Probing Anomalous Top Quark Couplings at the Future Linear Colliders*

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Abstract

In terms of an effective Lagrangian we investigate the possibilities of probing anomalous top quark couplings, $t\bar{t}H$, $\gamma t\bar{t}$, $Zt\bar{t}$ and $tWb$ at the future linear colliders. It is found that at a linear collider with a c. m. energy $\sqrt{s} \approx 0.5 - 1.5$ TeV and a high luminosity of $10 - 1000$ fb$^{-1}$, $e^+e^- \to t\bar{t}H$ is an ideal process in probing anomalous $t\bar{t}H$ couplings. We also study in detail the effects of anomalous couplings on $t\bar{t}$ spin correlations in the top pair production as well as the top quark decay processes with three bases (helicity, beam line and off-diagonal bases). Our results show that with a c. m. energy $\sqrt{s} \approx 0.5 - 1$ TeV and a high luminosity of $1 - 100$ fb$^{-1}$, the anomalous couplings $\gamma t\bar{t}$, $Zt\bar{t}$ and $tWb$ may be sensitively probed.

One believes that the large top-quark mass, which is close to the order of the weak scale ($m_t \approx v/\sqrt{2}$), makes the third generation to play a significant role in probing the new physics beyond the Standard Model (SM). Thus the linear collider (LC) will have a potential to explore the new physics associated with the Higgs and the top-quark sector.

In order to explore the possibility, we take a model-independent approach by using a linearly realized effective Lagrangian to dimension-6 operators including CP violation. We discuss the process $e^+e^- \to t\bar{t}H$ and the top quark spin correlation to probe the non-standard couplings. Particularly, the process $e^+e^- \to t\bar{t}H$ is an ideal one for probing anomalous coupling $t\bar{t}H$ and hopefully gains some insight for the new physics beyond the SM. The observability of the signal from anomalous couplings $t\bar{t}H$, $\gamma t\bar{t}$, $Zt\bar{t}$ and $tWb$ is studied.

In the case of linear realization, the new physics is parameterized by higher dimensional operators which contain the SM fields and are invariant under the SM gauge group. Below the new physics scale $\Lambda$, the effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

where $\mathcal{L}_0$ is the SM Lagrangian. $O_i$ are dimension-6 operators which are $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant and $C_i$ are coefficients which represent the coupling strengths of $O_i$.

All the operators $O_i$ are hermitian and the coefficients $C_i$ are real and the order of unity. If we assume that the new physics is of the origin associated with the electroweak symmetry breaking, then it is natural to identify the cut-off scale $\Lambda$ to be the order of $\mathcal{O}(4\pi v)$. Alternatively, based on unitarity argument for massive quark scattering, the scale for new physics in the top-quark sector should be below about 3 TeV. There are twelve dimension-six CP even operators. All the operators, which give new contributions to the couplings of $t\bar{t}H$, $\gamma t\bar{t}$, $Zt\bar{t}$ and $tWb$, are listed in Refs. [3, 4, 5].

Among them, some of the operators are energy independence such as

$$O_{1i} = (\Phi^\dagger \Phi - \frac{v^2}{2}) \left[\bar{q}_L t_R \tilde{\Phi} + \bar{\Phi}^\dagger t_R q_L\right]$$

and some are energy-dependent, such as

$$O_{Di} = (\bar{q}_L D_\mu t_R) D^\mu \tilde{\Phi} + (D^\mu \Phi)^\dagger (D_\mu t_R q_L)$$

due to the deviative. The energy-dependence of all dimension-6 operators are listed in the table of Refs. [3, 4, 5].

Generally, we can examine the possible constraints on the operators from the measurement $Z \to bb$. The observable $R_b$ at the $Z$ pole is calculated to be

$$R_b = \frac{\Gamma(Z \to bb)}{\Gamma(Z \to \text{hadrons})} = R_b^{SM} \left[1 + 2 \frac{v_b \delta V}{\sqrt{a_b^5 + a_b^6}} (1 - R_b^{SM})\right],$$

where $v_b$ and $a_b$ represent the SM couplings and $\delta V, \delta A$ the new physics contributions. If we attribute the difference between $R_b^{SM}$ and $R_b^{exp}$ as the new physics contribution, we obtain the limit at the $1\sigma$ (3$\sigma$) level

$$-4 \times 10^{-3} \lesssim (\delta V) < -5 \times 10^{-3} (4 \times 10^{-3}).$$

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Assuming that there is no accidental cancellation between different operators and noting that 
\[ 2 s_{\mu} C m_{Z}/e\nu \simeq 1, \] we obtain the bound for each of them at the 1σ (3σ) level as
\[
5 \times 10^{-5} \left( -4 \times 10^{-3} \right) < \frac{v^2}{\Lambda^2} c_{\Phi q}^{(1)} \quad \text{or} \quad \frac{v^2}{\Lambda^2} c_{\Phi q}^{(3)} < 4 \times 10^{-3} \left( 8 \times 10^{-3} \right). (6)
\]

On other hand, the constraints from \( A_{FB}^{(b)} \) are weaker than \( R_b \).

