Chapter 1

INFLUENCE OF THE NUCLEAR SYMMETRY ENERGY ON THE STRUCTURE AND COMPOSITION OF THE OUTER CRUST

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Abstract
We review and extend with nonrelativistic nuclear mean field calculations a previous study of the impact of the nuclear symmetry energy on the structure and composition of the outer crust of nonaccreting neutron stars [Roca-Maza(2008)]. First, we develop a simple “toy model” to understand the most relevant quantities determining the structure and composition of the outer crust: the nuclear symmetry energy and the pressure of the electron gas. While the latter is a well determined quantity, the former —specially its density dependence— still lacks an accurate characterization. We thus focus on the influence of the nuclear symmetry energy on the crustal composition. For that, we employ different nuclear models that are accurate in the description of terrestrial nuclei. We show that those models with stiffer symmetry energies —namely, those that generate thicker neutron skins in heavy nuclei and have smaller symmetry energies at subnormal nuclear densities— generate more exotic isotopes in the stellar crust than their softer counterparts.

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1. Introduction

Massive enough stars at the final stage of their evolution, produce the so-called supernovae explosion whose remnant is a neutron star or a black hole. Because all the nuclear fuel has been exhausted in fusion reactions during the lifetime of the progenitor star, the neutron star cannot compensate the gravitational pressure by burning light elements into heavier ones. Therefore, as there is no nuclear reaction equilibrating the gravitational pressure, the stability has to be ensured via the stiffness of the nuclear strong interaction when the baryonic matter is compressed to extreme densities by gravity. This new type of equilibrium leaves formidable imprints on the neutron star as compared to an average star. For instance, a typical neutron star with a mass like our Sun has a radius of about only 10 km, i.e., almost five orders of magnitude smaller than the radius of the Sun. Table 1 collects some typical values for a few properties of a neutron star. Owing to the huge densities and pressures present in these neutron-rich celestial bodies, the study of neutron stars requires a deep knowledge of the structure of matter under extreme conditions of density and isospin asymmetry [Lattimer(2001), Lattimer(2004)].

Figure 1 is believed to represent a plausible rendition of the structure and composition of a neutron star. The outer crust is organized into a Coulomb lattice of neutron-rich nuclei embedded in a uniform electron gas [Baym(1971)]. As the density increases, nuclei become more neutron rich until the neutron drip region is reached. This density defines the outermost part of the inner crust. The inner crust also consists of a Coulomb lattice of neutron-rich nuclei, but now, embedded in a uniform electron gas and a neutron vapor that permeates the system. As the density continues to increase in the inner crust, the system is speculated to morph into a variety of complex and exotic structures, such as spheres, cylinders, rods, plates, etc.—collectively known as nuclear pasta [Ravenhall(1983), Hashimoto(1984), Lorenz(1993)]. As the density increases even further, uniformity is eventually restored. Finally, at ultra high densities it has been established that the ground state of hadronic matter becomes a color superconductor in a color-flavor-locked (CFL) phase [Alford(1999), Rajagopal(2000)]. It is unknown, however, if the density at the core of a neutron star may reach the extreme values required for the CFL phase to develop. Thus, other exotic phases—such as meson condensates, hyperonic matter, and/or quark matter—may be likely to harbor the core of neutron stars.

Observers have recently discovered high-frequency oscillations in the tails of giant flares [Thompson(1995), Kouveliotou(1998), Kouveliotou(2003)] which are believed to be associated with seismic vibrations of the neutron star crust [Piro(2005), Watts(2007b)]. The frequencies of such vibrations are known to be very sensitive to the composition of the crust and to the nuclear symmetry energy and its density dependence [Strohmayer(2006)].

| $R$ (km) | $\bar{\rho}$ (g/cm$^3$) | $v_{esc}/c$ | $g/g_{Earth}$ (surface) | $P$ (dyn/cm$^2$) |
|---------|-----------------|-------------|------------------------|-----------------|
| 10      | $10^{14} - 10^{15}$ | 0.5         | $10^{11}$              | $0 - 10^{35}$   |

Table 1. Properties of a typical 1 solar-mass neutron star. Here $R$ is the radius, $\bar{\rho}$ is the mean density, $v_{esc}$ is the escape velocity, $c$ is the speed of light, $g$ is the acceleration of gravity and $P$ is the pressure.
Watts (2007a), Steiner (2009). Moreover, oscillating neutron stars are likely emitters of gravitational waves. It has been shown that the density dependence of the nuclear symmetry energy also has a notable influence on the emission of gravitational radiation from axial modes of oscillating neutron stars [Wen (2009)]. This fact may have observational consequences, as it is expected that gravitational waves emitted by neutron stars can be detected by the Laser Interferometer Gravitational Wave Observatory (LIGO) and the future Laser Interferometer Space Antenna (LISA) mission.

The nuclear symmetry energy $E_{\text{sym}}(\rho)$ represents the energy cost involved in converting the protons into neutrons in symmetric nuclear matter at density $\rho$. This quantity entails important consequences for a myriad of properties of neutron-rich matter both on Earth and in the Cosmos. Actually, neutron stars can synthesize very neutron-rich nuclei, far more isospin asymmetric than we can produce in terrestrial laboratory conditions. All in all, neutron stars are a privileged source for observational studies that may help in constraining nuclear properties under very extreme conditions. We expect that as the observational techniques improve, neutron stars will provide stringent limits on the equation of state of neutron-rich matter. Motivated by this possibility, we study the sensitivity of the structure and composition of the outermost layer —apart from the “atmosphere”, see Fig. 1— of a nonaccreting neutron star to the model dependence of the nuclear symmetry energy.

The outer crust comprises about seven orders of magnitude in density; from about $10^4$ g/cm$^3$ up to a neutron-drip density of about $4 \times 10^{11}$ g/cm$^3$ [Baym (1971)]. At these densities the electrons —present to maintain charge neutrality— are no longer bound to nuclei and move freely throughout the crust. Moreover, at these low nuclear densities it is energetically favorable for the nuclei to arrange themselves in a crystalline lattice. At the lowest densities, the electronic contribution is negligible so the Coulomb lattice is populated in good approximation by $^{56}$Fe nuclei. However, as the density increases and the electronic contribution becomes important, the more energetically advantageous process for the system is to lower its electron fraction by allowing the energetic electrons be captured by protons. The excess energy is carried away by neutrinos. The resulting nuclear lattice is then formed by nuclei having a slightly lower proton fraction than $^{56}$Fe. As the density continues to increase, the nuclear system evolves into a Coulomb lattice of pro-
gressively more neutron-rich nuclei until the critical neutron-drip density is reached. The essential physics of the outer crust is then nicely captured by a competition between the electronic contribution, which favors the formation of neutron-rich nuclei, and the nuclear symmetry energy, which favors a more symmetric configuration of the lattice nuclei.

