Estimation of mean energy characteristics of atmospheric turbulence at various heights from reanalysis data

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Abstract. This paper summarizes the results of investigations of energy spectra of atmospheric turbulence over a wide range of scales. A spectral method of estimating mean energy characteristics of atmospheric turbulence at various heights from reanalysis data is proposed. Height profiles of optical turbulence are obtained using the spectral method. The atmospheric optical turbulence characteristics calculated with this method are in good agreement with the data of measurements of the turbulent distortions of the wavefront.

1. Introduction

Considerable attention is paid to the energy spectra of fluctuations of the air flow velocity and temperature in the study of the atmospheric processes including estimating the realism of mesoscale meteorological models [1, 2]. Although numerical simulation reflects the dynamics of processes and phenomena in the atmospheric boundary layer, the resolution of the models often does not correspond to the parameterization of small-scale turbulence [3]. Understanding the features of turbulent air mixing under a wide range of spatial scales plays a key role in the modeling of atmospheric processes. One of the approaches to improving the models is to enquire height deformations of the energy spectra of atmospheric turbulence in a wide range of spatial scales under different external atmospheric conditions. Theoretical and experimental studies of the energy spectra of atmospheric flows have been carried out for more than 70 years [4,5,6,7,8]. Numerous results of studies of the energy spectra of multiscale atmospheric turbulence indicate the existence of two large-scale spatial ranges. The power spectral density (PSD) of wind speed fluctuations $E_V(k)$ depends on the wavenumber $k$ to the power of $-3$ in the spatial range from $2 \cdot 10^6$ rad / m to $8 \cdot 10^6$ rad / m. The index of the power decreases (on module) to $-5/3$ in the mesoscale range whose wavenumbers vary from $10^5$ rad / m to $10^3$ rad / m. The transition interval from the dependence $E_V(k) \sim k^{-3}$ to $E_V(k) \sim k^{-5/3}$ is near the Rossby deformation radius and covers a range of spatial scales from 800 km to 600 km. These power laws are well satisfied for the free atmosphere [5,6]. Deviations from this power law in individual spectral ranges may be caused by the action of mountain waves, convective instability, wind shear, adaptation of unstationary air movements, jet streams, frontal systems, or other factors. Observations performed under the Global Atmospheric Sampling Program as well as later studies have shown statistically the effect of mountain waves over a uniform and rough underlying surface on the shape of the turbulence spectrum. Fluctuations of the horizontal wind speed and air temperature were approximately 2 - 3 times higher over a rough underlying surface compared to fluctuations over plains and the water surface. The greatest deviations of the intensity of the fluctuations were observed in the spatial range between 10 and 80 km. Investigations of the energy spectra of atmospheric turbulence in the surface layer show that the large scale range with a slope "-3" significantly narrows with decreasing altitude [9]. A slope close to "-3" of the energy spectrum at a height of 301 m is observed in the range from 4.7...
\[ 10^{-6} \text{ Hz} \text{ (} \sim 60 \text{ h) to} 10^{-5} \text{ Hz} \text{ (} \sim 28 \text{ h). The slope } -3 \text{ is not observed below } 8 \text{ m, and the spectrum can be approximated by the dependence } E_v(f) \sim f^{-5/3} \text{ in the frequency range from } 2.3 \cdot 10^{-6} \text{ Hz to } 3.1 \cdot 10^{-5} \text{ Hz (or even higher)}. \]

According to the physical concepts, atmospheric large-scale air flows are supported by the generation of potential energy due to the spatial air temperature gradients. The potential energy generated is converted into kinetic energy of the air flow at zonal wavenumbers from 2 to 10 [1]. This mechanism gives hope that ensemble averaged energy spectra of fluctuations of wind speed and air temperature in the large-scale range have similar shape.

2. Method of estimating the energy characteristics of atmospheric turbulence at different heights from reanalysis data

There is a question of stability of the shape of the turbulence spectrum over a wide range of spatial and temporal scales for different total energies of atmospheric flows. In particular, atmospheric conditions in which the shape of the energy turbulence spectrum holds or slightly varies over a wide range of spatial and temporal scales are of interest. It is also important to know how far (long) atmospheric large-scale disturbances can affect the spectral components of turbulence of higher orders. Statistically, any atmospheric disturbance and its decay products, for which a certain spatial scale or spectral range can be determined, have a finite scale of correlation (consistency). From this point of view the energy spectrum should tend to some stable (mean) shape, especially for a statistical ensemble of stochastic states.

The steadiness of the shape of the energy spectrum of turbulence over a wide range of spatial and temporal scales forms a basis of the proposed method for estimating the energy characteristics of atmospheric turbulence from reanalysis data. Knowing the shape of the mean energy spectrum of turbulence over a wide range of scales and the characteristics of fluctuations in the large-scale region of the spectrum, one can estimate the mean characteristics of small-scale atmospheric turbulence based on the radiosonde data (for example, the NCEP / NCAR Reanalysis archive [10]).

