Analysis methods to search for transient events in ground-based Very High Energy $\gamma$-ray astronomy

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Abstract

Transient and variable phenomena in astrophysical sources are of particular importance to understand the underlying gamma-ray emission processes. In the very-high energy gamma-ray domain, transient and variable sources are related to charged particle acceleration processes that could for instance help understanding the origin of cosmic-rays. The imaging atmospheric Cherenkov technique used for gamma-ray astronomy above $\sim 100$ GeV is well suited for detecting such events. However, the standard analysis methods are not optimal for such a goal and more sensitive methods are specifically developed in this publication. The sensitivity improvement could therefore be helpful to detect brief and faint transient sources such as Gamma-Ray Bursts.

Keywords: $\gamma$-rays: general, Cherenkov Telescopes, Methods: statistical, Methods: data analysis, Transient phenomena

1. Introduction

Transient or variable astrophysical sources are particularly interesting to understand charged particle acceleration processes in the Universe. They are of great importance to study the origin of cosmic-rays or to test the validity of some of the most fundamental laws of physics such as Lorentz invariance. The most violent events in the Universe such as Gamma-Ray Bursts (GRBs) or outbursts from Active Galactic Nuclei (AGNs) accelerate particles to high energies which are expected to radiate in High and Very-High Energies (HE: $> 100$ MeV, VHE: $> 100$ GeV) gamma-rays.

At VHE, the Imaging Atmospheric Cherenkov Telescopes (IACTs) detect and study variability at minute to week time scales from a variety of astrophysical sources such as AGN, pulsars or binary systems (e.g. Abdalla et al. 2017, focusing on the AGN PKS 2155-304). GRBs are intensely studied since their discovery in 1967 at all wavelengths (e.g. Gehrels et al. 2004, Ackermann et al. 2013b, Narayana Bhat et al. 2016). However, due to the limited number of expected photons and the limited field-of-view and duty cycles of IACTs, detection of such objects at VHE is challenging. VHE gamma-ray emission from GRBs has recently been detected for the first time by H.E.S.S. (Abdalla et al. 2019) and MAGIC (Acciari et al. 2019). Increasing the detection statistics of GRBs in the VHE gamma-ray domain is now crucial to understand the processes at play in these objects.

The determination of the statistical significance of a VHE gamma-ray source in IACT analysis is usually done by accumulating data towards a test position and comparing the number of detected events at this position to the number obtained in (source-free) background control regions (e.g. Li and Ma 1983, Berge et al. 2007). However, in case of a transient source, this method is not optimal since the test position may only contain signal events during a limited fraction of observation and only background events afterwards. Some methods, specifically developed to search for transient events, are explored in this article. These methods aim at improving the sensitivity of IACT data analysis to such events and at recovering the detection of some of the weak and brief signals that standard analyses could miss.

The methods presented in this document have been implemented in the analysis frameworks used to analyze data from the H.E.S.S. experiment (Holler et al. 2015). They will also be useful for the data analysis of the next major instrument: the Cherenkov Telescope Array (e.g. Acharya et al. 2013).

The statistical tests we describe rely on a good knowledge of the instrument response at each time and observed position and its evolution with time. Therefore, in section 2 we first describe the methods used (i) to estimate the detector acceptance and (ii) to construct a new time reference in which the event rate is expected to be constant. Once this step is done, a set of (acceptance corrected) times is available, to which statistical tests can be applied to search for variability. The tests we developed or adapted from the literature to search for transient events are described in section 3 and their performances are estimated and compared in section 4.
nature (gamma-like or hadron-like events) according to the reconstruction algorithm. Gamma-like events are events that are reconstructed by the analysis software as potential gamma rays due to their image properties. In regions were no sources are present, these events are mainly originating from hadrons or electrons. Hadron-like events on the other hand are the events that are identified by the reconstruction method as the least similar to gamma-ray events. For a steady source or for background events, the arrival times are supposed to follow a Poisson process with constant rate. However, this behavior is affected by the detector response variations with time. For instance, if the acceptance of the instrument increases (due to a lower zenith angle or better atmospheric conditions), the event rate will increase and may create a fake transient event. It is therefore necessary to correct the time intervals for these acceptance variations.

