Wheel vehicle dynamics real-time simulation for on-board stand-alone moving control system realization

N V Buzunov¹, G O Kotiev² and B V Padalkin³
Bauman Moscow State Technical University, 105005 Baumanskaya 2th st., 5, Moscow, Russian Federation
E-mail: ¹ buzunovnv@bmstu.ru, ² kotievgo@yandex.ru, ³ padalkin@bmstu.ru

Abstract. The main problem of design on-board stand-alone moving control system is necessity of prototype object during all design stages. The real-time simulation models of wheel vehicle dynamics help to cope with such difficulties. These models must be similar to real wheel vehicle object sufficiently. The research describes the order of creation real-time simulation models of wheel vehicle dynamics using implicit numerical implementation method with application of higher order derivatives for solving differential equations system. Also the verification and proof of adequacy method is presented. It is noticed that the created real-time model corresponds to physical wheel vehicle object if the solving step of numerical implementation method no more than 0.001 second. Such real-time models can be used for design on-board stand-alone moving control systems.

Current development trends for vehicle on-board network are responsible for advanced level of automation of both separate systems and entire object control and diagnostic processes. On-board devices conjunction into single hardware-software system represents on-board information-control system. The presented approach into on-board network construction is a basis for stand-alone moving control system construction.

One of the most prospective approaches to the construction of on-board control systems is application of real-time operating controlled objects at stages of debugging and calibration of simulation mathematical models. Virtual prototypes significantly simplify stand-alone moving control system construction process, they make it possible to avoid a number of errors at stages of prototyping, which leads to significant reduction of control system development time and significant cost reduction in the future.

Study object real-time dynamic simulation mathematical model performance is characterized by the following criterion: performance quality depends not only on logical validity of calculations, but also on calculations time [1]. Model functions in real-time, if operating speed corresponds to physic process speed on controlled objects.

Real-time simulation process can be described with the use of diagram, which is illustrated on Figure 1 [2].
Figure 1. Real-time implementation of mathematical model:

\( T_j, T_{j+1} \) – current values (samples) of “real-time”, s;

\( T_{st} \) – time step, s;

\( t_n...t_{m+1} \) – current value of simulation time, s;

\( t_{stn}...t_{stm} \) – numerical integration steps, s;

\( t_{del} \) – duration of delay during numerical integration, s;

\( n...m+1 \) – sequence numbers of numerical integration steps;

\( j...j+1 \) – sequence numbers of “real-time” samples.

Implementation of fixed numerical integration step in cases of non-linear systems study is not possible [3], [4]. This fact defines requirements for selection of non-linear system mathematical model realization numerical method: method-operating algorithm has to provide integration step changing.

During stand-alone moving control system and wheeled vehicle real-time dynamic model cooperative work implementation, one of the main tasks in study of object respond on different control actions. For mathematical model adequacy to real object in defined area, transient process has to be simulated more accurately, than stationary process. Therefore, integration numeric method has to provide dynamic changing of implementation step during simulation. Runge-Kutta method, Miln method and implicit integration method with the use of higher derivatives meet the given requirement [5].

Another requirement to the numerical integration methods is based on the fact that given simulation mathematical model of wheeled vehicle has to operate in real-time mode to make its interoperability with physical control system possible. In order to meet this requirement, numerical integration method has to provide sufficient computational performance, considering solution stability check for recurrent iteration. Approximate solution stability check with the use of Runge-Kutta method significantly reduces computational performance of numerical integration.

Miln method and implicit integration method with the use of higher derivatives are free from this disadvantage. While using Miln method, it is necessary to preliminary estimate four first function samples and corresponding derivatives («starting section»), where after received data is put into approximation formulas for fifth and following samples of the required function. Estimation of initial values can be performed with any other numerical method. When it is needed to reduce given solution
step in order to evaluate first required function sample, following after “starting section”, “starting section” have also to be recalculated with the modified integration step.

In this context, implicit integration method with the use of higher derivatives is represented as an optimal one for realization of real-time dynamic models, which are to be implemented in the framework of development of stand-alone moving control systems of wheeled vehicles.

Numerical simulation can be represented with the following scheme.

