Unruh effect and macroscopic quantum interference

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We investigate the influence of Unruh radiation on matter-wave interferometry experiments using neutral objects modeled as dielectric spheres. The Unruh effect leads to a loss of coherence through momentum diffusion. This is a fundamental source of decoherence that affects all objects having electromagnetic interactions. However, the effect is not large enough to prevent the observation of interference for objects of any size, even when the path separation is larger than the size of the object. When the acceleration in the interferometer arms is large, inertial tidal forces will disrupt the material integrity of the interfering objects before the Unruh decoherence of the centre of mass motion is sufficient to prevent observable interference.

There is wide interest in discovering whether or not quantum interference can be observed for large objects. There are many ways in which this issue can be stated precisely. One of the natural ways is simply to take a lump of ordinary matter of larger and larger size, and discover whether or not interference fringes can be observed when the lump is made to pass through two slits, or, more generally, pass through an interferometer, the arms of which are separated by a distance larger than the diameter of the lump. As the size of the lump grows, so does its mass, and therefore its de Broglie wavelength at any given velocity falls. Consequently the interferometer gets more and more sensitive to small effects which may randomize the interference phase—the problem of decoherence. Therefore one expects to encounter limits which make it not feasible to observe such interference above some size of the lump. The question then arises, whether these limits are purely technological and could in principle be overcome, or whether the natural world itself poses intrinsic limits to quantum coherence. One may, for example, conjecture that the natural free motion of all physical entities has a stochastic component, or some other non-linear decohering process, not described by the existing formulation of quantum mechanics (quantum field theory). Or, one may argue that the existing standard model of physics already places fundamental limits to quantum coherence.

In this paper we address a mechanism which restricts the visibility of interference in matter-wave interferometers, within the existing standard formulation of physics. That ‘standard formulation’ we take to be quantum field theory on a background of classical spacetime described by general relativity. The mechanism we shall consider is Unruh radiation [1,4]. This may be said to be ‘fundamental’, i.e. implicit in the laws of motion, and unavoidable. A related phenomenon is the dynamical Casimir effect, in which the vacuum radiation pressure dissipates the kinetic energy of a moving mirror [5]; we comment on that connection at the end of the paper.

We consider a generic interferometer with two interfering ‘arms’. We study interference of the de Broglie waves associated with a sphere of proper radius \( R \), and consider the case where the separation of the arms is \( 2R \) (see figure 1). This captures the idea of a macroscopic separation of paths, such that the interfering object might be said to have been ‘separated from itself’ in the middle of the interferometer.

The argument of the present paper may be summarized as follows. Any matter-wave interferometer in which the arms enclose a non-zero area of spacetime involves acceleration. Owing to the Unruh effect, the accelerating object experiences a fluctuating force which leads to momentum diffusion. This in turn leads to fluctuation of the interference phase of the interferometer. We estimate the size of this effect for a wide class of physical objects. We find that the blurring effect is there, but it does not does not limit the size \( R \) of object for which interference can be observed. Also, we find the observation of interference does not place any new constraint on the acceleration of the object, over and above the one already imposed by the requirement that it is not torn apart by tidal inertial forces.

We now turn to this argument.

The Fulling-Davies-Unruh effect can be presented in more than one way [6, 7, 8, 9]. At the heart of it is Unruh’s observation that, in the coordinate system of the Rindler frame (constantly accelerating frame in flat
spacetime), the vacuum state of quantum field theory takes the form of a thermal state with temperature

$$k_B T = \frac{\hbar a}{2\pi c},$$  \hspace{1cm} (1)$$

where \(a\) is the proper acceleration. This statement is made in a more thoroughly well-defined way in the literature, and we shall elaborate on it shortly. There is not yet a complete consensus on the precise meaning and the physical implications\textsuperscript{[1,8]}, but it will be sufficient to our purpose to take the following broadly standard point of view. We consider an inertial reference frame, with the electromagnetic field initially in its vacuum state. In this frame we suppose there exists a system A which provides a force on system B, causing B to accelerate. B may be any system having electromagnetic interactions, such as a charged body, a detector, a dipole, a lump of fused silica. In this situation, we claim, the net force on B will fluctuate. If B has internal structure, then it will undergo internal excitations, and subsequently emit photons, in a stochastic way. The energy is provided by system A; the stochastic nature of the process is owing to the fluctuating vacuum and therefore is unavoidable. If the proper acceleration is constant on average and goes on for long enough, then the fluctuation is the same as if B were bathed in thermal\textsuperscript{[20]} radiation at the Unruh temperature in its instantaneous rest frame. If B has no internal structure but is an accelerating charged particle, the QED treatment of radiation reaction equally leads to a fluctuating force\textsuperscript{[7,9]}.

