A case study of 2019-nCoV in Russia using integer and fractional order derivatives

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In this article, we define a mathematical model to analyze the outbreaks of the most deadly disease of the decade named 2019-nCoV by using integer and fractional order derivatives. For the case study, the real data of Russia is taken to perform novel parameter estimation by using the Trust Region Reflective (TRR) algorithm. First, we define an integer order model and then generalize it by using fractional derivatives. A novel optimal control problem is derived to see the impact of possible preventive measures against the spread of 2019-nCoV. We implement the forward–backward sweep method to numerically solve our proposed model and control problem. A number of graphs have been plotted to see the impact of the proposed control practically. The Russian data-based parameter estimation along with the proposal of a mathematical model in the sense of Caputo fractional derivative that contains the memory term in the system are the main novel features of this study.

KEYWORDS
2019-nCoV, Caputo fractional derivative, forward–backward sweep method, mathematical model, optimal control problem

MSC CLASSIFICATION
26A33, 34C60, 92C60, 92D30

1 | INTRODUCTION

Nowadays, 2019-nCoV or Covid-19 or Coronavirus has become the most deadly disease of the decade. This epidemic has been spread out all over the world. Many strong economies like the United States, United Kingdom, China, Russia, and India have been badly disturbed because of this pandemic. China is identified as the origin of this virus. Not only in humans but this virus has been identified in animals also. The vaccine is now available to protect people from this virus, but still, in many countries, the cases continue. In many nations like India, and Brazil, the virus has been recorded in a number of waves.

Medical authorities have given their valuable efforts to control the outbreaks of this virus and have done it well in some regions. Mathematics has also played a vital role by giving time-to-time future predictions on the transmission of the virus. Different types of mathematical models have been introduced for forecasting the outbreaks of 2019-nCoV. Here we have a short literature survey on the mathematical modeling of 2019-nCoV epidemic: Kumar and Erturk have
proposed a novel fractional order mathematical model by using a generalized version of the Caputo derivative. In Kumar and Suat Erturk, the authors proposed a delay-type model in a fractional sense to examine the infection dynamics of 2019-nCoV with the effects of delay. The outbreaks of 2019-nCoV in Brazil are investigated in Kumar et al. In Kumar and Suat Erturk, the authors derived the solution of a 2019-nCoV model by using the parameter values based on the actual data of India. In Danane et al, the action of the government and public reaction are considered important factors to formulate a mathematical model by using fractional derivatives. Ahmad et al proposed a strong and efficient model to justify the dynamics of 2019-nCoV. Some novel theoretical results related to the existence and stability of the fuzzy-type 2019-nCoV model in terms of Caputo–Fabrizio derivative have been derived in Verma and Kumar. In Alghamdi et al, the authors used two various fractional derivatives to simulate the dynamics of a 2019-nCoV model by using real statistical data. Zeb et al have introduced two novel fractional order models containing vaccine terms and solved the models by using various methods, respectively. Some novel control analyses on the spread of 2019-nCoV by using a model have been given in Misra and Sisodiya. In Omar et al, the case study of Egyptian 2019-nCoV data has been given by using a stochastic fractional mathematical model. The researchers in Kumar et al have given a brief novel mathematical analysis of the outbreaks of 2019-nCoV in Argentina by using the real-data range from March 2020 to March 2021. In Khan and Atangana, a study is introduced to identify the transmission dynamics of a new variant of 2019-nCoV called Omicron. To show the role of vaccination in the 2019-nCoV outbreaks, the authors in Banerjee and Biswas derived a SIR epidemic model. In Nabi et al, the integer and fractional order derivatives are used to solve a mathematical model of 2019-nCoV. Nabi et al have studied the data projection of 2019-nCoV for India and Bangladesh by using novel mathematical modeling. Atangana and Araiz explored the 2019-nCoV outbreaks in Turkey and South Africa by using novel mathematical analysis. In Nabi et al, the researchers solved a 2019-nCoV model by using a neural networking approach. Chatterjee and Ahmad introduced a fractional order model of 2019-nCoV infection of epithelial cells. In Das and Samanta, the stability analysis of a fractional ordered 2019-nCoV model is given. Some other modeling-related studies on 2019-nCoV epidemic can be seen from previous works. Some recent advancements in mathematical modeling can be learned from previous works.

