On reggeization of vertex of three reggeized gluons in high energy QCD

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Abstract

The unitarity corrections to the propagator of reggeized gluons calculated in the framework of QCD RFT require a knowledge of the expressions for Reggeon propagator and vertices of interaction of three reggeized gluons (Reggeons) to one QCD loop precision, see [9]. In this paper we calculate the vertex of interactions of $A_+A_+A_-$ Reggeon fields, i.e. vertex of transition of $A_-$ Reggeon field to $A_+A_+$ fields, to this precision. We demonstrate, that all loop leading logarithmic order contributions to the vertex can be summed through the integro-differential equation similarly to the BFKL one, [14]. The solution of this equation leads to the reggeized form of the vertex with the trajectory twice larger then the trajectory of reggeized gluon propagator. The application of the obtained result is also discussed.

1 Introduction

The high energy, effective QCD action for an interaction of the reggeized gluons (Reggeons), introduced in [12], see also [3–8], describes quasi-elastic amplitudes of high-energy scattering processes in the multi-Regge kinematics. The applications of the approach to the description of high energy processes and calculation of unitarity corrections to the different production amplitudes can be found in [10–12] for example, whereas the generalization of the formalism for a case of the description of arbitrary production amplitudes and impact factors is presented in [5] with the prescription of the calculation of $S$-matrix elements accordingly to an approach of [13]. This effective action formalism, based on the reggeized gluons as main degrees of freedom, see [14], can be considered as reformulation of the RFT (Regge Field Theory) calculus introduced in [15], see also [16–24], for the case of high energy QCD. It was underlined in [12] that the main purposes of the approach is the construction of the $S$-matrix unitarity in the direct and crossing channels of the scattering processes through the multi-Reggeon dynamics described by the vertices of multi-Reggeon interactions, see simirar approaches in [25–31], the connections between the different formalisms were clarified in [3, 8, 32]. The unitarity of the Lipatov’s formalism, therefore, is related to the unitarity corrections in both RFT and QCD sectors of the theory.

Similarly to the phenomenological theories of interacting Reggeons, see [15–19] and references therein, we can separately calculate the corrections to the amplitudes which come from the pure RFT sector of the formalism. Namely, let us consider Lipatov’s effective action for reggeized gluons $A_\pm$ formulated as RFT (Regge Field Theory) in the form of generation functional obtained by an integration out of the gluon fields $v$ from the $S_{eff}[v, A]$: 

$$e^{i\Gamma[A]} = \int Dv e^{iS_{eff}[v, A]}$$

(1)
where
\[ S_{\text{eff}} = - \int d^4 x \left( \frac{1}{4} G_{\mu \nu}^a G_a^{\mu \nu} + tr \left[ \left( T_+(v_+) - A_+ \right) j^+_\text{reg} + \left( T_-(v_-) - A_- \right) j^-_\text{reg} \right] \right), \] (2)

with
\[ T_\pm(v_\pm) = \frac{1}{g} \partial_\pm O(v_\pm) = v_\pm O(v_\pm), \quad j^\pm_\text{reg} = \frac{1}{C(R)} \partial^2 A^\pm_a, \] (3)

here \( C(R) \) is eigenvalue of Casimir operator in the representation \( R \), \( tr(T^a T^b) = C(R) \delta^{ab} \) see [14]. The form of the Lipatov’s operator \( O \) (and correspondingly \( T \)) depends on the particular process of interests, see [3], we take it in the form of the Wilson line (ordered exponential) for the longitudinal gluon fields in the adjoint representation:
\[ O(v_\pm) = P e^g \int_{-\infty}^{v_\pm} dx^\pm v_\pm(x^+, x^-, x_\pm), \quad v_\pm = i T^a v^a_\pm, \] (4)

see also [12]. There are additional kinematical constraints for the reggeon fields
\[ \partial_- A_+ = \partial_+ A_- = 0, \] (5)

corresponding to the strong-ordering of the Sudakov components in the multi-Regge kinematics, see [12, 26]. The action is constructed by the request that the LO value of the classical gluon fields in the solutions of equations of motion will be fixed as
\[ v^c_\pm = A_\pm. \] (6)

In the light-cone gauge \( v_- = 0 \), the equations of motion can be solved and the general expressions for the gluon fields can be written in the following form:
\[ v_i^a \to v_i^a cl(A_\pm) + \varepsilon_i^a, \quad v^a_\pm \to v^a_\pm, \] (7)

The integration in respect to the fluctuations around the classical solutions provides QCD loop corrections to the effective vertices of the Lipatov’s action which now can be written as functional of the Reggeon fields only [1]:
\[ \Gamma = \sum_{n,m=1} \left( A^a_{+1} \cdots A^a_{+n} \left( K_{+,\cdots,+}^{a_1 \cdots a_n} b_{1 \cdots b_n} A_{-,\cdots,-}^{b_1 \cdots b_m} A_{-,\cdots,-} \right) = - A^a_{+x} \partial^2_x A^a_{+x} + A^a_{+x} \left( K^{a b}_{+x} \right)_+ A^b_{+y} + \cdots, \] (8)

in general the summation on the color indexes in the r.h.s of the equation means the integration on the corresponding coordinates as well. Now we see, that the theory have two different sources of any perturbative/unitarity corrections. The first one comes from the QCD sector of the formalism, it affects on the precision of the effective vertices (kernels) calculated in the pure QCD. Another source of the corrections is described by the processes formulated in terms of RFT sector degrees of freedom only, i.e. these corrections are constructed entirely in terms of the Reggeon fields and Eq. (8) vertices known to some QCD precision. These type of the RFT corrections were considered many times in the previous phenomenological RFT approaches, see [16, 19] and references therein for example; in the effective high energy QCD formalism there are the RFT corrections to the propagator and vertices of the Eq. (8) action as well. In the paper [8] the Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluons was derived in the framework of the formalism that allows to determine the calculation scheme for these corrections to any correlator of interests. We also note, that the obtained hierarchy formally is similar to the Balitsky hierarchy of equations and BK-JIMWLK approaches , see [25, 27, 30], and there is a correspondence between different degrees of freedom such as reggeized gluons and Wilson line operators, see details in [8, 32].

In order to make the notations shorter, we change the position of the color and other indexes of the vertices further in the article, preserving only the overall number of the indexes.
In our previous paper, \[9\], we first time calculated the one RFT loop correction to the propagator of reggeized gluons. It turns out, that this correction is large and does not suppressed perturbatively. Namely, beginning from the NLO there are terms of the same order in the correction which are involved in the full answer with the sign opposite to the leading order term sign, see details in \[9\]. The answer obtained is an example of the non-linear correction to the propagator of reggeized gluons, similar in some extend to the non-linear corrections of \[30\] formalism. The non-linear correction calculated in \[9\], nevertheless, was obtained with the use of the bare triple Reggeon vertices only. In order to provide the full one loop QCD precision for the RFT correction, we have to know the one QCD loop expression for the triple Reggeon vertices as well\[2\]. Therefore, in this paper we calculate the one QCD loop correction to the $A_+ A_- A_-$ vertex of interaction of three reggeized gluons\[2\]. It is found, that we can write for the vertex the equation of the Bethe - Salpeter type which sum up all loops contributions to the vertex to the LLA precision. The solution of this equation leads to the reggeization of the vertex with the trajectory twice large than the trajectory of the propagator of reggeized gluons, this is a main result of the article. Consequently, the paper is organized as follows. In the next section we remind some basic definitions from the \[4\], this Section as well as Appendices A-C intended to facilitate the understanding of the technical calculations. The Section 3 is dedicated to the calculation of the one loop QCD correction to the vertex of interests, the main technical details of the calculations are located in Appendix D. The last Section is the Conclusion of the article where we discuss the result of the calculations and their possible applications.

## 2 One loop effective action

The general expressions for the gluon fields can be written in the following form:

$$ v_i^a \rightarrow v_i^a_{\text{cl}} + \varepsilon_i^a, \quad v_i^+ \rightarrow v_i^{+\text{cl}} + \varepsilon_i^+, $$  \(9\)

at the next step we expand the Lagrangian of the effective action around this classical solution. Preserving in the expression only terms which are quadratic with respect to the fluctuation fields, we obtain for this part of the action:

$$ S_{e2} = -\frac{1}{2} \int d^4x \left( \varepsilon_i^a \left( \delta_{ac} \left( \partial_i \square + \partial_i \partial_j \right) - 2g f_{abc} \left( \delta_{ij} v_i^{b\text{cl}} \partial_k - 2v_j^{b\text{cl}} \partial_i + v_i^{b\text{cl}} \partial_j - \delta_{ij} v_j^{c\text{cl}} \partial_- \right) \right) - g^2 f_{abc} f_{cib} \left( \delta_{ij} v_i^{b\text{cl}} v_k^{b\text{cl}} - v_i^{b\text{cl}} v_j^{b\text{cl}} \right) \right) \varepsilon_j^c + \varepsilon_i^a \left( -2 \delta^{ac} \partial_- \partial_i - 2g f_{abc} \left( v_i^{b\text{cl}} \partial_- - \left( \partial_- v_i^{b\text{cl}} \right) \right) \right) \varepsilon_j^c + \varepsilon_i^a \delta_{ac} \partial_- \varepsilon_j^c + g \varepsilon_i^a \int d^4y \left( U_1^{abc} \right)_{xy} \left( \partial_i \partial_- \rho_0 \right)_x \varepsilon_j^c = -\frac{1}{2} \varepsilon_i^a \left( \left( M_0 \right)_{\mu\nu}^{ac} + \left( M_1 \right)_{\mu\nu}^{ac} + \left( M_2 \right)_{\mu\nu}^{ac} + \left( M_L \right)_{\mu\nu}^{ac} \right) \varepsilon_i^c. $$ \(10\)

Here we defined \((M_i)_{\mu\nu}^{ac}\) \(\propto g^i\) and note that

$$ (M_1)_{\mu\nu}^{ac} = -g f_{abc} \left( v_i^{b\text{cl}} \partial_- - \left( \partial_- v_i^{b\text{cl}} \right) \right), \quad (M_1)_{\mu\nu}^{ac} = -g f_{abc} \left( \partial_i \partial_- v_i^{b\text{cl}} - \left( \partial_- v_i^{b\text{cl}} \right) \right). $$ \(11\)

The last term in Eq. (10) expression, denoted as \((M_L)_{\mu\nu}^{ac}\) represents contribution of the Lipatov’s effective current into the action. This term is defined trough the following function:

$$ \left( U_1^{abc} \right)_{xy} = \text{tr} \left[ f_a G_{xy}^+ f_c O_y f_b O_x^T \right] + \text{tr} \left[ f_c G_{xy}^+ f_a O_x f_b O_y^T \right], $$ \(12\)

\(\text{\textsuperscript{2}}\)See [33] where similar vertex was considered.

