Evaluation of RSA based formula for critical responses and incidence angles of a concrete MRF subjected to ground motions scaled to different hazard levels

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Abstract. Seismic analysis of structures often requires applying a pair of ground motions simultaneously in the directions of the structures since an earthquake can excite with any incidence angle. Critical response and incident angle analysis therefore play a significant role in seismic structural design by predicting maximum structural responses. Existing procedures are often based on response spectrum analysis methods (RSA) to obtain critical angles and responses. The main objective of this study is to compare the critical incidence angles and responses obtained with response history analysis (RHA) and RSA for a concrete moment resisting frame (MRF). Unlike previous studies, this study considers the effect of different hazard levels by using different scaling methods for a total of fourteen selected pairs of recorded ground motions. Response history analyses of a nine-story concrete MRF are conducted under different seismic hazard levels. The studies demonstrate the significance of critical angle analysis as well as some discrepancy between RHA and RSA.

1. Introduction
Earthquakes can act along any horizontal direction and a well-designed structure should be able to resist an earthquake from all directions. Predicting which way an earthquake will strike or placing a structure in any other configuration other than the one proposed in the layout is out of the question since the configuration of a structure is usually determined even before a design has been made. Instead of taking into account the angle of incidence and adjusting for that, the best solution would be to account for the maximum response or part of the maximum response if it is not in the default direction.

Current designs of structures have yet to find a way to implement critical angle analysis. The current trend for seismic analysis of structures is to apply one ground motion in the x-direction of the structure and one ground motion in the y-direction. The default analysis direction is the 0 degree analysis of a structure. Code based design has not incorporated critical angles into seismic analysis for design purposes due to the lack of knowledge on the subject. Critical angle analysis is quite important despite not being in design procedures. Conducting critical angle analysis will further reinforce the design of the structure. In some cases, the maximum response may occur at the default analysis angle which means there will not be a change in the design but in other cases it can be 100% higher than the default analysis response.

The analysis of critical angles was first conducted in Three-Dimensional Dynamic Analysis for Multi-Component Earthquake Spectra by Wilson and Button [1]. The critical angle analysis procedure was improved to the most recent analysis procedures in The Critical Angle of Seismic Incidence and
the Maximum Structural Response by Lopez and Torres [2]. A comparison between the Response History Analysis and the Response Spectrum Analysis was analysed in Evaluation of Critical Responses and Critical Incidence Angles Obtained with RSA and RHA by Marinilli and Lopez [3].

The progression of response spectrum analysis research over the last 30 years has shown that not only the equations to determine critical response and angles need to be reworked but also the type of response used in the calculations. The first response used by Wilson and Button [1] was stress in elements, the second response used by Lopez and Torres [2] was torsion in the base of the structure and the third response used by Marinilli and Lopez [3] was axial force in the base of the structure. Wilson and Button [1] developed the first method which was then revised by Lopez and Torres [2] to the current method used in this paper and compared the response history analysis and response spectrum analysis with axial force as the response.

2. Properties of structures for analysis
The structure to be analysed is a 9-story concrete moment resisting frame located at 1455 Market Street in San Francisco, California. The structure is on top of class D soil in a high activity seismic zone. The structure is symmetrical with 5 bays in the east-west direction and 4 bays in the north-south direction. The structure contains 6 interior gravity columns and post-tensioned slabs and girders as the floor system as shown in Figures 1 and 2.

![Figure 1. Member for moment frames in E-W direction.](image1)

![Figure 2. Member for moment frames in S-N direction.](image2)

Perform-3D [4-6] is a structural analysis program created by Computer & Structures Inc. for nonlinear analysis and performance based design of structures. Perform-3D features a wide variety of elements and analysis methods but does not assist in the design of the elements. The design strengths and plastic rotations of the elements must be inputted manually which allows for easy customization for analysis. Parameters \(a\) and \(b\) define the strain hardening and plastic deformation of the curve. Parameter \(c\) defines the post strain hardening section of the curve that has reduced resistance as shown in Figure 3.

