Simple decay-lepton asymmetries in polarized $e^+e^- \rightarrow t\bar{t}$ and $CP$-violating dipole couplings of the top quark

P. Poulose
Theory Group, Physical Research Laboratory
Navrangpura, Ahmedabad 380009, India

Saurabh D. Rindani
Departament de Física Teòrica, Universitat de València
Av. Dr. Moliner 50, 46100 Burjassot, València, Spain

Abstract

We study two simple $CP$-violating asymmetries of leptons coming from the decay of $t$ and $\bar{t}$ in $e^+e^- \rightarrow t\bar{t}$, which do not need the full reconstruction of the $t$ or $\bar{t}$ for their measurement. They can arise when the top quark possesses nonzero electric and weak dipole form factors in the couplings to the photon and $Z$, respectively. Together, these two asymmetries can help to determine the electric and weak dipole form factors independently. If longitudinal beam polarization is available, independent determination of form factors can be done by measuring only one of the asymmetries. We obtain estimates of 90% confidence limits that can be put on these form factors at a future linear $e^+e^-$ collider operating at $\sqrt{s} = 500$ GeV.
Experiments at the Tevatron have seen evidence for the top quark with mass in the range of about 170-200 GeV [1]. Future runs of the experiment will be able to determine the mass more precisely and also determine other properties of the top quark. $t\bar{t}$ pairs will be produced more copiously at proposed $e^+e^-$ linear colliders operating above threshold. It would then be possible to investigate these properties further.

While the standard model (SM) predicts $CP$ violation outside the $K^-, D^-$ and $B$-meson systems to be unobservably small, in some extensions of SM, $CP$ violation might be considerably enhanced, especially in the presence of a heavy top quark. In particular, $CP$-violating electric dipole form factor of the top quark, and the analogous $CP$-violating “weak” dipole form factor in the $t\bar{t}$ coupling to $Z$, could be enhanced. These $CP$-violating form factors could be determined in a model-independent way at high energy $e^+e^-$ linear colliders, where $e^+e^- \rightarrow t\bar{t}$ would proceed through virtual $\gamma$ and $Z$ exchange.

Since a heavy top quark ($m_t \geq 120$ GeV) is expected to decay before it hadronizes [4], it has been suggested [5] that top polarization asymmetry in $e^+e^- \rightarrow t\bar{t}$ can be used to determine the $CP$-violating dipole form factors, since polarization information would be retained in the decay product distribution. Experiments have been proposed in which the $CP$-violating dipole couplings could be measured in decay momentum correlations [5, 6] or asymmetries [7, 8], even with beam polarization [5, 8]. These suggestions on the measurement of asymmetries have concentrated on experiments requiring the reconstruction of the top-quark momentum (with the exception of lepton energy asymmetry [5, 6, 8]). In this note we look at very simple lepton angular asymmetries in $e^+e^- \rightarrow t\bar{t}$ which do not require the experimental determination of the $t$ or $\bar{t}$ momentum. Being single-lepton asymmetries, they do not require both $t$ and $\bar{t}$ to decay leptonically. Since either $t$ or $\bar{t}$ is also allowed to decay hadronically, there is a gain in statistics.

The two asymmetries we study here are as follows. We look at the angular distributions of the charged leptons arising from the decay of $t$ and $\bar{t}$ in $e^+e^- \rightarrow t\bar{t}$. In terms of the polar angle distribution of the leptons with respect to the $e^-$ beam direction in the centre-of-mass (cm) frame, we can define two $CP$-violating asymmetries. One is simply the total lepton-charge asymmetry, with a cut-off $\theta_0$ on the forward and backward polar angles of the leptons, with respect to the beam direction as $z$ axis. The other is the leptonic forward-backward asymmetry combined with charge asymmetry, again with the angles within $\theta_0$ of the forward and backward directions excluded. (See
later for details).

