Bogoliubov compensation principle in the electro-weak interaction: value of the gauge constant, muon g-2 anomaly, predictions for Tevatron and LHC

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Bogoliubov compensation principle in the EW interaction

We apply Bogoliubov compensation principle to the gauge electro-weak interaction to demonstrate a spontaneous generation of anomalous three-boson gauge invariant effective interaction. The non-trivial solution of compensation equations uniquely defines values of parameters of the theory and the form-factor of the anomalous interaction. The contribution of this interaction to running EW coupling $\alpha_{ew}(p^2)$ gives its observable value $\alpha_{ew}(M_W^2) = 0.0374$ in satisfactory agreement to the experiment. The anomalous three-boson interaction gives natural explanation of the well-known discrepancy in muon $g-2$. The implications for EW studies at Tevatron and LHC are briefly discussed.

1 Introduction

In previous works [1, 2, 3, 4, 5] N.N. Bogoliubov compensation principle [6, 7, 8] was applied to studies of spontaneous generation of effective non-local interactions in renormalizable gauge theories. Spontaneous generation of Nambu – Jona-Lasinio like interaction was studied in works [2, 3, 5] and the description of low-energy hadron physics in terms of initial QCD parameters turns to be quite successful including values of parameters: $m_\pi, f_\pi, m_\sigma, \Gamma_\sigma, <\bar{q}q>, M_\rho, \Gamma_\rho, M_{a_1}, \Gamma_{a_1}$.

In work [4] the approach was applied for calculation of infrared behaviour of the QCD running coupling constant. In particular, in QCD a possibility of spontaneous generation
of anomalous three-gluon interaction of the form

\[- \frac{G}{3!} \cdot f_{abc} F^a_{\mu\nu} F^b_{\nu\rho} F^c_{\rho\mu}; \]  

was shown.

The main principle of the approach is to check if an effective interaction could be generated in a chosen variant of a renormalizable theory. In view of this one performs "add and subtract" procedure for the effective interaction with a form-factor. Then one assumes the presence of the effective interaction in the interaction Lagrangian and the same term with the opposite sign is assigned to the newly defined free Lagrangian. This transformation of the initial Lagrangian is evidently identical. However such free Lagrangian contains completely improper term, corresponding to the effective interaction of the opposite sign. Then one has to formulate a compensation equation, which guarantees that this new free Lagrangian is a genuine free one, that is effects of the uncommon term sum up to zero. Provided a non-trivial solution of this equation exists, one can state the generation of the effective interaction to be possible. Now we apply this procedure to our problem.

In the present work we consider a possibility of generation of interaction analogous to (1) in the electro-weak theory.

### 2 Compensation equation for anomalous three-boson interaction

We start with EW Lagrangian with 3 lepton and colour quark doublings with gauge group \(SU(2)\). That is we restrict the gauge sector to triplet of \(W^a_\mu\) only. Thus we consider \(U(1)\) abelian gauge field \(B\) to be decoupled, that means approximation \(\sin^2 \theta_W \ll 1\).

\[
L = \sum_{k=1}^{3} \left( \frac{i}{2} \left( \bar{\psi}_k \gamma_{\mu} \partial_\mu \psi_k - \partial_\mu \bar{\psi}_k \gamma_{\mu} \psi_k \right) - m_k \bar{\psi}_k \psi_k + \frac{g}{2} \bar{\psi}_k \gamma_{\mu} \tau^a W^a_{\mu} \psi_k \right) + \]

\[
+ \sum_{k=1}^{3} \left( \frac{i}{2} \left( \bar{q}_k \gamma_{\mu} \partial_\mu q_k - \partial_\mu \bar{q}_k \gamma_{\mu} q_k \right) - M_k \bar{q}_k q_k + \frac{g}{2} \bar{q}_k \gamma_{\mu} \tau^a W^a_{\mu} q_k \right) -
\]

\[- \frac{1}{4} \left( W^a_{\mu\nu} W^a_{\mu\nu} \right); \quad W^a_{\mu\nu} = \partial_\mu W^a_{\nu} - \partial_\nu W^a_{\mu} + g \epsilon_{abc} W^b_{\mu} W^c_{\nu}. \]

where we use the standard notations and \(\psi_k\) and \(q_k\) correspond to leptons and quarks respectfully. In accordance to the Bogoliubov approach [6, 7, 8] in application to QFT [1] we look for a non-trivial solution of a compensation equation, which is formulated on the basis of the Bogoliubov procedure add – subtract. Namely let us write down the initial
expression (2) in the following form

\[ L = L_0 + L_{int}; \]

\[ L_0 = \sum_{k=1}^{3} \left( \frac{1}{2} \left( \bar{\psi}_k \gamma_\mu \partial_\mu \psi_k - \partial_\mu \bar{\psi}_k \gamma_\mu \psi_k \right) - m_k \bar{\psi}_k \psi_k \right) + \frac{1}{2} \left( \bar{q}_k \gamma_\mu \partial_\mu q_k - \partial_\mu \bar{q}_k \gamma_\mu q_k \right) - \]

\[ - M_k \bar{q}_k q_k \right) - \frac{1}{4} W_{\mu\nu}^a W_{\mu\nu}^a + \frac{G}{3!} \cdot \epsilon_{abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c; \]

\[ L_{int} = \frac{g}{2} \sum_{k=1}^{3} \left( \bar{\psi}_k \gamma_\mu \tau^a \psi_k + \bar{q}_k \gamma_\mu \tau^a q_k \right) - \frac{G}{3!} \cdot \epsilon_{abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c. \]

