One hadron exchange non-relativistic model for $K^+ p$ potential

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Abstract. $K^+ p$ potential is modeled as one hadron exchange. The hadrons being exchanged include scalar meson ($\sigma$), vector mesons ($\omega, \rho$), hyperons ($\Lambda, \Sigma$), and their resonances ($\Lambda'(1600), \Sigma'(1385)$). This interaction model is derived for non-relativistic calculations and for that purpose the potential model is formulated within Blankenbecler-Sugar reduction. The values of the potential parameters are fixed by fitting to experimental data of $K^+ p$ spin-averaged differential cross section for kaon laboratory energies from 88.13 MeV to 380.58 MeV. Scattering calculations are performed by using a 3D technique without partial wave expansion.

1. Introduction

Theoretical calculations are needed to analyze experimental results and to predict values of observables of particle scattering processes and of bound systems, as well as to understand interaction between particles. Here we make an attempt to model an interaction between positive Kaon and Proton ($K^+ p$).

We derived the interaction as one hadron exchanges based on the works in References [1, 2]. This interaction model is meant for non-relativistic calculations and we apply the Blankenbecler-Sugar reduction [3] for that purpose. We add the resonance $\Lambda'(1600)$ to a previous model in Reference [4], thus, the hadrons included in this work are a scalar meson $\sigma$, vector mesons $\omega, \rho$, hyperons $\Lambda, \Sigma$, and resonances $\Lambda'(1600), \Sigma'(1385)$. Parameters values are determined by fitting to the experimental data of $K^+ p$ differential cross section for kaon laboratory energy range of 88.13 MeV to 380.58 MeV [5, 6]. To calculate $K^+ p$ scattering we use a three-dimensional (3D) technique without partial wave expansion [7].

2. $K^+ p$ scattering in 3D formulation

2.1 Basic state

According to the 3D technique[7], we define the basis state $|p,\lambda\tau\nu\rangle$ as a combination of the free state $|p\rangle$, the spin state $|\lambda\rangle$, and the total-isospin state $|\tau\nu\rangle$ of the system:

$$|p,\lambda\tau\nu\rangle \equiv |p\rangle|\lambda\rangle|\tau\nu\rangle,$$

with $p$ being the relative momentum, $\lambda = \pm \frac{1}{2}$ the spin projection in an arbitrary fixed z axis, $\tau$ the total isospin, and $\nu$ the z-component of the total isospin. The basis state in Equation (1) is normalized as
(p'λ'τν|pλτν) = δ(p' - p)δ_{λλ'}δ_{ττ'}δ_{νν'}, \hspace{1cm} (2)
and its completeness relation is as follows

\[ \sum_{λτν} \int dp|pλτν⟩⟨pλτν| = 1. \hspace{1cm} (3) \]

### 2.2 Potential Matrix Elements

In the 3D basis state given in Equation (1), we define the potential matrix elements \( V^{TV}_{λλ'}(p', p) \) as

\[ V^{TV}_{λλ'}(p', p) \equiv ⟨p'λ'τν|V|pλτν⟩ = ⟨λ'|⟨p'V|p⟩|τν⟩|λ⟩ = ⟨λ'|V^{TV}(p', p)|λ⟩, \hspace{1cm} (4) \]

with \( V^{TV}(p', p) \) being the potential matrix elements in momentum space for a given total isospin. The general structure of \( V^{TV}(p', p) \) can be written as

\[ V^{TV}(p', p) = f^0_{i}V^{TV}(p', p, \hat{p}' \cdot \hat{p}) + f^1_{i}V^{TV}(p', p, \hat{p}' \cdot \hat{p})(s \cdot \hat{p})(s \cdot \hat{p}). \hspace{1cm} (5) \]

with \( f^i_{(i = 0,1)} \) being spin-independent functions and \( s = \frac{1}{2}σ \), where \( σ \) is the Pauli matrix. Substituting Equation (5) into Equation (4) we get the potential matrix elements \( V^{TV}(p', p) \) as

\[ V^{TV}_{λλ'}(p', p) = \delta_{λλ'}f^0_{i}V^{TV}(p', p, \hat{p}' \cdot \hat{p}) + \delta_{λ, -λ} \frac{1}{2}e^{2iλφ^i}f^1_{i}V^{TV}(p', p, \hat{p}' \cdot \hat{p})\{cosθ'cosθ + e^{-2i(φ'-φ)}sinθ'sinθ}\] \hspace{1cm} (6)

