An enhancement in thermal performance of partially ionized fluid due to hybrid nanostructures exposed to magnetic field

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ABSTRACT
This article considers ethylene glycol as a partially ionized base fluid whose rheological characteristics can be exhibited by Carreau stress-strain relations. This dispersion of nanoparticles (MoS$_2$) and hybrid nanoparticles (a combination of (MoS$_2$ and SiO$_2$)) in ethylene-glycol is considered and thermal performance of MoS$_2$-Carreau nanofluid and hybrid nanofluid (MoS$_2$-SiO$_2$-ethylene glycol) are investigated numerically using FEM. The results are validated. The present theoretical analysis has shown that thermal performance of working fluid can be enhanced by the use of hybrid nano fluid rather than nano fluid. Unfortunately, shear stress on elastic surface exerted by hybrid nanofluid is greater than the shear stress exerted by nanofluid. Although the thermal performance of hybrid nano fluid is greater than the thermal performance of nanofluid but one must be cautious about strength of surface as it can afford sufficient stress otherwise thermal system may experience failure. Failure analysis prediction while using hybrid nanonfluid must be in mind. As ethylene glycol is partially ionized and its interaction with applied magnetic field induces Hall and ion slip currents. Due to Hall and ion slip currents, ethylene glycol experiences Hall and ion slip forces which are opposite to the Lorentz force of applied magnetic field. This Lorentz force is reduced Hall and ion slip forces. Consequently, the flow of ethylene glycol is accelerated when Hall and ion slip parameters are increased.

I. INTRODUCTION
Transportation of heat energy is a key process in industrial manufacturing. The efficiency of a thermal cooling system depends upon the speed of transportation amount of heat transported. An enhancement of heat or cooling in natural or processes affect the money factors like energy storage, reduction of process time, etc. Therefore, an enhancement in the transportation of heat has become a major concern has thermal performance is the main focus. There are several techniques/methods of transportation in the process of heat. These methods include the use of the extended surface, cooling fins, metallic and non-metallic coating, displacement insertion, wireless of flow, vibration, injection suction, jet impingement, the use corrugated tubes and cooling tower packings, etc. Although above-mentioned methods for good for the enhancement of transportation of heat but the most recent and extensively used technique is the inclusion of nano-sized metallic structures in the base fluid. An enhancement in technology and synthesis of nanofluid at the industrial level has increased the imported of the nanofluid. In view of the applications of nanofluid several practical and experimental studies have been published. For example, Sheikholeslami et al. considered Lorentz force and porous medium resistance to developed mathematical theory for an enhancement in the thermal performances due to aluminium particles and computed the solutions of modelled problems using Lattice Boltzmann method. Zhixiong et al. numerically analysed the role of Copper oxide on the thermal performance of water in a duct Lattice Boltzmann method. Sadiq et al. studied the combined effects of nano-sized metallic structures and free stream velocity. Saleem et al. developed mathematical modelling to incorporate the simultaneous effects of Brownian motion and thermophoresis on transport of heat energy and concentration and solved the resulting models to examine the impact of Walter B parameter, rotation of cone and unsteadiness of the transportation mechanism. Ramzan et al. discussed an enhancement in thermal
performance of couple stress fluid containing nano-structures under the mass flux conditions. Saleem et al. performed optimized analysis for parameters affecting dissipation phenomenon in nanofluid. Dogonchi et al. investigated the impact of nanoparticles on the transport of heat energy in natural convection in cavity subjected to elliptical heater. They also studied the role of shape of nano-structures on the thermal performance of nanofluid. Dogonchi et al. considered combined role of nano-structures, thermal radiations, resistance of porous medium, magnetic field during the transport of heat energy and solved the problems by FEM. Seyyedi et al. performed numerical simulations in order to capture the role of nano-structures on the entropy generation in the presence of magnetic field. Dogonchi et al. discussed the role of FEM.

For instance Chamkha et al. used proposed models to examine the impact of hybrid nanoparticle content on the thermal performance of working fluid. Afridi et al. performed numerical simulations in order to capture the role of hybrid nanofluid to model the transport of heat in fluid enclosed in circular cavity and solved problems by FEM.

Partially ionized fluid exposed to the magnetic field exhibit totally different to characteristics than the characteristics of a natural fluid. Due to these different dynamics of partially ionized nanofluid, several investigators have heat and mass transport in partially ionized fluids.

