Multiple extended stealth target tracking from image observation based on non-linear sub-random matrices approach

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Abstract
The detection and tracking of extended stealth targets (ESTs) is a challenging task in radar technology, especially if from image observations because of the fluctuations of radar cross section. To overcome this challenge, multi-Bernoulli (MB) filter can be used to extract the extended target (ET) states in efficient and reliable manner. Recently, the MB-filter-based random matrices model (RMM) approach has been proposed for ellipsoidal ET tracking with additional state variables. However, RMM-MB filter is demonstrated with known detection profile, which is unsuitable for EST tracking. Thus, a joint detection and tracking of multiple ESTs based on track-before-detect (TBD) approach, which is an efficient way to track low-observable ESTs, is proposed. In EST-RMM-TBD scenarios, although the extension ellipsoid is efficient, it may not be accurate because of some missing useful information, such as size, shape, and orientation. To address this, an EST-sub-RMM-TBD composed of sub-ellipses is introduced, each representing an RMM. Based on such models, a sub-RMM-MB-TBD filter is used to estimate the kinematic states and extensions of sub-objects for each EST. Furthermore, a sequential Monte Carlo (SMC) implementation to estimate non-linear kinematic EST state is applied. The results indicate that the proposed SMC-sub-RMM-MB-TBD filter has more accurate cardinality estimation and smaller optimal sub-pattern assignment errors than the recent extended tracking filters.

1 | INTRODUCTION

During the last decades, the counter-stealth technology has proven to be one of the most effective approaches in modern radar systems and is ongoing in the world [1,2]. Joint detection and tracking for multiple stealth targets (STs) is always a challenging problem in modern radar systems, especially from image observations with an unknown detection profile because of the fluctuation of radar cross section (RCS) [1–3]. In multiple ST tracking, the objective is to simultaneously estimate the number of STs and their states from a sequence of noisy measurements. Generally, each ST is assumed to be a point target, which produces at most one measurement per scan. This assumption is valid when the ST is far away from the radar position or when the resolution of the radar is low. For the high-resolution radar, or the distance between the ST and radar is small, the radar may be able to resolve individual features on the ST. Each ST may generate more than one measurement per scan, and the assumption of point STs is not appropriate. Hence, extended ST (EST) tracking arises [4,5]. Modern sensors (e.g., phased-array radar) with increased resolution can generate strong fluctuating number of measurements from EST, which provides not only target kinematic information but also more information about the target extension including size, shape, and orientation. The tracking of EST that might occupy more than one sensor cell leads to the so-called EST tracking problem. In EST tracking, the objects give rise to a varying number of potentially noisy measurements from different spatially distributed measurement sources, also referred to as reflection points. The shape of the EST is usually unknown and can even vary over time, and the objective is to recursively determine the shape of the EST and its kinematic parameters. Due to the non-linearity of the resulting estimation problem, tracking a single EST is a highly complex problem for which elaborate non-linear estimation techniques are required. The conventional EST
tracking approaches depend on measurements (position and range rate), which are extracted by thresholding the received signal in a surveillance radar [2,4,5]. The primary role of thresholding is to reduce the data flow. For a certain signal-to-noise ratio (SNR), the threshold determines detection probability and false alarm, while the probability of false alarm affects the complexity of the data association problem. Generally, a higher density of false alarms requires more sophisticated data association algorithms. In this case, the common approach applies a threshold and treats those cells of exceeding the threshold. It is acceptable if SNR is high [6,7]. For low-SNR targets, the threshold must be less than the sufficient probability to avoid a high rate of false detections. For low-observable targets such as ST, there is a considerable advantage of using unthresholded data in simultaneous detection and tracking from image observations, which is known as track-before-detect (TBD) [3]. The TBD algorithms for ET tracking mainly have three implementations: particle-filter-based TBD (PF-TBD) [8], dynamic-programming-based TBD (DP-TBD) [9], the histogram probabilistic multi-hypothesis-tracker-based TBD (H-PMHT-TBD) [10], and Hough-transformation-based TBD (HT-TBD) [11]. These algorithms are the parametric approaches for multiple ET tracking, which modelling to TBD. These extended tracking filters introduced the stochastically evolving shape matrix that defines the target extent, which allows for targets with unknown and changing size and shape. The model follows the random matrix model (RMM) as this integrates more readily with the mathematical framework of these filters. The RMM approach was initiated by Koch in [12] and then extended in [13]. This approach estimates the kinematic state and physical extension (shape, size, and orientation) considering that the ET can be treated as one target. The simple form of this approach is presented with one-step prediction and update being linear in the state and the extension. Its one-cycle computation is comparable to that of the Kalman filter.

In linear tracking scenario, the ET observations travel along straight lines and do not overlap. Hence, H-PMHT-TBD and DP-TBD perform well, allowing easy and accurate predictions. On the other hand, these filters perform considerably worse and degraded, when the ET observations travel close and overlaps. The RMM assumes an elliptical shape for the target's extent, and for targets with irregular, non-ellipsoidal extents, the shape can be approximated as a combination of several elliptically shaped sub-objects. Using multiple instances of a simpler shape alleviates the limitations posed by the implied elliptic object shape, and also retains, on a sub-object level, the relative simplicity of the RMM [14]. The approaches of using an RMM with TBD technique, initiated in Refs.[9,10], have limitations in tracking practical EST because they simplify the extension of an ET as an RMM. This simplification is valid only when the true extension can be adequately approximated by an ellipsoid. However, in practice, the scattering of electromagnetic energy from an EST is rather complicated and the extension cannot be approximated by one ellipsoid. Therefore, the object orientation cannot be identified. It becomes more severe when tracking a group of ESTs. In these cases, the extension details become important for identification and recognition. Recently, a Bayesian approach of extended object tracking by sub-RMM has been presented in Refs. [15,16]. In [15], a single ET tracking is given where the ET is a combination of sub-RMM with kinematic state vector and extent matrices. In [16], the non-ellipsoidal ET model [15] is used in a joint tracking and classification framework. This algorithm performs well only under specific detection parameters; otherwise, it can significantly reduce the tracking performance. With these approaches, the single-target extensions can be modelled by sub-RMM, and more detailed information about size, shape, and orientation can also be obtained. But, thus far, only a single-target solution has been presented. In Ref. [17–20], the authors proposed the multi-Bernoulli (MB) filter for tracking multi-target. This MB filter is the exact Bayes filter that propagates the parameters of a Bernoulli random finite set (RFS) for a dynamic system, which can randomly switch on and off. In Ref. [21], the authors showed that the MB filter overestimates the number of targets and proposed a cardinality-balanced MB filter to reduce the cardinality bias. Then, the sequential Monte Carlo (SMC) and Gaussian mixture (GM) implementations for the MB filter were proposed for generic and linear Gaussian dynamic and measurement models. The MB recursion propagates not only the moments and cardinality distributions but also the approximate MB posterior density. Therefore, the key advantage of the SMC-MB filter over the other recent filters is that the MB representation of the posterior density allows reliable and inexpensive extraction of state estimates. Therefore, in [2], the authors presented an approach of multiple EST tracking based on sub-RMM, derived a new variant of EST-MB filter that simultaneously estimates sub-ellipsoidal shape and kinematics for each EST, and gave its Beta-GM (BGM) implementation for a linear dynamic model of ESTs. Further, they also showed that the proposed MB filter can extract EST states efficiently and reliably. However, the MB-EST-BGM filter cannot be directly applied to non-linear non-Gaussian models, especially from image observations with TBD approach. With the assumption of point STs, the SMC implementation of MB-TBD filter for non-linear ST models [3] is proposed, which obtains a higher estimation accuracy than other filters, as the SMC-MB-TBD filter does not need the extra clustering method to extract the ST states. In Refs [22,23], the authors proposed SMC implementation of MB filter for ET tracking with known detection profile. However, these filters are not suitable for EST tracking because of its unknown detection profiles. Therefore, in this work, we will mainly study the SMC implementation of sub-RMM-MB filter with TBD approach for ET tracking and propose a novel method to choose the proposal density. Finally, we will evaluate the proposed filter’s performance in comparison with RMM-MB and H-PMHT filters through a non-linear example.

The rest of this article is structured as follows: Section 2 gives an EST tracking problem formulation, Section 3 proposes the model of sub-RMM-MB-TBD recursions, Section 4 outlines SMC implementation of the sub-RMM-MB-TBD filter, Section 5 presents the implemented pruning and merging scheme,
Section 6 presents the results of simulations, and Section 7 contains conclusions.

# PROBLEM FORMULATION

In the most recent research for ET tracking \cite{2,9,10,14,15,22,24,26}, the shape model of an ET is usually assumed by the RMM because it is an optimum combination of an informative model for ET and low computational complexity. In the framework of RMM, the extended state is modelled by a symmetric-positive definite (SPD) random matrix to simplify the extension of ET as an ellipsoid. This simplification is valid only when the true extension can be adequately approximated by an ellipsoid. However, the ESTs are characterized by a number of reflection points spreading over the extensions \cite{2}. Therefore, the SPD-RMM may not be accurate enough because of some missing useful information about the complexity of EST such as shape and orientation. In addition, the SPD-RMM does not address the following problems adequately:

- The time variation: the EST extension can change in shape, orientation, and size over tracking time.
- Distortion: practically, the observations of an EST may be distorted from the true extension in shape, size, and orientation, depending on the radar-to-target geometry. In Ref. \cite{2}, we discussed the estimation problem of an extended linear state for ESTs based on sub-RMM approach. In Section 2.1, we will discuss the estimation problem of an extended non-linear state for ESTs based on sub-RMM.

