Diffraction of Transient Cylindrical Waves by a Rigid Oscillating Strip

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Abstract: This investigation portrays the transient cylindrical wave diffraction by an oscillating strip. Mathematical analysis of the problem is carried out with the help of an integral transforms and the Wiener–Hopf technique. Using far zone approximation, the scattered field is evaluated by the method of steepest descent. This study takes into consideration the transient cylindrical source and an oscillating strip such that both the source and a scatterer have different oscillating frequencies $\omega'_1$ and $\omega'_0$, respectively. The situation under consideration is well supported by graphical results showing the effects of emerging parameters.

Keywords: diffraction; transient cylindrical wave; oscillating strip; Wiener–Hopf technique; steepest descent method

1. Introduction

Transient diffraction problems have been the focus of interest for a long time because of their practical utilization in many fields of modern science and technology. The transient nature of the incident/diffracted wavefield requires a more detailed investigation because of the fact that the involved mathematical analysis depends upon the spatial and temporal integral transforms simultaneously. The transient diffraction phenomenon is of vital importance in determining the mechanical properties of solids and liquids. Detection of cracks and their position, nature, extent and orientation is also analyzed by using transient scattered field measurements and such applications can be found in the works of Adrianus and Maarten [1], which analyzes the canonical problem of seismic waves using a perfect slip fracture. The propagation of transient elastic waves is studied by Wu and Gong [2], in the presence of internal interference that may arise due to steel ball impact, which are used to detect a void or inclusion in a plate-like structure for a point source or array receiver. In [3], diffraction of longitudinal pulse with cylindrical symmetry with a semi-infinite crack are studied to study the effects of an incident wave on upon the diffracted field. Ing and Ma [4], studied the effect of a finite crack, which is subjected to dynamic anti-plane and having a Heaviside function time dependency in an anisotropic material. Transient wave diffraction has also been of considerable interest
in geophysical applications including borehole sounding and minerals exploration. The sharp edge and corner geometries encountered in petroleum extraction, in particular, generate diffraction patterns as discussed in the works of Alford et al. [5], and Li et al. [6]. Another significant domain in which transient waves are studied rigorously is diffraction/scattering of acoustic/elastic/electromagnetic waves. A rich amount of discussion on this topic is available in the textbooks of Friedlander [7], Jones [8], Achenbach [9], and Harris [10]. Noteworthy contributions on diffraction of transient waves may be credited to Kriegsmann et al. [11], Ishii & Tanaka [12], Rienstra [13], Asghar et al. [14–18], Ahmad [19,20], Ayub et al. [21,22] and Mann et al. [23].

A literature survey reveals the fact that, due to tedious and at the same time the laborious nature of the transient diffraction problems, much more room is available for the study of these problems. It is apposite to state that examining a fluctuating half-plane despite of constant one posses a more challenging problem of the diffraction theory. Due to the importance of diffraction of waves produced by time dependent sources and a fluctuating half plane, in this manuscript, we studied Ayub et al.’s work [22] for transient line source diffraction by an oscillating strip with the help of the Wiener–Hopf (WH) technique [24]. Some important practical applications of strip/slit/slot/stub geometries can be found in optical and microwave instrumentation, coupling structures, reflectors, antenna covers and frequency selective surfaces [25,26]. Another potential application of strip geometry has been reported by Lizzi et al. [27] i.e., for improving public safety, emergency management and disaster recovery, a suitable and reliable communication system is required which depends upon a radiating structure supporting multiple standards. On such a radiating structure is a microstrip antenna which has the advantages of being lightweight, low profile, and is easy to fabricate. Thus, multiband behavior can be obtained by a geometry of a reference shape consisting of strips/stubs. Recently, [28] demonstrated that dielectric backed conducting strips can be used as inductive elements to implement a bandpass filter. These strips are placed at a half-wavelength interval and act as resonators. The wideness and spacing of each strip is varied to obtain the desired response. Fabrications of low loss microstrip triplexer and step impedance cells [29] are other applications of the presented analysis such that reduction in insertion loss is achieved by alignment of couple lines (used as strips which are swinging) performed by analysis over LC equivalent circuits and resonators.

