Spinfluid Phase Transitions

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Abstract
We start with the spinfluid: a nearly-homogeneous, 8-spinor medium, with small local spin eddies and twists. As it expends, these seed a raft of intersecting codimension $J$ singularities: a spinfoam, $\Sigma$. As $\Sigma$ expands, the energy trapped in each $(4-J)$ brane varies as $\gamma^J$, where $\gamma(t) \equiv \frac{\alpha(t)}{n_j}$ is the scale factor. Summing on $J = (0,1,2,3,4)$ creates a quartic dilation potential $V(n_J; \gamma) \sim n_J \gamma^J$, with either 1 or 2 minima: preferred length and mass scales. $\Sigma$ expands forever for $n_4 < 0$, but recontracts for $n_4 > 0$.

To quantize $\Sigma$, we take a canonical ensemble $\hat{\Sigma}$ of spinfoams, with mean $J$-brane populations $N_J$ immersed in a heat bath of $N_0$ vacuum spinors, whose microstates vastly outnumber the matter states. Its evolution is governed by a free energy $g(N; \gamma)$. This admits phase transitions at two critical scales, $\gamma_1$ and $\gamma_3$, separated by a triple point, $\gamma_2$. Their critical droplets correspond to the varieties of leptons and hadrons, respectively. We identify $\gamma_1$ as inflation, $\gamma_2$ as decoupling, $\gamma_3$ as baryogenesis; and the heat bath of vacuum spinors as dark energy.

1 The Spin World

Could all the structures we see today evolve from a nearly-homogeneous, maximally-symmetric spinfluid? Could the length and mass scales of particles, organisms, galaxies and the expanding universe emerge from a conformally-invariant action?

We show here how conformal-symmetry breaking can occur, provided that the initial state contains topological dislocations; phase singularities of different codimension, $J = (0,1,2,3,4)$. We employ the same 8-spinor action that gave us the particles as singularities and the fields in the regular regime surrounding them; but the results hold for the class of models with projective singularities.

Does this mean that the fields and singularities of today’s world must be ”programmed in” on some initial surface, as in holographic models [Banks, Fischler]; [Hertog et.al.]?
This is like asking if exactly the right shapes of dust motes must be present to nucleate the snow crystals in a snow storm. Any set of initial shapes, followed by almost any microscopic fluctuations in updrafts and vapor pressure, will radiate an enormous variety of snowflakes near the critical point for the phase transition.

We find two phase transitions in the spinfluid vacuum here, which we identify as inflation and baryogenesis.

This is the 2nd paper in a 3-part series. In part 1) [M.C.6], we discovered that the families of elementary particles correspond to the classes of codimension-J singularities or caustics, \((p,q,r)_J^d\) in a spin\((4,\mathbb{C})\) phase flow, and their symmetry types to images in 3-mirror Kaleidoscopes with dihedral angles \((\frac{\pi}{p},\frac{\pi}{q},\frac{\pi}{r})\). Their rest energies, \(m_s = (\frac{s}{2})^3\), appeared in terms their multiplicities, \(s\): the number of reflections it takes to close a cycle of null zigzags. These calculated rest energies agreed with the observed particle masses to within a few percent. The starting point there was the Spin Principle, Pl. Spinors, the fundamental spin-\(1/2\) representations of spin isometries, are the primary physical objects. Spacetime, geometry, gauge, and matter fields, along with their interactions, emerge as spin tensors in 4 left and 4 right chirality spinors: 8 in all, because it takes 8 spinors to make a natural 4 form.

But spinors are lightlike. How can lightlike rays make a massive particle?

Mass is the ability of energy to stay in one place—i.e. for its lightlike chiral components to weave a timelike worldtube, instead of zooming off at the speed of light. The key is that counterpropagating left and right helicity spinors can make a standing wave, if their 3-momenta cancel, but their energies add. Counterpropagating chiral pairs with opposite helicities but the same spins make massive particles with spin, like electrons, \(e_- \in (l_- \oplus r_-)\), and positrons, \(e^+ \in (l^+ \oplus r^+)\). Copropagating pairs with the same helicities make massless particles, like neutrinos, \(\nu_e \in (l_+ \oplus r_-)\) and photons, \(\gamma \in (l_+ \oplus r_-)\). A reaction like pair annihilation is just an exchange of chiral partners in the 8-spinor basis:

\[
e_- \otimes e_+ = (l_- \otimes r_-) \otimes (l^+ \otimes r^+) \rightarrow l_- \otimes r^+ \otimes r_- \otimes l^+ = \gamma \oplus \gamma.\]

On a microscopic scale, each spinor or cospinor propagates along a piecewise lightlike ray segment: a "zig" outward in the direction of cosmic expansion, \(\Delta T > 0\), or a "zag" inward, towards the "big bang", \(\Delta T < 0\):

\[(\Delta T, \Delta x) \in [r(1) + su(2)]_{\text{diag}} : |\Delta x| = \pm \Delta T = \Delta t,\]

where \(t\) is arctime: the arclength parameter along rays, and \(T\) is cosmic time: the logradius in \(R_4\), or "imaginary time".

