A $\Delta \Sigma$-Modulation Feedforward Network for Non-Binary Analog-to-Digital Converters

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SUMMARY A feedforward (FF) network using $\Delta \Sigma$ modulators is investigated to implement a non-binary analog-to-digital (A/D) converter. Weighting coefficients in the network are determined to suppress the generation of quantization noise. A moving average is adopted to prevent the analog signal amplitude from increasing beyond the allowable input range of the modulators. The noise transfer function is derived and used to estimate the signal-to-noise ratio (SNR). The FF network output is a non-uniformly distributed multi-level signal, which results in a better SNR than the estimation of quantization noise. A moving average is adopted to prevent the accumulation of quantization noise. Expressions representing the signal-to-noise ratio (SNR) are derived based on the noise transfer function (NTF) and a guideline for improving the SNR is explained. Our behavioral simulations show that the SNR is improved by more than 30 dB, or equivalently a bit resolution of 5 bits, compared with a conventional first-order $\Delta \Sigma$ modulator.

key words: delta-sigma modulator, A/D converter, artificial neural network, SNR, noise shaping, non-binary

1. Introduction

Analog-to-digital (A/D) converters play an essential role as interfaces between digital systems and our real world, where analog signals carry information. Along with the dramatic improvement in the digital system performance, the A/D converter specification has become more advanced. Various conversion architectures, including successive approximation, pipelined, and $\Delta \Sigma$ modulation, have been investigated to satisfy demanding requirements. Aiming at higher performance, the development of A/D converters incorporating hybrid configurations or digital calibration schemes is also progressing. Meanwhile, an A/D converter using a Hopfield-type neural network was proposed as an alternative. Parallel processing and learning capability in such an artificial neural network have been expected to improve the A/D converter performance.

In general, however, Hopfield-type neural networks had a drawback to overcome; i.e., a local minimum of the energy function associated with the system can lead to incorrect results. To solve the problem, introducing the $\Delta \Sigma$ modulation was attempted, though it was not proposed for A/D converters but other applications. The idea behind this was to use a somewhat random nature existing in $\Delta \Sigma$-modulated signals. This effectively recovered the system from the situation where it had fallen into a wrong local minimum, allowing us to obtain a collect solution. Based on this idea, an A/D converter including $\Delta \Sigma$ modulators was proposed. A sampling scheme and weighting coefficients, as well as a network configuration, were carefully designed to improve the signal-to-noise ratio (SNR), or the conversion bit-resolution, which was confirmed by behavioral simulations.

This paper focuses on a $\Delta \Sigma$-modulator feedforward network for non-binary A/D conversion with an improved signal-to-noise ratio (SNR). In particular, weighting coefficients in the network proposed previously are systematically re-examined, and the noise transfer function associated with the network is derived to estimate the SNR quantitatively. The impact on performance degradation due to the analog component mismatch is also discussed. Behavioral simulations are carried out to confirm the analytical results. In Sect. II, proposed feedforward networks are described. An SNR expression is derived as a function of the weighting coefficients and the number of channels that make up the network. In Sect. III, behavioral simulation results are presented and compared with the predictions mentioned in Sect. II.

2. Feedforward Network with $\Delta \Sigma$ Modulators

This section briefly reviews conventional networks used for the A/D converter first. Next, our FF network structures are described. In the previous works, the weighting coefficients were semi-empirically determined. In this paper, they are determined systematically to reduce quantization noise. Expressions representing the signal-to-noise ratio (SNR) are derived based on the noise transfer function (NTF) and a guideline for improving the SNR is explained. Finally, the impact of the mismatch on the SNR is discussed.

2.1 Conventional Neural Networks

Figure 1 (a) depicts a conventional A/D converter based on a Hopfield-type neural network. For simplicity, a 3-bit configuration is shown. In the following explanation, the horizontal lines corresponding to the output bits from $D_1$ (the most significant bit, MSB) to $D_3$ (the least significant bit, LSB) are referred to as channels. This configuration is thus called a 3-channel network. CP is the 4th-comparator (quantizer). The numbers attached to wiring intersections are weighting coefficients. For example, the input to CP$_3$ is $4V_{in} - 2D_2 - 4D_1$.

