Quantum impurity spin in Majorana edge fermions

Ryuichi Shindou,1 Akira Furusaki,1 and Naoto Nagaosa2,3

1Condensed Matter Theory Laboratory, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
2Department of Applied Physics, University of Tokyo, 7-3-1 Hongo, Tokyo 113-8656, Japan
3CMRG and CERG, RIKEN-ASI, Wako 351-0198, Japan

(Dated: November 5, 2010)

We show that Majorana edge modes of two-dimensional spin-triplet topological superconductors (superfluids) have Ising-like spin density whose direction is determined by the $d$-vector characterizing the spin-triplet pairing symmetry. Exchange coupling between an impurity spin ($S = \frac{1}{2}$) and Majorana edge modes is thus Ising-type. Under external magnetic field perpendicular to the Ising axis, the system can be mapped to a two-level system with Ohmic dissipation, which is equivalent to the anisotropic Kondo model. The magnetic response of the impurity spin can serve as a local experimental probe for the order parameter.

PACS numbers:

Majorana fermions are fermionic particles that are their own antiparticle. Originally proposed long ago to describe neutrinos in high energy physics,1 Majorana fermions have recently been a subject of intensive studies in condensed matter physics.2,3 Their mixed nature of being particle and antiparticle implies that Majorana fermions may emerge as elementary excitations in superconductors and superfluids where the number of particles is not well-defined. Indeed they are theoretically predicted to appear as gapless boundary excitations of topological superconductors/superfluids with spin triplet pairing.4,5 Candidates of such topological materials include superfluid phases of $^3$He−, the superconducting states of Sr$_2$RuO$_4$,2,3 and possibly some non-centrosymmetric superconductors.6 However, being "real-part" of ordinary (complex) fermions, Majorana fermions are charge neutral and only weakly interacting with other particles; they are hard to detect. To probe and control Majorana fermions is therefore a great challenge.

In this paper we study a quantum impurity spin coupled with Majorana edge modes with spin degree of freedom. We first argue that a distinct signature of Majorana edge fermions of a generic two-dimensional (2D) spin-triplet topological superconductor (superfluid) is Ising character of their spin density.6,10 To probe this Ising spin, we consider a spin-$\frac{1}{2}$ magnetic impurity coupled to the Majorana edge modes. We show that the impurity spin has strongly anisotropic and singular magnetic response which is due to quantum dissipation from the Majorana edge modes. We propose that electron spin resonance (ESR) can serve as a novel local probe for the Majorana fermions in spin-triplet topological superconductors and superfluids.

Let us take a closer look at the Majorana edge modes of 2D spin-triplet topological superconductors. We first consider the BdG Hamiltonian of a prototypical topological superconductor, a spinless chiral $p$-wave superconductor with $p_x \pm ip_y$ symmetry:7

$$\mathcal{H}_\pm^0 = \begin{pmatrix} -\hbar^2 \partial^2 - \mu & e^{i\vartheta} & \frac{e^{i\vartheta}}{2ik_F} \Delta(r), \partial_\mp \\ e^{-i\vartheta} & \frac{e^{-i\vartheta}}{2ik_F} & \frac{\hbar^2}{2m} \partial^2 + \mu \end{pmatrix}.$$ 

Here $r = (x, y)$, $\partial_\pm = \partial_x \pm i \partial_y$, $\partial^2 = \partial_x \partial_x + \partial_y \partial_y$, $k_F$ the Fermi wave number, $\mu = \hbar^2 k_F^2 / 2m$, and $\vartheta$ is a U(1) phase. In the bulk superconductor with a spatially uniform pair potential $\Delta(r) = \Delta$, quasiparticle spectrum is fully gapped. When the superconductor has a boundary, the BdG Hamiltonian has a gapless edge mode with linear energy dispersion $E(k) = \pm kF$ ($|k| < k_F$). For a straight boundary defined by $X = x \cos \phi + y \sin \phi = 0$, we solve the BdG Hamiltonian on the half plane $X < 0$, assuming fixed boundary conditions at $X = 0$. We then obtain the edge mode’s wave function

$$\begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = e^{\pm i kX} w_k(X) \begin{pmatrix} e^{i(2\vartheta \pm 2\phi + \pi)/4} \\ e^{-i(2\vartheta \pm 2\phi + \pi)/4} \end{pmatrix}.$$ 

