INTRODUCTION

Characterizing fluid flow behavior in the unconventional tight formations is of fundamental importance in geo-engineering projects, such as gas exploration and production,\textsuperscript{1-3} CO\textsubscript{2} sequestration,\textsuperscript{4-6} radioactive waste isolation,\textsuperscript{7,8} and geothermal energy recovery.\textsuperscript{9-12} Permeability of these formations must be measured effectively and accurately to understand and predict the fluid flow behavior in the reservoir.\textsuperscript{13,14} The conventional steady-state method is widely used for the permeability measurement on reservoir rocks with relatively high permeability, but fails to measure permeability of those
with low permeability, such as highly stressed coal, tight gas sands, and shales, due to extremely small flowrates and the long time period required to attain a steady flow state.\textsuperscript{15-17} Sander et al\textsuperscript{2} reviewed the commonly used permeability measurement methods and provided the possible permeability measurement range through statistical analysis. From their study, the pulse decay method is the optimal to measure low-permeability rocks in micro- to nano-Darcy scale. Jones\textsuperscript{18} also pointed out that the pulse decay method is effective for rock permeability in the range of 0.1 md to 0.01 μd.

Brace et al\textsuperscript{19} was the first one to present a double-reservoir pulse decay method (PDM) for measuring the permeability of Westerly granite. However, it is worth noting that a considerable error would occur due to the assumption that porosity of the sample is negligible in their study. In reality, some amount of gas would be stored in the sample due to the phenomenon of “compressive storage” of the reservoir,\textsuperscript{18,20} which is defined as the change in volume of fluid per unit change in gas pressure. The effect of compressive storage leads to a disturbance of the pressure response in both the upstream and downstream reservoir, and thus an error in the measured permeability. To correct the compressive storage, Jones\textsuperscript{18} introduced a correction factor “$f_1$” into the original analytical permeability solution to improve the efficiency and accuracy of the measured permeability. Although the effect of gas compressive storage can be somewhat corrected, the results are still less reliable because the porosity of the sample cannot be estimated precisely.\textsuperscript{17,21-23} Following Jones's work, Cui et al\textsuperscript{24} modified “$f_1$” to account for the effect of gas sorption by incorporating Langmuir constants into the analytical solution, which further extends the applicability of the pulse decay method in permeability measurements of rock reservoirs with gas sorption. If no gas sorption occurs, the calculated permeability from Jones’s is equal to that from Cui et al’s solution. Feng et al\textsuperscript{25} presented an experimental setup with two equal size reservoirs to account for compressive storage when calculating permeability from pressure pulse decay data. In their setup, two pressure pulses of the same magnitude were generated by increasing the upstream pressure and decreasing the downstream pressure simultaneously. Gas pressure then attained equilibrium at the same pressure as that prior to the pressure pulse generation. However, such method still cannot estimate porosity from the pressure pulse decay data.

Therefore, the objective of this study is to develop a novel PDM to measure the effective porosity and permeability of gas reservoir formations of tight structure under replicated in situ fluid flow conditions. We aim to get in-depth insights into the following five aspects:

- Development of a single downstream reservoir PDM for simultaneous determination of effective porosity and permeability of gas reservoirs.
- Establishment of a mathematical model for numerically investigating the novel PDM.
- Derivation of an analytical solution of the mathematical model for data interpretation from the PDM tests.
- Experimental verification of the proposed PDM.
- Effects of gas compressive storage and sorption on the measured permeability.

2  |  THEORETICAL WORK

2.1  |  Mathematical model development

Figure 1 depicts the schematic of the proposed single downstream reservoir PDM. Considering a small control volume of length $dx$ bounded by two cross sections at two positions $x$ and $x + dx$, a nonlinear partial differential equation can be derived to describe the fluid pressure distribution over time, mathematically expressed as:

$$\nabla \cdot (p \nabla p) = \frac{\mu \phi \partial p}{k \partial t}$$  \hspace{1cm} (1)$$

where $p$ is the fluid pressure along the sample with time, $\mu$ is the dynamic viscosity of the test fluid, $\phi$ is the sample porosity, $k$ is the sample permeability, and $t$ is the measurement time. This governing equation has also been formulated by other
where \( p_d \) is the pressure in the downstream reservoir, \( A \) is the cross-sectional area, and \( V_d \) is the downstream reservoir volume.

The initial conditions at time \( t = 0 \) are given by:

\[
\begin{align*}
p(x, 0) &= p_0 \quad \text{for } 0 \leq x < L \quad (3) \\
p_d(0) &= p_0 - \Delta p \quad \text{for } x = L \quad (4) \\
p(L, t) &= p_d(t) \quad \text{for } t > 0 \quad (5)
\end{align*}
\]

where \( \Delta p \) is the pressure pulse created at \( t = 0 \) and \( p_0 \) is the initial pore pressure.

