Inelastic Coulomb scattering rates due to acoustic and optical plasmon modes in coupled quantum wires.

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Abstract

We report a theoretical study on the inelastic Coulomb scattering rate of an injected electron in two coupled quantum wires in quasi-one-dimensional doped semiconductors. Two peaks appear in the scattering spectrum due to the optical and the acoustic plasmon scattering in the system. We find that the scattering rate due to the optical plasmon mode is similar to that in a single wire but the acoustic plasmon scattering depends crucially on its dispersion relation at small $q$. Furthermore, the effects of tunneling between the two wires are studied on the inelastic Coulomb scattering rate. We show that a weak tunneling can strongly affect the acoustic plasmon scattering.
I. INTRODUCTION

Recently, single-particle properties of electrons in quasi-one-dimensional (Q1D) electron systems have attracted considerable interest. With the theoretical calculations of quasiparticle renormalization factor [1] and the momentum distribution function around the Fermi surface, Hu and Das Sarma [2] have clarified that a clean 1D electron system shows the Luttinger liquid behavior, but even a slightest amount of impurities restores the Fermi surface and the Fermi-liquid behavior remains. Within a one-subband model, they evaluated the self-energy due to electron-electron Coulomb interaction in unclean Q1D systems by using the leading-order GW dynamical screening approximation [1,3]. Within such an approximation, Hwang and Das Sarma [4] obtained the band-gap renormalization in photoexcited doped-semiconductor quantum wire in the presence of plasmon-phonon coupling. In particular, the inelastic Coulomb scattering rate plays an important role in relaxation processes of an injected electron in the conduction band. The lifetime of the injected electron, determined by this scattering rate, can be measured by femtosecond time-resolved photoemission spectroscopy [5]. The relaxation processes of an injected electron occur through the scattering channels due to different excitations in the system, such as quasiparticle excitations, plasmons, and phonons [6,7]. Its lifetime provides information on the interactions between the electron and the different excitations. The relaxation mechanism is important because of its technological relevance, as most semiconductor-based devices operate under high-field and hot-electron conditions [8].

On the other hand, tunneling effects have provide new devices formed by coupled Q1D doped semiconductors [9] and attracted considerable theoretical interest because of their fundamental applicability. In this work we present a theoretical study on the inelastic Coulomb scattering rates in coupled bi-wire electron gas systems. A particular attention will be devoted to the effects of weak resonant tunneling. We find that a weak resonance tunneling can introduce a strong intersubband inelastic Coulomb scattering by emitting an acoustic plasmon. The emission of optical plasmon, on the other hand, is provided by
intrasubband scattering of injected electrons.

The rest of the paper is organized as follows. In Sec. II, we present the theoretical formalism of the inelastic Coulomb scattering rates in a multisubband Q1D system of coupled quantum wires. Sec. III is devoted to analyze the inelastic-scattering rates for a bi-wire system in the absence of tunneling between the wires. As an extension of such calculations we show in Sec. IV the numerical results in the presence of weak resonant tunneling. Finally, we summarize our results in Sec. V.

II. THEORETICAL FORMULATION

We consider a two-dimensional system in the $xy$ plane subjected to an additional confinement in the $y$-direction which forms two quantum wires parallel to each other in the $x$-direction. The confinement potential in the $y$-direction is taken to be of square well type of barrier height $V_0$ and well widths $W_1$ and $W_2$ representing the first and the second wire, respectively. The potential barrier between the two wires is of width $W_b$. The subband energies $E_n$ and the wave functions $\phi_n(y)$ are obtained from the numerical solution of the one-dimensional Schrödinger equation in the $y$-direction. We restrict ourselves to the case where $n = 1, 2$ and define $\omega_0 = E_2 - E_1$ as being the gap between the two subbands. The interpretation of the index $n$ depends on tunneling between the two wires. When there is no tunneling, the wavefunction $\phi_n(y)$ of the subband $E_n$ is localized in quantum wire $n$. Clearly, it is wire index. For two symmetric quantum wires, i.e., $W_1 = W_2$, one has $E_2 = E_1$ or $\omega_0 = 0$. When tunneling occurs, the wavefunction of each subband spreads in two quantum wires. In this case, $n$ is interpreted as subband index. For two symmetric quantum wires with tunneling, the wavefunctions of the two lowest eigenstates are symmetric and antisymmetric. In this case, the two wires are in the resonant tunneling condition and the gap between the two subbands is denoted by $\Delta_{SAS} = \omega_0$.

