A new quantum phase between the Fermi glass and the Wigner crystal in two dimensions

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For intermediate Coulomb energy to Fermi energy ratios \( r_s \), spinless fermions in a random potential form a new quantum phase which is nor a Fermi glass, neither a Wigner crystal. Studying small clusters, we show that this phase gives rise to an ordered flow of enhanced persistent currents, for disorder strength and ratios \( r_s \) where a metallic phase has been recently observed in two dimensions.

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An important parameter for a system of charged particles is the Coulomb energy to Fermi energy ratio \( r_s \). In a disordered two-dimensional system, the ground state is obvious in two limits. For large \( r_s \), the charges form a kind of pinned Wigner crystal, the Coulomb repulsion being dominant over the kinetic energy and the disorder. For small \( r_s \), the interaction becomes negligible and the ground state is a Fermi glass with localized one electron states. There is no theory for intermediate \( r_s \), while many transport measurements following the pioneering works of Kravchenko et al \[1\] and made with electron and hole gases give evidence of an intermediate metallic phase in two dimensions, observed \[2\] for instance when \( 6 < r_s < 9 \) for a hole gas in GaAs heterostructures. A simple model of spinless fermions with Coulomb repulsion in small disordered 2\( d \) clusters exhibits a new ground state characterized by an ordered flow of enhanced persistent currents for those values of \( r_s \). In a given cluster, as we turn on the interaction, the Fermi ground state can be followed from \( r_s \approx 0 \) up to a first level crossing. A second crossing occurs at a larger threshold after which the ground state can be followed to the limit \( r_s \rightarrow \infty \). There is then an intermediate state between the two crossings. In small clusters, the location of the crossings depends on the considered potentials, but a study over the statistical ensemble of the currents supported by the ground state gives us two well defined values \( r_{sF}^l \) and \( r_{sW}^l \): Mapping the system on a torus threaded by an Aharonov-Bohm flux, we denote respectively \( I_l \) and \( I_l^F \) the total longitudinal (direction enclosing the flux) and transverse parts of the driven current. One finds for their typical amplitudes \( |I_l| \approx \exp-(r_s/r_{sF}^l) \) and \( I_l \approx \exp-(r_s/r_{sW}^l) \) with \( r_{sF}^l < r_{sW}^l \). Below \( r_{sF}^l \), the flux gives rise to a glass of local currents and the sign of \( I_l \) can be diamagnetic or para-magnetic, depending on the random potentials. Above \( r_{sF}^l \), the transverse current is suppressed while an ordered flow of longitudinal currents persists up to \( r_{sW}^l \), where charge crystallization occurs. The sign of \( I_l \) can be paramagnetic or diamagnetic depending on the filling factor (as for the Wigner crystal), but does not depend on the random potentials (in contrast to the Fermi glass). One finds \( r_{sF}^l \) and \( r_{sW}^l \) in agreement with the values delineating the new metallic phase when \( 0.3 < k_F l < 3, k_F \) and \( l \) denoting the Fermi wave vector and the elastic mean free path respectively. For \( k_F l \geq 1 \), \( I_l \) is strongly increased between \( r_{sF}^l \) and \( r_{sW}^l \). This suggests that the intermediate phase of our model is related to the new metal observed in two dimensions by transport measurements which we shortly review.

In exceptionally clean GaAs/AlGaAs heterostructures, an insulator-metal transition (IMT) of a hole gas results \[3\] from an increase of the hole density induced by a gate. This occurs at \( r_s \approx 35 \), in close agreement to \( r_{sW}^W \approx 37 \), where charge crystallization takes place according to Monte Carlo calculations \[4\], and makes highly plausible that the observed IMT comes from the quantum melting of a pinned Wigner crystal. The values of \( r_s \) where an IMT has been previously seen in various systems (Si-Mosfet, Si-Ge, GaAS) are given in Ref. \[5\], corresponding to different degrees of disorder (measured by the elastic scattering time \( \tau \)). Those \( r_s \) drop quickly from 35 to a constant value \( r_s \approx 8 - 10 \) when \( \tau \) becomes smaller. This is again compatible with \( r_{sW}^W \approx 7.5 \) given by Monte Carlo calculations \[6\] for a solid-fluid transition in presence of disorder. If the observed IMT are due to interactions, it might be expected that this metallic phase will cease to exist as the carrier density is further increased. This is indeed the case \[7\] for a hole gas in GaAs heterostructures at \( r_s \approx 6 \) where an insulating state appears, characteristic of a Fermi glass with electron-electron interactions.

