MARKOV MODELS OF PARAMETRIC CONTROL OF ELECTRICAL SYSTEMS CONDITION AT THE OPERATION STAGE

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Abstract. The paper considers the state of control electrical systems (ES). It shows that the most critical element of this process is the parametric control of the ES state, which implies the solution of tasks related to ES parametric synthesis, control and prediction. Key definitions used in work are formulated. The parametric control strategy of the ES state at various stages of their life cycle is considered. It is shown that the real ES operation has two features. The first one is that the transition from one state to another mainly depends on a certain state of the ES operating in a given mode, and practically does not depend on the modes that the system has already passed through. The second feature is caused by the fact that the exposure time in each mode may be both deterministic and accidental. The conclusion is drawn on the possibility to model the ES operation by semi-Markov models with a discrete set of states. Models of changing the ES state in the modes of storage and intended application are synthesized. The process of creating the models of operating control modes of automatic control system of a ship power plant is considered as an example.

1. Introduction
The ES state control includes a broad range of tasks related to analysis and synthesis at various phases and stages of their life cycle [1, 2, 4]. Until recently these tasks were considered separately irrespective of each other, though generally all of them are closely connected. They are united by the fact that their solution covers the study of focused and random processes of changing the structure and parameters of the ES elements defining the quality of their performance, as well as methods and principles of controlling the above. The most important element of the ES state control is the parametric control ensuring the solution of tasks related to the ES parametric synthesis, control and prediction. The problem of parametric control of the ES state formulated in works [1] implies complex solution of these tasks on the basis of information on the performance limit of a system [5].

The task of designing the mathematical models that describe the processes of changing the ES state in the modes of storage and intended application is based on statements and analysis of basic definitions related to parametric control of the ES state and the use of a mathematical apparatus of the Markov chains. At the same time, the basic generalized model of the ES multiduty operation revealing possible states of a system, which differ in quality, is considered in works [1].
2. Materials and methods
Let us define the basic concepts used in this work. From information and energy point of view, the ES is understood as a technical system with electric energy being its main energy and data carrier. From functional and morphological points of view, the ES is understood as a technical system aimed to generate, distribute, convert, utilize the electric energy and ensure control of these processes. The ES elements represent electrical devices (functional representation) or electrical products (morphological representation), for example an electrical complex.

The state is the ES internal determination (substratum) is characterized within a considered moment by features established in the ES technical documentation, which serve as initial conditions for processes of their further change.

The ES state in any fixed moment is characterized by a set or a vector of parameters. They include the following:

- input parameters \( \mathbf{u} = (u_1, u_2, \ldots, u_k) \) characterizing the setting action \( \mathbf{u}(t) \), which are observed at the inlet to the systems. These actions are divided into control actions characterizing the ES operating modes and test actions present within the setup and technical diagnosis modes;

- external parameters \( \mathbf{V} = (v_1, v_2, \ldots, v_f) \) characterizing properties of external environment in relation to the ES (temperature, humidity, vibration, radiation) and affecting its performance;

- internal parameters \( \mathbf{X} = (X_1, X_2, \ldots, X_n) \) characterizing the state of the ES fitting elements and also called the initial parameters, which include the parameters of elements, such as resistance, induction, capacity, mass, inertia moment, rigidity of elastic coupling, and functions resulting from these parameters and having certain physical indexes (gain ratio, time constants);

- output parameters \( \mathbf{Y} = (Y_1, Y_2, \ldots, Y_m) \) characterizing the ES properties required by the consumer. These are called functional parameters, i.e. the functional relation of the ES phase variables \( \mathbf{Z} = (Z_1, Z_2, \ldots, Z_c) \) and parameters being the boundary values of external variable ranges ensuring system operability. These parameters usually serve as indicators of the ES quality.

The relation of the ES outputs with its inputs may be presented by the equation

\[ \mathbf{Y} = F(\mathbf{X}, \mathbf{u}, \mathbf{V}, t), \]

where \( F \) – telecoms operator.

The ES state control is understood as a targeted process of changing the control action, internal parameters and structure in order to prevent and eliminate failures of system elements and to achieve the optimum performance according to the specified criterion.

There are three control loops and, hence, three types of the state control – parametric, coordinate and structural (Fig. 1) [1]. The necessary condition to ensure any control loop is the possibility to define the state of the ES and its elements at any given time. The parametric control implies a deliberate action on the ES internal (initial) parameters \( \mathbf{X} \).

