Double-Lepton Polarization Asymmetries in
$B_s \rightarrow \phi \ell^+ \ell^-$ Decay in the Fourth-Generation Standard Model

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Abstract

In this paper, we investigate the effects of the fourth generation of quarks on the double-lepton polarization asymmetries in the $B_s \rightarrow \phi \ell^+ \ell^-$ decay. It is shown that these asymmetries in $B_s \rightarrow \phi \ell^+ \ell^-$ decay compared with those of $B \rightarrow K \ell^+ \ell^-$ decay are more sensitive to the fourth-generation parameters. We conclude that an efficient way to establish the existence of the fourth generation of quarks could be the study of these asymmetries in the $B_s \rightarrow \phi \ell^+ \ell^-$ decay.

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I. INTRODUCTION

Although Standard Model (SM) is a successful theory, there is no clear theoretical argument within this model to restrict the number of generations to three, and therefore the possibility of a new generation should not be ruled out. Based on this possibility, a number of theoretical and experimental investigations have been performed. The measurement of the $Z$ decay widths restricts the number of light neutrino for $m_\nu < m_Z/2$ to three [1]. However, if a heavy neutrino exits, the possibility of extra generations of heavy quarks is not excluded from the experiment. Moreover, the electro weak data [2] supports an extra generation of heavy quarks, if the mass difference between the new up and down-type quarks is not too large.

Many authors who support the existence of fourth-generation studied those effects in various areas, for instance Higgs and neutrino physics, cosmology and dark matter [3]–[8]. For example, in [8] it is argued that the fourth generation of quarks and leptons can be generated in the Higgs boson production at the Tevatron and the LHC, before being actually detected. By the detailed study of this process at the Tevatron and LHC, the number of generations in the SM can be determined. Moreover, the flavor democracy (Democratic Mass Matrix approach) [9] favors the existence of the nearly degenerate fourth SM family, while the fifth SM family is disfavored both by the mass phenomenology and precision tests of the SM [10]. The main restrictions on the new SM families come from the experimental data on the $\rho$ and $S$ parameters [10]. However, the common mass of the fourth quark ($m_\nu'$) lies between 320 GeV and 730 GeV considering the experimental value of $\rho = 1.0003^{+0.0007}_{-0.0004}$ [11]. The last value is close to upper limit on heavy quark masses, $m_q \leq 700 \, GeV \approx 4m_t$, which follows from partial-wave unitarity at high energies [12]. It should be noted that with preferable value $a \approx g_w$ Flavor Democracy predicts $m_\nu \approx 8m_w \approx 640 \, GeV$.

One of the promising areas in the experimental search for the fourth-generation, via its indirect loop effects, is the rare B meson decays. Based on this idea, serious attempts to probe the effects of the fourth-generation on the rare B meson were made by many researchers. The fourth-generation can affect physical observables, i.e. branching ratio, CP asymmetry, polarization asymmetries and forward–backward asymmetries. The study of these physical observables is a good tool to look for the fourth generation of up type quarks [13]–[29].
Recently, the sensitivity of the double-lepton polarization asymmetries to the fourth-generation in the transition of $B$ to a pseudo scalar meson ($B \rightarrow K\ell^{+}\ell^{-}$) has been investigated and it is found out that this observable is sensitive to the fourth-generation parameters ($m_{t'}$, $V_{tb}V_{ts}^*$)\cite{24}. In this work, we investigate the effects of the fourth generation of quarks ($b', t'$) on the double-lepton polarizations in the transition of $B$ to a vector meson ($B_s \rightarrow \phi\ell^{+}\ell^{-}$) and compare our results with those of $B \rightarrow K\ell^{+}\ell^{-}$ decay presented in Ref.\cite{24}. It should be mentioned that both decays occur through $b \rightarrow s$ transition in which the sequential fourth generation of up quarks ($t'$), like $u, c, t$ quarks, contributes at the loop level. Hence, this new generation will change only the values of the Wilson coefficients via virtual exchange of the fourth-generation up quark $t'$ and the full operator set is exactly the same as in SM.

The paper is organized as follows. In Section II, the expressions for the matrix element and double-lepton polarizations of $B_s \rightarrow \phi\ell^{+}\ell^{-}$ in the SM have been presented. The effect of the fourth generation of quarks on the effective Hamiltonian and the double-lepton polarization asymmetries have been discussed in Section III. The sensitivity of these polarizations to the fourth-generation parameters ($m_{t'}, r_{sb}, \phi_{sb}$) have been numerically analyzed in the final Section.

II. THE MATRIX ELEMENT AND DOUBLE-LEPTON POLARIZATIONS OF $B_s \rightarrow \phi\ell^{+}\ell^{-}$ IN THE SM

In the SM, the relevant effective Hamiltonian for $B_s \rightarrow \phi\ell^{+}\ell^{-}$ decay which is described by $b \rightarrow s\ell^{+}\ell^{-}$ transition at quark level can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu),$$

(1)

where the complete set of the operators $\mathcal{O}_i(\mu)$ and the corresponding expressions for the Wilson coefficients $C_i(\mu)$ are given in \cite{30}. Using the above effective Hamiltonian, the one-loop matrix elements of $b \rightarrow s\ell^{+}\ell^{-}$ can be written in terms of the tree-level matrix elements of the effective operators as:

$$\mathcal{M}(b \rightarrow s\ell^{+}\ell^{-}) = <s\ell^{+}\ell^{-}|\mathcal{H}_{\text{eff}}|b>$$

$$= -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i}C_i^{\text{eff}}(\mu) <s\ell^{+}\ell^{-}|\mathcal{O}_i|b>^{\text{tree}}.$$
\[- \frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* \left( \tilde{C}_{9}^{\alpha} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \ell + \tilde{C}_{10}^{\alpha} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \gamma_5 \ell \\
- 2 C_7^{\text{eff}} \frac{m_b}{q^2} \tilde{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \bar{\ell} \gamma_\mu \ell \right), \tag{2}\]

where \( q^2 = (p_1 + p_2)^2 \) and \( p_1 \) and \( p_2 \) are the final leptons four–momenta and the effective Wilson coefficients at \( \mu \) scale, are given as \([30, 31]\):

\[
C_{7}^{\text{eff}} = C_7 - \frac{1}{3} C_5 - C_6 \\
C_{10}^{\text{eff}} = \frac{\alpha}{2\pi} \tilde{C}_{10}^{\text{eff}} = C_{10} \\
C_{9}^{\text{eff}} = \frac{\alpha}{2\pi} \tilde{C}_{9}^{\text{eff}} = C_9 + \frac{\alpha}{2\pi} Y(s). \tag{3}\]

