The Dynamic analysis of Planetary roller screw in Tree-climbing robot

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Abstract. In the tree-climbing robot, the sawing module is lifted and lowered by the planetary roller lead screw, so the dynamic analysis of the planetary roller lead screw is very important. In this paper, the stiffness of the mechanical joint of planetary roller lead screw is calculated according to the fractal theory, and the stiffness calculation formulas between roller and ring gear, roller and lead screw are obtained. In the dynamic simulation, the stiffness of each contact pair is set according to the formula, and two groups of models with consistent and inconsistent roller contact stiffness are established. The first six order modal analysis results of the two groups of models are compared, and the influence of uneven roller contact stiffness distribution on the whole planetary roller lead screw is obtained.

1. Introduction
In the transmission structure of tree-climbing robot, the planetary roller lead screw needs to ensure its stability in the operation process, reduce the impact of vibration on the sawing cutter head, and ensure its smooth operation to avoid the clamping of the telescopic movement of the cutter head. For the planetary roller lead screw, its stability is closely related to the uneven load on the planetary roller, and the difference of roller contact stiffness caused by machining and installation error is the main reason for the uneven load on the planetary roller. In this paper, the contact stiffness of a single roller is calculated by fractal theory. This result is used as the parameter of roller contact pair in simulation calculation. The first six modal results of two groups of models with uniform and uneven roller distribution are calculated. From this result, the influence of uneven roller contact stiffness distribution on the planetary roller lead screw is found.

2. Calculation of contact stiffness of planetary roller lead screw
In the contact pair analysis of planetary roller lead screw, Hertz principle is generally used to model and analyze the contact between roller and lead screw and between roller and nut. However, in the actual machining process, the surface of various parts is not completely smooth, so there is a certain error in using Hertz theory to analyze the contact pair of planetary roller lead screw. In this paper, the fractal theory is used to calculate the contact stiffness between roller and lead screw, roller and nut, the contact to surface roughness is introduced, the stiffness calculation formula of single micro convex body is obtained from the modeling of micro convex contact body, and the total stiffness calculation formula of contact surface is derived by integration. Finally, the running stability of planetary roller lead screw under different contact stiffness of rolling element is evaluated by finite element simulation analysis.
2.1 Fractal theory
As for Hertz theory, the two surfaces in contact with each other are smooth spheres, ignoring the influence of surface roughness, describing the contact state between two points, which belongs to the research content of Euclidean geometry. The actual mechanical contact part is not completely smooth. Compared with the research object of European geometry, the research object of fractal geometry is more complex, irregular and closer to reality. Its important feature is that it cannot be described by classical geometry and has irregularity [1]. When the fractal theory is used to calculate the contact stiffness, the model object is a single micro convex body. Firstly, the contact stiffness of a single micro convex body is calculated, and then the micro convex body distribution density function is introduced to integrally solve the contact stiffness of the whole contact pair.

2.2 Structure and motion parameters of planetary roller lead screw
The planetary roller lead screw mainly includes five parts (as shown in Figure 1), 1-cage, 2-retaining ring, 3-roller, 4-lead screw and 5-nut. The nut, roller and lead screw are the main transmission parts. The cage separates the rollers according to the same spacing and maintains the synchronous movement of each roller. The retaining ring fixes the cage on the lead screw. When the lead screw rotates and the rotation freedom of the nut is limited, the lead screw transmits the force to the roller through the thread pair. The meshing teeth at both ends of the roller ensure that the roller can rotate around the lead screw on the lead screw. At the same time, the force is transmitted to the nut through the thread, and the nut will move along the axial direction of the lead screw.

