Late transient acceleration of the universe in string theory on $S^1/Z_2$

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Abstract. Recently, in Gong et al (2008 Phys. Lett. B 663 147 [arXiv:0711.1597]) and Wang and Santos (2007 arXiv:0712.3938) we showed that the effective cosmological constant on each of the two orbifold branes can be easily lowered to its current observational value, by using the large extra dimensions in the framework of both M-theory and string theory on $S^1/Z_2$. In this paper, we study the current acceleration of the universe, using the formulae developed in Wang and Santos (2007 arXiv:0712.3938). We first construct explicitly a time-dependent solution to the ten-dimensional bulk of the Neveu–Schwarz/Neveu–Schwarz sector, compactified on a five-dimensional torus. Then, we write down the generalized Friedmann equations on each of the two dynamical branes, and fit the models to the 182 gold Supernova Ia data and the BAO parameter from SDSS, using both of our MINUIT and Monte Carlo Markov Chain (MCMC) codes. With the best fitting values of the parameters involved as initial
conditions, we integrate the generalized Friedmann equations numerically and find the future evolution of the universe. We find that it depends on the choice of the radion potentials \( V_4^{(I)} (I = 1, 2) \) of the branes. In particular, when choosing them to be Goldberger–Wise potentials, \( V_4^{(I)} = \lambda_4^{(I)} (\psi^2 - v_I^2)^2 \), we find that the current acceleration of the universe driven by the effective cosmological constant is only temporary. Due to the effects of the potentials, the universe will finally be in its decelerating expansion phase again. We also study the proper distance between the two branes and find that it remains almost constant during the whole future evolution of the universe in all the models considered.

**Keywords:** dark energy theory, string theory and cosmology, cosmological simulations, cosmological applications of theories with extra dimensions

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1. **Introduction**

One of the long-standing problems in particle physics and cosmology is the cosmological constant problem: its theoretical expectation values from quantum field theory exceed observational limits by 120 orders of magnitude [1]. Even if such high energies are suppressed by supersymmetry, the electroweak corrections are still 56 orders higher. This problem was further sharpened by recent observations of supernova (SN) Ia, which reveal the striking discovery that our universe has lately been in an accelerated expansion phase [2]. Cross-checks from the cosmic microwave background radiation and large scale structure all confirm this unexpected result [3].
In Einstein’s theory of gravity, such an expansion can be achieved by a tiny positive cosmological constant. In fact, such a constant is well consistent with all observations carried out so far [4]. Therefore, solving the cosmological constant problem now becomes more urgent than ever before. As a matter of fact, it is exactly because of this that a large number of ambitious projects have been proposed lately to distinguish the cosmological constant from dynamical dark energy models [5].

Since the problem is intimately related to quantum gravity, its solution is expected to come from quantum gravity, too. At the present, string/M-theory is our best bet for a consistent quantum theory of gravity, so it is reasonable to ask what string/M-theory has to say about the cosmological constant. In the string landscape [6], it is expected that there are many different vacua with different local cosmological constants [7]. Using the anthropic principle, one may select the low energy vacuum in which we can exist. However, many theorists still hope to explain the problem without invoking the existence of ourselves in the universe.

Townsend and Wohlfarth [8] considered a time-dependent compactification of pure gravity in higher dimensions with hyperbolic internal space to circumvent Gibbons’ no-go theorem [9]. Their exact solution exhibits a short period of acceleration. The solution is the zero-flux limit of spacelike branes [10]. If non-zero flux or forms are turned on, a transient acceleration exists for both compact internal hyperbolic and flat spaces [11]. Other accelerating solutions, by compactifying more complicated time-dependent internal spaces, can be found in [12].

In the same spirit, the cosmological constant problem was also studied in the framework of a brane world in five-dimensional spacetimes [13] and six-dimensional supergravity [14]. However, it turned out that in the five-dimensional case hidden fine-tunings are required [15], while in the six-dimensional case it is still not clear whether loop corrections can be as small as required [16].

Recently, we [17] studied the cosmological constant problem and late acceleration of the universe in the framework of the Horava–Witten heterotic M-theory on $S^1/Z_2$ [18]. In particular, using the Arkani–Hamed–Dimopoulos–Dvali (ADD) mechanism of large extra dimensions [19], we showed explicitly that the effective cosmological constant on each of the two orbifold branes can be easily lowered to its current observational value. Applying it to cosmology, we further found that the domination of the effective cosmological constant is only temporary. Due to the interaction of the bulk and the brane, the universe will be in its decelerating expansion phase again in the future, whereby all problems connected with string cosmology [20] are resolved. Such studies were also generalized to string theory, and found that the ADD mechanism can be used in the same way to solve the cosmological constant problem [21]. Therefore, the ADD mechanism for solving both the cosmological constant problem and the hierarchy problem is a built-in mechanism in the brane world of string/M-theory.

In this paper, we apply the theory developed in [21] in the framework of string theory on $S^1/Z_2$ to cosmology, and study the current acceleration of the universe. In particular, in section 2 we first give a brief review of the theory and then write down the field equations both in the bulk and on the two orbifold branes. In section 3, we present a particular time-dependent solution to these equations, and study its local and global properties. Then, we write down explicitly the generalized Friedmann equations on each of the two branes for any radion potentials $V_4^{(I)} (I = 1, 2)$ of the branes. Depending on the choice of...
V_4^{(I)} s, the future evolution of the universe is different. We study two different cases. We fit these models to the 182 gold Supernova Ia data [22] and BAO parameter from SDSS [23], using both of our MINUIT [24] and Monte Carlo Markov Chain (MCMC) [25] codes. With these best-fitting values as the initial condition, we integrate numerically the field equations on the branes to find the future evolution of the universe. In the latter case, we show that the current acceleration of the universe driven by the effective cosmological constant is only temporary. Due to the effects of the potentials, the universe will be in its decelerating expansion phase again. We also study the proper distance between the two orbifold branes and find that it remains almost constant during the whole future evolution of the universe in all these models. In section 4, we summarize our main results and present some concluding remarks.

Before turning to section 2, we would like to note that Sahni and Shtanov [26] found that transient acceleration of the universe happened in the DGP brane model [27], too.

2. Brany cosmology of string theory on $S^1/Z_2$

To begin with, in this section we give a brief review on the cosmological models of orbifold branes developed in [21]. In this paper we shall restrict ourselves directly to the case $D = d = 5$. For the case with arbitrary $D$ and $d$, we refer readers to [21].

2.1. The model

For the toroidal compactification of the Neveu–Schwarz/Neveu–Schwarz (NS–NS) sector in (5 + 5) dimensions, $\hat{M}_{10} = M_5 \times T_5$, where $T_5$ is a five-dimensional torus, the action takes the form [28, 29]

$$\hat{S}_{10} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|\hat{g}_{10}|} e^{-\hat{\Phi}} \left\{ \hat{R}_{10}[\hat{g}] + \hat{g}^{AB}(\hat{\nabla}_A \hat{\Phi})(\hat{\nabla}_B \hat{\Phi}) - \frac{1}{12} \hat{H}^2 \right\},$$

(2.1)

where $\hat{\nabla}_A$ denotes the covariant derivative with respect to $\hat{g}^{AB}$ with $A, B = 0, 1, \ldots, 9$, and $\hat{\Phi}$ is the dilaton field. The NS 3-form field $\hat{H}_{ABC}$ is defined as

$$\hat{H}_{ABC} = 3\partial_{[A}\hat{B}_{BC]},$$

(2.2)

where the square brackets imply total antisymmetrization over all indices. The ten-dimensional spacetimes to be considered are described by the metric

$$d\hat{s}_{10}^2 = \hat{g}_{AB} dx^A dx^B = \hat{g}_{ab}(x^c) dx^a dx^b + h_{ij}(x^c) dz^i dz^j,$$

(2.3)

where $\hat{g}_{ab}$ is the metric on $M_5$, parameterized by the coordinates $x^a$ with $a, b, c = 0, 1, \ldots, 4$, and $h_{ij}$ is the metric on the compact space $T_5$ with periodic coordinates $z^i$, where $i, j = 5, 6, \ldots, 9$.