In Eq.(3), \( s = q^2/m_b^2 \) and the function \( Y(s) \) contains the short-distance contributions due to the one-loop matrix element of the four quark operators, \( Y_{\text{per}}(s) \), as well as the long-distance contributions coming from the real \( c\bar{c} \) intermediate states, i.e., \( J/\psi, \psi', \ldots \). The latter contributions are taken into account by introducing Breit–Wigner form of the resonance propagator which leads to the second term in the following formula (see Eq.4) \([32]–[34]\).

As a result the function \( Y(s) \) can be written as:

\[
Y(s) = Y_{\text{per}}(s) + \frac{3\pi}{\alpha^2} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\
\times \sum_{V_i = \psi_i} \frac{\kappa_i}{m_{V_i}^2 - s m_b^2 - i m_{V_i} \Gamma_{V_i}}, \tag{4}\]

where

\[
Y_{\text{per}}(s) = g \left( \frac{m_c}{m_b}, s \right) (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\
- \frac{1}{2} g(1, s) (4C_3 + 4C_4 + 3C_5 + C_6) \\
- \frac{1}{2} g(0, s) (C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6). \tag{5}\]

The explicit expressions for the \( g \) functions can be found in \([30]\) and the phenomenological parameters \( \kappa_i \) in Eq.(4) can be determined from

\[
\mathcal{B}(B \rightarrow K^* V_i \rightarrow K^* \ell^+ \ell^-) = \mathcal{B}(B \rightarrow K^* V_i) \mathcal{B}(V_i \rightarrow \ell^+ \ell^-), \tag{6}\]

where the data for the right hand side is given in \([35]\). For the lowest resonances, \( J/\psi \) and \( \psi' \) one can use \( \kappa = 1.65 \) and \( \kappa = 2.36 \), respectively (see \([36]\)). In this study, we neglect the long-distance contributions for simplicity and like Ref.\([30]\), to have a scheme independent matrix
element, we use the leading order as well as the next-to-leading order QCD corrections to $C_9$ and the leading order QCD corrections to the other Wilson coefficients.

In order to compute the decay width and other physical observables of $B_s \rightarrow \phi \ell^+ \ell^-$ decay, we need to sandwich the matrix elements in Eq. (2) between the final and initial meson states. Therefore, the hadronic matrix elements for the $B_s \rightarrow \phi \ell^+ \ell^-$ can be parameterized in terms of form factors. For the vector meson $\phi$ with polarization vector $\varepsilon_\mu$ the semileptonic form factors of the V–A current is defined as:

\[
< \phi(p_\phi, \epsilon) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p_B) >= \frac{2V(q^2)}{m_{B_s} + m_\phi} \epsilon_{\mu \rho \sigma} p_\rho q^\sigma \epsilon^{\nu},
\]

\[
- i \left[ \epsilon_\mu^*(m_{B_s} + m_\phi) A_1(q^2) - (\epsilon^* q)(p_{B_s} + p_\phi)_\mu \frac{A_2(q^2)}{m_{B_s} + m_\phi} - q_\mu (\epsilon^* q) \frac{2m_\phi}{q^2} (A_3(q^2) - A_0(q^2)) \right],
\]

where $q = p_{B_s} - p_\phi$, and $A_3(q^2 = 0) = A_0(q^2 = 0)$ (this condition ensures that there is no kinematical singularity in the matrix element at $q^2 = 0$). Also, the form factor $A_3(q^2)$ can be written as a linear combination of the form factors $A_1$ and $A_2$:

\[
A_3(q^2) = \frac{1}{2m_\phi} \left[ (m_{B_s} + m_\phi) A_1(q^2) - (m_{B_s} - m_\phi) A_2(q^2) \right].
\]

The other semileptonic form factors coming from the dipole operator $\sigma_{\mu \nu} q^\nu (1 + \gamma_5) b$ can be defined as:

\[
\langle \phi(p_\phi, \epsilon) | \bar{s}i \sigma_{\mu \nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle = 4\epsilon_{\mu \rho \sigma} \epsilon^{* \nu} p^\rho q^\sigma T_1(q^2) + 2i \left[ \epsilon_\mu^* (m_{B_s}^2 - m_\phi^2) - (p_{B_s} + p_\phi)_\mu (\epsilon^* q) \right] T_2(q^2) + 2i (\epsilon^* q) \left[ q_\mu - (p_{B_s} + p_\phi)_\rho \frac{q^2}{m_{B_s}^2 - m_\phi^2} \right] T_3(q^2).
\]

As seen From Eqs. (7-9), we have to compute the form factors to obtain the physical observables at hadronic level. The form factors are related to the non-perturbative sector of QCD and can be evaluated only by using non-perturbative methods. In the present work, we use light cone QCD sum rule predictions for the form factors in which one-loop radiative corrections to twist-2 and twist-3 contributions are taken into account. The form factors

\[
F(q^2) \in \{ V(q^2), A_0(q^2), A_1(q^2), A_2(q^2), A_3(q^2), T_1(q^2), T_2(q^2), T_3(q^2) \},
\]

are fitted to the the following functions $[37, 38]$:

\[
F(q^2) = \frac{F(0)}{1 - a_F \frac{q^2}{m_{B_s}^2} + b_F \left( \frac{q^2}{m_{B_s}^2} \right)^2},
\]

(10)
where the parameters $F(0)$, $a_F$ and $b_F$ are listed in the Table I.