The governing Equation (1), associated with Equations (2)-(5), is the partial differential equation (PDE) model established to represent the proposed single downstream reservoir transient technique, by which the pressure responses over time and space can be readily described and investigated.

### 2.2 Permeability solution of the PDE model

Considering the different boundary and initial conditions applied on the governing equation (Equation 1), the analytical solution for the single downstream reservoir PDM must be derived for permeability calculation.\(^{17,27}\) The PDE model is strongly nonlinear, which makes it extremely difficult to derive its analytical solution. However, a small pressure pulse, which is less than 10% of the pore pressure at each pressure level, is usually introduced into the system during the pulse decay measurement; thus, both viscosity and compressibility of the fluid are considered constant during the experiment.\(^{18,28}\) The PDE model can then be rewritten as:

\[
\left\{ \begin{array}{ll}
\nabla \cdot (\nabla p) = \frac{\mu \beta \phi}{k} \frac{\partial p}{\partial t}, & 0 < x < L, \ t > 0 \\
\frac{\partial p_d}{\partial t} = -\frac{Ak}{\mu V_d} \frac{\partial p}{\partial x}, & x = L, \ t > 0 \\
p(x, 0) = p_0, & 0 \leq x < L \\
p(L, 0) = p_0 - \Delta p, & t = 0 \\
p(L, t) = p_d(t), & t > 0
\end{array} \right.
\]

where \( \beta \) is the isothermal compressibility of the test fluid.

The analytical solution for the gas pressure changes in the core sample \( p(x, t) \) through the proposed PDE model can be obtained by performing Laplace transform:

\[
\frac{p_0 - p}{p_0 - p_f} = 1 + \sum_{n=1}^{\infty} 2(1 + \alpha) \exp \left( \frac{-k \theta_n^2 t}{\phi \mu \beta L^2} \right) \cos \left( \theta_n \sqrt{\frac{\mu}{\beta \phi \mu L^2}} \right)
\]

where \( \alpha \) is the ratio of the downstream reservoir volume \( (V_d) \) to the sample pore volume \( (V_p) \):

\[
\alpha = \frac{V_d}{V_p}
\]

and \( \theta_n \) are the roots of a transcendental equation:

\[
\tan \theta_n = -\alpha \theta_n
\]

Substituting Equation (9) into Equation (7), the analytical solution to pressure changes in the core sample is obtained as:

\[
p(x, t) = p_0 - \frac{2(1 + \alpha) \exp \left( \frac{-k \theta_n^2 t}{\phi \mu \beta L^2} \right) \cos \left( \theta_n \sqrt{\frac{\mu}{\beta \phi \mu L^2}} \right)}{(1 + \alpha + \alpha^2 \theta_n^2)}
\]

Substituting Equations (10) and (11) into Equation (2) and integrating from \( t = 0 \) to infinity, we can get the pressure change as a function of time in the downstream reservoir:

\[
p_d(t) = p_f - \sum_{n=1}^{\infty} \frac{2 \alpha \Delta p}{1 + \alpha + \alpha^2 \theta_n^2} \exp \left( \frac{-k \theta_n^2 t}{\phi \mu \beta L^2} \right)
\]

Substituting Equation (9) into Equation (12), Equation (12) can be rewritten as:

\[
\frac{p_f - p_d(t)}{p_f - (p_0 - \Delta p)} = \sum_{n=1}^{\infty} \frac{2 \alpha (1 + \alpha)}{1 + \alpha + \alpha^2 \theta_n^2} \exp \left( \frac{-k \theta_n^2 t}{\phi \mu \beta L^2} \right)
\]

All terms with \( n > 1 \) in Equation (13) are neglected due to a quick decay of the exponential term. Equation (13) can then be approximated as:

\[
\frac{p_f - p_d(t)}{p_f - (p_0 - \Delta p)} = \frac{2 \alpha (1 + \alpha)}{1 + \alpha + \alpha^2 \theta_1^2} \exp \left( \frac{-k \theta_1^2 t}{\phi \mu \beta L^2} \right)
\]
Equation (14) can be simplified in logarithmic form:

$$\ln \left( \frac{p_f - p_d(t)}{p_f - (p_0 - \Delta p)} \right) = \ln \left( \frac{2\alpha (1 + \alpha)}{1 + \alpha + \alpha^2 \theta_1^2} \right) + \left( \frac{-k\theta_1^2}{\phi \mu \beta L^2} \right) t$$  \hspace{1cm} (15)

Equation (15) is a linear function, and the slope \( s \) can be expressed as:

$$s = -\frac{k\theta_1^2}{\phi \mu \beta L^2}$$  \hspace{1cm} (16)

Equations (15) and (16) are the analytical solution for permeability calculation through the simplified pressure transient technique. However, two parameters must be determined in Equation (16) prior to permeability calculation: the coefficient \( \theta_1 \) and the effective porosity \( \phi \). The coefficient \( \theta_1 \) can be obtained by solving the transcendental Equation (10). The established relationship between the coefficient \( \theta_1 \) and the volume ratio \( \alpha \) is shown in Figure 2, which shows that \( \theta_1 \) tends to a constant when \( \alpha \) is greater than 12 \( (\theta_1 = 1.61) \).

