Quantum simulation of a general anti-PT-symmetric Hamiltonian with a trapped ion qubit

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\textbf{A B S T R A C T}

Non-Hermitian systems satisfying parity-time (PT) symmetry have aroused considerable interest owing to their exotic features. Anti-PT symmetry is an important counterpart of the PT symmetry, and has been studied in various classical systems. Although a Hamiltonian with anti-PT symmetry only differs from its PT-symmetric counterpart in a global \(\pm i\) phase, the information and energy exchange between systems and environment are different under them. It is also suggested theoretically that anti-PT symmetry is a useful concept in the context of quantum information storage with qubits coupled to a bosonic bath. So far, the observation of anti-PT symmetry in individual quantum systems remains elusive. Here, we implement an anti-PT-symmetric Hamiltonian of a single qubit in a single trapped ion by a designed microwave and optical control-pulse sequence. We characterize the anti-PT phase transition by mapping out the eigenvalues at different ratios between coupling strengths and dissipation rates. The full information of the quantum state is also obtained by quantum state tomography. Our work allows quantum simulation of genuine open-system feature of an anti-PT-symmetric system, which paves the way for utilizing non-Hermitian properties for quantum information processing.

1. Introduction

In quantum mechanics, the Hamiltonian of a system is typically taken to be Hermitian in order to generate a real eigenenergy spectrum. However, as pointed out by Bender et al. [1], non-Hermitian Hamiltonians obeying parity-time (PT) symmetry could still give real eigenenergies. PT-symmetric Hamiltonians exhibit various exotic behaviors, one of which is PT-symmetry-breaking transitions that show up at an exceptional point (EP). At an EP, the eigenenergies and eigenstates of the Hamiltonian become degenerate [2–7]. Experimental studies on PT-symmetry in classical systems have stimulated various applications such as laser [8], optimal energy transfer [9] and enhanced sensing [10]. Recently, PT-symmetric Hamiltonians are also constructed in genuine quantum systems, e.g., ultra cold atoms [4], NV-centers [7], trapped ions [5,6], and superconducting quantum circuits [11]. These allow quantum signatures such as perfect quantum coherence at EP to be revealed [5]. Moreover, the topological structure of exceptional points is utilized to realize, e.g., robust quantum control [12].

Not limited to PT symmetry, EP also appears in systems with anti-PT symmetry [13–15]. As an important counterpart of the PT symmetry, anti-PT symmetry has been studied in various physical systems [16,17], including coupled waveguides [18], nanophotonics [19], microwavities [20], lossy resonators [21], optical fibres [22], optical systems with atomic media [23,24], and electrical circuit resonators [13]. An anti-PT-symmetric Hamiltonian \(H_{\text{APT}}\) satisfying \([PT,H_{\text{APT}}]=0\) is related to a PT-symmetric Hamiltonian \(H_{\text{PT}}\) by \(H_{\text{PT}}=\pm iH_{\text{APT}}\). Here P and T denote parity and time reflection operation, and \(\pm i\) denotes anticommutator [1]. As a result, properties similar to PT-symmetric Hamiltonians such as eigenstates coalescing at EP and spontaneous symmetry-breaking transition show up [13]. The information and energy exchange between anti-PT-symmetric systems and environments are different from the PT-symmetric counterpart. These result in different information-exchange schemes, e.g., coherence flow [25], between anti-PT-symmetric systems and the PT-symmetric counterpart. Interestingly, recent theoretical works [26,27] studied the evolution of qubits under \(H_{\text{APT}}\) when coupled to a bosonic bath, and claimed that these qubits decohere more slowly compared to those under Hermitian or PT-symmetric Hamiltonians. This suggests that the anti-PT-symmetric Hamiltonian as a whole is a useful concept in advancing quantum information processing under the decohering environment. Therefore, it

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is important to carry out further experimental research on the eigensystem structure and environmental energy-exchange scheme of a generic anti-PT-symmetric system. 

