Effect of planar impurities on the superfluid density of striped cuprates

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We propose a mechanism for superconductivity suppression in stripe-correlated cuprates based on the pinning of the stripes by impurities, such as Zn. The suppression of superfluid density occurs in the vicinity of the impurity due to the low dimensional character of superfluid carriers on the stripes. Simple geometric argument about the planar fraction of the carriers, affected by stripe pinning, leads to prediction for impurity critical concentration \( z_c \sim T_c^2 \) and for the linear \( T_c \) suppression by Zn doping. We show that our results reproduce the Uemura relation in the pure system and we predict the behavior of the superfluid density as a function of Zn concentration. We compare our results with available data.

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One the most striking properties of high temperature superconductors (HTC) is the sensitivity of the critical temperature, \( T_c \), and the planar superfluid density, \( \sigma_z \), to planar impurities which are introduced with the substitution of the Cu atoms in the CuO planes. \( T_c \) is suppressed with a few percent of doping almost independently of the magnetic nature of the impurity. In particular it has been shown experimentally that the HTC undergo a superconductor to insulator transition due to Zn doping. The fast depression of \( \sigma_z \) with Zn has been studied extensively by NMR, \( \mu \)SR and infrared studies and a heated debate has been developed on the interpretation of the experimental data. It has also been established that this suppression is more robust in the underdoped compounds. The reduction of \( T_c \) has been assigned to formation of magnetic defects, the electron scattering by disorder in the presence of a d-wave order parameter, unitary scattering and local variation of the superconducting gap.

In this paper we would like to introduce a new scenario for the destruction of superfluid density which is based on the stripe picture of HTC. We propose that Zn sites pin stripes creating voids in the superconducting state with Zn sites creating voids in the superconducting picture of HTC. We propose that Zn sites pin stripes, so that the stripes form a quasistatic mesh. Within these approximations, assuming that all superfluid carriers are localized on the stripes, we find the rate of \( T_c \) suppression and critical Zn concentration required to completely suppress superconductivity in the simplified stripe model. The theoretical results fit the experimental data for underdoped La\(_{2-x}\)Sr\(_x\)CuO\(_4\), see Figs. 2,3.

Striped phases of holes have been observe experimentally in the superconductor La\(_{1.6-x}\)Nd\(_{0.4}\)Sr\(_x\)CuO\(_4\) and La\(_{2-x}\)Sr\(_x\)NiO\(_{4+y}\) which is an antiferromagnetic insulator. Moreover, stripe formation provides a simple explanation for the observed magnetic incommensurability in La\(_{2-x}\)Sr\(_x\)CuO\(_4\) and Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_8-x\). The magnetic incommensurability appears in neutron scattering by the splitting of the commensurate peak at \( Q = (\pi/a, \pi/a) \) \((a \approx 3.8\AA\) is the lattice spacing) by a quantity \( \delta \) which in the stripe picture is inversely proportional to the interstripe distance, \( \ell \). In a recent paper one of us (A. H. C. N.), proposed an explanation for the observed relationship between \( T_c \) and \( \delta \), namely, \( T_c \propto \delta \), as seen in neutron scattering and the suppression of \( T_c \) with Zn doping towards a superconductor-insulator transition in terms of the pinning of stripes by Zn. As shown in this problem is then formally equivalent to a set of coupled shunted Josephson junctions which undergo a Kosterlitz-Thouless transition \((KT)\) at \( T_c \) where (we use units such that \( \hbar = k_B = 1 \))

\[
T_c(x) = \frac{c}{\kappa_c \ell(x)} = \frac{569}{\ell(x)} \propto \delta
\]  

where \( \ell \) is given in \( \AA \), \( c = 0.2 \text{eV} \) is the velocity of propagation of the charge degrees of freedom and \( \kappa_c = 0.35 \) is the KT coupling constant. In what follows we will consider the physics of underdoped compounds like La\(_{2-x}\)Sr\(_x\)Cu\(_{1-x}\)Zn\(_x\)O\(_4\) where the stripe formation is possible.

In the case of La\(_{1.6-x}\)Nd\(_{0.4}\)Sr\(_x\)CuO\(_4\) neutron scattering indicates that the stripes organize themselves into a set of parallel chains on the CuO\(_2\) planes at distance \( \ell \) from each other. This set of chains rotate \( \pi/2 \) from plane to plane. The formation of these structures is associated with the competition between energy scales generated by the antiferromagnet and the long range Coulomb repulsion.

In our approach below we will assume that Zn sites act as a strongly repulsive sites which pin stripes. It is a natural assumption since we know that the Zn\(^{2+}\) in CuO\(_2\) plane will have a closed d shell \((\pi^{10})\) and any change of electronic density on Zn sites, brought upon motion of stripes, will require high energy excitations. Hence our assumption that Zn will pin stripes and slow their fluctuation in the plane. One can think of Zn sites as an...
obstacle which prevents stripes from fluctuating across in the plane. This assumption has to be contrasted with the effect of Sr$^-$ ions. Sr sites are well separated from the CuO$_2$ planes and the Coulomb interaction with the charges in the plane is weak. That is why we believe the main effect of Sr$^-$ is doping of the carriers in the plane. The pinning of stripes by Sr is weak if any.

