Sound mechanics from squeaky and booming dune sands

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Abstract: It is verified that the source of the acoustic emissions when beach sand or other squeaky grains are stepped on or impacted by a pestle lies in a thin shear band directly under the pestle. The grain layers in this band slide one over another at a slow creepy pace giving rise to energy transfer, via the stick-slip effect, from the impacting pestle to the elastic vibrations in the shear bands at the grain contact areas. In turn, this vibration energy feeds the elastic modes of vibration along the vertical grain columns or equivalently, the elastic modes of vibration in the shear band under the pestle comprising the vibrating columns, with dominant frequency in the range of 1,000 Hz. In search of an explanation of the acoustic emissions when booming dune sand is pushed by a blade or is freely avalanching, we adopt the concept of the collision shear band, where, due to the high degree of fluidity of the sand mass, the grain layers slide one over another at a brisk pace so that the average collision frequency between grains in two adjacent grain layers defines the dominant frequency of the acoustic emission in the range of 100 Hz.

Keywords: Acoustic emissions, Squeaky grains, Booming sands, Pushed or in avalanche

1. INTRODUCTION

In an earlier report by Patitsas [1], it was argued that the origin of the acoustic emissions, when certain beach sands or other grains, such as silica gel grains, are stepped on or impacted by a pestle in a dish, can be found in the formation of elastic shear bands at the grain contact areas when the grains are subjected to compression and shear forces as can be seen in Fig. 1(a). In a vertical column of $N$ grains, with a mass load on top, there are in effect $N - 1$ short springs between the grains resulting in a column with well defined elastic properties and with modes of vibration characterized by the fundamental frequency, $f_1$, in the range from about 500 to 2,000 Hz, its harmonics and the low frequency, the pestle frequency, where the entire column oscillates like a single loaded spring. Furthermore, it was argued that all the vertical columns under the pestle form the slip shear band, or better the “impact shear band,” seen in Fig. 1(b), with the same modes of vibration as in an individual column.

An appreciable amount of work on the acoustic emissions from dropping glass or steel spheres on a dish of squeaky grains can also be found in Patitsas [1]. However, the drop height of the spheres was relatively low and the frequency shift of the harmonics was not observed. This important phenomenon is dealt with in Sect. 2 where the drop height is 90 cm. It is worthy of note at this stage that no musical sound is emitted when similar spheres are dropped on a dish of booming sand. The issue is discussed in Sect. 3.

The earlier work by Patitsas [1] lacked the concept of the “collision shear band” and the conclusions regarding the acoustic emissions when booming sand is pushed by a blade or allowed to avalanche freely are questionable. In this study, the collision shear band is assumed to be the source of the acoustic emission as are the contact shear bands in the case of the impacted sand. The collision shear band, between the avalanche sand band and the fixed sand below, drives the elastic modes of vibration in the avalanche band in a “resonance effect,” as outlined below. A resonance effect is consistent with the enormous amount of energy radiated and audible some two km away, with dominant frequency $f_d \approx 100$ Hz, when only a few square meters of sand avalanches on a dune slip face. At this stage, we can think of the collision shear band as an assembly of grain layers sliding over the ones below in a synchronized manner so that all grains collide with the ones below at the same time.

In the studies by Andreotti [2] and by Douady et al. [3] it is suggested that the entire avalanche band, some 2 cm in thickness, acts like a collision shear band that results in the acoustic emission. The difficulties with these suggestions are elaborated in Sect. 4. In the study by Vriend et al. [4],...
2. THE CREEPY SHEAR BAND AND THE SQUEAKY SOUNDS

The grain column shown in Fig. 1(a) contains eight grains of various shapes and sizes with the contact shear bands between the grains depicted by dark thin bands. Each band is characterized by radius \( r_b \), thickness \( b \) and Young’s modulus \( Y_b \). In the equations that follow the weight of the grains is assumed to be negligible compared with that of the impacting pestle. Furthermore, the grains are assumed infinitely rigid. The grain column can be simulated by a thin rod of length \( L \), some diameter \( d_b \) and Young’s modulus \( Y_b \). Furthermore, the adjacent column can be similarly simulated with nearly the same values for \( L, d_b \) and \( Y_b \). It is assumed that such columns are compacted inside the impact shear band under the impacting pestle shown in Fig. 1(b). The vibration of the grain columns outside the impact shear band falls off rapidly with the distance from the impact shear band under the pestle.

