Description of vibrational oscillations in the viscous concrete medium by cylindrical punches

I V Kirilov, S I Khanin and V P Voronov
Belgorod State Technological University named after V G Shukhov, 46, Kostyukova street, Belgorod, 308012, Russia
E-mail: sergiykhanin@gmail.com, hanin.si@bstu.ru

Abstract. Analytical expressions for describing changes in the amplitude and phase shifts of the velocity of propagation of oscillations in a viscous concrete medium as applied to vibroform with cylindrical punches were obtained. Using graphical dependences, the influence on the parameters of the distance from the surface of the punch to the oscillating particle, the frequency and amplitude of its oscillations were established.

1. Introduction
Today, production of concrete and reinforced concrete structures and products is in demand in the construction industry [1, 2]. In their production, various technological systems (aggregate-flow, conveyor, and bench) are widely used. They determine the range of products, its cost, performance [3]. In recent years, progressive technologies for the production of concrete products of high finishing readiness [4] are becoming increasingly common. Production of pre-stressed reinforced concrete produced using a continuous form-free method is particularly popular. Production is carried out by several units moving relative to a stationary bench and performing various functions. Molding machines for volumetric vibration (their share is 72%) are applied in factories of the Russian Federation [5]. One of the working bodies of these machines is a platform which oscillates a viscous concrete medium. Its improvement is studied by authors considering both structural and technological changes [6, 7]. Some of the studies relate to various modes of operation, including vibration exposure parameters and vibration duration. Conditions that ensure the process of transmitting vibra oscillations directly to the concrete medium by punches attached to the platform are analyzed. However, the influence of the design of the working parts of the forming machine on vibration is understudied which complicates the rational selection of parameters to obtain a dense structure of concrete.

The effectiveness of vibration exposure to the concrete environment is usually characterized by the following parameters: the amplitude of oscillations, their frequency and duration of exposure. However, effective compaction can be achieved by changing vibration velocity [8]. Values of these parameters should be determined depending on rheological properties of the concrete mixture.

2. General provisions
Propagation of oscillations transmitted by a vibrating platform (vibroform) with a given amplitude and frequency to a concrete viscous medium can be described on the basis of the Navier – Stokes equation. Let us consider the propagation of vibration oscillations in a concrete environment when producing empty products in the form of floor slabs. Let us describe the propagation of vibrations in a viscous
concrete medium emitted by the surface of a cylindrical puncher in concrete. Since the cylindrical surface vibrating in the concrete medium has an axial symmetry, the mathematical description of propagation of vibrations must be carried out in the cylindrical coordinate system (r, φ, z) centered on the axis of symmetry (Figure 1).

Due to the axial symmetry of an oscillating cylinder, the velocity vector of concrete particles in a viscous medium will have a radial component of velocity vector \( v_r \) whose change will depend on time “t” and radial size “r” – the distance of the particle from the center of the cylinder. Therefore, the change in the velocity of displacement of particles of a concrete mixture will be described by the following equation

\[
\frac{\partial v_r}{\partial t} = \nu A_r v_r - \nu \frac{v_r}{r^2},
\]

where \( \nu \) is the coefficient of kinematic viscosity; \( A_r \) is the radial part of the Laplace operator.

The speed of oscillation \( v_0 \) of the surface of the cylinder sets the initial value of velocity of propagation of oscillations of the viscous concrete medium. It is determined as

\[
v_0 = A_0 \cdot \omega_0,
\]

where \( A_0 \) and \( \omega_0 \) are the amplitude and frequency of oscillation of the cylinder surface, respectively.

The equation describing the propagation of oscillations is solved within the linear combination of trigonometric functions sine and cosine:

\[
v_r = A_1(r) \sin \omega_0 t + A_2(r) \cos \omega_0 t,
\]

where \( A_1(r) \) and \( A_2(r) \) – amplitudes of harmonic oscillations whose value changes only in the radial direction.

