The Framed Standard Model (I) - A Physics Case for Framing the Yang-Mills Theory?*

CHAN Hong-Mo
h.m.chan @ stfc.ac.uk
Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

TSOU Sheung Tsun
tsou @ maths.ox.ac.uk
Mathematical Institute, University of Oxford,
Radcliffe Observatory Quarter, Woodstock Road,
Oxford, OX2 6GG, United Kingdom

Abstract

Introducing, in the underlying gauge theory of the Standard Model, the frame vectors in internal space as field variables (framons), in addition to the usual gauge boson and matter fermions fields, one obtains:

- the standard Higgs scalar as the framon in the electroweak sector;
- a global $\tilde{u}(3)$ symmetry dual to colour to play the role of fermion generations.

Renormalization via framon loops changes the orientation in generation space of the vacuum, hence also of the mass matrices of leptons and quarks, thus making them rotate with changing scale $\mu$. From previous work, it is known already that a rotating mass matrix will lead automatically to:

- CKM mixing and neutrino oscillations,
- hierarchic masses for quarks and leptons,
- a solution to the strong-CP problem transforming the theta-angle into a Kobayashi-Maskawa phase.

Here in the FSM, the renormalization group equation has some special properties which explain the main qualitative features seen in experiment both for mixing matrices of quarks and leptons, and for their mass spectrum. Quantitative results will be given in (II). The paper ends with some tentative predictions on Higgs decay, and with some speculations on the origin of dark matter.

arXiv:1505.05472v1 [hep-ph]  20 May 2015
*Invited talk at the Conference on 60 Years of Yang-Mills Gauge Theories, IAS, Singapore, 25-28 May 2015.
When the Yang-Mills theory was discovered 60 years ago, its significance was immediately recognized, although it was unclear at that stage in what physics context it should be applied. Now, 60 years later, the theory has found itself enthroned as the theoretical basis of, among other things, the standard model of particle physics, namely of all known physical phenomena apart from gravity. And this standard model can justly claim to be the most successful theory ever, given the range of phenomena it covers and the resilience it has shown in surviving the many detailed experimental tests to which it has been subjected.

However, some things are still missing from this beautiful picture, at least to the fastidious theoretical mind, notably an understanding of the origin of:

- The Higgs boson needed to break the electroweak symmetry,
- Three generations of quarks and leptons needed to fit experiment,

neither of which is part of the original Yang-Mills structure or has any other theoretical explanation. The lack of understanding of the second, especially, is practically significant, since the masses and mixing matrices of quarks and leptons fall into a bizarre hierarchical pattern, and they account for about two-thirds of the standard model’s twenty-odd empirical parameters. There are besides some ominous clouds on the horizon, such as the unresolved strong CP-problem, the missing right-handed neutrino, and the mysterious dominance of dark matter in the universe, plus, of course, the wanting link to gravity already mentioned.

To address these shortcomings of the standard model, if indeed shortcomings they are, one will obviously have to enrich its starting assumptions in some way. One direction, the most popular one, is to keep the Yang-Mills structure as it is, but enrich the superstructure by enlarging the gauge symmetry beyond the standard $su(3) \times su(2) \times u(1)$ (GUT, Supersymmetry), or the dimension of space-time beyond the standard $3 + 1$ (Kaluza-Klein), or both (Superstring, branes). These extensions often open up grand vistas of things to come but are less successful in answering some of the detailed questions of immediate interest. For example, instead of reducing the number of parameters in the standard model by explaining the known values of some of them, supersymmetric models usually end up with a hundred parameters or more. For this reason, one thought, it might be worth trying a different path.
Suppose one keeps the same gauge symmetry $su(3) \times su(2) \times u(1)$ and the same 4-dimensional space-time as for the standard model, but enrich instead the underlying structure by requiring the Yang-Mills theory be “framed”. By “framing” a gauge theory here, we mean the introduction also as dynamical variables of the frame vectors in the internal symmetry space in addition to the usual gauge potential $A_\mu$ and the fermionic matter fields $\psi$. Such frame vector (or “framon”) fields are analogues of the vierbeins $e^a_\mu$ in the theory of gravity in which they are often used in place of the metric $g_{\mu\nu}$ as dynamical variables. By taking them as dynamical variables too in the particle theory makes it closer in spirit to the gravity theory, and may eventually facilitate the union of the two. Our immediate aim, however, is not to attempt this union but, while staying within particle physics itself, to address the shortcomings of the standard model listed above.

