Magnetic Screening in Hot Non-Abelian Gauge Theory

A. Cucchieri, F. Karsch and P. Petreczky
Fakultät für Physik, Universität Bielefeld,
P.O. Box 100131, D-33501 Bielefeld, Germany

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Abstract

We analyze the large distance and low-momentum behavior of the magnetic gluon propagator of the SU(2) gauge theory at finite temperature. Lattice calculations within the 4-dimensional as well as the effective, dimensionally reduced 3-dimensional gauge theories in generalized Landau gauges and MAG show that the magnetic propagator is strongly infrared suppressed in Landau gauges but stays large and finite in MAG. Despite these differences in the low-momentum behavior of the propagator calculated in different gauges the magnetic fields are exponentially screened in all gauges considered. From the propagator calculated in maximally Abelian gauge we find for the screening mass, \( m_M = (1.48 \pm 0.17)T \) at \( T = 2T_c \).

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Magnetic fields in hot non-abelian gauge theories have to be screened at high temperature for any perturbative description of the plasma phase to make sense [1]. In fact, there is plenty of evidence from analytic [2, 3] as well as numerical [4, 5] calculations for the screening of electric and magnetic fields in non-abelian gauge theories at high temperature. The way this is realized on the partonic level, however, is poorly understood. Our intuitive understanding of screening at high temperature is largely influenced by the perturbative concept of electric and magnetic screening masses [6]. However, while the former is calculable at least to leading order in perturbation theory the latter is intrinsically non-perturbative. Its existence has been postulated to render the perturbative expansion finite [1].

In the vacuum the gluon propagator in Landau gauge is expected to be infrared suppressed. The existing rigorous bounds on the unrenormalized gluon propagator in momentum space imply that it is less singular than \( p^{-2} \) in 4 dimension and
$p^{-1}$ in 3 dimension and it has further been argued that it is likely to vanish at zero momentum in the thermodynamic limit. Numerical evidence for this has recently been found in lattice simulations of the SU(N) gauge theory in 3 and in 4 dimensions. As it is well established that the 4d, SU(N) gauge theory at finite temperature is well described by an effective, 3d gauge theory with an adjoint Higgs field and that, moreover, the magnetic sector is little influenced by the scalar Higgs fields, the findings for the gluon propagator in 3 dimensions are of direct relevance for the behavior of the magnetic gluon propagator in the 4d gauge theory at finite temperature. In fact, it recently has been suggested that also at finite temperature the gluon propagator might be infrared suppressed.

It is the purpose of this letter to clarify the long-distance, low-momentum structure of the magnetic gluon propagator of the finite temperature SU(2) gauge theory in 4d, establish the connection to the 3d gauge theory and analyze to what extent these findings are gauge dependent. To this end we will analyze the magnetic gluon propagator in Landau as well as maximally Abelian gauges (MAG). Both gauges are complementary in so far as the bounds on the zero momentum limit of the gluon propagator derived in Landau gauge are not valid in MAG because in this case the Fadeev-Popov matrix turns out to be quadratic in the gauge fields. Although details of the infrared behavior will turn out to be different in different gauges we show that in all cases considered magnetic fields are exponentially screened at large distances. In particular, we will make contact to earlier numerical calculations, which gave evidence for exponential damping of the magnetic propagator at large distances and led to the determination of a magnetic screening mass.

We have performed numerical calculations for the 4d, SU(2) gauge theory at finite temperature as well as for the effective, 3d gauge theory with an adjoint Higgs field and the pure SU(2) gauge theory in 3d. The magnetic gluon propagator, $D_M(z)$, and its Fourier transform, $\tilde{D}_M(p)$, have been calculated in Landau gauge and MAG on lattices of size $\Omega = N^2 \sigma N$, in 3d and $\Omega = N^2 \sigma N \sigma N$ in 4d, respectively. In our 3d simulations we also have considered an anisotropic generalization of the Landau gauge defined by the gauge condition,

$$\partial_1 A_1 + \partial_2 A_2 + \lambda_3 \partial_3 A_3 = 0,$$

which for $\lambda_3 = 1$ reduces to the ordinary Landau gauge condition. For the gauge fields we use the straightforward lattice definition, $A_\mu(x) = \left[ U_\mu(x) - U_\mu^\dagger(x) \right] / 2i$, with $U_\mu(x) \in SU(2)$, which reproduces the continuum gauge fields up to $O(a^2)$ discretization errors. We have checked previously that, up to an overall normalization, other lattice definitions with formally smaller discretization errors lead to identical results.

