Secrecy Capacity of the Wiretap Channel with Noisy Feedback

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Abstract—In this work, the role of noisy feedback in enhancing the secrecy capacity of the wiretap channel is investigated. A model is considered in which the feed-forward and feedback signals share the same noisy channel. More specifically, a discrete memoryless modulo-additive channel with a full-duplex destination node is considered first, and it is shown that a judicious use of feedback increases the perfect secrecy capacity to the capacity of the source-destination channel in the absence of the wiretapper. In the achievability scheme, the feedback signal corresponds to a private key, known only to the destination. Then a half-duplex system is considered, for which a novel feedback technique that always achieves a positive perfect secrecy rate (even when the source-wiretapper channel is less noisy than the source-destination channel) is proposed. These results hinge on the modulo-additive property of the channel, which is exploited by the destination to perform encryption over the channel without revealing its key to the source.

I. INTRODUCTION

Wyner introduced the wiretap channel in [1] and established the possibility of creating an almost perfectly secure source-destination link without relying on private (secret) keys. In the wiretap channel, both the wiretapper and destination observe the source encoded message through noisy channels, while the wiretapper is assumed to have unlimited computational resources. Wyner showed that when the source-wiretapper channel is a degraded version of the source-destination channel, the source can send perfectly secure messages to the destination at a non-zero rate. The main idea is to hide the information stream in the additional noise impairing the wiretapper by using a stochastic encoder which maps each message to many codewords according to an appropriate probability distribution. By doing this, one induces maximal equivocation at the wiretapper. By ensuring that the equivocation rate is arbitrarily close to the message rate, one achieves perfect secrecy in the sense that the wiretapper is now limited to learn almost nothing about the source-destination messages from its observations. Csiszár and Körner generalized Wyner’s approach by considering the transmission of confidential messages over broadcast channels [2]. This work characterized the perfect secrecy capacity of the Discrete Memoryless Channel (DMC), and showed that the perfect secrecy capacity is positive unless the source-wiretapper channel is less noisy than the source-destination channel (referred to as the main channel in the sequel).

Positive secrecy capacity is not always possible to achieve in practice. In an attempt to transmit messages securely in these unfavorable scenarios, [3] and [4] considered the wiretap channel with noiseless feedback (The authors also considered a more general secret sharing problem.). These works showed that one may leverage the feedback to achieve a positive perfect secrecy rate, even when the feed-forward perfect secrecy capacity is zero. In this model, there exists a separate noiseless public channel, through which the transmitter and receiver can exchange information. The wiretapper is assumed to obtain a perfect copy of the messages transmitted over this public channel. Upper and lower bounds were derived for the perfect secrecy capacity with noiseless feedback in [3], [4]. In several cases, as discussed in detail in the sequel, these bounds coincide. But, in general, the perfect secrecy capacity with noiseless feedback remains unknown.

Our work represents a marked departure from the public discussion paradigm. In our model, we do not assume the existence of a separate noiseless feedback channel. Instead, the feedback signal from the destination, which is allowed to depend on the signal received so far, is transmitted over the same noisy channel used by the source. Based on the noisy feedback signal, the source can then causally adapt its transmission scheme, hoping to increase the perfect secrecy rate. The wiretapper receives a mixture of the signal from the source and the feedback signal from the destination. Quite interestingly, we show that in the modulo-additive DMC with a full-duplex destination, the perfect secrecy capacity with noisy feedback equals the capacity of the main channel in the absence of the wiretapper. Furthermore, the capacity is achieved with a simple scheme in which the source ignores the feedback signal and the destination feeds back randomly generated symbols from a certain finite alphabet. This feedback signal plays the role of a private key, known only by the destination, and encryption is performed by the modulo-additive channel. The more challenging scenario with a half-duplex destination, which cannot transmit and receive simultaneously, is considered next. Here, the active transmission periods of the destination will introduce erasures in the feed-forward
source-destination channel. In this setting, we propose a novel feedback scheme that achieves a positive perfect secrecy rate for any non-trivial channel distribution. The feedback signal in our approach acts as a private destination only key which strikes the optimal tradeoff between introducing erasures at the destination and errors at the wiretapper. Overall, our work proposes a novel approach for encryption where 1) the feedback signal is used as a private key known only to the destination; and 2) the encryption is performed by exploiting the modulo-additive property of the channel. This encryption approach is shown to be significantly superior to the classical public discussion paradigm.