Our results are based on a fully analytical calculation of single lepton distributions in the production and subsequent decay of $t\bar{t}$. We present here only the leptonic asymmetries obtained by an integration of these distributions. The details of the fully differential distribution as well as the distribution in the polar angle of the lepton with respect to the beam direction in the centre-of-mass (cm) frame can be found elsewhere [9].

We have also included the effect of electron longitudinal polarization, likely to be easily available at linear colliders. In an earlier paper [8], we had shown how polarization helps to put independent limits on electric and weak dipole couplings, while providing greater sensitivity in the case of asymmetries. We also demonstrate these advantages for the present case, strengthening the case for polarization studies.

We first describe the calculation of these asymmetries in terms of the electric and weak dipole couplings of the top quarks. These $CP$-violating couplings give rise to top polarization asymmetries in the production of $t\bar{t}$ in $e^+e^- \rightarrow t\bar{t}$ which in turn give rise to angular asymmetries in the subsequent decay $t \rightarrow b l^+ \nu_l$ ($\bar{t} \rightarrow \bar{b} l^- \bar{\nu}_l$). We adopt the narrow-width approximation for $t$ and $\bar{t}$, as well as for $W^\pm$ produced in $t$, $\bar{t}$ decay.

We assume the top quark couplings to $\gamma$ and $Z$ to be given by the vertex factor $ie\Gamma^j_{\mu}$, where

$$\Gamma^j_{\mu} = c^j_\nu \gamma_\mu + c^j_\alpha \gamma_\mu \gamma_5 + \frac{c^j_d}{2m_t} i\gamma_5 (p_\ell - \nu_\ell)_{\mu}, \quad j = \gamma, Z, \quad (1)$$

with

$$c^\gamma_\nu = \frac{2}{3}, \quad c^\gamma_\alpha = 0,$$

$$c^Z_d = \frac{\left(\frac{1}{4} - \frac{2}{3} x_w\right)}{\sqrt{x_w (1 - x_w)}}, \quad (2)$$

$$c^Z_\alpha = \frac{1}{4\sqrt{x_w (1 - x_w)}}$$

and $x_w = sin^2\theta_w$, $\theta_w$ being the weak mixing angle. We have assumed in (1) that the only addition to the SM couplings $c^{\gamma,Z}_{\nu,\alpha}$ are the $CP$-violating electric and weak dipole form factors, $ee_d/m_t$ and $ee_d^Z/m_t$, which are assumed small.
Use has also been made of the Dirac equation in rewriting the usual dipole coupling $\sigma_{\mu\nu}(p_t + p_\tau)^\nu\gamma_5$ as $i\gamma_5(p_t - p_\tau)_{\mu}$, dropping small corrections to the vector and axial-vector couplings. We assume that there is no $CP$ violation in $t, \bar{t}$ decay\(^2\).

The helicity amplitudes for $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow t\bar{t}$ in the cm frame, including $c_d^{\gamma,Z}$ couplings, have been given in \(^7\) (see also Kane et al., ref. \(^3\)). We have calculated the helicity amplitudes for

$$t \rightarrow bW^+, \quad W^+ \rightarrow l^+\nu_l$$

and

$$\bar{t} \rightarrow \bar{b}W^-, \quad W^- \rightarrow l^-\bar{\nu}_l$$

in the respective rest frames of $t, \bar{t}$, assuming standard model couplings and neglecting all masses except $m_t$, the top mass. The expressions for these can be found in \(^9\).

Combining the production and decay amplitudes in the narrow-width approximation for $t, \bar{t}, W^+, W^-$, and using appropriate Lorentz boosts to calculate everything in the $e^+e^-$ cm frame, we obtained the $l^+$ and $l^-$ distributions for the case of $e^-, e^+$ with polarization $P_e, P_{\tau}$, the expressions can again be found in \(^9\). We further carry out the necessary integrations to obtain only the polar angle distributions for the leptons, which we use to write down the expressions for the $CP$-violating asymmetries defined below.

