Application of parametric discrete Fourier transform in nondestructive testing of composite materials with a free oscillation method

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Abstract. The paper discusses the use of discrete parametric Fourier transform (DFT-P) in nondestructive testing (NDT) of composite materials using the free oscillation method. It is shown that the methods and algorithms of spectral and vector processing of information signals based on the DFT retain their leading role in NDT despite a number of negative effects inherent to them (aliasing, stockade, leakage and scalloping effects). Based on the systems analysis of the advantages and disadvantages of digital processing on the basis of the DFT of information signals in nondestructive testing, its generalization in the form of a parametric DFT (DFT-P) is proposed. The analytical properties of the basic system of the proposed discrete transform are investigated.

1. Introduction
Composite materials (CM) - multicomponent materials consisting of a base (matrix), reinforced with thin fibers of high-strength materials, are widely used in various industries, including astronautics, aviation, rocket, automotive, shipbuilding, nuclear energy and medicine. CM for operational and technological properties (weight, heat resistance, fatigue strength, vibration resistance, impact strength, noise absorption) most fully respond to the ever-increasing requirements for structural materials.

The wide using of CM in the creation of modern technology sets the task of developing effective and efficient methods and algorithms for objective quality control, both at the stage of their production and at the stage of operation of products made out of them. It should be noted that CM are, from the point of view of control, complex objects, due to the specificity of the properties of KM, which requires special attention when choosing methods of nondestructive testing (NDT) of the quality of CM. Nondestructive testing of CM is often carried out by the method of free oscillations (MFO). This acoustic method has a number of advantages over other low frequency acoustic methods. This is, first of all, the detection of defects at great depths, the control of materials with high attenuation coefficients of elastic vibrations and materials with small elasticity modulus.

MFO is based on the sequential execution of the following stages:
- shock excitation of impulses of freely damped elastic oscillations in a testing product (or its part);
- translation of primary information (oscillations) into electrical signals with the help of converters;
- analysis of changes in the frequency spectrum of signals in defective and defect-free zones;
- making decisions about the state of the controlled object.
In modern flaw detectors based on MFO, the determination of the frequency spectrum is carried out on the basis of the discrete Fourier transform (DFT) using the fast Fourier transform (FFT) method. The main information parameter of flaw detectors is the difference between the current and average spectrum for the defect-free zone (taking into account the phase of the DFT coefficients - vector analysis, or without taking into account the phase of the DFT coefficients - spectral analysis).

2. Digital processing of information signals in nondestructive testing based on discrete Fourier transform

Consider the features of digital signal processing. DFT is an indirect discrete measuring spectral transformation and occupies a leading role among the spectral methods of measuring the parameters of signals at finite intervals in the frequency and time-frequency domains.

\[
S_N(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) W_N^{kn};
\]

where \(x(n)\) – discrete measurement signal, \(n = 0, N-1\), \(S_N(k)\) – DFT coefficients (spectrum).

In the practical application of DFT, a number of problems arise, the appearance of which is associated with the manifestation of specific effects accompanying its use [1-15].

We note three main problems.

1. The DFT does not give an answer to the question: what are the values of the spectrum defined by the discrete-time Fourier transform sequence \(x(n), n = 0, N-1\), between samples \(\frac{2\pi}{N} k, k = 0, N-1\) on a unit circle, thereby creating a known stockade effect.

2. The use of DFT is also accompanied by the effect of blurring of spectral components (often called the leakage effect). The manifestation of this effect is due to the fact that when performing spectral analysis of the function under study, we measure the cyclical convolution of the spectrum of the signal under investigation with a function of the form \(\frac{\sin(N \cdot x/2)}{N \sin(x/2)}\) that is not localized, but blurred in frequency (hence the name of the effect).

3. When using the DFT, the effect of parasitic amplitude modulation of the spectrum (also called the scalloping effect) also occurs. The manifestation of this effect is due to the fact that since the frequency characteristics of the filters corresponding to the DFT coefficients (bins of the DFT) have the form \(\frac{\sin(N \cdot x/2)}{N \sin(x/2)}\), this leads to fluctuations in the overall amplitude spectrum at \(N\) -point DFT (fluctuations reach 4 dB).

The theory of digital processing of information signals at finite intervals is based on three main and interrelated positions: the definition of a signal at a finite interval (\(N\) -interval); determining the shift of a signal as a certain permutation of its samples; definition of a complete system of discrete basis functions.

In the framework of the DFT, all of the above provisions of the theory of digital processing are defined:

1. the information signal \(x(n)\) is set on a finite interval \(0, N-1\);
2. the shift of the information signal \(x(n)\) is defined as the cyclic permutation of its samples within the interval;
3. as a basic system defined a complete system of discrete exponential functions
\[ p(k,n) = \exp\left(-j\frac{2\pi}{N} k \cdot n\right); \quad k,n = 0,N-1. \]

From the point of view of practical application of the DFT apparatus, we note the following
important point. The DFT of the sequence \( X(n), n = 0,N-1 \) can be considered as some approximation
to the Fourier transform of the function generating the sequence \( X(n) \). However, and this should be
emphasized, the DFT properties are exact (i.e. they are not approximate, based on the Fourier
transform properties of continuous signals).

Despite the fact that the methods of digital spectral processing of information signals retain, as the
analysis showed, their leading role in NDT, the results could have been more significant if not for their
fundamental flaws, which arise both from the nature of the DFT and from its analytical
properties of the basis of discrete exponential functions (DEF).

