DETERMINATION OF $f_0(980)$ STRUCTURE
BY FRAGMENTATION FUNCTIONS

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We discuss internal structure of an exotic hadron by using fragmentation functions. The fragmentation functions for the $f_0(980)$ meson are obtained by a global analysis of $e^+e^- \rightarrow f_0 + X$ data. Quark configuration of the $f_0(980)$ could be determined by peak positions and second moments of the obtained fragmentation functions.

Keywords: exotic hadron, fragmentation function.

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1. Introduction
Internal quark structure of exotic hadrons is an interesting topic. Information on the structure would be obtained from parton distribution functions (PDFs), which are determined by using experimental data in deeply inelastic scattering (DIS). Stable targets are needed for DIS experiments, whereas exotic hadrons decay immediately and their PDFs could not be measured.

In $e^+e^-$ annihilation experiments, fragmentation functions (FFs) have been measured for produced several hadrons. These functions have information on hadronization, so that it is suitable for discussing the internal structure of the hadrons. The FFs have two types: favored and disfavored functions. The favored function means a
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fragmentation from a parton which exists as a constituent quark in the hadron, and the disfavored one means a hadronization from a sea quark. In general, these functions have characteristic features on their peak positions at small $Q^2$ ($\sim 1$ GeV$^2$). The favored ones correspond to the valence quark distributions in the PDFs, so that the peak positions of the favored ones exist at large $z$, but those of the disfavored ones do at small $z$. In fact, such a behavior can be seen in the FFs of the charged $K$ meson (see Fig. 11 in Ref. [1]). Investigating the fragmentation functions, we can find the flavor constituent in the exotic hadrons. The $f_0(980)$ meson is considered as a candidate for an exotic hadron, which has structure beyond naive $q\bar{q}$ configurations. Therefore, we determine the FFs for the $f_0(980)$ by a global analysis and discuss its possible internal structure.

2. Global analysis of fragmentation functions

The FFs are associated with the non-perturbative part separated by the factorization theorem, so that these functions should be determined by using experimental data. The differential cross section, which is normalized by the total cross section $\sigma_{\text{tot}}$, is given for the $e^+e^-\rightarrow hX$ process as:

$$F_h(z, Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^-\rightarrow hX)}{dz} = \sum_i C_i(z, \alpha_s) \otimes D_h^i(z, Q^2).$$

(1)

Here, the variable $z$ is defined as the energy ratio of the produced hadron and a parent parton, and it is given by the hadron energy $E_h$ and the center-of-mass energy $\sqrt{s}$ ($= \sqrt{Q^2}$) $: z \equiv E_h/(\sqrt{s}/2)$. In the theoretical calculation with the perturbative QCD, this cross section can be expressed by the sum of convolution integrals with the coefficient functions $C_i(z, \alpha_s)$ and the fragmentation functions $D_h^i(z, Q^2)$. Here, $\otimes$ denotes the convolution integral, $f(z) \otimes g(z) = \int_z^1 dy f(y) g(z/y)/y$, $\alpha_s$ is the running coupling constant, and $i$ indicates quark flavor or gluon, $i = u, d, s, \cdots, g$.

In this analysis, $Q^2$ dependence of the FFs should be taken into account because the experimental data exist at several $Q^2$ points. The $Q^2$ dependence is calculated by the DGLAP evolution equations. To solve the equations numerically, initial functions must be defined at initial $Q^2$ ($= Q_0^2$). A functional form is somewhat arbitrary, which is a cause of model dependence in the global analysis. We adopt a general functional form which is used in other parametrization groups:

$$D_{f_0}^i(z, Q_0^2) = N_i^{f_0} z^{\alpha_i} (1 - z)^{\beta_i},$$

(2)

where parameter $N_i$ can be rewritten by the second moment $M_i(\equiv \int dz z D_{f_0}^i(z))$ and the beta function of the parameters $\alpha_i$ and $\beta_i$, $N_i = M_i/B(\alpha_i + 1, \beta_i + 1)$. The second moments have a physical meaning of energy conservation, $\sum_i M_i^i = 1$. The $M_i$, $\alpha_i$, and $\beta_i$ are free parameters, which are optimized by a $\chi^2$ analysis.