Neutron scattering experiments indicate that the distance between the stripes is not affected by Zn doping [18]. It implies that the distortions of the stripe lattice, if exist, have to be local. Because of the repulsion between Zn and the stripes we assume, for simplicity, that the Zn atoms are located half distance between stripes as shown in Fig.1. This is equivalent to the formation of a topological defect. A similar effect would happen when two fluctuating stripes touch each other [21]. Since it has been demonstrated experimentally, as in the case of La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$, that when stripes are pinned by lattice distortions, superconductivity is suppressed it is natural to assume that superconductivity is associated with stripe fluctuations. In the presence of an impurity like Zn these fluctuations are suppressed close to the Zn site. This suppression extends over a circle of radius $R$ around the Zn position which sets the range of interaction between the Zn and the holes. This distance $R$ is set by the various energies scales in the problem including the string tension of the stripe and therefore should scale with the interstripe distance $\ell$. From Fig.1 it is easy to see that the linear region of the stripes that is affected is $\sqrt{R^2 - \ell^2}/4$. Thus, we parametrize $R = \gamma \ell/2$ with $1 < \gamma$. Here $\gamma$ is a doping independent phenomenological parameter which depends on the disorder and energetic details. From the neutron scattering experiments [18] we know that $\delta \propto 1/\ell \rightarrow 0$ when $x \approx 0.04$ at the metal-insulator transition implying that $R \rightarrow \infty$ in the insulating phase. This indicates that the effect of Zn is stronger in the insulating limit. In the underdoped regime ($0.04 < x < 0.12$) the data indicates $\delta \propto 1/\ell \propto x$ and therefore $R \propto 1/x$. In the optimally doped regime ($0.12 < x < 0.18$) it is observed that $\delta \propto 1/\ell \rightarrow$ constant. From that one concludes that $R \rightarrow$ constant and a crossover to a Fermi liquid regime where the density is uniform.

Assuming that the Zn atoms suppress superfluidity inside the circle of Fig.1 we find the following results: $i)$ in the pure compound the superfluid density is given by

$$\sigma_s(x, 0) \approx \frac{n}{509} T_c(x, 0)$$

(2)

which is the Uemura relation [22] ($n$ is the linear density of electrons on the stripes); $ii)$ we predict that in the zero temperature limit

$$\frac{\sigma_s(x, z)}{\sigma_s(x, 0)} = 1 - \frac{z}{z_c(x)};$$

(3)

where $z_c(x)$ is the critical Zn concentration for which the superfluid density vanishes which is given by

$$z_c(x) = \left( \frac{a(\bar{A})}{805} \right)^2 \frac{T_c^2(x, 0)}{\sqrt{\gamma^2 - 1}};$$

(4)

$iii)$ using the fact that the KT parameter $\kappa_c$ has to change with Zn content as

$$\frac{\kappa_c(z)}{\kappa_c(0)} = 1 + \alpha(x) z$$

(5)

where $\alpha(x)$ is a constant which depends on the geometry of the mesh. We obtain from (2) and (5), in the dilute limit,

$$\frac{T_c(x, z)}{T_c(x, 0)} \approx 1 - \frac{z}{z_c(x)}$$

(6)

where $\alpha(x) \approx 1/z_c(x)$. Notice that in the underdoped compounds ($x < 1/8$) the neutron scattering data show that $T_c(x, 0) \propto x$ [18] which implies from (2) that $z_c(x) \propto x^2$ which is a non-trivial prediction of our model.
Suppression given by Eq. (6) with $z_c$ is the size of the system then $L = \ell M$ in the direction perpendicular to the stripes. Each stripe has a linear density $n$ of electrons so that the Fermi momentum on each stripe is $k_F = \pi n / 2$. The total number of electrons in stripes on a plane is $M n L = n L^2 / \ell$. In this case the planar density of electrons which participate on stripe formation is

$$\sigma(x, 0) = \frac{n}{\ell(x)}.$$  \hfill (7)

Since the stripes are separated among themselves by antiferromagnetic regions\cite{17} we assume that only the electrons in stripes can form Cooper pairs. The other electrons are in an insulating state. This picture seems to agree with recent numerical calculations of the t-J model\cite{24} and the application of the stripe picture to the angle resolved photoemission spectra\cite{25}. Thus, at zero temperature and in the absence of planar impurities, the superfluid density is also given by\cite{7}, that is, $\sigma_s = \sigma$. By direct comparison of (6) with (7) we find

$$\sigma_s(x, 0) = \frac{n c T_c(x, 0)}{\ell}$$ \hfill (8)

and using (6) we derive (7). In La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$ it is found that $n = 1/4$ and it does not seem to change with doping\cite{13}.

