DESCRIPTION OF ELECTRIC DIPOLE EXCITATIONS
IN THE TIN ISOTOPES WITHIN THE
QUASIPARTICLE TIME BLOCKING APPROXIMATION

E. V. Litvinova\textsuperscript{1,2} and V. I. Tselyaev\textsuperscript{3}

\textsuperscript{1}Institute of Physics and Power Engineering, 249020, Obninsk, Russia
\textsuperscript{2}Physik-Department der Technischen Universität München,
D-85748 Garching, Germany
\textsuperscript{3}Nuclear Physics Department, V. A. Fock Institute of Physics,
St. Petersburg State University, 198504, St. Petersburg, Russia

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Abstract

The quasiparticle time blocking approximation (QTBA) is applied to describe E1 excitations in the even-even tin isotopes. Within the model pairing correlations, two-quasiparticle (2q), and 2q\text\oplus phonon configurations are included. Thus the QTBA is an extension of the quasiparticle random phase approximation to include quasiparticle-phonon coupling. Calculational formulas are presented in case of neutral excitations in the spherically symmetric system. The main equations are written in the coordinate representation that allows to take into account single-particle continuum completely. The E1 photoabsorption cross sections have been calculated in nuclei \textsuperscript{116,120,124}Sn. It has been obtained that the 2q\text\oplus phonon configurations provide noticeable fragmentation of the giant dipole resonance resulting in appearance of significant spreading width. The results are compared with available experimental data.

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1 INTRODUCTION

Theoretical description of giant multipole resonances (GMRs) and resonance structures in magic and open-shell nuclei has a long history and remains a problem of current importance. The main tools in solving this problem within the framework of microscopic approach are the random phase approximation (RPA) and the quasiparticle RPA (QRPA) which is straightforward generalization of the RPA to include pairing correlations. However, despite these models enable one to describe mean energies and total strengths of the GMRs, they fail to reproduce the total widths of the resonances and their fine structure (see, e.g., Ref. [1]). One of the reasons is that RPA and QRPA do not provide a mechanism producing spreading width $\Gamma^\downarrow$ which gives a considerable contribution in the total widths of the GMRs.

Simplest mechanism of this type is the coupling of the quasiparticles to phonons being superpositions of the one-particle-one-hole (1p1h) or the two-quasiparticle (2q) configurations. As applied to structure of the even-even nuclei, the concept of the quasiparticle-phonon coupling (QPC, see Ref. [2]) enables one to take into account 1p1h$\otimes$phonon and 2q$\otimes$phonon configurations in addition to 1p1h and 2q ones incorporated within the RPA and the QRPA. A series of models has been developed to go beyond the RPA and the QRPA by means of inclusion of this mechanism (see Refs. [1], [3]–[8] and references therein). Recently, new model has been formulated (see Ref. [9]) in which pairing correlations, 2q, and 2q$\otimes$phonon configurations are included. This model is based on the Green function (GF) formalism that mainly determines its name: quasiparticle time blocking approximation (QTBA). The QTBA is a generalization of the method of chronological decoupling of diagrams (MCDD) developed in Ref. [10] to describe excited states of the even-even nuclei without pairing.

The first aim of this paper is to present calculational formulas obtained from the general ones of the model [9] making use of certain approximations in case of neutral excitations in the spherically symmetric system. In particular, in the formulas presented zero-range forces are adopted as an effective interaction and the Bardeen-Cooper-Schrieffer (BCS) approximation is used to determine quasiparticle energies and wave functions. The equations obtained are a system of coupled equations corresponding to the excitations in particle-hole, particle-particle, and hole-hole channels. The basic equations are written in the coordinate representation that allows to take into account single-particle continuum completely. Notice that inclusion of the continuum is of particular importance to describe correctly the widths of the GMRs.
Our second aim is to test the QTBA in calculations of the electric dipole excitations in the nuclei with pairing. With that end in view we have chosen tin isotopes $^{116,120,124}\text{Sn}$. We present E1 photoabsorption cross sections calculated within QRPA and QTBA. Calculated integral characteristics of the giant dipole resonance (GDR) are compared with the experimental data.

The paper is organized as follows. In Sec. 2 general scheme of the approach is presented. In Sec. 3 the basic equations of the QTBA are transformed to channel form in the coordinate representation. In Sec. 4 these equations are further reduced to the equations for partial components. Formulas for the correlated propagator of the QTBA in the coordinate representation are presented in Sec. 5. In Sec. 6 the equations for the matrix elements of the amplitude of quasiparticle-phonon interaction are obtained. These quantities enter the equations for the correlated propagator of the QTBA in terms of the reduced matrix elements which are presented in Sec. 7. In Sec. 8 we describe our calculational scheme and present the results obtained for the photoabsorption cross sections in three tin isotopes. Conclusions are drawn in the last section.

2 GENERAL FRAMEWORK

The basic quantity, which determines the physical observables in the QTBA, is the nuclear polarizability $\Pi(\omega)$. More precisely, it determines distribution of the transition strength caused by an external field $V^0$. In the representation of the single-quasiparticle basis functions, which will be specified below, $\Pi(\omega)$ is defined as

$$\Pi(\omega) = -\frac{1}{2} \sum_{1234} (eV^0)_{21}^* R_{12,34}^{\text{eff}}(\omega) (eV^0)_{43},$$

where $e$ is the effective charge operator, $\omega$ is complex energy variable, $R^{\text{eff}}(\omega)$ is the effective response function. $R^{\text{eff}}(\omega)$ is a solution of the following Bethe-Salpeter equation

$$R_{12,34}^{\text{eff}}(\omega) = A_{12,34}(\omega) - \sum_{5678} A_{12,56}(\omega) F_{56,78} R_{78,34}^{\text{eff}}(\omega),$$

where $A(\omega)$ is a correlated propagator, $F$ is an amplitude of the effective interaction. In particular, the strength function $S(E)$ which is frequently used for the description of nuclear excitations is expressed in terms of the polarizability as

$$S(E) = -\frac{1}{\pi} \text{Im} \Pi(E + i \Delta),$$
where \( \Delta \) is a smearing parameter.

The Eq. (2) for the effective response function is quite general. It was obtained in Ref. [9] for the Fermi systems with pairing within the framework of generalized Green function formalism in which the normal and the anomalous GFs are treated in a unified way in terms of the components of generalized GFs in a doubled space. The physical content of the model is determined by the choice of the propagator \( A(\omega) \). In particular, the QRPA equation for the \( R_{\text{eff}}(\omega) \) has the same form (2) with uncorrelated propagator taken instead of \( A(\omega) \). The equations defining \( A(\omega) \) within the QTBA are given in Ref. [9]. In the present paper we consider the version of QTBA in which the correlated propagator includes contributions of the 2q and 2q\( \otimes \)phonon configurations.

As a first step we have to determine the single-quasiparticle basis functions \( \psi_1(y) \) which form matrix representation of the theory. For the Fermi systems with pairing these functions are defined in the doubled space spanned by the coordinates \( y = \{ x, \chi \} \), where symbol \( x = \{ r, \sigma, \tau \} \) includes the spatial coordinate \( r \), the spin \( \sigma \), and the isospin \( \tau \) variables, \( \chi = \pm 1 \) is an additional index introduced for denoting the different components of the single-quasiparticle functions in the doubled coordinate space. Index \( 1 = \{ \lambda_1, \eta_1 \} \) of the doubled configuration space includes index \( \lambda_1 \) of the usual single-particle configuration space and the index \( \eta_1 = \pm 1 \) which is the sign of the eigenvalue corresponding to \( \psi_1(y) \). Namely, we have:

\[
\mathcal{H} \psi_1 = E_1 \psi_1 \quad \text{where} \quad \mathcal{H} \text{ is single-quasiparticle Hamiltonian,} \quad E_1 = \eta_1 E_{\lambda_1}, \quad E_{\lambda_1} = |E_1|.
\]

In case of the spherically symmetric system, we are interested in, the index \( \lambda \) can be represented by the following set: \( \lambda = \{ (\lambda), m \} \) where \( (\lambda) = \{ \tau, n, l, j \} \), \( m \) is the projection of the total angular momentum, and we have: \( E_\lambda = E_{(\lambda)} \).

We will use the approximation corresponding to so-called canonical basis representation of the functions \( \psi_1(y) \) (see Ref. [11] for details). To determine functions \( \psi_1(y) \) within this approximation let us note that in the matrix form the Hamiltonian \( \mathcal{H} \) reads

\[
\mathcal{H} = \begin{pmatrix}
h - \mu & \Delta \\
-\Delta^* & \mu - h^*
\end{pmatrix}
\]

where \( h = h(x, x') \) is the single-particle Hamiltonian, \( \Delta = \Delta(x, x') \) is the operator of the pairing field, \( \mu \) is the chemical potential. Let \( \{ \varphi_\lambda(x) \} \) be the complete set of orthonormal eigenfunctions of the Hamiltonian \( h(x, x') \): \( h \varphi_\lambda = \varepsilon_\lambda \varphi_\lambda \). We will assume that the operator \( \Delta(x, x') \) has the canonical form in the same basis \( \{ \varphi_\lambda(x) \} \) that corresponds to the state-dependent version of the BCS approximation (see, e.g., Ref. [12]), or to the so-called
approximation of the diagonal pairing. In this case for the spherically symmetric system we have:

\[
\psi_{\lambda, +}(x, +) = u_\lambda \varphi_\lambda(x), \quad \psi_{\lambda, +}(x, -) = (-1)^l j + m v_\lambda \varphi^*_\lambda(x), \\
\psi_{\lambda, -}(x, -) = u_\lambda \varphi^*_\lambda(x), \quad \psi_{\lambda, -}(x, +) = (-1)^l j + m v_\lambda \varphi_\lambda(x),
\]

(5)

where \(\bar{\lambda} = \{\lambda\}, -m\), \(v_\lambda\) and \(u_\lambda\) are real numbers which satisfy the following conditions:

\(u_\lambda = \sqrt{1 - v_\lambda^2} \geq 0\), \(v_\lambda^2 = v_{\bar{\lambda}}^2 \leq 1\) [see Eqs. \(\text{[7]} - \text{[10]}\) for an explicit definition of \(v_\lambda\) and \(u_\lambda\) within the BCS approximation]. The choice of the phase factors is determined by the formulas:

\[
\varphi_\lambda(x) = \delta_{r_\lambda, \tau} R_{(\lambda)}(r) \phi_{jlm}(n, \sigma), \quad \phi_{jlm}(n, \sigma) = \sum_{\mu} (l \mu \frac{1}{2} \sigma | jm) Y_{l\mu}(n), \quad \varphi_\lambda(x) = (-1)^l j + m + \frac{1}{2} \sigma \varphi^*_\lambda(\bar{x}),
\]

(6)

(7)

(8)

where \(n = r/r, \quad \bar{x} = \{r, -\sigma, \tau\}\).

