The Thermal Model and the Transition from Baryonic to Mesonic Freeze-Out.

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Abstract. The present status of the thermal model is reviewed and the recently discovered sharp peak in the $K^+/\pi^+$ ratio is discussed in this framework. It is shown that the rapid change is related to a transition from a baryon dominated hadronic gas to a meson dominated one. Further experimental tests to clarify the nature of the transition are discussed. In the thermal model the corresponding maxima in the $\Xi/\pi$ and $\Omega/\pi$ ratios occur at slightly different beam energies.

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1. Introduction

It is by now well-known that particle yields integrated over all momenta can be calculated as if they originate from a fireball at rest, provided chemical freeze-out happens at the same temperature and chemical potential on all points of the freeze-out surface. This follows from a covariance argument using the Cooper-Frye [1] formula as a starting point. The number of particles of type $i$ is determined by:

$$E_\mu \frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3} \int d\sigma_\mu p^\mu \exp \left( -\frac{p^\mu u_\mu}{T} + \frac{\mu_i}{T} \right)$$
where $u^\mu$ is the flow velocity which, in general, depends on space and time. Integrating this over all momenta leads to,

$$N_i = \frac{g_i}{(2\pi)^3} \int d\sigma_{\mu} \int d^3p \frac{p^\mu}{E} \exp \left(-\frac{p^\mu u_\mu}{T} + \frac{\mu_i}{T}\right).$$

Lorenz invariance dictates that the integral over momenta depends only on the flow four-velocity $u^\mu$. Hence

$$N_i = \int d\sigma_{\mu} u^\mu n_i(T, \mu)$$

where $n_i(T, \mu)$ is the particle density in a fireball at rest. If the temperature and chemical potential are unique along the freeze-out curve then $n_i$ has the same value along the freeze-out curve and one obtains,

$$N_i = n_i(T, \mu) \int d\sigma_{\mu} u^\mu,$$

i.e. integrated $(4\pi)$ multiplicities are the same as for a single fireball at rest. The volume has, of course, a completely different interpretation and depends in a complex manner on the evolution of the fireball.

2. Present Knowledge of the Chemical Freeze-Out Diagram

The present knowledge of the freeze-out parameters in relativistic heavy ion collisions is summarized in Fig.1 and shows a remarkable regularity when going from the lowest to the highest energies [2, 3].

3. The Horn in the $K^+/\pi^+$ Ratio

The NA49 Collaboration has recently performed a series of measurements of Pb-Pb collisions at 20, 30, 40, 80 and 158 AGeV beam energies [5]. When these results are combined with measurements at lower beam energies from the AGS they reveal an unusually sharp variation with beam energy in the $\Lambda/\langle \pi \rangle$, with $\langle \pi \rangle \equiv 3/2(\pi^+ + \pi^-)$, and $K^+/\pi^+$ ratios. Such a strong variation with energy does not occur in pp collisions and therefore indicates a major difference in heavy-ion collisions. This transition has been referred as the “horn”.

To understand this behavior, which is not seen anywhere in $p-p$ or $e^+-e^-$ collisions, one uses the Wroblewski factor [4],

$$\lambda_s = \frac{2 \langle ss \rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle},$$

which compares the number of newly created quark-anti-quark pairs before strong decays, i.e. before $\rho$'s and $\Delta$'s decay. The limiting values are: $\lambda_s = 1$ all quark
pairs are equally abundant, and $\lambda_s = 0$ if no strange quark pairs are present.

Fig. 1. Values of $\mu_B$ and $T$ for different energies obtained from RHIC, SPS, AGS and SIS. The solid line has been drawn to guide the eye.

The change in the Wroblewski factor is accompanied by a change in the composition of the hadronic fireball. Below the maximum in the Wroblewski ratio, the fireball is dominated by baryons, above it is dominated by mesons. To determine the precise position of the change-over we use the entropy density, normalized to $T^3$. This is shown in Fig. 4.

In the statistical model [6] therefore a rapid change is expected as the hadronic gas undergoes a transition from a baryon-dominated to a meson-dominated gas. The
transition occurs at a temperature $T = 140$ MeV and baryon chemical potential $\mu_B = 410$ MeV corresponding to an incident energy of $\sqrt{s_{NN}} = 8.2$ GeV.

It is to be expected that if the maxima observed in the particle ratios do not all occur at the same temperature, i.e. at the same beam energy, then the case for a phase transition is not very strong. A comparison with the data is shown in Fig. (4).

The observed behavior seems to be governed by properties of the hadron gas. In order to distinguish between this smooth change-over from a baryon-dominated to a meson-dominated hadronic gas it would be useful to determine the positions of the maxima more precisely. In the thermal model these are given in Table I.

More detailed experimental studies of multi-strange hadrons will allow the verification or disproval of the trends shown in this paper. It should be clear that the $\Omega^-/\pi^+$ ratio is very broad and shallow and it will be difficult to find a maximum experimentally.

5. Summary

In conclusion, while the statistical model cannot explain the sharpness of the peak in the $K^+ / \pi^+$ ratio, its position corresponds precisely to a transition from a baryon-dominated to a meson-dominated hadronic gas. This transition occurs at a

- temperature $T = 140$ MeV,
Fig. 3. The entropy density normalised to $T^3$ as a function of the beam energy as calculated in the thermal model. The contributions from baryons and mesons are shown separately.

| Ratio          | Maximum at $\sqrt{s_{NN}}$ (GeV) | Maximum Value |
|----------------|----------------------------------|---------------|
| $\Lambda/\langle\pi\rangle$ | 5.1                             | 0.052         |
| $\Xi^-/\pi^+$     | 10.2                            | 0.011         |
| $K^+/\pi^+$       | 10.8                            | 0.22          |
| $\Omega^-/\pi^+$  | 27                              | 0.0012        |

Table 1. Maxima in particle ratios as predicted by the thermal model.

- baryon chemical potential $\mu_B = 410$ MeV,
- energy $\sqrt{s_{NN}} = 8.2$ GeV.

In the statistical model this transition leads to a sharp peak in the $\Lambda/\langle\pi\rangle$ ratio, and to moderate peaks in the $K^+/\pi^+$, $\Xi^-/\pi^+$ and $\Omega^-/\pi^+$ ratios. Furthermore, these peaks are at different energies in the statistical model. The statistical model predicts that the maxima in the $\Lambda/\langle\pi\rangle$, $\Xi^-/\pi^+$ and $\Omega^-/\pi^+$ occur at increasing beam energies.
Fig. 4. (a) The Λ/⟨π⟩ ratio as a function of energy. (b) The $K^+$/π$^+$ and $K^-$/π$^-$ ratios as a function of energy. The solid and dashed lines are the results of the thermal model.

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