## 2.1 Dynamic model for TBD-sub-random matrix

Since the RMM can only represent an ellipsoid, in \cite{2}, we used multiple ellipsoids for each EST, which is known by sub-RMM, to approximate the EST extension in linear dynamic scenario. This idea is illustrated in Figure 1. As observed, the sub-RMM gives more detailed information about the size, shape, features, classification, and orientation of different EST models. Thus, we describe an EST extension by a combination of sub-ellipses or sub-RMM, each represented by an RMM. The sub-RMMs have different extension dynamics and/or initial parameters. Belonging to the same EST, the sub-RMM also shares some common dynamic characteristics, which constrain the individual dynamics. This assumption might be true when ESTs are very close to the radar. In this idea, the EST features and classification are structured and represented accurately by the proposed sub-RMM. In this work, we extend the dynamic model of ESTs to be non-linear kinematic scenario. When the EST extensions are modelled as a non-linear sub-RMM, assuming \( n^e_k \) is the number of sub-RMMs used to estimate the non-linear model of sub-object extension. Choosing \( n^e_k \) is usually not difficult since a small \( n^e_k \) may be enough for a practical EST. For example, given six sub-RMMs for F-117 and F-35 EST approximations as shown in Figure 1, we may be able to tell an airliner from a fighter, so \( n^e_k = 6 \) is enough for F-22 EST model. Generally, \( n^e_k \) can be time-varying in EST non-linear tracking. Suppose at time \( k \), a set of the augmented EST state based on sub-RMM approach is denoted by

\[
X_k = \left\{ \left\{ \{ x^{(e)}_k \} \right\}_{s=1}^{N_{x,k}}, x^{(e)}_k \right\} \quad \left( x^{(e)}_k, X^{(e)}_k \right) \quad (1)
\]

where \( N_{x,k} \) is the unknown number of ESTs, \( x^{(e)}_k \) is referred to be the kinematical state of the \( e \)th EST, and \( X^{(e)}_k \) refers to the extension state. Let us denote the state augment \( \xi^{(e)}_k \) is composed of kinematic and extension states, and \( \xi \) denotes that the EST kinematics is modelled up to \((s-1)\)th derivative. Operation \( \cdot \) denotes set cardinality, that is, \( N_{x,k} = |X_k| \). In a complicated multi-EST tracking scenario, we approximate the \( e \)th EST by a combination of multiple ellipsoidal sub-objects known by sub-RMM, which is represented by a random matrix and a random vector. Next, we will give the EST non-linear kinematic state and extension state models. Based on this assumption, \( x^{(e)}_k \in \mathbb{R}^{s\times d} \) and \( X^{(e)}_k \in \mathbb{S}^d \) describe the unknown kinematic state interval vector and extension of sub-object \( s \) of the \( e \)th EST, respectively. Therefore, the non-linear dynamic motion model of sub-object \( s \) of the \( e \)th EST,

\[
x^{(e)}_k = \begin{bmatrix} x^{(e)}_k, \omega^{(e)}_k \end{bmatrix}^T, \quad \begin{bmatrix} P^{(e)}_k, \omega^{(e)}_k \end{bmatrix} \quad \begin{bmatrix} P^{(e)}_k, \omega^{(e)}_k \end{bmatrix} \quad \omega^{(e)}_k \quad \text{comprises the planar position and velocity} \quad x^{(e)}_k = \begin{bmatrix} \omega^{(e)}_k \end{bmatrix} \quad \text{and the turn rate} \quad \omega^{(e)}_k \quad \text{motions are modelled by a non-linear nearly constant turn model given by}
\]

\[
x^{(e)}_k = F_{k|k-1}\left( x^{(e)}_k \right) + G_{k, m^{(e)}_k}, \quad m^{(e)}_k \sim N(0, D_{k|k-1} \otimes \omega^{(e)}_k) \quad (2)
\]
where \( F_{k|k-1} \) denotes the non-linear dynamic model in one-dimensional (1D) (physical) space (e.g. the single model used in [15]), and \( d \) is the dimension of the EST extension, that is, \( \chi_k^{(f,s)} \) is a \( d \times d \) SPD matrix, and \( \mathbb{S}_+ \) is a set of SPD \( d \times d \) matrices, \( G_k \) is the process noise gain matrix, \( \chi_k^{(f,s)} \) is zero-mean Gaussian process noise, that is, \( \mathcal{N}(0,D_{k|k-1}^{(f,s)} \otimes \chi_k^{(f,s)}) \), ‘\( \otimes \)’ stands for Kronecker product, \( D_{k|k-1}^{(f,s)} = (\sum_{k|k-1}^{f,s})^2 \) diag \((0,0,1)\) is the covariance matrix of independent Gaussian process noise \( w_k^{(f,s)} \), and \( \sum_{k|k-1}^{f,s} \) is the variance of acceleration noise. This distribution of the process noise indicates that the kinematic state \( X_k^{(f,s)} \) of sub-object \( s \) of the \( f \)th EST is affected by the extent state \( \chi_k^{(f,s)} \).

### 2.2 | Observation model for TBD sub-random matrix

We now describe an observation model of ESTs for TBD approach based on sub-SPD-RMM. The following is a specification of the RFS for observation model in the TBD context. The TBD approach defines a model for the raw, unthresholded measurements in terms of a state hypothesis. The 2D images of the surveillance region provided by the sensor is divided into \( D \) cells denoted as \( V_1,\ldots,V_D \) with \( V_j \subset \mathbb{R}^{d/2} \), the measurement data at time \( k \) is collected into a set \( Z_k = \{ z_k^1, \ldots, z_k^D \} \in \mathbb{R}^m \), with \( z_k^i \) the intensity measurement obtained in the \( i \)th cell. In this study, the EST returns measured by the radar are assumed to fluctuate according to the return amplitude fluctuation model [2].

Specially, let \( X(z_k^{(f,s)}) \) denote the set of pixels influenced by a sub-EST with augmented state \( \xi_k^{(f,s)} \), which means a pixel \( i \in X(z_k^{(f,s)}) \) is illuminated by a sub-EST with augmented state \( \xi_k^{(f,s)} \). Thus, the 2D images of the surveillance region provided by the sensor have the measurement \( z_k^i \) in the \( i \)th cell condition on \( \xi_k^{(f,s)} \), which can be denoted as follows:

\[
\begin{align*}
    & z_k^i = \begin{cases} 
        A(x_k^{(f,s)})(\chi_k^{(f,s)} + v_k^{(f,s)}), & i \in X(\xi_k^{(f,s)}) \\
        v_k^{(f,s)}, & i \in \Xi(\xi_k^{(f,s)})
    \end{cases}
\end{align*} \tag{3}
\]

where \( i \in X(\xi_k^{(f,s)}) \) means if there are \( n_k^{(f,s)} \) sub-ESTs present from \( N_k \) ESTs and \( i \in \Xi(\xi_k^{(f,s)}) \) denotes if there are no sub-ESTs. \( A(x_k^{(f,s)}) \) is the complex echo of sub-object \( s \) of the \( f \)th target \( x_k^{(f,s)} \), \( \chi_k^{(f,s)} \) is \( \chi_k^{(f,s)} \otimes \mathbb{I}_d \), and \( \mathcal{N}(0,B_{k|k-1}(\xi_k^{(f,s)})) \) is a possibly non-linear function of \( x_k^{(f,s)} \) and represents the contribution intensity of sub-object \( s \) of target \( f \) to the cell \( i \),

\[
\begin{align*}
    & v_k^{(f,s)} \sim \mathcal{N}(0,\lambda \chi_k^{(f,s)} + R_k^{(f,s)}) \equiv \mathcal{N}(0,B_k^{(f,s)} \chi_k^{(f,s)} (B_k^{(f,s)})^T) \quad \text{is a Gaussian measurement noise cell, and the measurement covariance matrix} \  R_k^{(f,s)} \  \text{is diagonal.} \  v_k^{(f,s)} \  \text{follows a Gaussian white noise with covariance matrix given by unknown} \  \chi_k^{(f,s)} \  \text{and variance} \  R_k^{(f,s)} \  \text{as mentioned previously.} \  \lambda \  \text{is a scalar description effect of} \  \chi_k^{(f,s)} \  \text{.} \  \lambda \  \text{can incorporate approximately by letting} \  B_k^{(f,s)} = (\lambda \chi_k^{(f,s)} + R_k^{(f,s)})^{1/2} (\chi_k^{(f,s)})^{-1/2} \quad \text{which represents the distortion of observed extension from actual one in size, shape, and orientation.}
\end{align*}
\]