|    | $F(0)$ | $a_F$ | $b_F$ |
|----|--------|-------|-------|
| $A_0^{B_s \rightarrow \phi}$ | 0.382 | 1.77  | 0.856 |
| $A_1^{B_s \rightarrow \phi}$ | 0.296 | 0.87  | -0.061 |
| $A_2^{B_s \rightarrow \phi}$ | 0.255 | 1.55  | 0.513 |
| $V^{B_s \rightarrow \phi}$   | 0.433 | 1.75  | 0.736 |
| $T_1^{B_s \rightarrow \phi}$ | 0.174 | 1.82  | 0.825 |
| $T_2^{B_s \rightarrow \phi}$ | 0.174 | 0.70  | -0.315 |
| $T_3^{B_s \rightarrow \phi}$ | 0.125 | 1.52  | 0.377 |

**TABLE I**: The form factors for $B_s \rightarrow \phi \ell^+ \ell^-$ in a three–parameter fit [37].

Using Eqs. (7-9), the matrix element of the $B_s \rightarrow \phi \ell^+ \ell^-$ decay can be written as follows:

$$
\mathcal{M}(B_s \rightarrow \phi \ell^+ \ell^-) = \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ \bar{\ell}\gamma(1-\gamma_5)\ell \left[ -2B_0 \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\mu} p_{\phi} q^{\sigma} - iB_1 \epsilon_{\mu}^* \\
+ iB_2 (\epsilon^* q)(p_{B_s} + p_\phi)_\mu + iB_3 (\epsilon^* q)q_\mu \right] \\
+ \bar{\ell}\gamma(1+\gamma_5)\ell \left[ -2C_1 \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p_{\phi} q^{\sigma} - iD_1 \epsilon_{\mu}^* \\
+ iD_2 (\epsilon^* q)(p_{B_s} + p_\phi)_\mu + iD_3 (\epsilon^* q)q_\mu \right] \right\} ,
$$

where

$$
B_0 = (\tilde{C}_9^{\text{eff}} - \tilde{C}_10^{\text{eff}}) \frac{V}{m_{B_s} + m_\phi} + 4(m_{B_s} + m_s)C_7^{\text{eff}} T_1 \frac{T_1}{q^2} ,
$$

$$
B_1 = (\tilde{C}_9^{\text{eff}} - \tilde{C}_10^{\text{eff}})(m_{B_s} + m_\phi)A_1 + 4(m_{B_s} + m_s)C_7^{\text{eff}} (m_{B_s}^2 - m_\phi^2) \frac{T_2}{q^2} ,
$$

$$
B_2 = \frac{\tilde{C}_9^{\text{eff}} - \tilde{C}_10^{\text{eff}}}{m_{B_s} + m_\phi} A_2 + 4(m_{B_s} + m_s)C_7^{\text{eff}} \left( \frac{1}{q^2} \left[ T_2 + \frac{q^2}{m_{B_s}^2 - m_\phi^2} T_3 \right] \right) ,
$$

$$
B_3 = 2(\tilde{C}_9^{\text{eff}} - \tilde{C}_10^{\text{eff}}) m_\phi \frac{A_3 - A_0}{q^2} - 4(m_{B_s} - m_s)C_7^{\text{eff}} \frac{T_3}{q^2} ,
$$

6
\[ C_1 = B_0(C_{10}^{\text{eff}} \rightarrow -C_{10}^{\text{eff}}), \]
\[ D_i = B_i(C_{10}^{\text{eff}} \rightarrow -C_{10}^{\text{eff}}), \quad (i = 1, 2, 3). \]

From the above equations for the differential decay width, we get the following result:

\[
\frac{d\Gamma}{ds}(B_s \rightarrow \phi \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_{B_s}^2}{24 \pi^5} |V_{tb}V_{ts}^*|^2 \lambda^{1/2}(1, \hat{r}, \hat{s}) \nu \Delta(\hat{s}),
\]

with

\[
\Delta = \frac{2}{3 \hat{s} m_{B_s}^2} \text{Re} \{-12 m_{B_s}^2 \hat{\delta} \lambda \hat{s} (B_3 - D_2 - D_3) B_1^* - (B_3 + B_2 - D_3) D_1^* \}
+ 12 m_{B_s}^4 \hat{\delta} \lambda \hat{s} (1 - \hat{r}_\phi) (B_2 - D_2) (B_3^* - D_3^*)
+ 48 \hat{\delta} \lambda \hat{s} (3 B_1 D_1^* + 2 m_{B_s}^2 \lambda B_0 C_1^*)
- 16 m_{B_s}^4 \hat{\delta} \lambda \hat{s} \{(|B_0|^2 + |C_1|^2) \}
- 6 m_{B_s}^2 \hat{\delta} \lambda \hat{s} \{(2 + 2 \hat{r}_\phi - \hat{s}) B_2 D_2^* - \hat{s} |(B_3 - D_3)|^2 \}
- 2 m_{B_s}^2 \lambda \{(3 - v^2) \hat{s} \{ |B_1|^2 + |D_1|^2 \} \}
- 2 m_{B_s}^2 \lambda \{(7 - 3(1 - \hat{r}_\phi)^2) - \hat{s} \} \{(|B_2|^2 + |D_2|^2) \},
\]

where \( \hat{s} = q^2/m_{B_s}^2 \), \( \hat{r}_\phi = m_{\phi}^2/m_{B_s}^2 \) and \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc, \hat{m}_\ell = m_\ell/m_{B_s}, \)
\( \nu = \sqrt{1 - 4 \hat{m}_\ell^2/\hat{s}} \) is the final lepton velocity.

