Strangeness production in a statistical effective model of hadronisation

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Abstract

We suppose that overall strangeness production in both high energy elementary and heavy ion collisions can be described within the framework of an equilibrium statistical model in which the effective degrees of freedom are constituent quarks as used in effective lagrangian models. In this picture, the excess of relative strangeness production in heavy ion collisions with respect to elementary particle collisions arises from the unbalance between initial non-strange matter and antimatter and from the exact colour and flavour quantum number conservation over different finite volumes. The comparison with the data and the possible sources of model dependence are discussed.

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1 Introduction: strangeness production and statistical hadronisation

Recent observations have shown that hadron multiplicities in both $e^+e^-$ and hadronic high energy collisions agree very well with a statistical-thermal ansatz [1]. This finding has been interpreted in terms of a multi-cluster hadronisation process in which each cluster fills its relevant multi-hadronic phase space in a pure statistical fashion, at a critical value of energy density [1,2]. Within this framework, temperature and other thermodynamical quantities have an essential statistical meaning which does not imply the existence of a thermalisation process at hadronic level through multiple collisions; rather, hadronisation itself yields a statistically equilibrated hadronic population. One of the main features of this approach is the very low number of free parameters required. Under suitable assumptions about cluster masses and charges fluctuations at fixed volumes [1,2], there are essentially two free parameters, namely the sum $V$ of the volumes of the clusters, and the temperature $T$. Yet, in order to reproduce the yields of strange particles, the model has to be supplemented with one more phenomenological parameter, $\gamma_S$, which suppresses the production of particles containing $n$ strange quarks by a factor $\gamma_S^n$. From fits to the available data, $\gamma_S$ turns out to be $< 1$ in all examined collisions and strongly dependent on the initial colliding systems [1–3] so that full strangeness chemical equilibrium is never observed.

A further insight in strangeness production is achieved by calculating the ratio between newly produced valence strange quarks and $u, d$ quarks, the so-called Wroblewski factor $\lambda_S = \frac{\langle s\bar{s} \rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle}$, which is in fact fairly constant in all kinds of elementary collisions (EC) over a centre-of-mass energy range spanning about two orders of magnitude, while it shows a non-trivial behaviour in heavy ion collisions (HIC) and it is as twice as large at $\sqrt{s} \approx 20$ GeV [3] (see Fig. 1). It should be mentioned that this ratio is calculated by using the fitted primary (i.e. directly emitted from the hadronising source) hadron multiplicities before hadronic decays take place, which are not measurable. Thus, this parameter depend in fact on the fitted parameters $T$, $V$ and $\gamma_S$ and, in principle, is a model dependent quantity; still, the model accurately reproduces all measured multiplicities and the model dependence decreases as the number of measured final multiplicities increases, so that the estimated $\lambda_S$ is expected to be reasonably close to its actual value. The explanation of the strangeness production in high energy collisions, and especially $\lambda_S$, is a major goal for the understanding of hadronisation and of the possible QGP formation in HIC.

In the following, we will try to account for this behaviour by resorting to a statistical description in terms of constituent quarks rather than hadrons, or, more specifically, using microscopic models containing quarks as fundamental degrees of freedom. As full QCD cannot be handled, we use effective models (EM) at finite temperature, which are commonly employed to investigate QCD phase transition at high temperatures and low baryon chemical potential, that is our region of interest. Particularly, we will refer to low energy models with four-fermion interactions such as the Nambu-Jona-Lasinio model [4,5] and to what is sometimes referred to, in literature, as ladder-QCD [6].

1 Very recently [2] a new parametrisation of extra strangeness suppression has been proposed which is equivalent to that with $\gamma_S$ at very high multiplicity.
2 The model

In using effective models with quarks as fundamental degrees of freedom to calculate relative flavour production, essentially the same physical scheme of the statistical hadronisation model (SHM) both in HIC and EC is kept. This means that the formation of a set of hadronising clusters endowed with mass, volume and flavour quantum charges is assumed, in which every allowed quantum state is equally likely. Also the assumption of suitable maximum-disorder fluctuations of cluster flavour charges and masses for fixed volumes, enabling the introduction of a global temperature, is retained. Three further assumptions are introduced:

1. Each single cluster is a colour singlet

2. The produced s quarks, or at least the ratio s/u, survive in the hadronic phase

3. The temperature $T$ and the chemical potentials (in the grand-canonical framework) fitted in the SHM with hadron multiplicities are interpreted as the critical values for deconfinement and (approximate) chiral symmetry restoration

The physical process of single cluster hadronisation, in both EC and HIC is envisaged as an evolution towards a chaotic quantum state which finally leads to statistical multi-hadronic phase space population by coalescence of produced constituent quarks. Within this picture, the lack of complete strangeness chemical equilibrium at hadron level is the effect of a complete strangeness chemical equilibrium at constituent quark level. Whilst in HIC an early thermalisation and colour deconfinement over a large volume is expected, with consequent formation of relatively large colour singlet clusters, in EC the chaotic behaviour of quantum dynamics is supposed to set in at a late stage when the colour preconfinement mechanism has already brought about the formation of small colour singlet clusters. These distinctive features of hadronisation in HIC
with respect to hadronisation in EC imply a difference of relative strange quark production which is, hopefully (as we have assumed in (2)), reflected into final hadrons. Moreover, the existence of a characteristic single cluster volume in EC independent of colliding system and centre-of-mass energy is argued to be the main responsible for the constancy of \( \lambda_S \) because of the strange quark suppression entailed by the colour singlet constraint over a small spacial region (canonical colour suppression). The assumption (3) is certainly the strongest one, as it implies that hadronisation itself is assumed to be a critical process and, thence, hadronisation temperature is to be identified with the critical QCD temperature; this is suggested by the observed constancy of fitted temperature for many collisions [2] and by its value \( T \simeq 160 \text{ MeV} \) close to the calculated lattice value [7].

As has been mentioned, effective lagrangian models are a useful tool to deal with quark degrees of freedom at finite temperature. In fact, none of them can account for colour confinement but many of them embody chiral symmetry breaking (\( \chi_{SB} \)) and its restoration (\( \chi_{SR} \)), which is expected to occur at the same critical point [4]. The predictions of EM about temperature dependence of several physical quantities may strongly vary, nevertheless they also show striking common features. Specifically, the phase diagram for \( \chi_{SR} \) exhibits a tricritical point in the chiral limit \( m_q \to 0 \) separating second order from first order phase transitions [3] and low-\( \mu \) and high-\( T \) region is second order. Moreover, the expression for the number of quarks for the \( i^{th} \) flavour generally stems from the one-loop term and, in the mean-field approximation of four-fermion models, it can be derived from a free Dirac Hamiltonian with constituent quark masses replacing current masses, so that in the grand-canonical ensemble:

\[
\langle n_i \rangle = \frac{N_c V}{\pi^2} \int_0^\Lambda dp \frac{p^2}{\exp[\sqrt{p^2 + M_i^2/T} - \mu_i/T]} + 1
\]

where \( N_c = 3 \) and \( \mu_i \) are the relevant chemical potentials. In Eq. (1) \( \Lambda \) is an UV cutoff which is needed in EM with four-fermion interactions as these models are not renormalisable (630 MeV in ref. [5]). In ladder-QCD models essentially the same expressions holds below \( \Lambda \) whereas at higher momenta the one-loop Hamiltonian has to be modified. However, this modification usually implies a minor change of the ratio \( s/u \) since the largest contribution to \( \langle n_i \rangle \) comes from the integration region \( p < \Lambda \). Finally, commonly to most EM, the u and d quark constituent masses \( M_{u,d} \) steeply decrease to a value close to the current mass within a small interval of temperature (which is fairly identified with the critical region) whereas the strange quark constituent mass \( M_s \) decreases much more slowly and in fact, in the critical region, it has a value still much higher than current mass value.