In applications of the theory it is convenient to use another basis functions which differ from the functions \(\text{[6]}\) by a unitary transformation. Let us introduce a matrix

\[
O_{12} = O_{\lambda_1 \eta_1, \lambda_2 \eta_2} = \delta_{\eta_1, -1} \delta_{\lambda_1, \lambda_2} + (-1)^{l_1 + j_1 - m_1} \delta_{\eta_1, -1} \delta_{\lambda_1, \bar{\lambda}_2}.
\]

(9)

This matrix is real and orthogonal, and consequently it is unitary. So wave functions \(\tilde{\psi}_1(y)\) defined through the single-quasiparticle basis functions \(\psi_1(y)\) by the formula

\[
\tilde{\psi}_1(y) = \sum_2 O_{12} \psi_2(y)
\]

(10)

also form a complete set of the orthonormal functions. We will use just the set \(\{\tilde{\psi}_1(y)\}\) as the set of basis functions. This does not lead to an inconsistency since the single-particle GF \(\tilde{G}(\varepsilon) = (\varepsilon - \mathcal{H})^{-1}\) is diagonal both in \(\{\psi_1(y)\}\) and in \(\{\tilde{\psi}_1(y)\}\) representation, and hence the formulas for the correlated propagator (see Ref. \(\text{[2]}\) are the same in both representations.

To describe dynamics of the system and to calculate the polarizability and the strength function we start from the equation for linear response matrix (LRM) \(\Lambda\). Notice that in the present work the term LRM is used instead of the frequently used one “density matrix variation in an external field” because it is more correct in our notations. In the coordinate representation the equation for \(\Lambda\) is obtained by the convolution of the equation for effective response function \(R^{\text{eff}}\) with an operator of the renormalized external field \(eV^0\):

\[
\Lambda(y_1, y_2; \omega) = \Lambda^0(y_1, y_2; \omega) - \int dy_3 dy_4 dy_5 dy_6 A(y_1, y_2; y_3, y_4; \omega) \\
\times F(y_3, y_4; y_5, y_6) \Lambda(y_5, y_6; \omega),
\]

(11)
where

$$\Lambda(y_1, y_2; \omega) = -\sum_{1234} \tilde{\psi}_1^*(y_1) \tilde{\psi}_2^*(y_2) R_{1234}^{\text{eff}}(\omega) (eV^0)_{43},$$

(12)

\[
\Lambda^0(y_1, y_2; \omega) = -\int dx_3 dx_4 [A(y_1, y_2; x_3+, x_4+; \omega) - A(y_1, y_2; x_4 -, x_3-; \omega)] \\
\times \tilde{V}^0(x_4, x_3).
\]

(13)

It is assumed that the correlated propagator of the model $A(\omega)$ is initially calculated in configuration space and then is transformed to coordinate representation:

$$A(y_1, y_2; y_3, y_4; \omega) = \sum_{1234} \tilde{\psi}_1^*(y_1) \tilde{\psi}_2^*(y_2) \tilde{\psi}_3^*(y_3) \tilde{\psi}_4^*(y_4) A_{1234}(\omega).$$

(14)

Components of the external field are

$$\tilde{V}^0(x_1, x_2) = \tilde{V}^0(x_1+, x_2+)= -\tilde{V}^0(x_2-, x_1-)= \sum_{12} \tilde{\psi}_1(x_1+) \tilde{\psi}_2^*(x_2+) (eV^0)_{12},$$

(15)

and it is supposed that $\tilde{V}^0(x_1+, x_2-)= \tilde{V}^0(x_1-, x_2+)=0$.

In terms of the LRM the Eq. (11) for the polarizability reads:

$$\Pi(\omega) = \int dx_1 dx_2 \tilde{V}^0(x_2, x_1) \Lambda(x_1+, x_2+; \omega).$$

(16)

To determine general form of the effective interaction in our approach let us note that within a self-consistent scheme the amplitude $\mathcal{F}$ in Eq. (11) can be defined as a second order functional derivative of some energy density functional $\mathcal{E}[\mathcal{R}]$ where $\mathcal{R}$ is the extended density matrix ($\mathcal{R}(y_1, y_2) = \sum_1 \delta_{y_1, -1} \tilde{\psi}_1(y_1) \tilde{\psi}_1^*(y_2)$, see, e.g., Ref. [11]):

$$\mathcal{F}(y_1, y_2; y_3, y_4) = \frac{2 \delta^2 \mathcal{E}[\mathcal{R}]}{\delta \mathcal{R}(y_1, y_2) \delta \mathcal{R}(y_4, y_3)}.$$

(17)

If $\mathcal{E}[\mathcal{R}]$ is usual functional of the Hartree-Fock-Bogoliubov theory built up on the basis of the Hamiltonian which includes only two-particle density-independent interaction with the antisymmetrized amplitude $w^{(2)}$, we have:

$$\mathcal{F}(y_1, y_2; y_3, y_4) = \frac{1}{2} \delta_{x_1, x_2} \delta_{x_3, x_4}$$

\[
\times \left( \delta_{x_1, x_3} \left[ \delta_{\chi_1, +1} \mathcal{F}^+(x_1, x_2; x_3, x_4) + \delta_{\chi_1, -1} \mathcal{F}^+(x_2, x_1; x_4, x_3) \right] \\
- \delta_{\chi_1, -x_3} \left[ \delta_{\chi_1, +1} \mathcal{F}^+(x_1, x_2; x_4, x_3) + \delta_{\chi_1, -1} \mathcal{F}^+(x_2, x_1; x_3, x_4) \right] \right) \\
+ \delta_{\chi_1, -x_2} \delta_{\chi_3, -x_4} \delta_{\chi_1, \chi_3} \left[ \delta_{\chi_2, +1} \mathcal{F}^-(x_1, x_2; x_3, x_4) + \delta_{\chi_2, -1} \mathcal{F}^-(x_3, x_4; x_1, x_2) \right],
\]

(18)
where

$$F^+(x_1, x_2; x_3, x_4) = w^{(2)}(x_2, x_3; x_1, x_4), \quad F^-(x_1, x_2; x_3, x_4) = \frac{1}{2} w^{(2)}(x_1, x_2; x_3, x_4).$$  \tag{19}$$

In what follows we assume that Eq. (18) is fulfilled for the interaction $F$, however Eqs. (19) are not supposed to be fulfilled. In other words, the amplitudes $F^+$ and $F^-$ will be considered to be independent, and it will be supposed that no other independent components are contained in the amplitude $F$. These assumptions correspond to the approximation adopted in the Theory of Finite Fermi Systems with pairing correlations (TFFS PC, Ref. [13]).

### 3 Transformation of the Equation for Linear Response Matrix to Channel Form

The LRM defined by Eq. (12) contains information about excitations of the initial system in three different channels corresponding to the transitions to the states of the final systems with different numbers of particles. Suppose that the number of particles in the ground state of the initial system is conserved and is equal to $N_0$. Let $N$ be the number of particles in the final system. Then, in accordance with the standard terminology, we have: (i) ph channel if $N = N_0$; (ii) pp channel if $N = N_0 + 2$; (iii) hh channel if $N = N_0 - 2$. Notice that it is not necessary to introduce hp channel explicitly because of symmetry of the LRM and other quantities.

Let us introduce the channel index $c \in \{ph, pp, hh\}$ and define the projection operators $\Xi^{(c)}$:

$$\Xi^{(ph)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3, +1} \delta_{\chi_4, +1} \delta(x_1, x_3) \delta(x_2, x_4),$$  \tag{20}$$

$$\Xi^{(pp)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3, +1} \delta_{\chi_4, -1} (-1)^{\frac{1}{2} + \sigma_2} \delta(x_1, x_3) \delta(\bar{x}_2, x_4),$$  \tag{21}$$

$$\Xi^{(hh)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3, -1} \delta_{\chi_4, +1} (-1)^{\frac{1}{2} + \sigma_1} \delta(\bar{x}_1, x_3) \delta(x_2, x_4).$$  \tag{22}$$

The sense of these operators is obvious: acting on any quasiparticle operator they cut out its components with fixed $\chi$ values. Thus applying each of these projectors to LRM and correlated propagator we obtain the following components:

$$\Lambda^{(c)}(x_1, x_2; \omega) = \int dy_3 \ dy_4 \ \Xi^{(c)}(x_1, x_2; y_3, y_4) \Lambda(y_3, y_4; \omega),$$  \tag{23}$$

$$\Lambda^0^{(c)}(x_1, x_2; \omega) = \int dy_3 \ dy_4 \ \Xi^{(c)}(x_1, x_2; y_3, y_4) \Lambda^0(y_3, y_4; \omega),$$  \tag{24}$$
\[ A^{(c,ph)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5 dy_6 \Xi^{(c)}(x_1, x_2; y_5, y_6) \]
\[ \times \left[ A(y_5, y_6; x_3+, x_4+; \omega) - A(y_5, y_6; x_4-, x_3-; \omega) \right], \]  
(25)
\[ A^{(c,pp)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5 dy_6 \Xi^{(c)}(x_1, x_2; y_5, y_6) \]
\[ \times A(y_5, y_6; x_3+, \bar{x}_4-; \omega) (-1)^{\frac{1}{2}+\sigma_4}, \]  
(26)
\[ A^{(c,hh)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5 dy_6 \Xi^{(c)}(x_1, x_2; y_5, y_6) \]
\[ \times A(y_5, y_6; \bar{x}_3-, x_4+; \omega) (-1)^{\frac{1}{2}+\sigma_3}, \]  
(27)
where the second channel indices in the Eqs. (25)–(27) are fixed by the \( \chi \)-indices of the propagators on the right-hand sides of these equations.

Let us also denote:
\[ \mathcal{F}^{(c,c')}(x_1, x_2; x_3, x_4) = \delta_{c,c'} \mathcal{F}^{(c)}(x_1, x_2; x_3, x_4) + \mathcal{F}^{\text{rest}(c,c')}(x_1, x_2; x_3, x_4), \]  
(28)
\[ \mathcal{F}^{(ph)}(x_1, x_2; x_3, x_4) = \mathcal{F}^+(x_1, x_2; x_3, x_4), \]  
(29)
\[ \mathcal{F}^{(pp)}(x_1, x_2; x_3, x_4) = (-1)^{\frac{1}{2}+\sigma_2+\frac{1}{2}+\sigma_4} \mathcal{F}^-(x_3, \bar{x}_4; x_1, \bar{x}_2), \]  
(30)
\[ \mathcal{F}^{(hh)}(x_1, x_2; x_3, x_4) = (-1)^{\frac{1}{2}+\sigma_1+\frac{1}{2}+\sigma_3} \mathcal{F}^-(\bar{x}_1, x_2; \bar{x}_3, x_4). \]  
(31)
The additional restoring amplitude \( \mathcal{F}^{\text{rest}(c,c')} \) is introduced in Eq. (28) for the purpose of “forced consistency” and will be specified in the following.

Making use of the definitions (18), (20)–(31) one can rewrite the Eq. (11) in the channel form
\[ \Lambda^{(c)}(x_1, x_2; \omega) = \Lambda^{(0,c)}(x_1, x_2; \omega) - \sum_{c',c''} \int dx_3 dx_4 dx_5 dx_6 A^{(c,c')}(x_1, x_2; x_3, x_4; \omega) \]
\[ \times \mathcal{F}^{(c',c'')} (x_3, x_4; x_5, x_6) \Lambda^{(c'')} (x_5, x_6; \omega), \]  
(32)
where summation is performed over all the channels.

