JETS IN \( p - p \) COLLISIONS AT RHIC AND MONTE CARLO STUDY OF \( \tau \)-SCALING

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Abstract

Impact of the cone algorithm parameters \( E_{\text{cut}}, E_{\text{seed}}, R \) on the efficiency and characteristics of the reconstructed jets in \( p - p \) collisions at the energy \( \sqrt{s} = 200 \text{ GeV} \) is studied. The PYTHIA Monte Carlo generator is used for event generation. The dependence of dijet production fraction on the parton transverse momentum \( \hat{p}_T \) at different algorithm parameters is analyzed. The dependence of reconstruction efficiency of parton energy in dijet events and two leading jets in \( N \)-jet \((N_{\text{Jet}} \geq 2) \) events on \( E_{\text{cut}}, E_{\text{seed}}, R \) is studied. Monte Carlo results are compared with predictions made in the framework of \( \tau \)-scaling and experimental data obtained at RHIC. The independence of the slope parameter \( \beta \) of the scaling function, \( \psi(z) \sim z^{-\beta} \), on the algorithm parameters over the energy range \( E_{\text{Jet}}^{\text{jet}} = 25 - 60 \text{ GeV} \) is found. The strong dependence of the invariant cross section and the slope parameter on the algorithm parameters with decreasing of \( E_{\text{Jet}}^{\text{jet}} \) for \( E_{\text{Jet}}^{\text{jet}} < 25 \text{ GeV} \) is observed.

1 Jet-finding algorithm and jet properties

Perturbative Quantum Chromodynamics (QCD) predicts the production cross sections for parton-parton scattering in \( p - p \) collisions at high \( p_T \). The outgoing partons from the parton-parton scattering hadronize to form jets of particles. Calculations of high-\( p_T \) jet production involve the folding of parton scattering cross sections with experimentally determined parton distribution functions (PDFs). Measurements of the inclusive jet cross section, the dijet angular distribution, and the dijet mass spectrum, are usually used to test predictions of perturbative QCD.

We used the Monte Carlo code PYTHIA 5.7 for event generation. The mechanism of jet production involves the hard scattering process, the initial and the final state radiation and the multiple parton interaction \([1]\). The dependence of the jet reconstruction efficiency, the parton transfers momentum \( P_T^{\text{Part}} \) accuracy and the inclusive cross sections of jet production on the algorithm parameters are studied. The \( \tau \)-scaling predictions for transverse jet spectra are compared with MC-results and RHIC data.

1.1 Jet-finding algorithm

We used the adapted by D0 experiment algorithm for jet reconstruction \([2]\). The jet-finding algorithm consists of the following steps:
1. The particles with energy \( E_i > E_{seed} \) are assigned as ”seeds”.
2. Start with highest-\( P_T \) ”seed” and iterate over all ”seeds”. The ”seed”-direction gives the first approximation of the jet direction \((\eta_{Jet}, \phi_{Jet})\).
3. All particles with distance \( R_i \) to the jet axis in the \((\eta, \phi)\) space lesser than \( R \) are included into jet
   \[
   R_i = \sqrt{(\eta - \eta_i)^2 + (\phi - \phi_i)^2} < R, \tag{1}
   \]
   where \((\eta_i, \phi_i)\) are the particle direction.
4. The energy \( E_{T_i}^{Jet} \) and the direction of jet are calculated using (2):
   \[
   E_{T_i}^{Jet} = \sum_i E_{T_i}, \quad \phi_{Jet} = \sum_i E_{T_i} \phi_i / \sum_i E_{T_i}, \quad \eta_{Jet} = \sum_i E_{T_i} \eta_i / \sum_i E_{T_i} \tag{2}
   \]
5. The steps (3)-(4) are iterated until the jet-direction is stable.
6. If \( E_{T_i}^{Jet} \leq E_{cut} \), the jet is discarded.
7. Overlapping jets are merged or splitted depending on the energy fraction in the overlapping region.
8. Repeat these steps for all ”seeds”.
9. The jet directions are recalculated using an alternative definition as given in (3)
   \[
   \theta_{Jet} = \tan^{-1} \left[ \frac{\sqrt{(\sum_i E_{i_x})^2 + (\sum_i E_{i_y})^2}}{\sum_i E_{i_z}} \right], \quad \phi_{Jet} = \tan^{-1} \left[ \frac{\sum_i E_{i_y}}{\sum_i E_{i_x}} \right], \quad \eta_{Jet} = -\ln[\tan(\theta_{Jet}/2)] \tag{3}
   \]
   where \(i\) corresponds to all particles whose centers are within the jet radius \( R \), \( E_{i_x} = E_i \sin \theta_i \cos \phi_i, E_{i_y} = E_i \sin \theta_i \sin \phi_i, E_{i_z} = E_i \cos \theta_i \).