The rest of the paper is organized as follows. In Section II we introduce the system model and our notation. Section III describes and analyzes the proposed feedback scheme which achieves the capacity of the full duplex modulo-additive DMC. Taking the Binary Symmetric Channel (BSC) as an example, we then compare the performance of the proposed scheme with the public discussion approach. The half-duplex scenario is studied in Section IV. Finally, Section V offers some concluding remarks and outlines possible avenues for future research.

II. THE MODULO-ADDITIVE DISCRETE MEMORYLESS CHANNEL

The modulo-additive discrete memoryless wiretap channel is described by the following relations

\[ Y(i) = X(i) + N_1(i), \]
\[ Z(i) = X(i) + N_2(i), \]

where, at time \( i = 1, \ldots, n \), \( Y(i) \) is the received symbol at the destination, \( Z(i) \) is the received symbol at the wiretapper, \( X(i) \) is the channel input, \( N_1(i) \) and \( N_2(i) \) are the noise samples at the destination and wiretapper, respectively. Here \( N_1(i) \) and \( N_2(i) \) are allowed to be correlated, while each process is assumed to be individually drawn from an independent and identically distributed source. Also we have \( X(i) \in \mathcal{X} = \{0, 1, \ldots, |\mathcal{X}| - 1\}, Y(i), N_1(i) \in \mathcal{Y} = \{0, 1, \ldots, |\mathcal{Y}| - 1\} \) and \( Z(i), N_2(i) \in \mathcal{Z} = \{0, 1, \ldots, |\mathcal{Z}| - 1\} \) with finite alphabet sizes \( |\mathcal{X}|, |\mathcal{Y}| \) and \( |\mathcal{Z}| \) respectively. Here ‘+’ is understood to be modulo addition with respect to the corresponding alphabet size, i.e., \( Y(i) = [X(i) + N_1(i)] \mod |\mathcal{Y}| \) and \( Z(i) = [X(i) + N_2(i)] \mod |\mathcal{Z}| \) with addition in the real field.

In this paper, we focus on the wiretap channel with noisy feedback. More specifically, at time \( i \) the destination sends the causal feedback signal \( X_1(i) \) over the same noisy channel used for feed-forward transmission, i.e., we do not assume the existence of a separate noiseless feedback channel. The causal feedback signal is allowed to depend on the signal received to that point \( Y^{i-1} \), i.e., \( X_1(i) = \Psi(Y^{i-1}) \), where \( \Psi \) can be any (possibly stochastic) function. In general, we allow the destination to choose the alphabet of the feedback signal \( X_1 \) and the corresponding size \( |X_1| \). With this noisy feedback from the destination, the received signal at the source, wiretapper and destination are

\[ Y_0(i) = X(i) + X_1(i) + N_0(i), \]
\[ Y(i) = X(i) + X_1(i) + N_1(i), \]

and

\[ Z(i) = X(i) + X_1(i) + N_2(i), \]

respectively. Here \( Y_0(i) \in \mathcal{Y}_0 = \{0, 1, \ldots, |\mathcal{Y}_0| - 1\} \) is the received noisy feedback signal at the source and \( N_0(i) \) is the feedback noise, which may be correlated with \( N_1(i) \) and \( N_2(i) \). We denote the alphabet size of \( N_0(i) \) and \( Y_0(i) \) by \( |\mathcal{Y}_0| \).

Again, all ‘+’ operation should be understood to be modulo addition with corresponding alphabet size.

Now, the source wishes to send the message \( W \in \mathcal{W} = \{1, \ldots, M\} \) to the destination using an \((M, n)\) code consisting of: 1) a casual stochastic encoder \( f \) at the source that maps the message \( w \) and the received noisy feedback signal \( y_0^{i-1} \) to a codeword \( x \in \mathcal{X}^n \) with

\[ x(i) = f(i, w, y_0^{i-1}), \]

2) a stochastic feedback encoder \( \Psi \) at the destination that maps the received signal into \( X_1(i) \) with \( x_1(i) = \Psi(y_0^{i-1}) \), and 3) a decoding function at the destination: \( \hat{W}^n \rightarrow \mathcal{W} \). The average error probability of the \((M, n)\) code is

\[ P_e^n = \sum_{w \in \mathcal{W}} \frac{1}{M} \Pr(d(y) \neq w | w \text{ was sent}). \]

The equivocation rate at the wiretapper is defined as

\[ R_e = \frac{1}{n} H(W|Z). \]

We are interested in achievable perfectly secure transmission rates defined as follows.

