KeV Scale Frozen-in Self-Interacting Fermionic Dark Matter

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We present a model in which the dark matter particle is frozen-in at MeV scale. In this model the mediator between the standard model sector and the dark sector can automatically provide a self-interaction for dark matter. The interaction strength is naturally to be the in the region in favor of the cluster mass deficit anomaly. Due to the self-scattering the Lyman-α constraint can be relaxed to $m_D \gtrsim 2$ keV. In this region the self-interaction and the Fermi pressure both play roles on forming a dark matter core at the center of the dwarf galaxies.

**Introduction.** Cold dark matter (CDM) paradigm, although has been extremely successful in explaining the large scale structure of our universe, is challenged by small scale anomalies from dwarf galaxies to galactic clusters. In particular, simulations based on CDM show that the mass density profile for CDM halo increases as $\rho_{DM} \propto r^{-1}$ toward the center [13–18], whereas many observed rotation curves of disk galaxies prefer a constant cored density profile $\rho_{DM} \propto r^0$ [19–22]. CDM also predicts a greater number of galactic satellites than predicted [7]. As for galactic clusters observations show that there is a mass deficit in the inner $\mathcal{O}(10)$ kpc region [8] compared to the NFW profile. In [9] the multi-tracer technique was used and the size of the core of Fornax is determined to be $0.2$ kpc $< r_c < 2.6$ kpc. However, recent study in [10] shows that the multi-tracer technique can mis-identify a cuspy profile to a cored profile. A more recent study of the dynamical friction of the globular clusters in the Fornax system gives an upper bound of the core size $r_c < 282$ pc [11] , which favors a small core.

Dark matter (DM), although supported by various evidences from astrophysics to cosmology, appears only through gravitational effects. The small scale properties may provide us with opportunities to explore the particle physics nature of DM. An interesting observation is that if DM is composed of $\mathcal{O}(100)$ eV fermions, the Fermi pressure forces the core of dwarf galaxies to be larger than observed (the Tremaine-Gunn bound) [12]. This observation unavoidably leads people to consider the idea that if the mass of DM is just around the boundary of where the Tremaine-Gunn bound allows, the Fermi pressure may provide a solution to the core-cusp problem [13–18]. On the other hand, the Lyman-α forests observation shows that the mass of DM, $m_D$ has to be larger than about 5 keV if the DM particles are thermally produced. keV scale DM particles can also be copiously produced inside stars, and as a result change the life-time, neutrino flux and luminosities of the stars, which strongly constrain the parameter space of this kind of models.

In this work, we propose a DM model based on the freeze-in mechanism [19]. In this model we extend SM with a Dirac fermion $\chi$, the DM candidate with mass $m_D \sim$ keV and a massive vector boson $V$ (the dark photon) with a mass $m_V$ at MeV scale. The interaction between the dark sector and the SM sector is conducted by the kinetic mixing between $V$ and the photon field. The Lagrangian is

$$\mathcal{L} = \bar{\chi} \gamma^\mu (\partial_\mu - ie_D V_\mu) \chi - m_D \bar{\chi} \chi - \frac{1}{4} \kappa V_\mu V_\mu + \frac{1}{2} m_D^2 V_\mu V_\mu - \frac{1}{2} \kappa V_\mu F^{\mu\nu} F^{\nu\mu}.$$  (1)

We show that in this model the Fermi pressure together with the self-scattering can produce a small core in dwarf galaxies, while the self-scattering is naturally in the region in favor of the cluster mass deficit anomaly. We also show that in this model right after frozen-in the DM particles quickly replicate themselves induces a much lower temperature in the dark sector than in the SM sector. The self-scattering of $\chi$ turn free-streaming into Brownian motion, which shortens the distance the DM particles migrate. With these effects the constraint from Lyman-α forests observation can be relaxed. We also show that the stellar constraints can also be avoided in this model.

Other possible particle physics scenarios in solving the small scale anomalies are warm dark matter [20,21], self-interaction dark matter [22,23] (and [24] for a review), or boson degeneracy [26–28]. The DM models which can solve the anomalies at different scales are self-interaction model with light mediator [22,23], the self-scattering through $s$-channel resonance [29].

