Compactifications of Type IIB Strings to Four Dimensions with Non-trivial Classical Potential

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Abstract

Type IIB strings are compactified on a Calabi-Yau three-fold. When Calabi-Yau-valued expectation values are given to the NS-NS and RR three-form field strengths, the dilaton hypermultiplet becomes both electrically and magnetically charged. The resultant classical potential is calculated, and minima are found. At singular points in the moduli space, such as Argyres-Douglas points, supersymmetric minima are found. A formula for the classical potential in $N = 2$ supergravity is given which holds in the presence of both electric and magnetic charges.
I. INTRODUCTION

It has been known for many years that compactification of type IIA or B strings on Calabi-Yau three-folds has an $N = 2$, $D = 4$ field theory limit. (See for example [1,2,3,4] and for explicit constructions, [5,6,7,8].) The 10-dimensional bosonic field content, in the electric description of type IIB strings, consists of the Neveu-Schwarz-Neveu-Schwarz (NS-NS) metric ($\hat{g}_{\hat{\mu}\hat{\nu}}$), dilaton ($\hat{\varphi}$) and two-form potential ($\hat{B}_{(1)}^{(\hat{\mu}\hat{\nu})}$) and the Ramond-Ramond (RR) dilaton ($\hat{l}$), two-form potential ($\hat{B}_{(2)}^{(\hat{\mu}\hat{\nu})}$) and four-form potential ($\hat{D}_{(4)}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$). (The hats distinguish the 10-dimensional fields/indices from 4-dimensional ones.) Under compactification on a Calabi-Yau three-fold, the metric gives rise to $2h_{21} + h_{11}$ real scalars and the $D = 4$ spacetime metric; each dilaton gives another scalar; each two-form potential gives rise to $h_{11} + 1$ real scalars and the four-form potential gives $h_{11}$ real scalars and $h_{21} + 1$ vectors (and their duals) [6,7,8]. This is the bosonic field content of $N = 2$, $D = 4$ supergravity with $h_{21}$ vector multiplets, $h_{11} + 1$ hypermultiplets and a gravity multiplet which contains the graviphoton which comes from $\hat{D}_{(4)}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ aligned along the Calabi-Yau holomorphic three-form [7,9,10,11]. Here the $h_{pq}$ are the Hodge numbers of the complex, Kähler manifold.

In this paper the consequences of giving expectation values to the field strengths of the 10-dimensional fields are examined. From Lorentz invariance only the three-form field strengths can get expectation values, since on a generic (i.e. not $T^6$ or $K_3 \times T^2$) Calabi-Yau, $h_{10}=0$. In section [1] it is shown that, as in [12], giving the field strengths expectation values corresponds, under dimensional reduction, to giving electric and magnetic charges to the dilaton hypermultiplet. In principle, the consistency (under the 10-dimensional equations of motion) of the expectation values with the Calabi-Yau structure of the compactification should be examined. However, since string theory suppresses the interactions of RR fields by a factor of the string coupling constant $e^{\hat{\varphi}}$, if attention is restricted to the weak coupling limit, where string perturbation theory is valid, then the theory for non-zero RR expectation values is just a perturbation of the usual Calabi-Yau compactification. Similarly, the field equation coupling the NS-NS field to gravity is suppressed by the volume of the Calabi-Yau,
so the large Calabi-Yau volume limit will be taken.

When the dilaton hypermultiplet is charged, the classical potential of the theory becomes non-trivial [12,10,13,14]. In [12], it was shown that giving a Calabi-Yau-valued expectation value to the RR 10-form field strength in the IIA theory, resulted in a potential with no non-singular minima; the theory was driven either to conifold points—where including fields corresponding to massless black holes removes the singularity [15,16,17]—or to the infinite Calabi-Yau-volume limit. It was subsequently speculated in [18] that if an additional RR field strength was given an expectation value, that the potential would have a non-singular minimum. It will be shown in section V that on the IIB side, both RR and NS-NS field strengths must have expectation values in order for the potential to have a minimum. This can be understood as follows. As explained in more detail in section I, the RR and NS-NS three-form expectation values are elements of $H^3(CY; \mathbb{Z})$, the natural basis for which is defined up to an $Sp(h_{21} + 1; \mathbb{Z})$ transformation (see, for example, [19,7]). Thus, the basis can be rotated so that the RR field strength is aligned along a specific basis vector. If the NS-NS field strength vanishes, this theory is related by mirror symmetry to that described in [12] for which, as just stated, the potential has no minimum. Hence, having only RR expectation values is insufficient for the potential to have a minimum.

While the potential can have minima when both RR and NS-NS fields have expectation values, it turns out that the minima not only occur for values of the moduli that are outside the region of validity (described above) of the calculation, but are not supersymmetric and hence are not protected from quantum corrections. At a conifold singularity, or at the more general singularities (Argyres-Douglas points) of [21], the potential can have flat directions with $N = 2$ supersymmetry. No $N = 1$ supersymmetric minima were found in this paper. This contrasts with [22] where $N = 1, D = 4$ type IIB vacua were found with

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1The argument can be given without reference to the IIB theory by considering the $Sp(h_{11} + 1; \mathbb{Z})$ action on the anticommuting basis for $H^0 \oplus H^2 \oplus H^4 \oplus H^6$ [21].
The paper is organized as follows. In section II it is shown how electric and magnetic charges for the dilaton hypermultiplet arise from expectation values of the three-form field strengths. This uses some results from [8] which are reviewed in the appendix. These results are then used in section II to derive the classical potential for the theory. The formula for the classical potential in a general $N = 2$ supergravity theory is reviewed in section IIIA. The formulas in the literature [10,13,14] all hold only in the absence of magnetic charge; to the author’s knowledge, a magnetic formula does not exist in the literature. One is proposed at the end of section IIIA. It turns out that the pure electric potential contains, for the purposes of this calculation, many of the same features as the general one, but is much simpler. Therefore the electric potential is discussed in detail before the magnetic one. (Of course, the electric potential cannot be used for the analysis at Argyres-Douglas points.) Assumptions of the model (such as the absence of the holomorphic prepotential for the vector moduli) are discussed in section IIIB and then the electric potential for the model under consideration is given explicitly. An explicit expression for the general potential is then discussed in section IIIIC. The electric potential is minimized in section IV and the general potential is minimized in section V. Supersymmetric minima at conifold points and particularly Argyres-Douglas points are discussed in section VI. Section VII is the conclusion.

II. DILATON CHARGES

Although the self-duality of the five-form field strength in type IIB string theory implies that the latter cannot be described by a supersymmetric 10-dimensional action, the bosonic fields can be described by a non-self-dual action in which the equation of motion for the five-form field strength is replaced by its Bianchi identity [23]. This is consistent with self-
duality, but does not imply it. When self-duality is imposed as a compactification condition, the non-self-dual action yields the correct compactified theory \cite{23,24}. In the Einstein frame, the action is, \cite{23}

\[ S = \int d^{10} \tilde{x} \sqrt{-\tilde{g}} \left\{ \frac{1}{2} \hat{R} - \frac{1}{8} \text{Tr}(\partial_{\hat{\mu}} \hat{M} \partial^{\hat{\mu}} \hat{M}^{-1}) + \frac{3}{8} \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}^T \hat{M} \hat{H}^{\hat{\mu}\hat{\nu}\hat{\rho}} + \frac{5}{12} \hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} + \frac{1}{192} \varepsilon^{ij} \varepsilon^{j\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}\hat{\beta}\hat{\gamma}\hat{\delta}} \hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \hat{H}_{\hat{\tau}\hat{\lambda}\hat{\beta}\hat{\gamma}\hat{\delta}} \right\}. \tag{2.1} \]

The field definitions are

\[ \hat{M} = \frac{1}{\text{Im} \lambda} \begin{pmatrix} |\lambda|^2 & -\text{Re} \lambda \\ -\text{Re} \lambda & 1 \end{pmatrix}; \quad \lambda = \hat{l} + i e^{-\hat{\phi}}, \tag{2.2a} \]

\[ \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \begin{pmatrix} \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}^{(1)} \\ \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}^{(2)} \end{pmatrix}; \quad \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}^{(i)} = \partial_{[\hat{\mu}} \hat{B}_{\hat{\nu}\hat{\rho}]^{(i)}}, \tag{2.2b} \]

\[ \hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = \partial_{[\hat{\mu}} \hat{D}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} + \frac{3}{4} \varepsilon^{ij} \hat{B}_{[\hat{\mu}\hat{\nu}}^{(i)} \partial_{\hat{\rho}} \hat{B}_{\hat{\sigma}]}^{(j)} + \ldots. \tag{2.2c} \]

Also, \( \varepsilon_{0\ldots9} = \sqrt{-\tilde{g}} \). It is in the final term of equation (2.1) that the \( D = 4 \) vectors (from \( \hat{D} \)) interact with the \( D = 4 \) scalars from the three-form field strengths. \( \) Therefore it is this term that will be examined closely.

