BIREFRINGENCE PHENOMENA REVISITED

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Abstract

The propagation of electromagnetic waves is investigated in the context of the isotropic and nonlinear dielectric media at rest in the eikonal limit of the geometrical optics. Taking into account the functional dependence $\varepsilon = \varepsilon(E, B)$ and $\mu = \mu(E, B)$ for the dielectric coefficients, a set of phenomena related to the birefringence of the electromagnetic waves induced by external fields are derived and discussed. Our results contemplate the known cases already reported in the literature: Kerr, Cotton-Mouton, Jones and magnetoelectric effects. Moreover, new effects are presented here as well as the perspectives of its experimental confirmations.

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1 Introduction

Electromagnetic waves in nonlinear media propagate according to Maxwell’s equations complemented by certain phenomenological constitutive relations linking strengths and induced fields \cite{1}. Depending on the dielectric properties of the medium and also on the presence of applied external fields, a variety of optical effects can be found. One of such an effects which has received significant attention of the scientific community in the last years is the birefringence phenomenon (or double refraction) \cite{2,3}. This effect takes place when the electromagnetic waves propagate in a medium which exhibits two distinct refractive indexes, in a same wave vector direction. Birefringence was first described by Rasmus Bartholin in 1669, who observed it in calcite (natural birefringence) \cite{4}. Moreover, the application of external fields in a medium with nonlinear dielectrics properties, may generate an artificial optical axis which turn the material into optically anisotropic (artificial birefringence) \cite{5}.

Birefringence induced by external electromagnetic fields in nonlinear media has been reported in the literature since a long time ago. Historically, Kerr effect was discovered in 1875 where
a relation between optics and electrostatic fields was found through the observation of the birefringence in glasses and similar observations in liquids [6]. Some years later, Cotton and Mouton observed an identical phenomenologically effect for magnetic fields [7]. In both cases, birefringence effect is squared with respect to the intensity of the external field applied and the optical axis is parallel to the direction of these fields. The possibility of a combined effect with a simultaneously action of electric and magnetic field has led to the discovery of the Jones effect in 1948 [8]. Jones showed that this effect could exist only in uniaxial media and it represents an additional case of the standard birefringence. Since then, Jones formalism has been considered as an useful tool in order to describe the light propagation in media with one or more possible distinct optical effects. Recently, new kinds of magnetoelectric optical effects has been experimentally and analytically investigated [9, 10]. Several applications of the birefringence effects are regarded in the papers [11].

The main point in this work is the search for new effects. In order to present a more general description of the birefringence phenomenon, we deal with isotropic and nonlinear dielectric media at rest characterized by the dielectric coefficients \( \varepsilon = \varepsilon(E, B) \) and \( \mu = \mu(E, B) \) in the eikonal limit of geometrical optics [12]. The analysis is restricted to local electrodynamics, where dispersive effects are neglected. Only monochromatic waves are considered and therefore there are no ambiguities with the velocity of the waves.

The paper is organized as follows. In the next section, through the shock waves formalism, we derive the eigenvalues equation related to the light propagation in dielectric nonlinear isotropic media. Cumming up, we get the dispersion relations to the possible propagating polarization modes. We derived a birefringence equation which presents three new optical birefringent phenomena. Each effect is analyzed separately. Such an equation also contemplates the well known five cases reported in the literature: Kerr, Cotton-Mouton, Jones and magnetoelectric effects. In Sec. 4 we discuss the possible experimental perspectives in order to measure these three new effects. In Sec. 5 we draw our conclusions. Aspects about effective geometry and polarization modes are considered in the Appendix. A Minkowskian spacetime described in an adapted Cartesian coordinate system is used throughout this work. The units are such that \( c = 1 \). The spacetime metric is denoted by \( \eta_{\alpha\beta} = \text{diag}(+1, -1, -1, -1) \). All quantities are referred to as measured by the geodetic observer \( V^\alpha = \delta^\alpha_0 \), where \( \delta^\alpha_\beta \) denotes the Kronecker tensor.

