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Effect of the magnetic material on AC losses in HTS conductors in AC magnetic field carrying AC transport current

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This paper presents an investigation on the AC losses in several typical superconducting composite conductors using the H-formulation model. A single superconducting strip with ferromagnetic substrate or cores and a stack of coated conductors with ferromagnetic substrates are studied. We consider all the coated conductors carrying AC transport currents and simultaneously exposed to perpendicular AC magnetic fields. The influences of the amplitude, frequency, phase difference and ferromagnetic materials on the AC losses are investigated. The results show that the magnetization losses of single strip and stacked strips have similar characteristics. The ferromagnetic substrate can increase the magnetization loss at low magnetic field, and decrease the loss at high magnetic field. The ferromagnetic substrate can obviously increase the transport loss in stacked strips. The trends of total AC losses of single strip and stacked strips are similar when they are carrying current or exposed to a perpendicular magnetic field. The effect of the frequency on the total AC losses of single strip is related to the amplitude of magnetic field. The AC losses decrease with increasing frequency in low magnetic field region while increase in high magnetic field region. As the phase difference changes, there is a periodic variation for the AC losses. Moreover, when the strip is under only the transport current and magnetic field, the ferromagnetic cores will increase the AC losses for large transport current or field. © 2015 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1063/1.4936652]

I. INTRODUCTION

The AC loss induced by AC transport current or external magnetic field is a remarkable problem in the applications of type-II superconductor. It is well known that AC losses can generate heat which may lead to the magnetic-thermal instability in some superconducting devices, such as power-transmission cables, transformers, rotating machines, compact high-current cables and fault current limiters.1,2 The operation of superconducting devices is under high electromagnetic loading and at extremely low temperature provided by cooling system. The investigation on AC losses in magnetic field and transport current is a key issue for the engineering design and quench protection.

Some investigators gave a detailed summary to introduce the electromagnetic modeling for superconductors and the computation of AC losses. The applicability of the constitutive relation which describes the relationship between current and electric field in superconductor was also discussed.3 For the modeling of coated conductors, many different critical current models were proposed by investigators. For examples, the flux-line-pinning anisotropy, dependence of critical current on the magnitude and orientation of field, longitudinal and transverse non-uniformity of superconducting material and the flux cutting mechanism in 3-D modeling were also discussed.3

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The analytical solutions of the distributions of current density and magnetic field for superconducting strip with transport current or in magnetic field have been presented. Some analytical expressions for AC losses of a strip carrying AC transport current exposed to a perpendicular magnetic field were given. Then, the electromagnetic behaviors and AC losses in stack of superconducting strips were widely studied by investigators, such as vertical arrays, horizontal arrays, and matrix arrays. In addition, since the AC loss is dependent on many factors, some researchers also paid their attentions to the effects of frequency and defects on the AC losses of superconductors.

The above investigations mainly studied the AC losses in superconducting layers without ferromagnetic substrate. Recently, the YBCO coated conductor can be made by growing the superconducting layer on top of magnetic metallic substrate. The ferromagnetic substrate can concentrate the magnetic flux around it, which will increase the magnetic induction around superconducting layer. The distributions of current density and AC losses in coated conductor are also changed. Then, the magnetic substrate will affect the AC losses in superconducting layer. The field dependence of the relative magnetic permeability and constant relative magnetic permeability in the ferromagnetic substrate are used to study the AC losses of coated conductors. Furthermore, some researchers also employed the complex field method to study the electromagnetic behaviors in superconducting strips with magnetic substrate.

Some classic and novel methods available in the literature could be used to carry out electromagnetic modeling of superconductors. The H-formulation is one typical method which was usually adopted to simulate the electromagnetic behaviors of superconductors. This formulation has some advantages for the electromagnetic modeling, such as the excellent convergence for the high-aspect ratio case and easy implement in commercial finite element method software. However, the time consumed increases obviously with the number of elements. Thus, the appropriate number of elements and the type of element should be chosen to achieve accurate results with an acceptable computation time. Since magnetic field is the state variables in H-formulation, the dependence of critical current on the magnitude and orientation of field is easily implemented in the modeling process.

