Strong decay patterns of the hidden-charm pentaquark states $P_c(4380)$ and $P_c(4450)$

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Abstract

With the heavy quark symmetry and spin rearrangement scheme, we study the strong decay behavior of the hidden-charm pentaquark states with $J^P = \frac{3}{2}^-$, assuming they are molecular candidates composed of $D^*$ and $\Sigma^*$. We obtain several typical ratios of the partial decay widths of the hidden-charm pentaquarks. For the three S-wave $(\bar{D}^*\Sigma^*)$, $(\bar{D}'\Sigma^*)$, and $(D'\Sigma^*)$ molecular pentaquarks with $J^P = 3/2^-$, we have obtained the ratio of their $J/\psi N$ decay widths: $\Gamma((\bar{D}^*\Sigma^*)) : \Gamma((\bar{D}'\Sigma^*)) : \Gamma((D'\Sigma^*)) = 2.7 : 1.0 : 5.4$, which may be useful to further test the possible molecular assignment of the $P_c$ states.

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I. INTRODUCTION

Recently, the LHCB Collaboration announced the discovery of two hidden-charm resonances, $P_c(4380)$ and $P_c(4450)$, in the $J/\psi p$ invariant mass spectrum in the process $\Lambda_b \rightarrow J/\psi pK$ [1]. The masses and widths of $P_c(4380)$ and $P_c(4450)$ are [1]

$$
M_{P_c(4380)} = (4380 \pm 8 \pm 29) \text{ MeV}, \\
\Gamma_{P_c(4380)} = (205 \pm 18 \pm 86) \text{ MeV}, \\
M_{P_c(4450)} = (4449.8 \pm 1.7 \pm 2.5) \text{ MeV}, \\
\Gamma_{P_c(4450)} = (39 \pm 5 \pm 19) \text{ MeV}.
$$

Since they are observed in the final state $J/\psi p$, the isospin of $P_c(4380)$ and $P_c(4450)$ is $I = 1/2$. According to LHCB’s analysis, their angular momentum and the parity of the two $P_c$ states are either $J^P = \frac{3}{2}^-$ or $\frac{5}{2}^-$. At present the spin and parity of each $P_c$ state cannot be determined.

In the literature there exist some theoretical discussions of the possible hidden-charm pentaquark states [2–6]. Specifically, the possibility of hidden-charm molecular pentaquarks composed of an anticharmed meson and a charmed baryon was studied systematically in the framework of the one boson exchange model in Ref. [4]. In fact, the existence of the hidden-charm molecular pentaquarks was predicted [4].

Let us take the deuteron, which is an extremely loosely bound molecular state composed of one proton and a neutron with a binding energy around 2 MeV, as an example. Generally speaking, the binding energy of the hadronic molecular state is around several to several tens of MeV. Within the molecular scheme, it is quite natural to understand the masses of $P_c(4380)$ and $P_c(4450)$, which lie several tens of MeV below the $(\bar{D}^*\Sigma_c)$ and $(D'\Sigma_c^*)$ threshold. We want to emphasize that the mass difference between these two $P_c$ states is almost the same as the mass difference between $\Sigma_c$ and $\Sigma_c^*$, which is around 70 MeV.

Within the molecular scheme, the P-wave, D-wave or even higher orbital excitations may also exist if the binding energy of the lowest S-wave hadronic molecule reaches several tens of MeV. For example, the P-wave state may lie slightly above the S-wave ground state with an excitation energy around several to tens of MeV. In other words, the S-wave and P-wave states may completely overlap with each other. There may exist two or more resonant signals around 4380 MeV which are close to each other but may carry different parity. If the P-wave or higher excitation is very broad with a width around 600 MeV, such a state may easily be mistaken as the background. On the other hand, if the excitation lies several MeV within 4380 MeV but with a width as narrow as several MeV, then this state may probably be buried by the $P_c(4380)$ resonance with a width around 205 MeV. The same situation may also occur around 4450 MeV. The above speculation may partly explain why the different assignment of the spin and parity of these two $P_c$ states yields roughly the same good fit [1].

