Design of constant temperature bathtub model without external heating system

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Abstract. With the market demand for constant temperature, low cost and environmentally friendly bathtub is increasing. Our goal on this paper is to establish a water temperature model to meet the needs of users' comfort and design a water temperature bathtub without external heating system. After mathematically analysis, we quantify the factors that cause heat dissipation and heat absorption in the bathtub. Based on the Energy Conservation Law, we set up an optimization model of two factors in time and space. We ignore the water temperature stratification, so we transform the three-dimensional problem into a two-dimensional problem to simplify the model. Then, we use MATLAB to simulate the heat distribution of the bathtub temperature field to compare the different locations of injected water. By comparing, we think the optimal strategy is injecting water from all around of the bathtub. Finally, we test the correctness of the water temperature model. We take advantage of the heat energy distribution in the whole bathtub simulated by MATLAB to analyze the influence of the human state on the water temperature in the bathtub. As can be seen in the simulation, the human movement will promote the dissipation of heat and affect water temperature distribution of the bathtub which proves the reliability of the water temperature model. Compared with the bathtub on the market, on the premise of ensuring the comfort of the user, our bathtub and the proposed strategy are of great significance in reducing the cost and saving water resources.

Keywords: constant temperature bathtub, MATLAB, Energy Conservation Law.

1. Introduction
In modern times, the bath culture is combined with the background of the times, the design and the high-end technology. The bath culture centered on the bathtub has grown up and developed. A bathtub is regarded as a way of enjoying life and relaxing the mind and body. Therefore, the design of temperature control system and comfort of the bathtub are particularly important.

An ordinary bathtub without temperature control system is only a simple water containment vessel. Because of the heat transfer from the water in the bathtub to the indoor air, the temperature of the water is decreasing during the bath. This bathtub can't meet the comfort requirements of people while bathing. However, most of the advanced bathtub with thermostat system is heated by the temperature control
switch to control the heating power supply. This heating method greatly increases the cost of making the bathtub.

At present, research on how to maintain a constant temperature of the bathtub is not much. Today's bathtub can't meet people's demand for temperature and price at the same time the market of constant temperature bathtub needs to be developed. So, we will scientifically arrange the time to add and release the water to design a constant temperature system from the point of view of water conservation.

According to the results of the market survey, we need to design the bathtub from three aspects: Import and export position, Water temperature, Comfort. Meanwhile, from the viewpoint of environment, if there is no reasonable arrangement for the amount of water, it is bound to cause the waste of water resources. 

2. Assumptions and Symbols

2.1. Assumptions and Justifications

To simplify our problems, we make the following basic assumptions, each of which is properly justified.

   Assumed that the temperature of the room remains the same.
   Assumed that the bathtub is a cuboid in dimensions shown in Figure 1.

   Assumed that the bathtub is an acrylic cuboid. By comparing the properties of a variety of materials, we finally decided to use acrylic. This material has the advantages of good insulation, strong corrosion resistance, good load-bearing performance, light in weight etc.
   Assumed that the injected hot water is regarded as a constant heat source for the water system in the bathtub. There is no other heat source in the system.
   Additional assumptions will be described throughout the paper.
### 2.2. Notations

| Abbreviation | Description |
|--------------|-------------|
| $T$          | The temperature of the water in the bathtub. |
| $t$          | Time. |
| $\rho$       | The density of water. |
| $\lambda$    | Coefficient of thermal conductivity of water and air. |
| $c$          | Specific heat at constant pressure of water. |
| $Q_g$        | The rate of heat change. |

Additional notations will be described throughout the paper.

### 3. Water Temperature Model

For the most of model based on conservation of energy, we think that the water temperature in the bathtub is approximately equal when the hot water pours into the bathtub. This assumption is only to simplify the calculation, but in the actual situation, this assumption is difficult to achieve. Besides, we regarded the external heating water as the heat source of the bathtub water system, and didn’t specify the location of the water. Therefore, we are going to optimize it further.

We consider the heat transfer, including inlet water and outlet water, heat exchange of the surface of the bathtub and diffusion in the bathtub interior by establishing the basic principle of heat conduction. Then, we set up the basic equations by the principle of heat conduction.

#### 3.1. Establishment of the Model

Assumed that the heat dissipation of water is only related to the heat conduction of the air, and has nothing to do with other factors. 10% of the heat loss due to the water and the inner wall of the bathtub contact, and 90% due to the heat conduction from water and air. In order to simplify the model, we ignore the interactions between the water and the wall of bathtub.