Having obtained the matrix element for the \( B_s \rightarrow \phi \ell^+ \ell^- \), we can now calculate the double-polarization asymmetries. For this purpose, we define the orthogonal unit vectors \( s_i^{\pm \mu} \) in the rest frame of leptons, where \( i = \text{L}, \text{N}, \text{T} \) refer to the longitudinal, normal and transversal polarization directions, respectively:

\[
\begin{align*}
    s_{\text{L}}^{-\mu} &= (0, \hat{e}_{\text{L}}^-) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), & s_{\text{L}}^{+\mu} &= (0, \hat{e}_{\text{L}}^+) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \\
    s_{\text{N}}^{-\mu} &= (0, \hat{e}_{\text{N}}^-) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_-}{|\vec{p}_\phi \times \vec{p}_-|}\right), & s_{\text{N}}^{+\mu} &= (0, \hat{e}_{\text{N}}^+) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_+}{|\vec{p}_\phi \times \vec{p}_+|}\right), \\
    s_{\text{T}}^{-\mu} &= (0, \hat{e}_{\text{T}}^-) = \left(0, \frac{\vec{e}_{\text{N}} \times \vec{e}_{\text{L}}^-}{|\vec{e}_{\text{N}} \times \vec{e}_{\text{L}}^-|}\right), & s_{\text{T}}^{+\mu} &= (0, \hat{e}_{\text{T}}^+) = \left(0, \frac{\vec{e}_{\text{N}} \times \vec{e}_{\text{L}}^+}{|\vec{e}_{\text{N}} \times \vec{e}_{\text{L}}^+|}\right).
\end{align*}
\]

In the above equations \( \vec{p}_\pm \) and \( \vec{p}_\phi \) are the three–momenta of the leptons \( \ell^\pm \) and \( \phi \) meson, respectively. Then by Lorentz transformation these unit vectors are boosted from the rest frame of leptons to the center of mass (CM) frame of leptons. Under this transformation
only the longitudinal unit vectors \( s_L^{\pm \mu} \) change, but the other two vectors remain unchanged.

\( s_L^{\pm \mu} \) in the CM frame of leptons are obtained as:

\[
(s_L^{-\mu})_{CM} = \left( \frac{|\vec{p}|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell |\vec{p}_-|} \right), \quad (s_L^{+\mu})_{CM} = \left( \frac{|\vec{p}|}{m_\ell}, -\frac{E\vec{p}_-}{m_\ell |\vec{p}_-|} \right).
\]

The polarization asymmetries can now be calculated using the spin projector \( \frac{1}{2}(1 + \gamma_5 \hat{r}_\ell) \) for \( \ell^- \) and the spin projector \( \frac{1}{2}(1 + \gamma_5 \hat{r}_\ell^\dagger) \) for \( \ell^+ \).

Considering the above explanations, we can define the double–lepton polarization asymmetries as in [39]:

\[
P_{ij}(\hat{s}) = \frac{2}{\hat{s}^2} \left( \frac{d\Gamma}{ds}(\hat{s}_i, \hat{s}_j) - \frac{d\Gamma}{ds}(\hat{s}_i, -\hat{s}_j) \right) - \frac{d\Gamma}{ds}(\hat{s}_i, -\hat{s}_j) + \frac{d\Gamma}{ds}(\hat{s}_i, \hat{s}_j) - \frac{d\Gamma}{ds}(\hat{s}_i, -\hat{s}_j) + \frac{d\Gamma}{ds}(\hat{s}_i, \hat{s}_j)
\]

where \( i, j = L, N, T, \) and the first index \( i \) corresponds to lepton while the second index \( j \) corresponds to antilepton, respectively. After doing the straightforward calculation we obtain the following expressions for \( P_{ij}(\hat{s}) \):

\[
P_{LL} = \frac{m_B^2}{3\hat{s}^2} \text{Re} \left\{ -24m_B^2 \hat{m}_t^2 \hat{s} \lambda \left( (B_1 - D_1)(B_3^* - D_3^*) \right) \right\}
\]

\[
+ 12m_B^3 \hat{m}_t \hat{s} \lambda (1 - \hat{r}_\phi) \left[ 2m_B \hat{m}_t (B_2 - D_2)(B_3^* - D_3^*) \right]
\]

\[
- 8m_B^4 \hat{r}_\phi \hat{s}^2 \lambda (1 + 3v^2)(|B_0|^2 + |C_1|^2) + 12m_B^3 \hat{m}_t \hat{s}^2 \lambda |B_3 - D_3|^2
\]

\[
+ 8m_B^2 \hat{m}_t^2 \lambda (4 - 4\hat{r}_\phi - \hat{s})(B_1 D_1^* + B_2 D_1^*) - 32\hat{m}_t^2 \lambda (3 - 3\hat{r}_\phi \hat{s}) B_1 D_1^*
\]

\[
- 8m_B^4 \hat{m}_t^2 \lambda [\lambda + 3(1 - \hat{r}_\phi)^2] B_2 D_2^* - 64m_B^4 \hat{m}_t^2 \hat{s}\lambda B_0 C_1^*
\]

\[
+ 8m_B^2 \lambda [\hat{s} - \hat{s}(\hat{r}_\phi + \hat{s}) - 3\hat{m}_t^2 (2 - 2\hat{r}_\phi - \hat{s})](B_1 B_2^* + D_1 D_2^*)
\]

\[
- m_B^4 \lambda (1 + 3v^2) - 3(1 - \hat{r}_\phi)^2 (1 - v^2)(|B_2|^2 + |D_2|^2)
\]

\[
+ 4(6\hat{m}_t^2 (\lambda + 6\hat{r}_\phi \hat{s}) - \hat{s}(\lambda + 12\hat{r}_\phi \hat{s}))(|B_1|^2 + |D_1|^2) \right\},
\]

\[
P_{LN} = \frac{\pi m_B^2}{2\hat{s}^2} \sqrt{\hat{s}} \left\{ -4m_B^4 \hat{m}_t \lambda (1 - \hat{r}_\phi) B_2 D_2^*
\]

\[
+ 2m_B^4 \hat{m}_t \hat{s} \lambda B_2 B_3^* - 2m_B^4 \hat{m}_t \hat{s} \lambda [B_3 D_2^* + (B_2 + D_2) D_3^*]
\]

\[
- 2m_B^2 \hat{m}_t \hat{s}(1 + 3\hat{r}_\phi - \hat{s})(B_1 B_2^* - D_1 D_2^*) - 4\hat{m}_t (1 - \hat{r}_\phi - \hat{s}) B_1 D_1^*
\]

\[
- 2m_B^2 \hat{m}_t \hat{s} (1 - \hat{r}_\phi - \hat{s})(B_1 + D_1)(B_3^* - D_3^*)
\]

