Testing Supersymmetry in Weak Decays by Means of Time Reversal Invariance

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Abstract

The minimal supersymmetric extension of the standard model allows for some of the coupling strengths to be complex parameters. The presence of such imaginary phases can lead to violations of time reversal invariance, which can be tested if correlations in products of an odd number of polarizations and momenta are measured and found to be different from zero. As an example, we consider the triple product $J \cdot (p_1 \times p_2)$ in the $\beta$-decay of the neutron, of the $\Sigma^-$, and in the decay $K_{3\mu}^+$. For these low-energy decays, we find that the present experimental precision is not enough to provide useful bounds on combinations of such phases and the masses of the supersymmetric particles. At higher energies, the same time reversal violating correlation in the semileptonic decay of the $t$ quark is of the order of $\alpha_s/\pi$, made bigger by the large mass of the decaying quark.
1. The presence of coupling strengths, which cannot be made real by a suitable redefinition of the particle fields, is a feature of any, and in particular of the minimal, supersymmetric extension of the standard model [1].

An immediate physical consequence of the irreducible complex nature of such a Lagrangian is that time reversal invariance can be violated—and CP conservation as well if, as we assume here, CPT invariance holds—to a larger degree than in the standard model.

Such a violation, if present, gives to otherwise vanishing observables a finite size which can be probed in low-energy experiments of sufficient precision. The first example that comes to mind is, probably, a non-vanishing electric dipole moment of the electron and the neutron [2]. A distinct possibility, and the one we consider in this letter, is represented by correlations of an odd number of polarizations and momenta, an example of which is the triple product

\[ J \cdot (p_1 \times p_2) , \]  

measuring the polarization transverse to the plane of the two momenta. The quantity (1) is odd under the action of the time reversal operator \( T \) because both the momenta \( p_i \) and the polarization \( J \) change sign; it is even under the parity operator \( P \), which changes the sign of the momenta only.

Such \( T \)-odd correlations have been looked for in nuclear physics [3] as clues to new dynamical laws and with substantial improvements in the accuracy of the measurements over the years. It is then interesting, we believe, to try to establish how much of their size could in principle originate from the minimal supersymmetric extension of the standard model.

This kind of enquiry, in which the supersymmetric sector manifests itself only by radiative corrections in the low-energy interactions of ordinary particles, trades the high energies required in actually producing the supersymmetric partners for the high precision needed to probe their effect in loop corrections.

On the other hand, because these \( T \)-odd correlations induced by supersymmetry are suppressed by powers of the ratio between the mass of the polarized particle and the heaviest supersymmetric mass in the loop, it is interesting to look also for semileptonic decays at higher energies, where more massive particles are available. Here we have in mind, above all, the \( t \) quark, the mass of which now seems to be of the same order as the supersymmetric scale [4]. Therefore, the production of these
quarks at the LHC and SSC will open new possibilities for studying correlations such as (1) and thus detect the effect, if any, of supersymmetry.

2. It is useful, before discussing any physical process, to briefly recall to what extent the non-vanishing of $T$-odd correlations such as (1) is a genuine signal of time reversal invariance violations. Because of the anti-unitary nature of the time reversal operator $T$ \[5\], once loop amplitudes are included in the computation, it is not necessarily true that $T$-odd observables violate time reversal invariance. The reason can be readily understood as follows. The $T$-odd observable originates, for example, in the interference between the imaginary part of a one-loop amplitude and a tree diagram, which is real. However, such an imaginary part can occur in two distinct ways: either directly, from the imaginary phases in the coupling strengths, or, indirectly, from the imaginary part of the loop integral. This latter unitary phase—sometime also referred to as the final state interactions—contributes to any $T$-odd observable without representing a violation of time reversal invariance and therefore constitutes a sort of background that has to be taken into account \[6\]. As we shall see, a direct estimate of these effects is possible.

We also notice that at the supercolliders, the semileptonic decays we are interested in come together with their $CP$ conjugate, as in the reaction \[q\bar{q} \rightarrow t\bar{t}\], which gives both $b\ell^-\bar{\nu}_\ell$ and $b\ell^+\nu_\ell$ as final states; it is therefore possible to take the difference between the two $CP$-conjugate cross sections in such a way that the unitary phase background (which is the same for the two decays) is eliminated \[7\].

