Temperature distribution in a subduction zone

A G Kirdyashkin, A A Kirdyashkin and Yu M Nepogodina

Sobolev Institute of Geology and Mineralogy, SB RAS, 3 Ac. Koptyug Ave., Novosibirsk, 630090, Russia

E-mail: aak@igm.nsc.ru

Abstract. A model of the thermal and hydrodynamic structure of the subduction zone is proposed. This model includes free convection flows in the asthenospheric layer and layer C (mantle transition zone). Temperature profiles in the subducting lithospheric plate, as well as in the continental limb of the subduction zone, are presented. The heat flux due to friction at the contact between the subducting plate and the continental limb significantly affects the heat transfer and, consequently, the temperature field formation in the subduction zone. The temperature level in the crustal layer of the submerging plate implies that there is no melting in the crustal layer.

1. Introduction

Subduction is a process of submerging of an oceanic lithosphere plate into the mantle beneath a continent or island arc [1]. It significantly influences the flow structure and heat transfer in the Earth's mantle [1, 2]. In the laboratory experiments, the evolution of forced flows induced by subducting plate is studied [3]. In the majority of works devoted to 2D numerical modeling of subduction, the motion of subducted slab is prescribed kinematically. The mantle wedge flow is induced by the slab motion; and the thermal buoyancy in the wedge is not taken into account [4]. Another way of mantle flow modeling is thermophysical (laboratory and theoretical) modeling of the thermal and hydrodynamic structure of free convection flows in the subduction zone.

2. Model of thermal and hydrodynamic structure of the subduction zone

Let us turn to the continental limb, being the continental region to the right of a subducting plate (figure 1). The continental lithosphere thickness is \( l_{\text{cont}} \). In the rheological sense, the continental lithosphere can be considered as elastic or highly viscous medium with dynamic viscosity \( \mu_{\text{cont}} \to \infty \). Asthenosphere lies beneath the continental lithosphere. The asthenospheric layer thickness \( l_a \) depends on the continental lithosphere thickness: \( l_a = 420 \text{ km} - l_{\text{cont}} \). Layer C lies beneath the asthenosphere. The layer C thickness on the continental limb \( l_C \) approaches that on the oceanic limb. For the continental limb the coordinate \( x \) axis (the depth axis) is directed along the gravity vector. The \( y \) axis is horizontal, i.e., normal to the gravity vector. Free convection flows occur in the asthenosphere and layer C.

As is evident from the experiments in a horizontal liquid layer, the descending free convection flow incident on the lower boundary of the layer flows outward from the frontal point. The outward flow along the bounding surface occurs under the effect of horizontal forces resulting from the opposing horizontal temperature gradients. As applied to subduction, during the interaction between the submerging plate and the upper–lower mantle interface (670 km boundary) the subduction flow incident on the 670 km boundary flows outward from the frontal point (in opposite directions) [5, 6].
Relying on the laboratory and theoretical modeling [5, 6], in the first approximation we have constructed the model of convection flows at the contact between the subducting plate and 670 km boundary (figure 1).

Let us consider regions of conductive heat transfer and boundary conditions for subducting plate (figure 1). The $x_1$ axis is directed along the contact between the plate and the continental limb. The $x_1$ coordinate is determined using the relation $x_1 = x / \sin \alpha$, where $x$ is the depth, $\alpha$ is the angle between the subduction rate and the horizontal surface of the Earth. The plate moves between the mantle beneath a continent and the mantle beneath an ocean and sinks into the upper mantle. The continent is fixed relative to the subducting plate. A shear stress $\tau_0$ at the contact between the subducting plate and the continent can be estimated. It comprises $\tau_0 = 3 \cdot 10^7$ N m$^{-2}$. We consider the approximation of the flat viscous crustal layer. For Couette flow [7] in the crustal layer (figure 1) we have $\tau_0 = \mu_{cl} \mu / \delta_{cl}$, where $\mu_{cl}$ is the dynamic viscosity of the crustal layer, $\mu$ is the subduction rate, and $\delta_{cl}$ is the crustal layer thickness. Then we obtain $\mu_{cl} = \tau_0 \delta_{cl} / \mu$. For $\delta_{cl} = 5$ km and $\mu = 6$ cm year$^{-1}$ (1.9 $\cdot$ 10$^{-9}$ m s$^{-1}$) we have $\mu_{cl} = 8 \cdot 10^{19}$ N m$^{-2}$. The dynamic viscosity of the continent is $\mu_{cont} = 10^{21}$–$10^{23}$ N s m$^{-2}$ [1]. For $\mu_{cont} = 10^{22}$ N s m$^{-2}$ we obtain $\mu_{cl} / \mu_{cont} = 8 \cdot 10^{-3}$ and, hence, the motion rate at the crustal layer–continent boundary is $8 \cdot 10^{3}$ times less than the subduction rate.

