1. INTRODUCTION

After the success of the extraction of cosmological information from the cosmic microwave background (CMB) temperature anisotropies, much effort is being put toward building experiments for measuring the CMB polarized signal, which is about 1 order of magnitude smaller. There are many scientific goals of the CMB polarization studies. These include determining the reionization epoch from the large-scale polarization power spectrum, testing inflationary models by searching specific patterns due to the existence of the primordial gravitational waves, and improving the precision of cosmological parameters derived by CMB temperature anisotropies alone. Aiming at these objectives, the satellites WMAP (Kogut et al. 2003) and Planck, balloon-borne experiments like ARCHEOPS,1 BOOMERANG (Montroy et al. 2003), and MAXIPOL (Johnson et al. 2003), and ground-based instruments like AMiBA (Lo et al. 2001) and DASI (Leitch et al. 2002) have observed or will observe the polarized sky.

An important issue concerning the measurement of the primordial polarization power spectrum is that it is not the only polarized source in the sky. In addition to the contamination from the so-called foregrounds such as galactic dust (Benoit et al. 2004) and free-free and synchrotron emissions (de Oliveira-Costa et al. 2003), there are other sources of polarization due to large-scale structure (LSS) (reionization [Ng & Ng 1996], weak lensing [Zaldarriaga & Seljak 1998], etc.). In particular, when the CMB photons pass through the hot ionized gas trapped in the potential wells of galaxy clusters, a polarized signal is induced. The presence of the CMB temperature quadrupole induces a linear polarization in the scattered radiation. Sunyaev & Zel’dovich (1980) were the first to estimate the level of such a polarization induced by galaxy clusters. These authors pointed out that in addition to the primary CMB quadrupole, there are two other sources of temperature quadrupole seen by a cluster, quadrupoles due to (1) the transverse peculiar velocity of the cluster and (2) double scattering. Studying the induced CMB polarization due to clusters can open a whole new window for cosmology. Sunyaev and Zel’dovich proposed to use the polarization to estimate a cluster’s transverse velocity. Audit & Simmons (1999) gave a description of the effect due to the kinetic quadrupole, including the frequency dependence. Moreover, measuring the polarization toward distant clusters should in principle permit the observation of the evolution of the CMB quadrupole. The CMB quadrupole seen by clusters contains statistical information on the last scattering surface at the cluster position. Therefore, measuring the cluster polarization should help us to beat the cosmic variance (Kamionkowski & Loeb 1997; Cooray & Baumann 2003; Portsmouth 2004). Sazonov & Sunyaev (1999) revisited the detailed polarization signal due to the three sources of quadrupole given the constraint on the CMB quadrupole by COBE (Bennett et al. 1996), the electron density profiles of clusters and their transverse peculiar velocities. For the COBE data, their sky-average polarization induced by the primordial quadrupole was found to be 3.1 τ μK, with τ the cluster optical depth. In addition, if clusters possess an intracluster magnetic field, Faraday rotation will affect the linearly polarized CMB. This effect was investigated by Takada et al. (2001), who found an amplitude of the order of a microgauss at low frequencies. Conversely, the effect of Faraday rotation on the CMB polarization was proposed to extract the information of the cluster magnetic field (Ohno et al. 2003) when the cluster electron density is obtained from the Sunyaev-Zel’dovich (SZ) effect (Sunyaev & Zel’dovich 1972), the scattering of CMB photons off free electrons, and X-ray emission of the cluster.

