Transition strengths from $^{10}$B($e, e'$)$^{10}$B

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Abstract

Inelastic electron scattering form factors are fitted with polynomial times Gaussian expressions in the variable $y = (bq/2)^2$ to extract electromagnetic transition strengths at the photon point.

1 Introduction

The table of radiative widths from the 1979 Ajzenberg-Selove tabulation was based on the low-$q$ results of Spamer [1] (Darmstadt) for the 6.03-MeV $4^+$; 0 level and the 7.48-MeV $2^+$; 1 level, together with the $180^\circ$ results of Fagg et al. [2] (NRL) for a number of levels. The 1984 and 1988 tabulations added results based on the work of Ansaldo et al. [3] (Saskatoon) for $0.61 < q < 1.81$ fm$^{-1}$ but did not take into account the erratum to that work [3].

The more recent work of Cichocki et al. [4] (NIKHEF) gives longitudinal and transverse form factors in the range $0.48 < q < 2.58$ fm$^{-1}$ for most levels up to the 6.56-MeV $4^-; 0$ level. The analysis in this work includes extensive shell-model calculations and the extraction of B(C2) values for five levels of $^{10}$B. The analysis also includes data up to $q \sim 4$ fm$^{-1}$ taken at $180^\circ$ for the ground-state, the 1.74-MeV level, and the 5.17-MeV level [5] (Bates). For most transitions, the form factors are plotted as a function of the effective momentum transfer $q_{\text{eff}} = q(1 + 2.75/E_0)$, where the beam energy $E_0$ is in MeV. This way of relating form factors in the plane-wave and distorted-wave Born approximations must also be applied to the data from the earlier works.

Cichocki et al. used a polynomial times Gaussian ($e^{-y}$) in the variable $y = (bq/2)^2$, where $b$ is the harmonic oscillator parameter, to represent the form factors and extract B(C2) values. The procedure is spelled out by Millener et al. [6] who defined

$$B(C\lambda, q) = f^{-2} \frac{Z^2}{4\pi} \left[ \frac{(2\lambda + 1)!!}{q^{2\lambda}} \right] F_L^2,$$

$$B(M\lambda, q) = f^{-2} \frac{Z^2}{4\pi} \frac{\lambda}{\lambda + 1} \left[ \frac{(2\lambda + 1)!!}{q^{\lambda}} \right] F_T^2,$$

where $f = f_{\text{SN}} f_{\text{c.m.}} e^{-y}$ takes out the exponential dependences in the (theoretical) form factor. In the conventional definition of $B(C\lambda, q)$ and $B(M\lambda, q)$, we should set $f = 1$. Because nature does not know about $f_{\text{c.m.}}$ (and even in theory we don’t need it if we use an appropriate system of relative coordinates), we perform the fit, with $f = f_{\text{SN}} e^{-y}$, to

$$B(\lambda, q)^{1/2} = f (A + By + Cy^2 + ...)$$

$$= \left[ b^{\lambda} \right] (A' + B'y + C'y^2 + ...) .$$

1
2 Corrections for electron distortion

We use the effective momentum transfer \( q_{\text{eff}} = q(1 + 2.75/E_0) \) prescription [4] to approximately correct the original data for electron distortion so that we can use form factors calculated in the plane-wave Born approximation. This is especially important at low incident energies \( E_0 \) and needs to be performed for the Darmstadt [1], NRL [2], and Saskatoon [3] data as sketched in the next subsections.

2.1 Darmstadt data

The measured quantity is the ratio of inelastic to elastic cross section and we could use this data together with a modern parametrization of the elastic cross section. The derived quantity \( B(\lambda, q) \) is tabulated as a function of \( q^2 \) where

\[
q^2 = 2k_0^2 (1 - \cos \theta) (1 - k/k_0) + k^2
\]  