In this work, the H-formulation method is employed to estimate the AC losses in coated conductors with and without ferromagnetic substrate. In part II, we give a brief description of the H-formulation and show the AC losses of single strip with and without ferromagnetic substrate. The transport and magnetization losses of stacked conductors are shown in part III. In part IV, we analyze the AC losses of a superconducting strip with three magnetic and non-magnetic cores. We give some conclusions in part V.

II. SINGLE HTS STRIP AND COMPOSITE STRIP (SC/FM BILAYER)

We use the H-formulation method to simulate the electromagnetic behavior of HTS strip. The magnetic field \( H \) is the dependent variable. The superconductor/ferromagnetic bilayer (SC/FM) consists of the superconducting strip and the ferromagnetic substrate. We assume a linear constitutive relation for ferromagnetic material. Here, we present the basic equations, and Ampere’s Circuital Law and Faraday’s Law of Induction are

\[
\mathbf{J} = \nabla \times \mathbf{H} \tag{1}
\]

\[
\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{2}
\]

where \( \mathbf{J} \) is current density, \( \mathbf{H} \) is magnetic field, \( \mathbf{E} \) is electric field and \( \mu = \mu_0 \mu_r \) is magnetic permeability.

For the superconductors, the relationship between current and voltage can be characterized by the nonlinear \( E-J \) power law relation:

\[
E = E_0 (\frac{J_{\text{norm}}}{J_C})^{n-1} \frac{J}{J_C} \tag{3}
\]

where \( J_C \) is the critical current density. For the ferromagnetic substrate and air, we use the Ohm’s Law to describe the relationship of current density and voltage, that is

\[
\]
Thus, the governing equation can be given as
\[ \mu \frac{\partial \mathbf{H}}{\partial t} + \nabla \times (\rho \nabla \times \mathbf{H}) = 0 \] (5)

Appropriate boundary conditions are implemented to control the transport current and magnetic field. The AC losses of strip can be calculated by the formula
\[ Q = \int_T dt \int_S (\mathbf{J} \cdot \mathbf{E}) \, ds \] (6)

where \( T \) is the time cycle of applied transport current and external magnetic field, and \( S \) is the cross section of superconducting strip. In this present work, the relative magnetic permeability of the ferromagnetic substrate is assumed to be a constant.

We consider the infinitely long strip case. Two samples are presented to analyze the effects of ferromagnetic substrate on the AC losses of strips. The first sample is a single HTS strip, and the second one is a single superconducting strip attached to a ferromagnetic substrate (SC/FM bilayer), as shown in Fig. 1. The width of strip is \( w = 4 \text{mm} \), the thickness of HTS layer is \( d = 1 \mu\text{m} \) and the thickness of ferromagnetic substrate is \( d_s = 50 \mu\text{m} \). The cross section of strip is in the \( x - y \) plane with reference to the Cartesian coordinates. The transport current enters the strip along the \( z \) direction (longitudinal direction), and the external magnetic field is applied along the \( y \) direction simultaneously. The cross section of the superconducting strip occupies an area of \( |x| \leq w/2 \) and \( |y| \leq d/2 \), and the ferromagnetic component occupies an area of \( |x| \leq w/2 \) and \( - (d_s + d/2) \leq y \leq -d/2 \). Then, there are three different subdomains in this model, the superconducting strip, the ferromagnetic substrate and the surrounding air. For coated conductor without magnetic substrate, the ferromagnetic substrate is replaced by air. The parameters used during simulations are presented in Table I.

