The role of photon scattering in shaping the light curves and spectra of γ-ray bursts

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ABSTRACT

We analyse the power spectra of the light curves of long gamma-ray bursts (GRBs), dividing the sample into luminosity bins, using the recently discovered variability–luminosity correlation. We find that the value of the variability parameter strongly correlates with the frequency that contains most of the power in the burst comoving frame. We compute the average power spectra in luminosity bins. The average power spectrum is well described by a broken power law and the break frequency is a function of the variability parameter, while the two slopes are roughly constant. This allows us to conclude that scattering processes do not play a relevant role in modelling the light curves. We finally discuss under what conditions scattering may still play a relevant role in shaping the spectra of GRBs.

Key words: gamma-rays: bursts.

1 INTRODUCTION

The study of the BATSE γ-ray bursts (GRBs) light curves has recently gained new interest thanks to the discovery of the variability–luminosity (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001) and lag–luminosity (Norris, Marani & Bonnell 2000) correlations (see also Chang, Yoon & Choi 2002). These enable one to assign a tentative redshift to BATSE GRBs, allowing one to perform spectral and temporal analysis of the light curves in the burst comoving frame, where their properties are more closely linked to the physics of the burst itself. An example, the use of these correlations enabled the possible discoveries of an evolution of the luminosity function with redshift (Lloyd-Ronning, Fryer & Ramirez-Ruiz 2002) and of a correlation between the peak photon frequency and the variability measure (Norris, Marani & Bonnell 2000). Besides being extremely useful tools, these correlations also call for an explanation of their origin. Several possible interpretations have been discussed in the literature. In particular, it is shown that an underlying correlation between the isotropic equivalent luminosity and the Lorentz factor of the fireball can explain both the correlations (Ramirez-Ruiz & Lloyd-Ronning 2002; Kobayashi, Ryde & MacFadyen 2002; Salmonson 2000; Mészáros et al. 2002). In some of these works, however, a significant role is attributed to the modification of the temporal properties of the light curves as a consequence of scattering by cold or hot electrons (see also Panaitescu, Spada & Mészáros 1999; Spada, Panaitescu & Mészáros 2000). The role of scattering seems to be strengthened by the need to explain the peak photon frequency versus isotropic equivalent luminosity correlation (Lloyd-Ronning & Ramirez-Ruiz 2002), which is not naturally predicted in the internal shock scenario (Ghisellini, Celotti & Lazzati 2000).

In this paper, we concentrate on the variability–luminosity correlation, using power spectra as a diagnostic to investigate several related issues. First (Section 2), we compute the dominant frequency of the power spectrum for each GRB in our sample of 220 light curves. We find that this dominant frequency strongly correlates with the variability measure. Then we separate the sample into variability bins and compute the average power spectrum. We find (Section 3) that the power spectra are self-similar in all the variability bins, with a break at a frequency that correlates with the variability parameter. No sign of a cut-off in the spectrum at large frequencies is observed. We (Section 4) develop a shot-noise model for the light curve, taking into account the effect of scattering in Fourier space (Section 5). In Section 6 we discuss our results, showing how scattering processes cannot be responsible for the variability–luminosity correlation, and also constraining the regions of the parameter space where photon scattering on cold electrons may imprint detectable signatures on the photon spectra of GRBs without leaving a detectable trace in their power spectra.
2 DATA ANALYSIS

We have adopted the whole sample of GRBs for which Fenimore & Ramirez-Ruiz (2000) computed the variability parameter and a tentative redshift. For each light curve, we have computed the power spectrum (hereafter PSD) as \( \text{PSD}(\nu) = |\tilde{f}(\nu)|^2 \), where \( \tilde{f}(\nu) \) is the Fourier transform of the light curve. Light curves were binned to 64-ms resolution and considered from the BATSE trigger time for a total duration of \( t = 3T_9 \), where \( T_9 \) is the time containing 90 per cent of the total burst emission. A cubic function was fitted to the time interval before and after the one considered above in order to remove the background. The effect of background removal was, however, weak and only affected the smallest frequencies.