We define two independent $CP$-violating asymmetries, which depend on different linear combinations of $Im c_d^{\gamma}$ and $Im c_d^{Z}$. (It is not possible to define $CP$-odd quantities which determine $Re c_d^{\gamma,Z}$ using single-lepton distributions \(^9\)). One is simply the total lepton-charge asymmetry, with a cut-off of $\theta_0$ on the forward and backward directions:

$$A_{ch}(\theta_0) = \frac{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} - \frac{d\sigma^-}{d\theta_l} \right)}{\int_{\theta_0}^{\pi-\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l} \right)}. \quad (3)$$

The other is the leptonic forward-backward asymmetry combined with charge asymmetry, again with the angles within $\theta_0$ of the forward and backward
directions excluded:

\[ A_{fb}(\theta_0) = \int_{\theta_0}^{\pi} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} - \frac{d\sigma^-}{d\theta_l} \right) - \int_{\pi}^{\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} - \frac{d\sigma^-}{d\theta_l} \right) \]

\[ \int_{\theta_0}^{\pi-\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l} \right) - \int_{\pi-\theta_0}^{\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l} \right) \]

\[ \int_{\pi-\theta_0}^{\pi} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l} \right) - \int_{\theta_0}^{\pi-\theta_0} d\theta_l \left( \frac{d\sigma^+}{d\theta_l} + \frac{d\sigma^-}{d\theta_l} \right) \]

(4)

In the above equations, \( \sigma^+ \) and \( \sigma^- \) refer respectively to the cm \( l^+ \) and \( l^- \) distributions. \( \theta_l \) is used to represent the polar angle angle of either \( l^+ \) or \( l^- \), with the \( z \) axis chosen along the \( e^- \) momentum.

These asymmetries are a measure of \( CP \) violation in the unpolarized case and in the case when polarization is present, but \( P_e = -P_\tau \). When \( P_e \neq -P_\tau \), the initial state is not invariant under \( CP \), and therefore \( CP \)-invariant interactions can contribute to the asymmetries. However, to the leading order in \( \alpha \), these \( CP \)-invariant contributions vanish in the limit \( m_e = 0 \). Order-\( \alpha \) collinear helicity-flip photon emission can give a \( CP \)-even contribution. However, this background can be suppressed by a suitable cut on the visible energy.

The expressions for \( A_{ch}(\theta_0) \) and \( A_{fb}(\theta_0) \) are given below.

\[
A_{ch}(\theta_0) = \frac{1}{2\sigma(\theta_0)} \frac{3\pi\alpha^2}{4s} B_t B_\tau 2 \cos\theta_0 \sin^2\theta_0 \left( \frac{1 - \beta^2}{1 - \beta} \log \frac{1 + \beta}{1 - \beta} - 2\beta \right) \\
\times \left\{ \text{Im} \left( \frac{c_v}{c_v^*} \right) \left[ (2c_v + (r_L + r_R)c_v^z) (1 - P_e P_\tau) + (r_L - r_R)c_v^z (P_\tau - P_e) \right] \\
+ \text{Im} \left( \frac{c_d}{c_d^*} \right) \left[ (r_L + r_R)c_v^z + (r_L^2 + r_R^2)c_v^z \right] (1 - P_e P_\tau) + (r_L - r_R)c_v^z \\
+ (r_L^2 - r_R^2)c_v^z \right\} (P_\tau - P_e) \right\};
\]

(5)

\[
A_{fb}(\theta_0) = \frac{1}{2\sigma(\theta_0)} \frac{3\pi\alpha^2}{2s} B_t B_\tau 2 \cos^2\theta_0 \left( \frac{1 - \beta^2}{1 - \beta} \log \frac{1 + \beta}{1 - \beta} - 2\beta \right) c_a^z \\
\times \left\{ \text{Im} \left( \frac{c_d}{c_d^*} \right) \left[ (r_L - r_R)(1 - P_e P_\tau) + (r_L + r_R)(P_\tau - P_e) \right] \\
+ \text{Im} \left( \frac{c_d}{c_d^*} \right) \left[ (r_L^2 - r_R^2)(1 - P_e P_\tau) + (r_L^2 + r_R^2)(P_\tau - P_e) \right] \right\}. \]

