PION PRODUCTION IN THE INNER DISK AROUND CYGNUS X-1

CÉSAR MEIRELLES FILHO
Instituto de Astronomia, Geofísica e de Ciências Atmosféricas, Universidade de São Paulo, R. do Matão 1226, 05508-090 São Paulo, SP, Brazil; cmeirelf@astro.iag.usp.br

AND

CELSO L. LIMA, HIDEAKI MIYAKE, AND VARESE TIMOTEO
Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970 São Paulo, SP, Brazil

Received 2002 July 15; accepted 2003 June 3

ABSTRACT

Neutron production via $^4$He breakup and $p(p, n\pi^+\pi^-)p$ is considered in the innermost region of an accretion disk surrounding a Kerr black hole. These reactions occur in a plasma in Wien equilibrium, where (radiatively produced) pair production equals annihilation. Cooling of the disk is assumed to be due to unsaturated inverse Comptonization of external soft photons and to the energy needed to ignite both nuclear reactions. Assuming matter composition of 90% H and 10% He, we show that, close to the border of this region, neutron production is essentially from $^4$He breakup. Close to the horizon, the contribution from $p(p, n\pi^+\pi^-)p$ to the neutron production is comparable to that from the breakup. We show that the viscosity generated by the collisions of the accreting matter with the neutrons may drive stationary accretion, for accretion rates below a critical value. In this case, the solution to the disk equations is double-valued, and for both solutions protons outnumber the pairs. Our results apply whenever $M_{\text{ff}}/M < 1.74$. We suggest that these solutions may mimic the states of high and low luminosity observed in Cygnus X-1 and related sources. This could be explained either by the coupling of thermal instability to the peculiar behavior of the viscosity parameter $\alpha$ with the ion temperature that may intermittently switch accretion off, or by the impossibility of a perfect tuning for both thermal and pair equilibrium in the disk, a fact that forces the system to undergo a kind of limit cycle behavior around the upper solution.

Subject headings: accretion, accretion disks — elementary particles — nuclear reactions, nucleosynthesis, abundances — radiative transfer

1. INTRODUCTION

Accretion disks are currently thought to be the environment for hydrodynamic flows occurring in close binary systems, around galactic nuclei, quasars, and protostellar systems. However, despite all the attention given to them in the last almost three decades, models that allow for the linking of particle processes to the detailed nature of the flow and to physical processes that may act as the source of the required viscosity are still lacking. Progress in these areas has rather come in isolated and restrictive ways.

Concerning the microphysics of particle processes, most of the theoretical work in the field has focused on the production of positron-electron pairs. Under the assumption of the very special condition of production-annihilation equilibrium, pair plasmas were first studied by Bisnovatyi-Kogan et al. (1971), Pozdnyakov et al. (1977), Stoeger (1977), and Liang (1979). Subsequent studies, under less restrictive conditions, have contributed to a better knowledge of astrophysical pair plasmas in accretion disks (Lightman 1982; Svensson 1982, 1984; Zdziarski 1984; Takahara & Kusunose 1985; Lightman & Zdziarski 1987; White & Lightman 1989; Kusunose 1996; Liang 1991; Björnsson & Svensson 1991, 1992; Mineshige 1993; Kusunose & Mineshige 1992, 1995, 1996). However, the solutions of the disk equations are characterized by a topological highly dependent on $\alpha$, the poorly known viscosity parameter.

As far as the nature of the flow is concerned, there has been quite a lot of progress since the $\alpha$ standard model by Shakura & Sunyaev (1973). Starting with Shapiro et al. (1976), some authors developed models for thick accretion disks (Abramowicz et al. 1980; Paczynski & Abramowicz 1982) in which they kept the assumptions of local equilibrium between heat production and radiative cooling. Soon afterward, thick-disk models taking the advective cooling mechanism into account were considered (Ichimaru 1977; Liang & Thompson 1980; White & Lightman 1990; Paczynski & Bisnovatyi-Kogan 1981; Muchotrzeb & Paczynski 1982; Abramowicz et al. 1988; Honna et al. 1991; Wallinder 1991; Chen & Taam 1993). Although these works delineated the basic physics of advection in thick accretion disks, only recently has there has the importance of cooling in optically thin accretion disks been recognized (Narayan & Yi 1994, 1996; Narayan et al. 1995a, 1995b, 1996; Abramowicz et al. 1995; Lasota et al. 1996a, 1996b; Chen et al. 1995). Finally, the inclusion of pairs in these models has been considered (Björnsson et al. 1996; Kusunose & Mineshige 1996; Esin et al. 1996, 1997), showing that it does not modify the topology of the disk equation solutions, being, again, highly dependent on $\alpha$.

Many physical processes have been invoked to account for the viscosity in accretion disks: (1) shear turbulence generated by the Keplerian rotation of the disk, (2) turbulence driven by convection, (3) tangled magnetic fields sheared by differential rotation, (4) angular momentum transport by waves, (5) Velikov-Chandrasekhar magnetic instability, (6) ion and neutron viscosities, and (7) radiative viscosity (Shakura & Sunyaev 1973; Zel’dovich 1981; Dubrulle 1990; Zahn 1991; Lin & Papaloizou 1980; Tayler 1980; Bisnovatyi-Kogan & Blinnikov 1977; Shakura et al. 1978;
Ryu & Goodman 1992; Cabot & Pollack 1992; Stone & Balbus 1996; Meirelles 1991a, 1993; Meirelles et al. 1997; Lynden-Bell 1969; Eardley & Lightman 1975; Ichimaru 1977; Coroniti 1981; Papaloizou & Pringle 1984, 1985; Vishniac & Diamond 1989, 1992; Vishniac et al. 1990; Balbus & Hawley 1991, 1992; Balbus et al. 1996; Guessoum & Kazanas 1990; Loeb & Laor 1992). None of these mechanisms is exempt from criticism, and, as a matter of fact, there are many drawbacks to accepting any of them as a general mechanism for viscosity generation.

So far, we have talked about flows in which the plasma is ruled only by electromagnetic interactions. However, recently some authors have considered situations in which nuclear reactions play a major role. Chakrabarti & Mukhopadhyay (1999) and Mukhopadhyay & Chakrabarti (2000) have considered conditions in $\alpha - M$ space in which elements in the disk are photodissociated, resulting in an abundant production of neutrons that, being decoupled from protons, can stay longer in the disk, possibly forming a neutron disk. This possibility has also been investigated by Belyanin & Derishev (2001). Neutron production in accretion disks has also been considered by Guessoum & Jean (2002).

Therefore, a proposal for studying neutron production in the innermost region in accretion disks seems quite justifiable in the sense that it gives us a viscosity mechanism and a self-consistent calculation of the viscosity parameter $\alpha$. We therefore would like to consider neutron viscosity assuming neutron production through helium breakup and through pion production due to proton-proton interaction.

