Tunneling singularities in the open Hubbard chain

Gerald Bedürftig and Holger Frahm

Institut für Theoretische Physik, Universität Hannover
D-30167 Hannover, Germany

We study singularities in the $I$-$V$ characteristics for sequential tunneling from resonant localized levels (e.g. a quantum dot) into a one dimensional electron system described by a Hubbard model. Boundary conformal field theory together with the exact solution of the Hubbard model subject to boundary fields allows to compute the exponents describing the singularity arising when the energy of the local level is tuned through the Fermi energy of the wire as a function of electron density and magnetic field. For boundary potentials with bound states a sequence of such singularities can be observed.

PACS-Nos.: 05.70.Jk, 71.10.Fd, 71.10.Pm, 73.20.-r

Keywords: Luttinger liquid, tunneling, quantum wires, edge singularities

e-mail: frahm@itp.uni-hannover.de
1 Introduction

Electronic correlations together with strong quantum fluctuations are known to determine the low temperature properties of quasi one-dimensional conductors. Theoretical investigations using integrable lattice realizations of these Tomonaga-Luttinger liquids (TLL) together with numerical studies and field theory approaches such as bosonization have provided much of the insight into the peculiar properties of such systems. Experimental evidence for TLL behaviour on the other hand is still rare in spite of the tremendous progress in synthetization of quasi one dimensional materials or fabrication of nano-structures in which the transport of electrons is confined to a single one dimensional channel [1–3]. One reason for this is that the theoretical work on TLLs has concentrated on low energy bulk properties of perfect infinitely long system which are difficult to access experimentally. Recently, studies of the response of a TLL on local perturbations have become feasible due to the construction of integrable models with open boundaries and a better understanding of quantum field theories in the presence of a boundary. Local inhomogeneities may have a profound effect on the transport properties of one dimensional interacting electron gases, even leading to phases with vanishing transmission of a barrier [4–8]. Finite size effects in the resulting open chains have been studied to understand the possible experimental consequences of TLL properties in such systems (see e.g. [9–14]). Among possible consequences of local perturbations are Fermi edge singularities which may be observed in X-ray absorption amplitudes and — as will be discussed in this paper — in tunneling experiments [15–18]. In both cases the nature of the singularities is strongly affected by the properties of the TLL.

In this paper we study tunneling from a resonant localized level into a TLL in the limit of low barrier conductance as observed in the current-voltage characteristic at zero bias. As a specific representation of the latter we choose the one-dimensional Hubbard model. In this lattice model we find — similar as in previous work on optical absorption processes [19] — a rich spectrum of edge singularities due to the existence of bound states reflecting the separation of spin and charge in the TLL [20]. An experimental realization of the tunneling processes under investigation might be a quantum dot (providing the localized level) coupled to a quantum wire. The reservoir supplying charges to fill the state in the quantum dot is left unspecified. The energy $E_i$ of the local level can be tuned by varying a gate voltage of the dot. The only influence of the quantum dot on the Luttinger liquid considered below is the electrostatic interaction with its net charge (we consider the dot occupied with a single electron to be electrically neutral). This description is very similar to the model invoked to describe the X-ray edge singularity in
metallic systems \[21\] and has previously been used to study tunneling from a resonant local level into two- and three-dimensional systems \[22\]. These considerations lead to the following Hamiltonian

\[
\mathcal{H} = - \sum_{\sigma,j=1}^{L-1} \left( c_{j,\uparrow}^\dagger c_{j+1,\sigma} + h.c. \right) + 4u \sum_{j=1}^{L} n_{j\uparrow} n_{j\downarrow} + \mu \hat{N} - \frac{\hbar}{2} (\hat{N} - 2\hat{N}_z) \\
+ E_i b^\dagger b - bb^\dagger p(\hat{N}_{1,\uparrow} + \hat{N}_{1,\downarrow}) .
\] (1.1)