It is important to notice that the neutron-rich nuclei populating the Coulomb lattice in the outer crust are on average more dilute than their more symmetric stable counterparts because of the development of a neutron skin. Let us recall that the size of the neutron skin of a nucleus is usually characterized by the so-called neutron skin thickness

$$\Delta r_{np} = r_n - r_p ,$$

where \( r_n \) and \( r_p \) are the root-mean-square radii of the neutron and proton density distributions in the nucleus, respectively. Neutron-rich nuclei having thick neutron skins are critically sensitive to the symmetry energy below nuclear matter saturation density [Centelles(2010)]. If such a situation prevails in the outer crust of neutron stars, the predicted composition may be affected since the symmetry energy at subsaturation densities is less certain than at saturation density. Indeed, at the densities relevant for the description of the exotic (neutron-rich) nuclei forming the Coulomb lattice, the density dependence of the nuclear symmetry energy is still not well constrained by the available experimental information.

Our incomplete knowledge of the density dependence of the symmetry energy also shows up in the microscopic effective nuclear mean-field (MF) models. While they are accurate and reliable for the prediction of many ground-state properties and excitation properties of finite nuclei along the whole periodic table, the nuclear models usually differ in the description of isospin-sensitive properties such as the neutron skin thickness or the equation of state of asymmetric nuclear matter. It is both practical and insightful to characterize the behavior of \( E_{\text{sym}}(\rho) \) in nuclear models by using a handful of parameters. To this end, it has become customary to expand \( E_{\text{sym}}(\rho) \) around the saturation density \( \rho_0 \) according to the formula

$$E_{\text{sym}}(\rho) = J + L \left( \frac{\rho - \rho_0}{3\rho_0} \right) + \frac{1}{2} K_{\text{sym}} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 + \cdots ,$$

where \( J \) is the value of the bulk symmetry energy and \( L \) and \( K_{\text{sym}} \) describe, respectively, the slope and curvature of the symmetry energy at saturation. To emphasize the intimate connections existing between the observables of neutron-rich nuclei and the density-dependent symmetry energy in nuclear models, we show in Fig. 2 the correlation [Brown(2000), Furnstahl(2002), Centelles(2010)] between the neutron skin thickness \( \Delta r_{np} \) of a neutron-rich nucleus such as \( ^{208}\text{Pb} \) and the parameter

$$L \equiv 3\rho_0 \left. \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \right|_{\rho_0} ,$$

which characterizes the slope of the symmetry energy at the saturation density \( \rho_0 \). Note that the pressure of pure neutron matter at saturation and the \( L \) parameter are directly proportional in nuclear models [Piekarewicz(2009)]. The quantities \( \Delta r_{np} \) of \( ^{208}\text{Pb} \) and \( L \) displayed in Fig. 2 have been calculated in a large and representative set of nuclear MF models. These models are based on very different theoretical grounds. For example, in Fig. 2 we
Figure 2. The neutron skin thickness of $^{208}$Pb against the density slope of the nuclear symmetry energy at saturation as predicted by different nuclear mean-field interactions. The four interactions used in the calculations presented later in this chapter are shown by squares.

have Skyrme zero-range forces (all the models with names starting by S) and Gogny finite-range forces (the D1S and D1N models). These are nonrelativistic forces derived from an effective Hamiltonian. Also displayed in Fig. 2 are relativistic forces based on effective field theory Lagrangians with meson self-interactions, density-dependent meson-nucleon couplings, and point couplings. The four interactions used in the calculations presented later in this chapter are highlighted by squares. As it can be seen, these interactions cover the whole range of possible values for the $L$ parameter that is predicted by the large sample of MF calculations shown in Fig. 2. Irrespective of the nature of the models, all of them are able to describe similarly well general properties of nuclei such as binding energies and charge radii along the valley of stability. However, one clearly notes in Fig. 2 a large disagreement in the predictions for the size of the neutron skin thickness of $^{208}$Pb, which ranges from about 0 fm to about 0.3 fm in the various models. A large spread is also found in the values of the $L$ parameter. Such discrepancies among models stem mainly from the lack of accurate experimental information caused by the inherent difficulties of probing uncharged, strongly interacting particles (neutrons) model independently.

The novel—and successfully commissioned—Parity Radius Experiment (PREx) at the Jefferson Laboratory [Kumar] has provided the first model-independent evidence of the existence of a significant neutron skin in $^{208}$Pb. PREx probes the neutron density distribution in $^{208}$Pb via parity-violating elastic electron scattering [Kumar, Horowitz(2001)]. This experimental technique is free of most strong interaction uncertainties, similarly to elastic electron scattering for probing the proton density distribution in nuclei. In this way, the experiment can constrain the neutron skin thickness of $^{208}$Pb cleanly and, by means of the correlations shown in Fig. 2 and in Ref. [Roca-Maza(2011)], the density dependence of the nuclear symmetry energy. In recent years it has been demonstrated that a variety of neutron-star observables are correlated with the neutron skin thickness of $^{208}$Pb and the slope of the nuclear symmetry energy in nuclear models [Horowitz(2001a), Horowitz(2001b), Horowitz(2002), Carriere(2003), Steiner(2005), Steiner(2008)].
particularly interesting correlation of direct relevance to the crustal region is a “data-to-
data” relation between the neutron skin thickness of $^{208}$Pb and the crust-to-core transition density $[\text{Horowitz}(2001a), \text{Ducoin}(2011)]$. Therefore, one may safely conclude that an improved knowledge of the isospin properties of the nuclear interaction will impact deeply on the study of the crust of neutron stars. Moreover, the benefits of these studies for the fields of nuclear structure and nuclear astrophysics are mutual. For example, as pointed out at the beginning of this Introduction, information deduced from observations of giant flares in strongly magnetized neutron stars may prove to be an excellent opportunity for probing nuclei at extreme conditions of density and isospin asymmetry that are hardly accessible to terrestrial laboratories.

The present chapter has been organized as follows. The formalism required to compute the composition and the equation of state of the outer crust is reviewed in Sec. 2. In Sec. 3 we employ several realistic nuclear-mass models to compute the structure and composition of the outer crust and focus on the sensitivity to the nuclear symmetry energy. A more comprehensive study of the outer crust of nonaccreting cold neutron stars can be found in the work by Ruester, Hempel, and Schaffner-Bielich $[\text{Ruester}(2006)]$. In spite of the fact that we do not use a large number of nuclear models for our investigations, we include a simple “toy model” that provides critical insights into the role played by the symmetry energy. Moreover, in Sec. 3 we investigate the imprints of the density dependence of the symmetry energy on the sequence of neutron-rich nuclei occurring in the outer crust. Finally, our conclusions are laid in Sec. 4.

2. Formalism

In the present section, we review the most important features of the formalism necessary to describe the outer crust of a nonaccreting neutron star $[\text{Roca-Maza}(2008)]$. We closely follow the seminal ideas introduced by Baym, Pethick, and Sutherland $[\text{Baym}(1971)]$. We refer the interested reader to Refs. $[\text{Haensel}(1989), \text{Haensel}(1994), \text{Ruester}(2006)]$ for recent comprehensive studies.