Owing to the importance of transient diffraction phenomenon and the strip geometry, the present investigation is done for the case of transient cylindrical wave diffraction by a fluctuating strip instead of a plane wave source. Noble’s scheme [24] in conjunction with the geometrical theory of diffraction (GTD) [30], which assumes the strip length to be large as compared to the operating wavelength, has been adopted to solve the problem. The separated field in this case is derived by uncoupling and solving two WH equations. The separated field for various time dependent oscillations of the strip can be found by introducing its generalized Fourier coefficients related to the fluctuation involved [19]. An interesting feature of the forgoing investigation is the deduction of results for the transient cylindrical wave diffraction by a half plane [22] mathematically and graphically with the help of the limit concept of calculus which serves as a check of correctness of the mathematical analysis. Graphs describing the actions of important parameters on the separated field have been presented and discussed.

2. Mathematical Formulation

Scattering of transient cylindrical wave by a fluctuating strip residing at the position \(-l \leq x' \leq 0\) and fluctuating normal to the scatterer with velocity \(u_0\) is considered and is depicted in Figure 1.

Here \(g^s(t')\) is a periodic function of time which is represented in its generalized Fourier series as

\[
g^s(t') = \sum_{n} a_n e^{j\omega_0 n t'},
\]  
(1)
where $a_n$ are the Fourier coefficients which can be evaluated by
\[
a_n = \frac{1}{T_0} \int_{-T_0}^{T_0} g^*(t') e^{-i\omega_0 t'} \, dt',
\]
and $\omega_0' = \frac{2\pi}{T_0}$ is the elemental non-zero frequency of the swinging strip. Thus the boundary value problem at hand is:
\[
\frac{\partial^2 \varphi}{\partial x'^2} + \frac{\partial^2 \varphi}{\partial y'^2} - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t'^2} = \delta^*(x' - x'_0)\delta^*(y' - y'_0)e^{-i\omega_1 t'},
\]
so that $\delta^*$ is the Dirac delta function, $\omega_1$ is the frequency of the incident signal and $\omega_1 = \frac{2\pi}{T_1}$, $\varphi_t$ represents total velocity potential and $c^*$ is the speed of sound satisfying the continuity conditions at $(x' > 0 \cup - < x' < -l)$ as
\[
\varphi_t(x', y' = 0^+, t') = \varphi_t(x', y' = 0^-, t'),
\]
and the boundary condition at $-l \leq x' < 0$ is
\[
\frac{\partial \varphi_t}{\partial y'} = u_0 g^*(t')
\]
such that velocity profile stays continuous on $x' < 0, y' = 0$ [20]. In rest of the analysis we shall just refer to $0^+$ instead of $y' = 0^+$. To persue the analysis, following [24] introduce
\[
\varphi_t = \varphi_0 + \varphi,
\]
so that $\varphi_0$ stands for the non-homogeneous forcing term and $\varphi$ satisfies the wave equation. Incorporating Equation (6) into Equations (3)–(5) we arrive at:
\[
\frac{\partial^2 \varphi_0}{\partial x'^2} + \frac{\partial^2 \varphi_0}{\partial y'^2} - \frac{1}{c^2} \frac{\partial^2 \varphi_0}{\partial t'^2} = \delta^*(x' - x'_0)\delta^*(y' - y'_0)e^{-i\omega_1 t'},
\]
with boundary and continuity conditions
\[
\frac{\partial \varphi}{\partial y'} + \frac{\partial \varphi_0}{\partial y'} = u_0 g^*(t'),
\]
\[
\varphi(x', 0^+, t') = \varphi(x', 0^-, t'),
\]
\[
\frac{\partial \varphi(x', 0^+, t')}{\partial y'} = \frac{\partial \varphi(x', 0^-, t')}{\partial y'},
\]
with an additional assumption that velocity $\frac{\partial \varphi}{\partial y'}$ is uninterrupted at $(x' < 0, y' = 0)$. 
Introducing the Fourier transform pair over the time variable \( t' \) as

\[
\tilde{\phi}(x', y', \omega') = \int \phi(x', y', t') \exp \left( i \omega' t' \right) dt',
\]

\[
\phi(x', y', t') = \frac{1}{2\pi} \int \tilde{\phi}(x', y', \omega') \exp \left( -i \omega' t' \right) d\omega'.
\]

(11)

Equations (7)–(10), in light of Equation (11) will result into

\[
\frac{\partial^2 \tilde{\phi}_0}{\partial x^2} + \frac{\partial^2 \tilde{\phi}_0}{\partial y^2} + k_1^2 \tilde{\phi}_0 = 2\pi \delta^*(x' - x_0') \delta^*(y' - y_0') \delta' (\omega' - \omega_1'),
\]