To reduce the error that may be induced during the course of calibrating the total gas reservoir volume on the downstream side including the volume of the downstream reservoir, the volume of gas lines, and the dead volume of the valves in the testing system, a larger sized reservoir with the volume ratio \( (\alpha) \) greater than 12 is recommended. And also, determination of the effective porosity is presented in detail in the following section.

2.3 | Porosity determination

Porosity is an important parameter for understanding the migration of fluid in rocks, but it has not been reliably and accurately calculated from PDM data.\(^{20,21,28}\) Recalling the law of mass conservation, we get the following equation:

$$p_0 \left( V_d + \phi V_s \right) - \Delta p V_d = p_d(t) V_d + \phi A \int_0^L p(x,t) \, dx$$  \hspace{1cm} (17)

Combining Equation (17) and Equation (12), the effective porosity of the sample \( (\phi) \), contributed by the connected pores, can be calculated at any moment during the course of measurement from the PDM data. To simplify the calculation process, the final equilibrium pressure, \( p_f \), is used for porosity calculation. When pressure in the downstream reservoir attains a new equilibrium state, we have:

$$p_d(t) = p(x,t) = p_f$$  \hspace{1cm} (18)

Effective sample porosity can be calculated through Equation (20).

3 | NUMERICAL SIMULATION

The analytical solution (ie, Equation11) describes the gas pressure changes in the time and space domain; however, the complexity of the analytical solution makes it hard to derive a clear perception of its behavior in a timely manner. Therefore, it is necessary to develop a more effective and efficient method to solve the PDE model. In this section, the finite difference schemes are applied to solve the model. High efficiency is required for the solver due to the low flow velocity of the fluid in the sample of low/ultralow permeability, which leads to a much longer time period, \( T = 10^7 \text{-} 10^9 \) seconds. The popular method of lines (MOL) is adopted to solve the above nonlinear parabolic PDE,\(^{29,30}\) which has better efficiency than uniform time stepping methods.
For the sake of easier exposition, we rewrite the PDE model (downstream pressure responses), Equations (1)-(5), as:

\[
\begin{cases}
  c p_i = p_{xx} + (p_i)^2, & 0 < x < L, \ t > 0 \\
  \lambda_2 p_i(L, t) = -p(L, t) p_i(L, t), & x = L, \ t > 0 \\
  p(x, 0) = p_0, & 0 \leq x < L \\
  p_x(0, t) = 0, & t > 0 \\
  p(L, 0) = p_0 - \Delta p
\end{cases}
\]  

(21)

where \(c = \frac{\mu c}{k}\) and \(\lambda_2 = \frac{\nu c}{\mu k}\) are positive constants. Here, we assume \(p\) to be sufficiently smooth. Notice that corner discontinuities of the initial condition will be quickly smoothed by the diffusion operator.

The space domain \(\Omega = [0, L]\) is partitioned uniformly by grid points \(0 = x_0 < x_1 < \cdots < x_M = L\), where \(x_i = i\Delta x\) with a spatial step size \(\Delta x = L/M\). To approximate the spatial derivatives, we will use second-order accurate central and one-sided finite difference formulas for interior and boundary grid point, respectively. Let \(p_i(t) \approx p(x_i, t)\) be the semidiscrete solution approximations and define the temporal solution vector:

\[
\vec{p}(t) = \begin{bmatrix}
p_0(t) \\
p_1(t) \\
\vdots \\
p_{M-1}(t) \\
p_M(t)
\end{bmatrix}
\]

(22)

which satisfies a system of nonlinear ordinary differential equations (ODEs)

\[
\frac{d\vec{p}}{dt} = \mathbf{f}(\vec{p}(t))
\]

(23)

where the vector function \(\mathbf{f}\) is derived from finite difference schemes. More specifically, the semidiscrete ODEs read (in component-wise):

\[
\begin{align*}
  \frac{dp_i}{dt} &= p_{i+1} - 2p_i + p_{i-1} + \left(\frac{p_{i+1} - p_{i-1}}{2\Delta x}\right)^2, \quad 1 \leq i \leq M - 1 \\
  \frac{dp_0}{dt} &= 0 \\
  \frac{dp_M}{dt} &= -p_M - 4p_{M-1} + 3p_M \\
  \lambda_2 \frac{d^2p_M}{dt^2} &= -p_M - 4p_{M-1} + 3p_M
\end{align*}
\]

(24)

where the initial condition is given by:

\[
\vec{p}(0) = \begin{bmatrix}
p_0(0) \\
p_1(0) \\
\vdots \\
p_{M-1}(0) \\
p_M(0)
\end{bmatrix} = \begin{bmatrix}
p_0 \\
p_0 \\
\vdots \\
p_0 \\
p_0 - \Delta p
\end{bmatrix}
\]

(25)

Notice that we have rewritten the Dirichlet boundary condition \(p_0(t) = p_0\) into \(\frac{dp_0}{dt} = 0\) for a unified formulation.