What then will framing give us in particle theory? Like the vierbeins $e^a_\mu$ in gravity, the framons we need for the particle theory may be regarded as column vectors of a matrix relating the local gauge frame to a global reference frame, carrying hence both a local symmetry index (analogous to $\mu$ in $e^a_\mu$) and a global frame index (analogous to $a$ in $e^a_\mu$). They thus transform both under the local gauge transformations of $G = su(3) \times su(2) \times u(1)$ and under the global transformations of the reference frame, say $\tilde{G} = \tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1)$, but are just scalars under the Lorentz transformations in space-time. Two immediate consequences of framing then result:

- **(A)** In the electroweak sector, the framon is an $su(2)$ doublet but Lorentz scalar field, exactly as needed for the Higgs field to break the electroweak symmetry;

- **(B)** Since physics is independent of the choice of the reference frame, the framed theory has to be invariant not only under the local symmetry $G$, but also under its dual, the global symmetry $\tilde{G}$. Of this the 1st component $\tilde{su}(3)$, a 3-fold symmetry, can function as fermion generation, while the 2nd component $\tilde{su}(2)$ can act as up-down flavour, and the 3rd component $\tilde{u}(1)$ as $(B - L)$,

giving thus both the Higgs field and fermion generation each an hitherto lacking geometric significance.

To push these advantages further, there is some ambiguity first to settle as to the exact form that the framons should take. Minimality considerations
in the number of scalar fields to be introduced suggest then that the framons in FSM belong to the representation \((3 + 2) \times 1\) in \(G\) but to \(\bar{3} \times 2 \times 1\) in \(\bar{G}\) and that some of its components may be taken as dependent on others \([1, 2]\)

leaving just:

- a “weak framon” of the form:
  \[
  \alpha \otimes \phi
  \]
  \[\text{(1)}\]
  where \(\alpha\) is a triplet in \(\bar{s}u(3)\), which may be taken without loss of generality \([2]\) as a real unit vector in generation space, but is constant in space-time, while \(\phi\) is an \(su(2)\) doublet but Lorentz scalar field over space-time which has the same properties as, and may thus be identified with, the standard Higgs field;

- the “strong framon”:
  \[
  \beta \otimes \phi^\tilde{a}, \quad \tilde{a} = \tilde{1}, \tilde{2}, \tilde{3}
  \]
  \[\text{(2)}\]
  where \(\beta\) is a doublet of unit length in \(\bar{s}u(2)\) space but constant in space-time, while \(\phi^\tilde{a}\) are 3 colour \(su(3)\) triplet Lorentz scalar fields over space-time, which when taken as column vectors give a matrix \(\Phi\) transforming by \(su(3)\) transformation from the left but by \(\bar{s}u(3)\) transformations from the right.

As usual the mass matrices at tree-level of quarks and leptons are to be obtained from the Yukawa couplings of the fermions to the Higgs scalar field (weak framon) by replacing it with its vacuum expectation value. But now since the weak framon \([1]\) carries a factor \(\alpha\), the fermion mass matrices will also carry this factor. Then by a simple relabelling of the right-handed singlet fields,

- (C) The mass matrices of all quarks and leptons can be rewritten conveniently in the following form:
  \[
  m = m_T \alpha^\dagger \alpha,
  \]
  \[\text{(3)}\]
  where \(\alpha\), coming from the framon, is “universal”, i.e. the same for up-type quark (U), down-type quark (D), charges leptons (L) and neutrinos (N), and only the coefficient \(m_T\) depends on the fermions species.
Now such a mass matrix has long been coverted by phenomenologists \[3, 4\] as a starting approximation, since it gives only 1 massive generation for each species, which may be interpreted as embryonic mass hierarchy, and zero mixing between up- and down-states, which is not a bad approximation, at least for quarks.

The question now, of course, is what happens above the tree-level. This is the point at which the FSM first shows its power, beyond what can be done by just phenomenology. Because of the double invariance under both $G = su(3) \times su(2) \times u(1)$ and $\tilde{G} = \tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1)$, the action for framons is much restricted in form, which allows some radiative corrections to be calculated. In particular, since the strong framon in (2) above carries both the local colour $su(3)$ and global dual colour $\tilde{su}(3)$ (or generation) indices, renormalization by strong framon loops will change the orientation of the vacuum in generation space. This change will depend in general on the renormalization scale $\mu$, thus inducing a $\mu$-dependent $\tilde{su}(3)$ transformation (rotation) on the vector $\alpha$ which appears in the fermion mass matrix (3).

