Red and blue tilted tensor spectrum from Gibbons-Hawking temperature

Subhendra Mohanty a and Akhilesh Nautiyal b

aPhysical Research Laboratory, Ahmedabad 380009, India.

bInstitute of Mathematical Sciences, Taramani, Chennai 600113, India.

The scale invariant scalar and tensor perturbations, which are predicted from inflation, are eigenmodes in the conformal coordinates. The ‘out’ observer in the de Sitter space observes a thermal spectrum with a Gibbons-Hawking temperature \( H/2\pi \) of these ‘Bunch-Davies’ particles. The tensor power spectrum observed in experiments can have an imprint of the Gibbons-Hawking thermal distribution due to the mode mixing between ‘in’ state conformal coordinates and the coordinate frame of the observer. We find that the the Bunch-Davies modes appear as thermal modes to the asymptotic Minkowski observer in the future and the power spectrum of the gravitational waves is blue-tilted with a spectral index \( n_T \sim 1 \) even in the standard slow-roll inflation. On the other hand if the coordinate frame of the observer is taken to be static coordinates, the tensor spectrum is red-tilted with \( n_T \sim -1 \). A likelihood analysis shows and find the best fit values of the slow-roll parameters for both cases. We find that the blue-tilted tensor gives a better fit and reconciles the PLANCK upper bound on the tensor-to-scalar ratio, \( r < 0.11 \) with BICEP2 measurement of \( r = 0.2 \). This supports the idea of particle production due to the mode mixing between the initial Bunch-Davies vacuum modes and the asymptotic Minkowski vacuum of the post-inflation universe.

1. INTRODUCTION

The prediction of a scale invariant scalar and tensor perturbations [1, 2] from inflation [3] rest on the assumption of a Bunch-Davies initial state in conformal coordinates of de Sitter space [4]. An observer in a different coordinate system, for instance an inertial observer in the static coordinates, will see the same perturbations as a thermal distribution with a Gibbons-Hawking temperature \( T = H/2\pi \) [5] due to mode-mixing between the Bunch-Davies modes and eigenmodes of the static coordinates [6–13]. Another way by which the scale invariant perturbations produced during inflation can appear as a thermal distribution is when one considers the mode mixing due to the change in observer between the conformal observer during inflation and the asymptotic Minkowski observer in future which measure the perturbations [14–16]. Variations of the standard Bunch-Davies state can be of phenomenological interest as a way of reconciling the large value of the tensor-to-scalar ratio implied by the B-mode polarization measurement by the BICEP2 collaboration with the lower upper bound established by PLANCK from the temperature anisotropy [17].

The BICEP2 collaboration [18] reported a tensor-to-scalar ratio \( r = 0.2^{+0.07}_{-0.05} \) by the measurement of the B-mode polarization [19], which is in apparent contradiction with the upper bound \( r < 0.11 \) (at 95% CL) placed by PLANCK [20] from the measurement of the TT spectrum. There is no direct contradiction between these two measurements as BICEP2 is most sensitive at \( l \sim 150 \) corresponding to a hub of \( k = 0.01 \) Mpc\(^{-1} \) while the PLANCK 2013 measurement uses the hub \( k = 0.002 \) Mpc\(^{-1} \) which corresponds to \( l \sim 30 \). However explaining the two measurements in a model of inflation would require (1) a blue tilted tensor spectrum with spectral index \( n_T \sim 1 \) [21–23] or (2) a running of the scalar spectrum \( d\ln n_s/d\ln k = -0.02 \) [20]. Either of the possible ways to explain the PLANCK-2013 and BICEP2 data simultaneously would require going beyond the single field inflation with Bunch-Davies initial state. Subsequently the dust popularization measurement reported by PLANCK-2014 [24] has diminished the statistical significance of the BICEP2 measurement but not ruled it out [25]. There is a possibility that the measurement of the B-mode polarization in other experiments (like KECK, SPTpol etc) may result in a value of the tensor-to-scalar ratio which still calls for a non-standard interpretation of the inflationary power spectrum to evade the standard consistency relation \( n_T = r/8 \) of the standard single field inflation.

