Degradation of elastic properties of non-uniformly damaged composite laminates

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Abstract. Stiffness reduction model for laminates with non-uniformly distributed intralaminar cracks is presented and used to analyze the effect of non-uniformity and the accuracy of predictions based on uniform spacing. The values of the normalized crack opening displacement (COD) as affected by the presence of other cracks are used to calculate the axial modulus of cross-ply laminates with cracks. The COD is calculated using finite element method (FEM), considering two closest neighbors of the crack and using the smallest versus the average crack spacing ratio as non-uniformity parameter. Assuming uniform spacing the axial modulus reduction is overestimated. A “double-periodic” approach is presented to calculate the COD of a crack in a non-uniform case as the average of two solutions for periodic crack systems.

1. Introduction

Intralaminar cracking in cross-ply laminates is the first mode of damage. The crack plane is orthogonal to the laminate midplane, they run parallel to fibers in the layer and often cover the whole thickness and width of the layer.

Usually the extent of cracking (number of cracks and distance between them) is characterized by average crack spacing and crack density (cracks/mm). Most of the existing stiffness models, for example, [1-7] use this assumption. However, the crack distribution in the layer may be highly non-uniform. The reason is that the transverse failure strength along the longitudinal direction of the layer has statistical distribution. The discussion in this paper is focused on the possible inaccuracy introduced in laminate stiffness prediction by using assumption of uniform spacing. We will develop a theoretical framework and analyze two cases: a) the system of cracks is “non-interactive” in average (low crack density) but some cracks are close to each other and interact; b) the crack density is high and all cracks interact even “in average”.

There are only a few studies where the effect of non-uniformity is addressed, see for example [5]. In [5] hypothesis was introduced that for a non-uniformly cracked laminate, the deformation field in the “element” between two neighboring ply cracks separated by a distance \( l_k \) is identical to that in a uniformly cracked laminate where the crack spacing is \( l_k \). Then, for example, the axial strain of the whole Representative Volume Element (RVE) can be calculated by the “rule of mixtures” of average strains of “elements”. Good agreement of this approach with another high accuracy semi-analytical methodology applied to the RVE was demonstrated.

The average value of the axial stress change between two cracks is proportional to the average crack opening displacement COD and crack sliding displacements (CSD) normalized with far field stress [6]. Therefore the damaged laminate stiffness reduction depends on density of cracks and these parameters: average COD and CSD as expressed in the GLOB-LOC model [1, 7]. These two rather robust parameters depend on the normalized distance to neighboring cracks. Therefore for non-uniform crack distribution they are different for each individual crack. The values of COD and CSD in...
the commonly assumed uniform crack distribution case correspond to average spacing between cracks and are different than the calculated average over COD’s and CSD’s of all individual cracks. We will generalize the previously developed GLOB-LOC model [1] to non-uniform spacing case. Parametric analysis of the effect of geometrical non-uniformity in terms of COD and the laminate axial modulus will be performed for $[\theta_s/90_t]$ cross-ply laminates with cracks in 90-layers. To simplify stiffness calculations for an arbitrary non-uniform distribution, a routine allowing determination of COD for any crack as a sum of solutions for two periodic systems of cracks, will be formulated: one solution is for periodic system with spacing as on the “+” side of the crack and another one for a periodic system with spacing as on the “−” side of the crack.

2. Elastic constants of damaged symmetric cross-ply laminates with intralaminar cracks

2.1. Distance between cracks

We consider RVE of a layer (index omitted) with $M$ cracks as schematically shown in Fig 1. Crack with index $m$ has two neighbors located at different distances $l_{m-1}$ and $l_m$ from this crack. The RVE length is $L$, the average distance between cracks (spacing) is $l_{av}$, the crack density is $\rho$

$$L = \sum_{m=0}^{M-1} l_m, \quad l_{av} = \frac{L}{M}, \quad \rho = \frac{l}{l_{av}} \quad (1)$$

We will use the distance between cracks normalized with respect to the layer thickness $t$

$$l_{mn} = \frac{l_m}{t}, \quad m = 0, 1, ..., M - 1, \quad l_{avn} = \frac{l_{av}}{t} \quad (2)$$

We denote $u_{2an}^m, u_{1an}^m$ the average normalized COD and CSD of the $m$-th crack. Obviously

$$u_{2an}^m = u_{2an}^m \left( l_{(m-1)n}, l_{mn} \right), \quad u_{1an}^m = u_{1an}^m \left( l_{(m-1)n}, l_{mn} \right) \quad (3)$$

If $l_m > l_{m-1}$ the displacements on the “−” (right) side will be larger than on the “+” (left) side.

