Force sensitivity analysis and scale design of Stewart parallel manipulator

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Abstract
There are many indices for evaluating the operational performance of parallel robots, but most of them start from kinematics and rarely pay special attention to the force transmission process. After studying the relationship and characteristics of the forward and reverse force transmission of the Stewart parallel manipulator under quasi-static conditions, three new indices are proposed in this paper, which are forward force sensitivity, reverse force sensitivity, and overall force sensitivity. Subsequently, the influences of six parameters on the overall force sensitivity index are analyzed. Furthermore, this paper propose three new structural optimization design schemes based on the three newly proposed indices. Finally, it is verified experimentally that the unit reverse force sensitivity of a single joint is consistent with the theory and the error between the theoretical and experimental values is obtained.

Keywords
Force sensitivity, Stewart parallel manipulator, human-machine collaboration, force transmission

Introduction

Collaborative robot

In recent years, with the continuous progress of automatic control technology, electronic technology, and internet technology, industrial robot technology has seen considerable development and is widely used in manufacturing. With the increasing demand for robots in human society, the safety and reliability of robotics also increases. People are no longer satisfied with the off-line programming of robots for control, but desire greater cooperation with robots. The robot can achieve or surpass its desired function or work better through gestures, direct contact, drag and teach, and other simpler and more intuitive information transmission methods. The concept of collaborative robots was formally proposed in 1996 by two professors, J. Edward and Michael Peshkin of Northwestern University, United States. They applied for a patent, which defined collaborative robots as a type of object that can be operated in cooperation with human operators.¹

The most notable obstacle to achieving cooperation between people and robots in the same working environment is the inevitable safety problems produced when people approach robots, such as contact, collision, and squeezing.¹,² These are key problems that restrict the progress of collaborative robot technology and pose important obstacles to further popularization and the application and expansion of production.

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In order to achieve the purpose of collaborative work between robots and humans, the most commonly used method is addition of sensors to sense external information for traditional industrial robots, and collect environmental information, including human actions, the force exerted on the robot, and whether collisions occur. Common external sensors include cameras that capture visual information, sound sensors that obtain auditory information, piezoelectric sensors that gauge touch, and force/torque sensors that obtain force measurements. Among the above methods, installing the force/torque sensor is the most popular method to improve the safety of the robot. It is possible to realize human-machine collaboration by using force perception technology. Collaborative robot collision detection technology, robot virtual force sensor and flexible joint technology, collaborative robot torque control, and impedance control technology.

Parallel collaborative robot

There is an increasing number of collaborative robots that can realize human-machine interaction. However, the number of collaborative robots based on the parallel structure is still relatively small. The parallel robot has the advantages of high rigidity, compact structure, large load-bearing ratio, and high dynamic performance. Therefore, it can be implemented in important applications, such as motion simulation, orientation adjustment, vibration suppression, and load bearing. Addition of a force sensor to a parallel robot can enable people and robots to interact more directly and cooperate, thereby enhancing the benefits of the parallel robot.

At present, the mainstream application of parallel robots still involves traditional independent work forms without contact with people. Research and application development of collaborative robots is primarily focused on the field of serial structure robots. With the continuous development and progress of parallel robot theory and engineering technology, there is a consequent improvement in reliability and safety, and parallel robots with built-in force sensors are becoming increasingly common.

The use of force sensors in parallel robots combined with dynamics can improve the safety of parallel robots in cooperating with people. This protects workers from injury when cooperating with robots, and also avoids robot damage caused by excessive joint force from contact with people.

As the theory and safety evaluation indices of the force transmission relationships of parallel robots continue to improve and the industrial production technology of force-aware parallel robots continues to advance, parallel robots promise to become an essential part of the field of collaborative robots.

**Evaluation indices of collaborative robot**

This study primarily focuses on the evaluation indices of parallel collaborative robots, and proposes an index for evaluating the strength and weakness of the parallel robot configuration in terms of force perception and force application, so as to analyze the force sensitivity or insensitivity of parallel robots, and guide the new robot scale design.

Many scholars focus on the Jacobian matrix of robots, and study the indices and related theories of force perception. Furthermore, they proposed indices, such as operability and minimum singular value from the eigenvalues, eigenvectors, and condition numbers of the Jacobian matrix. This enables evaluation of the force transmission characteristics of the robot, which provides information about the robot orientation adjustment and configuration parameter design. Yoshikawa proposed the concept of manipulability, which is used to evaluate the comfort level of the robot when receiving the force of the end-effector. Klein and Blaho compared a number of indices for the quantification of dexterity of manipulators. Gosselin and Angeles proposed the global conditioning index, and it displays a larger value when the joint force difference caused by the end effector is smaller. Pashkevich et al. considered operability to be the optimization goal of analysis and design. A scale design method for an orthogonal sliding mechanism was proposed. Kucuk designed a new hybrid parallel robot manipulator, and optimized the dexterous workspace based on the condition number. Inner and Kucuk developed a general Stewart platform simulation tool based on the optimization index of the condition number. Another category of indices is based on force transmission and spiral theory. Another category of indices is based on force transmission and screw theory rather than the Jacobian matrix. For example, Xie et al. proposed the global transmission index under determinate constraint of workspace and optimized the design of Stewart platform.