The dominant frequency \( \nu_d \) was detected as the frequency at which the power per unit decade frequency is maximized [similar to the peak of the \( \nu F(\nu) \) photon spectrum], i.e. the maximum of the function \( \nu \text{PSD}(\nu) \). We adopted this definition since it does not involve any fitting procedure and is therefore the most objective. The definition of the dominant frequency must, however, confront two problems. First, we must consider that the time series is limited (see below) and secondly the presence of a white noise component, which is caused by the Poisson noise of the light curve (which is inevitably affected by photon count statistics). This component results in a flat power spectrum (see, e.g., Leahy et al. 1983 and references therein), which produces a false peak at the highest frequency considered when the \( \nu \text{PSD}(\nu) \) function is considered for light curves in which the signal is weaker than a certain threshold. In order to avoid this contamination of spurious peaks, the spectra were all visually inspected, and every time the peak frequency was in the flat part of the PSD, the considered frequency range was shrunk in order to recompute the maximum over a shorter frequency range, where the signal-to-noise ratio was larger. This procedure, unavoidably, reduces the objectivity of the analysis, and for this reason the light curves in which the peak selection was modified by hand were flagged. In Fig. 1 we show the comoving frame dominant frequency versus the variability parameter for all the best class light curves, in which human intervention was either not required or clearly unbiased. A clear correlation is present. The correlation, according to the correlation coefficient statistics, is highly significant, with a vanishing probability of being spurious of \( P = 3 \times 10^{-22} \). It may be argued that the correlation is artificially made by a correlation between the redshift and the variability (which is indeed present in the data). In order to test whether this is true, we also computed the above probability for the observed frame frequency. Again, a strong correlation was found, with a probability of being spurious of \( P = 8 \times 10^{-12} \). The lower class bursts, in which human intervention was necessary and possibly biased, or in which the dominant frequency was the lowest one, are plotted in light grey in Fig. 1. It is possible to see how these points do not invalidate the correlation, even though the dominant frequencies tend to be, as expected, larger than for the best cases. In the following we will perform our analysis only on the subsample of 143 best light curves shown in Fig. 1, even though extending it to the whole sample does not affect the conclusions significantly. It is worth stressing that such a correlation is not entirely surprising, since it simply says that the variability parameter has something to do with the mean frequency over the PSD. What is relevant, as we are going to discuss in much more detail in the following, is that this correlation tells us that a change in the variability parameter \( V \) reflects a change in the whole PSD rather than the suppression of high- (or low-) frequency power.

When computing the PSD of an experimental time series, one has to take into account all the effects caused by the data binning and to the finite duration of the data. In particular, for GRBs, this second issue is particularly tricky, since the data are truncated not by hand, but by the intrinsic duration of the event. Consider an infinite function \( f_\infty(t) \). The binning process can be modelled as the convolution with a square filter \( b(t) \) and the multiplication with a lattice \( \sigma(t) \) (a series of Dirac \( \delta \) functions) with spacing equal to the full width of the square filter. The truncation of the data is represented as a multiplication by a window function \( w(t) \), which is usually assumed to be the characteristic function of the interval considered, but may have different shapes in the case of GRBs (see below). Thanks to the properties of multiplication and convolution in Fourier space, the Fourier transform of \( f \) can be written as (Lazzati & Stella 1997, hereafter LS97):

\[
\tilde{f}(\nu) = [\tilde{f}_\infty(\nu) * \tilde{w}(\nu)] \tilde{\delta}(\nu) * \tilde{b}(\nu) ,
\]

(1)

where the symbol \( * \) indicates a convolution. Let us analyse the implications of equation (1) from right to left. The Fourier transform of the lattice of spacing \( \Delta t \) (in our case 64 ms) is the reciprocal lattice, i.e. a lattice with spacing \( 2\pi/\Delta t \sim 100 \) s. This term is important if periodicities are present in the light curve, since it gives rise to the well-known phenomenon of aliasing. In our case, since the power spectra are monotonically decreasing with frequency, it is irrelevant. The Fourier transform of the binning function has a form (see, e.g., LS97)

\[
\tilde{b}(\nu) \propto \frac{\sin (\pi \Delta t \nu)}{\pi \Delta t \nu} .
\]

(2)

This is the most dangerous term, since it introduces a break at a frequency \( \nu \sim 1/\Delta t \sim 16 \) Hz in the observer frame. The largest of our dominant frequencies, in the observer frame, is \( \nu_{max}(t) = 0.5 \) Hz.

We conclude that this effect does not play a relevant role in our analysis. Finally, the effect of the window function is to smooth the observed PSD. In the most common case of a square function, the smoothing kernel is similar to the function in equation (2) but
with a smaller width. Again, this term is important where small-scale features overlaid on the spectrum are concerned. For any reasonable shape of the window function we can ignore its effect.