Quantum Field theory is statistical mechanics in imaginary time. The statistical mechanics of the ensemble of null zigzags histories is greatly simplified when the local system is immersed in a heat bath with a vastly greater number of states: a stochastic background of vacuum spinors, with temperature \(\beta^{-1} \sim \hbar\). Instead of summing over creations and annihilations of intermediate particles,
sum over the microhistories of null zigzags on a lightlike lattice in "imaginary

time", \( T \), connecting initial and final states.

To compute these path integrals we start with a Lagrangian that is a natural
(external) transformations: translations, rotations, boosts, and P (space) and
T (cosmic time) reversal- together with their spin space (internal) representa-
tions. It takes 8 spinors - 4 column spinors, \( \psi_I = \{ l^+, r^+, l^-, r^- \} \), and 4
provisionally independent row spinors, \( \psi^I = \{ l^+, r^+, l^-, r^- \} \), each of conformal
weight (dimension) \( \frac{1}{2} \), to make a natural 4 form in
spin space: the 8-spinor
position-velocity phase space \( (\psi, d\psi) \).

In complex coordinates on the cotangent
bundle, \( \Psi = \bigcup_{I=1}^{4} (\psi^I d\psi^J), (\psi_I + id\psi_J) \in \mathbb{C}T^*\psi \),

where \( d \equiv e^\alpha (x) \partial_\alpha (x) \) is the generalized exterior differential operator.

The simplest natural 4 form, with only one kind of term and no coupling
constants, is the 8-spinor factorization of the Maurer-Cartan 4 form; the E-
invariant (scalar) measure on phase space [M.C.1]:

\[
\mathcal{L}_g \equiv \bigwedge_{I,J=1}^{4} (\psi^I d\psi^J) \equiv (\psi^I d\psi^J)^\wedge^4.
\]

Its action gives the volume in spin space:

\[
S_g = \int (\psi^I d\psi^J)^\wedge^4 = \int r^\pm d\ell^\mp \wedge \ell^\pm dr^\mp \wedge r^\mp d\ell^\pm
\]

(sum over neutral sign combinations: i.e. whose \( \pm \) signs sum to 0). In the
stationery regime, the cospinors are the Dirac (P) conjugates of the spinors:
\( \psi^I \to \psi^I \sigma_2 \). The potential energy, \( V \), is minimized when each spinor \( \psi_I \)
pairs with a \( P-\)conjugate cospinor \( \psi^I \). It is this attraction between opposite-chirality
spinors that stabilizes particles.

Spacetime enters as the parameter space for the action integral, \( S_g \). \( S_g \) is
stationarized in either the PT symmetric (gravitostrong) or PT antisymmetric
(electroweak) case; in both, it gives the covering number of the compactified
internal group, \( g = [U(1) x SU(2)] \), over a compactified world-tube, or 4-brane,
\( B_4 \sim S_1 \times S_3 \):

\[
S_g \to \frac{1}{2} \int_{B_4} \text{Tr} [g^{-1} dg] = 16\pi^3 N
\]

In the regular regime, \( S_g \) yielded the proper effective actions for electroweak
and gravitostrong fields, when all 8 spinor fields ride on a nontrivial global
background of vacuum spinors: the null spinors on compactified Minkowsky
space, \( M_- \sim S_1 \times S_3 \). Accordingly, we expand each of the 8 spinor
fields here as a sum of a global, order-\( k^\pm \) vacuum (\( \mathcal{g} \)) distribution, plus a local
perturbation, or envelope modulation:
\[ \ell^{\pm}(x) = k^{\pm \lambda^{\pm}}(x) + \xi^{\pm}(x) \]
\[ r^{\pm}(x) = k^{\pm \rho^{\pm}}(x) + \eta^{\pm}(x) \]
\[ \ell^{\pm}(x) = k^{-\frac{1}{2}}\lambda^{\pm}(x) + \xi^{\pm}(x) \]
\[ r^{\pm}(x) = k^{-\frac{1}{2}}\rho^{\pm}(x) + \eta^{\pm}(x) \].

In paper 1 [M.C.6], we classified the Lagrangian singularities, or spin caustics \( \gamma_{4-J} \) of codimension \( J = (1, 2, 3, 4) \): focal loci where rays cross in the spacetime projection of the 8-spinor flow, and geometrical optics breaks down [M.C.2]. Caustics come in the discrete families, with the symmetry types in internal space of images in 3-mirror Kaleidoscopes, with dihedral angles \( \left( \frac{\pi}{p}, \frac{\pi}{q}, \frac{\pi}{r} \right) \); the Coxeter groups, of multiplicity-s:

\[ \gamma_{s} \equiv (p, q, r)_{s}. \]

The multiplicity, \( s \), or Coxeter number, is the minimal number of reflections it takes to close a zig-zag cycle of rays.

The surprising discovery in paper 1 was

D1. Not only do the types of 8-spinor caustics parallel the families of leptons \( (J = 1) \), mesons and photons \( (J = 2) \), and hadrons \( (J = 3) \) respectively, but their multiplicities \( s \) give their correct masses: \( m_{s} = (\frac{\xi}{3})^{3} \) (within a few percent).

