A reliability index for fuzzy expert group criterion

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Abstract. In the paper, a fuzzy expert group criterion is presented in the form of a linguistic variable with the properties of completeness and orthogonality. The expert group criterion is created based on individual expert criteria of evaluation a quantitative or qualitative characteristic. The construction is carried out on the basis of the minimum loss of information contained in the individual criteria. For neighboring terms (concepts) of the criterion, analogues of probabilities of errors of the first and the second kinds are determined, as well as the reliability index, taking into account the zones of uncertainty of their membership functions. The reliability index of fuzzy expert group criterion is determined based on the reliability indexes of all the neighboring terms.

1. Introduction
Linguistic variables, first identified in 1975 by Prof. Lotfi Zadeh [1], gave the beginning of a new stage in the expert information processing, harmonizing the mathematical modeling of the features of human mental activity and the possibility of their software implementation.

Linguistic variables formed the basis of the theoretical foundation of fuzzy logic and its practical applications [2-10]. The development of fuzzy logic has opened up solutions of approximate computational problems using linguistic variables. This development has resulted in numerous practical applications in various fields of human activity [2, 4, 6, 7, 10]. For the successful solution of problems in these areas, theoretical research is needed, primarily related to the properties of linguistic variables.

The researchers noticed that not all linguistic variables allow to adequately model evaluative processes and this was due to the properties of their membership functions. So, for example, for two adjacent terms the choice of one of them was not always clear. If membership functions of non-neighboring terms intersected, then the choice of the term was even more difficult. Significant difficulties arose if some point or segment of the universal set did not belong to any of the terms (that is, all the membership functions of a linguistic variable at this point or on a segment took zero value).

Over time, a natural question has become the question of the number of values of a linguistic variable. These values should not have duplicated each other, but, on the other hand, they should be enough so that a situation does not arise when some subsets of the universal set cannot be identified with any of the terms, because they do not belong to any of them.

Paper [3] was in fact the first to deal with theoretical substantiation of the properties of linguistic variables. It also determines the degree of fuzziness of the linguistic variable, which is associated with the degree of difficulty experienced by an expert using it for descriptive or evaluative processes.
The results [3] were used in [11] to determine the optimal linguistic scale based on linguistic variables, which would give experts a minimum of difficulties in using it.

Studies of the properties of linguistic variables made it possible to formulate logical requirements for their membership functions, which are that each point of the universal set must belong to one term or two neighboring terms, and the sum of the membership functions of all terms at any point is equal to one. Each term must have a typical point or segment for which its membership function takes a single value. The presence of typical points or segments simplifies the estimation process since they play the role of a reference for the corresponding term.

Linguistic variables with such properties were chosen in [12] as expert criteria for evaluating quantitative or qualitative characteristics. In this paper, the same requirements are imposed on the membership functions of the fuzzy expert group criterion. The construction of the fuzzy expert group criterion is carried out on the basis of the minimum loss of information contained in the individual expert criteria.

There are no similar models for constructing a fuzzy expert group criterion. Also, the problem of assessing the reliability of an expert group criterion, constructed on the basis of individual expert criteria, has not been solved.

The purpose of the paper is to construct a fuzzy expert group criterion and determine a reliability index for it.

2. Basic concepts and definitions

A triple \((X, U, \tilde{A})\) is a fuzzy variable with title \(X\), universal set \(U\) and fuzzy set \(\tilde{A}\), describing possible values of this variable [1]. On the basis of concept of a fuzzy variable the concept of a linguistic variable is introduced.

A quintuple \(\{X, T(X), U, V, S\}\) is a linguistic variable with title \(X\), a term-set \(T(X)\), universal set \(U\), syntactic rule \(V\) and semantic rule \(S\). A linguistic variable with the fixed \(T(X)\) is a semantic scope.

A semantic scope is a Full Orthogonal Semantic Scope, if according to [3], its membership functions have the properties: all the functions equal to one at a point or at segment and increase to the left of the point or segment on which they take the unit value, and decrease to the right of it; have a maximum of two break points of the first kind; the sum of all the functions at any point of \(U\) is equal to one.

