Control of atomic state decay in cavities and microspheres

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Abstract. Novel methods are discussed for the state control of atoms coupled to single-mode and multi-mode cavities and microspheres. (1) Excitation decay control: The quantum Zeno effect, i.e. inhibition of spontaneous decay by frequent measurements, is observable in high-Q cavities and microspheres using a sequence of evolution-interrupting pulses or randomly-modulated CW fields. By contrast, in ‘bad’ cavities or open space, frequent measurements can only accelerate the decay, causing the anti-Zeno effect. (2) Location-dependent interference of decay channels: Control of two metastable states is feasible via resonant single-photon absorption to an intermediate state, by engineering spontaneous emission in a multimode cavity. (3) Decoherence control by conditionally interfering parallel evolutions: An arbitrary internal state of an atomic wavepacket can be protected from decoherence by interference of its interactions with the cavity over many different time intervals in parallel, followed by the detection of appropriate atomic-momentum observables. The arsenal of control methods described above can advance the state-of-the-art of quantum information storage and manipulation in cavities.

1. Introduction

It is our purpose here to demonstrate that effective control of atomic states is possible in the presence of spontaneous relaxation into non-Markovian reservoirs, characterized by non-smooth spectra. Cavities, waveguides and periodic dielectric structures are environments that may give rise to such reservoirs for electromagnetic (EM) field modes, provided the spectral range wherein their mode density strongly varies is narrow enough [1]–[4], as elaborated in this paper. Condensed media or multi-ion traps may exhibit analogous non-Markovian features as reservoirs of phonon modes.
In cavities, one may employ the following arsenal of means for the control of radiative decay: (i) **mode density control**, which allows the suppression or enhancement of the decay rate in selected spectral ranges; (ii) **atomic location control**, which allows the adjustment of the atomic coupling to a cavity mode, by varying the atomic position relative to its field nodes (for atoms deposited on a thin film or cold, trapped atoms); (iii) **mode polarization control** (e.g. in microspheres); and (iv) **measurement control**, namely, non-selective or conditional measurements (CMs) of an *auxiliary variable* that is correlated to the internal state of the atom.

In what follows, we shall demonstrate how this arsenal can be implemented in several methods we have developed for quantum state control in cavities and microspheres: (a) strong-coupling control in microspheres [1] (section 2); (b) population decay control by frequent or continuous measurements, realizing the quantum Zeno or anti-Zeno effects [2, 3] (section 3); (c) complete control of state preparation by location-dependent interference of decay channels [4] (section 4); and (d) control of decoherence by CM-induced interference of evolution histories [5] (section 5). The aims will be to consolidate and integrate the presentation of the different methods, clarify their essence, by means of computer movies and other graphical illustrations, and finally compare their advantages and disadvantages (section 6).

### 2. Strong-coupling QED in a microsphere

#### 2.1. Background

Among the resonator configurations that may lead to the realization of strong-coupling QED effects in the optical domain, dielectric microspheres are particularly promising and important for the following reasons.

(a) The ability of dielectric microspheres to act as high-quality optical resonators [6]–[10] has been proven in a variety of experiments [6, 9], which have indicated sufficiently long mode lifetimes (*Q* values up to $10^9$) to allow the observation of strong-coupling QED in the microsphere, with negligible dissipative effects (well within the mode lifetime).

(b) The evanescent tail of a high-*Q* field mode in a dielectric microsphere can be selectively and strongly coupled to a resonance of an atom located up to a few wavelengths outside the surface [6]. Hence, strong-coupling QED effects should be observable in an atomic beam passing near a microsphere. Such effects can be augmented by binding cold atoms in an orbit around a dielectric microsphere via an off-resonant two-photon interaction with its field [7].

(c) From the conceptual point of view, both classical and QED nonlinear processes in dielectric microspheres are intriguing because of their unique features: (i) the spherical symmetry, which implies *mode degeneracy* and angular-momentum conservation; (ii) the inseparability of the optical fields inside and outside the sphere (leaky modes) [6, 8, 9].

#### 2.2. The modes and field–atom interaction

We are primarily interested in high-$l$ modes in spheres whose radii are much larger than an optical wavelength, since these modes couple predominantly to atoms close to the sphere surface [6]. The azimuthal mode number $m$ can be specified as the projection of the angular momentum on the $z$-axis. In an ideal sphere all $m$ modes with given $l$, $n$, and polarization (TM or TE) are degenerate. In reality, however, this degeneracy is always lifted to some extent due to
distortions that result in a spheroidal shape, which, for the sake of concreteness, is taken here as oblate. This distortion splits the \((2l + 1)\)-degenerate modes from the resonance frequency \(\omega_\lambda\) \((\lambda = l, n, \beta, \text{ where } \beta = \text{TM or TE})\) of an ideal-sphere mode into a manifold of \(m\)-dependent frequencies \(\omega_\lambda(m)\). For a small eccentricity \(\epsilon\), which is the fractional difference of the polar and equatorial radii, and \(l \gg 1\), these \(m\)-dependent frequencies are approximately given by \([10]\)

\[
\omega_\lambda(m) \approx \omega_\lambda \{1 + (\epsilon/6)[3m^2/|l(l + 1) - 1]\}.
\]

The fractional splitting \(\omega_\lambda(m)/\omega_\lambda\) is independent of the polarization (TM or TE), the radial order \(n\), and the radius.

It is possible to selectively populate only a few \(m\) modes out of the \((2l + 1)\)-fold multiplet by using the spatial confinement of the different \(m\) modes (figure 1(a)) and the \(m^2\) scaling of their splitting. For non-zero eccentricity, the atom will be chosen to lie near the polar axis of the degenerate modes with \(\lambda = l\). For \(m\gg1\), these \(m\)-dependent frequencies are approximately given by \([10]\)

\[
\Delta \omega_\lambda \approx \frac{\omega_\lambda}{\omega_\lambda} \left\{1 + \frac{\epsilon}{6}\left[\frac{3m^2}{|l(l + 1) - 1|}\right]\right\}.
\]

In the case of an ideal-sphere cavity, we shall restrict our treatment to \(m = 0, \pm 1\), by considering an atom near the sphere centre, which is resonant with the \(l = 1\) multiplet.

In either case, as the \(\Delta m_j = 0\) (\(\pi\)-polarized) atomic transition implies a radially polarized (\(\parallel z\)-axis) electric field, it cannot couple to the TE mode. Hence, only the \(\sigma\)-polarized transitions with \(\Delta m_j = \pm 1\) couple to the \(m = \pm 1\) TE modes, defining a \(\Lambda\) (or \(V\)) degenerate configuration (figure 1(b)). By contrast, a TM mode couples to both \(\sigma\)- and \(\pi\)-polarized transitions, and therefore gives rise to two \(\text{coupled } \Lambda\) (or \(V\)) systems.

