INSTABILITIES IN A SELF-GRAVITATING MAGNETIZED GAS DISK

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ABSTRACT

A linear stability analysis has been performed onto a self-gravitating magnetized gas disk bounded by external pressure. The resulting dispersion relation is fully explained by three kinds of instability: a Parker-type instability driven by self-gravity, usual Jeans gravitational instability and convection. In the direction parallel to the magnetic fields, the magnetic tension completely suppresses the convection. If the adiabatic index $\gamma$ is less than a certain critical value, the perturbations trigger the Parker as well as the Jeans instability in the disk. Consequently, the growth rate curve has two maxima: one at small wavenumber due to a combination of the Parker and Jeans instabilities, and the other at somewhat larger wavenumber mostly due to the Parker instability. In the horizontal direction perpendicular to the fields, the convection makes the growth rate increase monotonically upto a limiting value as the perturbation wavenumber gets large. However, at small wavenumbers, the Jeans instability becomes effective and develops a peak in the growth rate curve. Depending on the system parameters, the maximum growth rate of the convection may or may not be higher than the peak due to the Jeans-Parker instability. Therefore, a cooperative action of the Jeans and Parker instabilities can have chances to over-ride the convection and may develop large scale structures of cylindrical shape in non-linear stage. In thick disks the cylinder is expected to align its axis perpendicular to the field, while in thin ones parallel to it.

1. Introduction

The Parker instability is one of the most important processes through which the Galactic disk may have generated large scale structures. When one suggests the instability as a candidate mechanism for making a large scale structure in the Galaxy, one should be careful about destructive roles of convection (Kim & Hong 1998). Since growth rate of the convective instability increases with decreasing wavelength of perturbation, interstellar medium (ISM) in the Galactic disk may get shredded into filamentary pieces by the convection before fully developing a structure (Asséo et al. 1978). In most of the previous studies on the Parker instability, externally given gravity was taken as a sole source of its driving force. In the present study we instead take the self-gravity as the driving force and ignore the external gravity from stars. Nagai et al. (1998) also took the self-gravity into account, but they used uniform magnetic fields and considered only isothermal case. In making large scale structures the self-gravity ought have played a constructive role by triggering the Jeans instability in the medium.

In this study we model the Galactic ISM as an infinite disk of magnetized gas under the influence of its own gravity, and carefully follow up the competition among the Jeans,
Parker, and convective instabilities. We first give adiabatic perturbations to the ISM in an isothermal equilibrium, and then perform a linear stability analysis onto the perturbed disk to derive the dispersion relation. The $z$-axis is taken perpendicular to the disk plane, $y$-axis in the plane along the direction of un-perturbed magnetic fields, and $x$-axis perpendicular to the fields. All the lengths are normalized to the scale height $H$ of the equilibrium disk. The Galactic halo is supposed to bind the ISM disk between $z=\pm \zeta_a$, or equivalently $\pm \zeta_a(\equiv z_a/H)$. The normalized wavenumber is denoted by $\nu_x$ and $\nu_y$ for the perturbations in the $x$- and $y$-directions, respectively. The growth rate $|\omega|$ is normalized by the free-fall time, and we denote the dimensionless rate by $|\Omega|$. The system is fully described by the disk thickness, $\zeta_a$, the ratio of magnetic to gas pressure, $\alpha$, the adiabatic index, $\gamma$, boundary conditions, and finally the perturbation wavenumbers, $\nu_x$ and $\nu_y$.

2. Dispersion Relations for Thick and Thin Disks

Aiming at the gravitational instability, we assigned an odd symmetry to the perturbation at $z=0$. Perturbations with even symmetry do not trigger the gravitational instability. For the case of a thick disk with $\zeta_a=5.0$, $\alpha=0.1$, $\gamma=0.8$, and the odd symmetry, we have shown, in Figure 1, how the growth rate varies with $\nu_x$ and $\nu_y$. This particular set of system parameters is chosen in such a way that we could see all the features of the Parker, Jeans and convective instabilities in the resulting dispersion relation.