(5)

and \( k = E_x/\hbar c \) and \( k_0 = E_0/\hbar c \) (\( \hbar c = 197.32696 \text{ MeV.fm} \)). We recalculate \( q \) and calculate \( q_{\text{eff}} \) using the tabulated values of \( E_0 \) and \( \theta \). The units for \( B(\lambda, q) \) are given as \( 10^{-51} \text{ cm}^4 = 10^{-4} \text{ fm}^4 \) for \( C2 \) and \( 10^{-28} \text{ cm}^2 = 10^{-2} \text{ fm}^2 \) for \( M1 \). We absorb the factor of \( \alpha \hbar c = e^2 \) from the expressions of Spamer [1] so that \( B(\lambda, q) \) is expressed in the conventional units of \( e^2 \text{ fm}^{2\lambda} \). Then, from Eqs. (1) and (2) (with \( f = 1 \)) in terms of the \( B(\lambda, q) \) tabulated in Ref. [1]

\[
F_L^2 = 10 B(C2) \times 2.234 \times 10^{-3} \times q^4
\]  

(6)

for the longitudinal form factor of the 6.025-MeV \( 4^+; 0 \) level, and

\[
F_T^2 = 10^{-2} B(M1) \times 1.117 \times 10^{-1} \times q^2
\]  

(7)

for the transverse form factor of the 7.477-MeV \( 2^+; 1 \) level.

2.2 NRL data

Fagg et al. [2] give the 180° cross sections in \( \text{nb/sr} (= 10^{-7} \text{ fm}^2/\text{sr}) \) for three incident energies (40.5, 50.6, and 60.6 MeV and we have \( (e^2 = 1.44 \text{ MeV.fm}) \)

\[
\frac{d\sigma}{d\Omega} = \frac{Z^2 e^4}{4 E_0^2 R} \cdot F_T^2
\]  

(8)

with the recoil factor \( R = (1+2E_0/M) \) (\( M \) = nuclear mass, and e.g., \( R = 1.013 \) for \( E_0 = 60.6 \text{ MeV} \)) and \( q \) from Eq. (5).

2.3 Saskatoon data

The data are already tabulated as form factors and we simply change \( q \) to \( q_{\text{eff}} \).
3 C2 transitions

In their appendix, Cichocki et al. [4] extract B(C2) values for five states using Eq. (3), perhaps without the inclusion of the single-nucleon form factor $f_{SN}$ but this is essentially irrelevant at the photon point. For the 6.025-MeV level, the low-$q$ data from Darmstadt and the Saskatoon data were also included. In the following subsections, we discuss fits to each level starting with the 6.025-MeV $4^+; 0$ level. As in Ref. [4], the oscillator parameter is fixed at 1.60 fm. In principle, we could include $b$ in the fit but it turns out that $b = 1.60$ fm is close to the optimum value. Besides a change of $b$ in Eq. (4) is compensated for by a change in $A'$ when fitting to data. Of course, theoretical B(C2) values calculated with harmonic oscillator wave functions scale as $b^4$.

3.1 The 6.025-MeV $4^+; 0$ level

The original Darmstadt value for the B(C2) is $24.4 \pm 2.5$ e$^2$fm$^4$ (the inclusion of 5 data points from Orsay lead to a slightly smaller value of $23.4 \pm 2.5$ e$^2$fm$^4$) - this value is quite well reproduced in the first line of Table 1. Taking the effect of distortion into account via the $q_{eff}$ prescription results in a considerably lower value of $17.34 \pm 1.97$ e$^2$fm$^4$ (note that $B(C2) = (b/2)^4 A^2$) as pointed out by Cichocki et al. [4] - taking the $q$ values from Table 1 of Spamer [1] instead of recomputing them gives $B(C2) = 17.66 \pm 1.98$ e$^2$fm$^4$. Note that the parameter $B$ is not well determined and that the $e^{-y}$ term in the oscillator form factor pretty much takes into account the terms involving the transition radius in the original Darmstadt paper. As the third line of Table 1 shows one can obtain a one-parameter fit of similar quality but with a smaller error because of the restrictive nature of the fitting function. The fact that essentially a p-shell form factor fits so well is surprising because the transition is very strong and the higher-order terms responsible for this should lead to a $B$ coefficient which is