In this article, we derive a nonlinear mathematical model for the 2019-nCoV epidemic by using integer and fractional order derivatives. The motivation to go from integer to fractional order case is to capture the memory effects by using Caputo fractional operator. The parameter values used in the study are based on the real data of daily outbreaks of 2019-nCoV in India. The study is formulated as follows: In Section 2, we propose the integer order model which is taken from Sarkar et al to define the dynamical structure of 2019-nCoV. In that section, the necessary features of the model have been defined for a better understanding of the proposed problem. After that, the parameter estimation and model fitting have been done by using the daily reported cases of India from January 1, 2022, to April 30, 2022. In Section 3, the integer order model is generalized in a fractional order sense by using the Caputo operator to include the memory in the system. In Section 4, some optimal control strategies are discussed to find the possible control options to decrease the spread of the virus at the earliest stage of time. To derive the optimal control problem, the well-known Pontryagin’s minimum principle is used in the sense of the Caputo derivative. In Section 5, the necessary graphs are plotted to check what will be the structure of 2019-nCoV in India in the upcoming days; that is, how does the proposed model behave for a particular time period? In that section, the impact of proposed disease controls is clearly visible. Also, it is clearly justified that the model fitting is correct and matches the real row data. The proposed Caputo fractional derivative generates more varieties of solutions to predict the future happenings of 2019-nCoV in Russia. In the end, a conclusion finished the paper.

2 | MATHEMATICAL MODEL

First of all, for investigating the spread of 2019-nCoV in Russia, we recall an integer order model proposed by Sarkar et al. In this model, the total population is divided into six classes, namely, susceptible population $S(t)$, susceptible quarantined $S_q(t)$, asymptomatic infected individuals $A(t)$, infectious individuals $I(t)$, isolated infected peoples $I_q(t)$, and recovered individuals $R(t)$. Total size of the population is $N(t) = S(t) + S_q(t) + A(t) + I(t) + I_q(t) + R(t)$. The dynamical model in the ordinary differential equations is given by
Parameter estimation and data fitting

**FRACTIONAL ORDER MATHEMATICAL MODEL**

Fractional order models are advanced tools that have been used a number of times to define real-life problems. Fractional order derivatives are non-local, whereas integer order derivatives are local in nature. Thus, the basic reproduction number \( R_0 \) can be calculated. For this, we use the non-negative matrix \( F \) and the non-singular \( M \)-matrix \( V \) for the system (1), which express the formation of new infection and the transition component, respectively. For the system (1), the matrices \( F \) and \( V \) are described by

\[
F = \begin{bmatrix}
\beta_s (1-\gamma_s) \mu_s \frac{S}{N} \\
0 \\
\beta_a \gamma_s S \frac{I}{N}
\end{bmatrix}, \quad V = \begin{bmatrix}
(\delta_a + \xi_a) A \\
-\delta_a A + (\delta_i + \xi_i) I \\
\delta_i I + \xi_q I_q
\end{bmatrix}.
\]

Thus, the basic reproduction number \( R_0 \) is defined by

\[
R_0 = \frac{\beta_s (1-\gamma_s) \mu_s \delta_a}{(\delta_a + \xi_a)(\delta_i + \xi_i)}.
\]

**Remark 1.** Some other dynamical properties of the given mathematical model like equilibrium points and their stability can be explored from Sarkar et al.\(^{30}\)

### 2.1 Parameter estimation and data fitting

Now, we investigate the data fitting and parameter estimation of our model (1) with daily infectious cases of Russia from January 1, 2022 to April 30, 2022. The sources of real data are taken from Worldometer website.\(^{31}\) To calculate the values of the model parameters, we use the Trust Region Reflective (TRR) algorithm by using the lsqcurvefit function in MATLAB. According to recent statistics, Russia’s overall population in 2022 is estimated as 146,000,000. For initial conditions, we take \( S(0) = 145,723, 849 \), \( S_q(0) = 200,900 \), \( A(0) = 50,000 \), \( I(0) = 19751 \), \( I_q(0) = 5000 \), and \( R(0) = 500 \).

In Figure 1, we can see that the proposed model fits well and closely match with the actual data of Russia. The corresponding parameter values are obtained by using TRR algorithm and given in Table 1.

### 3 FRACTIONAL ORDER MATHEMATICAL MODEL

We know that the integer order derivatives are local in nature, while the fractional derivatives are non-local. Fractional order models are advanced tools that have been used a number of times to define real-life problems. Fractional order
FIGURE 1 Outcomes of the model fitting for daily infectious cases in Russia from January 1, 2022 to April 30, 2022 [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 The parameter values of the given model (1) by using TRR algorithm and daily infected cases in Russia from January 1, 2022 to April 30, 2022

| Parameters | Probable range | Base value | TRR output | References |
|------------|----------------|------------|-------------|------------|
| $\Gamma_s$ | $-\epsilon \times N(0)$ | 5479.452   | Estimated   |            |
| $\beta_s$  | 0–1            | 0.8799     | 0.571245    | Fitted     |
| $\gamma_s$ | 0–1            | 0.3199     | 0.500707    | Fitted     |
| $\mu_s$   | 0–100          | 14.83      | 0.017091    | Fitted     |
| $\epsilon$| $1/(73 \times 365)$ | -         | -           | Estimated   |
| $v_s$      | 0–1            | 0.04167    | 0.020532    | Fitted     |
| $\delta_a$| 0–1            | 0.0168     | 0.020085    | Fitted     |
| $\delta_i$| 0–1            | 0.116980   | 0.020085    | Fitted     |
| $\xi_a$   | 0–1            | 0.515234   | 0.020085    | Fitted     |
| $\xi_i$   | 0–1            | 0.457035   | 0.020085    | Fitted     |
| $\xi_q$   | 0–1            | 0.202866   | 0.020085    | Fitted     |

