Mechanical mass separator: an integrating approach in teaching Mechanics for Engineering and Physics majors

Américo Cuchillo Flórez$^{1,2}$

$^1$Departamento de Física, Universidad de Atacama (UDA), Copiapó, Chile
$^2$Centro para el Desarrollo Educacional Yachachiy (CEDEYA), Cusco, Perú

e-mail: americo.cuchillo@uda.cl

Abstract. An Integrating Approach (IA) in the teaching and learning of a first formal course of Mechanics for future engineers and physicists is presented. In this approach, the course is introduced as a practical and technological problem to resolve that delineates a single route during its development. The need to solve the different aspects of the problem leads progressively to establish the contents and concepts of the subject; i.e., these arise as tools to understand and resolve the aforementioned problem. The approach allows to elucidate and assess the approximations, limitations and corrections that must be performed to an own model of a scientific discipline such as physics. The approach potentiality relies in allowing the student, through a concrete and simple problem, to articulate Kinematics, Newton's laws, and Energy, appreciating the subject in a holistic and non-fragmentary fashion. Moreover, it allows the students value Physics and to estimate its usefulness, because they find it useful and necessary for understand the basics behind all technological development, even although it seems very simple. To this purpose, is proposed use a simple device that shoots different masses, through a spring placed on a horizontal surface at a certain height, and measure the distance to which they fall to the floor. This device facilitates the explanation of the physical concepts that emerge in a first analysis of the problem. This approach, as well, facilitates to present in an elegant way the influence of the mass on motion as well as the distinction between constant and variable forces. The experimental results show mass effects according with theoretical predictions, allowing going beyond the usual kinematics taught at this level.

1. Introduction

Physics teaching seeks the abstraction of reality by translating it into simple and universal models that allow describing and predicting the behavior of the natural phenomena that these models pretend to represent [1]. However, this same abstraction may interfere with students’ understanding of Physics, especially if the students are facing for the first time a formal course of Mechanics at university level. Concepts such as "point" object or particle, ideal string, ideal pulley, to name a few may induce to think they are aspects disconnected from reality and with little or none practical application. This may generate lack of interest for Physics and a decreasing motivation for subject learning. Textbooks, as professors in general, try to neutralize this by relating these abstract models with practical and real applications. This is mainly done through examples and exercises where concepts and mathematical relations are used in a wide range of common contexts and even interdisciplinary ones. While these practical applications allow students to value its usefulness, the mere fact the used model came from the above-mentioned simplifications may generate suspicion and confusion on the student about if these practical examples are directly related to reality or they are just "practical situations in paper". In
other words, it is necessary to apply the model to a concrete and real situation during the teaching of Mechanics and in general Physics.

In general, lab activities account for the latter work and can be carried out prior to, during or after theory teaching. They allow the student to link different activities to the concepts and models they have previously studied.

Both applications of the model through exercises and lab activities are oriented towards diversifying its use and applications. This is necessary since we are dealing with a basic science with an interdisciplinary reach. However classroom exercises are not always articulated within the same topic, least with exercises related to other topics within the same subject, save for concepts and mathematical relations. The same situation occurs with lab activities for a variety of reasons, such as time, equipment access at institutions with less money or others.

When there is no clear articulation between the abstraction activities (modelling), application of a given model (exercises) and lab (experiments), these are seen as fragments of something called Mechanics, which is good and useful, but lacking something concrete which can articulate the different activities carried out during Mechanics teaching.

On the other hand, even if the student performs well, not knowing when and why a given activity is carried out may lead him/her to gradually lose both direction and initial motivation. Lacking a define direction and something concrete that articulate the subject as a whole the student sees himself/herself in a sea of contents and activities, achieving learning without a clear purpose other than passing the subject and the hope of using this knowledge in the future in his/her own field of expertise. A few of above-mentioned problems has been extensively studied in the past by Physics teaching researchers, by means of robust tools, with findings such as that described here [2-6].

This paper sets a direction by using a concrete and real problem for the student to solve while, along the subject, a mechanical system (simple device) to solve the problem is designed by him/her. In this way the modelling and abstraction of a real physical system may be made easier and more understandable, presenting models as useful and predictive tools, with validity limited by the imposed approximations and, therefore, open to future corrections. Using this approach the future engineer learns why he/she needs Physics while the future physicist learns how a Physics theory is built.

The use of a concrete problem, being it real and practical, allows to organize the subject highlighting that solution requires understanding and interconnection of concepts being developed with already studied ones. This allows to achieve an integral articulation of different topics of the Mechanics through a unique and practical problem.

Finally this approach suggests to the Physics teacher he/she can always set the direction of the subject through a problem adequate to the educational context.

2. The Problem

The problem is to separate solid spheres of equal radius according to their different masses. This situation may occurs, for example, when separating beads with bimodal or even multimodal mass distribution within defined mass ranges. For the given case, it is considered a system composed by spheres of equal volume with different masses \(m\) and \(M\). As a device to achieve the stated purpose, a “mass launcher” consisting of a tube with a spring inside it is proposed. The masses are shot from the end of the device, located at a height \(H\) of the ground, and collected at some point at ground level, located a horizontal distance \(D\) from the launch point as shown in Figure 1. In principle, this device allows to give a complete solution to the proposed problem and sets the direction of the subject. It is also the concrete element of application and articulation.