In [3] the definition of fuzziness degree \(\zeta = \frac{1}{|U|} \int_U f(\mu_1(x) - \mu_2(x))dx\) of Full Orthogonal Semantic Scopes have been done, where \(\mu_k(x) = \max_{1 \leq j \leq m} \mu_j(x)\), \(\mu_j(x) = \max_{1 \leq j \leq m} \mu_j(x), f(0) = 1, f(1) = 0\).

If \(f(x) = 1 - x\), then \(\zeta = \frac{1}{|U|} \int_U \left(1 - \left(\mu_k(x) - \mu_j(x)\right)\right)dx = \frac{|U|}{2|U|}\), where \(\bar{U} = U - \bigcup_{l=1}^{m} \bar{U}_l\), \(\bar{U}_l = \{x \in U : 0 < \mu_k(x) < 1\}\).

In [9, 10], methods of creating individual expert criteria in the form Full Orthogonal Semantic Scopes are described in detail. These methods use statistical information or information of direct survey of a single expert.

If a characteristic \(X\) assessed by the expert is qualitative, then the universal set \(U\) is the segment \([0,1]\), if a characteristic \(X\) assessed by the expert is quantitative, then the universal set \(U\) is the segment \([a,b]\). To construct the Full Orthogonal Semantic Scope with a title \(X\) and term-set \(T(X) = \{X_1, X_2, ..., X_m\}\), an expert defines the typical points or intervals \((b^1_j, b^2_j)\) (for which membership functions equal to one) for all the terms \(X_j, j = 1, m\). Based on the information obtained,
each term $X_j$, $j=1,m$ can be formalized using a trapezoidal or triangle fuzzy number $\tilde{X}_j$, $j=1,m$ with membership function $\mu_j(x)$, $j=1,m$, which is determined by four or three parameters. For example, for trapezoidal number $\tilde{B}$ ($\mu_B(x) = (b_1, b_2, b_1, b_2)$) parameters $b_1, b_2$ are abscissas of the ends of the upper base and $b_1, b_2$ are the lengths of the left and right trapezium wings correspondingly.

If $X$ is a qualitative characteristic, then: $\mu_i(x) = \left(0, b_i^2, 0, \frac{b_i^2 - b_i^2}{2}\right)$.

If $X$ is a quantitative characteristic, then: $\mu_i(x) = \left(a_i x^2, 0, \frac{b_i^2 x^2 - b_i^2 x^2}{2}\right)$.

3. Problem formulation and solution

Let us denote as $X_i = \left\{\eta_i, l=1,m, i=1,k\right\}$ - individual expert criteria of evaluation a quantitative or qualitative characteristic $X$, $\eta_i(x) = (b_{i1}, b_{i2}, b_{i3}, b_{i4}, l=1,m, i=1,k$ and denote as $X = \left\{g_i(x), l=1,m\right\}$, $g_i(x) = (b_i, b_i, b_i, b_i, l=1,m$ - a fuzzy expert group criterion of evaluation of characteristic $X$.

Information loss between the group criterion $X = \left\{g_i(x), l=1,m\right\}$ and the criterion of $i$-th expert $X_i = \left\{\eta_i, l=1,m, i=1,k\right\}$ we determine as follows:

$$d(X_i, X) = \frac{1}{2} \sum_{i=1}^{m} \int_0^1 [\eta_i(x) - g_i(x)]dx.$$  \hspace{1cm} (1)

Average value of information losses between the group criterion $X = \left\{g_i(x), l=1,m\right\}$ and all the individual expert criteria $X_i = \left\{\eta_i, l=1,m, i=1,k\right\}$ [12] let us determine as follows:

$$\sigma = \frac{1}{k} \sum_{i=1}^{k} d(X_i, X), i=1,k.$$  \hspace{1cm} (2)

Let $b_{i1} = b_i^2 - b_i^2, b_{i2} = b_i^2 - b_i^2, b_{i3} = b_i^2 + b_i^2, b_{i4} = b_i^2 + b_i^2, b_{i1} = b_i^2 - b_i^2, b_{i2} = b_i^2 + b_i^2, b_{i3} = b_i^2, b_{i4} = b_i^2 + b_i^2$.

