Application of Prony Algorithm Based on EEMD for Identifying PSS Parameters

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Abstract. Some problems may caused by controller poor optimal control performance because of PSS parameters are hard for debugging, this paper proposes EEMD (Ensemble Empirical Mode Decomposition) and Prony algorithm combined online identification method for power system low-frequency oscillation. Firstly the EEMD is used to deal with the unstable signal and select the IMF (Intrinsic Mode Function). Secondly the model parameters of low-frequency oscillation in power systems are obtained by analyzing the IMF with Prony algorithm. Finally the PSS parameters have calculated through the residue method base on oscillation system dominant mode. Simulative results of case study show that it is better to identify the dominant mode of the signal. The proposed method has a certain reference value for engineering application.

1. Introduction

With the gradual expansion of the national power network structure, it is an inevitable trend to strengthen the interconnection between them. Every year, a large number of high-speed excitation systems are put into practical application, which causes many low-frequency oscillations between major power grids. Low-frequency oscillation has become one of the important factors affecting the safe operation of the power grid and limiting the transmission capacity of the interconnected power grid. In order to solve this problem, the Power System Stabilizers (PSS) are generally added to the excitation control system to improve the damping capacity of the system. Practical application shows that PSS is one of the most economical and effective ways to suppress low frequency oscillation at present. However, there are some problems in the debugging of PSS parameters in engineering, which makes the PSS not able to suppress the system oscillation well, so it is of great significance to correctly set the parameters.

At present, the signal analysis method is widely used in the analysis of low frequency oscillation. The common methods are: wavelet algorithm, Prony analysis, Hilbert-Huang transforms (HHT), Ensemble Empirical Mode Decomposition (EEMD), etc. The Prony algorithm is independent of the establishment of the mathematical model, so it is better to determine this block in the model parameters than other methods. But the classical Prony algorithm is sensitive to white noise in the signal, that is, when the sampled signal is non-stationary, its fitting effect is not good [1-3]. The core of HHT algorithm is a process of mode decomposition. The dynamic oscillation modes of signals can be extracted well by HHT. But in the core part of EMD mode decomposition [4-6], there are some
difficulties in decomposing two similar oscillation modes. The EEMD algorithm is an improvement of the modal aliasing problem in the process of processing signals in the EMD algorithm. By processing the original signal and replacing the white noise signal, the problem of the interference of the sampled signal that EMD usually appears is solved. Substitutional processing makes decomposition faster and more accurate [7].

In engineering applications, rough equivalent signals are usually used to adjust the system's uncompensated characteristics, which can not effectively tune the parameters of PSS. The Literature [8-12] introduces how to calculate the identified parameters of signals by residue method and obtain the corresponding controller parameters.

Based on the above analysis, this paper combines EEMD and Prony to identify and decompose the oscillation signal of the system. The EEMD-Prony effectively avoids the Prony algorithm from being affected by white noise signals, improves the accuracy and stability of signal identification, and provides a good basis for optimizing the PSS parameters by the residual method. The effectiveness of the method is verified by analyzing the examples.

2. Basic Principles of EEMD-Prony Algorithms

2.1. Principle of EEMD

EEMD algorithm is improved on the basis of EMD. In order to overcome the problem of identifying modal aliasing caused by signal discontinuity in EMD process, Huang proposed a noise-assisted decomposition method based on research, namely integrated empirical mode decomposition. This method mainly uses the IMF component as the average value of the signal, and uses the spectrum of the white noise signal to uniformly distribute the signals of different scales to the appropriate reference scale. Each component is added with a finite amplitude white noise signal in advance, and EMD decomposition is performed to obtain an average value of the IMF:

By adding the uniformly distributed noise signal to the original signal \( x(t) \), the independent signals of different scales \( x_i(t) \) can be obtained.

\[
x_i(t) = x(t) + n_i(t) \quad i = 1,2,...,N
\]

By using EMD algorithm to decompose independent signals with additional noise \( x_i(t) \):

\[
x_i(t) = \sum_{j=1}^{M} c_{ij}(t) + r_i(t)
\]

Among them, \( c_{ij}(t) \) is the jth intrinsic mode function (IMF) component decomposed by adding white Gaussian noise. \( r_i(t) \) is the component after EMD decomposition of \( x_i(t) \).