In a multisubband Q1D system, the inelastic Coulomb scattering rate for an injected electron in subband $n$ with momentum $k$ can be obtained by the imaginary part of the
screened exchange self-energy $\Sigma_n [k; \xi_n(k)]$, \(^{[1]}\) where, $\xi_n(k) = \hbar^2 k^2 / 2m^* + E_n - E_F$ is the electron energy with respect to the Fermi energy $E_F$ and $m^*$ the electron effective mass. At zero temperature, this self-energy can be obtained from the leading terms of the Dyson’s equation for the dressed electron Green’s function, \(^{[10]}\) and given by

$$\Sigma_n [k; \xi_n(k)] = \frac{i}{(2\pi)^2} \int dq \int d\omega' \sum_{n_1} V_{nn_1n,n_1n}(q, \omega') G^{(0)}_{n_1}(k + q, \xi_n(k) - \omega'),$$

(1)

where $G^{(0)}_{n_1}(k, \omega)$ is the Greens function of noninteracting electrons and $V_{nn_1n,n_1n}(q, \omega)$ is the dynamically screened electron-electron Coulomb potential. The screened Coulomb potential is related to the dielectric function $\varepsilon_{nm'm'n}(q, \omega)$ and the bare electron-electron interaction potential $V_{nn'm'n}(q)$ through the equation

$$\sum_{ll'} \varepsilon_{ll'n'm'}(q, \omega) V_{ll'n'm'}(q, \omega) = V_{nn'm'n}(q).$$

(2)

Similarly to the one-band model \(^{[2]}\), the self-energy in Eq. (1) can be separated into the frequency-independent exchange and the correlation part, $\Sigma_n [k; \xi_n(k)] = \Sigma_{ex}^n(k) + \Sigma_{cor}^n[k; \xi_n(k)]$. The exchange part is given by

$$\Sigma_{ex}^n(k) = -\frac{1}{2\pi} \int dq \sum_{n_1} V_{nm_1n_1n}(q) f_{n_1}(\xi_{n_1}(k + q)),$$

(3)

where $f_n(\xi)$ is the Fermi-Dirac distribution function. Notice that $\Sigma_{ex}^n(k)$ is real because the bare electron-electron Coulomb potential $V_{nm_1n_1n}(q)$ is totally real. Therefore, one only needs to analyze the imaginary part of $\Sigma_{cor}^n[k; \xi_n(k)]$, since it gives rise to the imaginary part of the self-energy which we are interested in. After some algebra, we find that the Coulomb inelastic-scattering rate for an electron in a subband $n$ with momentum $k$ is given by

$$\sigma_n(k) = -\text{Im} \Sigma_{cor}^n[k; \xi_n(k)] = \sum_{n'} \sigma_{n,n'}(k),$$

(4)

with

$$\sigma_{n,n'}(k) = \frac{1}{2\pi} \int dq \text{Im} \{V_{nn'n'n}(q, \xi_{n'}(k + q) - \xi_n(k))\}$$

$$\times \{\theta (\xi_n(k) - \xi_{n'}(k + q)) - \theta (-\xi_{n'}(k + q))\},$$

(5)
where $\theta (x)$ is the standard step function. In the above equation, the frequency integration has already been carried out, since the bare Green’s function $G^{(0)}_{n_1}$ can be written as a Dirac delta function of $\omega$.

For the present coupled quantum wire systems with two occupied subbands, the multi-subband dielectric function within the random-phase approximation (RPA) is given by

$$\varepsilon_{nn'nn'}(q, \omega) = \delta_{nn'}\delta_{nn'} - \Pi_{nn'}(q, \omega)V_{nn'nn'}(q). \quad (6)$$

