Sound and Complete Query Answering in Intensional P2P Data Integration

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Abstract. Contemporary use of the term ‘intension’ derives from the traditional logical doctrine that an idea has both an extension and an intension. In this paper we introduce an intensional FOL (First-order-logic) for P2P systems by fusing the Bealer’s intensional algebraic FOL with the S5 possible-world semantics of the Montague, we define the intensional equivalence relation for this logic and the weak deductive inference for it. The notion of ontology has become widespread in semantic Web. The meaning of concepts and views defined over some database ontology can be considered as intensional objects which have particular extension in some possible world: for instance in the actual world. Thus, non invasive mapping between completely independent peer databases in a P2P systems can be naturally specified by the set of couples of intensionally equivalent views, which have the same meaning (intension), over two different peers. Such a kind of mapping has very different semantics from the standard view-based mappings based on the material implication commonly used for Data Integration. We show how a P2P database system may be embedded into this intensional modal FOL, and how we are able to obtain a weak non-omniscient inference, which can be effectively implemented. For a query answering we consider non omniscient query agents and we define object-oriented class for them which implements as method the query rewriting algorithm. Finally, we show that this query answering algorithm is sound and complete w.r.t. the weak deduction of the P2P intensional logic.

1 Introduction

Ontologies play a prominent role on the Semantic Web. An ontology specifies a conceptualization of a domain in terms of concepts, attributes and relations. However, because of the Semantic Web distributed nature, data on it will inevitably come from many different ontologies. A key challenge in building the Semantic Web, one that has received relatively little, attention, is finding semantic mappings among the ontologies (peers). Given the de-centralized nature of the development of the Semantic Web, there will be an explosion in the number of ontologies. Many of these ontologies will describe similar domains, but using different terminologies, and others will have overlapping domains. To integrate data from disparate ontologies, we must know the semantic correspondence between their elements [1]. Recently are given a number of different architecture solutions [2,3,4,5,6,7]. The authors provided more information about different P2P systems...
and a comparative analysis in [8,9].

In what follows we will consider a reach ontology of peer databases, formally expressed as a global schema of a Data Integration System (DIS). A DIS [10] is a triple $I_i = (G_i, S_i, M_i)$, where $G_i = (\mathcal{O}_i, \Sigma_T i)$ is a global schema, expressed in a language $\mathcal{L}_G$, $\Sigma_T i$ are the integrity constraints, $S_i$ is a source schema and $M_i$ is a set of mappings between a global relational database schema (ontology) $\mathcal{O}_i$ and a source relational schema $S_i$ of data extracted by wrappers. In what follows we will consider the case of Global-As-View (GAV) mappings between source and global schema, with existential quantifiers also (for mapping an incomplete information from source to global schema). A DIS with constraints (for example, key-constraints) can become locally inconsistent w.r.t. data sources extracted by wrappers. Such local inconsistencies can be avoided by inconsistency repairing technics [11].

We conceive a peer $P_i$ as a software module, which encapsulates a DIS $I_i$. The internal structure of a peer database is hidden to the user, encapsulated in the way that only its logical relational schema $\mathcal{O}_i$ can be seen by users, and is able to respond to the union of conjunctive queries by known answers (true in all models of a peer-database).

We consider a view definition $q_k(x)$ as a conjunctive query, with a tuple of variables in $x$, $\text{head}(q_k) \leftarrow \text{body}(q_k)$ where $\text{body}(q_k)$ is a sequence $b_1, b_2, ..., b_m$, where each $b_j$ is an atom over a global relation name of a peer $P_i$. In what follows we will consider a view as a virtual predicate with a tuple $x$ of free variables in the head of a query.

In P2P systems, every node (peer) of the system acts as both client and server and provides part of the overall information available from an Internet-scale distributed environment. In this paper we consider a formal framework based on the following considerations [12]: what we need here is

- a mechanism that is able, given any two peer databases, to define mappings between them, without resorting to any unifying (global) conceptual structure.
- a completely decentralized network of database peers: all peers serve as entry points for search, offering their relational schema in order to formalize user queries.
- query answering, fundamentally based on interactions which are strictly local and guided by locally defined mappings of a considered peer w.r.t. other peers.
- not limit a-priori the topology of the mapping assertions between peers in the system: we do not impose acyclicity of assertions.
- a semantic characterization that leads to setting where query answering is decidable, and possibly, polynomially tractable.

The last two considerations, decidability and non acyclicity enforce the reason to use epistemic modal instead of FOL (First-Order-Language; see for example [7]) to model a peer database, with an epistemic operator "peer $P_i$ knows", $K_i$, and with relational database schema (ontology) $\mathcal{O}_i$ for conjunctive query language. Such an epistemic semantics of peers has been presented originally in [6] based on the hybrid mono-modal language with a unique universal modal operator $\Box$ [13], so that where $K_i = \Box_i \square$, where new modal operator, $\Box$, for this hybrid logic enables to "retrieve" worlds: A formula of the form $\Box_i \varphi$ is an instruction to move to the world labeled by the variable $i$ and evaluate $\varphi$ there. How this relational, view based, approach to peer ontologies relates to the actual semantic Web RDF language is explained in [14,15,16,17].

In what follows we abbreviate $A \Rightarrow B \land B \Rightarrow A$ by $A \equiv B$. 
Let \( q_{P_i}(x) \) and \( q_{P_j}(x) \) be two views (conjunctive queries) over \( P_i \) and \( P_j \) respectively, with the same set of free variables \( x \), then we can have two P2P scenarios:

1. The strong (extensional) multi-modal mapping, introduced by a formula \( K_i q_{P_i}(x) \Rightarrow K_k q_{P_k}(x) \), where \( \cdot \Rightarrow \cdot \) is the logic implication, used in a single S5 modality \([8, 7, 19]\), and in K45 multi-modality \([19]\). It tells that the knowledge of the peer \( P_i \) contained in its view \( q_{P_i}(x) \) must be contained in the view \( q_{P_k}(x) \) of the peer \( P_k \). In this case this forced transfer of the local data of one peer to other peers can render inconsistent knowledge of other peers, and from the semantic point of view, reassembles the kind of strong data integration system, with a global logic of the whole P2P system and a recursive Datalog for query rewriting \([7]\).

2. The weak (intensional) mapping, defined by the “formula” \( K_i q_{P_i}(x) \approx_{in} K_k q_{P_k}(x) \), where \( \approx' \) is the informal symbol for the intensional equivalence \([8, 6, 20, 21, 22]\), and formally in Definition \([6]\) by the logic modal formula \( \diamond q_{P_i}(x) \equiv \diamond q_{P_k}(x) \) of the intensional FOL introduced in this paper. This mapping tells only that these two peers have the knowledge about the same concept, without any constraint for extensions of this concept in these two peers respectively.

The more complete comparative analysis of these two different approaches can be found in \([8]\). In what follows we will consider only this new intensional version for P2P mapping better suited for fully independent peers \([22]\). Consequently, in order to be able to share the knowledge with other peer \( P_j \) in the network \( N \), each peer \( P_i \) has also an export-interface module \( M^{ij} \) composed by groups of ordered pairs of intensionally equivalent views (conjunctive queries over peer’s ontologies), denoted by \((q_i, q_j)\).

**Definition 1.** \([6]\) The P2P network system \( N \) is composed by \( 2 \leq N \) independent peers, where each peer module \( P_i \) is defined as follows: \( P_i := \langle O_{i}, M_{i} \rangle \), where \( M_{i} = \langle M_{i1}, ..., M_{iN} \rangle \) is an interface tuple with \( M^{ij} \), \( 1 \leq j \leq N \) (possibly empty) interface to other peer \( P_j \) in the network, defined as a group of intensionally equivalent query connections, denoted by \((q^{ij}_{ik}, q^{ij}_{jk})\) where \( q^{ij}_{ik}(x) \) is a conjunctive query defined over \( O_{i} \), while \( q^{ij}_{jk}(x) \) is a conjunctive query defined over the ontology \( O_{j} \) of the connected peer \( P_j \) : \( M^{ij} = \{ (q^{ij}_{ik}, q^{ij}_{jk}) \mid 1 \leq k \leq n_{ij} \} \), where \( n_{ij} \) is the total number of query connections of the peer \( P_i \) toward a peer \( P_j \).

Intuitively, when an user defines a conjunctive query over the ontology \( O_{i} \) of the peer \( P_i \), the intensionally equivalent concepts between this peer and other peers will be used in order to obtain the answers from a P2P system.

They will be the "bridge" which a query agent can use to rewrite the original user query over a peer \( P_i \) into intensionally equivalent query over other peer \( P_j \) which has different (and independent) ontology from the peer \( P_i \).

The answers of other peers will be epistemically considered as possible answers because the are based on the belief which has the peer \( P_i \) about the knowledge of a peer \( P_j \) this belief is formally represented by supposition of a peer \( P_i \) that the pair of queries \((q^{ij}_{ik}, q^{ij}_{jk}) \in M^{ij}\) is intensionally equivalent.

**Motivation:** The main motivation for the introduction of intensional logic for the mappings between peers is based on the desire to have the full epistemic independency of peer databases: we consider that they can change their ontology and/or extension of their knowledge independently from other peers and without any communication to
other peers. So, we intend to use the mappings between peers that are not controlled by any centralized system, which are not permanently correct during evolution of a P2P system in time but express only assumptions based on their local belief about knowledge of other peers [8]. Here there is no any transfer of the local knowledge of a peer to knowledge of other peers, which can possibly generate inconsistency of these other peers, but only a belief based assumption that they can speak about intensionally equivalent concepts. From a practical point of view, we assume that there is no any omniscient query agent, able to know the whole global P2P system. Consequently, as in human communications, based on the fact that the same concepts have the same meaning for people, but not the same extensions for every human being, a query answering must be based on the weaker form of deduction that the omniscient deduction which uses Modus Ponens and Necessity rule (for normal modal logic) to derive all possible deductions.

The formalization of this non omniscient intensional contextual reasoning for the query-agents in P2P database systems is presented in [23]. In this way we intend to obtain the very robust P2P systems but also the possibility to map naturally P2P database systems into grid computations: if the peers are fully independent it is enough to associate each pair (peer, query formulae) to a particular resource of grid computing, in order to obtain known answer from such a peer.