**Freeze-in.** The most important freeze-in channels are the $e^+e^-$ annihilation channel and the plasmon decay channels. The production rate of the dark matter number density $n_\chi$ in the $e^+e^-$ annihilation channel can be written as

$$\frac{d\Gamma_{e^+e^-}}{d\omega} = \frac{2\kappa^2 a_\alpha D}{3\pi^2} \int dq \frac{q^2(s + 2m_e^2)s}{(s - m_V)^2 + \frac{\alpha_D^4 q^4 s^4}{5}} f \left( \frac{\omega}{T}, \frac{q}{T}, s \right),$$  (2)

where $s = \omega^2 - q^2$ is the center-of-mass energy of the $e^+e^-$ pair,

$$f(x, y, s) = \frac{1}{2\pi y} \left[ \frac{4 \tanh^{-1} \left( \frac{a+1}{a+1} \tanh \left( \frac{s}{2} \right) \right)}{(a-1)(a+1)} \right],$$  (3)

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with $a = e^{2\pi/2}$ and $b = \frac{y}{2} \left(1 - 4m^2_s\right)^{1/2}$. In the case $m_V > 2m_\chi$, this process is dominated by the production of on-shell $V$, and the production rate is the same as discussed in [30].

Due to the plasma effect transverse photons develop a non-trivial dispersion relation and therefore can decay into a pair of dark matter particles. The collective motion of the charged particles in the thermal plasma behaves like a longitudinal mode of the photon field, which also decays into a $\chi\bar{\chi}$ pair. The production rate of the DM number density can be approximately written as [31]

$$\frac{d\Gamma^\chi}{d\omega} = \frac{\kappa^2 \alpha_D}{3\pi^2} \frac{qZ_t}{e^{\omega/T} - 1} \left(s - m_\chi^2\right) s^3 + \frac{1}{5} \alpha_D^2 s^2$$

$$\frac{d\Gamma^\chi}{d\omega} = \frac{\kappa^2 \alpha_D}{6\pi^2} \frac{qZ_t}{e^{\omega/T} - 1} \left(s - m_\chi^2\right) s^2 \omega^2$$

(4)

where $Z_t, t$ are the wave function renormalization factors. The magnitude of the three-momentum of the plasmon $q$ can be calculated from the dispersion relations of the transverse and longitudinal plasmons given in [31].

**The replication.** Right after frozen-in the scattering processes $\chi\bar{\chi} \rightarrow \chi\chi$, $\chi\chi \rightarrow \chi\chi$ and $\chi\chi \rightarrow \chi\chi$ with a much faster rate than the replication processes establish a “thermal” distribution

$$f_\chi(\tilde{\chi}) = \frac{2}{e^{(\mu - \mu)/T_D} + 1} ,$$

(5)

for $\chi$ and $\tilde{\chi}$ with a chemical potential because the $2 \rightarrow 2$ processes do not change the numbers of DM and anti-DM particles. In Eq. [5], $T_D$ is the temperature of the DM particles. The average kinetic energy of $\chi$ and $\tilde{\chi}$ are around $T_D$. When $T_D$ is larger than $2m_\chi$ the replication processes are dominated by the on-shell production of a pair of dark photon $V$, $\chi\chi \rightarrow VV$, with $V$ later decays into a $\chi\bar{\chi}$ pair. Then when $T_D$ is lowered to be around $m_\chi$ the $\chi\chi \rightarrow \chi\chi + V$ process dominates. In the case that $T_D < m_\chi$, the replication can only go through off-shell dark photons and it is easy to see that cross section in this case is like $T_D/m_\chi^4$, which diminishes fast with the expansion of the Universe. Therefore the dominant contribution of the replication happens around $T_D \gtrsim m_\chi$. The typical Feynman diagrams for the 2 to 4 processes are shown in Fig. 2. In practice we simulate the cross section of the replication processes with CALCHEP [52]. The replication processes push the dark sector to the true thermal equilibrium with zero chemical potential and with $T_D$ smaller than $T_{SM}$.

**The Boltzmann equations.** It turns out that in the parameter space in favor of solving the small scale anomalies the $2 \rightarrow 4$ processes are fast enough that a full thermal equilibrium ($\mu = 0$) can be established in the dark sector. The replication processes stop (with a rate smaller than the Hubble expansion rate) when the DM particles are still relativistic. Therefore the DM number density in this case is controlled by the total energy density transferred to the dark sector, one don’t need to trace the details of the $2 \rightarrow 4$ redistribution processes. The Boltzmann equation for freezing-in the DM energy density can be written as

$$\frac{d\rho_\chi}{dt} + 4H\rho_\chi = \Gamma_\rho + \Gamma_t + \Gamma_\ell ,$$

(6)

where $\Gamma_\rho$ ($i = e^\pm, t, \ell$) can be calculated from Eqs. 2 and 4, and their numerical values at $T = 10 m_\chi$ as functions of $m_\chi$ are shown in the left panel of Fig. 1. We can see that at the case $m_\chi > 1$ MeV the $e^\pm e^\mp$ annihilation process dominates since $V$ can be produced on-shell. Once $m_\chi$ is smaller than 1 MeV, the transverse photon decay starts to dominate. At the region that $V$ can be produced on-shell the rate of the freeze-in processes depends little on $\alpha_D$.

