Quantum correlations of an atomic ensemble via a classical bath

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(Dated: November 13, 2018)

PACS numbers: 42.50.Fx, 42.50.Ar, 42.50.Lc

Somewhat surprisingly, quantum features can be extracted from a classical bath. For this, we discuss a sample of three-level atoms in ladder configuration interacting only via the surrounding bath, and show that the fluorescence light emitted by this system exhibits non-classical properties. Typical realizations for such an environment are thermal baths for microwave transition frequencies, or incoherent broadband fields for optical transitions. In a small sample of atoms, the emitted light can be switched from sub- to super-poissonian and from anti-bunching to super-bunching controlled by the mean number of atoms in the sample. Larger samples allow to generate super-bunched light over a wide range of bath parameters and thus fluorescence light intensities. We also identify parameter ranges where the fields emitted on the two transitions are correlated or anti-correlated, such that the Cauchy-Schwarz inequality is violated. As in a moderately strong baths this violation occurs also for larger numbers of atoms, such samples exhibit mesoscopic quantum effects.

Initiated by Dicke [1], ensembles of few-level emitters interacting collectively with an environmental reservoir have been shown to be a source for many remarkable effects and applications [1, 2, 3, 4, 5, 6, 7, 8, 9]. In the recent past, this interest was renewed by the possible applications of such samples to quantum communications and logic devices. For example, the production of non-classical light has been a subject of several recent experiments [11, 12]. On the other hand, it was demonstrated that long-time entanglement between two arbitrary qubits can be generated if they interact with a common bath [13]. Thus, quantum features can be induced through the interactions with a classical electromagnetic field. This is not obvious, as, usually, it is believed that an interaction with a large environmental reservoir leads to decoherence. Thermal light may also produce effects like ghost imaging or sub-wavelength interference [14], which otherwise are known to occur for entangled light [15]. These results are of especial interest, as non-classical driving fields are often hard to produce experimentally with adequate intensities. Therefore, schemes which allow to extract quantum features from atoms in an otherwise classical external setup are highly desirable.

In this Letter, we demonstrate that such a conversion scheme may be implemented with an ensemble of atomic few-level systems subject to a classical bath. For this, we discuss a three-level setup in ladder configuration, and show that the fluorescence light emitted spontaneously on the two transitions has non-classical properties. The bath could e.g. be a thermal bath for atoms with microwave transition frequencies, or incoherent broadband driving for the optical frequency region. In particular, we demonstrate that in a small sample of atoms, the mean number of atoms in the sample allows to switch the light emitted on one of the transitions from sub- to super-poissonian statistics, and from anti- to super-bunching. Here, both super- and anti-bunched light with super-poissonian statistics can be produced. Larger samples, on the other hand, can be used to generate super-bunched light over a wide range of bath parameters, thus enabling a control of the intensity of the strongly correlated light. We also identify parameter ranges where the fields emitted on the two transitions are correlated or anti-correlated. As an application for this, we show that the Cauchy-Schwarz inequalities are violated for a moderately strong reservoir and large samples, thus demonstrating a mesoscopic quantum effect.

The basic element of our study is a sample of $N$ identical non-overlapping three-level radiators which interact with a classical reservoir, see Fig. 1. The emitters are located within a volume with linear dimensions smaller than the relevant emission wavelengths $\lambda_{12}, \lambda_{23}$, and densities low enough to avoid collisions (Dicke model) $|1\rangle, |2\rangle, |3\rangle, |4\rangle$. The excited atomic level $|1\rangle$ ($|2\rangle$) spontaneously decays to the state $|2\rangle$ ($|3\rangle$) with a decay rate $2\gamma_1$ ($2\gamma_2$). The only external driving is via the surrounding bath, which induces transitions among the atomic levels. In the microwave region, the bath could be a thermal bath, where the rates are proportional to the mean thermal photon number at

