User Preference Learning Based Edge Caching for Fog-RAN

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Abstract

In this paper, the edge caching problem in fog radio access networks (F-RAN) is investigated. By maximizing the overall cache hit rate, the edge caching optimization problem is formulated to find the optimal edge caching policy. We consider content popularity in terms of time and space from the perspective of regional users. Taking into account that users request the contents they are interested in, we propose an online content popularity prediction algorithm by leveraging the content features and user preferences, and an offline user preference learning algorithm by using the “Online Gradient Descent” (OGD) method and the “Follow The (Proximally) Regularized Leader” (FTRL-Proximal) method. Our proposed edge caching policy not only can promptly predict the future content popularity in an online fashion with low computational complexity, but it also can track the content popularity with spatial and temporal popularity dynamics in time without delay. We theoretically derive the upper bound of the popularity prediction error, the lower bound of the cache hit rate, and the regret bound of the overall cache hit rate of our proposed edge caching policy. Furthermore, to implement our proposed edge caching policy, we design two learning based edge caching architectures for F-RAN, which have the capability of flexibly setting the monitoring cycle and is effective in various edge caching scenarios. Simulation results show that the overall cache hit rate of our proposed policy is superior to those of the traditional policies and asymptotically approaches the optimal performance.

Index Terms

Fog-RAN, edge caching, cache hit rate, content popularity, user preference.

I. INTRODUCTION

With the continuous and rapid proliferation of various intelligent devices and advanced mobile application services, wireless networks have been suffering from an unprecedented data traffic surge in
recent years. Although centralized cloud caching and computing in cloud radio access networks (C-RAN) [1] or heterogeneous C-RAN (H-CRAN) [2] can provide reliable and stable service for end users during off-peak periods, ever-increasing mobile data traffic brings tremendous pressure on C-RAN or H-CRAN with capacity-limited fronthaul links and centralized baseband unit (BBU) pool, which may cause communication interruptions or traffic congestions especially at peak traffic moments. The main reason is that as location-based social applications become more and more popular, data traffic over fronthaul links surges with a lot of redundant and repeated information, which further worsens the fronthaul constraints. In this case, a feasible solution is to shift a small amount of resources such as communications, computing, and caching to network edge, and enable most of the frequently requested contents being served locally. At this point, fog radio access networks (F-RAN) as a complementary network architecture was proposed, which can effectively reduce fronthaul load by placing most popular contents as close as possible to the requesting users and extending traditional cloud computing paradigm to the network edge [3]–[5]. Up until now, F-RAN has attracted more and more attention from researchers and engineers. In F-RAN, traditional access points are turned into fog access points (F-APs) equipped with limited caching and computing resources, where edge caching is a key component to improve the performance of F-RAN. Due to storage constraints and fluctuating spatio-temporal traffic demands, however, F-APs face a myriad of challenges. For example, how, what and when to strategically store contents in their local caches in order to achieve a higher cache hit rate?

Traditional caching policies such as first in first out (FIFO) [6], least recently used (LRU) [7], least frequently used (LFU) [8] and their variants [9] have been widely applied in wired networks, where there are abundant caching and computing resources and the served area is usually very large. However, these traditional caching policies may not be applied well in wireless networks due to the characteristics of edge nodes such as smaller coverage areas and limited caching resources, and they may suffer major performance degradation since they may not be able to predict future content popularity correctly. Most of the existing works on edge caching in wireless networks assumed that the content popularity is already known or subject to a uniform distribution, and then explored the optimal caching policy under this assumption [10]–[12]. The edge caching problem was formulated as a many-to-many matching game between small base stations and service providers’ servers in [10], it is converted to an approximation facility location problem in [11], and successful transmission probability was maximized to obtain a local optimal caching and multicasting design in a general region in [12]. However, content popularity is
unknown and varies with time and space in reality.

By considering the time-varying nature of contents, recent works have turned to exploring sophisticated edge caching policies by learning future content popularity. In [13], the content popularity matrix was estimated through transfer learning by leveraging user-content correlation and information transfer between time periods. Nevertheless, the content popularity matrix remains typically to be large, which needs a great deal of calculation in the estimation. Meanwhile, the transfer learning approach has a poor performance for the case of low information correspondence ratio. In [14], the cache content placement problem was modeled as a contextual multi-arm bandit problem and an online policy was presented to learn the content popularity. This policy learns the content popularity independently across contents whereas ignores the content similarity and impact of user preference on content popularity, thereby resulting in high training complexity and slow learning speed. A low complexity online policy was proposed in [15], where content popularity was learned based on the assumption that the expected popularities of similar contents are similar. It performs well for video caching but may be ineffective for other types of content caching. However, all the above studies assume that the content popularity remains unchanged for a certain time period, and ignore to consider spatial changes of content popularity, and therefore cannot truly reflect the changes of content popularity. In real communications scenarios, both the coverage area of an edge node and the number of users that it can serve are limited. Correspondingly, the change of global content popularity for all the regions may not be consistent with that of local content popularity for a certain region. In order to maximize the cache hit rate to the greatest extent, learning based edge caching policies which can track real changes of local content popularity need to be studied.

Motivated by the aforementioned discussions, our main contributions are summarized below.

• We put forward a new idea of content popularity prediction. Unlike the static approach, we consider content popularity in terms of time and space from the perspective of regional users and propose an online content popularity prediction algorithm, which can predict the future content popularity of a certain region in an online fashion without any restriction on content types and track the popularity change in real time.

• We propose an offline user preference learning algorithm, which can discover the user’s own preference through its historically requested information. By monitoring the average prediction error in real time, it can be initiated automatically for relearning of user preference and continuous offline learning can thus be avoided.
• We analyze the performance of our proposed edge caching policy. We first derive the upper bound of the popularity prediction error of our proposed online content popularity prediction algorithm, and reveal the sub-linear relationship between the cumulative prediction error and the total number of content requests. We then derive the regret bound of the overall cache hit rate of our proposed edge caching policy, and show through theoretical analysis that our proposed policy has the capability to achieve the optimal performance asymptotically.

• We design two learning based edge caching architectures for F-RAN. They differ in that the offline user preference learning functionality is transferred from the F-APs in the first architecture to the smart user equipments (UEs) in the second one. Specifically, we introduce cooperative caching among regional F-APs and make more efficient usage of limited caching resources. Besides, we introduce a new module to enable regular monitoring of regional users by considering the impact of user mobility on cache decisions.

• We validate our theoretical results with real data. Simulation results show that our proposed edge caching policy can predict the content popularity with high precision, and real-timely track the real local popularity with spatial and temporal popularity dynamics. Simulation results also show the superior performance of our proposed policy in comparison with the other traditional policies and verify its asymptotical optimality.

The rest of this paper is organized as follows. In section II, the system model including the edge caching scenarios in F-RAN and the edge caching problem formulation is described. Our proposed edge caching policy including online content popularity prediction and offline user preference learning is presented in Section III. The performance analysis of our proposed edge caching policy is provided in Section IV. The two learning based edge caching architectures are described in Section V. Simulation results are shown in Section VI. Final conclusions are drawn in Section VII.

II. SYSTEM MODEL

A. Edge Caching Scenarios in F-RAN

We consider the edge caching scenarios in F-RAN as illustrated in Fig. [1]. Large amounts of F-APs with limited storage capacity are deployed in different scenarios, for example, a town with dense crowds and moderate user mobility, a park with relatively dense crowds and low user mobility, or a stadium with ultra-dense crowds and relatively low user mobility. In each edge caching scenario, according to the
corresponding design criteria (for example, location), the F-APs with close distance can cooperate and belong to the same region, which are also called regional F-APs. The users in the same region, also called regional users, can request contents of interest from their associated regional F-APs if the contents are stored in the local caches or from neighboring F-APs in the other regions or the cloud content center through fronthaul links otherwise [16].

In practice, different users may have different degrees of interest, i.e., different user preferences, in the same content. Correspondingly, the request possibilities for the same content among different regions gathered by users with different user preferences are different. This will result in a content popularity difference for the same content among different regions. In addition, the set of current users in a specific region may change dynamically over time due to user mobility, and the regional content popularity may thus be fluctuating. Taking account of spatial and temporal dynamics of content popularity, we propose the following edge caching design rule: The F-APs deployed in different regions should regularly monitor the regional users in consideration of user mobility, and set a monitoring cycle matched with the characteristics of different regions.

