Improved axion emissivity from a supernova via nucleon-nucleon bremsstrahlung

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Abstract. The most efficient axion production mechanism in a supernova (SN) core is the nucleon-nucleon bremsstrahlung. This process has been often modeled at the level of the vacuum one-pion exchange (OPE) approximation. Starting from this recipe, we revise the calculation including systematically different effects, namely a non-vanishing mass for the exchanged pion, the contribution from the two-pions exchange, effective in-medium nucleon masses and multiple nucleon scatterings. Moreover, we allow for an arbitrary degree of nucleon degeneracy. A self consistent treatment of the axion emission rate including all these effects is currently missing. The aim of this work is to provide such an analysis. We find that the axion emissivity is reduced by over an order of magnitude with respect to the basic OPE calculation, after all these effects are accounted for. The implications for the axion mass bound and the impact for the next generation experimental axion searches will be discussed.
1 Introduction

One of the most puzzling and long-standing problems in particle physics is related to the absence of an expected CP violation in the strong interactions: *the strong CP problem*. The most elegant solution to this problem is based on the Peccei-Quinn (PQ) mechanism [1–4], in which the Standard Model is enlarged with an additional global U(1) \( A \) symmetry, known as the PQ symmetry. The axion is the Nambu-Goldstone boson of the PQ symmetry, a low-mass pseudoscalar particle with properties similar to those of neutral pions. The axion mass is given by [5, 6]

\[
m_a = \frac{5.7 \text{ eV}}{f_a/10^8 \text{ GeV}},
\]

where \( f_a \) is the axion decay constant or PQ scale. The axion interactions with photons, electrons, and hadrons are also controlled by the PQ constant and scale as \( f_a^{-1} \). Therefore, the PQ scale controls the axion phenomenology and is constrained by different experiments and astrophysical arguments that involve interactions with photons, electrons, and hadrons (see [7–9] for recent reviews).

Among the astrophysical systems, core-collapse supernovae constitute a valuable laboratory to constrain the axion properties [10]. The most relevant process of axion emission in a SN core is the nucleon–nucleon (N–N) axion bremsstrahlung, which involves the axion-nucleon coupling [11]. This interaction strength has been constrained by two different arguments based on the observed neutrino signal of supernova 1987A [12–15]. It has been proposed that axion nucleon couplings which predict an axion energy loss greater or equal than the neutrino luminosity should be excluded as they would spoil the observed neutrino signal from SN 1987A.

According to the current results, which we are going to revise in the present work, axions with mass \( m_a \lesssim 16 \text{ meV} \), corresponding to \( f_a \gtrsim 4 \times 10^8 \text{ GeV} \), interact so weakly that the changes in the neutrino emission of SN 1987A, would be unobservable [10]. Analogously, very strongly interacting axions, with \( f_a \lesssim 2 \times 10^6 \text{ GeV} \), would guarantee a standard SN neutrino signal. In this case, axions would be so efficiently trapped in the SN core that their luminosity would be suppressed below any observable level. As discussed in [16], however, the
region with slightly lower axion decay constant, \( f_a \lesssim 3 \times 10^5 \) GeV, should be excluded since
axions would have caused too many events in the Kamiokande water Cherenkov detector
that observed the SN 1987A neutrino signal. As a result, it was argued that the window
\( 3 \times 10^5 \) GeV \( \lesssim f_a \lesssim 2 \times 10^6 \) GeV, dubbed “hadronic axion window”, could not be excluded
by the SN argument alone [17]. In particular, for hadronic axion models one has to invoke
cosmological mass bounds to close this window [18, 19], corresponding to axion masses of
few eV.

A renewed interest towards the axion from SNe arose recently, partially because the SN
1987A bound overlaps with the sensitivity range of the planned experiments, in particular
the International Axion Observatory (IAXO), which searches for solar axions [20]. IAXO is
sensitive to generic axion-like particles coupled to photons and has the potential to probe
the QCD axion region up to masses \( m_a \gtrsim 10^{-2} \) eV. Therefore, it is interesting to assess how
robust the SN 1987A bound is, in relation to the IAXO potential.

In addition to the impact on the current experimental potential, the axion mass region
allowed by the SN argument has other interesting phenomenological implications. In partic-
ular, it has been recently shown that for such values of the axion mass it would be possible
to detect peculiar modifications in the neutrino signal from a future galactic SN [21]. Ad-
ditionally, a dedicated study [22] predicted a non-negligible diffuse axion background from
past core-collapse supernovae. Finally, the existence of eV axions in the hadronic axion
window is certainly an appealing possibility, as planned experiments such as IAXO can
probe those masses. In the case of hadronic axion models, most notably the KSVZ ax-
ions [23, 24], the strong astrophysical constraints on the axion electron coupling [25, 26]
provide only a weak bound on the PQ constant, since the coupling to electrons is suppressed
by loop effects. Therefore, one has to rely on the weaker horizontal branch bound [27, 28],
\( g_{a\gamma} \lesssim 0.65 \times 10^{-10} \) GeV\(^{-1}\), which translates into \( f_a \gtrsim 1.8 C_\gamma \times 10^7 \) GeV, where \( C_\gamma \)
is a model dependent constant. Thus, a value of \( C_\gamma \sim 0.1 \), contemplated by recent studies [29],
would allow a PQ constant in the hadronic window. Moreover, the cosmological constraint on
the hadronic axion window can also be relaxed in nonstandard low-temperature-reheating cosmo-
logical scenarios, in which the thermal axion relic abundances is suppressed and cosmological
limits are significantly loosened [30]. It is thus relevant to investigate the true impact of the
SN bound on the hadronic axion window since this is not related to the model-dependent
strength of axion-photon coupling, and unaffected by cosmological arguments.

In order to have a realistic determination of axion impact on the SN neutrino signal
one needs an accurate determination of the axion production rate. As stated above, the
most relevant process of axion production in a SN core is the nucleon-nucleon (NN) axion
bremsstrahlung, which involves the axion-nucleon coupling. This process has been originally
modeled at the level of the vacuum one-pion exchange (OPE) approximation [31–35]. In
literature it has been realized that different effects might influence and somehow reduce the
emissivity with respect to the OPE approximation. In particular, an attempt to account for
these effects has been recently proposed in [36]. There, different factors were introduced to
schematically modify the OPE emission rate. As a results, it was found a sizable reduction
of the axion emissivity. However, that analysis ignored some relevant effects such as the
modification of the nuclear mass in the dense plasma and, more importantly, neglected the
relation between the multiple nucleon scatterings and the other contributions to the emission
rate. As we shall see, this last effect compensates partially the contribution of the other
corrections. At any rate, motivated by the results in [36] and because of the great importance
of a quantitative assessment of the axion bremsstrahlung emission rate, we decided to take a
closer look at the axion emissivity by including in a self-consistent calculation the different corrections to OPE. Some of the effects we are including have, to the best of our knowledge, never been considered in other calculations of the axion emission rate. More specifically, we are including the effect of a finite mass in the pion propagator, the contribution of the $\varrho$ meson exchange, which mimic the effects of a two-pions exchange [37], the medium modification of the nucleon mass, and the impact of nucleon multiple scatterings. Moreover, we provide numerical evaluations of the axion emissivity valid for generic nucleon degeneracy.

The plan of our work is as follows. In Section 2 we present the theoretical framework for the calculation of the axion emission rate, including modifications beyond the OPE and we discuss the impact of the different effects on the axion emissivity. We also calculate the axion luminosity based on our new emissivity in the case of free-streaming axions. In Section 3 we discuss the impact of the corrections to the axion emissivity on the axion mass bound and we compare with previous literature. In Section 4 we calculate the axion opacity for trapped axions and calculate their luminosity in this case. Finally, in Section 5 we summarize our results and we conclude.

