MULTI-COEFFICIENT OPTIMIZATION OF HOMOGENEOUS ANISOTROPIC HARDENING MODEL FOR AHSS

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Abstract. The goal of this research is to find out which test is best to predict the hardening of advanced high strength steel (AHSS) sheets and what kind of optimization method is most efficient. In this work, tension-compression and cyclic simple shear tests were conducted on AHSS sheets. Both experiments were carried out in the format of several cycles. Based on the experimental results, the distortional plasticity model (HAH) coefficients were optimized through the Nelder-Mead simplex algorithm and a genetic algorithm. The results were discussed to assess which test is best for the prediction of the stress-strain curve during forward-reverse loading and which method leads for lower computing time and higher accuracy.

1. Introduction
With the reinforcement of global environmental regulations and a requirement for improved car fuel efficiency, the demand for high strength and lightweight materials has increased. However, these advanced materials lead to unexpected problems during forming processes compared to conventional materials. For example, advanced high strength steels (AHSS) could satisfy the need for fuel efficiency due to their outstanding mechanical properties. But, it is difficult to control the final shape of the product after drawing, punching or bending of AHSS [1]. Therefore, it is necessary to predict accurate shapes to optimize forming processes but this requires to consider the strain path change influence, which is an important factor in sheet metal forming simulations. Many constitutive modeling descriptions have been introduced for this purpose but, in the present work, the homogeneous anisotropic hardening (HAH) model proposed by Barlat et al [2] is considered. The HAH model has been recognized as a valid approach to investigate strain path change effects in AHSS during forming simulations, in particular for the prediction of springback [3]. The HAH model has the potential for solving other forming issues efficiently.

An important aspect of using advanced constitutive equations such as developed in the HAH model is to calculate the coefficients efficiently and accurately. Although the empirical trial-and-error method has been employed by a few authors, e.g. [4], this is not suitable for real applications in the industrial environment. Optimization methods based on the minimization of an error function are more efficient and reliable but they require valid testing procedure and data for this purpose. The most studied strain path change is forward-reverse loading, in particular, tension-compression, because it allows the characterization of the Bauschinger effect and related phenomena. Choi et al. [5] used the forward-reverse simple shear test and showed that this procedure is also acceptable to acquire valid
data for coefficient optimization. However, the optimized coefficients obtained by tension–compression and forward-reverse simple shear tests for a given material have not been compared. Therefore, this step appears to be valuable because the material coefficients might be different for a number of reasons, including a different hardening behavior in these two stress states. It is also important to decide which optimization method is the most efficient. Since the HAH model activates five coefficients to describe forward–reverse loading behaviors the optimization algorithm might take a long time to complete or converge towards local minima of the error function. Therefore, in selecting the optimization method, time and accuracy are important factors to consider. The combination of a reliable testing procedure and a suitable optimization method should make the determination of the HAH coefficients more robust and easier to employ for industrial applications.

In this work, the coefficients of the HAH model are calculated for a DP780 sheet sample supplied by POSCO from tension-compression and simple shear cycle test data. Two optimization methods are selected, namely, the Nelder-mead simplex approach and a genetic algorithm, and assessed in terms of computation time and accuracy. The hardening characteristics of the two stress states mentioned above will also be discussed.

2. Experiments

The forward-reverse loading experiments are carried out using tension-compression and simple shear cycle tests in which materials exhibit the Bauschinger effect, transient stages of high hardening rate and permanent softening, as already shown for DP780 [4]. In previous researches, one or several reversals were conducted [3, 5-7]. The advantage of multiple cycles is that larger effective strains can be achieved and, therefore, this approach was selected in this work.

2.1. Tension-compression test

Since standards for the tension-compression test are not established yet, an experimental procedure was developed using a custom-built tension-compression machine in Figure 1. The 205 mm long specimen was employed in this work (Figure 1). The relative displacement rate of the grips was set to 0.001 mm/s, leading to a strain rate of approximately 0.001/s, and a laser extensometer was used to measure the longitudinal strains. A normal holding force of 700 kgf was applied on the gauge length of the specimen to prevent buckling. Multiple of 2% strain intervals with eight reversals were conducted in this test. The resulting experimental stress-strain curve, shown in Figure 2, exhibits a Bauschinger effect and related phenomena after each reloading. This figure also indicates that a total accumulated strain of approximately 62% was achieved.