In this work, we take advantage of exact diagonalization techniques for large sparse matrices (Lanczos method) where tiny changes of energy can be precisely studied. This restricts us to small clusters and low filling factors. Fortunately, the dependence on particle number has proved to be remarkably weak in many cases. In the clean limit, calculations \[8\] with 6-8 particles give the condensation of the electron gas into an incompressible quantum fluid when a magnetic field is applied. Pikus and Efros \[9\] have obtained \( r_{sW}^W \approx 35 \) from 6 \( \times \) 6 clusters with 6 particles, close to \( r_{sW}^W \approx 37 \) obtained by Tanatar and Ceperley for the thermodynamic limit. In the disordered limit which we consider, there is another reason for expecting weak finite size effects. When the energy levels do not depend very much on the boundary condi-
tions, the periodic repetition of the same cluster cannot drastically differ from the thermodynamic limit obtained from an ensemble of different clusters. This usual localization criterion applies for insulators as the Fermi glass or the pinned Wigner crystal. Small cluster approximations should then be sufficient for small and large $r_s$. This explains why the critical factors $r_s$ which we will discuss are close to the thermodynamic limit given by the experiments. Finite size effects can be important only if one has a metal for intermediate $r_s$.

We consider a simple model of $N = 4$ Coulomb interacting spinless fermions in a random potential defined on a square lattice with $L^2 = 36$ sites. The Hamiltonian reads:

$$ H = -t \sum_{<i,j>} c_i^\dagger c_j + \sum_i v_i n_i + U \sum_{i \neq j} n_i n_j. \tag{1} $$

$c_i^\dagger (c_i)$ creates (destroys) an electron in the site $i$, $t$ is the strength of the hopping terms between nearest neighbors (kinetic energy) and $r_{ij}$ is the inter-particle distance for a $2d$ torus. The random potential $v_i$ of the site $i$ with occupation number $n_i = c_i^\dagger c_i$ is taken from a box distribution of width $W$. The interaction strength $U$ yields a Coulomb energy to Fermi energy ratio $r_s = U/(2t\sqrt{\pi n_e})$ for a filling factor $n_e = N/L^2$. The disorder to hopping energy ratio $W/t$ is chosen such that $k_F l$ takes values where the IMT has been observed \cite{7}. A Fermi golden rule approximation for $\tau$ gives $k_F l \approx 192\pi n_e (t/W)^2$. One has $n_e = 1/9$, $W/t = 5, 10, 15$ corresponding to $k_F l = 2.7, 0.67$ and 0.3 respectively.

The boundary conditions are always taken periodic in the transverse $y$-direction, and such that the system becomes a torus enclosing an Aharonov-Bohm flux $\phi$ in the longitudinal $x$-direction. Imposing $\phi = \pi/2$ ($\phi = \pi$ corresponds to anti-periodic condition), one drives a persistent current of total longitudinal and transverse components given by

$$ I_l = -\frac{\partial E(\phi)}{\partial \phi} \big|_{\phi=\pi/2} = \sum_i I_l^i \frac{L}{L} \tag{2} $$

and $I_t = \sum_i I_t^i / L$ respectively. The local current $I_l^i$ flowing at the site $i$ in the longitudinal direction is defined by $I_l^i = 2\text{Im}(\langle \Psi_0 | c_{i+1,x}^\dagger c_{i,x}^\dagger \Psi_0 \rangle)$ and by a corresponding expression for $I_t^i$. The response is paramagnetic if $I_l > 0$ and diamagnetic if $I_l < 0$. We begin by showing behaviors characteristic of a single cluster when $r_s$ varies.

Fig. 1 corresponds to $k_F l \leq 1$ ($W/t = 15$). Looking at the low energy part of the spectrum, one can see that, as we gradually turn on the interaction, classification of the levels remains invariant up to first avoided crossings, where a Landau theory of the Fermi glass is certainly no longer possible. Looking at the electronic density $\rho_i = \langle \Psi_0 | n_i | \Psi_0 \rangle$ of the ground state $| \Psi_0 \rangle$, we have checked that it is mainly maximum in the minima of the site potentials for the Fermi glass. After the second avoided crossing, $\rho_i$ is negligible except for four sites forming a lattice of charges as close as possible to the Wigner crystal triangular network in the imposed square lattice. The degeneracy of the crystal is removed by the disorder, the array being pinned in 4 sites of favorable energies.