2 Axion emissivity

2.1 Axion-nucleon interaction and emissivity in one-pion-exchange

The dominant axion production channel in a SN core is the $N-N$ bremsstrahlung [11]:

\[ N_1 + N_2 \rightarrow N_3 + N_4 + a, \]  

(2.1)

where $N_i$ are nucleons (protons or neutrons) and $a$ is the axion field. The process (2.1) is induced by the axion-nucleon interaction described by the following Lagrangian term [32],

\[ \mathcal{L}_{aN} = \sum_{i=p,n} \frac{g_{ai}}{2m_N} N_i \gamma_\mu \gamma_5 N_i \partial^\mu a, \]  

(2.2)

with axion-nucleon couplings defined as follows,

\[ g_{ai} = C_{ai} \frac{m_N}{f_a} = 1.65 \times 10^{-7} \left( \frac{m_a}{eV} \right) C_{ai}, \]  

(2.3)

where $f_a$ is the Peccei-Quinn energy scale and $m_N$ is the nucleon mass (we assume $m_n \simeq m_p$). The coupling constants, $C_{ai}$, for the benchmark axion models have been recently calculated [5]. Their most accurate values, including the uncertainties, are

\[ C_{ap}^{\text{KSVZ}} = -0.47(3), \quad C_{an}^{\text{KSVZ}} = -0.02(3), \]  

(2.4)

\[ C_{ap}^{\text{DFSZ}} = -0.435 \sin^2 \beta + (-0.182 \pm 0.025), \quad C_{an}^{\text{DFSZ}} = 0.414 \sin^2 \beta + (-0.16 \pm 0.025). \]  

(2.5)

Here, $\tan \beta \equiv v_u/v_d$ represents the ratio of the two Higgs bosons in the DFSZ axion model [38, 39]. It is theoretically constrained from both ends by the requirement of perturbative unitarity of the Yukawa couplings $0.28 < \tan \beta < 140$.

The nuclear axion bremsstrahlung rate is highly uncertain, mostly because of the lack of understanding of the nuclear interactions; approximations are commonly applied based on vacuum physics. The matrix elements can only be calculated in specific frameworks,
the most widely used being the OPE potential, which describes the two nucleon interaction
with the exchange of a pion. In this approximation, the axion emission rate (in units of
energy/time x volume) can be calculated as [34]

\[ Q_a = \int \frac{d^3p_a}{2\omega_a(2\pi)^3} \prod_{i=1,4} \frac{g_{i} d^3p_i}{2E_i(2\pi)^3} \omega_a f_1 f_2 (1 - f_3) (1 - f_4) \sum_{\text{spins}} S |\mathcal{M}|^2 \delta^4(p_1 + p_2 - p_3 - p_4 - p_a) , \]

(2.6)

where the nucleon degeneracy factor \( g_i = 2 \), \( \omega_a \) is the axion energy, \( S \) is the usual symmetry
factor for identical particles in the initial and final states, and the matrix element squared
\(|\mathcal{M}|^2\) is summed over initial and final spins.

Before proceeding, let’s introduce the convenient notation \( g_a = m_N/f_a \) for the axion-
nucleon couplings. This notation encapsulates the model dependent constants [Eq. (2.3)]
which can be extracted as

\[
C_{ap} = g_{ap}/g_a , \\
C_{an} = g_{an}/g_a .
\]

(2.7)

The matrix-squared element for the nucleon axion bremsstrahlung in the one-pion-
exchange (OPE) can be conveniently written separating out the medium contribution \( \mathcal{M} \) [34]

\[ S \times \sum |\mathcal{M}|^2 = \frac{1}{4} \times \frac{g_a^2}{4m_N^2 \omega_a^2} \mathcal{M} , \]

(2.8)

with

\[ \mathcal{M} = 4 \times \frac{256}{3} m_N^4 \omega_a^2 \left( \frac{f}{m_\pi} \right)^4 (A_{nn} + A_{pp} + 4A_{np}) , \]

(2.9)

where \( f \approx 1 \) is the pion-nucleon “fine-structure constant,” and

\[
A_{NN} = C_{aN}^2 \left( \left| k \right|^2 / \left| k \right|^2 + m_\pi^2 \right)^2 + \left( \left| l \right|^2 / \left| l \right|^2 + m_\pi^2 \right)^2 + (1 - \xi) \left( \left| k \right|^2 / \left| k \right|^2 + m_\pi^2 \right) \left( \left| l \right|^2 / \left| l \right|^2 + m_\pi^2 \right) \]
\]

(2.10)

\[
A_{np} = (C_1^2 + C_2^2) \left( \left| k \right|^2 / \left| k \right|^2 + m_\pi^2 \right)^2 + (4C_1^2 + 2C_2^2) \left( \left| l \right|^2 / \left| l \right|^2 + m_\pi^2 \right)^2 + \]
\[
+ 2 \left( (C_1^2 + C_2^2) - (3C_1^2 + C_2^2) \xi \right) \left( \left| k \right|^2 / \left| k \right|^2 + m_\pi^2 \right) \left( \left| l \right|^2 / \left| l \right|^2 + m_\pi^2 \right) ,
\]

with

\[
\xi = 3 \left( \hat{k} \cdot \hat{l} \right)^2 ,
\]

(2.11)

\[
C_\pm = \frac{1}{2} \left( C_{an} \pm C_{ap} \right) ,
\]

(2.12)

while \( k = p_1 - p_3 \) and \( l = p_1 - p_4 \) are the exchanged nucleus momenta. In the above
expression, the notation \( A_{NN} \) indicates either \( A_{nn} \) or \( A_{pp} \). Analogously \( C_{aN} \) indicates \( C_{an,p} \).

It has been customary, in the previous literature, to neglect the pion mass \( m_\pi \), so that
the previous expressions in Eq. (2.10) become constants. As we shall see, this approximation
is justified at high temperatures, when the pion mass is subdominant with respect to the
exchanged momenta \( |k|^2 \sim |l|^2 \approx 3m_N T \). With this approximation, the integration over the
nucleons momenta is considerably simplified and the most complete expression for the axion emissivity, valid for generic nucleon degeneracy, is [34]

\[
Q_a^{(0)} \simeq 64 \left( \frac{f}{m_a} \right)^4 m_N^{5/2} T^{13/2} \left\{ \left( 1 - \frac{\xi}{3} \right) g_{nn} I(y_n, y_n) + \left( 1 - \frac{\xi}{3} \right) g_{nn} I(y_p, y_p) \right. \\
+ \frac{4(15 - 2\xi)}{9} \left( g_{nn}^2 + g_{np}^2 \right) I(y_n, y_p) + \frac{4(6 - 4\xi)}{9} \left( g_{nn} + g_{np} \right)^2 I(y_n, y_p) \right\}
\] (2.13)

The fitting functions \( I(y_1, y_2) \) are given in Ref. [34], with nucleon degeneracy \( y_i = \mu_i/T \) and nucleon chemical potentials \( \mu_i^0 = \mu_i - m_i \), where bare nucleon masses are assumed. The factor \( \xi \) parameterizes the degree of degeneracy, with \( \xi = 0 \) for degenerate matter and \( \xi = 1.3078 \) in the non-degenerate case [40].

### 2.2 Nucleon structure function

It is useful to express the axion emissivity \( Q_a \) of Eq. (2.6) in terms of the structure function [40, 41]

\[
S_\sigma = \frac{1}{n_B} \int \prod_{i=1,4} \frac{d^3 p_i}{2m_N(2\pi)^3} f_1 f_2 (1 - f_3)(1 - f_4) M \\
\times \delta(E_1 + E_2 - E_3 - E_4 + \omega) \delta^3(p_1 + p_2 - p_3 - p_4) ,
\] (2.14)

where \( n_B \) is the baryon density, and \( \omega = -\omega_a \) for axion emission, i.e. is the transfer of energy to the nuclear medium. The structure function obeys to the detailed balance \( S_\sigma(-\omega) = S_\sigma(\omega)e^{-\omega/T} \).