2.1. Acceptance estimation

In order to correct for the effect of acceptance variations, the instrument response is determined for each (observed) position on the sky and for each time during the observations in the following way. First, an exposition map is calculated in the detector frame. This map represents the fraction of time each position is outside regions known to contain gamma-ray sources, which are not used for background estimation and are hereafter called exclusion regions. As the sky is moving in the detector frame, the trajectory of each exclusion region in this frame has to be taken into account for a proper determination of the exposure. This can be done by using the fraction of hadron-like events in exclusion regions as a function of the position in the detector frame. Then, the exposition maps of all observation runs are summed up, weighted by the total number of (non-excluded) events in each run. At the same time, the map of gamma-like events outside any exclusion regions in the detector’s frame are summed up. Dividing this map of events by the exposition map gives an acceptance map in the detector’s frame, as illustrated in Fig. 1. This map can be re-projected on the sky to determine, for the given set of observation runs, the expected rate of gamma-like background events at a given position in the sky. Zenith angle effects are taken into account by applying this procedure in predefined zenith angle bands. Finally, once the acceptance map in the detector’s frame is computed, the acceptance at a given sky position and observation time can be determined. It is done by retrieving the acceptance value (in the detector’s frame) at the position corresponding to that of the requested sky position and time, and by scaling it by the gamma-like event rate of the corresponding observation run. The instrument response in this case is then the expected instantaneous event rate from gamma-like events. It is used in the following parts to correct the observed time intervals for the acceptance variations.

Even though it depends on the observation setup, observation conditions and analysis performances, typically a few (∼3–5) hours of observation is needed to be able to correctly estimate the acceptance from the data. In particular, it is technically necessary to observe a test position with different wobble offset to avoid the exclusion regions to always stand at the same position in the detector’s frame. In case this method cannot be used (for instance for shorter observation times such as a single observation run), it is possible to assume a radially symmetric acceptance and estimate it using tables (also binned in zenith angle) built from dedicated observations. This should lead to a slightly less accurate estimate of the acceptance but for both cases, the acceptance can be determined at the level of a few percent which is sufficient when dealing with the level of fluxes that can be probed (see Sect. 4).

2.2. Acceptance correction

Once the acceptance has been estimated, the time intervals can be corrected for the instrumental induced variations. The gaps between observation runs are taken into account by only considering the time intervals between detected events in the same observation run. The acceptance-corrected time intervals \( \Delta t \) can be computed from the observed time intervals \( \Delta t \) and the acceptance \( a(t) \) with the formula: \( \Delta t = a(t) \Delta t \). For a given set of \( N \) recorded times \( \{t_i\}_{i=1,..N} \), the corrected time intervals can be expressed as follows (taking the approximated acceptance value at the middle of the time interval):

\[
\Delta t_i = t_{i+1} - t_i = \frac{a(t_i) + a(t_{i+1})}{2} \Delta t_i
\]

where \( \Delta t_i = t_{i+1} - t_i \).

The acceptance-corrected time intervals can then be computed using the integrated acceptance. If we note \( \bar{a}_i \) the integral acceptance computed using the integrated acceptance.
of the acceptance from the beginning of the observation to the
time \( t_i \), \( \tilde{a}_i = \int_0^{t_i} a(t) \, dt \), then :

\[
\tilde{a}_{i+1} = \int_0^{t_{i+1}} a(t) \, dt - \int_0^{t_i} a(t) \, dt \\
= \int_0^{t_i} a(t) \, dt + \int_{t_i}^{t_{i+1}} a(t) \, dt \\
\approx \tilde{a}_i + \frac{a(t_i) + a(t_{i+1})}{2} \Delta t_i \\
\approx \tilde{a}_i + \Delta \tau_i
\]

then : \( \Delta \tau_i = \tilde{a}_{i+1} - \tilde{a}_i \).