In the case of fully thermalization the number density $n_\chi$ has a simple relation with $\rho_\chi$ that (e.g. see [33])

$$n_\chi = 6 \times 2^{1/4} (15/7)^{3/4} \zeta(3) \pi^{-7/2} T_\chi^{3/4} .$$

(7)

Then $m_D$ can be fixed by the DM relic abundance that

$$\frac{2n_\chi m_D}{n_{BM} m_{proton}} = \frac{\Omega_D}{\Omega_B} \approx 5 .$$

(8)

Then to get the relic abundance the kinetic mixing $\kappa$ satisfies

$$\kappa = \tilde{\kappa}(\alpha_D, m_\chi) \times T_\chi/\left(200 \text{ eV}\right)^{-2/3} .$$

(9)

$\tilde{\kappa}$ for $\alpha_D = \alpha_{EM}$ and $\alpha_D = 2\alpha_{EM}$ as a function of $m_\chi$ is shown in the right panel of Fig. 1 where one can see that the typical value of $\kappa$ to generate the observed relic abundance of DM is about $10^{-11}$ to $10^{-10}$ and the value...
of $\tilde{\kappa}$ depends mildly on $\alpha_D$, since $V$ is preferred to be produced on-shell.

**Stellar constraints.** Dark matter particles with $\mathcal{O}(100)$ eV mass can be copiously produced inside stars. At the center of the horizontal branch (HB) stars and the red giant (RG) stars the plasma frequency $\omega_p$ is at $\mathcal{O}(10$ keV), which is larger than twice of $m_D$. Therefore the dominant production channel of DM particles is the resonant decay of the transverse photons $\bar{\chi}$ processes [34, 35]. The energy loss rate to the dark sector is

$$\frac{dp^{T,L}_\chi}{dt} \approx \frac{\kappa^2 \alpha_D \omega_p^9}{3\pi^2 m_V^4} \left[ f \left( \frac{\omega_p}{T} \right) + \frac{1}{30} \left( \frac{1}{\omega_p/T - 1} \right) \right]. \quad (10)$$

where

$$f(x) = \int_1^\infty dx \frac{x(x^2 - 1)^{1/2}}{e^{(x - 1)/30} - 1}. \quad (11)$$

At the center of the HB stars $\omega_p \approx 5$ keV, $T \approx 10$ keV and the density $\rho \approx 3 \times 10^4$ gram cm$^{-3}$ [36], and the constraint is that the energy loss rate to the dark sector to be smaller than 8 erg/gram/sec [37]. For the RG stars we use $\omega_p \approx 20$ keV, $T \approx 8.6$ keV, $\rho \approx 10^6$ gram cm$^{-3}$ [38], and require that the dark radiation rate to be smaller than 10 erg/gram/sec [37].

Supernova1987A (SN) with $\omega_p \approx 10$ MeV and $T \approx 20$ MeV at the center can copiously produce $V$ on-shell if $m_V$ is around $\omega_p$. On the other hand if $m_V \ll \omega_p$ the production rate of the transverse and the longitudinal modes of $V$ are further suppressed by $(m_V/\omega_p)^4$ and $(m_V/\omega_p)^2$, respectively. In this region the dominant dark radiation production channel is again the decay of plasmon into $\chi \bar{\chi}$ through an off-shell $V$. We re-interpret the result in [39, 40] and its constraint turns out to be weaker than the constraint from RG stars at the region $m_V \sim 1$ MeV, but it becomes stronger with larger values of $m_V$ as shown in Fig. 3.

**Lyman-alpha forests observation.** Observations of the absorption lines in the spectra of quasars due to small hydrogen clouds - the so-called Lyman-$\alpha$ forests show the matter power spectrum is not suppressed at Mpc scale. This gives a strong constraint on the free-streaming length of DM. From Ref. [43] the most aggressive Lyman-$\alpha$ forests bound on the warm DM mass is $m_{\text{WDM}} > 5.3$ keV, if an alternative model of the evolution of the inter-galactic matter is taken the constraint can be weaken to $m_{\text{WDM}} > 3.5$ keV. In our model the dark sector reaches thermal equilibrium due to the 2 to 4 processes. The constraint on $m_D$ can be weaker than the warm DM. The reason is that the DM particles frequently scatter with each other and their path become a random walk before decoupling, such random walk significantly delay the starting of the free-streaming of the DM particles.