Now, we have studied the consequences of a rotating rank-one mass matrix (R2M2) for some years and it has been shown, e.g. in [5], that a mass matrix of the form (3), with $\alpha$ rotating with changing scale, will automatically give rise to the following effects:

- **(D1)** Mixing between the up and down states, i.e. a nontrivial CKM matrix between up and down quarks, and a PMNS matrix between the charged leptons and neutrinos leading to neutrino oscillations. [This can be easily seen in, for example, the CKM matrix element $V_{tb}$ which is the dot product between the state vectors $t$ and $b$ of respectively $t$ and $b$ in generations space. For (3), $t$ is the value of $\alpha$ at the scale $\mu = m_t$ while $b$ is the value of $\alpha$ at $\mu = m_b$, and since $m_t > m_b$ and $\alpha$ rotates, it follows that $t$ and $b$ are not aligned and $V_{tb} \neq 1$, or that there is mixing.]

- **(D2)** Fermion mass hierarchy in each species, with the mass in the heaviest generation in (3) of each species (e.g. $t$) “leaking” to the lower ones (e.g. $c$ and $u$), giving each a small but nonzero mass. [This can be seen as follows. The state vector $t$ for $t$ is the vector $\alpha$ at $\mu = m_t$, and the state vector $c$ is a vector orthogonal to $t$, and having a zero eigenvalue for (3) at $\mu = m_t$. But this is not the mass $m_c$ for $c$, which is to be measured at $\mu = m_c$, where $\alpha$ will have rotated already to a different direction with nonzero component in $c$, and hence $m_c \neq 0.$]
(D3) A solution to the strong-CP problem by transforming away a nonzero theta-angle in the QCD action by turning it, via rotation, into a nonzero Kobayashi-Maskawa phase in, and giving CP-violation to, the CKM matrix. [At every $\mu$, the mass matrix [3] has 2 zero eigenvalues, so that a chiral transformation can be performed to eliminate the theta-angle from the QCD action without making the mass matrix complex. The effects of this chiral transformation, however, is transmitted by rotation to other $\mu$ values and make the CKM matrix complex leading to a KM phase.]

Any R2M2 scheme with a rotating rank-one mass matrix will give (D1) - (D3) but the details will depend on the rotation trajectory of $\alpha$, i.e how it actually changes with scale $\mu$. Let us see now what FSM has to say about this trajectory. Recall that $\alpha$ is itself a global quantity with no gauge interactions, and therefore not subject directly to radiative corrections. But it is coupled to the strong vacuum, and if that rotates with scale $\mu$, $\alpha$ will rotate also. Now, information of how the strong vacuum rotates can be obtained by studying the renormalization of any quantity which depends on the strong vacuum. We have, mainly for historical reasons, focussed on the Yukawa coupling of the strong framon, and obtained therefrom the renormalization group equations; hence also the equations governing the rotation of $\alpha$. The implications of the rotation equation can be divided conveniently into 2 bits:

- The shape of the curve $\Gamma$ traced out by $\alpha$ on the unit sphere in generation space,
- The variable speed with respect to scale $\mu$ at which this curve $\Gamma$ is traced.

The shape of the curve $\Gamma$ turns out to be a consequence just of symmetry residual in the problem and depends only on a single integration constant, say $a$. This is shown in Figure [1] where it is seen that $\Gamma$ bends sharply near $\theta = 0, \phi = \pi$, thus giving it there a considerable local value for the geodesic curvature $\kappa_g$, especially for a small value of $a$. But $\kappa_g$ needs not be large elsewhere, and indeed changes sign further along the curve. [Notice that for a $\Gamma$ on the unit sphere, the torsion $\tau_g = 0$ and the normal curvature $\kappa_n = 1$, and only the geodesic curvature $\kappa_g$ is variable.]