The importance of Unruh’s calculation is that it suggests the effect under consideration is owing to basic kinematics of the electromagnetic field, and therefore is universal. The response of B is very like the one it would have if it were at rest and bathed in thermal radiation at the Unruh temperature, and that temperature depends only on the acceleration, not on other details of either A or B. We say ‘very like’ rather than ‘identical to’ because it is not necessary to our argument that the two cases be identical, only that they be like. Following Boyer\textsuperscript{[10]}, we make the following claim. A physical entity (often called ‘observer’) accelerating through the vacuum will undergo internal excitations similar to those it would undergo if it were moving inertially and subject to radiation such that the density matrix of the electromagnetic field is diagonal in the Fock basis, with a mean excitation per mode

$$\bar{n}(\omega) = \frac{1}{e^{2\pi \omega/a} - 1} \left( 1 + 2 \left( \frac{a}{\omega c} \right)^2 \right).$$  \hspace{1cm} (2)$$

This spectrum is not quite the ordinary thermal (Planck) form but is closely related to it, and we shall refer to this radiation as ‘thermal’ in the following. We say the excitations of the accelerating entity are similar, not identical, to those of the corresponding inertial entity because it is not possible to make a more precise statement unless one investigates how the internal dynamics of the entity are affected by acceleration. By arranging the forces so as to accelerate B while keeping its internal stress to a minimum, one may arrange that no large discrepancy will arise by this route, as long as the B is small enough for tidal effects to be negligible.

A further source of imprecision is the fact that the motion under consideration will only involve acceleration for finite periods of time, so the Unruh result does not apply exactly. Acceleration for finite periods is discussed in\textsuperscript{[3,11,12]}. The approximation that the Unruh temperature applies to the majority of the elapsed proper time is good when the product of proper acceleration and proper time is of order \(c\).

Now consider a generic matter-wave interferometer whose arms enclose a non-zero area in spacetime. As we already commented, such an interferometer involves acceleration and therefore the Unruh effect will come into play. The only way to avoid this is if both arms of the interferometer are geodesic (that is, they both represent free fall motion). This is in principle possible for a large part of the motion, for example if the arms pass either side of a massive gravitating object, but it is not clear whether or not the action of the beam splitters must involve non-gravitational forces. In any case, we will restrict the treatment to the case of flat spacetime in the following.

It will emerge that the decoherence scales as a high power of \(a\), and therefore to minimize the decoherence, a long period of low \(a\) is better than inertial motion combined with a short period of large \(a\). Therefore we will study a trajectory made of three periods of constant proper acceleration, as shown in figure\textsuperscript{[1,21]}. We model the interfering object as a dielectric sphere of relative permittivity \(\epsilon\) at low frequencies. Such a model applies to a wide class of objects as long as the dominant wavevectors of the electromagnetic radiation under consideration satisfy \(kR \ll 1\). For the motion under consideration, the spectrum of the radiation is thermal with typical wavevector \(k \approx k_B T / \hbar c = a/2\pi c^2\) so the model is valid when \(aR/c^2 \lesssim 2\pi\); we will show at the end that this condition holds.

According to our interpretation of the Unruh effect, the excitation of the sphere causes it to behave, in the instantaneous rest frame, as if it were scattering thermal radiation. When an incident photon of wavevector \(k\) is scattered, the momentum of the sphere changes by \(\delta p = h(k - k')\). For a heavy sphere \((mc \gg \hbar k)\), \(k' = k\) and so \(\delta p^2 = \hbar^2 k^2 (1 - \cos \theta)\). The photons arrive from random directions\textsuperscript{[22]} and therefore the momentum undergoes a random walk, such that after proper time \(\tau\) the momentum variance is \(\Delta p_\theta = \hbar^2 k^2 (1 - \cos \theta)\tau\) where \(\theta\) is the scattering angle. The momentum density is

$$\Gamma_k = \int \frac{d\Omega}{4\pi} \frac{\bar{n} c}{V} 2(1 - \cos \theta) = \frac{16\pi \bar{n} c}{3 V} k^4 R^6 \left( \frac{\epsilon - 1}{\epsilon + 2} \right)^2$$  \hspace{1cm} (3)$$

Here we have used the classical cross-section for scatter-
ing by a dielectric sphere in the limit \( kR \ll 1 \). \( V \) is the volume of space containing the electromagnetic field; it will go to infinity at the end of the calculation when we integrate over \( k \). \( \bar{n} \) is given by Eqn (2) with \( \omega = ck \).

