HALO COLD DARK MATTER
AND MICROLENSING

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ABSTRACT

There is good evidence that most of the baryons in the Universe are dark and some evidence that most of the matter in the Universe is nonbaryonic with cold dark matter (cdm) being a promising possibility. We discuss expectations for the abundance of baryons and cdm in the halo of our galaxy and locally. We show that in plausible cdm models the local density of cdm is at least $10^{-25}$ g cm$^{-3}$. We also discuss what one can learn about the the local cdm density from microlensing of stars in the LMC by dark stars in the halo and, based upon a suite of reasonable two-component halo models, conclude that microlensing is not a sensitive probe of the local cdm density.
Introduction. While the quantity and composition of matter in the Universe is still not known with certainty, it is known that: (i) luminous matter (stars, etc.) contributes much less than 1% of critical density; (ii) based upon primordial nucleosynthesis baryons contribute between about 1% and 10% of critical density; and (iii) based upon numerous dynamical measurements the total mass density is at least 10% of critical density \[1\], with several determinations indicating that it is close to the critical density \[2\]. Thus, there is overwhelming evidence that most of the matter in the Universe is “dark,” compelling evidence that most of the baryons are dark, and mounting evidence that most of the matter is nonbaryonic \[3\]. If the mean mass density is greater than about 10% of critical there are two dark matter problems, the nature of the baryonic and nonbaryonic dark matter.

The case for a critical Universe with nonbaryonic dark matter receives further support from studies of structure formation: The most successful models rely upon nonbaryonic dark matter, and the cold-dark matter models of structure formation (inflation-produced density perturbations and non-baryonic dark matter with negligible velocity dispersion) are very attractive \[4\]. The best motivated cold dark matter candidates are an axion of mass around \(10^{-5}\) eV and a neutralino of mass between 10 GeV and 1 TeV \[5\], and large-scale experiments are underway to directly detect the axions or neutralinos in the halo \[6\]. Needless to say, theoretical expectations for, and observational information about, the local mass density of cold dark matter are of great importance.

The flat rotation curves of spiral galaxies indicate that the luminous, disk shaped portion of a typical spiral sits in a dark halo that is roughly spherical with density that decreases as \(1/(r^2 + a^2)\) and extent that is undetermined (\(r\) is distance from the center of the galaxy and \(a\) is the core radius). The fact that galactic halos are more spherical and extended than the luminous parts of spirals strongly suggests that the dark halo material has probably not undergone significant dissipation. For the Milky Way, galactic modeling indicates that the core radius is between 2 kpc and 8 kpc and that the halo density nearby \((r \equiv r_0 \approx 8.5 \text{ kpc})\) is about \(5 \times 10^{-25} \text{ g cm}^{-3}\) (to within a factor of two) \[7\]. At our position, the halo material supports around 130 km s\(^{-1}\) of the 220 km s\(^{-1}\) circular rotation velocity (the various contributions—halo, disk, etc.—to the rotation velocity add in quadrature.)

In a cold dark matter Universe there are at least two forms of dark matter, baryons and cdm particles, and both are expected to contribute to the halo
mass density. The concerns of this Letter are twofold: first, the theoretical expectations for the local cdm density, and second, what one can learn about the local cdm density from microlensing experiments which can probe the baryonic component that exists in dark stars of mass $10^{-6}M_{\odot}$ to $100M_{\odot}$ \[8\]. Since the EROS and MACHO microlensing searches now have candidate microlensing events \[9\], this is a very timely issue.

**Expectations.** Consider a cold dark matter universe with $\Omega_{\text{cdm}} + \Omega_B = 1$. Based upon primordial nucleosynthesis $0.01h^{-2} \leq \Omega_B \leq 0.02h^{-2}$; further, in a critical, matter-dominated Universe the Hubble constant must be near its lower extreme, $h \sim 0.5$, in order to accommodate a sufficiently aged Universe ($H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ is the present Hubble constant and $\Omega_i$ is the fraction of critical density contributed by species $i$). This means that the universal baryon fraction of the matter density is $f_B = \Omega_B = 0.04 - 0.1$ and $f_{\text{cdm}} = 1 - f_B \approx 0.9 - 0.96$.

