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Gauge invariance, Lorentz covariance and canonical quantization in nucleon structure studies

Received: date / Accepted: date

Abstract There are different operators of quark and gluon momenta, orbital angular momenta, and gluon spin in the nucleon structure study. The precise meaning of these operators are studied based on gauge invariance, Lorentz covariance and canonical quantization rule. The advantage and disadvantage of different definitions are analyzed. A gauge invariant canonical decomposition of the total momentum and angular momentum into quark and gluon parts is suggested based on the decomposition of the gauge potential into gauge invariant (covariant) physical part and gauge dependent pure gauge part. Challenges to this proposal are answered.

Keywords Physical and pure gauge potentials; Gauge invariant canonical quark and gluon momenta, orbital angular momenta and spins; Homogeneous and non-homogeneous Lorentz transformations; Gauge invariant decomposition and gauge invariant extension; Classical and quantum measurements.

1 Introduction

After more than a quarter century of measurements by different groups, the quark spin contribution to the nucleon spin has been found to be only about one third of its known value, while the contribution from polarized gluons seems to be quite small. Attention therefore turns to the contributions from the orbital motion of quarks, and the spin and orbital motion of antiquarks and gluons. It is then relevant to ask if the spin and orbital angular momentum (OAM) of gluons inside nucleons are separately measurable.

The first issue is gauge invariance. This means in QED that calculations using different vector potentials that generate the same electromagnetic (em) fields give the same physical results. Many formulas describing the motion of electrons in em fields appear at first sight to be gauge-dependent, with non-unique possible values for the vector potential used, and hence of questionable measurability. Yet the momentum and OAM of electrons moving in em fields, and especially the energies of atomic electrons have all been successfully measured experimentally at a time when the corresponding theoretical descriptions were not explicitly gauge invariant. These measurements owe their success to a hidden or implicit gauge invariance in the formulas used. We shall describe, in Section 2, how hidden gauge invariance arises not just in free space and in QED in the Coulomb gauge, but also in other members of a certain family of gauges including the Lorenz gauge.

The confusion concerning gauge invariance and measurability would not have appeared if all formulas are first made explicitly gauge-independent. Explicitly gauge invariant expressions for the momentum and for the decomposition of the total AM of a gauge boson into its spin and OAM parts
seem to be unknown before our proposals for them. We shall review our proposals, and a few other related proposals in Section 3.

A number of objections have been raised against our operators by X. Ji. We list some of them below, together with our response to each in the nutshell drawn as parentheses:

1. Our momentum operator does not contain the gauge interaction. (It should not.)
2. Our decomposition is not Lorentz covariant. (It is Lorentz covariant.)
3. Our operators are nonlocal, and therefore unacceptable. (They are nonlocal but measurable.)

In the final Section 4, we shall explain our responses in more detail and discuss a number of related issues.

This short review gives only a schematic analysis of these issues. More details will be given elsewhere.

2 Hidden or implicit gauge invariance

In the standard Yang-Mills gauge theory, a physical gauge invariant interaction can be added to a free Dirac equation by introducing a physical gauge field. We are here also interested in constructing a gauge invariant theory that contains no physical interaction at all. The free Dirac equation for an electron of charge $e$ then takes the gauge invariant form (in natural units $\hbar = c = 1$)

$$[i\gamma^\mu(\partial_\mu + ieA^\prime_{\mu,\text{pure}}) - m] \psi' = 0,$$

where the “pure gauge” potential $A^\prime_{\mu,\text{pure}}$ is introduced solely to cancel an arbitrary local gauge transformation $U(x)$ added to a Dirac wave function $\psi(x)$

$$\psi'(x) = U(x)\psi(x), \quad U(x) = e^{-ie\omega(x)}; \quad U^{-1}(x) [\partial_\mu + ieA_{\mu,\text{pure}}(x)] U(x) = \partial_\mu, \quad \text{if} \quad A_{\mu,\text{pure}}(x) = \partial_\mu\omega(x).$$

Then $\psi(x)$ satisfies an unprimed version of Eq. (1) with $A_{\mu,\text{pure}} = 0$, and is free of any arbitrary local gauge function (i.e., $\omega(x) = 0$). A vector potential $A_{\mu,\text{pure}}$ if present is called pure gauge or nondynamical because it does not give rise to a physical gauge interaction:

$$F^{\mu\nu}_{\text{pure}} = \partial^\mu A^\nu_{\text{pure}} - \partial^\nu A^\mu_{\text{pure}} = 0.$$  \hspace{1cm} (3)

The unprimed Dirac equation for $\psi(x)$, with the immaterial $A_{\mu,\text{pure}} = 0$ left out, appears at first sight to be gauge dependent, but it is not. Its gauge invariance is only hidden or implicit.

