Nanofiber based displacement sensor

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We report on the realization of a displacement sensor based on an optical nanofiber. A single gold nano-sphere is deposited on top of a nanofiber and the system is placed within a standing wave which serves as a position ruler. Scattered light collected within the guided mode of the fiber gives a direct measure of the nanofiber displacement. We calibrated our device and found a sensitivity up to 1.2 nm/√Hz. A mechanical model based on the Mie scattering theory is then used to evaluate the optically induced force on the nanofiber by an external laser. With our sensing system, we demonstrate that an external force of 1 pN applied at the nanofiber waist can be detected.

I. INTRODUCTION

Position sensing with micro or nano-meter resolution is widely used both in scientific research and for industrial applications. [1, 2]. In order to probe weak forces the miniaturization of the devices towards the nanoscale is one of the main approaches [3, 4]. Nanowires [5], carbon nanotubes [6], and graphene [7] were successfully demonstrated as potential materials for position sensing. Several hybrid systems were also used including nanomechanical oscillators coupled to a whispering gallery mode resonator [8], superconducting microwave cavities cooled by radiation pressure damping [9], silicon nanowire mechanical oscillators for NMR force sensing [10] and nanomechanical oscillators combined with single quantum objects [11].

Pico-meter sensitivity has been achieved using a quantum point contact as a scanned charge-imaging sensor (sensitivity of 3 pm/√Hz) [12]) or using a nanorod placed within a Fabry-Prot microcavity (sensitivity of 0.2 pm/√Hz [13]).

Here, we propose to use an optical nanofiber as a nanometric displacement sensor which is all optical and can operate at room temperature and in free space. In the recent years, the optical properties of nanofibers have been widely used for atomic fluorescence investigation [14], atom trapping [15, 16], quantum optics and quantum information [17], chiral quantum optics [18], and optical resonators [19].

Interestingly, a nanofiber has also remarkable mechanical properties, including a small mass, on the order of 0.1 µg/m and a high sensitivity to vibrations and weak forces. In this work, we combine the optical and mechanical properties of a nanofiber to create a unique sensing device for displacement measurements and optomechanical applications. By depositing a nanoparticle on top of a nanofiber and placing this system in an optical standing wave positioned transversely as a ruler, we report an absolute sensitivity up to 1.2 nm/√Hz. Using the Mie scattering theory, we evaluate the force induced on the nanofiber by an external laser and demonstrate that a sensitivity of 1 pN can be achieved.

The article is organized as follows. We first introduce the experimental setup and describe the calibration procedure used to measure the sensitivity. We then present the experimental Allan deviation of our device, measured up to 10 minutes with a 100 µs integration time. In the second part, we propose a mechanical model to estimate the displacement of our sensor under an optically induced force, applied externally. We show that the full Mie scattering theory must be taken into account to precisely predict the force and we model the two orthogonal polarisations as a signature of the anisotropy of the system.

II. MEASURING THE ABSOLUTE DISPLACEMENT OF AN OPTICAL NANOFIBER

The general idea of our approach is to place an optical nanofiber within a transverse standing wave and to monitor the amount of scattered light in the guided mode of the fiber in order to infer its position. Optical nanofibers are optical waveguides with a sub-wavelength diameter. By carefully tapering a commercial single-mode fiber to sub-wavelength diameter, the adiabatic conversion of the HE_{11} mode can be achieved [20–22]. In the sub-wavelength region, the optical mode is guided in air and a strong evanescent field is present outside of the nanofiber. This has the crucial advantage that light can be coupled in and out with a scattering object deposited, in the evanescent field, on the surface of the nanofiber.

We fabricated a tapered optical nanofiber with a diameter of 300 nm and more than 95% transmission (at 532 nm), following the method detailed in [20] and used in [23]. To observe scattering, we overlapped a transverse
Standing wave with the nanofiber, by retro-reflecting a standard laser diode at 532 nm on a mirror (see Fig. 1a)). The mirror is mounted on a piezo-transducer, so that we can scan it in the longitudinal axis to move the standing wave envelop around the nanofiber position. In order to guarantee a high contrast of the standing-wave, we verified that the coherence length of the laser is much longer than the distance between the mirror and the nanofiber. In practice, this is not a stringent condition for standard narrow linewidth laser diode.

In this configuration the light scattered inside the guided mode of the nanofiber is only due to the rugosity of the nanofiber and is extremely low. To guarantee a high signal to noise ratio on the detection and there-

III. SENSITIVITY OF THE SYSTEM

The sensitivity of our system is linked to two main factors: at short integration time, the noise essentially comes from the fluctuations of the scattered light and the mechanical oscillations of the nanofiber and at long time scale we observed a drift caused by a temperature shift in the environment which displaces the standing wave.

