The Bosonic Sector of the Electroweak Interactions, Status and Tests at Present and Future Colliders

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Abstract

The status of the Standard Model (SM) is reviewed. We emphasize the fact that in spite of the success of the SM for the description of the fermionic sector, the status of the bosonic sector (gauge and scalar) suffers from many theoretical deficiencies and from the lack of empirical support. This situation, which leaves room for several types of extensions or alternatives to SM, strongly motivates the pursuit of intense efforts for finding hints of New Physics (NP) effects. We present a phenomenological description valid for energies lying below the NP scale. We discuss the indirect constraints established from high precision tests at LEP1, as well as the direct tests that could be performed at future machines.
1 Introduction, the status of the Standard Model

It is a common leitmotiv to say that the Standard Model (SM) is largely successful. On the one hand it is already remarkable that this model is able to make definite and unambiguous predictions for all processes involving usual particles. This property is the consequence of the gauge principle which allows to predict the dynamics once a classification group has been chosen. The simpler QED case with the $U(1)_{EM}$ has been extended to the non-abelian cases of QCD with $SU(3)_{colour}$ and to the electroweak interactions with $SU(2) \times U(1)$. However the specific feature of electroweak interactions is the fact that $W$, $Z$ bosons are massive. The gauge principle has to be completed with a mass generation mechanism. In the Standard Model it is chosen as the Higgs mechanism of spontaneous symmetry breaking (SSB). It is this last property that makes the SM a renormalizable theory which allows to compute high order effects and to make the accurate predictions mentioned above. These predictions practically agree with all available experimental results. In spite of this success many questions arise.

Let us first quickly review the status of the SM by clearly separating the characteristics of its three sectors:

a) The **fermionic sector** contains the constituents of matter i.e., the three families of leptons and quarks.

b) The **gauge sector** consists in the $\gamma$, $W^\pm$, $Z$ and the 8 gluons, as generated by the $SU(3) \times SU(2) \times U(1)$ gauge group.

At this stage both fermions and bosons are massless states. They are coupled through gauge interactions. Self-boson interactions appear through the non-abelian Yang-Mills kinetic terms of the $W^{\pm,3}$ gauge bosons.

c) The **scalar sector** is constructed with a complex doublet of Higgs fields. The gauge couplings of this scalar doublet provide the $W^\pm$ and $Z$ masses proportional to the vacuum expectation value $v$ (the Fermi scale). Fermion masses can be described at the expense of introducing by hand a set of Yukawa couplings between fermion and Higgs fields. The Higgs potential generates the Higgs mass and Higgs self-couplings.

The empirical status of these three sectors is now the following. The fermionic sector is well described by the $SU(2) \times U(1)$ classification with left-handed doublets and right-handed singlets. The description of their interactions mediated by the gauge bosons, agrees with the high precision tests performed in particular at LEP1, in some cases up to a few permille accuracy. This agreement may be surprising when one has in mind the broad spectrum of fermions (from the extremely light neutrinos up to the very heavy top quark), its peculiarities (special quantum numbers, chiralities, family replication with a spectacular hierarchy structure, absence of right-handed neutrinos, similarities but differences between leptons and quarks). One could expect to find some deviations from universality. Maybe the heavy quark sector, not yet tested at the same accuracy as the light fermion sector, will reveal some specific features. These questions may seem rather aesthetical, nevertheless the proliferation of 90 basic states is a strong motivation for the search of a simpler and more fundamental theory. To understand these various points one may need so-called ”New Physics” (NP) structures like the ones which extend or modify
the SM (Technicolour mechanism (TC), Grand Unified Theories (GUT), substructures). New concepts like those introduced with superstrings may also be necessary. In any case it is not obvious at what energy scale (between the TeV range and the Planck mass) these features may originate.

The status of the bosonic sector is not yet empirically established because it is still not possible to perform significant direct tests. The agreement of the SM predictions with experiments in fermionic processes (LEP1, low energy experiments) is often taken as a sign of general validity of the SM including the bosonic sector, because high order terms indirectly involve gauge boson and also higgs boson self-interactions. However as we will see in Sect.4, these indirect tests first suffer from a lack of accuracy, but also from many ambiguities which prevent to give well-defined model-independent statements. Many extensions of, or alternatives to, SM are also consistent with the fermionic results.

On another hand the bosonic sectors suffer from much more serious questions and deficiencies. They concern the origin of three a priori independent gauge couplings (that one would like to unify), the origin of the scalar potential (not generated by the gauge principle but put by hand), and of the Fermi scale $v$ (the basic mass scale of the electroweak interactions), the restricted choice of Higgs doublets, as well as the unpredicted value of the Higgs mass (seen either as an unpredicted coupling constant for the $\phi^4$ term or as a new mass scale).

Even more serious are the following two problems concerning the Higgs sector. One is called ”triviality” and expresses the fact that the renormalized coupling constant of the $\phi^4$ term of the Higgs potential tends to zero when one wants to get rid of the cut-off introduced for regularization (as one usually does in renormalizable field theories). This would imply that SSB disappears in this limit. The second one is called ”the naturalness problem” and corresponds to the fact that, at 1-loop, the Higgs mass depends quadratically of the cut-off and is no longer controlled by the tree level value $M_0^2$. Its value then only depends on an outside scale. This is opposite to what happens in the ”natural” fermion mass case where the mass shift is proportional to the tree level mass term and only weakly (logarithmically) depends on the cut-off. This seems to indicate that the description of the scalar sector of the SM is not in a fundamental stage but must be considered as an effective one, valid below a certain NP scale $\Lambda$ (identified with the cut-off).

It is the two facts, absence of direct tests and existence of hard questions, which render further studies of the bosonic sectors of primary importance. In the next Sect.2 we list in more details the precise pragmatical questions to be asked about $W^\pm$, $Z$, $\gamma$ and $H$ properties. In Sect.3 we present phenomenological tools which should allow to describe these properties in a rather model-independent way. Sect.4 is devoted to the discussion of the indirect tests performed at LEP1 and Sect.5 to the direct tests that can be achieved at future machines. Some perspectives are outlined in Sect.6.
2 Questions to be asked about the bosonic sector

The $W^\pm$ and $Z$ bosons have been discovered in a range of mass which precisely agrees with the one expected from the properties of the weak interactions found in low energy experiments. The high precision tests which followed their discovery have confirmed that their couplings to leptons and quarks agree up to a few permille with the SM predictions [1]. Can one from that conclude that $W$ and $Z$ have exactly the gauge nature that the SM assumes for them? Does it also mean that the Higgs mechanism is necessarily responsible for mass generation?

Certainly not! In fact many options for non-standard (NP) models are still allowed by the presently limited empirical knowledge and one can ask the following questions, classified into three types.

a) The nature of the $W^\pm$, $Z$ bosons.

Are they true gauge bosons? In that case what is the precise gauge group? $SU(2) \times U(1)$ or a larger one like $SU(2) \times SU(2) \times U(1)$ or $SU(2) \times U(1) \times U(1)$? Such extensions like Left-Right symmetry, E6 symmetry [4] are obtained on the way of a Grand Unified Theory (GUT) [3] or in certain alternative mass generation mechanisms based on a strongly interacting sector [3],[4].

More drastically departing from the SM picture, $W^\pm$ and $Z$ may be kind of massive vector states (hadron like) whose interactions respect some global symmetry. This is what happens in compositeness schemes where the global symmetry originates from the subconstituent structure. This ensures that the couplings to leptons and quarks are similar to the SM ones[3]. Mass may here simply originate from confinement effects.

b) The precise spectrum of weak bosons.