4 | APPLICATION OF THE MODIFIED PULSE DECAY METHOD

In order to interpret the analytical solution (Equations 15 and 16), a set of experiments were conducted by the simplified single downstream reservoir PDM on a coal sample retrieved from the San Juan basin in the United States. The diameter of the coal sample is 5.1 cm, and the height is 7.6 cm.

4.1 | Experimental setup

The experimental apparatus used in the study is schematically shown in Figure 3, which was a self-modified test instrument in our laboratory. The main component of the apparatus was a triaxial cell, being capable of performing triaxial compression tests at the lateral confining stress up to ~ 68.9 MPa and the vertical pressure up to ~ 2000 kN. The triaxial cell was connected to two syringe pumps of ~ 68.9 MPa capacity, enabling independent loading in the vertical and horizontal directions. Three high pressure transducers with a resolution of ~ 0.01 MPa were used to monitor the changes in the vertical stress, horizontal stress, and pore pressure.

4.2 | Experimental procedure

The tests were performed under three different stress conditions, as listed in Table 1, while the horizontal effective stress, defined as the difference between the horizontal stress and pore pressure, was kept constant as 0.34 MPa for all the tests. The temperature was maintained constant throughout the experiments at 40.5°C to best replicate the in situ temperature condition.

The sample was firstly stressed incrementally to 6.89 MPa and 4.82 MPa in the vertical and horizontal directions, respectively. A vacuum pump was then used to bleed off the air remaining in the sample to avoid any potential...
influence of the foreign gases. Next, methane was injected into the system at 0.50 MPa to flush the sample for 1 day, and the gas pressure was increased to 4.48 MPa. Adequate time, typically 5 days for San Juan basin coal, was given for the sample to attain stress, strain, and sorption equilibrium. After reaching equilibrium, a negative pulse with the magnitude less than 10% of the initial pore pressure (0.45 MPa) was created by decreasing the pressure in the downstream reservoir. The valve between the bottom end of the sample and the downstream reservoir was opened to enable gas to flow from the sample to the downstream reservoir, while pressure increase in the downstream reservoir was recorded for permeability estimation. Based on the pressure response in the downstream reservoir, core permeability was calculated through the analytical solution, Equations (15) and (16).

The properties of the core and the test fluid used for numerical simulation are presented in Table 2. Numerical simulation was performed by the PDE model (i.e., Equations 1-5) to depict the pressure responses. Figure 4 shows the pressure responses in the space and time domain at a pore pressure of 3.5 MPa, porosity of 20%, and permeability of 0.01 mD. From the figure, we can see that a pressure pulse with the magnitude of 0.35 MPa was established in the downstream reservoir at \( t = 0 \), and then pressure was built up exponentially with time. Pressure distribution along the sample is further plotted as a function of time for the convenience of analysis, as shown in Figure 5.

\[
\frac{\partial p}{\partial x} = 0 \quad \text{is assumed when deriving the analytical solution, which means that fluid pressure changes linearly along the sample. Such assumption is numerically verified to demonstrate the applicability of the analytical solution. The numerical simulation results are shown in Figure 5. It can be seen that a strong nonlinear change in pore pressure exists along the sample in the early stage of the test (e.g., \( t = 20,000 \) seconds), which can also be clearly observed in Figure 4. However, as time elapses, pore pressure diffuses further in the sample and a smooth pressure profile is gradually developed. This means that pressure along the sample changes linearly, and the pressure gradient becomes constant. When \( t = 50,000 \) seconds, the data obtained from the numerical simulation can be fitted by a straight line with \( R^2 = 0.95 \). Therefore, the late-time behavior should be used for permeability calculation through the derived analytical solution because of its linear variation along the sample.}

When using the double-reservoir PDM, small-volume reservoirs have been suggested to use for the purpose of

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**TABLE 1** Prescribed stress conditions for three pulse decay tests

| Vertical stress, MPa | Horizontal stress, MPa | Pore pressure, MPa | Pulse size, MPa |
|----------------------|------------------------|-------------------|----------------|
| Test 1 6.89          | 4.82                   | 4.48              | 0.45           |
| Test 2 6.89          | 3.79                   | 3.45              | 0.34           |
| Test 3 6.89          | 2.75                   | 2.41              | 0.24           |

**FIGURE 3** Core flooding apparatus for permeability measurement using the novel transient technique
achieving fast permeability measurements. It can be noticed that our single-reservoir PDM can be treated as a special case of the double-reservoir PDM where one reservoir with the zero volume is involved. Hence, even though the late-time behavior is used for permeability calculation, fast permeability measurements can be expected compared to the double-reservoir PDM. Besides, when using the double-reservoir PDM, considerable errors in the measured permeability could be raised by the dead volumes from gas lines, valves, and void space between the up-/downstream reservoirs and the two ends of the sample. Use of the single-reservoir PDM could greatly reduce the errors induced by the dead volumes.