The obtained set of acceptance-corrected time intervals \( \{\Delta \tau_i\}_{i=1..N-1} \) can be normalized to have a mean value of 1 in the following way : if \( N \) is the total number of events, then \( \int a(t) \, dt = \int \Delta \tau = \beta N \), which, for discrete times is : \( \sum \Delta \tau = \beta N \).

The normalization factor \( \beta \) is thus calculated by comparing the sum of the corrected time intervals and the number of events. The normalized acceptance-corrected time intervals are then : \( \tilde{\Delta} \tau = \frac{1}{\beta} \cdot \Delta \tau \). The expected corrected time intervals distribution is then an exponential with an expected slope of 1. This has proven to be helpful to check whether the procedure is working correctly.

Once this step is performed, the normalized acceptance-corrected time series can be constructed from the sum of the normalized acceptance-corrected time intervals. The time span between two observations is taken care of by suppressing the gap between the last event of an observation and the first event of the next one. The total time series is therefore built from the time intervals between consecutive events, purposely removing the intervals between events from different observations. The acceptance being estimated and corrected for each observation, the presence of gaps and their size has no influence on the final time series. In the case of a steady source or background, the time series constructed following this procedure follows a Poisson process with a steady event rate of 1, no matter the observation conditions or observation setup. Statistical tests can be applied to this time series to search for transient events. In addition, the acceptance determination being possible on the whole field-of-view, this allows to perform blind searches for transient events at any observed position on the sky.

### 2.3. Event selection and Map generation

The arrival date of the recorded events are stored during the analysis together with the information on the direction in the sky. Given these information and applying the acceptance correction procedure described above, a list of arrival times is built for a direction in the sky within a given integration radius. With IACT, observations are usually separated in observation runs of typically 30 minutes. For a given source or a given region in the sky, consecutive observation runs can be separated by a time ranging from a few minutes to a few years - emphasizing the need for an acceptance estimation and correction procedure. By construction, the set of acceptance-corrected arrival times has the following properties : the mean time interval between two events is equal to 1 and the arrival time of the last event is equal to the number of events in the set.

For map generation, a set of acceptance-corrected arrival times is determined at each sky position of a pre-defined grid (defining pixels in the map). In the applications described below, we applied an integration radius which is the same as the \( \theta^2 \)-cut optimizing the detection of point-like sources (see de Naurois and Rolland (2009) for more details on this aspect). In order to avoid statistical bias and as mentioned in the next section, we also apply the condition that at least 20 events are in the sample at each sky position.

### 3. Variability tests description

Several tests for small time scales flux variations are available in the literature. In this article, we describe and use three tests, original or adapted from the literature, which apply to the acceptance-corrected event times if not specified otherwise. As the tests could also apply to the raw measured times in case of a constant acceptance, they are described in the following sections using the generic notation \( T \) for the times.

#### 3.1. An adaptation of the Exp-Test

The Running Exp-Test is built upon the Exp-Test described in Prahl (1999). For the convenience of the reader, this test is reminded here and the Running Exp-Test is then described.

**Exp-Test**

For event arrival times \( \xi_i \), following Poisson statistics, there is a constant \( C \) such that for any \( \Delta \xi > 0 \) dividing the total time interval \( \Xi \) in evenly spaced intervals \( \Delta \xi \), the number of events per interval follow a Poisson law with \( \lambda = \Delta \xi / C \) (the Poisson distribution is \( P_\lambda(n) = e^{-\lambda} \frac{\lambda^n}{n!} \), where \( \lambda \) is the expectation value). The probability density function of time intervals \( \Delta \xi \) is then a decreasing exponential:

\[
f_C(\Delta \xi) = \frac{1}{C} \cdot \exp \left( -\frac{\Delta \xi}{C} \right) \quad (2)
\]