A problem associated with the network shown in Fig. 1 (a) is that the conversion error can be substantial be-
cause of the local minima of the network energy function. To solve the problem, an asymmetry structure shown in Fig. 1 (b) was proposed [4]. This is equivalent to the feedforward (FF) network shown in Fig. 1 (c), and there exist no local minima. Since this structure does not include a feedback loop, it is relatively easy to analyze and incorporate learning functions in the future.

The input of CP 2 in Fig. 1 (c) is twice as large as the quantization error, which is defined by the difference between the input and the output of CP 1. The same applies to the input of CP 3. In other words, the lower bits are sequentially determined by doubling the quantization error generated in the previous stage and passing it to the next stage. Therefore, the structure shown in Fig. 1 (c) is equivalent to conventional multi-step A/D converters based on the binary search algorithm, which is conventionally used in successive-approximation and pipelined architectures [2].

2.2 Proposed Feedforward Networks

2.2.1 Simple Two-Channel FF Network

A noble FF network was proposed based on the structure shown in Fig. 1 (c) [9]–[11]. First, consider a simple two-channel FF network shown in Fig. 2 (a), where the comparators CP i in Fig. 1 (c) were replaced with ΔΣ modulators, DSMi. H 2 is a digital filter, the impulse response of which is h 2(n). In this study, a first-order modulator shown in Fig. 2 (b) [13] was assumed. The output v(n) is expressed as

\[ v(n) = u(n) + [e(n) - e(n - 1)]. \] (1)

By performing the z-transform, the following equation is obtained:

\[ V(z) = U(z) + \left(1 - z^{-1}\right)E(z). \] (2)

In Fig. 2 (a), e 1 and e 2 are quantization errors generated in the comparators in DSM 1 and DSM 2. u x is the weighted sum of the input u 1 and the DSM 1 output v 1 such that

\[ u_x = 2u_1 - v_1. \] (3)

The input of DSM 2, u 2, is the moving average calculated by using the n-th sampled value and N samples before and after it as follows:

\[ u_2(n) = \frac{1}{2N + 1} \sum_{k=n-N}^{n+N} u_x(k). \] (4)

The output v T is expressed as

\[ v_T(n) = v_1(n - N) + h_2(n) * v_2(n). \] (5)

The reason to choose this structure is now explained. Figure 3 (a) shows the transfer characteristics of the conventional 2-step A/D converter shown in Fig. 1 (c). The input v in is assumed within the allowable input range of

\[ \frac{-v_{ref}}{2} < v_{in} < \frac{v_{ref}}{2}, \] (6)

where v ref is the reference voltage. The analog signal transferred to the second stage, v x, is expressed as

\[ v_x = 2v_{in} + v_{ref}\left(\frac{1}{2} - D_1\right). \] (7)

Here, D 1 is the most significant bit (MSB) and equal to either 0 or 1. If v in ≥ 0, then D 1 = 1, and if v in < 0, then D 1 = 0. Thus, v x remains in the region between −v ref/2 and v ref/2 so that the binary search algorithm can be further continued.

For the 2-channel feedforward network shown in Fig. 2 (a), the input signal u 1 is also assumed to be within the full scale range

