Determination of Effective Diameter of Solid Particles for the Eulerian–Eulerian Modelling Approach of Fluidized Bed.

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Abstract. The experimental and numerical study of common used effective diameters for fluidized bed at a wide range of gas velocities was carried out. This work was motivated by Eulerian approach to modelling multiphase media. In the conditions of the experiments carried out and the Gidaspowa drag model, the best agreement with the experiment was shown by the so-called modal effective diameter $D_{mod}$. For minor gas velocities, the diameter $D_{32}$ was corrected in order to best coincide with the experiment.

1. Introduction

One of the most important advantages of the fluidized bed apparatus is the high efficiency of heat-mass transfer processes. This fact makes such devices the most suitable for catalytic reactions [1, 2]. In this practical application, the solid particles used as the catalyst usually have different sizes, thus forming a polydisperse discrete phase.

Multi-phase Eulerian-Eulerian model in which the phases are treated as interacting and interpenetrating continua with semiempirical closure relations is widely used in numerical simulations of the fluidized bed. This approach requires the discretization of the particle characteristics. Particle characteristics can be discretized using the multi-group approach, where the probability distribution function (PDF) becomes a set of scalars which corresponds to an entire group of particle sizes. Thus, polydispersity can be simulated, in this case, the solving of transport equations is necessary for each group. The multi-group approach was used for numerical modelling of gas–solids flowin [3, 4]. In the previous work it is shown that the hydrodynamic characteristics of the flow can depending on the method of polydispersity accounting [2] moreover, the necessity of solving additional equations causes computational difficulties. In work [5] was conducted an analysis of the effective mean diameter for fluidized beds and for which the drag and gravitation forces are the primary determinants of particle motion.

Summarizing the foregoing, we can conclude that to improve the accuracy of calculations, and to reduce the computational costs, correct choice of the effective diameter is needed. Consequently, the purpose of this article is to improve accuracy of calculations via corrections of common used effective diameters and determine the modes of fluidization in which the calculation with one fraction of solid particles most accurately characterizes the flow parameters.
2. Problem formulation and experimental study

2.1. Experimental setup
Fig.1 shows a scheme of fluidized bed experimental setup using in present work. The experimental setup constitutes of a tall glass tube with an inner diameter of 2.2 cm. At the bottom of the tube is a porous material providing a uniform upward flow of air which supplies from compressor. Such a simple vessel helps to minimize an influence of bounds on hydrodynamic behaviour of fluidized bed, it is important if we want to study effective diameter by measuring height of bed. Volumetric flow rate was measured at the outlet of the tube by rotameter. Fluidization process was filmed on high-speed camera, height of a bed was estimated by shots from film.

![Figure 1. Scheme of experimental setup.](image)

2.2. Material properties
Polydisperse system of chromia-alumina microsperical catalyst particles meant for dehydrogenation of isobutane, belonging to Geldart Group B, were investigated. Cumulative distribution of particle size shown on Table 1. CDF, PDF and discrete probabilities of particle size shown on Figure 2. Air was used as a fluidizing agent.
Table 1. Cumulative distribution of particle size

| Particle size (mm) | % in a bed |
|-------------------|------------|
| 22.909            | 0          |
| 26.303            | 0.2        |
| 30.20             | 0.3        |
| 34.674            | 2.26       |
| 39.811            | 8.17       |
| 45.709            | 20.28      |
| 52.481            | 38.55      |
| 60.256            | 59.62      |
| 69.183            | 78.38      |
| 79.433            | 91.16      |
| 91.201            | 97.54      |
| 104.713           | 99.72      |
| 120.226           | 100        |
| 138.038           | 100        |

Figure 2. Particle characteristics.

The advantage of the Eulerian approach is it's relatively small computational cost. This approach requires the discretization of the particle characteristics. This problem is often solved with the help of the so-called generalized Sauter mean diameter (SMD) of the particles mixture. This is a common mean diameter is defined via the PDF of particles diameters, \( f(d) \), as
The PDF satisfies \( \int_{0}^{\infty} f(D) dD = 1 \). In work [5] was proposed \( D_{31} \) for the creeping flow regime \( (\text{Re}_f \ll 1) \), \( D_{3j} \) for intermediate Reynolds numbers, where \( 1 < j < 2 \) is a function of Reynolds number and \( D_{32} \) for inertial dominated regime \( (\text{Re}_f > 2000) \), \( \text{Re}_f = \frac{\rho_f |V_p - V_f| D}{\mu_f} \). Also modal effective diameter \( D_{\text{mod}} = \arg \max_{D} (f(D)) \) was considered.

2.3. Operating conditions
Tests were conducted with wide range of fluid velocities. This allowed us to obtain detailed information about the expansion of the bed for bubbling, turbulent, fast and transport regimes of fluidization. All experiments were carried out for 30 grams of catalyst and initial volume fraction of fixed bed was \( \alpha_0 = 0.41639 \).

2.4. Experimental results
The results of measurements of heights are given in Table 1. It should be noted that for velocities 0.019 and 0.0307 m/s there was bubbling regime of fluidization with a distinct boundary of the dense solid phase. Starting with the velocity of 0.0453 m/s, the entrainment of fines is observed. For velocities 0.1088 and 0.1213 m/s, there are significant fluctuations in the height of the bed.

