A Multiple-Reference Complex-Based Controller for Power Converters

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Abstract—A multiple-reference complex-based controller is proposed for three-phase power converters feeding nonlinear and unbalanced loads. The control scheme incorporates a stable multiple-complex coefficient filter with bandwidths that are set arbitrarily and independently for every harmonic under consideration. The multiple-reference complex-based control scheme is applied to an uninterruptible power supply system with a voltage source converter. Each harmonic is controlled using a standard complex proportional-resonant controller that is designed for stability and robustness using the Nyquist criterion. Similar stability and robustness properties follow for the overall system due to the frequency-domain properties of the filter and of the controllers. The proposed methodology is validated in simulation and experimental tests.

Index Terms—Complex Nyquist plots, complex-valued transfer functions, multiple-complex coefficient filter (MCCF), proportional-resonant (PR) complex controller, three-phase voltage-source converter (VSC).

I. INTRODUCTION

THREE-PHASE power converters often operate in environments with nonlinear elements and under unbalanced conditions. Nonlinearities induce higher harmonic components so that periodic signals are not longer sinusoidal, while unbalanced systems involve positive and negative sequences. Control algorithms for power converters have to deal with the control of multiple (positive and negative) frequencies [1].

The representation of physical systems using complex-valued dynamic models was considered for induction machines in [2] and, in general, for three-phase electrical systems in [3], considering its benefits when used in real applications. The main advantage of this representation is that the analysis is considerably simplified, especially when two-input two-output systems are converted to single-input single-output systems. Extensions of classic control techniques have been proposed in the last years to deal with complex-valued systems. Furthermore, control theory tools for complex-valued systems have been generalized such as the Routh–Hurwitz test for complex polynomials [4]–[6], the root locus technique [7], frequency-domain analysis methods [8], stability conditions for time-varying complex-valued systems [9], or the sliding-mode control technique [10], [11]. Increasingly, the complex domain representation is used not only to simplify the equations of a system, but also to design and analyze control systems in industrial applications. Examples include electrical machines [12]–[14], power converters [15]–[17], and others [8]. Complex controllers also have advantages in applications where harmonics with different sequences need to be controlled, because each sequence component can be treated independently [18], [19].

In applications, the use of an intermediate filter stage to separate the individual components from noisy signals is often advantageous. Complex filters that are commonly used for signal processing [20] have been recently proposed for electrical power systems. One of the most successful scheme is a set of first-order complex-coefficient filters (CCF) known as the multiple-CCF (MCCF) [21]. The main advantage of using an MCCF compared to classical band-pass filters using real coefficients is the polarity-selective property, which makes it possible to extract the positive or negative sequence components in a straightforward manner [22]. This algorithm is used for grid synchronization, but is also valuable for obtaining magnitude and phase information on the respective harmonic components. Applications of the MCCF algorithm include shunt active filters [23], [24] and DSTATCOMs [25].

In this article, the multiple-reference complex-based control (MRCC) scheme is proposed for the control of three-phase power converters. The control algorithm uses the ability of the MCCF of extracting the harmonic components, including positive and negative sequences. For each harmonic component, a complex proportional-resonant (PR) controller is proposed. The overall scheme looks similar to the set of reduced-order generalized integrators (ROGIs) presented in [26], but enables the tuning of the control gains for each harmonic independently. As an example, the MRCC scheme is applied to an uninterruptible power supply (UPS) consisting of a three-phase voltage-source converter (VSC) feeding a nonlinear unbalanced load. This example makes it possible to demonstrate the applicability of the proposed controller. The stability of the closed-loop system, including the filter dynamics that are disregarded in many papers,
is analysed via the Nyquist criterion applied to transfer functions with complex coefficients [8].

The main contributions are as follows.

1) A new proof of stability of the MCCF. The MCCF is shown to be stable for all values of the cutoff frequencies of the CCFs. This theoretical result is obtained through a relatively simple proof made possible by the complex-domain representation. The result removes the constraint that a single cutoff frequency be used for filtering all harmonics.

2) The main difference with respect to other papers in the literature is the inclusion of the MCCF dynamics as a part of the control loop for design purposes. Then, the design and tuning procedure is made by considering the overall dynamics, including the filter. This is not common and, usually, the filter dynamics are disregarded for stability analysis, see recent examples using the MCCF in [27] and [28].

3) An MRCC strategy for three-phase power converters. Thanks to the use of the MCCF that extracts the information for each harmonic, the proposed methodology makes it possible to individually analyse the dynamics of each harmonic and, compared with traditional approaches, facilitates the design by using the complex Nyquist criterion.

4) A practical example is illustrated by a three-phase VSC. The example includes a detailed design description, as well as simulation and experimental results.

This article is organized as follows. In Section II, the complex notation is presented. The MCCF and its stability properties are introduced in Section III. Section IV presents the overall MRCC scheme, and is applied to a VSC in Section V. Then, in Section VI, the proposed control algorithm is validated by simulations and experimental tests. Finally, the conclusion is summarized in Section VII.

II. COMPLEX REPRESENTATION OF THREE-PHASE SIGNALS

Consider a set of three-phase variables (voltages) \( \mathbf{v}_{abc}(t) = (v_a(t), v_b(t), v_c(t))^T \). Then, \( \mathbf{v}_{abc}(t) \) can be transformed into a complex variable, \( v(t) \), as

\[
v(t) = T \mathbf{v}_{abc}(t)
\]

where \( T \in \mathbb{C}^3 \) is defined as

\[
T = c \left( 1 - \frac{j}{2} \sqrt{3} - \frac{1}{2} - j \frac{\sqrt{3}}{2} \right) e^{-j\theta(t)}
\]

for a constant \( c \) and an angle \( \theta(t) \) to be defined. Different values of \( c \) are made in the literature, with \( c = \sqrt{\frac{2}{3}} \) preserving the definition of power and \( c = \frac{2}{3} \) maintaining the signal amplitude. The transformation (1) is not bijective and reconstruction of \( \mathbf{v}_{abc}(t) \) from \( v(t) \) generally requires additional information. In electrical three-phase systems, this information comes from the homopolar component that is the sum of the components of \( \mathbf{v}_{abc}(t) \)

\[
v_h(t) = v_a(t) + v_b(t) + v_c(t).
\]

Often, this component is assumed to be zero (and this is forced to be true for currents in three-wire devices).