\[ \frac{-v_{ref}}{2} < u_1 < \frac{v_{ref}}{2}. \] (8)
The input $u_1$ doubled is shown by the dashed line marked (i) in Fig. 3 (b). The output of DSM$_1$ is not uniquely determined by the input-signal sign but can be either $-v_{\text{ref}}/2$ or $v_{\text{ref}}/2$. In other words, the output can be $-v_{\text{ref}}/2$ even if $v_{\text{in}} \geq 0$ and vice versa. Therefore, $u_2$ can be plotted on the lines represented by (ii) or (iii). Obviously, the range of $u_s$ is beyond the allowable input range of DSM$_2$, which should be between $-v_{\text{ref}}/2$ and $v_{\text{ref}}/2$. However, by taking the moving average of the output, $u_2$ converges approximately to the line marked (iv), because it is likely that $v_1$ is $v_{\text{ref}}/2$ if $u_1$ is close to $v_{\text{ref}}/2$, and $v_1$ is $-v_{\text{ref}}/2$ if $u_1$ is close to $-v_{\text{ref}}/2$. Therefore, taking the average is desirable for $u_2$ to remain in the allowable input range of DSM$_2$. This is the reason why the weighting coefficients were chosen 2 and -1, as shown in Fig. 2 (a), and why the moving average shown in Eq. (4) is introduced. Another average with lowpass characteristics can be used, but this was selected for simplicity. The delay element denoted by $z^{-N}$ in Fig. 2 (a) was incorporated to compensate for the delay associated with the moving average.

Now, let us discuss how to select $H_2$ in Fig. 2 (a). By taking the $z$-transform of Eq. (5) and considering Eq. (2),

$$V_T(z) = z^{-N}V_1(x) + H_2V_2(z)$$
$$= (z^{-N} + H_2\text{MA}_N(z))U_1(z)$$
$$+ (z^{-N} - H_2\text{MA}_N(z))(1 - z^{-1})E_1$$
$$+ H_2(1 - z^{-1})E_2$$

is obtained, where $\text{MA}_N$ is the transfer function representing the moving average.

$$\text{MA}_N(z) = \frac{1}{2N+1} \frac{1 - z^{-(2N+1)}}{1 - z^{-1}}.$$  \hspace{1cm} (10)

If

$$z^{-N} = H_2\text{MA}_N(z),$$ \hspace{1cm} (11)

then the $E_1$ term in Eq. (9) disappears, and the following equation is obtained:

$$V_T(z) = z^{-N} \left[2U_1(z) + \frac{1}{\text{MA}_N(z)}(1 - z^{-1})E_2(z) \right].$$ \hspace{1cm} (12)

Therefore, by adding the second channel, the quantization error generated in the first (lower) channel can be eliminated, such that the entire quantization noise can be reduced. This is explained in detail below.

2.2.2 Extended Two-Channel FF Network

Figure 4 shows an extended 2-channel configuration, where the weighting coefficients, 2 and -1, in Fig. 2 (a) were replaced with $\alpha + 1$ and $-\alpha$, respectively. The difference between these coefficients should be equal to 1 so that $u_2$ remains within the acceptable range after taking the average. The output $v_T(n)$ is represented by Eq. (5). The following equation can be derived in the same manner as described above

$$V_T(z) = \left(z^{-N} + \text{MA}_N H_2 \right)U_1(z)$$
$$+ (z^{-N} - \alpha\text{MA}_N H_2)(1 - z^{-1})E_1(z)$$
$$+ H_2(1 - z^{-1})E_2.$$ \hspace{1cm} (13)

If

$$z^{-N} = \alpha\text{MA}_N H_2$$ \hspace{1cm} (14)

is assumed, then the $E_1(z)$ term vanishes and

$$V_T(z) = z^{-N} \left[1 + \frac{1}{\alpha} \right]U_1(z) + \frac{1}{\alpha \text{MA}_N} \left(1 - z^{-1}\right)E_2.$$ \hspace{1cm} (15)

is obtained.

The noise transfer function (NTF), which is defined by the ratio of $V_T$ to $E_2$, is then expressed as

$$\text{NTF}(z) = \frac{z^{-N}}{\alpha \text{MA}_N} \left(1 - z^{-1}\right).$$ \hspace{1cm} (16)

Replacing $z$ with $e^{j\omega}$ to obtain the frequency characteristic

Fig. 3 Transfer characteristics of (a) conventional 2-step A/D converter and (b) present feedforward network.