One can write the axion emissivity in terms of the structure function as [40]

\[
Q_a = \frac{g_a^2}{16\pi^2 m_N^2} \frac{n_B}{T} \int_0^{+\infty} d\omega e^{-\omega/T} \omega^4 S_\sigma(\omega) ,
\] (2.15)

where \( S_\sigma(\omega) \) reads

\[
S_\sigma(\omega) = \frac{\Gamma_\sigma}{\omega^2} s(\omega/T) ,
\] (2.16)

in terms of the nucleon “spin fluctuation rate” \( \Gamma_\sigma \), while \( s(x) \) is a dimensionless function. The factorization between \( \Gamma_\sigma \) and \( s(x) \) is not unique. Following [40, 41] we take \( \Gamma_\sigma \) such that \( s(0) = 1 \) for a medium of only one non-degenerate nucleon species and when the \( \pi \) and the \( \rho \) mass exchange is neglected, leading to [40, 41]

\[
\Gamma_\sigma = 4\pi^{-1.5} \rho \left( \frac{f}{m_\pi} \right)^4 T^{-0.5} m_N^{0.5} m_{\rho_{14}}^{0.5} m_{938}^{0.5} = 29.6 \text{ MeV} T_{\text{MeV}}^{0.5} m_{938}^{0.5} ,
\] (2.17)

where \( \rho_{14} = \rho/10^{14} \text{ g cm}^{-3} \), and \( T_{\text{MeV}} = T/1 \text{ MeV} \) and \( m_{938} = m_N/938 \text{ MeV} \). In this case, the explicit form of the \( s(x) \) function for a medium composed of protons and neutrons is given in Eq. (5.24) in the Appendix A.

Assuming that the nucleon spins evolve independently of each other, i.e. ignoring possible spin-spin correlations, the full scattering kernel must obey the normalization requirement, that for a medium composed by a mixture of protons and neutrons reads [41, 42]

\[
\int_{-\infty}^{+\infty} d\omega \frac{2\pi}{S_\sigma(\omega)} = \frac{1}{n_B} \sum_{i=p,n} C_{ai}^2 \int \frac{2d^3 p}{(2\pi)^3} f_i(1 - f_i) .
\] (2.18)

In case of a single non-degenerate species, one can neglect the Pauli blocking factor \( (1 - f_i) \) obtaining that the normalization is one.
2.3 Corrections to emissivity beyond ope-pion-exchange

Different improvements have been discussed in the literature, beyond the OPE. However, a reliable calculation is highly nontrivial. Difficulties include the description of an appropriate approximation for the nucleon-nucleon interaction potential, accounting for intermediate degree of nucleon degeneracy (which is in general different for protons and neutrons), nucleon-nucleon correlations, and multiple nucleon scattering effects. We will not be able to resolve all of these issues. In particular, we will ignore NN correlations in this work. However, we will include several improvements over the previous approximation. More specifically, we are including the following corrections:

Nonzero pion mass in the pion propagator

At a temperature $T$ the typical nucleon momentum is $(3m_N T)^{1/2}$ so that a typical momentum exchange in a collision is of a similar magnitude. At $T = 10$ MeV this is about 170 MeV, only slightly larger than the pion mass of 135 MeV. This implies that the pion mass cannot be ignored in the denominators in Eq. (2.10), while its impact would decrease at higher temperature (see [40, 41, 43] for early investigations).

Two-pions exchange

Two-pions exchange effects become important at distances below $2\text{ fm} \simeq 1.5 m_\pi^{-1}$. It has been proposed in [37] to evaluate the impact on the axion emissivity by mimicking the two-pions exchange contribution by a one–meson exchange, with mass $m_\varrho = 770$ MeV. One then modifies the propagator in Eq. (2.10) as

$$\left( \frac{|k|^2}{|k|^2 + m_\pi^2} \right)^2 \rightarrow \left( \frac{|k|^2}{|k|^2 + m_\pi^2} - C_\varrho \frac{|k|^2}{|k|^2 + m_\varrho^2} \right)^2,$$

with $C_\varrho = 1.67$. Taking $q^2 \simeq 3T m_N$, at $T = 10$ MeV one gets a $\sim 35\%$ reduction of the squared matrix element (see also [41]).

Effective nucleon mass

In the relativistic mean-field treatment, the reduction of the nucleon masses due to the scalar interactions at high density leads to effective nucleon masses $m_N^*$, which replace the vacuum mass $m_N$ in Eq. (2.10) (see, e.g. [33, 44]). Figure 1 shows the ratio of the effective nucleon mass $m_N^*$, based on the nuclear equation of state (EOS) given in [45–47], with respect to its vacuum value as a function of the density $\rho$ (left panel) and in the plane $t_{pb}$-$r$ (right panel), where $t_{pb}$ is the post-bounce time and $r$ the radial distance from the center of the SN. One realizes that for densities $\rho \lesssim 10^{14}$ g cm$^{-3}$ at $r \gtrsim 20$ km, the effective nucleon mass approaches the vacuum value. Conversely, at larger densities the correction becomes sizable and rather insensitive to the temperature. In particular, for $\rho \gtrsim 3 \times 10^{14}$ g cm$^{-3}$ one should expect a $\sim 40\%$ reduction in the nucleon mass. In principle one should take into account also the density-dependence of $m_\pi$ and $m_\varrho$. However, due to the large uncertainties in this dependence we neglect this effect [48].

Nucleon multiple scatterings

As discussed in [40, 49–52], many-body effects caused by multiple nucleon scatterings can also reduce the axion emissivity. If nucleons constantly scatter with each other in the medium, their individual energies become uncertain so that the (energy dependent) classical structure
function acquires a finite width. As shown in the references cited above, one can take the multiple scattering effects by a phenomenological ansatz

\[ S_\sigma(\omega) = \frac{\Gamma_\sigma}{\omega^2 + \Gamma^2} s(\omega/T) \]  

The scattering kernel is obtained determining the \( s(x) \) function from the bremsstrahlung calculation including the different approximations. Then the \( \Gamma \) function in Eq. (2.20) is determined in such a way that the normalization condition in Eq. (2.18) is fulfilled. One also has to take into account that the structure function saturates for \( \Gamma_\sigma/T \gtrsim 10 \) [50].

We give more details on how to implement the axion emissivity including all the previous corrections in Appendix A. We remark that our calculation represents the state-of-the-art beyond OPE, including all corrections discussed separately in the previous literature. Moreover, it is valid for arbitrary degeneracy. Therefore, it can be considered the generalization of the OPE calculation of Eq. (2.13).

2.4 Impact of the different corrections

SN reference model

In this Section we analyze the impact of the different corrections introduced beyond the OPE calculation of the axion emissivity. In order to connect these results in relation to realistic SN models, we consider as SN reference a model with 18 \( M_\odot \) progenitor simulated in spherical symmetry with the AGILE-BOLTZTRAN code [53, 54]. The matter density and temperature of our model shown in Fig. 2 and 3 correspond to the simulation of the protoneutron star deleptonization of [55] without considering axions. The left panels show the radial behavior of temperature and density at different post-bounce times \( t_{pb} \), while the right panels present contours of \( T \) and \( \rho \) in the \( t_{pb} \) and radial coordinate \( r \) plane. We can see that the temperature \( T \) presents a peak of \( \sim 40 \) MeV at \( r \approx 10 \) km. Given the steep temperature dependence of the axion emission rate, one expects the higher axion production around this peak temperature.
One also realizes that at larger $t_{pb}$ the peak in the temperature recedes, due to the cooling of the proto-neutron star. Concerning the matter density, we realize that it exceeds the nuclear density ($\rho_0 \simeq 3 \times 10^{14}$ g cm$^{-3}$) in the deepest SN regions ($r \lesssim 10$ km). There the density increases with time due to the contraction of the proto-neutron star.

In Fig. 4 we show the contour plots of the chemical potential $\mu^0$ over temperature $T$ in the plane $t_{pb}$-$r$ for protons (left panels) and neutrons (right panels). We realize that for typical SN conditions protons are always non-degenerate $\mu^0_p/T \ll -1$, while neutrons can be mildly degenerate ($\mu^0_n/T \gtrsim 1$). Therefore, the non-degenerate approximation often used in literature is not completely justified. When needed, we take as representative values of
Figure 4. Contour plot of the chemical potential $\mu^0$ over temperature $T$ in the plane $t_{ph}-r$ for protons (left panels) and neutrons (right panels).

Figure 5. Ratio of axion emissivity $Q_a/Q_a^{(0)}$ with pion mass correction. Left panel: Ratio in function of $T$. Right panel: Plane $t_{ph}-r$.

degeneracy for our estimation below $\mu^0_p/T = -3$ and $\mu^0_n/T = 1$. Moreover, in the following for simplicity we assume $g_{ap} = g_{an}$. The generalization to different couplings is discussed in Section 3.

