Seismic Damage Assessment of Steel Reinforced Concrete Members by a Modified Park-Ang Model

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Abstract
Because the seismic performance of steel reinforced concrete (SRC) members are usually different from that of common reinforced concrete members, a modified Park-Ang damage model for SRC members is proposed in this paper. The combination coefficient of the modified damage model is derived using the related experimental results of SRC columns. Then, a formula is developed to describe the relationship between the coefficient and the parameters of the SRC columns such as the axial load ratio, steel ratio and slenderness radio. The results indicate that at the failure state of the SRC members, the modified damage index has a mean value close to 1.0 and limited scatter. The damage response of SRC members can be better represented by the modified damage model. Finally, based on the features of the skeleton curves and related research on performance classification of SRC members, the damage indexes at the principle damage states are calculated using the modified model.

Keywords: SRC members; seismic damage; Park-Ang damage model; combination coefficient; performance level

1. Introduction
The displacement-based seismic design method was first carried out after the theoretical proposal of the performance-based seismic design. Researchers started to realize that the deformation capacity in the elastic-plastic state is of significant importance to the anti-collapse performance of structures. According to Priestley (2002), the damage state of structures is closely related to their sectional deformation and ultimate strain. Because the displacement can be obtained based on the sectional deformation, the damage state of structures can be controlled by displacement. Hence the structural performance target can be determined by the displacement index through displacement-based seismic design. However, the seismic performance and failure characteristics of structures cannot be reliably described using only the displacement index or displacement ductility index (Ye et al., 2012, Teran et al., 1996). With the development of the seismic design, it is commonly acknowledged that the double-control damage criterion based on the first passage maximum displacement response and accumulated plastic damage satisfies practical seismic damage tests. (Fajfar et al., 1994)

After analyzing the test results of a large number of reinforced concrete columns and beams, Park and Ang (1985) proposed an assessment formula for the linear combination of the maximum displacement and hysteretic energy dissipation, which conceptually illustrated the contribution of the maximum displacement and hysteretic energy dissipation to the structural damage. The formula has had a substantial impact on various studies of the seismic damage assessment of structural components. The formula is as follows:

$$D_{PA} = \frac{\delta_m}{\delta_u} + \beta_{PA} \int dE$$

where $\delta_u$ is the ultimate displacement under monotonic loads, $\delta_m$ is the maximum displacement under loading, $F_y$ is the yield strength, and $dE$ is the incremental absorbed plastic hysteretic energy. The empirical formula of the parameter $\beta_{PA}$ is obtained by regression analysis:

$$\beta_{PA} = (-0.447 + 0.073 \frac{L}{d} + 0.24n_s + 0.314\rho_s) \cdot 0.7$$

where $L/d$ is the shear span ratio of the components, $n_s$ is the axial compression ratio, $\rho_s$ is the longitudinal tensile reinforcement ratio (including the longitudinal reinforcement in the central section), and $\rho_s$ is the stirrup reinforcement ratio per unit volume in the reinforced area. Obtained by a regression curve from
extensive experimental results, Cosenza et al. (1993) reported that the coefficient ranged from -0.3 to +1.2, with a median of approximately 0.15.

This model, which has a simple form, has been verified by numerous experimental damage statistics. $D_{pa}\geq 1.0$ represents total collapse, and a $D_{pa}$ value of approximately 0.4–0.5 can be regarded as the limit of damage that can be repaired (Ang and De Leon 1994). It has been widely used by researchers around the world and imbedded in the well-known non-linear reinforced concrete damage analysis program IDARC (Kunnath et al., 1992). However, there are several drawbacks: (1) when structure failure occurs under a monotonic load, the damage index is not 1.0, and (2) when the structure is subjected to elastic cyclic loading, the damage index is not 0. Kunnath (1992) revised the Park-Ang model by removing the recoverable deformation from the first term as follows:

$$D_k = \frac{\theta_r - \theta_0}{\theta_r} + \beta \int \frac{dE}{F_0 \delta_0}$$

where $\theta_r$ is the recoverable deformation after unloading. Although the displacement term is revised, the damage index obtained from this formula is not 1.0 when failure occurs under monotonic loads. The model proposed by Bracci (1979), which is defined as the area between the force-deformation curve under monotonic loads and the fatigue damage skeleton curve, is difficult to apply in practical design.