where \(b^\dagger (b)\) are canonical fermionic creation (annihilation) operators for a spin-\(\uparrow\) electron the localized state and \(c_{j,\sigma}^\dagger\) creates an electron of spin \(\sigma\) on site \(j\) of the one-dimensional chain. The chemical potential \(\mu\) and magnetic field \(h = g \mu_B H\) allow to control the filling factor and magnetization of the quantum wire. Upon variation of the gate voltage tunneling between the local level and the wire becomes possible if the energy \(E_i\) of the local level exceeds the Fermi energy. We restrict ourselves to the case where the barrier conductance is low. Hence, the transport is dominated by incoherent sequential tunneling processes and we can neglect Coulomb blockade effects and higher order processes such as "cotunneling" \[23\]. Within the "orthodox theory" \[24\] the current due to sequential tunneling is computed by application of the golden rule leading to

\[
I(E_i) \propto \sum_n \left| \langle n | c_{1,\uparrow}^\dagger b \langle 0 | \right|^2 \delta(E_n - E_0 - E_i) .
\] (1.2)

Here \(\langle 0 \rangle = b^\dagger |0\rangle\) denotes the ground state of the open Hubbard chain in the \(N_e\)-particle sector with the local level occupied and hence vanishing boundary potential \(p\). The sum in (1.2) extends over all eigenstates \(|n\rangle\) of the chain in the \((N_e + 1)\)-particle sector in the presence of the boundary potential \(p\). Eq. (1.2) can be rewritten as a Fourier integral:

\[
I(E_i) \propto \text{Re} \int_0^\infty dt \ e^{iE_i^+ t} \langle \tilde{0} | b^\dagger(t) c_{1,\uparrow}(t) c_{1,\uparrow}^\dagger(0) b(0) | \tilde{0} \rangle \] (1.3)

where \(E_i^+ = E_i + i0\). Near the threshold \(E_i \approx E_{th}\) the intensity exhibits a characteristic singularity:

\[
I(E_i) \propto \frac{1}{|E_i - E_{th}|^{\alpha}} .
\] (1.4)

For non interacting electrons the exponent \(\alpha\) can be expressed in terms of the phase shift at the Fermi surface \[22\]. As in the case of the X-ray edge singularity one expects several thresholds if the electrostatic potential \(p\) is strong enough to form bound states in the TLL \[25, 26\]. In this paper we want to study this problem for tunneling into a TLL where an additional dependence of the exponent \(\alpha\) on the interaction parameters (i.e. electron density, magnetization and strength of the Hubbard interaction \(4u\)) in (1.1) of the Luttinger liquid is to be expected from the results
obtained for the related X-ray problem (see Refs. [19, 27, 28]). In the following section we summarize the relevant properties of the model (1.1) obtained from its Bethe Ansatz solution. From this solution combined with results from boundary conformal field theory (BCFT) [28–30] we extract the spectrum of thresholds and the corresponding exponents $\alpha$.

2 Bethe Ansatz Solution of the model

The Bethe Ansatz equations (BAE) determining the spectrum of $\mathcal{H}$ with empty local state (i.e. with boundary chemical potential $p$) in the $N_e$-particle sector with magnetization $M = \frac{1}{2}N_e - N_\downarrow$ read [31–33]:

$$e^{ik_j(2L+1)}B_c(k_j) = \prod_{\beta=-N_\uparrow}^{N_\uparrow} \frac{\sin k_j - \lambda_\beta + iu}{\sin k_j - \lambda_\beta - iu}, \quad j = -N_e, \ldots, N_e$$

$$B_s(\lambda_\alpha) \prod_{j=-N_e}^{N_e} \frac{\lambda_\alpha - \sin k_j + iu}{\lambda_\alpha - \sin k_j - iu} = \prod_{\beta=-N_\downarrow}^{N_\downarrow} \frac{\lambda_\alpha - \lambda_\beta + 2iu}{\lambda_\alpha - \lambda_\beta - 2iu}, \quad \alpha = -N_\downarrow, \ldots, N_\downarrow$$