Let us assume, that displacement vector $x_i$ and velocity vector $\dot{x}_i$, as projections on generalized coordinates, are known at the moment $t_i$. Position and velocity at the moment $t_{i+1} = t_i + h$, where $h$ is forecast step, are forecasted. Herewith, vectors of time derivatives of miscellaneous orders are estimated in generalized coordinates for the moment $t_i$.

Process of numerical integration of differential equation is based on Taylor expansion of functions $x(t)$ and $\dot{x}(t)$ at a point $t_i$; in consists of three stages.

1. Tentative forecast of displacement and velocity with order of accuracy of $h^4$ and $h^3$, correspondingly:

   $x_{i+1} = x_i + h \cdot \dot{x}_i + \frac{h^2}{2} \cdot \ddot{x}_i + \frac{h^3}{3!} \cdot \dddot{x}_i,$ \( (1) \)

   $\dot{x}_{i+1} = \dot{x}_i + h \cdot \ddot{x}_i + \frac{h^2}{2} \cdot \dddot{x}_i.$  \( (2) \)

2. Tentative forecast correction with accuracy to $h^5$. Based on results, received on first stage, displacement and velocity of object are estimated [5]:

   $x_{i+1} = x_i + h \cdot \frac{(\dot{x}_i + \ddot{x}_i)}{2} - \frac{h^2}{10} \cdot (\dddot{x}_i - \dddot{x}_i) = \frac{h^3}{120} \cdot (\dddot{x}_{i+1} + \dddot{x}_i),$  \( (3) \)

   $\dot{x}_{i+1} = \dot{x}_i + h \cdot \frac{(\ddot{x}_i + \dddot{x}_i)}{2} + \frac{h^2}{12} \cdot (\dddot{x}_i - \dddot{x}_i).$  \( (4) \)

Values, received according to equation (3) and (4), are implemented for modification and correction of second and first derivative values at a point $i+1$. Received at this stage function values and also values of its first, second and third derivatives represent initial approximation.

3. Stability check of estimated values $x_{i+1}, \dot{x}_{i+1}$. According to equation (3) and (4), with the use of second approximation, displacement and velocity at a point $i+1$ are estimated. These values represent third approximation. Then absolute values of differences of second and third approximations are estimated correspondingly. The received difference values should not exceed accuracy, defined before computations had started. In case of fulfillment of this condition, third approximations are used for next simulation step. Otherwise, computation, starting from paragraph 1, should be repeated for a moment $t_{i+1}$, assuming for this iteration step $h/2$.

In the presented paper realization of real-time dynamic model of plane non-linear motion of multi-axle wheeled gear (MWG) with axle arrangement 6x6 is performed on C++ high-level programming language. Implicit integration method with the use of higher derivatives is implemented as an integration method. Presented in papers [6], model of non-linear vehicle motion is implemented as an initial (“reference”) model. This model is realized with Simulink software, which is a component of program complex Matlab™.

Regarded virtual object is a base for development of stand-alone moving control system hardware and software.
In investigated real-time model, motion of vehicle, as of rigid body, is analyzed in horizontal plane of even non-deformable supporting surface and it consists of translational motion of mass center and of rotational motion around mass center (Figure 2) [11].

**Figure 2.** Systems of coordinates, used while simulating of non-linear motion of wheeled vehicle with axle arrangement 6x6:
- $x' - O - y'$ – fixed coordinate system (FCS);
- $x - C - y$ – moving coordinate system (MCS), attached to object mass center;
- $x'' - O_i - y''$ – moving coordinate system (MCS), attached to axis of rotation of $i$-th wheel;
- $\Theta$ – heading angle of wheeled vehicle model;
- $\theta_i$ – angular displacement of $i$-th wheel.