In this paper we show that if we assume a mode mixing between the Bunch-Davies initial vacuum and the post-inflation final vacuum and the Bogoliubov coefficients \( \alpha \) and \( \beta \) of the mode-mixing is of the thermal form \( |\beta|^2 = \frac{\eta}{T} \) with the Gibbons-Hawking temperature \( T = \beta^{-1} = H/2\pi \), then the spectral index of tensor modes will be blue-tilted with \( n_T = 1 - 2\epsilon \). On the other hand if we assume that the ‘out’ observer is the one with the static coordinates, then the Bogoliubov coefficients again give the same thermal distribution with identical \( |\beta|^2 \), however, the spectral index in this case is red-tilted \( n_T = -1 - 2\epsilon \). The difference in the spectral tilt between the two cases is due to the fact that when we transform the initial state from the conformal to static coordinates we have \( \alpha \beta^* < 0 \) while the transformation between the conformal coordinates and the asymptotic Minkowski coordinates of the late time observer gives \( \alpha \beta^* > 0 \). The difference in sign of \( \alpha \beta^* \) with the same \( |\beta|^2 \) results in a different spectral tilt. In order to avoid the successful prediction of the scale invariant scalar power spectrum, we will assume that the slow roll parameter \( \eta \) of the scalar potential is negative so that the scalar modes are tachyonic and the Hawking radiation of scalar modes is suppressed [26].

We do a likelihood analysis for the values of the tensor-to-scalar ratio for the case of red and blue tilted
spectra and determine the slow roll parameters of the model which would be reconcile the B-mode and TT anisotropy data. We conclude that mode mixing between the Bunch-Davies vacuum and the vacuum state of the observer, may resolve the tension between the PLANCK-2013 bound and BICEP2 measurement and the accurate experimental measurement of the spectral index can determine the nature of the initial state of the inflation generated perturbations.

2. BOGOLIUBOV TRANSFORMATION OF BUNCH-DAVIES VACUUM

We can express the tensor perturbations \( h(x,t) \) as a quantum field in terms of the mode functions \( \phi_{in,k} \) (which satisfies the minimally coupled Klein-Gordon equation) as

\[
h(x,t) = \frac{\sqrt{2}}{M_p} \int [dk] \left( a_k \phi_{in,k} + a_k^\dagger \phi_{in,k}^* \right),
\]

where \( a_k^\dagger \) (\( a_k \)) are the creation (annihilation) operators of the 'particles' in the conformal vacuum, also called the Bunch-Davies vacuum, which we will denote by \( |0_{in} \rangle \) and which is defined by \( a_k |0_{in} \rangle = 0 \). Eqn. \( \text{(1)} \) is written in terms of the spherical polar coordinates.

The Bunch-Davies vacuum is defined in conformal coordinates \((\eta, \rho, \theta, \phi)\) with the line element

\[
ds^2 = \frac{1}{H^2 \eta^2} (d\eta^2 - d\rho^2 - \rho^2 d\Omega^2) \quad \eta \in (-\infty, 0), \quad \rho \in (0, \infty).
\]

The mode functions \( \phi_{in,k} \) are solutions of the Klein-Gordon equation in the conformal coordinates,

\[
\frac{\partial^2 \phi_{in,k}}{\partial \eta^2} - \frac{2}{\eta} \frac{\partial \phi_{in,k}}{\partial \eta} - \nabla \rho^2 \phi_{in,k} = 0.
\]

This equation has the exact solution

\[
\phi_{in,k}(\eta, \rho, \theta, \phi) = \frac{i H}{\sqrt{2k^3}} e^{-ik\eta} (1 + i k \eta) j_l(k\eta) \frac{Y_{l,m}(\theta, \phi)}{\sqrt{4 \pi}}.
\]

The zero-point fluctuations during inflation are assumed to be eigenmodes of the KG equations in conformal coordinates, as the mode functions \( \phi_{in,k} \) in the high \( k \) limit have the same form \( \phi_{in,k} \sim \frac{1}{\sqrt{2k}} e^{-ik\eta} \) as positive frequency modes in Minkowski space.

