Numerical study on the influence of suspension damping on the bogie vertical vibration

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Abstract. The paper presents a study on the influence of suspension damping (SD) on bogie vertical vibrations generated by vertical track irregularities. The study is based on the results of the numerical simulations developed on the basis of a linear model with seven-degree of freedom of the bogie–track system, that takes into account the rigid vertical vibration modes of the bogie – bounce and pitch, vertical displacements of the wheels and rails, and the wheel-rail contact elasticity. The bogie vibration behaviour is assessed based on the dynamic response of the bogie chassis, expressed as the power spectral density of the vertical acceleration (PSD acceleration) at three reference points – at the bogie chassis centre and over the axles, and root mean square of vertical acceleration (RMS acceleration). It is highlighted the main characteristics of the bogie vibration behaviour according to the SD in correlation with the speed and quality of the railway track.

1. Introduction

Railway vehicle is a complex mechanical system, which presents a vibration regime with specific characteristics. Generally, the geometric irregularities of the track and the defects of the rolling surfaces of the wheel/rail system are the main causes for the vehicle vibrations [1]. The track geometry is affected by many irregularities especially due to the constructive tolerances, train-track interactions, alteration in the infrastructure because the effect of the environment factors or the ground settlement [2]. Irregularities of the track are errors from the designed geometry resulted from the vertical or lateral displacement of the two rails from the design position [3]. Defects of the rolling surfaces originate in the manufacturing process, which are then amplified during operation, or are defects due to the wear caused by either wheel-rail interaction or wheel-brake blocks interaction [4].

Running on a track with vertical irregularities will generate vibrations of the wheelset, which are transmitted to the bogie and the carbody, thus generating and maintaining the vibration on the entire vehicle. The suspension has the role to dampen or limit the amplitude of the vibrations conveyed from the axles to the bogie frame [5 - 7] and assists with the reduction in the dynamic load to which the carrying structure of the vehicle is subjected to, on the one hand, and the track, on the other hand [8, 9].

The paper presents a numerical study on the influence of SD on the vertical vibrations of a two-axle bogie generated at speeds between 50 km/h and 200 km/h on a track with imperfect geometry in vertical plane. Vertical track irregularities are described using the PSD defined for both high quality track, as well as a low-quality track, corresponding to the average statistical properties of the European railways [10].
2. The bogie-track system mechanical model

The bogie-track system model of used for the study of the influence of the SD on the vertical vibrations of bogie excited by the irregularities of the track when uniform moving at V speed is presented in figure 1.

The bogie model has 3 rigid bodies corresponding to chassis of the bogie and the two wheelsets, respectively. The suspension corresponding to each axle is modelled via a Kelvin-Voigt system, with the elastic constant $2k_{zb}$ and the damping constant $2c_{zb}$.

![Fig. 1. The bogie-track system mechanical model.](image)

The rigid-body modes of the bogie in the vertical plane are considered, respectively the bounce ($z_b$) and the pitch ($\theta_b$). The parameters for the bogie are: $m_b$ - mass of the bogie, $2a_b$ – wheelbase of the bogie, $J_b$ - moment of inertia. The wheelsets with the mass $m_w$ have a translational motion in the vertical direction ($z_w$).

The track is reduced to two one degree of freedom oscillators which have the vertical displacements $z_{r1,2}$. The parameters for the track model are: $m_r$ - mass, $2k_r$ - stiffness and $2c_r$ - the damping coefficient. The functions $\eta_{1,2}$ describe the irregularities of the track corresponding to the two axles.

To model the wheel/rail elastic contact, an elastic element of stiffness $k_H$ has been inserted between the wheel and rail. The contact stiffness is calculated according to the linear Hertz’s theory.

The equations describing the bounce and the pitch movements are:

$$m_b\ddot{z}_b = \sum_{i=1}^{2} F_{bi} - F_s,$$  \hspace{1cm} (1)

$$J_b \ddot{\theta}_b = a_b \sum_{i=1}^{2} (-1)^{i+1} F_{bi}.$$  \hspace{1cm} (2)

where $F_s$ is the force in the car body suspension of the railway vehicle and $F_{bi}$ is the force from the suspension of the axle $i$, (for $i = 1, 2$),

$$F_{b1,2} = -2c_{zb1,2} (\dot{z}_b \pm a_b \dot{\theta}_b - \dot{z}_{w1,2}) - 2k_{zb} (z_b \pm a_b \theta_b - z_{w1,2}).$$  \hspace{1cm} (3)