Fig. 4 Extended 2-channel feedforward network.
Eq. (18), the moving average $\text{MA}_N$ respectively, the SNR (in dB) can be derived as

$$\text{SNR} = \text{SNR}_1 + 20 \log (\alpha + 1).$$

Eq. (15) are the signal and quantization noise components, respectively, the SNR (in dB) can be derived as

$$\text{SNR} = \text{SNR}_1 + 20 \log (\alpha + 1).$$

where $\omega = 2\pi f_s T_s$. $T_s$ is the sampling period. Therefore, it is possible to reduce the quantization noise by a factor of $\alpha$ by adding the second channel to the conventional $\Delta\Sigma$ modulator. This is valid even if high-order $\Delta\Sigma$ modulators would be used in the present FF network.

Since the first and second terms in the right side of Eq. (15) are the signal and quantization noise components, respectively, the SNR (in dB) can be derived as

$$\text{SNR}_2 \approx \text{SNR}_1 + 20 \log (\alpha + 1).$$

Here, $\text{SNR}_1$ is the SNR obtained for the conventional first-order $\Delta\Sigma$ modulator shown in Fig. 2(b). In the derivation of Eq. (18), the moving average $\text{MA}_N$ is approximated to be 1. Since $\text{MA}_N$ has the $\text{sinc}$ characteristic, this approximation is valid if the signal bandwidth is narrow enough compared with $f_s/(2N)$, where $f_s$ is the sampling frequency. This equation clearly shows that increasing $\alpha$ improves $\text{SNR}_2$. However, it should be noted that if $\alpha$ becomes too large, it is less likely that $u_2$ remains in the acceptable input range of DSM$_2$, and the SNR deteriorates.

### 2.2.3 Three- and Four-Channel FF Networks

A three-channel FF network can be constructed by adding another path to DSM$_2$ and connecting the output through another digital filter $H_3$, as shown in Fig. 5. The total output $V_T$ can be expressed as

$$V_T(z) = \left( z^{-N} V_1 + H_2 V_2 \right) z^{-N} + H_3 V_3. \tag{19}$$

To eliminate the $E_1$ and $E_2$ terms, the following equations should be satisfied:

$$\begin{align*}
    z^{-N} H_2 &= H_3 \alpha \text{MA}_N \tag{20} \\
    z^{-N} &= H_2 (\alpha + 1) \text{MA}_N. \tag{21}
\end{align*}$$

Then,

$$V_T(z) = z^{-2N} \left[ \left( 1 + \frac{1}{\alpha} \right) U_1(z) \right. $$

$$\left. + \frac{1}{\alpha (\alpha + 1)} \frac{1}{\text{MA}_N^2} \left( 1 - \frac{1}{\alpha} \right) E_3(z) \right]. \tag{22}$$

is obtained. The noise transfer function for the 3-channel network, $\text{NTF}_3(z)$, is expressed as

$$\text{NTF}_3(z) = \frac{z^{-2N}}{\alpha (\alpha + 1) \text{MA}_N^2} \left( 1 - \frac{1}{\alpha} \right) \tag{23}$$

For a low frequency region, the frequency response of the NTF is written as

$$|\text{NTF}_3(\omega)| \approx \frac{\omega}{\alpha (\alpha + 1)} \tag{24}$$

This results in the SNR of

$$\text{SNR}_3 \approx \text{SNR}_1 + 40 \log (\alpha + 1). \tag{25}$$

Again, the moving average $\text{MA}_N$ is approximated to be 1. Comparison with Eq. (18) reveals that the SNR in the 3-channel network is better than that in the 2-channel case.

The discussion above can be extended to a 4-channel FF network. The total output is obtained as

$$V_T(z) = z^{-3N} \left[ \left( 1 + \frac{1}{\alpha} \right) U_1(z) \right. $$

$$\left. + \frac{1}{\alpha (\alpha + 1)^2} \frac{1}{\text{MA}_N^3} \left( 1 - \frac{1}{\alpha} \right) E_4(z) \right], \tag{26}$$

where $E_4$ is the quantization noise generated by the quantizer of the $\Delta\Sigma$ modulator in the fourth channel. Then, the NTF and the SNR are obtained as

$$\text{NTF}_4(z) = \frac{z^{-3N}}{\alpha (\alpha + 1)^2 \text{MA}_N^3} \left( 1 - \frac{1}{\alpha} \right), \tag{27}$$

and

$$\text{SNR}_4 \approx \text{SNR}_1 + 60 \log (\alpha + 1). \tag{28}$$

Therefore, increasing $\alpha$ and the number of channels improves the SNR. It should be noted, however, that for the same reason that the SNR starts to deteriorate when $\alpha$ becomes too large, the SNR deteriorates when the number of channels is too large.