Here $Y$ is the coordinate along the edge, $Y = -x \sin \phi + y \cos \phi$, and $w_k(X)$ is the normalized real-valued wave function localized at the edge. The particle-hole symmetry, $\mathcal{P}\mathcal{H}_\pm^0\mathcal{P} = -\mathcal{H}_\pm^0$, with $\mathcal{P} = \sigma_1 K$, guarantees that $(v^*_k, u_k) = (u_{-k}, v_{-k})$ is also an eigenmode with energy $-v_k$, where $K$ is complex conjugation and $\sigma_j$ is the $j$th component of the Pauli matrices. The mode expansion of the Nambu field $(\psi, \psi^\dagger)^t$ for $|E| < \Delta$ is then given by

$$\begin{pmatrix} \psi(r) \\ \psi^\dagger(r) \end{pmatrix} = \int_{-k_F}^{k_F} dk \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} + \begin{pmatrix} v^*_k(r) \\ u^*_k(r) \end{pmatrix},$$

which leads to the following condition of Majorana type

$$\psi(r) = ie^{i\vartheta \pm i\phi} \psi^\dagger(r).$$

There are two distinct types of 2D spin-triplet topological superconductors: (a) chiral type and (b) helical type. They can be obtained, for example, by combining two copies of the spinless chiral $p$-wave superconductors of same or opposite chiralities; their BdG
Hamiltonians are given by $H^{\delta}_k \oplus H^{\delta}_{\perp}$ and $H^{\delta}_k \oplus H^{\delta}_{\parallel}$, respectively. The two types of 2D spin-triplet topological superconductors are characterized by the order parameter $\Delta_k = d_k \cdot \sigma \sigma_2$ with the $d$-vector $d_k$.

$$d_k = \left\{ \begin{array}{ll} (\hat{x} \sigma_3 + \hat{y} \sigma_2)(k_x + i k_y), & \text{(chiral)}, \\ (\hat{x} \sigma_3 + \hat{y} \sigma_2)k_x + (2 \hat{x} - \hat{y} \sigma_2)k_y, & \text{(helical)}. \end{array} \right. \quad (5)$$

Here $(\sigma_3, \sigma_2) \equiv (\cos \theta, \sin \theta)$, $\theta = \frac{1}{2}(\theta_1 - \theta_2)$, and $\hat{x}$ and $\hat{y}$ are unit vectors in the spin space. Notice that the direction of $d_k$ is independent of $k$ in the chiral case, while it constitutes a coplanar spin texture in the $k$-space in the helical case.

The two types of 2D spin-triplet topological superconductors support gapless Majorana edge modes, each spin component of which obeys Eq. (1): (a) $\psi_\sigma(r) = i e^{i \theta_\sigma + i \phi} \psi_\sigma^\dagger(r)$ ($\sigma = \uparrow$ and $\downarrow$) for the chiral case, and (b) $\psi_\uparrow(r) = i e^{i \theta_\uparrow + i \phi} \psi_\uparrow^\dagger(r)$ and $\psi_\downarrow(r) = i e^{i \theta_\downarrow - i \phi} \psi_\downarrow^\dagger(r)$ for the helical case. These Majorana conditions lead to the operator identities for the edge mode’s spin density, $2 \hat{s}_z(r) = \psi_\uparrow^\dagger \psi_\downarrow - \psi_\downarrow^\dagger \psi_\uparrow = 0$, (6a) $\hat{s}_+(r) = \psi_\uparrow^\dagger \psi_\downarrow = \left\{ \begin{array}{ll} -e^{-2i \theta} \hat{s}_-(r) & \text{(chiral)}, \\ -e^{-2i (\theta + \phi)} \hat{s}_-(r) & \text{(helical)}. \end{array} \right. \quad (6b)$

These equations imply that the spin density is always Ising-like: $(\hat{s}_z, \hat{s}_x) \propto (s_\sigma, c_\sigma)$ for the chiral case and $(\hat{s}_z, \hat{s}_y) \propto (s_\theta + \phi, c_\theta + \phi)$ for the helical case. Comparing Eq. (5) with Eq. (6b), we can see that this Ising direction is dictated by the $d$-vector in the bulk as,

$$\hat{s}(r) \propto \left\{ \begin{array}{ll} d_k & \text{(chiral),} \\ d_X & \text{(helical),} \end{array} \right. \quad (7)$$

where $X = (\cos \phi, \sin \phi)$ is a vector normal to the boundary. In the helical case, $\hat{s}(r)$ depends on the direction of the boundary, as the rotation in the real space is transcribed into that in the spin space. Otherwise, it does not depend on the shape of the boundary.