The identification of the nearly degenerate resonances with different parities and widths may require a huge amount of experimental data.

The discovery of $P_c(4380)$ and $P_c(4450)$ opens a new window to study exotic hadronic matter. The recent discovery of $P_c(4380)$ and $P_c(4450)$ has inspired theorists’ extensive interest in these two states. With the one pion exchange model, Chen, Liu, Li, and Zhu performed a dynamical calculation of the $\Sigma_c D_c^*$ and $\Sigma_c' D_c^*$ systems. The results confirm that there do exist two S-wave $\Sigma_c D_c^*$ and $\Sigma_c' D_c^*$ molecular states around the mass regions of $P_c(4380)$ and $P_c(4450)$, respectively [7].

More recently, the authors of Ref. [8] constructed the spin-3/2 and spin-5/2 hidden-charm local pentaquark interpolating currents to investigate $P_c(4380)$ and $P_c(4450)$ within the framework of the QCD sum rule formalism. The spectra of the two newly observed $P_c$ states can be reproduced well, and two extra hidden-charm pentaquarks and two hidden-bottom pentaquarks are predicted. Unfortunately, these currents do not distinguish a tightly bound pentaquark structure.
or a molecular structure composed of an anticharmed meson and a charmed baryon.

In Ref. [9], Mironov and Morozov analyzed four possibilities of the configuration of pentaquarks qualitatively. They claimed that the internal color components of the pentaquark may play a crucial role in forming the two \( P_c \) states [9]. The Bethe-Salpeter equation was applied to studying the interactions of \( D \Sigma_c^* \) and \( \bar{D} \Sigma \), where \( P_c(4380) \) and \( P_c(4450) \) were explained as the \( D \Sigma_c^* \) state with \( J^P = 3/2^- \) and \( \bar{D} \Sigma \) with \( J^P = 5/2^+ \), respectively [10].

Lebed investigated the hidden-charm pentaquark through the dynamical diquark picture carefully [11], where the two \( P_c \) states were composed of color-antitriplet diquark \( cu \) and color-triplet triquark \( \bar{c}(ud) \). In Ref. [12], the authors suggest that the two \( P_c \) states have a configuration of diquark-diquark-antiquark. The total spin of the light diquark and the orbital excitation in the pentaquark states combine to explain the mass difference. In Ref. [13], the diquark-diquark-antiquark-type interpolating currents were introduced to study the two \( P_c \) states using QCD sum rules. The same formalism was extended to study the hidden-charm pentaquark with \( J^P = 1/2^± \) in Ref. [14].

With the bound state version of the topological soliton model for baryons, Scoccola, Riska, and Rho noticed the existence of a bound (or quasibound) \( \bar{c} \) and a charmed baryon. In Ref. [15], the authors suggest that the two \( P_c \) states have a configuration of diquark-diquark-antiquark. The total spin of the light diquark and the orbital excitation in the pentaquark states combine to explain the mass difference. In Ref. [13], the diquark-diquark-antiquark-type interpolating currents were introduced to study the two \( P_c \) states using QCD sum rules. The same formalism was extended to study the hidden-charm pentaquark with \( J^P = 1/2^± \) in Ref. [14].

With the bound state version of the topological soliton model for baryons, Scoccola, Riska, and Rho noticed the existence of a bound (or quasibound) \( \bar{D} \)-soliton state which is compatible with the two \( P_c \) states [15]. The quark delocalization color screening model was adopted to study the hidden-charm molecular pentaquarks, where \( P_c(4380) \) is suggested to be a mixed structure of \( \Lambda, D^*, \Sigma, D^*, \Sigma^* \), and \( \bar{D}^* \) with \( I(J^P) = 1/2(3/2^-) \), while \( P_c(4450) \) can be a \( \Sigma^*D^* \) state with \( I(J^P) = 1/2(5/2^+) \) [16]. Zhu and Qiao used a constituent diquark-triquark model to explain the two \( P_c \) states in Ref. [17]. \( P_c(4450) \) was proposed as a \( \chi_{c1}p \) resonance in Ref. [18].