The observation is an image consisting of an array of cells with a scalar intensity. The imaging sensor measurement corresponding to a sub-EST is modelled using a Gaussian spread function with a sub-EST assumption. For a sub-EST present at the position \( x_k^{(f,s)} \), this non-linear sub-object spread function \( H_k^{(f,s)}(x_k^{(f,s)}) \) in the measurement model in Equation (3) can be approximated according to the point spread function [6]:

\[
\begin{align*}
    & H_k^{(f,s)}(x_k^{(f,s)}) = \frac{\Delta_k^{(i,j)} (i - x_k^{(f,s)})^2 + (\Delta_k^{(i,j)} - y_k^{(f,s)})^2}{2\Sigma^2} \tag{4}
\end{align*}
\]

where \( \Sigma \) is the amount of blurring introduced by the sensor giving cell side lengths of \( \Delta_k^{(i,j)} = \Delta_k^{(i,j)} \) of the Gaussian spread function with source intensity \( I_k^{(f,s)} \). In this study, we mainly focus on the EST-TBD problem based on the non-linear measurements and sub-RMM approach. The most existing RMM methods are based on the ‘linear’ measurements, where the positions of the scatters are adopted in the formulation directly. Therefore, we need to transfer them to linear observations before carrying out the sub-EST tracking as mentioned previously. The non-linear function part of the non-linear observation formula is expanded at the state of the prediction, ignoring the high-order term and shifting item. We can then get the new linear measurement model as follows [27]:

\[
\begin{align*}
    & z_k^{(i)} = b \left( H_k^{(i,j)} x_k^{(f,s)} \right) + g_k^{(i)} = b \left( H_k^{(i,j)} x_k^{(f,s)} \right) + G_s(x_k^{(f,s)}) - x_k^{(f,s)} + g_k^{(i)},
\end{align*}
\]

where \( G \) is the Jacobian matrix of the nonlinear measurement equation, that is,

\[
\begin{align*}
    & G = \frac{\partial b(x)}{\partial x} \mid_{x = H_k^{(i,j)} x_k^{(f,s)}} . \tag{6}
\end{align*}
\]
Then, we can get the linear sub-EST observations as

\[ \tilde{z}_k^{(i)} = G^{-1} \left( z_k^{(i)} - b \left( H_k^{(i)} x_{k-1} \right) + G H_k^{(i)} x_{k-1}^{(i)} \right) = H_k^{(i)} x_k^{(i)} + G^{-1} g_k^{(i)}. \]  

(7)

Assume that the non-linear measurement noise distribution is \( g_k^{(i)} \sim N(0, Q_k^{(i)}) \), then we have the measurement noise distribution of \( \tilde{z}_k^{(i)} \) with \( G^{-1} g_k^{(i)} \sim N(0, G^{-1} Q_k^{(i)} G^{-T}). \) In the initial linear case, the measurement noise distribution is \( \psi_k^{(i)} \sim N(0, B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T). \) The non-linear sub-EST observations’ measure covariance matrix is [27]

\[ Q_k^{(i)} = G B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T G^{-1}. \]  

(8)

In addition, it also should be noted that the covariance matrix of noise has also changed from \( R_k^{(i)} \) to \( GR_k^{(i)} G^{-1}. \)

Assuming an independent measurement noise from cell to cell, then, the likelihood function for the measurement model can be obtained as

\[ p(Z_k | x_k^{(i)}) = \prod_{i=1}^D \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} \right), \quad i \in X \left( x_k^{(i)} \right) \]

\[ \prod_{i=1}^D \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} \right), \quad i \notin X \left( x_k^{(i)} \right). \]  

(9)

Here the fluctuation models are incorporated into the likelihood function to account for the target-return fluctuations. Under the further assumption of Gaussian background noise, the probability density functions (PDFs) can be expressed as

\[ \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} \right) = N \left( z_k^{(i)} ; A(x_k^{(i)}) H_k x_k^{(i)} , (G B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T G)^{-1} \right), \]

\[ \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} \right) = N \left( z_k^{(i)} ; 0, G R_k^{(i)} G^{-1} \right). \]  

(10)

where \( N \left( z_k^{(i)} ; A(x_k^{(i)}) H_k x_k^{(i)} , (G B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T G)^{-1} \right) \) implies that \( z_k^{(i)} \) is a Gaussian random variable with mean \( H_k x_k^{(i)} \) and variance \( (G B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T G)^{-1} \). The intensity of each bin's output is either Riceian distributed if there is an ET presenting in noise or Rayleigh distributed if there is only noise present [6,7]. Thus, for a given complex amplitude of ET, \( A(x_k^{(i)}) \approx A_k \), in a given bin(i), the corresponding likelihoods can be rewritten as

\[ \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} , A_k \right) = \frac{2 \tilde{z}_k^{(i)} (G B_k^{(i)} x_k^{(i)} (B_k^{(i)})^T G)^{-1}}{\sum_{i=0}^{\infty} i^2/\Gamma(i+1)} \exp \left( -\frac{\tilde{z}_k^{(i)}^2}{\sigma_R^2} \right). \]  

(11)

where \( \sum_{i=0}^{\infty} i^2/\Gamma(i+1) \) is the Bessel function and \( \sigma_R^2 = GR_k^{(i)} G^{-1} \) is the measurement variance. Let the measurement likelihood in cell (i) in the presence of at least one target be denoted by \( \phi_i^{(i)} \left( z_k^{(i)} | x_k \right) \) and the likelihood under the hypothesis of no targets be \( \phi_i^{(i)} \left( z_k^{(i)} \right) \). A given sub-object \( s \) of the \( \ell \)th target with extended state \( x_k^{(i)} \in X_k \), at time \( k \), illuminates a set of cells denoted by \( C(x_k^{(i)}) \). Here, we only consider the case that sub-objects are rigid bodies. It means that the regions are affected by different sub-objects without overlap. For example, \( C(x_k^{(i)}) \) could be the set of cells whose centre fall within a certain distance from the position of the target. Since conditioned on the multi-target state \( X_k \), the values of the cells are independently distributed and the multi-target likelihood of the whole observation \( Z_k \) is simply the product over the cells given by

\[ g(Z_k | X_k , A_k) = \prod_{x_k^{(i)} \in X_k} \prod_{\ell \in C(x_k^{(i)})} \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} , A_k \right) \]

\[ = \prod_{x_k^{(i)} \in X_k} \prod_{\ell \in C(x_k^{(i)})} \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} , A_k \right) \]

\[ \times \prod_{i \in U \cup C(x_k^{(i)})} \phi_i^{(i)} \left( z_k^{(i)} , x_k^{(i)} \right) \]

\[ = f(Z_k) \prod_{x_k^{(i)} \in X_k} \prod_{\ell \in U \cup C(x_k^{(i)})} \mathcal{L} \]  

(12)

Under extended state \( x_k^{(i)} \) assumption, the measurement pseudo-likelihood function for sub-object \( s \) with expected number of measurements is
where \( \gamma(\cdot) \) is the expected number of measurements (measurement rate) generated by a sub-object \( s \) of the \( \ell \)th ET and is assumed to be a function of extended sub-object \( s \) volume, \( \nu(\xi^{(e)}_s) \equiv \pi \sqrt{\chi^{(e)}_k} = \pi A_k^{(e)} d_k^{(e)} \), where \( A_k^{(e)} \) and \( d_k^{(e)} \) are the major and minor axes of the ellipse, respectively. This assumption is reasonable in many real scenarios, where a smaller target (only one part or sub-object of EST is represented by one ellipse) would occupy fewer sensors’ resolution cells than a larger EST, thus it yields fewer measurements. The true extension of sub-object \( s \) based on the size \( (A_k^{(e)} , d_k^{(e)}) \) and orientation \( (A_k^{(e)} , \ell) \) for each \( s \) is given by [2]

\[
\chi^{(e)}_k = A_k^{(e)} \text{ diag} \left( \left( A_k^{(e)} \right)^2 \left( d_k^{(e)} \right)^2 \right) (A_k^{(e)})^T.
\]

In [28], the authors introduced a simple model for an expected number of measurements. This model is equivalent to a uniform expected number of measurements in surveillance area per square root. The number of measurements generated by the target at each time step is a Poisson distributed with rate \( \nu(\xi^{(e)}_s) \) measurements per scan. Each sub-EST generates a Poisson distributed number of measurements, where the Poisson rate is a function of the augmented state. In general, based on this Poisson model, the shape of the distribution of measurements for each ET is described as an ellipse. In this work, we will use the same model for each sub-object to generate the expected number of measurements as [28]

\[
\nu(\xi^{(e)}_s) = \left\lfloor \frac{4}{15} \lambda^{(e)}_k + 0.5 \right\rfloor = \left\lfloor 2 \sqrt{A_k^{(e)} d_k^{(e)}} + 0.5 \right\rfloor.
\]

where \( \left\lfloor \cdot \right\rfloor \) is floor function and \( \lfloor x + 0.5 \rfloor \) rounds \( x \) to the nearest integer.

In [1,3], the authors presented an improvement in non-parametric detection technique by using Legendre orthogonal polynomials (LOP) to reconstruct the PDF of real-target RCS data. In this work, we will use this statistical model to achieve more accurate estimation of real EST. Here the improved LOP fluctuation model is incorporated into the likelihood function to account for the EST return fluctuations. The details of LOP fluctuation model are explained in [20] (see Equations [10]–[13] in [3]). Using this amplitude fluctuation model, the sub-

EST likelihood ratio based on the probability of a track and a PDF of real-target RCS data can be written in terms of the real return \( A_k \) as follows:

\[
\mathcal{L}_{Z_k}(\xi_k^{(e)}, A_k) = \int \mathcal{L}_{Z_k}(\xi_k^{(e)}, A_k)p(A_k | \bar{A})dA_k\quad (16)
\]

Referring to the form (see Equations [10]–[13] in [3]), it requires a numerical integration scheme. Here, an MC integration is adopted to calculate the sub-EST likelihood ratio, approximately as

\[
\mathcal{L}_{Z_k}(\xi_k^{(e)}, A_k) \approx \frac{1}{M} \sum_{m=1}^M \mathcal{L}_{Z_k}(\xi_k^{(e)}, A_k^m)\quad (17)
\]

where \( M \) is sample number and \( A_k^m \) is the random sample from distribution \( p(A_k | \bar{A}) \).