The correlation function for the $\mu$-component of the gauge fields $A_\mu$ is defined
in terms of the sum over gauge fields in a hyperplane orthogonal to \( x_3 \),

\[
D_{\mu\nu}(z) = \frac{1}{\Omega} \left\langle \sum_{b=1}^{3} \sum_{x_3} Q^b_{\mu}(x_3 + z) Q^b_{\mu}(x_3) \right\rangle, \tag{2}
\]

with \( Q^b_{\mu}(x_3) = \sum_{x_\perp} \text{Tr}[A_{\mu}(x_\perp, x_3) \sigma^b]/2 \). Here \( \sigma^b \) are the usual Pauli matrices; \( x_\perp = (x_1, x_2) \) in 3d and \( x_\perp = (x_0, x_1, x_2) \) in 4d, respectively. In terms of this we define the magnetic gluon propagator in coordinate space,

\[
D_M(z) = \frac{1}{2} (D_{11}(z) + D_{22}(z)) \quad , \tag{3}
\]

and the momentum space propagator

\[
\tilde{D}_M(p) = \frac{\beta a^2}{12} \sum_{\mu=1}^{3} \sum_{z=0}^{N_z-1} e^{2\pi i k z/N_z} D_{\mu\mu}(z) \quad , \tag{4}
\]

with \( p = 2 |\sin(\pi k/N_z)| \) and \( k = 0, 1, ..., N_z - 1 \). Here \( \beta \) is the coupling appearing in the Euclidean lattice action and \( a \) is the lattice spacing.

The 4d calculations at finite temperature have been performed in the high temperature phase at twice the critical temperature for the deconfinement transition. All calculations have been performed with the standard Wilson gauge action with gauge coupling \( \beta = 2.512 \) for \( N_\tau = 4 \) and \( \beta = 2.740 \) for \( N_\tau = 8 \). These values correspond to \( T = 2 T_c \) \[18\]. Corresponding calculations in 3d have been performed at \( \beta = 8 \) on lattices of size \( 16^2 \times 32 \), \( 24^2 \times 48 \), \( 28^2 \times 56 \), \( 32^2 \times 64 \), \( 48^2 \times 64 \) and \( 64^3 \). Moreover, we have performed 3d calculations at a smaller coupling, \( \beta = 5 \), on lattices of size \( 32^3 \) up to \( 96^3 \). This choice of parameters for calculations within the reduced theory allows to analyze the behavior of correlation functions in large physical volumes which are 40 times larger than what is feasible within our 4d simulations. The gauge coupling of the pure gauge theory was chosen according to \( g_3^2(T) = g^2(\mu)T \), where \( g^2(\mu) \) is 1-loop running coupling constant of the 4d \( SU(2) \) gauge theory in the \( \overline{MS} \) scheme, \( \mu = 18.86T \) and for \( \Lambda_{\overline{MS}} \) the numerically determined relation to the deconfinement temperature has been used, i.e. \( T_c = 1.06\Lambda_{\overline{MS}} \) \[4\]. This choice of \( g_3^2 \) ensures a perfect description of the temperature dependence of the spatial string tension of the 4d finite temperature theory in terms of the effective 3d theory \[15\] down to temperatures \( T \simeq 2T_c \). Furthermore, we have checked on our smaller lattices that the magnetic propagators calculated in the pure 3d \( SU(2) \) gauge theory and in the complete 3d effective theory, i.e. in the presence of an adjoint Higgs field, agree within statistical errors. We therefore will restrict our discussion here to results coming from the pure gauge sector alone. Our 3d calculations have been performed at \( \beta = 5 \) and 8 which correspond to lattice spacings \( a = 0.2752/T \) and \( 0.172/T \), respectively. Varying \( \beta \) in our 3d calculations and converting to physical units with the above relation for \( g_3^2 \) we have checked that cut-off effects are small.
To verify the cut-off independence of our results in 4d the calculations have been performed at two different values of the lattice cut-off, i.e. we used lattices of size $N^2_\sigma N_z N_\tau$ with $N_\tau = 4$ and 8, $N_z = 2N_\sigma$ and $8 \leq N_\sigma \leq 32$. Keeping the physical volume $VT^3 = 2(N_\sigma/N_\tau)^3$ constant we have verified that $D_M(z)$ is cut-off independent within the statistical errors of our calculation.

