A method for optimizing electromagnetic ultrasonic probe based on the spatial distribution of permanent magnets

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Abstract. The external magnetic field of a single permanent magnet is modeled by the vector magnetic potential method based on the Ampere hypothesis, which is compared with the finite element method to analyze the external magnetic field of the permanent magnet. Based on this, the mathematical models of the external magnetic field of multiple permanent magnets are obtained. By analyzing the external magnetic field of the permanent magnet of an electromagnetic ultrasonic probe proposed in this paper, the influence of the parameters of the permanent magnet on the external magnetic field is studied. The optimized structure containing multiple permanent magnets can increase the vertical component of magnetic field strength by about 32.3% compared with that of a single permanent magnet.

1. Introduction
Nondestructive testing refers to the monitoring and analysis of the internal defects of the tested material by using the changes in the reactions of light, electricity, magnetism, heat and sound caused by the defects in the material without damaging the material structure and with the help of modern equipment and instruments and signal processing technology. All kinds of nondestructive testing techniques have been applied in many fields, such as mechanical parts testing, tunnel engineering testing, water conservancy engineering quality testing, concrete structure testing. Among them, ultrasonic testing is a kind of nondestructive testing technology based on pulse echo method, which is a routine testing method for flaw detection and quality assessment of materials and components in various fields. Compared with other nondestructive testing technologies, ultrasonic testing technology has the advantages of less restrictions, thicker materials, low cost, safety and convenience, etc. Ultrasonic testing is mainly divided into piezoelectric ultrasonic and electromagnetic ultrasonic two kinds, including electromagnetic ultrasonic because of its no coupling agent, and materials required for testing without contact, can be used in high temperature environment test, etc, but the biggest problem is that conversion efficiency is not high, weak signal detection, thus attracting the attention of many scholars in recent years and the case for how to improve the efficiency of its change to study.
The complete electromagnetic ultrasonic transducer mainly contains three parts: 1. Excitation and detection coil; 2. External magnetic field (usually provided by permanent magnet or electromagnet); 3. Metal specimen to be tested. The main principle is that a high-frequency alternating current is introduced into the coil close to the surface of the metal specimen to induce an alternating eddy current on the surface of the specimen. Under the action of an external magnetic field, the alternating Lorentz force causes vibration excitation of ultrasonic wave on the metal specimen, and the ultrasonic wave reaches the inside of the metal specimen and is reflected back to the metal surface. Under the action of an external magnetic field, the voltage signal generated in the detection coil is detected.

In recent years, many scholars’ research on electromagnetic ultrasound mainly focuses on the establishment of its mathematical model, the selection of external magnetic field, the shape and size of the coil, the selection of excitation signal frequency, ultrasonic focusing technology and so on. In the last century, some foreign scholars deduced the governing equation of electromagnetic ultrasonic system based on Lorentz force, and introduced the solving method of this model in detail [1]. In the following years, D MacLauchlan et al. introduced the expression of electromagnetic ultrasonic signal-to-noise ratio under the condition of ignoring ultrasonic diffraction and ideal matching [2]. At the same time, has a strong magnetic field of the use of permanent magnet NdFeb material can obtain larger plus a magnetic field, and the structure of the electromagnetic ultrasonic probe can be more compact[3]. In contrast, the electromagnet can be adjusted flexibly and magnetic field intensity, but due to the magnetic field strength as NdFeb and will bring the problem of electromagnetic interference, less in the electromagnetic ultrasonic applications. In the research process, researchers found that coils with different shapes and magnetic fields in different directions can stimulate different types of ultrasonic waves on the metal to be tested. Different types of ultrasonic waves can detect different objects, such as body waves, which are mainly used to measure thickness and examine internal defects, and Lamb waves, which are mainly used to detect surface defects. The main types of existing coils are spiral type, track type, loop type, snake type, etc. In terms of excitation signals, Jiang Nian et al. found that when the frequency of coil excitation signal is the same as the natural frequency of electromagnetic ultrasonic transducer, energy transfer efficiency and signal-to-noise ratio can be improved [4]. H Ogi et al. used coils of unequal spacing to focus the ultrasonic wave inside the specimen to improve the detection accuracy [5]. Zhou Qi et al. used phased array technology and multiple ultrasonic probes to achieve ultrasonic focusing [6].