The threshold for helium breakup is about 29 MeV,

$$^{4}\text{He} + 28.296 \text{ MeV} \rightarrow 2p + 2n,$$

and about 290 MeV for the production of pions (and neutrons) in proton-proton collisions,

$$p + p + 290 \text{ MeV} \rightarrow p + n + \pi^+.$$

Close to the horizon, the energy in the rest frame of two colliding protons is about 2 GeV, which means that, at least theoretically, these reactions are energetically viable. The real difficulty is to find conditions in parameter space, if they exist, such that the plasma is no longer ruled by electromagnetic interactions alone and nuclear interactions start playing a role.

In a previous paper (Meirelles 1993) we considered neutron production through the $^{4}\text{He}$ breakup reaction, equation (1), for ion temperatures greater than 3 MeV. However, in addition to not considering pair production, the results obtained are highly dependent on the assumption of equality between drift and nuclear reaction timescales. Pair production lowers the plasma temperature, leaving less energy available to the nuclear reactions. Essentially, we consider the production of neutrons through $^{4}\text{He}$ breakup and through proton-proton collisions in a plasma with the concomitant occurrence of pair production. We claim that taking into account both reactions in the inner parts of an accretion disk together with an allowance for thermal instability may explain the transitions observed in Cyg X-1 between the states of high and low luminosity.

The plan of the paper is as follows. Section 2 presents some observational aspects of Cyg X-1, and § 3 outlines the model and the main assumptions. Section 4 describes the notation we employ as well as the disk equations, and § 5 describes the main approximations used to treat pair production. In § 6 we discuss $^{4}\text{He}$ breakup and pion and neutron production, obtaining reaction rates, neutron abundance, neutron cooling, and neutron viscosity. In § 7 we give some order-of-magnitude estimates, as well as arguments justifying the relevance of the effects included in the model. In § 8 we discuss some aspects of pion production, and § 9 gives our main conclusions.

2. OBSERVATIONAL ASPECTS: A BRIEF REVIEW

There is a widely accepted suspicion that the unseen compact object in the binary system Cyg X-1 is a black hole (Liang 1998). Support for this suspicion is found in detailed analyses carried out on radial velocity measurements as well as in recent analyses based on spectrum synthesis (Herrero et al. 1995), both of which give a mass of about 10 $M_\odot$, comfortably larger than the 3 $M_\odot$ upper limit of a neutron star mass. The luminosity is sub-Eddington, but greater than 0.01$L_{\text{Edd}}$ (Liang & Nolan 1984). Cyg X-1 exhibits a very characteristic transient behavior, being found on the hard or in the soft state (Oda et al. 1974; Liang & Nolan 1984; Tanaka & Lewin 1995). Most of the time, Cyg X-1 is found in the hard state, where the soft X-ray (2–10 keV) spectrum is relatively low and the hard X-ray spectrum is hard. After being in this state for a few years, the system undergoes a transition to the soft state. In this transition, its soft X-ray flux is increased by a factor of about 10, softening its X-ray spectrum. It remains in the soft state from weeks to months before returning to the hard state. The transition between the two states lasts from less than a day to more than a week (Wen et al. 2001). In the hard state, Cyg X-1 has a power-law spectrum characterized by a photon index $\approx$1.4. During the hard state the soft X-ray spectrum below 10 keV is often a simple continuation of the hard X-ray spectrum power law, with increasing flattening below 3 keV. In that case, the entire X-ray continuum is likely produced by a single hot component. However, during some episodes of the Cyg X-1 hard state, the best-fit model still requires a small blackbody component of temperature a few tenths of keV on the top of the power law, presuming the existence of another region of lesser temperature (Liang 1998). This is highly suggestive of a soft photon source, as expected in the inverse Compton model. In addition, there is often evidence of Fe K edge absorption at $\approx$7 keV (Liang 1998; Esin et al. 1998). When Cyg X-1 is in the soft state, its spectrum switches to one that is dominated by an ultrasoft component, with a temperature $\approx$0.3–0.4 keV. The spectrum above 10 keV becomes much softer than in the hard state, with a variable photon index $\approx$1.9–2.5 (Gierliński et al. 1997; Cui et al. 1997; Dotani et al. 1997). It also exhibits pair annihilation features in the region 500 keV $< E < 1000$ keV (Liang 1993a).

3. THE MODEL

The system we consider is a Keplerian disk kept in hydrostatic equilibrium in the direction perpendicular to the plane of the disk. The region of our concern is the innermost one, extending away from $R_i$, the inner radius of the disk, to approximately 10–15 $R_g$.

Here $R_i$ will be taken as the radius of the last stable orbit of a maximally synchronously rotating Kerr black hole, i.e., $R_i \approx 0.5R_g$, where $R_g$ is the gravitational radius (Bardeen et al. 1972). This assumption is meant to bring the disk closer to the hole, where it will possibly be hotter. Protons
and electrons are out of thermal equilibrium, with $T_e$, the proton temperature, much greater than $T_i$, the electron temperature. The pressure is given by the ions. In the outermost parts of the disk, matter composition is hydrogen (nine-tenths) and helium (one-tenth). In its way down to the hole, He depletion starts and the produced neutrons gradually contributes to the viscosity. When matter reaches the outer radius of the innermost region, the depletion is complete. As the incoming matter comes closer to the hole, charge-exchange reactions start to contribute to neutron production. Concomitant to these reactions, electron-positron pairs are radiatively produced in production-annihilation equilibrium. Somewhere, an external source supplies the disk with soft photons. Some of these photons succeed in being upscattered in energy, reaching the Wien peak, where they interact and produce pairs. The remaining are also upscattered, but leave the system before reaching the Wien peak. Both nuclear reactions we are considering are endothermic, so they lower the plasma temperature by taking the energy they need to ignite. For the He breakup, the number of resulting particles is greater than the number of interacting particles. Thermalization of these extra particles acts as an additional cooling source. The pion produced by the charge-exchange reaction rapidly decays, producing a positron that has to be thermalized and a neutrino that leaves the system, with further cooling of the disk.

Heating the disk by viscous processes locally balances cooling. Out of this neighboring region, we take for granted the existence of some unknown process that heats the gas and makes He breakup viable.

Since this innermost region is thick, ideally we should take advection into account. However, the more important advection is, the less important the radiative cooling. This increases the nucleon temperature, favoring the onset of nuclear reactions. Were advection taken into account, more energy would be advected and available to ignite nuclear reactions.

