Scaling of magnetic monopoles in the pure compact QED

J. Jersák, T. Neuhaus, and H. Pfeiffer

aInstitut für Theoretische Physik E, RWTH Aachen, Germany

In the pure U(1) lattice gauge theory with the Villain action we find that the monopole mass in the Coulomb phase and the monopole condensate in the confinement phase scale according to simple power laws. This holds outside the coupling region in which on finite toroidal lattices the metastability phenomena occur. A natural explanation of the observed accuracy of the scaling behaviour would be the second order of the phase transition between both phases in the general space of couplings not far away from the Villain action.

1. MOTIVATION

The phase transition between the confinement and Coulomb phases of the strongly coupled pure U(1) lattice gauge theory (pure compact QED) remains to be puzzling. For the extended Wilson and Villain action, the presence of the two-state signal on finite lattices has been recently confirmed. On the other hand, a scaling behaviour of various bulk quantities and of the gauge-ball spectrum consistent with a second order phase transition and universality has been observed outside the narrow region in which the two-state signal occurs. (See ref. [1] for references.)

Because the order of the phase transition for these actions is unknown, the extrapolation of these phenomena to the thermodynamic limit is uncertain. However, even if the scaling behaviour is only a transient phenomenon, it indicates that there is a region of the phase diagram described by an interacting effective field theory. It is of interest to investigate the properties of such a theory even if it is “only” effective, as effective theories are useful in physics. Here we address the question whether such a theory includes monopole degrees of freedom.

2. MAIN RESULTS

We have observed scaling of some observables related to the magnetic monopoles in the pure compact QED with Villain action.

In the Coulomb phase we find at various values of the coupling $\beta$ a very clean exponential decay of the monopole correlation function in a large range of distances. This demonstrates the dominance of a single particle state in this correlation function, the monopole, whose mass we determine. Due to its Coulomb magnetic field, the monopole mass strongly depends on the finite lattice size. However, we find [1] that it can be reliably extrapolated to the infinite volume.

The scaling behaviour of the extrapolated monopole mass $m_\infty$ at the phase transition follows a simple power law (fig. [4])

$$m_\infty(\beta) = a_m(\beta - \beta_{Coul})^{\nu_m},$$

with the critical exponent

$$\nu_m = 0.49(4).$$

The inverse mass achieves at least the magnitude of three lattice spacings.

The monopole condensate in the confinement phase shows a much weaker $L$ dependence. Its value extrapolated to the infinite volume, $\rho_\infty$, scales with the power law

$$\rho_\infty = a_\rho(\beta_{conf} - \beta)^{\beta_{exp}},$$

with the magnetic exponent

$$\beta_{exp} = 0.197(3).$$

As shown in fig. [4] the function (3) describes extremely well the data in a broad interval and the scaling behaviour of the condensate is thus well established.
The superscripts “Coul” and “conf” indicate that the corresponding values of $\beta_c$ have been determined by the power law fits using data only from one phase. Their values are

$$\beta_c^{\text{Coul}} = 0.6424(9)$$  \hfill (5)

and

$$\beta_c^{\text{conf}} = 0.6438(1).$$  \hfill (6)

Both values are consistent within two error bars.

Further results and technical details of our calculations are published in [1]. We have adopted the methods of ref. [2]. A quantity related to $\rho_\infty$ has been studied also in refs. [3].

3. INTERPRETATION OF RESULTS

The monopole mass in the Coulomb phase scales with the same Gaussian exponent $\nu_m$ which is also observed for the scalar gauge ball. This holds at least until the inverse mass of the latter achieves five lattice spacings. This implies that if one chooses the scalar gauge ball to become massless while the other gauge balls, whose $\nu$ is about 1/3, have finite non-vanishing masses, the monopoles will be massless and therefore important. Even if the scalar mass is chosen finite nonzero, and other gauge balls thus decouple, the monopoles stay present. Therefore the effective field theory would include monopole degrees of freedom, being thus a very interesting abelian gauge theory. This may be a sufficient motivation for further investigation of compact QED by the lattice community.

Now let us try to interpret our results from the point of view of Statistical Mechanics. The coexistence of first and second order phenomena is a typical property of tricritical points (TCP) [4].

As indicated schematically in fig. [5] in their vicinity crossover regions (shaded) separate regions of different behaviour even in the thermodynamic limit.

Approaching the phase transition along the path $\mathbf{a}$ may reveal first a second-order-like behaviour determined by the tricritical point, and only very close to the phase transition the presence of the two-state signal shows up. In finite systems, a two-state signal can appear even at the end of the path $\mathbf{b}$.

The observed properties of the compact QED with various actions can be explained by assum-
ing the existence of a tricritical part of the manifold separating the confinement and Coulomb phases in the multidimensional space of possible couplings. Thus under this hypothesis a genuine continuum limit of the compact QED would exist.

Such a manifold may, but does not need to include the couplings which have been already used for the investigation of compact QED. Therefore, the search for this manifold may require an introduction and investigation of new types of coupling terms. As the monopoles are relevant, the space of generalized couplings in which the TCP is to be located, is likely to include the monopole degrees of freedom. Their influence on the transition has been studied in refs. [6]. A possible TCP in this context has been discussed by Kleinert [6].

However, even if the finding of such a manifold may be challenging, the indications for its existence are remarkable: (i) the clean scaling behaviour like that of the monopole observables, and (ii) several universal phenomena in some intervals of couplings close to the phase transition points. In fact, these properties allow to investigate the corresponding continuum limit without an actual localization of the tricritical points.

Another scenario is that the coexistence of first and second order phenomena is due to a rare, but not impossible hybrid situation depicted in fig. [4]. A continuum limit would then exist in spite of the latent heat present in the thermodynamic limit.