Eccentricity growth of planetesimals in a self-gravitating protoplanetary disc

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ABSTRACT

We investigate the orbital evolution of planetesimals in a self-gravitating circumstellar disc in the size regime (\(\sim 1\)–5000 km) where the planetesimals behave approximately as test particles in the disc’s non-axisymmetric potential. We find that the particles respond to the stochastic, regenerative spiral features in the disc by executing large random excursions (up to a factor of 2 in radius in \(\sim 1000\) yr), although typical random orbital velocities are of the order of one tenth of the Keplerian speed. The limited time frame and small number of planetesimals modelled do not permit us to discern any net direction of planetesimal migration. Our main conclusion is that the high eccentricities (\(\sim 0.1\)) induced by interaction with spiral features in the disc is likely to be highly unfavourable to the collisional growth of planetesimals in this size range while the disc is in the self-gravitating regime. Thus if, as recently argued by Rice et al., the production of planetesimals gets under way when the disc is in the self-gravitating regime (either at smaller planetesimal size scales, where gas drag is important, or via gravitational fragmentation of the solid component), the planetesimals thus produced would not be able to grow collisionally until the disc ceases to be self-gravitating. It is unclear, however, given the large amplitude excursions undergone by planetesimals in the self-gravitating disc, whether they would be retained in the disc throughout this period, or whether they would instead be lost to the central star.

Key words: accretion, accretion discs – gravitation – instabilities – stars: formation – planetary systems: formation – planetary systems: protoplanetary discs.

1 INTRODUCTION

It is currently unclear whether the process of planet formation begins during the earliest phases of star formation (i.e. in circumstellar discs that are strongly self-gravitating). The high disc mass at this stage (with gas mass \(\sim 10\) per cent of the central star’s mass; e.g. Eisner & Carpenter 2006) is favourable to this hypothesis and provides the only opportunity for gas giants to form via (gas phase) gravitational instability (Boss 2000). A more subtle effect – again strongly favoured during the self-gravitating case – arises through the interplay between pressure gradients in spiral arms and gas drag on solid particles and results in the accumulation of dust in spiral arms (Rice et al. 2004). This dust may then be able to accumulate further through the action of collisions and/or self-gravity in the dust phase (Rice et al. 2006).1

Once the disc is no longer self-gravitating in the dominant (gas) component, it is evidently impossible to form planets through either gas phase Jeans instability or dust accumulation in spiral arms and thus the most successful models invoke the collisional growth of dust (Pollack et al. 1996) followed possibly by the accretion of a gaseous envelope. Despite the lower surface densities during later phases (estimates of gas disc mass from sub-millimetre dust measurement suggest typical masses that are around an order of magnitude lower than in the self-gravitating case; Andrews & Williams 2005), the great advantage for planet formation is simply the longer duration of this later phase. Discs with sufficient gas and dust to form planets typically survive for \(10^6\)–\(10^7\) yr (Haisch, Lada & Lada

1 Note that such rapid accumulation, which might result in the rapid formation of large size planetesimals, is required in order for the growing planetesimals to escape the dangerous metre-size range, where they are subject to very fast migration down into the hottest inner disc (Weidenschilling 1977).
2001; Armitage, Clarke & Palla 2003; Wyatt et al. 2005), whereas the existence of dusty debris discs around older stars suggests that (potentially planet building) collisions between rocky planetesimals proceed for $10^8$ yr or more (Kenyon & Bromley 2004). These numbers should be contrasted with the brief window of opportunity (~$10^5$ yr) in the self-gravitating phase.

In summary, then, it is unclear without detailed calculation which of these regimes is favoured in the trade-off between lifetime and disc mass.

The viability of planet formation in the self-gravitating regime, however, requires that a number of conditions are satisfied beyond the initial phases of accumulation outlined above. In particular, we need to understand the dynamical evolution of the protoplanetary components (whether at the scale of dust aggregates, planetesimals or protogiant planets) with a view of answering the following questions: (i) does orbital migration result in such bodies spiralling into the central star, or can they be retained in the disc at least over the initial self-gravitating phase, and (ii) in the case of dust or planetesimals) are the kinematics of such bodies conducive to further collisional growth?

