CEEX Exponentiation in QED*

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The aim is to summarize briefly on: (a) Main features of YFS Coherent Exclusive Exponentiation in QED, (b) Examples of recent results relevant for LEP/LC physics program.

This presentation is based mainly on three recent papers [13]. Standard Model calculations for LEP with YFS exponentiation are implemented in many Monte Carlo event generators: for $e^+e^- \rightarrow f\bar{f} + n\gamma$, $f = \tau, \mu, d, u, s, c$ process; (1a) YFS1 (1987-1989) $O(\alpha^1)_{\exp}$ IR [3], (1b) YFS2<KORALZ (1989-1990) $O(\alpha^1 + h.o.LL)_{\exp}$ IR [4], (1c) YFS3<KORALZ (1990-1998) $O(\alpha^1 + h.o.LL)_{\exp}$ ISR+FSR [5], (1d) KKMCI [6].

Examples of recent results relevant for LEP/LC physics program.

• Truncation of the IR-finite $\tilde{\beta}_s$ to finite $O(\alpha^n)$ and calculation of them from Feynman diagrams recursively.

Apart from disentangling properly ultraviolet divergences from the infrared (IR) ones YFS deals consistently with the overlapping and sub-leading IR singularities. To illustrate this point let is show the “map of IR singularities” for $2\gamma$ distribution $D_2(k_1, k_2)$ in the plane of the energies of two real photons:

The point $R_{12}$ is the location of the leading double IR singularity and the lines $R_1$ and $R_2$ host non-leading single IR singularities. The above topological picture translates into a decomposition of the differential distributions: $D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \tilde{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4})$, $D_1(p_f; k_1) = \tilde{\beta}_1(p_f)S(k_1)$ and $D_2(k_1, k_2) = \tilde{\beta}_2(k_1)S(k_2)$

where typically $p_{f_1} + p_{f_2} = p_{f_3} + p_{f_4}$ for $\tilde{\beta}_0$ in $D_0$.
and $p_{f_1} + p_{f_2} \neq p_{f_3} + p_{f_4}$ for $\beta_0$ in $D_1$ and $D_2$. As we see the IR-finite part $\beta_1$ in the decomposition of the 1-photon $D_1$ distribution into IR-leading and subleading parts is present as an element in the subleading 2-photon $D_2$ distribution. This nontrivial fact allows one to find out the IR-finite $\beta_1$ functions order by order from standard perturbative calculations. In this particular case the recursive, order-by-order definition of IR-finite $\beta$s looks as follows:

$$
\begin{align*}
\beta_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) &= D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}), \\
\beta_1(p_f; k_1) &= D_1(p_f; k_1) - \beta_0(p_f) S(k_1) \\
\beta_2(k_1, k_2) &= D_2(k_1, k_2) - \beta_0 S(k_1) S(k_2) - \beta_1(k_1) \bar{S}(k_2) + \beta_1(k_2) \bar{S}(k_1)
\end{align*}
$$

Having at hand the IR-free $\bar{\beta}$ the YFS scheme provides immediately for the multi-photon differential distributions all over the phase space and the phase space is integrated without any approximations using MC method.

Below we show an example of the ISR $O(\alpha^3)$ exponentiated multi-photon fully differential distributions for the CEEX variant of YFS exponentiation which is implemented in terms of the IR-free $\bar{\beta}$ objects constructed at the amplitude level, rather than using the spin-summed squared $\bar{\beta}$s. The CEEX we present here very schematically for the process $e^- (p_1, \lambda_1) + e^+ (p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + f(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + ... \gamma(k_n, \sigma_n)$:

$$
\sigma = \sum_{n=0} \int d\Phi_{n+2} \sum_{\lambda, \sigma_1,...,\sigma_n} [\alpha^2(m_r) M_{\alpha, \sigma_1, ..., \sigma_n}(k_1, ..., k_n)]^2,
\quad M_{\alpha, \sigma_1, ..., \sigma_n}(k_1, ..., k_n) = \beta_0(k_1) + \beta_1(k_1) \beta_0(k_2) + \beta_2(k_1, k_2) + \beta_3(k_1, k_2, k_3) + ... + \beta_n(k_1, k_2, ..., k_n)
$$

IR-finite building blocks are:

$$
\begin{align*}
\beta_0^\lambda &= (e^{-\alpha B_\lambda} M^{\text{Born+Virt}})_{\alpha(1)} \\
\beta_1^\lambda, \sigma(k) &= M_{\alpha, \sigma}(k) - \beta_0^\lambda \bar{s}(k)
\end{align*}
$$

It is to be stressed that everything here was done in terms of the spin $M$-amplitudes! Multiphoton differential distributions are positive by construction (which is not the case for EEX). In KKMC the above CEEX is implemented up to $O(\alpha^3)$ for ISR and FSR\footnote{Multiperipheral contributions and non-IR ISR@FSR interactions were also included in the implementation}. The example of the CEEX application is shown in the figure above from ref.\cite{1} where we present the “state of art” comparison of KK MC vs. ZFITTER for ISR+FSR, including IFI=ISR+FSR. We conclude that the precision $ds / \sigma = 0.2\%$ was reached, which is enough for LEP2 data analysis. Similar high precision was obtained for the forward-backward asymmetry $f$. 

Recently, the CEEX matrix element was extended in KKMC to the neutrino channel. The contributions from following diagrams are now included:

- The systematic error is estimated to be $1.3\%$ for $\nu_\mu \bar{\nu}_\mu \gamma$ and $0.8\%$ for $\nu_\mu \bar{\nu}_\mu \gamma$ and $\nu_\tau \bar{\nu}_\tau \gamma$.
- For observables with two observed photons we estimate the uncertainty to be about $5\%$.
- These new improved results were obtained thanks to the inclusion of non-photonic electroweak corrections of the ZFITTER package.
and due to newly constructed, exact, single and double emission photon amplitudes in the KKMC for the contribution with the $t$-channel $W$ exchange.

- The virtual corrections for the $W$ exchange are at present introduced in the approximated form. The exponentiation scheme CEEX is the same as in the original KKMC program.

In the first version of KKMC of ref. [9] the $O(\alpha^2)$ next-to-leading contributions were incomplete. In ref. [3] the $O(\alpha^2)$ contribution from the following diagrams were evaluated

and partly included in the development of KKMC – and the remaining contributions were found to be negligible. The contributions in questions [3] are due to one real and one virtual photon emission and the new numerical results are shown in the following figure

where the new calculation is shown to agree with the older calculations of the refs. [20][21]. NB. These contributions were also recently (re)calculated in ref. [22] for the purpose of the low energy electron collider experiments.

Conclusions The YFS inspired EEX and CEEX schemes are successful examples of the Monte Carlo based directly on the factorization theorem (albeit for IR soft case for Abelian QED only). They Work well in practice: KORALZ, BHLUMI, YWSWW3, BHWIDE, KK MC. Extension (as far as possible) to all collinear singularities would be very desirable and practically important! The KKMC program is extended to the neutrino channel. Missing fully differential $2f + \gamma_{\text{virt}} + \gamma_{\text{real}}$ distributions for $O(\alpha^2)$ CEEX are now available.

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