Vector bosons

Are there higher vector bosons? They could be either additional gauge bosons associated to an extended gauge group ($W_R^\pm$, $Z_R$, $Z'$, $V^{\pm,0}$,...)[4],[5],[6] or partners of $W^\pm$ and $Z$, like isoscalar vector bosons ($Y$, $Y_L$,...) or excited states ($W^{+*}$, $Z^*$), in alternative (for ex. composite) schemes[4].

Scalar bosons

Does the Higgs boson exist at all? This question arises because there exist alternative models without Higgs (Technicolour-like [5] or compositeness inspired [4]). If the Higgs exists, is it an elementary or a composite state [4]? If it exists as an elementary state, mainly because of the naturalness problem[2], the question arises whether it is light (close to $M_Z$) or heavy (close to the unitarity limit in the TeV range). If it is light, is it accompanied by other neutral $H^0$ states and charged $H^\pm$ states (as claimed by Supersymmetry in order to cancel the quadratic divergences) [4]?

One can also raise the question weather there exist higher spin ($J \geq 2$) bosonic states?

c) The precise structure of the bosonic interactions?

This question is motivated by the fact that any extension or modification of the SM should lead to "anomalous" interactions among usual bosons. In the vector boson subsector, the basic $W$, $Z$, $\gamma$ self-interactions can be different from the Yang-Mills ones. In
particular new forms and new multi-boson interactions could appear. Couplings involving longitudinal $W_L$ states may have special features related to the fact that they are created by the mass generation mechanism (MGM). This feature is a genuine one as compared to the QED or QCD cases where SSB does not occur. Within the SM structure it is already known that a very heavy Higgs is a source of strong $W_L W_L$ interactions \[7\]. New Physics structures may also introduce further differences between $W_T$ and $W_L$ interactions \[10\].

Obviously the Higgs sector should be directly affected by the existence of a different MGM, especially Higgs self-interactions because they reflect the structure of the potential. Scalar boson-Vector boson couplings would also be modified if the origin of the scalar boson is non standard, for example like in TC\[8\] or in any other compositeness schemes\[9\].

In order to answer these questions, precision tests of the bosonic sectors (gauge and scalar) have to be performed. Because of the rich variety of possible NP schemes, the analyses of present and future experiments must be done in the most possible unbiased and model independent way. This is the aim of the phenomenological description presented in the next Section.
3 Phenomenological description of the bosonic sector

Searches for NP effects can be divided into 2 classes:

(A) search for new particles which cannot fit into the SM classification (not a new family of leptons and quarks, not a Higgs scalar), and

(B) search for anomalous interactions among usual particles due to residual effects of NP.

In both cases we can look for direct as well as for indirect effects of these new particles or interactions. The characteristic scale of NP is generally expected to lie in the TeV range (following arguments based on unitarity, on the TC mechanism or simply on present experimental limits). If this is true, then new particles should more probably have masses in this TeV range so that their direct production requires high energy colliders. It is however not excluded that some states have lower masses and can be found earlier. If this is not the case one can nevertheless indirectly try, from their virtual effects in certain processes (mixing effects with usual particles, effects through loop diagrams), to find hints of their existence. Similarly the existence of new interactions can be directly observed in processes involving gauge bosons and Higgs bosons. But they could also be detected through indirect effects in fermionic processes (like loops involving self-boson couplings), measured with a very high accuracy as it is the case at Z peak.

A1) Direct production of new particles

The rate for new particle production in a collider is essentially controlled by the product $\sigma \times B$ of the production cross section times the branching ratio of the new particle decay mode into the channel that is detected. When no candidate event is observed a mass limit for the new particle is given. This is significant only if the coupling of the new particle to the initial and to the final states consisting of usual particles is sufficiently strong so that $\sigma \times B$ reaches the observability limit of the experiment. This is a very model dependent question and it explains why mass limits given in the literature are so strongly process dependent and why the results are so largely spread out. As one essentially uses fermionic processes, limits appear to be especially low for those states that are weakly coupled to usual leptons or quarks, i.e. $M_H \geq 60GeV$ from LEP1 [11], $M_V \geq 250GeV$ for the V bosons generated by the strongly interacting sector [5],[6]. On the opposite, in other cases they approach the TeV range [3]. The low values quoted above illustrate the fact that indeed, at present, the bosonic sector is still very weakly constrained.

A2) Indirect effects of new particles

As an example of indirect effect of heavier particles we shall treat the $Z - Z'$ mixing case which has been extensively studied at LEP1 [12]. We shall first present a rather general model-independent description and then look at specific models.

If the $Z^0$ mixes with a higher $Z'^0$ vector boson with a mixing angle $\theta_M$

$$Z = Z^0 \cos \theta_M + Z'^0 \sin \theta_M$$

its vector $g_{Vf}$ and axial $g_{Af}$ couplings get modified as follows

$$\delta g_{Vf} = G' \theta_M \frac{c_f + d_f}{2} \quad \delta g_{Af} = G' \theta_M \frac{c_f - d_f}{2}$$

(2)
depending on the $Z^0 f\bar{f}$ couplings defined as

$$-\frac{ieG'}{4sc}\gamma^\mu [\frac{1}{2}-\gamma^5 c_f + \frac{1}{2} + \gamma^5 d_f]$$

(3)

From eq.(2) one sees that the description will involve 7 independent parameters ($c_f$ and $d_f$) when one assumes family universality, i.e. $f_{L,R}$ representing $\nu_L, l_{L,R}, u_{L,R}, d_{L,R}$ states. In $Z$ peak experiments the disentangling of these 7 parameters will require the largest set of observables.

For the three parameters of the leptonic sector one has the three following observables: the charged leptonic $Z$ partial width $\Gamma_{l}$, the neutral one $\Gamma_{\nu}$, and the leptonic asymmetry $A_{l}$, defined as $A_{L,R}$, but also measurable through the tau lepton final polarization asymmetry or through the forward-backward asymmetry $A_{FB,l} = \frac{3}{4}A_{l}$.

For the 4 parameters of the quark sector one can take the following two partial widths $\Gamma_{4} = \Gamma_{u} + \Gamma_{d} + \Gamma_{c} + \Gamma_{s}$, $\Gamma_{b}$ and the two asymmetries $A_{c}$, $A_{b}$.

Only $\Gamma_{4}$ is presently available with a high accuracy. It is in fact more convenient [12] to use the combination

$$D = \frac{\Gamma_{4}}{\Gamma_{l}} - 2(3 - \frac{20}{3}s^2)A_{l}$$

(4)

The forward-backward asymmetries $A_{FBq} = \frac{3}{4}A_{l}A_{q}$ are not accurate enough to determine $A_{q}$, so that one needs measurements of the polarized asymmetries $A_{FBq}^{pol.q} = \frac{3}{4}A_{q}$ for $q = c, b$ in order to get a meaningful result [12], [13], [16].

Application to specific models

Various types of extensions of the SM (like $E_6$ or $L - R$ symmetry) or of alternative models can be treated in this manner. In each specific case, $c_f$ and $d_f$ are fixed by the classification group and in some cases $G'$ is related to the electroweak strength by unification conditions. The only free parameter is then $\theta_M$ which can also be related to the mass ratio $\frac{M_{Z'}}{M_{Z}}$. In Sect.4 we will see how LEP1 results allow to give upper limits for $\theta_M$ and hence to give lower mass limits for the $Z'$.

B) Residual bosonic interactions below New Physics threshold.