The outer crust can be treated in good approximation as a cold system ($T \approx 0$) that is composed by nuclei arranged in a Coulomb lattice and embedded in a uniform free Fermi gas of electrons. The densities comprised by the outer crust go from the complete ionization of the electrons preserving electric neutrality ($\rho \approx 10^4\text{g/cm}^3$), to the neutron drip line ($\rho \approx 10^{11}\text{g/cm}^3$) which defines the outer-inner crust interface (see Fig. 1). Therefore, the composition of the outer crust is determined by the nucleus (with neutron number $N$, proton number $Z$, and baryon number $A = N + Z$) that minimizes —for each density— the total energy per nucleon of the system. The energy per nucleon consists of the nuclear, electronic and lattice contributions:

$$\varepsilon(A, Z; n) = \varepsilon_n + \varepsilon_e + \varepsilon_\ell,$$

where the baryon density is denoted by $n \equiv A_{\text{total}}/V$. The nuclear contribution to the total energy per nucleon is

$$\varepsilon_n(N, Z) = \frac{M(N, Z)}{A}, \text{ with } M(N, Z) = Nm_n + Zm_p - B(N, Z).$$
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Note that within our description, the nuclear mass, $M(N, Z)$, does not depend on the density and, therefore, it will not contribute to the pressure of the system. $B(N, Z)$ is the binding energy of the nucleus and $m_n$ and $m_p$ are neutron and proton masses, respectively.

The electronic contribution at the densities of interest is modeled as a degenerate free Fermi gas [Baym(1971)]. That is,

$$\varepsilon_e(A, Z; n) = \frac{\varepsilon_e}{n} = \frac{1}{m^2_e} \int_{0}^{p_{Fe}} p^2 \sqrt{p^2 + m^2_e} \, dp ,$$

where $\varepsilon_e$, $m_e$, and $p_{Fe}$ are the electronic energy density, mass, and Fermi momentum, respectively. Note that the electronic Fermi momentum and baryon density are related as follows:

$$p_{Fe} = \left( 3 \pi^2 n_e \right)^{1/3} = \left( 3 \pi^2 y n \right)^{1/3} \equiv y^{1/3} p_F ,$$

where the electron fraction $y \equiv Z/A$ has been defined. In addition, for future convenience the following definition of the overall Fermi momentum has been introduced:

$$p_F = \left( 3 \pi^2 n \right)^{1/3} .$$

The integral in Eq. (6) can be evaluated analytically and computed using the closed form

$$\varepsilon_e(A, Z; n) = \frac{m_e^4}{8 \pi^2 n} \left[ x_F y_F \left( x_F^2 + y_F^2 \right) - \ln(x_F + y_F) \right] ,$$

where a dimensionless Fermi momentum and energy have been defined as follows:

$$x_F \equiv \frac{p_{Fe}}{m_e} \quad \text{and} \quad y_F \equiv \frac{\varepsilon_{Fe}}{m_e} = \sqrt{1 + x_F^2} .$$

Finally, the last term in Eq. (4) corresponds to the Coulomb lattice contribution to the total energy per particle. The completely ionized nuclei populating the outer crust feel the Coulomb repulsion among them and, at the conditions present in this layer, crystallize in a body-centered-cubic lattice. Such a behavior has been demonstrated by Wigner for the case of an electron gas [Wigner(1934), Wigner(1938), Fetter(1971)]. Accurate numerical calculations for the electron gas have been available for a long time [Coldwell-Horsfall(1960), Sholl(1967)] and the results can be readily generalized to the present case [Baym(1971)]. We will take advantage of the previous investigations and use the expression for the lattice energy per nucleon as written in [Roca-Maza(2008)]:

$$\varepsilon_l(A, Z; n) = -C_l \frac{Z^2}{A^{4/3}} p_F \quad \text{(with} \ C_l = 3.40665 \times 10^{-3} \text{)} .$$

The full expression for the energy per baryon in terms of the nuclear, electronic, and lattice contributions is

$$\varepsilon(A, Z; n) = \frac{M(N, Z)}{A} + \frac{m_e^4}{8 \pi^2 n} \left[ x_F y_F \left( x_F^2 + y_F^2 \right) - \ln(x_F + y_F) \right] - C_l \frac{Z^2}{A^{4/3}} p_F .$$

Note that all but the nuclear contribution, $M(N, Z)$, to the total energy per baryon are well known by the current theory. Such an uncertainty is due to the fact that it is very difficult
to deal with a many-body system of strongly interacting fermions even though the underlying theory, Quantum Chromodynamics, was established many years ago. This affects the calculations of the structure and composition of the outer crust since one cannot avoid the use of nuclear mass models. Experimental data on nuclear masses are available for a large number of nuclei around the line of stability but, unfortunately, masses of nuclei with large isospin asymmetries such as some of the nuclei that may exist in the outer crust are unknown. Therefore, observational information sensitive to crustal properties of neutron stars can provide valuable insights into exotic nuclei unexplored in terrestrial laboratories. Reciprocally, the advent of new facilities capable to produce rare ion beams for experimental studies in the laboratory may place strong constraints on the crustal properties.

In modeling the outer crust, the central assumption is that of thermal, hydrostatic, and chemical equilibrium. For this reason, we also calculate the equation of state (namely, the relation between pressure and density) and the chemical potential, since complete equilibrium demands the equality of temperature, pressure, and chemical potential at each layer of the outer crust. As we have mentioned, the individual nuclei do not contribute to the pressure and, then, only the electronic and lattice terms contribute:

$$P(A, Z; n) = -\left(\frac{\partial E}{\partial V}\right)_{A,Z}$$

$$= \frac{m_e^4}{3\pi^2} \left[ x_F^3 y_F - \frac{3}{8} x_F y_F \left( x_F^2 + y_F^2 \right) - \ln(x_F + y_F) \right] - \frac{n}{3} e^{-4/3} \frac{Z^2}{A^{4/3}} p_F .$$  

(13)

The temperature of the system is in good approximation assumed to be equal to zero (on the scale of nuclear energies) and, therefore, the only remaining thermodynamic observable to calculate is the chemical potential. At zero temperature, the Gibbs free energy and the total energy of the system are related by a Legendre transform ($G = E + PV$). That is,

$$\mu(A, Z; P) = \frac{G(A, Z; P)}{A_{\text{total}}} = \frac{\varepsilon(A, Z; n)}{A} + \frac{P}{n} = \frac{M(N, Z)}{A} + \mu_e - \frac{4}{3} C_e Z^2 A^{4/3} p_F ,$$  

(14)

where $\mu_e = \sqrt{p_{F_e}^2 + m_e^2}$ is the electronic chemical potential. Note that the chemical potential is a function of the pressure whereas the energy per baryon is a function of the baryon density. The transformation from one into the other is accomplished through Eq. (13). Actually, it is convenient to compute the composition of the outer crust by minimizing the chemical potential at a constant pressure rather than by minimizing the energy per particle at constant baryon density. This is because the equilibrium conditions demands the pressure and chemical potential be continuous throughout the outer crust but this do not necessarily imply the same for the baryon density and energy.