Both the AC transport current \( I(t) \) and AC magnetic field \( H(t) \) applied are cosine functions of time \( t \). They have the same angular frequency \( \omega = 2\pi f \). Here, \( I(t) = I_a \cos(\omega t) \) and

### TABLE I. Electrical and magnetic parameters.

| Parameter                                | Value                           |
|------------------------------------------|---------------------------------|
| Resistivity for superconductor           | \( \rho_{sc} = \frac{E_0}{J_{C}} J_{C}^{-1} \) |
| Critical current criterion               | \( E_0 = 1 \times 10^{-3} \text{ V/m} \) |
| Critical current density                 | \( J_{C} = 1 \times 10^{10} \text{ A/m}^2 \) |
| Creep exponent [42]                      | \( n = 21 \)                     |
| Relative permeability for superconductor | \( \mu_{r,sc} = 1 \)            |
| Resistivity for ferromagnetic material [49] | \( \rho_f = 9 \times 10^{-7} \text{ \Omega \cdot m} \) |
| Relative permeability for magnetic substrate | \( \mu_{r,f} = 30 \)          |
| Resistivity for air                       | \( \rho_a = 10^3 \text{ \Omega \cdot m} \) |
| Relative permeability for air             | \( \mu_{r,a} = 1 \)             |
\( H(t) = H_a \cos(\omega t + \alpha) \), and \( \alpha \) is the phase difference between the transport current and external magnetic field. The critical current of single strip is \( I_C = I_{Cwd} \), the characteristic field is \( H_0 = J_C d/\pi \), and the normalized coefficients are defined as \( i_a = I_a/I_C \) and \( h_a = H_a/H_0 \).

The AC losses of a single strip in external electromagnetic field are presented in the following part. The parameter \( \mu_0 I_C^2 \) is used to normalize the AC losses. The frequency of transport current and magnetic field is 20Hz except for considering the frequency dependence of AC losses. Our computational model is established in Comsol Multiphysics software package.

**A. Total losses of single strip**

We first consider the effects of the amplitudes of field and transport current on the total AC losses of superconducting strip and SC/FM bilayer. From our calculations, we can find that the eddy current loss of the substrate is negligible in most cases. Therefore, we only take into account the AC losses generated in the superconducting region. The AC loss is systematically discussed for two different situations. One is that the transport current changes with fixed external magnetic fields, and the other one is that the external magnetic field changes with fixed transport currents.

Figure 2 shows the calculated AC losses with different transport currents and magnetic fields, where the solid symbols represent the calculated results for the HTS strip and the open symbols stand for the results for the SC/FM bilayer. Form Fig. 2(a), it can be found that our numerical results have

**FIG. 2.** (a) The \( i_a \) dependence of AC losses, (b) the \( h_a \) dependence of AC losses. HTS stands for the HTS strip and SC/FM stands for the SC/FM bilayer.
a little difference with the results given by Norris. This deviation is mainly due to the different superconducting electromagnetic constitutive relations. In our numerical simulation, the power law model is used to calculate the AC losses, and the influence of the thermally activated flux motion on the loss characteristic is considered. However, the Bean critical model is applied to analyze the AC loss without considering the flux motion for the analytical results. Since the flux motion is closely associated with the flux-flow resistance, the AC loss calculated using the power-law model is relatively larger. In addition, the obtained numerical results are qualitatively in agreement with the results of Norris. From our calculations, we find that with the increase of the exponent $n$, the calculated AC loss will gradually approach the analytical results. In addition, for the transport case, it can be found that the ferromagnetic substrate can increase the transport losses of superconducting strip. This result is same with some previous reports. From Fig. 2(a), one can also see that for fixed magnetic fields, the AC losses always increase with increasing transport current.

Figure 2(b) shows the AC losses with different magnetic fields. For a small magnetic field, the SC/FM bilayer has a larger loss value relatively to one single strip. While for a large magnetic field, the HTS strip produces greater losses comparing to the SC/FM bilayer. Some theoretical and experimental results gave similar characteristics for the AC losses in superconducting strip on magnetic substrate. With the increasing of the applied field, magnetic flux starts to penetrate into the superconducting strip from the edges. When the magnetic field is in the low field region, the ferromagnetic substrate induces a larger penetration region at the edges. Therefore, more dissipation is generated in the superconductors. However, as the field is increased, more magnetic flux lines are concentrated and restricted in the edge areas due to the presence of ferromagnetic substrate. This causes a smaller AC losses compared to the losses generated in the strip with non-magnetic substrate. If the applied field is further increased to the high field region, the entire strip is fully penetrated. The AC losses in the strip with and without ferromagnetic substrate are almost identical. It is interesting that AC losses in HTS strip and SC/FM strip with different transport currents only have a little difference for $h_{tr} = 0.2$. This is due to the reason that ferromagnetic substrate in the coated conductor will lead to larger AC losses in low magnetic field region ($h_{tr} = 0$) and smaller AC losses in high magnetic field region ($h_{tr} = 0.6$). The AC loss curves for two different conductors will intersect at the field $H^*$ which is close to $h_{tr} = 0.2$ (see Fig. 2(b)). The crossing point $H^*$ is denoted in Fig. 2(b).

