FINITE-TIME SLIDING MODE CONTROL FOR UVMS VIA T-S FUZZY APPROACH

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ABSTRACT. In order to solve the control problem of Underwater Vehicle with Manipulator System (UVMS), this paper proposes a finite-time sliding mode control strategy via T-S fuzzy approach. From the general dynamic model of UVMS and considering the influence between the manipulator and the underwater vehicle, hydrodynamic damping, buoyancy and gravity as the fuzzy items, we establish global fuzzy dynamic model and design a closed-loop fuzzy sliding mode controller. We prove the model in theory from two aspects: the reachability of sliding domain and the finite-time boundedness. We also give the solution of the controller gain. A simulation on the actual four joint dynamic model of UVMS with two fuzzy subsystems is carried out to verify the effectiveness of this method.

1. Introduction. Underwater robots have been widely used in underwater inspection, surveillance, exploration, scientific investigation and other fields. Especially in the aspect of life detection and rescue, it is different from the current way which is mainly relying on manual search and rescue. Through the rescuers diving to the bottom of the incident waters for manual detection, the divers need carrying too many equipments, and often face unknown underwater dangers such as bad environment in the incident area. These problems will affect the progress of rescue and are not conducive to the rescue [6, 14]. Underwater Vehicle with Manipulator System (UVMS) has become an important means of underwater rescue because of its long working time, stable working state, strong load and operation ability, and large amount of information of human-computer real-time interaction.

Underwater robot belongs to high inertia and low frequency system, and manipulator belongs to low inertia and high frequency system, which makes UVMS belong to dynamic coupling system. Considering the base of the robot is floating, the movement of the manipulator will affect the pose of the robot, which directly affects the positioning and tracking accuracy of the manipulator. Meanwhile, the working environment of UVMS is generally in the ocean environment with unknown model, and the effect of seawater and current on the system is nonlinear, so it is
difficult to model it accurately. All these factors make UVMS belong to strong coupling, high nonlinear and time-varying complex mechanical system.

For the control of UVMS, it needs the cooperation between the manipulator and the underwater robot to complete the assigned tasks. The performance of each subsystem also affects the operation effect of the general UVMS system. Combined with the characteristics of UVMS, the main factors that limit its control are as follows [19, 5]:

(1) it is difficult to establish accurate models of the underwater robot, the manipulator and the working environment of the UVMS, especially the hydrodynamic models.

(2) UVMS is a nonlinear and strong coupling multi input and multi output system, so it is difficult to find a more effective method for attitude and mechanical control.

(3) it is difficult to describe the influence between the manipulator and the underwater robot body by model.

(4) UVMS is a kinematic redundant system, and the underwater robot body is more difficult to control than the manipulator system.

In view of the fact that UVMS itself is a complex system with strong nonlinearity and strong coupling, and the current research progress on hydrodynamic mechanism is relatively slow, it is impossible to establish an accurate system model in the underwater environment. In general, researchers need to choose different control strategies according to different tasks. At present, the control strategies used in UVMS control mainly include PID and its derivative improved algorithm, fuzzy algorithm, adaptive algorithm, sliding mode derivative algorithm [15, 9, 20, 3], and control algorithm based on robust characteristics [29, 27].

Sliding mode control (SMC) is a kind of nonlinear control strategy, and the nonlinear performance is the discontinuity of control. The SMC strategy is used to control the system whose structure is not fixed, and can purposefully change according to the current state of the system, forcing the system to move according to the predetermined “sliding mode” state trajectory [24, 26, 25]. SMC has the advantages of relatively simple design, rapid response, insensitive to the change of corresponding parameters and disturbance, and no need of on-line system identification [23, 4, 10]. However, the disadvantage of this method is that it is prone to chattering, which affects the control performance [11].

Therefore, this paper studies and designs a SMC method of UVMS based on the dynamic model combined of underwater robot and the manipulator. At present, there are many related research approaches. The basic control of sliding variable structure model can be designed and realized through the transformation and derivation of UVMS dynamic model. The switching gain of the sliding mode is adjusted through fuzzy control logic [22, 17, 12], and the fluid interference and the coupling of body manipulator are realized [7, 28]. The effect and other additional mass disturbance are processed by fuzzy function to eliminate the influence on the pose and trajectory control of the manipulator [1].

It is worth noting that in the working process of the manipulator, that is, in the process of search and rescue, the operation of the underwater manipulator is often required to be completed in finite time [8, 18, 16]. For example, in the case of the underwater human being trapped, the golden rescue time is measured in seconds, and the demolition and rescue operation of the manipulator must be completed in a very short time, so as not to delay the best rescue time, which requires the control
of the underwater manipulator is in line with the finite time control principle \[13\]. Therefore, in the design process of this controller, we need to take the finite time control into account.

