Estimation and ways of mechanical efficiency upgrading of planetary rotary hydraulic machines

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Abstract. Planetary rotary hydraulic machines (PRHM) with wavelike sun wheels and floating satellite wheels have high technical characteristics but they are understudied. The article is devoted to the estimation of energy efficiency of PRHM at the stage of their projecting. Calculated formulas for the pump and hydraulic engine are offered. The comparative estimation of schemes with the numbers of waves $M \times N$: $2 \times 2; 3 \times 3; 4 \times 4; 2 \times 4; 4 \times 6,$ has shown efficiency values close to each other a little bit more than 0.9. The reserve for efficiency upgrading for the considered hydraulic machines is the use of the teeth with half-round convex-concave side surfaces.

Volumetric pumps and hydraulic machines are considered to be one of the widely spread and actively developed types of machines. Among them planetary rotary hydraulic machines (PRHM) with floating satellite wheels are known. This type of hydraulic machines has “record” specific throughput in the conditions of sufficiently high pressure (about 200-250 atm.) [1, 2]. The important and understudied characteristic of PRHM is their mechanical efficiency.

One of the schemes of PRHM [3] is shown in figure 1. The machine has rotor 1 with external teeth, stator 2 with internal teeth and floating satellite wheels 3 situated between them. Closed voids are made with mated gear links 1, 2, 3 and flat surfaces of end plates 5. Thanks to wavelike form of the rotor and stator, cavity space changes while rotor 1 spinning. Satellite wheels 3, rolling over between rotor 1 and stator 2, open and block corresponding channels 4.

The numbers of waves of rotor $M$ and stator $N$ are proportional to corresponding numbers of teeth $Z_1$ and $Z_2$. The number of satellite wheels is $K = M + N$.

Taking into account the fact that in high pressure hydraulic machines, only symmetric schemes of PRHM can be used, the number of variants $M \times N$, to be analyzed is small: $2 \times 2, 3 \times 3, 4 \times 4, 2 \times 4, 4 \times 6, 6 \times 8$. 
Mechanical efficiency of a hydraulic machine $\eta$ is a ratio of the useful yield towards spent work, taken for example, of one rotation of the rotor. The efficiency can also be calculated through coefficient of loss $\psi$. It is the most convenient first to express loss coefficient $\psi_d$ of a hydraulic machine, working in the regime of the motor:

$$\psi_d = \frac{A_G}{A_H},$$  \hspace{1cm} (1)

where $A_H$ is “hydraulic” work, spent on charging operating environment, or released after its discharge; $A_G$ is loss work in engagements.

Work $A_H$ is equal to the production of capacity $W_1$, expelled in one revolution of the rotor, by environment pressure $p$

$$A_H = W_1 \cdot p.$$  \hspace{1cm} (2)

The value of working capacity $W_1$:

$$W_1 = S_1 \cdot \pi \cdot b \cdot \left( \frac{Z_2 \cdot m}{2} \right)^2,$$  \hspace{1cm} (3)

where $Z_2$ is the number of teeth of the annulus; $m$ is engagement module; $b$ is axis size of satellite wheel; $S_1$ is a ratio of actual working capacity to some calculated capacity, locked inside the crown of the annulus. The value of coefficient $S_1$ for the main schemes of PRHM with characteristic ratio of parameters [4] is given in table 1.
Table 1. Relative useful capacity of PRHM

| M×N  | 2×2 | 3×3 | 4×4 | 2×4 | 4×6 | 6×8 |
|------|-----|-----|-----|-----|-----|-----|
| $S_i$ | 0.46 | 0.69 | 0.90 | 0.32 | 0.49 | 0.51 |

Total loss work $A_G$ in engagements of PRHM is a production of loss work $A_i$ in one engagement by the number of engagements, which is equal to doubled number $K$ of satellite wheels:

$$A_G = 2 \cdot (M + N) \cdot A_i.$$  \hspace{1cm} (4)

Loss work $A_i$ in one engagement of the satellite wheel with one (any) central wheel in one rotation of the rotor

$$A_i = f \cdot R \cdot L_1 \cdot \frac{V_s}{V_p},$$  \hspace{1cm} (5)

where $f$ is friction coefficient;  
$R$ is normal force in the engagement;  
$L_1$ is path (movement) of the contact of satellite wheel with the central wheel in a circular direction in one rotation of the rotor;  
$\frac{V_s}{V_p}$ is ratio of average sliding speed $V_s$ in the engagement to calculated circular speed $V_p$ with the stopped carrier.