Although the electromagnetic ultrasonic detection technology in recent years gradually got the attention of some scholars, but people in improving the efficiency of its change can focus more on the research of in replacement of permanent magnet materials, optimization of the coil structure, the optimal excitation signal and to increase the number of probe, etc, and largely using only a single rectangle or cylinder permanent magnets, the use of multiple permanent magnets and optimize the spatial distribution of research seldom mentioned. Therefore, this paper proposes a method that utilizes multiple rectangular permanent magnets and optimizes their spatial structure to enhance the intensity of the external magnetic field to improve the detection sensitivity, aiming at the technology of electromagnetic ultrasonic thickness measurement and material internal defects detection.

2. Model of the external magnetic field generated by a permanent magnet
From D MacLauchlan’s electromagnetic ultrasonic signal-to-noise ratio expression:

\[
\frac{V_{EMAT}}{V_{noise}} = P_0 B^2 AExp(-\frac{\alpha G}{D})(W^2Z_{LS}^2(4KT\beta)^2R_0)^2
\]

(1)

It can be seen that the SNR and magnetic field strength square relationship, magnetic field strength has an important impact on the SNR, so the use of NdFeB permanent magnet with strong magnetic field can greatly improve the SNR. At the same time, the direction of the magnetic field also has a great influence on the type of ultrasonic wave generated on the metal specimen, so it is of great significance to accurately know the external magnetic field distribution of the permanent magnet to guide the design of the electromagnetic ultrasonic probe. With the help of theoretical analysis, this
paper analyses the distribution of the external magnetic field of NdFeB permanent magnet. It is an ideal method to calculate the numerical solution of the external magnetic field of permanent magnet by computer.

At present, the main methods used for numerical simulation of the external magnetic field of permanent magnets include equivalent magnetic charge method, vector magnetic potential method based on the Ampere hypothesis and finite element method, etc [7]. In this paper, the vector magnetic potential method based on molecular circulation view is used to calculate the external magnetic field of rectangular permanent magnet. The results are compared with the finite element method to verify the correctness and feasibility of the method.

The Ampere hypothesis holds that there is a circular current inside the atoms and molecules that make up objects, which turns each particle of matter into a tiny magnet. In a uniform magnetic medium, the direction of molecular circulation is the same, and any pair of adjacent current elements inside the medium have opposite directions. The magnetic fields generated cancel each other, except the magnetic fields generated by the current elements at the edge of the medium [8]. The positions of rectangular permanent magnets whose length, width and height are respectively a, b and c in the three-dimensional space coordinate system are assumed as shown in Fig.1.

![Fig.1 Rectangular permanent magnet model](image)

It is assumed that the permanent magnet is uniformly magnetized and saturated in the vertical direction. Based on the Ampere hypothesis, we can regard the external magnetic field of the permanent magnet as a closed circulation on the surface of the permanent magnet. A thin layer current ring with thickness of $dz_0$ is selected, and its surface current density is set as $J$. Assume that the magnetic field generated by current $I = J dz_0$ at any point $P(x, y, z)$ outside the permanent magnet is:

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} = \int_0^b dB_x \mathbf{i} + dB_y \mathbf{j} + dB_z \mathbf{k}$$

(2)

$dB_x$, $dB_y$, and $dB_z$ three components of the magnetic field at point $P(x, y, z)$. These magnetic field components along different directions on the right can be viewed as the superposition of the magnetic fields generated by the currents at the four boundaries of the thin current layer. According to Biot-Savart's law, the magnetic field generated at P can be obtained respectively:

$$d\mathbf{B}_{AB} = \mu_0 \frac{\mathbf{I} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} = \mu_0 \frac{J dz_0}{4\pi} \int_0^b \mathbf{I} \times (\mathbf{r} - \mathbf{r}') \frac{d\mathbf{r}}{|\mathbf{r} - \mathbf{r}'|^3}$$

(3)
\[ dB_{A'B'x} = \frac{\mu_0 I}{4\pi} \int_0^b \frac{z - z_0}{\left[ (x-a)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{A'B'y} = 0 \]

\[ dB_{A'B'z} = \frac{\mu_0 J_z}{4\pi} \int_0^b \frac{a-x}{\left[ (x-a)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{A'B'} , dB_{A'B'} , \text{ and } dB_{A'B'} \text{ represent the three components of the magnetic field generated by the current in segment } A'B' \text{ at the point } P(x, y, z). \text{ In the same way, the magnetic field generated by the current in segment } B'C' \text{ can be obtained:} \]