The Ising-like spin density (7) is a hallmark of the Majorana edge modes. Its strong anisotropy reflects the spin-triplet pairing symmetry in the bulk topological superconducting order. In the following we will study quantum dynamics of a magnetic impurity (probe spin) interacting with this Ising spin density.

The Hamiltonian for the coupling between the Majorana Ising spin density $\hat{s}(r)$ and a spin $\frac{1}{2}$ probe at $r = 0$, $S = (\hat{S}_z, \hat{S}_y, \hat{S}_x)$, is given by

$$H_{\text{ex}} = iJ \hat{S}_x \psi_\uparrow^\dagger(t) \psi_\downarrow(0). \quad (8)$$

Here the probe spin’s $S_z$ direction is taken to be parallel to $\hat{s}(0)$, and $\psi_\uparrow(Y)$ is a one-dimensional (1D) Majorana field satisfying $\psi_\uparrow^\dagger = \psi_\downarrow$ and $\{\psi_\sigma(Y), \psi_\sigma^\dagger(Y')\} = \delta_{\sigma, \sigma'} \delta(Y - Y')$. The 1D Majorana field is obtained from $\psi_\sigma(r)$ by appropriate U(1) gauge transformation and dropping unimportant $X$-dependence [i.e., $w_k(X) \rightarrow 1$].

The Kondo coupling in Eq. (8) can be obtained from the Anderson impurity model,

$$H_{\text{imp}} = \sum_{\sigma, \epsilon} \left\{ \epsilon \sigma n_{\epsilon, \sigma} + t \left[ \tilde{d}_\sigma \tilde{\psi}_\sigma(0) + \text{h.c.} \right] \right\} + U n_{\downarrow} n_{\uparrow},$$

with $n_{\downarrow, \uparrow} = d^\dagger \sigma d_{\sigma}$, by the standard procedure. This yields $S_z \equiv -2 d^\dagger_\sigma \sigma_y \sigma_z d_\sigma$ and $J = 2 eU/[\{U - e\phi\} \epsilon_d] > 0$. The kinetic energy of the Majorana edge modes reads

$$H_{\text{kin}} = i\nu \int_{-\infty}^{\infty} dY \left( \dot{\psi}_\uparrow \gamma_1 \psi_\uparrow \pm \dot{\psi}_\uparrow \gamma_2 \psi_\downarrow \right), \quad (9)$$

where the $+/-$ signs in the integrand are for the chiral/helical superconductors, respectively. The ground state of $H_{\text{kin}} + H_{\text{ex}}$ is doubly degenerate, $S_z = \pm \frac{1}{2}$.

We will make full use of the knowledge from earlier studies on dissipative two-state systems and Kondo effect to show that the impurity spin, when subjected to external magnetic fields, exhibits anisotropic dissipative quantum dynamics due to the “background” Majorana edge modes.

Consider first that a magnetic field is applied perpendicular to the Majorana Ising spin, say along the $x$-direction; the Hamiltonian reads

$$H = H_{\text{kin}} + H_{\text{ex}} + h \hat{S}_x. \quad (10)$$

Interestingly, the system undergoes a quantum phase transition, depending the exchange coupling $J$ (Fig. 1). To see this, let us map the Hamiltonian onto the Ohmic dissipative two-state system \cite{16,17},

$$H_{\text{en}} = \nu \int_{-\infty}^{\infty} (\partial_\tau \Phi)^2 dy + \frac{J}{\sqrt{2\pi}} \dot{S}_z (\partial_\tau \Phi)|_{\tau=0} + h \dot{S}_z, \quad (11)$$

where the bosonic field $\Phi(x)$ obeys $[\Phi(x), \partial_\tau \Phi(y)] = i\delta(x - y)$. We have combined the two species of chiral Majorana fields, $\{\psi_\uparrow, \psi_\downarrow\}$, into a single spinless chiral fermion, $\Psi(y) = [\psi_\uparrow(y) + i \psi_\downarrow(\pm y)]/\sqrt{2}$, where the $+/-$ sign refers to the chiral/helical superconductors, respectively. The Ising spin density at $r = 0$ is then reduced to the fermion number density $\Psi^\dagger(y) \Psi(y)$. The complex fermionic field is readily bosonized in terms of the phase operator, $\Psi(x) \propto \exp[i\sqrt{2\pi} \Phi(y)]$, which transforms Eq. (10) into Eq. (11). The Ising exchange coupling $J$ and the transverse field $h$ control the coupling to the Ohmic bath and tunneling between the two states ($S_z = \frac{1}{2}, -\frac{1}{2}$), respectively. The dissipative two-state system is related to the anisotropic Kondo model \cite{16,17},