By assuming that all the matter fields are functions of $x^a$ only, it can be shown that the effective five-dimensional action is given by

$$S_5 = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{|\hat{g}_5|} e^{-\hat{\phi}} \left\{ \hat{R}_5[\hat{g}] + (\hat{\nabla}_a \hat{\phi})(\hat{\nabla}^a \hat{\phi}) + \frac{1}{4}(\hat{\nabla}_a h^{ij})(\hat{\nabla}^a h_{ij}) - \frac{1}{12} \hat{H}_{abc} \hat{H}^{abc} - 12 \hat{H}_{abc} \hat{H}^{abc} \right\},$$

(2.4)
where
\[ \tilde{\phi} = \Phi - \frac{1}{2} \ln |h|, \]  
(2.5)

\[ \kappa_5^2 \equiv \frac{\kappa_{10}^2}{V_0}, \]  
(2.6)

with the five-dimensional internal volume given by
\[ V(x^a) \equiv \int d^5z \sqrt{|h|} = |h|^{1/2}V_0. \]  
(2.7)

Note that in writing the action (2.4) we had assumed that the flux is block diagonal:
\[ (\hat{B}_{CD}) \equiv (\tilde{B}_{ab} 0 0 B_{ij}). \]  
(2.8)

The action (2.4) is usually referred to as written in the string frame. To go to the Einstein frame, we make the following conformal transformations:
\[ g_{ab} = \Omega^2 \tilde{g}_{ab}, \]
\[ \Omega^2 = \exp \left( -\frac{2}{3} \tilde{\phi} \right), \]
\[ \phi = \sqrt{\frac{2}{3}} \tilde{\phi}. \]  
(2.9)

Then, the action (2.4) takes the form
\[ S_{5,(E)} = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{|g_5|} \left[ R_5[g] - \frac{1}{2} (\nabla \phi)^2 + \frac{1}{4} (\nabla_a h^{ij}) (\nabla^a h_{ij}) - \frac{1}{12} e^{-\sqrt{\frac{8}{3}}\phi} H_{abc} H^{abc} \right. \]
\[ \left. - \frac{1}{4} h^{ik} h^{jl} (\nabla_a B_{ij}) (\nabla^a B_{kl}) \right], \]  
(2.10)

where \( \nabla_a \) denotes the covariant derivative with respect to \( g_{ab} \). It should be noted that, since the definition of the 3-form \( \tilde{H}_{ABC} \) given by (2.2) is independent of the metric, it is conformally invariant. In particular, we have \( H_{abc} = \tilde{H}_{abc} \) and \( B_{ab} = \tilde{B}_{ab} \). However, we do have
\[ H^{abc} = g^{ad} g^{be} g^{cf} H_{def} = \Omega^{-6} \tilde{H}^{abc}, \]
\[ H_{abc} H^{abc} = \Omega^{-6} \tilde{H}_{abc} \tilde{H}^{abc}. \]  
(2.11)

Considering the addition of a potential term [29], in the string frame we have
\[ \tilde{S}_{10}^m = - \int d^{10}x \sqrt{|g_{10}|} V_{10}. \]  
(2.12)

Then, after the dimensional reduction we find
\[ S_{5,m} = -V_0 \int d^5x \sqrt{|g_5|} |h|^{1/2} V_{10}. \]  
(2.13)

where
\[ \tilde{g}_5 = \exp \left( \sqrt{\frac{50}{3}} \phi \right) g_5. \]  
(2.14)
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Changed to the Einstein frame, the action (2.13) finally takes the form

$$S_{5,m}^{(E)} \equiv -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{|g_5|} V_5,$$

where

$$V_5 \equiv 2\kappa_5^2 V_0 V_{10}^s \exp \left( \frac{5}{\sqrt{6}} \phi \right) |h|^{1/2}. \quad (2.16)$$

If we further assume that

$$h_{ij} = -\exp \left( \frac{1}{2} \psi \right) \delta_{ij},$$

$$h^{ij} = -\exp \left( -\frac{1}{2} \psi \right) \delta^{ij}, \quad (2.17)$$

we find that

$$S_5^{(E)} + S_{5,m}^{(E)} = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{|g_5|} \left\{ R_5[g] - \frac{1}{2} \left( (\nabla \phi)^2 + (\nabla \psi)^2 - 2V_5 \right) - \frac{1}{4} e^{-\sqrt{8/5}\psi} \delta^{ik} \delta^{jl} (\nabla_a B_{ij}) (\nabla^a B_{kl}) - \frac{1}{12} e^{-\sqrt{8/3}\phi} H_{abc} H^{abc} \right\}, \quad (2.18)$$

where the effective five-dimensional potential (2.14) now becomes

$$V_5 \equiv V_{(5)}^0 \exp \left( \frac{5}{\sqrt{6}} \phi + \sqrt{\frac{5}{2}} \psi \right), \quad (2.19)$$

where $V_{(5)}^0 \equiv 2\kappa_5^2 V_0 V_{10}^s$.

To study orbifold branes, we consider the brane actions

$$S_{4,m}^{(I)} = -\int_{M_4^{[I]}} d^4x \sqrt{|g_4^{[I]}|} \left( \epsilon_1 V_4^{(I)}(\phi, \psi) + g_s^{(I)} \right) d^4\xi^{(I)} + \int_{M_4^{[I]}} d^4\xi^{(I)} \sqrt{|g_4^{(I)}|} \right\} \cdot L_{4,m}^{(I)}(\phi, \psi, B, \chi), \quad (2.20)$$

where $I = 1, 2$, $V_4^{(I)}(\phi, \psi)$ denotes the potential of the scalar fields $\phi$ and $\psi$, and $\xi^{(I)}$ are the intrinsic coordinates of the $I$th brane with $\mu, \nu = 0, 1, 2, 3$ and $\epsilon_1 = -\epsilon_2 = 1$. $\chi$ denotes collectively the matter fields and $g_s^{(I)}$ is a constant, which is related to the four-dimensional Newtonian constant via the relation given by equation (2.39) below. The variation of the total action:

$$S_{\text{total}} = S_5^{(E)} + S_{5,m}^{(E)} + \sum_{i=1}^{2} S_{4,m}^{(I)}, \quad (2.21)$$

with respect to the metric $g_{ab}$ yields the field equations

$$G_{ab}^{(5)} = \kappa_5^2 T_{ab}^{(5)} + \kappa_5^2 \sum_{I=1}^{2} T_{\mu\nu}^{(I)} e_a^{(I,\mu)} e_b^{(I,\nu)} \sqrt{|g_4^{(I)}|/g_5} \delta (\Phi_I), \quad (2.22)$$

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where $\delta(x)$ denotes the Dirac delta function normalized in the sense of [30], and the two branes are localized on the surfaces:

$$\Phi_I(x^a) = 0.$$  

(2.23)

