Constraints From Extended Supersymmetry in Quantum Mechanics

Sonia Paban*¹, Savdeep Sethi*² and Mark Stern†³

*School of Natural Sciences
Institute for Advanced Study
Princeton, NJ 08540, USA

and

†Department of Mathematics
Duke University
Durham, NC 27706, USA

We consider quantum mechanical gauge theories with sixteen supersymmetries. The Hamiltonians or Lagrangians characterizing these theories can contain higher derivative terms. In the operator approach, we show that the free theory is essentially the unique abelian theory with up to four derivatives in the following sense: any small deformation of the free theory, which preserves the supersymmetries, can be gauged away by a unitary conjugation. We also present a method for deriving constraints on terms appearing in an effective Lagrangian. We apply this method to the effective Lagrangian describing the dynamics of two well-separated clusters of D0-branes. As a result, we prove a non-renormalization theorem for the $v^4$ interaction.
1. Introduction

Quantum mechanical theories play an important role in recent attempts to define M theory and field theories in various dimensions. One of the more important systems is the quantum mechanical gauge theory that describes the low-energy dynamics of zero-branes in type IIA string theory [1,2]. The system can be obtained by a dimensional reduction of supersymmetric Yang-Mills from ten dimensions [3]. The theory has sixteen supersymmetries and a $U(N)$ gauge symmetry. BFSS have conjectured that this matrix model describes M theory in eleven dimensions in a limit where $N \to \infty$ [4]. For finite $N$, the matrix model is believed to describe M theory quantized in the discrete light-cone formalism (DLCQ) [5,6].

The full gauge theories that appear in these matrix models are difficult to study, particularly when $N$ becomes large. Fortunately, a key feature of these gauge theories is the existence of flat directions on which scattering states localize. For many of the questions that we might wish to answer, it is sufficient to control the physics of the light modes propagating along these flat directions. There are two distinct ways to go about analyzing the effective dynamics on the flat directions: in an operator approach, we can use an integration procedure of the sort developed in [7] and further developed in [8,9]. This integration procedure requires some knowledge of the bound state wavefunctions, which makes it difficult to extend to large $N$.

A second approach involves the perturbative construction of an effective Lagrangian in a velocity expansion. To date, there have been a number of computations of loop corrections to the bosonic part of the effective action [10,11,12], which takes the form:

$$S = \int dt \left( f_1(r)v^2 + f_2(r)v^4 + \ldots \right).$$  \hspace{1cm} (1.1)

To order $v^2$, this theory can be canonically quantized so there is always an effective Hamiltonian corresponding to this effective Lagrangian. This is generally not the case when higher velocity terms are included. To obtain a supersymmetric completion of (1.1), we typically need to add terms involving accelerations and terms with more than a single time derivative acting on a fermion. In this case, the Lagrangian is not related to an unconstrained supersymmetric Hamiltonian in any straightforward way.

The aim of this paper is to explore the extent to which supersymmetry constrains both the form of the Hamiltonian and the terms that can appear in the effective Lagrangian (1.1). At order $v^2$, the two questions are closely related and in the Lagrangian approach,
we need to determine which metrics, specified by $f_1$, are compatible with supersymmetry. We will show in section two that supersymmetry actually constrains the metric to be flat.\footnote{A fact often stated in the literature, but proven in no previous work of which we are aware.}

Before going further, it is worth clarifying some points. The low-energy description of a gauge theory differs from a conventional sigma model. In more conventional sigma models, the fermions are sections of the tangent bundle, while in the case that we wish to understand, the fermions transform as sections of the spin bundle. The condition that we have sixteen supersymmetries corresponds to the existence of sixteen Dirac operators which act on the Hilbert space. We will take our Dirac operators or supercharges $Q_a$ to be functions of bosons $x^i$ and fermions $\psi_a$, where $i = 1, \ldots, 9$ and $a = 1, \ldots, 16$. If we use $V$ to denote the representation space of $Spin(9)$, which is a sixteen-dimensional space, then our wavefunctions are spinors of the bundle $V$. Note that our conclusion about the metric does not rule out the existence of a sigma model with a non-trivial metric, sixteen supersymmetries, and a different fermion content.

For theories with less supersymmetry, the metric is less constrained. With four supersymmetries, the metric is determined by an arbitrary real function of $r$. With eight supersymmetries, it was shown in an interesting paper $[13]$ using superspace techniques that the only allowed metrics are harmonic functions in five dimensions.

The next step is to consider terms to order $v^4$. In the operator approach discussed in section three, we will consider small deformations of the supercharges. We require that these deformations be compatible with a symmetry inherited from the full non-abelian gauge theory. This symmetry is essentially CPT. To this order in a derivative expansion, we will show that any small deformation of the supercharge can actually be gauged away by a unitary transformation. Since we are interested physically in interactions that fall off sufficiently fast at infinity, it is enough to study small deformations of the supercharge. Our results then show that in this class of theories, the free theory is the unique gauge theory with sixteen supersymmetries. It would be interesting to extend these results in two ways: first by considering higher order terms in the derivative expansion and second by considering non-abelian gauge theories.

In the Lagrangian approach, we know from loop computations that there are non-trivial higher order interactions. In section four, we examine constraints on the effective Lagrangian. We wish to know what choices of $f_2(r)$ admit a supersymmetric completion. To find a constraint, we study the eight fermion term in the supersymmetric completion of
This leads to the constraint that \( f_2 \) must be harmonic in nine dimensions which is the desired non-renormalization theorem. This ensures that the interaction between two gravitons in matrix theory agrees with supergravity [4].

This method for finding constraints on the effective action can be extended to both higher velocity terms in the quantum mechanics and to field theories in various dimensions. In four dimensions, a non-renormalization theorem for the four derivative terms in the Yang-Mills effective action was proven in [14] using arguments of a quite different flavor. Our results imply corresponding non-renormalization theorems for all higher dimensional Yang-Mills theories, except three-dimensional Yang-Mills. With sixteen supersymmetries, it might well be the case that there are some restrictions on which terms can appear at every order in the velocity expansion. It would be very interesting to see whether this is the case. Finally, we should point out that our results have close parallels with recent restrictions on higher order corrections to effective actions in string theory [15,16] and field theory [17]. After we completed this project, an interesting paper appeared with further evidence of a non-renormalization theorem for the \( v^4 \) terms [18].