In this paper we concentrate on the case in which no magnetic field is present in the intracluster gas. We study the polarized signal due to LSS by calculating the polarized angular power spectrum. Several authors have done some research on the polarization due to ionized gas in clusters or at the reionization. Hu (2000) investigated the secondary polarization induced by several physical processes using an analytic method. For the polarization in galaxy clusters, the author considered a density modulation of the kinetic and primordial polarization sources similar to the Vishniac effect (Vishniac 1987). Liu et al. (2001) computed the polarized signal at reionization using N-body simulations in combination with an analytic description to model the gas distribution. More recently, Santos et al. (2003) analytically
investigated the contribution from ionized patches at reionization. They assumed the ionizing sources reside in the dark matter halos and used the dark matter correlations in linear approximation. Both latter papers deal with the polarization induced by the primordial CMB quadrupole since it typically dominates over the kinetic and double scattering contribution at scales of interest. Concerning galaxy clusters, Baumann et al. (2003) computed the polarization induced by primordial and kinetic quadrupole using a halo-clustering approach to describe fluctuations in the electron density. Lavaux et al. (2004) focused on the polarized signal due to the kinetic quadrupole and the double scattering by the use of hydrodynamic simulations. More recently, Amblard & White (2004) computed large-scale polarization maps using N-body simulations.

In the present work, we revisit the polarization signal induced by LSS. We use the analytical method developed in Liu et al. (2001), combined for the first time with state-of-the-art hydrodynamic simulations. More recently, Amblard & White (2004) computed large-scale polarization maps using N-body simulations.

Throughout this paper we assume a flat $\Lambda$CDM cosmology with cosmological parameters matching present observations: matter density $\Omega_m = 0.3$, cosmological constant density $\Omega_{\Lambda} = 0.7$, baryon density $\Omega_b = 0.044$, and Hubble parameter $h = 0.71$.

2. BACKGROUND

2.1. Analytic Formulae

The polarized CMB signal is usually described by two of the Stokes parameters $Q$ and $U$. If we consider a wave traveling in the $\hat{z}$-direction, $Q$ is the difference in intensity in the $\hat{x}$- and $\hat{y}$-directions, while $U$ is the difference in $(\hat{x} + \hat{y})/\sqrt{2}$ and $(\hat{x} - \hat{y})/\sqrt{2}$ directions. The circular polarization parameter, $V$, cannot be produced by scattering, so we do not discuss it further. Polarization calculations are more complicated than temperature calculations because the values for $Q$ and $U$ depend on the choice of coordinate system. Therefore, it is more convenient to decompose the polarization on the sky into a divergence-free component, the so-called $E$ mode, and a curl component, the so-called $B$ mode, both of which are coordinate-independent.

We are interested in computing the $E$- and $B$-mode polarization power spectra due to galaxy clusters and filaments. For this, we use the analytic formulae detailed in Liu et al. (2001), to which we refer the reader. In this work, the electron density fluctuation modulated quadrupole at any given scale $k$ and conformal time $\eta$ is

$$S^{(m)}(k, \eta) \equiv \int d^3p \delta_e(k - p, \eta) \Delta_{T^2}^{(m)}(p, \eta),$$

where $\delta_e$ is the electron density fluctuation, $\Delta_{T^2}^{(m)}$ is the primordial CMB quadrupole with angular momentum $m = 0, \pm 1$, and $\pm 2$, and $Q_{2}^{(m)}(\eta) \equiv \int d^3p \Delta_{T^2}^{(m)}(p, \eta)$. The approximation in the second line assumes that the first-order temperature quadrupole $\Delta_{T^2}^{(m)}$ is uncorrelated with $\delta_e$, which is valid below $\sim 10^2$ (the coherence scale of the primordial quadrupole). In other words, the dominant contributions to the modulated quadrupole come mainly from the CMB quadrupole at large scales and from the electron density fluctuations at small scales; see Hu (2000). The temperature anisotropy field produced by scalar mode perturbations has axial symmetry. Therefore, the quadrupole field is decomposed into the $m = 0$ component in the $p$-basis parallel to the axis of symmetry; i.e., the temperature anisotropy quadrupole is $\Delta_{T^2}^{(0)} Y_2^0(\beta, \alpha)$, where $\beta$ and $\alpha$ are the polar and azimuthal angles defining $\hat{\theta}$ in this basis. Using the addition theorem, we can project the component in the $p$-basis onto the $k$-basis (see Ng & Liu 1999).