For \( O_{11}, O_{12}, O_{D1}, O_{W\Phi} \) and \( O_{tB\Phi} \), they are not constrained by \( R_b \) at tree level and bounds on them can be studied from the argument of partial wave unitarity in Ref. [8]. It is informative to see the ranges of the unitarity bounds for \( \Lambda \approx 3 \sim 1 \) TeV:
\[
|C_{11}| \frac{v^2}{\Lambda^2} \simeq 1.0 \sim 3.0, \quad |C_{12}| \frac{v^2}{\Lambda^2} \approx 0.29 \sim 2.6, \quad |C_{D1}| \frac{v^2}{\Lambda^2} \approx 0.07 \sim 0.63 \quad \text{or} \quad |C_{D1}| \frac{v^2}{\Lambda^2} \approx -0.04 \sim 0.40, \quad |C_{tW\Phi}| \frac{v^2}{\Lambda^2} \text{ or } |C_{tB\Phi}| \frac{v^2}{\Lambda^2} \approx 0.02 \sim 0.15. (7)
\]

At present, there is no significant experimental constraint on the CP-odd couplings involving the top-quark sector.

The relevant Feynman diagrams for \( e^+e^- \rightarrow t\bar{t}H \) production are depicted in Fig. 1, where (a)–(c) are those in the SM and the dots denote the contribution from new interactions. The four-particle vertex (Fig. 1d) should be paid more attention since there is no such vertex in the SM but exists in the effective couplings due to the gauge invariance. We evaluate all the diagrams including interference effects, employing a helicity amplitude package (FDG) developed in [8]. This package has the flexibility to include new interactions beyond the SM. We have not included the QCD corrections to the signal process, which are known to be positive and sizable.

![Figure 1](image-url)

**Figure 1.** Feynman diagrams for \( e^+e^- \rightarrow t\bar{t}H \) production. (a)–(c) are those in the SM. The dots denote the contribution from new interactions.

Due to the strong constraints on \( O_{\Phi q}^{(1)} \) and \( O_{\Phi q}^{(3)} \) from the \( Z \rightarrow b\bar{b} \) measurement, (see Eq. (3)), the effects of these operators at colliders will be rather small and can be neglected. Following the energy-dependence behavior, we expect that modifications to the SM prediction from different operators would be distinctive at high energies. For the purpose of illustration, we only present results for the operators \( O_{11} \) (energy-independent) and \( O_{D1} \) (most sensitive to energy scale) to represent to others.

The results include the production cross sections versus \( \sqrt{s} \), the Higgs mass \( m_H \) and the couplings. The effect due to the operator \( O_{D1} \) is insignificant at \( \sqrt{s} = 0.5 \) TeV, which at higher energies the contribution from \( O_{D1} \) is substantial. To establish the sensitivity limits on the anomalous couplings that may be probed at the future LC experiments, one needs to consider the identification of the final state from \( t\bar{t}H \), including the branch ratios and the detection efficiencies. The branching ratio of the leading decay mode \( H \rightarrow b\bar{b} \) is about 80% ~ 50% for the mass range of \( 100 \sim 130 \) GeV. We assume 65% efficiency for a single b-tagging to identify four b-jets in the final state. With the desirable consideration we estimate an efficiency factor \( \epsilon_s \) for detecting \( e^+e^- \rightarrow t\bar{t}H \) to be \( \epsilon_s = 10 - 30\% \) and a factor \( \epsilon_B \) for reducing QCD and EW background to be \( \epsilon_B = 10\% \). In order to estimate the luminosity (L) needed for probing the effects of the non-standard couplings, we define the significance of a signal rate (S) relative to a background rate (B) in terms of the Gaussian statistics
\[
\sigma_S = \frac{S}{\sqrt{B}} (8)
\]
for which a signal at 95% (99%) confidence level (C.L.) corresponds to \( \sigma_S = 2 \) (3). They are calculated as
\[
S = L(\sigma - \sigma_{SM}) \epsilon_s, \quad B = L \left( \sigma_{SM} \epsilon_s + (\sigma_{QCD} + \sigma_{EW}) \epsilon_B \right). (9)
\]

Then we obtain the luminosity required for observing the effects of \( O_{11} \) and \( O_{D1} \) at 95% C.L. for 0.5 TeV and 1 TeV for \( O_{11} \) and for 1 TeV and 1.5 TeV for \( O_{D1} \) in Fig. 2 where the two curves are for 10% and 30% of signal detection efficiency, respectively. It can be seen that at a 0.5 TeV collider, one would need rather high integrated luminosity to reach the sensitivity to the anomalous couplings; while at a collider with a higher c.m. energy one can sensitively probe those couplings with a few hundred \( fb^{-1} \) luminosity.