B. Frequency dependence of AC losses of coated conductor

Figure 3 shows the dependence of AC losses on frequency. From Fig. 3(a), one sees that if the magnetic field is relatively small, such as $h_{tr} = 0$ or 0.2, the penetration of magnetic field is restricted in outer region of the superconductor. Increasing the frequency will reduce the amount of fluxes contributing to the dissipation. Therefore, the AC losses are suppressed. However, if the magnetic field is relatively large, such as $h_{tr} = 3$ or 4, the magnetic fluxes fully penetrate the whole superconductor. Increasing frequency enhances $E \cdot J$ and thus causes larger AC losses. From Fig. 3(b) and 3(c), it is found that for smaller magnetic field, the AC losses decrease with frequency, while for larger magnetic field, the AC losses increase with frequency. Obviously, the ferromagnetic substrate changes the distribution and movement of magnetic fluxes, and changes the AC loss characteristics in the strip. It is clear for the results considering the effect of ferromagnetic substrate at $h_{tr} = 3$. Moreover, we find that the effect of ferromagnetic substrate is closely dependent on the amplitude of the external magnetic field. This is consistent with the previous results.

C. Effect of phase difference on AC losses

The effects of the phase difference between the transport current and magnetic field on the AC losses are shown in Fig. 4. When $\alpha$ is in the interval $[0, \pi/2]$, the AC losses decrease with phase difference. The AC losses of both conductors have the minimums as phase difference is $\pi/2$. In addition, the AC losses of superconducting strip are smaller than those of SC/FM bilayer in high current and low field cases ($h_{tr} = 0.2$ and $i_a = 0.8$), and larger than those of SC/FM bilayer in low current and high field cases ($h_{tr} = 0.8$, $i_a = 0.2$ and $h_{tr} = 1$, $i_a = 0.2$). These results are consistent with those
FIG. 3. Calculated AC losses for different frequencies. (a) $I_a = 0.6I_c$, (b) $H_a = 0.2H_0$, (c) $H_a = 4H_0$. Solid and open symbols respectively represent the HTS strip and SC/FM bilayer.

given in Fig. 2. As the applied field and transport current are applied simultaneously, the AC losses can be reduced by adjusting the phase difference.

III. AC LOSSES OF THE STACKED COATED CONDUCTORS

In the section, we establish a geometry consisting of a stack of conductors with infinite length in the $z$ direction. The superconducting strips are electrically insulated from each other and are
arranged in face-to-face with each other. The cross section of the stack is shown in Fig. 5, where the superconducting layers are separated by the insulation layer or ferromagnetic layer. From top to bottom, the midlines of the superconducting layers are respectively denoted as AA’, BB’ and CC’, and the midlines of the ferromagnetic substrates are respectively denoted as aa’, bb’ and cc’. Left part of Fig. 5 displays the configuration of stacked coated conductors. Right part displays the dimensions of superconducting layer, ferromagnetic substrate and insulation layer. During the computation, the characteristic magnetic field is $H_0 = nJ_Cd/\pi$ and critical current is $I_C = nJ_Cwd$, where $n$ is the numbers of superconducting strips in the stacked conductors. The transport current and magnetic field are normalized as $i_a = I_a/I_C$ and $h_a = H_a/H_0$. For the stacked SC/FM conductors, the separation between two conductors is $d_2 = 59 \mu m$ which is equal to the thickness of insulation layer. In this section, total AC losses are the sum of AC losses in all superconducting layers.

