Line-driven winds in the presence of strong gravitational fields

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ABSTRACT

We propose a general physical mechanism which could contribute to the formation of fast line-driven outflows at the vicinity of strong gravitational field sources. We argue that the gradient of the gravitational potential plays the same role as the velocity gradient plays in the Sobolev approximation. Both the Doppler effect and gravitational redshifting are taken into account in the Sobolev approximation. The radiation force becomes a function of the local velocity gradient and the gradient of the gravitational potential. The derived equation of motion has a critical point that is different from that of Castor, Abbott and Klein (CAK). A solution, which is continuous through the singular point, is obtained numerically. A comparison with CAK theory is presented. It is shown that the developed theory predicts terminal velocities which are greater than those obtained from the CAK theory.

Key words: hydrodynamics – radiation mechanisms: general – quasars: general.

1 INTRODUCTION

Acceleration of matter due to radiation pressure in lines plays an important role in the formation of outflows from hot stars and possibly from active galactic nuclei, e.g. quasi-stellar objects (QSOs), Seyfert galaxies, etc. This paper is the first in a series of papers on the effects that strong gravitational fields play in the formation and structure of winds driven by radiation pressure in lines.

The theory of winds from O-type stars is well developed and agrees well with observations. In a pioneering paper by Sobolev (1960), it was recognized that radiation transfer in an accelerated medium is simplified drastically in comparison with that of static atmosphere. The importance of the line opacity for the formation of winds from hot stars was pointed out in a paper by Lucy & Solomon (1970). A prominent step forward in this field was made by Castor, Abbott & Klein (1975, hereafter CAK). They have shown that the absorption of the radiation flux in lines can be an effective mechanism of redistributing the momentum from radiation to the wind. The presence of the velocity gradient produces an additional effect on the acceleration because of the Sobolev effect. CAK have shown that the resultant radiation force, which is due to absorption in an ensemble of optically thin and optically thick lines, may be several orders of magnitude greater than that due to electron scattering. The ideas and the technique of CAK have been developed by Abbott (1980) and in the works of many other authors.

The theory of winds accelerated by the radiation pressure in lines is usually applied to explain outflows from active galactic nuclei (AGNs). The many puzzling, observational characteristic features of AGNs are: the blueshifted, relative to the emission line rest frequency, broad absorption lines (BALs) – the most convincing evidence of outflows with velocities as large as 0.1c; narrow absorption lines (NALs) – seen in ultraviolet (UV) and X-ray spectra associated with ~1000 km s^{-1} outflows; broad emission lines (BELs) – observed in UV indicating the flow with the characteristic velocity of about several thousand km s^{-1}. From the short time-scale of the X-ray variability (Tennant et al. 1981) it is concluded that the size of the emitting region is ~10^{14} cm; although there is an uncertainty in the estimation of the column densities, see Arav, Korista & Martijn de Kool (2002) and references therein. Only the black hole (BH) as a central object can couple this small length-scale with the total luminosity L \sim 10^{46} egr s^{-1}. The existence of BALs together with large luminosity makes the radiation-driven mechanism plausible. Line-locking, observed in the spectra of some QSOs, gives additional evidence of the importance of radiation in the acceleration of the wind. Murray et al. (1995) model the wind that originates not far from the BH \sim 10^{16} cm. Simplifying assumptions meant that the equations of motion in the radial and polar angle directions could be solved separately. Two-dimensional time-dependent hydrodynamic calculations of radiation-driven winds from luminous accretion discs were made in Proga, Stone & Drew (1998) and Proga, Stone & Kallman (2000).

In all papers where the radiation pressure in lines is assumed to be a main driving force forming outflows from AGNs, the authors have adopted modifications of the CAK theory (although effects which are not included in the CAK theory have been taken into account). A serious difference between a hot star wind and AGN outflow is due to different geometry. Wind from an O-type star is nearly spherically-symmetric whereas an outflow from an AGN originates from an accretion disc and thus is closely axially-symmetric. From observations we know that wind from an AGN is exposed to a very high X-ray and UV flux. The AGN spectra are harder than those...
of O-type stars, thus the radiation force could be smaller due to a higher ionization state. On the other hand, the observations of spectral lines from moderately and highly ionized species pose a problem of how the wind avoids overionization. This problem can be overcome if there exists some gas that shields the wind from the ionizing radiation. The second way is to assume that the wind is in the form of dense clouds which are confined by some mechanism. The ionizing effects of the X-ray flux in the winds from massive X-ray binary systems have been considered in Stevens & Kallman (1990).

The main underpinning assumption of CAK is that the velocity gradient leads to enhancement of the radiation force. If there is no velocity gradient, then the radiation force is simply due to absorption of the fraction of the radiation flux which is blocked by lines. If there is a velocity gradient, the absorbing ions are exposed to the unattenuated radiation flux due to Doppler shifting.

