Agent-based Model and its Potential in Simulating Some Physical Systems

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Abstract. Agent-based model (ABM) is very flexible in defining types of interaction between agents including non-physical ones, which can be used to simulate social interaction, economy, until city development. Only our abstraction in defining the interactions is the limitation. For a physical system, an agent can be considered as a particle and interaction between them is simply the interaction force, e.g. nuclear force between nucleons, electrostatic force between ions, gravitation force between asteroids, and contact force between granular particles. In this work, simple rule for the interaction between particle is proposed and several materials phases such as solid, liquid, gas, and granular can be obtained, but only qualitatively.

Keywords: simulation, agent-based model, physical system, system of particles.

1. Introduction
Since its birth agent-based model (ABM) has already been used for various types of system, such as impact of out-of-stocks and supply chain design in manufactures [1], multi-lane traffic behavior [2], profit and risk for offshore hake trawl industry [3], the impact of antibiotic use on transmission of resistant bacteria in hospitals [4], and prediction of a bioeconomy system [5]. One of the advantages of ABM is its flexibility in defining interactions between agents, which is used in this work to obtained various materials phases.

2. Model
Agents or particles are placed in a rectangular grid, where one particle can only occupy one cell in the grid. Visual representation of the grid is shown in figure 1 (left), where its matrix representation is given in figure 1 (right). An empty cell will have the value of 0, while an occupied cell has the value of 1, and the wall is with the value of -1. There area eights agents or $N = 8$ in figure 1.

Figure 1. A system of particles in a 10x10 grid constrained by a surrounding wall and consisted of eight agents: a visual representation (left) and its matrix representation (right).
An agent is allowed to move in eight directions as shown in Table 1, where each direction is numbered as shown.

| Direction | Abbreviation | Number |
|-----------|--------------|--------|
| ↑         | NO (north)   | 0      |
| ↗         | NE (northeast)| 1     |
| →         | EA (east)    | 2      |
| ↘         | SE (southeast)| 3     |
| ↓         | SO (south)   | 4      |
| ↙         | SW (southwest)| 5     |
| ←         | WE (west)    | 6      |
| ↖         | NW (northwest)| 7     |

At time $t$ an agent is currently staying at cell $(i, j)$ and will move to another cell $(i + di, j + dj)$, if the destination cell is empty or not a wall cell, otherwise, it will remain at the same position. A random number $z$ is generated from a uniform random function $f_U$, which has a probability density function of

$$f_U(z) = \begin{cases} \frac{1}{b-a}, & a \leq z < b, \\ 0, & z < a \cup z \geq b, \end{cases}$$

where, in this work, it is set that $a = 0$ and $b = 1$. Since there are eight directions, eight probability values are required, one for each direction. Using Table 1 we can define $p_0$ until $p_7$, that should meet the condition of normalization

$$\sum_{k=0}^{7} p_k = 1.$$  

(2)

Value of $p_k$ means the probability of an agent to take the direction $k$. According to equation (1), generated value $z$ will be $0 \leq z < 1$, the value of $l$ will be produced through

$$l = \begin{cases} 0 \leq z < \sum_{k=0}^{l} p_k, & l = 0, \\ \sum_{k=0}^{l-1} p_k \leq z < \sum_{k=0}^{l} p_k, & 1 \leq l \leq 7, \end{cases}$$

(3)

where $l$ is simply the direction number in Table 1. In region consisted of some cells, we can define a probability vector $P$

$$P = [p_0 \; p_1 \; p_2 \; p_3 \; p_4 \; p_5 \; p_6 \; p_7]$$

(4)

which represents the probability of an agent to move in the eight allowed directions. In the system of $G_y \times G_x$ grid (Figure 1 is or $G_y = 10$ and $G_x = 10$) we can have several regions with different probability vector. Assume that $G_y = G_x = G$ then there will be $H_y \times H_x$ region with $H_y = H_x = H$, where

$$G < H$$

(5)
and

\[
\frac{G}{H} = \left[ \begin{array}{c} G \\ H \end{array} \right]. \tag{6}
\]

System direction probability will be in the form of a probability matrix

\[
P_{G=1} = [P] \tag{7}
\]

for \( G = 1 \),

\[
P_{G=2} = \begin{bmatrix} P_0 & P_1 \\ P_2 & P_3 \end{bmatrix} \tag{8}
\]

for \( G = 2 \), and until

\[
P_{G=5} = \begin{bmatrix} P_0 & P_1 & P_2 & P_3 & P_4 \\ P_5 & P_6 & P_7 & P_8 & P_9 \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ P_{15} & P_{16} & P_{17} & P_{18} & P_{19} \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} \end{bmatrix} \tag{9}
\]

for \( G = 5 \). Equation (7) if for the homogeneous region where the system has only one type of direction probability and equation (8) can accommodate the inhomogeneous region with various types of direction probability.