### A. Transport and magnetization losses of the stacked coated conductors

The transport and magnetization losses of coated conductors consist of two, three and four conductors are presented in Fig. 6. The solid symbols represent the losses of stacked superconducting conductors and the open symbols represent the losses of stacked SC/FM bilayers. The number of superconducting layers will vary in the stacked coated conductors. For the transport losses in Fig. 6(a),

![Diagram of stacked coated conductors](image-url)
one can find that the transport losses increase with increasing of the number of superconducting layers. The behaviors of stacked strips are similar to those of one thicker strip. As the number of superconducting strips increases, the strip becomes thicker and the transport losses also increase. However, the increased values of transport loss are related to the magnitude of transport current, and the trend is complex at high current region.

It can be found that the transport losses in single strip and SC/FM bilayer merge at high current region, while the transport current losses are different for the stacked superconducting strips and stacked SC/FM bilayers with high current. For the single strip with transport current, the effect of ferromagnetic substrate decreases with the increasing of the amplitude of transport current. Thus, the transport losses of single superconducting strip are almost equal to the losses of SC/FM bilayer at higher current region. However, for the stacked coated conductors, the difference of transport losses between the stacked strips and stacked SC/FM bilayers is obvious. This may be due to the reason that for the stacked conductors with transport current, one superconducting strip will be subjected to the small magnetic field induced by other strips. Since the magnetic substrate will concentrate the magnetic fluxes and enhance the magnetic flux density around the superconducting layers, it is known that small field can increase the losses of superconducting layer in SC/FM bilayer. Then, AC losses in stacked SC/FM bilayers are still larger than those in stacked superconducting strips for large transport current.
The magnetization losses of stacked superconducting conductors and SC/FM bilayers are shown in Fig. 6(b). It can be found the magnetization losses of stacked conductors also increase with increasing of the number of strips at low magnetic field region. The influences of the numbers of strips on the magnetization losses become smaller for large magnetic field. The magnetization losses of stacked SC/FM bilayers are also higher than those of stacked superconducting strips at low magnetic field region. The trend is opposite at high magnetic field, and the characteristic is also presented in section II.

B. Total losses of the stacked coated conductors

Figure 7(a) shows that the variation of total AC losses of the stacked conductors with the amplitude of transport current increasing from $0.1I_c$ to $0.9I_c$ in external magnetic field. Here, the number of stacked conductors is three. The amplitude of field is $0$, $0.2H_0$ and $0.8H_0$, respectively. The AC losses of the stack of SC/FM bilayers are also larger than those of the stack of HTS strips in low magnetic field. As the amplitude of magnetic field increases to $0.8H_0$, the difference of AC losses is not obvious. Figure 7(b) shows that the total losses of the stacked conductors change with the external magnetic field, and the amplitude of transport current is $0$, $0.2I_c$ and $0.6I_c$, respectively. The total AC losses of the stack of SC/FM bilayers are larger than the losses of the stack of HTS strips in small magnetic field. However, in large magnetic field, the former is less than the latter. As the external magnetic field is sufficiently large, the effects of the transport current and ferromagnetic substrate on

![Image of Figure 7(a) and Figure 7(b)]

FIG. 7. AC losses of the stack of conductors (a) the external magnetic field is fixed and the transport current changes (b) the transport current is fixed and the external magnetic field changes.
total losses of stack are very small. As the amplitude of magnetic field is $10H_0$, the total loss of stack is nearly equal to those for two other stacks with different transport currents. In this situation, the effect of the ferromagnetic substrate on the total AC losses can be neglected.

C. Effect of phase difference on AC losses of the stacked conductors

The influence of phase difference on the AC losses is considered for the stack of conductors. From Fig. 8(a), it can be found that the trend of AC losses is similar to that of single strip. The AC losses of the stack of HTS strips are also smaller than those of the stack of SC/FM bilayers in high current and low field cases ($h_a = 0.2, i_a = 0.2$ and $h_a = 0.2, i_a = 0.8$), and larger than those of the stack of SC/FM bilayers in low current and high field case ($h_a = 0.8, i_a = 0.2$). These results are consistent with those shown in Fig. 7. Figure 8(b) shows the normalized AC losses with those of $\alpha = 0$. We can see that the influence of phase difference is obvious in small transport current and magnetic field case. When the transport current or magnetic field increases, this influence becomes smaller. The ferromagnetic substrate reduces the sensitivity of AC losses on the phase difference. The AC loss variations of stacked SC/FM bilayers are smaller than those of stacked HTS strips.