![FIG. 1: The system setup. A sample of $N$ atoms, each with three atomic states in ladder configuration, is confined to a region small as compared to the emission wavelengths $\lambda$. The whole setup is placed in a classical bath. The excited states decay spontaneously with rates $2\gamma_1, 2\gamma_2$, giving rise to fluorescence lights with non-classical features.](image-url)
the corresponding transition frequencies. In the optical
frequency region, thermal excitations are negligible, and
the bath can be realized by a pseudo-thermal bath, i.e.
broadband incoherent driving fields perpendicular to the
observation direction. Then, the rates depend on the field
strength of the incoherent fields. In the usual mean-field,
Born-Markov and rotating-wave approximations, the sys-
tem is described by the following master equation [1, 2]:
\[
\dot{\rho}(t) = -\gamma_1(1 + \bar{n}_1)|S_{12}, S_{21}\rho\rangle - \gamma_2(1 + \bar{n}_2)|S_{23}, S_{32}\rho\rangle \\
- \gamma_1\bar{n}_1|S_{21}, S_{12}\rho\rangle - \gamma_2\bar{n}_2|S_{32}, S_{23}\rho\rangle + \text{h.c.}.
\]
(1)

Here, an overdot denotes differentiation with respect to
time. For thermal baths, \(\bar{n}_i = \exp(\beta\hbar\omega_{i,i+1}) - 1\)^{-1}
is the mean thermal photon number at transition fre-
quency \(\omega_{i,i+1} = \omega_i - \omega_{i+1}\) and for temperature \(T\), where
\(\beta = (k_B T)^{-1}\) with \(k_B\) as the Boltzmann constant. For
pseudo-thermal baths, \(\bar{n}_i = R_{i,i+1}d_{i,i+1}^2/(\gamma_i\hbar^2)\), where
\(R_{i,i+1}\) describes the strength of the incoherent pump-
ing [3]. It is important to note that Eq. (1) contains
collective atomic operators \(S_{ij} = \sum_{k=1}^{N}d_{k,i}^† d_{k,j}\), which
describe populations for \(i = j\), transitions for \(i \neq j\), and
which obey the commutation relation \([S_{ij}, S_{ij'}]\) =
\(\delta_{jj'}\delta_{ii'} - \delta_{ii'}\delta_{jj'}\) \((i, j \in \{1, 2, 3\})\) [3].

The steady-state limit of Eq. (1) can conveniently be
evaluated with the help of coherent atomic states
\(|N, n, m\rangle\) for the su(3) algebra, which denote a symmet-
ic collective state of \(N\) atoms with \(n\) atoms in bare state
\(|1\rangle\), \(m - n\) in bare state \(|2\rangle\), and \(N-m\) atoms in bare state
\(|3\rangle\) with \(0 \leq n \leq N\), \(n \leq m \leq N\) [4, 5, 6]. The diagonal elements
\(P_{nm} = \langle N, n, m|\rho_{ss}|N, n, m\rangle\) of the steady-state
density operator \(\rho_{ss}\) evaluate to:
\[
P_{nm} = (1 - \eta_2)\eta_1^n\eta_2^m \\
\times \left[ 1 - (\eta_1\eta_2)^N \right]^{-1} \left[ 1 - \eta_1\eta_2 \right]^{-1} \left[ 1 - \eta_1\eta_2 \right]^{-1} \\
= \left(1 - \eta_2\right)\eta_1^n\eta_2^m \\
\times \left[ 1 - (\eta_1\eta_2)^N \right]^{-1} \left[ 1 - \eta_1\eta_2 \right]^{-1} \left[ 1 - \eta_1\eta_2 \right]^{-1},
\]
(2)

with \(\eta_i = \bar{n}_i/(1 + \bar{n}_i)\), \((i \in \{1, 2\})\). From Eq. (2), the
atomic expectation values can easily be evaluated.

