We show in this work the existence of gravity-free thermal convection in a 2D granular gas, and describe its peculiar properties. The gas is enclosed in a rectangular region with dissipative sidewalls but no thermal gradient from external energy sources. Strikingly, and contrary to what would be expected from previous knowledge, thermal convection appears, due in this case to the 2D thermal gradient coming from wall-particle inelastic collisions and thermal walls. Our results are obtained by means of an event driven algorithm for inelastic hard disks.

Granular dynamics has been an interesting test ground in the last decades for non-equilibrium statistical mechanics and complex fluid mechanics [1, 2]. We know from previous works that much of the phenomenology observed in molecular gases and condensed matter [3] arises in granular matter as well, but in general with added complexity. Phenomena like jamming [4–7], crystallization [8–10], glass transitions [11, 12], fluid flow and convection [13, 14], memory effects [15, 16] etc., appear also in granular matter systems. But, furthermore, there is also a rich phenomenology which is intrinsic to granular media, such as clustering instabilities in low density systems [17, 18] or inelastic collapse in denser systems [19].

Granular convection and pattern formation in systems under gravity has been known for quite some time now [1, 20, 21]. It has been observed for instance in experiments with vertically oscillated particles [22, 23]. In the case of horizontally unbounded low density systems, previous works have provided complete descriptions of the different types of patterns that can be observed by means of computer simulations [23, 25] and theoretical studies [20, 27]. These works have shown the existence of a formal analogy with the classical Rayleigh-Bénard convection in molecular fluids [13]. However, experimental work shows, to a certain degree, a mismatch with the theory; for instance, in the threshold values of the buoyancy driven convection [28]. Part of the origin of this disagreement stems from the existence of another type of convection mechanism due to dissipation at the sidewalls. And of course, sidewalls are inherently present in granular dynamics experiments. In fact, the presence of sidewalls is known to have impact on hydrodynamic instabilities in general, even if they are physically inert [13, 25, 29]. More specifically, thermal convection induced by sidewall energy leaks is well known in molecular fluids [30]. For granular materials, a similar mechanism is triggered by wall-particle inelastic collisions, rather than a thermal leak [31, 33].

In any case, and to our knowledge, thermal convection in granular dynamics has always been detected in the presence of a gravitational field (see references above and their bibliographies) [35]. We now prove the existence of zero-gravity granular thermal convection [36]. Furthermore, as we will show, the intrinsic thermal gradient (induced by particle-particle inelastic collisions [33]) is not strictly necessary to produce convection, requiring only sidewalls energy dissipation (and the presence of thermal walls, that set the steady state temperature value). This result obviously has an impact in granular matter applications under microgravity or no gravity conditions (involving mining, storage and transportation of granular matter), and for instance in experiments planned by space agencies – our results could eventually be observed in a forthcoming VIP-GRAN experiment planned for the International Space Station (ISS) [21]–.

Let us describe our system in more detail. We deal with a 2D granular gas enclosed in a square-shaped system, as sketched in Figure 1. Particles are identical smooth hard disks with mass $m$ and diameter $\sigma$, used as mass and length units in this work. Particle density $n$ remains sufficiently low at all times in the system, so that collisions are always binary and instantaneous. Since we use the smooth hard particle collisional model, rotational degrees of freedom are neglected [20]. Coefficients of normal restitution $\alpha$ and $\alpha_w$ characterize the degree of inelasticity upon particle-particle and sidewall-particle collisions respectively [33]. A pair of thermal walls injects...
As seen in a previous work\[,33\] if a system like the one in Figure 1 is under the action of gravity, there is no mathematical solution to the corresponding hydrodynamic equations for a hydrostatic state, \(u = 0\). Therefore, the action of gravity combined with dissipative sidewalls leads automatically to convection (this happens in molecular fluids as well\[,30\]). But this is not so in the absence of gravity. In effect, the hydrodynamic equations corresponding to a system like the one pictured in Figure 1 for \(u = 0\) read \[37\]

\[
\begin{align*}
\frac{\partial p}{\partial x} &= 0, \quad \frac{\partial p}{\partial y} = 0, \\
\sqrt{T} \frac{\partial}{\partial y} \left( \sqrt{T} \frac{\partial T}{\partial y} \right) &= \frac{d \zeta^*(\alpha)}{2 \kappa^*(\alpha)},
\end{align*}
\]

which now admit a non-trivial solution. Hence, in general, the base state (the simplest hydrodynamic state) for \(g = 0\) is a hydrostatic one. In \[2\], \(p\) is the hydrostatic pressure, \(T\) is the granular temperature, \(\kappa^*(\alpha)\) is the transport coefficient associated to the heat flux and \(\zeta^*(\alpha)\) is the cooling rate due to particle-particle inelastic collisions\[,38\]. This means that an eventual gravity-free convection would appear only under certain conditions.

In order to analyze the problem, we perform event driven simulations of inelastic smooth hard disks\[,37\,39\]. We use \(\sigma, \tau = \sqrt{m \sigma^2}/T_0\), \(m\) and \(T_0\) as units of length, time, mass and temperature, respectively; where \(T_0\) is the thermal walls temperature. Throughout this work, we use the two-dimensional packing fraction value \(\bar{\phi} = N \pi \sigma^2/4 L^2 = 10^{-3}\), being \(N\) the total number of disks in the system. Therefore the mean free path is \(\lambda = (2 \sqrt{\pi} \bar{n} \sigma)^{-1} = 221.5567\), where \(\bar{n} = N/L^2\) is the system average particle density. The specifics of the simulation are discussed in the Supplementary Material file\[,37\].