The aim of this paper is to provide the clear semantics for such P2P database systems with intensional mappings between peers, and to provide the clear mathematical framework for its query answering computation which, successively, can be implemented into an massive grid computing framework.

The main contributions in this paper are the following:

1. We define a modal logic framework: we define an intensional S5 modal FOL $\mathcal{L}_\omega$ with intensional identity for a P2P system, by fusing Bealer’s algebraic and Montague’s possible world approaches, and enrich it with the intensional equivalence. We define a weak deduction inference for this intensional logic to be implemented by query answering algorithms of non omniscient query agents. This logic is S5 modal logic where the set of possible worlds is the set of all possible evolutions in a time of a given P2P system (when the peers modify their ontologies or their extensions).

2. Finally, we define an object-oriented class for query agents which implements, as method, a query rewriting algorithm able to reformulate the original user conjunctive query specified over a peer $P_i$, in intensionally equivalent queries for other peers. We show that this algorithm is sound and complete w.r.t. the weak deduction of the intensional logic $\mathcal{L}_\omega$.

This paper is written to be self-contained, so the original part w.r.t the previous publications cited by author is presented in the last Section 5.

The Plan of this work is the following: in Section 2 is presented the formal semantics for intensional FOL and intensional equivalence, used to define a non invasive semantic mappings between epistemically independent peers, based on relational views of different peers. This Section is a fundamental background for the rest of the paper, and distinguish this approach from all other currently used for definition of mappings between peers, as remarked in the introduction. It combines the Bealer’s algebraic and Montague’s possible worlds semantics for intensional logic FOL: The Bealer’s algebra is useful in order to define the abstraction of logic formulae, in order to be used as terms
in other logic formulae, while the Montague’s possible worlds semantics is used to define the intensional equivalence by existentially quantified modal formulae. In Section 3 we define an embedding of P2P systems into this intensional FOL with standard S5 modal omniscient inference. In Section 4 we define the weak (non-omniscient) deduction inference for it, different from the omniscient inference of the intensional FOL, which can be effectively used by query agents for computation considering only the actual Montague’s world. Finally, in Section 5 we define the weak deduction solution for query answering in this intensional logic, and we define the non omniscient query agent object-oriented class which implements sound and complete query rewriting algorithm w.r.t. the weak intensional deduction.

2 Intensional equivalence and Intensional FOL language

Contemporary use of the term ‘intension’ derives from the traditional logical doctrine that an idea has both an extension and an intension. Intensional entities are such things as concepts, propositions and properties. What make them ‘intensional’ is that they violate the principle of extensionality: the principle that extensional equivalence implies identity. All (or most) of these intensional entities have been classified at one time or another as kinds of Universals [24].

The fundamental entities are intensional abstracts or so called, ‘that’-clauses. We assume that they are singular terms; Intensional expressions like 'believe', 'mean', 'assert', 'know', are standard two-place predicates that take 'that' -clauses as arguments. Expressions like 'is necessary', 'is true', and 'is possible' are one-place predicates that take 'that' -clauses as arguments. For example, in the intensional sentence "it is necessary that A", where A is a proposition, the 'that A' is denoted by the (A), where (📸) is the intensional abstraction operator which transforms a logic formula into a term. So that the sentence "it is necessary that A" is expressed by the logic atom N(A), where N is the unary predicate 'is necessary'. In this way we are able to avoid to have the higher-order syntax for our intensional logic language (predicates appear in variable places of other predicates),as, for example HiLog [25] where the same symbol may denote a predicate, a function, or an atomic formula. In the First-order logic (FOL) with intensional abstraction we have more fine distinction between an atom A and its use as a term "that A", denoted by ⟨A⟩ and considered as intensional "name", inside some other predicate, and, for example, to have the first-order formula ¬A ∧ P(t, ⟨A⟩) instead of the second-order HiLog formula ¬A ∧ P(t, A).

Definition 2. The syntax of the First-order Logic language with intensional abstraction ⟨⟩, called Łω, in [26], is as follows:

Logic operators (∧, ¬, Ǝ); Predicate letters in P (functional letters are considered as particular case of predicate letters); Variables x, y, z,.. in Var; Abstraction ⟨⟩, and punctuation symbols (comma, parenthesis). With the following simultaneous inductive definition of term and formula:

1. All variables and constants (0-ary functional letters in P) are terms.
2. If t1, ..., tk are terms, then A(t1, ..., tk) is a formula (A ∈ P is a k-ary predicate letter).
3. If A and B are formulae, then (A ∧ B), ¬A, and (Ǝx)A are formulae.
4. If $A$ is a formula and $\alpha = < x_1, \ldots, x_n >$, is a sequence (tuple) of distinct variables (a subset of free variables in $A$), then $\langle A \rangle_\alpha$ is a term. The externally quantifiable variables are the free variables not in $\alpha$. When $n = 0$, $\langle A \rangle$ is a term which denotes a proposition, for $n \geq 1$ it denotes a $n$-ary relation-in-intension.

An occurrence of a variable $x_i$ in a formula (or a term) is bound (free) iff it lies (does not lie) within a formula of the form $(\exists x_i)A$ (or a term of the form $\langle A \rangle_{x_1 \ldots x_i \ldots x_n}$). A variable is free (bound) in a formula iff it has (does not have) a free occurrence in that formula.

A sentence is a formula having no free variables. The binary predicate letter $F_i$ is introduced as a distinguished logical predicate and formulae of the form $F_i(t_1, t_2)$ are to be rewritten in the form $t_1 = t_2$. The logic operators $\forall, \exists, \Rightarrow$ are defined in terms of $(\land, \lnot, \exists)$ in the usual way.

For example, "$x$ believes that $A$" is given by formula $B(x, \langle A \rangle)$ ($B$ is binary 'believe' predicate), "Being a bachelor is the same thing as being an unmarried man" is given by identity of terms $B(x)_x = (U(x) \land M(x))_x$ (with $B$ for 'bachelor', $U$ for 'unmarried', and $M$ for 'man', unary predicates).

Thus, analogously to Boolean algebras which are extensional models of propositional logic, we introduce an intensional algebra as follows. We consider a non empty domain $D = D_{-1} \cup D_1$, where a subdomain $D_{-1}$ is made of particulars (extensional entities), and the rest $D_1 = D_0 \cup D_1 \ldots \cup D_n \ldots$ is made of universals ($D_0$ for propositions (the 0-ary relation-in-intensions), and $D_n, n \geq 1$, for $n$-ary relations-in-intension).

**Definition 3.** Intensional algebra is a structure

$$\text{Alg}_{int} = < D, \text{conj}, \text{disj}, \text{impl}, \text{neg}, \text{pred}, \tau, f, t >,$$

with binary operations

$$\text{conj} : D_i \times D_i \rightarrow D_i, \quad \text{pred} : D_i \times D \rightarrow D_{i-1}, \quad \text{for } i \geq 1,$$

and unary operation

$$\text{neg} : D_i \rightarrow D_0,$$

for each $i \geq 0$; the disjunctions and implications are defined in a standard way by

$$\text{disj}(u, v) = \text{neg}(\text{conj}(\text{neg}(u), \text{neg}(v))), \quad \text{impl}(u, v) = \text{disj}(\text{neg}(u), v),$$

for any $u, v \in D_1$;

$\tau$ is a set of auxiliary operations [27] intended to be semantic counterparts of the syntactical operations of repeating the same variable one or more times within a given formula and of changing around the order of the variables within a given formula;

$f, t$ are empty set $\{\}$ and set $\{<>\}$ (with the empty tuple $<> \in D_{-1}$ i.e. the unique tuple of 0-ary relation) which may be thought of as falsity and truth, as those used in the relational algebra, respectively.

**Remark:** This definition differs from the original work in [27] where $t$ is defined as $D$, and $\text{conj} : D_i \times D_i \rightarrow D_i$, $i \geq 0$, here we are using the relational algebra semantics for the conjunction. So that we are able to support also structural composition for abstracted terms necessary for supporting relational conjunctive queries, as, for example, $\langle A(x, y) \land B(y, z) \rangle_{xyz}$, which is not possible in the reduced syntactic version of the Bealer’s algebra. In the original work [27] this "algebraization" of the intensional FOL is extended also to logic quantifiers, but for our purpose it is not necessary, because in the embedding of a P2P system into the intensional FOL for the query answering, we will use only the predicates from the global schema of each peer databases both with the queries (virtual predicates) used for intensional mapping between peers. The rest of peer’s ontology (a Data Integration System) can use also existential quantifiers for
internal mappings between source and global schema of a peer database (in the case of incomplete information which comes from some source into relations of a global schema (in GAV mappings), or in the case of particular integrity constraints over a global schema of a peer database). But as we noted in the introduction, this part of a peer ontology is encapsulated into the peers and is responsible only to define the exact extension, in a given instance of time (possible world for the intensional S5 modal FOL), for predicates used in the intensional FOL. In order to compute this extension, independently for each peer database, we will use the ordinary extensional FOL logic for encapsulated peers, based on the extensional S5 epistemic FOL (Subsection 3.1). The mapping $V$, used in the following Montague’s based approach, is a high-level result of the data semantics encapsulated into each peer database. It is logically specified in this extensional S5 epistemic FOL for a peer database.

The distinction between intensions and extensions is important especially because we are now able to have and equationial theory over intensional entities (as $\langle A \rangle$), that is predicate and function "names", that is separate from the extensional equality of relations and functions. Thus, intensional FOL has the simple Tarski-first-order semantics, with a decidable unification problem, but we need also the actual world mapping which maps any intensional entity to its actual world extension. In what follows we will identify a possible world by a particular mapping which assigns to intensional entities their extensions in such possible world. It is direct bridge between intensional FOL and possible worlds representation [28, 29, 30, 31, 32], where the intension of a proposition is a function from a set of possible worlds $\mathbb{W}$ to truth-values, and properties and functions from $\mathbb{W}$ to sets of possible (usually not-actual) objects.