Nonzero pion mass in the pion propagator

In order to investigate the impact of the different corrections on the emissivity we plot the ratio of the modified $Q_a$ with respect to the OPE prescription $Q_a^{(0)}$ of Eq. (2.13), where for definitiveness, we fix the value of $\xi$ to the non-degenerate case. In Fig. 5 we consider the impact of the finite pion mass $m_\pi$ in the propagator. From the left panel, one realizes
Figure 6. Ratio of axion emissivity $Q_a/Q_a(0)$ with $\rho$ mass correction [see Eq. (2.19)]. Left panel: Ratio in function of $T$. Right panel: Plane $t_{pb}-r$.

Figure 7. Ratio of axion emissivity $Q_a/Q_a(0)$ including the effective nucleon mass $m_N^*$. Left panel: Ratio in function of the density $\rho$. Right panel: Plane $t_{pb}-r$.

that the pion mass has a sizable effect for $T \lesssim 10$ MeV, when the pion mass correction becomes larger than the thermal energy of the nucleon. In this case the maximum reduction of the emissivity with respect to OPE is $\sim 40\%$. As expected the correction decreases with the temperature. For typical SN conditions, one expects that the overall reduction of the emissivity at $r \lesssim 15$ km would be $\sim 30\%$ with respect to the OPE for post-bounce times $t_{pb} \lesssim 4$ s, becoming even larger than $\sim 50\%$ at later times.
Two-pions exchange

In Fig. 6 we show the impact of $\varrho$-meson exchange in the propagator [see Eq. (2.19)]. This effect suppresses the OPE rate in a sizable way up to $\gtrsim 70\%$ for $T \gtrsim 30$ MeV (right panel). Notice that the $\varrho$-meson exchange produces an opposite trend in the emissivity with respect to a finite $m_\pi$. Indeed, the rate suppression due to $\varrho$-exchange increases with the temperature. Therefore, it is more important in the deepest regions of the star, i.e. $r \lesssim 10$ km (right panel). Observe also that this suppression is rather stable in time.

Effective nucleon mass

In Fig. 7 we consider the medium correction of the nucleon mass $m_N^\ast$. As shown before, the effective nucleon mass decreases as the density increases. As a consequence we find a reduction of the emission rate $\gtrsim 50\%$ for $\rho \gtrsim 1.5 \times 10^{14}$ g cm$^{-3}$ (left panel). This happens in the inner SN core, i.e. $r \lesssim 10$ km (right panel).

Multiple nucleon scatterings

In Fig. 8 we consider the impact of the multiple nucleon scatterings. Since the square of the spin-fluctuation rate entering the correction term, is quadratic in $\rho$ and linear in $1/T$ [cfr. Eq. (2.17) and Eq. (2.20)] one sees that the correction is larger at higher densities and lower temperatures. In particular, for $\rho \gtrsim 10^{14}$ g cm$^{-3}$ one may reach a reduction of the OPE emissivity up to 70%. This reduction of the emissivity with respect of OPE including multiple scattering effects was already pointed out in early papers on the subject, e.g. in [51]. However, we remark that the impact of the multiple spin fluctuations is reduced once the different corrections beyond OPE are included in the emissivity. Indeed, as shown in Fig. 9, the $s(x)$ function (with $x = \omega/T$) in Eq. (2.20) taken at $t_{\text{pb}} = 1$ s and $r = 10$ km, decreases with respect to the OPE case once the effect of $\pi$ and $\varrho$ mass exchange are included. As a result, due to the normalization condition of Eq. (2.18), the width of the Lorentzian function in Eq. (2.20) is also smaller. To be more quantitative, let’s define the Lorentzian width $g \equiv \Gamma/(\Gamma_\sigma/2)$ so that $g = 1$ for a single non-degenerate nucleon species [41]. In our case of mixed proton and neutron medium with intermediate degeneracy, one finds $g = 0.5$, for OPE, $g = 0.45$ when the effects of a finite $\pi$ mass are included, and $g = 0.1$ when also the effects of the $\varrho$ exchange are taken into account. Because of this fact, comparing the ratio $Q_a/Q_0$ including all the corrections except multiple scatterings (Fig. 10) with the one including also the multiple scatterings (Fig. 11) one realizes that the differences are minimal.

Including all the corrections

From our analysis we find a sizable reduction of the OPE emissivity when all the OPE corrections are accounted for. This is shown in Fig. 11, where we see that the OPE emissivity may be reduced by one order of magnitude when all these corrections are accounted for. This reduction of the emissivity rate with respect to OPE might be compared with the study performed in [56] where the axion emissivity was calculated with chiral perturbation theory and the results were calibrated using the nucleon-nucleon scattering data. Comparing the results with the OPE it was found that the emissivity could be reduced by a up to a factor of six. However, the reduction factor could be even larger. Indeed, as quoted in footnote 3 of Ref. [57], once the correct emission rate for OPE is considered, the reduction would be larger by 1.5 with respect the estimation presented in [56]. Therefore, our calculation is essentially in agreement with what obtained from scattering data in [56].
Figure 8. Ratio of axion emissivity $Q_a/Q_a^{(0)}$ including multiple nucleon scatterings. Left panel: Plane $T$-$\rho$. Right panel: Plane $t_{\text{pb}}$-$r$.

Figure 9. Function $s(x)$ with $x = \omega/T$ defined in Eq. (5.24) for the SN model at at $t_{\text{pb}} = 1$ s and $r = 10$ km.

We also see that this reduction has a very mild dependence on the temperature, so that it is rather flat in time as the star cools. Finally, in Fig. 12 we compare the axion emissivity $Q_a$ (in units $g_{\alpha n}^2 \times 10^{32}$ erg cm$^{-3}$s$^{-1}$) in the plane $t_{\text{pb}}$-$r$ in the OPE approximation (left panel) and including all the corrections (right panel). We realize that the bulk of the axion production occurs at low-radii $r \lesssim 10$ km, and there the rate is suppressed by roughly one order of magnitude with respect to the OPE, once all corrections are taken into consideration.
Figure 10. Ratio of axion emissivity $Q_a/Q_a^{(0)}$ including all the corrections except multiple nucleon scatterings. Left panel: Plane $T$-$\rho$. Right panel: Plane $t_{pb}$-$r$.

Figure 11. Ratio of axion emissivity $Q_a/Q_a^{(0)}$ including all the corrections. Left panel: Plane $T$-$\rho$. Right panel: Plane $t_{pb}$-$r$.

Axion luminosity

Let us finally calculate the axion luminosity, integrating the axion emissivity over the SN model [21]

$$ L_a = \int dr 4\pi r^2 Q_a. $$

(2.21)

In Figure 13, we compare the axion luminosity $L_a$ for $g_{ap} = g_{an} = 5 \times 10^{-10}$ for $t_{pb} > 1$ s, in the case of OPE (black continuous curve), and including the effective nucleon mass (black dot-dashed curve), a finite pion mass (black dotted curve), the exchange of the $\rho$ meson (red dotted curve), and multiple nucleon scatterings (red continuous curve), compared with the
Figure 12. Axion emissivity $Q_a$ (in units $g_{an}^2 \times 10^{52}$ erg cm$^{-3}$s$^{-1}$) in the plane $t_{pb}$-$r$ for OPE (left panel) and including all the corrections (right panel).

Figure 13. Time evolution of the axion luminosity $L_a$ for $g_{an} = g_{ap} = 5 \times 10^{-10}$ for OPE (black continuous curve), and including the effective nucleon mass (black dot-dashed curve), a finite pion mass (black dotted curve), the exchange of the $\rho$ meson (red dotted curve), and multiple nucleon scatterings (red continuous curve), compared with the $\bar{\nu}_e$ luminosity (black dashed curve). We see that the inclusion of all the corrections to OPE causes a reduction of the axion luminosity by one roughly an order of magnitude, the major impacts coming from the effective nucleon mass $m_N^*$ and the exchange of the $\rho$ meson. On the other hand, the effects of the non-zero pion mass and of multiple nucleon scatterings are subleading. Moreover, we see that for the chosen value of the axion-nucleon coupling the axion signal is larger than the neutrino one. This means that the axion feedback effect on the neutrino signal cannot be ignored and should be self-consistently included in SN simulations. We leave this important task for a future work.
In Fig. 14 we show the ratio of the axion luminosity $L_a$ with respect to the total neutrino luminosity $L_\nu$ at $t_{pb} = 1$ s as a function of $g_{an} = g_{ap}$ for the complete axion emissivity (continuous curve) and for the OPE (dashed curve). The limit $L_a/L_\nu = 1$ is shown as a horizontal dotted line. We consider a relative early post-bounce time, since the feedback on the neutrino signal would be less important than for later times. Cases with $g_{ap} = g_{an} \lesssim 10^{-7}$ correspond to the free-streaming case. Note that in the free-streaming case the luminosity increases at larger couplings, since it scales as $g_a^2$. As suggested in [14], axion-nucleon couplings that imply $L_a/L_\nu \gtrsim 1$ are excluded due to an excessive energy loss. For our SN model, in the case of the complete emission rate the condition of equality between axion and neutrino luminosity is reached for $g_{ap} = g_{an} \approx 10^{-9}$ (to be compared with $g_{ap} = g_{an} \approx 5 \times 10^{-10}$ for OPE). So, we expect that couplings larger than this value would cause an excessive energy loss and should be excluded. The limit on the axion-nucleon coupling (and on the axion mass) will be discussed in the next Section.

