Instanton Effects in Hadron Spectroscopy Revisited

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We use an optimised clover action to study spectroscopy on an instanton ensemble reconstructed from smoothed Monte Carlo configurations. Due to the better chirality of the clover action, the artificial configurations show a marked difference from the free field behaviour obtained with the Wilson action. They however still fail to reproduce the physics observed on the smoothed configurations. The presence of freely propagating quark modes is found to be responsible for this.

According to the instanton liquid model most of the low-energy properties of QCD can be explained by instantons [1]. This however has never been fully tested starting from first principles, in particular on the lattice. In a previous study hadron spectroscopy was performed on instanton ensembles reconstructed from smoothed Monte Carlo configurations [2]. The instanton ensemble was found to exhibit chiral symmetry breaking but the lightest states in the pion and rho channel remained degenerate down to small quark masses; a feature typical of free field theory (no gluons).

Now we would like to shed some more light on the origin of this peculiar admixture of “free field like” and “QCD like” properties of the instanton ensemble. We shall compare the following two ensembles of gauge configurations:

• Quenched Monte Carlo gauge configurations produced with the fixed point action of [2] at a lattice spacing of $a = 0.144$fm and then cycled (smoothed with a renormalisation group based procedure) 9 times, up to the point where instantons could be reliably identified.

• Artificially constructed instanton configurations reproducing the instanton content (location and size but not the relative orientation in internal space) of the above (for more details on the construction, see Ref. [2]).

We shall refer to the first (smoothed) and the second (instanton) ensemble as SM and IN respectively. The SM ensemble is so smooth that 70% of its action is carried by the instantons (assuming no interaction between them), still it has been shown to contain all the relevant long distance features of QCD.

An important feature of the instanton liquid model is the presence of near zero quark eigen-modes that are approximately linear combinations of instanton and antiinstanton zero modes. Their formation is possible only if the zero modes are (at least nearly) degenerate. Only then can small perturbations — e.g. gauge field fluctuations — mix them. This is not the case with the Wilson action. Since it breaks chiral symmetry explicitly, the would be fermion zero modes spread in an interval of the real axis.

To check how important the chirality of the fermion action is, we repeated the spectroscopy with the clover action ($c_{sw}=1.2$) optimised to produce “zero modes” in a narrow range around zero [3]. Fig. 1 shows the result for $m_\pi$ vs. $m_\rho$. This markedly differs from the $m_\pi = m_\rho$ free field line (also shown) that was obtained with the Wilson action. This clearly shows the importance of the chiral symmetry of the fermion action. On the other hand, the $\pi - \rho$ splitting on the IN ensemble still fails to reproduce that on the SM ensemble. In view of the good chirality of the optimised clover action, it is hard to imagine that this can be fully attributed to the remaining explicit chi-

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Figure 1. $m_\rho$ versus $m_\pi$ obtained with the optimised clover action on the instanton ensemble (crosses) and on the smoothed ensemble (boxes). The $m_\pi = m_\rho$ line is the free field result, given also by the Wilson action on the SM ensemble.

Figure 2. $m_\pi^2$ versus the bare quark mass, $m_q$, for the optimised clover action on the instanton ensemble (crosses) and on the smoothed ensemble (boxes). The line is the best fit to the SM data.

Field modes from the lowest topological modes and makes the computation faster. For orientation, in Fig. 2 we plotted the eigenvalues that we used from both ensembles, together with the location of the lowest antiperiodic free field mode.

The mixing of the free field modes into the eigenmodes of SM and IN can be characterised by $P_{\pm}(\lambda) = \|P_\pm \psi_\lambda\|$, where $P_\pm$ is the projection of the normalised eigenmode $\psi_\lambda$ onto the eigenspace corresponding to the eigenvalue $\mu_\pm$. This quantity is not gauge invariant therefore we work in Lorentz gauge. Any generic fermion mode on a non-flat but locally smooth configuration will have a nonzero projection on the $\mu_\pm$ eigenspaces. The question is whether this is just an accidental mixing or the given ensemble really contains close to free field modes. To decide this, a good quantity to look at is $P(\lambda) = P_+(\lambda)/P_-(\lambda)$. If the mixing is accidental, we expect $P(\lambda)$ to fluctuate around 1, independently of the corresponding eigenvalue $\lambda$. On the other hand, if there are free field like modes on a given configuration then $P(\lambda)$ will increase substantially when $\lambda$ approaches the value $\mu_+$.

In Fig. 4 we plot $P(\lambda)$ versus the distance of the given eigenvalue from the free field mode $\mu_+$. In the SM ensemble $P(\lambda)$ fluctuates around 1 ev-
Figure 3. The eigenvalues of the optimised clover Dirac operator on the smoothed (diamonds) and the instanton (boxes) ensemble. The cross shows the lowest antiperiodic free field mode.

everywhere, there is no trace of the free field mode. On the other hand, in the IN ensemble, the modes close to $\mu_+$ have a substantially larger projection on the $\mu_+$ eigenspace than on the $\mu_-$ subspace. In fact, these modes, close to $\mu_+$ have $\|P_+(\lambda)\| \approx 1$, so they are essentially free field modes. A detailed study of the chiral density $\psi_+^\dagger \gamma_5 \psi_\lambda$ reveals that all the $P(\lambda) \approx 1$ modes look like mixtures of instanton zero modes with the density concentrated in several lumps. This is to be contrasted with the chiral density of the modes with $P(\lambda) \gg 1$ that spreads roughly homogenously over the whole lattice, as expected of the lowest free field eigenmode.

In conclusion, we found that the instanton ensemble reconstructed from smoothed Monte Carlo configurations shows a peculiar mixture of free field and interacting QCD like behaviour. This is a consequence of the presence of both types of quark eigenmodes in this ensemble. The scattering of quarks by instantons is apparently not enough to eliminate free propagation completely. In contrast, we have not found any trace of the free field modes on the smoothed configurations. This is presumably due to some long wavelength fluctuations also responsible for confinement which is absent from the IN ensemble.

Finally we note that the density of modes around zero goes as $\propto V^{1/4}$ in free field theory, whereas the density of instanton modes in the instanton ensemble is $\propto V$, the lattice volume. It is thus conceivable that in the absence of confinement, there are always some free field modes which contribute to quark propagation but in the $V \to \infty$ limit they might be overwhelmed by the instanton modes. It would be interesting — although probably not very cheap — to test this directly on the lattice. The other possibility is that instantons alone might not be enough to reproduce low energy QCD, confinement is also needed. In this case it is confinement that eliminates free quark propagation and makes the hopping of quarks from instanton to instanton the dominant long distance mode of propagation.

REFERENCES

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