For the electron, \( e^{-} \equiv (l- \oplus r-) \), \( s = 2 \); the muon, \( s = 6 \), and the tauon, \( s = 30 \). (see table 2, in paper 1).

The key to the correspondence of caustics and particles lies in the Dirac propagator: the sum over null zigzags connecting the initial and final states [Ord]. A null zig zag is a double reflection \( l \rightarrow r \rightarrow l \) of (lightlike) spin rays (characteristics of the massless Dirac operators, \( D \) and \( \overline{D} \)) off Clifford mirrors: interference patterns with the 6 remaining vacuum spinor fields. Penrose [Pen] called these reflections mass scatterings. Mass scatterings at the boundary are what keep null zigzags confined to a world tube in cosmic time, \( T \)-i.e. what give a bispinor wavefunction mass.

The statisticalcal mechanics of matter envelopes in "imaginary time", \( T \), is the quantum field theory of particles in "real time", \( t \). Since the vacuum spinors - the dark matter and energy, make up about 90% of the total energy of the universe, the stochastic vacuum acts like a heat bath for the matter spinors, which ride on the vacuum like waves on the surface of the ocean. We derive a free energy, or dilation potential \( g(N; \gamma) \), for the vacuum seeded homogeneously with \( N_{J} \) topological defects of codimension \( J = (1, 2, 3, 4) \) which has either 1 or 2 minima: preferred length and mass scales. This dilation potential will act as an effective Higgs field, whose hills and valleys are sculpted by the nonlocal effects of all the topological defects in compactified spacetime: the "distant masses" of Mach’s principle.
2 Topological Trapping of Currents on Dual Branes

The flux integral,
\[ \int_{\gamma_2} K_{0r} e^0 \wedge e^r \equiv \int_{\gamma_2} *K = 4\pi N, \]
seems to quantize electric field, \( K_{0r} \), over dual (transverse) surfaces, \( S_{\theta\phi} \), spanned by the surface element \( e^0 \wedge e^r \).

Now the magnetic field, \( K_{\theta\phi} \), is quantized over spatial 2 surfaces by deRham cohomology. Why should the dual field be quantized?

Every contribution to \( S_g \) must be a Clifford \((C)\) scalar, built of all 8 spinors. The matter bispinors make the \( C^- \) valued \( J \)-form current, \( \omega_J \). This must be multiplied by some \((4 - J)\) form, \( \Omega^{4-J} \), that is both \( C \) and Hodge dual to it to get a \( C^- \)-scalar \((\sigma_0)\) valued 4 form- the only kind that can be invariantly integrated. Where does this dual come from?

The key is quantization of the topological action \([M.C.2]\),
\[ S_g = \int_{M_g} \Omega^4 = 16\pi^3 N. \]

Table I (below) shows how the vacuum spinors on \( S_1 \times S_3 \) automatically provide \((4 - J)\) surface elements dual to any \( J^- \) bispinor matter current; the ones needed to quantize the normal flux.

| **Table I** |
|-------------|
| **Vacuum Spin Forms:** Exterior Products \( \hat{\Omega}^J \) of the vacuum spin connections on \( S_1 \times S_3 \) with compactification radius \( a = \gamma a_\# \), and Minkowsky metric. The upper sign on \( \sigma_j \) is the \( L \)-sign, the lower sign, the \( R \). The upper and lower signs on \( \sigma_0 \) apply to the analytic and conjugate-analytic representations. |
| \( \hat{\Omega} = \pm \left( \frac{ik}{2a_\#} \right) \sigma_\alpha e^\alpha \) |
| \( \hat{\Omega}^2 = \left( \frac{ik}{2a_\#} \right)^2 \sigma_\ell \left[ \epsilon^{jk} e^j \wedge e^k \pm e^0 \wedge e^\ell \right] \) |
| \( \hat{\Omega}^3 = \pm \left( \frac{ik}{2a_\#} \right)^3 \sigma_\ell \epsilon^{jk} e^j \wedge e^k \wedge e^0 \pm i\epsilon^{jkl} \sigma_0 e^j \wedge e^k \wedge e^\ell \) |
| \( \hat{\Omega}^4 = \left( \frac{ik}{2a_\#} \right)^4 \sigma_0 \left[ \epsilon_{\alpha\beta\gamma\delta} e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta \right] = \frac{3}{2} \left( \frac{i^4}{a_\#^4} \right) d^4v \) |

The vacuum spin forms, \( \hat{\Omega}^{(4-J)} \), make Clifford line, surface, and volume elements dual to the \( J \) pairs of matter spinors and \( PT \)-conjugate differentials \( (\psi^I d\psi_I)^J \); the ones they multiply to give the Clifford scalar volume element \( \sigma_0 e^0 \wedge e^1 \wedge e^2 \wedge e^3 d^4v \).
For stationery solutions, our 8-spinor Lagrangian reduces to the Maurer-Cartan 4 form - the volume form in \( U(1) \times SU(2) \), whose action is topologically quantized over \( M_\# \sim S_1 \times S_3 \).