Movement $L_1$ is less than the length of rotor centroid, as it is calculated with the stopped imaginary carrier. Taking into account the fact that in PRHM the numbers of teeth of central wheels $Z_1$ and $Z_2$ are proportional to the numbers of their waves $M$ and $N$

$$L_1 = \frac{\pi \cdot m \cdot Z_1 \cdot N}{M + N}. \hspace{1cm} (6)$$

Normal force $R$ in the engagement will be expressed through circular force $F$

$$R = \frac{F}{\cos(a_w)}, \hspace{1cm} (7)$$

where $a_w$ is angle of engagement (in involute tooth systems of PRHM $a_w=20-25^\circ$).  
$F$ is circular force, which depends on pressure of operating environment affecting the satellite wheel:

$$F = \frac{p \cdot b \cdot Z_3 \cdot m}{2}. \hspace{1cm} (8)$$

Applying (4) and (5) into formula (1), we get the expression for calculation of loss coefficient $\psi_d$

$$\psi_d = \frac{4 \cdot f \cdot M \cdot Z_3 \cdot \frac{V_s}{V_p}}{Z_2 \cdot S_1 \cdot \cos(a_w)}. \hspace{1cm} (9)$$
For involute tooth system sliding speed $V_s$ can be calculated through its parameters in the following way [5]:

$$V_s = 2 \cdot V_p \cdot \left( \frac{1}{Z_3} \pm \frac{1}{Z_K} \right) \cdot k,$$

(10)

where $Z_3$ is the number of gear teeth; $Z_K$ is the number of wheel teeth; $k$ is coefficient, depending on the dislocation of the counterpart rack profile. In PRHM with $M<N$ let us assume $k = 1$, with $M=N$ we shall accept $k = 1.2$.

Considering that in PRHM $Z_3$ (i.e. satellite wheel) is always small (about 10), and $Z_K >>> Z_3$, as well as the fact that one of the engagements of the satellite wheel is external and the other is internal, then we get as the average:

$$\frac{V_s}{V_p} = 2 \cdot \left( \frac{1}{Z_3} \pm \frac{1}{Z_K} \right) \cdot k = \frac{2 \cdot k}{Z_3}.$$

(11)

So, for involute tooth system of PRHM formula (9) is converted into the form

$$\psi_d = \frac{8 \cdot f \cdot M \cdot k}{Z_2 \cdot S_1 \cdot \cos(a_w)}.$$

(12)

Applying the corresponding values of parameters into formula (10), we shall get the values of loss coefficient $\psi_d$ for different schemes of PRHM – table 2. The friction coefficient in all the cases is accepted as $f=0.1$.

In table 2 there are also calculated loss coefficients of hydraulic machines, working in the regime of pump $\psi_H = 1 + \psi_d$; their efficiency in the regime of motor $\eta_d = 1 - \psi_d$ and efficiency in the regime of pump $\eta_H = 1 - \psi_H$.

Table 2. Calculated energy efficiency of PRHM (with the friction coefficient $f=0.1$)

| $M \times N$ | $\psi_d$ | $\psi_H$ | $\eta_d$ | $\eta_H$ |
|-------------|----------|----------|----------|----------|
| 2×2         | 0.07     | 0.065    | 0.93     | 0.935    |
| 3×3         | 0.07     | 0.065    | 0.93     | 0.935    |
| 4×4         | 0.07     | 0.065    | 0.93     | 0.935    |
| 2×4         | 0.08     | 0.075    | 0.92     | 0.925    |
| 4×6         | 0.11     | 0.090    | 0.89     | 0.91     |
| 6×8         | 0.13     | 0.115    | 0.87     | 0.885    |

Let us analyze the possible ways of increasing efficiency of PRHM:

A) The decrease of the module and the increase of the number of teeth $Z_3$. However, in this situation, side force, affecting the satellite wheel, increases. In the end, this measure will limit maximum fluid pressure.

B) The increase of angle $\lambda$ (see figure 1) of holding of satellite wheel as much as it is possible. The restrictions are connected in some cases with interference (co-occurrence condition) of central wheels, in other cases with the risk of satellite wheel “falling out”.

C) The change of the form of the teeth – transformation from involute to jointed engagement [6] – see figure 2.
For PRHM with such engagements the ratio of sliding speed $V_s$ to the calculated circular speed $V_p$ in the engagement:

\[
\frac{V_s}{V_p} = \frac{\pi}{2 \cdot Z_3}.
\]  

(13)

It means that only thanks to the decrease of sliding speed along teeth profiles losses will be by $\frac{4 \cdot k}{\pi} = 1.25 + 1.5$ times smaller than in involute PRHM (12). Extra decrease of mechanical losses will be due to the decrease of friction coefficient $f$ in the teeth contact close to surface. The restriction of the use of jointed engagement is their disconjugacy hence vibro-activity. The dynamics of such hydraulic machine needs researching. Let us note that the use of such engagements is possible only in schemes $M < N$: i.e. : $2 \times 4$, $4 \times 6$, $6 \times 8$.

D) One more way of increasing efficiency is joining two sections of PRHM in series [7] see figure 3. In this case enlarged passageway channels unload satellite wheels, locking work chambers at the moment of minimum intensity of environment displacement.

Dead centres are present only in PRHM of group 1×1–P. Coping with these centres require significant complication of the mechanism of PRHM.

Figure 2. PRHM with jointed engagements [6]

Figure 3. Joining-up in series of two sections of PRHM [7]
This method gives the increase of theoretical efficiency by 1.3 ÷ 1.4 times without the increase of mechanical loss. The drawbacks of the method realization are in the complication of the construction and accompanying increase of hydraulic losses.

Conclusion
The carried-out work allows calculating mechanical losses at the stage of projecting of PRHM. Characteristic schemes of pumps and hydraulic motors have close efficiency – a little bit over than 0.9. Thus the choice of this or that scheme will depend, most of all, not on the efficiency but on other characteristics of hydraulic machines. Promising is the use of PRHM, which used teeth with a semicircular convex-concave lateral surface.

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