The force-perception Stewart manipulator mentioned in this paper pays much attention to the force transmission performance. However, a class of indices based on the Jacobian matrix are often concerned with evaluating kinematic relations, and seldom specifically consider mechanical transmission. Their common characteristic is that the relative size of the index is very meaningful, but the physical meaning of the value of the index itself is difficult to explain. Although the indices based on the screw theory that focus on the process of robot force transmission have a clear physical meaning, due to higher requirements on mathematical foundations and more complex mathematical operations, they are difficult to be widely promoted in industrial applications.
The first contribution in this paper is: considering the characteristics of the two types of indices, this paper will propose a new evaluation index system that starts from the Jacobian matrix, but specifically considers the force transmission process instead of kinematics, and is relatively convenient to calculate. The second contribution in this paper is: combining the different application scenarios of the Stewart manipulator, this paper will propose three new structural optimization design schemes based on the three newly proposed indices.

In this study, the mechanism of the parallel robot is first modeled. The kinematics model and basic static equations of the parallel robot are deduced from the aspects of kinematics and statics, and the Jacobian matrix of force transmission is obtained. Based on the Jacobian matrix of force transfer of parallel robots, the transfer relationship between the end effector force of the parallel robot and the joint force is studied. Subsequently, an evaluation index is proposed to determine whether the joint force of the robot is sensitive and vulnerable to damage. Similarly, another two evaluation indices is proposed, which comprehensively consider the joint force and the end effector force, to determine the degree of the amplification effect.

Subsequently, in this study, the proposed force sensitivity indices are used to analyze the sensitivity of the parallel robot joints to stress, so as to determine whether the parallel robot being studied is safe, and whether its joints are easily damaged. They are also used to guide the parallel robot to adjust its position and orientation in order to improve joint stress.

Finally, in this study, the force sensitivity indices of parallel robots is comprehensively investigated, and the safety of humans and machines is considered in combination with the actual design target application scenarios. Subsequently, several parallel robot scale designs are proposed.

Force sensitivity indices

Stewart parallel manipulator with force sensors

A Stewart parallel manipulator is a typical parallel robot. In this study, a Stewart parallel manipulator will be used to represent the parallel collaborative robot. Figure 1 depicts a Stewart parallel manipulator, which consists of a static platform and moving platform connected to each other by six telescopic joints. The telescopic joint is connected to the static platform through a Hook hinge, and to the moving platform through a ball hinge.

The ball screw converts the rotary motion of the servo motor into the linear motion of the telescopic joint, thereby realizing the movement ability of the six telescopic joints of the Stewart parallel manipulator.

At the location where each telescopic joint is connected to the ball hinge, a tension/compression force transducer is installed to measure the internal force of the six joints. A six-dimensional force sensor is installed in the center of the moving platform in order to measure the contact force between the moving platform and the external environment.

Overview of force sensitivity

In this study, the mechanics of Stewart parallel manipulator are investigated during interaction with people or other objects. This can be divided into two aspects. The first is the magnitude of force and moment exerted by the end effector on the external environment when the six joints of the Stewart parallel manipulator apply thrust. This reflects the degree of amplification of the pressure exerted by the Stewart parallel manipulator on the outside world and is qualitatively termed as the forward force sensitivity. The second aspect is the response of the internal force of the six joints when the moving platform of the Stewart parallel manipulator experiences external force or moment. This reflects the degree to which the external force exerted on the joint is amplified when the Stewart parallel manipulator experiences external force and is qualitatively defined as reverse force sensitivity.

When the thrust applied by the six joints is small and the force or torque output by the moving platform is large, it indicates that the forward force sensitivity of the Stewart parallel manipulator is very large. The advantage of this is that it can be used in scenarios where pressure is applied. However, the disadvantage is that a small joint thrust will be amplified to a large output force, during the human-machine interaction, which will easily injure the operator.

When the force experienced by the moving platform is small and the internal forces of the six joints are large, it indicates that the reverse force sensitivity of the Stewart parallel manipulator is very large. The
advantage of this is that it can be used to sense external force as a force sensor. However, the disadvantage is that during the human-machine interaction, a small pressure applied by an operator will be amplified into a large joint force, thereby damaging the machine.

In real working conditions, especially in work scenarios involving human-machine interaction, injury to the operator or damage to the machine is undesirable. Therefore, we wish to design a Stewart parallel manipulator with small forward force sensitivity and small reverse force sensitivity. In addition, we hope to ensure the safety of human-machine interaction to a feasible extent through the size design or the design of the working space.

The factors that affect the force sensitivity of Stewart parallel manipulators are segregated into the following two categories. The first includes the position and orientation parameters of the moving platform. The second involves the dimensions of the scale design of the Stewart parallel manipulator, which include the radius of the static platform, radius of movable platform, static platform hinge point circle center half angle, and moving platform hinge point circle center half angle.

We wish to establish a quantitative force sensitivity index to analyze how each influencing factor specifically affects the sensitivity of the Stewart parallel manipulator. In addition, we aim to obtain a suitable scale design to provide better security for humans and robots in human-machine interaction.