$$\sum_m Y_m^{m*}(\hat{\theta}) Y_m^p(\hat{\phi}) = \sqrt{\frac{2m + 1}{4\pi}} Y_m^0(\beta, \alpha).$$

It then follows that

$$Q^{(m)}(\eta) = \frac{4\pi}{5} \int d^3p \Delta_{T^2}^{(0)}(p, \eta)Y_2^{m*}(\hat{p})Y_2^m(\hat{p}).$$

Provided the modulated quadrupole, $S$ (eq. [1]), the polarized power spectrum can be obtained by integrating the Boltzmann equation along the photon past light cone. Here we skip the detailed derivation given in Liu et al (2001). For this, we also refer the readers to Kamionkowski et al. (1997), Zaldarriaga & Seljak (1997), and Hu & White (1997). We begin by writing down the time integral solution of the perturbed Stokes parameters,

$$\Delta_{Q \pm iU}(k, \hat{n}, \eta_0) = \frac{6\pi}{5} \int_0^{\eta_0} d\eta \int d^3p \Delta_{T^2}^{(0)}(p, \eta) \times \sum_m \bar{Y}_2^m(\hat{n})S^{(m)}(k, \eta),$$

where $\hat{n}$ is the direction of photon propagation. The visibility function $g(\eta)$ is the probability that a photon had its last scattering at epoch $\eta$ and reaches the observer at the present time, $\eta_0$. It is defined as

$$g(\eta) \equiv -\frac{d^3 \hat{\Omega}_{\eta}}{d\eta} e^{\tau(\eta) - \tau(0)},$$

where $\tau(\eta) \equiv \int_0^\eta d\eta' a_n\sigma_T$ is the Thomson electron-scattering optical depth at time $\eta$. $\hat{\Omega}_{\eta}$ represents the source term of polarization. The term $\bar{Y}_2^m(\hat{n})$ is the spherical harmonic with spin $\pm 2$. The perturbations $\Delta_{Q \pm iU}$ have spin $\pm 2$ because $Q$ and $U$ transform as $(Q \pm iU)(\hat{n}) = e^{i2\theta}(Q \pm iU)(\hat{n})$ under a right-handed rotation $\phi$ around the $z$-axis. In order to make the rotationally invariant measure of the polarization field $E$ and $B$ modes (with spin 0), we introduce the spin raising and lowering operators $\partial^\ell$ and $\partial^\ell$ (Newman & Penrose 1966)

$$\partial^\ell f = - (\sin \theta)^\ell \left( \frac{\partial}{\partial \theta} + i \csc \theta \frac{\partial}{\partial \phi} \right) (\sin \theta)^{-\ell} f,$$

$$\partial^\ell f = - (\sin \theta)^{-\ell} \left( \frac{\partial}{\partial \theta} - i \csc \theta \frac{\partial}{\partial \phi} \right) (\sin \theta)^\ell f,$$

with $f$ a spin-$s$ field. The $E$- and $B$-mode perturbations are then defined as

$$\Delta_E \equiv - \frac{1}{2} \left( \partial^2 \Delta_{Q \pm iU} + \partial^2 \Delta_{Q \mp iU} \right),$$

$$\Delta_B \equiv - \frac{1}{2} \left( \partial^2 \Delta_{Q \pm iU} - \partial^2 \Delta_{Q \mp iU} \right).$$
Following Zaldarriaga & Seljak (1997), the power spectra of the E and B modes can be written as

$$C_{E,B}^{(E,B)}(\ell) \equiv \frac{1}{2\ell + 1} (\ell - 2)! \sum_{m} \int d^{3}k \left\{ |\Delta_{E,B}^{(m)}(k, \eta)\Delta_{E,B}^{(m)}(k)|^{2} \right\} ,$$

where $\Delta_{E,B}^{(m)}(k)$ can be extracted using

$$\Delta_{E,B}^{(m)}(k) = \int d\Omega \ Y_{l}^{m}(\theta, \phi) \Delta_{E,B}(k).$$