Because of the energy dissipation of the structure under monotonic loads, the Park-Ang damage index exceeds 1.0 at the point of failure. Chai et al. (1995) introduced plastic energy under monotonic loading into the second term of Eq. 1 to solve the non-normalization problem of the Park-Ang model:

$$D_k = \frac{\delta_m - \delta_0}{\delta_m} + \beta \int \frac{dE - E_{sm}}{F_0 \delta_0}$$

where $E_{sm}$ is the plastic dissipated energy of the structure under monotonic loads. In this formula, the damage index at the failure point under monotonic loads is 1.0. However, the damage index is not 0 when the structure is subjected to elastic cyclic loads. Moreover, this modified model was only verified by a series of small-scale notched steel cantilever beams subjected to large inelastic displacement cycles. Thus, it is uncertain whether this model is appropriate for other structural members.

Jiang et al. (2015) found that the deformation of RC members at initial cracking was significantly smaller than deformation at yielding. Thus, a modified damage model can be simplified as follows:

$$D_k = (1 - \beta) \frac{\mu_m}{\mu_u} + \beta \int \frac{dE}{F_0 \delta_0}$$

where $\mu_m = \delta_m / \delta_y$, $\mu_u = \delta_u / \delta_y$, and $\delta_y$ is the yield displacement. This modified damage model corrected the non-convergence issues at the upper and lower limits. However, when it is applied to less ductile members, it may cause larger errors.

The combination coefficients $\beta$, which are important in the combined model of maximum displacement and hysteric energy, should reflect the influence of the entire displacement time history of the components (including the value of the displacement amplitudes, the occurrence sequences of different amplitudes and the deviation degree of the displacement) on the accumulated damage. The empirical formula of the combination coefficients is generally determined experimentally or using seismic damage cases, which cannot or only partially reflect the influence of the displacement time history on the accumulated damage. The strictly defined damage index should converge to 0 in the undamaged state and to 1.0 in the state of complete damage.

In this study, a modified form of the Park-Ang damage model is proposed to eliminate its non-normalization problem. The combination coefficient of the modified damage model is derived using the related experimental results of SRC columns. Then, a formula is developed to describe the relationship between the combination coefficient and the parameters of the SRC columns such as the axial load ratio, steel ratio and slenderness ratio. Finally, the damage indexes under different performance levels are discussed based on the deformation characteristics of the steel reinforced concrete components.

2. Modified Park-Ang Damage Model

To correct the non-convergence of the Park-Ang damage model at the upper and lower limits, the original formula is modified as follows:

$$D = (1 - \beta_{SRC}) \frac{\delta_m - \delta_y}{\delta_m} + \beta_{SRC} \int \frac{dE}{F_0 (\delta_m - \delta_y)}$$

where $\delta_y$ is the yield displacement, and $\beta_{SRC}$ is the combination coefficient which is different from $\beta_{pa}$ in Eq. 1. In this model, the structural member is considered non-damaged before the yield of concrete. $\delta_m$ is determined as follows:

$$\delta_m = \begin{cases} \delta_m & \delta_m \leq \delta_y \\ \delta_m & \delta_m > \delta_y \end{cases}$$

If the member is loaded within the elastic range ($\delta_m = \delta_y$), the damage index $D$ calculated by Eq. 6 equals 0 ($\delta_m = \delta_y$ and $dE = 0$). When the system is loaded monotonically, the value of damage index $D$ will be 1 ($\delta_m = \delta_y$ and $F_0 (\delta_m - \delta_y) = dE$) at collapse.

The combination coefficient $\beta_{SRC}$ was derived from Eq. 6 based on the assumption that the damage index equals 1 ($D = 1$) at the ultimate limit state, expressed by Eq. 8.
\[ \beta_{\text{SRC}} = \frac{F_y (\delta_u - \delta_m)}{\int dE - F_y (\delta_m - \delta_s)} \]  

(8)

The combination coefficient \( \beta_{\text{SRC}} \) is referred to as the surplus energy dissipation ratio. To determine the value of \( \beta_{\text{SRC}} \) for the proposed damage model, a large number of experimental results were analyzed.

### 3. Experimental Database

The test results of 43 SRC columns were selected from the publications of different researchers. All of them meet the following four principles: I-shaped or H-shaped steel is required; the columns are dominated by bending failure or bond-slip failure; the columns are loaded to failure with a complete test curve; and the detailed test statistics, such as the dimensions, material properties and loading model are provided. The parameters of the selected samples are shown in Table 1.