In the paper we consider \( E_{seed}, R, E_{cut} \) as the parameters of the jet-finding algorithm and analyze their influence on characteristics of reconstructed jets.

### 1.2 Jet reconstruction efficiency

We define the jet reconstruction efficiency as a probability to find two or three jets in event. Figure [ ] shows the probability to find two or three jets as a function of the transfers momentum \( \hat{p}_\perp \) of the hard process for different sets of parameters. The left frame is for the fixed \( E_{seed}, R \) and for different values of \( E_{cut} \), and the central frame is for the fixed \( E_{seed}, E_{cut} \) and for different values of \( R \). One can see that values of the algorithm parameters \( E_{cut}, R \) define the low limit \( P_T^{lim} \) of the \( \hat{p}_\perp \) spectrum and the ground boundary \( P_T^{sat} \) of saturation region for dijet events. For example, the dijet reconstruction probability drops very fast for \( \hat{p}_\perp < 13 \text{ GeV} \) at \( E_{cut} \approx 7 \text{ GeV} \). The low limit of the \( \hat{p}_\perp \) spectrum increases with \( E_{cut} \) and as \( R \) decreases. The probability to find two jets for given \( \hat{p}_\perp \) has a maximum at \( E_{cut} \approx \hat{p}_\perp/2 \). It diminishes with decreasing of \( E_{cut} \). Part of events are reconstructed as three-jets. The right frame is for fixed \( E_{cut} \) and \( R \) and different values of \( E_{seed} \). As seen from Figure [ ] the influence of \( E_{seed} \) on jet reconstruction is very small while \( E_{seed} \ll \hat{p}_\perp \). Therefore, following results are presented for \( E_{seed} = 1 \text{ GeV} \). The parameters \( E_{cut} \) and \( R \) have an influence on the number of reconstructed jets. Therefore the following results are presented for dijet events and for two leading jets of \( N \)-jet \((N_{Jet} > 1)\).

### 1.3 Parton transverse momentum reconstruction

We characterize the parton transverse momentum \( P_T^{Part} \) reconstruction efficiency via the dependence of the mean jet transverse energy \( < E_{T_i}^{Jet} > \) and the \( E_{T_i}^{Jet} \) distribution
Figure 1: The dependence of the probability to find two or three jets in $p-p$ collisions on the transfers energy of the hard process $\hat{p}_\perp$ at the different values of the algorithm parameters: (a) $E_{\text{seed}}=1.0$ GeV, $R=0.7$, $E_{\text{cut}}=5, 7, 14$ GeV; (b) $E_{\text{cut}}=7.0$ GeV, $E_{\text{seed}}=1.0$ GeV, $R=0.4, 0.7, 1.1$; (c) $E_{\text{cut}}=7.0$ GeV, $R=0.7$, $E_{\text{seed}}=0.5, 1.0, 1.5$ GeV.

width (RMS) on parton transverse momentum $P^\text{Part}_T$. These dependencies are obtained from analysis of the dijet distribution events as a function of jet energy $E^\text{Jet}_T$ for narrow parton momentum bins. One of them is presented in Figure 2(a). Figure 2(b) shows the dependence of the mean jet transverse energy as a function on $P^\text{Part}_T$ for a fixed value of $E_{\text{cut}}$ and $E_{\text{seed}}$ and different values of $R$. The linear dependence of $<E^\text{Jet}_T>$ on parton transverse momentum is observed at $P^\text{Part}_T > P^\text{lim}_T$. Similar results are found for the another sets of the jet finding algorithm parameters.

Figure 2: (a) The dependence of number of dijet events on the jet transverse energy $E^\text{Jet}_T$ for $20 < P^\text{Part}_T < 21$ (GeV/c). (b) The dependence of the mean jet transverse energy $<E^\text{Jet}_T>$ on the parton transverse energy $P^\text{Part}_T$ for two leading jets in $N$-jets events ($N_{\text{Jet}} \geq 2$) at $E_{\text{cut}}=7$ GeV and $R=0.4, 0.7, 1.1$.