The corresponding field–atom interaction Hamiltonian can be written in the rotating-wave approximation as

\[
\mathcal{H}_{\text{int}} = \hbar d \sum_\lambda a_\lambda \chi_\lambda \cdot \Pi_{+\lambda} + \text{h.c.}
\]

Here \(a_\lambda\) is the \(\lambda\)-mode annihilation operator satisfying the commutation relation \([a_\lambda, a_{\lambda'}^\dagger] = \delta_{\lambda,\lambda'}\), \(d\) is the dipole moment, \(\Pi_{+\lambda}\) is the appropriate dipolar raising operator, and

\[
\chi_\lambda(r) = \begin{cases} R_{l,n}^{\text{TE}}(r) \hat{L} Y^m_l(\theta, \phi) & \beta = \text{TE} \\ (ic/\omega_\lambda) \nabla \times R_{l,n}^{\text{TE}}(r) \hat{\mathcal{L}} Y^m_l(\theta, \phi) & \beta = \text{TM} \end{cases}
\]

are the mode eigenfunctions. Here the functions \(R_{l,n}^{\beta}\) are real solutions of the radial wave equation

\[
\left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + k^2 \epsilon(r) - \frac{l(l + 1)}{r^2} \right] R_l(r) = 0
\]

\(Y^m_l\) are the spherical harmonics, and \(\hat{\mathcal{L}}\) is the angular momentum operator. It should be stressed that for small eccentricities \((\epsilon \lesssim 10^{-4})\), the radial character of \(R_l(r)\) is practically the same as in the ideal sphere.

In particular, the TE modes with \(l = 1\) can be expressed as

\[
\chi_{\pm 1}^{\text{TE}}(r) = \mp i \sqrt{2} c R_{1,n}^{\text{TE}}(r) \hat{e}_{\pm 1} \quad \chi_0^{\text{TE}}(r) = 0
\]

and the \(l = 1\) TM modes can be expressed as

\[
\chi_{\pm 1}^{\text{TM}}(r) = \pm \left( -\sqrt{2} c r \right) \frac{\partial}{\partial r} [r R_{1,n}^{\text{TM}}(r)] \hat{e}_{\pm 1} \quad \chi_0^{\text{TM}}(r) = -2 \sqrt{2} c r R_{1,n}^{\text{TM}}(r) \hat{e}_0
\]
Figure 1. (a) Atomic coupling to $m = 0, \pm 1$ modes at the spheroidal surface. The modes can be externally excited. (b) The coupling of atomic $j = 0 \leftrightarrow j = 1$ transitions to the electric field at the microsphere surface: the $\pi$-polarized transition ($|g\rangle \leftrightarrow |e_0\rangle$) couples only to the radially polarized TM mode ($m = 0$), whereas the $\sigma$-polarized transition ($|g\rangle \leftrightarrow |e_{\pm 1}\rangle$) couples to both TE and TM tangentially polarized modes ($m = \pm 1$). Inset: The levels corresponding to the above degenerate transitions.
where the unit vector components are \( \hat{e}_{\pm 1} = 2^{-1/2}(\hat{x} \pm i\hat{y}) \), \( \hat{e}_0 = \hat{z} \).

In open (leaky) dielectric spheres, there are distinct Mie resonances labelled \((\beta, l, n)\), with \( l \gg 1 \), corresponding to high-\( Q \) ‘quasimodes’ localized near the sphere surface with evanescent ‘tails’ outside \([6]\). A quantization procedure for such ‘quasimodes’ \([9]\) shows that they can be effectively treated as nearly discrete, 

\[ h_{\ell}(r) = C h_{l}^{(1)}(k ln r) \]

where \( \beta = \text{TM or TE} \), \( C \) is the normalization constant, and \( k ln \) belongs to a discrete set of wavenumbers (Mie resonances). We reiterate that for very small eccentricities, \( h_{l}^{(1)} \) is nearly identical with the spheroidal eigenfunction \( h_{le}^{(1)} \).

### 2.3. Diagonalization of the field–atom Hamiltonian

The RWA Hamiltonian of interaction (equation (1)) between a \( j = 0 \leftrightarrow j = 1 \) dipole transition and a triplet of effectively degenerate angular-momentum eigenmodes, \( m = 0, \pm 1, l \geq 1 \), in the cases discussed above, can be diagonalized in the four-dimensional subspace of field–atom product states:

\[
\begin{align*}
|e_1; N_1 - 1, N_0, N_{-1}\rangle & \quad |e_0; N_1, N_0 - 1, N_{-1}\rangle \\
|e_{-1}; N_1, N_0, N_{-1} - 1\rangle & \quad |g; N_1, N_0, N_{-1}\rangle.
\end{align*}
\]  

(7a)

Here the relevant atomic eigenstates are \( \{|e_1\rangle, |e_0\rangle, |e_{-1}\rangle, |g\rangle\} \) corresponding to the three excited states \((j = 1, m_j = 0, \pm 1)\) and one ground state \((j = 0, m_j = 0)\). The photon numbers in modes with \( m = 0, \pm 1, l \geq 1 \), are denoted by \( N_{\pm 1}, N_0 \), respectively. We restrict our basis to photon numbers such that

\[
\sum_{m=\pm 1,0} N_m \equiv N \geq 1
\]  

(7b)

\( N \) being the total photon number when the atom is in the \( |g\rangle \) state.

Diagonalization of the Hamiltonian yields the eigenfrequencies

\[
E_{1,2} = \hbar (N \omega - \Delta) \quad E_{3,4} = \hbar (N \omega - \Delta/2 \pm W_N)
\]

(8)

where \( \omega \) is the mode frequency, \( \Delta = \omega - \omega_0 \) is the detuning from the atomic resonance frequency, and, using the fact that \( |\chi_1\beta|^2 = |\chi_{-1}\beta|^2 \), we express the field-induced energy shift as

\[
W_N \equiv \sqrt{(\Delta/2)^2 + N_+|\chi_1\beta|^2 + N_0|\chi_0\beta|^2}
\]  

(9a)

with

\[
N_+ \equiv N_1 + N_{-1}.
\]

(9b)

Consistent with (7b), if \( \chi_0 = 0 \) (as in the TE case), then \( N_+ \geq 1 \), and if both \( \chi_0, \chi_1 \neq 0 \) then \( N = N_+ + N_0 \geq 1 \). The corresponding eigenvectors are given by

\[
|u_{1,2}\rangle = \begin{pmatrix} e_1 \\ e_0 \\ c_0 \\ c_{-1} \\ 0 \end{pmatrix}
\]