models contain memory effects in the system which is very useful in disease modeling. Also, a fractional order model contains extra parameters as an order of the operator, which is useful for brief numerical simulations. These advantages of fractional order operators encourage us to generalize the proposed classical model (2) into Caputo-type fractional order sense.

3.1 Preliminaries

First, we recall the following preliminaries which are needed for the further investigation:

**Definition 1.** A real-valued function $f(r)$, $r > 0$ contains in the space

(a) $C_v$, $v \in \mathbb{R}$ if there exists a real number $q > v$, such that $f(r) = r^q f_1(r)$, $f_1 \in C[0, \infty)$. Clearly, $C_v \subset C_y$ if $y \leq v$.

(b) $C^n_v$, $n \in \mathbb{N} \cup \{0\}$ if $f^n \in C_v$.

**Definition 2.** The definition of left- and right-sided Caputo fractional derivatives for the function $f \in C^n_{m-1}$ with order $\alpha \in (m - 1, m]$, $m \in \mathbb{N}$ is given by

$$
^{C}D_{a+}^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau,
$$

and

$$
^{C}D_{b-}^\alpha f(t) = (-1)^m \frac{1}{\Gamma(m-\alpha)} \int_{t}^{t_0} (\tau-t)^{m-\alpha-1} f^{(m)}(\tau) d\tau,
$$

provided they exist almost everywhere on $[t_0, t_n]$. 
Therefore, the fractional order generalization of the proposed classical model (2) by using Caputo derivative is given as follows:

\[ \begin{align*}
\frac{CD^\alpha_{t_0+}}{S(t)} &= \Gamma_s - (\beta_s + \gamma_s (1 - \beta_s)) \mu_s \frac{SI}{N} - \epsilon S + \nu_s S_q, \\
\frac{CD^\alpha_{t_0+}}{S_q(t)} &= \gamma_s (1 - \beta_s) \mu_s \frac{SI}{N} - (\nu_s + \epsilon) S_q, \\
\frac{CD^\alpha_{t_0+}}{A(t)} &= \beta_s (1 - \gamma_s) \mu_s \frac{SL}{N} - (\delta_u + \xi_u + \epsilon) A, \\
\frac{CD^\alpha_{t_0+}}{I(t)} &= \delta_u A - (\delta_i + \xi_i + \epsilon) I, \\
\frac{CD^\alpha_{t_0+}}{I_q(t)} &= \beta_s \mu_s \gamma_s \frac{SI}{N} + \delta_i I - (\xi_q + \epsilon) I_q, \\
\frac{CD^\alpha_{t_0+}}{R(t)} &= \xi_u A + \xi_i I + \xi_q I_q - \epsilon R.
\end{align*} \tag{2} \]

**Remark 2.** Some other dynamical properties of the given fractional order model (2) like boundedness and positivity, existence and uniqueness, and stability of the system can be achieved by using the procedure followed in Kumar et al.\textsuperscript{34}

### 4 | OPTIMAL CONTROL PROBLEM

To reduce the transmission of the 2019-nCoV epidemic, the respective governments have used possible controls with high financial investments. In this section, we derive the optimality system of an optimal control problem by using Pontryagin’s minimum principle. Here, our objective is to determine two optimal controls \( u_1(t) \) and \( u_2(t) \) that minimize the cost functional

\[ J(u_1, u_2) = \frac{1}{2} \int_{t_0}^{T_{max}} \left( a_1 A^2 + a_2 I^2 + a_3 I_q^2 + a_4 u_1^2 + a_5 u_2^2 \right) dt. \]