The energy–momentum tensors $T_{ab}^{(5)}$ and $T_{\mu\nu}^{(I)}$ are given by

$$\kappa_5^2 T_{ab}^{(5)} = \frac{1}{2} \left[ (\nabla_a \phi)(\nabla_b \phi) + (\nabla_a \psi)(\nabla_b \psi) + \frac{1}{2} e^{-\sqrt{s/5} \phi} (\nabla_a B^{ij})(\nabla_b B_{ij}) + \frac{1}{2} e^{\sqrt{s/3} \phi} H_{abc} H^{bcd} \right]$$

$$- \frac{1}{2} g_{ab} \left[ (\nabla \phi)^2 + (\nabla \psi)^2 - 2 V_5 + \frac{1}{2} e^{-\sqrt{s/5} \phi} (\nabla_a B^{ij})(\nabla^c B_{ij}) \right]$$

$$+ \frac{1}{6} e^{\sqrt{s/3} \phi} H_{cde} H^{cde} ,$$

(2.24)

$$T_{\mu\nu}^{(I)} = \tau_{\mu\nu}^{(I)} + \left( g_{s}^{(I)} + \tau_{(\phi, \psi)}^{(I)} \right) g_{\mu\nu}^{(I)} ,$$

$$\tau_{\mu\nu}^{(I)} = 2 \frac{\delta L_{4,m}^{(I)}}{\delta g^{(I)\mu\nu}} - g^{(I)\mu\nu} L_{4,m}^{(I)} ,$$

where $B^{ij} \equiv \delta^{ij} \delta_{kl} B_{kl}$:

$$\tau_{(\phi, \psi)}^{(I)} = \epsilon_1 V_4^{(I)} (\phi, \psi) ,$$

$$e_{(\mu)}^{(I)a} \equiv \frac{\partial x^a}{\partial \xi_{(\mu)}^{(I)}} ,$$

$$e_{(\mu)}^{(I)a} \equiv g_{ab} g_{(I)\mu\nu} e_{(\nu)}^{(I)b} ,$$

and $g_{\mu\nu}^{(I)}$ is the reduced metric on the $I$th brane, defined as

$$g_{\mu\nu}^{(I)} \equiv \left. g_{ab} e_{(\mu)}^{(I)a} e_{(\nu)}^{(I)b} \right|_{\mathcal{M}_I^{(I)}}.$$

(2.25)

Variation of the total action equation (2.21) with respect to $\phi, \psi$ and $B$, respectively, yields the following equations of the matter fields:

$$\Box \phi = - \frac{\partial V_5}{\partial \phi} - \frac{1}{12} \sqrt{\frac{8}{3}} e^{-\sqrt{s/5} \phi} H_{abc} H^{abc} - 2 \kappa_5^2 \sum_{I=1}^{2} \left( \epsilon_1 \frac{\partial V_4^{(I)}}{\partial \phi} + \sigma_\phi^{(I)} \right) \sqrt{\left| g_4^{(I)} \right| \left| g_5 \right|} \delta (\Phi_I) ,$$

(2.27)

$$\Box \psi = - \frac{\partial V_5}{\partial \psi} - \sqrt{\frac{1}{10}} e^{-\sqrt{s/5} \phi} (\nabla_a B^{ij})(\nabla^a B_{ij}) - 2 \kappa_5^2 \sum_{I=1}^{2} \left( \epsilon_1 \frac{\partial V_4^{(I)}}{\partial \psi} + \sigma_\psi^{(I)} \right)$$

$$\left. \sqrt{\frac{g_4^{(I)}}{g_5}} \right| \delta (\Phi_I) ,$$

(2.28)

$$\Box B_{ij} = \sqrt{\frac{8}{5}} (\nabla_a \psi)(\nabla^a B_{ij}) - \sum_{I=1}^{2} \Psi_{ij}^{(I)} \left. \sqrt{\frac{g_4^{(I)}}{g_5}} \right| \delta (\Phi_I) ,$$

(2.29)

$$\nabla^c H_{cab} = \sqrt{\frac{8}{3}} H_{cab} \nabla^c \phi - \sum_{I=1}^{2} \Phi_{ab}^{(I)} \left. \sqrt{\frac{g_4^{(I)}}{g_5}} \right| \delta (\Phi_I) ,$$

(2.30)
where $\Box \equiv g^{ab}\nabla_a \nabla_b$ and

$$
\sigma_{\phi}^{(I)} \equiv -\frac{\delta L_{4,m}^{(I)}}{\delta \phi},
$$

$$
\sigma_{\psi}^{(I)} \equiv -\frac{\delta L_{4,m}^{(I)}}{\delta \psi},
$$

(2.31)

To write down the field equations on the branes, one can first express the delta function part of $G_{\mu\nu}^{(5)}$ in terms of the discontinuities of the first derivatives of the metric coefficients, and then equal the delta function parts of the two sides of equation (2.22), as shown systematically in [31]. The other way is to use the Gauss–Codacci equations to write the (4)-dimensional Einstein tensor as [32]

$$
G_{\mu\nu}^{(4)} = G_{\mu\nu}^{(5)} + E_{\mu\nu}^{(5)} + F_{\mu\nu}^{(4)},
$$

(2.32)

where

$$
G_{\mu\nu}^{(5)} \equiv \frac{2}{3} \left\{ G_{ab}^{(5)} e_{(\mu)}^a e_{(\nu)}^b - [G_{ab} n^a n^b + \frac{1}{3} G^{(5)}] g_{\mu\nu} \right\},
$$

$$
E_{\mu\nu}^{(5)} \equiv C_{abcd} n^a e_{(\mu)}^b n^c e_{(\nu)}^d,
$$

$$
F_{\mu\nu}^{(4)} \equiv K_{\mu}^\lambda K_{\nu}^\lambda - K K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (K_{\alpha\beta} K^{\alpha\beta} - K^2),
$$

(2.33)

where $n^a$ denotes the normal vector to the brane, $G^{(5)} \equiv g^{ab} G_{ab}^{(5)}$, and $C_{abcd}$ the Weyl tensor. The extrinsic curvature $K_{\mu\nu}$ is defined as

$$
K_{\mu\nu} \equiv e_{(\mu)}^a n_a \nabla_{(\nu)} n_b.
$$

(2.34)

A crucial step of this approach is the Lanczos equations [33]:

$$
[K_{\mu\nu}^{(I)}]^- - g_{\mu\nu} [K^{(I)}]^- = -\kappa_5^2 T_{\mu\nu}^{(I)},
$$

(2.35)

where

$$
[K_{\mu\nu}^{(I)}]^- \equiv \lim_{\phi_{\nu} \to 0^+} K_{\mu\nu}^{(I)+} - \lim_{\phi_{\nu} \to 0^-} K_{\mu\nu}^{(I)-},
$$

$$
[K^{(I)}]^- \equiv g_{\mu\nu}^{(I)} [K_{\mu\nu}^{(I)}]^-.
$$

(2.36)

Assuming that the branes have $Z_2$ symmetry, we can express the intrinsic curvatures $K_{\mu\nu}^{(I)}$ in terms of the effective energy–momentum tensor $T_{\mu\nu}^{(I)}$ through the Lanczos equations (2.35). Then, we find that $G_{\mu\nu}^{(4)}$ given by equation (2.32) can be cast in the form

$$
G_{\mu\nu}^{(4)} = G_{\mu\nu}^{(5)} + E_{\mu\nu}^{(5)} + \kappa_5^2 T_{\mu\nu} + \Lambda g_{\mu\nu} + \kappa_5^4 \pi_{\mu\nu},
$$

(2.37)
where
\[ \pi_{\mu\nu} \equiv \frac{1}{4} \left\{ \tau_{\mu\lambda} \tau^\lambda_{\nu} - \frac{1}{3} \tau_{\nu\mu} - \frac{1}{2} g_{\mu\nu} \left( \tau^{\alpha\beta} \tau_{\alpha\beta} - \frac{1}{3} \tau^2 \right) \right\}, \]
\[ \mathcal{E}^{(4)}_{\mu\nu} \equiv \frac{\kappa^4_5}{6} \tau_{(\phi,\psi)} \left[ \tau_{\mu\nu} + \left( g_s + \frac{1}{2} \tau_{(\phi,\psi)} \right) g_{\mu\nu} \right], \]  
(2.38)

and
\[ \kappa^2_4 = \frac{1}{6} g_s \kappa^4_5, \]
\[ \Lambda = \frac{1}{12} g^2_s \kappa^4_5. \]  
(2.39)

For a perfect fluid
\[ \tau_{\mu\nu} = \left( \rho + p \right) u_\mu u_\nu - pg_{\mu\nu}, \]  
(2.40)

where \( u_\mu \) is the 4-velocity of the fluid, we find that
\[ \pi_{\mu\nu} = \frac{\rho}{6} \left[ \left( \rho + p \right) u_\mu u_\nu - \left( p + \frac{1}{2} \rho \right) g_{\mu\nu} \right]. \]  
(2.41)

Note that, in writing equations (2.37)–(2.41), without causing any confusion, we had dropped the superscript \( (I) \).