| Data | N  | $\chi^2$/DF | A    | B     | C     | D     | B(C2) |
|------|----|-------------|------|-------|-------|-------|-------|
| D    | 9  | 0.392       | 7.707(410) | -2.18(231) |       |       | 24.33(259) |
| D b  | 9  | 0.346       | 6.507(369) | 2.14(178)  |       |       | 17.34(197) |
| D    | 9  | 0.365       | 6.746(139) |       |       |       | 18.64(77)   |
| N+S  | 28 | 1.59        | 6.707(131) | -0.945(290) | 0.435(173) | -0.010(30) | 18.42(91)   |
| N+S c| 25 | 1.69        | 6.600(194) | -0.545(556) | 0.032(452) | 0.013(109) | 17.84(137)  |
| N+S c| 25 | 1.62        | 6.619(108) | -0.607(194) | 0.085(71)  |       | 17.95(74)   |
| N+S+D| 37 | 1.28        | 6.761(106) | -1.052(244) | 0.491(71)  | -0.108(27) | 18.72(66)   |
| N+S+D c| 34 | 1.33        | 6.727(137) | -0.875(417) | 0.275(360) | -0.040(90) | 18.53(87)   |
| N+S+D c| 34 | 1.30        | 6.682(91)  | -0.707(169) | 0.118(64)  |       | 18.29(57)   |

$^a$ Uncorrected Darmstadt data.
$^b$ Darmstadt data vs. $q_{eff}$ with $q$ from $E_0$, $\theta$.
$^c$ $q_{eff} < 2$ fm$^{-1}$. 

Table 1: Fits to C2 form factors for the 6.025-MeV $4^+; 0$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations. When $\chi^2$/DF is greater than one, the error on B(C2) is inflated by the square root of this quantity.
negative (e.g., the harmonic oscillator form factor for the $2\hbar \omega$ giant quadrupole resonance is of the form $y(1 - 1/3y)e^{-y}$ and coherence at low $q$ means a negative coefficient for the next term).

If we fit the NIKHEF data using a 3-parameter polynomial, the $\chi^2$/DF is 1.77; for the NIKHEF + Saskatoon data, it is 2.02. Adding an extra term to take care of the high-$q$ behavior leads to some improvement (line 4 of Table 1). Removing the three data points with $q_{\text{eff}} > 2$ fm$^{-1}$ doesn’t lead to much change, although a three-parameter fit is now possible, as the next two lines of Table 1 show. The final three lines of Table 1 show fits to the complete data set. The four-parameter fit gives $B(C2) = 18.7 \pm 0.7$ e$^2$fm$^4$. To compare with the electromagnetic value for the $4^+ \to gs$ transition, we multiply by $7/9$ and convert to Weisskopf units (1 W.u. = 1.2797 e$^2$fm$^4$) getting $11.4 \pm 0.4$ W.u. This agrees with the electromagnetic value of $12.4 \pm 1.8$ W.u., which is derived from the $\omega \gamma$ value from the $^6\text{Li}(\alpha, \gamma)$ reaction and an E2/M1 mixing ratio.

3.2 The 0.718-MeV $1^+; 0$ level

The lifetime for this long-lived level is precisely known, $\tau = 1.020 \pm 0.005$ nsec. This corresponds to a $B(C2)$ for electron scattering of $1.796(9)$ e$^2$fm$^4$. The value of $1.71(14)$ e$^2$fm$^4$ in the first line of Table 2 derived from the NIKHEF data is in good agreement. Therefore including the electromagnetic value as a data point changes the $\chi^2$ only slightly. A three-parameter fit gives a significant increase in $\chi^2$.

Table 2: Fits to C2 form factors for the 0.718-MeV $1^+; 0$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations. The error on $B(C2)$ is inflated by $\sqrt{\chi^2/DF}$.

| Data   | N  | $\chi^2$/DF | A      | B                | C         | D         | B(C2) |
|--------|----|-------------|--------|------------------|-----------|-----------|-------|
| N      | 14 | 1.29        | 2.045(73) | -1.042(155) | 0.390(94) | -0.058(17) | 1.71(14) |
| N+EM   | 15 | 1.21        | 2.093(5)  | -1.144(51)  | 0.450(47) | -0.068(11) | 1.795(9) |

3.3 The 2.154-MeV $1^+; 0$ level

The first line of Table 3 shows a 3-parameter fit to all the NIKHEF data points while the next line shows the same fit with the highest $q$ data point removed. The 2-parameter fit in the third line shows very little deterioration in $\chi^2$. The last line shows a 1-parameter fit which is still acceptable in terms of $\chi^2$ but is certainly not as good as the other fits. The $\chi^2$ doesn’t change for $b = 1.56$ fm or $b = 1.66$ fm and neither does $B(C2)$ to any significant extent.