subject to the control system

\[ \begin{align*}
\frac{CD^\alpha_{t_0+}}{S(t)} &= \Gamma_s - (\beta_s + \gamma_s (1 - \beta_s)) \mu_s \frac{SI}{N(t)} - \epsilon S(t) + \nu_s S_q(t) - \nu_s u_1(t) S(t), \\
\frac{CD^\alpha_{t_0+}}{S_q(t)} &= \gamma_s (1 - \beta_s) \mu_s \frac{SI}{N(t)} - (\nu_s + \epsilon) S_q(t) - \nu_s u_1(t) S_q(t), \\
\frac{CD^\alpha_{t_0+}}{A(t)} &= \beta_s (1 - \gamma_s) \mu_s \frac{SI}{N(t)} - (\delta_u + \xi_u + \epsilon) A(t) - m_r u_2(t) A(t), \\
\frac{CD^\alpha_{t_0+}}{I(t)} &= \delta_u A(t) - (\delta_i + \xi_i + \epsilon) I(t) - m_r u_2(t) I(t), \\
\frac{CD^\alpha_{t_0+}}{I_q(t)} &= \beta_s \mu_s \gamma_s \frac{SI}{N(t)} + \delta_i I(t) - (\xi_q + \epsilon) I_q(t) - m_r u_2(t) I_q(t), \\
\frac{CD^\alpha_{t_0+}}{R(t)} &= \xi_u A(t) + \xi_i I(t) + \xi_q I_q(t) - \epsilon R(t) + \nu_s u_1(t) (S(t) + S_q(t)) + m_r u_2(t) (A(t) + I(t) + I_q(t)),
\end{align*} \]

where \( 0 < a \leq 1, a_j > 0 \) for all \( j = 1, 2, 3, 4, 5 \) and \( \nu_s, m_r \) represents the vaccination and treatment rate respectively.

To reduce the infection of Coronavirus disease, we apply the time-dependent optimal controls \( u_1(t) \), with \( 0 \leq u_1(t) \leq 1 \) denotes the fraction of susceptible and quarantined susceptible requires vaccination and \( u_2(t) \), with \( 0 \leq u_2(t) \leq 1 \) denotes the fraction of asymptomatic infected, infectious and isolated individuals who are notified and will be treated. Consider the adjoint vector \( \dot{\lambda}(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t)) \in \mathbb{R}^6 \) and the Hamiltonian of the given system is
\[ H = \frac{1}{2} \left( a_1 A^2(t) + a_2 I^2(t) + a_3 I^2_q(t) + a_4 u_1^2(t) + a_5 u_2^2(t) \right) \]
\[ + \lambda_1(t) \left( \Gamma_s - (\beta_s + \gamma_s(1 - \beta_s)) \mu_s \frac{S(t)I(t)}{N(t)} - cS(t) + v_s S_q(t) - v_u u_1(t)S(t) \right) \]
\[ + \lambda_2(t) \left( \gamma_s(1 - \beta_s) \mu_s \frac{S(t)I(t)}{N(t)} - (v_s + c) S_q(t) - v_u u_1(t)S(t) \right) \]
\[ + \lambda_3(t) \left( \beta_s(1 - \gamma_s) \mu_s \frac{S(t)I(t)}{N(t)} - (\delta_a + \xi_a + c) A(t) - m_r u_2(t)A(t) \right) \]
\[ + \lambda_4(t) (\delta_a A(t) - (\delta_i + \xi_i + c) I(t) - m_r u_2(t)I(t)) \]
\[ + \lambda_5(t) \left( \beta_s \mu_s \frac{S(t)I(t)}{N(t)} + \delta_i I(t) - (\xi_q + c) I_q(t) - m_r u_2(t)I_q(t) \right) \]
\[ + \lambda_6(t) (\xi_a A(t) + \xi_i I(t) + \xi_q I_q(t) - c R(t) + v_u u_1(t)(S(t) + S_q(t)) \]
\[ + m_r u_2(t)(A(t) + I(t) + I_q(t)) \].

Using Pontryagin’s minimum principle, we get the optimal controls \( u_1^* \) and \( u_2^* \) are derived by

\[ \frac{\partial H}{\partial u_1} = 0 \Rightarrow u_1^*(t) = \frac{v_s}{a_4} (S(t) \lambda_1(t) + S_q(t) \lambda_2(t) - S(t) \lambda_6(t) - S_q(t) \lambda_6(t)), \]

\[ \frac{\partial H}{\partial u_2} = 0 \Rightarrow u_2^*(t) = \frac{m_r}{a_5} (A(t) \lambda_3(t) + I(t) \lambda_4(t) + I_q(t) \lambda_5(t) - A(t) \lambda_6(t) - I(t) \lambda_6(t) - I_q(t) \lambda_6(t)). \]

the state equations are

\[ C D^\sigma_{t_0^+} S(t) = \Gamma_s - (\beta_s + \gamma_s(1 - \beta_s)) \mu_s \frac{S(t)I(t)}{N(t)} - cS(t) + v_s S_q(t) - v_u u_1(t)S(t), \]

\[ C D^\sigma_{t_0^+} S_q(t) = \gamma_s(1 - \beta_s) \mu_s \frac{S(t)I(t)}{N(t)} - (v_s + c) S_q(t) - v_u u_1(t)S_q(t), \]

\[ C D^\sigma_{t_0^+} A(t) = \beta_s(1 - \gamma_s) \mu_s \frac{S(t)I(t)}{N(t)} - (\delta_a + \xi_a + c) A(t) - m_r u_2(t)A(t), \]

