Recent developments in radiative $B$ decays

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Abstract. We report on recent theoretical progress in radiative $B$ decays. We focus on a calculation of logarithmically enhanced QED corrections to the branching ratio and forward-backward asymmetry in the inclusive rare decay $\bar{B} \to X_s \ell^+\ell^-$, and present the results of a detailed phenomenological analysis. We also report on the calculation of NNLO QCD corrections to the inclusive decay $\bar{B} \to X_s \gamma$. As far as exclusive modes are concerned we consider transversity amplitudes and the impact of right-handed currents in the exclusive $\bar{B} \to K^*\ell^+\ell^-$ decay. Finally, we state results for exclusive $\bar{B} \to V$ decays, notably the time-dependent CP-asymmetry in the exclusive $\bar{B} \to K^*\gamma$ decay and its potential to serve as a so-called “null test” of the Standard Model, and the extraction of CKM and unitarity triangle parameters from $\bar{B} \to (\rho, \omega)\gamma$ and $\bar{B} \to K^*\gamma$ decays.

1. Introduction
As flavor changing neutral current (FCNC) processes, radiative $b \to (s, d)$ transitions are very sensitive to physics beyond the Standard Model (SM) since virtual effects due to heavy degrees of freedom are not overwhelmed by large tree-level contributions and hence their impact is not necessarily suppressed with respect to the SM contributions. Therefore these transitions represent an ideal testing ground for an indirect search for new physics (NP), and they are examined with unprecedented precision both on the theoretical and experimental side. Remarkable results have been already achieved in this field, and more data still to come from the $B$ factories, from LHC and from a possible SuperB factory is awaited with excitement.

2. The inclusive decay $\bar{B} \to X_s \ell^+\ell^-$
Alike many transitions in flavor physics, the dynamics of the decay $\bar{B} \to X_s \ell^+\ell^-$ is most conveniently described by an effective Hamiltonian with the top quark and the heavy gauge bosons being integrated out. Within this framework, occurring large logarithms which stem from widely separated scales $\mathcal{O}(m_t, m_W, Z)$ and $\mathcal{O}(m_b)$ can be efficiently resummed order by order in perturbation theory. The corresponding effective operators and their associated Wilson coefficients can e.g. be found in [1]. The QCD corrections to the decay $\bar{B} \to X_s \ell^+\ell^-$ have achieved NNLO accuracy [2-7], and also non-perturbative [8-14] and higher order electroweak corrections [1,7] have been derived. The latter give rise to logarithmically enhanced corrections proportional to $\alpha_{em} \ln(m_b^2/m_t^2)$ which vanish upon integration over the entire phase space but are numerically relevant if one is restricted to certain regions of the invariant mass $q^2$ of the leptons [1]. The two most important quantities in $\bar{B} \to X_s \ell^+\ell^-$ are the differential branching ratio (BR) and forward backward asymmetry (FBA). In particular, the value $q_0^2$ for which the differential FBA vanishes is one of the most precise predictions in flavor physics with a
Figure 1. Differential BR as a function of the lepton inv. mass. Second curve from bottom (blue): SM. Second curve from top (thin black): Sign of $C_7$ reversed w.r.t. SM. See [21] for more details.

Figure 2. Forward backward asymmetry as a function of the lepton inv. mass. Curve 2: Reversed sign of $C_7$ w.r.t. SM. Curves 1, 3: Sign of $C_{10}$ reversed in addition to curves SM, 2 respectively [22].

Theoretical uncertainty of order 5%. A thorough phenomenological analysis which includes all known corrections yields for the BR integrated over the low-$q^2$ ($q^2 \in [1, 6]$ GeV$^2$) region

$$B(B \to X_s \mu \mu) = (1.59 \pm 0.11) \cdot 10^{-6},$$

where the indicated uncertainty includes only the parametric and perturbative ones [1]. No additional uncertainty for the unknown subleading non-perturbative corrections has been included. In particular, the uncalculated $\mathcal{O}(\alpha_s(\mu_b) \Lambda_{QCD}/m_{c,b})$ non-perturbative corrections imply an additional uncertainty of around 5% in the above formula. The current experimental world average of this quantity is $(1.60 \pm 0.51) \cdot 10^{-6}$ [15, 16]. For the BR integrated over the high-$q^2$ ($q^2 > 14.4$ GeV$^2$) region one obtains [17]

$$B(B \to X_s \mu \mu) = (2.42 \pm 0.65) \cdot 10^{-7}. $$

The corresponding experimental values are $(4.18 \pm 1.17_{\text{stat}} ^{+0.61}_{-0.68_{\text{sys}}}) \cdot 10^{-7}$ [15] and $(5 \pm 2.5_{\text{stat}} ^{+0.8}_{-0.7_{\text{sys}}}) \cdot 10^{-7}$ [16]. As far as the FBA is concerned, we find for the position $q^2_0$ for which the FBA vanishes [17]