The subducting plate is heated under conditions of non-stationary conductive heat transfer. Let us introduce the characteristic time for heat transfer in the plate and at the contact between the plate and the continental limb. This is the diving time $t_d = x_r / u \sin \alpha$, where $u = \text{const}$. The time $t_d$ ranges from subduction initiation to the time when the subducting plate comes in contact with 670 km boundary ($x_0 = 6.7 \cdot 10^3$ m), i.e., $0 \leq t_d \leq x_0 / u \sin \alpha$. The subduction lifetime is much above the time $t_d$. For time $t > t_d$ the heat transfer in the boundary layer of the plate is considered in the non-stationary heat transfer approximation. The boundary layer thickness is $\delta_1 = y_{1\text{min}}$ (figure 1).

On the continental limb of the subduction zone there is a boundary layer at the crustal layer, surrounding mantle boundary. The boundary layer thickness is $\delta_2$. The conductive heat transfer in this boundary layer is non-stationary. The plate comes in contact with the mantle of continental limb after
a time \( t_d = x/u \sin \alpha \) reckoned from a time of subduction initiation. The boundary layer develops from the time \( t_a \), that is the time of initiation of the boundary layer is \( t_c = t - x/u \sin \alpha \), where \( t \) is the time reckoned from subduction initiation.

The plunging plate is heated from the oceanic limb, i.e., a heat flux \( q_a \) is directed from the plate–oceanic limb boundary (\( y_1 = l_q \)) to the plate, where \( l_q \) is the plate thickness. The following designations are used: \( T_0 \) is the temperature at the plate–continental limb boundary (\( y = 0 \)), \( T_n \) is the temperature in the continental limb at a distance from the subduction zone \( y_2 \gg \delta_2 \), \( \delta_1 = y_{1\text{min}} \) is the thickness of the boundary layer at plate–oceanic limb boundary (\( y_1 = 0 \)), and \( T_{\text{min}} \) is the minimum temperature in the plate. The temperature of the boundary \( y_1 = 0 \) is \( T_0 \) for time \( t > 0 \). The heat flux \( q_a \) results from a friction at the boundary \( y_1 = 0 \). The friction heat is transferred to the subducting plate as its temperature is lower than that in the continent.

The aim of the present paper is to (1) find the temperature distribution in the subducting plate and in the continental limb of the subduction zone in relation to time; (2) determine the heat fluxes at the boundary between the subducting plate and lithosphere, asthenosphere and layer \( C \); and (3) determine the temperatures \( T_0 \) and \( T_{\text{min}} \).

3. Heat transfer between subducting plate and mantle on the continental limb of the subduction zone

The \( y_1 \) axis is normal to the plate surface. The \( x \) axis is directed along the plate surface; thus, the coordinate \( x \) is longitudinal and is reckoned from the day surface. As indicated above, the line \( y_1 = 0 \) is the boundary between the subducting plate and the continental limb of the subduction zone. Let us consider the subducting plate, i.e., the domain \( y_1 > 0 \). Heat transfer is analyzed in two domains of the plate.

1) The domain \( 0 \leq y_1 \leq y_{1\text{min}} \), where heat is transferred to the plate from the continental limb. This is due to the cooling of the upper mantle massif and heat resulting from friction at the plate–upper mantle boundary.