3 | MULTI-BERNOULLI TBD SUB-RMM RECURSIONS

In [2], we proposed an MB filter for tracking ESTs based on sub-RMM, which is capable of estimating the EST extent augment with estimated detection probability and kinematic state. EST tracking requires measurement partitioning. Partitioning is the key of the measurement update step of the proposed filter. Therefore, this filter [2] is unsuitable for TBD of ESTs from image observation; the amount of video data has been considered in the measurement updating. Thus, we improved this filter to be suitable for TBD approach. The main objective of this study is to reduce the size of the measurement set that the tracking filter needs to process and thus to limit computational requirements. On the other hand, we try to keep the validation gate as small as possible, so as to restrict the amount of video data. In the following, the proposed sub-RMM-MB-TBD filter’s measurement-update equations and the corresponding partitioning methods based on sub-RMM are simply presented, respectively. The focus in this study is on the prediction update of EST within the sub-RMM framework. We derive compact closed-form expressions for a recursive sub-RMM-MB-TBD update of the augmented state \( \xi_k^{(e)} \). This augmented state consists of the kinematic state \( (x_k^{(e)}, \ell) \) and the extended state \( \chi_k^{(e)} = [A_k^{(e)}, A_k^{(e)}, d_k^{(e)}]^T \) with the shape parameters such as the orientation \( (A_k^{(e)}) \) and semi-axis lengths \( (A_k^{(e)}, d_k^{(e)}) \) for each sub-EST.

3.1 | Sub-RMM-MB-TBD prediction

Assume that the combination of multiple sub-objects \( n_k^{(e)} \) of the \( \ell \)th target at time \( k-1 \) is an MB with parameters,
π_{k-1} = \left\{ \left\{ r^{(e,f)}_{k-1}, p^{(e,x)}_{k-1}(\xi_{k-1}, A_{k-1}, A_{k-1}, a_{k-1}) \right\} \right\}_{f=1}^{M_{k-1}} \right. ,

where \( r^{(e,f)}_{k-1} \) is the existence probability and \( p^{(e,x)}_{k-1}(\cdot) \) is the state distribution of sub-object \( s \) of the \( \ell \)th Bernoulli component for each EST. \( M_{k-1} \) denotes the number of posterior hypothesized tracks at time \( k-1 \), then the predicted density of ESTs is given by

\[
\pi^{(e,f)}_{k|k-1} = \left\{ \left\{ r^{(e,f)}_{k,k-1}(\xi_{k-1}, A_{k-1}, A_{k-1}, a_{k-1}) \right\} \right\}_{f=1}^{M_{k-1}} \left. \right. ,
\]

(18)

and \( f_{k|k-1}(\xi_{k} | \xi_{k-1}) \) is the transition density at time \( k \), for sub-EST augmented state with unknown parameters such as \( x_{k}, x_{k}, A_{k}, A_{k}, a_{k} \) and given value. \( U \) is the union symbol, \( p_{k; k} \) is the survival probability, and \( \{ r^{(e,f)}_{k,k-1}, p^{(e,x)}_{k,k-1} \} \) are parameters of an MB for birth targets at time \( k \), where \( \cdot \) represents inner product operation. The total number of predicted hypothesized tracks is \( M_{k|k} = M_{k-1} + M_{f;k} \).

3.2 Sub-RMM-MB-TBD update

Assume that MB predicts the density of ESTs is an MB with parameter \( \pi^{(e,f)}_{k|k-1} = \left\{ \left\{ r^{(e,f)}_{k,k-1}, p^{(e,x)}_{k,k-1}(\xi_{k,k-1}, A_{k,k-1}, A_{k,k-1}, a_{k,k-1}) \right\} \right\}_{f=1}^{M_{k-1}} \).

Since the origins of an EST measurement are only partially resolvable, we also assume that no further information is available about the correspondences between sub-objects and measurements set \( Z_{k} \), and the measurement can be obtained from any possible sub-object (see Equation [21]), where \( \varphi^{(P)}_{k} \) is the \( P \)th partition of the measurement set \( Z_{k} \), \( W^{(P,J)}_{k} \in \varphi^{(P)}_{k} \) is the \( J \)th cell of partition \( \varphi^{(P)}_{k} \), \( \varphi^{(P)}_{k} \) is the number of cells from \( \varphi^{(P)}_{k} \) and \( W^{(P,J)}_{k} \) does not include null set, \( N_{P,k} \) is the number of partitions at time step \( k \), the notation \( \varphi^{(P)}_{k} \) is the partitioning shorthand that means \( \varphi^{(P)}_{k} \) to partition the measurement set \( Z_{k} \) into non-empty cells \( W^{(P,J)}_{k} \). The updated MB filter, in the same cases, requires a likelihood of measurements in each cell \( W^{(P,J)}_{k} \). Thus, the measurement pseudo-likelihood of the sub-object \( s \) of the \( \ell \)th target is a function of \( (W^{(P,J)}_{k}, \xi^{(e,f)}_{k}) \) and given by Equation (17):

\[
\mathcal{L}_{\mathbf{W}^{(P,J)}_{k}}(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k}) = e^{-\gamma^{(e,f)}_{k}(\xi^{(e,f)}_{k})} | \mathbf{W}^{(P,J)}_{k} \rangle \times \prod_{z^{(i)}_{k} \in W^{(P,J)}_{k}} \frac{\phi^{(i)}_{(z^{(i)}_{k})}}{\phi^{(i)}_{(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k})}} .
\]

(20)

Thus, the updated TBD-MB parameters are

\[
\begin{align*}
\varphi^{(P)}_{k} & = \left\{ \left\{ \left\{ \left\{ r^{(e,f)}_{k,k-1}(\mathbf{W}^{(P,J)}_{k}), p^{(e,x)}_{k,k-1}(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k}) \right\} \right\}_{f=1}^{M_{k-1}} \right. \right. \\
& \times \prod_{z^{(i)}_{k} \in W^{(P,J)}_{k}} \frac{\phi^{(i)}_{(z^{(i)}_{k})}}{\phi^{(i)}_{(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k})}} .
\end{align*}
\]

(21)

\[
d_{\varphi^{(P)}_{k}} = \delta_{1,|\mathbf{W}^{(P,J)}_{k}|} \left| M_{k-1} \right| \\
\times \prod_{f=1}^{M_{k-1}} \frac{\left| r^{(e,f)}_{k,k-1}(\mathbf{W}^{(P,J)}_{k}, \xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k}), \mathcal{L}_{\mathbf{W}^{(P,J)}_{k}}(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k}) \right|}{1 - r^{(e,f)}_{k,k-1}(\mathbf{W}^{(P,J)}_{k}, \xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k}), \mathcal{L}_{\mathbf{W}^{(P,J)}_{k}}(\xi^{(e,f)}_{k}, A_{k}, A_{k}, a_{k})} .
\]

(22)

where the quantities \( m^{(P)}_{\varphi^{(P)}_{k}} \) and \( d_{\varphi^{(P)}_{k}} \) are, respectively, the non-negative coefficients of the partition \( \varphi^{(P)}_{k} \) and cell \( W^{(P,J)}_{k} \), \( \delta_{k,g} \) is the Kronecker delta function (i.e. if \( b = g \), \( \delta_{k,g} = 1 \); otherwise, \( \delta_{k,g} = 0 \). The presented sub-RMM-MB-
TBD filter for each sub-object \( s \) of the \( \ell \)th EST requires all partitions \( \varphi \) of the current measurement set \( Z_k \) for its update. To elaborate the importance of partitioning in the sub-RMM-MB-TBD filter, the partitioning process with a measurement set that contains the 2D images of the surveillance region provided by the sensor is divided into \( D \) cells, the measurement data at time \( k \) are collected into a set \( Z_k = \{ z_k^1, \ldots, z_k^D \} \in \mathbb{R}^m \), is here considered. Ideally, all the measurements contained by each cell stem from the same source, either an ET or a noise source. Thus, one cell only contains an ET or noise. In other words, the uncertainty of extension state \( \xi_k \) is equivalent to that of cell including this ET. Therefore, we can use the interval cell \( W_k^{(\varphi, \ell)} \) to approximate the uncertainty of extension state. A partition is a division of the measurement set into non-empty subsets called cells, \( W_k \). Each cell \( W \) can be interpreted as containing measurements that all stem from the same sub-object \( s \) of the \( \ell \)th EST, either a sub-object \( s \) or a noise source. Assume that there are \( N_k^E = (\sum_{\varphi=1}^{N_k} n_k^{(\varphi, s)}) N_k^E \) possibilities (partitions) to associate measurements with \( n_k^{(\varphi, s)} \) sub-objects and \( N_k^E \) EST at time \( k \). Let \( \varphi^{(i)} \), \( 1 \in \{ 1, \ldots, N_k^E \} \) be a possible association event. Without any further information, the events (partitions) can be reasonably assumed equally probable a priori, \( P(\varphi^{(i)}) = 1/N_k^E \). The measurement pseudo-likelihood (17) requires a summation over all possible partitions, which quickly becomes intractable because the number of possible partitions increases very rapidly as the size of \( Z_k \) increases. To solve this problem of association event reduction, finding a correct partitioning subset of the current measurement set is the crux of solving this problem. In [2], we used distance partition to solve this problem. However, this method is unsuitable for TBD work because this method is based on BGIW implementation, not for particle implementation with non-linear model track, which is used in this work. In order to obtain a computationally tractable solution to the EST filter, only a subset of all possible partitions can be considered.