To resolve the above mentioned problems, this paper proposes a finite-time sliding mode control method via T-S fuzzy approach. The innovations are as follows:

(1) We establish a global fuzzy dynamic model from the general dynamic model of UVMS and design a closed-loop fuzzy sliding mode controller.
(2) We prove the reachability of sliding domain and the finite-time boundedness of the closed-loop fuzzy sliding mode controller in theory, and derive the solution of the controller gain.
(3) We verify the effectiveness of the controller on an actual four joint dynamic model of UVMS with two fuzzy subsystems.

The rest of this paper is organized as follows. In section 2, the dynamic model of UVMS is given. In section 3, the main designs of the controller are presented. And in section 4 and section 5, the simulation results and the summaries are shown respectively.

2. System formulation. The general dynamic model of UVMS could be expressed as follows.

\[
M(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + D(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) + F(q(t), \dot{q}(t)) + \tau_d(t) = \tau(t) \tag{1}
\]

where \(q(t) = [f, g, h, \phi, \psi, \theta_1, \theta_2, \ldots, \theta_n]^T\) denotes the displacements and the angles vector of the underwater robot and the manipulator. \([f, g, h]^T\) is the linear displacements of the underwater robot body. \([\phi, \psi]^{T}\) is the heading angles of the underwater robot body. \([\theta_1, \theta_2, \ldots, \theta_n]^{T}\) is the joint angles of the manipulator with the number of the joints \(n\). In this paper, we consider the UVMS with four joints, i.e., \(n = 4\). The skew symmetric positive definite matrix \(M(q(t))\) represents the inertia of the robot. \(C(q(t), \dot{q}(t))\) represents the Coriolis and centripetal force. \(D(q(t), \dot{q}(t))\) denotes hydrodynamic damping part. \(G(q(t))\) is the sum of buoyancy and gravity. \(F(q(t), \dot{q}(t))\) denotes the coupling force between the robot body and the manipulator. \(\tau_d(t)\) is the external disturbance and \(\tau(t)\) is the system input.

For designing the sliding mode controller more convenient, we assume \(x_1(t) = q(t), x_2(t) = \dot{q}(t), x(t) = [x_1(t), x_2(t)]^T\). Then, the equation 1 could be reformulated as:

\[
\dot{x}(t) = Ax(t) + B_1\tau(t) + B_2\tau_d(t) + f(x(t)) \tag{2}
\]

where

\[
A = \begin{bmatrix}
0 & 1 \\
0 & -M^{-1}(x_1(t))[C(x_1(t), x_2(t)) + D(x_1(t), x_2(t))]
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0 \\
-M^{-1}(x_1(t))
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
0 \\
-M^{-1}(x_1(t))
\end{bmatrix},
\]

\[
f(x(t)) = \begin{bmatrix}
0 \\
-M^{-1}(x_1(t))[G(x_1(t)) + F(x_1(t), x_2(t))]
\end{bmatrix}.
\]

Considering the nonlinear term \(f(\bullet)\) in the system, we can describe equation 2 as the following F-S fuzzy model:

**Plant Rule n:**

IF \(\rho_1(t)\) is \(\Delta^n_1\), \(\rho_2(t)\) is \(\Delta^n_2\), ..., \(\rho_m(t)\) is \(\Delta^n_m\), Then
\[
\begin{align*}
\dot{x}(t) &= A_n x(t) + B_{1n} \tau(t) + B_{2n} \tau_d(t) \\
x(0) &= x_0, \quad t = 0 \\
n &= 1, 2, ..., b
\end{align*}
\]

where \(\Delta_i^n(n = 1, 2, ..., b, l = 1, 2, ..., m)\) are the fuzzy sets, \(b\) is the number of subsystems, \(\rho(t)\) is the \(l_{th}\) premise variable vector, which could be the state variable, the external disturbance, or other vectors.

For equation 3, which represents the F-S fuzzy dynamic model of UVMS, We use single point fuzzification, product reasoning and weighted average defuzzification methods. Then we can get the global fuzzy dynamic model as below:

\[
\begin{align*}
\dot{x}(t) &= \sum_{n=1}^{b} p_n(\rho(t))[A_n x(t) + B_{1n} \tau(t) + B_{2n} \tau_d(t)] \\
n &= 1, 2, ..., b
\end{align*}
\]

where \(\rho(t) = [\rho_1(t), \rho_2(t), ..., \rho_m(t)]^T\), \(p_n(\rho(t)) = \frac{\gamma_n(\rho(t))}{\sum_{n=1}^{m} \gamma_n(\rho(t))}\), \(\gamma_n(\rho(t)) = \prod_{q=1}^{m} \Delta_q^n(\rho_q(t))\) is the membership function of \(\rho_q\) with respect to fuzzy set \(\Delta_q^n\). \(\gamma_n(\rho(t))\) is the membership of rule \(n\) and satisfies \(\gamma_n(\rho) \geq 0\), \(\sum_{n=1}^{b} \gamma_n(\rho) > 0\), therefore, we can get

\[
\begin{align*}
p_n(\rho(t)) &\geq 0, n = 1, 2, ..., b \\
\sum_{n=1}^{b} p_n(\rho(t)) &= 1
\end{align*}
\]

This paper is aimed to design a sliding mode controller for the global fuzzy UVMS dynamic model 4 and guarantee the finite-time bounded (FTB) of the model. Before giving the main results, the following definitions and assumptions are necessary.