We therefore conclude that the correlation of Fig. 1 is real. It is worth noting that the typical frequencies, which seem to dominate the determination of the variability parameter $V$, are small, even in the comoving frame. This suggests that it is the $\sim$1-s variability that is physically linked to the luminosity of the GRB rather than its shorter time-scale fluctuations (see also below). There is also a suggestion, in the data, that the very small $V$ bursts belong to a different family in terms of their temporal properties. We will discuss this in the following, finding that the average power spectra seem to be more consistent with a continuation of the average spectral properties rather than with a different subclass.

3 AVERAGE POWER SPECTRA

We have divided the sample of 143 GRBs into six bins of the variability parameter $V$. Each bin contains $\sim$25 light curves, with the exception of the smallest $V$ bin, in which only 12 light curves are contained. This difference is caused by the need to keep these bursts isolated since they seem to belong to a different class. The background-subtracted light curves were normalized to contain the same number of photons and the power spectra averaged at the same comoving frequency. The errors were derived from the dispersion of the sample.

This process is similar to what was done by B98 and B00. Their average power spectrum, however, presents three fundamental differences. First, they performed the average at the same observed frequency, instead of the comoving frequency. This was caused by the fact that redshift estimates were not available at that time. Secondly, they decided to normalize their light curves to the same peak photon luminosity rather than to the same photon fluence. We decided to use this second normalization since the scatter in the sample of spectra was lower in this case (see B98 for a discussion). Finally, they built the average of all the light curves, while we subdivide our sample into variability bins.

The resulting average power spectra are shown in Fig. 2. The variability parameter increases from left to right and from top to bottom. The spectra seem to be made by a broken power law, sinking into the white noise at large frequencies. It is worth noting that our normalization criterion does not produce uniform white noise values. For this reason the subtraction of this component from the average spectra is not possible, and it is also dangerous to investigate

![Figure 2](https://example.com/figure2.png)

Figure 2. Average power spectra of the burst in the six variability subclasses defined in the text (see also Fig. 1). The spectra have been binned in frequency in order to have a constant number of points per logarithmic unit frequency. The grey line shows the best broken power law (plus constant) fit. Error bars are derived from the dispersion of the sample.
Table 1. Results of the fit of the average power spectra in Fig. 2. The rightmost column shows the significance for the existence of the break.

| (V)  | $\alpha_1$      | $\alpha_2$      | $v_0$ (Hz) | Prob. (\sigma) |
|------|-----------------|-----------------|------------|-----------------|
| 0.0032 | 0.6 \pm 0.18  | 3. \pm 0.15 | 0.071 \pm 0.016 | 7.2              |
| 0.0077 | 0.62 \pm 0.1   | 2.5 \pm 0.1  | 0.098 \pm 0.017 | >8.3             |
| 0.0125 | 0.58 \pm 0.09  | 2.3 \pm 0.08 | 0.145 \pm 0.03 | >8.3             |
| 0.018  | 0.7 \pm 0.13   | 1.9 \pm 0.06 | 0.16 \pm 0.04 | 8.1              |
| 0.027  | 0.67 \pm 0.09  | 2.16 \pm 0.07 | 0.35 \pm 0.07 | >8.3             |
| 0.053  | 0.75 \pm 0.08  | 2.22 \pm 0.3  | 2.0 \pm 0.6  | 8.2              |

Figure 3. The break frequency and high-energy slopes of the average power spectra versus variability. The upper panel shows the knee frequency as a function of variability. The upper point (open circle) is for the lowest variability class. The lower panel shows the high-frequency spectral index versus the variability parameter.

The frequency region where the average spectra are dominated by white noise.

We fit the average spectra with a smoothly broken power-law function of the form

$$\text{PSD}(v) = \frac{2F_0}{\left(v/v_0\right)^{2\alpha_1} + \left(v/v_0\right)^{2\alpha_2}}^{1/2} + K,$$

where $\alpha_1$ is the slope at frequencies smaller than the break $v_0$, $\alpha_2$ is the slope at larger frequencies and $K$ is a constant that takes into account the contribution of the white noise. $F_0$ is the value of the PSD at the break frequency.

