A loophole-free Wheeler-delayed-choice experiment

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Wheeler’s delayed-choice experiment investigates the indeterminacy of wave-particle duality and the role played by the measurement apparatus in quantum theory. Due to the inconsistency with classical physics, it has been generally believed that it is not possible to reproduce the delayed-choice experiment using a hidden variable theory. Recently, it was shown that this assumption was incorrect, and in fact Wheeler’s delayed-choice experiment can be explained by a causal two dimensional hidden-variable theory [R. Chaves, G. B. Lemos, and J. Pienaar, Phys. Rev. Lett. 120, 190401 (2018)]. Here, we carry out an experiment of a device-independent delayed-choice experiment using photon states that are space-like separated, and demonstrate a loophole-free version of the delayed-choice protocol that is consistent with quantum theory but inconsistent with any causal two-dimensional hidden variable theory. This salvages Wheeler’s thought experiment and shows that causality can be used to test quantum theory in a complementary way to the Bell and Leggett-Garg tests.

After the two famous Bohr-Einstein debates of 1927 and 1930 on the validity of the quantum theory [1], Einstein had to accept that quantum mechanics was correct. However, in his paper with Podolsky and Rosen (EPR) [2], EPR claimed that quantum theory, while not incorrect, was incomplete. That paper showed that quantum-entangled states had elements of nonlocality, un-reality, and uncertainty that no “sensible” theory should have. The EPR paper opened the door for a replacement theory for quantum mechanics, now called a hidden variable (HV) theory, that would be more like a classical statistical theory, where the statistics were governed by HVs that were either unknown or unaccessible. Von Neumann then provided a proof that no HV theory could reproduce all the predictions of quantum theory [3]. Later, Bohm produced a HV theory that reproduced all the predictions of nonrelativistic quantum theory. To solve this apparent paradox, Bell showed that von Neumann had made an explicit assumption that the HV theory was local, but that the HV theory of Bohm was nonlocal — that is actions at one place could affect outcomes far away in apparent violation of Einstein causality [4].

Thus, it is important to make the locality requirement explicit and show whether local HV theory could reproduce the predictions of quantum theory.

In an effort to challenge quantum mechanics, and make quantum predictions consistent with common sense, it was suggested that quantum particles can actually know in advance to which experiment they are going to be confronted through a HV, and thus can determine which behavior to show. For example, the photon could “decide” whether it was going to behave as a particle or behave as a wave before it reach the detection device in the double-slit experiment. However, Wheeler published two theory papers, now called Wheeler’s delayed-choice experiments (WDCE), that claimed to exclude the causal link between the experimental setup and a HV that predefines the photons behavior, and point out that complementarity and wave-particle duality alone contained an element of Einstein nonlocality [5, 6]. Fig. [1(a)] shows an example of an implementation of WDCE, where a photon enters a Mach-Zehnder interferometer. Wave-like behavior or particle-like behavior can be dictated according to whether the second beamsplitter (controlled by Bob) is put in place or not. So far, WDCE has been implemented experimentally in various quantum systems [7, 8]. Interestingly, a recent extension, quantum delayed-choice experiment (QDCE), suggested using a quantum beam splitter at the interferometer’s output [9, 10], enabling us to project the test photon into an arbitrary coherent wave-particle superposition, which motivated many QDCE experiments [11–13]. In short, until recently experimental demonstrations of WDCE (or QDCE) were thought to have perfectly ruled out the possible of quantum behavior induced by HV.

Chaves, Lemos, and Pienaar recently revisited WDCE, and showed using methods in causal inference [19, 20] that the original WDCE and QDCE can in fact be modeled by a causal HV theory [21]. The HV theory they suggest follows the same causal structure of the experiment shown in Fig. [1(a)], such that statistics produced by the final detection can be determined from a HV and the information of the type of measurement being performed. To overcome this shortcoming, the authors suggest a modified version of WDCE (MWDCE, Fig. [1(b)]) that cannot be explained by a causally structured HV theory, assuming a HV dimension of two. In the modi-
fied setup, the second beamsplitter is always put in place, and the two types of detection is controlled by the phase shifter $\beta_j$. The additional component is that Alice has the ability to prepare a set of states, controlled by her own phase shifter $\alpha_i$. Here, an important aspect is that Alice’s setting $\alpha_i$ cannot be directly communicated to Bob. In Ref. [21] it is shown that this new setup can no longer be modeled by a causally structured HV theory, as long as the HV has a dimension of two. Such a causal HV theory can be distinguished from a genuine quantum theory by comparing the statistics to a device-independent prepare and measure (PAM) scenario [22][23] (Fig. [1]c)). Using the statistics of the measurements and the setting values, a device-independent witness [22] can be constructed that is capable of distinguishing between a causally structured HV theory. The dimensional witness can, in principle, work with any transmittance strictly larger than zero, thus making MWDC specially suited to loophole-free experimental implementations [24][25].