The equations of motion for the wheelsets are

$$m_w \ddot{z}_{w1,2} = 2Q_{d1,2} - F_{b1,2}.$$  \hspace{1cm} (4)

where $Q_{d1,2}$ are the dynamic components of the wheel/rail contact forces; it assumes that the contact forces on both wheels of a wheelset are equal.
For determining the dynamic components of the contact forces, the linear form of the Hertz’s equation is used in that case:

\[ Q_{d1,2} = -k_H [\eta - \tilde{z}_{r1,2} - \tilde{z}_{l1,2}] \tag{5} \]

where \( k_H \) is depending on the curvatures in longitudinal and cross-section of the wheel/rail profiles and the static load on wheel.

Displacements of the rails are described by the equations:

\[ m_w \ddot{z}_{r1,2} = F_{r1,2} - 2Q_{d1,2} \tag{6} \]

with

\[ F_{r1,2} = -2c_r \dot{z}_{r1,2} - 2k_r z_{r1,2} \tag{7} \]

Finally, the motion of the bogie-track system is given by the following equations:

\[ m_b \ddot{z}_b + 2c_b [2 \dot{z}_b - (\dot{z}_{w1} + \dot{z}_{w2})] + 2k_b [2z_b - (z_{w1} + z_{w2})] - F_z = 0 \tag{8} \]

\[ F_{\dot{b}} + 2c_b \dot{\theta}_b [2 \dot{\theta}_b - (\dot{\theta}_{w1} + \dot{\theta}_{w2})] + 2k_b [2\theta_b - (\theta_{w1} + \theta_{w2})] = 0 \tag{9} \]

\[ m_{\omega} \ddot{\omega}_{w1,2} + 2c_b (\omega_{w1,2} - \omega_b - a_b \Theta_b) + 2k_b (\omega_{w1,2} - \omega_b + a_b \Theta_b) + 2k_H (\omega_{w1,2} - \eta_{l1,2}) = 0 \tag{10} \]

\[ m_p \ddot{\eta}_{r1,2} + 2c_r \ddot{\eta}_{r1,2} + 2k_z \eta_{r1,2} + 2k_H (\eta_{r1,2} - \omega_{w1,2} + \eta_{l1,2}) = 0 \tag{11} \]

Hence results a 6-equation system with ordinary derivatives (eq. 8 - 11), which can be solved numerically using MATLAB code.

3. The dynamic response of the bogie

Dynamic response of the bogie to the vertical track irregularities is evaluated based on the PSD acceleration and the RMS acceleration calculated at three reference points on the bogie. According to figure 1, the reference bogie points are thus defined: the point \( B \) – at the centre of the bogie chassis, the points \( B_{w1,2} \) over the axles, respectively over the supporting points of the bogie chassis on the suspension.

The PSD acceleration at the reference points is

\[ G_B (\omega) = G(\omega)\left| a^2 \overline{H}_{\omega_b} (\omega) \right|^2 \tag{12} \]

\[ G_{\omega_{w1,2}} (\omega) = G(\omega)\left| a^2 \overline{H}_{\omega_b} (\omega) \pm a_c \overline{H}_{\omega_c} (\omega) \right|^2 \tag{13} \]

where \( \overline{H}_{\omega_b} (\omega) \) and \( \overline{H}_{\omega_c} (\omega) \) are the frequency response functions for the bounce and pitch (\( \omega_c \) și \( \theta_c \)) of the bogie, and \( G(\omega) \) is the PSD of the vertical track irregularities,

\[ G(\omega) = \frac{A \Omega_c^2 V^3}{[\omega^2 + (\Omega_c \omega)^2]^2[\omega^2 + (\Omega_c \omega)^2]^2} \tag{14} \]

where the track parameters \( \Omega_c, \Omega_c \) and \( A \) are given in [10]; there are two values for \( A \), the lower is for good quality of the track and the higher is for low quality.