In the rest of this paper, we shall turn off the flux, i.e. \( \hat{B}_{CD} = 0 \), which is consistent with the field equations, provided that \( \Psi^{(I)}_{ij} = 0 \) and \( \Phi^{(I)}_{ab} = 0 \).

### 2.2. The general metric of the five-dimensional spacetimes

Since we shall apply such spacetimes to cosmology, let us first consider the embedding of a three-dimensional spatial space that is homogeneous, isotropic and independent of time. It is not difficult to show that such a space must have a constant curvature and its metric takes the form [34]
\[ d\Sigma_k^2 = \frac{d\tau^2}{1 - k\tau^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right), \]  
(2.42)

where the constant \( k \) represents the curvature of the 3-space, and can be positive, negative or zero. Without loss of generality, we shall choose coordinates such that \( k = 0, \pm 1 \). Then, one can see that the most general metric for the five-dimensional spacetime must take the form
\[ ds_5^2 = g_{ab} \, dx^a \, dx^b = g_{MN} \, dx^M \, dx^N - e^{2\omega(x^N)} \, d\Sigma_k^2, \]  
(2.43)

where \( M, N = 0, 1 \). Clearly, the metric (2.43) is invariant under the coordinate transformations
\[ x^N = f^N \left( x^M \right). \]  
(2.44)

Using these two degrees of freedom, without loss of generality, we can always set
\[ g_{00} = g_{11}, \quad g_{01} = 0, \]  
(2.45)

so that the five-dimensional metric finally takes the form
\[ ds_5^2 = e^{2\sigma(t, y)} \left( dt^2 - dy^2 \right) - e^{2\omega(y)} \, d\Sigma_k^2. \]  
(2.46)
It should be noted that metric (2.46) is still subjected to the gauge freedom
\[ t = f(t' + y') + g(t' - y'), \quad y = f(t' + y') - g(t' - y'), \] (2.47)
where \( f(t' + y') \) and \( g(t' - y') \) are arbitrary functions of their indicated arguments.

It is also interesting to note that in [31] a different gauge was used. Instead of setting \( g_{00} = g_{11} \) it was chosen that the two branes are comoving with the coordinates, so that they are located on two fixed hypersurfaces \( y = 0, y_c \). For details, see [31].

### 2.3. The field equations outside the two orbifold branes

The non-vanishing components of the Ricci tensor outside of the two branes are given by

\[
\begin{align*}
R^{(5)}_{tt} &= \sigma_{yy}^2 + 3\sigma_{yy}\omega_{tt} - [\sigma_{tt} + 3\omega_{tt} + 3\omega_t (\omega_t - \sigma_t)], \\
R^{(5)}_{ty} &= -3 [\omega_{ty} + \omega_t\omega_y - (\sigma_{ty} + \sigma_y\omega_t)], \\
R^{(5)}_{yy} &= \sigma_{tt} + 3\sigma_t\omega_t - [\sigma_{yy} + 3\omega_{yy} + 3\omega_y (\omega_y - \sigma_y)], \\
R^{(5)}_{mn} &= -e^{-2\sigma} g_{mn} \{\omega_{tt} + 3\omega_t^2 - (\omega_{yy} + 3\omega_y^2) + 2k e^{2(\sigma - \omega)}\},
\end{align*}
\] (2.48)

where now \( m, n = r, \theta, \phi, \sigma_t \equiv \partial \sigma / \partial t \) and so on. Then, it can be shown that outside of the two branes the field equations have four independent components, which can be cast into the form

\[
\begin{align*}
\omega_{tt} + \omega_t (\omega_t - 2\sigma_t) + \omega_{yy} + \omega_y (\omega_y - 2\sigma_y) &= -\frac{1}{6} \left[ (\phi_{,t}^2 + \phi_{,y}^2) + (\psi_{,t}^2 + \psi_{,y}^2) \right], \\
2\sigma_{tt} + \omega_{tt} - 3\omega_t^2 - (2\sigma_{yy} + \omega_{yy} - 3\omega_y^2) - 4k e^{2(\sigma - \omega)} &= -\frac{1}{2} \left[ (\phi_{,t}^2 - \phi_{,y}^2) + (\psi_{,t}^2 - \psi_{,y}^2) \right], \\
\omega_{ty} + \omega_t\omega_y - (\sigma_{ty} + \sigma_y\omega_t) &= -\frac{1}{6} (\phi_{,t}\phi_{,y} + \phi_{,t}\psi_{,y}), \\
\omega_{tt} + 3\omega_t^2 - (\omega_{yy} + 3\omega_y^2) + 2k e^{2(\sigma - \omega)} &= \frac{1}{3} e^{2\sigma} V_5,
\end{align*}
\] (2.49)

where \( V_5 \) is given by equation (2.19). On the other hand, the Klein–Gordon equations (2.27) and (2.28) outside the two branes take the form

\[
\begin{align*}
\phi_{,tt} + 3\phi_{,t}\omega_t - (\phi_{,yy} + 3\phi_{,y}\omega_y) &= -\frac{5}{12 \sqrt{6}} V_5 e^{2\sigma}, \\
\psi_{,tt} + 3\psi_{,t}\omega_t - (\psi_{,yy} + 3\psi_{,y}\omega_y) &= -\sqrt{\frac{5}{2}} V_5 e^{2\sigma}.
\end{align*}
\] (2.50)

### 2.4. The field equations on the two orbifold branes

Equations (2.48)–(2.54) are the field equations that are valid in between the two orbifold branes, \( y_2(t_2) < y < y_1(t_1) \), where \( y = y_1(t_1) \) denote the locations of the two branes. The proper distance between the two branes is given by