It is convenient to rewrite this (up to an overall constant) as

\[ \varepsilon^{ij} \int \hat{F} \wedge \hat{H}^{(i)} \wedge \hat{B}^{(j)}. \tag{2.3} \]

(The Chern-Simons terms in equation (2.2c) don’t contribute because of the (anti)-symmetry of the wedge product.) To compactify to four dimensions use \[7,23,11\]

\[ \hat{F} = F^A \wedge A_A - G_A \wedge B^A + \ldots, \tag{2.4} \]

\(^2\)Another contribution, of opposite sign but not equal magnitude, of this form arises from the \( \hat{F}^2 \) term because of self-duality of \( \hat{F} \) and the Chern-Simons term in equation (2.2c): \( \hat{F}^2 \propto \hat{F} \wedge \hat{F} \propto \varepsilon^{ij} \hat{F} \wedge \hat{H}^{(i)} \wedge \hat{B}^{(j)} + \ldots; \) see equation (2.3). The remaining terms do not involve \( \hat{H}^{(i)} \) and therefore will not yield a new interaction.
where \((\alpha_\Lambda, \beta_\Lambda), \Lambda = \{0, 1, \ldots, h_{21}\}\) are some choice of symplectic basis for \(H^3(CY)\), \(F^\Lambda_{\mu\nu}\) are the 4-dimensional vector field strengths and \(G_{\Lambda\mu\nu}\) are the magnetic field strengths and the dual relationship between \(F^\Lambda_{\mu\nu}\) and \(G_{\Lambda\mu\nu}\) is due to self-duality of \(\hat{F}\); the terms that have been left out of equation (2.4) are those which will not contribute to the integral in equation (2.3). The 3-form field strengths are given Calabi-Yau expectation values via

\[
\begin{align*}
\langle \hat{H}^{(1)} \rangle &= \nu^{(1)}_{e\Lambda} \beta^\Lambda - \nu^{(1)A}_{m} \alpha_\Lambda, \\
\langle \hat{H}^{(2)} \rangle &= \nu^{(2)}_{e\Lambda} \beta^\Lambda - \nu^{(2)A}_{m} \alpha_\Lambda,
\end{align*}
\]

where the \(\nu_{m(e)}\) are constants that have been prematurely identified as values of the magnetic (electric) charges.

Using equations (2.3), integration of equation (2.3) over the Calabi-Yau gives

\[
\varepsilon^{ij} \int \left( \nu^{(i)}_{e\Lambda} F^\Lambda \land B^{(j)} - \nu^{(i)A}_{m} G_{\Lambda} \land B^{(j)} \right).
\]

Writing \(F^\Lambda\) and \(G_{\Lambda}\) in terms of electric and magnetic vector potentials \(A_{\mu}^\Lambda\) and \(\tilde{A}_{\Lambda\mu}\), gives, after an integration by parts, (again up to a constant)

\[
\varepsilon^{ij} \int d^4x \sqrt{-g} \left( \nu^{(i)}_{e\Lambda} A_{\mu}^\Lambda H^{(j)\mu} - \nu^{(i)A}_{m} \tilde{A}_{\Lambda\mu} H^{(j)\mu} \right),
\]

where

\[
H^{(i)\mu} = \frac{1}{6} \varepsilon^{\nu\sigma\tau\mu} \partial_\nu B^{(i)}_{\sigma\tau}.
\]

To understand this, it is necessary to relate the \(H^{(i)}_{\mu}\) to the 4-dimensional scalars. This is done in the appendix. The result is that to lowest order in the coupling constant, with, for simplicity, the fields corresponding to the \(h_{21}\) data set to zero,

\[
\begin{align*}
H^{(1)}_{\mu} &= \frac{2}{3} e^{\frac{i}{2} \hat{K}} \frac{-4}{3} \hat{K} \partial_\mu \text{Im}S, \\
H^{(2)}_{\mu} &= \frac{2\sqrt{2}}{3} e^{\frac{i}{2} \hat{K}} \frac{-4}{3} \hat{K} \partial_\mu \text{Im}C_0;
\end{align*}
\]

also, the string coupling constant is

\[
e^{\hat{\phi}} = \sqrt{2e^{\frac{i}{2} \hat{K}} \frac{-4}{3} \hat{K}}.
\]
Here $S$ and $C_0$ are the $N = 1$ superfields which form the dilaton hypermultiplet; as in [12] the four dimensional dilaton has been generalized to

$$\phi = \frac{1}{2} e^{-\tilde{K}}. \quad (2.10)$$

The Kähler potential of the special geometry precursor to the quaternionic manifold [120] is denoted by $K$, while the Kähler potential for the rest of the quaternionic manifold is denoted by $\tilde{K}$. (In particular, the metric on the hypermultiplets is determined by $\tilde{K}$.) In equation (2.10), the non-dilaton multiplets are omitted from the Kähler potential (see also equation (3.19d)). That is, the above equations were derived by explicitly compactifying on a “minimal” Calabi-Yau manifold with $h_{11} = 1$ and $h_{21} = 0$ and even ignoring much of this Calabi-Yau data. For this compactification there is a relation between $\text{Im} Z$ ($Z$ being the complex coordinate on the one complex-dimensional special geometry precursor to the quaternionic manifold), $K$ and $\mathcal{R}^{00}$ (where $\mathcal{R}^{00}$ is defined in the appendix). Since for more generic manifolds, there are many $\text{Im} Z$s while there is only one $K$ (or $\mathcal{R}^{00}$), it is preferable to use the latter in these formulas.

Substituting equations (2.9) into equation (2.7) gives, after a Weyl rescaling $g_{\mu\nu} \rightarrow \sqrt{2} e^{\frac{3}{2}} \tilde{K} g_{\mu\nu}$ (to go to the $D = 4$ Einstein metric),

$$\frac{2\sqrt{2}}{3} \int d^4x \sqrt{-g} \left\{ \sqrt{2} e^{\tilde{K} + K} \nu_{\epsilon\alpha}^{(1)} A_\mu^\Lambda \partial^\mu \text{Im} C_0 - \sqrt{2} e^{-\tilde{K} + K} \nu_{m(1)}^{(1)\Lambda} \tilde{A}_\mu^\Lambda \partial^\mu \text{Im} C_0 - e^{2\tilde{K}} \nu_{\epsilon\alpha}^{(2)} A_\mu^\Lambda \partial^\mu \text{Im} S + e^{2\tilde{K}} \nu_{m(2)}^{(2)\Lambda} \tilde{A}_\mu^\Lambda \partial^\mu \text{Im} S \right\}. \quad (2.11)$$

This can be recognized as the interaction terms of the vector potentials with charged fields $e^S$ and $e^{C_0}$. Hence, as in [12], completing the square with the kinetic terms for the hypermultiplets [20,8] gives (with an appropriate numerical rescaling of $\nu_{\epsilon(m)\alpha}$)

$$S = \int d^4x \sqrt{-g} \left\{ 8 e^{\tilde{K} + K} \left( \nu_{\epsilon\alpha}^{(1)} A_\mu^\Lambda - \nu_{m(1)}^{(1)\Lambda} \tilde{A}_\mu^\Lambda + \partial_\mu \text{Im} C_0 \right)^2 + 2 e^{2\tilde{K}} \left( \nu_{\epsilon\alpha}^{(2)} A_\mu^\Lambda - \nu_{m(2)}^{(2)\Lambda} \tilde{A}_\mu^\Lambda + \partial_\mu \text{Im} S \right)^2 + \ldots \right\}. \quad (2.12)$$

From this equation, it is seen that $\text{Im} C_0$ carries electric (magnetic) charges $\nu_{\epsilon\alpha}^{(1)}$ ($\nu_{m(1)\alpha}$) and that $\text{Im} S$ carries electric (magnetic) charges $\nu_{\epsilon\alpha}^{(2)}$ ($\nu_{m(2)\alpha}$). Note that this coincides with the
type IIA calculation of \[12\] where ImS carried electric and magnetic charges proportional to the expectation values of the RR field strengths of the IIA theory (the authors of \[12\] did not consider NS-NS expectation values).

Charge quantization \[27,12\] requires that \(\nu_e^{(i)}\) and \(\nu_m^{(i)A}\) be integers.\(^3\)

Finally, note that an \(Sp(h_{21} + 1, \mathbb{Z})\) transformation on the basis \((\beta^A, \alpha^A)\) will rotate the charge vectors \((\nu_m^{(i)A}, \nu_e^{(i)})\). By performing \(SL(2, \mathbb{Z})\) electromagnetic duality transformations on each vector independently, followed by a perturbative \(Sp(h_{21} + 1, \mathbb{Z})\) transformation of the form \[
\begin{pmatrix}
A_{h_{21}+1} & 0 \\
0 & (A_{h_{21}+1}^T)^{-1}
\end{pmatrix},
\]
\(A_{h_{21}+1} \in SL(h_{21} + 1, \mathbb{Z})\) it is always possible to perform a rotation so that the NS-NS charge vector \((\nu_m^{(1)A}, \nu_e^{(1)})\) is pure electric and aligned with respect to only one \(U(1)\), say \(A^0\), with positive charge. It is then possible to perform further electromagnetic duality transformations on all the vectors but \(A^0\) so that the only potentially non-zero magnetic charge is \(\nu_m^{(2)0}\). This can be followed by a perturbative \(Sp(h_{21} + 1, \mathbb{Z})\) transformation with \(A_{h_{21}+1} = \begin{pmatrix} 1 & 0 \\ 0 & A_{h_{21}} \end{pmatrix}\) so that all \(\nu_e^{(2)}\) vanish except possibly \(\nu_e^{(2)}\) and \(\nu_e^{(2)}\).