For \( p = 2 \), we get

\[ V^{TV}_{λλ'}(p', p) = \delta_{λλ'}f^0_{i}V^{TV}(p', p, cosθ') + \frac{1}{4}f^1_{i}V^{TV}(p', p, cosθ')cosθ'\] \hspace{1cm} (7)

We put \( e^{-i(λ'-λ)φ'} \) as an overall factor in Equation (7) and, thus, obtain the potential matrix elements \( V^{TV}_{λλ'}(p', p) \) as

\[ V^{TV}_{λλ'}(p', p) = e^{-i(λ'-λ)φ'}V^{TV}_{λλ'}(p', θ', p), \hspace{1cm} (8) \]

with

\[ V^{TV}_{λλ'}(p', θ', p) = \delta_{λλ'}[f^0_{i}V^{TV}(p', p, cosθ') + \frac{1}{4}f^1_{i}V^{TV}(p', p, cosθ')cosθ'] \hspace{1cm} (9) \]

The potential matrix elements \( V^{TV}_{λλ'}(p', p) \) and \( V^{TV}_{λλ'}(p', θ', p) \) have symmetry relations as follows

\[ V^{TV}_{λλ'}(p', p) = (-)^{λ-λ}e^{-2i(λ'-λ)φ}V^{TV}_{-λ,-λ}(p', p) \hspace{1cm} (10) \]

\[ V^{TV}_{λλ'}(p', θ', p) = (-)^{λ-λ}V^{TV}_{-λ,-λ}(p', θ', p). \hspace{1cm} (11) \]

### 2.3 T-Matrix elements and spin-averaged differential cross section

We define the T-matrix elements in the 3D basis state as

\[ T^{TV}_{λλ'}(p', p) \equiv ⟨p'λ'τν|T|pλτν⟩. \hspace{1cm} (12) \]
The T-matrix elements defined in Equation (12) satisfies the Lipmann-Schwinger equation for T-matrix that in the 3D basis state is written as

\[
T^{TV}_{\lambda;\lambda'}(p', p) = \frac{1}{E_{p'} - E_p + i\varepsilon} T_{\lambda;\lambda'}(p', p).
\]  

with \( E_p = \frac{p^2}{2\mu} \) and \( E_{p'} = \frac{p'^2}{2\mu} \). The T-matrix elements have the same azimuthal behaviour as the potential matrix elements given in Equation (8), thus

\[
T^{TV}_{\lambda;\lambda'}(p', p\hat{z}) = e^{-i(\lambda' - \lambda)\phi} T^{TV}_{\lambda;\lambda'}(p', \theta', p).
\]  

The matrix elements \( T^{TV}_{\lambda;\lambda'}(p', \theta', p) \) satisfy the following integral equation

\[
T^{TV}_{\lambda;\lambda'}(p', \theta', p) = V^{TV}_{\lambda;\lambda'}(p', \theta', p) + 2\mu \lim_{\varepsilon \to 0} \sum_{\lambda' = -1/2}^{1/2} \int_0^\infty dp'' \frac{p''^2}{p^2 + i\varepsilon - p''^2} 
\]

\[
\times \int_{-1}^1 d\cos \theta'' V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \theta'') T^{TV}_{\lambda;\lambda'}(p'', \theta'', p),
\]

with \( V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \theta'') \) being defined as

\[
V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \theta'') = \int_0^{2\pi} d\phi'' V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \phi'') e^{i(\lambda\phi'' - \lambda'\phi''')} e^{-i(\phi' - \phi'')}.
\]

