1. INTRODUCTION

A knowledge of the star formation rate (SFR) is fundamental to our understanding of the formation and evolution of galaxies. In a seminal paper Madau et al. (1996) connected star formation in the distant universe with that estimated from low-redshift surveys by plotting the estimated star formation rate per unit comoving volume against redshift. A wide variety of techniques have contributed toward populating this diagram with observational points. Nonetheless, an unacceptable degree of uncertainty still attaches to our overall understanding of the evolution of star formation in the universe, and much of this uncertainty results from the effects of dust obscuration in and around the star-forming regions, particularly for those techniques that depend on optical or UV data (see the review by Calzetti 2001).

A fairly direct technique, which has enjoyed extensive use in the determination of star formation rates, is the measurement of hydrogen recombination line fluxes (Bell & Kennicutt 2001; Gallego et al. 1995; Jones & Bland-Hawthorn 2001; Moorwood et al. 2000; Tresse & Maddox 1998; Pascual et al. 2001; Yan et al. 1999; Dopita et al. 2002b, to name but a few). Provided that the \( \text{H} \, \beta \) region can absorb all the EUV photons produced by the central star, this should be a reliable technique since the flux in any hydrogen line is simply proportional to the number of photons produced by the hot stars, which is proportional to the birthrate of massive stars. This relationship has been well calibrated at solar metallicity for the \( \text{H} \, \alpha \) line (Dopita & Ryder 1994; Kennicutt 1998),

\[
\text{SFR}_{\text{H} \, \alpha} = 7.9 \times 10^{-42} \frac{L_{\text{H} \, \alpha}}{\text{ergs s}^{-1}}.
\]  

(1)

There exist a number of ways in which this relationship might break down. First, the \( \text{H} \, \beta \) region might be optically thin to the EUV photons in some directions. In this case the recombination flux will provide an underestimate of the star formation rate, and the appropriate correction factor is difficult to estimate. Second, there may be dust in front of the \( \text{H} \, \beta \) region, which both reddens and absorbs the recombination lines. In this case, the corrections for dust absorption can be made using a foreground screen approximation. This appears to work extraordinarily well in many galaxies (Bell & Kennicutt 2001; Dopita et al. 2002b; Kewley & Dopita 2002). Third, dust may be contained in dense, optically thick clouds within or surrounding the \( \text{H} \, \beta \) region. In this case the optical recombination lines are entirely absorbed in lines of sight passing through foreground clouds. However, the radio thermal continuum is transmitted by such clouds, so this factor can be estimated as a “gray” screen by comparison of the optical and the radio data. Such clouds will emit the incident radiation in the far-IR, and this emission may also be used to estimate the stellar EUV flux.

A fourth possibility is that dust within the ionized gas itself competes with the gas to absorb the EUV photons from the central star(s). This possibility, first seriously quantified by Petrosian, Silk, & Field (1972), has been discussed by a number of authors since (Panagia 1974; Mezger, Smith, & Churchwell 1974; Natta & Panagia 1976; Sarazin 1977; Smith, Biermann, & Mezger 1978; Shields & Kennicutt 1995; Bottorff et al. 1998). More recently its effect has been investigated and quantified (as far as possible by direct observation) in a series of recent papers by Inoue and his collaborators (Inoue, Hirashita, & Kamaya 2000, 2001; Inoue 2001). Here we attempt to calibrate these correction factors using theoretical models of dusty \( \text{H} \, \beta \) regions.

2. THE MODELS

2.1. The Dust Model

The MacOS version of the MAPPINGS III codes was used to generate spherical-shell dusty photoionized models...
appropriate to H II regions excited by clusters of OB stars. MAPPINGS IIId includes a number of advances in the treatment of dust physics and absorption. The dust model includes three types of grains, “astronomical” silicates and graphite or amorphous carbon grains having a Mathis, Rumpl, & Nordie (1977) distribution for grain sizes between 50 and 2500 Å. For these grains the extinction curve is based on the Laor & Draine (1993) data for silica and graphite. In addition, we have the option of including complex polycyclic aromatic hydrocarbon (PAH)–like organic molecules. For these, the photoionization cross section per carbon atom follow the yield factors given by Vestaete et al. (1990), which are based on pyrene and coronene. These have been extrapolated (using the same slope as the Vestaete curve) to \( N_C = 80 \), which seems to be appropriate for interstellar PAHs (Allain, Leach, & Sedlmayr 1996a, 1996b; Hollenbach & Tielens 1999; Peeters et al. 2002). For PAH molecules of these dimensions, the ionization potential is about 6 eV. The PAH opacities are taken from the recent publication by Li & Draine (2001).