![Figure 1. Non-uniform distribution of M cracks in damaged layer shown in its local coordinate system](image)

2.2. Laminate’s stress-strain relationship with Valulenko-Kachanov tensor

We consider symmetric N-layer laminate. The k-th layer of the laminate has thickness $t_k$, fiber orientation angle with respect to the global x-axis $\theta_k$ and stiffness matrix $[Q]$ in the material symmetry axes, calculated from elastic constants $E_1, E_2, G_{12}, v_{12}$. The total thickness of the laminate, $h = \sum_{k=1}^{N} t_k$. Each layer can be damaged and the crack density in a layer, $\rho_k$ is calculated using (1) where the average distance between cracks $l_{av}^k$ is measured transverse to the fiber direction in the k-th layer. Dimensionless crack density $\rho_{kn}$ in the layer is introduced as
Using divergence theorem, it is easy to show [8] that for stress states that satisfy equilibrium equations the average stress applied to external boundary is equal to volume averaged stress. For laminated composites with applied average in-plane stress vector, $\{\sigma_0\}^{LAM}$ this equality can be written as

$$
\{\sigma_0\}^{LAM} = \{\sigma\}^0 = \frac{1}{N} \sum_{k=1}^{N} \{\sigma\}^a_k \frac{t_k}{h}
$$

In (5) the volume average is calculated expressing the integral over the laminate volume as a sum of volume integrals over $N$ layers. Upper index $a$ is used to indicate volume averages. The bar over stresses indicates global coordinates. In global coordinates the Hook's law in averaged form is the same as in any point

$$
\{\sigma\}^a_k = [\mathbf{Q}]_{\mathbf{a}} \{\varepsilon\}^a_k
$$

Substituting (6) in (5) and using the relationship (see [1])

\[
\begin{bmatrix}
\varepsilon_x^a \\
\varepsilon_y^a \\
\gamma_{xy}^a
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} + \begin{bmatrix}
\beta_x \\
\beta_y \\
2\beta_{xy}
\end{bmatrix}
\]

we obtain

$$
\{\sigma_0\}^{LAM} = [Q]^LAM \{\varepsilon\}^{LAM} + \sum_{k=1}^{N} [Q]^a_k \frac{t_k}{h} \{\beta\}_k
$$

In (8) $\{\beta\}$ is the Vakulenko-Kachanov tensor [8] written in Voigt notation. In Cartesian coordinates

$$
\beta_{ij} = \frac{1}{V} \int_{S_C} \frac{1}{2} (u_i n_j + u_j n_i) dS
$$

Integration in (9) involves the total surface $S_C$ of all cracks in the layer, $u_i$ are displacements of the points on the crack surface, $n_i$ is outer normal to the crack surface, $V$ is the volume of the layer. $\{\beta\}$ in (8) represents the effect on stiffness of the crack face displacements (COD and CSD). Since $\beta_{ij}$ and strain are tensors for both of them we have the same transformation expressions between local and global coordinates

$$
\{\beta\}_k = [T]^T \{\beta\}_k
$$

2.3. Incorporation of COD and CSD in Vakulenko-Kachanov tensor (local coordinates)

The cracked layer is considered in its local coordinates with indexes 1, 2 and 3 corresponding to longitudinal, transverse and thickness directions. Index denoting k-th layer is omitted to simplify explanation in this section. In local coordinates the normal vector to the two faces of crack surface has coordinates

$$
n_1 = n_3 = 0 \quad n_2 = \pm 1
$$

Using the definition (9) for $\beta_{ij}$ we see that it contains only two non-zero elements: $\beta_{12}$ and $\beta_{22}$

$$
\beta_{22} = \frac{1}{L_t} \sum_{m=1}^{M} \int_{-\frac{1}{2}}^{+\frac{1}{2}} [u_m^m(z) - u_m^{-m}(z)] dz \\
\beta_{12} = \frac{1}{L_t} \sum_{m=1}^{M} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \frac{1}{2} [u_m^m(z) - u_m^{-m}(z) ] dz
$$