Suppose next that an em interaction is also present. Then the associated gauge potential will contain a physical part as well as a nonphysical pure gauge part: $A_{\mu} = A_{\mu,\text{pure}} + A_{\mu,\text{phys}}$, where

$$\partial^2 A_{\mu,\text{phys}} - \partial_\mu\partial_i A_{i,\text{phys}} = \partial_\mu F_{\mu}(x).$$

We are interested in the solution for $A_{\mu,\text{phys}}$ satisfying the physical transverse wave condition

$$\partial_i A_{i,\text{phys}} = 0.$$  \hspace{1cm} (5)

This condition also defines the traditional Coulomb gauge, one of many choices of gauge that allows $A_{\mu}$ to be calculated uniquely from a given em field $F$. In the Coulomb gauge, the physical em field $F$ of a photon in free space reside in the 2D transverse “physical subspace” perpendicular to the photon momentum.

Gauge transformations cannot change the em fields in the photon’s 2D physical space, by definition. (Technically, this happens because $A_{\text{phys}}$ commutes with $U(x)$. Such a condition is easily satisfied in QED, but is harder to realize in non-Abelian gauge theories.) They can only change $A_{\mu,\text{pure}}$. That is, $A_{\mu,\text{pure}}$ carries the whole gauge degree of freedom, while $A_{\mu,\text{phys}}$ is gauge invariant:

$$A_{\mu,\text{phys}} = A_{\mu,\text{phys}}, \quad A_{\mu,\text{pure}} = A_{\mu,\text{pure}} + \partial_\mu\omega'(x).$$  \hspace{1cm} (6)

Thus through a gauge transformation, one can eliminate the pure gauge part $A_{\mu,\text{pure}}$ completely, while leaving the gauge invariant part $A_{\mu,\text{phys}}$ intact to give Dirac’s gauge invariant result. Formulas that show $A$ alone might still be gauge invariant, though only implicitly.
Physically the simplest and most interesting $A_{\mu,\text{pure}}$s for photons are those that do not intrude into the photon physical subspace: For them $A_{\mu,\text{pure}}$s have only $\parallel$ and timelike components in the 2D subspace orthogonal to the physical subspace. For all these gauges, $A_{\text{phys}} = A_{\perp}$. These gauge transformations include many gauge choices of physical interest, including the Lorenz gauge.

If one’s emphasis is not on photons, other physically motivated subsidiary conditions different from Eq. (5) might be more useful. The resulting $A_{\mu,\text{phys}}$s are in general different from each other, and different from the photon quantity defined here. To each choice of physically motivated subsidiary conditions there is a family of pure gauge transformations that are nondynamical, with $F_{\mu\nu}^{\text{pure}} = 0$.

3 Gauge invariant decomposition of momentum and AM of a gauge system

To study the spin structure of nucleon, an $SU(3)$ color gauge field system, one wants to separate quark and gluon spin and OAM contributions. Jaffe and Manohar (JM) first obtained such a decomposition \[ J = \int d^3x \bar{\psi} \frac{1}{2} \Sigma \psi + \int d^3x \bar{\psi} x \times \frac{1}{i} D \psi + \int d^3x E \times A + \int d^3x E^i x \times A^i . \] (7)

The advantage of this decomposition is that the individual terms all satisfy the $SU(2)$ AM algebra; they are proper quark and gluon spins and OAMs. However, the terms not involving the quark spin $\Sigma$ are not gauge invariant.

