Line shifts in accretion disks—the case of Fe Kα

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Abstract Here we present a short overview and main re-
results of our investigations of several effects which can in-
duce shifts in the broad Fe Kα line emitted from relativis-
tic accretion disks around single and binary supermassive
black holes. We used numerical simulations based on ray-
tracing method in the Kerr metric to study the role of classi-
cal Doppler shift, special relativistic transverse Doppler shift
and Doppler beaming, general relativistic gravitational red-
shift, and perturbations of the disk emissivity in the forma-
tion of the observed Fe Kα line profiles. Besides, we also in-
vestigated whether the observed line profiles from the binary
systems of supermassive black holes could be affected by
the Doppler shifts due to dynamics of such systems. The pre-
sented results demonstrate that all these effects could have
a significant influence on the observed profiles of the broad
Fe Kα line emitted from relativistic accretion disks around
single and binary supermassive black holes.

Keywords Black holes: black-hole binaries · Galaxies:
active · Accretion and accretion disks · Spectral lines ·
X-Ray emission spectra

1 Introduction

It is now widely accepted that most galaxies harbor a su-
permassive black hole (SMBH) in their centers and that the
formation and evolution of galaxies is fundamentally influ-
enced by properties of their central SMBHs (see e.g. Kor-
mendy and Ho2013, and references therein). Active galax-
ies most likely represent one phase in galactic evolution, and
they derive their extraordinary luminosities from energy re-
lease by matter accreting towards, and falling into, a cen-
tral SMBH through a relativistic accretion disk which repre-
sents an efficient mechanism for extracting the gravitational
potential energy and converting it into the electromagnetic
radiation (see e.g. Jovanović and Popović 2009; Jovanović
2012, and references therein)

Since its discovery in Seyfert 1 galaxy MCG-6-30-15
by Tanaka et al. (1995), the broad Fe Kα emission line at
6.4 keV was observed in a number of Active Galactic
Nuclei (AGNs) (see e.g. Nandra et al. 2007, and refer-
cence therein). Based on the observations with the X-ray satel-
lites like Ginga, ASCA, RXTE, BeppoSAX, XMM-Newton,
Suzaku and Chandra, the fact that X-ray irradiation of the
accretion disk surface in a class of AGNs known as Seyfert 1
galaxies gives rise to the fluorescent Fe Kα line emission via
the X-ray reflection is now well established (see e.g. Fabian
et al. 2000). The broad and asymmetric profile of this line
with narrow bright “blue” and wide faint “red” peak is most
commonly attributed to the relativistic effects due to a very
fast rotation of the emitting material in the innermost regions
of the accretion disk (see e.g. Fabian et al. 2000, and refer-
ces therein), although there are some alternative attempts
to explain such profile by scattering/reflection/absorption by
the disk wind and/or ionized outflows which could mimic
relativistic effects (see Turner and Miller 2009, and refer-
ces therein). Since this line is emitted from a very com-
pact region in the accretion disk near the central SMBH, it
represents a powerful diagnostic tool for studying the effects of strong gravitational field (see e.g. Jovanović and Popović 2008), accretion physics and space-time geometry in the vicinity of SMBHs (see Jovanović 2012 for an overview and Jovanović et al. 2011 for a case study).

So far, the extensive theoretical studies of the line profiles emitted from relativistic accretion disks around SMBHs have been carried out, in which the line profiles have been modeled using different approaches (see e.g. Reynolds and Nowak 2003, for a review). In the weak field limit such line profiles can be evaluated analytically (Chen and Halpern 1989; Chen et al. 1989). Fabian et al. (1989) were the first who calculated the Fe Kα line profiles emitted by the accretion disk in the Schwarzschild metric. A full general relativistic approach based on “the transfer-function” which contains all relativistic effects is also used for modeling relativistically broadened lines (Laor 1991). The exact line profiles can be also computed by the direct integration of the photon trajectories in the Kerr metric (Karas et al. 1992; Bromley et al. 1997) which enabled studies of the complex disk models, such as geometrically thick or warped accretion disks (Hartnoll and Blackman 2000) or disks with non-axisymmetric structures in form of spiral arms (Karas et al. 2001). The standard X-ray spectral fitting code, XSPEC (Arnaud 1996) is often used for evaluating the general relativistic models of accretion disks and simulating the corresponding line profiles (see e.g. Dovčiak et al. 2004).

