Learning Hierarchical Information Flow
with Recurrent Neural Modules

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Abstract

We propose a deep learning model inspired by neocortical communication via the thalamus. Our model consists of recurrent neural modules that send features via a routing center, endowing the modules with the flexibility to share features over multiple time steps. We show that our model learns to route information hierarchically, processing input data by a chain of modules. We observe common architectures, such as feed forward neural networks and skip connections, emerging as special cases of our architecture, while novel connectivity patterns are learned for the text8 compression task. We demonstrate that our model outperforms standard recurrent neural networks on three sequential benchmarks.

1 Introduction

Deep learning models make use of modular building blocks such as fully connected layers, convolutional layers, and recurrent layers. Researchers often combine them in strictly layered or task-specific ways. Our method loosens this choice and learns how to route information as part of learning to solve the task. We achieve this using recurrent modules that communicate via a routing center that is inspired by the thalamus.

Warren McCulloch and Walter Pitts invented the perceptron in 1943 as the first mathematical model of neural information processing [20], laying the groundwork for modern research on artificial neural networks. Since then, researchers have continued looking for inspiration from neuroscience to propose new deep learning architectures [10, 12, 15, 31].

While some of these efforts have been directed to learning biologically plausible mechanisms in an attempt to explain brain behavior, our interest is to achieve a flexible learning model. In the neocortex, communication between areas can be broadly distinguished into two pathways: Direct communication and communication via the thalamus [28]. In our model, we borrow this latter notion of a centralized routing system to connect specializing neural modules.

In our experiments, our model learns to form connection patterns that process input hierarchically, including skip connections as known from ResNet [11], Highway networks [29], and DenseNet [13] and feedback connections, which are known to both play an important role in the neocortex and improve deep learning [6, 18]. The connectivity structure is learned for the task, allowing the model to trade-off computational width and depth. In this paper, we study such properties with the goal to build an understanding of the interactions between recurrent neural modules.

*Work done during an internship with Google Brain.
Section 2.1 defines our computational model. We explore two critical design axes, module structure and reading mechanism, in Section 3.1. We perform this study on a sequential benchmark, where our model outperforms a baseline of multi-layer recurrent networks, suggesting the viability of the approach. In Section 3.2, we apply the best performing design to the medium-scale language modeling task text8, where we observe that the model automatically learns hierarchical connectivity patterns.

2 Thalamus Gated Recurrent Modules

We find inspiration for our work in the neurological structure of the neocortex. Areas of the neocortex communicate via two principal pathways: The cortico-cortico-pathway comprises direct connections between nuclei, and the cortico-thalamic-cortico comprises connections relayed via the thalamus. Inspired by this second pathway, we develop a sequential deep learning model in which modules communicate via a routing center. We name the proposed model ThalNet.

2.1 Model Definition

Our system comprises a tuple of computation modules $F = (f^1, \cdots, f^I)$ that route their respective features into a shared center vector $\Phi$. An example instance of our ThalNet model is shown in Figure 1a. At every time step $t$, each module $f^i$ reads from the center vector via a context input $c^i_t$ and an optional task input $x^i_t$. The features $\phi^i_t = f^i(c^i_t, x^i_t)$ that each module produces are directed into the center $\Phi$. Output modules additionally produce task output from their feature vector as a function $o^i(\phi^i_t) = y^i$.

All modules send their features to the routing center, where they are merged to a single feature vector $\Phi_t = m(\phi^1_t, \cdots, \phi^I_t)$. In our experiments, we simply implement $m$ as the concatenation of all $\phi^i$. At the next time step, the center vector $\Phi_t$ is then read selectively by each module using a reading mechanism to obtain the context input $c^i_{t+1} = r^i(\Phi_t, \phi^i_t)$. This reading mechanism allows modules to read individual features, allowing for complex and selective reuse of information between modules. We define the initial center vector $\Phi_0$ as a zero vector.

In practice, we experiment with both feed forward and recurrent implementations of the modules $f^i$. For simplicity, we omit the hidden state used in recurrent modules in our notation.