We now present the description of residual interactions among usual particles. We anticipate the discussion of results from $Z$ peak physics which strongly constrain (at the permille level) all non SM effects involving light fermions. We restrict to couplings involving $W^\pm$, $Z$, $\gamma$ and Higgses, avoiding those which involve lepton and quark fields. The case of couplings involving a heavy top quark is still an opened question which is under study[17]. Let us start by recalling the basic SM bosonic couplings.

SM self-couplings at tree level

In the SM, self-gauge boson couplings are given by the Yang-Mills structure of the kinetic terms of the W triplet (no B field is involved). It produces 3-boson and 4-boson couplings involving at least one $W^+ W^-$ pair (no pure neutral coupling exist).

$$L_W = -\frac{1}{2} < W_{\mu \nu}\bar{W}^{\mu \nu} >$$

(5)
Higgs boson couplings with gauge bosons are given by the covariant derivative of the scalar kinetic terms.

\[ L_\Phi = (D_\mu \Phi^\dagger)(D^\mu \Phi) = \frac{v^2}{2} < D_\mu U D^\mu U^+ > \]  

(6)

Three- and four-Higgs couplings are given by the potential term

\[ L_V = -V = \frac{M_H^2}{2v^2}(\Phi^+ \Phi - \frac{v^2}{2})^2 = C + \mu^2 \Phi^+ \Phi + \lambda(\Phi^+ \Phi)^2 \]  

(7)

\[ C = \frac{M_H^2 v^2}{2} \quad \mu^2 = -\frac{M_H^2}{2} \quad v^2 = -\frac{\mu^2}{\lambda} \]  

(8)

Our notations are the following ones:

\[ W^a_{\mu\nu} = \partial_\mu W^a_{\nu} - \partial_\nu W^a_{\mu} - g\epsilon^{abc}W^b_{\mu}W^c_{\nu} \]  

(9)

\[ W_\mu = \frac{\tilde{W}^\dagger}{2}, \quad W_{\mu\nu} = \frac{\tilde{W}^\dagger_{\mu\nu}}{2}, \]  

(10)

\[ \Phi = \left( \frac{1}{\sqrt{2}}(v + H + i\phi^0) \right), \]  

(11)

\[ D_\mu = (\partial_\mu + ig_1 Y B_\mu + ig_2 W_\mu), \]  

(12)

\[ \hat{U} = \frac{v}{\sqrt{2}} U = (\Phi^\dagger, \Phi), \]  

where \( \Phi = i\tau^2 \Phi^* \) and \( \langle A \rangle \equiv TrA \).

Standard radiative corrections, at 1-loop (fermion and boson loops) generate form factors associated to each of the SM tree level terms but also new coupling forms which do not exist at tree level[13]. We shall illustrate the case of 3-gauge boson couplings (analogous studies have been done for 4-boson and for Higgs couplings).
General Lorentz and U(1) invariant forms for $ZW^+W^-$ and $\gamma W^+W^-$ couplings

The complete set has been established in [19]. It involves seven independent $VW^+W^-$ forms for both $V = Z, \gamma$ which are listed below:

1) $-ie g_V V_\mu \tilde{W}^{-\mu\nu} W_\nu^+ - \tilde{W}^{+\mu\nu} W_\nu^-$ (13)

2) $-ie g_{VK} \tilde{V}_\mu W^{+\mu} W^{-\nu}$ (14)

3) $+ie g_{SM}^{SM} \lambda_V \frac{\tilde{V}_{\nu\lambda} W^{-\lambda\mu} \tilde{W}_\nu^{+\mu}}{M_W^2}$ (15)

4) $rac{e z_V}{M_W^2} \partial_\alpha \hat{Z}_{\rho\sigma} (\partial^{\nu} W^{-\sigma} W^{+\alpha} - \partial^{\nu} W^{-\alpha} W^{+\sigma} + \partial^{\nu} W^{+\sigma} W^{-\alpha} - \partial^{\nu} W^{+\alpha} W^{-\sigma})$ (16)

5) $ie g_V^{SM} \tilde{\kappa}_V \tilde{Z}_{\mu\nu} W^{+\mu} W^{-\nu}$ (17)

6) $\frac{\hat{\lambda}_V}{M_W^2} \hat{Z}^{\mu\lambda} \hat{W}^{+\mu} \tilde{W}_{\nu}^{-\mu}$ (18)

7) $eg_{V}^{SM} K_V (\partial^{\mu} Z^{\nu} + \partial^{\nu} Z^{\mu}) W^+_\mu \tilde{W}^-_{\nu}$ (19)

where the abelian $\tilde{W}^{a}_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu$ is used as well as the dual

$\hat{Z}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} (\partial_\alpha Z_\beta - \partial_\beta Z_\alpha)$ (20)

In ref. [20] and [21] the following combinations of couplings are defined

$\delta_V = g_V - g_V^{SM}$ $x_V = (\kappa_V - 1) g_V$ $y_V = \lambda_V g_V^{SM}$ (21)

$z'_{1V} = g_V^{SM} K_V$ $z'_{2V} = g_V^{SM} (\tilde{\kappa}_V - \tilde{\lambda}_V)$ $z'_{3V} = g_V^{SM} \tilde{\lambda}_V / 2$ (22)

The SM case corresponds to

$g_{\gamma}^{SM} = 1$ (23)

$g_{Z}^{SM} = \cot\theta_W$ (24)

and all other terms being absent.
As summarized in Table 1 the three first terms are C- and P-conserving (charge, magnetic moment and quadrupole moment), the fourth one is C- and P-violating but CP-conserving (anapole term) and the last three ones are CP-violating. The specific helicity properties \[20,21\] of the \(W^+W^-\) state for each type of coupling are also given in the last three lines of Table 1. The identification of these properties is particularly useful for experimental analysis as it gives a way to disentangle the various forms.

**Table 1: Space-time properties of the seven 3-boson coupling forms**

| \(\delta V\) | \(xV\) | \(yV\) | \(zV\) | \(z'_{1V}\) | \(z'_{2V}\) | \(z'_{3V}\) |
|---|---|---|---|---|---|---|
| P | P | P | P | | | |
| C | C | C | C | | | |
| CP | CP | CP | CP | | | |
| TT | TT | TT | TT | | | |
| LL | LL | | | | | |
| LT | LT | LT | LT | LT | LT | |

SM radiative corrections feed all these terms with \(q^2\)-dependent form factors. Departures due to NP can contribute to such new couplings and form factors. This may happen in various ways. The basic \(W,Z\) structure may differ from the SM one if one uses an alternative description, for ex. if \(W,Z\) are massive vector bosons not directly generated by the gauge principle (composite states like hadronic \(\rho, \omega\)....vector mesons)\[27,29\]. In these cases tree level modifications of the self-boson couplings (finite \(\delta \kappa, \lambda\)....) may exist. In less drastic pictures in which the \(SU(2) \times U(1)\) system is kept but extended or coupled to a new additional sector, tree level modifications may still appear through mixing of \(W, Z\) with higher vector bosons (especially if these ones pertain to a strongly interacting sector like \(SU(2)_V\))\[5,6\]. In any case at 1-loop, NP effects will always appear through contributions of virtual states. They can even be enhanced by non-perturbative effects(hypercolour factors, resonant effects,...). The peculiarities of the terms generated in this way \[37\] (for example the specific sectors that they affect, charged versus neutral states, transverse versus longitudinal ones, Higgs versus no-Higgs final states,...) and the symmetries that they respect should reflect their origin and help to identify the nature of NP through detailed analyses of the processes.