Here isotopic summation is performed inside of each quark bi-linear combination, and notation \(- \frac{G}{3!} \cdot \epsilon_{abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c\) means corresponding non-local vertex in the momentum space

\[
(2\pi)^4 G \epsilon_{abc} (g_\mu_\nu (p_\rho q_\nu - p_\nu q_\rho) + g_{\rho\nu} (k_\mu q_\rho - k_\rho q_\mu) + + q_\mu k_\nu p_\rho - k_\mu p_\rho q_\nu) F(p, q, k) \delta(p + q + k) + \ldots;
\]

where \(F(p, q, k)\) is a form-factor and \(p, \mu, a; q, \nu, b; k, \rho, c\) are respectfully incoming momenta, Lorentz indices and weak isotopic indices of \(W\)-bosons. We mean also that there are present four-boson, five-boson and six-boson vertices according to expression for \(W_{\mu\nu}^a\).

Effective interaction

\[- \frac{G}{3!} \cdot \epsilon_{abc} W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c\]

is usually called anomalous three-boson interaction and it is considered for long time on phenomenological grounds [9]. Note, that the first attempt to obtain the anomalous three-boson interaction in the framework of Bogoliubov approach was done in work [10].

Our interaction constant \(G\) is connected with conventional definitions in the following way

\[ G = \frac{g \lambda}{M_W^2}. \]

The current limitations for parameter \(\lambda\) read [11]

\[
\lambda = -0.016^{+0.021}_{-0.023}; \quad -0.059 < \lambda < 0.026 (95\% C.L.);
\]

\[
\lambda = -0.024^{+0.025}_{-0.021}; \quad -0.068 < \lambda < 0.023 (95\% C.L.);
\]

where the first line corresponds to one-parameter fit (\(\lambda\)) with zero anomalous electric quadrupole moment of \(W\)-boson \(\kappa\), while the second line corresponds to two-parameter fit, including also \(\kappa\). Due to our approximation \(\sin^2 \theta_W \ll 1\) we use the same \(M_W\) for both charged \(W^\pm\) and neutral \(W^0\) bosons and assume no difference in anomalous interaction for \(Z\) and \(\gamma\), i.e. \(\lambda_Z = \lambda_\gamma = \lambda\).
Let us consider expression (3) as the new free Lagrangian \( L_0 \), whereas expression (4) as the new interaction Lagrangian \( L_{\text{int}} \). It is important to note, that we put into the new free Lagrangian the full quadratic in \( W \) term including self-boson interaction, because we prefer to maintain gauge invariance of the approximation being used. Indeed, we shall use both quartic term from the last term in (3) and triple one from the last but one term of (3). Then compensation conditions (see for details [1]) will consist in demand of full connected three-boson vertices of the structure (5), following from Lagrangian \( L_0 \), to be zero. This demand gives a non-linear equation for form-factor \( F \).

Such equations according to terminology of works [6, 7, 8] are called compensation equations. In a study of these equations it is always evident the existence of a perturbative trivial solution (in our case \( G = 0 \)), but, in general, a non-perturbative non-trivial solution may also exist. Just the quest of a non-trivial solution inspires the main interest in such problems. One can not succeed in finding an exact non-trivial solution in a realistic theory, therefore the goal of a study is a quest of an adequate approach, the first non-perturbative approximation of which describes the main features of the problem. Improvement of a precision of results is to be achieved by corrections to the initial first approximation.

Thus our task is to formulate the first approximation. Here the experience acquired in the course of performing works [1, 2, 3, 4] could be helpful. Now in view of obtaining the first approximation we would make the following assumptions.

1) In compensation equation we restrict ourselves by terms with loop numbers 0, 1.

2) We reduce thus obtained non-linear compensation equation to a linear integral equation. It means that in loop terms only one vertex contains the form-factor, being defined above, while other vertices are considered to be point-like. In diagram form equation for form-factor \( F \) is presented in Fig. 1. Here four-leg vertex correspond to interaction of four bosons due to our effective three-field interaction. In our approximation we take here point-like vertex with interaction constant proportional to \( gG \).

3) We integrate by angular variables of the 4-dimensional Euclidean space. The necessary rules are presented in paper [2].

At first let us present the expression for four-boson vertex

\[
V(p, m, \lambda; q, n, \sigma; k, r, \tau; l, s, \pi) = gG \left( \epsilon^{amn} \epsilon^{ars} (U(k, l; \sigma, \tau, \pi, \lambda) - U(k, l; \lambda, \tau, \pi, \sigma) - U(l, k; \sigma, \pi, \lambda) + U(l, k; \lambda, \pi, \tau, \sigma)) + \epsilon^{arn} \epsilon^{ams} (U(p, l; \sigma, \lambda, \pi, \tau) - U(p, l; \lambda, \sigma, \pi, \tau) + U(q, p; \pi, \sigma, \lambda, \tau) - U(q, p; \tau, \sigma, \lambda, \pi) + U(q, k; \pi, \sigma, \tau, \lambda) - U(q, k; \lambda, \sigma, \tau, \pi) + U(k, q; \pi, \tau, \sigma, \lambda) - U(k, q; \lambda, \tau, \sigma, \pi) + U(k, q; \lambda, \sigma, \tau, \pi) \right) +
\]

\[
\epsilon^{asn} \epsilon^{amr} \left( U(k, p; \sigma, \lambda, \tau, \pi) - U(p, k; \sigma, \tau, \lambda, \pi) + U(p, k; \pi, \tau, \lambda, \sigma) - U(k, p; \pi, \lambda, \tau, \sigma) - U(l, q; \lambda, \pi, \sigma, \tau) + U(l, q; \tau, \pi, \sigma, \lambda) - U(q, l; \tau, \sigma, \pi, \lambda) +
\]

\[
\right)
\]
Here triad \( p, m, \lambda \) etc means correspondingly incoming momentum, isotopic index, Lorentz index of a boson.