Besides the mass spectrum, the productions of the hidden-charm pentaquarks were investigated in the weak decays of \( \Lambda, D^*, \Sigma, D^*, \Sigma^* \), and \( \bar{D}^* \) with \( I(J^P) = 1/2(3/2^-) \), while \( P_c(4450) \) can be a \( \Sigma^*D^* \) state with \( I(J^P) = 1/2(5/2^+) \) [16]. Zhu and Qiao used a constituent diquark-triquark model to explain the two \( P_c \) states in Ref. [17]. \( P_c(4450) \) was proposed as a \( \chi_{c1}p \) resonance in Ref. [18].

As an approximate symmetry, heavy quark symmetry is applied to study the structures of hadrons which contain heavy quarks. In the heavy quark limit, the total angular momentum \( J \) of a system can be decomposed into two parts, i.e., the heavy spin \( S_H \) and light spin \( S_l \), which satisfy the relation \( S_l + S_H = J \). Here, the light spin denotes the light degrees of freedom including all of the orbital angular momenta and the spin of light quarks within a hadron, while the heavy spin is the total spin of the heavy quarks.

In general, under the spin arrangement scheme, the hidden-charm molecular pentaquark states composed of an anticharmed meson \( \bar{D}^{(*)} \) and a charmed baryon \( Q_c^{(*)} \), which have the total angular momentum \( J \), can be decomposed as
For the S-wave molecular state, monium and a baryon, the final state can generally be written as 

\[
|\bar{\Phi}(q_1 q_2 q_3)\rangle = \left[\left[|\bar{\Phi}(q_1 q_2 q_3)\rangle \otimes \left[c \otimes (q_2 q_3)\right]_{I_g} \otimes \left[l \otimes (q_2 q_3)\right]_{I_g} \right]_{J_0} \otimes \left[l' \otimes (q_2 q_3)\right]_{J_0}\right]_J
\]

where \( J = \sqrt{2J + 1} \). Here, \( L \) denotes the orbital angular momentum within the anticharmed meson, while \( L' \) is the orbital angular momentum between the anticharmed meson and the charmed baryon. \( s \) and \( g \) represent the light spin and the total angular momentum of the anticharmed meson, respectively. \( m \) and \( k \) are the light spin and the total angular momentum of the charmed baryon, respectively. \( J_0 \) denotes the total angular momentum of the anticharmed meson and the charmed baryon. \( J \) is the angular momentum of the pentaquark state. For the S-wave molecular state, \( J = J_0 \).

We also define \( h \) and \( T \) as the heavy spin and total light spin of the system. \( R \) is the total spin of the three light quarks. We denote the sum of the spin \( R \) and the orbital angular momentum \( L \) within the anticharmed meson as the angular momentum \( n \). In addition, \( |\bar{\Phi}(q_1 q_2 q_3)\rangle \) and \( |\bar{\Phi}(q_1 q_2 q_3)\rangle \) denote the flavor wave functions, where the general expressions \( |(ab)\rangle \) and \( |(abc)\rangle \) are the abbreviations of \(|ab + ba|\sqrt{2}\) and \(|ab + bac + acb + cba + bca|/\sqrt{6}\), respectively.

In the following, we focus on the S-wave \( \bar{\Phi}^*\Lambda_c^+ \), \( \bar{\Phi}^*\Sigma_c^+ \) molecular states with \( J^P = 3/2^- \) and \( J^P = 5/2^- \), and the P-wave \( \bar{\Phi}^*\Lambda_c^0 \) and \( \bar{\Phi}^*\Sigma_c^0 \) molecular states with \( J^P = 3/2^+ \) and \( J^P = 5/2^+ \). According to Eq. (1), we can perform the decomposition of the initial hidden-charm molecular pentaquarks, where the relevant terms and the corresponding coefficients are listed in Table I.