3 Axion mass bound and comparison with previous works

In order to assess the impact of our calculation on the axion mass bound based on emissivity from SN, here we compare our result with the previous literature. First of all, however, we should notice that previous works used different approximations and so there is a large range of variability in the quoted limits. For the sake of definitiveness we take as benchmark for our comparison, the result obtained in [10] and quoted in the latest edition of the Particle Data Group (PDG) [7]. This bound was based on a simple criterium to estimate the impact of

Figure 14. Ratio of the axion luminosity $L_a$ with respect to the total neutrino luminosity $L_\nu$ at $t_{pb} = 1$ s for the case of $g_{aN} \equiv g_{an} = g_{ap}$. Cases with $g_{aN} \gtrsim 10^{-6}$ corresponds to trapped axions, while for $g_{aN} \lesssim 10^{-7}$ one has free-streaming axions. Note that in the intermediate range of couplings one finds a mixed regime, where axions are neither free-streaming out of the star, nor properly trapped. This regime is quite more challenging to evaluate numerically. For this reason, in the plot we have left this coupling range empty. Couplings with $L_a/L_\nu \gtrsim 1$ (horizontal dotted line) are excluded due to an excessive energy loss.
Table 1. Bounds on axion couplings and mass for KVSZ model in a schematic SN model with $\varrho = 3 \times 10^{14}$ g cm$^{-3}$ and $T = 30$ MeV.

| $C_{ap}$ | $C_{an} = 0$ | $g_{ap}$ ($\times 10^{-10}$) | $m_a$ (meV) | $f_a$($\times 10^8$ GeV) |
|----------|--------------|-------------------------------|-------------|---------------------|
| OPE      | -0.47        | 7                            | 8           | 7.3                 |
| OPE+MS   |              | 9                            | 11          | 5.3                 |
| OPE+corr. (no MS) | 21         | 26                           | 2.3         |
| OPE+corr.+MS |           | 22                           | 28          | 2.1                 |

Table 2. Bounds on axion couplings and mass for KVSZ model in our SN model at $t_{pb} = 1$ s.

| $C_{ap}$ | $C_{an} = 0$ | $g_{ap}$ ($\times 10^{-10}$) | $m_a$ (meV) | $f_a$($\times 10^8$ GeV) |
|----------|--------------|-------------------------------|-------------|---------------------|
| OPE      | -0.47        | 9                            | 12          | 5.0                 |
| OPE+MS   |              | 12                           | 15          | 4.0                 |
| OPE+corr. (no MS) | 18          | 23                           | 2.6         |
| OPE+corr.+MS |           | 19                           | 24          | 2.5                 |

the axion emissivity from supernovae proposed in [15]. Namely, the observation of neutrinos from the SN 1987A implies that the energy-loss rate $\varepsilon_a = Q_a/\varrho$, associated to exotic particles such as axions, evaluated at $\varrho = 3 \times 10^{14}$ g cm$^{-3}$ and $T = 30$ MeV, should satisfy

$$\varepsilon_a \lesssim 1 \times 10^{19} \text{erg g}^{-1} \text{s}^{-1}. \quad (3.1)$$

Assuming a SN with a core mass of $1 M_\odot = 2 \times 10^{33}$ g, this gives an axion luminosity $L_a = \varepsilon_a M_\odot = 2 \times 10^{52}$ erg s$^{-1}$. Setting $\xi = 0$ in Eq. (2.11), assuming the non-degenerate limit of the axion emissivity in the squared-matrix element in Eq. (2.10), and including only multi-scattering effects as a correction of the OPE, in [10] it was derived the mass bound $m_a \lesssim 16$ meV (corresponding to $f_a \gtrsim 4 \times 10^8$ GeV) for the case of an hadronic axion model with $C_{an} = 0$, $C_{ap} = -0.4$ and a proton fraction $Y_p = 0.3$. In order to compare with this result, in Table 1 we present our bound on axion-nucleon coupling and on the mass for the schematic SN model described before, assuming non-degenerate emissivity and for KVSZ hadronic axion model with $C_{ap} = -0.47$ [cfr. Eq. (2.4)]. Note that for the case of OPE and multiple nucleon scatterings (OPE+MS) we find $m_a \lesssim 11$ meV, which is slightly different from the value quoted in [15]. This difference is due to the fact that we have not assumed $\xi = 0$ in the squared matrix element and that we assumed $C_{ap} = -0.47$, instead of $-0.4$. We note that the ratio of the square of the coupling constant $g_{ap}^2$ in the case of OPE+MS with respect to OPE is $\sim 0.5$, which is consistent with the reduction of the axion emissivity discussed in the previous section. If we include the other corrections to the emissivity, the axion bound relaxes by a factor of $\sim 3$ with respect to OPE. The additional inclusion of the multiple nucleon scatterings changes this results by only $\sim 5\%$.

In order to study the impact of the SN model on the axion mass bound, we consider the KSVZ hadronic axion model and, rather than referring to the simplified SN model discussed above, we calculate the axion luminosity from our realistic SN reference model at $t_{pb} = 1$ s. We show these results in Table 2. Note that in this case the OPE results already account for the effective nucleon mass. The differences with respect to the schematic SN model are less
than a factor 1.5. This result is reassuring since it implies that the derivation of the axion mass bound is not strongly affected by the details of the SN model. In this case, including all the corrections beyond OPE the relaxation of the axion mass bound is a factor \( \sim 2 \), leading to \( m_a \lesssim 24 \) meV.

We remark that a similar trend of relaxation of the bound on the axion-nucleon coupling in the free-streaming regime has been recently presented in [36]. In their calculation, the authors of [36] discuss some of the effects we also include, such as the finite pion mass and the multiple nucleon scatterings. Moreover, they attempted to go beyond OPE including also deviations coming from chiral perturbation theory [56, 58, 59]. When we compare their single correction beyond OPE with ours, we find a quantitative agreement. In particular, their correction coming from chiral perturbation theory (called \( \gamma_h \) in their paper) has a similar behavior and strength as ours coming from the \( \rho \) exchange. However, their implementation of the overall correction responsible for the emissivity suppression is more schematic than ours, since their different corrections are simply taken as fudge factors on top of the OPE. However, as we noticed above, the amplitude of the multiple scatterings effects (called \( \gamma_f \) in their work) gets significantly reduced once other corrections are included, since they diminish the width of the Lorentzian function, Eq. (2.20), fixed by the normalization condition in Eq. (2.18). Therefore, their final result that may reach \( m_a \lesssim 50 \) meV for the KSVZ model may be too optimistic.

In order to extend the previous analysis to generic axion models, we need to extract the axion luminosity dependence on the separate nucleon couplings, \( g_{an} \) and \( g_{ap} \). Considering our reference SN model at \( t_{pb} = 1 \) s we get

\[
L_a \simeq 1.60 \times 10^{70} \text{ erg s}^{-1}(g_{an}^2 + 0.36g_{ap}^2 + 0.04g_{an}g_{ap}) .
\]  

(3.2)

Notice that a similar dependence on the coupling is found also using the OPE emissivity. Imposing that this luminosity does not exceed the SN 1987A bound, the allowed parameter space in the plane \( g_{an} - g_{ap} \) is shown in Fig. 15. Imposing the condition \( L_a \lesssim L_\nu \simeq 2 \times 10^{52} \) erg s\(^{-1}\),
10^{52}\text{erg s}^{-1} one finds the constraint on the axion-nucleon couplings

\[ g_a^2 + 0.36g_{ap}^2 + 0.04g_ag_{ap} \lesssim 1.25 \times 10^{-18}; \]  

or, equivalently, the bound on the axion mass

\[ m_a \lesssim 6.8 \text{ meV} \times (C_{an}^2 + 0.36C_{ap}^2 + 0.04C_{an}C_{ap})^{-1/2}. \]  

Figure 16. In cyan, DFSZ axion parameters that satisfy Eq. (3.4). The RGB and the HB limits, discussed in the text, are also included. The areas filled in color are excluded by stellar evolution considerations. The two dashed lines delimit the allowed tan \( \beta \) region. In blue is the IAXO potential.