![Figure 3.](image3)
3. Ground motions

The earthquakes have a magnitude between 6 to 8 and a distance from epicenter between 20km to 100km. Selecting and scaling earthquakes are very important parts in the evaluation of a structure. Different earthquakes and different scaling methods cause the structure to respond differently. Table 1 lists the ground motions used in this study. American Society of Civil Engineers (ASCE) has multiple guidelines for the selection of earthquakes used for analysis and also provides a guide for scaling earthquakes. ASCE suggests using 5% damped response spectrums, a minimum of 7 pairs of ground motions and scaling each ground motion according to the *Seismic Loads: Guide to the Seismic Load Provisions of ASCE7-05*.

| Earthquake Name | Station                          | PGA (g) |
|-----------------|---------------------------------|---------|
| Aqaba           | Eilat                           | 0.097   |
| Aqaba           | Eilat                           | 0.086   |
| Cape Mendocino  | 89509 Eureka-Myrtle & West      | 0.154   |
| Cape Mendocino  | 89509 Eureka-Myrtle & West      | 0.178   |
| Chalfant Valley | 54214 Long Valley Dam           | 0.082   |
| Chalfant Valley | 54215 Long Valley Dam           | 0.074   |
| Chi Chi         | CHY054                          | 0.097   |
| Chi Chi         | CHY054                          | 0.094   |
| Coalinga        | 46314 Cantua Creek School       | 0.227   |
| Coalinga        | 46314 Cantua Creek School       | 0.281   |
| Imperial        | 5061 Calipatria Fire Station    | 0.078   |
| Imperial        | 5061 Calipatria Fire Station    | 0.128   |
| Kern County     | 1095 Taft Lincoln School        | 0.156   |
| Kern County     | 1095 Taft Lincoln School        | 0.178   |
| Kobe            | 0 TOT                           | 0.075   |
| Kobe            | 0 TOT                           | 0.076   |
| Kocaeli         | Goynuk                          | 0.132   |
| Kocaeli         | Goynuk                          | 0.119   |
| Loma Prieta     | 57064 Fremont                   | 0.124   |
| Loma Prieta     | 57064 Fremont                   | 0.106   |
| Morgan Hill     | 56012 Los Banos                 | 0.051   |
| Morgan Hill     | 56012 Los Banos                 | 0.057   |
| Northridge      | 90033 LA - Cypress Ave          | 0.21    |
| Northridge      | 90033 LA - Cypress Ave          | 0.149   |
| San Fernando    | 994 Gormon - Oso Pump Plant     | 0.084   |
Three scaling methods are used to scale the earthquakes in the analysis: ASCE 7’s scaling procedure, Geometric Mean Method and Distribution Scaling Method. The ASCE 7 method and the Geometric Mean method use similar procedures in determining scaling factors. These two methods use a target spectrum created by USGS hazard maps to scale each earthquake while the third method is based solely on probability. Each method provides different scale factors to provide us with a decent understanding of how different scale factors affect the response history analysis and the critical angles.

4. Response history analysis results
Response history analysis for 3D structures is conducted by applying a ground motion in both the x and y direction. For this structure, the ground motion with the larger peak acceleration is applied in the x direction. The structure is analyzed using 3 scaling methods for 14 pairs of ground motions from 0º to 180º at 10º intervals. The structure is only analyzed for 180º because of symmetry. A total of 756 response history analyses were conducted.

The roof drift, the drift of the structure from base to the roof, will be the main response used to determine the critical angles of the structure. The x, y and r direction roof drifts will be analyzed for the structure. The r-direction roof drift is the square root sum of squares of the x and y drifts. The r-direction drift will produce the highest values of roof drift but is not the most important in the design of the frames. The values for the r-direction are only used to determine the critical angles that are produced from combining both the x and y components of the response. The evaluation of the structure is not based off the r-direction response.

5. Response spectrum analysis results
The response spectrum analysis method for determining critical angles is still currently in development and has yet to see results accurate enough for use in practical design and analysis. If response spectrum analysis was accurate, it would overcome the cons of response history analysis. Response spectrum analysis would speed up the process of analyzing a structure for its critical angles significantly, making it possible to include in code design. Figure 4 illustrates geometrix mean scaled average response spectrums.

