Extraction of the light quark mass ratio from the decays $\psi' \rightarrow J/\psi\pi^0(\eta)$

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Abstract

Light quark masses are important fundamental parameters of the Standard Model. The decays $\psi' \rightarrow J/\psi\pi^0(\eta)$ were widely used in determining the light quark mass ratio $m_u/m_d$. However, there is a large discrepancy between the resulting value of $m_u/m_d$ and the one determined from the light pseudoscalar meson masses. Using the technique of non-relativistic effective field theory, we show that intermediate charmed meson loops lead to a sizeable contribution to the decays and hence make the $\psi' \rightarrow J/\psi\pi^0(\eta)$ decays not suitable for a precise extraction of the light quark mass ratio.

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The decays of $\psi'$ into $J/\psi\pi^0$ and $J/\psi\eta$ were suggested to be a reliable source for extracting the light quark mass ratio $m_u/m_d$ [1,2] (for reviews, see Refs. [3,4]). The decay $\psi' \rightarrow J/\psi\pi^0$ violates isospin symmetry. Both the up-down quark mass difference and the electromagnetic (em) interaction can contribute to isospin breaking. However, it was shown that the em contribution to the decay $\psi' \rightarrow J/\psi\pi^0$ is much smaller than the effect of the quark mass difference [3,7]. Based on the QCD multipole expansion and the axial anomaly, the relation between the quark mass ratio

$$\frac{1}{R} \equiv \frac{m_d - m_u}{m_s - m}, \quad (1)$$

where $\hat{m} = (m_d + m_u)/2$, and the ratio of the decay widths of these two decays was worked out up to the next-to-leading order in the chiral expansion [3,7]. At leading order, the relation reads [10]

$$R_{\pi^0/\eta} \equiv \frac{B(\psi' \rightarrow J/\psi\pi^0)}{B(\psi' \rightarrow J/\psi\eta)} = \frac{27}{16R^2} \frac{|\bar{q}_\pi|^3}{|\bar{q}_\eta|^3} (1 + \Delta_{\psi'}), \quad (2)$$

where $\bar{q}_{\pi(\eta)}$ denotes the momentum of the pion (eta) in the rest frame of the $\psi'$ and $\Delta_{\psi'}$ represents SU(3) breaking effects. Assuming $\Delta_{\psi'} < 0.4$, an upper limit of $R$ was determined through Eq. [3,10]. It can also be obtained by constructing a chiral effective Lagrangian for charmonium states and light mesons in a soft-exchange-approximation [11]. The amplitude for the $\psi' \rightarrow J/\psi\pi^0$ scales as

$$M(\psi' \rightarrow J/\psi\pi^0) \sim (m_d - m_u)|\bar{q}_\pi|.$$

Using the relation between the masses of quarks and mesons [12,13], Eq. (2) may be rewritten as

$$R_{\pi^0/\eta} = 3 \left( \frac{m_d - m_u}{m_d + m_u} \right)^2 \frac{2 F_{\pi}\ M_{\pi}^2}{2 F_{\eta}\ M_{\eta}^2} \left( \frac{|\bar{q}_\pi|^3}{|\bar{q}_\eta|^3} \right), \quad (4)$$

where $F_{\pi(\eta)}$ and $M_{\pi(\eta)}$ are the decay constant and mass of the pion (eta), respectively. Using Eq. (4) and the most recent measurement of the decay-width ratio [14]

$$R_{\pi^0/\eta} = (3.88 \pm 0.23 \pm 0.05)\%,$$  \quad (5)

the up-down quark mass ratio is obtained as [39]

$$\frac{m_u}{m_d} = 0.40 \pm 0.01. \quad (6)$$

This value is much smaller than the result obtained from the time-honored formula [12]

$$\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^+}^2 - M_{K^0}^2 + M_{\pi^0}^2 - M_{\pi^+}^2} = 0.56, \quad (7)$$

and it is also smaller than the large $N_c$ bound, $m_u/m_d \geq 1/2$, derived in Refs. [10,17]. Note that Eq. (7) is very little affected by higher order corrections. It is therefore of fundamental interest to understand theoretically the discrepancy between the values of up-down quark mass ratio determined from different sources. This Letter is devoted to show that the $\psi'$ decays into $J/\psi\pi^0(\eta)$ are not suitable for extracting the quark mass ratio, and hence the seeming discrepancy between Eq. (6) and Eq. (7) is meaningless. The reason underlying this statement is that the earlier analysis neglected effects from intermediate (virtual) charmed mesons. Those loops were shown to be important in some charmonium decays in phenomenological models, see, for instance, Refs. [18,20].