Here we study several phenomena which can induce line shifts, and thus cause the variability of the profile of the Fe Kα line emitted from a relativistic accretion disk around a SMBH, using a code based on ray-tracing method in the Kerr metric (see e.g. Čadež et al. 1998; Jovanović 2012, and references therein). This paper is organized as follows: in Sect. 2 we briefly describe the method used to obtain the simulated line profiles, in Sect. 3 we describe several types of line shifts and study different effects which can induce them, and finally in Sect. 4 we point out our conclusions.

2 Simulated line shapes

We studied the radiation from relativistic accretion disks around SMBHs using numerical simulations based on ray-tracing method in the Kerr metric (see Fanton et al. 1997; Čadež et al. 1998, for more details), in which we used the pseudo-analytical integration of the geodesic equations describing the photon trajectories in the general case of a rotating SMBH with some angular momentum (spin) $a$. Due to several effects which will be discussed below, photons emitted from a disk at energy $E_{em}$ (or wavelength $\lambda_{em}$) will be observed at energy $E_{obs}$ (or wavelength $\lambda_{obs}$) by an observer at infinity, causing the energy shift $g$ or, equivalently, the usual redshift in wavelength $z$:

$$g = \frac{E_{obs}}{E_{em}} = \frac{\lambda_{em}}{\lambda_{obs}} = \frac{1}{1 + z}. \quad (1)$$

In the ray-tracing method one takes into account only those photon trajectories reaching the observer’s sky plane. An observer’s sky plane is divided into a number of small elements (pixels), and for each pixel the photon trajectories are traced backward from the observer by following the geodesics in a Kerr space-time, until their intersection with the disk. We assumed an optically thick and geometrically thin disk for which the spectrum of emitted radiation depends on its structure, and therefore on the distance to the central SMBH (Shakura and Sunyaev 1973). Thus, assuming that disk radiates according to an emissivity law $\varepsilon(r)$, for each pixel at observer’s sky plane it is possible to calculate the flux density and the energy shift $g$ of the photons emitted by the disk (see Jovanović 2012, for more details). Usually, a power law for disk emissivity is assumed: $\varepsilon(r) \propto r^{-q}$, where $q$ is an emissivity index. In that way, one can obtain a simulated image of a relativistic accretion disk as would be seen by a distant observer using a powerful high resolution telescope. An example of such simulated image of a relativistic accretion disk is presented in Fig. 1, where $R_g = GM/c^2$ is the gravitational radius of the central SMBH with mass $M$ ($G$ being gravitational constant and $c$ speed of light), and $R_{ms}$ represents the radius of the marginally stable orbit which depends on spin $a$ of the SMBH.

The corresponding simulated profile of the line emitted from the simulated accretion disk can be calculated by taking into account the total observed flux and observed photon
Fig. 2 Two shapes of the lines emitted from the outer regions of a slightly inclined ($i = 10^\circ$) relativistic accretion disk around a maximally rotating Kerr SMBH, extending between 7000 and 8000 $R_g$ (solid line) and 1000 and 2000 $R_g$ (dashed line).

energies of all pixels over the disk image:

$$F_{\text{obs}}(E_{\text{obs}}) = \int_{\text{image}} \varepsilon(r) g^4 \delta(E_{\text{obs}} - gE_0) d\Omega,$$

(2)

where $d\Omega$ is the solid angle subtended by the disk in the observer’s sky and $E_0$ is the rest energy. Several examples for the simulated line profiles emitted from different regions of the relativistic accretion disks with different parameters are given in Figs. 2–3.

3 Results: Fe Kα line shifts

In this section we will discuss several phenomena which can induce shifts in the lines emitted from relativistic accretion disks around SMBHs. For that purpose we simulated the line profiles emitted from different regions of accretion disks, which parameters are given in Table 1. In the following paragraphs we describe the obtained results in more details. However, one should take into account that some other phenomena, such as gravitational microlensing could also induce line shifts (see e.g. Popović et al. 2003a,b, 2006; Jovanović et al. 2008; Jovanović and Popović 2009), but they will not be discussed here.