The main strategy of parametric control of the ES state is to ensure proper performance of these systems at all stages of their life cycle. At the same time, the design stage is intended to solve the task of choosing the optimum initial parameters according to the performance limit and the task to set the allowable limits changing these parameters, which results in the approximation of the performance limit boundary. The production stage is aimed to solve the tasks of the ES setup, which to some extent comprises a special case of tasks related to parametric synthesis implying the control of the adjustable parameters only. The operation stage defines the ES state, including the assessment of its performance limit and the ES setup [2-4].
Figure 1. The ES state control model

The real process of the ES operation has two features. The first one is that the transition from one state to another mainly depends on a certain condition of the ES operating in a given mode, and practically does not depend on the modes that the system has already passed through. The second feature is caused by the fact that the exposure time in each mode may be both deterministic and accidental. Hence, it is possible to model the ES operation via the semi-Markov models with a discrete set of states. Since the graph (Fig. 2) contains only the communicating states and is strongly connected, the semi-Markov process is considered ergodic.

Let us consider the task of controlling the ES multiduty operation. The mode may be controlled by changing the transition probabilities \( Q_{ij}(t) = \pi_{ij} F_0(t) \) of the semi-Markov process. Here \( \pi_{ij} = P(\nu_{n+1} = j \mid \nu_n = i) \) – conditional probability of the process transition to \( j \) state from \( i \) state at the
n-stage; $\nu_n = \nu(\tau_n) \in D$ – number of the current mode after a regular transition at the timepoint $T_n$; $D = \{1,2,...,m\}$ – set of operation mode numbers; $\sum_{j=1}^{m} \pi_{ij} = 1$.

The evolution of the semi-Markov process concerning transition points $T_n$, $n = 0,1,2,...$ is described by the equation of state $P(T_n + t) = Q(t)P(T_n)$, where $P(T_n) = p_i(T_n)_{m \times 1}$ – process state probability vector after transition at the timepoint $T_n$; $p_i(T_n) = P(\nu(T_n) = i)$. Thus, its discrete part $\nu_n = \nu(T_n)$, $n = 0,1,2,...$ forms the imbedded Markov chain with transition probability matrix $\bar{\Pi} = (\pi_{ij})_{m \times m}$. Due to ergodicity, the evolution of this process at a relatively long observation interval is defined by the evolution of the imbedded Markov chain. Thus, instead of the process the task of controlling the ES state may consider the Markov chain with nonrandom transition points $t_n$ [1, 5].

Let $D(n) \in D$ be the subset of states providing for the transition of the Markov chain at the timepoint $t_n$ from the state $i \in D(n-1)$. Then the timepoint of $n$-transition will equal:

$$t_n = \min_{i \in D(n-1)} \{t_{n-1} + \tau_i\}, \quad n = 1,2,...; \quad t_0 = 0,$$

where $\tau_i = \sum_{j \in D(n)} \pi_{ij} \tau_{ij}$ – average time of the process in $i \in D(n-1)$ state before transition to the subset $D(n)$. Since the transitions on the graph are possible only along the arcs, the set $D(n)$ contains only those graph nodes, which are connected by arcs with the node $i \in D(n-1)$:

$$D(n) = \{j : (i,j) \in \mathcal{E}; \quad i \in D(n-1)\}.$$ Setting the initial node $j_0$ thus initiating the random walk along the graph in initial timepoint $t_0 = 0$ it is possible to determine the sequence of sections $D(n)$ and transition moments $t_n$ by the above formulas. For nodes belonging to sections $D(n-1)$, $D(n)$, the transition probability $Q_{ij}(t_n)$ is defined by the expression:

$$Q_{ij}(t_n) = \pi_{ij}(t_{n-1} + \tau_i)/t_{n-1} + \tau_i$$

and for other transitions it equals zero. Provided the statistical information is available such approach allows solving the task of synthesis to control the ES state.

In the storage modes $J_1$, $J_2$, $J_3$, $J_4$ and the intended application modes $J_7$, $J_{10}$ the ES state changes continuously under the influence of gradual and sudden failures of random intensity. To design the mathematical model let us present the initial set of the ES states $S = \{\rho_0,\rho_1,\rho_2,...,\rho_d\}$ as three disjoint subsets: $S_0 = \{\rho_0\}$ – operational condition with initial performance limit $\rho_0$; $S_1 = \{\rho_1,\rho_2,...,\rho_{d-1}\}$ – set of operating states, each of which corresponds to a certain performance limit; $S_2 = \{\rho_d\} = S_{1p}$ – nonoperable state. The combination $S_p = S_0 \cup S_1$ forms the set of the operating ES states. Under the influence of gradual failures, the process $s(t) \in S$ takes place in all operating states $S_p = \{\rho_0,\rho_1,...,\rho_{d-1}\}$ consistently before reaching the nonoperable state $S_{1p} = \{\rho_d\}$. For simplification of the model it is assumed that the ES has no changeover and under the influence of sudden failures the process $s(t)$ at once shifts from any operating state $r \in S_p$ into a nonoperable state $S_{1p}$. The graph of this process is shown in Fig. 3. Since the transition intensity $\Lambda_j$ at each observation interval $[t_{n-1}, t_n]$ is constant, then the changes of the ES state are described by the conditional Markov process.