2 Nonlinear electrodynamics: the Fresnel generalized equation

Let us begin by mentioning that the dynamics of the electromagnetic Maxwell’s theory is described, in the absence of sources, by the set of coupled first order hyperbolic partial differential equations [2, 3]

\[
\partial_\beta P^{\alpha\beta} = 0, \quad \eta^{\alpha\beta\gamma\delta} \partial_\gamma F_{\alpha\beta} = 0,
\]

where the Levi-Civita tensor \( \eta^{\alpha\beta\gamma\delta} \) is defined as \( \eta^{0123} = +1 \) and the antisymmetric tensors \( F^{\alpha\beta} \) and \( P^{\alpha\beta} \) represent the total electromagnetic field [2]. Such fields can be expressed in terms of the intensities \( (E^\alpha, B^\alpha) \) and excitations \( (D^\alpha, H^\alpha) \) of the electromagnetic field in the following

Electromagnetic fields shall be here represented by 4-vectors space-like \( E^\alpha = (0, \vec{E}) \) and \( B^\alpha = (0, \vec{B}) \), where internal and external products are defined as \( \eta_{\alpha\beta}E^\alpha B^\beta = -\vec{E} \cdot \vec{B} \) and \( \eta^{\alpha\beta\gamma\delta}E_\beta V_\gamma B_\delta = (\vec{E} \times \vec{B})^\alpha \), respectively.
way

\[ F^{\alpha \beta} = V^\alpha E^\beta - V^\beta E^\alpha - \eta^{\alpha \gamma \beta \delta} V^\gamma B^\delta, \quad P^{\alpha \beta} = V^\alpha D^\beta - V^\beta D^\alpha - \eta^{\alpha \gamma \beta \delta} V^\gamma H^\delta. \] (2)

In the last equation, \( V^\alpha = \delta^\alpha_0 \) represents the velocity 4-vector in the framework of an observer at rest in the laboratory, where the fields are measured. According to the material media approach, the dielectric properties of the medium are determined by the matrices \( \varepsilon^{\alpha \beta}(E^\gamma, B^\gamma) \) and \( \mu^{\alpha \beta}(E^\gamma, B^\gamma) \). These matrices are related to the intensity fields and to the excitations by the phenomenological constitutive relations [3]

\[ D^\alpha = \varepsilon^{\alpha \beta} E^\beta, \quad H^\alpha = \mu^{\alpha \beta} B^\beta. \] (3)

In the nonlinear isotropic media context, the dielectric matrices may be written as [13]

\[ \varepsilon^{\alpha \beta} = \varepsilon h^{\alpha \beta} = (\varepsilon_0 + \varepsilon_{\text{ind}}) h^{\alpha \beta}, \quad \mu^{\alpha \beta} = \mu^{-1} h^{\alpha \beta} = (\mu_0 + \mu_{\text{ind}})^{-1} h^{\alpha \beta}, \] (4)

where \( h^{\alpha \beta} := \delta^{\alpha \beta} - V^\alpha V_\beta \) is the projector in the observer’s 3-dimensional space \( V^\alpha \). The dielectric coefficients are composed by two parts \( (\varepsilon_0, \mu_0) \) and \( (\varepsilon_{\text{ind}}, \mu_{\text{ind}}) \). The first one is a background contribution part (inherent to the material) and the second one represents an induced contribution. For the model considered here, the quantities \( \varepsilon_{\text{ind}} \) and \( \mu_{\text{ind}} \) depend on the external electromagnetic fields applied to the material in the following way

\[ \varepsilon_{\text{ind}}(E, B) = \varepsilon_1 E^2 + \varepsilon_2 B^2 + \varepsilon_3 (\vec{E} \cdot \vec{B}), \quad \mu_{\text{ind}}(E, B) = \mu_1 E^2 + \mu_2 B^2 + \mu_3 (\vec{E} \cdot \vec{B}), \] (5)

where \( \varepsilon_1, \varepsilon_2, \varepsilon_3, \mu_1, \mu_2 \) and \( \mu_3 \) are related to the nonlinear medium dependence caused by the external fields presence. These quantities correspond to small disturbances when compared to the background values \( \varepsilon_0 \) and \( \mu_0 \) and they shall be considered, here and so on, only in the first order.