As the initial functions of $f_0(980)$, we assume the relation, $D_{q_0}^i(z) = D_{\bar{q}_0}^i(z)$. Then, the following five functions $D_{u_0}^i(z, Q_0^2) (= D_{d_0}^i(z, Q_0^2), D_{s_0}^i(z, Q_0^2), D_{c_0}^i(z, Q_0^2), D_{t_0}^i(z, Q_0^2), D_{\bar{q}_0}^i(z, Q_0^2)).$
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Determination of $f_0(980)$ structure by fragmentation functions

Table 1. Possible $f_0(980)$ configurations and their relations to the second moments and the peak positions for the fragmentation functions of $f_0(980)$.

| Type                  | Configuration               | Second moments | Peak positions          |
|-----------------------|----------------------------|----------------|-------------------------|
| Strange $q\bar{q}$    | $s\bar{s}$                 | $M_u < M_s \lesssim M_g$ | $z_{u}^{\text{max}} < z_{s}^{\text{max}}$ |
| Tetraquark (or $KK$)  | $(u\bar{s}s + d\bar{s}s)/\sqrt{2}$ | $M_u \sim M_s \lesssim M_g$ | $z_{u}^{\text{max}} \sim z_{s}^{\text{max}}$ |

$D_f^0(z, Q_0^2)$, and $D_c^0(z, m_c^2)$, $D_b^0(z, m_b^2)$, are expressed by a number of parameters. Here, the functions for up-quark, strange-quark, and gluon are defined at $Q_0^2=1$ GeV$^2$, and heavy flavor functions are inserted into the calculation at the mass thresholds, $m_c=1.43$ GeV and $m_b=4.3$ GeV. We note that differences between the functions of the up- and strange-quarks would come from its mass difference. In principle, a threshold should be also provided for the strange quark at its mass threshold, $m_s \sim 0.2$ GeV. Since perturbative QCD would not be applicable at this low scale, which is of the order of $\Lambda_{\text{QCD}}$, the initial scale is defined at $Q_0^2=1$ GeV$^2$ as the lower limit in the theory. For this reason, the function of the strange-quark should differ intrinsically from that of the up-quark at the initial scale. However, flavor separation cannot be made only by the $e^+e^-$ experimental data. We need data from other processes, e.g., semi-inclusive DIS.

To discuss the internal structure for $f_0(980)$, four configurations are considered, non-strangeness $q\bar{q}$, naive $s\bar{s}$, tetraquark, and glueball. For simplicity, two configurations are listed in Table 1. This configuration can be determined by the relations in the peak positions and the second moments of the FFs. In the tetraquark configuration, for example, the functions of the up- and strange-quarks become the favored functions. These peak positions are the same: $z_{u}^{\text{max}} \sim z_{s}^{\text{max}}$. In addition, the relations of the second moments are suggested by order counting of the coupling in schematic diagrams. Hadronizations from the up- and strange-quarks can be expressed by the same diagrams as shown in Fig. 1. Their contributions to the cross sections are reflected in the second moments, therefore these moments will roughly become the same: $M_u \sim M_s$. By the same estimation, relations for other

Fig. 1. Schematic diagrams for $f_0$ production in the tetraquark picture.
Fig. 2. Obtained fragmentation functions for $f_0(980)$. The functions of the up-quark, strange-quark, and gluon are shown at $Q^2=1$ GeV$^2$, and heavy flavor functions of the charm- and bottom-quarks are at $Q^2 = m_c$ and $m_b$, respectively.

configurations are obtained (see Table. 1 in Ref. [2]).

3. Results and discussion
The fragmentation functions are determined by the global analysis with the $f_0(980)$ production data in the $e^+e^-$ annihilation experiments. Total $\chi^2$/DOF is 0.907, which is a reasonable value in the $\chi^2$ analysis. Obtained FFs are shown in Fig. 2. We find that the peak positions of the functions for the up- and strange-quarks exist at large $z$. These are the favored functions, and these peaks are located at the same points, $z_{u}^{max} \approx z_{s}^{max}$. It indicates the tetraquark configuration as the internal structure of $f_0(980)$. On the other hand, values of the obtained second moments are $M_u = 0.0012 \pm 0.0107$, $M_s = 0.0027 \pm 0.0183$, and $M_g = 0.0090 \pm 0.0046$. The second moment of the up-quark is less than that of the strange-quark, which indicates the naive $s\bar{s}$ configuration. This conflicting result is due to low accuracy of the current experimental data. In fact, uncertainties of the second moments are large. At this stage, the internal configuration cannot be determined clearly. To discuss the quark configuration in details, these uncertainties must be reduced by including precise data in the analysis. Therefore, we hope to get high accuracy data, which could be provided from the BELLE and BarBar experiments.

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