We obtain a symmetry relation for \( V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \theta'') \) as

\[
V^{TV}_{\lambda;\lambda'}(p', \theta', p'', \theta'') = (-)^{\lambda' - \lambda''} V^{TV}_{-\lambda';-\lambda''}(p', \theta', p'', \theta'').
\]

By means of Equation (17), we obtain a symmetry relation for \( T^{TV}_{\lambda;\lambda'}(p', \theta', p) \) being similar to that for \( V^{TV}_{\lambda;\lambda'}(p', \theta', p) \) given in Equation (11), that is

\[
T^{TV}_{\lambda;\lambda'}(p', \theta', p) = (-)^{\lambda' - \lambda} T^{TV}_{-\lambda';-\lambda}(p', \theta', p).
\]

As usual it is chosen that \( \hat{p} = \hat{z} \) and the xz plane is the scattering plane. Therefore \( \phi' = 0 \) and

\[
T^{TV}_{\lambda;\lambda'}(p', \hat{z}) = T^{TV}_{\lambda;\lambda}(p', \theta', p).
\]

Applying Equation (18), we obtain the spin-averaged differential cross section as

\[
\frac{d\sigma}{dp''} = (2\pi)^4 \mu^2 \left( |T^{TV}_{\frac{1}{2};\frac{1}{2}}(p, \theta', p)|^2 + |T^{TV}_{\frac{1}{2};\frac{1}{2}}(p, \theta', p)|^2 \right).
\]

where all possible total isospin states are included in \( T^{TV}_{\lambda;\lambda}(p, \theta', p) \) as

\[
T^{TV}_{\lambda;\lambda}(p, \theta', p) = \sum_{\tau} C^2 (\tau_1\tau_2 \tau' \tau'') T^{TV}_{\lambda;\lambda}(p, \theta', p). \]

But for K^p system there is only one value for total isospin, which is 1.
2.4 $K^+p$ interaction model

![Feynman diagram of the exchange of particles](image)

**Figure 1.** Feynman diagram of the exchange of particles (a) mesons $\omega, \rho,$ and $\sigma$ (b) baryons $\Lambda, \Sigma, \Sigma^*$ and $\Lambda^*$.

We construct the $K^+p$ interaction model as a one-hadron-exchange potential. The Feynman diagram of the interaction is shown in Figure 1. We also include the resonances $\Lambda^*(1600)$ and $\Sigma^*(1385)$. After applying the Blankenbecler-Sugar reduction[3], we obtain the interaction model in operator form as

$$V_{\sigma}(p', p) = \frac{-g_{KK\sigma}g_{NN\sigma}}{32\pi^2 m_N m_K \sqrt{E'_N + \omega'_K \sqrt{E_n + \omega_K}}} \left(\frac{m_N + m_K}{(p' - p)^2 + m_\sigma^2}\right),$$

$$V_{\rho}(p', p) = \frac{g_{KK\rho}g_{NN\rho}}{64\pi^2 m_N m_K \sqrt{E'_N + \omega'_K \sqrt{E_n + \omega_K}}} \left(\frac{m_N + m_K}{(p' - p)^2 + m_\rho^2}\right),$$

$$V_{\Sigma}(p', p) = \frac{-g_{{NYK}}}{128\pi^2 m_N m_K \sqrt{E'_N + \omega'_K \sqrt{E_n + \omega_K}}} \left(\frac{m_N + m_K}{(p' + p)^2 + m_\Sigma^2}\right),$$

$$V_{\Lambda}(p', p) = \frac{g_{{NAK}}}{128\pi^2 m_N m_K \sqrt{E'_N + \omega'_K \sqrt{E_n + \omega_K}}} \left(\frac{m_N + m_K}{(p' + p)^2 + m_\Lambda^2}\right),$$