In the classical hyperbolic partial differential equations theory, it is possible to analyze the propagation phenomenon by using the well-known Hadamard-Papapetrou method [14]. In such method, the propagation is considered by studying the evolution of the characteristic surfaces where the field is continuum. However, in general, the derivatives of the fields are not continuous. These surfaces can be understood as discontinuity surfaces which split the space-time into two globally separated domains. Let \( f^{(1)} \) and \( f^{(2)} \) be the two values of any function \( f \) taken in the two domains. The Hadamard discontinuity of the function \( f \) across a surface \( \Sigma \) is defined as \( [f(x)]_\Sigma := \lim_{\delta \to 0^+} \left\{ f^{(1)}(x + \delta) - f^{(2)}(x - \delta) \right\} \). In electrodynamics, for instance, the electromagnetic frontwave \( \Sigma \) is supposed to be a surface through which the discontinuities of the electromagnetic field (photons) propagate.

Taking into account the electromagnetic fields as smooth functions across \( \Sigma \) and their first derivatives with non-null discontinuities, it is possible to apply the procedure described by the Hadamard-Papapetrou technique and get the following discontinuity conditions for each point \( P \in \Sigma \) [15]

\[ [E^{\alpha}]_\Sigma = 0, \quad [\partial_\beta E^{\alpha}]_\Sigma = \varepsilon^{\alpha \beta} k_\beta, \quad [\partial_\beta D^{\alpha}]_\Sigma = (\varepsilon \varepsilon^{\alpha \beta} - \varepsilon E^{\alpha} E_\gamma e^\gamma - \varepsilon E^{\alpha} B_\gamma b^\gamma) k_\beta, \]  

\[ ^\star \text{Such a contribution is related to the external agents applied to the medium.} \]
Moreover, \( k \) by \( \omega \) frequency of the electromagnetic waves and the 3-dimensional wave vector are given respectively in order to describe the light rays propagation inside the material media.

\[
[B^\alpha]_{\Sigma} = 0, \quad [\partial_\beta B^\alpha]_{\Sigma} = b^\alpha k_\beta, \quad [\partial_\beta H^\alpha]_{\Sigma} = \left( \frac{1}{\mu} b^\alpha + \frac{\dot{\mu}}{\mu^2} B^\alpha E_\gamma e^\gamma + \frac{\ddot{\mu}}{\mu^2} B^\alpha B_\gamma b^\gamma \right) k_\beta, \tag{6}
\]

where we define \( \dot{f} := (1/E)(\partial f/\partial E) \) and \( \dot{f} := (1/B)(\partial f/\partial B) \). In the expressions presented above, the polarization vectors \( e^\alpha \) and \( b^\alpha \) are defined as \( e^\alpha := [\partial E^\alpha / \partial \Phi]_{\Sigma} \) and \( b^\alpha := [\partial B^\alpha / \partial \Phi]_{\Sigma} \).

Applying the boundary conditions (6) in the equations (1), one can obtain the eigenvalues equation \( Z^\alpha \beta e^\beta = 0 \), where the matrix \( Z^\alpha \beta \) can be written as [13]

\[
Z^\alpha \beta = \epsilon h^\alpha \beta - \epsilon E^\alpha E_\beta + v^{-1} \left[ \epsilon (\hat{q} \times \vec{B})_\beta E^\alpha + \frac{\dot{\mu}}{\mu^2} (\hat{q} \times \vec{B})^\alpha E_\beta \right] - \frac{v^{-2}}{\mu} (h^\alpha \beta + \hat{q}^\alpha \hat{q}_\beta)
+ \frac{v^{-2} \dot{\mu}}{\mu^2} \left[ (B^2 - (\hat{q} \cdot \vec{B})^2) h^\alpha \beta + B^2 \hat{q}^\alpha \hat{q}_\beta + B^\alpha B_\beta - (\hat{q} \cdot \vec{B}) \hat{q}^\alpha B_\beta - (\hat{q} \cdot \vec{B}) \hat{q}_\beta B^\alpha \right], \tag{7}
\]

where \( v := \omega / q \) is the phase velocity of the electromagnetic waves and \( q^\alpha := h^\alpha \beta k_\beta = (0, \vec{q}) \). Nontrivial solutions \( (e^\alpha \neq 0) \) can be obtained if, and only if, \( \det | Z^\alpha \beta | = 0 \), [15]. The determinant is calculated only through the 3-dimensional structure \( Z^\alpha \beta \). The reason is that the time components of such matrix are identically null. This result is known as Fresnel generalized equation [2][12] and it gives effective dispersion relations. These relations are a very useful tool in order to describe the light rays propagation inside the material media.

