Information Spreading in Interacting String Field Theory

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The commutator of string fields is considered in the context of light cone string field theory. It is shown that the commutator is in general non-vanishing outside the string light cone. This could have profound implications for our understanding of the localization of information in quantum gravity.

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1. Introduction

Conventional string theory provides us with a perturbative $S$–matrix. However, it is of interest to compute more general observable quantities, such as probability amplitudes at finite times. In this paper, we will be primarily interested in the causal properties of such amplitudes, and we will use string field theory to try and answer these questions.

In the known covariant formulations of quantized string field theory, the interactions are nonlocal in the center of mass coordinate $x^\mu$, and, as emphasized in [1], the initial value problem breaks down. Therefore, the theory cannot be canonically quantized in the conventional way. Fortunately, one is able to fix light cone gauge, where the interactions are local in the center of mass coordinate $x^+ = \tau$. This allows a conventional canonical quantization, and a second–quantized operator formulation exists [2–5], which allows one to perform the kind of calculations referred to in the preceding paragraph.

Our main concern will be the calculation of the commutator of two string fields on a flat spacetime background. In the free theory, this commutator vanishes outside the “string light cone” [6,7], but it will be shown in this paper that this is no longer the case when interactions are included.

The plan of the paper is as follows. We begin by reviewing some basic facts about second–quantized light cone string field theory [2–5]. Then we consider the commutator of two string fields in coordinate representation, and prove that it does not vanish identically outside the string light cone. To elucidate the behavior of this commutator, we calculate a matrix element of the commutator of tachyon component fields, and show that it is exponentially damped as the transverse separation increases. For certain kinematical configurations, the matrix element can be made large far outside the light cone. However, the rate of oscillation in light cone time of the matrix element is computed, and is shown to be large in these situations. The conclusion is that measurements could detect the information carried by a string state outside the light cone of the center of mass, but only if these measurements could be performed with resolution times smaller than the string scale. This result supports recent arguments by one of the authors concerning the nature of information in string theory [8], and could have profound implications for our understanding of localization of information in a theory of quantum gravity.

2. Calculation of Commutator

Let us introduce the light-cone coordinates

$$X^+ = (X^0 + X^{D-1})/\sqrt{2}, \quad X^- = (X^0 - X^{D-1})/\sqrt{2}$$

and parametrize the worldsheet of the string by the variables $\sigma$ and $\tau$. Light-cone gauge corresponds to fixing $X^+(\sigma) = x^+ = \tau$. In the following we will consider open bosonic strings; the generalization to closed strings and to superstrings should be similar. The transverse coordinates are expanded as

$$\vec{X}(\sigma) = \vec{x} + 2 \sum_{l=1}^{\infty} \vec{x}_l \cos(l\sigma).$$
In light-cone gauge, the string field is a physical observable and can be decomposed in terms of an infinite number of component fields. In the absence of interactions, the string field takes the form \[3–4\]

\[H = T(0) + \sum \int dp^+ \frac{d^{D-2}p}{(2\pi)^{D-1}} \sum \alpha_l \left[ A(p^+, \vec{p}, \{\vec{n}_l\}) e^{i(-\vec{p} \cdot \vec{x} + p^- - p^+ \tau)} f_{\{\vec{n}_l\}}(\vec{x}_l) + h.c. \right]. \tag{2.3}\]

Here the light-cone energy of a string state is given by

\[p^-(p^+, \vec{p}, \{\vec{n}_l\}) = \frac{\vec{p}^2 + 2 \sum_i l_i n_i^2 + m_0^2}{2p^+}, \tag{2.4}\]

where \(m_0^2\) is the mass squared of the ground state of the string. For bosonic strings, the ground state is a tachyon and \(m_0^2\) is negative. In order to effect the light cone quantization and calculate the commutator, we will regard \(m_0^2\) as a positive adjustable parameter [3,4]. Of course, this is inconsistent with Lorentz invariance for the bosonic string, but our results will be essentially unchanged in the superstring case where \(m_0^2 = 0\).

