How galaxies lose their angular momentum

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ABSTRACT
The processes are investigated by which gas loses its angular momentum during the protogalactic collapse phase, leading to disk galaxies that are too compact with respect to the observations. High-resolution N-body/SPH simulations in a cosmological context are presented including cold gas and dark matter. A halo with quiet merging activity since redshift $z\sim3.8$ and with a high spin parameter is analysed that should be an ideal candidate for the formation of an extended galactic disk. We show that the gas and the dark matter have similar specific angular momenta until a merger event occurs at $z\sim2$ with a mass ratio of 5:1. All the gas involved in the merger loses a substantial fraction of its specific angular momentum due to tidal torques and falls quickly into the centre. Dynamical friction plays a minor role, in contrast to previous claims. In fact, after this event a new extended disk begins to form from gas that was not involved in the 5:1 merger event and that falls in subsequently. We argue that the angular momentum problem of disk galaxy formation is a merger problem: in cold dark matter cosmology substantial mergers with mass ratios of 1:1 to 6:1 are expected to occur in almost all galaxies. We suggest that energetic feedback processes could in principle solve this problem, however only if the heating occurs at the time or shortly before the last substantial merger event. Good candidates for such a coordinated feedback would be a merger-triggered star burst or central black hole heating. If a large fraction of the low angular momentum gas would be ejected, late-type galaxies could form with a dominant extended disk component, resulting from late infall, a small bulge-to-disk ratio and a low baryon fraction, in agreement with observations.

Key words: galaxies: haloes – cosmology:theory, dark matter, gravitation – methods: numerical, N-body/SPH simulation

1 INTRODUCTION
An important goal of cosmology is to understand how galaxies form. Currently, the most popular scenario for structure formation in the Universe is based on the inflationary cold dark matter (CDM) theory (Blumenthal et al. 1984), according to which cosmic structures arise from small Gaussian density fluctuations composed of non-relativistic collisionless particles. The dark matter aggregates into larger and larger clumps as gravity amplifies the weak density perturbations, produced at early times in the universe. Gas cools and condenses within these dark halos, eventually forming the galaxies we see today (White & Rees 1978). This hierarchical picture of structure formation is an elegant and well defined theory that naturally explains the growth of large-scale structures from density fluctuations as small as those detected in the Cosmic Microwave Background to present-day galaxies. The CDM model however also has problems that might at the end lead to a deeper understanding of the nature and origin of cold dark matter and its interaction with baryonic matter. One of the most interesting puzzles at the moment is the so called cosmological angular momentum problem (for a review see e.g. Burkert & D’Onghia 2004, Primack 2005).

In spiral galaxies, almost all the stars and the gas move on circular orbits. The structure of these galaxies is therefore governed by angular momentum. Where does this angular momentum come from? According to the standard picture, the angular momentum of a dark matter fluctuation grows by tidal torques from neighboring structures (Peebles 1969, White 1984), until the protogalaxy decouples from the Hubble flow and collapses. Fall & Efstathiou (1980) argued that

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gas and dark matter should initially have been well mixed. In this case, the specific angular momentum distribution of the gas is initially equal to that of the dark halo. During the protogalactic collapse phase the gas component dissipates its kinetic energy, decouples from the collisionless and violently relaxing dark matter component and settles into a flattened, fast rotating disk that subsequently turns into stars. If the specific angular momentum were preserved during the infall into the equatorial plane, the scale length of the galactic disk would be directly related to the specific angular momentum of the halo. Fall & Efstathiou (1980) and later on Mo, Mao & White (1998) indeed showed that in this case the expected disk scale lengths are in good agreement with the observations (e.g. Courteau 1997).

In contrast to these analytical estimates, numerical simulations produce disks that are too small and much more centrally concentrated than observed. This is the so called angular momentum catastrophe of the gas (Navarro & Benz 1991; Navarro & Steinmetz 1997). Navarro & Steinmetz (2000) suggested that the loss of angular momentum is a result of dense gaseous clumps falling into the inner regions of dark halos and transferring large amounts of their angular momentum to the dark matter component by dynamical friction. However this process has not been quantified and investigated in details.

