Anatomy of an Inversion - Argon Isotopes

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Abstract.
Two different interactions give similar results for excitation energies and g factors of 2⁺\textsuperscript{1} states for most even-even argon isotopes except ⁴⁶Ar. This is explained in terms of an inversion in J=\frac{1}{2}\textsuperscript{+} and J=\frac{3}{2}\textsuperscript{+} levels in ⁴⁷K which is successfully obtained by one of the interactions but not the other. This example shows the possible dangers of nuclear astrophysics extrapolations.

1. Even-Even Argon Isotopes-Inversion
We here discuss some works which were previously published [1], but here emphasize certain points relevant to Nuclear Physics in Astrophysics. In that work we examined the properties of the even A isotopes of Argon between A=38-46. There we found that with two different interactions WBT[2] and SPDF-U[3, 4] we get very similar results for the properties of all even-even Ar isotopes except A=46. We show these results below in Table 1. We also include for the sake of completeness results from new calculations on ³⁶Ar.

Table 1. Excitation energies (in MeV) and g factors of the first 2⁺ state for the even mass Argon isotopes.

| Mass (Ar) | 36 | 38 | 40 | 42 | 44 | 46 |
|-----------|----|----|----|----|----|----|
| E(2⁺\textsuperscript{1}) | WBT | 1.927 | 2.010 | 1.424 | 1.292 | 1.172 | 1.143 |
| SPDF-U | 1.909 | 2.022 | 1.281 | 1.154 | 1.087 | 1.592 |
| Experiment | 1.970 | 2.167 | 1.464 | 1.208 | 1.144 | 1.550 |
| g(2⁺\textsuperscript{1}) | WBT | 0.487 | 0.308 | -0.197 | -0.095 | -0.002 | +0.100 |
| SPDF-U | 0.490 | 0.319 | -0.228 | -0.084 | -0.040 | +0.513 |
| Expt | 0.52(18) | 0.24(12) | -0.02(2)[5, 6] |

The change of sign in the g factor is due to shell structure.
To understand the big difference in the excitation energy and g factor for A=46 we should look at the odd K isotopes. Here we examine the J=\frac{3}{2}\textsuperscript{+}/J=\frac{1}{2}\textsuperscript{+} energy splitting. At A=47 there is an experimental inversion (J=\frac{1}{2}\textsuperscript{+} higher than J=\frac{3}{2}\textsuperscript{+}) that is correctly obtained by SPDF-U but not by WBT. This odd-even proton inversion leads to important changes in even-even properties.
Note that only when one is very close to the inversion does one get significant differences\textsuperscript{46}Ar is strongly affected but not \textsuperscript{44}Ar or the lighter isotopes. This shows possible dangers of nuclear astrophysics extrapolations. At the very least it underlies the importance of finding and being aware of these regions of inversion on the nuclear periodic table.

\textbf{Table 2.} \(J=\frac{3}{2}^+\) \(J=\frac{1}{2}^+\) proton splitting (MeV) in the K isotopes.

|        | Expt | WBT | SPDF-U |
|--------|------|-----|--------|
| 41     | 0.980| 1.106| 0.854  |
| 43     | 0.561| 1.109| 0.672  |
| 45     | 0.475| 0.871| 0.345  |
| 47     | -0.360| 0.507| -0.320 |
| 49     | -0.092| 0.729| 0.087  |

With the WBT interaction we get good agreement for even-even Argon isotopes \(A=36,38,40,42,44\) but the extrapolation to \(A=46\) could yield wrong results.

It would be great of one could measure the g factor \(g(2^+_1)\) for \(A=46\) and compare with the two interactions. We can understand however why are the computed g factors of \(J=2^+_1\) states in \(^{46}\text{Ar}\) so different.

There are two competing proton configurations for the \(2^+\) state,

1. \(d_{3/2}d_{3/2}\) This has a very small g factor +0.083 (Schmidt g factor)
2. \(s_{1/2}d_{3/2}\) this has a very large g factor \(g=|\mu(s_{1/2})+\mu(d_{3/2})|/2=1.46\)

If there is an inversion (\(s_{1/2}\) higher than \(d_{3/2}\)) configuration 2 is favored over configuration 1. Interaction SPDF-U correctly gives the inversion and hence a larger g factor (+0.513) than WBT (+0.100).