Esin et al. (1997) have studied the behavior of advection as a function of the combined effect of the accretion rate and viscosity parameter, concluding its importance for small values of $m$ (in Eddington units). As $m$ increases, from 0.01 to a critical $a$-dependent value, the advection zone becomes very luminous, shrinking in size with further increase of the accretion rate. For sufficiently large values of $m$, it disappears, with the thin disk extending down to the marginally stable orbit. For consistency, were advection included, we should also have to include conduction, a process that, in the inner disk of Cyg X-1, is by far more important than radiative cooling (Meirelles 1991a).

4. DISK EQUATIONS

Here we give the standard structure equations of disk theory, i.e., hydrostatic equilibrium pressure, mass conservation, and heat generation equations. For a more detailed derivation, the reader is referred to Shapiro et al. (1976) and Meirelles (1993).

Hydrostatic equilibrium in the vertical direction reads

$$P = \frac{\rho}{3}\Omega^2 H^2,$$

where $\rho$ is the density, $\Omega$ is the Keplerian angular velocity, and $H$ is the semi–scale height of the disk. The total pressure, $P$, is mainly given by the pressure due to the protons, i.e.,

$$P = N 10^{11} kT_i,$$

where $N$ is the proton number density, and $k$ is the Boltzmann constant. Ion temperature is in units of $10^{11}$, and electron temperature in units of $10^9$. Unless otherwise stated, physical variables are expressed in the cgs system of units. Combining equations (3) and (4) we obtain

$$H = 4.93 \times 10^9 \Omega^{-1} T_i^{0.5}.$$

Mass conservation, together with conservation of angular momentum and the definition of the viscosity parameter, yield

$$\frac{\dot{M} S}{4\pi} = \alpha \rho H,$$

which implies for the column density

$$\Sigma = 1.96 \times 10^{-20} \alpha^{-1} T_i^{-1} \Omega \dot{M} S,$$

where $\alpha$ is the viscosity parameter, $\dot{M}$ is the accretion, and $S$ is a function that depends on the boundary condition imposed on the torque at the inner radius. For $Q^+$, the heat flux, viscously generated, going into the disk at a distance $R$ from the central object,

$$Q^+ = \frac{3}{2} \pi \dot{M} H^2.$$

In the two-temperature regime, energy is collisionally transferred from protons to electrons and pairs at a rate, corrected for the inclusion of neutrons, given by (Guilbert & Stepney 1985; Spitzer 1962)

$$Q^- = 1.72 \times 10^{28} \rho^2 (T_i - T_e)(1 + 0.41 T_e^{0.5})$$
$$\times (1 + 2z + y_+ y_n^{-1})^{-2} T_e^{-3/2} H,$$

where $y_n$ is the total neutron abundance, $y_+$ is the abundance due to pion production, and $z$ is the positron density to proton density ratio. The electrons and pairs, in turn, lose energy through unsaturated inverse Comptonization of externally supplied soft photons. Like Shapiro et al. (1976), we assume that these photons are copiously supplied and that the Kompaneetz $y$ parameter is constant and equal to 1 (Rybicki & Lightman 1979), i.e.,

$$y = (0.674T_e)^{1/4},$$

where $\tau_{\text{es}}$ is the electron scattering depth, $i = 1$ for $T_e \leq 5.9$ and 2 otherwise, and $j = 1$ for $\tau_{\text{es}} \leq 1$ and 2 otherwise. We assume that the intensity of the soft photons coming from the external source overwhelms the intensity due to photons internally produced (bremsstrahlung). With the inclusion of neutrons, the electron scattering depth changes to

$$\tau_{\text{es}} = \frac{\sigma_T}{2m_p} \Sigma (1 + 2z + y_+)(1 + y_n)^{-1},$$

where $\sigma_T$ is the Thomson cross section for electron scattering, and $m_p$ is the proton mass. Clearly, this way of treating the radiative problem in the disk relies heavily on the existence of an external soft photon source, which we assume, from the very beginning, as granted. We should emphasize, since our main concern is not the radiative problem, that we adopt this procedure for practical reasons; the spectrum of these soft photons, after being upscattered in energy,
through unsaturated inverse Comptonization in the inner region, reproduces fairly well the observed spectrum of Cyg X-1 in the 8–500 keV region (Shapiro et al. 1976).

For the moment we postpone the discussion of the cooling of the disk due to breakup and pion production. We next discuss pair production in the disk.

5. PAIR PRODUCTION

As we have emphasized before, our main concern in this paper is to investigate the possibility of the occurrence of nuclear reactions in the inner region of the disk. We therefore construct a reasonable and realistic environment of photons and pairs, in which the nuclear reactions take place. As we have done in the case of radiative transport, we make some simplifications when treating the pairs. We assume that pairs are created mainly by Wien photons interacting with Wien photons. However, instead of writing down the formal pair equilibrium equation, we adopt a procedure similar to that of Liang (1979). This essentially consists of relating the photon chemical potential to the soft photon flux. This kind of procedure is valid as long as the plasma is in Wien equilibrium (Svensson 1984), which implies \( \tau_{\gamma\gamma} > 1 \).

Below, we briefly review this procedure.

For \( kT_e < m_e c^2 \), \( f_\gamma \), the photon occupation number, obeys the Kompaneetz equation. In that limit

\[
f_\gamma = 24 e^{\mu_\gamma} x^{-\gamma} (1 + x + x^2/2! + x^3/3! + x^4/4!) ,
\]

where \( \mu_\gamma \) is the photon chemical potential and \( x = h\nu/kT_e \).

The occupation number \( f_+ \) for positrons and electrons, respectively, can be written

\[
f_+ = e^{\mu_+} e^{-E_+} , \tag{13a}
\]

\[
f_- = e^{\mu_-} e^{-E_-} . \tag{13b}
\]

Since the number of particles is conserved in the reaction

\[
\gamma + \gamma \rightarrow e^+ + e^- , \tag{14}
\]

the chemical potential should satisfy

\[
2\mu_\gamma = \mu_+ + \mu_- . \tag{15}
\]

For Cyg X-1 and related sources \( n \approx 1 \), which implies \( s = 4 \). The radiative cooling will be

\[
F_r = \pi \int_{\nu_0}^{3kT_e} \frac{2\nu^3 f_\gamma}{c^2} d\nu . \tag{16}
\]

Now, using equations (12), (13a), (13b), (14), (15), and (16) we obtain

\[
N^2 z (1 + z) = \left[ \frac{J c^2 F_r}{(kT_e)^4 278 \pi} \right]^2 , \tag{17}
\]

where \( J \) is given by

\[
J = 4\pi cm_e^2 kT_e K_2 \left( \frac{m_e c^2}{kT_e} \right) , \tag{18}
\]

where \( K_2 \) is the modified Bessel function of the second kind and we have substituted in the slowly varying function \( \ln(kT_e/h\nu_0) \) for canonical values appropriate for Cyg X-1.