As a first step in answering this question, we here undertake pilot simulations of a small number of test particles in a self-gravitating disc, with a view to both considering the issue of their orbital migration and to establishing the velocity dispersion that is set up through interaction with spiral structure in the disc. It should be stressed that the calculation differs in several important ways from most calculations of orbital migration and planetesimal kinematics reported in the literature. First, conventional planetary migration, whether classified as Type I or Type II, is the consequence of the gravitational torque exerted on the planet (or planetesimal) by non-axisymmetric structure induced by the planet/planetesimal. Since this torque contribution can have significant contributions at the size scale of the Hill radius (or below; Bate et al. 2003), it is evidently important that this scale is well resolved in numerical calculations, which makes the modelling of the migration of low-mass objects particularly challenging, especially with codes such as Smoothed Particle Hydrodynamics (SPH) (de Val-Borro et al. 2006). In the present case, however, we are interested in a regime where the disc has pronounced non-axisymmetric structure in the absence of the planet/planetesimal (due to the presence of self-gravitating spiral modes in the disc) and we are therefore not obliged to resolve the Hill radius in order to capture the dominant torque contribution. Evidently, as the mass of the planetesimal is increased, the contribution to the torque which it experiences from self-induced spiral structure becomes significant and we will assess this a posteriori in order to set an upper mass scale on the applicability of our calculations. Second, in conventional calculations of the kinematics of planetesimal swarms (Kokubo & Ida 2000; Thommes, Duncan & Levison 2003), the velocity dispersion of planetesimals is set by an equilibrium between what is sometimes termed ‘viscous stirring’ (i.e. the transfer of kinetic energy from larger bodies to the planetesimal swarm via two-body scattering) and eccentricity damping by gas drag. In the present case, however, the velocity dispersion of our particles is set by the level to which eccentricity is pumped through interaction with the fluctuating potential of the self-gravitating disc. In this pilot calculation, we omit gas drag, which places a lower limit to the size scale of objects to which the calculation is applicable; again, we set this lower limit a posteriori.

Although we have stressed the qualitative differences between this calculation and that reported in the large body of literature on planetary migration and planetesimal kinematics, we point out that it bears close comparison with the calculation of Nelson (2005), which studies the response of planetesimals to the fluctuating non-axisymmetric structure in a disc that is subject to the magnetorotational (MRI) instability. In this study, it was found that the planetesimals underwent stochastic migration, but it was not possible to discern the mean magnitude (or even sign) of the migration rate over the limited time frame of the simulation (a few hundred orbits). In our experiment also, conducted over a similar time frame to Nelson (2005), we are unable to discern a net direction of migration, but we find (perhaps unsurprisingly given the fact that our disc is about an order of magnitude larger in mass) that the amplitude of the stochastic variations in the orbital radius is much larger, with several planetesimals undergoing radial excursions of a factor of 2 or more in both directions. Evidently, this behaviour has important implications for the ability of planetesimals to grow by collisions during the self-gravitating phase.

The structure of this paper is as follows. In Section 2, we describe the modelling of the self-gravitating disc, which is maintained in a ‘self-regulated’ (i.e. non-fragmenting) state through the imposition of an ad hoc cooling law for which the cooling time is a suitably large multiple of the local dynamical time. In Section 3, we analyse the orbital response of 12 test particles placed in a disc of mass 0.1 and 0.5 $M_\odot$. We also analyse the location of the dominant torque contribution on the planetesimals and assess the range of planetesimal mass and size scales for which we expect ‘test particle’ behaviour and for which gas drag can be ignored. In Section 4, we discuss the implications of our numerical findings for the migration and collisional growth of planetesimals in self-gravitating discs. Section 5 contains our chief conclusions.

2 NUMERICAL SIMULATIONS

2.1 Numerical code

In this paper, we simulate the interaction of a gaseous self-gravitating disc with a small number of planetesimals using SPH, a Lagrangian particle based method (Lucy 1977; Benz 1990; Monaghan 1992). This allows the simulation of gaseous and N-body particles, using individual time-steps (Bate, Bonnell & Price 1995), thus resulting in a large saving in computational time when a large dynamic range of time-steps is involved.

The general setup of our simulations is similar to Lodato & Rice (2004, 2005). We consider a massive gaseous disc orbiting around a 1 $M_\odot$ central star. The disc is taken to extend initially from 0.25 to 25 au, and we have considered two cases, where the disc mass is equal to 0.1 $M_\odot$ (standard runs) and 0.5 $M_\odot$ (massive runs), respectively.

Our simulations employ a number $N = 250 000$ particles and are thus intermediate–high resolution simulations. Previous simulations (Lodato & Rice 2004, 2005) at the same resolution have been shown to reproduce reasonably well the internal disc dynamics. In particular, once the disc reaches a quasi-steady self-regulated state (see below), the disc thickness $H$ is resolved with an average of two smoothing lengths for the low disc mass case and with four smoothing lengths for the hotter, high disc mass case.