Averaged the value of each IMF variables:

\[
c_j(t) = \frac{1}{N} \sum_{i=1}^{N} c_{ij}(t) \quad (3)
\]

\[
r(t) = \frac{1}{N} \sum_{i=1}^{N} r_i(t) \quad (4)
\]

Among them, \( c_j(t) \) is the jth IMF component obtained by EEMD the original signal. \( r_j(t) \) is the component after EMD decomposition.
2.2. Improve the principle of Prony algorithm
The Prony algorithm is an algorithm proposed by the French mathematician Prony in the 1970s based on the gas expansion theorem of his research, which uses a linear combination of complex exponential functions to fit the sampled signal.

The main idea is to transform the signal \((x(0), x(1), ..., x(N-1))\) decomposed by EEMD into a linear combination of corresponding exponential functions. The function form is:

\[
\hat{x}(n) = \sum_{i=1}^{p} A_i e^{i\theta_i} e^{(\alpha_i + i\beta_i)\Delta t}, n = 0, 1, \Lambda , N-1
\]

(5)

In the formula: \(A_i\) is amplitude; \(\theta_i\) is phase; \(f_i\) is frequency; \(\alpha_i\) is attenuation factor; \(\Delta t\) is sampling time interval;

By changing the function expression, the corresponding difference equation can be deduced in turn as follows:

\[
\hat{x}(n) = \sum_{i=1}^{p} a_i \hat{x}(n-i), n = 0, 1, \Lambda , N-1
\]

(6)

A set of linear matrix equations can be obtained by least square estimation of the parameters in the equation to minimize the sum of squares of errors.

\[
\begin{pmatrix}
    x(p) & x(p-1) & \Lambda & x(0) \\
    x(p+1) & x(p) & \Lambda & x(1) \\
    M & M & M & M \\
    x(N-1) & x(N-2) & \Lambda & x(N-P-1)
\end{pmatrix}
\begin{pmatrix}
    1 \\
    a_i \\
    a_i \\
    a_i
\end{pmatrix} =
\begin{pmatrix}
    x(0) \\
    x(1) \\
    a_i \\
    a_i
\end{pmatrix}
\]

(7)

\[
\begin{align*}
    r(i, k) &= \sum_{k=0}^{N-1} x(n-k)x^*(n-i), i, k = 0, 1, \Lambda, p
\end{align*}
\]

(8)

The normal equation of Prony algorithm:

\[
\begin{pmatrix}
    r(0,0) & r(0,1) & \Lambda & r(0,p) \\
    r(1,0) & r(1,1) & \Lambda & r(1,p) \\
    M & M & M & M \\
    r(p,0) & r(p,1) & \Lambda & r(p,p)
\end{pmatrix}
\begin{pmatrix}
    a_0 \\
    a_1 \\
    a_i \\
    a_i
\end{pmatrix} =
\begin{pmatrix}
    a_p \\
    0 \\
    0 \\
    0
\end{pmatrix}
\]

(9)

By solving the normal equation, we can get the estimated coefficients and the root of the polynomial (the pole of Prony) from the following equation:

\[
1 + a_i z^{-1} + a_i z^{-2} + \Lambda + a_p z^{-p} = 0
\]

(10)

According to the recursive formula (11) and (12), the parameters can be obtained.

\[
\hat{x}(n) = -\frac{\varepsilon}{i=1} a_i \hat{x}(n-i), n = 0, 1, \Lambda , N-1
\]

(11)
So the amplitude $A_i$, phase $\theta_i$, frequency $f_i$ and attenuation factor $\alpha_i$ can be obtained by the following formula:

$$
\begin{align*}
A_i &= |b_i| \\
\theta_i &= \arctg \left( \frac{\text{Im}(b_i)}{\text{Re}(b_i)} \right) \\
\alpha_i &= \frac{\Delta}{\pi} \\
f_i &= \frac{\arctg \left( \frac{\text{Im}(z_i)}{\text{Re}(z_i)} \right)}{2\pi\Delta}
\end{align*}
$$

3. **Principle of Residual Method Based on Signal Parameters**

Laplace transformation for a linear time-invariant system:

$$Y(s) = G(s)I(s)$$

$Y(s)$, $G(s)$, $I(s)$ is the Laplace transform of output, transfer and input functions respectively.

