Correlating prompt GRB photons with neutrinos

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ABSTRACT

It is standard in theoretical neutrino astrophysics to use a broken power law approximation, based on the Band function, to describe the average photon flux of the prompt emission of Gamma-Ray Bursts. We will show that this approximation overestimates the contribution of high energy $\gamma$-rays (and underestimates low energy $\gamma$-rays). As a consequence models that rely on this approximation overestimate neutrino event rate by a factor of $\approx 2$ depending on Earth’s column density in the direction of the GRB. Furthermore the characteristic energy of neutrinos that trigger a km$^3$ detector is typically $10^{16}$ eV, higher than previously predicted. We also provide a new broken power law approximation to the Band function and show that it properly represents the photon spectra.

Subject headings: Gamma-Ray Burst, Neutrinos, Neutrino Telescopes

1. Introduction

It is commonly believed that prompt emission by GRBs is due to synchrotron radiation by electrons accelerated in internal shocks associated with relativistic jets (with a bulk Lorentz boost $\Gamma$ of 100-1000). A review of the theoretical and observational status of GRBs is beyond the scope of this paper. The reader is referred to Meszaros (2006). The average prompt photon emission is often described by fitting a Band function (Band et al 1993). High energy neutrino emission by GRBs in coincidence with the prompt $\gamma$-ray photons has been proposed as a consequence of GRBs being a candidate to produce (UHE) ultra high energy (up to $\approx 10^{20}$ eV) cosmic rays (Waxman 1995; Vietri 1995).

Waxman & Bahcall (1997) have calculated the diffuse flux due to GRBs. Their work has been further improved by calculating neutrino emission for individual bursts (Guetta et al 2004) and by
performing detailed GEANT4 simulations of proton-photon interactions in the internal shocks (Murase & Nagataki 2006). This body of work supports the hypothesis that neutrino detection by km$^3$ Cherenkov detectors will probably be in coincidence with a handful of bright GRBs.

Previous studies have correlated photon emission with neutrino emission by approximating the photon spectra with a broken power law. We will show that this approximation overestimates the contribution of high energy photons (and conversely underestimates the contribution from low energy photons). This has consequences in the number of events expected by neutrino telescopes such as IceCube (Achterberg et al 2007c) and KM3NET (Kappes et al 2007) and in the characteristic energy of the neutrinos detected.

In section §2 we describe the Band function. In section §3 we follow the standard calculation of HE neutrinos emission from GRBs. In section §4 we compare the Band function with its broken power law approximation. In section §5 we calculate neutrino event rates. And in section §6 we discuss the consequences of the calculation shown here.

### 2. Prompt photon spectra

The time-averaged prompt GRB photon flux is often described by fitting a Band function (Band et al 1993):

\[
\frac{dN_\gamma}{dE_\gamma} (E_\gamma) = A_\gamma \begin{cases} 
\frac{E_\gamma}{100 \text{keV}}^{\alpha_\gamma} e^{-E_\gamma/E_{pk}} 
& \text{if } E_\gamma < E_{b}^b \\
\left(\frac{E_\gamma}{100 \text{keV}}\right)^{\alpha_\gamma - \beta_\gamma} \left(\frac{E_{pk}}{100 \text{keV}}\right)^{\beta_\gamma} e^{(\alpha_\gamma - \beta_\gamma) E_{pk}/100 \text{keV}} 
& \text{if } E_\gamma > E_{b}^b
\end{cases}
\]

(1)

The parameters of the Band function are the amplitude $A_\gamma$, the (asymptotic) low-energy spectral index $\alpha_\gamma$, the high-energy spectral index $\beta_\gamma$ and the peak energy $E_{pk}$ of the $\nu F_\nu$ distribution. Typical values for the parameters above are: $\alpha_\gamma \approx -1$, $\beta_\gamma \approx -2.2$, $E_{b}^b \approx 30$ keV–1000 keV and $A_\gamma \approx 0.001–1$ photons cm$^{-2}$ s$^{-1}$ keV$^{-1}$. The break energy $E_{b}^b$ and the peak energy $E_{pk}$ are related by:

\[
E_{b}^b = \frac{\alpha_\gamma - \beta_\gamma}{2 + \alpha_\gamma} E_{pk}.
\]

(2)

### 3. Photo-pion production of neutrinos

There are many good descriptions of the processes that lead to HE neutrinos from prompt GRB photons in the literature. We will not repeat the full derivation here. Instead we refer the reader to Guetta et al (2004) and we only present here the major features relevant to this paper.
Protons accelerated in internal shocks interact with GRB photons via the process:

\[ p + \gamma \rightarrow \Delta^+ \rightarrow \pi^+[+n] \rightarrow \mu^+ + \nu_{\mu} \rightarrow e^+ + \nu_{\mu} + \bar{\nu}_{\mu} + \nu_e \]  