The scalar field is quantized in terms of the creation and annihilation operators of the \( \phi_{out,k} \) quantum modes which are the elementary excitations in the different coordinate system dependent on the observer,

\[
h(x,t) = \int [d\omega] \left( b_\omega \phi_{out,\omega} + b_\omega^\dagger \phi_{out,\omega}^* \right).
\]

Here \( b_\omega^\dagger \) and \( b_\omega \) are the creation and annihilation operators acting on a different vacuum \( |\phi_{out} \rangle \). The two sets of modes \( \phi_{in,k} \) and \( \phi_{out,\omega} \) can be linearly related in terms of Bogoliubov coefficients \((\alpha_{\omega,k}, \beta_{\omega,k})\) as

\[
\phi_{out,\omega} = \int [dk] (\alpha_{\omega,k} \phi_{in,k} + \beta_{\omega,k} \phi_{in,k}^*) \quad \phi_{in,k} = \int [d\omega] \left( \alpha_{\omega,k}^* \phi_{out,\omega} - \beta_{\omega,k} \phi_{out,\omega}^* \right).
\]

The relation \( \text{(1)} \) in terms of cartesian coordinates \((\eta, x, y, z)\) can be written as

\[
h(x, \eta) = \frac{\sqrt{2}}{M_p} \int \frac{d^3k}{(2\pi)^3/2} h(k, \eta) e^{ik \cdot x} = \frac{\sqrt{2}}{M_p} \int \frac{d^3k}{(2\pi)^3/2} a_k^\dagger \phi_{in,k} + a_{-k} \phi_{in,k}^* \right) e^{ik \cdot x}.
\]

The power spectrum of tensor perturbation is given in terms of two-point correlation function of the field \( h(x, \eta) \), which in the out-vacuum \( |0_{out} \rangle \) can be obtained as

\[
\langle 0_{out}| h(x, \eta) h(y, \eta)|0_{out} \rangle = \frac{2}{M_p^*} \int d^3k d^3k' \delta(k-k') \langle 0_{out}| h(k, \eta) h(k', \eta)|0_{out} \rangle e^{i(k \cdot x + k' \cdot y)}.
\]

To compute the two-point correlation function \( \langle 0_{out}| h(k, \eta) h(k', \eta)|0_{out} \rangle \) we will again use the spherical coordinates \((\eta, \rho, \theta, \phi)\). Now we can express the creation and annihilation operators of the in-vacuum \( a_k, a_k^\dagger \) in terms of the creation and annihilation operators of the out-vacuum \( b_\omega, b_\omega^\dagger \) using Eqn. \( \text{(1)} \) as

\[
\alpha_k = \int [d\omega] \left( \alpha_{\omega,k} b_\omega + \beta_{\omega,k} b_\omega^\dagger \right) \quad \beta_k^\dagger = \int [d\omega] \left( \beta_{\omega,k} b_\omega + \alpha_{\omega,k} b_\omega^\dagger \right).
\]

So we obtain

\[
\langle 0_{out}| h(k, \eta) h(k', \eta)|0_{out} \rangle = \delta(k-k') \int [d\omega] [d\omega'] \left[ \langle \alpha_{\omega,k} \alpha_{\omega',k} + \beta_{\omega,k} \beta_{\omega',k} \rangle |\phi_{in,k}|^2 \right.
\]

\[
+ \langle \alpha_{\omega,k} \beta_{\omega',k} |\phi_{in,k}|^2 \rangle + \langle \beta_{\omega,k} \alpha_{\omega',k} |\phi_{in,k}|^2 \rangle \left. \right] .
\]

Now for the choices of out-vacua we consider in the next sections, \( \alpha_{\omega,k} \) and \( \beta_{\omega,k} \) are diagonal in \( \omega \) and the frequency \( \omega \) of the out-vacuum corresponds to \( \frac{\pi}{2} \) so the integrals in above expressions can be done by using \( \delta(\omega-\frac{\pi}{2}) \). Now using the temporal part of \( \phi_{in,k} \) given be Eqn. \( \text{(1)} \)

\[
\phi_{in,k} = \frac{i H}{\sqrt{2k^3}} (1 + i k \eta) e^{-ik \eta},
\]
we get for the super-horizon \((k\eta \ll 1)\) perturbations
\[
\langle 0_{\text{out}} | h(k, \eta) h(k', \eta) | 0_{\text{out}} \rangle = \delta (k-k') \frac{H^2}{k^3 M_p^2} [|\alpha_{\omega k}|^2 + |\beta_{\omega k}|^2 + 2\text{Re} (\alpha_{\omega k} \beta_{\omega k}^*)] .
\]