In this paper we propose a general physical mechanism which could play an important role in the formation of fast outflows at the vicinity of strong gravitational field sources. We argue that the gradient of the gravitational potential $d\phi/dr$ (in the case of sufficiently strong gravitational fields) plays the same role that $d\nu/dr$ plays in the acceleration of line-driven winds from hot stars.

Taking into account the gravitational redshift of frequency, we conclude that the gradient of the gravitational potential allows a line to shift out from behind its own shadow. The line is shifted to the extent where the continuum flux is not reduced by absorption, hence $dp/d\nu$ exposes the line to powerful radiation. Thus we call the resultant wind ‘gravitationally exposed flow’ (GEF).

In this paper, we adopt the following model. Consider a wind accelerated by the radiation pressure in lines at the vicinity of the BH. For simplicity we assume that the wind is spherically-symmetric and we skip the ionization problem. These crude assumptions are justified by the question we are going to answer, which is to compute the ionization problem. These crude assumptions are justified by the question we are going to answer, which is to compute the two-dimensional (2D) modelling of the problem in a separate paper.

The plan of the paper is as follows. In Section 1 we discuss relations for the optical depth $\tau_f$, and radiation force $f_i$ in standard CAK theory. In Section 2 we derive a relation for the radiation force taking into account gravitational redshifting. We derive the equation of motion describing stationary, spherically-symmetric wind. The properties of the solution at the critical point are investigated, and a numerical solution is provided. The results are summarized in the discussion and conclusions.

2 RADIATION FORCE

We assume that a radiation flux $F_f(\nu_f)$ ($F_i(\nu_i)$) is the continuum flux per interval of frequency on the line frequency $\nu_i$ emitted by the photosphere (or disc) is radially directed. The radiation force that results from absorption in a single line with the Doppler width $\Delta \nu_0 \approx \nu_0 \kappa l / c$ may be represented in the following form (Castor et al. 1975):

$$f_i = \kappa_i \left[ \frac{\Delta \nu F(\nu_i)}{c} \right] \frac{1 - e^{-\tau_i}}{\tau_i}. \quad (1)$$

The effective optical depth $\tau_i$ is calculated on the line frequency, which is obtained taking into account the Doppler shifting. $\kappa_i$ (cm$^2$ g$^{-1}$) is the opacity at the line centre:

$$\kappa_i = \frac{\kappa e^2}{mc} \frac{g_f}{\rho} \frac{N_i/s_i - N_i/s_i}{\Delta \nu_0}. \quad (2)$$

In the Sobolev approach $d\nu/dr$ is taken into account when calculating $\tau_i$. A relation for the optical depth was derived by Castor (1970):

$$\tau_i = \int \psi \kappa d\nu \simeq \frac{\rho v_0 \kappa L}{d\nu/dr}. \quad (3)$$

When calculating $\tau_i$ it is necessary to take into account only those absorbers which are in a section of the column across which the velocity changes by $v_0$. Relation (3) was obtained assuming that the line profile $\psi$ is a delta function. CAK introduced an optical depth parameter independent of line opacity

$$t \equiv \sigma_i \rho \frac{v_0}{d\nu/dr}. \quad (4)$$

where $\sigma_i = 0.4$ cm$^2$ g$^{-1}$. If the velocity gradient is large, the optical depth parameter $t$ may be much less than the corresponding electron optical depth. If $t \sim 1$ the radiation force is determined by the part of the radiation flux that is blocked by lines. In the opposite case of small $t$, the radiation force may exceed gravity by several orders of magnitude for a typical O-star atmosphere. Summing equation (1) over an ensemble of lines, CAK obtained the following relation:

$$f_i = \frac{L \sigma_i}{4\pi^2 c} M(t). \quad (5)$$

Here $M(t)$ is the force multiplier

$$M(t) = \sum \frac{F_f(\nu_f) \Delta \nu_0}{F} \min \left( \beta \frac{L(t)}{t}, t^{-1} \right). \quad (6)$$

where $\beta = L \kappa / \sigma_i$ (note that $t = \tau_L / \beta L$). $F$ is the total radiation flux, $\tau_i / \kappa_i$ is the column of mass which absorbs radiation, and the $(1 - e^{-\tau_i})$ multiplier in equation (1) gives the probability of absorption. If there are only strong lines, then the radiation force is proportional to the number of strong lines. Thus, in the optically thick case $f_i$ is independent of the line strength and $f_i \sim d\nu/dr$. If lines are all optically thin then each ion absorbs the unattenuated radiation flux, and thus the radiative acceleration is independent of the wind dynamics. For the ensemble of optically thin and thick lines, CAK found that $M(t)$ can be fitted by the power law:

$$M(t) = kt^{-\alpha}. \quad (7)$$

$\alpha = 1$ and $\alpha = 0$ represent the cases when all lines are optically thick and optically thin, respectively. With equations (7) and (4), equation (5) reads

$$f_i = \frac{L \sigma_i}{4\pi^2 c} (\sigma_i v_0)^{\alpha} \left( \frac{1}{\rho} \frac{d\nu}{dr} \right)^{\alpha}. \quad (8)$$