\(\text{\textsuperscript{3}}\)The calculation of the $A_- A_- A_+$ vertex is in the progress.
see definitions of the quantities in Appendix B. We underline, that Eq. (5) function can be expanded as an infinite series in respect to $g$ coupling constant and $v^a_{+}$ fields, see again Appendix B. Now we can perform the integration obtaining the one loop effective action:

\[
\Gamma = \int d^4x \left( L_{YM}(v^d_0, v^d_+) - v^a_{+} J^a_+(v^d_+) - A^a_+ \left( \partial_\theta^2 A^a_+ \right) \right) +
\]

\[\frac{i}{2} \text{Tr} \ln \left( \delta_{\rho\nu} + G_{0\rho\nu} \left( (M_1)_{\mu\nu} + (M_2)_{\mu\nu} + (M_L)_{\mu\nu} \right) \right) +
\]

\[\frac{1}{2} \int d^4x \int d^4y j^a_{\mu x} G^a_{\mu \nu}(x, y) j^b_{\nu y}. \tag{13}\]

Here we have: $G_{0\nu\mu}$ as bare gluon propagator

\[
(M_0)^{\mu\nu} G_{0\nu\rho} = \delta_{\mu\rho}, \tag{14}\]

see Appendix A; the full gluon propagator is defined as

\[
G^{ac}_{\mu\nu} = \left[ (M_0)^{ac}_{\mu\nu} + (M_1)^{ac}_{\mu\nu} + (M_2)^{ac}_{\mu\nu} + (M_L)^{ac}_{\mu\nu} \right]^{-1} \tag{15}\]

and can be written in the form of the following perturbative series:

\[
G^{ac}_{\mu\nu}(x, y) = G^{ac}_{0\mu\nu}(x, y) - \int d^4z G^{ab}_{0\mu\rho}(x, z) \left( (M_1(z))^{bd}_{\rho\gamma} + (M_2(z))^{bd}_{\rho\gamma} + (M_L(z))^{bd}_{\rho\gamma} \right) G^{de}_{\gamma\mu}(z, y); \tag{16}\]

the auxiliary currents $j^a_{\mu x}$ and $j^b_{\nu y}$ are requested for the many-loops calculations of the effective action, in our case of calculation of one loop precision we take them equal zero from the beginning.

### 3 Reggeization of $K^{+++}$ triple Reggeon vertex

Any vertex of the three Reggeon fields interactions is defined as follows:

\[
(K^{abc}_{xyz})_{\mu\nu\rho}^{+++} = \int d^4w \left( \frac{\delta^3 \Gamma(v^d_0(A), v^d_+(A))}{\delta A^a_+(y) \delta A^b_+(x) \delta A^c_+(z)} \right)_{A=0}, \tag{17}\]

The bare triple Reggeon vertices were calculated in [9], the one loop corrections to the $K^{+++}$ vertex are presented in Appendix D. Summing up Eq. (D.31), Eq. (D.46) and Eq. (D.54) expressions we obtain for the vertex to one QCD loop precision:

\[
-2i \left( K^{abc}_{xyz} \right)^{+++}_1 = \tag{18}\]

\[
-\frac{i g^3 N}{2(2\pi)^6} f_{abc} \delta^2_{z_+ z} \theta^2_{t_y} \left( \theta(z^+ - x^+) \int \frac{dk_+}{k_=} \int d^2k_{1-} \int d^2k_{1+} \frac{k^2_{1+}}{k^2_{1+} (k_+ - k_{1-})^2} e^{-i(y^+ - z^+) k_{1-}} - \right.
\]

\[
\theta(x^+ - z^+) \int \frac{dk_-}{k_-} \int d^2k_{1-} \int d^2k_{1+} \frac{k^2_{1+}}{k^2_{1+} (k_+ - k_{1-})^2} e^{-i(y^+ - x^+) k_{1-}}. \]

Correspondingly, performing Eq. (D.18) variable’s change, we write the vertex in the following form:

\[
\left( K^{abc}_{xyz} \right)^{+++}_1 = \frac{g^3 N}{2(2\pi)^6} f_{abc} \delta^2_{z_+ z} \eta \delta^2_{t_y} \left( \theta(z^+ - x^+) \int d^2k_{1-} \int d^2k_{1+} \frac{k^2_{1+}}{k^2_{1+} (k_+ - k_{1-})^2} e^{-i(y^+ - z^+) k_{1-}} - \right.
\]

\[
\theta(x^+ - z^+) \int d^2k_{1-} \int d^2k_{1+} \frac{k^2_{1+}}{k^2_{1+} (k_+ - k_{1-})^2} e^{-i(y^+ - x^+) k_{1-}}. \tag{19}\]

4
Using it’s bare value
\[
\left( K_{\alpha \beta \gamma ; \eta}^{abc} \right)_{0}^{++-} = \frac{1}{2} g f^{abc} \left( \theta(x^+ - y^+) - \theta(z^+ - x^+) \right) \delta^2(y_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \partial_{y^+}, \tag{20}
\]
see [9] for the details of the calculation, we can write the following integral equation for the vertex to the LLA precision:
\[
\left( K_{\alpha \beta \gamma ; \eta}^{abc} \right)_{0}^{++-} = \left( K_{\alpha \beta \gamma ; \eta}^{abc} \right)_{0}^{+++} - \frac{2 \alpha_s N}{(2\pi)^4} \int_0^\pi ds \int d^2w_\perp \left( K_{x \hat{y} z ; \eta}^{abc} \right)_{0}^{++-} \int d^2k_\perp \frac{k_\perp^2}{(k_\perp - k_\perp)_\perp} e^{-i(y^+ - w^+) k_\perp}, \tag{21}
\]
\[
\partial_\eta \int dt D_{++-0}(w_\perp, t_\perp) D_{++-0}(t_\perp, y_\perp) D_{++-0}(t_\perp, w_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{k_\perp^2}{(k_\perp - k_\perp)_\perp} e^{-i(y^+ - w^+) k_\perp}
\]
is an one loop correction to the bare vertex, see Appendix C for the definition of $D_{++-0}$. Now, performing Fourier transform with respect to the transverse variables of the full vertex
\[
\left( K_{x \hat{y} z ; \eta}^{abc} \right)_{0}^{++-} = \int \frac{d^2p_{\perp}}{(2\pi)^2} \int \frac{d^2p_{\perp}}{(2\pi)^2} \int \frac{d^2p_{\perp}}{(2\pi)^2} \tilde{K}_{x \hat{y} z ; \eta}^{abc}(x^+, z^+, p_{\perp}, p_{\perp}, p_{\perp}, \eta) e^{-i p_{\perp} x^+ i p_{\perp} z^+ p_{\perp} y^+} \tag{23}
\]
and taking derivative on $\eta$ we obtain:
\[
\frac{\partial \tilde{K}_{x \hat{y} z ; \eta}^{abc}}{\partial \eta} = 2 \varepsilon(p_{\perp}) \tilde{K}_{x \hat{y} z ; \eta}^{abc} \tag{24}
\]
with
\[
\varepsilon(p_{\perp}) = - \frac{\alpha_s N}{4 \pi^2} \int d^2k_\perp \frac{p_{\perp}^2}{k_\perp^2 (k_\perp - p_{\perp})^2}, \tag{25}
\]
The solution of the Eq. (23) with given precision is the following function:
\[
\tilde{K}_{x \hat{y} z ; \eta}^{abc}(x^+, z^+, p_{\perp}, p_{\perp}, p_{\perp}, \eta) = \tilde{K}_{0}^{abc}(x^+, z^+, p_{\perp}, p_{\perp}, p_{\perp}, \eta) e^{2 \eta \varepsilon(p_{\perp})}, \tag{26}
\]
which can be considered as reggeization of the bare vertex Eq. (20), see Eq. (C.27) expression as well.

4 Conclusion

In the formalism of Lipatov’s effective action, formulated as RFT, there is an additional source of perturbative and unitarity corrections to high energy QCD amplitudes based on the diagrams constructed entirely in terms of Reggeon fields and their vertices of interactions. The completeness of the correction to the propagator of reggeized gluon, calculated in [9], requires the knowledge of the vertices of the interaction of three Reggeon fields to one QCD loop precision. In the paper we calculated the $A_+ A_+ A_- \nu$ vertex with all loop LLA precision, see Eq. (26) expression which is the main result of the article.