In (12) $t$ is the cracked layer thickness, $u_m^m(z)$ and $u_m^{-m}(z)$ are sliding and opening displacements of the m-th crack, symbol + or – denotes the particular crack face as defined in 2.1. If the crack density is
high the crack face displacements depend on the distance between cracks. We define the average value of the opening and sliding on each crack surface

\[
\begin{align*}
    u_{1a}^{m+} &= \frac{1}{t} \int_{-l/2}^{l/2} [u_1(z) - u_1^{m+}(z)] dz \\
    u_{1a}^{m-} &= \frac{1}{t} \int_{-l/2}^{l/2} [u_1(z) - u_1^{m-}(z)] dz \\
    u_{2a}^{m+} &= \frac{1}{t} \int_{-l/2}^{l/2} [u_2(z) - u_2^{m+}(z)] dz \\
    u_{2a}^{m-} &= \frac{1}{t} \int_{-l/2}^{l/2} [u_2(z) - u_2^{m-}(z)] dz
\end{align*}
\] (13)

The average value of the average opening and the average sliding of both faces is

\[
\begin{align*}
    u_{1a}^m &= \frac{1}{2} (u_{1a}^{m+} + u_{1a}^{m-}) \\
    u_{2a}^m &= \frac{1}{2} (u_{2a}^{m+} + u_{2a}^{m-})
\end{align*}
\] (15)

Using (13), (14) and (15) the expressions for \( \beta_{12} \) and \( \beta_{22} \) are

\[
\begin{align*}
    \beta_{12} &= -\frac{1}{\rho} \sum_{m=1}^{M} u_{1a}^m (l_{(m-1)n}, l_{mn}) \\
    \beta_{22} &= -2 \rho u_{2a}
\end{align*}
\] (17)

We indicate in (16) that the displacements will be mostly affected by normalized distances to the two closest neighboring cracks. Expressions (16) can be rewritten in terms of crack density and average (over all cracks) displacements

\[
\begin{align*}
    \beta_{12} &= -\rho u_{1a} \\
    \beta_{22} &= -2 \rho u_{2a}
\end{align*}
\] (17)

We normalize \( u_{1a} \) and \( u_{2a} \) with respect to CLT stresses \( \sigma_{20}, \sigma_{120} \) and ply thickness \( t \)

\[
\begin{align*}
    u_{1an} &= \frac{G_{12}}{\rho \sigma_{120}} \frac{1}{M} \sum_{m=1}^{M} u_{1a}^m (l_{(m-1)n}, l_{mn}) \\
    u_{2an} &= \frac{E_2}{\rho \sigma_{20}} \frac{1}{M} \sum_{m=1}^{M} u_{2a}^m (l_{(m-1)n}, l_{mn})
\end{align*}
\] (19)

For normalized values the expressions for \( \beta_{ij} \) in (17) are slightly modified. They can be presented in the following matrix form

\[
\begin{align*}
\{ \beta \} &= \begin{bmatrix} 0 \\ \beta_{22} \\ 2\beta_{12} \end{bmatrix} = -\frac{\rho}{E_2} \begin{bmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{120} \end{bmatrix} [U] = 2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & u_{2an} & 0 \\ 0 & 0 & E_2 \end{bmatrix} \begin{bmatrix} u_{1an} \end{bmatrix}
\end{align*}
\] (20)

In (20) \( \rho_n \) is normalized crack density in the layer defined by (4).

2.4. Homogenized stiffness of the damaged laminate

Using the classical laminate theory (CLT), the stress \( \{ \sigma_0 \}_k \) in the \( k \)-th layer in local coordinates can be expressed through the applied laminate stress \( \{ \sigma_0 \}^{LAM} \) as follows

\[
\begin{align*}
\left[ \begin{array}{c} \sigma_{10} \\ \sigma_{20} \\ \sigma_{120} \end{array} \right]_k &= \left[ T \right]_k \left[ \sigma_0 \right]_k^{LAM} \\
\left[ \begin{array}{c} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{array} \right]_k &= \left[ T \right]_k \left[ \varepsilon_0 \right]_k^{LAM}
\end{align*}
\] (21)
Substituting (20) with (21) in (10) and further in (8) we obtain after arranging the result in the following form

$$\{\sigma_0\}^{LAM} = [Q]^{LAM} \{\epsilon\}^{LAM}$$  \hspace{1cm} (22)

with the following expressions for the stiffness and the compliance matrices of the damaged laminate

$$[Q]^{LAM} = \left( [I] + \sum_{k=1}^{N} \rho_{kn} \frac{t_k}{h} [P]_k [S]_0^{LAM} \right)^{-1} [Q]_0^{LAM}$$  \hspace{1cm} (23)

$$[S]^{LAM} = [S]_0^{LAM} \left( [I] + \sum_{k=1}^{N} \rho_{kn} \frac{t_k}{h} [P]_k [S]_0^{LAM} \right)$$  \hspace{1cm} (24)

