I. INTRODUCTION

The recent advances in the cooling, trapping, and manipulation of ultra-cold atomic gases using mostly optical beams have given birth to the new field of “condensed matter physics with light and atoms”. Key issues of the physics of strongly correlated quantum systems can now be addressed in this new context from a completely different perspective. Remarkable milestones in this respect have been the observation of the superfluid to Mott-insulator transition in a system of bosons in an optical lattice [1], the direct imaging of the Fermi surface in a degenerate Fermi gas [2], the demonstration of superfluidity in an interacting Fermi gas [3,4,5,6,7], and the recent observation [8,9] of an incompressible Mott insulating regime of fermionic atoms [10,11] trapped in the periodic potential of optical lattices.

Central objects in the theoretical and experimental study of quantum many-body systems are the low-energy excited states, and the possibility of describing those states in terms of quasi-particle excitations having a long excited states, and the possibility of describing those states in terms of quasi-particle excitations having a long life time and a well-defined dispersion relation for the excitation energy as a function of momentum [12,13]. There is in fact abundant experimental and theoretical evidence that quasi-particle states can be highly unconventional in strongly correlated systems, and significantly depart from the ones predicted by Landau Fermi-liquid theory [14,15]. An understanding of quasi-particle states is therefore an essential step in the direction of building a complete description of the peculiar electronic, magnetic, and optical properties that have been observed in a variety of strongly correlated materials.

A commonly used probe of the electronic states in solid materials is the so-called angle-resolved photoemission spectroscopy (ARPES) [16]. This technique consists in measuring the energy and momentum distribution of the electrons that are emitted from the solid when it is exposed to a beam of energetic photons. In the simplest approximation, the distribution of the emitted electrons is in fact proportional to the one-body spectral function of the electrons in the solid.

Inspired by the success of ARPES in solid-state systems, we recently proposed momentum-resolved stimulated Raman spectroscopy as a probe of quasi-particle states in strongly correlated atomic Fermi gases [17,18]. In a stimulated Raman process, atoms are transferred from the gas into a different internal state by absorbing a photon from a laser beam and immediately reemitting it into another beam of different frequency and wavevector. For a given intensity and duration of the Raman pulses, the momentum distribution of the atoms can be measured as a function of the wavevector and frequency of the Raman beams: analogously to electronic ARPES, this distribution is proportional to the one-body spectral function of the atoms in the gas. Alternative schemes that use Raman processes to probe the microscopic properties of atomic gases have been proposed in [19,20,21,22,23,24,25]. Experimental applications of the related Bragg scattering technique have been reported in [26,27,28,29].

This momentum-resolved stimulated Raman technique is a powerful extension of the RF spectroscopy technique that has been recently used to study strongly interacting atomic Fermi gases [30,31,32]. A pioneering experimental demonstration of the application of momentum-resolved RF spectroscopy to a strongly correlated atomic gas has recently appeared [33]. The energy dispersion of the occupied quasi-particle states has been measured at several points along the BEC-BCS crossover and has shown clear evidence of a pairing gap. Momentum-resolved RF spectroscopy in atomic Fermi gases has also been discussed recently in Ref. [34].

In this paper we extend the discussion of Ref. [17] and we give a comprehensive discussion of the promise of momentum-resolved stimulated Raman spectroscopy to investigate the microscopic properties of strongly interacting Fermi gases. On one hand, RF spectroscopy
naturally arises as a special case of Raman spectroscopy with a vanishing transferred momentum: the theoretically calculated spectra are in good agreement with the experimental observations of Ref.33. On the other hand, Raman spectroscopy offers several significant advantages as compared to the RF one, e.g., spatial selectivity to eliminate inhomogeneous broadening due to the trap potential, tunability of the transferred momentum from below to well above the Fermi momentum, weaker sensitivity to final-state interactions.

The paper is organized as follows: in Sec.II we review the general theory of the Raman spectroscopy and we provide explicit formulas for the observable signal in terms of the one-body spectral function of the gas. In Sec.III the advantage of the Raman spectroscopy over the RF one are illustrated on the simplest case of an ideal Fermi gas. In Sec.IV we show how the general theory is able to reproduce the experimental observations of Ref.33 for a strongly interacting Fermi gas along the BEC-BCS crossover. The promise of an inverse Raman spectroscopy to investigate the dispersion of the empty quasi-particle states is discussed in Sec.V. Concluding remarks are given in Sec.VI.

II. GENERAL THEORY OF THE RAMAN PHOTO-EMISSION TECHNIQUE

A. The optical process

In this section, we review the general theory of the stimulated Raman spectroscopy technique to probe the one-particle excitations of a many-atom gas that was originally proposed in [17]. Throughout the whole paper we will concentrate our attention on the case of a two-component mixture of fermionic atoms in two internal states $\alpha$ and $\alpha'$, but the extension to a generic system of fermions or bosons is straightforward. Generally, the $\alpha$ and $\alpha'$ states are long-lived hyperfine components of the electronic ground state of the atom. Atoms are transferred from the $\alpha$ state to a third hyperfine component $\beta$ by a pair of laser beams in the Raman configuration schematically shown in Fig.1. The laser fields are assumed to be far from resonance with the intermediate excited state $\gamma$ so that spontaneous emission events can be neglected.