Using (3), we have

\[
\omega_0 A_1(r) \cos \omega_0 t - \omega_0 A_2(r) \sin \omega_0 t = v \left[ \frac{d^2 A_1(r)}{dr^2} + \frac{1}{r} \frac{dA_1(r)}{dr} \frac{A_1(r)}{r^2} \right] \cdot \sin \omega_0 t + v \left[ \frac{d^2 A_2(r)}{dr^2} + \frac{1}{r} \frac{dA_2(r)}{dr} \frac{A_2(r)}{r^2} \right] \cdot \cos \omega_0 t.
\]

Based on equation (4), the following equations can be obtained:

\[
\omega_0 A_1(r) = v \left[ \frac{d^2 A_1(r)}{dr^2} + \frac{1}{r} \frac{dA_1(r)}{dr} \frac{A_1(r)}{r^2} \right],
\]

\[
\omega_0 A_2(r) = v \left[ \frac{d^2 A_2(r)}{dr^2} + \frac{1}{r} \frac{dA_2(r)}{dr} \frac{A_2(r)}{r^2} \right].
\]
\[- \omega_0 A_2(r) = \nu \left[ \frac{d^2 A_1(r)}{dr^2} + \frac{1}{r} \cdot \frac{dA_1(r)}{dr} - \frac{A_1(r)}{r^2} \right]. \quad (6)\]

Equations (5), (6) are a system of two ordinary second-order differential equations for finding unknown functions \( A_1(r) \) and \( A_2(r) \). Let us find solutions in the field of complex numbers.

\[ \alpha^2 \cdot \frac{d^2 z}{d\eta^2} + \frac{\alpha^2}{\eta_1} \cdot \frac{dz}{d\eta} - \alpha^2 \cdot \frac{z}{\eta_1^2} + i \cdot \frac{\omega_0}{\nu} \cdot z = 0, \quad (7) \]

where

\[ z = A_2(r) + iA_1(r); \quad (8) \]

Let us insert the parameter:

\[ \alpha^2 = -\frac{i\omega_0}{\nu}. \quad (9) \]

Equation (7) can be written as

\[ \eta_1^2 \cdot \frac{d^2 z}{d\eta_1^2} + \eta_1 \cdot \frac{dz}{d\eta_1} - \left( \eta_1^2 + 1 \right) \cdot z = 0, \quad (10) \]

where \( \eta_1 \) – dimensionless argument related to the radial size ratio:

\[ \eta_1 = \sqrt{-\frac{i\omega_0}{\nu}} \cdot r = \sqrt{\frac{2\omega_0}{\nu}} \cdot (1 - i) \cdot \frac{r}{2}. \quad (11) \]

Equation (10) is the Bessel equation whose solution is a linear combination of modified Bessel functions \( I_1(\eta_1) \) and \( K_1(\eta_1) \):

\[ z(\eta_1) = c_1 \cdot I_1(\eta_1) + c_2 \cdot K_1(\eta_1), \quad (12) \]

where \( c_1, c_2 \) – arbitrary constants.

According to [9], the modified Bessel function \( I_1(\eta_1) \) is an infinitely increasing function with an increasing argument, and the Bessel function \( K_1(\eta_1) \) is a monotonically decreasing function with an increasing argument.

As far as function \( z(\eta_1) \) has to meet the relation \( \eta_1 \to \infty < \infty \), where the inequality means the boundedness of function (12), and its application to expression (12) makes it necessary to equate the first term in (12) to zero, the equality should be zero for arbitrary constant \( c_1 \).

The solution of (10) is

\[ z(r) = c_2 K_1 \left( \sqrt{\frac{2\omega_0}{\nu}}(1 - i)\cdot \frac{r}{2} \right). \quad (13) \]

The constant \( c_2 \) can be found from the initial condition:

\[ z(r = R_0) = u_0 = A_0\omega_0, \quad (14) \]

where \( R_0 \) – the radius of the base of the cylindrical surface of the punch.

