Can the angular momentum of $u$-quarks in the nucleon be accessed at HERMES?

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Abstract. We investigate the possibility to acquire information on the generalized parton distribution $E$ and, through a model for $E$, also on the $u$-quark total angular momentum $J_u$ by studying deeply virtual Compton scattering and hard exclusive $\rho^0$ electroproduction on a transversely polarized hydrogen target at HERMES. It is found that a change in $J_u$ from zero to 0.4 corresponds to a $4\sigma$ ($2\sigma$) difference in the calculated transverse target-spin asymmetry in deeply virtual Compton scattering ($\rho^0$ electroproduction), where $\sigma$ is the total experimental uncertainty.

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1 Introduction

Over more than two decades, inclusive and semi-inclusive charged lepton scattering has been used as a powerful tool to successfully study the longitudinal momentum structure of the nucleon, which was parameterized in terms of parton distribution functions (PDFs). Hard exclusive reactions can be described in the theoretical framework of generalized parton distributions (GPDs) \[1,2,3,4,5\]. Their application became apparent after it had been shown \[6\] that measurements of the second moment of the sum of the ‘unpolarized’ GPDs $H$ and $E$ open, for the first time, access to the total angular momentum of partons in the nucleon:

$$J_a(Q^2) = \frac{1}{2}\lim_{t\to 0} \int_{-1}^{1} x [H_a(x, \xi, t, Q^2) + E_a(x, \xi, t, Q^2)] dx.$$  \hspace{1cm} (1)

In this relation $H_a(x, \xi, t, Q^2)$ and $E_a(x, \xi, t, Q^2)$ denote parton spin non-flip and spin flip GPDs ($a = u, d, s$), respectively\textsuperscript{1}. GPDs depend on the fractions $x$ and $\xi$ of longitudinal momentum of the proton carried by the parton and on $t = (p_1 - p_2)^2$, the square of the 4-momentum transfer between initial and final protons (see. Fig. 1). As ordinary PDFs, also GPDs are subject to QCD evolution. Their $Q^2$ dependence has been perturbatively calculated up to next-to-leading order in $\alpha_s$ \[5\] and is omitted in the notations throughout the paper.

\textsuperscript{1} Throughout this paper the GPD definitions of a recent review \[7\] are used.

Recently, a simultaneous description of the transverse spatial and the longitudinal momentum structure of the nucleon was shown to be an appealing interpretation of GPDs \[9,10,11,12\]. The concept of GPDs covers several types of processes, ranging from inclusive deeply inelastic lepton scattering to hard exclusive Compton scattering and meson production. Measurements of GPDs are expected to shed light especially on the hitherto theoretically uncharted territory of long-range (‘soft’) phenomena where parton-parton correlations are known to play an important role.

First steps towards the extraction of the GPD $H$ have already been performed by scattering leptons off unpolarized protons through measurements of either cross sections \[13,14\], or cross section asymmetries with respect to beam charge \[15\] or beam spin \[16,17\]. Future measure-
ments of the transverse target-spin asymmetry (TTSA) in hard exclusive electroproduction of a real photon (deeply virtual Compton scattering, DVCS) or a vector meson offer the possibility to acquire information on the spin-flip GPD $E$. The most promising experiments to access it are those running at intermediate energy, where the spin-flip amplitude is expected to be sizable, while at higher energies it is suppressed due to $s$-channel helicity conservation. Thus at present a realistic program may be envisaged for HERMES, CLAS and possibly COMPASS. In this paper the prospects are discussed for HERMES measurements of TTSA in DVCS and $\rho^0$ electroproduction, and in particular their sensitivity to the $u$-quark total angular momentum.

2 Modeling Generalized Parton Distributions

GPDs are most commonly parameterized using an ansatz based on double distributions [15,19] complemented with the D-term [20]. Factorizing out the $t$-dependence, the non-forward GPDs can be related to the ordinary PDFs and the proton elastic form factors. In this framework [21], the spin non-flip GPD $H$ is given by

$$H_q(x, \xi) = \frac{1}{1-t/4m^2} \frac{H_{q,q}(x, \xi)}{(1-t/0.71)^2}, \quad (2)$$

where $\kappa_p=1.793$ is the proton anomalous magnetic moment and $m$ is the proton mass. The neutron Dirac form factor is neglected compared to the one of the proton.

For quarks, the $t$-independent part of the GPDs $H_q$ is written as

$$H_q(x, \xi) = H_{q,DD}(x, \xi) + \theta(x - |x|) D_q \left( \frac{x}{\xi} \right), \quad (3)$$

where $D_q \left( \frac{x}{\xi} \right)$ is the D-term, and $H_{q,DD}$ is the part of the GPD that is obtained from the double distribution (DD) $F_q$:

$$H_{q,DD}(x, \xi) = \int_{-1}^{1} \frac{d|\beta|}{1+|\beta|} \int_{-1+|\beta|}^{1} d\alpha \delta(x - \beta - \alpha x) F_q(\beta, \alpha). \quad (4)$$

For the double distributions the suggestion of Ref. [18] is used,

$$F_q(\beta, \alpha) = h(\beta, \alpha) q(\beta), \quad (5)$$

where the profile function is given by [19]:

$$h(\beta, \alpha) = \frac{\Gamma(2b+2)}{2^{2b+1} \Gamma(b+1)} \frac{[1 - |\beta|^2 - \alpha^2]^b}{(1 - |\beta|^2)^{2b+1}}. \quad (6)$$

For $\beta > 0$, $q(\beta) = q_{val}(\beta) + q(\beta)$ is the ordinary quark density for the flavor $q$. The negative $\beta$ range corresponds to the antiquark density: $q(-\beta) = -\bar{q}(\beta)$. The parameter $b$ characterizes to what extent the GPD depends on the skewness $\xi$. In the limit $b \rightarrow \infty$ the GPD is independent on $\xi$, i.e., $H(x, \xi) = q(x)$. Note that $b$ is a free parameter for valence quarks ($b_{val}$) or sea quarks ($b_{sea}$) and thus can be used as a fit parameter in the extraction of GPDs from hard electroproduction data [22].

