DepQBF: A Dependency-Aware QBF Solver
(System Description)

Florian Lonsing and Armin Biere

Institute for Formal Models and Verification (FMV)
Johannes Kepler University, Linz, Austria
http://fmv.jku.at

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This Talk:

- DepQBF 0.1 system overview.
- Selected features: restarts, removal of learnt constraints.
- Experimental evaluation.

http://www.qbflib.org/index_eval.php
Overview

DepQBF:
- Input: QBFs in Prenex-CNF (PCNF).
- QDPLL with conflict-driven clause and solution-driven cube learning.
- Analysis of variable dependencies.

Variable Dependencies in QBFs:
- PCNF $Q_1 Q_2 \ldots Q_n. \phi$: linearly ordered sets of quantified variables.
- Left-to-right prefix order: strong dependencies.
- DepQBF: relaxing prefix order by dependency schemes.

Example

Quantifier ordering matters:
- $\forall x \exists y. (x = y)$ is satisfiable: value of $y$ depends on value of $x$.
- $\exists y \forall x. (x = y)$ is unsatisfiable: value of $y$ is fixed for all values of $x$. 
Dependency Schemes: \( D \subseteq (V_\exists \times V_\forall) \cup (V_\forall \times V_\exists) \). [SS09, LB09, LB10, Ben05]

- \((x, y) \notin D\): \(y\) independent from \(x\).
- \((x, y) \in D\): conservatively regard \(y\) as depending on \(x\).

DepQBF: \textit{standard dependency scheme} \( D^{\text{std}} \subseteq D^{\text{triv}} \).

- Previous work: \( D^{\text{std}} \) as dependency-DAG over equivalence classes.
- Efficient integration.

Example: \( \exists a, b \forall x, y \exists c, d. (a \lor x \lor c) \land (a \lor b) \land (b \lor d) \land (y \lor d) \).

\[
\begin{align*}
\exists a \quad & \exists b \\
\downarrow & \quad \downarrow \\
\forall x \quad & \forall y \\
\downarrow & \quad \downarrow \\
\exists c \quad & \exists d \\
& \\
\end{align*}
\]

\[ D^{\text{std}} \]

\[
\begin{align*}
\exists a \quad & \exists b \\
\downarrow & \quad \downarrow \\
\forall x \quad & \forall y \\
\downarrow & \quad \downarrow \\
\exists c \quad & \exists d \\
& \\
\end{align*}
\]

\[ D^{\text{triv}} \]

Standard dependency scheme \( D^{\text{std}} \), quantifier prefix \( D^{\text{triv}} \).
Figure: DepQBF workflow.

Boolean Constraint Propagation (BCP):

- Propagation of unit and pure literals.
- Watched data-structures for efficient detection.
Figure: DepQBF workflow.

Initialize Dependency-DAG:

- Top-most decision level 0.
- All assignments at top-level are permanent.
- Permanent simplifications (satisfied clauses).
- Potential reduction of dependencies.
Retrieve Decision Candidates (DC):

- Get possible decision variables (candidates) from dependency-DAG.
- Candidate: all “preconditions” (predecessors in DAG) assigned.
- Candidate set is maintained incrementally and lazily.
**Decision Making:**

- Select decision variable from candidate set.
- Activity-based priority queue of variables (VSIDS, like MiniSAT 2).
- Assignment caching.
Figure: DepQBF workflow.

Constraint Learning (Result Analysis):

- Conflict/solution: generate asserting learnt clause/cube.
- Augmented CNF: \( \phi := \phi_{OCL} \land (\phi_{LCL} \lor \phi_{LCU}) \).
- Learnt clauses \( \phi_{LCL} \) and cubes \( \phi_{LCU} \).
- Q-resolution/consensus to derive learnt clauses/cubes.
- See also our SAT’10 paper.
**Learnt Constraint Removal and Restarts:**

- Check each time when adding a new learnt constraint.
- Capacity exhausted: remove half of learnt constraints.
- Heuristically try to keep “useful” constraints, increase capacity.
- Inner-outer restart schedule (like PicoSAT).
System Overview (7/7)

**Backtracking:**

- General (frequent) case: backtrack to asserting level of learnt constraint.
- Special case: backtrack to restart level.

**Figure:** DepQBF workflow.
Learnt Constraints: \{GNT02, Let02, ZM02, GNT06, BKF95, GS08, ES03, GN02\}

- Clauses $\phi_{LCL}$ and cubes $\phi_{LCU}$, stored in doubly-linked lists.
- Initial capacities depend on formula size: [2500, 10000].