We now turn to our main interests in this study, the
coherence properties of the collective fluorescent light
generated on the transitions \(|1\rangle \rightarrow |2\rangle\) and \(|2\rangle \rightarrow |3\rangle\). The
photons emitted on the two transitions are distinguish-
able by their polarizations and frequencies, and can be
detected e.g. by a pair of single-photon detectors or by
atomic state detection [4, 5, 6]. The second-order coherence
function is defined as [7, 8].
\[
g_{ij}^{(2)}(\tau) = \frac{\langle J_i^†(t)J_j^†(t + \tau)J_j(t + \tau)J_i(t) \rangle}{\langle J_i^†(t)J_i(t) \rangle \langle J_j^†(t)J_j(t) \rangle},
\]
(3)

where \(i, j \in \{1, 2\}\) with \(J_1 = S_{21}\) and \(J_2 = S_{32}\). The
quantity \(g_{ij}^{(2)}(\tau)\) can be interpreted as a measure for the
probability for detecting one photon emitted on transi-
tion \(i\) and another photon emitted on transition \(j\) with
time delay \(\tau\). \(g_{ij}^{(2)}(0) < 1\) characterizes sub-poissonian,
\(g_{ij}^{(2)}(0) > 1\) super-poissonian, and \(g_{ij}^{(2)}(0) = 1\) poissonian
photon statistics. \(g_{ij}^{(2)}(\tau) > g_{ij}^{(2)}(0)\) is the condition for
photon anti-bunching, whereas \(g_{ij}^{(2)}(\tau) < g_{ij}^{(2)}(0)\) means
bunching. We further define super-bunching as bunching
with \(g_{ij}^{(2)}(0) > 2\) [13, 20]. More specific, correlation
functions with \(i = j\) describe the photon statistics of the fluo-
rescence light emitted on a single atomic transition, and
\(g_{ij}^{(2)}(0)\) the cross-correlations between the photon emis-
sion on two different transitions. We also need to consider
the fluorescence intensities \(g_{11}^{(1)}(0) = \langle J_1^† J_1 \rangle_s\) of the two
transitions. For example, applications may require partic-
sularly strong or weak non-classical fields. On the other
hands, in the microwave region, the signal from the sample
competes with noise from the surrounding heat bath
proportional to \(\bar{n}_1\), which especially for small samples
of atoms with low signal can render experimental veri-
fications difficult. For bath parameters \(0 < \eta_1, \eta_2 < 1\)
and larger samples \(N \gg 1\) such that \(\eta_1^N \approx 0\), one has
\(g_{11}^{(1)}(0) \approx \bar{n}_1 \bar{n}_2/[1 + \bar{n}_1/(1 + \bar{n}_2)], \ g_{22}^{(1)}(0) \approx \bar{n}_2N\).
Thus \(g_{11}^{(1)}(0)\) does not depend on \(N\) explicitly, while
\(g_{22}^{(1)}(0)\) increases linearly with \(N\). In the strong field limit
\((\eta_1, \eta_2 \rightarrow 1)\), one has \(g_{11}^{(1)}(0) = N(3 + N)/12 \sim N^2\).

The first results are shown in Fig. 2 where the corre-
lation function \(g_{11}^{(2)}(0)\) of the light emitted on transi-
tion \(|1\rangle \rightarrow |2\rangle\) is plotted against \(\eta_1\) and \(\eta_2\) for a sample
of \(N = 150\) atoms. It can be seen that photons with
super-poissonian statistics are generated on this transition
for moderate baths. In this region, the emitted light is
super-bunched except for small values of \(\eta_2\), where the
light is anti-bunched. This range of \(\eta_2\) for anti-bunching
depends on the number of atoms \(N\) and \(\eta_1\). In Fig. 2 for
\(\eta_1 < 0.6\), anti-bunching requires \(\eta_2 < 0.01\). For smaller
samples, however, the range for anti-bunching increases.
If \(\eta_2\) is small, then almost all atoms are in the ground
state \(|3\rangle\) due to collective effects, and on average at most
one atom gets excited to \(|1\rangle\) to emit light contributing to
\(g_{11}^{(2)}(0)\). Thus the light is anti-bunched. For higher \(\eta_2\),
more atoms can be excited to \(|1\rangle\) simultaneously, and the