**Definition 1.** For a given time interval \([t_1, t_2]\), positive constants \(\pi_1, \pi_2\) satisfy \(0 < \pi_1 < \pi_2\), weight matrix \(R > 0\), the input-free global fuzzy UVMS dynamic model 4 (\(\tau(t) = 0\)) is FTB with respect to \((\pi_1, \pi_2, [t_1, t_2], R, G_{[t_1, t_2], \omega})\), if for \(\forall t \in [t_1, t_2]\), we have:

\[x^T(t_1) Rx(t_1) \leq \pi_1 \Rightarrow x^T(t) Rx(t) \leq \pi_2\]

**Assumption 1.** We assume that the external disturbance is a bounded peak signal with a specified finite-time interval \([t_1, t_2]\), that is, there is a positive constant \(\omega > 0\), which satisfies \(G_{[t_1, t_2], \omega} \triangleq \tau_d^T(t) \tau_d(t) \leq \omega^2\).

3. **Main results.** By considering the parallel distributed compensation algorithm, we design the sliding mode function for the global fuzzy UVMS dynamic model 4 as follows:

**Sliding Function Rule n:**

IF \(\rho_1(t)\) is \(\Delta_1^n\), \(\rho_2(t)\) is \(\Delta_2^n\), ..., \(\rho_m(t)\) is \(\Delta_m^n\), THEN

\[S(x(t)) = B_1^T Lx(t)\]

and the sliding mode controller

**Controller Rule n:**

IF \(\rho_1(t)\) is \(\Delta_1^n\), \(\rho_2(t)\) is \(\Delta_2^n\), ..., \(\rho_m(t)\) is \(\Delta_m^n\), THEN

\[
\tau(t) = \begin{cases} 
Kx(t), & S(x(t)) = 0 \\
Kx(t) - \delta(t) \frac{S(x(t))}{\|S(x(t))\|}, & S(x(t)) \neq 0
\end{cases}
\]
where $L$ and $K$ are the parameters to be designed later, $L$ is designed that the $B_1^T LB_1$ be nonsingular. The concrete expression of the time-varying term $\delta(t)$ will be given in Theorem 1. From equations 7-8, we can obtain the following global fuzzy sliding mode function and global fuzzy sliding mode controller:

$$S(x(t)) = \sum_{n=1}^{b} p_n(\rho(t)) B_1^T L_n x(t)$$  \hspace{1cm} (9)

$$\tau(t) = \begin{cases} 
\sum_{n=1}^{b} p_n(\rho(t)) K_n x(t), & S(x(t)) = 0 \\
\sum_{n=1}^{b} p_n(\rho(t)) [K x(t) - \delta(t) \frac{S(x(t))}{\|S(x(t))\|}], & S(x(t)) \neq 0
\end{cases}$$ \hspace{1cm} (10)

By substituting the global fuzzy sliding mode controller 10 into the global fuzzy UVMS dynamic model 4, the following closed-loop global fuzzy UVMS dynamic model could be obtained:

$$\dot{x}(t) = \overline{A}_{ni} x(t) - \overline{B}_{1ni} \frac{S(x(t))}{\|S(x(t))\|} + B_{2ni} \tau_d(t)$$ \hspace{1cm} (11)

where $\overline{A}_{ni} = \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t)) p_i(\rho(t)) A_{n} + B_{1ni} K_i$, $\overline{B}_{1ni} = \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t)) p_i(\rho(t)) B_{1ni}$, $B_{2ni} = \sum_{n=1}^{b} p_n(\rho(t)) B_{2ni}$.

The difficulty and challenge of this work is to design appropriate fuzzy sliding mode function $S(x(t))$ and fuzzy sliding mode controller $\tau(t)$ for the global fuzzy UVMS dynamic model 4, so that the closed-loop global fuzzy UVMS dynamic model 11 satisfies the following conditions:

(i) **Reachability of sliding domain** ($T^* < T$). For the given time interval $[0, T]$, how to design the corresponding fuzzy sliding mode controller 8 to drive the closed-loop global fuzzy UVMS dynamic model 11 onto the sliding surface $S(x(t)) = 0$ during the finite-time interval $[0, T^*]$ on the condition of $T^* < T$, and stay on the sliding surface in the later time interval $[T^*, T]$?

(ii) **Finite-time boundedness**. For the given time interval $[0, T]$, how to guarantee the FTB of the closed-loop global fuzzy UVMS dynamic model in the arrival phase $[0, T^*]$ and the sliding phase $[T^*, T]$?

(iii) **Solution of the controller gain**. How to get the corresponding controller gain parameter $K$ on the basis of conditions (i) and condition (ii)?

Next, we will analyze and discuss the above conditions respectively, and give the corresponding conditions to make condition (i) and condition (ii) tenable, and further give the solution method for controller gain $K$ satisfying the conditions.