The electromagnetic data in the current tabulation gives $0.75(9)$ e$^2$fm$^4$ for the $B(C2)$ up. This depends on a number of values for lifetime ($2.13 \pm 0.20$ ps) and the ground-state branch ($21.1 \pm 1.6 \%$). Probably, the previous lifetime average of $2.30 \pm 0.02$ ps should be used but this only gets the the $B(C2)$ down to $0.69$ e$^2$fm$^4$ (the lowest $\gamma$-ray branch of $17.5\%$ would give $0.57$ e$^2$fm$^4$).
Table 3: Fits to C2 form factors for the 2.154-MeV $1^+; 0$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations.

| Data | N  | $\chi^2$/DF | A       | B       | C       | D       | B(C2)   |
|------|----|-------------|---------|---------|---------|---------|---------|
| N    | 13 | 0.84        | 0.963(50)| 0.091(74)| -0.015(22)| 0.380(36)|         |
| N a  | 12 | 0.40        | 1.005(52)| 0.010(81)| 0.013(25)| 0.413(43)|         |
| N a  | 12 | 0.39        | 0.981(27)| 0.052(17)|         | 0.394(22)|         |
| N a  | 12 | 1.15        | 1.047(14)|         |         | 0.449(14)|         |

*a Highest q data point removed.

3.4 The 3.587-MeV $2^+; 0$ level

The first line of Table 4 shows a 3-parameter fit to all the NIKHEF data points which yields $B(C2) = 0.616 \pm 0.044 \text{ e}^2\text{fm}^4$ which is in reasonable agreement with the electromagnetic value of $0.85 \pm 0.25 \text{ e}^2\text{fm}^4$. The latter depends on lifetime, branch, and mixing ratio.

Table 4: Fits to C2 form factors for the 3.587-MeV $2^+; 0$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations.

| Data | N  | $\chi^2$/DF | A       | B       | C       | D       | B(C2)   |
|------|----|-------------|---------|---------|---------|---------|---------|
| N 16 | 1.16| 1.226(42)   | -0.130(64)| 0.061(21)|         | 0.616(44)|         |
| N 16 | 1.21| 1.261(61)   | -0.226(139)| 0.129(91)| -0.014(18)| 0.652(63)|         |

3.5 The 5.920-MeV $2^+; 0$ level

The first line of Table 5 shows a 3-parameter fit to all the NIKHEF data points while the next line shows the same fit with the highest q data point removed. The 2-parameter fit in the third line shows very little deterioration in $\chi^2$. The same can be said of the 1-parameter fit in the last line but the $B(C2)$ changes from 0.164 to 0.202 as $b$ changes from 1.55 fm to 1.65 fm. In 2-parameter or 3-parameter fits the $\chi^2$ and $B(C2)$ vary little with modest changes in $b$.

Table 5: Fits to C2 form factors for the 5.920-MeV $1^+; 0$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations.

| Data | N  | $\chi^2$/DF | A       | B       | C       | D       | B(C2)   |
|------|----|-------------|---------|---------|---------|---------|---------|
| N    | 12 | 0.89        | 0.555(91)| 0.279(182)| -0.157(83)| 0.126(35)|         |
| N a  | 11 | 0.87        | 0.602(100)| 0.161(213)| -0.089(105)| 0.148(49)|         |
| N a  | 11 | 0.85        | 0.679(38)| -0.015(44)|         | 0.189(21)|         |
| N a  | 11 | 0.78        | 0.667(13)|         |         | 0.182(7) |         |

*a Highest q data point removed.
4 M3 transitions

In addition to the transition to the 1.74-MeV 0\(^+\) 1 level, the transverse form factor to the 5.164-MeV 2\(^+\) 1 level is dominantly M3 with a small correction for M1 at low \(q\). The \(q_{\text{eff}}\) prescription can be used on the Saskatoon data but the NIKHEF data for the 0\(^+\) 1 level is given as a function of \(q\) and can’t be corrected without a knowledge of \(E_0\) for each point. However, a B(M3) is available from a DWBA analysis of the data. Note that for \(A = 10\), 1 W.u. = 35.548 \(\mu^2\text{fm}^4 = 0.3932 e^2\text{fm}^6\).