Definition 1: A secrecy rate \( R_f \) is said to be achievable over the wiretap channel with noisy feedback if for any \( \epsilon > 0 \), there exists an integer \( n(\epsilon) \) and a sequence of codes \((M, n)\) such that for all \( n \geq n(\epsilon) \), we have

\[ R_f = \frac{1}{n} \log_2 M, \]

and

\[ \frac{1}{n} H(W|Z) \geq R_f - \epsilon. \]

Definition 2: The secrecy capacity with noisy feedback \( C_f' \) is the maximal rate at which messages can be sent to the destination with perfect secrecy; i.e.

\[ C_f' = \sup_{f, \psi} \{ R_f : R_f \text{ is achievable} \}. \]

Note that in our model, the wiretapper is assumed to have unlimited computational resources and to know the coding scheme of the source and the feedback function \( \Psi \) used by the destination. We believe that our feedback model captures realistic scenarios in which the terminals exchange information over noisy channels.

III. THE WIRETAP CHANNEL WITH FULL-DUPEX FEEDBACK

A. Known Results

The secrecy capacity of the wiretap DMC without feedback \( C_s \) was characterized in [2]. Specializing to our modulo-additive channel, one obtains

\[ C_s = \max_{V \rightarrow X \rightarrow YZ} [I(V; Y) - I(V; Z)]^+. \]
The wiretap DMC with public discussion was introduced and analyzed in [3], [4]. More specifically, these papers considered a more general model in which all the nodes observe correlated variables. The wiretap channel model is a particular mechanism for the nodes to observe the correlated variables, and corresponds to the “channel type model” studied in [4], and there exists an extra noiseless public channel with infinite capacity, through which both the source and the destination can send information. Combining the correlated variables and the publicly discussed messages, the source and the destination generate a key about which the wiretap has negligible information. Please refer to [4] for rigorous definitions of these notions. Since the public discussion channel is negligible, the wiretapper is assumed to observe a noiseless version of the information transmitted over it. It is worth noting that some of the schemes proposed in [3], [4] manage only to generate an identical secret key at both the source and destination. The source may then need to encrypt its message using the one-time pad scheme which reduces the effective secret key capacity of the public discussion paradigm.

Theorem 1 ([3], [4]): The secret key capacity of the public discussion approach satisfies the following conditions:

\[
\max_{P_X} \{\max[I(X;Y) - I(X;Z)], \max[I(X;Y) - I(Y;Z)]\} \leq C_s^P \leq \min_{P_X} \{\max[I(X;Y), \max I(X;Y|Z)]\}.
\]

Proof: Please refer to [3], [4].

These bounds are known to be tight in the following cases [3], [4].

1) \( P_{YZ|X} = P_{Y|X} P_{Z|X} \), i.e., the main channel and the source-wiretapper channel are independent; in this case

\[
C_s^P = \max_{P_X} \{I(X;Y) - I(Y;Z)\}.
\]

2) \( P_{XZ|Y} = P_{X|Y} P_{Z|Y} \), i.e., \( X \rightarrow Y \rightarrow Z \) forms a Markov chain, and hence the source-wiretapper channel is a degraded version of the main channel. In this case

\[
C_s^P = \max_{P_X} \{I(X;Y) - I(X;Z)\}.
\]

This is also the secrecy capacity of the degraded wiretap channel without feedback. Hence public discussion does not increase the secrecy capacity for the degraded wiretap channel.

3) \( P_{XY|Z} = P_{X|Z} P_{Y|Z} \), i.e., \( X \rightarrow Z \rightarrow Y \), so that the main channel is a degraded version of the wiretap channel. In this case

\[
C_s^P = 0.
\]

Again, public discussion does not help in this scenario.

B. The Main Result

The following theorem characterizes the secrecy capacity of the wiretap channel with noisy feedback. Moreover, achievability is established through a novel encryption scheme that exploits the modulo-additive structure of the channel and uses a private key known only to the destination.

Theorem 2: The secrecy capacity of the discrete memoryless modulo-additive wiretap channel with noisy feedback is

\[
C_s^P = C,
\]

where \( C \) is the capacity of the main channel in the absence of the wiretapper.