The transformation (1) includes the well-known \( \alpha \beta \) transformation as a special case with \( \theta(t) = 0 \) and where \( v_\alpha(t), v_\beta(t) \) are the real and imaginary parts of \( v(t) \). The \( dq \)-transformation is obtained with \( \theta(t) \) corresponding to the angle of the \( d \)-axis and \( v_d(t), v_q(t) \) being the real and imaginary parts of \( v(t) \) [17]. In particular, with \( \frac{d\phi}{dt} = \omega, v(t) \) is constant if and only if \( v_\alpha(t), v_\beta(t), \) and \( v_c(t) \) are positive-sequence balanced sinusoidal signals of frequency \( \omega \). However, \( v(t) \) is defined regardless of steady-state or balanced conditions, and generalizes the concept of phasor for \( v_\alpha(t) \) (as well as \( v_h(t), v_c(t) \) with \( \frac{2\pi}{3} \) phase shifts).

A periodic three-phase voltage \( \mathbf{v}_{abc}(t) \) can be expressed as a sum of multiple frequencies

\[
\mathbf{v}_{abc}(t) = \sum_{l \in \mathcal{H}} \mathbf{v}_l(t) = \sum_{l \in \mathcal{H}} \begin{pmatrix} V_{al} \cos(l\omega_0 t + \phi_{al}) \\ V_{bl} \cos \left( l\omega_0 t - \frac{2\pi}{3} + \phi_{bl} \right) \\ V_{cl} \cos \left( l\omega_0 t + \frac{2\pi}{3} + \phi_{cl} \right) \end{pmatrix}
\]

where \( \omega_0 \) is the fundamental angular frequency, \( \mathcal{H} = \{ h_1, h_2, \ldots, h_{pr} \} \subset \mathbb{N}^p \) is the set of harmonics of \( \mathbf{v}_{abc}(t) \), and \( V_{xt}, \phi_{xt} \), with \( x = a, b, c \), are the amplitude and phase of the \( l \)th harmonic, respectively.

Using (1), with \( \theta = 0 \) and \( c = \frac{2}{3} \) the (scalar) complex variable corresponding to the (vector) three-phase symmetric positive frequency signal in (4) can be written as

\[
v(t) = \sum_{l \in \mathcal{H}} (a_le^{jl\omega_0 t} + b_le^{-jl\omega_0 t})
\]

where

\[
a_l = \frac{V_{al}e^{j\phi_{al}} + V_{bl}e^{j\phi_{bl}} + V_{cl}e^{j\phi_{cl}}}{3} \quad b_l = \frac{V_{al}e^{-j\phi_{al}} + V_{bl}e^{-j\phi_{bl} + \frac{2\pi}{3}} + V_{cl}e^{-j\phi_{cl} - \frac{2\pi}{3}}}{3}
\]

are the coefficients of the well-known positive and negative sequences. Then, the complex signal in (5) can be written as the sum of \( v_{k\xi} \) complex signals, with positive and negative frequencies, as

\[
v(t) = \sum_{k \in \mathcal{H}_c} v_k(t)
\]

with a new set of (positive and negative) harmonics defined as \( \mathcal{H}_c = \mathbb{Z}^{2p} \), and

\[
v_k(t) = v_k \mathbf{e}^{j\phi_k e^{j k \omega_0 t}} \in \mathbb{C}
\]

where the complex values \( V_k \mathbf{e}^{j\phi_k} \) correspond to (6) or (7), depending if \( k = l \) or \( k = -l \).

Remark 1: In a symmetric and balanced case, \( V_{al} = V_{bl} = V_{cl} = V_l \) and \( \phi_{al} = \phi_{bl} = \phi_{cl} = \phi_l \). Then, coefficients in (6) and (7) are

\[
a_l = V_l e^{j\phi_l}, \quad b_l = 0
\]

and no negative frequencies appear.

III. MULTIPLE-COMPLEX COEFFICIENT FILTER

The MCCF is composed by a set of CCF that process \( \alpha \beta \) signals to extract the components for each harmonic. In the
complex notation used in Section II, this approach corresponds to estimating some terms of \( \nu(t) \) defined in (8). In [21], all the CCF modules have the same feedback gains, resulting in identical cutoff frequencies. In this article, the MCCF scheme is generalized considering different cutoff frequencies for the CCF modules. Fig. 1 shows the MCCF scheme using the complex notation, where \( \mathcal{H} = \{h_1, b_2, \ldots, b_n\} \subset \mathbb{Z}^n \) is the set of considered harmonics. \( \mathcal{H} \) is usually defined as a subset of \( \mathcal{H}_c \), including the most representative harmonics in the system, i.e., \( n \leq 2p \).

A. State-Space Description of the MCCF

Consider an MCCF for \( n \) harmonics defined by the set \( \mathcal{H} \subset \mathbb{Z}^n \). Then, the CCF for the \( m \)th harmonic is

\[
\frac{\dot{v}_m(s)}{v_m(s)} = \frac{\gamma_m}{s - jm\omega + \gamma_m}, \quad m \in \mathcal{H} \tag{11}
\]

where \( v_m \) and \( \dot{v}_m \) denote the complex input and output signals of the filter for the \( m \)th harmonic, respectively, \( \omega \) is the fundamental frequency and \( \gamma_m \) is the cutoff frequency, which impacts the bandwidth associated with the \( m \)th harmonic.