### 5.2 Influence of the reservoir volume on pressure responses

The volume of the downstream reservoir can significantly affect the pressure build-up rate and is a critical factor that should be considered in order to accurately and effectively determine rock permeability. A small reservoir is preferred because the pressure change is much faster and less time is required for the pressure plot to level off, as can be seen in Figure 6A. Although the measurement time can be shortened, nonlinear pressure decay behavior shows up in the early time. For ease of presentation and comparison, a dimensionless pressure is defined as:

$$\Delta p_D = \ln \left( \frac{p_f - p_d(t)}{p_f - (p_0 - \Delta p)} \right)$$  \hspace{1cm} (26)$$

where $\Delta p_D$ is the dimensionless pressure. A straight line would appear if $\Delta p_D$ is plotted against time, as mathematically expressed by Equation (15). The absolute value of the slope of the line equals to $\frac{k_\phi}{\mu L^2}$. Figure 6B depicts the changes in the dimensionless pressure against time when using different sized downstream reservoirs, where $\alpha$ represents the ratio of the reservoir volume and the sample pore volume. It can be seen that more time is needed for the dimensionless pressure to convert to a linear state when a small sized reservoir is used, that is, the time required for gas flow to reach a pseudosteady state is longer. Many researchers have also recommended using the late-time behavior of the pressure response for permeability estimation. A shorter time period is needed for the fluid to reach steady-state flow when larger sized reservoirs are adopted, mainly because the response of samples is treated to be homogeneous. But the pressure response deviations from that of the homogeneous samples can be accentuated when using smaller sized reservoirs.

### Table 2

| Parameter | Physical meaning | Value               |
|----------|------------------|---------------------|
| $A$      | Cross-sectional area of the core sample | 20.3 cm$^2$ |
| $L$      | Length of the sample | 7.6 cm |
| $V_u$    | Upstream reservoir volume | 50 cm$^3$ |
| $V_d$    | Downstream reservoir volume | 50 cm$^3$ |
| $\mu$    | Dynamic viscosity (methane) | $1.22 \times 10^{-5}$ Pa s (40.5°C) |
| $\phi$   | Rock porosity | 20%$^{31}$ |
| $k$      | Rock permeability | 0.01-0.1 mD |

**Figure 4** Pressure responses in time and space domain (the initial pore pressure of the measurement system is 3.5 MPa and a pressure pulse with the magnitude of $\Delta p$ is created at the downstream boundary at $t = 0$. As time elapses, the pressure pulse declines and pore pressure is redistributed along the sample)

**Figure 5** Pressure responses along the sample as time proceeds
5.3 Porosity and permeability determination via experimental measurement

Pressure responses at each prescribed pore pressure level are presented in Figure 7. Effective porosity of the sample at each pressure is calculated (via Equation 20) to be 12.3%, 14.6%, and 15.6% at the prescribed pore pressure of 4.48 MPa, 3.45 MPa, and 2.41 MPa, respectively. The experimental results show that the effective porosity is a dynamic parameter, which increases as the gas pressure reduces, and the simplified pulse decay method provides a way to estimate the effective porosity.

It is also found that the coal permeability is constantly enhanced as gas pressure decreases, indicated by the increase of the absolute value of the slope of the fitted lines. Coal permeability evolution during primary production has been investigated over the past two decades, and both experimental and theoretical work have shown that coal permeability evolution is determined by two factors: effective stress and sorption-induced matrix shrinkage. Since a constant effective stress state was maintained during the permeability measurements, the increased coal permeability can be attributed to sorption-induced coal matrix shrinkage.

5.4 Comparison of different single-reservoir PDMs

In this section, two different PDMs with a single up-/downstream reservoir are investigated and compared theoretically and experimentally. Three groups of experiments are conducted with a single upstream reservoir PDM using the same core sample under the same test conditions including boundary conditions, temperature, and pore pressure. Results will be used for comparison with the experimental results from the single downstream reservoir PDM.