Therefore, a novel fast partitioning algorithm with fuzzy adaptive resonance theory (ART) model for the ET-PHD filter is proposed in [29]. It can fast partition the measurement set by iteratively updating covariance matrix and mean vector, and obtain a robust subset that approximates all the possible partitions. In this work, we will use this method to find a correct partitioning subset of the current measurement set \( Z_k \) for its update. Let \( W_k^{(\varphi, \ell)} \) represent an interval of a 2D vector transformed by extension state \( \xi_k^{(\varphi, \ell)} \), which is an approximation. Ideally, all the measurements contained by each cell stem from the same source, either a sub-object \( s \) of the \( \ell \)th EST or a noise source. Thus, only one cell contains a sub-EST or noise.

In other words, the uncertainty of extension state \( \xi_k^{(\varphi, \ell)} \) is equivalent to that of cell including this sub-EST. The cell \( W_k^{(\varphi, \ell)} \) can be obtained by

\[
W_k^{(\varphi, \ell)} \triangleq f (\mu_k^{(\varphi, \ell)}, \Omega_k^{(\varphi, \ell)}), \quad \ell = 1, \ldots, |\varphi_k^{(\varphi)}|, \quad \varphi = 1, \ldots, N_{\varphi, k}, \quad (23)
\]

where \( \mu_k^{(\varphi, \ell)} \) and \( \Omega_k^{(\varphi, \ell)} \) are mean vector and covariance matrix of the \( \ell \)th category from the \( \varphi \)th partition at time step \( k \), and \( f (\cdot) \) describes the elliptical shape function of the distribution of measurements from cell \( W_k^{(\varphi, \ell)} \).

The function \( f (\mu_k^{(\varphi, \ell)}, \Omega_k^{(\varphi, \ell)}) \) describes the elliptical shape function of the distribution of measurements from cell \( W_k^{(\varphi, \ell)} \). Therefore, we can use the cell \( W_k^{(\varphi, \ell)} \) to approximate the extension state \( \xi_k^{(\varphi, \ell)} \) for each sub-object. Therefore, we consider validating the measurements for each sub-object. That is, for the measurements falling inside only one cell for a sub-object, they need not to be associated with other sub-objects. Furthermore, if there are some overlaps of measurements falling inside sub-ESTs, these measurements do not need to be associated again. Thus, only one cell contains a sub-EST or noise. Since the number of groups is no more than the number of sub-objects, the algorithm makes high requirements for partitioning algorithm and the right partition results must be included. The forming example of sub-RMM-TBD partition for EST (1F-117) with five cells \( \{ W_k^{(1,1)}, \ldots, W_k^{(5,5)} \} \), \( \ell = 1, \ldots, 5 \), and \( |\varphi_k^{(\varphi)}| = 5 \) is illustrated by Figure 2, while the measurement set that contains 11 individual measurements, \( Z_k = \{ z_k^1, \ldots, z_k^{11} \} \), is considered here. Thus, a measurement set is sub-RMM-TBD-partitioned as

\[
\varphi_k^{(1)} : \{ W_k^{(1,1)} = \{ z_k^1, z_k^2, z_k^3, z_k^4 \}, \ \ W_k^{(1,2)} = \{ z_k^5, z_k^6 \}, \ \ W_k^{(1,3)} = \{ z_k^7, z_k^{11} \}, \ \ W_k^{(1,4)} = \{ z_k^8 \}, \ \ W_k^{(1,5)} = \{ z_k^9, z_k^{10} \} \}
\]

4 | SMC IMPLEMENTATION OF SUB-RMM-MB-TBD

Note that we adopt the measurement means of cells from the partitioning subset as the approximations of the possible starting positions of extended sub-objects (9) of the \( \ell \)th EST in the sub-RMM-MB-TBD filter. Thus, we can overcome the limitation that sets the extended-sub-object starting positions, which usually appears in the RFS-based filters. In this work, the mean of a given cell \( W_k^{(\varphi, \ell)} \) is defined by

\[
Z_k^{(\varphi, \ell)} \triangleq \frac{1}{|W_k^{(\varphi, \ell)}|} \sum_{z_k^{(i)}} W_k^{(\varphi, \ell)} W_k^{(\varphi, \ell)} Z_k^{(i)}, \quad \ell = 1, \ldots, |\varphi_k^{(\varphi)}|, \quad \varphi = 1, \ldots, N_{\varphi, k}, \quad (24)
\]

where \( |W_k^{(\varphi, \ell)}| \) represents the number of measurements from the cell \( W_k^{(\varphi, \ell)} \).
4.1 SMC-sub-RMM-MB-TBD prediction

In this section, an SMC implementation of the sub-RMM-MB-TBD recursion is described. The proof of concept is to propagate multiple sets of weights and samples, where each set represents each hypothesized track, and to propagate these recursively in time. Assume that the combination of multiple sub-object \( \pi_k \) of the \( \ell \)-th EST at time \( k-1 \) is a posterior density,

\[
\pi_{k-1} = \left\{ \left\{ \begin{array}{c} \ell \\ g_{k-1}^{(\ell s)}(\xi_{k-1}; A_{k-1}, A_{k-1}, d_{k-1}) \end{array} \right\}_{\ell=1} \text{ obtained by the SMC-sub-RMM-MB-TBD filter at time step } k-1, \right. 
\]

and each \( p_{k-1}^{(\ell s)} \) for \( \ell = 1, \ldots, M_{k-1}, s = 1, \ldots, n_{k-1}^{(\ell)} \) is comprised of the set of resampled particles \( \{ w_{k-1}^{(\ell s)}, \xi_{k-1}^{(\ell s)} \}_{j=1}^{n_{k-1}^{(\ell)}} \). Thus, in this respect, each \( p_{k-1}^{(\ell s)} \) of the sub-object \( \pi_k \) at time \( k-1 \) can be approximated by

\[
p_{k-1}^{(\ell s)}(\xi_{k-1}; A_{k-1}, A_{k-1}, d_{k-1}) 
\approx \sum_{j=1}^{n_{k-1}^{(\ell)}} w_{k-1}^{(\ell s)} \delta_{\left( \xi_{k-1}^{(\ell s)}, A_{k-1}^{(\ell s)}, A_{k-1}^{(\ell s)}, d_{k-1}^{(\ell s)} \right)}(\xi_{k-1}) 
\]  (25)

Then we proposed the new densities \( q_k^{(\ell s)}(\cdot; \xi_{k-1}^{(\ell s)}, A_{k-1}^{(\ell s)}, A_{k-1}^{(\ell s)}, d_{k-1}^{(\ell s)}, Z_k) \), and \( b_k^{(\ell s)}(\cdot; Z_k) \) with support (i.e., points where the function is non-zero) satisfying as

\[
\text{support } \left( p_{k-1}^{(\ell s)} \right) \subseteq \text{support } \left( q_k^{(\ell s)} \right). 
\]

The posterior of birth model of the sub-object \( \pi_k \) of the \( \ell \)-th EST at time \( k \) is a sub-RMM-MB-TBD distribution with parameter set of \( \left\{ \{ f_{\ell k}^{(\ell s)}(\xi_{k}; A_{k}, A_{k}, d_{k}) \}_{\ell=1} \right\} \), where \( f_{\ell k}^{(\ell s)}(\xi_{k}; A_{k}, A_{k}, d_{k}) \) is SMC-sub-RMM-MB-TBD density and \( L_{k-1}^{(\ell s)} \) is the number of the particles to approximate the probability density \( p_{k-1}^{(\ell s)} \).