The differential form of (11) is

\[
\frac{d\dot{v}_m}{dt} = (jm\omega - \gamma_m)\dot{v}_m + \gamma_m\dot{v}_m \tag{12}
\]

and, from Fig. 1, the input for each CCF module is

\[
\dot{v}_m = \dot{v}_m - \sum_{k \in \mathcal{H}} \dot{v}_k + v \tag{13}
\]

where \( v \) is the complex input signal. From (12), the overall model of the MCCF in compact form is

\[
\dot{\nu} = A\nu + Bv \tag{14}
\]

where \( \dot{\nu} = (\dot{v}_1, \ldots, \dot{v}_n)^T \), and the matrices \( A \in \mathbb{C}^{n \times n} \) and \( B \in \mathbb{C}^{n \times 1} \) are given by

\[
A = \begin{pmatrix}
-\gamma_{h_1} & -\gamma_{h_1} & \cdots & -\gamma_{h_1} \\
-\gamma_{h_2} & -\gamma_{h_2} + j\beta_2 & \cdots & -\gamma_{h_2} \\
\vdots & \vdots & \ddots & \vdots \\
-\gamma_{h_n} & -\gamma_{h_n} & \cdots & -\gamma_{h_n} + j\beta_n
\end{pmatrix} \tag{15}
\]

and

\[
B^T = \begin{pmatrix}
\gamma_{h_1} \\
\gamma_{h_2} \\
\vdots \\
\gamma_{h_n}
\end{pmatrix} \tag{16}
\]

respectively.

Equation (14) is a complex description and a generalization of the state-space form of the MCCF presented in [21].

B. Transfer Function of the MCCF

The transfer function of a single CCF has been studied in [21], and references therein. However, the MCCF structure has a more complicated behavior than the CCF. In this section, the transfer function relating the \( m \)th output \( \dot{v}_m \) to the filter input \( v \) is obtained, and the stability properties of the MCCF are analyzed.

Define the \( m \)th output equation for the MCCF as

\[
\dot{v}_m = C_m \dot{\nu} \tag{17}
\]

where

\[
C_m = \begin{pmatrix}
c_{m,1} & c_{m,2} & \cdots & c_{m,n}
\end{pmatrix} \in \mathbb{R}^{1 \times n} \tag{18}
\]

and

\[
c_{m,k} = \begin{cases}
1 & \text{if } k = m \\
0 & \text{otherwise}.
\end{cases} \tag{19}
\]

Let \( H_m(s) \) be the transfer function relating the filter input \( v \) to the \( m \)th output \( \dot{v}_m \), so that

\[
\dot{v}_m(s) = H_m(s)v(s). \tag{20}
\]

The transfer function is obtained from (14), with (15) and (16) and (18), resulting in

\[
H_m(s) = \frac{N_{H_m}(s)}{D_H(s)} \tag{21}
\]

where

\[
N_{H_m}(s) = \gamma_m \prod_{k \in \mathcal{H}, k \neq m} (s - jh_k\omega_o) \tag{22}
\]

\[
D_H(s) = \prod_{k \in \mathcal{H}} (s - jh_k\omega_o) + \sum_{k \in \mathcal{H}} \gamma_k \prod_{l \in \mathcal{H}, l \neq k} (s - jh_l\omega_o). \tag{23}
\]

Therefore

\[
H_m(jh_l\omega_o) = \begin{cases}
0 & \text{if } l \neq m \\
1 & \text{if } l = m.
\end{cases} \tag{24}
\]

In other words, the \( m \)th harmonic is estimated exactly by the \( m \)th component while other harmonics are completely rejected.
This property can be observed in Fig. 2 that shows the gain plots of transfer functions (21) with \( \mathcal{H} = \{-11, -5, -1, 1, 7, 13\} \). Note that, since the transfer functions \( H_m(s) \) have complex coefficients, plots are not symmetric for positive and negative frequencies. The total number of harmonics, as well as the harmonic components, shape the gain plots, but the properties of (24) and (25) are always satisfied.

The stability of the MCCF is given by the eigenvalues of (15). Fig. 3 shows the result of calculating the \( n \) eigenvalues of \( A \) for 5000 different cutoff values. The parameters used for the computation were \( \omega_0 = 100\pi \text{ rad/s} \), six harmonics \( \mathcal{H} = \{-11, -5, -1, 1, 7, 13\} \) and \( \gamma_m > 0 \) for all \( m \in \mathcal{H} \). One finds that the MCCF remains stable for all values of the parameters \( \gamma_m \). This observation suggests that it may be possible to prove the stability of the algorithm for arbitrary cutoff frequencies. Indeed, the following result specifies that, if the MCCF (14) does not contain any repeated harmonic, the filter is stable for all choices of cutoff frequencies \( \gamma_k > 0 \), \( k \in \mathcal{H} \), and the result is true even if the gains are different for distinct harmonics.

**Proposition 1:** Assume that \( h_k \neq h_l \) for all \( k \neq l \) and \( \gamma_{h_k} > 0 \) for all \( k \). Then, the polynomial \( D_H(s) \) has all roots in the open left half-plane.

This property can be observed in Fig. 2 that shows the gain plots of transfer functions (21) with \( \mathcal{H} = \{-11, -5, -1, 1, 7, 13\} \). The harmonic spectrum in Fig. 16 of the experimental validation.