**Move-To-Front (MTF) Strategy:** approximating clause activities.
- Want to keep “used” (i.e. important?) constraints: units, learning.
- Move used constraints $C_i$ to head of list:

$$\{C_1, \ldots, C_{i-1}, C_i, C_{i+1}, \ldots, C_n\} \xrightarrow{MTF(C_i)} \{C_i, C_1, \ldots, C_{i-1}, C_{i+1}, \ldots, C_n\}$$

most-recently used \hspace{2cm} least-recently used \hspace{2cm} deletion order

**Deletion:**
- Capacity exhausted: remove half of constraints, starting at tail of list.
- Least-recently used ones are deleted (hopefully: least-important ones).
- Increase capacity by constant 500.
Inner-Outer Restart Schedule: when to restart? [Bie08]

- Inspired by PicoSAT: separate inner/outer restarts.
- Inner restart after $i$ backtracks, outer restart after $o$ inner restarts.
- Initially $i := 100$, $o := 10$.
- Before $i$th ordinary backtrack: jump to restart level instead, $i := i + 10$.
- After $o$ inner restarts: $i := 100$, $o := o + 5$ (outer restart).
**Restart Level:** where to jump to?
- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**
- Assignment stack, in order of decision levels.
- Conflict/solution at level 4.
- Restart is scheduled, where to jump to?
**Restart Level:** where to jump to?

- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**

- Current learnt constraint asserting at level 3.
- Last universal decision at level 2.
**Restart Level:** where to jump to?

- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**

- Current learnt constraint asserting at level 3.
- Last universal decision at level 2.
- **Restart:** take the longer jump.
**Restart Level:** where to jump to?
- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**
- Current learnt constraint asserting at level 1.
- Last universal decision at level 2.
**Restart Level:** where to jump to?

- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**

- Current learnt constraint asserting at level 1.
- Last universal decision at level 2.
- **Restart:** take the longer jump.
### Table: QBFEVAL’10 Main Track (568 Formulae). Ranking by Number of Solved Formulae.

| Solver          | All solved | All avg.time | Solved SAT solved | Solved SAT avg.time | Solved UNSAT solved | Solved UNSAT avg.time |
|-----------------|------------|--------------|-------------------|---------------------|---------------------|-----------------------|
| DepQBF          | 370        | 337.10       | 165               | 54.58               | 205                 | 20.82                 |
| DepQBF-nr       | 360        | 352.33       | 154               | 51.36               | 206                 | 24.35                 |
| DepQBF-nc       | 350        | 384.66       | 157               | 107.48              | 193                 | 28.05                 |
| DepQBF-np       | 345        | 398.12       | 141               | 114.72              | 204                 | 45.37                 |
| DepQBF-ncnr     | 340        | 400.24       | 147               | 124.10              | 193                 | 20.19                 |
| QuBE7.0-nopp    | 332        | 425.44       | 135               | 147.71              | 197                 | 47.27                 |
| QuBE6.6-nopp    | 301        | 468.51       | 113               | 136.48              | 188                 | 55.27                 |

**Setup:**

- Ubuntu 9.04, Intel® Q9550@2.83 GHz, 3 GB/900 sec.
- DepQBF: version 0.1 which participated in QBFEVAL’10.
| Solver          | All            | Solved SAT     | Solved UNSAT |
|----------------|----------------|----------------|--------------|
|                | solved | avg.time | solved | avg.time | solved | avg.time |
| DepQBF         | 370    | 337.10   | 165    | 54.58    | 205    | 20.82    |
| DepQBF-nr      | 360    | 352.33   | 154    | 51.36    | 206    | 24.35    |
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Table: QBFEVAL’10 main track (568 formulae). Ranking by number of solved formulae.

**Important:**
- Restarts (disabled in DepQBF-nr).
## Experiments: QBFEVAL’10 Main Track

| Solver           | All solved | All avg.time | Solved SAT solved | Solved SAT avg.time | Solved UNSAT solved | Solved UNSAT avg.time |
|------------------|------------|--------------|-------------------|--------------------|--------------------|----------------------|
| DepQBF           | 370        | 337.10       | 165               | 54.58              | 205                | 20.82                |
| DepQBF-nr        | 360        | 352.33       | 154               | 51.36              | 206                | 24.35                |
| **DepQBF-nc**    | **350**    | **384.66**   | **157**           | **107.48**         | **193**            | **28.05**            |
| DepQBF-np        | 345        | 398.12       | 141               | 114.72             | 204                | 45.37                |
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### Important:
- **Restarts.**
- **Assignment caching (disabled in DepQBF-nc).**
## Table: QBFEVAL'10 main track (568 formulae). Ranking by number of solved formulae.