The \(f_{\{\vec{n}_l\}}(\vec{x}_l)\) are harmonic oscillator wave functions given by

\[f_{\{\vec{n}_l\}}(\vec{x}_l) = \prod_{i=1}^\infty \prod_{l=1}^{D-2} H_{\vec{n}_i}(x_i^i) e^{-l(x_i^i)^2/(4\pi)}, \tag{2.5}\]

with \(H_{\vec{n}_i}(x_i^i)\) a Hermite polynomial. The \(A\) operators obey the canonical commutation relations

\[[A(p^+, \vec{p}, \{\vec{n}_l\}), A^\dagger(p^{+\prime}, \vec{p}', \{\vec{n}_l\}')] = 2p^+ (2\pi)^{D-1} \delta(p^+ - p^{+\prime}) \delta^{D-2} (\vec{p} - \vec{p}') \delta(\{\vec{n}_l\}, \{\vec{n}_l\}'), \tag{2.6}\]

A component field is obtained from \(\Phi\) by multiplying by the appropriate wave function (2.3) and integrating over the normal mode coordinates \(\vec{x}_l\). For example, the tachyon field is given by

\[T(\tau, x^-, \vec{x}) = \int \frac{d^{D-2}p}{(2\pi)^{D-1}} \int dp^+ \frac{d^{D-2}p}{(2\pi)^{D-1}} [a_T(\vec{p}, p^+) e^{i[\vec{p} \cdot \vec{x} - p^- + p^+ \tau]} + a_T^\dagger(\vec{p}, p^+) e^{-i[\vec{p} \cdot \vec{x} - p^- + p^+ \tau]}], \tag{2.7}\]

where \(a_T(\vec{p}, p^+) = A(\vec{p}, p^+, \{\vec{0}\})\) and \(p^-\) is given by (2.4).

Now we want to include a cubic interaction. The light cone Hamiltonian becomes \(H = H_0 + H_3\), where \(H_0\) is the Hamiltonian for free string field theory and the cubic interaction term \(H_3\) is given by

\[H_3 = g \int \Phi_{\alpha_1}(\vec{X}_1(\sigma)) \Phi_{\alpha_2}(\vec{X}_2(\sigma)) \Phi_{\alpha_3}(\vec{X}_3(\sigma)) \delta \left( \sum_{r=1}^3 \alpha_r \right) \Delta(\vec{X}_1(\sigma) - \vec{X}_2(\sigma) - \vec{X}_3(\sigma)) \]

\[\times \mu(\alpha_1, \alpha_2, \alpha_3) \prod_{r=1}^3 d\alpha_r \prod_{r=1}^3 D\vec{X}_r(\sigma), \tag{2.8}\]
where $g$ is the open string coupling, $\alpha_r = 2p_r^+$, and the measure factor is

$$
\mu(\alpha_1, \alpha_2, \alpha_3) = (\det \Gamma)^{(D-2)/2} \exp\left(-\frac{\tau_0 m_0^2}{2} \sum_{r=1}^3 \frac{1}{\alpha_r}\right).
$$

(2.9)

The infinite-dimensional matrix $\Gamma$ is defined by

$$
\Gamma = \sum_{r=1}^3 A^{(r)} A^{(r)T},
$$

(2.10)

$$
A^{(1)}_{mn} = \delta_{mn},
$$

$$
A^{(2)}_{mn} = -\frac{2}{\pi} \sqrt{mn} (-1)^m \frac{(\beta + 1) \sin(m\pi \beta)}{n^2 - m^2 (\beta + 1)^2},
$$

$$
A^{(3)}_{mn} = -\frac{2}{\pi} \sqrt{mn} (-1)^{m+n} \frac{\beta \sin(m\pi \beta)}{n^2 - m^2 \beta^2},
$$

with $\beta = \alpha_3/\alpha_1$ and $\tau_0 = \sum_{r=1}^3 \alpha_r \log |\alpha_r|$. This interaction corresponds to the splitting of one string into two, as shown in the light-cone diagram fig. 1.