(2.1)

where one should identify $k_{-j} \equiv -k_j$ and $\lambda_{-\alpha} \equiv -\lambda_\alpha$. The boundary phase shifts appearing in the BAE read

$$B_c(k) = \left( \frac{e^{ik} - p}{1 - pe^{ik}} \right) \frac{\sin k + iu}{\sin k - iu}, \quad B_s(\lambda) = \frac{\lambda + 2iu}{\lambda - 2iu}.$$  

(2.2)

The energy of the eigenstate of Eq. (1.1) corresponding to a solution of the BAE is given by

$$E = \sum_{j=1}^{N_e} \left( \mu - \frac{h}{2} - 2 \cos k_j \right) + hN_\downarrow.$$  

(2.3)

In Refs. [31–33] the ground state and the low–lying excitations of this model where studied for small boundary fields. In [20] the existence of boundary states for $|p| > 1$ has been established. In the Bethe Ansatz solution these bound states manifest themselves as additional complex solutions for the charge and spin rapidities. In Fig. 1 the spectrum of bound states for $u = 1$ is shown. Using standard procedures, the BAE for the ground state and low–lying excitations in the thermodynamic limit can be rewritten as linear integral equations for the densities $\rho_c(k)$ and $\rho_s(\lambda)$ of real quasi-momenta $k_j$ and spin rapidities $\lambda_\alpha$, respectively:

$$
\begin{pmatrix}
\rho_c(k) \\
\rho_s(\lambda)
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{\pi} + \frac{1}{\pi} \rho_c^0(k) \\
\frac{1}{\pi} \rho_s^0(\lambda)
\end{pmatrix} + K \ast 
\begin{pmatrix}
\rho_c(k') \\
\rho_s(\lambda')
\end{pmatrix}
$$

(2.4)

with the kernel $K$ given by

$$
K = 
\begin{pmatrix}
0 & \cos k \ a_{2a}(\sin k - \lambda') \\
\cos k \ a_{2a}(\sin k - \lambda') & -a_{4a}(\lambda - \lambda')
\end{pmatrix}.
$$

(2.5)
Here we have introduced \( a_y(x) = \frac{1}{2\pi} \frac{y}{y^2 + x^2} \), and \( f \ast g \) denotes the convolution \( \int_{-A}^A dy f(x-y)g(y) \) with boundaries \( A = k^{(0)} \) in the charge and \( A = \lambda^{(0)} \) in the spin sector. These boundaries are functions of the external chemical potential \( \mu \) and magnetic field \( h \). Alternatively, in a canonical approach the values of \( k^{(0)} \) and \( \lambda^{(0)} \) are fixed by the conditions

\[
\int_{-k^{(0)}}^{k^{(0)}} dk \rho_c = \frac{2}{L} [ N_c - C_c ] + 1, \quad \int_{-\lambda^{(0)}}^{\lambda^{(0)}} d\lambda \rho_s = \frac{2}{L} [ N_s - C_s ] + 1, \tag{2.6}
\]

where \( C_c (C_s) \) denotes the number of particle hole excitations at the Fermi points and the \( \nu \) is defined in terms of the integral equation

\[
\rho = \int \frac{d\lambda}{2\pi} \xi_s(\lambda) Z_{cs},
\]

where \( Z_{cc}, Z_{cs}, Z_{sc} \) are non negative integers counting the number of particle hole excitations at the Fermi points and \( \nu_{c,s} \) are the Fermi velocities of the massless charge and magnetic modes. \( \Delta N_{c,s} \) specify the quasi-particle content of the state, in a TLL these are holons/anti-holons in the charge sector and spinons in the magnetic sector of the theory. The dressed charge matrix \( Z \) is defined in terms of the integral equation

\[
\begin{pmatrix}
\xi_{cc}(k) & \xi_{sc}(k) \\
\xi_{cs}(\lambda) & \xi_{ss}(\lambda)
\end{pmatrix} = 
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} + K^\top \ast 
\begin{pmatrix}
\xi_{cc}(k') & \xi_{sc}(k') \\
\xi_{cs}(\lambda') & \xi_{ss}(\lambda')
\end{pmatrix}. \tag{2.10}
\]
3 Tunnel exponents