During the development of the subject the student will design and build, at least partially, the device. The most important thing is that the student will learn in the subject the concepts of mechanics through the construction and operation of the apparatus, being able to perform this in a group or individual way.

At the end of the course (Mechanics), the student must know in detail both the operation of the device and the physical principles that underlie it.
Figure 1. Right: Mechanical mass separator. For simplicity, the masses will always be launched under the same geometric conditions: height and angle (horizontal throw for the case). Right: Shooting done by one undergraduate student. It was done at angle different to zero by picture facilities during the flight.

The problem and the proposed solution do not limit, nor much less replace, the varied applications that should be taken both in theoretical classes and laboratory activities. However, the subject is oriented to achieve the final task that has been proposed, articulating implicitly the intermediate actions.

3. Subject Development

When observing the device and the mass selection mechanism the first problem that arises is the need to describe the movement of a body, in this case the sphere. It is necessary to introduce the physical concepts of: observer, reference system, position, speed and acceleration, among others; that is, the kinematics of a particle.

The application of the theoretical model of the movement of a projectile allows to predict that the horizontal distance to which the sphere falls from the launch point is given by:

\[ D = v_0 \cdot \sqrt{\frac{2H}{g}} \]  

(1)

Where \( H \), \( v_0 \) and \( g \) are the height of the launch, the initial velocity of the sphere, and the acceleration of gravity, respectively.

With respect to this result, the only reason that two different mass spheres launched from the same point can impact at two different positions is because of the difference between their initial velocities. Therefore, the different speeds that they acquire only depend on their masses. Then, the initial velocity of the sphere is a function of its mass: \( v_0 = v_0(m) \).

At this stage the urgent need of introducing the concept of mass appears. It should be noted that a mass cannot reach, on its own, a greater or lesser speed. The action of one or more external agents is required to cause any change in velocity. This, then, leads to the introduction of the concept of force and the laws of Newton, i.e., the study of dynamics. However, it should be remembered that in the above case the mass only affects the initial velocity of the projectile but not its subsequent movement; this will be discussed later.

At this stage, it is important to introduce the mass launcher mechanism. This consists of a smooth tube in which there is a spring with an elastic constant \( k \) whose natural length reaches the point where the sphere leaves the mass launcher. The spring is compressed into a deformation \( \Delta x = L \) and when released moves the mass \( m \) a length \( L \) from A to B, as shown in Figure 2.
The net force on the sphere is due only to the restoring force of the spring which is a variable force. However, in a first tentative approach to solve the problem, the force will be considered constant and equal to its initial value at A (see footnote)\(^1\).

With this assumption, the acceleration is given by:

\[
a = \frac{k \Delta x}{m}
\]  
(2)

and applying the equations of constant acceleration, the speed with which the sphere comes out of the tube is:

\[
v_{\text{salida}} = \sqrt{\frac{2k \Delta x}{m} L} = v_0(m)
\]  
(3)

Recalling that \(\Delta x = L\) results in:

\[
v_0(m) = \sqrt{\frac{2k}{m} L}
\]  
(4)

with which the dependence of the initial velocity, as a function of mass, and hence the expression for the horizontal range \(D\), is determined by the equation (1). By substituting (4) into (1) we can see the kinematics and dynamics articulated since \(D(k,m,L,H,g)\) contains elements of both but only roughly.

However, it is clear that there is an erroneous consideration in the previous application, which facilitates discussion, reinforces and makes note of the differences between a variable force, such as that manifested in an ideal spring, and the constant forces usually studied up until this stage.

\[\text{Figure 2. Mass launcher schematic representation. The points A and B correspond to the points of maximum compression and null elongation of the spring respectively.}\]

This difficulty, when working with variable forces, gives reason for noting the need to reformulate Newton’s second law, which is discussed later, introducing the physical concepts of work and mechanical energy, without which it is very difficult to solve the problem. Once work, mechanical energy and its conservation have been studied, it is possible to apply the theoretical model and perform the correct calculation of the exit speed of the sphere.

Applying the conservation of energy, with the approximation of disregarding friction, results in the corrected initial velocity of the sphere:

\[
v'_{\text{salida}} = v'_0(m) = \sqrt{\frac{k}{m} L}
\]  
(5)

\(^1\)This consideration is a fairly common mistake for a student who has only worked with constant forces; however, it can also be induced by the mathematical limitations of their level of training.
which differs from the result previously obtained. Finally, by substituting (5) into (1) we can see the kinematics, dynamics and energy articulated, now allowing for an exact solution to the problem.

From the correction made, it can be argued and emphasized that the variable forces are not applied as the constant forces and when not properly considered, the output velocity is overestimated by a factor of \( \sqrt{2} \). This demonstrates the need and utility of formulating new models with different potentialities such as energy.

In this way, it is possible to present the contents and the physical principles underlying the operation of the proposed device, by means of an approach which helps develop an integrated learning focused on a single problem in all its aspects indicated and discussed above.