B. Edge Caching Problem Formulation

We consider the edge caching problem in a specific region served by $M$ F-APs, which constitute an F-AP set $\mathcal{M} = \{1, 2, \cdots, m, \cdots, M\}$. It is assumed that the regional users can fetch the contents of interest from the local caches of the $M$ F-APs. Without loss of generality, we assume that all the contents have the same size and each F-AP has the same storage capacity and can store up to $\varphi$ contents from the content library $\mathcal{F} = \{1, 2, \cdots, f, \cdots, F\}$ which may vary over time and is located in the cloud content center. By considering user mobility, the F-APs monitor the users in the specific region during discrete
time periods $t = 1, 2, \cdots, T$, where $T$ is set to be a finite time horizon. Let $U_t$ denote the number of the regional users served by the $M$ F-APs during the $t$th time period with $U_{\text{min}} \leq U_t \leq U_{\text{max}}$, and $U_t = \{1, 2, \cdots, U_t\}$ the set of regional users monitored by the $M$ F-APs. It is assumed that the regional users remain unchanged during the considered time period. In the way of cache content placement, for description convenience, we adopt the partition-based caching content placement method in [17], where each content is separated into $M$ equal-sized subfiles, and the F-APs store its different subfiles. We remark here that more sophisticated caching content placement methods can be adopted.

Let $D_t$ denote the number of the requests during the $t$th time period, $D_t = \sum_{t=1}^{T} D_t$ the overall number of requests in the finite time horizon $T$, and $\text{req}_t = \{\text{req}_{t,1}, \text{req}_{t,2}, \cdots, \text{req}_{t,d}, \cdots, \text{req}_{t,D_t}\}$ the set of requests which come in sequence during the $t$th time period. The request $\text{req}_{t,d}$ can be further expressed as $\text{req}_{t,d} = \langle f(d), t(d), x(d) \rangle$, $\forall 1 \leq t \leq T, \forall 1 \leq d \leq D_t$, where $f(d) \in \mathcal{F}$ denotes the requested content, $t(d)$ the requesting time, $x(d) \in \mathbb{R}^N$ the feature vector with dimension $N$ describing the features of the requested content. Take movie as an example: $x(d)$ may include features like the movie rating, the movie type, the keyword frequency of movie critics, etc. Without loss of generality, we normalize the various dimensions of $x(d)$ and set $x(d) \in [0, 1]^N$.

During the $t$th time period, for each arriving request $\text{req}_{t,d}$, the regional F-APs first check whether $f(d)$ has been stored locally. Let $\theta_{t,d}(f(d)) \in \{0, 1\}$ denote the cache status of $f(d)$ at the requesting time $t(d)$, where $\theta_{t,d}(f(d)) = 1$ if $f(d)$ is stored locally, and $\theta_{t,d}(f(d)) = 0$ otherwise. If $f(d)$ has been stored in the local caches, a cache hit happens and the requesting user can then be served locally. Otherwise, a cache miss happens and $f(d)$ will be fetched from neighboring F-APs in the other regions or the cloud content center, and a caching decision will be further made to determine whether to store $f(d)$ locally. If the F-APs decide to store $f(d)$ and replace one of the existing contents in the local caches, denoted by $f_{\text{old}} \in \{f | \theta_{t,d}(f) = 1, \forall f \in \mathcal{F}\}$, a cache update happens. Then, the cache status can be updated according to the following rule,

$$
\theta_{t+1,d}(f) = \begin{cases} 
0, & f = f_{\text{old}}, \\
1, & f = f(d), \\
\theta_{t,d}(f), & f \in \mathcal{F} \setminus \{f(d), f_{\text{old}}\}.
\end{cases}
$$

In addition, a caching decision may be made that $f(d)$ not to be stored, and the cache status remains unchanged, i.e., $\theta_{t+1,d}(f) = \theta_{t,d}(f), \forall f \in \mathcal{F}$.

For convenience, we use $\theta_{t,d} = [\theta_{t,d}(1), \theta_{t,d}(2), \cdots, \theta_{t,d}(f), \cdots, \theta_{t,d}(F)]^T$ to indicate the cache
status of all the contents at the requesting time \( t(d) \). Generally, an edge caching policy can be represented by a function \( \Phi : (\theta_{t,d}, x(d), \mathcal{U}_t) \mapsto \theta_{t,d+1} \), which maps the current cache status vector, the current feature vector, and the set of regional users to the next cache status vector. After a request \( \text{req}_{t,d} \) is served, the cache status vector should be updated according to the edge caching policy. We use the overall cache hit rate to evaluate the caching performance, which is defined as the number of cache hits over the whole requests during the finite time horizon \( T \) as follows

\[
\mathcal{H}(\Phi) = \frac{1}{D} \sum_{t=1}^{T} \sum_{d=1}^{D_t} \theta_{t,d}(f(d)). \tag{2}
\]

Then, the corresponding edge caching optimization problem can be expressed as follows [18]

\[
\max_{\Phi} \mathcal{H}(\Phi), \tag{3}
\]

\[
\text{s.t. } \theta_{t,d}(f) \in \{0, 1\}, \text{ for } 1 \leq d \leq D_t, 1 \leq t \leq T, \forall f \in \mathcal{F},
\]

\[
\theta^T_{t,d} \theta_{t,d} \leq M\varphi, \text{ for } 1 \leq d \leq D_t, 1 \leq t \leq T.
\]

Our objective in this paper is to find the optimal edge caching policy by maximizing the overall cache hit rate over the finite time horizon \( T \) with the limited total cache size \( M\varphi \).

For convenience, a summary of major notations is presented in Table I.

### III. THE PROPOSED USER PREFERENCE LEARNING BASED EDGE CACHING POLICY

In order to maximize the cache hit rate, we propose a novel edge caching policy which includes an online content popularity prediction algorithm and an offline user preference learning algorithm. The proposed policy can continuously cache popular contents based on the content features and user preferences.

#### A. Policy Description

The detailed edge caching policy is shown in Algorithm 1. The considered \( M \) F-APs serving in the region set a fixed monitoring period and periodically monitor the current user set in the region. During the \( t \)th time period, the \( M \) F-APs first obtain the current user set \( \mathcal{U}_t \). For each arriving request \( \text{req}_{t,d} \) from the user \( u \in \mathcal{U}_t \), its request information will then be recorded. Meanwhile, the features of \( f(d) \) are extracted and recorded. The recorded data will be used to train or update the user preference model in order to improve the prediction precision of the content popularity. Let \( \mathcal{G}_{t,d} = \{f|\theta_{t,d}(f) = 1, \forall f \in \mathcal{F}\} \) denote the set of current contents in the local cache. It will be explored to see whether \( f(d) \) has already been
| Notation | Description |
|----------|-------------|
| $M, M$ | Number of regional F-APs, set of the $M$ regional F-APs |
| $t, T$ | Discrete time periods, finite time horizon |
| $\varphi$ | Cache size of each F-AP |
| $U_t, U_{\text{max}}, U_{\text{min}}, U_t$ | Number of regional users during the $t$th time period, maximum $U_t$, minimum $U_t$, set of the $U_t$ regional users |
| $D_t, D$ | Number of requests during the $t$th time period, number of overall requests in the finite time horizon $T$ |
| $\text{req}_t, \text{req}_{t,d}$ | Set of requests coming in sequence during the $t$th time period, the $d$th request during the $t$th time period |
| $f, f(d), f_{\text{least}}$ | Content, the $d$th requested content, content to be removed, content with the smallest popularity in the current local cache |
| $t(d), t_f, t_{\text{least}}$ | The $d$th requesting time, initial caching time of the content $f$, initial caching time of the content $f_{\text{least}}$ |
| $G_{t,d}, P_{t,f}^{\text{cur}}, P_{\text{least}}$ | Set of current contents in the local cache at the requesting time $t(d)$, current popularity of the caching content $f \in G_{t,d}$ after it is requested, the smallest content popularity |
| $Q_{t,d}$ | Priority queue that stores the caching contents and information sequentially |
| $\mathcal{F}, \Phi$ | Edge caching policy, the optimal edge caching policy |
| $\mathcal{H}_{t}(\Phi), \mathcal{H}_{t}(\Phi^*)$ | Cache hit rate of $\Phi$ during the $t$th time period, the optimal cache hit rate during the $t$th time period |
| $\mathcal{H}(\Phi), \mathcal{H}(\Phi^*)$ | Overall cache hit rate of $\Phi$ in the finite time horizon $T$, the optimal overall cache hit rate in the finite time horizon $T$ |
| $w_u, w_u^{(k)}$ | Vector of user preference model parameters of the user $u$, vector of user preference model parameters of the user $u$ for the $k$th iteration |
| $\hat{p}_{t,u,d}, p_{t,u,d}$ | Predicted possibility that the user $u$ requests $f(d)$ at $t(d)$ during the $t$th time period, real possibility that the user $u$ requests $f(d)$ at $t(d)$ during the $t$th time period |
| $P_{t,d}, P_{t,d}^{\text{cur}}$ | Predicted popularity of $f(d)$ at $t(d)$, real popularity of $f(d)$ at $t(d)$ |
| $y(d), y^{(k)}$ | Category label of the $d$th requested content, category label of the $k$th sample |
| $K_{t,u,d}, K$ | Cumulative number of samples for the user $u$ from the last model update to the time when the request $\text{req}_{t,d}$ arrives, number of collected samples for offline user preference learning |
| $\ell(w_u, x(d), y(d)), \xi_{t,u,d}$ | Logistic loss for the user $u$ at $t(d)$, average logistic loss for the user $u$ at $t(d)$ |
| $\gamma, \eta^{(k)}$ | Predefined threshold, non-increasing learning-rate schedule |
| $g^{(k)}, g^{(1:k)}$ | Gradient vector of the logistic loss of the $k$th sample with respect to $w_u$, sum of the gradient vectors of the logistic loss of the previous $k$ samples |