We make the same decomposition of the final states when these hidden-charm pentaquarks decay into a charmonium plus a baryon. If there exists an \( L'' \) excitation between a charmonium and a baryon, the final state can generally be written as follows under the spin rearrangement scheme:

\[
|\text{Charmonium} \rangle \otimes |\text{Baryon} \rangle
\]

stands for the total angular momentum of the baryon. The coupling of \( Q \) and \( L'' \) forms \( J'_{0} \). \( T' \) and \( g' \) are the light spin and heavy spin, respectively. We list the kinematically allowed final states in Table II when the hidden-charm pentaquarks lie in the mass range of 4430 ~ 4450 MeV.

For a molecular state with the configuration \( \bar{\Phi}^*Q_c^+ \), \( J_0 \) is the sum of the spins of the meson and baryon, which further couples with the orbital angular momentum \( L \) to form the total angular momentum \( J \) of the pentaquark. For a fixed \( J \), there exist different combinations of \( J_0 \) and \( L \) if \( L \) is nonzero. In Table I, we mark the corresponding \( J_0 \) values for some P-wave \( \bar{\Phi}^*Q_c^+ \) molecular states. In a similar way, we can also deal with the coupling of the angular momentum of the final states. For an initial \( \bar{\Phi}^*Q_c^+ \) molecular state, we have

\[
|\bar{\Phi}^*Q_c^+\rangle = \sum_{J_0} \sum_{L_m} \langle J_0 (g_m + k_m) |Q_m L_m\rangle \langle Q_m L_m J_0 |\bar{\Phi}^*Q_c^+\rangle
\]

where we adopt the same notations as in Eq. (1). For the final states, we have

\[
|\text{Charmonium} \rangle \otimes |N\rangle
\]

\[
= \langle (c\bar{\phi})_e | L_0^e \rangle \otimes |Q_0 \rangle \otimes |L''_n\rangle
\]

\[
= \sum_{J_0} \sum_{L_0} \langle J_0 (L_m + Q_m) |Q_0 \rangle \langle Q_0 |L''_n\rangle \langle L''_n |J_0\rangle
\]

\[
\]

\[
\]

where \( g' \), \( L \), and \( k' \) are the spin, orbital, and angular momentum quantum numbers of the charmonium, respectively. \( Q \) is the total spin of the three light quarks.

\[
|\text{Charmonium} \rangle \otimes |N\rangle
\]

\[
= \langle (c\bar{\phi})_e | L_0^e \rangle \otimes |Q_0 \rangle \otimes |L''_n\rangle
\]

\[
= \sum_{J_0} \sum_{L_0} \langle J_0 (L_m + Q_m) |Q_0 \rangle \langle Q_0 |L''_n\rangle \langle L''_n |J_0\rangle
\]

\[
\]

\[
\]

\[
\]

where \( N \) represents the nucleon. The notations are the same as in Eq. (2).

For a hidden-charm pentaquark decay into the charmonium and nucleon

\[
\bar{\Phi}^*Q_c^+ \rightarrow \text{Charmonium} + N,
\]

its \( T \) matrix element reads
TABLE I: The decomposition of the hidden-charm molecular pentaquarks. We list the coefficients $G_{h,R,T}^{L_p,L,q}$ in Eq. (1) for the combination $[h,R,T]$, which stands for $[[([\bar{q}c]_L \otimes [(qJ_2)]_R)_{10} \otimes L_L] \otimes L_R]$. We use the subscripts $H$ and $L$ to label the heavy and light spins, respectively. The notation $\cdots$ indicates that this combination is forbidden for the S-wave and P-wave molecular pentaquarks. We use 0 to denote the combination which is suppressed by the heavy quark symmetry.