The atom-laser interaction can be described by the standard dipolar Hamiltonian

$$V_{\text{dip}} = -\mathbf{d} \cdot [\mathbf{E}_1(r,t) + \mathbf{E}_2(r,t)].$$

where the laser fields are assumed to be classical and have the form

$$\mathbf{E}_{1,2}(r,t) = \tilde{\mathbf{E}}_{1,2}(r)e^{i[k_{1,2}\cdot r - \omega_{1,2}t]} + \text{c.c.}$$

The envelopes $\tilde{\mathbf{E}}_{1,2}(r)$ are slowly varying on the scale of the carrier wavevectors $k_{1,2}$ and account for the transverse profile of the laser beams. The corresponding Rabi frequencies are defined in terms of the electric field components of the optical transitions connecting the $\alpha, \beta$ states to the common excited state $\gamma$ as $\hbar \Omega_1(r) = \mathbf{d}_{\gamma \alpha} \tilde{\mathbf{E}}_1(r)$ and $\hbar \Omega_2(r) = \mathbf{d}_{\gamma \beta} \tilde{\mathbf{E}}_2(r)$.

Once we perform the rotating wave approximation and we eliminate the intermediate excited state $\gamma$ under the condition $\Omega_1, \Omega_2, \Gamma_\gamma \ll \Delta$ ($\Gamma_\gamma$ is here the natural linewidth of the excited $\gamma$ state), the light-matter interaction can be reduced to the simple form:

$$\hat{V}(t) = \hat{V} e^{-i\omega t} + \text{h.c.}$$

in which $\hat{V} = \int d\mathbf{r} \Omega_e(r,t) \psi_\alpha \psi_\beta e^{i\mathbf{q} \cdot \mathbf{r}}$. The basic process described by the Hamiltonian (3) consists of the transfer of atoms from the $\alpha$ to the $\gamma$ state by conservative Raman processes. $\Omega_e(r,t) = \Omega_1(r,t)\Omega_2(r,t)/\Delta$ is the Rabi frequency of the effective Raman coupling, $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ is the transferred momentum, and $\omega = \omega_1 - \omega_2$ is the transferred energy.

Note that RF spectroscopy with a single electromagnetic field at frequency $\omega$ is described by a Hamiltonian of exactly the same form (3) with a Rabi frequency $\Omega_e(r) = \mathbf{d}_{\beta \gamma} \tilde{\mathbf{E}}(r)$ and a vanishing transferred momentum $\mathbf{q} = 0$. All the discussion that follows is then straightforwardly extended to the case of RF spectroscopy by simply setting $\mathbf{q} = 0$.

B. The Raman emission rate

We first calculate the Raman transfer rate in the simplest case in which atoms in the final state $\beta$ do not interact with the atoms left behind in the $\alpha, \alpha'$ states and furthermore do not feel any trapping potential. Under these assumptions, their momentum is conserved while they freely propagate in space and one can envisage to perform a momentum-resolved measurement along the lines of the RF experiment of Ref.33.

FIG. 1: Upper panel: geometrical configuration of the Raman laser beams. Lower panel: scheme of the atomic levels involved in the Raman process.
In terms of the many-body states of the gas, the Raman process then consists in the excitation of the initial \( N \) body state \( |\phi_i^N\rangle \) to an excited \( N - 1 \) body state \( |\phi_{i-1}^N\rangle \) and the simultaneous out-coupling of one atom into the \( \beta \) state with momentum \( \mathbf{k} \). The rate \( R_q(k, \omega) \) for this process depends on the wavevector \( \mathbf{q} = k_1 - k_2 \) and frequency \( \omega = \omega_1 - \omega_2 \) that are transferred by the Raman beams to the atoms. The total Raman transfer rate \( R_q^T(\omega) \) is then obtained by integrating over final momenta \( \mathbf{k} \) as follows:

\[
R_q^T(\omega) = \int d^3k R_q(k, \omega). \tag{4}
\]

Direct application of the Fermi golden rule in the grand-canonical ensemble leads to

\[
R_q(k, \omega) = \frac{2\pi}{\hbar} \sum_{i,f} \frac{e^{-E_i/k_BT}}{Z} |M_{ki}^f|^2 \times \delta(E_f - \epsilon_{k\beta} - \epsilon_i - \hbar \omega). \tag{5}
\]

The initial state is assumed to be at thermal equilibrium at a temperature \( T \) with a chemical potential \( \mu \); \( Z = \sum_i e^{-E_i/k_BT} \) is the corresponding Grand-Canonical partition function. The sums over \( i \) and \( f \) refer to all the many-body states of the system. The energy \( E_i = E_i - \mu N_i \) is rescaled by the number of particles. The matrix element \( M_{ki}^f \) has the form:

\[
M_{ki}^f = \int d^3r \langle \mathbf{k}\beta | \hat{a}_i^\dagger | 0 \rangle \langle \phi_f | \hat{a}_{\alpha\mathbf{r}} | \phi_i \rangle \Omega_e(r) e^{i\mathbf{q}\cdot\mathbf{r}} = \int d^3r \langle \phi_f | \hat{a}_{\alpha\mathbf{r}} | \phi_i \rangle \Omega_e(r) e^{i\mathbf{q}\cdot\mathbf{r}} \tag{6}
\]

