Spontaneous Supersymmetry Breaking
from Extra Dimensions

Makoto Sakamoto\(^{(a)}\), Motoi Tachibana\(^{(b)}\) and Kazunori Takenaga\(^{(c)}\)

\(^{(a)}\) Department of Physics, Kobe University, Rokkodai, Nada, Kobe 657-8501, Japan
Graduate School of Science and Technology, Kobe University, Rokkodai, Nada, Kobe 657-8501, Japan
\(^{(c)}\) I.N.F.N, Sezione di Pisa, Via Buonarroti, 2 Ed. B, 56127 Pisa, Italy

Abstract

We propose a new spontaneous supersymmetry breaking mechanism, in which extra compact dimensions play an important role. To illustrate our mechanism, we study a simple model consisting of two chiral superfields, where one spatial dimension is compactified on a circle \(S^1\). It is shown that supersymmetry is spontaneously broken irrespective of the radius of the circle, and also that the translational invariance for the \(S^1\)-direction and a global symmetry are spontaneously broken when the radius becomes larger than a critical radius. These results are expected to be general features of our mechanism. We further discuss that our mechanism may be observed as the O’Raifeartaigh type of supersymmetry breaking at low energies.

\*E-mail: sakamoto@oct.phys.kobe-u.ac.jp
\^E-mail: tatibana@oct.phys.kobe-u.ac.jp
\|^E-mail: takenaga@ibmth.df.unipi.it
1 Introduction

Recently, a very challenging possibility of large-scale compactification has been pointed out in ref.[1]. The authors have discussed consequences of extra large-scale dimensions and have proposed phenomenologically interesting scenarios. So, an extremely exciting situation will be that not only extra dimensions but also supersymmetry might be observed in the near future. If the scale of extra dimensions is not far from that of supersymmetry breaking, it will be natural to think that supersymmetry breaking and compactification of extra dimensions may have a common dynamical origin.

The purpose of this paper is to propose a new spontaneous supersymmetry breaking mechanism by compactification. Let us first present a key idea of our mechanism below. Suppose that an (effective) potential $V(A_i)$ for scalar fields $A_i$ vanishes at some values $\bar{A}_i$ of $A_i$, i.e.

$$V(A_i)\big|_{A_i=\bar{A}_i} = 0.$$  \hfill (1)

One might then conclude that supersymmetry would be unbroken because of the vanishing vacuum energy. However, it is not always true. This is an essential point in our mechanism. Our spontaneous supersymmetry breaking mechanism will be a realization of the following simple idea: If there exist some mechanisms to force vacuum expectation values of $A_j$ for some $j$ not to take the values $\bar{A}_j$, then the vanishing vacuum energy solution $\bar{A}_i$ in eq.(1) will not be realized as a supersymmetric vacuum. A simple mechanism to force vacuum expectation values not to take nonzero constants has been proposed in ref.[3]. We shall apply this mechanism for supersymmetric field theories to break supersymmetry spontaneously.

In the next section, to illustrate our mechanism, we shall study a 3+1-dimensional Wess-Zumino type model in which one spatial dimension is compactified on a circle $S^1$ and show that supersymmetry is spontaneously broken. In Sect.3, it is shown that the translational invariance for the $S^1$-direction and a global symmetry are spontaneously broken when the radius of the circle becomes larger than a critical radius. In Sect.4, our mechanism is contrasted with the O’Raifeartaigh mechanism [4]. Some comments are given in the last section.

\footnote{An interesting example of such mechanism has been found in a special class of supersymmetric models [5], in which would-be supersymmetric vacuum configurations have been removed from quantum moduli spaces due to quantum deformed constraints.}
2 A Model

To illustrate our spontaneous supersymmetry breaking mechanism, let us consider a 3+1-dimensional Wess-Zumino type model consisting of two chiral superfields $\Phi_0$ and $\Phi_1$. The superpotential we take is

$$W(\Phi_0, \Phi_1) = g\Phi_0 \left( \frac{\Lambda^2}{g^2} - \frac{1}{2}(\Phi_1)^2 \right),$$

