Abstract

We discuss the appearance of time-asymmetric behavior in physical processes in cosmology and in the dynamics of the Universe itself. We begin with an analysis of the nature and origin of irreversibility in well-known physical processes such as dispersion, diffusion, dissipation and mixing, and make the distinction between processes whose irreversibility arises from the stipulation of special initial conditions, and those arising from the system’s interaction with a coarse-grained environment. We then study the irreversibility associated with quantum fluctuations in cosmological processes like particle creation and the ‘birth of the Universe’. We suggest that the backreaction effect of such quantum processes can be understood as the manifestation of a fluctuation-dissipation relation relating fluctuations of quantum fields to dissipations in the dynamics of spacetime. For the same reason it is shown that dissipation is bound to appear in the dynamics of minisuperspace cosmologies. This provides a natural course for the emergence of a cosmological and thermodynamic arrow of time and suggests a meaningful definition of gravitational entropy. We conclude with a discussion on the criteria for the choice of coarse-grainings and the stability of persistent physical structures.
1 Introduction

In this talk I would like to discuss the nature and origin of irreversibility in
time, or, the ‘arrow of time’ in cosmology. This includes physical processes in
the Universe, as well as the dynamics of the Universe itself. I will use examples
from modern cosmological theories since the sixties: i.e. the ‘standard’
cosmology (Peebles, 1971; Weinberg, 1972); the chaotic (Bianchi) cosmology
(Misner, 1969; Ryan and Shepley, 1975), the inflationary cosmology (Guth,
1981; Albrecht and Steinhardt 1982; Linde 1982), the semiclassical cosmologies
(Hu, 1982; Parker, 1982; Hartle, 1983) and to a lesser extent, quantum cosmol-
gy (Wheeler, 1967; DeWitt, 1968; Misner, 1972; Hartle and Hawking, 1983;
Vilenkin, 1986; Halliwell 1993). (For a layman’s introduction to these theo-
ries, see, e.g., Hu, 1987.)

There are many ways irreversibility shows up in ordinary physical processes.
I shall in the first part of my talk present some well-known examples (such
as dispersion, diffusion, dissipation and phase mixing) and discuss the nature
and origin of irreversibility in them. Distinction between dissipative processes
(which are always irreversible) and irreversible – or ‘apparently’ irreversible
processes (which are not necessarily dissipative) is highlighted. I’ll then use
the insights gained here to discuss certain aspects of chaotic and inflationary
cosmology. In the second section I’ll discuss some not-so-well-known but im-
portant examples involving quantum field processes such as vacuum fluctuation
and particle creation and discuss the origin of time-asymmetry in them. This
touches on basic questions like the statistical nature of the vacuum, which un-
derlies novel processes like the Hawking and Unruh effects discovered in the
seventies (Bekenstein, 1973, 1974; Hawking, 1975; Davies, 1975; Unruh, 1976).
In the third and fourth sections I shall discuss how these quantum processes
influence the structure and dynamics of the early Universe. We show that a sta-
tistical mechanical interpretation of these so-called cosmological ‘backreaction’
processes is possible: they are manifestations of a fluctuation-dissipation rela-
tion involving quantum fields. In this semiclassical theory it is the fluctua-
tion of the quantum field which brings about dissipation in the spacetime dynamics.
With this understanding I shall suggest some ways to examine the notion of
gravitational entropy (Penrose, 1979)– from the entropy of gravitational fields
to that of spacetimes. As for quantum cosmology, where spacetime and matter
are both quantized, I only indicate how the basic ideas and methods in statistical
mechanics adopted above to discuss irreversibility in cosmological processes can
also be fruitfully applied to address issues in quantum cosmology (Hu, 1991a),
but I’ll shy away from extrapolations, because many concepts remain ill-defined
or ambiguous (See, e.g. Ashtekar and Stachel, 1991; Isham, 1991). On the
issue of the origin of time in quantum gravity, see, e.g., Kuchar, 1992. For a
discussion of time asymmetry in quantum cosmology, see the contributions of
Halliwell, Hartle, and Hawking in this volume.

In the conclusion I summarize the key observations. The emphasis of this
talk is to put many cosmological phenomena on the same footing as ordinary statistical processes and to try to understand their meaning in terms of basic concepts in theoretical physics. We reach the conclusion that time asymmetry in cosmology is attributable to the same origins as those observed in ordinary physical processes; i.e., they are determined by the way one stipulates the boundary conditions and initial states, the time scale of observation in comparison with the dynamical time scale, how one decides what the relevant variables are and how they are separated from the irrelevant ones, how the irrelevant variables are coarse-grained, and what assumptions one makes and what limits one takes in shaping the macroscopic picture from one's imperfect knowledge of the underlying microscopic structure and dynamics. Note that here I try only to explain HOW time-asymmetry arises from the imposition of certain conditions or taking certain approximations, but do not pretend to explain WHY the Universe had to start in some particular condition, e.g., smooth, or low gravitational entropy state according to Penrose (1979) or a state defined by the no-boundary condition of Hartle and Hawking (1983), which can by design hopefully 'explain' time-asymmetry. When it comes to comparing philosophical inclinations my personal preference is that there should be no special initial state (Misner, 1969). The challenge would be to explain the present state of our Universe as a plausible and robust consequence of evolution from a wide variety of arbitrary initial states.

The material in the first part of my talk is old, as old as non-equilibrium statistical mechanics itself. The second part's results are known but more recent—from the work of quantum field theory in curved spacetimes applied to semiclassical cosmology. So I shall spend less time on them. The third and fourth parts contain new results, specifically, i) the existence of a fluctuation-dissipation relation for dynamical quantum fields at zero-temperature (thus under non-equilibrium conditions and detached from thermal considerations, where most previous discussions of this relation are premised upon) (Hu, Paz and Zhang, 1992, 1993a). ii) the appearance of dissipative dynamics in an effective Wheeler-DeWitt equation for the minisuperspace variables in quantum cosmology (Sinha and Hu, 1991; Hu, Paz and Sinha, 1993). Dissipation in quantum fields and semiclassical gravity has been discussed before (Hu, 1989, where references to earlier work on these issues can be found). The main emphasis in this talk is dissipation and irreversibility, the properties of noise and fluctuation which underline many important quantum statistical field processes are only briefly touched on. Because of space limitation, some ideas mentioned in my talk are not discussed here. These are: decoherence and dissipation in quantum cosmology (for a general discussion of the interrelation of these processes, see Hu 1991a; for specific models, see Calzetta 1991, Calzetta and Mazzitelli, 1991, Paz and Sinha, 1991, 1992), noise and fluctuations in semiclassical cosmology (Hu, Paz and Zhang, 1993c), coarse-graining in spacetime and gravitational entropy (Hu, 1983, 1984; Hu, 1993; Hu and Sinha, 1993).
2 Irreversibility and Dissipation: Examples from well-known processes

Let me begin by examining a few text-book type examples of irreversible processes to illustrate their different natures and origins.

A. Dispersion

Consider the trajectory of a particle colliding with fixed hard spheres (Ma, 1985, Sec. 26.5). Assume that the spheres are disks with radius $a$. The particle moves with constant velocity $v$ and has mean free distance $\lambda >> a$ (dilute gas approximation). The trajectories of this particle is of course reversible in time. However, if the incident angle of the particle on the first scattering is changed by $\delta \theta (0)$ initially at $t = 0$, then after many collisions

$$|\delta \theta(t)| \geq e^{t/\tau} |\delta \theta(0)|, \quad \tau = (\lambda/v)/ln(2\lambda/a)$$

At sufficiently long time, $|\delta \theta(t)| \approx 1$, the exit direction is randomized by the accumulated error. The asymmetry in the initial and final conditions of the congruence comes from the accumulation and magnification of the uncertainty in the initial conditions due to the collisions, even though the dynamical laws governing each trajectory are time-symmetric. To trace a particular trajectory backwards in time after a large number of collisions requires an exponentially increasing degree of precision in the specification of the initial condition.

This situation occurs in the inflationary cosmology, in which the scale factor of the Universe grows rapidly $a(t) \sim e^{Ht}$ for a certain period of time in the early history. Any initial small disturbance with some functional dependence on $a(t)$ will differ exponentially in time. Indeed this is what gives the desirable properties of inflationary cosmology in, say, addressing the flatness and horizon problems. The apparent irreversibility of inflation is also of this nature: not in the dynamics, but in the inbalance of the initial and final conditions. (See, e.g., Page, 1984.)

This simple phenomenon is amply illustrated by the many sophisticated results of modern chaotic dynamics. There, the divergence of neighboring trajectories in phase space or parameter space is an intrinsic property of the nonlinear Hamiltonian of the system, not a result of coarse-graining (which is implicit in, say, the postulate of molecular chaos in Boltzmann’s treatment of gas kinetics.) The evolution of an ensemble of such systems at some finite time from the initial moment is often ‘forgetful’ of their initial conditions, not because the individual systems are insensitive to the initial conditions (as in dissipation) but because they are overly sensitive to them to make an accurate prediction of each system almost impossible. It is in this sense that these systems manifest irreversibility.