It is worth mentioning that the radiative cooling now will be given by

\[
F_r = Q^+ - Q_n , \tag{19}
\]

where \( Q_n \) is the nuclear cooling. Adopting the procedure as we did to treat radiative cooling and pair production, we have overestimated both processes. Since the energy production is constant, this implies underestimating the nuclear reaction rate.

6. \( ^8\text{He}, \text{ PION, AND NEUTRON PRODUCTION} \)

In the chain of reactions leading to the \( ^8\text{He} \) breakup,

\[
\begin{align*}
18.35 \text{ MeV} & + p + ^4\text{He} \rightarrow ^3\text{He} + D \\
20.578 \text{ MeV} & + p + ^4\text{He} \rightarrow ^3\text{He} + p + n \\
19.844 \text{ MeV} & + p + ^4\text{He} \rightarrow ^3\text{H} + p + p \\
5.494 \text{ MeV} & + p + ^3\text{He} \rightarrow D + p + p \\
0.739 \text{ MeV} & + p + ^3\text{He} \rightarrow ^3\text{He} + n \\
2.224 \text{ MeV} & + p + D \rightarrow p + p + n ,
\end{align*}
\]

the one that takes longest to occur is the first one. Once this reaction has ignited, the other follow very rapidly. So, we assume that the breakup is dominated by this reaction (Guessoum & Dermer 1988), and for the nuclear reaction rate

\[
R_b = N_{^4\text{He}} \sigma_{\text{b}th} ,
\]

we use a prescription (Gould 1982, 1986; Guessoum & Gould 1989; Guessoum & Kazanas 1990) to write

\[
R_b = 1.42 \times 10^{-17} T_i^{-0.5} e^{-2.56/T_i} N_i^2 , \tag{20}
\]

where \( N_{^4\text{He}} \) is the \( ^4\text{He} \) particle density, \( N_i \) is the hydrogen density, \( \sigma_{\text{th}} \) is the cross section for the breakup, and \( T_i \) is the thermal velocity. This implies a neutron abundance, \( y_n \), due to this reaction, given by

\[
y_n = \frac{R_b}{M} (1 + y_n) = 1.05 \times 10^9 T_i^{-0.5} e^{-2.56/T_i} \Sigma^2 R^2 M^{-1} V^{-1} (1 + y_n)^{-1} , \tag{21}
\]

where \( V \approx \pi R^2 H \) is the volume.

The total neutron abundance \( y_n \) is given by

\[
y_n = y_+ + y_+ . \tag{22}
\]

Using equation (20) we can express the nuclear cooling due to this reaction as

\[
Q_b = R_b H \left( \Delta E_b + \frac{2}{3} kT_i \right)
\]

\[
= 2.84 \times 10^{20} (1 + 1.37 T_i) T_i^{-1} e^{-2.56/T_i} \Sigma^2 (1 + y_n)^{-2} , \tag{23}
\]

where \( \Delta E_b \) is the \( ^4\text{He} \) breakup energy and \( \frac{2}{3} kT_i \) is the energy needed to thermalize the excess of produced particles as compared to the reacting ones.

For the reaction

\[
p + p \rightarrow p + n + \pi^+ ,
\]
we use the results by Engel et al. (1996), who have carried out a very detailed study of the $p(p, \pi^+)p$ and $p(p, \pi^0)p$ reactions using a fully relativistic Feynman diagram technique. Their calculations have been carried out under the prescription of the one boson exchange (OBE) model, with allowance for the inclusion of both nucleon and delta isobar excitations in the intermediate states, as well as for the exchange of $\pi$, $\sigma$, $\rho$, and $\omega$ mesons. Accounting for the exchange of $\sigma$ and $\omega$ makes their results very reliable close to the threshold, where these contributions presumably dominate. Most of the parameters of the OBE model are determined by fitting to the $n-n$ scattering data over the energy range of 300 MeV to 2 GeV. A fitting to their data for the total cross section leads to

$$
\sigma_{pp} = 10^{f(E)},
$$
(24)

where

$$
f(E) = \begin{cases} 
0.0371785E - 14.781764 & \text{for } 290.0 \leq E \leq 378.8 \text{ MeV,} \\
-4.96247 + 0.0148283E - 0.0000839163E^2 & \text{for } 378.8 \leq E \leq 1060 \text{ MeV,} \\
-0.000064893617 + 1.430787 & \text{for } 1060 \leq E \leq 2000 \text{ MeV,}
\end{cases}
$$

where $E$ is expressed in MeV and the cross section in mbarn. The reaction rate for this reaction can be written as

$$
R_+ = 0.5N^2\left(\frac{kT_i}{m_p}\right)^{0.5}\int_{w_s}^{\infty} \sigma_{pp}e^{-w} dw,
$$
(25)

where $w$ is the energy in units of $kT_i$, and $w_s$ is the threshold energy for this reaction. Then $y_+$, the contribution of this reaction to the neutron abundance, follows straightforwardly,

$$
y_+ = 9.24 \times 10^{-4} R^2 \Sigma^2 \left(\frac{kT_i}{m_p}\right)^{0.5} t(w)T^{-1}H^{-1}(1 + y_n)^{-1},
$$
(26)

where $t(w)$ is the integral defined in equation (25), now in units of $10^{-27}$ mbarn. Figure 1 shows $t(w)$ as a function of $T_i$.

To find the cooling of the disk due to this reaction, we remind ourselves that the pion has a mean life of about $2.6 \times 10^{-8}$ s, decaying through

$$
\pi^+ \rightarrow \mu^+ + \nu_+ + 34 \text{ MeV}.
$$
The $\mu^+$, in turn, has a mean life of about $2.2 \times 10^{-6}$ s, decaying through

$$
\mu^+ \rightarrow \nu_e + e^+ + \bar{\nu}_e + 105 \text{ MeV},
$$
which allows us to write for the cooling

$$
Q_{pp} = 1.23 \times 10^{38}(2.40 \times 10^{-2} + 2.07 \times 10^{-7} T_e) \\
\times \Sigma^2 T_i^{0.5} t(w)H^{-1}(1 + y_n)^{-2}.
$$
(27)

Finally, we can verify whether neutrons can account for the viscosity in the disk. In order to do so, we write for the neutron viscosity (Weaver 1976)

$$
\eta = \frac{1}{3} \rho_n v_n \lambda_n,
$$
(28)

where $\rho_n$, $v_n$, and $\lambda_n$ are respectively the neutron density, neutron velocity, and neutron mean free path. Taking the average for a Maxwellian distribution, we obtain (Bond et al. 1965)

$$
v_n = 1.08 y_n \left(\frac{kT_i}{m_n}\right)^{0.5} \ell_n,
$$
(29)

where $\ell_n$ is the effective mean free path given by

$$
\ell_n = \frac{\lambda_n}{1 + (\lambda_n/H)^2},
$$
(30)

with $\lambda_n$ given by

$$
\lambda_n = \frac{m_p}{\sigma_{np}}.
$$
(31)

