) are the proportional coefficients of the two sensitive axes.

As shown in Figure 1, \( \text{o-xyz} \) is the CCD image coordinate system in the tilted state, and \( \text{o-x'y'z'} \) is the CCD image coordinate system in the horizontal state. The axis \( \text{oz'} \) points to the zenith direction. \( T_x \) and \( T_y \) are the projections of the \( \text{oX} \) and \( \text{oY} \) axes on a horizontal plane, \( T_x \) and \( T_y \) are not necessarily vertical, \( AB \) is the intersection between the inclined plane and the horizontal plane, so is perpendicular to the \( \text{oz'} \) axis and \( \text{oZ} \) axis, \( \angle xoT_x = \theta_1, \angle yoT_y = \theta_2, \angle Box = \theta_2, \angle AoY = \theta_4 = \frac{\pi}{2} - \theta_3 \). \( \Theta \) is the dihedral angle between the two coordinate systems, \( M \) and \( N \) are the focus of the optical axis of the digital zenith camera.

**Figure 1.** Calculation model
According to the correlation theorem of the dihedral angle, it can be obtained (Chonghui Li, Zheng Yong, 2013):

\[
\theta = \arcsin\left(\sqrt{\sin^2 \theta_1 + \sin^2 \theta_2}\right)
\]

\[
\theta_1 = \arcsin\left(\frac{\sin \theta_1}{\sin \theta}\right)
\]

\[
\theta_4 = \arcsin\left(\frac{\sin \theta_4}{\sin \theta}\right)
\]

(5)

In the CCD image coordinate system \(o'\-xyz\), there is a unit vector \(u = (\cos \theta_3, -\cos \theta_4, 0)\), and the coordinate system \(o'-x'y'z'\) can be rotated around the unit vector \(u\) by the \(\theta\) to coincide with the coordinate system \(o\-xyz\).

According to the basic knowledge of quaternions, it can be obtained (QIN Yongyuan, 2014):

\[
Q = \cos \frac{\theta}{2} + u \sin\frac{\theta}{2} = \cos \frac{\theta}{2} + (\cos \theta_3 i - \cos \theta_4 j) \sin \frac{\theta}{2}
\]

(6)

Let \(q_0 = \cos \frac{\theta}{2}\), \(q_1 = \cos \theta_3 \sin \frac{\theta}{2}\), \(q_2 = -\cos \theta_4 \sin \frac{\theta}{2}\). The coordinate transformation matrix \(C\) from the coordinate system \(o'-x'y'z'\) to the coordinate system \(o\-xyz\) can be determined by the quaternion \(Q\):

\[
C = \begin{bmatrix}
1 - 2q_2^2 & 2q_1q_2 & 2q_0q_2 \\
2q_1q_2 & 1 - 2q_1^2 & -2q_0q_1 \\
-2q_1q_2 & 2q_0q_1 & 1 - 2(q_1^2 + q_2^2)
\end{bmatrix}
\]

(7)

Then the coordinate transformation matrix from coordinate system \(o\-xyz\) to standard \(o\-x'y'z'\) is \(C^{-1}\). In coordinate system \(o\-xyz\), there is a vector \(r = (x, y, -f)\), It satisfies:

\[
\begin{bmatrix}
1 & c_{11} & c_{12} \\
c_{21} & 1 & c_{22} \\
c_{31} & c_{32} & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
-f
\end{bmatrix} =
\begin{bmatrix}
x_{c_{11}} + yc_{12} - fc_{13} \\
x_{c_{21}} + yc_{22} - fc_{23} \\
x_{c_{31}} + yc_{32} - fc_{33}
\end{bmatrix}
\]

(8)

Because the light emitted by the star can be considered as parallel light, In coordinate system \(o\-x'y'z'\), there is a vector \(r' = (x', y', -f)\), It satisfies:

\[
\frac{x'}{x_{c_{11}} + yc_{12} - fc_{13}} = \frac{y'}{x_{c_{21}} + yc_{22} - fc_{23}} = \frac{-f}{x_{c_{31}} + yc_{32} - fc_{33}}
\]

(9)

Simplified form can be obtained:

\[
x' = \frac{-f(x_{c_{11}} + yc_{12} - fc_{13})}{x_{c_{31}} + yc_{32} - fc_{33}}
\]

\[
y' = \frac{-f(x_{c_{21}} + yc_{22} - fc_{23})}{x_{c_{31}} + yc_{32} - fc_{33}}
\]

(10)
Formula 10 is called Calculation model of horizontal state image coordinates.

