Transport through a superconductor-interacting normal metal junction: a phenomenological description

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(Dated: January 24, 2022)

We propose a phenomenological description of electronic transport through a normal metal/superconductor interface of arbitrary transparency, which accounts for the presence of electron-electron interaction in the normal metal. The effect of interactions is included through an energy-dependent transmission probability that is inserted in the expression for the current-voltage characteristics of a non-interacting system. This results in a crossover from the Andreev to the tunneling limit as a function of the energy at which transport is probed. The proposed description reproduces qualitatively the results obtained with formally correct theories as well as experimental observations. In view of its simplicity, we expect our approach to be of use for the interpretation of future experiments.

PACS numbers: 73.23.-b, 73.40.-c, 73.63.Fg, 74.45.+c, 74.78.Na

Low-energy electronic transport through the interface between a normal metal (N) and a superconductor (S) can be understood in terms of normal and Andreev reflection. For systems where the effect of the Coulomb repulsion between electrons is negligible, the interplay between these two processes is governed by the transmission of the NS interface. If the transmission is high (close to unity) Andreev reflection dominates and the low-energy conductance of the interface is higher than the conductance measured at energies above the gap. On the contrary, if the transmission is low (tunneling regime), Andreev reflection is strongly suppressed and so is the low-energy conductance of the NS interface. The crossover from high to low interface transparency is well described by the theory and it is in good agreement with experimental results on a variety of systems.

When the normal conductor is a low-dimensionality system, however, electron-electron interactions play an important role and the situation is considerably more complex. In this case, electronic transport through the NS interface is not only determined by the transmission coefficient, but also by the strength of the interaction. It is understood quite in general that electron-electron interactions suppress the probability for Andreev reflection. For interacting systems, however, a simple and intuitive picture of the interplay between normal and Andreev reflection does not exist. Nevertheless, such a picture would be extremely useful for the qualitative interpretation of experiments.

The purpose of this paper is to present a simple phenomenological description that captures the important aspects of electrical transport through a NS interface in the presence of repulsive electron-electron interactions in the normal conductor. To this end, we consider the simplest possible case of interacting electrons in a one-dimensional (1D) ballistic conductor connected to a superconductor with an arbitrary interface transparency. Motivated by recent theoretical studies on interacting normal mesoscopic conductors, the effect of electron-electron interaction is included as an energy-dependent transmission probability $T(E)$, which we substitute into the theoretical expression for the current-voltage characteristics of a system with no electron-electron interaction. We substantiate the validity of the proposed phenomenological picture by performing explicit calculations for the case in which $T(E)$ is given by the expression valid for a barrier in an infinite 1D interacting conductor. The outcome of these calculations reproduces qualitatively the trends found in formally correct theories. We also illustrate how different aspects of our results can explain different aspects of the transport behavior of carbon nanotubes connected to superconducting electrodes.

Our work is stimulated by the description of electron transport through a tunnel barrier of arbitrary transmission in a 1D interacting conductor, which was first proposed by Matveev and coworkers. Within this picture, the tunnel barrier induces a Friedel oscillation of the electron density in the one-dimensional conductor. In the presence of electron-electron interaction, such a density oscillation creates an electrostatic potential that contributes to scattering the electrons. Thus, to find the final transmission coefficient for the tunnel barrier, the effect of multiple reflections on the barrier and on the Friedel oscillation needs to be considered. It was shown that the net effect of the interaction is to introduce an energy dependence in the final transmission probability, such that the transmission decreases with decreasing energy.

This description of electron-electron interaction in terms of a renormalized, energy-dependent transmission coefficient has been recently extended to deal with more general (i.e., not only one dimensional) mesoscopic conductors. Similarly to the 1D case, it was found that the dominant effect on low-energy electron transport is due to elastic scattering and it can be accounted for by an energy renormalization of the transmission coefficients. At sufficiently low energy, this renormalization...
where $0 < \alpha < 1$ quantifies the electron-electron interaction strength ($\alpha = 0$ corresponds to no interactions) and $D_0$ is a high-energy cut-off determined by the energy bandwidth of the electronic states. In what follows, we take $T(E)$ as given by Eq. (1) and insert it in the known expression for the $I - V$ characteristics of a NS interface (with bare transmission $T_0$) of a non-interacting system.