The cross section for neutron scattering, averaged over a Maxwellian distribution, is (Gammel 1963)

$$
\sigma_n = 5.47 T_i^{-0.85} \text{ barn}.
$$
(32)

Finally, using the definition of the viscosity parameter

$$
\alpha = \frac{\rho_n v_n \lambda_n}{\rho v_n H} = \frac{y_n \lambda_n}{1 + y_n \lambda_n} H,
$$

together with equation (29), we can write

$$
\alpha \left[1 + \left(\frac{\lambda_n}{H}\right)^2\right] = \frac{y_n \lambda_n}{1 + y_n \lambda_n} H,
$$

and using equations (31) and (32) we obtain

$$
\alpha^2 = 3.65 \times 10^{-2} M^2 R^2 \Omega T_i^{-1.85}(1 + y_n)^{-1} \\
- 10^{-5} M^2 R^2 \Omega^2 T_i^{-3.7},
$$
(33)

the accretion rate expressed in units of $10^{17}$ g s$^{-1}$. 
We are now in a position to ask ourselves if neutron collisions with the accreting matter can indeed act as a source for the viscosity in the innermost regions of an accretion disk. Previous results (Meirelles 1993) relied heavily on the constraint imposed assuming equality between nuclear reaction time and dynamical time. Dropping this assumption leaves practically no restriction as far as the accretion rate is concerned. To not be burdened by additional complexities, let us unravel things a little bit by neglecting, for the moment, the contribution to the neutrons due to pion production. Under this procedure, using equations (5) and (7), obtaining $\alpha^2$ from equation (21), and equating this expression with that of equation (33) yields

$$y_n^2 \left( 3.65 \times 10^{-2} \dot{M}_{17} \Omega T_i^{1.15} - 10^{-5} \dot{M}_{17}^2 \Omega^2 T_i^{-0.7} \right) - 10^{-3} y_n \dot{M}_{17}^2 \Omega^2 T_i^{-0.7} \frac{138}{M_{17}} e^{-2.56/T_i} = 0. \quad (34)$$

Now specializing for $r = 10$, we obtain

$$\dot{M}_{17} \leq 5.77 T_i^{1.85}. \quad (35)$$

This, however, is not the final result, since we have to investigate the effects of cooling and pair production. Figure 2 shows the neutron abundance $y_n$ as a function of $T_i$ for $r = 10$ and $\dot{M}_{17} = 1.0$.

In Figure 3 we present the viscosity parameter $\alpha$ as a function of $T_i$, in the same conditions as Figure 2. We see that the viscosity parameter may reach very high values. The maximum $\alpha$ occurs for $T_i \approx 3.0$.

We finally reduce our system of equations for the disk to only two equations involving $T_e$ and $T_i$. The first is the thermal equilibrium equation obtained straightforwardly from equations (8), (9), (19), and (23), which after due simplifications reads

$$(1 + 1.37 T_i) e^{-2.56/T_i} \alpha^{-2} T_i^{-2} (1 + y_n)^{-2} + 12.14 (1 + 0.41 T_e^{0.5}) \alpha^{-1} T_i^{0.5} T_e^{2.5} (1 + y_n)^{-1} - 10.92 = 0, \quad (36)$$

and the second is the pair equilibrium equation, obtained from equation (17), again with concomitant use of equations (8), (9), (19), and (23), and reads

$$\Delta^2 - 1 - 252.5 T_i^2 \left( 1 + 0.41 T_e^{0.5} \right)^2 e^{-11.87/T_i} T_e^{-10} = 0, \quad (37a)$$

where $\Delta$ is given by

$$\Delta = 6.26 \alpha T_i T_e^{-1} (1 + y_n)^{-1} - y_n, \quad (37b)$$

and $\alpha$ and $y_n$ are given by equations (33) and (34), respectively.

A glance at Figure 4 reveals the existence of a solution for the two-temperature soft photon Comptonized accretion disk with pairs and viscosity generated by neutron collisions with the accreting matter, neutrons supplied by $^4$He breakup.
To check for the consistency of our solution, we also plot $(1 + 2z)$ in Figure 5, in both the thermal equilibrium and pair equilibrium situations.

7. HIGHLIGHTING PION PRODUCTION

We have seen in the previous section, neglecting contributions from pion production to the neutrons in the disk, the existence of a solution to the disk equations at $r = 10$, $T_i / C_25^2$.5. Were pion production included, the results would not change so much, since at this temperature the contribution would be negligible. However, at that temperature the depletion of $^4$He is complete. Therefore, for $r < 10$, breakup no longer contributes to the cooling of the disk, its contribution being restricted to the viscosity through the neutrons already produced.

To underline the pion contribution relative to the neutron production, let us calculate it at the point where the ion temperature is largest, i.e., $r = 1$. Calculating physical variables at this point and inserting them into equation (26) yields

$$
\frac{1}{2} \frac{\partial}{\partial \beta} y_+ - 0.364 \alpha^{-2} T_i^{-2} \frac{d}{d \beta} \ln w ,
$$

and $\alpha$ is now (eq. [33])

$$
\alpha^2 = 730 \left( \frac{1 + 19 y_+}{20 + 19 y_+} \right) T_i^{-1.85} - 4 \times 10^3 T_i^{-3.7} .
$$

To see how $\alpha$ and $y_+$ behave in that region, we plot them in Figures 6 and 7. In Figure 6, we see that the production of neutrons due to $p(p, \pi^+ n)p$ may be comparable to the contribution due to the breakup.

Looking at equation (35), we realize that the constraint on the accretion rate now reads

$$
M_{17} \leq 0.183 T_i^{1.85} .
$$

As we have done previously, we now investigate the effects of cooling and pair production, and for that, we reduce our system of equations to only two equations involving $T_e$ and $T_i$. The thermal equilibrium and pair equilibrium equations become, respectively,

$$
\left( \frac{20}{79} + y_+ \right)^2 - 3.86 \alpha^{-2} T_i^{-2} \frac{d}{d \beta} \ln w - 1.1 \alpha^{-1} T_i^{-0.5} T_e^{-2.5} \times \left( 1 + 0.41 T_e^{0.5} \right) \left( \frac{20}{79} + y_+ \right) = 0
$$

and

$$
\Delta^2 - 1 - 204 T_i^2 \left( 1 + 0.41 T_e^{0.5} \right)^2 e^{-11.87/T_e} T_e^{-10} = 0 ,
$$

where $\Delta$ is now given by

$$
\Delta = 0.1975 \alpha T_e^{-1} \left( \frac{20}{79} + y_+ \right) - y_+ ,
$$

and $y_+$ and $\alpha$ are given by equations (38) and (39), respectively.