To the best of author’s knowledge, no study on thermal performance partially ionized Carreau liquid flow by hybrid nanoparticles exposed to magnetic field investigated so far. The complex mathematical models are solved by finite element method (FEM) and an efficient method. This method has been successfully implemented to complex problems of computational fluid dynamics (CFD). Simulations are carried out by indigenously developed computer program on personal computer. Eventually, the remarkable observations are noted and discussed.

II. MODELLING AND DIMENSIONAL ANALYSIS

We considered MoS₂ and (MoS₂, SiO₂) as hybrid nano solid structures in ethylene glycol (Carreau liquid) over the horizontal sheet moving with velocity \( V_\infty = [a(x + y)^m, b(x + y)^n] \). A non-uniform temperature and magnetic field are considered as \( T_w = T_\infty + A_i T_0(y + x)^m, B_0(x + y)^n \). Furthermore, MoS₂ and (MoS₂, SiO₂) in ethylene glycol is determined to be plasma and non-Newtonian in form of shear thinning. The schematic representation is given by Fig. 1.

The boundary layer equations under consideration are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, 
\]

\[
\frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\nu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{hvf} \beta_i^{(x+y)}}{\rho_{hvf}(1 + \beta_i \beta_e)^2 + (\beta_e)^2} [v \beta_e - (1 + \beta_i \beta_e) u], 
\]

\[
\frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{\nu}{\rho} \frac{\partial^2 v}{\partial y^2} - \frac{\sigma_{hvf} \beta_i^{(x+y)}}{\rho_{hvf}(1 + \beta_i \beta_e)^2 + (\beta_e)^2} [u \beta_e + (1 + \beta_i \beta_e) v], 
\]

\[
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k_{hvf}}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_{hvf} \beta_i^{(x+y)}}{\rho_{hvf}(1 + \beta_i \beta_e)^2 + (\beta_e)^2} (u^2 + v^2) + \frac{\beta_{hvf}}{\rho C_p} \left[1 + \beta_i \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} \right] \left[ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} \right].
\]  

In above equations, \( [u, v, w], \Gamma^*, m, \beta_i, \beta_e, \beta_0 \) and \( hnf \) are the velocity, time constant, index number, Hall parameter, ion slip parameter, magnetic induction and hybrid nanofluid respectively. It is also mentioned that a Carreau liquid is Newtonian for case \( m = 1 \), shear
thinning for the case $0 < m < 1$ and shear thickening for the case of $m > 1$.

The boundary conditions are developed by taking the velocity of Carreau liquid equals to the velocity of solid and hence

\[ u = a(x+y)^m, \quad v = b(x+y)^m, \quad w = 0 \]
\[ T = T_w = T_\infty + A_1 T_0 (y+x)^{2m} \text{ at } z = 0 \]
\[ u = 0, \quad T \to T_\infty, \quad v = 0, \quad w \to \infty, \quad z \to \infty. \]

Here $nf$ stands the nanofluid and $hnf$ stands hybrid nanofluid.

The change of variables

\[
\begin{align*}
    u &= a(y+x)^m f', \
    v &= b(y+x)^m g', \
    \eta &= \sqrt{\frac{a}{\nu_f}} (y+x)^{2/3} z, \\
    w &= -\sqrt{\frac{a}{\nu_f}} (y+x)^{2/3} \left( \frac{n+1}{2} (f+g) + \frac{n-1}{2} \eta (f'+g') \right), \\
    T &= A_1 T_0 (y+x)^{2m} \theta(\eta) + T_\infty.
\end{align*}
\]