![Figure 1. Sectional view of planetary roller lead screw](image)

2.2.1 Structural parameters
The structural parameters of the planetary roller lead screw are shown in table 1:

| Name                      | Value | Name                | Value |
|---------------------------|-------|---------------------|-------|
| Pitch                     | 5     | Number of rollers   | 5     |
| Number of lead screw heads| 5     | pressure angle      | 20    |
| Number of Nut Heads       | 5     | Tooth angle         | 90    |
| Lead screw                | 25    | Number of roller teeth | 10   |
| Nut Guide                 | 25    | Number of screw teeth | 50  |
| Roller Guide              | 1     | modulus             | 1.5   |

2.2.2 Motion parameters
The planetary roller lead screw pair includes two parts: thread pair and gear pair. The thread pair is composed of five thread pairs between roller and screw rod and five thread pairs between roller and nut. The gear pair is composed of five roller gears and lead screw gears. The mechanism movement diagram [2] is shown in Figure 2.
It can be derived from the kinematic relationship that when the nut is fixed, the axial displacement $S$ of the nut with respect to the roller is \[ S = \frac{k n_s p}{2(k+1)} \pm \frac{k (k+2) p}{2(k+1)} \] \[ (1) \]

$k = \frac{d_s}{d_r}$, $d_s$ is the Middle meridian of screw thread, $d_r$ is the Medium meridian of roller thread, $n_r$ is the Number of screw thread heads, $p$ is pitch. Tables 1 are taken into account in the above formulas to obtain the axial displacement of 5 mm for each rotation of the screw when the nut is fixed.

The angular velocity relation between roller and screw can be derived from the motion condition of planetary gear \[ [4][5] \]:

\[ \omega_r = \frac{\omega_s k}{2(k+1)} \quad, \quad \omega_\mu = \frac{\omega_s k(k+2)}{2(k+1)} \] \[ (2) \]

Where $\omega_r$ is the Roller angular velocity, $\omega_\mu$ is the Roller rotation angular speed, $\omega_s$ is the Angular speed of screw.

2.3 Calculation of contact stiffness of planetary roller screw

When calculating the stiffness of two contact surfaces with Hertz theory, it is considered that both contact surfaces are smooth surfaces, ignoring the influence of surface roughness on the contact stiffness. In order to calculate the contact surface stiffness accurately, the contact surface is dispersed into several micro-convex bodies. The contact model of a single micro-convex body is established by using fractal theory and the contact stiffness of a single micro-convex body on the contact surface is obtained. Then the contact stiffness of the whole contact pair is calculated by means of the density function of the micro-convex distribution.

2.3.1 Micro-convex body model

In general, the roller, the screw and the nut are in an elastic deformation state during operation. On the surface of the contact pair, the contact pair of the micro-convex body is formed by squeezing two arbitrary surfaces together. The simplified model is shown in Figure 3.
The elastic deformation of the micro-convex body is equivalent to the standard ellipse on the contact surface, with the short axle \( a \) and the long axle \( b \). To simplify the calculation, it is assumed that the two micro-convex bodies are spherical surfaces with equal radii. According to the principle of elasticity, the stiffness \( k_n \) between the single micro-convex bodies can be obtained \([1]\):

\[
k_n = \frac{\sqrt{ab}}{1.571}
\]

### 2.3.2 Contact stiffness calculation

In order to calculate the stiffness of the entire contact pair, a size distribution function is introduced here \( \xi(s) \). The function describes the distribution rule of micro-convex on contact surface according to fractal theory. By integrating the stiffness of single micro-convex on the whole contact surface, the total contact stiffness on the contact surface can be obtained.

\[
\xi(s) = \frac{D}{2} \psi^{\frac{2-D}{2}} \frac{D}{2} \frac{2+D}{2} \frac{s_{\text{max}}}{s}
\]

\( D \): fractal dimension, \( \psi \): Expansion factor, \( s_{\text{max}} \): Maximum contact area of micro-convex body, \( s \): Contact area of micro-convex body. For the integration of the single contact stiffness of the micro-convex body, there are:

\[
K_n = \int_{s_c}^{s_{\text{max}}} k_n \xi(s) \, ds
\]