With our present state of knowledge of the physics of PAH molecules, it is not at all clear whether such molecules can survive for significant lengths of time in the hostile environment offered by the ionized zones of an H II region. Provided that the heating rate by the absorption of photons in the ISM can be at least matched by the infrared radiative rate, then the survival of PAHs is set by the competition between photodissociation (by the ejection of an acetyleneic group) and its repair through accretion of carbon atoms (Allain et al. 1996a, 1996b). If \( \tau_{\text{diss}} \) is the radiative dissociation timescale and \( \tau_{\text{acc}} \) is the C atom accretion timescale, then these are given by

\[
\tau_{\text{diss}} = (F_{\text{FUV}} \sigma_{\text{diss}})^{-1},
\]

and

\[
\tau_{\text{acc}} = (n_H X_C k_{\text{acc}})^{-1},
\]

where \( F_{\text{FUV}} \) is the far-UV radiation field, \( \sigma_{\text{diss}} \) is the photodissociation cross section per PAH molecule, \( n_H \) is the number density of hydrogen atoms, \( X_C \) is the abundance of C in the ISM, and \( k_{\text{acc}} \) is the reaction rate for sticking of a carbon atom onto such a molecule. For the far-UV radiation field, we take the total radiation field above the ionization potential adopted for these molecules (≥6 eV). Putting together equations (1) and (2), we see that the PAHs will be destroyed when

\[
F_{\text{FUV}} \sigma_{\text{diss}} > n_H X_C k_{\text{acc}},
\]

or

\[
\mathcal{H}_* = \frac{F_{\text{FUV}}}{c n_H} \frac{X_C k_{\text{acc}}}{\sigma_{\text{diss}}}.
\]

We define \( \mathcal{H}_* \) as the Habing photodissociation parameter by analogy with the dimensionless ionization parameter \( \mathcal{H}_\star \) used in H II region theory. The advantage of the use of this parameter is that all photodissociation rates will scale in this way, and therefore the local value of this dimensionless parameter will also determine the local chemistry of photodissociation regions to the first order. The actual shape of the photodissociating spectrum will determine the chemistry to the second order.

Since we do not know the absolute value of \( \mathcal{H}_* \) above which PAHs are destroyed, we have to ask, what are the most extreme values of \( \mathcal{H}_* \) observed in regions that still contain PAHs? From Allain et al. (1996a), such extreme regions can be identified as in the diffuse ISM, high above the Galactic plane where \( n_H \sim 0.1 \) cm\(^{-3}\), \( F_{\text{FUV}} \sim 1.5 \times 10^8 \) photons cm\(^{-2}\) s\(^{-1}\), or in the planetary nebula NGC 7027 where \( n_H \sim 7 \times 10^4 \) cm\(^{-3}\) and \( F_{\text{FUV}} \sim 7.6 \times 10^{13} \) photons cm\(^{-2}\) s\(^{-1}\), corresponding to \( \mathcal{H}_* \sim 0.05 \) and \( \mathcal{H}_* \sim 0.04 \), respectively. We therefore adopt a threshold of \( \mathcal{H}_* \sim 0.05 \) for the destruction of PAHs. This threshold is not reached for any of the photoionization models presented here, so it is at least conceivable that charged PAH-like molecules may survive in the environment of an H II region.

### 2.2. The Photoionization Models

The photoionization modeling procedure adopted here closely follows that described in Dopita et al. (2000) and Kewley et al. (2001). For the central star cluster, we adopt a STARBURST99 (Leitherer et al. 1999) instantaneous burst with a total luminosity of \( 10^{40} \) ergs s\(^{-1}\) and having an initial mass function (IMF) with a power-law slope of the Salpeter form (\( \alpha = 2.35 \)). The lower mass cutoff was set at \( 0.1 M_\odot \) and the upper mass cutoff at \( 120 M_\odot \). For the purpose of the models presented here, the actual form of the assumed IMF is relatively unimportant in determining the final results, since the strength of the EUV field and the metallicity of the gas prove to be more important than the shape of the ionizing spectrum in determining the EUV absorption by dust.

