Power-like corrections and the determination of the gluon distribution

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Abstract

Power-suppressed corrections to parton evolution may affect the theoretical accuracy of current determinations of parton distributions. We study the role of multigluon-exchange terms in the extraction of the gluon distribution for the Large Hadron Collider (LHC). Working in the high-energy approximation, we analyze multi-gluon contributions in powers of $1/Q^2$. We find a moderate, negative correction to the structure function’s derivative $dF_2/d\ln Q^2$, characterized by a slow fall-off in the region of low to medium $Q^2$ relevant for determinations of the gluon at small momentum fractions.
The estimate of the theoretical accuracy on the determination of parton distributions is relevant for phenomenology at the Large Hadron Collider (LHC) [1–3]. A potential source of theory uncertainty is given by power-suppressed contributions to parton evolution. In particular, the extraction of the gluon distribution for small momentum fractions depends on data in the region of high energies and moderate to low \( Q^2 \), in which power-like corrections from multiple parton scatterings are potentially significant.

The purpose of this note is to derive an estimate of these corrections, based on high-energy amplitudes for multigluon exchange [4]. We concentrate on the power correction \( \Delta \) to the \( Q^2 \) derivative of the structure function \( F_2 \)

\[
\frac{dF_2}{d\ln Q^2} = K \otimes G \ [1 + \Delta] + \text{quark term ,} \tag{1}
\]

where \( K \) is the perturbative kernel, \( G \) is the gluon distribution, and \( \Delta \) is \( \mathcal{O}(1/Q^2) \). This correction contributes to the theory uncertainty on \( G \), since below \( x \lesssim 10^{-2} \) this is mostly determined from data for \( dF_2/d\ln Q^2 \).

The theoretical framework to treat multiple scatterings is based on the \( s \)-channel picture of DIS [4], and its basic degrees of freedom are described by matrix elements of eikonal lines. The corresponding predictions, incorporating nonperturbative dynamics, are valid down to small \( Q^2 \). To enforce consistency with the framework of standard parton analyses [2,3,5] we will expand the answer in powers of \( 1/Q^2 \). We study the behavior of the power expansion at low \( Q^2 \) and \( x \), and identify the power corrections by subtraction of the leading-power contribution.

We find moderate, but non-negligible negative corrections, increasing in size as \( x \) decreases. We find that with a physically natural choice of parameters in the eikonal matrix elements we can achieve a sensible description of data and still have power corrections to \( dF_2/d\ln Q^2 \) that do not exceed the leading power already at \( Q^2 \gtrsim 0.5 \text{ GeV}^2 \). However we find that for small \( x \) the corrections have a slow fall-off with \( Q^2 \) in the region of intermediate \( Q^2 \), \( Q^2 \sim 1 \div 10 \text{ GeV}^2 \), behaving effectively like \( 1/Q^\lambda \) in this region, with \( \lambda \) close to 1 for \( x \lesssim 10^{-3} \). This behavior results from summing the power expansion. As a consequence the power corrections stay larger than 10\% up to \( Q^2 \) of a few GeV\(^2\) for \( x \) below \( 10^{-3} \).

The contents of the paper is as follows. We first give the \( s \)-channel results that provide the basic elements for the evaluation of the power expansion. Then we focus on the \( Q^2 \) derivative of the structure function. We examine the next-to-leading-power contribution and the sum of the power series, and present numerical results.

We begin by recasting the result [4] for multi-gluon contributions to DIS structure functions in the Mellin-transform representation. This is convenient to analyze the \( 1/Q^2 \) expansion. In the high-energy approximation we describe gluon exchange in terms of eikonal-line operators

\[
V(z) = \mathcal{P} \exp \left\{ -ig_s \int_{-\infty}^{+\infty} dz^- A_a^+(0, z^-, z)t_a \right\} , \tag{2}
\]

where \( A \) is the color potential and \( \mathcal{P} \) is the path-ordering. Following [6] we notate the matrix elements of eikonal operators at transverse positions \( b \) and \( b + z \) as
Here \([dP'] = dP' + d^2P'/2P' + (2\pi)^3\), \(z\) is the transverse separation between the eikonal lines, and \(b\) is the impact parameter. The result [4] for the transverse structure function \(F_T\) can be written as

\[
x F_T(x, Q^2) = \int_{c-i\infty}^{c+i\infty} \frac{du}{2\pi i} \int d^2b \hat{\Xi}(u, b) \Phi(u) ,
\]

where \(0 < c < 1\), \(\hat{\Xi}\) is the Mellin transform of \(\Xi\)

\[
\hat{\Xi}(u, b) = \int \frac{d^2z}{\pi z^2} (z^2)^{u-1} \Xi(z, b) ,
\]

and \(\Phi\) is a calculable coefficient. To lowest order

\[
\Phi(u) = \sum_a e_a^2 \frac{N_c}{16\pi^4 u} \frac{\pi^2}{4^u} \frac{\Gamma(3-u)\Gamma(2-u)\Gamma(1-u)\Gamma(2+u)}{\Gamma(5/2-u)\Gamma(3/2+u)} ,
\]

where \(N_c = 3\), and \(e_a\) is the electric charge of quark of type \(a\).