(12)

where \( k_1 = \frac{\omega_1'}{c} \)

\[
\frac{\partial^2 \tilde{\phi}}{\partial x^2} + \frac{\partial^2 \tilde{\phi}}{\partial y^2} + k^2 \tilde{\phi} = 0,
\]

(13)

\[
\frac{\partial \tilde{\phi}}{\partial x} + \frac{\partial \tilde{\phi}_0}{\partial y} = u_0 F(\omega'),
\]

(14)

\[
\tilde{\phi}(x', 0^+, \omega') = \tilde{\phi}(x', 0^-, \omega'),
\]

\[
\frac{\partial \tilde{\phi}(x', 0^+, \omega')}{\partial y} = \frac{\partial \tilde{\phi}(x', 0^-, \omega')}{\partial y}.
\]

(15)

To solve the problem further we need the Fourier integral transform pair on the variables \( x' \) and \( \beta' \) as

\[
\kappa(\beta', y', \omega') = \int \kappa(x', y', \omega') e^{i \beta' x'} dx' = \kappa_-(\beta') + \kappa_1(\beta') + \kappa_+(\beta'),
\]

(16a)

and

\[
\tilde{\kappa}(x', y', \omega') = \int \kappa(\beta', y', \omega') e^{-i \beta' x'} d\beta',
\]

(16b)

with

\[
\kappa_-(\beta') = \int_{-1}^{0} \tilde{\kappa}(x', y', \omega') e^{i \beta' x'} dx',
\]

(17)

\[
\kappa_1(\beta') = \int_{0}^{1} \tilde{\kappa}(x', y', \omega') e^{i \beta' x'} dx',
\]

(18)

\[
\kappa_+(\beta') = \int_{-1}^{0} \tilde{\kappa}(x', y', \omega') e^{i \beta' x'} dx',
\]

(19)

Transforming Equations (12)–(15) in the \( \beta' - \) plane, result in

\[
\left( \frac{d^2}{d^2 y} + k^2 \right) \kappa_0 (\beta', y', \omega_1') = \delta^* \left( y' - y_0' \right) \delta^* \left( \omega' - \omega_1' \right) \exp i \beta' x_0',
\]

(20)

where

\[
k^2 = k_1^2 - \beta'^2,
\]
and

\[ \frac{d^2 \kappa}{dy^2} - \gamma^2 \kappa (\beta', y', \omega') = 0, \]  

\[ \frac{d\kappa_1(\beta',0,\omega')}{dy} - \frac{d\kappa_0(\beta',0,\omega_1)}{dy} = - \frac{im_0F(\omega')}{\beta'} \left[ 1 - \exp \left( -i\beta' t \right) \right], \]  

and

\[ \kappa_+ (\beta', 0^+, \omega') = \kappa_+ (\beta', 0^-, \omega'), \]  

\[ \frac{\partial \kappa_+ (\beta', 0^+, \omega')}{\partial y} = \frac{\partial \kappa_+ (\beta', 0^-, \omega')}{\partial y}, \]  

with \( k = \frac{\omega'}{c} = k_1 + ik_2 \) with \( k_2 > 0 \) and also \( \gamma^2 = \beta'^2 - k^2 \), such that real \( \gamma \) is positive in strip of analyticity \( \text{Img} - k < \text{Img} \beta' < \text{Img} \lambda \). Equation (20) can be solved to determine \( \kappa_0 (\beta', y', \omega_1) \) \[24\] as:

\[ \kappa_0 (\beta', y', \omega_1) = - \frac{\delta^+ (\omega' - \omega_1)}{2} \int \frac{\exp \left[ -i\beta' \left( x' - x_0' \right) - \kappa \left| y' - y_0' \right| \right]}{\sqrt{\beta'^2 - k^2}} \, d\beta', \]  

Letting

\[ \beta' = -k_1 \cos \left( \theta + it' \right), \text{ with } 0 < \theta < \pi, \, -< t' < , \]  

\[ \left| x' - x_0' \right| = R \cos \theta, \, \left| y' - y_0' \right| = R \sin \theta \text{ and } R = \left( \left| x' - x_0' \right|^2 + \left| y' - y_0' \right|^2 \right)^{\frac{1}{2}}, \]  

in Equation (26) we arrive at

\[ \tilde{\phi}_0 (x', y', \omega_1) = - \frac{\delta^+ (\omega' - \omega_1)}{2} \sqrt{\frac{2\pi}{k_1R}} e^{ik_1l + i\frac{\pi}{4}}. \]  