It is more appropriate, especially when it comes to the mass density in a grain column, to study the modes of vibration of the impacted sand using the impact shear band under the pestle, rather than a given grain column. The surface of the shear band at \( z = 0 \) in Fig. 1(b) is assumed to be fixed while a pestle load, the sphere mass in this case, is placed at \( z = L \). We assume that the impact shear band is characterized by Young’s modulus \( Y \), compression phase velocity \( c \) and sand mass density \( \rho \). Then, the solution for the particle displacement along \( z \) is \( \xi = (A \cos \alpha z + B \sin \alpha z)e^{int} \), where \( \alpha = \omega/c \) and the boundary condition at \( z = 0 \) implies that \( A = 0 \). As in Eq. (3.22b) in Kinsler et al. [9], we write at \( z = L \),

\[
-\pi a^2 Y \frac{\partial^2 \xi}{\partial z^2} = m_p \frac{\partial^2 \xi}{\partial t^2} \tag{1}
\]

Thus we obtain, \( \cot(\alpha L) = m_p \rho \sigma^2/(\pi a^2 Y \alpha) \) and then with \( Y = c^2 \rho \) and \( \omega = ca \), we obtain,

\[
\cot(\alpha L) = \frac{m_p}{m_s} \alpha L \tag{2}
\]

where \( m_p \) is the pestle mass and \( m_s \) is the sand mass in the impact shear band in Fig. 1(b). The solutions of Eq. (2) are at \( \alpha L = \epsilon, \pi + \epsilon_1, 2\pi + \epsilon_2 \text{ etc.} \), resulting in,

\[
f_p = 1/(2\pi)(\epsilon/L)c, \quad f_1 = 1/(2\pi)(\pi + \epsilon_1)/L)c, \quad f_2 = 1/(2\pi)(2\pi + \epsilon_2)/L)c \tag{3}
\]

where \( f_p \) is the low frequency oscillation when the shear band oscillates as a single spring. Generally \( \epsilon_2 < \epsilon_1 \) resulting in \( f_2 < 2f_1 \) but for \( m_p/m_s \gg 1, \epsilon, \epsilon_1, \epsilon_2 \) are near zero resulting in \( f_2 \approx 2f_1 \). Later, we will see that the results in Fig. 2 imply \( m_p/m_s \approx 1.0 \).

When the sand mass is impacted by the falling sphere, the grains in a given column are forced to slide over the ones below as they move away from the center. The contact shear bands are thus created and energy is transferred from the impact sphere to the vibrations in the contact shear bands as outlined in Patitsas [10]. Effectively, if the friction coefficient decreases with slip velocity between the grains, the rubbing energy serves to excite the vibration in the grain contact shear bands and by extension the modes of vibration in the impact shear band. In the case of the squealing chalk when rubbed on a blackboard, Patitsas [10], the rubbing energy serves to excite the modes of vibration in a thin somewhat fluidized chalk layer between the chalk and the blackboard.

Similarly, we can argue that the full spectrum of the acoustic emission is developed inside a given contact shear.
band. In Eq. (2) the pestle mass $m_p$ is very small but so is the mass $m_s$ of the contact shear band that could result in values of $m_p / m_s$ in the neighborhood of unity. It can be argued that during the acoustic emission there is a resonance effect in progress where the length of the grain column $L$ and the thickness $b$ of the contact shear bands are self adjusted so as the spectrum of the modes of vibration in the former coincide with that of the latter. Furthermore, the phase of the fundamental mode in a given contact shear band is adjusted so as to coincide with that of the fundamental mode in the entire impact band at the location of the given contact band. Thus, the vibration energy in the contact shear bands is completely transferred to the vibration energy of the entire impact shear band.