For events observed at times \( (T_i)_{i=1..N+1} \), the distribution of time intervals between two consecutive events is:

\[
(\Delta T_i)_{i=1..N} := \left( (T_{i+1} - T_i) \right)_{i=1..N} \quad (3)
\]

with a mean value \( \Delta T = C^* \). If the constant \( C \) is fixed at the observed value, the \( \Delta T_i \) then follow the distribution \( f_C(\Delta \xi) \). A test can then be applied on the observed \( \Delta T_i \) to determine whether they follow the distribution \( f_C(\Delta \xi) \). The estimator built by Prahl (1999) is the following:

\[
M(F) := \int_0^{C^*} \left( 1 - \frac{\Delta \xi}{C} \right) F(\Delta \xi) \, d\Delta \xi \quad (4)
\]

where \( C^* = \int \Delta \xi F(\Delta \xi) \, d\Delta \xi \) and \( F(\Delta \xi) \), which correspond to the fraction of intervals equals to \( \Delta \xi \) is defined as:

\[
F(\Delta \xi) := \frac{1}{N} \sum_{i=1}^{N} \delta(\Delta \xi - \Delta T_i) \quad (5)
\]
The estimator can be expressed in the following way:

\[ M = \frac{1}{N} \sum_{\Delta t < C^*} \left( 1 - \frac{\Delta T_i}{C^*} \right) \]  

(6)

It has the property to be equal to \(1/e\) for \(F = f_C\), which is the expected value for a Poisson behaviour. The estimator will be greater than the expected value for a burst-like behaviour (excess of small \(\Delta T_i\)) and smaller than the expected value for a periodic behaviour.

This estimator can be normalized to follow a normal distribution:

\[ M_e = \frac{M - (1/e - \alpha/N)}{\beta/\sqrt{N}} \]  

(7)

where \(N\) is the number of events and the values of \(\alpha = 0.189 \pm 0.004\) and \(\beta = 0.2427 \pm 0.0002\) were adjusted in Prahl (1999) by the means of simulations, assuming a steady Poisson process. In the same publication, it is noted that at least 20 events should be in the sample in order to avoid statistical bias. It is therefore possible to express the result of the test in terms of number of standard deviations or significance.

**Running Exp-Test**

For a transient event of fixed duration, it is expected that the detection efficiency of the Exp-Test is reduced if the total observation time increases. The fixed number of “smaller-than-expected” time intervals corresponding to the flare may be significant in a small dataset while, as the dataset increases, it would be hidden in the global distribution of time intervals expected from the steady Poisson process.

The Running Exp-Test aims at resolving this issue by applying the same estimator as for the Exp-Test to subsets of the total number of events. In practice, the number of events on which the test is performed is fixed to a value chosen by the user which can be physically motivated (the number of events corresponds to a time scale that may be expected from a given physical process). The Exp-Test is performed on a sliding window of the chosen size (in number of events), the starting event of each test window being increased by one at each step. For each window, the test is performed with the mean value of the time interval \((C^*)\) of the whole dataset. The result of the Running Exp-Test is then a set of significances and we choose to use the maximum value of this set as the output of the test.

As the test windows overlap, they are not independent and trial factors have to be corrected for. This is done with simulations: for a given dataset, the same number of events is simulated assuming a pure Poisson behaviour and the Running Exp-Test is applied. The simulation is performed a large number of times, giving a distribution of expected values for a pure Poisson process. From this distribution, the significance of the Running Exp-Test obtained on the real dataset can be corrected (using the cumulative distribution for instance).

The advantages of this test is that its efficiency is much less affected by the size of the dataset and can be adapted to the physical process one aims to detect. On the other hand, a window size has to be chosen by the user which will have an impact on the results of the test. Due to the acceptance variations and the Poisson process itself, this window size does not correspond to a constant time-scale probed.

### 3.2. ON-OFF Test

The ON-OFF test for time series was developed in order to probe variability on a given time scale without it being affected by the acceptance variations. With this test, the probed time scale is the real physical one and it is not affected by the acceptance correction procedure. This method – similar to the method used at high energies by the Fermi-LAT (Ackermann et al. 2013) – is the analogous in the time domain to the standard ON-OFF method used to compute excess and significance maps (Berge et al. 2007) in very high energy gamma-ray astronomy.