The gauge non-invariance originates from the quantization of the momentum and OAM of a charged particle moving in an em field \[ p = m v + e A , \quad L = x \times p . \] (8)

Classically, they are gauge dependent and so are not measurable. Canonical quantization quantizes them as canonical momentum and OAM no matter which gauge is used. Feynman had explained why we quantize the canonical momentum $p$ rather than the mechanical momentum $m v$. Canonical quantization appears to be gauge invariant. However, since the wave function of a charged particle is still gauge dependent, the MEs of both canonical momentum and OAM remain gauge dependent. That is, they are still not measurable.

To remedy the gauge non-invariance of JM’s decomposition, both our group and Ji obtained a gauge invariant decomposition in 1997 \[ J = \int d^3x \bar{\psi} \frac{1}{2} \Sigma \psi + \int d^3x \bar{\psi} x \times \frac{1}{i} D_{\text{pure}} \psi + \int d^3x E \times A_{\text{phys}} + \int d^3x E^i x \times A^i_{\text{phys}} , \] (9)

where $D = \nabla - ieA$. The advantage of this decomposition is that each term is individually gauge invariant. So it has been used in theoretical studies of the nucleon spin in recent years. However we had pointed out from the very beginning that, excepting the quark spin term, the individual term does not satisfy the $SU(2)$ AM algebra. In addition, the term for the gluon total AM has not been decomposed further into spin and orbital parts \[ J = \int d^3x \bar{\psi} \frac{1}{2} \Sigma \psi + \int d^3x \bar{\psi} x \times \frac{1}{i} D_{\text{pure}} \psi + \int d^3x E \times A_{\text{phys}} + \int d^3x E^i x \times A^i_{\text{phys}} . \] (10)

In order to obtain a decomposition in which the individual term is not only gauge invariant but also satisfies the Poincaré algebra and keeps the standard physical meaning as much as possible, we proposed a new gauge invariant canonical decomposition in 2008 \[ J = \int d^3x \bar{\psi} \frac{1}{2} \Sigma \psi + \int d^3x \bar{\psi} x \times \frac{1}{i} D_{\text{pure}} \psi + \int d^3x E \times A_{\text{phys}} + \int d^3x E^i x \times A^i_{\text{phys}} . \] (10)
Here $D_{\text{pure}} = \nabla - ieA_{\text{pure}}$. So $-iD_{\text{pure}}$ is the gauge invariant version of the canonical momentum that reduces to the standard canonical momentum when $A_{\text{pure}} = 0$. The three components of $-iD_{\text{pure}}$ commute with each other, the same as the canonical momentum. The second term satisfies the standard OAM algebra. The commutators between this new momentum and OAM is the same as those of canonical ones in the Poincaré algebra. Due to these properties they are the gauge invariant canonical momentum and OAM. We also obtain a corresponding decomposition of the total momentum,

$$P = \int d^3x \, \psi^\dagger D_{\text{pure}}^i \psi + \int d^3x \, E^i \nabla A_{\text{phys}}^i.$$  \hspace{1cm} (11)

The momenta and the OAMs in Eqs. (10) and (11) keep the standard relation $\ell = x \times p$ in the integrand. In contrast, the gauge invariant decomposition (9) gives instead the density $x \times p = j = \ell + s$, where $p = E \times B$ is the Poynting vector. It is not hard to check that the individual terms in the last two decompositions, Eqs. (10) (11), are all gauge invariant and satisfy the canonical momentum and AM quantization rule. Complications might arise if one uses the helicity representation for massless bosons, here photons or gluons, due to the transversality of $A_{\text{phys}}$.

The decomposition (10) reduces to the JM decomposition (7) in the Coulomb gauge. So the JM operators are perfectly acceptable when used in the Coulomb gauge. Likewise, the usual gauge-dependent canonical operators for electron momentum and OAM when used in the Coulomb gauge give the same results as gauge invariant operators. The use of Coulomb gauge explains why the many results calculated with “gauge dependent” canonical momentum and OAM are consistent with measurements. If one goes beyond the Coulomb gauge, however, one can get wrong results [13].