The bare QCD value of the vertex of interests also was calculated in [9], Writing precise one QCD loop expression for the vertex, see Eq. (19), we can put attention that the many loops contribution to the vertex are reproduced by iterations expressed finally in the form of Bethe-Salpeter equation. The solution of this equation, Eq. (23), have a form of reggeized vertex with the trajectory twice larger than the trajectory of the Reggeon’s propagator, this is a new and unexpected result of the calculations. The high energy QCD reformulated as RFT, therefore, becomes non-local in rapidity.
space with both vertices and propagators as functions of rapidity intervals. Of course, this is result of the addition of the Lipatov’s effective currents to the pure QCD Lagrangian, whereas these currents are absent these non-local terms disappear as well. Also, we note, that the calculated terms describe the high-energy asymptotic behavior of the theory, there are additional QCD type contributions to any functions of interests which provide sub-leading corrections to both vertices and propagators.

We also note, that in general the Dyson-Schwinger hierarchy for the vertices of the formalism exists as well, similarly to the hierarchy of the theory’s correlators obtained in [8]. The derivation of this system of equations, as well as calculation of the $A_A A_A A_+$ vertex to one QCD loop precision will allow to determine the next leading order non-linear corrections to the propagator of reggeized gluon, which is important task in high energy QCD. Also, the next important step to be considered is the calculation of the BFKL Pomeron on the base of new expression for the reggeized gluons propagator, see [8]. Indeed, the infrared divergence of obtained in [9] propagator is different from the divergence of the usual propagator’s trajectory function, the situation will be even worse when the both triple vertices will be included in the answer with one QCD loop precision. Additionally, an interesting question arises about the possible reggeization of the four Reggeon vertex in the framework of the theory. Therefore, the interesting subjects of the future research are the non-linear corrections to the Reggeons propagator and Pomeron calculated in the framework with possible reggeization of the vertices of the theory included. For example, the very interesting question to investigate is about the form and rapidity dependence of this modified Pomeron.

In conclusion we emphasize, that the article is considered as an additional step to the developing of the high energy QCD RFT which will help clarify the non-linear unitarity corrections to the amplitudes of high energy processes.
Appendix A: Bare gluon propagator in light-cone gauge

In order to find the expression for the gluon fields bare propagator in the light-cone gauge we solve the following system of equations

\[ M^{0 \mu \nu} G_{0 \nu \rho} = \delta^\mu_\rho \]  

(A.1)

with

\[ g^\nu_\nu = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mu, \nu = (+, -, \perp), \]  

(A.2)

see also Eq. (A.21) below. The expression for the \( M_{0 \mu \nu} \) matrix can obtained from the bare gluon’s Lagrangian for the gluon’s fluctuations field, in light-cone gauge it has the following form:

\[ L_0 = -\frac{1}{2} \varepsilon_i^a \delta_{ab} (\delta_{ij} \Box + \partial_i \partial_j) \varepsilon_j^b + \varepsilon_i^a \partial_+ \partial_i \varepsilon_i^a - \frac{1}{2} \varepsilon_i^a \partial_-^2 \varepsilon_i^a = -\frac{1}{2} \varepsilon_i^a M_{0 \mu \nu} \varepsilon_j^b \delta_{ab}, \]  

(A.3)

In the following system of equations

\[ M_0^{i+} G_{0+j} + M_0^{ik} G_{0kj} = \delta_j^i \]  

\[ M_0^{-i+} G_{0+i} + M_0^{-ik} G_{0kj} = \delta^i_+ \]  

\[ M_0^{i+} G_{0ij} + M_0^{++} G_{0+j} = 0 \]  

\[ M_0^{p+} G_{0p+} + M_0^{++} G_{0++} = 0, \]  

(A.4)

the last two equations we can consider as definitions of corresponding Green’s functions:

\[ G_{0+i} = -M_{0++}^{-1} M_0^{+j} G_{0ji}, \]  

(A.5)

and

\[ G_{0i+} = -M_{0++}^{-1} M_0^{+i} G_{0+i}. \]  

(A.6)

Here for

\[ M_{0pj} = \delta_{pj} \Box + \partial_p \partial_j, \quad M_{0p-} = -\partial_p \partial_-, \quad M_{0-} = \partial_-^2 \]  

(A.7)

we have

\[ M_{0ij}^{-1}(x, y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \left( \delta_{ij} - \frac{p_i p_j}{2(p_+ p_-)} \right), \]  

(A.8)

and correspondingly

\[ M_{0+i}^{-1}(x, y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2}. \]  

(A.9)

Therefore, for the two remaining Green’s functions we obtain:

\[ \left( M_0^{++} - M_0^{+i} M_{0ij}^{-1} M_0^{+j} \right) G_{0++} = \delta^+_+, \]  

(A.10)

and

\[ \left( M_0^{ik} - M_0^{i+} M_{0++}^{-1} + M_0^{++} \right) G_{0kj} = \delta^i_j. \]  

(A.11)

Performing Fourier transform of the functions, we write the Eq. (A.11) in the following form:

\[-\int \frac{d^4p}{(2\pi)^4} \left( \delta^{ik} p_2^2 + p_i^p p_k^p \right) e^{-ip(x-y)} \tilde{G}_{0kj}(p) + \int \frac{d^4p}{(2\pi)^4} p_2^2 p_i^p p_k^p \frac{e^{-ip(x-y)}}{p_2^2} \tilde{G}_{0kj}(p) = \delta^i_j \]  

(A.12)

We suppress color and coordinate notations in the definition of the propagators below.
that provides:

$$G_{0ij}(x, y) = - \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \delta_{ij}.$$  \hspace{1cm} (A.13)

Correspondingly, for Eq. (A.9) we have:

$$- \int \frac{d^4p}{(2\pi)^4} p_+^2 e^{-ip(x-y)} \tilde{G}_{0++}(p) + \int \frac{d^4p}{(2\pi)^4} \frac{p_+^2 p_-^j}{p^2} \left( \delta_{ij} - \frac{p_i p_j}{2(p_- p_+)} \right) e^{-ip(x-y)} \tilde{G}_{0++}(p) = \delta_+^+,$$

that can be rewritten as

$$\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \left( -p_-^2 + \frac{p_+^i p_-^i}{p^2} - \frac{p_-^2}{2p^2} \left( \frac{p_+^i p_-^i}{p^2} \right) \right) \tilde{G}_{0++}(p) = \delta_+^+.$$  \hspace{1cm} (A.14)

Writing this expression as

$$\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \left( -p_-^2 - \frac{p_+^i p_-^i}{p^2} - \frac{p_-^2}{2p^2} \left( \frac{p_+^i p_-^i}{p^2} \right) \right) \tilde{G}_{0++}(p) = \delta_+^+.$$  \hspace{1cm} (A.15)

we obtain finally for the Green’s function

$$G_{0++}(x, y) = - \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \frac{2p_+}{p_-}.$$  \hspace{1cm} (A.17)

Inserting Eq. (A.13) and Eq. (A.17) functions in Eq. (A.5)-Eq. (A.6) definitions we obtain for the last two Green’s functions:

$$G_{0i+} = G_{0+i} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \frac{p_i}{p_-}.$$  \hspace{1cm} (A.18)

Now, introducing the following vector in light-cone coordinates

$$n_\mu = (1, 0, 0), \quad \mu = (+, -, \perp)$$  \hspace{1cm} (A.19)

we can write the whole propagator as

$$G_{0\mu\nu}(x, y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2} \left( g_{\mu\nu} - g_{\mu\sigma} g_{\nu\rho} \frac{p_+^\sigma n_-^\rho + p_-^\rho n_+^\sigma}{p^2 n_+^\rho} \right).$$  \hspace{1cm} (A.20)

with

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \mu, \nu = (+, -, \perp)$$  \hspace{1cm} (A.21)

and where Kogut-Soper convention, [], for the light-cone notations and scalar product is used:

$$px = p_+ x^+ + p_- x^- + p_i x^i.$$  \hspace{1cm} (A.22)
Appendix B: Lipatov’s effective current

For the arbitrary representation of $v_+ = i T^a v_+^a$ with $D_+ = \partial_+ - g v_+$, we can consider the following representation of $O$ and $O^T$ operators:

$$O_x = \delta^{ab} + g \int d^4 y G_{zy}^{+a_1 a} (v_+(y))_{a_1 b} = 1 + g G_{xy}^+ v_+$$

(B.1)

and correspondingly

$$O^T_x = 1 + g v_+ v_y^+$$

(B.2)

which is redefinition of the operator expansions used in [3] in terms of Green’s function instead of integral operators, see Appendix B above. The Green’s function in above equations we understand as Green’s function of the $D_+$ operator and express it in the perturbative sense as:

$$G_{xy}^+ = G_{xy}^{+0} + g G_{xy}^{+0} v_+ G_{zy}^+$$

(B.3)

and

$$G_{yx}^+ = G_{yx}^{+0} + g G_{yx}^{+0} v_+ G_{xx}^{+0},$$

(B.4)

with the bare propagators defined as (there is no integration on index $x$ in expressions)

$$\partial_{+x} G_{xy}^{+0} = \delta_{xy}, \quad G_{yx}^{+0} \partial_{+x} = -\delta_{xy}.$$  