### 3.1. Reachability of sliding domain ($T^* < T$).

**Theorem 1.** For the given time interval $[0, T]$ and a set of global fuzzy sliding mode function 9, the closed-loop global fuzzy UVMS dynamic model 11 could be driven onto the specified sliding mode surface $S(x(t)) = 0$ in finite-time interval $[0, T^*]$ on the condition of $T^* < T$ under action of the global fuzzy sliding mode controller 10, and then stay on the sliding mode surface in the next time interval $[T^*, T]$. If the parameter of the global fuzzy sliding mode controller 10 satisfies the following condition:

$$\delta_{ni}(t) = \mu_{1ni} + \mu_{2ni} + \mu_{3ni}\|x(t)\|$$ \hspace{1cm} (12)
where \( \mu_{2ni} = \| (B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i B_{1n} \| \), \( \mu_{3ni} = \| (B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i \| \) and \( \| K_i \| \), \( \varepsilon \) is a positive constant and satisfies \( 0 < \varepsilon < T \). Nonlinear matrix \( B_{1n}^T L_i B_{1n} \) can guarantee \( L_i \geq 0 \).

**Proof.** Select the following Lyapunov function:

\[
V_{3}(x(t)) = S^{T}(x(t))(B_{1n}^T L_i B_{1n})^{-1}S(x(t))
\]

From the global fuzzy sliding mode function 9, we have:

\[
\dot{S}(x(t)) = \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))B_{1n}^T L_i (A_n x(t) + B_{1n} \tau(t) + B_{2n} \tau_d(t))
\]

Getting the derivative of equation 14 and taking into account equation 10, it yields:

\[
\dot{V}_{3}(x(t)) = 2 \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))S^{T}(x(t)) (B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i (A_n x(t) + B_{1n} \tau(t) + B_{2n} \tau_d(t)) + B_{2n} \tau_d(t)\]

\[
= 2 \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))S^{T}(x(t)) (B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i A_n x(t) + 2 \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))S^{T}(x(t)) (B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i B_{2n} \tau_d(t) + 2 \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))S^{T}(x(t))(K_i x(t) - \delta_{ni}(t))
\]

From equations 12-13 and \( \| S(x(t)) \| < \| S(x) \| \), we have:

\[
\dot{V}_{3}(x(t)) \leq -2 \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t))p_i(\rho(t))\mu_{1ni}\| S^{T}(t) \| < 0
\]

Therefore, we can see that if equations 12-13 hold, the closed-loop global fuzzy UVMS dynamic model 11 could be driven to the specified sliding surface \( S(x(t)) = 0 \) by the global fuzzy sliding mode controller 10.

In the following part, we will prove that the closed-loop global fuzzy UVMS dynamic model 11 can be driven to the specified sliding surface \( S(x(t)) = 0 \) in finite-time interval \([0, T^*]\) with \( T^* < T \). From equation 14, we can get:

\[
V_{3}(x(t)) \leq \sum_{n=1}^{b} \sum_{i=1}^{b} \lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1}] \| S(x(t)) \|^2
\]

From inequality 17, we can get:

\[
\dot{V}_{3}(x(t)) \leq -2 \sum_{n=1}^{b} \sum_{i=1}^{b} \sqrt{\lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1}] | \sqrt{V_{3}(x(t))} |}
\]
For the integral of inequality 19 from 0 → t, we can obtain:

\[ \sqrt{V_1(x(t))} - \sqrt{V_1(x(0))} \leq - \sum_{n=1}^{b} \sum_{i=1}^{b} \frac{t \mu_{1ni}}{\sqrt{\lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1} ]}} \]  

(20)

From inequality 20, we can get that there exists a time point \( T^* < T \) satisfies

\[ T^* \leq \sum_{n=1}^{b} \sum_{i=1}^{b} \frac{\sqrt{\lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1} ]} \sqrt{V_1(x(0))}}{\mu_{1ni}} \]

(21)

for that \( V_1(x(t)) = 0 \), and then \( S(x(t)) = 0 \) tenable.

From inequality 18, we can see that:

\[ V_1(x(0)) \leq \sum_{n=1}^{b} \sum_{i=1}^{b} \lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1}] \|S(x(0))\|^2 \]

(22)

In combination with inequality 21, we can get:

\[ T^* \leq \sum_{n=1}^{b} \sum_{i=1}^{b} \frac{\sqrt{\lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1}] \|S(x(0))\|}}{\mu_{1ni}} \]

(23)

Considering \( S(x(t)) = \sum_{n=1}^{b} \sum_{i=1}^{b} p_n(\rho(t)) p_i(\rho(t)) B_{1n}^T L_i x(0) \), we can obtain:

\[ T^* \leq \sum_{n=1}^{b} \sum_{i=1}^{b} \frac{\lambda_{\text{max}}[(B_{1n}^T L_i B_{1n})^{-1}] \|B_{1n}^T L_i x(0)\|}}{\mu_{1ni}} \]

