Electric/magnetic flux tube on the background of magnetic/electric field

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It is argued that the phenomenon of a flux tube in quantum chromodynamics is closely connected with a spontaneously symmetry breakdown of gauge theory. It is shown that in the presence of a mass term in the SU(2) gauge theory the Nielsen-Olesen equations describe the flux tube surrounded by an external field.

I. INTRODUCTION

One of the more fascinating aspects of quantum chromodynamics (QCD) is a quark confinement. The phenomenon of the confinement of quarks is connected with a hypothesized flux tube filled with the color electric field and stretched between quark and antiquark. This situation is in contrast with the electron and positron pair where the force lines in the whole space are spread. The confinement is a non-perturbative effect, it means that it can not be derived using traditional theoretical methods of the quantum field theory, i.e. Feynman diagram technique. The main problem for the derivation of confinement from the first principles is the absence of quantization methods for such strongly nonlinear field theories as SU(3) gauge field. The lattice simulations confirm the point of view that in QCD there is such phenomenon as the flux tube filled with a chromoelectric field. Another confirmation of this point of view is a dual QCD [1] where the electric and magnetic fields switch places and a dual Meissner effect with respect to the chromoelectric force lines takes place. In this theory there is a famous Nielsen-Olesen flux tube solution [2] filled with longitudinal magnetic field which can imitate the flux tube stretched between quark and antiquark.

Evidently one of the most essential problems in the confinement phenomenon is the derivation of a field distribution for the flux tube from first principles. At first, similar problem was set up by Heisenberg [3]: he has calculated some characteristics of a quantized nonlinear spinor field. His basic idea is that the fundamental equation for the quantized nonlinear spinor field is the corresponding classical equation where the classical spinor field is replaced by an operator of the spinor field. This equation is applied for the derivation of an infinite equations set for Green’s functions which give us all information relevant for quantized spinor field.

The idea presented here is the same [4]: we assume that the fundamental equations in QCD is Yang-Mills equations where the gauge potential $A_\mu$ is replaced by an operator $\hat{A}_\mu^a$. Later, according to Heisenberg, we derive equations for field correlators that is similar to ideas proposed in Ref. [5]. In the first approximation we will assume that $\langle Q|f(A_\mu^a)|Q\rangle = f(\langle Q|A_\mu^a|Q\rangle)$ here $|Q\rangle$ is a quantum state. But this is not all: we assume that the non-linearity of the SU(2) gauge theory leads to symmetry breaking, i.e. to the appearance of some additional terms in the Yang-Mills equations. We presuppose that these terms is like to $(-m^2(a)A_\mu^a)$ (here the brackets $\langle\ldots\rangle$ are omitted). This assumption is similar to a Coleman-Weinberg mechanism [6] in $\lambda\phi^4$ theory. The essence of the Coleman-Weinberg mechanism is that the nonlinear terms like $\phi^4$ produce some corrections by such a way that a spontaneously spontaneously breakdown occurs. In the present letter we explicitly incorporate mass terms of some components of the gauge potential into the SU(2) Yang-Mills equations to describe dynamics of the flux tube gauge fields in the presence of an external field. In Ref. [7] similar idea is presented to derive an effective Abelian gauge theory of a modified SU(2) gluodynamics. Although the mass-generation mechanism is not well understood analytically at present we accept the mass generation as true in the beginning of our discussion without questioning its mechanism. We expect that the mass terms of gauge potential are dynamically induced by a non-perturbative effect of gluodynamics.

In the next section we will obtain equations set [8]-[11] which describes a cylindrically symmetric distribution of the SU(2) gauge field coupled with the Higgs scalar field. These equations contain the abelian Nielsen-Olesen flux tube $\phi_{M}$ equations with $f = v = 0$. One can hope that these equations can involve another interesting flux tube solutions with non-abelian gauge fields. In this note we do the first step in this direction: we show that these equations actually have solutions which can be interpreted as electric/magnetic flux tube on the background of magnetic/electric field. We hope that there is the next step on which we will obtain the more complicated flux tube solutions with longitudinal non-abelian chromoelectric field which is absent in the Nielsen-Olesen flux tube.