As we will show, based on a power counting argument in the spirit of heavy quark effective field theory (HQEFT), which is supported by an explicit calculation, these contributions overwhelm the one directly related to the quark masses.

To be specific, we calculate the pertinent diagrams for the decays $\psi' \rightarrow J/\psi\pi^0(\eta)$ involving the lowest lying pseudoscalar and vector charmed mesons, see Fig. 1. The couplings of pion and eta to the charmed mesons follow from heavy quark symmetry and chiral symmetry [21,23]. In the two-component notation of Ref. [24], the charmed mesons are represented by $H_a = \tilde{V}_a \cdot \bar{\sigma} + P_a$ with $V_a$ and $P_a$ denoting the vector and pseudoscalar charmed mesons, respectively, $\bar{\sigma}$ is the Pauli matrix, and $a$ is the flavor index. The lowest order axial coupling Lagrangian is [24]

$$\mathcal{L}_\phi = -\frac{g}{2} \bar{x} \gamma \cdot \left[ I_a H_b \bar{\sigma} \cdot \bar{u}_{ba} \right], \quad (8)$$
where the axial current is \( \bar{u} = -\sqrt{2}\bar{\sigma}D_0/F + O(\phi^3) \). \( F \) is the pion decay constant in the chiral limit, and the \( 3 \times 3 \) matrix \( \phi \) collects the octet Goldstone bosons. The leading order Lagrangian for the coupling of the \( J/\psi \) to the charmed and anti-charmed mesons can be constructed considering parity, charge parity and spin symmetry. In two-component language, it is given by

\[
L_\psi = i\frac{g_2}{2} \text{Tr} \left[ J^I H_a \bar{\sigma} \cdot \vec{D} \bar{H}_a \right] + \text{H.c.},
\]

with \( \bar{\sigma} \cdot \vec{D} B = A(\bar{D} B) - (\bar{D} A) B \). The charmonium field is given by \( J = \psi \cdot \bar{\sigma} + \eta_c \), with \( \psi \) and \( \eta_c \) annihilating the \( \psi \) and \( \eta_c \) states, and \( H_a = -\bar{V}_a \cdot \bar{\sigma} + P_a \) is the field for anti-charmed mesons \([26]\). This Lagrangian was first introduced in Ref. \([25]\) in four-component notation with the same coupling \( g_2 \). Since \( \psi' \) is the first radial excitation of the \( J/\psi \), the Lagrangian for the \( \psi' \) coupling to the charmed and anti-charmed mesons has the same form as Eq. \((9)\) with the coupling constant \( g_2 \) replaced by the one for \( \psi \), \( g'_2 \).

Because the \( \psi' \) and \( J/\psi \) are SU(3) singlets, it is obvious that the decay \( \psi' \to J/\psi \pi^0 \) violates isospin symmetry, and the decay \( \psi' \to J/\psi \eta \) violates SU(3) flavor symmetry \([40]\). Accordingly, the decay amplitudes reflect the flavor symmetry breaking. Here, all the charmed mesons in a flavor multiplet can contribute, and it is the mass differences within the multiplet that generates the isospin or the SU(3) breaking. Similar effects have been studied in \( a_0 - f_0 \) mixing \([27, 28]\), and the isospin breaking hadronic decay of the \( D_s^* (2317) \) \([29, 31]\).

Explicitly, the \( \psi' \to J/\psi \pi^0 \) decay amplitude is proportional to the difference of the charged and neutral mesons loops

\[
\mathcal{M}(\psi' \to J/\psi \pi^0) \propto e^{ijk} q_{\pi} q_{\psi'} q_{J/\psi} (I_c - I_n),
\]

where \( e^{ijk} \) denotes the spatial component of the polarization vector of the \( J/\psi (\psi') \), \( I_c \) and \( I_n \) are the loop integral expressions which will be given below in Eq. \((14)\) for charged and neutral charmed mesons. Denoting the expression for the strange charmed-meson loop by \( I_s \), one obtains the decay amplitude for the \( \psi' \to J/\psi \eta \)

\[
\mathcal{M}(\psi' \to J/\psi \eta) \propto e^{ijk} q_{\eta} q_{\psi'} q_{J/\psi} \frac{1}{\sqrt{3}} (I_c - I_n - 2I_s).
\]