As mentioned above, our code follows the dynamics of both SPH particles and point masses, which are able to accrete gas particles, if they come closer than a given accretion radius to the point mass. For the central object, such accretion radius is set to 0.25 au. At the end of the simulations, typically a fraction no larger than 10 per cent of the initial disc mass has been accreted on to the central star. The planetesimals have a nominal mass equal to the default gas particle mass, that is $4 \times 10^{-7} M_\odot$ for the standard runs and $2 \times 10^{-6} M_\odot$ for the massive run. This choice ensures that the
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The gravitational force of individual planetesimals does not influence the overall disc structure and the dynamics of the planetesimals. In order to prevent planetesimals from accreting gaseous particles, we have set their accretion radius to a very small value. The gas particles are allowed to heat up via $pdV$ work and artificial viscosity and to cool down with the following simple prescription

$$\frac{du}{dt}_{\text{cool}} = -\frac{u}{t_{\text{cool}}},$$

where the cooling time-scale $t_{\text{cool}} = \beta \Omega^{-1}$ and $\beta$ is fixed in space and time. While this is not a realistic prescription for the cooling in actual protostellar discs, it provides a convenient way of parametrising the cooling physics in order that one can ensure that one is modelling discs that remain in the self-gravitating regime without fragmenting. In particular, if the value of the parameter $\beta$ is large enough ($\beta \gtrsim 5$–6; Gammie 2001; Rice, Lodato & Armitage 2005), it allows the realization of a feedback loop where the disc evolves into a quasi-steady, self-regulated case, and is maintained close to marginal stability. For smaller values of $\beta$, the disc undergoes fragmentation. In our simulations, we have set $\beta = 7.5$. We note that since this is close to the critical value of $\beta$, we expect such a disc to be relatively ‘active’ (in the sense that the maintenance of thermal equilibrium requires spiral modes to be of relatively large amplitude). We revisit this point in Section 4.

2.2 Initial conditions

Initially, the gas in the disc is relatively hot, with a temperature profile $T(R) \propto R^{-1/2}$ and a minimum value of the stability parameter (Toomre 1964) $Q = 2$, attained at the outer disc edge. Since marginal stability corresponds to $Q = 1$, the whole disc is initially gravitationally stable. We initially follow the evolution of the gaseous disc only, as it cools down and becomes gravitationally unstable. The initial evolution is very similar to the one described in several other papers, for example Lodato & Rice (2004, 2005), Mejía et al. (2005): when $Q \approx 1$, the disc develops a spiral structure, inducing dissipation through compression and mild shocks and therefore heating up the disc and establishing a feedback loop which keeps the disc close to marginal stability, so that the Toomre $Q$ parameter is close to unity over the radial range $\sim 1$–20. This initial evolution takes roughly 60 outer dynamical times for the standard runs and 45 outer dynamical times for the massive runs. In this state, the surface density distribution is dominated by pronounced (of the order of unity in amplitude) spiral features, which, although regenerative, are individually transient structures (i.e. with lifetimes that are of the order of the local dynamical time-scale).

An ensemble of 12 planetesimals was then introduced between 10.8 and 18.8 au, aligned in four equispaced radial spokes (see Fig. 1 for the 0.1 $M_\odot$ disc and Fig. 2 for the 0.5 $M_\odot$ disc). The default velocities were chosen to be Keplerian with a small correction for the gravitational potential of the disc, although we also investigated the case where the planetesimals were introduced with a non-zero initial eccentricity.

3 RESULTS

3.1 Orbital evolution in the 0.1 solar mass disc case

In Fig. 3, we show the time evolution of the orbital radii of all the planetesimals for the case where $M_{\text{disc}} = 0.1 M_\star$ and where the planetesimals are initially set in nearly circular orbits. Although some planetesimals undergo large amplitude excursions (a factor of 2 or more), the average orbital radius of the ensemble varies by no more than 10 per cent during the simulation, indicating that planetesimals in a given region are apparently as likely to go inwards as outwards. The thick dashed line in Fig. 4 shows the mean eccentricity of the planetesimals as a function of time, showing that $e \sim 0.05$ is typical...
(although individual planetesimals may temporarily attain still higher eccentricities, \( \sim 0.3 \)). Thus, although much of the fine structure in Fig. 3 is simply a consequence of elliptical orbital motion, it is also evident that in addition some planetesimals undergo abrupt and/or large-scale changes in orbital radius (e.g. one planetesimal migrates over 20 au in 4000 yr, and another follows a period of relative orbital quiescence by migrating \( \sim 5 \) au in about 200 yr). Such migration rates imply radial velocities which are at times a significant fraction of the Keplerian speed.

In Fig. 4, we also show the results of starting the planetesimal ensemble with non-zero initial eccentricity, and see that the results are unchanged for initial eccentricity less than \( \sim 0.1 \). On the other hand, if a larger initial eccentricity is employed, the mean eccentricity of the ensemble remains around this value for the duration of the experiment.

3.2 Orbital evolution in the 0.5 solar mass disc case

Fig. 5 illustrates the rich variety of orbital histories of planetesimals in the massive disc case. The amplitude of orbital migration is somewhat larger than in the standard case, although again the change in average orbital radius for the ensemble is small, \(<20\) per cent. We note that, as in the standard case, the planetesimals undergo stochastic migration events even when they are located at radii (\( \sim 20 \) au) where \( Q \) is generally somewhat larger than unity. These events coincide with spiral structures temporarily extending out into these relatively quiescent regions.