The shape of $\Gamma$ in Figure [1] then immediately implies:
Figure 1: The curve \( \Gamma \) traced out by the rotating \( \alpha \) on the unit sphere in generation space, for various values of the integration constant \( a \)
(E1) The corner elements, \( V_{ub}, V_{td} \) of the CKM matrix for quarks, and similarly \( U_{e3}, U_{\tau 1} \) of the PMNS matrix for leptons, both due to twist in \( \Gamma \), are much smaller than the other off-diagonal elements because \( \tau_g = 0 \).

(E2) The elements \( V_{us}, V_{cd} \) in the CKM matrix for quarks, due to sideways bending of \( \Gamma \) (i.e. governed by \( \kappa_g \)) can be much larger than the elements \( V_{cb}, V_{ts} \) governed by \( \kappa_n = 1 \), although the corresponding elements in the PMNS matrix can all be of similar magnitudes.

and also the following, though less obviously, from the already noted fact that \( \kappa_g \) eventually changes sign:

(E3) \( m_u < m_d \), despite that for the two heavier generations, \( m_t \gg m_b, m_c \gg m_s \).

The points (E1) and (E2) are seen to be borne out by experiment which give the approximate CKM and PMNS matrices as:

\[
V_{CKM} = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}; \quad U_{PMNS} = \begin{pmatrix} 0.82 & 0.55 & 0.17 \\ 0.50 & 0.52 & 0.70 \\ 0.30 & 0.66 & 0.70 \end{pmatrix}
\]

(E4)

The last (E3) is of course a crucial empirical fact, without which the proton would be unstable, and we ourselves would not be here, but a fact which is, at first sight, theoretically very hard to understand.

Apart from other things, the equation governing the speed at which \( \Gamma \) is traced shows that \( \alpha \) has a fixed point at \( \mu = \infty \), so that its rotation starts slowly as \( \mu \) lowers from \( \infty \), but will accelerate with decreasing \( \mu \). This then immediately implies the following results:

(F1) Since lower generation masses according to (D)(ii) comes from “leakage” through rotation and will increase with rotation speed, it follows from the fact that \( m_t > m_b > m_\tau \) that:

\[
m_c/m_t < m_s/m_b < m_\mu/m_\tau.
\]

This agrees with experiment which obtained the mass ratios as respectively 0.0074, 0.0227, 0.0595.
Since mixing, according to (D) (i), also comes from rotation and will increase with rotation speed, it follows from the fact that quarks are generally heavier than leptons that the off-diagonal mixing elements in the CKM matrix for quarks will generally be larger than the corresponding elements in the PMNS matrix for leptons. That this also agrees with experiment can be seen in (4).

Since the rotation is generally slow at quark mass scales, one can make a small angle approximation, which allows one to estimate the amount of CP-violation in the CKM matrix obtained via rotation as per (D3) from any given value the theta-angle in the QCD action, giving a Jarlskog invariant as [5]:

$$|J| \sim 7 \times \sin(\theta/2)10^{-5},$$

(6)

which, for $\theta$ of order unity, is of the same order of magnitude as the experimentally measured value of $J \sim 2.95 \times 10^{-5}$.

Since, however, one has already derived the renormalization group equations governing the rotation for $\alpha$, there is no need at all to stop at just this qualitative level. True, the rotation equations still depend at present on a number of parameters, but if one is willing to supply some experimental information to determine these parameters, then one can proceed to evaluate essentially all the masses and mixing angles which are taken as inputs from experiment in the usual formulation of the standard model. What the FSM does essentially is to replace 17 independent parameters of the standard model by 7.

By fitting the 7 adjustable parameters of the rotation equations in FSM to 6 pieces of data, we have then calculated the values of 17 independent parameters of the standard model. Not all the 17 quantities have been measured experimentally, but of those 12 that have been measured, the agreement is good. [6]

I shall leave you to judge for yourselves the significance of this result which you will hear from my collaborator Tsou Sheung Tsun in the next talk (II). But it does seem that, with the freedom still left in theory, there will not be much difficulty in reproducing the data as are known at present.
However, what proves a theory, of course, is not its ability to reproduce known results, but its predictions which can be tested against and then confirmed by experiment. Here, one is unfortunately hampered by not knowing how to calculate in general with a theory where the mass matrix rotates. To deduce the results reported, we had some patchy rules for calculating single particle properties like masses and mixing angles, but we have not worked out logically the rules for more general calculations, so any predictions we can make at this stage are only tentative.