$$q^2_0 \mu \mu = (3.54 \pm 0.09) \text{GeV}^2.$$  

By the end of the $B$ factories the fully differential shape of the BR and FBA will not be accessible, contrary to their integrals over one or two bins in the low-$q^2$ region. However, these quantities will already allow to discriminate between different NP scenarios (see Figures 1 and 2 as well as Ref. [18]), and their SM predictions are given in Ref. [17]. Quite recently two additional quantities related to $B \to X_s \ell^+ \ell^-$ have been proposed. One is related to the structure of the double differential decay width and represents a third linearly independent quantity in addition to the BR and FBA [19]. The other one is the differential decay width integrated over the high-$q^2$ region and normalized to the semileptonic $b \to u \ell \nu$ rate with the same cut [20] in order to significantly reduce the error due to parameters in the non-perturbative $1/m_b$ corrections [17, 20]. Moreover, an experimental determination of this quantity might become feasible by the end of the $B$ factories.

3. The inclusive decay $B \to X_s \gamma$

The inclusive rare decay $B \to X_s \gamma$ is the most prominent among the radiative $b \to s$ transitions. Its branching ratio has been measured at several accelerator facilities [23–27], yielding a world
average of [28] $B(B \to X_s\gamma)^{\text{exp.}} = (3.55 \pm 0.24^{+0.03}_{-0.10} \pm 0.03) \times 10^{-4}$ for a photon energy cut of $E_{\text{cut}} > 1.6$ GeV in the restframe of the $B$. The errors are combined statistical and systematic, systematic due to extrapolation in $E_{\text{cut}}$, and due to the $b \to d\gamma$ fraction. By the end of the $B$ factories these errors are expected to decrease to around 5% due to larger statistics and possible lower cuts on $E_{\gamma}$. This also calls for precise predictions on the theoretical side. Alike in $B \to X_s\ell^+\ell^-$ this decay is described in the framework of the effective Hamiltonian. The different steps of calculating matching conditions for the Wilson coefficients at the electroweak scale [6, 29], determining the anomalous dimensions matrix which governs the running of the Wilson coefficients [30–32], and finally the extraction of on-shell matrix elements [33–40], have now achieved the NNLO level and involve multi (two to four)-loop calculations. The resulting SM prediction, which includes also electroweak corrections [41–45] as well as non-perturbative $\Lambda^2/m_c^2$ corrections [9,46–52], is $B(B \to X_s\gamma)^{E_{\gamma}>1.6\text{GeV}} = (3.15\pm0.23)\times10^{-4}$ [53]. The unknown $\mathcal{O}(\alpha_s\Lambda/m_b)$ corrections are estimated to be of order 5% [53, 54]. The other uncertainties which contribute to the total error are parametric uncertainties (3%), scale uncertainties (3%), and an uncertainty (3%) due to an interpolation in $m_b$ in the computation of the three-loop matrix elements of $P_{1,2}$. In going from NLO to NNLO accuracy, the scale dependence gets tremendously reduced, which is in particular true for the charm scale $\mu_c$. This scale first enters at NLO and hence one needs NNLO precision in order to tame its dependence [53]. For other recent work on this decay mode see Refs. [55,56] and references therein.

4. Transversity amplitudes in $B \to K^*(K\pi)\ell^+\ell^-$

The matrix element of the $B \to K^*$ transition can be parameterized in terms of seven a priori independent form factors. However, in the limit of a heavy quark and a large $E_{K^*}$ the seven $B \to K^*$ form factors reduce to two universal ones [57,58]. Those form factors in turn cancel out in specific transverse asymmetries, which then depend on short-distance information only [59,60], and SM prediction on these transverse asymmetries can be found in Refs. [59,60]. However, there are NP scenarios where there can be huge deviations from the SM values of these asymmetries. In Ref. [60] one can find examples for the case of the MSSM with $R$-parity and non-MFV in down-squarks soft-breaking terms. Transverse asymmetries therefore provide a theoretically clean way to analyse the chiral structure of the $b \to s$ current. For another very recent analysis on angular distributions in $B \to K\ell^+\ell^-$ see Ref. [61].