| $J^P = \frac{3}{2}^-$ | $J^P = \frac{3}{2}^-$ | $J^P = \frac{3}{2}^-$ |
|----------------|----------------|----------------|
| $[0_H, \frac{1}{2}, \frac{1}{2}]$ | $[1_H, \frac{1}{2}, \frac{1}{2}]$ | $[1_H, \frac{1}{2}, \frac{1}{2}]$ |
| $|\bar{D}^* \Lambda_c\rangle$ | 0 | 1 | 0 |
| $|\bar{D} \Sigma_c\rangle$ | $\frac{1}{\sqrt{3}}$ | $-\frac{1}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{3}}$ |
| $|\bar{D} \Sigma_c\rangle$ | $-\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{3}}$ |
| $|\bar{D} \Sigma_c\rangle$ | $\frac{\sqrt{2}}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |

| $J^P = \frac{1}{2}^+$ | $J^P = \frac{1}{2}^+$ |
|----------------|----------------|
| $[0_H, \frac{1}{2}, \frac{1}{2}]$ | $[1_H, \frac{1}{2}, \frac{1}{2}]$ |
| $|\bar{D}^* \Lambda_c\rangle$ | $0 \ 0 \ 0$ |
| $|\bar{D} \Sigma_c\rangle$ | $0 \ 0 \ 0$ |
| $|\bar{D} \Sigma_c\rangle$ | $0 \ 0 \ 0$ |
| $|\bar{D} \Sigma_c\rangle$ | $0 \ 0 \ 0$ |

| $J^P = \frac{5}{2}^-$ | $J^P = \frac{5}{2}^-$ |
|----------------|----------------|
| $[0_H, \frac{5}{2}, \frac{1}{2}]$ | $[1_H, \frac{5}{2}, \frac{1}{2}]$ |
| $|\bar{D}^* \Lambda_c\rangle$ | $0 \ 1 \ 0 \ 0$ |
| $|\bar{D} \Sigma_c\rangle$ | $\frac{1}{\sqrt{3}}$ | $-\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| $|\bar{D} \Sigma_c\rangle$ | $-\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| $|\bar{D} \Sigma_c\rangle$ | $\frac{\sqrt{2}}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{3}}$ |

$|T|^2 = A \sum_{g_m \otimes Q_m} \sum_{k_m \otimes L_m} \left| \left< \text{Charmonium} \otimes N \right| H \left| \bar{D} \Sigma_c^{(*)} \right> \right|^2$

\begin{align*}
&\sum_{g_m \otimes Q_m} \sum_{k_m \otimes L_m} \left( <J_0(k_m + Q_m) | k' k''_m Q Q_m | J_0(k_m + Q_m) L' L'_m> <J_0(k_m + Q_m) | g g_m k k_m> \right) \\
&\times (J_0(g_m + k_m + L_m) | J_0(g_m + k_m + L_m) L' L'_m) \delta(g_m + k_m + L_m - k_m - Q_m - L_m) \\
&\times \left( <[\bar{c}q]_c \otimes L_L \otimes [Q \otimes L']_L | H | [\bar{D} \Sigma_c^{(*)}]_{J_0} \otimes L> \right) \left( [\bar{c}q]_c \otimes L_L \otimes [Q \otimes L']_L | H | [\bar{D} \Sigma_c^{(*)}]_{J_0} \otimes L> \right)
\end{align*}

where $A$ is the normalization factor and $J_{10}$ is a quantum number similar to $J_0$. Since this matrix element does not depend
TABLE II: The coefficients $F_{g,T}^{J,L,K',J_0}$ in Eq. (2) corresponding to different combinations of $[g,T]$, which is the abbreviation of $\left(\langle cc\rangle_g \otimes [L \otimes J_0']_T\right)$. We use $\cdots$ to mark the forbidden combination for the S-wave and P-wave decays, while we use 0 to denote the combination which is suppressed by the heavy quark symmetry.

