Spin Susceptibility of an Ultra-Low Density Two Dimensional Electron System

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We determine the spin susceptibility in a two dimensional electron system in GaAs/AlGaAs over a wide range of low densities from $2\times10^9$cm$^{-2}$ to $4\times10^{10}$cm$^{-2}$. Our data can be fitted to an equation that describes the density dependence as well as the polarization dependence of the spin susceptibility. It can account for the anomalous g-factors recently reported in GaAs electron and hole systems. The paramagnetic spin susceptibility increases with decreasing density as expected from theoretical calculations.

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The low-density ground state of a degenerate electron system is one of the oldest questions of many particle physics. In two dimensions, it is expected that in the dilute limit, the electron Fermi liquid undergoes a phase transition to a solid known as the Wigner crystal[1, 2, 3]. Furthermore, driven by the exchange coupling between electrons, a ferromagnetic state may arise at densities slightly above the critical density of Wigner crystalization[4, 5, 6, 7]. None of these phases have yet been detected due to the lack of sufficiently high-quality, low-density specimens. Recent measurements of relevant system parameters such as the spin susceptibility $\chi$ and the effective g-factor $g^*$ have shown considerable deviations from their standard, non-interacting values[8, 9, 10, 11, 12]. This is believed to be the result of strong interaction[11, 12], but disorder-led enhancement cannot be ruled out. Such explorations of the pre-transition regime provide valuable data against which to check the same theoretical calculations that determine the transition point to either a ferromagnetic state or to the Wigner solid. A possible connection between the ferromagnetic properties and the metal-to-insulator transition (MIT) in low density 2DESs has added to the complexity of the problem and prompted intense recent studies on this subject in Silicon MOSFETs.[8, 9, 10, 11, 12] These results generally lend support to the possibility of a ferromagnetic state. On the other hand two groups have recently reported anomalous density dependencies of the g-factors in the GaAs/AlGaAs electron and hole systems that are at odds with results in Silicon MOSFETs and disfavor the predicted ferromagnetic transition[11, 12].

In this paper, we report measurements of $\chi$ in a variable density 2DES of exceedingly high quality to unprecedented low densities. In addition to the density dependence of $\chi$, our measurements determine, for the first time, the explicit polarization dependence of $\chi$. This polarization dependence can account for the "anomalous" g-factors recently reported in GaAs 2DESs and 2D hole systems (2DHG) [11, 12].

Our specimen is a heterojunction-insulated gate field effect transistor (HIGFET). The specimen consists of a (001) GaAs substrate, overgrown by molecular beam epitaxy with 0.5µm of GaAs, followed by a 200-fold superlattice of 10nm of GaAs and 3nm of Al$_{0.32}$Ga$_{0.68}$As. Subsequently, 2µm of GaAs are deposited as a channel, followed by 600nm of Al$_{0.32}$Ga$_{0.68}$As as an effective insulator, and capped by a heavily doped GaAs n$^+$ layer, serving as a top gate. The specimen is processed into a 600 µm square mesa. Sixteen Ni-Ge-Au contact pads are spaced evenly along the edges of the mesa using standard photolithography. One corner pad provides the contact to the top gate, which allows for a continuous and in situ change of the 2DES density. The density range available for measurement extends from $1.7\times10^9$cm$^{-2}$ to $6.4\times10^{10}$cm$^{-2}$. The interaction parameter $r_s$, defined as the ratio of the inter-electron spacing to the Bohr radius, $a/a_B=1/\sqrt{\pi n_B}$, spans $2.2<r_s<13.4$. Screening of the e-e interaction via the top metallic gate can be neglected, since its distance exceeds the electron spacing by a factor of 4.4 to 27 in our experiments. The MIT occurs at $2\times10^9$cm$^{-2}$, which is the lowest transition density ever reported in a 2D system, attesting to the low disorder of the specimen. Our FET gives us the unique advantage of wide density tunability within one specimen and its reproducible gate-voltage/density relation lets us sweep density at fixed magnetic field, which is an essential method in our $\chi$ determination.

Our measurements were performed in a dilution refrigerator equipped with a rotating sample platform reaching a base temperature of 30mK. A standard low frequency (3-23Hz) lock-in technique was used with excitation currents ranging from 100pA to 100nA. All experiments were performed during one cooldown, facilitating quantitative comparison between our data.