Converts Eqs. (2)–(6) into the dimensionless form which are

\[
\begin{align*}
    \left[ 1 + mW^2 (f'')^2 \right] f''' - \nu \eta f'' &= n (f'+g') f' - \frac{n+1}{2} (f+g) f'', \\
    f(0) &= f'(\infty) = 0, \quad f'(0) = 1, \\
    \left[ 1 + mW^2 (g'')^2 \right] g''' - \nu \eta g'' &= n (f'+g') g' - \frac{n+1}{2} (f+g) g'', \\
    g(0) &= g'(\infty) = 0, \quad g'(0) = \lambda, \\
    \beta f' - (1 + \beta \beta_i) g' &= 0, \\
    \beta g' - (1 + \beta \beta_i) f' &= 0, \\
    \theta(0) &= 1, \quad \theta(\infty) = 0.
\end{align*}
\]
in which $We = \left( \frac{\rho c_p v}{\sigma} \right)^{1/2}$ is the Weissenberg number, $Pr = \left( \frac{\nu}{\mu} \right)$ is the Prandtl number, $M = \left( \frac{\rho c_p}{\mu} \right)$ is the Hartmann number and $Ec = \left( \frac{\mu v}{\sigma} \right)$ is the Eckert number. The thermal properties $\rho$, $c_p$, $k$, $\sigma$, $\rho_f$, $k_{hf}$, $\sigma_{hf}$, $\rho_{hf}$, ($c_p)_f$, $k_{hf}$, $\sigma_{hf}$, $\mu$, $\nu$ are given in Table II.

The wall shear stresses are expressed as

$$C_s = \frac{\mu_{hf}(x+y)\phi}{\rho_f u_f} = \left[ \frac{1}{1 - \phi} \left( \frac{\sigma}{\sigma_f} \right) \frac{\mu}{\mu_{HF}} \right],$$

The wall heat transfer rate is given by

$$Nu = \frac{-k_{hf}(x+y)\phi}{(Re)^{1/2}k_f(T_w - T_\infty)} = \frac{\phi}{k_{hf}}(Re)^{1/2}g(0).$$

Where $Re = \left( \frac{\mu (x+y)}{\nu} \right)$ is the local Reynolds number. Further, the numerical values of thermophysical properties are given in Table I.

### III. SOLUTION STRATEGY

The computational procedure is briefly described as under.

**Step I:** Problem domain is converted into finite number of the elements and gets linear type polynomial solution over each element. The weighted residual function is implemented to transformed strong form of the flow problem into weak form. Shape functions are used in flow problem to compute the approximate solution.

**Step II:** Stiffness matrices, force vector and boundary integral vector are computed according to the shape functions over each element. Finally, the global stiffness matrix is calculated over a whole domain and Picards scheme is used to transform system of non-linear equations into linear equations.

**Step III:** The solution of system of linear equations is computed iteratively under computational tolerance $10^{-8}$.

**Step IV:** FEM code is developed in MAPLE. The mesh-free analysis is carried out for the computational domain [0, 1]. Several experiments are run for different grids and obtained numerical data is recorded in Table II. Table II predicts that computed solutions become mesh-free when domain is broken down into 500 elements.

### Validation of results

The results in special cases are compared with already published work. This comparison is given in Tables III and IV.

### TABLE I. The models for thermal properties

| Properties          | Ethylene glycol and MoS$_2$ | Ethylene glycol, MoS$_2$, and SiO$_2$ |
|---------------------|------------------------------|---------------------------------------|
| Density             | $\rho_{hf} = (1 - \phi)\rho_f + \phi_0\rho_s$ | $\rho_{hf} = [(1 - \phi_f)\rho_f + \phi_0\rho_s]$ |
| Heat capacity       | $(\rho c_p)_{hf} = (1 - \phi)\rho c_p + \phi (\rho c_p)_f$ | $(\rho c_p)_{hf} = [(1 - \phi_f)\rho c_p + \phi (\rho c_p)_f]$ |
| Viscosity           | $\mu_{hf} = \frac{\mu}{(1 - \phi)^2}$ | $\mu_{hf} = \frac{\mu}{(1 - \phi_f)^2}$ |
| Thermal conductivity| $k_{hf} = \left( \frac{k_{hf}}{k} \right)$ | $k_{hf} = \left( \frac{k_{hf}}{k} \right)$ |
| Electrical conductivity | $\sigma_{hf} = \left( \frac{\sigma}{\sigma_f} \right)$ | $\sigma_{hf} = \left( \frac{\sigma}{\sigma_f} \right)$ |