Now that interactions have been included, we wish to determine whether the commutator of two string fields vanishes when the arguments of the string fields lie outside the string light cone \[6,7\]. Suppose that

$$
[\Phi(x_1^+, x_1^-, \vec{X}_1(\sigma)), \Phi(x_2^+, x_2^-, \vec{X}_2(\sigma))] = 0
$$

(2.11)

when

$$
\frac{1}{\pi} \int d\sigma (X_1(\sigma) - X_2(\sigma))^2 < 0,
$$

(2.12)

where we are using the mostly minus convention for the spacetime metric. For fixed $X_2(\sigma)$, equation (2.11) can be regarded as a function of $X_1(\sigma)$, which vanishes in the entire region in which equation (2.12) is satisfied. Differentiating equation (2.11) with respect to $x_1^+$ and setting $x_1^+ = x_2^- = \tau$, one obtains

$$
[\Phi(\tau, x_1^-, \vec{X}_1(\sigma)), \Phi(\tau, x_2^-, \vec{X}_2(\sigma))] = 0
$$

(2.13)
in the region in which equation (2.12) holds. Here $\Phi$ denotes $\partial \Phi / \partial x^+$. At equal light cone times, equation (2.12) reduces to
\[
\frac{1}{\pi} \int d\sigma (\vec{X}_1(\sigma) - \vec{X}_2(\sigma))^2 > 0 .
\] (2.14)
Therefore, to prove that the string field commutator does not vanish identically outside the string light cone, it is sufficient to prove that equation (2.13) fails to hold when equation (2.14) is satisfied.

To proceed, note that we can use the Heisenberg equation of motion to express the field $\Phi$ as
\[
\Phi = i[H, \Phi] .
\] (2.15)
Consider now the matrix element
\[
\langle 0 | \Phi(p_3^+, \vec{X}_3(\sigma)) [\Phi(x_1^-, \vec{X}_1(\sigma_1)), \Phi(x_2^-, \vec{X}_2(\sigma_2))] | 0 \rangle
= i \langle 0 | \Phi(p_3^+, \vec{X}_3(\sigma_3)) [[H, \Phi(x_1^-, \vec{X}_1(\sigma_1))], \Phi(x_2^-, \vec{X}_2(\sigma_2))] | 0 \rangle ,
\] (2.16)
where all fields are evaluated at $\tau = 0$. Expanding $H = H_0 + H_3$, the terms involving $H_0$ all vanish by orthogonality. This is a reflection of the fact that the commutator does in fact vanish outside the string light cone in free string field theory [6]. The remaining terms can be expressed as
\[
\langle 0 | \Phi(p_3^+, \vec{X}_3(\sigma_3)) [\Phi(x_1^-, \vec{X}_1(\sigma_1)), \Phi(x_2^-, \vec{X}_2(\sigma_2))] | 0 \rangle = \frac{2ig(2\pi)^{(D-1)/2}}{(2\pi)^3p_3^+} \int_0^{\infty} \frac{dp_1^+}{2p_1^+} \int_0^{\infty} \frac{dp_2^+}{2p_2^+}
\left( e^{i(p_1^+x_1^- + p_2^+x_2^-)} V(2p_1^+, \vec{X}_1(\sigma_1); 2p_2^+, \vec{X}_2(\sigma_2); -2p_3^+, \vec{X}_3(\sigma_3))
- e^{i(x_1^- p_2^- - x_2^- p_1^-)} V(-2p_1^+, \vec{X}_1(\sigma_1); 2p_2^+, \vec{X}_2(\sigma_2); -2p_3^+, \vec{X}_3(\sigma_3))
- e^{i(x_1^- p_2^+ - x_2^- p_1^+)} V(2p_1^+, \vec{X}_1(\sigma_1); -2p_2^+, \vec{X}_2(\sigma_2); -2p_3^+, \vec{X}_3(\sigma_3)) \right)
\] (2.17)
where $V$ is the vertex factor obtained from (2.8). The three terms represent the three possible kinematical situations, in which the center of mass of string 3, 2, or 1 lies between the centers of mass of the other two, respectively. The $p_1^+$ integral may be performed by using the $\delta(\sum_{r=1}^3 \alpha_r)$ factor. Then one notes that the functional $\delta$-function in (2.8) contains a zero mode piece $\delta^{D-2}(\sum_{r=1}^3 \alpha_r, \vec{x}_r)$. Using one of these delta functions, say for the $x^1$ component, allows the integral over $p_2^+$ to be performed, and sets
\[
\alpha_2 = -\alpha_3 s ,
\]
\[
\alpha_1 = (s-1)\alpha_3 ,
\] (2.18)
where
\[
s = \frac{(x_3^1 - x_1^1)}{(x_2^1 - x_1^1)} .
\] (2.19)
The crucial point to notice is that one is left with a \((D - 3)\)-dimensional \(δ\)-function requiring the \(\vec{x}_r\) to be collinear, and that each term in (2.17) has support on a distinct ordering of the \(\vec{x}_r\) on the line connecting them. We therefore find that the commutator of two string fields is in general non–vanishing outside the string light-cone, \(\int dσ δX^μ(σ))^2 = 0\), when interactions are included. This also implies the commutator is non–vanishing when the centers of mass of the strings are spacelike separated. It should be stressed here that the non–vanishing of the commutator at spacelike separations has nothing to do with the fact the bosonic string has a tachyon. The same will be true in the tachyon–free superstring case.