Before performing the explicit calculation of the loops it is important to first understand the power counting of the system. As was just derived, each vertex in the triangle diagrams is of \( p \)-wave character and is thus linear in the momentum. Due to parity conservation, one momentum has to appear as external parameter (c.f. Eq. \((8)\)). Thus the loops themselves scale as \( v^3/(v^2)^2 = v \), where \( v \) replaced momentum factors by the dimensionless velocities — the proper expansion parameter of HQEFT — and the factors denote the non-relativistic integral measure and propagators as well as the vertex factors just described, in order. The typical heavy meson velocity in the loops may be estimated via

\[
v \approx \sqrt{2(M_D - M_\psi)/M_D} \simeq 0.53,
\]

where \( M_D \) is the averaged charmed-meson mass, and \( M_\psi = (M_{J/\psi} + M_{\psi'})/2 \). The quantities of interest here are differences of loops with the remaining terms proportional to \( m_q = \) this is an energy scale of \( O(v^2) \). We therefore expect the heavy meson loops to scale as \( m_q/v |q| \) which gives some enhancement compared to Eq. \((4)\).

To confirm this power counting estimate and allow for a more quantitative statement, we now evaluate the diagrams of Fig. \(\text{(a)}\) explicitly using the non-relativistic technique. Let us consider diagram (b) in Fig. \(\text{(a)}\) as an example of these calculations. The decay amplitude in \( d \) dimensions is given by

\[
\mathcal{M}_{(b)} = 2 \frac{g}{F} g_{\psi DD} g_{\psi DD} \cdot e^{ijk} q_{\psi'} q_{J/\psi} \sqrt{M_D M_D} \frac{l^k (2l^k - q^k)}{8 M_D^2 M_D} \int_0 d^d q^i \int_0 (2\pi)^d \left( t^0 - \frac{\vec{q}^2}{2 M_D} - i\epsilon \right) \left( t^0 + b_{DD}^2 + \frac{\vec{q}^2}{2 M_D} - i\epsilon \right) \left( t^0 - q^0 + \Delta_D - \frac{(\vec{q}^2)}{2 M_D} + i\epsilon \right) \frac{l^k (2l^k - q^k)}{8 M_D^2 M_D} \int_0 d^d-1 l^k \int_0 (2\pi)^d-1 \frac{l^k (2l^k - q^k)}{8 M_D^2 M_D} \frac{1}{\sqrt{\Delta(b)}},
\]

\[
= -\frac{g}{2 F} g_{\psi DD} g_{\psi DD} \cdot e^{ijk} q_{\psi'} q_{J/\psi} \sqrt{M_D M_D} \frac{M_D + M_D}{M_D + M_D} \int_0 d^d q^i \int_0 (2\pi)^d \frac{l^k (2l^k - q^k)}{8 M_D^2 M_D} \frac{1}{\sqrt{\Delta(b)}},
\]

\[
= \frac{g}{8 F} g_{\psi DD} g_{\psi DD} \cdot e^{ijk} q_{\psi'} q_{J/\psi} \sqrt{M_D M_D} \frac{M_D + M_D}{M_D + M_D} \int_0 d^d q^i \int_0 (2\pi)^d \frac{l^k (2l^k - q^k)}{8 M_D^2 M_D} \frac{1}{\sqrt{\Delta(b)}},
\]

\[
(12)\]
where \( q \) is the \( \pi^0(\eta) \) momentum, \( \Delta_D = M_{D^*} - M_D \) is the mass difference between the vector and the pseudoscalar charmed mesons, \( \Delta_D = -a x^2 + (c - c') x + c' \), and 
\[ a = \frac{q^2}{4}, \quad c = b_{DD} M_D + \frac{q^2}{2}, \quad \text{and} \quad c' = 2 \mu_{D^*}, b_{DD}. \]
Further, \( \mu_{D^*} = M_D M_{D^*}/(M_D + M_{D^*}) \) is the reduced mass of the \( D \) and \( D^* \). \( b_{DD} = M_D + M_{D^*} - M_{\psi} \) and 
\[ b_{DD} = 2 M_D - E_{J/\psi} \] with \( E_{J/\psi} \) being the energy of the \( J/\psi \) in the \( \psi' \) rest-frame. The amplitude in Eq. (12) has been multiplied by a factor of \( \sqrt{M_{\psi'} M_{J/\psi} M_D^2 M_{D^*}^2} \) to account for the non-relativistic normalization of the heavy meson fields in the Lagrangians given in Eqs. (8) and (9). The dimensionless coupling constants \( g_{\psi DD} \) and \( g_{\psi' D^* D} \) are related to the dimensional ones \( g_2 \) and \( g_2' \) via \( g_{\psi DD} = g_2 \sqrt{M_{J/\psi} M_D^2} \) and 
\[ g_{\psi' D^* D} = g_2' \sqrt{M_{\psi'} M_D M_{D^*}}. \]
Note in the last step, we have taken \( d = 4 \). The integral is finite when evaluated with dimensional regularization for only a power divergence appears. The integral appearing in Eq. (12) for \( c > a > 0, c' > 0 \), which is satisfied here, is given by