Using the definition of the real space spectral function at finite temperature \[13\],

\[
A(r, r'; \omega') = \sum_{i,f} \frac{e^{-E_i/k_BT} + e^{-E_f/k_BT}}{Z} \times \langle \phi_i | \hat{a}_{\alpha\mathbf{r}}^\dagger | \phi_f \rangle \langle \phi_f | \hat{a}_{\alpha\mathbf{r}} | \phi_i \rangle \delta(h\omega' + \tilde{E}_f - \tilde{E}_i) \quad \tag{7}
\]

the Raman transfer rate can be rewritten as

\[
R_q(k, \omega) = \frac{2\pi}{\hbar} \int d^3r \int d^3r' \Omega_e(r) \Omega_e^*(r') e^{i\mathbf{q}\cdot(r-r')} \times n_F(\epsilon_{k\beta} - \hbar \omega - \mu) A(r, r'; \epsilon_{k\beta} - \hbar \omega - \mu). \tag{8}
\]

Here, the Fermi factor has the usual definition \( n_F(E) = 1/[1 + \exp(E/k_BT)] \).

**C. Spatially homogeneous geometry**

In the simplest case of a spatially homogeneous system with no trapping potential and spatially uniform Raman beams \( \Omega_e(r) = \Omega_e \), the Raman rate per unit volume can be rewritten in a simplified form:

\[
\frac{dR_q(k, \omega)}{dV} = \frac{2\pi}{\hbar} |\Omega_e|^2 n_F(\epsilon_{k\beta} - \hbar \omega - \mu) \times A(k - \mathbf{q}, \epsilon_{k\beta} - \hbar \omega - \mu). \tag{9}
\]

that only involves the momentum space spectral function

\[
A(k, \omega) = \sum_{i,f} \frac{e^{-E_i/k_BT} + e^{-E_f/k_BT}}{Z} \times |\langle \phi_f | \hat{a}_{\alpha\mathbf{r}} | \phi_i \rangle|^2 \delta(h\omega + \tilde{E}_f - \tilde{E}_i). \quad \tag{10}
\]

In the simplest case of a non-interacting gas at zero temperature \( T = 0 \), the spectral function \[10\] has the form

\[
A(k, \omega) = \delta(h\omega + \mu - \epsilon_{k\alpha}) \tag{11}
\]

The energy and momentum conservation condition that underlie \[9\] have a simple physical interpretation. Initially, the many-body system is in the state \( i \) of energy \( E_i^N \) of momentum \( \mathbf{k}_i \); the laser beams consist of \( N_{1,2} \) photons in the two beams of respectively frequencies \( \omega_{1,2} \) and momenta \( \mathbf{k}_{1,2} \). At the end of the Raman process, the many-body system has an energy \( E_f^N \) and a momentum \( \mathbf{k}_f^{N-1} \), while the laser beams contain respectively \( N_1 - 1 \) and \( N_2 + 1 \) photons. By energy and momentum conservation, one therefore has that

\[
E_i^N + N_1 \hbar \omega_1 + N_2 \hbar \omega_2 = E_f^{N-1} + (N_1 - 1) \hbar \omega_1 + (N_2 + 1) \hbar \omega_2 + \epsilon_{k\beta} \tag{12}
\]

\[
\mathbf{k}_i^N + N_1 \mathbf{k}_1 + N_2 \mathbf{k}_2 = \mathbf{k}_f^{N-1} + (N_1 - 1) \mathbf{k}_1 + (N_2 + 1) \mathbf{k}_2 + \mathbf{k}. \tag{13}
\]

which reduces to:

\[
E_i^N - E_f^{N-1} = \epsilon_{k\beta} - \hbar \omega \]

\[
\mathbf{k}_i^N - \mathbf{k}_f^{N-1} = \mathbf{k} - \mathbf{q} \quad \tag{14}
\]

As expected, these conditions perfectly correspond to the frequency and momentum values at which the spectral function \[10\] is evaluated.

**D. Local density approximation**

In view of concrete applications, it is useful to derive approximate formulas that can accurately describe the case of spatially selective Raman processes in trapped systems. Spatial selectivity is in fact a key advantage of Raman techniques over RF spectroscopy, as it allows to avoid inhomogeneous broadening effects by restricting the Raman out-coupling process to a limited region of space where the system can be seen as almost uniform. This possibility is even more important if the system presents several different phases with macroscopically different properties.
The simplest way to include the trapping potential \( V_{\alpha}(R) \) acting on the \( \alpha \) atoms and/or of the spatial profile of the Raman beams is to perform the so-called Local Density Approximation (LDA). This approximation is generally accurate as long as the characteristic length on which the properties of the system vary is much larger than all microscopic scales of the system.

Under this approximation, the Raman rate can be written as an integral over different regions of space,

\[
R_q(k, \omega) = \frac{2\pi}{\hbar} \int d^3 R \sum_{\alpha} n_F(\varepsilon_k - \hbar \omega - \mu) \times A(k - q, \varepsilon_k - \hbar \omega - \mu; \mu_R). \tag{15}
\]

the contribution of each spatial region is approximated by the prediction \( \phi \) for a homogeneous system with an effective chemical potential \( \mu_R = \mu - V_{\alpha}(R) \) that includes the effect of the trap. Here, \( A(k, \hbar \omega; \mu_R) \) is the spectral function \( \triangleq \) with the local chemical potential \( \mu_R \).