### 3 Birefringence: some new effects

The standard method to solve the Fresnel eigenvalues equation takes into account the eigenvector \( e^\alpha \) expansion in a convenient base in the 3-dimensional space [16]. A first aspect that arises here, despite the simplicity of this technique, is whether the set of chosen vectors is really linearly independent. In order to avoid this subtle aspect we shall apply here another method [17] which deal only with the algebraic structure of the matrix \( Z^\alpha \beta \). In this approach, the 3-dimensional determinant of this matrix can be written in a covariant way

\[
\det | Z^\alpha \beta | = -\frac{1}{6} (Z^\alpha \alpha)^3 + \frac{1}{2} Z^\alpha \alpha Z^\beta \gamma Z^\gamma \beta - \frac{1}{3} Z^\alpha \beta Z^\beta \gamma Z^\gamma \alpha. \tag{8}
\]

The Fresnel generalized equation for the model studied here is given by the product of two polynomial as follows

\[
\left[ v^2 \varepsilon - \frac{1}{\mu} \right] \left[ v^2 (\varepsilon + \varepsilon E^2) + v \left( \dot{\varepsilon} + \frac{\dot{\mu}}{\mu^2} \right) E B \sin \theta \sin \vartheta - \frac{\varepsilon}{\varepsilon \mu} E^2 \cos^2 \phi + \frac{\dot{\mu}}{\mu^2} B^2 \sin^2 \varphi - \frac{1}{\mu} \right] = 0, \tag{9}
\]

where \( \hat{q} \cdot \vec{E} = \cos \phi, \hat{q} \cdot \vec{B} = \cos \varphi, \vec{E} \cdot \vec{B} = \cos \theta \) and \( (\vec{E} \times \vec{B}) \cdot \hat{q} = \sin \theta \sin \vartheta \). We detach that \( \vartheta \) is the angle formed by the vector \( \hat{q} \) and the plane formed by the vectors \( \vec{E} \) and \( \vec{B} \).

One can notice from the last equation that there are, in general, two dispersion relations for the possible polarization modes which propagate in the medium. This phenomenon is known as birefringence or double refraction. In what follows, we denote the polarization modes mentioned above as ordinary and extraordinary rays. Hence, the first and the second polynomial in Eq.
correspond to the dispersion relations for the ordinary and extraordinary rays, respectively. We derive in the Appendix, using the dispersion relations previously commented, the effective metric structures as well as the polarization modes for each ray.

As one can check, the phase velocity for the ordinary ray is written as

\[ v_o^\pm = \pm \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \left[ 1 - \frac{1}{2} \left( \frac{\varepsilon_1}{\varepsilon_0} + \frac{\mu_1}{\mu_0} \right) B^2 - \frac{1}{2} \left( \frac{\varepsilon_2}{\varepsilon_0} + \frac{\mu_2}{\mu_0} \right) E^2 - \frac{1}{2} \left( \frac{\varepsilon_3}{\varepsilon_0} + \frac{\mu_3}{\mu_0} \right) EB \cos \theta \right]. \] (10)