In particular, for the particularly interesting case of DFSZ axions [see Eq. (2.5)] our analysis indicates \( m_a \lesssim 30 - 50 \text{ meV} \), the exact value depending on \( \tan \beta \). This bound is shown in Fig. 16. As evident from the figure, the SN bound is comparable with the RGB one, and dominates only in regions of small \( \tan \beta \). It is also evident that the SN and RGB bounds leave a part of the axion parameter space accessible to the next generation of axion experiments such as IAXO [9, 20].

We finally mention that a stronger bound on the axion-neutron coupling, namely \( g_{an} < 2.8 \times 10^{-10} \), has been placed in [60] from the thermal evolution of the hot young neutron star in the supernova remnant HESS J1731-347. However, just like the SN bound, also this one is affected by uncertainties associated with the phase structure and linear response properties of matter at supra-nuclear density. Therefore, it is always important to use more than one argument to constrain axion-nucleon couplings.
4 Axion opacity

In the case of axions with mass exceeding $10^{-1}$ eV or so, the axion mean free-path for absorption (at the temperatures and densities of a proto-neutron star) becomes less than the radius of a proto-neutron star. Therefore, such axions would not be free-streaming but “trapped” and emitted from an axion-sphere in analogy with neutrinos [13]. We closely follow the treatment of the axion trapping given in [62]. To calculate the axion luminosity, one has to compute the reduced Rosseland mean opacity $\kappa$, defined as (see, e.g., Sec. 4.4 of [62])

$$\frac{1}{\rho \kappa} = \frac{15}{4\pi^2 T^3} \int_0^\infty d\omega \lambda_\omega (1 - e^{-\omega/T})^{-1} \partial_T B_\omega(T) ,$$

(4.1)

where $\rho$ is the mass density of the medium, $\lambda_\omega$ is the axion mean free-path against absorption and

$$B_\omega(T) = \frac{1}{2\pi^2} \frac{\omega^3}{e^{-\omega/T} - 1} ,$$

(4.2)

is the axion spectral density for one degree of freedom. Applying the operator $\partial_T = \partial / \partial T$ one gets

$$\frac{1}{\rho \kappa} = \frac{15}{8\pi^4} \int_0^\infty dx \lambda_x \frac{x^4 e^{2x}}{(e^x - 1)^3} ,$$

(4.3)

where $x \equiv \omega / T$.

One can determine the axion mean free-path, starting from the axion emissivity $Q_a$ of Eq. (2.6) writing it as

$$Q_a = \frac{T^4}{2\pi^2} \int dx x^3 e^{-x} \lambda_x^{-1} .$$

(4.4)

In Appendix B we give details of the calculation of the mean free-path starting from our emissivity. Finally the optical depth is defined as

$$\tau(r) = \int_0^r \kappa \rho dr .$$

(4.5)

Setting $\tau(r_{ax}) = 2/3$ we determine the radius of the axion-sphere $r_{ax}$. Using the temperature and density profiles shown in Fig. 2 and 3, we obtain the radius and temperature of the axion-sphere in function of the post-bounce time $t_{pb}$ as shown in Fig. 17 in the left and right panels, respectively, for axions with $g_{an} = g_{ap} = 9 \times 10^{-7}$ for the complete emissivity (continuous curve) and for OPE (dotted curve), compared with the analogous quantities for non-electron neutrinos $\nu_x$ (dashed curve). For the complete recipe for axion emissivity, one realizes that the axion-sphere radius is $\sim 10$ km after the core-bounce and declines with time. This radius corresponds to a temperature $T \sim 40$ MeV (see also Fig. 2). We see that the OPE prescription, having a larger axion absorption rate, would have given a larger axion-sphere radius and a smaller decoupling temperature (Fig. 2). For comparison, the $\nu_x$'s whose mean free path is determined by neutral currents, decouple at larger $r \simeq 20$ km and then have a lower temperature, i.e. $T \sim 5$ MeV.

Finally, the axion luminosity can be determined by the Stefan’s law

$$L_a = \frac{\pi^2}{120} 4\pi r_{ax}^2 T^4(r_{ax}) ,$$

(4.6)

and shown in Fig. 18 for the complete emissivity (continuous curve) and for OPE (dotted curve). Notice that, contrarily to the free-streaming case, the complete calculation in the
trapping regimes gives a larger axion luminosity than the OPE. This is easily explained since in the complete calculation we have seen that the axion-sphere has a larger temperature. Note also that for the chosen axion-nucleon coupling, the axion luminosity would be much larger than the unperturbed neutrino one (see Fig. 14). In Fig. 14, which shows the ratio $L_a/L_a$, the trapping case corresponds to $g_{an} = g_{ap} \lesssim 10^{-6}$. Notice that increasing the nucleon couplings the axion luminosity decreases. This should not be surprising. In fact, the corrections to OPE reduce the axion interaction rate implying a larger axion-sphere radius, where the temperature is higher. The luminosity being proportional to the square of the axion-sphere radius and the fourth power of the axion-sphere temperature, gets contribution from both effects but the lowering of the temperature dominates for essentially the full range of couplings.

5 Discussions and Conclusions

Axions can be produced in a SN core by nucleon-nucleon bremsstrahlung, providing an additional source of energy loss or transfer with respect to the standard neutrino emission. This process has been often modeled at the level of the vacuum OPE approximation. Starting from this recipe, we have refined the calculation of the axion emissivity, systematically including different effects: a non-zero pion mass, the contribution of the two-pions exchange, effective in-medium nucleon masses and multiple nucleon scatterings. Moreover, we have obtained axion emission rates valid for generic nucleon degeneracy. We find that the axion emissivity is significantly reduced (by about an order of magnitude) with respect to the OPE. From the SN 1987A neutrino observations (more specifically, requiring that the axion energy loss does not exceed the energy loss in neutrinos), we obtain a constraint on the axion-nucleon coupling $g_{an} = g_{ap} \lesssim 10^{-9}$, to be compared with $g_{an} = g_{ap} \lesssim 5 \times 10^{-10}$, found in the simple OPE approximation. In the case of KSVZ model, we find $g_{ap} \lesssim 2 \times 10^{-9}$, corresponding to $m_a \lesssim 24$ meV. This corresponds to a relaxation by a factor of 2 with respect to the OPE result. In the case of DFSZ axions our analysis indicates $m_a \lesssim 30 – 50$ meV, the exact value depending on $\tan \beta$. This bound is shown in Fig. 16. As evident from the figure, the SN bound is competitive with the RGB bound, however not dominating, except for low
values of $\tan \beta$. When compared with the expected IAXO potential [9, 20], it is evident that helioscopes of the next generation have the capability to explore large regions of the axion parameter space that are not affected by the SN analysis. As stressed above, the results of our analysis should not be considered as a robust bound as we are not including the axion feedback on the star, which at these couplings is most likely non-negligible. In fact, since axions are assumed to be transporting energy away from the SN at a rate comparable with that of neutrinos, it is likely that the stellar model with axion would be colder and therefore the axion rate reduced. Based on this, we can expect the axion bound to be further reduced when the axion feedback is taken into account in a self-consistent simulation of the SN evolution.