By pushing further the LDA approximation, one can obtain a simple formula for the Raman rate also in the presence of an external potential \( V_{\beta}(R) \) acting on the atoms in the \( \beta \) state,

\[
R_q(k, \omega) = \frac{2\pi}{\hbar} \int d^3 R \sum_{\alpha} n_F(\varepsilon_k - \hbar \omega - \mu) \times A(k - q, \varepsilon_k - \hbar \omega - \mu; \mu_R). \tag{16}
\]

The only difference with \( \phi \) is that the energy dispersion of \( \beta \) atoms now depends on position according to

\[
\varepsilon_k = \varepsilon_k + V_{\beta}(R). \tag{17}
\]

It is important to stress that the validity of \( \phi \) requires more stringent conditions on the measurement process than a standard LDA. First, the out-coupled \( \beta \) atoms must not significantly move during the Raman process nor be accelerated. Second, the trap potential for \( \beta \) atoms should be switched off as soon as possible after the Raman process in order to minimize the distortion of the \( k \)-space distribution of the out-coupled atoms induced by the evolution in the trapping potential.

Equation \( \phi \) is the key result of this section and will be the base of the calculations that we shall present in the following Sections.

E. Localized probe

For the sake of simplicity, we restrict our attention to the case of a pair of Raman beams at an angle \( \phi \) with transverse Gaussian envelopes (Fig.1),

\[
\begin{align*}
\mathbf{E}_1(r) &= E_1 e^{-z^2/2\sigma^2-y^2/2\sigma^2} \quad \text{(18)} \\
\mathbf{E}_2(r) &= E_2 e^{-z^2/2\sigma^2-(x \sin \phi - y \cos \phi)^2/2\sigma^2}. \quad \text{(19)}
\end{align*}
\]

The Raman coupling amplitude has therefore a Gaussian shape localized around \( r = 0 \)

\[
\Omega^\omega_c(r) = \frac{E_1^* \cdot d_{\gamma \alpha}(E_2^* \cdot d_{\beta \gamma})}{\Delta}. \tag{20}
\]

with a peak amplitude

\[
\Omega^\omega_c = \frac{(E_1^* \cdot d_{\gamma \alpha})(E_2^* \cdot d_{\beta \gamma})}{\Delta}. \tag{21}
\]

and a Gaussian matrix

\[
L = \begin{pmatrix}
\sin^2 \phi & \sin \phi \cos \phi & 0 \\
\sin \phi \cos \phi & 1 + \cos^2 \phi & 0 \\
0 & 0 & 2
\end{pmatrix}. \tag{22}
\]

Inserting this formulas into the general expression for the Raman rate \( \phi \) and then changing to relative \( \rho = r - r' \) and center-of-mass variables \( R = (r + r')/2 \), one obtains

\[
R_q(k, \omega) = \frac{2\pi}{\hbar} \int d^3 R d^3 \rho e^{-R^T L R/\sigma^2} e^{-\rho^T L \rho/4\sigma^2} \\
\times e^{i(q-k)\mu} n_F(\varepsilon_{k\beta} - \hbar \omega - \mu) A(R, \rho; \varepsilon_{k\beta} - \hbar \omega - \mu). \tag{23}
\]

For the sake of completeness, it is interesting to briefly discuss the case of a spatially homogeneous system in the absence of any trapping potential that is probed by tightly focussed Raman beams. This formula can be further simplified by performing the Gaussian integrals, which leads to

\[
R_q(k, \omega) = \frac{(2\pi)^4}{\hbar \det[L]} \int d^3 q' e^{-\sigma^2 (q - q')^T L^{-1} (q - q')} \times n_F(\varepsilon_{k\beta} - \hbar \omega - \mu) A(k - q', \varepsilon_{k\beta} - \hbar \omega - \mu). \tag{24}
\]

As compared to the result \( \phi \) for the spatially homogeneous geometry, the momentum distribution of the Raman out-coupled atoms is now smeared out by the convolution with the Fourier transform of the Raman extraction profile. As expected, this latter has a width of the order \( 1/\sigma \).

A faithful image of the spectral and momentum features of the system is then obtained simply by choosing a value of \( \sigma \) much larger than all microscopic length scales of the system. This condition is compatible with the spatial selection of an almost homogeneous region as soon as the system is macroscopic, i.e. has a much larger size than all microscopic length scales.

It is interesting to note that the Gaussian factor in \( \phi \) tends to a delta-function in the \( \sigma \to \infty \) limit, which perfectly recovers the results of Sec. \( \alpha \) for the spatially homogeneous system. On the other hand, all information on the momentum distribution is lost in an extremely localized Raman probe of size \( \sigma \ll k_F^{-1} \) which provides momentum-integrated images for a given \( \omega \).

III. NON-INTERACTING FERMIONS: RF VS. RAMAN

Once we have established a general theory of Raman spectroscopy, it is important to validate its performances on some specific examples whose physics is well-known
and under control. In this Section, we shall start from the simplest case of a non-interacting degenerate Fermi gas. In particular, we shall discuss the effect of the trapping potential and we shall demonstrate the advantages of Raman spectroscopy over RF spectroscopy in order to obtain spatially resolved information on the microscopic properties of the system even in the presence of a trap.