where $\omega, \rho,$ and $\sigma$ are mesons and $\Lambda, \Sigma, \Sigma^*$ and $\Lambda^*$ are baryons.
\[
V_{\gamma'}(p', p) = \frac{g_{N\pi N}^2}{256\pi^3m_Nm_K^3} \frac{(m_N + m_K) F^2_{N\pi N}[(p' + p)^2]}{E'_N + \omega'_K - E_N + \omega_K} \left[ (3 - \tau_1 \cdot \tau_2) \right] (p + p')^2 + m^2_{\pi'} \times \frac{\mathcal{D}_{\gamma'}(p', p)}{(\omega'_K \omega_K W W')^{1/2}},
\]

with \(\mathcal{D}_{\sigma}(p', p), \mathcal{D}_{v1}(p', p), \mathcal{D}_{v2}(p', p), \mathcal{D}_\gamma(p', p), \mathcal{D}_{A'}(p', p), \text{ and } \mathcal{D}_{\pi'}(p', p)\) being given as

\[
\mathcal{D}_\sigma(p', p) = m_K(W W' - \sigma \cdot p' \sigma \cdot p),
\]

\[
\mathcal{D}_{v1}(p', p) = W p'^2 + W' p^2 + W W' (\omega'_K + \omega_K) + (W' + W + \omega_K + \omega_K') \sigma \cdot p' \sigma \cdot p,
\]

\[
\mathcal{D}_{v2}(p', p) = \frac{1}{2m_n} (E'_N + E_N)(\omega'_K + \omega_K) + (p' + p)^2 (\sigma \cdot p' \sigma \cdot p - W W'),
\]

\[
\mathcal{D}_\gamma(p', p) = (2m_N - m_N - \omega_K - E_N)W W' + (m_N - 2m_N - \omega_K - E_N)\sigma \cdot p' \sigma \cdot p,
\]

\[
\mathcal{D}_{A'}(p', p) = \sigma \cdot p' \sigma \cdot p.
\]

In Equations (22) – (26) the form factors are given as

\[
F_{\alpha}(q^2_r) = \left( \frac{\Lambda^2_{\alpha} - m^2_{\pi'}}{\Lambda^2_{\alpha} - q^2_r} \right)^{n_{\alpha}}, F_{\beta}(q^2_r) = \left( \frac{\Lambda^4_{\beta} - m^4_r}{\Lambda^4_{\beta} - (q^2_r)^2} \right),
\]

with \(q^2_r(\sigma, \omega, \rho)\) and \(q^2_r(\nu)\) being the momentum transfers:

\[
q^r_r = p' - p,
\]

\[
q^\nu_{\nu} = (p'_N - p_K)^{\nu} = (E'_N - \omega_K, p'_N - p_K) = (E'_N - \omega_K, p' + p),
\]

and there are additional isospin factors of \(\tau_1 \cdot \tau_2\) for \(\rho\) exchange, \(\frac{1}{2}(1 + \tau_1 \cdot \tau_2)\) for \(\Lambda\) exchange, and \(\frac{1}{2}(3 - \tau_1 \cdot \tau_2)\) for \(\Sigma\) exchange.

3. Results and discussion

Some constant parameters are given in Table 1. For the fitting process the model needs initial parameters values shown in Table 2, which we take from Reference [1, 8]. The mass of \(\sigma\) is also fitted with an initial value of 720 MeV.

We fit to the experimental data, namely the differential cross section for kaon laboratory energy range of 88.13 MeV to 380.58 MeV. The fitting process yields new parameter values given in Table 3, \(\sigma\) mass of 700 MeV, and the \(\chi^2/N\) value of 18.47 according to the following formula.
Table 1. Unfitted parameters

| Particle | Mass (MeV) | f/g | Exponent |
|----------|------------|-----|----------|
| σ        | -          | 0   | 1        |
| ρ        | 769        | 6.1 | 2        |
| ω        | 782.6      | 0   | 2        |
| Λ        | 1116       | 0   | 1        |
| Σ        | 1193       | 0   | 1        |
| Λ*       | 1600       | 0   | 1        |
| Σ*       | 1385       | 0   | 1        |