In order to make an optimal caching decision, the feature vector $x(d)$ and the vectors of well-trained user preference model parameters of all the users $\{w_u | \forall u \in \mathcal{U}_t\}$ are extracted to predict the popularity $P_{t,d}$ of the requested content $f(d)$. In addition, considering that the content popularity will change over time, in order to track popularity changes, we let $P_{t,f}^{\text{cur}}$ denote the current popularity of the caching content $f \in G_{t,d}$ after it is requested (also called the residual request rate). We know that users may have a certain request delay on the same content. In order to ensure timely and reasonable cache updating, we propose to select the content with the characteristics of the smallest content popularity $P_{\text{least}}$ and relatively earlier initial cache time $t_{\text{fleast}}$, denoted by $f_{\text{least}}$, as the content to be removed from the local caches. In order to locate $f_{\text{least}}$ quickly, we propose to reserve a priority queue $Q_{t,d}$ that stores the caching contents along
Algorithm 1 The Proposed Edge Caching Policy

1: procedure EDGE_CACHING(req_t,d)
2: for t = 1, 2, · · · , T, do
3: The considered M F-APs monitor $U_t$;
4: for d = 1, 2, · · · , D_t, do
5: Record the request information of req_t,d;
6: Read the set of the current caching content $G_{t,d}$;
7: if $f(d)$ has been stored locally, then
8: The users are served locally;
9: $P_{cur}^{t,f(d)} = P_{cur}^{t,f(d)} - 1/U_t$;
10: else
11: Fetch $f(d)$ from the cloud content center or the neighboring F-APs in the other regions;
12: Extract $x(d)$ and $\{w_u | \forall u \in U_t\}$;
13: $\hat{P}_{t,d} \leftarrow$ Predict ($x(d), \{w_u | \forall u \in U_t\}$);
14: Sort $Q_{t,d}$ based on $P_{cur}^{t,f}$ and $t_f$ for $f \in G_{t,d}$;
15: Get $P_{\text{least}}$, $f_{\text{least}}$ from the top of $Q_{t,d}$;
16: if $\hat{P}_{t,d} > P_{\text{least}}$ then \hspace{1cm} \triangleright 
\hspace{1cm} Cache update
17: Remove the top element from $Q_{t,d}$;
18: $t_{f(d)} = t(d)$; $P_{cur}^{t,f(d)} = \hat{P}_{t,d} - 1/U_t$;
19: Insert ($P_{cur}^{t,f(d)}, f(d), t_f$) into $Q_{t,d}$;
20: Replace $f_{\text{least}}$ by $f(d)$.
21: end if
22: end if
23: end for
24: end for
25: end procedure

with their current content popularity $P_{cur}^{t,f}$ and their initial caching time $t_f$ for $f \in G_{t,d}$. The elements of $Q_{t,d}$ are sorted in sequence when the request $req_t,d$ arrives, whose top element is composed of $f_{\text{least}}$, $t_{f_{\text{least}}}$ and $P_{\text{least}}$. A caching decision is made by comparing the predictive popularity $\hat{P}_{t,d}$ and $P_{\text{least}}$. If $\hat{P}_{t,d}$ is larger than $P_{\text{least}}$, the existing content $f_{\text{least}}$ will be replaced by $f(d)$, the initial caching time of $f(d)$ will be recorded, and the current popularity of $f(d)$ and the priority queue $Q_{t,d}$ will be updated accordingly. After that, a cache update process is completed. Otherwise, nothing will be done to the local caches. The key here, obviously, is to obtain $\hat{P}_{t,d}$, which will be described in the next subsection.

B. Online Content Popularity Prediction

In this subsection, we propose an online content popularity prediction algorithm based on the content features and user preferences. During the $t$th time period, for each requesting user $u \in U_t$, the requested content can be classified into a favorite category or an unfavorite one for this user based on its user preference. Generally, a user prefers to request contents of its favorite category. The problem of whether a user will request a certain content can be converted into a simple two-category one. We use the sigmoid function to approximate the correspondence between the feature vector and the category label of the requested content [19], and construct a logical regression model to approximate the user preference model. For the arriving request $req_t,d$, it is characterized by the feature vector $x(d)$. Let $y(d)$ denote
the corresponding category label with \( y(d) = 1 \) if the requested content is the favorite category of the user and \( y(d) = 0 \) otherwise. Let \( p_{t,u,d} \) denote the possibility that the user \( u \in U_t \) requests the content \( f(d) \) at the requesting time \( t(d) \) during the \( t \)th time period. Specifically, we assume that a user will not have a second request to the same content. If the user \( u \) has already requested \( f(d) \) previously, then \( \hat{p}_{t,u,d} = 0 \). Otherwise, \( p_{t,u,d} \) can be predicted based on the following user preference model

\[
\hat{p}_{t,u,d} = p_{w_u}[y(d) = 1 | x(d)] = \frac{1}{1 + e^{-(w_u \cdot x(d))}}.
\] (4)

Furthermore, the regional content popularity \( P_{t,d} \) can be predicted by using the average requested possibility of the regional users to \( f(d) \) as follows

\[
\hat{P}_{t,d} = \frac{1}{U_t} \sum_{u=1}^{U_t} \hat{p}_{t,u,d}.
\] (5)

After that, if the requested content is determined to be stored into the local cache or it has been stored locally and is requested again, according to our previous assumption that a user will not have a second request to the same content, its current popularity at the requesting time can be calculated respectively as follows

\[
P_{t,f(d)}^{\text{cur}} = \hat{P}_{t,d} - \frac{1}{U_t},
\] (6)

\[
P_{t,f(d)}^{\text{cur}} = P_{t,f(d)}^{\text{cur}} - \frac{1}{U_t}.
\] (7)

Besides, if the requested content is determined not to be stored locally and is requested by another regional user again, its current popularity at the next requesting time can be predicted by (5) since we have set the possibility that the previous requesting user will request this content again to zero. In this way, our proposed online content popularity algorithm can track the popularity changes in real time.

To measure the prediction performance, we introduce the logistic loss \( \ell(w_u, x(d), y(d)) \) for the user \( u \), which is defined as the negative log-likelihood of \( y(d) \) given \( p_{w_u}(y(d) | x(d)) \) and can be expressed as follows

\[
\ell(w_u, x(d), y(d)) = -y(d) \log p_{w_u}(y(d) | x(d)) - (1 - y(d)) \log[1 - p_{w_u}(y(d) | x(d))].
\] (8)

Since user preference may change over time, we need to capture the moment when the user preference changes and timely update the user preference model. For this purpose, we collect the samples
Algorithm 2 The Online Content Popularity Prediction Algorithm

1: procedure PREDICT(x(d), {w_u|∀u ∈ U_t})
2: for u ∈ U_t, do
3: if User u has requested the content previously, then
4: \( \hat{p}_{t,u,d} = 0; \)
5: else
6: Obtain the well-trained \( w_u \) from the M F-APs;
7: \( \hat{p}_{t,u,d} = \frac{1}{1 + e^{-w_u \cdot x(d)}}; \)
8: end if
9: end for
10: Get \( (x(K_{t,u,d}), y(K_{t,u,d})) \), \( K_{t,u,d} = K_{t,u,d} - 1 + 1 \);
11: \( \xi_{t,u,d} = \xi_{t,u,d} - 1 + \frac{1}{K_{t,u,d}} \ell(w_u, x(K_{t,u,d}), y(K_{t,u,d})); \)
12: if \( \xi_{t,u,d} \geq \gamma \), then
13: \( w_u \leftarrow \text{Learn} \{ (x(k), y(k)) \}_{k=1}^{K_{t,u,d}} \).
14: end if
15: end for
16: return \( \frac{1}{U_t} \sum_{u=1}^{U_t} \hat{p}_{t,u,d} \).
17: end procedure

\[
\{(x^{(k)}, y^{(k)})\}_{k=1}^{K_{t,u,d}} \text{ and monitor the prediction performance in real time, where } x^{(k)} \text{ and } y^{(k)} \text{ denote the feature vector and category label of the } k \text{th sample, respectively, and } K_{t,u,d} \text{ the cumulative number of samples for the user } u \text{ from the last model update to the time when the request } req_{t,d} \text{ arrives during the } t \text{th time period. Then, the average logistic loss for the user } u \text{ can be expressed as follows}
\]

\[
\xi_{t,u,d} = \frac{1}{K_{t,u,d}} \sum_{k=1}^{K_{t,u,d}} \ell(w_u, x^{(k)}, y^{(k)}). \tag{9}
\]

Let \( \gamma \) denote a predefined threshold with \( 0 \leq \gamma \leq 1 \). When \( \xi_{t,u,d} \) exceeds \( \gamma \), the update of the user preference model will be initiated.