In order to obtain quantitative predictions for EC within a given effective model the canonical expressions of quark numbers are needed, hence we have to implement exact colour and flavour conservation over finite volumes. This task can be accomplished by means of well known methods based on group theory [8]. In our case the involved symmetry group is \( G = SU(3)_c \times U(1)_u \times U(1)_d \times U(1)_s \) and physical states to be counted in the partition function of a single cluster should be projected onto the irreducible invariant 1-dimensional subspace associated with the colour singlet representation with given initial flavour numbers. The overall number of quarks for a given flavour \( i \) can be obtained by taking the derivative of the overall partition function (i.e. the partition function of the multi-cluster system) with respect to fictitious fugacities \( \lambda_i \):

\[
\langle n_i \rangle = \frac{\partial \log Z(\lambda_i)}{\partial \lambda_i} \bigg|_{\lambda_i=1}
\]
where the overall partition function reads (the proof is given in ref. [9]):

\[
Z(\lambda_1, \lambda_2, \lambda_3) = \prod_{i=1}^{3} \int_{-\pi}^{\pi} \frac{d\phi_i}{2\pi} \exp[iN_i\phi_i]\left\{ \int d\mu(\theta_1, \theta_2) \exp\left[\sum_{i=1}^{3} \frac{2V_c}{(2\pi)^3} \right. \right.
\]
\[
\times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \chi_{1,0}(n\theta_1, n\theta_2) e^{in\phi_i} \lambda_i^n \int_{0}^{\Lambda} d^3p \exp[-\sqrt{p^2 + M_i^2/T} + c.c.] \left. \right\} V/V_c \right)^{V/V_c}
\]

(3)

where \(d\mu\) is the normalised SU(3)\(_c\) group measure [8] and \(\chi_{1,0}\) is the character of the fundamental SU(3)\(_c\) representation. Two different volumes show up in Eq. (3) owing to the fact that colour singlet constraint applies to each single cluster (with volume \(V_c\)) whereas the flavour constraint (equal to that of initial state) applies to the system of clusters overall; indeed \(V\) is meant to be the sum of all clusters proper volumes.

3 Analysis and Results

In our numerical analysis we essentially determine \(\lambda_S\) on the basis of Eqs. (2), (3). Generally speaking, \(\lambda_S\) depends on the value of constituent quark masses at a fixed temperature \(T\), on the cutoff \(\Lambda\) and on the two volumes \(V\) and \(V_c\). According to statement (3) in previous section, the temperature \(T\) has been set equal to the value fitted within SHM, which is about 160 MeV in EC and in Pb–Pb collisions [3].

As far as HIC are concerned, \(\lambda_S\) calculation can be performed in the grand-canonical ensemble on the basis of Eq. (1) because of the very large involved overall volumes \(V\). Colour canonical suppression (see below) sets in only at very small \(V_c\) volumes, which are excluded because of the assumption of colour deconfinement. Thereby, since \(\lambda_S\) is a ratio of particle numbers, the dependence on volumes vanishes and, by setting \(\Lambda \to \infty\) in Eq. (1), it is found that one needs \(M_s \sim 500\) MeV and \(M_q < 100\) MeV to match the fitted \(\lambda_S\) value in Pb–Pb (\(\simeq 0.4\)) at \(T = 160\) MeV. This means that an eligible effective model should have a critical region (at very low baryon chemical potential) around 160 MeV with constituent u and d masses essentially dropped from their value at \(T = 0\). These features can be precisely found in one version (case 2) of the NJL model in ref. [5] which has thus been used to make a quantitative comparison with the data. In particular, this model has an UV cutoff \(\Lambda = 630\) MeV, \(M_{u,d}(T = 160\) MeV, \(\mu = 0) = 64\) MeV and \(M_s(T = 160\) MeV, \(\mu = 0) = 449\) MeV. The agreement between calculated and fitted \(\lambda_S\) values in Pb–Pb is very good as it can be seen in Table 1.

| Collision     | \(T\) (MeV) | \(\mu_q\) (MeV) | \(\lambda_S\) | \(\lambda_S\) (calc.) |
|---------------|-------------|-----------------|---------------|-------------------|
| Pb–Pb SPS     | 158.1 ± 3.2 | 79.3 ± 4.3      | 0.447 ± 0.025 | 0.455             |
| Au–Au RHIC    | 165 ± 7     | 13.7 ± 1.7      | 0.335         |                   |

Table 1: Comparison between \(\lambda_S\) fitted with SHM [3] and the calculated \(\lambda_S\) in high energy heavy ion collisions by using the central fitted values of \(T\) and \(\mu_q \simeq \mu_B/3\) [3,10].

