Metric-Field Approach to Gravitation and the Problem of the Universe Acceleration

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1 Introduction

The geometrical properties of space-time can be described only by means of measuring instruments. At the same time, the description of the properties of measuring instruments, strictly speaking, requires the knowledge of space-time geometry. One of the implications of it is that geometrical properties of space and time have no experimentally verifiable significance by themselves but only within the aggregate "geometry + measuring instruments". We got aware of it owing to Poincaré \[1\]. It is a development of the Berkley - Leibnitz - Mach idea about relativity of space-time properties, which is an alternative to the well known Newtonian approach.

If we proceed from the conception of relativity of space-time, we assume that there is no way of quantitative description of physical phenomena other than attributing them to a certain frame of reference which, in itself, is a physical device for space and time measurements. But then the relativity of the geometrical properties of space and time mentioned above is nothing else but relativity of space-time geometry with respect to the frame of reference being used\[1\]. Thus, it should be assumed that the concept of the reference frame as a physical object, whose properties are given and are independent of the properties of space and time, is approximate, and only the aggregate "frame of reference + space-time geometry" has a sense. The Einstein theory of gravitation demonstrates relativity of space-time with respect to distribution of matter. However, space-time relativity with respect to measurement instruments hitherto has not been realized in physical theory. An attempt to show that there is also space-time relativity to the used reference frames has been undertaken for the first time in \[3\], \[4\].

2 Fundamental Metric Form in NIFRs.

At present we do not know how the space-time geometry in inertial frames of reference (IFRs) is connected with the frames properties. Under the circumstances, we simply postulate (according to special relativity) that space - time in IFRs is

\[^{1}\)]There is an important difference between a frame of reference (as a physical device) and a coordinate system (as the way to parameterize points of space-time) \[6\]
pseudo-Euclidean. Next, we find a space-time metric differential form in noninertial frames of reference (NIFRs) from the viewpoint of an observer in the NIFR who proceeds from the relativity of space and time in the Berkley - Leibnitz - Mach - Poincaré (BLMP) sense.

By a noninertial frame of reference we mean the frame, whose body of reference is formed by the point masses moving in the IFR under the effect of a given force field. It would be a mistake to identify “a priori” a transition from the IFR to the NIFR with the transformation of coordinates related to the frames. If we act in such a way, we already assume that the properties of space-time in both frames are identical. However, for an observer in the NIFR, who proceeds from the relativity of space and time in the BLMP sense, space-time geometry is not given “a priori” and must be ascertained from the analysis of experimental data. We shall suppose that the reference body (RB) of the IFR or NIFR is formed by the identical point masses $m_p$. If the observer is at rest in one of the frames, his world line will coincide with the world line of some point of the reference body. It is obvious to the observer in the IFR that the accelerations of the point masses forming the reference body are equal to zero. Of course, this fact occurs in relativistic sense too. Let $d\eta$ and $ds$ be denote the differential metric forms in the IFR and NIFR. Then, if $\nu^\alpha = dx^\alpha/d\eta$ is the 4-velocity vector of the point masses forming the reference body of the IFR, the absolute derivative of the vector $\nu^\alpha$ is equal to zero, i.e.

$$D\nu^\alpha/d\eta = 0. \tag{1}$$

From the viewpoint of an observer in a NIFR who proceeds from the relativity of space-time in the BLM sense the points of his body reference are the points of his physical space, they are not exposed any forces. Consequently, their accelerations are equal to zero both in nonrelativistic and relativistic sense. This means that 4-velocity $\zeta^\alpha = dx^\alpha/ds$ of the point masses forming the reference body of the NIFR obeys equality

$$D\zeta^\alpha/ds = 0 \tag{2}$$

In other words, since for the observer in the IFR according to eq. (1), world lines of the IFR points are geodesic lines, then for the observer in the NIFR world lines of the NIFR points of the reference body are also geodesic lines in his space-time, which can be expressed by eq. (2).