The detailed description of our proposed online content popularity prediction algorithm is shown in Algorithm 2. We remark here that our proposed algorithm not only can predict content popularity in an online fashion, but it also can determine when to update the user preference model proactively. We also remark here that the computational complexity of our proposed algorithm is very low.

C. Offline User Preference Learning

When the update of the user preference model is initiated, we assume that there are \( K \) samples collected for the considered user and extracted from the recorded data, which are denoted by \( \{ (x^{(k)}, y^{(k)}) \}_{k=1}^{K} \).

Based on the collected samples, we propose to learn the user preference model parameters iteratively by minimizing the logistic loss of each sample as follows

\[
w_u^{(k+1)} = \arg \min_{w_u} \ell \left( w_u, x^{(k)}, y^{(k)} \right), \quad k = 1, 2, \cdots, K, \tag{10}
\]
where \( \mathbf{w}_u^{(k+1)} \) denotes the vector of the learned user preference model parameters of the \( k \)th iteration for the user \( u \). By using the “online gradient descent” (OGD) method [20], the solution of the above optimization problem can be obtained through the iteratively updated model parameters as follows

\[
\mathbf{w}_u^{(k+1)} = \mathbf{w}_u^{(k)} - \eta^{(k)} \mathbf{g}^{(k)}, \quad k = 1, 2, \cdots, K.
\]  

(11)

where \( \mathbf{g}^{(k)} = \nabla_{\mathbf{w}_u} \ell (\mathbf{w}_u, \mathbf{x}^{(k)}, y^{(k)}) = [p_{\mathbf{w}_u} (y^{(k)} | \mathbf{x}^{(k)}) - y^{(k)}] \mathbf{x}^{(k)} \) denotes the gradient vector of the logistic loss of the \( k \)th sample with respect to \( \mathbf{w}_u \), and \( \eta^{(k)} \) a non-increasing learning-rate schedule with \( \sigma^{(k)} \) satisfying \( \sum_{k'=1}^{k} \sigma^{(k')} = 1/\eta^{(k)} \). Then, we have the following theorem.

**Theorem 1:** The solution of the optimization problem in (10) can also be obtained by solving the following equivalent optimization problem

\[
\mathbf{w}_u^{(k+1)} = \arg \min_{\mathbf{w}_u} \left( \mathbf{g}^{(1:k)}^T \mathbf{w}_u + \frac{1}{2} \sum_{k'=1}^{k} \sigma^{(k')} \left\| \mathbf{w}_u - \mathbf{w}_u^{(k')} \right\|_2^2 \right), \quad k = 1, 2, \cdots, K.
\]  

(12)

where \( \mathbf{g}^{(1:k)} = \sum_{k'=1}^{k} \mathbf{g}^{(k')} \) denotes the sum of the gradient vector of the logistic loss of the previous \( k \) samples.

**Proof:** Please see Appendix A.

Due to the sparse and unbalanced data and high-dimensional feature vector, there may exist over-fitting and high computational complexity problems [21]. Inspired by the “follow the (proximally) regularized leader” (FTRL-Proximal) method [22], which is an online optimization method based on the OGD method, the \( L_1 \) - and \( L_2 \)-regularization terms are introduced simultaneously in the optimization problem in (12) in order to avoid the above mentioned potential problems whereas obtain the optimal model parameters. The introduction of the \( L_1 \)-regularization term is beneficial for realizing feature selection and producing sparse model, while the introduction of the \( L_2 \)-regularization term is conductive to the smooth solution of the corresponding optimization problem. Correspondingly, the model parameters can be updated according to the previous samples by solving the following optimization problem

\[
\mathbf{w}_u^{(k+1)} = \arg \min_{\mathbf{w}_u} \left\{ \mathbf{g}^{(1:k)} - \sum_{k'=1}^{k} \sigma^{(k')} \mathbf{w}_u^{(k')} \right\}^T \mathbf{w}_u + \frac{1}{2} \sum_{k'=1}^{k} \sigma^{(k')} \left\| \mathbf{w}_u - \mathbf{w}_u^{(k')} \right\|_2^2 + \lambda_1 \left\| \mathbf{w}_u \right\|_1
\]

\[
+ \frac{1}{2} \sum_{k'=1}^{k} \sigma^{(k')} \left\| \mathbf{w}_u^{(k')} \right\|_2^2 \right), \quad k = 1, 2, \cdots, K.
\]  

(13)

where \( \lambda_1 \) and \( \lambda_2 \) denote the regularization parameters with positive values, \( \| \cdot \|_1 \) and \( \| \cdot \|_2^2 \) the \( L_1 \)-norm
and L2-norm, respectively.

It can be readily seen from (13) that \(1/2 \sum_{k'=1}^{k} \sigma^{(k')} \|w^{(k')}\|_2^2\) is irrespective with \(w_u\). Let \(z^{(k)} = g^{(1:k)} - \sum_{k'=1}^{k} \sigma^{(k')} w^{(k')}\). Then, an iterative relationship between \(z^{(k)}\) and \(z^{(k-1)}\) can be established as follows
\[
z^{(k)} = z^{(k-1)} + g^{(k)} - \left(\frac{1}{\eta^{(k)}} - \frac{1}{\eta^{(k-1)}}\right) w^{(k)},
\]
which implies that we only need to store \(z^{(k-1)}\) after using the last sample for learning. Correspondingly, the optimization problem in (13) can be further expressed as follows
\[
\begin{aligned}
\mathbf{w}^{(k+1)}_u &= \arg\min_{\mathbf{w}_u} \left\{ (z^{(k)})^T \mathbf{w}_u + \lambda_1 \| \mathbf{w}_u \|_1 + \frac{1}{2} \left( \lambda_2 + \sum_{k'=1}^{k} \sigma^{(k')} \right) \| \mathbf{w}_u \|_2^2 \right\}, \quad k = 1, 2, \cdots, K.
\end{aligned}
\]

We know that there exists a difference for the change rate of the weight of each feature dimension for the requested content, and that the gradient value with respect to each feature dimension can reflect this change rate. Therefore, different learning rates are preferred for different feature dimensions. Define
\[
\begin{aligned}
\mathbf{g}^{(k)} &= [g_1^{(k)}, g_2^{(k)}, \cdots, g_n^{(k)}, \cdots, g_N^{(k)}]^T, \\
z^{(k)} &= [z_1^{(k)}, z_2^{(k)}, \cdots, z_n^{(k)}, \cdots, z_N^{(k)}]^T, \\
\mathbf{w}_u &= [w_{u,1}, w_{u,2}, \cdots, w_{u,n}, \cdots, w_{u,N}]^T, \\
\mathbf{w}^{(k+1)}_u &= [w^{(k+1)}_{u,1}, w^{(k+1)}_{u,2}, \cdots, w^{(k+1)}_{u,n}, \cdots, w^{(k+1)}_{u,N}]^T.
\end{aligned}
\]
Let
\[
\sum_{k'=1}^{k} \sigma^{(k')} = \frac{1}{\eta_n^{(k)}},
\]\(\text{where } \eta_n^{(k)} = \alpha / [\beta + \sqrt{\sum_{k'=1}^{k} \left(g_n^{(k')}\right)^2}]\) denotes the learning-rate schedule of the \(n\)th feature dimension with \(\alpha\) and \(\beta\) being the adjusting parameters which are chosen to yield good learning performance. Then, the optimization problem in (15) can be decoupled into the following \(N\) independent scalar minimization problems
\[
\begin{aligned}
\mathbf{w}^{(k+1)}_{u,n} &= \arg\min_{\mathbf{w}_{u,n}} \left\{ z_n^{(k)} w_{u,n} + \lambda_1 |w_{u,n}| + \frac{1}{2} \left( \lambda_2 + \sum_{k'=1}^{k} \sigma^{(k')} \right) w_{u,n}^2 \right\}, \quad n = 1, 2, \cdots, N,
\end{aligned}
\]
It can be easily verified that the optimization problem in (17) is an unconstrained non-smooth one, where the second item \(\lambda_1 |w_{u,n}|\) is non-differentiable at \(w_{u,n} = 0\). Let \(\eta = \partial |w_{u,n}| \bigg|_{w_{u,n}=w^{(k+1)}_{u,n}}\) denote the
Algorithm 3 The Offline User Preference Learning Algorithm