FIG. 2: Plot of the second-order correlation function \(g_{11}^{(2)}(0)\)
against bath parameters \(\eta_1, \eta_2\). The number of atoms in the
sample is \(N = 150\).
light is bunched. As the super-bunched photons are produced over a wide range of values for \( \eta \), the intensity of the generated light can be controlled via the bath parameters \( \bar{n}_1, \bar{n}_2 \) from very weak up to intense flux. In other words, by modifying the reservoir characteristics, we can obtain a low or an intense flux of strongly correlated photons. This setup is particularly suitable for microwave transitions, as then thermal baths with high values for \( \eta \) can be achieved, and larger samples of atoms allow for a good signal-to-noise ratio (SNR). An optical realization, however, is also possible. We now turn to the “channel” around \( \eta_1 = 1, \eta_2 \geq 1 \) in Fig. 2. For \( \eta_1 = \eta_2 = 1 \), one finds \( g_{11}^{(2)}(0) = 8(N - 1)(N + 4)/[5N(N + 3)] \), which for \( N \rightarrow \infty \) goes to 8/5. The corresponding limit for \( \eta_1 = 1, \eta_2 > 1 \) yields 6/5. This “channel” can be understood by noting that for these parameters the sample acts collectively, i.e. \( G_{\tau}^{(1)}(0) \propto N^{2i}, (i \in \{1, 2\}) \), resulting in a close to coherent photon statistics that corresponds to a superfluorescent atomic sample \( 1 \). It should be emphasized here that while a thermal reservoir or a direct incoherent pumping of the transitions only admits for values \( 0 < \eta_1, \eta_2 \leq 1 \), we have also used larger values for these parameters in Figs. 3(b). Such situations may occur if additional driving fields are applied to the sample of atoms, e.g. incoherent repumping from the lower to the upper atomic states \( 2 \). We stress, however, that our main results are obtained without such driving.

In the following, we discuss the special case of a small collection of atoms. This is of particular interest for an experimental verification in the optical region, whereas in the microwave region the small number of atoms makes it hard to obtain a decent SNR against the thermal background. In Fig. 4(a), we show \( g_{11}^{(2)}(0) \) against the number of atoms in the sample for different values of \( \eta_1 = \eta_2 \). Consider, for example, an experimental setup with an atomic beam passing through a low quality cavity. Then, depending on the mean number of atoms which are simultaneously inside the cavity and on the bath parameters, switching between sub- and super-poissonian statistics or anti-bunching and super-bunching of the emitted light can be observed \( 3 \). In the figure, squares (triangles) denote anti-bunching (bunching) of the emitted photons. Note that together with a super-poissonian statistics, both bunching and anti-bunching can be observed. Only collective states \( |N, 1, m\rangle \) with a single atom in state \( |1\rangle \) may lead to anti-bunching. All other states \( |N, n > 1, m\rangle \) contribute to bunching. The total system behavior depends on the ratio of these two contributions. With increasing bath strength and increasing number of atoms, more bunching states \( |N, n > 1, m\rangle \) are available and populated, such that the switching from anti-bunching to super-bunching occurs. Some examples for \( g_{11}^{(2)}(\tau) \) versus delay time \( \tau \) are shown in Fig. 4(b). Curve (i) shows super-bunching for \( N = 15 \) and \( \eta_1 = \eta_2 = 0.6 \). Starting from an initial value close to 4, the correlation function drops rapidly to unity with increasing \( \tau \). Example (ii) shows anti-bunching with super-poissonian photon statistics and large intermediate values of \( g_{11}^{(2)}(\tau) \). The maximum value of the correlation function can be further increased, however, at the cost of intensity. As reference, (iii) shows an evolution for the single-atom case.