### 4. Calibration of the Combination Coefficient

Based on the damage indexes in the case of \( D=1 \), the combination coefficient \( \beta_{\text{SRC}} \) can be obtained by reversing the formula. According to Park (1989), the yield deformation \( \delta \), and yield load \( F_y \), are the displacement and the corresponding carrying capacity.

| Tester | Specimen | No. | \( L \)/mm | \( B \)/mm | \( H \)/mm | Steel | \( f_y \)/MPa | \( \lambda \) | \( \alpha_u \)/% | \( \beta_c \)/% | \( \rho_s \)/% | \( \rho_r \)/% |
|--------|----------|-----|----------|--------|--------|------|-------------|------|-------------|-------------|-------------|-------------|
| **Guo (2010)** | | | | | | | | | | | | |
| 17 | S1 | 1000 | 300 | 300 | H200×150×6×10 | 25.9 | 2.75 | 0.15 | 1.19 | 0.236 | 4.53 | 0.75 |
| 18 | S2 | 1000 | 300 | 300 | H200×150×6×10 | 25.9 | 2.75 | 0.3 | 1.19 | 0.236 | 4.53 | 0.75 |
| 19 | S3 | 1000 | 300 | 300 | H200×150×6×10 | 25.9 | 2.75 | 0.4 | 1.19 | 0.236 | 4.53 | 0.75 |
| 20 | S4 | 1000 | 300 | 300 | H200×150×6×10 | 29 | 2.75 | 0.3 | 0.78 | 0.153 | 4.53 | 0.75 |
| 21 | S5 | 1000 | 300 | 300 | H200×150×6×10 | 29 | 2.75 | 0.3 | 1.86 | 0.299 | 4.53 | 0.75 |
| 22 | S6 | 1000 | 300 | 300 | H200×150×6×10 | 29 | 2.75 | 0.45 | 1.86 | 0.298 | 4.53 | 0.75 |
| **Li (2005)** | | | | | | | | | | | | |
| 23 | SRC-5 | 370 | 160 | 220 | I14 | 67.3 | 1.5 | 0.36 | 1.2 | 0.063 | 6.11 | 1.28 |
| 24 | SRC-6 | 370 | 160 | 220 | I14 | 70.4 | 1.5 | 0.36 | 1.6 | 0.08 | 6.11 | 1.28 |
| 25 | SRC-10 | 480 | 160 | 220 | I14 | 81.8 | 2 | 0.2 | 0.8 | 0.045 | 6.11 | 1.28 |
| 26 | SRC-11 | 480 | 160 | 220 | I14 | 81.8 | 2 | 0.2 | 1.2 | 0.052 | 6.11 | 1.28 |
| 27 | SRC-13 | 480 | 160 | 220 | I14 | 83.1 | 2 | 0.28 | 0.8 | 0.044 | 6.11 | 1.28 |
| 28 | SRC-14 | 480 | 160 | 220 | I14 | 81.8 | 2 | 0.28 | 1.2 | 0.052 | 6.11 | 1.28 |
| 29 | SRC-15 | 480 | 160 | 220 | I14 | 84.9 | 2 | 0.28 | 1.6 | 0.123 | 6.11 | 1.28 |
| 30 | SRC-18 | 480 | 160 | 220 | I14 | 84.4 | 2 | 0.36 | 1.6 | 0.124 | 6.11 | 1.28 |
| **Ricles (1994)** | | | | | | | | | | | | |
| 31 | S1 | 1930 | 406 | 406 | H210×205×9×14 | 31.1 | 4.75 | 0.29 | 0.84 | 0.15 | 4.5 | 1.6 |
| 32 | S3 | 1930 | 406 | 406 | H210×205×9×14 | 30.9 | 4.75 | 0.29 | 0.84 | 0.151 | 4.5 | 2.8 |
| 33 | S4 | 1930 | 406 | 406 | H210×205×9×14 | 31.1 | 4.75 | 0.29 | 0.84 | 0.15 | 4.5 | 1.6 |
| 34 | S7 | 1930 | 406 | 406 | H210×205×9×14 | 30.9 | 4.75 | 0.29 | 0.84 | 0.151 | 4.5 | 2.8 |
| **Chen (2014)** | | | | | | | | | | | | |
| 35 | SRC1-1-2 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.3 | 1.43 | 0.108 | 6.15 | 1.68 |
| 36 | SRC1-2-2 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.3 | 1.7 | 0.088 | 6.15 | 1.68 |
| 37 | SRC1-3-2 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.3 | 0.95 | 0.072 | 6.15 | 1.68 |
| 38 | SRC2-1-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.4 | 1.77 | 0.134 | 6.15 | 1.68 |
| 39 | SRC2-2-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.4 | 1.43 | 0.108 | 6.15 | 1.68 |
| 40 | SRC2-3-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.4 | 1.17 | 0.088 | 6.15 | 1.68 |
| 41 | SRC3-1-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.45 | 2.05 | 0.155 | 6.15 | 1.68 |
| 42 | SRC3-2-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.45 | 1.7 | 0.088 | 6.15 | 1.68 |
| 43 | SRC3-3-1 | 1000 | 180 | 260 | H160×80×8×10 | 37.5 | 4.23 | 0.45 | 1.43 | 0.108 | 6.15 | 1.68 |

