Beam-plasma instability, the features of HXR spectra and polarization in solar flares

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Abstract. A one-dimensional quasi-linear beam relaxation leads to the formation of a plateau-like electron distribution function. Of greatest practical interest is the case when the initial velocity of the beam $v_0$ greatly exceeds the thermal velocity of the plasma electrons $V_T$, and the initial velocity spread in the beam $\Delta v_0$ is small compared with $v_0$. These conditions are usually met for solar flares. The article deals with the features of the energy spectra and polarization of hard x-ray radiation (HXR) in the energy range of 20-150 keV generated by electrons with a plateau-like electron distribution function. Calculations in the framework of a thick target take into account not only the energy losses of electrons interacting with plasma particles, but also changes in their angular distribution. It is shown that the break down energy of the electron spectrum leads to changes in HXR spectra, as well as the degree of linear polarization. There is a characteristic softening of the spectrum near the break-down energy. HXR polarization increases with quanta energy and can reach tens of percent.

1. Introduction
It is generally accepted that HXR flux in the energy range between 20 and 150 keV is produced mostly through dipole electron-ion ($e$-$i$) bremsstrahlung. The distribution function of such electrons can be obtained by analyzing electromagnetic radiation and simulating electron propagation in the plasma of the flare loops [1-5]. Based on the results of HXR observations, the energy part of electron distribution function is restored by a methods of forward-fitting or solving the inverse problem [6-8]. At the same time, the pitch-angle part of the distribution function remains obscure, and we can only judge the degree of anisotropy qualitatively. When solving the inverse problem of restoring electron distributions function, we can detect step-type features in electron energy spectra and even inversions of the spectra in local energy band that are not observed in HXR spectra as a result of their smoothing when integrating the accelerated electron distribution function. The purpose of this article is to study the features of X-rays under the assumption of a specific distribution function of accelerated electrons with a plateau velocity spectrum [9] with a sharp decline at a certain boundary velocity.

2. Hard X-Ray spectra. Plateau-like velocity distribution function
HXR flux in the energy band from $\varepsilon$ to $\varepsilon+\varepsilon d\varepsilon$ radiated by a unit plasma volume with a radius vector $r$ to a unit solid angle is determined by the expression (see, for example, [10])

$$dI_{\varepsilon}(\alpha, r) = Q_{\varepsilon}(\alpha, r)d\varepsilon$$  \hspace{1cm} (1)
\[ Q(E, \alpha, r) = \int_{E}^{\infty} dE \int_{\Omega} N(r) \sigma(E) v(E) f(E, n, r) d\Omega \]  \hspace{1cm} (2)

where \( E \), \( \alpha \) – the energy of x-ray photons and electrons, \( v \) is the electron velocity, \( n \) is a unit vector along the impulse of the electron, \( d\Omega \) - elementary solid angle along the direction of the vector \( n \), \( \sigma(E, \alpha, \theta) \) is the relativistic bremsstrahlung cross-section [11] with polarization vector \( e_\lambda \), which has a projection \( e_1 \) in the plane of the vectors \( B \) (along the Oz axis) and \( k \) (wave vector), and perpendicular to this plane - \( e_2 \), \( \alpha \) is the viewing angle, the angle between the x-ray quantum wave vector \( k \) and the Oz axis; \( \theta \) is the angle between vectors \( k \) and \( n \), \( N \) - a plasma density. For solar flares at the center of the Sun the viewing angle is equal 0\(^0\) practically and it is 90\(^0\) for limb flares.

As a distribution function, we select a plateau-like model function in the phase space of the pulse

\[ f(E, \theta) = A \cdot F(E) \cdot \chi(\theta) \]  \hspace{1cm} (3)

where \( F(E) = \left( E_0 / E \right)^{\delta} \) at \( E < E_0 \) and \( F(E) = \left( E_0 / E \right)^{\delta} \) at \( E > E_0 \)

A is the normalizing multiplier.

This energy distribution corresponds to a plateau-like velocity distribution function that transitioned to the power spectrum starting with \( v_o (E_0) \). Since the question of the angular distribution of fast electrons remains uncertain, we consider both an isotropic distribution and anisotropic distributions of the type \( \chi \sim \cos^{2}\theta \) at \( 0 < \theta < 180^0 \) (recalculate for a symmetric distribution).