The mean eccentricity of the ensemble is about double its value in the standard case \( \sim 0.15 \), with individual planetesimals temporarily attaining eccentricities of up to 0.5. Although some of the quasi-periodic structure in Fig. 5 is simply attributable to elliptical orbital motion (i.e. at roughly constant angular momentum), we also see planetesimals in which, temporarily, the energy and angular momentum are subject to correlated quasi-periodic variations. Such behaviour suggests that these planetesimals are at this stage being driven by a quasi-steady rotating non-axisymmetric potential. We have investigated this possibility by noting that in this case the Jacobi constant \( (E_i) \) of the planetesimal’s motion (i.e. the total energy measured in the frame of the rotating pattern) should be conserved, and that since \( E_i \) is related to the energy, \( E \), in the inertial frame, and angular momentum, \( L \), via

\[
E_i = E - L \Omega_p
\]

(where \( \Omega_p \) is the pattern speed), we can test for such behaviour by plotting \( E \) versus \( L \). This exercise indicates that there are indeed periods when certain planetesimals appear to be exhibiting this behaviour. This can be evidenced by a linear relation between \( E \) and \( L \), as shown in Fig. 6 for three of the simulated planetesimals. From the
analysis of this relation, we can then determine the pattern speed of the driving pattern. The planetesimals illustrated in Fig. 6 appear to be interacting with a mode with co-rotation at around 14 au: inspection of an animated series of snapshots of the simulation indicates that these three planetesimals are indeed roughly corotating with a spiral feature during this period, but that, as the feature dissolves shortly thereafter, they then migrate and stall close to another spiral arm at a different radius. This behaviour is not seen in the standard (0.1 M⊙ disc) simulations, which is consistent with the expectation (see e.g. Lodato & Rice 2005) that the more massive disc produces rather more coherent and long-lived spiral features. Nevertheless, we find only occasional evidence for planetesimals that are being driven by a dominant quasi-steady spiral mode so that the analysis of orbital families in this situation (see e.g. Vorobyov & Shchekinov 2006) is of limited applicability here.

3.3 Planetesimal–planetesimal encounters and gas densities close to the planetesimals

The inspection of animations of the simulations suggests that planetesimals tend to spend time preferentially in spiral structures in the disc. As such structures dissolve on a roughly dynamical time-scale, the planetesimals then migrate and find further temporary lodging in a new spiral structure. (The strongly fluctuating nature of the spiral potential means that, as discussed above, the planetesimals usually have insufficient time to attain steady orbits in the frame corotating with the local spiral pattern.) This predilection for being in the spiral arms suggests that the planetesimals are expected to come closer to each other and to sample higher gas densities than in the case of particles orbiting in an axisymmetric disc.

We illustrate the increased tendency for planetesimal–planetesimal encounters in Table 1, where we contrast the number of encounters between the planetesimals in various separation ranges with the corresponding statistics from a control run in which the planetesimals orbit in a smooth, non-self-gravitating disc. We see that the number of encounters with closest approach smaller than 0.5 au is significantly enhanced in the self-gravitating disc case. As expected, the effect of planetesimals lingering near spiral features is to decrease the minimum distance between planetesimals in the simulation. We stress that, given the very low masses of the planetesimals (i.e. equal in mass to a single gas particle), the effect of mutual encounters on the planetary dynamics is negligible – as intended – even for the closest encounters in the simulation.

We illustrate the effect of non-axisymmetric structure in the disc on the gas density field sampled by the orbiting planetesimals in Fig. 7. We plot – for a given planetesimal – the density excess, which we define as

\[ \eta = \frac{\rho_p - \rho_p^\text{ave}}{\rho_p^\text{ave}} \]

where \( \rho_p \) is the gas density in the vicinity of the planetesimal and \( \rho_p^\text{ave} \) is the azimuthally averaged density at the instantaneous orbital radius of the planetesimal. This planetesimal appears to spend most of its time in the spiral arms and thus at larger gas densities. This is typical, although some planetesimals also undergo phases where they sample lower gas densities as can be seen in Fig. 8. Such periods in underdense regions are less apparent in the case of the massive disc simulation.

We see from Fig. 7 that although this planetesimal samples gas densities larger than the local azimuthal average, \( \eta \) is always less than or of the order of unity. This is to be expected: the amplitude of the spiral features in the gas is only of the order of unity and thus even if planetesimals spent all their time in spiral arms, they would only experience a density that exceeded the local mean by a modest factor.

| Table 1. Number of encounters with closest approach smaller than \( d \) (measured in au) between planetesimals during the first 15 000 time units in the 0.1 M⊙ disc run and the non-self-gravitating disc run. |

| \( d < 0.5 \) | \( 0.5 < d < 1 \) | \( 1 < d < 1.5 \) | \( 1.5 < d < 2 \) |
|-----------------|-----------------|-----------------|-----------------|
| 0.1 M⊙ disc    | 10              | 17              | 17              | 19              |
| Non-self-grav.  | 0               | 17              | 13              | 16              |

Figure 7. Comparison between the density in the vicinity of one planetesimal in the 0.1 M⊙ disc with the mean gas density at the current radius of the planetesimal.