The cross-correlations \( g_{i\neq j}^{(2)}(0) \) also show non-classical behavior. For an atomic sample in a weak bath, \( g_{12}^{(2)}(0) \) is much larger than unity as shown in Fig. 3 indicating super-poissonian light statistics, which is accompanied by strong correlation between the fluorescence light radiated on both atomic transitions, i.e. cross super-bunching. The reason is that then atoms which decay from \( |1\rangle \) to \( |2\rangle \) also decay further to \( |3\rangle \) with a high probability. For stronger baths, however, larger samples exhibit bunched sub-poissonian light. Then \( \lim_{\eta_1, \eta_2 \rightarrow 1} g_{12}^{(2)}(0) = 4(N + 2)(N + 4)/[5N(N + 3)] \), with limit \( 4/5 < 1 \) for \( N \rightarrow \infty \). In this case, atoms decaying from \( |1\rangle \) to \( |2\rangle \) are repumped by the bath rather than decaying further to \( |3\rangle \).

The other cross-correlation, \( g_{23}^{(2)}(0) \), is below unity for a weak bath, as a transition \( |2\rangle \rightarrow |3\rangle \) cannot directly be followed by a transition \( |1\rangle \rightarrow |2\rangle \) without an extra excitation. For small samples \( (N < 8) \), for or medium samples \( (N \sim 40) \) with strong bath on transition 1 \( (\eta_1 \rightarrow 1) \), the emitted light is anti-bunched, thus showing collective cross anti-bunching. If both transitions
are driven strongly, one finds cross anti-bunching with
\( G_z^{(1)}(0) \propto N^2 \). For larger samples and smaller \( \eta_1 \), the light is anti-bunched for low values of \( \eta_2 \), but switches to bunched light with increasing \( \eta_2 \).

As an application for the non-classical features, we now show that the light emitted from the sample of atoms violates the Cauchy-Schwarz inequalities (CSI) [21]. The CSI are violated if

\[
\chi_{(2)} = g_{12}^{(2)}(0)g_{22}^{(2)}(0)\left[ g_{12}^{(2)}(0) \right]^2 < 1,
\]

i.e., if the cross-correlations between photons emitted on two different transitions are larger than the correlation between photons emitted from the individual levels. Fig. 5 shows the violation of the CSI function for moderately strong baths. Within the Dicke model, this violation is present for any number of atoms in the sample, thus demonstrating a mesoscopic quantum effect. In addition, \( \chi_1 \) is always smaller than unity for \( N \leq 3 \) and the entire range of \( 0 \leq \eta_1, \eta_2 \leq 1 \) \((\lim_{\eta_1, \eta_2 \rightarrow 1} \chi_1 = 4/(N-1)/(N+2)^2)\), while \( \chi_2 \) is larger than unity for any number of atoms and for any values of \( \eta_1, \eta_2 \). The CSI violation can best be observed in the optical region, as low values of \( \eta_1 \) are favorable, and as it is more difficult to obtain a decent SNR in the microwave region.

In summary, we have demonstrated quantum features in the fluorescence light of a sample of atoms driven only by a surrounding classical bath. We discussed both thermal baths with microwave atomic transition frequencies and pseudo-thermal baths for optical transition frequencies as realizations of the bath. For small samples, a change of the mean number of atoms in the sample induces sensitive switching between sub- or super-poissonian statistics and anti-bunching or super-bunching of the light emitted on one of the transitions. For appropriate bath parameters, even mesoscopic samples exhibit anti-bunching. As an application, we have shown that the Cauchy-Schwarz inequalities are violated in our system over a wide range of parameters.

\[ \text{FIG. 5: The Cauchy-Schwarz parameters } \chi_1 \text{ shown versus bath parameters } \eta_1 = \eta_2 \equiv \eta. \text{ The solid, long-dashed and short-dashed curves are plotted for } N = 4, 10 \text{ and 150.} \]

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[20] L. Davidovich, Rev. Mod. Phys. 68, 127 (1996).
[21] J. F. Clauser, Phys. Rev. D 9, 853 (1974); R. Loudon, Rep. Prog. Phys. 43, 58 (1980).