(10)
with coefficients that satisfy the equation
\[ \sum_{m=\pm 1,0} c_m \chi_m^* \sqrt{N_m} = 0 \]  
(11)

and
\[ |u_{3,4}\rangle = \frac{1}{\sqrt{2W_N^2 \pm \Delta W_N}} \begin{pmatrix} \chi_1 \sqrt{N_1} \\ \chi_0 \sqrt{N_0} \\ \chi_{-1} \sqrt{N_{-1}} \\ \Delta/2 \pm W_N \end{pmatrix}. \]  
(12)

Equations (10) and (11) show that there are two ‘dark’ (trapping) states \( |u_{1,2}\rangle \) in which the atom is excited and stable against emission of the photon, because the two emission processes (with left- and right-circular photon polarizations) in these states exhibit destructive interference. The corresponding eigenvalues \( E_{1,2} \) (equation (8)) are not affected by the field–atom interaction; i.e. the atom is essentially decoupled from the field. As is well known, in a \( \Lambda \) configuration there is only one dark (trapping) state.

Note that the TE case \( (\chi_{0,TE} = 0) \) is equivalent to the TM case with either \( N_0 = 0 \) or with the atom located at a \( \chi_{0,TM} \) node. In the following, the indices \( \beta \) and \( j \) will be suppressed unless required.

2.4. Generalized Rabi oscillations and damping (decay)

The eigenenergies and eigenvectors derived above allow us to calculate the time-dependent atomic transition amplitudes for a given photon number. When the atom is initially in the ground state, and the state at \( t = 0 \) is \( |g; \{N_m\} = \{N_1, N_0, N_{-1}\}\rangle \), the \( |g\rangle \rightarrow |e_m\rangle \) transition probabilities are
\[ P_{\{N_m\}}^{g \rightarrow e_m} = N_m |\chi_m|^2 \sin^2(W_N t)/W_N^2. \]  
(13)

Hence, Rabi oscillations occur with a frequency \( W_N = \sqrt{(\Delta/2)^2 + |\chi|^2 N} \) that depends on the total number of photons (when the atom is in \( |g\rangle \)). In the TE case, \( N_+ = N_1 + N_{-1} \) is the relevant photon number. The system then behaves as a two-level atom oscillating between the ground state \( |g\rangle \) and the superposition of excited states \( |e_+\rangle \equiv (1/\sqrt{N_+})(\sqrt{N_1}|e_1\rangle + \sqrt{N_{-1}}|e_{-1}\rangle) \), interacting effectively with a single mode having \( N_+ \) photons. The reason for this behaviour is that by taking the appropriate linear combinations of the \( m = \pm 1 \) modes, we can construct two orthogonal elliptically polarized modes, such that one mode is in a \( N_+ \)-photon state and the other is in its vacuum state.

To conclude, atomic decay in the strong-coupling regime associated with an elliptic Mie-resonant mode can be treated along the same lines as decay into a leaky cavity mode. An atom, initially in the ground state, can only exchange one \textit{elliptically polarized} photon with such a mode. The broadening of Mie resonances by field leakage from the sphere endows each resonance frequency \( \omega_\chi \) with width \( i\Gamma_\chi \).

3. The Zeno and anti-Zeno effects on decay in cavities

3.1. Background

The quantum Zeno effect (QZE) is a manifestation of the influence of measurements on the evolution of a quantum system. The original QZE prediction has been that \textit{irreversible decay}...
of any excited state can be inhibited [11] by repeated measurements (e.g. the interaction of an unstable particle with its environment) [12, 13]. The interruption of Rabi oscillations has been at the focus of interest [15]–[22]. Recently, the QZE has been considered for tunnelling from a potential well into the continuum [23], as well as for photoionization [24]. Tacit assumptions have been made that the QZE is in principle attainable for radiative decay in open space, but is technically difficult.

We have recently demonstrated [2] that the energy uncertainty (spread) incurred by frequent projections fundamentally restricts the QZE observability and renders decay acceleration by frequent measurements far more ubiquitous than its inhibition. We refer to this universal effect (first discovered by us for spontaneous emission in cavities [3]) as the anti-Zeno effect (AZE). By contrast, a fundamental limitation on the observability of the QZE emerges from our theory: the intervals \( \tau \) between consecutive measurements required for the QZE are so short for many processes (such as radiative and radioactive decay) that the resulting energy spread may destroy the observed system by coupling it to unwarranted channels, worst of all the creation of new particles (e.g. electron–positron pair creation by a spontaneously emitting atom). We have shown, by means of our unified theory of spontaneous emission into arbitrary reservoirs [25] that two-level emitters in cavities or in waveguides are in fact adequate for radiative decay control by the QZE [3]. Condensed media or multi-ion traps are their analogues for vibrational decay control (phonon emission) by the QZE [4].

Here we wish to demonstrate that the QZE is indeed achievable, either by frequent impulsive measurements or by continuous measurements of the excited state, but only in reservoirs whose spectral response rises up to a frequency which does not exceed the resonance (transition) frequency. Specifically, we shall consider high-\( Q \) optical cavities and dielectric microspheres, where the strong-coupling QED regime prevails. By contrast, in open-space (or bad-cavity) decay, where the reservoir response has a relativistic cut-off frequency, non-destructive frequent measurements are much more likely to accelerate decay, causing the anti-Zeno effect.

3.2. Impulsive measurements (Cook’s scheme)

Consider an initially excited two-level atom coupled to an arbitrary density-of-modes (DOM) spectrum \( \rho(\omega) \) of the electromagnetic field in the vacuum state. At time \( \tau \) its evolution is interrupted by a short optical pulse, which serves as an impulsive quantum measurement [14]–[22]. Its role is to break the evolution coherence, by transferring the populations of the excited state \( |e\rangle \) to an auxiliary state \( |u\rangle \), which then decays back to \( |e\rangle \) incoherently.

A possible realization of this scheme is as follows. Within an open cavity the atoms repeatedly interact with a pump laser, which is resonant with the \( |e\rangle \rightarrow |u\rangle \) transition frequency. The resulting \( |e\rangle \rightarrow |g\rangle \) fluorescence rate is collected and monitored as a function of the pulse repetition rate \( 1/\tau \). Each short, intense pump pulse of duration \( t_p \) and Rabi frequency \( \Omega_p \) is followed by spontaneous decay from \( |u\rangle \) back to \( |e\rangle \), at a rate \( \gamma_u \), so as to destroy the coherence of the system evolution, on the one hand, and reshuffle the entire population from \( |e\rangle \) to \( |u\rangle \) and back, on the other (figure 2). The demand that the interval between measurements should significantly exceed the measurement time, yields the inequality \( \tau \gg t_p \). The above inequality can be reduced to the requirement \( \tau \gg \gamma_u^{-1} \) if the ‘measurements’ are performed with \( \pi \) pulses:
28.8

\[ \Omega_p t_p = \pi, \quad t_p \ll \gamma_u^{-1}. \]

This calls for choosing a \( |u \rangle \rightarrow |e \rangle \) transition with a much shorter radiative lifetime than that of \( |e \rangle \rightarrow |g \rangle \).