3.1 Classical Doppler shift due to Keplerian rotation in accretion disk

Rotation of emitting material in the outermost regions of an accretion disk, located thousands $R_g$ from the central SMBH, is practically Keplerian since the relativistic effects are negligible at such large distances. Therefore, the lines emitted from these regions are broadened depending on Keplerian rotational velocity and have two symmetric peaks (see the solid line profile in Fig. 2): the “blue” one emitted by the material located on the side of the disk which is approaching towards an observer (denoted by blue shades in the simulated accretion disk image presented in the top panel of Fig. 1), and the “red” one, emitted by the material on the receding side of the disk with respect to the observer (denoted by red shades in the top panel of Fig. 1).
3.2 Transverse Doppler shift and Doppler beaming due to special relativistic effects

However, in the regions which are located less than a few thousand $R_g$ from the SMBH, the rotational velocities are much higher (on the order of several thousand km s$^{-1}$) inducing, not only the wider line shapes, but also the special relativistic effects which produce an asymmetry in the line shapes by shifting the emission from the inner regions of the disk to the “red” and beaming its radiation in the direction of motion (see e.g. Fabian 1999), and thus enhancing the “blue” peak with respect to the “red” one (see the dashed line profile in Fig. 2). These two special relativistic effects are known as transverse Doppler shift and Doppler beaming, respectively.

The transverse Doppler shift originates when the observer is displaced in a direction perpendicular to the direction of the motion of emitters in the accretion disk, and is a consequence of the relativistic Doppler effect which includes the time dilation. Namely, if in the observer’s frame the angle between the direction of the emitter at the time of emission and the observed direction of the light at the time of observation is equal to $\pi/2$, the relativistic Doppler effect reduces to the transverse Doppler shift, due to which the observed radiation from a rapidly rotating accretion disk is redshifted by the Lorentz factor, as it is the case with the dashed simulated line profile in Fig. 2.

Doppler beaming (boosting) modifies the apparent brightness of the rapidly moving emitting matter in the sense that, if it is moving towards the observer then it will appear brighter than if it is at rest, and vice versa, if it is moving away from the observer it will appear fainter than if it is at rest. When this is applied to the rapidly rotating emitters in the accretion disk, it will enhance the apparent brightness of the approaching side of the disk with respect to its receding side (see the bottom panel in Fig. 1), and thus it will also enhance the “blue” peak of the line with respect to its “red” peak, as it can be seen from the dashed simulated line profile in Fig. 2.

Since the velocities of material, as well as the radii of the emitting region of the dashed simulated line profile in Fig. 2, correspond to the parts of the disk from which the optical spectral lines like $H\alpha$ and $H\beta$ are observed, one can conclude that the transverse Doppler shift and Doppler beaming significantly affect even the optical lines which originate from outer regions of the accretion disk. In the case of such lines, as it can be seen from Table 1, the maximum redshift of the photons ($1/g_{\text{min}} - 1$) is significantly larger by absolute value than their maximum blueshift ($1/g_{\text{max}} - 1$), which quantitatively represents the influence of transverse Doppler shift in this case.

However, since the Fe K$\alpha$ line originates in the innermost regions of an accretion disk, its emitting material rotates with the fastest velocities (on the order of tens of thousands of km s$^{-1}$). Therefore, the previously mentioned classical and special relativistic effects are even more significant in the case of Fe K$\alpha$ line (see the both panels in Fig. 3, as well as the corresponding rows in Table 1). As it can be seen from Fig. 3, when compared to the simulated profile of an optical line (dashed profile from Fig. 2), the Fe K$\alpha$ line profile is much wider (due to the larger classical Doppler shift), more asymmetric (due to the larger Doppler beaming) and more redshifted (due to the larger transverse Doppler shift, as well as due to the gravitational redshift which is discussed in more details in the following paragraph).

