Predictions for $B \to K\gamma\gamma$ decays

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Abstract

We present a phenomenological study of the rare double radiative decay $B \to K\gamma\gamma$ in the Standard Model (SM) and beyond. Using the operator product expansion (OPE) technique, we estimate the short-distance (SD) contribution to the decay amplitude in a region of the phase space which is around the point where all decay products have energy $\sim m_b/3$ in the rest frame of the $B$-meson. At lowest order in $1/Q$, where $Q$ is of order $m_b$, the $B \to K\gamma\gamma$ matrix element is then expressed in terms of the usual $B \to K$ form factors known from semileptonic rare decays. The integrated SD branching ratio in the SM in the OPE region turns out to be $\Delta\mathcal{B}(B \to K\gamma\gamma)^{\text{OPE}}_{\text{SM}} \simeq 1 \times 10^{-9}$. We work out the di-photon invariant mass distribution with and without the resonant background through $B \to K\{\eta_c, \chi_c\} \to K\gamma\gamma$. In the SM, the resonance contribution is dominant in the region of phase space where the OPE is valid. The present experimental upper limit on $B_s \to \tau^+\tau^-$ decays, which constrains the scalar/pseudoscalar Four-Fermi operators with $\tau^+\tau^-$, leaves considerable room for new physics in the one-particle-irreducible contribution to $B \to K\gamma\gamma$ decays. In this case, we find that the SD $B \to K\gamma\gamma$ branching ratio can be enhanced by one order of magnitude with respect to its SM value and the SD contribution can lie outside of the resonance peaks.

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1 Introduction

Rare $B$-decays mediated by flavor changing neutral currents (FCNC) provide a rich laboratory to study the flavor structure of the Standard Model (SM) and its extensions. Such processes are forbidden in the Born approximation and are generated by loops, which gives them a high sensitivity to new physics (NP). A prominent example is the both experimentally and theoretically well-studied $b \to s\gamma$ transition, exemplified by the inclusive $B \to X_s\gamma$ and the exclusive $B \to K^{(*)}\gamma\gamma$ decays (for a detailed review, see [1]).

Radiative $B$-decays involving two photons, such as $B \to \gamma\gamma$, $B \to X\gamma\gamma$ or $B \to K^{(*)}\gamma\gamma$ decays, are of further interest. Among the Cabibbo-favored $b \to s$ transitions, the decays $B_s \to \gamma\gamma$ and $B \to X_s\gamma\gamma$ have been theoretically analyzed in the SM some time ago [2] and later on by taking into account the leading-order QCD corrections in the framework of the electroweak effective Hamiltonian [3–6].

Recalling that at the quark level the FCNC double-photon decay is mediated by $b \to s\gamma\gamma$, it receives one-particle-reducible (1PR) contributions from the $b \to s\gamma$ transition plus an additional photon and a one-particle-irreducible (1PI) term from a fermion loop with the two photons emitted from that loop. The correlation with $b \to s\gamma$ implies that NP, which predominantly induces contributions to the 1PR piece, gives no drastic effects in the double radiative decays because of the strong constraints from $B \to X_s\gamma$ data. This has been shown, for example, for the two-Higgs-doublet model [7]. On the other hand, NP in the 1PI contribution, that is in Four-Fermi operators, is subleading in $b \to s\gamma$ decays and hence can be sizable in $b \to s\gamma\gamma$ transitions where it enters at the same order as the 1PR contributions. So far only experimental upper limits on double radiative decays have been set (at 90% C.L.):

$$B(B_d \to \gamma\gamma) < 1.7 \times 10^{-6} \text{ (BaBar)} \quad [8],$$
$$B(B_s \to \gamma\gamma) < 1.48 \times 10^{-4} \text{ (L3)} \quad [9],$$

which are about two orders of magnitude above their respective SM values, e.g. [6].

While many theoretical investigations have been dedicated to $B_s \to \gamma\gamma$ and $B \to X_s\gamma\gamma$, less attention has been devoted to exclusive $B \to K^{(*)}\gamma\gamma$ decays. For instance, only two groups [10,11] have estimated the $B \to K\gamma\gamma$ branching ratio, with the result given as $\sim (0.5 - 5.6) \times 10^{-7}$, depending on cuts on the photon energies. Note that $B \to K\gamma\gamma$ decays have a complicated matrix element due to the non-local 1PR contribution. In the aforementioned studies the 1PR piece has been solely parametrized in terms of a long-distance contribution using hadronic models, i.e. vector meson dominance. There is also a sizable resonant background from charmonia through $B \to K(c\bar{c}) \to K\gamma\gamma$. Since $B \to K\gamma\gamma$ decays provide a reasonable experimental signal for not too soft photons and with branching ratios in the $10^{-7}$ range, they are in reach of the currently operating $B$-factories at SLAC and KEK and at experiments at the hadron colliders, Tevatron and LHC. Therefore, a detailed study of these decays in the SM and beyond is of great interest, in particular due to the complementarity with $b \to s\gamma$ decays with respect to NP searches.
Our main objectives in this paper are (i) to write down a model-independent description of exclusive $B \to K \gamma \gamma$ decays and (ii) to find out to what extent one can extract short-distance (SD) physics from these decays, that is, how big is the SD contribution and can it be experimentally isolated from the resonance contributions. The idea here is to use the freedom of a 3-body decay and choose the photon energies such that the virtualities of the intermediate quarks in the 1PR $b \to s \gamma \gamma$ diagrams are hard. Then, these scales of order $m_b$ are integrated out with the effective field theory methods using heavy-quark-effective-theory (HQET) \cite{12} and the soft-collinear-effective-theory (SCET) \cite{13,14}. The resulting operator product expansion (OPE) enables us to express the matrix element in terms of the $B \to K$ form factors in some region of the phase space. There are other mechanisms contributing to $B \to K \gamma \gamma$ decays, such as when one photon gets emitted from the spectator quark or annihilation diagrams, which we comment on. We further incorporate the $\eta_c, \chi_{c0}$-resonance contributions using a phenomenological parametrization à la Breit-Wigner to see whether they can be distinguished from the SD ones obtained from the OPE. We work out possible NP effects in the $B \to K \gamma \gamma$ spectra and branching ratios.