We shall discuss these questions in a precise manner through the effective lagrangian method. If the characteristic scale \(\Lambda\) of NP is sufficiently larger than \(M_W\), effective lagrangians among usual particles are obtained by integrating out all heavy degrees of freedom. They can be written in the form

\[
L = \Sigma_i \frac{f_i}{\Lambda^{d-4}} O_i^{(d)}
\]

in which \(O_i^{(d)}\) are operators of dimension \(d\) constructed with usual fields, \(\Lambda^{d-4}\) is a scale factor ensuring that \(L\) has the correct dimension 4 when the coupling constants \(f_i\) are
dimensionless. A priori such a series can be infinite and one needs restrictions in order to
have in practice a useful description. These restrictions must be done on a physical basis
because often an apparently "harmless" mathematical property can have very important
physical consequences. As already said and motivated by LEP1 results we restrict $O^i$
to not involve lepton and quark fields. The next restriction comes from the dimension.
If $\Lambda >> M_W$ it is natural to expect observable effects only from the lowest dimensions
d = 4, 6, perhaps 8.

Global symmetries

The above method leads to a large set of possible coupling forms. One can try to reduce
this list by demanding that the lagrangian satisfy certain global symmetries which are
empirically known to be essential. For example the global $SU(2)_{Weak}$ symmetry ensures
the correct form of the W-fermion couplings[27]. Broken by electromagnetism through
$\gamma - W^3$ junction, after mixing, it produces the physical photon and the physical $Z$. In this
picture if one considers the Lorentz invariant, $U(1)_{EM}$ invariant and global $SU(2)_{Weak}$
invariant, d=4 forms constructed without Higgs, one obtains a set of couplings involving
four free parameters (two for the 3-boson part and two for the 4-boson one)[27]. The two
parameters of the 3-boson part contribute to the departure of the Yang-Mills $ZW^+W^-$
coupling constant from the SM value, $\delta_Z = g_Z - \cot \theta_W$ and to the anomalous magnetic
moment couplings $ZW^+W^-$ and $\gamma W^+W^-$ satisfying the relation

$$x_Z = -\frac{s}{c} \gamma$$

(26)

No quadrupole coupling is generated at this level. This set of free parameters can be
further reduced if one considers the high energy behaviour of boson-boson scattering
amplitudes. Because of these non-standard terms, they grow like $s^2$. Demanding that
these terms cancel, one obtains certain relations among the four free parameters which
finally reduce to only one[28].

$$x_Z = -\frac{s}{c} \gamma = -s^2 \delta_Z$$

(27)

The amplitudes then only grow like s. This is the level at which Higgs contributions would
appear. We shall come back to this point later on.

If one wants to generate a quadrupole coupling, d=6 terms have to be allowed. Only
one free parameter is generated if one demands the exact validity of the global $SU(2)_L$
symmetry[32], so that one obtains

$$\lambda_\gamma = \lambda_Z$$

(28)

This requirement of global $SU(2)_L$ is motivated by the concept of custodial symmetry[33].
It is this concept which explains why, in spite of the large symmetry breaking (for ex.
$m_t >> m_b$) the ratio $\rho$ remains very close to one (up to radiative correction effect whose
leading term is $1/m_Z^2$). This means that there is essentially no violation of the $W$ triplet
structure (i.e. $SU(2)_L$ global symmetry). The custodial symmetry can be defined as any
$SU(2)_c$ global symmetry under which the W fields transforms as a triplet. Before SSB
the scalar sector of the SM satisfies a global $SU(2)_L \times SU(2)_R$ symmetry. SSB breaks this
symmetry down to $SU(2)_c$ and this is what ensures $\rho = 1$. Note that $SU(2)_c$ is broken
by the gauge coupling of the B field. This is why it is often discussed only in the limit $g' \to 0$.

Local symmetries

The simplest case that we shall develop here is the one in which the $SU(2) \times U(1)$ SM structure is extended by group factors whose degrees of freedom are associated to a heavy NP scale and will be integrated out. When SSB occurs the effective lagrangian $L(W, B, \Phi)$ which satisfies local $SU(2) \times U(1)$ produces the effective $L(W, Z, \gamma, H)$.

There are other possibilities. For example the basic group $SU(2) \times SU(2) \times U(1)$ can be directly broken to $U(1)_{EM}$ without passing through $SU(2) \times U(1)$. In such cases the effective lagrangian explicitly breaks $SU(2) \times U(1)$ gauge invariance. Another example is the one in which the Higgs mass is very large so that the Higgs field practically disappears from the spectrum, leading also to breaking of $SU(2) \times U(1)$ gauge invariance.

For simplicity and also because $SU(2) \times U(1)$ gauge invariance ensures many good properties for high order effects, we shall concentrate on this gauge invariant case. However one must realize that this requirement of gauge invariance by itself does not restrict the number of independent couplings. It is always possible to find a suitable combination of scalar field which render gauge invariant a given self-boson coupling written in the unitary gauge. It is only when one simultaneously restrict the dimension that one gets an appreciable reduction of the number of free parameters. For $d = 4$ all the $SU(2) \times U(1)$ gauge invariant terms are already provided by SM. Only at $d = 6$ start the new effective terms. In terms of pure bosonic fields there are 11 independent CP-conserving such terms. They are listed below.

\[
\begin{align*}
\mathcal{O}_{DW} & = 4 \langle ([D_{\mu}, W^{\mu\rho}])([D^{\nu}, W_{\nu\rho}]) \rangle, \\
\mathcal{O}_{DB} & = (\partial_{\mu} B_{\nu\rho})(\partial^{\mu} B^{\nu\rho}), \\
\mathcal{O}_{BW} & = \Phi^{\dagger} B_{\mu\nu} W^{\mu\nu} \Phi, \\
\mathcal{O}_{\Phi_1} & = (D_{\mu} \Phi)^{\dagger} \Phi \Phi^{\dagger} (D^{\mu} \Phi).
\end{align*}
\]

\[
\begin{align*}
\mathcal{O}_W & = \frac{1}{3!} \left( \tilde{W}^{\mu} \times \tilde{W}^{\nu} \times \tilde{W}^{\lambda} \right) \cdot \tilde{W}^{\mu} \Phi = \frac{2i}{3} \langle W^{\mu\nu} W_{\lambda\mu} W^{\mu\nu} \rangle, \\
\hat{\mathcal{O}}_{UU} & = \frac{1}{2} \langle \tilde{U} \tilde{U} \rangle \langle W_{\mu\nu} W_{\mu\nu} \rangle, \\
\hat{\mathcal{O}}_{UB} & = \langle \tilde{U} \tilde{U} \rangle B^{\mu\nu} B_{\mu\nu}, \\
\mathcal{O}_{W\Phi} & = 2 (D_{\mu} \Phi)^{\dagger} W^{\mu\nu} (D_{\nu} \Phi), \\
\mathcal{O}_{B\Phi} & = (D_{\mu} \Phi)^{\dagger} B^{\mu\nu} (D_{\nu} \Phi).
\end{align*}
\]
\[ O_{\Phi_2} = \langle \partial_\mu (\hat{U} \hat{U}^\dagger) \rangle (\partial^\mu \langle \hat{U} \hat{U}^\dagger \rangle) \quad , \]
\[ O_{\Phi_3} = \langle \hat{U} \hat{U}^\dagger \rangle^3 \quad . \]

As presented in Table 2, one can regroup the operators into sets which have basically different physical consequences and behaviours under certain symmetries. The first four of them are called non-blind because they involve 2-point gauge boson functions. They would then directly affect the observables measured at LEP1. Consequently their coupling constants must have a strongly reduced strength in order to avoid direct observation. The next five ones are the "blind" ones in the sense that LEP1 is blind to them at tree level. They can only affect the LEP1 observables through 1-loop. The resulting constraints are very mild and allow for large values of the coupling constants. The last two ones only involve Higgs fields and has been dubbed "super-blind" because they are almost unconstrainable by present and future machines.