**Static model**

Figure 2 illustrates the structure of a Stewart parallel manipulator. The upper platform and lower platform are moving and static, respectively. The circle center of the static platform is defined as $O$, and the $\{A : O \rightarrow xyz\}$ rectangular coordinate system is established with $O$ as the origin. Similarly, the circle center of the moving platform is defined as $P$, and the $\{B : P \rightarrow uvw\}$ rectangular coordinate system is established with $P$ as the origin. When the moving platform is in the initial position and orientation, the $x$-axis is in the same direction as the $u$-axis, the $y$-axis is in the same direction as the $v$-axis, and the $z$-axis is in line with the $w$-axis.

On the static platform, hinge points $A_1, A_2, A_3, A_4, A_5,$ and $A_6$ are distributed in a counterclockwise direction in sequence. Similarly, on the moving platform, hinge points $B_1, B_2, B_3, B_4, B_5,$ and $B_6$ are also distributed in sequence. $A_i$ and $B_i$ ($i = 1, 2, ..., 6$) are two hinge points distributed on the same telescopic joint.

The red mark in the schematic of the Stewart parallel manipulator in Figure 2 indicates the application of thrust. The six telescopic joint each apply a thrust, and finally the moving platform is transformed into a force and moment, which acts on the external environment.

Figure 3 presents the top view of the Stewart parallel manipulator. The radius of the static platform hinge point distribution circle, radius of movable platform hinge point distribution circle radius, static platform hinge point circle center half angle, and moving platform hinge point circle center half angle are denoted by $R_a, R_b, \gamma_a,$ and $\gamma_b$.

The position of $A_i$ in coordinate system $\{A\}$ can be represented by vector $\mathbf{a}_i$. Similarly, the position of $B_i$ in coordinate system $\{B\}$ can be represented by vector $\mathbf{b}_i$. The angle between $OA_i$ and $x$-axis is $\theta_{A_i}$. The angle between $OB_i$ and $x$-axis is $\theta_{B_i}$. The length of $A_iB_i$ is $d_i$.
and the direction is $s_i$. The travel range of the telescopic joint is $[p_{\text{min}}, p_{\text{max}}]$. In addition, the original configuration parameters of the Stewart parallel robot investigated in this study are presented in Table 1.

According to the differential kinematics of the parallel robot, the following equation can be obtained:

$$\dot{d} = J^{-1} \dot{x} \quad (1)$$

Where $J^{-1}$ is the inverse Jacobian matrix with six rows and six columns, and its specific form is as follows:

$$J^{-1} = \begin{bmatrix} s_1^T, (Rb_1 \times s_1)^T \\ \vdots \\ s_6^T, (Rb_6 \times s_6)^T \end{bmatrix} \quad (2)$$

Where $R$ is the rotation transformation matrix of the coordinate system $\{B\}$ relative to $\{A\}$.

Note that under the singular position of the Stewart platform, the Jacobian matrix will appear singular. At this time, the matrix is not invertible. The mechanisms involved in this article are all discussed in a non-singular position. For the coherence of the discussion, I will not repeat the proof that the Jacobian matrix involved is not singular.

The statics force transmission relationship of the Stewart parallel manipulator can be given as follows, and the derivation process is detailed in Appendix.

$$F = J^{-T} \tau \quad (3)$$

Where, $F$ is the six-dimensional force-torque combined vector, and $\tau$ is the joint force variable.

To promote ease of analysis, $J^{-T}$ is assumed to be reversible in the following discussion. This can be achieved by limiting the end trajectory of the Stewart parallel manipulator to a non-singular workspace. So far, we have obtained the force transfer relationship between the moving platform of the parallel robot and the telescopic joint, that is, the static model.

**Definition of force sensitivity**

Observing equation (3), we find that the six elements in $\tau$ are all forces, while the first three elements in $F$ are forces and the last three elements are moments. For the convenience of subsequent discussions and to make the dimensions uniform, the last three elements in $F$ are divided by $R_b$, and every element in the last three rows of $J^{-T}$ is also divided by $R_b$, making $F$ six forces. In this way, all elements in $J^{-T}$ are dimensionless.

Equation (3) after unified dimensions can be expressed as follows:

$$\bar{F} = J_f^T \bar{\tau} \quad (4)$$

$$\bar{\tau} = J_f^T \bar{F} \quad (5)$$

Where,

$$\bar{F} = \begin{bmatrix} F_x, F_y, F_z, M_x, M_y, M_z \end{bmatrix}^T \quad (6)$$

$$J_f^{-T} = \begin{bmatrix} j_1, j_2, j_3, j_4, j_5, j_6 \end{bmatrix}^T \quad (7)$$

Where $j_i$ is the $i$th row vector of $J_f^{-T}$.

For the convenience of subsequent expressions, we replace $J_f^{-T}$ with $J_f$ to express the forward force transfer matrix of the Stewart parallel manipulator. In addition, we replace $J_f^T$ with $J_f$ to express the reverse force transfer matrix of the Stewart parallel manipulator. This is expressed as follows:

$$\bar{F} = J_f \bar{\tau} \quad (8)$$

$$\bar{\tau} = J_f^T \bar{F} \quad (9)$$

When we focus on the forward force transmission characteristics of the robot, we need to extract a feature quantity from $J_f$ to represent the amplification of the robot joint force $\tau$ transmission to the end force $F$.