The remainder of this paper is organized as follows: After a brief description of the effective Hamiltonian formalism for double radiative $b \to s \gamma \gamma$ decays in Section 2 we evaluate in Section 3 the (non-local) matrix element of $B \to K \gamma \gamma$ decays with the OPE and present the resulting decay amplitude and distributions. Additional contributions to the $B \to K \gamma \gamma$ amplitude, namely photon radiation off the spectator, annihilation and contributions from intermediate charmonia are explored in Section 4. In Section 5 a phenomenological discussion of the $B \to K \gamma \gamma$ Dalitz region, in which the OPE is valid, and the di-photon invariant mass distributions is given in the SM and with NP. We summarize in Section 6.

2 The lowest order $b \to s \gamma \gamma$ amplitude

We treat the effects of heavy degrees of freedom at the weak scale with an effective Hamiltonian known from $b \to s \gamma$ decays, e.g. \cite{15}.

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_i C_i(\mu)\mathcal{O}_i(\mu),$$

which contains four-fermion operators

$$\mathcal{O}_1 = (\bar{s}_\alpha \gamma^\mu L c_\beta)(\bar{c}_\beta \gamma_\mu L b_\alpha), \quad \mathcal{O}_2 = (\bar{s}_\alpha \gamma^\mu L c_\alpha)(\bar{c}_\beta \gamma_\mu L b_\beta),$$

and the electromagnetic dipole operator

$$\mathcal{O}_7 = \frac{e}{16\pi^2}m_b \bar{s}_\alpha \sigma^\mu\nu R b_\alpha F_{\mu\nu}.$$
Figure 1: Leading order Feynman diagrams contributing to $b \rightarrow s\gamma\gamma$ decays. The 1PI diagram illustrates the insertion of four-fermion operators (left plot), while the 1PR diagrams (right two plots) represent the insertion of $O_7$. The indices $i, j = 1, 2$ (with $i \neq j$) denote both photons and their interchanged counterparts.

Here, $\alpha$ and $\beta$ are color indices, $L, R = (1 \mp \gamma_5)/2$ are chiral projectors and $F_{\mu\nu}$ denotes the QED field strength tensor. In Eq. (2) we neglect the penguin operators $\bar{s}\gamma_\mu L b \sum \bar{q}\gamma_\mu L/R q$ and double Cabibbo-suppressed contributions proportional to $V_{ub}^* V_{us}$. We further neglect the strange quark mass throughout this paper. Analytical formulas for the Wilson coefficients $C_i(\mu)$ can be seen in [15]. Using the equations of motion, one can show that there are not further independent, gauge invariant operators with two photons and with dimension equal or less than six. Hence, the Hamiltonian in Eq. (2) is the same for $b \rightarrow s\gamma$ and $b \rightarrow s\gamma\gamma$ decays.

The lowest order Feynman diagrams contributing to $b \rightarrow s\gamma\gamma$ decays are shown in Fig 1. Only the sum of the two 1PR diagrams is gauge invariant [2]. The corresponding amplitude can be written as [5]

$$A(b \rightarrow s\gamma\gamma) = -\frac{i e^2 G_F}{\sqrt{2\pi^2}} V_{tb}^* \bar{u}_s(p') \cdot \left[ (N_C C_1 + C_2) Q_u^2 \kappa_e W_{2}^{\mu\nu} + C_7 Q_d W_{7}^{\mu\nu} \right] \cdot u_b(p) \epsilon_\mu(k_1) \epsilon_\nu(k_2),$$

where $p(p')$ represents the momentum of the $b(s)$-quark. We denote by $k_1, k_2$ the 4-momenta and by $\epsilon(k_{1,2})$ the polarization vectors of the photons. The auxiliary tensors are written as

$$W_{2}^{\mu\nu} = -i \left\{ \frac{1}{k_1 \cdot k_2} \left[ k_1^\nu \epsilon^{\mu\rho\sigma\lambda} \gamma_\rho k_1^\sigma k_2^\lambda - k_2^\nu \epsilon^{\rho\sigma\lambda\mu} \gamma_\rho k_1^\sigma k_2^\lambda \right] + \epsilon^{\mu\rho\lambda\nu} \gamma_\rho (k_2 - k_1)_\lambda \right\} L,$$

$$W_{7}^{\mu\nu} = \frac{m_b}{2} \left[ \frac{k_1}{} \gamma_\mu R (p - k_2 + m_b) \gamma_\nu + \frac{\gamma_\nu (p' + k_2) \cdot k_1}{} \gamma_\mu R \right] (k_1, \mu \leftrightarrow k_2, \nu),$$

where $N_C = 3$ denotes the number of colors, $Q_u = 2/3$ and $Q_d = -1/3$ are the up-type and down-type quark electric charges. Note that $W_{2}^{\mu\nu}$ is proportional to the Adler-Rosenberg tensor [16]. We further suppress the renormalization scale dependence of the Wilson coefficients. To the order we are working, the Wilson coefficients are evaluated at leading log at $\mu \simeq m_b$. We further use $\epsilon^{0123} = -1$. The
loop function \( \kappa_q \) is defined as \([17]\)

\[
\kappa_q = \kappa(q^2, m_q^2) = \frac{1}{2} + \frac{1}{z_q} \int_0^1 \frac{dx}{x} \ln \left(1 - z_q x + z_q x^2\right),
\]

\[
= \begin{cases} 
\frac{1}{2} - \frac{2}{z_q} \left( \arctan \sqrt{\frac{z_q}{z_q - 1}} \right)^2 \\
\frac{1}{2} + \frac{1}{z_q} \left[ -\pi^2/2 + 2 \left( \ln \frac{\sqrt{z_q} \sqrt{z_q - 1}}{2} \right)^2 - 2i\pi \ln \left( \frac{\sqrt{z_q} \sqrt{z_q - 1}}{2} \right) \right]
\end{cases}
\text{if } z_q < 4 \text{, otherwise,}
\] (8)

where \( z_q = q^2/m_q^2 \) and \( q^2 = (k_1 + k_2)^2 \). It assumes the following limits

\[
\kappa_c = \begin{cases} 
-\frac{q^2}{24m^2} + \mathcal{O}(q^4/m^4) & \text{for } q^2 \ll m_c^2, \\
\frac{1}{2} + \mathcal{O}(m_c^2/q^2, m_c^2/q^2 \times \log s) & \text{for } q^2 \gg m_c^2.
\end{cases}
\] (9)