Let us formulate compensation equations in this approximation. For free Lagrangian \( L_0 \) full connected three-boson vertices with Lorentz structure (5) are to vanish. One can succeed in obtaining analytic solutions for the following set of momentum variables (see Fig. 1): left-hand legs have momenta \( p \) and \(-p\), and a right-hand leg has zero momentum. However in our approximation we need form-factor \( F \) also for non-zero values of this momentum. We look for a solution with the following simple dependence on all three variables

\[
F(p_1, p_2, p_3) = F\left(\frac{p_1^2 + p_2^2 + p_3^2}{2}\right);
\]

Really, expression (10) is symmetric and it turns to \( F(x) \) for \( p_3 = 0, p_1^2 = p_2^2 = x \). We consider the representation (10) to be the first approximation and we plan to take into account the corresponding correction in forthcoming studies.

Now according to the rules being stated above we obtain the following equation for form-factor \( F(x) \)

\[
F(x) = -\frac{G^2 N}{64 \pi^2} \left( \int_0^Y F(y) y dy - \frac{1}{12} \int_0^x F(y) y^3 dy + \frac{1}{6x} \int_0^x F(y) y^2 dy + \frac{x}{6} \int_x^Y F(y) dy \right)
+ \frac{G g N}{16 \pi^2} \left( \int_{3x/4}^x \frac{(3x - 4y)^2(3x - 8y)}{x^2(x - 2y)} F(y) dy + \int_x^Y \frac{(5x - 6y)}{(x - 2y)} F(y) dy \right)
+ \frac{G g N}{32 \pi^2} \left( \int_{3x/4}^x \frac{3(4y - 3x)^2(x - 4xy + 2y^2)}{8x^2(2y - x)^2} F(y) dy + \int_x^Y \frac{3(x^2 - 2y^2)}{8(2y - x)^2} F(y) dy \right)
+ \int_0^x \frac{5y^2 - 12xy}{16x^2} F(y) dy + \int_x^Y \frac{3x^2 - 4xy - 6y^2}{16y^2} F(y) dy \right).
\]

Here \( x = p^2 \) and \( y = q^2 \), where \( q \) is an integration momentum, \( N = 2 \). The last four terms in brackets represent diagrams with one usual gauge vertex (see three last diagrams at Fig. 1). We introduce here an effective cut-off \( Y \), which bounds a “low-momentum” region where our non-perturbative effects act and consider the equation at interval \([0, Y]\) under condition

\[
F(Y) = 0.
\]

We shall solve equation (11) by iterations. That is we expand its terms being proportional
to \( g \) in powers of \( x \) and take at first only constant term. Thus we have

\[
F_0(x) = -\frac{G^2 N}{64 \pi^2} \left( \int_0^X F_0(y) \, dy - \frac{1}{12} \int_0^x F_0(y) \, dy + \frac{1}{6} \int_0^x F_0(y) \, dy \right) + \frac{x}{6} \int_0^X F_0(y) \, dy - \frac{x^2}{12} \int_0^X \frac{F_0(y)}{y} \, dy + \frac{87 G g N}{512 \pi^2} \int_0^X F_0(y) \, dy. \tag{13}
\]

Expression (13) provides an equation of the type which were studied in papers \(^1[1] 2, 3, 4\), where the way of obtaining solutions of equations analogous to (13) are described. Indeed, by successive differentiation of Eq. (13) we come to Meijer differential equation \(^2[2]\)

\[
\begin{align*}
\left( x \frac{d}{dx} + 2 \right)\left( x \frac{d}{dx} + 1 \right)\left( x \frac{d}{dx} - 1 \right)\left( x \frac{d}{dx} - 2 \right) F(x) + \frac{G^2 N x^2}{64 \pi^2} F(x) &= 0,\tag{14}
\end{align*}
\]

which solution looks like

\[
F_0(z) = C_1 G_{04}^{10} \left( z |^{1/2, 1, -1/2, -1} \right) + C_2 G_{04}^{10} \left( z |^{1, 1/2, -1/2, -1} \right) - \frac{G N}{128 \pi^2} G_{15}^{31} \left( z |^{0, 0, 0, -1/2, -1} \right) \int_0^X \left( G y - \frac{87 g N}{8} \right) F_0(y) \, dy;
\]

\[
G_{15}^{31} \left( z |^{0, 0, 0, -1/2, -1} \right) = \frac{1}{2 z} - G_{04}^{30} \left( z |^{1, 1/2, -1, -1/2, -1} \right); \quad z = \frac{G^2 N x^2}{1024 \pi^2};
\]

where

\[
G_{q \mu}^{nm} \left( z |^{a_1, \ldots, a_q}_{b_1, \ldots, b_p} \right);
\]

is a Meijer function \(^2[2]\). In case \( q = 0 \) we write only indices \( b_i \) in one line. Constants \( C_1, C_2 \) are defined by the following boundary conditions

\[
\begin{align*}
\left[ 2 z^2 \frac{d^3 F_0(z)}{dz^3} + 9 z \frac{d^2 F_0(z)}{dz^2} + \frac{d F_0(z)}{dz} \right]_{z = z_0} = 0; \\
\left[ 2 z^2 \frac{d^2 F_0(z)}{dz^2} + 5 z \frac{d F_0(z)}{dz} + F_0(z) \right]_{z = z_0} = 0; \quad z_0 = \frac{G^2 N Y^2}{1024 \pi^2}. \tag{16}
\end{align*}
\]