So far the experiments on anti-PT-symmetric Hamiltonian construction are mainly limited to classical systems [13] and ensemble spin systems [28]. Here, we demonstrate the implementation of an individual quantum system acquiring anti-PT symmetry. We realize the evolution under \( H_{\text{APT}} \) by designing appropriate pulse sequences, as a standard technique in quantum simulation experiments [29]. The sequence consists of an evolution under a passive PT-symmetric Hamiltonian \( H_M \) sandwiching between two \( \pi/2 \) pulses with opposite phases. The evolution under \( H_M \) is achieved by a dissipation scheme as demonstrated in [4–6], in the context of PT-symmetric Hamiltonian construction. We experimentally verify the anti-PT-symmetric Hamiltonian by studying its anti-PT phase transition behavior. We obtain the eigenvalues at different coupling strengths by preparing a certain initial state, evolving it under \( H_{\text{APT}} \) for some known durations, and measure the overlap between the evolved state and the initial state, similar to ref. [6]. The results clearly show the transition from the anti-PT symmetry region to the anti-PT symmetry broken region. The full information of the quantum states, i.e., the population as well as the coherence, is also obtained by quantum state tomography. This enables further studies on anti-PT-symmetric systems, e.g., the information flow [28] and the topological state transition near an EP [30] to be conducted. Our work could also serve as a first step towards harnessing non-Hermitian PT or anti-PT physics to advance the field of quantum information processing [11,31].

### 2. Anti-PT-symmetric Hamiltonian construction

The anti-PT-symmetric Hamiltonian we want to construct reads

\[
H_{\text{APT}} = -2\hbar \mathbf{I}_z + 2\hbar \mathbf{I}_y - i\Gamma \mathbf{I}_x
\]

(1)

where

\[
I_x = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I_y = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad I_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

are angular momentum operators, and \( \Gamma \), \( J \) are real parameters. Indeed, \( \{P, H_{\text{APT}}\} = 0 \), satisfying the anti-PT requirement [24], where \( P = 2I_x, T = \sigma \) denotes complex conjugation operation. Given a qubit with eigenstates \( |0\rangle = (1, 0)^T \) and \( |1\rangle = (0, 1)^T \), \( H_{\text{APT}} \) in Eq. 1 could be realized as follows.

We first realize a passive PT-symmetric Hamiltonian through a spin-dependent dissipation scheme, which is first realized in cold atoms [4], then in trapped ions [5]. This is achieved by coupling \( |0\rangle \) and \( |1\rangle \) by a control field (e.g., a microwave field) with coupling strength \( J \), and generating a loss of population on \( |1\rangle \) with effective loss rate \( 4\Gamma \). The population loss could be achieved by adding a dissipative beam [6] as will be explained in the following section. Now we have constructed the passive PT-symmetric Hamiltonian \( H_M = 2\hbar \mathbf{I}_z + 2\hbar \mathbf{I}_y - i\Gamma \mathbf{I}_x \). We then realize the final anti-PT-symmetric Hamiltonian Eq. 1 by designing appropriate pulse sequences, as is typically done in quantum simulation experiments [29]. According to the identities:

\[
R_\alpha R_\beta = e^{i\alpha R_\beta}, \quad \text{if } RR^\dagger = I
\]

(2)

for arbitrary square matrices \( A \) and

\[
e^{-i\theta J}e^{i\theta J} = J \cos \theta + i J \sin \theta
\]

(3)

where \( (x, y, z) \) are cyclic permutations of \( (x, y, z) \), the evolution of the system under (1), \( e^{-iH_{\text{APT}}t} \), could be realized by sandwiching the evolution under \( H_M \) between two \( \pi/2 \) pulses along \( \pm \gamma \)-axis. That is, denote \( R_{\pm\gamma}(\theta) = e^{-i\gamma t/2} \), the rotation along \( \gamma \) for \( \theta \), then by taking \( R = R_{\pm\gamma}(\pi/2) \) and \( A = -iH_{\text{APT}} \), we have

\[
R_{\pm\gamma}(\frac{\pi}{2}) e^{-iH_{\text{APT}}t} R_{\pm\gamma}(\frac{\pi}{2}) = e^{-iR_{\pm\gamma}(\frac{\pi}{2})H_M R_{\pm\gamma}(\frac{\pi}{2})t} = e^{-i(2\hbar \mathbf{I}_z - 2iI_y + i\Gamma I_x)t} = e^{-iH_{\text{APT}}t}
\]

The pulse sequence is illustrated in Fig. 1a.