| $J$ | $J = \frac{1}{2}^-$ | $J = \frac{1}{2}^+$ | $J = \frac{3}{2}^-$ | $J = \frac{3}{2}^+$ |
|---|---|---|---|---|
| $[J/\psi N]$ | 1 | $\cdots$ | $\cdots$ | $\cdots$ |
| $[\chi_{c0}(1^3P_0)N]$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $[\chi_{c1}(1^3P_1)N]$ | $\cdots$ | $\cdots$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ |
| $[\chi_{c2}(1^3P_2)N]$ | $\cdots$ | $\sqrt{\frac{5}{3}}$ | $\frac{1}{\sqrt{6}}$ | 0 |
| $[\eta_c(1^3S_1)N]$ | $\cdots$ | $\cdots$ | 0 | 1 |

| $J = \frac{1}{2}^-$ | $J = \frac{1}{2}^+$ | $J = \frac{3}{2}^-$ | $J = \frac{3}{2}^+$ |
|---|---|---|---|
| $[0_J, \frac{1}{2}]$ | $[1_J, \frac{1}{2}]$ | $[1_J, \frac{3}{2}]$ | $[1_J, \frac{5}{2}]$ |
| $[J/\psi N](J_0 = \frac{1}{2})$ | 0 | 1 | 0 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $|J/\psi N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | 0 | 0 | 1 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $|\chi_{c0}(1^3P_0)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 0 | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $\cdots$ | $\cdots$ |
| $|\chi_{c1}(1^3P_1)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 0 | $\frac{1}{\sqrt{6}}$ | $\frac{\sqrt{7}}{\sqrt{10}}$ | $\frac{1}{\sqrt{6}}$ | $\sqrt{\frac{7}{10}}$ |
| $|\chi_{c2}(1^3P_2)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 0 | $\frac{\sqrt{5}}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | 0 | 0 | 1 | 0 |
| $|\chi_{c3}(1^3P_3)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 0 | $\frac{\sqrt{5}}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | 0 | 0 | 1 | 0 |
| $|\eta_c(1^3S_0)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | 1 | 0 | 0 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $|\eta_c(1^3P_1)N(J_0 = \frac{1}{2})|0_J, \frac{1}{2}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | 1 | 0 | 0 | 0 | $\cdots$ | $\cdots$ |

on $g_m, k_m, L_m, Q_m, k_m',$ or $L_m'$, Eq. (5) can be further simplified with as

$$|T|^2 = \sum_{J_0, J_{10}} \sum_{J_{00}, J_{10}} C(g, k, L, J_0, J_{10}, J; k', Q, L', J_{00}, J'_{10}, J')$$

\[
\times \left(\left(\langle cc\rangle_g \otimes L\right)_{J_0} \otimes \left[Q \otimes L'\right]_{J_{00}}\right)^{\dagger} \left\|H\right\| \left(\left|\vec{B}_g^{(\epsilon)\Sigma_k^{(\epsilon)}}\right|_{J_0} \otimes L\right) \left(\left|\vec{B}_g^{(\epsilon)\Sigma_k^{(\epsilon)}}\right|_{J_{00}} \otimes L'\right)^{\dagger}
\]"
Through the derivation of Eqs. (6) and (7), we prove that different \( J_0 \) (or \( J'_0 \)) contributions in the initial (or final) state for a fixed \( J \) are equivalent when \( J_0 = J_{10} \) and \( J'_0 = J'_{10} \). If \( J_0 \neq J_{10} \) or \( J'_0 \neq J'_{10} \), the coefficient \( C \left( g, k, L, J_0, J_{10}, J'_{10}, J'_0 \right) \)
vanishes, which shows that there is no mixing between different \( J_0 \) and \( J_{10} \) or \( J'_0 \) and \( J'_{10} \) states. Now Eq. (6) becomes quite simple,

\[
|T|^2 = \sum_{j_0} \left( \sum_{j'_0} \frac{1}{(2j_0 + 1)(2j'_0 + 1)} \right) \left| \left[ (c\bar{c})_k \otimes [Q \otimes L'_c]_{j_0} \right] [H]| \left[ (D^*\Sigma^c)_k \right]_{j_0} \otimes L' \right|^2. \tag{8}
\]

Combining Eqs. (1) and (2) with Eq. (8) and using the heavy quark symmetry, we can obtain useful information about the \( T \) matrix element for the hidden-charm decays.