Furthermore, if \(\nu_e^{(1)}\nu_m^{(2)A} - \nu_m^{(1)A}\nu_e^{(2)} = 0\) then in the new basis \(\nu_m^{(2)0} = 0\) and all charges are pure electric. This is known \[21\] as the local case. Otherwise, \(\nu_m^{(2)0} \neq 0\); this is the non-local case.

To summarize the last paragraph, it is always possible to choose a symplectic basis so that the NS-NS charge vector \((\nu_m^{(1)A}, \nu_e^{(1)})\) is pure electric, with respect to only one \(U(1)\) and the RR charge vector is, at most, magnetically and electrically charged with respect to that \(U(1)\) and electrically charged with respect to one other \(U(1)\). In fact, there is more freedom in special cases. In the local (vanishing magnetic charge) case, if \(\nu_e^{(2)} = m\nu_e^{(2)}\), \(m \in \mathbb{Z}\), then

\(^3\)Presumably this result can also be obtained directly from equations \([2,3]\) and quantization of RR charge in ten dimensions \([28,29,27]\) (S duality extends this quantization to the NS-NS three-form charge).
choosing } \mathbf{A}_{h_{21}+1} = \begin{pmatrix} 1 & -m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ makes the RR charge vector electrically charged under only one } U(1), \text{ different from that under which the NS-NS is charged. (This does not work in the non-local case as the magnetic charge transforms non-trivially.) In the non-local case if } \nu^{(2)}_{eA} \text{ are integer multiples of } \nu^{(2)}_m, \text{ then they can be eliminated using the symplectic matrix (all but } \Lambda = 0, 1 \text{ components are suppressed)}

\begin{pmatrix}
1 & 0 & \nu^{(2)}_{eA} & -\nu^{(2)}_v \\
0 & 1 & -\nu^{(2)}_v & \nu^{(2)}_m \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}

\text{so that the RR charge vector is pure magnetic under the same } U(1) \text{ that the NS-NS charge vector is pure electric.}

III. CLASSICAL POTENTIAL

A. Review of } N = 2 \text{ Supergravity with Electrically Charged Matter and Generalization to Magnetically Charged Matter

Recall [30] that there are essentially three types of } N = 2 \text{ multiplets: gravitational, vector and hypermultiplets. The gravitational multiplet consists of the graviton } g_{\mu\nu}, \text{ two gravitini } \psi^{A}_\mu \text{ and the graviphoton } A_\mu. \text{ The gravitini form a doublet under the } SU(2) \text{ which relates the two supersymmetries; hence they are labelled by the index } A = 1, 2. \text{ Each of the } h_{21} \text{ vector multiplets consists of a vector } A^a_\mu, \text{ two gauginos } \lambda^{aA} \text{ and a complex scalar } z^a, \text{ } a = 1 \ldots h_{21}. \text{ (Vector multiplets can also be written in } N = 1 \text{ superfield notation as a chiral multiplet plus an } N = 1 \text{ vector multiplet.) The } h_{11} + 1 \text{ hypermultiplets consist of } 4(h_{11} + 1) \text{ scalars } q^u \text{ and } 2h_{11} \text{ hyperini } \zeta_\alpha, \text{ } u = 1 \ldots 4(h_{11} + 1) \text{ and } \alpha = 1 \ldots 2(h_{11} + 1). \text{ The natural way in which the index } \alpha \text{ arises will be discussed shortly. (As used in the previous section, in } N = 1 \text{ superfield notation, a hypermultiplet consists of two chiral superfields.)}

The vector multiplet scalars map out a special Kähler manifold. This will only be described briefly here; for more detail the reader is referred to the litera-
ture \[9,10,13,14,31,32,25\]. Roughly, a special Kähler manifold is a complex Kähler manifold whose Kähler potential is derived from a holomorphic prepotential \( F(z^a) \). It is convenient to define special coordinates via projective coordinates \( X^\Lambda, \Lambda = 0 \ldots h_{21}, z^a = \frac{X^a}{X^0} \).

Then, \( F(X^\Lambda) \) is required to be a homogeneous function of degree 2. Defining
\[
F_\Lambda = \frac{\partial F(X^\Lambda)}{\partial X^\Lambda},
\]
the Kähler potential can be written as
\[
K_V = -\ln i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda).
\]

Equation (3.2) is \( Sp(h_{21}+1, \mathbb{R}) \) invariant, where \( (X^\Lambda, F_\Lambda) \) transform as a symplectic vector. In special coordinates it is natural, especially given the superspace Bianchi identities \[9\], to define the graviphoton to be \( A^0 \), so that, in addition to the \( X^\Lambda \)s there are \( A^\Lambda \)s. (In fact, this argument in reverse is the usual reason for introducing \( X^0 \).) Thus, it is seen how the symplectic formulation of special Kähler geometry is the natural one. In fact, the \( Sp(h_{21}+1, \mathbb{R}) \) transformation is the same one that mixes the basis vectors \( \alpha_\Lambda, \beta^\Lambda \) of \( H^3(CY) \).

Charge quantization, and/or the requirement that \( (\beta^\Lambda, \alpha_\Lambda) \) be in \( H^3(CY, \mathbb{Z}) \), requires the restriction to \( Sp(h_{21}+1, \mathbb{Z}) \). It follows immediately that in a general basis \( A^0 \) will be the graviphoton only if the Calabi-Yau holomorphic three-form is aligned with \( \alpha_0 \); in general this is not only false but impossible. Also, the \( X^\Lambda(z^a) \) are not necessarily projective versions of the coordinates \( z^a \), but are general holomorphic functions. This fact and the fact that the holomorphic prepotential \( F(X^\Lambda) \) is not guaranteed to exist in a general basis, makes it necessary to find symplectic invariant, prepotential independent, formulas for quantities. This has been done in \[31,25\].

It is sometimes useful to define
\[
L^\Lambda = e^{\frac{K_V}{2}} X^\Lambda, M_\Lambda = e^{\frac{K_V}{2}} F_\Lambda.
\]

The natural derivative to use for these is the covariant derivative
\[
\nabla_a L^\Lambda = \partial_a L^\Lambda + \frac{1}{2} (\partial_a K_V) L^\Lambda \equiv f_a^\Lambda, \partial_a = \frac{\partial}{\partial z^a}.
\]
\[ \nabla_a M_\Lambda = \partial_a M_\Lambda + \frac{1}{2}(\partial_a K_V)M_\Lambda \equiv h_{a\Lambda}, \quad (3.4b) \]
\[ \nabla_a L^\Lambda = \partial_a L^\Lambda - \frac{1}{2}(\partial_a K_V)L^\Lambda \equiv 0, \quad (3.4c) \]
\[ \nabla_a M_\Lambda = \partial_a M_\Lambda - \frac{1}{2}(\partial_a K_V)M_\Lambda \equiv 0, \quad (3.4d) \]

where the last two equations follow from holomorphicity of \( X^\Lambda \) and of \( F_\Lambda \). Supersymmetry implies the existence of a matrix \( \mathcal{N}_{\Lambda \Sigma} \) so that \[ M_\Lambda = \mathcal{N}_{\Lambda \Sigma} L^\Sigma, \quad (3.5a) \]
and
\[ h_{a\Lambda} = \mathcal{N}_{\Lambda \Sigma} f^\Sigma_a, \quad (3.5b) \]

Finally, it is convenient to define
\[ U^{\Lambda \Sigma} = g^{ab} f^\Lambda_a f^\Sigma_b, \quad (3.6) \]
where \( g^{ab} \) is the inverse of the Kähler metric \( g_{ab} = \partial_a \partial_b K_V \).

The hypermultiplets parametrize a quaternionic manifold. A quaternionic manifold has three almost complex structures that obey the quaternionic \( (Sp(1) \sim SU(2)) \) algebra and whose Kähler forms are covariantly closed using an \( SU(2) \) connection whose field strength is proportional to the Kähler form triplet. That is,
\[ \Omega^{xu}_v \Omega^{yw}_w = -\delta^{xy} \delta^u_v - \varepsilon^{xyz} \Omega^{zu}_w, \quad (3.7a) \]
\[ \nabla \Omega^x \equiv d\Omega^x + \varepsilon^{xyz} \omega^y \wedge \Omega^z = 0, \quad \text{and} \quad (3.7b) \]
\[ d\omega^x + \frac{1}{2} \varepsilon^{xyz} \omega^y \wedge \omega^z = \Omega^x, \quad (3.7c) \]

using the canonical normalization \[ \text{[32,11,13,14]}, \] where \( \Omega^{xu}_v \) is the triplet of complex structures and \( \omega^x_u \) is the \( SU(2) \) connection, \( x = 1, 2, 3 \). The holonomy of a quaternionic manifold is \( SU(2) \times H \) with \( H \subset Sp(h_{11} + 1) \). The \( SU(2) \) factor is that whose curvature is the Kähler form triplet and is also the \( SU(2) \) that rotates the supersymmetries. From the holonomy of the manifold, the natural flat metric is the \( SU(2) \times Sp(h_{11} + 1) \) one; i.e. the vielbein is \( U^u_{Aa} \)
where again \( A = 1, 2 \) is the \( SU(2) \) index and \( \alpha = 1, \ldots, 2(h_1 + 1) \) is the \( Sp(h_1 + 1) \) index. Because each hyperino is an \( SU(2) \) singlet, the hyperini are labelled only by the \( \alpha \) index, as indicated above.