Table 2: Properties of the eleven bosonic operators

| operator | non-blind | blind | super-blind | SU(2)_c |
|----------|-----------|------|-------------|---------|
| \( \mathcal{O}_{DW} \) | x         |      |             | x       |
| \( \mathcal{O}_{DB} \) | x         |      |             |         |
| \( \mathcal{O}_{BW} \) | x         |      |             |         |
| \( \mathcal{O}_{\Phi_1} \) | x         |      |             |         |
| \( \mathcal{O}_W \) | x         |      |             | x       |
| \( \hat{\mathcal{O}}_{BW} \) | x         |      |             | x       |
| \( \mathcal{O}_{UW} \) | x         |      |             | x       |
| \( \mathcal{O}_{UB} \) | x         |      |             |         |
| \( \hat{\mathcal{O}}_{\Phi} \) | x         |      |             |         |
| \( \hat{\mathcal{O}}_{\Phi_2} \) | x         | x    |             |         |
| \( \mathcal{O}_{\Phi_3} \) | x         | x    |             |         |

Let us concentrate on the 5 blind ones. It is interesting to examine how they contribute to self-boson couplings.

\( \mathcal{O}_W \) leads to the famous quadrupole type of coupling with the relation given in eq.(28).

\( \mathcal{O}_{W\Phi} \) and \( \mathcal{O}_{B\Phi} \) contribute to \( \delta_Z, \kappa_\gamma \) and \( \kappa_Z \), satisfying the relations(26) previously obtained in the general \( SU(2)_W \) invariant schemes[27], [29]. In particular \( \mathcal{O}_{W\Phi} \) reproduces the special case[28] satisfying relations (27).

The two other operators \( \hat{\mathcal{O}}_{UW} \) and \( \hat{\mathcal{O}}_{UB} \) only contribute to anomalous Higgs-gauge boson couplings[37].

Custodial symmetry

Demanding a strict application of the custodial symmetry for the NP effects, strongly restricts the list of operators, see Table 2. From the 11 above ones only five of them
are SU(2)\textsubscript{c} invariant namely, \( \hat{O}_{DW}, \hat{O}_W, \hat{O}_{UW} \) and the two superblind \( \mathcal{O}_{\phi 2} \) and \( \mathcal{O}_{\phi 3} \). SU(2)\textsubscript{c} symmetry restricts the 5 blind ones to only two. Remember that one of them, \( \hat{O}_W \) was already obtained from the SU(2)\textsubscript{L} global symmetry which is for the pure W sector, a remnant of the full SU(2)\textsubscript{c}. The other one is \( \mathcal{O}_{UW} \) which also involves Higgs fields. The justification for this strict use of custodial symmetry is that NP is supposed to be intimately related to the origin of the scalar sector and should therefore respects the same symmetries.

Chiral descriptions

Let us consider a situation in which the SU(2)\times U(1) electroweak symmetry is broken by a strongly interacting sector, all new particles including the Higgs boson being much heavier than \( M_W \). In such a situation it is convenient to use a non-linear representation in which the \( U \) matrix of eq.(12) containing the three goldstone degrees of freedom is written as

\[ U = e^{i \frac{\mathbf{q} \cdot \mathbf{p}}{2}} \]  

(40)

Effective lagrangians invariant under SU(2)\times U(1) resulting from integrating out the effects of this sector can be constructed as combinations of gauge boson fields, \( U \) matrices and their covariant derivatives. At present energies it is meaningful to make an expansion with respect to the number \( p \) of derivatives or of gauge fields (\( U \) being dimensionless). At lowest \( (p^2) \) order one finds the SM part eq.(6). New couplings appear at order \( p^4, p^6, \ldots \) etc. In this way one can again generate all possible bosonic operators. In the physical gauge, they produce the set of anomalous 3-boson couplings listed above as well as higher multi-boson couplings. However the difference with the linear representation presented before is the absence of a physical Higgs field and a different ordering in magnitude of the anomalous self-boson couplings. For example \( \delta_Z, \kappa_\gamma \) and \( \kappa_Z \) appear at order \( p^4 \), through the operators called \( L_{9L} \) and \( L_{9R} \) and satisfy eq.(26)

\[ L = -ig L_{9L} \langle W^{\mu \nu} D_\mu D_\nu U^\dagger \rangle - ig' L_{9R} \langle B^{\mu \nu} D_\mu D_\nu U^\dagger \rangle \]  

(41)

\[ \delta_Z = \frac{e^2}{2cs^3} L_{9L} \]  

(42)

\[ x_Z = \frac{s}{c} x_\gamma = \frac{e^2}{2cs} (L_{9L} + L_{9R}) \]  

(43)

On another hand the quadrupole coupling \( \lambda \) appears at order \( p^6 \) through \( L_\lambda \) which is just the usual operator \( O_W \) of eq.(33). For more details and specific applications see ref [43].

Unitarity constraints.

When operators with \( d > 4 \) are considered, they generally lead to boson-boson scattering amplitudes which grow fastly with the center of mass energy. For example \( d = 6 \) terms lead to partial wave amplitudes growing like \( s \) or \( s^2 \). This means that for a given value of the coupling constant the amplitudes reach the unitarity limit at a certain energy scale. At this point unitarity saturation effects (resonances or new particle creation,...)
must occur. So the unitarity relations which are obtained for each of the operators have two meanings.

1. For a given coupling constant one obtains a value for the scale at which unitarity saturation occurs (this can be considered as a practical definition of the NP scale),

2. For a given NP scale one can set upper limits for the coupling constants in order to satisfy unitarity in the whole \( s \leq \Lambda^2 \) domain.

For the 5 blind operators the unitarity constraints read\[39, 40\]

\[
|f_B| \leq 98 \frac{M_W^2}{s}, \quad |f_W| \leq 31 \frac{M_W^2}{s}, \quad |\lambda_W| \lesssim 19 \frac{M_W^2}{s}
\]

\[
|\lambda_W| \lesssim 19 \frac{M_W^2}{s}
\]

\[
|d| \lesssim 17.6 \frac{M_W^2}{s} + 2.43 \frac{M_W}{\sqrt{s}}
\]

\[
-236 \frac{M_W^2}{s} + 1070 \frac{M_W^3}{s^{3/2}} \lesssim d_B \lesssim 192 \frac{M_W^2}{s} - 1123 \frac{M_W^3}{s^{3/2}}.
\]

If one fixes the NP scale at 1 TeV, the coupling constants have to satisfy the bounds

\[
|f_B| \approx 0.6, \quad |f_W| \leq 0.2, \quad |\lambda_W| \approx 0.12
\]

\[
|d| \approx 0.3 - 0.8 \lesssim d_B \lesssim 0.6.
\]

We shall see in the next Sect.4 that these bounds are highly non trivial as compared to the indirect constraints obtained from LEP1. On the opposite, similar bounds obtained for non-blind operators are totally useless as they lie far above the very stringent LEP1 limits.
4 Status after the high precision tests at LEP 1

It is interesting to discuss how far the high precision tests done at Z peak with fermionic processes can be used to test the bosonic sector. In order to achieve this goal it is essential to use a description of the Z exchange processes which is sufficiently general in order to account for possible NP effects but also to cover in an accurate way the SM radiative correction effects (W, Z self-energies, vertex and box corrections). For this reason the usual description [45], [50] of the effective Z exchange amplitude in $e^+e^- \rightarrow f\bar{f}$ has been somewhat generalized [24].