Formally this procedure is not correct, since the interaction-induced renormalization of the transmission coefficient in an infinite 1D conductor differs from that of a tunnel barrier at a NS interface. This is because the presence of the superconductor changes the details of the Friedel oscillations and thus the specific dependence of $T$ on $E$. Nevertheless, calculations similar to those of Matveev and coworkers performed for a 1D interacting normal metal/superconductor junction have shown that also in this case the effect of the interaction is to renormalize the scattering coefficients$^{16}$. More importantly, Eq. (1) captures the trend of an interaction-induced suppression of the transmission probability that quite in general accounts for the effect of electron-electron interaction on transport, as discussed above. Therefore, since our work only aims at providing a transparent phenomenological description of transport across a NS interface and not at discussing specific details, the use of Eq. (1) is qualitatively justified$^{14}$.

The second ingredient of our description is the expression for the current $I$ flowing through a NS interface of arbitrary transparency as a function of the applied bias $V$. This was found long ago by Blonder, Tinkham, and Klapwijk (BTK)$^{25}$ to be:

$$I = G_0 \int_{-\infty}^{\infty} dE [f(E-eV)-f(E)][1+A(E,z)-B(E,z)]$$

(2)

Here $A(E,z)$ and $B(E,z)$ are the energy dependent Andreev and normal reflection probability, respectively, and $G_0$ is a constant. The parameter $z$ is a constant that quantifies the amplitude of a delta-like potential barrier at the interface. It is related to the transmission probability $T_0$ as

$$z^2 = \frac{1-T_0}{T_0}$$

(3)

FIG. 1: Temperature dependence of the zero-bias differential resistance calculated for a non-interacting ($\alpha = 0$, dashed lines) and an interacting ($\alpha = 0.15$, dotted lines) 1D conductor connected to a superconductor, for different values of the contact transparency ($z = 0, 0.22$, and $1.5$). For $z = 0$ (continuous line) the behavior is identical for the interacting and the non interacting case.

FIG. 2: The $dV/dI - V$ characteristic calculated at different temperatures below $T_c$, for a high transparency contact ($z = 0.03$) in the presence of interaction for the same interaction strength (corresponding to $\alpha = 0.6$). Note that at higher temperature (e.g., $T = 0.32 \Delta$) the $dV/dI - V$ is similar to what is observed in the non-interacting case, whereas at lower temperature (e.g., $T = 0.02 \Delta$) a zero bias peaks emerge that cannot be accounted for in terms of the BTK model.
\( z = 0 \) corresponds to a perfectly transparent interface, resulting in \( A(E, 0) = 1 \) and \( B(E, 0) = 0 \) for \( E < \Delta \) (Andreev limit); \( z \gg 1 \) corresponds to the tunneling limit in which \( A(E, z) \ll 1 \) and \( B(E, z) \simeq 1 \).

Whereas in the non-interacting case the \( z \) parameter is a constant, after substituting \( T(E) \) for \( T_0 \) in Eq. 2 to account for the presence of interactions \( z \) acquires an energy dependence. We label the energy dependent \( z \) parameter as \( z_{e-c} \). By direct substitution we have:

\[
z_{e-c}^2 = \frac{E}{D_0} \left( 1 - \frac{T_0}{T_0} \right)^{-\alpha} \left( \frac{E}{D_0} \right)^{-\alpha} z^2 \tag{4}
\]

It is clear that now, even for values of \( T_0 \) close to unity, it is the energy \( E \) which determines whether \( z_{e-c} \) is approximately 0 or much larger than one. If, for \( E < \Delta \), \( z_{e-c} \) varies considerably, electron-electron interactions can induce a crossover from the Andreev limit to the tunneling limit in an individual sample, depending on the energy scale on which transport is probed. This crossover is responsible for the qualitatively different behavior of systems in which electron-electron interaction plays an important role, as compared to systems that can be described in terms of independent electrons. Note, in passing, that if \( z = 0 \) (i.e., \( T_0 = 1 \)), \( z_{e-c} \) is also 0, implying that for a perfectly ballistic system, electron-electron interactions do not change the DC conductance. This is a well-known result that has been demonstrated from formally correct theories and which is reproduced by our phenomenological approach.