4. Discussions

The mass of the particle in the movement of a projectile or kinematics is usually ignored. However for the inquisitive student it is conflicting to accept that the mass of the object does not affect the movement of the body, since the experience of throwing an object suggests that with a larger mass a smaller reach is achieved. Kinematics does not manage to explain this to the student, but the texts at this level do not clarify it either. However, through the approach presented this behavior emerges clearly and at the same time naturally through equations (5) and (1). Although the mass does not affect the movement during its flight, it only affects the initial velocity and therefore the horizontal range. In the studied system the launch velocity is dependent on the mass of the particle and its effect is seen in the achieved reach. The approach considered does not avoid this reality, since it makes imperative to consider the mass, making it clear that it only intervenes in a stage of the movement, which does not generate any conflict with everyday life.

It is also immediately appreciated that the same force (mass launcher) produces different effects \( (v_0) \) on different masses. This forces the immediate joint (articulation) between the kinematic and Newton’s laws.

In Figure 3 are shown results from experimental measurements under the same conditions done by some students, as a way to illustrate what would be expected from measurements made by all physics students. For this, a set of spheres with masses concentrated in two regions was considered. As can be seen, those with the largest mass obtain lower (initial) output velocity as opposed to the smaller ones. As a consequence of this, the larger masses achieve a lower (horizontal) range, as can be seen in the upper box. On the other hand the experimental range is compared with the theoretically expected value in the lower box and the adjusted straight line shown has slope 0.94. In an ideal situation (hypothetical) the line should have slope 1.00, however, the obtained result is quite clear in showing that there is an expected correspondence between the theoretical and experimental values. This should be sufficient to illustrate the usefulness (validity) of the theory and the performance of the apparatus as a mass separator.

On the other hand, the approach given to the problem allows to clarify a very common problem in variable forces that is the confusion between position, displacement and deformation, noting that this can lead to erroneous conclusions similar to the one observed in the relations (2) and (3).

Finally, it should be noted that in the usual transition to the concepts of work and energy, these usually arise as new topics unrelated to Newton’s laws except to define the concepts of kinetic and potential energy (work-energy theorem). However, the beauty of the approach allows them to be introduced in a natural, useful and necessary way to overcome the difficulty imposed by the variable forces, thus articulating the dynamics with energy and work, allowing a different view of the problem through tools with other potentialities and limitations with respect to Newton’s laws.
5. Conclusions

Through a visual problem, real, simple, but also challenging for the student, it is possible on the one hand to articulate the contents and activities of the subject; and on the other hand establishes a heading, central course, from beginning to end, during the development of the subject. This allows us to conceive an integral vision of Mechanics and specially avoid getting lost in the contents and the Mathematics.

The presented approach allows to reflect and to go beyond the second law of Newton with constant force, articulating it with the concepts of energy as an alternative formulation for the study of the movement. This is a valuable aspect since a progressive transition is made from a concrete (problem) object to one of the most successful abstractions of physics and at the same time one of the most difficult concepts to understand, that is energy.

Finally, the approach facilitates the student, to follow and evaluate the different stages and tools that are required when constructing a model or a physical theory of something concrete, and at the same time allows him to confirm that a physical model, in spite of the simplifications, is useful in a real problem and transcends its limitations (especially crucial in a first course of abstraction and/or physical modeling).

Regarding the impact of the Integrating Approach (IA) on student learning, research will be carried out and reported in a timely manner.
Acknowledgments
The author gives special thanks to Professors Gonzalo Fuster Roa, Nicolás Porras Rojas and Roberto Rojas Vega, collaborators with a long history in teaching subjects from the Department of Physics of the Universidad Técnica Federico Santa María, for: a) to have corrected and suggested important modifications presented in this article; b) to be great managers of the initiation and conclusion of the author in the investigation in this matter; and c) to have generously provided material requirements and their knowledge. My students have been, over the years, my best systems of study, learning as well as inspiration, for which I thank you for their willingness. The work was partially funded through the INCUBA project of the Faculty of Natural Sciences UDA.

References
[1] Young H D, Freedman R A and Ford A L 2008 Sears and Zemansky’s University Physics with Modern Physics 12th ed, (San Francisco: Pearson Education, Inc.)
[2] Hammer D 1994 Epistemological beliefs in introductory physics Cogn. Instruct. 12 151-83
[3] Redish E F, Steinberg R N and Saul J M 1997 The Distribution and Change of Student Expectations in Introductory Physics (in The Changing Role of Physics Departments in Modern Universities) AIP Conf. Proc. 399 689-98
[4] Redish E F, Steinberg R N and Saul J M 1998 Student Expectations in an Introductory Physics Am. J. Phys. 66 212-24
[5] Adams W K, Perkins K K, Podolefsky N S, Dubson M, Finkelstein N D and Wieman C E 2006 A new instrument for measuring student beliefs about physics and learning physics: The Colorado Learning Attitudes about Science Survey Phys. Rev. ST Phys. Educ. Res. 2 010101
[6] Sahin M 2009 Exploring university students’ expectations about physics and physics learning in a problem-based learning context Eurasia J. Math. Sci. Technol. Educ. 5(4) 321–33