3. Let us now consider the minimal supersymmetric extension of the standard model.

We neglect generation mixing. Hence, only three terms in the supersymmetric Lagrangian can give rise to $CP$-violating phases which cannot be rotated away \[8\]: The superpotential contains a complex coefficient $\mu$ in the term bilinear in the Higgs superfields. The soft supersymmetry breaking operators introduce two further complex terms, the gaugino masses $\tilde{M}_i$ and the left- and right-handed squark mixing term $A_q$. We consider only the latter two, which are carried by truly supersymmetric particles, and leave out the additional contribution of the Higgs sector.
As long as we are not committed to any specific model of supersymmetry breaking, it is not important to evaluate all possible diagrams which can give a contribution. Among them, we concentrate on the gluino Penguin diagram, in which a gluino line is attached to two scalar-quark vertices (see Fig.1). We leave out contributions in which neutralinos and charginos enter the loop, they are slightly suppressed by a factor $\alpha_w/\alpha_s$ with respect to the gluino loop.

The squark mass eigenstates $\tilde{q}_{\alpha,n}^j$ are related to the weak eigenstates $\tilde{q}_{\alpha,L}^j$ and $\tilde{q}_{\alpha,R}^j$ through the mixing matrix:

$$
\tilde{q}_{\alpha,L}^j = \exp(-i\phi_{A_q}/2) \left[ \cos \theta \tilde{q}_{\alpha,1}^j + \sin \theta \tilde{q}_{\alpha,2}^j \right] = \sum_m a_m^L \tilde{q}_{\alpha,m}^j
$$

$$
\tilde{q}_{\alpha,R}^j = \exp(i\phi_{A_q}/2) \left[ \cos \theta \tilde{q}_{\alpha,2}^j - \sin \theta \tilde{q}_{\alpha,1}^j \right] = \sum_n a_n^R \tilde{q}_{\alpha,n}^j
$$

where

$$
\tan 2\theta = \frac{2|A_q|m_q}{(L^2 - R^2)\tilde{m}}
$$

and

$$
A_q\tilde{m}m_q = \xi_q v_2 + \mu^* h_q v_1 \quad A_q = |A_q| \exp i\phi_{A_q}.
$$

$\tilde{m}L$ and $\tilde{m}R$ are the squark mass parameters, $v_i$ the vacuum expectation values of the Higgses, and $\xi_q$ the coefficient in the cubic term of the soft breaking operator. The diagonalization of the squark masses gives the eigenvalues

$$
\tilde{m}_{1,2}^2 = \frac{1}{2} \left\{ (L^2 + R^2)\tilde{m}^2 + 2m_q^2 \mp \left[ (L^2 - R^2)^2\tilde{m}^4 + 4m_q^2|A_q|^2\tilde{m}^2 \right]^{1/2} \right\}.
$$

The gluino majorana mass

$$
\tilde{M}_g = \tilde{m}_g \exp(i\phi_g)
$$

gives an additional phase shift once it has been rotated into the interaction to make the masses real.

The relevant terms in the Lagrangian are the following two:

$$
L_{\tilde{q}\tilde{q}W} = -\frac{ig_w}{\sqrt{2}} W^-_{\alpha} \left( \tilde{d}_L^* \tilde{q}_{L}^\alpha \tilde{u}_L \right) + h.c. ,
$$

$$
L_{\tilde{q}\tilde{g}} = \frac{g_s}{\sqrt{2}} T_{jk}^a \sum_{\alpha=u,d} \left[ \tilde{g}_a^L (1 - \gamma_5) \tilde{q}_{\alpha,m}^k \Gamma_a L \tilde{q}_{\alpha,m}^j + \tilde{q}_{\alpha,m}^j (1 + \gamma_5) \tilde{g}_a^L \Gamma_a^{\mu} \tilde{q}_{\alpha,m}^k 
- \tilde{g}_a^R (1 + \gamma_5) \tilde{q}_{\alpha,n}^k \Gamma_a^{\mu} \tilde{q}_{\alpha,n}^j - \tilde{q}_{\alpha,m}^j (1 - \gamma_5) \tilde{g}_a^R \Gamma_a^{\mu} \tilde{q}_{\alpha,n}^k \right]
$$
where
\[ \Gamma^m_L = a^L_m \exp(-i\phi_g) \quad \Gamma^n_R = a^R_n \exp(-i\phi_g). \] (10)