(3)

and

\[ p + \gamma \rightarrow \Delta^+ \rightarrow \pi^0 + p . \]  

(4)

Because the photon-proton interaction has to create a \( \Delta \) resonance, given a photon energy there is a minimum proton energy for which Eqs.3 and 4 can take place. In Earth’s reference frame:

\[ E_p \geq \frac{1}{(1+z)^2} \frac{m_\Delta^2 - m_p^2}{4 E_\gamma} \Gamma^2 . \]  

(5)

Correspondingly the neutrinos resulting from Eq.3 have a minimum energy:

\[ E_\nu \geq \frac{1}{4} < x_{p\rightarrow\pi^+} > \frac{1}{(1+z)^2} \frac{m_\Delta^2 - m_p^2}{4 E_\gamma} \Gamma^2 , \]  

(6)

where \( < x_{p\rightarrow\pi^+} > \approx 1/5 \) is the fraction of the energy transferred to the charged pion from the initial proton energy and the factor of 1/4 arises because on average each one of the four final leptons in Eq.[3] has the same energy.

It is customary to approximate the Band function fit to the average photon spectra as a broken power law (which we label approximation \( A \)):

\[ \frac{dN_\gamma}{dE_\gamma} = A_\gamma \left\{ \begin{array}{ll} \left( \frac{E_\gamma}{100 \text{keV}} \right)^{\alpha_\gamma} & E_\gamma < \epsilon^b_\gamma \\ \left( \frac{E_\gamma}{100 \text{keV}} \right)^{\beta_\gamma} & E_\gamma \geq \epsilon^b_\gamma \end{array} \right. \]  

(7)

Supposing that protons have a power law spectrum \( dN_p/dE_p \sim E_p^{-2} \), the neutrino spectrum traces Eq.[7]

\[ \frac{dN_\nu}{dE_\nu} = A_\nu \left\{ \begin{array}{ll} (E/\epsilon^b_\nu)^{\alpha_\nu} & E < \epsilon^b_\nu \\ (E/\epsilon^b_\nu)^{\beta_\nu} & \epsilon^b_\nu \leq E \leq \epsilon^s_\nu \\ (E/\epsilon^b_\nu)^{\beta_\nu}(E/\epsilon^s_\nu)^{-2} & E > \epsilon^s_\nu \end{array} \right. , \]  

(8)

where \( \alpha_\nu = -\beta_\gamma - 3, \beta_\nu = -\alpha_\gamma - 3 \) and the neutrino break energy \( \epsilon^b_\nu \) is taken from the minimum energy in Eq.[6]

\[ \epsilon^b_\nu = \frac{1}{20} \frac{1}{(1+z)^2} \frac{m_\Delta^2 - m_p^2}{4 \epsilon^s_\gamma} \Gamma^2 . \]  

(9)

The spectrum is steeper above \( \epsilon^s_\nu \) because of synchrotron energy losses by charged pions. Typically \( \epsilon^s_\nu \approx 10^{16} \text{ eV} \). Muons also suffer from synchrotron losses, but following usual approximations we ignore this effect.
The neutrino flux normalization $A_\nu$ is obtained by supposing that the neutrino fluence (ignoring synchrotron losses) is proportional to the bolometric photon fluence, $S_\gamma$:

$$A_\nu \propto \int_{E_{\nu \text{min}}}^{E_{\nu \text{max}}} E_\nu \frac{dN_\nu}{dE_\nu} dE_\nu \propto S_\gamma = \int_0^\infty dE_\gamma \frac{dN_\gamma}{dE_\gamma}.$$ (10)

Here the choice of minimum neutrino energy, $E_{\nu \text{min}}$ is unimportant and the maximum neutrino energy $E_{\nu \text{max}}$ can be set so that the maximum proton energy is comparable to the highest energy cosmic rays ($E_{p \text{max}} \sim 10^{20} \text{ eV}$ & $E_{\nu \text{max}} \sim 5 \cdot 10^{18} \text{ eV}$). Depending on the spectral indices $\alpha_\nu$ and $\beta_\nu$, the neutrino normalization $A_\nu$ is independent or a very weak function of $E_{\nu \text{max}}$.

It is possible to calculate the neutrino flux without approximating the Band function. In this case the neutrino spectra is:

$$\frac{dN_\nu}{dE_\nu}^\text{Band} = A_\nu \begin{cases} 
\left(\frac{E_\nu}{\epsilon_\nu^b}\right)^{\alpha_\nu} e^{-\left(\alpha_\nu - \beta_\nu\right)\left(\epsilon_\nu^b/E_\nu - 1\right)} & E_\nu < \epsilon_\nu^b \\
\left(\frac{E_\nu}{\epsilon_\nu^b}\right)^{\beta_\nu} e^{-\left(\alpha_\nu - \beta_\nu\right)\left(\epsilon_\nu^b/E_\nu - 1\right)^{-2}} & E_\nu > \epsilon_\nu^b
\end{cases}.$$ (11)

Here we have approximated the flux above the synchrotron energy to a power law. This approximation is correct as long as $\epsilon_\nu^s$ is sufficiently larger than $\epsilon_\nu^b$. The normalization, $A_\nu$, is obtained using Eq.(10).

