Are Galactic Rotation Curves Really Flat?*

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Abstract

In this paper we identify a new regularity in the systematics of galactic rotation curves, namely we find that at the last detected points in galaxies of widely varying luminosity, the centripetal acceleration is found to have the completely universal form $v^2/R = c^2(\gamma_0 + \gamma^* N^*)/2$ where $\gamma_0$ and $\gamma^*$ are new universal constants and $N^*$ is the amount of visible matter in each galaxy. This regularity points to a role for the linear potentials associated with conformal gravity, with the galaxy independent $\gamma_0$ term being found to be generated not from within individual galaxies at all, but rather to be of cosmological origin being due to the global Hubble flow of a necessarily spatially open Universe of 3-space scalar curvature $k = -(\gamma_0/2)^2 = -2.3 \times 10^{-60}$ cm$^{-2}$.

In discussions of the dynamics of galactic rotation curves it is usually assumed that rotation curves are asymptotically flat at large radial distances, and that whatever is responsible for this non-Keplerian behavior is itself just a purely local phenomenon which arises solely from within the galaxies themselves. In this paper we challenge these two widely accepted notions, and show that once the luminous Newtonian contribution is subtracted out, the resulting velocity discrepancies in individual galaxies are not merely actually growing (and quite rapidly in fact) with distance at the largest available radial distances, but, moreover, they are actually growing in a universal manner. Beyond being an interesting model independent phenomenological regularity in and of itself, and beyond being one which dark matter models of rotation curves should therefore be expected to account for, this regularity also points to a role for cosmology in the elucidation of rotation curves, as well as to the possible relevance of the conformal gravity theory of Weyl which is currently being explored.

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by Mannheim and Kazanas as a candidate alternative to the standard dark matter paradigm. Moreover, with the apparent failure so far of the epochal gravitational microlensing observations to conclusively confirm the existence of the copious spherical dark matter halo that the Milky Way galaxy is widely believed to possess, the issue of alternate gravity even appears to have acquired some urgency. However, since the whole issue of alternate gravity remains controversial at the present time, we shall begin by first looking for possible model independent clues in the data themselves.

In their analysis of available HI rotation curve data, Casertano and van Gorkom (1991) pointed out that the data basically fell into three broad categories: the rotation curves of low luminosity galaxies were found to generally be rising at the last detected points, those of intermediate to high luminosity galaxies to be flat, and those of the highest luminosity galaxies to be (mildly) falling. Out of the available 28 galaxy set Begeman, Broeils, and Sanders (1991) identified a particularly reliable 11 galaxy subset, and since these 11 galaxies range by more than 1000 in luminosity (see Table (1)) while clearly exhibiting the Casertano and van Gorkom trend (see Fig. (1)), the 11 galaxy subset should indeed be regarded as typical.\footnote{While none of the low luminosity galaxies currently show any flat rotation curve region at all, there is a noticeable turnover in one of these galaxies, viz. DDO 154. However, since this is the most gas dominated galaxy in the entire sample, random gas pressures could be making a substantial contribution to motions in the turnover region. We shall thus ignore any possible ramifications of these last few points here, though clearly if this turnover proves to be a real trend which is then reproduced in other low luminosity galaxies, it would eventually have to be accounted for, not only here in fact but even in the standard dark matter theory and in the MOND theory both of which anticipate a flattening not a drop in the DDO 154 rotation curve.}