Following the same process for deriving the analytical solutions of rock porosity and permeability for the single downstream reservoir PDM, the analytical solutions for porosity and permeability estimation for the single upstream reservoir PDM are derived as:

\[
\phi = \frac{\left[\left(p_0 + \Delta p\right) - p_f\right] V_u}{p_f - p_0} V_s \quad (27)
\]

\[
\ln \left(\frac{p_u(t) - p_f}{p_0 + \Delta p - p_f}\right) = \ln \left(\frac{2\alpha (1 + \alpha)}{1 + \alpha + \alpha^2 \theta^2_1}\right) + \left(\frac{-k\theta^2_1}{\phi \mu \beta L^2}\right) t \quad (28)
\]

\[
s = -\frac{k\theta^2_1}{\phi \mu \beta L^2}
\]

Figure 8 shows pressure responses in the upstream reservoir at different initial gas pressures. By interpreting the pulse decay data via Equation (27), the effective porosity can be calculated as 12.2%, 13.4%, and 14.8% at the prescribed pore pressure of 4.48 MPa, 3.45 MPa, and 2.41 MPa. By comparing with the measured effective porosity from the single downstream reservoir PDM, the differences at the each pore pressure are 0.8%, 8.1%, and 5.1%, respectively. The little difference in the measured porosity suggests that single-reservoir PDMs have high accuracy and are reliable for porosity determination.

The effective porosity, slope, and permeability at different pressures measured by different single-reservoir
Figure 7 Experimental results from permeability measurements using the single downstream reservoir PDM: (A) pressure build-up vs time at the initial pore pressure of 4.48 MPa; (B) dimensionless pressure vs time for slope fitting at the initial pore pressure of 4.48 MPa; (C) pressure build-up vs time at the initial pore pressure of 3.45 MPa; (D) dimensionless pressure vs time for slope fitting at the initial pore pressure of 3.45 MPa; (E) pressure build-up vs time at the initial pore pressure of 2.41 MPa; (F) dimensionless pressure vs time for slope fitting at the initial pore pressure of 2.41 MPa.
FIGURE 8  Experimental results from permeability measurements using the single upstream reservoir PDM: (A) pressure build-up vs time at the initial pore pressure of 4.48 MPa; (B) dimensionless pressure vs time for slope fitting at the initial pore pressure of 4.48 MPa; (C) pressure build-up vs time at the initial pore pressure of 3.45 MPa; (D) dimensionless pressure vs time for slope fitting at the initial pore pressure of 3.45 MPa; (E) pressure build-up vs time at the initial pore pressure of 2.41 MPa; (F) dimensionless pressure vs time for slope fitting at the initial pore pressure of 2.41 MPa
PDMs are presented in Table 3. As pressure reduces, coal matrix shrinks, making the fractures open up; thus, the effective porosity increases with gas depletion. Correspondingly, it can be expected that gas flow rate increases, as reflected by the increased permeability. However, a detailed comparison suggests that: Firstly, the effective porosity measured by the single upstream reservoir PDM at a certain pore pressure is slightly lower than that of the downstream reservoir PDM. This is because the mean pore pressure in the sample after generating a positive pulse when employing the single upstream reservoir PDM is higher than the initial pore pressure, leading to coal matrix swelling and thus resulting in a reduction in effective porosity. When using the single downstream reservoir PDM, the mean pore pressure is lower than the initial pore pressure after a negative pressure pulse is created in the downstream reservoir, resulting in matrix shrinkage and thus an increase in effective porosity. Secondly, a larger effective porosity leads to a lower permeability value. As displayed in Table 3, the measured permeability by the single upstream PDM is larger than that by the single downstream PDM at a certain pore pressure. This phenomenon can be attributed to the gas compressibility. At a certain pore pressure, gas compressibility for the upstream reservoir PDM becomes smaller after a positive pressure pulse is generated, causing a faster gas flow rate. This can be reflected by the fact that the absolute value of the slope obtained by the single upstream reservoir PDM is larger than that obtained by the single downstream reservoir PDM at a certain pore pressure. In our study, the initial pore pressure is used to estimate the gas compressibility and the rock permeability, while some researchers estimated gas compressibility and rock permeability by the mean pore pressure after the pressure pulse generation. Given that only a small pressure pulse with the magnitude of less than 10% of the initial pore pressure is used, the difference in gas compressibility estimated for those two cases is negligible, especially at high pressures. However, if gas compressibility is evaluated at the mean pore pressure, as done by other researchers, the difference in measured permeability between those two different PDMs would increase. By comparing the two PDMs, it is recommended that the single downstream reservoir PDM should be used, as it is capable of improving the accuracy of the measured porosity and permeability by better replicating the in situ gas flow behavior.