Combining equations (4), (6), (7), (8), and (9), the power spectra for the E and B modes are

$$C_{E,B}^{(E,B)}(\ell) \equiv (4\pi)^{2} \frac{9}{16} \frac{1}{(\ell - 2)!} \times \sum_{m} \int d^{3}k \left\{ |d\eta g(\eta)S^{(m)}(k, \eta)\Delta_{E,B}^{(m)}(kr)|^{2} \right\} .$$

The geometric factor $T_{E,B}^{(m)}(kr)$, with $r = c(\eta_{0} - \eta)$ and $c$ the speed of light, is given by the combination of the spherical Bessel functions. Here we skip the derivation (Liu et al. 2019) and just list the results of $T_{E,B}^{(m)}(kr)$ in Table 1. Note that the expression $T_{E,B}^{(m)}(kr)$ in the table are valid only in flat universes. The hyperspherical Bessel functions should be used in curved universes (Hu et al. 1998).

### 2.2. Simulation Details

From equation (10) we easily see that the polarization amplitude depends on the density of the electrons that scatter the CMB photons through $g(\eta)$ and $S^{(m)}$. We therefore expect that different gas phases contribute differently to the overall signal. One important step of our study is thus to quantify these contributions, using hydrodynamic N-body simulations to model the gas dynamics.

The simulations were generated with the public version of the Hydra code (Couchman et al. 1995; Pearce & Couchman 1997), which implements an adaptive particle-particle-particle-mesh (AP3M) algorithm (Couchman 1991) to calculate gravitational forces and smoothed particle hydrodynamics (SPH) (Monaghan 1992) to estimate hydrodynamic quantities. In the present study, we consider a nonradiative model; i.e., the gas component evolves under the action of gravity, viscous forces, and adiabatic expansion. The effects of nongravitational heating and radiative dissipation of energy will be addressed in a subsequent study.

We present results from three simulation runs of the ΛCDM cosmology adopted in this paper (see § 1). The initial density and velocity fields were constructed with 160^3 particles of both baryonic and dark matter, perturbed from a regular grid of fixed comoving size $L = 100 h^{-1}$ Mpc, using the Zel’dovich approximation. With this choice of parameters, the masses of the baryonic and dark matter were $m_{gas} = 2.6 \times 10^9 h^{-1} M_{\odot}$ and $m_{dark} = 2.1 \times 10^{10} h^{-1} M_{\odot}$, respectively. We assumed a matter power spectrum described by the CDM transfer function in Bardeen et al. (1986) (known as the BBKS transfer function for CDM models) with shape parameter, $\Gamma$, given by the formula in Sugiyama (1995). The normalization of the matter power spectrum in each simulation run was set such that the present-day rms matter fluctuations in spheres of 8 $h^{-1}$ Mpc radius, $\sigma_8$, were equal to 0.8, 0.9, and 1, respectively. We used the same realization of the power spectrum in each run to permit a direct comparison between forming structures in the three simulations.

The runs were started at redshift $z = 49$. The gravitational softening was fixed at 25 $h^{-1}$ kpc in physical units between redshifts 0 and 1 and held constant above this redshift to 50 $h^{-1}$ kpc in comoving coordinates.