Chaotic dynamics also appears in cosmology, one example is the dynamics of the mixmaster (diagonal Bianchi Type IX) Universe (Misner, 1969). The chaotic
behavior is associated with the divergence of trajectories which describe different world histories in the minisuperspace (Misner, 1972) parametrized by the shape parameters \((\beta_+, \beta_-)\) (while the deformation parameter \(\alpha\) plays the role of time in quantum cosmology). This was pointed out by Lifshitz and Kalatnikov (1971), Barrows (1982), Bogoliubovskii (1985), and many others (for a recent work, see, e.g. Berger, 1992). The collision of the ‘world particle’ is now with the moving ‘walls’ arising from the anisotropic 3-curvature of the homogeneous space. One can define quantities like ‘topological entropy’ to measure the trajectory instability of this nonlinear system. It is of interest to see if the trajectories in the minisuperspace will exhibit mixing properties, in which case all configurations of the Universe at a later time can be equally accessed from arbitrary initial conditions. If the trajectories distribute unevenly in certain regions it will also be interesting to distinguish the set of initial conditions which give rise to such distinct behaviors. Notice that, by contrast, in the presence of dissipative mechanisms, as we will discuss in Example C, the trajectories in the minisuperspace will indeed evolve to a particular region around the origin, which corresponds to the Friedmann Universe. This signifies the dissipation of anisotropy, a necessary condition for the implementation of the chaotic cosmology program.

B. Diffusion

Let us look at some simple examples in kinetic theory: gas expansion, ice melting and ink drop in water. These are irreversible processes simply because the initial states of \(10^{23}\) molecules on one side of the chamber and a piece of ice or ink drop immersed in a bath of water are highly unlikely configurations out of all possible arrangements. These initial conditions are states of very low entropy. The only reason why they are special is because we arrange them to be so. For these problems, we also know that the system-environment separation and interaction make a difference in the outcome. In the case of the expanding gas, e.g., for free expansion: \(\delta S_{\text{system}} > 0, \delta S_{\text{environ}} = 0, \delta S_{\text{tot}} > 0\) whereas for isothermal expansion: \(\delta S_{\text{system}} = -\delta S_{\text{environ}} > 0, \delta S_{\text{tot}} = 0\).

Another important factor in determining whether a process is irreversible is the time scale of observation compared to the dynamic time scale of the process. We are all familiar with the irreversible process of an ink drop dispersing in water which happens in a matter of seconds, but the same dye suspension put in glycerin takes days to diffuse, and for a short duration after the initial mixing (say, by cranking the column of glycerin with a verticle stripe of dye one way) one can easily ‘unmix’ them (by reversing the direction of cranking, see, e.g., Heller, 1960). Diffusion is nevertheless an intrinsically irreversible process.

In evolutionary cosmology, the significance of any physical processes is evaluated in comparison with the Hubble expansion \((H = \dot{a}/a, \text{ where } a \text{ is the scale factor})\). Those with characteristic time scales shorter than the Hubble time \((H^{-1})\) could have enough time to come to equilibrium with the environment, whence one can assign some temperature to the mixture and use thermodynam-
ical descriptions. Thus in the radiation-dominated era \( (a \sim t^{1/2}) \) one usually refers to the temperature of the ambient photon gas as the temperature of the Universe. However, for weakly interacting particles like neutrinos and gravitons which are rarely collision-dominated, kinetic equations are needed to describe their transport processes. For quantum processes such as particle creation from the vacuum occurring at the Planck time \( t_{pd} = 10^{-43} \text{sec} \), they are intrinsically nonequilibrium quantum processes which require a statistical field-theoretical description. By the same token, when the background spacetime expands very rapidly, as during the vacuum-energy-dominated inflation epoch \( (a \sim e^{Ht}) \), the ordinary practice of describing the phase transition with finite temperature theories may prove to be rather inadequate. Such are the ways how time-scales and the time dependence of the scale factor enter in cosmological processes. Now what about the time-reversible behavior of \( a(t) \) itself?

It is often assumed that the dynamics of the Universe in the contraction phase (say, in a closed Friedmann model) is identical with the expansion phase, because the Einstein equation is time-reversal invariant. (Of course more coalescing and greater inhomogeneity will appear in the contraction phase due to the phase-space difference). One can ask: How about deflation—Is deflation during the contracting phase just as likely to happen as inflation in the expanding phase? The answer to this question depends not on the dynamics, as all cosmological models based on Einstein’s theory are time-reversal invariant, but on the initial conditions. Specifically, can the conditions conducive to these different behaviors exist with equal likelihood in the expansion and contraction phases for these universes? The radiation-dominated condition responsible for the Friedmann-class of behavior can be assumed to hold approximately at the beginning of the contracting phase just as in the expanding phase. However, the vacuum-dominated condition may not be so. This is because inflation is associated with phase transition—be it via nucleation (‘old’) or spinodal decomposition (‘new’) which is not necessarily time-symmetric. To answer this question one should analyze the probability for vacuum energy dominance to occur as the temperature of the Universe increases during contraction, as the broken symmetries are restored, and as the curvature and inhomogeneities of spacetime grow in the approach towards the big crunch. Recent results suggest that deflation is less likely (Goldwirth 1991).

C. Dissipation

There are two basic models of dissipation in non-equilibrium statistical mechanics: the Boltzmann kinetic theory of dilute gas, and the (Einstein-Smoluchowsky) Langevin theory of Brownian motion. Each invokes a different set of concepts, and even their relation is illustrative. In kinetic theory, the equations governing the \( n \)-particle distribution functions (the BBGKY hierarchy) preserve the full information of an \( n \) particle system. It is the truncation of this hierarchy, a procedure justified when one is only interested in the behavior of the low-order correlation (usually the one-particle distribution) functions, that
dissipation appears. It is in ignoring (more often restricted by the precision of one’s observation than by choice) the information contained in the higher-order correlations which brings about dissipation and irreversibility in the dynamics of the lower-order correlations. (Zwanzig, 1961; Prigogine, 1962; Balescu, 1975; de Groot, van Leeuven and van Weert, 1980; Calzetta and Hu, 1988). For the Brownian motion problem modeled, say, by a set of coupled oscillators with one oscillator (mass $M$) picked out as the Brownian particle and the rest (with mass $m$) serving as the bath (Rubin, 1960; Ford, Kac and Mazur, 1963; Feynman and Vernon, 1963; Caldeira and Leggett, 1983). Dissipation in the dynamics of the system arises from ignoring details of the bath variables but only keeping their averaged effect on the system (this also brings about a renormalization of the mass and the natural frequency of the Brownian particle). Usually one assumes $M \gg m$ and weak coupling of the system and the bath to simplify calculations. The effect of the bath can be summarized by its spectral density function, which is not unique to any particular bath. In both of these models, as well as in more general cases, the following conditions are essential for the appearance of dissipation (Hu, 1989, 1990; Calzetta, 1990, 1991):

a) **system-environment separation.** This split depends on what one is interested in: it could be the slow variables, the low modes, the low order correlations, the mean fields; or what one is restricted to: the local domain, the late history, the low energy, the asymptotic region, outside the event horizon, inside the particle horizon, etc. We shall bring up this issue again at the end of this talk.
b) **coupling.** The environment must have many degrees of freedom to share and spread the information from the system; its coupling with the system must be effective in the transfer of information (e.g., non-adiabatic) and the response of the coarse-grained environment must be ‘sluggish’ (sufficiently degrading) in that it will only react to the system in a non-systematic and retarded way.
c) **coarse-graining.** One must ignore or down-grade the full information in the environmental variables to see dissipation appearing in the dynamics of the open system. (The time of observation enters also, in that it has to be greater than the interaction time of the constituents but shorter than the recurrence time in the environment). Coarse-graining can be the truncation of a correlation hierarchy, the averaging of the higher modes, the ‘integrating out’ of the fluctuation fields, or the tracing of a density matrix (discarding phase informations). See the last section for more discussions on this point.
d) **initial conditions.** Whereas a dissipative system is generally insensitive to the initial conditions in that for a wide range of initial states dissipation can drive the system to the same final (equilibrium) state, the process is nevertheless possible only if the initial state is off-equilibrium. The process manifests irreversibility also because the initial time is singled out as a special reference point when the system is prepared in that particular initial state. Thus in this weaker sense, dissipation is also a consequence of specially prescribed initial conditions.  

\(^1\)Note the distinction between these cases: If one defines $t_0$ as the time when a dissipative
While the original combined system and environment still preserve the unitarity of motion, and its entropy remains constant in time, under these approximations, the subsystem becomes an open system, the entropy of the open system (constructed from the reduced density matrix by tracing out the environmental variables) increases in time, and irreversibility appears in its dynamics.

Both irreversible (but non-dissipative) processes and dissipative (and irreversible) processes depend on the stipulation of special initial conditions. The difference is that the former depends sensitively so, the latter insensitively. Dissipative processes involve coarse-graining while non-dissipative processes do not. However, both type of irreversible processes (Case B and C) can entail entropy generation (even in Case A one can associate some mathematical entropy to describe the divergence of the trajectories). Irreversible processes described by the second law is what usually defines the thermodynamic arrow of time.

In the context of dissipative processes, it is important to distinguish dissipation from phase mixing, which, though sometimes called damping (e.g. Landau damping) and has the appearance of an irreversible process, is actually reversible.