The results of the fit are reported in Table 1 and in Fig. 3. Several interesting remarks can be derived from the results. First, all the power spectra are well-fitted by the model. Moreover, the slopes of the broken power law before and after the break are roughly consistent with being constant, with the break frequency $v_0$ being the only quantity that evolves with the variability parameter $V$. Marginal evidence of evolution of the high-frequency power-law index is also present, especially if the smallest variability bin is included in the sample. A second important remark is that the high-frequency slope is larger than that found by B98. This difference is not caused by the redshift correction nor by the different normalization. In fact, if we average among them the six power spectra of Fig. 6 (see Section 5), we recover with good accuracy the $-\frac{2}{3}$ slope in B98. It therefore appears that the slope can be attributed to the convolution of the break frequencies in different variability bins. Finally, even though the lowest variability bin values are different from the rest of the sample, it is not possible to define them as a separate class given the present data. The apparent segregation of the dominant frequencies in Fig. 1 may be caused by the fact that the dominant frequencies of the very low variability bursts approach the lower cut in the analysed frequencies, and therefore include an additional noise term.

4 SHOT NOISE MODEL

In the internal shock model for GRBs (Rees & Mészáros 1994) the light curve is modelled as the random superposition of a number of pulses of similar shape stretched in shape and scaled in luminosity according to some prescription for the ejection of shells by the inner engine (Kobayashi, Piran & Sari 1997; Panaitescu et al. 1999; Spada et al. 2000, 2001; Ramirez-Ruiz & Lloyd-Ronning 2002). This light curve is similar to the shot noise model supposed to play a relevant role in the red-noise component observed in the power spectra of X-ray pulsars (LS97 and references therein).

Consider a normalized pulse of shape $p(t - t_0, \tau_r)$, with rise time $\tau_r$ and unit decay time, with fluence peaking at time $t_0$. A GRB light curve can be described as a random superposition of $N$ such pulses:

$$l(t) = \sum_j f_j p \left( \frac{t - t_j}{\alpha_j}, \tau_r \right),$$

where the pulse fluence $f_j$, the stretching factor $\alpha_j$ and the peak time $t_j$ are randomly selected according to some prescription (for example, simulating the hierarchical shock evolution of an inhomogeneous flow). The Fourier transform of the above equation can be written (up to a normalization factor) as

$$\hat{l}(\omega) \propto \sum_j f(\alpha_j \omega) e^{-\omega^2 \tau_r^2}.$$  

(5)

When we compute the average power spectrum of a sample of light curves, under the assumption that each light curve is a random realization of the same underlying process, we compute the ensemble average of the square of the modulus of equation (5). This (if the peak times of the pulses are not correlated with their properties and are uniform in time) is given by (LS97)

$$\langle \text{PSD} \rangle \propto \langle f^2(\omega) \rangle.$$  

(6)

i.e. the average power spectrum is independent of the time history of the pulse ejection and the pulse fluence distribution, but depends on the distribution of the pulse durations. To understand the effect

1 It is a well-known result that the average width of the pulses does not evolve during the bursts (Ramirez-Ruiz & Fenimore 2000), but their frequency and/or fluence may be larger in the early phase of bursts. This correlations takes the form of a non-square window function in Fourier space, and are not relevant here, as discussed in Section 2. Finally, it may be wrong to assume that the pulse fluence does not depend on the pulse duration (Ramirez-Ruiz & Fenimore 2000). Should this assumption be wrong, it would be reflected in a different numeric value for equation (16).
of the above conclusion, we consider a simplified exponential pulse profile:

\[ p(t) = e^{-\chi_{(0,\infty)}}. \]  

where \( \chi_{(a,b)} \) is the characteristic function of the interval \((a, b)\). In addition, we consider a power-law distribution\(^2\) of \( \alpha \) values between a minimum value \( \alpha_m \) and a maximum value \( \alpha_M \):

\[ n(\alpha) \propto \alpha^{\alpha} \chi_{(\alpha_m, \alpha_M)}. \]  