$$H_K = -2iv \int dy \left[ \Psi_\uparrow^\dagger(y) \partial_\tau \Psi_\uparrow(y) + \Psi_\downarrow^\dagger(y) \partial_\tau \Psi_\downarrow(y) \right] \quad (12)$$

$$+ J_\perp \left( \hat{S}_+ \Psi_\uparrow^\dagger(0) \Psi_\uparrow(0) + \hat{S}_- \Psi_\downarrow^\dagger(0) \Psi_\downarrow(0) \right)$$

$$+ J_z \dot{S}_z \left[ \Psi_\uparrow^\dagger(0) \Psi_\uparrow(0) - \Psi_\downarrow^\dagger(0) \Psi_\downarrow(0) \right],$$

where $\Psi_\uparrow, \Psi_\downarrow$ are complex spinful fermionic fields and $\hat{S}_\pm = i \hat{S}_x \pm \hat{S}_y$. The coupling constants are related by

$$h \tau = \frac{J_\perp}{4\pi v}, \quad \frac{J_z}{2\pi v} = \sqrt{2} \left( \frac{J_z}{2\pi v} - 2 \right). \quad (13)$$
When Ising exchange coupling \( J_z \) and transverse field \( h \) are related to the transverse field \( h \) and the Ising exchange coupling \( J \), respectively. With the help of these mappings, the renormalization group (RG) equations for \( \eta \equiv \hbar \tau \) and \( \epsilon \equiv (J/4\pi v)^2 \) can be readily obtained:

\[
\frac{d\epsilon}{d\ln \tau} = -4\epsilon \eta^2, \quad \frac{d\eta}{d\ln \tau} = \frac{1}{2}(\epsilon + 2\eta). \tag{14}
\]

These scaling equations lead to the RG flow diagram in Fig. 1 which has a fixed point at \((J, h) = (0, \infty)\) and a line of fixed points at \( h = 0 \) for \(|J| > 4\sqrt{2}\pi v \equiv J_c (\epsilon > 2)\). When \(|J| < J_c\), the transverse field \( h \) is a relevant perturbation, and the system flows toward the high-field fixed point. In this case the ground state is a quantum mechanical superposition of \( S_z = +\frac{1}{2} \) and \(-\frac{1}{2}\) states, corresponding to the antiferromagnetic Kondo singlet. When \(|J| > J_c\), on the other hand, the orthogonality catastrophe of Majorana edge excitations suppresses the tunneling, thereby making a small transverse field \( h \) irrelevant. For small fields \( h < h_c \), where

\[
h_c = \frac{\Delta}{\sqrt{2}} \left( \frac{|J|}{4\pi v} - \sqrt{2} \right), \tag{15}
\]

the RG flows end at the zero-field fixed point; this corresponds to the ferromagnetic Kondo regime, \( J_z \sim -|J| \). For large enough fields \( h > h_c \), the system is eventually renormalized to the strong-coupling regime (see Fig. 1). The crossover from weak- to strong-field regime occurs around the Kondo temperature \( T_K \) given as:

\[
T_K = \begin{cases} 
\Delta \exp(-b/\sqrt{h-h_c}), & |J| > J_c \land h > h_c, \\
(h/\Delta)^{1/(2-\epsilon)}, & |J| < J_c,
\end{cases} \tag{16}
\]

where \( b = \pi\Delta/\sqrt{8h_c} \).

To see this, let us consider the dynamical susceptibility \( \chi_{zz}(\omega) = -\text{Im} \chi_{zz}(\omega) = \int_0^\infty dt e^{i(\omega+it)} \langle [\hat{S}_z(t), \hat{S}_z(0)] \rangle \), which is a Fourier transform of the time-evolution of the magnetization, \( P(t) = \langle \hat{S}_z(t) \rangle \), studied in the context of the dissipative two-state system.\(^{16,17}\) The transition spectrum is obtained from the imaginary part of \( \chi_{zz}(\omega) \) and has a universal scaling form which depends on \( \epsilon \), \( S_{zz}(\omega) := \text{Im} \chi_{zz}(\omega)/\omega = f_\epsilon(\omega/T_K) \). Its qualitative feature is well understood (for small \( h/\Delta \)\(^{16,17,19}\)) as we briefly summarize below.