\textbf{2. N=Z Nuclei-Comparison with Sulfer.}

In ref [7] measured g factors of 2+ states of N=Z nuclei \(^{32}\text{S}\) and \(^{36}\text{Ar}\) are given. The results expt.[theory] are respectively 0.45(7) [0.501] and 0.52(18) [0.488]. These are isoscalar g factors. It has been noted by many that these g factors always come out to be very close to 0.5. This is consistent with an \(L=2\ S=0\ J=2^+\) state or with the collective result \(g=Z/A\). There is a remarkable insensitivity to the details of the nuclear interaction that is used.

As shown in [7] the singly magic nucleus \(^{36}\text{S}\) has a very large g factor 1.3 (5) [1.16]. This was explained by using the configuration 2 in the previous section, \(s_{1/2}d_{3/2}\). On the other hand the semi-magic \(^{38}\text{Ar}\) has a small positive g factor because the dominant proton configuration is 1 from above \(d_{3/2}\). In references [8, 9] it is noted that \(^{40}\text{S}\) has a very small g factor, very far away from the collective value \(Z/A\). However the authors say that other aspects of \(^{40}\text{S}\) are consistent with it being deformed, especially a lower lying \(2^+\) state compared with neighbors and a calculated negative static quadrupole moment of the \(2^+\) state indicating a prolate deformation. However in [5] the other authors conclude that \(^{40}\text{Ar}\) displays near spherical behaviour. This in itself is not a contradiction, but what is surprising is that the measured \(B(E2)\) (see [7]) for the near spherical nucleus is larger than or at best equal that of the deformed one. 374(13) vs 334(36) \(e^2fm^4\). The calculated values of \(Q\) in [9] with SPDF (SDF) in units of \(efm^2\) are -19.3(-9.8) for \(^{40}\text{S}\) (prolate) and +5.2(+13.5) for \(^{40}\text{Ar}\) (oblate).
3. The B(E2) problem in $^{46}\text{Ar}$

Whereas many of the calculated properties of $^{46}\text{Ar}$ for the 2 different interactions- WBT and SPDF-U are quite different (due to the inversion) the B(E2)’s from ground to the $J = 2^+$ state are nearly equal -about 535 $e^2f m^4$ [1]. In that paper we quoted and compared our results to an experimental value due to Scheid et al.[10] which was much smaller 196 $e^2f m^4$. Since in the single $j$ shell model $^{46}\text{Ar}$ has a closed $f_{7/2}$ neutron shell our large calculated result appears to be counterintuitive. However the large calculated result has been obtained by others including Scheid et al in the same paper where the small value was measured [10]. Most recently however a large measured value for this B(E2), consistent with our calculation [1], was published by Mengoni et al.[11] While this is quite gratifying for us it would be nice to have a confirming measurement. The Mengoni result for $^{46}\text{Ar}$ 570 $e^2f m^4$ is larger than the measured value for $^{44}\text{Ar}$ 334 $e^2f m^4$. However from isotope shift measurements it is known that the charge radius of $^{46}\text{Ar}$ is smaller than that of $^{44}\text{Ar}$, see the paper of Blaum et al. [12]. Usually the radius increases with deformation. This is not a definitive argument against a larger B(E) for $^{46}\text{Ar}$ but all of these things will have to be sorted out.

Besides the full space (0 $\hbar\omega$) calculations we also performed, for comparison, sinlge j shell calculations. For these we found some amusing results. The spectra, g facors and B(E2)’s in the small model space were the same for $^{40}\text{Ar}$ and $^{44}\text{Ar}$. This could be explained by the fact that the two protons are at midshell and we have two $f_{7/2}$ neutrons in $^{40}\text{Ar}$ and two $f_{7/2}$ neutron holes in $^{44}\text{Ar}$. Deviaiotns from this symmetry are seen in in Table 1.

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