\[
D \equiv \int_{y_2}^{y_1} \sqrt{-g_{yy}} \, dy.
\] (2.55)
On each of the two branes, the metric reduces to
\[ \text{d} s^2 |_{M^I_4} = g^{(t)}_{\mu \nu} \text{d} \xi^\mu_{(t)} \text{d} \xi^\nu_{(t)} = \text{d} \tau_1^2 - a^2 (\tau_1) \text{d} \Sigma_k^2, \]
where \[ \xi^\mu_{(s(I))} \equiv \{ \tau_I, r, \theta, \varphi \}, \] and \( \tau_I \) denotes the proper time of the \( I \)th brane, defined by
\[ \text{d} \tau_I = e^\sigma \left( 1 - \left( \frac{\dot{y}_I}{\dot{t}_I} \right)^2 \right) \text{d} t_I, \]
with \( \dot{y}_I \equiv \text{d} y_I / \text{d} \tau_I \), etc. For the sake of simplicity, and without causing any confusion, from now on we shall drop all the indices \( I \), unless some specific attention is needed. Then, the normal vector \( n_a \) and the tangential vectors \( e^\alpha_{(\langle \nu \rangle)} \) are given, respectively, by
\[ n_a = \epsilon e^{2\sigma} (-\dot{y}_d^t + i \dot{y}_d^y), \quad n^a = -\epsilon (i \dot{y}_a^t + \dot{y}_a^y), \]
\[ e^a_{(\tau)} = i \dot{y}_a^t + \dot{y}_a^y, \quad e^a_{(r)} = \delta^a_r, \]
\[ e^a_{(\theta)} = \delta^a_\theta, \quad e^a_{(\varphi)} = \delta^a_\varphi, \]
where \( \epsilon = \pm 1 \). When \( \epsilon = +1 \), the normal vector \( n^a \) points toward the increasing direction of \( y \), and when \( \epsilon = -1 \), it points toward the decreasing direction of \( y \). Then, the four-dimensional field equations on each of the two branes take the form
\[ H^2 + \frac{k}{a^2} = \frac{8 \pi G}{3} \left( \rho + \tau_{(\phi, \psi)} \right) + \frac{1}{3} \Lambda + \frac{1}{3} G_5^{(S)} + E_5^{(S)} + \frac{2 \pi G}{3 \rho_L} \left( \rho + \tau_{(\phi, \psi)} \right) ^2, \]
\[ \frac{\dot{a}}{a} = -\frac{4 \pi G}{3} \left( \rho + 3 p - 2 \tau_{(\phi, \psi)} \right) + \frac{1}{3} \Lambda - E_5^{(S)} - \frac{1}{6} \left( G_5^{(S)} + 3 G_5^{(5)} \right) - \frac{2 \pi G}{3 \rho_L} \left( \rho + 3 p - \tau_{(\phi, \psi)} \right) \tau_{(\phi, \psi)}, \]
where \( H \equiv \dot{a}/a, \rho_L \equiv \Lambda_4 / (8 \pi G_4), \) and
\[ G^{(S)}_5 \equiv \frac{1}{8 \pi} e^{-2\sigma} \left[ \left( \phi_r^2 + \psi_r^2 \right) - \left( \phi_y^2 + \psi_y^2 \right) \right] - \frac{1}{2} \left( 5 \left( \nabla \phi \right)^2 + \left( \nabla \psi \right)^2 \right) - 6 V_5, \]
\[ G^{(5)}_\phi \equiv \frac{1}{2 \pi} \left\{ 8 \left( \phi_r^2 + \psi_r^2 \right) - 6 V_5 + \frac{5}{2} \left( \nabla \phi \right)^2 + \left( \nabla \psi \right)^2 \right\}, \]
\[ E_5^{(S)} \equiv \frac{1}{6} e^{-2\sigma} \left[ \left( \sigma_{,tt} - \omega_{,t} \right) - \left( \sigma_{,yy} - \omega_{,y} \right) \right] + k e^{2(\sigma - \omega)}, \]
with \( \phi_n \equiv n^a \nabla a^\phi \). If the typical size of the extra dimensions is \( R \), then it can be shown that
\[ \rho_\Lambda = \frac{\Lambda_4}{8 \pi G_4} = 3 \left( \frac{R}{l_{pl}} \right)^{10} \left( \frac{M_{10}}{M_{pl}} \right)^{16} M_{pl}^4, \]
where \( M_{pl} \) and \( l_{pl} \) denote the Planck mass and length, respectively. If \( M_{10} \) is of the order of TeV \([35]\), we find that, in order to have \( \rho_\Lambda \) be of the order of its current observational value \( \rho_\Lambda \simeq 10^{-47} \) GeV\(^4\), the typical size of the extra dimensions should be \( R \simeq 10^{-22} \) m, which is well below the current experimental limit of the extra dimensions \([36]\).

### 3. A particular model

In this section, we consider a specific solution of the five-dimensional bulk and the corresponding Friedmann equations on the orbifold branes.
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3.1. Exact solutions in the bulk

It can be shown that the following solution satisfies the field equations in the bulk:

$$
\sigma(t) = \frac{1}{9} \ln(t) + \frac{1}{2} \ln \left( \frac{7}{6} \right), \\
\omega(t) = \frac{10}{9} \ln(t), \\
\phi(t) = -\frac{5}{18} \sqrt{6} \ln(t) + \phi_0, \\
\psi(t) = -\frac{\sqrt{10}}{6} \ln(t) + \psi_0,
$$

for $k = -1$, where

$$
\phi_0 = \frac{\sqrt{6}}{5} \left\{ \ln \left( \frac{2}{3\sqrt{5}} \right) - \sqrt{\frac{5}{2}} t_0 \right\},
$$

with $\psi_0$ being an arbitrary constant. Then, the corresponding five-dimensional metric takes the form

$$
ds_5^2 = \left( \frac{7}{6} \right) t^{2/9} (dt^2 - dy^2) - t^{20/9} d\Sigma_{-1}^2.
$$

Clearly, the spacetime is singular at $t = 0$ where all the four spatial dimensions collapse into a point singularity, like a big bang. This can be seen more clearly from the expression

$$
\psi^a \psi^a = \frac{3}{5} \phi_0^2 = \frac{5}{21} t^{-20/9}.
$$

The corresponding Penrose diagram is given by figure 1.

Lifting the solution to the ten-dimensional superstring spacetime, we find that in the string frame the metric (2.3) takes the form

$$
ds_{10}^2 = \hat{g}_{AB} dx^A dx^B = e^{\sqrt{2/5} \phi_0} \left\{ \left( \frac{7}{6} \right) t^{-1/3} \left( dt^2 - dy^2 \right) - t^{5/3} d\Sigma_{-1}^2 \right\}
- e^{\sqrt{2/5} \phi_0} t^{-1/3} \delta_{ij} dz^i dz^j.
$$

The corresponding dilaton field is given by

$$
\hat{\Phi} = -\frac{5}{3} \ln(t) + \hat{\Phi}_0,
$$

Figure 1. The Penrose diagram for the metric given by equation (3.3) in the text, where the spacetime is singular at $t = 0$. The curves OPA and OQA describes the history of the two orbifold branes located on the surfaces $y = y_I(\tau_I)$ with $I = 1, 2$. The bulk is the region between these two lines.
where $\hat{\Phi}_0 \equiv \sqrt{3/2} \phi_0 + \sqrt{5/2} \psi_0$, from which we find
\[
\hat{\Phi}_A \hat{\Phi}^A = \frac{50}{21} e^{-\sqrt{2/3} \phi_0 t - 5/3}.
\] (3.7)
Clearly, it is also singular at $t = 0$, but with a weaker strength in comparison to that of the five-dimensional spacetime given by equation (2.19). A critical difference is that in the string frame the proper distance along the $y$ direction becomes decreasing as $t$ increases, in contrast to that in the Einstein frame, as can be seen clearly from equations (3.3) and (3.5).

### 3.2. Generalized Friedmann equations on the branes

On the other hand, from equation (2.61) we find that
\[
E^{(5)} = -\frac{1}{42a^2}, \quad G^{(5)}_\tau = \frac{31}{126a^2}, \quad G^{(5)}_\theta = \frac{20}{81a^{9/5}} \dot{y}^2 - \frac{13}{378a^2},
\] (3.8)
where now $a(\tau) = t^{10/9}(\tau)$ and $\dot{y}$ is given by
\[
\dot{y} = \epsilon_y a^{9/10} \left[ \left( \frac{9}{10} \right)^2 H^2 - \frac{6}{7a^2} \right]^{1/2},
\] (3.9)
with $\epsilon_y = \pm 1$. Inserting equations (3.8) and (3.9) into equations (2.59) and (2.60), we find that
\[
H^2 = \frac{8\pi G}{3} \left( \rho + \tau_{(\phi,\psi)} + \rho_\Lambda \right) + \frac{200}{189a^2} + \frac{2\pi G}{3\rho_\Lambda} \left( \rho + \tau_{(\phi,\psi)} \right)^2,
\] (3.10)
\[
\ddot{a} = \frac{4\pi G}{5} \left( 3\rho_\Lambda + 3\dot{\tau}_{(\phi,\psi)} - 2\rho - 5p \right) - \frac{2\pi G}{3\rho_\Lambda} \times \left[ \frac{1}{10} \left( \rho + \tau_{(\phi,\psi)} \right)^2 + \rho \left( 2\rho + 3p \right) \left( \rho + 3p - \tau_{(\phi,\psi)} \right) \tau_{(\phi,\psi)} \right].
\] (3.11)
It is remarkable to note that these two equations do not depend on both $\epsilon$ defined in equation (2.58) and $\epsilon_y$ defined in equation (3.9). Combining equations (3.10) and (3.11), we obtain
\[
(\rho + \dot{\tau}_{(\phi,\psi)}) + 3H (\rho + p) = -\frac{H}{20\Delta} \left[ 4 \left( \rho + \rho_\Lambda + \tau_{(\phi,\psi)} \right) + \frac{(\rho + \tau_{(\phi,\psi)})^2}{\rho_\Lambda} \right],
\] (3.12)
where
\[
\Delta \equiv 1 + \frac{1}{2\rho_\Lambda} \left( \rho + \tau_{(\phi,\psi)} \right).
\] (3.13)
Equation (3.12) shows clearly the interaction among the matter fields confined on the branes and the bulk. This can also be seen from equation (3.8).
3.3. Current acceleration of the universe