Alternatively, as Witten has pointed out, the Weiss-Zummo 4 form may be quantized over the boundary, \( \gamma_4 \sim \partial B_5 \) of the 5-manifold, \( B_5 \sim \mathbb{C} \times S_3 \), obtained by complexifying time: \( s^0 \equiv t + iT \in \mathbb{C} \) — and then imposing periodic (or matched asymptotic) boundary conditions on the inertial and final hypersurfaces [Witten 1,2]:

\[
\int_{\partial B_5} \zeta (d\zeta)^4 = 32\pi^4 N.
\]

The intensity, \( k \), of each chiral pair of vacuum spinors must scale as \( k \sim \gamma^{-1} \) for the action of the 4 global pairs, quantized over volume

\( V (T) = \gamma^4 V (0) \), to remain constant. Then the integral of the \( J \) matter-spinor pairs, \( \Omega^J \), over their singular loci, \( B_J \), remains constant, while the dual vacuum spin forms \( \hat{\Omega}^{4-J} \), contribute a factor of \( k^{4-J} \sim \gamma^{4-J} \). After multiplying by the uniformly expanded 4-volume element, \((dV)^4 = \gamma^4 [d\nu]^4\), this gives a net factor of \( \gamma^J \) in the action contributed by the \( D^J \) stratum:

\[
\begin{align*}
J = 0 \text{ (vacuum spinors);} & \quad \int_{\hat{\Omega}^4} = 16\pi^3 n_0; \\
J = 1 \text{ (leptons);} & \quad \sum_{D^1} \int_{S_3 \times \gamma_1} \hat{\Omega}^3 \wedge \hat{\Omega}^1 \\
& \quad -i8\pi^2 \gamma \sum_{D^1} 2\pi m = 16\pi^3 n_1 \gamma \\
J = 2 \text{ (photons; mesons);} & \quad \sum_{D^2} \int_{S_2 \times \gamma_1} \hat{\Omega}^2 \wedge \hat{\Omega}^2 \\
& \quad = 4\pi \gamma^2 \sum_{D^2} 4\pi \eta \equiv 16\pi^3 n_2 \gamma^2 \\
J = 3 \text{ (hadrons);} & \quad \sum_{D^3} \int_{S_1 \times \gamma_1} \hat{\Omega}^1 \wedge \hat{\Omega}^3 \\
& \quad = i2\pi \gamma^3 \sum_{D^3} i8\pi^2 \beta \equiv 16\pi^3 n_3 \gamma^3 \\
J = 4 \text{ (spin (4, \mathbb{C}) vortices; atoms);} & \quad \sum_{D^4} \int_{\gamma_4} \hat{\Omega}^4 = \sum_{D^4} 16\pi^3 n_4 \gamma^4
\end{align*}
\]

The sum of terms (??) gives the net action expressed in powers of \( \gamma \) (??):

\[
\mathcal{S}_g (\gamma) = \int_{B_6} \hat{\Omega}^4 + \gamma \int_{B_3} \hat{\Omega}^4 \wedge (\psi^I d\psi_I) + \gamma^2 \int_{B_2} \hat{\Omega}^2 \wedge (\psi^I d\psi_I)^2 \\
+ \gamma^3 \int_{B_2} \hat{\Omega} \wedge (\psi^I d\psi_I)^3 + \gamma^4 \int_{B_1} (\psi^I d\psi_I)^4 \\
= 16\pi^3 \left[ n_0 + n_1 \gamma + n_2 \gamma^2 + n_3 \gamma^3 + n_4 \gamma^4 \right].
\]

The Euclidean action integral (1) over \((T, \mathbf{x})\) serves as a Lagrangian

\[
\left( \gamma, \dot{\gamma} \right) = T (\dot{\gamma}) - V (\gamma),
\]

6
governing the evolution of the scale factor \( \gamma(t) \) in Minkowsky time: the "many-fingered" time parameter common to all particle trajectories and piece-wise null rays (characteristics), \(|\Delta T| = |\Delta x| = \Delta t\).

The potential energy,

\[
V(\gamma) = - \left[ n_0 + n_1 \gamma + n_2 \gamma^2 + n_3 \gamma^3 + n_4 \gamma^4 \right],
\]

contains the standard pressure terms. The radiation pressure, \( p_r \), varies as \( \gamma^{-4} \). Its 4-volume integral is contained in the constant \( (n_0) \) term: the total energy stored in the background radiation: cosmic neutrinos and photons. The matter pressure, \( p_m \), varies as \( \gamma^{-3} \), and contributes to the linear \( (n_1) \) term.