3 GRAVITATIONALLY EXPOSED FLOW

3.1 The radiation force due to the gradient of gravitational field

If a wind is accelerated in a strong gravitational field we can expect that a photon emitted by the disc may become resonant with the opacity of some line, not only because of the Doppler effect but also because of the gravitational redshifting. The redshifting of the photon’s frequency is the only effect of general relativity (hereafter GR) that we take into account in our approximate treatment. A photon with the frequency $\nu_0$, emitted at the point with the gravitational potential $\phi_0$, will be registered at the point where $\phi = \phi_1$ [$|\phi_1| > |\phi_2|$, $\phi < 0$] redshifted according to the well-known approximate formula (see, for example, Landau & Lifshitz 1960):
\[ \Delta v \equiv v_2 - v_4 = \frac{\phi_1 - \phi_2}{c^2} v_4. \] (9)

If there is a velocity difference between these two points the photon will be additionally redshifted due to the Doppler effect. The resultant frequency, seen by the absorber at its rest frame, reads

\[ v \simeq v_4 + \nu_4 \left( \frac{v}{c} + \frac{\Delta \phi}{c^2} \right). \] (10)

To derive a relation for the radiation force we need to calculate the corresponding Sobolev optical depth, but now taking into account the gravitational redshifting. Assuming that absorption occurs in a line with the delta-function profile, for the optical depth in a radial direction we obtain

\[ \tau_f = \int_r^{\infty} \kappa_1 \rho \, dr \simeq \frac{\rho \nu_0 \kappa_1}{c \, \nu} \frac{\dot{\tau} \nu_0 \kappa_1}{c \, \nu} \text{d} \phi/\text{d}r. \] (11)

A characteristic length, \( \delta r \sim v_\theta / \left[ (\text{d}v/\text{d}r + (1/c) \, \text{d}\phi/\text{d}r) \right] \), gives the thickness of a shell where the absorption due to a single line takes place.

In our calculations, we neglect the influence of the gravitational field on the geometry of space–time, because in that case our treatment will be too complicated. Introducing an optical depth parameter, which is analogous to equation (4), we have

\[ t \equiv \frac{\sigma_0 \rho \nu_0}{\text{d}v/\text{d}r + (1/c) \, \text{d}\phi/\text{d}r}. \] (12)

Note that relations (5) and (7) are derived in such a way that all information about wind dynamics (the distribution of \( v \) which is responsible for redshifting) is imbedded into parameter \( t \). It is assumed that \( \tau \) must be calculated taking into account redshifting. We make no assumptions about the particular physical mechanism that produces redshifting. In our case, \( \tau \) also contains information about additional redshifting which is due to the gradient of the gravitational potential. Note that equation (12) may be represented in the form

\[ t = \frac{\sigma_0 \rho \nu_0}{\dot{r}/r}, \]

where \( \nu' = v + (1/c) \phi \). The distribution of \( \nu' \) contains all the information about redshifting. These considerations hold so far as the Sobolev approximation is assumed and until the influence of the gravitational field on the geometry of space–time is neglected. We are justified in making use of equation (12) instead of equation (4) when calculating equation (7).

By substituting equation (12) into equations (5) and (7), the radiation force is obtained

\[ f_L = \frac{L \sigma_k}{4 \pi r^2 c^2} \left( \frac{4 \pi}{|M\sigma v_\theta|} \right)^a \left[ \nu' r^2 \left( \frac{\text{d}v}{\text{d}r} + \frac{1}{c} \frac{\text{d}\phi}{\text{d}r} \right) \right]^a, \] (13)

where the radiation force was transformed making use of the continuity equation

\[ |M| = 4 \pi \sigma r v^2. \] (14)

### 3.2 Basic equations

An equation of motion describing the stationary, spherically-symmetric, isothermal, line-driven wind reads

\[ \frac{\text{d}v}{\text{d}r} = \frac{1}{\rho} \frac{\text{d}P}{\text{d}r} - \frac{\text{d}\phi}{\text{d}r} \left[ \frac{L \sigma_k}{4 \pi r^2 c^2} + \frac{L \sigma_k}{4 \pi r^2 c^4} \left( \frac{4 \pi}{|M\sigma v_\theta|} \right)^a \right] \nu' \left( \frac{\text{d}v}{\text{d}r} + \frac{1}{c} \frac{\text{d}\phi}{\text{d}r} \right)^a. \] (15)

As well as the CAK equation of motion, equation (15) is nonlinear with respect to the velocity gradient. This behaviour complicates the structure of equation (15) and slows the numerical solution. Investigating equation (15) we hope to find a transcritical solution, which starts subsonically, proceeds through the critical point and goes to infinity approaching a terminal velocity \( v^\infty \). Equation (15) resembles the CAK equation of motion except for the radiation pressure term. Instead of the \( (\text{d}v/\text{d}r)^a \) term, equation (15) includes a combination: \( \{\text{d}v/\text{d}r + (1/c) \, \text{d}\phi/\text{d}r\}^a \). As we will see, this property of equation (15) changes the structure of the transcritical solution.