To study the current acceleration of the universe, we first set

\[ p = 0, \]  

and then introduce the quantities

\[ \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_\tau = \frac{\mathcal{T}(\phi, \psi)}{\rho_{cr}}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \quad \Omega_k = \frac{200}{189H_0^2a^2} = \frac{\Omega_k^{(0)}}{a^2}, \]  

where \( \rho_{cr} \equiv 3H_0^2/8\pi G \). It should be noted the slight difference between \( \Omega_k \) defined here and the one normally used, \( \Omega_k = -\kappa/(H_0^2a^2) \). Then, equations (3.10), (3.12) and (3.9) can be written as

\[ E^2 = \Omega_\Lambda + \Omega_\tau + \Omega_k + \frac{\Omega_t^2}{4\Omega_\Lambda}, \]  

\[ \Omega_t^* = -\frac{E}{\Delta} \left\{ \frac{1}{5} \left( \Omega_\Lambda + 16\Omega_\tau - 15\Omega_t \right) + \frac{\Omega_t}{20\Omega_\Lambda} (31\Omega_t - 30\Omega_\tau) \right\}, \]  

\[ y^* = \epsilon_y \left( \frac{9}{10} \right) \left( \frac{\Omega_k^{(0)}}{\Omega_k} \right)^{9/20} \sqrt{\Omega_\Lambda + \Omega_t + \frac{\Omega_t^2}{4\Omega_\Lambda}}, \]

where \( E \equiv H/H_0, y^* \equiv d\psi/d(H_0\tau) \) and

\[ \Omega_t = \Omega_m + \Omega_\tau, \]  

with the constraint

\[ 1 = \Omega_k^{(0)} + \Omega_\Lambda + \Omega_t^{(0)} + \frac{\Omega_t^{(0)}^2}{4\Omega_\Lambda}, \]

where \( \Omega_N^{(0)} \)’s denote their current values. On the other hand, in terms of \( \Omega \)’s, we find

\[ \frac{a^{**}}{a} = \frac{3}{10} \left( 3\Omega_\Lambda - 2\Omega_\tau + 5\Omega_t \right) + \frac{3\Omega_t}{40\Omega_\Lambda} \left( 7\Omega_t - 10\Omega_\tau \right). \]

To study equations (3.16)–(3.18) and (3.21) further, we need to specify \( \Omega_\tau \). In the following, we shall consider two different cases.

3.3.1. \( V_4^{(I)} = V_4^{(0)} \exp\{(n/2)((5/\sqrt{6})\phi + \sqrt{5/2}\psi)\} \). If we choose the potential \( V_4^{(I)}(\phi, \psi) \) on each of the two branes as (cf equation (2.19))

\[ V_4^{(I)} = V_4^{(0)} \exp \left\{ \frac{n}{2} \left( \frac{5}{\sqrt{6}} \phi + \sqrt{5/2} \psi \right) \right\}, \]

where \( V_4^{(0)} \) and \( n \) are arbitrary constants, we find that

\[ \Omega_\tau = \epsilon_I \frac{V_4^{(0)}}{\rho_{cr}} \left( \frac{2}{3V_4^{(0)(5)}} \right)^{n/2} \frac{1}{a^n} = \frac{\Omega_t^{(0)}}{a^n}. \]
Figure 2. The marginalized contour of $\Omega_m - \Omega_{\Lambda}$ for the potential given by equation (3.22) with $n = 1$.

Figure 3. The marginalized contour of $\Omega_m - \Omega_k$ for the potential given by equation (3.22) with $n = 1$.

Then our fitting parameters in this case can be chosen as

$$\left\{ \Omega_{\Lambda}, \Omega_m^{(0)}, \Omega_k^{(0)} \right\},$$

for any given $n$.

Fitting the above model to the 182 gold Supernova Ia data [22] and the BAO parameter from SDSS [23], by using our numerical code [24], based on the publicly

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available MINUIT program of CERN, we find that, for $n = 1$, the best fitting is $\Omega_m = 0.24 \pm 0.03$, $\Omega_\Lambda = 0.76 \pm 0.27$ and $\Omega_k = 0.00 \pm 0.05$ with $\chi^2 = 172.4$. Figures 2–4 show the marginalized contours of the $\Omega$s, from which we can see that the effect of the interaction between the bulk and the brane is negligible, and the later evolution of the universe follows more or less the same pattern as that of the $\Lambda$ CDM model in the Einstein theory of gravity.
Figure 6. The marginalized contour of $\Omega_m - \Omega_k$ for the potential given by equation (3.22) with $n = 3.5$.

Figure 7. The marginalized contour of $\Omega_k - \Omega_\Lambda$ for the potential given by equation (3.22) with $n = 3.5$.

For $n = 3.5$, we find that the best fitting is $\Omega_m = 0.27 \pm 0.03$, $\Omega_\Lambda = 0.58 \pm 0.11$ and $\Omega_k = 0.00 \pm 0.06$ with $\chi^2 = 164.2$. Figures 5–7 show the marginalized contours of the $\Omega$s.

The above shows clearly that the case with $n = 3.5$ is observationally more favorable than that of $n = 1$. We have also fitted the data with various values of $n$ and found that the best-fitting value of $n$ is about $n = 3.5$. 
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Figure 8. The evolution of the matter components, $\Omega_i$, for the potential given by equation (3.22) with $n = 3.5$.

With the above best-fitting values of the $\Omega_i$s and $n$ as initial conditions, the future evolution of the universe is shown in figures 8 and 9, from which we can see that all of them, except for $\Omega_\Lambda$, decrease rapidly, and $\Omega_\Lambda$ soon dominates the evolution of the universe, whereby a de Sitter universe results.

From the metrics of equations (3.3) and (3.5), on the other hand, one may naively conclude that the radion in the present case is not stable, as the proper distance given by equation (2.55) seems either to increase to infinity (in the Einstein frame, given by equation (3.3)) or to decrease to zero (in the string frame, given by equation (3.5)), as $t \to \infty$. A closer investigation shows that the problem is not as simple as it looks. In particular, since $y_I = y_I(\tau_I)$, equation (2.55) makes sense only when the relation $\tau_1 = \tau_1(\tau_2)$ is known. In the present case, we transform such a dependence to the expansion factor $a$ and plot it out in figure 10, together with $y_I(a)$, from which we can see clearly that the distance between the two branes remains almost constant. This indicates that the radion might be stable. Certainly, before a definitive conclusion is reached, more detailed investigations are needed.

We also fit the above model with $n = 3.5$ by using our Monte Carlo Markov Chain (MCMC) code [25], based on the publicly available package COSMOMC [37], and find that the best fitting is $\Omega_m = 0.27 \pm 0.04\,0.03$, $\Omega_\Lambda = 0.61 \pm 0.09\,0.10$ and $\Omega_k = -0.0026 \pm 0.23\,0.2396$ with $\chi^2 = 164.10$, where

$$\Omega_k \equiv \tilde{\Omega}_k^2.$$  

(3.25)

The corresponding marginalized probabilities and contours are given in figure 11. Clearly, these best-fitting values are consistent with those obtained above by using our MINUIT code [24].
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3.3.2. $V^{(I)}_4 = \lambda^{(I)}_4 (\psi^2 - v_I^2)^2$. To stabilize the radion, Goldberger and Wise proposed to choose the potential $V^{(I)}_4$ as [38]

$$V^{(I)}_4(\phi, \psi) = \lambda^{(I)}_4 (\psi^2 - v_I^2)^2,$$

(3.26)

where $\lambda^{(I)}_4$ and $v_I^2$ ($I = 1, 2$) are constants. Then, we find that

$$\Omega^{(I)}_\tau = \Omega^{(0,I)}_\tau \left( \left( \frac{3}{40} \ln(a) \right)^2 - v_I^2 \right)^2,$$

(3.27)

where $\Omega^{(0,I)}_\tau \equiv \epsilon_I \lambda^{(I)}_4 / \rho_{cr}$. Note that, in writing the above expressions, without any loss of generality, we had set $\psi_0 = 0$. Then, the fitting parameters can now be taken as

$$\left\{ \Omega_\Lambda, \Omega^{(0)}_m, \Omega^{(0)}_k, v_I \right\}.$$

(3.28)

Fitting the above model to the 182 gold Supernova Ia data [22] and the BAO parameter from SDSS [23], we first study the dependence of $\chi^2$ on $v_I$. Table 1 shows such a dependence and the best-fitting values of $\Omega_i$s for each given $v_I$.