4 EQUATION FOR PARTIAL COMPONENTS OF THE LRM
IN CASE OF NEUTRAL EXCITATIONS

In the present paper we solve the LRM equation (32) for neutral excitations. To separate the angular dependence in a spherically symmetric system we use decompositions in terms of spherical tensor operators. In particular, it is assumed that the operator of the renormalized
external field in the Eqs. \((13), (15), (16)\) has the form

\[
\tilde{V}^0(x_1, x_2) = \tilde{V}^0_{JM}(x_1, x_2) = \delta(r_1 - r_2) \delta_{\tau_1, \tau_2} \sum_{LS} \tilde{V}^0_{JLS\tau_1}(r_1) T_{JLS\mu}(n_1)_{\sigma_1, \sigma_2},
\]

(33)

where

\[
T_{JLS\mu}(n)_{\sigma_1, \sigma_2} = \sum_{m_\mu} (LmS\mu|JM) Y_{LM}(n)(\sigma_{S\mu})_{\sigma_1, \sigma_2},
\]

(34)

\(\sigma_{S\mu}\) are the Pauli matrices in the tensor representation:

\[
(\sigma_{S\mu})_{\sigma_1, \sigma_2} = \sqrt{2(2S + 1)} (-1)^{1 - \sigma_1} \left( \begin{array}{c} \frac{1}{2} \\ \frac{1}{2} \\ \sigma_2 \\ -\sigma_1 \\ \mu \end{array} \right).
\]

(35)

For electric multipole excitations we use the standard ansatz:

\[
\tilde{V}^0_{JLS\tau}(r) = \delta_{JL} \delta_{S0} r^L e^{(L)}_r,
\]

(36)

where \(e^{(L)}_r\) is an effective charge in the center-of-mass reference frame: \(e^{(L)}_n = Z (-A^{-1})^L, e^{(L)}_p = (1 - A^{-1})^L + (Z - 1) (-A^{-1})^L\).

Let us denote in accordance with Eqs. \((13), (24), (25),\) and \((33)\):

\[
\Lambda^{(c)}_{JM}(x_1, x_2; \omega) = -\int dx_3 dx_4 A^{(c, ph)}(x_1, x_2; x_3, x_4; \omega) \tilde{V}^0_{JM}(x_4, x_3).
\]

(37)

Let \(\Lambda^{(c)}_{JM}\) be the solution of the Eq. \((32)\) with \(\Lambda^{(c)} = \Lambda^{(c)}_{JM}\). In this case the partial components of the LRM \(\Lambda^{(c)}_{JM}\) and of the related quantities are defined as

\[
\Lambda^{(c)}_{JLS\tau}(r; \omega) = \int dn dx_1 dx_2 T_{JLS\mu}^{(c)}(n)_{\sigma_1, \sigma_2} \delta(x_1, x_2; r, \tau) \Lambda^{(c)}_{JM}(x_1, x_2; \omega),
\]

(38)

\[
\Lambda^{(c)}_{JLS\tau}(r; \omega) = \int dn dx_1 dx_2 T_{JLS\mu}^{(c)}(n)_{\sigma_1, \sigma_2} \delta(x_1, x_2; r, \tau) \Lambda^{(c)}_{JM}(x_1, x_2; \omega),
\]

(39)

\[
A^{(c, e')}_{JLS\tau, L'S\tau'}(r, r'; \omega) = \delta_{\tau, \tau'} \int dn dn' dx_1 dx_2 dx_3 dx_4 \delta(x_1, x_2; r, \tau) \delta(x_3, x_4; r', \tau')
\times T_{JLS\mu}^{(c)}(n)_{\sigma_1, \sigma_2} A^{(c, e')}(x_1, x_2; x_3, x_4; \omega) T_{JLSM}(n')_{\sigma_1, \sigma_2},
\]

(40)

where

\[
T_{JLS\mu}^{(c)}(n)_{\sigma_1, \sigma_2} = (-1)^{J + L + S + M} T_{JLS\mu}^{(c)}(n)_{\sigma_1, \sigma_2},
\]

(41)

\[
\delta(x_1, x_2; r, \tau) = \delta_{\tau, \tau} \delta_{\tau_2, \tau} \delta(r_1 - r) \delta(r_2 - r).
\]

(42)
Further, we assume that the effective interaction is determined by the following decomposition:

\[
F^{(c,c')}(x_1, x_2; x_3, x_4) = \delta(r_1 - r_2) \delta(r_3 - r_4) \delta_{\tau_1, \tau_2} \delta_{\tau_3, \tau_4} \\
\times \sum_{LSL'S'M} T_{JLSM}(n_1)_{\sigma_2 \sigma_4} T_{JL'S'M}^\dagger(n_3)_{\sigma_3 \sigma_4} \\
\times F_J^{(c,c')}(LS\tau_1, L'S\tau_3)(r_1, r_3),
\]

where

\[
F_J^{(c,c')}(LS\tau, L'S\tau')(r, r') = \delta_{c,c'} \delta_{LL'} \delta_{SS'} \delta(r - r') F_S^{(c)}(r) \\
+ \delta_{LL'} \delta_{L1} \delta_{J1} \sum_{k=1}^2 \kappa_k F_S^{(c)}(r) F_{S',k}^{(c)}(r'),
\]

\[
F_{S,\tau\tau'}^{(ph)} = C_0 \left( \delta_{S0} \left[ f + (2 \delta_{\tau,\tau'} - 1) f' \right] + \delta_{S1} \left[ g + (2 \delta_{\tau,\tau'} - 1) g' \right] \right),
\]

\[
F_{S,\tau\tau'}^{(pp)} = F_{S,\tau\tau'}^{(hh)} = \delta_{S0} \delta_{\tau,\tau'} F^\xi.
\]

In Eq. (44), the first term represents Landau-Migdal zero-range interaction of the TFFSPC. In the standard parametrization, the quantities \(C_0, g,\) and \(g'\) in Eq. (45) are constants. The functions \(f(r)\) and \(f'(r)\) are determined by the parameters \(f_{ex}, f_{in}, f'_{ex}, f'_{in}\), and by the nuclear density in the ground state \(\rho_0(r)\) by means of the ansatz:

\[
f(r) = f_{ex} + (f_{in} - f_{ex}) \rho_0(r)/\rho_0(0),
\]

and analogously for \(f'(r)\). For the interaction in the pp and hh channels we have:

\[
F^\xi = C_0 / \ln(c_p/\xi),
\]

where \(c_p\) is a constant, \(\xi\) is an energy cutoff parameter.

The second term in the Eq. (44) is a correction corresponding to the amplitude \(F^{rest(c,c')}\) in the Eq. (28). The similar correction arises in the calculational scheme with “forced consistency” which was developed in Ref. [14] to obtain the spurious isoscalar 1\(^-\) state at zero energy in the non-self-consistent approach for the case when only ph channel is taken into account. Here the straightforward generalization of this scheme is presented in which the pp and hh channels are also included. Making use of the same method as in Ref. [14] we obtain:

\[
F^{(c)}_{S,\tau,1}(r) = \delta_{c,ph} \delta_{S0} \frac{dU^\tau(r)}{dr}, \quad F^{(c)}_{S,\tau,2}(r) = \sum_{\tau'} F^{(c)}_{S,\tau,\tau'}(r) Q^{(c)}_{S\tau}(r),
\]

\[
F^{(c)}_{S,\tau,1}(r) = \delta_{c,ph} \delta_{S0} \frac{dU^\tau(r)}{dr}, \quad F^{(c)}_{S,\tau,2}(r) = \sum_{\tau'} F^{(c)}_{S,\tau,\tau'}(r) Q^{(c)}_{S\tau}(r),
\]

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\[ \kappa_k^{-1} = - \sum_{cS\tau} \int_0^\infty dr \, r^2 F_{St,k}^{(c)}(r) Q_{St}^{(c)}(r), \]  

where

\[ Q_{St}^{(c)}(r) = \sum_{c'S'} \int_0^\infty dr' \, r'^2 A_{1St,1S't}^{(c,c')}(r, r'; \omega_0) F_{St_1}^{(c')}(r'), \quad \omega_0 \to 0. \]

In Eqs. (49), \( U^\tau(r) \) is an auxiliary potential, the well depth of which is chosen to satisfy the condition:

\[ \int_0^\infty dr \, r^3 \left[ ZQ^{(ph)}_0(n) - NQ^{(ph)}_0(p) \right] = 0, \]

which ensures the spurious state excitation probability to be equal to zero.

Making use of the above definitions we can reduce the Eq. (32) in case of the spherically symmetric system to the following equation for the partial components:

\[ \Lambda_{JLS\tau}^{(c)}(r; \omega) = \Lambda_{JLS\tau}^{(0)}(r; \omega) - \sum_{L'S'c'c''} \sum_{L'S'c''} \int_0^\infty dr' \, r'^2 \int_0^\infty dr'' \, r''^2 A_{L'S'L'S''}^{(c,c')}(r, r'; \omega) \times \mathcal{J}_{L'S'c', L'S''c''}(r', r'') \Lambda_{JLS\tau}^{(c'')} \left( r''; \omega \right). \]

where

\[ \Lambda_{JLS\tau}^{(0)}(r; \omega) = - \sum_{L'S'c'c''} \int_0^\infty dr' \, r'^2 A_{L'S'L'S''}^{(c,c')}(r, r'; \omega) \tilde{V}_{JLS\tau}^{(c')} \left( r' \right), \]

\[ \tilde{V}_{JLS\tau}^{(c)}(r) = \delta_{c,ph} \tilde{V}_{JLS\tau}^{(0)}(r). \]

In terms of the partial components the Eq. (16) for the polarizability takes the form:

\[ \Pi_J(\omega) = (2J + 1) \sum_{LStc} \int_0^\infty dr \, r^2 \tilde{V}_{JLS\tau}^{(c)}(r) \Lambda_{JLS\tau}^{(c)}(r; \omega). \]

It is important to note that the amplitudes \( u_\lambda \) and \( v_\lambda \), which define the single-quasiparticle basis functions according to Eq. (5), have to be determined from the solution of the gap equation with the same interaction \( \mathcal{F}^{\xi} \) which enters the Eq. (53) for the LRM through the Eqs. (44) and (46). Namely, we have:

\[ u_\lambda = \sqrt{\frac{1}{2} \left( 1 + \frac{\varepsilon_\lambda - \mu_{\tau_\lambda}}{E_\lambda} \right)}, \quad v_\lambda = \text{sgn}(\Delta_\lambda) \sqrt{\frac{1}{2} \left( 1 - \frac{\varepsilon_\lambda - \mu_{\tau_\lambda}}{E_\lambda} \right)}, \]

where \( \varepsilon_\lambda \) is the eigenvalue of the single-particle Hamiltonian corresponding to the eigenfunction \( \varphi_\lambda(x) \), \( \mu_\tau \) is the chemical potential,

\[ E_\lambda = \sqrt{(\varepsilon_\lambda - \mu_{\tau_\lambda})^2 + \Delta_\lambda^2}. \]
The values of the energy gap $\Delta_\lambda$ are determined within the BCS approximation from the equation:
\[ \Delta_\lambda = -\sum_{(\lambda')} \frac{2j_{\lambda'} + 1}{4\pi} F_{(\lambda',\lambda')}^\xi \frac{\Delta_{\lambda'}}{2E_{\lambda'}}, \]  
(59)
where
\[ F_{(\lambda',\lambda')}^\xi = \delta_{\tau_{\lambda'}, \tau_{\lambda'}} \int_0^\infty dr r^2 R_{(\lambda')}^2 (r) R_{(\lambda')}^2 (r) F_{r}(r). \]  
(60)