In a normal Fermi liquid, the spin susceptibility $\chi = d\Delta n/dB = g^*\rho/2\pi$, where $g^*$ is the effective g-factor, $\rho$ is the density of states (DOS) at the Fermi level, and $\Delta n=n^{\uparrow}-n^{\downarrow}$. In a 2D system, $\rho = m^*/\pi\hbar^2$, therefore
\[ \chi = g^* m^* \mu_B / 2\pi \hbar^2. \] Generally, theories find that \( \chi \) increases with growing interaction due to spin-exchange coupling. To investigate the experimental situation, we measure the spin susceptibility \( \chi \) as a function of the 2DES density \( n \). We express it as a relative spin susceptibility \( \chi / \chi_0 = m^* g^* / m_0 g_0 \), where \( m_0 = 0.067 m_e \) and \( g_0 = 0.44 \) are the band values of mass and \( g \)-factor in GaAs and \( \chi_0 \) the Pauli susceptibility determined by these band values. In the remainder of the paper, we use values for \( \chi \) and \( m^* g^* \) normalized to \( \chi_0 \) and \( m_0 g_0 \) respectively, so that \( \chi = m^* g^* \). We employ two different methods to measure \( m^* g^* \). First, we follow and extend the tilted field method introduced by Fang and Stiles in Silicon MOSFET \(^{16}\). Secondly, we follow the parallel-field method recently utilized by Refs. \(^{11, 12}\) to derive \( m^* g^* \) from the full polarization condition of the 2DES.

In a magnetic field, spin-up and spin-down electrons form two separate sets of Landau levels. As the magnetic field is tilted, the two sets of Landau levels move with respect to each other. The energy diagrams are schematically shown as insets in Fig. 1(a). Solid and dotted lines represent spin-up and spin-down Landau levels, respectively. The spacing between Landau levels, \( \hbar \omega_c = e \hbar B_{\text{perp}} / m^* \), depends on the perpendicular component of the field. The shift between both sets, on the other hand, is the Zeeman energy, \( \Delta E_z = g^* \mu_B B_{\text{tot}} \), which depends on the total field. By adjusting the tilt angle, \( \theta \) and the total field, \( B_{\text{tot}}, \hbar \omega_c \) and \( \Delta E_z \) can be independently changed. Particularly useful configurations arise when \( g^* \mu_B B_{\text{tot}} = i \hbar \omega_c \), where \( i \) is an integer or a half-integer. At half-integer configurations, the Landau levels from both spins interleave and form a set of uniformly spaced levels. At integer configurations the Landau levels coincide and form again a set of uniformly spaced levels, however, this time with double spacing compared to the half-integer case (see insets Fig. 1(a)).

The magneto-resistance of the 2DES reflects the configuration of the energy levels. Distinctive signatures from different configurations are observed when the 2DES density is swept at fixed \( B_{\text{tot}} \) and \( \theta \). \( B_{\text{perp}} \), and hence \( \theta \), is accurately determined from the period of the oscillations and the Landau level degeneracy \( e B_{\text{perp}} / \hbar \). At half-integer configurations, the depths of successive minima are just equal. This is also the case for the integer configurations, but here each second minimum disappears. The different configurations are realized only transiently at particular densities \( n_0 \) that satisfy \( g^* \mu_B B_{\text{tot}} = i \hbar \omega_c \), equivalent to \( m^* g^*(n_0) = i e \hbar \cos \theta / \mu_B \), since \( m^* g^* \) depends on density. Integer configurations are easily identified by the disappearance of a minimum. At half-integer configurations, the depth of neighboring minima interchange strength at \( n_0 \), which is identified by the crossing point of two smooth envelopes drawn along alternate minima, see Fig. 1(a) for 85.71°. The trace at 88.40° of Fig. 1(a) shows, as examples, several configurations as the density is swept and indicated by the neighboring diagrams. Increasing \( \theta \) slowly and tracking the indices carefully, we are able to identify and label events that belong to configurations with index 1/2, 1, 3/2, 2, and 5/2. The product \( m^* g^* \) is calculated according to \( m^* g^* = i e \hbar \cos \theta / \mu_B \). The solid symbols in Fig. 2 represent data derived with this method, using different symbols for different indices.