If the hypermultiplet is electrically charged, then there must be a symmetry of the theory that is gauged. In other words the vector multiplets gauge isometries of the quaternionic manifold. (This is also true of the special Kähler manifolds; however, the vectors considered here are abelian and hence uncharged.) The covariant derivative of the coordinate (hypermultiplet scalar) is (compare to equation (2.12))

\[
\nabla_\mu q^u = \partial_\mu q^u + k_\Lambda^u A_\mu^\Lambda
\]

where \( k_\Lambda^u \) is the Killing vector that generates the isometry. The isometries of the quaternionic manifold must respect the quaternionic nature of the manifold. So, the Lie derivatives of the Kähler forms and the \( SU(2) \) connection, with respect to the Killing vector, must vanish up to an \( SU(2) \) gauge transformation \([10,13,14]\). Then, Killing prepotentials, \( P_\Lambda^x \), can be found which satisfy \(3.9\)

\[
\Omega^x(k_\Lambda, \cdot) = -dP_\Lambda^x - \varepsilon^{xyz} \omega^y P_\Lambda^z.
\]

The general formula for the classical potential in an \( N = 2 \) supergravity theory was given in \([35,36]\). Note that the derivation therein is very general and should hold in the presence of both electric and magnetic charges. The potential is given by the Ward identity

\[
V \delta^A_B = g_{ab} W^{aAC} \bar{W}^b_{BC} + N^{\alpha A} N_\alpha B - 12 S_{BC} S^{CA},
\]

where \( W^{aAC}, N_\alpha^A \) and \( S_{AB} \) are respectively the matrices governing the SUSY transformations of the gaugino, the hyperino and the gravitino mass matrix. Specifically,

\[
\delta \lambda^a_A = \ldots + W^{aAB} \epsilon_B,
\]

\[
\delta \zeta_\alpha = \ldots + N_\alpha^A \epsilon_A,
\]

\[
\delta \psi_{A\mu} = D_\mu \epsilon_A + \ldots + i S_{AB} \gamma_\mu \epsilon^B.
\]
where $\epsilon$ is the SUSY transformation parameter, $D_\mu$ is the spacetime covariant derivative, and the missing terms are those which vanish in the Lorentz invariant, bosonic background.

These matrices were worked out for the case of vanishing magnetic charge in [10,13,14]. They are given by

\begin{align}
W^{aAB} &= i(\sigma_x)C^BP_x^A\xi^{ab}\bar{f}^{\Lambda}_{b}, \\
N^A_{\alpha} &= 2U^A_{\alpha\mu}k^L_{\lambda}\Lambda^\lambda, \\
S_{AB} &= \frac{1}{2}(\sigma_x)_{A}^{C}\xi_{BC}(P_x^{L}\Lambda^\lambda - \bar{\mathcal{P}}^{L}\Lambda^\lambda).
\end{align}

This gives

\[ V = 2h_{uv}k^L_{\lambda}\Lambda^\lambda\bar{L}_\Sigma + (U^\Lambda\Sigma - 3L^\Lambda\bar{L}_\Sigma)\mathcal{P}_{\Lambda\Sigma} \]

for the potential, upon insertion into equation (3.10). The quaternionic metric is denoted by $h_{uv}$.

To generalize this to the case of non-vanishing magnetic charge, it is necessary (though not necessarily sufficient) to find symplectic invariant versions of e.g. equations (3.12) and (3.13), that reduce to these when the magnetic charge vanishes. To attempt this, note first that equation (2.12) suggests that equation (3.8) be replaced by

\[ \nabla_{\mu}q^u = \partial_{\mu}q^u + k^u_{\Lambda}\xi_{\mu} - \bar{k}^u_{\Lambda}A_{\mu}, \]

where $\bar{k}^u_{\Lambda}$ is the Killing vector gauged by the magnetic vectors, $\bar{A}_{\Lambda}$, and the minus sign comes from the symplectic metric. (Of course this only works on-shell—the off-shell Lagrangian is necessarily either non-local or non-Lorentz covariant—see e.g. [37,38].) It is clear, then, (or at least natural to assume) that ($\bar{k}^u_{\Lambda}$, $k_{\Lambda}$) and ($A^\Lambda$, $\bar{A}_{\Lambda}$) are symplectic vectors. It is also clear that the magnetic Killing vectors suffer from the same restrictions as the electric ones, and hence the analogue of equation (3.9) holds for $\bar{P}^{Lx}$. So the generalization of equations (3.12) is

\begin{align}
W^{aAB} &= i(\sigma_x)C^BP_x^A\xi^{ab}(\mathcal{P}_x^{L}\bar{f}^{\Lambda}_{b} - \bar{\mathcal{P}}^{Lx}\bar{h}_{\Lambda}), \\
N^A_{\alpha} &= 2U^A_{\alpha\mu}(k^L_{\lambda}\bar{L}^\lambda - \bar{k}^\Lambda^u\bar{M}_{\lambda}), \\
S_{AB} &= \frac{1}{2}(\sigma_x)_{A}^{C}\xi_{BC}(P_x^{L}\Lambda^\lambda - \bar{\mathcal{P}}^{Lx}\Lambda^\lambda).
\end{align}
Inserting this into equation (3.10) gives

\[ V = g^{ab}(\mathcal{P}_{\Lambda}^x f_a^\Lambda - \tilde{\mathcal{P}}^{\Lambda x} h_{a\Lambda})(\mathcal{P}_{\Sigma}^x \tilde{f}^\Sigma - \tilde{\mathcal{P}}^{\Sigma x} \tilde{h}_{b\Sigma}) - 3(\mathcal{P}_{\Lambda}^x L^\Lambda - \tilde{\mathcal{P}}^{\Lambda x} M_{\Lambda})(\mathcal{P}_{\Sigma}^x \tilde{L}^{\Sigma} - \tilde{\mathcal{P}}^{\Sigma x} \tilde{M}_{\Sigma}) + 
\]

\[ 2h_{uv}(k_\Lambda L^\Lambda - \tilde{k}^{\Lambda u} M_{\Lambda})(k_{\Sigma} L^{\Sigma} - \tilde{k}^{\Sigma u} \tilde{M}_{\Sigma}). \]  

(3.16)

Deriving these using the approach of [9,10] would be the ultimate justification of these formulas.

\[ \text{B. The Local Case} \]

Returning to the theory at hand, if \( \nu^{(1)}_{e\Lambda} \nu^{(2)}_{m\Lambda} - \nu^{(1)}_{m\Lambda} \nu^{(2)}_{e\Lambda} = 0 \) then, as discussed at the end of section II, it is possible to perform an \( Sp(h_{21} + 1; \mathbb{Z}) \) transformation to a basis in which the magnetic charges vanish. The more general case of nonvanishing magnetic charge will be considered in the next subsection.

From [26,12,8], the quaternionic structure is given by

\[ \Omega^x = i e^x \sigma^x e, \quad \text{and} \]

\[ \omega^x \sigma^x = 2i \begin{pmatrix} \frac{1}{4}(v - \bar{v}) & -u \\ \bar{u} & -\frac{1}{4}(v - \bar{v}) \end{pmatrix}, \]

(3.17)

(3.18)

where

\[ e = \begin{pmatrix} u \\ v \end{pmatrix}; \]

(3.19a)

\[ u = 2e^{\frac{\hat{K}}{2} + \frac{K}{2}} dC_0, \quad \text{and} \]

(3.19b)

\[ v = e^{\hat{K}} dS - 4(C_0 + \bar{C}_0)e^{\hat{K} + K} dC_0, \quad \text{with} \]

(3.19c)

\[ \hat{K} = - \ln[S + \bar{S} - 2(C_0 + \bar{C}_0)^2 e^K], \]