**FIG. 8.** (a) The phase difference dependence of AC losses in the stack of conductors (b) the results are normalized by the AC losses of $\alpha = 0$. 


D. The distributions of current density and instantaneous AC losses in superconducting layers or substrate

Figure 9 shows the distributions of $J_z/J_C$ in the stack of three HTS strips and three SC/FM bilayers at AA', BB', CC', where, u, m and b stand for the upper, middle and bottom superconducting layers, respectively. In Fig. 9(a), due to the symmetry of stack of HTS strips about $x$ axis, the current density $J_z/J_C$ at AA' and CC' are identical. $J_z/J_C$ at BB' is different from that at AA' and CC', because of the shielding effect of the upper and bottom superconducting layers. We can see that the flux-free region ($J_z/J_C$ equals zero) occupies most part of the central line in the middle strip. However, there are no flux-free region in the upper and bottom strips. In Fig. 9(b), due to the effect of ferromagnetic substrate, the distributions of $J_z/J_C$ at AA', BB' and CC' are obviously different. The size of flux-free region in middle superconducting layer is smaller. The distributions of current density $J_z/J_C$ at upper and bottom superconducting layers are not identical. This is due to the reason

![Figure 9](image-url)
FIG. 10. The transport current and instantaneous AC losses in upper, middle and bottom superconducting layers (a) the stack of HTS strips (b) the stack of SC/FM bilayers. The average stands for the mean value.

that ferromagnetic substrate can concentrate magnetic fluxes and affect the distribution of current density.

Figure 10 shows the changes of transport current and instantaneous AC losses with time in upper, middle and bottom superconducting layers. It can be expected that the transport current and instantaneous AC losses in upper and bottom strips in the stacked HTS strips are equal (see Fig. 10(a)). In Fig. 10(b), the transport current and instantaneous AC losses in upper, middle and bottom superconducting layers of the stacked SC/FM bilayers are different. The transport current and instantaneous AC losses in the upper superconducting layer are the largest and those in the middle layer are the smallest.

From Fig. 9 and Fig. 10, we can see that the current density and transport current in the stack of three HTS strips have identical values in the upper and bottom superconducting layers, and these values in the middle layer are smaller. This is due to the coupling effect between each strip and the shielding effect of the upper and bottom strips. The vertical array of superconducting strips was investigated by Pardo et al. They found that as the interval of the vertical array is small, the strong coupling effect exists. The current profiles are very similar to the case that there is no gap between
FIG. 11. (a) the eddy current and instantaneous AC losses in upper, middle and bottom ferromagnetic substrates of the stacked SC/FM bilayers, (b) distributions of current density ($J_z/J_C$) in ferromagnetic substrate. u stands for $J_z/J_C$ at line $aa'$, m stands for $J_z/J_C$ at line $bb'$ and b stands for $J_z/J_C$ at line $cc'$.

the superconducting strips. Therefore, the array can be regarded as a single thicker strip. However, when the interval of the vertical array is large, the magnetic coupling becomes weaker. Then, the current penetration for each strip is almost the same, and it looks as if the other strips are not present. The similar coupling effect can be also observed for the case of the vertical array carrying transport current. This coupling effect between superconducting strips decreases with increase the interval between strips. In our simulations, the vertical array consists of superconducting strips and the interval is very small. The distributions of transport current and current density are dependent on the coupling effect. Then, the current density may be different in each superconducting layer.