5 CORRELATED PROPAGATOR
IN THE COORDINATE REPRESENTATION

Let us obtain an explicit formula for the partial components of the correlated propagator in terms of the reduced matrix elements. First of all, notice that the following relations are fulfilled for the matrix elements $A_{1234}(\omega)$ entering Eq. (14) in case of the spherically symmetric system:

\[ A_{1234}(\omega) = \sum_{JM} (2J + 1) A_{[1234]}^J(\omega) \]
\times (1) I_{j_2 - m_2} \left( \begin{array}{ccc} j_1 & j_2 & J \\ m_1 & -m_2 & M \end{array} \right) (-1)^{j_4 - m_4} \left( \begin{array}{ccc} j_3 & j_4 & J \\ m_3 & -m_4 & M \end{array} \right), \]  
(61)

\[ A_{1234}^J(\omega) = \sum_{m_1m_2m_3m_4} (2J + 1) A_{1234}(\omega) \]
\times (1) I_{j_2 - m_2} \left( \begin{array}{ccc} j_1 & j_2 & J \\ m_1 & -m_2 & M \end{array} \right) (-1)^{j_4 - m_4} \left( \begin{array}{ccc} j_3 & j_4 & J \\ m_3 & -m_4 & M \end{array} \right), \]  
(62)

where $1 = \{[1], m_1\}, [1] = \{(1), n_1\}, (1) = (\lambda_1) = \{\tau_1, n_1, l_1, j_1\}$. Notice that with these abbreviated notations we have: $u_{(1)} = u_{\lambda_1}$, $v_{(1)} = v_{\lambda_1}$, $\varepsilon_{(1)} = \varepsilon_{\lambda_1}$, $E_{(1)} = E_{\lambda_1}$. We will use the antisymmetric form of equations for the correlated propagator (see Ref. [9] for details) which allows to reduce the dimensions of matrices entering these equations. In this case the following relations are fulfilled:

\[ A_{[1234]}^J(\omega) = -\eta_1 \eta_2 (-1)^{J+l_1-l_2+j_1-j_2} A_{[2134]}^J(\omega) \]
\[ = -\eta_3 \eta_4 (-1)^{J+l_3-l_4+j_3-j_4} A_{[1233]}^J(\omega), \]  
(63)

where $[1] = \{(1), -n_1\}$.

Making use of Eqs. (5), (7), (8), and (10) it is easy to show that the functions $\tilde{\psi}_1(y)$ entering Eq. (14) obey the equalities:

\[ \tilde{\psi}_1(x; +) = w_1 \varphi_{\lambda_1}(x), \quad (-1)^{\frac{1}{2} + \sigma} \tilde{\psi}_1(x; -) = \eta_1 w_1 \varphi_{\lambda_1}(x), \]  
(64)
where
\[ w_1 = w_{[1]} = \delta_{\eta_1, +1} u_{(1)} + \delta_{\eta_1, -1} v_{(1)}. \]  

Let us define the reduced matrix elements of the operator \( T_{JLSM} \) by the relation:
\[
\langle j_1 l_1 m_1 | T_{JLSM} | j_2 l_2 m_2 \rangle = (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & j_2 & J \\ m_1 & -m_2 & -M \end{pmatrix} \langle j_1 l_1 || T_{JLS} || j_2 l_2 \rangle. \]  

In the explicit form we have:
\[
\langle j_1 l_1 || T_{JLS} || j_2 l_2 \rangle = \frac{1}{2} \left[ 1 + (-1)^{L+l_1+l_2} \right] (-1)^{S+j_2 - \frac{1}{2}} \\
\times \sqrt{\frac{(2J+1)(2L+1)(2j_1+1)(2j_2+1)}{4\pi}} \begin{pmatrix} j_1 & j_2 & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \\
\times \left\{ \begin{pmatrix} J & L & S \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{\frac{S (S+1)}{J (J+1)}} \begin{pmatrix} J & L & S \\ 1 & 0 & -1 \end{pmatrix} \right\} \times \left( l_1 - j_1 \right) \left( 2j_1 + 1 \right) + (-1)^{J+L+S} \left( l_2 - j_2 \right) \left( 2j_2 + 1 \right). \]  

Making use of Eqs. \( \{6, 7, 14, 20-22, 25-27, 61, 63, 64, 66\} \), we obtain from the Eq. \( \{40\} \) the following ansatz:
\[
A^{J(c, c')}_{LSLM,S'}(r, r'; \omega) = \left( 1 + \delta_{c', ph} \right) \delta_{r, r'} \sum_{[1234]} \delta_{r_1, r} \delta_{r_2, r} \delta_{r_3, r'} \delta_{r_4, r'} \theta_{(21)} \theta_{(43)} \\
\times A^{J(c, c')}_{[12, 34]}(r, r'; \omega), \]  

where
\[
A^{J(c, c')}_{[12, 34]}(r, r'; \omega) = R_{(1)}(r) R_{(2)}(r) R_{(3)}(r') R_{(4)}(r') T^{(J)}_{(12, 34)}(r, r'; \omega) \\
\times \left( \alpha_{[12]}^c - \eta_1 \eta_2 \left( -1 \right)^S \alpha_{[21]}^c \right) A^{J}_{[12, 34]}(\omega) \\
\times \left( \alpha_{[34]}^c - \eta_3 \eta_4 \left( -1 \right)^{S'} \alpha_{[43]}^c \right), \]  

\[
T^{(J)}_{(12, 34)}(r, r'; \omega) = \frac{1}{2J+1} \left( j_2 l_2 || T_{JLS} || j_1 l_1 \right) \left( j_4 l_4 || T_{JL'S'} || j_3 l_3 \right), \]  

\[
\alpha_{[12]}^{ph} = w_1 w_2, \quad \alpha_{[12]}^{pp} = \eta_2 w_1 w_2, \quad \alpha_{[12]}^{hh} = \eta_1 w_1 w_2. \]  

Theoretically, summation in the Eq. \( \{68\} \) is supposed to be over complete ordered set of the states forming the doubled configuration space. This summation is facilitated due to
the symmetry defined by the Eq. \((63)\). The order-bounding factors are defined as follows: 
\[ \theta_{(21)} = 1 \] if the ordinal number of the state \((1)\) is lesser than the number of \((2)\) \([\{(1) < (2)\}]\), 
\[ \theta_{(21)} = \frac{1}{2} \] if \((1) = (2)\), \[ \theta_{(21)} = 0 \] if \((1) > (2)\).

However, in our calculational scheme, summation in the Eq. \((68)\) is restricted by the discrete and quasidiscrete states entering a valence zone near the Fermi level. It is supposed that the remaining part of the sum can be approximated fairly well by the RPA-like propagator \(A_{cont}\), which contains transitions from the quasiparticle levels to the continuum. Thus, we use the scheme which is analogous to the ones described in Refs. \([8, 14]\). According to this scheme the total correlated propagator is represented as a sum of two terms:

\[
A_{LS\tau,L'S'\tau'}^{(c,c')} (r,r'; \omega) = A_{LS\tau,L'S'\tau'}^{(c,c')}^{cont} (r,r'; \omega) + A_{LS\tau,L'S'\tau'}^{(c,c')}^{disc} (r,r'; \omega),
\]

where

\[
A_{LS\tau,L'S'\tau'}^{(c,c')}^{cont} (r,r'; \omega) = - \delta_{c,c'} \delta_{c,ph} \delta_{\tau,\tau'} \left\{ \sum_{j=1}^{disc} \sum_{l=1}^{disc} \delta_{\tau_1, \tau} \varepsilon_{(2)}^2 R_{(2)} (r) R_{(2)} (r') \right.
\times T_{(12,12)}^{(j) LSL'S'} \left[ \tilde{G}_{j11_1 \tau}^{nor} (r', r'; \mu_\tau - E_{(2)} + \omega) + (-1)^{S+S'} \tilde{G}_{j11_1 \tau}^{nor} (r, r'; \mu_\tau - E_{(2)} - \omega) \right]
\left. - \sum_{(12)} \delta_{\tau_1, \tau} \delta_{\tau_2, \tau} \theta_{(21)} R_{(1)} (r) R_{(2)} (r) R_{(1)} (r') R_{(2)} (r') \right.
\times T_{(12,12)}^{(j) LSL'S'} \left[ \frac{\varepsilon_{(1)}^2}{\omega + \mu_\tau - E_{(2)} - \varepsilon_{(1)}} - \frac{\varepsilon_{(2)}^2}{\omega - \mu_\tau + E_{(1)} + \varepsilon_{(2)}} \right] \left. + (-1)^{S+S'} \left( \frac{\varepsilon_{(1)}^2}{\omega + \mu_\tau - E_{(1)} - \varepsilon_{(1)}} - \frac{\varepsilon_{(2)}^2}{\omega - \mu_\tau + E_{(2)} + \varepsilon_{(1)}} \right) \right\},
\]

\[
A_{LS\tau,L'S'\tau'}^{(c,c')}^{disc} (r,r'; \omega) = \delta_{c,c'} \delta_{c,ph} \delta_{\tau,\tau'} \sum_{(12)} \delta_{\tau_1, \tau} \delta_{\tau_2, \tau} \theta_{(1)}^b \left\{ 1 - \theta_{(2)}^b \right\} T_{(12,12)}^{(j) LSL'S'}
\times R_{(1)} (r) R_{(2)} (r) R_{(1)} (r') R_{(2)} (r') \left[ \varepsilon_{(1)}^2 (1 - \varepsilon_{(2)}^2) + (-1)^{S+S'} \varepsilon_{(2)}^2 (1 - \varepsilon_{(1)}^2) \right]
\times \left[ \frac{1}{\omega + E_{(1)} + E_{(2)}} - \frac{(-1)^{S+S'}}{\omega - E_{(1)} - E_{(2)}} \right]
\times \left. + \left( 1 + \delta_{c,c'} \right) \delta_{\tau,\tau'} \sum_{\{1234\}} \delta_{\tau_1, \tau} \delta_{\tau_2, \tau} \delta_{\tau_3, \tau} \delta_{\tau_4, \tau} \theta_{(21)} \theta_{(43)} A_{12,34}^{(c,c')}^{LSL'S'} (r,r'; \omega) \right). \]

In Eq. \((73)\), \(\tilde{G}_{j11_1 \tau}^{nor} (r', r'; \varepsilon)\) is the partial component of the normal single-particle GF without pairing. It is calculated via the regular and irregular solutions of the Schrödinger equation.
that allows to take into account continuum completely on the RPA level (see, e.g., Ref. \[15\]). The relation between the Eqs. (72)–(74) and the initial Eqs. (68), (69) is determined by the formal decomposition:

\[
\tilde{\tilde{G}}_{j \tau r}^{nor}(r, r'; \varepsilon) = \sum_{(1)} \delta_{j_1, j} \delta_{l_1, l} \delta_{\tau_1, \tau} \frac{R_{(1)}(r) R_{(1)}(r')}{\varepsilon - \varepsilon_{(1)}}.
\]

In Eq. (75), \(\theta^b_{(1)}\) are the bounding factors defined as: \(\theta^b_{(1)} = 1\) if \((1) < (\lambda_b)\), \(\theta^b_{(1)} = 0\) if \((1) \geq (\lambda_b)\), where \(\lambda_b\) is the bottom level of the valence zone which coincides in our calculations with “pairing window”, i.e. with zone where the energy gap is not equal to zero (see Sec. 8 for details). The superscript “\(\text{disc}\)” in the \(\sum\) symbols means summation over all discrete and quasidiscrete states of the single-particle spectrum. The superscript “\(\text{wind}\)” means summation over the discrete and quasidiscrete states only inside the “pairing window”, i.e. the same summation as in the Eq. (59).