(B.5)

The following properties of the operators now can be derived:

1. 

$$\delta G_{xy}^+ = g G_{xx}^{+0} (\delta v_+) + G_{xx}^{+0} v_+ \delta G_{xy}^+ = g G_{xx}^{+0} (\delta v_+) G_{zy}^+ + G_{xx}^{+0} v_+ (\delta G_{xy}^+) D_+ + D_+ G_{xy}^+ =$$

$$= g (G_{xx}^{+0} (\delta v_+) G_{zy}^+ - G_{xx}^{+0} v_+ G_{zy}^+ (\delta D_+)) + g (G_{xx}^{+0} v_+ G_{zy}^+ (\delta D_+)) =$$

$$= g G_{xp}^+ \delta v_+ G_{py}^+.$$  

(B.6)

2. 

$$\delta O_x = g G_{xy}^+ (\delta v_+) + g G_{xp}^+ (\delta v_+) v_+ = g G_{xp}^+ \delta v_+ (1 + g G_{py}^+ v_+) = g G_{xp}^+ \delta v_+ O_p;$$  

(B.7)

3. 

$$\partial_{+x} \delta O_x = g \left( \partial_{+x} G_{xp}^+ \right) \delta v_+ O_p = g \left(1 + g v_+ G_{xp}^+ \right) \delta v_+ O_p = g O_x^T \delta v_+ O_x;$$  

(B.8)

4. 

$$\partial_{+x} O_x = g \left( \partial_{+x} G_{xy}^+ \right) \delta v_+ v_+ = g v_+ \left(1 + g G_{xy}^+ v_+ \right) = g v_+ O_x;$$  

(B.9)

5. 

$$O_x^T \partial_{+x} = g v_+ \left( G_{yx}^{+} \partial_{+x} \right) = -g \left(1 + v_+ G_{yx}^{+} \right) v_+ = -g O_x^T v_+.$$  

(B.10)

We see, that the operator $O$ and $O^T$ have the properties of ordered exponents. For example, choosing bare propagators as

$$G_{xy}^{+0} = \theta (x^+ - y^+) \delta_{xy}^3, \quad G_{yx}^{+0} = \theta (y^+ - x^+) \delta_{xy}^3,$$

(B.11)

Due the light cone gauge we consider here only $O(x^+)$ operators. The construction of the representation of the $O(x^-)$ operators can be done similarly. We also note, that the integration is assumed for repeating indexes in expressions below if it is not noted otherwise.
we immediately reproduce:

\[
O_x = P e^{\frac{1}{g} \int_{-\infty}^{x^+} dx^+ v_+(x^+)} , \quad O_x^T = P e^{\frac{1}{g} \int_{x^+}^{\infty} dx^+ v_+(x^+)}.
\] (B.12)

The form of the bare propagator \( G_{xy}^{\pm 0} = \frac{1}{2} \left[ \theta(x^+ - y^+) - \theta(y^+ - x^+) \right] \delta_3^{\pm 0} \) will lead to the more complicated representations of \( O \) and \( O^T \) operators, see in [1] and [3]. We note also that the Green’s function notation \( \tilde{G}_{x^\pm y^\pm} \) in the paper is used for the designation of the only theta function part of the full \( G_{x^\pm y^\pm} \) Green’s function.

Now we consider a variation of the action’s full current:

\[
\delta \text{tr} \left[ v_{++} O_x \partial_i^2 A^+ \right] = \frac{1}{g} \delta \text{tr} \left[ (\partial_{++} O_x) \partial_i^2 A^+ \right] = \frac{1}{g} \text{tr} \left[ (\partial_{++} \delta O_x) \partial_i^2 A^+ \right] = \text{tr} \left[ O_x^T \delta v_{++} O_x \left( \partial_i^2 A^+ \right) \right],
\] (B.13)

which can be rewritten in the familiar form used in the paper:

\[
\delta (v_+ J^+) = \delta \text{tr} \left[ (v_{++} O_x \partial_i^2 A^+) \right] = -\delta v_+^a \text{tr} \left[ T_a O T_b O^T \right] \left( \partial_i^2 A^+_b \right).
\] (B.14)

We also note, that with the help of Eq. (A.3) representation of the \( O \) operator the full action’s current can we written as follows

\[
\text{tr} \left[ (v_{++} O_x - A_+) \partial_i^2 A^+ \right] = \text{tr} \left[ (v_+ - A_+ + v_{++} G_{xy}^+ v_{++} + v_{++} G_{xy}^+ v_{++}) \left( \partial_i^2 A^+ \right) \right].
\] (B.15)
Appendix C: NLO vertex of interactions of reggeized gluons

The NLO one-loop vertex of reggeized gluons interactions is defined in the formalism as

\[-2i K^{a b}_{x y 1} = \left( \frac{\delta^2 \ln (1 + G_0 M)}{\delta A^a_{\perp x} \delta A^b_{\perp y}} \right)_{A_+, A_-, v_f \perp = 0} = \]

\[= \left[ G_0 \frac{\delta^2 M}{\delta A^a_{\perp x} \delta A^b_{\perp y}} (1 + G_0 M)^{-1} - G_0 \frac{\delta M}{\delta A^b_{\perp y}} (1 + G_0 M)^{-1} G_0 \frac{\delta^2 M}{\delta A^a_{\perp x}} (1 + G_0 M)^{-1} \right]_{A_+, A_-, v_f \perp = 0} \quad (C.1)\]

where the trace of the expression is assumed. With the help of Eq. (13), see also [4], we have correspondingly:

\[-2i K^{a b}_{x y 1} = \left[ G_0 \frac{\delta^2 M}{\delta A^a_{\perp x} \delta A^b_{\perp y}} - G_0 \frac{\delta M}{\delta A^b_{\perp y}} G_0 \frac{\delta^2 M}{\delta A^a_{\perp x}} \right]_{A_+, A_-, v_f \perp = 0}. \quad (C.2)\]

Taking into account the asymptotically leading contributions of \(g^2\) order, that means the \(M_L\) term presence in the expressions, see [4][25], we obtain:

\[-2i K^{a b}_{x y 1} = \left[ G_0 \frac{\delta^2 M_L}{\delta A^a_{\perp x} \delta A^b_{\perp y}} - G_0 \frac{\delta M_L}{\delta A^b_{\perp y}} G_0 \frac{\delta M_L}{\delta A^a_{\perp x}} \right]_{A_+, A_-, v_f \perp = 0}. \quad (C.3)\]

For the first term we have:

\[-2i K^{a b}_{x y 1, 1} = G_0 \frac{\delta^2 M_L}{\delta A^a_{\perp x} \delta A^b_{\perp y}} = G_0^{z+} \frac{g}{N} \frac{\delta (U^{cd}_{1a})}{\delta A^a_{\perp x}} \frac{\delta A^d_{\perp y}}{(C.4)}\]

where the following identity was used:

\[\partial_a \partial_\perp \rho^i_a = -\frac{1}{N} \delta^a_i \partial^2 A^a, \quad (C.5)\]

see Eq. (13) and [4]. Using the following expressions

\[\frac{\delta (U^{cd}_{1a})}{\delta A^a_{\perp x}} = g \left( U^{cd}_{1a} \right)^{++} - \frac{\delta \eta^{a1 cl}_{cl}}{\delta A_{\perp x}} \quad (C.6)\]

and

\[\frac{\delta \eta^{a1 cl}_{cl}}{\delta A^a_{\perp x}} = \delta^{a1} (\partial^2_{x_{\perp} w_{\perp} x_{\perp} w_{\perp}}) \quad (C.7)\]

to requested accuracy, we obtain for Eq. (C.4):

\[-2i K^{a b}_{x y 1, 1} = \frac{g^2}{N} G_0^{z+} \left( U^{cl\perp}_{2} \right)^{++} \left( \delta^{a1} \partial^2_{x_{\perp} w_{\perp} x_{\perp} w_{\perp}} \right) \left( \delta^2_{y_{\perp} z_{\perp} y_{\perp} z_{\perp}} \right) \left( \delta^2_{z_{\perp} z_{\perp}} \right), \quad (C.8)\]

where the NNLO term of the Lipatov’s current series expansion reads as

\[\left( \left( U^{cl\perp}_{2} \right)^{++} \right)_{A_+, A_- = 0} = \frac{1}{2} N^2 \delta^{ab} \left[ \left( G^{+0}_{zw} G^{+0}_{wz} + G^{+0}_{tw} G^{+0}_{wz} \right) + \right.\]

\[\left. + 2 \left( G^{+0}_{tw} G^{+0}_{tw} + G^{+0}_{lz} G^{+0}_{lz} + G^{+0}_{zw} G^{+0}_{lw} + G^{+0}_{tw} G^{+0}_{lw} \right) \right]. \quad (C.9)\]