(24)

Therefore, from equation 13 and equation 24, we know that \( T^* \leq T - \varepsilon \leq T \); that is, if equations 12-13 hold, the closed-loop global fuzzy UVMS dynamic model 11 could be driven to the specified sliding surface \( S(x(t)) = 0 \) by the global fuzzy controller 10 in a finite-time interval.

\[ \square \]

3.2. FTB analysis. The state trajectory of the closed-loop global fuzzy UVMS dynamic model 11 has two motion stages under the action of the fuzzy controller during the time interval \([0, T]\), namely, the arrival stage of the interval \([0, T^*]\) and the stay sliding stage of the interval \([T^*, T]\). Therefore, we will consider the FTB of the arrival stage of the interval \([0, T^*]\) and the FTB of the stay sliding stage of the interval \([T^*, T]\), respectively when we analyze the FTB of the closed-loop global fuzzy UVMS dynamic model. The following lemma is very useful for FTB analysis of different stages before giving the main conclusion.

**Lemma 1.** [21, 2] For a given time interval \([0, T]\), positive constants \( \pi_1, \pi_2 \) satisfy \( 0 < \pi_1 < \pi_2 \), weight matrix \( R > 0 \), if and only if there is an auxiliary constant \( \pi^* \) satisfying \( \pi_1 < \pi^* < \pi_2 \) such that the closed-loop global fuzzy UVMS dynamic model 11 is FTB with respect to \((\pi_1, \pi_2; [0, T], R, G_{[t_1, t_2], \omega})\), if the closed-loop global fuzzy UVMS dynamic model 11 is FTB with respect to \((\pi^*, \pi_2, [0, T^*], R, G_{[t_1, t_2], \omega})\) in the arrival stage and FTB with respect to \((\pi^*, \pi_2, [T^*, T], R, G_{[t_1, t_2], \omega})\) in the stay sliding stage.

In Theorem 2 and Theorem 3, the corresponding sufficient conditions will be given to guarantee the FTB of the closed-loop global fuzzy UVMS dynamic model 11 in the arrival stage and the stay sliding stage, respectively.
Theorem 2. For the given positive constants $\pi_1 > 0, T > 0, \alpha > 0$, the closed-loop global fuzzy UVMS dynamic model 11 is FTB in the arrival stage with respect to $(\pi_1, \pi_*, [0, T^*], R, G_{t_1, t_2], \pi)$, if there exist symmetric positive definite matrix $P > 0$ and constant $\pi^* > 0$, so that:

$$
\begin{bmatrix}
\bar{A}_{ni}P + P\bar{A}_{ni} - \alpha P & PB_{1n} & PB_{2n} \\
0 & 0 & 0 \\
0 & 0 & -\alpha I
\end{bmatrix} < 0
$$

(25)

$$
e^{\alpha t}(\bar{v}(p)\pi_1 + \alpha \bar{\omega}^2 T) < \bar{\psi}(p)\pi^*
$$

(26)

$$
\pi_1 < \pi^* < \pi_2
$$

(27)

where $\bar{v}(p) = \lambda_{\max}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$, $\bar{\psi}(p) = \lambda_{\min}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$.

Proof. Selecting the following Lyapunov function:

$$
V_2(x(t)) = x^T(t)Px(t)
$$

(28)

Following the trajectory of the closed-loop global fuzzy UVMS dynamic model 11, the derivation of the 28 is given as:

$$
\dot{V}_2(x(t)) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)
$$

$$
= x^T(t)(P\bar{A}_{ni} + \bar{A}_{ni}P)x(t) + x^T(t)PB_{2n}\tau_d(t) + \tau_d^T(t)PB_{2n}^TPx(t)
$$

$$
- x^T(t)PB_{1n}S(x(t)) - \frac{S^T(x(t))B_{1n}^TPx(t)}{\|S(x(t))\|}
$$

(29)

Considering the Definition 1, we introduce the following auxiliary inequality

$$
\dot{V}_2(x(t)) < \alpha V_2(x(t)) + \alpha \tau_d^T(t)\tau_d(t)
$$

(30)

The inequality 30 can be guaranteed by inequality 25. Multiply both sides of the above inequality by $e^{-\alpha t}$ and then integrate the inequality from 0 to t with $t \in [0, T^*]$, we can get:

$$
e^{-\alpha t}V_2(x(t)) < V_2(x(0)) + \alpha \int_0^t e^{-\alpha t}\tau_d^T(t)\tau_d(t)dt
$$

(31)

Letting $\bar{v}(p) = \lambda_{\max}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$ and $\bar{\psi}(p) = \lambda_{\min}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$, we can get the following inequality from inequality 31:

$$
e^{-\alpha t}V_2(x(t)) < \bar{v}(p)\pi_1 + \alpha \bar{\omega}^2 T
$$

(32)

On the other hand, we can get the following relation from equation 28:

$$
e^{-\alpha t}V_2(x(t)) \geq e^{-\alpha t}\bar{\psi}(p)x^T(t)Rx(t)
$$

(33)

Combining inequalities 32-33, we can get:

$$
e^{-\alpha t}\bar{\psi}(p)x^T(t)Rx(t) < \bar{v}(p)\pi_1 + \alpha \bar{\omega}^2 T
$$

(34)

that is, $x^T(t)Rx(t) < \frac{e^{\alpha t}(\bar{v}(p)\pi_1 + \alpha \bar{\omega}^2 T)}{\bar{\psi}(p)}$ can be tenable for any $t \in [0, T^*]$.