While the lack of flatness of the low luminosity galaxies is quite apparent, nonetheless the flatness (or near flatness) of all the other galaxies is so striking that the rise in the low luminosity galaxies has essentially been discounted by the general community as being in any way suggestive of a trend, and it is generally assumed that these curves will eventually flatten off, with asymptotically flat rotation curves now being the standard paradigm. However, closer examination of the data reveals a possibly different outcome. Instead of looking at the actual rotation velocities, it is instructive to look at the velocity discrepancy, viz. the excess of the measured rotation velocity over the luminous Newtonian
contribution, a quantity whose overall normalization is essentially uniquely fixed once and for all by the inner region rotation curve alone. Indeed, as we see from Fig. (1), this velocity discrepancy is itself far from flat, and in fact is actually rising in each and every galaxy in the sample at the last detected data points. Since this discrepancy itself is usually explained by a spherical dark matter halo, we see that while such halos may eventually lead to asymptotic flatness, their contributions in the detected regions are necessarily still rising at the farthest points, with the flat total velocities that they produce in the bright galaxies actually being achieved by carefully fine tuning the halo contribution galaxy by galaxy (through the use of two free parameters per halo and thus no less than 22 in total for our 11 galaxy sample) to rise at just the same rate as the luminous matter contribution is falling. Even for isothermal sphere halos asymptotia is thus still some way off, with the case for flat velocity discrepancies not yet being mandated by any available rotation curve data.

Beyond the issue of the shape of the velocity curves one can also ask if there is any regularity in the magnitudes of the velocities. For the flat rotation curve galaxies there is indeed such a regularity, viz. the Tully-Fisher law, a phenomenologically established universal relation between the luminosity and the fourth power of the velocity dispersion in the observed flat rotation curve region. Moreover, these same galaxies also appear to possess a second form of universality which was first noted by Freeman, namely that the most prominent spiral galaxies all seem to have a common central surface brightness, $\Sigma_F^0$. (In passing we note that while there also exist low surface brightness galaxies with $\Sigma_0 < \Sigma_F^0$, there do not appear to be any galaxies with $\Sigma_0 > \Sigma_F^0$, thus making $\Sigma_F^0$ an empirical upper bound on galaxies). While the brighter galaxies thus possess a great deal of universality in addition to having flat rotation curves, this universality is not enjoyed by the non-flat low luminosity galaxies. Thus it would be of interest to find a universality which also involves the low luminosity ones as well. Given the suggestive fact that the velocity discrepancies are actually rising in all the galaxies, we thus evaluate the centripetal acceleration at the last data point in each galaxy (except for DDO 154 for which we use the last point before
the turnover. As we can see from the fourth column in Table (1) the total \((v^2/c^2R)_{tot}\) is remarkably universal, varying only by a factor of 5 or so over the sample and certainly not by the factor of 1000 by which the luminosity varies in the same sample. Even more interesting is the net value \((v^2/c^2R)_{net}\) obtained after the Newtonian contribution is extracted out. As we see from Table (1) this quantity only varies by a factor of 4. Moreover, we see a small but clear trend with increasing mass in the centripetal acceleration. And in fact, as will become more apparent below, we find that we can parameterize this net acceleration according to the two component relation \((v^2/c^2R)_{net} = (\gamma_0 + \gamma^* N^*)/2\) where the two universal constants \(\gamma_0\) and \(\gamma^*\) take numerical values \(3.06 \times 10^{-30} \text{cm}^{-1}\) and \(5.42 \times 10^{-41} \text{cm}^{-1}\) respectively, and where \(N^*\) is the total amount of stellar (and gaseous) material in solar mass units in each galaxy. (While the present author was drawn to this regularity via the conformal gravity study presented below, this regularity is an interesting one in and of itself which now serves as a new constraint on all theories of rotation curves.) As regards this regularity, it is important to realize that there is nothing in any way significant about the actual magnitudes of the radial coordinates, \(R\), of the last detected points in the 11 galaxies, since their locations are fixed purely by the instrumental limits of the various detectors used in measuring the various gas surface brightnesses and not fixed by any dynamics associated with the galaxies themselves. Thus the magnitude of each last measured radial \(R\) (a quantity which varies from 8 kpc to 40 kpc or so over the sample) is essentially arbitrary for the galaxies, and yet once the Newtonian contribution is removed, \(v^2/R\) can nonetheless still be universally parameterized. As far as we can see, the only obvious way that this could in fact happen would be if \(v^2\) were in fact growing universally with \(R\) so that the magnitude of \(v^2/R\) would not in fact depend on where the last detected points just happened to be located within galaxies. This pattern is clearly not one that one would expect with flat rotation curves, or even in fact think to look for in such a paradigm, and would instead seem to point to potentials which if anything are actually growing (linearly) with distance rather than falling in the familiar Newtonian manner.
Since our phenomenological analysis points to a role for linear potentials in elucidating rotation curves, it is immediately suggested to consider conformal gravity which contains such linear potentials to see whether it can account for the rotation curve phenomenology we have now identified. While conformal gravity dates back to Weyl and Eddington and to the early days of relativity, that it might enable us to dispense with dark matter was recognized only recently by Mannheim and Kazanas on finding (Mannheim and Kazanas 1989; see also Riegert 1984) the exact metric outside of a star in the conformal theory, viz. \[ ds^2 = B(r)c^2dt^2 - dr^2/B(r) - r^2d\Omega \] where \[ B(r) = 1 - 2\beta^*c^2/r + \gamma^*c^2r. \] Since this metric generalizes not only Newton but Schwarzschild also, it thus not only meets the classic solar system General Relativity tests, but it also provides for departures from Newton-Einstein on distances large enough that the linear potential term might first make itself manifest. Indeed, integrating the stellar potentials \[ V^*(r) = -\beta^*c^2/r + \gamma^*c^2r/2 \] over the visible galactic disk provides a luminous matter galactic potential (characterized by acceleration \[ v^2/R = g_{\text{lum}} = g_{\beta} + g_{\gamma} \]) which nicely fits the shapes of the rotation curves of our 11 galaxy sample (Mannheim 1993, Mannheim and Kmetko 1996, Carlson and Lowenstein 1996), but not their overall normalizations, since such a galactic disk would on its own only generate an asymptotic contribution \[ v^2/c^2R = \gamma^*N^*/2 \] and thus lack the \( N^* \) independent \( \gamma_0/2 \) term found above in our phenomenological analysis of centripetal accelerations.