(12)

Now the tensor power spectrum is defined as
\[
4 \times \frac{k^3}{2 \pi^2} \langle 0_{\text{out}} | h(k, \eta) h(k', \eta) | 0_{\text{out}} \rangle = \delta (k-k') P_T(k).
\]

(13)

So
\[
P_T = \frac{8}{M_p^2} \frac{H^2}{2 \pi} \left( \frac{k}{aH} \right)^{2-2\epsilon} [|\alpha_{\omega k}|^2 + |\beta_{\omega k}|^2 + 2\text{Re} (\alpha_{\omega k} \beta_{\omega k}^*)] .
\]

(14)

3. TENSOR POWER SPECTRUM MEASURED FOR THE STATIC OBSERVER

The coordinate system which describes the static observer in de Sitter space with coordinate \((t, r, \theta, \phi)\) and the metric given by
\[
ds^2 = (1-r^2 H^2)dt^2 - \frac{1}{(1-r^2 H^2)} dr^2 - r^2 d\Omega^2
\]
\(t \in (-\infty, \infty), \ r \in (0, H^{-1}) .\)

(15)

The time evolution of the quantum state with respect to an observer located at \(r=0\) is determined by a Hamiltonian operator defined by the time-like Killing vector \(\partial_t\). The static coordinate system has a coordinate singularity at \(r = H^{-1}\) which is the event horizon for the observer at \(r = 0\).

The two coordinate systems overlap in the region \(\eta \in (-\infty, 0)\) and can be related as
\[
\eta = -\frac{1}{H (1-H^2 r^2)^{1/2}} e^{-H t},
\]
\(\rho = -r \eta .\)

(16)

We consider first the two-dimensional spacetime to obtain the Bogoliubov coefficients for an static observer. We will also set \(H = 1\) to make the notation simple and at the end of the calculation we will restore \(H\) by substituting \((r, t) \rightarrow (H r, H t)\) and \(k \rightarrow \frac{k}{H}\). The metric \([15]\) for two-dimensional static coordinates becomes
\[
ds^2 = (1-r^2 H^2)dt^2 - \frac{1}{(1-r^2 H^2)} dr^2
\]
\(t \in (-\infty, \infty), \ r \in (-H^{-1}, H^{-1}) .\)

(17)

The solution of Klein-Gordon eqn. \(\Box \phi = 0\) in this coordinate system is given as
\[
\phi_{\text{out } \omega}(t, r) = \frac{1}{\sqrt{2 \omega}} e^{-i \omega t} \left[ \frac{1}{1+r} \right]^{\frac{1}{2}},
\]
and the solution of the KG eqn. \((3)\) in sub-Hubble limit in two-dimension is given as
\[
\phi_{\text{in } \omega}(t, \rho) = \frac{1}{\sqrt{2k}} e^{-i k (\eta - \rho)} .
\]

(19)

Now the Bogoliubov coefficients defined in Eqn. \((6)\) can be obtained by Klein-Gordon inner product \([27]\)
\[
\alpha_{\omega,k} = \langle \phi_{\text{out } \omega} \phi_{\text{in } \omega} \rangle
\]
\(\beta_{\omega,k} = -\langle \phi_{\text{out } \omega} \phi_{\text{in } \omega} \rangle .\)

(20)

Now using the metric \((17)\) and integrating over the constant time hypersurface we can obtain \(\alpha_{\omega,k}\) as
\[
\alpha_{\omega,k} = -i \int_{-1}^{1} \frac{dr}{1-r^2} \left( \phi_{\text{out } \omega} \phi_{\text{in } \omega} \right) .
\]

(22)

The Bunch-Davies mode \(\phi_{\text{in } \omega}\) can be expressed in terms of the static coordinates \(t, r\) using the transformations \([16]\) as
\[
\phi_{\text{in } \omega} = \frac{1}{\sqrt{2k}} e^{i k \left[ \frac{1}{1+r} \right]^{1/2} z - i t} .
\]

(23)

Now changing variable to \(z = \left( \frac{1}{1+r} \right)^{1/2}\) the above integral becomes
\[
\alpha_{\omega,k} = \frac{1}{2 \sqrt{\omega k}} \int_{-1}^{1} \frac{dr}{1-r^2} \left[ k \left( \frac{1+r}{1-r} \right)^{1/2} e^{-i k \left( \frac{1+r}{1-r} \right)^{1/2}} ight. \\
\left. + \omega \left( \frac{1+r}{1-r} \right)^{1/2} e^{-i k \left( \frac{1+r}{1-r} \right)^{1/2}} \right] .
\]