From inequalities 26-27, we can see that $x^T(t)Rx(t) < \pi^*$ holds for any $t \in [0, T^*]$. Therefore, the closed-loop global fuzzy UVMS dynamic model 11 is FTB within $(\pi_1, \pi^*, [0, T^*], R, G_{t_1, t_2], \pi)$ in the arrival stage $[0, T^*]$ according to Definition 1. We know that $\dot{S}(x(t)) = 0$ in the sliding stage $[T^*, T]$ from the sliding mode theory if the closed-loop global fuzzy UVMS dynamic model 11 reaches the sliding surface. Therefore, we can obtain the equivalent controller as:

$$
\tau^*(t) = -[(B_{1n}^TLiB_{1n})^{-1}B_{1n}^TL_iA_nx(t) + (B_{1n}^TL_iB_{1n})^{-1}B_{1n}^TL_iB_{2n}\tau_d(t)]
$$

(35)
We substitute the equivalent controller 35 into the global fuzzy UVMS dynamic model 4 and get:

\[
\dot{x}(t) = (I - C_{ni})(\tilde{A}_n x(t) + \tilde{B}_{2n} \tau_d(t))
\]

where \(C_{ni} = B_{1n}(B_{1n}^T L_i B_{1n})^{-1} B_{1n}^T L_i, \tilde{A}_n = \sum_{n=1}^{b} p_n(\rho(t)) A_n, \tilde{B}_{2n} = \sum_{n=1}^{b} p_n(\rho(t)) B_{2n} \).

**Theorem 3.** For the given positive constants \(\pi^* > 0, T > 0, \alpha > 0\), the closed-loop global fuzzy UVMS dynamic model 36 is FTB in the sliding stage \([T^*, T]\) with respect to \((\pi^*, \pi_2, [T^*, T], R, G_{[t_1, t_2], \varpi})\), if existing symmetric positive definite matrix \(P > 0\) and constant \(\pi_2 > 0\), such that:

\[
\begin{bmatrix}
P(I - C_{ni})\tilde{A}_n + [(I - C_{ni})\tilde{A}_n]^T P - \alpha P & P(I - C_{ni})\tilde{B}_{2n} \\
-\alpha I & 0
\end{bmatrix} < 0
\]

The inequality 41 can be guaranteed by inequality 37. Multiply both sides of the inequality 41 by \(e^{-\alpha t}\), then integrate the inequality from \(T^* \to t\) with \(t \in [T^*, T]\), we can get:

\[
e^{-\alpha t}V_2(x(t)) < e^{-\alpha T^*}V_2(x(T^*)) + \alpha \int_{T^*}^{t} e^{-\alpha t} \tau_d(t) \tau_d(t) dt
\]

Letting \(\tilde{\nu}(p) = \lambda_{\max}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})\) and \(\underline{\nu}(p) = \lambda_{\min}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})\), we can get:

\[
e^{-\alpha t}V_2(x(t)) < \tilde{\nu}(p) x^T(t) R x(T^*) + \alpha \varpi^2 T
\]

From Theorem 2, we know that \(x^T(t) R x(t) < \pi^*\) holds. Thus, the inequality 43 is equivalent to:

\[
e^{-\alpha t}V_2(x(t)) < \tilde{\nu}(p) \pi^* + \alpha \varpi^2 T
\]

In the same way, we can get inequality 45 from equation 28:

\[
e^{-\alpha t}V_2(x(t)) \geq e^{-\alpha t} \underline{\nu}(p) x^T(t) R x(t)
\]

Combining inequalities 44-45, we can obtain:

\[
e^{-\alpha t} \underline{\nu}(p) x^T(t) R x(t) < \tilde{\nu}(p) \pi^* + \alpha \varpi^2 T
\]

that is, \(x^T(t) R x(t) < \frac{\tilde{\nu}(p) \pi^* + \alpha \varpi^2 T}{\underline{\nu}(p)}\) can be tenable for any \(t \in [T^*, T]\).