4.1 The 1.740-MeV 0\(^+\) 1 level

We first note that \(\Gamma_\gamma = (1.05 \pm 0.25) \times 10^{-9}\) from the original analysis of the Saskatoon data (see erratum of Ref. [3]) corresponds to \(B(\text{M3} \uparrow) = (8.27 \pm 1.97) e^2\text{fm}^6 = (748 \pm 178) \mu^2\text{fm}^4\).

The first line of Table 6 gives \(B(\text{M3} \uparrow) = (804 \pm 110) \mu^2\text{fm}^4\) for a fit to the data as a function of \(q\). This is reduced to \((688 \pm 101) \mu^2\text{fm}^4\) for a fit to the data as a function of \(q_{\text{eff}}\). The value from a DWBA fit to the complete data set shown in Ref. [4] is 633 \(\mu^2\text{fm}^4\) (R. Hicks, private communication).

| Data | N | \(\chi^2/\text{DF}\) | \(A\) | \(B\) | \(B(\text{M3} \uparrow)\) |
|------|---|-----------------|------|------|------------------|
| S    | 8 | 0.37            | 5.823(400) | 0.213(400) | 8.89(122) |
| S    | 8 | 0.36            | 5.386(396) | 0.522(388) | 7.61(112) |

4.2 The 5.164-MeV 2\(^+\) 1 level

The original analysis of the Saskatoon data [3] gave \(B(\text{M3} \uparrow) = (21.6 \pm 2.2) e^2\text{fm}^6 = (1953 \pm 19) \mu^2\text{fm}^4\). This fit included an M1 contribution.

The first two lines of Table 7 contain no correction for the M1 contribution at low \(q\). The first line fits the Saskatoon and NIKHEF data as a function of \(q_{\text{eff}}\) while the second line also contains the Catholic University of America low \(q\) data. A significant difference in the extracted \(B(\text{M3})\) can be seen when the two points with \(q_{\text{eff}} < 0.8 \text{ fm}^{-1}\) are removed from the S+N data set (third line).

Finally, we subtract an M1 contribution calculated by normalizing the computed M1 shell-model form factor to the B(M1) obtained from electromagnetic data. Because there is such a large cancellation for the lowest \(q\) data point of the CUA data set, we omit this point entirely. This results in \(B(\text{M3} \uparrow) = 19.4 \pm 2.0 e^2\text{fm}^6\) or \((1756 \pm 181) \mu^2\text{fm}^4\); \(B(\text{M3} \downarrow) = 27.2 \pm 2.8 e^2\text{fm}^6\) or \((69.1 \pm 7.1)\) W.u. This corresponds to \(\Gamma_\gamma = (1.00 \pm 0.10) \times 10^{-6}\) eV.

5 M1 transition for the 7.48-MeV level

Ansaldo et al. [3] give \(\Gamma_\gamma = 11.75 \pm 0.75\) eV for this strong M1 transition while Spamer [1] gives \(\Gamma_\gamma = 12.0 \pm 2.2\) eV. However, Chertok [7] corrected the later value to 11.0 \(\pm 2.2\) eV.
Table 7: Fits to M3 form factor for the 5.164-MeV $2^+; 1$ level using Eq. (4). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations. The unit for B(M3) is $e^2fm^6$.

| Data     | N  | $\chi^2$/DF | A      | B       | C       | B(M3↑)  |
|----------|----|-------------|--------|---------|---------|---------|
| S+N      | 17 | 0.51        | 9.611(543) | -3.059(945) | 1.608(364) | 24.2(27) |
| S+N+CUA  | 20 | 0.68        | 9.913(397) | -3.571(726) | 1.793(294) | 25.8(21) |
| S+N $^a$ | 15 | 0.37        | 8.672(838) | -1.569(1380) | 1.090(502) | 19.7(38) |
| S+N+CUA $^b$ | 19 | 0.39 | 8.612(452) | -1.466(805) | 1.052(319) | 19.4(20) |

$^a q_{\text{eff}} > 0.81$ fm$^{-1}$.