(3.19d)

ignoring all but the dilaton multiplet. This gives,
\( \Omega^1 = i(\bar{u} \land v + \bar{v} \land u) \)
\[
= 2ie^{\frac{\bar{K}}{2} + \frac{K}{2}}(d\bar{C}_0 \land dS + d\bar{S} \land dC_0) - 16i(C_0 + \bar{C}_0)e^{\frac{\bar{K}}{2} + \frac{K}{2}}d\bar{C}_0 \land dC_0, \tag{3.20a}
\]
\( \Omega^2 = (\bar{u} \land v - \bar{v} \land u) \)
\[
= 2e^{\frac{\bar{K}}{2} + \frac{K}{2}}(d\bar{C}_0 \land dS - d\bar{S} \land dC_0), \tag{3.20b}
\]
\( \Omega^3 = i(\bar{u} \land u - \bar{v} \land v) \)
\[
= 4ie^{\bar{K} + K}(1 - 4(C_0 + \bar{C}_0)^2)e^{\bar{K} + K}d\bar{C}_0 \land dC_0 - ie^{2\bar{K}}d\bar{S} \land dS +
4i(C_0 + \bar{C}_0)e^{2\bar{K} + K}(d\bar{C}_0 \land dS + d\bar{S} \land dC_0), \tag{3.20c}
\]
\[\omega^1 = i(\bar{u} - u) = 2ie^{\frac{\bar{K}}{2} + \frac{K}{2}}(d\bar{C}_0 - dC_0), \tag{3.20d}\]
\[\omega^2 = u + \bar{u} = 2e^{\frac{\bar{K}}{2} + \frac{K}{2}}(dC_0 + d\bar{C}_0), \tag{3.20e}\]
\[\omega^3 = \frac{i}{2}(v - \bar{v}) = \frac{i}{2}e^{\bar{K}}(dS - d\bar{S}) - 2i(C_0 + \bar{C}_0)e^{\bar{K} + K}(dC_0 - d\bar{C}_0). \tag{3.20f}\]

It is readily verified that equations (3.20) satisfy equations (3.7), where the quaternionic metric is derived from equation (3.19d) (see equations (3.25) below).

The Killing vectors can be read off of equation (2.12). They are
\[
k^u_{\Lambda} = i\nu^{(1)}_{\Lambda \Lambda} \left( \frac{\partial}{\partial C_0} - \frac{\partial}{\partial \bar{C}_0} \right)^u + i\nu^{(2)}_{\Lambda \Lambda} \left( \frac{\partial}{\partial S} - \frac{\partial}{\partial \bar{S}} \right)^u. \tag{3.21}
\]

It is readily verified that
\[
\mathcal{L}_{k_{\Lambda}} \omega^x = 0 \tag{3.22a}
\]
which implies
\[
\mathcal{L}_{k_{\Lambda}} \Omega^x = 0, \tag{3.22b}
\]
where \( \mathcal{L}_X \) denotes the Lie derivative with respect to the vector field \( X \). These together imply the vanishing of the \( SU(2) \) compensator associated with the \( k_{\Lambda} \), which, in turn, gives \( 10 \)
\[
\mathcal{P}^x_{\Lambda} = \omega^x_{\Lambda \Lambda} k^u_{\Lambda}, \tag{3.23}
\]
or

15
\[ \mathcal{P}^1_\Lambda = 4e^{\hat{K} + K}\nu_{e\Lambda}^{(1)}, \quad \text{(3.24a)} \]
\[ \mathcal{P}^2_\Lambda = 0, \quad \text{and} \quad \mathcal{P}^3_\Lambda = 4(C_0 + \bar{C}_0)e^{\hat{K} + K}\nu_{e\Lambda}^{(1)} - e^{\hat{K}}\nu_{e\Lambda}^{(2)}. \quad \text{(3.24c)} \]

These, of course, satisfy equation (3.9). Again, this coincides with the IIA result of [12] when \( \nu_{e\Lambda}^{(1)} = 0 \).

From equation (3.19d) the quaternionic metric components are found to be
\[ h_{\bar{S}S} = e^{2\hat{K}}, \quad \text{(3.25a)} \]
\[ h_{\bar{C}0} = h_{C0\bar{S}} = -4(C_0 + \bar{C}_0)e^{2\hat{K} + K}, \quad \text{(3.25b)} \]
\[ h_{C0\bar{C}0} = 4e^{\hat{K} + K} + 16(C_0 + \bar{C}_0)^2e^{2\hat{K} + 2K}, \quad \text{(3.25c)} \]

and so
\[ h_{uv}k^u_{\Lambda\Lambda}k^v_{\Sigma\Sigma} = 8e^{\hat{K} + K}[1 + 4(C_0 + \bar{C}_0)^2e^{\hat{K} + K}]\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(1)} + 2e^{2\hat{K}}\nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(2)} - 8(C_0 + \bar{C}_0)e^{2\hat{K} + K}(\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(2)} + \nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(1)}). \quad \text{(3.26)} \]

However, from equations (3.24), it is found that
\[ \mathcal{P}^x_\Lambda\mathcal{P}^x_\Sigma = 16e^{\hat{K} + K}[1 + (C_0 + \bar{C}_0)^2e^{\hat{K} + K}]\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(1)} + e^{2\hat{K}}\nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(2)} - 4(C_0 + \bar{C}_0)e^{2\hat{K} + K}(\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(2)} + \nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(1)}). \quad \text{(3.27)} \]

Inserting equations (3.26) and (3.27) into equation (3.13) gives the classical potential:
\[ V = \left\{ 16e^{\hat{K} + K}[1 + 4(C_0 + \bar{C}_0)^2e^{\hat{K} + K}]\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(1)} + 4e^{2\hat{K}}\nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(2)} - 16e^{2\hat{K} + K}(C_0 + \bar{C}_0)(\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(2)} + \nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(1)}) \right\} L^\Lambda L^\Sigma + \]
\[ (U^{\Lambda\Sigma} - 3L^\Lambda \bar{L}^\Sigma) \left\{ 16e^{\hat{K} + K}[1 + (C_0 + \bar{C}_0)^2e^{\hat{K} + K}]\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(1)} + \right. \]
\[ \left. e^{2\hat{K}}\nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(2)} - 4e^{2\hat{K} + K}(C_0 + \bar{C}_0)(\nu_{e\Lambda}^{(1)}\nu_{e\Sigma}^{(2)} + \nu_{e\Lambda}^{(2)}\nu_{e\Sigma}^{(1)}) \right\}. \quad \text{(3.28)} \]

It is fairly obvious from this equation that the classical potential will not vanish for generic moduli and non-zero \( \nu_{e\Lambda}^{(i)} \) (see also section [V]). This distinguishes this model from that of
for which the classical potential vanished identically, and, for an appropriate choice of charges, there was partial supersymmetry breaking to $N = 1$. This difference can be understood as arising from the quaternionic structure of the manifold. Specifically, the difference comes from the fact that in the current model, $h_{uv}k^u_{\Lambda}k^v_{\Sigma} \neq \mathcal{P}_\Lambda^x\mathcal{P}_\Sigma^y$, while there was equality in the model of [39]. This can be seen in more detail by using the fact that equation (3.23) holds for both models; hence, (since $\omega^x_{(u}\omega^y_{v)} = 0$)

$$\mathcal{P}_\Lambda^x\mathcal{P}_\Sigma^y = \omega^x_{(u}\omega^y_{v)}k^u_{\Lambda}k^v_{\Sigma}. \quad (3.29)$$

So, $\omega^x_{(u}\omega^y_{v)}$ behaves like a metric in equation (3.29). In fact, the quaternionic manifold in [39] is sufficiently trivial that $\omega^x_{(u}\omega^y_{v)}$ is the metric (parallel to $k^u_{\Lambda}$); however, in the current case, equations (3.20) shows that $\omega^x_{(u}\omega^y_{v)}$ is not even hermitian (with respect to the complex structure defining $S$ and $C_0$ as complex variables)! The resultant mismatched factors in equation (3.28) are then not surprising.

C. The General Case

The only change from the previous subsection, is that for non-zero $\nu_m^{(i)\Lambda}$, equation (2.12) gives, in addition to equation (3.21),

$$\tilde{k}^{u\Lambda} = i\nu_m^{(1)\Lambda} \left( \frac{\partial}{\partial C_0} - \frac{\partial}{\partial \bar{C}_0} \right)^u + i\nu_m^{(2)\Lambda} \left( \frac{\partial}{\partial S} - \frac{\partial}{\partial \bar{S}} \right)^u. \quad (3.30)$$

As above, this gives

$$\tilde{P}_1^\Lambda = 4e^{\frac{3}{2} + \nu_m^{(1)\Lambda}}, \quad (3.31a)$$

$$\tilde{P}_2^\Lambda = 0, \quad \text{and} \quad (3.31b)$$

$$\tilde{P}_3^\Lambda = 4(C_0 + \bar{C}_0)e^{K + K}\nu_m^{(1)\Lambda} - e^{K}\nu_m^{(2)\Lambda}. \quad (3.31c)$$

As discussed at the end of section [4], it is always possible to choose a symplectic basis so $\nu_m^{(1)\Lambda} = 0$.

The classical potential is obtained by substituting equations (3.21), (3.30), (3.24) and (3.31) into equation (3.16).
IV. MINIMA OF THE ELECTRIC POTENTIAL

In the local case, there is no loss of generality in taking the charge vectors all electric, and only $\nu_i^{(1)}$, $\nu_0^{(2)}$ and $\nu_{e1}^{(2)}$ non-zero. If $L^1$ and $L^0$ are linearly independent, then it is straightforward to show that the potential is extremized only when the charge vectors vanish identically. Therefore the case where $L^1$ and $L^0$ are linearly dependent is examined; this is equivalent to demanding the non-existence of the holomorphic prepotential for the Kähler potential.