The set of events arrival times is binned with a binning size being the probed time scale (the first event defines the starting time of the first bin). The times considered here are the real ones and not the acceptance-corrected ones. The number of events in each time bin is compared to the number of events in all the others bins as shown in Figure 2. The number of events in a bin is composed of background events, possibly gamma-rays if we are looking at a steady source, and additional gamma-rays if an increase in the source activity occurs or if a transient source appears. The “steady” number of events in a considered bin is estimated using all the other bins. The number of gamma-rays, also called excess of gamma-rays, is therefore computed in a given bin (the ON region) with respect to all the other bins (the OFF region). The significance of this excess can then be computed from eq. 17 of the Li&Ma publication (Li and Ma 1983). The OFF region should not contain bins in which a significant excess is measured. The procedure is thus iterative and bins containing significant signal (above 5 \(\sigma\)) are not used for background estimation. As prescribed by Li and Ma (1983), there should be at least 10 events in the ON and OFF regions. In order to declare a detection significant, one can follow the rather conservative prescription from Bernl¨ohr et al. (2013) : the significiance from eq. 17 of Li and Ma (1983), the number of excess events and the number of excess over background events should be above 5, 10 and 0.05, respectively. The latter value being mainly guided by the level of control of the systematics. The corresponding minimal timescale that can be probed depends on the instrument and the overall performances of the analysis. In the application described in this publication, the minimal timescale is ~ 2 minutes.

For \(N\) time bins, the excess of gamma-rays in a given time bin is \(N_{\text{on},i} = N_{\text{ON},i} - \alpha_i \times N_{\text{OFF},i}\), where:

- \(N_{\text{ON},i}\) is the number of events in the \(i^{th}\) bin,
- \(N_{\text{OFF},i}\) is the total number of events outside of the \(i^{th}\) bin (and from bins which have not been recognized as containing signal),
- \(\alpha_i\) is the ratio of the total acceptance in the ON and in the OFF bins.
Again, we consider the maximum significance value over all the bins as the output of the test. Since a number of bins is tested for an excess, trial factors should be taken into account. The final, post-trials significance value is the pre-trials significance of the excess (determined after the iterative procedure) and corrected by the number of time bins tested. Since the significance follows a normal distribution in the absence of signal, the pre-trials significance can be converted into a probability, corrected for trial factors (using $P_{\text{post}} = 1 - (1 - P_{\text{pre}})^N$) and converted back in a post-trials significance value.

An additional step can be achieved with this test: knowing the observation conditions in each time bin, the number of expected events can be computed from Monte-Carlo simulations. Under the assumption of a spectral shape for a putative gamma-ray emitter, this number can be compared to the observed excess in this bin and a value (or an upper limit) for the flux contained in this bin can be derived. This value (or upper-limit) can be interpreted as the additional flux due to the transient process for a steady source or as the flux of a source emitting in the considered energy range only during the outburst.

Even though a time-scale has to be chosen by the user, this test has the advantage of allowing for a real time-scale to be probed (as opposed to an acceptance corrected time scale for the Running Exp-Test). It relies on a robust method and can easily be used to the estimation of physical parameters (flux or upper-limit on the flux).

3.3. Cumulative Sum test

The Cumulative Sum test is originally a method to detect and monitor change points, often used in the industry for quality control. It was first developed in Page (1954). The version proposed here is based on the fact that the statistics of the time interval between two events is known for a Poisson behaviour. The principle is to compute the cumulative sum of the time intervals, retrieving at each step the global mean value in order to get a variable with a null mean value.