2. Some Generalities

2.1. Constraining the metric

The Spin(9) Clifford algebra can be represented by real symmetric matrices \( \gamma_{ab}^i \), where \( i = 1, \ldots, 9 \) and \( a = 1, \ldots, 16 \). These matrices obey:

\[
\{ \gamma^i, \gamma^j \} = 2 \delta^{ij}.
\]

(2.1)

All the operators that we wish to study can be constructed in terms of the basis \( \{ I, \gamma^i, \gamma^{ij}, \gamma^{ijk}, \gamma^{ijkl} \} \), where we define:

\[
\gamma^{ij} = \frac{1}{2!}(\gamma^i \gamma^j - \gamma^j \gamma^i)
\]

\[
\gamma^{ijk} = \frac{1}{3!}(\gamma^i \gamma^j \gamma^k - \gamma^j \gamma^i \gamma^k + \ldots)
\]

\[
\gamma^{ijkl} = \frac{1}{4!}(\gamma^i \gamma^j \gamma^k \gamma^l - \gamma^j \gamma^i \gamma^k \gamma^l + \ldots).
\]

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2 Three-dimensional Yang-Mills is exceptional because of vector-scalar duality and will be studied elsewhere.
Note that \( \{ I, \gamma^i, \gamma^{ijkl} \} \) are symmetric while \( \{ \gamma^{ij}, \gamma^{ijk} \} \) are antisymmetric. The normalizations in (2.2) are chosen so that the trace of the square of a basis element is \( \pm 16 \).

To fix the metric, it is easiest to work in the Lagrangian approach. The most general \( Spin(9) \) invariant metric takes the form:

\[
d s^2 = g_{\mu \nu} dx^\mu dx^\nu = g_1(r) dx^i dx^i + g_2(r) x^i x^j dx^i dx^j,
\]

(2.3)

The last step in (2.3) is always possible by a coordinate choice. The Lagrangian is then,

\[
L = \int dt f_1(r) v^2 + \ldots
\]

(2.4)

The supersymmetry transformations are highly constrained at this order. Since this system is a special case of a theory with \( N = 4 \) supersymmetry which has a superspace formulation (at this order) [13], the supersymmetry transformations can only be a more restricted form of the \( N = 4 \) transformations. To be compatible with \( Spin(9) \) invariance, the supersymmetry transformations must take the form (at this order):

\[
\delta x^i = -i \epsilon \gamma^i \psi
\]

(2.5)

\[
\delta \psi_a = (\gamma^i v^i \epsilon)_a + (M \epsilon)_a.
\]

The Grassmann parameter \( \epsilon \) is a 16 component real spinor and \( M \) is an order \( v \) expression containing two fermions. To show the metric is flat, we need to show that \( M \) must vanish. The algebra must close on time translations in the Lagrangian approach. Since,

\[
[d_1, d_2] x^i = -i(\epsilon_2 \gamma^i (v^k \gamma^k + M) \epsilon_1 - \epsilon_1 \gamma^i (v^k \gamma^k + M) \epsilon_2)
\]

\[
= -i \epsilon_2 (\{ \gamma^i, \gamma^k \} v^k + \{ \gamma^i M + M^T \gamma^i \}) \epsilon_1
\]

\[
= -2i \epsilon_2 \epsilon_1 v^i - i \epsilon_2 \{ \gamma^i M + M^T \gamma^i \} \epsilon_1,
\]

(2.6)

the second term containing \( M \) must vanish. It is not hard to check that satisfying the condition \( (\gamma^i M + M^T \gamma^i) = 0 \) requires \( M \) to vanish. Therefore the metric must be flat.

2.2. The momentum expansion

Let us now turn to the operator approach. As operators, the sixteen real fermions \( \psi_a \) satisfy the anti-commutation relations,

\[
\{ \psi_a, \psi_b \} = 2 \delta_{ab}.
\]

(2.7)
The supercharge $Q_a$ is hermitian with respect to the flat metric and must obey the algebra,

$$\{Q_a, Q_b\} = 2\delta_{ab}H,$$

(2.8)

with no additional central terms on the right side. The requirement of a flat metric derived in the previous section can also be derived from the requirement that the supercharge satisfy (2.8), but the argument is more involved.

The algebra (2.8) is to be contrasted with quantum mechanical gauge theories with charged fields which close on the Hamiltonian only up to gauge transformations. Obeying the algebra (2.8) provides a strong constraint on the allowed deformations of the charges.

The supercharge $Q_a$ can be expanded in the number of fermions,

$$Q_a = f_{ac}\psi_c + f_{acde}\psi_e\psi_d\psi_c + \ldots,$$

(2.9)

where $f_{acde}$ is antisymmetric in the last three indices etc. Let us introduce momenta conjugate to $x$ obeying the usual relation,

$$[x^i, p_j] = i\delta^i_j.$$

The basic Spin$(9)$ invariant combinations are $(x^2, p^2, x \cdot p)$. Then the most general possible form for $f_{ac}$ is,

$$f_{ac} = \delta_{ac}D + \gamma^i_{ac}D_i + \gamma^{ij}_{ac}D_{ij},$$

(2.10)

where we have specified nothing about the particular dependence of the operators $D, D_i, D_{ij}$ on the momenta. We can then express each of these operators in terms of unknown functions of $(x^2, p^2, x \cdot p)$:

$$D = h_1$$

$$D_i = h_2p_i + h_3x_i$$

$$D_{ij} = h_4(x_ip_j - x_jp_i).$$

(2.11)

Now it should be clear why $\gamma^{ijk}$ did not appear in (2.10). That would require the existence of an operator $D_{ijk}$ anti-symmetric in $i, j, k$ but such an operator cannot be constructed.