The above expression (28), after substituting \( x_0' = r_0 \cos \theta_0, y_0' = r_0 \sin \theta_0 \) and for large value of \( r_0 \) simplifies as

\[ \tilde{\phi}_0 (x', y', \omega_1) = \tilde{b} (r_0) e^{-ik_1 \left( x' \cos \theta_0 + y' \sin \theta_0 \right)}, \]  

with

\[ \tilde{b} (r_0) = \delta^+ (\omega' - \omega_1) \sqrt{\frac{\pi}{2k_1r_0}} e^{ik_1l + i\frac{\pi}{4}}. \]  

The solution of Equation (21) fulfilling radiation conditions and after utilizing Equations (22)–(24), may be written as

\[ \kappa (\beta', y', \omega') = \left\{ \begin{array}{lcl} A_1 (\beta', \omega') e^{-\gamma y'} & y' & \geq 0, \\ -A_1 (\beta', \omega') e^{\gamma y'} & y' & \leq 0. \end{array} \right. \]  

Now in view of Equations (16a) and (31), we have

\[ A_1 (\beta') = \kappa_- (\beta', 0^+, \omega') + \kappa_1 (\beta', 0^+, \omega') + \kappa_+ (\beta', 0^+, \omega'), \]  

and

\[ -A_1 (\beta') = \kappa_- (\beta', 0^-, \omega') + \kappa_1 (\beta', 0^-, \omega') + \kappa_+ (\beta', 0^-, \omega'). \]
For detailed treatment of Equations (32) and (33) we refer the readers to the Appendix A of the paper and the unknown coefficient $A_1 (\beta')$ is calculated to be:

$$A_1 (\beta') = -\frac{1}{\gamma} \left[ \frac{b m \gamma (\beta') \left[ \exp -i (\beta' - k_1 \cos \theta_0) l \right]}{\gamma (k_1 \cos \theta_0) (\beta' - k_1 \cos \theta_0)} \right] - b m \gamma (\beta') \tilde{R}_2 (-\beta') \exp (-i \beta' l)$$

$$+ b m \gamma \left( \beta' \right) \gamma (k) \tilde{T} (-\beta') \tilde{C}_2 \exp (-i \beta' l) + \frac{i u_0 g^*(\omega') \gamma (\beta') \exp (-i \beta' l)}{\beta' \gamma (0)}$$

$$+ \frac{i u_0 g^*(\omega') \gamma (k) \gamma (\beta') \exp (-i \beta' l)}{k (1 - \gamma \gamma (k) \tilde{T}^2 (k))}$$

$$- \frac{\tilde{T} (-\beta') \tilde{T} (k) \gamma (k) \gamma (\beta') \left[ \exp (-i \beta' l) \right]}{k (1 - \gamma \gamma (k) \tilde{T}^2 (k))} - \frac{b m \gamma (\beta')}{\gamma (k_1 \cos \theta_0) (\beta' - k_1 \cos \theta_0)}$$

$$- \frac{\gamma (k) \gamma (\beta') \exp (-i \beta' l)}{k (1 - \gamma \gamma (k) \tilde{T}^2 (k))} + b m \gamma (k) \gamma (\beta') \tilde{T} (k) \tilde{C}_1. \quad (34)$$

On inverting the Fourier transform over the variable $\beta'$ in Equation (16b), the function $\tilde{q}(x', y', \omega')$ can be obtained as

$$\tilde{q}(x', y', \omega') = \frac{1}{2\pi} \int \chi(\beta', y', \omega') \exp \left( -i \beta' x' \right) d\beta' = \frac{1}{2\pi} \int \chi(\beta', \omega') \exp \left( -\gamma |y'| - i \beta' x' \right) d\beta'. \quad (35)$$