From inequalities 38-39, we can see that \(x^T(t) R x(t) < \pi_2\) holds for any \(t \in [T^*, T]\). Thus, the closed-loop equivalent global fuzzy UVMS dynamic model 36 is FTB with respect to \((\pi^*, \pi_2, [T^*, T], R, G_{[t_1, t_2], \varpi})\) in the arrival stage \([T^*, T]\). \(\square\)
3.3. Solution of the controller gain. From Lemma 1, if and only if the conditions in Theorem 2 and Theorem 3 are satisfied at the same time, we can get that the closed-loop global fuzzy UVMS dynamic model is FTB with respect to \((\pi_1, \pi_2, [0, T], R, G_{[t_1, t_2], \omega})\) in the whole time interval \([0, T]\). Therefore, the parameters \(L_i\) and \(K_i\) of the sliding mode controller solved by us should also satisfy the conditions in Theorem 2 and Theorem 3. In Theorem 4, we will derive the corresponding sufficient conditions to solve the parameters of the sliding mode controller.

**Theorem 4.** For the given constants \(\pi_1 > 0, T > 0, \alpha > 0\), the closed-loop global fuzzy UVMS dynamic model 11 is FTB with respect to \((\pi_1, \pi_2, [0, T], R, G_{[t_1, t_2], \omega})\) on the condition of the fuzzy sliding mode controller 10 with controller gain of \(K_i = Y_iX^{-1}\), if existing symmetric positive definite matrix \(P > 0\) and constant \(\pi_2 > 0\), such that the following LMI holds:

\[
\Phi_{ni} < 0, \quad n = 1, 2, ..., b
\]

\[
\Lambda_{ni} < 0, \quad n = 1, 2, ..., b
\]

\[
\Phi_{ni} + \Phi_{in} < 0, \quad n = 1, 2, ..., b, i = 1, 2, ..., b
\]

\[
\Lambda_{ni} + \Lambda_{in} < 0, \quad n = 1, 2, ..., b, i = 1, 2, ..., b
\]

\[
\sum_{n=1}^{b} \eta_n R^{-1} < X < R^{-1}
\]

\[
e^{\alpha t}(\bar{v}_i(p)\pi_1 + \alpha \bar{\omega}^2 T) < \text{underline}v_i(p)\pi^*
\]

\[
e^{\alpha t}(\bar{v}_i(p)\pi^* + \alpha \bar{\omega}^2 T) < \text{underline}v_i(p)\pi_{2}
\]

\[
\pi_1 < \pi^* < \pi_2
\]

where \(\Phi_{ni} = \begin{bmatrix} A_n X + X A_n^T + B_{1n} Y_i + Y_i^T B_{1n}^T - \alpha X & B_{1n} X & B_{2n} X \\ * & 0 & 0 \\ * & * & -\alpha I \end{bmatrix}\)

\(\Lambda_{ni} = \begin{bmatrix} A_n X + X A_n^T - C_{ni} A_n X - X A_n^T C_{ni}^T & B_{2n} X - C_{ni} B_{2n} X \\ * & * & -\alpha I \end{bmatrix}\), \(X = P^{-1}\), \(Y_i = K_i X\), \(\bar{\omega}(\lambda) = \lambda_{\text{max}}(X)\), \(\eta_{\text{min}}(X)\), \(\eta_{\text{min}} < \lambda_{\text{max}}(X)\).

**Proof.** Substituting \(\overline{A}_{ni} = \sum_{n=1}^{b} p_n(\rho(t)) A_{ni} + B_{1n} K_i\), \(\overline{B}_{1ni} = \sum_{n=1}^{b} p_n(\rho(t)) B_{1n}\) into inequality 25, and substituting \(\bar{A}_n = \sum_{n=1}^{b} p_n(\rho(t)) A_n\) and \(\bar{B}_{2n} = \sum_{n=1}^{b} p_n(\rho(t)) B_{2n}\) into inequality 37, we can get:

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) \Gamma'_{ni} < 0
\]

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) \Theta'_{ni} < 0
\]

where \(\Gamma'_{ni} = \begin{bmatrix} (A_n + B_{1n} K_i)^T P + P(A_n + B_{1n} K_i) - \alpha P & \alpha P & \alpha P \\ * & 0 & 0 \\ * & * & -\alpha I \end{bmatrix}\),

\(\Theta'_{ni} = \begin{bmatrix} P(I - C_{ni}) A_n + [(I - C_{ni}) A_n]^T P - \alpha P & \alpha P & \alpha P \\ * & 0 & 0 \\ * & * & -\alpha I \end{bmatrix}\).
Multiply both sides of inequalities 55-56 by diag\([P^{-1}, I, I]\) and let \(x = P^{-1}, Y_i = K_iX\), we can get:

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) \Gamma''_{ni} < 0 \quad (57)
\]

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) \Theta''_{ni} < 0 \quad (58)
\]

where \(\Gamma''_{ni} = \begin{bmatrix} A_nX + XA_n^T + B_{1n}Y_i + Y_i^TB_{1n}^T - \alpha X & B_{1n}X & B_{2n}X \\ * & 0 & 0 \\ * & * & -\alpha I \end{bmatrix}\)

\(\Theta''_{ni} = \begin{bmatrix} A_nX + XA_n^T - C_{ni}A_nX - XA_n^TC_{ni}^T & B_{2n}X - C_{ni}B_{2n}X \\ * & -\alpha I \end{bmatrix}\).