1: procedure Learn(\{ (x^{(k)}, y^{(k)}) \}_{k=1}^{K})
2: Initialize: \( \alpha, \beta, \lambda_1, \lambda_2, w_u^{(1)}, z^{(0)} = q^{(0)} = 0 \in \mathbb{R}^N; \)
3: for \( k = 1, 2, 3, \ldots, K, \)
4: \( g^{(k)} = \nabla \ell_{w_u} (w_u, x^{(k)}, y^{(k)}) \mid_{w_u = w_u^{(k)}}, \)
5: for \( n = 1, 2, 3, \ldots, N, \)
6: \( \sigma_n^{(k)} = \frac{1}{2} \left( \sqrt{q_n^{(k-1)} + (g_n^{(k)})^2} - \sqrt{q_n^{(k-1)}} \right); \)
7: \( z_n^{(k)} = z_n^{(k-1)} + g_n^{(k)} - \sigma_n^{(k)} w_u^{(k)}; \)
8: \( q_n^{(k)} = q_n^{(k-1)} + (g_n^{(k)})^2; \)
9: Calculate \( w_u^{(k+1)} \) according to (20) by setting \( \sum_{r=1}^{k} \sigma_n^{(r)} \) to \( (\beta + \sqrt{q_n^{(k)})}/\alpha. \)
10: end for
11: end for
12: return \( w_u^{(K+1)} \)
13: end procedure

partial differential of \( |w_{u,n}| \) at \( w_{u,n}^{(k+1)} \). Then, we have

\[
\begin{cases} 
-1 < \eta < 1, & \text{if } w_{u,n}^{(k+1)} = 0, \\
\eta = -1, & \text{else if } w_{u,n}^{(k+1)} < 0, \\
\eta = 1, & \text{otherwise}.
\end{cases}
\]  

Correspondingly, the optimal solution \( w_{u,n}^{(k+1)} \) should satisfy the following relationship

\[
z_n^{(k)} + \lambda_1 \eta + (\lambda_2 + \sum_{k'=1}^{k} \sigma_n^{(k')}) w_{u,n}^{(k+1)} = 0, \quad n = 1, 2, \ldots, N.
\]  

We have known previously that \( \lambda_1 > 0 \). Correspondingly, by classifying \( z_n^{(k)} \) into three cases, i.e., \( |z_n^{(k)}| < \lambda_1 \), \( z_n^{(k)} > \lambda_1 \) and \( z_n^{(k)} < -\lambda_1 \), the closed-form solution of the optimization problem in (17) can be obtained from (19) as follows

\[
w_{u,n}^{(k+1)} = \begin{cases} 
0, & \text{if } |z_n^{(k)}| < \lambda_1, \\
\frac{\lambda_1 \text{sgn}(z_n^{(k)}) - z_n^{(k)}}{\lambda_2 + \sum_{k'=1}^{k} \sigma_n^{(k')}}, & \text{otherwise},
\end{cases} \quad n = 1, 2, \ldots, N.
\]  

The entire user preference learning algorithm with the nice property of self-starting is described in Algorithm 3. Note that our proposed algorithm only needs to store the last \( z^{(k)} \) after performing a user preference updating process, and the previously recorded data can be cleared which is helpful to save storage space. Furthermore, the computational complexity of our proposed algorithm is not an issue due to its offline property.

We remark that our proposed policy can asymptotically approach the optimal solution of the optimization problem in (5) as the user requests increase, whose proof will be presented in the next section.
IV. PERFORMANCE ANALYSIS

In this section, the performance of our proposed edge caching policy will be analyzed. Firstly, we derive the upper bound of the popularity prediction error of our proposed online content popularity prediction algorithm. Secondly, we derive the lower bound of the cache hit rate of our proposed edge caching policy. Finally, we derive the regret bound of the overall cache hit rate of our proposed edge caching policy.

A. The Upper Bound of the Popularity Prediction Error

Let \( W_u = \max_{w_u, w_u' \in \mathcal{E}} \| w_u - w_u' \| \). (21)

Let \( L_d(w_u) \) denote a sequence of convex loss functions, and define

\[
G_u = \max_{w_u \in \mathcal{E}, 1 \leq d \leq D_t, t=1, 2, \ldots, T} \| \nabla L_d(w_u) \|. \tag{22}
\]

Then, according to Corollary 1 in [23], the following relationship can be readily established for the optimization problem in (13) in the finite time horizon \( T \)

\[
\sum_{t=1}^{T} \sum_{d=1}^{D_t} L_d(w_u) - \sum_{t=1}^{T} \sum_{d=1}^{D_t} L_d(w_u^*) \leq W_u G_u \sqrt{2D} \tag{23}
\]

where \( w_u^* \) denotes the vector of the optimal user preference model parameters. Let \( \tau_u \) denote a sufficiently small constant with a positive value, which can meet \( \sum_{t=1}^{T} \sum_{d=1}^{D_t} L_d(w_u^*) \leq \tau_u \). Define

\[
W_{\text{max}} = \max_{u \in U, t=1, 2, \ldots, T} W_u, \quad G_{\text{max}} = \max_{u \in U, t=1, 2, \ldots, T} G_u, \quad \tau_{\text{max}} = \max_{u \in U, t=1, 2, \ldots, T} \tau_u. \tag{24}
\]

Then, we have the following theorem.

Theorem 2: The expected popularity prediction error for the overall \( D \) requests in the finite time horizon \( T \), i.e., \( \mathbb{E} \sum_{t=1}^{T} \sum_{d=1}^{D_t} \left| \hat{P}_{t,d} - P_{t,d} \right| \), is upper bounded by \( \frac{T_{\text{max}}}{T_{\text{min}}} \left( W_{\text{max}} G_{\text{max}} \sqrt{2D} + \tau_{\text{max}} \right) \).

Proof: Please see Appendix B. \( \square \)

It is clear that an upper bound exists for the cumulative prediction error of the content requested probability of one single user and that of the regional content popularity within a limited time periods. Specifically, by using the user preference model that is obtained through self-learning, the upper bound of the cumulative prediction error of the content requested probability of one single user has a sub-linear relationship with the total number of requests \( D \) in the finite time horizon \( T \). Similar relationship can be
found for the cumulative prediction error of the content popularity in the considered region, which means that
\[
\frac{1}{D} \mathbb{E} \sum_{t=1}^{T} \sum_{d=1}^{D_{t}} \left| \hat{P}_{t,d} - P_{t,d} \right| \to 0 \text{ as } D \to \infty.
\]

The above analytical results imply that the learned user preference model will asymptotically approach the real user preference model through sufficient learning with the collection of more user requesting information and the increased requests. Correspondingly, the proposed online content popularity prediction algorithm can make the content popularity prediction more accurate. On the other hand, the analytical results also reveal that the performance of our proposed policy can be improved with the increased requests. After a certain number of content requests, the prediction precision of the proposed policy can achieve the ideal value.

B. The Lower Bound of the Cache Hit Rate

In this subsection, we first show the cache hit rate of the optimal edge caching policy which knows the real popularities of all the contents and caches the most popular contents during each time period, and then derive the lower bound of the cache hit rate of the proposed edge caching policy and reveal their relationship.

1) The cache hit rate of the optimal edge caching policy: In the ideal case, the optimal cache hit rate can be achieved by caching the most popular contents of the current time period based on the known content popularity. Let \( \Phi^* \) denote the optimal edge caching policy. In practice, we note that the cache hit rate depends not only on the edge caching policy \( \Phi \) but also on the degree of concentration of the same content requests. Generally speaking, a more concentrated content request process implies a higher potential cache hit rate. For the ease of analysis, we assume that the requests of the same content is concentrated in one time period. Let \( P'_{t,d'} \) denote the content popularity of the \( d' \)th most popular content when it is requested firstly during the \( t \)th time period, \( F_t \) the number of the requested different contents during the \( t \)th time period. Then, by using the relationship in (5), the optimal cache hit rate during the \( t \)th time period can be calculated as follows

\[
H_t (\Phi^*) = \frac{\sum_{d'=1}^{M_{\Phi}} U_t P'_{t,d'}}{\sum_{d'=1}^{F_t} U_t P'_{t,d'}} = \frac{\sum_{d'=1}^{M_{\Phi}} P'_{t,d'}}{\sum_{d'=1}^{F_t} P'_{t,d'}}.
\]

(25)

Note that \( H_t (\Phi^*) \) has no relation with the number of requested users during the current time period, and may vary with different time periods. Besides, we make no assumption about the popularity distribution, and \( H_t (\Phi^*) \) is just the cache hit rate that is calculated based on the real content popularity during the current time period.
2) The lower bound of the cache hit rate of the proposed edge caching policy: In practice, there always exist popularity prediction errors. Let \( \hat{P}_{t,d'} \) denote the predicted popularity with respect to \( P_{t,d'} \). Let \( \Delta P_t = \max_{d'=1,2,\cdots,F_t} |\hat{P}_{t,d'} - P_{t,d'}| \). Then, we have the following theorem.