Equation (8) can be written as a force component form as follows:

$$\bar{j}_i = a_i \cdot \bar{\tau} = |a_i| |\bar{\tau}| \cos(a_i, \bar{\tau}), \quad i = 1, 2, \ldots, 6 \quad (10)$$

Where $\bar{j}_i$ is the $i$th element in $\bar{F}$, $a_i$ is the $i$th row vector of $J_f$, $|a_i|$ is the Euclidean norm of $a_i$, and $|\bar{\tau}|$ is the Euclidean norm of $\bar{\tau}$. $\langle a_i, \bar{\tau} \rangle$ is the angle between $a_i$ and $\bar{\tau}$.

We separately focus on the transmission of six joint forces to a single element of end force $\bar{F}$. In actual working conditions, the value and direction of $\bar{\tau}$ are random. However, when the configuration, position, and orientation of the robot are determined, $J_f$ is

**Table 1. Original configuration parameters of the manipulator.**

| Configuration parameter | Specific value or expression |
|-------------------------|-----------------------------|
| $R_a$                   | 450mm                       |
| $R_b$                   | 225mm                       |
| $\gamma_a$              | 10°                         |
| $\gamma_b$              | 25°                         |
| $\rho_{\text{min}}$     | 752mm                       |
| $\rho_{\text{max}}$     | 982mm                       |
| $\theta_{a_1}, \ldots, \theta_{a_6}$ | $\frac{\pi}{2} - \gamma_a + \frac{\pi}{2} - \gamma_a$ |
| $\theta_{b_1}, \ldots, \theta_{b_6}$ | $\frac{\pi}{6} + \gamma_b + \frac{\pi}{6} + \gamma_b - \frac{\pi}{2} - \gamma_b$ |


determined. Although the degree of amplification of each force depends on the angle $\langle a, \tau \rangle$. $|a|$ represents the average degree of amplification of the joint force of any value and direction.

The unit forward force sensitivity index is defined as follows:

$$
\eta_i = |a_i|, \quad i = 1, 2, ..., 6 \quad (11)
$$

The unit forward force sensitivity direction is defined as follows:

$$
P_i = \frac{a_i}{|a_i|}, \quad i = 1, 2, ..., 6 \quad (12)
$$

The unit forward force sensitivity index $\eta_i$ characterizes the degree of amplification of six joint force amplification to the $i$th end force element.

The unit forward force sensitivity direction $P_i$ characterizes the direction in which the $i$th end force element is most sensitive to joint force.

When the joint forces $\tau$ and $P_i$ are in the same direction, the joint force $\tau$ is amplified to the $i$th end force element $f_i$ by a factor of $\eta_i$ and this causes the largest positive end force element $f_i$.

When the direction of the joint forces $\tau$ and $P_i$ is reversed, the joint force $\tau$ is amplified to the $i$th end force element $f_i$ by a factor of $-\eta_i$, and this causes the largest negative end force element $f_i$.

When the joint directions of $\tau$ and $P_i$ are vertical, the joint force $\tau$ will not cause the end force $f_i$. That is, the passage of the joint force $\tau$ to the end force element $f_i$ is blocked, but other end force element caused by $\tau$ can be large.

The forward force sensitivity index represents the average magnification of the joint force transferred to the end force. The value of the forward force sensitivity represents the maximum value of the Euclidean norm of the six-dimensional end force vector when the direction of the unit six-dimensional joint force is random. The definition is provided directly as follows:

$$
\eta_r = \frac{1}{6} \sum_{i=1}^{6} |b_i| \quad (14)
$$

Where $|b_i|$ is the $i$th row vector of $J_r$.

In order to comprehensively judge the situation of the forward and reverse force sensitivity of the entire mechanical structure, we calculate the average of the forward force sensitivity index and reverse force sensitivity index to obtain the overall force sensitivity index as follows:

$$
\eta = \frac{1}{2} (\eta_r + \eta) = \frac{1}{12} \sum_{i=1}^{6} |a_i| + \frac{1}{12} \sum_{i=1}^{6} |b_i| \quad (15)
$$

The overall force sensitivity represents the average sensitivity of force transmission of the robot, which can be used to judge whether the force transmission process of the robot is stable and safe for application in actual work.

If the overall force sensitivity index shows a very small reading, no matter whether the robot is applying force in the forward direction or feeling force in the reverse direction, significant safety problems will not occur.

If the overall force sensitivity index displays a very large reading, it means that in some positions and orientations of the robot, a small contact force at certain angle may exert a large force in some joints of the robot, which can easily damage the joints. It could also entail that at certain angles, a small joint force will be amplified into a large end force, which could cause injury to people when the parallel robot interacts with them.

In order to illustrate the main contributions of three proposed indices, this paper compares them with the similar indices in the literature, as shown in Tables 2 to 4.

**Analysis of force sensitivity indices**

According to the definition of force sensitivity, there are two main types of influencing factors. One is the position and orientation of the moving platform, and the other is the configuration parameters of the Stewart parallel manipulator.

In the following studies, except for the parameters under study, the other parameters are taken from the initial configuration parameters in Table 1 and the initial position, orientation parameters, which is $x = [x, y, z, \psi, \theta, \phi]^T = [0m, 0m, -0.81m, 0^\circ, 0^\circ, 0^\circ]^T$.