3. Results and discussion

Several types of direction probability are tested and the results are discussed. The system is specified with \( G = 100 \) and \( N \approx 1681 \) or less (since it is randomly generated in \( 41 \times 41 \) cells with \( N_0 = 10000 \) as the initial state).

3.1. Single phase system (\( G = 1 \))

There are four types of system phase that can be obtained using \( P_{G=1} \), which are solid, liquid, gas, and granular

\[
P_{G=1} = [P_{sp}]. \tag{10}
\]

with probability vector single phase \( P_{sp} \) are

\[
P_{sol} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}, \tag{11}
\]

\[
P_{liq} = \begin{bmatrix} 0.060 & 0.110 & 0.110 & 0.125 & 0.250 & 0.125 & 0.110 & 0.110 \end{bmatrix}, \tag{12}
\]

\[
P_{gas} = \begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \end{bmatrix}, \tag{13}
\]

\[
P_{gra} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.250 & 0.500 & 0.250 & 0.000 & 0.000 \end{bmatrix}. \tag{14}
\]

Implementation of equations (11) – (14) and the results are given in figure 2.
Figure 2. Single phase system (after) initial and final state for various types of materials (a) solid, (b) liquid, (c) gas, and (d) granular.

Different probability vector will give different final state as shown in figure 2. $P_{\text{Sol}}$ can only allow agent to move according to the direction of gravitation acceleration $\vec{g}$, $P_{\text{Liq}}$ has most direction probability to the direction of gravitation acceleration $\vec{g}$ and still has for the other directions, $P_{\text{Gas}}$ has an isotropic direction probability to the eight directions, while $P_{\text{Gra}}$ has only three direction probabilities, which are NE, NO, and NW.

3.2. System with $G = 2$

In this part, a system consists of four regions, where each region can have its own probability vector $P_{\text{dir}}$ which could be in the form of $P_{\text{NO}}, P_{\text{NE}}, P_{\text{EA}}, P_{\text{SE}}, P_{\text{SO}}, P_{\text{SW}}, P_{\text{WE}}, P_{\text{NW}}$, whose value are given in following table 2. In the table probability vector for isotropic direction, $P_{\text{IS}}$ is the same as $P_{\text{Gas}}$ in the previous part of this work.

Table 2. Defined probability vector $P_{\text{dir}}$ for eight directions

| Dominant direction | Symbol | Value |
|--------------------|--------|-------|
| ↑                  | $P_{\text{NO}}$ | [0.650 0.050 0.050 0.050 0.050 0.050 0.050 0.050] |
| ↗                  | $P_{\text{NE}}$ | [0.050 0.650 0.050 0.050 0.050 0.050 0.050 0.050] |
| →                  | $P_{\text{EA}}$ | [0.050 0.050 0.650 0.050 0.050 0.050 0.050 0.050] |
| ↘                  | $P_{\text{SE}}$ | [0.050 0.050 0.050 0.650 0.050 0.050 0.050 0.050] |
| ↓                  | $P_{\text{SO}}$ | [0.050 0.050 0.050 0.050 0.650 0.050 0.050 0.050] |
| ↖                  | $P_{\text{SW}}$ | [0.050 0.050 0.050 0.050 0.050 0.650 0.050 0.050] |
| ←                  | $P_{\text{WE}}$ | [0.050 0.050 0.050 0.050 0.050 0.650 0.050 0.050] |
| ↼                  | $P_{\text{NW}}$ | [0.050 0.050 0.050 0.050 0.050 0.050 0.650 0.050] |
| ⋅                  | $P_{\text{IS}}$ | [0.125 0.125 0.125 0.125 0.125 0.125 0.125 0.125] |

There are to examples of implementation of probability vectors $P_{\text{dir}}$ from table 2 as shown in figure 3.
Figure 3. A system with $G = 2$ (after) initial and final state for fluid-like materials: (a) to center and (b) to corner flow.

Probability matrices for figure 3 are

\[
P_{\text{to center}}^{2 \times 2} = \begin{bmatrix} P_{SE} & P_{SW} \\ P_{NE} & P_{NW} \end{bmatrix}
\]

and

\[
P_{\text{to corner}}^{2 \times 2} = \begin{bmatrix} P_{NW} & P_{NE} \\ P_{SW} & P_{SE} \end{bmatrix}
\]

To center flow will constraint agents in the center region, while to corner flow will bring about a quarter of agents to each corner. Agents can be trapped in the corner since there are two walls constructing a corner and probability vector in the region is toward the corner.