The power spectrum of the single pulse of equation (7) is a Lorentzian function, i.e. (roughly speaking) a constant for \( \nu < \frac{\alpha}{\pi} \) and a power law \( \nu^{-2} \) for \( \nu > \frac{\alpha}{\pi} \). For the stretched pulse, the PSD is flat for \( \nu < 1/(2\pi \alpha) \) and a power law afterwards. Consider now the average PSD of equation (6) for the \( \alpha \) distribution defined in equation (8). If \( a \ll -1 \) the PSD will be dominated by the shortest pulse, and will retain a Lorentzian shape. For \( a \gg 1 \), it will be dominated by the longest pulse, again retaining a Lorentzian shape. For \( -1 < a < 1 \), a new power-law branch will appear, in the range \( 1/(2\pi \alpha) < \nu < 1/(\pi \alpha) \), with a slope \( \nu^{-1/(\alpha+1)} \). Since for \( a \gg 1 \) the PSD is dominated by the longest pulse, here and in the following we will define the ‘shortest relevant pulse’ as the shortest pulse that has an influence on the PSD shape. This pulse may not be the shortest observed in the light curve. For example, consider a broken power-law distribution of pulse durations, with \( -1 < a < 1 \) at short durations and \( a \gg 1 \) at short durations. The break in the power spectrum will be related to the pulse at the break of this distribution. We call this the shortest relevant pulse.

The shape of the fundamental pulse in equation (7) can be made more complicated. Consider as an example, a double exponential pulse, of functional shape

\[ p(t, \tau_l, \tau_a) = \frac{1}{\tau_l + \tau_a} \left[ e^{2/\nu} \chi_{(-\infty, 0)} + e^{-2/\nu} \chi_{(0, +\infty)} \right]. \]

Its PSD, can be written as

\[ |\tilde{p}(\nu)|^2 \propto \frac{1}{1 + \omega^2 \tau_a^2 \left( \omega^2 \tau_l^{-4} + 1 + \tau_l^{-2} / \tau_a^{-2} \right)} \]

where \( \omega \equiv 2\pi \nu \) is the pulsation. If \( \tau_l \ll \tau_a \), equation (10) can be approximated as a double broken power law:

\[ |\tilde{p}(\nu)|^2 \propto \begin{cases} \nu^0 & \text{for } \nu < 1/2\pi \tau_a \\ \nu^{-2} / 2\pi \tau_l^{-2} & \text{for } \nu < 1/2\pi \tau_l \\ \nu^{-4} & \text{for } \nu > 1/2\pi \tau_l \end{cases} \]

Again, the ensemble average of equation (6) can add different power-law branches if \( -2 < a < 0 \). Finally, let us consider a stretched double exponential (Norris et al. 1996) equation:

\[ p(t, \tau_l, \tau_a) \propto \left[ e^{\nu/\nu} \chi_{(-\infty, 0)} + e^{-\nu/\nu} \chi_{(0, +\infty)} \right]. \]

For \( s < 1 \) this gives a pulse with a very spiky core and broad wings, while for \( s > 1 \) the resulting pulse has a square shape. The PSD cannot be analytically computed. In Fig. 4 we show the shape of the PSD for a set of values of \( s \), compared with the PSD of the single and double exponential pulses. For \( s < 1 \) the effect is to smooth out the breaks and extend the power-law branch \( \nu^{-2} \), while the effect of \( s > 1 \) is more complex. For \( 1 < s < 2 \) the break frequency is moved to larger values, while the slope of the power-law decay is increased. For \( s > 2 \) the PSD starts to deviate from the simple form, with the appearance of ‘absorptions’. In an ensemble average these small-scale features will be erased and we are left with the envelope, which again yields a very steep power-law slope.

### 5 PHOTON SCATTERING

It has been proposed that photon scattering may play a relevant role in shaping both the temporal (Panaitescu et al. 1999; Spada et al. 2000; Kobayashi et al. 2002; Ramirez-Ruiz & Lloyd-Ronning 2002) and spectral (Mészáros & Rees 2000; Mészáros et al. 2002) properties of GRB light curves. The effect of photon scattering has a distinctive signature in Fourier space, and the power spectrum of the light curves is then the best way in which the importance of scattering can be evaluated.

Consider a source of photons producing a flash (ideally a Dirac \( \delta \) function in time) in the centre of a cloud of ideal scattering particles.\(^3\) Even though the central source emitted a photon impulse, an observer located outside the cloud would detect a light pulse with a finite duration and a particular shape. This is caused by the fact that different photons make different paths, finally reaching the observer after being scattered many times in random directions. The shape of this pulse will be a function of the opacity of the cloud to photon scattering only, up to a scalefactor that takes into account the size of the cloud.