![Fig. 5.—Relation between $(1 + 2z)$ and $T_i$, for $M_{17} = 1.0$. The upper curve is obtained using $T_i - T_e$ data from the solution of the thermal equilibrium equation; the lower, using data from the pair equilibrium equation.](image1)

![Fig. 6.—Plot of $y_+$, the $p(p, \pi^+ n)p$ contribution to the neutron abundance.](image2)

![Fig. 7.—Viscosity parameter $\alpha$, taking pion production into account at $r = 1$.](image3)
In Figure 8 we plot the solutions to the thermal and pair equilibrium equations. To see how the solutions of these equations behave as a function of the accretion rate, we have plotted them for $M_{17} = 0.9$ in Figure 9. In this figure can be seen the emergence of two solutions, instead of only one as in the $M_{17} = 1.0$ case. In Figure 10 we plot $(1 + 2z)$ as a function of $T_i$. In contrast to Figure 5, we now have two solutions, both with $z < 1.0$. Finally, to see the importance of nuclear cooling due to $\pi^+$ production, we plot $q = Q_{pp}/Q_{ei}$ in Figure 11.

8. MASS AND ACCRETION RATE DEPENDENCE OF THE RESULTS

Here we think it would be of interest to present some quantitative arguments to justify the relevance of the effects we are including in the model. A discussion of this kind certainly implies a comparison between the infall and nuclear reaction times, $t_{\text{inf}}$ and $t_{\text{nr}}$. For a more detailed analysis of the timescales involved, see Gould (1982), Stepney (1983), Takahara & Kusunose (1985), Guessoum & Gould (1989), and Begelman et al. (1987).

Assuming a thick disk, we have $H \approx R$, which implies

$$V_r \approx -\alpha v_K,$$

where $H$ is the semi-scale height of the disk, $R$ is the radial distance, $V_r$ is the radial (infall) velocity, and $v_K$ is the Keplerian velocity. Therefore, the particle number density, ion temperature, infall, and nuclear reaction times are,
respectively,
\[ N = 5.1 \times 10^{16} \frac{\dot{M}_{17}}{M^2} \alpha^{-1} r^{-3/2}, \]
\[ T_i = 72 r^{-1}, \]
\[ t_{\text{inf}} = 3.47 \times 10^{-5} M^{\alpha-1} r^{3/2}, \]
\[ t_{\text{nr}} = 3.5 \times 10^{-2} \frac{\dot{M}_{17}}{M^2} \alpha^{3/2} T_i^{1/2}. \]

The ratio between the nuclear reaction time and the infall time is
\[ \frac{t_{\text{nr}}}{t_{\text{inf}}} \approx 10^3 \frac{M}{\dot{M}_{17}} \alpha^2 T_i^{1/2}. \]

If \( \alpha \) grows, so does \( t_{\text{nr}} \), with opposite behavior for \( t_{\text{inf}} \). If \( t_{\text{inf}} \) decreases too much, there will not be enough time for nuclear reactions to take place. If \( \alpha \) decreases too much, the nuclear reactions will be ineffective. However, nuclear reactions supply the disk with neutrons, generating viscosity, keeping accretion going on. A compromise is found when \( t_{\text{nr}}/t_{\text{inf}} \approx 1 \).

To calculate this ratio, we must estimate \( \alpha \). Assuming full depletion of \(^4\text{He} \), we have two neutrons produced for each reaction. So, for cosmic abundance,
\[ y_n = \frac{2}{18} \approx 0.05. \]

Inserting this value into the ratio calculated at \( r = 10 \) yields \( t_{\text{nr}}/t_{\text{inf}} \approx 1-10 \) for values appropriate for Cyg X-1. Although we have applied this to Cyg X-1, our results are general, as may be seen by considering the expression for \( y_n \) and the results for \( t_{\text{nr}}/t_{\text{inf}} \).

Using the definition of \( y_n \), we arrive at
\[ y_n \approx \frac{4\pi R_{\text{nr}} R_i^3}{M} = 0.02 \frac{\dot{M}_{17}}{M} \alpha^{-2} T_i^{-1/2} e^{-2.56/T_i}, \]
where \( R_{\text{nr}} \) is the nuclear reaction rate. In that formulation \( y_n \approx \alpha \), and therefore
\[ y_n \approx \left( 0.02 \frac{\dot{M}_{17}}{M} \right)^{1/3} T_i^{-1/6} e^{-2.56/(3T_i)}. \]

A similar result holds for \( t_{\text{nr}}/t_{\text{inf}} \), i.e.,
\[ \frac{t_{\text{nr}}}{t_{\text{inf}}} = f_2 \left( \frac{M}{\dot{M}} \right) g_2(r). \]

The \( f_i \) and \( g_i \) are, respectively, scale and universal functions, and have a kind of similarity. If the disk thickens so as to have almost virial ion temperature, it does not matter if we are dealing with disks in active galactic nuclei (AGNs) or around black hole candidates. For example, if the ratio \( \dot{M}/M \) is the same, we will have the same neutron abundance and the same ratio of times no matter where the disk is.

So far, we have assumed \( T_i \) close to the virial value. For this to occur, the existence of conditions for a two-temperature regime is needed.

Following Takahara & Kusunose (1985) and Begelman et al. (1987) a two-temperature flow will develop in the disk whenever the following condition is satisfied:
\[ \dot{m} < 50 \left( \frac{\dot{m}_{\text{ff}}}{L_E} \right)^2, \]
where \( \dot{m} = \frac{\dot{M}}{M} \), \( \dot{m}_{\text{ff}} \) is the free-fall velocity, and \( L_E \) is the Eddington luminosity. In our units, with all the simplifications, this criterion reads
\[ \frac{\dot{M}_{17}}{M} < 1.74. \]

Therefore, if this criterion is met, with the inner radius as close as possible to the hole, our results apply, independently of the values of the mass and accretion rate.

9. ANALYSIS AND CONCLUSIONS

We have considered neutron production in the innermost region of the accretion disk in Cyg X-1. The inner radius of the accretion disk was assumed to be at the radius of the last stable orbit of a maximally synchronously prograde rotating Kerr black hole, \( R_i \approx 0.5 R_g \), where \( R_g \) is the gravitational radius given by \( 2GM/c^2 \) (Bardeen et al. 1972). This region extends up to \( 20R_g \). Outside this region, some unknown process generates viscosity and drives accretion. As matter flows inward, it heats up, reaching high temperatures to ignite \(^4\text{He} \) breakup. This reaction produces neutrons, contributing to the viscosity. As matter comes close to the outer radius of the inner region, it is almost fully depleted of \(^4\text{He} \). This fact roughly defines \( R_2 \), the outer radius. Farther away from \( R_2 \), inside this region, charge-exchange reactions start to contribute to neutron production.