### 3. METHOD

We study the relative contribution to the polarization power spectrum from clusters (hot, high-density gas) and filamentary structures (warm, low-density gas). We define the gas phases based on the baryon collapsed density and temperature. We call hot ionized gas the phase consisting of all gas particles with temperatures above the threshold $T_{th} = 10^5 K$. Gas above this temperature threshold is assumed fully ionized and is considered in the calculations to be the only gas scattering the CMB photons. Hot ionized gas particles with overdensities $\delta_i$ larger than the density contrast at collapse, $\Delta_c = 178\Omega_m(z)^{-0.53}$ (Eke et al. 1998), are considered to be part of the intracluster medium (ICM) high-density phase. Low-density ionized gas particles with overdensities ranging in the interval $5 < \delta_i < \Delta_c$ encompass a warm intergalactic medium (IGM) phase (see, e.g., Valageas et al. 2001), which we call “filamentary structures.” Figure 1 shows the overdensity-temperature distribution (phase space) of all gas particles in our $\sigma_8 = 0.9$ simulation run at redshifts $z = 0$ (top left panel), $z = 1$ (top right panel), $z = 3.1$ (bottom left panel), and $z = 6.1$ (bottom right panel). In each case the phase space was discretized in bins of size $dx = 0.1$ per logarithmic decade, which gives the gas fraction density (gas fraction per dex^3). The horizontal dashed lines represent the temperature threshold, $T_{th}$, above which the gas is considered to be ionized. The ICM and IGM phases are defined in each panel by the vertical dashed lines. We should note that our simulations do not include photoionization heating due to UV background radiation, which explains the significant fraction of very cold low-overdensity gas. In our present simulations, resolution together with a simplified physical model prevent us from properly describing this gas phase.
In order to calculate the $E$ and $B$ power spectra using equation (10), we need to evaluate the electron density, $\delta_e(k, \eta)$, and the primordial CMB quadrupole, $\Delta_{22}$, which appears in equation (3). We modify the publicly available code CMBFast (Zaldarriaga & Seljak 1997) to output the time evolution of the primordial CMB quadrupole at different $k$ modes. This is then integrated in $k$-space at each epoch to obtain $Q_2^m(\eta)$ in equation (1). The next step is to compute the visibility function $g(\eta)$ and the polarization source term in equation (1), for which the electron density fluctuations $\delta_e(k, \eta)$ are needed. The first quantity is obtained directly from our simulation runs by evaluating the ionization fraction at different redshifts. For the electron density fluctuations, we start by evaluating this in real space, $\delta_e(r, \eta)$, on a regular grid with $N_{\text{grid}}^3$ cells inside the comoving box, making use of the SPH formalism (Monaghan 1992). In this step we assume a primordial gas composition with 24% helium abundance. The computation of $\delta_e(k, \eta)$ follows by performing the Fourier transform of $\delta_e(r, \eta)$.

4. POWER SPECTRA OF POLARIZATION OF CLUSTERS AND FILAMENTS

We compute the $E$ and $B$ power spectra for filaments, galaxy clusters, and all ionized gas. Results obtained adopting a regular grid with $600^3$ cells (corresponding to a fixed cell separation of $0.17 \, h^{-1} \, \text{Mpc}$ in comoving coordinates) are shown in Figure 2 for the $\sigma_8 = 0.9$ case. The error bars ($1 \, \sigma$) account only for the error on the optical depth, obtained from our simulation. They do not contain the major source of uncertainty, which is due to
We illustrate the difference between cluster and filament polarized power spectra by plotting in Figure 3 the evolution of the visibility function (thick dashed line) and density perturbations at two typical scales, 3 and 10 h\(^{-1}\) Mpc for clusters and filaments, respectively. Recall from equation (10) that the power spectrum is the integral of the electron density weighted by the visibility function times the primordial quadrupole. The latter is practically constant down to \(z \sim 1\), below which it increases with time. At the typical cluster scale, the power does not exceed that of filaments, down to \(z \sim 1\), below which the quadrupole boosts the cluster contribution. At these redshifts, the visibility function from our simulation is relatively small. It peaks at \(z \sim 3.2\) and its FWHM is bounded by \(z \sim 1.1\) and 6.2. In this range of redshifts, the power from filaments is much larger than from clusters. As a consequence, most of the scattering producing the linear polarization occurs for \(z \gtrsim 1.2\) when the relative contribution of clusters is still small.