\[ dB_{B'C'} = \frac{\mu_0 I}{4\pi} \int_0^a \frac{z - z_0}{\left[ (x-x_0)^2 + (y-b)^2 + (z-z_0)^2 \right]^{3/2}} dx_0 \]

\[ dB_{B'C'} = 0 \]

\[ dB_{B'C'} = \frac{\mu_0 J_z}{4\pi} \int_0^b \frac{z - z_0}{\left[ (x-x_0)^2 + (y-b)^2 + (z-z_0)^2 \right]^{3/2}} dx_0 \]

\[ dB_{B'C'} = \frac{\mu_0 J_z}{4\pi} \int_0^b \frac{a-x}{\left[ (x-a)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{B'C'} , dB_{B'C'} , \text{ and } dB_{B'C'} \text{ represent the three components of the magnetic field generated by the current in segment } B'C' \text{ at the point } P(x, y, z). \text{ In the same way, the magnetic field generated by the current in segment } C'D' \text{ can be obtained:} \]

\[ dB_{C'D'} = \frac{\mu_0 I}{4\pi} \int_0^a \frac{z - z_0}{\left[ (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{C'D'} = 0 \]

\[ dB_{C'D'} = \frac{\mu_0 J_z}{4\pi} \int_0^b \frac{z - z_0}{\left[ (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{C'D'} = \frac{\mu_0 J_z}{4\pi} \int_0^b \frac{a-x}{\left[ (x-a)^2 + (y-y_0)^2 + (z-z_0)^2 \right]^{3/2}} dy_0 \]

\[ dB_{C'D'} , dB_{C'D'} , \text{ and } dB_{C'D'} \text{ represent the three components of the magnetic field generated by the current in segment } C'D' \text{ at the point } P(x, y, z). \text{ In the same way, the magnetic field generated by the current in segment } D'A' \text{ can be obtained:} \]
The total magnetic field at point \( P(x, y, z) \) can be obtained by adding the magnetic field components generated by each current segment, which can be simplified by calculation as follows:

\[
B_x = \frac{K}{2} \left[ -\Gamma(a-x, b-y, z) - \Gamma(a-x, y, z) + \Gamma(x, b-y, z) + \Gamma(x, y, z) \right]
\]

\[
B_y = \frac{K}{2} \left[ -\Gamma(b-y, a-x, z) - \Gamma(b-y, x, z) + \Gamma(y, a-x, z) + \Gamma(y, x, z) \right]
\]

\[
B_z = K \left[ -\Psi(b-y, a-x, z) - \Psi(y, a-x, z) - \Psi(a-x, b-y, z) - \Psi(x, b-y, z) - \Psi(b-y, x, z) - \Psi(y, x, z) - \Psi(a-x, y, z) - \Psi(x, y, z) \right]
\]

Where, \( K = \frac{\mu_0 J_s}{4\pi} \) and :

\[
\Gamma(x, y, z) = \ln \frac{\sqrt{x^2 + y^2 + (z-z_0)^2} - y}{\sqrt{x^2 + y^2 + (z-z_0)^2} + y} \bigg|_{z_0=0}^{z_0=h}
\]

\[
\Psi(x, y, z) = \arctan \left[ \frac{x}{y} \sqrt{\frac{z-z_0}{x^2 + y^2 + (z-z_0)^2}} \right] \bigg|_{z_0=0}^{z_0=h}
\]

However, the magnitude of \( J_s \) is only related to the magnetization intensity of the permanent magnet, which is related to the material of the permanent magnet, so \( K \) can be regarded as a constant. Therefore, based on Equation 11, we can obtain the external magnetic field distribution of the rectangular permanent magnet magnetized in the z direction.

3. Comparison between the numerical method and the finite element method

Based on the three-dimensional space coordinates and permanent magnet positions shown in Fig.1 and Equation 11, the computer software MATLAB is used to calculate the external magnetic field distribution of the rectangular permanent magnet with length, width and height of 40mm. At the same time, the finite element simulation software is used to simulate the permanent magnet with the same shape to check the external magnetic field distribution.
Firstly, MATLAB is used to calculate the distribution of horizontal and vertical magnetic fields above the permanent magnet. Take \( x = 20 \text{mm} \), \( z = 45 \text{mm} \) and \( y = [-80 \text{mm}, 120 \text{mm}] \) to calculate \( B_y \) and \( B_z \) respectively, and the results are shown in Fig.2 and Fig.3:

![Fig.2 Value of \( B_y \)](image1)

![Fig.3 Value of \( B_z \)](image2)