\[
+ 2m_B^2 \hat{m}_t [\lambda (1 - \hat{r}_\phi)(1 - \hat{r}_\phi - \hat{s})](B_2 D_1^* + B_1 D_2^*) \right\},
\]

\[
(14)
\]

\[
(15)
\]

\[
(16)
\]

\[
(17)
\]
\[ P_{NL} = -P_{LN} \]  

\[ P_{LT} = \frac{\pi m^2}{\tilde{\tau}_\phi} \sqrt{\frac{\lambda}{s}} \Re \left\{ m^4 B_2 \tilde{m}_\ell \lambda (1 - \tilde{r}_\phi) |B_2 - D_2|^2 \right\} - 8m^2 B_2 \tilde{m}_\ell \tilde{s}B_0 B_1^* - C_1 D_1^* \right\} + m^4 B_2 \tilde{s} \lambda \tilde{m}_\ell B_2 B_3^* \right\} + m_B \tilde{s} (1 - \tilde{r}_\phi - \tilde{s}) [ - m_B \tilde{m}_\ell (B_1 - D_1)(B_3^* - D_3^* \right\} ] - m^2 B_2 \tilde{m}_\ell [\lambda + (1 - \tilde{r}_\phi)(1 - \tilde{r}_\phi - \tilde{s})](B_1 - D_1)(B_2^* - D_2^*) \right\} , \]  

\[ P_{TL} = \frac{\pi m^2}{\tilde{\tau}_\phi} \sqrt{\frac{\lambda}{s}} \Re \left\{ m^4 B_2 \tilde{m}_\ell \lambda (1 - \tilde{r}_\phi) |B_2 - D_2|^2 \right\} + 8m^2 B_2 \tilde{m}_\ell \tilde{s}B_0 B_1^* - C_1 D_1^* \right\} + m^4 B_2 \tilde{s} \lambda \tilde{m}_\ell B_2 B_3^* \right\} + m_B \tilde{s} (1 - \tilde{r}_\phi - \tilde{s}) [ - m_B \tilde{m}_\ell (B_1 - D_1)(B_3^* - D_3^* \right\} ] - m^2 B_2 \tilde{m}_\ell [\lambda + (1 - \tilde{r}_\phi)(1 - \tilde{r}_\phi - \tilde{s})](B_1 - D_1)(B_2^* - D_2^*) \right\} , \]  

\[ P_{NT} = \frac{2m^2}{3\tilde{\tau}_\phi} \Im \left\{ 4\lambda (B_1 D_1^* + m^4 B_2 \lambda B_2 D_2^*) - 16m^4 B_2 \tilde{s} \lambda \tilde{r}_\phi B_0 C_1^* \right\} \]  

\[ P_{TN} = -P_{NT} \]  

\[ P_{NN} = \frac{2m^2}{3\tilde{\tau}_\phi} \Re \left\{ - 24\tilde{m}_\ell^2 \tilde{r}_\phi (|B_1|^2 + |D_1|^2) + 16m^4 B_2 \tilde{s} \lambda \tilde{r}_\phi v^2 B_0 C_1^* \right\} + 6m^2 B_2 \tilde{m}_\ell^2 \lambda [ - 2B_1 (B_2^* + B_3^* - D_3^* + 2D_1 (B_3^* - D_2^* - D_3^* \right\} ] + 6m^3 B_2 \tilde{m}_\ell (1 - \tilde{r}_\phi) [2m_B \tilde{m}_\ell (B_2 - D_2)(B_3^* - D_3^*) \right\] + 6m^4 B_2 \tilde{m}_\ell^2 \lambda (2 + 2\tilde{r}_\phi - \tilde{s})(|B_2|^2 + |D_2|^2) + 6m^4 B_2 \tilde{m}_\ell^2 \tilde{s} \lambda |B_3 - D_3|^2 \right\} + m_B^2 \lambda (3(2 - 2\tilde{r}_\phi - \tilde{s}) - v^2(2 - 2\tilde{r}_\phi + \tilde{s})(B_1 D_2^* + B_2 D_1^*) \right\} + m_B^4 \lambda (3 + v^2) \lambda + 3(1 - v^2)(1 - \tilde{r}_\phi)^2 \right\} B_2 D_2^* \]  

\[ - 2[6\tilde{r}_\phi \tilde{s}(1 - v^2) + \lambda (3 - v^2)] B_1 D_1^* \right\} , \]
\[
P_{TT} = \frac{2m_{B_s}^2}{3r_\phi s}\Delta \text{Re}\left\{8m_{B_s}^4 \hat{r}_\phi \hat{s} \lambda \left[4m_t^2(|B_0|^2 + |C_1|^2) + 2\hat{s}B_0 C_1^* \right] - 6m_{B_s}^2 \hat{m}_t^2 \hat{s} \lambda \left[-2(B_1 - D_1)(B_3^* - D_3^*) \right] - 6m_{B_s}^3 \hat{m}_t \hat{s} \lambda (1 - \hat{r}_\phi) \left[2m_{B_s} \hat{m}_t (B_2 - D_2)(B_3^* - D_3^*) \right] - 6m_{B_s}^4 \hat{m}_t^2 \hat{s} \lambda |B_3 - D_3|^2 + 4m_{B_s}^2 \hat{m}_t^2 \lambda (4 - 4\hat{r}_\phi - \hat{s})(B_1 B_2^* + D_1 D_2^*) + 2\hat{s}[2\hat{r}_\phi \hat{s}(1 - v^2) + \lambda (1 - 3v^2)]B_1 D_1^* - 2m_{B_s}^4 \hat{m}_t^2 \lambda [\lambda + 3(1 - \hat{r}_\phi)^2](|B_2|^2 + |D_2|^2) - m_{B_s}^2 \hat{s} \lambda [2 - 2\hat{r}_\phi + \hat{s} - 3v^2(2 - 2\hat{r}_\phi - \hat{s})](B_1 D_2^* + B_2 D_1^*) - 8m_t^2 \lambda (\lambda - 3\hat{r}_\phi \hat{s})(|B_1|^2 + |D_1|^2) - m_{B_s}^4 \hat{m}_t^2 \lambda [(1 + 3v^2)\lambda - 3(1 - v^2)(1 - \hat{r}_\phi)^2]B_2 D_2^* \right\}. \tag{24}\]

The analytical dependence of the double-lepton polarizations on the fourth quark mass \(m_{t'}\) and the product of quark mixing matrix elements \(V_{tb}^* V_{ts} = r_{sb} e^{i\phi_{sb}}\) are studied in the next section.