3 \( B \to K \gamma \gamma \) decays

In this section we obtain explicit expressions for the \( B \to K \gamma \gamma \) decay distributions. A general parametrization of the amplitude and decay rate is given in Section 3.1. We perform an expansion in inverse scales of order \( m_b \) in Section 3.2 to render the matrix element of the 1PR contribution local. This enables us to express the \( B \to K \gamma \gamma \) matrix element in terms of \( B \to K \) form factors. The resulting spectrum is presented in Section 3.3.

3.1 The general amplitude and decay spectra

We write the general \( B \to K \gamma \gamma \) amplitude as

\[
A(B \to K \gamma \gamma) = T_{\mu\nu} \epsilon^\mu(k_1) \epsilon^\nu(k_2),
\] (10)

where

\[
T_{\mu\nu} = A(g_{\mu\nu} - \frac{k_1\mu k_2\nu}{k_1 \cdot k_2}) + iB \epsilon_{\mu \nu \alpha \beta} k_1^\alpha k_2^\beta
\] (11)

\[
+ D \left[ p_B \cdot k_1 p_{B\nu} k_{2\mu} + p_B \cdot k_2 p_{B\nu} k_{1\mu} - p_B \cdot k_1 p_{B\mu} \cdot k_2 k_{1\nu} k_{2\mu} - k_1 \cdot k_2 p_{B\mu} p_{B\nu} \right]
\]

\[
+ iC^+ \left[ k_1 \cdot k_2 \epsilon_{\mu \nu \alpha \beta} (k_1^\alpha + k_2^\alpha) p_B^\beta + (k_2 \mu \epsilon_{\nu \alpha \beta \gamma} + k_1 \nu \epsilon_{\mu \alpha \beta \gamma}) k_1^\alpha k_2^\beta p_B^\gamma \right]
\]

\[
+ iC^- \left[ k_1 \cdot k_2 \epsilon_{\mu \nu \alpha \beta} (k_2^\alpha - k_1^\alpha) p_B^\beta + (k_2 \mu \epsilon_{\nu \alpha \beta \gamma} - k_1 \nu \epsilon_{\mu \alpha \beta \gamma}) k_1^\alpha k_2^\beta p_B^\gamma \right]
\]

\[
+ iD^+ \left[ p_B \cdot k_1 \epsilon_{\nu \mu \alpha \beta} k_2^\alpha p_B^\beta + p_B \cdot k_2 \epsilon_{\nu \mu \alpha \beta} k_1^\alpha p_B^\beta + (p_B \nu \epsilon_{\mu \alpha \beta \gamma} + p_B \mu \epsilon_{\nu \alpha \beta \gamma}) k_1^\alpha k_2^\beta p_B^\gamma \right]
\]

\[
+ iD^- \left[ p_B \cdot k_1 \epsilon_{\nu \mu \alpha \beta} k_2^\alpha p_B^\beta - p_B \cdot k_2 \epsilon_{\nu \mu \alpha \beta} k_1^\alpha p_B^\beta + (p_B \nu \epsilon_{\mu \alpha \beta \gamma} - p_B \mu \epsilon_{\nu \alpha \beta \gamma}) k_1^\alpha k_2^\beta p_B^\gamma \right].
\]

\(^{1}\)Since we are not concerned about CP violation and to avoid clutter we use the convention \( B \equiv b\bar{q} \) throughout this work.
which is manifestly gauge invariant. Here, we denote by $p_B$ the 4-momentum of the $B$-meson. The form factors $A, B, D, C^\pm, D^\pm$ are functions of two kinematical variables. We choose later to use the photon energies $E_1, E_2$ defined in the $B$-meson rest frame or the invariant mass $q^2$. The amplitude squared is then given as

$$|A(B \to K\gamma\gamma)|^2 = 2|A|^2 - 2\text{Re}(AD^*) \left( m_B^2 k_1 \cdot k_2 - 2k_1 \cdot p_B k_2 \cdot p_B \right)$$

$$+ \left( |D|^2 + 2|D^*|^2 \right) (m_B^2 k_1 \cdot k_2 - 2k_1 \cdot p_B k_2 \cdot p_B)^2$$

$$+ 2(k_1 \cdot k_2)^2 |B - m_B^2 D^- + k_1 \cdot p_B (C^+ - C^-) - k_2 \cdot p_B (C^+ + C^-)|^2,$$

and the double differential rates (which includes a factor of 1/2 for identical photons)

$$\frac{d\Gamma}{dq^2dE_1} = \frac{|A(B \to K\gamma\gamma)|^2}{256m_B^2\pi^3}.$$  

(13)

The full photon invariant mass spectrum $d\Gamma/dq^2$ can be obtained by integration with

$$E_1^{\text{min/\text{max}}} = \frac{1}{4m_B} \left( m_B^2 + q^2 - m_K^2 \mp \sqrt{\lambda(q^2, m_B^2, m_K^2)} \right).$$

(14)

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. We also use

$$\frac{d\Gamma}{dE_1dE_2} = \frac{|A(B \to K\gamma\gamma)|^2}{128m_B\pi^3}.$$  

(15)