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Therefore the main goal of this letter is to show that spontaneously symmetry breakdown of the SU(2) gauge theory coupled with the Higgs scalar field leads to a variant of the SU(2) flux tube filled with electric/magnetic field surrounded by an external magnetic/electric field.

II. ANSATZ AND EQUATIONS

The equations for the SU(2) gauge potential $A_a^\mu$ are

$$D_\nu F_{a\mu} = e \epsilon^{abc} D_\mu \phi^b - m^2 (a) A_a^\mu, \quad (1)$$

$$D_\mu D_\mu \phi^a = - \lambda \phi^a (\phi^b \phi^b - \phi^2_\infty), \quad (2)$$

here $e$ and $\phi_\infty$ are some constants; $D_\mu (\ldots)^a = \partial_\mu (\ldots)^a + e \epsilon^{abc} A_b^\mu (\ldots)^c$ is the covariant derivative; $\phi^a$ is a scalar field which is by analogy with the Nielsen-Olesen flux tube. Let us remind that the term $m^2 (a) A_a^\mu$ is the consequence of the quantization (which leads to a gauge symmetry breakdown) and equations (1), (2) are some approximation for the calculation of the averaged gauge potential $\langle A_a^\mu \rangle$. We consider the cylindrically symmetric case and consequently ansatz for the gauge potential we choose in the form

$$A_1^t (\rho) = \frac{f (\rho)}{e}, \quad (3)$$

$$A_2^z (\rho) = \frac{v (\rho)}{e}, \quad (4)$$

$$A_3^\varphi (\rho) = \frac{\rho w (\rho)}{e}, \quad (5)$$

$$\phi^1 (\rho) = \frac{\phi (\rho)}{e}, \quad (6)$$

here $\rho, z$ and $\varphi$ are the cylindrical coordinates. The corresponding equations are

$$f'' + \frac{f'}{\rho} = f \left( v^2 + w^2 \right) - m^2 f, \quad (7)$$

$$v'' + \frac{v'}{\rho} = v \left( - f^2 + w^2 + \phi^2 \right) - m^2 v, \quad (8)$$

$$w'' + \frac{w'}{\rho} - \frac{w}{\rho^2} = w \left( - f^2 + w^2 + \phi^2 \right), \quad (9)$$

$$\phi'' + \frac{\phi'}{\rho} = \phi \left( v^2 + w^2 \right) + \lambda \phi (\phi^2 - \phi^2_\infty), \quad (10)$$

here we have replaced $e\phi \to \phi$, $\lambda/e^2 \to \lambda$, $e\phi_\infty \to \phi_\infty$, $m(1) = m(2) = m$, $m(3) = 0$. The condition $m_3 = 0$ means that the U(1) gauge symmetry remains unbroken. Noticing that the system (7)-(10) is close to that of the spherically symmetric case in the Prasad-Sommerfeld limit, we look for the general solution for $f, v, \phi$ in the form

$$v (\rho) = K (\rho), \quad \phi (\rho) = K (\rho) \cosh \gamma, \quad f (\rho) = K (\rho) \sinh \gamma \quad (11)$$

here $\gamma$ is an arbitrary constant. Additionally we will consider the limit

$$\lambda \to 0, \text{ but } \lambda \phi^2_\infty = m^2. \quad (12)$$

After which we have the following equations

$$K'' + \frac{K'}{\rho} = K w^2 + K \left( K^2 - m^2 \right), \quad (13)$$

$$w'' + \frac{w'}{\rho} - \frac{w}{\rho^2} = 2 w K', \quad (14)$$

Let us introduce the dimensionless coordinate $x = \rho m \sqrt{2}$ and replace $K/m \to K$, $w/(m \sqrt{2}) \to w$

$$K'' + \frac{K'}{x} = K w^2 + \frac{1}{2} K \left( K^2 - m^2 \right), \quad (15)$$

$$w'' + \frac{w'}{x} - \frac{w}{x^2} = w K, \quad (16)$$
This is the famous Nielsen-Olesen equations for the flux tube and with special choice of \( \lambda = 1/2 \). These equations are good investigated, see for example Ref’s. \[10\]. It is well known that \(15\) and \(16\) equations are equivalent to the first order equations

\[
\frac{F'}{x} + \frac{s_{\pm}}{2} (K^2 - 1) = 0, \tag{17}
\]
\[
K' + s_{\pm} \frac{FK}{x} = 0 \tag{18}
\]

here \( F = xw \) and \( s_{\pm} = \pm 1 \). These equations are some special case of BPS strings which describe flux tube solutions in dual \( \text{SU}(N) \) theories \[13\].