In what follows we will use one simplified S5 modal logic framework (we will not consider the time as one independent parameter as in Montague’s original work) with a model $\mathcal{M} = (\mathbb{W}, \mathcal{R}, \mathcal{D}, V)$, where $\mathbb{W}$ is a set of possible worlds, $\mathcal{R}$ is a reflexive, symmetric and transitive accessibility relation between worlds ($\mathcal{R} = \mathbb{W} \times \mathbb{W}$), $\mathcal{D}$ is a non-empty domain of individuals given by Definition $\mathcal{3}$ while $V$ is a function defined for the following two cases:

1. $V : \mathbb{W} \times F \to \bigcup_{n \in \omega} \mathcal{D}^{D^n}$, with $F$ a set of functional symbols of the language, such that for any world $w \in \mathbb{W}$ and a functional symbol $f \in F$, we obtain a function $V(w, f) : D^{\text{arity}(f)} \to \mathcal{D}$.
2. $V : \mathbb{W} \times P \to \bigcup_{n \in \omega} 2^{D^n}$, with $P$ a set of predicate symbols of the language and $2 = \{t, f\}$ is the set of truth values (true and false, respectively), such that for any world $w \in \mathbb{W}$ and a predicate symbol $p \in P$, we obtain a function $V(w, p) : D^{\text{arity}(p)} \to 2$, which defines the extension $[p] = \{a | a \in D^{\text{arity}(p)} \text{ and } V(w, p)(a) = t\}$ of this predicate $p$ in the world $w$.

The extension of a formula $A$. w.r.t. a model $\mathcal{M}$, a world $w \in \mathbb{W}$ and an assignment $g : \text{Var} \to \mathcal{D}$ is denoted by $[A]_{\mathcal{M}, w, g}$ or by $[A/g]_{\mathcal{M}, w}$ where $A/g$ denotes the formula obtained from $A$ by assigning (with $g$) the values to all its free variables. Thus, if $p \in F \cup P$ then for a given world $w \in \mathbb{W}$ and the assignment function for variables $g$, $[p]_{\mathcal{M}, w, g} = V(w, p) : D^{\text{arity}(p)} \to 2$, that is, for any set of terms $t_1, \ldots, t_n$, where $n$ is the arity of $p$, we have $[p(t_1, \ldots, t_n)]_{\mathcal{M}, w, g} = V(w, p)([t_1]_{\mathcal{M}, w, g}, \ldots, [t_n]_{\mathcal{M}, w, g}) \in 2$.

For any formula $A$, $\mathcal{M} \models_{w, g} A$ is equivalent to $[A]_{\mathcal{M}, w, g} = t$, means ’$A$ is true in the
world $w$ of a model $\mathcal{M}$ for assignment $g'$.

The additional semantic rules relative to the modal operators $\Box$ and $\Diamond$ are as follows:

$\mathcal{M} \models_{w, g} \Box A$ iff $\mathcal{M} \models_{w', g} A$ for every $w'$ in $W$ such that $wRw'$.

$\mathcal{M} \models_{w, g} \Diamond A$ iff there exists a $w'$ in $W$ such that $wRw'$ and $\mathcal{M} \models_{w', g} A$.

A formula $A$ is said to be true in a model $\mathcal{M}$ if $\mathcal{M} \models_{w, g} A$ for each $g$ and $w \in W$.

A formula is said to be valid if it is true in each model.

Montague defined the intension of a formula $A$ as follows:

$$[A]_{\text{in}}^{\mathcal{M}, g} = \text{def} \{ w \mapsto [A]^{\mathcal{M}, w, g} \mid w \in W \},$$

i.e., as graph of the function $[A]_{\text{in}}^{\mathcal{M}, g} : W \to \bigcup_{w \in W} [A]^{\mathcal{M}, w, g}$.

One thing that should be immediately clear is that one can determine its extension with respect to a particular world but not vice versa, i.e., $[A]^{\mathcal{M}, w, g} = [A]_{\text{in}}^{\mathcal{M}, g}(w)$.

In particular, if $c$ is a non-logical constant (individual constant or predicate symbol), the definition of the extension of $c$ is $[c]^{\mathcal{M}, w, g} = \text{def} V(w, c)$. Hence, the intensions of the non-logical constants are the following functions: $[c]_{\text{in}}^{\mathcal{M}, g} : W \to \bigcup_{w \in W} V(w, c)$.

The extension of a variable is supplied by the value assignment $g$ only, and thus does not differ from one world to the other; if $x$ is a variable we have $[x]^{\mathcal{M}, g} = g(x)$.

Thus the intension of a variable will be a constant function on worlds which corresponds to its extension. Finally, the connection between Bealer’s non-reductionistic and Montague’s possible world approach to intensional logic can be given by the isomorphism (its meaning is that basically we can use the extensionalization functions in the place of Montague’s possible worlds):

$$F : W \simeq E,$$

where $E$ is a set of possible extensionalization functions which can be considered as possible worlds (up to the previous isomorphism): Each extensionalization function $h \in E$ assigns to the intensional elements of $D$ an appropriate extension as follows:

for each proposition $p \in D_0$, $h(u) = \{ f, t \}$ if $f \in p$ (true value); for each n-ary relation-in-intension $u \in D_n$, $h(u)$ is a subset of $D^n$ (n-th Cartesian product of $D$); in the case of particulars $u \in D_{-1}$, $h(u) = u$. We require that operations $\text{conj}, \text{disj}$ and $\text{neg}$ in this intensional algebra behave in the expected way with respect to each extensionalization function (for example, for all $u \in D_0$, $h(\text{neg}(u)) = t$ if $h(u) = f$, etc.), that is

$$h = h_{-1} + h_0 + \sum_{i \geq 1} h_i : \sum_{i \geq -1} D_i \longrightarrow D_{-1} + 2 + \sum_{i \geq 1} P(D^i)$$

where $h_{-1} = id : D_{-1} \rightarrow D_{-1}$ is identity, $h_0 : D_0 \rightarrow 2$ assigns truth values in $2 = \{ f, t \}$, to all propositions, and $h_i : D_i \rightarrow P(D^i), i \geq 1$, assigns extension to all relations-in-intension, where $P$ is the powerset operator. Thus, intensions can be seen as names of abstract or concrete entities, while extensions correspond to various rules that these entities play in different worlds. Among the possible functions in $E$ there is a distinguished function $\kappa$ which is to be thought as the actual extensionalization function: it tells us the extension of the intensional elements in $D$ in the current actual world. In what follows we will use the join operator $\triangleright$, such that for any two relations $r_1, r_2$ their join is defined by: $r_1 \triangleright r_2 = \{ (a, c, b) \mid (a, c) \in r_1 \text{ and } (c, b) \in r_2 \}$, where $a, c, b$ are tuples (also empty) of constants, so that $r_1 \triangleright \{ \} = \{ \}$ and $r_1 \triangleright \{ <\} = r_1$. 
Definition 4. (SEMANTICS): The operations of the algebra \( Alg_{int} \) must satisfy the following conditions, for any \( h \in \mathcal{E} \), with \( f = \{ \} \), \( t = \{ < > \} \), and \( u_1, \ldots, u_i \in \mathcal{D} \):

1. \( h(\text{conj}(u, v)) = h(u) \otimes h(v) \), for \( u, v \in D_1 \).
2. \( h(\text{neg}(u)) = t \iff h(u) = f \), for \( u \in D_0 \).
3. \( h(\text{pred}(u, u_1)) = t \iff u_1 \in h(u) \), for \( u \in D_1 \).
4. \( h(\text{pred}(u, u_1)) = t \iff u_1 \in h(u) \), for \( u \in D_1, i \geq 1 \).

Notice that this definition for the semantics of the conjunction operation is different from the original work in [26] where

1. \( < u_1, \ldots, u_i > \in h(\text{conj}(u, v)) \iff < u_1, \ldots, u_i > \in h(u) \), for \( u, v \in D_1, i \geq 1 \).
2. \( h(\text{conj}(u, v)) = t \iff h(u) = h(v) = t \), for \( u, v \in D_0 \).

Once one has found a method for specifying the denotations of singular terms of \( \mathcal{L}_\omega \) (take in consideration the particularity of abstracted terms), the Tarski-style definitions of truth and validity for \( \mathcal{L}_\omega \) may be given in the customary way. An intensional interpretation \( I \) [27] maps each i-ary predicate letter of \( \mathcal{L}_\omega \) to i-ary relations-in-intention in \( D_1 \). It can be extended to all formulae in usual way. What is being sought specifically is a method for characterizing the denotations of singular terms of \( \mathcal{L}_\omega \) in such a way that a given singular term \( \langle A \rangle_{x_1, \ldots, x_m} \) will denote an appropriate property, relation, or proposition, depending on the value of \( m \). We denote by \( A_{BS} \) the set of intensional abstracts (terms, so that \( A_{BS} \subset \mathcal{L}_\omega \). Thus, the mapping \( \text{den} : A_{BS} \rightarrow \mathcal{D} \) given in the original version of Bealer [27] will be called denotation, such that the denotation of \( \langle A \rangle \) is equal to the meaning of a proposition \( A \), that is, \( \text{den}(\langle A \rangle) = I(A) \in D_0 \). In the case when \( A \) is an atom \( F^m(x_1, \ldots, x_m) \) then \( \text{den}(F^m(x_1, \ldots, x_m))_{x_1, \ldots, x_m} = I(F^m) \in D_m \). The denotation of a more complex abstract \( \langle A \rangle_o \) is defined in terms of the denotation(s) of the relevant syntactically simpler abstract(s) [27].

For example \( I(A(x) \land B(x)) = \text{conj}(I(A(x)), I(B(x))) \), \( I(\neg p) = \text{neg}(I(p)) \). A sentence \( A \) is true relative to \( I \) and the intensional algebra, iff its actual extention is equal to \( t \), that is, \( Tr(\langle A \rangle) \iff k(I(\langle A \rangle)) = t \), where \( Tr \) is unary predicate for true sentences. For the predicate calculus with individual constants (variables with fixed assignment, proper names, and intensional abstracts) we introduced an additional binary algebraic operation \( \text{pred} \) (singular predication, or membership relation), such that for any two \( u, v \in \mathcal{D} \), for any extensionalization function \( h \) holds \( h(\text{pred}(u, v)) = t \iff v \in h(u) \).

So we are able to assign appropriate intensional value (propositional meaning) to a ground atom \( A(c) \in \mathcal{L}_\omega \) with individual constant \( c \).