3.3. System with $G = 3$

There are three systems with $G = 3$, where their probability matrices are given as follow

\[
P_{\text{two center} \backslash}^{2 \times 2} = \begin{bmatrix} P_{SE} & P_{WE} & P_{SW} \\ P_{NO} & P_{IS} & P_{SO} \\ P_{NE} & P_{EA} & P_{NW} \end{bmatrix},
\]

\[
P_{\text{two center} /}^{2 \times 2} = \begin{bmatrix} P_{SE} & P_{EA} & P_{SW} \\ P_{SO} & P_{IS} & P_{NO} \\ P_{NE} & P_{WE} & P_{NW} \end{bmatrix}
\]

and

\[
P_{\text{one center}}^{2 \times 2} = \begin{bmatrix} P_{SW} & P_{SO} & P_{SW} \\ P_{EA} & P_{IS} & P_{WE} \\ P_{NE} & P_{NO} & P_{NW} \end{bmatrix}
\]

Results from equations (17) – (19) are given in figure 4.
Figure 4. System with $G = 3$ (after) initial and final state for fluid-like materials: (a) two center \, (b) two center / and (c) one center.

Figure 4(a) and 4(b) are mirrored results to each other, where agents are segregated to opposite corners, while in figure 4(c) agents are confined in the center of the system.

3.4. System with $G = 4$
Four systems with $G = 4$ are observed, whose probability matrices are given as follow

\[
\mathbf{p}_{4\times4}^{(a)} = \begin{bmatrix}
P_{NW} & P_{NO} & P_{NO} & P_{NE} \\
P_{WE} & P_{IS} & P_{IS} & P_{EA} \\
P_{WE} & P_{IS} & P_{IS} & P_{EA} \\
P_{SW} & P_{SO} & P_{SO} & P_{SE}
\end{bmatrix}, \quad (20)
\]

\[
\mathbf{p}_{4\times4}^{(b)} = \begin{bmatrix}
P_{SE} & P_{SO} & P_{SO} & P_{SW} \\
P_{WE} & P_{IS} & P_{IS} & P_{EA} \\
P_{WE} & P_{IS} & P_{IS} & P_{EA} \\
P_{NE} & P_{NO} & P_{NO} & P_{NW}
\end{bmatrix}, \quad (21)
\]

\[
\mathbf{p}_{4\times4}^{(c)} = \begin{bmatrix}
P_{NE} & P_{EA} & P_{EA} & P_{SE} \\
P_{NO} & P_{NW} & P_{NE} & P_{SO} \\
P_{NO} & P_{SW} & P_{SE} & P_{SO} \\
P_{NW} & P_{WE} & P_{WE} & P_{SW}
\end{bmatrix}, \quad (22)
\]

\[
\mathbf{p}_{4\times4}^{(d)} = \begin{bmatrix}
P_{SW} & P_{NW} & P_{NE} & P_{SE} \\
P_{SO} & P_{NO} & P_{NO} & P_{SO} \\
P_{SO} & P_{NO} & P_{NO} & P_{SO} \\
P_{SE} & P_{NE} & P_{NW} & P_{SW}
\end{bmatrix}, \quad (23)
\]

Results from equations (20) – (23) is given in figure 5.
3.5. Discussion
In single phase system (system with $G = 1$), where the probability vector in all cells has the same value, four phases of materials, i.e. solid, liquid, gas, and granular, can be produced qualitatively. Then, by introducing regions with different vector probability ($G > 1$) richer phases of fluid-like materials can be obtained such as center- and corner-deposition ($G = 2$), two- and one-cluster agglomeration ($G = 3$), and outside- and horizontal-deposition, and single- and double-circular flow ($G = 4$). Further investigation with a higher value of $G$, e.g. $G > 4$, could lead to other interesting phases. From the results it can be said that this method has a very good potential in simulating a physical system, but with the physical reason why a probability vector $P$ should have such value. It would be better when $P$ can be derived from interactions between agents and also between agents and their environment.

4. Summary
Some materials phases, e.g. solid, liquid, gas, granular, deposition, flow, have been simulated using ABM for a single- and multi-region system. This approach can show how initial state evolving to final stable static or dynamics state, but only qualitatively due to the absence of the connection between interaction form and probability vector.

5. References
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