The effect of volume finiteness on \(\lambda_S\) (canonical suppression) is mainly relevant to EC and has been accurately studied by enforcing colour and flavour constraints either separately or
Figure 2: Calculated $\lambda_S$ in $e^+e^-$ and pp collisions at $T=160$ MeV within the NJL model as a function of the total volume $V$ for single cluster volume $V_c$ varying from 5 fm$^3$ (lowest curves) to 40 fm$^3$ in steps of 5 fm$^3$. The horizontal bands are the ranges of fitted $\lambda_S$ values in the SHM (see Fig. 1).

simultaneously. Generally speaking, the requirement of an exact conservation of some global quantity (such as colour or charge) suppresses heavier particles more than lighter ones with respect to the grand-canonical limit because, at finite $T$, less energy can be spent to compensate the unbalance created by one particle generation. Indeed, for a given volume $V = V_c$, the canonical suppression of s quarks with respect to u, d quarks is expected, and has been found, to be predominantly determined by the net zero strangeness constraint rather than by colour singlet constraint as colour can be compensated by the generation of two light u, d quarks instead of one heavier $\bar{s}$ quark. Unlike the number of quarks, the masses of constituent quarks have been determined by minimising the free energy in the grand-canonical limit, i.e. neglecting the effect of finite volume, at $T = 160$ MeV and $\mu_i = 0$.

The effect of combined colour and flavour conservation over different volumes on $\lambda_S$ is shown in Fig. 2 as a function of $V$ and $V_c$ for colliding systems such as $e^+e^-$ (flavour neutral) or pp, along with conservatively estimated ranges of $\lambda_S$ determined with the multiplicity fits within the SHM (see Fig. 1). The colour canonical suppression of $\lambda_S$ clearly shows up for single-cluster volumes below $\approx 15$ fm$^3$. Apparently, a $V_c$ between 5 and 10 fm$^3$ can account for the observed constant $\lambda_S$ value in $e^+e^-$ collisions whereas the predicted value in pp is too high. In fact, the relative strangeness enhancement due to the presence of initial u and d quarks in the colliding protons which inhibits the creation of uu, dd pairs, seems to prevail over the relative strangeness suppression entailed by colour and $S = 0$ constraint.
4 Conclusions

We have studied strangeness production within a statistical model of hadronisation by using effective models with constituent quarks. The basic idea is that full statistical equilibrium is achieved at the level of quark degrees of freedom in both elementary and heavy ion collisions. Besides the effect of different initial light flavour content and density, the smaller relative strangeness production in EC with respect to HIC is supposed to be related to the smaller system size and to colour confinement over small distances. A quantitative study in this regard gives a satisfactory agreement with the data in heavy ion and $e^+e^-$ collisions but a significant disagreement in pp, which might be cured by taking more complex assumptions about quantum numbers distribution among the hadronising clusters.

References

[1] F. Becattini, Z. Phys. C69 (1996) 485; F. Becattini and U. Heinz, Z. Phys. C76 (1997) 269.

[2] F. Becattini, L. Bellucci, G. Passaleva, Nucl. Phys. Proc. Suppl. 92 (2001) 137; F. Becattini, G. Passaleva, in preparation.

[3] F. Becattini, M. Gazdzicki, J. Sollfrank, Eur. Phys. J. C5 (1998) 143; F. Becattini et al., Phys. Rev. C64 (2001) 024901.

[4] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; 124 (1961) 246.

[5] T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221.

[6] A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto and G. Pettini, Phys. Rev. D41, (1990) 1610; A. Barducci, R. Casalbuoni, R. Gatto and G. Pettini, Phys. Rev. D49, (1994) 426 and references therein.

[7] F. Karsch, this conference.

[8] K. Redlich and L. Turko, Z. Phys. C5, (1980) 201; M.I. Gorenstein et al., Phys. Lett. B123 (1983) 437; G. Auberson et al., J. Math. Phys. 27 (6) (1986) 1658.

[9] F. Becattini and G. Pettini, in preparation.

[10] W. Florkowski, W. Broniowski and M. Michalec, nucl-th/0106009.