In order to investigate the effects of ionization parameter, \( \mathcal{H}_* \), and the chemical abundances (which effectively determine the gas-to-dust ratio), we ran three sets of models for the abundances \( Z = 0.4, 1.0, \) and \( 2.0 Z_\odot \) and covering a range of initial dimensionless ionization parameters from \( \mathcal{H}_* = 0.000 \) up to 0.0066. This encompasses the full range that is normally encountered in range of ionization parameters encountered in bright H II regions (see Dopita et al. 2000). Each model had the chemical abundances of the central cluster set the same as for the gas in the H II region. In

### Table 1

| Element   | log \( Z_\odot \) | log \( D \) |
|-----------|------------------|------------|
| H         | 0                | 0.00       |
| He        | -1.01            | 0.00       |
| C         | -3.44            | -0.30      |
| N         | -3.95            | -0.22      |
| O         | -3.35            | -0.07      |
| Ne        | -3.91            | 0.00       |
| Cl        | -5.75            | 0.00       |
| Mg        | -4.42            | -0.70      |
| Si        | -4.45            | -1.00      |
| S         | -4.79            | 0.00       |
| Ar        | -5.44            | 0.00       |
| Ca        | -5.64            | -2.52      |
| Fe        | -4.36            | -2.00      |
| Ni        | -5.68            | -2.00      |
spherical H ii region models, the spherical divergence of the radiation field ensures that the mean ionization parameter is not well defined when the H ii region becomes thick in comparison to its radius. We have therefore ensured that the models remain geometrically thin with a well-defined \( \mathcal{U} \) by raising the assumed hydrogen density from \( n = 10 \text{ cm}^{-3} \) up to \( n = 100 \text{ cm}^{-3} \) for the models with the highest ionization parameter.

The absorption of the PAH molecules is very important in determining the EUV extinction, since they have a very large absorption cross section per carbon atom above 13.6 eV. Since we have no way of directly determining whether such molecules survive in the H ii region itself, we have run two sets of models, one that did not include PAH-like molecules and another in which we have set the abundance of PAH molecules equivalent to 20% of the total carbon atoms. This is probably an underestimate of their true abundance, since only a maximum of 40% of interstellar carbon is locked up in carbonaceous grains (Duley & Seahra 1999), and studies of PAH emission suggests that perhaps only about 10% of the interstellar carbon is actually locked up in PAH-like molecules (Li & Draine 2002).

For most models, we have also run a comparison dust-free H ii region model with the same gas-phase abundances as in our dusty models in order both to check the absolute value of the dust-free H ii recombination line flux and to ensure that this is independent of the ionization parameter for a given input spectrum, as required by theory. We find that, over the metallicity range covered by these models, the EUV blanketing of the central star cluster has very little effect on the number of Lyman continuum photons produced per unit of stellar luminosity (less than 0.02 dex).

For reasons fully explained in the next section, we expect that the fraction of ionizing photons absorbed by the dust should scale as the product \( \mathcal{U}(O/H) \), for a given grain model. Therefore, in Figure 1 we have plotted this quantity against the computed ratio of Balmer H\( \beta \) recombination line fluxes, with and without the inclusion of dust. The two families of curves are shown, one with 10% of the C locked in PAH-like molecules and the other with 20% of the C locked in such molecules. The families of models with 0.4 times solar abundance (open circles), 1.0 times solar abundance (crossed circles) and 2 times solar abundance (filled circles) lie close to each other for each grain composition. This gives us confidence that the \( \mathcal{U}(O/H) \) scaling factor is correct, to first order.

The largest dust absorption is found for the models with extreme metallicity and ionization parameter (\( \mathcal{U} = 0.0133 \) and \( Z = 2 Z_\odot \)). In practice, H ii regions are rarely encountered with such a high ionization parameter (Dopita et al. 2000; Kewley & Dopita 2002). Therefore, we conclude on the basis of our models that accounting for the effect of dust absorption in determining star formation rates is both a significant and important correction, but that dust rarely dominates in the absorption of the EUV photons in normal H ii regions.

### 2.3. A Simple Analytic Fit

It is self-evident that greater dust content will enable the dust to compete more efficiently with the gas for the EUV Lyman continuum photons. The dust content is determined by the balance of grain formation and grain destruction processes, but it should, to first order, scale as the metallicity, \( Z \). We have taken this as an assumption of our models by using the same set of depletion factors in all models. However, it is not so immediately apparent that for a high ionization parameter, the dust again becomes relatively more important in the competition for absorption of the ionizing photons. This result is readily established. After Dopita et al. (2002a), the local absorption of ionizing photons by the ionized plasma is simply equal to the local recombination rate:

\[
\frac{dS_s}{dx} = -\alpha(T_e)n^2,
\]

