Behavior of helical coil with water cooling channel and temperature dependent conductivity of copper winding used for MFH purpose

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Abstract. The following paper presents coupling of electromagnetic and thermal modelling of water-cooled inductors for magnetic fluid hyperthermia (MFH) purpose. In numerical analysis a nonlinear conductivity of the copper coil winding and turbulent water flow inside the cooling channel have been taken into account. The induced current in the coil windings generates heat, which affects the temperature dependent conductivity of copper. As shown by the conducted simulations, this influences resulted magnetic field and temperature distribution inside the inductor as well as actual resistance and inductance of the coil. Interestingly, in model without circulating water inside copper winding, these quantities are characterized by the long-term transient state that virtually disappears under the water cooling condition. For simplicity the described problem, including coupling of the Helmholtz-type equation for magnetic vector potential and classical heat transfer equation based on energy balance, has been solved for an axis-symmetric model in a cylindrical coordinate system using finite element method.

1. Introduction
Electromagnetic radiation is widely used in many important environmental, agricultural and health applications[1-4]. In recent years, electromagnetic fields have found a special place in treatment of cancerous tissues where nanoparticle hyperthermia is increasingly popular method nowadays [5,6]. The energy of the external AC electromagnetic field transmitted to the magnetic nanoparticles causes their heating by the Néel and Brown relaxation mechanisms occurred inside ferrofluid [7]. In this therapeutic process, magnetic nanoparticles are placed inside the tumor and then inductively heated up to high temperatures causing permanent destruction of malignant cells [5,8]. At the same time, such nanoparticles may function as nanocarriers, that are equipped with chemotherapeutics and special enzymes to localize such particles in the target tissue and increase result of combination cancer therapy including drug delivery and hyperthermia treatment [8,9].

Heating efficiency of magnetic fluids with different physicochemical properties is usually tested under laboratory conditions [10], during the so-called magnetic fluid hyperthermia (MFH) procedure, using special coils to generate magnetic field of sufficient intensity [11]. In most cases, the coils have helical shape and the tested ferrofluid samples are localized inside them [12]. The MFH inductors, due to large currents flowing through them, should be equipped with efficient water cooling systems[13], which task is to prevent overheating the copper coil windings, reduce non-specific heating effects of the ferrofluid sample and create beneficial conditions for temperature measuring of the magnetic fluid.
with professional sensors [14]. It is worth emphasized that coils of various sizes and shapes are widely used in a variety of biomedical applications, such as electromagnetic inductive heating of magnetic nanoparticles or metal implants during hyperthermia and ablation therapies [15], peripheral nerve stimulation [16] as well as transcranial magnetic stimulation of the brain [17]. At the same time, researchers utilize different optimization procedures to obtain a homogeneous magnetic field in the region of interest [18], maximize tissue temperature [15], optimize shape of the coil [16] and coil configurations [17] or distances between coil loops [19].

The current paper presents the behavior of helical water-cooled inductors for MFH procedure with a nonlinear conductivity model of copper coil winding and cold circulating water flow inside the cooling channel. The induced current in the copper coil windings generates heat, which affects the temperature-dependent electrical conductivity of copper. As shown below, this influences resulted magnetic field, temperature distribution inside the inductor as well as some important coil parameters.

2. Model definition and basic equations

From mathematical point of view current paper presents coupling of electromagnetic and thermal modeling of water-cooled induction coil for magnetic fluid hyperthermia procedure. To achieve significant currents as well as high temperatures within tested ferrofluid samples, such coils are powered in real measurement systems using parallel resonance LC circuits [12] as shown in figure 1. Schematic view of the water-cooled inductor containing N coil loops, as analyzed in this publication, shows figure 2. To simplify, described problem uses an axis-symmetric model in a cylindrical coordinate system (r, φ, z). All specific dimensions of the coil with cooling water channel hollowed inside the copper coil winding are detailed shown in table 1.

| Elements of the inductor | Dimensions (mm) |
|--------------------------|-----------------|
| length of the solenoid   | h = a + b = 30  |
| half of the solenoid     | a = b = 15      |
| radius of the solenoid   | R₀ = 20         |
| inner radius of the solenoid | R₁ = R₀ − r₂ = 18.34 |
| outer radius of the solenoid | R₂ = R₀ + r₂ = 21.66 |
| diameter of the water cooling channel | d₁ = 2r₁ = 1.66 |
| diameter of the coil winding | d₂ = 2r₂ = 3.32 |

Figure 1. Example of LC resonance circuit for magnetic fluid hyperthermia purpose [12].