Figure 1. Energy spectra of wind speed fluctuations corresponding to the atmospheric boundary layer over a wide frequency range.

To determine the energetic characteristics of small-scale turbulence, we should consider deformations of the shape of the temporal spectra of atmospheric turbulent flows. The energy spectra of wind speed fluctuations corresponding to the atmospheric boundary layer over a wide frequency
range are shown in Figure 1. The energy spectra of air temperature fluctuations over a wide frequency range are shown in Figure 2. The frequencies are plotted along the abscissa and the values of the power spectral density of the fluctuations are plotted along the ordinate. These spectra are plotted in bilogarithmic coordinate systems.

The shape of the energy spectra of atmospheric turbulence (both wind speed fluctuations and air temperature fluctuations) averaged over a statistical ensemble of states in the atmospheric boundary layer varies slightly. The variations of the specific energy of high-frequency (small-scale) turbulence are consistent both with variations of the total energy of the turbulent flow spectrum and the energy of large-scale atmospheric inhomogeneities.

Analysis of the deformations of the statistically averaged spectra of wind speed fluctuations and air temperature fluctuations allows us to assume that large-scale inhomogeneous flows are energetically structured by small-scale turbulence.

The steadiness of the energy spectra of turbulence is also confirmed by the height dependence of the disturbance of atmospheric flows:

\[ E_p(z) = \sigma_v(z) / \langle V \rangle(z), \]  

(1)

where \( \langle V \rangle(z) \) is the mean wind speed at height \( z \), \( \sigma_v(z) \) is the root mean square deviation of the wind speed at height \( z \).

The dependences of \( E_p(z) = \sigma_v(z) / \langle V \rangle(z) \) calculated using data obtained on a 301-m high-altitude meteorological mast (Obninsk) under clear sky for 2016 differentiated by ranges of the mean wind speed changes are shown in Figure 3. The values were calculated over a three-minute interval from 0.2 s wind speed measured. A gradual decrease in the values with height can be an evidence of the steadiness of the energy spectra of turbulence.

The method developed by us takes into account the shape of the energy spectrum of turbulence in a wide range of spatial and temporal scales: from macro-scale flows (with typical sizes from 600 km to 3000 km and lifetimes from several hours to 5-7 days) to small-scale turbulence. Time intervals of 5-7 days were defined by Multanovsky as “a natural synoptic period during which such a thermobaric field in the troposphere is preserved which causes a certain orientation of the displacement of baric
formations at the Earth's surface and preserves the general picture of the location of their centers in the space of the so-called natural synoptic region.

Figure 3. Dependences of $E_p(z)$ calculated using data of a high-altitude meteorological mast (Obninsk) under clear sky for 2016 differentiated by ranges of mean wind speed changes.

Experimental data on the structure of air flows in the free atmosphere show that the spatial energy spectrum of air temperature fluctuations can be approximated by two dependences: $E(f) \sim f^{-3}$ in the lowest frequency spatial range and $E(f) \sim f^{-5/3}$ in the mesoscale and high frequency ranges. Taking into account the shape of the energy spectrum of turbulence in a wide space-time range, the PSD of turbulent fluctuations in the high-frequency range for the free atmosphere can be parameterized as:

$$E_T(f_t) = E(f_L) \exp\left(-3\left(\ln \frac{f_L}{f_t}\right) - \frac{5}{3}\ln \frac{f_L}{f_t}\right),$$

where $E(f_L) = \sigma_T^2(f_L)/f_L$ is the amplitude of the spectrum in the low-frequency range, $f_L$ is the spatial frequency in the low-frequency range, $f_t$ is the spatial frequency in the high-frequency range, $f_T$ is the spatial frequency of the transition region from the slope "-3" to the slope "-5/3," and $\sigma_T^2(f_L)$ is the characteristic air temperature dispersion of the low-frequency range.

In studies of the energy spectra of the atmospheric turbulence one often has to deal with time series data. From this point of view, we use both spatial and temporal frequencies. The calculated temporal $\sigma_T^2(f_L)$ are equivalent to the space values:

$$\sigma_T^2(\tau) = \int_{f_0}^{f_T} E(f_0) df_0 = \int_{f_m}^{f_L} E(f) df,$$
where \( f_0 \) are the temporal frequencies \( f_{r_0} = \frac{1}{24 \text{hours}} \), and \( f_m \) is the spatial frequency corresponding to 24 hours.

Thus, knowing the dependence of the PSD of the air temperature fluctuations on the frequency in a wide range of spatial frequencies, we can estimate the statistically averaged energy characteristic of the small-scale turbulent flows using the archive radiosonde data of reanalysis.