The feature of this configuration is that it has a multilevel output. In a conventional multi-bit $\Delta\Sigma$ modulator, the number of comparators required increases exponentially with the number of output bits. However, our method can avoid this exponential growth. The number increases linearly: $2^N$ levels are obtained with only $N$ comparators. However, this is achieved at the expense of a linear increase in the number of integrators. Second-order noise shaping can also be obtained by using a second-order modulator in the final channel. For example, second-order noise shaping in a four-channel, $2^4$-level configuration requires four comparators and five integrators. In contrast, a conventional 4-bit single-loop second-order $\Delta\Sigma$ modulator requires 15 comparators and two integrators. The small number of integrators may be advantageous to reduce the overall power consumption. However, recent efforts to reduce the power consumption of integrators, such as inverter-based ones, make
the proposed approach a practical option. Besides, this structure offers a significant advantage of quantization noise leakage, which will be discussed later.

2.2.4 Non-Binary Output Levels

With an approximation of $\text{MA}_N = 1$, Eq. (5) can be written as

$$v_T(n) = v_1(n - N) + \frac{1}{\alpha} v_2(n - N)$$

(29)

for a 2-channel FF network. Since $v_1(n - N)$ and $v_2(n - N)$ are either $v_{\text{ref}}/2$ or $-v_{\text{ref}}/2$, $v_T(n)$ is expressed as

$$v_T(n) = \left( \pm \frac{1}{\alpha} \right) \frac{v_{\text{ref}}}{2}.$$  

(30)

For simplicity, $v_{\text{ref}} = 2$ was assumed below. If $\alpha = 2$, the output $v_T(n)$ is thus one of four values: $\pm 3/2$ or $\pm 1/2$. In this respect, the present network operates like a 2-bit $\Delta\Sigma$ modulator. It should be noted, however, if $\alpha \geq 3$, the output is still a 4-level signal, but the output is no longer binary, or the levels are not evenly distributed. For example, they are $\pm 4/3$ and $\pm 2/3$ for $\alpha = 3$.

Similarly, the numbers of output levels for the 3- and 4-channel networks are $2^3$ and $2^4$, respectively, for $\alpha \geq 2$. Also, in this case, the output levels are not evenly distributed. For example,

$$v_T(n) = \pm 1 + \frac{1}{\alpha} \pm \frac{1}{2(\alpha + 1)}$$

(31)

and

$$v_T(n) = \pm 1 + \frac{1}{\alpha + 1} \pm \frac{1}{(\alpha + 1)^2} \pm \frac{1}{\alpha(\alpha + 1)^2}$$

(32)

are satisfied for 3- and 4-channel cases. If $\alpha = 1$ for the 4-channel case, the output is 9-valued: $\pm 2, \pm 3/2, \cdots, 1/2$ or 0. If $\alpha = 2$ for the 4-channel case, the output is 16-valued, but it is $\pm 3/2, \pm 25/18, \cdots, \pm 11/18$ or $1/2$. The levels are not evenly distributed. These will be demonstrated by our simulation below.

In conventional single-loop multi-bit $\Delta\Sigma$ modulators, the SNR is degraded due to nonlinearity of the DAC used in the feedback path, so linearization techniques such as the dynamic element matching (DEM), are necessary. In contrast, this configuration does not require feedback DACs, except for 1-bit DSMs’s, and avoids the linearization problem. However, characteristic mismatch in the analog network remains a problem, which will be discussed later.