Throughout the whole section we shall use the parameters of the Boulder experiment: (i) a Fermi energy $E_F = h \times 9.4$ kHz, (ii) a Fermi wave vector $k_F = 8.6 \mu m^{-1}$, a total number of atoms $N = 3 \times 10^5$, (iv) a total size of the cloud $L = 190/k_F$, and (v) a temperature $T/T_F = 0.18$. As the total size of the cloud $L = 190/k_F$ is much larger than the only microscopic length scale $k_F^{-1}$, the system can be safely considered in the macroscopic regime where the Local Density Approximation is accurate.

In order to help the reader to extract the significant information on the dispersion of $\alpha$-state quasi-particles, all plots from now on (except the left panel of Fig. 1) will be given as a function of $E_s = \varepsilon_{k\beta} - \hbar \omega$ and $\mathbf{k} = \mathbf{k} - \mathbf{q}$.

### A. RF spectroscopy

Simulated plots for the RF signal are given in Fig. 2 for different trapping configurations. As usual for RF spectroscopy, the RF field acts uniformly on the whole atomic cloud and the transferred momentum is $\mathbf{q} = 0$, which implies $\mathbf{k} = \mathbf{k}$.

![Figure 2](image)

**FIG. 2:** Contour plot of the calculated RF intensity for a degenerate gas of non-interacting Fermions at $T/T_F = 0.18$. The color scale indicates the rate of transfer into a state of momentum $\mathbf{k}$ of the $\beta$ state as a function of the (rescaled) RF frequency $E_s = \varepsilon_{k\beta} - \omega$. The left (a) panel refers to a spatially homogeneous system. The (b,c) panels have been calculated within the Local Density Approximation: The central (b) panel refers to the case where both $\alpha$ and $\beta$ states experience the same harmonic trap potential. The right (c) panel refers to the case in which only the $\alpha$-atoms are trapped. The RF field is spatially uniform over the whole system.

The left panel Fig. 2(a) refers to the case of a spatially homogeneous system in the absence of any trapping potential. As the dispersion of the $\alpha$ and $\beta$ states of non-interacting Fermions are exactly parallel, the momentum-resolved RF signal peaks at the same value $\omega_{res} = \varepsilon_{k\beta} - \varepsilon_{k\alpha}$ independently of $\mathbf{k}$. Once we move to the $(\mathbf{k}, E_s)$ variables considered in the plot, the peaks lie on the dispersion $\varepsilon_{k\alpha}$ of the $\alpha$ state. The peak intensity is independent of $k$ and extends up to the edge of the Fermi sphere at $k_F$. The abrupt edge at $k_F$ is here smoothened by the finite temperature value $T/T_F = 0.18$.

![Figure 3](image)

**FIG. 3:** Momentum-integrated RF signal for a trapped gas; as in the central panel of Fig. 2(b), the trap potential is assumed to act in the same way on the two $\alpha$, $\beta$ atomic states. Differently from 1, the momentum integration is here performed along lines of fixed $E_s$. The calculated spectrum is in good agreement with the experimental observation by the Boulder group (Fig. 5b of Ref. [33]).

The effect of trapping is included in the central and right panels, Fig. 2(b,c). In the central panel, the $\alpha$ and $\beta$ states are assumed to experience the same trapping potential as in the Boulder experiment. Under this condition, the dispersions of the $\alpha$ and $\beta$ states remain parallel at a fixed distance, so that it is still possible to extract the $\alpha$ state dispersion following the peak of the signal intensity in the $(\mathbf{k}, E_s)$ plane. In contrast to the previous case, the peak intensity is however strongly dependent on $k$: while the low-momentum states are filled at all positions of the trap, only the center of the trap contributes in fact to the high-momentum states close to $k_F$. This implies that the peak intensity strongly decreases with $k$ and, in particular, it can be hardly visible in the region around $k_F$. This can be a serious issue when one is to assess the effect of interactions, which is generally most pronounced in the vicinity of the Fermi edge.

Note the qualitative agreement of the calculated signal of Fig. 2(b) with the experimental observations by the Boulder group shown in Fig. 3 of Ref. [33]. The agreement is even more striking when we compare the theoretical plot for the $\mathbf{k}$-integrated signal shown in Fig. 3 and the experimental result in Fig. 5b of Ref. [33].

The effect of the trap is even more dramatic if we consider the case where only the $\alpha$ state feels the trap potential while the $\beta$ atoms are untrapped. The distance between the $\varepsilon_{k\alpha}^R$ and $\varepsilon_{k\beta}^R$ dispersions is now strongly dependent on the position $\mathbf{R}$, which produces the sizable inhomogeneous broadening that is visible in Fig. 2(c): the lower boundary corresponds to the contribution of the central region of the trap, while the upper boundary corresponds to the edges of the cloud. In this case, it ap-
pears to be difficult to extract useful information on the 
\( \varepsilon_{\mathbf{k}\alpha} \) dispersion from such a broadened RF signal.

**B. Raman spectroscopy**

The advantage of the Raman spectroscopy technique over the RF one is clearly visible in the simulated Raman signal that is plotted in Fig. 4. For the parameters of the Boulder experiment of Ref. 33, a transverse size of the Raman beams \( \sigma = 15/k_F \approx 1.74 \mu m \) well satisfies the inequalities \( L \gg \sigma \gg k_F^{-1} \). In particular, we consider the Raman beams to be focussed onto the central region of the trap and to have an angle \( \phi = \pi/2 \) between them. Their wavelength is assumed to be in the optical range, e.g., \( \lambda = 0.7 \mu m \). The momentum transferred to the atoms during the Raman process is then \( q = 2 \sin(\phi/2)/\lambda \approx 2 \mu m^{-1} \), much smaller than the Fermi momentum \( q/k_F \approx 0.2 \).