From this approach the parton-model framework is recovered through an expansion in powers of \(gA\), valid for small \(z\). In particular, the coefficient of the quadratic term in the expansion of \(\Xi\) can be related to the gluon distribution \(G\) [4],

\[
\int d^2b \Xi \simeq \frac{\pi g_s^2 T_R}{4N_c} (x G) z^2 (1 + \mathcal{O}(|z|)) ,
\]

where \(T_R = 1/2\).

The quark distribution can be dealt with by the same method. The main difference is that while in the structure function case the ultraviolet region is naturally regulated by the physical scale \(Q^2\), in the case of the quark distribution we need to treat the ultraviolet divergences. The result in dimensional regularization is [7]

\[
x q(x, \mu) = (\mu^2)^{-2\epsilon} \int d^2z \ d^2b \ w(z) \Xi(z, b) - \text{UV} ,
\]

where

\[
w(z) = \frac{N_c}{3\pi^4} \frac{1}{z^4} \left( \frac{\mu^2 z^2}{4\pi} \right)^{2\epsilon} \frac{\Gamma(2 - \epsilon)^2}{1 - 2\epsilon/3} ,
\]

and we have indicated by UV the ultraviolet subtraction. This is required since in Eq. (3) \((1 - V^\dagger V) \to 0\) for \(z \to 0\) and \(\Xi \propto z^2\). Using Eqs. (7),(9) the \(x \ll 1\) form of the evolution equation for the quark is reobtained from Eq. (8). Power-suppressed contributions arise from the difference between the terms left over from the ultraviolet regularization in the quark-distribution and structure-function case [7]. In general, these depend on the scheme used for the ultraviolet subtraction. This dependence does not enter in the correction to the structure function’s derivative which we consider next.
The matrix element $\Xi$ is nonperturbative and is to be determined from experiment. For the numerical calculations that follow we model its functional form according to the model $[8,9]$

$$\tilde{\Xi}(u, b) = \frac{\Gamma(u)}{1-u} \left( \frac{\mu_s^2(b)}{4} \right)^{1-u},$$

(10)

where $\mu_s$ is the saturation scale, with $b$ dependence as in [8]. The operator relation (7) implies for model (10) that

$$\int d\mu_s^2 \frac{1}{N_c} \alpha_s(\mu_r) xG(x, \mu_f).$$

(11)

In Eq. (11) we have indicated explicitly the dependence of the running coupling and gluon distribution on the renormalization/factorization scales $\mu_r, \mu_f$. In the present context the choice of these scales amounts to specifying the model for $\Xi$.

Consider now the derivative of $F_T$ with respect to $Q^2$, $F'_T \equiv dF_T/d\ln Q^2$. Taking the derivative cancels the factor $1/u$ in Eq. (6). We determine the expansion of $F'_T$ in powers of $1/Q^2$ by closing the integration contour in the complex $u$-plane to the left and evaluating the residues at the poles of the integrand. First we verify that the result from the leading pole (LP) $u = 0$ coincides with that from Eq. (8) for the quark distribution. We get

$$xF_{T,LP} = \sum_a e_a^2 \frac{1}{3\pi} \int d^2b \ \text{Res}_{u=0} \tilde{\Xi}.$$  

(12)

By inserting Eqs. (10),(11), Eq. (12) yields the perturbative leading-power coefficient. We identify the power-suppressed correction by subtracting off this contribution. Next we consider the contribution from the next-to-leading pole (NLP) $u = -1$, $F'_{T,\text{NLP}}$. This is proportional to the $u = -1$ residue of $\tilde{\Xi}$. In Fig. 1 we compute the ratio

$$\delta^{(\text{NLP})} \equiv F'_{T,\text{NLP}}/(F'_{T,\text{LP}} + F'_{T,\text{NLP}})$$

(13)

versus $Q^2$ at $x = 10^{-2}$ and $x = 10^{-4}$ for two different choices of $\mu_r, \mu_f$. The natural scale for $\mu_r$ and $\mu_f$ should be set by the inverse of the mean transverse distance $z$. For the illustration in Fig. 1 we take this scale to be on the order of $Q$, and plot results for $\mu_f = Q$, $\mu_r = Q$ and $\mu_f = 2Q$, $\mu_r = Q/2$. We use the CTEQ parton distributions [3].