D. Phase Mixing

Two well-known effects fall under this category: Landau damping and spin echo (e.g., Balescu, 1975, Sec.12.2; Ma, 1985, Sec. 24.3). Let us examine the first example. In the lowest order truncation of the BBGKY hierarchy valid for the description of dilute gases, the Liouvillian operator $L$ acting on the one-particle distribution function $f_{1}(r_{1}, p_{1}, t)$ is driven by a collision integral involving a two-particle distribution function $f_{2}(r_{1}, p_{1}, r_{2}, p_{2}, t)$:

$$\frac{\partial}{\partial t} + \frac{p_{1}}{m} \cdot \nabla_{r_{1}} + F(r_{1}) \cdot \nabla_{p_{1}} f_{1}(r_{1}, p_{1}, t) = \left( \frac{\partial f_{1}}{\partial t} \right)_{\text{coll}}$$

$$\left( \frac{\partial f_{1}}{\partial t} \right)_{\text{coll}} = \left( \frac{N}{V} \right) \int [\nabla_{r_{1}} V(r_{1}, r_{2})] \cdot \nabla_{p_{1}} f_{2}(r_{1}, p_{1}, r_{2}, p_{2}, t) \, d^{3}r_{2} \, d^{3}p_{2}$$

The molecular chaos ansatz assumes an initial uncorrelated state between two particles (a factorizable condition): $f_{2}(1, 2) = f_{1}(1)f_{1}(2)$, i.e., that the probability of finding particle 1 at $(r_{1}, p_{1}, t)$ and particle 2 at $(r_{2}, p_{2}, t)$ at the same time $t$ is equal to the product of the single particle probabilities. When this condition is assumed to hold initially and finally in a collision processes, (but the two collision partners are assumed to be correlated within the short range dynamics begins and $t_{1}$ as when it ends, then the dynamics from $t_{0}$ to $-t$ is exactly the same as from $t_{0}$ to $t$, i.e., the system variable at $-t_{1}$ is the same as at $t_{1}$. This is expected because of the special role assigned to $t_{0}$ in the dynamics with respect to which there is time-reversal invariance, but it is not what is usually meant by irreversibility in a dissipative dynamics. The arrow of time there is defined as the direction of increase of entropy and irreversibility refers to the inequivalence of the results obtained by reversing $t_{0}$ and $t_{1}$ (or, for that matter reversing $t_{0}$ and $-t_{1}$), but not between $t_{1}$ and $-t_{1}$. The time-reversal invariance of the H-theorem has the same meaning.
of the interaction force), one gets the Boltzmann equation. However, for long-ranged forces such as the Coulomb force in a dilute plasma gas where close encounters and collisions are rare, the factorizable condition can be assumed to hold throughout. In such cases the kinetic equation becomes a Vlasov (or collisionless-Boltzmann) Equation: (e.g., Balescu, 1975; Kreuzer, 1981)

\[
\frac{\partial}{\partial t} + \frac{p_{1}}{m} \nabla r_{1} + [F(r_{1}) - \nabla r_{1} \tilde{\Phi}(r_{1}, t)] \cdot \nabla p_{1} \big] f_{1}(r_{1}, p_{1}, t) = 0
\]  

(3)

Here

\[
\tilde{\Phi}(r_{1}, t) = \left( \frac{N}{V} \right) \int V(r_{1}, r_{2}) f_{1}(r_{2}, p_{2}, t) d^{3}r_{2} d^{3}p_{2}
\]

(4)

is the mean field potential experienced by any one particle produced by all other particles. It is determined by the density excess over the equilibrium value. The effect of the mean field potential is similar to the Debye-Huckel screening in dilute electrolyte systems. The dependence on \( f_{1} \) makes the Vlasov equation nonlinear: Equations (3) and (4) have to be solved in a self-consistent way. This is analogous to the Hartree approximation in many-body theory. Note that the Vlasov equation which has a form depicting free streaming is time-reversal invariant: The Vlasov term accounting for the effect of the averaged field does not bring about entropy generation. This mean-field approximation in kinetic theory, which yields a unitary evolution of reversible dynamics, is, however, only valid for times short compared to the relaxation time of the system in its approach to equilibrium. This relaxation time is associated with the collision-induced dissipation process.

Landau damping in the collective local charge oscillations described by the Vlasov equation is only an apparently irreversible processes. The appearance of ‘damping’ depends critically on some stipulated special initial conditions. This damping is different from the dissipation process discussed in Case C, in that the latter has an intrinsic time scale but not the former, and that while dissipation depends only weakly on the initial conditions, mixing is very sensitive to the initial conditions. A more appropriate name for these processes is ‘phase mixing’ (Balescu 1975). Spin echo is a somewhat different example of phase mixing.

From all of the above examples we see that irreversibility and dissipation involve very different processes. The effect of interaction, the role of coarse-graining, the choice of time-scales, and the specification of initial conditions in these processes can give rise to very different results. In the next section we shall use these examples to illustrate the statistical properties of quantum field processes in the early Universe.
3 Fluctuations and Irreversibility: Examples from cosmological particle creation

We see in the above the many origins of irreversibility and the distinction between dissipative and irreversible processes. Let us continue exploring these conceptual issues now by adding an additional dimension, fluctuations – both quantum and thermal fluctuations. These refer to statistical variations from the mean – the vacuum or the background field in the case of quantum fluctuations, the equilibrium state or the mean field in the case of thermal fluctuations. Only quantum fluctuations exist at zero temperature. (Their relation is an interesting issue in itself, involving the viability of background separation, applicability of the notion of ensembles, and the definition of classicality, to name just a few. See, Hu and Zhang, 1992, 1993; Calzetta and Hu, 1993b) Processes involving fluctuations play important roles in cosmology. Examples are: Fluctuations in background spacetimes induce density contrasts as seeds for galaxy formation (Hu, Paz and Zhang, 1993b); parametric amplification of vacuum fluctuations leads to particle creation in the early Universe (Parker, 1969; Zel’’dovich, 1970); fluctuations of quantum fields bring about phase transitions in the inflationary cosmology (Guth, 1981; Sato, 1981; Linde, 1982; Albrecht and Steinhardt, 1982); thermal fluctuation (noise)-induced phase transitions (the Kramer process). Even the creation of the Universe (and its babies!) has been attributed to fluctuations of spacetime geometry and topology. (Vilenkin, 1986; Coleman et al, 1991)

For a description of fluctuations, at least two factors, the number of samples taken and the time of observation, usually enter into the consideration: For $N$ samples of a system in equilibrium, the fluctuations of physical quantities associated with the system are of the magnitude $N^{-1/2}$, and can be made arbitrarily small by making $N$ large. Thus in taking the thermodynamic limit of the system, i.e., letting $N$ and $V$ large but keeping $N/V$ constant, or, by looking at the system at longer time spans, the occurrence of large fluctuations are statistically suppressed. The former operation forfeits the Poincare recurrence, while the latter operation (made equivalent to averaging over a large number of copies) assumes the validity of ergodicity. By contrast, for finite nonequilibrium systems, large fluctuations can arise more readily. Because non-equilibrium systems have intrinsic time-scales, one cannot hope to get an ensemble-averaged suppression by taking a long enough waiting time, as in the equilibrium cases. As for the issues of time-reversibility of events involving fluctuations, although the appearance of a fluctuation and its disappearance are time-symmetric, the set-up of problems involving fluctuations is often such that the chronicle of interesting events starts at the time when the fluctuation first comes into existence, or becomes eventful. This imparts the subsequent history an apparent arrow of time. Thus we talk about the ‘beginning’ of a new phase, or the ‘genesis’ of the Universe, as if time only exists after that particular moment.
Irreversibility and thermal fluctuations are studied in many textbooks of non-equilibrium statistical mechanics. Here I want to focus on the statistical properties of vacuum fluctuations, especially in cosmological processes involving vacuum fluctuations. Let us first analyze entropy-generation and irreversibility in a simple but basic process, particle creation from the vacuum.

Pair creation involves the spontaneous or stimulated release of energy in the amount of the threshold or above from the vacuum or from existing particles. Note that the mechanism according to the basic physical laws is time-symmetric. Thus, given equal initial and final conditions, pair annihilation should be equally probable. However, the initial condition is usually arranged differently from the final conditions, and this is where the problem arises. It is easier for a pair to be created than for them to annihilate, because only particles-antiparticle pairs with $\pm k$ can do it and the two have to be brought together at the same point in spacetime for this to happen. (This is what is usually referred to as the phase space factor difference).

One of the reasons for our interest in vacuum particle creation processes is to try to get a handle on the nature of the ubiquitous, omnipotent, but mysterious and often ambiguous entity called the vacuum. Note that by comparison with the particles it creates, which carry precise and reproducible information content, the vacuum understood in a naive way contains little information. However, the vacuum is far more complex than a simple ‘nothing’. It is made to play many different roles and perform many difficult tasks: The vacuum is every rich man’s garbage dump (witness all the divergences) and every poor man’s Messiah (“The Universe is a free lunch”, Guth, 1981) It is far from devoid of information, because everything can in principle be obtained from it, given some viable mechanism (e.g., pair production) and some luck (probability and stochasticity). Therefore the mechanisms which transform the vacuum into physical reality is of special interest. It is for this reason that some understanding of the statistical properties of the vacuum is essential to launching the adventurous but noble quest to ‘get everything from nothing’, otherwise known as Don Quixote’s ‘free lunch’.

Cosmological particle creation adds into consideration an additional factor of the influence of background spacetimes on the vacuum (Parker, 1969). We shall look at just the dynamical effects here but not those effects associated with the global structures of spacetime such as the event horizon (Hawking, 1975). We have in earlier work analyzed the problem of entropy generation from cosmological particle creation and interaction processes. Let us try to understand the different nature of irreversibility in these processes.

Assuming that at an initial time $t_0$ the system is in a mixed state described by a density matrix $\rho$ which is diagonal in the representation whose basis consists of the eigenstates of the number operators at $t_0$, then the number of particles in mode $k$ in a unit volume at a later time $t$ is given by (Parker, 1969)

$$< N_k(t) > = |\beta_k(t)|^2 + a_k < N_k(t_0) >$$

(5)
where $\beta_k$ is the Bogolubov coefficient measuring the mixing of the positive and negative frequency components, and $a_k = 1 + 2|\beta_k(t)|^2$ is the parametric amplification factor for mode $k$. The two parts in this expression can be understood as the parametric amplification of vacuum fluctuations and that of particles already present in mode $k$. The first part (spontaneous creation) always increases while the second part (induced creation) can increase or decrease depending on the correlation and phase relation of the initial state and on whether the particle is a boson or a fermion.