For gluons, the $t$-independent part of the GPD $H_g$ is directly given by the double distribution,

$$H_g(x, \xi) = H_{g,DD}(x, \xi) = \int_{-1}^{1} \frac{d\beta}{1-|\beta|} \int_{-1+|\beta|}^{1} d\alpha \delta(x - \beta - \alpha x) \beta F_g(\beta, \alpha) \quad (7)$$

with the same form of the profile function in the double distribution

$$F_g(\beta, \alpha) = h(\beta, \alpha) g(\beta). \quad (8)$$

The $t$-dependence for gluons is taken to be the same as that for quarks.

The factorized ansatz [2] is the simplest way of modeling GPDs. However, experimental studies of elastic diffractive processes indicate that the $t$-dependence of the cross section is entangled with its dependence on the photon-nucleon invariant mass [23]. Recent evidence comes from lattice QCD calculations [21,25] and phenomenological considerations [20,24]. The non-factorized ansatz can be based on soft Regge-type parameterizations. In this case, the $t$-dependence is not factorized out and not controlled by a form factor as in Eq. (2). Instead, it is kept in Eqs. (3), (4) and (7). The $t$-dependence of double distributions is then modeled as [21]:

$$F_{q,g}(\beta, \alpha, t) = F_{q,g}(\beta, \alpha) \frac{1}{|\beta|^2 t}, \quad (9)$$

which is referred to as Regge ansatz in the following. Here $\alpha'$ is the slope of the Regge trajectory, $\alpha'_q = 0.8$ GeV$^{-2}$ for quarks and $\alpha'_g = 0.25$ GeV$^{-2}$ for gluons.

In the factored ansatz the spin-flip quark GPDs $E_{q}$ are given by [21]:

$$E_{q}(x, \xi, t) = \frac{E_{q}(x, \xi)}{(1-t/0.71)^2}. \quad (10)$$

In the Regge ansatz the $t$-dependence is modeled in analogy to Eq. (9).

The $t$-independent part is parameterized using the double distribution ansatz:

$$E_{q}(x, \xi) = E_{q,DD}(x, \xi) - \theta(x - |x|) D_q \left( \frac{x}{\xi} \right). \quad (11)$$

Note that the $D$-term has the same size, but the opposite sign in Eqs. (11) and (3). Therefore, it drops out when calculating $J_q$ according to Eq. (1).

The double distribution has a form analogous to the spin-nonflip case:

$$E_{q,DD}(x, \xi) = \int_{-1}^{1} \frac{d\beta}{1-|\beta|} \int_{-1+|\beta|}^{1} d\alpha \delta(x - \beta - \alpha x) K_q(\beta, \alpha). \quad (12)$$
with:

\[ K_q(\beta, \alpha) = h(\beta, \alpha) e_q(\beta). \] (13)

The spin-flip parton densities \( e_q(x) \) cannot be extracted from deep-inelastic scattering (DIS) data, unlike the case of spin non-flip ones. Based on the chiral quark soliton model [21], the spin-flip density is taken as a sum of valence and sea quarks contributions. Since in this model the sea part was found to be very narrowly peaked around \( x = 0 \), the whole density is written as:

\[ e_q(x) = A_q q_{val}(x) + B_q \delta(x). \] (14)

In this expression, the shape of the valence quark part is given by that of the spin non-flip density. The coefficients \( A_q \) and \( B_q \) are constrained by the total angular momentum sum rule [11] and the normalization condition

\[ \int_{-1}^{+1} dx \ e_q(x) = \kappa_q, \] (15)

where \( \kappa_q \) is the anomalous magnetic moment of quarks of flavor \( q \) (\( \kappa_u = 2 \kappa_p + \kappa_n = 1.67, \kappa_d = \kappa_p + 2 \kappa_n = -2.03 \)). The constraints yield:

\[ A_u = \frac{2 J_u - M_u^{(2)}}{M_{q,val}^{(2)}}, \] (16)

\[ B_u = 2 \left[ \frac{1}{2} \kappa_u - \frac{2 J_u - M_u^{(2)}}{M_{q,val}^{(2)}} \right], \] (17)

\[ B_d = \kappa_d - \frac{2 J_d - M_d^{(2)}}{M_{q,val}^{(2)}}. \] (18)

Here \( M_{q,val}^{(2)} \) and \( M_{q,val}^{(2)} \) are the parton momentum contributions to the proton momentum:

\[ M_{q,val}^{(2)} = \int x q_{val}(x) dx, \quad M_{q,val}^{(2)} = \int x [q_{val}(x) + 2 \bar{q}(x)] dx. \] (19)

In the given scenario the total angular momenta carried by \( u \) - and \( d \)-quarks, \( J_u \) and \( J_d \), enter directly as free parameters in the parameterization of the spin-flip GPD \( E_q(x, \xi, t) \). Hence the parameterization [11] can be used to investigate the sensitivity of hard electroproduction observables to variations in \( J_u \) and \( J_d \).