Now $\tilde{q}(x', y', \omega')$ can be written as

$$\tilde{q}(x', y', \omega') = q^{\text{separated}}(x', y', \omega') + q^{\text{interacted}}(x', y', \omega'), \quad (36)$$

$$q^{\text{separated}}(x', y', \omega') = \frac{1}{2\pi} \int \left[ \frac{b m \gamma (\beta') \left[ \exp -i (\beta' - k_1 \cos \theta_0) l \right]}{\gamma (k_1 \cos \theta_0) (\beta' - k_1 \cos \theta_0)} \right] - \frac{b m \gamma (\beta')}{\gamma (k_1 \cos \theta_0) (\beta' - k_1 \cos \theta_0)}$$

$$+ \frac{i u_0 g^*(\omega') \gamma (\beta') \exp (-i \beta' l)}{\beta' \gamma (0)} - \frac{i u_0 g^*(\omega') \gamma (\beta') \exp (-i \beta' l)}{\beta' \gamma (0)} e^{-\gamma |y'| - i \beta' x'} d\beta', \quad (37)$$

and

$$q^{\text{interacted}}(x', y', \omega') = \frac{1}{2\pi} \int \left[ \frac{-1}{\gamma} \left( -b m \gamma (\beta') \tilde{R}_2 (-\beta') \exp (-i \beta' l) \right) \right]$$

$$+ b m \gamma (\beta') \gamma (k) \tilde{T} (-\beta') \tilde{C}_2 \exp (-i \beta' l)$$

$$+ \frac{\gamma (k) \gamma (\beta') \left[ \exp (-i \beta' l) \right]}{k (1 - \gamma \gamma (k) \tilde{T}^2 (k))} \left( \frac{1}{\gamma (k)} - \frac{1}{\gamma (0)} \right)$$

$$- \frac{\gamma (k) \gamma (\beta') \tilde{T} (k) \exp (-i \beta' l)}{k (1 - \gamma \gamma (k) \tilde{T}^2 (k))} - b m \gamma (\beta') e^{ik_1 \cos \theta_0} \tilde{R}_1 (\beta')$$
\[ + \frac{\gamma_\oplus(k) \gamma_\ominus \left( \beta' \right) iu_0 g^*(\omega') T \left( \beta' \right) \bar{T}(k)}{k \left( 1 - \gamma_\oplus(k) \bar{T}^2(k) \right)} \left( \frac{1}{\gamma_\oplus(k)} - \frac{1}{\gamma_\ominus(0)} \right) \]
\[ - \frac{\gamma_\oplus(k) \gamma_\ominus \left( \beta' \right) iu_0 g^*(\omega') T \left( \beta' \right) \bar{T}(k)}{k \left( 1 - \gamma_\oplus(k) \bar{T}^2(k) \right)} + b m \gamma_\ominus(k) \gamma_\oplus \left( \beta' \right) \bar{C} \left( \beta' \right) e^{-\gamma|y| - i\delta''x'd\beta}. \]

Here \( \bar{q}^{\text{separated}}(x', y', \omega') \) gives the diffracted field produced by the edges at \( x' = 0 \) and \( -l \) and \( \bar{q}^{\text{interacted}}(x', y', \omega') \) accounts for an interaction field which is the impact of one edge over the other edge.

![Cylindrical wave diffraction](image)

**Figure 1.** Geometry of the problem.

### 3. Far-Field Solution

The far-field can be derived asymptotically by evaluating the integral in Equation (37). For that substitute \( x' = r \cos \theta, |y'| = r \sin \theta \) and transform the path of integration using the transformation \( \beta' = -k \cos \tau, \) with \( 0 < \theta < \pi, -\pi < q_3 < \). Hence for \( kr \rightarrow \), Equation (37) becomes

\[ \bar{q}^{\text{separated}}(x', y', \omega') = \frac{\delta^+(\omega' - \omega') \sin \theta_0 \sin \theta \exp(ik(r + r_0))}{2\sqrt{rr_0} \left( \cos \theta + \cos \theta_0 \right)} \]
\[ \times \left[ \exp \left( -i \left( k \cos \theta + k_1 \cos \theta_0 \right) l \right) \right. \]
\[ + \frac{i u_0 \sum_{n=-\infty}^{\infty} \sin \theta C_n \delta^+ \left( \omega' - n \omega' \right) \left[ \exp \left( ik \cos \theta \right) + \frac{1}{i \sqrt{1 + \cos \theta}} \right] \exp \left( i k r - \frac{\pi}{4} \right) \left( \frac{2\pi}{k r} \right)^{\frac{1}{2}}. \]