Results from boundary conformal field theory allow to extract the exponent $\alpha$ in Eq. (1.4) from the finite size spectrum (2.8) [28–30]: the Green’s function of an operator $O$ with dimension $x$ on the complex half plane is given by:

$$\langle A|O(\tau_1)O^\dagger(\tau_2)|A\rangle = \frac{1}{(\tau_1 - \tau_2)^2x}$$

(3.1)

Conformal mapping of the half plane onto a strip of finite width $L$ allows to extract the scaling dimension of the boundary changing operator $O$ from the finite size spectrum (2.8) by taking differences of the energy $E_A^0$ of the system’s ground state $|A\rangle$ in the $N_e$-particle sector without boundary potential and the energy $E_B^0$ of the lowest excitation $|B, n\rangle$ in the $(N_e + 1)$-particle sector with boundary potential $p$ and non vanishing form factor $|\langle B, n|O^\dagger|A\rangle|^2$ (see Refs. [19,20,35,36] for details on the CFT approach to the asymptotics of correlation functions in the Hubbard model). In the present problem $E_A^0$ is obtained from (2.8) by choosing $\Delta N^0_{\alpha} = \theta^c_0$ while for $E_B^0$ we have to choose $\Delta N^0_{e} = 1 + \theta^c_0$ and $\Delta N^0_{s} = \theta^s_0$ corresponding to an extra spin-$\uparrow$ electron created by $O^\dagger$. Writing the $O(L^{-1})$ terms in the energy difference as $(\pi/L)(v_c x_c + v_s x_s)$ the corresponding edge exponent in Eq. (1.4) is given as

$$\alpha = 1 - 2(x_c + x_s).$$

(3.2)

We now want to study the exponents at the several possible thresholds. To gain some more insight into the role of the boundary states we begin with a discussion of noninteracting fermions.

3.1 Ferromagnetic case

For sufficiently large magnetic field the electrons are polarized ferromagnetically, hence an explicit expression is available for the wave function in terms of a Slater determinant of single-particle states. Considering $|p| < 1$ these are plane waves corresponding to real wave numbers $k$ exist and we expect a single threshold. The corresponding edge exponent is a function of $p$ and the density of electrons $n_e = N_e/L$ (see also Ref. [37]):

$$\alpha^r = 1 - 2x^r_p = 1 - \left(1 + \frac{1}{\pi} \arctan \left(\frac{p + 1}{p - 1} \tan \frac{\pi n_e}{2}\right) + \frac{n_e}{2}\right)^2.$$

(3.3)

For $|p| > 1$ the boundary potential can bind an electron which leads to the existence of two thresholds, depending on whether the bound state is occupied in the final state. For an empty
bound state we obtain the result $\alpha^c$ of Eq. (3.3) for the edge exponent. A different exponent is found if the bound state occupied:

$$\alpha^c = 1 - 2x_p^c = 1 - \left( \frac{1}{\pi} \arctan \left( \frac{p + 1}{p - 1} \tan \frac{\pi n_e}{2} \right) + \frac{n_e}{2} \right)^2. \quad (3.4)$$