We adopt two types of potentials, the Newtonian potential (NP) and the Paczynski–Witt (PW) potential:

\[ \phi = \frac{GM}{r - r_g}. \] (16)

We make use of the modified potentials to model approximately the GR effects. The PW potential for the Schwarzschild BH correctly reproduces the positions of both the last stable orbit and the marginally bound orbit. The many properties of the modified potentials are discussed in Artemova, Bjørnsson & Novikov (1996). The introduction of the modified potentials allows us approximately to take into account some properties of the exact relativistic description. For simplicity hereafter we use the NP. The case of the PW potential is described in the appendix. For Newtonian gravity \( \phi = -GM/r \), equation (15) reads

\[ F = \left( 1 - \frac{a}{v^2} \right) \frac{\text{d}v}{\text{d}r} = \frac{-GM}{r^2} - \frac{L \sigma_k}{4 \pi r^2 c^2} \left( \frac{4 \pi}{|M\sigma v_\theta|} \right)^a \nu'^2 \left( \frac{\text{d}v}{\text{d}r} + \frac{1}{c} \frac{\text{d}\phi}{\text{d}r} \right)^a = 0, \] (17)

where \( a = \left( \frac{\partial P}{\partial \rho} \right)^{1/2} \) is the sound velocity. When obtaining equation (17) we assume that the wind is isothermal \( T = \) const. and \( P = \rho v^2 \).

We introduce non-dimensional variables

\[ x = \frac{r}{r_c}, \quad \tilde{v} = \frac{v}{v_c}, \] (18)

where \( r_c \) is the critical point radius and \( v_c \) is the velocity of matter at the critical point. Taking into account equation (18), equation (17) reads

\[ F(p, v, x) = \left( 1 - \frac{a_1^2}{v^2} \right) v x^2 p - 2a_1^2 v x + \frac{\beta^2}{2} \left( 1 - \Gamma \right) \] (19)

\[ -\mu \beta^2 \left[ \frac{v}{\sqrt{\beta^2 x^2}} \left( p + \frac{\beta}{2} \right)^a \right]^a = 0, \]

where \( p = \frac{\text{d}v}{\text{d}r} \) and \( \Gamma = \frac{x}{1 + \mu \beta \Delta v}. \) For simplicity hereafter we omit the tilde. The following non-dimensional parameters were introduced:

\[ a_1 = \frac{a}{\nu_c}, \quad \beta = \frac{\nu_0}{v_c}. \] (20)

Note that \( \beta = \tilde{a} a_1 \) and there is only one independent parameter \( a_1 \). A constant \( \mu \) is determined according to the relation:

\[ \mu = \Gamma k \left[ \frac{8 \pi GM}{|M\sigma v_\theta|} \right]^a. \] (21)

Equation (19) is non-linear with respect to \( p \). The point where the speed of sound is equal to velocity of the flow is no longer the singular point of the equation of motion. To treat such equations, a special technique must be applied. We are interested of the solution that starts subsonic near the BH, is continuous through the
singular point and goes supersonically to infinity. The singular point is defined by the condition:

\[
\left( \frac{\partial F}{\partial \rho} \right)_{c} = 0.
\]

The second condition comes from the fact that, at the critical point, the velocity gradient is continuous which requires that \( \nu^\ast \) is defined at the critical point. The regularity condition reads

\[
\left( \frac{\partial F}{\partial x} \right)_{c} + p_c \left( \frac{\partial F}{\partial \nu} \right)_{c} = 0,
\]

where

\[ p_c = \frac{r_s}{v_c} \left( \frac{dv}{dr} \right) \bigg|_{c} \]

From equations (19) and (22) the following useful relation is obtained

\[
\mu = \alpha^{-1} \left( 1 - a_1^2 \right) \left[ \frac{x_c^2}{\beta^2} \left( p_c + \frac{\beta}{2x_c^2} \right) \right]^{1-\alpha},
\]

where \( x_c = \frac{r_c}{r_g} \). Substituting equation (19) into equation (22), and taking into account equation (24), will result in the following equation:

\[
2a_1^2x_c^2p_c^2 + \frac{1}{2} (a_1^2 - 1) \beta p_c - 2a_1^2 = 0.
\]

The momentum equation (19) is valid only for the domain of investigation. Thus, calculating it at the critical point and substituting \( \mu \) from equation (24) will provide the following equation:

\[
\frac{\alpha - 1}{\alpha} \left( 1 - a_1^2 \right) x_c^2 p_c - 2a_1^2 x_c + \frac{\beta^2(1-\Gamma)}{2} - \left( 1 - a_1^2 \right) \frac{\beta}{2\alpha} = 0.
\]

Equations (25) and (26) form the system of equations from which \( p_c \) and \( a_1 \) can be calculated. For example, assuming that the critical point is situated at 100\( r_g \), from equations (25) and (26) we find:

\[ a_1 = 0.0393124, \quad p_c = 10.48311. \]

The introduced term from equation (25) will reveal the result of the CAK solution of the isothermal wind: \( p_c = \pm 1/x_c \), or \( (dv/dr)x_c = \pm v_c/r_c \). To solve equations (25) and (26) for \( a_1 \) and \( p_c \), we must take into account that \( \beta = (c/v_{\text{th}})a_1 \), where \( v_{\text{th}} = \sqrt{kT/m_a} \).