Are these processes time-asymmetric? Is there entropy generation in a vacuum particle creation process? The search for an answer to these seemingly simple questions teaches us something interesting. Let us separate the time-asymmetry question into two parts: one referring to the time-reversed process of pair annihilation, the other referring to the probability of particle creation in the Universe’s contraction phase.

Assume the Universe is in the expansion phase. Consider first the more complicated but conceptually easier case of particle creation with interaction. If we measure only the one-particle distribution, the entropy function constructed from the reduced density matrix will under general conditions (assuming bosons with initial state an eigenstate of the number operator) increase. (For details see Hu and Kandrup, 1987). The primary reason is that one has ignored the information in the higher-order correlation functions. The presence of interaction is such that even if one starts with an initial state with no correlation between the relevant and irrelevant variables, interaction can change the correlations and bring about entropy generation. This case is similar in nature to our example above of dissipations in an interacting gas. These dissipative processes are irreversible, and their outcomes usually do not depend or depend only weakly on the initial conditions.

The other case of particle creation from the vacuum with no interaction is more subtle (Hu and Pavon, 1986; Kandrup, 1988). On the one hand we know that both the initial vacuum and the final particle pair are in a pure state, so there cannot be any entropy generation. On the other hand we clearly see an increase of particles in time, and one is tempted to use the particle number as a measure of entropy and conclude that entropy is generated in the process of particle creation. (Indeed, in the thermodynamic approximation, $S \sim N^3$, but this relation is only valid for collision-dominated gas, which assumes interaction, from which entropy generation is expected). The resolution of this paradox lies in the fact that usually in calculating particle creation one works in a Fock space representation where the initial state (e.g., the vacuum or the thermal state) is assumed to be an eigenstate of the number operator (N-representation). However, an uncertainty relation exists between the number and the phase information. It is at the sacrifice of the phase information that one sees an increase of the number in time. Had one chosen the initial state to be of definite phase ($P$-representation), particle number will not be monotonically increasing. Therefore it is only for the customary choice of an eigenstate
of the number operator as the initial state that the non-dissipative process of particle creation with no interaction appears to be irreversible. As in the case of phase mixing in Example D above, this apparent ‘irreversibility’ is also highly sensitive to the choice of the initial state.

Now consider the situation where these processes take place at the contraction stage of the Universe and ask the question whether they will take place with the same probability. Let us take the simplest case of cosmological particle creation, assuming that the in-vacuum and the out-vacuum are well defined (e.g., statically-bounded dynamics, or work with some conformal-vacuum) and symmetric. Since the Bogolubov transformations which relate a set of creation and annihilation operators at one time to another is time-reversal invariant, the process should be time-symmetric. That is, one should expect to see particle creation just as likely to happen in the contraction phase. However, except for steady state models, cosmological conditions are not symmetric between the in and the out states in the expanding and the contracting phases. In the expanding phase, the in-state for particle creation processes of any cosmological significance is usually taken to be at the singularity (‘big bang’) or at least around the Planck time, while the out-state is defined at late times before re-contraction when curvature and field effects are weak. There is asymmetry in the in and out states between the expanding and the contracting phases which affects the production rates. Despite these differences, there is entropy generation associated with particle creation and interaction in both the expanding and the contracting phases. Thus the thermodynamic arrow of time defined by the direction of entropy increase will see no change at the turnaround point. To the extent that the thermodynamic arrow of time can be traced to be the root of many other arrows of time (including the psychological), entropy generation in particle creation can play a fundamental role in the problem of time-asymmetry.

We see in the above cosmological examples the workings of the differences between irreversible and dissipative processes as manifested in vacuum fluctuations and particle creation. We shall see next how these processes can affect the dynamics of the early Universe, and manifest as a relation between fluctuation in the quantum fields and dissipation in the dynamics of spacetime.

4 Fluctuation and Dissipation: Example from cosmological backreaction processes

Cosmological particle creation comes from the amplification of vacuum fluctuations by the dynamics of the background spacetime. It is the transformation of a microscopic random process into macroscopic proportions. At late times like today’s Universe this process is rather insignificant (Parker, 1969). However, near the Planck time ($t_{pl} \sim 10^{-43} \text{sec}$ from the Big Bang), for non-conformal fields, or for non-conformally flat universes, production of particles might have
been so copious that they could have exerted a strong influence on the dynamics of the early Universe (Zel’dovich 1970). In particular, anisotropies in the early Universe can be dissipated away in fractions of $t_{Pl}$ (Zel’dovich and Starobinsky, 1971; Hu and Parker, 1978; Hartle and Hu, 1980). Backreaction processes like these have been studied extensively for cosmological (origin of isotropy in the Universe), philosophical (chaotic cosmology program), and theoretical (quantum to classical transition) inquiries. Here we’d like to view it as an example of the fluctuation-dissipation relation relating the fluctuations of the vacuum to the dissipative dynamics of the Universe (Hu, 1989). Take for example a massless conformal scalar field in an anisotropic, homogeneous Bianchi Type-1 Universe with line element

$$ds^2 = a^2(\eta) [d\eta^2 - \sum_{i,j=1}^{3} e^{2\beta_{ij}(\eta)} dx_i dx_j].$$

(6)

The equation of motion for the anisotropic expansion rates $q_{ij} \equiv \beta'_{ij} \equiv d\beta_{ij}/d\eta$ calculated in the Schwinger (1961) - Keldysh (1964) (or closed time-path, or in-in) formalism is given by (Calzetta and Hu, 1987)

$$\frac{d}{d\eta}(Mq'_{ij}) + 3(2880\pi^2)^{-1}Kq'_{ij} + kq_{ij} = c_{ij},$$

(7)

where $c_{ij}$ is a constant measuring the initial anisotropy, $M$ and $k$ are functions of $a, a'$ and $a''$. The nonlocal kernel $K(\eta - \eta')$

$$Kq'_{ij} = \int_{-\infty}^{\eta} d\eta' (\frac{d^3}{d\eta'} q_{ij}(\eta')) \ln(|\eta - \eta'|).$$

(8)

linking the ‘velocities’ $q'_{ij}$ at different times gives a non-local viscosity function $\gamma$ (in Fourier space)

$$\gamma(\omega) = \frac{\pi}{60(4\pi)^2} |\omega|^3.$$

(9)

which is responsible for the dissipation of anisotropy in the background dynamics. The energy density dissipated in the background dynamics is shown to be exactly equal to the energy density of the particles created:

$$\rho(\text{particle creation}) = \rho(\text{anisotropy dissipation})$$

(10)

This relation, as we pointed out earlier (Hu, 1989), embodies the fluctuation-dissipation relation in the cosmological context, but does not yet have the correct form (F-D relation for black holes and de Sitter spacetimes have been proposed by Candelas and Sciama, 1977; Sorkin, 1986; and by Mottola, 1986 respectively).

Notice that velocity $\beta'$ enters in the equation of motion (7) instead of displacements $\beta$. This is because the coupling between the field and the background dynamics via the Laplace-Beltrami operator is of a derivative kind. This equation is in the form of a Langevin equation, except for the absence of explicit
random forces. This is because in the above example we worked with pure states to begin with and there is no explicit coarse-graining of the environment fields. Technically the ordinary effective action which takes into account the averaged effect of quantum fluctuations can be generalized to a coarsened-grained one, where the environment field is averaged away. The coarse-grained (closed time-path) effective action (Hu and Zhang, 1990; Hu, 1991b; Sinha and Hu, 1991) is intimately related to the influence action (Feynman and Vernon, 1963) which is needed for a full display of backreaction effects in quantum statistical systems (Hu, 1991a, 1993). The coarse-grained effective action has a real part which is responsible for particle production, while the influence action has also an imaginary part responsible for noise. The equation of motion derived from the influence action is the master equation for the reduced density matrix of the system after details of the environment are traced out. In the semiclassical limit the Wigner function associated with the reduced density matrix obeys the Fokker-Planck equation, while, equivalently, the system variable obeys a Langevin equation with an explicit noise term whose distribution function depends on the nature of and the system’s coupling with the environment. This extended formalism in terms of the influence functional provides a more complete platform for the discussion of both dissipation and fluctuation processes.

Many physical processes in the macroscopic world manifest dissipative behavior, which is time-asymmetric. This is contradictory to the basic laws governing the microscopic world, which is time-symmetric. To resolve this difference is one of the central tasks of statistical mechanics. One way is to conceive of a natural transformation (or spontaneous evolution, see Calzetta and Hu, 1993a) of a closed system to an open system involving the procedures outlined in Example C, i.e., separation of the system (the relevant variables) from the environment (the irrelevant variables), choice of boundary conditions, and averaging (coarse-graining) of the irrelevant variables. Backreaction of the averaged effect of the irrelevant variables modifies the dynamics of the relevant variables with a dissipative contribution. It is through this means that random microscopic reversible processes can bring forth irreversible behavior in the systematic macroscopic dynamics. The connection between these two aspects is best captured in the fluctuation-dissipation (FD) relation. In a concrete form, it provides a microscopic derivation of the kinetic coefficients (e.g. viscosity function). It is also one of the means that the quantum world described by wave functions and interference effects can be related to the classical world described by the classical equations of motion. We will discuss the meaning of the fluctuation-dissipation relation and the environment-induced decoherence effect here (Zurek, 1981; Joos and Zeh, 1985; Zeh, 1986), but leave the discussion of its relation with noise and classical structure elsewhere (Hu, 1991a, 1993; Gell-Mann and Hartle, 1993). They are interrelated.