From the assumption, we think the heat dissipation of water is only related to the heat conduction of the air. Temperature changes with the time and three space coordinates called three-dimensional unsteady heat conduction which can be described by the heat equation,

$$
\rho c \frac{\partial T}{\partial t} - \lambda (\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) = Q_g
$$

Since we keep the bath temperature constant, we have,

$$
\frac{\partial T}{\partial t} = 0
$$

Considering that the height distribution is more uniform, we ignore the influence of this dimension and don’t consider the water temperature stratification. We will study the influence of the inlet position on the temperature distribution from the horizontal. And the three-dimensional model is simplified by a two-dimensional model. We arrive at,

$$
- \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = Q_g
$$

#### 3.2. Calculation and Simulation

It is generally believed that when the gap between the inlet and the outlet is the farthest, the less the heat is lost. So, we install the water inlet on the surface of the water, and the outlet at the bottom of the bathtub.

In the optimized water temperature model, we think the injection of hot water is the heat source. We drop this thought in the following model, we no longer use the injected hot water as a source of heat, but make the boundary value of the temperature distribution map as a fixed value, which means that hot water keeps flowing into the bathtub. Therefore, in the heat conduction formula, we think,
\[ Q_g = 0 \]

Then, we arrive at,

\[-\lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0 \]

We set up a coordinate axis shown in Figure 2.

\[ y=b \] is the heat source with a higher temperature which is defined to be \( U \). \( x=0, y=0 \) and \( x=a \) have the lower temperature which is defined to be \( u \). Through the investigation, we have known that the best bath temperature is 39°C, so we let \( U=45°C \), \( u=39°C \), then, we obtain,

Where we let \( a=1.7m \), \( b=0.8m \).

We use the control variable method to keep the amount of hot water constant. With the above method, the heat source is added from the length, width, corner and all around. Then, we use MATLAB to simulate the temperature distribution of the water temperature model to compare and analyze the best import Position shown in the following figures.

\[ \text{a. Injection of water from length. b. Injection of water from width.} \]

\[ \text{c. Injection of water from angle. d. Injection of water from all around.} \]

**Figure 2.** The coordinate.

**Figure 3.** A heat distribution map of the water in the surface of a bathtub.
Observed from above figures that the position of hot water into the bathtub will affect the distribution of the water temperature of the bathtub.

Add water from the corner. The hot water added to the bathtub can only keep the corner area of the bathtub at a higher temperature. The temperature decreases in the direction of heat transfer. As can be seen that the range of heat radiation is very small.

Add water from the width and length. The heating range is wider than that from the corner, but the heat distribution in the bathtub is uneven. The closer the edge is, the more heat it distributes. However, the center of the bathtub doesn't get more heat.

Add water from the four sides of the bathtub. The hot water spreads around the bathtub and meets in the middle. Compared with the other three ways of adding water, the cross-section temperature of this method is higher and more balanced. From the angle of human comfort, the hot water injected into the bathtub affects the human back, the foot and the abdomen, and the heat spreads along the surface of the body. Therefore, the human body is relatively comfortable.

Compared to the four heating methods, whether from the heat distribution or the human comfort, it is better to add water from all around.

3.3. Test of the model

In water temperature model, we mentioned that the movement of people in the bathtub will widen the range of human activities in the water, increase the contact area between water and human, and increase the forced convection heat transfer coefficient to increase the heat dissipation.

In the model, we discussed the influence of water inlet position on temperature distribution. Now we select the optimal water inlet position and reuse the control variable method to verify the effect of people's movement on the temperature distribution.

We let hot water enter from the surface all around. We want to consider the influence of human's movement on the distribution of water temperature, and we choose the longitudinal section as the research object, so we can only see the influence of the heat source from the upper boundary.

![Figure 4. The coordinate system of people in the bathtub.](image)

We set up the following coordinate system shown in Figure 8. Figure a and Figure b are the range of activities at the time of the static and moving state of human, respectively.

We obtain the temperature distribution diagram of human being at rest and during human's movement respectively by MATLAB simulation shown in Figure 9.
We assume that the temperature of the human body is stable at 37°C. We can see that the person has a small range of activity at rest, so the range of 37°C in Figure a is small. On the contrary, the range of activity becomes larger when people are moving, so the dark blue area is obviously bigger as shown in the Figure b. On the whole, the temperature of the bathtub when the person at rest is lower than the temperature when the person is moving because the area of heat exchange between people and water is different.

As we can see that the temperature of people during human's movement is somewhat lower. From this we can also prove that the human movement will promote the dissipation of heat.

4. Conclusion
We analyze many conditions and study the factors that affect the temperature of bathtub. We build the water temperature model based on the Energy Conservation Law, and the heat conduction law. We get the following conclusions: Our model analyzes a variety of influencing factors and a relatively comprehensive analysis of the problem. We can calculate the rate and the temperature of hot water injected into the bathtub at a constant temperature. The loss of water heat is related to the surface area of the bathtub, the movement of people and so on. The import position of hot water will affect the distribution of water temperature.

According to our water temperature model, the users can adjust the bath temperature of a bathtub which is comfortable to them by controlling the hot water injection rate and the initial temperature of the hot water. We get the water temperature distribution map by MATLAB simulation, which proves our conclusion.

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