To consider the measurement-affected decay of the excited state \( |e \rangle \), we cast its amplitude in the form \( \alpha_e(\tau) e^{-i\omega_a \tau} \), where \( \omega_a \) is the atomic resonance frequency. Restricting ourselves to sufficiently short interruption intervals \( \tau \) such that \( \alpha_e(\tau) \simeq 1 \), yet long enough to allow the rotating wave approximation, we obtain

\[
\alpha_e(\tau) \simeq 1 - \int_0^\tau dt (\tau - t) \Phi(t) e^{i\Delta t}. \tag{14}
\]

Here

\[
\Phi(t) = \int_0^\infty d\omega G(\omega) e^{-i(\omega - \omega_s) t} \tag{15}
\]

is the Fourier transform of \( G(\omega) \), the spectral response, i.e. \( (2\pi)^{-1} \) times the emission rate into the reservoir at frequency \( \omega \),

\[
G(\omega) = |g(\omega)|^2 \rho(\omega) \tag{16}
\]

\( \rho(\omega) \) being the DOM, \( \hbar g(\omega) \) the field–atom coupling energy, and \( \Delta = \omega_a - \omega_s \) is the detuning of the atomic resonance from the peak (or cut-off) \( \omega_s \) of \( G(\omega) \).

To first order in the atom–field interaction, the excited-state probability after \( n \) interruptions (measurements), \( W(t = n\tau) = |\alpha_e(\tau)|^{2n} \), can be written as

\[
W(t = n\tau) \approx [2 \Re \alpha_e(\tau) - 1]^n \approx e^{-\kappa t} \tag{17}
\]

where

\[
\kappa = \frac{2}{\tau} \Re[1 - \alpha_e(\tau)] = \frac{2}{\tau} \Re \int_0^\tau dt (\tau - t) \Phi(t) e^{i\Delta t}. \tag{18}
\]

The QZE is obtained if \( \kappa \) decreases with \( \tau \) for sufficiently short \( \tau \). This essentially means that the correlation (or memory) time of the field reservoir is longer (or, equivalently, \( \Phi(t) \) falls off slower) than the chosen interruption interval \( \tau \).

Equation (18) can be rewritten as

\[
\kappa = 2\pi \int G(\omega) \left\{ \frac{\tau}{2\pi} \sin^2 \left[ \frac{(\omega - \omega_a)\tau}{2} \right] \right\} d\omega \tag{19}
\]

where the interruptions are seen to cause dephasing whose spectral width is \( \sim 1/\tau \).

**Figure 2.** Cook’s scheme for impulsive measurements (see description in text).
3.3. Noisy-field dephasing: random Stark shifts

Instead of disrupting the coherence of the evolution by a sequence of ‘impulsive’ measurements, as above, we can achieve this goal by noisy-field dephasing of $\alpha_e(t)$: random ac-Stark shifts by an off-resonant intensity-fluctuating field result in the replacement of equation (19) by (figure 3)

$$\kappa = \int G(\Delta + \omega_a) \mathcal{L}(\Delta) d\Delta. \quad (20)$$

Here the spectral response $G(\Delta + \omega_a)$ is the same as in equation (16), whereas $\mathcal{L}(\Delta)$ is the Lorentzian-shaped relaxation function of the coherence element $\rho_{eg}(t)$, which for the common dephasing model decays exponentially. This Lorentzian relaxation spectrum has a HWHM width $\nu = \langle \Delta \omega^2 \rangle \tau_c$, the product of the mean-square Stark shift and the noisy-field correlation time. The QZE condition is that this width be larger than the width of $G(\omega)$ (figure 3). The advantage of this realization is that it does not depend on $\gamma_u$, and is realizable for any atomic transition. Its importance for molecules is even greater: if we start with a single vibrational level of $|e\rangle$, no additional levels will be populated by this process.

3.4. Continuous measurements

The random ac-Stark shifts described above cause both shifting and broadening of the spectral transition. If we wish to avoid the shifting altogether, we may employ a CW driving field that is nearly resonant with the $|e\rangle \leftrightarrow |u\rangle$ transition [15, 16]. If the decay rate of this transition, $\gamma_u$, is larger than the Rabi frequency $\Omega$ of the driving field, then one can show that $\kappa$ is given again by equation (20), where the Lorentzian (dephasing) width is

$$\nu = \frac{2\Omega^2}{\gamma_u}. \quad (21)$$

This scheme is a realization of continuous measurements with the effective rate $\nu$.

3.5. Universal formula

All of the above schemes are seen to yield the same universal formula for the decay rate

$$\kappa = 2\pi \int G(\omega) F(\omega - \omega_a) d\omega \quad (22)$$
where $F(\omega)$ expresses the relevant measurement-induced dephasing (sinc- or a Lorentzian-shaped): its width relative to that of $G(\omega)$ determines the QZE behaviour.

3.6. Spectrally finite reservoirs: QZE and AZE for Lorentzian lines

The simplest application of the above analysis is to the case of a two-level atom coupled to a near-resonant Lorentzian line centred at $\omega_s$, characterizing a high-$Q$ cavity mode [3, 26]. In this case,

$$G_s(\omega) = \frac{g_s^2 \Gamma_s}{\pi [\Gamma_s^2 + (\omega - \omega_s)^2]}$$

(23)

where $g_s$ is the resonant coupling strength and $\Gamma_s$ is the linewidth (figure 4). Here $G_s(\omega)$ stands for the sharply-varying (nearly-singular) part of the DOM distribution, associated with narrow cavity-mode lines or with the frequency cut-off in waveguides or photonic band edges. The broad portion of the DOM distribution $G_b(\omega)$ (the ‘background’ modes) always coincides with the free-space DOM $\rho(\omega) \sim \omega^2$ at frequencies well above the sharp spectral features. In an open cavity, $G_b(\omega)$ represents the atom coupling to the unconfined free-space modes. This gives rise to an exponential decay factor in the excited-state probability, regardless of how short $\tau$ is, i.e.

$$\kappa = \kappa_s + \gamma_b$$

(24)

where $\kappa_s$ is the contribution to $\kappa$ from the sharply-varying modes and $\gamma_b = 2\pi G_b(\omega_a)$ is the effective rate of spontaneous emission into the background modes. In most structures $\gamma_b$ is comparable to the free-space decay rate $\gamma_f$.