As to the gluons, there exists no hint how the spin-flip GPD \( E_g \) could be described. There is an expectation that \( E_g \) is not large compared to \( E_u \) and \( E_d \) [22]. Hence, for simplicity throughout the present study \( E_g \) is neglected (“passive” gluons, i.e. \( E_g = 0 \)).

As an example, Fig. 2 shows the \( t \)-independent part of quark and gluon GPDs at \( Q^2 = 4 \) GeV\(^2\), \( \xi = 0.1 \) (MRST98 PDFs are used).

\( d \)-quark PDFs can be safely neglected. Since gluons are absent in leading-order DVCS, uncertainties resulting from gluon PDFs are of little influence for DVCS asymmetries and have been found to lead to a fractional change of up to 15% for the \( J^2_q \) asymmetries. For the following calculations the MRST98 PDF set is taken.

### 3 Sensitivity of DVCS to the \( u \)-quark Total Angular Momentum

#### 3.1 Cross Section and Asymmetries

The 5-fold cross section for the process \( e(k) + p(p_1) \rightarrow e(k') + p(p_2) + \gamma(q_2) \) is given by:

\[
\frac{d\sigma}{dx_B dQ^2 dt d\phi d\phi_S} = \frac{\alpha_m^2 x_B y}{16\pi^2 Q^2 \sqrt{1 + 4x_B^2 Q^2 / Q'^2}}, \quad |T|^2 = \frac{|T_{BH} + T_{DVCS}|^2}{|T_{BH}|^2 + |T_{DVCS}|^2 + I}, \quad (21)
\]

where \( Q' = -q_2^2 \) is the negative squared 4-momentum of the virtual photon, \( x_B = Q^2 / (2p_1 \cdot q_1) \) is the Bjorken variable, \( t = (p_1 - p_2)^2, y = (p_1 \cdot q_1) / (p_1 \cdot k) \). \( T \) denotes the photon production amplitude and \( e \) is the electron charge. Since the DVCS and Bethe-Heitler (BH) processes have an identical final state, in which the photon is radiated either from a parton or from a lepton, respectively, \( T \) is given by the coherent sum of the BH amplitude \( T_{BH} \) and the DVCS amplitude \( T_{DVCS} \):

\[
|T|^2 = |T_{BH} + T_{DVCS}|^2 = |T_{BH}|^2 + |T_{DVCS}|^2 + I, \quad (21)
\]

in which

\[
I = T_{BH} T_{DVCS} + T_{BH} T_{DVCS}^* \quad (22)
\]

describes the interference between both processes.

The coordinate system is defined in the target rest frame, as explained in Fig. 3. The theoretical formulae used below refer to the target being transversely polarized w.r.t. the virtual photon direction, while in the experiment the target polarization is transverse w.r.t. the incident lepton direction. At HERMES kinematics, these two directions are approximately parallel and the small
The leading twist and leading order $c_{1, unp}$ contributions of the DVCS-BH interference term to the total cross section can be approximated as:

$$A_{UT}(\phi - \phi_S) \approx \Im \frac{M}{c_{0, unp}} \sin (\phi - \phi_S) \cdot \sin \phi.$$ 

Here $+$ ($-$) stands for a negatively (positively) charged lepton beam and $f(x_B, y, Q^2)$ is a kinematic pre-factor independent of azimuthal angles. The full expressions for $c_{1, unp}$ can be found in Eqs. (53-56) of Ref. [33]. $\hat{M}_N$ and $\hat{M}_S$ are certain linear combinations of the Compton form factors $H$, $E$, $\bar{H}$ and $\bar{E}$, which are convolutions of the respective twist-2 GPDs $H$, $E$, $\bar{H}$ and $\bar{E}$ with the hard-scattering kernels as defined in Eq. (9) of Ref. [33].

The full expressions for $\hat{M}_N$ and $\hat{M}_S$ can be found in Eq. (71) in Ref. [33] or in Eq. (60) in Ref. [34]. Since $x \approx x_B/(2-x_B)$ is small in a wide range of experimentally relevant kinematics, terms with pre-factor $x_B$ or $x_B$ can be neglected, except for the GPD $\bar{E}$ because the pion pole contribution to $\bar{E}$ scales like $\xi^{-1}$, so that $\hat{M}_N$ and $\hat{M}_S$ can be approximated as:

$$\hat{M}_N \approx -\frac{t}{4M^2} \cdot [F_2 H - F_1 \bar{E}],$$

$$\hat{M}_S \approx -\frac{t}{4M^2} \cdot [F_2 \bar{H} - F_1 \xi \bar{E}].$$

(26)

Here $F_1$ and $F_2$ are the Dirac and Pauli form factors of the proton, respectively.

In order to constrain the GPDs involved in Eq. (26), the transverse polarization component of the interference term, $I_{TP}$, has to be singled out. This can be accomplished by forming the transverse (T) target-spin asymmetry with unpolarized (U) beam:

$$A_{UT}(\phi - \phi_S) = \frac{d\sigma(\phi - \phi_S) - d\sigma(\phi + \phi_S + \pi)}{d\sigma(\phi - \phi_S) + d\sigma(\phi + \phi_S + \pi)}$$

$$\approx A_{UT}^\sin (\phi - \phi_S) \sin \phi + A_{UT}^\cos (\phi - \phi_S) \cos \phi.\cos \phi.$$ 

As $|T_{unp}^BH|^2$ and $I_{unp}$ are independent on $\phi - \phi_S$, they do not appear in the numerators of Eq. (27). Since their dominant contribution to the denominator in Eq. (27) is given by $e_{0, unp}$, the two amplitudes of the TTSA, $A_{UT}^\sin (\phi - \phi_S) \sin \phi$ and $A_{UT}^\cos (\phi - \phi_S) \sin \phi$, can be approximated as:

$$A_{UT}^\sin (\phi - \phi_S) \sin \phi \approx \pm f(x_B, y, Q^2) \cdot \Im \hat{M}_N,$$

$$A_{UT}^\cos (\phi - \phi_S) \sin \phi \approx \pm f(x_B, y, Q^2) \cdot \Im \hat{M}_S.$$ 

(29)

Note that the approximations used in this section are for illustrative purposes only and are not used in the numerical calculations described below.