This fact leads to important consequences. The differential equations of these world lines at the same time are the Lagrange equations of motion of the NIFR RB points. The Lagrange equations, describing the motion of the identical RB point masses in the IFR, can be obtained from the Lagrange action $S$ by the principle of least action. Therefore, the equations of the geodesic lines can be obtained from the differential metric form $ds = k\,dS(x, dx)$, where $k$ is the constant, $dSdt = L(x, \dot{x})dt$ and $L$ is the Lagrange function. The constant $k = -(m_pc)^{-1}$, as it follows from the analysis of the case when the frame of reference is inertial. Thus, if we proceed from relativity of space and time in the BLMP sense, the differential metric form
of space-time in the NIFR can be expected to have the following form \[^{3},^{4}\].

\[ ds = -(m_p c)^{-1} dS(x, dx). \]  

(3)

So, the properties of space-time in the NIFR are entirely determined by the properties of the used frame in accordance with the idea of relativity of space and time in the BLMP sense.

Consider two examples of the NIFR.

1. The reference body is formed by noninteracting electric charges moving in a constant homogeneous electric field \( E \). The motion of the charges is described in Cartesian coordinates by the Lagrangian

\[ L = -m_p c^2 (1 - V^2 / c^2)^{1/2} + E e x, \]  

(4)

where \( V \) is the speed of the charges. According to eq. (4) the space-time metric differential form in this frame is given by

\[ ds = d\eta - (wx/c^2) dx^0, \]  

(5)

where \( d\eta = (c^2 dt^2 - dx^2 - dy^2 - dz^2)^{1/2} \), is the metric differential form of the pseudo-Euclidean space-time in the IFR and \( w = e E / m \) is the acceleration of the charges.

2. The reference body consists of noninteracting electric charges in a constant homogeneous magnetic field \( H \) directed along the axis \( z \). The Lagrangian describing the motion of the particles can be written as follows

\[ L = -m_p c^2 (1 - V^2 / c^2)^{1/2} - (m_p \Omega_0/2)(\dot{x}y - \dot{y}x), \]  

(6)

where \( \dot{x} = dx/dt, \dot{y} = dy/dt \) and \( \Omega_0 = e H / 2mc \). The points of such a system rotate in the plane \( xy \) around the axis \( z \) with the angular frequency \( \omega = \Omega_0[1 + (\Omega_0 r/c)^2]^{-1/2} \), where \( r = (x^2 + y^2)^{1/2} \). The linear velocities of the BR points tend to \( c \) when \( r \to \infty \). For the given NIFR

\[ ds = d\eta + (\Omega_0/2c) (ydx - xdy). \]  

(7)

In the above NIFR \( ds \) is of the form \( ds = F(x, dx) \) where \( F(x, dx) = d\eta + f_\alpha(x) dx^\alpha \), \( f_\alpha \) is a vector field. The function \( F \) is a homogeneous of the first degree in \( dx^\alpha \). Therefore, generally speaking, the space-time in NIFR is Finslerian \[^{3}\] with the sign - indefinite differential metric form.

One of the consequences of the above result is a natural explanation of the Sagnac effect and the fact of the existence of the inertial forces in NIFRs \[^{3},^{4}\].

### 3 Experimental Verification

A clock, which is in a NIFR at rest, is unaffected by acceleration in space-time of the frame. The change in rate of the ideal clock is a real consequence of the difference between the space-time metrics in the IFR and NIFR. It is given by the factor \( \sigma = ds/d\eta \) from the equation \( ds = \sigma d\eta \). For the rotating with the angular
velocity $\Omega$ disk of the radius $R$ the factor $\sigma = 1 - \Omega^2 R^2/2c^2$ which gives rise to the observed red shift in the well known Pound - Rebka - Snider experiments.

Another experimentally verifiable consequence of the above theory is some difference between the inertial mass $m_p^{eq}$ of a body on the Earth’s equator and the mass $m_p^{pol}$ of the same body on the pole. It is given by

$$\frac{(m_p^{eq} - m_p^{pol})}{m_p^{pol}} = 1.2 \cdot 10^{-12}$$

(8)

The dependence of the inertial mass of particles on the Earth’s longitude can be observed by the Mössbauer effect. Indeed, the change $\Delta \lambda$ in the wave length $\lambda$ at the Compton scattering on particles of the masses $m_p$ is proportional to $m_p^{-1}$. If this value is measured for gamma - quanums with the help of the Mössbauer effect at a fixed scattering angle, then after transporting the measuring device from the longitude $\varphi_1$ to the longitude $\varphi_2$ we have

$$\frac{(\Delta \lambda)^{-1}_{\varphi_1} - (\Delta \lambda)^{-1}_{\varphi_2}}{(\Delta \lambda)^{-1}_{\varphi_1}} = \Theta [\cos^2(\varphi_1) - \cos^2(\varphi_2)],$$

(9)

where $\Theta = 1.2 \cdot 10^{-12}$.