3. Results

Following the formalism presented in the previous section, we provide here the results obtained for the structure and composition of the outer crust of a nonaccreting neutron
star. First, we develop a “toy-model” with the aim of understanding the role of the different terms in the energy and chemical potential. Secondly, we use for the calculations two of the most accurate mass models available in the literature. For this reason, we will take them as a reference along this section. These two models are the one by Duflo and Zuker [Duflo(1994), Zuker(1994), Duflo(1995)] and the finite range droplet model of Möller, Nix and collaborators [Moller(1995), Moller(1996)]. Both models are based on sophisticated microscopic/macroscopic approaches that yield root-mean-square (rms) errors of only a fraction of an MeV when compared to large databases of available experimental nuclear masses [Audi(1993), Audi(1995)]. First and foremost, we are interested in understanding how successful models in describing ground-state properties of stable nuclei differ in their predictions of exotic (neutron-rich) nuclei and in correlating such differences to the little constrained isovector channel of those models. Unfortunately, the microscopic/macroscopic mass models do not provide predictions for the nuclear symmetry energy at different densities. This fact motivates the use of nuclear MF models that are also accurate in the description of nuclear masses —typical rms deviations are of a few MeV when compared to large databases of known masses— and predict a specific density dependence of the nuclear symmetry energy. Our set of MF models has been selected to represent a broad range of values for the $L$ parameter (see Fig. 2) that describes the density slope of the nuclear symmetry energy at saturation.

### 3.1. A Toy Model of the Outer Crust

We first introduce a simple “toy model” that captures the essential physics of the outer crust. That is, the competition between an electronic density that drives the system towards more neutron-rich nuclei and a nuclear symmetry energy that opposes such a change. The toy model is based on the following two approximations. First, a simple liquid-drop model will be used to compute nuclear masses [see Eq. (5)]. Second, the electronic contribution will be assumed to be that of an extremely relativistic ($m_e/p_{Fe} \to 0$) Fermi gas. Both of these approximations will allow an analytic treatment and provide a not too simplified approach to the outer crust. For these reasons, valuable information on the physics taking place in the outer crust will be deduced.

In the absence of pairing correlations, the liquid-drop mass formula may be written as follows:

$$
\varepsilon_n(x, y) = m_p y + m_n (1 - y) - a_v + \frac{a_s}{x} + a_c x^2 y^2 + a_a (1 - 2y)^2,
$$

where $x \equiv A^{1/3}$ and $y \equiv Z/A$. The various empirical constants ($a_v$, $a_s$, $a_c$, and $a_a$) represent volume, surface, Coulomb, and asymmetry contribution, respectively. Using a least-squares fit to 2049 nuclei (available online at the UNEDF collaboration website [http://www.unedf.org/]) one obtains the following values for the four empirical constants:

$$
a_v = 15.71511 \text{ MeV}, \ a_s = 17.53638 \text{ MeV}, \ a_c = 0.71363 \text{ MeV}, \ a_a = 23.37837 \text{ MeV}.
$$

At zero density, the only term contributing to the total energy of the outer crust is $M(N, Z)$. In this situation, the optimal values of $x$ and $y$ using the simple liquid-drop
The simple analytic solution is

\[ A = x^3 = \left( \frac{a_n}{2a_c} \right) \frac{1}{y^2} , \]  

\[ y = \frac{1 + \left( \frac{\Delta m}{4a_n} \right)}{2 + \left( \frac{a_c}{2a_n} \right)^2} \approx \frac{1/2}{1 + \left( \frac{a_c}{4a_n} \right)^2} , \]

(17a, 17b)

where we have defined \( \Delta m \equiv m_n - m_p \). The above solutions suggest that for a fixed proton fraction \( y = Z/A \), the optimal value of \( x \) emerges from a competition between surface (which favors large \( x \)) and Coulomb contributions (which favors small \( x \)). Conversely, if \( A = x^3 \) is held fixed, then the optimal proton fraction \( y \) results from the competition between Coulomb and asymmetry contribution, with the former favoring \( y = 0 \) and the latter \( y = 1/2 \).

When both equations are solved simultaneously, one finds the most stable nucleus for this parameter set: \( x_0 = 3.906 \) and \( y_0 = 0.454 \), or equivalently: \( A_0 = 59.598, Z_0 = 27.060, N_0 = 32.538 \) and \( (B/A)_0 = 8.784 \) MeV.

The second assumption defining the toy model is that of an ultrarelativistic Fermi gas of electrons (i.e., \( p_F \gg m_e \)). In this limit one obtains simple expressions for the total energy per baryon, chemical potential, and pressure in terms of the adopted set of variables. That is,

\[ \varepsilon(x, y, p_F) = \varepsilon_n(x, y) + \frac{3}{4} y^{4/3} p_F - C_L x^2 y^2 p_F , \]  

\[ \mu(x, y, p_F) = \varepsilon_n(x, y) + y^{1/3} p_F - \frac{4}{3} C_L x^2 y^2 p_F , \]  

\[ P(x, y, p_F) = \frac{n}{4} y^{4/3} p_F - \frac{n}{3} C_L x^2 y^2 p_F . \]

(18a, 18b, 18c)

Assuming a neutron-drip density of about \( 4 \times 10^{11} \text{g/cm}^3 \) consistently found in all calculations (see Table 2 below), the overall Fermi momentum is approximately equal to \( p_F^{\text{drip}} \approx 40 \) MeV. This provides a large electronic contribution at the outer-inner crust interface of about \( \varepsilon_e^{\text{drip}} \approx 30 y_0^{4/3} p_F^{\text{drip}} \approx 6.2 \) MeV when one takes the proton fraction of the conventionally accepted drip nucleus \(^{118}\text{Kr}\). Therefore, the electrons will drive the system to small values of \( y \) as compared to the solution at zero density \( (y_0 = 0.454) \) since the lattice contribution has a minor effect on the total energy: \( \varepsilon_l^{\text{drip}} \approx -0.14 y_0^{4/3} x_0^{2} \approx -0.3 \) MeV. Unsurprisingly, the lattice contribution has the same dependence in the parameters \( x \) and \( y \) as the Coulomb term in the liquid-drop mass formula. Hence, it can be included through a density dependent redefinition of the Coulomb coefficient: \( a_c \rightarrow a_c(p_F) \equiv a_c - C_L p_F \).

The analytical equations to be solved when the electronic and lattice contributions are incorporated to the liquid-drop mass formula in order to describe the total energy of the
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System are, at a fixed density and in terms of the \( x \) and \( y \) variables,

\[
\left( \frac{\partial \varepsilon}{\partial x} \right)_{y,p_F} = -\frac{a_s}{x^2} + 2\tilde{a}_c x y^2 = 0 , \tag{19a}
\]

\[
\left( \frac{\partial \varepsilon_n}{\partial y} \right)_{x,p_F} = -\Delta m + 2\tilde{a}_c x^2 y - 4a_a (1 - 2y) + y^{1/3} p_F = 0 . \tag{19b}
\]

Therefore, the optimal value of the proton fraction \( y \) (for fixed \( x \)) will emerge from a competition between the redefined Coulomb and asymmetry terms where the lattice contribution corrects the solution at zero density towards a more symmetric \( y = 1/2 \) configuration. The electron gas is responsible for driving the system towards progressively more neutron-rich nuclei. Thus, the outer crust represents a unique laboratory for the study of neutron-rich nuclei in the \( Z \approx 20-50 \) region. With the advent of new Rare-Isotope Facilities worldwide that aim to provide a detailed map of the nuclear landscape, observational and theoretical studies of the neutron star crust can constitute a very important complement.