6 THE AMPLITUDE OF THE QUASIPARTICLE-PHONON INTERACTION AND ITS MATRIX ELEMENTS

Matrix elements of the amplitude of quasiparticle-phonon interaction \(g_{12}^m\) belong to the basic quantities entering formulas for the correlated propagator of the QTBA (see Ref. \[9\] and Sec. 7 of the present paper). In accordance with the general definition of this amplitude in the microscopic approach we have:

\[
g^m(y_1, y_2) = \int dy_3 dy_4 \mathcal{F}(y_1, y_2; y_3, y_4) \rho^{m0}(y_3, y_4),
\]

where the interaction amplitude \(\mathcal{F}\) is defined by the Eq. (18), \(\rho^{m0}\) is the transition amplitude:

\[
\rho^{m0}(y_1, y_2) = \langle m \mid b^\dagger(y_1) b(y_2) \mid 0 \rangle.
\]

The operators \(b(y) = b(x, \chi)\) are defined by the relations:

\[
b(x, +) = a(x), \quad b(x, -) = a^\dagger(x),
\]

where \(a^\dagger(x)\) and \(a(x)\) are creation and annihilation operators of particles.

As a first step we consider the coupling of the quasiparticles with core excitations only in the ph channel. So, substituting Eq. (18) into Eq. (76), we keep only the contributions
of the amplitude $\mathcal{F}^+$. Taking into account the symmetry: $\rho^{m_0}(y_1, y_2) = -\rho^{m_0}(y_2, y_1)$, we obtain from Eqs. (18) and (76):

$$g^m(y_1, y_2) = \delta_{\chi_1, \chi_2} \left[ \delta_{\chi_1, +1} g^m(x_1, x_2) - \delta_{\chi_1, -1} g^m(x_2, x_1) \right], \quad (79)$$

where

$$g^m(x_1, x_2) = \int dx_3 dx_4 \mathcal{F}^+(x_1, x_2; x_3, x_4) \rho^{m_0}(x_3+, x_4+). \quad (80)$$

In what follows we assume that the function $g^m(x_1, x_2)$ has the form:

$$g^m(x_1, x_2) = g^{qM}(x_1, x_2) = \delta(r_1 - r_2) \delta_{\tau_1, \tau_2} \sum_{\ell S} g^{qLS}_{\tau_1 \tau_2} T_{J_qLSM}(n_1)_{\sigma_2 \sigma_1}, \quad (81)$$

where we separate the projection of the total angular momentum $M$ from the remaining phonon quantum numbers, which are denoted by the index $q$ (including the total angular momentum $J_q$). Notice that Eq. (81) is in accordance with Eq. (80) and with the decomposition of the effective interaction within the TFFSPC (13).

Let us denote

$$g^{qM}_{12} = \int dy_1 dy_2 \tilde{\psi}_1(y_1) \tilde{\psi}_2^*(y_2) g^{qM}(y_1, y_2), \quad (82)$$

$$g^{qM}_{\lambda_1 \lambda_2} = \int dx_1 dx_2 \varphi_{\lambda_1}^*(x_1) \varphi_{\lambda_2}^*(x_2) g^{qM}(x_1, x_2). \quad (83)$$

From Eqs. (6), (66), (81), and (83) it follows that

$$g^{qM}_{\lambda_1 \lambda_2} = (-1)^{j_2-m_2} \left( \begin{array}{ccc} j_2 & j_1 & J_q \\ m_2 & -m_1 & -M \end{array} \right) \sum_{S} g^{qS}_{(12)}, \quad (84)$$

where

$$g^{qS}_{(12)} = \sum_{L} g^{qLS}_{(12)} \mathcal{J}_{JqLS} \mathcal{J}_{J1L1} \mathcal{J}_{J2L2}, \quad (85)$$

$$g^{qLS}_{(12)} = \delta_{\tau_1, \tau_2} \int_{0}^{\infty} dr r^2 R_{(1)}(r) R_{(2)}(r) g^{qLS}_{\tau_1 \tau_2}(r). \quad (86)$$

Making use of Eqs. (61), (79), (82)–(84), and the symmetry of the operator $T_{JLSM}$ we obtain the following two main formulas for the matrix elements of the amplitude of quasiparticle-phonon interaction:

$$g^{qM}_{12} = (-1)^{j_2-m_2} \left( \begin{array}{ccc} j_2 & j_1 & J_q \\ m_2 & -m_1 & -M \end{array} \right) g^{q}_{[12]}, \quad (87)$$

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where
\[
g_{12}^q = \sum_S \left( w_1 w_2 - \eta_1 \eta_2 (-1)^S w_1 w_2 \right) g_{12}^{qS} .
\] (88)

The Eq. (88) can be easily generalized to take into account the coupling of the quasi-particles with core excitations in the pp and the hh channels. Proceeding in the same way, we obtain:
\[
g_{12}^q = \sum_{LSC} \left( \alpha_{c12} - \delta_{c,ph} \eta_1 \eta_2 (-1)^S \alpha_{c21} \right) \langle j_2 l_2 \mid T_{qLS} \mid j_1 l_1 \rangle g_{12}^{qLS(c)} ,
\] (89)
where the amplitudes \( \alpha_{c12} \) are defined by the Eqs. (71),
\[
g_{12}^{qLS(c)}(c) = \delta_{\tau_1, \tau_2} \int_0^\infty dr r^2 R_{(1)}(r) R_{(2)}(r) g_{12}^{qLS(c)}(r) .
\] (90)

Notice that in the Eqs. (88) and (89) we have: \( g_{qLS}(r) = g_{qLS(ph)}(r) \). To determine the function \( g_{qLS(c)}^{LS}(r) \) entering Eq. (90) the following method is possible. In terms of the partial components the Eq. (76) reads:
\[
g_{qLS(c)}^{LS}(r) = \sum_{\tau \tau'} \mathcal{J}_{S, \tau \tau'}(c) \rho_{qLS(c)}^{LS}(r) ,
\] (91)
where functions \( \mathcal{J}_{S, \tau \tau'}(r) \) are determined by the Eqs. (45) and (46), \( \rho_{qLS(c)}^{LS}(r) \) is the partial component of the transition amplitude, and the contribution of the restoring amplitude in Eq. (44) is omitted. From the spectral decomposition of the partial component of the LRM one can find that up to irrelevant constant phase factor the following equality is fulfilled:
\[
\rho_{qLS(c)}^{LS}(c) = \frac{2 J_q + 1}{B(q)} \int_{C_q} \frac{d\omega}{2\pi i} \Lambda_{qLS_{\tau}}^{(c)}(r; \omega) ,
\] (92)
where \( \Lambda_{qLS_{\tau}}^{(c)} \) is a solution of Eq. (53), \( B(q) \uparrow = B(\text{g.s.} \rightarrow q) \) is reduced excitation probability which is determined by the formula
\[
B(q) \uparrow = \int_{C_q} \frac{d\omega}{2\pi i} \Pi_{J_q}(\omega) .
\] (93)

The polarizability \( \Pi_{J}(\omega) \) in Eq. (93) is determined by Eq. (56). In the Eqs. (92) and (93) the integration is performed along the contour \( C_q \) in the complex energy plane which encloses the poles of the functions \( \Lambda_{qLS_{\tau}}^{(c)}(r; \omega) \) and \( \Pi_{J_q}(\omega) \) corresponding to the phonon energy \( \omega_q \). This method automatically ensures correct normalization of the functions \( \rho_{qLS(c)}^{LS}(r) \) and \( g_{qLS(c)}^{LS}(r) \). However, to determine these functions in case of the excitations in the pp and the
hh channels one has to solve Eq. (53) for the different external fields \( \tilde{V}_{JLS}^{(c)}(r) \) which take non-zero values for \( c = pp \) and \( c = hh \) in contrast to the definition (55).

Notice that in order to calculate the function \( g_{JLS}^{(c)}(r) \) by means of the formulas (91)–(93) it is not necessary to know the exact value of the phonon energy. It is sufficient that the pole \( \omega = \omega_q \) (and only this pole) would be inside the contour \( C_q \). The exact value of \( \omega_q \) can be determined by the same method with making use of the formula

\[
\omega_q = \frac{1}{B(q) \uparrow} \int_{C_q} \frac{d\omega}{2\pi i} \omega \Pi_{JLS}(\omega) .
\]

Another method to determine matrix elements \( g_{\ell \ell'}^{q} \) consists in solving QRPA equation for the transition amplitudes making use of the basis restricted by discrete and quasidiscrete single-particle states entering a valence zone near the Fermi level. In terms of the reduced matrix elements this equation reads:

\[
(\omega_q - \eta [E(1) + E(2)]) \rho_{(12)\eta}^{q} = \sum_{\eta'} \sum_{(34)}^{\text{wind}} \eta \mathcal{F}_{(12)\eta,(34)\eta'}^{q} \theta_{(43)} \rho_{(34)\eta'}^{q} ,
\]

where

\[
\mathcal{F}_{(12)\eta,(34)\eta'}^{q} = \sum_{\eta_1,\eta_2,\eta_3,\eta_4} \delta_{\eta_1,\eta} \delta_{\eta_2,-\eta} \delta_{\eta_3,-\eta'} \delta_{\eta_4,-\eta'} \mathcal{F}_{(12),(34)}^{J},
\]

\[
\mathcal{F}_{(12),(34)}^{J} = \frac{1}{2} \sum_{\text{LSc}} \left( \alpha_{\ell 2}^{c} - \eta_1 \eta_2 (-1)^{S} \alpha_{\ell 1}^{c} \right) \left( \alpha_{\ell 3}^{c} - \eta_3 \eta_4 (-1)^{S} \alpha_{\ell 4}^{c} \right) \times \left( 1 + \delta_{c\text{ph}} \right) T_{(12),(34)}^{(J)\text{LS,LS}} \mathcal{F}_{(12),(34)}^{(c)} ,
\]

\[
\mathcal{F}_{(12),(34)}^{(c)} = \delta_{\tau_1,\tau_2} \delta_{\tau_3,\tau_4} \int_{0}^{\infty} dr r^2 R_{(1)}(r) R_{(2)}(r) R_{(3)}(r) R_{(4)}(r) \mathcal{F}_{S,\tau_1,\tau_3}^{(c)}(r) .
\]

The quantities entering these formulas are defined by the Eqs. (45), (46), (70), (71). Notice that QRPA equation (95) includes contributions of all the channels: ph, pp, and hh.