3.2 Expected Value of TTSA and Projected Statistical Uncertainty

Since in the DVCS process the gluons enter only in NLO in $\phi_S$, their contributions to cross section and TTSA are neglected. For the quarks, it can be seen from Eq. (20)
that, besides the GPDs $H$ and $E$ which have been discussed in Sect. 2 there are two other GPDs, $H$ and $E$, involved in the TTSA for DVCS. Since they are not the main interest of this paper, in the calculations below they are always included and kept unchanged. In their model description, the forward limit of the GPD $H$ is fixed by the quark helicity distributions $\Delta q(x, \mu^2)$, while the GPD $E$ is evaluated from the pion pole which only provides a real part to $\tilde{M}_S$ in Eq. (26).

At present, there exists a code [35] designed to calculate observables in the exclusive reaction $ep \to e\gamma p$. It has been used (see App. 5) to evaluate the TTSA arising from the DVCS-BH interference. The TTSA is calculated at the average kinematic values per bin in $x_B$, $Q^2$ and $t$ taken from a measurement of the beam-spin asymmetry in DVCS at HERMES [22] (see Tab. 1).

The statistical error of an asymmetry is independent on its size if the asymmetry itself is small. For a single beam (target) spin asymmetry it is obtained as:

$$\sigma^2_{\text{stat}} \propto \frac{1}{N} \frac{1}{P^2_{\text{beam}(\text{target})}},$$ (30)

where $N$ is the total number of events that is proportional to the integrated luminosity, and $P_{\text{beam}(\text{target})}$ is the beam (target) polarization. The following projection is based on a future HERMES data set of 8 million DIS events to be taken with an unpolarized positron beam and a transversely polarized hydrogen target. Using the known statistical errors of the beam-spin asymmetry measurement at HERMES on an unpolarized hydrogen target (7 million DIS events, $P_{\text{beam}} \simeq 50\%$) [22], the projected statistical error for the TTSA is obtained.

### Table 1. Average kinematic values for $Q^2$, $x_B$, $-t$ bins and statistical errors, taken from a measurement of the beam-spin asymmetry at HERMES [22].

| $Q^2$ bin (GeV$^2$) | 1.00-1.50 | 1.50-2.30 | 2.30-3.50 | 3.50-4.00 | 6.00-10.00 |
|-------------------|---------|---------|---------|---------|---------|
| $(Q^2)$ (GeV$^2$) | 1.2     | 1.8     | 2.8     | 4.4     | 7.1     |
| $(x_B)$           | 0.06    | 0.08    | 0.10    | 0.14    | 0.24    |
| $(-t)$ (GeV$^2$)  | 0.07    | 0.09    | 0.12    | 0.17    | 0.24    |
| stat. $\delta_{MT}^{(x_B)}$ | 0.053 | 0.059 | 0.064 | 0.070 | 0.163 |
| $x_B$ bin         | 0.05-0.07 | 0.07-0.10 | 0.10-0.13 | 0.15-0.20 | 0.20-0.35 |
| $(Q^2)$ (GeV$^2$) | 2.4     | 2.6     | 3.4     | 4.5     | 6.7     |
| $(x_B)$           | 0.05    | 0.07    | 0.12    | 0.17    | 0.24    |
| $(-t)$ (GeV$^2$)  | 0.08    | 0.10    | 0.12    | 0.17    | 0.22    |
| stat. $\delta_{MT}^{(x_B)}$ | 0.048 | 0.053 | 0.060 | 0.099 | 0.145 |
| $-t$ bin (GeV$^2$) | 0.00-0.06 | 0.06-0.14 | 0.14-0.30 | 0.30-0.50 | 0.50-0.70 |
| $(Q^2)$ (GeV$^2$) | 2.0     | 2.5     | 3.0     | 3.6     | 3.9     |
| $(x_B)$           | 0.08    | 0.10    | 0.11    | 0.12    | 0.12    |
| $(-t)$ (GeV$^2$)  | 0.03    | 0.04    | 0.20    | 0.37    | 0.57    |
| stat. $\delta_{MT}^{(-t)}$ | 0.041 | 0.052 | 0.066 | 0.126 | 0.263 |

The projections for $A_{UT}^{\sin(\phi_2-\phi)} \cos \phi$ and $A_{UT}^{\cos(\phi_2-\phi)} \sin \phi$ in the Regge ansatz for $b_{u\!\!\!t} = 1$, $b_{s\!\!\!t} = \infty$, $J_d = 0.4$ (0.2, 0.4), $J_d = 0.0$. $E = 0$ denotes zero effective contribution from the quark GPDs $E_q$. The calculations are done at the average kinematic values as listed in Tab. 1. Projected statistical errors are shown.