The kinetic energy, \( k(\dot{\gamma}) \), of a Friedmann 3-brane expanding in the ambient space comes from the balance between the analytic (+) and conjugate analytic (-) spin waves, whose interference pattern strikes off this 3-brane physically as the level surface of the \( U(1) \) phase, \( \theta^0 \equiv \theta^0_+ + \theta^0_- = 0 \): the bulk-neutrality condition for the background \( u(1) \) flux, as measured in a coexpanding frame. Here

\[
\zeta^0_\pm \equiv (\theta^0_+ + i \phi^0_+ \pm i \phi^0_-) (t \pm iT)
\]

are the complex phase factors of the analytic (+) and conjugate-analytic (-) spinors respectively. They obey the Cauchy-Riemann conditions

\[
\partial_t \phi^0_\pm = \mp \partial_T \theta^0_\pm; \quad \partial_T \phi^0_\pm = \pm \partial_t \theta^0_\pm.
\]

Bulk-neutrality, together with analyticity, gives a net dilation exponent of

\[
\varphi \equiv \frac{1}{2} [\varphi^0_+ - \varphi^0_-] = \varphi^0_+ = -\varphi_-
\]

for our Friedman 3-brane solution, \( S_3(t) \). It is governed by the effective Lagrangian

\[
L(\varphi, \dot{\varphi}) = \varphi + \ln P(\varphi),
\]

where

\[
P(\varphi) = n_0 e^{-2(\varphi)} + n_1 e^{-\varphi} + n_2 + n_3 e^{\varphi} + n_4 e^{2\varphi}.
\]

Varying \( L \) with respect to \( \varphi \), we obtain the differential equation governing the Minkowsky-time evolution of our expanding shell, \( \dot{M} = (\dot{T}(t) , S_3(t)) \):

\[
\dot{\varphi} = -\partial_\varphi \ln P(n; \varphi);
\]

\[
P(n; \varphi) = n_0 e^{-2\varphi} + n_1 e^{-\varphi} + n_2 + n_3 e^\varphi + n_4 e^{2\varphi},
\]

where \( (\varphi^I + \varphi^T) \equiv \varphi(n; t) \) is the net dilation exponent. Here \( n \equiv (n_0, n_1, n_2, n_3, n_4) \), the population vector, is the number of cells in each dimension. We consider \( n \) constant first. Below, we assume that \( n(\gamma) \) adjusts more quickly to \( \gamma \) than \( \gamma \) can change in \( t \) — i.e. that \( n \) is "slaved" to \( \gamma(t) \). A more detailed model could include the connectivity of the cells, say written as an incidence matrix. Like a raft of soap bubbles, the connectivity of the complex could change with \( t \) as
tubes $\gamma_4$, $\gamma_3$, $\gamma_2$, $\gamma_1$, and $\gamma_0$ divide and reconnect. But there is a constraint: the alternating sum of the coefficients, or the index of the Spin$^c$-4 complex,

$$I = 16\pi^3 [n_0 - n_1 + n_2 - n_3 + n_4],$$

is a homotopy invariant, that must be conserved over $t$ evolution.

The dilation rate $\varphi$ oscillates in the effective potential well:

$$U(n; \varphi) \equiv -\ln \left[ n_0 e^{-2\varphi} + n_1 e^{-\varphi} + n_2 + n_3 e^{\varphi} + n_4 e^{2\varphi} \right] \equiv -\ln P(n; \varphi).$$

The potential gradient, $\partial \gamma U$, will pull $\varphi(t)$ downhill in $U(\gamma)$, to end up in a local minimum; or in a global minimum, if there is enough kinetic energy $\dot{\varphi}(t)$ to "coast" over any intervening uphill sections.

For example, for cell population vector $n = (e^{-4}, e^{-1}, 1, e^{-1}, e^{4})$, the system has a stable equilibrium at $\varphi_s \simeq -2.35$, $U_s \simeq -0.135$, and an unstable equilibrium at $\varphi_u \simeq 2.35$, $U_u = 1.518$ (Figure 1).

The first equilibrium, $\varphi_s$, represents a classical soliton; a standing wave bound state inside its own potential well. Physically, it is a 3-sphere $S_3(a_s)$, covered $n_0$ times by $SU(2)_L \times SU(2)_R$, for which the attractive force between hadrons ($n_3$) is stably balanced by the quantum-mechanical preference of leptons ($n_1$) for delocalization. This topologically-nontrivial field configuration cannot be captured by ad-hoc cutting and pasting of general relativity and quantum field theory; it lives within the domain of the unified theory, which gives rise to them both.

We expect oscillations about the local minimum $\varphi_s$. But if $\varphi$ exceeds local maximum $\varphi_u$, the state will escape its basin of attraction and $\varphi$ will inflate rapidly to the next branch, which is attractive for $n_3 > 0$. For $n_4 < 0$, this branch is unstable, and the radius expands forever. For $n_4 \geq 0$, however, the state gets "caught" in the basis of a second stable equilibrium.

We show below that the statistical ensemble of Spin$^c$-4 complexes exhibits an inflationary phase transition at the critical radius $a_c = a_\#$, before the classical system becomes unstable. As $a$ increases further, there is a "critical cascade" of $J = (1, 2, 3, and 4)$ pairs of matter spinors condensing on the codimension-$J$ singular loci, $D^J$.

### 3 Pattern Nucleation Near the Critical Points

Quantum mechanics is statistical mechanics in "imaginary time," i.e. cosmic time: the logradius $T = a_\# \ln \gamma$ of our spatial hypersurface.