These predictions can be checked by studying the finite-size behaviour of the form factors. From the conformal mapping mentioned above one expects \[20, 26\]

$$\langle p | c_{1,\uparrow}^\dagger | 0 \rangle \sim \left( \frac{1}{L} \right)^{x_p^c} \quad \text{and} \quad | \langle \overline{p} | c_{1,\uparrow}^\dagger | 0 \rangle | \sim \left( \frac{1}{L} \right)^{x_r^p}. \quad (3.5)$$

where $| p \rangle$ and $| \overline{p} \rangle$ denote the ground state and the lowest state with empty bound state in the $(N_e + 1)$-particle sector with boundary potential $p > 1$. Note that exponent $x_p^c$ vanishes in the limit $p \to 0$. This coincides with exact result, $I(E_i) \propto \sqrt{1 - \frac{E_i^2}{4}} \Theta (E_i + 2 \cos(\pi n_e))$, which does not exhibit a singularity.

Eq. (1.2) is now evaluated numerically. To avoid the use of explicit representations of the $\delta$-function we use the integral

$$J(E) \equiv \int_{-\infty}^{E} dE_i \, I(E_i). \quad (3.6)$$

Typical numerical results for $J(E)$ are shown in Figs. 2 and 3. To make the numerical analysis of (1.2) feasible we had to restrict the sum $\sum_n$ to the most important states. The error of this approximation has been estimated using the sum rule $J(E \to E_{max}) = 1 - \langle N_1 \rangle$, which is satisfied to >99% in all cases. After smoothing of $J(E)$ and numerical differentiation it is possible to obtain $I(E_i)$. The result is shown in Fig. 4 for several potentials. For positive $p$ the exponent is always positive at the absolute threshold and negative at the second one for $p > 1$.

The situation changes completely for $p < 0$ where the ground state is always parametrized by real wave numbers $k$ giving a negative exponent $\alpha$ at the absolute threshold. Occupation of the anti-bound state corresponding to a complex $k$ leads to a positive exponent.

For the X-ray edge problem one can show that the functional dependence of $J(E)$ is nearly unchanged by increasing the system size \[38, 39\]. This allows to extract quantitative informations of rather small systems ($L = 80$ in the present case) by fitting of $J(E)$ to the trial function:

$$f(E) = a(E + b)^c. \quad (3.7)$$

The resulting exponent $c$ can be compared to the BCFT-results. Using the 600 lowest states contributing to the sum (1.2) we obtain a good agreement with the CFT-results (see Fig. 5) for $p > 1$ and electron densities $n_e \lesssim 0.4$. For larger densities a bigger discrepancy between the numerical results and the BCFT predictions is found due to stronger finite size effects.
The second threshold due to the presence of a bound state is most pronounced for \( p < -1 \). Here the jump of \( J(E) \) characteristic of a positive edge exponent occurs at the second threshold (see Fig. 3). While the BCFT result for the edge exponent at the absolute threshold is \( \alpha_{abs} = \alpha^c = -0.33 \) the fit to the numerical data gives \( \alpha_{abs} = -0.46 \) — this indicates that a genuine singularity is strongly affected by finite size effects. On the other hand the numerical value for the positive exponent at the threshold corresponding to occupied anti-bound state, \( \alpha^c = 0.988 \), is in very good agreement with the CFT-result \( \alpha^c = 0.977 \).

### 3.2 Magnetic field dependence of edge exponents

For vanishing magnetic field one has \( \lambda^{(0)} = \infty \) allowing to solve the spin part of the integral equations by Fourier transformation. As a consequence the dressed charge matrix \( Z \) (2.9) is function of a single variable \( \xi = \xi(k^{(0)}) \) [34], which is defined by the following integral equation:

\[
\xi(k) = 1 + \int_{-k^{(0)}}^{k^{(0)}} dk' \cos k' G(\sin k - \sin k')\xi(k') ,
\]

(3.8)

where \( G(\lambda) = \frac{1}{4\pi u} \text{Re} \{\Psi(1 + i\frac{\lambda}{2u}) - \Psi(\frac{1}{2} + i\frac{\lambda}{2u})\} \) (\( \Psi(x) \) is the digamma function). Furthermore, one finds \( \theta_p^s = \frac{1}{2}\theta_p^c \) yielding

\[
\alpha_{abs} = 1 - 2(x_c + x_s) = \frac{1}{2} - \frac{1}{\xi^2} \left(1 + \theta_0^c - \theta_p^c\right)^2
\]