2) The domain \( y_{1\text{min}} \leq y_1 \leq l_q \), where heat is transferred to the plate from the asthenosphere and layer \( C \) of the oceanic limb.

The domain (1) mates with the domain (2) at \( y = y_{1\text{min}} \), where the minimum temperature \( T = T_{\text{min}} \) is achieved, \( \partial T/\partial y \big|_{y=y_{1\text{min}}} = 0 \) and specific heat flux to the right of coordinate \( y = y_{1\text{min}} \) is equal in magnitude but opposite in sign to that to the left of \( y = y_{1\text{min}} \). On the continental limb the upper mantle with the initial temperature \( T_0 \) is cooled. The temperature at the boundary \( y = 0 \) is \( T = T_{\text{min}} \). The total heat flux from the upper mantle (for \( y_2 > 0 \)) is \( q = q_a + q_{\text{clm}} \), where \( q_{\text{clm}} \) is the heat flux from the continental limb to the subducting plate. The initial (at \( t = 0 \)) superadiabatic temperature distribution in the subducting plate is linear: \( T_q = q_a \lambda x \), where \( q_a \) is the specific heat flux from the oceanic limb into the subducting plate at \( y_1 = l_q \), and \( \lambda \) is the thermal conductivity.

Let us consider heat transfer in the submerging oceanic lithosphere in the domain (1) \( 0 \leq y_1 \leq y_{1\text{min}} \). At the submerging plate boundary, the plate is heated while the surrounding mantle is cooled. In a geological sense the plate boundary \( y_1 = 0 \) undergoes instantaneous heating because the plate comes in contact with the continent. In this case we use the model of an instantaneous change in temperature of the surface of a semi-infinite half-space [2].

In the first approximation, the temperature profile in the thermal boundary layer is assumed to correspond to the solution for \( \Delta T = T_0 - T_{\text{min}} = \text{const} \), i.e., to quasi-stationary heat transfer. Since \( \partial^2 T/\partial y_1^2 \gg \partial^2 T/\partial x^2 \), the heat transfer equation has the form \( u(\partial T/\partial x) = a(\partial^2 T/\partial y_1^2) \), where \( a \) is the thermal diffusivity. At constant subduction rate \( (u = \text{const}) \) we introduce a new variable, namely, the time \( t_a = x/u \sin \alpha \). The temperature is presented in a dimensionless form: \( \theta_1 = T - T_{\text{min}} / T_0 - T_{\text{min}} \). Then the heat transfer equation takes the form: \( \partial \theta_1/\partial t_a = a(\partial^2 \theta_1/\partial y_1^2) \). The boundary conditions are as follows: \( \theta_1(y_1, 0) = 0 \), \( \theta_1(0, t_a) = 1 \), \( \theta_1(t_a, \infty) = 0 \).

The only parameter that has the dimension of length is the characteristic thermal diffusion distance \((a t_a)^{1/2}\) [2]. Using the dimensionless quantity \( \eta_1 = y_1/2(a t_a)^{1/2} \), we transform the equation (2) and obtain
\[
-\eta_i \frac{d\theta_i}{dt} = \frac{1}{2} \frac{d^2 \theta_i}{d\eta_i^2}.
\]

(1)

The boundary conditions for the equation (1) are \(\theta_i(0) = 1\), \(\theta_i(\infty) = 0\). The solution to the equation (1) with boundary conditions given above is as follows:

\[
\theta_i = 1 - \text{erf} \eta_i = \text{erfc} \eta_i,
\]

(2)

where \(\text{erf} \eta_i = \frac{2}{\sqrt{\pi}} \int_0^{\eta_i} \exp(-t^2)dt\eta_i\) is the error function, and \(\text{erfc} \eta_i\) is the complementary error function.