**TABLE II. Mesh-free study for We = 3.0, n = 1.5, $\beta_s = 0.3$, $\beta_f = 0.7$, Pr = 3, $\phi_1 = 0.0075$, M = 0.5, $m = 0.5$, $Ec = 0.7$, $\phi_s = 0.002$, $x = 2.3$ for different grids.**

| Number of elements | $f'(\frac{\alpha_{hf}-\alpha_0}{\alpha_0})$ | $g'(\frac{\alpha_{hf}-\alpha_0}{\alpha_0})$ | $\theta(\frac{\alpha_{hf}-\alpha_0}{\alpha_0})$ |
|--------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 30                 | 0.3541580568                             | 0.1294932517                             | 0.4573329735                             |
| 60                 | 0.3231914922                             | 0.1066804743                             | 0.4425279120                             |
| 90                 | 0.3133274105                             | 0.09974095029                             | 0.437696256                              |
| 120                | 0.3084829185                             | 0.0963920167                             | 0.4352701510                             |
| 150                | 0.3056042211                             | 0.09442067580                             | 0.4338289136                             |
| 180                | 0.3036970247                             | 0.0931220679                             | 0.4328700147                             |
| 210                | 0.3023402245                             | 0.0922037313                             | 0.4321859117                             |
| 240                | 0.3013492123                             | 0.0922713751                             | 0.4328318109                             |
| 270                | 0.3005380199                             | 0.0909857951                             | 0.4312747792                             |
| 300                | 0.3005300954                             | 0.0909826397                             | 0.4312563333                             |

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The mathematical models governing the transport phenomenon in the partially ionized Carreau liquid fluid containing nanoparticles (MoS$_2$) and hybrid nanoparticles (MoS$_2$ and SiO$_2$) are solved numerically in order to investigate their thermal performances. Numerical runs have provided very useful information regarding impact of hybrid nanoparticles on thermal efficiency of working fluid mixture (mixture of Carreau fluid, MoS$_2$ and SiO$_2$). The extracted information is displayed graphically and in the form of numerical data.

### IV. GRAPHICAL AND NUMERICAL DATA AND ITS DESCRIPTION

The impact of Weissenberg number $We$ on the flow of nano and hybrid fluids is studied and obtained dynamics of flow is displayed by Figs. 2a and 2b. The solid curves are velocity curves for hybrid nanofluid whereas the dotted curves are velocity curves for nanofluid. Figs. 2a and 2b are lateral velocities under the influence of $We$. These Figs. predict that the flow of nano and hybrid nanofluids slows down when $We$ is increased. It is also evident that velocity of Newtonian nano and hybrid nanofluid are greater than the velocity of Carreau nano and hybrid nanofluids. The momentum boundary layer thickness can be shortened by the use of Carreau fluids. This fact can be read by Figs. 2a and 2b. The behavior of nano and hybrid nanofluid under the change of magnetic intensity is also investigated and obtained behavior is represented by Fig. 3a and 3b. These Figs. clearly predict that the flow in both $x$- and $y$- directions is slowed down when intensity of magnetic field is increased. This increase in the intensity of magnetic field also reduces the momentum boundary layer thickness see Figs. 3a and 3b. The behavior of Hall and ion

![Graphical representation](image-url)

**FIG. 2.** a, b. Impact of $We$ on the flow of hybrid nanofluid (MoS$_2$/SiO$_2$) for $M = 0.3$, $Ec = 0.7$, $\phi_1 = 0.0075$, $\phi_2 = 0.002$, $Pr = 2$, $m = 0.7$, $n = 0.3$, $\beta_e = 0.3$, $\beta_i = 0.3$. 

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**TABLE III.** Comparison of numerical results of $-f''(0)$ and $-g''(0)$ for special case studied by Khan et al. when $We = m = \phi_1 = \phi_2 = M = 0$, $\beta_e = 0.7$, $\beta_i = 0.3$, $Ec = 0.1$.

| $n$ | $\lambda$ | Present results $f''(0)$ | Khan et al. $^{36}$ | Present results $g''(0)$ | Khan et al. $^{36}$ |
|-----|-----------|--------------------------|-------------------|--------------------------|-------------------|
| 1   | 0         | 0.5                      | 1.223961          | 1.224745                 | 0.613010          | 0.612372          |
|     |           | 1                        | 1.411397          | 1.414214                 | 1.414261          | 1.414214          |
| 3   | 0         | 0.5                      | 1.623812          | 1.624356                 | 0                   | 0                 |
|     |           | 1                        | 1.989513          | 1.989422                 | 0.993127          | 0.994711          |
| 1   | 0         | 1                        | 2.297017          | 2.297186                 | 2.298731          | 2.297186          |