The ratio $\delta^{(\text{NLP})}$ goes like $1/Q^2$. We see from Fig. 1 that the size of the NLP contribution is rather sensitive to the scales $\mu_f$ and $\mu_r$. This is not surprising. The definition of the model for $\Xi$, embodied in the choice of $\mu_f, \mu_r$, corresponds to defining the (otherwise arbitrary) separation of perturbative and nonperturbative effects. In particular, varying these scales amounts to effectively simulating contributions of higher perturbative order. The variation of the nonperturbative power correction in Fig. 1 says that this is unambiguously defined only once we specify what we include in the perturbative part of the calculation. See [10] for an extensive study of the issue of scale-setting and running coupling effects in high-energy evolution. In what follows we simply set the scales from comparison with experimental data.

Beyond the next-to-leading power, the poles in the $u$-plane have multiplicity higher than 1, leading to $\ln Q^2$ enhancements of the power corrections. The order-$n$ term is of the form
\[ C(n, \ln Q^2) \frac{\xi_n}{(Q^2)^n}, \quad (14) \]

where \( \xi_n \) give the dimensionful nonperturbative scale in terms of the \( b \)-integral of the moments (5) of \( \Xi \), and \( C \) are coefficients determined from Eq. (6). Through the moments of \( \Xi \) the correction (14) receives contribution from the exchange of any number of gluons via eikonal operators.

We now proceed to evaluate numerically the contribution of all powers in \( 1/Q^2 \). We go to \( z \)-space via the inverse Mellin transform and let the scales \( \mu_f \) and \( \mu_r \) vary with the distance \( z \). Details for the numerical integration in coordinate space are given in [7]. Adding to Eq. (4) the contribution of longitudinal \( F_L \), we compute the structure function \( F_2 \) and tune the factorization/renormalization scales, \( \mu_f = c_1/|z| \) and \( \mu_r = c_2/|z| \), by comparing the answer with the experimental data [11]. The result of doing this is reported in Fig. 2, where \( c_1 = 4 \), \( c_2 = 0.32 \).

Using these values for the model parameters, we turn to the derivative \( dF_2/d\ln Q^2 \). We calculate the power correction by subtraction of the leading power as described around
Eq. (12). In Fig. 3 we plot the result for the power correction normalized to the full answer and multiplied by $(-1)$. We see from Fig. 3 that above $Q^2 = 1$ GeV$^2$ the corrections are below 20% for $x > 10^{-4}$. We take this as an indication that the power expansion is not breaking down, at least to such values of $x$. In fact, we find that corrections do not exceed the leading power already at $Q^2 \simeq 0.5$ GeV$^2$ for $x$ above $10^{-5}$.

Fig. 3 also shows, however, that as $x$ decreases the power corrections remain non-negligible up to higher and higher $Q^2$. For $x \approx 10^{-3}$ it takes $Q^2$ of a few GeV$^2$ before the correction is less than 10%. This is associated with the curves having a rather slow decrease in the range of medium $Q^2$ in the figure, much slower than the asymptotic $1/Q^2$. For instance, for $x \approx 10^{-3}$ the behavior in the region $Q^2 \approx 1 \div 10$ GeV$^2$ is closer to an effective power $1/Q^\lambda$ with $\lambda \simeq 1.2$. This slow fall-off results from summing the terms (14).

We leave to a future study the question of whether this behavior can be interpreted in terms of an effective, $x$-dependent semihard scale on the order of the GeV.

In summary, the results above indicate that the power expansion should still work at $x$ and $Q^2$ as low as in the region of the data presently used for determinations of the gluon distribution $G$ for the LHC, but subleading corrections to $dF_2/d\ln Q^2$ from multigluon exchange are non-negligible in this region and contribute to the theory uncertainty on $G$. We have also computed the analogous corrections for the derivative of the transverse structure
function [7], and we find that these are generally smaller than in the case of $F_2$. This may be regarded as yet another motivation for the importance of a separate measurement of the longitudinal component $F_L$ [12].

A word of caution is needed in interpreting these results. Multigluon amplitudes are treated in the high-energy approximation. Also, the modeling of the nonperturbative matrix elements and the summation of the power series expansion call for a firmer understanding. Besides, power corrections from sources other than that considered here may be relevant as well. In particular, corrections from self-energy graph insertions are still largely unexplored for flavor-singlet observables [13].

Nevertheless, the method presented above allows one to obtain an estimate of multi-gluon corrections which can be made consistently with perturbative evolution order by order. It is based on subtraction of the leading pole in Mellin space. This serves to specify the definition of the power correction. In this work we have been concerned with the contribution to the $Q^2$ derivative of the structure function, relevant for the extraction of $G$. But the approach can in principle be extended to evaluate corrections to $F_2$ itself, and to processes directly coupled to gluons. The latter will be especially interesting for studying multiple-scattering effects in the production of jet final states.

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