### III. EFFECTS OF THE FOURTH-GENERATION

As we mentioned in the introduction, the inclusion of the fourth-generation in the Standard Model (SM4) does not lead to new operators in the \(\mathcal{H}_{\text{eff}}\) and all Wilson coefficients receive additional terms as \(\frac{\lambda_{t'}}{\lambda_t} C_{i}^{\text{SM4}}\) either via virtual exchange of the fourth-generation up-type quark \(t'\) \((C_3, ..., C_{10})\) or via using the unitarity of CKM matrix \((C_1, C_2)\). Consequently, one can write the new effective Hamiltonian as:

\[
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \sum_{i=1}^{10} C_{i}^{\text{new}}(\mu) \mathcal{O}_i(\mu), \tag{25}\]

where \(C_{i}^{\text{new}}\) are:

\[
C_{i}^{\text{new}}(\mu) = C_i(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{i}^{\text{SM4}}(\mu), \quad i = 1 \ldots 10. \tag{26}\]

In the above equation, \(\lambda_f = V_{fb}^* V_{fs}\) and \(\lambda_{t'}\) can be parameterized as:

\[
\lambda_{t'} = V_{t'b} V_{t's}^* = r_{sb} e^{i\phi_{sb}}. \tag{27}\]

Now by using the above effective Hamiltonian, we can reobtain the one-loop matrix elements of \(b \to s\ell^+\ell^-\) by replacing \(C_{i}^{\text{eff}}(\tilde{C}_{i}^{\text{eff}})\) with \(C_{i}^{\text{eff new}}(\tilde{C}_{i}^{\text{eff new}})\) in Eq.\(2\), where \(C_{i}^{\text{eff new}}\)
and \( \tilde{C}_{\text{eff new}} \) are given as:

\[
C_{\text{eff new}}^i(\mu) = C_{\text{eff}}^i(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{\text{SM4}}^i(\mu), \quad i = 7, \\
\tilde{C}_{\text{eff new}}^i(\mu) = \tilde{C}_{\text{eff}}^i(\mu) + \frac{\lambda_{t'}}{\lambda_t} \tilde{C}_{\text{SM4}}^i(\mu), \quad i = 9, 10.
\] (28)

Here the effective Wilson coefficients \( C_{\text{eff SM4}}^i \) and \( \tilde{C}_{\text{eff SM4}}^i \) are defined in the same way as Eqs.(3) by substituting \( C_i \) with \( C_{\text{SM4}}^i \). It is worth noting that the explicit forms of \( C_{\text{eff SM4}}^i \) and \( \tilde{C}_{\text{eff SM4}}^i \) can also be found from the corresponding Wilson coefficients in SM by replacing \( m_t \rightarrow m_t' \) [30]. Based on the preceding explanations, in order to obtain the matrix element and the double-lepton polarization asymmetries for \( B_s \rightarrow \phi \ell^+ \ell^- \) decay in the presence of the fourth-generation, one should replace \( C_{\text{eff}}^i(\tilde{C}_{\text{eff}}^i) \) with \( C_{\text{eff new}}^i(\tilde{C}_{\text{eff new}}^i) \) in all equations of the previous section.

The unitary quark mixing matrix is now \( 4 \times 4 \) which can be written in terms of 6 mixing angles and 3 CP violating phases. The relevant elements of this matrix for \( b \rightarrow s \) transition satisfy the relation:

\[
\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0.
\] (29)

Consequently, as required by GIM mechanism, the factor \( \lambda_t C_{\text{new}}^i \) should be modified to \( \lambda_t C_i \) when \( m_{t'} \rightarrow m_t \) or \( \lambda_{t'} \rightarrow 0 \). We can easily check the validity of this condition by using Eq.(29):

\[
\lambda_t C_{\text{new}}^i = \lambda_t C_i + \lambda_{t'} C_{\text{SM4}}^i = -(\lambda_u + \lambda_c) C_i + \lambda_{t'} (C_{\text{SM4}}^i - C_i) = -(\lambda_u + \lambda_c) C_i = \lambda_t C_i.
\] (30)

The numerical analysis of the dependence of the double-lepton polarizations on the fourth quark mass (\( m_{t'} \)) and the product of quark mixing matrix elements \( (V_{t'b} V_{t's} = r_{sbe^{i\phi_ab}}) \) are presented in the next section.

IV. RESULTS AND DISCUSSIONS

The main input parameters in the calculations are the form factors for which we have chosen the predictions of light cone QCD sum rule method [37, 38], as pointed out in section

11
II. Besides the form factors, we use the other input parameters as follow:

\[ m_{B_s} = 5.37 \text{ GeV}, \ m_b = 4.8 \text{ GeV}, \ m_c = 1.5 \text{ GeV}, \ m_{\tau} = 1.77 \text{ GeV}, \]
\[ m_{\mu} = 0.105 \text{ GeV}, \ m_{\phi} = 1.020 \text{ GeV}, |V_{tb}^{\prime}V_{ts}^{\prime*}| = 0.0385, \ \alpha^{-1} = 129, \]
\[ G_f = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \ \tau_{B_s} = 1.46 \times 10^{-12} \text{ s}. \] (31)

In order to present a quantitative analysis of the double-lepton polarization asymmetries, the values of fourth-generation parameters are needed. Considering the experimental values of \( B^- \to X_s \gamma \) and \( B^- \to X_s \ell^+ \ell^- \) decays the value of the \( r_{sb} \) parameter is restricted to the range \{0.01 - 0.03\} for \( \phi_{sb} \sim \{0^\circ - 360^\circ\} \) and \( m_{\ell'} \sim \{200 - 600\} \text{ GeV} \)\cite{17, 27}. Using the \( B_s \) mixing parameter \( \Delta m_{B_s} \), a sharp restriction on \( \phi_{sb} \) has been obtained \( (\phi_{sb} \sim 90^\circ) \)\cite{13}. Therefore in our following numerical analysis, the corresponding values of above ranges are:

\[ r_{sb} = \{0.01, 0.02, 0.03\}, \ \phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}, \ m_{\ell'} = 175 \leq m_{\ell'} \leq 600. \]