(3.9)

for the exponent at the absolute threshold. In Fig. 3 we present regions where the exponent \( \alpha_{abs} \) is positive as a function of electronic density \( n_e \) and boundary potential \( p \) together with the density dependence of the exponent for some fixed values of \( p \). In a finite magnetic field \( h \) the bulk state of the Hubbard model is ferromagnetic below a critical particle density \( n_c \). This density can be calculated from

\[
h = \frac{2u}{\pi} \int_{-\pi n_c}^{\pi n_c} dk' \cos k' \cos k' - \cos(\pi n_c) \frac{u^2 + \sin^2 k'}{u^2 + \sin^2 k'}. \]

(3.10)

For non vanishing magnetic field we will only consider electron densities above \( n_c \). For \( h > 0 \) the exponent is given by:

\[
\alpha_{abs} = 1 - \frac{\left[(1 + \theta_0^c - \theta_p^c) Z_{ss} - (\theta_0^c - \theta_p^s) Z_{cs}\right]^2 + \left[(\theta_0^s - \theta_p^s) Z_{cc} - (1 + \theta_0^c - \theta_p^c) Z_{sc}\right]^2}{\det^2(Z)}. \]

(3.11)

In Fig. 7 the magnetic field dependence of this exponent is shown for several values boundary potentials \( p \). Note the characteristic change of this curve near \( p = p_1 = u + \sqrt{u^2 + 1} \) where a low lying excited bound state for a charge and a spinon (corresponding to a complex quasi momentum \( k \) and a complex spin rapidity \( \lambda \) in the set of roots of (2.1) [20]) appears, see Fig. 4.
For $h$ approaching the saturation field $(3.10)$ the exponents can be calculated explicitly giving

$$\alpha_{\text{abs}}(n_e \to n_c(h)) > 0 \ 	ext{for} \ 0 < p < p_1 \quad \text{and} \quad \alpha_{\text{abs}}(n_e \to n_c(h)) < 0 \ \text{for} \ p > p_1 .$$

The difference of the limiting values at $p = p_1$ is $\Delta \alpha_{\text{abs}} = 1$. The crossover due to this behaviour is clearly seen in Fig. 6. As before additional edge singularities arise as a consequence of bound states in the boundary potential. The corresponding exponents can be computed from the Bethe Ansatz equations as above (for details see Ref. [20]). Each of the boundary states seen in Fig. 1 gives rise to a singularity $(1.4)$ in the $I$–$V$ curve.

4 Summary and Conclusion

We have studied the $I$–$V$ characteristics for tunneling from a resonant localized level into a one dimensional interacting electron gas described in terms of a Hubbard model $(1.1)$. Compared to tunneling into a higher dimensional system one finds a rich structure of thresholds due to the presence of various bound states in the many particle spectrum, each of them leading to a possible resonance for the tunneling charge. Their appearance may be understood as a consequence of the separation of charge and spin degrees of freedom of the electrons allowing to generate a current either by the electrons decaying into their holon and spinon constituents in the wire or by holons alone leaving the spinon bound by the electrostatic potential of the tunnel contact. Furthermore, the electronic correlations within the quantum wire strongly affect the nature of the singularities, i.e. the exponents $\alpha$ in $(1.4)$. While we have considered correlation functions strictly at zero temperature, the above singularities are still observable at small finite $T$: for sufficiently long wires the current at the threshold depends on temperature as $I(E_{th}) \propto T^{-\alpha}$. Hence, the dependence of $\alpha$ on parameters such as filling factor of the wire or the external magnetic field should be accessible experimentally thereby allowing to determine the properties of the potential due to a vacancy of the localized level.