**TABLE IV.** Comparison of numerical values of $-\theta'(0)$ for special case studied by Khan et al. when $m = \phi_1 = \phi_2 = M = 0$, $\beta_e = 0.7$, $\beta_i = 0.3$, $Ec = 0.1$, $We = 3.5$, $Pr = 0.7$.

| $n$ | $\lambda$ | Khan et al. $^{36}$ | Present results $\theta'(0)$ |
|-----|-----------|---------------------|-----------------------------|
| 1   | 0         | 0.783668            | 0.788154                    |
|     | 0.5       | 0.962033            | 0.969012                    |
| 3   | 1         | 1.122406            | 1.122567                    |
|     | 0         | 1.292193            | 1.291989                    |
|     | 0.5       | 1.582607            | 1.589700                    |
| 1   | 1         | 1.826437            | 1.826838                    |
FIG. 3. a, b. Impact of $M$ on the flow of hybrid nanofluid ($\text{MoS}_2/\text{SiO}_2$) for $We = 3.5$, $Ec = 0.5$, $\phi_1 = 0.0075$, $\phi_2 = 0.002$, $Pr = 3.0$, $m = 0.7$, $n = 0.5$, $\beta_e = 0.7$, $\beta_i = 0.5$.

FIG. 4. a, b. Impact of $\beta_i$ on the flow of hybrid nanofluid ($\text{MoS}_2/\text{SiO}_2$) for $M = 3.7$, $Ec = 1.7$, $\phi_1 = 0.0075$, $\phi_2 = 0.002$, $Pr = 4.0$, $m = 0.5$, $n = 0.9$, $\beta_e = 0.3$, $We = 2.5$.

FIG. 5. a, b. Impact of $\beta_e$ on the flow of hybrid nanofluid ($\text{MoS}_2/\text{SiO}_2$) for $M = 0.5$, $Ec = 0.8$, $\phi_1 = 0.0075$, $\phi_2 = 0.002$, $Pr = 3.5$, $m = 0.7$, $n = 0.8$, $We = 3.0$, $\beta_i = 0.5$. 
slip currents on the velocity ($f', g'$) are studied and information is displayed in Figs. 4a–5b. One can easily study that the Hall and ion slip forces are favourable for flow in $x$-direction. It is also noted that enhancement in momentum boundary due to an increase in Hall and ions slip currents. However, this increasing impact of Hall and ion slip currents on nanofluid is more than the impact on hybrid nanofluid (see Figs. 4a, 4b, 5a and 5b).

B. Impact of parameters on temperature

The impact of Hall and ion slip parameters ($\beta_e, \beta_i$) on the temperature of fluid is also examined. Corresponding numerical data is represented by Figs. 6a and 6b. This graphical representation of data reveals that temperature decreases when Hall and ion slip parameters are increased. This decreasing trend is based on the fact that $\beta_e$ and $\beta_i$ appear in denominator of Joule heating terms and an increase in $\beta_e$ and $\beta_i$ results a decrease in the heat generated due to Ohmic dissipation phenomena. The thermal boundary layer thickness has also shown a decreasing behavior when $\beta_e$ and $\beta_i$ are increased. This implies that the use of partially ionized liquid in the presence of magnetic field helps in controlling the thermal boundary layer thickness. Thermal boundary layer thickness associated with natural liquid is greater them thermal boundary layer thickness in partially ionized liquid. The Eckert number ($Ec$) appears as coefficient of viscous dissipation term in the energy equation and increase in ($Ec$) corresponding to enhancement in the dissipation heat. Consequently, temperature of the fluid rises. This fact is shown by Fig. 7. Since $Ec$ appears in the viscous dissipation term of energy equation and an increase $Ec$ corresponds to an increase in rate of work done by the friction force. According to first law of thermodynamics, this work done is used to increase the internal energy of the fluid. Consequently temperature increases.

C. Wall shear stresses and wall heat flux

The role of Hall and ion slip parameters and power law index on the wall shear stress, wall heat and mass flux are studied for nano and hybrid nanofluids. Wall shear stresses have shown reduction when the parameter ($We$) is varied. However, an enhancement in wall heat flux is analysed (see Table V). Furthermore, wall shear stresses for Newtonian fluid are less than the wall shear stresses for Carreau fluid ($We \neq 0$). However, the case of Newtonian fluid is less than that for Carreau fluid. The above-noted observations are valid for both nano and hybrid nanofluid. It is also noted from that Table V. That wall shear stresses for nanofluid have low values when compared with those for hybrid nanofluid. Moreover, the wall heat flux for the case of hybrid nanofluid is greater than that for the case of nanofluid (see Table V). Table V predicts that the wall shear stresses and wall heat flux for nano and hybrid nanofluid increase when the hall parameter $\beta_e$ is enhanced whereas...
\( \beta_i \) has shown an increasing trend on the wall shear stresses and decreasing trend on wall shear stresses. The impact of power law index (\( m \)) on the wall shear stresses and wall heat flux is also given by Table V.