Apart from the fact that the \( \gamma^*N^*/2 \) term arises from a non-Newtonian potential, it is otherwise a completely standard, local non-relativistic term which arises from the local galactic matter distribution and which scales as the total galactic luminosity. However, the additional \( \gamma_0/2 \) term we require is on a very different footing since it is luminosity independent. Since, moreover, its magnitude given above is of order the inverse Hubble radius, it would thus appear to have to have a global, cosmological origin, with cosmology thus needing to provide galaxies with a second linear potential in addition to the one that they themselves internally generate. Now, quite remarkably, it was noted by Mannheim and Kazanas in their original 1989 paper (where they found the generalized exterior Schwarzschild
solution discussed above) that cosmology does precisely that. Specifically, they noted the kinematic fact that the general coordinate transformation
\[
\rho = \frac{4r}{(2 + \gamma_0 r)^{1/2} + 2 + \gamma_0 r} \quad , \quad t = \int d\tau / R(\tau)
\] (1)
effects the metric transformation
\[
(1 + \gamma_0 r) c^2 dt^2 - \frac{dr^2}{(1 + \gamma_0 r)} - r^2 d\Omega \rightarrow \frac{(1 + \rho \gamma_0 / 4)^2}{R^2(\tau)(1 - \rho \gamma_0 / 4)^2} \left( c^2 d\tau^2 - \frac{R^2(\tau)(d\rho^2 + \rho^2 d\Omega)}{(1 - \rho^2 \gamma_0^2 / 16)^2} \right)
\] (2)
to thus yield a metric which is conformal to a RW metric with scale factor \(R(\tau)\) and (explicitly negative) 3-space scalar curvature \(k = -\gamma_0^2 / 4\). Now, and this is the key point, in a geometry which is both homogeneous and isotropic about all points, any observer can serve as the origin for the coordinate \(\rho\); thus in his own local rest frame each observer is able to make the general coordinate transformation of Eq. (1) involving his own particular \(\rho\). Moreover, since the observer is also free in conformal gravity to make arbitrary conformal transformations as well, that observer will then be able to see the entire Hubble flow appear in his own local static coordinate system as a universal linear potential with a universal acceleration \(\gamma_0 c^2 / 2\) coming from the spatial curvature of the Universe. Now in that specific static coordinate system any other Hubble flow observer would see something entirely different and not recognize anything that would look like a simple universal linear potential at all. Only in his own explicit rest frame would any other observer be able to recognize such a universal linear potential. Thus, while the transformations of Eqs. (1) and (2) would not be useful for describing the Hubble flow motions of the individual galaxies themselves, they appear to be ideally suited for describing the internal orbital motions of the stars and gas within each galaxy, since each internal motion can be discussed independently in each galaxy’s own rest frame. Thus it would appear that in conformal gravity each observer sees the general Hubble flow metric as a local universal linear potential with a strength fixed by the scalar curvature of the Universe (a nicely time independent quantity unlike the time dependent Hubble parameter