Acknowledgments

We are grateful to R. Haug for discussions. This work has been supported by the Deutsche Forschungsgemeinschaft under Grant No. Fr 737/2–3.
References

[1] C. Schlenker, J. Dumas, M. Greenblatt, and S. van Smaalen (eds.), *Physics and Chemistry of Low-Dimensional Inorganic Conductors*, NATO ASI Series B: Physics Vol. 354 (Plenum, New York, 1996)

[2] P. A. Serena and N. García (eds.), *Nanowires*, NATO ASI Series E: Applied Sciences Vol. 340 (Kluwer, Dordrecht, 1997).

[3] P. M. Petroff (ed.), *Proceedings of the Eighth International Conference on Modulated Semiconductor Structures*, Physica E 2 (1998).

[4] C. L. Kane and M. P. A. Fisher, Phys. Rev. B 46, 15233 (1992).

[5] A. Furusaki and N. Nagaosa, Phys. Rev. B 47, 3827 (1993).

[6] A. Furusaki and N. Nagaosa, Phys. Rev. B 47, 4631 (1993).

[7] E. Wong and I. Affleck, Nucl. Phys. B 417, 403 (1994), cond-mat/9311040.

[8] I. Safi and H. J. Schulz, Phys. Rev. B 52, R17040 (1995), cond-mat/9505079.

[9] M. Fabrizio and A. O. Gogolin, Phys. Rev. B 51, 17827 (1995), cond-mat/9504011.

[10] R. Egger and H. Grabert, Phys. Rev. Lett. 75, 3505 (1995), cond-mat/9509100.

[11] H. Frahm and A. A. Zvyagin, Phys. Rev. B 55, 1341 (1997).

[12] R. Egger and H. Grabert, Phys. Rev. B 55, 9929 (1997), cond-mat/9701057.

[13] A. E. Mattsson, S. Eggert, and H. Johannesson, Phys. Rev. B 56, 15615 (1997), cond-mat/9711204.

[14] G. Bedürftig, B. Brendel, H. Frahm, and R. M. Noack, Phys. Rev. B 58, 10225 (1998), cond-mat/9805123.

[15] J. M. Calleja et al., Sol. St. Comm. 79, 911 (1991).

[16] A. K. Geim et al., Phys. Rev. Lett. 72, 2061 (1994).

[17] J. Smoliner, Semicond. Sci. Technol. 11, 1 (1996).

[18] C.-T. Liang et al., Phys. Rev. B 55, 6723 (1997).

[19] F. H. L. Eßler and H. Frahm, Phys. Rev. B 56, 6631 (1997), cond-mat/9702234.

[20] G. Bedürftig and H. Frahm, J. Phys. A 30, 4139 (1997), cond-mat/9702227.

[21] P. Nozières and C. T. de Dominicis, Phys. Rev. 178, 1097 (1969).

[22] K. A. Matveev and A. I. Larkin, Phys. Rev. B 46, 15337 (1992).
[23] H. Grabert and M. H. Devoret (eds.), *Single Charge Tunneling: Coulomb blockade phenomena in nanostructures*, NATO ASI Series B: Physics Vol. 294 (Plenum, New York, 1992).

[24] D. V. Averin and K. K. Likharev, Single electronics: A correlated transfer of single electrons and Cooper pairs in systems of small tunnel junctions, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb, chap. 6 (North-Holland, Amsterdam, 1991).

[25] M. Combescot and P. Nozières, J. Physique **32**, 913 (1971).

[26] I. Affleck, Nucl. Phys. B (Proc. Suppl.) **58**, 35 (1997), [hep-th/9611064](http://arxiv.org/abs/hep-th/9611064).

[27] T. Ogawa, A. Furusaki, and N. Nagaosa, Phys. Rev. Lett. **68**, 3638 (1992).