**Proof.** Using the matrix determinant lemma (see [29, Th. 18.1.1]), the characteristic polynomial of (15) given by (23) can be written as

\[
D_H(s) = P(s) (s - j h_1 \omega_0) \cdots (s - j h_n \omega_0)
\]

where

\[
P(s) = 1 + \frac{\gamma_{h_1}}{s - j h_1 \omega_0} + \cdots + \frac{\gamma_{h_n}}{s - j h_n \omega_0}
\]

The remainder of the proof has two parts:

**Part 1:** The roots of \( D_H(s) \) are distinct from \( s = j h_k \omega_0 \) for all \( k \). Indeed, for \( s = j h_1 \omega_0 \), \( D_H(j h_1 \omega_0) = \gamma_{h_1} (j \omega_0)^{n-1} (h_1 - h_2) \cdots (h_1 - h_n) \), which is different from zero under the assumptions. Repeating for other values of \( k \), the result is obtained.

**Part 2:** The roots of \( D_H(s) \) are in the open left half-plane. Given Part 1, the roots of \( P(s) \) are the same as the roots of \( D_H(s) \) and are in the open left half-plane if and only if \( P(s) \neq 0 \) for all values of \( s \) with \( \text{Re}(s) \geq 0 \). Values of \( s \) such that \( s = j h_k \omega_0 \) do not need to be considered. In other words, one needs \( P(a + jb) \neq 0 \) for all \( a \geq 0 \), \( b \) arbitrary, and \( a + jb \neq j h_k \omega_0 \) for all \( k \). One has

\[
P(a + jb) = 1 + \frac{\gamma_{h_1} (a - j(b - h_1 \omega_0))}{a^2 + (b - h_1 \omega_0)^2} + \cdots + \frac{\gamma_{h_n} (a - j(b - h_n \omega_0))}{a^2 + (b - h_n \omega_0)^2}.
\]

Under the assumptions, \( a^2 + (b - h_1 \omega_0)^2 \neq 0 \) for all \( i \)

\[
\text{Re} (P(a + jb)) = 1 + \frac{\gamma_{h_1} a}{a^2 + (b - h_1 \omega_0)^2} + \cdots + \frac{\gamma_{h_n} a}{a^2 + (b - h_n \omega_0)^2} \geq 1
\]

which implies that \( P(a + jb) \neq 0 \).

IV. MULTIPLE-REFERENCE COMPLEX-BASED CONTROL

A. Controller Structure

The MRCC is based on Fig. 4. It consists in extracting the harmonic components of a signal, \( \hat{v}(t) \), by means of the MCCF

\[
v(t) \approx \hat{v}(t) = \sum_{k \in \mathcal{H}} \hat{v}_k(t).
\]
Then, the control action is composed of the sum of individual control actions corresponding to each harmonic

\[ u(s) = \sum_{k \in \mathbb{H}} C_k(s)(v_k^d(s) - \hat{v}_k(s)). \]  

(31)

The reference signal for the \( m \)th harmonic is

\[ v_m^d(s) = \frac{V_m}{s - jm\omega_o} \]  

(32)

where \( V_m \) is the desired amplitude of the \( m \)th harmonic. The main difference with respect to the set of ROGI presented in [26] is the inclusion of the CCF modules in Fig. 4 to extract the harmonic components and facilitate the control design.

The controller \( C_m(s) \) for harmonic \( m \) consists of a standard complex PR controller with the form [30]

\[ C_m(s) = K_{pm} + \frac{K_{im}}{s - jm\omega_o} \]  

(33)

where \( K_{pm} \) and \( K_{im} \in \mathbb{C} \) are the proportional and resonant gains, respectively. The second term, also known as an ROGI [26], is used to ensure zero error when tracking sinusoidal signals [31] and rejecting disturbances, thanks to the internal model principle. Defining \( K_m = K_{pm} \) and \( \alpha_m = \frac{K_{im}}{K_{pm}} \), the PR controller in (33), can be written as

\[ C_m(s) = K_m - \frac{s + \alpha_m - jm\omega_o}{s - jm\omega_o} \]  

(34)

where \( K_m, \alpha_m \in \mathbb{C} \) are the complex gains that will be selected in the design of the controller.

**B. Closed-Loop Dynamics**

Using (20) in (31), the control input can be written as

\[ u(s) = \sum_{k \in \mathbb{H}} C_k(s)\hat{v}_k(s) - \sum_{k \in \mathbb{H}} C_k(s)H_k(s)v(s) \]  

(35)

so that the output signal is

\[ v(s) = \frac{\sum_{k \in \mathbb{H}} C_k(s)G(s)v_k^d(s)}{1 + \sum_{k \in \mathbb{H}} C_k(s)H_k(s)G(s)} \]  

(36)

where \( G(s) \) is the transfer function of the power converter (that may have complex coefficients). Closed-loop stability of (36) is determined by the roots of

\[ 1 + L(s) = 0 \]  

(37)

where \( L(s) \) is the loop transfer function

\[ L(s) = \sum_{k \in \mathbb{H}} L_k(s) \]  

(38)

where for a given harmonic \( m \)

\[ L_m(s) = C_m(s)H_m(s)G(s). \]  

(39)

\( L_m(s) \) is the loop transfer function that would be obtained if a single harmonic was canceled. The overall loop transfer function is the sum of the loop transfer functions associated with the different harmonics.