Proof: (Sketch) For the converse part, we first upper bound the secrecy capacity with feedback by the channel capacity with feedback in the absence of the wiretapper, by ignoring the equivocation condition (3). Since feedback does not increase the channel capacity of the DMC, the channel capacity with feedback in the absence of the wiretapper is \( C \), which completes the converse part.

For the achievability part, we use the following novel scheme. The source ignores the feedback signal and uses a channel coding scheme for the ordinary channel without a wiretapper. The destination sets \( X_1 = Z \), and at any time \( i \) sets \( X_i(i) = a \), where \( a \) is chosen randomly and equiprobably from \( \{0, \ldots, |Z| - 1\} \). Hence \( X_i \) is uniformly distributed over \( Z^n \). After receiving \( Y \), the destination first sets \( Y = Y - X_1 \), and then obtains an estimate of the codeword by finding an \( X \) that is jointly typical with \( Y \). It can be shown that the error probability at the destination can be made arbitrarily small. The wiretapper receives \( Z = X + X_1 + N_2 \), which can be shown to be uniformly distributed over \( Z^n \) for any realizations of \( X \) and \( N_2 \). Hence \( I(X;Z) = 0 \), which further implies \( I(W;Z) = 0 \) by the data processing inequality.

Please refer to [5] for further details.

Our scheme achieves \( I(W;Z) = 0 \). This implies perfect secrecy in the strong sense of Shannon [6] as opposed to Wyner’s notion of perfect secrecy [1], which has been pointed out to be insufficient for certain cryptographic applications [7]. The enabling observation behind our achievability scheme is that, by judiciously exploiting the modulo-additive structure of the channel, one can render the channel output at the wiretapper independent of the codeword transmitted by the source. Here, the feedback signal \( X_1 \) serves as a private key and the encryption operation is carried out by the channel. Instead of requiring both the source and destination to know a common encryption key, we show that only the destination needs to know the encryption key, hence eliminating the burden of secret key distribution. Remarkably, the secrecy capacity with noisy feedback is shown to be larger than the secret key capacity of public discussion schemes. This point will be further illustrated by the binary symmetric channel example discussed next. This presents a marked departure from the conventional wisdom, inspired by the data processing inequality, which suggests the superiority of noiseless feedback. This result is due to the fact that the noiseless feedback signal is also available to the wiretapper, while in the proposed noisy feedback scheme neither the source nor the wiretapper knows the feedback signal perfectly. In fact,
the source in our scheme ignores the feedback signal, which is used primarily to confuse the wiretapper. Our result shows that complicated feedback functions $\Psi$ are not needed to achieve optimal performance in this setting (i.e., a random number generator suffices). Also, the alphabet size of the feedback signal can be set equal to the alphabet size of the wiretapper channel and the coding scheme used by the source is the same as the one used in the absence of the wiretapper.

C. The Binary Symmetric Channel Example

To illustrate the idea of encryption over the channel, we consider in some details the wiretap BSC shown in Figure 1 where $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \{0, 1\}$, $\Pr\{N_1(i) = 1\} = \epsilon$ and $\Pr\{N_2(i) = 1\} = \delta$. The secrecy capacity of this channel without feedback is known to be [3]

$$C_s = [H(\delta) - H(\epsilon)]^+, \quad$$

with $H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$. We differentiate among the following special cases.

1) $\epsilon = \delta = 0$.

In this case, both the main channel and wiretap channel are noiseless, hence $C_s = 0$. Also we have $C_s^p = 0$, since the wiretapper sees exactly the same as what the destination sees. Specializing our scheme to this BSC channel, at time $i$, the destination randomly chooses $X_1(i) = 1$ with probability 1/2 and sends $X_1(i)$ over the channel. This creates a virtual BSC at the wiretapper with $\delta = 1/2$. On the other hand, since the destination knows the value of $X_1(i)$, it can cancel it by adding $X_1(i)$ to the received signal. This converts the original channel to an equivalent BSC with $\epsilon = 0$. Hence, through our noisy feedback approach, we obtain an equivalent wiretap BSC with parameters $\epsilon = 0$ and $\delta = 1/2$ resulting in $C_s^f = H(\delta) - H(\epsilon) = 1$.

2) $0 < \delta < \epsilon < 1/2$, $N_1(i)$ and $N_2(i)$ are independent.