From the table we can see that $\chi^2$ decreases until $v_I \simeq 3.0$ and then starts to increase, as $v_I$ is continuously increasing. However, $\Omega_\Lambda$ and its uncertainty also increase as $v_I$ is increasing, while $\Omega_m$ and $\Omega_k$ remain almost the same. Since $\Omega_\tau$ acts as a varying cosmological constant, table 1 shows that the total effective cosmological constant $\Omega^{\text{eff}}_\Lambda \equiv \Omega_\Lambda + \Omega_\tau$ is between 0.47 and 0.73.

Figure 12 shows the marginalized probabilities and contours for the potential given by equation (3.26) with $v_I = 0.5$, and figure 13 shows the future evolution of the corresponding acceleration of the universe. From there we can see that the acceleration increases to a maximal value and then starts to decrease. As the time is continuously
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**Figure 10.** The locations of the two branes $y_I(a)$, and the proper distance $D$ between the two branes for the potential given by equation (3.22) with $n = 3.5$. The initial conditions are chosen so that $y_1(a_0) = 3$ and $y_2(a_0) = 1$. The choice of $\epsilon_y = +1$ ($\epsilon_y = -1$) corresponds to the case where the branes move towards the increasing (decreasing) direction of $y$.

**Table 1.** The best-fitting values of $\Omega_i$ for a given $v_I$ of the potential given by equation (3.26).

| $v_I$ | $\chi^2$ | $\Omega_m$ | $\Omega_k$ | $\Omega_\Lambda$ | $\Omega_\Lambda + \Omega_\tau$ |
|------|----------|------------|------------|-------------------|-----------------------------|
| 0.1  | 171.28   | 0.25±0.04  | -0.0009±0.22| 0.72±0.05         | 0.73                        |
| 0.3  | 168.10   | 0.29±0.05  | -0.0006±0.41| 1.06±0.15         | 0.47                        |
| 0.5  | 157.50   | 0.29±0.04  | -0.008±0.46 | 1.28±0.31         | 0.70                        |
| 1.0  | 156.69   | 0.29±0.03  | -0.002±0.52 | 1.64±0.71         | 0.74                        |
| 3.0  | 156.38   | 0.28±0.04  | -0.008±0.53 | 1.93±1.01         | 0.57                        |
| 10.0 | 166.35   | 0.28±0.05  | -0.002±0.62 | 1.97±2.17         | 0.56                        |

increasing, it will pass the zero point and then becomes negative. Thus, in the present model, the domination of the cosmological constant is only temporary. Due to the presence of the potential term, represented by $\Omega_\tau$, the universe will be in its decelerating expansion phase again in the future, whereby all problems connected with a far-future de Sitter universe are resolved [20]. The effects of $\Omega_\tau$ can be seen clearly from figure 14, from which we can see that both $\Omega_m$ and $\Omega_k$ decrease rapidly and soon $\Omega_\tau$ dominates the evolution of the universe.

These are the common features for any given value of $v_I$. Figures 15–17 show, respectively, the marginalized probabilities and contours, the future evolution of $a^{**}/a$ and of $\Omega_i$ for $v_I = 0.1$. 
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**Figure 11.** The marginalized probabilities and contours for the potential given by equation (3.22) with $n = 3.5$.

**Figure 12.** The marginalized probabilities and contours for the potential given by equation (3.26) with $v_I = 0.5$.

In addition, we also find that the proper distance between the two orbifold branes defined by equation (2.55) is not sensitive to the choice of $v_I$, and remains almost constant during the future evolution of the universe, as shown in figure 18. This also indicates that the radion might be stable in the present case, too.
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**Figure 13.** The acceleration $a^{**}/a$ for the potential given by equation (3.26) with $v_f = 0.5$.

**Figure 14.** The future evolution of $\Omega_i$ for the potential given by equation (3.26) with $v_f = 0.5$.

4. Conclusions and remarks

Recently, we studied the cosmological constant problem in the framework of both M-theory [17] and string theory [21] on $S^1/Z_2$ and showed that, among other things, the effective cosmological constant on the branes can be easily lowered to its current
observational value using the ADD large extra-dimension mechanism [19]. Thus, brany cosmology of string/M-theory seems to have a built-in mechanism for solving both the cosmological constant and the hierarchy problems.

In this paper, we have studied a particular cosmological model in the framework of string theory on $S^1/Z_2$, developed in [21]. We have first solved the field equations in the bulk and then studied its local and global properties. In particular, we have found that the ten-dimensional bulk has a big-bang-like singularity at $t = 0$. 

Figure 15. The marginalized probabilities and contours for the potential given by equation (3.26) with $v_I = 0.1$.

Figure 16. The acceleration $a''/a$ for the potential given by equation (3.26) with $v_I = 0.1$. 

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Figure 17. The future evolution of $\Omega_i$ for the potential given by equation (3.26) with $v_I = 0.1$.

After we obtained explicitly the generalized Friedmann equations on each of the two branes for any radion potentials $V_4^{(I)}$ ($I = 1, 2$) of the branes, we have studied two different cases where $V_4^{(I)} = V_4^{(0)} \exp\{(n/2)((5/\sqrt{6})\phi + \sqrt{5}/2\psi)\}$ and $V_4^{(I)} = \lambda_4^{(I)}(\psi^2 - v_I^2)^2$, respectively. For each of these potentials, we have fitted the corresponding models to the 182 gold Supernova Ia data [22] and the BAO parameter from SDSS [23] and obtained the best fitting values of the parameters involved. To doubly check our numerical codes, we have used both our MINUIT [24] and Monte Carlo Markov Chain (MCMC) [25] codes and gotten the same results within the allowed errors. With these best-fitting values as the initial conditions, we have integrated numerically the field equations on the branes and found the future evolution of the universe. In particular, for $V_4^{(I)} = \lambda_4^{(I)}(\psi^2 - v_I^2)^2$, we have found that the current acceleration of the universe driven by the effective cosmological constant is only temporary. Due to the effects of the potentials, the universe will be in its decelerating expansion phase again in the future. We have also studied the proper distance between the two orbifold branes and found that it remains almost constant during the whole future evolution of the universe in all these models.

In the framework of orbifold branes, an important question is the radion stability. The considerations of the proper distance between the two orbifold branes indicate that in these cases the radion might be stable, although further studies are highly demanded. Recently, two of the authors (NOS and AW) studied the problem in a static background with a four-dimensional Poincaré symmetry:

$$d\sigma_5^2 = e^{2\sigma(y)} \left( \eta_{\mu\nu} \, dx^\mu \, dx^\nu - dy^2 \right),$$  \hspace{1cm} (4.1)
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Figure 18. The locations of the two branes, $y_I(a)$, and the proper distance, $D$, between the two branes for the potential given by equation (3.26) with $v_I = 0.5$. The initial conditions are chosen so that $y_1(a_0) = 3$ and $y_2(a_0) = 1$. The choice of $\epsilon_y = +1$ ($\epsilon_y = -1$) corresponds to the case where the branes move towards the increasing (decreasing) direction of $y$.

where

$$\sigma(y) = \frac{1}{9} \ln |y + y_0|,$$
$$\phi(y) = -\frac{5}{\sqrt{54}} \ln |y + y_0| + \phi_0,$$
$$\psi(y) = -\sqrt{\frac{5}{18}} \ln |y + y_0| + \psi_0,$$

(4.2)

and $y_0, \phi_0$ and $\psi_0$ are the integration constants with

$$\psi_0 = \sqrt{\frac{2}{5}} \left[ \ln \left( \frac{2}{9\sqrt{3}} \right) - \frac{5}{\sqrt{6}} \phi_0 \right].$$

(4.3)

Following [39], we are currently investigating the radion stability and hope to report our findings somewhere else soon.