Normalization condition for the transition amplitudes \( \rho_{(12)\eta}^{q} \) has the form

\[
\sum_{\eta} \sum_{(12)}^{\text{wind}} \theta_{(21)} \eta \left| \rho_{(12)\eta}^{q} \right|^2 = 2J_q + 1 .
\]

The reduced matrix elements \( g_{[12]}^{q} \) and the reduced probability \( B(q) \uparrow \) of excitation induced by the external field (33) are determined by the found values of \( \rho_{(12)\eta}^{q} \) through the formulas:

\[
g_{[12]}^{q} = \sum_{[34]}^{\text{wind}} \mathcal{F}_{[12],[34]}^{J} \theta_{(43)} \delta_{\eta_2,-\eta_4} \rho_{(34)\eta_3}^{q} ,
\]
\[ B(q) = \frac{1}{2J_q + 1} \left| \sum_{\eta} \sum_{(12)}^{wind} \theta_{(21)}(\tilde{V}^0_{j_\eta})_{(21)\eta} \rho^0_{(12)\eta} \right|^2, \]  

where

\[(\tilde{V}^0_j)_{(21)\eta} = \sum_{LS} \eta^s \left[ u_{(1)} v_{(2)} + (-1)^s v_{(1)} u_{(2)} \right] \langle j_2 l_2 || T_{JLS} || j_1 l_1 \rangle (\tilde{V}^0_{jLS})_{(21)}, \]  

\[ (\tilde{V}^0_{JLS})_{(12)} = \delta_{\tau_1, \tau_2} \int_0^\infty dr r^2 R_{(1)}(r) R_{(2)}(r) \tilde{V}^0_{JLS\tau_1}(r). \]

7 CORRELATED PROPAGATOR OF THE QTBA IN TERMS OF THE REDUCED MATRIX ELEMENTS

In this section we draw the formulas for the reduced matrix elements of the correlated propagator \( A^J_{(12,34)}(\omega) \) which are obtained within the QTBA in case of the QPC model. In what follows, the summation over single-quasiparticle indices means summation over the discrete states inside the “pairing window”. The general formula for the propagator of the QTBA satisfying Eq. (33) reads:

\[ A^J_{(12,34)}(\omega) = \sum_{[5678]} \left[ \delta_{15} \delta_{26} + Q^J_{(12,34)}(\omega) \right] A^J_{(12,56)}(\omega) \left[ \delta_{73} \delta_{84} + Q^J_{(78,34)}(\omega) \right] \]

\[ + \frac{1}{2} \left[ P^J_{(12,34)}(\omega) - (-1)^{j_1 + l_1 - j_2 + l_2} P^J_{(21,34)}(\omega) \right], \]  

where

\[ Q^J_{(12,34)}(\omega) = \theta_{(43)} \left[ Q^J_{(12,34)}(\omega) + (-1)^{j_1 + l_1 - j_2 + l_2} Q^J_{(12,34)}(\omega) \right], \]  

\[ Q^J_{(12,34)}(\omega) = \theta_{(21)} \left[ Q^J_{(12,34)}(\omega) + (-1)^{j_1 + l_1 - j_2 + l_2} Q^J_{(12,34)}(\omega) \right], \]

\[ A^J_{(12,34)}(\omega) = \delta_{\eta_1 - \eta_2} \delta_{\eta_3 - \eta_4} A^J_{(12)\eta_1, (34)\eta_3}(\omega). \]

The quantity \( A^J_{(12)\eta_1, (34)\eta_3}(\omega) \) is a solution of the equation

\[ A^J_{(12)\eta_1, (34)\eta_3}(\omega) = \tilde{A}^J_{(12)\eta_1, (34)\eta_3}(\omega) + \sum_{\eta''} \sum_{(56)} \theta_{(65)} K^J_{(12)\eta_1, (56)\eta''}(\omega) A^J_{(56)\eta'', (34)\eta_3}(\omega), \]  

where

\[ \tilde{A}^J_{(12)\eta_1, (34)\eta_3}(\omega) = -\frac{\eta \delta_{\eta, \eta''} \left[ \delta_{(13)} \delta_{(24)} + (-1)^{j_1 + l_1 - j_2 + l_2} \delta_{(14)} \delta_{(23)} \right]}{2(\omega - \eta (E_{(1)} + E_{(2)}))}, \]
of the Eqs. (62), (87), (113)–(115) we obtain:

\[
\mathcal{K}_{\eta_1\eta_2\eta_3\eta_4}^j(\omega) = \eta \left[ \Phi_{(12)\eta_1(34)\eta_4}^j(\omega) + (-1)^{J_1-l_2} l_3-j_2 \Phi_{(21)\eta_2(34)\eta_4}^j(\omega) \right] \frac{\omega - \eta [E_{(1)} + E_{(2)}]} {\omega - \eta [E_{(1)} + E_{(2)}]},
\]

(110)

\[
\Phi_{(12)\eta(34)\eta_4}^j(\omega) = \sum_{\eta_1\eta_2\eta_3\eta_4} \delta_{\eta_1,\eta} \delta_{\eta_2,-\eta} \delta_{\eta_3,-\eta} \delta_{\eta_4,-\eta} \Phi^{J}_{(12,34)}(\omega).
\]

(111)

It is easy to see that solution of the Eq. (108) possesses the following symmetry:

\[
A^{J}_{(12)\eta(34)\eta_4}(\omega) = (-1)^{J_1-l_2} l_3-j_2 A^{J}_{(21)\eta,\eta_4}(\omega)
\]

\[
= (-1)^{J_1+l_4-j_3-j_4} A^{J}_{(12)\eta_1\eta_2(34)\eta_4}(\omega).
\]

(112)

It enables one to determine all the elements of the matrix \(A^{J}_{(12)\eta(34)\eta_4}(\omega)\) by solving Eq. (108) for the non-zero block of the matrix \(\theta^{J}_{(21)} A^{J}_{(12)\eta(34)\eta_4}(\omega) \theta^{(43)}\). However, as follows from Eqs. (62), (74), (104)–(107), for the construction of propagator in the coordinate representation it is sufficient to determine the elements only of this block.

In order to define the remaining quantities which enter Eqs. (104)–(106), (111) in case of the quasiparticle-phonon coupling model let us introduce notations:

\[
\Omega_{12} = E_{12} + \eta_1 \omega_q, \quad E_{12} = \eta_1 \left[ E_{(1)} + E_{(2)} \right],
\]

(113)

\[
D_{[12,34]\eta}^q = \delta_{\eta,+1} g_{[13]}^q g_{[24]}^{q*} + \delta_{\eta,-1} (-1)^{j_1-j_2-j_3} g_{[31]}^q g_{[42]}^q,
\]

(114)

\[
X_{[12,34]\eta}^J = (-1)^{J_1-l_2-j_3} \begin{pmatrix} j_1 & j_2 & J \end{pmatrix} \begin{pmatrix} j_4 & j_3 & J \end{pmatrix} D_{[12,34]\eta}^q, \quad Y_{[12,3]}^q = \frac{\delta_{j_1,j_2} \delta_{l_1,l_2}}{2j_1 + 1} D_{[12,33]\eta_3},
\]

(115)

where the reduced matrix elements \(g_{[12]}^q\) are defined by the formulas of Sec. (6). Making use of the Eqs. (62), (87), (113)–(115) we obtain:

\[
\Phi_{[12,34]}^J(\omega) = \Phi_{[12,34]}^{J(\text{res})}(\omega) + \Phi_{[12,34]}^{J(\text{GSC})}(\omega) + \Phi_{[12,34]}^{(\text{comp})}(\omega),
\]

(116)

\[
\Phi_{[12,34]}^{J(\text{res})}(\omega) = \eta_1 \delta_{\eta_1,-\eta_2} \delta_{\eta_1,-\eta_4} \sum_q \left[ \delta_{\eta_1,\eta_3} \frac{X_{[12,34]\eta_3}^q}{\omega - \Omega_{12 q}} + \frac{X_{[12,34]\eta_4}^q}{\omega - \Omega_{14 q}} \right]
\]

\[
+ \delta_{[24]} \sum_{[5]} \delta_{\eta_6,\eta_1} \frac{Y_{[13,5]}^q}{\omega - \Omega_{52 q}} + \delta_{[13]} \sum_{[6]} \delta_{\eta_6,\eta_2} \frac{Y_{[42,6]}^q}{\omega - \Omega_{16 q}},
\]

(117)
\[
\Phi_{[12,34]}^{(GSC)}(\omega) = -\eta \sum_{q} \delta_{q_{1},q_{2}} \delta_{q_{3},q_{4}} \sum_{q} \left[ \frac{\delta_{q_{4},q_{3}}}{\Omega_{12}^{q_{3}} \Omega_{24}^{q_{4}}} + \frac{\delta_{q_{4},q_{1}}}{\Omega_{13}^{q_{1}} \Omega_{24}^{q_{4}}} \right] + \delta_{[24]} \sum_{[5]} \delta_{q_{5},q_{2}} Y_{13,5}^{q} \left( \omega - E_{12} - \Omega_{35}^{q} \right) \\
+ \delta_{[13]} \sum_{[6]} \delta_{q_{6},q_{1}} Y_{42,6}^{q} \left( \omega - E_{12} - \Omega_{64}^{q} \right) \right], \tag{118}
\]

\[
\Phi_{[12,34]}^{(comp)}(\omega) = -\eta \sum_{q} \delta_{q_{1},q_{2}} \delta_{q_{3},q_{4}} \sum_{q} \left[ \frac{\delta_{q_{5},q_{2}}}{\Omega_{15}^{q_{1}} \Omega_{35}^{q_{4}}} + \frac{\delta_{q_{5},q_{1}}}{\Omega_{62}^{q_{2}} \Omega_{64}^{q_{4}}} \right] \times \left( \omega + E_{56} - \Omega_{24}^{q_{2}} - \Omega_{34}^{q_{4}} \right), \tag{119}
\]

\[
P_{[12,34]}^{(++)}(\omega) = \eta \sum_{q} \left[ \frac{\delta_{q_{5},q_{3}}}{\Omega_{13}^{q_{2}} \Omega_{24}^{q_{4}}} \left( \frac{1}{\omega - \Omega_{32}^{q_{2}}} - \frac{1}{\omega - \Omega_{14}^{q_{4}}} \right) + \sum_{[5]} \frac{\delta_{q_{5},q_{1}}}{\Omega_{51}^{q_{1}} \Omega_{53}^{q_{2}}} \left( \omega - \Omega_{52}^{q_{2}} \right) \right] + \delta_{[24]} \sum_{[5]} \left[ \frac{\delta_{q_{5},q_{1}}}{\Omega_{13}^{q_{3}} \Omega_{53}^{q_{2}}} \left( \frac{1}{E_{13}^{q_{3}}} + \frac{1}{\omega - \Omega_{52}^{q_{2}}} \right) \right] \\
+ \delta_{[24]} \sum_{[5]} \delta_{q_{5},q_{2}} Y_{13,5}^{q} \left( \Omega_{13}^{q_{3}} \Omega_{24}^{q_{4}} \right) \left( \frac{1}{E_{13}^{q_{3}}} + \frac{1}{\omega - \Omega_{52}^{q_{2}}} \right) \right] \tag{120}
\]

\[
Q_{[12,34]}^{(+-)}(\omega) = \delta_{q_{1},q_{2}} \delta_{q_{3},q_{4}} \left[ \frac{\delta_{q_{5},q_{3}}}{\Omega_{31}^{q_{2}} \Omega_{53}^{q_{4}}} \left( \omega - \Omega_{32}^{q_{2}} \right) \right] + \delta_{q_{5},q_{1}} \left( \frac{1}{E_{13}^{q_{3}}} + \frac{1}{\omega - \Omega_{52}^{q_{2}}} \right) \right] \\
+ \delta_{[24]} \sum_{[5]} \left[ \frac{\delta_{q_{5},q_{1}}}{\Omega_{31}^{q_{3}} \Omega_{53}^{q_{2}}} \left( \frac{1}{E_{13}^{q_{3}}} + \frac{1}{\omega - \Omega_{52}^{q_{2}}} \right) \right] \tag{121}
\]

\[
Q_{[12,34]}^{(-+)}(\omega) = \delta_{q_{1},q_{2}} \delta_{q_{3},q_{4}} \left[ \frac{\delta_{q_{5},q_{3}}}{\Omega_{42}^{q_{2}} \Omega_{32}^{q_{4}}} \left( \omega - \Omega_{32}^{q_{2}} \right) \right] + \delta_{q_{5},q_{1}} \left( \frac{1}{E_{13}^{q_{3}}} + \frac{1}{\omega - \Omega_{52}^{q_{2}}} \right) \right] \tag{122}
\]
The Eqs. (104)–(122) completely define the reduced matrix elements $A^{J}_{12,34}(\omega)$ which enter Eqs. (69) and (74) for the “discrete” part of the total correlated propagator in the coordinate representation. Notice that in the Eq. (69) the matrix elements $A^{J}_{12,34}(\omega)$ are the same for all the channels which differ from each other only by the amplitudes $\alpha_{c}^{\nu}$. Notice also that the form of the above equations for the reduced matrix elements $A^{J}_{12,34}(\omega)$ are the same both for magic and for open-shell nuclei. In the former case, however, the index $\eta_1$ in the set $[1] = \{(1), \eta_1\}$ is not independent, but is determined by the occupation number $n_{(1)} = 0$ or 1 of the state $(1) = (\lambda_1)$ as: $\eta_1 = 1 - 2 n_{(1)}$.