A quantitative description of the sensitivity is given in Fig. 3 by presenting the Allan deviation. The Allan deviation is typically used to measure frequency stability in oscillators but it can also be implemented in the time domain. Here we used the modified output rate sample method [24]. \( \Omega(\tau_0) \) is the measured time history of photon counts with a sample period of \( \tau_0 \). The averaging time is set as \( \tau = m\tau_0 \), where \( m \) is the averaging factor.

![FIG. 1.](image1)

- a) Sketch of the experimental setup. A gold nano-
sphere on the waist of an optical nanofiber is placed in a stand-
ing wave (\( \lambda_{sw} \) is 532 nm). The diffusion signal is collected via the fiber. b) Ruler resolution. Maximum and minimum intensity are marked as \( I_{max} \) and \( I_{min} \), with 133 nm interval. c) MEB image of a single gold nano-sphere deposited on nanofiber. d) Optical microscope image of a gold-nano-sphere on the nanofiber within the standing wave.

![FIG. 2.](image2)

- Collected photon counts per second while scanning the standing wave. Integration time is \( \tau_{int} = 100 \mu s \) for grey lines and \( \tau_{int} = 100 \) ms for green dots. The solid green line is a sinusoidal fit to determine the visibility. \( I_{max} = 4.7 \) and \( I_{min} = 0.87 \). Inset: the maximum resolution region is fitted with a linear trend.
We divide the data sequence (photon arrival time on the photodiode) into clusters of time $\tau$. Then, we compute the Allan deviation $\sigma(\tau)$ using averages of the output rate samples over each time cluster.

$$\sigma(\tau) = \sqrt{\frac{1}{2\tau^2} < (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 >}$$ \hspace{1cm} (1)

where $\theta(t) = \int^t \Omega(t') dt'$ is the output angle.

On fig.3, three main regions can be identified: i) at high frequency (higher than 1 kHz) where the sensitivity increases with the integration time (lower $\sigma$), ii) at intermediate frequency (between 1 kHz and 1 Hz) where we observe a sinusoidal noise with characteristics bumps in the Allan deviation signature of the mechanical modes of the nanofiber, and iii) at low frequency where the sensitivity increases again. Finally, at longer integration times than 3 minutes, the sensitivity decreases again (higher $\sigma$) due to the thermal drift in the standing wave phase. We can extract two key figures of merit from these data. The resolution at 1s integration time is 5.1 nm/$\sqrt{\text{Hz}}$, and the maximum absolute resolution is 1.2 nm/$\sqrt{\text{Hz}}$.

### IV. Displacement of the Nanofiber Driven by Externally Applied Force

In the second part of this paper, we theoretically estimate the displacement of the nanofiber when it is driven by an externally applied force and we investigate optically-induced forces based on the Mie theory.

We model the nanofiber as an elastic string of circular section (radius $a$) with the two ends fixed. The location of the applied force on the nanofiber is an important parameter for the sensitivity to the force. Here, we assume that the force is applied at the center of the nanofiber, i.e. in the most sensitive region, however the model can be extended easily to other positions. First, we take the initial tension on the nanofiber to be zero. The expression of the displacement as function of the applied force is then given by:

$$\delta = \left[ \frac{F}{8\sigma a^2 E_Y} \right]^{1/3} L,$$ \hspace{1cm} (2)

where $E_Y$ is the Young modulus of silica, and $L$ is the length of the nanofiber region.[25, 26].

It is interesting to evaluate if a radiation pressure force applied by an external pushing laser could be observed using this system with the sensitivity measured earlier. A coarse evaluation can be made using a simple model by considering the nanofiber as a beam with squared section $(2a)^2$ and estimate the force by the differential Fresnel reflections on the sides of the silica beam. In this geometrical approach, the pushing laser is described by its diameter $d$, its power $P$ and its intensity $I = P/(\pi d^2/4)$. If $d \gg 2a$, the illuminated section of the nanofiber is $2a \times d$ and the force is

$$F_{og} = \frac{2I}{c} 2R 2ad = 8 \frac{P}{c\pi d^2} 2R 2a,$$ \hspace{1cm} (3)

with $c$ the speed of light in vacuum and $R$ the normal incidence reflection coefficient. With this model, we can estimate the force of a flat top beam of $P = 100$ mW and diameter $d = 10$ $\mu$m on a nanofiber of squared section $2a = 500$ nm made in silica (optical index of 1.5 which induces a Fresnel reflection coefficient $R = 0.04$):

$$F_{go} \sim 3 \text{ pN}.$$  

However, if this simple model is instructive to obtain an order of magnitude of the applied force, it must be refined to take into account interferences and polarization effects. In the following, we use the Mie theory of diffusion to give a more precise estimation of the optical force. We assume a cylindrical nanofiber (radius $a$ and index $n_1$) illuminated on normal incidence by a plane wave (with Gaussian envelope of width $d$). This configuration is represented in fig. 4.