4. Approximations of the Band function

Figure 1 shows the Band function with the parameters: $\alpha_\gamma = -1$, $\beta_\gamma = -2$, $\epsilon_\nu^b = 300 \text{ keV}$ and $A_\gamma = 0.01 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1}$. Also shown is the corresponding approximation A from Eq.7. The normalization of approximation A has been chosen so that the bolometric fluence matches that of the Band function. This is a natural choice for setting the normalization, because the photon fluence is used in Eq.10.

It is quite clear that approximation A overestimates the contribution from high energy photons and underestimates the contribution from low energy photons. The reason why this occurs is because the break energy of the broken power law has been forced to match that of the Band function.

It may be desirable (e.g. to simplify calculations) to have an alternative broken power law approximation. Also we will use this new approximation, $B$, to illustrate the deficiencies of approximation A. A better choice for approximating the Band function is to require that the asymptotic behavior (for very large and very small $E_\gamma$) of the broken power law matches the Band function,
while leaving the break energy for the broken power law a free parameter:
\[
d\frac{N_{\gamma}}{dE_\gamma}^B = A_\gamma \left\{ \begin{array}{cc}
\left( \frac{E_\gamma}{100\text{keV}} \right)^{\alpha_\gamma} & E_\gamma < \bar{\epsilon}_\gamma \\
\left( \frac{E_\gamma}{100\text{keV}} \right)^{\beta_\gamma} \left[ \frac{(\alpha_\gamma - \beta_\gamma)}{2 + \alpha_\gamma} \frac{E_{\text{pk}}}{100\text{keV}} \right]^{\alpha_\gamma - \beta_\gamma} e^{\beta_\gamma - \alpha_\gamma} & E_\gamma \geq \bar{\epsilon}_\gamma
\end{array} \right.
\]
(12)

The value of the effective break energy \(\bar{\epsilon}_\gamma\) is given by the energy at which the two branches of the broken power law are equal to each other:
\[
\left( \frac{\bar{\epsilon}_\gamma}{100\text{keV}} \right)^{\alpha_\gamma} = \left( \frac{\bar{\epsilon}_\gamma}{100\text{keV}} \right)^{\beta_\gamma} \left[ \frac{\alpha_\gamma - \beta_\gamma}{2 + \alpha_\gamma} \frac{E_{\text{pk}}}{100\text{keV}} \right]^{\alpha_\gamma - \beta_\gamma} e^{\beta_\gamma - \alpha_\gamma}
\]
(13)

Which results in an effective break energy that is independent of the spectral indices:
\[
\bar{\epsilon}_\gamma = \frac{\epsilon^b}{e}
\]
(14)

where \(e\) is Euler’s number. Figure II also shows approximation B with the same parameters as before.

5. Effect of expected number of events

Given a neutrino spectrum \(dN_\nu/dE_\nu\), the expected number of events in a neutrino telescope is:
\[
N_{\text{evt}} = A^\mu \int_{E_{\text{min}}^\mu}^{E_{\text{max}}} dE P_\mu(E_\nu; E_{\text{min}}^\mu) S(E_\nu, \theta) \frac{dN_\nu}{dE_\nu}
\]
(15)

where \(A^\mu\) is the muon effective area (1 km\(^2\) for IceCube/KM3NET), \(P_\mu(E_\nu; E_{\text{min}}^\mu)\) is the probability of a neutrino of energy \(E_\nu\) to produce a muon with energy equal or greater than the neutrino telescope threshold \(E_{\text{min}}^\mu\) (we assume 100 GeV) and \(S(E_\nu, \theta)\) is Earth’s attenuation. The probability \(P_\mu(E_\nu; E_{\text{min}}^\mu)\) is given by:
\[
P_\mu(E_\nu; E_{\text{min}}^\mu) = N_A \sigma_{cc}(E_\nu) < R_\mu(E_\nu; E_{\text{min}}^\mu),
\]
(16)

where \(< R_\mu(E_\nu; E_{\text{min}}^\mu) >\) is the average muon range given a neutrino energy \(E_\nu\) and a muon threshold \(E_{\text{min}}^\mu\). Earth’s attenuation factor is given by:
\[
S(E_\nu, \theta) = e^{-z(\theta)N_A \sigma_T(E_\nu)}
\]
(17)

where \(z(\theta)\) is Earth’s column density as a function of angle and \(\sigma_T\) is the total \(\nu\)-matter cross-section.
We have calculated the expected number of events for a km$^3$ detector. We have used CTEQ5 for the neutrino-matter cross-section (Lai et al 2000). We follow the calculation by (Lipari & Stanev 1991) of average muon range. The Earth column density is taken from the Preliminary Earth Reference Model (Dziewonski & Anderson 1981).