**Theorem 3:** During the \( t \)th time period, the achievable cache hit rate \( H_t(\Phi) \) can be lower bounded by 

\[
H_t(\Phi^*) - M \varphi (\Delta P_t U_t + 2)/D_t.
\]

**Proof:** Please see Appendix C.

It is clear that a lower bound exists for the cache hit rate of the proposed edge caching policy during each time period. The analytical result from Theorem 3 gives us the insight that there exists a certain performance gap, i.e., \( M \varphi (\Delta P_t U_t + 2)/D_t \), between the cache hit rate of our proposed edge caching policy and that of the optimal edge caching policy. The first component of the performance gap, i.e., \( M \varphi \Delta P_t U_t/D_t \), is mainly caused by the popularity prediction error \( \Delta P_t \) and principally determined by the accuracy of the learned user preference model. The second component of the performance gap, i.e., \( 2M \varphi /D_t \) is mainly caused by the operational mechanism of our proposed edge caching policy, which decides to cache a content only after but not before its first request (i.e., an initial cache miss will happen).

Although our proposed edge caching policy may result in an initial cache miss, it can actually avoid the extremely high computational load that may bring about to F-APs due to the need of continuously predicting all the content popularities otherwise. We point out here that this type of performance gap will gradually approach zero with the increased number of content requests, and hence the benefit outweighs the cost. Moreover, the analytical result from Theorem 3 also reveals that the overall performance gap will approach zero when the prediction error of the content popularity approaches zero, i.e., \( \Delta P_t \to 0 \), and the number of content requests during the \( t \)th time period is much larger than the overall cache size of all the F-APs in the considered region, i.e., \( D_t \gg M \varphi \). Correspondingly, the cache hit rate of our proposed edge caching policy during the \( t \)th time period will approach that of the optimal edge caching policy if the above two conditions are satisfied.

**C. The Regret Bound of the Overall Cache Hit Rate**

In order to measure the performance loss of the proposed edge caching policy in comparison with the optimal one, we will analyze and bound the regret of the overall cache hit rate.

Let \( H(\Phi^*) \) denote the overall cache hit rate of the optimal edge caching policy in the finite time horizon.
Then, from (25), it can be calculated as follows

\[ H(\Phi^*) = \frac{\sum_{t=1}^{T} \sum_{d'=1}^{M\varphi} U_t P'_{t,d'} \sum_{d'=1}^{2} U_t P'_{t,d'} \sum_{t=1}^{T} \mathcal{D}_t}{\sum_{t=1}^{T} D_t}. \]  \tag{26}

Let \( \mathcal{H}(\Phi) \) denote the overall cache hit rate of the proposed edge caching policy in the finite time horizon \( T \). Then, we have

\[ \mathcal{H}(\Phi) = \frac{\sum_{t=1}^{T} \mathcal{H}_t(\Phi) D_t}{\sum_{t=1}^{T} D_t}. \]  \tag{27}

Then, the regret of the overall cache hit rate of the proposed edge caching policy for the total \( D \) requests in the finite time horizon \( T \) can be defined as follows

\[ R(D) = E[H(\Phi^*) - H(\Phi)]. \]  \tag{28}

Utilizing the analytical results from Theorem 2 and Theorem 3, we have the following theorem.

**Theorem 4:** The regret of the overall cache hit rate for the total \( D \) requests in the finite time horizon \( T \), i.e., \( R(D) \), is upper bounded by

\[ U_{\max} M\varphi \left[ \frac{U_{\max}}{U_{\min}} \left( W_{\max} G_{\max} \sqrt{2D} + \tau_{\max} \right) + 2T/U_{\max} \right] / D. \]

**Proof:** Please see Appendix D.

According to the above theorem, with the limited \( U_{\max}, M\varphi \) and \( T \), the following relationship can be naturally obtained: \( \lim_{D \to +\infty} R(D) = 0 \), which shows that the performance loss of our proposed edge caching policy can be gradually reduced to zero as the number of requests is increased, i.e., our proposed edge caching policy has the capability to achieve the optimal performance asymptotically. The reason is that the learned user preference model gradually approaches the real one with the increased request samples, which makes the prediction errors even smaller.

**V. THE PROPOSED LEARNING BASED EDGE CACHING ARCHITECTURES**

In this section, we propose two learning based edge caching architectures which can implement the functionality of our previously proposed edge caching policy. In our proposed first architecture, by considering that not all UEs are equipped with artificial intelligence (AI) chipsets and support offline learning, both the online popularity prediction algorithm and the offline user preference learning algorithm are implemented inside the F-APs. In our proposed second architecture, by considering future UEs equipped with AI chipsets supporting offline learning in smart wireless communications scenarios, the online popularity prediction algorithm is implemented inside the F-APs and the offline user preference
A. Learning Based Edge Caching Architecture (I)

For the proposed first architecture as illustrated in Fig. 2, its fundamental modules are as follows: Local Cache, Cache Management, Request Processor, and User Interface, which have functions similar to the traditional caching architectures [14], [15]. In order to learn user preference and predict content popularity, our proposed architecture also includes the following modules: Information Monitoring and Interaction, Popularity Prediction, Offline Learning, Data Updater, Cache Information, and Cache Monitor. Their functions are described as follows.

- The Information Monitoring and Interaction module is mainly responsible for realizing regular information monitoring and interaction between regional F-APs. On the one hand, this module periodically collects the current user set of the serving F-AP and the current user information (including the regional user set and user preference model) of the other regional F-APs, and stores them into the Learning Database. On the other hand, this module periodically sends the current user information of the serving F-AP to the other regional ones, and finally realizes the monitoring and sharing of the current user information among the regional F-APs.
- The Popularity Prediction module is mainly responsible for predicting the popularity of the current
Fig. 3. Illustration of the learning based edge caching flowchart.

requested content based on the Learning Database. Note that the Offline Learning module will be initiated if the average prediction error is larger than a predefined threshold.

- The Offline Learning module is mainly responsible for learning the current user preference model parameters based on the collected information from the Feature Database and Request Database.
- The Data Updater module is mainly responsible for updating the content feature data, the requested content, and the requested time to the Feature Database and Request Database, and realizing the collection and update of the requested information.
- The Cache Information module is mainly responsible for storing and updating the current content popularity information, the initial cache time, and the cache content ID.
- The Cache Monitor module is mainly responsible for monitoring the cache information to capture the contents which need to be removed from the local cache.

The flowchart of our proposed first learning base edge caching architecture consists of five phases as illustrated in Fig. 2 and Fig. 3, and is presented below.

(i) Initializing and periodic information monitoring

(a) The Information Monitoring and Interaction module periodically extracts the current user information of the serving F-AP and regional ones from their User Interface modules. (b) This module regularly updates the collected regional user information to the Learning Database. (c) This module extracts the current user information of the serving F-AP from the Learning Database. (d) This module delivers the current user information of the serving F-AP to the regional ones.
(ii) Direct local request responding

(1) The User Interface module delivers the user requesting information to the Request Processor module. (2) Then, the Request Processor module initiates a data updating procedure. (3) The Data Updater module carries on numerical processing to the requested content feature and writes the processed feature data into the Feature Database, and updates the requested content information in the Request Database. (4) If the requested content is stored locally, the Cache Management module delivers the stored content from the local cache to the Request Processor module. (A) The Request Processor module serves the user request.

(iii) Dynamic content caching and updating

(5) If the requested content is not stored locally, the Request Processor module triggers the Popularity Prediction module to make online content popularity prediction of the requested content. (6) The Popularity Prediction module extracts the user preference model parameters and the requested content features from the Learning Database, and then predicts the popularity of the requested content. (7) The Popularity Prediction module feeds back the predicted popularity of the requested content first to the Request Processor module, and then to the Cache Management module through the Request Processor module. (8) The Cache Management module triggers the Cache Monitor module to initiate monitoring of the cache content information. (9) The Cache Monitor module extracts the information of the content to be removed from the Cache Information module, and then feeds back the information to the Cache Management module. (10) The Cache Management module makes cache decision based on the feedback information, and notifies the Request Processor module to serve the user request. (11) The User Interface module broadcasts the local cached content information to the regional users.

(iv) Cooperative caching and information synchronizing

(B) The Cache Management module executes the received cache decision of the current F-AP, and notifies the other regional F-APs to execute the same cache decision. If the requested content is to be cached, the Cache Management module retrieves the content from the neighboring F-APs in the other regions or the cloud content center, and then stores it locally by means of partition-based caching. The Cache Management module updates the cache information of the current F-AP, and synchronizes the cache information of the other regional F-APs.