Inverting the Fourier transform of Equation (39) will recast the diffracted field in time domain as

\[ \bar{q}^{\text{separated}}(x', y', t') = \frac{\sin \theta_0 \sin \theta}{4\pi \sqrt{rr_0} \left( \cos \theta + \cos \theta_0 \right)} \]
\[ \times \left[ \exp \left( -i k_1 \left( \cos \theta + \cos \theta_0 \right) l \right) \right. \]
\[ + \frac{i u_0 \sin \theta \sum_{n=-\infty}^{\infty} C_n \left[ \exp \left( i k l \cos \theta \right) + \frac{1}{i \sqrt{1 + \cos \theta}} \right] \exp \left( i k r - \frac{\pi}{4} - i \omega_1 t' \right) \]
\[ + \frac{i u_0 \sin \theta \sum_{n=-\infty}^{\infty} C_n \left[ \frac{\exp \left( i k l \cos \theta \right)}{\sqrt{1 - \cos \theta}} + \frac{1}{i \sqrt{1 + \cos \theta}} \right] \exp \left( i k r - \frac{\pi}{4} - i \omega_1 t' \right) \left( \frac{2\pi}{k r} \right)^{\frac{1}{2}}, \]

where \( k = \frac{n c}{\omega} \). It is important to remark here that we can deduce the results for cylindrical source diffraction by a half-plane situation [22] mathematically from the present analysis. From Equation (38),
expressions for $\tilde{T}(-\beta')$, $\tilde{R}_{1,2}(\beta')$ and $\tilde{C}_{1,2}$ vanish and value of $\tilde{E}_{-1}$ can be evaluated under the limit $l \to \left( \frac{\sqrt{l}}{\pi} \right)$. By virtue of these values, Equation (38) vanishes, and we are left with only separated field, containing the contribution of diffracted field for the half-plane in it. Now on extending the finite barrier to a half plane, the diffracted field of [22] can be recovered exactly which serves as a check of correctness of the mathematical results of this manuscript.

4. Graphical Results

Now we shall present graphical illustrations of the diffracted field for different parameters and investigate the trends of diffraction characteristics of the diffracted field.

From Figure 2a, we note that the modulus of higher frequency of swinging strip attains the prominent amplitude of the diffracted field compared to the modulus of lower frequency of the swinging strip. Figure 2b portrays that the incidence wave with the higher modulus of frequency will yield an elevated amplitude of the diffracted field which is in accordance with the physics of the problem. Figure 2c,d confirm the fact that as the source and the observer are gradually taken far from the origin, the amplitude of the separated field decreases. Figure 3a illustrates that on increasing the incidence angle, the amplitude of the diffracted field also becomes adulatory. Figure 3a–d given below are in close agreement with Figures 2–6 of [22], respectively. As the strip length is increased gradually from $l = 10^1$, $l = 10^{14}$ to $l = 10^{22}$ as depicted in Figures 2 and 3 the trends of the graphs are the same as of the half-plane situation [22]. Furthermore, Equation (56) shows the generalized expression for the separated field. It can be observed that it contains the term $C_n$ for Fourier coefficients. The problem at hand can be extended and analysis can be performed on the expression for varying input signals present in nature based on these Fourier coefficients. Such signals can include unit gate function, ramp function, unit impulse function, complex exponential, sawtooth wave, triangular wave, square wave, sinc function, etc. Furthermore, this type of solution has practical applications in geophysics in the exploration of mineral, analysis of capacitive elements formed due to soldering in electronics, acoustic signals analysis in sound transmission, miniature antenna design for public use, medical imaging such as in ultrasound for disease detection, etc. [31–38]. The graph Figure 2a matches with Figure 5 of [22] and Figure 3c matches with Figure 4 of [22]. This is sufficient evidence to show that the graphical results of [22] can be obtained from the present analysis. Figures 2b–d and 3a,b,d do not match the other figures of [22] because the parameters considered are different. Those figures can also be obtained if the parameters were matched.