### 3.3 Numerical solution

For a given position of the critical point \( x_c = r_c/r_g \), equations (25) and (26) are used to obtain the velocity gradient at the critical point. In contrast to the CAK wind, the GEF case is more difficult to analyse. In the isothermal limit of the CAK wind, equations (25) and (26) could be solved analytically. Unfortunately, it is not possible (although see the end of this section) in the GEF case. In order to obtain \( p_c \) and \( a_1 \), equations (25) and (26) must be solved numerically. All calculations in this paper are made for a 10\(^7\) M\(_{\odot}\) BH, the temperature of the flow is \( T = 4 \times 10^8 \) K and other parameters are \( \Gamma = 0.5, \alpha = 0.6 \) and \( k = 0.1 \).

In the case of stellar winds, a photospheric boundary condition is usually adopted. Adjusting the position of the critical point, we obtain a solution that gives the position of the photosphere \( (r = T_{\text{eff}} \) at \( r_c = 2/3 \) at some prescribed radius \( R \), which is identified with the radius of a star (Bisnovatyi-Kogan 2001). A similar procedure was adopted by CAK. In the case of AGNs, such ‘photospheric’ conditions are clearly nonphysical. The solution for the wind should be continuously fitted with the solution for the accretion disc. This requires 2D modelling which is beyond the scope of this paper. In the spherically-symmetric approximation adopted here it is not possible to fit self-consistently a wind solution with that of the accretion disc.

To compare the GEF solution with that of CAK we start deeply subsonic \( (v \ll v_c) \), starting from some initial density \( \rho_{\text{in}} \) and calculating both CAK and GEF solutions. Mathematically, this is equivalent to the problem of fitting the wind solution with that of a static core when calculating a structure of a star when mass loss is taken into account. In such a case, a solution for the outflowing envelope is continuously fitted with that of a static core. As it results from a stellar wind theory (for a stationary sphericallysymmetric wind) to fit continuously a solution for a stationary outflowing envelope with that of a static core, only \( \rho \) and \( T \) should be fitted. In the case of stellar wind, this condition reads \( \rho_{\text{in}} = \rho_{\text{core}} \), \( T_{\text{in}} = T_{\text{core}} \) where the fitting point must lie in a deep subsonic region \( v \ll v_c \); see Bisnovatyi-Kogan & Dorodnitsyn (1999) and references therein. The adopted procedure is equivalent to that if we start from some static configuration \( (v \ll v_c) \) with the density \( \rho_{\text{in}} \) and then relax to the stationary wind solution (CAK and GEF). Prescribing \( \rho_{\text{in}} \) at the very bottom of the wind \( (v \ll v_c) \) at \( r_{\text{in}} \) we may find the solution which satisfies the boundary condition. This requirement allows us to compare the GEF solution with that obtained from CAK theory.

We step out from the critical point \( x_c \) by means of the approximate formula, \( v = 1 \pm |p_c| \Delta x \), where \( \Delta x = x_c = x_c \). On every integration step, equation (19) is solved numerically in order to obtain \( dv/dx \). The fourth-order, adaptive Runge–Kutta scheme was used to obtain the solutions shown in Figs 1–4.

Adjusting \( x_{\text{CAK}}^{\ast} \) and \( x_{\text{GEF}}^{\ast} \), we select those solutions which both satisfy inner density condition. The following densities at the inner boundaries were adopted: \( \rho_{\text{in}} = 10^{-12} \) g cm\(^{-3} \) for \( x_c = 50, \rho_{\text{in}} = 10^{-14} \) g cm\(^{-3} \) for \( x_c = 100, \rho_{\text{in}} = 10^{-16} \) g cm\(^{-3} \) for \( x_c = 500, \rho_{\text{in}} = 10^{-21} \) g cm\(^{-3} \) for \( x_c = 8000 \). The comparative results of the numerical integration of equation (19) for \( s = 1 \) (GEF solution) and for \( s = 0 \) (CAK wind theory) are shown in Fig. 1. These solutions were obtained for the set of parameters shown in Table 1.

The difference between GEF and CAK wind is more pronounced when the considerable portion of the wind is accelerated at a distance less than 100\( r_g \) from the BH. Note that the terminal velocity changes from \( \Delta v^\infty/v^\infty \) at \( x_c = 50 \) to \( \Delta v^\infty/v^\infty \) at \( x_c = 500 \). The obtained results show that the GEF flow can be sufficiently more fast than the flow which is described by CAK theory.