A) Formalism

We write it in the form

$$A^Z = \frac{\sqrt{2G_\mu M_Z^2}}{q^2 - M_Z^2 - iM_Z\Gamma_Z(q^2)}[1 + \delta^{s.e.}] [\bar{u}_f\gamma^\mu ((g_{Vf} + \Delta g_{Vf}) - \gamma^5(g_{Af} + \Delta g_{Af}))v_f] \times$$

$$\times [\bar{u}_f\gamma^\mu ((g_{Vf} + \Delta g_{Vf}) - \gamma^5(g_{Af} + \Delta g_{Af}))v_f]$$

(50)

The SM part at 1-loop is fully taken into account through the three inputs $\alpha(0)$, $G_\mu$, $M_Z$, and through the shifts $\delta^{s.e.}$, $\Delta g_{Vf}$ and $\Delta g_{Af}$. From the inputs one derives

$$s_1^2 c_1^2 = \frac{\pi\alpha(0)}{\sqrt{2G_\mu M_Z^2}}$$

(51)

and the basic $g_{Vf}$ and $g_{Af}$ couplings

$$g_{Vf} = \frac{\sqrt{2}}{C_f} - 2s_1^2Q_f, \quad g_{Af} = \frac{\sqrt{2}}{C_f}.$$  

(52)

The shifts contain the SM radiative correction effects (in particular the large $m_t$ and $M_H$ dependent terms) and the NP contributions. We have already seen in Sect.3 how $Z - Z'$ mixing effects modify the Z couplings, i.e. add $\delta g_{Vf}$ and $\delta g_{Af}$ for $f = \nu, l, u, d$ (assuming universality). Non universal effects (i.e. b quark terms different from s quark terms) already appear within SM because of large $m_t^2$ effects in $Z\bar{b}b$ couplings [26]. NP can add further non universal terms which can be described by eq(50). This leads us to separately discuss the various subsectors.

a) charged leptonic processes $e^+e^- \rightarrow Z \rightarrow l^+l^-$

It is convenient to embed the two parameters $\Delta g_{Vl}$ and $\Delta g_{Al}$ into two gauge invariant parameters, namely $\epsilon_1'$ and $s_1'^2$. They are precisely defined through\[45]\n
$$\epsilon_1' = \epsilon_1 = \delta^{s.e.} - 4\Delta g_{Al}$$

(53)

$$s_1'^2 = s_1^2(1 + \Delta \kappa_l')$$

(54)

$$\Delta \kappa_l' = \frac{1}{2s_1'}(\Delta g_{Vl} - (1 - 4s_1'^2)\Delta g_{Al})$$

(55)
and can be experimentally measured through two "good" observables

\[ \Gamma_l = \frac{G_\mu M_Z^3}{24\pi \sqrt{2}} [1 + \epsilon_1^l][1 + (1 - 4s_l^2)^2] \]  

(56)

and

\[ A_l = \frac{2(1 - 4s_l^2)}{1 + (1 - 4s_l^2)} \]  

(57)

that can be measured through the polarized asymmetry \( A_{LR} = A_l \) (or through the \( \tau \) asymmetry) or through the unpolarized forward-backward asymmetry \( A_{FB,l} = \frac{3}{4}A_l^2 \).

b) light quark processes \( e^+e^- \rightarrow Z \rightarrow q\bar{q} \)

We assume universality for the first two families, i.e. \( q = u \) or \( c \) and \( q = d \) or \( s \). In this case one obtains 4 parameters (the generalization to the non universal case with 8 parameters can be done in a straightforward manner). The four parameters \( \Delta g_{V,u,d} \) and \( \Delta g_{A,u,d} \) are now replaced by the gauge invariant ones \[ \epsilon_{1}^{u,d} = \epsilon_1 + \delta_{u,d}^{(1)} \]  

(58)

\[ \bar{s}_{u,d} = s_1^2(1 + \Delta \kappa'_{u,d}) \]  

(59)

with the parameters \( \delta_{u,d}^{(1)} \) and \( \delta_{u,d}' \) describing the differences with respect to the leptonic case

\[ \delta_{u,d}^{(1)} = 4[\delta g_{Al} \pm \delta g_{Au,d}] \]  

(60)

\[ \Delta \kappa'_{u,d} = \Delta \kappa'_{u} + \delta_{u,d}' \]  

(61)

\[ \delta_{u} = -\frac{1}{2s^2}[\delta g_{VL} - v\delta g_{Al} + \frac{3}{2}\delta g_{ Vu} - (\frac{3}{2} - 4s^2)\delta g_{Au}] \]  

(62)

\[ \delta_{d} = -\frac{1}{2s^2}[\delta g_{VL} - v\delta g_{Al} - 3\delta g_{ Vu} - (3 - 4s^2)\delta g_{Ad}] \]  

(63)

They could in principle be determined by the four observables \( \Gamma_{u,d} \) or \( \Gamma_{c,s} \) and \( A_{u,d} \) or \( A_{c,s} \).

In practice the situation is slightly less simple as one can measure in an accurate way only \( \Gamma_4 \) or the combination \( D \) given in eq.(4) and at a weaker level maybe also \( \Gamma_c \).

Asymmetry factors \( A_q \) are involved in the forward-backward asymmetries \( A_{FB,q} = \frac{3}{4}A_l A_q \) but can only be measured with a sufficient accuracy through polarized \( e^\pm \) beams with \( A_{FB,q}^{pol(q)} = \frac{3}{4}A_q \) for \( q = c \) and at a weaker accuracy for \( q = s \).

c) heavy quark sector

The only process available at \( Z \) peak is \( e^+e^- \rightarrow Z \rightarrow b\bar{b} \). It contains two additional parameters \[ \delta_{gV} = \delta_{gVd} + \delta_{gV}^{ Heavy} \]  

(64)

\[ \delta_{gA} = \delta_{gAd} + \delta_{gA}^{ Heavy} \]  

(65)
that can be determined through the two new observables

\[ \Gamma_b = \Gamma_d[1 + \delta_{bV}] \]  
\[ A_b = A_d[1 + \eta_b] \]

where the coefficients correspond to

\[ \delta_{bV} = \frac{4}{1 + v_d^2} \left[ v_d \delta g_{Vb}^{Heavy} + \delta g_{Ab}^{Heavy} \right] \]  
\[ \eta_b = -\frac{2(1 - v_d^2)}{v_d(1 + v_d^2)} \left[ \delta g_{Vb}^{Heavy} - v_d \delta g_{Ab}^{Heavy} \right] \]

with \( v_d = 1 - \frac{4}{3}s^2 \). They are measurable through

\[ R_b = \frac{\Gamma_b}{\Gamma_{had}} \]

and

\[ A_{pol(b)} = \frac{3}{4} A_b \]

Note that the parameter \( \epsilon_b \) introduced in [45] corresponds to a restricted scheme in which only pure left-handed effects appear

\[ \delta g_{Vb}^{Heavy} = \delta g_{Ab}^{Heavy} = \frac{\epsilon_b}{2} \]