The FD relation is often written for equilibrium (finite temperature $T$) conditions and derived via linear-response theories (Callen and Welton, 1951; Kubo, 1959). We believe that, owing to its general nature, a relation should exist for
non-equilibrium, and for quantum \((T = 0)\) processes. In a recent work (Hu, Paz and Zhang, 1992, 1993a; see also Sinha and Sorkin, 1992) we have proven at least the latter case in quantum Brownian motion models. This provides the theoretical basis for a statistical interpretation of quantum backreaction processes, which include the well-known radiation-reaction problem in electrodynamics, as well as the backreaction problems in semiclassical cosmology. For the purpose of extending the fluctuation-dissipation relation to quantum fields, we used path-integral methods. Our results are summarized as follows.

Consider a Brownian particle with mass \(M\) interacting with a thermal bath at temperature \(T = (k_B \beta)^{-1}\). The classical action of the Brownian particle is

\[
S_S[x] = \int_0^t ds \left\{ \frac{1}{2} m \ddot{x}^2 - V(x) \right\}
\]

(11)

The bath consists of a set of harmonic oscillators with mass \(m_n\) whose motion is described by the classical action

\[
S_E[[q_n]] = \int_0^t ds \sum_n \left\{ \frac{1}{2} m_n \dot{q}_n^2 - \frac{1}{2} m_n \omega_n^2 q_n^2 \right\}
\]

(12)

Assume as an example that the system and environment interacts via a biquadratic coupling with action

\[
S_{int}[x, \{q_n\}] = \int_0^t ds \sum_n \left\{ -\lambda C_n x^2 \ddot{q}_n^2 \right\}
\]

(13)

Here \(\lambda\) is a coupling constant multiplied to each \(C_n\), which is assumed to be small for perturbation calculations. The case of linear coupling has been derived by many authors (e.g., Feynman and Vernon, 1963; Caldeira and Leggett, 1983; Unruh and Zurek, 1989; Hu, Paz and Zhang, 1992). For biquadratic coupling, the fluctuation-dissipation relation between the second noise kernel \(\tilde{\nu}(s_1 - s_2)\) and the dissipation kernel \(\gamma(s_1 - s_2)\) can be written down explicitly as (Hu, Paz and Zhang, 1993a)

\[
\bar{h} \tilde{\nu}(s) = \int_0^{+\infty} ds' K(s - s') \gamma(s')
\]

(14)

where the time convolution kernel \(K(s)\) is given by

\[
K(s) = \hbar \int_0^{+\infty} \frac{d\omega}{\pi} \left\{ \frac{1 + \coth \frac{1}{2} \beta \hbar \omega}{2 \coth \frac{1}{2} \beta \hbar \omega} \right\} \omega \cos \omega s
\]

(15)

except for the temperature dependent factor (the term within the curly brackets), this has the same form as the linear coupling case (which is given by
\( \coth(\beta \hbar \omega/2) \). For higher order couplings with an action \( \lambda C_n f(x) q_n^* \), the FD relation again has the same form as (14) and (15), only that the temperature-dependent factor is different. In the high temperature limit \( k_B T \gg \hbar \Gamma \), where \( \Gamma \) is the cutoff frequency of the bath oscillators, \( K(s) = 2k_B T \delta(s) \) and the fluctuation-dissipation relation reduces to

\[
\hbar \nu (s_1 - s_2) = 2k_B T \gamma (s_1 - s_2)
\]

which is the famous Einstein formula (Einstein, 1905).

In the zero temperature limit \( \beta \hbar \omega \to +\infty \),

\[
K(s) = \hbar \int_0^{+\infty} \frac{d\omega}{\pi} \frac{\omega \cos \omega s}{\omega^2}
\]

It is interesting to note that the fluctuation-dissipation relations for the linear and the nonlinear coupling models we have studied are identical both in the high temperature and the zero temperature limits. This insensitivity to the different system-bath couplings reflects that it is a categorical relation (backreaction) between the stochastic stimuli (fluctuation-noise) of the environment and the averaged response of a system (dissipation-relaxation) which has a much deeper and broader meaning than that associated with the special cases studied in the literature. We have also derived the influence action for field theory models with non-linear coupling and colored noise environments (Zhang, 1990; Hu, Paz and Zhang, 1993b) and found that a set of FD relations exist which are identical in form to the quantum mechanical results given above. This seems to confirm our earlier suggestion about the universality of such relations (Hu, 1989). The FD relation suggests how macroscopic irreversibility can arise from microscopic reversible processes. It is in this capacity that it is relevant to the time-asymmetry problem.

The extension of the quantum Brownian motion results to quantum cosmology is under investigation. This requires first an upgrading in the treatment of the cosmological backreaction problem described above from the semiclassical to the full quantum level (describing wave functions of the Universe). One also needs to generalize this problem to statistical ensembles (of quantum states of the Universe) and study the evolution of the reduced density matrix of the Universe obtained by tracing out, say, the scalar fields viewed as the environment variables. (See, e.g., Paz and Sinha, 1991, 1992) Consideration of this cosmological backreaction problem in the statistical context pushes the domain of validity of the fluctuation-dissipation relation to a new level, that which involves fluctuations of quantum fields and dissipative spacetime dynamics (Hu and Sinha, 1993a). This relation viewed in the cosmological context has direct implications on the notion of gravitational entropy and the time-asymmetry issue, as we now show.
5 Coarse-Graining and Dissipation in Spacetime: Example in minisuperspace cosmology

In a statistical-mechanical interpretation of the problem of backreaction due to particle creation, the background spacetime plays the role of the system while the scalar field that of the environment. The backreaction can be calculated by the effective action method in loop expansions. In the ordinary approach, a background-fluctuation field decomposition is assumed, and the backreaction is due to the radiative correction effects $O(\hbar)$ of the matter field like vacuum fluctuation and particle creation. One can generalize this method to treat quantum statistical processes involving coarse-graining. Suppose one separates the field of the combined system $\phi$ into two parts: the system field $\bar{\phi}$ and the environment field $\tilde{\phi}$, i.e., $\phi = \bar{\phi} + \tilde{\phi}$, and assumes that they are coupled weakly with a small parameter $\lambda$. One can then construct a coarse-grained effective action $\Gamma[\bar{\phi}]$ by integrating away the environment variables. This procedure has been used in a renormalization group theory treatment of critical phenomena in the inflationary Universe (Hu and Zhang, 1990, Hu, 1991b). For quantum cosmology, one can use this method to study the effect of truncation in the gravitational degrees of freedom, and discuss the validity of the minisuperspace approximation. (A more comprehensive discussion of viewing minisuperspace as a quantum open system in quantum cosmology is given in Hu, Paz and Sinha, 1993)

5.1 Minisuperspace Approximation

Those cosmological models most often studied, like the Robertson-Walker, de Sitter, and the Bianchi universes, which possess high symmetries are but a small class of a large set of possible cosmological solutions of the Einstein equations. In terms of superspace, the space of all three-geometries, (Wheeler, 1968; DeWitt, 1967) these are the lower-dimensional minisuperspaces (Misner, 1972) (e.g., the mixmaster Universe with parameters $\alpha, \beta_+, \beta_-$ is a three-dimensional minisuperspace). In quantizing just the few lowest modes, as is often done in quantum cosmology studies, one ignores by fiat all these other modes. Is the minisuperspace quantization justified? (Kuchar and Ryan, 1986, 1989) Under what conditions is it justified? What is the backreaction effect of the inhomogeneous modes on the homogeneous mode? One can view the homogeneous geometry as the system and the matter fields (or the inhomogeneous perturbations of spacetime, the gravitons) as the environment, and use the coarse-grained effective action to calculate the averaged effect of the environment on the system. Notice the similarity with the statistical mechanical problems we have treated above. In one illustrative calculation (Sinha and Hu, 1991) we used a model of self-interacting quantum fields to mimic the nonlinear coupling of the gravitational waves modes (WKB time is used as it provides correct semiclassical results) and obtained an effective Wheeler-DeWitt equation for the minisuper-
space sector with a new term containing a nonlocal kernel. Similar in form to Eq.(7) in the particle creation backreaction problem, it signifies the appearance of dissipative effects in the dynamics of the minisuperspace variables due to their interaction with the inhomogeneous modes. Thus one can conclude that the minisuperspace approximation is valid only if this dissipation is small. In the same sense as the other statistical processes we have considered above, the appearance of dissipation creates an arrow of time in the minisuperspace sector. This also provides one way to define gravitational entropy. Notice that in this view, as long as one limits one’s observation to a subset of all possible geometrodynamics, and allows for some special initial conditions, dissipative behavior and the emergent arrow of time are unavoidable consequences.

5.2 Gravitational Entropy

The entropy of gravitational fields has been studied in connection with self-gravitating matter (Lynden-Bell and Wood, 1967; Lynden-Bell and Lynden-Bell, 1977; Sorkin, Wald and Zhang, 1984), with black holes (Hawking, 1975; Sorkin, 1986), with cosmology (Penrose, 1979) and with gravitons (Smolin 1984). We shall consider it for cosmological spacetimes without event horizon (See Davies, Ford and Page, 1989 for the case of de Sitter universe, which has an event horizon). Gravitational entropy of the Universe has also been discussed before in conjunction with quantum dissipative processes in the early Universe (Hu, 1983, 1984). Here I want to discuss it in the context of quantum cosmology (see also Kandrup, 1989) and the theme of the present conference, time-asymmetry.

Following the idea of minisuperspace approximation in quantum cosmology discussed above as a backreaction problem and generalizing the wave functions of the Universe to density matrices of the Universe, we can work with the reduced density matrix of the Universe constructed by tracing out the matter fields or the higher gravitational modes and define a gravitational entropy of the homogeneous Universe as

\[ S = -\text{Tr}\rho_{\text{red}}\ln\rho_{\text{red}} \]  

(18)

From the theory of subdynamics, we know that \( S \) increases with time. (Note again that some notion of time has to be introduced beforehand, e.g., the WKB time, or the 4-volume time of Sorkin, 1993). The arrow of time arises as the direction of information flow from the relevant (spacetime, or the homogeneous gravitational modes) to the irrelevant (the matter fields, or the inhomogeneous modes) degrees of freedom. (See Hu, 1993; Hu and Sinha, 1993b for details.)