\textsuperscript{2}In passing we note that in the cosmology discussed in Mannheim 1992, 1995b an open Universe with explicitly (very) negative \(k\) was in fact realized, with such a Universe not suffering from the flatness problem found in the standard cosmology.
itself), with the matter in each galaxy now acting as test particles which are being swept through the Hubble flow.\footnote{We thus see a crucial difference between relativistic and non-relativistic reasoning. In strictly Newtonian physics the only effect of any background would be to put tidal forces on individual galaxies, forces that would not account for the rotational motions of stars and gas but only to a departure therefrom. Relativistically however, since the background produces an effect at the center of each galaxy, the background therefore contributes to the explicit rotational motions of the stars themselves, to thus yield a previously unappreciated but nonetheless quite general consequence of curvature.}

In order to now combine the local and global linear potentials we need to embed each local galaxy into the Hubble flow and solve the gravitational equations of motion in the presence of $T^{\mu\nu}_{\text{local}} + T^{\mu\nu}_{\text{global}}$.\footnote{It is the very presence of $T^{\mu\nu}_{\text{local}}$ and its associated local geometry (viz. standard static Schwarzschild coordinates) which dictates the appropriate general coordinate transformation needed for Eq. (1).} Given the fact that gravity is weak within galaxies, we shall as a first approximation simply add the local and global metrics given above to yield the total weak gravity acceleration $v^2/R = g_{\text{tot}} = g_{\text{gal}}^{\text{lum}} + \gamma_0 c^2/2$ which can now be fitted to data. With $\gamma_0$ and $\gamma^*$ taking fixed numerical values (found to be $3.06 \times 10^{-30} \text{cm}^{-1}$ and $5.42 \times 10^{-41} \text{cm}^{-1}$ respectively in the fitting), the fits reduce to just one free parameter per galaxy, viz. the standard optical disk mass to light ratio (or equivalently the total amount of stars and gas per galaxy, $N^*$, in solar mass units). Since, unlike dark matter theory, our theory is based on parameters with an absolute scale, it is thus very sensitive to distance determinations to galaxies. Consequently, we first calculate the total velocity predictions (the dotted curves) in Fig. (1) using the distances (listed in Table (1)) quoted by Begeman, Broeils and Sanders (1991) (this paper also gives complete data references). Then, again following Begeman, Broeils and Sanders, we allow for typical uncertainties in the adopted distances to give modest distance shifts of up to $\pm 15\%$ or so.\footnote{While larger shifts can actually improve the fits a little in some cases, we have not allowed for shifts of more than this except for NGC 1560 for which a distance estimate of $3.7 \text{ Mpc} (+23\%)$ has actually been reported in the literature.} With the indicated percentage shifts in adopted distance, with the fitted $M/L$ ratios listed in Table (1), and with $g_{\text{gal}}^{\text{lum}}$ calculated solely from the known luminous galactic matter (stars and gas), we then obtain the full curve fits of Fig. (1), with the dashed and dash-dotted curves showing the velocities that the Newtonian $g_{\gamma}^{\text{lum}}$ and linear $g_{\gamma}^{\text{lum}} + \gamma_0 c^2/2$ terms would separately produce.\footnote{That cosmology might impact on rotation curves had already been suggested by Mannheim (1995a) in}
dark matter is assumed, and as we can see from the fits, none would appear to be needed. Despite the fact that our model is a highly constrained one with very few free parameters, it nonetheless appears to have captured the essence of the data (our fits have smoothed out some of the structure in the data since we treated the optical disks as single exponentials for simplicity), and phenomenologically our fitting would appear to be competitive with that of both the standard dark matter model (with its unsatisfactory plethora of free parameters) and the MOND (Milgrom 1983) alternative. Of course, beyond the question of fitting, unlike either dark matter or MOND, our theory is a fully motivated output to a fully covariant theory rather than being merely a phenomenologically motivated input, and for that reason alone it is already to be preferred over the other contenders. Moreover, if our theory is in fact correct, then it provides us with an actual measurement of the scalar curvature of the Universe, something which despite years of intensive work has yet to be achieved in the standard theory.