Let us introduce the probability vector $P(t)$ of the ES state at timepoint $t$:

$$P(t) = (P_0(t), P_1(t),...,P_d(t))^T, \quad P_i(t) = P(s(t) = r), \quad \sum_{r=1}^{d} P_r(t) = 1, \quad t \in [t_{n-1}, t_n].$$

The general solution under initial conditions $p(0) = p(t_{n-1})$ will be as follows: $p(t) = e^{\Lambda(t-t_{n-1})}p(0) = \Pi_{\Lambda}(t)p(0), \quad$ where $\Pi_{\Lambda}(t) = e^{\Lambda(t-t_{n-1})}$ – functional matrix of the ES state transformation; $\Lambda = \lambda_{ij}^{(d+1) \times (d+1)}$. 

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\[ \tilde{\lambda}_{ij} = \sum_{k=1}^{n} \tilde{\pi}^{(k)}_{ij} / \sum_{k=1}^{n} \tau_k \] - estimates of intensity matrix elements determined according to the results of the periodic ES state control; \( \tilde{\pi}^{(k)}_{ij} \) - statistical estimation of transition probability \( P\left(s_k = j \mid s_{k-1} = i\right) \) at \( k \)-observation interval; \( \tau_k = t_k - t_{k-1} \) - duration of this interval.

![Figure 3. Graph of the ES state change at storage and intended application modes](image)

If the performance limit, which is understood as the approximation of an actual state vector of a system to its maximum permissible value [1] is not defined, then the ES may have either of the three states: \( S_0 (\rho_0 = 1) \); \( S_1 (\rho_1 \in [1,0]) \); \( S_2 (\rho_0 = 0) \). The transition from state \( S_0 \) and \( S_1 \) into \( S_2 \) takes place under the influence of a Poisson flow of failures with intensities \( \lambda_{01}, \lambda_{02}, \lambda_{12} \). Let us find the probabilities of the ES states through time \( t \) if at \( t=0 \) the ES is in \( S_0 \) condition. According to the state graph, let us write the equations of the Markov process for initial conditions \( P_0(0) = P_0; P_1(0) = P_1; P_2(0) = 0 \).

\[
\frac{dP_0(t)}{dt} = -(\lambda_{01} + \lambda_{02})P_0(t); \quad \frac{dP_1(t)}{dt} = \lambda_{01}P_0(t) - \lambda_{12}P_1(t);
\]
\[
\frac{dP_2(t)}{dt} = \lambda_{02}P_0(t) + \lambda_{12}P_1(t); \quad P_0(t) + P_1(t) + P_2(t) = 1.
\]

By solving the set of equations we get:

\[
P_0(t) = P_0(0)e^{-\left(\lambda_{01} + \lambda_{02}\right)t}; \quad P_2(t) = 1 - P_0(t) - P_1(t)
\]
\[
P_1(t) = P_1(0)e^{-\lambda_{12}t} + \frac{\lambda_{01}P_0}{\lambda_{12} - \lambda_{01} - \lambda_{02}} e^{-\left(\lambda_{01} + \lambda_{02}\right)t}.
\]

The equations describe the evolution of the ES state at storage and intended application modes for the given conditions. The transition intensity is defined by continuous change of their initial parameters. Similarly, it is possible to obtain the equations describing the evolution of the ES state in other operating modes [1].

3. Results and discussion

Let us consider the design of models to control the performance of the automated control system (ACS) of a ship power plant (SPP). This system may be presented as a set of control subsystems (CSS), each of which ensures a specific control function. Such CSS include the following: SBPSS – subsystem of backup power startup and shutdown; SASG – subsystem of automatic synchronization of generators; SDALSNF – subsystem of distribution of active loading and stabilization of network frequency; SGSS – subsystem of generator sets (GS) startup at supply failure on feeder switchboard strip of the SPP. Let \( EC_0 \) be the operating state and \( PF_0 \) – correct operation of all ACS SPP at a given time; \( EC_1 \) and \( PF_1 \), \( EC_2 \) and \( PF_2 \), \( EC_3 \) and \( PF_3 \), \( EC_4 \) and \( PF_4 \) – operating state and correct operation
according to SBPSS, SASG, SDALSNF, SGSS. The conditions of performance and correct operation are as follows:

$$EC_0 = EC_1 \land EC_2 \land EC_3 \land EC_4 = \bigcap_{i=1}^{4} EC_i, \quad PF_0 = PF_1 \land PF_2 \land PF_3 \land PF_4 = \bigcap_{i=1}^{4} PF_i .$$

The ACS SPP operation is characterized by the change of its operation modes under certain conditions. Here such modes are understood as the implementation of a control function of the corresponding subsystem.