4. Analysis of experimental results

In November 2017, a large number of experiments were carried out using a digital zenith camera. The CCD sensor used in the instrument has a size of 4096 pixels×4096 pixels and a single pixel size of 9 μm × 9 μm. The asterisk of the two-axis inclinometer is Leica Nivel210. The output value of the inclinometer of the digital zenith camera in a positioning cycle is shown in Table 1.

| No. | m/*  | n/*  | Δθ1/* | Δθ2/* | No. | m/*  | n/*  | Δθ1/* | Δθ2/* |
|-----|------|------|--------|--------|-----|------|------|--------|--------|
| 1   | -7.631 | 0.618 | 2.245 | -7.307 | 9   | -11.138 | -0.412 | 4.497 | -10.177 |
| 2   | -12.582 | 1.031 | 3.69  | -12.052 | 10  | -7.425  | -1.856 | 4.467 | -6.198  |
| 3   | -8.663  | 6.187 | -2.547 | -10.329 | 11  | -6.806  | 4.537  | -1.7  | -7.995  |
| 4   | -8.25   | 0.206 | 2.856 | -7.728 | 12  | -12.375 | 2.887  | 1.89  | -12.548 |
| 5   | -12.788 | 2.681 | 2.234 | -12.855 | 13  | -7.219  | 0.618  | 2.093 | -6.925  |
| 6   | -7.425  | 4.331 | -1.28 | -8.493 | 14  | -7.425  | 7.425  | -4.154 | -9.64   |
| 7   | -7.631  | -2.681 | 5.31  | -6.083 | 15  | -12.375 | 2.887  | 1.89  | -12.548 |
| 8   | -11.138 | -0.618 | 4.689 | -10.101 | 16  | -8.044  | 2.062  | 1.056 | -8.225  |

It can be seen from the analysis in Table 2 that the tilt of the instrument causes the position of the image point to be offset integrally. And the offset is more significant. The offset of the X coordinate fluctuates around the average value of 53.287 pixels, and the offset of the Y coordinate fluctuates around the average value of 19.657 pixels.

| No. | the CCD image coordinate in tilted state | the CCD image coordinate in horizontal state | Difference |
|-----|----------------------------------------|------------------------------------------|------------|
|     | CCD x/pixel | CCD y/pixel | CCD x/pixel | CCD y/pixel | Δx | Δy |
| 1   | 1689.295 | 2641.995 | 1742.573 | 2661.65 | 53.278 | 19.654 |
| 2   | 1167.233 | 839.967 | 1220.524 | 859.64 | 53.291 | 19.674 |
| 3   | 1120.551 | 1343.26 | 1173.841 | 1362.924 | 53.29 | 19.665 |
| 4   | 2405.67 | 1242.288 | 2458.948 | 1261.943 | 53.278 | 19.654 |
| 5   | 1875.846 | 486.843 | 1929.125 | 506.511 | 53.279 | 19.668 |
| 6   | 620.459 | 2668.37 | 673.756 | 2688.016 | 53.297 | 19.646 |
| 7   | 2607.68 | 692.685 | 2660.958 | 712.338 | 53.278 | 19.654 |
| 8   | 1404.238 | 2256.797 | 1457.58 | 2276.451 | 53.282 | 19.654 |
| 9   | 3408.001 | 1207.817 | 3461.296 | 1227.462 | 53.296 | 19.644 |
| 10  | 701.156 | 1975.763 | 754.455 | 1995.419 | 53.299 | 19.656 |
|     | Average/pixel |                             |             |             | 53.287 | 19.657 |

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It can be seen from Section 2 that the distance $d$ between the image points needs to be calculated during the star map matching process. Due to the limitation of this paper, only the distance $d$ between the first image point and the remaining nine image points is calculated in Table 1. As shown in Table 3.
Table 3. Comparison of distance values

| No. | Distance in tilted state | Distance in horizontal state | Difference |
|-----|--------------------------|-----------------------------|------------|
| 1   | 1876.127                | 1876.106                    | 0.021      |
| 2   | 1417.809                | 1417.796                    | 0.013      |
| 3   | 1572.378                | 1572.378                    | 0          |
| 4   | 2163.211                | 2163.198                    | 0.013      |
| 5   | 1069.161                | 1069.142                    | 0.019      |
| 6   | 2154.818                | 2154.82                     | -0.002     |
| 7   | 479.2025                | 479.2009                    | 0.002      |
| 8   | 2238.485                | 2238.505                    | -0.019     |
| 9   | 1191.757                | 1191.739                    | 0.018      |

According to the analysis in Table 3, the distance between the CCD image points calculated by formula 10 is basically the same as the distance between the image points in tilted state, and the maximum difference of the distance is not more than 0.1 pixels. In the process of star map matching, the matching threshold $\epsilon$ is usually set to 1~2 pixels, so the change of the distance between image points caused by the tilt of the instrument will not affect the matching process.

5. Conclusion
The tilt of the instrument has a significant effect on the position of the star image on CCD, but does not affect the calculation of the distance between image points and the result of star image identification.

References
[1] Hirt C, Bürki B, Somieski A, et al. Modern determination of vertical deflections using digital zenith cameras[J]. Journal of Surveying Engineering,2010,136(1): 1-12
[2] Zhaofa ZHOU, Xianyi LIU, Zhili ZHANG, et al. Research on Two-axis Tilt Sensor Based on Digital Zenith Camera[J], ACTA PHOTONICA SINICA. 2015,44(8): 08120021-08120026
[3] Xianyi LIU, Zhao-fa ZHOU, Zhili ZHANG, et al. Research on the Transformation of Coordinates Astronomical Fixation[J]. Electronics Optics&Control,2016,23(1)11-14.
[4] QIN Yongyaun. Inertial Navigation[M]. Beijing: Science Press,2014:244-252
[5] Chonghui Li, Zheng Yong, ZHANU Chao, et al. A Robust Celestial Positioning Method without Precise Leveling[J]. Acta Geodaetica et Cartographica Sinica,2013,42(6):810-816