The low-lying Scalar Mesons and Related Topics

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After presenting the motivations to explore the low-lying scalar mesons such as the $\sigma$, $a_0$ and $\kappa$ in the unquenched as well as quenched lattice QCD, we review the works done by our collaboration (SCALAR Collaboration) with a what-to-do-next list. We briefly mention the importance to explore the $N_c$ dependence of and possible effects of the $U_A(1)$ anomaly to the properties the low-lying scalar mesons.

1. Introduction

The low-lying scalar mesons have now acquired much renewed interest since 1990’s when extensive analyses claimed the existence of the $\sigma$ meson pole of the $\pi\pi S$-matrix in the complex energy plane in the $I = J = 0$ channel. In these analyses, the significance of respecting chiral symmetry, unitarity and crossing symmetry has been recognized and emphasized to reproduce the phase shifts both in the $\sigma$ (s) and $\rho$ (t)-channels with a low-mass $\sigma$ pole.\cite{1}. One of the most elaborated analyses\cite{2} identify the $\sigma$ pole at $M_{\sigma} = 441 - i272$ MeV. The low-lying $\sigma$ meson is also seen in decay processes from heavy particles\cite{3}. Such a low-lying isoscalar and scalar meson has been called for and known to play a significant role in nuclear and hadron physics\cite{4}.

The significance of the $\sigma$-degrees of freedom may become more apparent at finite temperature($T$) and/or density, where chiral restoration is believed to occur\cite{5}. The order parameter of the chiral transition is the chiral condensate $\langle \bar{\psi}\psi \rangle$. To identify the critical temperature $T_c$ of chiral transition, the chiral susceptibility, $\chi_m \equiv \frac{\partial}{\partial m} \langle \bar{\psi}\psi \rangle = \langle : (\bar{\psi}\psi)^2 : \rangle$, is calculated; the temperature at which $\chi_m$ takes the peak is identified as $T_c$. It is noteworthy that $\chi_m$ describes the fluctuations of the order parameter in the $\sigma$ direction. The lattice simulations\cite{6} do show that $\chi_m$ takes a peak structure, which means that the $\sigma$ degrees of freedom is physically relevant around the critical point, at least. The development of the peak of $\chi_m$ is equivalently describable as a softening of the $\sigma$-like excitation around $T_c$: the lattice simulations show that the generalized $\sigma$ mass as defined by $m_{\sigma}^{\text{gen}} \equiv \chi_m^{-1/2}$ takes the minimum around $T_c$. One should notice, however, that $m_{\sigma}^{\text{gen}}$ is not the dynamical mass defined as a pole of the time-correlator.

So far, we have seen the significance of the $\sigma$ degrees of freedom. However, the existence of the low-lying scalar mesons could be a puzzle in QCD: In the constituent quark model; the meson with the quantum number $J^{PC} = 0^{++}$ is in the $3P_0$ state in the nonrelativistic quark model, which normally implies that the mass lies
in the region from 1.2 to 1.6 GeV region. So some mechanism is needed to lower the mass with as large as 600 $\sim$ 800 MeV: (i) The most popular idea is the tetraquark structure proposed by Jaffe[7], who showed that the color magnetic interaction between the di-quark and the anti-di-quark gives a large enough attraction to down the masses of the scalar mesons around 600 MeV. (ii) Another time-honored idea is attributing to the possible collective nature of the scalar mesons as the pion, as the Nambu-Jona-Lasinio model[8] describes. It is well known that the scalar meson appears as a consequence of the chiral symmetry and its dynamical breaking as the pion does, and the mass of the sigma satisfies the Nambu relation, $m_\sigma = 2 M_f$, with $M_f$ being the dynamically generated fermion(quark) mass, which should be valid within any Nambu-Jona-Lasinio type model. If we put $M_f = 300$ MeV, $m_\sigma$ becomes 600 MeV. It is shown that this feature essentially persists with the $U_A(1)$ anomaly term incorporated[9]. (iii) The wave function of scalar mesons should also have components of the meson molecule states as these states are seen through the $\pi-\pi$ or $\pi-K$ scattering.