The quantity \( v_o^\pm \) in last equation is isotropic, because it does not depend on the direction of propagation defined by the vector \( \hat{q} \). On the other hand, the phase velocity of the extraordinary ray is composed by an isotropic part \( v_o^\pm \) and an anisotropic one \( \delta v^\pm \). The phase velocity of the extraordinary ray \( v_e^\pm \) depends on the direction of propagation in the following way

\[ v_e^\pm = v_o^\pm + \delta v^\pm, \] (11)

where \( \delta v^\pm = \left[ -\frac{\varepsilon_3 \sin 2\theta \sin \vartheta}{4\varepsilon_0} \pm \frac{\varepsilon_1 \sin^2 \phi}{\varepsilon_0 \sqrt{\varepsilon_0 \mu_0}} \right] E^2 + \left[ -\frac{\mu_3 \sin 2\theta \sin \vartheta}{4\varepsilon_0 \mu_0^2} \pm \frac{\mu_2 \sin^2 \varepsilon}{\mu_0 \sqrt{\varepsilon_0 \mu_0}} \right] B^2 + \frac{1}{4\varepsilon_0} \left( \frac{\varepsilon_2}{\varepsilon_0} + \frac{\mu_1}{\varepsilon_0 \mu_0} \right) \sin \theta \sin \vartheta \pm \frac{\cos \theta}{2\sqrt{\varepsilon_0 \mu_0}} \left( \frac{\varepsilon_3}{\varepsilon_0} \sin^2 \phi + \frac{\mu_3}{\mu_0} \sin^2 \varphi \right) \right] EB. \] (12)

Note that there are three distinct solution for the phase velocities. One of them corresponds to the ordinary ray. The another ones, which are not symmetric with respect to the direction of propagation, correspond to the extraordinary ray and are expected to represent two different velocities in the same direction.

According to our previous discussion, \( \varepsilon_i \) and \( \mu_i \) represent small disturbances when compared to \( \varepsilon_0 \) and \( \mu_0 \). Hence, a straightforward comparison between the equations (10) and (12) shows that \( |v_o| \gg |\delta v| \). Consequently \( v_o^+ > 0 \) and \( v_o^- < 0 \). This means that the phase velocity presented by the extraordinary ray, when it propagates in the direction defined by the vector \( (\hat{q}) \), is different from that one related to the propagation in the opposite direction defined by \( (-\hat{q}) \). Besides, since the ordinary ray propagates isotropically, birefringence phenomena will take place in any possible direction, except in the case where the phase velocity of both rays coincide.

Let us remind the usual refraction index definition \( n := v^{-1} \). Let us also define \( \delta n^\pm := (n_o - n_e)^\pm \), where \( n_o \) and \( n_e \) correspond to the refraction indexes of the medium related to the ordinary and extraordinary rays, respectively. From this point one can get \( \delta n^\pm \approx \varepsilon_0 \mu_0 \delta v^\pm \). Using these definitions and Eq. (12) one derive the following equation

\[ \delta n^\pm \cong -\frac{\varepsilon_3 \mu_0 \sin 2\theta \sin \vartheta}{4} E^2 - \frac{\mu_3 \sin 2\theta \sin \vartheta}{4\mu_0} B^2 \pm \frac{\mu_3}{2\sqrt{\mu_0}} \varepsilon_0 \cos \theta \sin^2 \varphi EB \]

\[ \pm \frac{\varepsilon_1 \sqrt{\mu_0} \sin^2 \phi}{\sqrt{\varepsilon_0}} E^2 \pm \frac{\mu_2 \sqrt{\varepsilon_0} \sin^2 \varphi}{\sqrt{\mu_0}} \frac{B^2 - \varepsilon_2 \mu_0 \sin \theta \sin \vartheta EB}{2} \]

\[ + \frac{\mu_1}{\mu_0} \sin \theta \sin \vartheta EB \pm \frac{\varepsilon_3 \sqrt{\mu_0} \cos \theta \sin^2 \phi}{2} EB. \] (13)

The magnitude of the birefringence phenomena is expressed by this last equation for nonlinear dielectric isotropic media. The last five terms in Eq. (13) represent the known birefringence
effects: Kerr [6], Cotton-Mouton [7], linear magnetoelectric [10] and Jones [8]. However, three new optical effects are represented by the three first terms. They emerge only due to the fact that we deal with the dielectric coefficients in the form (1) and (5).