Another, probably related problem, encountered by numerical simulations of galaxy formation is that without energetic heating star formation is already very efficient in low mass structures at high redshifts (Navarro, Frenk & White 1991; Steinmetz & Müller 1994). The dense, compact stellar systems, formed in that way, later are collected in the innermost regions of larger galaxies, forming compact bulge-dominated systems that do not resemble real late-type spiral galaxies. Subsequent work tried to solve some of these shortcomings by employing energetic feedback, leading to more promising results (Sommer-Larsen, Götz & Portinari 2003; Abadi et al. 2003; Governato et al. 2004; Robertson et al. 2004). Here, the idea was that heating would reduce early star formation and, at the same time, decouple gas from the dark haloes in an early phase, thus also reducing the effect of dynamical friction and angular momentum loss on the gas component when the dark matter substructures merge. Later on, gas cools and falls back into the equatorial plane in a monolithic-like collapse.

Although some cases have now been reported to form realistic disk galaxies in this way, most disks are still denser and contain more massive bulges than observed in late-type galaxies, indicating that the specific angular momentum problem is not solved. It also has been suggested that only halos with a quiet merging history after $z \geq 2$ can host disk galaxies (but see Springel & Hernquist 2005). However N-body simulations indicate that halos with a quiet merging history since $z \sim 3$ (those expected to host bulgeless disks), should in general have too low an angular momentum to reproduce the observed scale lengths of pure disks (D’Onghia & Burkert 2004).

The aim of this paper is to better understand the origin of the cosmological angular momentum problem. Our hope is that a more detailed insight could help to find a realistic solution for this unsolved puzzle. The plan of this paper is as follows. In §2 we present some numerical simulations which are analysed in §3 where we identify the dominant process that leads to the angular momentum loss of the gas. §4 discusses the implications for our current understanding of the formation of disk galaxies in a hierarchical universe.

2 THE NUMERICAL METHOD

We performed a set of high-resolution simulations of a region that evolves to form, at $z = 0$, galaxy-sized dark matter halos in a low-density, flat, “concordance” Cold Dark Matter ($\Lambda$CDM) scenario: $\Omega_0 = 0.3$, $h = 0.7$, $\Omega_b = 0.039$, $\Omega_{\Lambda} = 0.7$, $\sigma_8 = 0.9$. 1 The region was first identified in a low resolution cosmological simulation of a periodic box (10 $h^{-1}$ Mpc on a side) that was evolved until the present time ($z = 0$) using the Tree+SPH code GADGET2 (Springel 2005) and then resimulated at higher resolution. At $z = 0$ the target dark matter halo under consideration has a circular velocity, $V_{200} \sim 150$ km/s, and a total mass of $M_{200} = 5 \times 10^{11} h^{-1}$ $M_\odot$, measured at the virial radius, $r_{200} = 150 h^{-1}$ kpc, where the mean inner density contrast (relative to the critical density for closure) is 200. This halo was selected from a list of clumps compiled using a friend-of-friends algorithm with linking parameter set to 15% of the mean interparticle separation. The particles of the identified region were then traced back to the initial conditions, where a box containing all of them was drawn. The high-resolution region was filled with 256$^3$ particles on a cubic grid and the appropriate small-scale power was added up to the Nyquist frequency of the new particle grid, preserving the Fourier amplitudes and phases of the low resolution box. The outer regions were coarse-sampled using shells filled with particles of increasing mass, up to the original resolution, in order to reproduce the original tidal field. The re-simulation was performed using the multi–mass technique initial condition implementation of the package ART (Klypin, Kravtsov & Khokhlov 1997; Klypin et al. 2001).

A gas component was included by placing the same number of gas particles on top of the dark matter (DM) particles in the high-resolution box at the starting redshift $z_i \sim 74$. The gas particles were then shifted by half the grid size and their velocities were calculated by averaging the velocities of the eight neighbouring DM particles. “Border” grid cells (i.e. cells with less than eight high-resolution DM particles) were left gas–empty. For this simulation the number of high–resolution SPH+DM particles was $N_p \sim 1.100.000$. The gas and dark matter particle mass was $m_p = 6.45 \times 10^5 h^{-1}$ $M_\odot$ and $m_{dm} = 4.3 \times 10^8 h^{-1}$ $M_\odot$, respectively. This corresponds to a baryonic fraction $F_b = 0.13$ consistent with $\Omega_0 = 0.04$. We adopted a comoving Plummer softening scale length of $1h^{-1}$ kpc for all gravitational interactions between pairs of particles. The minimum SPH softening for the gas was fixed to half this value.