Both the \( \Gamma^m_L \) and \( \Gamma^n_R \) are determined by the supersymmetry breaking mechanism and are in general complex numbers \[1, \tilde{8}\] that cannot be made real by a redefinition of the phases. However, the presence of such an imaginary phase is not sufficient. Inasmuch as we want to compute terms proportional to the polarization of one of the external fermions, also the chiral structure of the diagram is important. It must be such that the fermion of which we measure the polarization changes its helicity as compared to the tree level diagram. The relevant diagrams are depicted in Fig.2, with crosses representing the point of chirality flip due to the mass term; there are then three of them.

Yet, these one-loop diagrams by themselves cannot contribute to a correlation such as (11) because there are not enough independent momenta. In fact, (11) is nothing but the covariant quantity
\[ \epsilon^{\alpha\beta\gamma\delta} J_{\alpha} P_{\beta} P_{\gamma} P_{\delta} \] (11)
in the rest frame of one of the momenta. Clearly, (11) requires at least three independent momenta. It is therefore necessary to insert the diagrams of Fig.1 in some decay or scattering process.

To summarize: in order for a Feynman diagram to lead, by interference with the tree diagram, to a time reversal violation in the cross section, it must satisfy two independent requirements: it must contain an imaginary phase and it must flip chirality with respect to the tree level diagram.

4. We now turn to a specific example, namely the \( \beta \)-decay of the neutron, which we would like to analyze in some detail because it can be used as a template for the other processes we are interested in.

Since we are after new physics induced by the supersymmetric sector, we must first of all estimate the potential contribution of the standard model itself. The imaginary phase, which comes from the three-family quark mixing in the corresponding one-loop diagram, is suppressed by the unitary constraint, which makes the leading contribution roughly eight orders of magnitude smaller than the current experimental bounds.
Next, it is important to isolate the unitary phase background. It originates in the final state interactions, which in this case are only electromagnetic. Even though one would expect them to be of the order of \( Z \alpha \), this turns out to be incorrect because for the standard model and its minimal supersymmetric extension, and for any chiral theory, such a correction vanishes and the first unitary phase comes only as a recoil effect of order \( Z \alpha E_e/M \), where \( M \) is the mass of the nucleon or of the nucleus. It has been estimated to be \( 2.6 \times 10^{-4} \). 

Because the error in the best experimental bound is still larger than the unitary phase background, we can compare the supersymmetric result directly with the experiments.

Let us now insert the diagrams of Fig.2 in the \( \beta \)-decay amplitude

\[
n \rightarrow pe\bar{\nu}_e .
\]

As we have pointed out, only the chiral structure in which a right-handed fermion ends up as a left-handed one is needed. The potentially relevant terms in the vertex are thus:

\[
\bar{u}(p_p) \Gamma^\mu u(p_n) = \bar{u}(p_p) \left[ (1 + \gamma_5) P^\mu A + (1 - \gamma_5) P^\mu B \right. + \left. \gamma^\mu (1 + \gamma_5) C \right] u(p_n),
\]

where \( P \equiv p_p + p_n \).

Unfortunately, a direct evaluation of the hadronic matrix elements of these operators is not possible. Naive dimensional analysis suggests that they are of the same order as the ones computed by means of quark matrix elements.

In the \( \beta \)-decay of the neutron all three operators are about of the same order because of the closeness of the masses of the \( u \) and \( d \) quark. The Penguin diagram gives

\[
A \equiv -i g_w \sin \theta \sum_{m,n} \Gamma^m \Gamma^n \left[ I_0^{m,n} - I_1^{m,n} \right] \xi_A ,
\]

\[
B \equiv -i g_w \sin \theta \sum_{m,n} \Gamma^m \Gamma^n \left[ I_0^{m,n} - I_1^{m,n} \right] \xi_B ,
\]

and

\[
C \equiv -i g_w \sum_{m,n} \Gamma^m \Gamma^n \left[ I_2^{m,n} \right] \xi_C .
\]
The coefficients $I_{0}^{m,n}$ and $I_{1}^{m,n}$, which appear in (14) and (15), depend on the masses of the gluinos and squarks (indices $n$ and $m$); they come from the integration over the loop momentum and are defined as follows:

$$I_{0}^{m,n} = i\pi^2 \int_{0}^{1} dx \int_{0}^{1} dy \frac{1}{M^2 - p^2}$$

$$I_{1}^{m,n} = i\pi^2 \int_{0}^{1} dx \int_{0}^{1} dy \frac{1 - xy}{M^2 - p^2},$$

where

$$p^\mu = (1 - x)p^\mu_u + x(1 - y)p^\mu_d$$

$$M^2 = x(1 - y)(m^2_u - \tilde{m}^2_n) - (m^2_u - (1 - x)\tilde{m}^2_m) + xy\tilde{m}^2_g.$$ (18)

Similarly, $I_2^{m,n}$ comes from the finite part of the vertex renormalization and is given by

$$I_2^{m,n} = i\pi^2 \int dx \int dy \ln \frac{M^2 - p^2}{4\pi\mu^2},$$ (19)

where $\mu$ is the renormalization point.

The coefficients $\xi$’s come from the renormalization group running of these operators in going from the supersymmetric scale—at which we have computed the loop correction—to the nuclear scale. They can be estimated by means of their anomalous dimension [10]. They are however of order of one and we will simply carry a common factor $\xi$ to indicate this correction.

The relevant cross section can now be computed and is

$$\frac{d\Gamma}{dE_c d\cos\theta_{\nu_e}} = \frac{C_{\mu}^2}{\pi^3} E^2 E^2_{\nu_e} \left[ 1 + D \mathbf{n} \cdot \frac{\mathbf{P}_e \times \mathbf{P}_{\nu_e}}{E_c E_{\nu_e}} \right],$$ (20)

where $E_{\nu_e} = m_n - m_p - E_e$. In (20) the first term is the leading tree level contribution and the second one, the one we are interested in, is obtained by the interference of the one-loop amplitude and the tree. This term violates time reversal invariance. The coefficient in front of it is defined to be

$$D \equiv \frac{2\sqrt{2}}{g_w} \left\{ m_N \text{Im} \mathcal{A} - m_N \text{Im} \mathcal{B} + \text{Im} \mathcal{C} \right\}$$ (21)

with $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ given respectively by (14), (15) and (16) in the framework of the minimal supersymmetric extension of the standard model. $\mathbf{n} \equiv J/J$ points to the direction of the polarization of the decaying neutron.
In order to obtain an overall estimate of the effect, we consider the case in which the squark masses $\tilde{m}_1$ and $\tilde{m}_2$ are almost degenerate and equal to $\tilde{m}_g \simeq \tilde{M}$. Because of the orthogonality of the coefficients $\Gamma^{L,R}$ we must however keep the dependence on the masses before summing over $m$ and $n$. This gives a factor

$$\frac{\tilde{m}_2^2 - \tilde{m}_1^2}{M^4} \simeq \cos \theta \sin \theta \frac{2|A_g|m_q}{M^3}$$

in front of the integrals, that are now dimensionless quantity that can be computed. In the same approximation $\cos \theta \sin \theta = 1/2$.

This way, we obtain

$$D \simeq \frac{\alpha_s}{12\pi} \xi \left[ \left( \frac{m_N m_d}{M^2} \right) |A_d| \sin (\phi_{A_d} - \phi_g) + \left( \frac{m_N m_u}{M^2} \right) |A_u| \sin (\phi_{A_u} - \phi_g) \right. + \left. \left( \frac{m_u m_d}{M^2} \right) |A_u||A_d| \sin (\phi_{A_d} - \phi_{A_u}) \right].$$

(23)

The result (23) can be compared with the best experimental bound available, which is [11]:

$$D = (4 \pm 8) \times 10^{-4},$$

(24)

obtained in the nuclear decay $^{19}\text{Ne} \rightarrow ^{19}\text{F}$. For a typical supersymmetric mass of 100 GeV, and $\alpha_s \simeq .1$, the estimate (23) is too small to be useful. Notice that the phases $\phi_{A_d} - \phi_g$ and $\phi_{A_u} - \phi_g$ are already constrained to be smaller than $10^{-3}$ from bounds on the electric dipole moment of the neutron [12]. The phase $\phi_{A_d} - \phi_{A_u}$ is unconstrained but the coefficient in front of it is even more suppressed than the other two by the two quark masses.