That is, \( I(A(c)) = \text{pred}(I(A(x))), I(c) \) is an expression in this intensional algebra with \( I(A(x)) \in D_1 \) and \( I(c) \in D_{-1} \). So that \( h(I(A(c))) = h(\text{pred}(I(A(x))), I(c)) \) is \( t \iff I(c) \in h(I(A(x))) \). That is, in the ‘world’ \( h \), \( A(c) \) is true (that is, the extension of the propositional meaning of \( A(c) \) is equal to \( t \)) iff the interpretation of \( c \) is in the extension of the interpretation of the predicate \( A(x) \). Or, for example, for a given formula with intensional abstract, \( B(\langle A(x, y) \rangle_{x, y}) \in \mathcal{L}_\omega \), we have that \( h(I(B(\langle A(x, y) \rangle_{x, y}))) = h(\text{pred}(I(B(z))), \text{den}(\langle A(x, y) \rangle_{x, y})) = t \iff \text{den}(\langle A(x, y) \rangle_{x, y}) \in h(I(B(z))) \), where \( I(B(z)) \in D_1 \) and \( \text{den}(\langle A(x, y) \rangle_{x, y}) \in D_2 \).
actual world in which intensional elements have the extensions defined by $\mathcal{K}$. Such a correspondence, not present in original intensional theory [24], is a natural identification of intensional logics with modal Kripke based logics.

**Definition 5. (Model):** A model for the intensional FOL is the S5 Kripke structure $\mathcal{M}_{int} = (W, R, D, V)$ where $W = \{F^{-1}(h) \mid h \in \mathcal{E}\}$, $R = W \times W$.

Intensional identity $\approx$ between ground intensional terms $\langle A \rangle /g$ and $\langle B \rangle /g$, where all free variables (not in $\alpha$) are instantiated by $g \in D^{Var}$, is defined as follows:

\[
\langle A \rangle /g = \langle B \rangle /g \quad \text{iff} \quad \Box (A^\alpha /g \equiv B^\alpha /g)
\]

where $A^\alpha /g$ denotes the logic formula where all free variables not in $\alpha$ are instantiated by the assignment $g$. If $\alpha$ are all free variables in $A$ then instead of $\Box A^\alpha /g$ we write simply $\Box A$. The symbol $\Box$ is the universal "necessity" S5 modal operator.

Let $A/g$ denote the ground formula obtained from a formula with free variables $A$ and an assignment $g : \text{Var} \rightarrow D$. Then the satisfaction relation $\models$ for this Kripke semantics is defined by, $\mathcal{M} \models_{w,g} A$ iff $\mathcal{F}(w)(I(A/g)) = t$.

Remark: this semantics is equivalent to the algebraic semantics for $\mathcal{L}_\omega$ in [26] where intensional entities are considered to be identical if and only if they are necessarily equivalent. Intensional identity is much stronger than the standard extensional equality in the actual world, just because requires the extensional equality in all possible worlds, in fact, if $\langle A \rangle /g = \langle B \rangle /g$ then $h(\text{den}(\langle A \rangle /g)) = h(\text{den}(\langle B \rangle /g))$ for all extensionalization functions $h \in \mathcal{E}$ (that is possible worlds $F^{-1}(h) \in W$).

But we can have the extensional equality in the possible world $w = F^{-1}(h)$, while $\text{den}(\langle A \rangle /g) \neq \text{den}(\langle B \rangle /g)$, that is, when $A$ and $B$ are not intensionally equal, so that each intensional identity class of elements is the subset of the extensional equivalence class.

Moreover, for this intensional FOL holds the soundness and completeness: For all formulae $A$ in $\mathcal{L}_\omega$, $A$ is valid if and only if $A$ is a theorem of this First-order S5 modal logic with intensional equality [26].

It is easy to verify that the intensional equality means that in every possible world $w \in \mathcal{W}$ the intensional entities $A$ and $B$ have the same extensions (as in Montague’s approach), moreover:

**Proposition 1 (Bealer-Montague connection):** For any intensional entity $\langle A \rangle /g$ its extension in a possible world $w \in \mathcal{W}$ is equal to $\mathcal{F}(w)(\text{den}(\langle A \rangle /g))) = [A]_{in}^{\mathcal{M},g}(w)$.

**Proof:** Directly from the definition of the identification of a possible world $w$ of Montague’s approach with the extensional function $h = \mathcal{F}(w) \in \mathcal{E}$ in the Bealer’s approach, where $[A]_{in}^{\mathcal{M},g}$ is the "functional" intension of Montague, and $\langle A \rangle$ is an intensional term of Bealer’s logic.

**Definition 6. (Intensional Equivalence $\approx$):** The intensional ground terms $\langle A \rangle /g$ and $\langle B \rangle /g$, where the assignment $g$ is applied only to free variables not in $\alpha$, are intensionally equivalent, $\langle A \rangle /g \equiv \langle B \rangle /g$ iff $\Diamond A^\alpha /g \equiv \Diamond B^\alpha /g$, where $\Diamond = \neg \Box \neg$, and $A^\alpha /g$ denotes the logic formula where all free variables not in $\alpha$ are instantiated by the assignment $g \in D^{Var}$. If $\alpha$ are all free variables in $A$ then
In what concerns this paper we will consider

sion does not depend on possible worlds.

That is, the concept

minated real-world entity because its extension is constant, i.e. fixed for every world, as

entity because its extension depends on a particular world, while the second in denom-

It is easy to verify that the n-ary concepts

\begin{equation}
\begin{array}{c}
\{<g(x_1),...,g(x_n)> | \exists w_1\{\langle w, w_1\rangle \in \mathcal{R} \text{ and } \mathcal{M} \models w_1, g_1 A^n/g \} \\
\end{array}
\end{equation}

This definition of equivalence relation is the flat-accumulation case presented in

[22]: if the first predicate is true in some world then the second must be true in some world also, and vice versa. Each equality is also intensional equivalence, but not vice versa.

Let the logic modal formula \(\Box A^n/g\), where the assignment \(g\) is applied only to free variables of a formula \(A\) not in the list of variables in \(\alpha = < x_1,...,x_n >\), \(n \geq 1\), represents an n-ary intensional concept such that \(I(\Box A^n/g) \in D_n\) and \(I(A^n/g) =\)

\begin{equation}
\begin{array}{c}
\{<g(x_1),...,g(x_n)> | \exists w_1\{\langle w, w_1\rangle \in \mathcal{R} \text{ and } \mathcal{M} \models w_1, g_1 A^n/g \} \\
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
\bigcup_{h_1} I(\Box A^n/g) = \bigcup_{h_1} I(\Box A^n/g).
\end{array}
\end{equation}

It is easy to verify that the n-ary concepts \(A^n/g\) and \(\Box A^n/g\) are intensionally equiv-

cient (i.e., it holds that \(A^n/g \equiv \Box A^n/g\)), the first is denominated contingent-world entity because its extension depends on a particular world, while the second in denominated real-world entity because its extension is constant, i.e. fixed for every world, as explained in [22]. That is, the concept \(\Box A^n/g\) is a built-in (or rigid) concept whose extension does not depend on possible worlds, and can be considered as representative element (with maximal extension) for each class of intensionally equivalent concepts. Analogously, the "necessity" intensional operator \(\necess\) \(\alpha \rightarrow \alpha\) for each \(i \geq 1\) is a new operation of the intensional algebra

\begin{equation}
\begin{array}{c}
\{<g(x_1),...,g(x_n)> | \forall w_1\{\langle w, w_1\rangle \in \mathcal{R} \text{ and } \mathcal{M} \models w_1, g_1 A^n/g \} \\
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
\bigcup_{h_1} I(\necess A^n/g) = \bigcup_{h_1} I(\necess A^n/g).
\end{array}
\end{equation}

Consequently, the concept \(\necess A^n/g\) is a built-in (or rigid) concept as well, whose extension does not depend on possible worlds.

For example, for two intensionally equal ground terms \(\langle A\rangle_{\alpha}/g\) and \(\langle B\rangle_{\alpha}/g\), we have that

\begin{equation}
\begin{array}{c}
\bigcup_{h_1} I(\necess A^n/g) = \bigcup_{h_1} I(\necess A^n/g).
\end{array}
\end{equation}

In what concerns this paper we will consider only the actual world \(w_0 = F^{-1}(k)\).
Moreover, the set of basic intensional equivalences are designed by users, and we will not verify if they satisfy the modal formula used to define the intensional equivalence: the definition above has a theoretical interest, but useful to understand the meaning of the intensional equivalence and the "omniscient" inference relation \( \vdash_{in} \), able to deduce all other intensional equivalences from a given basic set.

**Proposition 2** Let \( C(x) \) be a logic formula defined from built-in predicates (ex, \( \leq, \geq \), etc.), then \( \langle A(x) \rangle_x \approx \langle B(x) \rangle_x \) implies \( \langle A(x) \land C(x) \rangle_x \approx \langle B(x) \land C(x) \rangle_x \).

**Proof:** immediately from the fact that a built-in formula \( C(x) \) has constant extension in any possible world in \( W \). \( \square \)

The quotient intensional FOL \( \mathcal{L}_\omega / \approx \) (its algebraic counterpart is a Lindenbaum-Tarski algebra) is fundamental for query answering in intensional P2P database mapping systems: given a query \( q(x) \) over a peer \( P_i \), the answer to this query is defined as the extension of the denotation of the intensional concept \( \langle q(x) \rangle_x \), in the intensional P2P logic \( \mathcal{L}_\omega / \approx \).

### 3 Embedding of P2P database systems into intensional FOL

The formal semantic framework for P2P database systems, presented also in [6] as a hybrid modal logic, in this paper will be defined as quotient (by intensional equivalence) intensional FOL.

We will consider only the actual world \( w_0 = F^{-1}(\mathcal{X}) \), correspondent to the extensionalization function \( k \) of the quotient intensional FOL \( \mathcal{L}_\omega / \approx \): the actual world for \( \mathcal{L}_\omega / \approx \) corresponds to the actual extension of peer databases. When an user defines a conjunctive query \( q(x) \) over an ontology \( O_i \) of a peer database \( P_i \), the answer to this query is computed in this actual world \( w_0 \), that is in the actual extension of all peer databases in a P2P network \( \mathcal{N} = \{ P_i \mid 1 \leq i \leq N \} \).