From the inequalities 55-56, we can obtain:

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) [\Phi_{ni} + \Phi_{ni}] + \sum_{n=1}^{b} p_n^2(\rho(t)) \Phi_{nn} < 0 \quad (59)
\]

\[
\sum_{n=1}^{b} p_n(\rho(t)) \sum_{i=1}^{b} p_i(\rho(t)) [\Lambda_{ni} + \Lambda_{ni}] + \sum_{n=1}^{b} p_n^2(\rho(t)) \Lambda_{nn} < 0 \quad (60)
\]

where \(\Phi_{ni} = \Gamma''_{ni}, \Lambda_{ni} = \Theta''_{ni}\).

From the inequalities 26-27, 38-39 and letting \(\overline{X} = R^{-\frac{1}{2}}X R^{-\frac{1}{2}}, \overline{\eta}(\overline{X}) = \lambda_{max}(\overline{X}), \overline{\eta}(\overline{X}) = \lambda_{min}(\overline{X}), \overline{\eta}_1 > \lambda_{max}(\overline{X}), \overline{\eta}_1 < \lambda_{min}(\overline{X})\), we can conclude that inequalities 51-54 hold.

4. Simulation. In order to illustrate the effectiveness of the sliding mode controller designed in this paper, in this section, we will simulate on the actual four joint dynamic model of UVMS with two fuzzy subsystems. Combined with the actual UVMS model 4, we can get:

\[
\dot{x}(t) = \sum_{n=1}^{2} p_n(\rho(t)) [A_nx(t) + B_{1n}\tau(t) + B_{2n}\tau_d(t)] \quad (61)
\]

In order to obtain the fuzzy subsystem, we let \(\rho(t) = \sin^2(x(t))\).

The main parameters of UVMS model are given in Table 1.

| Parameter | Value   |
|-----------|---------|
| \(l_1\)  | 1.12m   |
| \(l_2\)  | 0.89m   |
| \(m_1\)  | 4.32kg  |
| \(m_2\)  | 3.43kg  |
| \(g\)     | 9.8m/s² |

Where \(l_1\) and \(l_2\) are the lengths of link 1 and link 2, \(m_1\) and \(m_2\) are the masses of link 1 and link 2, \(g\) is the acceleration of gravity. The standard membership functions are \(\rho_1(t) = 1 - \sin^2(x(t))\) and \(\rho_2(t) = \sin^2(x(t))\) respectively. We define the initial vector \(x(0) = 0\), external disturbance \(\tau_d(t) = 2\sin^2(x(t))\) and assume the
other initial conditions are $\pi_1 = 0.8$, $R = I$, $T = 10$. By solving the LMIs 47-54, we can get the controller gain parameters and the required solution parameters as:

$$K_1 = \begin{bmatrix} -3.257 & -1.436 & -0.548 & 1.267 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -3.726 & -2.078 & -0.895 & 1.374 \end{bmatrix},$$

$$\pi^* = 3.48, \quad \pi_2 = 8.76.$$  

(62)

By simulating the global fuzzy UVMS dynamic model, the results are shown in Fig.1 and Fig.2. Fig.1 is the global fuzzy sliding mode variable designed in this paper. It can be seen that the global fuzzy sliding mode function reaches a stable state after a short time adjustment in a finite short time $[0, 1]$. This means that if the global fuzzy UVMS dynamic model can be driven to the sliding mode surface and move along the sliding mode surface under the action of the sliding mode controller, the dynamic model can be guaranteed to be bounded in a given finite short time. Fig.2 shows the open-loop dynamic trajectory without control and the closed-loop dynamic trajectory under the action of the sliding mode controller. The red track denotes that the open-loop dynamic trajectory without control is divergent and exceeds the given bound in a given time interval. And the blue track denotes that the closed-loop dynamic trajectory under the action of the sliding mode controller is bounded in a given finite short time. Combined with the simulation results in Fig. 1 and Fig. 2, it can be clearly seen that the closed-loop global fuzzy UVMS dynamic model is FTB and effective.

![The global fuzzy sliding mode variable](image)

**Figure 1.** The global fuzzy sliding mode variable

5. **Conclusion.** In this paper, a finite-time sliding mode control method via T-S fuzzy approach is designed to solve the difficult control problem of UVMS. We give the general dynamic model of UVMS, and by the approach of fuzzifying the influence between the manipulator and the underwater vehicle and other disturbances, we establish a global fuzzy dynamic model and design the closed-loop fuzzy sliding mode controller. We prove the reachability of sliding domain and the FTB of this model and give the solution of the controller gain. Then we choose an actual four joint dynamic model of UVMS with two fuzzy subsystems, and verify the effectiveness of the system.
For future research work, we will explore the application of the controller in complex underwater operation tasks, such as dual arms cooperative operation, continuous finite time operation and multi task operation of UVMS, explore the gain adjustment method of adaptive controller, make full use of the advantages of sliding mode controller and fuzzy control strategy, and carry out practical test and application in underwater detection and rescue.

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