The power spectrum of polarization induced by total ionized particles (Fig. 2, solid curve) peaks at a scale of about \(\ell \sim 10,000\) with an amplitude much smaller than the primary E-mode polarization. We now compare the power spectrum of the secondary polarization and the power spectrum of the secondary temperature anisotropies due to thermal SZ effect (inverse Compton scattering of CMB photons off free electrons) for the ionized phase. We base our comparison on the results of da Silva et al. (2001). They estimated the SZ temperature power spectrum using similar nonradiative simulations. The amplitude of the thermal SZ temperature power spectrum is 5 orders of magnitude larger than the polarization signal (see Fig. 3 in da Silva et al. 2001). In Figure 4 of da Silva et al. (2001), the power spectrum of temperature anisotropies associated with clusters, in the nonradiative case, dominates that of filaments by almost a factor of 2, whereas the power spectrum of polarized signal is dominated by filaments. This is because the amplitude of the thermal SZ temperature anisotropy is an integral of the free electron density weighted by the gas temperature. Typically, the temperature in clusters is 100 times higher than the temperature in filaments. The secondary polarization signal, although weak, can provide complementary information to what we get from secondary temperature fluctuations, especially for the filaments.

5. DISCUSSION

This is the first time we are able to simultaneously investigate the polarization contribution from ICM and IGM gas phases using hydrodynamic simulations. Therefore, direct comparisons with previous studies, based either on analytic computations or on N-body simulations, are not straightforward especially for the filamentary structures but also for clusters. As an example, our angular power spectrum for clusters matches the one given by Santos et al. (2003), particularly below \(\ell \sim 1000\). However, they consider, in their halo model, structures within 100 K < \(T_{\text{vir}}\) < 10\(^4\) K, which we do not model here (see § 2.2). By nature our results from the ICM phase are easier to compare with those from Liu et al. (2001), which are based on N-body computations. Our
polarization power spectrum for clusters is in very good agreement with the relevant models (A and B) in Figure 4 of Liu et al. (2001). Finally, we find that the amplitude of the power spectrum induced by galaxy clusters, at its maximum, is consistent with the estimate of Sazonov & Sunyaev (1999) for the sky-averaged polarization: 
\[ \frac{1}{C_{28}^2 C_{22}^2} K_0 \approx 30 \text{nK}, \] 
with a typical optical depth for clusters \( \tau = 0.01 \). From our hydrodynamic simulation we find an average value of 24 nK.

As suggested by equation (10), the amplitude of the secondary polarized signal from clusters is expected to have a strong dependence on the amplitude of the density fluctuations, which is usually parameterized by \( \sigma_8 \). To estimate the effect of \( \sigma_8 \) on our results, we compare in Figure 4 the power spectra obtained with the three simulations: \( \sigma_8 = 0.8 \) (long-dashed line), 0.9 (solid line), and 1 (dashed line). Figure 4a shows the total power spectrum from all ionized particles. Figures 4b and 4c exhibit the power from the IGM and ICM, respectively. Assuming that the power spectra from clusters, filaments, and all ionized particles vary like \( C_{E,B} \propto \sigma_8^2 \), we get \( n = 3.2 - 4.0 \) for all ionized particles, 3.2 - 4.0 for filaments, and 3.5 - 4.6 for clusters. The amplitude of the polarization power spectrum is sensitive to \( \sigma_8 \); however, the value of the index \( n \) is smaller than that obtained for the thermal SZ power spectrum variation \( 6 \leq n \leq 9 \) (see, e.g., Zhang et al. 2002; Komatsu & Seljak 2002).