(In the two popular variants of cold dark matter, hot + cold dark matter, where $\Omega_{\nu} \sim 0.3$, and cold dark matter + cosmological constant, where $\Omega_{\Lambda} \sim 0.8$, $\Omega_{\text{cdm}} \sim 0.2$ and $h \sim 0.7$, the situation is only quantitatively different. In the former case, phase-space considerations limit the neutrino fraction in galaxies like our own to be less than about 5% \[10\], and so the baryonic fraction of matter that can clump in galaxies, $f_B \approx \Omega_B/(1 - \Omega_{\nu}) \approx 0.06 - 0.14$, is slightly higher. In the latter case, the baryonic fraction of matter, $f_B = \Omega_B/(1 - \Omega_{\Lambda}) \approx 0.1 - 0.2$, is even larger. All that follows is applicable to these models by taking account of the larger value of $f_B$.)

So long as gravity alone shapes the evolution of the Universe the baryonic fraction of matter remains at its universal value. Once dissipative forces (e.g., electromagnetic interactions) become important, baryons can lose energy and become more condensed, increasing the local baryonic fraction. The formation of the disks of spiral galaxies provides a good example: Through collisional processes baryons lose their energy, but not angular momentum, ultimately forming a thin, rotationally supported disk. The local mass density of the disk is about a factor of 20 greater than that of the halo.

In turn we now consider three models for the formation of the halo of our galaxy, from a very simple scenario where only gravity is involved to an extreme scenario where hydrodynamical forces form the galaxy and the cold dark matter particles are captured subsequently by accretion.

(1) The simplest and most plausible scenario is one where the halo of
our galaxy formed through the action of gravity alone, after which a small fraction of the baryons dissipated energy forming the disk. One then expects an isothermal halo, with density decreasing as $r^{-2}$, whose extent is dictated by the “sphere of influence” of our galaxy, at most about half the distance to M31 ($\approx 400$ kpc) \(^\square\). The baryonic fraction of the halo mass density should be about equal to the universal value, $f_B \approx 0.04 - 0.1$, or smaller, if a larger fraction of the baryons dissipated their energy and reside in the disk.

(2) Suppose further that a substantial fraction of halo baryons undergo moderate dissipation (though not enough to collapse to form a disk), so that the baryonic halo remains roughly spherical but shrinks in size. Specifically, assume that both cdm and baryons exist in isothermal halos truncated at different radii, $R_B$ and $R_{cdm} = \beta R_B$. If most of the baryons are in the halo, the ratio of the halo mass in baryons to that in cdm is $f_B/(1 - f_B) \approx f_B$, and interior to $R_B$ the ratio of baryons to cdm is $\beta f_B/(1 - f_B) \approx \beta f_B$. That is, the local baryon to cdm ratio is increased by the ratio of their truncation radii. To be concrete, if $R_{cdm} \sim 200$ kpc and $R_B \sim 30$ kpc, then interior to 30 kpc the ratio of baryons to cdm is about $7 f_B \sim 0.3 - 0.7$.

(3) Consider a very radical scenario, one where our galaxy formed by nongravitational forces (e.g., hydrodynamical shock waves), so that the baryonic part of the galaxy was assembled first and the cdm halo accreted subsequently. By using the spherical accretion model \(^\square\) one can estimate the mass of cdm halo that is added. Suppose at time $t_i$ a point mass $M_0$ is placed at the origin of an otherwise smooth critical Universe comprised of cold dark matter. Thereafter, all cdm particles in the Universe are gravitationally bound to $M_0$ and ultimately cease moving away (clearly the model is only applicable within the sphere of influence of $M_0$, say half way to M31.)