III. MAGNETIC AND ELECTRIC FLUX TUBES ON THE BACKGROUND OF ELECTRIC AND MAGNETIC FIELDS

It is well known that equations \(17\)–\(18\) describe the Nielsen-Olesen flux tube. This solution at the origin is

\[
F = 1 - \frac{x^2}{4} + \cdots, \tag{19}
\]
\[
K = bx + \cdots, \tag{20}
\]
\[
s_- = -1 \tag{21}
\]

here \( b \) is an arbitrary constant. The asymptotical behavior is

\[
F(x) \bigg|_{x \to \infty} \to 0, \tag{22}
\]
\[
K(x) \bigg|_{x \to \infty} \to 1. \tag{23}
\]

This solution describes the magnetic field localized in a tube. Both functions are presented on Fig. 1. The flux of magnetic field is quantized which means that the solution exists only for a special choice of \( b \approx 0.6031985 \).

![Fig. 1: The first case. The functions \( F(x) \) and \( K(x) \).](image)

![Fig. 2: The first case. The electric \( E_z \) and magnetic \( H_z \) fields.](image)

But our interpretation of this solution for the \( \text{SU}(2) \) gauge theory with symmetry breaking is different. In our case
we have the following color fields

\[ E_z^3 = F_{t_z}^3 = e e^{3 b_c A_t^b A_c^e} = \frac{f v}{e} = \frac{K^2 \sinh \gamma}{e}, \]  
\[ H_z^3 = -\rho F^{3 \rho \varphi} = -\frac{F'}{\rho}. \]  
\[ H_\varphi^2 = F^{2 \rho z} = \partial _\rho A^2_z = \frac{v'}{e} = \frac{K'}{e} \]  
\[ E_\varphi^2 = F^{2 t \varphi} = -A_t^1 A_\varphi^3 = -\rho f w = -\rho \omega K \sinh \gamma, \]  
\[ E_\rho^1 = F_{\rho z}^1 = \partial _\rho A^1_\rho = \frac{f'}{e} = \frac{K' \sinh \gamma}{e}, \]  
\[ H_\rho^1 = \rho F^{1z \varphi} = v w = Kw. \]  

For us interesting is only \( E_z \) and \( H_z \) fields which are presented on Fig. 2.

The second type of solution has the following form at the origin

\[ F = \frac{1}{4} (1 - K_0^2) x^2 + \ldots, \]  
\[ K = K_0 - \frac{1}{8} K_0 \left( 1 - K_0^2 \right) x^2 + \ldots, \]  
\[ s_+ = +1 \]  

here \( K_0^2 < 1 \) is an arbitrary constant. The solution has the following asymptotical behavior

\[ F(x) \bigg|_{x \to \infty} \to \frac{x^2}{4}, \]  
\[ K(x) \bigg|_{x \to \infty} \to K_\infty e^{-\frac{x^2}{4}}. \]

here \( K_\infty \) is some constant. In contrast with the Nielsen-Olesen flux tube this solution describes a flux of electric field on the background of the magnetic field. The solution exists for arbitrary \( K_0^2 < 1 \) and this means that the flux of electric field is non-quantized. The typical solution is presented on Fig. 3. On Fig. 4 the electric and magnetic fields are plotted.

We see that in both cases one field (electric/magnetic) pushes out another one (magnetic/electric).

**FIG. 3:** The second case. The functions \( F(x) \) and \( K(x) \).

**FIG. 4:** The second case. The electric \( E_z \) and magnetic \( H_z \) fields.

**IV. DISCUSSION AND CONCLUSIONS**

Our interpretation of presented solutions is the following: we have obtained the flux tube filled with the longitudinal electric/magnetic field on the background of the longitudinal external magnetic/electric field. The word "external"
demands an explanation. The non-Abelian gauge theories are nonlinear and consequently an external field must be a part of the solution of Yang-Mills equations which describes simultaneously the flux tube and external fields. Thus presented solutions is a nonlinear superposition of the flux tube and external fields.