Of more direct physical interest is the analogous calculation for the component fields. For simplicity, we will do the calculation for the tachyon field, though the generalization to an arbitrary mass eigenstate is straightforward. Following the previous line of reasoning, we compute the matrix element

\[
\langle 0|T(p_3^+, \vec{x}_3) [\bar{T}(x_1^-, \vec{x}_1), T(x_2^-, \vec{x}_2)]|0⟩ = \frac{i(2π)^{5(D−1)/2}g}{p_3^+(2π)^3} ∫ \frac{dp_1^+}{2p_1^+} ∫ \frac{dp_2^+}{2p_2^+} \left( e^{i(x_1^+ p_1^- + x_2^+ p_2^+)} V(2p_1^+, \vec{x}_1; 2p_2^+, \vec{x}_2; -2p_3^+, \vec{x}_3) \right)
\]

(2.20)

where, as before, all operators are at time \(τ = 0\). The vertex appearing in equation (2.20) is the Mandelstam vertex [2], which has the momentum space representation

\[
V(α_r, \vec{p}_r) = δ^{D−2} (\sum_{r=1}^{3} \vec{p}_r) \delta(\sum_{r=1}^{3} α_r) \exp\left(\frac{τ_0}{2} \sum_{r=1}^{3} \frac{\vec{p}_r^2 + m_r^2}{α_r}\right).
\]

(2.21)

Fourier transforming to coordinate representation, one obtains

\[
V(α_r, \vec{x}_r) = δ^{D−2} (\sum_{r=1}^{3} α_r \vec{x}_r) \delta(\sum_{r=1}^{3} α_r) \left(\frac{α_1 α_2 α_3}{8π^3 τ_0}\right)^{(D−2)/2} \exp\left(\frac{τ_0 m_0^2}{2} \sum_{r=1}^{3} \frac{1}{α_r} + \frac{α_1 α_2 α_3}{8τ_0} (\vec{x}_1 − \vec{x}_2 \vec{α}_3 - \vec{x}_1 − \vec{x}_3)^2\right).
\]

(2.22)

As was the case for the general string field vertex, equation (2.22) contains the factor \(δ^{D−2}(\sum_{r=1}^{3} α_r \vec{x}_r)\), so the result is non–vanishing only when the points \(\vec{x}_r\) are collinear. The off-shell vertex corresponds to, say, one tachyon splitting into two others such that all transverse centers of mass lie along the same line at equal times. In addition there is a Gaussian factor depending on the separation of the particles.
Consider a configuration in which \( \vec{x}_3 \) lies between \( \vec{x}_1 \) and \( \vec{x}_2 \), so that only the first term in equation (2.20) is non-zero. A simple calculation then gives