(24)

This integral can be solved using the \(\Gamma\) functions and finally we get
\[
\alpha_{\omega,k} = \frac{1}{2 \sqrt{\omega k}} \int_{0}^{\infty} dz \left( k z^2 + \omega z^2 - 1 \right) e^{-ikz} .
\]

(25)

Similarly we obtain the another coefficient \(\beta_{\omega,k}\) using the inner product \((20)\) as
\[
\beta_{\omega,k} = -\sqrt{\frac{\omega}{k}} k^{-i \omega} e^{-\frac{i \pi}{4}} \Gamma(i \omega) .
\]

(26)

Now in four-dimension to solve the Klein-Gordon equation for the modes of static observer \(\phi_{\text{out } \omega}\) we separate wave function in the \((t, r, \theta, \phi)\) coordinates
\[
\phi_{\text{out } \omega} = \frac{f(r)}{r} Y_{l,m}(\theta, \phi) e^{-i \omega t} .
\]

(28)
The equation for radial wave function can be written in a simple form
\[
\frac{d^2}{dr^2} f(r) + (1-r^2) \left( \frac{l(l+1)}{r^2} - 2 \right) f(r) + k^2 f(r) = 0,
\]
(29)
in terms of the tortoise coordinates
\[
r_* = \int \frac{dr}{(1-r^2)^{1/2}} = \frac{1}{2} \log \left( \frac{1+r}{1-r} \right).
\]
(30)
The radial equation can be solved exactly in terms of Hypergeometric functions that can be written in terms of Legendre functions of second kind and the solution is
\[
\phi(t, r, \theta, \phi) = \frac{1}{\sqrt{2\omega}} \exp(-i\omega t) Q_l^0 \left( \frac{1}{r} \right) Y_{lm}(\theta, \phi).
\]
(31)

The sub-Hubble limit of the Bunch-Davies mode functions \( \phi_{\text{in}} \) is given as
\[
\phi_{\text{in}}(t, r, \theta, \phi) = -\frac{1}{\sqrt{2\omega}} \exp(-i\omega t) j_l(k\eta) Y_{lm}(\theta, \phi).
\]
(32)

It can be shown (see \[7\]) that the Bogoliubov coefficients for \( l = 0 \) in four-dimensions are same as for two-dimensions except normalization factors. So we can write the Bogoliubov transformations for \( l = 0 \) case in four-dimensions as
\[
\alpha_{\omega, k} = N k^{-i\omega} e^{-\frac{\pi}{4}} \Gamma(i\omega),
\]
\[
\beta_{\omega, k} = -N k^{-i\omega} e^{-\frac{\pi}{4}} \Gamma(i\omega).
\]
(33)

Here \( N \) is normalization constant. Putting back \( k = k/H \) and \( \omega = \omega/H \), using the identity
\[
|\Gamma(i\omega)|^2 = \frac{\pi}{\omega \sinh(\pi\omega)}
\]
(34)
and the normalization condition for Bogoliubov coefficients
\[
\int dk (\alpha_{\omega, k} \alpha_{\omega, k}^* - \beta_{\omega, k} \beta_{\omega, k}^*) = \delta(\omega - \omega'),
\]
(35)
we obtain the expressions for \( \alpha_{\omega, k} \) and \( \beta_{\omega, k} \) from Eqn. (33) as
\[
|\alpha_{\omega, k}|^2 = \frac{e^{\beta\omega}}{e^{2\beta\omega} - 1},
\]
\[
|\beta_{\omega, k}|^2 = \frac{1}{e^{2\beta\omega} - 1},
\]
\[
\alpha_{\omega, k} \beta_{\omega, k}^* = -\frac{e^{\beta\omega}}{e^{2\beta\omega} - 1},
\]
(36)
where
\[
\beta = \frac{2\pi}{H}.
\]
(37)