In the weakly dissipative regime \( \epsilon < \frac{\Delta}{2} \), \( S_{zz}(\omega) \) has a peak at \( \omega \sim T_K \), which signifies coherent transitions between the bonding and anti-bonding states split by the renormalized transverse field (Fig. 2b). Stronger dissipation destroys the coherent tunneling; when \( \epsilon > \frac{\Delta}{2} \), the coherence is lost and \( S_{zz}(\omega) \) shows only a diffusive peak at \( \omega = 0 \) (Fig. 2c). The half-value width of this peak is on the order of \( T_K \).\(^{19}\) As the dissipation is further increased toward the ferromagnetic Kondo region, the diffusive peak gets sharper, as \( T_K \to 0 \) at \( \epsilon = 2 \).

In the ferromagnetic Kondo region, the spectral function at \( T = 0 \) has a \( \delta \)-function peak at \( \omega = 0 \).\(^{16,17}\) At finite temperature, or in the presence of a finite dc “bias” field \( h_z \), the width of the diffusive peak at \( \omega = 0 \) is finite, as \( T \) and \( h_z \) play a role of low-energy cutoff.\(^{16,17}\) The half-value width \( \omega_0 \) can be obtained by 2nd order perturbation in the transverse field \( h \), yielding \( \omega_0 \propto (h^2/\Delta)(T/\Delta)^{c-1} \) or \( (h^2/\Delta)(h_z/\Delta)^{c-1} \). A diffusive peak with the width of the same \( T \) and \( h_z \)-dependences also appears in the antiferromagnetic Kondo regime, when \( T > T_K \) or \( h_z > T_K \).

Next, we discuss response of the probe spin \( S \) to the transverse field \( h \) (while we set \( h_z = 0 \)), such as the magnetization \( M_x = \langle \hat{S}_x \rangle \) and the susceptibility \( \chi_{xx} = \partial M_x/\partial h \). This response is unique to our system and has not been considered in the related two models\(^{11,12}\) and \(^{12}\), as the Zeeman term \( h\hat{S}_z \) corresponds to the tunneling and the Kondo exchange interaction, respectively.

At the critical field \( h = h_c \) separating the ferromagnetic and antiferromagnetic Kondo phases, the transverse

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{(color online) Renormalization group flows.}
\end{figure}
magnetization $M_z$ shows only weak (essential) singularity, $M_z(h) - M_z(h_c) \propto \exp(-b/\sqrt{|h| - h_c})$. This singularity is like the specific heat anomaly at Kosterlitz-Thouless transition and is too weak to be observed. Thus $M_z$ increases monotonically as a function of $h$ without any hint of anomaly at $h = h_c$.

In the limit of small transverse field $h \ll \Delta$, the transverse magnetization $M_x$ increases linearly with $h$, when dissipation is strong, $\epsilon > 1$. The susceptibility $\chi_{xx}$ is diverging as $\epsilon^{-1}$ for $\epsilon \rightarrow 1$, and at the point $\epsilon = 1$, the linear $h$ dependence acquires a logarithmic correction, $M_x \propto -4h\Delta^{-1}\ln(h/\Delta) + \cdots$. This result can be obtained from the exact ground state energy in the Toulouse limit. In the weakly dissipative case $0 < \epsilon < 1$, the $h$ dependence becomes sublinear, $M_x \propto (h/\Delta)^{\epsilon/(2-\epsilon)}$. This indicates that the linear susceptibility diverges in the low-$T$ limit. However, this singularity is weaker than that of the Curie behavior and is given by

$$\chi_{xx}|_{h=h_c=0} = \frac{\Gamma\left(\frac{1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{3}{2}\right)} \sin\left(\frac{\pi T}{\Delta}\right) T^{-1+\epsilon}. \tag{17}$$

In the presence of dc longitudinal field $h_z\hat{z}$, a small ac transverse field $h_{\perp}\hat{x}$ induces transitions between the $S_z = \pm \frac{1}{2}$ states. The transition spectrum has a divergent edge singularity,

$$\text{Im} \chi_{xx}(\omega)|_{T=h=0} = \frac{c\pi}{\Gamma(\epsilon)} e^{-c'|\omega-h_c|/\Delta} \Theta(\omega-h_c). \tag{18}$$

for positive $\omega$, where $c$ and $c'$ are some positive constants. Equation (18) is also valid for $\epsilon \geq 1$.