In Table (1) we also list the values for the velocity discrepancy \( (v^2/c^2R)_{\text{net}} \) at the last detected points as calculated at the shifted adopted distances by subtracting out the associated luminous Newtonian contribution. As we can see from Fig. (1), \( (v^2/c^2R)_{\text{net}} \) is indeed remarkably well fitted by \( g_{\text{lum}}^\gamma + \gamma_0 c^2/2 \sim (\gamma^* N^* + \gamma_0)c^2/2 \) for each and every galaxy in our sample; and that even while the quantity \( \gamma^* N^*/2 \) does vary enormously with luminosity over our sample (see Table (1)), nonetheless the \( \gamma_0/2 \) term overwhelms it in all but the largest galaxies, so that \( (v^2/c^2R)_{\text{net}} \) only shows a mild (but nonetheless significant) dependence on galactic mass. Given the values for \( \gamma_0 \) and \( \gamma^* \) that we obtain from the fits, we see that these two terms would contribute the same amount for galaxies with \( N^*_{\text{crit}} = 5.65 \times 10^{10} \) stars which is indeed toward the high end of our sample.\footnote{Since our theory is based on rising potentials, a paper where only the \( \gamma_0 \) term was considered in addition to \( g_{\beta}^{\text{lum}} \). It is only with the inclusion of the local \( g_{\gamma}^{\text{lum}} \) as well that the fits can be brought completely into line with the data.}

\footnote{In passing it is intriguing to note that with \( \gamma^* \) being identifiable as the coefficient of the linear potential put out by a typical star such as the sun, and with \( N^*_{\text{crit}} \) falling right in the range where the prominent galaxies are located, the asymptotic linear potential produced by a typical galaxy will be of the form \( V_{\gamma}^{\text{lum}}(r) = c^2 \gamma^* N^*_{\text{crit}} r/2 \), i.e. numerically of order \( V_{\gamma}^{\text{lum}}(r) = c^2 \gamma_0 r/2 \). Since each local galactic potential becomes of order one on distance scales of order \( r = 1/\gamma_0 \), the cooperative effect of all of the galaxies in actually...}
it is at first sight puzzling that it is able to (universally) fit the flat high luminosity rotation curves at all. To explain this intriguing aspect of our theory we recall that for an exponential disk spiral with surface brightness $\Sigma(R) = \Sigma_0 \exp(-R/R_0)$ the pure luminous Newtonian contribution causes the rotation curve to peak at around $2R_0$ with a normalization which depends on $\Sigma_0$. If we now universally match $\gamma_0$ to the Freeman limit value $\Sigma_0^F$, then in a Freeman limit galaxy with $N^*_\text{crit}$ stars (i.e. a galaxy whose entire linear term is then also universally normalized to $\Sigma_0^F$), the value of the velocity at, say, $10R_0$ or so (a region where the linear term is already dominant) will be equal to its value at the $2R_0$ Newtonian peak. Further, since at around $6R_0$ the Newtonian contribution has dropped to about half its peak value while the linear contribution there is at about half of its value at $10R_0$, and thus a flat rotation curve from $2R_0$ all the way out to about $10R_0$. Freeman limit, $N^*_\text{crit}$ galaxies thus naturally balance the falling Newtonian contribution against the rising linear one and allow flatness to obtain out to around $10R_0$ or so before the ultimate rise required of the linear potential finally sets in. Further, since we have tuned $\gamma_0$ to $\Sigma_0^F$ in the same galaxies at around $10R_0$ the velocity obeys $v^4 \sim R_0^2(\gamma_0)^2 \sim R_0^2(\Sigma_0^F)^2 \sim \Sigma_0^F L$, which we recognize as the Tully-Fisher relation. The universal matching of $\Sigma_0^F$ to $\gamma_0$ thus leads to both flatness and Tully-Fisher in $N^*_\text{crit}$ galaxies. Moreover, recognizing the special status enjoyed by $\Sigma_0^F$ producing the Hubble flow in the first place can thus reasonably be expected to produce a Universe whose natural distance scale is in fact $1/\gamma_0$.