**Remark 2:** Notice the difference of the closed-loop dynamics in (36) with respect to the standard approach consisting of a set of PR (without the MCCF) that would result in

\[ v(s) = \sum_{k \in \mathbb{H}} C_k(s)G(s) \frac{v_k^d(s)}{1 + \sum_{k \in \mathbb{H}} C_k(s)G(s)} \]  

(40)

\[ a) \quad \text{In the Denominator: The filter transfer function, } H_k(s), \text{ appears in the closed-loop transfer function (36) affecting only the poles and, thus, the stability compared to (40).} \]

\[ b) \quad \text{In the Numerator: In (40), the overall reference } v^d(s) \text{ multiplies the closed-loop transfer function. In contrast, in (36) each reference component } v_k^d(s) \text{ only multiplies its corresponding PR controller, } C_k(s). \text{ This feature offers more possibilities for adjusting the response to each harmonic component.} \]

**C. Stability Analysis**

The Nyquist Criterion is a useful tool to determine the stability of feedback systems. Through the complex transformation, the system has become a single-input single-output system, where the Nyquist diagram is most powerful for stability and robustness analysis. The criterion, based on the argument principle which is valid for any meromorphic function, applies to transfer functions with complex coefficients mostly as in the real case, but with the difference that the complex Nyquist curve is no longer symmetric for negative and positive frequencies. See further details in [32].

Consider first the case where a single harmonic is controlled, and \( L(s) = L_m(s) \). Assume that the converter transfer function \( G(s) \) is stable. Given that \( H_m(s) \) is stable for all \( \gamma_m > 0 \), the loop transfer function \( L_m(s) \) is stable, except for a pole at \( s = jm\omega_o \). A modified Nyquist contour must be used, where the portion of the imaginary axis close to \( \omega = m\omega_o \) is replaced by a semicircle of a small radius. Specifically, the segment \( \omega = m\omega_o + \delta\omega \), where \( \delta\omega \) varies from \(-\varepsilon \) to \( \varepsilon \) and \( \varepsilon \) is small is replaced by a semicircle where \( \delta\omega = \varepsilon e^{j\alpha} \) and \( \alpha \) varies from \(-\pi/2 \) to \( \pi/2 \). The number of encirclements is counted using the image of the modified contour, noting that the loop transfer function does not have any pole within the modified contour.

Let

\[ L_m(s) = \frac{L_{mo}(s)}{s - jm\omega_o} \]  

(41)

where \( L_{mo}(s) \) is stable with no poles on the \( j\omega \)-axis. For infinitesimal \( \varepsilon \), the image of the semicircle is

\[ L_m(jm\omega_o + \varepsilon e^{j\alpha}) \approx \frac{L_{mo}(jm\omega_o)}{\varepsilon} e^{-j\alpha} \]  

(42)

which is also a semicircle, but with large radius and connecting the frequency response from \( \omega = m\omega_o - \varepsilon \) to \( \omega = m\omega_o + \varepsilon \) in the clockwise direction.

A necessary and sufficient condition for closed-loop stability is that the Nyquist curve does not encircle the \(-1 + j0\) coordinate. A sufficient condition is that the Nyquist curve does not cross the real axis for \( \text{Re}(s) < -1 \). In particular, the large semicircle crosses the real axis in the right-hand side of the plane if the angle of the asymptote for \( \omega \to m\omega_o \) belongs to \((0, \pi)\) (or,
The dynamics associated with the VSC are described in abc coordinates by

\[ \frac{d\mathbf{i}_{abc}}{dt} = -R\mathbf{i}_{abc} - \mathbf{M}\mathbf{v}_{abc} + \mathbf{M}\mathbf{u}_{abc} \]  
(43)

\[ \frac{d\mathbf{v}_{abc}}{dt} = \mathbf{i}_{abc} - \mathbf{i}_{Labc} \]  
(44)

where \( \mathbf{v}_{abc}^T(t) = (v_a(t), v_b(t), v_c(t)) \) is the vector of output voltages (i.e., the voltages on the filter capacitors), \( \mathbf{i}_{abc}^T(t) = (i_a(t), i_b(t), i_c(t)) \) is the vector of currents in the inductors, \( \mathbf{u}_{abc}^T(t) = (u_a(t), u_b(t), u_c(t)) \) is the vector of voltages computed by the switching policy (that will be used as control inputs), \( \mathbf{v}_{Labc}^T(t) = (i_{La}(t), i_{Lb}(t), i_{Lc}(t)) \) is the vector of load currents, and the \( \mathbf{M} \) matrix is

\[
\mathbf{M} = \frac{1}{3} \begin{pmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{pmatrix}.
\]  
(45)

Remark 3: The model (43) and (44) assumes that the load currents are independent from the voltages, so that the load currents can be treated as disturbances. Practically, load currents depend on the voltages and constitute an additional feedback path. The effect of this feedback will be considered in Section V-D.

The control objective is to ensure balanced three-phase voltages on the capacitors, that is

\[
\mathbf{v}_{abc}^d(t) = V^d \begin{pmatrix}
\cos(\omega_o t) \\
\cos(\omega_o t - \frac{\pi}{6}) \\
\cos(\omega_o t + \frac{\pi}{6})
\end{pmatrix}
\]  
(46)

where \( V^d \) and \( \omega_o \) are the desired amplitude and frequency, respectively.

The three-phase to complex transformation (1) is then used not only to define equivalent complex signals but also a complex equivalent VSC system. In particular, assuming a three-phase connection without neutral (i.e., the homopolar component is zero), the VSC dynamics in (43) and (44) can be transformed into the complex equivalent system (letting \( \theta = 0 \))

\[ \frac{di}{dt} = -Ri - v + u \]  
(47)

\[ \frac{dv}{dt} = i - i_L \]  
(48)

where \( v, i \in \mathbb{C} \) are the complex voltage and current, \( u \in \mathbb{C} \) is the complex control input, and \( i_L \in \mathbb{C} \) is the complex load current (all are scalar variables). Similarly to (8), the load current is written as

\[ i_L(t) = \sum_{k \in H} i_{Lk}(t) = \sum_{k \in H} I_{Lk}e^{jk\omega_o t} \]  
(49)

where the modulus and phase of \( I_{Lk} \in \mathbb{C} \) are the amplitude and phase of \( i_{Lk}(t) \), respectively.