System of equations (5), describing presented motion, makes it possible to calculate current accelerations, based on values of forces and moments, which are interacting a vehicle:

\[
\begin{align*}
    a_x &= \frac{dV_x}{dt} - \omega_z \cdot V_y = \frac{1}{m} \left( P_{xx} + \sum_{i=1}^{6} R_{xi} \right) \\
    a_y &= \frac{dV_y}{dt} + \omega_z \cdot V_x = \frac{1}{m} \left( P_{yy} + \sum_{i=1}^{6} R_{yi} \right) \\
    J_z \cdot \frac{d\omega_z}{dt} &= \sum_{i=1}^{6} M_{xi} + \sum_{i=1}^{6} \bar{M}(\bar{R}_i) \tag{5} \\
    V_x &= \frac{dx'}{dt} = V_x \cdot \cos \Theta - V_y \cdot \sin \Theta \\
    V_y &= \frac{dy'}{dt} = V_x \cdot \sin \Theta + V_y \cdot \cos \Theta \\
    \omega_z &= \frac{d\Theta}{dt}
\end{align*}
\]
where $m$ – vehicle mass; $J_z$ – vehicle mass moment of inertia about an axis $z$; $\vec{V}$ - vehicle mass center velocity vector; $\vec{\alpha}$ - vehicle mass center acceleration vector (absolute derivative of vehicle mass center velocity vector); $\frac{d\vec{V}}{dt}$ - relative derivative of vehicle mass center velocity vector; $\vec{\omega}$ - vehicle turning rate vector; $\theta$ - vehicle turning rate about an axis $x'$; $x'$, $y'$ - coordinates of vehicle mass center in fixed coordinate system; $x$ - $y$ - moving coordinate system, attached to vehicle; $x''$, $y''$ - coordinate system, attached to vehicle $i$-th wheel; $\vec{R}_i$ - vector of undercoat interaction force, affecting $i$-th wheel; $\vec{p}_w$ - vector of air resistance; $M_{tri}$ – moment of resistance to $i$-th wheel turn.

Figure 3 shows wheel analytical model [12], [15], which is implemented for the presented real-time model. The following conventional signs are used:

$\vec{V}_{sl}$ – vector of sliding velocity of wheel contact point relative to supporting surface;

$\vec{V}_{relative}$ – vector of wheel relative velocity at a point of contact area;

$\vec{V}_{port}$ – vector of transport velocity of wheel rotational center $O_i$ (vector of velocity of point MCS $x$ - $C$ - $y$, which is currently connected to $O_i$ of MCS $x''$ - $O_i$ - $y''$, relative to FCS $x'$ - $O$ - $y'$);

$\vec{V}_{wsp_{wv}}$ – vector of linear velocity of translational motion of mass center of wheeled vehicle in FCS $x'$ - $O$ - $y'$;

$\vec{V}_{rotsp_{wv}}$ – vector of velocity of rotational motion of $i$-th wheel center around mass center of wheeled vehicle;

$\alpha$ – rotational displacement of vector of sliding velocity of wheel relatively to $x''$ axis of $x''$ - $O_i$ - $y''$ system;
\( \omega_k \) – angular velocity of \( i \)-th wheel rotation;
\( \vec{R}_i \) – force of \( i \)-th wheel interaction with supporting surface;
\( \theta_i \) – rotational angle of \( i \)-th wheel;
\( M_{f_i} \) – modulus of rolling resistance of \( i \)-th wheel;
\( M_{c_i} \) – modulus of cornering resistance of \( i \)-th wheel.

Table 1 presents input data and model parameters.

| Parameter                                         | Value  |
|---------------------------------------------------|--------|
| Vehicle mass, kg:                                 | 24500  |
| Length, m:                                        | 5,9    |
| Wheel track, m:                                   | 2,1    |
| Mass center height, m:                            | 1,66   |
| Maximum velocity, km/h:                           | 110    |
| Wheel rolling radius, m:                          | 0,615  |
| Coefficient of engagement in \( \chi \) axis direction, \( max \) | 0,6    |
| Coefficient of engagement in \( \gamma \) axis direction, \( max \) | 0,6    |
| Coefficient, which is defining shape of \( \phi(s) \) diagram: | 0,005  |
| Initial linear velocity of mass center \( v_x \), m/s: | 2      |
| Displacement angle of drop arm (const), \( \circ \) | 5      |
| Minimum turn radius during accelerated motion, m  | 77,3   |
| Minimum turn radius during slow down motion, m    | 42,3   |

Research and verification of the presented real-time model is realized in two stages. In the frame work of first stage vehicle moves with constant displacement angle of drop arm with acceleration, during the second stage – with slow down. Simulation is realized with different values of maximum decision step (0,001 s; 0,003 s; 0,005 s). Duration of each heat is 30 s. On completion of all the heats, the received results (of linear and angular model parameters) are compared with Simulink-model verified data.