![Image](315x360 to 453x496)

**FIG. 4:** Contour plot of the Raman rate for a trapped gas with the same parameters as in the Boulder experiment and a beam transverse size \( \sigma = 15/k_F \). Left panel: raw data in the \( (k, \omega) \) plane. Right panel: Translated data in the \( (|\mathbf{k} - \mathbf{q}|, E_s) \) plane.

Raw data for the momentum-resolved Raman rate as a function of \( \omega \) and \( k \) are plotted in the left panel Fig. 4(a). In order to facilitate physical understanding of the quasiparticle dispersion, the same data are plotted in the right panel as a function of the rescaled variables \( E_s = \varepsilon_{\mathbf{k}3} - \omega \) and \( \mathbf{k} = \mathbf{k} - \mathbf{q} \). Even though we are dealing with a trapped gas, the observed signal is now very similar to the one of the homogeneous system shown in Fig. 2(a): this proves the utility of the spatial selectivity of Raman spectroscopy in view of extracting information on the local microscopic properties of a trapped gas. The broadening due to the finite beam size \( \sigma = 15/k_F \) is almost irrelevant on the scale of the graphs.

**IV. STRONGLY CORRELATED FERMIONS**

To better understand the advantage of Raman spectroscopy over the RF technique, we now extend our analysis to a remarkable example of strongly interacting fermionic system. Inspired by the Boulder experiment of Ref. 33, we consider (i) a Fermi gas in the unitary limit \((k_F a_s)^{-1} = 0\) at a temperature close to the superfluid critical temperature where the physics is dominated by a pseudogap, and (ii) a molecular gas in the BEC regime \((k_F a_s)^{-1} > 1\). In both cases, we will show how the Raman spectroscopy is able to provide useful information on the quasi-particle spectrum of the system.

**A. Preformed pairs at unitary limit**

At a temperature just below the superfluid critical temperature \( T/T_c = 0.9 \), the superfluid fraction and the superfluid order parameter are still very small. Yet, a sizable pairing gap appears in the spectral function due to the presence of preformed pairs. Remarkably, such a pseudo-gap has a width of the order of the Fermi energy in the critical region and persists even at temperatures well above the critical point.

![Image](b)

**FIG. 5:** Left (a) panel: Pseudogap model for a Fermi gas in the unitary limit. The black line is the free particle dispersion while the blue and red curves are the BCS quasi-particle branches. Central (b) and right (c) panels: contour plots of the RF rate for a spatially homogeneous system at low \( T/T_F = 0.05 \) and intermediate \( T/T_F = 0.4 \) temperatures.

A reasonable picture of the spectral function in the presence of a pseudogap is obtained using the standard BCS theory with the pseudogap \( \Delta_{ps} \) replacing the standard superfluid gap \( \Delta_{SF} \),

\[
A(k, \omega) = u_k^2 \delta(\hbar \omega + \mu - E_k^+) + v_k^2 \delta(\hbar \omega + \mu - E_k^-) \tag{25}
\]

The quasiparticle energy dispersion \( E_k^\pm \) has the usual BCS form

\[
E_k^\pm = \mu \pm \sqrt{(\varepsilon_{\mathbf{k}\alpha} - \mu)^2 + \Delta_{ps}^2}, \tag{26}
\]

as well as the Bogoliubov coefficients \( u_k^2 \) and \( v_k^2 \),

\[
u_k^2 = \frac{1}{2} \left[ 1 \pm \frac{\varepsilon_{\mathbf{k}\alpha} - \mu}{\sqrt{(\varepsilon_{\mathbf{k}\alpha} - \mu)^2 + \Delta_{ps}^2}} \right]. \tag{27}
\]
The RF and Raman signals for a spatially homogeneous Fermi gas at unitarity are readily obtained by inserting the spectral function (25) into the general expression (26).

At very low temperatures $k_B T/\Delta_{ps} \ll 1$, only the second term contributes to (26): the density of unpaired atoms is in fact exponentially suppressed by the factor $\exp(-\Delta_{ps}/k_B T) \ll 1$. As shown in Fig.5(b), the RF rate then peaks on a single branch in the $(k, E_s)$ plane corresponding to the lower BCS branch at $E_s^-$. This branch is strong up to the edge of the Fermi sphere. The long tail at high momenta is due to the particle-hole mixing characteristic of BCS theory that is visible in the expression (27) of the $\nu^r$ Bogoliubov coefficient.