Figure 8. Evolution of the density excess \( \eta \) (see equation 3) as in Fig. 7, but for a different planetesimal.

3.4 Numerical issues

First, our experiments have aimed to study the response of each planetesimal to the fluctuating disc potential and have only modelled the evolution of an ensemble of planetesimals for reasons of computational economy. Therefore, we need to be satisfied that the stochastic behaviour we see is a result of planetesimal–disc and not planetesimal–planetesimal interactions. We find, in fact, that the torques experienced by the planetesimals due to other planetesimals are typically two to three orders of magnitude less than those arising from the disc. This is to be expected. The closest planetesimal–planetesimal interactions occur at distances of \( \sim 1 \) au: given that a typical smoothing length in the disc is \( \sim 0.2 \) au and that a smoothing kernel contains 50 particles, it follows that even at closest approach there are about 6000 gas particles that are closer to a planetesimal
the simulations cannot be trusted. Nevertheless, the inclinations stay from close to the planetesimal actually can make up a large fraction perpendicular to $z$ how the migration depends on planetesimal mass in Section 3.5.

Second, we are interested here in examining the regime where the dominant torques experienced by the planetesimal derive from the non-axisymmetric density distribution that the disc would have in the absence of the planetesimal (i.e. that due to the spiral structure in the disc) rather than the non-axisymmetric structure induced in the disc by the planetesimal. Note that in non-self-gravitating discs, and thus in all discussions of planetesimal migration in the literature (with the exception of that in Nelson (2005), it is the latter effect that is responsible for planet migration. In this case, it is essential that the calculation properly resolves the Hill radius of the planetesimal

$$R_h = R_p \left( \frac{M_p}{3M_\star} \right)^{1/3}$$

where $R_p$ is the radial coordinate of the planetesimal and $M_p$ the mass of the planet, since this is the region from which the bulk of the torques on the planetesimal arise.

In the present case, by contrast, where we examine migration driven by spiral structure in the self-gravitating disc, it is no longer important to resolve the Hill radius and we can thus examine the migration of low-mass planetesimals for which resolution of the Hill radius would be impracticable. Nevertheless, we need to check that our assumption is correct, i.e. that the dominant torques indeed arise at relatively large distances from the planet. Fig. 9 illustrates a typical snapshot of the cumulative distribution for the $z$ component of the torque of the disc on the planetesimal as a function of distance from the planetesimal. We see that the dominant contribution to the torque is around 1 au from the planetesimal and that the contribution to the torque from the poorly resolved region around the planetesimal (i.e. from within a typical particle smoothing length in that region, around 0.2 au) is small (<20 per cent). We are thus satisfied that the orbital evolution is not driven by the disc’s response to the planetesimal and that the planetesimal should be behaving approximately as a test particle. We examine this assumption further by investigating how the migration depends on planetesimal mass in Section 3.5.

On the other hand, in the case of the component of the torque perpendicular to $z$, i.e. acting on the inclination, the contributions from close to the planetesimal actually can make up a large fraction of the torque (i.e. around 40 per cent of the torque originating from within a smoothing length). Thus, the inclination development of the simulations cannot be trusted. Nevertheless, the inclinations stay relatively small (typically less than 0.02 radians in the standard and 0.03 for the massive disc). This is small compared with the axis ratio of the disc $H/R \sim 0.1$, and so the planetesimals remain mainly confined within the disc.

3.5 Valid regime

We now discuss the range of planetesimal masses for which the current calculation is valid. As noted in Section 2.1, we are modelling the regime in which the planetesimal behaves as a test particle in the gravitational potential of the star-disc system and where the results should be independent of planetesimal mass. In reality, at low masses the effect of gas drag (omitted in the present simulations) would break this mass independence, whereas at high masses, the disc response to the presence of the planetesimal affects the torques on the planetesimal. We consider each of these issues in turn.

A lower limit for the mass range of our simulation can be obtained by calculating the mass scale at which the stopping time due to drag is comparable with the time-scale on which the planetesimal undergoes stochastic migration due to interaction with spiral structure in the disc. From the inspection of Fig. 3, we see that planetesimals typically reverse their direction of migration on time-scales of about $10^4$ yr. Thus, if we demand that the time-scale for gas drag to significantly perturb the orbit should exceed say $10^4$ yr (~58 orbits), we should safely be in the regime where gas drag plays a minor role in the dynamics of the system.

The drag force on solid bodies within a protostellar disc $F_D$ is given by (Whipple (1972); Weidenschilling (1977))

$$F_D = \frac{1}{2} C_D \pi a^2 \rho u^2,$$  

where $a$ is the radius of the planetesimal, $u$ is the velocity of the planetesimal relative to the gas, $\rho$ is the gas density and $C_D$ is the drag coefficient.