Figure 2. Analyzed model of the water-cooled inductor with 9 turns.
Starting from Maxwell's equations in the frequency domain, a Helmholtz-type equation for magnetic vector potential $A$ describing electromagnetic field distribution within the computational area can be derived, namely [13]:

$$\nabla \times \left( \frac{1}{\mu_0 \mu_r} \nabla \times A \right) - \omega^2 \varepsilon_0 \left( \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right) A = J_e$$

(1)

where $J_e$ means the excitation current density, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and $\mu_0 = 4\pi \cdot 10^{-7}$ H/m stand for the electric and magnetic constants, respectively, $\sigma$ indicates electric conductivity and $\omega = 2\pi f$ is an angular frequency of applied EM field. Importantly, the complex relative permittivity is given by [20]:

$$\varepsilon_r' = \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}$$

(2)

where $\varepsilon_r'$ and $\varepsilon_r''$ are real and imaginary part of such defined complex permittivity also called dielectric constant and dielectric loss factor, respectively, and $j = \sqrt{-1}$ is imaginary unit. Moreover, nonlinear temperature-dependent electrical conductivity of copper coil winding is governed by [21,22]:

$$\sigma(T) = \frac{1}{\rho(T)} = \frac{1}{\rho_0 \left[ 1 + \alpha (T - T_0) \right]} = \frac{\sigma_0}{1 + \alpha (T - T_0)}$$

(3)

where $\rho(T)$ means temperature-dependent resistivity of the copper at actual temperature $T, \rho_0$ is electrical resistivity at ambient temperature $T_0 = 20^\circ C$ and $\alpha$ – copper electrical resistivity coefficient. It should be emphasized that in the case when current coil winding temperature is equal to $T = T_0$, the copper conductivity achieves constant value of $\sigma = \sigma_0$. What is important, the magnetic vector potential can be easily used to specify the magnetic field inside solenoid by the formula [20]:

$$B = \nabla \times A$$

(4)

In addition, electro-thermal analysis of presented problem requires the solution of classical heat transfer equation based on energy balance as follows [15,22]:

$$\rho C_e \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = Q_{\text{ext}} + Q_{\text{cool}}$$

(5)

where $T$ denotes actual temperature of the object, $\rho$ – is its density, $C_e$ – specific heat, $k$ – thermal conductivity and $t$ expresses the time. What is more, the term $Q_{\text{ext}}$ refers to average over time inductive heating generated by the helical induction coil, namely [13]:

$$Q_{\text{ext}} = \frac{1}{2} \sigma(T) |E|^2 = \frac{1}{2} \omega^2 \sigma(T) |A|^2$$

(6)

where $E$ indicates electric field strength. The total convective heat losses due to turbulent cooling water flow inside the hollow channel of the copper winding was obtained from the equation [13,23]:

$$Q_{\text{cool}} = \frac{dm_w}{dt} C_w \left( T_{\text{w,in}} - T \right)$$

(7)

where $dm_w/dt$ states for the mass flow rate of the cooling water, $T_{\text{w,in}}$ means the inlet temperature of the water inside the hollow channel, $R_0$ – the radius of the helical coil and $S_w = \pi r_1^2$ – the cross section of the circular channel with radius $r_1$.

Knowing magnetic induction $B$ or magnetic vector potential $A$ we are able to determine the inductance of the water-cooled coil as below [24]:
where $\Phi_B$ correspond to the total magnetic flux penetrating surface area of single coil turn, $\mathbf{n}$ is the unit vector normal to the loop surface $S_0 = \pi R_0^2$, $\mathbf{t}$ is the unit tangent vector to a curve $\Gamma$ enclosing the $S_0$-area and $I$ is exciting current passes through the $N$-turned coil. Moreover, a coil resistance is estimated by the integral equation\cite{24,25}:

$$
R = \frac{\int \frac{\rho(T)Nl}{S_0}dS}{\int \frac{2Nr}{\sigma(T)R_0^2}dS}
$$

where $l = 2\pi r$ is the mean length of the single coil turn and $r$ is the first coordinate in the cylindrical system. Of course, reactance $X$ and complex-valued impedance $Z$ of the water-cooled inductor are calculated using well-known formulas, namely $X = \omega L$ and $Z = R + jX$.