When the temperature gets higher, a second branch appears that corresponds to the upper branch of the BCS dispersion at $E_s^+$. The weight of this branch rapidly grows with temperature as $\exp(-\Delta_{ps}/k_B T) \ll 1$. As one can see in Fig.5(c), the combined effect of the $\nu^r$ coefficient and of the Fermi factor makes the intensity of this branch to be concentrated in the region close to the pseudo-gap where most of the unpaired particles are found.

\begin{align}
\Delta_{ps}(r) &= \Delta_{ps}(0) \left(1 - \frac{V_\alpha(r)}{\mu}\right).
\end{align}

(28)

The broadening of the line that is apparent at $k = 0$ results from the spatial variation of the minimum of the quasi-particle dispersion, $E_{min} = \mu - V_\alpha(r) - \sqrt{\mu - V_\alpha(r)^2 + \Delta^2}$. The second branch that around $k/k_F \approx 0.8$ emerges from the Bogoliubov dispersion in the upwards direction is due to the reduced gap amplitude in the outer part of the cloud and to the corresponding unpaired atoms. The qualitative agreement of theoretical data with the experiment is satisfactory. A more quantitative comparison between theory and experiment (Fig.5c of Ref.33) is successfully made on the momentum-integrated spectral density shown in Fig.7.

The advantage of the Raman spectroscopy technique over the RF one is apparent in Fig.6(b), where the simulated signal for a Raman probe localized in the central part of the trap is plotted. As expected, the inhomogeneous broadening effects disappear and the signal closely follows the Bogoliubov branch. Furthermore, the intensity of the branch is everywhere determined by the Bogoliubov $\nu^r$ coefficient and does not involve any spatial average. For the parameters considered here ($T/T_F = 0.18, \Delta \sim E_F$), the intensity of the upper Bogoliubov branch of unpaired atoms due to the thermal excitations is relatively small.

B. Tightly bound molecules in BEC limit

The same BCS theory that we have used in the previous section to describe the unitary gas can be extended to the BEC side where the gas is constituted by tightly bound molecules. The spectral function has the form (25), but the gap $\Delta$ and the chemical potential $\mu$ appearing in the quasi-particle branches (26) have to be calculated by means of the self-consistency relations of BCS theory.

In the simplest case far in the BEC limit $k_F a_s \ll 1$ at
\[ T \simeq 0, \text{one has:} \]
\[
\mu = -\frac{E_b}{2} + \frac{2}{3\pi} E_F (k_F a_s) \tag{29}
\]
\[
\Delta = \sqrt{\frac{16}{\pi}} \frac{E_F}{\sqrt{k_F a_s}} \tag{30}
\]
\[
E_b = \frac{\hbar^2}{ma_s^2}. \tag{31}
\]
which corresponds to a pair of quasi-particle branches
\[
E^+ \kappa \simeq \varepsilon_{\kappa a} + \frac{12\pi \hbar^2 n a_s}{m} \tag{32}
\]
\[
E^- \kappa \simeq -E_b - \varepsilon_{\kappa a} - \frac{10\pi \hbar^2 n a_s}{m}. \tag{33}
\]

The upper branch corresponds to thermally excited unpaired atoms. As a consequence of the (repulsive) potential of molecules, the branch is slightly blue-shifted with respect to the free atom dispersion. The lower branch corresponding to molecules is separated by the upper one due to trapping that would otherwise disturb the RF signal and has an inverted dispersion with negative effective mass. As suggested in [33], this peculiar fact has a transparent physical interpretation: eliminating a \(\alpha\) atom of wavevector \(k\) means that one has to break a molecule and eventually leaves the system with an unpaired \(\alpha'\) atom at a momentum \(-k\). Not only this requires the binding energy \(E_b\) of the molecule, but also the kinetic energy \(\varepsilon_{-k\alpha'}\) of the unpaired atom.

\[ \text{V. INVERSE RAMAN SPECTROSCOPY} \]

All the discussion so far has been focussed on the case where the state \(\beta\) is initially empty: in this case, momentum-resolved photo-emission spectroscopy is able to provide detailed information only on those bands that are initially occupied. This fact is apparent in the \(n_F(\varepsilon_{k\beta} - \hbar \omega - \mu)\) factor multiplying the spectral function \(A\) in the equation for the Raman rate (9).

In many cases of current experimental interest, it is however desirable to have experimental access to the dispersion of empty bands as well. This is particularly interesting at very low temperatures, where only the “negative energy” states below the chemical potential are occupied, but much interesting physics is contained in the “positive energy” ones above the chemical potential. For instance, in the BCS-like case discussed in Sec [IV], observation of both \(E^\pm_k\) branches would provide unambiguous information on the amplitude of the superconducting gap.

Information on the empty bands can be obtained if both direct \(\alpha \to \beta\) and reverse \(\beta \to \alpha\) Raman processes can be induced by the same pair of laser beams. This is the case if some incoherent population of atoms is already present in the \(\beta\) state at the beginning of the Raman experiment: the relative occupation of the initial and final states determines whether the direct or the reverse Raman process will dominate. The resulting signal results from the difference of the two photo-emission and photo-absorption processes and is quantified by the rate of increase/decrease of the population of the \(k\) momentum state of the \(\beta\) level. Within the same approximation performed in the previous sections of the paper, the rate \(\rho\) is easily generalized to include also reverse Raman processes:

\[
\frac{dR_q(k, \omega)}{dV} = \frac{2\pi}{\hbar} \left| \Omega_\varepsilon \right|^2 A(k - q, \varepsilon_{k\beta} - \hbar \omega - \mu) \times \left[ n_F(\varepsilon_{k\beta} - \mu) - n_F(\varepsilon_{k\beta} - \mu_{\beta}) \right]. \tag{34}
\]

The ideal Fermi gas in the \(\beta\) state is here assumed to have a thermal distribution at same temperature \(T\) as the gas in the \(\alpha\) state, and a chemical potential \(\mu_{\beta}\). Generalization of (34) to the case of an arbitrary occupation law is done by simply replacing the occupation factor \(n_F(\varepsilon_{k\beta} - \mu_{\beta})\). In the \(n_F(\varepsilon_{k\beta} - \mu_{\beta}) = 0\) limit, equation (34) reduces to (9).