### 3.1.1. First-order Solution

The exact solutions of Eqs. (19) are, unfortunately, not analytical. For that reason, we first compute approximate solutions accurate to first order in \( p_F \). The analytic first-order solutions have the advantage of providing valuable insights into the composition of the outer crust. Those solutions are

\[
x(p_F) = x_0 \left[ 1 + \left( \frac{(C_1 - 1) C_\ell + 2 C_2}{3 C_1 - 1} \right) \frac{p_F}{a_c} \right] = (3.90610 + 0.03023 p_F) , \tag{20a}
\]

\[
y(p_F) = y_0 \left[ 1 - \left( \frac{3 C_2 - C_\ell}{3 C_1 - 1} \right) \frac{p_F}{a_c} \right] = (0.45405 - 0.00419 p_F) . \tag{20b}
\]

Note that in the above expressions the Fermi momentum is given in MeV. Moreover, for simplicity, the following two dimensionless quantities were introduced:

\[
C_1 \equiv \frac{4a_a}{x_0^2 \tilde{a}_c} \approx 8.58843 \quad \text{and} \quad C_2 \equiv \frac{1}{2 x_0^2 y_0^{2/3}} \approx 0.05547 . \tag{21}
\]

What was previously discussed is now qualitatively confirmed by the first-order solutions of the “toy-model”. That is, the proton fraction \( y \) decreases with density in an effort to minimize the “repulsive” electronic contribution. An excellent approximation of Eq. (20b) is

\[
y(p_F) \approx y_0 - \frac{p_{Fe}}{8a_a} \approx y_0 - y_0^{1/3} \frac{p_F}{8a_a} = 0.45405 - 0.00411 p_F . \tag{22}
\]

In the latter expression, the denominator \( (8a_a) \) is significantly larger than the electronic Fermi momentum over the entire region of interest. Specifically, \( p_{Fe}/8a_a \leq 0.00411 P_F^{drip} \approx 1/6 \). For this reason, we expect that the first-order approximation is fairly accurate over the entire outer crust. Indeed, assuming the last approximated expression and
a drip density of $\rho_{\text{drip}} = 4 \times 10^{11} \text{ g/cm}^3$ one finds a proton fraction of $y_{\text{drip}} = 0.298$ which represents a 2% deviation from the value of $y^{(118}\text{Kr}) = 0.305$ —for the conventionally accepted drip nucleus $^{118}\text{Kr}$. Thus, if Eq. (22) is confirmed to be accurate when compared to the exact solution of the “toy-model” and to the more realistic calculations of the following section, it will provide a very simple and clear picture for the evolution of the proton fraction throughout the outer crust. That is, the larger the value of the asymmetry energy coefficient $a_a$, the slower the evolution away from $y_0$ and, therefore, the more symmetric the nuclei in the crustal lattice will remain.

![Figure 3](image)

**Figure 3.** Proton fraction $y = Z/A$ (a) and baryon number $x = A^{1/3}$ (b) are displayed as a function of the Fermi momentum $p_F \equiv (3\pi^2 n)^{1/3}$. The black solid lines represent the exact solution to the toy-model problem given in Eqs. (19), while the red dashed lines display the corresponding solution in the $C_\ell \equiv 0$ (no lattice) limit [see Eqs. (23)]. Finally, the low-density solution [Eqs. (20)] is displayed by the blue dot-dashed lines.

### 3.1.2. Exact Solution

Although numerically simple, the exact solution of the toy-model problem for $x$ and $y$ at a fixed density cannot be displayed in an analytical form. However, there is the possibility of presenting an approximate closed solution slightly different from the exact one in terms of $x$ and the overall Fermi momentum $p_F$ when the lattice contribution is neglected. That is,

$$x(y) = \left(\frac{a_s}{2a_c y^2}\right)^{1/3}, \quad (23a)$$

$$p_F(y) = \frac{\Delta m - 2a_c x^2 y + 4a_s (1 - 2y)}{y^{1/3}}. \quad (23b)$$

These equations suggest that one can solve analytically the problem for $x$ and $p_F$ as a function of $y$, with the maximum value of $y$ given by $y_{\text{max}} = y_0 = 0.45405$ and the minimum
value of \( y \) given by the neutron drip-line condition \( \mu(y_{\min}) = m_n \).

In Fig. 3 the baryon number \( x = A^{1/3} \) and proton fraction \( y = Z/A \) are displayed as a function of the Fermi momentum \( p_F \equiv (3\pi^2 n)^{1/3} \). The black solid lines display the exact numerical solution to the toy-model problem [see Eqs. (19)]. In this simple model, the drip line density is predicted to be at \( \rho_{\text{drip}} = 4 \times 10^{11} \text{ g/cm}^3 \) with the drip-line nucleus being \(^{154}\text{Cd} \) (i.e., \( Z = 48 \) and \( N = 106 \)). The solution obtained by ignoring the lattice contribution is displayed by the red dashed lines. Because the lattice contribution to the chemical potential is negative, the \( C_\ell \equiv 0 \) solution reaches the drip line faster, i.e., at a lower density. Moreover, as the lattice contribution “renormalizes” the Coulomb term in the semi-empirical mass formula (or equivalently, enhances the role of the symmetry energy) the \( C_\ell \equiv 0 \) solution predicts a lower proton fraction than the exact solution. Finally, the dot-dashed blue lines display the solution correct to first-order in \( p_F \). In the particular case of the proton fraction \( y \), the approximate linear solution \( y = y_0 - p_F e / 8a_a \) [Eq. (22)] reproduces fairly accurately the behavior of the exact solution.

The equation of state (i.e., pressure vs density) predicted by the toy model is displayed in Fig. 4. As the lattice provides a negative contribution to the pressure [Eq. (18c)], the equation of state for the \( C_\ell \equiv 0 \) case is slightly stiffer than the exact one. The first-order solution in \( p_F \) provides a quantitatively accurate description of the equation of state up to fairly large values of the density. Note that the first-order approximation to the pressure is defined as follows:

\[
\frac{P}{n p_F} = \frac{1}{4} y^{4/3} - \frac{1}{3} C_\ell x^2 y^2 \approx (0.08367 - 0.00106 p_F).
\]  

(24)
3.2. Realistic Models of the Outer Crust

In this section we employ realistic nuclear mass models to compute the structure and composition of the outer crust. Two of the models [Moller(1995), Moller(1996), Duflo(1994), Zuker(1994), Duflo(1995)] are based on sophisticated mass formulas that have been calibrated to the around two thousand available experimental masses throughout the periodic table [Audi(1993), Audi(1995)]. The other four models are based on accurately-calibrated microscopic approaches that employ a handful of empirical parameters to reproduce the ground-state properties of finite nuclei and some nuclear collective excitations [Lalazissis(1997), Lalazissis(1999), Todd-Rutel(2005), Berger(1991), Chabanat(1997), Dobaczewski(2004)].