The compliance matrix of the undamaged laminate is $[S]_0^{LAM} = \left([Q]_0^{LAM}\right)^{-1}$. In (23) and (24) the matrix-function $[P]_k$ for a layer with index $k$ is defined as

$$[P]_k = \frac{1}{E_2} [Q]_k [T]_k [U]_k [T]_k [Q]_k$$  \hspace{1cm} (25)

The involved matrices $[T]_k$ and $[Q]_k$ are defined according to CLT, upper index T denotes transposed matrix and bar over stiffness matrix indicates that it is written in global coordinates. The influence of cracks in k-th layer is represented by matrix $[U]_k$ given by (20). Index $k$ is added because the matrix depends on the constraint of surrounding layers and therefore $u_{2an}, u_{1an}$ may be different for different layers. Elements of this matrix $u_{2an}^{k}, u_{1an}^{k}$ are calculated, see (19), using normalized and averaged crack face opening (COD) and sliding displacements (CSD) of all cracks as affected by varying spacing between them.

### 2.5. Elastic modulus of balanced laminates with cracks in 90-layer

In case of balanced laminates with damage in 90-layers only, expressions for $[P]_k$ and for $[S]^{LAM}$ have been obtained calculating analytically the matrix products in (23) - (25). For example, the obtained expression for laminate normalized axial modulus is

$$\frac{E_x}{E_x^0} = \frac{1}{l + 2 \rho_{90n} \frac{t_{90}}{h} u_{2an}^{90} c}$$

$$c = \frac{E_2}{E_x^0} \left( \frac{l - \nu_{12}v_{xy}^0}{l - \nu_{12}v_{12}^0} \right)^2$$  \hspace{1cm} (26)

Subscript 90 is used to indicate 90-layer. The quantities with lower index x,y are laminate constants, quantities with additional upper index 0 are undamaged laminate constants and (19) has to be used to calculate $u_{2an}^{90}$. In the case of uniform crack distribution all CODs in (19) are equal. In this paper we analyze damaged laminates with axial modulus given by (26) and therefore only COD is of interest.
3. Results and discussions

3.1. Formulation of calculation examples

The effect of the non-uniform crack distribution on $u_{2an}^{90}$ was analyzed using FEM for damaged $[0_n/90_8]^S$ laminates (n=1,8) at fixed dimensionless crack density $\rho_{90n}$, see Fig 2 where the repeating “super-element” with two cracks is shown. To characterize the non-uniformity of the spatial distribution parameter $K$ is introduced

$$K = \frac{l_0}{l_{av}}$$

(27)

The same value of $K$ holds for normalized spacing. For uniform crack distribution $K = 1$. The average normalized crack spacing, $l_{av}$ was kept constant $l_{avn} = \frac{l}{2}(l_{0n} + l_{1n})$.

CF/EP and GF/EP composites with constants given in Table 1 were analyzed. All plies are considered to be transversally isotropic with $E_2 = E_3$, $G_{12} = G_{13}$ and $\nu_{23} = \nu_{13}$ (1 is the longitudinal direction and 2 and 3 are the transverse directions of the fiber). All results are presented in terms of normalized crack spacing and normalized crack density. Stiffness depends on layer thickness ratio, not on the absolute value of ply thickness.

The commercial code ABAQUS was used to find the solution for the ”super-element” in Fig. 2. 3D continuum elements (C3D8) 8-node linear brick were used. The same mesh with total number of 86400 elements was used in each FE model. To the right boundary $x=0$ of the model a given constant displacement in x-direction corresponding to 1% average strain was applied keeping a symmetry boundary condition at the left side. The top surface was free of tractions. On the front edge ($y=0$) and the edge $y=w$ ($w$ is the width of the laminate) coupling conditions were applied ($U_y$ =unknown constant). To cover large variation in elastic constants both CF/EP and GF/EP composites with constants given in Table 1 were analyzed. Studying the effect of non-uniform distribution the normalized spacing $l_{0n}$, see Fig. 2, was used as a parameter which was lower or equal to the average spacing.

| Material | $E_1$(GPa) | $E_2$(GPa) | $\nu_{12}$ | $\nu_{23}$ | $G_{12}$ (GPa) | $G_{23}$ (GPa) |
|----------|------------|------------|-------------|-------------|----------------|----------------|
| GF/EP    | 45         | 15         | 0.3         | 0.4         | 5              | 5.36           |
| CF/EP    | 150        | 10         | 0.3         | 0.4         | 5              | 3.57           |

Table 1. Composite elastic constants used in simulations.