Therefore, writing explicitly all integrations in the expression, we obtain:

\[-2i K^{a b}_{x y 1, 1} = \frac{1}{2} g^2 N \delta^{ab} \int d^4 z d^4 t d^4 w \left( \left( \partial^2_{z_{\perp} t_{\perp} w_{\perp} z_{\perp} w_{\perp}} \right) \left( \delta^2_{y_{\perp} z_{\perp} y_{\perp} z_{\perp}} \right) \right) \left( \delta^2_{z_{\perp} z_{\perp}} \right) \left( \delta^2_{z_{\perp} z_{\perp}} \right) \cdot \left[ \left( G^{+0}_{zw} G^{+0}_{wz} + G^{+0}_{tw} G^{+0}_{wz} \right) + 2 \left( G^{+0}_{tw} G^{+0}_{tw} + G^{+0}_{lz} G^{+0}_{lz} + G^{+0}_{zw} G^{+0}_{lw} + G^{+0}_{tw} G^{+0}_{lw} \right) \right]. \quad (C.10)\]
Formally, there are three additional terms present in Eq. (C.2). The first one
\[-2i K^{ab}_{xy, 1, 2} = -G_{0^+} \frac{\delta M_L}{\delta A^{-y}_y} G_{0^+} \frac{\delta M_{1^+}}{\delta A^+_x}, \tag{C.11} \]
the second one
\[-2i K^{ab}_{xy, 1, 3} = -G_{0^i} \frac{\delta M_L}{\delta A^{-y}_y} G_{0^+} \frac{\delta M_{1^-}}{\delta A^+_x}, \tag{C.12} \]
and the third one
\[-2i K^{ab}_{xy, 1, 4} = -G_{0^i} \frac{\delta M_L}{\delta A^{-y}_y} G_{0^+} \frac{\delta M_{1^i}}{\delta A^+_x}. \tag{C.13} \]
Nevertheless, only the third one contributes to the kernel in the limit of zero Reggeon fields, we have
\[\frac{\delta M^{cd}_{\text{reg}}}{\delta A^x} = \frac{g}{N} \left( \left( U^{\text{reg}}_{cd} \right)^+ \right)_{\delta\beta\gamma\delta\gamma} \frac{\partial^2}{\partial^2 t} \tag{C.14} \]
where
\[\left( \left( U^{\text{reg}}_{cd} \right)^+ \right)_{\delta\beta\gamma\delta\gamma} = \frac{1}{2} N f_{cd} (G^+_{lw} - G^+_{ut}) \tag{C.15} \]
Also we have:
\[\frac{\delta M^{dc}_{\text{reg}}}{\delta A^x} = 2 g f_{dc} \delta_{y} \delta z \frac{\partial^2}{\partial^2 t} \tag{C.16} \]
The final expression for this term reads, therefore, as:
\[-2i K^{ab}_{xy, 1, 4} = -g^2 N \frac{\delta^2}{8\pi} \int d^4t d^4w d^4z \left( G^{+}_{lw} - G^{+}_{ut} \right) \frac{\partial^2}{\partial^2 t} \frac{\partial^2}{\partial^2 z} e^{-iz(x-\eta)} \tag{C.17} \]
We notice that both Eq. (C.17) and Eq. (C.10) contributions are precisely the same as obtained in [4] paper. Therefore, we immediately write the full contribution from [4] which is
\[K^{ab}_{xy, 1} = -\frac{g^2 N}{8\pi} \frac{\partial^2}{\partial^2 t} \left( \int \frac{dp}{2(\pi)^2} \left( \frac{d^2k}{2(\pi)^2} \frac{k^2}{p^2} \right) \right) e^{-iz(x-\eta)} \tag{C.18} \]
We can rewrite this expression redefining the vertex as
\[K^{ab}_{xy, 1} \rightarrow K^{ab}_{xy, 1} \delta_{y} \frac{\partial^2}{\partial^2 t} \tag{C.19} \]
with
\[\tilde{K}(p, \eta) = -\frac{N \pi g^2}{2} \delta(p^+) \delta(p^-) \int_0^\eta d\eta' \int \frac{d^2k}{(2\pi)^2} \frac{k^2}{p^2} \frac{p^2}{(p^- - k^-)^2} \tag{C.20} \]
where the physical cut-off \( \eta \) in rapidity space \( y = \frac{1}{2} \ln(A k^-) \) is introduced. Now, introducing the bare propagator of the Reggeon as
\[D^{ab}_{+0}(x, y) = D^{ab}_{0}(x, y) = \delta^{ab} \frac{\partial^2}{\partial^2 z} e^{-iz(x-\eta)} \tag{C.21} \]
the full propagator to the leading order precision can be written as the following equation
\[D^{ac}_{xz} = D^{ac}_{xz0} - \int d^4z \int d^4w \left( \partial^2_{x} D^{ab}_{x0} \right) \frac{k^2}{k^2} e^{-iz(x-\eta)} \tag{C.22} \]
Introducing
\[D^{ac}_{xy} = \delta^{ac} \delta(y^- - x^-) \delta(x^+ - y^+) \int \frac{d^2p}{(2\pi)^2} \tilde{D}(p, \eta) e^{-iz(x-\eta)} \tag{C.23} \]
we obtain finally:

\[ \tilde{D}^{ab}(p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} + \epsilon(p_\perp^2) \int_0^\eta d\eta' \tilde{D}^{ab}(p_\perp, \eta') \]  

(C.24)

with

\[ \epsilon(p_\perp^2) = -\frac{\alpha_s N}{4\pi^2} \int d^2k_\perp \frac{p_\perp^2}{k_\perp^2 (p_\perp - k_\perp)^2} \]  

(C.25)

as trajectory of the propagator of reggeized gluons. Rewriting this equation as the differential one:

\[ \frac{\partial \tilde{D}^{ab}(p_\perp, \eta)}{\partial \eta} = \tilde{D}^{ab}(p_\perp, \eta) \epsilon(p_\perp^2) \]  

(C.26)

we obtain the finally the propagator:

\[ \tilde{D}^{ab}(p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} e^{\eta \epsilon(p_\perp^2)} , \]  

(C.27)

with \( \eta \) defined in some rapidity interval \( 0 < \eta < Y = \ln(s/s_0) \) of interest; it is the BFKL propagator for reggeized gluons, see [14].
Appendix D: NLO vertex of interactions of $A_+ A_+ A_-$ Reggeon fields

In the following calculations we omit the production fields in the expressions taking them equal to zero from the beginning. We also do not denote but mean zero limit of the all Reggeon fields in the end of the functional derivatives calculations, also the trace in the expressions is assumed. Therefore, for the NLO (one loop or $g^3$ order) vertex of interaction of $A_+ A_+ A_-$ Reggeon fields we have:

$$- 2 \Gamma_{xy}^{abc} = \frac{\delta^3 \ln (1 + G_0 M)}{\delta A^a_x \delta A^b_y \delta A^c_z} = G_0 \frac{\delta^3 M}{\delta A^a_x \delta A^b_y \delta A^c_z} - G_0 \frac{\delta^2 M}{\delta A^a_x \delta A^b_y} - G_0 \frac{\delta M}{\delta A^a_x} - G_0 \frac{\delta M}{\delta A^c_z} +$$

$$+ G_0 \frac{\delta M}{\delta A^b_y} G_0 \frac{\delta M}{\delta A^a_x} G_0 \frac{\delta M}{\delta A^c_z}.$$  \hspace{1cm} (D.1)

Similarly to the done in the previous Appendix, we keep in the Eq. (D.1) expression only terms which are arising from the $M_L$ term in Eq. (10) and which provide leading asymptotic contributions:

$$- 2 \Gamma_{xy}^{abc} = \frac{\delta^3 \ln (1 + G_0 M)}{\delta A^a_x \delta A^b_y \delta A^c_z} = G_0 \frac{\delta^3 M_L}{\delta A^a_x \delta A^b_y \delta A^c_z} - G_0 \frac{\delta^2 M_L}{\delta A^a_x \delta A^b_y} G_0 \frac{\delta M_1}{\delta A^c_z} +$$

$$+ G_0 \frac{\delta M_L}{\delta A^b_y} G_0 \frac{\delta M_1}{\delta A^a_x} G_0 \frac{\delta M_1}{\delta A^c_z}.$$  \hspace{1cm} (D.2)

Further we consider all relevant terms one by one.

First contribution

For the first term in the r.h.s. of Eq. (D.2) we obtain:

$$- 2 \Gamma_{xy}^{abc} = G_0 \frac{\delta^3 M_L}{\delta A^a_x \delta A^b_y \delta A^c_z} = G_0 \frac{\delta M}{\delta A^a_x} G_0 \frac{\delta M}{\delta A^b_y} G_0 \frac{\delta M}{\delta A^c_z}.$$  \hspace{1cm} (D.3)

see Eq. (10) in the previous Appendix. Now, using Eq. (C.6) expression, we have:

$$\frac{\delta^2 (U_1^{bda})_{pt}^{++}}{\delta A^a_{+x} \delta A^b_{+y}} = g \frac{\delta^2 (U_2^{bda})_{ptw}^{++}}{\delta A^a_{+x}} \frac{\delta v_{+w}^{a_1c}}{\delta A^b_{+y}} + g \frac{\delta^2 v_{+w}^{a_1c}}{\delta A^a_{+x} \delta A^b_{+y}}.$$  \hspace{1cm} (D.4)