| Solver         | All solved | All avg.time | Solved SAT solved | Solved SAT avg.time | Solved UNSAT solved | Solved UNSAT avg.time |
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| **DepQBF-np**  | **345**    | **398.12**   | **141**           | **114.72**         | **204**             | **45.37**             |
| DepQBF-ncnr    | 340        | 400.24       | 147               | 124.10             | 193                 | 20.19                 |
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**Important:**
- Restarts.
- Assignment caching.
- Pure literal detection (disabled in DepQBF-np).
### Table: QBFEVAL’10 main track (568 formulae). Ranking by number of solved formulae.

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| **DepQBF-ncnr** | **340**    | **400.24**   | **147**           | **124.10**          | **193**             | **20.19**             |
| QuBE7.0-nopp    | 332        | 425.44       | 135               | 147.71              | 197                 | 47.27                 |
| QuBE6.6-nopp    | 301        | 468.51       | 113               | 136.48              | 188                 | 55.27                 |

**Important:**
- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching (disabled in DepQBF-ncnr).
|                   | All       | Solved SAT | Solved UNSAT |
|------------------|-----------|------------|--------------|
|                  | solved    | avg.time   | solved       | avg.time | solved   | avg.time |
| QuBE7.0-pre⇒DepQBF | 424       | 254.23     | 197          | 48.17    | 227      | 23.42    |
| QuBE7            | 414       | 310.29     | 187          | 130.52   | 227      | 58.33    |
| QuBE6.6          | 387       | 341.91     | 168          | 98.97    | 219      | 67.03    |

**without preprocessing**

|                   | All       | Solved SAT | Solved UNSAT |
|------------------|-----------|------------|--------------|
|                  | solved    | avg.time   | solved       | avg.time | solved   | avg.time |
| DepQBF           | 370       | 337.10     | 165          | 54.58    | 205      | 20.82    |
| DepQBF-nr        | 360       | 352.33     | 154          | 51.36    | 206      | 24.35    |
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| DepQBF-np        | 345       | 398.12     | 141          | 114.72   | 204      | 45.37    |
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| QuBE7.0-nopp     | 332       | 425.44     | 135          | 147.71   | 197      | 47.27    |
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Table: QBFEVAL’10 main track (568 formulae). Ranking by number of solved formulae.

**Important:**

- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching.
- Preprocessing (**not** part of DepQBF 0.1, disabled in QuBE*-nopp).
Experiments: QBFEVAL’10 Main Track

DepQBF: A Dependency-Aware QBF Solver (System Description)

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Experiments: QBFEVAL’10 Main Track

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DepQBF: A Dependency-Aware QBF Solver (System Description)
DepQBF:

- Search-based QBF solver with clause- and cube-learning.
- Relaxing prefix order by dependency-DAG for $D^{\text{std}}$.
- Approaches from SAT domain.
- Development:
  - Fuzz testing using QBFuzz: [http://fmv.jku.at/qbfuzz/](http://fmv.jku.at/qbfuzz/)
  - Delta-debugging using QBFDD: [http://fmv.jku.at/qbfdd/](http://fmv.jku.at/qbfdd/)
  - Cross-checking against other solvers, mainly QuBE.

Performance:

- Top-ranked solver in QBFEVAL’10.
- DepQBF 0.1 does not include preprocessing.
- But: preprocessing is very important.

Future Work:

- Preprocessing, parameter tuning, decision heuristics, . . .

DepQBF 0.1 is open source: [http://fmv.jku.at/depqbf/](http://fmv.jku.at/depqbf/)
Unit Clauses: Clause $C$ is unit iff

- no $l \in C$ is true.
- exactly one $l_e \in L_\exists(C)$ is unassigned.
- for all unassigned $l_u \in L_\forall(C)$: $l_u \not\approx l_e$, i.e. $\text{Var}(l_u), \text{Var}(l_e)$ independent.
- Dependency checking $\prec$ with respect to dependency scheme.
- Dual definition for cubes.

Two-Literal-Watching:

- Watch two unassigned literals $l_1, l_2 \in C$ such that
  1. either $q(l_1) = q(l_2) = \exists$, or
  2. $q(l_1) = \forall, q(l_2) = \exists$ and $l_1 \prec l_2$.

Watcher Update:

- Dependency checking needed only in case (2).
- Stop when finding satisfying literal.
- No work needed during backtracking.
Pure Literals (PL):

- Variable has only positive/negative literals left.
- Assigning ∀-PLs/∃-PLs can trigger new units/further PLs.
- Drawback: expensive detection in $\phi_{OCL} \land (\phi_{LCL} \lor \phi_{LCU})$.