In this and earlier sections I have only sketched the statistical nature of certain quantum processes in semiclassical gravity and quantum cosmology, but I hope this array of examples and questions – from billiard balls to ink drops to plasma waves to particle creation to anisotropy damping to density matrix
of the Universe – has demonstrated to you, despite the great disparity of their
context, the universality of the issues involved and the conceptual unity in our
understanding.

Up to now I have only discussed HOW one can see dissipation and the arrow
of time arising in the system from coarse-graining the environment. I have
not mentioned anything about the more fundamental and difficult questions
in the system-environment approach to these issues in statistical mechanics,
i.e., WHY? Why should the system be regarded as such? Why should the
separation be made as such? Why should a sector be viewed as the system
and get preferential treatment over the others. The answer to these questions
when raised in the cosmological context can be more meaningfully sought with
the open-system perspective if the spacetime has some distinguished global or
physical structures like event horizon, particle horizon, non-trivial topology,
etc. One can then define an objectively meaningful domain for the system and
study its effective dynamics. The outcome also depends on how the coarse-
graining (measurement, observation, participation) is taken, and how effective
it is in producing persistent robust structures (Woo, 1989). Consistency in the
behavior of the system after these procedures are taken (such as how stable
any level of structure is with respect to iteration of the same coarse-graining
routines, and how sensitive the open system is with respect to variations of
coarse-graining) is certainly an important criterion in any consideration. I will
now say a few things on these issues to conclude my talk.

6 Coarse-Graining and Persistent Structure in
the Physical World

Let me summarize the main points of this talk and suggest a few questions to
explore on the issue of irreversibility in cosmology.

On the whole, there are two different causes for the appearance of irre-
versibility: one due to special initial conditions, the other due to dissipation.
The first class is \textit{a priori} determined by the initial conditions, the other is \textit{a
posteriori} rather insensitive to the initial conditions. Of the examples we have
given, the first class includes chaotic dynamics, Landau damping, vacuum par-
ticle creation, the second class includes molecular dynamics, diffusion, particle
creation with interaction, anisotropy dissipation, decoherence. Appearance of
dissipation is accompanied by a degradation of information via coarse grain-
ing (such as the molecular chaos assumption in kinetic theory, restriction to
one-particle distribution in particle creation with interaction, ‘integrating out’
some class of histories in decoherence). An arrow of time appears either because

\footnote{As discussed earlier, dissipation also requires the stipulation of a somewhat special initial
condition, i.e., that the system is not in an equilibrium state; but ‘not more special than it
needs to be’ – in the words of R. Sorkin.}
of some special prearranged conditions or as a consequence of coarse-graining introduced to the system. The issues we have touched on involve the transformation of a closed to an open system, the relation between the microscopic and the macroscopic world, and the transition from quantum to classical realities. Many perceived phenomena in the observable physical world, including the phenomenon of time-asymmetry, can indeed be understood in the open-system viewpoint via the approximations introduced to the objective microscopic world by a macroscopic observer. We have discussed the procedures which can bring about these results. However, what to me seems more important and challenging is to explore under what conditions the outcomes become less subjective and less sensitive to these procedures, such as the system–environment split and the coarse-graining of the environment. These procedures provide one with a viable prescription to get certain general qualitative results, but are still not specific and robust enough to explain how and why the variety of observed phenomena in the physical world arise and stay in their particular ways. To address these issues one should ask a different set of questions:

1) By what criteria are the system variables chosen? –collectivity and hierarchy of structure and interactions

In a model problem, one picks out the system variables – be it the Brownian particle or the minisuperspace variables – by fiat. One defines one’s system in a particular way because one wants to calculate the properties of that particular system. But in the real world, certain variables distinguish themselves from others because they possess a relatively well-defined, stable, and meaningful set of properties for which the observer can carry out measurements and derive meaningful results. Its meaningfulness is defined by the range of validity or degree of precision or the level of relevance to what the observer chooses to extract information from. In this sense, it clearly carries a certain degree of subjectivity– not in the sense of arbitrariness in the will of the observer, but in the specification of the parameters of observation and measurement. For example, the thermodynamic and hydrodynamic variables are only good for systems close to equilibrium; in other regimes one needs to describe the system in terms of kinetic-theoretical or statistical-mechanical variables. The soundness in the choice of a system in this example thus depends on the time scale of measurement compared to the relaxation time. As another example, contrast the variables used in nuclear collective model and the independent nucleon models. One can use the rotational-vibrational degrees of freedom to depict some macroscopic properties of the motion of the nucleus, and one can carry out meaningful calculations of the dissipation of the collective trajectories (in the phase space of the nucleons) due to stochastic forces. In such cases, the non-collective degrees of freedom can be taken as the noise source. However, if one is interested in how the independent nucleons contribute to the properties of the nucleus, such as the shell structure, one’s system variable should, barring some simple cases, not be the elements of the $SO(3)$ group, or the $SU(6)$ group. At a higher still
energy where the attributes of the quarks and the gluons become apparent, the system variables for the calculation of, say, the stability of the quark-gluon plasma should change accordingly. The level of relevance which defines one’s system changes with the level of structure of matter and the relative importance of the forces at work at that level. The improvement of the Weinberg-Salam model with $W, Z$ intermediate bosons over the Fermi model of four point interactions is what is needed in probing a deeper level of interaction and structure which puts the electromagnetic and weak forces on the same footing. Therefore, one needs to explore the rules for the formation of such relatively distinct and stable levels, before one can sensibly define one’s system (and the environment) to carry out meaningful inquiries of a statistical nature.

What is interesting here is that these levels of structures and interactions come in approximate hierarchical order (so one doesn’t need QCD to calculate the rotational spectrum of a nucleus, and the Einstein spacetime manifold picture will hopefully provide most of what we need in the post-Planckian era). One needs both some knowledge of the hierarchy of interactions (e.g., Weinberg 1974) and the way effective theories emerge— from ‘integrating out’ variables at very different energy scales in the hierarchical structure (e.g., ordinary gravity plus grand-unified theory regarded as a low energy effective Kaluza-Klein theory) The first part involves fundamental constituents and interactions and the second part the application of statistical methods. One should also keep in mind that what is viewed as fundamental at one level can be a composite or statistical mixture at a finer level. There are system-environment separation schemes which are designed to accomodate or reflect these more intricate structures, such as the mean field-fluctuation field split, the dynamics of correlations (Balescu, 1975; Calzetta and Hu, 1988) and the multiple source formalism (Cornwall, Jackiw and Tomboulis 1974; Calzetta and Hu, 1993a). The validity of these approximations depends on where exactly one wants to probe in between any two levels of structure. Statistical properties of the system such as the appearance of dissipative effects and the associated irreversibility character of the dynamics in an open system certainly depend on this separation.

2) How does the behavior of the subsystem depend on coarse-graining?— sensitivity and variability of coarse-graining, stability and robustness of structure

Does there exist a common asymptotic regime as the result of including successively higher order iterations in the same coarse-graining routine? This measures the sensitivity of the end result to a particular kind of coarse-graining. How well can different kinds of coarse-graining measure produce and preserve the same result? This is measured by its variability. Based on these properties of coarse-graining, one can discuss the relative stability of the behavior of the resultant open system after a sequence of coarse-grainings within the same routine, and its robustness with respect to changes to slightly different coarse-graining routines.

Let me use some simple examples to illustrate what this problem is about.
When we present a microscopic derivation of the transport coefficients (viscosity, heat conductivity, etc) in kinetic theory via the system-environment scheme, we usually get the same correct answer independent of the way the environment is chosen or coarse-grained. Have we ever wondered why? It turns out that this is the case only if we operate in the linear-response regime. (Feynman and Vernon 1963). The linear coupling between the system and the environment makes this dependence simple. This is something we usually take for granted, but has some deeper meaning. For nonlinear coupling, the above problem becomes nontrivial. Another aspect of this problem can be brought out in the following consideration (Balian and Veneroni, 1987). Compare these two levels of structure and interaction: hydrodynamic regime and kinetic regime. Construct the relevant entropy (in the information theory sense) from the one-particle distribution \( \rho \) under the constraint that the average of any physical variable \( O \) is given by 

\[
\langle O \rangle = Tr \rho O.
\]

\( \rho \) changes with different levels of coarse-graining. In terms of the one-particle classical distribution function \( f_1 \) the entropy function \( S \) is given by

\[
S_B = \int d\vec{r}d\vec{p}f_1(\vec{r}, \vec{p})[1 - ln h^3 f_1(\vec{r}, \vec{p})] \tag{19}
\]

in Botzmann’s kinetic theories, and

\[
S_H \sim N^3, \quad N = \int d\vec{r}d\vec{p}f_1(\vec{r}, \vec{p}) \tag{20}
\]

in hydrodynamics. Notice that \( S_H > S_B \) is a maximum in the sequence of different coarse-graining procedures. In the terminology we introduced above, by comparison with the other regimes, the hydrodynamic regime is more robust in its structure and interactions with respect to varying levels of coarse-graining. The reason for this is, as we know, because the hydrodynamic variables describe systems in equilibrium. Further coarse-graining on these systems is expected to produce the same result. Therefore, a kind of ‘maximal entropy principle’ with respect to variability of coarse-graining is one way where thermodynamically robust systems can be located.

While including successively higher orders of the same coarse-graining measure usually gives rise to quantitative differences (if there is a convergent result, that is, but this condition is not guaranteed, especially if phase transition intervenes), coarse-graining of a different nature will in general result in very different behavior in the dynamics of the open system. Let us look further at the relation of variability of coarse-graining and robustness of structure.