In view of this effect and the role of the sliding grain layers in the study of the acoustic emissions from moving booming sands, it is appropriate here to think of sliding grain layers, below the pestle, as the source of the acoustic emissions as opposed to vibrating grain columns. According to Bolton [11], a few grain layers of wet sand pressed together between the thumb and the fore finger could evoke the singing sound. We can estimate that the impact shear band between the fingers had a diameter of 5 to 10 mm and a thickness of about 6 grain layers with the grains in the two central layers moving away from the center somewhat faster than the grains in the other layers. If the grains in one of the central layers traveled the distance of 1 mm during the typical emission time of 30 ms, then the same grains traveled the distance of 0.33 mm relative to the grains in the layer below during the same time. That is, the overtaking grains remained in contact with the same grains below during the acoustic emission, assuming an average grain diameter $d = 0.3$ mm. Such slow motion of the grain layers relative to one another has to be the major characterization of the “squeaky sound emissions” and the impact shear band is henceforth referred to as the “creepy shear band.”

Figure 2 depicts the frequency spectrum when a 16 mm glass sphere impacted a beach sand collected from the mouth of the Brevort River flowing into the north shore of Lake Michigan, USA, about 25 km west of the city of St. Ignace. We assume that during the acoustic emission the pressure was maximum in a circular area directly under the impacting sphere where the creepy shear band was formed as depicted in Fig. 1(b). Furthermore we assume that the creepy shear band comprised about ten grain layers. It can be seen in Fig. 2 that the fundamental frequency $f_1$ remained nearly constant during the emission time $T_e \approx 30$ ms. Evidently, $(\pi + \varepsilon_1)/L$ and the phase velocity $c$ increased nearly at the same rate during the time $T_e$ or these parameters had reached final values when the acoustic emission commenced. In the experimental results shown in Figs. 2, 4 and 6 in Patitsas [1], where 11, 16 and 25 mm steel spheres were dropped on the beach sand from the north shore of Lake Michigan from the relatively low heights $H$ equal to 10 and 20 cm, $f_2 \approx 2f_1$ and no excitation at the pestle frequency $f_p$ is present. Evidently, the low impact velocity resulted in low impact band length $L$ and low sand mass $m_s$ and in $m_p / m_s \gg 1$. Similar spectra can be seen in Figs. 8 and 10 in Patitsas [1] where a hand held 1.5 cm diameter rod was pushed slowly into the sand bed. The same can be seen in Takahara [12] where the sand bed was struck by a large pestle.

However, in Fig. 2 here with $H = 90$ cm, the frequencies are as follows; $f_p \approx 137$ Hz, $f_1 \approx 558$ Hz and $f_2 \approx 1,052$ Hz resulting in $2f_1 - f_2 = 64$ Hz. Evidently, the
larger height $H$ resulted in larger $L$ and $m_s$ and $m_p/m_s$ not much greater than 1.0. In a similar experiment a 19 mm steel sphere was dropped from $H = 90$ cm resulting in $f_2 \approx 74$ Hz, $f_1 \approx 426$ Hz and $f_3 \approx 811$ Hz and in $2f_1 - f_2 = 41$ Hz. There is a hint of the same effect in Figs. 12, 13 and 14 in Miwa et al. [13] where a large pestle was hand pushed gently into the sand bed. A close examination of the positions of the frequency peaks reveals that $f_2$ is slightly lower than $2f_1$. In Appendix A we argue that Eq. (2) can account for the ratio $f_1/f_2 \approx 4$ and for the frequency shift $2f_1 - f_2$ in Fig. 2 for $m_p/m_s = 1$.

According to Eq. (3), $f_1$ decreases as $1/L$, where $L$ increases with $H$. However, $\epsilon_1$ and possibly $c$ increase with $H$ as well and this can explain the weak decrease of $f_1$ with $H$. In Fig. 6 in Patitsas [1] a 25 mm steel sphere was dropped from $H = 20$ cm resulting in $f_1 = 467$ Hz, but in other similar experiments the same sphere was dropped from $H = 90$ cm resulting in an average value of $f_1 = 426$ Hz.

Similarly, $f_1$ decreases weakly with the sphere diameter $D_s$ since $L$, $\epsilon_1$ and possibly $c$ increase with $D_s$. In particular, in Figs. 4 and 6 in Patitsas [1], 16 and 25 mm steel spheres were dropped on the same beach sand bed from $H = 20$ cm resulting in $f_1$ equal to 578 and 467 Hz respectively. The frequency ratio, 1.24, is nearly equal to $\sqrt{25/16}$ implying that $f_1$ decreased as $D_s^{-1/2}$ and as $M_s^{-1/6}$ where $M_s$ is the sphere mass. Such slow decrease of $f_1$ with $m_p$ can be seen in Fig. 2 in the study by Nishiyama and Mori [14], where 25 mm diameter metallic rods were dropped, from $H = 10$ cm, on the sand bed in a large container so as wall effects could be neglected. The value of $f_1$ would also depend on the size and shape of the container of the sand bed as seen in Fig. 2 in the above reference.