Consider now the central source itself producing a signal of finite duration with a given shape \( s(t) \). This can be approximated as an infinite series of spikes, each of them being detected by the outside observer as a pulse of shape \( s(t) \). In mathematical terms, the observer

\[^2\text{It has been found that the duration distribution of ‘well-separated’ pulses is well described by a lognormal function (McBreen et al. 1994; Li & Fenimore 1996). We prefer to adopt a power-law distribution since it fits our average power spectra better and since our PSDs do include non-well-separated pulses.}\]

\[^3\text{In this context ideal means that the photons are never absorbed and that the scattering properties of the cloud do not depend on the photon energy.}\]
will detect a signal $S(t)$ given by the convolution of the original function $s(t)$ times the transfer function $k(t)$:

$$S(t) = s(t) * k(t) \equiv \int_{-\infty}^{\infty} s(t - t') k(t') \, dt'.$$

(13)

Thanks to the properties of Fourier transforms, the PSD of the detected signal $S$ is the product of the spectra of $s$ and $k$.

In the case of photon scattering by free electrons, we consider a cloud of radius $R$ and uniform density $n$, with opacity $\tau = R n \sigma_T$. We neglect the angular dependence of the Thompson cross-section. In Fig. 5, we show the transfer functions obtained by Monte Carlo calculations of scattering for a set of optical depths. For $\tau \lesssim 1$ the transfer functions are Dirac $\delta$ functions with a small tail at large times, but for $\tau > 1$ the vast majority of photons undergo at least one scattering and the transfer function becomes much smoother. For $\tau > 10$ the opacity itself is no longer a parameter but just a scalefactor:

$$k_{\tau > 10,R/c}(t) = \frac{c}{R \tau} k_{10,1} \left( \frac{ct}{R \tau} \right).$$

(14)

This behaviour is reflected by the shape of the transfer functions in Fourier space. In Fig. 6 we show the PSD of the same transfer functions shown in Fig. 5. For small opacity, as expected, the PSD of the transfer function is almost a unit constant, so that $S(\omega) \approx \delta(\omega)$. As the opacity increases a pronounced break appears at a frequency $v_b = \frac{c}{\pi R \tau}$.

(15)

For intermediate opacities, the PSD can be roughly described as a power law for $\nu > v_b$, while for large opacity the break takes the form of an exponential cut-off. In conclusion, the effect of photon scattering is to create a clear break in the power spectrum, suppressing all the frequencies larger than $v_b$. In Fig. 7 we show the result of a set of simulations that include scattering at various degrees. The left-hand column shows simulated light curves with a single exponential shot noise model (equation 7) with 20 pulses. The pulse fluence has a lognormal distribution, while the pulse duration $\tau_p$ is distributed according to equation (8) with $\alpha = -\frac{1}{2}$, $\sigma_\alpha = 10$ and $\sigma_M = 250$ s. The top light curve has no scattering, while the second one has $\tau = 3$ and the third one has $\tau = 10$. In all cases $R/c = 1$. The last curve has been made by smoothing with $\tau = 10$ only the seven shortest pulses. This may simulate more closely the effect of scattering in the external shock scenario, where shorter pulses are produced closer to the inner engine, where the opacity of the flow is larger (see, e.g., Spada et al. 2001 and references therein). The second column shows the PSD of the light curves shown in the first column, while the third column shows the average PSD of 100 curves generated according to the same pulse process. The difference between the scattered and unscattered PSDs is shown by overlaying the unscattered PSD with a light grey curve.

As mentioned above, the effect of photon scattering in defining the duration of GRB pulses has been considered in a number of works (Panaitescu et al. 1999; Spada et al. 2000; Kobayashi et al. 2002; Ramirez-Ruiz & Lloyd-Ronning 2002). It is worth mentioning, however, that the way in which this was done is not completely accurate, especially when the power spectrum is concerned. In fact, in numerical works, the duration of a pulse is computed as the sum in quadrature of the intrinsic duration (owing to the shell width and the angular spreading times) plus the scattering duration. This total duration is then used as a scale parameter for the pulse duration (the same as $\sigma$ in equation 4), assuming a fixed functional shape for the pulse. In contrast, when the diffusion time-scale is comparable to or larger than the intrinsic time-scale, the shape of the pulse should be changed, smoothing out its sharp features but leaving the long time-scale features unaffected. When simulated light curves are observed in Fourier space, in both cases one finds, as

\[ \frac{\nu}{R \tau / c} \]
expected, that the average frequency has decreased. However, the real scattering decreases the mean frequency by suppressing the high frequencies, while the above approximation rescales the whole spectrum at smaller frequencies. The result is that, while real scattering produces a prominent break in the spectrum, this approach leaves the spectral shape unaffected.