(2)

where the parameters $g$ and $\Lambda$ are chosen to be real and positive for simplicity. This model has a global $Z_2$ symmetry

$$\Phi_0 \rightarrow +\Phi_0,$$
$$\Phi_1 \rightarrow -\Phi_1. \quad (3)$$

It turns out that this global symmetry plays an important role in this model. The scalar potential is given by

$$V(A_0, A_1) = |F_0|^2 + |F_1|^2,$$

(4)

where $A_0$ and $A_1$ denote the lowest scalar components of $\Phi_0$ and $\Phi_1$, respectively, and

$$F_0 = -\left( \frac{\partial W(A_0, A_1)}{\partial A_0} \right)^* = -\frac{\Lambda^2}{g} + \frac{g}{2}(A_1^*)^2,$$
$$F_1 = -\left( \frac{\partial W(A_0, A_1)}{\partial A_1} \right)^* = gA_0^*A_1^*. \quad (5)$$

Since the scalar potential $V(A_0, A_1)$ would vanish at $A_0 = 0$ and $A_1 = \pm \frac{\sqrt{2}\Lambda}{g}$, supersymmetry might be unbroken, while the $Z_2$ symmetry be broken, spontaneously. This is, however, a hasty conclusion, as we will see below.

Let us suppose that one of the space coordinates, say, $y \equiv x^3$ is compactified on a circle $S^1$ whose radius is $R$. Since $S^1$ is multiply-connected and the action has the $Z_2$ symmetry (3), we can impose the following nontrivial boundary conditions associated with the $Z_2$ symmetry:

$$\Phi_0(x^\mu, y + 2\pi R) = +\Phi_0(x^\mu, y),$$
$$\Phi_1(x^\mu, y + 2\pi R) = -\Phi_1(x^\mu, y), \quad (6)$$

where $x^\mu$ denote the coordinates of the uncompactified 2+1-dimensional Minkowski spacetime. It should be stressed that the boundary conditions (6) are consistent with supersymmetry and that the action is still single-valued thanks to the $Z_2$ symmetry (3).
important consequence of the nontrivial boundary conditions (3) is that any vacuum expectation value of $\Phi_1(x^\mu, y)$ (or $A_1(x^\mu, y)$) cannot be a ($y$-independent) nonzero constant. It immediately follows that (would-be) supersymmetric vacuum configurations $A_0 = 0$ and $A_1 = \pm \sqrt{2\Lambda} g$ should be ruled out. If we assume that the vacuum would translationally be invariant the vacuum expectation value of $A_1(x^\mu, y)$ has to vanish, i.e.

$$\langle A_1(x^\mu, y) \rangle = 0. \quad (7)$$

Replacing $A_0$ and $A_1$ by their vacuum expectation values in $V(A_0, A_1)$, we find

$$V(\langle A_0 \rangle, \langle A_1 \rangle = 0) = \frac{\Lambda^4}{g^2} > 0, \quad (8)$$

which implies that supersymmetry is spontaneously broken, as expected. In this model, there is a flat direction in vacuum configurations since the potential (8) is independent of $\langle A_0 \rangle$.

Another way to see the supersymmetry breaking more explicitly may be to expand the component fields in the Fourier-series according to the boundary conditions (3).

$$A_0(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} a_0^{(2n)}(x^\mu) e^{i2n \frac{y}{2R}},$$

$$A_1(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{l=-\infty}^{\infty} a_1^{(2l-1)}(x^\mu) e^{i(2l-1) \frac{y}{2R}},$$

$$\psi_0(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \chi_0^{(2n)}(x^\mu) e^{i2n \frac{y}{2R}},$$

$$\psi_1(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{l=-\infty}^{\infty} \chi_1^{(2l-1)}(x^\mu) e^{i(2l-1) \frac{y}{2R}}. \quad (9)$$