(6)

Here \( \sigma(\theta_0) \) is the cross section for \( l^+ \) or \( l^- \) production with a cut-off \( \theta_0 \), and is given by

\[
\sigma(\theta_0) = \frac{3\pi\alpha^2}{8s} B_t B_\tau 2 \cos\theta_0 \left\{ \left( \frac{1 - \beta^2}{1 - \beta} \right) \log \frac{1 + \beta}{1 - \beta} \sin^2\theta_0 \right\}.
\]
\[ +2\beta \left[ 1 + \left( 1 - \frac{2}{3} \beta^2 \right) \cos^2 \theta_0 \right] \]
\[ \times \left\{ 2c_{\gamma}^2 + 2c_{\gamma}^2 c_{\alpha}^2 (r_L + r_R) + c_{\gamma}^2 (r_L^2 + r_R^2) \right\} (1 - P_eP_\tau) \]
\[ + c_{\alpha}^2 \left[ (r_L - r_R)c_{\alpha}^2 + (r_L^2 - r_R^2)c_{\alpha} \right] (P_\tau - P_e) \]
\[ + \left\{ (1 - \beta^2) \log \frac{1 + \beta}{1 - \beta} \sin^2 \theta_0 + 2\beta \left[ 2\beta^2 - 1 + \left( 1 - \frac{2}{3} \beta^2 \right) \cos^2 \theta_0 \right] \right\} \]
\[ \times c_{\alpha}^2 \left\{ (r_L^2 + r_R^2)(1 - P_eP_\tau) + (r_L^2 - r_R^2)(P_\tau - P_e) \right\} - 2(1 - \beta^2) \]
\[ \times \left( \log \frac{1 + \beta}{1 - \beta} - 2 \right) \sin^2 \theta_0 c_{\alpha}^2 \left\{ (r_L + r_R)c_{\alpha}^2 + (r_L^2 + r_R^2)c_{\alpha}^2 \right\} \]
\[ \times (1 - P_eP_\tau) + \left[ (r_L - r_R)c_{\alpha}^2 + (r_L^2 - r_R^2)c_{\alpha}^2 \right] (P_\tau - P_e) \right\} \right) . \tag{7} \]

In these equations, \( \beta \) is the \( t \) (or \( \bar{t} \)) velocity: \( \beta = \sqrt{1 - 4m_t^2 / s} \), and \( \gamma = 1 / \sqrt{1 - \beta^2} \), and \( B_t \) and \( B_{\bar{t}} \) are respectively the branching ratios of \( t \) and \( \bar{t} \) into the final states being considered. \( -e r_{L,R} / s \) is the product of the \( Z \)-propagator and left-handed (right-handed) electron couplings to \( Z \), with

\[ r_L = \frac{\left( \frac{1}{s} - x_w \right)}{\left( 1 - \frac{m_e^2}{s} \right) \sqrt{x_w (1 - x_w)}}. \]
\[ r_R = \frac{-x_w}{\left( 1 - \frac{m_e^2}{s} \right) \sqrt{x_w (1 - x_w)}}. \tag{8} \]

We note the curious fact that \( A_{ch}(\theta_0) \) vanishes for \( \theta_0 = 0 \). This implies that the \( CP \)-violating charge asymmetry does not exist unless a cut-off is imposed on the lepton production angle. \( A_{fb}(\theta_0) \), however, is nonzero for \( \theta_0 = 0 \).

We now describe the numerical results for the calculation of 90\% confidence level (CL) limits that could be put on \( \text{Im} c_{\alpha}^{\gamma,Z} \) using the asymmetries described earlier, as well as the \( CP \)-odd part of the angular distribution in eq. (9).