3. **Parametric discrete Fourier transform**

We introduce discrete functions of the form:
\[ p(k,n,\theta) = W_N^{k+\theta n} = \exp\left[-j\frac{2\pi}{N} (k + \theta)n\right], \quad \theta \neq 0, \]
which we define as parametric discrete exponential functions (DEF-P).

The DEF-P basis is a generalization of the DEF basis and is identical with the value of the parameter \( \theta = 0 \).

**The main properties of DEF-P.**
1. DEF-P, unlike DEF, are not functions of two equivalent variables \( k \) and \( n \). Consequently, the
   DEF-P matrix is asymmetric.
2. DEF-P are periodic functions with respect to the variable \( k \) and parametric periodic functions
   with respect to the variable \( n \) with a period \( N \):
   \[ p(k \pm pN,n,\theta) = p(k,n,\theta), \]
   \[ p(k,n \pm pN,\theta) = p(k,n,\theta)W_N^{\pm \theta p}. \]
3. The DEF-P system is not multiplicative with respect to the variable \( k \)
   \[ p(k,n,\theta) \cdot p(l,n,\theta) \neq p((k+l),n,\theta), \quad k,l = 0,N-1; \quad k \neq l \]
   \[ p(k,n,\theta) \cdot p(k,m,\theta) = p(k,(n+m),\theta), \quad n,m = 0,N-1; \quad n \neq m. \]
4. The average value of DEF-P
   with respect to the variable \( k \) is zero when \( n \neq 0 \):
   \[ \frac{1}{N} \sum_{k=0}^{N-1} p(k,n,\theta) = \exp\left(-j\frac{2\pi}{N} \theta n\right) \frac{1 - \exp(-j2\pi n)}{1 - \exp(-j2\pi/N)} \]
   and with respect to the variable \( n \) is not zero:
   \[ \frac{1}{N} \sum_{n=0}^{N-1} p(k,n,\theta) = \frac{1 - \exp(-j2\pi n(k+\theta))}{1 - \exp(-j2\pi/N (k + \theta))} \]
5. The DEF-P system is orthogonal with respect to both variables
\[
\sum_{n=0}^{N-1} W_N^{(k+\theta)n} W_N^{(l+\theta)n} = \begin{cases} 1 - W_N^{-(k-l)N} & N, \ k = l \\ 0 & N, \ k \neq l \end{cases}
\]

\[
\sum_{k=0}^{N-1} W_N^{(k+\theta)n} W_N^{(k+\theta)m} = \begin{cases} 1 - W_N^{-(m-n)N} & N, \ m = n \\ 0 & N, \ m \neq n \end{cases}
\]

6. The DEF-P system is a complete system, since the number of linearly independent functions is equal to the dimension of the set of discrete signals.

The expansion in the DEF-P basic system is called the parametric discrete Fourier transform (DFT-P), which is defined in the matrix form by the following relation:

\[
S_{N,\theta} = \frac{1}{N} F_{N,\theta} X_N; \quad 0 \leq \theta < 1,
\]

where

\[
F_{N,\theta} = \begin{bmatrix}
0 & 1 & \ldots & (N-1) \\
0 & 1 & \ldots & W_N^{\theta(N-1)} \\
\vdots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & \ldots \\
(N-1) & \ldots & \ldots & W_N^{(N-1+\theta)(N-1)}
\end{bmatrix},
\]

The authors proved the validity for the DFT-P theorems of linearity, shift, correlation, and Parseval equality.

4. The problem of spectral peak location in nondestructive testing by acoustic methods

Acoustic methods (including MFO), according to the nature of the registration of the parameter of the information signal, are classified as amplitude, frequency and spectral. In other words, the search for informative parameters of signals is carried out in the spectral, frequency and time-frequency domains. The search for informative parameters of signals includes the solution of the so-called problem of localization of spectral peaks.

Let a harmonic signal with a frequency of \(k = 14.3\) be given and the task of measuring its frequency (the problem of localizing spectral peaks) is posed. It is easy to see that when using the DFT (even using the signal addition operation with zero samples in the time domain), it is impossible to “combine” the frequency grid of the DFT filters with the frequency of the harmonic component. Indeed, an increase in the analysis interval by adding zero samples to the original signal allows changing the interval between samples only a multiple of times. At the same time, with the help of the DFT-P, the problem of localization of spectral peaks is effectively solved by a simple variation of the DFT-P parameter \(\theta\) (Figure 1, a, b, c).
5. Conclusions
1. In nondestructive testing by low-frequency acoustic methods, the DFT occupies a leading role among the existing methods for processing information signals in the frequency and time-frequency domains.
2. In the practical application of the DFT in nondestructive testing, a number of problems arise, the appearance of which is associated with the manifestation of undesirable effects. The known methods of dealing with the fundamental shortcomings of the DFT (operation of zero padding, using time, frequency and spectral windows, etc.), on the one hand have limited possibilities for suppressing undesirable effects of the DFT, on the other hand generate new problems.
3. To solve the problems of processing information signals of a complex structure, which are the signals in low-frequency acoustic methods, including MFO, the authors proposed a new class of basic systems based on parametric discrete exponential functions.
4. A parametric DFT (DFT-P), which is a generalization of the discrete Fourier transform, has been developed, investigated and introduced into the practice of processing information signals in nondestructive testing. The authors proved for this transformation the validity of the theorems of linearity, shift, correlation, and Parseval's equality.
5. DFT-P expands the whole apparatus of nondestructive testing methods, allows to increase the effectiveness, efficiency and accuracy of measurement results of information parameters and characteristics of signals in the frequency, time-frequency and spectral domains.

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