4 Gravitation in Inertial and Proper Reference Frames

Consider a frame of reference whose reference body is formed by identical material points $m_p$ moving under the effect of the field $\psi_{\alpha\beta}$. These frames will be called the proper frames of reference (PRFs) of the given field. Any observer, located in the PFR at rest, moves in space-time of this frame along the geodesic line of his space-time. This implies that the space-time metric differential form in the NIFR is given by eq. (3) where $S$ is the action describing in a IFR the motion of particles forming the reference body of the NIFR. Now suppose, following Thirring [6], that in pseudo- Euclidean space-time gravitation can be described as a tensor field $\psi_{\alpha\beta}(x)$, and the Lagrangian describing motion of a test particle with the mass $m_p$ is of the form

$$L = -m_p c [g_{\alpha\beta}(\psi) x^\alpha x^\beta]^{1/2},$$

(10)

where $x^\alpha = dx^\alpha/dt$ and $g_{\alpha\beta}$ is the symmetric tensor whose components are the function of $\psi_{\alpha\beta}$. Then, according to (3) the space-time metric differential form in the PFR is given by

$$ds^2 = g_{\alpha\beta}(\psi) dx^\alpha dx^\beta$$

(11)

Thus, the space-time in the PFR is a Riemannian with the curvature other than zero. Viewed by an observer in the IFR, the motion of the test particle forming the reference body of the PFR is affected by the force field $\psi_{\alpha\beta}$. But the observer located in the PRF will not observe the force properties of the field $\psi_{\alpha\beta}$ since he moves in space-time of the PRF along the geodesic line. For him the presence of
the field $\psi_{\alpha\beta}$ will be displayed in another way — as space-time curvature differing from zero in these frames, e.g. as a deviation of the world lines of the neighbouring points of the reference body. For example, when studying the Earth’s gravity, a frame of reference fixed to the Earth can be considered as an inertial frame if the forces of inertia are ignored. An observer located in this frame can consider motion of the particles forming the PRF reference body in flat space-time on the basis of eq. (10) without running into contradiction with experiments. However, the observer in the PFR (in a comoving frame for free falling particles) does not find the Earth’s gravity as some force field. If he proceeds from the relativity of space-time, he believes that point particles, forming the reference body of his reference frame, are the point of his physical space. They are not affected by a force field and, therefore, their accelerations in his space-time are equal to zero. In spite of that, he observes a change in the relative distances of these particles. Such an experimental fact has apparently the only explanation as non-relativistic display of the deviations of the geodesic lines caused by space-time curvature. So, we observe an important fact that only in proper frames of reference we have an evidence for gravitation identification with space-time curvature.

Thus, we arrive at the following hypothesis. In inertial frames of reference, where space-time is pseudo-Euclidean, gravitation is a field $\psi_{\alpha\beta}$. In the proper frames of reference of the field $\psi_{\alpha\beta}$, where space-time is Riemannian, gravitation manifests itself as curvature of space-time and must be described completely by the geometrical properties of the latter.

Of course, eq. (3) refers to any classical field. For instance, space-time in the PRF of an electromagnetic field is Finslerian. However, since $ds$ depends on the mass $m$ and charge $e$ of the point masses forming the reference body, this fact is not of great significance.