5. Time-dependent CP-asymmetry in $B \to K^*\gamma$

The time-dependent CP-asymmetry (TDCPA) in $B \to K^*\gamma$ [62],

$$A_{\text{CP}}(t) = \frac{\Gamma(B^0(t) \to K^{*0}\gamma) - \Gamma(B^0(t) \to K^{0*}\gamma)}{\Gamma(B^0(t) \to K^{*0}\gamma) + \Gamma(B^0(t) \to K^{0*}\gamma)} = S \sin(\Delta m_B t) - C \cos(\Delta m_B t),$$

is believed to be small in the SM. Due to the operator $Q_7 = \bar{s}\sigma^{\mu\nu}F_{\mu\nu}(m_bP_R + m_sP_L)b$, which follows from the structure of the weak interaction, the emitted photon is predominantly left-handed in $b$ and right-handed in $b$ decays. Hence the TDCPA is suppressed by a factor of $\mathcal{O}(m_s/d/m_b)$. On the other hand, the TDCPA can be enhanced by terms of $\mathcal{O}(m_{\text{heavy}}/m_b)$ due to a helicity flip on heavy internal lines in NP models. This quantity is therefore considered a prime candidate for a so-called “null-test” of the SM [63]. However, there is a possible enhancement [64,65] also in the SM due to gluon emission from a quark loop generated by operators like $Q_2 = (\bar{c}\gamma^\mu P_Lb)(\bar{s}\gamma_\mu P_Lc)$ [66,67]. If these contributions turn out to be small, one can interpret a possible large value of the TDCPA as a signal for NP [68]. A value for $S(B \to K^*\gamma)$ in Eq. (4) has been derived at several places in the literature. The analyses in Refs. [68,69], which combine QCD-factorisation with QCD sum rules on the light-cone to estimate long-distance photon emission and soft-gluon emission from quark loops yield...
\[ S = -0.022 \pm 0.015^{+0.005}_{-0.01} \text{ and } S_{\text{soft gluons}} = 0.005 \pm 0.01, \] whereas a conservative dimensional estimate (from a SCET based analysis) gives \( |S_{\text{soft gluons}}| \approx 0.06 \) [64, 65]. There are, however, arguments that for the \( B \to K^* \gamma \) channel the number extracted in Refs. [64, 65] can be smaller, see Ref. [55] and references therein for a recent discussion. The calculation in pQCD yields \( S_{\text{pQCD}} = -0.035 \pm 0.017 \) [70], where the effect is mainly from hard gluons, and soft ones are treated in a model dependent way. The experimental world average reads [28, 71, 72] \( S_{\text{HFAG}} = -0.28 \pm 0.26 \). While LHC will have better performance in decays like \( B_s \to \phi \gamma \), a SuperB factory can measure \( S(B \to K^* \gamma) \) with an uncertainty as low as 0.04 at 50 ab\(^{-1}\) [21].

6. Extraction of CKM and UT parameters from \( B \to (\rho, \omega) \gamma \) and \( B \to K^* \gamma \) decays

We finally would like to report on an analysis which was performed in Ref. [68]. One considers ratios of branching ratios (BR) of exclusive radiative \( B \) decays since the ratios of the occurring form factors are much better known than the individual form factors themselves. The following two observables are particularly interesting, \( R_{\rho/\omega} = \frac{\bar{B}(B \to (\rho, \omega) \gamma)}{\bar{B}(B \to K^* \gamma)} \), \( R_\rho = \frac{\bar{B}(B \to \rho \gamma)}{\bar{B}(B \to K^* \gamma)} \), where the BRs are CP and isospin averaged. The knowledge of these two quantities — and a few other parameters — allows one to extract \( |V_{td}/V_{ts}| \) as well as the UT angle \( \gamma \), where the extraction of the latter involves a twofold degeneracy \( 2\pi \leftrightarrow 2\pi - \gamma \). The extraction of \( \gamma \) from tree-level CP asymmetries in \( B \to D^{(*)} K^{(*)} \) [73] on the other hand carries a twofold degeneracy \( \pi \leftrightarrow \pi + \gamma \). Combining these two different degeneracies hence allows one to unambiguously determine the UT angle \( \gamma \). With the most recent results from the \( B \)-factories [28, 74--76] for the above ratios, the authors of Ref. [68] find, under the assumption of a unitary CKM matrix, that the solution \( \gamma < 180^\circ \) is clearly favored,

\[
\text{BaBar: } \frac{|V_{td}/V_{ts}|}{\rho/\omega} = 0.199^{+0.022}_{-0.025} \text{ (exp) } \pm 0.014 \text{ (th)} \quad \leftrightarrow \quad \gamma = (61.0^{+13.5}_{-16.0} \text{ (exp)} + 8.9 \text{ (th)})^\circ \\
\text{Belle: } \frac{|V_{td}/V_{ts}|}{\rho} = 0.207^{+0.028}_{-0.033} \text{ (exp) } \pm 0.015 \text{ (th)} \quad \leftrightarrow \quad \gamma = (65.7^{+17.3}_{-20.7} \text{ (exp)} + 8.9 \text{ (th)})^\circ.
\]

Acknowledgments

I would like to thank the organizers of EPS 2007 and the convenors of the flavor session for the excellent organization of the conference and for creating an inspiring atmosphere. I am grateful to Tobias Hurth, Enrico Lunghi, and Mikolaj Misiak for a careful reading of the manuscript and for valuable comments. This work was supported by Deutsche Forschungsgemeinschaft SFB/TR 9 “Computergestützte Theoretische Teilchenphysik”.

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