\( K_n \) is the contact-to-total stiffness and \( s_c \) is the critical contact area. Where the critical state from elastic deformation to plastic deformation is neglected, the calculation formulas of total stiffness can be obtained by simultaneous equations (4), (5), (6):

\[
K_n = \frac{\sqrt{DE \psi}}{1.571(1-D)}
\]

According to the relationship between the actual contact area and the cross-section area of the contact point, the formula of the maximum contact area can be obtained:

\[
S_{\text{max}} = \frac{2(2-D)}{D} A_r
\]

\( A_r \) is the contact area of the microconvex body. When the two microconvex bodies contact within the range of elastic deformation, the amount of contact deformation of the two micro-convex bodies is considered as \( \delta \). For small amounts, the contact surface can be considered to have a radius of \( \delta \). The contact area formula can be obtained from the circular area of the:

\[
A_r = \pi \delta^2
\]

Fractal dimension \( D \) and expansion coefficient of machined surface when analyzing contact surface with fractal theory \( \psi \). The following empirical relationships are obtained with the conventional roughness characterization parameter \( R_a^{[3]} \):

\[
D = \frac{1.528}{R_a^{0.044}}
\]

\[
\psi^{(2-D)/2} - \left( 1 + \psi^{-D/2} \right)^{-(2-D)/D} = (2 - D)/D
\]

During the calculation process in this paper, the surface roughness of each part is set at 0.2, and the fractal dimension \( D=1.635 \) and expansion coefficient can be calculated \( \psi=1.9055 \). The formula of contact stiffness of planetary roller screw and the deformation of micro-convex body are obtained by bringing fractal dimension, expansion coefficient, formula (7) and formula (8) into formula (6):

\[
K_n = -3.6972 \delta E
\]
When the roughness of the two contact surfaces is 0.2, the curvature radius of the micro-convex body is set at 0.2 \( \mu \)m. In the calculation of deformation amount in this paper, the maximum deformation amount on the micro-convex body should be the deformation amount in the critical state. The stiffness of the two contact surfaces in the critical state can be obtained by quoting the formula of deformation amount of the micro-convex body under the critical condition [1].

### 3. Dynamic simulation calculation of planetary roller screw

In the mode simulation, contact pairs of roller and screw shaft, roller and holder, holder and screw, snap and screw, and snap and holder are set. The contact parameters are set according to the parameters calculated in the previous section. In the solution analysis, the maximum mode order is set to order 6 with one end of the screw fixed and no external load. In the contact stiffness inconsistent model, the contact stiffness of roller 1 is set at 50% of the normal contact stiffness. \( K_R = 0.893 \text{N/mm} \). The total displacement results of the first six modal analysis of the roller contact stiffness consistent model are derived, as shown in Figure 4:

![Figure 4](image1.png)
(a) First order mode 3500.8Hz
(b) Second order mode 3500.9Hz
(c) Third order mode 8236.9Hz
(d) Fourth order mode 8237.4Hz
(e) 5th mode 9113.3Hz
(f) 6th mode 11651Hz

Figure 4. Uniform modal analysis results of roller stiffness

The first six modal analysis results of the inconsistent roller contact stiffness model are derived, as shown in Figure 5:

![Figure 5](image2.png)
(a) First order mode 3500.7Hz
(b) Second order mode 3500.9Hz
4. Conclusions
Comparing the two groups of results, when the contact stiffness of No. 1 roller is inconsistent with that of the other four rollers, No. 1 roller will produce large deformation in the sixth mode, indicating that No. 1 roller will have fatigue damage earlier during operation. Therefore, the contact stiffness of the roller of the planetary roller lead screw should be consistent in order to maintain a good running state. At the same time, the displacement difference caused by roller contact stiffness in the first three modal results is small, which also shows that the difference of roller contact stiffness will not have a great impact on the overall motion at low speed.

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