Let us discuss about the three first terms in the r.h.s. of Eq. (13). The first one is an electric-optical contribution. One can see that an analogy between this term and the Kerr effect is straightforward, once the squared dependence on the electric field is observed in both of them. The second and the third ones are magneto-optical and magnetoelectric contributions, respectively. They can be understood as the analogue terms of the Cotton-Mouton and Jones effects, respectively. However, there is a difference between these three terms mentioned above and their analogue ones. If one consider the first term in r.h.s. for example, it is possible to observe that birefringence effect vanishes when the propagation is parallel to the plane formed by the electromagnetic fields and also when such fields are parallel. On the other hand, in the Kerr effect case, the direction of the magnetic field does not affect the measure of the electric-optical birefringence. The same observation can be extended for analogue cases of Cotton-Mouton and Jones effects. Furthermore, the birefringence magnitude will be maximizes when \( \theta = \pi/4 \) (Kerr and Cotton-Mouton analogue terms) and \( \theta = 0 \) (Jones analogue term).

For the sake of completeness let us comment that the other five terms of Eq. (13) are nothing else but the well known birefringence effects already commented above. For example, the fourth and fifth terms in the r.h.s. correspond to Kerr and Cotton-Mouton effects, respectively. Observe that the intensity of such effects will be maximizes when the propagation is perpendicular to the direction defined by the applied field and it will be null if propagation is parallel to the electric and magnetic fields. In addition, the sixth and seventh terms in the r.h.s. correspond to the linear magnetoelectric birefringence. They achieve their maximum value in the crossed external fields situation (\( \theta = \pi/2 \)) and their minimum (birefringence disappear) with parallel external fields (\( \theta = 0 \)). Moreover, the last term correspond to Jones effect with maximum value for crossed fields (\( \theta = 0 \)) and null for (\( \theta = \pi/2 \)).

It should be remarked that magnetoelectric birefringence is composed by a pair of terms. The first pair (proportional to \( \sin \theta \)) represents magnetoelectric terms. The second pair (proportional to \( \cos \theta \)) is composed by the Jones effect and its analogue term (new one). Hence, as far as the angle \( \theta \) between the external electromagnetic fields varies (\( 0 \leq \theta \leq 2\pi \)), one of these pairs decreases while the another one increases. Therefore, it is possible to infer from the previous analysis that the magnetoelectric birefringence will always take place.

4 Experimental tests, a perspective

We present in this section a brief perspective about experimental tests, taking into account the whole birefringence scenario, including the three new terms presented in the Sec. 3. Some aspects of a consistent experimental framework about birefringence tests can be found in [9]. Such a framework incorporates a successful experimental description of birefringence phenomenon.

\[ \text{Electric-optical and magneto-optical birefringence occurs when the birefringence depends on the magnitude of the applied external electric and magnetic fields, respectively.} \]

\[ \text{The Kerr analogue term depends on the direction of the magnetic field, but it does not depends on its magnitude.} \]
However, this subject is very much extensive and a deep approach here would not be compatible with the context of this work.

In order to keep a better correspondence with the experimental scope, we begin by rewriting the quantity $\delta n$, described by the relation (13), in terms of the birefringence magnitude factor $K(\lambda)$. For example, the $\delta n$ contribution for the Kerr, Cotton-Mouton, Jones and linear magnetoelectric effects may be written as follows

$$
\delta n_K = K_K(\lambda) \lambda \sin^2 \phi E^2, \quad \delta n_{CM} = K_{CM}(\lambda) \lambda \sin^2 \varphi B^2,
$$

$$
\delta n_J = K_J(\lambda) \lambda \cos \theta \sin^2 \phi E B, \quad \delta n_{ME} = K_{ME}(\lambda) \lambda \sin \theta \sin \vartheta E B,
$$

where $\lambda$ is the wavelength of the light propagating in the medium. A comparison between relations (13), (14) and (15) allows one to write the birefringence magnitude factors presented in the last equations, in the form

$$
K_K = \frac{\varepsilon_1 \sqrt{\mu_0}}{\lambda \sqrt{\varepsilon_0}}, \quad K_{CM} = \frac{\mu_2 \sqrt{\varepsilon_0}}{\lambda \sqrt{\mu_0}}, \quad K_J = \frac{\varepsilon_3 \sqrt{\mu_0}}{2 \lambda \sqrt{\varepsilon_0}}, \quad K_{ME} = \frac{\mu_0 \varepsilon_2}{\lambda}.
$$

All these effects represented in (14) and (15) has already been experimentally confirmed [9]. However, the search for material media where birefringence phenomena are detectable with good accuracy is not quite trivial. Usually, for a particular effect, a set of measurements are performed for different materials. The effects are better observed in the medium which presents a greater value for the quantity $K(\lambda)$.