Conditions (12) (16) defines set of parameters

\[
z_0 = \infty; \quad C_1 = 0; \quad C_2 = 0. \tag{17}
\]

The normalization condition for form-factor \( F(0) = 1 \) here is the following

\[
-\frac{G^2 N}{64 \pi^2} \int_0^\infty F_0(y) \, dy + \frac{87 G g N}{512 \pi^2} \int_0^\infty F_0(y) \, dy = 1. \tag{18}
\]
However the first integral in (18) diverges due to asymptotics

\[ G_{15}^{31}\left( z_{1,1/2,0,-1/2,-1}^{0} \right) \rightarrow \frac{1}{2z}, \quad z \rightarrow \infty; \]

and we have no consistent solution. In view of this we consider the next approximation. We substitute solution (15) with account of (18) into terms of Eq. (11) being proportional to gauge constant \( q \) but the constant ones and calculate terms proportional to \( \sqrt{z} \). Now we have bearing in mind the normalization condition

\[
F(z) = 1 + \frac{85 \sqrt{N} \sqrt{z}}{96 \pi} \left( \ln z + 4 \gamma + 4 \ln 2 + \frac{1}{2} G_{15}^{31}\left( z_{1,0,0,1/2,-1/2,-1}^{0} \right) - \frac{3160}{357} \right) + \frac{2}{3} F(t) t dt + \frac{4}{3 \sqrt{\pi}} \int_{0}^{z} F(t) \sqrt{t} dt - \frac{4 \sqrt{z}}{3} \int_{z}^{z_0} F(t) \frac{dt}{\sqrt{t}} + \frac{2z}{3} \int_{z}^{z_0} F(t) \frac{dt}{t};
\]

(19)

where \( \gamma \) is the Euler constant. We look for solution of (19) in the form

\[
F(z) \approx \frac{1}{2} G_{15}^{31}\left( z_{1,1/2,0,-1/2,-1}^{1/2} \right) - \frac{85 \sqrt{N}}{312 \pi} G_{15}^{31}\left( z_{1,1/2,0,-1/2,-1}^{1/2} \right) + C_1 G_{04}^{10}\left( z_{1/2,1,-1/2,-1}^{1/2} \right) + C_2 G_{04}^{10}\left( z_{1/2,1,-1/2,-1}^{1/2} \right).
\]

(20)

We have also conditions

\[
1 + 8 \int_{0}^{z_0} F(z) dz = \frac{87 \sqrt{N}}{32 \pi} \int_{0}^{z_0} F_0(z) \frac{dz}{\sqrt{z}}; \quad (21)
\]

\[
F(z_0) = 0; \quad (22)
\]

and boundary conditions analogous to (16). The last condition (22) means smooth transition from the non-trivial solution to trivial one \( G = 0 \). Knowing form (21) of a solution we calculate both sides of relation (19) in two different points in interval \( 0 < z < z_0 \) and having four equations for four parameters solve the set. With \( N = 2 \) we obtain the following solution, which we use to describe the electro-weak case

\[
g(z_0) = -0.43014 ; \quad z_0 = 205.42535 ; \quad C_1 = 0.003687 ; \quad C_2 = 0.005821. \]

(23)

We would draw attention to the fixed value of parameter \( z_0 \). The solution exists only for this value (23) and it plays the role of eigenvalue. As a matter of fact from the beginning the existence of such eigenvalue is by no means evident.

Note that there is also solution with a smaller value of \( z_0 \) and large positive \( g(z_0) \), which with \( N = 3 \) presumably corresponds to strong interaction. This solution is similar to that considered in work [1] and it will be studied elsewhere.

We consider the neglected terms of equation (11) as perturbations to be taken into account in forthcoming studies.
3 Running EW coupling

We use Schwinger-Dyson equation for \(W\)-boson polarization operator to obtain a contribution of additional effective vertex to the running EW coupling constant \(\alpha_{ew}\). The corresponding diagram is presented at Fig. 2 a. Due to this vertex being gauge invariant, there is no contribution of ghost fields. So the contribution under discussion reads

\[
\Delta \Pi_{\mu\nu}(x) = \frac{g G N}{2 (2 \pi)^4} \int \frac{\Gamma^\alpha_{\rho\sigma}(p, -q + \frac{p}{2}, q - \frac{p}{2}) \Gamma^\text{eff}_{\rho\sigma}(-p, q + \frac{p}{2}, -q + \frac{p}{2}) F(q^2 + 3p^2) dq}{(q^2 + p^2/4)^2 - (pq)^2};
\]

where \(\Gamma^\alpha_{\rho\sigma}(p, q, k) = g_{\rho\sigma}(p_{\sigma} - q_{\sigma}) + g_{\rho\sigma}(q_{\mu} - k_{\mu}) + g_{\sigma\mu}(k_{\rho} - p_{\rho})\) and \(\Gamma^\text{eff}_{\rho\sigma}(p, q, k)\) is the Lorentz structure of effective vertex (5).