It is illustrative to demonstrate that the CAK solution may be obtained from equation (17) by ‘switching off’ smoothly the gravitational redshifting. To obtain the continuous transition from the GEF solution to the CAK solution of equation (17), the following procedure was adopted. We have assumed that the radiation force in equation (17) is \( f_L \sim dv/dr + (1/c) sG M/v^2 \). The introduced parameter \( s \) continuously changes from 1 (GEF case) to 0 (CAK wind). Numerically calculating the solutions of equation (17) for different values of \( s \), applying the inner density boundary conditions, it is possible to demonstrate the continuous transition of these solutions from the limiting cases of CAK and GEF solutions. The results are shown in Fig. 2.

The introduction of the PW potential (16) allows us to simulate the effects of GR. Equations analogous to equations (19), (25) and (26) are derived in the appendix. The results of the numerical integration are shown in Fig. 4. The introduction of the modified potential can produce an increase in \( v^\ast \) that varies from \( \Delta v^\infty/v^\infty \) to 0.1 for \( x_c = 30 \) to \( \Delta v^\infty/v^\infty \) to 0.03 for \( x_c = 100 \).
We are looking for the solution with \( \Delta \epsilon \) where it is supposed that \( \epsilon \). To obtain an approximate solution of equation (27) we take into account that this will result in the following equation for determining \( a_1 \): 

\[
2a_1^3(1 - \Gamma) - 2a_1^2\epsilon + \frac{\alpha - 1}{\alpha} \epsilon = 0.
\]

(32)

The second term in equation (32) is small compared with other two. It allows us to obtain the following simple relation for \( a_1 \):

\[
a_1 \simeq \left[ \frac{\epsilon}{2(1 - \Gamma)} \frac{1 - \alpha}{\alpha} \right]^{1/3}.
\]

(33)

Calculating \( a_1 \) and \( p_c \) numerically from equations (25) and (26), we have found that the value of \( a_1 \) depends rather weakly on \( x_c \).

To obtain an approximate relation for \( a_1 \) we substitute \( p_{c,2} \) from equation (29) into equation (26). Taking into account that \( \beta = a_1/\epsilon \), the resulting equation reads

\[
-2a_1^2x_c\epsilon^2 + \frac{\alpha - 1}{\alpha} \left( 1 - a_1^2 \right)^2 \epsilon + a_1^2(1 - \Gamma) - \frac{\epsilon a_1}{2\alpha} (1 - a_1^2) = 0.
\]

(31)

If, in equation (31), we neglect terms which contain \( a_1^4 \) and \( a_1^3\epsilon^2 \), this will result in the following equation for determining \( a_1 \):

\[
2a_1^3(1 - \Gamma) - 2a_1^2\epsilon + \frac{\alpha - 1}{\alpha} \epsilon = 0.
\]

(32)

Approximate relations for \( a_1 \), \( p_c \).

Introducing the following non-dimensional combination \( \epsilon = a/c \), equation (25) will read

\[
4a_1x_c^2\epsilon^2 p_c^2 + \left( a_1^2 - 1 \right) p_c - 4a_1 \epsilon = 0.
\]

(27)

To obtain an approximate solution of equation (27) we take into account that \( \epsilon \ll 1 \). Equation (27) has two roots:

\[
(p_c)^\pm \simeq \frac{1}{8a_1x_c^2} \left( 1 - a_1^2 + 1 + \frac{32a_1x_c^2\epsilon^2}{(a_1^2 - 1)^2} \right). \tag{28}
\]

From equation (28) we obtain

\[
p_{c,1} \simeq - \frac{4a_1 \epsilon}{1 - a_1^2}, \quad p_{c,2} \simeq \frac{1 - a_1^2}{4a_1x_c^2 \epsilon}.
\]

(29)

We are looking for the solution with \( p_c > 0 \) and with \( a_1 = a_1/v_{ce} \ll 1 \), thus the second root of equation (29) should be considered

\[
p \simeq \frac{1}{4a_1x_c^2 \epsilon}, \tag{30}
\]

where it is supposed that \( a_1 \ll 1 \). The accuracy of equation (30) is \( \Delta p_c/p_c \sim 10^{-3} \) (for \( x_c = 1 \), \( p_c = 12.0027 \), \( p_{c,\text{approx}} = 12.0169 \)).