The reading mechanism is conditioned on both $\Phi_t$ and $\phi^i_t$ separately as the merging does not need to preserve $\phi^i_t$ in the general case.
In summary, ThalNet is governed by the following equations:

Module features: \[ \phi_t^i = f(c_t^i, x_t^i) \] (1)
Module output: \[ y_t^i = o(\phi_t^i) \] (2)
Center features: \[ \Phi_t = m(\phi_1^t, \cdots, \phi_I^t) \] (3)
Read context input: \[ c_{t+1}^i = r(\Phi_t, \phi_t^i) \] (4)

The choice of input and output modules depends on the task at hand. In a simple scenario (e.g., single task), there is exactly one input module receiving task input, some number of side modules, and exactly one output module producing predictions. The output modules get trained using appropriate loss functions, with their gradients flowing backwards through the fully differentiable routing center into all modules.

Modules can operate in parallel as reads target the center vector from the previous time step. This suggest a sequential nature of our model, even though application to static input is possible by allowing multiple glimpses at the input. An unrolling of the multi-step process can be seen in Figure 1b. This figure illustrates the ability to arbitrarily route between modules between time steps.

We hypothesize that modules will use the center to route information through a chain of modules before producing the final output (see Section 3.2). To allow task input to be processed in this way, we simulate our model for multiple sub time steps for each frame of the task input. Please refer to Graves [7] for a study of a similar approach.

2.2 Reading Mechanisms

We now discuss implementations of the reading mechanism \( r(\Phi, \phi^i) \) and modules \( f(c^i, x^i) \), as defined in Section 2.1. We draw a distinction between static and dynamic reading mechanisms for ThalNet. For static reading, \( r^i(\Phi) \) is conditioned on independent parameters. For dynamic reading, \( r^i(\Phi, \phi^i) \) is conditioned on the current corresponding module state, allowing the model to adapt its connectivity within a single sequence. We investigate the following reading mechanisms:

- **Linear Mapping.** In its simplest form, static reading consists of a fully connected layer \( r^i(\Phi) = W^i\Phi \), illustrated in Figure 2. This simple approach performs reasonably well, but can exhibit unstable learning dynamics and produces noisy weight matrices that are hard to interpret.

- **Weight Normalization.** We found linear mappings with weight normalization [25] parameterization \( r^i(\Phi) = \beta \frac{W^i}{||W^i||_2} \Phi \) to be effective. Normalization causes interpretable weights since increasing one weight pushes other, less important weights closer to zero as demonstrated in Section 3.2. We also experimented with L1 and L2 penalties on the reading weights, but this sometimes caused side models to not get read from.
• **Fast Softmax.** To achieve dynamic routing, we condition the reading weight matrix on the current module features $\phi_i$. This can be seen as a form of fast weights, providing a biologically plausible method for attention [2, 26]. We then apply softmax normalization so that each element of the context is computed as a weighted average over center elements. While this allows for a different connectivity pattern at each time step, it introduces $|\phi_i + 1| \times |\Phi| \times |c_i|$ learned parameters per module.

• **Fast Gaussian.** As an efficient parameterization, we consider choosing each context element as a Gaussian weighted average of $\Phi$, with only mean and variance vectors learned conditioned on $\phi_i$. This reading mechanism only requires $|\phi_i + 1| \times 2 \times |c_i|$ parameters per module and thus makes dynamic reading more practical. The density is evaluated for each index in $\Phi$ based on its distance to the mean to read weighted averages over the center.

Reading mechanisms could also select between modules on a high level, instead of individual feature elements. We do not explore this direction since it seems biologically implausible. Moreover, we demonstrate that such knowledge about feature boundaries is not necessary, and hierarchical information flow emerges with fine-grained routing (see Figure 4). Theoretically, this also allows our model to perform a wider class of computation. We compare the predictive performance of these reading mechanisms on a sequential task in Section 3.1.

2.3 Interpretation as Recurrent Mixture of Experts

ThalNet can route information from the input to the output over multiple time steps. This enables it to trade off shallow and deep computation paths. To understand this, we view ThalNet as a smooth mixture of experts model [14], where the modules $F = (f_1, \ldots, f_I)$ are the recurrent experts. Each module outputs its features to the center vector $\Phi_t$. A linear combination of $\Phi_t$ is read at the next time step, which effectively performs a mixing of expert outputs. Compared to the recurrent mixture of experts model presented by Shazeer et al. [27], our model can recurrently route information through the mixture of multiple times, increasing the number of mixture compounds.