Note: concrete strength \( f_c \), shear span ratio \( \lambda \), axial compression ratio \( \alpha_u \), volume stirrup reinforcement ratio \( \rho_s \), eigenvalue of stirrup \( \beta_c \), steel ratio \( \rho_r \), reinforcement ratio \( \rho_c \).
when the load reaches 75% of the maximum carrying capacity of the test samples. As shown in Fig.1, the maximum or failure displacement $\delta_m$ corresponds to the point at which the maximum carrying capacity decreased by 15%. The ultimate displacement $\delta_u$ of the samples under monotonic loading is obtained from the test results if a monotonic load is applied. Otherwise, the formula for the displacement ductility coefficient $\mu$, which was derived based on the test results of 50 SRC columns according to Zhang (2006), is employed:

$$\mu = \frac{0.47\beta + 0.065(-2.34\lambda^2 + 10.58\lambda + 8.08)(-0.172\rho_\lambda^2 + 1.59\rho_\lambda - 2.66)}{n_0 + 0.26}$$  \tag{9}$$

where $\beta$, is the eigenvalue of the stirrup, $\lambda$ is the shear span ratio, $\rho_\lambda$ is the steel ratio, and $n_0$ is the axial compression ratio.

![Fig.1. Yield Displacement and Ultimate Displacement](image1)

Because $\beta_{SRC}$ is generally considered related to the structural ductility, when the ductility is larger, the energy dissipation of the structure is stronger, and $\beta_{SRC}$ is smaller. Conversely, $\beta_{SRC}$ is larger when the ductility coefficient $\mu$ is smaller. By analyzing the values of $\beta_{SRC}$ and the characteristic parameters of the test samples, several results can be found. Generally, the value of $\beta_{SRC}$ shows a decreasing trend when the axial compression ratio $n_0$ increases. Moreover, when $\lambda$ increases, both $\beta_{SRC}$ and $\rho_\lambda$ increase. Compared with $\rho_\lambda$, the eigenvalue of the stirrup $\beta$, and the reinforcement ratio $\beta$, have lesser effects on $\beta_{SRC}$. Based on the value of $\beta_{SRC}$ and the characteristic parameters of the displacement ductility coefficient $\mu$, a formula is derived via regression analysis:

$$\beta_{SRC} = (0.142 - 0.01447\lambda + 0.49614n_0^{1.72424}) \cdot 0.90971^{\lambda^3}$$  \tag{10}$$

where $\lambda$ is the shear span ratio, $\rho_\lambda$ is the steel ratio, and $n_0$ is the axial compression ratio.

The regression combination coefficient is compared to the experimental combination coefficient of the damage index as presented in Fig.2. The plot indicates that the regression formula proposed in this paper can effectively predict the combination coefficient to a certain extent.

![Fig.2. Regression and Experimental Coefficient Values](image2)

To evaluate the damage index determined by the regression formula of $\beta_{SRC}$, the damage index of the test samples at the failure point is calculated using the modified model proposed in this paper and the Park-Ang model. The results are shown in Fig.3. The damage index at the failure point based on the modified damage model exhibits greater accuracy than the index produced by the Park-Ang model.