We followed the strategy of keeping our model as simple as possible, hence focusing our investigation on the effects of the presence of a collisional component (gas) on the dynamical properties of the target halo, and in particular on the evolution of the angular momentum in the collisional and in the collisionless (DM) component. The affect of star formation and thermal feedback were neglected in this work and

\[ H(z = 0) = H_0 = 100 \hbar \text{ km s}^{-1} \text{ Mpc}^{-1} \]
will be discussed in a subsequent paper. Instead, the temperature of each gas particle was fixed at a constant value of $10^5$ K. Due to the high density of the cold gas at late times, all runs including gas were stopped at $z = 0.3$, since the computational time would have been too expensive. This choice does not affect the conclusions of this work as all the important action which we plan to discuss happens at higher redshifts.

3 RESULTS

3.1 Properties of the target halos

Previous work has explored the importance of mergers in explaining galactic morphologies and has shown that major mergers with mass ratios 1:1 to 4:1, setup with cosmological self-consistent orbital parameters (Khochfar & Burkert 2006), produce remnants with properties in agreement with observed elliptical galaxies (Naab & Burkert 2001; 2003; Burkert & Naab 2004, 2005; Naab & Trujillo 2005). Disk galaxies are in contrast expected to form from mergers of substructures with larger mass ratios (so called “minor mergers” and “smooth accretion”). The more quiescent the merger history, the more likely it should be to form extended disks. In addition, a high specific angular momentum would help. We therefore identified dark halos in the low-resolution disks. In addition, a high specific angular momentum would be suitable for the formation of an extended, high-angular momentum disk. For comparison, figure 1 also shows two additional halos, B and C, with masses $10^{12}h^{-1}M_\odot$ and $4\times10^{11}h^{-1}M_\odot$ and high spin parameters of $\lambda \sim 0.04$ and $\lambda \sim 0.03$ respectively. Both have also been selected in isolated regions of the original cube and resimulated with the same technique for comparison. Their merging activities however are more violent. Halo B experiences a 2:1 major merger at $z=2.2$. Halo C undergoes a 1:1 merger at $z \sim 2$. For both halos, the merger activity is largely over at $z \sim 1$ when most of the mass is in place.

Note that the selected halo A is a very rare exception due to its large specific angular momentum of $\lambda \approx 0.04$, in addition to its smooth merging history. If in this halo again a disk with small scale length would form, the problem would be even worse in all other cases. In a recent paper D’Onghia & Burkert (2004) explored the angular momentum properties of halos that did not experience any major mergers since $z=3$. They found that those quiescent halos have low specific angular momenta (spin values) that peak around a value of $\lambda \sim 0.02$, a factor of two lower than the average in the log-normal distribution of spin values $\lambda = 0.04$ (Bullock et al. 2001) and substantially smaller than expected for disk-dominated late-type galaxies (van den Bosch, Burkert & Swaters 2001).

3.2 The origin of the compact disk component

Does the gas retain its initial specific angular momentum during the evolution of halo A without a major merger or is its angular momentum lost?

To answer this question let us focus on the evolution of object A. Figure 2 shows the gas disk that has formed at $z = 0.3$. 80% of the gas particles within the virial radius are concentrated in the central $3.5 h^{-1}$ kpc, where they form a very compact disk of mass $3.59\times10^{10}h^{-1}M_\odot$ (blue dots in Figure 2), and scale length $1 h^{-1}$ kpc, surrounded by a massive dark halo. The remaining 20% of the gas particles have settled into a thin disk of small total mass ($\sim 8.45\times10^{9}h^{-1}M_\odot$) (red dots in Figure 2) but scale length $2.5 h^{-1}$ kpc, which extends to $10 h^{-1}$ kpc from the center of mass of the remnant. It is interesting that the outer lower-mass disk has a scale length that is in good agreement with observations. The inner disk however is much more compact than observed.