The phenomenon of diffraction that takes place in an incident signal from a strip has been studied and it is applicable to many real-world problems. The applications include electronics, acoustic signals, antenna design, medical imaging, geophysics, etc. Some physical and practical applications for the derived relationships are as follows. In [31], discontinuity in a microstrip such as soldering, gaps, etc. in electronic circuits is studied to determine their capacitive effects. The derived field is the addition of two fields generated by two edges of the strip and an interaction field and plays a vital role in the study of electromagnetic theory of antennas [32]. In [33], analysis of acoustic signals that are diffracted by slits and ribbons has been performed, which can help in medical imaging using ultrasound. The optical activity and diffraction on a plane from a split aperture can take place in many biological materials such as bacteria, seashells, amino acids, an array of sugars, DNA, laurel, turpentine oil, and aqueous solutions of tartaric acids and the study of such materials can help in medical sciences [34]. The transient of seismic wave, which includes diffraction of waves from a solid strip has been analyzed for a geological region in [35]. The study of seismic waves for geological exploration has been studied in [36]. Brain wave patterns using diffraction theory has been studied in [37] for detection of migraines. In [38], diffraction theory has been applied for the detection of liver disease.
Figure 2. Variation of diffracted potential with observation angle for various values of (a) fluctuating strip frequency (b) incident wave frequency (c) distance from the fluctuating source to the fluctuating strip (d) distance from the observer to the fluctuating strip.

Figure 3. (a) Variation of the diffracted field with observation angle for different values of (a) incidence wave angles (b) oscillating strip frequency when $l = 10$. (c) Oscillating strip frequency for $l = 10^{14}$. (d) Oscillating strip frequency when $l = 10^{22}$. 
5. Final Remarks

The present mathematical analysis accounts for the diffracted field obtained by the diffraction of a transient cylindrical wave from a swinging strip. Diffracted fields that are separated and interacted are obtained. It is noted that the results of the plane wave diffraction of acoustic waves by a swinging half-plane can be recovered from the present analysis by shifting the line source far away and extending the swinging strip to a half-plane both mathematically and graphically. The presented analysis has practical applications in antenna design and the construction of a frequency selective surface. The far-field situation and some graphs showing the effects of various parameters on the diffracted field are also plotted and discussed.

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Appendix A

In the appendix, a brief overview of construction of the WH functional equation is presented here. By adding and subtracting Equations (32) and (33), we arrive at

\[ S_- e^{-i\beta' l} + \kappa_+ (\beta', 0, \omega') + F_1 (\beta', 0, \omega') = 0, \quad (A1) \]

where

\[ 2S_- = \kappa_+ (\beta', 0^+, \omega') + \kappa_- (\beta', 0^-, \omega'), \quad (A2) \]

\[ 2F_1 (\beta', 0, \omega') = \kappa_1 (\beta', 0^+, \omega') + \kappa_1 (\beta', 0^-, \omega'), \quad (A3) \]

\[ 2\kappa_+ (\beta', 0, \omega') = \kappa_+ (\beta', 0^+, \omega') + \kappa_+ (\beta', 0^-, \omega'), \quad (A4) \]

and

\[ A_1 (\beta') = f^* (\beta', 0, \omega'), \quad (A5) \]

where

\[ f^* (\beta', 0, \omega') = \frac{1}{2} \left[ \kappa_1 (\beta', 0^+, \omega') - \kappa_1 (\beta', 0^-, \omega') \right]. \quad (A6) \]

With the help of Equations (17) and (31) we arrive at

\[ \gamma A_1 (\beta') = \kappa_+ (\beta', 0, \omega') + \kappa_1 (\beta', 0, \omega') + \epsilon^{-i\beta' l} \kappa_- (\beta', 0, \omega'). \quad (A7) \]

Equation (A7) can be simplified with the help of continuity condition on \( \kappa_+ \) across \( y' = 0 \) and using Equations (17) and (31) and we yield

\[ - \gamma f^* (\beta', 0, \omega') = \kappa_+ (\beta', 0, \omega') + \kappa_0 (\beta', 0, \omega') \]

\[ + \frac{u_0 g^*(\omega')}{i\beta'} \left[ 1 - \exp (-i\beta' l) \right] + e^{-i\beta' l} \kappa_- (\beta', 0, \omega'). \quad (A8) \]
Equation (A8) can be rearranged to give

\[
e^{-i \kappa z'} (\beta', 0, \omega') + \gamma J^* (\beta', 0, \omega') + \kappa z' (\beta', 0, \omega') =
\]

\[
= \kappa_0 (\beta', 0, \omega') + \frac{u_0 G^*(\omega')}{i \beta} [1 - \exp (-i \beta l)].
\] (A9)