Since $\delta < \epsilon$, we have $C_s = 0$. Also, $N_1(i)$ and $N_2(i)$ are independent, so $P_{V_i|X} = P_{V_i|X} P_{Z_i|X}$. Then from [5], one can easily obtain that [3] $C_s^p = H(\epsilon + \delta - 2\epsilon \delta) - H(\epsilon)$. Our feedback scheme, on the other hand, achieves $C_s^f = 1 - H(\epsilon)$. Since $H(\epsilon + \delta - 2\epsilon \delta) \leq 1$, we have $C_s^f \geq C_s^p$ with equality if and only if $\epsilon + \delta - 2\epsilon \delta = 1/2$.

3) $0 < \delta < \epsilon < 1/2$ and $N_1(i) = N_2(i) + N'(i)$, where $\Pr\{N'(i) = 1\} = (\epsilon - \delta)/(1 - 2\delta)$.

The main channel is a degraded version of the source-wiretapper channel, $X \rightarrow Y \rightarrow Z \rightarrow Y$, as shown in Figure 2.

$$X \rightarrow Y \rightarrow Z \rightarrow Y$$

Fig. 2. The BSC Wiretap Channel with a Degraded Main Channel.

Hence, from [7], we have $C_s = C_s^p = 0$, while $C_s^f = 1 - H(\epsilon)$.

4) $0 < \epsilon < \delta < 1/2$, and $N_2(i) = N_1(i) + N'(i)$, where $\Pr\{N'(i) = 1\} = (\delta - \epsilon)/(1 - 2\epsilon)$.

$$X \rightarrow Y \rightarrow Z$$

Fig. 3. The BSC wiretap Channel with a Degraded Source-Wiretapper Channel.

In this case, the source-wiretapper channel is a degraded version of the main channel as shown in Figure 3, $X \rightarrow Y \rightarrow Z$, so from [6] $C_s = C_s^p = H(\delta) - H(\epsilon)$. But $C_s^f = 1 - H(\epsilon) \geq C_s^p$ with equality if and only if $\delta = 1/2$.

5) $N_1(i)$ and $N_2(i)$ are correlated and the channel is not degraded.

In this case $C_s = [H(\delta) - H(\epsilon)]^+$, The value of $C_s^p$ is unknown in this case but can be bounded by

$$C_s = [H(\delta) - H(\epsilon)]^+ \leq C_s^p \leq 1 - H(\epsilon) = C_s^f.$$ In summary, the secrecy capacity with noisy feedback is always larger than or equal to that of the public discussion paradigm when the underlying wiretap channel is a BSC. More strongly, the gain offered by the noisy feedback approach, over the public discussion paradigm, is rather significant in many relevant special cases.

IV. EVEN HALF-DUPLEX FEEDBACK IS SUFFICIENT

It is reasonable to argue against the practicality of the full duplex assumption adopted in the previous section. For example, in the wireless setting, nodes may not be able to transmit and receive with the same degree of freedom due to the large difference between the power levels of the transmit and receive chains. This motivates extending our results to the half duplex wiretap channel in which the terminals can either transmit or receive but never both at the same time. Under this situation, if the destination wishes to feed back at time $i$, it loses the opportunity to receive the $i^{th}$ symbol transmitted by the source, which effectively results in an erasure (assuming that the source is unaware of the destination’s decision). The proper feedback strategy must, therefore, strike a balance between confusing the wiretapper and degrading the source-destination
In order to simplify the following presentation, we first focus on the wiretap BSC. The extension to arbitrary modulo-additive channels is briefly outlined afterwards.

In the full-duplex case, at any time $i$, the optimal scheme is to let the destination send $X_1(i)$, which equals 0 or 1 equiprobably. But in the half-duplex case, if the destination always keeps sending, it does not have a chance to receive information from the source, and hence, the achievable secrecy rate is zero. This problem, however, can be solved by observing that if at time $i$, $X_1(i) = 0$, the signal the wiretapper receives, i.e., $Z(i) = X_1(i) + N_2(i)$, is the same as in the case in which the destination does not transmit. The only crucial difference in this case is that the wiretapper does not know whether the feedback has taken place or not, since $X_1(i)$ can be randomly generated at the destination and kept private.

