On dipole compression modes in nuclei

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Abstract

Isoscalar dipole strength distributions in spherical medium- and heavy-mass nuclei are calculated within random phase approximation (RPA) or quasiparticle RPA. Different Skyrme-type interactions corresponding to incompressibilities in the range 200 - 250 MeV are used. The results are discussed in comparison with existing data on isoscalar giant dipole resonances. Two main issues are raised, firstly the calculated giant resonance energies are somewhat higher than the observed ones, and secondly a sizable fraction of strength is predicted below 20 MeV which needs to be experimentally confirmed.

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The isoscalar giant dipole resonance (ISGDR) is a compressional mode, like the well-known isoscalar giant monopole resonance (ISGMR), and its energy is related to the nuclear incompressibility $K_\infty[1]$. For this reason it has been studied both experimentally and theoretically for many years, as reviewed in [2]. It can be associated with the operator

$$\hat{D} = \sum_{i=1}^{A} r_i^3 Y_{1\mu}(\hat{r}_i)$$

and it can be viewed as a non-isotropic compression mode.

Although some first indications about the energy location of this mode date back to the beginning of the eighties, more recent evidence of the ISGDR in $^{208}$Pb has been reported [3] from the $0^0$ measurements of 200 MeV inelastic $\alpha$-scattering at the Indiana University cyclotron facility. Further evidence based on extensive angular distributions at near-$0^0$ angles has since come from 240 MeV inelastic $\alpha$-scattering experiments at Texas A&M University. The ISGDR strength has been extracted in a large number of nuclei using a multipole decomposition of the observed inelastic scattering spectra. The results for medium- and heavy-mass nuclei like $^{90}$Zr, $^{116}$Sn, $^{144}$Sm and $^{208}$Pb are reported in [4]. The measured strength is spread over a wide energy range between 15 and 30 MeV and it is claimed to exhaust nearly 100% of the appropriate energy-weighted sum rule (EWSR). The values of the centroid energy $E_0 \equiv m_1/m_0$ are 26.3(4), 24.3(3), 23.0(3) and 20.3(2) MeV respectively in the four nuclei quoted above (the moments $m_k$ of the strength distribution are defined as $m_k \equiv \sum_n |<n|\hat{D}|0>|^2(E_n - E_0)^k$).

Here, we report the ISGDR results calculated with effective Skyrme interactions, within self-consistent Hartree-Fock (HF) plus Random Phase Approximation (RPA) in the case of $^{208}$Pb, and Hartree-Fock-BCS (HF-BCS) plus quasi-particle RPA (QRPA) for the other, non double-magic nuclei. The Skyrme forces used in this work are: SkP [5], SGII [6], SKM* [7], SLy4 [8] and SkI2 [9]. They span a range of values of $K_\infty$ from 200 to 250 MeV. The HF mean field is first calculated in coordinate space, then the single-particle spectrum of occupied and unoccupied states is built by diagonalizing the mean field on a harmonic oscillator basis. Details of RPA calculations can be found in Ref. [10]. The dimension of the 1particle-1hole (1p-1h) space is fixed by requiring the exhaustion of the RPA $m_1$ sum rule. In the HF-BCS calculations constant pairing gaps $\Delta$ are introduced according to the usual
12 MeV/\sqrt{A} parametrization. The QRPA matrix equations are solved with a procedure which parallels what has been said for RPA, with the two quasiparticle configurations replacing the 1p-1h ones. The method is the same as that of Ref. [11].

In the ISGDR problem, one has to face the question of the spurious state associated with the center-of-mass motion which carries the same quantum numbers \( J^\pi = 1^- \). In a bona fide self-consistent RPA the spurious state would appear as an eigenstate at zero energy, exhausting the whole strength of the operator

\[
\hat{S} = \sum_{i=1}^{A} r_i Y_{1\mu}(\hat{r}_i)
\]

and orthogonal to all other physical states. However, in actual calculations the spurious state is at low but not zero energy because of small numerical inaccuracies and therefore, strength associated with the operator \( \hat{S} \) will be shared among the physical states. Starting from the actual RPA set of states \(|n'\rangle\), we construct a new set of normalized states \(|n\rangle\),

\[
|n\rangle = N_n(|n'\rangle - \alpha_n |S\rangle),
\]

where the state \(|S\rangle\) is defined as

\[
|S\rangle \equiv \hat{S}|0\rangle,
\]