Finally, we obtain the evolution under anti-PT-symmetric Hamiltonian for time \( \tau \) equals the evolution time under \( H_M \), i.e., the duration of the middle pulse in Fig. 1a. One could change the duration of the middle pulse to obtain evolutions under \( H_{\text{APT}} \) for arbitrary times. It is noted that, a Floquet scheme [6,32] has used similar pulse sequence, but needs many cycles to generate the desired evolution.

Starting from \( |0\rangle \), without the dissipative beam, the state is fixed during the middle pulse and will return to \( |0\rangle \) at the end of the sequence. The addition of dissipation during the middle pulse evokes the state to a different point in the Hilbert space, which essentially result in the evolution of \( H_{\text{APT}} \). To better understand the evolution of the quantum state in the Hilbert space under dissipation, we construct a non-Hermitian Bloch sphere using CPT symmetry [33] as illustrated in Fig. 2. The system is evolving under \( H_M \) during the middle pulse, for such a passive PT-symmetric non-Hermitian Hamiltonian, a linear operator \( C \) exists, which satisfies \( \{H_M, C\} = 0 \) and \( [PT, C] = 0 \) [33,34]. Replacing the complex conjugate with the CPT-conjugate, the new inner product for an arbitrary state \( |\psi\rangle \) can be written as the form of Dirac inner product [35]: \( \langle \psi|\psi\rangle^{\text{CPT}} = \langle \psi|\psi\rangle^{C} \). The structure of the resulting new Hilbert space (CPT-inner-product space) depends on \( C \), which could be written as \( C = 2/\sqrt{1 - r^2(I_x + irI_y)} \), where \( r := \Gamma/\mathcal{J} \). Then, taking the normalized eigenstates \( |e_+\rangle \) and \( |e_-\rangle \) of \( H_M \) as a basis, an arbitrary state \( |\psi\rangle \) could be written as \( |\psi\rangle = R \cos \Theta |e_+\rangle + \sin \Theta e^{i\phi} |e_-\rangle \). As \( \langle e_+|\psi\rangle^{\text{CPT}} = 1 \) and \( \langle e_-|\psi\rangle^{\text{CPT}} = 0 \), the evolution of \( |\psi\rangle \) under \( H_M \) could be demonstrated on a newly constructed non-Hermitian Bloch sphere with radius \( R \). The axes are chosen such that the point \( (x = 0, y = 0, z = 2R) \) represents \( R|e_\pm\rangle \), \( \Theta \) equals the angle spanned by the state vector and z axis, and \( \Phi \) equals the angle between the state vector and x axis. As an example, the trajectory under \( H_M \) with initial state \( |\psi\rangle | = 1/\sqrt{2}(|0\rangle + |1\rangle) \), \( J = 0.06 \) MHz, \( r = 50 \) ps, \( \Gamma = 0.03 \) MHz is plotted on the unit \( (R = 1) \) non-Hermitian Bloch sphere, as illustrated in Fig. 2a. If \( \Gamma \) is changed to 0.12 MHz, while other parameters remain unchanged, the trajectory is plotted on another non-
Hermitian Bloch sphere, as shown in Fig. 2b. If $\Gamma = 0$, the state is fixed on the sphere. As $\Gamma$ gets larger, the state starts to evolve and trajectories appear on the corresponding CPT sphere. This helps to better understand the evolution under both Hermitian coupling and non-Hermitian dissipation.

3. Experimental implementation and verification of the anti-$PT$-symmetric Hamiltonian

We conduct the experiment on a trapped $^{171}$Yb$^+$ quantum processor. Qubit levels $|0\rangle$ and $|1\rangle$ correspond to the two hyperfine ground states $|F = 0, m = 0\rangle$ and $|F = 1, m = 0\rangle$, as demonstrated in Fig. 1b. The qubit splitting $\delta_{\text{qubit}} \approx 12.6$ GHz. The two levels are coupled by microwave fields with coupling strength $J$. The population loss is realized by a dissipation scheme, as demonstrated in ref. [6]. The ion is excited from $|F = 1, m = 0\rangle$ to $2P_{1/2}$ state by a 369.5 nm dissipative beam, which contains only $x$-polarization components. This ensures that excitations from states $|F = 1, m = \pm 1\rangle$ are forbidden according to selection rules. Through spontaneously emitting $\sigma$ or $\pi$ polarized photons, the excited $2P_{1/2}$ state will decay to the $|F = 1, m = 0, \pm 1\rangle$ states in the $2S_{1/2}$ manifold. This effectively generates a population loss on $|1\rangle$ at a loss rate $4\Gamma$ [6]. The whole scheme is illustrated in Fig. 1b.