According to this model, cdm is accreted in a self-similar way, with a density profile $\rho \propto r^{-9/4}$ for $r \ll r_*$; at time $t$ the “turn-around” radius $r_*(t) = 0.771(GM_0)^{1/3}t^{8/9}/t_i^{2/9}$. The mass accreted by time $t$ and interior to radius $r \ll r_*$ is: $M(r,t) = 1.39(M_0r)^{3/4}/G^{1/4}t_i^{1/2}$. Taking $M_0 \sim 10^{11} M_\odot \sim$ (baryonic mass of our galaxy), the turn-around radius today is around 1 Mpc, and the mass accreted within 100 kpc is about $7 \times 10^{10} M_\odot (1 + z_i)^{3/4} h^{1/2} \sim 10^{11} M_\odot$ for $z_i \sim 1 - 2$ and $h \sim 0.5$ ($z_i$ is the red shift corresponding to time $t_i$). The density of cdm particles 8.5 kpc from the galactic center is $0.7 \times 10^{-25}$ g cm$^{-3}(1 + z_i)^{3/4} h^{1/2} \sim 10^{-25}$ g cm$^{-3}$ for $z_i \sim 1 - 2$ and $h \sim 0.5$.

Even in this radical model, the amount of cold dark matter accreted within
100 kpc is about equal to the baryonic mass, and the local density of cold dark matter is only slightly lower than the estimates in scenarios (1) and (2).

(One can consider a more extreme version of this scenario: suppose that the Milky Way resides in an underdense region of the Universe, which, in the absence of the point mass \(M_0\), behaves like a small portion of an \(\Omega_0 < 1\) Universe. In this case, the total cdm mass accreted is \((1 + z_i)\Omega_0 M_0/(1 - \Omega_0)\), again comparable to \(M_0\), and so the previous comments apply.)

**Microlensing.** Now we turn to what one can hope to learn experimentally from the study of microlensing of stars in the LMC by dark stars in the halo of our galaxy. Such experiments can only "detect" halo baryons if they exist in the form of \(10^{-6} M_\odot\) to \(100 M_\odot\) dark stars. Little is known about the kind of objects halo baryons would form (in part, because of our poor understanding of star formation in general). The strongest statements that can be made concern in what form halo baryons cannot exist and lead to the suggestion that halo baryons are likely to be stars of mass \(10^{-3} M_\odot\) to \(0.1 M_\odot\), a range that can be probed by microlensing.

In modeling a two-component halo we assume that: (i) baryons and cdm exist in separate, spherically-symmetric isothermal halos, with core radii \(a_B\) and \(a_{\text{cdm}}\) and density profiles,

\[
\rho_i(r) = \rho_{\text{local},i} \left( \frac{a_i^2 + r_0^2}{a_i^2 + r^2} \right),
\]

where \(\rho_{\text{local},i}\) is the local density; (ii) the core radii are between 2 kpc and 8 kpc; (iii) halo baryons are dark stars of mass \(M\). Both the baryonic and cdm halos play a role in determining the galactic rotation curve, but of course only the baryonic halo determines the microlensing rate.

Next we compute the microlensing rate for stars in the Large Magellanic Cloud (LMC) as a function of the assumed local density of cdm for a suite of reasonable halo models (see below). That rate depends upon the distribution of baryonic matter in the halo and is given by

\[
\Gamma(\rho_{\text{local, cdm}}, a_i) = \omega_0 u_T \int_0^{x_{\text{max}}} \frac{dx \sqrt{x(1-x)}}{A' + Bx + x^2},
\]

where \(\omega_0 = \sqrt{8\pi G L^3 \rho_{\text{local,B}} v_H A'}/3 M c^2\), \(A' = (a_B^2 + r_0^2)/L^2\), and \(B = -2(r_0/L) \cos b \cos l\), \(v_H\) is the halo-velocity dispersion, and \(L \simeq 50\) kpc, \(b = -33^\circ\), and \(l = 281^\circ\).
are respectively the distance, and galactic latitude and longitude of the LMC. The quantity $u_T$, the threshold impact parameter in units of the Einstein radius, is set by the minimum amplification that can be detected; e.g., $u_T = 1$ corresponds to a amplification threshold of 1.34 which is typical of current searches [4]. The quantity $x_{\text{max}}$ is the lesser of 1 and the distance to the edge of the baryonic halo along the line of sight to the LMC in units of $L$.