The phase of the de Broglie waves is Lorentz-invariant and is most conveniently calculated in an inertial frame \( F \) that moves in the \( z \) direction relative to the beam-splitters shown in figure 1 such that the sphere moves along the \( x \) axis of \( F \). In \( F \), the momentum of the sphere is related to that in the instantaneous rest frame by a Lorentz transformation. Consequently it is distributed with a standard deviation given by \( \Delta p = \gamma \Delta p_0 = \gamma \hbar k(\Gamma_k \tau)^{1/2} \) where \( \gamma \) is the Lorentz factor.

In the absence of momentum diffusion, the phase accumulated along one interferometer arm is given by a path integral along the classical trajectory \( x(\tau) \). In the presence of momentum diffusion, the phase gradually acquires a spread given by \( 15, 16 \).

\[
\Delta \phi_k = \int \frac{\Delta p \, dx}{\hbar} = \int \frac{\gamma \Delta p_0}{\hbar} \left| \frac{dx}{d\tau} \right| d\tau
= k \int \gamma^2 |v| \sqrt{\Gamma_k \tau} \, d\tau
(4)
\]

Let \( \tau_1 \) be the duration of the first period of constant proper acceleration. The worldline is given by \( x = x_0 + (c^2/a) \cosh(a\tau/c) \) with \( x(\tau_1) - x(0) = R/2 \) hence

\[
\tau_1 = \frac{c}{a} \cosh^{-1} \left( 1 + \frac{aR}{2c^2} \right) \approx \sqrt{R/a}
(5)
\]

and \( \gamma = \cosh(a(\tau - 2j\tau_1)/c) \), \( v = c \tanh(a(\tau - 2j\tau_1)/c) \) where \( j = \{0, 1, 2\} \) for the three parts of the worldline between the beam splitters, of proper duration \( \tau_1, 2\tau_1, \tau_1 \) respectively. The integrand in Eqn (4) is shown in figure 2. The integral is \( k c \sqrt{\Gamma_k} c^3/a^3 \) multiplied by a function of \( (a\tau_1/c) \) which we obtained by numerical integration. Since the Unruh effect is only expected when the acceleration goes on for long enough, such that \( a\tau_1 \) is of order \( c \), we are only interested in studying the integral for values of \( a\tau_1/c \) in the range \( 0.5 < a\tau_1/c < \cosh^{-1}(3/2) \). In this range the result can be approximated, to 0.15% accuracy, by

\[
\Delta \phi_k^2 \approx 7.325 \Gamma_k \tau_1 \left( \frac{k c^2}{a} \right)^2 \sinh^4 \left( \frac{a\tau_1}{c} \right).
(6)
\]

Here we have also introduced the approximation that \( \Gamma_k \) is independent of \( \tau \), which means we ignore the complications in the Unruh effect associated with a finite period of acceleration. The formula for the cross-section in Eqn (6) is also approximate since the sphere is accelerating. The assumption of rigid motion (constant proper dimensions) makes this a reasonable first approximation, but a more thorough analysis would be needed to check the degree of approximation involved. Using (5), we have

\[
\Delta \phi_k^2 \approx 7.3 \Gamma_k \tau_1 (kR)^2 \left( 1 + \frac{aR}{4c^2} \right)^2.
(7)
\]

So far we have obtained the variance of the phase owing to fluctuations at one frequency. Using (2), (3) and (7), the variance owing to fluctuations at all frequencies is

\[
\Delta \phi^2 = \int \Delta \phi_k^2 V^3 k (2\pi)^3
\approx 0.04 \left( \frac{\epsilon - 1}{\epsilon + 2} \right)^2 \left( 1 + \frac{aR}{4c^2} \right)^2 \frac{\tau_1 c}{R} \left( \frac{aR}{c^2} \right)^9.
(8)
\]

Here, we took \( \epsilon \) as independent of frequency, which is valid for an ordinary material since we assumed \( kR \ll 1 \) and we are interested in values of \( R \) of the order microns or larger. Therefore \( \epsilon \) is the low-frequency (d.c.) relative permittivity.

Finally, we need to consider the combination of both arms of the interferometer. If the interferometer were really bathed in thermal radiation, then the low-frequency contribution to the fluctuations in the two arms would be correlated. However, here there is no incoming radiation (in the Minkowski frame). Rather, the scattering calculation is a mathematical method that is being used to estimate fluctuations of the forces that are causing acceleration in the two arms. Since these forces are acting in spacelike separated regions it is not self-evident that their fluctuations would be correlated, but in view of the fact that the effect involves the quantum vacuum one cannot rule out correlation. This is an open question. Here we shall treat the case of no correlation; in this case when both arms of the interferometer are included, the interference phase uncertainty is \( \Delta \phi_{\text{tot}} = \sqrt{2} \Delta \phi \). Correlations would be expected to result in less decoherence, so we thus obtain an upper bound on the decoherence.