More to the point, when the rods in Fig. 2 in the above reference, were dropped in a large container, $40 \times 40$ by 30 cm deep, the measured sound frequency, $f_3$, decreased a lot slower with the rod mass $m_1$ than $m_1^{-1/2}$, more like $m_1^{-1/6}$. But, when the rods were dropped on the sand in a porcelain mortar, 10 cm in diameter and with sand depth only equal to 3 cm, $f_3$ decreased nearly as $m_1^{-1/2}$ and more so when the rods were dropped on the sand in a cylindrical vessel with diameter of only 5.4 cm. On the basis of the results in the last two cases, it was concluded that the frequency $f_3$ was due to rod oscillations where the sand mass under the impacting rod acted as a single spring loaded with the mass $m_1$ of the rod. However, such a conclusion cannot account for the harmonics of $f_3$ as can be seen in the experiments by Miwa et al. [13], Takahara [12] and Qu et al. [15].

We argue that the measured frequency $f_3$ was the frequency $f_1$ and that the relatively rapid decrease of $f_1$ with $m_1$ was due to insufficient stiffness in the creepy shear band due to slippage at the nearly vessel wall. Effectively, the thickness $L$ of the creepy shear band was considerably larger than in the case of the large container. Thus, the $m_1^{-1/2}$ decrease of $f_3$ with $m_1$ in Fig. 10 in Andreotti [16] is in need of revision. The issue was addressed also in Patitsas [1] where it was also recognized that the single spring model could not account for the observed harmonics as seen in Takahara [17]. Furthermore, it was argued that according to this model the dominant frequency $f_3$ ought to be a lot lower when the impacting rod was hand held as opposed to freely dropped on the sand bed, but that was not the case.

3. FLUIDITY OF BEACH AND BOOMING SANDS

During the process of collecting singing sands, some 17 years ago, at the mouth of the Brevort River on the north shore of Lake Michigan, some 40 kg of the sand was placed in a plastic container and dumped sharply on the side of a nearby steep ridge. There was no liquid-like-flow downhill as the sand mass moved downhill by sliding on top of one or two rupture slip channels. Evidently, there was not sufficient pressure in the channels for the formation of creepy shear bands. On the contrary, the flow of booming sand has been compared to that of a water stream, Sholtz et al. [18]; Bagnold [19]; Humphries [20]. In Lindsay et al. [21], it is stated that “booming grains posses extremely smooth surfaces.” It is thus not difficult to argue that booming sands don’t squeak since the fluidity of the grains does not allow for the formation of creepy shear bands unless there is complete confinement of the grains as will be seen below.

4. BEYOND THE CREEPY SHEAR BAND

In an experiment by Takahara [17], singing sand from Kotobikihama beach was placed in a glass funnel and struck by the end of a smooth rounded wood rod. The signal is characterized by dominant frequency $f_3 \approx 510$ Hz but it is far from sinusoidal, rather like a saw tooth variation with time, where it rises sharply to a high peak, then decreases somewhat linearly to its lowest value and rises again sharply. If only a few grams of sand were placed in a small funnel, it could be argued that there was not sufficient sand compactness (stiffness) below the pestle to bring about the formation of a creepy shear band. We argue that somewhere in the funnel the conditions were right for the creation of two nearly rigid grain layers sliding one over the other and that the source of the acoustic emission was the grain collisions during the sliding of the two grain layers.

In what follows we will define the collision frequency between the sliding layers as $f_c$, where $f_c = \Delta u/d$, where $\Delta u$ is the relative velocity between the sliding grain layers. Thus, an average grain diameter $d = 0.3$ mm implies the
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reasonable value $\Delta u = 15\text{ cm s}^{-1}$. In Fig. 2 in [17], only six oscillations are shown. So, in ten oscillations the two grain layers were sliding for only the time $10/f_c = 20\text{ ms}$ and the sliding distance amounted to only 3 mm.