Table 1. Parameters of GEF and CAK wind.

| \( \lambda_{\text{GEF}} \) | \( \lambda_{\text{CAK}} \) | \( v_{\infty,\text{GEF}} \) cm s\(^{-1} \) | \( v_{\infty,\text{CAK}} \) cm s\(^{-1} \) | \( M_{\text{GEF}} \) \( g \) s\(^{-1} \) | \( M_{\text{CAK}} \) \( g \) s\(^{-1} \) |
|---|---|---|---|---|---|
| 50 | 75.0 | 5.0 \times 10^9 | 3.67 \times 10^9 | 2.56597 \times 10^{26} | 2.55692 \times 10^{26} |
| 100 | 155.2 | 3.3 \times 10^9 | 2.6 \times 10^9 | 2.56597 \times 10^{26} | 2.55693 \times 10^{26} |
| 500 | 751.7 | 1.37 \times 10^9 | 1.15 \times 10^9 | 2.56599 \times 10^{26} | 2.55697 \times 10^{26} |

Figure 1. Solution of the wind equation (19): solid line, GEF solution; dashed line, CAK solution. The crosses indicate GEF critical points; circles, CAK critical points. The curves for \( x = 8000 \) are graphically indistinguishable.
Figure 2. The continuous transition of the solution of equation (19) from GEF ($s = 1$) to the CAK regime ($s = 0$); see explanations in the text.

Figure 3. Density distribution in the wind. The solution curves are calculated for $x_{c}^{\text{GEF}} = 50$ and $x_{c}^{\text{CAK}} = 75$.

(obtained for $\alpha = 0.6$, $\Gamma = 0.5$): $x_c = 50$, $a_1 = 0.034347$, $p_c = 48.0109$; $x_c = 500$, $a_1 = 0.034348$, $p_c = 0.480112$; $x_c = 5000$, $a_1 = 0.034369$, $p_c = 0.004806$. Relation (33) has a relatively good accuracy: for $\epsilon = 6 \times 10^{-5}$ equation (33) gives $a_1^{\text{approx}} \simeq 0.03430$ with accuracy $\Delta a_1/a_1 = 10^{-3}$. The velocity of the wind at the critical point is approximately $30a_s$ ($v_c = a_s^{-1} a_1$).

Above the critical point, the influence of $d\phi/dr$ (GEF) on the additional acceleration of the radiationally-driven wind is small compared to the effect due to $dv/dr$. 

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is proportional to \(\frac{d\nu}{dr}\) that is due to an ensemble of optically thin and optically thick lines. Powerful X-ray and UV radiation from the disc pose a problem of overionization of the outflowing plasma. In the Sobolev approximation, which is in the background of the CAK theory, the radially streaming radiation \(\phi\) overionizes the outflow. The line-driven wind from the O-type star is spherically-symmetric. Outflows in AGNs are assumed to originate from the luminous accretion discs and are approximately axially-symmetric. Outflows (Arav & Li 1994; Arav, Li & Begelman 1994; Murray et al. 1995; Proga et al. 1998; Proga et al. 2000). The CAK theory was applied to AGNs in order to explain fast (up to \(v = 10^{5}\)) outflows from close to the BH. An exact self-consistent solution of this problem is possible only in GR. We avoid this sophisticated task by considering all equations in flat space-time.

Based on the considerations of Sobolev, we conclude that the greater \(\frac{d\phi}{dr}\), the more effectively a line is shifted to the extent where the radiation flux is unattenuated. In such a case, the gravitational field ‘exposes’ a wind to the unattenuated continuum. Note that this effect is independent of the wind dynamics; it works also in a medium with \(v = 0\). We call a wind accelerated in this regime, ‘gravitationally exposed flow’.

The main goal of this paper has been to compare a solution for GEF with that obtained from a standard line-driven wind theory (CAK). To solve this problem a very simple input physics was assumed: spherical symmetry, constant temperature and no ionization balance. We found that, in such a case, the gravitational field ‘exposes’ a wind to the unattenuated continuum. Note that this effect is independent of the wind dynamics; it works also in a medium with \(v = 0\). We call a wind accelerated in this regime, ‘gravitationally exposed flow’.

In this paper, we have developed a theory of winds that takes into account the effects of the strong gravitational fields. We point out that, if a wind is accelerated near a supermassive BH, a gravitational change of the photon frequency must be taken into account. In a strong gravitational field, a photon emitted by a disc will be redshifted as a result of both the Doppler effect \(\sim v/c\) and gravitational redshifting \(\Delta v/v = \Delta \phi/c^2\). We argue that taking into account gravitational redshifting can substantially change the wind dynamics and structure. However, it should be mentioned that the developed theory (in the adopted model) cannot be directly applied to explain outflows from close to the BH. An exact self-consistent solution of this problem is possible only in GR. We avoid this sophisticated task by considering all equations in flat space-time.

\[v^- = 0.115 c\]
\[v^- = 0.176 c\]
\[v^- = 0.221 c\]
\[v^- = 0.244 c\]

Figure 4. The solid line indicates the GEF solution for the NP, and the dashed line the GEF solution for the PW potential.

4 DISCUSSION

The most successful model describing winds from hot stars presented so far is that of Castor (1970) and CAK. The geometry and ionization balance differentiate the O-star wind from the AGN outflow. The line-driven wind from the O-type star is spherically-symmetric. Outflows in AGNs are assumed to originate from the luminous accretion discs and are approximately axially-symmetric. The most successful model describing winds from hot stars presented so far is that of Castor (1970) and CAK. The geometry and ionization balance were studied in detail simultaneously with 2D hydrodynamical calculations. However, it should be mentioned that the developed theory (in the adopted model) cannot be directly applied to explain outflows from close to the BH. An exact self-consistent solution of this problem is possible only in GR. We avoid this sophisticated task by considering all equations in flat space-time.