Figure 4 shows assemblage of linear velocities \( V_x \) of vehicle mass center for different values of maximum simulation step and also velocity \( V_x \) of mass center of verified Simulink-model. Figure 5 shows simulation errors for different maximum steps. Tables 2 and 3 present maximum error values of all the analyzed parameters during accelerated and slowdown motion.
Figure 4. Time dependencies of linear velocity $V_x$ of vehicle mass center:
1 – dependency, received with verified Simulink-model;
2 – dependency, received with analyzed real-time model with maximum simulation step 0.001 s;
3 – dependency, received with analyzed real-time model with maximum simulation step 0.003 s;
4 – dependency, received with analyzed real-time model with maximum simulation step 0.005 s.

Figure 5. Time dependencies of relative errors $\varepsilon$ of velocities $V_x$ of mass center of analyzed model in conditions of different model parameters:
1 – linear velocity error for maximum simulation step 0.001 s;
2 – linear velocity error for maximum simulation step 0.003 s;
3 – linear velocity error for maximum simulation step 0.005 s.
Table 2. Maximum relative simulation error values for non-linear accelerated vehicle motion.

| Model parameter          | Error, $\text{max}_{\text{step 0,001 s}}$ | Error, $\text{max}_{\text{step 0,003 s}}$ | Error, $\text{max}_{\text{step 0,005 s}}$ |
|--------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| Mass center velocity, $v_x$ | 4 %                                      | 16 %                                     | 42 %                                     |
| Mass center velocity, $v_y$ | 5 %                                      | 23 %                                     | 90 %                                     |
| Angular velocity, $\omega$ | 6 %                                      | 45 %                                     | 57 %                                     |
| Mass center $X$-coordinate | 0,45 %                                   | 16 %                                     | 41 %                                     |
| Mass center $Y$-coordinate | 9,5 %                                    | 55 %                                     | 85 %                                     |
| Angular displacement, $\Theta$ | 3 %                                      | 19 %                                     | 54 %                                     |

Table 3. Maximum relative simulation error values for non-linear slowdown vehicle motion.

| Model parameter          | Error, $\text{max}_{\text{step 0,001 s}}$ | Error, $\text{max}_{\text{step 0,003 s}}$ | Error, $\text{max}_{\text{step 0,005 s}}$ |
|--------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| Mass center velocity, $v_x$ | 0,65 %                                   | 5 %                                      | 40 %                                     |
| Mass center velocity, $v_y$ | 3 %                                      | 15 %                                     | 53 %                                     |
| Angular velocity, $\omega$ | 5 %                                      | 26,5 %                                   | 50 %                                     |
| Mass center $X$-coordinate | 9,5 %                                    | 35 %                                     | 72 %                                     |
| Mass center $Y$-coordinate | 1 %                                      | 30 %                                     | 51 %                                     |
| Angular displacement, $\Theta$ | 2 %                                      | 31 %                                     | 54 %                                     |

Figure 6 shows vehicle mass center tracks, which are plotted as a result of slowdown heats of “reference” and analyzed objects in conditions of different maximum values of simulation step. It should be noticed, that increase in simulation step results in “delayed” motion of simulation object in relation to verified model, but for the most part tracks match in superimposition, except final vehicle position.
Figure 6. Vehicle motion tracks in conditions of different model parameters:
1 – motion track of «reference» model;
2 – motion track for maximum simulation step 0.001 s;
3 – motion track for maximum simulation step 0.005 s;
a – vehicle position of «reference» model simulation;
b – vehicle position of simulation with maximum simulation step 0.001 s;
c – vehicle position of simulation with maximum simulation step 0.005 s.

The computational experiments performed show that simulation real-time mathematical model perform and adequacy significantly depends on selected maximum solution step. Based on data, presented in tables 2 and 3, the conclusion is that analyzed model can be implemented as a vehicle emulator, while solution steps are no greater than 0.001 s. Such conclusions are compatible with native and foreign researches results, presented in [18], [19], [20] and [21].

The presented results show, that it is possible to apply real-time dynamic models of wheeled vehicle motion in development of on-board stand-alone moving control systems.