### III. NUMERICAL RESULTS

With the preparation discussed in Sec. II, we focus on the ratios of the partial decay widths of the hidden-charm molecular pentaquarks using the spin rearrangement scheme. The spin structure of the initial state has been decomposed into heavy and light spins and labeled by three quantum numbers, \( h, R', \) and \( T \), which are introduced in Eq. (1). For the final states, the total spin \( R \) is fixed as 1/2 since we only consider the decay modes which are involved with a nucleon and a charmonium.

Since the D-wave or higher partial waves are strongly suppressed by phase space, we focus on the S-wave and P-wave decay of these states. Throughout our calculation, we redefine the matrix element as

\[
H_{R,R'} = \left[ (h, R, T) \right] \left| (h, R', T) \right|. \tag{9}
\]

which corresponds on the matrix element to the right-hand side of Eq. (8), where \( \left[ (h, R, T) \right] \) and \( \left[ (h, R', T) \right] \) are the abbreviations of the initial and final states, respectively. In Table III, we collect the \( H_{R,R'} \) related \( T \) matrix element, where the results depend on two unknown matrix elements, \( H_{\frac{1}{2}, \frac{1}{2}} \) and \( H_{\frac{3}{2}, \frac{1}{2}} \). These matrix elements depend on the specific decay dynamics. Hence, they cannot be determined by the symmetry analysis alone.

In Table III, we first illustrate the decay patterns of the \( J^P = 3/2^- \) hidden-charm molecular pentaquarks composed of \((\bar{D}\Sigma^c), (\bar{D}^*\Sigma^c), \) and \((\bar{D}^*\Sigma^c)\), respectively. In the heavy quark symmetry limit, the \( D \) and \( D^* \) mesons have the same spatial wave function. Similarly, \( \Sigma^c \) and \( \Sigma^d \) have the same spatial wave function. For the \( J/\psi N \) decay mode of the above three types of molecular states, we notice that the decay process depends on one common matrix element \( H_{\frac{1}{2}, \frac{1}{2}} \). Therefore, we get the ratio

\[
\Gamma \left( \left\{ (\bar{D}\Sigma^c) \right\} : \Gamma \left( \left\{ (\bar{D}^*\Sigma^c) \right\} : \Gamma \left( \left\{ (\bar{D}^*\Sigma^c) \right\} \right) = 2.7 : 1.0 : 5.4, \tag{10}
\right.
\]

where we have included the contribution of the corresponding phase spaces. Compared with the \( J/\psi N \) decay mode, the other P-wave decay modes, like \( \chi_{c0} N, \chi_{c1} N \left( J_0 = \frac{1}{2} \right), \) and \( \chi_{c1} N \left( J_0 = \frac{1}{2} \right) \), depend on two unknown matrix elements. Moreover, they are strongly suppressed by the limited phase space by the factor \( |\mathbf{p}_N|^3 \), where \( \mathbf{p}_N \) is the decay momentum of the nucleon.

For the hidden-charm molecular pentaquarks with \( J^P = 3/2^- \), their decay pattern is quite different than that of the \( J^P = 3/2^- \) states. In Table III, we use 0 to denote the decay channels which are suppressed due to the heavy quark symmetry. This feature reflects the inner structure of the pentaquark state with \( J^P = 3/2^- \). For these hidden-charm molecular pentaquarks with the same \( I(J^P) \) quantum number and the same configuration, the corresponding matrix elements are still different if the corresponding \( J_0 \) quantum number is different, as shown in Table III. For the \( \eta N \) decay mode, the ratio of the partial decay widths of the \( (\bar{D}^*\Sigma^c) \left( J_0 = \frac{1}{2} \right) \) and \( (\bar{D}^*\Sigma^c) \left( J_0 = \frac{1}{2} \right) \) molecular states is

\[
\Gamma \left( \left\{ (\bar{D}^*\Sigma^c) \left( J_0 = \frac{1}{2} \right) \right\} : \Gamma \left( \left\{ (\bar{D}^*\Sigma^c) \left( J_0 = \frac{1}{2} \right) \right\} = 1.97, \tag{11}
\right.
\]

and the ratio of the partial decay widths of the \( (\bar{D}\Sigma^c) \left( J_0 = \frac{3}{2} \right), \)
TABLE III: The relevant $H_{R\bar{c}}$ matrix elements of the hidden-charm molecular pentaquarks with $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^-$. Here, $\cdots$ indicates that the decay channel is forbidden for the S-wave and P-wave decays.