Figure 2. ”Super-element” model for COD studies with non-uniformly cracked 90-layers.
3.2. COD parametric analysis at low crack density
Results are presented for average normalized spacing \( l_{\text{av}} = 10 \) (\( \rho_{90n} = 0.1 \)). The profiles of normalized crack face displacements \( u_{z_n}^{-}(z_n) \), \( u_{z_n}^{+}(z_n) \) along the thickness coordinate \( z_n = z \frac{2}{l_{90}} + 1 \) are shown in Fig. 3. The “+” face of the crack has smaller displacements than the “-“ face and the difference is larger when the \( l_{0n} \) is smaller than 1 (the neighboring crack to the left is very close). The neighbor to the “-“ face is at larger distance than the average spacing and therefore the displacement profile is almost unaffected. For the same geometry the CODs in CF composites are always significantly smaller than the CODs in GF. The CODs are significantly smaller when the relative thickness of the neighboring layer is higher. This effect is more pronounced for GF composite.

![Figure 3. COD profiles of cracks in [0/90]s laminate with normalized crack density \( \rho_{90n} = 0.1 \)](image)

Using crack face displacements the average normalized CODs, \( u_{2n}^{90} \) are calculated by numerical integration. The obtained dependence on the non-uniformity parameter \( K \) is shown in Fig. 4. The \( u_{2n}^{90} \) is larger if the spacing is uniform. However, the effect is negligible for \( K > 0.2 \) (\( l_0 > 2t_{90} \)).

![Figure 4. Effect of non-uniform spacing on COD of cracks in cross-ply laminates \( \rho_{90n} = 0.1 \)](image)
3.3. Approximate COD determination from periodic solutions

The average normalized COD, \( u_{2an}^{90} \), of a crack in a layer with non-uniform crack distribution can be found considering separately the average normalized COD of the “-” face of the crack and “+” of the crack.

\[
\frac{u_{2an}^{90}}{2} = \frac{1}{2} \left( u_{2an}^{-} + u_{2an}^{+} \right)
\]

(28)

In this section the following hypothesis will be inspected:

“The opening of a given crack face (“+” or “-“ in Fig. 2) depends only on the distance \( l_{0n} \) or \( l_{1n} \) to the closest neighboring crack on that side. It can be determined considering the corresponding region between these two cracks as a periodic element.”

According to this “double-periodic” approach \( u_{2an}^{90} \) is obtained as

\[
\frac{u_{2an}^{90}}{p} \approx u_{2an}^{p}, \quad u_{2an}^{p} = \frac{1}{2} \left( u_{2an}^{-p} + u_{2an}^{+p} \right)
\]

(29)

The two values \( u_{2an}^{p+}, u_{2an}^{p-} \) are solutions of the two periodic models. This hypothesis is equivalent to statement that in Fig. 2 symmetry conditions on the plane \( x = \pm l/2 \) can be applied. If the “double-periodic” approach is accurate enough, the \( u_{2an}^{90} \) for any crack location with respect to other cracks could be calculated from a master curve for uniform crack distribution. This curve, which is expression of \( u_{2an}^{90} \) as a function of crack spacing in a layer with uniformly distributed cracks, would be used twice to read the \( u_{2an}^{p+}, u_{2an}^{p-} \) values of the left and the right face of the crack.

In order to check the accuracy and validity of the “double-periodic” assumption, the \( u_{2an}^{90} \) for each value of non-uniformity parameter was calculated in two different ways: a) directly applying FEM to the non-uniform geometry; b) applying FEM two times and using (29).

From Fig. 5 where displacement profiles according to a) and b) are presented we conclude that the trends in the double-periodic approach are described correctly but the values of face displacements are not accurate. On the left face where the interaction is strongest the \( u_{2an}^{p+} \) is too small but on the right face, where the next crack is further away, \( u_{2an}^{p-} \) is too large.