Then the finite element simulation software was used to calculate the horizontal magnetic field and the vertical magnetic field at the same position. The simulation geometric model was shown in Fig.4:

![Fig.4 Finite element model of permanent magnet](image3)

![Fig.5 Permanent magnet meshing model](image4)

Since this paper is concerned about the spatial magnetic field distribution close to the permanent magnet, a more precise mesh division is adopted for the space 10mm outside the permanent magnet, as shown in Fig.5. In addition, since this paper focuses on the distribution of the external magnetic field of the permanent magnet, \( K = 1 \) is used in MATLAB to simplify the calculation results. In the finite element simulation, the setting of magnetization intensity of permanent magnet is also different from the actual permanent magnet, so the magnetic field intensity in the figure will be different from the actual external magnetic field intensity of permanent magnet. The magnetic field distribution calculated by the finite element method is shown in Fig.6 and Fig.7:

![Fig.6 Value of \( B_y \) in the finite element method](image5)

![Fig.7 Value of \( B_z \) in the finite element method](image6)
From Fig.2, Fig.3, Fig.6 and Fig.7, you can see that based on Ampere hypothesis of vector magnetic potential method and finite element method of permanent magnet external magnetic field distribution, and vertical and horizontal magnetic field magnetic field on the edge of the permanent magnet and center to obtain the maximum value respectively, verified in this paper, the distribution of the external magnetic field of permanent magnet mathematic analytical model is correct.

4. Influence of multiple rectangular permanent magnets on the external magnetic field distribution of electromagnetic ultrasonic probe

According to Equation 1, the external magnetic field distribution of a single rectangular permanent magnet can be obtained. The external magnetic field distribution of multiple rectangular permanent magnets can be regarded as the superposition of external magnetic fields of each permanent magnet. In Fig.1, the point where the rectangular permanent magnet coincides with the origin of the coordinate system is defined as point \( O \), then the magnetic field generated by the rectangular permanent magnet with an arbitrary size of \( a \times b \times h \) and a point \( O(x_0, y_0, z_0) \) representing its spatial position at any point \( P(x, y, z) \) in space is:

\[
\begin{aligned}
B_x &= \frac{K}{2} [-\Gamma(a-x+x_0, b-y+y_0, z-z_0) - \Gamma(a-x+x_0, y-y_0, z-z_0)
+ \Gamma(x-x_0, b-y+y_0, z-z_0) + \Gamma(x-x_0, y-y_0, z-z_0)] \\
B_y &= \frac{K}{2} [-\Gamma(b-y+y_0, a-x+x_0, z-z_0) - \Gamma(b-y+y_0, x-x_0, z-z_0)
+ \Gamma(y-y_0, a-x+x_0, z-z_0) + \Gamma(y-y_0, x-x_0, z-z_0)] \\
B_z &= K [-\Psi(b-y+y_0, a-x+x_0, z-z_0) - \Psi(y-y_0, a-x+x_0, z-z_0)
- \Psi(b-y+y_0, x-x_0, z-z_0) - \Psi(x-x_0, y-y_0, z-z_0) - \Psi(b-y+y_0, x-x_0, z-z_0)
- \Psi(y-y_0, x-x_0, z-z_0) - \Psi(a-x+x_0, y-y_0, z-z_0) - \Psi(x-x_0, y-y_0, z-z_0)]
\end{aligned}
\]

Based on Equation 13, the spatial magnetic field distribution with multiple rectangular permanent magnets can be obtained as shown in Fig.8. It is assumed that the material types of multiple rectangular permanent magnets are the same. Since the coefficient \( K \) is only related to the material properties of permanent magnets, they have the same coefficient \( K \). In order to simplify the calculation, the coefficient \( K = 1 \) is taken.

Fig.8 Geometric model of multiple rectangular permanent magnets

Considering that the distance between the coil of the electromagnetic ultrasonic probe and the permanent magnet is generally less than 3mm in order to obtain a larger external magnetic field, the magnetic field below the central permanent magnet in Fig.8 is calculated in this paper. In Fig.8, there is a rectangular permanent magnet with a side length of 40mm surrounded by four 40×20×10mm
rectangular permanent magnets, magnetizing in the opposite direction to the central magnetic field. After calculation, the vertical magnetic field is shown in Fig.9:

As can be seen from Fig.9, in the case of multiple permanent magnets, the vertical magnetic field intensity under the central permanent magnet is significantly enhanced. When $y = 20mm$, the vertical component of the magnetic field containing multiple permanent magnets is about 23% higher than that of a single permanent magnet, indicating that the magnetic field intensity in the vertical direction can be significantly improved by the structure of multiple permanent magnets as shown in Fig.8. This paper continues to study the effect of the side lengths and positions of the four permanent magnets next to the central permanent magnet on improving the magnetic field intensity.