Table 2. Initial values of the parameters being fitted

| Particle | $g_N/\sqrt{4\pi}$ | $g_K/\sqrt{4\pi}$ | $\Lambda_N$ (MeV) | $\Lambda_K$ (MeV) |
|----------|--------------------|--------------------|-------------------|-------------------|
| σ        | 2.9906             | 0.3777             | 1900              | 1400              |
| ρ        | 0.9487             | 0.857              | 1850              | 2250              |
| ω        | 4.4947             | 1.394              | 1850              | 1250              |
| Λ        | -3.944             | -3.944             | 1400              | 1400              |
| Σ        | 0.759              | 0.759              | 1400              | 1400              |
| Λ*       | 0.2                | 0.2                | 2000              | 2000              |
| Σ*       | -0.193             | -0.193             | 2000              | 2000              |

Table 3. New values of the parameters

| Particle | $g_N/\sqrt{4\pi}$ | $g_K/\sqrt{4\pi}$ | $\Lambda_N$ (MeV) | $\Lambda_K$ (MeV) |
|----------|--------------------|--------------------|-------------------|-------------------|
| σ        | 3.928              | 1.690              | 1936              | 1400              |
| ρ        | 1.162              | 1.700              | 1600              | 1550              |
| ω        | 3.010              | 0.7297             | 2000              | 2000              |
| Λ        | 0.2                | 0.2                | 1215              | 1215              |
| Σ        | 0.2024             | 0.2024             | 1995              | 1995              |
| Λ*       | 0.1                | 0.1                | 1920              | 1920              |
| Σ*       | 1.702              | 1.702              | 1976              | 1976              |

Here we show and discuss the differential cross section using the obtained parameter values. We choose as example kaon laboratory energies of 88.13 Mev, 232.37 Mev, and 380.58 Mev. The differential cross sections for the 3 energies are shown in Figure 2. It can be seen that calculation still cannot agree with the data well enough.
Figure 2. Differential cross sections for energy of (a) 88.13 MeV, (b) 232.37 MeV, and (c) 380.58 MeV.

Next the contributions of each particle exchanges to the interaction model are studied by eliminating the corresponding terms, that is by setting the coupling constant $g_N = 0$ and $g_K = 0$. The calculated cross sections are then compared with those obtained for the full potential. Large discrepancies mean that the contributions of the corresponding exchanges are significant. Figure 3 shows contributions from meson exchanges, Figure 4 from hyperon exchanges, and Figure 5 from resonance exchanges.
Figure 3. Differential cross sections for energy of (a) 88.13 MeV, (b) 232.37 MeV, and (c) 380.58 MeV without mesons $\sigma$, $\rho$, and $\omega$ exchanges.
Figure 4. Differential cross sections for energy of (a) 88.13 MeV, (b) 232.37 MeV, and (c) 380.58 MeV without hyperons $\Lambda$ and $\Sigma$ exchanges.

Figure 5. Differential cross sections for energy of (a) 88.13 MeV, (b) 232.37 MeV, and (c) 380.58 MeV without resonances $\Lambda^*(1600)$ and $\Sigma^*(1385)$ exchanges.

It is shown in Figure 3 that the contributions from $\sigma$, $\rho$, $\omega$ particle exchanges are very large especially in forward angles and higher energies. Figure 4 shows that the contributions from $\Lambda$ and $\Sigma$ exchanges occur only in higher energies and forward directions. In Figure 5, the contributions from $\Lambda^*$ exchange
are observed only in higher energies and in forward angle, while those from Σ* exchange occur at all the energies being chosen and in forward directions.

4. Conclusion
This research attempts to make a one hadron exchange nonrelativistic model for K+p interaction. The parameters are determined by fitting processes to experimental data of differential scattering for kaon laboratory energies from 88.13 MeV to 380.58 MeV. The model still needs to be improved as indicated by, for example, the large $\chi^2/N$ value of 18.47. Contributions from each particle exchanges are discussed. The $\Sigma$ and $\Lambda$ exchanges contribute only on higher energies, while other exchanges contribute in all energies being considered. All exchanges are getting more important as energy increases. Contributions are observed especially in forward directions.

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