In this paper, we have developed a theory of winds that takes into account the effects of the strong gravitational fields. We point out that, if a wind is accelerated near a supermassive BH, a gravitational change of the photon frequency must be taken into account. In a strong gravitational field, a photon emitted by a disc will be redshifted as a result of both the Doppler effect \(\sim v/c\) and gravitational redshifting \(\Delta v/v = \Delta \phi/c^2\). We argue that taking into account gravitational redshifting can substantially change the wind dynamics and structure. However, it should be mentioned that the developed theory (in the adopted model) cannot be directly applied to explain outflows from close to the BH. An exact self-consistent solution of this problem is possible only in GR. We avoid this sophisticated task by considering all equations in flat space-time.
Numerically solving the equations describing spherical symmetric, stationary outflowing winds, we found that gravitational redshifting can make the acceleration up to 35 per cent (for $r_\alpha = 50 r_\theta$) more efficient.

In order to take into account approximately the effects of GR, we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential. Numerical analysis demonstrated that if the critical point is located as far as we made use of a modified potential, the PW potential.

5 CONCLUSIONS

(i) We have developed a theory of winds accelerated by the radiation pressure in lines taking into account gravitational redshifting of photons. A system of equations describing stationary, spherically-symmetric, isothermal flow is derived.

(ii) A solution of these equations is obtained numerically for two cases: a standard line-driven wind (CAK theory) and GEF; a wind that is accelerated by the radiation pressure in lines if taking into account gravitational redshifting. It is shown that an increase of up to 35 per cent in $\Delta v^\infty$ can be obtained.

(iii) To take into account approximately the effects of GR, the PW potential is adopted. A wind solution is calculated for this type of potential and a comparative analysis with the NP is presented.

(iv) The developed theory can be used to explain fast outflows from AGNs.

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APPENDIX

Substituting the PW potential into equation (15), we obtain the equation of motion:

$$F = \left(1 - \frac{a_1^2}{v^2}\right) \frac{dv}{dr} - \frac{2a_1^2}{r} + \frac{GM}{r(r - r_\theta)^2} - \frac{L\sigma}{4\pi r^2 c}$$

$$- \frac{L\kappa}{4\pi\alpha^2 c} \left(\frac{4\pi}{M|v_n|}\right)^a \left\{\frac{v^2}{\alpha} \left[\frac{dv}{dr} + \frac{1}{c} \left(\frac{GM}{r(r - r_\theta)^2}\right)^a\right]\right\} = 0.$$  

(34)

Scaling all variables according to equation (18), and omitting the tilde, equation (34) reads

$$F(p, v, x) = \left(1 - \frac{a_1^2}{v^2}\right)p v x^2 - 2a_1^2 p x + \frac{\beta_1^2}{2} \left(\frac{x^2}{(x - 1)^2} - \Gamma^2\right)$$

$$- \mu \beta^2 \left\{\frac{v^2}{\beta^2} \left[p + \frac{\beta}{2(x - 1)^2}\right]\right\}^a = 0,$$  

(35)

where $p \equiv dv/dx$, $\mu$ is determined by equation (21), and $a_1$ and $\beta$ by equation (20). Making use of equations (22) and (23) will result in the following relations:

$$\alpha - 1 = \alpha_1 \left(1 - a_1^2\right) x^2_0 p_0 - 2a_1^2 x_0 + \frac{\beta_1^2}{2} \left(\frac{x^2_0}{(x_0 - 1)^2} - \Gamma\right)$$

$$- \left(1 - a_1^2\right) \frac{\beta}{2(x_0 - 1)^2} x^2_0 = 0,$$  

(36)

$$c_1 p_0^2 + c_1 p_0 + c_0 = 0.$$  

(37)

Coefficients $c_1$ are determined from the following relations:

$$c_2 = 2a_1^2 x^2_0,$$  

(38)

$$c_1 = \frac{(a_1^2 - 1)x^2_0 \beta}{2(x_0 - 1)^2};$$  

(39)

$$c_0 = -x_0 (\beta - \beta_1) \beta + a_1^2 \left(2(x_0 - 1)^2 - \beta x_0\right).$$  

(40)

The equation for $\mu$ reads

$$\mu = \alpha^{-1} \left(1 - a_1^2\right) \left[\frac{x^2_0}{\beta^2} \left(p_0 + \frac{\beta}{2(x_0 - 1)^2}\right)\right]^{1-a}.$$  

(41)

Equation (41) is analogous to equation (24) except for the last term in brackets. For given $x_0$, equations (36) and (37) are solved numerically to determine $p_0$ and $a_1$. Then equation (35) is integrated numerically as described in the text. The comparative results of the numerical solution of equation (35) are shown in Fig. 4.

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