**Influence of position of the moving platform**

Figure 4 illustrates the change in the overall force sensitivity index $\eta$ with position parameters $x$ and $y$ of the
moving platform. It is seen that $\eta$ takes the smallest value $1.4773$ when $[x, y]$ is equal to $[0m, 0m]$, and as the absolute value of $x$ or $y$ increases, $\eta$ also increases. Therefore, the mechanical structure is the safest when the value of $[x, y]$ is $[0m, 0m]$.

Figure 5 displays the change in the overall force sensitivity index $\eta$ with the position parameters $z$ of the moving platform. It is evident from the figure that in the range of $-1m < z < 0m$, $\eta$ first falls and then rises. And the speed becomes slower and slower during the
descent, and the minimum value $h = 1.1595$ is obtained when $z = -0.24m$.

However, under actual working conditions, in fact, $z$ cannot obtain $-0.24m$. For the Stewart parallel manipulator used in this study, during the experiment, the value of $z$ was only $-0.93m$ to $-0.70m$. Therefore, for this robot, the larger $z$ is, the smaller $h$ is, and the safer it is.

Influence of orientation of the moving platform

Figure 6 illustrates the change in the overall force sensitivity index $\eta$ with roll angle $\psi$, pitch angle $\theta$, and yaw angle $\phi$ of the moving platform. As can be seen from the figure, no matter whether it is $\psi$, $\theta$, or $\phi$, when they change from $-45^\circ$ to $45^\circ$, $\eta$ is first reduced and then increased, and when they are equal to $0^\circ$, the minimum value $\eta = 1.4773$ is obtained.

$$\eta \text{ with roll angle } \psi, \text{ pitch angle } \theta, \text{ and yaw angle } \phi$$

Therefore, when $\psi$, $\theta$, and $\phi$ all take $0^\circ$, the mechanical structure is the safest. In all three angles, the larger the angle is, the more sensitive the mechanism mechanics is and the less safe it is.

Influence of configuration parameter of the manipulator

Figure 7 shows the change of overall force sensitivity index $\eta$ with the radius $R_a$ and the radius $R_b$. With the change of $R_a$ and $R_b$, $\eta$ is not monotonous, and has experienced the process of decreasing first and then increasing. When $R_a$ or $R_b$ is close to $0m$ or close to positive infinity, $\eta$ is positive infinity. In order to express concisely and conveniently, Figure 7 intercepts

$$\eta \text{ with the radius } R_a \text{ and the radius } R_b$$
Figure 8. Change of overall force sensitivity index $\eta$ with the angle $\gamma_a$ and the angle $\gamma_b$.

Figure 9. Change of overall force sensitivity index $\eta$ with the radius $R_b$ and $z_0$ when $R_a$ is 1m.

a change curve near the minimum value of $\eta$ to represent its overall change trend. During the process of $R_a$ changing from 0.3m to 2m, $\eta$ obtains the minimum value $\eta = 1.3077$ at $R_a = 0.91m$. In the process of $R_b$ changing from 0.1m to 1m, $\eta$ takes the minimum value $\eta = 1.4298$ at $R_b = 0.41m$.

Figure 8 shows the change of overall force sensitivity index $\eta$ with the angle $\gamma_a$ and the angle $\gamma_b$. It can be known from the definition of $\gamma_a$ that the value range of $\gamma_a$ is only $0^\circ \sim 60^\circ$, and so is $\gamma_b$. Several phenomena can be seen from the figure. First, all $\eta$ on the line of $\gamma_a + \gamma_b = C$ (C is a constant) are equal, that is, when $\gamma_a + \gamma_b$ is equal, $\eta$ is also equal. Second, as $C$ increases, $\eta$ increases first and then decreases. When $\gamma_a + \gamma_b = 60^\circ$, $\eta$ tends to positive infinity. Third, the value of $\eta$ is symmetrically distributed about the line $\gamma_a + \gamma_b = 60^\circ$.

When designing an manipulator, $\gamma_a$ and $\gamma_b$ usually assume values within a small range of less than $30^\circ$. Therefore, in this range, $\eta$ will increase with an increase in $\gamma_a$ or $\gamma_b$. Considering that the actual mechanical structure may interfere, it is impossible to obtain $0^\circ$ for $\gamma_a$ or $\gamma_b$. Therefore, considering the safety and the practical feasibility of the mechanical structure, $\gamma_a$ and $\gamma_b$ should be as small as possible without interference.

**Scale design based on force sensitivity**

The force sensitivity index can be used to address the following three aspects of scale design. First, we can minimize the overall force sensitivity, so that the mechanism will not have a large forward or reverse force amplification effect during the human-machine interaction process, which makes the mechanism scale design extremely safe. Second, we can maximize the forward force sensitivity. Such a mechanism can amplify a small joint force into a large force applied by moving platform, which can be used as a press. Third, we can maximize the reverse force sensitivity. Such a mechanism can amplify a small contact force into a large joint force, which is very suitable for force sensors.

**Safest mechanism scale design with minimal overall force sensitivity**

We wish to design an optimal mechanical structure so that the parallel robot can minimize the overall force sensitivity as long as it meets the working conditions, thereby ensuring the safety of people and machines.