![Fig. 1. Behavior of a single cluster for $k_F l \leq 1$ as a function of $r_s$. Top: Low energy spectrum (a 1.9$r_s$ term has been subtracted); Bottom left: jumps of $\gamma$ at the second crossing where the ground state (filled diamonds) and the first excited state (empty diamonds) are interchanged. Bottom right: $I_l$ (left scale, empty circles) and number of occupied sites $\xi_s$ (right scale, filled circles).](image)

For the same cluster, we have calculated $C(r) = N^{-1} \sum \rho_i \rho_{i-r}$ and the parameter $\gamma = \max, C(r) - \min, C(r)$ used by Pikus and Efros \cite{4} for characterizing the melting of the crystal. $\gamma = 1$ for a crystal and 0 for a liquid. Calculated for the ground state and the first excited state, $\gamma(r_s)$ allows us to identify the second crossing with the melting of the crystal. Moreover, one can see that the crystal becomes unstable in the intermediate phase, while the ground state is related to the first excitation of the crystal (Fig. 1 bottom left). Around the crossings, the longitudinal current $I_l$ and the participation ratio $\xi_s = N^2 (\sum \rho_i^2)^{-1}$ of the ground state (i.e. of the number of sites that it occupies) are enhanced (Fig. 1 bottom right). The general picture is somewhat reminis-
percent of strongly disordered chains \[8\] where level crossings associated to charge reorganizations of the ground state are accompanied by enhancements of the persistent currents. Fig.\[4\] is representative of the ensemble, with the restriction that the location of the crossings fluctuates from one sample to another as well as the sign (paramagnetic or diamagnetic) of \(I_t\) below the first crossing, in contrast to \(1d\).

![Diagram](image1.png)

**FIG. 2.** Behavior of a single cluster for \(k_f l \geq 1\) as a function of \(r_s\). Left coordinates: currents \(I_t\) (circle) and \(I_l\) (triangle). Right coordinates: \(\xi_s\) (filled circle).

Fig.\[4\] corresponds to \(k_f l \geq 1\) \((W/t = 5)\). The previous level crossings are now almost suppressed by a stronger level repulsion and charge crystallization occurs more continuously. There is instead a broad enhancement of \(I_l\) which, in contrast to Fig.\[4\], is not accompanied by a corresponding increase of \(\xi_s\), which smoothly decreases from 20 of the 36 possible sites down to 4 when charge crystallization becomes perfect. A transition of the persistent current, from a disordered array of loops towards an ordered flow as \(r_s\) increases has been noticed \([1]\) by Berkovits and Avishai. To illustrate this phenomenon, the total transverse current \(I_t\) is shown in Fig.\[2\]. One can see that \(I_t\) is suppressed at \(r_s \approx 5\) while \(I_l\) continues to increase up to \(r_s \approx 15\). We have checked that a disordered array of loops persists up to \(r_s \approx 5\), followed by an ordered flow of enhanced longitudinal currents persisting up to \(r_s \approx 15\). The disordered array of loops gives rise to a diamagnetic or paramagnetic current \(I_l\), depending on the microscopic disorder. The ordered flow gives rise to a paramagnetic \(I_l\). However, Coulumb repulsions do not always yield a paramagnetic response. For instance, \(4 \times 6\) clusters with \(N = 6\) become always diamagnetic at large \(r_s\). One can only conclude that the sign of the response in \(2d\) does not depend on the random potential when \(r_s\) is sufficient for suppressing \(I_t\). In \(1d\), Legett’s theorem \([8]\) states that the sign of \(I_t\) depends on the parity of \(N\) only, for all disorder and interaction strength. The proof is based on the nature of “non symmetry dictated nodal surfaces”, which is trivial in \(1d\), but which has a quite complicated topology in higher \(d\). It is likely that such a theorem could be extended in \(2d\) when the transverse flow is suppressed.

![Diagram](image2.png)

**FIG. 3.** Statistical study of an ensemble of clusters for \(W/t = 5\), \((\text{circle}) 10\) (square) and 15 (triangle) as a function of \(r_s\). Top left: mean value \(<I_t>\>; \text{Top right: fraction } C_d \text{ of diamagnetic samples; Middle left: distribution of the logarithms of the paramagnetic current } I_{p,l} \text{ at } r_s = 1.7 \text{ and } W/t = 15. \text{ Middle right: mean number of occupied sites by the ground state. Bottom left: longitudinal paramagnetic (empty symbols) and diamagnetic (filled symbols) currents. Bottom right: transverse currents. The straight lines are exponential fits giving } r_s^e \text{ and } r_s^w \text{ shown in Fig.}\[4].