3. Obtained results

A helical air coil with $N = 9$ copper windings are cooled from the interior with turbulent flowing water of temperature 20°C and mass flow rate equal to 320 mL/min. Moreover, within the copper coil windings, an excitation current of amplitude 65 A and frequency 532.1 kHz was applied. All material constants used in numerical analysis have been gathered together in table 2. To solve this pretty complex problem, the appropriate boundary and initial conditions should be taken into account. Firstly, the magnetic insulation $\mathbf{n} \times \mathbf{A} = 0$ and temperature of 20°C were assumed on the outer edge of the calculation area. The same temperature was defined as initial temperature $T_0$ in all domains and as inlet temperature of cooling water $T_{w,in}$ inside the water channel. In addition, zero magnetic vector potential $\mathbf{A} = 0$ was assumed within the entire computational space at initial time moment. What is important, all performed calculations for two models of copper winding conductivity as well as under cooling water condition or without it were based on finite element method (FEM). All computations have been done in Comsol Multiphysics software.

Figure 3 shows the dependence of copper conductivity on temperature for two analyzed models of coil winding. As shown below, in general the $\sigma(T)$-dependence of copper conductivity is greatly nonlinear but for small temperature changes it can be successfully linearized (see figure 4).
Table 2. Physical parameters used in electro-thermal simulation.

| Quantity                              | air     | copper | water |
|---------------------------------------|---------|--------|-------|
| relative permittivity $\varepsilon_r$ | 1       | 1e6    | 80    |
| electrical conductivity $\sigma$ (S/m) | 0       | 5.998e7| 5.5e-6|
| electrical resistivity $\rho_0$ (Ω·m) | –       | 1.754e-8| –    |
| electrical resistivity coefficient $\alpha$ (1/K) | –       | 0.0039 | –    |
| relative magnetic permeability $\mu_r$ | 1       | 1      | 1     |
| mass density $\rho$ (kg/m$^3$)       | 1.2     | 8 700  | 998   |
| specific heat $C$ (J/kg/K)            | 1385    | 4 183  |       |
| thermal conductivity $k$ (W/m/K)      | 0.025   | 400    | 0.59  |

Figures 5–8 illustrate behavior of magnetic field within the analyzed copper coil with constant and temperature-dependent electrical conductivity and in the absence of circulating water cooling. Figure 5 represents contours of magnetic flux density $B$ inside the inductor with linear $\sigma_0$-parameter. The largest magnetic field occurs in the vicinity of copper windings at the inner side of the coil, while it practically disappears outside of the inductor. Figure 6 compares the time dependences of magnetic induction in the central point of solenoid $B_0 = B(0,0,0)$ for linear and nonlinear cases of copper windings. Interestingly, for model with $\sigma(T)$-conductivity the long-term growth of $B$-field values from 13.99 mT (equivalent for $\sigma_0$-conductivity) to steady state of 14.67 mT is observed. Following figures 7 and 8 show the magnetic induction distributions along the $r$ and $z$ axes respectively. Also here an increase of magnetic field in the case of nonlinear copper conductivity in the interior as well as exterior of the coil is clearly visible. Detailed data including $B_{\text{max}}$ values is given in table 3.

Figure 5. Contour plot for magnetic flux density inside induction coil obtained from FEM analysis.

Figure 6. Transient distribution of magnetic induction in the center of the inductor for two models of copper electrical conductivity.
Figures 7 and 8 illustrate inductor heating for two analyzed models of copper winding conductivity both in the center of the helical coil as well as inside the hollow winding of the coil. Time-dependent temperature curves excluding the cooling water effects are depicted in figure 9. The similar plots for the water-cooled coil are presented in figure 10. In the absence of circulating water cooling, the temperature profiles both for induction coil center and hollow channel in the coil winding take significant values, but for model with temperature-dependent conductivity of the copper winding they have much greater values and stabilize much slower than in the case of $\sigma_0$-conductivity. Moreover, for inductor under water cooling condition, the temperatures in the middle of coil do not exceed the value of 25.5°C and they are established within just 3 min time-period. In this case, similar temperature levels indicates that assumed mass flow rate of water of 20°C at 320 mL/min is sufficient for effective cooling of the copper coil winding with $\sigma(T)$-conductivity just like in model with constant copper conductivity. Comparison of all data is presented in table 3.

**Figure 7.** Distribution of magnetic flux density along $r$-axis of the inductor for two models of copper winding electrical conductivity.

**Figure 8.** Distribution of magnetic flux density along $z$-axis of the inductor for two models of copper winding electrical conductivity.