To sum up the previous study, we studied a total of 10 parameters including $R_a$, $R_b$, $\gamma_a$, $\gamma_b$, $x$, $y$, $z$, $\psi$, $\theta$, and $\phi$. Among them, the mechanical structure design includes four structural parameters $R_a$, $R_b$, $\gamma_a$, $\gamma_b$, and $z_0$ (the initial value of $z$, which represents the influence of $\rho_{min}$ and $\rho_{max}$). From the previous research, we can conclude that it is safer for $\gamma_a$ and $\gamma_b$ to be as small as possible. It is explained below how $R_a$, $R_b$, and $z_0$ get their values.

The method we study is to first take a fixed value of $R_a$, and change $R_b$ and $z_0$ to find the optimal value to make $\eta$ the smallest. Then, change $R_a$, repeat the above process, and finally obtain the proportional ratio of $R_a$, $R_b$, and $z_0$.

Figure 9 shows the change in the overall force sensitivity index $\eta$ with the radius $R_b$ and $z_0$ when $R_a$ is 1m. You can see the general trend of $\eta$ in the figure with the change of $R_b$ and $z_0$. You can see that there is a concave point in the figure. The farther away from the concave point, the larger $\eta$ is. In addition, when $z_0$ is relatively small, the change of $\eta$ is not obvious. When $z_0$ is close to 0m, the change of $\eta$ is more evident for a larger value of $\eta$. Through numerical calculation, we can find that when $R_a$ is 1m,
the minimum value of $\eta$ is 1.0696, which is obtained near $R_b = 0.9m$ and $z_0 = -0.4226m$.

In order to obtain more accurate values of $R_b$ and $z_0$, we continue to narrow the scope and improve the calculation progress. Figure 10 depicts the enlarged view of the vicinity of the concave point. From this, we calculated that when $\eta$ takes the minimum value, $R_b$ is 0.9063m and $z_0 = -0.4226m$.

Subsequently, we change the value of $R_a$ and continue our calculations in order to obtain the results shown in Table 5. Through observation, it is found that regardless of the change in the value of $R_a$, although the optimal values of $R_b$ and $z_0$ fluctuate, the ratio $R_b/R_a$ and $z_0/R_a$ remains unchanged. From this, we obtain the optimal scale configuration parameters of the mechanical structure that minimizes the overall force sensitivity $\eta$ as follows:

1. $\gamma_a = \gamma_b = 0^\circ$
2. $\frac{R_b}{R_a} = 0.9063$
3. $\frac{z_0}{R_a} = -0.4226$

Through further calculation and research, these optimal size ratio Stewart parallel manipulators were found to have a common feature. That is, the angle between the telescopic joint and the direction of gravity is approximately about $45^\circ$. The simple mechanical explanation for this feature is that the forward or reverse force transmission is more symmetrical, so the integrated force sensitivity is better.

The schematic of the optimal structural scale design of the safest mechanism is presented in Figure 11. When designing a new Stewart parallel manipulator, we can choose a radius ratio $R_b/R_a = 0.9063$, and the length of the telescopic joint can also refer to the ratio $z_0/R_a = -0.4226$. They ensure the safety of operator and machines.

**Press scale design with maximum forward force sensitivity**

In accordance with the previous study, we consider the five factors of $R_a, R_b, \gamma_a, \gamma_b$, and $z_0$ in the structural design here.

In order to design the press, we use the previously defined forward force sensitivity as an optimization index. According to the previous analysis, if the value of the forward force sensitivity is large, the joint force can also cause greater end pressure. Therefore, we

| $R_a$ (m) | $R_b$ (m) | $z_0$ (m) | $\eta$ | $\frac{R_b}{R_a}$ | $\frac{z_0}{R_a}$ |
|----------|-----------|-----------|-------|-------------------|-------------------|
| 0.1      | 0.0906    | -0.0423   | 1.0696| 0.9063            | -0.4226           |
| 0.45     | 0.4078    | -0.1902   | 1.0696| 0.9063            | -0.4226           |
| 0.8      | 0.7250    | -0.3381   | 1.0696| 0.9063            | -0.4226           |
| 1        | 0.9063    | -0.4226   | 1.0696| 0.9063            | -0.4226           |
| 2        | 1.8126    | -0.8452   | 1.0696| 0.9063            | -0.4226           |
| 5        | 4.5315    | -2.1131   | 1.0696| 0.9063            | -0.4226           |
| 10       | 9.0631    | -4.2262   | 1.0696| 0.9063            | -0.4226           |

Figure 10. Enlarged view of the vicinity of the concave point.

Figure 11. Schematic of the optimal structural scale design of the safest mechanism.
varies, but it is basically between $-0.86$ and $-0.9$, we take $-0.88$ here.

Therefore, in order to obtain the press structure with the most obvious pressure amplification effect, the optimal size configuration is as: $\gamma_a = \gamma_b \rightarrow 0^\circ, R_b/R_a = 0.5, z_0/R_a = -0.88$. In addition, the schematic diagram of the optimal structural scale design of the press is shown in Figure 14.

**Force sensor scale design with maximum reverse force sensitivity**

Similarly, we design the optimal sensor scale design based on the reverse force sensitivity. If the reverse force sensitivity is large, then the small end force can be amplified into a large internal joint force. Such a structure is highly suitable for the working situation of the sensors. Therefore, we configure the parameters according to the reverse force sensitivity. We want it to be as large as possible.