where \( n \) is the density in the ionized plasma, \( S_s = (hc)^{-1} \int \lambda I(\lambda) d\lambda \) is the local photon density from the ionizing source (cm\(^{-2}\) s\(^{-1}\)), and \( \alpha(T_e) \) is the recombination coefficient. Here we have implicitly assumed that the nebula is fully ionized and that \( n = n_e = n_H \). The absorption of photons by dust is given by

\[
\frac{dS_d}{dx} = -\kappa n S_s,
\]

where \( \kappa \) is the effective dust opacity (per atom). It follows that dust absorption becomes relatively more important as the strength of the ionizing field increases and dominates the absorption of photons in the photoionized plasma when

\[
\mathcal{U} > \frac{\alpha(T_e)}{\tilde{c} \kappa}.
\]
where $\mathcal{U} = S_\alpha / cn$ is the dimensionless ionization parameter and $c$ is the speed of light. Substituting numerical values appropriate for solar abundance, we find that the critical ionization parameter above which dust dominates the absorption is $\sim 0.01$. This is in good agreement with what is indicated by detailed modeling.

In fact, we can quite readily compute the fraction of ionizing photons absorbed by a dusty H II region model relative to a dust-free model. Dropping the explicit dependence of $\alpha$ on $T_e$, the radiative transfer equation for a plane-parallel (geometrically thin) nebula including both the gas (eq. [5]) and the dust (eq. [6]) terms is

$$\frac{dS_\alpha}{dX} = -\alpha(T_e) n^2 - \kappa n S_\alpha .$$

(8)

Solving this in terms of the ionization parameter at the inner edge of the nebula $\mathcal{U}_0$,

$$\ln \frac{\alpha}{c \kappa} - \ln \left( \frac{\alpha}{c \kappa} + \mathcal{U}_0 \right) = x_d \kappa n = \tau_d ,$$

(9)

where $x_d$ is the thickness of the ionized layer (with dust), and $\tau_d$ is the optical depth in dust through the ionized layer. In the absence of dust, we can integrate equation (5) to solve for the thickness of the ionized layer, $x_\alpha = \mathcal{U}_0 / cn$. Therefore, in a nebula of uniform density, the ratio of the recombination line flux with and without dust is simply

$$f = \frac{F_{\mathrm{H}_\alpha}}{F_{\mathrm{H}_\beta}(0)} = x_d / x_\alpha .$$

Thus it follows from equation (9) that

$$f = \frac{F_{\mathrm{H}_\beta}}{F_{\mathrm{H}_\beta}(0)} = y^{-1} \ln \left( \frac{1}{1 + y} \right),$$

(10)

where $y = (c/\alpha) \mathcal{U}_0 c$. This fraction has the same meaning as the fraction $f$ defined by Inoue et al. (2001) or Petrosian et al. (1972) for spherical filled H II regions.

Since the dust opacity, $\kappa$, scales as the metallicity (i.e., as the O/H abundance by number of atoms) it follows that the fraction of EUV photons absorbed by the dust is a function only of the product of the metallicity, $Z$, or (equivalently) O/H, and the initial ionization parameter, $\mathcal{U}_0$. This scaling is the physical reason why Inoue et al. (2001) found an inverse correlation not only between $f$ and log(O/H), but also an inverse correlation between $f$ and the number of Lyman photons estimated from the central star(s). It is also significant that the H II regions having the most luminous clusters also tend to have low densities, consistent with them having high values of $\mathcal{U}_0$. These H II regions also tend to have the lowest inferred values of $f$.

Equation (10) is strictly valid if the effect of absorption of EUV photons on dust is only to remove photons from the EUV field. However, grains can also produce photoelectrons, and some of these may recombine with a proton to produce an additional contribution to the Balmer emission. Provided that only a small percentage of dust absorptions produce a photoelectron, then to the first order the corrected absorption fraction $f'$, is

$$f' = (1 - f) Y ,$$

(11)

where $Y$ is the photoelectric yield, of the order of 0.1. The importance of photoelectric emission is suggested by the fact that, in Figure 1, the models having twice solar abundance lie systematically higher than the models with lower abundance. This is the result of two effects. First, the higher dust-to-gas ratio ensures that there are more dust-produced photoelectrons per unit mass of gas. Second, at high heavy element abundance, the electron temperature of the H II region is much lower, and the recombination coefficient of hydrogen is therefore higher, ensuring a greater production of Balmer photons by recombining dust-produced photoelectrons.