It is known that real atmospheric flows, especially near the underlying surface, are not homogeneous and isotropic; however, atmospheric turbulence in the small-scale range is well described by the Kolmogorov-Obukhov theory (power law "\(-5/3\)"). According to this theory, the PSD of turbulent air temperature fluctuations depends on the frequency to the power "\(-5/3\)" in the range \( 1/L_0 \gg f_i \gg 1/l \):

\[
E_i(f_i) = 0.125C_f^2f_i^{-5/3},
\quad (4)
\]

where \( l \) is the internal scale of turbulence, \( L_0 \) is the outer scale of turbulence, and \( C_f^2 \) is the structural characteristic of the air temperature fluctuations.

Substituting the expression (2) in (4), we can obtain:

\[
C_T^2 = \frac{E_i(f_L)\exp\left(-\frac{3}{5}\left(\ln \frac{f_i}{f_L}\right) - \frac{5}{3}\left(\ln \frac{f_i}{f_L}\right) \cdot \frac{5}{f_i^3}\right)}{0.125}.
\quad (5)
\]

The last relation makes it possible to estimate the mean value of the structural characteristic of the air temperature fluctuations at different heights in the free atmosphere from reanalysis data.

3. Verification of the method using an example of calculating the characteristics of optical turbulence

To measure the characteristics of atmospheric turbulence at different heights, remote optical methods in addition to contact methods are often used [11, 12]. The main parameters of atmospheric turbulence estimated from optical methods are the structural characteristic of the air refraction index fluctuations \( C_n^2 \) and the Fried radius \( r_0 \). It can be assumed that the spatial distribution of \( C_n^2 \) is equivalent to the distribution of the intensity of optical turbulence, and of the Fried radius, to its integral (in the layer) scale.

In order to verify the method of estimating the mean energy characteristics of turbulence, a comparison between the calculated Fried radius and \( r_0 \) estimated from the measurements of the wavefront parameters of radiation propagating in the atmospheric layer from 20 to 0 km is performed. From the point of view of atmospheric physics, the Fried radius is determined by the profile of optical turbulence:

\[
r_0 = \left[ 0.423(2\pi\lambda^{-1})^2 \sec \alpha \int_0^H C_n^2(z)dz \right]^{-3/5},
\quad (6)
\]

where \( z \) is the height, \( \alpha \) is the zenith angle, and

\[
C_n^2 = \left( \frac{AP}{< T >} \right) C_T^2,
\quad (7)
\]

\( P \) is the atmospheric pressure, and \( < T > \) is the mean air temperature.

The data of the NCEP / NCAR reanalysis archive (www.esrl.noaa.gov) allowed us to estimate the air temperature dispersion and calculate \( C_n^2 \) at different heights using (7), (5). The height profiles of \( C_n^2 \) at the Large Solar Vacuum Telescope site for summer and winter are shown in Figure 4.
values of $C_n^2(z)$ obtained at different heights as well as the height changes are consistent with both the physics and the data of measurements (4).

In addition, long-term direct measurements of the integral characteristics of optical turbulence were also performed to compare these statistics. The physical foundations of the remote method of measuring the wavefront are described in [13]. The calculation of the Fried radius was carried out on the basis of measurements of local wavefront slopes:

$$r_0 = 0.528 \left( \frac{\sigma_{\alpha_1-\alpha_2}}{D} \right)^{-3/5} \lambda^{6/5} D^{-1/5} \left( 1 - 0.562 \frac{d}{D} \right)^{1/3},$$

where $\sigma_{\alpha_1-\alpha_2}$ is dispersion of the differences of the local wavefront slopes, $D$ is the size of the subaperture, $d$ is the distance between the centers of the subapertures, and $\lambda$ is the wavelength of light. The results of the measurements show that the values of the mean Fried radius estimated from (6) and (8) differ by not more than 1 cm.

4. Summary
In this paper, a method of estimating mean energy characteristics of atmospheric turbulence at various heights based on spectral properties of turbulence was proposed. It has been shown that the shape of the statistically averaged energy spectrum of atmospheric turbulence over a wide range of frequencies is stable. The observed energy spectra show a «3» slope for the low frequencies and a «5/3» slope for the meso- and high frequencies. This means that the large-scale inhomogeneous flows are energetically structured by small-scale turbulence. Deviations from the observed shape can be clarified for the selected site. The steadiness of the energy spectra of turbulence in the atmosphere is also confirmed by the small scattering of the height profiles of disturbance of atmospheric flows. Using the spectral method we have recovered height profiles of $C_n^2(z)$ for the Large Solar Vacuum Telescope site, which are consistent with the data of optical measurements. Also, it is possible to estimate the mean energy characteristics of atmospheric turbulence at various heights for other sites.

Acknowledgments
This study of the height structure of turbulence, as well as optical turbulence, was supported by the RFBR under project no. 18-35-00033. The study of deformations of statistically averaged energy
spectra of atmospheric turbulence has been carried under FR program II.16 using the Unique Research Facility Large Solar Vacuum Telescope, http://ckp-rf.ru/usu/200615/. Particularly the study is supported by the program of the Presidium of RAS №56 "The fundamental principles of breakthrough technologies for the interests of national security"

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