$^b$ Theoretical $F_T^2(M1)$ normalized to $B(M1) = 0.023 \pm 0.006$ W.u. subtracted and CUA $q = 0.41$ fm$^{-1}$ point omitted because of a large cancellation.

The fit as a function of $q_{\text{eff}}$ in the first line of Table 8 yields $\Gamma_0^\gamma = 10.84 \pm 1.58$ eV. Adding the CUA data points gives $\Gamma_0^\gamma = 11.00 \pm 1.14$ eV. The Saskatoon contains three points around the second maximum of the M1 form factor. Adding these data points gives a worse fit and $\Gamma_0^\gamma = 11.35 \pm 0.37$ eV. Increasing the number of parameters to three improves the fit but gives a substantially larger $B(M1)$ value corresponding to $\Gamma_0^\gamma = 12.55 \pm 0.58$ eV.

As far as the $B(M1)$ is concerned, it is preferable to stick with the value derived from the low-$q$ data.

Table 8: Fits to M1 form factor for the 7.48-MeV $2^+; 1$ level using Eq. (3). The harmonic oscillator parameter is fixed at $b = 1.60$ fm. The quantities in parentheses are standard deviations. The unit for B(M1) is $e^2fm^2$.

| Data     | N  | $\chi^2$/DF | A      | $B$     | B(M1↑)  |
|----------|----|-------------|--------|---------|---------|
| D        | 12 | 0.56        | 0.133(10) | -0.0126(945) | 0.0177(26) |
| D+CUA    | 15 | 0.54        | 0.134(7) | -0.0123(45) | 0.0180(19) |
| D+CUA+S  | 23 | 0.95        | 0.136(2) | -0.162(5) | 0.0185(6) |
| D+CUA+S  | 23 | 0.58        | 0.1432(33) | -0.196(13) | 0.023(8) | 0.0205(10) |

6 C3 transitions

Cichocki et al. [4] present data on the form factors for the $2^−$, $3^−$, and $4^−$ levels at 5.110 MeV, 6.127 MeV, and 6.561 MeV. The longitudinal form factor for the isolated 6.56-MeV level is best defined. The C3 Weisskopf unit is $5.94 e^2fm^6$.

6.1 The 6.56-MeV $4^−; 0$ level

The C1 and C3 harmonic oscillator form factors for $1/\hbar \omega$ transitions cannot be distinguished. However, the shell-model calculations in Ref. [4] indicate the the C3 transition is dominant for the $4^−$ level.
The first line of Table 9 shows a 4-parameter fit to the full NIKHEF data set which shows that B, C, and D are not determined and that there is a large error on B(C3). The second line shows a 3-parameter fit and a significant change in B(C3) (but within errors). Because we are interested in pinning down a low-\(q\) parameter, the third line shows the effect of omitting the two highest \(q\) data points. Now C is undetermined and the final line shows a 2-parameter fit to the reduced data set (there is no change in the overall \(\chi^2\)). Then B(C3↑) = 21.8 ± 1.1 e²fm⁶ and B(C3↓) = 17.0 ± 0.9 e²fm⁶ = 2.9 ± 0.2 W.u.

Table 9: Fits to C3 form factor for the 6.560-MeV 4⁻; 0 level using Eq. (4). The harmonic oscillator parameter is fixed at \(b = 1.60\) fm. The quantities in parentheses are standard deviations. The unit for B(C3) is e²fm⁶.

| Data | N | \(\chi^2/DF\) | A       | B         | C         | D         | B(C3↑)     |
|------|---|--------------|---------|-----------|-----------|-----------|------------|
| N    | 14| 0.87         | 8.642(1180) | -0.115(1880) | -0.597(871) | 0.112(122) | 19.6(54)   |
| N    | 14| 0.86         | 9.621(479)  | -1.764(522)  | 0.189(126)  |           | 24.3(24)   |
| N     | 12| 0.89         | 9.115(651)  | -1.077(793)  | -0.004(210) |           | 21.7(31)   |
| N     | 12| 0.80         | 9.127(233)  | -1.093(121)  |           |           | 21.8(11)   |

\(a\) \(q_{\text{eff}} < 2.2\) fm⁻¹.