In particular, choose,

$$L^1 = e^{i\alpha} L^0.$$  (4.1)

Equation (4.1) implies, via equation (3.6),

$$U^{00} = e^{-i\alpha} U^{01} = e^{i\alpha} U^{10} = U^{11}. \quad (4.2)$$

This is consistent with the hypothesis that

$$U^{00} = \lambda L^0 \bar{L}^0. \quad (4.3)$$

Note that since both $U^{00}$ and $L^0 \bar{L}^0$ are positive, that $\lambda > 0$. It is worth noting that equations (4.1)-(4.3) hold in the $SU(1,1)/U(1)$ models of [31,41], with $\alpha = \frac{\pi}{2}$ and $\lambda = 1$. In fact, equation (4.1) can be derived, in the absence of charge quantization (i.e. allowing for $Sp(h_{21} + 1, \mathbb{R})$ transformations) from the linear dependence of the $L^A$s.

To look for minima of the potential on the hypermultiplet moduli space, the variation of the potential with respect to the dilaton multiplet is taken, and set to zero. The variation is (recall that in the chosen basis, all $\nu_{eA \geq 2}^{(i)} = 0$ and $\nu_{e1}^{(1)} = 0$)

---

$^4$Actually, Brian Greene has pointed out to me that the prepotential always exists as one can calculate it in a basis where the $X^A$s are linearly independent. However, as is common in the literature, I will use the term nonexistence to mean that in the chosen basis, $\frac{1}{2} F_A X^A$ is not a prepotential.
\[ \delta V = L^0 \bar{L}^0 \left\{ 16 \left( (2 - \lambda) - 2(1 + \lambda) (C_0 + \bar{C}_0)^2 e^{K + \bar{K}} \right) e^{2(K + \bar{K}) (\nu_{e_0}^{(1)})^2} - 2(1 + \lambda) e^{3\bar{K}} |\nu_{e_0}^{(2)}| + e^{i\alpha}\nu_{e_1}^{(2)} |^2 + 16(1 + \lambda)(C_0 + \bar{C}_0)e^{3K + \bar{K}} \nu_{e_0}^{(1)} (\nu_{e_0}^{(2)} + \nu_{e_1}^{(2)} \cos \alpha) \right\} (\delta S + \delta \bar{S}) + L^0 \bar{L}^0 \left\{ 32 \left( (3\lambda - 3) + 4(1 + \lambda)(C_0 + \bar{C}_0)^2 e^{K + \bar{K}} \right) (C_0 + \bar{C}_0) e^{2(K + \bar{K}) (\nu_{e_0}^{(1)})^2} - 8(1 + \lambda)(C_0 + \bar{C}_0) e^{3K + \bar{K}} |\nu_{e_0}^{(2)}| + e^{i\alpha}\nu_{e_1}^{(2)} |^2 - 8(1 + \lambda)(1 + 8(C_0 + \bar{C}_0)^2 e^{2K + \bar{K}}) e^{2(K + \bar{K}) (\nu_{e_0}^{(2)} + \nu_{e_1}^{(2)} \cos \alpha)} \right\} (\delta C_0 + \delta \bar{C}_0), \quad (4.4) \]

and so generically \( \nu_{e_A}^{(i)} = 0 \). That is just the usual Calabi-Yau compactification and so is uninteresting for this paper. At special points on the moduli space, however, specifically those for which

\[ e^K = \beta e^{\bar{K}}, \quad (4.5a) \]

and

\[ (C_0 + \bar{C}_0) = \frac{1}{\gamma} e^{-\bar{K}}, \quad (4.5b) \]

where \( \beta > 0 \) and \( \gamma \) are real constants, minimizing the potential corresponds to solving the two equations

\[ \begin{align*}
0 &= 8\beta \left( \frac{2 - \lambda}{1 + \lambda} - 2\frac{\beta}{\gamma^2} \right) (\nu_{e_0}^{(1)})^2 - (\nu_{e_0}^{(2)})^2 + (\nu_{e_1}^{(2)})^2 + 2\nu_{e_0}^{(2)} \nu_{e_1}^{(2)} \cos \alpha) + 8\frac{\beta}{\gamma} (\nu_{e_0}^{(1)} (\nu_{e_0}^{(2)} + \nu_{e_1}^{(2)} \cos \alpha)), \\
0 &= 4\frac{\beta}{\gamma} \left( 3\frac{\lambda - 1}{\lambda + 1} + 4\frac{\beta}{\gamma^2} \right) (\nu_{e_0}^{(1)})^2 + \frac{1}{\gamma} (\nu_{e_0}^{(2)})^2 + (\nu_{e_1}^{(2)})^2 + 2\nu_{e_0}^{(2)} \nu_{e_1}^{(2)} \cos \alpha) - (1 + 8\frac{\beta}{\gamma^2}) (\nu_{e_0}^{(1)} (\nu_{e_0}^{(2)} + \nu_{e_1}^{(2)} \cos \alpha). \quad (4.6a, 4.6b) \end{align*} \]

These have solutions

\[ \begin{align*}
\nu_{e_0}^{(2)} &= 4\frac{\beta}{\gamma} \nu_{e_0}^{(1)} \pm 2\sqrt{\frac{2\beta(2 - \lambda)}{1 + \lambda}} \nu_{e_0}^{(1)} \cot \alpha, \\
\nu_{e_1}^{(2)} &= \mp 2\sqrt{\frac{2\beta(2 - \lambda)}{1 + \lambda}} \nu_{e_0}^{(1)} \csc \alpha. \quad (4.7a, 4.7b) \end{align*} \]

Note that this is only well defined for \( -1 < \lambda \leq 2 \) (and, as noted above, \( \lambda > 0 \)). These can be integer valued only for special values of \( \alpha, \beta, \gamma, \lambda \). Also, if \( \nu_{e_A}^{(1)} = 0 \), then \( \nu_{e_A}^{(2)} = 0 \).
This contradicts the prediction made in [18]; however, this makes sense because for this set of models, any set of \( \nu^{(2)}_e \Lambda_s \) can be symplectically transformed into a new basis in which, say, only \( \nu^{(2)}_{e_0} \neq 0 \), and it was shown in [12] that the potential has no minimum in this case. Rather, to have a minimum for the potential requires NS-NS expectation values in addition to the RR ones of [12] (as predicted therein). However, from equation (2.9c) and (1.5a), the string coupling constant is

\[
e^{\hat{\phi}} = \sqrt{\frac{2}{\beta}}, \tag{4.8}\]

and so \( e^{\hat{\phi}} \langle H^{(2)} \rangle \) is \( O(1) \) (for small but non-zero integral \( \nu^{(1)}_e \Lambda \)). This contradicts the statement that the expectation values act only as perturbations of the Calabi-Yau compactification. That is, the solution of equation (4.7) is just outside the validity of the perturbative approximation, and so cannot be trusted.

The value of the potential at the minima of equations (4.7) is

\[
8\frac{\lambda - 2}{\beta} (\nu^{(1)}_{e_0})^2 L^0 \bar{L}^0 e^{2K}. \tag{4.9}\]

This vanishes for non-zero integral charges only for \( \lambda = 2 \).

Also, the determinant of the matrix governing the supersymmetry transformation of the gaugino, at the minima of the potential, is

\[
\det W^a = \beta^{-1} e^{2K} (g^{a\bar{b}} f^{\bar{b}}) (4 + e^{2i\alpha}) (\nu^{(1)}_{e_0})^2 \tag{4.10}\]

which is never zero and so implies that the gaugino transforms under both supersymmetries and hence that there are no unbroken supersymmetries. Thus, there is no partial supersymmetry breaking.

**V. MINIMA OF THE GENERAL POTENTIAL**

In the previous section it was assumed that the symplectic section can be chosen so that equations (4.1) and (4.3) hold. In addition, it will now be convenient to make analogous assumptions for the \( M \Lambda_s \); specifically it will be assumed that
\[ M_1 = e^{i\tilde{\alpha}} M_0 \]  

(5.1)

and

\[ g^{ab} h_{a0} \tilde{h}_{b0} = \tilde{\lambda} M_0 \bar{M}_0. \]  

(5.2)

In addition it will be assumed that

\[ g^{\tilde{a}b} f^0_a \tilde{h}_{b0} = \sqrt{\lambda \tilde{\lambda}} \bar{L}_0 \bar{L}_0. \]  

(5.3)

These formulas are again justified by their validity for the \( SU(1,1)/U(1) \) model of [39], with \( \tilde{\alpha} = \alpha = \frac{\pi}{2} \) and \( \tilde{\lambda} = \lambda = 1 \).