![Fig. 4. Expected DVCS TTSA amplitudes $A_{UT}^{\sin(\phi_2-\phi)} \cos \phi$ and $A_{UT}^{\cos(\phi_2-\phi)} \sin \phi$ in the Regge ansatz for $b_{u\!\!\!t} = 1$, $b_{s\!\!\!t} = \infty$, $J_d = 0.4$ (0.2, 0.4), $J_d = 0.0$. $E = 0$ denotes zero effective contribution from the quark GPDs $E_q$. The calculations are done at the average kinematic values as listed in Tab. 1. Projected statistical errors are shown.](image)

As expected from Eqs. 26 and 29, variations in the parameter settings for the GPD $E$ become manifest in $A_{UT}^{\sin(\phi_2-\phi)} \cos \phi$ while $A_{UT}^{\cos(\phi_2-\phi)} \sin \phi$ shows only minor modifications. The latter are apparent only in the kinematic regime of large $x_B$ or correspondingly large $Q^2$ since the contribution of the GPDs $E_q$ to $\tilde{M}_S$ is suppressed by $x_B$ and thus has been neglected in Eq. 26. Within these model calculations $A_{UT}^{\sin(\phi_2-\phi)} \cos \phi$ turns out to be sizable even when the calculation is done for $E_q = 0$. Thus a solid knowledge about the GPD $H_u$ is needed in order to con-
strain \( J_u \). It has been shown \[27\] that the model parameters for the GPD \( H_u \), in particular the size of the profile parameters \( b_{\text{ca}} \) and \( b_{\text{sea}} \), can be well constrained by the envisaged HERMES DVCS measurements until 2007, using an unpolarized hydrogen target. Since in addition the profile parameters are assumed to be the same for the GPD \( E_u \), the only remaining free parameter is \( J_u \). Hence the projected measurement of \( A_{UT}^{\sin(\phi - \phi_0) \cos \phi} \) has a clear potential to constrain \( J_u \), as can be seen from the left panels of Figs. 4 and 5.

The discriminative power of the envisaged TTSA measurement can be enhanced by combining all data into one point, since all considered models show the same kinematic dependences. The corresponding statistical power of a HERMES data set based on 8 million DIS events is shown in Fig. 6 for \( b_{\text{sea}} \) equal to one or infinity, and for three different values of the total \( u \)-quark angular momentum \( J_u \) plus the special case \( E_q = 0 \).

It appears that for \( b_{\text{sea}} = 1 \) (\( b_{\text{sea}} = \infty \)) the amplitude ranges between values of \(-0.17 \) and \(-0.27 \) (-0.19 and -0.29) when \( J_u \) ranges between 0.4 and 0. The projected statistical error for these integrated TTSA amplitudes is 0.017. Extrapolating the knowledge on the systematic uncertainty from the analysis of 2000 HERMES data \[22\], its size can be expected to not exceed the statistical error, such that a total experimental uncertainty below 0.025 appears as a realistic estimate. Altogether, the difference in the size of the TTSA due to a change of \( J_u \) between zero and 0.4 corresponds to a 4\( \sigma \) effect, where \( \sigma \) denotes the total experimental uncertainty. Thus, based on the GPD model used it can be expected that the upcoming DVCS results from HERMES\(^3\) will provide a constraint on the size of \( J_u \).

4 Sensitivity of Elastic \( \rho^0 \) Electroproduction to the \( u \)-quark Total Angular Momentum

Also a measurement of the TTSA in elastic vector meson electroproduction can be a source of information about the spin-flip generalized parton distribution \( E \). An estimate for the asymmetry was obtained in Ref. \[21\], using the factorized model of GPDs described in Sect. 2 without inclusion of gluons. The scope of this section is to also include the Regge ansatz, to check the assumption that the gluon contribution to the \( \rho^0 \) electroproduction cross section is small, and to eventually calculate the size of the TTSA at HERMES kinematics. The issue is raised since in contrast to DVCS, in vector meson elastic electroproduction gluons enter at the same order of magnitude ranges between values of \(-0.17 \) and \(-0.27 \) (-0.19 and -0.29) when \( J_u \) ranges between 0.4 and 0. The projected statistical error for these integrated TTSA amplitudes is 0.017. Extrapolating the knowledge on the systematic uncertainty from the analysis of 2000 HERMES data \[22\], its size can be expected to not exceed the statistical error, such that a total experimental uncertainty below 0.025 appears as a realistic estimate. Altogether, the difference in the size of the TTSA due to a change of \( J_u \) between zero and 0.4 corresponds to a 4\( \sigma \) effect, where \( \sigma \) denotes the total experimental uncertainty. Thus, based on the GPD model used it can be expected that the upcoming DVCS results from HERMES\(^3\) will provide a constraint on the size of \( J_u \).

4.1 Cross Section and Gluonic Contribution

It was shown \[25\] that the leading twist contribution to exclusive electroproduction of vector mesons requires both the virtual photon and the vector meson to be longitudinal, \( i.e. \) transversely polarized. Therefore the present

\[3\] The recent switch of the accelerator (HERA) to an electron beam will require to also perform the above calculations for the negative beam charge. However, the sensitivity to \( J_u \) of the combined electron and positron measurements is expected to be similar to the one calculated here for a positron beam only.
The cross section of the reaction $\gamma_L^* p \to \rho^0 p'$ is given by \[ \frac{d\sigma_L}{dt} = \frac{1}{8m^2(W^2-m^2)^2|q_1|^4} \left( |T_A|^2 + |T_B|^2 \right), \] where $q_1$ is the momentum of the virtual photon in the center of mass system of this photon and the initial proton, while $W$ is their invariant mass. The spin-flip amplitude reads

\[ T_A = -ie^{2\sqrt{2}\pi\alpha_s} A \bar{u}(p_2)n^\mu\gamma_\mu u(p_1) \int \frac{dz}{z} \Phi(z), \]

and the spin-flip one is:

\[ T_B = e^{2\sqrt{2}\pi\alpha_s} B \bar{u}(p_2)\sigma^{\mu\nu}n_\mu\Delta_\nu u(p_1) \int \frac{dz}{z} \Phi(z). \]