The function $\Pi_{nn'}(q, \omega)$ is the 1D non-interacting irreducible polarizability at zero temperature for a system free from any impurity scattering. In the presence of impurity scattering, we use Mermin’s formula \[11\]

$$\Pi^{\gamma}_{nn'}(q, \omega) = \frac{(\omega + i\gamma)\Pi_{nn'}(q, \omega + i\gamma)}{\omega + i\gamma \left[ \Pi_{nn'}(q, \omega + i\gamma)/\Pi_{nn'}(q, 0) \right]} \quad (7)$$

to obtain the polarizability including the effect of level broadening through a phenomenological damping constant $\gamma$. The Coulomb potential

$$V_{nn'nn'}(q) = \frac{2e^2}{\epsilon_0} \int dy \int dy' \phi_n(y)\phi_{n'}(y)K_0(q |y - y'|)\phi_m(y')\phi_{m'}(y')$$

is calculated by using the numerical solution of the electron wavefunction $\phi_n(y)$. Here, $\epsilon_0$ is the static lattice dielectric constant, $e$ is the electron charge, and $K_0(q |y - y'|)$ is the zeroth-order modified Bessel function of the second order. The electron-electron Coulomb interaction describes two-particle scattering events. We observe the following characteristics of the electron-electron Coulomb interaction in the coupled quantum wires representing different physical scattering processes: $V_{1111}(q) = V_A$, $V_{2222}(q) = V_B$, and $V_{1122}(q) = V_{2211}(q) = V_C$ represent the scattering in which the electrons keep in their original wires or subbands; $V_{1212}(q) = V_{2121}(q) = V_{1221}(q) = V_{2112}(q) = V_D$ represent the scattering in which both electrons change their wire or subband indices; $V_{1112}(q) = V_{1121}(q) = V_{1211}(q) = V_{2111}(q) = V_J$ and $V_{2212}(q) = V_{2221}(q) = V_{1222}(q) = V_{2122}(q) = V_H$ indicating the scattering in which only one of the electrons suffers the interwire or intersubband transition. When there is no tunneling, $V_D = V_H = V_J = 0$. Clearly, they are responsible for tunneling effects. We also notice that, for two symmetric quantum wires in resonant tunneling, $V_J$ and $V_H$ vanish.
III. BI-WIRES WITHOUT TUNNELEDING

In the following, we will analyze the inelastic Coulomb scattering rate of electrons in two coupled symmetric quantum wires \((W_1 = W_2 = W)\) in the absence of tunneling. As we discussed before, when there is no tunneling between two quantum wires, \(V_D = V_H = V_J = 0\). Only the Coulomb interactions \(V_A\), \(V_B\), and \(V_C\) contribute to the electron-electron interaction. Furthermore, the potential \(V_A\) and \(V_B\) are responsible for the intrawire interaction and \(V_A = V_B\) due to the symmetry property of the two wires. The potential \(V_C\) is responsible for the interwire Coulomb interaction. If we assume that the two wires have an identical electron density \(n_1 = n_2 = n_e\), the total electron density in the system is \(N_e = 2n_e\). In this case, the two quantum wires have the same Fermi level \(E_F\) so that \(\Pi_{11} = \Pi_{22} = \Pi_0\). Therefore, from Eqs. (3) and (3), we obtain the screened intrawire Coulomb potential \(V_{1111}^s = V_{2222}^s = V^s\) given by

\[
V^s = \frac{V_A - (V_A + V_C)(V_A - V_C)\Pi_0}{[1 - (V_A + V_C)\Pi_0][1 - (V_A - V_C)\Pi_0]}.
\]

The denominator in the above equation is the determinant of the dielectric matrix \(\det |\epsilon(q, \omega)|\). The equation \(\det |\epsilon(q, \omega)| = 0\) yields the plasmon dispersions of the electron gas system. The plasmons result in singularities in the screened Coulomb potential which are of the most important contribution to the inelastic Coulomb scattering rate.

According to Eq. (5), the intrawire scattering rate of the symmetric bi-wires with identical electron density becomes

\[
\sigma_{n,n}(k) = \frac{1}{2\pi} \int dq \left\{ \text{Im} \left[ V^s (q, 2kq + q^2) \right] \right\} \left\{ \theta \left( -2kq - q^2 \right) - \theta \left( E_{F_n} - k^2 - q^2 - 2kq \right) \right\},
\]