4.1. The effect of position on magnetic field strength
The relative horizontal and vertical positions of the four permanent magnets and the central permanent magnet were respectively changed to compare the vertical component of magnetic field strength at the center of the lower surface of the central permanent magnet at different positions, as shown in Fig.10.

In Fig.10, $x_0$ and $y_0$ of point $O(x_0, y_0, z_0)$ are kept unchanged, and only $z_0$ is changed. Negative value indicates that the permanent magnet is shifted in the negative direction along the $z$ axis. From the beginning, as the four permanent magnets move down, the vertical component of the central magnetic field strength below the central permanent magnet first increases and then decreases. When $z_0 = -10mm$, change the horizontal distance between the four permanent magnets and the center permanent magnet, and calculate the vertical magnetic field component at the same position. The result is shown in Fig.11. Obviously, the vertical component of the central magnetic field intensity decreases with the increase of horizontal spacing. Therefore, the horizontal distance between the center permanent magnet and the four nearby permanent magnets should be minimized in the practical application of the substructure. At the same time, in order to ensure that the coil is close to the center
magnet as well as the tested material, the vertical positions of the four permanent magnets should be selected according to the actual situation.

4.2. Effect of side length on magnetic field intensity

The length, width and thickness of the four permanent magnets were changed respectively, and the vertical component of the magnetic field at 3mm below the center of the lower surface of the permanent magnet at the center was calculated. Considering the practical application, in order to ensure that the coil is close to the center magnet and the material to be measured, the vertical displacement $z_0$ of the four permanent magnets should not be too large. Here, $z_0 = -5\text{mm}$ is selected, and the horizontal distance between the permanent magnet and the center is set as 0. The calculated results are shown in Fig.12, Fig.13 and Fig.14:

![Fig.12 Influence of length on magnetic field](image1)

![Fig.13 Influence of width on magnetic field](image2)

![Fig.14 Effect of thickness on magnetic field](image3)

It can be seen from Fig.12 and Fig.13, within a certain range, the vertical component of magnetic field intensity at the measured place can be increased by appropriately increasing the length and width of the surrounding permanent magnet. In order to avoid the excessive size of the electromagnetic ultrasonic probe, the length and width of the surrounding permanent magnet should be designed as appropriate in practice. As can be seen from Fig.14, when the width of the permanent magnet is 22.5mm, the magnetic field intensity of the measured point has a maximum value. Therefore, the appropriate thickness of the four surrounding permanent magnets in the design can have a certain effect on enhancing the magnetic field intensity.

According to the above calculation and analysis, in the case that the permanent magnet in the center is $40 \times 40 \times 40\text{mm}$, the permanent magnet around the center is $40 \times 20 \times 22.5\text{mm}$. The vertical component of magnetic field strength under the permanent magnet at the center was recalculated, and the result is shown in Fig.15:
As can be seen from Fig.15, compared with a single permanent magnet, the vertical component of magnetic field strength below the central permanent magnet can be increased by about 32.3% by using the structure of multiple permanent magnets. Moreover, the vertical component of the magnetic field at the lower edge of a single permanent magnet will decrease, while the vertical component of the magnetic field at the lower edge of the central permanent magnet will not decrease for the structure of multiple permanent magnets.

5. Conclusion
The numerical calculation model of the external magnetic field of permanent magnet established in this paper is compared with the finite element method, and the correctness of the model is verified. At the same time, the proposed structure using multiple permanent magnets in the electromagnetic ultrasonic probe can significantly enhance the vertical component of the external magnetic field (about 32.3%). It is found that the enhancement effect is related to the space position of the surrounding permanent magnet and the length of each side. When other conditions remain unchanged, the magnetic field strength is positively related to the length and width of the surrounding permanent magnet, and the vertical component of magnetic field strength has a maximum value under a certain thickness. However, considering the limitations of the size of the actual electromagnetic ultrasonic probe and the detection requirements, the parameters and space positions of the surrounding permanent magnets need to be selected according to the actual situation. This paper can provide reference for the design of electromagnetic ultrasonic probe.

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