It is apparent that the general potential contains terms proportional to \( L_0 \bar{L}_0, L_0 \bar{M}_0, M_0 \bar{L}_0, M_0 \bar{M}_0 \) and that the coefficients of the terms proportional to \( L_0 \bar{M}_0 \) and \( M_0 \bar{L}_0 \) are complex conjugate. It is also apparent that the coefficient of \( L_0 \bar{L}_0 \) is the same as in the electric case, and is the same as the coefficient of \( M_0 \bar{M}_0 \) with \( n^{(i)}_{e\Lambda} \rightarrow n^{(i)\Lambda}_{e\Lambda} \). So, as \( M_0 \) is linearly independent of \( L_0 \), the minimum of the potential is given by a subset of equations (4.5) and two copies of equation (4.7), with \( n^{(i)}_{e\Lambda} \rightarrow n^{(i)\Lambda}_{e\Lambda} \) in one of those copies. Varying the coefficients of \( L_0 \bar{M}_0 \) and its complex conjugate, leads to an inconsistency, unless either \( \nu^{(i)\Lambda}_m = 0 \) (and/or \( \nu^{(i)}_{e\Lambda} = 0 \)) or \( \lambda = \tilde{\lambda} \). The first case has been discussed in section [V]; the second then leads only to a correlation between the choice of sign in both copies of equation (4.7) (the same choice of sign must be made). But, since the choice of basis was such that \( \nu^{(i)\Lambda}_m = 0 \), this means that all magnetic charges vanish, and the situation is that of section [V].

**VI. SINGULARITIES**

Points where \( X^\Lambda \) and/or \( F_\Lambda \) vanish are conifold singularities. At these points the theory appears to be singular, but this is because black holes become massless at these points, and so need to be “integrated in.” [13,16] The 10-dimensional description of the black holes is that of 3-branes wrapped around Calabi-Yau 3-cycles. The black holes are massless when
the 3-cycle volume vanishes. These conifold singularities appear on complex codimension one surfaces in the moduli space. The black holes have unit charge with respect to the $U(1)$ and electric or magnetic, corresponding to the vanishing period of the degenerating 3-cycle. $((X^\Lambda, F_\Lambda)$ is the symplectic period vector.) More complicated singularities (which are called Argyres-Douglas points in this paper) occur on complex codimension two surfaces where two surfaces on which there are conifold singularities intersect. These singularities were first discovered in a field-theoretic context in [21] and their relevance to string theory was given in [17]. At these points two black holes become massless and their charge vectors can be non-local.

A massless black hole is included in the low energy theory as a hypermultiplet. As in [12], only black holes with the same types of charges as the dilaton, will be considered. Of course, even with this restriction, not all sets of charges will be physically realizable at conifold and/or Argyres-Douglas points, but this will not affect the discussion. Also, since the black hole charges are associated with the vanishing periods, equation (3.15c) shows that the gravitino mass matrix is continuous. Similarly, the matrix governing the hyperino variation is continuous at the singularities (see equation (3.15f)); however, the gaugino variation matrix, equation (3.15a) is not continuous at a singularity, because the appropriate $f^\Lambda_a$s and $h_{a\Lambda}$s will not vanish there.

In [12], it was shown that on the IIA side with only 10-dimensional RR Calabi-Yau expectation values, there are flat directions with $N = 2$ supersymmetry at conifold points. These flat directions were those for which it was possible to set

$$\mathcal{P}^x_\Lambda = 0.$$  \hspace{1cm} (6.1)

A similar result will be shown here, in the more general case of both NS-NS and RR expectation values, and also at Argyres-Douglas points.

First consider Argyres-Douglas points. The black hole hypermultiplets are each doublets, $B_1$ and $B_2$. At least to lowest order in an expansion in $B_1$ and $B_2$, and about the singularity, each element of the black hole doublet has the same charge, so the Killing vectors,
equations (3.21) and (3.30) become

\[ k^u_0 = i \nu^{(1)}_e \left( \frac{\partial}{\partial C_0} - \frac{\partial}{\partial C_0} \right)^u + i \nu^{(2)}_e \left( \frac{\partial}{\partial S} - \frac{\partial}{\partial S} \right)^u + \]

\[ i \left( B_1^T \frac{\partial}{\partial B_1} - B_1^T \frac{\partial}{\partial B_1} \right) + i n_{Be0} \left( B_1^T \frac{\partial}{\partial B_2} - B_2^T \frac{\partial}{\partial B_2} \right), \quad (6.2a) \]

\[ k^u_1 = i \nu^{(2)}_e \left( \frac{\partial}{\partial S} - \frac{\partial}{\partial S} \right)^u + i n_{Be1} \left( B_1^T \frac{\partial}{\partial B_2} - B_2^T \frac{\partial}{\partial B_2} \right), \quad (6.2b) \]

\[ \tilde{k}^{0u} = i \nu^{(2)}_m \left( \frac{\partial}{\partial S} - \frac{\partial}{\partial S} \right)^u + i n_{Bm} \left( B_1^T \frac{\partial}{\partial B_2} - B_2^T \frac{\partial}{\partial B_2} \right), \quad (6.2c) \]

where \((n^{(i)}_m, n^{(i)}_e)\) are the black hole charge vectors. To lowest order, the \(SU(2)\) connection on the black hole quaternionic manifold can be ignored, and the triplet of Kähler forms can be taken to be \(\Omega^x = -idB_i^T \wedge \sigma^x dB_i\), giving

\[ \mathcal{P}_x^\Lambda = \mathcal{P}_x^\Lambda |_{B_i=0} + B_1^T \sigma^x B_1^\dagger \delta^\Lambda_A + n^{(2)}_{Be1} B_2^T \sigma^x B_2, \quad (6.3a) \]

\[ \tilde{\mathcal{P}}^{0x} = \tilde{\mathcal{P}}^{0x} |_{B_i=0} + n^{(2)}_{Bm} B_2^T \sigma^x B_2. \quad (6.3b) \]

where \(\mathcal{P}_x^\Lambda |_{B_i=0}\) and \(\tilde{\mathcal{P}}^{0x} |_{B_i=0}\) were given in equations (3.24) and (3.31). Taking \(L^0\) and \(M^0\) to be the vanishing periods in a symplectic basis where \(L^0\) and \(L^1\) are linearly independent, it can be shown that the hyperino variation cannot have any zero eigenvectors, and hence that there will be no supersymmetric minima of the potential. (This analysis requires the quaternionic vielbein which was given in [8] and is essentially equation (3.19a).) If there is linear dependence, then \(L^1\) also vanishes, and so the gravitino mass matrix vanishes. Then, the classical potential is non-negative, so the only vacua have non-negative cosmological constant; i.e. the vacua are (asymptotically) flat or de Sitter. Since de Sitter spaces do not admit a supersymmetry algebra, any supersymmetric minimum of the potential (if such a minimum exists) must occur at points where the potential vanishes. These minima have \(N = 2\) supersymmetry since for non-positive cosmological constant the number of supersymmetries is equal to the number of massless gravitini [35]. This requires

\[ \mathcal{P}_x^\Lambda = \tilde{\mathcal{P}}^{0x} = 0. \quad (6.4) \]

These have solutions when, for example, the black hole charges are proportional to the dilaton charges. This result has also been obtained via explicit calculation.
In that case, equations (6.4) are six equations in eight (real) unknowns. Thus the flat directions are parametrized by two real numbers, which correspond to the overall phase of the hypermultiplets. These are the would-be goldstone bosons that are eaten by the vectors; as in [19], there is a transition at the Argyres-Douglas points from a Calabi-Yau compactification with Hodge numbers \((h_{11}, h_{21})\) to one with Hodge numbers \((h_{11}, h_{21} - 2)\).

The above discussion also holds for conifold points by taking either \(B_1 = 0\) or \(B_2 = 0\), in addition to the above. Again, equation (6.4) may not have solutions for all choices of charge vectors. Of course, this time only one black-hole hypermultiplet is being eaten by a vector multiplet, so the transition is to a Calabi-Yau with Hodge numbers \((h_{11}, h_{21} - 1)\).

VII. CONCLUSION

When Type IIB strings are compactified on a Calabi-Yau manifold (with \(h_{21} \geq 1\)) and Calabi-Yau valued expectation values are given to the NS-NS and RR 3-forms, the dilaton is given electric and magnetic charges. The classical potential was derived in this situation. Under the assumption that the special Kähler moduli space of complex structures of the Calabi-Yau has a symplectic basis for which there is no prepotential (and some auxiliary assumptions, most of which would be unnecessary if \(Sp(h_{21} + 1, \mathbb{R})\) transformations were allowed instead of just \(Sp(h_{11} + 1, \mathbb{Z})\)) it was shown that for certain values of the charges, the potential could be minimized, though not while remaining within the validity of the calculation. \(N = 2\) supersymmetric minima are obtained at conifold points, Argyres-Douglas points and, as in [12], in the infinite Calabi-Yau volume limit. It is interesting that the \(N = 0\) minima are below the \(N = 2\) minima. In fact, from equations (4.9), (4.3), (3.19d) and (A10e), it is seen that the global minimum of the potential \((V \to -\infty)\) occurs in the limit of vanishing Calabi-Yau volume. It has been shown in [11] that \(N = 0\) vacua are classically stable if they occur at global minima of the potential. Unfortunately, it is neither clear that this would hold quantum mechanically, nor likely, since the vanishing Calabi-Yau volume limit is both well outside the limit of validity of the calculation and well inside the
region where significant quantum and stringy effects are expected.