4 Discussion

Ji tried to use his gauge invariant mechanical or kinematic momentum to explain why the gauge dependent canonical momentum should be used in quantum mechanics to compare theory with experiment [14]. He argued that the vector potential is $(\alpha_e^2)_{\text{em}}$ suppressed when compared to the canonical momentum, and so the latter is approximately gauge invariant. Gauge invariance is an exact symmetry. There is only gauge invariance or non-invariance; approximate gauge invariance is still gauge non-invariance. He also misinterpreted Feynman’s idea about the quantum mechanical momentum. Feynman wrote, “I’d like to make a brief digression to show you what this is all about - why there must be something like Eq.(21.15) ($p$ momentum = $m$v + $q$A) in the quantum mechanics” [7]. Here Feynman clearly asserted that only the $p$-momentum (canonical) and not the kinematical $mv$-momentum should be quantized in quantum mechanics. Because canonical quantization quantizes the canonical momentum and coordinate, the three components of the mechanical (kinematical) momentum do not commute; they cannot be diagonalized simultaneously. They then do not make up a complete commuting set to give a momentum representation that is completely equivalent to the coordinate representation based on a position operator whose components commute.

Up to now nobody has solved the eigenvalue equation for the kinematic momentum operator. Even if it would be solved, the eigenfunctions will not be the plane waves now used in quantum physics. The resulting $x \leftrightarrow p$ representation transformations will not be the usual Fourier transformations that connect a flat position space without interactions to a flat momentum space without interaction. With the kinematic momentum operator for different physical systems containing different vector gauge potentials, one will not have a universal momentum representation for all physical systems. It would be very confusing indeed if the very meaning of physical momentum changes as soon as the interaction changes. The quark “OAM” introduced by Ji in the gauge invariant decomposition (9) has similar problems.

There is still no consensus about the decomposition of the total momentum and AM of gauge systems. One critique to the decompositions (10) and (11) is that it uses non-local operators $A_{\text{phys}}^i$ and $A_{\text{pure}}^i$. We believe that such non-local operators are perfectly acceptable. First, the decomposition in the Coulomb gauge gives standard results. Second, the renowned Aharonov-Bohm (A-B) effect is a non-local effect that is perfectly described by the decomposition of the gauge potential into $A_{\text{phys}}^i$ and $A_{\text{pure}}^i$ [15]. Finally, non-local operators are already used in nucleon structure studies. For example, parton distributions all come from non-local operators.
A second critique of the decompositions (10) and (11) is that they are not Lorentz covariant. This criticism is due to a misunderstanding of Lorentz covariance. The physical 4-coordinate and 4-momentum can be measured in different Lorentz frames between which they are known to transform with the well-known homogeneous Lorentz transformation law. The 4-vector potentials \( A' \) is not measurable because they are not uniquely defined due to the gauge degree of freedom. Gauge invariance requires that any gauge fixing must be Lorentz frame independent, otherwise the two fundamental principles, gauge invariance and Lorentz covariance, will interfere with each other. Such a Lorentz frame independence can be realized by including the gauge degree of freedom in the Lorentz transformation: In general, the 4-vector potential transforms with the non-homogeneous Lorentz transformation law,

\[
A'_\mu(x') = A'^\nu A_\nu(x) + \partial'_\nu \omega(x').
\] (12)

Even the Lorentz gauge fixing, which is usually assumed to be transformed with homogeneous Lorentz transformation law, can actually contain the non-homogeneous term \( \partial'_\nu \omega(x') \) due to the residual gauge degree of freedom. By means of this gauge degree of freedom, one can retain the Lorentz covariance of any gauge fixing in every Lorentz frame.

A typical example is the Coulomb gauge fixing \( \nabla \cdot \mathbf{A}(x) = 0 \). In this gauge, the usual homogeneous Lorentz transformation will add an unphysical pure gauge part to the vector potential in the new Lorentz frame. The non-homogeneous term \( \partial'_\nu \omega(x') \) gives a second contribution that exactly cancels the unphysical pure gauge part to enforce the transversality condition \( \nabla' \cdot \mathbf{A}'(x') = 0 \) in the new Lorentz frame. Such a non-homogeneous Lorentz transformation had been used in well known text books for quite some time [14; 17].

