Broadband continuous supersymmetric transformation: a new paradigm for transformation optics

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Abstract: Supersymmetry provides a new paradigm for transformation optics. We experimentally demonstrate broadband continuous supersymmetric transformation by designing a novel metamaterial on a Si platform for advanced control of the spatial characteristics of light. © 2023 The Author(s)

OCIS codes: (160.3918) (250.5300) (350.4238)

Transformation optics [1,2], with the aid of metamaterials [3], has established a versatile framework to mold the flow of light and tailor its spatial characteristics at will. However, in order to achieve richer functionality other than bending the trajectories, a paradigmic shift beyond traditional coordinate transformation is further required. Another intrinsic principle to formulate the transformation of a physical system is observing its Hamiltonian under transformation. For example, supersymmetry (SUSY) [4], which originated from the description of the transformation between bosons and fermions, features the degenerate eigenenergy spectra between two distinct Hamiltonians, and has facilitated advanced control of the spatial characteristics of light [5,6].

Here, we report the first experimental demonstration of continuous SUSY transformation by designing a novel gradient-index (GRIN) metamaterial on a Si platform. We utilize the synergy of supersymmetry and the metamaterial to design spatially varying dielectric permittivity, which constitutes a two-dimensional map where arbitrary transformations are prescribed simultaneously to multiple optical states for routing, switching, and spatial mode shaping, while strictly maintaining their original propagation constants.

Owing to the mathematical correspondence between the Schrödinger equation and the Helmholtz equation, we formulate the SUSY transformation by describing the potential of the Hamiltonian using the inhomogeneously distributed refractive index $n(x)$ in the transverse dimension of an optical system. Likewise, the eigenvalue spectrum of the Hamiltonian is represented by the spectrum of the propagation constants calculated from $n(x)$. For an original optical potential $n_0(x)$, SUSY transformation [7] leads to a family of isospectral optical potentials $n_f(x)$ with a free parameter $\alpha_i$:

$$ n_f^2(x; \alpha_i) = n_0^2(x) + \frac{2}{k_0} \frac{d}{dx} \left( \frac{1}{l_m(x)} \frac{d l_m(x)}{dx} \right), $$

where $l_m(x) = \int_{-\infty}^{x} \psi_0^*(x') dx'$ + $\alpha_i$, and $\psi_0(x)$ is the $m$th eigenstate of $n_0(x)$. By this means, one can conveniently delete the eigenstate $\psi_m$ and reinstate a new $\psi_m$ with the same propagation constant but different spatial characteristics of light depending on parameter $\alpha_i$. Hence, this mathematical operation can be understood as a reshaping process of $n_0(x)$, enabling the isospectral transformation of light by continuously varying the parameter $\alpha_i$ (and thus $n_f(x; \alpha_i)$) in the propagation direction (z-axis). Since the choice of $\psi_m$ is associated with the $\alpha_i$, Eq. 1 can be iterated $n$ times with different $\psi_m$ and $\alpha_i$ to generate a $n$-parameter (i.e., $\{\alpha_1, \alpha_2, ..., \alpha_n\}$) family of isospectral potentials, which offer sufficient design flexibility appropriate for transformation optics. Note that for a normalizable $\psi_m$, $n_f(x; \alpha_i)$ is guaranteed to be non-singular as long as $\alpha_i < -1$ or $\alpha_i > 0$.

As shown in Figure 1a, we design a Si waveguide system where the eigenstates bound in the transverse plane propagate along the z direction. This full 2D map of spatially dependent refractive index $n(x,z)$, in which the trajectories and the transverse mode profiles of three guided eigenstates are controlled, is obtained by substituting

$$ n_0^2(x) = a + b e^{-(x-d)^2/c^2} $$

with $a = 2.31, b = 1.4, c = 0.8$, and $d = 6.1875 \mu m$, and the wavelength of light $\lambda = 1550 nm$ into Eq. 1. Since we consider three eigenstates well guided in the given $n_0(x)$. SUSY transformation is applied to $n_0(x)$ at each z coordinate twice: the first with $\psi_m$ and parameter $\alpha_1$, and the second with $\psi_n$ and $\alpha_2$, where $m$ and $n$ denote the order numbers of the selected eigenstates. After such two-parameter transformation, as shown in Figure 1b, $n_0(x)$ is transformed to $n_f(x)$ with respect to $\psi_m$ and $\psi_n$ as well as $\alpha_1$ and $\alpha_2$. In this scenario, a series of $n_f(x)$ with continuously changing $\alpha_1(z)$ and $\alpha_2(z)$ are connected along the propagation direction z to form the 2D map of $n(x,z)$. The virtue of SUSY transformation in optics is the controllability of light with increased spatial degrees of...
freedom in association with the eigenstates (i.e., optical modes), which allows them to be arbitrarily routed, switched, and spatially evolve while preserving their propagation constants secured by SUSY. Figure 1c shows the simulation result in which all the eigenstates displayed at $z = 0$ are fundamental modes, but they are transformed along the $z$ direction to possess the spatial characteristics of the high-order modes at $z = 0.5L$, while their propagation constants remain identical to those of the corresponding fundamental modes at $z = 0$, respectively. At $z = L$, the eigenstates revert to fundamental modes, but they are directed toward different $x$ coordinates, indicating that their trajectories can also be arbitrarily maneuvered at will.

Broadband light steering and spatial switching through SUSY transformations were experimentally validated by characterizing the normalized transmission spectra of the Si GRIN metamaterial corresponding to the design in Figure 1. Figure 2a displays the transmission spectra measured at three outputs at $z = L$ (O1, O2, O3) for three different inputs at $z = 0$ (I1, I2, I3) using a tunable laser with a wavelength range from 1460 nm to 1570 nm. The result shows high transmission of 0.94 on average with negligible crosstalk between neighboring channels at the wavelength of 1550 nm. In addition, as shown in Figure 2b, broadband transformation of spatial characteristics of light enabled by the SUSY transformation was verified by imaging the intensity profile of guided light at the midpoint along $z$ of the GRIN metamaterial. The clear nodes observed in the images agree well with the calculated modal field profiles at $z = 0.5L$ as shown in Figure 1c, definitively confirming the efficacy of isospectral SUSY transformation.

In conclusion, we have experimentally demonstrated an on-chip Si GRIN metamaterial that features the continuous SUSY transformation through an isospectral family of optical potentials, corresponding to the continuous transformation of the Hamiltonian of a physical system but strictly maintaining its eigenvalues. Our continuous SUSY transformation approach is scalable to a higher number of eigenstates and free parameters, and also applicable to more complicated index distribution, thereby creating an ideal platform for on-chip space-division multiplexing in information technologies.

Figure 1. a. GRIN metamaterial designed to substantiate $n(x, z)$, where different optical states (blue, red, green) with different propagation constants can propagate with spatial characteristics and directions dictated by the SUSY transformation. b. The optical potentials and the corresponding mode profiles of three eigenstates (blue, red, green) at five different $z$ coordinates at $z = 0$, 0.25L, 0.5L, 0.75L, and L. c. Simulated electric field intensity in the GRIN metamaterials with prescribed gap distributions as in a.

Figure 2. a. Normalized transmittance spectra measured for the device in Figure 1. The transmission spectra are measured at three outputs (O1, O2, and O3) for different inputs from left to right: I1, I2, and I3. b. Measured intensity profiles in the $xy$ plane when the light was launched at different inputs from left to right: I1, I2, and I3.

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