6 DISCUSSION

How does shot-noise theory compare with the average spectra derived in Section 3? The observed spectra show a clear break, which we show is strongly correlated with the variability parameter. The power-law slope changes from $-\frac{2}{3}$ before the break to $-2$ after the break.

Let us for the moment neglect photon scattering. The fact that the largest slope is $-2$ suggests that an exponential shot noise model can easily reproduce the observations, with a typical pulse decay time longer than the rise time (Norris et al. 1996). The fact that the low-frequency slope is not flat, can be easily accommodated by invoking a power-law distribution of pulse durations (cf. equation 8)

$$n(t_\delta) \propto t_\delta^{-1/3}.$$  \hspace{1cm} (16)

The correlation between the break frequency and the variability (or the GRB luminosity) can therefore be interpreted as a correlation between the typical shortest relevant pulse duration and the luminosity. This does not mean that the light curve cannot have shorter pulses. In fact, if the distribution of pulse durations has a break, shorter pulses can be present but not contribute to the PSD.

Let us now consider the effect of photon scattering. In a very simple scenario, all the pulses may go through a scattering screen with opacity $\tau$ and radius $R$. In this case, a clear break should be present in the PSD at a frequency $v = c/(\pi R \tau)$. It is straightforward to show that the observed break cannot be caused by scattering but must, instead, be attributed to the intrinsic properties of the unscattered pulses. In fact, the change of slope before and after the break is only of unity, while scattering would require either a jump of 2 in slope (intermediate opacity) or an exponential cut-off ($\tau > 10$). Alternatively, one can consider a scenario in which the opacity and size of the screen are different for different pulses. In order to be consistent with the lack of a pronounced break in the observations, this requires the scattering time-scale $\tau R/c$ to be smaller than $t_\delta$. In this case, the PSD of the single pulse would only be affected in the power-law tail and its effect would not be observed in the average PSD. In order to explain the break in the average PSD, however, one should consider a broken power law of pulse durations. With such conditions one can explain the observed average power spectrum. However, the role of scattering is not relevant in the shape of the PSD, and so it cannot be important in the measure of the variability parameter $V$. A case may also be envisaged in which $\tau R/c \gg t_\delta$ for all pulses, with a broken power-law distribution of the values of $\tau R$ suited to mimic the average PSD shape. In this case, however, the light curve would be entirely dominated by scattering. The pulses should have the shape of the transfer functions $k(t)$, in contradiction with what is found in their direct analysis (Norris et al. 1996).

A more detailed model of an internal shock process should, however, take into account the fact that different pulses may not be scattered with the same value of $\tau R/c$, since the shortest pulses are likely to be produced closer to the engine, where the relativistic wind is denser and more opaque. To mimic such a case, we consider a light curve in which only the shortest pulses underwent scattering.

In this case, the PSD should show a steepening break followed by a flattening break when the unscattered pulse component becomes dominant (see the lowest right-hand panel of Fig. 7). Again, this is not observed in real data (see Fig. 2).

It may be argued that these models are too simplistic, since in real simulations each pulse is smeared with its own value of the parameter $\tau R/c$. What emerges from our analysis is that no clear signature of scattering is present in the PSD data. It may still be possible that a more detailed numerical simulation for the evolution of the flow can include scattering in such a way that its effect is relevant but does not produce a cut-off in the PSD. However, this requires a fine tuning of the distribution of the parameter $\tau R/c$ and that the shape of at least part of the pulses be dominated by the transfer functions of Fig. 5. Such simulations, with a proper treatment of the scattering, are called for if the proposed link between scattering and variability has to be believed.