The microscopic MF models, although not as accurate as the microscopic/macroscopic ones in the description of nuclear masses, provide useful insights on various details of the underlying physics. The MF models typically have around 10 parameters adjusted to reproduce some selected nuclear data. In contrast to the microscopic/macroscopic models, which have usually a much larger number of parameters, the microscopic models are solved self-consistently and, therefore, the bulk and surface contributions to the total energy are closely related. This situation allows one to unravel possible correlations between the predicted finite nuclei properties and the properties of the infinite system, such as the correlation between the neutron skin thickness of lead and the slope of the symmetry energy shown in Fig. 2. In this sense, the critical role played by the symmetry energy in the evolution of the proton fraction with density can be studied within the framework of the mean-field approach.

We do not know from experiment how the symmetry energy coefficient $a_0$ changes as nuclei approach the neutron drip line. Nuclear mean field models predict the development of a significant neutron skin that renders these nuclei more diffuse. If so, one needs to extrapolate the symmetry energy to lower densities, a procedure that is highly uncertain because of our poor knowledge of the density derivative of the symmetry energy. To illustrate this uncertainty, the equation of state of pure neutron matter predicted by NL3 (green dashed line) and FSUGold (blue solid line) is displayed in Fig. 5. For comparison, we also show the predictions from the microscopic model of Friedman and Pandharipande based on realistic two-body interactions [Friedman(1981)] (purple upside-down triangles) and the model-independent result based on the physics of resonant Fermi gases by Schwenk and Pethick [Schwenk(2005)] (red hatched region). Note that to a very good approximation, the equation of state of pure neutron matter equals that of symmetric nuclear matter plus the symmetry energy [Piekarewicz(2009)]. The differences between NL3 and FSUGold displayed in Fig. 5 are all due to the large uncertainties in the symmetry energy. In particular, as NL3 predicts a stiffer equation of state than FSUGold, namely, one whose energy increases faster with density at suprasaturation densities, the symmetry energy of NL3 is lower than that of FSUGold at subsaturation densities (see Fig. 6). Thus, FSUGold has been shown to reach the neutron-drip lines earlier than NL3 [Todd(2003)]. By the same token, NL3 should predict a sequence of more neutron-rich nuclei (lower $y$) in the outer crust than FSUGold.

We depict in Fig. 6 the behavior of the symmetry energy at subnormal nuclear densities as predicted by the microscopic mean-field models, including the nonrelativistic interac-
Influence of the symmetry energy on the outer crust

Figure 5. Energy per particle for pure neutron matter as a function of the neutron Fermi momentum. Shown are the microscopic model of Friedman and Pandharipande \cite{Friedman1981} (purple triangles) and the model-independent result based on the physics of resonant Fermi gases by Schwenk and Pethick \cite{Schwenk2005} (red region). Also shown are the predictions from the accurately calibrated NL3 \cite{Lalazissis1997,Lalazissis1999} (green dashed line) and FSUGold \cite{Todd-Rutel2005} (blue line) models.

The bulk symmetry energy at saturation density is \( J = 37.4, 32.6, 32.0, \text{ and } 31.2 \text{ MeV} \) for NL3, FSUGold, SLy4, and D1S, respectively, whereas the value of the slope of the symmetry energy at saturation \( [\text{recall Eqs. (2) and (3)}] \) is \( L = 118, 60, 46, \text{ and } 22 \text{ MeV} \) for the same four models. We observe that in spite of the fact that NL3 has the largest symmetry energy at saturation \( (J = 37.4 \text{ MeV}) \), it predicts the lowest symmetry energy at subsaturation densities \( \rho \lesssim 0.1 \text{ fm}^{-3} \), owing to the fact that NL3 has (by far) the largest \( L \) value in the four models. Indeed, in the region \( \rho \lesssim 0.1 \text{ fm}^{-3} \) the relativistic NL3 parameter set and the SLy4 Skyrme interaction yield a quite similar symmetry energy. The D1S Gogny force, having the lower \( J \) and \( L \) values in the considered parameter sets, predicts the largest symmetry energy in the regime \( \rho \lesssim 0.1 \text{ fm}^{-3} \). The results by the relativistic parameter set FSUGold in the alluded density region are not far from the D1S curve. Thus, we advance that one may expect more similar predictions for the composition of the outer crust between NL3 and SLy4, on the one hand, and between FSUGold and D1S, on the other hand.

Shown in the left-hand panel of Fig. 7 is the proton fraction predicted by the two microscopic models FSUGold (blue solid line) and NL3 (green dashed line). Also shown is the estimate obtained from the liquid-drop formula [Eq. (22)]. In spite of its simplicity, Eq. (22) nicely averages the result of the realistic calculations, which confirms its usefulness for a general and qualitative understanding of the composition of the outer crust of a neutron star. The proton fraction predicted with the FSUGold parameter set is consistently higher than for the NL3 set. This is a reflection of the stiffer penalty imposed on the FSUGold set for...
Figure 6. Density dependence of the symmetry energy predicted at nuclear subsaturation by the indicated microscopic mean-field models.

departing from the symmetric ($N = Z$) limit. The right-hand panel shows the corresponding behavior for the case of the microscopic/macroscopic models of Moller-Nix (red solid line) and Duflo-Zuker (purple dashed line). Differences among these models are small.

Similar trends may be observed in Figs. 8–10 where the composition of the outer crust is displayed as a function of density. As the system makes a rapid jump in neutron number (say to magic number $N = 50$) the proton number jumps with it. Along this neutron plateau, the proton fraction decreases systematically with increasing density in an effort to reduce the electronic contribution to the chemical potential. Eventually, the neutron-proton mismatch is too large and the jump to the next neutron plateau ensues; a jump that is driven by the symmetry energy. Clearly, the larger the symmetry energy at low densities, the smaller the neutron-proton mismatch and the early the jump to the next neutron plateau. These features are clearly displayed in Fig. 8 as one contrasts the behavior of FSUGold to that of NL3.

Considering Fig. 8 together with the results predicted by the nonrelativistic microscopic models SLy4 and D1S, shown in Fig. 9 brings into notice the importance of the nuclear shell effects in the composition of the outer crust. Indeed, with the sole exception of $^{58}$Fe ($Z = 26$) in the case of the NL3 model at low crustal density, the ground state of the outer crust consists systematically of nuclei that have either a neutron magic number ($N = 28, 50, 82$) or a proton magic number ($Z = 28, 50$), or both.