\[ C D^\sigma_{t_0^+} I(t) = \delta_a A(t) - (\delta_i + \xi_i + c) I(t) - m_r u_2(t)I(t), \]

\[ C D^\sigma_{t_0^+} I_q(t) = \beta_s \mu_s \xi_q \frac{S(t)I(t)}{N(t)} + \delta_i I(t) - (\xi_q + c) I_q(t) - m_r u_2(t)I_q(t), \]

\[ C D^\sigma_{t_0^+} R(t) = \xi_a A(t) + \xi_i I(t) + \xi_q I_q(t) - c R(t) + v_u u_1(t)(S(t) + S_q(t)) \]
\[ + m_r u_2(t)(A(t) + I(t) + I_q(t)), \]

and the adjoint equations are
Let us take the weighting quantities $a_j = 1$ for $j = 1, 2, 3, 4, 5$ and the time-dependent optimal controls $u^*_1(t)$ and $u^*_2(t)$ are

$$
\begin{aligned}
& u^*_1(t) = \min \left(1, \max \left(0, \nu_r(S(t)\lambda_1(t) + S_\rho(t)\lambda_2(t) - S(t)\lambda_6(t) - S_\rho(t)\lambda_6(t))\right)\right), \\
& u^*_2(t) = \min \left(1, \max \left(0, m_r(S_\rho(t)\lambda_4(t) + S_e(t)\lambda_5(t) + S(t)\lambda_6(t))\right)\right).
\end{aligned}
$$

**Lemma 1.** The following equations are equivalent:

$$
\begin{aligned}
\mathcal{C}D^\alpha_{T_{\max}} \lambda_1(t) &= \frac{\partial H}{\partial y}(t, y(t), u(t), \lambda(t)), \\
\mathcal{C}D^\alpha_{T_{\max}} \lambda(T_{\max} - t) &= \frac{\partial H}{\partial y}(T_{\max} - t, y(T_{\max} - t), u(T_{\max} - t), \lambda(T_{\max} - t)),
\end{aligned}
$$

where $0 < \alpha \leq 1$.

**Proof.** The proof of the above lemma is available in the recent studies.\textsuperscript{35,36} $\square$

From the above lemma, we can rewrite the adjoint Equation (4) in terms of left fractional derivatives

$$
\begin{aligned}
\mathcal{C}D^\alpha_{t_1} \lambda_1(T_{\max} - t) &= \frac{\partial H}{\partial s}(T_{\max} - t, y(T_{\max} - t), u(T_{\max} - t), \lambda(T_{\max} - t)), \\
&= -\left(\beta_i + \gamma_i(1 - \beta_i)\right)\mu_i \frac{I(\lambda_1 - \epsilon)\lambda_1 - \nu_r u_1(T_{\max} - t)\lambda_1(T_{\max} - t)}{N} \\
&+ \gamma_i(1 - \beta_i)\mu_i \frac{I\lambda_1 - \nu_r u_1(T_{\max} - t)\lambda_1(T_{\max} - t)}{N} \\
&+ \frac{I(T_{\max} - t)}{N} \lambda_2(T_{\max} - t) + \beta_i(1 - \gamma_i)\mu_i \frac{I\lambda_2(T_{\max} - t)}{N} \\
&+ \beta_i\mu_i\gamma_i \frac{I\lambda_5(T_{\max} - t)}{N} + \nu_r u_1(T_{\max} - t)\lambda_6(T_{\max} - t),
\end{aligned}
$$

with the terminal conditions

$$
\lambda_i(T_{\max}) = 0, \text{ for all } i = 1, 2, 3, 4, 5, 6.
$$
\[ C_d^r \lambda_2(T_{\text{max}} - t) = \frac{\partial H}{\partial S_q} = v_3 \lambda_2(T_{\text{max}} - t) - (v_3 + \varepsilon) \lambda_2(T_{\text{max}} - t) - v_r u_1(T_{\text{max}} - t) \lambda_2(T_{\text{max}} - t) + v_r u_1(T_{\text{max}} - t) \lambda_2(T_{\text{max}} - t), \]

\[ C_d^r \lambda_3(T_{\text{max}} - t) = \frac{\partial H}{\partial A} = -(\delta_a + \xi_a + \varepsilon) \lambda_3(T_{\text{max}} - t) - m_r u_2(T_{\text{max}} - t) \lambda_3(T_{\text{max}} - t) + \delta_a \lambda_4(T_{\text{max}} - t) \]

\[ + \xi_a \lambda_6(T_{\text{max}} - t) + m_r u_2(T_{\text{max}} - t) \lambda_6(T_{\text{max}} - t) + a_1 \lambda_6(T_{\text{max}} - t), \]