\[
\langle 0| T(p^+_3, \vec{x}_3) \left[ \hat{T}(x_1^-, \vec{x}_1), T(x_2^-, \vec{x}_2) \right]|0 \rangle = -ie^{ip_3^+(1-s)x_1^-} \frac{\delta^{D-3}(\vec{x}_3 - \vec{x}_1 - s(\vec{x}_2 - \vec{x}_1))}{8\sqrt{2\pi}(p^+_3)^2 s(1-s)|x_2^2 - x_1^2|} \\
\times \left( \frac{(2\pi)^2 s(1-s)}{\gamma(s)} \right)^{(D-2)/2} \exp \left( -\frac{s(1-s)}{2\gamma(s)}(\vec{x}_1 - \vec{x}_2)^2 - m_0^2 \gamma(s)(s^2 - s + 1) \right),
\]

where \( s \) is given in equation (2.19) and

\[
\gamma(s) = -\left[ s \log(s) + (1-s) \log(1-s) \right].
\]

Note that because of our choice of configuration, \( s \in [0,1] \), and that \( \gamma \) is non-negative. The matrix element (2.23) depends on the transverse displacement \( |\vec{x}_1 - \vec{x}_2| \) through a Gaussian factor with variance

\[
\sigma^2 = \frac{\gamma(s)}{s(1-s)}. \tag{2.25}
\]

One therefore finds that the matrix element has support over a distance of order \( \sigma^2 \) outside the light–cone of the center of mass. This spread can be made quite large. Indeed, for small \( s \), we have

\[
\lim_{s \to 0} \frac{\gamma(s)}{s(1-s)} \sim -\log(s), \tag{2.26}
\]

so for \( s \sim \exp\left(-|\vec{x}_1 - \vec{x}_2|^2\right) \), the matrix element is appreciable. This can always be achieved by choosing \( x_3^1 \) sufficiently close to \( x_1^1 \).

The question is whether one is able to resolve this information in practice. To get an estimate of how quickly the matrix element is oscillating in light cone time, we can calculate the matrix element

\[
\langle 0| T(p^+_3, \vec{x}_3) \left[ \hat{T}(x_1^-, \vec{x}_1), T(x_2^-, \vec{x}_2) \right]|0 \rangle, \tag{2.27}
\]

and divide by the matrix element (2.20). This is proportional to the frequency of oscillation. To do this carefully, we must multiply both (2.20) and (2.27) by a slowly varying function \( f \) and then integrate over \( \vec{x}_2, \ldots, \vec{x}_{D-2} \) to eliminate the \( \delta^{D-3}(\sum \alpha_r \vec{x}_r) \) factors. Performing this calculation leads to the following oscillation time scale

\[
\delta t \sim \frac{p^+_3 s}{(\vec{x}_2 - \vec{x}_1)^2 \log^{-2}(s) + (D - 2 + m_0^2)} \tag{2.28}
\]

valid for \( s \to 0 \). For the case of interest, \( s = \exp\left(-|\vec{x}_1 - \vec{x}_2|^2\right) \), this becomes

\[
\delta t \sim \frac{p^+_3 \exp\left(-|\vec{x}_1 - \vec{x}_2|^2\right)}{(\vec{x}_1 - \vec{x}_2)^2 + (D - 2 + m_0^2)}. \tag{2.29}
\]

The conclusion is that in order to observe the spread of information over more than a string length, one must perform measurements involving time scales much smaller than the string time.
3. Discussion

Two questions concerning the results of the previous section should be addressed. The first concerns the meaning of the result (2.23). The fact that the commutator is not identically zero for \((\vec{x}_1 - \vec{x}_2)^2\) spacelike does not signify a breakdown of causality in string theory, but is merely the result of trying to describe extended objects by local fields. Indeed, one should expect that much of the information carried by a string state is outside the light cone of the center of mass. This does not necessarily mean that signals can propagate faster than the speed of light.