The evolution under $H_{\text{APT}}$ is realized by the sequence depicted in Fig.1a: (1) apply a $\pi/2$ microwave pulse along $-y$ axis in the rotating frame. (2) Apply a microwave field along $x$ axis with strength $J$ and a dissipative field with dissipation rate $4\Gamma$ simultaneously, for a duration $\tau$. This creates an evolution under $H_M$ for $\tau$. (3) Finally, a $\pi/2$ pulse along $y$ is implemented. The resulting whole evolution is $U(\tau) = e^{-iH_{\text{APT}}\tau}$, i.e., an evolution under $H_{\text{APT}}$ for time $\tau$. Denote the initial state $|\Psi(0)\rangle$, and the density matrix at time $\tau$, $\rho(\tau) = |\Psi(\tau)\rangle\langle\Psi(\tau)|$, where $|\Psi(\tau)\rangle = e^{-iH_{\text{APT}}\tau}|\Psi(0)\rangle$. Starting from $|0\rangle$, the density matrix elements $\rho_{ij}(\tau) = \langle i|\rho(\tau)|j\rangle$ read

$$
\rho_{00}(\tau) = e^{-2\Gamma\tau}\frac{\cos^2\left(\frac{\tau\sqrt{J^2 - \Gamma^2}}{\sqrt{J^2 - \Gamma^2}}\right)} + \frac{J\sin\left(2\tau\sqrt{J^2 - \Gamma^2}\right)}{\sqrt{J^2 - \Gamma^2}}
$$
$$
\rho_{11}(\tau) = e^{-2\Gamma\tau}\frac{\sin^2\left(\frac{\tau\sqrt{J^2 - \Gamma^2}}{\sqrt{J^2 - \Gamma^2}}\right)}{\sqrt{J^2 - \Gamma^2}} + \frac{J\sin\left(2\tau\sqrt{J^2 - \Gamma^2}\right)}{\sqrt{J^2 - \Gamma^2}}
$$
$$
\rho_{10}(\tau) = e^{-2\Gamma\tau}\frac{\sin\left(2\tau\sqrt{J^2 - \Gamma^2}\right)}{\sqrt{J^2 - \Gamma^2}}
$$

where $\rho_{ij}(\tau)$ is measured for both $J/\Gamma < 1$ and $J/\Gamma > 1$, as shown in Fig. 3a. $J = 0.06$ MHz is obtained separately by fitting the Rabi oscillation. $\Gamma$ could be obtained by fitting according to $\rho_{00}(\tau)$ in Eq. 4, e.g., $\Gamma = 0.004$ (0.53) MHz for the solid (dashed) curve. Similarly, $\rho_{11}(\tau)$ and $\rho_{01}(\tau)$ could be obtained by standard quantum state tomography technique. $\Gamma$ could also be independently calibrated by preparing the system in $|1\rangle$, let it evolve under pure dissipation (the corresponding Hamiltonian is $-2\Gamma|1\rangle\langle1|$) for $\tau$, measure the final population on $|1\rangle$ and fit it with $e^{-\Gamma \tau}$. For example, in Fig. 3b, two such processes are plotted and $\Gamma$ is fitted to be 0.022 (0.050) MHz for the solid (dashed) curve.