In the \(q = 0\) case of RF spectroscopy, probing the upper BCS band requires that a sizable population is present in the \(\beta\) state up to high momenta. At low temperatures, this requires that the Fermi momentum \(k_{F\beta}\) of the \(\beta\) state is larger than the one \(k_F\) of the \(\alpha\) state, i.e. that there are initially more atoms in the \(\beta\) state than in the \(\alpha\) one. Examples of combined photo-emission and photo-absorption spectra of the \(q = 0\) case are shown in Fig [V] for the BCS-pseudogap model of strongly correlated Fermions discussed in Sec [V] and described by the spectral function (25). Different panels refer to different values of the initial population of the \(\beta\) state (left to right) and to different temperatures (up to down).
FIG. 9: Photo-emission spectroscopy with \( q = 0 \) for different temperatures \( T = 0.05T_F \) (upper panels), \( T = 0.4T_F \) (lower panels) and for different initial populations of \( \beta \)-atoms \( k_{F\beta} = 0 \) (left panels), \( k_{F\beta} = 1.73k_F \) (right panels). In all panels, the purple background color corresponds to a vanishing photo-emission and photo-absorption intensity. The yellow color indicates the occurrence of direct, photo-emission processes from \( \alpha \) into \( \beta \); the blue color indicates the occurrence of reverse, photo-absorption processes from \( \beta \) into \( \alpha \).

The left panels correspond to an initially empty \( \beta \) state: at \( T/T_F = 0.05 \) (upper panel), only the lower BCS branch at \( E_k^- \) is visible as a positive, photo-emission signal (yellow). At higher \( T/T_F = 0.4 \) (lower panel), also the upper BCS branch at \( E_k^+ \) appears in the spectrum again as a positive signal. The right panels correspond to a highly degenerate \( \beta \) state where the lowest \( \beta \) states have an almost unity filling \( k_{F\beta} = 1.73k_F \). At low temperatures \( (T/T_F = 0.05) \), photo-emission from the \( E_k^- \) is strongly suppressed by Pauli blocking, while the upper band at \( E_k^+ \) clearly appears as a negative, photo-absorption signal (blue). At higher temperatures, both bands are visible due to thermal broadening.

The difficulty of having a high initial density of atoms in \( \beta \) state can be overcome by adopting a Raman scheme with a transferred wave vector \( q \) comparable to \( k_F \) (see Fig.10). In this case, the population in the almost full lowest \( \beta \) states can be effectively transferred into the empty \( \alpha \) states in the most interesting region \( k \simeq k_F \) around the superconducting gap.

Simulated spectra for higher values of the transferred momentum \( q = 0.5k_F \) (left) and \( q = k_F \) (right) are shown in Fig.10. At low temperature \( T/T_F = 0.05 \) (upper panel), the photo-emission processes from the lower \( E_k^- \) branch are suppressed by Pauli blocking in the momentum range \( [-k_{F\beta}-q,k_{F\beta}-q] \). By contrast, the photo-absorption processes into the upper \( E_k^+ \) branch clearly appear in this momentum range as a negative signal. At a higher temperature \( T/T_F = 0.4 \) (lower panel), both processes are broadened over the whole range of momentum by thermal effects.

The comparison between \( q = 0.5k_F \) (left panels) and \( q = k_F \) (right panels) shows that the double branch structure is most visible when the transferred momentum \( q \) is closest to the Fermi momentum \( k_F \): in this case, the value of the pseudo-gap amplitude of the BCS model can be easily extracted from the distance in frequency between the two features.

VI. CONCLUSIONS AND PERSPECTIVES

In conclusion, we have proposed a momentum-resolved Raman spectroscopy technique that, in analogy to the Angle-Resolved Photo-Emission Spectroscopy of solid state physics is able to probe the one-body properties of an atomic gas. The power of this technique to measure the dispersion of the filled quasi-particle states has been illustrated on a number of simple systems ranging from an ideal Fermi gas to a strongly correlated one and has been validated on existing experimental data.

Several advantages over previous techniques (in particular RF spectroscopy) have been pointed out: the use of focussed Raman beams enables one to eliminate the inhomogeneous broadening due to the trap potential by restricting the optical process to a limited spatial region. A large value of the transferred momentum can also help to purify the measured spectra by suppressing the effect...
of interactions between the photo-emitted atoms and the rest of the cloud.

A direct extension of the technique to the case where an incoherent population is already present in the final state of the Raman process is finally proposed: Raman processes take place in both directions and reverse Raman spectroscopy can be used to obtain information on the empty quasi-particle states lying above the chemical potential.

Thanks to these remarkable features, we expect that momentum-resolved Raman spectroscopy will play an important role in the experimental characterization of the variety of novel quantum states of matter that can be obtained in systems of strongly interacting ultracold atoms, e.g., fermionic Mott insulator states, d-wave superconductors, anti-ferromagnetic states.

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