### Table V. Dynamics of wall shear stress and wall heat flux for \( \text{MoS}_2 \)-ethylene glycol \( \text{MoS}_2 \) and \( \text{SiO}_2 \)-ethylene glycol when \( M = 0.3, Ec = 1.5, \phi_1 = 0.0075, \phi_2 = 0.002, Pr = 2.7, m = 0.5, n = 0.3, \omega_0 = 1.5, \beta_e = 0.3, \beta_i = 0.6. \)

| \( We \) | Ethylene glycol (Nanofluid MoS\(_2\)) | Ethylene glycol (Hybrid nanofluid MoS\(_2\)/SiO\(_2\)) |
|---|---|---|
| - | \( (Re)^{-\frac{1}{2}} C_f \) | \( (Re)^{-\frac{1}{2}} C_f \) | \( (Re)^{-\frac{1}{2}} Nu \) |
| 1.0 | 0.75403829 | 0.87075248 | 2.0612875 |
| 1.5 | 0.27432544 | 0.51342104 | 2.1135903 |
| 2.5 | 0.81232621 | 0.91477483 | 2.0723568 |
| 0.3 | 0.89664574 | 0.95157594 | 2.0811255 |
| 0.5 | 0.88557883 | 0.99649784 | 2.0537989 |
| 1.5 | 0.86972423 | 0.9536129 | 2.0728395 |
| 0.9 | 0.8576361 | 0.91563929 | 2.0857695 |
| 0.5 | 0.86739374 | 0.88743661 | 2.0916335 |
| 1.0 | 0.8672949 | 0.88727664 | 2.0908764 |
| 1.5 | 0.86377551 | 0.88631506 | 2.0903349 |

#### V. CONCLUSION

The ethylene glycol is partially ionized and exhibits shear thinning/shear thickening behaviour. Therefore, thermal performance of nano-ethylene glycol and hybrid nano-ethylene glycol by considering ethylene glycol as a partially ionized liquid is investigated numerically. Several numerical experiments are conducted to analyse the behaviour of key parameters on the thermal performances of nano-ethylene glycol and hybrid nano-ethylene glycol. The key observations are listed below.

- The present theoretical analysis has shown that thermal performance of working fluid can be enhanced by the use of hybrid nano fluid rather than nano fluid
- Unfortunately, shear stress on elastic surface exerted by hybrid nano fluid is greater than the shear stress exerted by nano fluid. Although the thermal performance of hybrid nano fluid is greater than the thermal performance of nano fluid but one must be cautious about strength of surface as it can afford sufficient stress otherwise thermal system may experience failure. Failure analysis prediction while using hybrid nanofluid must be in mind.
- As ethylene glycol is partially ionized and its interaction with applied magnetic field induces Hall and ion slip currents. Due to Hall and ion slip currents, ethylene glycol experiences Hall and ion slip forces which are opposite to the Lorentz force of applied magnetic field. This Lorentz force is reduced Hall and ion slip forces. Consequently, the flow of ethylene glycol is accelerated when Hall and ion slip parameters are increased
- Joule heating and viscous dissipation have shown an increasing impact on the temperature because heat dissipated as a result of Ohmic and friction phenomenon adds to the fluid and its temperature rises

- It is observed that thermal performance of partially ionized hybrid nanofluid is greater than that of partially ionized nanofluid