Spurious Pure Literals (SPL):

- Def.: Variable is pure (SPL) if it is pure in original clauses $\phi_{OCL}$ only.
- SPL-Detection neglects all learnt constraints in $(\phi_{LCL} \lor \phi_{LCU})$.
  - Advantage: more efficient detection.
- Variable might be pure in $\phi_{OCL}$ but not in $\phi_{OCL} \land (\phi_{LCL} \lor \phi_{LCU})$.
  - Drawback: must ignore such SPL-implications in $(\phi_{LCL} \lor \phi_{LCU})$.

Clause Watching:

- Positive/negative occurrences $C(x), C(\bar{x}) \subseteq \phi_{OCL}$.
- Watch two unsatisfied clauses $C_x \in C(x)$ and $C_{\bar{x}} \in C(\bar{x})$. 
Clause Watcher Update:

- Assign $x/\overline{x}$: all clauses in $C(x)/C(\overline{x})$ will be satisfied.
- Update watchers of variables $y$ watching clauses in $C(x)/C(\overline{x})$.

Notification Lists:

- Goal: avoid searching for variables which need watcher update.
- Lists $NL_x/NL_{\overline{x}}$ of variables $y$ watching clauses in $C(x)/C(\overline{x})$.
- Assign $x/\overline{x}$:
  - exactly all variables in $NL_x/NL_{\overline{x}}$ must update their watcher.
  - update $NL_x/NL_{\overline{x}}$ of variables $x$ occurring in old and new watched clauses.

- No work needed during backtracking.
Activity-Based Variable Priority Queue: [MMZ⁺01, ES03]

- DepQBF: straight-forward generalization of idea from SAT domain.
- Maintain VSIDS score (activity) for each variable.
- Increase activity of variables encountered during learning.
- Periodically down-scale activities.
- Implementation follows MiniSAT 2.
- Decision making: select candidate with highest activity.
- Lazy priority queue maintenance (like in MiniSAT): Discard assigned variables and non-candidates on the fly upon removal.
Also called: Phase Saving

- DepQBF: straight-forward generalization of idea from SAT-domain.
- Each variable has a cached assignment (possibly undefined).
- All assignments (unit, pure literals, decisions) update cache.
- Decision variables: assign cached value, if any.
- No distinction between different quantifiers.
## Suite mqm (136 formulae)

| Solver            | solved | avg.time |
|-------------------|--------|----------|
| DepQBF            | 136    | 39.83    |
| QuBE7             | 117    | 306.43   |
| QuBE7.0-nopp      | 115    | 304.82   |
| QuBE6.6           | 100    | 393.93   |
| QuBE6.6-nopp      | 97     | 399.55   |

### Table: Solvers sorted by number of solved formulae.

## Benchmark Suite \textit{mqm}:

- Minimal Query Inseparability Module Extraction in DL-Lite.
- Newly submitted to QBFEVAL’10 by Roman Kontchakov.
- As the only solver, DepQBF solved entire suite in QBFEVAL’10.
### QBFEVAL’10: solved formulae only

| solved | SAT-\(\cap\) | UNSAT-\(\cap\) |
|--------|---------------|-----------------|
| solved | 328           | 132             |
| avg.time | 84.97         | 21.87           |
|         | 140.16        | 32.43           |
|         | 47.81         | 14.75           |

### QBFEVAL’10: unique results

| solved | SAT-\(\leftrightarrow\) | UNSAT-\(\leftrightarrow\) |
|--------|--------------------------|---------------------------|
| solved | 86                       | 42                        |
|        | 55                       | 33                        |
|        | 31                       | 9                         |

**Table:** QuBE7 (left columns) vs. DepQBF (right columns).

### QBFEVAL’10: solved formulae only

| solved | SAT-\(\cap\) | UNSAT-\(\cap\) |
|--------|---------------|-----------------|
| solved | 308           | 115             |
| avg.time | 80.14         | 17.49           |
|         | 114.17        | 23.23           |
|         | 59.86         | 14.07           |

### QBFEVAL’10: unique results

| solved | SAT-\(\leftrightarrow\) | UNSAT-\(\leftrightarrow\) |
|--------|--------------------------|---------------------------|
| solved | 79                       | 62                        |
|        | 53                       | 50                        |
|        | 26                       | 12                        |

**Table:** QuBE6.6 (left columns) vs. DepQBF (right columns).
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