As $b_{i1} = 0, b_{i2} = 0, i=1,k$, we suppose that $b_1 = 0, b_2 = 0$. As $b_{i3} = 1, b_{i4} = 1, i=1,k$, we suppose that $b_{i3} = 1, b_{i4} = 1$.

If $b_{i3} > b_{i3}, b_{i4} > b_{i4}$, then loss of the information within the boundaries of $l$-th and $(l+1)$-th terms is equal to square of trapezoid with base $b_{i3} - b_{i3}, b_{i4} - b_{i4}$ and unit height, that is $\frac{1}{2}(b_{i3} - b_{i3} + b_{i4} - b_{i4})$.

If $b_{i3} < b_{i3}, b_{i4} < b_{i4}$, then loss of the information within the boundaries of $l$-th and $(l+1)$-th terms is equal to square of trapezoid with base $b_{i3} - b_{i3}, b_{i4} - b_{i4}$ and unit height, that is $\frac{1}{2}(b_{i3} - b_{i3} + b_{i4} - b_{i4})$.

If $b_{i3} < b_{i3}, b_{i4} > b_{i4}$, then loss of the information within the boundaries of $l$-th and $(l+1)$-th terms is equal to square of trapezoid with base $b_{i3} - b_{i3}, b_{i4} - b_{i4}$ and unit height, that is $\frac{1}{2}(b_{i3} - b_{i3} + b_{i4} - b_{i4})$.
equal to the sum of squares of two triangles. One triangle has its base equal to \(-b_{i3} + b_{i4}\), and another triangle has its base equal to \(b_{i4} - b_{i3}\). Let us determine heights of these triangles.

As triangles with bases \(-b_{i3} + b_{i4}\), \(b_{i4} - b_{i3}\) and heights \(h_1, h_2\) are similar, we have

\[
\begin{align*}
\frac{h_1}{h_2} &= \frac{b_{i3} - b_{i4}}{b_{i4} - b_{i3}} \\
\frac{h_1}{h_2} &= \frac{b_{i3} - b_{i4}}{b_{i3} - b_{i4}} + \frac{b_{i3} - b_{i4} + b_{i3} - b_{i4}}{b_{i4} - b_{i3} + b_{i3}} \\
\frac{h_1}{h_2} &= 1.
\end{align*}
\]

Then

\[
h_1 = \frac{b_{i3} - b_{i4}}{b_{i4} - b_{i3}(b_{i4} - b_{i3} - b_{i4} + b_{i3})} + \frac{b_{i3} - b_{i4} + b_{i3} - b_{i4}}{b_{i4} - b_{i3} + b_{i3}} = \frac{b_{i3} - b_{i4}}{b_{i4} - b_{i3}}.
\]

\[
h_2 = 1 - \frac{b_{i3} - b_{i4}}{b_{i4} - b_{i3} + b_{i3}} = \frac{b_{i4} - b_{i3}}{b_{i4} - b_{i3} + b_{i3}}.
\]

In this case information loss is equal to \(\frac{(b_{i3} - b_{i4})^2 + (b_{i4} - b_{i3})^2}{2(b_{i4} - b_{i3} + b_{i3})}\). If \(b_{i3} > b_{i3}, b_{i4} < b_{i4}\), then information loss is equal to \(-\frac{(b_{i3} - b_{i3})^2 + (b_{i4} - b_{i4})^2}{2(b_{i4} - b_{i3} + b_{i3})}\).