Two tentative predictions, however, stand out, both concerned with the Higgs boson. The rotating mass matrix for quarks and leptons were deduced, from a Yukawa coupling. So, if $\alpha$ in the mass matrix rotates, it would seem that it should do the same in the Yukawa coupling too, and so affect the decay of the Higgs boson to fermion-antifermion pairs. An analysis along these lines then suggests that:

- **(H1?)** Branching ratios of the Higgs boson to the second generation fermions such as $H \rightarrow \mu^+\mu^-$, $c\bar{c}$ would be suppressed compared to the standard model predictions.

- **(H2?)** There can be flavour-violating decays, such as $H \rightarrow \tau\mu$ with branching ratios at the $10^{-4}$ level.

Experimentally, present sensitivity for these decays are still a couple of orders of magnitude higher than is required for these effects to be detected.

The other tentative prediction involve the mixing between the weak framond (Higgs boson) and the strong. Given that the FSM has to be invariant under the local gauge symmetry $G$ and also under its dual the global symmetry $\tilde{G}$, the form of the interaction potential between the framons can be worked out, which has already been used above in the calculations reported. It is then an easy step to evaluate the mass matrix of the framons to tree-level, and this shows:

- **(H3?)** There is mixing between the Higgs boson with several of its strong analogues. This mixing depends on a couple of yet unknown parameters and so its details are still unknown, but could make the Higgs’ decays depart from the standard model predictions.

The predictions **(H1?)-(H3?)** all come from the weak framond [1]. What is likely to be even more interesting, however, is how the strong framons of
will manifest themselves. After all, they are the truly new ingredients added by the FSM to the standard model. We recall that it is strong framons loops which gave rise, via renormalization, to rotation in the mass matrix of quarks and leptons, and hence to their mixing and mass hierarchy. So these important effects should already be regarded, in the FSM context, as indirect manifestations of the strong framon. Thus, rather, the question that one is actually asking now is whether strong framons could manifest themselves more directly in some other experimentally detectable physical phenomena. The answer to this, however, is not obvious and is at present perforce speculative, the strong framon being an entirely new type of field with special properties, including probably some unusual soft (nonperturbative) colour physics. But the question is nevertheless sufficiently intriguing, with potentially far-reaching consequences, to deserve some speculations, to which let us then, for a little while, indulge.

We recall that the strong framon carries colour, and so, because of colour confinement, cannot exist as a particle in free space, only inside hadronic matter. But it can combine with other constituents of hadronic matter with the opposite colour to form a colour neutral state, which then appear as a hadron in free space. For example, a framon can combine with an antiframon to form bosons, which in the lowest s-wave state are colour analogues of the standard Higgs boson in electroweak $su(2)$ and mix with it, as mentioned in (H3?) above.

There are altogether 9 such states, and they are likely to be the lowest batch of the new hadrons which contain a strong framon as constituent. As also mentioned, their mass matrix at tree-level has already been worked out from the known framon self-interaction potential, and it is straightforward to extend the calculation to their mutual couplings, which has now also been completed. These show that they can readily decay into one another, but the lowest among them, called, for historical reason, $H_-, H_4, H_5$ are, of course, stable against such decays. These $H_K$’s are electrically neutral, but have for constituents the strong framons (2) with charge $-1/3$, which would allow, among them, $H_-$, made from a framon and an antiframon with the same generation $\tilde{s}\tilde{u}(3)$ index, to decay into photons. But this does not apply to $H_4$ and $H_5$ which are made from a framon and antiframon carrying different $\tilde{s}\tilde{u}(3)$ indices. Further, since even their framon constituents carry no weak charge, these last $H_f$’s ($f =$ four or five) also cannot decay weakly, and would thus seem to be stable altogether.

Now, if these $H_f$ states do exist and are stable, an obvious question would
be why they have not been seen. Presumably, like quarks and gluons, strong framons would be present in the sea of hadronic matter inside a proton. And since they are both coloured and charged, they too, like quarks, can be “knocked out” by a hard kick from a gluon or a photon. A quark so “knocked out” will pick up and combine with an antiquark in the sea and emerge as a meson. Cannot then a “knocked out” framon pick up and combine with an antiframon from the sea and emerge as a Hf? If so, why do we not see it?