8Given this correlation, it is plausible that the Freeman limit itself may ultimately arise as an upper bound on the galaxies which are generatable as fluctuations out of the cosmological background, a background which is indeed controlled by the $\gamma_0$ scale.

9At this juncture it is interesting to point out that it is possible to make some contact with Milgrom’s MOND alternative. Specifically, for Freeman limit, $N^*_\text{crit}$ galaxies, we note that in the region (near $6R_0$) where the total $\beta$ and total $\gamma$ terms are approximately equal (i.e. where $g_{\text{tot}}^\text{lum} \sim (g_{\gamma}^\text{lum} + \gamma_0 c^2/2) \sim \gamma_0 c^2$), $g_{\text{tot}}$ takes the numerical value $2(\gamma_0 c^2 g_{\beta}^\text{lum})^{1/2}$, an expression which we recognize as being of the MOND form on identifying $4\gamma_0 c^2 (=1.1 \times 10^{-8}\text{cm sec}^{-2})$ with Milgrom’s $a_0$ (a phenomenologically introduced parameter whose fitted numerical value is typically found to be $1.2 \times 10^{-8}\text{cm sec}^{-2}$). With this equivalence we see that while conformal gravity and MOND give very different predictions in the region beyond $10R_0$, they nonetheless give quite similar predictions in the region below $10R_0$ where most of the current measurements have been made. From a theoretical viewpoint we note that while Milgrom developed MOND in order to be able to use a universal acceleration to explain the universal Tully-Fisher relation, the particular $a_0$ dominated region form for MOND that he chose (viz. $g_{\text{tot}} = (a_0 g_{\beta}^\text{lum})^{1/2}$) was motivated by the additional assumption of asymptotically flat rotation curves (and thus asymptotic flatness even for the low luminosity rotation curves.
and \( N_{crit}^* \) in our theory, we are now able to explain the trend found by Casertano and van Gorkom. Since the low luminosity galaxies are both sub Freeman and sub \( N_{crit}^* \), the \( \gamma_0 \) term wins and the rotation curves start to rise immediately. (This parallels the trend identified in dark matter fits where the low luminosity galaxies are found to be overwhelmingly dark.) Since the intermediate galaxies are both close to Freeman and close to \( N_{crit}^* \), their rotation curves are both very flat and Tully-Fisher. And since the highest luminosity galaxies have \( N^* \) greater than \( N_{crit}^* \), the \( \gamma_0 \) term is temporarily overcome so that the curves actually display a mild initial fall (and the galaxies will still be close to Tully-Fisher unless \( N^* \) is altogether larger than \( N_{crit}^* \)). In this regard a particularly interesting high luminosity case is NGC 2841 whose data go out to twice as many scale lengths as the other high luminosity galaxies. For it the rotation velocity is actually seen to peak in the inner region at \( 326 \pm 3 \) km sec\(^{-1} \), to drop to a low of \( 271 \pm 2 \) km sec\(^{-1} \) in the intermediate region and to then rise back to \( 294 \pm 6 \) km sec\(^{-1} \) at the largest distances, a behavior which is quite suggestive of the onset of a delayed rise\(^{10} \). The conformal gravity theory would thus appear capable of explaining the general systematics of galactic rotation curves in a completely natural manner, and our study suggests that rising rather than flat rotation curves is actually the paradigm, with the luminous Newtonian contribution having inadvertently masked that fact in the higher luminosity galaxies. Moreover, through the cosmological connection we have presented, we believe we have made a case for the existence of a universal linear potential associated with the cosmological Hubble flow, an intriguing possibility which appears to eliminate the need for dark matter. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