![Figure 1. Tension – compression test specimen size and tester](image-url)
2.2. Cyclic simple shear test

The simple shear test was conducted with repetitive reversals as well. This test has no standard either and was conducted following the procedure in [5, 8] on a jig and installed on a 500 kN MTS tension-compression machine. The 16 x 50 mm rectangular specimens were machined as shown in Figure 3. The holding pressure to clamp the specimens was 30 MPa. The relative displacement rate of the grips was set to 0.05 mm/s. A digital image correlation system (DIC) was used to measure the displacement field and calculate the strain. The shear strain (γ) was calculated in the same region for all the specimens by designating a homogeneous area of interest (AOI) in the camera image. The DIC image area is a rectangular shape of 4 x 20 mm and the data was extracted in that area. The shear stress was obtained from load divided by the side area of the specimen.
In this test, the amplitudes of the strain cycles are not that as those for the uniaxial test because the equipment proceeds by displacement control. Therefore, it is impossible to control the strain cycle accurately. Figure 4 shows the experimental stress-strain curve cycles, which leads to a total accumulated strain of approximately 48%.

![Figure 4. Simple shear test cycles for DP780](image)

3. Optimization results

Before the HAH model coefficient can be optimized, the constitutive models for classical anisotropic plasticity under isotropic hardening are needed. The Yld2000-2d anisotropic yield function [9] with the Swift-modified Voce hardening law, defined as

\[ \bar{\sigma} = K(\varepsilon_0 + \bar{\varepsilon})^n + \rho \bar{\varepsilon} + \sigma_y + \sigma_b(1 - \exp(-\eta \bar{\varepsilon})) \]  

(1)

are selected. The rolling direction uniaxial tension and bulge test data are used to approximate the isotropic hardening equation. The experimental data for the tension-compression and the simple shear tests are assumed to be properly account for the isotropic hardening contribution because the bulge test effective strain reaches the value of 50%. Table 1 lists the yield function and strain hardening coefficients for DP780.

| Table 1. Coefficients of Yld2000-2d and Swift-modified Voce equation |
|--------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Yld2000-2d               | α_1           | α_2           | α_3           | α_4           | α_5           | α_6           | α_7           | α_8           | a             |
|                          | 0.922         | 1.015         | 1.050         | 0.993         | 1.013         | 0.934         | 1.030         | 0.970         | 5             |
| Swift-modified Voce      | K             | \varepsilon_0 | n             | ρ             | \sigma_y      | \sigma_b      | η             |
|                          | 1184.6        | 0.0004        | 0.1           | 1.0           | 1.0           | 123.2         | 53.4          |
Several versions of the HAH model have been published, the first version of which [2] was used in this paper. The yield function of the HAH model and evolution of model parameters are as follows.

\[
\phi(s) = \left[ \phi_0(s) + f_1^q |\hat{h} : s| - |\hat{h} : s|^{q_1} + f_2^q |\hat{h} : s| + |\hat{h} : s|^{q_1} \right]^{\frac{1}{q_1}} = \bar{\sigma}
\]  

\[
f_1 = \left[ \frac{1}{g_1} - 1 \right]^{\frac{1}{q_1}} \text{ and } f_2 = \left[ \frac{1}{g_2} - 1 \right]^{\frac{1}{q_1}}
\]

Case 1: \( \hat{h} : s \geq 0 \)

\[
\begin{align*}
\frac{dg_1}{d\bar{\varepsilon}} &= k_2 \left( k_3 \frac{\sigma_R(0)}{\sigma_R(\varepsilon)} - g_1 \right) \\
\frac{dg_2}{d\bar{\varepsilon}} &= k_1 \frac{g_3 - g_2}{g_2} \\
\frac{dg_4}{d\bar{\varepsilon}} &= k_5(k_4 - g_4)
\end{align*}
\]

Case 2: \( \hat{h} : s < 0 \)

\[
\begin{align*}
\frac{dg_1}{d\bar{\varepsilon}} &= k_1 \frac{g_4 - g_1}{g_1} \\
\frac{dg_2}{d\bar{\varepsilon}} &= k_2 \left( k_3 \frac{\sigma_R(0)}{\sigma_R(\varepsilon)} - g_2 \right) \\
\frac{dg_3}{d\bar{\varepsilon}} &= k_5(k_4 - g_3)
\end{align*}
\]