![Figure 2](image-url)

**Figure 2.** (a) Sensitivity to the anomalous couplings versus the integrated luminosity for a 95% confidence level limits with \( m_H = 120 \) GeV for (a) \( O_{11} \) at \( \sqrt{s} = 0.5 \) TeV and \( \sqrt{s} = 1 \) TeV and for (b) \( O_{D1} \) at \( \sqrt{s} = 1 \) TeV and \( \sqrt{s} = 1.5 \) TeV.

If there exist effective CP-odd operators beside the SM interaction, then CP will be violated in the Higgs and top-quark sector. By using the similar discussion, one can
try to observe the effects of the operators beyond the SM expectation. The CP-violating effect can be parameterized by a cross section asymmetry as

\[ A_{CP} \equiv \frac{\sigma((p_1 \times p_3) \cdot p_4 < 0) - \sigma((p_1 \times p_3) \cdot p_4 > 0)}{\sigma((p_1 \times p_3) \cdot p_4 < 0) + \sigma((p_1 \times p_3) \cdot p_4 > 0)} \]  

(10)

where \( p_1, p_3 \) and \( p_4 \) are the momenta of the incoming electron, top quark and anti-top quark, respectively. The luminosity required for detecting the effects on the total cross sections and \( A_{CP} \) is shown in Fig. 3 versus CP-odd operators with 95\% C.L. for \( m_H = 120 \text{ GeV} \) and \( \sqrt{s} = 1 \text{ TeV} \). The solid curves are for the cross sections with efficiency factor \( \epsilon_S = 30\% \) and \( \epsilon_B = 10\% \) according to Eq. (9). Apparently, the effects on the total cross section due to CP-odd operators are much stronger than that on \( A_{CP} \). In other words, the direct observation of the CP asymmetry would need much lighter luminosity to reach.

Figure 3. Sensitivity to the anomalous CP-odd couplings versus the integrated luminosity for a 95\% confidence level limits and for 30\% of detection efficiency at \( \sqrt{s} = 1 \text{ TeV} \), with \( m_H = 120 \text{ GeV} \). The solid line is for the total cross section and the dash line is for the CP asymmetry \( A_{CP} \).

We now discuss the corrections from the anomalous couplings in the top-quark pair production and the top-quark decays. In order to evaluate the possible effects from new physics and study which spin basis is more sensitive to anomalous couplings, we use the generic spin basis suggested by Parke and Shadmi [8]. They are helicity basis, beamline basis and off-diagonal basis. With the above bases and center of mass energy \( \sqrt{s} \), we calculate the differential polarized cross section at the tree level and the differential cross section of \( e^- e^- \rightarrow t\bar{t} \rightarrow (b\bar{l}l')(b'\nu\nu') \). In order to see the effects of the anomalous coupling, we define \( <S_t>, <S_t'> \) and \( <S_tS_t'> \) are \( t, t' \) and \( tl \) correlation functions and calculate the relevant observables. Our results show that with a c.m. energy \( \sqrt{s} \approx 0.5 - 1.5 \text{ TeV} \) and a higher luminosity of \( 1 - 100 \text{ fb}^{-1} \), the anomalous couplings \( \gamma t\bar{t}, Zt\bar{t} \) and \( tWb \) may be sensitively probed [8].

In summary, we have considered a general effective lagrangian to dimension-6 operators including CP violation effects. The constraints on these anomalous couplings has been derived from the \( Z \rightarrow b\bar{b} \) data and unitarity consideration.

In order to explore the effects of these non-standard couplings, we have studied the process \( e^- e^- \rightarrow t\bar{t}H \), anomalous couplings \( \gamma t\bar{t}, Zt\bar{t} \) and \( tWb \) on \( t\bar{t} \) spin correlations in the top pair production as well as the top-quark decay process with three bases (helicity, beamline and off-diagonal basis). We find that the future linear collider experiments should be able to probe those couplings well below their unitarity bounds. To reach a good sensitivity, the higher integrated luminosity needed is about several hundred \( \text{fb}^{-1} \) for a c.m. \( \sqrt{s} \approx 0.5 - 1.5 \text{ TeV} \).

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