\[
\int_0^1 dx \sqrt{\Delta_D(x)} = \frac{1}{4a} \left\{ 2a \sqrt{c-a} + (c-c') \left( \sqrt{c'} - \sqrt{c-a} \right) + \frac{(c-c')^2 + 4ac'}{2\sqrt{a}} \right\} \times \arctan \left\{ \frac{2a \sqrt{c-a} + (c-c') \left( \sqrt{c-a} - \sqrt{c'} \right)}{(c-c')^2 + 2a(c-c') + 4a \sqrt{c(c-c')}} \right\} = \frac{2c + c' + \sqrt{c'} c}{3\sqrt{c + c'}} \left[ 1 + O \left( \frac{a}{c} \right) \right]. \quad (13)
\]

We checked that neglecting the \( a \) term, which is proportional to \( q^2 \), only makes a difference of several percent. Thus, neglecting the \( O(q^2) \) terms, the decay amplitude of any loop shown in Fig. 1 scales as

\[
I \equiv \frac{2 \mu_{D} + 2 \mu' b' + \sqrt{2} \mu_{D} b' \mu'}{\sqrt{2} \mu_{D} + \sqrt{2} \mu' b'}, \quad (14)
\]

where \( \mu(\mu') \) is the reduced mass of the charmed mesons connected to the \( J/\psi(\psi') \) in the loop, and \( b(b') \) is the difference between the charmed meson threshold and \( E_{J/\psi}(M_{\psi'}) \).

We may now compare the explicit expressions to the power counting argument presented above. Since \( \sqrt{2 \mu_{D}} \) and \( \sqrt{2 \mu_{D} b'} \) are approximately the momenta of the charmed mesons in the loops, we count them as \( M_{D} \) with \( M_{D} \) and \( v \) being the mass and velocity of the charmed meson. For the purpose of the power counting analysis, one can neglect the difference between \( \sqrt{2 \mu_{D}} \) and \( \sqrt{2 \mu_{D} b'} \). Then one has \( I \sim \sqrt{2 \mu_{D}} \). Denoting the charged and neutral charmed meson mass difference by \( \delta \), we have \( \mu_{\psi'} = \mu_{\psi} + \delta/2 \) and \( b_{\psi(\psi')} = b_{\psi(\psi')} + 2\delta \) where the lower index \( c \) or \( n \) means charged or neutral. Thus,

\[
M(\psi' \to J/\psi \pi^0) \sim |q_{\psi'}|^2 \left( \frac{\sqrt{2} \mu_{\psi} b_{\psi}}{\sqrt{2} \mu_{\psi} b_{\psi}} - \frac{\sqrt{2} \mu_{\psi} b_{\psi}}{\sqrt{2} \mu_{\psi} b_{\psi}} \right)
\]

\[
= \frac{1}{2} \delta^2 \frac{b_{\psi} + b_{\psi}/2}{\sqrt{2} \mu_{\psi} b_{\psi}} + O(\delta^2)
\]

\[
\sim |q_{\psi'}| \delta \frac{\mu_{\psi}}{v}. \quad (15)
\]

The mass difference \( \delta \) may be divided into the strong (quark-mass difference) and the em contributions as \( \lambda(m_d - m_u) + \beta e^2 \), see Ref. [31], therefore \( M(\psi' \to J/\psi \pi^0) \) scales as \( (m_d - m_u)|q_{\psi'}|/v \) in line with the estimate given above.

The validity of Eq. (4) is based on the assumption that the light mesons are produced through soft gluons, and hence at a distance much larger than the size of the charmonium, which is the basic assumption of the QCD multipole expansion [32–34]. Then the matrix element of the soft gluon operator between the vacuum and a light meson can be worked out using the axial anomaly and chiral symmetry. In the mechanism considered in this Letter, the light mesons are produced through their coupling to the virtual intermediate charmed mesons. This kind of mechanism was not included in the QCD multipole expansion. These contributions are genuine, i.e. there is no underlying double counting. This can be seen from the fact that the corresponding integrals are finite in dimensional regularization and that the leading terms are non-analytic in the quark masses. Comparing Eq. (15) with Eq. (4), one sees that the charmed-meson loop effects in the amplitude are enhanced by a factor of \( 1/v \sim 2 \). Therefore, they are more important.