3.3 Gravitational redshift due to general relativistic effects

According to the general relativity, the observed frequency of electromagnetic radiation emitted by a source in a strong gravitational field in vicinity of a SMBH will be reduced with respect to the emitted frequency, due to gravitational time dilation. This effect is called the gravitational redshift (Misner et al. 1973), and in the Newtonian limit it is approximately proportional to the mass of a SMBH and inversely proportional to the distance between it and a point in the disk at which a photon is emitted.

In order to study the influence of the gravitational redshift on the Fe K$\alpha$ line profile, we simulated the line profiles originating from two emitting regions in accretion disk around a non-rotating Schwarzschild SMBH (top panel in Fig. 3) and a maximally rotating Kerr SMBH (bottom panel in Fig. 3), having two different inner radii: $R_{\text{in}} = 10 \, R_g$ (denoted by solid lines) and $R_{\text{in}} = R_{\text{ms}}$ (denoted by dashed lines). The outer radii in both cases are $R_{\text{out}} = 20 \, R_g$. As it can be seen from Fig. 3, in the case of both metrics the profiles originating from emitting regions with smaller inner radii are redshifted with respect to the profiles from emitting regions with larger inner radii, due to the gravitational redshift. This is even more obvious if one compare the corresponding maximum redshifts ($1/g_{\text{min}} - 1$) and blueshifts ($1/g_{\text{max}} - 1$) presented in Table 1, from which it can be seen that the maximum redshifts are much bigger in the cases with the smaller inner radii, while the blueshifts are the same in both cases. Therefore, the gravitational redshift causes the further deformations of the Fe K$\alpha$ line profile by smearing its “blue” emission into the “red” one.

This effect is stronger in the case of a maximally rotating Kerr SMBH than in the case of a non-rotating Schwarzschild SMBH due to the fact that the radius of the innermost stable circular orbit (also known as radius of marginally stable orbit, $R_{\text{ms}}$) strongly depends on SMBH spin $a$, so that in the case of a non-rotating Schwarzschild SMBH (i.e. for $a = 0$) it is equal to $6 \, R_g$ (top panel in Fig. 3), and in the case of an extremely rotating Kerr SMBH (i.e. for $a = 1$) it is equal...
to $1 \ R_g$ (bottom panel in Fig. 3). Thus, in the case of an extremely rotating SMBH it is possible to detect the radiation which originates closer to the SMBH itself, and which is therefore more subject to the gravitational redshift (see the last row in Table 1), than in the case of a non-rotating Schwarzschild SMBH.

In that way the SMBH spin $a$ affects the radiation originating from the innermost regions of the disk which are closest to the SMBH (see e.g. Jovanović and Popović 2008) and, as it can be seen from both panels in Fig. 3 and the corresponding rows of Table 1, it significantly affects the profile of the Fe $\alpha$ line, especially its “red” wing by extending it towards the lower energies for the higher values of $a$ (see also Jovanović et al. 2011 for a case study and Jovanović 2012 for more details).