From the current-time curve in Fig. 11(a), we can see that there are a little eddy current in ferromagnetic substrate, and the amplitude of eddy current is much less than the amplitude of transport current in superconducting layer. The instantaneous AC losses are also much less than those of superconducting layer. Due to the effect of superconducting layer on the upper and middle ferromagnetic substrates, the eddy current and instantaneous AC losses in them are less than those in the bottom ferromagnetic substrate. From the distributions of the current density in Fig. 11(b), we also can see
FIG. 12. The cross section of superconducting strip with three ferromagnetic cores.

the shielding effects of superconducting layer on the upper and middle ferromagnetic substrates are obvious.

IV. AC LOSSES OF STRIP WITH FERROMAGNETIC CORES

In this section, we consider the AC losses of a carrying current superconducting strip with three ferromagnetic cores exposed to external perpendicular magnetic field, as shown in Fig. 12.

In this model, we choose the thickness of superconducting strip is fifty times the thin HTS strip. The thickness of strip is \( d = 50 \mu m \) and the width of strip is \( w = 4d \). The radius of ferromagnetic core is \( r = d/3 \), and the center distance of the adjacent two cores is \( a = 1.2d \). For this case, the critical current of this strip is \( I_C = J_C w d - 3\pi r^2 \). During the computation, we still choose the characteristic field as \( H_0 = J_C d / \pi \). In our simulation, there are three domains, superconductor, ferromagnetic material and air. The electrical and magnetic parameters are given in Table. II.

For the case of superconducting strip with ferromagnetic cores, the AC losses in this strip with magnetic cores and non-magnetic cores are very close, as shown in Fig. 13. The AC losses are identical when \( i_a \leq 0.5 \) for the transport case, and the AC losses are also identical when \( h_a \leq 0.4 \) for the magnetization case. However, when the transport current is larger than \( 0.5I_c \) or the magnetic field is larger than \( 0.4H_0 \), the AC losses in this strip with magnetic cores and non-magnetic cores are not equal.

It can also be found that the AC losses in strip with magnetic cores are larger than those in strip with non-magnetic cores for larger field or transport current. The effects of magnetic cores on AC loss are not significant for small magnetic field and transport current. This is due to the reason that for the superconducting strip with ferromagnetic cores, the magnetic field is shielded by the outer superconducting region and only a few fluxes enter the inner region in small field. When the transport

| TABLE II. Electrical and magnetic parameters. |
|------------------------------------------------|
| Critical current density \( J_C = 1 \times 10^8 A/m^2 \) | \( \mu_r = 30 \) |
| Relative permeability for magnetic core | \( \mu_r = 1 \) |
| Relative permeability for non-magnetic core | |
FIG. 13. The AC losses change with the amplitude of transport current (a) and magnetic field (b). The dashed line stands for the AC loss with magnetic cores and the solid line stands for the AC loss with non-magnetic cores.

current or magnetic field is large enough, the fluxes enter the strip increase and effect of ferromagnetic cores becomes larger.

V. CONCLUSIONS

In this work, we present the total AC losses of the single coated conductor and the stacked coated conductors carrying AC transport current simultaneously exposed to the external magnetic field. After computing the AC losses in single strip, SC/FM bilayer and the stack of them, the influences of ferromagnetic substrate on the AC losses are obtained. The ferromagnetic substrate can increase the AC losses in the small magnetic field and decrease the AC losses in the large magnetic field. As the magnetic field is larger enough, the AC losses is dominated by the external magnetic field and the influences of the transport current and ferromagnetic substrate are not obvious. As the effect of frequency is considered, we can find that the AC losses decrease with frequency in small magnetic field and increase with frequency in large magnetic field. The influence of phase difference between transport current and external magnetic field on the AC losses is also shown. The AC losses reach minimum when the phase difference is $\pi/2$. The transport and magnetization losses of stacked conductors with different number of strips are also considered. These ferromagnetic substrates can significantly enhance the transport losses of stacked conductors. For the magnetization losses, the effect of ferromagnetic substrate decreases with the increasing of external magnetic field. The superconducting strip
with ferromagnetic cores is also considered and the trend of AC loss is a little different with that of the superconducting strip with ferromagnetic substrate. For the strip with ferromagnetic cores, the AC losses for the magnetic cores are equal to those for non-magnetic cores at low current and magnetic field. The magnetic cores will enhance the AC losses at high current or magnetic field.

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