It is actually easy to construct charges that satisfy the supersymmetry algebra. For example, we could simply set $D_i = D_{ij} = 0$ and take any hermitian $D$. The resulting Hamiltonian $H = D^2$ contains no fermion terms. These are essentially trivial solutions and we would like to rule out these cases. Actually, the charges that appear in physical
contexts, such as the abelian theories describing the low-energy dynamics of quantum mechanical gauge theories, usually have $D = D_{ij} = 0$. We will not, however, impose so strong a constraint quite yet. Rather, to rule out uninteresting solutions, we will require $h_2$ to be non-zero.

We will also eventually require that as $|x| \to \infty$, the supercharge reduce to the charge $Q^0_a$ for a $U(1)$ gauge theory:

$$Q^0_a = \gamma^i_{ac} \psi_c p^i.$$  \hfill (2.12)

This weak restriction is completely natural for most physical models, where interactions become weak as the distances become large.

The functions $h_i$ can be expanded in powers of $p$. The usual counting parameter is the number of momenta plus half the number of fermions. This gives an expansion in powers of $h$ which we have generally set to one. For example to lowest order in $h$, the supercharge is schematically,

$$Q \sim \psi p + \psi^3 + \psi h,$$

where we have included the possibility of a static potential with the $h$ term. Let us conveniently normalize our supercharges so that these lowest order terms are $O(h)$. Then every additional momentum brings a power of $h$ as do every two fermions. If there are non-trivial choices for some of the functions in (2.11) then there will exist non-trivial solutions for $Q$ which satisfy the supersymmetry algebra. What do we mean by a non-trivial solution? At first sight, the answer seems self-evident, but that is actually not the case.

2.3. Gauge transformations

We need to discuss a class of deformations of the supercharge that do not change the physical system. The deformations correspond to unitary transformations of the supercharge,

$$Q_a \to e^{iC} Q_a e^{-iC},$$  \hfill (2.13)

where $C$ is hermitian. Any conjugation of this kind automatically gives a new set of charges that satisfy the supersymmetry algebra. To leading order in $C$, this transformation takes the form:

$$Q_a \to Q_a + i [C, Q_a].$$  \hfill (2.14)
For example, the charge

\[ Q_a = (\gamma^i \psi)_a (p_i - p_j f_i p_j), \]  
(2.15)

is hermitian and obeys the algebra,

\[ \{ Q_a, Q_b \} = 2 \delta_{ab} (p^2 - 2p_j f_i p_j + ip_j f_i p_j) + \ldots, \]  
(2.16)

for any \( f_i = \partial_i f(r) \) and where the omitted terms are higher order in \( \hbar \). However, to leading order in \( \hbar \), this deformation can be written as a gauge transformation where \( C = p_j f p_j \) and so does not result in a physically distinct system. We should therefore only consider deformations modulo these ‘gauge’ transformations.

We can consider the possible \( C \) operators both compatible with \( Spin(9) \) invariance and first order in powers of momenta. Up to first order, the only allowed operators are:

\[ C = p_i k_i + k_i p_i + t(r). \]  
(2.17)

As usual, \( k_i \) can written as \( x_i k(r) \) for some radial function \( k \). This deformation seems to generate a ‘metric’ at leading order (2.14) but the rescaling of the kinetic term is compensated by the induced scalar potential so the physical metric is unchanged. The key point is that we will meet certain deformations like (2.15) which appear to give new physical systems where the Hamiltonian has terms higher order in momenta, but are actually gauge transformations. Our goal is to study deformations of the supercharge which cannot be undone by some unitary transformation.

3. Constraining the Supercharge

3.1. The terms leading in \( \hbar \)

To study the leading order terms, we can truncate the expansion (2.9) at three fermions. Note that we will use the fact that the Hilbert space metric is flat; if this were not the case then, for example, \( p_i \) would not be hermitian. The three fermion term cannot appear with any momentum operators at this order. We can then plug the charge into the supersymmetry algebra to obtain,

\[ \{ Q_a, Q_b \} = \{ f_{ac}, f_{bc} \} - 6 \{ f_{acde}, f_{bcde} \} \]
\[ + ([f_{ak}, f_{bl}] + 3 \{ f_{ac}, f_{bcket} \} + 3 \{ f_{bc}, f_{ackl} \} - 18 [f_{amnk}, f_{bmn}] \) \psi_{kl} \]
\[ + ([f_{ak}, f_{bmn}] + [f_{bk}, f_{amnl}] + 9 \{ f_{ackl}, f_{bcmn} \}) \psi_{klmn} \]
\[ + [f_{acde}, f_{bmn}] \psi_{cdeemr} \].

(3.1)
A summation over repeated indices is assumed and we have defined:

\[ \psi_{kl} = \frac{1}{2!}[\psi_k, \psi_l] \]
\[ \psi_{klm} = \frac{1}{3!}(\psi_k \psi_l \psi_m - \psi_k \psi_l \psi_m - \psi_k \psi_l \psi_m + \ldots) \]
\[ \psi_{klmn} = \frac{1}{4!}(\psi_k \psi_l \psi_m \psi_n - \psi_k \psi_l \psi_m \psi_n + \ldots) \]

We wish to answer the following question: what are the allowed operators in (2.9)? Note that the six fermion term and the \([f_{amnk}, f_{bml}]\) contribution to the two fermion term in (3.1) automatically vanish at this order.

To evaluate the terms appearing in (3.1), we can use the following observation. The right hand side is an operator \(T_{ab}\) symmetric in \(a\) and \(b\). We can therefore expand it in basis elements,

\[ T_{ab} = \frac{1}{16}\left(\delta_{ab}\delta_{cd}T_{cd} + \gamma^{ij}_a\gamma^{ij}_bT_{cd} + \frac{1}{4!}\gamma^{ijkl}_a\gamma^{ijkl}_bT_{cd}\right). \]

An analogous expansion can be performed for antisymmetric operators in terms of \((\gamma^{ij}, \gamma^{ijk})\). The constraints come from requiring that the coefficients of \((\gamma^i, \gamma^{ijkl})\) vanish. We can start by evaluating the first term with no fermions in (3.1),

\[ \{f_{ac}, f_{bc}\} = 2\delta_{ab}(D^2 + D_iD_i + D_{ij}D_{ij}) + 2\gamma^i_{ab}\{D_j, \delta_{ij}D + 2D_{ij}\}. \]