\(^{10}\) NGC 2841 is also of interest in another regard, since it does not actually appear to obey the Tully-Fisher relation. (To be a Tully-Fisher galaxy NGC 2841 would have to be at an adopted distance of about 18 Mpc (Begeman, Broeils and Sanders 1991) rather than at the Hubble law determined distance of 9.5 Mpc which we have used in this paper, and it is thus one of the few galaxies for which the Hubble law and Tully-Fisher distance determinations differ markedly.) Since NGC 2841 is the only galaxy in our sample for which \( N^* \) is altogether larger than \( N_{crit}^* \), it is thus the only high luminosity galaxy (or high rotation velocity - its velocities actually being altogether bigger than those of any of the other galaxies) in our sample which according to our theory should then not in fact be Tully-Fisher.
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Figure Caption

Figure (1). The predicted rotational velocity curves associated with conformal gravity for each of the 11 galaxies in the sample. In each graph the bars show the data points with their quoted errors, the full curve shows the overall (adopted distance adjusted) theoretical velocity prediction (in km sec$^{-1}$) as a function of distance from the center of each galaxy (in units of $R/R_0$ where each time $R_0$ is each galaxy’s own optical disk scale length), while

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the dashed and dash-dotted curves show the velocities that the Newtonian and the linear potentials would separately produce. The dotted curves shows the total velocities that would be produced without any adopted distance modification. No dark matter is assumed.

**Table (1)**

| Galaxy | Distance (Mpc) | Luminosity ($10^9 L_{\odot}$) | ($v^2/c^2 R_{tot}$) $(10^{-30} cm^{-1})$ | Shift (%) | ($M/L_{\odot}L_{\odot}^{-1}$) | ($v^2/c^2 R_{net}$) $(10^{-30} cm^{-1})$ | $\gamma^* N^*/2$ $(10^{-30} cm^{-1})$ |
|--------|----------------|-------------------------------|-------------------------------|-----------|----------------|-------------------------------|------------------|
| DDO 154 | 3.80           | 0.05                          | 1.51                         | -11       | 0.71           | 1.49 ± 0.04                 | 0.01             |
| DDO 170 | 12.01          | 0.16                          | 1.63                         | -07       | 5.36           | 1.47 ± 0.07                 | 0.04             |
| NGC 1560 | 3.00           | 0.35                          | 2.70                         | +23       | 2.01           | 1.68 ± 0.13                 | 0.08             |
| NGC 3109 | 1.70           | 0.81                          | 1.98                         |           | 0.01           | 1.74 ± 0.19                 | 0.03             |
| UGC 2259 | 9.80           | 1.02                          | 3.85                         | +15       | 3.62           | 1.99 ± 0.26                 | 0.15             |
| NGC 6503 | 5.94           | 4.80                          | 2.14                         |           | 3.00           | 1.58 ± 0.15                 | 0.46             |
| NGC 2403 | 3.25           | 7.90                          | 3.31                         | +15       | 1.76           | 2.04 ± 0.17                 | 0.66             |
| NGC 3198 | 9.36           | 9.00                          | 2.67                         | -15       | 4.78           | 2.23 ± 0.13                 | 0.97             |
| NGC 2903 | 6.40           | 15.30                         | 4.86                         | +14       | 3.15           | 2.83 ± 0.19                 | 1.80             |
| NGC 7331 | 14.90          | 54.00                         | 5.51                         | -16       | 3.03           | 4.42 ± 0.50                 | 3.39             |
| NGC 2841 | 9.50           | 20.50                         | 7.25                         |           | 8.26           | 5.75 ± 0.30                 | 4.76             |