**Definition 7.** Let \( \mathcal{N} = \{ P_i \mid 1 \leq i \leq N \} \) be a P2P database system. The intensional FOL \( \mathcal{L}_\omega \) for a query answering in a P2P network \( \mathcal{N} \) is composed by:

1. The set of basic intensional entities is a disjoint union of entities of peers \( \mathcal{S}_I = \bigsqcup_{1 \leq i \leq N} \{ r(y) \mid r(y) \in O_i \} \). The intensional interpretation of the set of all intensional entities define the domains \( D_n, n \geq 1 \):
2. The extensional part of a domain, \( D_{-1} \), corresponds to the disjoint union of domains of peer databases. The intension-in-proposition part, \( D_0 \), is defined by disjoint union of peer’s Herbrand bases.
3. The basic set of the equivalence relation \( \approx \) is defined as disjoint union for each peer \( P_i \) as follows (\( x \) is a tuple of variables of queries):
   - if \( \langle q^{ij}_{1k}(x), q^{ij}_{2k}(x) \rangle \in \mathcal{M}^{ij} \), then \( \langle q^{ij}_{1k}(x) \rangle_x \approx \langle q^{ij}_{2k}(x) \rangle_x \).

The **COMPLETE P2P** answer to a conjunctive query \( q(x) \) over a peer \( P_i \) is equal to the extension of the quotient-intensional concept \( \langle q(x) \rangle_x \), whose equivalence class is determined by the deductive omniscient closure of \( \vdash_{in} \), in the quotient intensional P2P logic \( \mathcal{L}_\omega / \approx \).
Notice that in this embedding of a P2P system, into the intensional FOL $\mathcal{L}_{\omega}$, we do not use any existential quantifier, so that the intensional algebra in Definition 3 is sufficient for a P2P query answering. We need to make complete the model of the intensional logic $\mathcal{L}_{\omega}/\approx$ by defining its extensionalization function $k$ for the actual world $w_0$.

For this aim we will consider as actual world $w_0$, of the intensional logic, the actual extensional FOL multi-modal P2P database system:

What we obtain is a two-level modal framework: the higher, or P2P query answering, level is the Bealer’s intensional logic (without quantifiers) with S5 Montague’s possible-worlds $\mathcal{W}$ modal structure, where $w_0 \in \mathcal{W}$ is actual world for a P2P system; the lower, "computational", level is the extensional FOL multi-modal epistemic logic (with existential quantifiers also) of each peer database. We can see this "computational" level as a sophisticated wrapper (based on the FOL logic of a Data Integration System which is encapsulated into a peer as an Abstract Data Type (ADT) [8]). Each peer is considered as an independent (from other peers) sophisticated wrapper, which extracts the exact extension (of only known facts) for all predicates used in upper intensional P2P query answering logic layer. The extension of each peer database models the extensionalization function for intensional FOL with intensional equivalence, used for intensional embedding of a P2P database system given by Definition 7. The details of the computation of this extensionalization function $k$, for the actual Montague’s world $w_0$, can be found in [33,34].

**Remark:** This is very important observation. What we obtained is relatively simple intensional logic without quantifiers, with only a subset of predicates used in the global schema of peer databases with the set of views (virtual predicates) defined for intensional mapping between peers. The extension of these predicates is wrapped by ADT of each peer independently. The logic specification for these sophisticated wrappers can be obtained by using the epistemic FOL logic [8] of each single peer database.

Each peer database architecture uses the strong (extensional) semantic Global-As-View (GAV) mapping, based on views, inside each peer database, as in standard Data Integration Systems [35,36], with the possibility to use also the logic negation [37]. The weak (intensional) semantic mapping based on views, is used for the mapping between the peers. This architecture takes advantage of both semantical approaches: extensional for building independent peer databases (a development of any particular per database can be done by a compact group of developers, dedicated to developing and to maintaining its functionalities), with intensional, robust and non invasive, mapping between peers, based on beliefs of developers of one peer about the intensionally equivalent knowledge of other peers (which are not under their control).

The actual world $w_0$, with correspondent extensionalization function $k = F(w_0)$, is represented as an extensional FOL multi-modal logic theory for a P2P database system, composed by a number of peers $\{P_i \mid 1 \leq i \leq N\}$, defined as follows:

**Definition 8.** We consider a model $\mathcal{M}$, for the extensional multi-modal logic translation of a P2P database system composed by $N$ peers, a four-tuple $(\mathcal{W}, \{R_i\}, \mathcal{D}, \mathcal{V})$, where:

- The set of points is a disjoint union $\mathcal{W} = \sum_{1 \leq i \leq N} (\mathcal{W}_i \cup \{P_i\})$, with $\mathcal{W}_i = \text{Mod}(P_i)$, where:
1. Each point $P_i$ is considered as a FOL theory with incomplete information, composed by an extensional (ground atoms/facts) and, possibly, an intensional part (logic formulae with variables).

2. For each peer database $P_i$, the set of points $W_i = Mod(P_i), 1 \leq i \leq N$ is the set of all preferred Herbrand models of such peer database. Each $w \in Mod(P_i)$ can be seen as a logical theory also, composed by only ground terms (only extensional part).

- $\mathcal{R}_0$ is a binary accessibility relation between peers, such that $(P_i, P_j) \in \mathcal{R}_0$ if a mapping exists from peer $P_i$ to peer $P_j$. Then we close this relation for its reflexivity and transitivity properties.

- $\mathcal{R}_i = \{P_i\} \times W_i, 1 \leq i \leq N$ is a binary accessibility relation for the $i$-th peer universal modal operator $K_i$, so that, for a given view $q(x)$ over a peer $P_i$, and assignment $g$, $M \models_{P_i,g} K_i q(x)$ iff $\forall w((P_i, w) \in \mathcal{R}_i)$ implies $M \models_{w,g} q(x)$.

- $\mathcal{V}$ is a function which assigns to each pair consisting of an $n$-place predicate constant $r$ and of an element $w \in W$ a function $\mathcal{V}(r, w)$ from $D^n$ to $\{0,1\}$.

So, the extensionalization function $\kappa = \mathcal{F}(w_0)$ for basic intensional entities of the intensional P2P logic $L_{\omega/\omega}$ is defined as follows: for any $\langle r(y) \rangle$, where $r$ is an $n$-ary (virtual) predicate of a peer $P_i$, and $y, c$ are $n$-tuples of variables and constants in $D$ respectively, we define

- for any $n$-ary relation-in-intension $\text{den}((r(y))_y) \in D_n, n \geq 1$, $\kappa(\text{den}((r(y))_y)) = \{g(y) \mid M \models_{P_i,g} K_i r(y), \text{ and assignment } g : \text{Var} \rightarrow D\}$.

- for intensional propositions $\text{den}((r(c)))$ in $D_0$, $\kappa(\text{den}((r(c))) = t$ if $M \models_{P_i,g} K_i r(c)$ ; $f$, otherwise.

In this way the binary relation of each partition (peer database), $\mathcal{R}_i, i \geq 1$, models the local universal epistemic modal operator $K_i$ for each peer database. In fact it holds that $M \models_{P_i,g} K_i q(x)$ iff $\forall w \in Mod(P_i)(M \models_{w,g} q(x))$, i.e., $K_i q(g(x))$ is true iff $q(g(x))$ is true in all preferred models of a peer $P_i$. The binary relation $\mathcal{R}_0$, instead, models the global epistemic P2P modal operator $K$ [3] in this extensional multi-modal logic.

Context-dependent query answering: notice, that the answer to any query depends on the topology of the P2P network, that is, it depends on the peer’s accessibility relation $\mathcal{R}_0$, so that for equivalent queries, but formalized over different peers we will generally obtain different answers. Now we are able to synthesize the definition of intensionally equivalent views used for mappings between peers, in this two-leveled Kripke model framework:

**Definition 9.** (Intensional FOL for P2P systems): A two-level Kripke model for the intensional FOL of a P2P database system $\mathcal{N}$, given in Definition[7] is the S5 Kripke structure $\mathcal{M}_{int} = (\mathcal{W}, \mathcal{R}, D, V)$, where each Montague’s possible world $w_n \in \mathcal{W}$ is the multi-modal translation of a P2P database system in that world, given by Definition[8] that is $w_n = (\mathcal{W}, \{\mathcal{R}_i\}, D, V) \in \mathcal{W}$, so that an intensional equivalence of views, $q_i(x)$ and $q_j(x)$, defined as conjunctive queries over peers $P_i$ and $P_j$ respectively, is formally given by the following modal formulae of the intensional FOL:

$$\Diamond q_i(x) \equiv \Diamond q_j(x) \quad \text{i.e.,} \quad (\Diamond q_i(x) \Rightarrow \Diamond q_j(x)) \land (\Diamond q_j(x) \Rightarrow \Diamond q_i(x))$$
This definition tells us, intuitively, that any possible world (for a given time-instance) of the intensional logic for P2P database system $\mathcal{N}$, represents (that is models) a particular state of this P2P database, that is, the structure and the extensions of all peer databases in such a time instance. The set of possible worlds $w_n \in \mathcal{W}$ corresponds to the whole evolution in time of the given P2P system. Such an evolution is result of all possible modifications of an initially defined P2P database system: a simple modification of extensions of peer databases, an inserting of a new peer, or a deleting of an existing peer in this network $\mathcal{N}$.

4 Weak intensional inference relation

In real Web applications we will never have the omniscient query agents that will contemporarily have the complete knowledge about all ontologies of all peers. Such a supposition would generate the system with a global and centralized knowledge, in contrast with our pragmatic and completely decentralized P2P systems with completely independent peers, which can change their local ontology in any instance of time without informing any other peer or ”global” system about it. Thus, what we will consider that a query agent reasoning system has a weaker form of deduction than $\vdash_{\text{omniscient}}$ (of this ideal omniscient intensional logic inference), more adequate for limited and local knowledge of query agents about the peers. What we consider is that a query agent will begin its work for a given user query $q(x)$ over a peer $P_i$, and, by using only the local knowledge about this peer’s ontology and the set of its local intensional mappings towards other peers, it will be able to move to the locally-next peers to obtain answers from them also. This context-sensitive query answering is analog to the human query answering: interviewer will ask the indicated person and will obtain his known answer, but this person can tell also which other people, he believes, will be able to respond to this question. It will be the task of the interviewer to find other people and to reformulate the question to them. It is, practically impossible to have all people who know something about this question to be in common interaction one with all other to combine the partial knowledge of each of them in order to provide possibly complete answer to such a question. This, context dependent and locally-based query answering system, for practical query agents in P2P systems, is partially described in the Example 1. In what follows we will define the weaker deductive inference relation also $\triangleright$, denoted by $\triangleright$, for the intensional FOL $\mathcal{L}_{\text{int}}$, such that the query answering algorithm used by these non-omniscient query agents, is complete w.r.t. this intensional logic deductive system.