It turns out to be convenient to divide the Fourier mode $a_1^{(2l-1)}$ into two parts as

$$a_1^{(2l-1)} = a_+^{(2l-1)} + ia_-^{(2l-1)}$$

with $a_+^{(2l-1)*} = a_-^{(-2l+1)}$. Then the squared masses for $a_0^{(2n)}$, $\chi_0^{(2n)}$, $a_\pm^{(2l-1)}$ and $\chi_1^{(2l-1)}$ (in a viewpoint of the 2+1-dimensional Minkowski spacetime) are given by $m^2 = \left(\frac{n}{R}\right)^2, \left(\frac{n}{R}\right)^2, \pm \Lambda^2, |M|^2 + \left(\frac{l-\frac{1}{2}}{R}\right)^2, |M|^2 + \left(\frac{l-\frac{1}{2}}{R}\right)^2$, respectively, where $M = g\langle A_0 \rangle$. Here we would like to make several comments on the mass spectrum. The first comment is that the

---

*This assumption is true only when the radius of the circle is smaller than a critical radius $R^* = \frac{1}{2\Lambda}$. See the next section.
supersymmetry breaking scale is found, from the mass splitting, to be of the order of $\Lambda$. The second comment is that the mass spectrum satisfies the following relations [3]:

$$
\begin{align*}
    m^2_{a_0^{(2n)}} &= m^2_{\chi_0^{(2n)}}, \\
    m^2_{a_{-}^{(2l-1)}} + m^2_{a_{+}^{(2l-1)}} &= 2m^2_{\chi_{1}^{(2l-1)}}.
\end{align*}
$$

(11)

The third comment is that the fermionic mode $\chi_0^{(0)}$ is massless and corresponds to the Nambu-Goldstone fermion associated with the spontaneous supersymmetry breaking. The bosonic partner $a_0^{(0)}$ is also massless but its origin is quite different. A part of it will correspond to the Nambu-Goldstone boson associated with the spontaneous breaking of a $U(1)_R$ symmetry (with $\langle A_0 \rangle \neq 0$). The masslessness of $a_0^{(0)}$ is also guaranteed by the existence of a flat direction of $\langle A_0 \rangle$ (at least at the tree level). The last comment is that choosing $\langle A_0 \rangle = 0$ we find that some of the bosonic modes $a_{-}^{(2l-1)}$ might have negative squared masses for $R > R^* = \frac{1}{2\Lambda}$. This observation suggests that the configuration [7] may become unstable for $R > R^*$ and that a phase transition can occur at $R = R^*$. This is the subject of the next section.

3 Spontaneous Breakdown of Translational Invariance

In the previous section, we have assumed the translational invariance would be unbroken. It turns out that this assumption is not true for $R > R^*$, as suggested in the previous section. We shall here discuss spontaneous breakdown of the translational invariance for the $S^1$-direction. To this end, we should take account of kinetic terms as well as potential terms since the vacuum configuration might be coordinate-dependent. The vacuum configuration will then be obtained by solving a minimization problem of the functional [7]

$$
E[A_0, A_1; R] = \int_0^{2\pi R} dy \left\{ \left| \frac{\partial A_0}{\partial y} \right|^2 + \left| \frac{\partial A_1}{\partial y} \right|^2 + V(A_0, A_1) \right\},
$$

(12)

with the boundary conditions

$$
A_0(y + 2\pi R) = +A_0(y), \\
A_1(y + 2\pi R) = -A_1(y).
$$

(13)

\[^{[7]}\text{The} E[A_0, A_1] \text{ may be thought of as a potential in a viewpoint of the 2+1-dimensional Minkowski spacetime.}\]
In the following, we ignore the $x^\mu$ dependence since we are interested in the vacuum configuration, for which the translational invariance of the 2+1-dimensional Minkowski spacetime is assumed to be unbroken. We first note that the vacuum configuration for $A_0(y)$ and $A_1(y)$ should satisfy the following field equations:

$$ 0 = \frac{\delta E[A_0, A_1; R]}{\delta A_0(y)} = -\frac{d^2 A_0(y)}{dy^2} + g^2 A_0(y) |A_1(y)|^2, $$

$$ 0 = \frac{\delta E[A_0, A_1; R]}{\delta A_1(y)} = -\frac{d^2 A_1(y)}{dy^2} - \Lambda^2 A_1^*(y) + g^2 \left( |A_0(y)|^2 + \frac{1}{2} |A_1(y)|^2 \right) A_1(y). $$