### 3.2 The OPE

The matrix element of $B \to K\gamma\gamma$ decays induced by the dipole operator $O_\gamma$ is a non-local one, see Fig. [1] and difficult to estimate model-independently. Here our strategy is to go to a kinematical region where both the internal $s$- and $b$-quarks are far off-shell, with virtualities of order $m_b$. To see that there exists such a region of phase space, we investigate the propagators of the 1PR diagrams in Eq. [16], which are given as (in the bottom rest frame)

$$Q_1^i = (p' + k_2)^2 = m_b^2 - 2m_b E_1,$$

$$Q_2^i = (p' + k_1)^2 = m_b^2 - 2m_b E_2,$$

where

$$Q_1^i = (p' + k_2)^2 = m_b^2 - 2m_b E_1,$$

$$Q_2^i = (p' + k_1)^2 = m_b^2 - 2m_b E_2.$$

(16)

(17)

For photon energies neither too soft nor too hard with energies near their maximal value all $Q_i^j$ are hard. For example, if both photons have energies $m_b/3$, then $Q_{1,2}^i = m_b/\sqrt{3}$ and $Q_{1,2}^h = m_b\sqrt{2/3}$. Note that here the light quark has large energy $m_b/3$ as well, and $q^2 = m_b^2/3$.

As a next step, we integrate out the large scales associated with the intermediate quark propagators. We construct the resulting local operators out of a bottom heavy HQET quark $h_b$ and a strange collinear SCET quark $\chi [13][14]$. Here, $v = p_B/m_B$ and a light-like vector $n = p_K/E_K$, where $E_K, p_K$ denotes the energy, 4-momentum of the kaon. Hence, we perform an OPE in $\Lambda_{QCD}/Q$ where $Q = \{m_b, E_K, Q_{1,2}^{s,b}, \sqrt{q^2}\}$. In the following we neglect $m_K$ and $m_B - m_b$. 
For the lowest order matching onto the \( b \to s\gamma\gamma \) amplitude, the following gauge invariant operators with dimension 8 with a heavy and a collinear quark and two photons are needed after using the equations of motion (Wilson lines are understood in the definition of the field \( \chi \)):

\[
\begin{align*}
Q_1 &= \frac{m_b}{4} \bar{\chi} \sigma_{\mu \nu} \sigma_{\alpha \beta} R h_v F_{\alpha \beta} F_{2 \mu \nu}, \\
Q_2 &= -2i m_b \bar{\chi} \sigma_{\mu \nu} R h_v F_{1 \mu \nu} F_{2 \alpha \beta} v_{\alpha \beta}, \\
Q_3 &= \bar{\chi} \gamma^{\mu} L h_v F_{1 \alpha \beta} D_\alpha \tilde{F}_{2 \beta \mu},
\end{align*}
\]

where \( \tilde{F}_{\mu \nu} = 1/2 \epsilon_{\mu \nu \alpha \beta} F^{\alpha \beta} \). The primed operators are related to the un-primed ones by interchanging photons. Then we obtain

\[
\bar{s} W_1^{\mu \nu} b e(k_1)^{\mu} e(k_2)^{\nu} \rightarrow - \frac{1}{2} \left\{ \left( \frac{1}{Q_1^2} - \frac{1}{Q_1^2} \right) Q_1 + \left( \frac{1}{Q_2^2} - \frac{1}{Q_2^2} \right) Q_2 + \frac{m_b E_K}{(Q_1^2)(Q_2^2)} Q_3 \right\}.
\]

The matching of the 1PI contribution from 4-Fermi operators \( O_{1,2} \) is readily computed:

\[
\bar{s} W_2^{\mu \nu} b e(k_1)^{\mu} e(k_2)^{\nu} \rightarrow 2 \frac{Q_3 + Q_3'}{q^2}.
\]

Note that all 1PR and 1PI diagrams are of the same order in \( 1/Q \). This is in contrast to the behavior in \( B \to \gamma\gamma \) decays, where the 1PR contribution with intermediate \( s \)-quark gives the leading power result [6].

### 3.3 The \( B \to K\gamma\gamma \) matrix element and decay distribution

The matrix elements of the local operators contributing to \( B \to K\gamma\gamma \) decays are obtained from tree level matching of the QCD onto the SCET currents [13,18,19] as

\[
\begin{align*}
\langle K(n)|\bar{\chi} h_v|B(v) \rangle &= 2 E_K \zeta(E_K), \\
\langle K(n)|\bar{\chi} \gamma_\mu h_v|B(v) \rangle &= 2 E_K \zeta(E_K) n_\mu, \\
\langle K(n)|\bar{\chi} \sigma_{\mu \nu} h_v|B(v) \rangle &= -2i E_K \zeta(E_K) (v_\mu n_\nu - v_\nu n_\mu), \\
\langle K(n)|\bar{\chi} \sigma_{\mu \nu} \gamma_5 h_v|B(v) \rangle &= -2 E_K \zeta(E_K) \epsilon_{\mu \nu \alpha \beta} v^\alpha n^\beta,
\end{align*}
\]

where the form factor \( \zeta \) can be identified with the QCD form factor in the usual parametrization, see e.g. [20], as \( \zeta = f_+ \). This is the only form factor remaining in the symmetry limit, which enters all \( B \to K \) transitions such as \( B \to K \ell^+\ell^- \) or \( B \to K\nu\bar{\nu} \) decays.
We obtain then the following $B \to K\gamma\gamma$ amplitude in terms of the parametrization given in Eq. (11) as

$$A = -\kappa\zeta(E_K)Q_d m_b C_7 \frac{1}{8E_1E_2}(2E_1 + 2E_2 - m_B)(8E_1E_2 - E_1m_B - E_2m_B),$$  

$$B = -\kappa\zeta(E_K) \left( \frac{Q_d m_b C_7 (E_1 + E_2)^2}{8E_1E_2 m_B} + (C_1 N_C + C_2)Q_u^2 \kappa_c \right),$$  