We compare our secondary polarization spectra (solid, dotted, and long-dashed lines in Fig. 2) with the primary \( E \) and \( B \) modes due to scalar and tensor mode perturbations (dashed and dot-dashed lines, respectively). The \( E \)-mode signal from clusters and filaments dominates the primary signal at \( \ell \gtrsim 5000 \). Below this scale, the secondary polarized signal is always small and therefore does not significantly contaminate the primary signal. Gravitational waves, i.e., tensor mode perturbations, induce primary \( B \)-mode polarization, the amplitude of which depends on the ratio of tensor to scalar quadrupole, \( r = C_T^2 / C_S^2 \), which is not well constrained at present. We compute the primary \( B \)-mode signal, with \( 0 < r < 0.9 \) within the upper limits (at 95% confidence level) set by WMAP+CBI+ACBAR+2dFGRS+Ly\( \alpha \) observations (Spergel et al. 2003) and the tensor spectral index \( n_T = -r/8 \), and compare it with our secondary \( B \)-mode signal. We conclude that within \( 0 < r < 0.9 \) the secondary signal does not prevent us from detecting gravitational wave induced polarization at scales \( \ell \lesssim 100 \).

Gravitational lensing by LLSs slightly modifies the primary \( E \)-mode power spectrum. Most noticeably it generates, through mode coupling, \( B \)-mode polarization out of pure \( E \)-mode signal (Benabed et al. 2001; Zaldarriaga & Seljak 1998). The gravitational lensing-induced signal, which peaks around \( \ell \sim 1000 \), is a major issue for primary \( B \)-mode measurement. We compare in Figure 2 the \( B \)-mode power spectrum of the polarized signal from gravitational lensing by LLSs to that associated with clusters and filaments. We find the latter is about 2 orders of magnitude smaller and it dominates at very small angular scales (\( \ell \gtrsim 10,000 \)). The polarized signal from clusters and filaments might be observed at larger angular scales if the lensing-induced \( B \) modes can be significantly cleaned by appropriate techniques as those proposed for example by Seljak & Hirata (2004). We
note that the dependence of lensing-induced polarization on \( \sigma_8 \) is different from that of clusters and filaments. An increase of \( \sigma_8 \) would enhance more the latter with respect to the former.

6. CONCLUSION

In this paper, we present the first computation of the \( E \) - and \( B \)-mode quadrupole-induced polarization power spectra of galaxy clusters and filaments using hydrodynamic large-scale structure (LSS) simulations.

We find that the power spectrum from filaments dominates the power from clusters by a factor of about 2 on small scales and by 1 order of magnitude on large scales. The secondary polarization signal can thus provide complementary information to those we get from secondary temperature fluctuations especially for filaments. Assuming the power spectra vary like \( \sigma_8^n \) we find \( n \) in the range 3.2–4.6, depending on the gas phase.

As expected, the secondary polarization power spectra dominate at small angular scales. Although this first computation of the polarization from filaments shows that the signal can be 1 order of magnitude larger on large scales, the secondary polarization is still subdominant. Therefore, it is not a major problem for the primordial \( B \)-mode detection.

The authors thank an anonymous referee for his/her comments. They also thank N. Sugiyama for fruitful discussions. G. C. L. acknowledges support by The National Science Council of Taiwan NSC92010P. A. D. S. acknowledges support from CMBnet EU TMR network and JSPS for partial fundings. N. A. acknowledges NAOJ for support. The authors further thank the NAOJ for hospitality during the finalization of the paper.