For the five parameters $R_a, R_b, \gamma_a, \gamma_b$, and $z_0$. First we study the two parameters $\gamma_a, \gamma_b$, and their effects are shown in Figure 15. Observing this figure, it can be found that the same as before is that when $\gamma_a + \gamma_b$ is a fixed value, $\eta_r$ is also a fixed value. The difference is that $\eta_r$ here increases as the value of $\gamma_a + \gamma_b$ increases. Therefore, we choose larger $\gamma_a, \gamma_b$, which is $\gamma_a = \gamma_b = 30^\circ$.

After that, take $R_a$ as a fixed value and observe the law. First, $R_a = 1m$, the change of reverse force sensitivity index $\eta_r$ with the radius $R_b$ and $z_0$ is shown in Figure 16. It can be seen from the figure that within a certain range, the closer $R_b$ and $z_0$ are to $0m$, the larger $\eta_r$ is. Therefore, we should take $R_b$ as small as possible, and $z_0$ should be as close to $0m$ as possible.

In order to increase the sensitivity of the sensor force, the optimal scale design is configured as: $\gamma_a = \gamma_b = 30^\circ, R_b/R_a \rightarrow 0, z_0/R_a \rightarrow 0$. The schematic of
the optimal structural scale design of the force sensor is presented in Figure 17.

To sum up, the optimal size design of Stewart parallel manipulator for three different application scenarios is shown in Table 6.

**Force sensitivity experiment**

This chapter verifies the influence of the different positions and orientations of the moving platform on the unit reverse force sensitivity when the structure configuration parameters of the Stewart parallel manipulator are determined.

**Experimental method**

Figure 18 shows a Stewart parallel manipulator with force sensing function for experiment. The MCL-L tension pressure sensor is installed at each joint of the Stewart parallel manipulator, the range of which is 500 N. The analog voltage signal of the sensor is converted into a digital signal through the ACC-36P data conversion module.
acquisition card, and is stored in the variable storage area of the Turbo PMAC1 card. The computer software instructions are converted into hardware instructions through the Turbo PMAC1 card and sent to the Sinano H15C servo drive. The servo drive drives the Sinano 6CC201G-3DEBE AC servo motor to control the movement of the robot. The motion information read by the drive from the motor encoder is also stored in the Turbo PMAC1 card, and the co-controller reads from the Turbo PMAC1 card and the moving platform orientation and joint force.

In the experiment, the interactor applied pressure at the origin point of the driven platform with a spring tension gauge.

Figure 19 represents a physical map of the force-applying process in the force-sensing experiment. The interactive personnel use a precision spring tension gauge to apply horizontal tension in the plane of the moving platform. The magnitude of the tension is guaranteed by the precision spring and scale, and the error does not exceed 0.5N.

Figure 20 is a schematic of the direction change of the external force sequence during the experiment (top view of the moving platform). The size of the external force is constantly controlled and maintained at 50N. A horizontal pure tensile force $F$ of 50N is applied to the origin point $P$ of the moving platform in the plane of the moving platform. The $k$-th applied $F$ is recorded as $F_k$, and the external force sequence is applied at intervals of 30°. During this whole process, the changes of the sensibility of the six joints with time were recorded as a set of experimental data.

An experiment is conducted under the original position and orientation conditions. Subsequently, the position and orientation of the six parameters are changed successively, and the experiment is conducted again. After that, this data is compared with the experimental data of the original parameters and the difference is observed.

**Experimental results**

Figure 21 depicts the measured change values of the six joint forces during the application of the external force sequence $\{F_1, ..., F_{12}\}$ at the original position and orientation parameter $x_0 = [x, y, z, \psi, \theta, \phi]^T = [0m, 0m, -0.86m, 0°, 0°, 0°]^T$ (joint force excluding systematic errors), where $f_i$ represents the schematic diagram of the force change of the $i$th joint with time. It can be seen from the figure that for a series of external forces with constant magnitude and variable direction, the maximum value of the six joint forces is approximately $f_{max} = 52N$, so the unit reverse force sensitivity is about $\eta_{ri} = 52N/50N = 1.04$.

Figure 22 depicts the change in the joint force with the external force sequence $\{F_1, ..., F_{12}\}$ when the moving platform is located at $x_1 = [0m, 0m, -1m, 0°, 0°, 0°]^T$ in the working space. It can be deduced from the figure that the unit reverse force sensitivity is $62N/50N = 1.24$. Comparing Figures 21 and 22, it can be seen that when the parameter $z$ decreases, the unit reverse force sensitivity of the six joints increases, which is consistent with the conclusions of the previous study. The theoretical values of the unit reverse force sensitivity of the six joints of the moving platform at $[0m, 0m, -0.86m, 0°, 0°, 0°]^T$ and $[0m, 0m, -1m, 0°, 0°, 0°]^T$ are 1.02 and 1.17, respectively, and the errors of the experimental and theoretical values are 1.7% and 6.4%, respectively.