On the basis of the work by Douady et al. [3], we argue that when booming sand is pushed by a blade, the single interfacial grain collision process discussed above is replaced by a thin shear band comprising $N$ grain layers. The $N$ grain layers become locked into a synchronized sliding motion where the time required for one grain to overtake the grain below is equal to $d/\Delta u = 1/f_c$. The band with the $N$ grain layers is labeled as, “the collision shear band.” The frequency $f_c$ is defined by the geometry surrounding the collision shear band and the forces acting on it. According to Douady et al. [3], frequencies as low as 25 Hz were produced by softly manipulating booming sand. We argue that the sound emitted by the sliding grain layers in the collision band would not be sinusoidal but rather of the saw tooth form.

5. THE PUSHED BOOMING SAND

In the report by Douady et al. [3], the sand was pushed by a blade rotating inside a circular chamber with 1 m and 0.75 m outside and inside diameters respectively as seen in Fig. 1. in the same reference. The frequency of the emitted sound increased nearly linearly with blade velocity and remained greater than about 100 Hz. The authors argue that there is “a sheared layer between a static and a pushed pile where the flowing grains in one grain layer have enough energy to collide with those in the layer below at the collision frequency $f_c$ equal to the fundamental frequency of the acoustic emission.” They argue that “the grains synchronize their motion through a coupling wave propagating through the sheared layer.” However, no mathematical description of such a wave was provided and neither was the geometry of the sheared layer. Here, we assume that the layer collisions are limited to a collision band, sandwiched between the avalanching band and the static sand below.

The collision shear band can be visualized as follows: A rectangular array of about 10 glass beads on either side, with diameter $D_b$, is formed on a flat horizontal surface and the beads are glued together at the contact points. Then, another similar layer is forced to slide over the first at a constant speed $V_b$ resulting at a sound signal with dominant frequency $f_c = V_b/D_b$. When several similar layers slide over one another, we have a collision shear band if all layers are synchronized so as all grains in all layers collide with the ones below at the same time. The emitted sound signal is not sinusoidal but rather of a saw-tooth form. It is assumed that the collision shear band maintains its elastic properties and can support elastic modes of vibration normal to its grain layers.

6. FREELY AVALANCHEING BOOMING SAND

According to the model introduced by Andreotti [2] and Douady et al. [3], the sound emitted during a sand avalanche on the slip face of a dune is to be found in the collisions in some 100 grain layers superimposed and locked into synchronization, while sliding downhill, so that all grains collide with the ones below at the same time. We are looking at a huge collision shear band with thickness of about 2 cm, an area of several $m^2$ and producing the sonorous sound for up to two minutes. The preservation and integrity of such a complex structure while avalanching downhill is highly questionable especially so when the sand surface may contain foreign objects and impurities. Furthermore, the acoustic signal would have to be of saw tooth form, not sinusoidal, as seen in Fig. 4 below. Finally, this model could not predict a low frequency content at around 30Hz, also seen in Fig. 4, since the avalanche collision band is not loaded with a pestle mass.

As in the previous section, we assume that the source of the acoustic emission, when booming sand avalanches downhill, is a collision band sandwiched between the avalanche sand band and the static sand below and more or less attached to the sand below as depicted in Fig. 3. It is possible to get an estimate of the number of grain layers in the collision band by using the report by Haff [22], regarding freely avalanching sand plates. It is stated that “As they gain speed eventually moving at perhaps 15 cm/s, a low pitched groaning builds in magnitude from the disturbed dune mass.” With a shear rate of $\Delta u/d = 100\text{ s}^{-1} = f_c$ and grain diameter $d = 0.18\text{ mm}$, $\Delta u =$
0.018 m/s. So, the number of grain layers in the collision band is \( N = 0.15/0.018 = 8.3 \). Such a relatively low number is consistent with the stability of the rather complex operation of the collision shear band.