If $\gamma < 1 + \alpha$, the convection arises in the system. In the $x$-direction, the growth rate of the convection increases with increasing wavenumber. This is the reason why the ridge height in Figure 1 slowly increases, as $\nu_x \to \infty$, upto the limiting value, $-\Omega_{\max}^2 = (2/\alpha) \left[ 1 + \alpha + \gamma - 2\sqrt{\gamma(1+\alpha)} \right] \tanh^2 \zeta_a$. However, in the $y$-direction, the convection gets completely suppressed by magnetic tension. If $1 - \alpha < \gamma < 1 + \alpha$, the magnetic Rayleigh-Taylor instability wouldn’t have a chance to develop. Since $\gamma < 1 - \alpha$ in our case, the magnetic Rayleigh-Taylor instability can be triggered and yields non-zero growth rates all
A rather sharp peak in the dispersion curve at \((\nu_x \approx 0.50, \nu_y = 0)\) is clearly due to the Jeans instability. One can see a similar peak at \((\nu_x = 0, \nu_y \approx 0.61)\). The latter is higher than the former, because it is due to a combined effect of the Jeans and Parker instabilities. The Parker instability driven by the self-gravity has brought about the third maximum at around \((\nu_x = 0, \nu_y \approx 1.4)\).

Thick disks have enough space for the magnetic fields to bend over so that matter can easily slide down. Therefore, the gravity gets an extra boost from the fields. This is the reason why the \(\nu_y\)-axis peak is higher than the \(\nu_x\)-axis one in Figure 1. In thin disks, however, there is not enough leeway for the fields to buckle up. Consequently, the fields tightly confined in narrow layer hinder, instead of boosting, the system not to develop the gravitational instability along the \(y\)-direction. Without being hindered, the system can still develop the Jeans instability along the \(x\)-direction. This makes the \(\nu_y\)-axis peak lower than the \(\nu_x\)-axis one in Figure 2. Because of the \(\tanh^2 \zeta\) factor, the gravity always over-rides the convection in thin disks. This is how Figure 2 becomes so different from Figure 1.

### 3. Competition between the Jeans-Parker and the Convection

In order to see under what conditions the Jeans instability assisted by the Parker may win the convection, we have compared their maximum growth rates with each other. As can be seen from the left panel of Figure 3, for a given \(\alpha\), one may find a critical value for \(\gamma\), above which the Jeans-Parker instability (solid line with open circles) dominates the system over the convection (dotted line). Three instability criteria are compared with each other in the right panel of the figure: the solid line is for the convection, the dashed one for the magnetic Rayleigh-Taylor instability, and the dotted line with open circles for the Jeans-Parker instability. In the domain below the dashed line both the magnetic Rayleigh-Taylor and convection may develop; while in the domain bounded by the dashed and solid lines the magnetic Rayleigh-Taylor may not occur (cf. Newcomb 1961; Parker 1967). Above the
Fig. 3. (left) Comparison of the maximum growth rate between the convection and the Jeans-Parker instability. The ordinate and abscissa are for the growth rate and the adiabatic index, respectively. When $\gamma = \gamma_{\text{crit}}(\alpha)$, the Jeans-Parker (solid) and the convection (dotted) will grow at the same rate. (right) Instability domains in the ($\alpha, \gamma$) plane. The solid and dashed lines are for the criteria of convection and magnetic Rayleigh-Taylor instability, respectively. The dotted line represents the $\alpha$-dependence of the critical adiabatic index.

dotted line with open circles the system forms a large scale structure via the Jeans-Parker instability.

4. Conclusion

The linear stability analysis has marked out in the system parameter space those domains where the magnetized gas disk under self-gravity would become unstable against the perturbations of large wavelength. If the disk is thick, the Parker instability could assist the gravity to generate large scale structures of cylindrical shape. Because undular perturbation should be applied to the magnetic fields to trigger the Parker instability, the structure formed through the Jeans-Parker instability tends to align its axis perpendicular to the magnetic fields. In thin disks the undular perturbation ought be of short wavelength, and the resulting strong magnetic tension wouldn’t allow any structures to form perpendicular to the magnetic fields. Consequently, if the disk is thin, the self-gravity would drive the system to develop large scale structures along the field direction.

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