\(|0\rangle\) being the RPA vacuum. According to [12] we associate to \(|S\rangle\) the transition density

\[
\alpha_S \frac{d\rho_0}{dr}
\]

where \(\rho_0\) is the HF ground state density. The state \(|n\rangle\) is required to satisfy the condition \(\langle n|\hat{S}|0\rangle = 0\), i.e.,

\[
\int drr^3(\delta \rho_{n'} - a_n \frac{d\rho_0}{dr}) = 0,
\]

where \(\delta \rho_{n'}\) is the transition density of the RPA state \(|n'\rangle\) defined in the usual way. The problem of the spurious state normalization \(\alpha_S\) is circumvented by the use of Eq. (3) since \(a_n \equiv \alpha_n \alpha_S\) is well behaved (i.e., not divergent).

The difference between the strength distributions associated with the states \(|n'\rangle\) and \(|n\rangle\) is shown in the top-left corner of Fig. 1, for the typical case of \(^{208}\)Pb with the force SGII. The strengths are essentially the same.
in the energy range which will be denoted “giant resonance (GR) region” (this range is evident from the plot but it is explicitly indicated in Tables 1 and 2). At lower energies, omitting the projection procedure can lead to a serious overestimation of the ISGDR strength. It is clear nevertheless from Fig. 1 that a non-negligible amount of non-spurious strength is present in the energy range which will be called “low-energy region”. This low-lying strength is due to $1 \hbar \omega$ excitations, which of course can contain strength associated with the $\hat{D}$ operator.

Another way of eliminating the spurious strength is to keep the $|n'\rangle$ states and to replace the operator (1) by

$$\hat{D}_{\text{modif}} = \sum_{i=1}^{A} (r_i^3 - \eta r_i)Y_{1\mu}(\hat{r}_i), \quad (7)$$

where $\eta = \frac{5}{3} \langle r^2 \rangle$. This prescription was derived and used in Ref. [13]. Although the derivation is based on hydrodynamical-type arguments, one thus obtains strength distributions which are almost indistinguishable to those calculated with the present projection procedure. One may also note that in Ref. [14] a different prescription was used for the subtraction of spurious strength resulting in an almost disappearance of strength in the low-energy region.

In Fig. 1 we also show center-of-mass corrected strength distributions for the other nuclei calculated with a typical interaction, namely SGII. The general features are: a) a large fraction of the strength lies in the GR region, and b) a non negligible amount of strength is in the low-energy region. The latter region contains about 20% of the ISGDR energy-weighted sum rule. These features are common to the results obtained with the other interactions. A more detailed analysis in terms of the moments $m_0$ and $m_1$ is reported in Table 1 for $^{208}$Pb and all interactions, and in Table 2 for all 4 nuclei and the SGII interaction.

In comparison with the existing data, there are two main issues to be faced. First, there is a large discrepancy between predicted and measured GR energies, much larger than in all other GR cases. This is the more puzzling that the same model employed here was used successfully to describe the ISGMR in $^{208}$Pb [15]. Second, the calculations predict a sizeable amount of strength at low-energy, which needs to be experimentally confirmed [16]. These features are common to the calculated strength distributions of the
operator $j_1(qr)Y_{1\mu}(\hat{r})$, which is a generalization of Eq. (1). In particular, they remain peaked at the same energies as the strength distribution of $\hat{D}$ for values of $q$ up to 0.6 fm$^{-1}$.

In Fig. 2 we show the predicted peak and centroid energies of the GR region for various nuclei as a function of $K_{\infty}$. The experimental values of $E_0$ for the GR region quoted above [4] would be outside the figure, except for $^{90}$Zr. The discrepancy appears very severe in Pb and Sn. In what follows we concentrate on Pb because it is the nucleus where the HF+RPA model should work better.