Table 2 Measurements of heights.

| Fluid velocities (m/s) | Observed minimum height (m) | Mean height (m) | Observed maximum height (m) |
|------------------------|-----------------------------|----------------|-----------------------------|
| 0.0190                 | 0.09                        | 0.1011         | 0.115                       |
| 0.0307                 | 0.10                        | 0.1084         | 0.15                        |
| 0.0453                 | 0.09                        | 0.1104         | 0.15                        |
| 0.0556                 | 0.10                        | 0.1118         | 0.16                        |
| 0.0716                 | 0.107                       | 0.1386         | 0.19                        |
| 0.0892                 | 0.12                        | 0.1433         | 0.18                        |
| 0.1088                 | 0.13                        | 0.1744         | 0.23                        |

3. Numerical study

3.1. Fluidized bed numerical model
The numerical simulation of a fluidized bed was performed via continuous multiphase Eulerian-Eulerian model coupled with the kinetic theory to include the interaction between solid particles. In present study the following equations were solved:

Conservation of mass

\[
\frac{\partial (\alpha \rho)}{\partial t} + \nabla \cdot (\alpha \rho \vec{v}) = 0.
\]
where $\alpha_i$ is the volume fraction of the $i$-th phase, $\rho_i$ is the real density, $v_i$ is the velocity.

**Conservation of momentum**

$$\frac{d}{dt} \alpha_i \rho_i \vec{v}_i + \nabla \cdot (\alpha_i \rho_i \vec{v}_i \vec{v}_i) = -\alpha_i \nabla p + \nabla \cdot \vec{F}_i + \alpha_i \rho_i \dot{g} + \sum \vec{R}_j,$$

where $\vec{F}_i = \alpha_i \mu_i \left( \nabla \vec{v}_i + \nabla \vec{v}_i^T \right) + \alpha_i \left( \lambda_i - (2/3) \mu_i \right) \nabla \cdot \vec{v}_i \vec{I}$ is the stress tensor, $p$ is the pressure, $\mu_i, \lambda_i$ are shear and bulk viscosity, $\vec{I}$ is a unity tensor, $\vec{R}_j = K_j \left( \vec{v}_j - \vec{v}_j \right)$ is the force of interphase interaction.

The equation for granular temperature takes the form [6]

$$\frac{3}{2} \frac{d}{dt} \left[ \alpha_i \rho_i \Theta_i \right] + \nabla \cdot \left( \alpha_i \rho_i \vec{v}_i \Theta_i \right) = \left( -p_i \vec{I} + \vec{F}_i \right) : \nabla \vec{v}_i + \nabla \cdot \left( k_{\Theta_i} \nabla \Theta_i \right) - \gamma_{\Theta_i} + \phi_{\Theta_i},$$

where $k_{\Theta_i}$ is the diffusion of energy coefficient, $\Theta_i$ is granular temperature, $\gamma_{\Theta_i}$ is collisional dissipation of energy, $\phi_{\Theta_i}$ is the energy exchange between the gas phase and the solid phase.

The system was closed by semiempirical models of interphase interaction. For liquid (gaseous) phase ($i$-th) – solid granular phase ($j$-th) interaction were used from [7], in the case of two solid granular phases is the model of [8]. The energy dissipation $\gamma_{\Theta_i}$ due to the particles collision is written by the model obtained in [11].

**4. Results and discussion**

Numerical simulations of the experiment described above with $D_{31}$, $D_{32}$ and $D_{mod}$ were carried out. Averaged by time heights of the bed from experiments and calculation are shown on the Figure 3.

![Figure 3. Particle characteristics.](image-url)
All three considered effective diameters reflect the behavior of the two-phase flow, but we note that for several experiments, the results of calculations for $D_{31}$ and $D_{32}$ do not fall within the boundaries of the observed heights of the bed. On the contrary, $D_{\text{mod}}$ for all the cases considered is within the limits of observed maxima and minima. The root-mean-square deviation of averaged heights obtained in the calculations and experiments is given in the Table 4. It is obvious that $D_{\text{mod}}$ shows the best fit for the experiment under the conditions considered.

| Effective diameter | Root-mean-square deviation (m) |
|--------------------|-------------------------------|
| $D_{31}$           | 0.0220                        |
| $D_{32}$           | 0.0269                        |
| $D_{\text{mod}}$  | 0.0139                        |

Effective diameters selected for each experiment shown on the Figure 4. For the interval of gas velocities at which there is a distinct boundary of a bed and minor entrainment of fines, it was possible to successfully find effective diameters and approximate it by $D_{32}$ and linear function.

$$D_{\text{eff}} = D_{32}(35.0069 + 645.152 V), \ V \in (0.019,0.0556)$$

(5)
With the increase in gas velocity, the entrainment of fines and fluctuations of the bed height becomes more significant. Although it was possible to find the effective diameter at which the average height of calculations and experiments coincide, calculation with one fraction does not reflect the real behavior of the bed, because entrainment of fines can't be taken into account.

5. Conclusion
Although usually the Eulerian simulation of the fluidized bed is carried out using the diameter \( D_{32} \), in our work the best agreement with the experiment was shown by the diameter \( D_{\text{mod}} \). For minor gas velocities correction of \( D_{32} \) was carried out. As the gas velocity increases, the effects associated with polydispersity become more pronounced, hence calculation with one fraction cannot reflect all the peculiarity of the process. The problem of polydisperse system approximation by a finite number of fractions with accounting of fines entrainment is an object of a new study.

6. References
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