Even though scattering processes are shown to be not important in determining the variability parameter $V$, it is still possible that they have some relevance in shaping the spectra of GRBs. In fact, if the scattering screen is small and thick, the cut-off frequency can be large, in a range in which the PSD is dominated by the white noise. This kind of scattering would not influence the temporal pulse profile, but the photon energy may be shifted by

$$\Delta \epsilon \simeq \frac{\tau^2 \epsilon}{m_e c^2} \left(4kT - \epsilon\right),$$  \hspace{1cm} (17)

where $T$ is the temperature of the scattering electrons. This mechanism for modifying the typical energy of the photons in GRB spectra has been proposed and discussed, for example, in Ramirez-Ruiz & Lloyd-Ronning (2002) and Mészáros et al. (2002). In particular, assuming that the comoving electron temperature is small and asking that the photon energy be significantly modified ($\Delta \epsilon \sim \epsilon$), one can show that the opacity must be

$$\tau \sim \sqrt{\frac{m_e c^2}{\epsilon}} \approx \sqrt{T},$$  \hspace{1cm} (18)

where $\Gamma$ is the bulk Lorentz factor of the flow and the rightmost term holds if the initial observed energy of the photon was close to $m_e c^2$.

Before discussing this issue in detail, we consider that all the computations described above are relevant if the scattering screen is comoving with the relativistic flow. In fact, in order to preserve the burst variability, a scattering screen at rest in the frame of the host galaxy needs to have an optical depth much smaller than unity. For this reason, the screens that we consider here are comoving with the flow – they can be the same shell in which the radiation is produced, or the total contribution of previously ejected shells.

In order for the scattering to be important in shaping the spectrum of GRBs, one have to assume a large opacity, so that the cut-off in the power spectrum should be exponential. Consider the bursts with largest variability (lower left-hand panel of Fig. 2). The spectrum is well described by a power law up to observed comoving frequencies $\nu \sim 10^6 \Gamma c$. We can then constrain the comoving size of the scattering screen to be

$$R' < 10^9 \Gamma^{1/2}.$$  \hspace{1cm} (19)

Within the framework of the internal shock, the comoving width of the shell is given by $\Delta = r_0 / \Gamma \sim r_0 \Gamma$, where $r_0$ is the size of the shell at the moment of ejection. The constraint (19) then implies

$$r_0 \lesssim 10^8 \Gamma^{-1/2},$$  \hspace{1cm} (20)

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which is consistent with the internal shock picture only if the inner engine produces many shells, of the order of 1000 shells in a burst lasting for 10 s. Without being confined in the standard internal shock model, one can envisage a scenario in which the energy is liberated in the fireball at radii \( r < r_{0}/\Gamma^{2} \). In this case, it must be considered that the photons will be advected by the flow, which will become optically thin to radiation (owing to the shell expansion) in a time-scale smaller than the diffusion time-scale of the photons. In this case, the break frequency would be expected to be at \( \nu \simeq c\Gamma^{2}/(2\pi R) \), which is always above the measured 10 Hz.

7 CONCLUSIONS

We have computed the average power spectrum density for a set of GRB light curves, divided into six bins according to their variability properties and we have developed a shot noise model to be compared with them. Photon scattering is self-consistently included in the model. We find the following.

(i) The variability parameter \( V \) as computed by Fenimore & Ramirez-Ruiz (2000) strongly correlates with the dominant frequency of the spectra, being defined as the maximum of the
(v) The PSDs can be easily interpreted as being caused by the superposition of random similar shots (with a distribution of decay times and fluences) with double exponential shape (possibly stretched, see Norris et al. 1996). In this case the position of the break frequency (and therefore the variability parameter $V$) is related to the shortest relevant pulses in the duration distribution of pulses.

(iv) We show that photon scattering should imprint a detectable break or cut-off in the PSD. The lack of such a signature makes untenable all models in which the luminosity variability correlation is ascribed to the smoothing of the shortest time-scales in low-luminosity GRBs.

(v) Under certain circumstances (a compact engine or deviations from the standard internal shock picture) it is possible to find models in which photon scattering plays a relevant role in shaping the spectra of GRBs consistent with the measured PSD.

To conclude, it must be emphasized that the results we have presented do depend at some level on the existence of a tight correlation between the burst variability and luminosity. In particular, all the spectral analysis is made on the rest frame frequency, which is calculated thanks to the redshift guessed from the above-mentioned correlation. However, the results are still valid and relevant even if such a correlation should prove not to be real. In that case, the variability would not be related to the luminosity, but still we should conclude that scattering is not relevant in shaping the light curves of GRBs. To test this, we performed the same PSD analysis on the observed frequencies. The average spectral shapes do remain the same, even if the values of the brake frequencies are different and, again, there is no sign of a cut-off which may indicate that scattering is relevant in determining the degree of variability of the light curves.

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