The article is written based on the results of work carried out with the financial support of the Ministry of Education and Science of the Russian Federation under the agreement No. 14.574.21.0178 (Unique work identifier: RFMEF57417X0178).

References
[1] Scherbin M A 2015 Modern on-board information and control system of automotive vehicles J Automobile engineers association 3 (92) pp 26-29
[2] Belanger J Vennie P and Paquin J N 2009 The what, where and why of real-time simulation J Transactions on Power Delivery 24 pp 390-399
[3] Liu C S and Peng H 1998 A state and parameter identification scheme for linearly parameterized systems ASME J of Dynamic Systems, Measurement and Control 120 pp 524-528
[4] Tavoosi V, Kazemi R and Hosseini S M 2014 Vehicle handling improvement with steer-by-wire system using hardware in the loop method J of Applied Research and Technology 12 pp 769-781
[5] Demidovich B P, Maron I A and Shuvalova E Z 1967 Numerical methods of analysis
(Moscow: Science) p 368

[6] Gorelov V A, Kotiev G O and Beketov A A 2008 Movement mathematical model of all-wheeled vehicle *J Automobile engineers association* 1(48) pp 50-54

[7] Keller A, Aliukov S 2015 Effectiveness of Methods of Power Distribution in Transmissions of All-Wheel-Drive Trucks *SAE Technical Paper* 2015-01-2732

[8] Keller A, Aliukov S, Anchukov V, et al. 2016 Investigations of Power Distribution in Transmissions of Heavy Trucks *SAE Technical Paper* 2016-01-1100

[9] Zhileykin M M, Kotiev G O and Nagatsev M V 2018 Synthesis of the adaptive continuous system for the multi-axle wheeled vehicle body oscillation damping *IOP Conf Series: Materials Science and Engineering* **315** issue 1 article number 012031

[10] Novikov V V, Pozdeev A V and Diakov A S 2015 Research and testing complex for analysis of vehicle suspension units *International Conf on Industrial Engineering, ICIE* **129** pp 465-470

[11] Skotnikov G I, Jileykin M M and Komissarov A I 2017 Increasing the stability of the articulated lorry at braking by locking the fifth wheel coupling *IOP Conf Series: Materials Science and Engineering* **315** issue 1 article number 012027

[12] Gorelov V A and Komissarov A I 2016 Mathematical model of the straight-line rolling tire - Rigid terrain irregularities interaction *2nd International Conf on Industrial Engineering, ICIE 2016* **150** pp 1322-28

[13] Keller A V, Gorelov V A, Vdovin D S, et al. 2015 Mathematical model of all-terrain truck *Proceedings of the ECCOMAS Thematic Conference on Multibody Dynamics* pp 1285–96

[14] Keller A V, Gorelov V A and Anchukov V V 2015 Modeling truck driveline dynamic loads at differential locking unit engagement *Procedia Engineering* **129** pp 280–287

[15] Vol’skaya N S, Zhileykin M M and Zakharov A Y 2017 Mathematical model of rolling an elastic wheel over deformable support base *IOP Conf Series: Materials Science and Engineering* **315** issue 1 article number 012028

[16] Keller A and Aliukov S 2015 Efficient power distribution in an all-wheel drive truck *Lecture Notes in Engineering and Computer Science* **2218** pp 1201-1206

[17] Keller A and Aliukov S 2015 Methodology of System Analysis of Power Distribution among Drive Wheels of an All-wheel-drive Truck *SAE Technical Paper* 2015-01-2788

[18] Choi G J, Yoo Y M and Lees K P 2000 A real-time multibody vehicle dynamic analysis method using suspension composite joints *International J of Vehicle Design* **24** pp 259-273

[19] Gyoojae C 2001 Efficient solving methods exploiting sparsity of matrix in real-time multibody dynamic simulation with relative coordinate formulation *J of Mechanical Science and Technology* **15** pp 1090-1096

[20] Sung-Soo K and Jeong W 2007 Subsystem synthesis method with approximate function approach for a real-time multibody vehicle model *J Multibody System Dynamics* **17** pp 141-156

[21] Weidong P and Papelis Y E 2005 Real-time dynamic simulation of vehicles with electronic stability control: Modeling and validation *International J of Vehicle Systems Modelling and Testing* **1** pp 143-167