In the short-time approximation, taking into account the fact that the Fourier transform of the Lorentzian $G_s(\omega)$ is $\Phi_s(t) = g_s^2 e^{-\Gamma_s t}$, equation (14) yields (without the background-modes contribution)

$$\alpha_e(\tau) \approx 1 - \frac{g_s^2}{\Gamma_s - i\Delta} \left[ \tau + \frac{e^{(i\Delta - \Gamma_s)\tau} - 1}{\Gamma_s - i\Delta} \right].$$

(25)

The QZE condition is then

$$\tau \ll (\Gamma_s + |\Delta|)^{-1}, g_s^{-1}.$$  

(26)

On resonance, when $\Delta = 0$, equations (18) and (25) yield

$$\kappa_s = g_s^2 \tau.$$  

(27)

Thus the background-DOM effect cannot be modified by QZE. Only the sharply-varying DOM contribution $\kappa_s$ may allow for QZE. Only the $\kappa_s$ term decreases with $\tau$, indicating the

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Figure 5. Evolution of excited-state population $W$ in a two-level atom coupled to the cavity mode with Lorentzian lineshape on resonance, $(\Delta = 0)$: red curve, uninterrupted decay in cavity with $F \equiv (1 - R)^{-2} = 10^4$, $L = 15$ cm, and $f = 0.02$; green curve, interrupted evolution at intervals $\tau = 3 \times 10^{-8}$ s; yellow dots denote the interruption moments. Here $\gamma_b \simeq \gamma_f = 10^6$ s$^{-1}$. See animation.

Figure 6. As in figure 5, for detuning $\Delta = 10^8$ s$^{-1}$ and $F = 10^5$. See animation.

QZE inhibition of the nearly-exponential decay into the Lorentzian field reservoir as $\tau \to 0$. Since $\Gamma_s$ has dropped out of equation (27), the decay rate $\kappa$ is the same for both strong-coupling ($g_s > \Gamma_s$) and weak-coupling ($g_s \ll \Gamma_s$) regimes. Physically, this comes about since for $\tau \ll g_s^{-1}$ the energy uncertainty of the emitted photon is too large to distinguish between reversible and irreversible evolutions.

The evolution inhibition, however, has a rather different meaning for the two regimes. In the weak-coupling regime, where, in the absence of the external control, the excited-state population decays nearly exponentially at the rate $g_s^2/\Gamma_s + \gamma_b$ (at $\Delta = 0$), one can speak about the inhibition of irreversible decay, in the spirit of the original QZE prediction [11]. By contrast, in the strong-coupling regime in the absence of interruptions (measurements), the excited-state population
undergoes damped Rabi oscillations at the frequency $2g_s$. In this case, the QZE slows down the evolution during the first Rabi half-cycle ($0 \leq t \leq \pi/2g_s^{-1}$), the evolution on the whole becoming irreversible.

Figure 5, describing the QZE for a Lorentz line on resonance ($\Delta = 0$), has been programmed for feasible cavity parameters: $\Gamma_s = (1 - R)c/L$, $g_s = \sqrt{cf\gamma_f/(2L)}$, $\gamma_b = (1 - f)\gamma_f$, where $R$ is the geometric-mean reflectivity of the two mirrors, $f$ is the fractional solid angle (normalized to $4\pi$) subtended by the confocal cavity, and $L$ is the cavity length. It shows that the population of $|e\rangle$ decays nearly-exponentially well within interruption intervals $\tau$, but when those intervals become too short, there is significant inhibition of the decay.

Figure 6 shows the effect of the detuning $\Delta = \omega_a - \omega_s$ on the decay: the decay now becomes oscillatory. The interruptions now enhance the decay, in the spirit of the anti-Zeno effect (AZE) [2]. The degree of enhancement depends on the phase between interruptions.

3.7. The bad-cavity limit and open-space reservoirs: AZE

The spectral response for hydrogenic-atom radiative decay via the $\vec{p} \cdot \vec{A}$ free-space interaction is given by [27]

$$G(\omega) = \frac{\alpha \omega}{[1 + (\omega/\omega_c)^2]^{\frac{3}{4}}}$$

where $\alpha$ is the effective field–atom coupling constant and the cut-off frequency is

$$\omega_c \approx 10^{19} \text{ s}^{-1} \sim \frac{c}{\alpha_b}. \quad (29)$$

Using measurement control that produces Lorentzian broadening (equation (20)) we then obtain

$$\kappa = \frac{\alpha \omega_c}{3} \text{Re} \left[ \frac{f(2f^4 - 7f^2 + 11)}{2(f^2 - 1)^3} - \frac{6f \ln f}{(f^2 - 1)^4} - \frac{3i\pi(f^2 + 4f + 5)}{16(f + 1)^4} \right]$$

where

$$f = \frac{\nu - i\omega_a}{\omega_c}. \quad (31)$$

In the range

$$\nu \ll \omega_c \quad (32)$$

we obtain from equation (30) the anti-Zeno effect of accelerated decay. This comes about due to the rising of the spectral response $G(\omega) \approx \alpha \omega$ as a function of frequency (for $\omega \ll \omega_c$). The Zeno effect can hypothetically occur only for $\nu \sim \omega_c \sim 10^{19} \text{ s}^{-1}$. However, this range is well beyond the limit of validity of the present analysis, since $\Delta E \sim \hbar \nu \sim \hbar \omega_c$ may then induce other channels, in addition to spontaneous transitions to $|g\rangle$, notably the breakdown of the rotating-wave approximation for $\nu \sim \omega_c$ and electron–positron pair creation at $\nu \sim \omega_c$ [2].

4. State control by location-dependent interference of decay channels

The conventional approaches to state control in a multilevel system are based on interference of absorbed or emitted radiation in two or more channels (transitions), which allows for coherences between field-dressed states of the system [28]–[36]. In these approaches, spontaneous
Figure 7. (a) A scheme of location-dependent interference of decay channels in a cavity. The vertical line denotes a thin film, $f_{\text{in(out)}}$ denote an incoming and outgoing photon, $r(l)$ and 1(2) subscripts denote photon propagation to the right (left) near $\omega_1$-green ($\omega_2$-red). (b) A level scheme for atoms in a film.

emission has an adverse effect on interference and its suppression is highly desirable. The adiabatic population transfer method (STIRAP), for example, achieves this goal by preventing the intermediate decaying state from being populated [34]. Here, by contrast, we outline a state-control method [4] wherein spontaneous emission from the intermediate state is, in fact, beneficial, since it exhibits interference between competing channels (transitions), owing to the boundary conditions on the cavity.