In this paper, we carry out a device-independent demonstration of the MWDC. By measuring a device-independent witness we verify that two-dimensional causally structured HV theory can indeed be ruled out. We furthermore rule out any a priori prepared correlated noise sources that may exist between Alice and Bob by violation of a dimension witness inequality. Finally, we quantify the degree of retrocausality that would be required to reproduce our experiments, placing bounds on such hypothetical scenarios. Our experiment tests the causal structure in a WDCE, and is complementary to tests of nonlocality in quantum theory such as in a Bell test or a Leggett-Garg test [26].

Figure[1]d) shows the schematic experimental implementation for realizing the MWDC. The path-based interferometer as shown in Fig. [1]b) is implemented in our experiment by a polarization-based interferometer. The horizontal ($H$) and vertical ($V$) polarizations correspond to the upper and lower paths in the MZIs in Fig. [1]b). To prepare the various initial states we use an EPR photon pair emitter located at Charlie’s location $C$. Charlie is located closer to Alice than to Bob to allow Alice to make her preparation step first by measuring her half of the entangled pair after applying her polarization rotation $\alpha_i$ first. This collapses Bob’s half of the entangled pair into a single photon to which Alice’s polarization rotation $\alpha_i$ has been applied. Then Bob applies his polarization rotation $\beta_j$, before any influence from Alice can reach him, and performs the interference by passing the photon through a polarization beam splitter (PBS), followed by a measurement. We note that this is much different than the usual Bell test where Alice and Bob are symmetric in the protocol and measure correlations between their outcomes as opposed to the current scenario, and Bob measures an outcome conditioned on Alice’s preparation state. Time ordering is now important in the current scheme.

Our experimental setup is shown in Fig. [2] At Charlie’s location, an ultraviolet laser pulse with a central wavelength of 394 nm, pulse duration of 150 fs, and repetition rate of 80 MHz passes through a $\beta$-barium borate (BBO) crystal to produce a polarization-entangled pairs $|\Psi^+\rangle_{AB} = |H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B$ [27]. A half-wave plate (HWP) is placed at an arm of the entangled pairs to produce the Bell state $|\Phi^+\rangle_{AB} = |H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B$. The two photons are then coupled to the into single mode fiber and sent to Alice (preparer) and Bob (measurer), respectively. The distance between Alice and Bob is 46 m, and the length of the fiber from the entanglement source (Charlie) to Alice is shorter than that to Bob. The electro-optic modulator (EOM) then applies a phase shift of $0$ or $\pi/2$ chosen by a quantum random number generator (QRNG) such that Alice’s photon is measured as one of four possibilities $|H\rangle_A \pm |V\rangle_A$ and $|H\rangle_A \pm i|V\rangle_A$. This causes Bob’s state to collapse to $|H\rangle_B \pm |V\rangle_B$ or $|H\rangle_B \pm i|V\rangle_B$ respectively. Bob then deploys his delayed-choice of applying two polarization rotations $\beta_j \in \{\beta_0 = \pi/2, \beta_1 = 0\}$ chosen by another QRNG. This is equivalent to measuring his photon on one of the two bases $\{\langle H\rangle_B \pm e^{i\phi}|V\rangle_B\}$ chosen by $\beta_j$. We note that the four preparation states are produced by a combination of Alice’s QRNG and the random collapse of the EPR state. Thus no postselection is performed to prepare the state.

In order that there is no causal connection between Alice’s choice of settings and Bob’s detection, they are space-like separated at large enough distances such that the QRNG and the
photons of a causal HV model without any causal assumptions, as shown in [21] and [29]. Figure 5 shows the data for this witness, with the FSA. The constraints on the two-dimensional witness are that the measurements are outside their mutual lightcone (Fig. 3(a)). In other words, in order to meet the delayed-choice condition, the measurement basis of Bob must be chosen sufficiently later than the measurement basis chosen by Alice. Two synchronized signals from a central clock are used to trigger the two QNRGs. In addition, the QNRGs must fire outside the light cone of the EPR source to ensure that Charlie cannot influence the outcome of the QNRGs, as shown in Fig. 3(b). Without this additional condition on the EPR source, the experiment does not meet all the assumptions of the PAM scenario.