We look at only semileptonic final states. That is to say, when \( t \) decays leptonically, we assume \( \bar{t} \) decays hadronically, and \textit{vice versa}. We sum over the electron and muon decay channels. Thus, \( B_tB_{\bar{t}} \) is taken to be \( 2/3 \times 2/9 \). The number of events for various relevant \( \theta_0 \) and for beam polarizations \( P_e = 0, \pm 0.5 \) are listed in Table 1.
In each case we have derived simultaneous 90% CL limits on \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \) that could be put in an experiment at a future linear collider with \( \sqrt{s} = 500 \text{ GeV} \) and an integrated luminosity of 10 fb\(^{-1}\). We do this by equating the asymmetry \( (A_{ch} \text{ or } A_{fb}) \) to \( 2.15/\sqrt{N} \), where \( N \) is the total number of expected events. In the unpolarized case, each of \( A_{ch} \) and \( A_{fb} \) gives a band of allowed values in the \( \text{Im} c^\gamma_d - \text{Im} c^Z_d \) plane. If both \( A_{ch} \) and \( A_{fb} \) are looked for in an experiment, the intersection region of the corresponding bands determines the best 90% CL limits which can be put simultaneously on \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \). These best results are obtained for \( \theta_0 = 35^\circ \) and are shown in Fig. 1(a) and Fig. 1(b), for two values of the top mass, \( m_t = 174 \text{ GeV} \), and \( m_t = 200 \text{ GeV} \) respectively.

We see from Fig. 1 that the 90% CL limits that could be put on \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \) simultaneously are, respectively, 2.4 and 17, for \( m_t = 174 \text{ GeV} \). The same limits are 4.0 and 28 for \( m_t = 200 \text{ GeV} \).

In the case where the \( e^- \) beam is longitudinally polarized, we have assumed the degree of polarization \( P_e = \pm 0.5 \), and determined 90% CL limits which can be achieved. In this case, the use of \( P_e = +0.5 \) and \( P_e = -0.5 \) is sufficient to constrain \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \) simultaneously even though only one asymmetry (either \( A_{ch} \) or \( A_{fb} \)) is determined. The 90% CL bands corresponding to \( P_e = \pm 0.5 \) are shown in Figs. 2 and 3, for \( A_{ch} \) with \( \theta_0 = 60^\circ \), and for \( A_{fb} \) with \( \theta_0 = 10^\circ \), respectively. Again, these values of \( \theta_0 \) are chosen to maximize the sensitivity\(^3\).

It can be seen from these figures that the simultaneous limits expected to be obtained on \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \) are, respectively, about 0.45 and 1.5 for \( m_t = 174 \text{ GeV} \) from both the types of asymmetries. These limits are about 0.78 and 2.5 for \( m_t = 200 \text{ GeV} \). We see thus that the use of polarization leads to an improvement of by a factor of about 5 in the sensitivity to the measurement of \( \text{Im} c^\gamma_d \), and by a factor of at least 10 in the case of \( \text{Im} c^Z_d \). Moreover, with polarization, either of \( A_{fb} \) and \( A_{ch} \), with a suitably chosen cut-off, suffices to get the same improvement in sensitivity.

Apart from simultaneous limits on \( \text{Im} c^\gamma_d \), we have also found out the sensitivities of one of \( \text{Im} c^\gamma_d \), assuming the other to be zero, using the \( CP \)-odd combination of angular distributions \( \frac{d\sigma}{d\cos\theta}(\theta_l) - \frac{d\sigma}{d\cos\theta}(\pi - \theta_l) \). We assume that the data is collected over bins in \( \theta_l \), and add the 90% CL limits obtained from individual bins in inverse quadrature. We find that the best individual limits are respectively 0.12 and 0.28 for \( \text{Im} c^\gamma_d \) and \( \text{Im} c^Z_d \), both in the case of \( P_e = -0.5, \) for \( m_t = 174 \text{ GeV} \). The corresponding limits for \( m_t = 200 \text{ GeV} \)
are 0.18 and 0.43. As expected, these limits are better than simultaneous ones. Even here, there is an improvement due to polarization, but it is not as dramatic as in the case of simultaneous limits.