**Figure 1.** HXR spectra for two values of viewing angles \( \alpha \), the boundary energy \( E_0 \), and power-law indexes \( \delta \). \( \chi \sim \cos^{2}\theta \) at \( 0 < \theta < 180^0 \), a) \( \delta = 3.5 \), \( E_0 = 50\text{keV} \), b) \( \delta = 3.5 \), \( E_0 = 80\text{keV} \), c) \( \delta = 5.5 \), \( E_0 = 50\text{keV} \), d) \( \delta = 5.5 \), \( E_0 = 80\text{keV} \)
The results of HXR spectra calculations are shown in figure 1 for different boundary energy values $E_0$, for two viewing angles $\alpha$ (0° and 90°) and for two indexes $\delta=3.5$ and $\delta=5.5$. We emphasize that HXR spectra do not correspond to the power law at energies less then value of $E_0$ at all parameters. When the energy value $E_0$ increases, the deviation of the spectrum from the power-law increases (figure 1a,b). In the case of a sharper drop in the electron spectrum ($\delta=5.5$), the spectrum is distorted from the power law even more (figures 1c, 1d). But, at the energies higher then the boundary energy $E_0=50$ keV and 80 keV HXR spectra correspond to the power law at all values of $\delta$ and viewing angles $\alpha$. Thus, a plateau-like distribution of electrons can be detected by characteristic features in the energy spectrum of HXR depending on the boundary energy $E_0$.

Even more interesting are the energy dependences of the degree of HXR linear polarization, the calculations of which for plateau-like energy distribution and angular distributions $\sim \cos^2 \theta$ and $\cos^6 \theta$ of electrons were performed similarly to [10,12] and are presented in figure 2. The degree of linear polarization of HXR is defined as

$$P = (J_2 - J_1)/(J_2 + J_1)$$

where $J_1$ and $J_2$ are HXR fluxes which has a projection in the plane of the vectors B (along the Oz axis) and k and perpendicular to this plane. For an isotropic distribution of electrons, the degree of HXR polarization turns to 0. For anisotropic distributions, the polarization degree is negative at all values of viewing angles $\alpha$, the boundary energy $E_0$, and power-law indexes $\delta$ (figure 2a,b). And HXR polarization degree increases with the growth of the quantum energy from 20 keV up to the energy $E_0$.

![Figure 2](image-url)

Figure 2. HXR polarization degree for various parameters of the electron spectrum $E_0$, $\delta$ and $\chi$

The value of the HXR polarization degree depends on the hardness of the energy spectrum and reaches a maximum value of -48% for $\delta=3.5$ and -55% for $\delta=5.5$ on the energy $E_0$. For the quantum energies greater than the boundary energy $E_0$, HXR polarization degree decreases with the growth of the quantum energy (figure 2a,b). Thus the analyses of measurements of HXR polarization degree allow us to select the type of energy and angular distribution of fast electrons.

3. HXR spectra. Thick target model.

Consider HXR characteristics of the solar flare in the thick target model. In this case, it is necessary to take into account the Coulomb collisions of electrons with solar plasma particles. The time-independent relativistic kinetic equation for the fast electron distribution function $f(E, \vec{r}, \theta)$ has the same form as in [13], but with the electron injection sources in the right side

$$\frac{1}{c} \nabla \text{grad} f = \sum_{i=1}^{n} \left( c_i \frac{\partial}{\partial E} \left( \frac{f}{\beta} \right) + \frac{c_2}{\beta^2 \gamma^2} \frac{\partial}{\partial (\cos \theta)} (\sin^2 \theta \frac{\partial}{\partial (\cos \theta)} \left( \frac{f}{\beta} \right) ) \right) + k \nu \langle E \rangle g(\vec{r}) \chi(\theta)$$

(5)
where \( \lambda_0^{-1} = 4 \pi N_i r_0^2 \ln \Lambda \), \( v \) is the electron velocity, \( r_0 \) is the classical electron radius, \( \ln \Lambda \) is the Coulomb logarithm, \( \gamma \) is the Lorentz factor, \( \beta = v/c \), \( c \) is the speed of light, \( \theta \) is the pitch angle of the electron, \( E = \gamma - 1 \) is the kinetic energy of the electron in \( mc^2 \) units, \( N_i \) is the plasma density, \( g(r) \) describes the spatial distribution of accelerated electron sources, \( \psi(E) \) sets the energy spectrum in the source, \( \chi(\theta) \) characterizes the pitch-angular distribution of electrons in the injected beam.