We proceed to construct a mathematical model that reflects the experimental reports on the avalanche of booming sand and then proceed to establish that such a model is consistent with the corresponding mathematical equations. We assume that during the acoustic emission there is a standing wave in both sand bands with fundamental frequency \( f_1 \) equal to the collision frequency \( f_c \) in the collision shear band and with fundamental frequency \( f_1 \) in the avalanche band. Furthermore, for the two bands to interact effectively, both standing waves must have antinodes at \( z = L \) in Fig. 3 and must be in phase. Effectively then, the wavelength in the avalanche band is, \( \lambda_1 = 2L \), and that in the collision band is, \( \lambda_{1b} = 4L_b \). In such a configuration, gravitational energy from the avalanche band is transferred to the collision band until it grows in size and \( f_{1b} = f_1 \). Then, the collision band drives the fundamental mode of vibration in the avalanche band, in a resonance effect, resulting in the intense musical acoustic emission. This interesting process continues during the acoustic emission, i.e., the avalanche mass feeds gravitational energy into the collision band which in turn feeds it back into the avalanche band in the form of vibration energy of its elastic modes.

In the avalanche band, we write for the particle displacement along \( z \), 
\[
\xi = (A \cos(\alpha z) + B \sin(\alpha z))e^{i\omega t} \quad \text{and in the collision shear band,} \quad \xi_b = (A_b \cos(\alpha_b z) + B_b \sin(\alpha_b z))e^{i\omega t}.
\]
At \( z = 0 \), \( \partial \xi / \partial z = 0 \), so, \( B = 0 \) and, 
\[
\xi = A \cos(\alpha z)e^{i\omega t}.
\]
At \( z = L + L_b \), \( \xi_b = 0 \), so that,
\[
A_b \cos(\alpha_b (L + L_b)) + B_b \sin(\alpha_b (L + L_b)) = 0 \quad (5)
\]
and at \( z = L \), \( \partial \xi / \partial t = \partial \xi_b / \partial t \) resulting in,
\[
A \cos(\alpha L) = A_b \cos(\alpha_b L) + B_b \sin(\alpha_b L) \quad (6)
\]
and finally at \( z = L \) we write as in Eq. (1),
\[
-\frac{8S}{L_b} \frac{\partial^2 \xi}{\partial z^2} = S \frac{\partial^2 \xi_b}{\partial z^2} \quad (7)
\]
where \( S \) is the area of the avalanching band, \( Y = \rho c^2 \), \( Y_b = \rho_b c_b^2 \), \( \omega = ca = c_b \alpha_b \). From Eq. (7) we can write,
\[
AY \sin(\alpha L) = -A_b Y_b \alpha_b \sin(\alpha_b L) + B_b Y_b \alpha_b \cos(\alpha_b L) \quad (8)
\]
We divide Eq. (8) into Eq. (6) and with the aid of Eq. (5) we arrive at the transcendental equation,
\[
cot(\alpha L) = \frac{AL}{\alpha_b Y_b L} K = (\alpha L) \frac{Y}{\alpha_b Y_b L} K \quad (9)
\]
where,
\[
K = -\frac{\cos(\alpha_b L) - \sin(\alpha_b L) \cot(\alpha_b (L + L_b))}{\sin(\alpha_b L) + \cos(\alpha_b L) \cot(\alpha_b (L + L_b))} \quad (10)
\]
Based on the assumption that \( \lambda_{1b} = 4L_b \), then, \( \alpha_b \) is equal to \( \pi/(2L_b) \) and then, \( \cot(\alpha_b (L + L_b)) = -\tan(\alpha_b L) \). Thus, the denominator in the above equation vanishes while the numerator is equal to \( 1/\cos(\alpha_b L) \). Thus, the mathematical equations imply an infinite value for \( K \) and an exact value \( \alpha = \pi/L \) in Eq. (4), implying in turn a nodal plane at \( z = L/2 \) in the avalanche band as observed experimentally by Andreotti and Bonneau [8]. However, such a value for \( \alpha_b \) does not allow for a mode with low frequency content. But, the assumption that the surface of the collision shear band at \( z = L \) is totally free, i.e., \( \lambda_{b1} = 4L_b \) and thus \( \alpha_b = 2\pi/\lambda_{b1} = \pi/(2L_b) \) cannot pass without
Larger values of $\alpha_b$ lead to lower values of $K$ and with $\alpha_b = \pi/L_b, \ K = 0.0$. From Eq. (9) we can estimate that $\Gamma \approx 1.0$ and that with $\alpha_b$ somewhat larger than $\pi/(2L_b)$ we could have $\Gamma K \approx 1.0$. Then, as per Sect. 2, there is a root in Eq. (9) at $\alpha L = \pi/3$ resulting in the low frequency content at $f_p \approx 27$ Hz, as seen in Fig. 4. As in the case of the sphere impact in Sect. 2, we interpret this mode of vibration as corresponding to the case where the elastic collision band vibrates as a single spring loaded with the mass of the avalanche band above it.