After angular integrations we have

\[
\Delta \Pi_{\mu\nu}(x) = (g_{\mu\nu} p^2 - p_{\mu} p_{\nu}) \Pi(x); \quad x = p^2; \quad y' = q^2 + \frac{3x}{4};
\]

\[
\Pi(x) = \frac{g G N}{32 \pi^2} \left( \frac{1}{x^2} \int_{3x/4}^{x} \frac{F(y')dy'}{y' - x/2} \left( 16 \frac{y'^3}{x^2} - 48 \frac{y'^2}{x} + 45y - \frac{27}{2} x \right) + \int_{x}^{Y} \frac{F(y')dy'}{y' - x/2} \left( -3y' + \frac{5}{2} x \right) \right).
\]

Here coupling constant \(g\) corresponds to \(g(Y)\). We calculate integrals in (25) with substitution of solution (15, 23) numerically.

So we have modified one-loop expression for \(\alpha_{ew}(p^2)\)

\[
\alpha_{ew}(x) = \frac{6 \pi \alpha_{ew}(x_0')}{6 \pi + 5 \alpha_{ew}(x_0') \ln(x/x_0') + 6 \pi \Pi(x)}; \quad x = p^2;
\]

where \(x_0'\) means a normalization point such that \(\Pi(x_0') = 0\). We normalize the running coupling by condition

\[
\alpha_{ew}(x_0) = \frac{g(Y)^2}{4 \pi};
\]

where Coupling constant \(g\) entering in expression (25) is just corresponding to this normalization point. However in expression (26) \(x_0'\) does not coincide with \(x_0\), because polarization operator (25) does not vanish at this point. It does vanish at \(x_0' = 4/3 x_0\). So we have to renormalize expression (26) and obtain

\[
\alpha_{ew}(x_0') = \alpha_{ew}(x_0) \frac{6\pi (1 + \Pi(x_0))}{6\pi + 5 \alpha_{ew}(x_0) \ln(4/3)};
\]

Using expressions (26, 27, 28) we calculate behaviour of \(\alpha_{ew}(x)\) down to values of \(x = p^2\) being by order of magnitude of \(M_W\).

On this stage we have to get an information on our \(G\). In the next section we define self-consistent value of this parameter so achieve unique definition of \(\alpha_{ew}(p^2)\).
4 Interaction with Higgs and muon g-2 anomaly

Let us consider a contribution of effective interaction in (4) to $g - 2$ anomaly. In the approach under consideration this problem is connected with new contributions to interaction of $W$-bosons and the Higgs particle. For the moment we do not consider the scheme for electro-weak symmetry breaking in our approach. For comparison with the actual physics we shall use the ready phenomenology according to which the $WWH$ interaction corresponds to the following vertex

$$ig_{\mu\nu} \delta_{ab} g_{M_W}.$$  (29)

From this moment we assume that $M_W$ is known. Additional contribution to vertex (29) is provided by our effective interaction due to diagram presented at Fig. 3. Substituting in the first approximation $F_0$ we obtain the following additional gauge invariant contribution

$$\iota 2 \sqrt{2} G g M_W \delta_{ab} (g_{\mu\nu} (q k) - q_{\nu} k_{\mu}) F_H(x);$$  (30)

where $F_H(x)$, $F_H(0) = 1$ is a form-factor, which one calculate from diagram Fig. 3. Calculation of an additional contribution to anomalous magnetic moment of muon due to vertex (30) needs knowledge of a form-factor of $WWH$ interaction. Let us formulate the equation for this form-factor, which is presented in diagrams at Fig. 4, where inhomogeneous part is just expression (30), where we take the first two terms of expansion of form-factor $F_H(x)$. Now the equation reads

$$\Phi(x) = \frac{2 G g M_W \sqrt{2}}{H} + \frac{g M_W G^2 x}{16 \pi^2 H} \left( \ln z + 4 \gamma + 4 \ln 2 - \frac{16}{3} + \right.$$

$$\left. + \frac{1}{2} G_{15}^2 (z_0^0, 0, 1/2) \right) + \beta \left( \frac{1}{2} \int_0^Y \Phi(y) dy + \frac{1}{12 x^2} \int_0^x y^2 \Phi(y) dy - \right. \left. \frac{1}{6 x} \int_0^x y \Phi(y) dy - \frac{x}{6} \int_x^Y \frac{\Phi(y)}{y} dy + \frac{x^2}{12} \int_x^Y \frac{\Phi(y)}{y^2} dy \right);$$  (31)

$$\beta = \frac{h^2}{16 \pi^2}; \quad h = \frac{4 \sqrt{2} G g M_W}{2 - \beta \int_0^Y \Phi(y) dy}; \quad \Phi(0) = 1.$$  

Here $h$ is a resulting constant of $WWH$ interaction and $\Phi(x)$ is the form-factor of the interaction with momentum $q \rightarrow 0$. Upper limit of integration $Y$ has to be the same as in (III), because our result in studying non-trivial solution of (III) demands $G = 0$ for $x > Y$. The same $G$ enters into expression (30) so that for $x > Y$ we also demand $\Phi(x)$ to vanish. Thus we have also condition

$$\Phi(Y) = 0.$$  (32)
This condition defines relation between variable $z$ of the previous section and dimensionless variable $u$, which is peculiar to Eq. (31)

$$u = \beta x = \frac{8\sqrt{2} g^2 M_W^2 G}{\pi \eta^2} \sqrt{z} = \xi \sqrt{z}; \quad (33)$$

$$\eta = 1 - \frac{1}{2} \int_{0}^{u_0} \Phi(u) du; \quad u_0 = \xi \sqrt{z_0}.$$  