Expression (3) can be reduced to the following form: \( \nu_r = A_p(r)\sin(\omega_0t + \varphi_0) \),

\[ A_p(r) = \sqrt{A_1^2(r) + A_2^2(r)}, \quad (15) \]

where \( \varphi_0 = \arctg \frac{A_2(r)}{A_1(r)}. \quad (16) \]

Applying (14) to solution (13) we find:

\[ c_2 = \frac{A_0\omega_0}{K_1 \left( \sqrt{\frac{2\omega_0}{\nu}}(1 - i)\cdot \frac{R_0}{2} \right)}. \quad (17) \]

On the basis of solution (13) in view of (18) we find:
where \( l \) is the shortest distance from the cylindrical surface of the punch with a radius \( R_0 \) to the particle under consideration oscillating in the concrete medium (depth of oscillation penetration).

Substituting (19) and (20) into (15) and (14) we have:

\[
A_p(r) = A_0 \omega_0 \sqrt{\frac{R_0}{r}} \cdot \exp\left(-\frac{\omega_0}{2\nu} \cdot (r - R_0)\right),
\]

(21)

\[
\varphi_0 = \arctg\left(\text{ctg}\left(\frac{\omega_0}{2\nu} (r - R_0)\right)\right).
\]

(22)

The relations (21) and (22) determine the change in the amplitude and phase shift of the velocity of propagation of oscillations in the viscous concrete medium.

3. Results

Let us apply the obtained expressions to the description of the phased vibration compaction of a concrete mixture by a vibroform with cylindrical punches, consisting of two parts, oscillating with different frequencies and amplitudes [10]. Let us construct graphical dependences of the change in the amplitude and phase shifts of the velocity of propagation of oscillations on the frequency of oscillations of the surface of the punches \( \omega_0 \) and the radial size \( r \). Let the cylindrical surface of the punch with a radius of \( R_0 = 0.078 \) m, and density of the concrete mix be equal to 2400 kg/m\(^3\).

**Figure 2.** Dependences of the change in the amplitude of velocity of propagation of oscillations \( A_p(r) \) on the frequency of oscillations of the surface of the cylindrical punch \( \omega_0 \) and the radial size \( r \): (a) for the first part of the vibromould at \( A_0 = 0.0007 \) m; (b) for the second part of the vibromould at \( A_0 = 0.0004 \) m.

The graphs of functions \( A_p(r) \) from \( \omega_0 \) at \( r = 0.078-0.100 \) m (Figure 2) have a monotonously increasing character up to some values \( \omega_0 \) which are different for each part of the vibroform. It follows from the dependencies that with an increase in the oscillation frequency \( \omega_0 \) of the first part of the vibroform from 35 Hz to 55 Hz, with amplitude \( A_0 = 0.0007 \) m (Figure 2a), the amplitude of the velocity of oscillation at the base of the punch surface increases linearly from 0.0255 to 0.0385 m/s i.e. by 50.9%. A smaller increase in the amplitude of the velocity of propagation of oscillations is observed for the second part of the vibroform. At \( A_0 = 0.0004 \) m, changes in the oscillation frequency \( \omega_0 \) from 55 Hz to
75 Hz and an increase in r from 0.022 to 0.030 m increase Ap(r) by 36.4%. The values of the amplitude of the velocity of propagation of oscillations Ap(r) depend on the value of the amplitude of oscillations A0.

Let us consider the change in the value Ap(r) in the range of values r. The change in the amplitude of the velocity of propagation of oscillations Ap(r) depending on the remoteness of the material particle from the punch surface is non-linear, sharply decreasing. This characterizes the damping of oscillations at a relatively small depth of their penetration. As the oscillations decay, the nature of the dependence Ap(r) on ω0 changes. For the first part of the vibroform with a radial size r = 0.085 m (oscillation penetration depth l = 0.007 m), there is no influence of the oscillation frequency of the surface of the cylindrical punch ω0 on the change in the amplitude value of the vibration propagation velocity which is Ap(r) = 0.0043 m/s (Figure 2a). For the second part of the vibroform, a similar phenomenon is observed at r = 0.084 m (l = 0.006 m) and the amplitude value of the velocity of propagation of oscillations is Ap(r) = 0.0034 m/s (Figure 2b). Moving away from the concrete surface, the amplitude of the velocity of propagation of oscillations decrease 8.2-fold and 8.8-fold for the first and second parts.