Concerning to Jones and magnetoelectric birefringence for example, the measurements were performed for a set of materials and a table relating these materials and their corresponding values for the quantities $K_J$ and $K_{ME}$ can be found in [9]. A sample of methylcyclopentadienyl-Mn-tricarbonyl presents the most relevant results: $K_J = 47 \times 10^{-12} \text{ V}^{-1} \text{ T}^{-1}$ and $K_{ME} = 51 \times 10^{-12} \text{ V}^{-1} \text{ T}^{-1}$. As the authors have detached in [9], such values confirm the validity of the equations (15), proving the existence of Jones and magnetoelectric birefringence. Furthermore, results for the Kerr and Cotton-Mouton effects agree reasonably with the literature values and the validity of the equations (14) has also been confirmed. In particular, a sample of nitrobenzene shows the following acceptable values $K_K = 3, 9 \times 10^{-12} \text{ m V}^{-2}$ and $K_{CM} = 2, 1 \times 10^{-2} \text{ m}^{-1} \text{ T}^{-2}$.

As pointed out in Sec. (3), the three new theoretical effects predicted by the approach considered in this paper represent the analogue terms of Kerr, Cotton-Mouton and Jones effects. We shall denote their corresponding birefringence magnitude factors as $K_{KA}$, $K_{CMA}$ and $K_{JA}$, respectively. In the same way as performed above, one can write these quantities as follows

$$
K_{KA} = \frac{\varepsilon_2 \mu_0}{4 \lambda}, \quad K_{CMA} = \frac{\mu_3}{4 \lambda \mu_0}, \quad K_{JA} = \frac{\mu_5 \sqrt{\varepsilon_0}}{2 \lambda \sqrt{\mu_0}}.
$$

Concerning the experimental observations of these effects, one has to take into account that the well understanding about Jones birefringence and molecular structure is not complete yet. Hence pure molecular liquids are good candidates to perform measurements of the quantity $K_{JA}$ presented in the last equation. Notwithstanding, methylcyclopentadienyl-Mn-tricarbonyl seems to be a remarkable sample when the birefringence effect (to be measured) is proportional to $EB$. For these reasons we believe that it presents good perspectives in order to confirm Jones
analogue effect. By similar reasons, samples of nitrobenzene seem to play the role as good candidates in order to perform measurements of the Kerr and Cotton-Mouton analogue effects.

As a last comment we detach that very technical issues are related to the descriptions and details about the experimental setup to observe the new effects described here and, as we already stressed, considerations about the merit of these questions would not be in the context of this work.

5 Conclusions and discussions

The propagation description of the electromagnetic monochromatic waves was considered in the context of isotropic nonlinear dielectric media in the eikonal limit of the geometric optics. By using the Hadamard-Papapetrou technique, the eigenvalues problem was presented and solved for isotropic material media represented by the dielectric coefficients $\varepsilon = \varepsilon(E, B)$ and $\mu = \mu(E, B)$. We found the dispersion relations and the effective optical metric structure related to each possible polarization mode.

It is worth noting that the birefringence study we performed, taking into account the dielectric coefficients mentioned above, produced a new result which is represented in Eq. (13). According to this equation, there are three new optical effects unknown until now. One of them presents a squared dependence on electric field and for this reason it can be understood as an analogue term of the Kerr effect. By the same way, the another two terms can be interpreted as the analogues ones of the Cotton-Mouton and Jones effects. Furthermore, Eq. (13) also contemplates the five well known effects presented in the literature: Kerr, Cotton-Mouton, Jones and linear birefringence magnetic-electric.

We have also addressed in Sec. (4) the possibility of performing experimental tests in order to confirm the three new theoretical effects presented in section Sec. (3). The scenario for pure molecular liquids seems to be promising. In particular, samples of methylcyclopentadienyl-Mn-tricarbonyl are good candidates in order to measure the Jones analogue effect. Nitrobenzene seems to be good samples in the measurements of the Kerr and Cotton-Mouton analogue effects.