The procedure to determine the critical angle and critical response of a structure is:
1. Apply spectrum 1 (Sa1) along the X direction of the structure and calculate the peak dynamic response which will be called R1x.
2. Apply Spectrum 1 (Sa1) along the Y direction of the structure and calculate R1y.
3. Apply spectrum 2 (Sa2) along the X direction of the structure and calculate the peak dynamic response which will be called R2x.
4. Apply Spectrum 2 (Sa2) along the Y direction of the structure and calculate R2y.
5. Apply Spectrum 3 (Sa3) along the Z direction to obtain R3.

The different modal values of $R^{1x}$, $R^{1y}$, $R^{2x}$ and $R^{2y}$ are combined using the Complete Quadratic Combination (CQC) method. The $C_{ij}$ value is the coefficient obtained from the CQC method for combining the different modal components. The CQC method was created to avoid the numerical errors that come with the Square Root Sum of Squares method used for modal combinations.
where, $C_{ij}$ is the CQC Coefficient for modal combination; $r$ is the Angular frequency ratio between modes; and $\zeta$ is the damping ratio between modes. The modal components are combined for x and y separately

$$R^{1x}_i = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{1x} R_{j}^{1x} \right]^{1/2}$$ (2)

$$R^{1y}_i = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{1y} R_{j}^{1y} \right]^{1/2}$$ (3)

$$R^{2x}_i = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{2x} R_{j}^{2x} \right]^{1/2}$$ (4)

$$R^{2y}_i = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{2y} R_{j}^{2y} \right]^{1/2}$$ (5)

The x and y components for each mode are combined separately via the CQC coefficient for modal combination.

$$R^{1}_i = R^{1x}_i \cos \theta + R^{1y}_i \sin \theta$$ (6)

$$R^{2}_i = R^{2x}_i \cos \theta - R^{2y}_i \sin \theta$$ (7)

The combined x and y components for each mode are then combined again to produce the overall response from response spectrum 1, 2 and 3.

$$R^1 = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{1} R_{j}^{1} \right]^{1/2}$$ (8)

$$R^2 = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{2} R_{j}^{2} \right]^{1/2}$$ (9)

$$R^3 = \left[ \sum_{j} \sum_{i} C_{ij} R_{i}^{3} R_{j}^{3} \right]^{1/2}$$ (10)

These values are plugged into the critical angle equation to determine the critical angle. The critical angle is independent of the third response spectrum which is applied in the vertical direction of the structure since it has no effect on the horizontal responses. The critical angle is calculated using the following equation and the opposite whether minimum or maximum is 90 degrees from the calculated angle.

![Figure 4. Geometric mean scaled average response spectrums.](image.png)
The maximum response equation is calculated by inputting the critical angle in the equation. The response at any other angle may also be calculated by inputting any angle in the equation.

\[
\theta_{cr} = \frac{\sum_i \sum_j C_{ij} \left[ R_{ij}^{2y} R_{ij}^{2x} - R_{ij}^{1x} R_{ij}^{1y} \right]}{\left( R_{ij}^{2y} \right)^2 + \left( R_{ij}^{2x} \right)^2 - \left( R_{ij}^{1x} \right)^2 - \left( R_{ij}^{2y} \right)^2}
\]

The maximum response equation is calculated by inputting the critical angle in the equation. The response at any other angle may also be calculated by inputting any angle in the equation.

\[
R(\theta) = \left\{ \left[ (R_{ix}^{1x})^2 + (R_{ix}^{2y})^2 \right] \cos^2 \theta + \left[ (R_{iy}^{1y})^2 + (R_{iy}^{2x})^2 \right] \sin^2 \theta + 2 \sin \theta \cos \theta \left( \sum_i \sum_j C_{ij} R_{ij}^{2x} R_{ij}^{1y} - \sum_i \sum_j C_{ij} R_{ij}^{2x} R_{ij}^{2y} \right) + (R^2)^2 \right\}^{1/2}
\]