$T$ combines with Minkowsky time, $t$, to make complex time,

$$z^0 \equiv t + iT.$$

They are related by the "Wick rotation", $W$; analytic continuation to imaginary time, $T$.
\[ W : t \rightarrow T \Rightarrow e^{-\beta E_S} \rightarrow e^{\hbar^{-1} S_P}; \]
\[ E = T(\dot{\gamma}) + V(\gamma) \rightarrow -T(\dot{\gamma}) + V(\gamma) \equiv -S(t, x). \]

But wait! This mathematical "sleight of hand", \( W \), obscures the underlying physics:

the interaction of every local system with an invisible heat bath of vacuum energy at "temperature" \( \beta^{-1} = \) \( \| \), that contains vastly many more microstates than any local system.

This enormously simplifies the sum over microstates, because we need only count the microstates \( \delta\hat{\Omega} \) gained or lost by the vacuum in exchanging a 4-dimensional energy increment, \( \delta\hat{E} \), with the local system.

By definition, \( \beta \equiv \frac{\delta \ln \hat{\Omega}}{\delta \hat{E}} \Rightarrow \frac{\delta\hat{\Omega}}{\delta\hat{E}} = \beta\hat{\Omega}, \)

where \( \delta\hat{E} \) is the incremental change in the energy of the heat bath need to create \( \delta\hat{\Omega} \) new vacuum states. In the continuum approximation,

\[ \frac{\delta\hat{\Omega}}{\delta\hat{E}} = -\beta\hat{\Omega} \Rightarrow \hat{\Omega}\left(\hat{E}_0 + \delta\hat{E}\right) = \hat{\Omega}\left(\hat{E}_0\right) e^{\beta\delta\hat{E}} \]
\[ = \hat{\Omega}_0 e^{-\beta E_S} \approx \hat{\Omega}\left(\hat{E}_0\right)\left(1 + \beta\delta\hat{E}\right), \]

since \( \delta E_s = -\delta\hat{E}. \)

However, states really come in integral units:

\[ \delta\hat{\Omega} = 1 = \beta\delta\hat{E} \Rightarrow \delta\hat{E} = \beta^{-1} = \hbar \rightarrow \delta S_P \]

The quantum of action is the Euclidean energy needed to create one more vacuum state; because 4-energy is action under analytic continuation to Minkowsky time,

\[ W : \beta\delta\hat{E} \rightarrow \hbar^{-1} \delta S_P. \]

Action is quantized because vacuum states are discrete; as are the microscopic states of any localized system, say, \( \Psi(t) \equiv \Psi(T, x; t) \): a microhistory of the wave-function on the lattice, \( (T, x) \in N \), as it evolves in arctime, \( t \). Wick rotation maps the energy \( E_C \) of a 4-dimensional state \( \varphi(T, X) \) to the action \( S(\varphi) \) for a path: a microhistory, \( \psi(t) \), parametrized by Minkowsky time, \( t \).

Now the macroscopic world seems to behave like the average over the ensemble, \( \sum_C \), of possible microstates compatible with the macroscopic state. If there are competing states- in this case macroscopic histories—it’s not just the energy \( E \) of each state, but also its entropy

\[ S \equiv k \ln \Omega, \]

the log of the number \( \Omega \) of microscopic realizations in its ensemble, that determines which macrohistory it will choose.
Energy and entropy combine in the *Gibbs free energy*

\[ G \equiv E + pV - TS \]

which is minimized at equilibria: \( dG = 0 \). It is the *entropy* term, \(-TS\), that distinguishes the statistical mechanics of an ensemble from the classical mechanics of any of its members. States flow downhill in \( G \); \( \Delta G \leq 0 \), in any spontaneous process, because \( \Delta S = k \Delta \ln \Omega > 0 \), because a given energy increment transferred from system to *vacuum* may fill many more possible vacuum states than the number of states lost to the system.

For a massive bispinor particle, the microhistories are *null zig-zag* paths, whose *edges* are segments of the light-like world lines of \( L \)- and \( R \)-chirality spinors, and whose *vertices* are mass-scatterings with the vacuum spinors. These edges \( \gamma_1 \) bound surfaces \( \gamma_2 \), which bound volumes \( \gamma_3 \); these bound world tubes, \( \gamma_4 \). We call the union of all of these simplices a *Spin\(^{L \cup R} \)-4 complex*. Each complex can be inscribed on a *null lattice*, \( N \), with null edges \( \gamma_1 \). The particle *propagator* is the sum over all null zig-zag paths connecting the initial and final vertices.

The count of possible histories of particles and interactions thus behaves as if each were restricted to a piecewise-null lattice \( | \Delta x \Delta T | = 1 \), with each cycle supporting integral holonomy of the \( u(1) \oplus su(2) \) phase in its spacetime projection,

\[
\Pi : z(t) \rightarrow \gamma_1(t) \equiv [T(t), x(t)].
\]

\[
\int_{\gamma_1} P = \int_{\gamma_1} P_\alpha e^{-\alpha} - \int P \cdot dx = \int_{\gamma_1} \left( T \partial_T \zeta - \partial_\zeta \right) dt = \int_{\gamma_1} d\zeta \equiv \Delta \zeta = n = 2\pi n \hbar
\]

\[
[N \equiv n + m; \ \text{an integer}].
\]

This *Integral Holonomy condition* enables wavefunctions to be *single-valued* on the null lattice, \( N \).