What is the origin of the compact disk? We find that during the 5:1 merger at redshift 1.8 gas loses a substantial amount of its specific angular momentum to the surrounding dark matter component due to strong gravitational torques. To quantify this process, we selected at $z \sim 0.3$ three groups of particles: the dark matter particles within the virial radius; the gas particles in the inner disk within $\sim 3.5 h^{-1}$ kpc from the galactic center (blue dots in Figure 2) and those between $3.5 < r < 10 h^{-1}$ kpc (red dots in Figure 2). We traced back in time the evolution of the specific angular momentum of each group with respect to its centre of mass. Figure 3 (left panel), shows the specific angular momentum for the dark matter (solid line), the inner gas particles (filled circles) and outer gas particles (open circles) as a function of redshift. We find that the gas and the dark matter have similar angular momentum distributions until the 5:1 merger event at $z \sim 2$, when a large fraction of the specific angular
disk at $z=0.3$ (blue disk in Fig.2) is formed from gas particles that form the disk of the major progenitor (green particles in figure 4) and its dark matter particles within the virial radius and again followed back in time their specific angular momenta. The right panel of figure 3 shows the evolution in redshift of the specific angular momentum of these dark matter (blue dots) and gas particles (red triangles), respectively. Interestingly, between $z=2$ and $z=5$, gas and dark matter have roughly the same, low specific angular momentum. No signature of catastrophic angular momentum loss is found at $z \sim 3.5$ which we attribute to the fact that due to their low angular momentum the two progenitors in this 1:1 merger collide head-on without generating gravitational torques that could remove angular momentum if the objects would spiral in more gradually. We conclude that the compact disk in the major progenitor at $z \sim 2$ is not a result of angular momentum loss. Instead at the time of its formation ($z \sim 3.5$) angular momentum build-up through gravitational interaction with neighboring structures had not yet been efficient. The angular momentum problem arises later, at $z = 2$, as a result of the violent merging epoch when the system had gained a substantial amount of angular momentum. Now the merger efficiently redistributes angular momentum from the inner to the outer regions.

3.3 The origin of the extended disk component
What is the origin of the extended thin disk at $z=0.3$ in Figure 2? It formed after the 5:1 merger event, by smooth accretion of gas ($\sim 30\%$) and by gas stripped from infalling dark matter substructures with mass ratios larger than 10:1 ($\sim 70\%$). As demonstrated in Figure 4 this gas (red dots) is still too far away at $z=1.8$ to lose angular momentum by the gravitational torques that arise in the course of the 5:1 merger. In fact, like the extended dark matter component, it gains angular momentum transferred from the inner, merging parts to the outer regions. This is shown by the red open circles in Figure 3 (left panel).

Similarly to halo A, the objects B and C also form a very compact disk of gas and subsequently accrete an extended low mass disk of gas around it. In Figure 4 the time evolution of specific angular momentum of the gas and the dark matter for object B (blue symbols in the top panel) and object C (green symbols, bottom panel) is compared with the quiescent halo A (red symbols, middle panel). Model B, during its 2:1 major merger at $z \sim 2.2$ assembles 50% of its final mass and 80% of gas loses its angular momentum in this process. The same happens to model C at $z \sim 2$ when a 1:1 merger occurs. The filled triangles (object B) and filled squares (model C) in Figure 5 show the time evolution of the specific angular momentum of the gas that forms the inner compact disk. The open symbols show the specific angular momenta of the extended disks (20% of total gas mass) that are accreted and gain angular momentum.