For a set of time intervals, ordered in time $\{T_i\}_{i=1,N}$ and $\{T_{i+1} - T_i\}_{i=1,N}$, one can compute the cumulative sum variable, which is for the time $i$:

$$\chi_i = \sum_{k=1}^{i} (\Delta T_k - \langle \Delta T \rangle)$$

where $\langle \Delta T \rangle = 1/N \sum_{i=1}^{N} \Delta T_i = C$. This estimator has a null mean as for each $i$:

$$\langle \chi_i \rangle = \sum_{k=1}^{i} (\langle \Delta T_k \rangle - \frac{1}{N} \sum_{j=1}^{N} \langle \Delta T_j \rangle) = 0$$

If the events follow a Poisson behaviour, then :

$$\langle \Delta T^2 \rangle = \int \Delta T^2 f_C(\Delta T) \, d\Delta T = 2\sigma^2$$

The variance of the $\chi_i$ variable is then:

$$\text{Var}(\chi_i) = \langle \chi_i^2 \rangle - \langle \chi_i \rangle^2 = \langle \chi_i^2 \rangle$$

$$= \left( \frac{i}{N} \sum_{k=1}^{i} \Delta T_k - \frac{i}{N} \sum_{j=1}^{N} \Delta T_j \right)^2$$

$$= \frac{iC^2}{N}(N - i)$$

The complete calculation can be found in the Appendix A. It is thus possible to determine at each step if the variable is close to the distribution expected for a steady source and compute the number of standard deviations from it. Since the estimator is bound to be null for the first and the last event, a reasonable minimum number of events should be used to avoid statistical bias. As such, the prescription from Prahl (1999) for the Exp-Test to have at least 20 events in the sample appears to be a safe choice in this case as well.

The advantage of this test is its fast computing time and the fact that it does not rely on any assumption regarding the time-scale to probe.

4. Performance of the tests

In order to test and compare the performances of the methods described in the section 3, a simple simulation is adopted. It is based on the knowledge of two quantities for a given observation run: the background rate and the expected rate of gamma rays coming from a source with a given spectrum.

4.1. Simulation procedure

From a real observation run, obtained with H.E.S.S. II in combined mode (i.e. using either monoscopic events from CT5 only or events seen with at least two telescopes, Holler et al. 2015 with the Loose set of cuts, we derived the average rate of
Figure 3: Exp-Test, Cumulative Sum, Running Exp-Test and ON-OFF Test performances for a run duration of 28 minutes and an off-axis angle of 0.5°. Simulations are performed as described in the text for burst durations between 0.5 and 5.5 minutes and flux scaling factor between 0.1 and 0.8. The map represents the significance of the tests, the white lines correspond to the position of the $5\sigma$ boundary (the dotted line corresponding to the $1\sigma$ contours) and the red lines correspond to the $5\sigma$ significance boundary of the standard Li&Ma test. For the Running Exp-Test, the number of events in the window is set to 20 (map and blue lines) and 50 (white lines). For the ON-OFF Test, the probed time-scales are 2 minutes (map and blue lines) and 5 minutes (white lines).

Figure 4: Comparison of the Cumulative Sum Test, the Running Exp-Test and the ON-OFF Test for a run duration of 28 minutes and an off-axis angle of 0.5°. Similarly to Fig. 3, simulations are performed for burst durations between 0.5 and 5.5 minutes and flux scaling factor between 0.1 and 0.8. The color maps represent the Li&Ma significance with the $5\sigma$ boundary line in black dashed. The $5\sigma$ boundary lines (with their confidence contours) are represented in white for the ON-OFF Test (with probed time-scale of 2 minutes (left) and 5 minutes (right)), in green for the Cumulative sum test and in blue for the Running Exp-Test (with a running window of 20 events (left) and 50 events (right)).
gamma-like background events (see Sect. 2) as a function of the angular distance to the observation position (the off-axis angle). The derived rate depends on the observation conditions and the telescope performances. In our case, the observation run was taken towards an empty extragalactic field with a mean zenith angle of $\sim 10^\circ$. We also assume that the acceptance is constant during the run. This is the case when applying the method described in section 2 and this is in any case a valid assumption if the observation is not too long and if the observation conditions are stable, which is supposed to be the case here. If a systematic shift in the determination of the acceptance occurs, this would be equivalent to a change in the event rate over the whole observation. By construction this would have no effect for the detection power of the Exp-Test, the Running Exp-Test or the Cumulative Sum test. Only the ON-OFF Test would have its sensitivity modified at the same level as the amplitude of the shift (which is not expected to happen at a level of more than a few percent). If the acceptance is incorrectly estimated and if this error in addition varies with time (for instance due to zenith angle gradient not properly taken into account), all the tests could be affected. However, this would still be at the level of a few percent and would only have a moderate effect on their detection efficiency.