We have shown that spontaneously symmetry breakdown probably is an essential ingredient of a quantum gauge theory leading to a flux tube field distribution. The question about the mechanism of this phenomenon is very complicated. The origin of symmetry breakdown can be connected with the non-linearity of non-Abelian theories, ghost condensation [11] or maybe even with properties of an algebra of field operators [12].

We have mentioned above that every electric/magnetic field in the flux tube pushes out the external magnetic/electric field. It can be connected with the fact that we group together $A^a_t, A^a_z$ components of the gauge potential and the scalar field $\phi$. From the ‘t Hooft-Polyakov monopole and Nielsen-Olesen flux tube we know that the scalar field pushes out the gauge fields. Thus in our case it may be that the separation of the flux tube and external fields is connected with the bundle (11) of some gauge potential components and scalar field. Probably that the breakdown of the bundle (11) will lead to a flux tube without any external gauge field and it should be the next step of the investigation in this direction.

In closing I want to underline that this consideration is not pure classical one as we add the mass term which is connected with the quantization of such non-linear theory as the SU(2) gauge theory.

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[1] M. Baker, James S. Ball and F. Zachariasen, Phys. Rep., 209, 73 (1991).
[2] H.B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
[3] W. Heisenberg, Introduction to the unified field theory of elementary particles, Max - Planck - Institut für Physik und Astrophysik, Interscience Publishers London, New York, Sydney, 1966; W. Heisenberg, Nachr. Akad. Wiss. Göttingen, N8, 111(1953); W. Heisenberg, Zs. Naturforsch., 9a, 292(1954); W. Heisenberg, F. Kortel and H. Mütter, Zs. Naturforsch., 10a, 425(1955); W. Heisenberg, Zs. für Phys., 144, 1 (1956); P. Askali and W. Heisenberg, Zs. Naturforsch., 12a, 177(1957); W. Heisenberg, Nucl. Phys., 4, 532(1957); W. Heisenberg, Rev. Mod. Phys., 29, 269(1957).
[4] V. Dzhunushaliev and D. Singleton, Phys. Rev., D65, 125007 (2002); V. Dzhunushaliev and D. Singleton, Mod. Phys. Lett., A18, 955(2003); V. Dzhunushaliev and D. Singleton, “Effective ‘t Hooft-Polyakov monopoles from pure SU(3) gauge theory”, in preparation.
[5] A. Di Giacomo, H. G. Dosch, V. I. Shevchenko, Yu. A. Simonov, “Field correlators in QCD. Theory and applications”, hep-ph/0007229, Yu. A. Simonov, “Selfcoupled equations for the field correlators”, hep-th/9712250.
[6] S. Coleman and E. Weinberg, Phys. Rev., D7, 1888 (1973).
[7] S. Deguchi and Y. Kokubo, "Abelian projection of massive SU(2) gluodynamics - towards color confinement“, hep-th/0302141.
[8] Y. N. Obukhov, Int. J. Theor. Phys. 37, 1455 (1998).
[9] M.K. Prasad and C.M. Sommerfeld, Phys. Rev. Lett., 35, 760 (1975).
[10] H.J. de Vega and F.A. Schaposnik, Phys. Rev. D14, 1100 (1976); L. Jacobs and C. Rebbi, Phys. Rev. B319, 4486 (1979); E.I. Weinberg, Phys. Rev. D19, 3008 (1979); C.H. Taubes, Commun. Math. Phys. 72, 277 (1980).
[11] D. Dudal and H. Verschelde, “On ghost condensation and Abelian dominance in the Maximal Abelian Gauge”, hep-th/0209025, V.E.R. Lemes, M.S. Sarandy, and S.P. Sorella, “Ghost number dynamical symmetry breaking in Yang-Mills theories in the maximal Abelian gauge”, hep-th/0206251.
[12] V. Dzhunushaliev, Found. Phys. Lett., 16, 265 (2003).
[13] M. A. Kneipp and P. Brockill, Phys. Rev. D 64 (2001) 125012, arXiv:hep-th/0104171, M. A. Kneipp, hep-th/0211146, M. A. Kneipp, hep-th/0211049.