Sometimes the stability of a system with respect to coarse-graining is an implicit criterion behind the proper choice of a system. For example, Boltzmann’s equation governing the one-particle distribution function which gives a very adequate depiction of the physical world is, as we have seen, only the lowest order equation in an infinite (BBGKY) hierarchy. If coarse-graining is
by the order of the hierarchy – e.g., if the second and higher order correlations are ignored, then one can calculate without ambiguity the error introduced by such a truncation. The dynamics of the open system which includes dissipation effects and irreversible behavior will change little as one coarse-grains further to higher and higher order (if the series convergences, see, e.g., Dorfman, 1981). In another approximation, for a binary gas of large mass discrepancy, if one considers the system as the heavy mass particles, ignore their mutual interactions and coarse-grain the effect of the light molecules on the heavy ones, one can turn the Boltzmann equation into a Fokker-Planck equation for Brownian motion, and get qualitatively very different results in the behavior of the system.

In general the variability of different coarse-grainings in producing a qualitatively similar result is higher (more variations allowed) when the system one works with is closer to a stable level in the interaction range or in the hierarchical order of structure of matter. The result is more sensitive to different coarse-graining measures if it is far away from a stable structure, usually falling in between two stable levels.

One tentative analogy may help to fix these concepts: robust systems are like the stable fixed points in a parameter space in the renormalization group theory description of critical phenomena: the points in a trajectory are the results of performing successive orders of the same coarse-graining routine on the system (e.g., the Kadanoff-Migdal scaling), a trajectory will form if the coarse graining routine is stable. An unstable routine will produce in the most radical situations a random set of points. Different trajectories arise from different coarse-graining routines. Neighboring trajectories will converge if the system is robust, and diverge if not. Therefore the existence of a stable fixed point where trajectories converge to is an indication that the system is robust. Only robust systems survive in nature and carry definite meaning in terms of their persistent structure and systematic evolutions. This is where the relation of coarse-graining and persistent structures enter.

So far we have only discussed the activity around one level of robust structure. To investigate the domain lying in-between two levels of structures (e.g., between nucleons and quark-gluons) one needs to first know the basic constituents and interactions of the two levels. This brings back our consideration of levels of structures above. Studies in the properties of coarse-graining can provide a useful guide to venture into the often nebulous and evasive area between the two levels and extract meaningful results pertaining to the collective behavior of the underlying structure. But one probably cannot gain new information about the fine structure and the new interactions from the old just by these statistical measures. (cf. the old bootstrapping idea in particle physics versus the quark model). In this sense, one should not expect to gain new fundamental information about quantum gravity just by extrapolating what we know about the semiclassical theory, although studying the way how the semiclassical theory takes shape (viewed as an effective theory) from a more basic quantum theory is useful. It may also be sufficient for what we can understand or care
about in this later stage of the Universe we now live in.

There are immediate consequences from these theoretical discussions for cosmology. Questions like, why the Universe should in its later stage settle into the highly symmetric state of isotropy and homogeneity? Is this a particular choice of the ‘system’ from the beginning, or is it a consequence of coarse-graining an initial larger set of possibilities both in the spacetime and the matter degrees of freedom? What are the stable coarse-graining routines? How different can the coarse-graining routines be to still produce robust results? I have just begun to explore these questions in a number of ways. They are, a) Viewing the homogeneous cosmology as the infrared sector of spacetime excitations, and using the rules of dimensional reduction as possible explanation for its prevalence. (Hu, 1990) b) Gravity as an effective theory and geometric structure as collective degrees of freedom (Sahkarov, 1968; Adler and Zee, 1984). c) Einstein’s gravity as the hydrodynamic limit of a nonlinear and nonlocal theory, drawing on the insight from the behavior of the Boltzmann equation, the BBGKY hierarchy and the long-wavelength hydrodynamic approximations. There is no time to describe them here, but I hope the discussions on the properties and origins of irreversible processes in cosmology, sketchy as they have been presented here, can help us gain a better perspective of the universality of these issues in physics and provide some theoretical basis for further discussions of their meanings.

7 Acknowledgements

I thank Raphael Sorkin for a careful and thoughtful reading of this manuscript and for evoking many interesting discussions on various issues raised in this paper. Research is supported in part by the National Science Foundation under grant PHY91-19726.

8 Questions and Comments

Cover: In his question, P. Davies has suggested that the entropy of a gravitational field might be replaced by Kolmogorov (or algorithmic) complexity. It should be noted that entropy, as well as algorithmic complexity, are descriptive complexities. Moreover, they usually agree. And in the special case of equipartition of energy (or probability), entropy and Kolmogorov complexity equal the logarithm of the number of microstates of the given macrostate.

Hartle: If we are going to consider complexity then we are going to have to ask “whose complexity is it” that is, what coarse-graining is going to be used to compute it?

Hu: Although coarse-graining has a strong element of subjectivity, those classes which lead to physical reality (including complexity) which is agreed upon by a large class of observers (including us) merit special attention. It is important
to study the *criteria* and *conditions* for these coarse-grainings to be favorably selected in the evolutionary process which give rise to persistent structures (persistent at least to the degree we can perceive them).

9 References

Adler, S. L. (1982) Rev. Mod. Phys., 54, 719.
Albrecht, A. and Steinhardt, P. J. (1982) Phys. Rev. Lett., 48, 1220.
Ashtekar, A. and Stachel, J. (1991) eds *Conceptual Problems in Quantum Gravity* (Birkhauser, Boston)
Balescu, R. (1975) *Equilibrium and Nonequilibrium Statistical Mechanics*, (Wiley, New York).
Balian, R. and Veneroni, M. (1987) Ann. Phys. (N.Y.), 174, 229-244.
Barrow, J. D. (1982) Phys. Rep., 85, 1.
Bekenstein, J. D. (1973) Phys. Rev., D7, 2333.
Bekenstein, J.D. (1974) Phys. Rev., D9, 3292.
Berger, B. K. (1992) in Proc. GR13, Cordoba, Argentina
Bogoiavlenskii, O. I. (1985) *Methods in the Qualitative Theory of Dynamical Systems in Astrophysics and Gas Dynamics* (Springer-Verlag, Berlin).
Caldeira, A. O. and Leggett, A. J. (1983) Physica, A121, 587.
Callen, H. B. and Welton, T. A. (1951) Phys. Rev. 83, 34.
Calzetta, E. (1989) Class. Quantum Grav., 6, L227.
Calzetta, E. (1991) Phys. Rev., D43, 2498.
Calzetta, E. and Hu, B. L. (1987) Phys. Rev., D35, 495.
Calzetta, E. and Hu, B. L. (1988) Phys. Rev., D37, 2878.
Calzetta, E. and Hu, B. L. (1989) Phys. Rev., D40, 656.
Calzetta, E. and Hu, B. L. (1993a) “Decoherence of Correlation Histories” in *Directions in General Relativity* Vol 2 (Brill Festschrift) eds. B. L. Hu and T. A. Jacobson (Cambridge Univ., Cambridge)
Calzetta, E. and Hu, B. L. (1993b) “From Kinetic Theory to Brownian Motion” unpublished.
Calzetta, E. and Mazzitelli, F. (1991) Phys. Rev., D42, 4066.
Candelas, P. and Sciama, D. W. (1977) Phys. Rev. Lett., 38, 1372.
Coleman, S., Hartle, J., Piran, T., and Weinberg, S. (1990) eds *Quantum Cosmology and Baby Universes* (World Scientific, Singapore).
Cornwall, J. M., Jackiw, R., and Tomboulis, E. (1974) Phys. Rev. D10, 2428.
Davies, P. C. W. (1975) J. Phys. A8, 609.
Davies, P. C. W., Ford L, and Page, D. (1987) Phys. Rev., D34, 1700.
De Groot, S. R., van Leeuwen, W. A. and van Weert, Ch. G. (1980) Relativistic Kinetic Theory (North-Holland, Amsterdam)
DeWitt, B. S. (1967) Phys. Rev., 160, 1113.
Dorfman, R. (1981) in Perspectives in Statistical Physics Vol. IX, Eds. H.J. Raveche (North-Holland, Amsterdam).
Einstein, A. (1905) Ann. Phys. (Leipzig) 17, 549
Feynman, R. P. and Vernon, F. L. (1963) Ann. Phys. (N.Y.), 24, 118.
Ford, G. W., Kac, M. and Mazur, P. (1963) J. Math. Phys. 6, 504
Gell-Mann, M. and Hartle, J. B. (1990) in Complexity, Entropy and the Physics of Information ed. W. H. Zurek (Addison-Wesley, N.Y.).
Gell-Mann, M. and Hartle, J. B. (1993) Phys. Rev., 47
Goldwirth, D. S. (1991) Phys. Lett. B256, 354.
Grabert, H. (1982) Projection Operator Techniques in Nonequilibrium Statistical Mechanics (Springer Verlag, Berlin).
Grabert H., Schramm, P. and Ingold, G. (1988) Phys. Rep., 168, 115 .
Guth, A., (1981) Phys. Rev., D23, 347.
Halliwell, J. J. (1993) Quantum Cosmology (Cambridge Univ. Press, Cambridge).
Hartle, J. B. (1983) in The Very Early Universe, eds. G. Gibbons, S. W. Hawking and S. Siklos (Cambridge Univ. Press, Cambridge)
Hartle, J. B. and Hawking, S. W. (1983) Phys. Rev., D28, 2960.
Hartle, J. B. and Hu, B. L. (1980) Phys. Rev., D21, 2756.
Hawking, S. W. (1975) Commun. Math. Phys., 87, 395.
Heller, J. P. (1960) Am. J. Phys., 28, 348-353.
Hu, B. L. (1982) in Proc. Second Marcel Grossmann Meeting 1979, Ed. R. Ruffini, (North-Holland, Amsterdam).
Hu, B. L. (1983) Phys. Lett., 97A, 368.
Hu, B. L. (1984) in Cosmology of the Early Universe, Ed. L. Z. Fang and R. Ruffini (World Scientific, Singapore).
Hu, B. L. (1987) “Recent Development in Cosmological Theories”, IASS Preprint, Princeton, IASSNS-HEP87/15.
Hu, B. L. (1989) Physica, A158, 399.
Hu, B. L. (1990) “Quantum and Statistical Effects in Superspace Cosmology” in Quantum Mechanics in Curved Spacetime, ed. J. Audretsch and V. de Sabbata (Plenum, London, 1990)