To obtain an estimation on the parameter space of this model detailed simulation of structure formation is needed, which is beyond the scope of this letter. Here we work on the following simplified treatment. We first use the package CAMB [41] to calculate the matter power spectrum for 5.3 (3.5) keV warm DM model and convert to the 1D matter power spectrum by integration over a k plane. Then similarly we calculate the matter power spectrum for our model, but the free-streaming is only turned on when the temperature is below temperature $T_{\text{fs}}$, or the free streaming velocity is simply set to zero when $T > T_{\text{fs}}$. $T_{\text{fs}}$ is defined as the temperature of the SM sector at which the scattering rate of the DM particles is equal to the Hubble expansion rate. In the NR limit the average cross section of the $\chi \chi$ and $\bar{\chi} \bar{\chi}$ processes reads

$$\sigma_m = \frac{15\pi\alpha_D^2 m_D^2}{m_V^4}. \quad (12)$$

Therefore, at temperature $T$ the collision rate can be estimated as

$$\Gamma_c(T) = \langle \sigma_m v_r \rangle n_D \approx \frac{30\zeta(3)\alpha_D^2 \zeta_D T^4 \Omega_D}{\pi m_V^2 \eta_r m_p \Omega_B}, \quad (13)$$

where $\eta_r \approx 6 \times 10^{-10}$ is the baryon-to-photon ratio, and

$$\zeta_D \equiv \frac{T_D}{T} \approx \left( \frac{10\eta_r m_p}{3m_D} \right)^{1/3} \approx 0.1 \times \left( \frac{m_D}{2 \text{keV}} \right)^{-1/3}. \quad (14)$$

Equating $\Gamma_c$ to the Hubble expansion rate $H = 1.66g_*/T^2/m_{pl}$, where $g_*$ is the effective degree of freedom and $m_{pl}$ is the Planck mass, we get

$$T_{fs} \approx 2 \text{ eV} \times \left( \frac{\alpha_D}{\alpha_{\text{EM}}} \right)^{-1} \left( \frac{\zeta_D}{0.1} \right)^{-1/2} \left( \frac{m_V}{1 \text{ MeV}} \right)^2. \quad (15)$$

The 1D matter power spectrum we simulated using the CAMB package is shown in Fig. 4, where the black dashed and dotted curves are for matter power spectrum with 5.3 keV and 3.5 keV warm DM model. The red and
from the center, within a mass deficit is observed. However, it is shown that the NFW process at the region outside of the bullet cluster. The dot-dashed line shows the lower limit from the Lyman-α forests observation. The purple region is excluded by the stellar constraints. The kinetic mixing \( \kappa \) in this figure is determined by the DM relic abundance. The red band shows the region in favor of the cluster mass deficit anomaly which will be discussed later.

The implication on small scale anomalies. It has been observed that the profile of clusters agrees well with the NFW process at the region outside \( O(10) \) kpc region from the center, within \( O(10) \) kpc on the other hand a mass deficit is observed. However, it is shown that this problem can be solved if the DM particles have a self-interaction with \( \sigma_T/m_D = 0.1^{+0.03}_{-0.02} \) cm\(^2\)/gram \[23\], where \( \sigma_T \) is the momentum-transfer cross section. But as in \[8\], due to the observation of the out flow and the severe baryonic process at the center of the cluster, observations of cluster alone cannot provide unambiguous support for DM theories. However, in a later study \[24\] numerical simulation shows that a self-interaction with \( \sigma_T/m_D > 0.1 \) cm\(^2\)/gram is disfavored. In our model the dark photon \( V \) conducts a self-interaction of \( \chi \) and \( \bar{\chi} \). Since \( m_V \gg m_D \) the scattering is s-wave and therefore \( \sigma_T \) equals the total cross section. From Eq. (12),

\[
\frac{\sigma_T}{m_D} = 0.125 \, \text{cm}^2/\text{g} \left( \frac{\alpha_D}{\alpha_{EM}} \right)^2 \left( \frac{m_D}{2 \, \text{keV}} \right) \left( \frac{m_V}{2 \, \text{MeV}} \right)^{-4} \tag{16}
\]

which is just in the right region. The region in favor of the cluster mass deficit problem for \( \alpha_D = 2\alpha_{EM} \) is shown as the red band in Fig. 5. One can see that with considering self-scattering the Lyman-\( \alpha \) constraint can be lowered to about 2.5 keV (1.4 keV) for the strong (weak) bound.