Now each macroscopic history, \( C \) admits an ensemble of microhistories, \( s \), with different cell *counts*, \( n_j \). We have assumed that all the microcells of codimension \( J \) contribute *equally* to the potential energy, \( V^s(\gamma) = -[n_j \gamma^J] \), of each state.

Each \((4 - J)\) cell \( B_{4-J} \) with energy \( \epsilon_J \) contributes a probability weight of \( e^{-\beta \epsilon_J} \) to the ensemble average. A complex, \( s \), containing \( n_j^s \) \( J \) cells has probability weight \( e^{-\beta n_j^s \epsilon_J^s} \).

We assume *equipartition*: \( T^s = V^s \), so

\[
e^s(\gamma) = -2n_j^s \gamma^J.
\]

Summing on \( J = (0, ..., 4) \), and over all the microhistories \( s \) in the ensemble, we obtain the *partition function*,

\[
Z \equiv \sum_s e^{-\beta (n_j^s \epsilon_j^s)}.
\]
Since the energy is extensive, the partition function factors:

\[ Z = \frac{\zeta_0 \zeta_1 \cdots \zeta_4}{N_0! N_1! \cdots N_4!}, \]

where \( \zeta_J = \sum e^{-\beta \epsilon_J} \).

Here \( N_J \equiv n_J = \beta^{-1} \frac{\partial \ln Z}{\partial n_J} \) is the mean (ensemble-average) population of the \( J \)th stratum.

The calculation now goes just like the one for reacting chemical species in equilibrium, except that conservation of atoms is replaced by conservation of the Index, \( I = (-1)^J N_J \), as \( \gamma \) grows and the simplices "react" to form a new minimal spin complex:

\[ \sum (-1)^J \mu_J = 0, \]

where \( \mu_J \equiv \frac{\partial G}{\partial N_J} = -\beta^{-1} \ln \left[ \frac{\zeta_J}{N_J} \right] \) is the chemical potential for the \( J \)th component: again, the capital letters indicate ensemble averages:

\[ X \equiv \bar{x} = \beta^{-1} \frac{\partial \ln Z}{\partial x}. \]

At equilibrium

\[ \zeta_0 \zeta_1 \cdots \zeta_4 = K, \]

the equilibrium constant.

For example, suppose that the mean populations of only two strata, say \( J \) and \( K \), were allowed to vary. Then

\[ \frac{\Delta N_J}{\Delta N_K} = -\gamma^{J-K}; \]

e.g. \( \frac{\Delta N_4}{\Delta N_0} = -\gamma^4 \).

As \( \gamma \) increases the population shifts from the \( J = 0 \) stratum, through the \( J = 2 \) and \( 3 \) strata, to the \( J = 4 \) stratum.

For any given temperature and pressure, it is the Gibbs free energy that is minimized at equilibrium:

\[ G(N_J, \gamma) = E + PV - TS; \]

\[ \Delta G \sim \Delta E + \Delta (PV) - T \Delta S = 0. \]

So far, we have only accounted for the entropy contribution, \( \Delta S \), from changes in the cell populations, \( N_J \). We must add the contribution of cosmic expansion, through the potential energy \( V(\gamma) \). Equipartition mandates an equal contribution from the kinetic energy,

\[ E = T + V = 2V = -2 \sum N_J \gamma^J, \]
\[ \frac{\partial G}{\partial N} \Delta N + \frac{\partial G}{\partial \gamma} N \Delta \gamma = \sum \mu^J \Delta N^J + \int \frac{\partial G}{\partial \varphi} d\varphi. \]

The effect of the integral is to create a phase transition in the statistical ensemble. The phase point \((\varphi, G(\varphi))\) moves past the local minimum \((\varphi_s, G_s)\) on the stable branch, but never reaches local maximum, \((\varphi_u, G_u)\), where the classical system (Figure 1) would have lost stability. Instead, when it reaches the critical point \((\varphi_c, G_c)\), it jumps into (or "tunnels" through to) the point \((\varphi_d, G_c)\), where a further increase in \(\varphi\) will again cause the integral to decrease. The phase transition occurs where the areas between the Maxwell line \(U = U_c\) and the curve \(U(\gamma)\) on the left and right sides are equal. Moreover, the fastest growing scale for fluctuations is the critical scale, \(\gamma_c\), for the inflationary phase transition.

### 3.1 Critical Pattern Growth and Selection

The stochastic dynamics of cell populations suggests the following picture of pattern formation with increasing scale factor.

1. For \(\gamma \leq \gamma_C\) the fundamental lightlike modes; the vacuum spinors, dominate the ensemble average. They carry the lowest energy, \(N_0 \gamma^4\), and thus the highest weight, \(N_0 \sim e^{-\beta \epsilon_0}\).

The fastest-growing wavelength for fluctuations remains the same as the domain inflates, causing the condensation of critical droplets. As \(\gamma\) increases, the \(N_1\) term comes into play, stabilising the first basin of attraction. Leptons bound in this basin stack up in higher and higher energy states. The leptonic phase loses stability at the first critical point, \(\gamma_c\), and inflation occurs.