Hu, B. L. (1991a) “Statistical Mechanics and Quantum Cosmology”, in Proc. Second International Workshop on Thermal Fields and Their Applications, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991)

Hu, B. L. (1991b) “Coarse-Graining and Backreaction in Inflationary and Minisuperspace Cosmology” in Relativity and Gravitation: Classical and Quantum, Proc. SILARG VII, Cocoyoc, Mexico 1990, eds. J. C. D’Olivo et al (World Scientific, Singapore, 1991)

Hu, B. L. (1993) “Quantum Statistical Processes in the Early Universe” in Quantum Physics and the Universe, Proc. Waseda Conference, Aug. 1992 ed. M. Namiki, K. Maeda, et al (Pergamon Press, Tokyo, 1993)

Hu, B. L. and Kandrup, H. E. (1987) Phys. Rev., D35, 1776.

Hu, B. L. and Parker, L. (1978) Phys. Rev., D17, 933.

Hu, B. L. and Pavon, D. (1986) Phys. Lett., 180B, 329.

Hu, B. L., Paz, J. P., and Sinha, S. (1993) “Minisuperspace as a Quantum Open System” in Directions in General Relativity Vol. 1, (Misner Festschrift) eds B. L. Hu, M. P. Ryan and C. V. Vishveswara (Cambridge Univ., Cambridge)

Hu, B. L., Paz, J. P. and Zhang, Y. (1992) Phys. Rev., D45, 2843.

Hu, B. L., Paz, J. P. and Zhang, Y. (1993a) “Quantum Brownian Motion in a General Environment II. Nonlinear coupling and perturbative approach” Phys. Rev. D47 (1993)

Hu, B. L., Paz, J. P. and Zhang, Y. (1993b) “Stochastic Dynamics of Interacting Quantum Fields” Phys. Rev. D (1993)

Hu, B. L., Paz, J. P. and Zhang, Y. (1993c) “Quantum Origin of Noise and Fluctuation in Cosmology” in Proc. Conference on the Origin of Structure in the Universe Chateau du Pont d’Oye, Belgium, April, 1992, ed. E. Gunzig and P. Nardone (NATO ASI Series) (Kluwer, Dordrecht, 1993)

Hu, B. L. and Sinha, Sukanya (1993a) ”Fluctuation-Dissipation Relation in Cosmology” Univ. Maryland preprint

Hu, B. L. and Sinha, Sukanya (1993b) ”Spacetime Coarse-Graining and Gravitational Entropy” Univ. Maryland preprint

Hu, B. L. and Zhang, Y. (1990) “Coarse-Graining, Scaling, and Inflation” Univ. Maryland Preprint 90-186 (1990)

Hu, B. L. and Zhang, Y. (1992) “Uncertainty Principle at Finite Temperature” Univ. Maryland preprint (1992)
Hu, B. L. and Zhang, Y. (1993) “Quantum and Thermal Fluctuations, Uncertainty Principle, Decoherence and Classicality” in Quantum Dynamics of Chaotic Systems: Proc. Third International Workshop on Quantum Nonintegrability, Drexel University, Philadelphia, May 1992, ed. J. M. Yuan, D. H. Feng, and G. M. Zaslavsky (Gordon and Breach, Langhorne, 1993)

Isham, C. J. (1991) “Conceptual and Geometrical Problems in Quantum Gravity” Lectures at the Schladming Winter School, Imperial College preprint TP/90-91/14

Joos, E. and Zeh, H. D. (1985) Z. Phys. B59, 223.

Kandrup, H. E. (1988) Phys. Rev. D37, 3505.

Kandrup, H. E. (1988) Class. Quantum Grav. 5, 903

Keldysh, L. V. (1964) Zh. Eksp. Teor. Fiz. 47, 1515 [Sov. Phys. JETP 20, 1018 (1965)]

Kreuzer, H. J. (1981) Nonequilibrium Thermodynamics and Its Statistical Foundations (Oxford Univ., Oxford).

Kubo, R. (1959) Lectures in Theoretical Physics, Vol 1, pp 120-203 (Interscience, N. Y. 1959)

Kuchar, K. (1992) “Time and Interpretations in Quantum Gravity” in Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics, eds. G. Kunstatter, D. Vincent and J. Williams (World Scientific, Singapore).

Kuchar, K. and Ryan, M. P., Jr., (1986) In Gravitational Collapse and Relativity Ed. H. Sato and T. Nakamura (World Scientific, Singapore).

Kuchar, K. and Ryan, M. P. Jr., (1989) Phys. Rev., D40, 3982.

Linde, A. (1982) Phys. Lett., 108B, 389.

Lynden-Bell D. and Wood, R. (1967) MNRAS 136, 101.

Lynden-Bell D. and Lynden-Bell, R. M. (1977) MNRAS 181, 405.

Ma, S. K. (1985) Statistical Mechanics (World Scientific, Singapore).

Misner, C. W. (1969) Phys. Rev. Lett., 22, 1071.

Misner, C. W. (1972) in Magic Without Magic, Ed. J. Klauder (Freeman, San Francisco).

Morikawa, M. (1989) Phys. Rev., D40, 4023.

Mottola, E. (1986) Phys. Rev., D33, 2126.

Page, D. M. (1984) private communication

Parker, L. (1969) Phys. Rev., 183, 1057.
Parker, L. (1986) in: The Quantum Theory of Gravity, S. Christensen, Ed. (Adam Hilger, S. Bristol, 1986).

Paz, J.P. (1990) Phys. Rev., D40, 1054.

Paz, J. P. and Sinha, Sukanya (1991) Phys. Rev. D44, 1038.

Paz, J. P. and Sinha, Sukanya (1992) Phys. Rev. D45, 2823.

Peebles, P. J.E. (1971) Physical Cosmology (Princeton Univ. Press, Princeton).

Penrose, R. (1979) “Singularities and Time-Asymmetry” in General Relativity: an Einstein Centenary Survey, eds. S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979)

Prigogine, I. (1962) Introduction to Thermodynamics of Irreversible Processes 2nd ed. (Wiley, New York).

Rubin, R. (1960) J. Math. Phys. 1, 309.

Ryan, M. P., Jr., and Shepley, L.C. (1975) Homogeneous Relativistic Cosmologies (Princeton Univ. Press, Princeton).

Sakharov, A. D. (1967) Dok. Akad. Nauk. SSR, 177, 70 [Sov. Phys. Dokl 12 (1968) 1040].

Sato, K. (1981) Phys. Lett. 99B, 66.

Schwinger, J. S. (1961) J. Math. Phys. 2, 407.

Sciama, D. W. (1979) in Centenario di Einstein Editrice Giunti Barbaras-Universitaria.

Sextl, R. U. and Urbantke, H. K. (1969) Phys. Rev., 179, 1247

Sinha, Sukanya (1991) Ph. D. Thesis, University of Maryland.

Sinha, Sukanya and Hu, B. L. (1991) Phys. Rev., D44, 1028-1037.

Sinha, Supurna and Sorkin, R. D. (1992) Phys. Rev., B45, 8123-8126.

Smolin, L. (1985) Gen. Rel. Grav., 7, 417-437.

Sorkin, R. D. (1986) Phys. Rev. Lett. 56, 1885-1888.

Sorkin, R. D. (1993) Int. J. Theor. Phys.

Sorkin, R. D., Wald, R. M. and Zhang, Z. J. (1981) Gen. Rel. Grav., 12, 1127.

Unruh, W. G. (1976) Phys. Rev., D14, 870.

Unruh, W. and Zurek, W. H. (1989) Phys. Rev. D40, 1071.

Vilenkin, A. (1986) Phys. Rev., D33, 3560.

Weinberg, S. (1972) Gravitation and Cosmology (John wiley, N.Y.).

Weinberg, S. (1980) Phys. Lett. B91, 51
Wheeler, J. A., (1968) in *Battelle Rencontres*, eds. C. DeWitt and J. A. Wheeler (Benjamin, New York).

Woo, C. H. (1989) Phys. Rev., *D39*, 3174.

Zee, A. (1979) Phys. Rev. Lett., *42* 417.

Zeh, H. D. (1986) Phys. Lett. *A116*, 9.

Zel’dovich, Ya. B. and Starobinsky, A. A. (1971) Zh. Eksp. Teor. Fiz *61*, 2161 [Sov. Phys. JETP, *34*, 1159 (1972)].

Zel’dovich, Ya. B. (1970) Pis’ma Zh Eksp. Teor. Fiz. *12*, 443 [JETP Lett., *12* (1970) 307].

Zhang, Yuhong (1990) Ph. D. Thesis, University of Maryland.

Zurek, W. H. (1981) Phys. Rev. *D24*, 1516.

Zurek, W. H. (1982) Phys. Rev. *D26*, 1861.

Zwanzig, R. (1961) in *Lectures in Theoretical Physics, Vol. III*, Eds. W.E. Britten, B.W. Downes and J. Downes (Interscience, New York).