We envisage a thin film of atoms or molecules located at a certain position relative to the cavity mirrors. The coupling to the cavity field in the ‘bad cavity’ regime effectively amounts to having a one-dimensional (1D) mode continuum, corresponding to spontaneous emission predominantly into one or several Lorentz-broadened modes along the cavity axis. Let us assume that one of the cavity mirrors admits a single photon, a spectrally narrow wavepacket around $\omega_1$. This input condition can be realized by unidirectional mirror transparency near $\omega_1$, or by a measurement verifying that the incoming photon has not been reflected by this mirror. Let us consider the atoms on the thin film to be in the $\Lambda$-configuration, such that $|1\rangle \leftrightarrow |3\rangle$ transition and $|2\rangle \leftrightarrow |3\rangle$ transition (figure 7).

Suppose that the task at hand is population transfer from $|1\rangle$ to $|2\rangle$, by means of resonant absorption of the $\omega_1$-photon together with spontaneous emission, which is concentrated in the bands around $\omega_1$ and $\omega_2$. The Wigner–Weisskopf solution for this system, at times much longer than the spontaneous lifetime in the cavity $\gamma^{-1}$, can be obtained upon taking account of the boundary conditions at the mirrors and neglecting the retardation of the photon (due to its travel between the mirrors) relative to the atomic response [4]. This solution satisfies the requirement for complete (100%) population transfer from $|1\rangle$ to $|2\rangle$, provided that

$$R_1 e^{i\theta_1} (1 + R_2 e^{i\phi_2}) (1 + r_2 e^{i\phi_2}) = \frac{g_1^2}{g_2^2}. \quad (33)$$

Here $g_{1(2)}$ are the vacuum field–dipole couplings (vacuum Rabi frequencies) for the $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ transitions, respectively; $R_{1(2)}$ and $r_{1(2)}$ are the $\omega_1$- or $\omega_2$-reflectivities of the left-hand and right-hand mirrors, respectively, whereas $\theta_{1(2)}$ ($\phi_{1(2)}$) are the corresponding phase delays accumulated by reflection and round-trip travel to and from the left-hand (right-hand) mirror. Note that $r_1$, $\phi_1$ do not appear in this expression, since the complete $|1\rangle \rightarrow |2\rangle$ transfer condition must be accompanied by complete conversion of the incident $\omega_1$-photon into the $\omega_2$-photon (figure 7).

Figure 8 traces the evolution of a $\omega_1$-photon wavepacket entering the cavity under condition (33). We can watch the synchronism between its disappearance and the build-up of the exiting $\omega_2$-photon wavepacket.
Figure 8. Wavepacket conversion from $\omega_1$ input (green envelope) to $\omega_2$ output (red envelope) in the cavity under condition (33). $\tau \sim 50\gamma_c^{-1}$ gives an error of $\sim 10^{-4}$ in population transfer. The distance between mirror and atom is assumed to be much less than the pulse envelope (shown not to scale), so that the steady-state approximation holds for the field between atom and mirror (multiple reflections occur within the pulse propagation time, and are not resolvable on this time scale). See animation.

If we take $R_1 = R_2 = 1$, $\theta_1 = \theta_2 = 2\pi$, i.e. an ideally-reflecting right-hand mirror and an anti-node position for the thin film, then we must choose $r_2^2 = |g_1^2 - g_2^2|/(g_1^2 + g_2^2)$ and $\phi_2 = 2\pi$ for $g_1 > g_2$ ($\phi_2 = \pi$ for $g_1 < g_2$). In turn, the $g_1/g_2$ ratio is determined by the height ratio of the mode Lorentzian around $\omega_1$ and $\omega_2$ and the dipole moments $\mu_1, \mu_2$ for these transitions. These conditions impose totally destructive interference between the $\omega_1$-emission and its reflection, and constructive interference between their $\omega_2$-counterparts.

In principle, it is possible to prepare any superposition of states $|1\rangle$ and $|2\rangle$ by this method, whereby the intermediate state $|3\rangle$ is excited by a single photon at $\omega_1$ or $\omega_2$, and decays into two channels with appropriately chosen interference between the emitted and reflected amplitudes. Errors in this state control will be incurred by the decay rate $\gamma_b$ of $|3\rangle$ into the background modes (unconfined by the cavity). In this respect, a cavity with large solid angle confinement, wherein $\gamma_b$ is partly suppressed, is helpful. In any case, the success of this method depends on the satisfaction of the following condition by the state preparation time (or incident pulse duration) $\tau$:

$$\gamma_b^{-1} \gg \tau \gg \gamma_c^{-1}$$

which implies that the spontaneous emission rate in the cavity, $\gamma_c$, must greatly exceed the decay rate $\gamma_b$ into unconfined modes.

5. Decoherence control by conditionally interfering parallel evolutions

5.1. Principles

The true challenge of protecting quantum states from decoherence is to devise a simple, low-cost method that would apply universally to any state, i.e. it would not require any prior knowledge...
of the state. Such a method could be of interest for quantum information studies in systems with many degrees of freedom, for example multimode cavities, microspheres or ion traps. Here we graphically illustrate the principles of such a method, which we denote by CIPE (conditionally interfering parallel evolutions), that can control the evolution of an internal atomic state that is coupled to many discrete modes in a cavity reservoir. An analogous method is possible for ion traps [5]. This method does not require the control or ‘engineering’ of the reservoir, but only atomic momentum control by atomic gratings and detectors, which is adapted to the given mode structure.

If the atom is initially in a superposition of its two internal states, and the field in the vacuum state,

$$|\psi^{(i)}_{A+F}\rangle = (B|e\rangle + C|g\rangle)|0\rangle$$  \hspace{1cm} (35)

then the atomic interaction with $M$ field modes during a time $t$ results in an entangled state

$$U(t)|\psi^{(i)}_{A+F}\rangle = B\left(h(t)|e\rangle|0\rangle + \sum_{m=0}^{M} f_m(t)|g\rangle|1_m\rangle\right) + C|g\rangle|0\rangle$$  \hspace{1cm} (36)

with certain $h(t)$ and $f_m(t)$ dictated by the unitary evolution operator $U(t)$ of the system, as elaborated below. The atom-field entanglement in (36) translates, upon tracing over the field degrees of freedom, into decoherence of the reduced atomic density matrix. In order to eliminate the decoherence in the atomic state, we suggest the superposition of many evolution paths of the atom–field system, differing in the duration of the atom–field interaction [5]. This can be achieved by a CM of ‘ancilla’ atomic variables which determine the field–atom interaction time, for example the atomic momentum.