The input/output response is determined by the complex transfer function of the system from \( u \) to \( v \), or

\[ G(s) = \frac{1}{LCs^2 + RCs + 1}. \]  
(50)
From (46) with (1), the control objective is to track the complex signal
\[ v^d(t) = V_1 e^{j\omega_c t} \]  
where \( V_1 \in \mathbb{C} \) accounts for both the amplitude and the phase of the reference, respectively

\[ V_1 = V^d e^{j\phi}. \]  

By applying the Laplace transform, (51) results in
\[ v^d(s) = \frac{V_1}{s - j\omega_c}. \]

The reference signals as defined in (32) are given by
\[ V_m = \begin{cases} V_1 & \text{if } m = 1 \\ 0 & \text{otherwise.} \end{cases} \]

### B. Control Design for the mth Harmonic

We begin with a study of the influence of the control parameters on the complex Nyquist plot. From (34), one has
\[ C_m(j\omega) = K_m \frac{\alpha_m + j(\omega - m\omega_0)}{j(\omega - m\omega_0)} \]
with the magnitude and phase equations
\[ |C_m(j\omega)| = |K_m| \sqrt{\text{Re}(\alpha_m)^2 + (\text{Im}(\alpha_m) + \omega - m\omega_0)^2} \]
\[ \angle C_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} \text{sign}(\omega - m\omega_0) \]
\[ + \arctan \left( \frac{\text{Im}(\alpha_m) + \omega - m\omega_0}{\text{Re}(\alpha_m)} \right) \]  

where \( \phi_{K_m} \) denotes the phase of the complex number \( K_m \). Note that the magnitude and the phase of \( K_m \) have important and independent impacts in (56) and (57), respectively. On the other hand, (57) suggests that \( \alpha_m \) can be used for a fine-tuning at frequencies away from \( m\omega_0 \).

The discussion is illustrated for \( m = 1 \) on Fig. 6, which shows complex Nyquist plots for the loop transfer function \( L_1(s) \) and various choices of control parameters. The VSC and MCCF parameters are the ones used in Section VI-A. The magnitude and phase of \( K_1 \) directly affect the magnitude and the phase of the loop transfer function, as shown in the top plots of Fig. 6. The effect of \( \alpha_1 \) is shown in the bottom plots. Since changing the imaginary part of \( \alpha_1 \) is similar to modifying the phase of \( K_1 \), the real part of \( \alpha_1 \) reduces the arc of the unit circumference defined by the crossing points with the curves for \( \omega < m\omega_0 \) and \( \omega > m\omega_0 \). On the other hand, \( \text{Re}(\alpha_1) \) can be used to adjust the gain margin.

To ensure the stability of the feedback system, one needs to calculate \( \lim_{\omega \to m\omega_0} \angle L_m(j\omega) \), where
\[ \angle L_m(j\omega) = \angle C_m(j\omega) + \angle G(j\omega) + \angle H_m(j\omega). \]

By definition of the MCCF, \( \lim_{\omega \to m\omega_0} \angle H_m(j\omega) = 0 \). Then, it follows from (50) and (57) that
\[ \lim_{\omega \to m\omega_0} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} + \arctan \left( \frac{m\omega_0 RC}{1 - m^2\omega_0^2 LC} \right) \]
and two different cases appear
\[ m^2\omega_0^2 < \frac{1}{LC} \Rightarrow \arctan \left( \frac{m\omega_0 RC}{1 - m^2\omega_0^2 LC} \right) \in (0, \pi) \]  
\[ m^2\omega_0^2 > \frac{1}{LC} \Rightarrow \arctan \left( \frac{m\omega_0 RC}{1 - m^2\omega_0^2 LC} \right) \in (0, -\pi). \]

Conditions (60) and (61) are directly related to the resonance frequency of the LC filter, given by \( \omega_r = \frac{1}{\sqrt{LC}} \). The result shows that \( \phi_{K_m} \) should be used to compensate the negative phase of the third term in (59), depending on whether the harmonic frequency is smaller or larger than the LC resonance frequency.

Table I shows the control gains for the numerical example used in Section VI-A. The set of harmonics considered, \( \mathcal{H} = \{-11, -5, -1, 1, 7, 13\} \), is representative for the load used in the experimental validation in Section VI, see Fig. 16(a).
Notice that, since \( \omega_0 = 100 \pi \) rad/s and the VSC has a resonance frequency at \( \omega_r = 3.1427 \times 10^3 \) rad/s, harmonics with \( |m| > 10 \) require \( \phi m = \pi \) rad, which is equivalent to a reversal of feedback polarity. The complex Nyquist plots for each harmonic are shown in Fig. 7.

C. Stability and Robustness of the Overall System

In Fig. 8, the complex Nyquist plot of \( L(j\omega) \) is shown. Given that there are no encirclements of \(-1 + j0\), the closed-loop system is stable. Besides, closed-loop poles on the imaginary axis are avoided because the Nyquist plot does not pass through the \((-1,0)\) point.

In Fig. 9, the complex Nyquist plots of \( L_m(j\omega) \) and \( L(j\omega) \) are compared. One finds that the loop frequency response appears as the combination of the loop frequency responses. In particular, \( L(j\omega) \) approaches one of the \( L_m(j\omega) \) along the asymptotes and close to the unit circle. This result is obtained each frequency response \( C_m(j\omega)H_m(j\omega) \) becomes large in a different frequency band, not only because of the resonant mode of \( C_m(s) \), but also because of the selectivity of the filter \( H_m(s) \) (see Fig. 2). The closed-loop poles of \( L(j\omega) \) and \( L_m(j\omega) \) are shown in Fig. 10, which suggests that the collective behavior of the individual loops is representative of the overall system in terms of pole locations as well.