From the instrument response function, and for a given assumed spectrum, we can also derive the rate of events expected from a source of gamma-rays at a given off-axis angle in this observation run. In the following, we use a source with a power-law spectrum $\frac{dN}{dE} = \phi(E_0) \times \left(\frac{E}{E_0}\right)^\Gamma$ with the parameters of the Crab nebula as measured in Aharonian et al. (2006) ($\phi(1 \text{ TeV}) = 3.45 \times 10^{-11} \text{ cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$ and $\Gamma = -2.63$). Since the event rate scales linearly with the flux, we compute this rate once and only scale the value in order to test for different normalizations.

With the knowledge of the background rate and the gamma rate (of a source with an assumed spectrum), we perform the simulations in the following way:

- We chose the duration of the observation run to 28 minutes (standard duration in H.E.S.S.) and an off-axis angle value. From the average background rate determined above, and under the assumption of a Poisson behaviour, we generate random time intervals between consecutive events and build the time series of gamma-like events.

- In addition to the gamma-like background, we assume that a burst of gamma rays from a transient source is observed, the lightcurve of which is a step-function. The source is assumed to be active during a given amount of time and quiet during the rest of the run. We chose the duration and the flux level of the burst and, as for the gamma like events, we determine the time series of gamma ray events from the transient source using the expected rate of gamma-rays.

- The two time series are then merged and the tests are applied to the global dataset.

Apart from observation run and analysis configuration, the parameters for the simulations are the off-axis angle, the run duration, the duration of the burst and its level of flux. For the tests, only the Running Exp-Test and the ON-OFF Test have a parameter that needs to be chosen (the number of events in the running window and the time-scale to probe for the Running Exp-Test and the ON-OFF Test respectively).

When simulating the gamma-like events, we simultaneously perform 10 different realisations of the simulation. These additional simulations serve as a background estimate to derive the significance of the source as would be done for a standard analysis using the reflected background method (Berge et al. 2007).

4.2. Results

In order to perform the tests and compare them, we set the run duration to 28 minutes and the off-axis angle to $0.5^\circ$. We then perform simulations as described above for different pairs of burst durations and flux scaling factor values. The tested values range from 0.5 to 5.5 minutes for the burst duration and from 0.1 to 0.8 for the flux scaling factor. The number of events in the running window for the Running Exp-Test is set to 20 and 50 and the time-scales probed by the ON-OFF Test are 2 and 5 minutes.

For each pair of values, we perform 1000 different realisations of the simulation. We obtain, for each test, a distribution of 1000 significance values, of which we derive the mean and the RMS. It is expected that at short (long) burst durations and for low (high) flux scaling factors, the significance of the test is low (high). The results obtained from the simulation presented here are compatible with those obtained using the Injector module which was developed and implemented in the H.E.S.S. analysis software to test the transient analysis tools performances with any kind of spectra and lightcurves (Bernhard et al. 2017).

In figure 3 the mean significance is represented as a function of the burst duration and the flux scaling factor. The white lines in figure 3 represent the $5\sigma$ contour and the white dashed lines represent $5\sigma$ contours when taking into account the RMS of the significance distribution. Similarly, the red lines show the $5\sigma$ contours of the significance obtained with eq. 17 of the Li&Ma publication (Li and Ma 1983). On the bottom maps of figure 3 the blue lines represent the $5\sigma$ contour of the Running Exp-Test (left) and the ON-OFF Test for different values of probed time-scales (see caption for details).