In similar ways, several other sets of position and orientation parameters were considered and the experiment was conducted. The results are shown in Table 7. It can be seen from the table that the experimental results are also consistent with the theoretically predicted impact trend and the error between the theoretically calculated value and the experimentally measured value is within 10%.
Conclusion

In this study, the forward and reverse force transmission characteristics of the Stewart parallel manipulator are investigated. Quantitative indices for measuring the force sensitivity of the robot are proposed, including unit forward force sensitivity, forward force sensitivity, unit reverse force sensitivity, reverse force sensitivity, and overall force sensitivity. The specific uses of these indices are as follows:

(i) When the configuration parameters of the robot have been determined, the following positions and orientations can make it safest for the robot to interact with humans: $x, y, \psi, \theta, \phi$ are all 0, and the value of $z$ is determined according to the $\eta - z$ curve and value range of $z$.

(ii) When the configuration parameters of the robot have not been determined, that is, when the robot structure is being designed, we can select a certain index as the basis for the optimal scale design and design the robot for different application scenarios.

(1) For example, in order to design the safest parallel structure for human-machine interaction, we should decrease the overall force sensitivity, and obtain the following scale design solution. If interference does not occur, $\gamma_a$ and $\gamma_b$ should be as small as possible, and $R_a$ is determined according to the working conditions. After $R_a$ is determined according to the operating conditions, the value of $R_b$ can be obtained from $R_b/R_a = 0.9063$, the value of $z_0$ can be derived from $z_0/R_a = -0.4226$, and the value of $\rho_{min}$ and $\rho_{max}$ are based on the value of $z_0$.

(2) In order to design a press, we should increase the forward force sensitivity, and obtain a scale design solution in which $\gamma_a$ and $\gamma_b$ should be as small as possible, $R_b/R_a$ should be 0.5, and $z_0/R_a$ should be $-0.88$.

(3) In order to design a force sensor, the reverse force sensitivity should be increased. In addition, the following scale design solution should be obtained: $\gamma_a + \gamma_b$ should be $60^\circ$, $R_b/R_a$ should be as small as possible, and $z_0/R_a$ should be close to 0.
In this study, an experimental method is used to verify the consistency of the unit reverse force sensitivity with the theoretical analysis. Furthermore, the error between the experimental value and the theoretical value is calculated.

The structure is limited to the Stewart parallel manipulator in this paper. In the future, the method in this paper will be further summarized and formed into a general method, which can be applied to various parallel robots.

This article is mainly based on statics and quasi-static conditions. In the future, the dynamics of the robot will be considered, so that this method will be applied to the conditions of high-speed robot movement.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Table 7. The error of unit reverse force sensitivity of joint 1 between theoretical value and experimental value.

| Position and orientation | Theoretical value | Experimental value | Error (%) |
|--------------------------|-------------------|--------------------|-----------|
| $x_0 = [0m, 0m, -0.86m, 0^\circ, 0^\circ, 0^\circ]^T$ | 1.02               | 1.04               | 2.0       |
| $x_0 = [0m, 0m, -1m, 0^\circ, 0^\circ, 0^\circ]^T$ | 1.17               | 1.24               | 6.0       |
| $x_0 = [0m, 0.1m, -0.86m, 0^\circ, 0^\circ, 0^\circ]^T$ | 1.05               | 1.08               | 2.9       |
| $x_0 = [0.1m, 0m, -0.86m, 0^\circ, 0^\circ, 0^\circ]^T$ | 1.02               | 1.06               | 3.9       |
| $x_0 = [0m, 0m, -0.86m, 0^\circ, 30^\circ, 0^\circ]^T$ | 1.06               | 1.10               | 3.8       |
| $x_0 = [0m, 0, -0.86m, 0^\circ, 15^\circ, 0^\circ]^T$ | 1.09               | 1.16               | 6.4       |

Figure 22. Variation of joint force with external force sequence under the position orientation $x_1 = [0m, 0m, -1m, 0^\circ, 0^\circ, 0^\circ]^T$. From (a) to (f), the forces on joints 1 to 6 are shown respectively.
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Appendix

Derivation process of statics force transmission relationship

The terminal interaction force variable is equivalent to a six-dimensional force-torque combined vector \( \mathbf{F} \) acting on the origin point \( P \) of the moving platform coordinate system, where \( \mathbf{F} \) is expressed as follows:

\[
\mathbf{F} = \left[ F_x, F_y, F_z, M_x, M_y, M_z \right]^T
\]  

(A1)

The value of the joint force of the telescopic joint \( i \) is \( f_i \) and the direction is \( s_i \). Since the joint can only perform one-dimensional linear motion, it can be regarded as a two-force rod that only receives tensile or compressive forces, so the joint force variable can be written as:

\[
\mathbf{\tau} = \begin{bmatrix} f_1, f_2, f_3, f_4, f_5, f_6 \end{bmatrix}^T
\]  

(A2)
For a mechanism under static or quasi-static conditions, at any time, the virtual work $\partial W_1$ performed by its end effector should be equal to the virtual work $\partial W_2$ performed by the drive joint.

The virtual displacement variables of the end and driving joints are $\partial x$ and $\partial d$, and we can get the following equation:

$$F \cdot \partial x = \tau \cdot \partial d \quad (A3)$$

we can also obtain the following equation:

$$\partial d = J^{-1} \partial x \quad (A4)$$

Substituting (A4) into (A3), we obtain the following:

$$F = J^{-T} \tau \quad (A5)$$