We illustrate the physical consequences of the energy-induced crossover from a transparent NS interface to the tunneling regime by looking at the behavior of the differential resistance calculated from Eq. 2 that we integrate numerically after replacing \( z \) with \( z_{e-c} \). In particular, we look at the dependence of the differential resistance on temperature and voltage for different values of \( z \) and \( \alpha \). The different \( dV/dI \) curves are normalized to \( R_N = (1 + z^2)/G_0 \), the normal state resistance in the non-interacting case (\( \alpha = 0 \)). Note that, in general and contrary to the case of non-interacting electrons, the differential resistance of the interacting system depends on the bias even when \( eV > \Delta \), because of the energy dependence of the transmission probability.

Fig. 3 shows the temperature dependence of the linear resistance of the NS interface for different values of the (bare) \( z \) parameter, enabling the direct comparison of the non-interacting (\( \alpha = 0 \)) and the interacting case (with \( \alpha = 0.15 \) in this example). As explained above, interactions do not have any influence if \( z = 0 \) and the temperature dependence of the linear resistance is identical in the two cases. At finite values of \( z \), however, the effect of the interaction is visible. In particular, at low \( z \) (\( z = 0.22 \)) and in the absence of interaction the resistance of the non-interacting system decreases with decreasing \( T \) below \( T_c \), so that at low temperature the resistance is smaller than that measured for \( T > T_c \). However, in the presence of interactions the behavior of the resistance is qualitatively different. After an initial decrease (just below \( T_c \)), the resistance increases again rapidly, so that at low temperature it is larger than in the normal state.

This difference in behavior is a clear manifestation of the crossover from the Andreev to the tunneling limit mentioned above. For the interacting system, the thermal energy of the electrons at \( T \simeq T_c \) is sufficiently large to prevent a strong renormalization of the transmission probability at the interface, and the observed behavior (i.e., the decrease in resistance) is similar to the non-interacting case in the Andreev limit. At lower temperature, however, the renormalized transmission probability becomes small and the interacting system goes into the tunneling limit. As a consequence the resistance increases. The precise temperature at which the upturn occurs depends on \( z \) and on the interaction strength \( \alpha \), shifting to higher values with increasing these parameters. As shown in the figure, for sufficiently large values of \( z \) the resistance increases with lowering \( T \) across \( T_c \) both in the interacting and the non-interacting case, and the difference in behavior becomes only of quantitative nature.

A second manifestation of the crossover from Andreev to tunneling behavior can be seen in the \( dV/dI-V \) curves measured at different temperatures. Also here, the case exhibiting qualitative differences for the interacting and non-interacting cases is that of a small (bare) \( z \) parameter. The results of the calculations are shown in Fig. 2 where \( z = 0.03 \) and \( \alpha = 0.6 \). At higher temperature

**FIG. 3:** The \( dV/dI – V \) calculated for a fixed interface transparency (\( z = 0.22 \)) and temperature \( T = 0.5 \Delta \), as a function of interaction strength (\( \alpha \) ranging from 0 to 0.6). Increasing the interaction results in the complete disappearance of the resistance suppression seen below the gap for the non-interacting system.
(approximately 0.3 \( \Delta \) in the present case), a pronounced broad minimum in \( dV/dI \) is observed when the DC bias is smaller than the superconducting gap, as it is characteristic for Andreev reflection. As the temperature is lowered, however, a sharp peak in the \( dV/dI \) curves appears around zero bias, causing the resistance to exceed the normal state value. This peak cannot be accounted for by the BTK-theory for non-interacting systems. Again, the \( dV/dI \) peak is a consequence of the renormalization of the transmission probability which brings the NS interface in the tunneling regime at sufficiently low energy.

Finally, it is instructive to look at the dependence of the \( dV/dI-V \) curves as a function of the electron-electron interaction strength \( \alpha \). The results of the calculations are shown in Fig. 4 for the case of \( z = 0.22 \) and \( T = 0.5 \Delta \), with \( \alpha \) ranging from 0 to 0.6. In the non-interacting case \((\alpha = 0)\), transport occurs in the Andreev regime for all values of the applied bias, resulting in a suppression of the resistance at low energy. However, as the interaction strength is enhanced, a peak in \( dV/dI \) appears around zero bias, which becomes broader for larger values of \( \alpha \). Eventually, this peak completely dominates the behavior of the differential resistance for bias voltages below the superconducting gap. In this case, the resistance suppression characteristic of Andreev reflection is not visible any more. This is because, even though the bare value of the transmission coefficient has remained the same, the actual value of the transmission has changed due to the energy dependent renormalization induced by electron-electron interaction. In a non-interacting picture the observation of such a peak in the \( dV/dI-V \) curve would be tentatively interpreted as due to a low transparency interface. One would however note that the shape of the curve is very different from what is normally observed in the tunneling regime, as indicated, for instance, by the absence of any feature at a bias corresponding to the superconducting gap.