Here $n = (1, 0, 0, -1)/(\sqrt{2}(p_1 + p_2)^\perp)$ is a light-like vector along the $z$-axis, $\Delta = p_2 - p_1$ is the 4-momentum transfer ($\Delta^2 = t$). The modulus of its transverse component is given by $|\Delta_T| = \sqrt{-t}(1 - \xi^2) - 4Q^2m^2$. The $\rho^0$-meson wave function is taken in the form

\[ \Phi(z) = 6z(1-z)f_\rho \]

with $f_\rho = 0.216$ GeV and $z$ being the meson longitudinal momentum fraction carried by a parton. The complex factors $A$ and $B$ are given by:

\[ A = \frac{1}{\sqrt{2}} \int \left( e_u H_u(x, \xi, t) - e_d H_d(x, \xi, t) - \frac{3}{8}(e_u - e_d) \frac{H_d(x, \xi, t)}{x} \right) \times \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\} dx, \]

\[ B = \frac{1}{\sqrt{2}} \int \left( e_u E_u(x, \xi, t) - e_d E_d(x, \xi, t) - \frac{3}{8}(e_u - e_d) \frac{E_d(x, \xi, t)}{x} \right) \times \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\} dx. \]

The TTSA is defined as

\[ A_{UT}(\phi - \phi_S) = \frac{d\sigma(\phi - \phi_S) - d\sigma(\phi - \phi_S + \pi)}{d\sigma(\phi - \phi_S) + d\sigma(\phi - \phi_S + \pi)} = A_{UT}^{\sin(\phi - \phi_S)} \sin(\phi - \phi_S). \]

The $A_{UT}^{\sin(\phi - \phi_S)}$ amplitude of the TTSA can be expressed in terms of the spin flip and spin non-flip amplitudes as

\[ A_{UT}^{\sin(\phi - \phi_S)} = \frac{Im(AB^*)|\Delta_T|}{m(1 - \xi^2)|A|^2 - (\xi^2 + \frac{4}{m^2})|B|^2 - 2\xi^2 Re(AB^*)}. \]

Note that using the Trento convention the sign of this equation is opposite to that in Ref. \[21\] and the normalization is larger by a factor of $\pi/2$.

The cross section is calculated using both factorized and Regge ansätze for GPDs. The value $b = 1$ is taken for the profile parameter both for sea and valence quarks. It is found that using $b_{sea} = \infty$ instead of $b_{sea} = 1$ leads to a rise of the cross section by a factor of about 1.15. The value $b_{val} = b_{sea} = 1$ is chosen to provide a direct comparison to previous calculations \[39, 21\]. The value of the profile parameter for gluons is chosen as $b = 2$ and it has been checked that choosing $b = 1$ or $\infty$ does not change the cross section by more than 20\%. The $W$-dependence of the cross section for $Q^2 = 4$ GeV$^2$ is shown in Fig. 7. For both ansätze the calculations overshoot considerably the experimental data from HERMES. However, a significant reduction of the calculated cross section might be expected if transverse motion effects are taken into account \[21, 39\]. On the other hand, also the double distribution based calculations of the DVCS cross section have been found to overshoot the data from H1 \[11, 12\].

An unexpected result of the calculation shown in Fig. 7 is a quite small (15-20\%) pure quark contribution to the cross section, while in Refs. \[21, 39\] the quark contribution was found to be dominant. Comparing the calculated quark contribution to experimental data (Fig. 7) it could also be concluded that the gluon contribution in the present calculation is substantially underestimated, while the quark contribution itself is reasonable and can explain alone (in the factorized ansätze) the value of the measured cross section. However, there exists experimental evidence that the gluon spin non-flip part is indeed large \[25\].

On the amplitude level, the cross sections of $\rho^0$ and $\phi$ mesons are given as:

\[ \sigma_{\rho^0} = C_{\rho^0}(|T_f + T_g|^2), \]

\[ \sigma_{\phi} = \frac{2}{9} C_{\phi}|T_g|^2. \]

\[ \sigma_{\rho^0} = C_{\rho^0}(|T_f|^2 + 2|T_f||T_g|\cos(\varphi_{fg}) + |T_g|^2), \]

\[ \sigma_{\phi} = \frac{2}{9} C_{\phi}|T_g|^2. \]

The principal values of the integrals in Eqs. (35) and (36) are calculated in the following way:

\[ \int_a^b f(x) dx = (f(b) \ln(b) - f(a) \ln(a) - \int_a^b f'(x) \ln(x) dx) + \int_a^b f(x) \ln(x) dx. \]

In this way the non-integrable singularity is exchanged by an integrable one.