Figure 3 shows the dependence of RMS as a function of $P^\text{Part}_T$ for fixed values of $R$ and $E_{\text{seed}}$ and different values of $E_{\text{cut}}$. The left frame is for dijet events. The slight growth of width is observed for $P^\text{Part}_T > P^\text{sat}_T$. We would like to note, that the momentum $P^\text{sat}_T$ defines the low boundary of the saturation region of the probability to find dijet events. The diminution of the dijet reconstruction probability for $P^\text{Part}_T < P^\text{sat}_T$ leads to decrease of RMS values. The lower $E_{\text{cut}}$ provides the smaller $E^\text{Jet}_T$ distribution width. The dijet reconstruction probability decreases in this case. The right frame is for two leading jet of
Figure 3: The dependence of RMS for dijet events (a) and two leading jets in N-jets events ($N_{\text{Jet}} \geq 2$) (b) on the parton transverse momentum $P_{\text{Part}}^{T}$ at $R=0.7$ and $E_{\text{cut}} = 14, 7, 5$ GeV.

$N$-jet events, where $N_{\text{Jet}} > 1$. The jet transverse energy distribution width is proportional to $P_{\text{Part}}^{T}$ in this case. The parton transverse momentum reconstruction efficiency is almost the same for different values of $E_{\text{cut}}$.

Figure 4: The dependence of RMS for dijet events (a) and two leading jets in N-jets events ($N_{\text{Jet}} \geq 2$) (b) on the parton transverse momentum $P_{\text{Part}}^{T}$ at $E_{\text{cut}} = 7$ GeV and $R = 0.4, 0.7, 1.1$.

Figure 4 demonstrates the dependence of RMS on $P_{\text{Part}}^{T}$ for fixed values of $E_{\text{cut}}$ and $E_{\text{seed}}$ and different values of $R$. The left frame is for dijet events. The growth of width for $P_{\text{Part}}^{T} > P_{\text{sat}}^{T}$ and alteration of RMS for $P_{\text{Part}}^{T} < P_{\text{sat}}^{T}$ are found. The higher $R$ provides the smaller $E_{T}^{\text{Jet}}$ distribution width. The difference of RMS for $R=0.7$ and $R=1.1$ is found to be not so large. Results of analysis with higher $R$ are potentially more sensitive to background. The value of the RMS raises substantially for $R=0.4$. The right frame is for two leading jet of $N$-jet events ($N_{\text{Jet}} > 1$). The jet transverse energy distribution width is proportional to $P_{\text{Part}}^{T}$. The reconstruction efficiency is different for various $R$.

1.4 Inclusive cross section of jet production

We study the dependence of jet cross section on the jet-finding algorithm parameters $E_{\text{seed}}, R, E_{\text{cut}}$. Figure 5 presented the dependence of inclusive jet cross section on the
jet transverse energy $E_{T}^{jet}$ for one, two, three and all-jet events (the left frame). The dependence of the cross section ratio on $E_{T}^{jet}$ is shown on the right frame. One see that events with one reconstructed jet dominate for $E_{T}^{jet} < 15\,\text{GeV}$. Their contributions are noticeable for $E_{T}^{jet} < 25\,\text{GeV}$. The dijet events dominate for $E_{T}^{jet} > 15\,\text{GeV}$. The fraction of events with three reconstructed jets is noticeable over a range $20 < E_{T}^{jet} < 50\,\text{GeV}$. Maximum contribution of such events is about 15%. Behavior of jet multiplicity versus $E_{T}^{jet}$ is almost the same for another set of parameters ($E_{cut} = 5\,\text{GeV}, E_{seed} = 0.5\,\text{GeV}, R = 0.4$). Maximum contribution of three-jet events is $\approx 20\%$.

Figure 5: The dependence of the inclusive cross section (a) and the cross section ratio $Jet_{N}/Jet_{All}$ (b) for all and $N$-jet events ($N = 1, 2, 3$) on the jet transverse energy $E_{T}^{jet}$ for PAR1 set of parameters in $p - p$ interaction at $\sqrt{s}=200\,\text{GeV}$. (PAR1: $E_{T}^{jet} = 7\,\text{GeV}, E_{seed} = 1.0\,\text{GeV}, R = 0.7$).