REFERENCES

Amblard, A., & White, M. 2004, preprint (astro-ph/0409063)
Audit, E., & Simmons, J. F. L. 1999, MNRAS, 305, L27
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Baumann, D., Cooray, A., & Kamionkowski, M. 2003, NewA, 8, 565
Benabed, K., Bernardcau, F., & van Waerbeke, L. 2001, Phys. Rev. D, 63, 043501
Bennett, C. L., et al. 1996, ApJ, 462, L49
Benoit, A., et al. 2004, A&A, 424, 517B
Cooray, A., & Baumann, D. 2003, Phys. Rev. D, 67, 063505
Couchman, H. M. P. 1991, ApJ, 368, L23
Couchman, H. M. P., Thomas, P. A., Pearce, F. R. 1995, ApJ, 452, 797
da Silva, A., et al. 2001, ApJL, 561, L14
de Oliveira-Costa, A., et al. 2003, Phys. Rev. D, 68, 083003
Eke, V. R., Navarro, J. F., & Frenk, C. S. 1998, ApJ, 503, 569
Hu, W. 2000, ApJ, 529, 12
Hu, W., Seljak, U., White, M., & Zaldarriaga, M. 1998, Phys. Rev. D, 57, 3290
Hu, W., & White, M. 1997, Phys. Rev. D, 56, 596
Johnson, B. R., et al. 2003, NewA Rev., 47, 1067
Kamionkowski, M., Kosowsky, A., & Stebbins, A. 1997, Phys. Rev. D, 55, 7368
Kamionkowski, M., & Loeb, A. 1997, Phys. Rev. D, 56, 4511
Kogut, A., et al. 2003, ApJS, 148, 161
Komatsu, E., & Seljak U. 2002, MNRAS, 336, 1256
Lavaux, G., Diego, J. M., Mathis, H., & Silk, J. 2004, MNRAS, 347, 729
Leitch, E. M., et al. 2002, Nature, 420, 763
Liu, G. C., Sugiyama, N., Benson, A. J., Lacey, C. G., & Nusser, A. 2001, ApJ, 561, 504

As expected, the secondary polarization power spectra dominate at small angular scales. Although this first computation of the polarization from filaments shows that the signal can be 1 order of magnitude larger on large scales, the secondary polarization is still subdominant. Therefore, it is not a major problem for the primordial \( B \)-mode detection.

The authors thank an anonymous referee for his/her comments. They also thank N. Sugiyama for fruitful discussions. G. C. L. acknowledges support by The National Science Council of Taiwan NSC92010P. A. D. S. acknowledges support from CMBnet EU TMR network and JSPS for partial fundings. N. A. acknowledges NAOJ for support. The authors further thank the NAOJ for hospitality during the finalization of the paper.

REFERENCES

Amblard, A., & White, M. 2004, preprint (astro-ph/0409063)
Audit, E., & Simmons, J. F. L. 1999, MNRAS, 305, L27
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Baumann, D., Cooray, A., & Kamionkowski, M. 2003, NewA, 8, 565
Benabed, K., Bernardcau, F., & van Waerbeke, L. 2001, Phys. Rev. D, 63, 043501
Bennett, C. L., et al. 1996, ApJ, 462, L49
Benoit, A., et al. 2004, A&A, 424, 517B
Cooray, A., & Baumann, D. 2003, Phys. Rev. D, 67, 063505
Couchman, H. M. P. 1991, ApJ, 368, L23
Couchman, H. M. P., Thomas, P. A., Pearce, F. R. 1995, ApJ, 452, 797
da Silva, A., et al. 2001, ApJL, 561, L14
de Oliveira-Costa, A., et al. 2003, Phys. Rev. D, 68, 083003
Eke, V. R., Navarro, J. F., & Frenk, C. S. 1998, ApJ, 503, 569
Hu, W. 2000, ApJ, 529, 12
Hu, W., Seljak, U., White, M., & Zaldarriaga, M. 1998, Phys. Rev. D, 57, 3290
Hu, W., & White, M. 1997, Phys. Rev. D, 56, 596
Johnson, B. R., et al. 2003, NewA Rev., 47, 1067
Kamionkowski, M., Kosowsky, A., & Stebbins, A. 1997, Phys. Rev. D, 55, 7368
Kamionkowski, M., & Loeb, A. 1997, Phys. Rev. D, 56, 4511
Kogut, A., et al. 2003, ApJS, 148, 161
Komatsu, E., & Seljak U. 2002, MNRAS, 336, 1256
Lavaux, G., Diego, J. M., Mathis, H., & Silk, J. 2004, MNRAS, 347, 729
Leitch, E. M., et al. 2002, Nature, 420, 763
Liu, G. C., Sugiyama, N., Benson, A. J., Lacey, C. G., & Nusser, A. 2001, ApJ, 561, 504