The gluon propagator has been calculated on 1000 to 8000 gauge fixed gauge field configurations in the case of 3d and about 400 configurations in 4d calculations. In Fig. 1 we show the magnetic propagator, $D_M(z)$, calculated in Landau and maximally Abelian gauges on various size lattices. We note that the 4d and 3d calculations performed on lattices of similar physical size are in good agreement. While the magnetic gluon propagator calculated in MAG shows no significant volume dependence the Landau gauge propagators are strongly volume dependent for $zT > \sim 1$. A similarly strong volume dependence we also observe in other generalized Landau gauges, i.e. gauges based on the gauge condition given in Eq. 1 with $\lambda_3 \neq 1$. In fact, at large distances the propagators in generalized Landau gauges, including the Coulomb gauge limit, become negative.

The virtue of the effective 3d theory is that we can reach here a much larger physical volume and thus can control the volume dependence still apparent in the comparison of 3d and 4d data shown in Fig. 1. We have analyzed the large distance behavior of $D_M(z)$ in these gauges in our 3d calculations on large lattices of size $L^3$, $L = 32, 40, 48, 56, 64, 72$ and $96$ at $\beta = 5$. This corresponds to physical volumes of $VT^3 = 683$ up to $VT^3 = 5464$. In all cases we clearly observe negative correlation functions at large distances. For some of these lattices we show the long-distance part of the $D_M(z)$ in Fig. 2 for $\lambda_3 = 1$. Similar results we find for $\lambda_3 \neq 1$. We note that the volume dependence is small on these large lattices. Moreover, the correlation functions become smaller with increasing lattice size. This is in accordance with our expectation that finite size effects are due to zero mode contributions which disappear in the infinite volume limit. On finite lattices, however, they add a volume-dependent, positive constant to the correlation function in coordinate space.

Our analysis thus gives clear evidence that the magnetic gluon correlation function in coordinate space calculated in generalized Landau gauges becomes negative at large distances, i.e. for $zT \gtrsim 1.5$. At $T = 0$ such a behavior is, in fact, expected to occur for the gluon propagator as a consequence of the conjecture made by Zwanziger that the Landau gauge gluon propagator vanishes in the infrared limit. In coordinate space it has been suggested that the propagator takes on the form

$$D_M(z) = Ae^{-m_M z} \cos(bm_M z + c),$$

(5)

Although we can at present not quantify the functional form of the correlation function in generalized Landau gauges, our results are consistent with this ansatz up to distances $zT \simeq 3$. To judge whether the correlation function will indeed
continue to oscillate as suggested by Eq. 3 will, however, require a more detailed
analysis of the large distance behavior of $D_M(z)$. In general, for $D_M(p)$ to vanish
at $p = 0$ in momentum space one should find $\tan(c) = 1/b$ with the above ansatz.
This conjecture, $\lim_{p \to 0} D_M(p) = 0$, may also be valid at non-zero temperature [15].

In contrast to the complicated structure of the Landau gauge propagator the
propagator calculated in MAG does show a simple exponential decay at large
distances and does not show any significant volume dependence. We could verify that
it stays positive at least up to distance $zT = 3.5$. For $zT > 1.5$ simple exponential
fits gave $\chi^2/(d.o.f) = 1.7$ and we also found that local masses became independent
of $z$ within statistical errors for $zT > 1$. Best fits with the ansatz given in Eq. 3
were thus obtained for $b = 0$. From this we find for the magnetic screening mass of
the magnetic propagator calculated in MAG,

$$\frac{m_M}{T} = 1.48 \pm 0.17 \quad \text{at} \quad T = 2T_c \quad .$$

(6)

When taking the absolute value of the Landau gauge propagator this is found to be
bounded by the exponentially decaying MAG-propagator. This confirms that also
in Landau gauge correlations of magnetic fields are exponentially screened.

The fact that the magnetic gluon propagator calculated in Landau gauge be-
comes negative at large distances translates into a suppression of the momentum
space propagator at small momentum. This is shown in Fig. 3 for the Landau gauge
propagators, similar results hold for $\lambda_3 \neq 1$. For $p < T$ the momentum space prop-
gagator is sensitive to the volume, while for large momenta ($p > T$) it is essentially
independent of the lattice size. Moreover, we note that for small momenta the finite
volume effects lead to a decrease of $D(p)$ with increasing volume while the volume
dependence is already negligible for $p/T \approx 1$. The occurrence of a maximum in
$D(p)$ for $p/T \approx 1$ and the infrared suppression of the momentum space propagator
at $p = 0$ thus is firmly established by our data.