5 Geodesic-invariant equations of gravitation

Gravitational equations should be some kind of differential equations for the function $\psi_{\alpha\beta}$ or $g_{\alpha\beta} (\psi)$, which are invariant under a certain kind of gauge transformations of the potentials $\psi_{\alpha\beta}$. Since $g_{\alpha\beta} = g_{\alpha\beta} (\psi)$, the Einstein equations are the equations both for $g_{\alpha\beta}$ and for $\psi_{\alpha\beta}$. Under the transformation $\psi_{\alpha\beta} \to \tilde{\psi}_{\alpha\beta}$ the quantities $g_{\alpha\beta} (\psi)$ undergo some transformations too and, as a consequence, the equations of the test particle motion resulting from eq. (10) and the Einstein’s equations do not remain invariant. The equations of motion resulting from eq. (10) are at the same time the equations of a geodesic line of the Riemannian space-time $V_n$ of the dimensionality $n$ with the metric tensor $g_{\alpha\beta} (\psi)$. That is why if the given gauge transformation $\psi_{\alpha\beta} \to \tilde{\psi}_{\alpha\beta}$ leaves the equations of motion invariant, then the corresponding transformation $g_{\alpha\beta} \to \tilde{g}_{\alpha\beta}$ is a mapping $V \to \tilde{V}$ of the Riemannian spaces leaving geodesic lines invariant, i.e. it is a geodesic, (projective) mapping. Let us assume that not only eq. (10) but also the field equations contain $\psi_{\alpha\beta}$ only in the form $g_{\alpha\beta} (\psi)$, then it becomes clear that the gauge-invariance of the equations of motion will be ensured if the field equations are invariant with respect to geodesic mappings of the Riemannian space $V_n$. Thus, if we start from eq. (10), then the gravitational field equations as well as the physical field characteristics must be invariant with respect to geodesic (projective) mappings of the
Riemannian space-time $V_n$ with the metric tensor $g_{\alpha\beta}(\psi)$.

The simplest equations of gravitation that can be considered as a realization of the above idea were proposed in paper [7]. (From another viewpoint). They are given by

$$B^\gamma_{\beta\gamma;\alpha} - B^\nu_{\beta\mu} B^\mu_{\gamma\nu} = 0,$$  \hspace{1cm} (12)

In these equations

$$B^\gamma_{\alpha\beta} = \Pi^\gamma_{\alpha\beta} - \circ_{\alpha\beta}^\gamma,$$  \hspace{1cm} (13)

where $\Pi^\gamma_{\alpha\beta}$ and $\circ_{\alpha\beta}^\gamma$ are the Thomas symbols for $V_n$ and $E_n$,

$$\Pi^\gamma_{\alpha\beta} = \Gamma^\gamma_{\alpha\beta} - (n + 1)^{-1} \left[ \delta^\gamma_{\alpha\beta} \Gamma^\mu_{\beta\mu} + \delta^\gamma_{\alpha\beta} \Gamma^\mu_{\alpha\mu} \right],$$  \hspace{1cm} (14)

$$\circ_{\alpha\beta}^\gamma = \circ_{\alpha\beta}^\gamma - (n + 1)^{-1} \left[ \delta^\gamma_{\alpha\beta} \circ_{\epsilon\beta}^{\epsilon\gamma} + \delta^\gamma_{\alpha\beta} \circ_{\epsilon\alpha}^{\epsilon\gamma} \right],$$  \hspace{1cm} (15)

$\Gamma^\gamma_{\alpha\beta}$ and $\circ_{\alpha\beta}^\gamma$ are the Christoffel symbols in $V_n$ and $E_n$, respectively.

They are bimetric geodesic invariant equations. Each solution $g_{\alpha\beta}(x)$ of (12) refers to some coordinate system and is determined up to arbitrary geodesic mappings, which play the role of gauge transformation in the theory under consideration. The physical meaning may have only geodesic invariant magnitudes, for example, the tensor $B^\gamma_{\alpha\beta}$. At the covariant gauge conditions $Q_\alpha = \Gamma^\beta_{\alpha\beta} - \circ_{\alpha\beta}^\gamma = 0$ eqs. 12 are equivalent to the system

$$R_{\alpha\beta} = 0,$$  \hspace{1cm} (16)

and

$$Q_\alpha = 0,$$  \hspace{1cm} (17)

where $R_{\alpha\beta}$ is the Ricci tensor.

The equations do not contain the functions $\psi_{\alpha\beta}(x)$ explicitly. The simplest way of obtaining equations for such a kind of the functions $\psi_{\alpha\beta}$ is to set

$$B^\gamma_{\beta\gamma;\alpha} = \nabla^\kappa \psi_{\beta\gamma} - (n + 1)^{-1} \left( \delta^\gamma_{\beta\gamma} \nabla^\sigma \psi_{\kappa\gamma} + \delta^\gamma_{\alpha\beta} \nabla^\sigma \psi_{\kappa\beta} \right),$$  \hspace{1cm} (18)

where $\nabla^\kappa$ is the covariant derivative in flat space-time. Then, at the gauge condition $\nabla^\kappa \psi_{\kappa\gamma} = 0$ eq. (12) are given by