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Appendix

We present in this Appendix two additional aspects related with the model treated here: the effective metric for the ordinary and extraordinary rays, as well as a brief description of the
polarization modes in the context of the Kerr effect.

Effective geometry
Let us begin mentioning that the development of methods which enable one to test several
cinematic aspects of General Relativity in the laboratory has been explored in some areas of
physics. For example, in nonlinear electrodynamics it has been considered as a possible scenario
to construct analogue models of General Relativity, in the context of nonlinear Lagrangian and
nonlinear material media. It is based on the fact that the trajectory of photons can be described
in terms of null geodesics in an effective geometry $G^{\alpha\beta}$, also known as optical geometry (see
[18, 19] for more information about effective geometry approach in the context of nonlinear
electrodynamics).

For several physical configurations, the dispersion relations can be written in a suggestive
way as $G^{\alpha\beta} k_{\alpha} k_{\beta} = 0$. In the model considered here, we write the dispersion relation for the
ordinary ray in the form

$$\omega^2 \varepsilon \mu - q^2 = 0 = (G_o)^{\alpha\beta} k_{\alpha} k_{\beta}, \quad (18)$$

where

$$(G_o)^{\alpha\beta} = \eta^{\alpha\beta} - (1 - \varepsilon \mu) V^{\alpha} V^{\beta}, \quad (19)$$

represents the optical metric related to the ordinary ray and corresponds to the Gordon’s metric
[20]. In the same way, for the extraordinary ray we get

$$(G_e)^{\alpha\beta} = (G_o)^{\alpha\beta} + \left( \varepsilon \mu E^2 + \varepsilon \mu B^2 \right) V^{\alpha} V^{\beta} - \frac{\varepsilon}{\varepsilon - \varepsilon_{\mu}} E^\alpha E^\beta - \frac{\mu}{\mu - \mu_{\mu}} B^\alpha B^\beta + \frac{1}{2} \left( \varepsilon \mu + \frac{\mu}{\mu_{\mu}} \right) (E \times B)^{(\alpha} V^{\beta)}. \quad (20)$$

Observe that such an effective geometries indeed correspond to the background Minkowskian
metric deviations. Hence, the nonlinear properties of the medium do affect the trajectories of
the light ray.

Polarization: Kerr effect
For any one of the cases cited in the Sec. (3), one can get the corresponding polarization vector $(e^\alpha, b^\alpha)$ by leading any specific solution in the generalized Fresnel equation. We
consider the following electromagnetic configuration \{$\varepsilon = \varepsilon(E); \mu = \mu_0; \hat{E} = \hat{x}; \hat{B} = \hat{y};$
$\hat{q} = \hat{z}$\}, which is the source of the Kerr effect with crossed external electromagnetic fields
and a perpendicular propagation to the plane formed by such fields. Hence, by writing the
polarization vector in the 3-dimensional representation $\hat{e} = c_1 \hat{x} + c_2 \hat{y} + c_3 \hat{z}$ and considering
the generalized Fresnel equation, together the dispersion relation for each ray, one can
find the following polarization vectors

$$(\hat{e}_o, \hat{b}_o) = (\hat{y}, -\hat{x}) = (\hat{B}, -\hat{E}), \quad (\hat{e}_e, \hat{b}_e) = (\hat{x}, \hat{y}) = (\hat{E}, \hat{B}), \quad (21)$$

where we got the vectors $\hat{b}$ by using the relation $\hat{b} = \omega^{-1}(\hat{q} \times \hat{e})$. The result presented
above can also be written in the matrix form

$$\begin{pmatrix} \hat{e}_o \\ \hat{b}_o \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \hat{e}_e \\ \hat{b}_e \end{pmatrix}. \quad (22)$$
Observe that in the Kerr effect, for example, one can get the corresponding polarization vectors for the ordinary ray in terms of the polarization vectors related to the extraordinary ray by using a rotation in the plane formed by the external electromagnetic fields. The extension of the procedure considered here to any electromagnetic configuration presented in the Sec. (3) is straightforward.

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