To measure the causal HV witness, we measure the conditional probability of the detection of the state $|H\rangle_B \pm e^{i\beta} |V\rangle_B$ as a function of the settings $(\alpha_i, \beta_j)$. The matrix elements of the $(2 \times 2)$ witness matrix $W$ is defined as [22]

$$W_{k,l} = p(d_{2k-2,l-1}) - p(d_{2k-1,l-1}),$$

(1)

where $k, l \in \{1, 2\}$, $p(d_{i,j})$ is the probability of the outcome $d_{i,j}$. For a causally structured two-dimensional HV theory one should find that $\text{det}(W) = 0$, whereas according to quantum mechanics for an ideal system $|\text{det}(W)| = 1$. Figure 4 shows a comparison of the theoretical predictions and the experimental measurements with and without the fair-sampling assumption (FSA), which performs a postselection only on coincidence events. With the FSA, the two-dimensional witness is calculated as $|\text{det}(W)| = 0.778 \pm 0.005$ according to the measurement results. Experimental errors mainly come from higher-order events in the SPDC and the control accuracy of EOM. Without the FSA, the two-dimensional witness is measured to be $|\text{det}(W)| = 0.0268 \pm 0.0006$, which is mainly due to the low collection efficiency. However, even without the FSA, the witness of the two dimensional HV, $|\text{det}(W)| = 0$, is still violated by 44 standard deviations. This means that we have — with a great degree of confidence — shown that our experiment is inconsistent with a causally structured HV theory with two dimensions. Thus, this scheme is highly resilient to detection inefficiencies.

We also test for HV models that are dependent upon hidden noise terms that could influence the output of the interferometer. While our experiment rules out the causal influence of the operations that Alice and Bob perform on each other, it is possible that such a noise variable could be prepared long before the start of the experiment, and would not be forbidden by causality. We use the dimension witness [23, 28]

$$I_{DW} = \langle D_{00} \rangle + \langle D_{01} \rangle + \langle D_{10} \rangle - \langle D_{11} \rangle - \langle D_{20} \rangle$$

(2)

where $\langle D_{i,j} \rangle = p(e_{i,j}) - p(d_{i,j})$, $p(e_{i,j})$ is the probability of the outcome $e_{i,j}$ (see Fig. 1(d)), and the measurement settings are $\alpha_i \in \{\pi/4, 3\pi/4, \pi/2\}$ and $\beta_j \in \{\pi/2, 0\}$ as before. Any HV theory that accounts for correlations between Alice and Bob hiding in the noise gives a strict bound of $I_{DW}^\text{HV} \leq 3$, with quantum theory predicting $I_{DW}^\text{Q} = 1 + 2\sqrt{2} \approx 3.828$ [21, 29]. The data for this witness is shown in Fig. 5 and yields, $I_{DW}^\text{Q} = 3.445 \pm 0.043$, with the FSA. The bound in this case is violated by 10 standard deviations.

The above shows that a causally structured two-dimensional HV model would be inconsistent with our experimental results. However, if one allows for the possibility of retrocausality (i.e. signaling backwards in time), it becomes possible to construct a HV theory that can account for statistics consistent with quantum experiment. Recall if full retrocausality is allowed, then a local HV model cannot be distinguished from quantum theory [30]. Interestingly, in Ref. [21] it is shown that quantum mechanics can give bounds on types of retrocausal HV models that are allowed, by quantifying the degree of retrocausality contained in them. A measure...
of retrocausality $R$ is given by

$$R = \max \left[ \frac{I_{DW} - 3}{4}, 0 \right],$$

where $I_{DW}$ is the same dimensional witness used above. Using our experimental estimate we obtain $R = 0.114 \pm 0.003$, in comparison to the ideal case where $R^Q = (\sqrt{2} - 1)/2 \approx 0.207$. The meaning of this is that any retrocausal model with $R \leq 0.207$ would not be able to reproduce the results that quantum theory is capable of producing. Taking our estimate, any retrocausal model with $R \leq 0.114$ could not give consistent results to our experiment.

In summary, we have demonstrated a modified WDCE experiment and measured a device-independent witness to evaluate its consistency with a causally structured two-dimensional HV model. We have found that the witness clearly shows that our experiment is inconsistent with such a HV model. A key component in our experiment is to preserve the same causal structure as the model provided in Ref. [21], which was achieved by separating Alice (preparer) and Bob (measurer) by space-like distances, which that there was no causal influence on each other. We have also excluded HV theories which assume noise between Alice and Bob has...
been correlated in advance of the experiment, and we put a bound on the amount of retrocausality needed to explain our data without quantum mechanics. Thus, the original WDCE experiment is salvaged, with no more reference to the rather heuristic notion of wave-particle duality, and the evidence for quantum theory against various classes of HV theory seems uncontestable.

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Note added. During the final stages of manuscript preparation, we became aware of a similar work by Polino et al., which was carried out simultaneously and independently.

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