| $J^P$ | Initial state | $J(\phi N)$ | $\chi_c N$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
|-------|---------------|-------------|-------------|-----------------------------|-----------------------------|
| $\frac{3}{2}^-$ | $\bar{D}^* \Lambda_c$ | $J(\phi N)$ | $\chi_c N$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\sqrt{\frac{1}{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |
| | $\bar{D}^* \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |
| | $\bar{D}^* \Sigma_c^*$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |

| $\frac{1}{2}^-$ | $\bar{D} \Lambda_c$ | $J(\phi N)$ | $\chi_c N$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |
| | $\bar{D}^* \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |

| $\frac{1}{2}^-$ | $\bar{D}^* \Lambda_c$ | $J(\phi N)$ | $\chi_c N$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} + \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |
| | $\bar{D}^* \Sigma_c$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{1}{\sqrt{2}} H_2 \frac{1}{2} - \frac{1}{\sqrt{2}} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ | $\frac{\sqrt{5}}{3} H_2 \frac{1}{2}$ |

| $\frac{1}{2}^-$ | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |

| $\frac{1}{2}^-$ | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
| | $\bar{D} \Sigma_c^*$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ | $\chi_c N(J_0 = \frac{1}{2})$ |
(\bar{D}'\Sigma_c)(J_0 = 1/2)$, and $(\bar{D}'\Sigma_c^*)(J_0 = 3/2)$ states is
\[
\Gamma [\bar{D}'\Sigma_c^*(J_0 = 3/2)] : \Gamma [\bar{D}'\Sigma_c(J_0 = 1/2)] : \Gamma [\bar{D}'\Sigma_c^*(J_0 = 3/2)] = 1.0 : 1.7 : 2.6,
\]
(12)

For the S-wave $\bar{D}'\Sigma_c^*$ states with $J^P = 5/2^-$, its $h_c(1P)N$ decay channel is strongly suppressed in the heavy quark symmetry. The ratio of the partial decay widths of its $\chi_{c1}(1P)N$, $\chi_{c2}(1P)N (J_0 = 1/2)$, and $\chi_{c2}(1P)N (J_0 = 3/2)$ modes reads
\[
\Gamma [\chi_{c1}(1P)N] : \Gamma [\chi_{c2}(1P)N (J_0 = 1/2)] : \Gamma [\chi_{c2}(1P)N (J_0 = 3/2)] = 1.5 : 1.4 : 1.0,
\]
(13)

where we have also considered the phase space correction. Unfortunately, for these states with $J^P = 5/2^-$, there does not exist any simple relation.

IV. SUMMARY

Inspired by the recent experimental observation of the two $P_c$ states [1], we have studied the decay behaviors of the hidden-charm pentaquarks with the $\bar{D}'\Sigma_c$ configuration and spin-parity $J^P = 3/2^+$, $5/2^+$ through the spin rearrangement scheme. Within this framework, we have obtained several ratios of the partial decay widths of some decay channels of the hidden-charm pentaquarks, which will be useful in the further experimental investigation of their inner structures.

We also notice that most of the partial decay widths of the hidden-charm pentaquarks depend on the unknown matrix element $H_{R,F}$, which is governed by strong decay dynamics. Such matrix elements can either be calculated with a phenomenological model or extracted through the experimental measurement of several partial decay widths. Once they are known, all of the other hidden-charm decay widths can be predicted.

The two observed $P_c$ states open a Pandora’s box of exotic state studies. In the coming years, more and more novel phenomena are expected with experimental progress, especially from LHCB and the forthcoming BelleII. Clearly, more theoretical explorations of the hidden-charm pentaquark states are desirable.

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