The radiative cooling of the disk was assumed to be due to unsaturated inverse Comptonization of soft external photons impinging on the disk. Some of those soft photons are upscattered in energy, reaching the Wien peak, where they interact with themselves, producing pairs.

Radiative cooling and pair production decrease the electron temperature, an effect that hinders the efficiency of nuclear reactions, even if the ion temperature is high enough for them to occur. When \( T_e \) decreases, the number of interacting electrons with one proton increases sharply, as a result of less shielding. Nuclear forces only act at short distances, and the high ion temperature needed to overcome the Coulomb barrier also hinders the occurrence of nuclear events because of the very short time in the nuclear range. As a matter of fact, the reaction rate due to electromagnetic interactions prevails over the rate due to nuclear interactions if \( T_e \) satisfies
\[ T_e < 28.56 T_i^{1/3}. \]

However, the energy transferred in the (electromagnetic) scattering will be in the keV range, while for the nuclear reaction it will be in the MeV range. Therefore, the previous inequality changes roughly to
\[ T_e < 0.2856 T_i^{1/3}. \]

when one also considers the energy transferred in the event.

Although this kind of reasoning is correct, it is somehow limited because it does not take into account the fact that nuclear reactions are driving accretion on. If they occur at a
very defective rate, so do all further reactions. In other words, nuclear reactions have to occur first. This can be seen by the ratio of nuclear reaction time to the infall time, where a compromise is found, giving a value \(\sim 1-10\).

Aiming at the application to Cyg X-1 and related sources, we have set \(M_{17} = 1\) and 0.9, which are much less than the critical value for \(M_1\) calculated close to the horizon, i.e.,

\[
M_{17} = 0.18371^{0.85}.
\]

For both values of the accretion rate, \(T_4\) is larger than the value given by the inequality (43). As a result, for every solution we have found, \(z < 1\). This is surprising, since it is known that, in the absence of these reactions, one of the solutions for the two-temperature disk has \(z > 1\). Another surprising result we have found concerns the sensitivity of the number of solutions to the value of the accretion rate. Calculating at \(r = 1\) and 10, for \(M_{17} = 1\) we have found that the solution is unique. Decreasing to \(M_{17} = 0.9\), there are two solutions. For these values of the accretion rate the neutron abundance in the inner region is fairly high, going from \(y_n = 1/19\) in the outer border, with total \(^4\)He depletion, to about twice that value close to the horizon. This implies \(\alpha\) quite large, i.e., large viscosity. This large viscosity, nevertheless, comes along with a high nuclear cooling, being comparable to the radiative cooling.

Although using data appropriate for Cyg X-1, our results are general because they come from expressions that obey scaling laws, depending only on the ratio \(M_c/M\). Once this ratio is constant, and the inner radius is the same, so as to have high enough temperatures, the results are independent of the mass and accretion rate individually.

It is known that the innermost region of the accretion disk is secular and thermally unstable (Shakura & Sunyaev 1976) in the absence of nuclear reactions. Taking these nuclear reactions into account, the disk behaves in the same manner at the very onset of these instabilities. However, it is worth observing the behavior of the viscosity parameter with \(T_4\), because two scenarios may emerge as far as the time evolution of the inner region is concerned. As \(\alpha\) starts growing, it reaches a maximum somewhere between \(T_4 = 15\) and 20, then decreases and may become very small, even null, with increasing \(T_4\). At that moment, accretion starts switching off and the cooling, which scales with \(\alpha^{-2}\), is practically instantaneous. Outside this region, however, some other mechanism for viscosity generation is still operating, and there accretion keeps going on. As the matter reaches the border of the inner region it gets piled up there. As this piling up continues, this region will be subject to several instabilities. Once one of these instabilities starts to grow, it triggers accretion in the inner region. As the accretion proceeds, the accretion rate decreases until it reaches a value for which there are two steady states for the disk. Which one will the disk choose? At that moment, do both thermal and pair equilibrium hold in that region? A less drastic scenario is the one somewhat similar to that proposed by Kusunose & Mineshige (1991). Physical processes in the disk are characterized by timescales, the one chosen by the system being that with the lesser timescale. As we have seen, thermal and pair equilibrium only hold under special circumstances. It may happen that because of the nuclear reactions and instabilities in the disk, the system may undergo a kind of limit cycle behavior around the upper solution.

However, to have a better understanding of the time evolution of these systems will require a better treatment of both radiative cooling and pair production, and a more detailed stability analysis of the problem as well. That is what we intend to do next, in a future contribution, also taking into account the reaction \(p(p, p^n)\).

The authors thank an anonymous referee for very pertinent comments that helped to improve the text. C. L. L. acknowledges support from FAPESP (project 1998/065902).

REFERENCES

Abramowicz, M. A., Calvani, M., & Nobili, L. 1980, ApJ, 242, 772
Abramowicz, M. A., Chen, X., Kato, S., Lasota, J. P., & Regev, O. 1995, ApJ, 438, L37
Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
———. 1992, ApJ, 392, 662
Balbus, S. A., Hawley, J. F., & Stone, J. M. 1996, ApJ, 467, 76
Bardeen, J. M., Press, W. H., & Teukolsky, S. A. 1972, ApJ, 178, 347
Beigelman, M. C., Sikora, M., & Rees, M. J. 1987, ApJ, 313, 689
Belyanin, A. A., & Derishev, E. V. 2001, A&A, 379, L25
Bisnovatyi-Kogan, G. S., & Blinnnikov, S. I. 1986, A&A, 59, 111
Bisnovatyi-Kogan, G. S., Zef dovich, Yu. B., & Sunyaev, R. A. 1971, Soviet Astron.—AJ, 15, 17
Bjornsson, G., Abramowicz, M., Chen, X., & Lasota, J. P. 1996, ApJ, 467, 99
Bjornsson, G., & Svensson, R. 1991, MNras, 249, 177
———. 1992, ApJ, 394, 500
Bond, J. W., Watson, K. M., & Welch, J. A. 1965, Atomic Theory of Gas Dynamics (Reading: Addison-Wesley)
Cabot, W., & Pollack, J. B. 1992, Geophys. Astrophys. Fluids, 64, 97
Chakrabarti, S. K., & Mukhopadhay, B. 1999, A&A, 344, 105
Chen, X. M., Abramowicz, M., Lasota, J. P., Narayan, R., & Yi, Y. 1995, ApJ, 443, L61
Chen, X. M., & Taam, R. E. 1993, ApJ, 412, 254
Coroniti, F. V. 1981, ApJ, 244, 587
Cui, W., Heindl, W. A., Rothschild, R. E., Zhang, S. N., Jahoda, K., & Focke, W. 1997, ApJ, 474, L57
Dotani, T., et al. 1997, ApJ, 485, L87
Dubrulle, B. 1990, Ph.D. thesis, Toulouse Univ.
Eardley, D. M., & Lightman, A. P. 1975, ApJ, 200, 187
Engel, A., Dutt-Mazumder, A. K., Shaym, R., & Mosel, U. 1996, Nucl. Phys. A, 603, 387