for \(n = 1\) and \(2\), where \(E_{F_n} = E_F - E_n\) is the subband Fermi energy. Notice that \(E_1 = E_2\) for two symmetric quantum wires. It is obvious that \(\sigma_{1,1}(k) = \sigma_{2,2}(k)\). In the absence of tunneling, interwire scattering rates \(\sigma_{1,2}(k)\) and \(\sigma_{2,1}(k)\) are zero because the transition of an electron from one wire to the other is impossible. Therefore, we have \(\sigma_1(k) = \sigma_{1,1}(k) = \sigma_2(k) = \sigma_{2,2}(k)\).
σ_{2,2}(k). But the interwire Coulomb interaction $V_C$ influences the collective excitations in the system leading to two different plasmon modes, i.e., the optical and acoustic modes. Subsequently, it affects the inelastic-scattering rates. We know that the zeros of the two parts $1 - (V_A + V_C)\Pi_0$ and $1 - (V_A - V_C)\Pi_0$ in the denominator in equation (3) yield the optical and acoustic plasmon mode dispersions, respectively. To understand better the scattering mechanism, we show in Fig. 1 the collective excitation dispersion relations of the two coupled symmetric GaAs quantum wires of width $W = 150 \text{ Å}$ with different barrier widths. In the calculations, we consider the barrier height $V_0 = \infty$, which does not permit tunneling between the wires. The plasmon modes in Fig. 1 correspond to the different scattering channels through which the injected electron can lose energy. We find a higher (lower) frequency plasmon branch which represents the optical (acoustic) plasmon mode $\omega_+ (\omega_-)$. Intrawire quasiparticle excitation continuum $QPE$ (shadow region) is also indicated in the figure. The thin-solid curve is the plasmon dispersion of a single quantum wire with electron density $n_e$. It corresponds to the situation in which the distance between the two wires is infinity ($W_b = \infty$) or $V_C = 0$. In this case, the plasmon mode is of dispersion relation $\omega(q) \sim \sqrt{n_e q |\ln qW|^{1/2}}$ at $q \to 0$. [2] As the distance between the wires decreases, the potential $V_C$ increases. A finite $V_C$ leads to a gap between the two plasmon modes. When the two wires are close enough, the acoustic mode develops a linear wave vector dependence. For $q \to 0$, $\omega_-(q) = vq$ with $v = [v_F + 4V_-(q = 0)/\pi]$ where $v_F$ is the Fermi velocity and $V_-(q) = V_A(q) - V_C(q)$, whereas the optical plasmon still keeps its well-know 1D dispersion relation, $\omega_+(q) \sim \sqrt{N_e q |\ln qW|^{1/2}}$. [4,12] Notice that, the interwire Coulomb interaction $V_C$, depending on the distance between the two wires, is responsible for the behavior of the wavevector dependence of the acoustic mode. As we will see, this affects significantly the inelastic Coulomb scattering rate due to the acoustic plasmons.

Fig. 2 shows the numerical results of inelastic plasmon scattering rate in the coupled wires corresponding to Fig.1(a) with a very small broadening constant $\gamma = 10^{-4}$ meV. We observe two scattering peaks. The lower (higher) one is due to the acoustic (optical) plasmon scattering. The abrupt increase of the scattering rate at threshold electron momenta $k^-$
and \( k^+_c \) correspond to the onset of the scattering of the acoustic and optical plasmon modes, respectively. The higher scattering peak due to the optical plasmon mode is always divergent at the onset \( k = k^+_c \) and \( \sigma_{1,1}(k) \propto (k - k^+_c)^{-1/2} \), similarly to that in the single wire. But the behavior of the lower scattering peak is dependent on the distance between the two wires which is directly related to the dispersion relation of the acoustic plasmon mode at small \( q \). For small \( W_b \), the acoustic mode is of a linear wavevector dependence leading to a finite scattering rate at the onset \( k = k^-_c \). With increasing \( W_b \), the acoustic mode loses its linear \( q \) dependence resulting in the divergency at the onset of the scattering. In order to clarify such a behavior, we show in the inset the energy- vs momentum-loss curve

\[
\omega_k(q) = 2kq - q^2
\]