Starting from the PSD acceleration, RMS acceleration is calculated at the reference points of the bogie, namely:

\[ a_B = \sqrt{\frac{1}{\pi} \int_0^\infty G_B d\omega}, \quad a_{\omega_{w1,2}} = \sqrt{\frac{1}{\pi} \int_0^\infty G_{\omega_{w1,2}} d\omega} \tag{15} \]
4. The outcomes of the numerical applications
In this section, the outcomes of the numerical applications on the dynamic response of the bogie moving along a track with irregularities are shown. The reference parameters of the model previously presented are shown in table 1; the natural frequencies of the bogie itself are: 5.9 Hz for bounce and 9.4 Hz for pitch. The damping ratio of the suspension is given \( \zeta_b = c_b/(k_b m_b)^{1/2} \). For reference parameters of bogie-track system \( \zeta_b = 0.22 \).

| Parameter | Value |
|-----------|-------|
| \( m_b \) | 2700 kg |
| \( k_{zb} \) | 1.10 MN/m |
| \( m_w \) | 1400 kg |
| \( c_{zb} \) | 13.05 kNs/m |
| \( m_r \) | 175 kg |
| \( k_r \) | 70 MN/m |
| \( 2a_b \) | 2.56 m |
| \( c_r \) | 60 kNs/m |
| \( J_b \) | \( 2.05 \times 10^3 \) kg\cdot m^2 |
| \( k_H \) | 1500 MN/m |

Table 1. The numerical parameters of the bogie-track system.

Figure 2 shows the PSD acceleration at the reference points of the bogie when the speed ranges from 50 km/h to 200 km/h, calculated for low quality track. First, the peaks corresponding to the bogie resonance frequencies are highlighted. Then, it should be noted that the vibration level of the bogie increases with the speed, and this manifests more strongly at the resonance frequencies of the bogie. Last but not least, it should be noted that the vibration level of the bogie is higher over axles and lower in the middle of the chassis, and that the dynamic response of the bogie over the two axles is asymmetric - the vibration level of the bogie over rear axle is greater than over the front axle. The asymmetry of the dynamic response of the bogie over the two axles is due to the SD (see figure 3).

Figure 2. PSD acceleration – influence of the velocity:
(a) at the bogie chassis centre; (b) over the axle 1; (c) over the axle 2.

Figure 3. PSD acceleration: (a) \( \zeta_b = 0.22 \); (b) \( \zeta_b = 0 \).
The diagrams in figure 4 show the impact of the SD on the dynamic response of the bogie at 200 km/h, expressed by the PSD acceleration. These out that the bogie vibration level increases significantly at bogie resonance frequencies for SD ratio less than 0.15. For example, in the centre of the bogie chassis at the frequency of 5.9 Hz, for the damping ratio reference value ($\zeta_b = 0.22$), the PSD acceleration is $0.13 [\text{m/s}^2/(1/\text{s})]$; when the damping ratio of the suspension decreases to 0.1, the PSD acceleration increases to $0.57 [\text{m/s}^2/(1/\text{s})]$.

Figure 4. PSD acceleration at 200 km/h – influence of the damping ratio of the suspension: (a) at the bogie chassis centre; (b) over the axle 1; (c) over the axle 2.

Figure 5 shows the RMS acceleration when the speed ranges between 50 km/h and 200 km/h for different values of the SD ratio, for track of low quality ($A = 1.080 \cdot 10^{-6}$ radm) - the diagrams (a), (b) and (c), and for track of high quality ($A = 4.032 \cdot 10^{-7}$ radm) - the diagrams (a'), (b') and (c'). On one hand, it is noted that RMS acceleration increases with speed, and on the other hand, that RMS
acceleration is reduced when the SD ratio increases. Track quality influences sensitively the RMS acceleration values. For instance, running on track of low quality with 200 km/h speed. RMS acceleration of the bogie over front axle is 1.67 m/s², and over rear axle is 2.17 m/s², and running on track of high quality, the RMS accelerations are $a_{b1} = 1.02 \text{ m/s}^2$ and $a_{b2} = 1.32 \text{ m/s}^2$; the SD ratio takes the reference value.

5. Conclusions
The paper presents a numerical analysis on the effect of SD on bogie vertical vibrations generated by vertical irregularities of the track. Assessment of the vibration behaviour of the bogie is based on the PSD acceleration and RMS acceleration, calculated at three reference points of the bogie chassis depending on the SD ratio in correlation with the speed and quality of the track. Thus, it has been shown that the bogie vibration increases with speed, especially at the resonance frequencies of the bogie bounce and pitch, and is more intense at the reference points located over the axles than at the centre of the bogie chassis. It has also been shown that the dynamic response of the bogie over the two axles is asymmetric, and this is due to the damping of the suspension. A low damping of the suspension or low-quality track causes an increased level of vibration at all bogie reference points. Reducing the vertical vibrations of the bogie can be achieved by increasing the damping ratio of the suspension. The damping ratio of the suspension cannot be increased as much as desired, due to the limitations brought about the vertical dynamic wheel/rail forces.

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