In the presence of impurities the situation is changed because stripes are one dimensional objects are therefore will be greatly affected by impurity presence. Neutron scattering data indicates that the interstripe distance is not strongly affected by planar impurities and therefore $\ell = \ell(x)$ only\cite{13}. As it is shown in Fig.3 the number of stripe electrons which do not participate on the superfluidity per Zn atom is $4n \sqrt{R^2 - \ell^2 / 4} = 2 \ell n \sqrt{\gamma^2 - 1}$. Moreover, we assume that all the $N_{Zn}$ Zn atoms take part in pinning the stripes. Thus, the total number of electrons which do not participate is $N_{Zn} 2 \ell n \sqrt{\gamma^2 - 1}$ which implies a suppressed planar density of $N_{Zn} 2 \ell n / \sqrt{\gamma^2 - 1} / L^2$. However, we have $z = N_{Zn} (a / L)^2$ so that the superfluid density which is suppressed is

$$\delta \sigma_s = \frac{2 \ell n \sqrt{\gamma^2 - 1}}{a^2} = \frac{2 \ell^2 \sqrt{\gamma^2 - 1}}{a^2} \sigma(x, 0)$$ \hfill (9)

where we used (3). Finally, the total superfluid density of the system is given by $\sigma_s(x, z) = \sigma_s(x, 0) - \delta \sigma_s$ which immediately leads to Eq. (5) where the critical amount of Zn for which the superfluid density vanishes is given in (2) as

$$z_c(x) = \frac{1}{2 \sqrt{\gamma^2 - 1}} \left( \frac{a}{\ell(x)} \right)^2.$$ \hfill (10)

Comparing (2) to (7) one finds a non-trivial result which is given by (3). This result comes out naturally in the stripe model and should not be generally expected within

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**FIG. 2.** $z_c \times T_c$: continuous line: results from (4); filled squares: data from ref \cite{11}; open pentagons: data from ref \cite{10} for the overdoped systems.

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Some of these results can be compared with the experimental data for the in-plane penetration depth, $\lambda_{ab}$, obtained in $\mu$SR\cite{24}. The relaxation rate, $\tau^{-1}$, is given by: $\tau^{-1} \propto \sqrt{\lambda_{ab}^2 \times n_s / m^*}$ where $n_s$ is the three dimensional superfluid density and $m^*$ is the electron effective mass. If $c$ is the distance between the CuO$_2$ planes then $n_s = \sigma_s / c$. In Fig.2 we show the experimental data for $z_c \times T_c$ in La$_{2-x}$Sr$_x$CuO$_4$\cite{11,10} and the theoretical result with (4) with $\gamma \approx 1.43$ so that $R \approx 0.71 \ell$. Close to $x = 1/8$ the critical temperature develops a plateau. This plateau is reminiscent of the suppression of $T_c$ due to structural transitions in La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$\cite{13}. Because of the relationship between $z_c$ and $T_c$ as given in (4) we expect the system to be very sensitive to Zn content in this region. Thus, the effect of Zn in suppressing the superfluid density is actually stronger at the commensurate filling where stripes fluctuate less due to lattice pining\cite{13}. Indeed, experiments on the suppression of $T_c$ and on the thermoelectric power in La$_{2-x}$Sr$_x$Cu$_{1-x}$Zn$_x$O$_4$ strongly supports this scenario\cite{23}. The agreement between theory and experiment is not applicable since the system is essentially homogeneous and resembles more a two dimensional Fermi gas of holes.

In Fig.3 we show the result of equation (6) with $z_c(x)$ as given by (4) for the same parameters as above for La$_{2-x}$Sr$_x$Cu$_{1-x}$Zn$_x$O$_4$ with $x = 0.1$ and $x = 0.16$. We find a good agreement between our theory and experimental data\cite{11,10} in the overdoped region with the $T_c$ suppression given by Eq. (6) with $z_c(x = 0.15) \approx 0.03$ and $z_c(x = 0.10) \approx 0.02$.

The demonstration of these results is straightforward. We assume that there are $M$ stripes in the CuO$_2$ planes. Thus if $L$ is the size of the system then $L = \ell M$ in the

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homogeneous BCS-like models, where \( z_c \propto T_c(x, z = 0) \). Indeed the superconductivity in Abrikosov-Gor’kov approach is suppressed when the scattering rate \( 1/\tau_c \propto z \) becomes of the order of the superconducting gap \( \propto T_c \).

\[ \frac{T_c(x, z)}{T_c(x, 0)} \] from [24]: continuos line: \( x = 0.16 \), dashed line: \( x = 0.1 \). Experimental data from [10]: filled squares: \( x = 0.16 \); empty squares: \( x = 0.1 \).

In conclusion, we presented a “geometrical” model for the suppression of superfluid density and \( T_c \) by impurities within the stripe approach. It is based on the assumption that Zn impurities work as local pairbreaker and suppresses the superfluid density on stripes in the immediate vicinity of Zn. We find that our model fit the data on the underdoped La\(_{2-x}\)Sr\(_x\)Cu\(_{1-x}\)Zn\(_x\)O\(_4\) compounds with linear \( T_c \) suppression, Eq. [20] and predicts that \( z_c(x) \propto T_c(x) \) which is consistent with the data.

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