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#### REFERENCES

1. M. Sheikholeslami, S. Saleem, A. Shafee, Z. Li, T. Hayat, A. Alsaedi, and M. I. Khan, “Mesoscopic investigation for alumina nanofluid heat transfer in permeable medium influenced by Lorentz forces,” Computer Methods in Applied Mechanics and Engineering 349, 839–858 (2019).
2. Z. Li, M. Sheikholeslami, A. S. Mittal, A. Shafee, and R. U. Haq, “Nanofluid heat transfer in a porous duct in the presence of Lorentz forces using the lattice Boltzmann method,” The European Physical Journal Plus 134(1), 30 (2019).
3. M. A. Sadiq, A. U. Khan, S. Saleem, and S. Nadeem, “Numerical simulation of oscillatory oblique stagnation point flow of a magneto micropolar nanofluid,” RSC Advances 9(9), 4751–4764 (2019).
4. S. Saleem, H. Firdous, S. Nadeem, and A. U. Khan, “Convective heat and mass transfer in magneto Walter’s B nanofluid flow induced by a rotating cone,” Arabian Journal for Science and Engineering 44(2), 1515–1523 (2019).
5. M. Ramzan, M. Sheikholeslami, M. Saeed, and J. D. Chung, “On the convective heat and zero nanoparticle mass flux conditions in the flow of 3D MHD couple stress nanofluid over an exponentially stretched surface,” Scientific Reports 9(1), 562 (2019).
6. S. Saleem, S. Nadeem, M. M. Rashidi, and C. S. Raju, “An optimal analysis of radiated nanomaterial flow with viscous dissipation and heat source,” Microsystem Technologies 25(2), 683–689 (2019).
7. A. S. Dogonchi, T. Armaghani, A. J. Chamkha, and D. D. Ganji, “Natural convection analysis in a cavity with an inclined elliptical heater subject to shape factor of nanoparticles and magnetic field,” Arabian Journal for Science and Engineering 44, 7919–7931 (2019).
8. A. S. Dogonchi, M. Waqas, S. M. Seyyedi, M. Hashemi-Tilehnoee, and D. D. Ganji, “Numerical simulation for thermal radiation and porous medium...
characteristics in flow of CuO–H₂O nanofluid,” Journal of the Brazilian Society of Mechanical Sciences and Engineering 41(6), 249 (2019).

8. M. Seyyedi, A. S. Dogonchi, R. Nuraei, D. D. Ganji, and M. Hashemi-Tilehnoee, “Numerical analysis of entropy generation of a nanofluid in a semi-annulus porous enclosure with different nanoparticle shapes in the presence of a magnetic field,” The European Physical Journal Plus 134(6), 268 (2019).

9. A. S. Dogonchi, T. Tayebi, A. J. Chamkha, and D. D. Ganji, “Natural convection analysis in a square enclosure with a wavy circular heater under magnetic field and nanoparticles,” Journal of Thermal Analysis and Calorimetry (2019).

10. Y. S. Daniel, Z. A. Aziz, Z. Ismail, A. Bahar, and F. Salah, “Stratified electromagnetohydrodynamic flow of nanofluid supporting convective role,” Korean Journal of Chemical Engineering 36(7), 1021–1032 (2019).

11. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Effects of thermal radiation, viscous and Joule heating on electrical MHD nanofluid with double stratification,” Chinese Journal of Physics 55(3), 630–651 (2017).

12. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Entropy analysis in electrical magnetohydrodynamic (MHD) flow of nanofluid with effects of thermal radiation, viscous dissipation, and chemical reaction,” Theoretical and Applied Mechanics Letters 7(4), 235–242 (2017).

13. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Double stratification effects on unsteady electrical MHD mixed convection flow of nanofluid with viscous dissipation and Joule heating,” Journal of Applied Research and Technology 15(5), 464–476 (2017).

14. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Thermal stratification effects on MHD radiative flow of nanofluid over nonlinear stretching sheet with variable thickness,” Journal of Computational Design and Engineering 5(2), 232–242 (2018).

15. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Effects of slip and convective conditions on MHD flow of nanofluid over a porous nonlinear stretching/shrinking sheet,” Australian Journal of Mechanical Engineering 16(3), 213–229 (2018).

16. Y. S. Daniel, Z. A. Aziz, Z. Ismail, and F. Salah, “Numerical study of Entropy analysis for electrical unsteady natural magnetohydrodynamic flow of nanofluid and heat transfer,” Chinese Journal of Physics 55(5), 1821–1848 (2017).

17. Y. S. Daniel, Z. A. Aziz, Z. Ismail, A. Bahar, and F. Salah, “Slip role for unsteady MHD mixed convection of nanofluid over stretching sheet with thermal radiation and electric field,” Indian Journal of Physics 1–3 (2019).