A third critique is that the decompositions (10) and (11) are not unique; infinitely many other gauge invariant decompositions can be constructed by using the so-called gauge invariant extension. Gauge invariance is a necessary condition for an operator to be measurable, but insufficient to fix the decomposition. One has to add a physical condition to fix the decomposition. We proposed to do this with the transverse wave condition \( \nabla \cdot \mathbf{A}_{\text{phys}} = 0 \). This photon condition and \( F_{\mu\nu} = 0 \) guarantee that \( \mathbf{A}_{\text{phys}} \) of Eq. (3) has a unique solution. The photon condition is just the well known Helmholtz theorem. Because an em wave is transverse, its physical vector potential has only two dynamical degrees of freedom that reside in the 2D transverse space. Coulomb gauge fixing thus includes only the physical degrees of freedom; there is no unphysical state in the Hilbert space. The optics community confirmed these results both theoretically and experimentally [11; 12]. Our treatment is also consistent with Dirac’s description of gauge invariance in QED [3].

The extension of the transverse wave condition to non-Abelian cases needs further study. We only proved that the perturbative solution of this separation is unique. For strong gauge transformations involving different winding number vacua, there might be additional complications, such as the Gribov ambiguity [13]. In other physical condition choices, the pure gauge part usually does not separate out completely from the physical part. For example, the light-cone gauge still contains a residual gauge degree of freedom. See the recent comprehensive review [10] for more information.

Perhaps the best argument against Ji’s idea that nonlocal operators are not measurable is provided by the photon spin. Because photons travel with light speed, the measurable photon spin operator is the helicity \( \mathbf{S} \cdot \kappa, \kappa = \mathbf{p}/|\mathbf{p}| \), where the projection into the momentum direction involves a nonlocal operation in ordinary space. The complete spin operator \( \mathbf{S}' = \kappa (\mathbf{S} \cdot \kappa) \) is Abelian: \( \mathbf{S}' \times \mathbf{S}' = 0 \), because projected values along the single momentum direction commute. Our spin operator is based on the density \( \mathbf{E} \times \mathbf{A}_{\perp} \) [10; 11] that points in the same momentum direction and is also nonlocal. The nonlocality of \( \mathbf{S}' \) has not prevented the helicity from being measured in atomic, nuclear, particle and optics physics.

There are still some issues concerning the photon OAM operator: The operator \( \mathbf{L}' = \mathbf{J} - \mathbf{S}' \) proposed in optics [11] does not satisfy the usual \( SO(3) \) AM algebra. Yet its expectation value in light wave has been measured [11]. Our OAM is based instead on the OAM density \( -\mathbf{E}_{\mathbf{L}}(\mathbf{iL})\mathbf{A}_{\perp} \), where \( \mathbf{L} = \mathbf{x} \times \mathbf{p} \) is the usual \( SO(3) \) OAM operator and \( \mathbf{p} = -i\nabla \) [12]. Two other terms in our decomposition, one each from the spin and OAM densities, cancel out. This is why our operator \( \mathbf{S}' \) does not contain any transverse part and is Abelian. There is a need to understand the difference between these two approaches.

Some final words on the momentum operator: Ji [14] and Wakamatsu [20] have insisted that in deep inelastic scattering, the measured quark momentum is the mechanical or kinematical momentum because of its appearance in the classical Lorentz equation. We agree that this classical mechanical
Feynman’s momentum is indeed measurable in classical physics. The situation in quantum mechanics is different, however: One can only measure the quantum canonical momentum because only this operator has components that commute, and are therefore simultaneously measurable. The same conclusion also follows from studies of the second moment of the quark parton distribution that is related to the matrix element \(\langle PS| p^+ - gA^+ |PS \rangle\) on the light cone. This form was initially misinterpreted as the ME of the light-cone mechanical momentum \(p - gA\). The misunderstanding was finally corrected by Ji et al. who showed that in the infinite momentum frame, the vector potential \(A\) does not include the physical transverse part \([21]\). A study of the gauge link in the collinear approximation also shows that only the longitudinal component of the gauge potential (which is a nonphysical pure gauge contribution) is included in the gauge link. Both results confirm that the measured quark momentum distribution is the ME of \(p^+ - gA^+\), the gauge invariant canonical momentum \(p - gA\) in the infinite momentum frame, in exact agreement with our decomposition \([11]\). Our proposed parton distributions \([10]\) thus express better the physics of quark momentum and gluon helicity parton distribution.

This work is supported by the NSFC grant 1175215, 11175088, 11035006.

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