Heat flux from the boundary \(y_1 = 0\) into the subducting plate is \(q_\theta = -\lambda (\partial T/\partial y_1)_{y_1=0}\) and, in view of (2), we obtain

\[
q_\theta = \lambda (T_0 - T_{\text{min}}) / (\pi x / u \sin \alpha)^{1/2}.
\]

(3)

Heat flux averaged over time \(t_d\) is

\[
\bar{q}_\theta = 2\lambda (T_0 - T_{\text{min}}) / (\pi x / u \sin \alpha)^{1/2}.
\]

The boundary layer thickness \(d_1\) is determined using a condition \(\theta_i(\eta_1) = 0.1\). Using the solution (2) and this condition, we find \(\eta_1 = 1.16\) and since \(d_1 = 2\eta_1(t_d)^{1/2}\) we obtain \(d_1 = 2.32(t_d)^{1/2} = 2.32(ax/u)^{1/2}\). Heat is released due to friction at the boundary between the plate and mantle of the continental limb. The frictional heat flux is defined by a relation \(q_{fr} = \tau_{fr} u\), where \(\tau_{fr}\) is the shear stress at \(y_1 = 0\) [6]. The plate driving force is \(F_{dr} = F_{tg} + F_{pt} + F_c\), where \(F_{tg}\) is the thermogravitational force, \(F_{pt}\) is the force caused by the olivine-wadsleyite phase transition, and \(F_c\) is the force caused by eclogitization of the crustal layer of the submerging plate. Friction force is equal to the plate driving force \(F_{dr}\) for the subduction rate \(u = \text{const}\). Thus, the shear stress at the plate–upper mantle boundary is \(\tau_{fr} = (F_{dr} \sin^2 \alpha)/x\). The frictional heat flux is

\[
q_{fr} = (u F_{tg} \sin^2 \alpha) / x.
\]

(4)

The driving force per running meter of the subducting plate has a value \(F_{dr} = 4.1 \cdot 10^{13} \text{ N m}^{-1}\) [6]. For \(u = 5 - 8 \text{ cm year}^{-1} (1.59 \cdot 10^{-9} - 2.54 \cdot 10^{-9} \text{ m s}^{-1})\), since \(= 0.707\) (\(\alpha = 45^°\)) and \(x_0 = 670 \text{ km}\) we find that \(q_{fr} = 0.049 - 0.078 \text{ W m}^{-2}\), i.e., heat flux \(q_{fr}\) compares with the average heat flux on the ocean floor. The effect of heat flux \(q_{fr}\) is manifested in the increase of temperature \(T_0\).

Let us consider heat transfer in the continent \((y_2 \geq 0, u_{\text{cont}} = 0)\). Cooling of continent commences from time \(t_d = x_1/u\) \((x_1 = x/\sin \alpha)\) for \(x_1 = \text{const}\). This time is a reference time when determining the temperature profile in the continent for coordinate \(x_1 = \text{const}\), i.e., \(t_c = t - t_d = t - (x/\sin \alpha)\), where \(t_c\) is the characteristic time of the boundary layer development at the continental limb. There is nonstationary conductive heat transfer in the continental limb \((y_2 \geq 0)\). Since \(\partial T/\partial y_2 \gg \partial T/\partial x\), the heat transfer equation has the following form:

\[
\partial T/\partial t_c = a(\partial^2 T/\partial y_2^2).\]

We find the solution to this equation for the continent \((y_2 \geq 0)\) with the boundary, subjected to an instantaneous temperature lowering. At \(t_c = 0\) the temperature is equal to \(T_{x}\) throughout the whole space. At \(t_c > 0\), the temperature of the surface \((y_2 = 0)\) equals \(T_0\). At \(T_0 < T_{x}\) heat flux, directed into the submerging lithospheric plate, arises. Then the temperature of a continent lowers in the vicinity of \(y_2 = 0\).

As above, we introduce a dimensionless temperature \(\theta_2 = T - T_0/T_{x} - T_0\) and dimensionless coordinate \(\eta_2 = y_2/(2t_c)^{1/2}\). Then we get an equation:

\[
-\eta_2 (d\theta_2/d\eta_2) = 1/2(\partial^2 \theta_2/\partial \eta_2^2).
\]

The boundary conditions for this equation are as follows: \(\theta_2(0) = 0, \theta_2(\infty) = 1\). The solution to the equation with these boundary conditions is \(\theta_2 = \text{erf} \eta_2\) or \((T - T_0)/(T_{x} - T_0) = \text{erf} \eta_2\). Heat flux for \(y_2 = 0\) is

\[
q_{fr} = -\lambda (T_{x} - T_0) / (\pi x)^{1/2}.
\]