(14)

If the translational invariance for the $S^1$-direction would be unbroken, the vacuum expectation value of $A_1$ has to vanish due to the boundary conditions (13) and then the functional $E[A_0, A_1; R]$ becomes

$$ E[A_0 = \text{const}, A_1 = 0; R] = \frac{2\pi R \Lambda^4}{g^2}. $$

(15)

Using the field equations (14) to eliminate the "kinetic" terms in eq.(12), we may find

$$ E[A_0, A_1; R]\big|bracket{_{\delta A_0 = \delta A_1 = 0}} = \frac{2\pi R \Lambda^4}{g^2} - \int_0^{2\pi R} dy \left\{ \frac{g^2}{4} |A_1|^4 + g^2 |A_0 A_1|^2 \right\} $$

$$ \leq E[A_0 = \text{const}, A_1 = 0; R]. $$

(16)

We have thus arrived at an important conclusion: If there would appear nontrivial solutions ($A_1 \neq 0$) to the field equations (14), then $A_1 = 0$ is no longer a vacuum configuration and the translational invariance for the $S^1$-direction would then be broken spontaneously (with the $Z_2$ symmetry breaking) since nonvanishing $A_1(y)$ inevitably has the $y$ dependence to be consistent with the boundary conditions (13). We should again emphasize that (would-be) $y$-independent solutions $A_0 = 0$ and $A_1 = \pm \frac{\sqrt{2} \Lambda}{g}$ to eqs.(14) are not consistent with eqs.(13). It turns out that for $R \leq R^* = \frac{1}{2\Lambda}$ there exists only the trivial solution ($A_0 = \text{const.}$ and $A_1 = 0$) to eqs.(14), while for $R > R^*$ there will appear many other (nontrivial) solutions. This result may be seen by noting that a vacuum configuration with $A_1 \neq 0$ can be realized only when $A_0 = 0$ and $\text{Im} A_1 = 0$, and then by solving eqs.(14) with the boundary conditions (13). The vacuum configuration for $A_0$ and $A_1$, which minimizes $E[A_0, A_1; R]$, has finally been found to be

$$ \langle A_0(x^\mu, y) \rangle = \begin{cases} \text{arbitrary constant} & \text{for } R \leq R^* \\ 0 & \text{for } R > R^* \end{cases}, $$

$$ \langle A_1(x^\mu, y) \rangle = \begin{cases} 0 & \text{for } R \leq R^* \\ \frac{2k \omega}{g} \text{sn} (\omega(y - y_0), k) & \text{for } R > R^* \end{cases}. $$

(17)
with \( \omega = \frac{\Lambda}{\sqrt{1 + k^2}} \). Here, \( \text{sn}(u,k) \) is the Jacobi elliptic function whose period is \( 4K(k) \), where \( K(k) \) denotes the complete elliptic function of the first kind. The parameter \( k \) (0 \( \leq k < 1 \)) and the radius \( R \) should be related through the equation

\[
R = \frac{K(k)}{\pi \omega}.
\]

(18)

Note that as \( k \) runs from zero to one the right hand side of eq.(18) increases monotonically from \( R^* = \frac{1}{2\Lambda} \) to infinity. As expected in the previous section, a phase transition occurs at \( R = R^* \) and the translational invariance for the \( S^1 \)-direction is spontaneously broken for \( R > R^* \).

We would like to comment on the normal modes of oscillation about the vacuum configuration for \( R > R^* \). In the previous section, we have observed that some of Fourier modes might have negative squared masses for \( R > R^* \). This is merely due to the fact that we have not taken the true vacuum configuration for \( R > R^* \). In fact, we can show that all squared masses are positive semi-definite, as they should be, if the fields are correctly expanded in the normal modes of oscillation about the true vacuum configuration (17). We may then find three massless modes (in a sense of real degrees of freedom): One is bosonic and two are fermionic. The massless fermionic (bosonic) modes correspond to the Nambu-Goldstone modes associated with spontaneous breakdown of supersymmetry (the translational invariance for the \( S^1 \)-direction).