As expected the volume dependence of the magnetic propagator in momentum
space is strongest at $p = 0$. On large lattices $D_M(p)$ reaches a maximum at a non-
zero but small value of $p/T$ and keeps decreasing at $p = 0$ with increasing lattice size.
This might indicate that $D_M(0)$ could actually vanish in the infinite volume limit.
The propagator at vanishing momentum reflects the contribution of zero momentum
fluctuations of the gauge fields,

$$\tilde{D}_M(0) = \frac{\beta a^2}{12} \Omega \sum_{\mu,b} \langle (\phi^b_\mu)^2 \rangle \quad \text{with} \quad \phi^b_\mu = \frac{1}{\Omega} \sum_x A^b_\mu(x) \ .$$

(7)

In order for it to vanish in the infinite volume limit the zero mode fluctuations
$\langle (\phi^b_\mu)^2 \rangle$ thus have to drop faster than $\Omega^{-1}$. To quantify this behavior we have fitted
$\tilde{D}_M(0)T^2$ to a simple ansatz that assumes a power-like dependence on the volume,
$\tilde{D}_M(0)T^2 = b(VT^3)^{-z} + c$. At present such fits to our $4d$ and $3d$ data can, however,
not rule out a non-vanishing value for the constant $c$. We stress, however, that the
value of $\tilde{D}_M(0)$ itself is of little importance for the observed complex structure of
the gluon propagator in Landau gauge in coordinate as well as in momentum space
and in any case is gauge dependent. Nonetheless, the behavior found here for the
finite-$T$ magnetic gluon propagator is consistent with findings of Ref. \[9\] for the
$T = 0$ gluon propagator of the $3d$, $SU(2)$ gauge theory. There it was shown that
the magnetic propagator calculated in Landau gauge is less singular then $p^{-1}$ in the
infrared and is likely to vanish in $p \to 0$, $V \to \infty$ limit.

The momentum dependence of $\tilde{D}_M(p)$ shown in Fig. 8 also makes clear why
earlier calculations on medium size lattices let to the determination of a non-zero
magnetic mass. The infrared suppression becomes significant only for momenta
$p/T \lesssim 0.5$. The existence of a simple pole mass becomes unlikely on the basis of our
analysis. However, we stress that magnetic fields in coordinate space are, of course,
strongly screened. The magnetic gluon propagator calculated in Landau gauge as
well as MAG is exponentially damped. The value for the screening mass extracted
from the propagator in MAG gauge is in good agreement with earlier in $4d$ \[4\] and
$3d$ \[5\] lattice calculations in Landau gauge, although the data analysis in these cases
was based on a simple pole ansatz for the magnetic mass.

In the course of the calculations reported here we also have reanalyzed the behav-
ior of the electric gluon propagator in different gauges. In that case no suppression
in the infrared has been observed and the electric screening mass has been found to
be gauge independent within statistical errors. Details of this calculation as well as
a more detailed discussion of our results for the magnetic gluon propagator will be
presented elsewhere.

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Figure 1: Volume dependence of the magnetic propagators in coordinate space calculated in Landau (top) and maximally Abelian (bottom) gauges. The propagators were normalized to 1 at $z = 0$. Shown are results from 3d and 4d simulations, which are compared at similar values of the physical volume in units of $T^3$. These volumes correspond to lattices of size $16^2 \times 32 \times 8$, $24^2 \times 48 \times 8$ and $16^2 \times 32 \times 4$ in 4d and to $16^2 \times 32$, $24^2 \times 48$, $32^2 \times 64$ and $48^2 \times 64$ in 3d.
Figure 2: The magnetic gluon propagator calculated in generalized Landau gauges with $\lambda_3 = 1$ at $\beta = 5$. Also shown are data from a calculation in MAG on a $48^2 \times 64$ lattice. We only show the propagators at large distances. They were normalized to 1 at $z = 0$.

Figure 3: The magnetic gluon propagator in momentum space calculated in Landau gauge on $3d$ lattices of various sizes at $\beta = 5$. 