Example 1: Let us consider the cyclic P2P system in a Fig.2, with a sound but generally incomplete deduction [23], which can be easily implemented by non-omniscient query agents: we have $P_i$, with the ontology $\mathcal{O}_i$ and the interface $\mathcal{M}^j_i = \{ (v_{im}, v_{jm}) \mid 1 \leq m \leq k_1 \}$ toward the peer $P_j$, and the peer $P_j$, with the ontology $\mathcal{O}_j$ and the interface $\mathcal{M}^j_j = \{ (w_{jm}, w_{im}) \mid 1 \leq m \leq n_1 \}$ toward the peer $P_i$. First we traduce a pair $(v_{im}, v_{jm})$ by the intensional equivalence $(v_{im}) \approx (v_{jm})$. In what follows, the subscript of a query identifies the peer relative to such a query. Let $q_i(x)$ be the original user’s conjunctive query over the ontology $\mathcal{O}_i$ of the peer database $P_i$. If this query can be rewritten [39], by the query rewriting algorithm $\text{id}_1$, in the equal query over the set of views $\{v_{i1}, \ldots, v_{ik}\} \subseteq \pi_1 \mathcal{M}^j_i$, where $\pi_1$ is the first pro-
queries is a subset of the extension of this quotient intensional entity

Thus we obtain the three intensionally equivalent queries

\( \exists x : q_1(x) \equiv \exists x : (\exists y : t(x, y)) \).

In the next step the conjunctive query formula

\( \forall j \subseteq \{1, \ldots, v\} \). In the same way (see the inverse bottom horizontal arrows of a diagram in Fig.1), based on the interface specification of the peer \( P_j \), \( M^{\Omega_j} = \{(w_j, w_{im}) | 1 \leq m \leq n_j\} \), we obtain that also

\( \exists x : q_j(x) \equiv \exists x : (\exists y : t(x, y)) \).

Thus we obtain the three intensionally equivalent queries \( q_i(x), q_j(x) \) and \( q_k(x) \), where two of them, \( q_i(x), q_k(x) \) are over the same peer \( P_i \): the first one is the original user query, while the second is the intensionally equivalent derived query (based on P2P interface intensional specification).

These three query formulae, \( \{q_i(x), q_j(x), q_k(x)\} \), are the subset of the equivalent class \( C \) for the given user query, which in the intensional FOL \( L_{\omega/\approx} \) is represented by the quotient intensional entity \( Q(x) \), whose extension (from Definition 3 in the actual world \( w_0 \) is defined by

\( \bigcup_{1 \leq i \leq m} \mathcal{F}(w_0)(\text{den}((Q(x))_x)) \).

\( = \{ t \in \mathcal{D}^k \mid Q(t, Q(x)) \in C \} = \{ (t, Q(x)) \mid Q(x) \in C \} \)
The semantics for this weaker form of deduction of intensional equivalences, i.e., of the intensional equivalent queries over other "contextual" peers, w.r.t. the user interrogated peer \( P_i \), can be formally expressed by deduction chains which begin from a peer \( P_i \).

**Proposition 3** Given an intensional logic \( \mathcal{L}_\omega \) for a P2P system (Definition 7), with a basic, user defined, set of intensional equivalences \( S_{eq} \), and its deductive inference relation \( \vdash_{in} \), then we define the weak intensional inference relation \( \models_{w} \), as follows

\[
\mathcal{L}_\omega \models_{w} \langle A \rangle_\alpha \equiv \langle B \rangle_\alpha \iff \text{there is a chain } A_1, A_2, A_3, ..., A_{3n+1} \text{ of the formulae with the same set of free variables but each of them expressed by relation symbols of only one particular peer's ontology, such that } A_1 = A, A_{3n+1} = B, \text{ and } \langle A_i \rangle_\alpha \equiv_{i+1} \langle A_{i+1} \rangle_\alpha, \text{ for } \equiv_{3i} \text{ equal to } \equiv; \text{ to } = \text{ otherwise, } A_{3i-2} \text{ is a query over a peer's ontology, and } A_{3i-3}, A_{3i-1} \text{ over peer's views, while } A_{3i} \text{ is a query over views contained in the interface of this peer but are views of some other peer, } 1 \leq i \leq n. 
\]

These chains for the intensional logic \( \mathcal{L}_\omega \) of a P2P database system are finite, and holds that \( \mathcal{L}_\omega \models_{w} \langle A \rangle_\alpha \equiv \langle B \rangle_\alpha \) implies \( \mathcal{L}_\omega \vdash_{in} \langle A \rangle_\alpha \equiv \langle B \rangle_\alpha \), but not vice versa.

**Proof:** This proposition tells us that two formulae, over any two peer’s ontologies, with the same free variables, are intensionally equivalent, if there is a chain of the formulae, identical or intensionally equivalent, and these two formulae are the initial and final formulae of such a chain. In fact, any two identical formulae can be reduced to only one, by eliminating other (substitution property for identity), that is, we are able to reduce such a chain to the sub chain with only intensional equivalent formulae, and based on the transitive property of the equivalence relation, we obtain that initial and final formula in this chain are intensionally equivalent.

The finite chain property for P2P systems is the result of the fact that, also in presence of cyclic mappings between peers, the number of *different* conjunctive queries but intensionally equivalent, which can be expressed by the finite set \( S \) of views of a peer \( P_i \), used as mapping toward the same peer \( P_j \) \((i \neq j)\), is always finite. The number of subsets of this set \( S \) of view, sufficient to formalize the intensionally equivalent conjunctive formula, is finite: thus, the passage from \( P_i \) to \( P_j \) during the derivation of new intensional equivalences, can be used only a finite number of times.

Moreover, this proposition explains the way in which weak deduction of the Intensional logic is able to derive the intensionally equivalent formulae from the basic set (explicitly defined by a peer’s developers) of intensionally equivalent formulae: in our case it is the set of intensionally equivalent views (conjunctive queries) over different peers.

**Proposition 4** Let \( \mathcal{L}_\omega \models \langle q_i(x) \rangle_x \approx \langle q'_i(x) \rangle_x \) be a weak deduction of the intensional equivalence, where the bottom index \( i \) of the conjunctive queries denotes the peer \( P_i \) relative to these queries, with its views \( v_1, ..., v_k \), and, from the Prop. 3 \( A_1, A_2, ..., A_n \) be a finite chain of the formulae with the same set of free variables, such that \( A_1 \equiv q_i(x), A_2 \equiv \Psi(v_1, ..., v_k), \langle A_1 \rangle_x = \langle A_2 \rangle_x \) and \( A_{n-1} \equiv q'_i(x), A_n \equiv \Phi(v_1, ..., v_k), \langle A_{n-1} \rangle_x = \langle A_n \rangle_x \), where conjunctive queries \( \Psi \) and \( \Phi \) have also the same free variables (view attributes).

Then \( k(den(\langle \Psi(v_1, ..., v_k) \rangle_x)) \subseteq k(den(\langle \Phi(v_1, ..., v_k) \rangle_x)) \).
Proof: It comes directly for all user conjunctive-queries without built-in predicates: two conjunctive queries with the same set of predicates and the same set of variables in the query head are identical. In the case when user query contains also a derived built-in predicate \( C(x) \), we can take out this formula from the rest of query, and consider the intensional equivalence only for such reduct without built-in predicates, based on the Proposition \( \square \) at the end of derivation of the intensional equivalence class w.r.t. this reduct query, we can add to each conjunctive formula of this equivalence class the formula ” \( \land C(x) \)”. The chain of derivations can only add some new conjunction of built-in predicates, thus we will obtain that \( \Psi(x) \equiv \Phi(x) \land C_1(x) \), where \( C_1(x) \) can be also empty.

\( \square \)

This proposition tells us that any two intensionally equivalent conjunctive queries, with the same set of virtual predicates (views \( v_1, ..., v_k \) of a peer \( P_i \)) and the same set of variables in the head of these two queries, the second derived query is extensionally contained in the first, so that we can stop the propagation of deductions and to discard \( \Psi(x) \). As a consequence of Propositions \( \square \) and \( \square \) given a query \( q(x) \) over a peer \( P_i \), the set of different conjunctive queries (such that one is not subsumed in other), but intensionally equivalent to \( q(x) \), over any peer \( P_k \) is a finite set: that means that in principle we are able to define a complete query rewriting algorithm for finite P2P database systems w.r.t. the weak deduction \( \models \) of the intensional FOL \( L_\omega \). More about this non omniscient inference can be found in \( [23] \) also.

5 Sound and complete query answering

The implementation of query answering in P2P systems needs a standard mathematical semantics based on an adequate (co)algebra: as for example, is the relational (co)algebra for SQL query answering in Relational Databases. Here, the computation is more intricate because of the complex epistemic logic structures of peers and the necessity of query rewriting algorithms \( Rew \). We consider that the rule of query agent is to start and to maintain complete query answering transaction: this transaction starts when is defined an user query \( q(x) \) over a peer \( P_i \). A query agent supports the \( Rew \) algorithm in order to construct intensionally equivalent rewritten queries over other peers and then calls grid computation network to calculate answers, by assigning to each grid computation node one peer with a union of rewritten conjunctive queries for it. The transaction ends when query agent receives the answers from all grid nodes, and presents collected answers to the user. The definition of this P2P query computing system, which abstracts all not necessary implementation details of a peer, has to be given in an abstract (co)algebraic mathematical language; so, this abstract mathematical specification (co-Algebraic Abstract Type) can be successively implemented in any current grid computing system.

But the query answering for intensionally based P2P mappings can not be embedded into recursive Datalog, as in the case of a standard view-based mappings based on a material implication \( [7] \), so we need more complex and general mathematical framework for it.
5.1 Final semantics for the weak deduction \( \vdash \)

The Kripke structure of the frame \( F = (\mathcal{W}, \{R_i\}) \), given in the Definition 8, is a prerequisite in order to obtain a coalgebraic semantics for query answering in P2P database framework. (Co)Algebras provide an unifying view on a large variety of dynamic systems such as transition systems, automata, data structures, and objects \([40,41]\) or Kripke models; they are especially useful for the dynamic query answering P2P systems. In order to render this paper more selfcontained we will introduce the following formal definition for coalgebras:

**Definition 10.** (Abstract Coalgebras) Let \( \text{Set} \) be a category with its objects all (small) sets and its arrows all functions, and \( T \) be an endofunctor (mapping) \( T : \text{Set} \to \text{Set} \). A \( T \)-coalgebra is a pair of a (small) set \( C \) and a \( \text{Set} \)-arrow \( \alpha : C \to TC \), that is, \( (C, \alpha : C \to TC) \). \( T \) is called a signature functor or type, and \( C \) a carrier set.