\[ \int_a^b f(x) dx = (f(b) \ln(b) - f(a) \ln(a) - \int_a^b f'(x) \ln(x) dx) + \int_a^b f(x) \ln(x) dx. \]
Here $T_q$ is the quark amplitude and $T_g$ the gluon amplitude. In the existing GPD-based calculations \cite{21,31}, both quark and gluon contributions are dominated by the imaginary parts which have the same sign, i.e. $\varphi_{qq} \approx 0$. In the present calculation $\varphi_{gg} \approx 30^\circ$ is obtained. Considering the wave functions of $\rho^0$ and $\phi$ mesons to be similar (as it is supported by the measured values of their decay widths), $C_{\rho^0} \simeq C_{\phi}$ follows, and the ratio of $\varphi$ to $\rho^0$ cross sections reads:

$$\frac{\sigma_{\varphi}}{\sigma_{\rho^0}} = \frac{2}{9} \left| \frac{T_q}{|T_q|^2 + 2|T_g|^2} \right| \cos(\varphi_{qq}) + \left| T_g \right|^2. \tag{40}$$

At HERMES, the ratio of $\sigma_{\varphi}/\sigma_{\rho^0}$ was measured \cite{41}. The experimental value was $0.08 \pm 0.01$, slightly increasing with $Q^2$. Inserting it to the l.h.s of Eq. (40) and taking $\varphi_{gg} = 0^\circ (30^\circ)$ yields $\left| \frac{T_q}{T_g} \right|_{HERMES} = 0.7 (0.78)$. This value is in good agreement with the results of the present calculation, where $\left| \frac{T_q}{T_g} \right|$ ranges between 0.8 and 0.5 (0.5 and 0.3) for the factorized (Regge) ansatz when $W$ increases from 4 to 6 GeV. This is in contrast to the above mentioned result of a dominant quark contribution, $\left| \frac{T_q}{T_g} \right| \approx 3 \, \tilde{3} \, 0 \to 21$. Hence, it is concluded that $H_g$ can not be neglected, i.e. to arrive at the measured cross section both quark and gluon amplitudes have to be scaled down in a similar proportion.

**4.2 Expected Value of TTSA and Projected Statistical Uncertainty**

The $A_{UT}^{\sin(\varphi_{qq})}$ amplitude of the TTSA in Eq. (8) is calculated at HERMES kinematics. The statistical error is extrapolated from a preliminary analysis of the HERMES longitudinal target-spin asymmetry measured on the deuteron \cite{35} that is based on 8 million DIS events. In the latter analysis the data is not split into parts corresponding to longitudinal and transverse virtual photons, while the present calculation is related to longitudinal photons only. At HERMES kinematics ($Q^2 \approx 2$ GeV$^2$), longitudinal photons constitute about 50\% of all virtual photons. Also, the transverse target polarization is 0.75 while the longitudinal one is 0.85. The projected statistical error for 8 million DIS events taken on a transversely polarized target is then larger by a factor $\sqrt{0.75/0.85} = 1.6$ compared to that of Ref. \cite{15}. Note that this error estimate may be considered ‘optimistic’, since it assumes that the contribution to the asymmetry from longitudinal and transverse photons can be completely disentangled.

The calculated $x_B$ and $t$-dependences of $A_{UT}^{\sin(\varphi_{qq})}$ are shown in Fig. 8 for different values of $J_d$. As in the case of DVCS, $J_d$ is fixed inspired by the fact that the $d$-quark contribution is still suppressed, although the suppression in $\rho^0$ production is half as strong as in DVCS. Again, the choice of $J_d = 0$ is based on the results of recent lattice calculations (see e.g. Ref. \cite{35}). Note that in contrast to DVCS, $E = 0$ results in a vanishing asymmetry. As it can be seen from comparing Fig. 8 to Figs. 4 and 5, the expected magnitude of $A_{UT}^{\sin(\varphi_{qq})}$ in $\rho^0$ production is much smaller than that in DVCS. This is due to a large gluonic contribution to the asymmetry, which is considered as “passive” ($E_g = 0$, $H_g \neq 0$), i.e. the gluons dilute the asymmetry in this case. It was found that the difference in $A_{UT}^{\sin(\varphi_{qq})}$ between the factorized and Regge ansatz is negligible. Also the variation of $b_{sea}$ only leads to a small difference as can be seen when comparing the left and right panels of Fig. 8, where $x_B$- and $t$-dependences of the $A_{UT}^{\sin(\varphi_{qq})}$ amplitude of the asymmetry are shown for $b_{sea} = 1$ and $b_{sea} = \infty$, respectively. The amplitude of the integrated TTSA is shown in Fig. 9 for the same two cases. It is essentially independent of $b_{sea}$ and ranges between values of 0.10 and 0.01 when $J_d$ ranges between zero and 0.4.

The projected statistical error for the integrated TTSA amplitudes is 0.034. Extrapolating the knowledge on the systematic uncertainty from Ref. \cite{15}, its size can be expected to be about 0.02, such that a total experimental uncertainty below 0.04 appears as a realistic estimate. Altogether, the difference in $A_{UT}^{\sin(\varphi_{qq})}$ due to a change of $J_d$ between zero and 0.4 corresponds to an about 2$\sigma$ effect, where $\sigma$ denotes the total experimental uncertainty. Thus it can be expected that the upcoming $\rho^0$ electroproduction measurements performed at HERMES will provide an additional constraint on the size of $J_d$.