5.5 | Effect of gas compressive storage and ad-/desorption on the measured permeability

Gas is known to be highly compressible, and hence its compressive storage in rock samples should be considered during gas permeability estimation. To correct the influence of gas compressive storage on the measured permeability, a factor “$f_1$” was defined by Jones and expressed as:

$$ f_1 = \frac{\theta_1^2}{a+b} $$

(29)

where $a$ and $b$ are the ratios of the sample’s pore volume to the upstream and downstream reservoir volume ($a = \frac{V_p}{V_u}$ and $b = \frac{V_p}{V_d}$), respectively. For generality, $f_1$ values as a function of the larger value of $a$ and $b$ are plotted for different combination of the up-/downstream reservoir volumes in Figure 9. $\theta_1$ is the first root of the following equation:

$$ \tan \theta_1 = \frac{(a+b) \theta}{\theta_1^2 - ab} $$

(30)

When the upstream reservoir volume is zero, Equation(30) is reduced to Equation(10). The single downstream reservoir PDM, hence, can be treated as a special case of the double-reservoir PDM, where the volume of the upstream reservoir is zero. For the single downstream reservoir PDM, $f_1$ values are dependent upon the volume ratio of $b$, as indicated by the red line shown in

| Pore pressure, MPa/psi | 4.48/650 | 3.45/500 | 2.41/350 |
|------------------------|----------|----------|----------|
| **Single downstream reservoir PDM tests** | | | |
| Effective porosity, % | 12.3 | 14.6 | 15.6 |
| Slope | $-1.12 \times 10^{-2}$ | $-1.36 \times 10^{-2}$ | $-2.10 \times 10^{-2}$ |
| Permeability, mD | $1.09 \times 10^{-3}$ | $2.02 \times 10^{-3}$ | $4.69 \times 10^{-3}$ |
| **Single upstream reservoir PDM tests** | | | |
| Effective porosity | 12.2 | 13.4 | 14.8 |
| Slope | $-1.48 \times 10^{-2}$ | $-1.68 \times 10^{-2}$ | $-2.27 \times 10^{-2}$ |
| Permeability, mD | $1.57 \times 10^{-3}$ | $2.52 \times 10^{-3}$ | $5.29 \times 10^{-3}$ |
| **Difference between two PDMs** | | | |
| Porosity, % | 0.8 | 8.1 | 5.1 |
| Permeability, % | 30.6 | 19.8 | 11.3 |
Figure 9. By taking gas compressive storage into account, the analytical solution for rock permeability estimation (Equation 16) can be revised as:

\[
s = \frac{f_1 k \theta^2}{\phi \mu \rho L^2}
\]  

(31)

It is evident that the influence of gas compressive storage on permeability calculation can be neglected \((f_1 \approx 1)\) when using a larger downstream reservoir \((b \approx 0)\). From Figure 9, it can be seen that \(f_1\) value can reach 0.97 when the downstream reservoir is 12 times larger than the pore volume of the sample.

Another feature of unconventional reservoir rocks that should be considered is the influence of the ad-/desorption effect on permeability estimation when using the PDM. Cui et al.\(^{24}\) corrected the “\(f_1\)” factor to account for the contribution of gas ad-/desorption for the double-reservoir PDM. In their study, the amount of ad-/desorbed gas \(V_a\) is approximately quantified using the Langmuir model\(^{35}\) and mathematically expressed as:

\[
V_a = \frac{V_{LP}}{p_L + p}
\]  

(32)

where \(V_a\) is the Langmuir volume and \(p_L\) is the Langmuir pressure. To incorporate the gas sorption into the analytical solution initially proposed by Brace et al.\(^{19}\) the volume ratios were defined as:

\[
\begin{align*}
a &= \frac{V_p \left(1 + \frac{\phi}{\theta}\right)}{\phi} \\
b &= \frac{V_p \left(1 + \frac{\phi}{\theta}\right)}{V_d}
\end{align*}
\]  

(33)

\[\phi_n = \frac{\rho_s (1 - \phi) p_L V_L}{\rho V_{std} (p_L + p)^2}
\]  

(34)

where \(\rho\) is the molar density of gas, \(\rho_s\) is the skeleton density of porous media, and \(V_{std}\) is the molar volume of gas at standard pressure and temperature. If no ad-/desorption occurs, the volume ratios, \(a\) and \(b\), become the same ratios as defined by the previous researchers.\(^{18,28}\) For the double-reservoir PDM, both the up- and downstream reservoir volume, including the volume of gas lines and the dead volumes of valves, should be carefully calibrated so that more accurate permeability can be estimated. However, only the volume ratio \(b\) needs to be considered when using the single downstream reservoir PDM, which reduces the uncertainties during permeability estimation compared to the double-reservoir PDM.

For the double-reservoir PDM, Brace et al.’s\(^{19}\) solution is an approximate solution without considering gas compressive storage and ad-/desorption effect; Jones’\(^{18}\) solution (or Dicker and Smits\(^{28}\)) only accounted for gas compressive storage, and Cui et al.’s\(^{24}\) solution took both effects into account for permeability calculation. The measured permeability by the single downstream reservoir PDM is shown in Figure 10, in which three sets of permeability solutions are presented. Since the single downstream reservoir PDM can be treated as a special case of the double-reservoir PDM, for simplicity, these three sets of solutions are also called Brace et al.’s solution, Jones’ solution, and Cui et al.’s solution, correspondingly.