As evident from Fig. 14, the shift in luminosity induced by the corrections to OPE has dramatic consequences for the case of trapped axions, $g_{an} = g_{ap} \gtrsim 10^{-7}$. In the standard OPE scenario, the axion luminosity curve intersects the neutrino luminosity at $g_{an} = g_{ap} \simeq 5 \times 10^{-5}$. In that case, larger couplings are allowed since the axion luminosity would be reduced below $L_\nu$. The OPE corrections lead to a qualitative change, with the result that axions in the trapped axion region contribute always more than neutrinos to the SN energy loss.\footnote{At least down to values of the PQ constant $f_a \sim \text{TeV}$. Such values are excluded by many other considerations. In particular, the hadronic axion window disappears.} This result is in agreement with the findings in [36] and strengthens the cosmological argument for the exclusion of eV axions in the so-called “hadronic axion window” [18, 19].

We stress that our calculations, though represent a definite improvement on the OPE case, are not complete. As already mentioned, we have not included the effects of the NN correlations. Moreover, in the case of neutrinos, the nucleon-nucleon bremsstrahlung calculations beyond OPE have been performed in the framework of modern nuclear interactions.

Figure 18. Trapping regime. Axion luminosity in function of post-bounce time $t_{pb}$ for $g_{an} = g_{ap} = 9 \times 10^{-7}$ for the complete emissivity (continuous curve) and for OPE (dotted curve).
based on chiral effective field theory [56, 58, 59, 63]. Such an approach has been proposed also for the case of axions [56]. However, a systematic investigation is still lacking. We believe that such approach would be welcome in future investigations and add confidence in the robustness of the results. Finally, and perhaps most importantly, our estimation of the axion luminosity is based on an unperturbed SN model. Neglecting the axion production feedback on the SN is not fully justified and we expect a further relaxation of the free streaming axion bound when such effects are taken into account. Moreover, though the requirement that the axion luminosity does not exceed the neutrino one is certainly very reasonable, it is not obvious that this should be an exact criterion to fix the SN axion bound. A better, though much lengthier, approach requires a study of the axion induced modifications of the neutrino signal, which need then to be compared with the available data. We do plan to improve the current result, implementing what discussed above, in a forthcoming work. In particular, we plan to include the axion emissivity in a fully self-consistent SN simulation, as done in [21], in order to characterize the feedback on the neutrino signal.

In any case, we believe that our calculation represent a step forward in the effort to characterize the axion bounds from SN and a starting point for future investigations.

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Appendix A. Calculation of the emission rate

We show here the details of the calculation of the axion emissivity including all the corrections described before. The axion emissivity can be calculated as [34]

\[
Q_a = \int \frac{d^3 p_a}{2 \omega_a (2 \pi)^3} \prod_{i=1,4} \frac{g_i d^3 p_i}{2 E_i (2 \pi)^3} \omega_a f_1 f_2 (1 - f_3) (1 - f_4) \sum_{\text{spins}} S \times |\mathcal{M}|^2 \delta^4 (p_1 + p_2 - p_3 - p_4 - p_a),
\]

(5.1)

where the nucleon degeneracy factor \( g_i = 2 \), \( \omega_a \) is the axion energy. The distribution functions for the different species are

\[
f_i = \left[ \exp\left(\frac{|p_i|^2}{2 m_N T} - \hat{\mu}_i\right) + 1\right]^{-1},
\]

(5.2)

with

\[
\hat{\mu}_i = \frac{\mu_i - m_N}{T},
\]

(5.3)

and \( \mu \) is the chemical potential. For nearly all nuclear EoS, in particular the relativistic mean filed models, for supernova studies the nucleon chemical potentials contain all kind of interactions, e.g., scalar and vector self energies, while the chemical potential in Eq. (5.3) are those of the non-interacting gas. In complete analogy to the weak interactions that involve nucleons [61] we have to correct for these interactions. The factors \((1 - f_i)\) in Eq. (5.1) take into account the nucleon Pauli blocking effect, that we inserted since we considered situations
of arbitrary nucleon degeneracy. The squared matrix element \(|\mathcal{M}|^2\) is summed over initial and final spins, and \(S\) is the usual symmetry factor for identical particles in the initial and final states. Assuming one-pion-exchange (OPE) one would get

\[
S \times \sum |\mathcal{M}|^2 = \frac{1}{4} \times \frac{g_a^2}{4m_N^2} \omega_a^2 M ,
\]

with

\[
M = 4 \times \frac{256}{3} m_N^4 \omega_a^{-2} \left( \frac{f}{m_\pi} \right)^4 (A_{nn} + A_{pp} + 4A_{np}) ,
\]

where \(f \simeq 1\) is the pion-nucleon “fine-structure constant,” and

\[
A_{NN} = C_{aN}^2 \left[ \left( \frac{|k|^2}{|k|^2 + m_\pi^2} \right)^2 + \left( \frac{|l|^2}{|l|^2 + m_\pi^2} \right)^2 + (1 - \xi) \left( \frac{|k|^2}{|k|^2 + m_\pi^2} \right) \left( \frac{|l|^2}{|l|^2 + m_\pi^2} \right) \right] \quad (5.6)
\]

\[
A_{np} = (C_+^2 + C_-^2) \left( \frac{|k|^2}{|k|^2 + m_\pi^2} \right)^2 + (4C_+^2 + 2C_-^2) \left( \frac{|l|^2}{|l|^2 + m_\pi^2} \right)^2 + 2 \left( C_+^2 - (3C_+^2 + C_-^2) \right) \xi \left( \frac{|k|^2}{|k|^2 + m_\pi^2} \right) \left( \frac{|l|^2}{|l|^2 + m_\pi^2} \right) ,
\]

with

\[
\xi = 3(\hat{k} \cdot \hat{l})^2 ,
\]

\[
C_\pm = \frac{1}{2} (C_{an} \pm C_{ap}) ,
\]

where \(k = p_1 - p_3\) and \(l = p_1 - p_4\). We introduce the variables [40]

\[
p_{1/2} = p_0 \pm p; \quad p_{3/4} = p_0 \pm q
\]

with

\[
p_0 = |p_0| (\sin \eta \cos \phi, \sin \eta \sin \phi, \cos \eta)
\]

\[
p = |p| (0, 0, 1)
\]

\[
q = |q| (\sin \theta, 0, \cos \theta)
\]

Using these variables we get

\[
\hat{p} \cdot \hat{q} = \cos \theta \quad \hat{p} \cdot \hat{p}_0 = \cos \eta \quad \hat{q} \cdot \hat{p}_0 = \sin \eta \sin \theta \cos \phi + \cos \eta \cos \theta .
\]

Finally we introduce

\[
u = \frac{|p|^2}{m_N T}; \quad v = \frac{|q|^2}{m_N T}; \quad w = \frac{|p_0|^2}{m_N T}
\]

\[
y = \frac{m_\pi^2}{m_N T}; \quad z = \cos \theta .
\]

Moreover, in the non-relativistic limit for the nucleon, one may neglect the axion radiation in the law of conservation of moments, so that Eq. (5.1) is simplified as

\[
\delta^4(p_1 + p_2 - p_3 - p_4 - p_o) \to \delta(E_1 + E_2 - E_3 - E_4 - \omega) \delta^4(p_1 + p_2 - p_3 - p_4) .
\]
We discuss now the inclusion of the different corrections beyond OPE.

Pion mass exchange. In terms of the new kinematical variables Eq. (5.7) reads

\[ A_{nn} + A_{pp} = (C_{an}^2 + C_{ap}^2) \left[ F_+^2 + F_-^2 + (1 - \xi) F_+ F_- \right] , \]

\[ A_{np} = (C_+^2 + C_-^2) F_+^2 + (4C_+^2 + 2C_-^2) F_-^2 + 2 \left[ (C_+^2 + C_-^2) - (3C_+^2 + C_-^2) \frac{\xi}{3} \right] F_+ F_- , \]

(5.15)

where

\[ F_+ = \frac{|l|^2}{|l|^2 + m_\pi^2} = \frac{u + v + 2\sqrt{uvz}}{u + v + 2\sqrt{uvz} + y} , \]

\[ F_- = \frac{|k|^2}{|k|^2 + m_\pi^2} = \frac{u + v - 2\sqrt{uvz}}{u + v - 2\sqrt{uvz} + y} , \]

\[ \xi = 3(\hat{k} \cdot \hat{l})^2 = \frac{(u - v)^2}{(u + v)^2 - 4uvz^2} , \]

(5.16)

having used

\[ |k|^2 = m_N T(u + v - 2z\sqrt{uv}) , \]

\[ |l|^2 = m_N T(u + v + 2z\sqrt{uv}) . \]

(5.17)