for \( k = k^+_c \simeq 2.13 \times 10^6 \text{ cm}^{-1} \) (thin-solid curve) and \( k^-_c \simeq 1.65 \times 10^6 \text{ cm}^{-1} \) (thin-dashed curve) in the system with \( W_b = 30 \text{ Å} \). Along these curves, the momentum and energy conservations are obeyed and the electron relaxation is allowed. The dispersions of the optical and acoustic plasmon modes \( \omega_+(q) \) and \( \omega_-(q) \) are also given by thick long-dashed curves in the same figure. For \( k = k^+_c \) \((k^-_c)\), the thin-solid (thin-dashed) curve intersects the optical (acoustic) mode dispersion curve at \( q = q^+_c \) \((q^-_c)\). This means that the injected electron with momentum \( k^+_c \) \((k^-_c)\) can emit one optical (acoustic) plasmon of frequency \( \omega_+(q^+_c) \) \((\omega_-(q^-_c))\). Notice that, the slopes of the curves \( \omega_k^+(q) \) \((\omega_k^-(q))\) and \( \omega_+(q) \) \((\omega_-(q))\) are equal at \( q = q^+_c \) \((q^-_c)\). For the optical plasmon mode, the intersection always occurs at finite \( q^+_c \) because the optical plasmon goes as \( \omega_+(q) \sim q \ln qW \) for small \( q \). The divergency due to the optical plasmon scattering is similar to that in the single quantum wire \(^2\) which is resulted from the coupling of the initial and final states via plasmon emission at \( k = k^+_c \). However, for the acoustic plasmon mode with linear \( q \)-dependence, \( q^-_c = 0 \) because \( \omega_k(q) \to 2kq \) at \( q \to 0 \). In this case, one can obtain \( k^-_c = v/2 \). Due to the fact that the plasmon mode is of vanished oscillator strength at \( q = 0 \), the emission of the acoustic plasmon of the wavevector \( q = q^-_c \) cannot produce divergency in the inelastic-scattering rate. With increasing the distance between the two wires, the acoustic plasmon mode loses its linear \( q \)-dependence and approaches to
the dispersion of the optical plasmon mode. Consequently, the $q_c^{-}$ becomes finite and the scattering rate is divergent at the threshold momentum $k_c^{-}$. In Fig. 3, we show the scattering rates in the same structures as in Fig. 2 but with higher electron density $n_e = 10^6$ cm$^{-1}$. We see that, in the systems of higher electron density, the scattering threshold shift to larger momentum and the scattering is enhanced.

In Fig. 2 and 3, we have not shown the inelastic-scattering rate due to virtual emission of quasiparticles which would occur below the threshold wavevector. It is known that, in a one-subband quantum wire, the contribution of the quasiparticle excitations to the inelastic Coulomb scattering rate is completely suppressed due to the restrictions of the energy and momentum conservations. Consequently, the scattering rate is zero until the onset of the plasmon scattering at a threshold $k_c > k_F$. The quasiparticle excitations contribute to the inelastic scattering only when the level broadening is introduced. These contributions are negligible when the broadening constant is small. Although, in the present case, we are dealing with two coupled quantum wires, the Coulomb interaction does not influence the quasiparticle excitations as well as their contributions to the inelastic scattering.

As far as the effect of the phenomenological broadening constant $\gamma$ is concerned, we show in Fig. 4 the dependence of the inelastic-scattering rate for different $\gamma$. Finite broadening values of $\gamma$ in the system give rise to the breaking of translational invariance due to the presence of impurity. This fact is responsible for relaxing the momentum conservation permitting inelastic scattering via quasiparticle and plasmon excitations for $k < k_c^{\pm}$. We show such a contribution in the inset of Fig. 4. For $k = k_F \simeq 1.6 \times 10^6$ cm$^{-1}$, conservation of energy and momentum does not permit opening of any excitation channels. This means that the injected-electron has infinite lifetime at the Fermi surface which has been restored by impurity effects.
IV. WEAK TUNNELING EFFECTS

In this remaining section, we are going to discuss the effect of weak tunneling on the inelastic Coulomb scattering rates in two coupled symmetric quantum wires as have been shown in the previous section. When the tunneling occurs, an energy gap $\Delta_{SAS}$ opens up between the two lowest subbands which have symmetric and antisymmetric wavefunctions in the $y$-direction about the center of the barrier. In this case, only the subband index is a good quantum number. As we have seen in section II, $V_J$ and $V_H$ vanish in two symmetric quantum wires in resonant tunneling. But, $V_D$ is finite and responsible for the tunneling effects on the Coulomb scattering. In weak resonant tunneling condition, one finds $V_A \simeq V_B \simeq V_C \simeq U$. After some algebra, we obtain

$$V_{1111}^s = \frac{1 + U(\Pi_{11} - \Pi_{22})}{1 - U(\Pi_{11} + \Pi_{22})} U,$$  \hspace{1cm} (11)

$$V_{2222}^s = \frac{1 - U(\Pi_{11} - \Pi_{22})}{1 - U(\Pi_{11} + \Pi_{22})} U,$$  \hspace{1cm} (12)

$$V_{1221}^s = \frac{1 + V_D(\Pi_{12} - \Pi_{21})}{1 - V_D(\Pi_{12} + \Pi_{21})} V_D,$$  \hspace{1cm} (13)

and

$$V_{2112}^s = \frac{1 - V_D(\Pi_{12} - \Pi_{21})}{1 - V_D(\Pi_{12} + \Pi_{21})} V_D.$$  \hspace{1cm} (14)