The only first term of Eq. (D.4) will remain in the limit of the zero Reggeon fields, see the NLO value of $v_{+}^{c}$ in (27), therefore:

$$\frac{\delta^2 (U_1^{bda})_{pt}^{++}}{\delta A^a_{+x} \delta A^b_{+y}} = g^2 \frac{\delta^2 (U_2^{bda})_{ptw}^{++}}{\delta A^a_{+x} \delta A^b_{+y}} \frac{\delta v_{+w}^{a_1c}}{\delta A^a_{+x}} \frac{\delta v_{+w}^{a_2c}}{\delta A^b_{+y}}.$$  \hspace{1cm} (D.5)

where in the expression

$$\left(U_3^{abcdef} \right)^{+++} = \sum tr [f_{[a} G^+ f_e G^+ f_c G^+ f_d] O_f O^T],$$  \hspace{1cm} (D.6)
the sum is performed on the permutations of the *a c d e* indexes, see Eq. (5) and Appendix A for the derivation of the expression. There are the following traces of the color matrices in the adjoint representation we need to know. The first one is the following one:

\[
C_{cdab}^5 = \text{tr}[\mathbf{f} c \mathbf{f} d \mathbf{f} c \mathbf{f} a \mathbf{f} b] = \text{tr}[\mathbf{f} c \mathbf{f} d \mathbf{f} c \mathbf{f} a \mathbf{f} b] - f_{cd} \text{tr}[\mathbf{f} c \mathbf{f} a \mathbf{f} b] = -N \text{tr}[\mathbf{f} d \mathbf{f} a \mathbf{f} b] - f_{cd} \text{tr}[\mathbf{f} c \mathbf{f} a \mathbf{f} b] =
\]

\[
= -\frac{N^2}{2} f_{dab} - f_{cd} \left( \delta_{ci} \delta_{ae} + \frac{1}{2} (\delta_{ci} \delta_{ab} + \delta_{ci} \delta_{de}) + \frac{N}{4} (f_{cbe} f_{eaa} + d_{cbe} d_{eaa}) \right) =
\]

\[
= \frac{N^2}{2} f_{dab} - f_{dab} + \frac{1}{2} f_{dab} - \frac{N}{4} f_{cd} f_{cbe} f_{eaa} - \frac{N}{4} f_{cd} d_{cbe} d_{eaa},
\]

(D.7)

here \(d\) is fully symmetric tensor:

\[
d_{abc} = 2 \text{tr} \{[T^a, T^b] T^c\}
\]

(D.8)

with \(T^a\) as a color matrix in a fundamental representation. Now, using the following identities:

\[
f_{dc} \text{tr} f_{cbe} f_{eaa} = -\frac{1}{2} N f_{dab}
\]

(D.9)

and

\[
f_{dc} d_{cbe} d_{eaa} = \left( \frac{N^2 - 4}{2N} \right) f_{dab},
\]

(D.10)

see [11], we obtain finally for the factor:

\[
C_{cdab}^5 = \text{tr}[\mathbf{f} c \mathbf{f} d \mathbf{f} c \mathbf{f} a \mathbf{f} b] = \frac{N^2}{4} f_{dab}.
\]

(D.11)

An another color factor we need is the following one:

\[
C_{cdab}^5 = \text{tr}[\mathbf{f} c \mathbf{f} d \mathbf{f} c \mathbf{f} a \mathbf{f} b] = \frac{N^2}{2} f_{dab}.
\]

(D.12)

Therefore, we obtain for the Eq. (D.6) expression:

\[
\left( U_{3a}^{blda_1a_2} \right)^{+++} = C_{ba_2ba_1d}^{5} \left[ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
- G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} + C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
\left. + G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right),
\]

(D.13)

or

\[
\left( U_{3a}^{blda_1a_2} \right)^{+++} = C_{ba_2ba_1d}^5 \left( G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right) + \right.
\]

\[
+ G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} - G_{w_1}^{+0} G_{w_1}^{+0} G_{w_1}^{+0} \right),
\]

(D.14)
In order to reduce the complexity of the calculations we note the following. For the symmetrical with respect to \( k_- \) (rapidity) contributions we can restrict the integrals only by positive values of \( k_- \). Namely, regularizing the integral over the \( k_- \) we in general obtain the following type of expressions

\[
\int \frac{dk_-}{k_-} I \rightarrow \left( \int_{e}^{1/e} \frac{dk_-}{k_-} I_1 + \int_{-1/e}^{-e} \frac{dk_-}{k_-} I_2 \right),
\]  

(D.15)

where \( I_i \) is a corresponding expressions obtained by integration in respect to other coordinates. Now, taking into account that for the large part of the contributions the following condition holds

\[
I_1 = -I_2, \tag{D.16}
\]

which is related to the different directions of the integration contours in the complex plane of \( k_- \) variable, we use the following regularization of \( k_- \) integrals:

\[
\int \frac{dk_-}{k_-} I \rightarrow \frac{1}{2} \left( \int_{e}^{1/e} \frac{dk_-}{k_-} - \int_{-1/e}^{-e} \frac{dk_-}{k_-} \right) I_1 = \int_{\Lambda}^{k_{\text{max}}} \frac{dk_-}{k_-} I_1. \tag{D.17}
\]

Therefore, in general, for the contributions which are symmetrical with respect to the \( k_- \) momenta in the sense of Eq. (D.16), we can restrict the integrals over \( k_- \) only by positive values introducing the rapidity variable as \( y = \frac{1}{2} \ln \left( \frac{k_+}{\Lambda} \right) \) and obtaining

\[
\int \frac{dk_-}{k_-} I = \int_{\Lambda}^{k_{\text{max}}} \frac{dk_-}{k_-} I_1 = 2 \int_0^\infty dy \, I_1 \tag{D.18}
\]

with \( \eta \) as ultraviolet cut-off related to the value of the particle’s cluster in the effective action approach. Consequently, we notice that the integrals which consist of \( \theta_{pt}^+ \) function and have no any singularities in integration with respect to the \( + \) components of the coordinates are zero due the fact that

\[
G_{0++} \propto \int dk_+ \frac{e^{-ik_+ \left( t^- - p^- \right)}}{k_+ - k_0^2/2k_- + i\varepsilon} \propto \theta(t^+ - p^+). \tag{D.19}
\]

There are the following terms which are zero because of that reason: \( G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+ \), \( G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+ \) and \( G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+, G_{ptw1}^+ \) and corresponding terms obtained after the \( w \equiv w_1 \) substitution. We also note, that after the integration with respect to all delta functions in Eq. (D.15) and \( G^{+0} \) functions, the remaining answer will depend only on transverse \( \delta_{y_1 x_1}^2 \) and \( \delta_{y_1 z_1}^2 \) functions. Therefore, further, for the shortening of notations, we will use \( G^{+0} \) functions as if it equivalent to the theta functions, remembering that all delta functions are remain in the final expressions with any number of theta functions \( (G^{+0} \) functions) in them. There are the following remaining terms we have to account:

\[
\left( \frac{t_3^{\text{old}a_2 \text{old}a_2}}{ptw_1} \right)^{+++} = -f_{a_1 a_2} \frac{N^2}{4} \left( 2 \left( G_{ptw1}^0 + G_{ptw1}^0 + G_{ptw1}^0 \right) + 2 G_{ptw1}^0G_{ptw1}^0 \right) + 2 G_{ptw1}^0G_{ptw1}^0 + G_{ptw1}^0G_{ptw1}^0 + G_{ptw1}^0G_{ptw1}^0 + G_{ptw1}^0G_{ptw1}^0 - \left( w \equiv w_1 \right). \tag{D.20}
\]

We used and will use here the following identity for theta functions:

\[
\theta_{xy}^+\theta_{yz}^+ = \theta_{xz}^+ - \theta_{zx}^+ - \theta_{xy}^+ - \theta_{yz}^+. \tag{D.21}
\]

and correspondingly we obtain:

\[
\left( \frac{t_3^{\text{old}a_2 \text{old}a_2}}{ptw_1} \right)^{+++} = -f_{a_1 a_2} \frac{N^2}{4} \left( 2 G_{ptw1}^0 G_{ptw1}^0 - 2 G_{ptw1}^0 G_{ptw1}^0 G_{ptw1}^0 + 2 G_{ptw1}^0 G_{ptw1}^0 \right) - 2 G_{ptw1}^0 G_{ptw1}^0 G_{ptw1}^0 + 2 G_{ptw1}^0 G_{ptw1}^0 G_{ptw1}^0 + 2 G_{ptw1}^0 G_{ptw1}^0 - \left( w \equiv w_1 \right). \tag{D.22}
\]
Again using Eq. (D.21) identity and preserving only non-zero terms we write Eq. (D.22) in the following form:
\[
\left(U_{a1}^{babb}a_2\right)_{ptw,w_1}^{+++} = - f_{a1} a_2 \frac{N^2}{4} \left(2 G_{w_{p1}}^{+0} G_{w_{1}w}^{+0} + 2 G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} + 2 g_{w_{1}t} G_{t_{w1}}^{+0} + 2 g_{w_{1}p} G_{p_{w1}}^{+0} - 
2 G_{w_{1}t}^{+0} G_{t_{w1}}^{+0} - 2 G_{w_{1}p}^{+0} G_{p_{w1}}^{+0} + G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} + G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} - 
G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} G_{t_{w1}}^{+0} G_{t_{w1}}^{+0} - G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} G_{t_{w1}}^{+0} G_{t_{w1}}^{+0} - G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} - (w = w_{1})\right). \quad (D.23)
\]

The contributions of all integrals in Eq. (D.23) expression are proportional to the same tadpole integral and we consider contributions of different terms in Eq. (D.23) one by one calculating the overall coefficient of the expression.

1. The first four terms in Eq. (D.23) provide together zero contribution to Eq. (D.3):
\[
I_{1-4} = - 16 \pi i k_{-} \theta(w_{1} - w) \int \frac{d^2 k_{\perp}}{k_{\perp}^2}, \quad (D.24)
\]

with only two first terms contributing to the answer.