A closed-loop system is constructed as shown in Figure 1:

![Figure 1. Transfer function diagram of close loop system.](image)

The Lassian transformation of the input function consists of a delay factor and its corresponding eigenvalue. The expression of the transformation is as follows:

$$I(s) = \frac{c_0 + c_1e^{-\lambda_1s} + c_2e^{-\lambda_2s} + \cdots + c_ne^{-\lambda_ns}}{s - \lambda_{n+1}}$$

The transfer function of the system is as follows:

$$G(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i}$$

Among them, $R_i$ is residue, $\lambda_i$ is pole.

The expression of the output function is as follows:
Then the signal is transformed into a set of linear combinations of P exponential functions with arbitrary amplitude, phase, frequency and attenuation factor by Prony algorithm. Its expression is as follows:

$$x(n) = \sum_{j=1}^{P} B_j \lambda_j^n$$

Through the analysis and solution of EEMD-Prony algorithm, $B_j$ and $\lambda_j$ are obtained. The residue expression of Ith mode in the transfer function can be deduced as follows:

$$R_j = \frac{B_j (\lambda_j - \lambda_{j+1})}{\sum_{i=1}^{P} c_i e^{\lambda_i (d_i - d_j)}}$$

The lag phase angle $\phi_{eq}$ of forward channel of excitation control can be obtained by the residue $R_j$. The corresponding compensation phases of PSS are as follows:

$$\varphi_{PSS} = -\pi / 2 - \varphi_{eq} = -\pi / 2 - R_j$$

Due to the adoption of two compensation links, $n = 2$. The angle of compensation for each link is as follows:

$$\beta_j = \frac{1 + \sin \varphi_j}{1 - \sin \varphi_j}$$

The compensation parameters of lead-lag can be obtained as follows:

$$\begin{align*}
T_1 &= \frac{1}{2\pi \cdot f_j \sqrt{\beta_j}} \\
T_2 &= \beta_j T_1
\end{align*}$$

4. Simulation analysis

4.1. The Simulation of Case
In order to verify the accuracy of EEMD-Prony algorithm in signal extraction, a compound value signal $y$ is constructed. Gauss white noise signal with signal-to-noise ratio of 20dB is added to the original signal $\omega(n)$.

$$\begin{align*}
y &= y1 + y2 + y3 \\
y1 &= e^{-0.25t} \cos(2\pi \ast 1.5 \ast t + \pi) \\
y2 &= 0.4e^{0.25t} \cos(2\pi \ast 1 \ast t + \pi / 2) \\
y3 &= 0.8e^{-0.78t} \cos(2\pi \ast 0.9 \ast t)
\end{align*}$$
The EEMD-Prony algorithm is used to identify the composite signal. Six IMF components can be obtained, and the correlation components are screened. The true and false IMF components and the component modes which have great influence on the composite signal are selected. Finally, the identification parameters of the main mode IMF are obtained.