It was reported by Patitsas [1] that on the basis of signals recorded directly above a field avalanche in Atlantic Sahara, Morocco, by Andreotti [2], a frequency spectrum was determined with dominant frequency $f_1 \approx 93$ Hz, a harmonic $f_2 \approx 2f_1$ and a low frequency content at $f_p \approx 27$ Hz as can be seen in Fig. 4. The secondary peaks superimposed on the frequency envelope are likely due to the lack of sufficient rigidity in the avalanche band. The signal was microphone, not geophone recorded, as reported earlier in Patitsas [1]. The frequency analysis was based on a signal running nearly from 22.0 to 22.6 s. During this short time interval, the signal was nearly sinusoidal as can be seen in Fig. 4. The geophone recorded signal was not well defined since riding on a floating raft it would render the sand surface not a free surface as required by Eq. (4).

There is a faint evidence of such a low frequency content in Fig. 1 in Dagois-Bohy et al. [7] at time equal to 6 and 17 s. There is no low frequency content in the reports by Hunt and Vriend [5] and by Vriend et al. [23] but that could be due to geophone cut-off frequency at around 25 Hz.

According to Sholtz et al. [18], the following authors, Poynting and Thomson [24]; Reynolds [25] argued that a shearing stress on a closed packed granular substance could result in one grain layer sliding over another grain layer and in the process create a periodic expansion-contraction of the substance. Then, Dagois-Bohy et al. [7], argued that grain layers in an avalanching band oscillate sinusoidally and that results in a musical sound without the aid of a standing or traveling mode of vibration. However, the acceleration signal shown in the lower inset in Fig. 3 in the same reference has more of a saw tooth than a sinusoidal form. Furthermore in Figs. 4 and 6 in the report by Dagois-Bohy et al. [6], the geophone signals have a saw tooth form while in Fig. 4 the microphone signal has a sinusoidal form. A simple explanation of such anomalies can be found in the absence of an effective harmonizing standing mode of vibration under the raft carrying the accelerometer. Effectively, the raft rendered the surface not a free surface as required in Eq. (4) resulting in a dominant frequency $f_1$ in the avalanche band that cannot be reconciled with the collision frequency $f_c$.

7. FREELY AVALANCHE OR NEARLY STANDING SAND PLATES

In the report by Sholtz et al. [18], it is stated that “Fully developed avalanches, in which sliding plates of sand remain intact for most of their motion, exploit the shear potential of booming dunes to the outmost extent.” According to Humphries [20] the plate thickness was about 10 cm. Then, there is the report by Haff [22], as stated above. We argue that the source of the acoustic emission is a thin collision shear band sandwiched between the plate and the static sand below. Furthermore, there are reports of sounds emitted when the sand, likely in plate form, is hardly in motion before or after an avalanche, Hunt and Vriend [5]; Vriend et al. [23]. Additionally, in Tales of Travel by Curzon [26] p. 261, it is stated that, “sound was emitted by booming sands even when apparently quiescent.” In this case we argue that the grain layers between the static sand and the hard plate above are confined sufficiently to form a creepy shear band and produce a musical sound as in Sect. 2.

8. VERTICAL ROD PULLED OVER THE SAND SURFACE

Figure 5 depicts a vertical rod some 25 mm in diameter, immersed in the booming sand bed by about 5 cm, drawn along the $x$-axis with constant velocity $V_d$. The arrows depict the sand flow, a few mm in thickness, on either side of the rod. As in the case of the avalanche sand,
we argue that the source of the emitted sound is a collision shear band under the flowing sand and more or less attached to the rod surface. In the case of the squeaky sand the source of the sound is the creepy shear band in front and under the rod where the stress level is highest.