![Graph](image)

Figure 3. Dependences of the change in the phase shift of velocity of propagation of oscillations φ0 on the radial size r: (a) for the first part of the platform at A0 = 0.0007 m; (b) for the second part of the platform at A0 = 0.0004 m.

Graphic dependences of the change in the phase of the velocity of propagation of oscillations φ0 from r presented in Figure 3 are linear decreasing. Their analysis suggests that as particles move away from the punch surface, the phase of the oscillation velocity changes. The value of the gap with a positive phase of the velocity of oscillation depends on the frequency of oscillation of the punch surface ω0. In the gap bounded by the punch surface and r = 0.0832 m with ω0 = 55 Hz, the velocity of propagation of oscillations has a positive phase; outside it, it is negative. For the second part of the vibroform with ω0 = 75 Hz, the intervals are separated by r = 0.0824 m.

4. Conclusion

Analytical expressions for describing changes in the amplitude and phase shifts of velocity of propagation of oscillations in the viscous concrete medium as applied to vibrating mold with cylindrical punches were obtained. Using the graphical dependencies for the vibroform with punches of R0 = 0.078 m in radius, the parameters were analyzed depending on the oscillation frequency of their surfaces ω0 (for the first part of the vibroform ω0 = 35.55 Hz, for the second one - 55.75 Hz), radial size r (r = 0.078..0.100 m) and amplitudes of oscillations A0 = 0.0004 m and A0 = 0.0007 m for the first and second
parts of the vibroform. The influence of the velocity of propagation of oscillations in the viscous concrete medium on the frequency and the amplitude of oscillations of punch surfaces, radial size \( r \) was established.

5. Acknowledgments
The work is realized in the framework of the Program of flagship university development on the base of the Belgorod State Technological University named after V.G. Shukhov, using equipment of High Technology Center at BSTU named after V G Shukhov.

References
[1] Suleymanov K A, Pogorelov I A, Suleymanova L A, Bazhenova O O 2016 The Impact of Coarse Aggregate on Concrete Creep Bulletin of BSTU named after V G Shukhov. 11 53–57
[2] Botsman L N, Ageeva M S, Botsman A N, Shapovalov S M 2018 Modified pavement cement concrete IOP Conf. Ser.: Mater. Sci. and Eng. 327 032011
[3] Huijben F, Van Herwijnen F & Nijssse R 2011 Concrete shell structures revisited: Introducing a new 'low-tech' construction method using vacuumics formwork Structural Membranes – 5th International Conference on Textile Composites and Inflatable Structures. p. 409
[4] Buswell R A, Soar R C, Gibb A G F, & Thorpe A 2007 Freeform construction: Mega-scale rapid manufacturing for construction Automation in Construction. 16(2) 224–231
[5] Kopsha S P 2013 The form-free molding technology is a key to industrial upgrading and cost reduction Concrete Technology. 11 29–33
[6] Palikhe S, Kim J J, Lim J, Kim S 2018 Systematic Analysis of Production Errors and Accuracy when using Pin-Bed Mold of Free-form Concrete KSCE J. of Civil Eng. 22(9) 3196–3203
[7] Bolotnik N N & Nguyen C 1985 Choice of parameters of vibration machines with inertial excitation Mechanics of solids. 20(1) 54–61
[8] Batyanovskiy E I 2018 Vibropressed concrete: technology and properties (Minsk: BNTU) 263 p.
[9] Abramovitsa M, Stigan I 1979 Reference book on special functions (Moscow: Science) 832 p.
[10] Kirilov I, Khanin S I, RU Patent No. 2014139138 (11.06.2018)