By passing the atom through an appropriately designed diffraction grating, before it enters the cavity, we obtain at the exit from the cavity a combined (field–atom) state of the form

$$|\psi^{(f)}_{A+F}\rangle = \sum_j \beta_j |p_j\rangle U(t_j)|\psi^{(i)}_{A+F}\rangle.$$  \hspace{1cm} (37)

Here the superposed intracavity $A-F$ evolution operators $U(t_j)$ are functions of the evolution time, $t_j = mL/p_{\perp j}$, which is the corresponding cavity traversal time determined by the momentum projections $p_{\perp j}$ perpendicular to the cavity axis (figure 9), while $\beta_j$ are the corresponding diffraction amplitudes. The CM projects this state onto an appropriate
superposition of momentum states. This can be done by passing the atom through a second grating after the cavity whose transmission function is centred at the same momenta \( |\vec{p}_j\rangle \), but with different amplitudes \( \tilde{\beta}_j \). The CM amounts to detecting the atom in the direction corresponding to \( |\vec{p}_l\rangle \). The system obtained by such a CM is then given by

\[
|\psi_A^{(f)}\rangle = \sum_j \alpha_j U(t_j) |\psi_A^{(i)}\rangle
\]  

with \( \alpha_j = P^{-1/2} \beta_j \beta_{l-j} \), where \( P \) is the success probability of the momentum state detection.

Tracing over the field degrees of freedom, we find that, in order for the final atomic state to be identical with the initial state, namely \( \rho_A^{(f)} = \rho_A^{(i)} \), it is necessary and sufficient that the following \((M+2)\) conditions be satisfied by the set of \( \alpha_j \)'s and \( t_j \)'s parameters:

\[
\begin{align*}
\sum_j \alpha_j &= 1 \\
\sum_j \alpha_j h(t_j) &= 1 \\
\sum_j \alpha_j f_m(t_j) &= 0 \quad m = 1, \ldots, M.
\end{align*}
\]

5.2. Explicit analysis of multi-mode decoherence

We shall illustrate the above conditions by considering an explicit Hamiltonian of our model system, comprising a two-level atom interacting with \( M \) modes of a cavity field, with frequencies \( \omega_m \), \( m = 1, \ldots, M \), nearly resonant with the atomic transition frequency \( \omega_a \), which is given in the rotating-wave approximation by

\[
H = \hbar \omega_a |e\rangle\langle e| + \sum_m \hbar \omega_m a_m^\dagger a_m + \sum_m \left[ \hbar \kappa_m^* a_m^\dagger |g\rangle\langle e| + \hbar \kappa_m a_m |e\rangle\langle g| \right].
\]  

Here \( a_m^\dagger \) and \( a_m \) are the creation and annihilation operators of the \( m \)th mode, and \( \kappa_m \) is the coupling constant of the \( m \)th mode with the atomic dipole.

We now set out to compute the evolution of the system from its initial state \((35)\). We note first that the Hamiltonian \((42)\) preserves the total excitation number, as \([H, N] = 0\), where the number operator is \( N = |e\rangle\langle e| + \sum_m a_m^\dagger a_m \).

Therefore, it leaves unaltered the ground state of the system, \( |g\rangle|0\rangle \), and it transforms the state \( |e\rangle|0\rangle \) at \( t = 0 \) into the state

\[
|\psi_{a+f}(t)\rangle = h(t)|e\rangle|0\rangle + \sum_m f_m(t) |g\rangle|1_m\rangle
\]

at later times, with amplitudes \( h(t) \) and \( f_m(t) \) being \( h(0) = 1 \) and \( f_m(0) = 0 \) initially and evolving in time as we now determine.

Inserting the form \((43)\) into the Schrödinger equation

\[
i\hbar \dot{\psi}_{a+f}(t) = H|\psi_{a+f}(t)\rangle
\]

we obtain the following equations:

\[
\begin{align*}
\dot{h}(t) &= -i \omega_a h(t) - i \sum_m \kappa_m f_m(t) \\
\dot{f}_m(t) &= -i \omega_m f_m(t) - i\kappa_m^* h(t)
\end{align*}
\]  

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whose Laplace transform reads

\[ s \hat{h}(s) - 1 = -i\omega_0 \hat{h}(s) - i \sum_m \kappa_m \hat{f}_m(s) \]

\[ s \hat{f}_m(s) = -i\omega_m \hat{f}_m(s) - i\kappa_m^* \hat{h}(s) \]  
(46)

using the notation \( u(t) \rightarrow \hat{u}(s) \equiv \int_0^\infty u(t) e^{-st} dt \) and the relation \( \hat{u}(t) = s\hat{u}(s) - u(0) \). From equation (46) it follows that

\[ \hat{f}_m(s) = \frac{-i\kappa_m^*}{s + i\omega_m} \hat{h}(s) \]  
(47)

and consequently

\[ \hat{h}(s) = \left(s + i\omega_a + \sum_m \frac{|\kappa_m|^2}{s + i\omega_m} \right)^{-1}. \]  
(48)

Presenting the right-hand side of equation (48) as a quotient \( P(s)/Q(s) \) of two polynomials in \( s \),

\[ P(s) = \prod_m (s + i\omega_m) \]

\[ Q(s) = (s + i\omega_a) \prod_m (s + i\omega_m) + \sum_m |\kappa_m|^2 \prod_{m' \neq m} (s + i\omega_{m'}) \]  
(49)

with \( \deg Q(s) = \deg P(s) + 1 = M + 1 \), we find an exact expression for the inverse Laplace transform of \( \hat{h}(s) \):

\[ h(t) = \sum_k \frac{P(s_k)}{Q'(s_k)} \exp(s_k t) \]  
(50)

where \( s_k \) are the \( M + 1 \) complex roots of \( Q(s) \) which are shown below to be distinct and purely imaginary. Substituting this expression into the inverse transform of equation (47),

\[ f_m(t) = -i\kappa_m^* \int_0^t \hat{h}(\tau) \exp(-i\omega_m(t - \tau)) d\tau \]  
(51)

we obtain an exact expression for the amplitudes \( f_m(t) \):

\[ f_m(t) = -i\kappa_m^* \sum_k \frac{P(s_k) \exp(s_k t) - \exp(-i\omega_m t)}{Q'(s_k) s_k + i\omega_m}. \]  
(52)