$$D = \kappa\zeta(E_K)Q_d m_b C_7 \frac{(2E_1 + 2E_2 - m_B)(4E_1E_2 - E_1m_B - E_2m_B)}{2E_1E_2(m_B - 2E_1)(m_B - 2E_2)m_B^2},$$  

$$C^+ = \kappa\zeta(E_K)Q_d m_b C_7 \frac{E_1 - E_2}{4E_1E_2m_B^2},$$  

$$C^- = -\kappa\zeta(E_K) \left( \frac{(C_1 N_C + C_2)Q_u^2 \kappa_c}{2E_1 + 2E_2 - m_B} \right),$$  

$$D^+ = \kappa\zeta(E_K)Q_d m_b C_7 \frac{(E_1 - E_2)(2E_1 + 2E_2 - m_B)}{4E_1E_2m_B(m_B - 2E_1)(m_B - 2E_2)},$$  

$$D^- = 0,$$

where

$$\kappa \equiv +i \frac{G_F}{\sqrt{2\pi}} V_{tb} V^*_{ts} \alpha_{em}. \quad (34)$$

Here we use $m_b \simeq m_B$ but keep explicitly a factor of $m_b$ next to $C_7$ from the definition of the dipole operator given in Eq. (11).

From Eqs. (9) and (12) it follows that the $B \to K\gamma\gamma$ rate vanishes for $q^2 \to 0$. This must be so since in the limit that one of the photon energies vanishes the double photon rate does because the decay $B \to K\gamma$ is forbidden. As a further consequence, phase space integration of the $B \to K\gamma\gamma$ amplitude squared is IR finite, as opposed to the inclusive $B \to X_s\gamma\gamma$ decays, which need the cancellation with the virtual electromagnetic corrections in the low photon energy limit [5].

We obtain for the differential decay spectrum

$$\frac{d\Gamma}{dE_2 dE_1} = \frac{\alpha_{em}^2 G_F^2 |V_{tb} V_{ts}|^2}{256m_B^7\pi^5} |\zeta(E_K)|^2 \left\{ m_b^2 Q_d^2 |C_7|^2 \left( \frac{m_B - 2(E_1 + E_2)^2}{E_1^2E_2^2} \right) \times \left( 24E_1^2E_2^2 - 4m_B E_1E_2(E_1 + E_2) + m_B^2(E_1^2 + E_2^2) \right) \right.$$  

$$+ 32m_b Q_d Q_u^2 \text{Re}(C_7 \kappa_c^*) (C_1 N_C + C_2) m_B (m_B - E_1 - E_2) (m_B - 2(E_1 + E_2))$$  

$$+ 32Q_u^4 |\kappa_c|^2 (C_1 N_C + C_2)^2 m_b^2 (m_B - E_1 - E_2)^2 \right\}. \quad (35)$$

We recall that Eq. (35) is valid in the domain of the Dalitz plot with a fast light meson with energy $E_K \gg \Lambda_{QCD}$ and $Q_{1,2}^b$ and $q^2$ hard.
4 Additional contributions

In this section we discuss contributions to $B \to K\gamma\gamma$ decays beyond Eq. (35). These are if a photon gets radiated from the spectator quark, which is discussed in Section 4.1 and annihilation contributions, see Section 4.2. Effects of this type can be experimentally accessed by comparing neutral versus charged $B$ decays. In Section 4.3 we further discuss contributions from intermediate hadronic states.

4.1 Photon radiation off the spectator

The OPE is invalidated by contributions where one of the photons is emitted by the spectator, for an example see the left diagram of Fig. 2. In the case of soft gluon exchanges the momenta of the final state $s$- and anti-quark are $\vec{p}_s \simeq -\vec{k}_1$ and $\vec{p}_{\bar{q}} \simeq -\vec{k}_2$ up to order $\Lambda_{QCD}$ in the decaying heavy meson rest frame. This configuration does not allow a Kaon to be formed if the angle between the photons is large $\sin \theta \simeq |\vec{k}_{\perp}|/E_K \gg \Lambda_{QCD}/m_B$. This condition is indeed satisfied in the OPE region around $E_{1,2,K} \sim m_b/3$, where the $B$-decay products form a Mercedes Benz star in the $B$-rest frame, i.e. have angles of 120° between each other. Therefore, the exchange of soft gluons in these types of diagrams is excluded by the special kinematics.

There are further hard scattering corrections from energetic gluon exchange, which are, however, suppressed by the strong coupling constant. They can produce the $K$ also in a highly asymmetric configuration where $p_s \simeq p_K$ and are not power suppressed. We assume here that the endpoint suppression is sufficient to render this contribution subdominant. Note that due to the existence of a hard line in these diagrams, in the effective theory they get contributions from SCET operators [21] involving collinear gluons. For example, in the left diagram shown in Fig. 2 the $s$-quark emitted from the FCNC vertex has virtuality $(Q_s^2) \sim O(m_b^2)$ and after integrating out these scales it is mediated by an operator with field content $\bar{\chi}h_v$ plus one photon and a collinear gluon. Analogous statements hold for the other gluon exchange topologies which are not shown in Fig. 2.

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Figure 2: Contributions to the $B \to K\gamma\gamma$ amplitude beyond the form factor-type OPE. Left hand side: radiation off the spectator, the blob denotes an insertion of the dipole operator $O_7$. Right hand side: Leading power annihilation contributions, the blob denotes a Four-Fermi operator and the second photon can be attached to any of the crosses. Diagrams with interchanged photons are not shown.
4.2 Annihilation topologies

If we take additional operators in the effective Hamiltonian Eq. (2) into account, annihilations topologies in $B \to K\gamma\gamma$ decays arise. Weak annihilation (WA) contributions in charged $B^\pm \to K^\pm\gamma\gamma$ decays are induced by the operators $O_{1,2}^{u} \simeq (\bar{s}\gamma^\mu Lu)(\bar{u}\gamma^\mu Lb)$, which are obtained from $O_{1,2}$ by replacing $c$ with $u$, and are CKM suppressed. In both neutral and charged $B$ decays penguin annihilation is possible. It is suppressed by the small Wilson coefficients of the penguin operators which are of order $10^{-2}$ in the SM.