**d) W mass**

The analysis of these precision tests often uses an additional observable, the W mass. This defines one more parameter that is taken as \( \delta \xi \) [48] or \( \Delta r_{ew} \) or \( \epsilon_2 \) [45],

\[ \frac{M_W^2}{c^2 M_Z^2} = 1 + \delta \xi \]

\[ \delta \xi = -\frac{s^2}{c^2 - s^2} \Delta r_{ew} \]

\[ \Delta r_{ew} = -\frac{c^2}{s^2} \epsilon_1 + 2\epsilon_2 + \frac{c^2 - s^2}{s^2} \epsilon_3 \]

One can check that these combinations are vertex correction independent. At this point it may be useful for the reader to have a look at Table 3.

| Ref. [45] | Ref. [46] | Ref. [47] | Vac. pol. |
|-----------|-----------|-----------|-----------|
| \( \epsilon_1 \) | \( \alpha T \) | \( \Delta \rho \) | \( \frac{A_{33}(0)-A_{11}(0)}{M_W^2} \) |
| \( \epsilon_2 \) | \( \frac{\alpha S}{4\pi^2} \) | \( -c^2 \Delta_{3Q} \) | \( \frac{\epsilon}{s} F_{30}(M_Z^2) \) |
| \( \epsilon_3 \) | \( \frac{-\alpha U}{4\pi^2} \) | \( \Delta_{1Q} - c^2 \Delta_{3Q} \) | \( F_{11}(M_W^2) - F_{33}(M_Z^2) \) |
This table contains a dictionary for the various notations which had been introduced in the past for the leading vacuum polarization (universal or "oblique") contributions written as

\[ \Pi^{ij}(q^2) = A_{ij}(0) + q^2 F_{ij}(q^2) \]  

Let us also recall the definition

\[ \Delta \alpha = F_{\gamma\gamma}(0) - F_{\gamma\gamma}(M_Z^2) \]  

and notice one recent notation\[30]\]

\[ \Delta x = \epsilon_1 - \epsilon_2 \quad \Delta y = -\epsilon_2 \quad \epsilon = -\epsilon_3 \]  

We emphasize that the inclusion of non universal SM or NP terms requires the use of the more general parametrization defined above with at least 7+1 free parameters.

**Brief summary of LEP1 constraints**

Within a pure SM analysis, limits on \( m_t \) and \( M_H \) were obtained from the sensitivity of the radiative correction terms to these masses (essentially the \( m_t^2 \) and \( \log M_H \) dependences). The sensitivity to \( m_t \) is large and has allowed to get a strong constraint

\[ m_t = 177^{+11+18}_{-11-19} \text{GeV} \]  

(\text{the first error is experimental, the second one corresponds to the unknown Higgs mass effect that is varied between 60 GeV and 1 TeV}) which is in perfect agreement with the observations made at Fermilab \[51], \[52]\]

\[ m_t = 174 \pm 10^{+13}_{-12} \text{GeV} \]  

The bosonic contribution to SM radiative corrections is mainly concentrated in the parameter \( \Delta y \) \[30\]. The accuracy is however not sufficient to significantly constrain the Higgs mass \[31\], although low values seem to be favored \[1\].

In a more general non-standard analysis one can eliminate the unknown SM parameters \[48\] and get constrains on the deviations from the standard \( Z f \bar{f} \) couplings \[12\]:

- in the leptonic sector (at two standard deviations)
  \[ \delta g_{Al} \leq 0.002 \quad \delta g_{Vl} \leq 0.006 \]  
  \[ \delta g_{A\nu} = \delta g_{V\nu} \leq 0.005 \]  

- in the light quark sector
  \[ \frac{4}{23}[3\delta g_{Vu} + 9\delta g_{Au} - 6\delta g_{Vd} + 4\delta g_{Ve} + 23\delta g_{Ae}] \leq 0.008 \]  

and in the heavy sector \[53\]

\[ \delta b_V = 0.0414 \pm 0.0110 \]
This last result is obtained for \( m_t = 175 \text{GeV} \), and it constitutes the first sign for a possible departure from SM predictions.

So in conclusion no NP effect appears in the light fermion sector at an accuracy reaching a few permille. Some effect may exist in the heavy quark sector at an accuracy of a few percent.

**B) Indirect constraints on the bosonic sector**

We are now able to discuss the influence of the 11 effective lagrangians describing NP effects in the bosonic sector.

Non blind operators contribute directly (at tree level) to the \( \epsilon_i \) parameters of the light fermionic sector. Consequently the constraints on the coupling constants are very strong\cite{34}, \cite{35}, i.e.

\[
|\bar{f}_i \frac{M_Z^2}{\Lambda^2}| \lesssim O(10^{-2} \text{ to } 10^{-3})
\]

Note nevertheless that there is not enough information to disentangle all possible contributions because of strong correlation effects.

Blind operators affect the LEP1 parameters only at 1-loop. The use of effective lagrangians for loop computations has raised a lot of technical and physical questions, especially because of the occurrence of strong divergences in some cases\cite{34}, \cite{35}. The physical meaning of these divergent terms is that the chosen effective lagrangian does not sufficiently specify the NP effects when \( q^2 \) approaches \( \Lambda^2 \). The model has to be completed by additional terms. Restricting to terms involving only usual particles, the gauge invariance prescription is a (non unique) way of choosing such additional terms (multi-boson terms, terms involving Higgs bosons,…). In this case the cancellation of the violent divergences is provided by diagrams involving the additional 4-boson couplings and/or the ones involving Higgs bosons. These features illustrate the model-dependence of these indirect effects. Another technical point is the fact that the domain of integration corresponding to the divergent part may correspond to a strong coupling regime (and even overpass the unitarity limit) so that non-perturbative effects should in principle be taken into account. This weakens the power of the constraints that has been derived from perturbative analyses. Nevertheless they give an orientation. In any case, because of the loop factor \( \frac{\alpha}{4\pi} \) the constraints are much weaker than in the case of nonblind operators

\[
|\bar{f}_i \frac{M_Z^2}{\Lambda^2}| \lesssim O(1 \text{ to } 10^{-1})
\]

for example\cite{35}

\[
|\lambda_W| \lesssim 0.6
\]

When more than one operator at a time is considered, again because of strong correlation effects no useful constraint remains.

A very special case has however been noticed \cite{53}. It has been shown that the \( Zb\bar{b} \) width allows to get a rather unambiguous constraint on the \( O_{W\Phi} \) operator (with coupling constant \( f_W \)), and (owing to specific counting factors and other numerical factors) a
negligible effect of the $O_{B\Phi}$ one (with coupling constant $f_B$). This arises because of the $m_t^2$ enhancement factor which selects the longitudinal modes of the $W$ couplings inside the loop.

\[-0.40 \lesssim \tilde{f}_W \frac{M_Z^2}{2\Lambda^2} - 0.04 \tilde{f}_B \frac{M_Z^2}{2\Lambda^2} \lesssim -0.15 \]  

(88)

The present experimental result which seem to indicate a non zero NP effect, if it is interpreted as a bosonic effect, would imply a rather strong departure to SM,

\[-0.7 \lesssim \delta_Z \lesssim -0.3 \]  

(89)

largely visible in direct tests at LEP2 (that we shall discuss in the next Sect.5).

In addition, if no anomalous effect is simultaneously observed in the light fermion sector ($\epsilon_i$ parameters), this means that the $O_{W\Phi}$ contribution has to cancel against other contributions to these parameters. In turn this implies an even richer set of observable effects at LEP2 due to these other sources.