To highlight two extreme cases, the modules could read from identical locations in the center. In this case, the model does a wide and shallow computation over 1 time step, analogous to Graves [7]. In the other extreme, each module reads from a different module, recovering a hierarchy of recurrent layers. This gives a deep but narrow computation stretched over multiple time steps. In between, there exist a spectrum of complex patterns of information flow with differing and dynamic computation depths. This is comparable to DenseNet [13], which also blends information from paths of different computational depth, although in a purely feed-forward model.

Using state-less modules, our model could still leverage the recurrence between the modules and the center to store information over time. However, this bounds the number of distinct computation steps that ThalNet could apply to an input. Using recurrent modules, the computation steps can change over time, increasing the flexibility of the model. Recurrent modules give a stronger prior for using feedback and shows improved performance in our experiments (Section 3.1).

3 Experiments

We investigate the properties and performance of our model on three benchmarks. First, we compare reading mechanisms and module designs on a simple sequential task based on the MNIST dataset. We find weight normalized reading to be most effective and interpretable, and choose this reading mechanisms for two further experiments on more challenging problems, permuted MNIST and sequential CIFAR. In all three experiments, we significantly outperform a recurrent neural networks with the same number of parameters.

3.1 Module Designs and Reading Mechanisms

We use a sequential variant of MNIST [17] to compare the reading mechanisms described in Section 2.2 along with implementations of the module function. In sequential MNIST, the model observes handwritten digits of $28 \times 28$ pixels from top to bottom, one row per time step. The prediction is given at the last time step, so that the model has to integrate and remember observed
Figure 3: Test performance on the sequential MNIST task grouped by module design (left) and reading mechanism (right). Plots show the top, median, and bottom accuracy over the dimension that is not grouped. Recurrent modules train faster than pure fully connected modules. Linear map (no weight norm) for reading and FF-GRU modules perform best. We use weight norm for reading for interpretability and stability at minor cost to performance. FF-GRU-FF modules perform similarly to FF-GRU while limiting the size of the center.

information over the sequence. This makes the task more challenging than in the static setting. A multi-layer recurrent network achieves 7% error on this task.

To implement the modules $f_i(c^i, x^i)$ we test various combinations of fully connected and recurrent layers of Gated Recurrent Units (GRU) [3]. Modules require some amount of local structure to allow them to specialize. We test with two fully connected layers (FF), a GRU layer (GRU), fully connected followed by GRU (FF-GRU), GRU followed by fully connected (GRU-FF), and a GRU sandwiched between fully connected layers (FF-GRU-FF). In addition, we compare performance to a stacked GRU baseline with 4 layers. For all models, we pick the largest layer sizes such that the number of parameters does not exceed 50,000.

We train for 100 epochs on batches of size 50 using RMSProp [30] with a learning rate of $10^{-3}$. Figure 3 shows the test accuracy of module designs and reading mechanisms. ThalNet outperforms the stacked GRU baseline in most configurations. We assume that the structure imposed by our model acts as a regularizer. Moreover, the amount of information paths in ThalNet results in a large number of gradient paths compared to the baseline. We perform a further performance comparison in Section 3.3.

We observe a benefit of recurrent modules as they exhibit faster and more stable training than fully connected modules. This could be explained by the fact that pure fully connected modules have to learn to use the routing center to store information over time, which is a long feedback loop. Having a fully connected layer before the recurrent layer significantly improves performance compared to having the recurrent layers first. A fully connected layer after the GRU let us produce compact feature vectors $\phi^i$ that scale better to large modules and produces similar results here, although we found FF-GRU to be beneficial in later experiments (Section 3.3).

For the reading mechanisms, we observe the best peak performance with linear learning. However, learning is less stable as can be seen from the shaded performance range in Figure 3b. Therefore, we use weight normalization for further experiments, due to both its stability and prediction performance.

We do not include accuracy curves of fast Gaussian reading as it did not match performance of other methods shown in the figures.

3.2 Hierarchical Connectivity Patterns

Using its routing center, our model is able to learn its structure as part of learning to solve the task. In this section, we explore the emergent connectivity patterns. We show that our model learns

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4Implementing the modules as a single fully connected layer recovers a standard recurrent neural network with one large layer.
to route features in hierarchical ways as hypothesized, including skip connections and feedback connections. For this purpose, we choose the text8 corpus, a medium-scale language modeling benchmark consisting of the first $10^8$ bytes of Wikipedia, preprocessed for the Hutter Prize [19]. The model observes one one-hot encoded byte per time step, and is trained to classify the byte one time step ahead.