\[ C_d^r \lambda_4(T_{\text{max}} - t) = \frac{\partial H}{\partial I_q} = -(\beta_s + \gamma_s(1 - \beta_s)) \mu_s \frac{S(T_{\text{max}} - t)}{N} \lambda_1(T_{\text{max}} - t) + \delta_i \lambda_5(T_{\text{max}} - t) \]

\[ + a_2 I(T_{\text{max}} - t) + \beta_s(1 - \gamma_s) \mu_s \frac{S(T_{\text{max}} - t)}{N} \lambda_3(T_{\text{max}} - t) \]

\[ + \beta_s \mu_s \gamma_s \frac{S(T_{\text{max}} - t)}{N} \lambda_5(T_{\text{max}} - t) + \gamma_s(1 - \beta_s) \mu_s \frac{S(T_{\text{max}} - t)}{N} \lambda_2(T_{\text{max}} - t) \]

\[ - (\delta_i + \xi_i + \varepsilon) \lambda_4(T_{\text{max}} - t), \]

\[ C_d^r \lambda_5(T_{\text{max}} - t) = \frac{\partial H}{\partial q} = -(\varepsilon + \varepsilon) \lambda_5(T_{\text{max}} - t) + \xi_q \lambda_5(T_{\text{max}} - t) - m_r u_2(T_{\text{max}} - t) \lambda_5(T_{\text{max}} - t) \]

\[ + a_1 I_q(T_{\text{max}} - t) + m_r u_2(T_{\text{max}} - t) \lambda_6(T_{\text{max}} - t), \]

\[ C_d^r \lambda_6(T_{\text{max}} - t) = \frac{\partial H}{\partial R} = -\varepsilon \lambda_6(T_{\text{max}} - t). \]

### 5 | NUMERICAL SIMULATIONS

In this part, we investigate the numerical solution of our model (2) and the optimal control problem by applying forward–backward sweep method. The numerical simulations and graphs are performed in MATLAB. The initial values are \( S(0) = 145,723,849, \ S_q(0) = 200,900, \ A(0) = 50,000, \ I(0) = 19751, \ I_q(0) = 5000 \) and \( R(0) = 500 \). The parameter values of the model (1) by using TRR algorithm are mentioned in the table. The vaccination rate \( v_r \) and treatment rate \( m_r \) are assumed as 0.01 and 0.02, respectively.

To simulate the state and adjoint equations, we apply the following forward–backward sweep algorithm

1. Split the time interval \([t_0, T_{\text{max}}]\) into \( n \) equal subintervals and set \( h = \frac{T_{\text{max}}}{n}, \ t_m = mh, \ m = 0,1, \ldots, n. \)
2. Guess/choose the initial value of the control \( u = (u_k), \ k = 0,1, \ldots, n. \)
3. Using the initial value of \( x \) and \( u \), solve the state Equation (3). The idea is to change the state Equation (3) into Volterra integral equation

\[ x(t) = x_0 + \frac{1}{\Gamma(a)} \int_0^t (t-s)^{a-1} f(s, x(s), u(s)) \, ds, \]

where

\[ x(t) = [S(t), S_q(t), A(t), I(t), I_q(t), R(t)], \]

\[ x_0 = [S(0), S_q(0), A(0), I(0), I_q(0), R(0)], \]

\[ f(t,x(t), u(t)) = \begin{cases} 
\Gamma_3 - (\beta_s + \gamma_s(1 - \beta_s)) \mu_s \frac{S(t)}{N(t)} - \varepsilon S(t) + v_3 S_q(t) - v_r u_1(t) S(t), \\
\gamma_s(1 - \beta_s) \mu_s \frac{S(t)}{N(t)} - (v_3 + \varepsilon) S_q(t) - v_r u_1(t) S_q(t), \\
\beta_s(1 - \gamma_s) \mu_s \frac{S(t)}{N(t)} - (\delta_a + \xi_a + \varepsilon) A(t) - m_r u_2(t) A(t), \\
\delta_a A(t) - (\delta_i + \xi_i + \varepsilon) I(t) - m_r u_2(t) I(t), \\
\beta_s \mu_s \gamma_s \frac{S(t)}{N(t)} + \delta_i I(t) - (\xi_q + \varepsilon) I_q(t) - m_r u_2(t) I_q(t), \\
\xi_q A(t) + \xi_i I(t) + \varepsilon I_q(t) - c R(t) + v_3 u_1(t)(S(t) + S_q(t)) \\
+ m_r u_2(t)(A(t) + I(t) + I_q(t)), 
\end{cases} \]
and then apply the Adams predictor–corrector method as follows

\[
x_{l+1} = x_0 + \frac{h^\alpha}{\Gamma(\alpha + 2)} \left[ f(t_{l+1}, x_{l+1}, w_{l+1}) + \sum_{j=0}^{l} a_{j,l+1} f(t_j, x_j, w_j) \right],
\]

\[
x_{p,l+1} = x_0 + \frac{h^\alpha}{\Gamma(\alpha + 1)} \left[ \sum_{j=0}^{l} b_{j,l+1} f(t_j, x_j, w_j) \right],
\]

where \( l = 0, 1, \ldots, n - 1 \).