4 CONCLUSION
The origin of the angular momentum loss of gas in cosmological simulations of galaxy formation has been investigated. We focussed on the most promising candidate for the formation of an extended disk component: a halo with high
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Figure 2. Gas particles projected such that the target object A is viewed face-on (top panel) and edge-on (lower panel) at z=0.3. Almost 80% of the gas particles within the virial radius are concentrated in the central disk of outer radius 3.5 h$^{-1}$ kpc (blue dots). The remaining 20% of the gas particles has settled into a thin, more extended and lower mass disk (red dots).

spin and no major merger since redshift 3. The gas and the dark matter initially have similar angular momenta. However at z ~ 2, the gas involved in a 5:1 merger loses most of its angular momentum by gravitational torques. Angular momentum loss during major mergers with mass ratios of 1:1 to 4:1 is a process that is well known and has been studied in simulations of elliptical galaxy formation (Mihos & Hernquist 1996, Naab & Burkert 2001). Interestingly however, even less violent (substantial) mergers with mass ratios of order 4:1 to 6:1 that are not considered to produce elliptical galaxies appear to be still very efficient in redistributing specific angular momentum (Bournaud et al. 2005). Semi-analytic models predict these mergers to be an order of magnitude more frequent than major mergers (Khochfar & Silk 2005).

Contrary to common believe, dynamical friction that would always act during clumpy gas infall is not the dominant cause of the angular momentum problem. For example, the gas that falls in after the last substantial merger, although it resides in dark matter substructures, does not lose its specific angular momentum, contrary to what one would expect if dynamical friction is the reason for angular momentum loss. Instead it is the 5:1 merger that occurs when a large fraction of the galaxy has been assembled that generates the angular momentum problem. We also checked that the dark matter in the inner region experiences a similar angular momentum loss as the gas which confirms that gravitational torques are the dominant process.

In the simulations presented here, there was no dark halo which had an even more quiet merging history. In a subsequent paper we will present a detailed statistical investigation of galaxy merging histories, based on semi-analytical models by Khochfar & Burkert (2001, 2005) that confirms the present conclusion and demonstrates that the likelihood for a disk galaxy to not experience at least one substantial merger event since redshift 3 is extremely small. Rather than being a dynamical friction issue, the angular momentum problem therefore appears to arise from the fact that in the cold dark matter scenario substantial mergers are expected to occur in almost all galaxies.

An interesting scenario of disk galaxy formation has been proposed by Birnboim & Dekel (2003) and Keres et al (2005) who suggested that gas in high redshift disk galaxies is accreted cold and without virial shocks directly from filaments, forming galactic disks. Our simulations demonstrate however that this scenario will also suffer from the same angular problem as other models, as a substantial merger is likely to destroy these early disks lateron.

We find that the gas outside the merging region that is accreted lateron keeps most of its angular momentum, forming an extended secondary disk with a scale length that is

Figure 3. The left panel shows the specific angular momentum of the dark matter within the virial radius (solid line), of the gas particles that are placed within 3.5 h$^{-1}$ kpc from the center of mass at z=0.3 (filled circles) and of the gas particles that lie at radii 3.5 < r < 10 h$^{-1}$ kpc at z=0 (open circles) as a function of redshift. The curves in the right panel show the angular momentum evolution of the dark matter (blue) and the gas (red) of the massive progenitor that at z=2 has already formed a compact disk.
This is consistent with the cosmological simulations e.g. of Abadi et al. 2003.

What is then the origin of the late-type spiral galaxy population with small bulge-to-disk ratios? We can think of several possible answers. First, most of the gas that settles into the inner disk might be blown out by a central supermassive black hole (Robertson et al. 2005, astro-ph/0503369), leaving behind a small bulge that however needs to be large enough to fit the observed black hole mass versus bulge mass correlation (Haring et al. 05). This bulge would be surrounded by the now dominant, extended disk component that formed from late infall. Note that this scenario could explain why spiral galaxies have baryon fractions that appear to be smaller than the cosmologically predicted universal baryon fraction (Yang et al. 2005). Unfortunately, the problem of generating extended disks in bulgeless galaxies cannot be solved in this way (D’Onghia & Burkert 2004).

Another process that could reduce the mass of the inner disk is energetic feedback that decouples the gas from the dark matter and prevents it from falling in before the last substantial merger episode. This might explain why some numerical simulations with feedback are successful in generating dominant large disks while others fail. If gas heating occurred just before the last substantial merger, an extended disk could form through subsequent monolithic gas infall. If gas heating would however act too early, the gas would cool again and fall back into the inner regions before the merger occurs, leading again to an angular momentum problem. Vice versa, if heating would be turned on too late, gas would already have lost its angular momentum in the merger. This suggests that the energetic feedback should best be coupled directly to the last merger event e.g. by triggered star bursts and/or nuclear activities, coupled with central black hole formation.