2.3 Effect of Mismatch on SNR

In designing analog circuits, it is necessary to consider the variation in component characteristics. In the present networks, this results in mismatches between the weighting coefficients, which can deteriorate the SNR. In this study, the mismatch in the FF network shown in Fig. 6 is assumed. Specifically, let $\Delta_i$, be the relative error in each $\alpha$, such that

$$\alpha_i = \alpha(1 + \Delta_i) (i = 1, \cdots, 4).$$

Then, the output $V_T(z)$ can be modified as

$$V_T(z) \approx \left( 1 + \frac{1}{\alpha} + (\Delta_1 - \Delta_2) \right) U_1(z)$$

$$- \left( \frac{\alpha}{\alpha + 1} (\Delta_3 - \Delta_4) \right) (1 - z^{-1}) E_1$$

$$+ \frac{1}{\alpha(\alpha + 1)} \frac{\text{MA}_N^2}{(1 - z^{-1})} E_3.$$  

(33)

In the above approximation, the mismatch terms of the order of $\Delta_i/\alpha$ and $\Delta_i^2$ were omitted for simplicity.

The following expression of the SNR for the 3-channel network can be derived in a similar manner described above:

$$\text{SNR}_{3m} = \text{SNR}_3 - 10 \alpha^2 (\alpha + 1)^2 \left[ 1 + 2 \left( \frac{\alpha}{\alpha + 1} \right)^2 \right] \Delta_i^2.$$  

(34)

Here, $\text{SNR}_3$ is the SNR of the three-channel FF network without the mismatch (Eq. (25)), and the second term in Eq. (34) represents the SNR degradation due to the mismatch. It is assumed that the square averages of $\Delta_i$’s are all the same as $\Delta^2$. The larger the coefficient $\alpha$, the more significant the effect of mismatch.

The proposed structure is similar to that known as MASH [14]. However, there is an essential difference in the signal to be transferred to the following channel. In conventional MASH, the signal used is the difference between the input and output of the comparator of the $\Delta\Sigma$ modulator, i.e., the quantization noise, and this noise can leak to the output if the matching between the analog and digital blocks is insufficient, decreasing the SNR [13]. On the other hand, in our proposal, the transferred signal is the difference between the input and output of the entire $\Delta\Sigma$ modulator, i.e., the shaped quantization noise, as shown in Eqs. (9) and (13). In other words, it is the shaped noise that leaks to the output. Therefore, even if the matching condition of Eq. (11), (14), (20), or (21) is not strictly fulfilled, the proposed structure can substantially suppress the SNR degradation.

3. Simulation Results

Behavioral simulations were carried out to evaluate the analysis result described in the previous section. The SNR was estimated using FFT of the output waveform from the network consisting of first-order $\Delta\Sigma$ modulators. The oversampling ratio (OSR), defined as the ratio of the sampling frequency to the Nyquist rate, was 128 unless otherwise mentioned. Also, the sampling period $T_s$ was assumed to be 1.
Fig. 7   Input signal waveforms to the DSM3 modulator ($u_3$) in Fig. 5 with (a) $N = 2$ and (b) $N = 8$. $\alpha = 3$.

Fig. 8   Input sinusoidal waves (——) and output multi-level signals (•’s) obtained from a 4-channel feedforward network with (a) $\alpha = 2$ and (b) $\alpha = 1$.

3.1 Waveforms

Figure 7 shows the input signal waveforms into the third $\Delta\Sigma$ modulator (DSM3) in Fig. 5 for different numbers of samples used in the moving average: $N = 2$ and $N = 8$. Although the input amplitude becomes large due to the multiplication by $\alpha + 1$ and $\alpha$, this figure indicates that taking the moving average is effective to suppress the growth, as $N$ increases, so that the resulting signal is within the allowable input range of ±1.

The multi-level output signals are plotted in Fig. 8 with input sinusoidal signals. Since the network consisted of four channels, the numbers of levels were $2^4$ and 9 for $\alpha = 2$ and $\alpha = 1$, respectively, as mentioned in Sect. 2.2.4. While the levels are uniform distributed for $\alpha = 1$, they were unevenly distributed for $\alpha = 2$. It will be shown below that the uneven distribution results in a better SNR than the uniform one. The improvement in SNR is probably due to the more accurate $\Delta\Sigma$ modulation of the quantization noise, generated in the upper bit channel, for $\alpha = 2$ than for $\alpha = 1$.