### 3.4 Line shifts due to perturbed disk emissivity

Beside the previously mentioned classical, special and general relativistic effects, several observational/theoretical studies indicated that the line shapes and shifts could be affected by the perturbations of the disk emissivity in the form of flares or orbiting bright spots, especially in the case of high accretion rate objects, such as Seyfert galaxies and QSOs (see e.g. Czerny et al. 2004; Czerny and Goosmann 2004; Jovanović et al. 2010). For instance, appearance of the redshifted Fe $\alpha$ emission feature due to co-rotating flare above the accretion disk which irradiates a spot on its surface was reported in the case of the Seyfert galaxies NGC 3516 (Iwasawa et al. 2004) and NGC 3783 (Tombesi et al. 2007), based on their observations by XMM-Newton. However, since the sensitivity of XMM-Newton was not sufficient to clearly resolve sub-orbital features in the observed X-ray spectra, the azimuthal irradiation structure of the inner accretion disk is modeled and simulated by Czerny et al. (2004), Czerny and Goosmann (2004), Goosmann et al. (2007a,b) in order to investigate the detectability of orbiting spots in nearby Seyfert galaxies with current and future X-ray observatories, such as X-Ray Evolving Universe Spectroscopy (XEUS), a planned successor to XMM-Newton which evolved to Advanced Telescope for High Energy Astrophysics (ATHENA+) that is currently under development, aiming to have several hundred times larger sensitivity than Chandra X-ray Observatory and XMM-Newton. For that purpose, Goosmann et al. (2007a,b) developed a method to compute the local spectra of the hot spot emission from the surface of an accretion disk underlying a strong co-rotating flare above the disk, and used it to model the X-ray reprocessing from a persisting flare lasting for a significant fraction of one orbital period at the given disk radius in some AGNs such as MCG-6-30-15 and NGC 5548, as well as the short-term flares with durations of a few hundreds of seconds. These studies showed that the future high precision observations will allow to disentangle the effects of the flares in the accretion disks of Seyfert galaxies from the intrinsic variability of the local emission, even at distances which are less than $5 \ R_g$ from the central SMBHs (Goosmann et al. 2007a). Also, the spectral variations induced by such flares depend on the viewing direction and are stronger for intermediate (edge-on) viewing angles, as supposed for Seyfert-2 galaxies, than for a face-on viewing direction, as assumed in Seyfert-1 galaxies, and thus, they could be used to put observational constraints on the disk inclination (Czerny et al. 2004; Czerny and Goosmann 2004). However, indications for the flares in the accretion disks were found not only in the X-ray band, but also in the optical spectra of some AGNs. For example, the variability of the observed $H\beta$ line profiles of quasar 3C 390.3 was recently successfully explained by a perturbation of the power law disk emissivity, revealing the appearance of two successive bright spots on the approaching side of the disk which were interpreted as the fragments of spiral arms in the accretion disk of 3C 390.3 (Jovanović et al. 2010).

Such perturbations of the power law disk emissivity can be modeled as (Jovanović et al. 2010):

$$
\varepsilon(x, y) \propto r(x, y)^{-q} \cdot \left(1 + \varepsilon_p \cdot e^{-\frac{(x-x_p)^2+(y-y_p)^2}{\sigma_p^2}}\right),
$$

where $q$ is power law emissivity index, $\varepsilon_p$ is emissivity of perturbing region, and $(x_p, y_p)$ and $\sigma_p$ are its position and width (both in $R_g$), respectively. An illustration of the perturbed disk emissivity is presented in the top panel of Fig. 4, and the perturbed and unperturbed Fe $\alpha$ line profiles for three different positions and widths of perturbing region are given in the bottom panel of the same figure. As one can see from Fig. 4, a perturbing region on the receding side of accretion disk, affects only the “red wing” of the Fe $\alpha$ line, while the “blue” one and the line core stay nearly unaffected. Moreover, the position of perturbing region is in a direct relation with the position of the “red peak” of the line. Recently, a model of accretion disk with perturbing region was used for fitting the observed spectra of some AGNs, resulting with a non-random distribution of bright spot positions along the receding side of the disk. This indicated that the most plausible physical mechanism which can explain such behavior is a fragmented spiral arm of the accretion disk (see e.g. Jovanović et al. 2010; Lewis et al. 2010, and references therein). Perturbing regions could be then explained by the emissivity lumps caused by fragments in spiral arms, such as isolated clumps which could pass through the arm and dominate in its emissivity. The fragments naturally travel with the arm as it precesses through the disk, but since the outer parts of an accretion disk are self-gravitating, the fragments of spiral arms could be also launched from the outer regions of the disk and propagate inward (Lewis et al. 2010). Besides, in the inner regions of the disk, the disk winds and highly ionized fast outflows could play an important role and could...
also cause the outward propagation of such clumps (see e.g. Popovi´c et al. 2011; Sim et al. 2010). Since the flux variability caused by a spiral arm is on timescales of a year to several years while its fragments cause shorter variations on timescales of several months, this model is able to explain the observed variability of some AGNs on the timescales ranging from several months to several years (Lewis et al. 2010). As it can be seen from Fig. 4, when a perturbation which originates in the inner regions of the disk and, due to some wind or outflow, spirals away along the receding side of the disk further from the central SMBH and toward the outer parts of the disk, it will cause the blueward shift of the “red peak” which will therefore move towards the line core. This demonstrates that in some cases, the deformations of the line profiles could arise due to shifts of some of their parts, caused by the variability of the disk emissivity (for more details see Jovanovi´c et al. 2010).