We use comparably small models to be able to run experiments quickly, comparing ThalNet models of 4 FF-GRU-FF modules with layer sizes 50, 100, 50 and 50, 200, 50. Both experiments use weight normalized reading. These models sizes do not perform on par with published results on the task, reaching scores in the range of 2.2 – 1.9 bits per character. However, in this section we focus on exploring learned connectivity patterns, and we expect that recent techniques in recurrent language modeling in combination with bigger models yield competitive results.

We simulate two sub time steps to allow for the output module to receive information of the current input frame as discussed in Section 2.1. Models are trained for 50 epochs on batches of size 10 containing sequences of length 50 using RMSProp with a learning rate of $10^{-3}$. In general, we observe different random seeds converging to similar connectivity patterns with recurring elements.

Figure 4 shows trained reading weights for various reading mechanisms, along with their manually deduced connectivity graphs. Each image represents a reading weight matrix for the modules 1 to 4 (top to bottom). Each pixel row shows the weight factors that get multiplied with $\Phi$ to produce a single element of the context vector of that module. The weight matrices thus has dimensions of $|\Phi| \times |c^i|$. White pixels represent large magnitudes, suggesting focus on features at those positions.

The weight matrices of weight normalized reading clearly resemble the boundaries of the four concatenated module features $\phi^1, \cdots, \phi^4$ in the center vector $\Phi$, even though the model has no notion of the origin and ordering of elements in the center vector.

A similar pattern emerges with fast softmax reading. These weight matrices are sparser than the weights from weight normalization. Over the course of a sequence, we observe some weights staying constant while others change their magnitudes at each time step. This suggests that optimal connectivity might include both static and dynamic elements. However as seen in Section 3.1, this
reading mechanism leads to less stable performance, and tends to perform worse. This problem could potentially be alleviated by applying weight normalization to the fast weight matrix.

With fast Gaussian reading, we see that the distributions occasionally tighten on specific features in the first and last modules, the modules that receive input and emit output. The other modules learn large variance parameters, effectively spanning all center features. This could potentially be addressed by reading using mixtures of Gaussians for each context element instead. We generally find that weight normalized and fast Softmax reading select features in a more targeted way.

The top row in Figure 4 shows manually deducted connectivity graphs between modules. Arrows represent the main direction of information flow in the model. For example, the two incoming arrows to module 4 in Figure 4a indicate that module 4 mainly attends to features produced by modules 1 and 3. We infer the connections from the larger weight magnitudes in the first and third quarters of the reading weights for module 4 (bottom row).

A typical pattern that emerges during the experiments can be seen in the connectivity graphs of both weight normalized and fast softmax reading (Figures 4a and 4b). Namely, the output module reads features directly from the input module. This direction connection is established early on during training, likely because this is the most direct gradient path from output to input. Later on, the side modules develop useful features to support the input and output modules.

In another pattern, one module reads from all other modules and combines their information. In Figure 4b, module 2 takes this role, reading from modules 1, 3, 4, and distributing these features via the input module. In additional experiments with more than four modules, we observed this pattern to emerge predominantly. This connection pattern provides a more efficient way of information sharing than cross-connecting all modules.

Both connectivity graphs in Figure 4 include hierarchical computation paths through the modules. They include learn skip connections, which are known to improve gradient flow from popular models such as ResNet [11], Highway networks [29], and DenseNet [13]. Furthermore, the connectivity graphs contain backward connections, creating feedback loops over two or more modules. Feedback connections are known to play a critical role in the neocortex, which inspired our work [6].

### 3.3 Performance Comparison

To gain insights into the performance of our model, we conduct a comparison on two tasks of higher complexity compared to the experiment in Section 3.1. Permutated MNIST is a more challenging variant of sequential MNIST where 2D grid of pixels is rearranged by a fixed random permutation. The permuted digits are fed the same way, as a sequence of 28 rows of sizes 28. For the sequential CIFAR task, the model observes one row of the $32 \times 32 \times 3$ pixel image of the CIFAR-10 dataset [16] per time step. We flatten the observation at each time step, giving an input vector of length 96 at each time step. In both tasks, the model must output its class prediction at the last time step of the image.