Performing substitutions (33) into (31) we obtain the following equation

$$\Phi(u) = 1 + \frac{\eta u}{2 \pi \xi} \left(2 \ln u - 2 \ln \xi + 4 \gamma + 4 \ln 2 - \frac{16}{3} + \right.\left. + \frac{1}{2} G_{15}^{31}(z_0 | 0, 1/2, -1/2, -1) \right) + \frac{1}{12 u^2} \int_{0}^{u} t^2 \Phi(t) dt - \frac{1}{6} u \int_{0}^{u} t \Phi(t) dt - \frac{u}{6} \int_{u}^{u_0} \frac{\Phi(t)}{t} dt + \frac{u^2}{12} \int_{u}^{u_0} \frac{\Phi(t)}{t^2} dt; \quad (34)$$

We look for a solution of (34) with condition (32) in the following form

$$\Phi(u) = 2 G_{15}^{31}(u | 0, 1/2, -1, -1) - \frac{12 \eta}{\pi \xi} G_{15}^{31}(u | 1, 1/2, -1, -1) +$$

$$+ C_2^\phi \left(G_{04}^{20}(-u | 1, 2, -2, -1) - i \pi G_{04}^{10}(u | 2, -2, -1, 1) \right) +$$

$$+ C_1^\phi G_{04}^{10}(u | 2, -2, -1, 1); \quad \Phi(0) = 0. \quad (35)$$

Now we substitute (35) into (34) and obtain the following solution for parameters

$$C_1^\phi = 0.721216; \quad C_2^\phi = -4.027240; \quad \eta = -0.077332; \quad \xi = 0.2872314. \quad (36)$$

Relation (33) allows to define coupling constant $G$ of the three-boson effective interaction

$$G = \frac{\Lambda}{M_W^2}; \quad \Lambda = 0.010312. \quad (37)$$

Note, that $g$ in relation (33) is just $g(z_0)$ from solution (23). With this result we completely define expression for $\alpha_{ew}$ of the previous section. So substituting (37) into relations (26, 27, 28) we obtain

$$\alpha_{ew}(M_W^2) = 0.0374; \quad (38)$$

what is only 10% larger than well-known value

$$\alpha_{ew}(M_W^2) = \frac{\alpha(M_W)}{\sin^2 \theta_W} = 0.0337. \quad (39)$$
We consider this result as strong confirmation of the approach. As a matter of fact the accuracy of the present approach was estimated to be just $(10 - 15)\%$ [1, 3].

From relations (37, 38) bearing in mind negative sign of $g$ we have

$$\lambda = \frac{\Lambda}{g(M_W^2)} = -0.0151; \quad (40)$$

that evidently agrees with limitations (8).

Now we have new effective interaction of Higgs with $W$ with form-factor (35, 36). Due to diagrams of Fig. 5 this interaction contributes to muon magnetic moment giving the following additional term in $a = g - 2$

$$\Delta a = -\frac{\Lambda \sqrt{2}}{8 \pi \eta} \left(\frac{m_\mu}{M_W}\right)^2 \int_0^{u_0} \frac{\alpha_{ew}(u) \Phi(u) u \, du}{(u + u_w)(u + u_h)}; \quad (41)$$

$$u_w = \frac{h^2 M_W^2}{16 \pi^2}; \quad u_h = \frac{h^2 M_H^2}{16 \pi^2} = u_w \frac{M_H^2}{M_W^2};$$

where $M_H$ is yet unknown mass of the Higgs particle. Behaviour (26) of $\alpha_{ew}(Q)$ is presented at Fig. 6.

We know everything but $M_H$ in expression (41) and e.g. for mass of Higgs $M_H = 114 GeV$ we obtain

$$\Delta a = 3.34 \cdot 10^{-9}; \quad (42)$$

that comfortably fits into error bars for well-known deviation [13, 14, 15]

$$\Delta a = (3.02 \pm 0.88) \cdot 10^{-9}. \quad (43)$$

With $M_H$ growing $\Delta a$ (41) slowly decreases inside the error bars down to $2.67 \cdot 10^{-9}$ for $M_H = 300 GeV$. Thus we can state, that our result agrees with experiment (43) for values $150 GeV < M_H < 300 GeV$, which we shall discuss in the next section in connection with experimental implications.

Contribution (41) to electron $g - 2$ is four orders of magnitude smaller and so it is far below experimental accuracy $\pm 4 \cdot 10^{-12}$.

### 5 Experimental implications

New interaction of $H$ with $W$-s

$$L_{HW} = \frac{h}{2} W^a_{\mu \nu} W^a_{\mu \nu} H. \quad (44)$$

leads to changes in usual branching ratios for $H$ decays. We use here the well-known expression for $W^0$ mixed state with physical value for $\sin^2 \theta_W$. There are unusually significant channels $H \rightarrow \gamma \gamma$ and $H \rightarrow \gamma Z$. Therefore there are additional restrictions
from existing experiments. Recent data from Tevatron on search for Higgs particle in $\gamma \gamma$ channel \cite{16} exclude Higgs particle with interaction (44) for $M_H < 150 GeV$. Thus in the framework of our approach we consider only

$$M_H > 150 GeV.$$ \hfill (45)

We calculate cross-sections of Higgs production at Tevatron with $\sqrt{s} = 1960 GeV$ and branching ratios of its decays for three values of the mass: 175, 200, 225 GeV. Results are presented in Table 1. We see, that presumably it could be possible to study our predictions with the existing Tevatron facilities in channels $\gamma \gamma, \gamma Z, Z Z, W^+ W^-$. A choice of the most promising channel depends on efficiency of registration.