It was found that partial supersymmetry breaking cannot occur. This agrees with \cite{1} where the conditions for Type IIB compactified to $D = 4$, to have supersymmetry were found and it was discovered that there was $N = 2$ or $N = 0$. This problem was also studied in \cite{2}, with a warp factor (Calabi-Yau-valued conformal factor for the space-time metric) included, but with the same conclusion. This remains true at singularities in the moduli space.

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APPENDIX: COMPACTIFICATION OF IIB ON A CALABI-YAU

In this appendix, the compactification of type IIB supergravity on a Calabi-Yau manifold is discussed, following \cite{8}. Therefore instead of using the non-self-dual action of equation (2.1), the type IIB equations of motion \cite{42} will be used. Also, as in \cite{8}, attention is restricted to an $h_{11} = 1, h_{21} = 0$ Calabi-Yau. The (uncomplexified) moduli space therefore is one-dimensional, and corresponds to the choice of metric; specifically a conformal factor $e^\sigma$. Furthermore, as RR fields are suppressed in string perturbation theory, and because only the structure of the dilaton multiplet is of interest, it will be convenient to take $\sigma$.

\footnote{It is interesting that if the two and four form field strengths are not assumed to vanish on the Calabi-Yau, then for a Calabi-Yau with $h_{11} > 1$, the fact that the wedge product of two harmonic}
\( \hat{I} = 0; \hat{B}_{\hat{i}\hat{j}} = 0; D_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\tau}} = 0. \) (A1)

The self-duality of the five-form field strength is then devoid of content \[8\]. (This is not inconsistent with equation (2.4) since the vectors do not mix with the scalars and only the scalars are being considered here.)

The equations of motion are usually written in terms of the fields \[42,24\]

\[
\hat{\psi} = \frac{1 + i\lambda}{1 - i\lambda} = \frac{1 - e^{-\varphi}}{1 + e^{-\varphi}},
\]

\[
\hat{P}_{\hat{\mu}} = \frac{\partial_{\hat{\mu}}\hat{\psi}}{1 - \hat{\psi}^*\hat{\psi}},
\]

\[
\hat{Q}_{\hat{\mu}} = \frac{\text{Im}(\hat{\psi}\partial_{\hat{\mu}}\hat{\psi}^*)}{1 - \hat{\psi}^*\hat{\psi}},
\]

\[
\hat{G}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} - \hat{\psi}\hat{H}^*_{\hat{\mu}\hat{\nu}\hat{\rho}}(1 - \hat{\psi}^*\hat{\psi})^{1/2}; \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \hat{H}^{(1)}_{\hat{\mu}\hat{\nu}\hat{\rho}} + i\hat{H}^{(2)}_{\hat{\mu}\hat{\nu}\hat{\rho}}. \quad (A2d)
\]

The equation of motion that will be most interesting is

\[
(\nabla_{\hat{\mu}} - i\hat{Q}_{\hat{\mu}})\hat{G}^{\hat{\mu}\hat{\nu}\hat{\rho}} = \hat{P}_{\hat{\mu}}\hat{G}^{\ast\hat{\mu}\hat{\nu}\hat{\rho}}. \quad (A3)
\]

Equation (A3) is satisfied trivially on the Calabi-Yau. After performing a 4-dimensional Weyl rescaling \( g_{\mu\nu} \rightarrow e^{-3\sigma}g_{\mu\nu} \), equation (A3) becomes (on the spacetime)\[1\]

\[
(\nabla_{\mu} + \frac{\psi^*\partial_{\mu}\psi - \psi\partial_{\mu}\psi^*}{2(1 - \psi^*\psi)})e^{3\sigma}G^{\mu\nu\rho} = \frac{\partial_{\mu}\psi}{1 - \psi^*\psi}e^{3\sigma}G^{*\mu\nu\rho}. \quad (A4)
\]

Subtracting \( \psi \) times the complex conjugate of equation (A4), from equation (A4), gives

\[6\] (1,1)-forms is not harmonic, means that the ansatz for the four form necessarily involves non-harmonic three-forms. Nevertheless, (and fortunately), it turns out that no residual effects of the three-forms appear in the four-dimensional action.

Note that this differs slightly from equation (2.18) of \[8\]; however, equations (A5) and (A6) agree with \[8\].

\[26\]
\[ (1 - \psi^* \psi)^{\frac{1}{2}} \nabla_\mu \left[ e^{3\sigma} \frac{G^\mu_{\nu \rho} - \psi G^*_{\mu \nu \rho}}{(1 - \psi^* \psi)^{\frac{1}{2}}} \right], \]  

which is satisfied by introducing a complex scalar field \( D \) such that

\[ \partial_\mu D = e^{3\sigma} \frac{G_\mu - \psi G^*_\mu}{(1 - \psi^* \psi)^{\frac{1}{2}}}, \]

where, as in equation (2.8),

\[ G_{\mu \nu \rho} = \varepsilon_{\mu \nu \rho \sigma} G_\sigma. \]

The other equation of motion that is used in [8] is

\[ \hat{R}_{\hat{\mu} \hat{\nu}} = 2 \hat{P}_{(\hat{\mu} \hat{\nu})} + \frac{9}{4} \hat{G}_{(\hat{\mu} \hat{\nu}) \hat{\sigma} \hat{\tau}} - \frac{3}{16} \hat{g}_{\hat{\mu} \hat{\nu}} \hat{G}^{\hat{\sigma} \hat{\tau}} \hat{G}^{\hat{\mu} \hat{\nu}} \]

By substituting the Calabi-Yau part of this equation into the space-time part of the equation, the four-dimensional action

\[ S = \int d^4 x \sqrt{-g} \left\{ \frac{1}{2} R + |P_\mu|^2 + 3(\partial_\mu \sigma)^2 + \frac{9}{4} |G_\mu|^2 \right\} \]

can be deduced.\(^7\) Alternatively, this can be found, almost by inspection, via dimensional reduction of the NSD action of equation (2.1). As mentioned above, there is a space-time dependent conformal factor of \( e^\sigma \) in the Calabi-Yau metric; hence \( \sqrt{-g} = e^{3\sigma} \sqrt{-\hat{g}} \) and so to remain in the Einstein frame required the Weyl rescaling of the four dimensional metric \( g_{\mu \nu} \rightarrow e^{-3\sigma} g_{\mu \nu} \). This is the same Weyl rescaling used in the derivation of equation (A5) and the reason for it.

To obtain the standard quaternionic geometry, make the field redefinitions:\(^8\)

\[ Z = -ie^{\sigma + \frac{1}{2} \varphi}; \]
\[ C_0 = i \frac{3\sqrt{2}}{4} \text{Im} D; \]

\(^7\)This equation differs from the corresponding formula in [8] in an essential way.

\(^8\)Equation (A10) differs from the corresponding formula in [8] in an essential way.
\[ \phi = e^{3\sigma - \frac{1}{2} \varphi}; \quad (A10c) \]

\[ \tilde{\phi} = \text{Re} D; \quad \text{and} \quad (A10d) \]

\[ S = \phi + i \tilde{\phi}. \quad (A10e) \]

Define also

\[ K = -\ln(-\frac{i}{8}(Z - \bar{Z})); \quad (A11a) \]

\[ \tilde{K} = -\ln[S + \bar{S}]; \quad (A11b) \]

\[ N_{00} = R_{00} = \frac{i}{32}(Z - \bar{Z})^3; \quad (A11c) \]

\[ D_\mu C_0 = \partial_\mu C_0; \quad \text{and} \quad (A11d) \]

\[ D_\mu S = \partial_\mu S \quad (A11e) \]

Then, the scalar part of the action of equation (A9) becomes (as in [26])

\[
S = \int d^4x \sqrt{-g} \left\{ K_{ZZ} \partial_\mu Z \partial^\mu \bar{Z} + \tilde{K}_{SS} D_\mu S D^\mu \bar{S} + \tilde{K}_{SC_0} D_\mu S D^\mu \bar{C}_0 + \tilde{K}_{\bar{C}_0 C_0} D_\mu \bar{C}_0 D^\mu C_0 \right\}, \quad (A12)
\]

where the subscripts on \( K, \tilde{K} \) denote differentiation. Note that, as defined above, \( C_0 \) is pure imaginary; this however, is a consequence only of the simplifying assumptions made above and is, of course, not general, and is not assumed in the main body of the paper.

Combining equations (A2d), (A7), (A6), (A10) and (A11) gives equations (2.9). Also, the Weyl rescaling used here can be reexpressed in terms of \( K \) and \( \tilde{\phi} \); this is the Weyl rescaling used in equation (2.11). These results can also be obtained from the slightly more general formulas of [8] (after the above corrections have been made) by keeping only terms of lowest order in the string coupling constant \( e^\tilde{\phi} \).
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