Our limits on $\text{Im} c_d^{\gamma,Z}$ are summarized in Table 2.

To conclude, we have obtained expressions for certain simple $CP$-violating angular asymmetries in the production and subsequent decay of $t\bar{t}$ in the presence of electric and weak dipole form factors of the top quark. These asymmetries are specially chosen so that they do not require the reconstruction of the $t$ or $\bar{t}$ directions or energies. We have also included the effect of longitudinal electron beam polarization. We have analyzed these asymmetries to obtain simultaneous 90% CL limits on the imaginary parts of the electric and weak dipole couplings which would be possible at future linear $e^+e^-$ collider operating at $\sqrt{s} = 500$ GeV and with a luminosity of 10 fb$^{-1}$. Figs. 1-3 show the allowed regions in the $\text{Im} c_d^{\gamma} - \text{Im} c_d^Z$ plane at the 90% CL. Table 2 summarizes the 90% CL limits on $\text{Im} c_d^{\gamma,Z}$ in various cases.

Our general conclusion is that the sensitivity to the measurement of dipole couplings is improved considerably if the electron beam is polarized, a situation which might easily obtain at linear colliders. Another general observation is that the sensitivity is better for a lower top mass than a higher one.

If we compare these results for sensitivities with those obtained in [8], where we studied asymmetries requiring the top momentum determination, we find that while the sensitivities with the asymmetries studied here are worse by a factor of about 3 in the unpolarized case, the limits in the polarized case are higher by a factor of about 2 as compared to those in [8]. It is likely that since in the experiments suggested here, only the lepton charges and direction need be determined, improvement in experimental accuracy can easily compensate for these factors. A detailed simulation of experimental conditions is needed to reach a definite conclusion on the exact overall sensitivities.

We have also compared our results with those of [6], where $CP$-odd momentum correlations are studied in the presence of $e^-$ polarization. With comparable parameters, the sensitivities we obtain are comparable to those obtained in [6]. In some cases our sensitivities are slightly worse because we require either $t$ or $\bar{t}$ to decay leptonically, leading to a reduced event rate. However, the better experimental efficiencies in lepton momentum measurement may again compensate for this loss.
As mentioned earlier, since we consider only the electron beam to be polarized, the asymmetries considered here can have backgrounds from order-α collinear initial-state photon emission, which, in principle, have to be calculated and subtracted. However, in case of correlations, it was found in [1] that the background contribution can be neglected for the luminosity we assume here. This is likely to be the case in the asymmetries we consider here.

The theoretical predictions for \( c_\gamma^Z \) and \( c_\gamma^Z \) are at the level of \( 10^{-2} - 10^{-3} \), as for example, in the Higgs-exchange and supersymmetric models of CP violation [4, 7, 12]. Hence the measurements suggested here cannot exclude these modes at the 90% C.L. However, as simultaneous model-independent limits on both \( c_\gamma^Z \) and \( c_\gamma^Z \), the ones obtainable from the experiments we suggest, are an improvement over those obtainable from measurements in unpolarized experiments.

Increase in polarization beyond ±0.5 can increase the asymmetries in some cases we consider. Also, a change in the \( e^+ e^- \) cm energy also has an effect on the asymmetries. However, we have tried to give here only the salient features of the outcome of a possible experiment in the presence of longitudinal beam polarization.