4 O’Raifeartaigh Mechanism vs. Ours

In this section, we would like to point out that our mechanism may be observed as the O’Raifeartaigh type of supersymmetry breaking at low energies, even though we will also point out several differences between the O’Raifeartaigh mechanism and ours.

We shall first summarize general settings to construct supersymmetric models based on our mechanism. Let \( \bar{A}_i \) be a (would-be) supersymmetric vacuum configuration satisfying \( V(\bar{A}_i) = 0 \) or \( \frac{\partial V(\bar{A}_i)}{\partial A_j} = 0 \) for all \( j \). Suppose that some of space dimensions are compactified on a manifold which should be translationally invariant and be multiply-connected, like \( S^1 \). We then impose nontrivial boundary conditions on superfields, which have to be consistent with global symmetries of the theory. The crucial point is that the boundary...
conditions have to be chosen to prevent some of vacuum expectation values of \( A_i \) from taking the values \( \bar{A}_i \). It turns out \( ^3 \) that at least two chiral superfields are required for our mechanism to work in Wess-Zumino type models and that the model presented in Sect. 2 is the minimum one. This may be contrasted with the O’Raifeartaigh mechanism, in which at least three chiral superfields are required. In the O’Raifeartaigh mechanism, superpotentials \( W(A_i) \) should be chosen such that there are no consistent solutions to the equations

\[
\frac{\partial W(A_i)}{\partial A_j} = 0 \quad \text{for all } j.
\] (19)

On the other hand, in our mechanism superpotentials will be chosen to have (would-be) solutions to eqs. (19) but to have no solutions if we further impose boundary conditions on \( A_i \), which have to be inconsistent with eqs. (19). In this point, our mechanism is apparently different from the O’Raifeartaigh one.

Let us next discuss a resemblance between the two mechanisms. To make our discussions simple, we will consider the model studied in Sect. 2 again. Let \( W(a_0^{(2n)}, a_1^{(2l-1)}) \) be the superpotential for the Fourier modes given in eq. (9). Since the mode expansions (14) have been done in a consistent way with the boundary conditions (13), we may not need to take care of boundary conditions any more, as long as the Fourier modes \( a_0^{(2n)} \) and \( a_1^{(2l-1)} \) are considered. Then, supersymmetry breaking might be observed by showing that the equations

\[
\frac{\partial W(a_0^{(2n)}, a_1^{(2l-1)})}{\partial a_0^{(2m)}} = \frac{\partial W(a_0^{(2n)}, a_1^{(2l-1)})}{\partial a_1^{(2k-1)}} = 0 \quad \text{for all } m \text{ and } k
\] (20)

have no consistent solutions. In this sense, our mechanism might be thought of as a kind of the O’Raifeartaigh one, though the model consists of infinitely many (Kaluza-Klein) modes. To see the resemblance between two mechanisms more explicitly, let us look at the model from a low energy point of view. To this end, we shall restrict our considerations to the light five bosonic modes, \( a_0^{(0)}, a_0^{(2)} \) and \( a_1^{(1)} \), \( ^11 \) and simply put other “heavy” modes to be zero. We may then find the equations (20) for \( a_0^{(0)}, a_0^{(2)} \) and \( a_1^{(1)} \) to be

\[
\begin{align*}
a_1^{(1)} & a_1^{(-1)} = \frac{2\pi R \Lambda^2}{g^2}, \\
(a_1^{(1)})^2 & = 0, \\
a_0^{(0)} a_1^{(1)} + a_0^{(2)} a_1^{(1)} & = 0.
\end{align*}
\] (21)

\( ^{11} \)For \( R > R^* \), the fields should appropriately be expanded in the normal modes of oscillation about the true vacuum configuration (7).
respectively. It is easy to see that these equations have no consistent solutions. This observation suggests that our mechanism may be observed as the O’Raifeartaigh type of supersymmetry breaking at low energies.