Clearly, \( m^* g^* \) increases with decreasing density and increasing index \( i \). More strikingly, for each fixed index \( i \), \( m^* g^* \) displays a power law dependence on \( n \) within the measured range, indicated by the dashed lines on a log-log scale. This dependence is best developed for \( i = 1/2 \) and 3/2. Furthermore, the parallel lines suggest a single exponent. We therefore fit a power law dependence to the data of \( i = 1/2 \) and use the exponent as a constraint in fitting data of the other indices. The coefficients from these five fits display a very good linear dependence on \( i \), as seen in Fig. 1(b). This gives us an empirical equation, \( m^* g^* = (2.73 + 0.66i) n^{-0.4} \), where \( n \) is in units of \( 10^{10} \text{cm}^{-2} \). This equation describes all our data points remarkably well.

Before discussing the implications of such an empirical equation, we proceed with the second method of determining \( m^* g^* \). With increasing in-plane magnetic...
field, $B_{||}$, the polarization, $P=(n^+-n^-)/n$ of the 2DES increases and saturates at unity at a threshold field $B_p$ when $g^*\mu_BB_p/2=\hbar\omega_c$. The derivative of $\Delta n (=Pn)$ with respect to $B_{||}$ is $\chi (=m^*g^*)$. Assuming $m^*g^*$ to be independent of $P$ it follows that $m^*g^*=2\pi\hbar^2n/\mu_BB_p$. As asserted by several groups, full polarization of the 2DES is signaled by the onset of an exponential behavior in a parallel-field magneto-resistance experiment [3, 17, 18, 19, 20]. We have performed such experiments on our HIGFET for different densities (Fig. 1(c)), determined $B_p$ according to Ref. [11], and show their values in Fig. 1(d). Within experimental error, $B_p$ is independent of the current direction relative to $B_{||}$. Translated into $m^*g^*$ the results are plotted as open circles in Fig. 2. Unlike the solid data points from the first determination of $m^*g^*$, these new data exhibit a non-monotonic dependence on density and are consistently larger in value than the solid data points, with the discrepancy increasing with increasing density.

The key to reconciling the discrepancy between these data sets is to recognize the dependence of $m^*g^*$ on the polarization, $P$. In the tilted field method, a higher index $i$ implies a higher Zeeman energy and therefore a higher degree of 2DES polarization. In the Fermi liquid limit ($B_{\text{perp}}=0$), $P=(n^+-n^-)/n = \Delta E_p/\hbar^2n=(g^*\mu_BB_{\text{tot}})(m^*/2\hbar^2)/n$. The tilted field method assumes that the introduction of a small $B_{\text{perp}}$ does not alter the values of $g^*$ and $m^*$. Using the relation $g^*\mu_BB_{\text{tot}}=i\hbar\omega_c$, a straightforward calculation yields $P=ieB_{\text{perp}}/\hbar n$. This direct relationship between $P$ and $n$, the susceptibility $\chi$ increases linearly with increasing polarization. This monotonic increase reflects an increasing spin-exchange energy with increasing population of like-spins.

Quite clearly, the assumption of a $P$-independent $\chi$, assumed in Refs. [11, 12] to derive $m^*g^*$ from $B_p$, does not apply. Our empirical interpolation equation can be used to make contact between the data obtained from the tilted-field method and from the parallel field method. Their relationship is best discussed referring to the inset of Fig. 2. This diagram shows, as an example, the evolution of the net spin $\Delta n=Pn=n^+-n^-$ with $B_{\text{tot}}$ for a fixed density, $n=1\times10^{10}\text{cm}^{-2}$. By definition, the slope of $\Delta n(B)$ is the susceptibility $\chi = m^*g^*$. The parallel-field method determines $\Delta n=n (=1\times10^{10}\text{cm}^{-2})$ at $B_p=4.9T$. The assumption of a $B$-independent $m^*g^*$ implies a linear rise of $\Delta n$ with $B$, shown as a straight line in the inset. One value, $B_p=4.9T$, determines $m^*g^*_p=2\pi\hbar^2n/\mu_BB_p=5.7$. However, the relationship between $\Delta n$ and $B$ is not linear but must be deduced from the tilted-field experiments. We derive the actual $\Delta n$ vs $B$ curve by integrating the empirical equation of $\chi(P)$ and obtain $\Delta n=0.69[\exp(0.138Bn^{-0.4})-1]$. The curve for $n=1\times10^{10}\text{cm}^{-2}$ is also shown in the inset. The solid portion represents the interpolated regime of $\chi$ while the dotted portion represents extrapolation of $\chi$ beyond our tilted field data. Requiring $\Delta n=n (=1\times10^{10}\text{cm}^{-2})$, we obtain $B_{\text{ext}}=6.5T$ for the full polarization field, which is 33% higher than the measured value of $B_p$. This $B_{\text{ext}}$ yields a ”nominal” $m^*g^*_\text{ext}=2\pi\hbar^2n/\mu_BB_{\text{ext}}=4.3$, equivalent to the slope of the dash-dotted line in the inset.