Using \( \alpha_{\omega, k} \) and \( \beta_{\omega, k} \) given by (36) in two-point function (12) the tensor power spectrum can be expressed as
\[
P_T = \frac{8}{M_P^2} \left( \frac{H}{2\pi} \right)^2 \frac{2}{(aH)^{2\epsilon}} \left[ \frac{e^{\frac{aH}{k}} + 1}{(e^{\frac{aH}{k}} - 1)} \right]^2
\]
\[
\sim \frac{8}{M_P^2} \left( \frac{H}{2\pi} \right)^2 \frac{2}{(aH)^{2\epsilon}} \left[ \frac{2}{\pi} \frac{(aH/k)}{k} \right] \text{ for } k \ll aH,
\]
(38)
which is a red-tilted spectrum for the tensor modes with spectral index \( n_T = -1 - 2\epsilon \).

\section{Particle Production in Late Universe}

The post inflation universe at the time when all the modes are sufficiently sub-horizon, the modes can be considered as plane waves in Minkowski space,
\[
\phi_{\text{out}}(t, r, \theta, \phi) = A e^{-i\omega t},
\]
(39)
while the Bunch-Davies modes \( \phi_{\text{in}} \) in the sub-horizon limit is
\[
\phi_{\text{in}}(t, r, \theta, \phi) = \frac{1}{\sqrt{2\omega}} e^{-i\omega t}.
\]
(40)
Using Bogoliubov transformations (13) we get
\[
\phi_{\text{in}}(t, r, \theta, \phi) = A \int_{-\infty}^{\infty} d\omega f(\omega, k) e^{-i\omega t}.
\]
(41)
So
\[
\alpha_{\omega, k}^* = f(\omega, k), \quad \beta_{\omega, k}^* = -f(-\omega, k).
\]
(42)
So the Bogoliubov coefficients for these 'in' and 'out' states can be obtained by doing inverse Fourier Transforms as
\[
\alpha_{\omega, k} = \frac{1}{A} \int_{-\infty}^{\infty} \phi_{\text{in}}(k) e^{i\omega t} dt
\]
\[
= B \frac{\omega}{A} \int_{-\infty}^{\infty} e^{-t} e^{ikz} dt.
\]
(43)
By substituting \( e^{-t} = z \) the integral reduces to
\[
\alpha_{\omega, k} = \frac{B}{A} \int_{0}^{\infty} z^{-i\omega} e^{ikz} dz
\]
\[
= \frac{B}{A} \frac{\omega}{k} k^{-i\omega} e^{\pi\omega/2} \Gamma(-i\omega).
\]
(44)
Similarly
\[
\beta_{\omega, k} = \frac{1}{A} \int_{-\infty}^{\infty} \phi_{\text{in}}(k) e^{-i\omega t} dt
\]
\[
= \frac{\omega}{k} k^{-i\omega} e^{\pi\omega/2} \Gamma(-i\omega).
\]
(45)
Normalizing $\alpha_{\omega k}$ and $\beta_{\omega k}$ we obtain

$$|\alpha_{\omega k}|^2 = \frac{e^{\beta_\omega}}{e^{\beta_\omega} - 1}$$

$$|\beta_{\omega k}|^2 = \frac{1}{e^{\beta_\omega} - 1}$$

$$\alpha_{\omega k}\beta_{\omega k}^\star = \frac{e^{\beta_\omega}}{e^{\beta_\omega} - 1},$$

where again $\beta = \frac{2\pi}{H}$, which is the Hawking-Gibbon temperature of the de Sitter space. Using $\alpha_{\omega k}$ and $\beta_{\omega k}$ given by (48) in two-point function (12) the tensor power spectrum can be expressed as

$$P_T = \frac{8}{M_p^2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{-2\epsilon} \left[ \left( \frac{e^{\frac{\pi}{2\eta}} - 1}{e^{\frac{\pi}{2\eta}} - 1} \right)^2 \right]$$

$$\simeq \frac{8}{M_p^2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{-2\epsilon} \left[ \frac{\pi}{2} \left( \frac{k}{aH} \right) \right] \text{ for } k \ll aH,$$

(49)

which is a blue-tilted spectrum for the tensor modes with $n_T = 1 - 2\epsilon$.