![Fig.3. The Damage Index at the Failure Point](image3)

As calculated by the proposed regression formula of $\beta_{SRC}$ and the modified damage model in this paper, the mean value, standard deviation and coefficient of variation of the damage index at the failure state are 1.032, 0.11 and 10.66%, respectively. As illustrated
in Fig.3(a), when the empirical formula of $\beta_{SRC}$ and the modified damage model are employed, the mean value of the damage index at the failure state is close to 1.0, with a little scatter. Thus, the modified damage model can calculate the damage index of SRC columns appropriately.

Given that the characteristic parameters, such as slenderness ratio and steel ratio of most SRC columns in engineering are within the limits, $\beta_{SRC}$ in the modified model is represented by a constant, which is used in structural design. As presented in Fig.2., the mean value of $\beta_{SRC}$ calculated based on the test results is 0.064, and the regression value of $\beta_{SRC}$ is between 0.05 and 0.07. Therefore, it is reasonable to set $\beta_{SRC}$ as 0.064 when the proposed modified damage model is adopted for generally designed SRC columns. Accordingly, the damage index at the failure state of the samples are presented in Fig.4. The mean and variation of the damage index values are 1.029 and 0.119, respectively.

5. Performance Level of SRC Columns

As shown in Table 2., Guo (2009) summarized four performance levels for SRC columns based on the classification methods of seismic performance levels recommended by researchers around the world.

Because there is no significant distinction between the performance features of material deformation for SRC components, based on the features of the skeleton curve and the performance levels of SRC columns shown in Table 2., the performance of SRC columns is classified into three points: a yield point, peak point and failure point. These points correspond to the performance levels of basic operation, repairable and avoiding collapse.

Based on the tests listed in Table 1. and the modified damage model proposed in this paper, the damage indexes of the test samples at three performance points (yield point, peak point and failure point) are calculated to determine the performance levels of the SRC columns. Figs.5.-7. show the damage index and its distribution for the SRC columns at the three performance points. The damage index determined

| Performance level       | Damage state          | Structure                        | Structural components                                                                 |
|-------------------------|-----------------------|----------------------------------|---------------------------------------------------------------------------------------|
| Operation               | Generally undamaged   | Structure undamaged; all functions unaffected. | Longitudinal reinforcement and steel not yielding; loaded to 30% - 60% of the ultimate load; crack width of 0.05 - 0.20 mm; no residual deformation. |
| Basic operation         | Slightly damaged      | Structure generally well; primary functions unaffected. | Compression reinforcement and compression flange of the steel allowed to yield; web of the steel not completely yielding; loaded to 50% - 90% of the ultimate load. |
| Repairable              | Moderately damaged    | Structure moderately damaged; primary functions repairable. | The maximum residual inter-story drift ratio after unloading is less than 1/100. |
| Avoiding collapse       | Significantly damaged | Structure significantly damaged; load-carrying system remaining; partial functions unrepairable. | Carrying capacity decrease to 70% - 90% of the maximum carrying capacity; local buckling of the longitudinal reinforcement and steel of the columns. |

![Fig.4. The Damage Index at the Failure Point Based on the Recommended Combination Coefficient](image)

![Fig.5. The Damage Index and Distribution at the Yield Point](image)
by the proposed modified damage model exhibited less scatter and can evaluate the damage performance of SRC columns reliably. Accordingly, the damage indexes of 0.03, 0.40 and 1.0 at the three performance levels, i.e., basic operation, repairable and avoiding collapse are recommended for SRC members in this study.

6. Conclusion
The modified form of the Park-Ang damage model is proposed in this paper based on its non-convergence at the upper and lower limits. The combination coefficient of the modified damage model is calculated using the related test results of SRC columns. Furthermore, a formula that describes the relationship between the combination coefficient and the characteristic parameters of the SRC components is derived. The following conclusions can be drawn:

(1) At the ultimate point of the components, the modified damage index is close to 1.0 with a smaller scatter, indicating that the proposed damage model can appropriately describe the damage performance of SRC components.

(2) The empirical formula of the combination coefficient is derived from the regression analysis of $\beta_{\text{SRC}}$ and the parameters of the test samples such as the axial compression ratio, steel ratio and slenderness ratio.

(3) Based on the skeleton curve of SRC components and the related research on the classification of the performance levels, damage indexes of 0.03, 0.40 and 1.0 at the three performance levels, i.e., basic operation, repairable and avoiding collapse are recommended for SRC members in this study.

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