8 CALCULATIONS: DETAILS AND DISCUSSION

As an application of our approach isovector electric dipole excitations in the GDR region have been calculated in semi-magic isotopes $^{116,120,124}$Sn. We started from a description of the independent single-particle motion in the standard phenomenological Woods-Saxon potential. Then the gap equation (59) was solved for neutron subsystem under the usual condition that the number of particles in valence zone is conserved on average. In our calculations for the tin isotopes this zone (“pairing window”) consists of all discrete ($\varepsilon_{\lambda} < 0$) and quasidiscrete states above the chemical potential $\mu$ and of the discrete states below $\mu$ starting from $1f_{7/2}$ subshell for neutrons and $1d_{5/2}$ subshell for protons. We emphasize that the same valence zone restricts summations both in the gap equation (59) and in the equations of Sec. 7 for the QTBA propagator.

As the quasidiscrete states we chose the discrete states with $\varepsilon_{\lambda} > 0$ calculated with a box boundary condition and having the last extremum of the radial wave function in the range of non-vanishing values of the discrete spectrum ($\varepsilon_{\lambda} < 0$) wave functions. According to this criterion the quasidiscrete states are selected quite well if size of the box is not too large. Notice that the partial components of the normal single-particle GF $\tilde{G}_{j\tau r}^{nor}(r, r'; \varepsilon)$ entering Eq. (63) were calculated with outgoing wave boundary condition except for the small number of components for which $j\ell$-values coincide with $j\ell$-values of the quasidiscrete states. For such partial components the box boundary condition was used in order to avoid inconsistency in the calculations.

The parameter $c_p$ in Eq. (48) was chosen so as to obtain the averaged solution of the gap equation $\bar{\Delta}_{\lambda}$ to be equal to the averaged empirical value (see Ref. [16]):

$$\bar{\Delta} = 12 \text{ MeV} \times A^{-1/2}. \quad (123)$$
For $^{120}$Sn we have obtained $\bar{\Delta} \approx 1.1$ MeV with $c_p = 0.719$ MeV. This parameter was used for the remaining two nuclei. The quantity $\xi$ in Eq. [18] is determined as follows: $\xi = \sqrt{\xi_1 \xi_2}$ where $\xi_1 = \mu - \min(\varepsilon_\lambda)$, $\xi_2 = \max(\varepsilon_\lambda) - \mu$.

We assumed that the observable energies of single-particle excitations in the neighbouring odd nuclei have to be reproduced in the framework of the mean field plus BCS model. These observable energies (experimental energy differences) are defined by the following equation:

$$\varepsilon^{\text{exp}}_\lambda = \pm \left[ E_{\lambda}(A \pm 1) - E_{\text{GS}}(A) \right],$$

where $E_{\text{GS}}$ is the ground state energy of the even-even nucleus (core) consisting of the $A$ nucleons, $E_{\lambda}$ is the energy of the ground or the excited state of the neighbouring odd nucleus consisting of the $A \pm 1$ nucleons. In order to get an agreement with the experimental energies the well depth of central part of the Woods-Saxon potential was slightly varied so as to obtain $\varepsilon^{\text{exp}}_\lambda = \varepsilon_\lambda$ for protons and $\varepsilon^{\text{exp}}_\lambda = \mu_\tau \pm E_\lambda$ for neutrons where $E_\lambda$ is connected with $\varepsilon_\lambda$ by the Eq. (58). Thus obtained energies and wave functions form the above mentioned basis set $\{\varepsilon_\lambda, \varphi_\lambda(x)\}$.

In the present calculations we included ground state correlations (GSC) only in the QRPA part of the correlated propagator. Another type of GSC caused by quasiparticle-phonon coupling (GSC/QPC) and originated from backward-going terms in time-ordered diagrams was not incorporated. This means that: (i) only the term $\Phi^{(\text{res})}_{[12,34]}(\omega)$ in the Eq. (111) is accounted for; (ii) associated components of the propagator containing the functions $P^{(++)}_{[12,34]}(\omega)$, $Q^{(+)}_{[12,34]}(\omega)$, and $Q^{(-)}_{[12,34]}(\omega)$ [see Eqs. (104)–(106), (120)–(122)] are excluded from the calculations.

Since our single-particle and single-quasiparticle energies are fitted to the experiment as it was mentioned above, these energies and corresponding wave functions already contain effectively the admixture of phonons. This phonon contribution should be removed from the mean field, energy gap, and the effective interaction to avoid double counting if the quasiparticle-phonon coupling is included explicitly. To solve this problem we use the method which corresponds to the self-consistent scheme of the QTBA (see Ref. [9]) if the GSC/QPC are not included. In this case the method consists of the replacement of the amplitude $\Phi^{J}_{[12,34]}(\omega)$ in the Eq. (111) by the difference amplitude $\Phi^{(\text{res})}_{[12,34]}(\omega)$ where

$$\Phi^{J}_{[12,34]}(\omega) = \Phi^{(\text{res})}_{[12,34]}(\omega) - \Phi^{J}_{[12,34]}(0). \quad (124)$$

Making use of this subtraction procedure, ground-state contributions of the quasiparticle-phonon coupling defined by the quantity $\Phi^{J}_{[12,34]}(0)$ are removed from both the mass operator and the effective interaction.
In the calculations of the GDR we neglected the contribution of pp and hh channels that is justified for this type of excitations (see Ref. [13]). The parameters of the Landau-Migdal zero-range interaction in the ph channel entering Eqs. (45) and (47) were taken in accordance with the standard set which is usually used in similar calculations (see, e.g., Refs. [1, 14]):

\[ f_{in} = -0.002, \quad f'_{ex} = 2.62, \quad f'_{in} = 0.76, \quad g = 0.05, \quad g' = 0.96, \quad C_0 = 300 \text{ MeV fm}^3. \]  

(125)

Notice, however, that the parameter \( f'_{ex} \) has been changed as compared to one used in Refs. [1, 14] in order to reproduce the experimental mean energy of the GDR in \(^{120}\text{Sn}\) within QTBA calculation. The parameter \( f_{ex} \) was slightly varied from \(-2.187\) to \(-1.957\) in the computation of the energies and amplitudes of low-lying collective \( 2^+ \) and \( 3^- \) phonons within QRPA in configuration space. This allowed us to fit the energies of these phonons to experimental values. Then the value of \( f_{ex} \) was averaged and used in the calculations of the remaining phonons and further in the QTBA calculations.

The phonon characteristics were calculated within the QRPA by making use of Eqs. (95)–(103). The configuration space was restricted by the “pairing window” defined above. In this calculation the matrix elements of the interaction \( F^{(c)c}_{(12,34)} \) with \( S = 1 \) were omitted since they do not give significant contribution in case of low-lying collective modes with natural parity. In addition, we have neglected contribution of pp and hh channels. These simplifications decrease dimension of the QRPA matrix in the configuration space by a factor of two. For all three chosen tin isotopes the collective modes with spin and parity \( 2^+, 3^-, 4^+, 5^- \), \( 6^+ \) and with energies below the neutron separation energy were included into the phonon space. A mode is assumed to enter the phonon space if its reduced transition probability is more than 10 % of the maximal one for fixed spin and parity. This value was taken as an approximate criterion for phonons selection. Characteristics of the low-lying vibrations taken into account in the calculations are collected in the Table I.

The dipole photoabsorption cross section \( \sigma_{E1}(E) \) is the basic observable computed in the present work. This value is expressed via the strength function \( S_{E1}(E) \) according to the well known formula:

\[ \sigma_{E1}(E) = \frac{16\pi^3 e^2}{9\hbar c} E S_{E1}(E). \]  

(126)

The strength function in turn is simply connected with the polarizability \( \Pi_{E1}(\omega) \) [see Eq. (3)]:
Table 1: Energies and reduced transition probabilities of the low-lying phonons used in QTBA calculations.

| J^π | 116Sn  | 120Sn  | 124Sn  |
|-----|--------|--------|--------|
|     | ω (MeV) | B(EL)↑ (e^2fm^2) | ω (MeV) | B(EL)↑ (e^2fm^2) | ω (MeV) | B(EL)↑ (e^2fm^2) |
| 2^+ | 1.29   | 2.97×10^3 | 1.17   | 3.53×10^3 | 1.13   | 2.55×10^3 |
| 3^- | 2.27   | 9.06×10^4 | 2.40   | 8.42×10^4 | 2.60   | 1.32×10^5 |
| 4^+ | 5.21   | 1.35×10^4 | 5.07   | 1.89×10^4 | 1.21   | 1.19×10^6 |
|     | 2.60   | 1.59×10^6 | 2.62   | 1.22×10^6 | 2.11   | 5.87×10^6 |
|     | 6.56   | 5.15×10^5 | 7.58   | 8.12×10^5 | 3.93   | 5.33×10^5 |
|     | 4.88   | 3.08×10^5 | 5.35   | 3.35×10^5 | 7.60   | 5.33×10^5 |
|     | 5.40   | 3.02×10^5 | 4.89   | 3.26×10^5 | 5.06   | 4.17×10^5 |
|     | 5.56   | 2.58×10^5 | 3.44   | 2.25×10^5 | 7.36   | 2.31×10^5 |
| 5^- | 5.91   | 2.95×10^7 | 2.67   | 1.98×10^7 | 6.85   | 2.85×10^7 |
|     | 2.75   | 1.46×10^7 | 7.04   | 1.80×10^7 | 2.40   | 1.47×10^7 |
|     | 6.13   | 6.53×10^6 | 5.52   | 1.41×10^7 | 5.49   | 1.41×10^7 |
|     | 3.49   | 5.52×10^6 | 6.53   | 9.98×10^6 | 4.03   | 1.14×10^7 |
|     | 8.39   | 4.15×10^6 | 3.56   | 8.57×10^6 | 8.48   | 9.66×10^6 |
|     | 8.98   | 3.57×10^6 | 8.59   | 8.43×10^6 | 6.09   | 5.05×10^6 |
|     | 3.82   | 3.13×10^6 | 8.59   | 8.43×10^6 | 6.09   | 5.05×10^6 |
| 6^+ | 5.38   | 7.63×10^8 | 7.44   | 9.74×10^8 | 4.78   | 1.04×10^9 |
|     | 5.70   | 7.51×10^8 | 5.34   | 7.56×10^8 | 7.24   | 8.79×10^8 |
|     | 4.84   | 5.93×10^8 | 4.27   | 5.63×10^8 | 5.06   | 5.59×10^8 |
|     | 6.41   | 5.48×10^8 | 4.88   | 4.33×10^8 | 2.40   | 5.47×10^8 |
|     | 3.35   | 4.29×10^8 | 2.94   | 3.93×10^8 | 5.31   | 5.46×10^8 |
|     | 3.99   | 2.27×10^8 | 7.83   | 1.80×10^8 | 7.67   | 2.58×10^8 |
|     | 6.74   | 1.32×10^8 | 4.98   | 1.67×10^8 | 3.70   | 1.58×10^8 |
\[ S_{E1}(E) = -\frac{1}{\pi} \text{Im} \Pi_{E1}(E + i \Delta). \] (127)

As follows from the Eq. (56), to determine the value of function \( \Pi_{E1}(\omega) \) at a given complex energy variable \( \omega \) and, consequently, to compute the values of \( S_{E1}(E) \) and \( \sigma_{E1}(E) \) one has to solve Eq. (53) for the LRM. This equation was solved within the framework of two models: QTBA and QRPA. The spurious isoscalar \( 1^- \) state has been eliminated using the “forced consistency” method presented in Sec. 4. The single-particle continuum was included as described in Sec. 5. Following calculational scheme which is usually used in the response function formalism, in the present work we introduced a smearing parameter \( \Delta \) which is actually an imaginary part of the energy variable \( \omega \). In the calculations of GDR the value \( \Delta = 250 \text{ keV} \) was used.