The leading annihilation diagrams are the ones with at least one photon emitted from the light anti-quark in the $B$-meson, see Fig. 2. Then one propagator is off-shell by order $\sim Q\Lambda_{QCD}$. Dimensional analysis indicates that this contribution is leading power with respect to the form factor-type contribution §

$$\frac{f_B f_K}{m_B \Lambda_{QCD} f_+} \propto \mathcal{O}(1) \quad \text{with} \quad f_B \propto m_b^{-1/2}, \quad f_+ \propto m_b^{-3/2}, \quad (36)$$

therefore it is only suppressed by small CKM elements or small Wilson coefficients. Such leading power annihilation contributions have also been found in $B \to K\ell^+\ell^-$ decays $^{[22]}$. However, their impact is bigger in the two-photon decays than in the semileptonic decays since the dominant Wilson coefficients associated with the form factor type contribution is larger in the latter. We estimate for $B \to K\gamma\gamma$ decays $A_{WA}/A_{\text{formfactor}} \sim V_{ub}^* V_{us}/(V_{tb} V_{ts}^*) C_2/C_7$, a correction of order ten percent at the amplitude level.

4.3 Long-distance effects

Resonance effects through $B \to (\eta_x \to \gamma\gamma)K$ where $\eta_x = \eta, \eta', \eta_c$ and $B \to (K^* \to K\gamma)\gamma$ have been discussed in $^{[10,11]}$. In particular, the latter decay chain has been used to model the $O_{\gamma}$-type contribution in these works. Relevant here to the kinematical OPE window are only the charmonium resonances, since invariant diphoton masses of $m_\eta^2, m_{\eta'}^2$ are too low and the decays with virtual $K^*$ have one photon with near maximal energy $\sim m_B/2$, which is excluded. To be specific, the $\eta_c$ and the $\chi_{c0,2}$ have masses in the signal region and sufficient branching ratios given as $^{[23,25]}$

$$\mathcal{B}(B \to K\eta_c) \times \mathcal{B}(\eta_c \to \gamma\gamma) \simeq 4 \cdot 10^{-7}, \quad (37)$$

$$\mathcal{B}(B \to K\chi_{c0}) \times \mathcal{B}(\chi_{c0} \to \gamma\gamma) \simeq 2 \cdot 10^{-7}, \quad (38)$$

$$\mathcal{B}(B \to K\chi_{c2}) \times \mathcal{B}(\chi_{c2} \to \gamma\gamma) < 1 \cdot 10^{-8}. \quad (39)$$

We neglect in our study the $\chi_{c2}$ since the production through $B \to K\chi_{c2}$ has not been seen yet and its impact on $B \to K\gamma\gamma$ is at least an order of magnitude below the other two charmonia.

§More precisely, the scaling law of the heavy-to-light form factor is $f_+ \propto m_b^{-3/2}/(1 - q^2/m_b^2)^2$ $^{[18]}$, which simplifies to the “standard” form at large recoil also at $q^2 = \mathcal{O}(m_{\eta_c}^2)$ as long as $q^2$ is order one away from $m_{\eta_c}^2$. 

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To estimate how severe the long-distance contamination is, we assume Breit-Wigner form for the scalar and pseudoscalar $X = \eta_c, \chi_c$ resonance amplitudes

$$A(B \to K(X \to \gamma \gamma)) = A(B \to XK) \frac{1}{q^2 - m_X^2 + im_X \Gamma(X)} A(X \to \gamma \gamma),$$

where

$$A(\eta_c \to \gamma \gamma) = ia(\eta_c) F_{\mu \nu} \tilde{F}^{\mu \nu}, \quad A(\chi_c \to \gamma \gamma) = a(\chi_c) F_{\mu \nu} F^{\mu \nu},$$

and take $|a(X)|$ and $|A(B \to XK)|$ from the measured rates

$$\Gamma(X) = \frac{|A(X \to \gamma \gamma)|^2 m_X^3}{16\pi}, \quad \Gamma(B \to KX) = \frac{|A(B \to XK)|^2 |\vec{p}_K|}{8\pi m_B^2}.$$  

This phenomenological parametrization has problems because the information about the strong phase is lost, a residual $q^2$-dependence in the partial amplitudes is neglected and there is double counting to some extend. However, the typical size of the effect is reproduced correctly. We use this model to see whether short-distance physics can be extracted from $B \to K \gamma \gamma$ decays in the OPE region. This is worked out numerically in Section 5.

The pollution from $c\bar{c}$ is also known from rare semileptonic decays, which get a sizable background through $B \to (\Psi(\ell^+ \ell^-) \to \ell^+ \ell^-)K(*)$ that has to be removed by kinematical cuts. If duality would be perfectly at work, the 1PI diagram with the charm loop would include the contributions from the charmonium resonances. For $q^2 \approx m_b^2 \gg m_c^2$ OPE methods have been suggested for $B \to K^* \ell^+ \ell^-$ decays for a model-independent calculation of the rate \cite{26}. For $B \to K \gamma \gamma$ decays, this amounts at lowest order to expanding $\kappa(q^2, m_c^2) = \kappa(q^2, 0) + \mathcal{O}(m_c^2/q^2)$, that is, using $\kappa_c = 1/2$, see Eq. (9). However, in the double photon decays we cannot have $q^2$ large enough and hence are too close to the charm threshold to use this expansion.

## 5 Phenomenology of $B \to K \gamma \gamma$ decays

We discuss the OPE allowed phase space and give estimates for the $B \to K \gamma \gamma$ branching ratio in the SM in Section 5.1 and in Section 5.2 with NP.