(5)
Heat flux averaged over time \( t_c \) is \( q_{clav} = -2\lambda(T_v - T_0)/(\pi\alpha t_c)^{1/2} \). The boundary layer thickness \( \delta_b \) is determined from the condition \( \theta_2(\eta_{max}) = 0.98 \). Then, using (2) we obtain \( \eta_{max} = 1.7 \) and, hence, with the equality \( \delta_b = 2\eta_{max}(at_c)^{1/2} \) we obtain \( \delta_b = 3.4(at_c)^{1/2} \). The temperature at the plate–continent boundary is determined by the heat balance:

\[
q_c = q_{clav} + q_{pr} .
\] (6)

Substitution of relations (3)-(5) into (6) and rearranging gives:

\[
T_0 = \left[ \frac{T_{max}}{t_d^{1/2}} + \frac{T_{\infty}}{t_c^{1/2}} + \frac{uF_0(\pi\alpha \sin \alpha)^{1/2}}{\lambda x_0} \right] \left[ \left( \frac{t_0}{t} \right)^{1/2} \right].
\] (7)

The temperature averaged over the plate thickness \( t_p \) in different sections \( x \) is determined using a relation

\[
T_{min} = \frac{(q_b + q_{pr} + q_{cl})}{t_p C \rho \sin \alpha} + \frac{T_{lu}}{2},
\] (8)

where \( C \) is the heat capacity, \( \rho \) is the density, and \( T_{lu}/2 = 600 \) °C is the mean temperature of submerging oceanic lithosphere at \( x = 0 \) [6]. We adopt the following values: \( t_p = 70 \cdot 10^3 \) m, \( C = 1.2 \cdot 10^3 \) J kg\(^{-1}\)°C\(^{-1}\), \( \rho = 3300 \) kg m\(^{-3}\), \( \lambda = 3 \) W m\(^{-1}\)°C\(^{-1}\), \( \alpha = \lambda/C\rho = 7.6 \cdot 10^{-7} \) m\(^2\) s\(^{-1}\), \( u = 1.9 \cdot 10^{-9} \) m s\(^{-1}\) (6 cm year\(^{-1}\)), \( \sin \alpha = 0.707 (\alpha = 45^\circ) \), \( x_0 = 6.7 \cdot 10^5 \) m, \( F_0 = 4.1 \cdot 10^{13} \) N m\(^{-1}\), \( q_b = 0.058 \) W m\(^{-2}\), and \( q_{pr} = 0.025 \) W m\(^{-2}\) [1, 5, 6]. Temperature \( T_0 \) is calculated for depth \( x = 100 \) km, 200 km, 400 km and 600 km. Superadiabatic temperatures away from the subduction zone \( T_0 \) are taken from the temperature distribution in the upper mantle beneath a continent [8]. We accept the following time values (in millions of years, Ma): \( t = 2.5 \) Ma, 5 Ma, 10 Ma, 20 Ma, 50 Ma and 100 Ma.

Calculations of temperature distribution in a subduction zone were performed using the method of successive approximations. The above formulae were used in our calculations. The calculations have shown that the second approximation is sufficient to find the temperatures \( T_{min} \) and \( T_0 \). Temperature distributions in the subducting plate as well as those in the continental limb have been obtained using calculated \( T_{min} \) and \( T_0 \) values. The temperature distribution in the subducting lithospheric plate has been determined using the relation \( T_{dp} = \text{erfc}_1(T_v - T_{min}) + T_{min} \), where the quantity \( \eta_1 \) lies in the interval from 0 to 1.16. The temperature distribution in the boundary layer of the mantle of continental limb has been found using the relation \( T_{dp} = \text{erfc}_2(T_v - T_0) + T_0 \), where the quantity \( \eta_2 \) lies in the interval from 0 to 1.7. The boundary layer thicknesses \( \delta_1 \) and \( \delta_2 \) have been calculated from corresponding relations obtained above.