To get a constraint, we need to say something about the three fermion term. Let us make a few general remarks. A tensor, \(T_{ab_1\ldots b_{2k+1}}\), antisymmetric in the \(b_j\) indices can be expressed in the following convenient way:

\[ T_{ab_1\ldots b_{2k+1}} = (B_{a[1}^{I_{1}} A_{b_2}^{I_2} \ldots A_{b_{2k+1]}^{I_{k+1}}}^{I_{k+1}})_{I_1I_2\ldots I_{k+1}}. \]

In this expansion, \(A \in (\gamma^{ij}, \gamma^{ijk})\) while \(B\) is any basis element. The index \(I_j\) stands for a collection of Spin(9) vector indices. In particular, the term \(f_{abcd}\) can be written:

\[ f_{abcd} = B_{a[1}^{I_{ij}} \gamma_{cd]}^{ij} t_{I_1I_2} + B_{a[1}^{I_{ij}'}} \gamma_{cd]}^{ij} t_{I_1I_2}' \]

Let us demand that our supercharges be invariant under the symmetry CPT which acts as complex conjugation and sends,

\[ x \rightarrow -x \quad \quad p \rightarrow p. \]
We can now list the possible structures that can appear in the first term of (3.4),

\[(1) \quad x^j \gamma^i_{a[b} \gamma^{j]}_{c]d}] \]
\[(2) \quad \gamma^i_{a[b} \gamma^{j}_{c]d}] \]
\[(3) \quad x^k \gamma^{ijk}_{a[b} \gamma^{j]}_{c]d}] \]

where we recall that no momenta can appear in (3.4) at this order. Using the identities in Appendix A, we see that (2) vanishes and that (1) and (3) are proportional. Similarly, the possible structures for the second term in (3.4) take the form,

\[(1') \quad x^k \gamma^{ijkl}_{a[b} \gamma^{j]}_{c]d}] \]
\[(2') \quad \gamma^{ijkl}_{a[b} \gamma^{j]}_{c]d}] \]
\[(3') \quad x^l \gamma^{ijkl}_{a[b} \gamma^{j]}_{c]d}] \]

Again (2') vanishes and (1') and (3') are proportional to (1). So there is only one unique structure at this order:

\[f_{abcd} = F(r)x^j \gamma^i_{a[b} \gamma^{j]}_{c]d}] \] (3.7)

To be hermitian and CPT invariant, the function \(F\) appearing in (3.7) must be imaginary. In evaluating the contribution of \(\{f_{acde}, f_{bcde}\}\) to the coefficient of \(\gamma^i_{ab}\), we want to compute \(\gamma^{ij}_{ab}\) \(\{f_{acde}, f_{bcde}\}\). This involves a trace of seven gamma matrices which automatically vanishes. This gives our first constraint:

\[\{D_j, \delta_{ij}D + 2D_{ij}\} = 0, \]

for every \(i\).

The second possible constraint comes from the coefficient of the \(\gamma^{ijkl}_{ab}\) term with no fermions. There is no contribution from \(\{f_{ac}, f_{bc}\}\). For \(\{f_{acde}, f_{bcde}\}\) to contribute, the trace \(\gamma^{ijkl}_{ab}\) \(\{f_{acde}, f_{bcde}\}\) must be non-zero. However, a quick inspection of this trace shows that it vanishes; therefore, there are no further constraints from the term with no fermions in (3.1).

Before examining the term with two fermions in (3.1), let us expand the \(h_k\) in powers of momenta. We can use the notation \(h_k^{\{n,2m\}}\) to denote a radial function that comes with the following combination of \(x\) and \(p\): \(x_{i_1} \cdots x_{i_n} p_{i_1} \cdots p_{i_n} (p^2)^m\). Then to leading order, we need to consider:

\[D = h_1^{\{0,0\}} + h_1^{\{1,0\}} (x \cdot p) \]
\[D_i = h_2^{\{0,0\}} p_i + h_3^{\{1,0\}} x_i (x \cdot p) + h_3^{\{0,0\}} x_i \]
\[D_{ij} = h_4^{\{0,0\}} (x_i p_j - x_j p_i). \]
CPT invariance together with hermiticity kills $h^{\{1,0\}}_1$, the real part of $h^{\{0,0\}}_3$ and $D_{ij}$. These symmetries further fix the imaginary part of $h^{\{0,0\}}_3$ in terms of $h^{\{0,0\}}_2$ and $h^{\{1,0\}}_3$. Our general constraint (3.8) then implies that $D = 0$. We are therefore left with,

$$D_i = \frac{1}{2} \left( h^{\{0,0\}}_2 p_i + h^{\{1,0\}}_3 x_i (x \cdot p) + p_i h^{\{0,0\}}_2 + (p \cdot x) h^{\{1,0\}}_3 x_i \right),$$

with unknown real functions $h^{\{0,0\}}_2, h^{\{1,0\}}_3$. We have finally reduced the possible form of the supercharge down to the form we would obtain by canonically quantizing the Lagrangian considered in section 2.1. Since the only metric compatible with supersymmetry is the flat metric, we can set $h^{\{1,0\}}_3 = 0, h^{\{0,0\}}_2 = 1$ and $F(r) = 0$ leaving $D_i = p_i$.

### 3.2. A quick death for terms of order $\bar{\hbar}^2$

Our supercharge is the free particle charge $Q^0_a$ to lowest order, and we can now consider deformations which are higher order in $\hbar$. At the next order, we can expand our operators in momenta and impose CPT to get:

$$D = \frac{1}{2} \left( h^{\{2,0\}}_1 (x \cdot p)^2 + h^{\{0,2\}}_1 p^2 + \text{h.c.} \right)$$
$$D_i = p_i$$
$$D_{ij} = h^{\{1,0\}}_4 (x \cdot p)(x_ip_j - x_j p_i).$$