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References

[1] Weinberg S, 1989 Rev. Mod. Phys. 61 1 [SPIRES]
Carroll S M, 2000 arXiv:astro-ph/0004075
Padmanabhan T, 2003 Phys. Rep. 380 235 [SPIRES]
Nobbenhuis S, 2004 arXiv:gr-qc/0411093
Polchinski J, 2006 arXiv:hep-th/0603249
Cline J M, 2006 arXiv:hep-th/0612129

[2] Riess A G et al, 1998 Astrophys. J. 116 1009 [SPIRES]
Perlmutter S et al, 1999 Astrophys. J. 517 565 [SPIRES]

[3] Riess A G et al, 2004 Astrophys. J. 607 665 [SPIRES]
Astier P et al, 2006 Astron. Astrophys. 447 31 [SPIRES]
Spergel D N et al, 2007 Astrophys. J. Suppl. 170 377 [arXiv:astro-ph/0603449]
Wood-Vasey W M et al, 2007 arXiv:astro-ph/0701041
Davis T M et al, 2007 arXiv:astro-ph/0701510

[4] Sullivan S, Cooray A and Holz D E, 2007 J. Cosmol. Astropart. Phys. JCAP09(2007)004 [SPIRES] [arXiv:0706.3730]
Mantz A et al, 2008 Mon. Not. R. Astron. Soc. 387 1179 [arXiv:0709.4294]

[5] Albrecht A et al, 2006 arXiv:astro-ph/0609591
Peacock J A et al, 2006 arXiv:astro-ph/0610906

[6] Susskind L, 2003 arXiv:hep-th/0302219
Bousso R and Polchinski J, 2000 J. High Energy Phys. JHEP06(2000)006 [SPIRES]
 Townsend P K and Wohlfarth N R, 2003 Phys. Rev. Lett. 91 061302 [SPIRES]
 Gibbons G W, 1985 Supersymmetry, Supergravity and Related Topics ed F de Aguila et al (Singapore: World Scientific) p 124

[7] Sullivan S, Cooray A and Holz D E, 2007 J. Cosmol. Astropart. Phys. JCAP09(2007)004 [SPIRES] [arXiv:0706.3730]
Mantz A et al, 2008 Mon. Not. R. Astron. Soc. 387 1179 [arXiv:0709.4294]
Dunkley J et al, 2008 arXiv:0803.0586

[8] Wohlfarth N R, 2003 Phys. Lett. B 567 322 [SPIRES]

[9] Sullivan S, Cooray A and Holz D E, 2007 J. Cosmol. Astropart. Phys. JCAP09(2007)004 [SPIRES] [arXiv:0706.3730]
Mantz A et al, 2008 Mon. Not. R. Astron. Soc. 387 1179 [arXiv:0709.4294]
Late transient acceleration of the universe in string theory on $S^1/Z_2$

[18] Horava H and Witten E, 1996 Nucl. Phys. B 460 506 [SPIRES]
[19] Arkani-Hamed N, Dimopoulos S and Dvali G, 1998 Phys. Lett. B 429 263 [SPIRES]
Arkani-Hamed N, Dimopoulos S and Dvali G, 1999 Phys. Rev. D 59 086004 [SPIRES]
Antoniadis I et al, 1998 Phys. Lett. B 436 257 [SPIRES]
[20] Fischler W et al, 2001 J. High Energy Phys. JHEP07(2001)003 [SPIRES]
Cline J M, 2001 J. High Energy Phys. JHEP08(2001)035 [SPIRES]
Horava H and Witten E, 1996 Nucl. Phys. B 475 94 [SPIRES]
[18] Horava H and Witten E, 1996 Nucl. Phys. B 460 506 [SPIRES]
[19] Horava H and Witten E, 1996 Nucl. Phys. B 475 94 [SPIRES]

[21] Wang A and Santos N O, 2007 arXiv:0712.3938
[22] Riess A G et al, 2006 arXiv:astro-ph/0611572
[23] Eisenstein D J et al, 2005 Astrophys. J. 633 560 [SPIRES]
[24] Wu Q, Gong Y-G, Wang A and Alcaniz J S, 2008 Phys. Lett. B 659 34 [SPIRES] [arXiv:0705.1006]
Gong Y-G and Wang A, 2007 Phys. Rev. D 75 043520 [SPIRES] [arXiv:astro-ph/0612196]
Gong Y-G and Wang A, 2006 Phys. Rev. D 73 083506 [SPIRES] [arXiv:astro-ph/0601453]
[25] Gong Y-G, Wu Q and Wang A, 2008 Astrophys. J. 681 27 [SPIRES] [arXiv:0708.1817]
[26] Sahni V and Shtanov Y, 2003 J. Cosmol. Astropart. Phys. JCAP11(2003)014 [SPIRES]
[arXiv:astro-ph/0202346]

[27] Dvali G R, Gabadadze G and Porrati M, 2000 Phys. Lett. B 484 112 [SPIRES]
Dvali G R, 2001 Phys. Lett. B 502 199 [SPIRES]
[28] Lissey J E, Wands D and Copeland E J, 2000 Phys. Rep. 337 343 [SPIRES]
Gasperini M, 2007 Elements of String Cosmology (Cambridge: Cambridge University Press)
[29] Battefeld T and Watson S, 2006 arXiv:astro-ph/0611572
[30] Leblond F, Myers R C and Winters D J, 2001 J. High Energy Phys. JHEP07(2001)031 [SPIRES]
[31] Wang A, Cai R-G and Santos N O, 2008 Nucl. Phys. B 797 395 [SPIRES] [arXiv:astro-ph/0607371]
[32] Cai R-G and Cao L-M, 2007 Nucl. Phys. B 785 135 [SPIRES] [arXiv:astro-ph/0202346]

[33] Lanczos C, 1922 Phys. Z. 23 539
Lanczos C, 1924 Ann. Phys. (Germany) 74 518
Israel W, 1967 Nuovo Cim. B 44 I
Israel W, 1967 Nuovo Cim. B 48 I (errata)
Wolf J A, 1984 Spaces of Constant Curvature 5th edn (Wilmington, DE: Publish or Perish)

[35] Antoniadis I, 2005 J. Phys. Conf. Ser. 8 112
Kumar V H S and Suresh P K, 2006 arXiv:hep-th/0606194

[36] Long J C et al, 2003 Nature 421 922 [SPIRES]
Kapner D J et al, 2007 Phys. Rev. Lett. 98 021101 [SPIRES]
[37] Lewis A and Bridle S, 2002 Phys. Rev. D 66 103511 [SPIRES]
[38] Goldberger W D and Wise M B, 1999 Phys. Rev. Lett. 83 4922 [SPIRES]
[39] Garriga J and Tanaka T, 2000 Phys. Rev. Lett. 84 2778 [SPIRES]
Tanaka T and Montes X, 2000 Nucl. Phys. B 582 259 [SPIRES]

[40] DeWolfe O, Freedman D Z, Gubser S S and Karch A, 2000 Phys. Rev. D 62 046008 [SPIRES]
Csáki C, Graesser M L and Kribs G D, 2001 Phys. Rev. D 63 065002 [SPIRES]
Lesgourgues J and Sorbo L, 2004 Phys. Rev. D 70 084010 [SPIRES]
Konikowska D, Olechowski M and Schmidt M G, 2006 Phys. Rev. D 73 105018 [SPIRES]