Suppose that the intensity of birth RFS is an unnormalized SMC-sub-RMM-MB-TBD distribution. At time step \( k \), to avoid a large number of additional particles of the extended sub-objects tracking, the new-born sub-object particles are produced by the similar sampling strategy suggested in [29]. Different from the point target, at a given time step \( k \), each extended sub-object can give rise to multiple measurements. In the sub-object particle, implementation of the SMC-sub-RMM-MB-TBD filter, at current time step \( k \), new-born sub-object particles are formed by the measurement-set partitioning subset at the previous time step \( k-1 \). In this work, we utilize the ART partition [29] to produce the partitioning subset of measurements. The detailed implementation process is described as follows. First, for the subset of partitions obtained at time step \( k-1 \), we select the most likely one partition as the most credible partition, that is,

\[
\psi_{k-1}^p \triangleq \arg \max_{\psi_{k-1}^p} \left\{ w_{\psi_{k-1}^p}, P = 1, \ldots, N p_{k-1} \right\}. 
\]  (26)
where $N_{p,k-1}$ is the number of partitions. The partitioning weight $w^{(p)}_{p,k-1}$ can be interpreted as the probability of partition $\varphi^{(p)}_{k-1}$ being true. It implies that the greater weight $w^{(p)}_{p,k-1}$, the more credible partition $\varphi^{(p)}_{k-1}$. Therefore, here we use the cells of partition $\varphi^{(p)}_{k-1}$, whose weight $w^{(p)}_{p,k-1}$ is maximal, to form new-born sub-object particles at time step $k$. Thus, we can avoid a large number of cells from partitions, obtained by ART partition [29], to produce new-born sub-object particles, leading to the reduction of the computational burden. Using the cell set $\{W^{(p,j)}_{k-1}\}_{j=1}^{J}$ from partition $\varphi^{(p)}_{k-1}$ at time $k-1$ forms new-born sub-object particles at time $k$, where $W^{(p,j)}_{k-1}$ and $\varphi^{(p)}_{k-1}$ represent the $j$th cell and the cell number of partition $\varphi^{(p)}_{k-1}$, respectively. For each cell $W^{(p,j)}_{k-1}$ of the sub-objects $n^{(p)}_k$ of the $\ell$th EST, $L^{(\epsilon)}_{1:k}$ new-born sub-object particles $\xi^{(p,j)}_k$, $j = 1, \ldots, L^{(\epsilon)}_{1:k}$ are drawn from the Gaussian distribution $N(\bar{Z}^{(p,j)}_k, S^{(p,j)}_k)$, where the mean $\bar{Z}^{(p,j)}_k$ is computed by Equation (24), and $\Omega^{(p,j)}_{k-1}$ is defined as

$$\Omega^{(p,j)}_{k-1} \triangleq \sum_{j=1}^{L^{(\epsilon)}_{1:k}} w^{(p,j)}_{k-1} \left( \bar{Z}^{(i)}_k - \bar{Z}^{(p,j)}_{k-1} \right) \times \left( \bar{Z}^{(i)}_k - \bar{Z}^{(p,j)}_{k-1} \right)^T$$

(27)

The existence probability $r^{(\epsilon)}_{1:k}$ and the weight of new-born sub-object particles $w^{(j,\epsilon)}_{1:k}$ are, respectively, set to

$$r^{(\epsilon)}_{1:k} = \text{parameter given by birth model},$$

(28)

$$w^{(j,\epsilon)}_{1:k} = \sum_{j=1}^{L^{(\epsilon)}_{1:k}} \bar{w}^{(j,\epsilon)}_{1:k} \delta(\xi^{(p,j)}_k, A^{(\epsilon)}_{1:k}, a^{(\epsilon)}_{1:k}, \xi^{(p,j)}_k, \xi^{(p,j)}_{k-1}), A^{(\epsilon)}_{1:k}, A^{(\epsilon)}_{1:k}, a^{(\epsilon)}_{1:k}, \bar{Z}^{(j,\epsilon)}_k), \forall j = 1, \ldots, L^{(\epsilon)}_{1:k}$$

(29)

$$\bar{w}^{(j,\epsilon)}_{1:k} = \sum_{j=1}^{L^{(\epsilon)}_{1:k}} w^{(j,\epsilon)}_{1:k},$$

(30)

For $\ell = 1, \ldots, \ell_{\text{max}}$, $s = 1, \ldots, n^{(\epsilon)}_{k}$, $\forall j = 1, \ldots, L^{(\epsilon)}_{1:k}$, $s = 1, \ldots, n^{(\epsilon)}_{k}$.

Finally, using $r^{(\epsilon)}_{1:k}$ and $p^{(\epsilon)}_{1:k}(\xi_k)$, we can compute the predicted SMC-sub-RMM-MB-TBD posterior density, which is defined by

$$\pi^{\epsilon}_{k|k-1} = \left\{ \left( r^{(\epsilon)}_{p,k|k-1}, p^{(\epsilon)}_{p,k|k-1}(\xi_k, A_{k-1}, A_{k-1}, a_{k-1}) \right)_{j=1}^{n^{(\epsilon)}_k} \right\}_{\ell=1}^{\ell_{\text{max}}},$$

(31)

where the persistent and new-born SMC-sub-RMM-MB-TBD components are calculated as follows:

$$r^{(\epsilon)}_{p,k|k-1} = r^{(\epsilon)}_{k-1} \sum_{j=1}^{n^{(\epsilon)}_k} w^{(j,\epsilon)}_{k-1} \delta(\xi^{(p,j)}_k, A^{(\epsilon)}_{k-1}, A^{(\epsilon)}_{k-1}, a^{(\epsilon)}_{k-1}, Z^{(j,\epsilon)}_k), \forall j = 1, \ldots, L^{(\epsilon)}_{1:k},$$

(32)

$$p^{(\epsilon)}_{p,k|k-1}(\xi_k; A_{k-1}, a_{k-1}) = \sum_{j=1}^{n^{(\epsilon)}_k} w^{(j,\epsilon)}_{k-1} \delta(\xi^{(p,j)}_k, A^{(\epsilon)}_{k-1}, A^{(\epsilon)}_{k-1}, a^{(\epsilon)}_{k-1}, Z^{(j,\epsilon)}_k),$$

(33)

where

$$w^{(j,\epsilon)}_{k-1} \sim f_{k|k-1}(\xi^{(j,\epsilon)}_{1:k-1}, \xi^{(j,\epsilon)}_{1:k-1}), \forall j = 1, \ldots, L^{(\epsilon)}_{1:k}.$$
where $\delta_k^{(f_{\ell})} > d - 1$ is the degrees of freedom and the invertible matrix $A_k^{(f_{\ell})} \in \mathbb{R}^{d \times d}$ can describe the dependence of the extension on orientation (if $A_k^{(f_{\ell})} = (\delta_k^{(f_{\ell})})^{-1/2} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is a rotation matrix with angle $\theta$), size (e.g., $A_k^{(f_{\ell})} = \lambda I_d$), or shape (if $A_k^{(f_{\ell})}$ is some other matrix). $w(Y, a, C)$ is density of Wishart distribution using SPD random matrix $Y \in \mathbb{S}_d^{++}$, as [15]

$$W(Y, a, C) = \frac{1}{C |C|^{d/2}} |Y|^{(a-d-1)} \text{etr} \left( -\frac{1}{2} C^{-1} Y \right)$$

(36)

with $a \geq d$, where $\text{etr}(A) = \exp(\text{Tr}(A))$ is the exponential of tracing matrix $A$, $C = 2^{ad/2} |a|^{d/2}$, and $|\Gamma|_1$ is the multivariate gamma function. In this study, we will use this density of Wishart distribution to model the unknown extension profiles for sub-EST based on RMM because this model has sufficient flexibility to capture various extension profiles and evolution in size, shape, and orientation.

### 4.2 SMC-sub-RMM-MB-TBD update

Assuming that the predicted sub-RMM-MB-TBD density at time step $k$ is represented as $\bar{\pi}_{k|k-1} = \left\{ \begin{pmatrix} p_{k|k-1}^{(f_{\ell})} \\ \pi_{k|k-1}^{(f_{\ell})} \end{pmatrix} \right\}_{\ell = 1}^{M_{k|k-1}}$, $\bar{\pi}_{k|k-1}^{(f_{\ell})} \triangleq (\bar{\pi}_{k|k-1}^{(f_{\ell})}, \bar{A}_{k|k-1}^{(f_{\ell})}, \bar{A}_k^{(f_{\ell})}, \bar{A}_k^{(f_{\ell})}, \bar{A}_k^{(f_{\ell})})$, and each $p_{k|k-1}^{(f_{\ell})}$ for $\ell = 1, \ldots, M_{k|k-1}$, is comprised of a set of weighted particles $\left\{ w_{k|k-1}^{(j_{\ell})}, \bar{x}_{k|k-1}^{(j_{\ell})}, \bar{A}_{k|k-1}^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})} \right\}_{j=1}^{L_{k|k-1}}$, that is,

$$p_{k|k-1}^{(f_{\ell})}(\bar{x}_k, A_k, A_k, a_k) = \sum_{j=1}^{L_{k|k-1}} w_{k|k-1}^{(j_{\ell})} \tilde{p}_{k|k-1}^{(f_{\ell})}(\bar{x}_{k|k-1}^{(j_{\ell})}, \bar{A}_{k|k-1}^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})})(\bar{x}_k, \cdot)$$

(37)

Then, the updated sub-RMM-MB-TBD density can be approximated by Equation (20), where the updated sub-RMM-MB-TBD parameters of sub-object are computed as follows:

$$\tilde{r}_k^{(f_{\ell})}(\bar{x}_k, A_k, A_k, a_k) = \sum_{j=1}^{L_{k|k-1}} \tilde{r}_{k|k-1}^{(f_{\ell})}(\bar{x}_{k|k-1}^{(j_{\ell})}, \bar{A}_{k|k-1}^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})})(\bar{x}_k, \cdot)$$

$$\tilde{p}_k^{(f_{\ell})}(\bar{x}_k, A_k, A_k, a_k, W_k^{(f_{\ell})}) = \frac{1}{\tilde{r}_k^{(f_{\ell})}(\bar{x}_k, A_k, A_k, a_k)} \prod_{j=1}^{L_{k|k-1}} \tilde{p}_{k|k-1}^{(f_{\ell})}(\bar{x}_{k|k-1}^{(j_{\ell})}, \bar{A}_{k|k-1}^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})}, \bar{A}_k^{(j_{\ell})})(\bar{x}_k, \cdot),$$