We present also in Table 2 predictions for forthcoming LHC search for Higgs particle. For the sake of further experimental implications we present behaviour of $\alpha_{ew}(Q)$ at Fig. 6, where also the usual perturbative dependence of $\alpha_{ew}(Q)$ is shown. The main difference of the solution from perturbative description consists in existence of triple gauge-boson interaction (6) with form-factor (20), (23). The dependence of this form-factor on $Q$ is presented at Fig. 7. From these pictures one sees that the difference of physical effects from the perturbative ones might occur for TeV range of energy.

First of all we calculate total cross-section of reaction $e^+ e^- \rightarrow W^+ W^-$ for our solution. The result is presented at Fig. 8 for c.m.r.f. total energy $0.4 TeV < Q < 9 TeV$. One sees, that the difference of the non-perturbative solution, which corresponds to the upper curve, from the perturbative one (lower curve) starts around $1.5 TeV$ and becomes maximal at $5 - 6 TeV$. Really for small $Q$ contribution of the three-boson interaction is small. Then the contribution increases with $Q$ increasing, but for much larger $Q$ the decrease of the form-factor (see Fig. 7) in line with decreasing $\alpha_{ew}(Q)$ lead to fast decrease of the effect.

Let us consider also reaction $e^+ e^- \rightarrow ZZ$. The behavior of the cross-section is presented at Fig. 9. Here the non-perturbative curve is below the perturbative one. The effect seems significant, but cross-section of the reaction in TeV region is small contrary to $e^+ e^- \rightarrow W^+ W^-$. Implication of our results for reactions at LHC

$$p + p \rightarrow W^+ + W^- + X; \quad p + p \rightarrow W^+ + Z + X; \quad p + p \rightarrow Z + Z + X; \quad (46)$$

needs special extensive study. One could expect here significant effects for high invariant masses of boson pairs.

For calculations of this section CompHEP package \cite{17} was used.

6 Conclusion

To conclude the author would emphasize, that albeit we discuss quite unusual effects, we do not deal with something beyond the Standard Model. We are just in the framework of
the Standard Model. What makes difference with usual results is non-trivial solution of compensation equation. There is of course also trivial perturbative solution. Which of the solutions is realized is to be defined by a stability condition. In view of this it would be desirable to estimate contribution of our solution to vacuum energy density. For example in our case we have non-zero non-perturbative boson condensate, which in diagram form is presented at Fig. 2 b. It is positive, that means negative contribution to vacuum energy density. So with account of only this fact we could state that non-trivial solution is stable and therefore it really exists. Of course real situation is much more difficult and there are other contributions to the vacuum energy density, which have to be taken into account. First of all the contribution of symmetry breaking mechanism, e.g. of the Higgs one, is without doubt quite important. For the moment we would say, that the problem of stability will be considered in forthcoming studies in more details.

With the present results we would draw attention to two important achievements provided by the non-trivial non-perturbative solution. The first one is unique determination of gauge electro-weak coupling constant (38) in close agreement with experimental value (39). At this point we would emphasize, that the existence of a non-trivial solution itself always leads to additional conditions for parameters of a problem under study. So the result on the coupling constant is by no means surprising. The second important point consists in agreement of our calculation for additional contribution to muon anomalous magnetic moment (42) with experimental number (43). So this effect does not need a hypothetical exit beyond the Standard Model. These two achievements strengthen the confidence in the correctness of applicability of Bogoliubov compensation approach to the principal problems of elementary particles theory.

Recent Tevatron data impose on mass of Higgs particle restriction $M_H > 150\, GeV$. It seems, that Tevatron facilities allow to check predictions of the work for $M_H$ starting from this value up to value $\approx 250\, GeV$.

We have also pointed to the possibility of verification of the present results in forthcoming experiments at LHC. The most promising processes are again Higgs production and presumably pair production of weak bosons $W^+W^-$ and $W^+Z$ with high invariant masses.

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**References**

[1] B.A. Arbuzov, Theor. Math. Phys., 140, 1205 (2004).

[2] B.A. Arbuzov, Phys. Atom. Nucl., 69, 1588 (2006).
[3] B.A. Arbuzov, M.K. Volkov and I.V. Zaitsev, Int. Journ. Mod. Phys. A, 21, 5721 (2006).

[4] B.A. Arbuzov, Phys. Lett. B, 656, 67 (2007).

[5] B.A. Arbuzov, M.K. Volkov and I.V. Zaitsev, Int. Journ. Mod. Phys. A (in press); arXiv:0809.4952 [hep-ph] (2008).

[6] N.N. Bogoliubov, Soviet Phys.-Uspekhi, 67, 236 (1959).

[7] N.N. Bogoliubov, Physica Suppl., 26, 1 (1960).

[8] N.N. Bogoliubov, Quasi-averages in problems of statistical mechanics. Preprint JINR D-781, (JINR, Dubna 1961).

[9] K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B, 282, 253 (1987).

[10] B.A. Arbuzov, Phys. Lett. B, 288, 179 (1992).

[11] LEP Electro-weak Working Group, arXiv:hep-ex/0612034v2 (2006).

[12] H. Bateman and A. Erdélyi, Higher transcendental functions. V. 1 (New York, Toronto, London: McGraw-Hill, 1953).

[13] G.W. Bennett et al., Phys. Rev. D 73, 072003 (2006).

[14] F. Jegerlehner, Acta Phys. Polon. B 38, 3021 (2007).

[15] M. Passera, W.J. Marciano and A. Sirlin, arXiv:0809.4062 [hep-ph] (2008).