\( k_1 \) is the material constant in HAH model and it does not have a unit. The five coefficients \( k_1 \) to \( k_5 \) of the HAH model [2], which account for the Bauschinger effect and permanent softening, can be optimized by reducing the difference between experimental and simulated flow stress at given amount of accumulated strain. For this purpose, the Nelder-Mead simplex algorithm, and a genetic algorithm built-in Matlab were employed. With the Nelder-Mead simplex algorithm, the key for obtaining proper coefficients is to provide a range of value and a suitable initial estimate of the set of the sought for coefficients because this algorithm might converge towards a local minimum of the error function. Therefore, the initial estimates must be set as close as possible from the solution which, in general, is a difficult task. However, in the HAH model, the coefficients have physical meanings, which allow a reasonable estimation of the ranges and initial values. In this work, the initial estimates were set to intermediate values in the ranges \( k_1 = 125, k_2 = 60, k_3 = 0.35, k_4 = 0.9 \) and \( k_5 = 37.5 \). The coefficient \( k \) is irrelevant for forward-reverse loading [2]. For a genetic algorithm, no initial values are required but the range of coefficients plays an important role in avoiding local minima. Again, the range of coefficients are set based on the physical meaning of the coefficients, which is as follows in this work:

\[
\begin{align*}
50 \leq k_1 &\leq 200 \\
20 \leq k_2 &\leq 100 \\
0.1 \leq k_3 &\leq 0.6 \\
0.8 \leq k_4 &\leq 1 \\
5 \leq k_5 &\leq 70
\end{align*}
\]

3.1. Optimization method comparison
Figure 5. Comparison by optimization methods

Table 2 lists the coefficients $k_1$ to $k_5$ determined for all the cases considered. A comparison of the optimization methods is shown in Figure 5. In the tension-compression test, compared to a genetic algorithm, the Nelder-Mead simplex algorithm leads to a slightly better stress-strain prediction at the beginning of cycles; however, there is almost no difference in large strain ranges. Overall, both algorithms lead to virtually reasonable fits in both tension-compression and cyclic simple shear test. A remarkable point is that a genetic algorithm consumes much more time than the Nelder-Mead simplex algorithm in both stress states as indicated in Table 2. Consequently, the Nelder-Mead simplex algorithm is a better method to get adoptable model coefficients with time efficiency.

| Tension-compression | Nelder-Mead | Genetic | Time |
|---------------------|-------------|---------|------|
| $k$                 | $k_1$       | $k_2$   | $k_3$ | $k_4$ | $k_5$ |    |
| 15                  | 77.39       | 94.30   | 0.57  | 0.82  | 16.51 | 481 sec |
| Genetic             |             |         |       |       |       |      |
| $k$                 | $k_1$       | $k_2$   | $k_3$ | $k_4$ | $k_5$ |    |
| 15                  | 72.91       | 39.98   | 0.59  | 0.81  | 16.54 | 6961 sec |

| Cyclic simple shear | Nelder-Mead | Genetic | Time |
|---------------------|-------------|---------|------|
| $k$                 | $k_1$       | $k_2$   | $k_3$ | $k_4$ | $k_5$ |    |
| 15                  | 73.43       | 99.61   | 0.22  | 0.84  | 46.72 | 508 sec |
| Genetic             |             |         |       |       |       |      |
| $k$                 | $k_1$       | $k_2$   | $k_3$ | $k_4$ | $k_5$ |    |
| 15                  | 66.67       | 93.83   | 0.21  | 0.83  | 21.76 | 6244 sec |

3.2. Stress state comparison
In Figure 6, the stress-strain curves obtained with the coefficients optimized with the Nelder-Mead algorithm for tension-compression (TC) and simple shear (SS) tests were compared with the experimental SS and TC stress-strain curves, respectively. Figure 6 indicates that each set reproduces the experimental results of the other stress state with reasonable accuracy. Therefore, the identification stress state does not play a significant role in the final predicted flow curves for this DP780 steel.

4. Conclusion
HAH model coefficients in loading – reverse loading behaviors are optimized based on experimental results which are the tension-compression cycle and the simple shear cycle of DP780 steel. Two optimization methods are used and compared in terms of accuracy and time efficiency. Also, stress state comparison proceeds after selecting the optimization method. As a result, the following conclusions were obtained.

- In terms of time efficiency and accuracy, it is shown that the Nelder-Mead simplex algorithm is more performant than a genetic algorithm. However, the Nelder-Mead simplex method requires the use of initial coefficients that might lead to different results. Therefore, proper initial coefficients are required, which might require several trials.

- The predicted forward-reverse loading stress-strain curves of DP780 in tension and simple shear were shown to be similar to the experimental curves. Therefore, both sets of HAH coefficients, identified either with the tension-compression or simple shear cycles lead to reasonable predictions.

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