The objectives of Scalar Collaboration[10,11] are summarized as follows. Seeing that the confidence level of the sigma meson and other scalar mesons has been increasing, and its physical significance in hadron physics and QCD is apparent, we have been (and will be) addressing the following questions about the scalar mesons using lattice QCD; i.e., are you a pole in QCD ? i.e., the $\sigma$ and other low-lying scalar mesons are resonances in QCD or something else ? Notice that these questions are issued as early as 2001-2002.

2. The Scalar mesons on the Lattice; a full QCD calculation

The Scalar Collaboration[10] performed a first exploratory work on the sigma in lattice QCD with dynamical quarks. Notice that a full QCD simulation is necessary to properly describe the sigma with the possible contents, i.e., the glueball, tetra quarks and so on.

They employed Wilson fermions and the plaquette gauge action, with a point source and sink. The lattice employed is rather small, $8^3 \times 16$ and coarse, $\beta = 4.8$ corresponding to a lattice spacing $a = 0.207(9)$ fm, which leads to larger masses due to a mixture of higher mass states. In other words, the masses to be obtained in our simulation should be considered as upper limits. The following three hopping parameters are employed, i.e., $\kappa = 0.1846, 0.1874$ and $0.1891$; the critical value is found to be $\kappa_c = 0.195(3)$.

The following operator was adopted $\tilde{\sigma}(x) \equiv \sum_{\alpha=1}^{3} \sum_{\beta=1}^{4} (\bar{u}_\alpha(x) u_\beta(x) + \bar{d}_\alpha(x) d_\beta(x))/\sqrt{2}$, for creating a meson state with $I = 0$ and $J^{PC} = 0^{++}$; here where $u$ (d) denotes the up-quark (down-quark) operator with $c$ and $\alpha$ being the color and Dirac-spinor indices, respectively. The $\sigma$ meson propagator consists of two terms, i.e., a connected diagram and a disconnected diagram. Since the vacuum expectation value $\langle \sigma(x) \rangle$ does not vanish, it should be subtracted from the $\sigma$ operator. The disconnected diagram is calculated using the $Z_2$ noise method. As for other simulation parameters, we refer to the original paper[10].

| $\kappa$ | $m_\sigma/m_\rho$ | $m_{\text{con.}}/m_\rho$ |
|------|-------------|-----------------|
| 0.1846 | 1.583$\pm$0.098 | 2.400$\pm$0.018 |
| 0.1874 | 1.336$\pm$0.071 | 2.436$\pm$0.025 |
| 0.1891 | 1.112$\pm$0.060 | 2.481$\pm$0.031 |

Table 1: Summary of the results. $m_{\text{con.}}$ is the $\sigma$ mass estimated only from the connected part.

We show in Table 1 the value of $m_\sigma/m_\rho$ for each hopping parameter together with the corresponding $m_{\text{con.}}/m_\rho$, where $m_{\text{con.}}$ denotes the scalar meson mass for which the disconnected diagram is not included. It should be noticed here that the mass $m_{\text{con.}}$ can be identified with the $a_0$ meson which is the isovector meson for which the disconnected diagram does not play any role.

The individual contributions of the connected and disconnected parts of the $\sigma$ propagator are shown in Fig.1, which tells us that the connected part only shows a rapid damping with small error bars, while the disconnected part overwhelming
the connected part and dominates the $\sigma$ propagator. Thus, we see that the $\sigma$ as a light meson results from the disconnected part of the $\sigma$ propagator with the background vacuum condensate subtracted.

Figure 1. Propagators of the connected and disconnected diagrams of the $\sigma$ for $\kappa = 0.1874$.

The calculated $m_{\pi}^2$, $m_\rho$, $m_\sigma$ and $2m_\pi$ are presented in Fig. 3 of ref.[10], which shows that as the chiral limit is approached, the $\sigma$ meson mass obtained from the $\sigma$ propagator decreases and eventually becomes smaller than the $\rho$ meson mass in the chiral limit.