Whether one of these scenarios will solve the angular momentum problem and can reproduce the observed frequency and luminosity function of late type galaxies is an interesting question that should be explored in greater details.

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REFERENCES

Abadi M., Navarro J. F., Steinmetz M., Eke V. R. 2003, ApJ, 591, 499
Birnboim, Y., Dekel, A. 2003, MNRAS, 345, 349
Blumenthal G. R., Faber S. M., Primack J. R., Rees M. J. 1984, Nature, 311, 517
Bournaud F., Jog C. J. & Combes F. 2005, AA, 437, 69
Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R. 2001, ApJ, 555, 240
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Burkert A. & D'Onghia, E. 2004, in "Penetrating Bars Through Masks of Cosmic Dust", eds. D. Block, E. Block, K. Freeman, K. Puerari, R. Gross (Springer), p. 341, astro-ph/0409540

Burkert A., Naab T. 2004, in “Coevolution of Black Holes and Galaxies”, eds. L. C. Ho, p. 421

Burkert A., Naab T. 2005, MNRAS, 363, 597

Courteau S. 1997, AJ, 114, 240

D'Onghia E. & Burkert A. 2004, ApJ, 612, L13

Fall S. M. & Efstathiou G. 1980, MNRAS, 193, 189

Governato F., Mayer L., Wadsley J., Gardner J. P., Willman B., Hayashi E., Quinn T., Stadel J., Lake G. 2004, ApJ, 607, 688

Haring N. & Rix H-W. 2004, ApJ, 604, L89

Keres D., Katz N., Weinberg D.H., Dave R. 2005, MNRAS, 363, 2

Khochfar S., & Burkert A. 2001, ApJ, 561, 517

Khochfar, S., & Burkert, A. 2003, ApJ, 597, L117

Khochfar, S., & Burkert, A. 2005, MNRAS, 359, 1379

Khochfar, S., & Silk, J. 2005, ArXiv Astrophysics e-prints, arXiv:astro-ph/0509375

Khochfar S. & Burkert A. 2006, A&A, 445, 403

Klypin A., Kravtsov A. V., Bullock J. S., Primack J. R. 2001, ApJ, 554, 903

Kravtsov A. V., Klypin A. A., Khokhlov Alexei M. 1997, ApJS, 111, 73

Mo H. J., Mao S. & White S. D. M. 1998, MNRAS, 295, 319

Naab T., Burkert A 2001, ApJ, 555, L9

Naab T., Burkert A 2003, ApJ, 597, 893

Naab, T., & Trujillo, I. 2005, ArXiv Astrophysics e-prints, arXiv:astro-ph/0508362

Navarro J. F., Benz W. 1991, ApJ, 380, 320

Navarro J. F., Frenk C. S. & White S. D. M. 1995, MNRAS, 275, 270

Navarro J. F., Steinmetz M. 1997, ApJ, 478, 13

Navarro J. F., Steinmetz M. 2000, ApJ, 538, 477

Peebles P. J. E. 1969, ApJ, 155, 393

Primack, J.R. 2005, NewAR, 49, 25

Robertson B., Yoshida N., Springel V., Hernquist L. 2004, ApJ, 606, 32

Robertson B., Hernquist L., Bullock J.S., Cox T.J., Di Matteo T., Springel V., Yoshida N. 2005, ApJL submitted, astro-ph/0503369

Sommer-Larsen J., Gotz M., Portinari L. 2003, ApJ, 596, 47

Springel V. 2005, MNRAS, 999

Springel V. & Hernquist L. 2005, ApJ, 622, L9

Steinmetz M. & Mueller E. 1994, A&A, 281, L97

van den Bosch, F.C., Burkert, A., Swaters, R.A. 2001, MNRAS, 326, 1205

White S.D.M. & Rees M. J 1978, MNRAS, 183, 341

White S. D. M 1984, ApJ, 286, 38

Yang X., Mo H.J., Jing Y.P., van den Bosch F.C. 2005, MNRAS, 358, 217