FIGURE 2  (A–F) Simulation graphs of the fractional order 2019-nCoV model for \( \alpha = 1, 0.95, 0.9 \) and 0.85 [Colour figure can be viewed at wileyonlinelibrary.com]
4. Use the terminal conditions $\lambda$ and the values of $x$ and $u$, solve the adjoint Equation (5). Similar to step 3, convert the adjoint equation into equivalent integral equation

$$
\lambda(T_{\text{max}} - t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1} \frac{\partial H(T_{\text{max}} - s, x(T_{\text{max}} - s), u(T_{\text{max}} - s), \lambda_{T_{\text{max}}-s})}{\partial x} \, ds
$$

and then apply the predictor–corrector method as follows

$$
\begin{align*}
\lambda_{n-1} &= \frac{h}{\Gamma(\alpha + 2)} \left[ \frac{\partial H(t_{n-1}, x_{n-1}, w_{n-1}, \lambda_{n-1})}{\partial x} + \sum_{j=0}^l a_{j+1} \frac{\partial H(t_{n-j}, x_{n-j}, w_{n-j}, \lambda_{n-j})}{\partial x} \right], \\
\lambda'_{n-1} &= \frac{h}{\Gamma(\alpha + 1)} \left[ \frac{\partial H(t_{n-j}, x_{n-j}, w_{n-j}, \lambda_{n-j})}{\partial x} \right],
\end{align*}
$$

where $l = 0, 1, \ldots, n - 1$. The coefficients of $a_{j+1}$ and $b_{l+1}$ are

$$
a_{j+1} = \begin{cases} 
\frac{h}{\Gamma(\alpha + 1)} \frac{\partial H(t_{n-j}, x_{n-j}, w_{n-j}, \lambda_{n-j})}{\partial x}, & \text{for } j = 0 \\
\frac{(l - j + 2)^{a+1} + (l - j)^{a+1} - 2(l - j + 1)^{a+1}}{\Gamma(\alpha + 1)}, & \text{for } 1 \leq j \leq n
\end{cases}
$$

5. Substitute the new $x$ and $\lambda$ values into the control equation $u$, then continue the same process with the updated $u$ value. Finally, if the values of the variable’s current and previous iterations are the same, then stop the process and take the current values as the solutions. Otherwise, return and repeat the process from step 3.

First, we identify the influence of the fractional order derivative with power $\alpha$ in the Coronavirus transmission model (2). Then we apply controls such as vaccination and treatment to the mathematical model to observe the impact of controls on the coronavirus disease spread in the human population.

In the cluster of Figure 2, all model classes are plotted at fractional orders $\alpha = 1, 0.95, 0.90, 0.85$ in the time range [0, 365]. In the case of $\alpha = 1$, we see that starting from $t = 0$ (January 1, 2022), the asymptomatic (Figure 2C), symptomatic (Figure 2D), and isolated (Figure 2E) 2019-nCoV infectious population increases till $t = 50$ and then starts to decrease for the later time period, which exactly fits to the last range of the given data set of Russia ($t = 120$; April 30, 2022). The predictions by using fractional orders are also given with different colors to capture the behavior of the infection under memory effects. Here we notice when the order of the derivative operator decreases then the infection peaks are shifted to the right side of the time scale, which shows the possibility that the diseases can take more time to disappear from the population. Variations in the population of recovered individuals can be observed from the Figure 2F.

Now we check the role of some specific parameters by changing their original values to see how those parameters are sensitive for the proposed model. For the integer order case (i.e., $\alpha = 1$), we modify the values of the parameters $\gamma$, $\delta$, and $\xi$ from 0.3 to 0.7 to explain the sensitivity of the model. The number of infectious people $I(t)$ decreases as the $\gamma$, $\delta$, and $\xi$ parameter values increase. From Figure 3, we see that when we change the quarantined rate $\gamma_q$ of susceptible individuals from $\gamma_q = 0.4$ to $\gamma_q = 0.7$ then the peak of the infectious population goes down. In Figure 4, variations in the infectious class

**FIGURE 3** Change in the infectious population $I(t)$ when the quarantined rate of susceptible individuals $\gamma_q$ varies [Colour figure can be viewed at wileyonlinelibrary.com]
From the given solution of the fractional order model (2), we see that the model fits well with the real data of Russia and gives accurate outputs for the given time range. But we notice that there is no specific optimal control considered yet to reduce the rate of the infection. Now, we use the following strategies to examine the importance of controls in the Coronavirus pandemic in Russia.