2. It is here that the entropy term \(-TS\) in \(G\) has a decisive effect, by catalyzing the formation of a Bose condensate. Unlike Fermions, an unlimited number of Bosons may be put into a state. This tends to raise the entropy. The entropy contribution to the Gibbs free energy varies as the cube of the scale factor,

\[ S_J \sim kN_4 \ln \left( \frac{N_3^3}{N_J} \right); \]

via the cubic term; it favors the coalescence of three bispinors (quarks) into a hadron.

3. A positive quartic \((N_4)\) term would create a second basin of attraction, stabilizing the association of the leptonic and hadronic states into \(Spin^c\)-4 vortex solitons [M.C.3]. A negative quartic term destroys the second basin. The states "slide down the hill", i.e. the scale factor would expand forever.
4 Conclusion

A growing body of evidence suggests that we are immersed in an ocean of "dark energy" deeper than the matter fields which ride on its surface.

In previous work, we found effective Lagrangians for the standard field equations- plus the varieties and masses of the particles- by expanding each of the $8$ spinor fields as a sum of a global, order $\gamma^{-\frac{1}{2}}$ distribution, plus a local envelope modulation. What happens is that the envelopes that vary on faster scales are effectively fixed at their average values, while the solutions that vary on slower scales appear as "parameters", with slowly- varying values. As the scale factor $\gamma(t)$ increases, the Clifford residues $(\psi^J d\psi_I)^J \equiv (d\zeta_J)^J$ become localized over $J$ branes, and their products with the vacuum spinors over $(4-J)$ branes [MC?]. This results in a spinfoam, $\Sigma(t)$.

If the incidence relations of the $J$ branes in $\Sigma$ are preserved, the resulting topological charges are conserved during the $t$ evolution, so each brane bounding a volume must have arisen from a small inhomogeneity in the initial conditions, wrapped nontrivially around its boundary, $\partial\gamma_{4-J} = \gamma_{4-J}$.

In principle, fields in a volume of spacetime may be found by the holographic method, given those on the boundary [refs...]- except near singularities of the classical spin map, where $|d\zeta| = 0$; or, near phase transitions of the quantum ensemble. Here microscopic inhomogeneities are "promoted" to the macroscopic level. Perhaps the galaxies and "great walls" we see today originated in microscopic vortex lines or planes in the primaeval spinfluid. But where did these come from?

Both the classical and quantum cases admit periodic solutions for $N_4 > 0$; i.e. for matter dominating antimatter. Here, initial conditions at the "big bang" are inherited from final conditions at the "big crunch". Neither of these are singular; both are identified with the global turning point $\gamma_{\text{min}}$, where small perturbations can nucleate large structures or events. But there are many local turning points-degenerate singularities, $|d^2\zeta| = 0$, where tiny perturbations- either random or conscious, can change history.

Some conceptual problems arise with the quantum evolution - the sum over histories in "imaginary time", $T$. The first involves ergodicity. This says that the time average equals the ensemble average; so, to calculate any function of state, average over all microstates with the same macroscopic value of the classical observable. For ergodicity to hold literally, the system must explore all the microstates compatible with the observed macrostate during the observation period. Some questions arise about this.

What does ergodicity mean when the "microstates" are different histories of the universe as a whole? Is the cosmic history we experience actually an average over all possible histories? Does history branch into many worlds at each decision point; or is one particular path in the ensemble always chosen (as in a random walk), but the invisible influences that determine which path are
so manifold and subtle, and their fluctuations so fast, that we can only predict ensemble averages?

Doesn’t Bell’s theorem rule out such a ”realistic” explanation of which path is chosen for the ”real” history? No; not if its nonlocal—i.e. if there is some globally—determined field; or if there are closed causal cycles, $\gamma_1 = \partial B_2$, that loop around some area in spacetime to return to the same place, $x$, at the same cosmic time, $T$:

$$(T, x)(t_0 + \Delta t) = (T, x)(t_0).$$

The evidence for such closed cycles $\gamma_1 : t \rightarrow (T, x)(t)$ has been staring us in the face for a century: quantization of action around spacetime cycles:

$$\int_{\gamma_1} Edt - Pdq = n\pi\hbar,$$

and of electric charge, the spacetime-component of the spin curvature [ccq].

Null geodesics, or photons (helicity -1) on $S_1 \times S_3$, propagate once around their twisted light cones, before coming back on their original values. But spinors (helicity-1/2) must go twice around $\gamma_1$ (or once in opposite directions) before coming back on themselves in value.

These global cycles together with all the local integral-homonymy cycles, make a dynamical history. In the 4-dimensional picture, history is geometry. Much like Kirchoffs laws for electric circuits, the physical ”laws”are incidence relations between $(4 - J)$ branes, each carrying a dual $J$ form current. Each microhistory must obey these constraints. But the question remains:

*Why does an observation ”collapse” the ensemble of allowed microstates into a single one?*

We shall see in the sequel that dynamical collapse of the wavepacket in Minkowsky time, $t$, can appear instantaneous in cosmic time.

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