At the lowest densities of the outer crust, FSUGold and NL3 favor the presence of the $^{64}$Ni ($Z = 28$) and $^{58}$Fe ($Z = 26$) isotopes, respectively. In contrast, at low crustal densities, the nonrelativistic models SLy4 and D1S favor the major neutron shell closure $N = 28$, allowing nuclei with lower atomic number to populate the ground state of the crust —i.e., Cr ($Z = 24$) in SLy4 and D1S, and also Ti ($Z = 22$) in SLy4. Compared with D1S, the fact that SLy4 has a lower symmetry energy at nuclear subsaturation densities (see Fig. 6) allows SLy4 to accommodate a larger neutron-proton mismatch, with the appearance of the $^{50}$Ti nucleus after $^{52}$Cr. This fact, in combination with the robustness of the $N = 28$
Figure 7. Panel (a) displays the proton fraction predicted by the accurately calibrated FSUGold (blue solid line) and NL3 (green dashed line) parameter sets. Also shown is the simple liquid-drop formula given in Eq. (22). Panel (b) shows the proton fraction predicted by the Moller-Nix (red solid line) and Duflo-Zuker (purple dashed line) mass formulas.

shell, delays in SLy4 the jump to the next neutron plateau till a much higher crustal density around $\rho = 2 \times 10^9 g/cm^3$; the highest density value found for the first jump to the next plateau in the four microscopic models studied here. Interestingly enough, SLy4 predicts that the next neutron plateau occurs at $N = 40$ instead of $N = 50$, which is at variance with the other three microscopic models. Indeed, the possible change of the shell structure and the replacement of the traditional magic numbers by new islands of stability for isotopes in the vicinity of the drip lines is a thrilling area of modern nuclear structure. Our understanding of the outer crust will benefit from the availability of more information from radioactive beam experiments on the evolution of the shell structure in exotic nuclei. Furthermore, some of these exotic isotopes are important also for other problems in astrophysics such as stellar nucleosynthesis and element abundances.

Having a similar symmetry energy below nuclear saturation (cf. Fig. 6), FSUGold and D1S predict a similar composition for the central plateau of $N = 50$. In both models, the $N = 50$ plateau starts with Sr ($Z = 38$) and ends with Ge ($Z = 32$). The jump to $N = 82$ takes place at practically the same crustal density $\rho = 3 \times 10^{10} g/cm^3$ in FSUGold and D1S. For the $N = 82$ plateau, the predicted nuclides are again the same in these two models, excepting that FSUGold terminates (namely, $\mu(A, Z; P) = m_n$) at $^{118}$Kr ($Z = 36$) and D1S terminates at $^{120}$Sr ($Z = 38$). We have seen in Fig. 6 that at nuclear subsaturation densities the NL3 and SLy4 models have a lower symmetry energy than FSUGold and D1S. Because of this reason, in both NL3 and SLy4 the plateau of $N = 50$ supports more neutron-rich nuclei than FSUGold and D1S, reaching up to $^{78}$Ni ($Z = 28$). As it happened in the case of FSUGold and D1S, the NL3 and SLy4 parameter sets jump to the $N = 82$ plateau both at a very similar crustal density. However, the density value is now higher; i.e., $\rho \sim 1.5 \times 10^{11} g/cm^3$, or 5 times larger than in FSUGold and D1S. As expected, the transition to $N = 82$ has been delayed with respect to FSUGold and D1S because NL3 and SLy4 have a reduced symmetry energy in the subsaturation region. Both NL3 and SLy4 begin the $N = 82$ plateau with Mo ($Z = 42$), while NL3 attains the bottom of the outer
Figure 8. Composition of the outer crust of a neutron star as predicted by the relativistic mean-field parameter sets FSUGold (a) and NL3 (b). Protons are displayed with the (lower) blue line while neutrons with the (upper) green line.

Figure 9. Composition of the outer crust of a neutron star as predicted by the nonrelativistic mean-field models SLy4 (a) and D1S (b). Protons are displayed with the (lower) blue line while neutrons with the (upper) green line.

crust at $^{120}\text{Kr}$ and SLy4 at $^{120}\text{Sr}$.

In contrast to the microscopic models, fewer differences are noticeable in Fig. 10 when comparing the microscopic/macrosopic model of Moller-Nix to that of Duflo-Zuker. First,
in both models the proton magic nucleus Ni largely dominates the composition of the ground state of the crust up to a density $\rho \sim 1 - 1.5 \times 10^9 \text{g/cm}^3$. This regime is followed by the $N = 50$ and $N = 82$ plateaus displaying the same composition in both models, with only slight differences in the density value where each isotope occurs. Indeed, in the two microscopic/macroscopic models the crustal density where the jump to $N = 82$ ensues is predicted at nearly the same value $\rho \sim 1.5 \times 10^{11} \text{g/cm}^3$. This density value was predicted also by the NL3 and SLy4 microscopic models discussed before.

We conclude this section by displaying in Fig. 11 the predictions for the equation-of-state (pressure vs density relation) of the outer crust of a neutron star. The left-hand panel shows results from calculations using the FSUGold (blue solid line) and NL3 (green dashed line) parameter sets. (The results obtained with D1S and SLy4 display the same general trends and therefore are not shown.) Although barely visible in the figure, the density shows discontinuities at those places where the composition changes abruptly. It is also noted that the FSUGold parametrization predicts a pressure that rises slightly faster with density than NL3. For the NL3 set, the symmetry energy admits lower values of the proton/electron fraction $y$ which, in turn, lowers the pressure of the system. Lower values of $y$ also yield lower values of the chemical potential, thereby delaying the arrival to the neutron-drip line. Indeed, whereas FSUGold predicts a drip-line density of $\rho_{\text{drip}} = 4.17 \times 10^{11} \text{g/cm}^3$, with NL3 the transition is delayed by about 8%, or until $\rho_{\text{drip}} = 4.49 \times 10^{11} \text{g/cm}^3$. A similar plot is shown for the microscopic/macroscopic models of Moller-Nix (red solid line) and Duflo-Zuker (purple dashed line). Differences among these two models are barely noticeable. Indeed, drip-line densities in both models are predicted at about $\rho_{\text{drip}} = 4.3 \times 10^{11} \text{g/cm}^3$. Model predictions for various observables at the base of the outer crust (i.e., in the drip-line region) are listed in Table 2 for all the studied models.

Figure 10. Composition of the outer crust of a neutron star as predicted using the mass formulas of Moller-Nix (a) and Duflo-Zuker (b). Protons are displayed with the blue (lower) line while neutrons with the green (upper) line.
Figure 11. Panel (a) displays the zero-temperature equation of state (pressure vs density) predicted by the accurately calibrated FSUGold (blue solid line) and NL3 (green dashed line) parameter sets. Also shown is the prediction from the simple liquid-drop formula. Panel (b) shows the corresponding expression as predicted by the Moller-Nix (red solid line) and Duflo-Zuker (purple dashed line) mass formulas.

4. Conclusions, final remarks and open questions

We studied the structure and composition of the outer crust of a nonaccreting neutron star focusing on the effects of the current uncertainties in the nuclear symmetry energy and its density derivative on the crustal properties. For that, we followed the seminal work by Baym, Pethick, and Sutherland, as well as the more recent comprehensive work by Ruester, Hempel, and Schaffner-Bielich. In our investigations, six different models were adopted. Two of these models, Moller-Nix and Duflo-Zuker, are based on a combined microscopic/macroscopic approach and yield the most accurate nuclear masses available in the literature. The other four models are of a purely microscopic nature. They are based on relativistic (NL3 and FSUGold) and nonrelativistic (Skyrme SLy4 and Gogny D1S) mean-field approaches. Although the microscopic/macroscopic models are significantly more accurate than the mean-field models in the description of nuclear masses, microscopic models have the advantage of making definite predictions on how the symmetry energy changes with density (see Figs. 5 and 6).