There is a difference, however, between a quark and a framon in that the latter has an imaginary mass, for like the Higgs scalar field, the quadratic term in the self-interaction potential has a negative coefficient. One can interpret this as meaning that the framon in hadronic matter, unlike the quark, has only a finite life-time. It thus has only a limited time to seek out a partner from the sea to form an $H_f$, and in this it may not succeed if the time is short. Hence, again unlike the quark, a “knocked out” framon may not succeed to emerge from the host proton at all. Whether this happens will depend on its life-time and the amount of hadronic matter that it can traverse inside the proton during its life-time. Let us say here, for the sake of argument, that the conditions are such that this will not happen, so that no $H_f$’s can be produced in ordinary hadronic reactions, and so explain their non-observation so far in experiment.

However, that no $H_f$’s are produced in ordinary hadron collisions in present experiments need not mean that the same is true under other circumstances. For example, in the primordial universe (or even now in, say, the galactic centre) both temperatures and densities are much higher than can be found under present experimental conditions in our laboratories. This may allow then these $H_f$ to be formed, and once formed, being stable according to our previous argument, they would still be around with us today.

The question then leaps out whether they may be candidates for dark matter. A preliminary investigation does indeed indicate that they may have rather little interaction with ordinary matter, and also with themselves. That the strong framon is short-lived suggests, by an argument similar to that given above for the non-production of $H_f$’s in present laboratory experiments, that they do not have the usual strong interactions of ordinary hadrons (although they are formally hadrons, being colour neutral bound states by colour confinement of coloured constituents). Besides, they are charged neutral, both electrically and in colour. And being relatively light (though not light enough to be hot dark matter) it may not be impossible for them to satisfy the otherwise very stringent bounds already set by recent
experiments such as Xenon 100 and LUX. Hence, perhaps:

- *(?) New constituents of Dark Matter (?)*

However, if indeed relatively light, it will take a lot of them to make up a sufficient mass so as to matter in the dark matter problem. Is there then any reason why they should be produced in the early universe in such an abundance, say, as compared to luminous baryonic matter? Amusingly, there is, or at least may be, a possible reason. At some stage in its development after the Big Bang, the universe is presumably just a large blob of hot, dense hadronic matter with, among other things, quarks and framons swimming around. As it cools and expands further, the quarks and framons inside would be frantically seeking partners to hitch up as colour neutral bound states so as to survive into the next epoch as hadrons. For the framon to survive as an $H_f$, all it needs is to find an antiframon of opposite colour. For the quark to survive as a nucleon, however, it will have to find 2 others of the right colour at the same time, which looks an altogether tougher proposition. Hence, the much greater abundance of $H_f$ than nucleons in our world today. Indeed, it would seem a very lucky chance that enough luminous baryonic matter managed to survive, or else most of the things we know would not be here.

As matters stand, of course, all this discussion about the $H_f$’s as dark matter is merely an exercise in imagination. But, since some of the parameters in FSM have already been determined in the work to be reported by Tsou in the next talk, there may be a chance that some of these imaginings can actually be investigated, and either substantiated or repudiated in the not too distant future.

If there is any truth in the these speculations, however, it would indeed seem an extraordinary stroke of good luck that enough luminous baryonic matter is left around from the early universe for us humans to come into existence. Then we are even luckier than we think, today, to be able to come together here to celebrate the 60th anniversary of the Yang-Mills theory. For this we have to thank, first Professor Yang for giving us the cause to celebrate, and secondly Professor Phua and the other organisers for giving us the opportunity to enjoy this celebration.

The work summarized in this talk has almost all been done in collaboration with Jose Bordes, and in part in collaboration with Mike Baker. We have benefitted also from discussions with, and constant interest and encouragement from, James Bjorken.
References

[1] Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A27 (2012) 1230002; arXiv:1111.3832.

[2] Michael J. Baker, Jose Bordes, Chan Hong-Mo and Tsou Shueng Tsun, Int. J. Mod. Phys. A27 (2012) 1250087; arXiv:1111.5591.

[3] H. Fritsch, Nucl. Phys. B155 (1978) 189.

[4] H. Harari, H. Haut and J. Weyers, Phys. Lett. B78 (1978) 459.

[5] Michael J. Baker, Jose Bordes, Chan Hong-Mo and Tsou Shueng Tsun, Int. J. Mod. Phys. A26 (2011) 2087-2124; arXiv:1103.5615.

[6] Jose Bordes, Chan Hong-Mo and Tsou Shueng Tsun, Int. J. Mod. Phys. A30 (2015) 1550051; arXiv:1410.8022.