In figure 4 the $5\sigma$ contours of the Cumulative Sum Test, the Running Exp-Test and the ON-OFF Test are represented on the same figure. The Running Exp-Test appears to be the most sensitive test but, like the ON-OFF Test, its result depends on an a-priori choice of time scale to be probed. The Cumulative Sum Test gives results similar to the other two tests but without any assumption on the time-scale. In any case, all of the tests...
presented in this publication perform significantly better than the standard method used for source detection for time scales shorter than approximately 8 minutes.

5. Conclusion

New methods were developed to search for transient events in VHE gamma-ray data from IACTs. The methods described in this paper allow to estimate the instrument response at any time and position of the observations. This in turn allows to apply statistical tests on the acceptance-corrected time series. The tests can be applied at any observed position, leading to the possibility to perform blind search of transient sources over the whole observed field-of-view.

Several tests were presented and, when applied to short time scale signals, they all perform significantly better than the standard LiMa test. The better sensitivity to transient events achieved with these methods show a clear interest to use them for weak transient sources searches. In addition, these methods allow to search for transient emissions such as prompt emissions from GRBs in archival data which could not be revealed by standard analysis methods.

The statistical tests described in this publication are already used for the analysis of H.E.S.S. data and will be of prime interest for CTA. They will be implemented in the analysis softwares of CTA: ctools [Knödlseder et al. 2016] and gammapy (Deil et al. 2017). The performance of the methods described in this publication applied to CTA analysis and for more realistic cases will be estimated in a forthcoming publication.

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Appendix A. Cumulative Sum variance derivation

The variance of the Cumulative Sum estimator described in section 3 is given by:

\[ \sigma^2 = \frac{2}{n} \]
\[ \text{Var}(\chi_i) = \langle \chi_i^2 \rangle - \langle \chi_i \rangle^2 = \langle \chi_i^2 \rangle \]
\[ = \left( \left( \sum_{k=1}^{i} \Delta T_k - \frac{i}{N} \sum_{j=1}^{N} \Delta T_j \right) \right)^2 \]
\[ = \left( \sum_{k=1}^{i} \Delta T_k \sum_{j=1}^{i} \Delta T_j - \frac{2i}{N} \left( \sum_{k=1}^{i} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) \right) \]
\[ + \frac{i^2}{N^2} \left( \sum_{k=1}^{N} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) \]

which gives:
\[ \text{Var}(\chi_i) = \left( \sum_{k=1}^{i} \Delta T_k \sum_{j=1}^{i} \Delta T_j \right) - \frac{2i}{N} \left( \sum_{k=1}^{i} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) \]
\[ + \frac{i^2}{N^2} \left( \sum_{k=1}^{N} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) \]

The three terms in this sum are:
\[ \left( \sum_{k=1}^{i} \Delta T_k \sum_{j=1}^{i} \Delta T_j \right) = \sum_{k=1}^{i} \sum_{j=1}^{i} \langle \Delta T_k \Delta T_j \rangle \]
\[ = \sum_{k=1}^{i} \sum_{j=1}^{i} \langle \Delta T_k \rangle \langle \Delta T_j \rangle + \sum_{k=1}^{i} \langle \Delta T_k^2 \rangle \]
\[ = i(i-1)C^2 + 2iC^2 \]

\[ \left( \sum_{k=1}^{N} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) = N(N-1)C^2 + 2NC^2 \]

\[ \left( \sum_{k=1}^{j} \Delta T_k \sum_{j=1}^{N} \Delta T_j \right) = \sum_{k=1}^{i} \sum_{j=1}^{i} \langle \Delta T_k \Delta T_j \rangle + \sum_{k=1}^{i} \sum_{j=i+1}^{N} \langle \Delta T_k \Delta T_j \rangle \]
\[ = i(i-1)C^2 + 2iC^2 + i(N-i)C^2 \]

Putting everything together gives:
\[ \text{Var}(\chi_i) = \frac{iC^2}{N} (N - i) \]