ESIN, A. A., Mc Clintock, J. E., & Narayan, R. 1997, ApJ, 489, 865
ESIN, A. A., Narayan, R., Cui, W., Grove, J. E., & Zhang, S. N. 1998, ApJ, 505, 854
ESIN, A. A., Narayan, R., Ostriker, E., & Yi, Y. 1996, ApJ, 465, 312
Gammel, J. 1963, in Fast Neutron Physics Part II, ed. J. B. Marion & J. L. Fowler (New York: Interscience), 2209
Gierlinski, M., Zdziarski, A. S., Done, C., Johnson, W., Ebisawa, K., Ueda, Y., Haardt, F., & Phillips, B. F. 1997, MNras, 288, 958
Gould, R. J. 1982, ApJ, 263, 879
———. 1986, Nucl. Phys. B, 266, 737
Guissoum, N., & Dermer, C. 1988, in Nuclear Spectroscopy of Astrophysical Sources, ed. N. Gehrels & G. M. Share (New York: AIP), 332
Guissoum, N., & Gould, R. J. 1989, ApJ, 345, 556
Guissoum, N., & Jean, P. 2002, A&A, 396, 157
Guissoum, N., & Kazanas, D. 1990, ApJ, 358, 525
Guilbert, P. W., & Stepney, S. 1985, MNras, 212, 523
Herrero, A., Kudritzki, R. P., Gabler, R., Vilchez, J. M., & Gabler, A. 1995, A&A, 297, 556
Honna, F., et al. 1991, PASJ, 43, 261
Ichimaru, S. 1977, ApJ, 214, 840
Kusunose, M. 1996, ApJ, 457, 813
Kusunose, M., & Mineshige, S. 1991, ApJ, 381, 490
———. 1992, ApJ, 392, 653
———. 1995, ApJ, 440, 100
———. 1996, ApJ, 468, 330
Lasota, J. P., Abramowicz, M., Chen, X., Krolik, J., Narayan, R., & Yi, Y. 1996a, ApJ, 462, 462
Lasota, J. P., Narayan, R., & Yi, Y. 1996b, A&A, 314, 813
Liang, E. P. 1979, ApJ, 234, 1105
———. 1991, ApJ, 367, 470
———. 1993a, Second Compton Symposium, Evolution of X-Ray Binaries Conference and Huntsville Gamma Ray Burst Workshop
———. 1998, Phys. Rep., 302, 67
Liang, E. P., & Nolan, P. L. 1984, Space Sci. Rev., 38, 353
Liang, E. P., & Thompson, K. A. 1980, ApJ, 240, 271
Lightman, A. P. 1982, ApJ, 253, 842
Lightman, A. P., & Zdziarski, A. A. 1987, ApJ, 319, 643
Lin, D. N. C., & Papaloizou, J. 1980, MNRAS, 191, 37
Loeb, A., & Laor, A. 1992, ApJ, 384, 115
Lynden-Bell, D. 1969, Nature, 233, 690
Meirelles, C. 1991a, ApJ, 378, 266
———. 1993, ApJ, 412, 288
Meirelles, C., Ruiz, M. R., & Luo, C. 1997, A&A, 317, 290
Mineshige, S. 1993, ApSS, 210, 83
Muchotrzeb, B., & Paczynski, B. 1982, Acta Astron., 32, 1
Mukhopadhyay, B., & Chakrabarti, S. K. 2000, A&A, 353, 1029
Narayan, R., & Yi, I. 1994, ApJ, 428, L13
———. 1996, ApJ, 457, 821
Narayan, R., Yi, I., & Mahadavan, R. 1995a, Nature, 374, 623
———. 1995b, Nature, 452, 710
Oda, M., Takagishi, K., Matsuoka, M., Miyamoto, S., & Ogawara, Y. 1974, PASJ, 26, 303
Paczynski, B., & Abramowicz, M. 1982, ApJ, 253, 897
Paczynski, B., & Binovatyi-Kogan, G. 1981, Acta Astron., 31, 283
Papaloizou, J. C. B., & Pringle, J. E. 1984, MNRAS, 208, 721
———. 1985, MNRAS, 213, 599
Pozdnyakov, L. A., Sobol', I. M., & Sunyaev, R. A. 1977, Soviet Astron.-AJ, 21, 708
Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley)
Ryu, D., & Goodman, J. 1992, ApJ, 388, 438
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
———. 1976, MNRAS, 175, 613
Shakura, N. I., Sunyaev, R. A., & Zilitinkevich, S. S. 1978, A&A, 62, 179
Shapiro, S., Lightman, A. P., & Eardley, D. M. 1976, ApJ, 204, 187
Spitzer, L., Jr. 1962, Physics of Fully Ionized Gases (New York: Wiley)
Stepney, S. 1983, MNRAS, 202, 467
Stoeger, W. R. 1977, A&A, 61, 659
Stone, J. M., & Balbus, S. A. 1996, ApJ, 464, 364
Svensson, R. 1982, ApJ, 258, 321
———. 1984, MNRAS, 209, 175
Takahara, F., & Kusunose, M. 1985, Progr. Theor. Phys., 73, 1390
Tanaka, Y., & Lewin, W. H. G. 1995, in X-Ray Binaries, ed. W. H. G. Lewin et al. (Cambridge: Cambridge Univ. Press), 126
Tayler, R. J. 1980, MNRAS, 191, 135
Vishniac, E., & Diamond, P. 1989, ApJ, 347, 435
———. 1992, ApJ, 398, 561
Vishniac, E. T., Jin, L., & Diamond, P. 1990, ApJ, 36, 648
Wallinder, F. H. 1991, A&A, 249, 107
Weaver, T. A. 1976, ApJS, 32, 233
Wen, L., Cui, W., & Bradt, H. V. 2001, ApJ, 546, L105
White, T., & Lightman, A. P. 1989, ApJ, 340, 1024
———. 1990, ApJ, 352, 495
Zahn, J. P. 1991, in Structure and Emission Properties of Accretion Disks (Gif-sur-Yvette: Editions Frontières), 87
Zdziarski, A. A. 1984, ApJ, 283, 842
Zel'dovich, Ya. B. 1981, Proc. R. Soc. London A, 374, 299