Our charge is then of the form,

$$Q = Q^0 + \delta Q,$$

where $\delta Q$ contains all terms of order at least $\hbar^2$. The most fermionic term has five fermions with no momenta. The general constraint (3.8) still applies since to order $\hbar^3$, the purely bosonic part of the Hamiltonian gets no contributions from terms with more than one fermion in $Q$. This constraint is easy to analyze,

$$\frac{1}{2} \{ D_j, \delta_{ij} D + 2D_{ij} \} = h^{\{2,0\}}_1 x^m x^n p_mp_np_i + h^{\{0,2\}}_1 p^2 p_i$$
$$+ h^{\{1,0\}}_4 (x^m x^ip^m p^2 - x^m x^np_mp_np_i) + \ldots,$$

where the omitted terms have fewer powers of momenta. A quick glance tells us that for this expression to vanish, $h^{\{1,0\}}_4 = 0$ and therefore $h^{\{2,0\}}_1 = h^{\{0,2\}}_1 = 0$. To find interesting deformations, we need to go to the next order.
3.3. A study of terms of order $\hbar^3$

By imposing CPT and hermiticity, we may set $D = D_{ij} = 0$ at this order. We are left with,

$$\begin{align*}
D_i &= p_i + \frac{1}{2} \left( h_2^{(2,0)} (x \cdot p)^2 p_i + x_i h_3^{(3,0)} (x \cdot p)^3 
+ h_2^{(0,2)} p_i + x_i h_3^{(1,2)} (x \cdot p)p^2 + \text{h.c.} \right) . 
\end{align*}$$

(3.9)

Only two of the four functions appearing in (3.9) are actually independent. Gauge transformations of the form,

$$C = c(r)(x \cdot p)^3 + \text{h.c.},$$

(3.10)

or,

$$C = c(r)(x \cdot p)^2 + \text{h.c.},$$

(3.11)

mix either $h_2^{(2,0)}$ with $h_3^{(3,0)}$ or $h_2^{(0,2)}$ with $h_3^{(1,2)}$.

Since we are considering terms in $H$ of at most order $\hbar^4$, we only need to consider:

$$\{Q^0, \delta Q\}.$$ 

That we do not need to consider terms quadratic in $\delta Q$ will simplify our computations considerably. The terms in (3.1) with no fermions give no constraints on $D_i$. We need to consider the two fermion terms in (3.1). There are three non-vanishing terms at this order,

$$T_{ab} = [f_{ak}, f_{bl}] + 3\{f_{ac}, f_{bckl}\} + 3\{f_{bc}, f_{ackl}\},$$

(3.12)

but when we trace with either $\gamma^{ij}_{ab}$ or $\gamma^{ijk}_{ab}$ to find relations on the $h$ functions, the last two terms give the same contribution.

The first term can be written as,

$$[f_{ak}, f_{bl}] = \frac{1}{16} \left( \frac{1}{4!} \gamma^{nmrs}_{ab} \{ \gamma^i, \gamma^{nmrs}, \gamma^j \}_{kl} + \delta_{ab} \gamma^{ij}_{kl} + \gamma^{m}_{ab} \gamma^{imj}_{kl} \right) [D_i, D_j],$$

(3.13)

where we take the terms antisymmetric in $k$ and $l$, and where we can replace $[D_i, D_j]$ by the general form $k(x, p)(x^i p^j - x^j p^i)$.

We need to examine the form of $f_{bckl}$ compatible with CPT and hermiticity. Let us start by determining the number of independent structures. The first structure is the unique vector structure $\gamma^{ij}_{bc} \gamma^{kl}_{i}$ with one $\text{Spin}(9)$ index that already appeared at lowest order in (3.7). This structure can appear with a number of CPT invariant functions such as $F(r)x^i p^2$ where $F$ is imaginary. The resulting three fermion term is hermitian. Note
that this vector structure, which is completely antisymmetric in c, k, l, can be expressed as a linear combination of the four structures,

$$\left\{ \gamma_{bc}^j \gamma_{kl}^i, \gamma_{bc}^{ijk}, \gamma_{bc}^j \gamma_{kl}^{ijk}, \gamma_{bc}^{ijkl} \right\},$$

which are only antisymmetric in k, l.

We can also consider tensor structures with two vector indices; for example, $\gamma_{ij}^a [\gamma_{bc}^j \gamma_{cd}^k]$, which could appear with $F(x^j p^k (x \cdot p))$ where F is now real. However, the resulting three fermion term is not hermitian for any of these structures. The last possibility is a tensor structure with three vector indices. There are four possible structures,

$$\left\{ \gamma_{a[b} \gamma_{c]d}^i, \gamma_{a[b} \gamma_{cd}^{ikm}, \gamma_{a[b} \gamma_{cd}^{jkm}, \gamma_{a[b} \gamma_{cd}^{jmn} kln} \right\},$$

which are not all independent. These tensor structures can give CPT invariant, hermitian three fermion terms so we must include them.

At this order, the three fermion term can be written in the general form:

$$f_{bc}^m \psi_c \psi_k \psi_l = \{ \gamma_{bc}^j \gamma_{kl}^i a_{ijt} + \gamma_{bc}^{ijm} b_{ijt} + \gamma_{bc}^{ijkm} c_{ijt} + \gamma_{bc}^{ijm} d_{ijt} \} \psi_c \psi_k \psi_l + \text{h.c.}$$

We have hidden the vector structures in the choice of tensors $a, b, c, d$ and we have also only presented an expression manifestly antisymmetric in $k, l$. We can now compute,

$$\{ f_{ac}, f_{bc} \} = \frac{1}{16} \left( \gamma_u^m \gamma_m^v \gamma_i^v \gamma_{jk}^r \gamma_{kl}^s \{ p_u, b_{ijt} \} + \frac{1}{4!} \gamma_{nmrs}^a \gamma_{ijq}^t \gamma_{kl}^q \{ p_u, c_{ijt} \} \right) + \frac{1}{4!} \gamma_{nmrs}^a \gamma_{ijqw}^t \gamma_{kl}^q \{ p_u, d_{ijt} \} + \ldots,$$

where the omitted terms are either proportional to $\delta_{ab}$ or antisymmetric in $a, b$. Note that $a_{ijt}$ does not appear in (3.16). Let us first analyze the constraint from $\gamma_{nmrs}^{ab}$.