The Nyquist curve can also be used to evaluate the robustness margins of the system. Gain and phase margins can be measured directly, but one should remember that they apply to the complex system, or to the real system if the same gain and phase changes are applied to the different components (\( a, b, \) and \( c \)). The delay
margin is defined as the maximum time delay that may be applied while preserving stability, and can be calculated as
\[ L(j\omega_{gc})e^{-j\omega_{gc}T_{dm}} = -1 \] (62)
where \( \omega_{gc} \) is the gain crossover frequency (where the magnitude of the loop frequency response is equal to 1). Note that both positive and negative frequencies can determine the delay margin. If multiple frequencies are present, the smallest value must be used. In Fig. 7, the result of the computation of the delay margin is given for each individual harmonic. The minimum is \( T_{dm} = 0.397 \) ms for \( m = 1 \). The value is close to the value obtained with the overall Nyquist plot of \( L(j\omega) \) in Fig. 8, which gives \( T_{dm} = 0.347 \) ms. The delay margin is more than sufficient for the sampling rate of 0.05 ms used in the experimental stage in Section VI-B.

D. Robustness to Load Feedback

As pointed out in Section V-A, the model (48) considers the load current \( i_L \) as a disturbance. In general, this current depends on the voltage \( v \) and constitutes a feedback path. It is not possible to consider all possible load dynamics, but the effect of a static (resistive) load is studied more carefully in this section. From the model (47) and (48) with \( i_L = \frac{1}{R_L} v \), the complex transfer function from \( u \) to \( v \) is
\[ G(s) = \frac{R_L}{R_L LC s^2 + (R_L RC + L)s + R + R_L}. \] (63)
Note that, when \( R_L \to \infty \), the transfer function (50) is obtained.

Following the analysis of the asymptotes in the previous section, we have now
\[
\lim_{\omega \to m\omega_o} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} + \arctan \left( \frac{m\omega_o(R_L RC + L)}{R + R_L(1 - m^2\omega_o^2LC)} \right)
\] (64)
that shows how different values of the resistive load affect the phase for frequencies close to the resonant frequency. In particular, note that when \( R_L \to \infty \), condition (59) is recovered. Conversely, for \( R_L = 0 \)
\[
\lim_{\omega \to m\omega_o} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} - \arctan \left( \frac{m\omega_o L}{R} \right). \] (65)

Fig. 11 shows how the asymptotes in (64) evolve with \( R_L \). For higher loads (when \( R_L \to 0 \)), the last term in (64) converges to the angle of the impedance \( Z_m = R + jm\omega_o L \), which is close to \( \frac{\pi}{2} \). This effect can also be observed in the complex Nyquist plots, see Fig. 12. In contrast, for high values of \( R_L \), the asymptotes of the complex Nyquist plots tend to move in the counterclockwise direction, for harmonics in the range defined by the resonance frequency, \( \omega_r \), (clockwise for other harmonics), reducing the phase margin. In conclusion, the worst-case scenario is when \( R_L \to \infty \), which was the one considered for design in Section V-B.

VI. EXPERIMENTAL VALIDATION

The MRCC proposed in the previous section was validated through simulations and experiments. The system parameters were \( v_{dc} = 200 \) V, \( L = 2.75 \) mH, \( R = 0.3 \) \( \Omega \), \( C = 45 \) \( \mu \)F, with a 20 kHz switching frequency. The desired three-phase voltages are balanced with phase amplitude 100 V and 50 Hz frequency.
Therefore, from (51), (52), and (54), \( V_d = 100 \text{ V} \), \( \phi = 0^\circ \), and \( \omega_0 = 2\pi 50 \text{ rad/s} \).

The VSC feeds a nonlinear unbalanced load, consisting of three single-phase unbalanced rectifiers connected to each phase and a three-phase rectifier feeding a resistive load. The main harmonic content is \( \mathcal{H} = \{-11, -5, -1, 1, 7, 13\} \), see Fig. 16(a).

A. Simulation Results

Simulations have been performed using SimPowerSystems of MATLAB/Simulink with a realistic model of the power converter including losses, switching effects, and noise in the measured signals. This model does not emulate the microcontroller (and its problems related to execution times or quantification), nor thermal or electromagnetic interferences.

The proposed test consists in three scenarios. First, only the fundamental component is controlled (controllers \( C_m(s) \) for \( m \neq 1 \) are switched OFF). Then, at \( t = 0.2 \text{ s} \), the complete MRCC is activated (with zero reference for the harmonic components). Finally, at \( t = 0.4 \text{ s} \), a reference change is applied to the fundamental component from 100 to 80 V amplitude. Figs. 13–16 show the results. For simplicity, the initial transient is omitted and only reference changes are discussed.

During the first part of the test, the fundamental component tracks its reference. See in Fig. 13(a) that \( e_1(t) \) remains close to zero, but other harmonic components are still present because they are not controlled. See also errors in Fig. 13 and voltage/current waveforms in Fig. 14. At \( t = 0.2 \text{ s} \), all the harmonic components are controlled and their errors converge to zero (see Fig. 13). In Fig. 14, the voltage waveforms are improved. Finally, when the reference changes at \( t = 0.4 \text{ s} \), the fundamental component achieves the desired value in approximately 30 ms (one cycle and a half of the output voltage), and the errors of other harmonics also come down to zero after a short transient (see Fig. 13).

Frequency spectrums for the three-phase output voltages are shown for the first and second scenarios in Fig. 16. The total harmonic distortion when the full MRCC is used drops from 14.12% to 3.01%, which is a low value compared with to common standards for UPS.