It is obvious that the success of our proposal depends crucially on proper identification of the \( t \bar{t} \) events and measurements of the polar angles and the charges of leptons. This will require cuts, and will lead to experimental detection efficiencies less than one as assumed here. Our results are quantitatively exact under ideal experimental conditions. Inclusion of experimental detection efficiencies may change our results somewhat. However, the main thrust of our conclusions, that we have identified rather simple observables for measurement of dipole form factors, and that longitudinal beam polarization improves the sensitivity, would still remain valid.
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Footnotes

1. The paper of Atwood and Soni [3] introduces optimal variables whose expectation values maximize the statistical sensitivity.

2. CP violation in top decays has been considered, for example, in [10].

3. In case of $A_{fb}$, the choice $\theta_0 = 0$ also gives very similar sensitivities. However, since this condition would be impossible to achieve in a practical situation, we choose a nonzero value of $\theta_0$. 
Figure Captions

Fig. 1. Bands showing simultaneous 90% CL limits on Im $c_γ^d$ and Im $c_Z^d$ using $A_{fb}$ and $A_{ch}$ with unpolarized electron beam at cm energy 500 GeV and cut-off angle 35°. Mass of the top quark is taken to be (a) 174 GeV and (b) 200 GeV.

Fig. 2. Bands showing simultaneous 90% CL limits on Im $c_γ^d$ and Im $c_Z^d$ using $A_{ch}$ with different beam polarizations, and at a cm energy of 500 GeV and cut-off angle 60°. Mass of the top quark is taken to be (a) 174 GeV and (b) 200 GeV.

Fig. 3. Bands showing simultaneous 90% CL limits on Im $c_γ^d$ and Im $c_Z^d$ using $A_{fb}$ with different beam polarizations, and at a cm energy of 500 GeV and cut-off angle 10°. Mass of the top quark is taken to be (a) 174 GeV and (b) 200 GeV.
Table Captions

Table 1. Number of $t\bar{t}$ events, with either $t$ or $\bar{t}$ decaying leptonically, for c.m. energy 500 GeV and integrated luminosity 10 fb$^{-1}$ for two different top masses with polarized and unpolarized electron beams at different cut-off angles $\theta_0$.

Table 2. Limits on dipole couplings obtainable from different asymmetries. In case (a) limits are obtained from $A_{ch}$ and $A_{fb}$ using unpolarized beams (Fig. 1), and in case (b) from either of $A_{ch}$ (Fig. 2) and $A_{fb}$ (Fig. 3) with polarizations $P_e = 0$, $\pm 0.5$. Charge-asymmetric angular distribution is used in case (c) where 0 and $\pm 0.5$ polarizations are considered separately. All the limits are at 90% CL.
\[ m_t = 174 \text{ GeV} \]
\[ m_t = 200 \text{ GeV} \]

| \( \theta_0 \) | \( P_e = -0.5 \) | \( P_e = 0 \) | \( P_e = +0.5 \) | \( P_e = -0.5 \) | \( P_e = 0 \) | \( P_e = +0.5 \) |
|---|---|---|---|---|---|---|
| 0° | 1003 | 845 | 687 | 862 | 723 | 585 |
| 10° | 988 | 832 | 675 | 849 | 712 | 576 |
| 35° | 826 | 689 | 553 | 711 | 593 | 475 |
| 60° | 507 | 419 | 330 | 438 | 362 | 286 |

Table 1

| Case | \( m_t = 174 \text{ GeV} \) | \( m_t = 200 \text{ GeV} \) |
|---|---|---|
| (a) unpolarized | | |
| (b) polarized(\( P_e = 0, \pm 0.5 \)) | 0.45 1.5 | 0.78 2.5 |
| (c) angular distribution: \( P_e = +0.5 \) | 0.13 0.74 | 0.21 1.21 |
| \( P_e = 0.0 \) | 0.13 0.81 | 0.20 1.30 |
| \( P_e = -0.5 \) | 0.12 0.28 | 0.18 0.43 |

Table 2
Fig 2(a)  

Fig 2(b)
Fig 3(a)  
Fig 3(b)
Fig 1(a)  Fig 1(b)