$$\Box \psi_{\alpha\beta} - \nabla^\kappa \psi_{\alpha\gamma} \nabla_{\kappa} \psi_{\gamma\beta} = 0; \hspace{0.5cm} \nabla^\kappa \psi_{\kappa\gamma} = 0,$$
where $\Box$ is the covariant Dalamber operator in pseudo-Euclidean space-time. It is natural to suppose that with the presence of matter these equations are given by

$$\Box \psi_{\alpha\beta} = \kappa (T_{\alpha\beta} + t_{\alpha\beta}); \quad \nabla^\sigma \psi_{\sigma\gamma} = 0,$$

where $\kappa = \frac{8\pi G}{c^4}$, $t_{\alpha\beta} = \kappa^{-1} \nabla^\sigma \psi_{\alpha\gamma} \nabla^\gamma \psi_{\sigma\beta}$ and $T_{\alpha\beta}$ is the matter tensor of the energy-momentum. Obviously, the equality

$$\nabla^\beta (T_{\alpha\beta} + t_{\alpha\beta}) = 0$$

is valid. Therefore, the magnitude $t_{\alpha\beta}$ can be interpreted as the energy-momentum tensor of a gravitational field.

$$t_{\alpha\beta} = \chi^{-1} B_{\alpha\gamma}^\beta B_{\beta\gamma}$$

6 Gravitational Energy of a Point Mass

If the Lagrangian of test particles is invariant under the mapping $t \to -t$, the fundamental metric form of space-time $V_4$ in the spherically-symmetric case can be written as

$$ds^2 = -Adr^2 - B[d\theta^2 + \sin^2 \theta d\varphi^2] + Cdx_0^2,$$

where $A, B$ and $C$ are the functions of the radial coordinate $r$. The general solution of the system (16) – (17) at the conditions

$$\lim_{r \to \infty} A = 1, \quad \lim_{r \to \infty} (B/r^2) = 1, \quad \lim_{r \to \infty} C = 1.$$

is of the form

$$A = (f')^2 (1 - Q/f)^{-1}, \quad B = f^2, \quad C = 1 - Q/f$$

where

$$f = (r^3 + K^3)^{1/3}$$

$f' = df/dr$, $Q$ and $K$ are constants.

The equations of the motion of a test particle resulting from Lagrangian (10) is given by

$$\ddot{x}^\alpha + (\Gamma^\alpha_{\beta\gamma} - c^{-1} \Gamma^0_{\beta\gamma} \dot{x}^\alpha) \dot{x}^\beta \dot{x}^\gamma = 0.$$

\footnote{It should be noted that, when we introduce it in some way, we cannot be sure a priori that the equation for $\psi_{\alpha\beta}$ yields all solutions of the equations for $B^\alpha_{\beta\gamma}$ We may introduce a potential $\psi_{\alpha\beta}$ also in another way.}
In the nonrelativistic limit \( \ddot{x} = -c^2 \Gamma^r_{00} \), where \( \Gamma^r_{00} = C'/2A = r^4C'/f^4C \). Therefore, to obtain the Newton gravity law it should be supposed that at large \( r \) the function \( f \approx r \) and \( Q = r_g = 2G M/c^2 \) is the classical Schwarzshild radius.

We can also argue that the constant \( K = r_g \). Indeed, consider the 00-component of the first of eq. (19). Let us set \( T_{\alpha\beta} = \rho c^2 u_\alpha u_\beta \), where \( \rho \) is the matter density and \( u_\alpha \) is the 4-velocity of matter points. At the small macroscopic velocities of the matter we can set \( u_0 = 1 \) and \( u_i = 0 \). Therefore, the equation is of the form

\[
\Box \psi_{\alpha\beta} = \chi (\rho c^2 + t_{00})
\]  

(26)

where \( \chi = 4\pi G/c^4 \) and \( t_{00} \) is the 00-component of the tensor (21). Let us find the energy of a gravitational field of the point mass \( M \) as the following integral in the pseudo-Euclidean space-time

\[
\mathcal{E} = \int t_{00} dV,
\]  

(27)

resulting from the above solution, where \( dV \) is the volume element. In the Newtonian theory this integral is divergent. In our case we have:

\[
\mathcal{E} = \frac{Q}{K} M c^2
\]  

(28)

We arrive at the conclusion that at \( K \neq 0 \) the energy of the point mass is finite and at \( K = Q \) the rest energy of the point particle in full is caused by its gravitational field: \( \mathcal{E} = M c^2 \).