The dust absorption curve without photoelectric emission (eq. [10]) and with photoelectric emission (Y = 0.1; eq. [11]) were scaled to fit the two families of models (with 0% and 20% of C in PAHs, respectively). These two families of curves probably represent the extremes allowed by the theoretical models. For the 0% PAH models, $y = 1$ corresponds to $\log[\mathcal{U}_0 \times (O/H)] = -5.377$, while for the 20% of C in PAH models, $y = 1$ is reached by $\log[\mathcal{U}_0 \times (O/H)] = -5.854$. This emphasizes how efficient PAHs (if present) are in increasing the opacity of the H II region to dust. This is because PAHs offer an essentially molecular opacity, with the carbon atoms arranged in flat sheets so that relatively few atoms can offer a large cross section to the EUV radiation field.

The dust absorption in spherical models is more severe than for the thin-shell models presented here. Inoue et al. (2001) and Petrosian et al. (1972) showed that, for such models,

$$f = \frac{F_{\mathrm{H}_\beta}}{F_{\mathrm{H}_\beta}(0)} = \frac{\tau_d^2}{3} \left[ (\tau_d^2 - 2 \tau_d + 2) \exp(\tau_d) - 2 \right].$$

(12)

If the initial slope of this function is fitted to our thin-shell H II region models, it rapidly becomes too steep at larger $\log[\mathcal{U}_0 \times (O/H)]$, and $f$ approaches zero. The reason for this is that the inner regions of filled spherical H II regions are characterized by a very high local radiation field, so that dust can compete much more efficiently for EUV photons in this region. Thus, overall, the filled H II regions will provide an upper limit to the dust absorption fraction. However, since the geometry of most extragalactic H II regions is better fitted with a central empty zone, such an empty zone is naturally produced by the strong stellar winds of the OB stars in H II regions, and since the strong-line diagnostics of extragalactic H II regions are all well fitted by shell models (Dopita et al. 2000; Kewley & Dopita 2002), we are of the opinion that spherical shell models should be more physically realistic than filled Strömgren spheres in most cases.

3. CONCLUSIONS

We have studied the theoretical effect of internal dust in absorbing the Lyman continuum photons and so in reducing the Balmer line fluxes produced by H II regions. We agree with the conclusion of Inoue et al. (2001) that “the effect of Lyman continuum extinction is not negligible relative to other uncertainties of estimating the star formation rates of galaxies.” However, the largest uncertainty in correcting the estimated star formation rates for internal dust absorption is the presence or absence of PAH-like organic molecules in the ionized gas. Since this is so theoretically uncertain, we have run models either without these molecules or else including these at the level of 20% of total carbon, which is probably a reasonable upper limit for their abundance.

We have also provided simple theoretical fits to the fraction of the EUV photons that are used by the ionized gas, $f$. 


On the assumption that the dust-to-gas ratio scales as the metallicity of the gas, this fraction \( f \) is shown to be only a function of the product of the initial ionization parameter, \( \mathcal{U} \), and the oxygen abundance, \( \mathrm{O/H} \). This is fortunate because each of these can be estimated independently for a given \( \text{H} \) ii region or starburst galaxy by measuring only the strong emission lines at optical wavelengths (ideally the \([\mathrm{O} \, \text{ii}]\), \([\mathrm{H} \alpha]/\mathrm{C}\) 12, \([\mathrm{O} \, \text{iii}]\), \([\mathrm{H} \beta]/\mathrm{C}\) 11, \([\mathrm{N} \, \text{ii}]\), and \([\mathrm{S} \, \text{ii}]\) lines, although subsets of these may be used in certain circumstances; Kewley & Dopita 2002).

Once both \( \mathcal{U} \) and \( \mathrm{O/H} \) have been determined, the \( f \)-factor can then be obtained either by reading directly from Figure 1 or else by solving for \( f \) using the analytic fits given by equations (10) and (11), with \( y = 1 \) corresponding to \(-5.854 < \log(\mathcal{U}_0 \times \mathrm{O/H}) < -5.377\). Within these theoretical uncertainties, the star formation can then be derived using the corrected Kennicutt (1998) expression;

\[
\text{SFR}_{\text{H} \alpha} = 7.9 \times 10^{-2} \frac{L_{\text{H} \alpha}}{\mathcal{U}_0 } \, \text{ergs s}^{-1}\text{.}
\]

We could also use an analogous expression involving another member of the Balmer, Paschen, or Brackett series of hydrogen.

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