5. BEST FIT VALUES OF R FOR RED AND BLUE TILTED TENSOR SPECTRUM

We use the power spectra (38) and (49) to compute the angular power spectra for $B$-mode polarization using CAMB [28]. We have taken the best-fit values of the parameters ($\Omega_b h^2$, $H_0$, $\Omega_c h^2$, $\tau$, $A_s$ and $n_s$) given by PLANCK [18] for the base $\Lambda$CDM model. The scalar amplitude $A_s$ and the scalar spectral index are taken at the pivot scale $k = 0.05\text{Mpc}^{-1}$. We have also taken into account the lensed $B$-modes generated from $E$-modes. $B$-modes due to the modified and the standard power spectra, along with BICEP2 data and WMAP bounds are shown in Fig. 1. The value of tensor-to-scalar ratio $r$ is taken at the pivot scale $k = 0.002\text{Mpc}^{-1}$. We see that with $n_T \simeq 1$ the BICEP2 data is in agreement with a low value of $r = 0.04$ consistent with the PLANCK’s upper bound. In Fig. 1, we have also shown the standard power spectra (taking $n_T \simeq 0$) with $r = 0.2$ and $r = 0.11$ for comparison. We see that in the standard power spectrum, the PLANCK upper bound $r = 0.11$ is not in agreement with the BICEP2 data.

The tensor power also contributes to the temperature anisotropy. In Fig. 2, we show the $TT$ anisotropy with the modified and the standard power spectra. As expected the $n_T = 1$, $r = 0.04$ power spectrum gives the lowest contribution to temperature anisotropy. With the temperature anisotropy the red-tilted power spectrum (38) is ruled out.

In Fig. 3 we plot the likelihood for the tensor-to-scalar ratio $r$ using the modified power spectra (49) and (38) for the BICEP2 data. The best fit value of tensor-to-scalar ratio $r$, the corresponding maximum likelihood ($\ln L$) and the slow-roll parameters for both blue-tilted and red-tilted spectra are given in Table 1.

As displayed in Table 2, the maximum likelihood is at $r = 0.042$ for the blue-tilted power spectrum and $r = 0.95$ for the red-tilted power spectrum. The value of $\ln L$ is highest for the blue-tilted power spectrum, which shows that the blue-tilted tensor power spectrum is a better fit to BICEP2 data compared to the red-tilted and scale invariant power spectra.

For the maximum likelihood value of $r_{0.002} = 0.042$ from a blue tilted spectrum, the slow-roll parameter $\epsilon = 0.002$. From the scalar spectral index $n_s = 1 - 6\epsilon + 2\eta = 0.9619$ we have $\eta \sim -0.014$.

On the other hand the maximum likelihood value of $r_{0.002} = 0.95$ from a red tilted spectrum, the slow-roll parameter $\epsilon = 0.093$. From the scalar spectral index $n_s = 1 - 6\epsilon + 2\eta = 0.9619$ we have $\eta \sim 0.2607$.

The accurate determination of the tensor spectrum in future experimental measurements of the $B$-model will determine the parameters of the inflation model and help
| $n_T$ | $r$  | $\ln \mathcal{L}$ | $\epsilon$ | $\eta$ |
|-------|-------|-------------------|-----------|-------|
| 1     | 0.042 | -2.6390           | 0.002     | -0.014 |
| 0     | 0.215 | -3.6830           | 0.013     | 0.019  |
| -1    | 0.95  | -4.7223           | 0.093     | 0.2607 |

**TABLE I:** Best fit tensor-to-scalar ratio and Maximum Likelihood for blue and red-tilted tensor power spectra using BICEP2 data.

in picking out the correct model of inflation.

6. CONCLUSION

The combined data from B-mode measurement by BICEP2 [18] with the temperature anisotropy measurement from PLANCK-2013 [20] implies that the slow roll inflation consistency relation $n_T \sim r/8$ is violated. It is well known that assuming a different initial state compared to the Bunch-Davies one can modify the relation between the slow-roll $\epsilon$ parameter derived from the potential and the observed tensor spectral index $n_T$ [17, 29]. In this paper we examine the modification to the tensor spectrum due to mode mixing between a Bunch-Davies 'in' vacuum and the 'out' vacuum of the (a) static coordinate observer and (b) the post inflation asymptotic Minkowski observer. Both the scenarios result in a Gibbons-Hawking thermal distribution as observed w.r.t the 'out' vacuum. The relative phases of the Bogoliubov coefficients are different in the two cases and these lead to quite different predictions for the tensor spectral index. The combined BICEP2 and PLANCK-2013 data gives a better fit for a blue-tilted tensor spectrum which supports the post-inflation particle production scenario. The Hawking-Gibbons temperature unlike temperature of perturbation form a possible pre-inflation radiation era does not go down exponentially during the course of inflation so the effect is not diluted after a few e-foldings [30, 31]. Measurements of the B-mode in future experiments may give a signature of the Gibbons-Hawking temperature.