[28] I. Affleck and A. W. W. Ludwig, J. Phys. A **27**, 5375 (1994), [cond-mat/9405057](http://arxiv.org/abs/cond-mat/9405057).

[29] J. L. Cardy, Nucl. Phys. B **324**, 581 (1989).

[30] I. Affleck, Conformal-field-theory approach to quantum-impurity problems, in *Correlation effects in low-dimensional electron systems*, edited by A. Okiji and N. Kawakami, Springer Series in Solid-State Sciences Vol. 118, p. 82 (Springer Verlag, Berlin, 1994).

[31] H. Asakawa and M. Suzuki, J. Phys. A **29**, 225 (1996).

[32] T. Deguchi and R. Yue, preprint (1997), [cond-mat/9704138](http://arxiv.org/abs/cond-mat/9704138).

[33] M. Shiroishi and M. Wadati, J. Phys. Soc. Japan **66**, 1 (1997).

[34] F. Woynarovich, J. Phys. A **22**, 4243 (1989).

[35] H. Frahm and V. E. Korepin, Phys. Rev. B **42**, 10553 (1990).

[36] H. Frahm and V. E. Korepin, Phys. Rev. B **43**, 5653 (1991).

[37] A. M. Zagoskin and I. Affleck, J. Phys. A **30**, 5743 (1997), [cond-mat/9704248](http://arxiv.org/abs/cond-mat/9704248).

[38] J. D. Dow and C. P. Flynn, J. Phys. C **13**, 1341 (1980).

[39] K. Ohtaka and Y. Tanabe, Rev. Mod. Phys. **62**, 929 (1990).

[40] G. Bedürftig, *Randeffekte und Störstellen in eindimensionalen integra len Elektronensystemen*, Dissertation, Universität Hannover (1998).

[41] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).

[42] S. Qin, M. Fabrizio, and L. Yu, Phys. Rev. B **54**, R9643 (1996), [cond-mat/9608024](http://arxiv.org/abs/cond-mat/9608024).
Figures

Figure 1: Spectrum of boundary bound states of (1.1) for \( u = 1 \) with chemical potential \( \mu = 0.5 \) and magnetic field \( h = 0.3 \), for (a) \( p < 0 \) and (b) \( p > 0 \). The thresholds \( p_1 \) for binding a charge and spin and \( p_2 \) for binding of a singlet pair of electrons are indicated.
Figure 2: Numerical results for $J(E)$ for a system of $L = 80$ sites with a density $n_e = 0.2$ of spin-$\uparrow$ electrons and $p = 3$. The dashed line is the fit to Eq. (3.7). The jump of $J(E)$ at the threshold is characteristic to a singularity with a positive exponent $\alpha$. Its height is given by the matrix element $\langle p|c_{1,\uparrow}^\dagger|0\rangle$ which vanishes in the thermodynamic limit according to Eq. (3.5). This is the well known orthogonality catastrophe [41, 42].

Figure 3: Same as Fig. 2 but for $p = -3$. 
Figure 4: Numerical results for $I(E_i)$ for systems of size $L = 80$ with 16 electrons for different $p < 0$ (a) and $p > 0$ (b). The singularities are suppressed due to the numerical differentiation.

Figure 5: Comparison of the fit (3.7) to the numerical data with the BCFT results (solid lines) for three different electron densities.
Figure 6: Region with positive absolute exponent $\alpha_{\text{abs}}$ resulting in a singularity in the $I-V$ characteristics for vanishing magnetic field for fixed $u = 1$. The insets show $\alpha_{\text{abs}}$ for fixed values of $p$ as a function of the electron density $n_e$. 
Figure 7: Magnetic field dependence of the edge exponent $\alpha_{abs}$ for $u = 1$ and fixed chemical potential $\mu = -0.2$ for several values of the boundary potential $p$ above and below the threshold $p_1 = 1 + \sqrt{2}$ for creation of the holon/spinon bound state.