Dynamic stress-strain state of earth dam considering an inhomogeneous base

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Abstract. The design, construction and operation of important water retaining structures (high earth dams) in seismic regions, such as the territory of the Republic of Uzbekistan, require constant improvement of the methods to calculate their behavior under various loads, including seismic ones. A variational statement of the problem of natural vibrations of a plane elastic earth dam with and without an adjacent base is given. The problem is solved numerically by the finite element method. The aim of this study is to determine the effect of a piecewise-inhomogeneous section at the base of an earth dam on the stress-strain state of the dam itself. The result of solving the problem lies in the analysis of eigenfrequencies, vibration modes, stresses of a concrete earth dam.

1. Introduction
The consequences of the accidents or damage in high earth dams can lead to large human casualties [1, 2]. The methods for calculating earth dams under static and dynamic impacts (including seismic ones) are regulated by a number of regulatory documents [4-8], created on the basis of research conducted by leading experts in the field of hydro-technical engineering. These norms are usually based on a one-dimensional model that does not take into account the structure geometry and the piecewise inhomogeneity of physical and mechanical characteristics of the structure and its base; this results in applying numerical methods for solving the problems of the mechanics of rigid bodies [9-12].

Seismic impacts are resonant in nature. The magnitude of seismic impact on a structure at a given earthquake intensity is determined by the structure's own dynamic characteristics (frequencies and modes of natural vibrations) - a kind of a “passport” of the structure. These characteristics are determined both by the design (configuration, dimensions, weight and physical and mechanical properties of the structure materials) and (to no less extent) by the rigidity or flexibility of the base. Seismic resistance and strength of the structure is checked by various calculation methods (static, linear-spectral and dynamic analysis) and by experimental methods.

2. Materials and methods
Mathematical statement of the plane problem for an elastic plane-deformable structure includes [9-11]:

1) The variational principle of the minimum total energy of the system:
\[
\delta IT - \delta W = 0
\]
where $\delta W$ is the increment of potential energy of the system, and $\delta W$ is the sum of the work of external forces on virtual displacements.

2) The equation of state expressing the relationship between the components of stresses $\sigma_{ij}$ and strains $\varepsilon_{ij}$ in an elastic medium (the Hooke's law)

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2 \mu \varepsilon_{ij}$$ (2)

3) Cauchy relations connecting strains with displacements

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$ (3)

4) Boundary conditions on a rigid base, meaning the absence of virtual displacements $\delta u$

$$\delta u = 0$$ (4)

Here $\bar{u} = \{u, u_i\}$ - are the horizontal and vertical displacements of the point of a body with coordinates $\{x, x_i\}$; $\sigma_{ij}, \varepsilon_{ij}$ are the components of the stress and strain tensor; $\lambda, \mu$ are the Lamé constants.

For the numerical solution of the problem (1-4), the finite element method is used, which is invariant with respect to the geometry of the object under study and mechanical characteristics of soils; it allows accounting the structure interaction with surrounding medium, various mechanical (static and dynamic) loads, various types of boundary conditions.

To carry out calculations, the options were considered for a plane earth structure (of trapezoidal cross-section) with a rigid base and with account for the piecewise-inhomogeneous characteristics of the base soil. It was necessary to choose a bounded design model "structure-base", which adequately represent the base settlement under the structure.

The boundary conditions are:
- there are no virtual linear displacements (vertical and horizontal ones) on the lower (rigid) face of the model, i.e. for $y=10$, $\delta u=0, \delta v=0$;
- the sliding hinges are installed on the side faces of the base, which do not impede vertical displacements, i.e. for $x=0$ and $x=L$, $\delta u=0$.

Such boundary conditions on the side faces of the base mean that only vertical displacements of the endless strip of base take place.

The first step in the dynamic analysis of a structure is to determine its dynamic characteristics – the frequencies and the modes of natural vibrations.

With the use of the developed methodology and a system of applied programs [9-11], the problem of natural vibrations has the form

$$[M] \{\ddot{q}(t)\} + [K]\{q(t)\} = 0,$$ (5)

and the solution to system (5) is sought in the form

$$\{q\} = \{q_0\} \sin \omega t.$$

As a result of substitution of solution into equation (5), a homogeneous system of algebraic equations is obtained; it has a unique solution only if the determinant of the system is equal to zero

$$([K] - \omega^2 [M]) \{q_0\} = 0$$ (6)

Thus, with the application of the finite element method based on the statement of variational problem; introduction of a vector of generalized displacements (displacements of nodes of finite element discretization mesh); determination of stiffness matrices (matrices of response to unit displacements of nodes); mass matrices (matrices of response to unit accelerations of nodal points); and arrangement of separate matrices of the elements of overall matrices of mass and stiffness for the entire plane area (of a dam), we obtain an algebraic problem for eigen values (6), where the following designations are taken: $[K], [M]$ are the stiffness and mass matrices of the structure; $\omega^2$ – is the eigenvalue (square of natural vibration frequency); $\{q\}$ is the mode of natural vibrations corresponding to the frequency (the vector of displacements of mesh nodes).
The eigen frequencies $\omega$ and the modes of natural vibrations $\{q\}$, representing the two-component displacements of the mesh nodes, corresponding to one or another vibration frequency, are determined using the Muller numerical method.

3. Results

The reliability of the results obtained was validated by a test problem, carried out by the authors when determining the first three eigenfrequencies and modes of vibrations of the Nurek dam. The results were compared with the data obtained by the All-Russian Research Institute of Hydraulic Engineering named after B.E. Vedeneev and NIS Hydoproject named after S.E. Zhuk, and a good agreement was observed.