6.2 The 6.13-MeV 3⁻; 0 level

This form factor is not so well defined because of the difficulty of separating the cross section from the strong 4⁺ level at 6.025 MeV. Again, the shell-model calculations of Ref. [4] indicate the the C3 transition is dominant but in this case a significant C1 contribution is also predicted.

The first line of Table 10 shows a 3-parameter fit to the full NIKHEF data set which is poor and gives a large error on B(C3). The second line shows the effect of omitting the two highest \(q\) data points and reducing the number of parameters by one. A better \(\chi^2\) is obtained in the last line by omitting two high data points. Then B(C3↑) = B(C3↓) = 33.1 ± 2.7 e²fm⁶ = 5.6 ± 0.5 W.u.

Table 10: Fits to C3 form factor for the 6.130-MeV 3⁻; 0 level using Eq. (4). The harmonic oscillator parameter is fixed at \(b = 1.60\) fm. The quantities in parentheses are standard deviations. The unit for B(C3) is e²fm⁶.

| Data | N  | \(\chi^2/DF\) | A       | B         | C         | B(C3↑)     |
|------|----|--------------|---------|-----------|-----------|------------|
| N    | 13| 3.55         | 10.27(81) | -1.30(134) | -1.20(50)  | 27.6(82)   |
| N     | 11| 2.85         | 11.21(38) | -0.96(33)  |           | 33.0(38)   |
| N     | 9  | 1.54         | 11.25(39) | -1.13(34)  |           | 33.1(27)   |

\(a\) \(q_{\text{eff}} < 1.7\) fm⁻¹.

\(b\) Points at \(q_{\text{eff}} = 1.08\) and 1.46 fm⁻¹ omitted.
6.3 The 5.11-MeV 2−; 0 level

Here, the shell-model calculations of Ref. [4] indicate that the C1 transition is dominant over C3. In addition, there exists a non-zero B(E1↓) of \( (5.0 \pm 1.0) \times 10^{-4} \) W.u. that arises from isospin mixing. This corresponds to \( B(E1↑) = (1.07 \pm 0.21) \times 10^{-4} \ e^2\text{fm}^2 \).

The first line of Table 11 assumes good isospin and therefore no A coefficient. Allowing A to be non-zero improves the fit (second line). Including the photon point in the fit worsens the \( \chi^2 \) somewhat but still gives a reasonable fit.

Table 11: Fits to the longitudinal form factor for the 5.110-MeV 2−; 0 level using Eq. (3). The harmonic oscillator parameter is fixed at \( b = 1.60 \) fm. The quantities in parentheses are standard deviations.

| Data | N | \( \chi^2/\text{DF} \) | A       | B         | C         | B(C1↑)                  |
|------|---|-----------------|---------|-----------|-----------|-------------------------|
| N    | 8 | 1.62            | 0.259(11) | -0.100(13) | 0.0       |                         |
| N    | 8 | 0.83            | -0.036(16) | 0.366(47) | -0.171(33) | \( (1.3 \pm 1.1) \times 10^{-3} \) |
| N \( ^a \) | 9 | 1.16            | -0.0106(11) | 0.290(12) | -0.121(13) | \( (1.09 \pm 0.23) \times 10^{-4} \) |

\( ^a \) Including electromagnetic data for the photon point.

7 Higher levels

Ansaldo et al. [3] give a longitudinal form factor for a level at 8.07 MeV with a width of 760 keV, which they assign as 2+; 0, and a transverse form factor for the 2+; 1/3−; 1 doublet at 8.9 MeV. Fitting the data for the 8.07-MeV level yields a \( B(C2↑) = 5.1 \pm 0.7 \ e^2\text{fm}^4 \). This is about a quarter of the strength of the very strong transition to the 6.025-MeV 4+ level. As Zeidman et al. [8] note, this should lead to a very strong excitation in inelastic pion scattering which is not seen. In fact, even adding the cross sections for states at 7.8 and 8.07 MeV still gives less than 25% of the expected cross section for an isoscalar C2 excitation.

For the 8.9-MeV doublet, it is difficult to say anything much without guidance from the shell-model as to the dominant multipoles expected. The state is stronger than expected for an isovector excitation in inelastic pion scattering [8].

This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

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