It is clear from the expressions of all nine double–lepton polarization asymmetries that they depend on the momentum transfer \( q^2 \) and the new parameters \( (m_{\ell'}, r_{sb}, \phi_{sb}) \). Consequently, it may be experimentally difficult to investigate these dependencies at the same time. One way to deal with this problem is to integrate over \( q^2 \) and study the averaged double-lepton polarization asymmetries. The average of \( P_{ij} \) over \( q^2 \) is defined as:

\[
\langle P_{ij} \rangle = \frac{\int_{4\hat{m}_{\ell'}^2}^{1-(1-\sqrt{\hat{\rho}})^2} P_{ij} \frac{dB}{d\hat{s}} d\hat{s}}{\int_{4\hat{m}_{\ell'}^2}^{1-(1-\sqrt{\hat{\rho}})^2} \frac{dB}{d\hat{s}} d\hat{s}}. \] (32)

We have used the above formula and depicted the dependency of \( \langle P_{ij} \rangle \) on the fourth-generation parameters in Fig.\cite{11, 17}. In the following, we compare our results for \( B_s \to \phi \ell^+ \ell^- \) decay with the results of Ref.\cite{24} for \( B \to K \ell^+ \ell^- \) decay. Since the overall behavior of \( \langle P_{ij} \rangle \) versus \( m_{\ell'}, r_{sb} \) and \( \phi_{sb} \) are almost the same as that of \( B \to K \ell^+ \ell^- \) decay, we discuss the differences of these two decays and some aspects which have not been discussed in Ref.\cite{24}:

- **Figure(1)**: Similar to the \( B \to K \mu^+ \mu^- \) decay, \( \langle P_{LL} \rangle \) is not sensitive to the fourth-generation quark parameters, therefore the \( \langle P_{LL} \rangle \) plots for \( \mu \) channel have been omitted. However, for the \( \tau \) channel, the maximum deviation from SM is about 50\% which can be seen at \( m_{\ell'} \sim 600 \text{ GeV} \). In comparison with the results of Ref.\cite{24}, it is understood that the deviation from SM for \( B_s \to \phi \tau^+ \tau^- \) is twice that of \( B \to K \tau^+ \tau^- \).
decay. Therefore, the magnitude of $\langle P_{LL} \rangle$ in $B_s \rightarrow \phi \tau^+ \tau^-$ compared with that in $B \rightarrow K \tau^+ \tau^-$ decay has more chance to show the existence of the fourth-generation.

- **Figure (2):** The value of $\langle P_{LN} \rangle_{\text{max}}$ for $\mu$ channel is about 0.04 which is four times greater than that for $B \rightarrow K$ decay. However, for $\tau$ channel such value is at most around 0.3 which is approximately equal to the maximum value of $\langle P_{LN} \rangle$ for $B \rightarrow K$ decay. Furthermore, in $\mu$ and $\tau$ channels by increasing $r_{sb}$ and keeping the values of $\phi_{sb}$ fixed, the maximum deviation from SM occurs at smaller values of $m_{t'}$. This result can be interesting since the maximum deviation from SM happens for $r_{sb} \sim \{0.02 - 0.03\}$ and $m_{t'} \sim \{300 - 400\}$ GeV. Therefore, the new generation has a chance to be observed around $m_{t'} \sim \{300 - 400\}$ GeV. Our analysis shows that to measure the effect of the fourth-generation in $\langle P_{LN} \rangle$, the $\tau$ channel of $B_s \rightarrow \phi$ and $B \rightarrow K$ are more important than $\mu$ channel of these decays, knowing that in the $\mu$ channel the $B_s \rightarrow \phi$ decay is more significant than the $B \rightarrow K$ decay.

- **Figure (3):** For $\mu$ channel, the magnitude of $\langle P_{LT} \rangle$ in $B_s \rightarrow \phi$ decay changes at most about 80% compared with the SM prediction, while the maximum change in $B \rightarrow K$ decay reaches up to 60%. For $\tau$ case, unlike $B \rightarrow K$ decay, the magnitude of $\langle P_{LT} \rangle$ in $B_s \rightarrow \phi$ transition exhibits the strong dependence on the fourth quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($|V_{t'b}V_{t's}^*| = r_{sb}$). As seen from Fig. (3) the maximum deviation from SM in $\tau$ channel is much more than that in $\mu$ channel. Therefore for establishing the fourth generation of quarks the measurement of $\langle P_{LT} \rangle$ for $B_s \rightarrow \phi \tau^+ \tau^-$ decay is more suitable than such measurement for $B_s \rightarrow \phi \mu^+ \mu^-$ and $B \rightarrow K \mu^+ \mu^-$ decays.

- **Figure (4):** It is seen from Eqs. (19) and (20) that contrary to $B \rightarrow K$ decay, $P_{TL}$ is neither symmetric nor anti-symmetric under the exchange of subscripts L and T which leads to different values for $P_{TL}$ and $P_{LT}$. For $\mu$ channel, the magnitude of $\langle P_{TL} \rangle$ in $B_s \rightarrow \phi$ decay changes at most about 40% compared with the SM prediction, while the maximum change in the case of $B \rightarrow K$ decay reaches up to 60%. For $\tau$ case, unlike $B \rightarrow K$ decay, the magnitude of $\langle P_{TL} \rangle$ in $B_s \rightarrow \phi$ transition changes at most about 60% compared with the SM prediction. Therefore, in the measurement of $\langle P_{TL} \rangle$, the decays $B_s \rightarrow \phi \ell^+ \ell^- (\ell = \mu, \tau)$ and $B \rightarrow K \mu^+ \mu^-$ have the same significance for finding the new generation of quarks.
• **Figure(5)**: By comparing this figure with Fig.(2), one can find out that the overall behavior of $\langle P_{TN} \rangle$ and $\langle P_{LN} \rangle$ are the same. Furthermore, the magnitude of $\langle P_{TN} \rangle_{\text{max}}$ for $\mu$ channel is about 0.22 which is four times smaller than that for $B \to K$ decay and for $\tau$ channel such value is at most around 0.0075 which is approximately ten times smaller than $\langle P_{TN} \rangle_{\text{max}}$ for $B \to K$ decay. Although the measurement of $\langle P_{TN} \rangle$ in $B \to K\tau^+\tau^-$ decay for finding the new generation is useful, such measurement in the decays $B_s \to \phi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ are more significant.