For the case of a fully ionized plasma \( C_1 = 1, C_2 = (3 + \gamma)/4 \) [13]. When solving the equation (5), we assume that the energy of electrons accelerated in a flare is limited by the value \( E_{\text{max}} \gg E_0 \). We will consider the following energy distribution of the injected electrons (plateau-like in the phase space of the pulse)

\[
\Psi = \begin{cases} \\
\sqrt{E_0/E} - \sqrt{E_0/E_{\text{max}}}, & \text{at } E < E_0 \\
\sqrt{E_0/E} \cdot \left(\frac{E_0}{E}\right)^{\delta} - \sqrt{E_0/E_{\text{max}}} \cdot \left(\frac{E_0}{E_{\text{max}}}\right)^{\delta}, & \text{at } E_0 < E \leq E_{\text{max}} \\
0, & \text{at } E > E_{\text{max}}
\end{cases}
\]  

(6)

As an initial angular distribution, consider a distribution of type \( \cos^2 \theta \), which can be represented as a sum over Legendre polynomials with numbers \( l \leq 2n \). The method for solving equation (5-6) is described in detail in the paper [12]. To find the characteristics of the integral bremsstrahlung of electrons in the model of a thick target, it is sufficient to use the electron distribution function integrated in the source volume. Within this approximation, we will neglect the output of electrons from the radiation region, i.e. we will assume that integration is performed over the surface that restricts the radiation region.

HXR flux generated from the entire radiation area per unit of solid angle at the viewing angle \( \alpha \) will be determined by the expression

\[
J_{\lambda}(\varepsilon, \alpha) = \int Q_{\lambda}(\varepsilon, \alpha, r) dr
\]

(7)

where \( Q_{\lambda} \) is calculated from (2).

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**Figure 3.** HXR spectra for different values of viewing angles \( \alpha \), the boundary energy \( E_0 \), and power-law indexes \( \delta \). \( \chi \sim \cos^2(\theta) \), a) \( \delta = 3.5, E_0 = 50 \text{ keV}, \) b) \( \delta = 3.5, E_0 = 80 \text{ keV}, \) c) \( \delta = 5.5, E_0 = 50 \text{ keV}, \) d) \( \delta = 5.5, E_0 = 80 \text{ keV} \)
Figures 3(a-d) show the bremsstrahlung HXR spectra calculated from (7) in the thick target model for $\delta = 3.5$ and $\delta = 5.5$, $\chi \sim \cos^2 \theta$, for two viewing angles $\alpha = 0^\circ$ and $90^\circ$ and for $E_0 = 50$ keV and 80 keV. For the energy spectrum with the index $\delta = 5.5$ in the energy band $\varepsilon = 100 - 500$ keV, these spectra can be described by the power-law with the exponent of the spectra $\gamma = 5.45$ and 4.87, respectively. Note that in the non-relativistic approximation, these indexes would be equal to 5. HXR spectrum restriction at high energies is determined by high energy limit of the electron distribution function at $E_{\text{max}} = 10m_e c^2$. Note that the similar dependencies of HXR spectrum on the viewing angle occur for other parameters $\delta$ and other angular dependencies. For lower energies the break of the spectrum is clearly visible on figures 3a - 3d.

![Figure 3](image)

Figure 4 shows the results of HXR polarization degree calculations. The polarization degree increases with increasing energy up to the value $E_0$, after which it decreases. Note that the higher the anisotropy, the greater HXR polarization degree (see, curves 1,2 versus 3-4). In the case of $\chi \sim \cos^2 \theta$ anisotropy the values of HXR polarization negative and reach (15-20) % for hard energy spectrum - index $\delta = 3.5$ and about 30% for softer spectra with $\delta = 5.5$. Although the maximum polarization value (-20% as a result of fast electron scattering on background plasma particles) is less than in the previous model (figure 2), it remains high enough to be measured in future experiments.

4. Conclusions

The features of HXR energy spectra and linear polarization degree of fast electrons are studied, taking into account that their energy distribution was formed when interacting with Langmuir waves. In this case, a plateau-like distribution is formed with a sharp drop at some characteristic energy $E_0$. It is shown that this break in the electron spectrum leads to a features of HXR spectra and HXR polarization. A spectrum break is also formed on the spectra of both local and integral radiation from the entire HXR flare region, and this break can be registered during measurements. The dependence of linear polarization degree on the energy also has a special feature – the polarization increases with the growth of energy to the boundary energy $E_0$, after which it decreases. In this case, HXR polarization can reaches tens of percent. This also shows the importance of polarimetry measurements of HXR of a flares in order to identify features of the distributions of electrons accelerated in a flares.

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