### 5.1 Dalitz plot and SM predictions

The $B \to K \gamma \gamma$ Dalitz plot in the $E_1 - E_2$ plane is shown in Fig. 3. The surviving phase space after cuts is the shaded triangle. We use $E_{1,2} < 2$ GeV (thin solid lines) which imply $E_K > 1.3$ GeV, and $q^2 > m_B^2/5$ (long-dashed line). As can be seen, other cuts required by our OPE analysis, such as a minimal angle between the photon momenta for example $\theta > 45^\circ$ (dotted line) and a low energy cut on the photon energies of 1 GeV (short-dashed lines), are ineffective in the presence of the previous ones. The reference point $E_{1,2}, E_K = m_B/3$ is in the surviving region. The corresponding $q^2$-range is numerically $5.6$ GeV$^2 < q^2 < 14.6$ GeV$^2$. Note that the
Figure 3: $B \to K\gamma\gamma$ phase space in the plane of the photon energies $E_1$ and $E_2$ in GeV. The thick solid line is the phase space boundary, the thin solid, dotted, short-dashed, long-dashed lines denote the cuts $E_{1,2} < 2$ GeV, $\theta > 45^\circ$, $1$ GeV $\leq E_{1,2}$ and $q^2 > m_B^2/5$. Also shown is the point $E_1 = E_2 = m_B/3$.

exclusion of the low di-photon mass events is not a big loss in the event rate since the $B \to K\gamma\gamma$ is suppressed by small $q^2$.

For our numerical analysis, we evaluate the SM Wilson coefficients at leading log at $\mu = m_b$ and take the $B \to K$ form factor $\zeta = f_+$ from Light cone-QCD sum rules $f_+(0) = 0.319$ [20], which is in good agreement with the corresponding lattice-QCD result [27]. To be specific, we use $C_1 = -0.25$, $C_2 = 1.11$ and $C_7 = -0.35$. We integrate Eq. (13) between $\max(E_{1,2}^{\text{min}}, 1.2$ GeV) and $\min(E_{1,2}^{\text{max}}, 2$ GeV), see Eq. (14), to obtain the di-photon mass spectrum $d\Gamma/dq^2$, which is shown in Fig. 4. As can be seen, the SD part of the spectrum (solid curve) is completely hidden behind the $\eta_c$ and the $\chi_{c0}$ resonance contribution (dash-dotted curve). The integrated SD branching ratio in the SM with cuts is small, $\Delta B(B \to K\gamma\gamma)^{\text{OPPE}}_{\text{SM}} \simeq 1 \times 10^{-9}$. It has an uncertainty from the form factor of about 20 % and about 50 % from the renormalization scale dependence, when $\mu$ is varied between $m_b/2$ and $2m_b$. The order of magnitude of the branching ratio is consistent with the findings of Ref. [28], where the 1PI contribution has not been taken into account and with no cuts.

We also show for comparison the contribution from 1PI SD diagrams only (dashed curve), which gives a somewhat reduced branching ratio $B(B \to K\gamma\gamma)^{\text{OPPE1PI}}_{\text{SM}} \simeq 0.5 \times 10^{-9}$. Even if we integrate this contribution over a larger phase space region, the resulting branching ratio is about two orders of magnitude smaller than the corresponding ones $\sim \text{few} \times 10^{-7}$ given in [10] (where a very large value of the form factor is used) and [11]. Such large $B \to K\gamma\gamma$ SD branching ratios are also in conflict with the one of the inclusive decays $B(B \to X_{s}\gamma\gamma) \simeq 3.7 \times 10^{-7}$ [5].

5.2 New physics

To be comparable to the resonance contributions at least in some region of phase space the SD contribution has to be lifted by roughly one order of magnitude above the SM one. We investigate here whether there exists such type of NP that is at the
Beyond the SM, contributions to the dipole operator $O_7$ are tightly constrained by data on $b \to s\gamma$ decays. In particular, $|C_7|$ is fixed at the 30 percent level \cite{29}. The QCD penguin operators have small Wilson coefficients in the SM $C_{SM}^{QCD}/C_2 \sim 10^{-2}$. Even an enhancement of a factor of 100, which is problematic because of data on $B \to K\pi$ and other rare decays would constitute only an $O(1)$ effect in $b \to s\gamma\gamma$. Hence, these types of NP are inefficient to enhance the SD contribution in $B \to K\gamma\gamma$ decays to be above the $\eta_c, \chi_{c0}$ background.

Let us consider scalar/pseudoscalar operators with leptons $\ell = e, \mu, \tau$

$$O_{S(P)}^\ell = \frac{\alpha_{em}}{4\pi} sRb(\gamma_5)\ell. \quad (43)$$

They contribute to the matrix element of double radiative decays through a lepton loop, similar to the charm loop shown in Figure 1. The contribution w.r.t. the charm loop scales roughly as $\alpha_{em}/(4\pi) \times C_{S(P)}^\ell/(Q_{u}^2 C_2)$. The Wilson coefficients of the muonic operators ($\ell = \mu$) are constrained by $B(B_s \to \mu^+\mu^-)$ data to be order one \cite{29}, hence way too small to raise the $B \to K\gamma\gamma$ SD contribution above the resonance contributions. Assuming lepton universality, i.e., $C_{S(P)}^\ell \propto m_\ell$, which is realized if the operators are induced for example by neutral Higgs exchange, the electron contribution is tiny and $|C_{S(P)}^{\tau}| \lesssim 30$, which is still not large enough. If one
relaxes lepton universality, however, one obtains model-independently $|C_{S(P)}^r| \lesssim 700$ from $\mathcal{B}(B_s \to \tau^+\tau^-) < 5\%$ [30]. Sizable enhancements of the electron couplings are excluded by data on $b \to se^+e^-$ decays. The operators $\mathcal{O}_{S(P)}^r$ enter $b \to s\gamma \gamma$ decays as a 2-loop electromagnetic radiative correction, hence give only tiny effects in the single photon channel. There is no one-loop mixing of $\mathcal{O}_S^r - \mathcal{O}_P^r$ onto $\mathcal{O}_g$, see [29] for details, hence no correction to $b \to se^+e^-$ and $b \to s\mu^+\mu^-$ decays at this level. Note that the decoupling of $b \to s\gamma$ decays from NP in $b \to s\gamma\gamma$ has also been discussed in the context of SUSY with broken R-parity in $B_s \to \gamma\gamma$ and $B \to X_s\gamma\gamma$ decays in Ref. [31].