[16] V.M. Abazov et al. (D0 Collaboration), arXiv:0901.1887 [hep-ex] (2009).

[17] E.E. Boos et al. (CompHEP Collaboration), Nucl. Instr. Meth. A 534, 250 (2004).
Table captions

Table 1. Predictions for Tevatron search for Higgs particle, $\sqrt{s} = 1.96 \, TeV$.

Table 2. Predictions for LHC search for Higgs particle, $\sqrt{s} = 14 \, TeV$. 
Figure captions

Fig. 1. Diagram representation of the compensation equation. Black spot corresponds to anomalous three-boson vertex with a form-factor. Empty circles correspond to point-like anomalous three-boson and four-boson vertices. Simple point corresponds to usual gauge vertex. Incoming momenta are denoted by the corresponding external lines.

Fig. 2. Loop contribution to boson polarization operator. Simple point corresponds to the perturbative vertex (a). Diagram corresponding to calculation of boson condensate; cross on the inside line describes "vertex" $W^a_{\mu\nu} W^a_{\mu\nu}$ (b).

Fig. 3. Diagram for the first step in calculation of $HWW$ vertex. Double line represents Higgs particle. Simple point represent usual $HWW$ vertex.

Fig. 4. Diagram representation of the equation for $HWW$ form-factor. Double circle with black internal one corresponds to anomalous $HWW$ vertex with a form-factor. The same with empty internal circle correspond to point-like anomalous $HWW$ vertex. Single circle corresponds to $HWW$ vertex calculated according diagram at Fig. 3.

Fig. 5. Diagrams for new contribution to muon magnetic moment. Dotted line represents muon (or other spin one-half particle), the line going up from the full vertex describes a photon.

Fig. 6. Behaviour of modified $\alpha_{ew}(Q)$ for $0 < Q < 40 TeV$ with $\alpha_{ew}(M_W) = 0.0374$. The upper line corresponds to usual electro-weak coupling with the same normalization.

Fig. 7. Behaviour of form-factor $F(Q)$, $0 < Q < 35.74 TeV$. For $Q > 35.74 TeV$ $F(Q) = 0$.

Fig. 8. Total cross-section $\sigma(Q)$ of reaction $e^+ e^- \rightarrow W^+ W^-$ for $0.6 TeV < Q < 9 TeV$. The lower line corresponds to usual electro-weak coupling.

Fig. 9. Total cross-section $\sigma(Q)$ of reaction $e^+ e^- \rightarrow ZZ$ for $0.6 TeV < Q < 7 TeV$. The upper line corresponds to usual electro-weak coupling.
Table 1.

|             | $M_H$ GeV | $175$ | $200$ | $225$ |
|-------------|-----------|-------|-------|-------|
| $\Gamma_H$ GeV |           | $0.287$ | $0.789$ | $1.714$ |
| $\sigma_t(p\bar{p} \rightarrow H + X) pb$ |           | $0.885$ | $0.586$ | $0.403$ |
| $BR(H \rightarrow \gamma \gamma)$ % |           | $7.41$ | $4.02$ | $2.63$ |
| $BR(H \rightarrow \gamma Z)$ % |           | $25.60$ | $17.84$ | $13.73$ |
| $BR(H \rightarrow ZZ)$ % |           | $0$ | $12.65$ | $17.12$ |
| $BR(H \rightarrow W^+ W^-)$ % |           | $64.38$ | $64.40$ | $65.95$ |
| $BR(H \rightarrow bb)$ % |           | $2.61$ | $1.09$ | $0.56$ |
| $\sigma_t BR(\gamma \gamma) pb$ |           | $0.066$ | $0.024$ | $0.011$ |
| $\sigma_t BR(\gamma Z) pb$ |           | $0.227$ | $0.105$ | $0.055$ |
| $\sigma_t BR(ZZ) pb$ |           | $0$ | $0.074$ | $0.069$ |
| $\sigma_t BR(W^+ W^-) pb$ |           | $0.570$ | $0.377$ | $0.266$ |
| $\sigma_t BR(bb) pb$ |           | $0.023$ | $0.006$ | $0.002$ |
Table 2.

| $M_H$ GeV | 175 | 225 | 275 | 325 | 375 |
|-----------|-----|-----|-----|-----|-----|
| $\Gamma_H$ GeV | 0.287 | 1.714 | 2.474 | 12.306 | 23.808 |
| $\sigma_t (pp \rightarrow H + X)$ pb | 6.54 | 4.48 | 3.44 | 2.60 | 2.11 |
| $\sigma_t BR(\gamma \gamma)$ pb | 0.485 | 0.118 | 0.052 | 0.029 | 0.019 |
| $\sigma_t BR(\gamma Z)$ pb | 1.674 | 0.615 | 0.326 | 0.201 | 0.138 |
| $\sigma_t BR(Z Z)$ pb | 0 | 0.766 | 0.827 | 0.720 | 0.597 |
| $\sigma_t BR(W^+ W^-)$ pb | 4.210 | 2.952 | 2.227 | 1.650 | 1.258 |
| $\sigma_t BR(bb)$ pb | 0.171 | 0.025 | 0.007 | 0.003 | 0.001 |
| $\sigma_t BR(t \bar{t})$ pb | 0 | 0 | 0 | 0 | 0.095 |
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6
Fig. 7

$F(Q)$

$Q$

$TeV$
$\sigma(Q) \, pb$

$e^+e^- \rightarrow W^+W^-$

Fig. 8
\[ \sigma(Q) \text{ pb} \]

\[ e^+e^- \rightarrow ZZ \]

Fig. 9