Thus, the time-scale on which relative motion between the planetesimal and the disc gas is damped by gas drag is

$$t_e = \frac{M_p u}{F_D}.$$  

In the case that we are considering here, the relative velocity between the planetesimal and the disc gas does not derive – as in the case usually considered (e.g. Weidenschilling (1977)) – from the difference between Keplerian planetesimal motion and sub-Keplerian gas motion subject to outward pressure forces. Instead, this relative velocity, $u$, derives from the eccentricity of the planetesimal orbits induced by the spiral structure in the disc, so that we have $u = e V_K$, where $V_K$ is the local Keplerian velocity.

Assuming a spherical planetesimal of density $\rho_p$, we can then write $t_e$ as

$$t_e = \frac{8 \rho_p R}{3 C_D \rho e V_K}.$$  

or, equivalently (and adopting $C_D$ in the range 0.5–1, which is appropriate to the case considered here where the Reynolds number exceeds a few hundred)

$$t_e \sim \frac{8}{1.32} \frac{\rho_p a}{\rho R} \Omega^{-1},$$

where $R$ is the orbital radius of the planetesimal. If we then require that $t_e > 10^4$ yr (~300 $\Omega^{-1}$), and adopting typical values of $e \sim 0.1, \rho \sim a \times 10^{-11}$ g cm$^{-3}$ and $\rho_p \sim$ a few g cm$^{-3}$, then we obtain the requirement that $a$ must be at least of the order of a kilometre or so before one can neglect the effect of gas drag.
We thus conclude that the effect of gas drag is negligible for planetesimals of kilometre scale or above (corresponding to bodies of masses $\sim 10^{16}$ g).

Our simulations are valid for all higher masses as long as the accelerations experienced by the planetesimals remain independent of mass. But these accelerations change as soon as the planetesimals have enough mass to gravitationally influence the gas density. Thus, we get an upper limit for the validity of our simulations by checking at what mass scale the torque per planetesimal mass $T/M_p$ changes with mass. In Fig. 10, we plot the evolution of $T/M_p$ for a single planetesimal in the case that it has the mass indicated in each of the curves. It can be seen in Fig. 10 that significant changes to the torques on the planetesimals only arise when the planetesimal mass is raised above 10 times the one used in our standard run, corresponding to about $8 \times 10^{27}$ g. We reach the same conclusion from the inspection of Fig. 11, where again we see a marked deviation in orbital evolution for the largest mass planetesimal.

In summary, we have as a region of validity of our simulations:

$$2 \times 10^{-8} M_\odot \sim 10^{16} \text{ g} \leq M_p < 8 \times 10^{27} \text{ g} = 1M_\odot$$

(9)

or in planetesimal radii

$$1 \text{ km} < a < 5000 \text{ km}.$$  

(10)

### 4 DISCUSSION

We can summarize our numerical findings as follows. Planetesimals in the size/mass range to which our calculations apply (see equation 10) are subject to large-scale fluctuations in orbital radius due to interaction with regenerative spiral structure in the disc. We can discern no net sign to the mean migration rate over the time-scale of the simulation and are thus unable to answer one of the goals of the present study, i.e. whether we expect planetesimals to migrate into the star during the self-gravitating phase of the disc. Since the disc’s lifetime in the self-gravitating phase exceeds the duration of the present experiment by about a factor of 30, it will be computationally challenging to answer this question, especially since (given the stochastic nature of the orbital evolution) it will be necessary to assess the fraction of planetesimals retained in the disc through following the evolution of a large ensemble of particles.

We can, however, make some comments about how the dynamical evolution of our particles is likely to affect their collisional growth during the self-gravitating phase. As noted by Nelson (2005) in the context of the evolution of planetesimals in a disc subject to MRI, the fluctuations in orbital radius and eccentricity could in principle either aid or hinder planetesimal growth. On the positive side, orbital migration provides a mechanism for avoiding the self-limitation of planetesimal growth through the clearing out of a protoplanet’s ‘feeding zone’. In conventional models, this imposes an ‘isolation mass’ limit which makes it difficult to grow beyond a terrestrial mass scale in the inner regions of the disc (Ida & Lin 2004). Although this limitation is bypassed in our self-gravitating models, since the feeding zone would be continuously replenished by planetesimals undergoing an orbital random walk, this factor is not very significant since in any case the high surface density during the self-gravitating phase means that isolation masses are high (higher, that is, than the upper limit for the applicability of our calculations). Another potentially positive aspect of evolution in a self-gravitating disc is the possibility of concentrating planetesimals in high-density spiral features, where they might be more prone to undergo collisional growth.