DM self scattering cross section will induce a cored profile. As the cross section goes small N body simulation is hard to have resolution to see such core, so here we use the analytical modeling of \[11\]. In the outer region of a halo the DM scattering count is statistically less than one in the history of the halo, so it will not be significant to change an NFW profile. But in the central region such scattering makes the halo isothermal. In that case in the partition function the momentum part will always gives a constant after integration for Maxwell distribution, so the density \( \rho(r) \propto e^{-\frac{r}{s}} \). Then Poisson equation

\[
\nabla^2 \Phi = 4\pi G \rho = 4\pi G (\rho_0 e^{-\frac{r}{s}} + \rho_B) \tag{17}
\]

can be solved with the observed baryonic distribution. At boundary \( r_M \) which is the transition point of the two
regions, we impose the physical condition that the enclosed mass as well as the local density are the same as their corresponding NFW values. In fact there are studies shown that to form a thermal distribution at least 2.7 collisions is required \[45\]. The required number of collisions to reach thermal equilibrium can induce considerable uncertainties in determining the size of the core \[45\]. In this study we adopt the criteria of 2.7 collisions. Moreover, in this study the NFW reference halo is taken according to the Placco CD halo concentration-mass relation, \(c_{200} = 10^{3.90.5\pm0.11} (M_{200}/10^{12} h^{-1} M_{\odot})^{-0.101}\) \[45\]. For our fermionic DM model we use Fermi-Dirac distribution instead which also incorporate the Fermi pressure, the difference for a fixed DM mass is here the chemical potential is a new parameter, while the central density \(\rho_0\) parameter in the Maxwell case now can be calculated.

For keV-scale fermionic dark matter the Fermi degeneracy pressure will also lead to a sizable core. In the NR limit the energy density of a Fermi-degenerate gas is \(\rho = m_D n = 4\pi g_D m_D p^3_f/3(2\pi)^3\), where \(p_f\) is the Fermi momentum and \(g_D\) is the degeneracy of DM \((g_D = 4)\). Then the pressure can be written as \(P = (4\pi g/3(2\pi)^3) \int_0^{p_f} (p^3/m_D) dp = (4\pi^2/5m_D^2)\rho^{8/3}/3(4\pi g_D)^2/r^{5/3}\), in terms of \(\rho\). The hydrostatic equilibrium equation of the halo gives

\[
\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2} \rho(r),
\]

(18)

where \(G\) is the Newton’s gravitational constant, \(M(r)\) is the mass enclosed within the radius \(r\) including the baryonic mass. Eq. [18] is an integro-differential equation of \(\rho\), which can reduce to the second order differential equation

\[
\frac{4}{3m_D^{8/3}} \frac{4\pi^2}{3} \left(\frac{3}{4\pi g_D}\right)^{2/3} \frac{1}{r^2} \frac{d}{dr} \left(\frac{r^2}{\rho^{3/3}} \frac{d\rho}{dr}\right) = 4\pi G (\rho + \rho_B)
\]

(19)

As such Fermi pressure just affect the inner region of the halo, we should also impose the same boundary condition as above at transition radius, \(M(r_M) = M_{\text{NFW}}(r_M)\) and \(\rho(r_M) = \rho_{\text{NFW}}(r_M)\).

In Fig. [6] we show the radius of the core as a function of \(m_D\) with \(\sigma_T/m_D\) fixed as 0.1 cm\(^2\)/gram. The radius of the core, \(r_c\), is defined as the radius where the density is half of the density at the center. In getting the plot for small DM mass we solve Eq. [18] and for large DM mass we solve Eq. [17], because the former strongly depends on \(m_D\) and the core solution gets smaller while using larger \(m_D\) whereas the latter one is dominated by scattering effect and is independent of \(m_D\). In the shaded region where \(m_D\) is between 1.5 to about 5 keV, the classical pressure becomes non-negligible. As a result a sizeable correction in this intermediate region of our estimation is expected. The black line on the other hand shows the size of core without consider the effect of the Fermi pressure, namely pure dark matter scattering effect with the Maxwell distribution. One can see that at \(m_D = 1.5\) keV the size of the core with the Fermi pressure is about 15 parsec, and at \(m_D = 2.5\) keV the size of the core can be about 10 parsec.

**Conclusion and discussions.** We have presented a model that the DM relic abundance is generated through the freeze-in mechanism. In this model the Lyman-\(\alpha\) constraint can be relaxed to \(m_D \gtrsim\) keV. In this region the Fermi pressure and the self-scattering can produce a small core \((\sim 10)\) pc in dwarf galaxies. We use the models in Refs. [17, 44] to analyze the properties of dwarf galaxies. To get a better understanding a detailed numerical simulation with the Fermi pressure included is needed.

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