The temperature and bias voltage dependence of the differential resistance that we find in the interacting case for high (bare) values of the transmission through the NS interface reproduce qualitatively what has been found previously from formally correct calculations for a Superconductor-Luttinger Liquid junction. This indicates that the phenomenological picture presented here is physically sound. We want to explicitly make clear, however, that a Luttinger liquid behavior of the normal metal is not a necessary requirement for the observation of the behavior that we have described. All is needed is a sufficiently rapid suppression of the transmission probability with lowering energy on the scale of the superconducting gap. As mentioned above, in low-dimensionality samples such a suppression can be caused by electron-electron interaction under fairly general circumstances.

Finally, our results exhibit qualitative agreement with the behavior observed experimentally in carbon nanotubes contacted with superconducting electrodes. The case of high transmission contacts has been studied in Ref. [12], where an anomalous (as compared to the non-interacting case) temperature and bias dependence of the differential resistance has been reported. The experimental observations are qualitatively very similar to the behavior shown in Fig. 1 and 2 for the low \( z \) case. The case of lower transmission contacts has been studied in Ref. [16]. In that experiment a tunneling-like BCS-density of state was expected and observed without any clear "BCS shoulders". The absence is attributed to the presence of interaction which decreases the magnitude of this large feature. This behavior is similar to the one predicted by our calculations, as illustrated in Fig. 3 (curves with \( \alpha = 0.5 \) and 0.6).

In conclusion, we have proposed a phenomenological description of the effect of electron-electron interaction on transport through a normal conductor/superconductor interface of arbitrary transparency. The results obtained from this description agree qualitatively with what is predicted by formally correct theories and reproduce the behavior observed experimentally in carbon nanotube/superconductor junctions. We expect that the simplicity of the picture proposed here will be useful in the interpretation of experimental data.

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1. A.F. Andreev, Zh. Eksp. Theor. Fiz. 46, 1823 (1964) [Sov. Phys. JETP 19, 1228 (1964)].
2. G.E. Blonder, M. Tinkham and T.M. Klapwijk, Phys. Rev. B 25, 4515 (1982).
3. see e.g. G.E. Blonder and M. Tinkham, Phys. Rev. B 27, 112 (1983); P. Szabo et al., Phys. Rev. Lett. 87, 137005 (2001); R.J. Soulen et al., Science 283, 85 (1998).
4. S. Vishvshewara, C. Bena, L. Balents and M.P.A. Fisher, Phys. Rev. B 66, 165411 (2002).
5. D.L. Maslov, M. Stone, P.M. Goldbart and D. Loss, Phys. Rev. B 53, 1548 (1996).
6. I. Affleck, J.-S. Caux, and A.M. Zagoskin, Phys. Rev. B 62, 1433 (2000).
7. H.-W. Lee et al., Phys. Rev. Lett. 90, 247001 (2003).
8. Y. Oreg, P.W. Brouwer, B.D. Simons, and Alexander Altland, Phys. Rev. Lett. 82, 1269 (1999).
9. K.A. Matveev, D. Yue and L.I. Glazman, Phys. Rev. Lett. 71, 3351 (1993).
10. M. Kindermann and Yu.V. Nazarov, Phys. Rev. Lett. 91, 136802 (2003).
11. D.A. Bagrets and Yu.V. Nazarov, cond-mat/0304339
12. A.F. Morpurgo, J. Kong, C.M. Marcus and H. Dai, Science 286, 263 (2001).
13. Y. Takane and Y. Koyama, J. Phys. Soc. Jpn. 66, 419 (1997).
14. In Eq. 10, \( T(E) \) vanishes at \( E = 0 \), whereas in different experimental systems the interaction induced suppression of \( T(E) \) can saturate at a finite value. Although it is important to bear this point in mind, this does not change the qualitative conclusions of our work.
15. D.L. Maslov and M. Stone, Phys. Rev. B 52, R5539 (1995).
16. V. Kastić et al., Phys. Rev. B 68, 205402 (2003).