The response spectrum analysis was conducted using the ASCE7 scale factors with procedures provided in Lopez and Torres [2]. The number of modes used for the response spectrum analysis was 3 because the mass factors exceeded 90% for each orthogonal direction as per ASCE 7-10 12.9.1. The third mode is a torsion mode to show that there is no torsion in the structure. The modal contribution factor also known as the mass participation factor shows how much of the total mass is participating in a mode. Table 2 lists the maximum and minimum drift and corresponding angle. In this case, 93.58% of the mass participates in the 1st mode for the y-direction and 94.16% of the mass participates in the 2nd mode for the x-direction. A larger number of modes may be used to account for closer to 100% of the mass but was not used.

| Earthquake     | Roof Drift vs. Critical Angle | Max (°) | Min (°) |
|----------------|-----------------------------|--------|--------|
| Aqaba          |                             | 154.15 | 3.84   |
| Cape           |                             | 64.15  | 3.692  |
| Mendocino      |                             | 0.056  | 877    |
| Chalfant Valley|                             | 0.906  | 1.880  |
| Chi Chi        |                             | 137.34 | 47.34  |
| Coalinga       |                             | 1.66   | 0.853  |
| Imperial       |                             | 47.34  | 8.18   |
| Kern County    |                             | 137.34 | 47.34  |
| Kobe           |                             | 137.29 | 47.34  |
| Kocaeli        |                             | 137.34 | 47.34  |
| Loma Prieta    |                             | 137.34 | 47.34  |
| Morgan Hill    |                             | 137.34 | 47.34  |
| Northridge     |                             | 137.34 | 47.34  |
| San Fernando   |                             | 137.34 | 47.34  |
| Superstition   |                             | 137.34 | 47.34  |
| Hills          |                             | 137.34 | 4.54   |

6. Comparison of RHA and RSA results
The results obtained from the response spectrum analysis methods are consistent for most earthquakes. The minimums and maximums occur at the same area for both responses used for the analysis method. The angles differ by 2°-3° for the two responses but overall they are consistent. The problem with the consistency is that the critical angles for the response history analysis are not all in the same areas. Although the responses behave like sine waves, it does not have minimums and maximums at the same place as the response spectrum analysis. The response history analyses are considered more accurate so accuracy will be based off how close the RSA is to the RHA. The difference between the two methods is shown for each scaling method and an average of all the scaling methods in Table 3.
The difference in angles between the two methods is large. These results show that the response spectrum analysis method for analyzing critical angles is inaccurate. A difference of 32° is high and it can go as high as 83°. The RSA cannot be used for the analysis of critical angles until it has been improved for accuracy.

### Table 3. Comparison of critical angles.

| Angles    | ASCE 7 (°) | Geometric Mean (°) | D-Scaling (°) | Average (°) |
|-----------|------------|--------------------|---------------|-------------|
| x-direction | 46.24      | 43.74              | 32.49         | 40.82       |
| y-direction | 70.41      | 49.17              | 67.86         | 62.48       |
| r-direction | 59.17      | 46.67              | 83.51         | 63.12       |

### 7. Summary and conclusions

The objective of this research was to evaluate critical angles for performance based earthquake engineering. The critical angle analysis methods were evaluated along with the behavior of critical angles when the earthquakes are scaled using different methods. The results from this research show the actual maximum responses of our structure frequently occur outside the default analysis angle. To properly analyze a structure’s performance we need to not only address the performance objective and seismic hazard level but cover all angles of incidence to ensure we are account for the maximum response. An earthquake can hit a structure at any angle and the response from the critical angle can exceed the response of the default analysis angle by over 100%.

A comparison between the response history analysis and response spectrum analysis results for critical angles showed that the response spectrum analysis results cannot properly predict the critical angles in a structure. However, the method does predict some features of the response correctly. It correctly predicts a sine wave type response graph and that a minimum or maximum will occur at 90° from another minimum or maximum.

### References

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