In the case of the more massive $-0.5 M_\odot$ disc (see Section 4), there is some evidence for the dominance at times of spiral modes which persist over a number of dynamical times, allowing particles to settle into orbits which conserve a Jacobi integral in the co-rotating frame. Some of these orbits involve a preference for particles to spend time in the spiral arms, an impression strengthened by the inspection of animations which show particles which co-rotate for a while in the vicinity of an arm and which then respond to changes in the spiral structure by passing rapidly to another spiral feature. This tendency will not lead to a significantly enhanced collision rate, however, unless there is a strong enhancement in the solid-to-gas ratio in the spiral arms: since the amplitude of the gaseous arms is around a factor of 2 at most, then even a particle that spent all its time in the arms would only, at fixed solid-to-dust ratio, experience an enhancement in the local planetesimal density of a factor of 2 at most. Since we have only studied the evolution of 10 planetesimals, we cannot, on the basis of the present study, comment on the degree to which planetesimals are differentially concentrated in the arms. Nevertheless, the results of Rice et al. (2004) suggest that the differential accumulation of solids in the arms is weak at size scales where gas drag is unimportant.

We now turn to the negative implication of stochastic migration noted by Nelson (2005), i.e. the growth of planetesimal velocity dispersion. This hinders planetesimal growth in two respects: higher collisional velocities are associated with disruption of parent bodies instead of collisional growth (e.g. Leinhardt, Stewart & Schultz...
t = \frac{\rho^{2/3} M_p^{1/3}}{\Sigma \Omega (1 + 4 \Omega M_p / (R_p \sigma^2))},
\tag{11}
\end{equation}

where \( \Sigma \) is the surface density of planetesimals, \( \sigma \) is the planetesimal velocity dispersion and \( \Omega \) is the local Keplerian orbital frequency. For a planetesimal of density \( \sim 3 \text{ g cm}^{-3} \) located at 10 au in a disc with surface density of planetesimals equal to 0.4 g cm\(^{-2}\), the growth time-scale via two-body collisions is
\[ t_{\text{grow}} = 5 \times 10^9 \frac{M_1^{1/3}}{1 + M_2^{2/3} \sigma^{-2}} \text{ yr} \tag{12} \]

where \( M = (M_p/6 \times 10^{-4} M_\oplus) \) and \( \sigma = (\sigma/1 \text{ km s}^{-1}) \). Evidently, this time-scale is far too long to be of interest unless the second term on the denominator (due to enhancement of the collisional cross-section by gravitational focusing) is large. If we simply set \( \sigma \) to be the product of the orbital eccentricity and local Keplerian velocity (an assumption we revisit below), it follows that at a given location in the disc and for a given mass scale of planetesimal, the growth time-scale in the gravitationally focused regime scales as the product of the eccentricity divided by the surface density normalization. In other words, for a planetesimal at a given mass scale to be able to grow significantly over a disc phase of duration \( t_{\text{disc}} \), we require
\[ \frac{t_{\text{grow}}}{t_{\text{disc}}} < 1, \tag{13} \]

which translates into a lower limit on the product \( t_{\text{disc}} \Sigma e^2 \) (where \( \Sigma \) is the surface density normalization of the disc).

We can now return to the issue raised in the Introduction that whether or not planetesimal growth is favoured in the high density but short-lived phase during which the disc is self-gravitating. Assuming for simplicity that the self-gravitating phase lasts \( \sim 10 \) per cent of the subsequent lifetime of the primordial gas/dust disc but that its surface density (mass) is typically 10 times greater, we see that these two factors roughly cancel. Thus, whether the growth of planetesimals is favoured during the self-gravitating phase actually boils down to the relative values of the eccentricity in the two phases.

Our simulations have shown that this factor is decisive in favouring the self-gravitating regime. In conventional models for the dynamical evolution of planetesimal swarms (e.g. Kokubo & Ida 2000), the equilibrium eccentricity (attained through the balance between stirring by larger bodies and damping by gas drag) is very low (of the order of 0.01). We have, however, seen that in the self-gravitating phase, the typical eccentricity is 0.1 or more. Since the growth time scales as the square of the eccentricity, this means that the self-gravitating phase is highly unfavourable (compared with the later more quiescent phase) for the collisional growth of planetesimals. We note that the above estimates for conventional protoplanetary discs assume a laminar disc structure; eccentricity driving by interaction with turbulent structures in a disc exhibiting MRI would further hinder collisional growth, whether or not the disc is self-gravitating.

There, however, remain two possible qualifiers of the above conclusion. First, it is only valid to set the local velocity dispersion equal to the product of the eccentricity and local Keplerian velocity in the case that the kinematics are well described as a set of elliptical orbits which are elongated in a random direction. Such a description does not, of course, allow for the possibility of local velocity coherence in a planetesimal swarm in which velocities are significantly noncircular. In this sense, therefore, the above estimates are pessimistic.