(v) Offline self-starting user preference learning
Fig. 4. Illustration of the second learning based edge caching architecture.

(C) The Popularity Prediction module initiates the Offline Learning module if the average prediction error cumulated under a user preference model is larger than a predefined threshold. (D) The Offline Model Update module retrieves the historical requested data of the considered user from the Feature Database and Request Database, generates a new training sample set, and then updates the user preference model parameters. (E) The Offline Model Update module refreshes the updated user preference model parameters to the Learning Database. (F) The Request Database releases the historical requested data of the considered user.

B. Learning Based Edge Caching Architecture (II)

For the proposed second architecture as illustrated in Fig. 4 part of the functionality, i.e., offline user preference learning, is transferred from the F-APs to the powerful and smart UEs. Therefore, both the signaling overhead among regional F-APs and the computational burden undertaken by the F-APs can be greatly reduced.

The corresponding processing flow of our proposed edge caching policy is presented in brief below. (a-d) The current F-AP is mainly responsible for monitoring the regional users in coordination with the regional F-APs and storing the corresponding user preference model parameters from the UEs into the Learning Database. (1-2, A) The current F-AP serves the requested user if it caches the requested content locally. (3-8) If the requested content is not stored in the local cache, the current F-AP predicts the content popularity, makes the corresponding caching decision, and broadcasts the caching information to the regional users. (B) The current F-AP notifies the other regional F-APs to execute the same cache decision and then it updates the corresponding cache information.
We remark here that our proposed first architecture is more suitable for wireless communications scenarios including intelligent F-APs and general UEs while our proposed second architecture is more suitable for wireless communications scenarios including general F-APs and intelligent UEs. With the rapid development of AI and smart UEs, the second architecture will show more advantages and dominate in future wireless communications. We also remark here that the signaling overhead can be further reduced by setting a cluster head for the clustered regional F-APs and the corresponding edge caching architecture is omitted here due to space limitation.

VI. SIMULATION RESULTS

To evaluate the performance of the proposed edge caching policy, we take movie content as an example and our main datasets are extracted from the MovieLens 200M Dataset [24], [25]. From the MovieLens, we choose the requesting dataset of the selected 30 users who request the contents from January 01, 2010 to October 17, 2016. The first part of the requesting dataset, whose requesting dates are from January 01, 2010 to December 31, 2015, is used for initializing the user preference, while the second part of the requesting dataset, whose requesting dates are from January 01, 2016 to October 17, 2016, is used for evaluating the performance. Considering that users will generally comment on the movie after they have just watched it, we take the movie rating from a user as the request for this movie [14], [15]. In our simulations, we set the number of the considered F-APs $M$ to 3, the finite time horizon $T$ to 6984 hours, the monitoring cycle to 1 hour, and the predefined threshold $\gamma$ to 0.2, respectively.

In Fig. 5, we show the predicted popularity by using our proposed edge caching policy in comparison with the real popularity at the moment when the contents are requested firstly by one of the regional users.

![Fig. 5. Comparison of the predicted popularity and the real popularity in the finite time horizon $T$.](image)
Fig. 6. Comparison of the predicted local popularity, the real local popularity, and the real global popularity for a certain requested content over time.

Fig. 7. Comparison of the predicted local popularity, the real local popularity for the considered region, and the real global popularity for all the regions.

for the preceding 5000 contents in the finite time horizon $T$, where the content ID is marked according to the first requesting time of its representing content in chronological order. It can be observed that the average error between the predicted popularity and the real popularity is very small. It can also be observed that the first requesting time of a regional popular content is random. Therefore, it is impractical for traditional edge caching policies to cache the most popular contents directly without consideration of the content requesting time and temporal popularity dynamic, and it is necessary for our proposed edge caching policy to consider the content requesting time and track popularity changes in real time.

In Fig. 6, we show the predicted local popularity, the real local popularity, and the real global popularity over time for a randomly selected content that is requested by the regional users. It can be observed that both the global real popularity and the local real popularity decrease with time whereas the latter fluctuates slightly, which verifies that the global popularity cannot precisely reflect the temporal changes of the local popularity. It can also be observed that the predicted local popularity changes with the real local popularity
in real time and the former approaches the latter, which verifies that our proposed edge caching policy can indeed track the real local popularity changes without delay.

Without consideration of the requesting time difference of the users and the duration difference of the continuous requests for the same content, we analyze the spatial changes of content popularity. In Fig. 7, we show the predicted local popularity, the real local popularity for the considered region in comparison with the real global popularity for all the regions, where the content ID is marked according to the real global popularity of its representing content in descending order. It can be observed that most of the contents with the real local popularity larger than 0.2 have a content ID smaller that 2000, which reveals that most of the local popular contents originate from the global popular ones. We can observe that the contents with the content ID smaller than 500 generally have a larger real global popularity but a fluctuant real local popularity. We can also observe that the real global popularity approximately follows a typical Zipf distribution whereas the real local popularity does not. These observations reveal that the local popularity changes with the spatial popularity dynamics and does not necessarily follow the global popularity, and confirm the necessity of our proposed learning based edge caching policy.

In Fig. 8, we show the overall cache hit rate of our proposed policy with different $M\varphi$ in the finite time horizon $T$. Also included in Fig. 8 are the overall cache hit rates of the four baseline policies, i.e., the FIFO [6], LRU [7], LFU [8] policies, and the optimal policy with real content popularity. The total cache size $M\varphi$ increases from $1.5\%F = 60$ to $11.97\%F = 4800$ contents with $F = 40110$. It can be observed that the overall cache hit rates of all the considered policies are gradually increased with $M\varphi$. It can also be observed that the overall cache hit rate of our proposed policy gradually approaches the optimal performance and is apparently superior to those of the other three baseline policies for all the considered cache sizes. The reason is that the latter can not predict future content popularity. Instead, our proposed policy not only can predict the content popularity online, but also can track its changes in real time. Specifically, it can be observed that our proposed policy only needs a cache size of approximately 2400 contents to achieve the cache hit rate of 0.6 whereas the other three baseline policies need a cache size of approximately 4200 contents.

In Fig. 9, we show the overall cache hit rates of our proposed policy and the four baseline policies until the current time period with $M\varphi = 1800$. It can be observed that the overall cache hit rate of our proposed policy follows consistently along with that of the optimal policy. The reasons are that both of them make caching decisions according to the content popularity, and that the distributions of the
predicted popularity and the real one are consistent during every time period. It can also be observed that the changes of the overall cache hit rates of all the policies are different. The FIFO, LRU and LFU policies have low overall cache hit rates during the initial time periods due to the inevitable cold-start problem, whereas our proposed policy can cache the predicted popular contents according to the already-learned user preference model during the initial time period and then achieve a higher cache hit rate accordingly. After that, the overall cache hit rates of all the policies gradually increase with the time period. The reason is that caching decisions can be made more accurate with the increase of user requests.

VII. CONCLUSIONS

In this paper, we have proposed two edge caching architectures and a novel edge caching policy by learning user preference and predicting content popularity. Our proposed policy can promptly detect the regional popular contents through online content popularity prediction, and timely store it to the local
caches. Specifically, we have proposed a self-starting offline user preference model updating mechanism by monitoring the average logistic loss in real time, which avoids frequent and blind training. Analytical results have shown that our proposed policy has the capability of asymptotically approaching the optimal performance. Simulation results have shown that our proposed policy achieves a better caching performance (i.e., overall cache hit rate) compared to the other traditional policies.