Lin\(^{16}\) indicated that considerable error may occur if the gas compressive storage is neglected when measuring rock permeability. The influence of the compressive storage on permeability estimation can be seen by comparing Brace et al.’s solution and Jones’ solution (Figure 10). The results show that permeability is underestimated without taking the compressive storage into account. It can be found that there
is a positive correlation between the effective porosity and the permeability increment when using Jones's solution over Brace et al's solution, which is consistent with previous studies. However, the permeability increments only change from 4.6% to 5.9% when the pore pressure reduces from 4.48 MPa to 2.41 MPa, indicating that permeability difference between the two solutions can be neglected.

By incorporating Langmuir model into permeability calculation, higher permeability is obtained by Cui et al's solution than the other two solutions, and the difference between Cui et al's and other two solutions increases with pressure reduction. As shown in Figure 10, Cui et al's solution is 15.1% higher than Brace et al's solution at 4.48 MPa pore pressure, and the difference between them rises up to 26.1% when pore pressure reduces to 2.41 MPa. Besides the small contribution of the compressive storage, the increasing discrepancy between Cui et al's and Brace et al's solution is mainly attributed to the ad-/desorption effect. As reported by many researchers, desorption-induced coal matrix shrinkage has a positive impact on coal permeability enhancement. Therefore, the role of ad-/desorption effect should be considered when testing unconventional gas reservoirs using the PDMs, especially for rocks with strong sorption potential.

5.6 Effect of gas slippage on the measured permeability

During gas production, permeability variation is mainly controlled by two factors: effective stress and desorption-induced matrix shrinkage. In our case, the effective stress was maintained constant, and hence the increase of rock permeability with gas pressure reduction can be attributed to the dominant role of the desorption effect. However, there is a possibility that gas slippage may play a role in the measured permeability. Therefore, the intrinsic permeability must be obtained to verify the dominant influence of gas desorption-induced matrix shrinkage in enhanced permeability. The relationship between the intrinsic and measured permeability is mathematically given as:

$$k_a = k_\infty \left( 1 + \frac{b_k}{p} \right)$$

where $k_a$ is the measured permeability, $k_\infty$ is the intrinsic permeability, and $b_k$ is the Klinkenberg coefficient. It has been experimentally obtained that $b_k$ is approximately equal to 0.27 MPa for San Juan basin coal. A comparison is only made between the intrinsic and measured permeability based on Cui et al's solution, since the other two solutions follow the similar behavior. As can be expected, the deviation of the measured permeability from the intrinsic permeability increases with gas pressure reduction due to the influence of Klinkenberg effect (Figure 11). This phenomenon is consistent with the finding for tight shales that the slip flow plays an increasing role with gas depletion. However, in our case, the difference between them only ranges from 5.7% to 10.1% when using the measured permeability over the intrinsic permeability. That illustrates the little influence of gas slippage on enhanced permeability and the determinant role of desorption effect in permeability evolution.

6 SUMMARY AND CONCLUSIONS

A modified transient technique is proposed in this study to determine the effective porosity and permeability by replicating the reservoir fluid flow conditions in time and space domain. Based on the work completed, the following conclusions are drawn:

- Accurate measurements on effective sample porosity and permeability can be achieved by the single downstream reservoir PDM due to its capability of better replicating the in situ fluid flow behavior. However, the positive pressure pulse generated in the single upstream reservoir PDM may cause coal matrix swelling and gas compressibility reduction, underestimate the effective porosity, and overestimate the permeability.
- The effect of the volume ratio of the gas reservoir to the pore volume of the sample is remarkable on the accuracy of the measured permeability when using the single downstream PDM. The numerical results suggest that the volume ratio greater than 12 should be used in order to reduce the error of the measured porosity and permeability.
• The nonlinear PDE model, closely representing the single downstream PDM, can be effectively and efficiently solved by Method of Lines. The numerical results from the nonlinear PDE model indicate that permeability can be more accurately estimated when gas flow along the sample reach a pseudosteady state.

• Higher accuracy in permeability estimation can be expected using the single downstream reservoir PDM than the double-reservoir PDM, since the single downstream PDM reduces the uncertainties involved in permeability measurements.

• Ad-/desorption effect on the measured permeability must be corrected when testing unconventional gas reservoirs with strong sorption potential, since sorption-induced matrix shrinkage plays an important role in permeability enhancement; however, the influence of gas compressive storage on the measured permeability can be neglected if the sample porosity is small.

CONFLICT OF INTEREST
The authors declare no conflict of interest.

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