5. As we have seen, the smallness of the $u$ and $d$ quark masses makes the observable (1) small. An improvement can be found in the weak decay of the strange baryons, because of the larger mass of the $s$ quark. For example, the observable (1) in the decay

$$\Sigma^- \rightarrow n e^- \bar{\nu}_e$$

(25)

has been studied [13]. The experimental bound is however far from been as good as for the $\beta$-decay and reads:

$$D = .11 \pm .10,$$

(26)

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to be compared to our estimate that in this case is
\[ D \simeq \frac{\alpha_s}{12\pi} \xi \left( \frac{m_{\Sigma} m_s}{M^2} \right) |A_s| \sin (\phi_{A_s} - \phi_g) \] (27)
which, even though it is almost one hundred times bigger than before, it is still five order of magnitude smaller than (26).

6. Another low-energy process in which supersymmetric loop corrections can play a role is the decay \( K^{+} \rightarrow \pi \mu \nu \mu \), in which
\[ K \rightarrow \pi \mu \nu \mu . \] (28)
The experimental bound on the coefficient of \( J_\mu \cdot (p_\mu \times p_\pi) \) in this case is [14]:
\[ D = (-3.0 \pm 4.7) \times 10^{-3} . \] (29)
The supersymmetric effect we have computed is
\[ \frac{\alpha_s}{12\pi} \xi \left( \frac{m_K m_s}{M^2} \right) |A_s| \sin (\phi_{A_s} - \phi_g) , \] (30)
which is roughly three order of magnitude too small. Nevertheless, such a semileptonic decay will be an interesting candidate for new bounds as there is going to be an improvement in the experimental sensitiveness [15].

7. Up to this point, we have considered \( T \)-odd correlations of polarizations and momenta only in low-energy processes to show that they are potentially interesting probes of new physics beyond the standard model and, in particular, of its minimal supersymmetric extension.

Such low-energy experiments should be pursued and considered as complementary to accelerator physics because a substantial improvement in the measurements could provide new and useful bounds on imaginary phases and supersymmetric masses.

At the same time, our results show that the decay of a heavier particle would have the advantage of being less suppressed by the mass ratio to which \( D \) is proportional.
The production of $t$ quarks at the LHC and SSC will make it possible to study the semileptonic decay

$$t \rightarrow b\bar{\nu}_l$$

and therefore test time reversal invariance by means of the observable

$$J_t \cdot \left( p_t \times p_{\nu_l} \right),$$

as in the $\beta$-decay, where now $J_t$ is the polarization of the $t$ quark [16].

The unitary background for this process has been recently estimated to be around $10^{-3} - 10^{-4}$ for $m_t = 100 - 200$ GeV [17].

The large mass of the $t$ quark greatly enhances the effect of supersymmetry, whereas the standard model contribution remains negligible. The computation of the supersymmetric contribution to (32) proceeds—but for replacing the $d$ by the $t$ quark and the $u$ by the $b$ quark—along the same lines as for the $\beta$-decay but because of the small mass of the $b$ quark with respect to the $t$, only the operator proportional to $A$ in (13) contributes. The $t$ quark is not expected to hadronize before decaying.

By retracing our steps in the previous sections, we obtain that

$$D \simeq \frac{\alpha_s}{12\pi} \left( \frac{m_t}{M} \right)^2 \left[ 1 + \frac{E_{\nu}}{m_t} (1 - \cos \vartheta) \right] \left[ 1 - 2 \frac{2E_{\nu}}{m_t} \right] |A_t| \sin (\phi_{A_t} - \phi_g),$$

where the terms in the square brackets come from the kinematics that cannot be neglected as we did before for the low-energy processes, $\vartheta$ being the angle between the lepton momenta.

The mass $m_t$ is larger than 94 GeV [4] and therefore (33) is roughly $\alpha_s/\pi$ for maximal $CP$ violation; a result that makes the decay of the $t$ quark a very promising candidate in the search for supersymmetric physics beyond the standard model by means of time reversal violations.

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Fig. 1: The gluino Penguin diagram.

Fig. 2: The relevant diagrams for, respectively, the term proportional to $A$, $C$ and $B$. Crosses denote the change of chirality because of the mass operator.