We note that our final result has some similarity with a prediction for the dynamical Casimir effect (DCE) considered below, which suggests that the Unruh effect does lead to decoherence.

Now, we have assumed all along that the sphere is substantially unaffected by its acceleration. For example, one may imagine that the forces on it are arranged such
that it retains fixed proper dimensions. However, if $a$ is the proper acceleration of the center of the sphere, then when $R = c^2/a$ the surface of the sphere extends to the horizon in Rindler space. In other words, this is the condition where the sphere can no longer move rigidly and tidal forces are large; not merely non-negligible but insurmountable. At this limiting value of $R$ we find

$$\Delta \phi_{\text{tot}} \simeq 0.35 \left( \frac{\epsilon - 1}{\epsilon + 2} \right).$$

(9)

Hence we find that $|\Delta \phi_{\text{tot}}| \ll \pi$ for $\epsilon > 0$. This means the interference fringes will be easily visible for any positive $\epsilon$. We therefore conclude that the decoherence owing to the Unruh effect is small: it is not sufficient to prevent interference even when the acceleration is so extreme that the sphere begins to break up. Schrödinger’s cat would be killed by the inertial forces before its acceleration was sufficient for this form of decoherence to be substantial.

Also, owing to the high power of $\alpha R/c^2$ in Eqn. (8), the decoherence is very small for any value of $R$ less than $c^2/2a$. But this condition is, within a numerical factor of order 1, the same as the one required by purely classical relativistic considerations if the interfering entity is not to experience extreme tidal forces.

We now compare the above with the conclusions for DCE as described in [3]. In the case of an oscillating mirror, Dalvit and Maia Neto find that the effect of DCE is to single out a pointer basis consisting of coherent states of mirror motion (c.f. [14]), and the off-diagonal elements of the density matrix for a superposition of coherent states decay at a rate $\Gamma$. In the perfectly reflecting limit, this decoherence rate is given by

$$\Gamma = |\alpha|^2 \omega_0^2 / 3\pi M c^2$$

where $\alpha$ is a coherent state parameter, $M$ is the mass of the mirror and $\omega_0$ is the oscillation frequency. The calculation is carried out in the low velocity limit, $v \ll c$. To compare this with our results, consider the case where the amplitude of the oscillation associated with the coherent state is $R$. Then $|\alpha|^2 = R^2 M \omega_0^2 / 2\hbar$ so $\Gamma = R^2 \omega_0^2 / 3\pi$. The situation comparable to our treatment above is when the mirror can oscillate for a half-period without losing coherence. Suppose the mirror can oscillate for $N$ half-periods before substantial decoherence occurs. The condition for this is $\Gamma \lesssim \omega_0^2 N$, which gives $R^2 \omega_0^2 \lesssim 6c^2/N$. Using that the acceleration is of order $a \sim \omega_0^2 R$, this can be expressed $Ra/c^2 \lesssim 1/N$. For $N = 1$ this agrees with our conclusions above, although the two calculations are very different, one involving a general estimate of the fluctuations and treating $v \sim c$, the other employing field theory for a more specific system in the limit $v \ll c$.

With the benefit of hindsight, one might claim that the limiting condition on $(aR)$ could be obtained by dimensional analysis, but it was not self-evident at the outset that Planck’s constant would not appear in the result, and indeed this simple condition was not remarked in [3]. Also, it emerged in the latter case in a calculation in the limit $v \ll c$ with no role for Special Relativity as such. Of course for $N = 1$ the DCE calculation is not valid near the upper bound, since the condition then gives $v \approx c$, and our Unruh effect calculation is not valid for $a \tau_1 \ll c$. The DCE calculation does not exhibit the strong scaling with $(aR/c^2)$ that we observe in eqn [3]. This implies that the DCE is only loosely related to the Unruh effect, or else that a qualitative change in the latter occurs when one passes from $a \tau_1 \sim c$ to $a \tau_1 \ll c$.

If one interest is in observing the influence of the Unruh effect, one could use particles with a large charge to mass ratio, such as single electrons, and then a much larger effect will be found [13] [14]. An enhancement might also be available by the use of plasmonic material or an exotic material with a relative permittivity approaching $-2$ over a significant frequency range. In this case one must allow for the frequency-dependence of the Unruh effect which might in principle be made to exhibit interference after passing either size of an obstacle.

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[21] Different parts of the sphere here experience different amounts of proper time, and this will lead to further decoherence unless the sphere is at zero temperature. To avoid this the sphere can be cooled, or the interferometer can be extended to a more symmetric configuration involving two areas traversed in opposite directions (sometimes called ’zero area’).

[22] We do not need to assume isotropic radiation here, only that the random component in the directions is well modelled by a random walk with the assumed step size.