Using approximations A and B and the neutrino spectrum derived from the Band function, we have studied a GRB with a photon break energy $\epsilon_b^\gamma = 300$ keV, located at a redshift $z=1$ and with Lorentz bulk boost $\Gamma = 300$. We also set the spectral indices to $\alpha_\gamma = -2$ and $\beta_\gamma = -1$. We have normalized the neutrino fluence of all three spectra to the same (arbitrary) value as described in section §3. For this GRB the effective break energy is $\bar{\epsilon}_\gamma = 110$ keV. The neutrino break energy for approximation A is $5.98 \times 10^{14}eV$ and for approximation B it’s $1.63 \times 10^{15}eV$. For all cases we have fixed the synchrotron energy break at $10^{16}eV$. Figure 2 shows the three neutrino spectra. Again it is clear that approximation A is inadequate because it overestimates the contribution of low energy neutrinos.

Figure 3 the ratio approximation A to B of expected number of events as a function $\cos(\theta)$ for this example GRB. For all GRB locations in the sky we see that approximation A overestimates the expected number of events. For steeper angles the ratio is larger, because for approximation B the characteristic neutrino energy is higher and therefore Earth’s attenuation is higher.

### 6. Discussion

We have shown that the usual choice to describe the photon spectra in the calculation of neutrino fluxes from GRBs overestimates the contribution of high energy photons (and therefore contribution of low energy neutrinos is overestimated). This results in a higher expected event rate by a factor of $\approx 2$ for all models that make this assumption. The actual value of the overestimation of the expected number of events depends on the matter column depth that neutrinos must cross through Earth in the direction of the GRB. Also we have shown that the typical neutrino energy is $\approx 10^{15}eV$. The characteristic energy of neutrinos is a factor of $e$ larger than the values obtained by Guetta et al (2004) and a factor of 10 than the average value used by Waxman & Bahcall (1997).

For back of the envelope calculations we provide a new approximation to the Band function that is adequate for the computation of neutrino spectra.

Kashti & Waxman (2005) have discussed the effects of muon and pion energy losses leading to neutrino flavor flux ratios (at Earth) different than 1:1:1. Previous AMANDA searches for neutrinos (Achterberg et al 2007a,b), based on Waxman-Bahcall-like assumptions have argued that this effect is not important experimentally because the typical energy of the neutrinos detected is close to the neutrino break energy. Because, as we have shown, the effective neutrino break energy...
is close to the typical synchrotron energy break, the effects described by Kashti & Waxman (2005) must indeed be relevant.

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Fig. 1.— The band function is compared to approximations A and B. The three plots show \( dN_\gamma/dE_\gamma \), \( E dN_\gamma/dE_\gamma \) and \( E^2 dN_\gamma/dE_\gamma \). The Integral of \( E dN_\gamma/dE_\gamma \) (fluence) for approximations A and B has been normalized to the fluence of the band function. For this example we have chosen \( A_\gamma = 10^{-2} \) photons \( \text{cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \), \( \alpha_\gamma = -1 \), \( \beta_\gamma = -2 \) and \( \epsilon_\gamma^b = 300 \) keV. For each plot, the two vertical lines indicate \( \epsilon_\gamma^b \) (right line) and \( \bar{\epsilon}_\gamma \) (left line).

Fig. 2.— Neutrino spectra corresponding to the Band function, approximation A and B with the parameters used in Fig. 1. The normalization \( A_\nu \) was chosen arbitrarily, but for approximations A and B the neutrino fluence, ignoring synchrotron radiation, is the same for all three spectra. The neutrino break energy is \( 5.98 \times 10^{14} \) eV for the neutrino spectra derived from the Band function and for the one derived from approximation A and \( 1.62 \times 10^{15} \) eV for the one derived from approximation B. The synchrotron energy loss has been arbitrarily set to \( 10^{16} \) eV.
Fig. 3.— Left: Ratio of expected number of events for approximation A to the neutrino spectrum derived from the Band function as a function of angle. The parameters chosen for the test GRB match those of Fig. 1 and 2. For $\cos(\theta) = -1$ neutrinos go across Earth, while for $\cos(\theta) = 0$ neutrinos glance Earth along the horizon. The structure in the plot is correlated with the density profile of Earth as a function of depth, e.g. Earth’s core effect is seen between $\cos(\theta) \approx -1$ and $-0.85$. Right: Ratio of expected number of events for approximation B to the spectrum derived using the Band function.