$$\left\{ \gamma^i \gamma^{nmrs} \gamma^j \right\}_{kl} k(x, p) (x^i p^j - x^j p^i) + 6 \text{tr}(\gamma_u^a \gamma_{nmrs}^i \gamma_{ijq}^m \gamma_{kl}^q \{ p_u, c_{ijt} \}) + 6 \text{tr}(\gamma_u^a \gamma_{nmrs}^i \gamma_{ijqw}^m \gamma_{kl}^q \{ p_u, d_{ijt} \}) = 0.$$

To cancel the piece proportional to $k(x, p)$, the tensor $c$ must contain a vector term:

$$c_{ijt} = \frac{1}{384} k(x, p) (\delta_{it} x^j - \delta_{jt} x^i) + \ldots.$$
The other tensor $d$ must contain a vector piece:

$$d_{ijt} = \frac{1}{1152} k(x,p)(\delta_{it}x^j - \delta_{jt}x^i) + \ldots.$$ 

We know that the allowed terms in (3.15), after antisymmetrizing the $c, k, l$ indices, contain a unique vector structure $\gamma^j_{b[c} \gamma^{ij}_{kl]}$. We can expand,

$$12 \gamma^j_{b[c} \gamma^{ij}_{kl]} = -7 \gamma_{bc}^j \gamma^j_{kl} - \gamma_{bc}^j \gamma^j_{kl} + \gamma_{bc}^j \gamma^j_{kl} - \frac{1}{6} \gamma_{bc}^j \gamma_{kl},$$

but the ratio of the coefficients of the terms that correspond to the vector parts of $c_{ijt}$ and $d_{ijt}$ (the second and fourth term, respectively) do not agree with the solutions for $c$ and $d$ found above. Therefore there is no solution unless we set $k(x,p) = 0$. That $k(x,p) = 0$ implies that,

$$[p_i, D_j] = 0,$$

to the order of interest. In turn, this implies that the $h$ functions are precisely of the form that can be removed by gauge transformations (3.10) and (3.11). We can then conclude that there are no deformations of the free abelian theory which generate non-gauge $p^4$ terms. It would be interesting to see whether this result extends to higher terms in the derivative expansion.

4. Constraining the Lagrangian

4.1. The structure of the supersymmetry transformations

In this section, we will examine the restrictions that supersymmetry and $Spin(9)$ invariance impose on the effective Lagrangian. In section (2.1), we showed that to order $v^2$ the Lagrangian contains the terms:

$$L_1 = \int dt \left( \frac{1}{2} v^2 + i \bar{\psi} \dot{\psi} \right).$$

(4.1)

The supersymmetry tranformations are those given in (2.5) with $M = 0$. At order $v^4$, we must consider all terms,

---

3 That the unique vector structure cannot cancel the curvature term proportional to $k(x,p)$ in (3.17) can be computed directly or confirmed with Mathematica.
\[ L_2 = \int dt \left( f_2^{(0)}(r) v^4 + \ldots + f_2^{(8)}(r) \psi^8 \right). \]

which are in the supersymmetric completion of \( v^4 \). The terms that we have not written generally contain accelerations as well as fermions \( \psi \) with more than a single time derivative. To find restrictions on these terms, it is necessary to first understand the general form of the supersymmetry transformations. It is clear that the free-particle supersymmetry transformations have to be modified by higher order terms when (4.2) is added to (4.1), if the supersymmetry algebra is to close on-shell; see, for example [19]. The new transformations can be expressed quite generally in the form:

\[
\begin{align*}
\delta x^i &= -i\epsilon \gamma^i \psi + \epsilon N^i \psi \\
\delta \psi_a &= (\gamma^i v^i \epsilon)_a + (M \epsilon)_a.
\end{align*}
\]

Note that it is impossible to find a solution where either \( N^i \) or \( M \) vanish when we consider \( L = L_1 + L_2 \) since the algebra will no longer close. At order \( v^2 \), we could set \( N^i \) and \( M \) to zero because the free-particle equation of motion, \( \dot{\psi} = 0 \), is sufficient to ensure closure of the algebra. Of course, the fermion equation of motion is considerably more complicated for \( L \). Constructing \( N^i \) and \( M \) is a formidable algebraic task. Fortunately, as we shall explain, we will not need to know very much more about (4.3) to constrain \( f_2 \).

The corrections to \( \delta x^i \), encoded in \( N^i \), are order \( n = 2 \) where \( n \) counts the number of time derivatives plus twice the number of fermions.\footnote{We count \( \epsilon \) as order \( n = -1/2 \).} The corrections to \( \delta \psi \), encoded in \( M \), are order \( n = 3 \). The variation of \( L \) contains two pieces: the first is order 2 and automatically vanishes for the variations (4.3). The second piece is order 4 and gets contributions from \( L_1 \) and \( L_2 \), where we act with the free-particle transformations on \( L_2 \) and with the corrections on \( L_1 \).

Our interest is primarily with the eight fermion term, which has quite magical properties. This is essentially the ‘top’ form in the supersymmetric completion of \( v^4 \) and studying this term (and its higher velocity analogues) is a natural way to look for constraints on the Lagrangian. The variation of this term in (4.2) schematically contains two pieces,

\[
\delta(f_2^{(8)}(r)\psi^8) = \delta f_2^{(8)}(r) \psi^8 + f_2^{(8)}(r) \delta \psi^8.
\]

The second term contains seven fermions to order 4 and mixes with the variation of \( L_1 \) and with the variation of the six fermion term in \( L_2 \). The first term contains nine fermions.
and is quite special. No other term in $L_2$ varies into a nine fermion term. Can any term from $L_1$ contain nine fermions? After noting that $M$ contains at most six fermions and $N^i$ at most four fermions, it is easy to see that the variation of $L_1$ cannot contain a nine fermion term. We can now conclude that the nine fermion term must vanish by itself. If the metric were not flat, as in the case with eight supersymmetries, then the corresponding variation of $L_1$ could mix with the nine fermion term. Even in that case, we would still obtain some equations that the eight fermion term would need to obey. However, we will not pursue that case further here.