To verify the constructed anti-$PT$-symmetric system, we experimentally study the anti-$PT$ phase transition behavior. The eigenvalues $E_{\pm}$ of the anti-$PT$-symmetric Hamiltonian (1), or equivalently $E_{\pm} = E_{\pm}/\Gamma$, could serve to characterize this transition:

$$
E_{\pm} = -i \pm \frac{\sqrt{J^2 - \Gamma^2}}{\Gamma}
$$

One way [7] to obtain $E_{\pm}$ and verify the constructed system would be calibrating $J$ and $\Gamma$ independently, and curve fitting $\rho_{01}(\tau)$ to the theoretical predictions. Here we utilize a more effective method: when measuring the overlap between a certain initial state and the corresponding evolved state [6], as

$$
P_{J/\Gamma}(\tau) = \frac{|\langle 0|\rho(\tau)|1\rangle|^2}{\sqrt{2}} \exp\left(-iH_{\text{APT}}\tau\right) \frac{|0\rangle - |1\rangle}{\sqrt{2}} \exp\left(-iH_{\text{APT}}\tau\right)^* = \cos^2\left(\frac{\sqrt{J^2 - \Gamma^2} \tau}{\sqrt{J^2 - \Gamma^2}}\right) e^{-2\Gamma\tau}
$$

one could first measure $P_{J/\Gamma}(\tau)$ by preparing the initial state $|0\rangle - |1\rangle/\sqrt{2}$, let it evolve under $H_{\text{APT}}$ for a known $\tau$, and measure the overlap between the evolved state and $|0\rangle - |1\rangle/\sqrt{2}$ (left multiply its conjugate transpose). Together with the independently measured $\Gamma$, one could obtain $\sqrt{J^2 - \Gamma^2}$ by calculating the arccos or arccosh function followed by a division over $\tau$, hence $E_{\pm}$ is also obtained.

$P_{J/\Gamma}(\tau)$ could be measured as follows: by starting from $|0\rangle$, apply an $R_y(\pi/2)$ pulse, let the system evolve according to $e^{-iH_{\text{APT}}\tau}$, then apply an $R_y(-\pi/2)$ pulse and readout the population on $|0\rangle$. For example, in Fig. 3c, we first set $\Gamma = 0.022$ MHz by an independent calibration (Fig. 2b, solid curve), and $J = 0.065$ MHz, which gives the theoretical curve; then $P_{J/\Gamma}(\tau)$ is measured by the above sequence. The theoretical curve and experimental data are close to each other. The experimental procedure to obtain $E_{\pm}$ at different $J/\Gamma$ is listed below.

(1) $\Gamma$ and $J$ are first calibrated through independent measurements. This gives us a correspondence between the dissipative beam strength (microwave field strength) and $\Gamma (J)$. In the following, we set the dissipative beam strength corresponding to $\Gamma = 0.050$ MHz (Fig. 2b, dashed curve), and vary the microwave field strength.

(2) The initial state is prepared in $|0\rangle$ via optical pumping. Set the microwave field strength corresponding to $J$.

(3) $P_{J/\Gamma}$ is measured at a certain time $\tau_0$, here we choose $\tau_0=1/J$. At this step, we also separately measure $\Gamma$ as in Fig. 2b.
(4) Plugging \( P_{\text{dist}} \), \( \tau_0 \) and \( \Gamma \) (obtained in (3)) into (6), we obtain \( \sqrt{J^2 - \Gamma^2} \). Plugging \( \sqrt{J^2 - \Gamma^2} \) and \( \Gamma \) (obtained in (3)) into (5), we obtain \( E_\pm \).

(5) By repeating steps (2) to (4) with varying \( J/\Gamma \), we obtain \( E_\pm \) at different \( J/\Gamma \). Here \( J/\Gamma \), i.e., the horizontal axis of Fig. 3d, is evaluated by the calibrated values in step (1).

(6) Repeat steps (2) to (5) three times to obtain the average and variance of \( E_\pm \). The result is shown in Fig. 3d, where only \( E_+ \) is plotted. The error bars reflect the fact that there are noises in \( J, \Gamma \), the constructed evolution under \( H_{\text{APT}} \), and the readout process.

The system stays in the anti-\( PT \)-symmetry region with purely imaginary eigenvalues when \( J/\Gamma < 1 \). At EP (\( J/\Gamma = 1 \)), the eigenvalues become degenerate and equal \(-i\Gamma\). The system enters the anti-\( PT \) symmetry broken region when \( J/\Gamma > 1 \), and the eigenvalues start to have real components. The experimental results agree well with the theoretical calculations, as illustrated in Fig. 3d.