**Figure 5.** Calculated COD profiles of cracks in [0/90]s laminate with normalized crack density \( \rho_{90n} = 0.1 \)
However, the $u_{2an}^p$ given by (29) is requested for stiffness predictions and not the value for each face separately. The values of $u_{2an}$ and $u_{2an}^p$ can be compared using results presented in Table 2 for all lay-ups, materials and non-uniformity parameter values. A very good agreement between values exists for all cases which validates the use of the “double-periodic” hypothesis.

Table 2. Average normalized COD of cracks from FEM and from “double-periodic” approach $\rho_{90n} = 0.1$

| $\rho_{90n}$ | [0/90]s GF/EP | [0/90]s CF/EP | [0/90]s CF/EP | [0/90]s GF/EP |
|--------------|----------------|----------------|----------------|----------------|
| 0.50         | 1.1027         | 0.6927         | 0.6941         | 0.5721         |
| 0.30         | 1.1027         | 0.6928         | 0.6928         | 0.5712         |
| 0.20         | 1.1039         | 0.6905         | 0.6771         | 0.5600         |
| 0.10         | 1.0510         | 0.5860         | 0.5910         | 0.4857         |
| 0.05         | 0.8588         | 0.4927         | 0.4841         | 0.3977         |

4. Elastic Modulus prediction and validation with FEM

The effect of the non-uniform crack distribution on axial modulus of cross-ply laminates is shown in Fig 6. All results are for the same normalized crack density $\rho_{90n} = 0.1$. The normalized axial modulus of the laminate is calculated in three different ways:

a) Calculating the average applied stress using FEM and then using definition of $E_x = \frac{\sigma_x}{\epsilon_x}$;

b) Applying (26) and using for $u_{2an}^{90}$ values of $u_{2an}$ obtained from FEM and presented in Table 2;

c) Applying (26) and using for $u_{2an}^{90}$ values of $u_{2an}^p$ obtained from “double-periodic” approach presented in Table 2;
The elastic modulus of the RVE with two non-equidistant cracks calculated directly from FEM coincides with the elastic modulus for this RVE calculated using (26) with $u_{2un}$ input from the same FEM solution (called “COD non-uniform” in Fig.6). Since (26) is an exact analytical expression this result was expected.

On the other hand, the “double-periodic” approach values practically coincide with the FEM values, proving the accuracy and potential of this approach for simulation of systems with multiple non-uniformly spaced cracks.

For the used crack density and all investigated materials and lay-ups the axial modulus reduction is the highest if cracks have uniform distribution. The effect of non-uniform distribution of internal cracks on laminate modulus can be neglected if the non-uniformity parameter $K > 0.2$.

Similar calculations as described above were performed for higher crack density $\rho_{90n} = 0.5$ Results are presented in Fig. 7.

**Figure 6.** Effect of non-uniform crack distribution on axial modulus of cross-ply laminates $\rho_{90n} = 0.1$

**Figure 7.** Effect of non-uniform crack distribution on axial modulus of cross-ply laminates, $\rho_{90n} = 0.5$
The effect of non-uniform distribution is even larger than at low crack density. In contrast to low crack density case, there is no plateau region in Fig. 7. The “double-periodic” approach at high crack density is still highly accurate.

5. Conclusions
Elastic properties of damaged symmetric cross-ply laminates can be predicted and the effect of non-uniform crack distribution can be analyzed using the derived exact expressions. However, the COD and the CSD in these expressions have to be calculated solving numerically the stress distribution problem for the RVE or using approximate solutions. The COD, CSD and the number of cracks per unit length in the layer are governing the laminate properties reduction.
This model was applied to cross-ply laminates with cracks in 90-layers. The effect of the non-uniform crack distribution on the damaged cross-ply laminate axial modulus was analyzed numerically. COD values needed as an input in the model were calculated using FEM and stiffness calculations were performed for GF/EP as well as CF/EP laminates with low and also with high crack density. The same trend was found for all crack densities and lay-ups: assuming uniform crack distribution the damaged laminate modulus is underestimated.
An approximate “double-periodic” approach was proposed stating that the COD of a crack with different distances to the closest neighbors can be calculated as the average of two solutions for equidistant cracks. It was shown numerically for cross-ply laminates that very accurate COD values for cracks with non-uniform spacing and elastic modulus values can be obtained using this approach.

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