**FIGURE 4** Change in the infectious population \( I(t) \) when the rate of infected to isolated humans \( \delta_i \) varies [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 5** Change in the infectious peoples \( I(t) \) when the recovery rate of infectious individuals \( \xi_i \) varies [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 6** Change in the recovery class \( R(t) \) when the recovery rate of infectious individuals \( \xi_i \) changes from \( \xi_i = 0.3 \) to \( \xi_i = 0.6 \) can be observed.

\( I(t) \) when the rate of infected to isolated humans \( \delta_i \) changes from \( \delta_i = 0.4 \) to \( \delta_i = 0.7 \) are plotted. In Figure 5, variations in the infectious class \( I(t) \) when the recovery rate of infectious individuals \( \xi_i \) changes from \( \xi_i = 0.3 \) to \( \xi_i = 0.6 \) are plotted. From Figure 6, the variations in the recovery class \( R(t) \) when the recovery rate of infectious individuals \( \xi_i \) changes from \( \xi_i = 0.3 \) to \( \xi_i = 0.6 \) can be observed.
FIGURE 7  Infectious individuals $I(t)$ with and without controls at $\alpha = 1$ (A) and 0.95 (B) [Colour figure can be viewed at wileyonlinelibrary.com]

5.1  |  Strategy A: Using only vaccination control (i.e., $u_1 \neq 0$ and $u_2 = 0$)

In this case, we use only vaccination control in both susceptible individuals $S(t)$ and susceptible quarantined individuals $S_q(t)$ to reduce the spread of Coronavirus outbreak in Russia. In Figures 7 and 8, we can easily understand that the vaccination control $u_1(t)$ is more effective than the treatment control $u_2(t)$. If we can increase the vaccination rate $v_r$ in susceptible and susceptible quarantine individuals, then it is evident that the spread of Coronavirus pandemic in Russia is under control.

5.2  |  Strategy B: Using only treatment control (i.e., $u_1 = 0$ and $u_2 \neq 0$)

In this case, we assume that the vaccination control is not available and treatment control is the only option to minimize the all infected population $A(t), I(t)$ and $I_q(t)$. In Figures 7 and 8, we can observe that the treatment $u_2(t)$ reduce the infectious population very well compare with without controls but not with vaccination control $u_1(t)$. Similarly, if we can increase the treatment rate $m_r$ in $A(t), I(t)$ and $I_q(t)$, then we can reduce the transmission of the Coronavirus disease.

5.3  |  Strategy C: Using both vaccination and treatment controls (i.e., $u_1 \neq 0$ and $u_2 \neq 0$)

In this case, we use both vaccination and treatment control in human population. It is evident that from Figures 7 and 8 if we apply vaccination and treatment controls, then it is reducing the infectious individuals $I(t)$ and it is more effective than the single control strategies A and B.
Infectious individuals $I(t)$ with and without controls at $\alpha = 0.90$ (A) and $0.85$ (B) [Colour figure can be viewed at wileyonlinelibrary.com]

From the given graphical interpretations, we observe that the proposed Caputo fractional derivative generates more varieties in the outputs and performs well to understand the dynamics of 2019-nCoV in Russia for current and upcoming days.

6 | CONCLUSION

In this paper, we have successfully studied the outbreaks of 2019-nCoV in Russia by using a nonlinear mathematical model in the sense of integer and fractional order operators. For performing the parameter estimation based on the real data of Russia, we have used the TRR algorithm. An optimal control problem has been derived to find some possible controls for reducing the transmission of 2019-nCoV. We have implemented a forward–backward sweep method to solve our proposed model and control problem. For the clear visualization, a number of graphs have been plotted. The following points are concluded from the given analysis: (i) The proposed model fits well with the given real-data set of Russia. (ii) The generalization of the integer order model into fractional order sense by using Caputo fractional derivative generates more varieties in the simulations. (iii) The fractional order values of the given operator identified a possible delay in the reduction of 2019-nCoV cases in Russia in the future. (iv) In the optimal control problem, the vaccination control is highly effective compared to general treatment control. The inclusion of combined control gives the best results to decrease the infection. (v) From the graphical outputs, we predict that very soon the 2019-nCoV epidemic will be under control in Russia.

In the future, the given model can be further utilized to simulate the 2019-nCoV dynamics in any other country. Also, for the same data set, any other fractional derivative can be used to predict the outcomes of the model.
CONFLICT OF INTEREST

This study does not have any conflicts of interest.

AUTHOR CONTRIBUTION

M. Vellappandi: Conceptualization, formal analysis, investigation, resources, visualization, software, and writing-review & editing. Pushpendra Kumar: Conceptualization, investigation, formal analysis, resources, visualization, and writing-original draft. Venkatesan Govindaraj: Investigation, software, formal analysis, and writing-review & editing.

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