Summary: For any $n > 1$ we determine the uniform and nonuniform lattices of the smallest covolume in the Lie group $\text{Sp}(n,1)$. We explicitly describe them in terms of the ring of Hurwitz integers in the nonuniform case with $n$ even, respectively, of the icosian ring in the uniform case for all $n > 1$.

MSC:

- 22E40 Discrete subgroups of Lie groups
- 11E57 Classical groups
- 20G30 Linear algebraic groups over global fields and their integers
- 51M25 Length, area and volume in real or complex geometry

Keywords:

- arithmetic lattices; quaternionic hyperbolic space; minimal volume; Prasad’s volume formula

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