Figure 6: The dependence of the $(Jet_{PAR2} - Jet_{PAR1})/Jet_{PAR1}$ ratio for two sets of parameters PAR1 PAR2 on the jet transverse energy $E_{T}^{jet}$ for one-jet (a) and $N$-jets ($N_{Jet} = 1, 2, 3$) (b) events. (PAR1: $E_{T}^{jet} = 7\,\text{GeV}, E_{seed} = 1.0\,\text{GeV}, R = 0.7$).

Figure 6 shows the dependence of the $(Jet_{PAR2} - Jet_{PAR1})/Jet_{PAR1}$ ratio for two sets of parameters PAR1 PAR2 on the jet transverse energy $E_{T}^{jet}$. The left frame is for one-jet events. The right frame is for $N$-jets ($N_{Jet} = 1, 2, 3$). As seen from Figure 6 the ratio is practically constant above the threshold $E_{Sh}^{th} > 15\,\text{GeV}$ for dijet events. The absolute value of the obtained jet cross sections depend on the algorithm parameters. But the energy dependence of cross sections is practically the same for any sets of parameters.
above \( E_T^{Sh} = (2 - 3)E_{cut} \). Below this threshold the shape of cross sections depend very strongly on parameter choice. The value of the threshold for inclusive jet cross sections is found to be \( E_T^{Sh} = (3 - 5)E_{cut} \).

Thus, above the threshold \( E_T^{Sh} \) the parameter choice results in the cross section normalization only. The value of cross section depends very strongly on the algorithm parameters below threshold.

2 \( z \)-Presentation of jet cross section

In this section we would like to remind the basic ideas of \( z \)-scaling [3, 4] dealing with the investigation of the inclusive process

\[ P_1 + P_2 \rightarrow p + X. \] (4)

Here the momenta and masses of the colliding and inclusive particles are denoted by \( P_1, P_2, p \) and \( M_1, M_2, m_1 \), respectively. It is assumed that the process can be described in terms of the corresponding kinematic characteristics of the exclusive subprocess written in the symbolic form

\[ (x_1M_1) + (x_2M_2) \rightarrow m_1 + (x_1M_1 + x_2M_2 + m_2). \] (5)

The parameter \( m_2 \) is introduced to take into account the internal conservation laws (for isospin, baryon number and strangeness). The quantities \( x_1 \) and \( x_2 \) are the fractions of incoming four-momenta \( P_1, P_2 \) of colliding objects. The energy of the parton subprocess is defined as

\[ \hat{s}^{1/2} = (x_1^2M_1^2 + 2x_1x_2(P_1P_2) + x_2^2M_2^2)^{1/2}. \] (6)

The elementary parton-parton collision is considered as a binary subprocess which satisfies the 4-momentum conservation law written in the following form

\[ (x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \] (7)

To determine the fractions \( x_1 \) and \( x_2 \) the principle of minimum resolution is used. It states that the measure of the constituent interaction \( \Omega^{-1} \) which connects kinematic and dynamic characteristics of the interaction

\[ \Omega^{-1}(x_1, x_2) = [(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}]^{-1}, \] (8)

should be minimum one under the condition (7). Here \( \delta_1 \) and \( \delta_2 \) are the factors relating the fractal structure of colliding objects [3, 4].

The scaling variable \( z \) and scaling function \( \psi(z) \) as suggested in [3, 4] are determined as follows

\[ z = \frac{\sqrt{s}_\perp}{m\rho(s)\Omega} = z_0\Omega^{-1}, \quad \psi(z) = -\frac{\pi s}{\rho(s, \eta)\sigma_{int}} \int_{d^3p} J^{-1}E d^3\sigma dp^3. \] (9)

Here \( m \) is the mass constant (nucleon mass), \( \sqrt{s}_\perp \) is the transverse energy of parton subprocess.

The scaling variable \( z \) is expressed via the transverse kinetic energy of subprocess \( \hat{s}_\perp \), the multiplicity density of charge particles \( \rho(s) \) and \( \Omega(x_1, x_2) \). The scaling function
ψ(z) is expressed via the colliding energy $\sqrt{s}$, the average charged particle multiplicity density $\rho(s, \eta) = d < N > / d\eta$, the inelastic cross section $\sigma_{\text{inel}}$, the inclusive cross section $Ed^3\sigma/dy^3$, and the Jacobian $J$ of transformation from the variables $\{p_z, p_T\}$ to $\{\eta, z\}$. The function $\psi$ is interpreted as the probability density to produce a particle with the formation length $z$. Authors of z-scaling concept argue that the z-scaling reflects features of particle substructure, constituent interaction and particle formation such as locality, self-similarity and fractality over a wide scale range.

