Experimental realization of the quantum metrological triangle experiment

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Abstract. The quantum metrological triangle experiment (QMTE) consists in realizing Ohm’s law with Josephson (JE), quantum Hall (QHE) and single electron tunneling (SET) effects. The aim is to check the consistency of the link among the phenomenological constants $K_J$, $R_K$ and $Q_X$ involved in these effects and theoretically expressed with the fundamental constants $e$ and $h$. Such an experiment could be a contribution for a new definition of the système international d’unités (SI) base units.

In the QMTE, a current generated by a SET device flows through a resistor calibrated against QHE standard and the voltage induced at its terminals is compared to the metrological voltage generated by a Josephson junctions array. At LNE, the studied SET devices are 3 junctions single electron pumps with on chip resistors. The quantized current generated by this pump is theoretically equal to $e f$ (f is the frequency of the driving signals applied on the gates) and is measured through a cryogenic current comparator (CCC), which allows to amplify the low pumping current with a metrological accuracy.

We will present and discuss the experimental set-up developed at LNE and the first results. In addition to the main aim of QMTE described above, these preliminary results are also a first step towards a determination of $e$.

1. Introduction

In the framework of a future revision of the international system of units (SI), national metrology institutes (NMIs) are carrying out experiments which aim is the redefinition of the base units in term of a reduced set of fundamental constants ($e$, $h$, $k_B$, ...). For this purpose, the three most important experiments are watt balance, quantum metrological triangle and the Thompson-Lampard calculable cross-capacitor. In the electrical domain, the quantum metrological triangle experiment (QMTEx) is the central contribution to this redefinition [1]. It consists in applying Ohm’s law with the three effects used in quantum electrical metrology : Josephson effect (JE), quantum Hall effect (QHE) and single electron tunneling effect (SET) (Figure 1). The Thompson-Lampard calculable cross-capacitor allows a direct determination of the von Klitzing constant $R_K$. The watt balance experiment consists in linking the kilogram, presently defined from a material artefact, to the electrical units through Josephson constant $K_J$. The QMTE aims to check the consistency of the phenomenological constants associated with the three quantum effects and theoretically linked to the fundamental constants $h$ and $e$ : the Josephson constant.
\(K_J = 2e/h\), the von Klitzing constant \(R_K = h/e^2\) and an estimate of the electron charge \(Q_X = e\). Practically, the closure of the quantum metrological triangle is a measurement of the product \(R_K K_J Q_X\), theoretically equal to 2. Checking the equality \(R_K K_J Q_X = 2\) at an uncertainty level of 1 part in \(10^8\) is the ultimate aim of this experiment and will be a significant contribution to the redefinition of electrical units.

Figure 1. Principle of the quantum metrological triangle experiment (QMTE). \(\varepsilon_J, \varepsilon_K\) and \(\varepsilon_e\) represents the discrepancies between \(K_J, R_K, Q_X\) and their theoretical values.

2. Direct realization of the QMTE

We have chosen to close the triangle directly, i.e. by applying Ohm’s law \(U = R I\) [2]. This is made possible by the use of a cryogenic current comparator (CCC) for the amplification of the SET current (see below). Nowadays, Josephson and quantum Hall effects provides representation of volt and ohm at a level of uncertainty below 1 part in \(10^8\). To achieve the target uncertainty of 1 part in \(10^8\) on the product \(R_K K_J Q_X\), the weak link of the triangle is the current branch, due to the low value of the current provided by the SET device, difficult to measure with an ultra-low uncertainty. Note that another way explored by other NMIs is the indirect closure, which consists in applying the relation \(Q = C V\) by charging a capacitor with an electron pump [3].

Our implementation of direct closure of the quantum metrological triangle is shown in figure 2. It consists in feeding a resistor \(R_{cal}\) calibrated against quantum Hall effect with the amplified current supplied by the SET source and comparing the voltage induced at the terminals of the resistor with the voltage \(V_J = n_J f_J/K_J\) generated by a Josephson array voltage standard (using \(n_J\) junctions irradiated by an electromagnetic wave with the frequency \(f_J\)). The used SET source is a metallic 3-junctions electron pump with on-chip resistors at its terminals to reduce co-tunneling. It generates a quantized current \(I_{SET} = e f_{SET}\), where \(f_{SET}\) is the pumping frequency. The maximum frequency at which this device can be driven is limited to 100 MHz, which corresponds to a maximum quantized current with an intensity of around 16 pA. This current is amplified and measured with the help of a cryogenic current comparator (CCC). This one is a metrological tool initially developed for the very accurate comparison of currents which can be used as an amplifier with a very accurate known gain [4]. The CCC is made of two windings of \(N_1 = 20000\) and \(N_2 = 1\) turns embedded in a superconducting toroidal shield associated with a DC SQUID, and its gain is exactly equal to the winding ratio \(N_1/N_2\). The first winding is directly connected to the pump and is flowed by the quantized current \(I_{SET}\) when the second winding is fed by a feedback current \(I_{FB} = N_1/N_2 I_{SET}\) supplied by a home-made external very stable current source.
In this configuration we measure, with a null detector, the experimental discrepancy to the theoretical equality:

\[ \frac{n_J f_J}{K_J} = R_{\text{cal}} \frac{N_1}{N_2} e f_{\text{SET}} \]  

(1)

where the pumping frequency \( f_{\text{SET}} \) and the Josephson parameters \( n_J, f_J \) are previously adjusted.

The measurement of this equality is the experimental test of the consistency between \( R_K, K_J \) and \( Q_X \).

![Figure 2. Experimental set-up for the direct closure of quantum metrological triangle at LNE.](image)

3. Results
First of all, we have totally characterized the electron pump and determined its parameters (stability diagram, capacitances, junction resistances,...) in order to optimize the electron pumping process by adjusting gate voltages (dc values and amplitudes and phases of the ac components). Then, we have performed \( I_{\text{SET}}(V_{\text{bias}}) \) measurements to study the deviation between the measured current value and the theoretical quantized value of the single electron current. We have also investigated the flatness of the current plateaus for different pumping frequencies. Some results are shown on figure 3.

![Figure 3. Current plateaus flatness for different frequencies.](image)
After this, the set-up previously described has been carried out to investigate directly the quantum metrological triangle by measuring the deviation between $Q_X$ and the CODATA recommended value $e$ [5].

Through these measurements, we have proven the direct QMT set-up is operational and makes it possible the experimental verification of the equality (1). The best achieved relative random uncertainty $u_r$ on this equality has been found to be few parts in $10^6$, as reported on figure 4 [6]. Nevertheless some irreproducibility problems are observed from a series of measurement to another. To solve it, improvements of the set-up are in progress especially concerning the ground references and the bias voltage of the devices.

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5. References
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