5 Comments

We have studied a simple 3+1-dimensional supersymmetric model, in which one spatial dimension is compactified on a circle, to illustrate our spontaneous supersymmetry breaking mechanism. It has been shown that supersymmetry is spontaneously broken for \( R > 0 \) and also that the translational invariance for the \( S^1 \)-direction with the global \( Z_2 \) symmetry is spontaneously broken for \( R > R^* \). These results will not be specific to this model but are expected to be general features of our mechanism.

We should make a comment on the Scherk-Schwarz mechanism \[7\]. One might impose nontrivial boundary conditions associated with a \( U(1)_R \) symmetry. Then, bosonic components of superfields may satisfy different boundary conditions from fermionic ones. A crucial difference between the Scherk-Schwarz mechanism and ours is that the breaking à la Scherk-Schwarz is \textit{explicit} rather than spontaneous at the level of global supersymmetry. Another difference is that the Scherk-Schwarz mechanism will work for any choice of superpotentials, just like supersymmetry breaking at finite temperature, while our mechanism will not.

The final comment is as follows: An interesting supersymmetry breaking mechanism by compactification has been proposed by Dvali and Shifman \[8\], who have called it dynamical compactification. The authors have suggested the idea that our Universe could spontaneously be generated in the form of a four-dimensional topological or non-topological stable defect in higher-dimensional spacetime and that the low-energy observers trapped in the core of the defect would not detect supersymmetry, although the vacuum of the original higher-dimensional theory is fully supersymmetric. What we would like to point out is that the supersymmetric model presented in Sect.2 may be thought of as an explicit realization of dynamical compactification in the limit of \( R \to \infty \). In this limit, the vacuum expectation value of \( \langle A_1(x^\mu, y) \rangle \) becomes

\[
\langle A_1(x^\mu, y) \rangle \bigg|_{R=\infty} = \frac{\sqrt{2} \Lambda}{g} \tanh \left( \frac{\Lambda}{\sqrt{2}} (y - y_0) \right). \tag{22}
\]

This is a single kink solution, which is just one of the topologically stable defects discussed
in the paper \cite{8}. A key difference from the Dvali-Shifman approach is that the topologically stable solution (22) has been chosen as the vacuum configuration in our model (but not chosen by hand). This could be an advantage of our approach.

We hope that our mechanism might shed new light on supersymmetry breaking. It would be of great importance to construct phenomenologically realistic supersymmetric models based on our approach.

\textit{ACKNOWLEDGMENTS}

We would like to thank to H. Hatanaka and C. S. Lim for useful discussions. K.T. would like to thank the I.N.F.N, Sezione di Pisa for hospitality.
References

[1] I. Antoniadis, S. Dimopoulos and G. Dvali, Nucl. Phys. B516 (1998) 70 (hep-ph/9710204);
    N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263 (hep-ph/9803313);
    K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436 (1998) 55 (hep-ph/9803466);
    I. Antoniadis, S. Dimopoulos, A. Pomarol and M.Quiros, hep-ph/9810410;
    A. Delgado, A. Pomarol and M. Quiros, hep-ph/9812489.

[2] K. Izawa and T. Yanagida, Prog. Theor. Phys. 95 (1996) 829 (hep-th/9602180);
    K. Intriligator and S. Thomas, Nucl. Phys. B473 (1996) 121 (hep-th/9603158).

[3] M. Sakamoto, M. Tachibana and K. Takenaga, preprint KOBE-TH-99-01 (1999), IFUP-TH 6/99, (hep-th/9902069).

[4] L. O’Raifeartaigh, Nucl. Phys. B96 (1975) 331.

[5] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403.

[6] M. Sakamoto, M. Tachibana and K. Takenaga, in preparation.

[7] J. Scherk and J. H. Schwarz, Phys. Lett. B82 (1979) 60;
    P. Fayet, Phys. Lett. B159 (1985) 121; Nucl. Phys. B263 (1986) 87;
    K. Takenaga, Phys. Lett. B425 (1998) 114 (hep-th/9710058); Phys. Rev. D58 (1998) 026004 (hep-th/9801075).

[8] G. Dvali and M. Shifman, Nucl. Phys. B504 (1997) 127 (hep-th/9611213).