We use the model configurations and training setup described in Section 3.1. Figures 5a and 5b show results on permuted MNIST, and Figures 5c and 5d show results on sequential CIFAR. ThalNet improves upon the multi-layer GRU baseline on both problems. As already observed on the simpler task, our model also appears to be less prone to overfitting. This can be seen from the similar training error but better test error compared to the baseline. We hypothesize the information bottleneck at the reading mechanism acting as an implicit regularizer that encourages generalization.

### 4 Related Work

We describe a recurrent mixture of experts model, that learns to dynamically pass information between the modules. Related approaches can be found in various recurrent and multi-task methods as outlined in this section.

**Modular Neural Networks.** ThalNet consists of several recurrent modules that interact and exploit each other. Modularity is a common property of existing neural models. [4] learn a matrix of tasks and robot bodies to improve both multitask and transfer learning. [11] learn modules specific to objects present in the scene, which are selected by an object classifier. These approaches specify modules corresponding to a specific task or variable manually. In contrast, our model
Figure 5: Performance on the permuted sequential MNIST and sequential CIFAR tasks. The stacked GRU baseline reaches higher training accuracy on CIFAR, but fails to generalize well. On both tasks, ThalNet outperforms the baseline in test accuracy. On CIFAR, we see how recurrency within the modules speeds up training.

automatically discovers and exploits the inherent modularity of the task and does not require a one-to-one correspondence of modules to task variables.

**Learned Computation Paths.** We learn the connectivity between modules alongside the task. There are various methods in the multi-task context that also connect between modules. Fernando et al. [5] learn paths through multiple layers of experts using an evolutionary approach. Rusu et al. [24] learn adapter connections to connect to fixed previously trained experts and exploit their information. These approaches focus on feed-forward architectures. The recurrency in our approach allows for complex and flexible computational paths. Moreover, we learn interpretable weight matrices that can be examined directly without performing costly sensitivity analysis.

The Neural Programmer Interpreted presented by Reed and De Freitas [22] is related to our dynamic gating mechanisms. In their work, a network recursively calls itself in a parameterized way to perform tree-shaped computations. In comparison, our model allows for parallel computation between modules and for unrestricted connectivity patterns between modules.

**Mixture of Experts.** Our network can be seen as a mixture of experts model (Section 2.3), a common approach for exploiting local structure in the task. Typically, a gating network blends between the outputs of multiple expert networks, sometimes applied to different sections of the input. Gating mechanisms vary from softmax gating to blend experts [21] to forcing sparsity in a recurrent framework [27]. Much of these areas of research are built on the foundational approaches outlined in [23]. However, our approach differs in that it can leverage pass information between experts repeatedly, over the course multiple time steps.
Memory Augmented RNNs. The center vector in our model can be interpreted as an external memory, with multiple recurrent controllers operating on it. Preceding work proposes recurrent neural networks operating on external memory structures. The Neural Turing Machine proposed by Graves et al. [8], and follow-up work [9], investigate differentiable ways to address a memory for reading and writing. In the ThalNet model, we use multiple recurrent controllers accessing the center vector. Moreover, our center vector is recomputed at each time step, and thus should not be confused with a persistent memory as is typical for model with external memory.

5 Conclusion

We presented ThalNet, a recurrent modular framework that learns to pass information between neural modules in a hierarchical way. Experiments on sequential and permuted variants of MNIST and CIFAR-10 are a promising sign of the viability of this approach. In these experiments, ThalNet learns novel connectivity patterns that include hierarchical paths, skip connections, and feedback connections.

In our current implementation, we assume the center features to be a vector. Introducing a matrix shape for the center features would open up ways to integrate convolutional modules and similarity-based attention mechanisms for reading from the center. While matrix shaped features are easily interpretable for visual input, it is less clear how this structure will be leveraged for other modalities.

A further direction of future work is to apply our paradigm to tasks with multiple modalities for inputs and outputs. It seems natural to either have a separate input module for each modality, or to have multiple output modules that can all share information through the center. We believe this could be used to hint specialization into specific patterns and create more controllable connectivity patterns between modules. Similarly, we an interesting direction is to explore the proposed model can be leveraged to learn and remember a sequence of tasks.

We believe modular computation in neural networks will become more important as researchers approach more complex tasks and employ deep learning to rich, multi-modal domains. Our work provides a step in the direction of automatically organizing neural modules that leverage each other in order to solve a wide range of tasks in a complex world.

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