The properties of the scaling function $\psi$ of jet production at ISR, Sp$^+$pS, and Tevatron were investigated in [2, 3]. The energy and angular independence of $\psi$ and power behavior, $\psi \sim z^{-\beta}$, of jet and dijet production at high energy were established. It was found that the values of the slope parameter $\beta$ in $p - p$ and $\bar{p} - p$ collisions are different. The properties of the scaling function $\psi$ were used to predict the transverse energy spectrum of jet production in $p - p$ and $\bar{p} - p$ collisions at RHIC, Tevatron and LHC energies.

The STAR collaboration presented [6] the first results on inclusive cross sections of jet production in $p - p$ collisions at RHIC. The transverse spectra are measured over the range $\sqrt{s} = 200$ GeV, $0.2 < |\eta| < 0.8$, and $5 < E_T^{\text{jet}} < 50$ GeV.

In the present paper we analyze MC-results and RHIC data [6] in $z$-presentation. The comparison with predictions of z-scaling is given as well.

![Figure 7](image-url)

Figure 7: The inclusive cross section of jet production in $p - p$ collisions at $\sqrt{s} = 200$ in $E_T$ (a,b) and $z$ (c) presentation.

Figure 7 (a) shows Monte Carlo results on the invariant inclusive jet cross section for sets of algorithm parameters PAR1 and PAR2. Predictions of z-scaling are shown by the solid line. We found that the shape of MC jet cross sections for different sets of parameters coincides with the curve predicted by z-scaling over a range $25 < E_T^{\text{jet}} < 55$ GeV. As seen from (b) the STAR data are in agreement with MC results. The strong dependence of the cross section shape on the algorithm parameters with decreasing $E_T^{\text{jet}}$ is observed for $E_T^{\text{jet}} < 25$ GeV.

The comparison of the MC results, STAR data and predictions of z-scaling in $z$-presentation are shown in (c). We see that both MC simulation and STAR data are in a good agreement with z-scaling predictions for $E_T^{\text{jet}} > 25$ GeV ($z > 180$). The value of the slope parameter $\beta$ is found to be $\beta = 6.01 \pm 0.06$ for Monte Carlo results at $\sqrt{s} = 200$ GeV. It is compatible with $\beta = 5.95 \pm 0.21$ obtained for the AFS data at $\sqrt{s} = 38.8, 45, \text{and } 63$ GeV [4]. Note that the shape of the scaling function $\psi(z)$ for $p_T < 10$ GeV/c can not be described by the power law $\psi(z) \sim z^{-\beta}$. The error bars of the experimental data [6]
are large enough for precise test of asymptotic behavior of $z$-scaling of jet production in $p - p$ collisions at RHIC energies.

## 3 Conclusions

The Monte Carlo study of jet production in $p - p$ collisions at $\sqrt{s} = 200$ GeV using the PYTHIA generator was performed. The impact of the one algorithm parameters $E_{cut}, E_{seed}, R$ on the efficiency and characteristics of the reconstructed jets was investigated. It was established that the $P_T^{Part}$ reconstruction efficiency is independent of $E_{cut}$ at $R = 0.7$ and depends on $R$ for two leading jets in $N$-jet events.

The probability of two-jet reconstruction was found to drop very fast if the transverse energy of hard process is less than the threshold $\hat{p}_T < P_T^{lim}$. It goes to saturation if $\hat{p}_T > P_T^{sat}$. These limits are controlled by the values of the algorithm parameters $E_{seed}, R, E_{cut}$. Results of Monte Carlo study were compared with predictions made in the framework of $z$-scaling and experimental data obtained by the STAR collaboration at RHIC. The independence of the slope parameter $\beta$ of the scaling function, $\psi(z) z^{-\beta}$, on the algorithm parameters over a range $E_{Jet}^{T} = 25 - 60$ GeV was established. The strong dependence of the transverse energy spectra of jet production and the slope parameter on algorithm parameters were found for $E_{Jet}^{T} < 25$ GeV.

Verification of $z$-scaling for jet production in $p - p$ collisions at RHIC and comparison with data at Tevatron and LHC energies are of interest for understanding the jet phenomena.

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