(38)

The final step of the SMC-sub-RMM-MB-TBD filter is to estimate the ET states. At time step $k$, for each $p_k^{(f_{\ell})}$ of $\pi_k$, the state $\bar{x}_k^{(f_{\ell})}$ can be estimated as $\bar{x}_k^{(f_{\ell})} = (\bar{x}_k^{(f_{\ell})}, \bar{x}_k^{(f_{\ell})})$, where

$$\hat{x}_k^{(f_{\ell})} = \sum_{j=1}^{L_{k|k-1}} w_{k|k-1}^{(j_{\ell})} \text{mid} (\bar{x}_k^{(j_{\ell})}),$$

$$\hat{A}_k^{(f_{\ell})} = \sum_{j=1}^{L_{k|k-1}} w_{k|k-1}^{(j_{\ell})} A_k^{(j_{\ell})},$$

$$\hat{A}_k^{(f_{\ell})} = \sum_{j=1}^{L_{k|k-1}} w_{k|k-1}^{(j_{\ell})} A_k^{(j_{\ell})},$$

$$\hat{A}_k^{(f_{\ell})} = \sum_{j=1}^{L_{k|k-1}} w_{k|k-1}^{(j_{\ell})} A_k^{(j_{\ell})},$$

$$\ell = 1, \ldots, M_k, s = 1, \ldots, n_{k|k-1}^{(f_{\ell})}$$

and $\text{mid} (\cdot)$ denotes an operator that takes the state centre.

### 4.3 Computational complexity of the proposed SMC-sub-RMM-MB-TBD filter

The main motivation of this work is to use fewer particles effectively approximating the intensity over the state space at the expense of less TBD accuracy in the hope of greatly reducing the computational complexity. To reduce the computational complexity, the authors proposed a fuzzy ART partition method [29]. This partition method is inspired by the fuzzy ART, that is, a neural network architecture of rapid stable learning. It can fast partition the measurement set by iteratively updating covariance matrix and mean vector, and obtain a robust subset that approximates all possible partitions. In ART partition, one possible clustering result is equal to one possible partition, and one category from one clustering result is equal to one cell from one partition. The computational complexity of ART partition for the proposed filter is investigated in this subsection. In the following, we analyse the computational cost of the proposed algorithm. The computational cost of ART partition is
approximately $O\left(N_{z,k}^2\right)$, $N_{z,k} \triangleq Z_k = \{z_1^k, \ldots, z_D^k\} \in \mathbb{R}^m$. In order to reduce the computational burden, the computational cost of its own is negligible compared to that of obtaining one partition. Here, we only concern how many alternative partitions are generated using the bisection method. Suppose that $N_{\phi,k}, N_{\phi,k} \ll N_{z,k}$ partitions are formed in the worst case. Assume the number of measurements after canceling the noise and clutter cells equal to $N_{z,k}$. Thus, the worst-case complexity of ART partition is approximated as $O\left(N_{z,k}^2 \log N_{z,k}\right)$ [29]. For distance partition, which are used for H-PMHT and Gamma Gaussian inverse Wishart (MB-GGIW), the worst-case complexity is approximated as $O\left(N_{z,k}^2\right)$ [10,28].

### 5 | PRUNING AND MERGING

To account for the non-overlapping assumption, estimates that would overlap on the image observation of the sub-objects $\tilde{h}_k^{\ell(z)}$ of the $\ell$th EST are merged. If the two or more hypothesized tracks of the sub-objects $\tilde{h}_k^{\ell(z)}$ of the $\ell$th EST are spatially very close, merging then ensues. A simple way of merging is to combine the existence probabilities $\tilde{p}_k^{\ell(z)}(W_k^{(P,J)})$ and densities $\tilde{p}_k^{\ell(z)}(\xi_k, W_k^{(P,J)})$ of each cell $W_k^{(P,J)}$ whose estimates fall within a given distance $T_{merge}$ of each other. The recursive processing of particles generally leads to a rapid growth of the number of particles owing to the EST birth in the state prediction step and the averaging of hypothesized tracks in the measurement update step. To reduce the growing number of particles for each cell $W_k^{(P,J)}$ with $\tilde{p}_k^{\ell(z)}(W_k^{(P,J)})$, pruning of hypothesized tracks at each time step is performed by discarding those with existence probabilities below a truncation threshold $T$. The details of the implemented pruning and merging scheme are given in Table 1.

### 6 | NUMERICAL RESULTS

According to the latest survey of multi-extended targets tracking in a TBD application [9,10], the histogram probabilistic multi-hypothesis tracker (H-PMHT) proposed in [10] is, by far, the best previously developed multi-ET TBD technique.
In this section, we demonstrate the performance of our proposed sub-ellipses-MB-SMC filter for ESTs in a TBD application and compare the results with H-PMHT-TBD and MB-GGIW filters. Two scenarios are used to illustrate their relative strengths and weaknesses. To evaluate the performances of EST tracking algorithms, the optimal sub-pattern assignment (OSPA) distance [30] is recently developed between the set of ESTs and the true one:
Scenario (1): Ground truths for two ESTs (F-117 and F-22 models) that move in parallel

Stealth target (1) F-117 model
Two closed ESTs moving in parallel
Stealth target (2) F-22 model

Scenario (2): Ground Truths for Nonlinear states tracking for 10 Extended stealth targets

X_k = \left\{ \left\{ \xi_k^{(i,j)} \right\}_{j=1}^{N_{g,k}}, \xi_k^{(i,j)} \right\}_{j=1}^{N_{g,k}} = \begin{bmatrix} X_k^{(i,j)} & \xi_k^{(i,j)} \end{bmatrix}^T

\begin{align}
d \left( \xi_k^{(i,j)}, \xi_k^{(i,j)} \right) &= \frac{w_x}{c_x} d_{ij,ls}^{(c_x)} + \frac{w_\chi}{c_\chi} d_{ij,ls}^{(c_\chi)} + \frac{w_A}{c_A} d_{ij,ls}^{(c_A)} \\
&+ \frac{w_{a}}{c_a} d_{ij,ls}^{(c_a)},
\end{align}

Here, we first calculate the distance between an estimated sub-object \( \xi_k^{(i,j)} \) and a true \( \xi_k^{(i,j)} \), which is decomposed as where \( w_x + w_\chi + w_A + w_{a} = 1 \), and
Table 4 Setting of parameter values related to simulation

| Parameters                        | Values          |
|-----------------------------------|-----------------|
| Range resolution                  | 2.5 m           |
| Doppler resolution                | 2 m/s           |
| Azimuth resolution                | 1°              |
| Sampling interval                 | \( T = 1 \text{s}, \theta = 1 \text{s}, \sum_{i=1}^{j} = 0.1 \text{m/s}^2 \) |
| State model for each EST          | Nearly constant velocity 260 m/s |
| Measurement distance error        | 0.5 m           |
| Measurement angle error           | 0.05°           |
| Nonlinear measurement model       | Rayleigh noise distribution |
| S1: physical distance              | 5 m             |
| between two closed ESTs.          |                 |
| S1: SNR                           |                 |
| Figure 5a: high SNR               | 20 dB, \( R_2 = \text{diag} ([1,1]) \text{m}^2 \) |
| Figure 5b: low SNR                | 7 dB, \( R_2 = \text{diag} ([4,4]) \text{m}^2 \) |
| S2: SNR                           |                 |
| Figure 8a: frame no. 43           | 13 dB, \( R_k = \text{diag} ([2,2]) \text{m}^2 \) |
| With high SNR                     |                 |
| Figure 8b: frame no. 65           | 4 dB, \( R_k = \text{diag} ([6,6]) \text{m}^2 \) |
| With low SNR                      |                 |
| The cell side lengths             | \( \Delta_s = \Delta_0 = 1 \text{ m} \) |
| The blurring factor               | \( \Sigma = 0.5 \) |
| S1: observation region and image  | (160 m \( \times \) 160 m), (160 pix \( \times \) 160 pix) |
| S2: observation region and image  | (200 m \( \times \) 200 m), (200 pix \( \times \) 200 pix) |
| Max/min particles/hypothesized    | \( L_{\text{max}} = 5000, L_{\text{min}} = 1000 \) particles. |
| Existence probabilities           | \( p = 10^{-2} \) |
| Maximum tracks                    | \( T_{\text{max}} = 100 \) |
| Merging threshold                 | \( T_{\text{merge}} = 0.75 \) times the pixel width |
| The performance evaluation        |                 |
| the parameters                    |                 |
| p = 1, \( c = c_x + c_y, c_z = 60, \) | |
| c_x = c_4 + c_y + c_z = 40, \( w_x = 0.8, \) | |
| \( w_y = 0.2 \) | |
| Survival probability              | \( P_{kk} = 0.99 \) |

Abbreviations: EST, extended stealth target; SNR, signal-to-noise ratio.