3.2 SNR

Figure 9 shows the noise shaping characteristics obtained by the FFT analysis of the 4-channel-network output signals. Two different numbers of samples used for the moving average were considered: $N = 2$ and $N = 8$. For $N = 2$, the slope was around 20 dB/dec, consistent with the prediction based on Eq. (27), and the same as that obtained for a conventional first-order $\Delta\Sigma$ modulator. For $N = 8$, however, the slope is steeper between $f/f_s = 10^{-2}$ and $f/f_s = 10^{-1}$. This is attributed to the moving average of $\text{MA}_N$, which was neglected in deriving Eq. (27) for simplicity. As shown in Eq. (10), $\text{MA}_N$ has the lowpass $\text{sinc}$ characteristics. Since Eq. (27) contains the inverse of $\text{MA}_N$, another highpass characteristic is added to the normal first-order shaping of $(1 - z^{-1})$, if $\text{MA}_N$ is considered. This is the reason why the shaping slope changes for $N = 8$. For $N = 2$, the highpass characteristic appears at a much higher frequency range than for $N = 8$, and the slope stays almost constant.

By using the spectrum shown in Fig. 9, the SNR was estimated as a ratio of the signal power to the quantization noise power in the signal band, which was defined here as a frequency range below the input signal frequency. In Fig. 10, the SNR is plotted as a function of the input signal amplitude. The amplitude is normalized by the full scale.
Peak SNR as a function of $\alpha$ for 2-, 3-, and 4-channel feedforward networks represented by $\times$’s, $\triangle$’s, and $\circ$’s, respectively. The arrow on the right shows the SNR obtained for a conventional first-order $\Delta\Sigma$ modulator, $N = 8$.

Peak SNR as a function of the number of samples used for the moving average for 2-, 3-, and 4-channel feedforward networks represented by $\times$’s, $\triangle$’s, and $\circ$’s, respectively. $\alpha = 4$.

The peak SNR increases as the OSR increases, as shown in Fig. 13, which compares the results obtained from two FF networks, $N = 2$ and $N = 8$, with that from a conventional first-order $\Delta\Sigma$ modulator. The slope obtained for the conventional first-order $\Delta\Sigma$ modulator is close to the theoretically predicted value of 9 dB/oct, and it is almost the same for the case of $N = 2$. This is because $|NTF|$ is proportional to the sampling frequency, as shown in Eq. (27). In other words, this is attributable to the slope of 20 dB/dec shown in Fig. 9. On the other hand, the slope for $N = 8$ is steep, as was shown in Fig. 9, which led to the improvement of SNR.

Finally, Fig. 14 shows the effect of mismatch on the peak SNR. The mismatch values were determined by generating random numbers and adding them to the weighting coefficients. If the mismatch (or the relative error) is less than 10% (corresponding to $10^{-1}$ in this figure), any significant SNR degradation was not observed. A mismatch of less than 10% is not difficult to achieve in present LSI technology. Therefore, Fig. 14 indicates that the proposed technique is reasonably robust against the mismatch.

4. Conclusion

The feedforward networks, including first-order $\Delta\Sigma$ modulators, were analyzed to evaluate its analog-to-digital converter performance. In particular, the signal-to-noise ratio...
tio (SNR) was estimated based on the noise transfer function (NTF). The network configuration and weighting coefficients necessary to improve the SNR were presented. A moving average was adopted to prevent the analog signal amplitude from increasing beyond the modulator allowable input range. Our behavioral simulation showed that increasing the weighting coefficients and the number of channels improved the SNR by more than 30 dB, or equivalently a bit resolution of 5 bits, compared with a conventional first-order ΔΣ modulator. The SNR deteriorated due to the mismatch. However, if the mismatch is as small as 10%, its adverse effect cannot be significant.

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