Let \( (C, \alpha) \) and \( (D, \beta) \) be \( T \)-coalgebras, and \( f : C \to D \) a \( \text{Set} \)-arrow. \( f \) is said to be a morphism of \( T \)-coalgebras or \( T \)-morphism, if \( \beta \circ f = Tf \circ \alpha \), where \( \circ \) is a composition of arrows. It is an isomorphism if it is a bijective mapping.

**Example 2:** Aczel’s semantics of CCS \([42]\), is described by the coalgebra \( k : \text{Prog} \to \mathcal{P}_{\text{fin}}(\text{Act} \times \text{Prog}) \), of the endofunctor \( T = \mathcal{P}_{\text{fin}}(\text{Act} \times _\vdash) \) with the set of actions \( a \in \text{Act} \), such that \( k(P) = \{ < a, P' > \mid P \rightarrow a, P' \} \) is the set of atomic transitions which the CCS program \( P \) can perform and pass to the new program \( P' \). The symbol \( \mathcal{P}_{\text{fin}} \) is the finite powerset operator. This semantics exploits the special final coalgebra theorem, that is a unique homomorphism \( k^\oplus : (\text{Prog}, k) \to (\text{gfp}(T), \simeq) \) to the final coalgebra, which is a isomorphic (bijective) coalgebra: \( \simeq : \text{gfp}(T) \to \mathcal{P}_{\text{fin}}(\text{Act} \times \text{Prog}) \) with \( \text{gfp}(T) \) the set of (infinite) labeled transition systems (labeled trees) which are greatest fixed points of the ‘behavioral functor’ \( T = \mathcal{P}_{\text{fin}}(\text{Act} \times _\vdash) \), that is for every program \( P \), \( k^\oplus(P) = \{ < a, k^\oplus(P') > \mid P \rightarrow a, P' \} \), such that the following diagram commutes.

\[
\begin{array}{ccc}
\text{Prog} & \xrightarrow{k^\oplus} & \text{gfp}(T) \\
\downarrow{k} & & \downarrow{\simeq} \\
T(\text{Prog}) & \xrightarrow{T(k^\oplus)} & T(\text{gfp}(T))
\end{array}
\]

Final coalgebras are ‘strongly extensional’, that is, two elements of the final coalgebra are equal iff they are \( T \)-bisimilar. □

The semantics above for CCS and its properties can be generalized to arbitrary behaviors: in our case, for a given conjunctive query language \( \mathcal{L}_Q \) over relational symbols of P2P database system, we consider the programs as pairs \( (P_i, q_i(x)) \) (a query over a peer \( P_i \) can be considered as a program: the execution of this program will return the known answers to this query) so that \( \text{Prog} = \mathcal{W}_0 \times \mathcal{L}_Q \), the set \( \text{Act} = \{ \| \leq \} \) as a singleton, with the only deductive action \( \| \leq \) (restriction of \( \| \) for chains of length 3 only), so that \( \mathcal{P}_{\text{fin}}(\text{Act} \times \text{Prog}) \) can be substituted by \( \mathcal{P}_{\text{fin}}(\text{Act} \times \text{Prog}) \), that is, in our case \( T = \mathcal{P}_{\text{fin}} \).
Thus, we can consider the *intensional deduction* process, defined in Proposition 3, as a coalgebra $k : W_0 \times \mathbb{L}_Q \rightarrow \mathcal{P}_{fin}(W_0 \times \mathbb{L}_Q)$, such that, given an initial query $q_i(x) = q(x) \in \mathbb{L}_Q$ over a peer $P_i$, that is a pair $(P_i, q_i(x)) \in W_0 \times \mathbb{L}_Q$, which corresponds to the intensional entity $\langle q(x) \rangle_x$, the inferential step will generate the complete set $k(P_i, q_i(x)) = \{ (P_j, q_j(x)) \mid (P_i, P_j) \in \mathcal{R}_0 \} \in \mathcal{P}_{fin}(W_0 \times \mathbb{L}_Q)$, where $\langle q(x) \rangle_x \approx \langle q_j(x) \rangle_x$ if it can be derived as an intensional-equivalent query $q_j(x)$ over a peer $P_j$ (that is if $\mathbb{L}_\omega \models_3 \langle q(x) \rangle_x \approx \langle q_j(x) \rangle_x$); $q_j(x) = \emptyset$, otherwise.

We can use $\mathcal{P}_{fin}$ because P2P system is composed by a finite number $N$ of peers, so that for any peer $P_i$ the number of accessible peers for it is finite.

By applying recursively the function $k$, equivalent to the single application of the unique homomorphism $k^0$, we obtain a possibly infinite transition relation (tree), see Fig. 2, with the root in the initial state (that is, a program $(P_i, q_i(x))$). We consider the general case of cyclic mappings of the P2P system as in Fig. 1 of the example 1.

This tree will have a lot of nodes with empty queries, and possibly infinite copies of nodes (with the same query over a given peer). So, we need to 'normalize' this unique solution of the weak deduction, by eliminating duplicates and nodes with empty query. Consequently we define the mapping $fl : gfp(\mathcal{P}_{fin}) \rightarrow \mathcal{P}_{fin}(W_0 \times \mathbb{P}(\mathbb{L}_Q))$, such that, given a unique solution (tree) $k^0(P_i, q_i^1(x))$,

$$fl(k^0(P_i, q_i^1(x))) = \bigcup_{1 \leq k \leq N} \{ (P_k, q_k^m(x)) \mid (P_k, q_k^m(x)) \in k^0(P_i, q_i^1(x)), m \in \mathbb{I} \},$$

where $\mathbb{I}$ is the set of integers. In this way we will obtain, for each peer, the set of all conjunctive queries, intensionally equivalent to the user query $q_i^1(x) = q(x)$. This set of queries for a given peer is complete w.r.t. the intensional FOL and its weak deductive inference for intensionally equivalent formulae. It is the largest equivalence relation class, i.e., the closure $C_\omega(\mathbb{L}_\omega, q_i^1(x))$ of the inference $\models^\omega$, defined by $q'(x) \in C_\omega(\mathbb{L}_\omega, q_i^1(x))$ iff $\mathbb{L}_\omega \models \langle q_i^1(x) \rangle_x \approx \langle q'(x) \rangle_x$.

### 5.2 Sound and complete w.r.t $\models^\omega$ query answering algorithm

In order to have a decidable complete query answering, we need to prove that the set of all rewritten queries for each peer is finite. After that we need to define this query
The polynomial functor 
the final coalgebra semantics \[40\], with
this list. If
in this list. Thus, every query agent object (instance of this object-oriented class) can be
while with

With

other peers, and passes (deterministically) to the next state in

Formally, for any state \( f : \mathcal{W}_0 \rightarrow \mathcal{L}_S \times \mathbb{N} \), the stop

The polynomial functor \( T(X) = B + X \), where + is the operation of the set union, has
the final coalgebra semantics \[40\], with \( gfp(B + _) = \mathcal{P}_{fin}(\mathcal{W}_0 \times \mathcal{P}(\mathcal{L}_Q)) \), the infinite extension of
\( B \) when for each peer in \( \mathcal{W}_0 \) we can have also infinite number of queries, so that \( B + gfp(B + _) = gfp(B + _) \) and holds the bijection \( \simeq \) for the final coalgebra
of the functor \( T = B + _{\cdot} \). We denote the unique solution, of the deterministic system
\( \text{st} : X \rightarrow B + X \), by the unique homomorphism between this coalgebra and the final
coalgebra, that is, \( \text{st}^{\ell} : (X, \text{st}) \rightarrow (gfp(B + _{\cdot}), \simeq) \); this homomorphism corresponds
to the top commutative diagram in the diagram below.

The coalgebra mapping, \( \text{st} \), specifies the method of this query agent class for the query
rewriting algorithm, as follows: given an initial state \( f_0 \in X \), the \( \text{st} \) terminates with
a result of query rewriting algorithm if all queries of every peer are elaborated, and are
not generated new queries; otherwise \( \text{st} \), when elaborates a first non-elaborated query
\( q_k^n(x) \) of a peer \( P_k \), can generate the set of new intensionally equivalent queries over
other peers, and passes (deterministically) to the next state in \( X = (\mathcal{L}_S \times \mathbb{N})^{\mathcal{W}_0} \).

Formally, for any state \( f : \mathcal{W}_0 \rightarrow \mathcal{L}_S \times \mathbb{N} \),

\[
\text{st}(f) =
\begin{cases}
\{(P_k, \{q_k^n \mid q_k^n \in \pi_1 f(P_k), 1 \leq n \leq \pi_2 f(P_k)) \mid P_k \in \mathcal{W}_0\} & \text{if } 0 = \sum_{P_i \in \mathcal{W}_0} ln(\pi_1 f(P_i)) - \pi_2 f(P_i), \\
\text{next}(f) = f_1 : \mathcal{W}_0 \rightarrow \mathcal{L}_S \times \mathbb{N} & \text{otherwise,}
\end{cases}
\]

such that \( f_1(P_k) = (S, n + 1) \), with \((S, n) = f(P_k) : \) increments pointer.