On another hand one can compare the order of magnitude of the indirect constraints obtained in this way with purely theoretical considerations like the unitarity relations that we mentioned earlier [40]. It appears that these LEP1 indirect tests can only feel effects associated to a scale which is weaker than 1 TeV (after all this not so surprising for 1-loop effects at $Z$ peak). However, if the effective scale is only of a few hundreds of GeV, the validity of the pure perturbative treatment is questionable, and it is not reasonable to take these numerical values too strictly.

The lesson of this discussion is that only direct tests can give unambiguous results and this is what we shall discuss in the next Sect.5.
5 Tests at future machines

The future machines that we shall consider are LEP2, LHC and NLC. There exist also projects for developing LEP1 (Polarized beams, high luminosities) and extending the Tevatron energy to 4 TeV. With these developments and these new machines one can expect to improve the indirect tests\cite{22}, \cite{23}; but the real progress in the empirical knowledge of the bosonic sector will only come from a copious production of boson pairs.

Up to now only very mild direct limits have been obtained at CERN\cite{54} and at Fermilab\cite{55}, with anomalous \(\kappa\) or \(\lambda\) of the order of the unity. The first really significant results should come from LEP2 when a few thousands of \(e^+e^- \to W^+W^-\) events will be observed\cite{21}. The standard reactions \(e^+e^- \to ZZ, Z\gamma, \gamma\gamma\) do not involve \(VW^+W^-\) couplings. Purely neutral non-standard 3-boson couplings \(\gamma ZZ, \gamma\gamma Z, ZZZ\) may exist but their effects are expected to be depressed \cite{56}. If by chance the Higgs boson is light enough to be produced through \(e^+e^- \to ZH\) or \(e^+e^- \to \gamma H\) the first meaningful tests of Higgs couplings could also be performed\cite{57}.

More possibilities will then be offered at LHC\cite{58}. Through quark-antiquark annihilation one can also produce \(W^+W^-, ZZ, Z\gamma, \gamma\gamma\) neutral pairs, but the first new feature is the existence of charged pair production \(W^\pm Z\) and \(W^\pm\gamma\) through \(W^\pm\) exchange diagram, which will allow to disentangle anomalous \(ZW^+W^-\) couplings from \(\gamma W^+W^-\) ones. Boson-boson fusion processes will also take place and give genuine new informations involving 4-boson couplings and Higgs exchanges\cite{41}.

At a linear \(e^+e^-\) collider (NLC) in the TeV range \cite{59} the same processes already studied at LEP2 will be pursued at higher energies and with a higher luminosity\cite{20}, \cite{49}. Boson-boson fusion processes will also appear \cite{60}, \cite{62}. A very appealing way to observe this set of processes is through laser induced \cite{61} photon-photon collisions and also photon-electron collisions. They should be especially interesting for direct Higgs production \((\gamma\gamma \to H)\)\cite{57}.

In all these direct observations of the bosonic sector, strategies have to be developed in order to identify the nature of a possible anomalous effect or to give sensible observability limits. An observation means a departure from the SM prediction in a given process. The sensitivity of any observable to an NP effect generally behaves like

\[
 f\left(\frac{S}{\Lambda^2}\right)^n
\]  

(90)

This applies to production rates, ratios of cross sections, angular distributions, polarization asymmetries,...

Particularly interesting cases are those where the SM contribution is depressed (for example when it occurs only at 1-loop like in \(\gamma\gamma H\) or in \(\gamma\gamma ZZ\)) so that any signal would be a candidate for NP.

Let us just mention a few highlights extracted from the phenomenological studies that have been recently made in these processes.

In \(e^+e^- \to W^+W^-\), precision tests require the analysis of the final \(W^\pm\) polarization. The separation of \(W_TW_T, W_LW_L, W_TW_L\) production allows to disentangle the 3 types of C
and $P$ conserving anomalous couplings $\delta$, $\kappa$ and $\lambda$\cite{21, 20}, see Fig.7,8 of \cite{21}. The anapole couplings lead to strong forward-backward asymmetries. The CP violating couplings can be isolated by doing an analysis of the $W^\pm$ spin density matrices measurable through their decay distributions \cite{42}.

The reaction $e^+e^- \rightarrow \gamma H$ is only observable if it is enhanced by anomalous Higgs couplings, for example those generated by the operators $O_{UB}$ and $O_{UW}$\cite{57}.

At LHC many processes with different initial states overlap and it will be difficult to identify the origin of an effect. In a restricted case with only $O_W$ and $O_{UW}$ involved, it has been shown that ratios of cross sections like $\frac{\sigma(WZ)}{\sigma(ZZ)}$ or $\frac{\sigma(W\gamma)}{\sigma(ZZ)}$ allow a clear disentangling of $O_W$ and of $O_{UW}$ effects\cite{11}. See Fig.7 of \cite{41}.

Finally we quote the laser induced $\gamma\gamma \rightarrow H$ process which can give the highest sensitivity to anomalous Higgs couplings\cite{57} see Fig.1 of \cite{57}.

Details about these preliminary analyses can be found in the quoted references. Further more elaborate studies are in progress \cite{63}.
6 Perspectives

The results of the preliminary analyses which have been done along the lines presented above are summarized in Table 4 and 5.

| Collider | $|\lambda_W|$ | $\Lambda_{sat}$ | Reference |
|----------|--------------|----------------|-----------|
| LEP2 170GeV | 0.14 | 0.9TeV | [21] |
| LEP2 230GeV | 0.06 | 1.4TeV | [21] |
| LHC | 0.01 | 3.5TeV | [10] |
| NLC 0.5TeV | 0.008 | 4TeV | [20] |
| NLC 1TeV | 0.002 | 8TeV | [20] |

| Collider | $|d|$ | $\Lambda_{sat}$ | Reference |
|----------|------|--------------------|-----------|
| LHC (WW) | 0.1 | 2.5TeV | [10] |
| NLC (WW) | 0.25-0.02 | 1.5-6TeV | [18] |
| NLC (HZ) | 0.005 | 11TeV | [18] |
| laser NLC (H) | 0.001 | 30TeV | [57] |

In these tables we show the sensitivity to two typical $SU(2)_c$ conserving bosonic couplings, the anomalous 3-gauge boson coupling $\lambda_W$ associated to the operator $O_W$ and the anomalous Higgs boson coupling $d$ associated to the $O_{UW}$ operator.

These results can be compared to the present indirect LEP1 constraints:\cite{35}

$$|\lambda_W| \lesssim 0.6 \quad |d| \lesssim 1.$$  \hfill (91)

and to the unitarity bounds for $\Lambda = 1$TeV: \cite{36}, [40]

$$|\lambda_W| \lesssim 0.12 \quad |d| \lesssim 0.3$$  \hfill (92)

In conclusion, with these analyses one observes that step by step the sensitivity will increase when going from LEP2 to LHC and to NLC, reaching finally the $10^{-3}$ level of accuracy. So at the end the bosonic sector should be tested at the same accuracy as the fermionic sector is tested at Z peak. In terms of NP scale as shown in Tables 4,5 this means an order of magnitude of about 10 TeV. This range of scales is interesting because it covers a domain in which several types of theoretical models predict NP effects.

It is also exciting to follow the way these progresses may arise:

—From the high precision direct tests of the fermionic sector at LEP1, one gets indirect hints about the gauge boson sector.

—The next step starts at LEP2 with direct tests of the gauge boson sector and some indirect hints about the Higgs sector.
—Finally at LHC, and better at NLC, direct tests of the Higgs sector should be achieved.

Should they give some indirect hints about a possible underlying sector at the origin of the mass generation mechanism?!
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