We now proceed to show that the roots of \( Q(s) \) are distinct and purely imaginary. Indeed, to each zero \( \hat{\omega}_k \) of the real function

\[ R(\hat{\omega}) \equiv iQ(-i\hat{\omega})/P(-i\hat{\omega}) = \hat{\omega} - \omega_a - \sum_m \frac{|\kappa_m|^2}{\hat{\omega} - \omega_m} \]  
(53)

there corresponds a purely imaginary root \( s_k \) of \( Q(s) \) through the relation \( s_k \equiv -i\hat{\omega}_k \), and since there are exactly \( M + 1 \) distinct zeros such as this, all the \( M + 1 \) roots of \( Q(s) \) are obtained in this way and are therefore distinct and purely imaginary. (The frequencies \( \hat{\omega}_k \) can be easily found numerically as the zeros of \( R(\hat{\omega}) \) (e.g. by the Mathematica command \texttt{FindRoot}[\texttt{R[\omega]==0,\{\omega,\{w0,w1\}\}],\texttt{AccuracyGoal->14}] upon choosing, say, \( w_0 = \omega_m + \epsilon \) and \( w_1 = \omega_{m+1} - \epsilon \).)
Figure 10. The coherence term $\rho_{eg}(t)$ of the atomic reduced density matrix as a function of evolution time $t$, showing decoherence of the atomic state with time. By the CIPE method (equations (39)–(41)) we can recreate $\rho_{eg} = 1$ at any desired time $t$ (using (54)–(56)) if the required CM is successful.

To gain clarity, we restate the above results in terms of the real frequencies $\tilde{\omega}_k$ associated with the roots of $Q(s)$: equations (50) and (52) assume the form

$$h(t) = \sum_k C_k \exp(-i\tilde{\omega}_k t)$$

$$f_m(t) = \kappa_m^* \sum_k C_k \frac{\exp(-i\tilde{\omega}_k t) - \exp(-i\omega_m t)}{\tilde{\omega}_k - \omega_m}$$

with real coefficients

$$C_k \equiv \frac{P(s_k)}{Q'(s_k)} = \left(1 + \sum_m \frac{|\kappa_m|^2}{(\tilde{\omega}_k - \omega_m)^2}\right)^{-1}.$$  

(Equation (56) follows from: $r(s) \equiv Q(s)/P(s)$, $Q(s_k) = 0$, $P(s_k) \neq 0 \implies Q'(s_k)/P(s_k) = r'(s_k) = R'(\tilde{\omega}_k)$.)

As a special case of the results here we obtain the evolution of the resonant Jaynes–Cummings model on taking $M = 1$ and $\omega_1 = \omega_a$: the frequencies associated with the roots of $Q(s)$ are then $\tilde{\omega}_\pm = \omega_a \pm |\kappa_1|$ and we have

$$h(t) = \cos(|\kappa_1| t) \exp(-i\omega_a t)$$

$$e^{-i\omega_a t} f_1(t) = -i \frac{\kappa_1^*}{|\kappa_1|} \sin(|\kappa_1| t) \exp(-i\omega_a t).$$

We can now substitute the results (54)–(56) into equations (39)–(41), in order to obtain the explicit conditions for the protection of an arbitrary atomic state from decoherence by means of CIPE. These conditions impose a ‘revival’ on the decohering state (figure 10) at any desired time (subject to successful realization of the $\alpha_j$ coefficients by a conditional measurement with probability $P$). As the number of modes $M$ grows, the imposition of a revival at any time requires more control parameters.

We note that for a leaky cavity, wherein the mode frequencies $\omega_m$ are complex, and the norm of $\sum_m |f_m(t)|^2$ decays in time, these conditions cannot be completely fulfilled at all times. Nevertheless, they may still be used to reduce atomic decoherence even in leaky cavities.
5.3. Conclusions

The above analysis demonstrates that, provided that the number of control (momentum-transmission) parameters is sufficiently large, it should be possible to satisfy them simultaneously and thus ensure that any internal atomic state remains unspoilt, in spite of the atom’s exposure to many interaction channels. Furthermore, additional control parameters can be used to optimize the CM success probability $P$.

In figure 10 we show that the entanglement caused by different evolutions $U(t_j)$, and the ensuing tracing over $M = 20$ field modes yield $t_j$-dependent decoherence of the initial atomic state $B|e\rangle + C|g\rangle$. By contrast, when this tracing follows the interference of $M + 2$ evolution operations $U(t_j)$ with the rightly chosen $\alpha_j$, it yields an atomic state that is identical to the initial one, i.e. completely protected from decoherence. Considerable deviations from the prescribed conditions on $\alpha_j$ and $t_j$, i.e. a ‘failed’ measurement, may result in worse decoherence than that associated with each $t_j$. Hence the need for accurate assignment of the control parameters and for optimization of the CM success probability, which is achievable by means of extra control parameters.

6. Conclusions

The present outline and graphical illustration of several methods for combating population decay and state decoherence in cavities underscores their essential traits and allows a comparison of their merits and limitations.

(a) **The quantum Zeno effect (QZE) and the anti-Zeno effect (AZE):** Our unified analysis of two-level system coupling to field reservoirs has revealed the general optimal conditions for observing the QZE and AZE in cavities and waveguides. This analysis also clarifies that QZE cannot combat the background-mode contribution to exponential decay. The best way to prevent this contribution is by an ac Stark shift of the resonance, well into a forbidden gap. This method achieves the inhibition of population decay at the expense of increasing decoherence.

(b) **State control by location-dependent interference of decay channels:** This method is remarkable in that, rather than try to avoid spontaneous emission by its inhibition (in cavities or by adiabatic off-resonance transfer [34]), it guides this decay into desired channels, and thereby can populate any desired superposition of two (or more) atomic (molecular) states and, correspondingly, of radiation frequencies in the photon wavepacket emerging from the cavity. The limitations of this method are: (i) it is appropriate only for thin films, whose location is controllable to within a fraction of a wavelength; (ii) as in the case of QZE, it requires the spontaneous emission rates into the axial cavity modes $\gamma_c$ to be much larger than the corresponding rates into the unconfined ‘background’ modes $\gamma_b$. The decay probability of the atom into unconfined modes, $\sim \gamma_b \tau$, during the time $\tau$ needed for the creation of the superposition state and the photon wavepacket transformation is the error probability of this state control method. The method is effective only if $1/\gamma_c \ll \tau \ll 1/\gamma_b$.

(c) **Conditionally interfering parallel evolutions (CIPE):** The main merit of this method is that it allows us to control the evolution of $M \gg 1$ coupled degrees of freedom over arbitrary time intervals by essentially as many $(M + 2)$ ancilla or control parameters. In particular, its applicability to arbitrary atomic superposition states implies that we can use this method...
as part of the quantum processing chain of operations. By optimizing the CM (adding more control parameters), we can achieve high success probability, or confidence in the method. Nevertheless, the probabilistic nature of this method remains a limitation. Furthermore, the method does not apply to decay into mode continua, but only to coupling with a discrete (albeit large) number of modes.

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