The calculated results are presented in figure 2 for depth \( x = 100 \) km and \( x = 400 \) km. As the upper–lower mantle boundary is approached, temperature \( T_0 \) increases. Our calculations have shown that temperature \( T_0 \) is 1720 °C at a depth of 600 km. This value is significantly below the melting points of basalt and peridotite KLB-1 \( T_m = 2130 \) °C at the above-mentioned depth [9, 10]. Consequently, there are no conditions for melting in the crustal layer at a depth of 600 km. At the top of the lower mantle (at a depth of 670 km) a temperature is \( T_{670} = 1970 \) °C, whereas the melting point of basalt is \( T_m = 2200 \) °C [9].

Figure 2(a) shows temperature profiles in the subducting plate and continental limb of the subduction zone for a depth \( x = 100 \) km. At the plate–continental lithosphere boundary the temperature \( T_0 \) is higher than the temperature of lithosphere. At a depth \( x = 400 \) km the heat flux distribution is the same that at the depth \( x = 100 \) km (figure 2(b)). The submerging plate is heated from oceanic and continental limbs of the subduction zone. The temperature of the plate increases. At a depth of 600 km the plate is heated due to heat, generated by friction at the plate–mantle boundary. The temperature \( T_0 \) \( \sim T_{\infty} \), i.e., heat flux from the continental limb is absent. The plate is also heated from the oceanic limb. The estimated heat fluxes show that the melting of the crustal layer is possible only at 670 km
boundary. Thus, the formation of thermochemical plumes is possible at this boundary. Thermochemical plumes form because of a decrease in melting temperature of the crustal layer. The

![Temperature distribution](image)

**Figure 2.** Temperature distribution in the subducting plate and in the boundary layer at the plate–continental limb boundary for $x = 100$ km (a) and $x = 400$ km (b) at different times $t$.

(a) - 1 - $t = 2.5$ Ma, 2 - $t = 5$ Ma, 3 - $t = 10$ Ma, 4 - $t = 100$ Ma; (b) - 1 - $t = 10$ Ma, 2 - $t = 20$ Ma, 3 - $t = 50$ Ma, 4 - $t = 100$ Ma.

melting temperature decreases owing to the presence of local chemical doping in the crustal layer of the subducting plate [11].

**Conclusions**
The model of thermal and hydrodynamic structure of subduction zone has been presented. Temperature distribution in the subducting plate as well as in the boundary layer at the plate–mantle boundary has been determined under the conditions of non-stationary conductive heat transfer. The obtained temperature profiles elucidate the main features of heat transfer between a subducting plate and ambient mantle for different depths ($x = 100–600$ km). The temperature level in the crustal layer of the subducting plate implies the absence of melting in this layer.

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**References**
[1] Dobretsov N L, Kirdyashkin A G and Kirdyashkin A A 2001 *Deep-Level Geodynamics* (Novosibirsk: SB RAS Publishing House, GEO Branch)
[2] Turcotte D L and Schubert G 2002 *Geodynamics* (Cambridge: Cambridge University Press)
[3] Strak V and Schellart W P 2016 *J. Geophys. Res.* 121 4641–54
[4] van Keken P E, Hacker B R, Syracuse E M and Abers G A 2011 *J. Geophys. Res.* 116 B01401
[5] Kirdyashkin A A and Kirdyashkin A G 2013 *Geotectonics* 47 156–66
[6] Kirdyashkin A A and Kirdyashkin A G 2014 *Geotectonics* 48 54–67
[7] Schlichting H and Gersten K 2000 *Boundary-Layer Theory* (Berlin, Heidelberg: Springer)
[8] Kirdyashkin A A, Kirdyashkin A G and Distanov V E 2020 *Transbaikal State University Journal* 26 14–22
[9] Yasuda A, Fujii T and Kurita K 1994 *J. Geophys. Res.* 99 9401–14
[10] Zhang J and Herzberg C 1994 *J. Geophys. Res.* 99 17,729–42
[11] Kirdyashkin A A, Kirdyashkin A G, Gladkov I N and Distanov V E 2019 *Transbaikal State University Journal* 25 13–24