From the above equations and equation (5), we can obtain the inelastic Coulomb scattering rates in the presence of tunneling. We also notice that the zeros of the denominators in equations (11) and (12) yield the optical plasmon dispersion and those in equations (13) and (14) yield the acoustic plasmon dispersion. It indicates that the optical plasmons only contribute to the intrasubband scattering $\sigma_{11}$ and $\sigma_{22}$, and the acoustic plasmons to the intersubband scattering $\sigma_{12}$ and $\sigma_{21}$.

We consider two coupled GaAs/Al$_{0.3}$Ga$_{0.7}$As ($V_b = 228$ meV) quantum wires of widths $W_1 = W_2 = 150$ Å separated by a barrier of $W_b = 70$ Å. In this case, we find $\Delta_{SAS} = 0.14$.
meV indicating a very weak resonant tunneling. We show in Fig. 5(a) both inter- and intra-subband scattering rates. The intrasubband scattering rates $\sigma_{11}$ and $\sigma_{22}$, induced by the emission of the optical plasmons, is very similar to that in the absence of tunneling. It is also not difficult to understand that $\sigma_{11} \simeq \sigma_{22}$ because, above the threshold of the optical plasmon emission, the plasmon frequency is much larger than $\Delta_{SAS}$ and consequently, $\Pi_{11} \simeq \Pi_{22}$.

On the other hand, the tunneling introduces the intersubband scattering rates $\sigma_{12}$ and $\sigma_{21}$ and modifies strongly the mechanism of the acoustic plasmon emission. In order to clarify such results, we plot the corresponding acoustic plasmon dispersion relation in thick-dashed curve in Fig. 5(b). The acoustic mode develops a plasmon gap at zero $q$ due to the tunneling effect. The thin lines indicate the intersubband energy- vs momentum-loss curves at the onsets of the acoustic plasmon scattering. They are determined by conservations of energy and momentum, given by

$$\omega_{k}^{12}(q) = 2qk - q^2 - \Delta_{SAS}$$  \hspace{1cm} (15)

for $k = k_{c}^{12}$ (thin-dashed curve), and

$$\omega_{k}^{21}(q) = 2qk - q^2 + \Delta_{SAS}$$  \hspace{1cm} (16)

for $k = k_{c}^{21}$ (thin-dotted curve), where $k_{c}^{12}$ and $k_{c}^{21}$ are threshold wavevectors above which the injected electron can be transferred to a different subband by emitting an acoustic plasmon. The $\omega_{k}^{21}(q)$ (thin-dotted curve) intersects the acoustic plasmon dispersion at small wavevector $q = q_{c}^{21} \simeq 0.05 \times 10^6 \text{cm}^{-1}$. The scattering process is similar to the acoustic plasmon scattering in the absence of tunneling as we discussed in the previous section. But now, the acoustic plasmon mode is of finite frequency with also a finite oscillator strength at $q \to 0$ resulting in a small divergency at $k_{c}^{21}$. On the other hand, the intersection between the $\omega_{k}^{12}(q)$ (thin-dashed curve) and the acoustic plasmon dispersion occurs at a quite larger wavevector $q = q_{c}^{12} \simeq 0.18 \times 10^6 \text{cm}^{-1}$. The scattering mechanism is more similar to that of the intrasubband scattering and produces a pronounceable divergence at $k_{c}^{12}$.

Finally, we would like to show the tunneling effects on the total inelastic Coulomb scattering rates $\sigma_n(k) = \sum_{n'} \sigma_{n,n'}(k)$. Fig. 6 gives the total scattering rates in (a) the absence
and (b) the presence of tunneling between two quantum wires with $W = 150 \text{ Å}$ and $W_b = 70 \text{ Å}$. We observe that a weak resonant tunneling does not influence much the optical-plasmon scattering, but it does affect strongly acoustic-plasmon scattering. The acoustic-plasmon scattering for the injected electron in the lowest subband is enhanced significantly and a quite strong scattering peak appears. For the injected electron in the second subband, tunneling introduces a small divergency in the scattering rate and shifts the scattering threshold to the lower wavevector.