Some dimensional analysis is in order. The coupling in this quantum mechanical theory, $g^2$, has mass dimension three. In matrix theory, $g^2 = M_{pl}^6 R_{\parallel}^3$, where $R_{\parallel}$ is the size of the longitudinal direction. For purposes of dimension counting, the action can be written in the following way:

$$\frac{1}{g^2} \int dt \sum_n v^{2n} f_n(r).$$

(4.5)

In perturbation theory, we can expand each $f_n$ in a power series in the coupling,

$$f_n = \left(\frac{1}{r^4}\right)^{n-1} \sum_l C_{nl} \left(\frac{g^2}{r^3}\right)^l,$$

where $l$ counts the number of loops; see, for example [12,20]. There could also be non-perturbative corrections to the functions $f_n$. To agree with classical long distance supergravity, $f_1 = 1$ as we showed in section two. The coefficient $C_{21}$ was computed in [10] and found to be non-vanishing in agreement with supergravity. In [12], $C_{22}$ was found to vanish supporting the conjecture that,

$$f_2(r) \sim \frac{1}{r^7}.$$

We will show that this conjecture is true non-perturbatively.

There have also been discussions of interactions involving spin dependence [21,22]. The latter paper [22] actually involves a quite non-trivial loop computation of the eight fermion term, which gives the following interesting result:

$$f_2^{(8)}(r) \psi^8 = -15 \left(\frac{1}{2r}\right)^{11} \left(\psi \gamma^{ij} \psi \psi \gamma^{jk} \psi \psi \gamma^{lm} \psi \psi \gamma^{mn} \psi\right) \times$$

\( \left(2\delta_{in}\delta_{kl} - \frac{44}{r^2}\delta_{kl}x_i x_n + \frac{143}{r^4} x_i x_l x_l x_n\right) \)

(4.6)

We will prove that this is indeed the only form of the eight fermion term compatible with supersymmetry up to an overall numerical factor. This immediately gives the desired non-renormalization theorem.
4.2. The eight fermion term

To prove that this is the only eight fermion term compatible with $\text{Spin}(9)$ and supersymmetry, we first need to prove that the structures that appear in (4.6) are the only possible structures. Then we will fix the functional dependence on $r$ using our observations about the variation of this term.

Since the fermions are real, there are only two possible fundamental building blocks for fermionic terms: $\psi \gamma^{ij} \psi$ and $\psi \gamma^{ijk} \psi$. At order $n = 4$, the eight fermion term only depends on $x^i$ and not on $v^i$. Let us then consider all possible terms that can be constructed from just $\psi \gamma^{ij} \psi$:

\begin{enumerate}
  \item $\psi \gamma^{ij} \psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{kl} \psi$
  \item $\psi \gamma^{ij} \psi \gamma^{jk} \psi \gamma^{kl} \psi \gamma^{li} \psi$
  \item $\psi \gamma^{ij} \psi \gamma^{jk} \psi \gamma^{kl} \psi \gamma^{lm} \psi \gamma^{imn} \psi \gamma^{xixm}$ (4.7)
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{kl} \psi \gamma^{jm} \psi \gamma^{xixm}$
  \item $\psi \gamma^{ij} \psi \gamma^{jk} \psi \gamma^{lm} \psi \gamma^{mn} \psi \gamma^{xixkxixn}$
\end{enumerate}

Using the identities in Appendix A, we can see that (1) and (4) vanish. The remaining three terms are independent and are actually the three structures that appear in (4.6).

We should also consider the terms that contain mixed products of $\psi \gamma^{ij} \psi$ and $\psi \gamma^{mnq} \psi$. Observe that terms that contain an odd number of each structure are forbidden by CPT. We are then left with terms of the form,

\begin{enumerate}
  \item $\psi \gamma^{ijk} \psi \gamma^{jk} \psi \gamma^{mn} \psi \gamma^{lmn} \psi$
  \item $\psi \gamma^{ij} \psi \gamma^{ij} \psi \gamma^{lmn} \psi \gamma^{lmn} \psi$
  \item $\psi \gamma^{ij} \psi \gamma^{jk} \psi \gamma^{kmn} \psi \gamma^{lmn} \psi$
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{lmn} \psi \gamma^{lmn} \psi \gamma^{xixk}$ (4.8)
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{lmn} \psi \gamma^{imn} \psi \gamma^{xixk}$
  \item $\psi \gamma^{ij} \psi \gamma^{jk} \psi \gamma^{mnq} \psi \gamma^{lmq} \psi \gamma^{xixm}$
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{mijn} \psi \gamma^{klm} \psi \gamma^{xixm}$
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{mn} \psi \gamma^{jln} \psi \gamma^{xixm}$
  \item $\psi \gamma^{ij} \psi \gamma^{kl} \psi \gamma^{mkn} \psi \gamma^{jln} \psi \gamma^{xixm}$
\end{enumerate}
Each of these structures either vanishes or can be reduced to terms appearing in (4.7) by using the relations in Appendix A.

The last group of terms can be made out of products of $\psi\gamma^{ijk}\psi$. It is not complicated to see that all the nonvanishing structures can be reduced again to terms in (4.7). The most general eight fermion term then takes the form:

$$L_{\psi^8} = (\psi\gamma^{ij}\psi\gamma^{jk}\psi\gamma^{lm}\psi\gamma^{mn}\psi)(g_1(r)\delta_{in}\delta_{kl} + g_2(r)\delta_{kl}x_ix_n + g_3(r)x_ix_kx_lx_n + \ldots).$$

(4.9)

Under a supersymmetry variation,

$$\delta_a \left( f_2^{(8)}\psi^8 \right) = -i\gamma^a_{bc}\psi_b \left( \psi\gamma^{ij}\psi\gamma^{jk}\psi\gamma^{lm}\psi\gamma^{mn}\psi \right) \times \delta_s (g_1(r)\delta_{in}\delta_{kl} + g_2(r)\delta_{kl}x_ix_n + g_3(r)x_ix_kx_lx_n) + \ldots,$$

where the omitted terms either contain seven fermions or are order 6. As we pointed out, the term with nine fermions cannot be cancelled by any other term, and must vanish by itself. Since (4.10) is zero, let us apply the operator $\gamma^a_{ac}\frac{d}{d\psi_c}\partial_q$ to (4.10),

$$-i\gamma^a_{ac}\frac{d}{d\psi_c}\partial_q \left\{ \gamma^a_{bc}\psi_b \left( \psi\gamma^{ij}\psi\gamma^{jk}\psi\gamma^{lm}\psi\gamma^{mn}\psi \right) \times \delta_s (g_1(r)\delta_{in}\delta_{kl} + g_2(r)\delta_{kl}x_ix_n + g_3(r)x_ix_kx_lx_n) \right\} = 0$$