From Equation (29), we have

\[
\kappa z' (\beta', 0, \omega') = \frac{ibk_1 \sin \theta_0}{(\beta' - k_1 \cos \theta_0)} \left[ \exp \left( -i (\beta' - k_1 \cos \theta_0 l) - 1 \right) \right],
\] (A10)

and Equation (A9) becomes

\[
e^{-i \kappa z'} (\beta', 0, \omega') + \gamma J^* (\beta', 0, \omega') + \kappa z' (\beta', 0, \omega') =
\]

\[
= \frac{ibk_1 \sin \theta_0}{(\beta' - k_1 \cos \theta_0)} \left[ \exp \left( -i (\beta' - k_1 \cos \theta_0 l) - 1 \right) \right] - \frac{u_0 G^*(\omega')}{i \beta} [1 - \exp (-i \beta l)].
\] (A11)

We now proceed towards the solution of the standard W-H Equation (A11) by using the WH procedure [24]. Factorizing \( \gamma (\beta') \) via product form as

\[
\gamma (\beta') = \gamma_\oplus (\beta') \gamma_\odot (\beta'),
\] (A12)

where \( \gamma_\oplus, \gamma_\odot \) are holomorphic in the upper and lower half-plane regimes respectively. Inserting Equation (A12) into Equation (A11) we yield

\[
e^{-i \kappa z'} (\beta', 0, \omega') + \gamma_\oplus (\beta') \gamma_\odot (\beta') J^* (\beta', 0, \omega') + \kappa z' (\beta', 0, \omega') =
\]

\[
= \frac{ibk_1 \sin \theta_0}{(\beta' - k_1 \cos \theta_0)} \left[ \exp \left( -i (\beta' - k_1 \cos \theta_0 l) - 1 \right) \right] - \frac{u_0 G^*(\omega')}{i \beta} [1 - \exp (-i \beta l)].
\] (A13)

Equation (A13) is a typical expression for a strip barrier and calculation of \( \kappa \) in this expression will complete the solution of the WH equation by following the procedure outlined in [24,39] and the results are:

\[
\kappa_0 (\beta', 0, \omega') = bm \gamma_\oplus (\beta') \bar{G}_1 (\beta') + bm \gamma_\odot (\beta') \bar{T} (\beta') \bar{C}_1 + \frac{i u_0 G^*(\omega') \gamma_\oplus (\beta')}{\beta'} \left( \frac{1}{\gamma_\oplus (\beta')} - \frac{1}{\gamma_\odot (\beta')} \right)
\]

\[
+ \frac{\gamma_\oplus (k) \gamma_\odot (\beta') \Gamma (\omega') \bar{T} (\beta') \bar{T} (k)}{k(1 - \gamma_\odot (k) \bar{T} (k))} \left( \frac{1}{\gamma_\oplus (k)} - \frac{1}{\gamma_\odot (k)} \right) - \frac{\gamma_\oplus (k) \gamma_\odot (\beta') \Gamma (\omega') \bar{T} (\beta') \bar{T} (k)}{k(1 - \gamma_\odot (k) \bar{T} (k))},
\] (A14)

and

\[
\kappa_0 (\beta', 0, \omega') = bm \gamma_\oplus (\beta') \bar{G}_2 (\beta') + bm \gamma_\odot (k) \bar{T} (\beta') \gamma_\odot (\beta') \bar{C}_2 - \frac{i u_0 G^*(\omega') \gamma_\odot (\beta')}{\beta'} \left( \frac{1}{\gamma_\odot (\beta')} - \frac{1}{\gamma_\odot (\beta')} \right)
\]

\[
- \frac{\gamma_\odot (k) \bar{T} (\beta') \Gamma (\omega') \bar{T} (\beta') \bar{T} (k)}{k(1 - \gamma_\odot (k) \bar{T} (k))},
\] (A15)

where

\[
\bar{\beta} = \sqrt{\frac{\pi}{2k_1 \theta_0}} \delta^* (\omega' - \omega_0) e^{i(k_1 \theta_0 + i \frac{\pi}{2})}, \quad m = k_1 \sin \theta_0,
\] (A16)

where \( \bar{G}_{1,2} (\beta'), \bar{C}_{1,2}, \bar{R}_{1,2}, \bar{T} (\alpha), \bar{E}_1, \bar{H}_1 \) and \( \bar{W}_{n-\frac{1}{2}} (z) \) are same as given in [22,24].
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