Explicit calculation of the $\tau$-loop gives the following corrections to Eqs. (27)-(33) as $A \to A + \delta A_r$, $B \to B + \delta B_r$ etc., where

$$\delta A_r = \kappa \frac{\alpha_{em}}{4\pi} \zeta(E_K) C_r^\tau E_K m_r \left( \kappa_r (4 - z_r) + \frac{z_r}{2} \right),$$

$$\delta B_r = \kappa \frac{\alpha_{em}}{4\pi} \zeta(E_K) C_r^\tau E_K \frac{m_r}{2} (1 - 2\kappa_r),$$

and all other $\delta C_{r,}^\pm$, $\delta D_{r,}$, $\delta D_{l,}^\pm$ vanish. Note that the matrix elements of $\mathcal{O}_{S(P)}^r$ can be deduced from the one of the fierzed QCD penguin $\mathcal{O}_6$, see e.g. [45]. The impact of $C_r^\tau = \pm 700$, $C_S^\tau = 0$ and all other Wilson coefficients assuming SM values is shown in Fig. 4 (dotted and double-dotted curves). As can be seen, even maximal NP enhancement brings the SD distribution barely above the resonance background outside the $\eta_c$ and $\chi_{c0}$ peaks. For $C_r^\tau = -700$, $C_S^\tau = 0$ we obtain $\Delta \mathcal{B}(B \to K\gamma\gamma)^{OPE}_{NP} \simeq (1 - 2) \times 10^{-8}$, an order of magnitude above the SM value.

6 Conclusions

We presented predictions for the exclusive $B \to K\gamma\gamma$ decay. In particular, we calculated the SD contribution to the matrix element by means of an OPE in $1/Q$, where $Q$ is a combined scale of order $m_b$ and contains the $b$-quark mass, the energy of the final meson in the $B$ rest frame, the photon invariant mass and the virtualities of the intermediate quarks in the 1PR $b \to s\gamma\gamma$ diagrams. Then, around the Dalitz region where all decay products have energy $\sim m_b/3$, the $b \to s\gamma\gamma$ amplitude can be expressed at lowest order in terms of local dimension 8 operators made out of a heavy HQET and a collinear SCET field plus two photons. The resulting local matrix element is given in terms of a form factor known from $B \to K\ell^+\ell^-$ decays.

We found that the resulting SD branching ratio in the SM is small, order $10^{-9}$, which is at variance with previous estimates [10,11]. Such small branching ratios for double radiative decays are not a complete surprise. In fact, being of the same order in $\alpha_{em}$ as semileptonic rare decays with branching ratios of few $\times 10^{-7}$ [20], we expect

$$\mathcal{B}(B \to K\gamma\gamma) \sim \left[ \frac{\kappa_e Q_e^2 C_2}{|C_9|^2 + |C_{10}|^2} \right] \frac{|C_1|^2}{|C_9|^2 + |C_{10}|^2} \times \mathcal{B}(B \to K\ell^+\ell^-) \simeq \mathcal{O}(10^{-9}).$$

*For the OPE this requires the operators $Q_4 = m_b \bar{c}Rh_v F \cdot F$ and $Q_5 = m_b \bar{c}Rh_v F \cdot \tilde{F}$.
The double photon decays are substantially suppressed with respect to the semileptonic ones, since the di-lepton operators with large coefficients \( |C_{9,10}^{SM}| \approx O(4 - 5) \) are not contributing to the former, see e.g. [15].

In our analysis we further took long-distance effects via \( B \to \eta c K \to \gamma \gamma K \) and \( B \to \chi_{c0} K \to \gamma \gamma K \) into account and found that the SM SD contribution is not accessible behind the resonance peaks, see Fig.4. We also discussed additional contributions from photon radiation off the spectator and annihilation diagrams, which induce sizable theoretical uncertainties and exhibit increased complexity compared to the ones in \( B \to K^* \gamma \) decays due to the second (energetic) photon. On the other hand, non-standard scalar/pseudoscalar couplings to taus can give enhanced \( B \to K \gamma \gamma \) branching ratios of the order \( 10^{-8} \). Note that such a NP scenario is consistent with data on other rare decays such as \( b \to s \gamma \) and \( b \to s \ell^+ \ell^- \). In particular, large couplings are allowed because FCNC decays into a tau pair are essentially unconstrained to date. In other words, decays such as \( B_s \to \tau^+ \tau^- \) and \( B \to K^{(*)} \tau^+ \tau^- \) have sizable room for NP. The corresponding maximal SD \( B \to K \gamma \gamma \) spectrum with NP in the \( \tau \) couplings leaks marginally outside the resonance background. We conclude that it is very difficult to test SD physics with exclusive \( B \to K \gamma \gamma \) decays.

Very similar arguments apply to \( B \to K^* \gamma \gamma \) decays. We obtained for the pure SD SM estimate of the 1PI induced contribution \( B(\mathcal{B}(B \to K^* \gamma \gamma)) \approx \text{few} \times 10^{-9} \), which is two orders of magnitude below the corresponding value quoted in [32].

**Note added:** By means of the Schouten identity [33] only four of the form factors in Eq. (11) correspond to independent Lorentz structures. In fact, \( C^\pm \) and \( D^- \) can be absorbed into \( B \), such that redefining \( B \to B - m_B^2 D^- + k_1 \cdot p_B (C^+ - C^-) - k_2 \cdot p_B (C^+ + C^-) \) with the remaining form factors \( A, D, D^+ \) left unchanged gives identical results for the \( B \to K \gamma \gamma \) amplitude.

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