Figure 1: \( E1 \) photoabsorption cross section for \(^{116}\text{Sn}\) calculated within QRPA (dashed line) and QTBA (solid line). The smearing parameter \( \Delta \) is equal to 250 keV.
Calculated photoabsorption cross sections for three above indicated tin isotopes are shown in Figs. 1–3. The solid and the dashed curves represent the QTBA and the QRPA correspondingly. In order to compare the calculated cross sections with experimental data the parameters of Lorentz function $\sigma_L(E)$ were found where

$$\sigma_L(E) = \sigma_0 \frac{\Gamma^2 E^2}{(E^2 - E_0^2)^2 + \Gamma^2 E^2}.$$  \hspace{1cm} (128)

The parameters $E_0$, $\Gamma$, and $\sigma_0$ in the Eq. (128) were obtained by making use of the following condition: the energy-weighted moments $m_0$, $m_{-1}$, and $m_{-2}$ of the functions (126) and (128) should coincide. This method is analogous to the one developed in Ref. [17] but in contrast to Ref. [17] we calculated the moments in the finite energy interval 0–30 MeV for which experimental data are available. Parameters of the Lorentz fit are compiled in the Table 2. In this table values of the depletion of the energy weighted sum rule (EWSR), i.e.
integrated cross sections $\int_{0}^{30\text{MeV}} \sigma_{E1}(E) \, dE$, are also presented as percentages with respect to the corresponding Thomas-Reiche-Kuhn (TRK) values $59.74 \text{N}\text{Z}/\text{A}\text{MeV mb}$.

In the Table 3 experimental Lorentz function parameters of the GDR are shown. Experimental mean energies demonstrate the well known property to decrease against neutron excess. Our QTBA results drawn in the Table 2 reproduce quite well these mean energies. Since we take into account a finite number of the low-lying phonons, our theoretical curves have the shapes which are rather far from single Lorentzians. Nevertheless, remaining parameters of the Lorentz function calculated within QTBA, i.e. $\Gamma$ and $\sigma_0$, are in a reasonable agreement with experimental values.

Consider the results obtained within QTBA and QRPA to analyze effect of the QPC on the integral characteristics of resonances. As one can see from the Table 2 QTBA gives significant increase of the total width as compared to QRPA ($\Gamma_{\text{QTBA}} \gtrsim 1.6 \Gamma_{\text{QRPA}}$) owing to

Figure 3: Same as Fig. 1 but for $^{124}\text{Sn}$.
Table 2: Lorentz function parameters of the GDR in $^{116,120,124}$Sn obtained within two microscopic approaches for 0–30 MeV energy interval. Values of the depletion of the EWSR are presented as percentages with respect to the corresponding TRK values.

|          | $^{116}$Sn |          | $^{120}$Sn |          | $^{124}$Sn |
|----------|------------|----------|------------|----------|------------|
|          | QRPA       | QTBA     | QRPA       | QTBA     | QRPA       | QTBA     |
| $E_0$ (MeV) | 14.74     | 15.44    | 14.65      | 15.39    | 14.35      | 15.10    |
| $\Gamma$ (MeV) | 2.4      | 4.0      | 2.7        | 4.4      | 2.6        | 4.4      |
| $\sigma_0$ (mb) | 452      | 302      | 423        | 288      | 452        | 298      |
| EWSR (%)   | 94        | 99       | 97         | 102      | 97         | 102      |

Table 3: Experimental Lorentz function parameters of the GDR. The results are taken from Refs. [18] and [19].

|          | $^{116}$Sn |          | $^{120}$Sn |          | $^{124}$Sn |
|----------|------------|----------|------------|----------|------------|
|          | [18]      | [19]     | [18]      | [19]     | [18]      | [19]     |
| $E_0$ (MeV) | 15.57 ±0.1 | 15.67±0.04 | 15.38±0.1  | 15.40±0.04 | 15.29±0.1  | 15.18±0.04 |
| $\Gamma$ (MeV) | 5.21±0.1  | 4.19±0.06 | 5.25±0.1   | 4.88±0.06 | 4.96±0.1   | 4.81±0.06 |
| $\sigma_0$ (mb) | 270±5    | 266±7    | 284±5      | 280±8    | 275±5      | 283±8    |

contribution of the spreading width $\Gamma^4$. Clearly this result could be expected from physical point of view. The EWSR values obtained within the QTBA for the investigated energy interval 0–30 MeV are rather close to the TRK ones but again there is a noticeable difference between the QTBA and the QRPA values. However this difference has another source which is the subtraction procedure described above [see Eq. (124)]. It can be rigorously proved that for the version of QTBA in which GSC/QPC are not taken into account and the subtraction procedure is not applied the equality $EWSR_{QTBA} = EWSR_{QRPA}$ is fulfilled exactly. Notice that the analogous equality is fulfilled between the values of EWSR defined within the MCDD (Ref. [10]) and within the RPA. In our calculations just the subtraction procedure determined by the Eq. (124) gives rise to increment of EWSR in the QTBA. Switching off this procedure we have obtained that the values of EWSR calculated within the QTBA in the “infinite” energy interval (0–200 MeV in our calculations) are equal to the corresponding...
QRPA values with sufficiently high accuracy. This result can be considered as a test of our calculational scheme. The mean energy shift of about 0.7–0.8 MeV obtained in QTBA with respect to QRPA has the similar nature: it arises mainly due to the subtraction procedure. Without this subtraction the QTBA mean energies decrease as compared to the QRPA ones by about of 0.2 MeV. Thus, the subtraction procedure results in the significant change of the averaged characteristics of the excited states calculated in the QTBA as compared to the QRPA. On the other hand, it ensures elimination of the QPC contributions from the ground-state characteristics leading to the equality between QTBA and QRPA response functions $R_{\text{eff}}(\omega)$ at $\omega = 0$.

Finally, notice that although we have taken into account the most important effects of the QPC, the neglected contributions of the GSC/QPC and of the dynamical coupling to the pp and hh channels may also affect the results. In case of magic nuclei, the role of the GSC/QPC in the description of nuclear excitations was investigated in a series of papers (see, e.g., Ref. [1] and references therein). The study of these effects in the nuclei with pairing is in progress.

9 CONCLUSION

The quasiparticle time blocking approximation (QTBA) is applied to describe E1 excitations in the even-even open-shell spherical nuclei. Within the QTBA pairing correlations, two-quasiparticle (2q), and 2q⊗phonon configurations are included. The model is based on the generalized Green function formalism in which the normal and the anomalous Green functions are treated in a unified way in terms of the components of generalized Green functions in a doubled space. To determine response of the spherically symmetric system against an external field within the QTBA the integral equation for the partial components of the linear response matrix in the coordinate representation has been obtained including coupling between particle-hole, particle-particle, and hole-hole channels. Configurations with a particle in the continuum are included into the QRPA part of the response function. This enables us to describe both spreading and escape widths of nuclear excited states. In our calculations we use phenomenological Woods-Saxon single-particle input and independently parametrized effective interaction of the Landau-Migdal form. So the additional procedure to eliminate spurious dipole mode has been formulated in terms of QTBA.

The developed method has been applied to calculate isovector E1 strength distribution
in nuclei $^{116,120,124}$Sn. The results for the photoabsorption cross sections in the indicated tin isotopes are presented. Since our main purpose was to test new approach, these first calculations have been performed assuming some additional simplifications of the model: ground state correlations caused by quasiparticle-phonon coupling and dynamical coupling to pp and hh channels were ignored. Nevertheless noticeably fragmented giant dipole resonances have been obtained for all three investigated nuclei. Calculated integral characteristics of the resonances are in a reasonable agreement with experimental values.

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References

[1] S. P. Kamerdzhiiev, G. Ya. Tertychny, and V. I. Tselyaev, Fiz. Elem. Chastits At. Yadra 28, 333 (1997) [ Phys. Part. Nucl. 28, 134 (1997) ].

[2] A. Bohr and B. Mottelson, Nuclear Structure, Vol. 2 (Benjamin, New York, 1975).

[3] V. G. Soloviev, Theory of Atomic Nuclei: Quasiparticles and Phonons (Institute of Physics, Bristol and Philadelphia, USA, 1992).

[4] G. F. Bertsch, P. F. Bortignon, R. A. Broglia, and C. H. Dasso, Phys. Lett. 80B, 161 (1979).

[5] P. F. Bortignon and R. A. Broglia, Nucl. Phys. A371, 405 (1981).

[6] G. Colò, P. F. Bortignon, Nguyen Van Giai, A. Bracco, and R. A. Broglia, Phys. Lett. 276B, 279 (1992).
[7] G. Colò and P. F. Bortignon, Nucl. Phys. A696, 427 (2001).

[8] S. P. Kamerdzhiiev and E. V. Litvinova, Yad. Fiz. 67, 180 (2004) [ Phys. At. Nucl. 67, 183 (2004) ].

[9] V. I. Tselyaev, arXiv:nucl-th/0505031.

[10] V. I. Tselyaev, Yad. Fiz. 50, 1252 (1989) [ Sov. J. Nucl. Phys. 50, 780 (1989) ].

[11] S. Krewald, V. B. Soubbotin, V. I. Tselyaev, and X. Viñas, arXiv:nucl-th/0412018.

[12] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).

[13] A. B. Migdal, Theory of Finite Fermi Systems and Applications to Atomic Nuclei (Interscience, New York, 1967).

[14] S. Kamerdzhiiev, R. J. Liotta, E. Litvinova, and V. I. Tselyaev, Phys. Rev. C 58, 172 (1998).

[15] S. Shlomo and G. Bertsch, Nucl. Phys. A243, 507 (1975).

[16] A. Bohr and B. Mottelson, Nuclear Structure, Vol. 1 (Benjamin, New York, 1969).

[17] V. I. Tselyaev, Izv. Ross. Akad. Nauk, Ser. Fiz. 64, 541 (2000) [ Bull. Russ. Acad. Sci., Phys. (USA) 64, 434 (2000) ].

[18] A. Leprêtre, H. Beil, R. Bergère, P. Carlos, A. De Miniac, A. Veyssi ère, and K. Kernbach, Nucl. Phys. A219, 39 (1974).

[19] S. C. Fultz, B. L. Berman, J. T. Caldwell, R. L. Bramblett, and M. A. Kelly, Phys. Rev. 186, 1255 (1969).