Based on the results discussed so far, we propose that ESR measurements of the impurity spin may be used to identify the direction of the Majorana Ising spin. A dc field smaller than the (lower) critical field $H_{c(1)}$ will cause precession of only those probe spins located around the boundary (edge). An additional ac field (perpendicular to the dc field) will then induce resonant transitions. Figure 2 shows qualitative picture of the absorption spectra for two complementary experimental geometries, i.e., the dc field applied parallel (a,c) and perpendicular (b,d) to the Majorana Ising spin. The upper two panels (a,b) assume weak dissipation. The spectra show the strongest edge singularity at the Larmor frequency $h_z$, when the dc field is parallel to the Majorana Ising spin (a). When the spin precession is driven by the transverse field $h$ (b), the singularity is replaced by a resonance peak at $\omega = T_K = h(h/\Delta)^{\epsilon/(2-\epsilon)} \ll h$. The clear difference between the spectra measured in the two experimental geometries persists at stronger dissipation (compare c and d). We thus expect that the Ising direction can be identified in principle by measuring the absorption spectra for various directions of the dc field. Incidentally, anisotropy in the static susceptibility may also serve as a good indicator; it shows Curie-law along the Ising direction ($\chi_{zz}$), while it remains constant for $\epsilon < 1$ or diverges as $T^{\epsilon-1}$ for $\epsilon > 1$ in the perpendicular direction ($\chi_{xx} \equiv \chi_{yy}$).

In summary, we proposed to probe the massless Majorana edge modes of spin-triplet topological superconductors and superfluids, by introducing a quantum impurity spin to their boundary. Anisotropic magnetic responses of the probe spin directly reflect the Majorana nature of the edge excitations.

This work was supported by Grants-in-Aid for Scientific Research (Grants No. 17071007, 17071005, 19048008, 19048015, 21244053, and 21540332) from the MEXT and the JSPS, Japan. N.N. is supported also by Funding Program for World-Leading Innovative R & D on Science and Technology (FIRST Program).

---

1. E. Majorana, Nuovo Cimento 5, 171 (1937).
2. F. Wilczek, Nature Physics 5, 614 (2009).
3. L. Fu and C. Kane, Phys. Rev. Lett. 100, 096407 (2008).
4. A. Schynder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B 78, 195125 (2008); AIP Conf. Prof. 1134, 10 (2009).
5. A. Kitaev, AIP Conf. Proc. 1134, 22 (2009).
6. G. E. Volovik, The Universe in a Helium Droplet (Oxford University Press, Oxford, 2003).
7. Y. Maeno et al., Nature 372, 532 (1994).
8. M. Sato and S. Fujimoto, Phys. Rev. B 79, 094504 (2009); Y. Tanaka, T. Yokoyama, A. V. Balatsky, and N. Nagaosa, Phys. Rev. B 79, 060505(R) (2009).
9. M. Stone and R. Roy, Phys. Rev. B 69, 184511 (2004).
10. S. B. Chung and S. C. Zhang, Phys. Rev. Lett. 103, 235301 (2009).
11. N. Read and A. Green, Phys. Rev. B 61, 10267 (2000); D. A. Ivanov, Phys. Rev. Lett. 86, 268 (2001).
12 X. L. Qi, T. L. Hughes, S. Raghu, and S. C. Zhang, Phys. Rev. Lett. 102, 187001 (2009).
13 R. Roy, [arXiv:0803.2868](https://arxiv.org/abs/0803.2868).
14 A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).
15 P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B 1, 4464 (1970).
16 A. J. Leggett et al. Rev. Mod. Phys. 59, 1 (1987)
17 U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).
18 A. Furusaki and K. Matveev, Phys. Rev. Lett. 88, 226404 (2002).
19 T. A. Costi and C. Kieffer, Phys. Rev. Lett. 76, 1683 (1996); F. Lesage, H. Saleur, and S. Skorik, Phys. Rev. Lett. 76, 3388 (1996).
20 G. D. Mahan, *Many-Particle Physics* (Kluwer Academic, New York, 2000).