Minor axes length \( \rightarrow d_{ij, s}^{(c)} = \min \left( c_A, \left\| A_k^{(ij)} - A_k^{(is)} \right\|_F \right) \),

Orientation \( \rightarrow d_{ij, s}^{(s)} = \min \left( c_A, \left\| A_k^{(is)} - A_k^{(ij)} \right\|_F \right) \) (42),

\( \| \cdot \|_2 \) is Euclidean norm and \( \| \cdot \|_F \) is Frobenius norm.

The EST tracking performance is presented as

\[
\bar{d}^{(c)}_p = \left( \frac{1}{N} \sum_{i=1}^{N} \left( d_{ij, s}^{(c)} \right)^p \right) + c^2 \left( \bar{N}_{kk} - N_{kk} \right). \quad (44)
\]

6.1 Modelling and tracking set-up of ESTs

In this study, we will use the same methodology as [2] to extract the stable scattering centre of each EST from the wideband measurements at sparsely distributed aspect angles shown in Figure 3. The real RCS values and stable scattering centres, such as the number of total measurements and the number of ellipses for each EST, are summarized in Table 2. The results can be used to reconstruct the EST signatures such as RCS data at any aspect in angular extent as shown in Figure 3. In Table 3, we summarized the profiles for each EST get reconstructed by fluctuation model. Two scenarios are used to illustrate their relative strengths and weaknesses as shown in Figure 4. The first scenario (S1) is used to illustrate the proposed filter performance for the difficult problem of tracking closely spaced ESTs. In the second scenario (S2), a challenging non-linear tracking with 10 ESTs to demonstrate the true capabilities of the proposed filter. The setting of parameter values related to simulation is summarized in Table 4. At time step \( k \), each sub-object \( s \) of the \( l \)th EST follows a non-linear nearly constant turn model in which the EST state is denoted by \( \mathbf{x}_k^{(s)} = [\mathbf{x}_k^{(s)}]^T, \omega_k^{(s)} \) , where \( \mathbf{x}_k^{(s)} = [x_{x,k}^{(s)}, y_{x,k}^{(s)}, \dot{x}_{x,k}^{(s)}, \dot{y}_{x,k}^{(s)}]^T, \) and the turn rate \( \omega_k^{(s)} \), motions are modelled by a non-linear nearly constant turn model given in Equation (2), where \( F_{k/k-1} \) and \( G_k \) are defined as...
The true sub-object $s$ of the $\ell$th EST extension based on the size $(A_k^{(s)}, a_k^{(s)})$ and orientation $(\hat{A}_k^{(s)}, \hat{a}_k^{(s)})$ for each $s$ are given in Equation (12). Here, we adopt the following simple model for an expected number of measurements generated by each sub-RMM-EST, the major axes, minor axes, and measurement rates. Assume that there are 10 ESTs emerging and disappearing successively. The real EST trajectories are shown in Figure 4b. The birth SMC-sub-RMM-MB-TBD process is a MB RFS with density:

$$P_{ik-1}(\omega_k^{(s)}) = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    \frac{\sin \omega_k^{(s)}}{a_k^{(k)}} & 0 & -\cos \omega_k^{(s)} & \Delta \\
    \cos \omega_k^{(s)} & 0 & \sin \omega_k^{(s)} & 0 \\
    0 & \frac{1 - \cos \omega_k^{(s)}}{a_k^{(k)}} & 1 & 0 \\
    0 & \frac{\sin \omega_k^{(s)}}{a_k^{(k)}} & 0 & \cos \omega_k^{(s)} \\
\end{bmatrix}, \ G_k = \begin{bmatrix}
    \Delta^2 & 0 & 0 & 0 \\
    0 & \Delta & 0 & 0 \\
    0 & 0 & \Delta^2 & 0 \\
    0 & 0 & 0 & \Delta \\
\end{bmatrix}$$

6.2 The first scenario (S1): tracking for two closed ESTs in different environments

In this scenario, the two ESTs move closer and then move in parallel, before separating again. For space consideration, the filter output for only the first six sampling periods in a single run is shown in Figure 5, where the estimated positions and extensions of ESTs by the proposed filter, H-PMHT and MB-GGIW filters, are given. The sample images from a scenario simulation with low and high noise effects are shown in

![Figure 5](image-url)
Figure 5a,b. It shows the simulated noisy data at various SNR levels based on RCS and noise effect. The two ESTs appear at the marked locations but are indistinguishable from the background noise. The single run for 100 time steps consist of two ESTs that are present for the entire scenario.

As shown in Figure 5a, in the case of low noise effect, as two ESTs moving in parallel, since the peak SNR is high, the EST position can be observed simply by the distinct pixels. The proposed filter can obtain detailed extension information about size, shape, and orientation, while the
other filters can only approximate the extension by using an ellipsoid (almost a circle). The results of different filters with high noise effect are shown in Figure 5b; in this case, it is not possible to detect the existence of the EST due to high noise effect. There is a downward trend while two ESTs are moving in parallel. The proposed algorithm suffers from delays in the initiation of tracks when all the sub-objects for each EST were initialized with the same circle (without any further shape and orientation information), as shown in Figure 5b. Since we are not sure which estimated extension corresponds to which true ellipse a priori (this correspondence is decided automatically by online adaptation using measurement information), we first try to find the permutation of the estimated sub-objects that makes the algorithms best approximate the true extension [15]. As shown in Figures 6 and 7, in the initiation of tracks, we can prevent the divergence of the proposed algorithm such as the shape and orientation information, and then the proposed algorithm corrects the extension for all the sub-objects for each EST at the next sampling periods ($k = 25$). In addition, other filters incorrectly obtain one large EST, instead of two smaller ones, in the most majority of sampling periods. This is what causes that the estimates of the major and minor axes by these two filters are much longer than the true ones, as shown in Figure 6. The proposed filter can almost accurately estimate the major and minor axes for each sub-object. The OSPA and the cardinality estimates are shown in Figure 5c,d. During the parallel motion, the other filters’ performance deteriorates, due to losing EST. The proposed filter performs well without cardinality errors and with smaller OSPA.

6.3 | The second scenario (S2): non-linear tracking

In the second scenario, we evaluate the performance of the proposed filter over 100 MC runs for non-linear TBD scenario with a higher number of ESTs (10 ESTs). This will be more fair to test the performance of proposed filter. The results of different filters with low and high noise effects are shown in Figure 8a,b. The estimated extension positions with accurate shape and size should be obtained from the true value by using the proposed filter comparing to other filters. The proposed algorithm suffers from delays in the initiation of tracks as we mentioned previously. As shown in Figure 8c,d, the proposed filter performs well without cardinality errors and with smaller OSPA. It is clear that the proposed filter outperforms H-PMHT filter whose performance is better than MB-GGIW filter with a higher number of ESTs, when the ESTs are overlapping at a time from 30 to 80 s. Note that while EST observations appear close to one another, they do not overlap. It can be seen that the proposed filter is able to maintain lock on all target tracks and accurately estimate their locations for the entire scenario. The H-PMHT performs well in this case since the targets travel along straight lines, thereby allowing easy and accurate predictions. On the other hand, it is clear that the H-PMHT filter performs considerably worse and degraded, when the ESTs are overlapping at a time from 30 to 80 s. The cardinality results of the proposed and other filters are shown in Figure 9. It can be seen from the averaged results that the proposed filter converges to a correct high number of ESTs, whereas H-PMHT and MB-GGIW filters appear to slowly self-correct due to their reliance on clustering.

From Table 5, we can see that the proposed filter has the best real-time performance. By calculating, for one MC run, the H-PMHT and MB-GGIW filters take 421.1467 and 534.3577 s on average, respectively, while the SMC-sub-RMM-MB-TBD filter only needs 117.6396 s.

**TABLE 5** Average running time for single Monte Carlo (s)

| SMC-Sub-RMM-MB-TBD | H-PMHT-TBD | MB-GGIW |
|---------------------|------------|---------|
| 117.6396            | 421.1467   | 534.3577 |

Abbreviations: GGIW, Gamma Gaussian inverse Wishart; MB, multi-Bernoulli; PMHT, Probabilistic Multi-Hypothesis-Tracker; RMM, random matrices model; TBD, track before detect.
**FIGURE 8** The second scenario (S2): the measurement frames in x and y directions at different time steps with Rayleigh noise effect: (a) low noise effect (frame no. 43 with high SNR = 13 dB, $R_k = \text{diag}([2, 2])m^2$); (b) high noise effect (frame no. 65 with low SNR = 4 dB, $R_k = \text{diag}([6, 6])m^2$); (c) mean OSPA (dashed lines) ± SD, (d) cardinality estimate (mean cardinality is dashed lines)

**FIGURE 9** Extended stealth target (EST) tracking in S2, and true EST tracks and measurement frames in x and y directions versus time
This is the biggest advantage of the proposed filter, namely the proposed filter can avoid several particles and reduce computational complexity, compared to the other filters, implying good application prospects for the real-time EST-TBD.

7 | CONCLUSION

In this study, we proposed SMC-sub-RMM-MB-TBD filter to improve the extent state estimation of multiple ESTs by using the sub-RMM to be accurate enough of useful information, such as size, shape, and orientation. In addition, we address a joint detection and tracking of ESTs and the TBD approach, which is an efficient way to track low-observable ESTs. We model the EST as a combination of multiple ellipsoidal sub-objects, each represented by a random matrix model (RMM). The simulation results show that the proposed algorithm has better performance than the recent H-PMHT and one-ellipse-MB-GGIW filters in low SNR applications.

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