It might be more informative to display the masses as functions of $m_\pi^2$, which could be extracted by some effective models[12]. The $m_\pi^2$ dependence of $m_\sigma$ and $m_\rho$ is shown in Fig.2, which is equivalent to Fig.3 of ref.[10].

Important points obtained in ref. [10] are that the $\sigma$ propagator exhibits a pole behavior and its mass is found to satisfy $m_\pi < m_\sigma \leq m_\rho$; for the sigma mass to become small, the disconnected diagram plays an essential role. The flavored scalar meson is not light as observed experimentally; $m_{\pi a} \sim 1.9$ GeV, which are much higher than the experimental masses, $0.6 \sim 0.8$ GeV.

Figure 2. The $m_\pi^2$ dependence of $m_\rho$ and $m_\sigma$ in the lattice unit. Explicitly, the respective dependence is well represented by a linear function of $m_\pi^2$: $m_\rho a = 0.3767 \times (m_\pi a)^2 + 0.8099$ and $m_\sigma a = 1.6818 \times (m_\pi a)^2 + 0.3219$, respectively, with $a$ being the lattice spacing.

3. The $\kappa$ meson in quenched approximation

The $\kappa$ meson is a scalar meson with the strangeness. Recent experimental candidates are reported [13,14] to have a mass in the range from 660 to 800 MeV.

The SCALAR collaboration[11] has performed a lattice QCD simulation of the $\kappa$ which has been performed in the quenched approximation. The simple Wilson fermion and plaquette gauge action were adopted for the $20^3 \times 24$ lattice, and the lattice spacing was found to be $a = 0.1038(33)$ fm. They extracted the mass of the $\kappa$ to be $\sim 1.7$ GeV, which is larger than even twice the experimental mass $\sim 800$ MeV. The relatively heavy mass of the $\kappa$ might be attributed to the absence of the disconnected diagram in the $\kappa$ propagator; notice that the disconnected diagram was essential for realizing the low-mass $\sigma$. Indeed, Table 1 shows that the mass of the valence $\sigma_v$ described solely with the connected propagator is far larger than the experimental value 500–600 MeV.

Thus it is concluded that the simple two-body
4. Summary and concluding remarks

The $\sigma$ meson and other low-lying scalar mesons are still a source of debates. The understanding of the nature or the even (non-)existence is important for a deep understanding of the QCD vacuum as well as the QCD/hadron dynamics. A full QCD lattice simulation suggests the existence of a low-lying $\sigma$ as a pole in QCD. However, its physics content, i.e., a tetra quark, a hybrid with the glue ball or the $q\bar{q}$ collective state, is still obscure, although the fact that the disconnected diagram gives the dominant contribution to the $\sigma$ propagator suggests that one or some of these exotic components are contained in the $\sigma$. A quenched lattice calculation suggests that the kappa can not be a normal $q\bar{q}$ state. There are many things to do for a better simulation, even apart from adopting larger lattices and taking $a$ as close continuum limit as possible. One should make an effort to reduce errors, say, by taking smearing of the sources. In fact, it is known that the glue ball states are strongly dependent on the lattice spacing. For studying the $\sigma$, one should try to reduce large errors coming from the disconnected diagrams. The $\sigma$ has several components and above the $\pi-\pi$ threshold, the variational method with multiple interpolating operators should be adopted, where interpolation fields include a tetraquark operator. To see whether the obtained state is a resonance but not a $2\pi$ scattering state, the volume dependence of the physical observables should be calculated. It is interesting to see the $N_c$-dependence of the mass and the width[15], which will tell us a hint on the physics content of the hadron. Maybe one should identify observables which are sensitive to the inner structure of the $\sigma$. Is there any role of the axial anomaly[9,16] to realize the low-lying $\sigma$? Exploration of the $\sigma$ at finite temperature may reveal the nature of the particle [5]. Of course, it would be desirable to use chiral fermions. The most of the above points are also true for the $\kappa$ meson.

Finally, we refer to nice review articles for more detailed accounts on the scalar mesons on the lattice[17,18].

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