---

[1] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D28, 679 (1983); S. W. Hawking, Phys. Lett. B115, 295 (1982); A. D. Linde, Phys. Lett. B116, 335 (1982).

[2] L. F. Abbott and M. B. Wise, Nucl. Phys. B244, 541 (1984); A. Starobinski, Sov. Astron. Lett. 11, 133 (1985); V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. B115, 189 (1982); R. Fabbri and M. D. Pollock, Phys. Lett. B125, 445 (1983); B. Allen, Phys. Rev. D37, 2078 (1988); V. Sahni, Phys. Rev. D42, 453 (1990).

[3] A. H. Guth, Phys. Rev. D 23, 347 (1981); A. D. Linde, Phys. Lett. B 108, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[4] V. Mukhanov and S. Winitzki, Cambridge, UK: Cambridge Univ. Pr. (2007).

[5] G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2738 (1977).

[6] A. S. Lapedes, J. Math. Phys. 19, 2289 (1978).

[7] T. Mishima and A. Nakayama, Phys. Rev. D 37, 348 (1988).

[8] R. H. Brandenberger and R. Kahn, Phys. Lett. B 119, 75 (1982).

[9] E. Mottola, Phys. Rev. D 31, 754 (1985).

[10] B. Allen and A. Folacci, Phys. Rev. D 35, 3771 (1987).

[11] M. Spradlin, A. Strominger and A. Volovich, hep-th/0110007.

[12] B. Greene, M. Parikh and J. P. van der Schaar, JHEP 0604, 057 (2006) [hep-th/0512243].

[13] I. Agullo, J. Navarro-Salas, G. J. Olmo and L. Parker, Phys. Rev. Lett. 101, 171301 (2008) [arXiv:0806.0034 [gr-qc]].

[14] A. M. Polyakov, arXiv:1209.4135 [hep-th].

[15] P. R. Anderson and E. Mottola, arXiv:1310.0030 [gr-qc].
[16] S. Singh, C. Ganguly and T. Padmanabhan, Phys. Rev. D 87, 104004 (2013) [arXiv:1302.7177 [gr-qc]].

[17] A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari and G. Shiu, Phys. Lett. B 737, 98 (2014) [arXiv:1403.6099 [hep-th]].

[18] P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].

[19] U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. 78, 2054 (1997) [astro-ph/9609169].

[20] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014) [arXiv:1303.5082 [astro-ph.CO]].

[21] M. Gerbino, A. Marchini, L. Pagano, L. Salvati, E. Di Valentino and A. Melchiorri, arXiv:1403.5732 [astro-ph.CO].

[22] K. M. Smith, C. Dvorkin, L. Boyle, N. Turok, M. Halpern, G. Hinshaw and B. Gold, arXiv:1404.0373 [astro-ph.CO].

[23] Y. Wang and W. Xue, arXiv:1403.5817 [astro-ph.CO].

[24] R. Adam et al. [Planck Collaboration], arXiv:1409.5738 [astro-ph.CO].

[25] M. J. Mortonson and U. Seljak, JCAP 1410, no. 10, 035 (2014) [arXiv:1405.5857 [astro-ph.CO]].

[26] H. Epstein and U. Moschella, arXiv:1403.3319 [hep-th].

[27] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, Cambridge Univ. Press, Cambridge (1982).

[28] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473 (2000) [astro-ph/9911177].

[29] L. Hui and W. H. Kinney, Phys. Rev. D 65, 103507 (2002) [astro-ph/0109107].

[30] K. Bhattacharya, S. Mohanty and R. Rangarajan, Phys. Rev. Lett. 96, 121302 (2006) [hep-ph/0508070].

[31] K. Bhattacharya, S. Mohanty and A. Nautiyal, Phys. Rev. Lett. 97, 251301 (2006) [astro-ph/0607049].