Let us define $A_1, A_2, A_3, A_4$ as the ACS operation modes, each of which is ensured by SBPSS, SASG, SDALSNF, SGSS respectively. Let us also define the transition conditions: $\lambda_{1-2}$ – from mode $A_1$ to mode $A_2$; $\lambda_{2-3}$ – from mode $A_2$ to mode $A_3$; $\lambda_{3-4}$ – from mode $A_3$ to mode $A_4$; $\lambda_{1-3}$ – from mode $A_1$ to mode $A_3$; $\lambda_{1-4}$, $\lambda_{2-4}$, $\lambda_{3-4}$ – from modes $A_1, A_2$ and $A_3$ to mode $A_4$. The transition from $A_4$ to $A_1$ is unconditional. In this case, the ACS SPP operation may be presented as an oriented graph (Fig. 4) where the operation modes correspond to the vertex set and the set of transitions at the interchange of these modes corresponds to the arc set.

The operation model of ACS SPP, making it possible to calculate the probabilities of a system in either state, is as follows:

$$\frac{dP(A_1(t))}{dt} = \lambda_{2-1}P(A_2(t)) + \lambda_{3-1}P(A_3(t)) + \lambda_{4-1}P(A_4(t)) + (\lambda_{1-2} + \lambda_{1-3} + \lambda_{1-4})P(A_1(t))$$

$$\frac{dP(A_2(t))}{dt} = \lambda_{1-2}P(A_1(t)) - (\lambda_{1-2} + \lambda_{2-3} + \lambda_{2-4})P(A_2(t))$$

$$\frac{dP(A_3(t))}{dt} = \lambda_{1-3}P(A_1(t)) + \lambda_{2-3}P(A_2(t)) - (\lambda_{1-3} + \lambda_{3-4})P(A_3(t))$$

$$\frac{dP(A_4(t))}{dt} = \lambda_{1-4}P(A_1(t)) + \lambda_{2-4}P(A_2(t)) + \lambda_{3-4}P(A_3(t)) - \lambda_{4-1}P(A_4(t))$$

$$P(A_1(t)) = 1 - P(A_2(t)) - P(A_3(t)) - P(A_4(t)).$$

![Figure 4. Operation model of ACS SPP presented as a graph](Image)

The system check in $A_1, A_2, A_3$ and $A_4$ modes indicates the correct operation of ACS SPP in general, however, for instance, the information on the compliance with transition conditions is required for its diagnostics up to the depth of a subsystem. Therein the transition conditions $\lambda_{1-2}$, $\lambda_{2-3}$ appear only due to the control function of SBPSS and SASG respectively. The correct performance of
SDALSNF functions does not directly influence the $\lambda_{3-1}$ formatting. The transition conditions have discrete nature, are described in the language of logical algebra and are similar for almost all vessels. The work considers similar examples of creating the Markov models to ensure the marine ES state control. The works [7] describe structural methods of the ES state control within control and performance restoration modes of marine electrical systems.

4. Conclusions
The obtained mathematical description of the parametric control of the ES state in storage and intended application modes provides for quantitative assessment of probabilities of a system in any state and allows predicting the behavior of a system in the future.

The practical application of the considered models requires the data on the ES transition intensity from one state to another. Such data may only be obtained based on statistical information on the pattern of change of the ES initial parameters. These issues are considered in works [1, 4, 6].

References
[1] Saushev A V 2014 Control methods of electrical systems states of marine transport (SPb.: AMSUMIS Publishing House)
[2] Yurkov N K 2014 Failure risks of complex technical systems. Reliability and quality of complex technical systems 1(5) 18-24
[3] Abramov O V 2016 Choice of optimum setting parameters of technical devices and systems. Automation and telemechanics 4 55-66
[4] Abramov O V 2017 Planning of preventive parameter change of technical devices and systems. Computer science and control systems 3 (53) 55-66
[5] Saushev A V 2013 Mathematical description of performance areas of electromechanical systems. Mechatronics, automation, control 6(147) 7-13
[6] KuznESov S E 2015 Fundamentals of operating the marine electrical equipment and automation equipment (SPb.: AMSUMIS Publishing House)
[7] Karakayev A B, Lukanin A V, Khekert E V 2016 Development of methodology, methods and models to analyze the influence of various options of structure and modes of maintenance and performance restoration of marine electrical systems (Part 1). Marine transport operation 3 (80) 54-60