We compare all microlensing rates to a fiducial model, a baryons-only, “best fit” halo model with $a_B = 5$ kpc, $v_H = 270$ km s$^{-1}$, normalized to have rotation velocity of 220 km s$^{-1}$ at our position. The microlensing rate for this model is $\Gamma_0 = 1.66 \times 10^{-6} u_T / \sqrt{M/M_\odot}$ events yr$^{-1}$. For further discussion of microlensing we refer the reader to Ref. [14].

Our suite of models was constructed as follows: For each assumed value of the local cdm density, we allow the core radii to vary separately between 2 kpc and 8 kpc; the value of $\rho_{\text{local},B}$ is determined by constraining the rotation velocity at our position to be $v_c(r_0) = 220 \pm 10$ km s$^{-1}$,

$$
\rho_{\text{local},B} = \frac{(a_B^2 + r_0^2)^{-1}}{1 - (a_B/r_0) \tan^{-1}(r_0/a_B)} \times \left[ \frac{v^2}{4\pi G} - \rho_{\text{local,cdm}}(a_{\text{cdm}}^2 + r_0^2)[1 - (a_{\text{cdm}}/r_0) \tan^{-1}(r_0/a_{\text{cdm}})] \right]
$$

where $v(r_0) \simeq 130 \pm 17$ km s$^{-1}$ is the portion of the local rotational velocity that is supported by the halo. To ensure that a given model is “reasonable” we construct the rotation curve; in so doing we also take into account the contributions of the disk, bulge, and central components of the galaxy by using results from Ref. [7]. For our “criterion of reasonableness” we follow Ref. [15]: the relative difference of the maximum and minimum rotation velocities over the interval 3 kpc to 18 kpc must be less than 14% (most of the models pass this test; see Fig. 2). In the limit of a single halo component, the local halo density in our models is $3 - 7 \times 10^{-25}$ g cm$^{-3}$, consistent with previous estimates [7].

The range of microlensing rates for our suite of reasonable models is shown in Fig. 1 as a function of the local cdm density. The microlensing rate is relatively insensitive to the local cdm density—the range of cdm densities consistent with a given $\Gamma$ spans about a factor of 3—and is relatively sensitive to the halo model parameters—for fixed cdm density $\Gamma$ varies by around $\pm 50\%$. This is not surprising; first, most of the microlensing is due to objects
between 10 kpc and 30 kpc from the galactic center, so only this part of the baryonic halo is probed. Second, the galactic halo is not well constrained by rotation-curve data, so the “phase space” of reasonable models is large [10].

Clearly, microlensing can only provide limited information about the local cdm density. For example, the microlensing rate can be as large as its value in the best fit, baryons-only (fiducial) model in a two-component model where the local cdm density is $3 \times 10^{-25}$ g cm$^{-3}$. Or, suppose that the microlensing rate were determined to be half the fiducial value; for our suite of halo models the local cdm density is $1 - 5 \times 10^{-25}$ g cm$^{-3}$. On the other hand, if the observed microlensing rate is found to be small, say 10% or less of the fiducial value, based on our set of models one could argue that the local cdm mass density is at least $2 \times 10^{-25}$ g cm$^{-3}$.

To be specific about the dependence of the microlensing rate on the halo model, for a given cdm density the higher rates occur in models with larger baryonic core radii and larger values of the local rotation speed. In models with a truncated baryonic halo (not shown) $\Gamma$ is insensitive to the truncation radius provided that it is greater than about 30 kpc; this is because most of the microlensing is due to halo objects between 10 kpc and 30 kpc from the galactic center. There is a small dependence upon the distance to the LMC which is not shown; changing the LMC distance by $\pm 10\%$ changes $\Gamma$ by about $\pm 5\%$. We should emphasize that our two-component models are very simple; one could easily imagine more complicated models, e.g., where the halos are not spherically symmetric. This increases further the range of plausible microlensing rate for a given cdm density [17].