Figure 2 displays flow velocity \((u)\) and granular temperature \((T)\) hydrodynamic fields for a system with \(L = 15 \lambda\) and \(\alpha = 0.9\), for three different \(\alpha_w\) values (\(\alpha_w = 1, 0.9\) and 0.6). As it can be seen, convection is initially absent for \(\alpha_w = 1\) (only remnant noise is observed) but develops for \(\alpha_w \leq \alpha_{w,th}(\alpha) < 1\), where \(\alpha_{w,th}(\alpha)\) is the convection critical value, that depends on \(\alpha\) and the other system parameters. Moreover, convection is strongly dependent on the parameter \(\alpha_w\), this being possibly due to the strong correlation between sidewalls dissipation and the temperature gradient\[,37\]. Notice that, in the presence of gravity, sidewall dissipation generates just one cell attached to each dissipative wall\[,38\], whereas now we find 2 convective cells per dissipative wall. This result makes sense since, for our geometry and with \(g = 0\), streamlines should have two perpendicular axes of specular symmetry, both passing through the center of the system.
Figure 3. Hydrodynamic fields $u$ and $T$ for $L = (15/8)\lambda$, with: (a) $\alpha = 1.0, \alpha_w = 1.0$; $u_{\text{max}} = 0.00450$, (b) $\alpha = 1.0, \alpha_w = 0.7$; $u_{\text{max}} = 0.02453$, (c) $\alpha = 0.9, \alpha_w = 0.7$; $u_{\text{max}} = 0.02254$, (d) $\alpha = 0.9, \alpha_w = 0.5$; $u_{\text{max}} = 0.03686$. Flow centers appear highlighted in clear blue.

Streamlines coming from dissipative sidewalls flow towards the center, then bending off the center onto the thermal walls. In fact, the orientation of the flow in the convection cells can be explained by looking at the behaviour in certain singular points, where streamlines in convection rolls meet. We mark these flow centers with circles in Figures 2 and 3. In this sense, we can see that in Figure 2 there is only one flow center, which coincides with the center of the system whereas in Figure 3 we find 3 flow centers.

In Figure 3 we show the results for a smaller system ($L = (15/8)\lambda$). As we see, the convection pattern is now very different. We now observe 4 cells next to dissipative walls plus 4 additional cells emerging near the system center, totalling 8 convection cells and three flow centers. Also notice that decreasing the value of $\alpha_w$ expands the area of the four central cells while decreasing the size of the cells next to the lateral walls. A reason for the appearance of the new central cells may be that the center is now hotter than in the bigger system in Figure 2.

This produces a new convection center from which streamlines flow out.

As we see, a 2D thermal gradient in the system corners – generated in this case by the combined action of thermal and dissipative walls – is sufficient to trigger convection (since for an elastic particle-particle collisions, $\alpha = 1.0$, there is convection, Figure 3b); i.e. natural convection can occur in a molecular gas even in the absence of gravity. Figures 2, 3 show a general trend of stronger convection for overall increasing inelasticity, this trend being more important for sidewall dissipation increase (decrease of $\alpha_w$).

A convenient way to analyze the convection intensity is by looking at the vorticity field, $\omega \equiv \partial_x u_y - \partial_y u_x$, as in Figure 4. Two distinctive convection patterns are found, as already mentioned, with either 4 or 8 cells with alternating vorticity sign, the 4 central cells disappearing for bigger systems. Notice that, in Figure 4b, vorticity nuclei are closer to the corners. As we commented above, an important aspect of gravity-free convection is that the
system allows for hydrostatic states and hence convection is not expected to appear in all cases. Figure [5] illustrates this point, where convection threshold lines are shown on the global vorticity surface $\langle |\omega| - |\omega_0| \rangle (\alpha, \alpha_w)$. We clearly detect non-convective (hydrostatic) regions. Moreover, the hydrostatic state tends to occupy wider regions in the parameter space as the system increases in size. Vorticity surface reveals clearly (Figure 5 b) that, although gravity-free convection is indeed stronger for more inelastic systems, the opposite trend is observed near the complete inelastic sidewall limit, where a significant region where vorticity drops for increasing inelasticity is observed.

In summary, in this work we have shown the existence of gravity-free granular convection. It is produced by the existence of sufficiently strong a 2D thermal gradient out of an initially hydrostatic base state at small gradients. Since 2D thermal gradients are actually characteristic of granular experiments (usually being produced at system boundary corners), this type of convection should be present in most zero gravity experiments. From a fundamental point of view, it is interesting to remark that molecular gases should also display gravity-free convection as long as they present analogous boundaries to those studied here. In fact, analogous gravity-free natural convection may be found in liquid droplets due to thermocapillary effects [40] [41] (the difference being here that now the 2D thermal gradient is generated by the boundary curvature). From a more general point view, our results constitute a rare example of natural thermal convection with no gravity, in a generic fluid system (the Grashof number [42] in the system we analyzed is strictly zero). Additionally, we think the results in this work may have an impact in other contexts such as horizontal systems (common in living matter for instance). Finally, the mechanism for the onset of gravity-free natural convection is outlined.

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Figure 5. Global average (i.e., averaged over all points in the system) of the vorticity absolute value, $\langle |\omega| \rangle - \langle |\omega_0| \rangle$ against $\alpha$ and $\alpha_w$. (a) For $L = 15\lambda$ and (b) For $L = (15/8)\lambda$. A diagram showing areas where convection/hydrostatic states is projected on the top of each figure. The curve separating both regions marks the $\alpha_w^{\nuB}(\alpha)$ critical values. The smaller framing box in (b) marks the parameter space represented in (a). $\langle |\omega_0| \rangle$ is the average base vorticity absolute value coming from noisy data at $\alpha = \alpha_w = 1$.

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[36] As noted in the reference by Aumaître et al. in our bibliography, even under low-gravity, convection has been detected just in a computational study with polydisperse granular materials, but not under strictly zero-gravity conditions.

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