Two linear polarisations can be used, either parallel or perpendicular to the nanofiber axis. The force given by the Mie theory is [27, 28]:

$$F_{Mie} = \frac{2I}{c} ad Y,$$ \hspace{1cm} (4)

where $Y$ is a dimensionless quantity which depends on the polarisation. For a linear polarisation parallel to the nanofiber $Y = Y_{||}(n_1,k,a)$ is given by

$$Y_{||}(n_1,k,a) = -\frac{1}{ka} \text{Re} \left[ \sum_{l=-\infty}^{+\infty} b_l (b_{l+1}^* + b_{l-1}^* + 2) \right].$$ \hspace{1cm} (5)

The geometric coefficient $b_l = b_l(n_1,k,a)$ is given by

$$b_l = \frac{n_1 J_l(n_1 ka) J_l(ka) - J_l(n_1 ka) J_l'(ka)}{n_1 J_l'(n_1 ka) H_l^{(1)}(ka) - J_l(n_1 ka) H_l^{(1)}(ka)}.$$ \hspace{1cm} (6)
with $H_{1}^{(1)}$ is the first order Hankel function and $J_{1}$ is the first order Bessel function. Similarly, for a linear polarisation perpendicular to the nanofiber, we have

$$Y_{\parallel}(n_1,k,a) = -\frac{1}{ka} \text{Re} \left[ \sum_{i=-\infty}^{+\infty} a_i (a_{i+1}^* + a_{i-1}^*) \right], \quad (7)$$

with

$$a_i = \frac{J_{1}(n_1ka)J_{1}(ka) - n_1J_{1}(n_1ka)J_{1}'(ka)}{J_{1}(n_1ka)H_{1}^{(1)}(ka) - n_1J_{1}(n_1ka)H_{1}^{(1)}(ka)}. \quad (8)$$

This model is compared numerically to the geometric model in Fig.4-a). It can be observed that the geometric model clearly under-estimate the force for both polarisations above 200 nm. Compared to the geometrical optics model, the Mie scattering model also predicts oscillations in the radiation force as a function of the increasing radius of the nanofiber, which corresponds to an interference phenomena. In addition, the radiation force shows dependence on the incident polarization which is characteristic of the anisotropy of the system [29].

In the following, we use the full model of Mie scattering to estimate the displacement. We inject the expression of the force in the displacement of the nanofiber given by Eq. 2 and we trace the displacement as function of the nanofiber radius in the absence of tension $T_0$. In this configuration, we predict a displacement on the order of 5 microns for both polarisations at fiber diameter around 400 nm. The minimal force that can be detected with our device is then on the order of 1 pN. Finally, to extend our model, we can include a non-zero initial tension on the fiber. In presence of an initial tension $T_0$, a correction term can be added to Eq. 2 to determine the force $F$ required to induce a displacement $\delta$ of the nanofiber:

$$F = 8\pi a^2 Y_E \left[ \frac{\delta}{L} \right]^3 + 4T_0 \frac{\delta}{L} \quad (9)$$

The first term is linked to the mechanical properties of silica as the second one is a consequence of the nanofiber tension. As $\delta/L \ll 1$ in our experiment, tension might play an important role in the ability of our system for detecting weak force applied on the fiber. Experimentally, we have anticipated this situation by adding a bending piezoelectric transducer to the nanofiber holding mount. This open the way to future investigations of the tension as well as the potential role of damping due to air around the nanofiber.

V. CONCLUSION

In conclusion, we have presented a position sensor based on a gold nano-sphere deposited on a nanofiber and placed within a standing wave. Our sensor can measure the displacement of a nanofiber driven by externally applied force and vibrations. We experimentally calibrated our sensing system and obtain a resolution of 1.2 nm/√Hz. We proposed a mechanical model to estimate the response of our sensor to optically-induced pressure force and we found that the sensitivity corresponding to the sensibility of an externally applied force at the nanofiber waist is 1 pN. This optical nanofiber sensor paves the way to realize integrated optomechanical research using this platform, such as the demonstration of superfluidity of light in a photon fluid [30, 31].

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