18. Y. S. Daniel, “MHD laminar flows and heat transfer adjacent to permeable stretching sheets with partial slip condition,” Journal of Advanced Mechanical Design and Systems 35(9-10), 2002–2018 (2007).

19. Y. S. Daniel, S. K. Daniel, “Effects of buoyancy and thermal radiation on MHD flow over a stretching porous sheet using homotopy analysis method,” Alexandria Engineering Journal 54(3), 705–712 (2015).

20. Y. S. Daniel, “Laminar convective boundary layer slip flow over a flat plate using homotopy analysis method,” Journal of The Institution of Engineers (India): Series F 97(2), 115–121 (2016).

21. A. J. Chamkha, A. S. Dogonchi, and D. D. Ganji, “Magneto-hydrodynamic flow and heat transfer of a hybrid nanofluid in a rotating system among two surfaces in the presence of thermal radiation and Joule heating,” AIP Advances (2019).

22. M. I. Afriy, M. Qasim, and S. Saleem, “Second law analysis of three dimensional dissipative flow of hybrid nanofluid,” Journal of Nanofluids 7(6), 1272–1280 (2018).

23. M. Sheikholeslami, S. A. Mehryan, A. Shafee, and M. A. Shermad, “Variable magnetic forces impact on magnetizable hybrid nanofluid heat transfer through a circular cavity,” Journal of Molecular Liquids 277, 388–396 (2019).

24. T. H. Qureshi, M. Nawaz, and A. Shahzad, “Numerical study of dispersion of nanoparticles in magnetohydrodynamic liquid with Hall and ion slip currents,” AIP Advances 9(2), 025219 (2019).

25. M. Nawaz, S. Rana, I. H. Qureshi, and T. Hayat, “Three-dimensional heat transfer in the mixture of nanoparticles and micropolar MHD plasma with Hall and ion slip effects,” AIP Advances 8(10), 105109 (2018).

26. M. Nawaz and T. Zubair, “Finite element study of three dimensional radiative nano-plasma flow subject to Hall and ion slip currents,” Results in Physics 7, 4111–4122 (2017).

27. Y. Hayat and M. Nawaz, “Hall and ion-slip effects on three-dimensional flow of a second grade fluid,” International Journal for Numerical Methods in Fluids 66(2), 183–193 (2011).

28. M. Nawaz, S. Rana, I. H. Qureshi, and T. Hayat, “Three-dimensional heat transfer in the mixture of nanoparticles and micropolar MHD plasma with Hall and ion slip effects,” AIP Advances 8(10), 105109 (2018).

29. M. Rafiq, H. Yasmin, T. Hayat, and F. Alsaadi, “Effect of Hall and ion-slip on the peristaltic transport of nanofluid: A biomedical application,” Chinese Journal of Physics 60, 208–227 (2019).

30. M. R. M. A. Apai, M. H. A. M. Noor, and B. Ahmad, “Hall current and Joule heating effects on peristaltic flow of viscous fluid in a rotating channel with convective boundary conditions,” Results in Physics 7, 2831–2836 (2017).

31. R. K. Tiwari and M. K. Das, “Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids,” International Journal of heat and Mass transfer 50(9–10), 2002–2018 (2007).

32. M. Aghamajid, M. Yaadi, S. Dinavand, and I. Pop, “Tiwari-Das nanofluid model for magnetohydrodynamics (MHD) natural-convective flow of a nanofluid adjacent to a spinning down-pointing vertical cone,” Propulsion and Power Research 7(1), 78–90 (2018).

33. H. Maurer and C. Kessler, “Identification and quantification of ethylene glycol and diethylene glycol in plasma using gas chromatography-mass spectrometry,” Archives of Toxicology 62(1), 66–69 (1988).

34. A. Mariano, M. J. Pastoriza-Gallego, L. Lugo, A. Camacho, S. Canzonieri, and M. M. Pinheiro, “Thermal conductivity, rheological behaviour and density of non-Newtonian ethylene glycol-based SnO₂ nanofluids,” Fluid Phase Equilibria 337, 119–124 (2013).

35. A. Khan, M. Mustafa, T. Hayat, and A. Alsaedi, “On three-dimensional flow and heat transfer over a non-linearly stretching sheet: Analytical and numerical solutions,” PloS one 9(9), e017287 (2014).