(4.11)

After summing over the index $a$ this equation gives:

$$\Delta (g_1(r)\delta_{in}\delta_{kl} + g_2(r)\delta_{kl}x_ix_n + g_3(r)x_ix_kx_lx_n) =$$

$$\left( \psi\gamma^{ij}\psi\gamma^{jk}\psi\gamma^{lm}\psi\gamma^{mn}\psi \right) \times$$

$$\left\{ \left( \frac{d^2g_1}{dr^2} + \frac{8}{r}\frac{dg_1}{dr} + 2g_2 \right)\delta_{in}\delta_{kl} + \left( \frac{d^2g_2}{dr^2} + \frac{12}{r}\frac{dg_2}{dr} + 8g_3 \right)\delta_{kl}x_ix_n + \left( \frac{d^2g_3}{dr^2} + \frac{16}{r}\frac{dg_3}{dr} \right)x_ix_kx_lx_n \right\} = 0.$$
Since the three terms appearing in (4.12) are actually independent, we obtain three conditions,
\[
\frac{d^2 g_1}{dr^2} + \frac{8}{r} \frac{dg_1}{dr} + 2g_2 = 0 \\
\frac{d^2 g_2}{dr^2} + \frac{12}{r} \frac{dg_2}{dr} + 8g_3 = 0 \\
\frac{d^2 g_3}{dr^2} + \frac{16}{r} \frac{dg_3}{dr} = 0.
\] (4.13)

The solutions to these equations are easily determined,
\[
g_1(r) = \frac{2}{143} \frac{c}{r^{11}} - \frac{1}{9} \frac{c_1}{r^9} + \frac{c_4}{r^7} + \frac{2}{143} c_0 r^4 - \frac{1}{9} c_2 r^2 + c_3 \\
g_2(r) = -\frac{4}{13} \frac{c}{r^{13}} + \frac{c_1}{r^{11}} - \frac{4}{13} c_0 r^2 + c_2 \\
g_3(r) = \frac{c}{r^{15}} + c_0.
\] (4.14)

The constraint that we imposed is actually weaker than invariance of the eight fermion term. Therefore, some of the solutions found in (4.14) may not satisfy the stronger invariance condition. On physical grounds, we know that \(c_0, c_2, c_3\) are zero, since the eight fermion term should go to zero as \(r \to \infty\). The coefficient \(c_4\) corresponds to a term that comes with a negative power of \(g^2\), so we can set it to zero as well. Lastly, \(c_1\) corresponds to a term that comes with a positive but fractional power of the coupling: \(g^{2/3}\). This term is clearly not perturbative and we would like to rule it out.

All these unwanted terms actually correspond to eight fermion terms that do not satisfy the invariance condition. To see this, let us apply \(\gamma_q \frac{d}{d\psi_c} \psi_q\) to (4.10). This gives three coupled first order differential equations which must be satisfied if the eight fermion term is to be supersymmetric. These equations give stronger constraints than just harmonicity. A similar analysis to the one described above shows that we actually need to set all the \(c_i\) coefficients in (4.14) to zero leaving only \(c\) non-zero. The remaining solution corresponds to the one-loop result computed in [22], up to an overall numerical factor \(c\). The key point is that there are no higher loop corrections to the eight fermion term. The only possible non-perturbative eight fermion term is given by the solution (4.14).

This same argument can be extended to higher velocity terms. In particular, there must exist relations on the twelve fermion term in the supersymmetric completion of \(u^6\). For the higher velocity terms, the equations will involve a mixing of the most fermionic term at a given order with the variation of lower order most fermionic terms. Unravelling these constraints and studying their implications for M theory should prove exciting.
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Appendix A. Fierz Identities

A collection of Fierz identities used in the text. In this appendix, the fermions obey the relation \(\{\psi_a, \psi_b\} = 0\) and anticommute with \(\epsilon\).

\[
\begin{align*}
\epsilon \gamma^{ij} \psi \gamma^{ij} \psi &= 0 \\
\epsilon \gamma^{ijk} \psi \gamma^{ijk} \psi &= 0 \\
\psi \gamma^{ij} \psi \gamma^{ijk} \psi &= 0 \\
\psi \gamma^i \psi \gamma^{ij} \psi &= 0 \\
\epsilon \gamma^{ijk} \psi \gamma^{jk} \psi &= 2 \epsilon \gamma^n \psi \gamma^{ni} \psi \\
\epsilon \gamma^{jkl} \psi \gamma^{jkl} \psi &= -6 \epsilon \gamma^n \psi \gamma^{ni} \psi \\
\epsilon \gamma^{jkl} \psi \gamma^{jki} \psi &= -2 \epsilon \gamma^n \psi \gamma^{ni} \psi \\
\psi \gamma^{ij} \psi \gamma^{jk} \psi &= \psi \gamma^{ij} \psi \psi \gamma^{jk} \psi - \psi \gamma^{ln} \psi \psi \gamma^{nk} \psi \\
\psi \gamma^{ipj} \psi \psi \gamma^{imk} \psi &= -3 \psi \gamma^{ip} \psi \psi \gamma^{mk} \psi - 2 \psi \gamma^{im} \psi \psi \gamma^{pk} \psi \\
&\quad + 2 \psi \gamma^{ik} \psi \psi \gamma^{pm} \psi + \delta^{pm} \psi \gamma^{ia} \psi \psi \gamma^{ak} \psi \\
&\quad + \delta^{ik} \psi \gamma^{pa} \psi \psi \gamma^{am} \psi - \delta^{im} \psi \gamma^{pa} \psi \psi \gamma^{ak} \psi \\
&\quad - \delta^{kp} \psi \gamma^{ia} \psi \psi \gamma^{am} \psi.
\end{align*}
\]
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