For any \( P_j \) locally connected with \( P_k \), and \((S, m) = f(P_j)\),
\( f_1(P_j) = (\text{push}(S, q_j^{l+1}), m) \), for \( l = ln(S) \)
The peer-to-peer query step-rewriting \( \text{Rew} = \text{Unfolding} \circ \text{Subst} \circ \text{MiniCon} \), where \( \circ \) is the sequential composition for algorithms, can be described as follows (see also Example 1):

Given a conjunctive query \( q(x) \) over a peer \( P_i \), by using the set of intensional equivalences in its interface \( \mathcal{M}^i = \{(v_{im}, v_{jm}) \mid 1 \leq m \leq k_i \} \) toward a peer \( P_j \) (see Def. 1), in the case when such set is not enough for the complete and equivalent rewriting [43], returns with the empty query \( q_j(x) = \emptyset \) for the peer \( P_j \). Otherwise it uses the MiniCon Algorithm [43] over the set of views in \( \{v_{i1}, ..., v_{ik}\} \subseteq \pi_1 \mathcal{M}^j \), to rewrite equivalently a query \( q(x) \) into a query \( \Phi(v_{i1}, ..., v_{ik}) \). After that it makes the substitution of views of \( P_i \) in \( \{v_{i1}, ..., v_{ik}\} \) by intensionally equivalent set of views of \( P_j \) in \( \{v_{i1}, ..., v_{ik}\} \), to obtain an intensionally equivalent query formula \( \Phi(v_{i1}, ..., v_{ik}) \) over views of \( P_j \). Finally, it uses the Unfolding Algorithm [43], to unfold \( \Phi(v_{i1}, ..., v_{ik}) \) and to obtain the query \( q_j(x) \) over the ontology of a peer \( P_j \). Notice that in the case when the ontology of \( P_j \) is changed, so that the set of views \( v_{i1}, ..., v_{ik} \) in the interface \( \mathcal{M}^j \) of the peer \( P_i \) does not match with this new ontology of \( P_j \), the algorithm returns with the empty query, that is, with \( q_j(x) = \emptyset \).

**Proposition 5** The mapping \( \text{next} : X \rightarrow X \) is monotonic w.r.t. the ordering \( \preceq \) such that for any \( f_1, f_2 \in X = (\mathcal{L}_{SQ} \times \mathbb{N})_{W_0} \),
\[
 f_1 \preceq f_2 \iff \forall P_i \in W_0 (\pi_1(f_1(P_i)) \subseteq \pi_1(f_2(P_i))).
\]
For the least fixpoint for this “next-consequence-operator” of \( f_0 = \text{new}(\ast) \), \( \text{lst}(f_0) \), holds that \( \text{st}^{\text{fin}}(f_0) = \text{st}((\text{lst}(f_0)) \).

Notice that this algorithm works well also for union of conjunctive queries: it works well for \( \text{Unfolding} \) and \( \text{MiniCon} [43] \), while for \( \text{Subst} \) works from the fact that for a normal modal logic holds \( \Diamond (A \lor B) \equiv \Diamond A \lor \Diamond B \). The \( \text{Subst} \) works for conjunctive queries and the class of peers defined as follows:

**Proposition 6** [22] Let us consider the class of peers with the integrity constraints which does not contain negative clauses of the form \( \neg A_1 \lor ... \lor \neg A_m \), \( m \geq 2 \). Then, the intensional equivalence is preserved by conjunction logic operation, that is, if \( \varphi \equiv (b_1 \land ... \land b_k), k \geq 1, \) is a conjunctive query over a peer \( P_i \), and \( b_i \equiv c_i, 1 \leq i \leq k, \) the set of intensionally equivalent views toward a peer \( P_j \), then \( \varphi \equiv \psi \), where \( \equiv \) is a logic equivalence and \( \psi \equiv (c_1 \land ... \land c_k) \) is the conjunctive query over a peer \( P_j \).

We are able to define the mapping \( \text{pop} : X \rightarrow W_0 \times \mathcal{L}_Q \) between domains of the query agent class coalgebra and deductive coalgebra, such that, for any \( f \in X \), that is, \( f : W_0 \rightarrow \mathcal{L}_{SQ} \times \mathbb{N}, \) \( \text{pop}(f) = \{(P_i, q_i) \mid P_i \in W_0, \text{ and } q_i = \pi_n(\pi_1(f(P_i))) \text{ if } n = 1 + \pi_2(f(P_i)) \leq \ln(\pi_1(f(P_i))): \emptyset \text{ otherwise} \}. \)

This mapping associate to any peer \( P_i \) the next query in its list to be elaborated.
Theorem 1 The query answering algorithm implemented by the query class \( st : X \to B + X \) will terminate for any user conjunctive query \( q(x) \) over a peer \( P_i \). It is sound and complete algorithm w.r.t. the weak deduction inference \( \models \) of the intensional logic \( \mathbb{L}_\omega \) for a P2P database system. That is, for any user query action \( \text{new} \), holds that 
\[
st^{\text{@}} \circ \text{new} = f_l \circ k^{\text{@}} \circ \text{pop} \circ \text{new}
\] or, equivalently, any user action \( \text{new} \), which defines a conjunctive query \( q(x) \) over a peer \( P_i \), is EQUALIZER of the functions \( st^{\text{@}} \) and \( f_l \circ k^{\text{@}} \circ \text{pop} \). Graphically

\[
\begin{array}{c}
I \xrightarrow{\text{new}} (L_{\mathbb{Q}} \times \mathbb{N})^W_0 \xrightarrow{st^{\text{@}}} gfP(B + \_)
\end{array}
\]

This theorem can be represented by the following commutative diagram: the top commutative square corresponds to the query agent with the (unique) solution for its query rewriting algorithm, while the bottom commutative square corresponds to the unique solution of the weak deduction inference. The dashed diagram in the middle corresponds to the equalizer of this theorem, and, intuitively, shows that each unique solution of query answering algorithm is equal to the unique solution set obtained by the weak deductive inference \( \models \) of the intensional logic \( \mathbb{L}_\omega \) for a P2P database system.

\[
\begin{array}{c}
B + (L_{\mathbb{Q}} \times \mathbb{N})^W_0 \xrightarrow{1_B + st^{\text{@}}} B + gfP(B + \_)
\end{array}
\]

Proof: The query rewriting algorithm is sound, because it derives intensionally equivalent queries. From the fact that it derives the subset of the intensionally equivalent queries over peers, w.r.t. the weak deductive inference \( \models \) which for a given finite P2P system derives only a finite number of equivalent queries (from Propositions 3, 4), we conclude that it must terminate. Let us sketch now the completeness proof for a given user action \( \text{new} \), which specifies a query \( q_i \) over a peer \( P_i \), such that \( f_0 = \text{new}(\_0) \), and \( (P_i, q_i) = \text{pop}(f_0) \). We can focus only on non empty queries,

1. Let \( (P_k, q_k), q_k \neq \emptyset \) be a node in the infinite tree \( k^\text{@}(P_i, q_i) \in gfP(P_{\text{fin}}) \). So, there is a chain (path) from the root of this tree \( (P_i, q_i) \) to this node: the set of mutually different nodes with non-empty queries in this path must be finite number \( n \). Thus,
there is a maximal number $m \leq n$ of consecutive executions of the query rewriting method $st$, denoted by $st^m$, so that $q_k \in \pi_1(f(P_k))$ for $f = st^m(f_0)$, so, there exists $(P_k, S) \in st^\theta(f_0)$ (a unique solution for the user query in $f_0$, with $f_0(P_l) = q_l$) such that $q_k \in S$.

2. Vice versa, let $(P_k, S) \in st^\theta(f_0)$ (a unique solution for the user query in $f_0$, with $f_0(P_l) = \{q_l\}$) such that $q_k \in S$. Let prove that $(P_k, q_k)$ must be a node in the tree $k@\pi(P_l, q_l) \in gfp(P_{fin})$:

From the fact that $(P_k, S) \in st^\theta(f_0)$ we conclude that there exists a finite number $n$ such that $f_n = st^n(f_0)$ and $q_k = \pi_m(\pi_1(f(P_k)))$, with $m = ln(\pi_1(f(P_k)))$. Thus there exists the following sequence-ordered subset of all recursive executions of the query algorithm method $st$ which begins from $f_0$ and ends with $f_n$, inductively defined in the backward direction: the step, immediately precedent to the step $n$ in this subset, is a step $m_1 \leq n - 1$ in which the method $st$ invokes the action $push(S, q_k)$ for a locally-connected peer $P_j$, i.e., $f_{m_1}(P_j) = (push(S, q_k), m')$, which inserts the query $q_k$ in the list $S$ of the peer $P_k$. Thus, also for some step $m_2 \leq m_1 - 1$ the method $st$ invokes the action $push(S', q_j)$ for a locally-connected peer $P_l$ to $P_j$, i.e., $f_{m_2}(P_l) = (push(S', q_j), m''')$, which inserts the query $q_j$ in the list $S'$ of the peer $P_j$, etc.. In this kind of a backward recursion we will reach the beginning element $f_0$ for the initial query couple $(P_l, q_l)$.

The chain of nodes $C = < (P_l, q_l), ..., (P_j, q_j), (P_k, q_k) >$ is a chain of intensionally equivalent queries, thus, must be a weak deduction inference chain, and, consequently, a part of the unique derivation tree $k@\pi(P_l, q_l)$ (with the root in the node $(P_l, q_l)$). Consequently, $(P_k, q_k)$ is a node in this tree.

6 Conclusion

As this paper has shown, the problem of defining the semantics for intensional ontology mappings between peer databases, can be expressed in an intensional FOL language. The extensionalization function, for the actual P2P world, can be computationally modeled by an epistemic logic of each peer database: P2P answers to conjunctive queries are based on the known answers of peers to intensionally equivalent queries over them. This intensional FOL for P2P system is obtained by the particular fusion of the Bealer’s intensional algebra and Montague’s possible-worlds modal logic for the semantics of the natural language. In this paper we enriched such a logic framework by a kind of intensional equivalence, which can be used to define an intensional view-based mapping between peer’s local ontologies. We conclude that the intensional FOL logic is a good candidate language for specification of such P2P database systems.

We have shown how such intensional mapping can be used during a query answering process, but we do not use the omniscient inference of this modal S5 Montague’s intensional logic. Such inference would use all possible worlds, thus would be very hard to obtain. Rather than it, we defined a weak non-omniscient inference, presented in [23] also, which can be computed in the actual Montague’s world only, and based on query-rewriting algorithms: Minicon and unfolding algorithms for conjunctive queries. Finally we show how this query-rewriting algorithm can be conveniently implemented by non omniscient query agents, and we have shown that for any P2P system it has a
unique final semantics solution. It is the responsibility of a query agent to rewrite the
original user query over an initial peer to all other intensionally equivalent queries over
other peers in a P2P network. We defined the object-oriented class for such query agents
and we have shown that its query rewriting method (algorithm) is sound and complete
w.r.t. the weak deduction of the intensional FOL.

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