The full density matrix of the system is further obtained through quantum state tomography. As an example, Fig. 4 shows the experimental density matrix \( \rho_{\text{exp}} \) at \( t = 10 \mu s, J/\Gamma = 0.15 \), together with the theoretical values \( \rho_0 \). Applying the state fidelity formula \( F = \frac{\text{Tr}(\rho_0 \rho_{\text{exp}})}{\sqrt{\text{Tr}(\rho_0^2) \text{Tr}(\rho_{\text{exp}}^2)}} \), where \( \rho_0 = \rho_0 \rho_{\text{exp}}/\text{Tr}(\rho_{\text{exp}}) \), \( \rho_0 = \rho_0 \rho_{\text{exp}}/\text{Tr}(\rho_0) \) are experimental and theoretical normalized density matrix [28,36], the fidelity is calculated to be 97.3\% \pm 1.1\%. Simulation results suggest that the errors might be caused by noises in the dissipative beam and readout pulses. The experimental results match theoretical predictions, which demonstrates the reliability of the constructed anti-\( PT \)-symmetric system.

While the passive \( PT \)-symmetric Hamiltonian is constructed by the state-dependent dissipation scheme in trapped-ions [5,6], the anti-\( PT \)-symmetric one hasn’t been directly implemented, due to the difficulties in constructing controllable dissipative-coupling between the two qubit states. Our method effectively implements the anti-\( PT \)-symmetric Hamiltonian by adding two additional pulses, arbitrary evolutions under \( H_{\text{APT}} \) could thus be simulated. Noting that in the experiment, the constructed Hamiltonian is exactly Eq. 1, which is anti-\( PT \)-symmetric, there is no need to remove the \( i\Gamma \) term artificially as is done when simulating PT-symmetric systems [4–6]. Although \( H_{\text{APT}} \) and \( H_{\text{PT}} = \pm iH_{\text{PT}} \) have similar eigensystem structures, a recent experimental work [25] suggests that they behave differently in the context of coherence flow between the system and environment. When taking the phonon degrees of freedom into account, evolutions under \( H_{\text{APT}} \) might decohere more slowly than those under \( H_{\text{PT}} \), as studied in ref. [35]. This suggests that anti-\( PT \) symmetry is a useful concept in trapped-ion quantum information processing.

4. Conclusion

To conclude, we have successfully implemented an anti-\( PT \)-symmetric quantum system by a single \( ^{171}\text{Yb}^+ \) ion. By sandwiching an evolution under a passive \( PT \)-symmetric Hamiltonian between two \( \pi/2 \) pulses with opposite phases, we realize the desired evolution under anti-\( PT \)-symmetric Hamiltonian. We experimentally verify the anti-\( PT \)-symmetric Hamiltonian by studying its anti-\( PT \)-symmetric phase transition behavior. By preparing a certain initial state, evolving it under \( H_{\text{APT}} \), and measuring the overlap between the final state and the initial state, we obtain the eigenvalues at different \( J/\Gamma \). The transition from the anti-\( PT \) symmetry region to the anti-\( PT \) symmetry broken region, together with the eigenvalue coalescing, is clearly revealed from the data. The full information of the quantum states, i.e., the population as well as the coherence, is also obtained by quantum state tomography. The experimental results agree well with theoretical predictions.

Based on the constructed anti-\( PT \)-symmetric Hamiltonian and the versatile quantum-control toolbox trapped ions offer [37,38], further experimental studies on non-Hermitian physics could be envisioned. For example, the information retrieval [28,32] and topological state transfer [9,12] in non-Hermitian quantum systems. Our work also paves the way for harnessing non-Hermitian physics in quantum information-processing applications, e.g., qubits with non-Hermitian \( PT \) or anti-\( PT \)-symmetric Hamiltonians could have superior coherence times compared to Hermitian qubits when coupled to a bosonic bath [26,27]. Note that after this work was finished, we became aware of a similar experiment done recently by Ding et al. [32]. They implemented a Floquet Hamiltonian requiring multi-cycles of pulses to create anti-\( PT \)-symmetric Hamiltonian by periodically driving a dissipative single trapped ion.

Declaration of competing interest

The authors declare that they have no conflicts of interest in this work.

Acknowledgments

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