We now present a statistical study of an ensemble of \(10^3\) clusters for \(W/t = 5, 10, 15\). At the left top of Fig.\[3\] one can see an increase of the mean \(I_t\) by about one order of magnitude when \(r_s \approx 7\) for \(W/t = 5\). We note that the persistent currents \([10]\) measured in an ensemble of mesoscopic rings are typically higher by a similar amount than the theoretical prediction neglecting the interactions. At the right top of Fig.\[3\], the fraction of diamagnetic clusters is given as function of \(r_s\), showing that the enhancement of the mean is partially related to the suppression of the diamagnetic currents. This suppression is faster for weak disorders. The mean number \(\xi_s\) of sites occupied
by the ground state is given at the middle right of Fig. 3, showing a negligible increase when \( W/t = 15 \) at low \( r_s \) and a regular decay otherwise. The paramagnetic \( I_{p} \) and diamagnetic \( I_{d} \) longitudinal currents, and \( |I| \) have log-normal distributions for all values of \( r_s \) when \( W/t \geq 5 \). The stronger is the disorder, the better is the log-normal shape of the distribution (see middle left of Fig. 3). The average of the logarithms give the typical values shown in the bottom part of Fig. 3. On the left, the longitudinal currents \( I_{t} \) are given, the diamagnetic responses \( I_{t,d} \) (filled symbols) being separated from the paramagnetic responses \( I_{t,p} \) (empty symbols), while the transverse currents \( I_{t} \) are given at the right side. The log-averages exponentially decay as \( I_{t,d} \propto |I| \propto \exp(-r_s/r_s^F) \) and \( I_{t,p} \propto \exp(-r_s/r_s^W) \) when \( r_s \) is large enough. The variances of \( \log |I| \) and \( \log I_t \) increase as \( r_s/r_s^F \) and \( r_s/r_s^W \) above \( r_s^P \) and \( r_s^W \), respectively. The values of \( r_s^F \) and \( r_s^W \) extracted from the exponential fits (straight lines of Fig. 3) are given in Fig. 4 where a sketch of the phase diagram is proposed.

![Proposed phase diagram for 2d spinless fermions in a random potential.](image)

**FIG. 4.** Proposed phase diagram for 2d spinless fermions in a random potential. \( r_s^W \) (filled circle) and \( r_s^F \) (empty circle) obtained from Fig. 3 bottom.

Fig. 3 and Fig. 4 show that a simple model of spinless fermions with Coulomb repulsion in a random potential can account for the critical carrier densities and disorder strengths where the IMT occurs. The comparison between the curve \( r_s(\tau) \) given in Ref. 3 (summarizing the factors \( r_s \) where the IMT has been observed) and the curve \( r_s^W(k_Fl) \) of Fig. 4 (characterizing the suppression of \( I_t \)) is very striking. The value \( r_s = 6 \) where the reentry has been observed in Ref. 3 is also compatible with the curve \( r_s^W(k_Fl) \) characterizing the suppression of \( I_t \). We have not indicated in the proposed phase diagram the difference between \( k_Fl \geq 1 \) (where \( I_t \) has a strong enhancement which may be the signature of a new metal at the thermodynamic limit) and \( k_Fl \leq 1 \) (where \( I_t \) persists up to \( r_s^W \) without noticeable enhancement). A study of the size dependence at a fixed filling factor will be necessary for studying a possible IMT driven by an increase of \( k_Fl \) at intermediary \( r_s \). This is unfortunately out of reach of exact diagonalization techniques. Another striking difference is that \( \xi_s \) and \( I_t \) convey similar information when \( k_Fl < 1 \) while the increase of \( I_t \) is accompanied by a decrease of \( \xi_s \) when \( k_Fl > 1 \). This suggests that transport for intermediary \( r_s \) results more from a collective motion of charges than from a delocalization of individual charges. The spin degrees of freedom are not included in our model, the orbital part of the wave function being totally anti-symmetrized. This restriction is quite important for short range screened interactions, but is certainly less severe for long range interactions and low densities. However, there are many experimental evidences 12 that spin effects play a role. A more complex phase diagram is possible when spins are included. But even for 2d spinless fermions, these small cluster studies allow us to conclude that there is a new quantum phase, clearly separated from the Fermi glass and from the Wigner crystal, identified by a plastic flow of currents without charge crystallization, which is likely to give a new metal in the thermodynamic limit when \( k_Fl \geq 1 \).

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