Performing this derivation for all densities using the interpolation equation we arrive at an $m^*g^*_\text{ext}$, which is plotted as a thin, curved line in Fig. 2. It shows a qualitative similarity to the $m^*g^*_p$ data derived from the parallel field measurements, particularly the non-monotonic density dependence. In fact, in the low density limit our derived curve matches $m^*g^*_p$ very well, indicating that our empirical equation extrapolates well into this
regime. At higher densities considerable discrepancies arise, which one may, at first glance, attribute to a discontinuity in $\Delta n$ on the polarization curve, indicating the existence of a first order phase transition. However, given the good agreement between $m^*g^*_p$ and $m^*g^*_\text{ext}$ at low densities, and hence the absence of a phase transition in this regime, it is unlikely that such a transition occurs at higher densities. Instead, we suspect that the extrapolation of our empirical equation becomes less accurate with increasing density due to the increasing range of extrapolation and the solid curve of the inset bends sharply, but continuously towards $B=4.9T$. A mass increase, caused by the in-plane field, could provide a mechanism for such an accelerated bending. This effect is negligible at low field and hence low density, but increases rapidly with $B_\parallel$ [22, 23]. Alternatively, at high polarization, terms of higher order in $\Delta n$ may increasingly contribute to $\chi$, leading to an accelerated bending as well. In spite of this discrepancy for extrapolations at high densities, the qualitative and partially quantitative agreement between $m^*g^*_\text{ext}$ and $m^*g^*_p$ strongly suggests that our empirical equation captures correctly the underlying physics of the system.

Our analysis of $m^*g^*$ provides a simple interpretation of the non-monotonic behavior of $m^*g^*$ derived from the parallel-field data. The unusual density dependence of $m^*g^*_p$ results from a combination of the polarization and density dependence of $\chi$. Clearly, this $m^*g^*$ does not reflect the actual susceptibility at any polarization value and consequently its density dependence cannot be used to assess the possibility of a ferromagnetic transition. By comparing measurement and extrapolation, we conclude that a first order transition is unlikely to occur in our 2DES within the regime of $r_s$ studied (3-12.4). In the remainder of the paper, we examine $\chi$ in the limit of vanishing polarization, which plays a central role in the context of a second order ferromagnetic transition.

Our empirical formula $\chi=m^*g^*=(2.73+3.9P_n)n^{-0.4}$, is readily extrapolated to $P=0$ to yield the spin susceptibility, $\chi$, of a normal Fermi liquid (see Fig. 2). From $n=5\times10^9\text{cm}^{-2}$ to $4\times10^{10}\text{cm}^{-2}$, $\chi = 2.73n^{-0.4}$ showing an enhancement factor of 1.6 to 3.6. This extrapolation is very reliable since it extends only slightly beyond the range of our data. Furthermore, from the excellent agreement at low densities between the $m^*g^*_p$ data and the extrapolated $m^*g^*_\text{ext}$ we confirm that it provides very good estimates for $\chi$ to densities as low as the MIT density $n=2\times10^9\text{cm}^{-2}$ ($r_s=12.4$), where $\chi$ reaches about 5.5. Whether a divergence occurs at yet lower density cannot be inferred from our data.

Recent quantum Monte Carlo (QMC) calculations have predicted a weakly first order ferromagnetic transition at $r_s=20-30$, corresponding to $3.4-7.6\times10^9\text{cm}^{-2}$ in our 2DES [16, 17]. This density range is currently beyond our reach. However, our measurement of $\chi$ as the 2DES approaches the phase transition can be compared to the-
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