B. Experimental Results

The experimental platform consists in the LC three-phase VSC described earlier. The converter was assembled from a SEMIX-101GD12E4s IGBT module of Semikron and the controller algorithm was implemented in a TMS320F28335 floating point DSP of Texas Instrument. The experimental testbed is shown in Fig. 17.

The testing procedure includes the same scenarios described with the numerical simulations. Fig. 18(a) shows the voltage signals during the first scenario (when only the control of the fundamental component is running with \( V_d = 100 \text{ V} \)). The amplitude and frequency of the three-phase voltages (CH1 to CH3) are controlled, but the waveforms are distorted because there is no harmonic compensation of the components of the load current (see phase b in CH4). In contrast, the voltage is improved when all the significant harmonic components are controlled [see Fig. 18(b)].

Finally, Fig. 18(c) and (d) shows the transient behavior when the full MRCC is engaged, and under an amplitude reference change, respectively. In Fig. 18(c), the waveform of the voltage is improved in approximately one cycle and a half, and the
VII. CONCLUSION

In this article, a new scheme is presented for controlling three-phase power converters. The methodology is based on complex-parameter models, which transform the systems to single-input single-output systems and enable powerful analysis tools. The proposed controller is based on an MCCF combined with multiple reference controllers. An intermediate contribution is a new proof of stability of the MCCF for arbitrary values of the gains. Filtering of separate harmonic components can be tuned independently. As opposed to traditional schemes, the design of the controllers is simplified by considering one harmonic at a time, a procedure that is made possible by the frequency selectivity of the combined MCCF and reference controllers. Original contributions are the inclusion of the dynamics of the MCCF during the control design, and a guarantee of stability of the overall feedback system. The analysis using the Nyquist diagram also makes it possible to evaluate the robustness margins of the system. Due to the frequency separation, the margins are close to the ones predicted from the designs carried out for separate harmonics. Simulation and experimental results validated the proposed control methodology in a practical converter design.

REFERENCES

[1] H. Nian and R. Zeng, “Improved control strategy for stand-alone distributed generation system under unbalanced and non-linear loads,” IET Renewable Power Gener., vol. 5, no. 5, pp. 325–331, 2011.
[2] D. Novotny and J. H. Wouterse, “Induction machine transfer functions and dynamic response by means of complex time variables,” IEEE Trans. Power App. Syst., vol. PAS-95, no. 4, pp. 1325–1335, Jul. 1976.
[3] L. Harnefors, “Modeling of three-phase dynamic systems using complex transfer functions and transfer matrices,” IEEE Trans. Ind. Electron., vol. 54, no. 4, pp. 2239–2248, Aug. 2007.
[4] E. Frank, “On the zeros polynomials with complex coefficients,” Bull. Amer. Math. Soc., vol. 5, no. 2, pp. 144–157, 1946.
[5] S. Agashe, “A new general Routh-like algorithm to determine the number of RHP roots of a real or complex polynomial,” IEEE Trans. Autom. Control, vol. AC-30, no. 4, pp. 406–409, Apr. 1985.

[6] M. Benidir and B. Piozinho, “The extended Routh’s table in the complex domain,” Control. Eng. Practice, vol. 1, no. 4, pp. 253–256, Feb. 1993.

[7] A. Dória-Cerezo and M. Bodson, “Root locus rules for polynomials with complex coefficients,” in Proc. 21st Mediterranean Conf. Control Autom., 2013, pp. 663–670.

[8] Y. Xu, “An improved Routh-like algorithm to determine the number of complex roots of a polynomial,” J. Frankl. Inst., vol. 352, no. 9, pp. 3459–3472, Sep. 2015.

[9] A. Dória-Cerezo and M. Bodson, “Design of controllers for electrical power systems using a complex root locus method,” IEEE Trans. Ind. Electron., vol. 63, no. 6, pp. 3706–3716, Jun. 2016.

[10] A. Dória-Cerezo, F. Serra, and M. Bodson, “Complex-based controller for a three-phase inverter with an LCL filter connected to unbalanced grids,” IEEE Trans. Power Electron., vol. 34, no. 4, pp. 3899–3908, Apr. 2019.

[11] F. Serra, A. Dória-Cerezo, C. D. Angelo, L. M. Fernández, and M. Bodson, “Complex pole placement control for a three-phase voltage source converter,” in Proc. IEEE Int. Conf. Ind. Technol., 2020, pp. 901–906.

[12] P. Zhong, J. Sun, Z. Tian, M. Huang, P. Yu, and X. Zha, “An improved impedance measurement method for grid-connected inverter systems considering the background harmonics and frequency deviation,” IEEE Trans. Emerg. Sel. Top. Power Electron., to be published.

[13] P. N. Babu, J. M. Guerrero, P. Siano, R. Peesapati, and G. Panda, “An improved adaptive control strategy in grid-tied PV system with active power filter for power quality enhancement,” IEEE Syst. J., vol. 15, no. 2, pp. 2859–2870, Jun. 2021.

[14] D. A. Harville, Matrix Algebra From a Statistician’s Perspective. Berlin, Germany: Springer-Verlag, 1997.

[15] D. Znood, D. Holmes, and G. Bode, “Frequency-domain analysis of three-phase linear current regulators,” IEEE Trans. Ind. Appl., vol. 37, no. 2, pp. 601–610, Mar./Apr. 2001.

[16] S. Fukuda and T. Yoda, “A novel current-tracking method for active filters based on a sinusoidal internal model,” IEEE Trans. Ind. Appl., vol. 37, no. 3, pp. 888–895, May/Jun. 2001.

[17] S. Gataric and N. Gajic, “Modeling and design of three-phase systems using complex transfer functions,” in Proc. 30th Annu. IEEE Power Electron. Specialists Conf., 1999, pp. 691–697.

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