Finally, a fine point, in computing the microlensing rate, we have followed Ref. [14] in assuming that the distribution of halo velocities is Maxwellian, which leads to the factor of $v_H$ in $\omega_0$, cf. Eq. (2). This is only strictly true for a galaxy model consisting solely of an untruncated, zero-core radius halo. In that regard, our models (like most) are not self-consistent. To explore the sensitivity of our results to this inconsistency, we replaced $v_H$ by $\sqrt{3/2}v_{\text{circ}}(20 \text{kpc})$; that is, we used the circular velocity at the radius where most of the microlensing occurs to characterize the halo velocity dispersion. This increased the sensitivity of the microlensing rate to halo model, though very slightly (by a few per cent). Further, we also computed the optical depth for microlensing, cf. Eq. (4) in Ref. [14], which does not depend upon the distribution of halo velocities, and it exhibits a similar variation for fixed
cdm density as Γ. Both of these facts suggest that our estimates of the model uncertainties in the microlensing rates—which is our main concern—have not been affected significantly by lack of self consistency. Absolute rates will of course depend more strongly on consistency [17].

Summary. If the density of the Universe is significantly greater than about 10% of critical density, as a number of observations indicate and several arguments suggest [2, 3], then there are two dark matter problems, baryonic and nonbaryonic. If the nonbaryonic dark matter is cold dark matter, which seems to be the most promising possibility, then the halos of spiral galaxies should contain both baryons and cdm particles. Provided the formation of the halo involved only gravity, the local baryonic fraction of the halo should be small, less than about 10%. If baryons in the halo underwent some dissipation, so that the baryonic halo contracted relative to the cdm halo, the local baryonic fraction is increased by the contraction factor and could be significantly higher. In the extreme, if the baryonic portion of the galaxy formed first through nongravitational forces and a cdm halo is accreted thereafter, the mass of cdm in the halo is comparable to that in baryons and the local cdm density is still $10^{-25}$ g cm$^{-3}$ or so.

Because of uncertainties inherent in modeling our halo and the fact that microlensing only probes the part of the halo between 10 kpc and 30 kpc from the center of the galaxy, it is difficult at present to learn much about the cdm content of our own halo from microlensing. For example, if the microlensing rate were found to be equal to that expected in the best fit, baryons-only halo model, the local cdm density could be as large as $3 \times 10^{-25}$ g cm$^{-3}$ when the uncertainties in halo models are taken into account. On the other hand, if the microlensing rate were found to be small, say 20% or less of the baryons-only model, based on our models one could argue that this is evidence for a local cold dark matter density larger than about $2 \times 10^{-25}$ g cm$^{-3}$. While the MACHO and EROS collaborations have yet to discuss the microlensing rate that can be inferred from their candidate events [9], a naive analysis of their data suggests that the rate could be as low as 20% of the baryons only model [18].

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1 Figure Captions

Figure 1: The microlensing rate $\Gamma$ for our suite of reasonable, two-component halo models as a function of the local cdm density, normalized to a best fit, one-component (baryons) halo model with core radius of 5 kpc. The heavy curves show the extreme range of the models; from top to bottom the lines correspond to models with $a_B = a_{\text{cdm}} = 6.5, 5, 3$ kpc.

Figure 2: A sample of galactic rotation curves for our models. The heavy curves correspond to extreme models (top: $a_{\text{cdm}} = 2$ kpc, $a_B = 8$ kpc, and $\rho_{\text{local,cdm}} = 0.8 \times 10^{-25}$ g cm$^{-3}$; bottom: $a_{\text{cdm}} = a_B = 2$ kpc). The two middle curves correspond to $a_{\text{cdm}} = a_B = 6.5$ kpc (upper), 5 kpc (lower).
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