Then the general loss of the information is equal to

\[
\sigma = \frac{1}{k} \sum_{i=1}^{m} \sum_{i=1}^{k} \left[ \frac{1}{2} \delta_i^1 \left( b_{i3} - b_{i3} + b_{i4} - b_{i4} \right) \right] + \frac{1}{k} \sum_{i=1}^{m} \sum_{i=1}^{k} \left[ \delta_i^2 \left( \frac{(b_{i3} - b_{i3})^2 + (b_{i4} - b_{i4})^2}{2(b_{i4} - b_{i3} + b_{i3})} \right) \right],
\]

\[
\delta_i^1 = \begin{cases} 
1, & b_{i3} \geq b_{i3}, b_{i4} \geq b_{i4} \\
-1, & b_{i3} \leq b_{i3}, b_{i4} \leq b_{i4} \\
0, & b_{i3} > b_{i3}, b_{i4} < b_{i4}, b_{i4} > b_{i4} \geq b_{i4} \geq b_{i4} = a_{i4} 
\end{cases}
\]

\[
\delta_i^2 = \begin{cases} 
1, & b_{i3} > b_{i3}, b_{i4} > b_{i4} \\
-1, & b_{i3} < b_{i3}, b_{i4} < b_{i4} < b_{i3} \geq b_{i4} \geq b_{i4} \leq a_{i4} 
\end{cases}
\]

Unknown parameters \(b_{i3}, b_{i4}, l = 1, m - 1\) are solutions of optimization problem

\[
\sigma = \frac{1}{2k} \sum_{i=1}^{m} \sum_{i=1}^{k} \left[ 2 \delta_i^1 \left( b_{i3} - b_{i3} + b_{i4} - b_{i4} \right) \right] + \frac{1}{2k} \sum_{i=1}^{m} \sum_{i=1}^{k} \left[ \delta_i^2 \left( \frac{(b_{i3} - b_{i3})^2 + (b_{i4} - b_{i4})^2}{(b_{i4} - b_{i3} + b_{i3})} \right) \right] \rightarrow \min.
\]

Solutions meet limits of known methods [13].

Let \(X = \{g_i(x), l = 1, m\}\) - expert group criterion of characteristic \(X\) evaluation, \(g_i(x) = (b_{i1}, b_{i2}, b_{i3}, b_{i4})\). For membership functions \(g_i(x), g_{i+1}(x), l = 1, m - 1\) of the adjacent terms there are uncertainty zones under \((x \in U : 0 < g_i(x) < 1, 0 < g_{i+1}(x) < 1)\), which potency is equal to \(b_{i}^l, l = 1, m - 1\).

Let consider a qualitative characteristic \(X\). Based on the properties of Full Orthogonal Semantic Scopes and the concept of a geometric probability, it is possible to consider \(\alpha_i = \beta_i = \frac{b_i^l}{2}\) as analogues of probabilities of errors of the first and second kinds for the terms \(X_i\) and \(X_{i+1}\). Then analogues of probabilities of errors of the first and second kinds for the group criterion we determine as
\[ \alpha = \beta = \max_{l=1,m-1} \{ \alpha_l \} \] and a reliability index of the group criterion we determine as
\[ V = (1 - \alpha)^2 = \left( 1 - \max_{l=1,m-1} \left\{ \frac{b^l - b^l_0}{2} \right\} \right)^2. \]

Let consider quantitative characteristic \( X \), defined on \( U = [a,b] \). Then the analogues of probabilities of errors of the first and second kinds for the terms \( X_i \) and \( X_{i,1} \) we determine as \( \alpha_i = \beta_i = \frac{b^l_i}{2(b-a)} \), analogues of probabilities of errors of the first and second kinds for the group criterion we determine as \( \alpha = \beta = \max_{l=1,m-1} \{ \alpha_l \} \).

Then a reliability index of the group criterion we determine as
\[ V = \left( 1 - \max_{l=1,m-1} \left\{ \frac{b^l_i}{2(b-a)} \right\} \right)^2. \]

4. Conclusion

In the paper, Full Orthogonal Semantic Scopes were selected as expert criteria for evaluating quantitative or qualitative characteristics, which are linguistic variables with the properties of completeness and orthogonality.

The construction of fuzzy expert group criterion is carried out on the basis of the minimum loss of information contained in the individual expert criteria, on the basis of which the expert group criterion is created.

For fuzzy expert group criterion, analogues of probabilities of errors of the first and the second kinds are determined, as well as the reliability index.

Taking into account the involvement of experts in solving numerous problems of industry, ecology, economics, education, respectively, the need to process the information received from them and take into account its reliability, we can conclude that the results obtained in the paper are relevant and in demand in all problem areas with the active participation of experts.

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