The composition and equation of state of the outer crust emerge from a competition among the nuclear, electronic, and lattice contributions to the energy, pressure and chemical potential of the system. The nuclear contribution is independent of the density of the system and, therefore, does not contribute to the total pressure; it appears exclusively in the form of nuclear masses. The electronic contribution is modeled as a zero-temperature free Fermi gas and dominates the behavior of the system with baryon density. Finally, the lattice contribution which also depends on the density, provides a modest correction, less than a 10%, to the total energy of the system. Hence, the competition between the different terms is basically driven by the energy of the electron Fermi gas and the nuclear symmetry energy. The former favors a small electron fraction and, to preserve charge neutrality, a small proton
| Model      | $\rho$ | $n$  | $P$  | $\mu_e$ | Element  | $B/A$ |
|------------|-------|------|------|---------|----------|-------|
| Moller-Nix | 4.34  | 2.60 | 4.93 | 26.22   | $^{118}$Kr | 7.21  |
| Duflo-Zuker| 4.32  | 2.58 | 4.89 | 26.17   | $^{118}$Kr | 7.19  |
| FSUGold    | 4.17  | 2.50 | 4.68 | 25.88   | $^{118}$Kr | 7.11  |
| NL3        | 4.49  | 2.69 | 5.06 | 26.39   | $^{120}$Kr | 7.13  |
| SLy4       | 4.10  | 2.46 | 4.81 | 26.06   | $^{120}$Sr | 7.42  |
| D1S        | 3.98  | 2.39 | 4.62 | 25.81   | $^{120}$Sr | 7.34  |

Table 2. Equation-of-state observables (mass density $\rho$ in $10^{11}$g/cm$^3$, baryon density $n$ in $10^{-4}$fm$^{-3}$, pressure $P$ in $10^{-4}$MeV/fm$^3$, and electronic chemical potential $\mu_e$ in MeV) and the predicted composition (nucleus and binding-energy per nucleon $B/A$ in MeV) at the base of the outer crust.

The fraction which leads the system toward more and more neutron-rich nuclei in the crust. The nuclear symmetry energy opposes such a change by driving the system to a more symmetric configuration in terms of the neutron and proton numbers.

For a better understanding of the competition between the electron contribution and the nuclear symmetry energy, we implemented a “toy model” of the outer crust by using a simple semi-empirical (“Bethe-Weizsäcker”) nuclear mass formula. Volume, surface, Coulomb, and asymmetry terms were extracted from a least-squares fit to 2049 nuclei. Such a simple model provided us with useful insights thanks to the analytic structure of the results. Indeed, a particularly transparent result was obtained that illustrates nicely the role of the electronic contribution and the nuclear symmetry energy in deciding the proton fraction:

$$y(p_F) = y_0 - \frac{p_{Fe}}{8a_a} + O(p_{Fe}^2)$$  \hspace{1cm} (25)

where $y_0$ is the zero-density proton fraction, $p_{Fe}$ is the electronic Fermi momentum and $a_a$ is the symmetry energy coefficient of the liquid-drop mass formula. While illuminating, this (first-order) result is also surprisingly accurate, as the electronic Fermi momentum at the base of the outer crust is very close in value to the symmetry energy coefficient ($p_{Fe} \approx 26$ MeV vs $a_a \approx 23$ MeV). In particular, the toy model predicts a value for the electron fraction at the base of the crust that differs by only a few percent from that of the drip-line nucleus $^{118}$Kr.

How the symmetry energy coefficient $a_a$ is modified as nuclei move far away from the line of stability is not known. Likely, the symmetry energy is reduced in neutron-drip nuclei due to the development of a dilute neutron skin. Mean field models of nuclear structure predict that the size of the neutron skin of neutron-rich nuclei is strongly correlated with the slope $L$ of the symmetry energy. The stiffer the symmetry energy, i.e., the larger the $L$ parameter, the thicker the neutron skin. To investigate the sensitivity of the structure and composition of the outer crust to the density dependence of the symmetry energy, we employed relativistic (NL3 and FSUGold) and nonrelativistic (SLy4 and D1S) mean-field models. Although these models have been accurately calibrated for the description of masses, charge radii, and other important ground-state properties, they predict a significantly different density dependence for the symmetry energy. Most of the relativistic models predict larger
values of the symmetry energy at saturation than the nonrelativistic ones. It is also quite common that those mean-field models show an opposite trend at subsaturation densities. That is, those models with a stiffer symmetry energy at saturation predict smaller values of the symmetry energy at subsaturation densities (see Figs. 5 and 6). One of the main goals of the present chapter was to document how such differences impact on the composition of the outer crust.

Quite generally, we show that the first substantial change to the ground-state configuration of the nuclei composing the lattice at very low densities corresponds to a jump of the neutron number that remains fixed at the magic number $N = 50$ for a wide range of densities. At the same densities, the proton fraction also suffers a jump and, then, decreases systematically in an effort to reduce the electronic contribution to the chemical potential. Eventually, the proton fraction becomes too low and the penalty caused by the symmetry energy at subsaturation densities drives the system into the next plateau at the magic number $N = 82$ which remains fixed until the outer-inner crust interface is reached. How low can the proton fraction get and, consequently, how exotic is the composition of the crust is, then, a question that must be answered by the symmetry energy: its accurate determination remains as one of the most outstanding problems in Nuclear Physics nowadays. Indeed, whereas NL3 predicts the formation of $^{78}\text{Ni}_{50}$, FSUGold (having a larger symmetry energy) leaves the $N = 50$ plateau with the formation of $^{82}\text{Ge}_{50}$ (or four protons earlier); similar conclusions could be drawn from the study of the nonrelativistic models SLy4 and D1S. This result may be stated in the form of a correlation between the neutron radius of $^{208}\text{Pb}$ and the composition of the outer crust: the larger the neutron skin of $^{208}\text{Pb}$, the more exotic the composition of the outer crust. Finally, and as it was done in Ref. [Ruester(2006)], we have computed crustal properties using two of the most accurate tables of nuclear masses available today, namely, those of Moller-Nix and Duflo-Zuker. Our results using the model of Moller and Nix agree well with those published in Ref. [Ruester(2006)]. These results are practically indistinguishable from the ones obtained using the Duflo-Zuker nuclear mass table; a table that includes 9210 nuclei! Both calculations served us as a reference and test of the formalism used for our study of the outer crust of a nonaccreting neutron star.

With the advent of new rare ion-beam facilities, the experimental study of exotic nuclei will be increasingly feasible in laboratories worldwide. This promising scenario, together with the recent and, hopefully, future observations of crustal modes in magnetars, and perhaps the detection of gravitational waves from oscillating neutron stars, will likely provide us with more stringent limits on the equation of state of neutron-rich matter and will contribute to a better understanding of the outer crust of neutron stars.

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