Further calculations, involving a large swarm of planetesimals from which one could derive the local velocity dispersion, are required in order to settle this issue. We, however, note that our difficulty in identifying coherent orbital families, interacting with a long-lived spiral mode, probably implies that this effect is unlikely to improve the prospects for particle growth significantly. The second possible qualifier is that, as noted in Section 2, our simulations employ a cooling time-scale which is close to the critical one for disc fragmentation and therefore represent conditions where spiral modes are of relatively high amplitude. In the inner regions of protoplanetary discs, however, cooling time-scales are longer than employed here and thus one would expect lower amplitude spiral structures, with a correspondingly lower amplitude of stochastic driving of planetesimal orbits. This possibility needs to be quantified through further numerical investigation.

If one leaves aside these possibilities, then we have shown that the self-gravitating phase is unfavourable for collisional growth in the regime of planetesimal scale studied. We have not ruled out the possibility of collisional growth at smaller scales (where gas drag can effect a significant enhancement of the local solid-to-gas ratio in spiral features; Rice et al. 2004) nor that (where such enhancement exists) it may not be possible to produce much larger bodies through the gravitational fragmentation of the solid component (e.g. Rice et al. 2006). We nevertheless consider it likely that whatever the size scale attained in this way, there will be no further collisional growth of planetesimals while the disc is self-gravitating.

This obviously limits the extent to which planet formation can get under way in the self-gravitating phase. Either a massive planet is produced via gas phase Jeans instability (Boss 2000), thus obviating the need for collisional growth, or else, if self-gravity aids the initial accumulation of solid bodies (Rice et al. 2004, 2006), then it is unlikely that such bodies can undergo further collisional growth once they become large enough for gas drag to be unimportant. In this latter case, further growth of such bodies could only resume once the disc had evolved to the point where its self-gravity (and the associated pumping up of the planetesimal velocity dispersion) has become unimportant. The vital unanswered question, therefore, is whether orbital migration would prevent the planetesimals from remaining in the disc throughout the self-gravitating phase. This question can only be answered through further, computationally challenging, calculations.

ACKNOWLEDGMENTS

MB gratefully acknowledges the hospitality of the IOA and financial support via the Marie Curie EARA Early Stage Training program. We thank the referee, Phil Armitage, for constructive criticism.

REFERENCES

Andrews S. M., Williams J. P., 2005, ApJ, 631, 1134
Armitage P. J., Clarke C. J., Palla F., 2003, MNRAS, 342, 1139
Bate M. R., Bonnell I. A., Price N. M., 1995, MNRAS, 277, 362
Bate M. R., Lubow S. H., Ogilvie G. I., Miller K. A., 2003, MNRAS, 213, 341
Benz W., 1990, in Buchler J., ed., The Numerical Modelling of Non-linear Stellar Pulsations. Kluwer, Dordrecht
Boss A. P., 2000, ApJ, 536, L101
de Val-Borro M. et al., 2006, MNRAS, 370, 529
Eisner J., Carpenter J., 2006, ApJ, 641, 1162
Gammie C. F., 2001, ApJ, 553, 174
Haisch K., Lada E., Lada C., 2001, ApJ, 553, L153
Ida S., Lin D. N. C., 2004, ApJ, 604, 388

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Kenyon S. J., Bromley B. C., 2004, AJ, 127, 513
Kokubo E., Ida S., 2000, Icarus, 143, 15
Leinhardt Z. M., Stewart S. T., Schultz P. H., 2007, preprint (astro-ph/07053943)
Lodato G., Rice W. K. M., 2004, MNRAS, 351, 630
Lodato G., Rice W. K. M., 2005, MNRAS, 358, 1489
Lucy L. B., 1977, AJ, 82, 1013
Mejia A. C., Durisen R. H., Pickett M. K., Cai K., 2005, ApJ, 619, 1098
Monaghan J. J., 1992, ARA&A, 30, 543
Nelson R. P., 2005, A&A, 443, 1067
Pollack J. B., Hubickyj O., Bodenheimer P., Lissauer J., Podolak M., Greenzweig Y., 1996, Icarus, 124, 62
Rice W. K. M., Lodato G., Pringle J. E., Armitage P. J., Bonnell I. A., 2004, MNRAS, 355, 543
Rice W. K. M., Lodato G., Armitage P. J., 2005, MNRAS, 364, L56
Rice W. K. M., Lodato G., Pringle J. E., Armitage P. J., Bonnell I. A., 2006, MNRAS, 372, L9
Thommes E. W., Duncan M. J., Levison H. F., 2003, Icarus, 161, 431
Toomre A., 1964, ApJ, 139, 1217
Vorobyov E. I., Shchekinov Y. A., 2006, New Astron., 11, 240
Weidenschilling S., 1977, MNRAS, 180, 57
Whipple F. L., 1972, in Elvius A., ed., From Plasma to Planets. Wiley, London
Wyatt M. C., Greaves J. S., Dent W. R. F., Coulson I. M., 2005, ApJ, 620, 492

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