Earlier RPA calculations [17] performed with the finite range Gogny interaction already found that the ISGDR energy was in the range of 26 MeV, in qualitative agreement with the present results and with Ref. [13]. One might expect that effects beyond RPA, like the coupling to 2p-2h excitations would somewhat lower the centroid energy. However, the calculations of Ref. [18] find a downward shift of less than 1 MeV. The ISGDR has also been calculated in the relativistic RPA approach [19] in $^{208}$Pb and $^{144}$Sm and it is found that, for effective lagrangian parametrizations corresponding to $K_{\infty}$ in the range 200-270 MeV the energy of the ISGDR is of the order of 25 MeV. Thus, the question of understanding the observed values of $E_0$ is still open.

As for the low-energy region, our analysis of the configurations involved for instance in $^{208}$Pb, shows that the strength comes from bound-to-bound neutron transitions like $h_{9/2} \rightarrow i_{11/2}$, $i_{13/2} \rightarrow j_{15/2}$ and, in some cases, $f_{5/2} \rightarrow g_{7/2}$. In the data reported in Ref. [4] no low-lying strength is present. Further analysis of the same data is currently in progress [19], which may reveal the presence of isoscalar strength around the region of the isovector dipole.

In conclusion, we report in this paper HF+RPA and HF-BCS+QRPA calculations of the ISGDR in $^{90}$Zr, $^{112}$Sn, $^{144}$Sm and $^{208}$Pb nuclei. Two general features appear from the calculated strength distributions: some large resonance-type distribution of strength in the $110A^{-1/3}$ MeV energy region and some smaller, but still sizeable fraction of the strength below 20 MeV. These two characteristic features do not seem to agree quantitatively with the observation.

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Fig. 1. ISGDR strength distributions calculated with the interaction SGII and corrected for center-of-mass effects. In the case of $^{208}$Pb the dashed line corresponds to a calculation without proper subtraction of the spurious center-of-mass state (see text).

Fig. 2. Centroid energies $E_0$ (black circles) and peak energies (open circles) of the GR regions, determined by using different Skyrme interactions and plotted as a function of $K_\infty$. The full and dashed lines are drawn as a guide to the eye.

Table 1: Integral properties of the ISGDR strength distribution in $^{208}$Pb calculated with different forces. The corresponding incompressibilities are indicated in MeV. Results for the low-energy region (0-17 MeV) and the GR region (17-30 MeV) are separated. For each region the values of $m_0$ and $m_1$ are given in units $10^5$fm$^6$ and $10^6$fm$^6$·MeV, respectively. The last column shows (in MeV) the centroid energies $m_1/m_0$ of the GR region, and the peak energies in parenthesis.

| Force  | $K_\infty$ | Low-energy region | GR region |
|--------|------------|------------------|----------|
|        | $m_0$  | $m_1$  | $m_0$  | $m_1$  | $m_1/m_0$ |
| SkP    | 201     | 0.85   | 0.89  | 1.86  | 4.25  | 22.8 (23.6) |
| SGII   | 215     | 0.90   | 0.98  | 1.61  | 3.85  | 23.9 (24.1) |
| SkM*   | 217     | 0.94   | 1.01  | 1.67  | 3.96  | 23.7 (24.2) |
| SLy4   | 230     | 0.92   | 0.99  | 1.51  | 3.67  | 24.3 (25.2) |
| SkI2   | 241     | 1.02   | 1.05  | 1.58  | 3.88  | 24.6 (25.3) |

Table 2: Same as in Table 1, for the 4 nuclei calculated with SGII.

| Nucleus | Range of GR region | Low-energy region | GR region |
|---------|--------------------|------------------|----------|
|         | $m_0$  | $m_1$  | $m_0$  | $m_1$  | $m_1/m_0$ |
| $^{208}$Pb | 17-30   | 0.90   | 0.98  | 1.61  | 3.85  | 23.9 (24.1) |
| $^{144}$Sm | 21-32   | 0.34   | 0.46  | 0.59  | 1.57  | 26.6 (26.5) |
| $^{116}$Sn | 20-35   | 0.20   | 0.25  | 0.40  | 1.10  | 27.5 (28.4) |
| $^{90}$Zr  | 22-40   | 0.11   | 0.16  | 0.21  | 0.63  | 30.0 (26.7,32.5) |