**APPENDIX**

**A. PROOF OF THEOREM 1**

It can be readily seen that the objective function of the optimization problem in (12) is convex. Therefore, the iteratively updated model parameters can be obtained by setting the first order partial derivative of the corresponding objective function with respect to \( w_u \) to zero as follows

\[
\frac{\partial}{\partial w_u} \left( (g^{(1:k)})^T w_u + \frac{1}{2} \sum_{k'=1}^{k} \sigma^{(k')} \| w_u - w_u^{(k')} \|_2^2 \right) / \partial w_u = g^{(1:k)} + \sum_{k'=1}^{k} \sigma^{(k')} (w_u - w_u^{(k')}) = 0. \tag{29}
\]

Correspondingly, the solution of the optimization problem in (12), i.e., \( w_u^{(k+1)} \), should satisfy the following relationship

\[
\sum_{k'=1}^{k} \sigma^{(k')} w_u^{(k+1)} = \sum_{k'=1}^{k} \sigma^{(k')} w_u^{(k')} - g^{(1:k)}. \tag{30}
\]

Utilizing the relationship \( \sum_{k'=1}^{k} \sigma^{(k')} = 1/\eta^{(k)} \), we can establish that

\[
\frac{1}{\eta^{(k)}} w_u^{(k+1)} = \sum_{k'=1}^{k} \sigma^{(k')} w_u^{(k')} - g^{(1:k)}. \tag{31}
\]

Replace \( k \) in (31) by \( k - 1 \). Then, we have

\[
\frac{1}{\eta^{(k-1)}} w_u^{(k)} = \sum_{k'=1}^{k-1} \sigma^{(k')} w_u^{(k')} - g^{(1:k-1)}. \tag{32}
\]

From (31) and (32), we can readily establish that

\[
\frac{1}{\eta^{(k)}} w_u^{(k+1)} - \frac{1}{\eta^{(k-1)}} w_u^{(k)} = \sigma^{(k)} w_u^{(k)} - g^{(k)}. \tag{33}
\]

Exploiting the relationship \( \sigma^{(k)} = (1/\eta^{(k)} - 1/\eta^{(k-1)}) \), we can further establish that

\[
\frac{1}{\eta^{(k)}} w_u^{(k+1)} - \frac{1}{\eta^{(k-1)}} w_u^{(k)} = \left( \frac{1}{\eta^{(k)}} - \frac{1}{\eta^{(k-1)}} \right) w_u^{(k)} - g^{(k)}. \tag{34}
\]
Correspondingly, we can finally obtain the solution of the optimization problem in (12), which is the same as (11), i.e., the solution of the optimization problem in (10).

This completes the proof.

**B. Proof of Theorem 2**

We first analyze the upper bound of the expected prediction error of the content requested possibility of one single user for the overall $D$ requests in the finite time horizon $T$. Without loss of generality, the convex loss function is chosen to be an absolute loss function, i.e., $L_d(w_u) = |\hat{p}_{t,u,d} - p_{t,u,d}|$. Then, from (23), the following relationship can be readily established

$$
\mathbb{E} \sum_{t=1}^{T} \sum_{d=1}^{D_t} |\hat{p}_{t,u,d} - p_{t,u,d}| \leq W_u G_u \sqrt{2D} + \tau_u.
$$

(35)

Furthermore, by using the relationship in (5), the expected popularity prediction error for the overall $D$ requests in the finite time horizon $T$ can be formulated as follows

$$
\mathbb{E} \sum_{t=1}^{T} \sum_{u=1}^{U_t} \sum_{d=1}^{D_t} \frac{1}{U_t} |\hat{p}_{t,u,d} - p_{t,u,d}|.
$$

(36)

By considering that $U_{\min} \leq U_t \leq U_{\max}$, the following inequality can be readily established

$$
\mathbb{E} \sum_{t=1}^{T} \sum_{d=1}^{D_t} |\hat{p}_{t,d} - p_{t,d}| \leq \frac{1}{U_{\min}} \mathbb{E} \sum_{u=1}^{U_{\max}} \sum_{t=1}^{T} \sum_{d=1}^{D_t} |\hat{p}_{t,u,d} - p_{t,u,d}|.
$$

(37)

Then, from (35), we can obtain

$$
\mathbb{E} \sum_{t=1}^{T} \sum_{d=1}^{D_t} |\hat{p}_{t,d} - p_{t,d}| \leq \frac{U_{\max}}{U_{\min}} \left(W_{\max} G_{\max} \sqrt{2D} + \tau_{\max}\right).
$$

(38)

This completes the proof.

**C. Proof of Theorem 3**

During each time period, the proposed edge caching policy always tries to cache the $M\varphi$ most popular contents. In the ideal case, the contents with the $M\varphi$ largest real popularities, i.e., $\{P'_{t,1}, P'_{t,2}, \cdots, P'_{t,M\varphi}\}$, will be cached. Due to the popularity prediction errors, the contents with the $M\varphi$ largest predicted popularities will however be cached in our proposed edge caching policy. Assume that the predicted popularities are sorted in descending order as follows: $\hat{p}'_{t,d_t^1} \geq \hat{p}'_{t,d_t^2} \geq \cdots \geq \hat{p}'_{t,d_t^{M\varphi}} \geq \cdots \geq \hat{p}'_{t,d_t^1}$. 


Obviously, \( \{\hat{P}'_{t,d'_1}, \hat{P}'_{t,d'_2}, \ldots, \hat{P}'_{t,d'_M:\varphi} \} \) represent the \( M\varphi \) largest predicted popularities. Then, the following relationship can be readily established

\[
\sum_{f=1}^{M\varphi} \hat{P}'_{t,d'_f} \geq \sum_{d'=1}^{M\varphi} \hat{P}'_{t,d'}.
\]  

(39)

According to the definition of \( \Delta P_t \), we have \(|\hat{P}'_{t,d'} - P'_{t,d'}| \leq \Delta P_t\). Correspondingly, we have

\[
\hat{P}'_{t,d'} \geq P'_{t,d'} - \Delta P_t,
\]

(40)

\[
\sum_{d'=1}^{M\varphi} \hat{P}'_{t,d'} \geq \sum_{d'=1}^{M\varphi} \left( P'_{t,d'} - \Delta P_t \right).
\]

(41)

From (25), the following relationship holds

\[
\sum_{d'=1}^{M\varphi} P'_{t,d'} = \mathcal{H}_t (\Phi^*) \sum_{d'=1}^{F_t} P'_{t,d'}.
\]

(42)

Correspondingly, from (39), (41) and (42), we can readily obtain

\[
\sum_{f=1}^{M\varphi} \hat{P}'_{t,d'_f} \geq \mathcal{H}_t (\Phi^*) \sum_{d'=1}^{F_t} P'_{t,d'} - \Delta P_t M\varphi.
\]

(43)

According to the previous descriptions in Section II, the achievable cache hit rate \( \mathcal{H}_t (\Phi) \) during the \( t \)th time period can be defined as follows

\[
\mathcal{H}_t (\Phi) = \frac{1}{D_t} \sum_{d=1}^{D_t} \theta_{t,d} (f (d)).
\]

(44)

We have assumed previously that the requests of the same content is concentrated in one time period. Therefore, the contents with the corresponding popularities \( \{\hat{P}'_{t,d'_1}, \hat{P}'_{t,d'_2}, \ldots, \hat{P}'_{t,d'_M:\varphi} \} \) will be cached after the first content request with an initial cache miss happens during the \( t \)th time period. Then, we have

\[
\sum_{d=1}^{D_t} \theta_{t,d} (f (d)) = \sum_{f=1}^{M\varphi} \left[U_t \hat{P}'_{t,d'_f} - 1\right],
\]

(45)

where \( \lfloor \cdot \rfloor \) denotes the floor operation. Then, by using the relationships \( D_t = \sum_{d'=1}^{F_t} U_t P'_{t,d'} \) and \( \lfloor x \rfloor \geq x - 1 \), we can further establish the following inequality

\[
\mathcal{H}_t (\Phi) \geq \frac{\sum_{f=1}^{M\varphi} \hat{P}'_{t,d'_f} - 2/U_t}{\sum_{d'=1}^{F_t} P'_{t,d'}}.
\]

(46)
By utilizing (43), the following relationship can be readily established

\[ H_t(\Phi) \geq H_t(\Phi^*) - \frac{M\varphi}{D_t}(\Delta P_t U_t + 2). \]  

(47)

This completes the proof.

D. PROOF OF THEOREM 4

From (26), (27) and (28), we have

\[ R(D) = \mathbb{E} \sum_{t=1}^{T} \sum_{t=1}^{T} [H_t(\Phi^*) - H_t(\Phi)] D_t. \]  

(48)

By utilizing the analytical results from Theorem 3, the following relationship can be readily established

\[ R(D) \leq \mathbb{E} \frac{M\varphi}{D} \sum_{t=1}^{T} (\Delta P_t U_t + 2). \]  

(49)

According to the definition of \( \Delta P_t \), we have \( \Delta P_t \leq \sum_{d=1}^{D_t} |\hat{P}_{t,d} - P_{t,d}'| \). Exploiting the above relationship and considering that \( U_t \leq U_{\text{max}} \), we can further obtain

\[ R(D) \leq \mathbb{E} \frac{U_{\text{max}}M\varphi}{D} \sum_{t=1}^{T} \left( \sum_{d=1}^{D_t} |\hat{P}_{t,d} - P_{t,d}'| + \frac{2}{U_{\text{max}}} \right). \]  

(50)

By using the analytical results from Theorem 2, the following relationship can be readily established

\[ R(D) \leq \frac{U_{\text{max}}M\varphi}{D} \left[ \frac{U_{\text{max}}}{U_{\text{min}}} \left( W_{\text{max}}G_{\text{max}}\sqrt{2D} + \tau_{\text{max}} \right) + \frac{2T}{U_{\text{max}}} \right]. \]  

(51)

This completes the proof.

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