The spatial components of the vector \( P_\alpha = t_{0\alpha} \) are equal to zero. Due to these facts we assume in the present paper that \( K = Q = r_g \) and consider the solution (24) in the spherical coordinate system at the used gauge condition as a basis for the subsequent analysis.

7 Acceleration of the Universe as a Consequence of Gravitation Properties

The analysis of the recent observations data gives evidence that the deceleration parameter \( q_o = -\ddot{a}(t) a(t)/\dot{a}(t) \) ( \( a \) is the scale factor) is negative at the moment \( [12], [13] \). It means that \( \ddot{a} > 0 \) i.e. the expansion is accompanied with acceleration, while according to classical insights the gravity force must retard the expansion.

Equations (12) were successfully testified by the classical tests and binary pulsar PSR1913+16 \([10], [9]\). The motion of test particles in the spherically-symmetric gravitational field differs very little from that in general relativity at distances from the center \( r \gg r_g \). However, they are completely different at \( r \leq r_g \). The solution of (12) has no the event horizon and physical singularity in the center. The gravitational force (as the mass multiplied by the acceleration) affecting escape particles is repulsive from \( r = 0 \) up to distances of the order of \( r_g \). The observed radius \( R_U \) of the Universe is about \( 10^{27} \text{cm} \), and the observed mass \( M_U \) is \( 10^{56} \div 10^{57} \text{g} \), so the magnitude \( 2GM_U/c^2 \) is of the order of \( R_U \). For this reason, we can expect some manifestation of the repulsive force for distant objects.
Consider in flat space-time a simple model of an expanding self-gravitating homogeneous dust-ball with the sizes of the observed Universe. According to [7], the equation of the motion of a test particle in the spherically-symmetric field are given by

\[ \ddot{r} = \left( \frac{c^2 C}{A} \right) \left[ 1 - \left( \frac{C}{E} \right) \left( 1 + \frac{r_g J^2}{B} \right) \right], \]  

\[ \dot{\varphi} = c C \frac{r_g}{(B E)} \]  

where \((r, \varphi, \theta)\) are the spherical coordinates (\(\theta\) is supposed to be equal to \(\pi/2\)), \(\dot{r} = dr/dt, \dot{\varphi} = d\varphi/dt, E = E/(mc^2), J = J/(amc), E\) is the particle energy, \(J\) is the angular momentum.

Consequently, the radial velocity of the specks of dust of the ball surface as a function of its radius \(R\) is given by

\[ V^2 = \frac{c^2 C}{A} \left[ 1 - \frac{C}{E^2} \right], \]

where \(V = \dot{R} = dR/dt,\)

\[ C = 1 - \frac{1}{\bar{f}}, \quad A = \frac{r_g^4}{\bar{f}^4} C^{-1}, \quad \bar{f} = (1 + r_g \bar{f}^3)^{1/3} \]  

and the dependence of the acceleration \(\ddot{V} = dV/dt\) on the radius \(R\) is given by

\[ \ddot{V} = V V' \]  

Figs. 1 and 2 show the plot of the velocity and the acceleration (arbitrary units) as the function of the radius at \(E^2 = 0.60 < 1\) and \(E^2 = 1.20 > 1\).

It follows from the figures that the acceleration is negative at \(R/r_g \gg 1\), and is positive if \(R/r_g\) is of the order of \(r_g\) or less than that. For example, if the radius \(R = R_U\) and the matter density \(\rho = 2 \cdot 10^{-28} g/cm^3\), the value of \(R_U/r_g = 0.9 < 1\) and the acceleration is equal to \(1 \cdot 10^{-8} cm/s^2\) which is half as large as this magnitude resulting from the value of \(q_0 = -1\) that was found in [12].

In paper [13] the Riess et al. results were studied in detail in view of the model above. A good compliance was found.

8 Conclusion

The key reason preventing a correct inclusion of the Einstein theory of gravitation in the interactions unification is that gravity is identified with space-time curvature. It is also a cause of such unsolved problems of the theory as an operational definition of the observables, the energy-momentum tensor problem and gravity quantization. So, if the analysed above possibility really takes place in nature, then it will remove an isolation of the geometrical gravitational theories from the theories of other fields.
Figure 1: The radial velocity on the ball surface vs. the radius $R$ at $E^2 < 1$.

Figure 2: The radial velocity and acceleration on the ball surface vs. the radius $R$ at $E^2 < 1$. 
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