3.5 Line shifts in the binary systems of SMBHs

Binary systems of supermassive black holes are formed in galactic mergers, and at some stage when two SMBHs become gravitationally bound and start to orbit around their center of mass, accretion of the surrounding matter on both SMBHs could be expected and as a result, a strong line emission could arise (see Popovi´c 2012 for theoretical aspects and Bon et al. 2012; Yan et al. 2015 for observational evidences). In such a case, the Fe Kα line might be also observed, and it would most likely arise from both accretion disks around primary and secondary SMBHs, located in a central low density cavity of a circumbinary disk (Jovanovi´c et al. 2014). Therefore, the Fe Kα line profiles emitted from both accretion disks might be affected by the Doppler shifts due to the orbital motion of the SMBH binary, and in that case the corresponding signatures would be imprinted in the observed composite line profile (Jovanovi´c et al. 2014).

Assuming that the radial velocities of the primary and secondary components are \(V_{1,2}^{\text{rad}} \ll c\) so that the corresponding Doppler shifts due to their orbital motion are \(\approx V_{1,2}^{\text{rad}}/c\), a composite profile \(F(g)\) of the Fe Kα line emitted from both accretion disks of a SMBH binary at some orbital phase can be calculated from two constituent unshifted line profiles \(F_1(g)\) and \(F_2(g)\) according to (Jovanovi´c et al. 2014):

\[
F(g) = F_1\left(1 - \frac{V_{1}^{\text{rad}}}{c}\right)^{-1} + F_2\left(1 - \frac{V_{2}^{\text{rad}}}{c}\right)^{-1}. \tag{4}
\]

Three such simulated composite Fe Kα line profiles emitted during three different phases along the Keplerian orbit of a SMBH binary with sub-parsec separation between the components are presented in Fig. 5. As it can be seen from Fig. 5, Doppler shifts due to the orbital motion of the components in the binary have significant influence on the observed composite Fe Kα line profiles and induce their ripple variability, as well as the shifts of their parts, especially of the line cores and their “blue” wings (for more details see Jovanovi´c et al. 2014). Such shifts and ripple variability of
the complex Fe Kα line profiles strongly depend on the orbital phase of the binary system, and therefore if observed in the spectra of some AGNs, they could represent the observational signatures of the existence of SMBH binary systems in the cores of these AGNs.

4 Conclusions

Here we discussed different types of the Fe Kα line shifts in AGNs, as well as several phenomena that can induce them. For that purpose, we performed numerical simulations of the X-ray radiation from the relativistic accretion disks around single and binary SMBHs, based on ray-tracing method in the Kerr metric. The obtained results showed that the following effects have a significant influence on the line profiles:

- classical Doppler shift which causes double-peaked profiles of the lines emitted over a whole relativistic accretion disk, from its innermost regions where X-ray radiation originates, to its outermost parts which emit in the optical band;
- special relativistic transverse Doppler shift and relativistic beaming which cause the redshift of the line profile, as well as the relative enhancement of its “blue” peak with respect to the “red” one, not only in the case of the broad Fe Kα line which originates from the innermost regions of the disk, but also in the case of the optical lines which originate from its outer parts;
- general relativistic gravitational redshift which causes the further deformations of the Fe Kα line profile by smearing its “blue” emission into the “red” one, as well as the SMBH spin $a$ which affects the “red” wing of the line by extending it towards the lower energies for the higher values of $a$;
- perturbations of the disk emissivity in form of flares or bright spots moving along the receding side of the disk, which can induce blueward shift of the “red peak” not only of the Fe Kα line, but also of the lines in other spectral bands;
- Doppler shifts due to the orbital motion in the SMBH binaries which could produce the ripple variability of the complex Fe Kα line profiles and induce shifting of their parts. Such variability of the observed Fe Kα line profiles strongly depends on the orbital phases of the SMBH binary systems, and could represent their observational signatures.

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