A tempting possibility provided by $\rho^0$ production is related to an estimate of the gluonic content of $E$. Strongly simplifying, Eq. (8) represents the ratio $E/H \propto (E_g +$
versely polarized hydrogen target. To check the accessibility of $E$ at HERMES, different parameterization ansätze and parameters of $H$ and $E$ are chosen. As the model for $E$ depends on the total angular momentum of the $u$-quarks in the proton, the possibility arises to check the sensitivity of the data to different values chosen as $J_u = 0.4, 0.2, 0.0$, while on the basis of $u$-quark dominance and recent lattice calculations (see e.g. Ref. [30]) a fixed value $J_d = 0$ is used. The calculations are performed at the HERMES average kinematic values [22]. The results show that the DVCS TTSA amplitude $A_{UT}^{sin(\phi - \phi_s)} \cos \phi$ is sensitive to the GPD $E$ and thus to the total $u$-quark angular momentum $J_u$, while $A_{UT}^{cos(\phi - \phi_s) \sin \phi}$ is not. It was found that aside from $J_u$ the amplitude $A_{UT}^{sin(\phi - \phi_s) \cos \phi}$ is largely independent on different parameterization ansätze and model parameters. Projected statistical errors for the asymmetries are evaluated by converting the ones from Ref. [22] to a data set corresponding to 8 million DIS events taken on a transversely polarized hydrogen target.

The same parameterizations are used to calculate the TTSA in $\rho^0$ electroproduction by longitudinal virtual photons. The main difference to the DVCS case is a large gluonic contribution to the amplitude. At present, only the spin-nonflip part of the gluonic amplitude can be reasonably described, while the spin-flip gluonic GPD $E_g$ is totally unknown. Therefore, throughout the calculation $E_g$ is set to zero (“passive” gluons). Under this assumption, the situation in $\rho^0$ electroproduction appears less favorable concerning the sensitivity of the expected TTSA amplitude to the total angular momentum $J_u$. However, should the value of the amplitude be measured larger than that predicted by these calculations, this would imply that $E_g$ can not be neglected, and thus indicate that gluons inside the proton carry significant orbital angular momentum.

Altogether, transverse target-spin asymmetries in both DVCS and $\rho^0$ electroproduction are studied to evaluate projected uncertainties for extracting the value of $J_u$ from future data. Considering all anticipated HERMES data to be taken for DVCS ($\rho^0$-production), the projected total experimental $1\sigma$-uncertainty is estimated to correspond to a range of about 0.1 (0.2) in $J_u$.

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### Appendix: TTSA Calculation in DVCS

A code [55] is used to estimate the TTSA related to DVCS. The coordinate system and angles defined in the code are
the same as depicted in Fig. 3. The polarization of the target in the code is defined according to the virtual photon direction. For a transversely polarized target, the target polarization direction can be chosen either in the lepton plane ($z$ direction) or perpendicular to it ($y$ direction). The former corresponds to $\phi_S = 0$ or $\pi$, the latter to $\phi_S = \pi/2$ or $3\pi/2$. Therefore the following intermediate asymmetries can be calculated:

$$A_x(\phi) = \frac{d\sigma_{\phi=0}(\phi) - d\sigma_{\phi=\pi}(\phi)}{d\sigma_{\phi=0}(\phi) + d\sigma_{\phi=\pi}(\phi)},$$

$$A_y(\phi) = \frac{d\sigma_{\phi=\frac{\pi}{2}}(\phi) - d\sigma_{\phi=\frac{3\pi}{2}}(\phi)}{d\sigma_{\phi=\frac{\pi}{2}}(\phi) + d\sigma_{\phi=\frac{3\pi}{2}}(\phi)}.$$  (41)

Defining the following functions

$$A_1(\phi) = A_x \cdot \sin \phi - A_y \cdot \cos \phi,$$

$$A_2(\phi) = A_x \cdot \cos \phi + A_y \cdot \sin \phi,$$  (42)

the contribution of the transverse target polarization component of the interference term $I_{TP}$ to the total cross section in Eq. 25 can be expressed as:

$$d\sigma_{TP} = d\sigma_{unp} \left[ A_1(\phi) \cdot \sin(\phi - \phi_S) + A_2(\phi) \cdot \cos(\phi - \phi_S) \right].$$  (43)

Therefore the asymmetries defined in Eq. 28 can be computed as:

$$A_{UT}^{\sin(\phi-\phi_S) \cos \phi} = A_1^{\cos \phi},$$

$$A_{UT}^{\cos(\phi-\phi_S) \sin \phi} = A_2^{\sin \phi}.$$  

References

1. F. M. Dittes et al. [H1 Collaboration], Phys. Lett. B 209 (1988) 325.
2. D. Müller et al., Fortschr. Phys. 42 (1994) 101.
3. X. D. Ji, Phys. Rev. D 55 (1997) 7114.
4. A. V. Radyushkin, Phys. Lett. B 385 (1996) 333.
5. A. V. Radyushkin, Phys. Rev. D 56 (1997) 5524.
6. X. D. Ji, Phys. Rev. Lett. 78 (1997) 610.
7. M. Diehl, Phys. Rep. 388 (2003) 41.
8. A. V. Belitsky, A. Freund and D. Müller, Nucl. Phys. B 574 (2000) 347.
9. M. Burkardt, Phys. Rev. D 62 (2000) 071503 [Erratum-ibid. D 66 (2002) 119903].
10. J. P. Ralston and B. Pire, Phys. Rev. D 66 (2002) 111501.
11. M. Diehl, Eur. Phys. J. C 25 (2002) 223 [Erratum-ibid. C 31 (2003) 277].
12. M. Burkardt, Int. J. Mod. Phys. A 18 (2003) 173.
13. C. Adloff et al. [H1 Collaboration], Phys. Lett. B 517 (2001) 47.
14. S. Chekanov et al. [ZEUS Collaboration], Phys. Lett. B 573 (2003) 46.
15. F. Ellinghaus [HERMES Collaboration], Nucl. Phys. A 711 (2002) 171.
16. A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. 87 (2001) 182001.