To illustrate what is meant here, consider the case of large \(N\) QCD without matter. In the \(N \to \infty\) limit, the theory describes noninteracting, extended objects, namely glueballs. One could choose to write down a free field theory for the glueballs, using operators which create and annihilate glueballs, and these fields will commute when their arguments are spacelike separated. Once \(N\) is allowed to be finite, however, interactions must be included. If one continues to use the glueball fields, one will find that the interactions appear highly nonlocal, and fields will fail to commute when their arguments are spacelike separated. Despite all this, we know that causality is not violated, because the underlying theory is QCD, which is a causal, local quantum field theory.

Quantum electrodynamics provides another example. If one chooses to perform calculations in Coulomb gauge, one finds the commutator of the gauge field is non–vanishing at spacelike separations. This is because there are only two physical degrees of freedom (the fields transverse to the direction of propagation of a photon, for example) and the third field must be expressed as a nonlocal function of the other two, using the gauge constraint. When one computes the commutator of gauge invariant quantities such as the electromagnetic field strength, however, the apparent acausality disappears. It is the nonlocality in the description of the \(A\) field that causes this problem.

Both of the above examples differ from string field theory in that for both cases there exists an underlying causal, local field theory. The above examples simply show that if one chooses to describe extended objects in these theories by local fields, one must be careful in interpreting what will appear to be acausal results. String field theory is different because the only known formulations of string field theory are inherently nonlocal. This is not to say that there does not exist a local formulation of string field theory—this remains an open question. There is, however, a fairly large amount of evidence showing that such a local formulation does not exist \([1]\), and we believe that this is probably the case. In any event, because strings are extended objects, it is incorrect to conclude from the above results that causality is violated in string theory. It should also be noted that the non–vanishing of the commutator is a result of the interactions between strings. As was shown in \([3,7]\), free string fields do commute outside the string light cone. This is also evidenced by the fact that the matrix element (2.23) is proportional to the open string coupling \(g\).

The second point to be addressed is the question of Lorentz covariance of the expression (2.23). Although Lorentz covariance is not manifest, this does not mean that the matrix element (2.23) is not Lorentz covariant (for \(m_0^2 = -2\)). If it were, this would contradict the statement made previously that there is no violation of causality in string field theory. To see why, remember that in order to even define the light cone string field theory, one must fix a light cone gauge. The fields then depend explicitly on this choice of
coordinates. In order to leave light cone gauge, one must understand what the light cone fields are in terms of the degrees of freedom of a covariant string field theory. In general, the light cone fields will be complicated functionals of the new degrees of freedom, and certainly should not be expected to have simple Lorentz transformation properties. Said another way, the light cone string fields are defined as functionals of loops which have a particular orientation in spacetime. When one makes a Lorentz transformation, the loops on which the new string fields are defined are a different set of loops with a different orientation, and the Lorentz transformation properties must include terms which transform one set of loops to another. The transformation will be very complicated.

The pathologies of the bosonic string introduce some ambiguity into the interpretation of our results. We again stress that the calculation of the commutator of bosonic string fields presented here can be done in the tachyon free superstring case, and one will obtain a similar answer.

The information content of these matrix elements exhibits precisely the same type of diffusive behavior as was described in [8], which was argued to provide a possible resolution of the black hole information paradox. Under conditions relevant to strings propagating near a horizon, the spread of the matrix elements (2.20) can become arbitrarily large. As an explicit example, we can replace the field $T(\tau, x^2, \vec{x}_2)$ in equation (2.20) by $T(\tau, p^+_2, \vec{x}_2)$, and treat this field as representing a string which falls toward the horizon of a large black hole of mass $M$. If the string starts off with light cone momentum $p^+_1$ at Schwarzschild time $t = 0$, then after a time $t$ has elapsed, the momenta will be given by $p^+_2 \sim p^+ e^{-t/4M}$.

The field $T(\tau, x^1, \vec{x}_1)$ can be interpreted as a string close to the horizon at a fixed position, and $T(\tau, p^+_3, \vec{x}_3)$ can be interpreted as a test string which remains far from the horizon. Setting $p^+_3 = p^+_1$, the matrix element contains the same Gaussian factor as equation (2.23), with $s = e^{-t/4M}$. One thus finds that the spread of the Gaussian is given by $\sigma^2 \sim \frac{t}{4M}$. This diffusive behavior is the same as that found in [8].

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