• **Figure(6)**: For both $\mu$ and $\tau$ channels in $B_s \to \phi$ decay, the values of $\langle P_{NN} \rangle$ show stronger dependence on the fourth-generation parameters ($m_{t'}, r_{sb}, \phi_{sb}$) in comparison with those in $B \to K$ decay. Furthermore, the situation for $\tau$ channel is even more interesting than $\mu$ channel, since for fixed values of $\phi_{sb}$ and $r_{sb}$, an increase in $m_{t'}$ changes the sign of $\langle P_{NN} \rangle$. So, for $B_s \to \phi$ decay, the study of the magnitude and the sign of $\langle P_{NN} \rangle$ for $\tau$ channel and the magnitude of this asymmetry in $\mu$ channel can serve as good tests for discovering the new physics beyond the SM. It should also be mentioned that for both $\mu$ and $\tau$ channels of $B \to K$ decay in general, and specially for the $\mu$ channel, the deviation of $\langle P_{NN} \rangle$ from SM can be a measurable quantity, even though it is less sensitive to the fourth generation of quarks compared with that of $B \to \phi$ decay (see Ref.[24]).

• **Figure(7)**: A comparison between this figure and an analogous figure for $B \to K\ell^+\ell^-$ shows that the values of $\langle P_{TT} \rangle$ for both $\mu$ and $\tau$ channels in $B_s \to \phi$ decay have considerable dependency on the fourth-generation parameters ($m_{t'}, r_{sb}, \phi_{sb}$). Therefore, compared with $B \to K\ell^+\ell^-$ decay in Ref.[24], the study of the magnitude of $\langle P_{TT} \rangle$ in $B_s \to \phi\ell^+\ell^-$ provides a better opportunity to see the effect of the new physics beyond the SM.

Finally, let us briefly discuss whether it is possible to measure the lepton polarization asymmetries in experiments or not. Experimentally, to measure an asymmetry $\langle P_{ij} \rangle$ of the decay with branching ratio $B$ at $n\sigma$ level, the required number of events (i.e., the number of $B\bar{B}$) is given by the formula

$$N = \frac{n^2}{Bs_1s_2\langle P_{ij} \rangle^2},$$
where $s_1$ and $s_2$ are the efficiencies of the leptons. Typical values of the efficiencies of the \( \tau \)–leptons vary from 50\% to 90\% for their different decay modes \[40\] and the error in \( \tau \)–lepton polarization is estimated to be about $(10 - 15)\%$ \[41\]. So, the error in measurement of the \( \tau \)–lepton asymmetries is approximately $(20 - 30)\%$, and the error in obtaining the number of events is about 50\%.

Looking at the expression of \( N \), it can be understood that in order to detect the lepton polarization asymmetries in the $\mu$ and $\tau$ channels at $3\sigma$ level, the minimum number of required events are (for the efficiency of $\tau$–lepton we take 0.5):

- for $B_s \to \phi \mu^+ \mu^-$ decay

\[
N \sim \begin{cases} 
10^6 & \text{(for } \langle P_{LL} \rangle \text{)}, \\
10^7 & \text{(for } \langle P_{NT} \rangle, \langle P_{TN} \rangle \text{)}, \\
10^8 & \text{(for } \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle, \langle P_{TT} \rangle \text{)}, \\
10^9 & \text{(for } \langle P_{LN} \rangle, \langle P_{NL} \rangle \text{)}.
\end{cases}
\]

- for $B_s \to \phi \tau^+ \tau^-$ decay

\[
N \sim \begin{cases} 
10^8 & \text{(for } \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle, \langle P_{TT} \rangle \text{)}, \\
10^9 & \text{(for } \langle P_{LL} \rangle, \langle P_{LN} \rangle, \langle P_{NL} \rangle \text{)}, \\
10^{12} & \text{(for } \langle P_{NT} \rangle, \langle P_{TN} \rangle \text{)}.
\end{cases}
\]

Considering the above values for \( N \) and the number of $B \bar{B}$ pairs which will be produced at LHC($\sim 10^{12}$), one can conclude that except $\langle P_{NT} \rangle$ and $\langle P_{TN} \rangle$ for $\tau$ channel, all double-lepton polarizations can be detected at the LHC.

In summary, in this paper we have presented the analyses of the double-lepton polarization asymmetries in $B_s \to \phi \ell^+ \ell^-$ decay using the SM with the fourth generation of quarks. We found out that these asymmetries have strong dependency on the fourth-generation parameters which can be detected at the LHC. We compared $B_s \to \phi \ell^+ \ell^-$ decay with $B \to K \ell^+ \ell^-$ decay, and showed that the double-lepton polarizations of $B_s \to \phi \ell^+ \ell^-$ are more sensitive to the fourth-generation parameters and therefore by looking at $B_s \to \phi \ell^+ \ell^-$ decay, one has more chance to investigate the correctness of the fourth generation of quarks hypothesis in the near future.
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FIG. 1: The dependence of the $\langle P_{LL} \rangle$ on the fourth-generation quark mass $m_t$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\tau$ channel.
FIG. 2: The dependence of the $\langle P_{LN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.
FIG. 3: The dependence of the $\langle P_{LT} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.
FIG. 4: The dependence of the $\langle P_{TL} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.
FIG. 5: The dependence of the $\langle P_{TN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.
FIG. 6: The dependence of the $\langle P_{NN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.
FIG. 7: The dependence of the $\langle P_{TT} \rangle$ on the fourth-generation quark mass $m_t'$ for three different values of $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the $\mu$ and $\tau$ channels.