The process of generating such a musical sound on a dune surface can be viewed in a video presentation with the title, “Song of the Dunes” prepared by the authors of the report by Douady et al. [3]. A similar video exists with the title, “Booming Sands” prepared by the authors of the report by Vriend et al. [23].

9. SINGING GRAINS SHAKEN IN A GLASS JAR

Experimental work on the subject can be found in the study by Leach and Rubin [27] where nearly half of a glass coffee jar of 675 mL volume was filled with squeeke or booming sand and shaken vertically or horizontally along the jar axis at the rate of about 1 Hz. A microphone attached to the jar recorded the sound emitted. We argue that in the case of the squeaky grains the creepy shear band was formed close to the wall where it experienced maximum pressure from the rest of the grain mass piling on top of it. In the case of the booming dune sand, more likely the collision band was formed near the glass wall and the rest of the sand mass on top of it avalanched in some direction as in a dune avalanche.

10. CONCLUSIONS

It is fairly well established that the source of squeaky sounds emitted when singing beach sands or other grains such as silica gel are impacted by a pestle, lies with the standing modes of vibration in the impact shear band directly under the impacting pestle. The standing modes of vibration comprise the fundamental mode with frequency $f_1$, its harmonics and the low frequency content with frequency $f_2$ where the sand mass under the pestle acts as a single spring loaded with the mass $m_1$ of the pestle. Contrary to published reports the intensity of the low frequency content is weak or even negligible compared with that of the fundamental mode of vibration. The mathematical model of the vibrations in the elastic impact band can account for the observed frequency shift in the harmonics of the fundamental $f_1$ when the sphere is dropped from considerable heights above the sand bed. During the acoustic emission the grain layers in the impact shear band slide one over another at extremely slow creepy rate, hence the name, “creepy shear band.”

In the case of the freely avalanche booming sand, the model formulated in this paper appears to present the best hope in explaining the acoustic emissions from the avalanche sand and also from sand pushed by a rod or shaken in a glass jar. In the case of the avalanche sand, the collision shear band is sandwiched between the avalanche band and the fixed sand below. Gravitational energy from the avalanche band is fed into the fundamental mode of vibration in the collision band which in turn drives, in a resonance effect, the fundamental mode of vibration in the avalanche band resulting in the intense musical sound. Elastic modes of vibration are also radiated from the fundamental mode in the avalanche band into the entire dune surface. The mathematical equations describing the physical process predict a nodal plane at the middle of the avalanche band in agreement with experimental observations. We tend to think that the existence of the collision shear band below the avalanche band depends critically on the fluidity of the sand mass. Then, if wind currents result in two neighboring hills with slightly different fluidities, the sand in one hill may be booming but not so in the other hill.

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APPENDIX

Calculations of the frequencies $f_p$, $f_1$, $f_2$, from Eq. (2), soon revealed that the best fit with the data shown in Fig. 2 was possible with $m_p/m_s = 1$. Thus, $\alpha_p L = \epsilon = (50/180)\pi$, $\alpha_1 L = \pi + (15/180)\pi$ and $\alpha_2 L = 2\pi + (9/180)\pi$, i.e., $\epsilon_1 = (15/180)\pi$ and $\epsilon_2 = (9/180)\pi$. So, $f_1/f_p \approx 3.9$, while from Fig. 2, $f_1/f_p \approx 4.0$. It is noteworthy that for $m_p/m_s = 2$, the same ratio is equal to a number somewhat larger than 5.

From Eq. (3) we arrive at

$$2f_1 - f_2 = (1/(2\pi))(c/L)(2\epsilon_1 - \epsilon_2)$$

$$= (1/2)(c/L)(21/180)$$  \hspace{1cm} (A-1)

We can obtain a value or $c/L$ by noting that $\alpha_1 L = \pi + \epsilon_1$, $\alpha_1 = 2\pi/\lambda_1$ where $\lambda_1$ is the fundamental wavelength in the creepy shear band. Then, $2\pi L/\lambda_1 = \pi + \epsilon_1$ and $\lambda_1 = 2\pi L/(\pi + \epsilon_1) = 2L/(1 + \epsilon_1/\pi) \approx 2L$. Then, $c/L = f_1\lambda_1/L = 2f_1$, resulting in $2f_1 - f_2 = 65$ Hz, close enough to 64 Hz from Fig. 2.