V. SUMMARY

We have calculated the inelastic Coulomb scattering rates of two coupled Q1D electron gas systems within the GW approximation. The screened Coulomb potential was obtained within the RPA. The Coulomb interaction between the two quantum wires leads to the optical and acoustic plasmon modes and, consequently, two scattering peaks appear due to the scattering of the two modes. We found that the scattering of the optical plasmons in two coupled quantum wires is very similar to the plasmon scattering in a single wire because both plasmon modes have similar dispersion relations at small $q$. The scattering rate is divergent at the onset of the optical plasmon scattering. However, the acoustic plasmon mode does not produce such a divergency when it is of a linear $q$-dependence at small $q$. This happens when two wires are close enough. Furthermore, we studied the tunneling effects on the inelastic scattering. A weak resonant tunneling was introduced between the wires. Such a tunneling lifts the degeneracy of the two subbands originated from two quantum wires and also produces a small plasmon gap on the acoustic mode at $q = 0$. Moreover, intersubband scattering appears. We show that, in this case, the optical plasmons are only responsible for the intrasubband scattering and the acoustic plasmons are for the intersubband scattering. A weak tunneling enhances significantly acoustic plasmon scattering for an injected electron in the lowest subband.
VI. ACKNOWLEDGMENTS

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FIGURES

FIG. 1. Dispersions of the collective excitations of two coupled quantum wires of (a) $n_e = 0.5 \times 10^6$ cm$^{-1}$ and (b) $n_e = 10^6$ cm$^{-1}$ with $W_1 = W_2 = 150$ Å and $W_b = 300$ Å (dotted curves), 70 Å (dashed curves), and 30 Å (long-dashed curves). Plasmon mode of the corresponding single wire ($W_b = \infty$) is presented by the thin-solid curves. The shadow areas indicate the quasiparticle continua.

FIG. 2. The inelastic Coulomb scattering rates corresponding to Fig. 1(a) with $n_e = 0.5 \times 10^6$ cm$^{-1}$. Inset shows the acoustic and optical modes (thick-dashed lines) for $W_b = 30$ Å, and the intrawire energy- vs momentum-loss curves at the onset of the optical (thin-solid line) and acoustic (thin-dashed line) plasmon scattering.

FIG. 3. The same as Fig. 2 but now with $n_e = 10^6$ cm$^{-1}$.

FIG. 4. The inelastic-scattering rate in coupled wires of $n_e = 10^6$ cm$^{-1}$, $W_1 = W_2 = 150$ Å, and $W_b = 70$ Å for different values of the broadening constant $\gamma = 10^{-4}$ (thin line), 0.01 (dotted line), 0.1 (dashed line), and 1 meV (long-dashed line).

FIG. 5. (a) The intra- and inter-subband inelastic-scattering rates in two coupled GaAs/Al$\textsubscript{0.3}$Ga$\textsubscript{0.7}$As quantum wires with tunneling. $W_1 = W_2 = 150$ Å, $W_b = 70$ Å and $N_e = 10^6$ cm$^{-1}$. The solid curves present $\sigma_{1,1}(k)$ and $\sigma_{2,2}(k)$. The dashed and dotted curves present $\sigma_{1,2}(k)$ and $\sigma_{2,1}(k)$, respectively. (b) The acoustic plasmon dispersion $\omega_{-}(q)$ (thick-dashed curve) in the system. The thin-dashed line indicates the $\omega_{k}^{12}(q)$ curve for $k_{c}^{12} \simeq 1.79 \times 10^6$ cm$^{-1}$ and the thin-dotted line indicates the $\omega_{k}^{21}(q)$ curve for $k_{c}^{21} \simeq 1.59 \times 10^6$ cm$^{-1}$. $n_1 = 0.51 \times 10^6$ cm$^{-1}$ and $n_2 = 0.49 \times 10^6$ cm$^{-1}$.

FIG. 6. Total inelastic-scattering rate $\sigma_n(k)$ of the bi-wire system (a) without ($V_0 = \infty$) and (b) with tunneling ($V_0 = 228$ meV). $W_1 = W_2 = 150$ Å, $W_b = 70$ Å and $N_e = 10^6$ cm$^{-1}$.
(a) \( n=1 \) and 2

(b) \( n=1 \)
\[ n=2 \]