The previous discussion inspires the following feedback scheme for the half-duplex channel. The destination first fixes a value $t \in [0, 1]$ which is revealed to both the source and wiretapper. At time $i$, the destination randomly generates $X_1(i) = 1$ with probability $t$ and $X_1(i) = 0$ with probability $1 - t$. If $X_1(i) = 1$, the destination sends $X_1(i)$ over the channel, which causes an erasure at the destination and a potential error at the wiretapper. On the other hand, when $X_1(i) = 0$, the destination does not send a feedback signal and spends the time on receiving from the channel. The key to this scheme is that although the source and wiretapper know $t$, neither is aware of the exact timing of the event \{$X_1(i) = 1$\}. The source ignores the feedback and keeps sending information. The following result characterizes the achievable secrecy rate with the proposed feedback scheme.

**Theorem 3:** For a BSC with half-duplex nodes and parameters $\epsilon$ and $\delta$, the scheme proposed above achieves

$$R_s^f = \max_{\mu, t} \left(1 - t\right) \left[H(\epsilon + \mu - 2\mu\epsilon) - H(\epsilon) - H(\tilde{\mu} + \mu - 2\mu\tilde{\mu}) - H(\tilde{\mu})\right]^+, $$

with $\tilde{\mu} = \delta + t - 2\delta t$.

In general, one can obtain the optimal values of $\mu$ and $t$ by setting the partial derivatives of $R^f$, with respect to $\mu$ and $t$ to 0, and solving the corresponding equations. Unfortunately, except for some special cases, we do not have a closed form solution for these equations at the moment. Interestingly, using the not necessarily optimal choice of $\mu = t = 1/2$, we obtain $R^f = [1 - H(\epsilon)]/2$ implying that we can achieve a nonzero secrecy rate as long as $\epsilon \neq 1/2$ irrespective of the wiretap channel conditions. Hence, even for half-duplex nodes, noisy feedback from the destination allows for transmitting information securely for almost any wiretap BSC.

Finally, we compare the performance of different schemes in some special cases of the wiretap BSC.

1) $\epsilon = \delta = 0$.

As mentioned above, here we have $C_s = C_s^p = 0$. It is easy to verify that the optimal choice of $\mu$ and $t$ are $1/2$, and we thus have $R^f_s = 1/2$.

2) $0 < \delta < \epsilon < 1/2$ and $N_1(i) = N_2(i) + N'(i)$, where $\Pr\{N'(i) = 1\} = (\epsilon - \delta)/(1 - 2\delta)$.

The main channel is a degraded version of the wiretap channel, so $C_s = C_s^p = 0$. But by setting $\mu = t = 1/2$ in our half-duplex noisy feedback scheme, we obtain $R^f_s = [1 - H(\epsilon)]/2$.

The extension to the general discrete modulo-additive channel is natural. The destination can set $X_1(i)$, which generates a certain distribution $P_{X_1}$. At time $i$, if the randomly generated $X_1(i) \neq 0$, the destination sends a feedback signal, incurring an erasure to itself. On the other hand, if $X_1(i) = 0$, it does not send the feedback signal and spends the time listening to the source. The achievable performance could be calculated based on the equivalent channels as done in the BSC. This scheme guarantees a positive secrecy capacity as seen in the case where $P_{X_1}$ is chosen to be uniformly distributed over $Z$. This is because a uniform distribution over $Z$ renders the output at the wiretapper independent from the source input, i.e., $I(W; Z) = 0$, while the destination can still spend $1/|Z|$ part of the time listening to the source. Finding the optimal distribution $P_{X_1}$, however, is tedious.

**V. Conclusion**

In this paper, we have obtained the secrecy capacity (or achievable rate) for several instantiations of the wiretap channel with noisy feedback. More specifically, with a full duplex destination, it has been shown that the secrecy capacity of modulo-additive channels is equal to the capacity of the source-destination channel in the absence of the wiretapper. Furthermore, the secrecy capacity is achieved with a simple scheme in which the destination randomly chooses its feedback signal from a certain finite alphabet. Interestingly, with a slightly modified feedback scheme, we are able to achieve a positive secrecy rate for the half duplex channel. We have shown that this paradigm significantly outperforms the public discussion approach for sharing private keys between the source and destination. Our results motivate several interesting directions for future research. For example, characterizing the secrecy capacity of arbitrary DMCs (and the AWGN channel) with feedback remains an open problem. From an algorithmic perspective, it is also important to understand how to exploit different channel structures (in addition to the modulo-additive one) for encryption purposes. Finally, extending our work to multi-user channels (e.g., the relay-eavesdropper channel [8]) is of considerable interest.

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