\( \varrho \) exchange. In order to take into account the \( \varrho \)-meson exchange, as in Eq. (2.19), we shift the quantities in Eq. (5.16) as

\[ F_+ \rightarrow \tilde{F}_+ = F_+ - C_\varrho G_+ , \]

\[ G_+ = \frac{u + v \pm 2\sqrt{uvz}}{u + v \pm 2\sqrt{uvz} + r} , \]

\[ r = \frac{m_\varrho^2}{m_N T} , \]

(5.19)

where \( C_\varrho = 1.67. \)

Structure function. It is useful to express the axion emissivity \( Q_a \) of Eq. (5.1) in terms of the structure function as [40]

\[ Q_a = \frac{g_a^2}{16\pi^2} \frac{n_B}{m_N^2} \int_0^{+\infty} d\omega e^{-\omega/T} \omega^4 S_\sigma(\omega) , \]

(5.20)

where \( S_\sigma(\omega) \) may be written as

\[ S_\sigma(\omega) = \frac{\Gamma_{\sigma}}{\omega^2 s(\omega/T)} , \]

(5.21)

where \( \Gamma_{\sigma} \) is called the nucleon “spin fluctuation rate”, while \( s(x) \) is a dimensionless function of \( x = \omega/T \). The factorization between \( \Gamma_{\sigma} \) and \( s(x) \) is not unique. Following [40, 41] we take \( \Gamma_{\sigma} \) such that \( s(0) = 1 \) for a medium of only one non-degenerate nucleon species and when the \( \pi \) and the \( \varrho \) mass exchange is neglected, leading to [40, 41]

\[ \Gamma_{\sigma} = 4\pi^{-1.5} \left( \frac{f}{m_\pi} \right)^4 T^{0.5} m_N^{0.5} = 29.6 \text{ MeV} \rho_{14} T_{0.5}^{0.5} m_{0.5}^{0.5} m_{0.5}^{0.5} , \]

(5.22)
where $\rho_{14} = \rho/10^{14} \text{g cm}^{-3}$, and $T_{\text{MeV}} = T/1 \text{ MeV}$ and $m_{938} = m_{N}/938 \text{ MeV}$.

For a medium composed of neutron and protons one would get

$$s(x) = s^{nn}(x) + s^{pp}(x) + s^{np}(x) \ ,$$

with

\[ s^{nn}(x) = \frac{1}{3} C_{\text{an}}^{2} Y_{n}^{2} (s_{k} + s_{l} + s_{kl} - 3s_{kl}) , \]

\[ s^{pp}(x) = \frac{1}{3} C_{\text{np}}^{2} Y_{p}^{2} (s_{k} + s_{l} + s_{kl} - 3s_{kl}) , \]

\[ s^{np}(x) = \frac{4}{3} Y_{n} Y_{p} (C_{+}^{2} + C_{-}^{2}) s_{k} + \frac{4}{3} Y_{n} Y_{p} (4C_{+}^{2} + 2C_{-}^{2}) s_{l} + \]

\[ + \frac{8}{3} Y_{n} Y_{p} [(C_{+}^{2} + C_{-}^{2}) s_{kl} - (3C_{+}^{2} + C_{-}^{2}) s_{kl}] \ , \]

where including all the correction described above and considering arbitrary degeneracy one obtains

\[
s_{k}(x) = \int \frac{d \cos \eta}{2} \frac{d \phi}{2 \pi} \frac{\sqrt{wdu}}{v^{1/2}} \frac{d \cos \theta}{2} \left[ \frac{\rho Y_{1}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left[ \frac{\rho Y_{2}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left( u(x - x) e^{w \mu - \mu} H_{u}^{+}(\mu_{1}) H_{u}^{-}(\mu_{2}) H_{v}^{+}(\mu_{3}) H_{v}^{-}(\mu_{4}) F_{u}^{2} \right)_{v=u-x} \]

\[
s_{l}(x) = \int \frac{d \cos \eta}{2} \frac{d \phi}{2 \pi} \frac{\sqrt{wdu}}{v^{1/2}} \frac{d \cos \theta}{2} \left[ \frac{\rho Y_{1}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left[ \frac{\rho Y_{2}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left( u(x - x) e^{w \mu - \mu} H_{u}^{+}(\mu_{1}) H_{u}^{-}(\mu_{2}) H_{v}^{+}(\mu_{3}) H_{v}^{-}(\mu_{4}) F_{u}^{2} \right)_{v=u-x} \]

\[
s_{kl}(x) = \int \frac{d \cos \eta}{2} \frac{d \phi}{2 \pi} \frac{\sqrt{wdu}}{v^{1/2}} \frac{d \cos \theta}{2} \left[ \frac{\rho Y_{1}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left[ \frac{\rho Y_{2}}{m_{N}} \left( \frac{2\pi}{m_{N}T} \right)^{1.5} \right]^{-1} \left( u(x - x) e^{w \mu - \mu} H_{u}^{+}(\mu_{1}) H_{u}^{-}(\mu_{2}) H_{v}^{+}(\mu_{3}) H_{v}^{-}(\mu_{4}) F_{u}^{2} \right)_{v=u-x} \]

where $Y_{i}$ are the numbers of nucleons $i$ per baryons. In the previous expressions, one introduces the functions

\[
H_{u}^{\pm}(\mu) = \left( e^{\mu \pm \mu} \sqrt{w \cos \eta} \right) ^{-1} , \]

\[
H_{v}^{\pm}(\mu) = \left( e^{\mu \pm \mu} \sqrt{w \sin \eta \sin \phi \cos \theta + \cos \eta \cos \theta} \right) ^{-1} . \]

In the OPE and non-degenerate limit we recover the results shown in [40].

As discussed before, many-body effects caused by multiple nucleon scatterings can also reduce the axion emissivity. One can take these effects into account by the ansatz

\[
S_{\sigma}(\omega) = \frac{\Gamma_{\sigma}}{\omega^{2} + \Gamma_{\sigma}^{2}} s(\omega/T) .
\]

[5.23]
The calculation of the axion emissivity requires the solution of six-dimensional integrals. For this purpose we use the subroutine D01GDF for multidimensional Gaussian quadrature from the Numerical Algorithms Group (NAG) [64].

Appendix B. Axion opacity

One can determine the axion mean free-path, starting from the axion emissivity $Q_a$, writing this latter as

$$Q_a = \frac{T^4}{2\pi^2} \int dx \, x^3 e^{-x} \lambda_x^{-1} . \tag{5.28}$$

For each channel $pp$, $nn$, $np$, from the emissivity one can extract a mean free-path

$$Q_{ij}^a = \frac{T^4}{2\pi^2} \int dx \, x^3 e^{-x} (\lambda_x^{ij})^{-1} \rightarrow (\lambda_x^{ij})^{-1} = \frac{g_a^2}{8} \frac{\rho}{m_N^3} \frac{\Gamma_\sigma}{T} x^2 s^{ij}(x) ; \tag{5.29}$$

where $i, j = n, p$. In conclusion, the opacity is

$$\frac{1}{k_a \rho} = \frac{15}{8\pi^4} \int dx \, \frac{x^4 e^{2x}}{(e^x - 1)^3} \left( \lambda_x^{nm} + \lambda_x^{pp} + \lambda_x^{np} \right) = \frac{15}{8\pi^4} \int dx \, \frac{x^4 e^{2x}}{(e^x - 1)^3} \left[ \frac{g_a^2}{8} \frac{\rho}{m_N^3} \frac{\Gamma_\sigma}{T} x^2 s^{nm}(x, \mu) + \frac{1}{s^{pp}(x, \mu)} + \frac{1}{s^{np}(x, \mu)} \right] \tag{5.30}$$

Including also the MS effect, the opacity becomes

$$\frac{1}{k_a \rho} = \frac{15}{8\pi^4} \int dx \, \frac{x^4 e^{2x}}{(e^x - 1)^3} \left[ \frac{g_a^2}{8} \frac{\rho}{m_N^3} \frac{\Gamma_\sigma}{T} x^2 + \Gamma^2/T^2 x^2 \right]^{-1} \left[ \frac{1}{s^{nm}(x, \mu)} + \frac{1}{s^{pp}(x, \mu)} + \frac{1}{s^{np}(x, \mu)} \right] . \tag{5.31}$$

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