The EEMD-Prony algorithm is used to identify the composite signal after denoising preprocessing. The identified frequency, damping ratio, amplitude and phase angle parameters are compared with the composite signal original parameters and Prony identification parameter. Relative errors between the signal and the composite signal are calculated. The identification results are shown in Table 1.

| Method     | Modality | Frequency  | Relative error % | Damping ratio | Relative error % | Amplitude value | Relative error % |
|------------|----------|------------|------------------|---------------|------------------|-----------------|------------------|
| Prony      | 1        | 1.5644     | 4.29             | 0.0249        | 6.03             | 0.93            | 6.6              |
|            | 2        | 1.0291     | 2.9              | 0.0410        | 3.01             | 0.38            | 4.5              |
|            | 3        | 0.8785     | 2.38             | 0.1143        | 17.94            | 0.73            | 8.12             |
| EEMD-Prony | 1        | 1.5032     | 0.21             | 0.0264        | 0.37             | 0.99            | 0.2              |
|            | 2        | 1.0033     | 0.33             | 0.0399        | 0.25             | 0.39            | 0.65             |
|            | 3        | 1.0033     | 0.34             | 0.1394        | 0.07             | 0.79            | 0.003            |

Prony algorithm is directly used to identify the signal with mixed noise. The identified parameters have larger errors than the original parameters of the composite signal, and the fitting curve obtained has larger offset in the front section of the composite signal. The identification method of Prony algorithm based on EEMD has more accurate fitting effect, which effectively eliminates the influence of noise signal on identification results, and verifies the validity of identification and fitting in noisy signals.

4.2. The Analysis of Sampling signal and PSS parameter tuning
The measured parameters of a generator are as follows: $x_d = 0.9694 \, \text{pu}$, $x_q = 0.659 \, \text{pu}$, $x_q' = 0.3582 \, \text{pu}$, $x_s = 0.4 \, \text{pu}$. The time constants are as follows: $T_{dx} = 3.2397 \, \text{s}$, $T_{j} = 2.16 \, \text{s}$, $D = 3$. The initial parameters of the system are as follows: $P_e = 0.9$, $Q_e = 0.1$, $U_s = 1$. The original setting parameters of PSS are as follows: $T_i = 0.4018$, $T_j = 0.02$, $K_p = 1.08$.

When the generator PSS is out of operation, the PSS superposition point of the generator excitation controller is applied with a small disturbance of 0.1s duration, the rotation speed deviation signal is weakly damped, and the generator rotation speed deviation is recorded and sampled. Then the sampled signal is decomposed by EEMD, and the IMF components are obtained as shown in Fig. 2.

![Figure 2. The IMF component of the signal.](image-url)
As can be seen from Fig. 4, the amplitudes of IMF2 and IMF3 are quite different from those of IMF1, and the waveform period of IMF1 is similar to the sampled signal. Therefore, it can be judged that IMF1 plays a leading role in the attenuation oscillation of generator speed deviation. The dominant mode IMF1 is analyzed by Prony algorithm, and the results are shown in Table 2. According to the residue method, two advanced links of PSS are designed, \( T_1 = 0.4379 \), \( T_2 = 0.0503 \), \( K_p = 3.09 \).

**Table 2. Parameters Fitted by EEMD-Prony method.**

| Modality | Amplitude value | Phase Angle | Damping ratio | Frequency |
|----------|-----------------|-------------|--------------|-----------|
| IMF1     | 0.1722          | 93.1558     | 0.1257       | 2.2664    |
| IMF2     | 0.0096          | 132.0104    | 0.0108       | 2.8701    |

4.3. **The Disturbance Test of Power Regulation**

In order to verify that the optimized PSS can suppress generator oscillation more effectively and quickly, the active power step-up is set up by 10%. Then, the state variables of the two modes are observed. The results are shown in Figure 3. Compared with the original PSS parameter model, the optimal parameters of PSS reduce the overshoot of oscillation and improve the ability to suppress oscillation, which reduces the frequency of power regulation disturbance oscillation.

![Figure 3. The system responses to power regulation disturbance.](image)

5. **Conclusion**

In this paper, a method of power system low-frequency oscillation mode identification based on EEMD and Prony algorithm is proposed. The parameters of PSS are optimized and tuned according to the identified signal parameters and residue method. This method can effectively solve the problem of difficult configuration of PSS parameters in engineering application and improve the ability of the controller to suppress oscillation. By using signal analysis method to identify system parameters, the influence of rough equivalence in traditional simplification process is reduced. This method has certain reference value for engineering application.

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