Calculations of dynamic characteristics of the operated earth dam of the Tupolang hydro power plant (HPP) were carried out, taking into account the design features and real physical and mechanical characteristics of soil of the structure and the underlying base. This study was carried out by order of JSC "Hydoproject" to assess dynamic characteristics and to predict the stress-strain state of earth dam and its inhomogeneous base under static and dynamic loads.

When calculating structures for seismic effects, the formula of seismic load was determined as [5]

$$S_k = K_1 K_2 Q_k A \beta \eta_{ik} K_\phi .$$

The formula includes various coefficients, among which the coefficient of the calculated seismicity $A$, which depends on the intensity of expected earthquake; $Q_k$ is the weight acting on the $k$-th node of the structure; $\eta_{ik}$ are the coefficients of natural vibration modes of the structure, obtained by solving the problem on eigenvalues; dynamic factor $\beta_i$ - inversely proportional to the $i$-th period of natural vibration taken in calculations.

Thus, the first step in dynamic design of a structure is to determine its dynamic characteristics – the frequencies, especially those that could cause intense vibrations of the structure due to their proximity to the frequency spectrum of seismic vibrations in a given region, and the corresponding vibration modes.

\[ a) \, \omega_1 = 1.01 \, \text{Hz} \quad b) \, \omega_2 = 1.6 \, \text{Hz} \quad c) \, \omega_3 = 1.7 \, \text{Hz} \]

![Figure 1](image_url)  
*Figure 1. Fundamental frequencies and corresponding modes of natural vibrations of the dam on a homogeneous base.*

When solving the eigenvalue problem, the dynamic characteristics of earth dam of the Tupolang HPP ($H = 165m$) were obtained in two options: on a homogeneous rock base without a weakened zone ($E = 13000$ MPa, $\rho = 2.56$ t/m$^3$) and on a rocky base in the presence of a weakened zone – an anhydrite zone with the following physical and mechanical characteristics of soil $E=16$ MPa, $\rho=1.58$ t/m$^3$. 
The parameters of the dam soil were selected as follows: slope ratios of 2 and 1.9 for the upstream and downstream slopes, respectively, $E=70$ MPa, $\rho=2.04$ t/m$^3$, Poisson's ratio for all options was taken equal to 0.3.

The resulting vibration modes with indication of frequencies are shown in Figs. 1 and 2. Here the deformed state by modes is shown against the finite element mesh. A weakened zone in the form of an anhydrite zone is located along the entire diagonal line of the base; it is uniform in thickness, of a height of 50 m in two finite elements. In the first option of calculations - in the case of a homogeneous base - the parameters of this strip are chosen similar to the parameters of surrounding soil. In the second option, the parameters of this strip correspond to the parameters of the anhydride zone.

![Figure 2](image_url)

**Figure 2.** Distribution of horizontal (a), vertical (b) displacements and equivalent stresses (c) in the section of a dam on homogeneous rock base during the first mode shear.

The modes shown in the figures are: the shear in transverse direction (the 1st mode), a vertical mode (settlement) (the 2nd mode) and a complex mode of transverse vibrations (the 3rd mode). The rigid rock base remains undeformable.
4. Discussion
The horizontal displacements occurring in the cross section (Fig. 2,a) and the equivalent stresses (Fig. 2,b) are determined from the basic mode - the most energy-intensive one - the transverse displacement of the dam. The choice of these parameters is explained by the fact that they determine the strain mode (a horizontal shear) and the virtual transition of the model from elastic state in plastic.

Dynamic characteristics of the dam located on an inhomogeneous base (with a weakened zone) were determined in calculations: \( \omega_1 = 0.148 \, \text{Hz} \); \( \omega_2 = 0.21 \, \text{Hz} \); \( \omega_3 = 0.46 \, \text{Hz} \) and the corresponding modes of natural vibrations were plotted. Comparison of fundamental frequencies and modes of vibrations of the "dam-base" plane system shows that the presence of a weakened zone at the base changes the spectrum of fundamental frequencies: it decreases noticeably (due to the weakened zone) along with the decrease in the base rigidity.

![Vertical strain](image)

**Figure 3.** Vertical strain distribution of the "dam-inhomogeneous base" model during the first mode shear.

While the fundamental modes of vibration on a homogeneous rock base include the structure strain as a less rigid part of the model, then in the presence of a weakened zone, the fundamental modes of vibration include non-uniform vertical strain of the base. In this case, the structure tilts like a solid body, almost without deformation, as indicated in Fig. 3 by the vertical component of strain at the first mode of natural vibrations of the “dam-inhomogeneous base” model. The maximum value of vertical strain (dark color) falls on the right side of the anhydrite zone, and the “+” sign indicates vertical extension of this area.

5. Summary
A methodology and a complex of applied programs for the dynamic calculation of plane elastic system "earth dam-base" have been developed numerically by the finite element method. The analysis of the stress-strain state of this system indicates the need to account for physical and mechanical characteristics of soil of the structure itself and its base under various kinds of loads, not provided for in the standards. The influence of the weakened zone at the base on the SSS was investigated.

It was revealed that the first mode and the corresponding frequency reflect the oscillatory processes in the base, which do not affect the structure itself (the dam). Therefore, a comparison of the stress state in a dam on a homogeneous base (without a weakened zone) and on a inhomogeneous base (with a weakened zone) must be carried out using identical modes, which are the first mode for a dam on a homogeneous base and the third mode for a dam on a heterogeneous base.

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