2. The next term, \(G_{w_{1}t}^{+0} G_{t_{w1}}^{+0} G_{t_{w1}}^{+0}\), we rewrite as following:
\[
G_{w_{1}t}^{+0} G_{t_{w1}}^{+0} G_{t_{w1}}^{+0} = G_{w_{1}t}^{+0} G_{t_{w1}}^{+0} - G_{w_{1}t}^{+0} G_{t_{w1}}^{+0} G_{t_{w1}}^{+0}, \quad (D.25)
\]

the integration on \(k_{\perp}\) variable gives zero for these contributions as well.

3. The next term we consider is the \(- 2 G_{t_{w1}}^{+0} G_{w_{1}p}^{+0} G_{p_{w1}}^{+0}\) expression. We have after the integration on + components of coordinates:
\[
2 \int \frac{d k_{\perp}}{k_{\perp}^2} \frac{1 - e^{-ik_{+}(w_{1}^{+} - w^{+})}}{(k_{+} - i \varepsilon)} \left(k_{+} - k_{\perp}^2/2 k_{-} + i \varepsilon\right) = - 4 \pi i k_{-} \frac{2k_{+}}{k_{\perp}^4} \left(1 - e^{-\frac{k_{+}^2}{2} (w_{1}^{+} - w^{+})}\right) \theta(w_{1}^{+} - w^{+}). \quad (D.26)
\]

that, following to ’t Hooft-Veltman conjecture, see [34], gives zero final contribution after an integration on transverse momenta.

4. The terms \(G_{w_{1}w}^{+0} G_{w_{w1}}^{+0}\) and \(G_{w_{1}w}^{+0} G_{w_{w1}}^{+0}\) after the integration on \(k_{\perp}\) equal to
\[
8 \pi i k_{-} \theta(w_{1} - w) \int \frac{d^2 k_{\perp}}{k_{\perp}^2}. \quad (D.27)
\]

5. The \(- G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} G_{w_{w1}}^{+0}\) expression in Eq. (D.23) is proportional to the following integral:
\[
\int \frac{d^2 k_{\perp}}{k_{\perp}^2} \int \frac{d k_{\perp}}{k_{\perp}^2} \frac{e^{-ik_{+}(w_{1}^{+} - w^{+})}}{(k_{+} - i \varepsilon)} \left(k_{+} - k_{\perp}^2/2 k_{-} + i \varepsilon\right) = - 2 \pi i k_{-} \frac{2k_{+}}{k_{\perp}^4} \theta(w_{1} - w) e^{-\frac{k_{+}^2}{2} (w_{1}^{+} - w^{+})}, \quad (D.28)
\]

Expanding the exponential and keeping only non-zero in the sense of ’t Hooft-Veltman conjecture terms, we obtain for this contribution:
\[
\int \frac{d^2 k_{\perp}}{k_{\perp}^2} \int \frac{d k_{\perp}}{k_{\perp}^2} \frac{e^{-ik_{+}(w_{1}^{+} - w^{+})}}{(k_{+} - i \varepsilon)} \left(k_{+} - k_{\perp}^2/2 k_{-} + i \varepsilon\right) = - 4 \pi i k_{-} \theta(w_{1} - w) \int \frac{d^2 k_{\perp}}{k_{\perp}^2}. \quad (D.29)
\]

6. The \(G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} G_{w_{w1}}^{+0}\) and \(G_{w_{1}w}^{+0} G_{w_{w1}}^{+0} G_{w_{w1}}^{+0}\) terms, in turn, provides the following integral
\[
-2 \int \frac{d^2 k_{\perp}}{k_{\perp}^2} \int \frac{d k_{\perp}}{k_{\perp}^2} \theta(w_{1} - w) \frac{d k_{+}}{k_{+} + i \varepsilon} \left(k_{+} - k_{\perp}^2/2 k_{-} + i \varepsilon\right) = 4 \pi i k_{-} \theta(w_{1} - w) \int \frac{d^2 k_{\perp}}{k_{\perp}^2}. \quad (D.30)
\]

Finally, performing all requested integration, we obtain for the Eq. (D.3) contribution:
\[
- 2 i K_{abc}^{xyz} x \cdot y = - i \frac{g^2 N}{8 \pi} f_{abc} \left(\theta(z^{+} - x^{+}) - \theta(x^{+} - z^{+})\right) \partial_{z}^2 \left(\delta_{x}^{2} y \cdot z \cdot z \cdot z \cdot \int \frac{d k_{-}}{k_{-}} \int \frac{d^2 k_{\perp}}{k_{\perp}^2}\right). \quad (D.31)
\]

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Second contribution

Now, consider the second term (the third one can be obtained from this term by \( a, x \rightarrow c, z \) substitution) in Eq. (D.2).

\[
-2t \, K_{x \, y \, z \, 1, \, 2}^{abc} = - G_{0+i} \frac{\delta^2 M^{++}_L}{\delta A^a_+ x \, \delta A^b_+ y} G_{0+j} \frac{\delta M_{1, ji}}{\delta A^c_+ z} - G_{0+i} \frac{\delta^2 M^{++}_L}{\delta A^a_+ x \, \delta A^b_+ y} G_{0+j} \frac{\delta M_{1, ji}}{\delta A^c_+ z} - G_{0+i} \frac{\delta^2 M^{++}_L}{\delta A^a_+ x \, \delta A^b_+ y} G_{0+j} \frac{\delta M_{1, ji}}{\delta A^c_+ z}.
\]

The only non-zero contribution comes from the last term in the r.h.s. of the expression, see Eq. (D.33) - Eq. (D.34). We have:

\[
G_{0+i} \frac{\delta^2 M^{++}_L}{\delta A^a_+ x \, \delta A^b_+ y} G_{0+j} \frac{\delta M_{1, ji}}{\delta A^c_+ z} = \frac{g}{N} c_{0+i}^{tw} \frac{\delta}{\delta A^a_+ x} \frac{\delta}{\delta A^b_+ y} \frac{\delta}{\delta A^c_+ z}, \quad (D.33)
\]

where \( (U_{2}^{a \, b \, c})^{++}_{wpw1} = \frac{\delta v_{a \, b \, c}}{\delta A^a_+ x} \frac{\delta}{\delta A^b_+ y} \frac{\delta}{\delta A^c_+ z} \) and

\[
\begin{align*}
(U_{2}^{a \, b \, c})^{++}_{wpw1} &= \text{tr}[f_{a \, i} G^{+}_{wp} f_{a \, j} G^{+}_{pw} f_{a \, k} O_{w \, l} f_{b} O_{w}^{T}] + \text{tr}[f_{a \, i} G^{+}_{wp} f_{a \, j} G^{+}_{pw} f_{a \, k} O_{w \, l} f_{b} O_{w}^{T}] + \\
&+ \text{tr}[f_{a \, i} G^{+}_{wp} f_{a \, j} G^{+}_{pw} f_{a \, k} O_{w \, l} f_{b} O_{w}^{T}] + \\
&+ \text{tr}[f_{a \, i} G^{+}_{wp} f_{a \, j} G^{+}_{pw} f_{a \, k} O_{w \, l} f_{b} O_{w}^{T}],
\end{align*}
\]

Now we use the following identities for the requested traces:

\[
f_{a \, b \, c} \, \text{tr}[f_{a \, i} f_{a \, j} f_{b \, k}] = - \frac{N^2}{4} f_{abc}, \quad (D.36)
\]

see Eq. (D.7) above. Correspondingly, taking into account that

\[
\text{tr}[f_{a \, i} f_{a \, j} f_{a \, k}] = \delta_{a \, i} \delta_{a \, j} \delta_{a \, k} + \frac{N}{4} (d_{a \, e} d_{a \, e} d_{a \, e} - d_{a \, e} d_{a \, e} d_{a \, e} - d_{a \, e} d_{a \, e} d_{a \, e})
\]

is symmetrical in respect to \( a_1 \) and \( a_2 \), we obtain

\[
f_{a \, b \, c} \, \text{tr}[f_{a \, i} f_{a \, j} f_{b \, k}] = 0. \quad (D.38)
\]

Therefore we have:

\[
f_{a \, b \, c} \left(U_{2}^{a \, b \, c}ight)^{++}_{wpw1} = \frac{N^2}{4} f_{abc} \left(G^{+}_{wp} G^{+}_{pw} + G^{+}_{wp} G^{+}_{pw} - G^{+}_{wp} G^{+}_{pw} - G^{+}_{wp} G^{+}_{pw} \right). \quad (D.39)
\]

Let’s again consider different contributions in Eq. (D.34) expression.
1. The first term consists of $\theta^{+}_{\omega_{10}}$, $\theta^{+}_{\omega_{20}}$ theta functions. After the integration on + components of coordinates we obtain the following integrals with respect to + components of momenta:

$$
\int dk_1 + \int dk_+ \frac{e^{-i(k_+-k_1_+)(x^+-z^+)}}{(k_+ + i\varepsilon) (k_+ - k_1_+ + i\varepsilon) (k_+ - k_1_+^2/2k_- + i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon)}, \quad (D.40)
$$

where the integration on $\delta(k_- - k_1_-)$ functions was performed. First of all, we perform integration on $k_1_+$ variable obtaining:

$$
- \int dk_1 + \frac{e^{i k_1_+ (x^+ - z^+)}}{(k_1_+ - k_1_+ - i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon)} = -2\pi i \left( \frac{\theta(x^+ - z^+) e^{ik_1_+ (x^+ - z^+)}}{(k_1_+ - k_1_+^2/2k_- + i\varepsilon)} + \frac{\theta(z^+ - x^+) e^{-ik_1_+^2/2k_- + i\varepsilon}}{(k_1_+ - k_1_+^2/2k_- + i\varepsilon)} \right). \quad (D.41)
$$

The integration on $k_+$ variable in the second term of Eq. (D.41) provides $\theta(x^+ - z^+)$ function in the final answer, therefore this contribution is zero. The remaining integral is the following one:

$$
\int dk_+ \frac{1}{(k_+ + i\varepsilon) (k_+ - k_1_+ - i\varepsilon) (k_+ - k_1_+^2/2k_- + i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon)} \propto \frac{1}{k_1^2 k_1^2} + \frac{1}{k_2^2 (k_1^2 - k_1^2)} + \frac{1}{k_2^2 (k_1^2 - k_2^2)} = 0. \quad (D.42)
$$

2. The expression for the third term in Eq. (D.39) can be obtained from the previous one by the $k_1, x \rightarrow k_1, z$ substitution, i.e. it is equal to zero as well.