Assuming the intermediate charmed-meson loop mechanism saturates the decay widths of the \( \psi' \to J/\psi \pi^0(\eta) \), we get

\[
R_{\pi^0/\eta} = 0.14 \pm 0.09, \quad (16)
\]

where the central value is what we get from a direct calculation, and the uncertainty is from neglecting the contribution of Eq. (5) using \( v \sim \sqrt{(2 M_D - M_{\psi})/M_D} \sim 0.53 \) and contains the one that originates from either using physical masses or averaged masses for the field normalizations. This value is within \( 2 \sigma \) of the experimental ratio. Note that in the ratio all the coupling constants \( g \), \( g_2 \) and \( g_2' \) disappear.

We cannot give a prediction for the corresponding decays \( Y' \to Y \pi^0(\eta) \) by naively extending the formalism to the bottom sector. This is because the strong and the em contributions to \( M_{B^0} - M_{B^+} \) interfere destructively [35], and make \( M_{B^0} - M_{B^+} \) as small as \( 0.33 \pm 0.06 \text{ MeV} \) [16]. Accord-
ingly, although the $B$-meson loop contribution to the decay $\Upsilon' \to \Upsilon \eta$ is more important than in the charm sector—$\psi$ is smaller—it’s contribution to the $\Upsilon' \to \Upsilon \pi^0$ is highly suppressed. On the contrary, in the charmed sector of relevance here, the strong and EM contributions to $M_{D^+} - M_{\psi^0}$ interfere constructively [31], and hence enhance the meson loop effects.

Further support of the proposed scheme is provided by analyzing the resulting absolute values of the decay widths. Using $\psi = 0.6$, which is extracted from a tree level calculation of the $D^+$ width, $F = 92.4$ MeV, and $g_{\psi D^i} = g_{\psi D^i} = G$, we obtain the absolute values of the decay widths as

$$
\Gamma(\psi' \to J/\psi \pi^0) = (3.6 \pm 1.3 \ [5.5 \pm 2.9]) \times 10^{-4} G^4 \text{ keV},
$$

$$
\Gamma(\psi' \to J/\psi \eta) = (2.5 \pm 1.3 \ [4.9 \pm 2.6]) \times 10^{-3} G^4 \text{ keV},
$$

(17)

where the numbers outside and inside the square brackets are obtained using the physical and averaged masses for the field normalizations, respectively. In order to reproduce the experimental values $\Gamma(\psi' \to J/\psi \pi^0) = 0.40 \pm 0.03 \text{ keV}$ and $\Gamma(\psi' \to J/\psi \eta) = 10.0 \pm 0.4 \text{ keV}$ [16], we need $G = 6.2 \pm 1.0 \ [5.5 \pm 0.9]$ and $G = 8.4 \pm 2.1 \ [7.1 \pm 1.1]$, respectively. These numbers are close to independent model estimates for $g_{\psi DD}$ existing in the literature, see e.g. Refs. [25, 36, 37].

In summary, in this Letter, utilizing the technique of non-relativistic effective field theory, we show that intermediate charmed mesons play an important role in the $\psi'$ decays into $J/\psi \pi^0$ and $J/\psi \eta$. They are enhanced by a factor of $1/\psi$ compared with the contribution directly from the quark mass differences. The light quark mass ratio can only be extracted from these decays after establishing a complete effective field theory up to next-to-leading order with the Goldstone bosons, charmonia and charmed mesons as the degrees of freedom in the future. What was done in this Letter can be regarded as the first step towards that goal.

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[30] There is a discrepancy between the CLEO result [14] given in Eq. 5 and the BES result [16], $R_{\psi^0/\eta} = (4.8 \pm 0.5)\%$. If we use the branching fractions of the $\psi^0 \to J/\psi \pi^0$ and $\psi^0 \to J/\psi \eta$ given by the Particle Data Group Ref. [16], which result in $R_{\psi^0/\eta} = (4.0 \pm 0.3)\%$, the result $m_u/m_d = 0.39 \pm 0.02$ is slightly smaller.
[31] Here we assume the $\eta$ is in the SU(3) octet, and the effect of the $\eta - \eta'$ mixing is assumed to be small [11].