**Figure 9.** Transient temperature distributions in the center of the inductor and the hollow coil winding without cooling water flow for two models of copper electrical conductivity.

**Figure 10.** Transient temperature distributions in the center of the inductor and the hollow coil winding under cooling water condition for two models of copper electrical conductivity.
Table 3. Comparison of magnetic field and temperature in the steady state for 9-turned coil with excitation current of 65 A and frequency of 532.1 kHz.

| Models of copper electrical conductivity | $B_0$ [mT] in the coil center | $B_{\text{max}}$ [mT] the highest value | $T$ [$^\circ$C] in the coil center | $T$ [$^\circ$C] in the winding |
|----------------------------------------|-------------------------------|-----------------------------------------|-----------------------------------|-----------------------------|
| constant conductivity $\sigma = \text{const}$ | 13.99                         | 22.30                                   | 6576.4                           | 7004.5                      |
| temperature-dependent conductivity $\sigma = \sigma(T)$ | 14.67                         | 20.53                                   | 44805.2                          | 47933.6                     |
| constant conductivity $\sigma = \text{const}$ | 13.99                         | 22.30                                   | 25.4                             | 20.0                        |
| temperature-dependent conductivity $\sigma = \sigma(T)$ | 14.00                         | 22.32                                   | 25.5                             | 20.0                        |

In the following part of the work, the basic parameters of analyzed inductor such as inductance $L$, resistance $R$, reactance $X$ and impedance $Z$ have been numerically determined (see figures 11 – 14). In all cases, time dependencies of studied parameters are similar to those in figure 6. For model with nonlinear conductivity of copper coil winding and without water cooling effects, the coil inductance fluctuates from about 2.17 µH, its resistance from 0.11 Ω, reactance from 7.26 Ω and module of impedance also from 7.26 Ω, and stabilize after long-term transient state. In other cases, mentioned parameters are virtually unchanged no matter whether the coil is under cooling water conditions or not. It should be noted that very low coil resistance values obtained in this study cause slight changes in the coil impedance module relative to its reactance. All detailed data of interest is gathered together in table 4.

Figure 11. Transient distribution of inductance of the coil without circulating water for two models of copper electrical conductivity.

Figure 12. Transient distribution of resistance of the inductor without circulating water flow for two models of copper electrical conductivity.
Figure 13. Transient distribution of reactance of the inductor without cooling water flow for two models of copper electrical conductivity.

Figure 14. Transient distribution of module impedance of coil without cooling water flow for two models of copper electrical conductivity.

| Models of copper electrical conductivity | Inductance L [µH] | Resistance R [Ω] | Reactance X [Ω] | Impedance | | Z | [Ω] |
|----------------------------------------|-----------------|-----------------|-----------------|------------|-----------------|-----------------|-----------------|
| coil without water cooling             |                 |                 |                 |            |                 |                 |                 |
| constant conductivity  \( \sigma = \text{const} \) | 2.171           | 0.107           | 7.259           | 7.260      |
| temperature-dependent conductivity  \( \sigma = \sigma(T) \) | 2.385           | 0.796           | 7.974           | 8.014      |
| coil with water cooling                |                 |                 |                 |            |                 |                 |                 |
| constant conductivity  \( \sigma = \text{const} \) | 2.173           | 0.109           | 7.265           | 7.266      |

4. Summary

Nowadays the magnetic fluid hyperthermia is highly popular technique because of the possibility to investigate the thermal efficiency of ferrofluids used in hyperthermia treatment of cancerous tissues. The coils utilized in this procedure generally have a helical shape and are cooled with circulating water to reduce non-specific heating effects of the tested samples of magnetic fluid and to protect expensive professional sensors typically localized inside them.

In this paper, the influence of temperature-dependent electrical conductivity model of copper coil windings on the resulting magnetic field and temperature distribution inside the inductor was evaluated. The coil behavior was examined both under and without water cooling conditions and then compared with a linear model with constant copper conductivity. It has been shown that coil model with nonlinear conductivity of copper winding and without cooling by circulating cold water is characterized by the long-term transient distributions of parameters of interest that disappears under water cooling condition. Interestingly, this is reflected both in temperature and magnetic field values occurring inside the inductor as well as in coil parameters such as inductance, resistance, reactance,
and impedance. It can be stated that usage of water-cooled inductors causes that the negative heating effects of the copper coil winding disappear. What is more, under cooling water conditions a model with nonlinear copper conductivity can be successfully replaced by a model with constant conductivity of copper which simplifies the numerical procedure.

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