3. The next term we consider is proportional to $\theta^+_{\omega_{10}}, \theta^+_{\omega_{20}}$ theta functions. This contributions is proportional to the following integral:

$$
- \int dk_1 + \int dk_+ \frac{e^{-i(k_+ - k_1_+)(x^+ - z^+)}}{(k_+ + i\varepsilon) (k_+ - k_1_+^2/2k_- + i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon)}
$$

or after the variables change to

$$
\int ds_+ \int dk_+ \frac{e^{is_+ (x^+ - z^+)}}{(k_+ + i\varepsilon) (s_+ + i\varepsilon) (k_+ - k_1_+^2/2k_- + i\varepsilon) (k_+ + s_+ - k_1_+^2/2k_- + i\varepsilon)}. \quad (D.43)
$$

An integration on $k_+$ and subsequent integration with respect to $s_+$ provides:

$$
- 2\pi i \frac{2k_-}{k_2} \int ds_+ \frac{e^{is_+ (x^+ - z^+)}}{(s_+ + i\varepsilon) (s_+ - k_1_+^2/2k_- + i\varepsilon) (k_+ + s_+ - k_1_+^2/2k_- + i\varepsilon) (k_1_+ - k_1_+^2/2k_- + i\varepsilon)} = (2\pi i)^2 \frac{4k_-}{k_2 k_1} \int \left( 1 - e^{i\frac{k_2}{2k_-} (x^+ - z^+)} \right) \theta(z^+ - x^+). \quad (D.45)
$$

4. The last term in Eq. (D.39) can be obtained from the Eq. (D.44) answer by changind the overall sign of the expression and $k_1, x \rightarrow k_1, z$ substitution.

Finally, taking second and third term’s contributions of Eq. (D.2) together, we obtain:

$$
\frac{-2i}{(2\pi)^2} K_{a b c} \int (2\pi)^2 \delta^2 z_y \left( \theta(z^+ - x^+) \delta^2_{x_y} \right) \int \frac{dk_-}{k_-} \int d^2 k_+ \int d^2 k_+ e^{-i(k_+ - k_-) (x^+ - z^+)} \left( \frac{k_1 k_1^i}{(k_1)^2 (k_1^i)^2} \right) \left( 1 - \frac{1}{2} e^{i\frac{k_2}{2k_-} (x^+ - z^+)} - \frac{1}{2} e^{i\frac{k_2}{2k_-} (z^+ - x^+)} \right) \left( 1 - \frac{1}{2} e^{i\frac{k_2}{2k_-} (z^+ - x^+)} - \frac{1}{2} e^{i\frac{k_2}{2k_-} (z^+ - x^+)} \right). \quad (D.46)
$$
Considering the integral with respect to \( k_- \) variable only, we make the change of the variables \( c, d \) and expanding the exponentials we obtain the following integral:

\[
\sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dk_- \left( \frac{-e^{i(n+1)k_-}}{(n+1)!k_-^{n+2}} \right) = 0, \tag{D.47}
\]

therefore the whole Eq. \( \text{(D.46)} \) contribution is zero as well.

### Third contribution

The only contribution of the last two terms in Eq. \( \text{(D.2)} \) can be written as:

\[
-2i K_{x \psi z} = G_{0k+} \frac{\delta M_{L}^{++}}{\delta A_{-y}^{a}} - G_{0j+} \frac{\delta M_{L}^{++}}{\delta A_{+x}^{-}} + G_{0k+} \frac{\delta M_{J}^{ij}}{\delta A_{-y}^{a}} + G_{0j+} \frac{\delta M_{J}^{pq}}{\delta A_{+x}^{a}} . \tag{D.48}
\]

The formulas we use here are Eq. \( \text{(C.14)} \)–Eq. \( \text{(C.16)} \) only, taking into account that only \( G_{ut}^{0} \) from Eq. \( \text{(C.15)} \) contributes in the final answer for the positive values of \( k_- \), we obtain for the momentum integrals in the first term of Eq. \( \text{(D.47)} \) expression:

\[
- \delta_{u+x+} \delta_{z+} \int \frac{d^{4}k_{-}}{(2\pi)^{4}} \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{k_{1}}{k_{-}} i \kappa_{1} \theta(w^{+} - t^{+}) e^{-ik(s-t) - i(k(w-u))} \int \frac{d^{4}k_{2}}{(2\pi)^{4}} \frac{k_{2}}{k_{-}} e^{-ik_{2}(s-u)}, \tag{D.49}
\]

the overall \( - \) sign here is from the sign of \( G_{ut}^{0} \) Green’s function. The integration on the following coordinate variables provides in turn:

\[
\int dt^{+} \int dw^{+} \int du^{-} \int ds^{-} (\cdots) = - \frac{(2\pi)^{4}}{k_{+} - i\varepsilon} \delta_{k_{+}k_{1+}} \delta_{k_{-}k_{2-}} \delta_{k_{-}k_{2-}} . \tag{D.50}
\]

Therefore, an integration in respect to \( k_{-} k_{+} k_{1+} k_{1-} \) gives:

\[
\frac{k_{1}k_{1}i}{(2\pi)^{8}} \int \frac{dk_{-}}{k_{-}} \int \frac{dk_{+}}{(2k_{-}^{2} - i\varepsilon) (k_{+} - k_{2+} + i\varepsilon) (k_{+} - k_{2+}^{2} + i\varepsilon)} = \frac{2\pi}{(2\pi)^{4}} k_{1}k_{1i} \int \frac{dk_{-}}{k_{-}} \theta(x^{+} - z^{+}) . \tag{D.51}
\]

Correspondingly, the integral on \( k_{2} \) variable provides:

\[
\frac{2\pi i}{(2\pi)^{4}} \frac{k_{2}k_{2i}}{k_{2+}^{2} - 2k_{2+}^{2} + 2k_{+}^{2} - i\varepsilon} = \frac{-\pi i}{(2\pi)^{4}} \theta(x^{+} - z^{+}) e^{-i\kappa_{2}(x^{+} - z^{+})} . \tag{D.52}
\]

Now, taking all two terms of Eq. \( \text{(D.48)} \) together, we obtain:

\[
-2i K_{x \psi z} = \frac{i}{2(2\pi)^{4}} f_{abc} \delta^{2}_{x y} \left( \theta(z^{+} - x^{+}) \int \frac{dk_{-}}{k_{-}} \int d^{2}k_{-} \int d^{2}k_{1+} \int d^{2}k_{2+} \frac{k_{1}k_{1i}}{k_{2+}^{2} + 2k_{2+}^{2}} \right) e^{-ik_{i}(x^{+} - y^{+}) - ik_{i}(y^{+} - z^{+}) - ik_{i}(z^{+} - x^{+})} \theta(x^{+} - z^{+}) \int \frac{dk_{-}}{k_{-}} \int d^{2}k_{-} \int d^{2}k_{1+} \int d^{2}k_{2+} \frac{k_{1}k_{1i}}{k_{2+}^{2} + 2k_{2+}^{2}} \left( \int d^{2}k_{1+} \int d^{2}k_{2+} \frac{k_{1}k_{1i}}{k_{2+}^{2} + 2k_{2+}^{2}} e^{-ik_{i}(z^{+} - y^{+}) - ik_{i}(y^{+} - x^{+}) - ik_{i}(x^{+} - y^{+})} \right) . \tag{D.53}
\]

We note, that for the case when \( x^{+} = z^{+} \) we obtain \( K_{x \psi z} = 0 \) as it must be. We also note, the first term in Eq. \( \text{(C.15)} \) will contribute to the final answer as well. Therefore, the whole Eq. \( \text{(D.53)} \)
contribution must be doubled and we will obtain for it:

\[-2 \imath K^{abc}_{xyz,1,3} = \frac{\imath g^3 N}{(2\pi)^3} f_{abc} \delta^2_{x,z} \partial^2_{i,y} \]

\[
\left( \theta(z^+ - x^+) \int \frac{dk_-}{k_-} \int d^2k_\perp \int d^2k_{1\perp} \frac{k_ik_{1i}}{k^2_\perp k^2_{1\perp}} e^{-\imath k_1(x^i - y^i) - \imath k_{1i}(y^i - z^i)} - \theta(x^+ - z^+) \int \frac{dk_-}{k_-} \int d^2k_\perp \int d^2k_{1\perp} \frac{k_ik_{1i}}{k^2_\perp k^2_{1\perp}} e^{-\imath k_1(z^i - y^i) - \imath k_{1i}(y^i - x^i)} \right). \]

where again the only first term of the expansion of $e^{-\imath k x}$ function remain in the final expression.
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