Transformation Method of Exterior Orientation Angular Elements Obtained via Position and Orientation System Under Gauss-Kruger Projection Coordinate System

YUAN Xiuxiao,  ZHANG Xueping,  FU Jianhong
School of Remote Sensing and Information Engineering, Wuhan University, 129 Luoyu Road, Wuhan 430079, China

© Wuhan University and Springer-Verlag Berlin Heidelberg 2010

Abstract  Data obtained via airborne position and orientation system (POS) is in WGS 84 global geocentric reference frame, while the national coordinate reference system for topographic mapping in China is generally Gauss-Kruger projection coordinate system. Therefore, data obtained via a POS must be transformed to national coordinate system. Owing to the effects of earth curvature and meridian deviation, there are some errors in the process of angle transformation from roll, pitch, and heading \((\Phi, \Theta, \Psi)\) obtained directly via a POS to the attitude angles of images \((\phi, \omega, \kappa)\) needed in photogrammetry. On the basis of effect theories of earth curvature and meridian deviation on exterior orientation angular elements of images, a method using a compensation matrix to correct the transformation errors from attitude angles obtained via the POS to exterior orientation angular elements of images is proposed in this paper. Moreover, the rigorous formula of the compensation matrix is deduced. Two sets of actual data obtained via a POS AV 510, which are different in scale and terrain, are selected and used to perform experiments. The empirical results not only indicate that the compensation matrix proposed in this paper is correct and practical but also show that transformation accuracy of exterior orientation angular elements obtained via the POS based on compensation matrix is relevant to the selection of vertical axis (a projection of central meridian) of Gauss-Kruger projection coordinate system; the proper vertical axis should be the Gauss-Kruger projection of the central meridian of projection zone in which the survey area locates. However, the transformation accuracy of exterior orientation angular elements is irrelevant to the choice of origin of coordinate system; it is appropriate that the origin of coordinate system locates at the center point of the survey area. Moreover, transformation accuracy of exterior orientation angular elements achieved based on the compensation matrix deduced in this paper is higher than that obtained via the existing POS processing software.

Keywords  position and orientation system (POS); exterior orientation elements of image; earth curvature; meridian deviation; compensation matrix

CLC number  P231.5

Introduction

An integrated global positioning system and inertial navigation system (GPS/INS) are used in aerial remote sensing to obtain the position and attitude of a sensor, that is, the image orientation parameters are determined at the time of exposure. The aim is to
perform direct georeferencing (DG) for aerial photogrammetry without aerotriangulation with ground control points (GCPs), which will have an expansive application foreground in the future.\cite{1-4} In order to implement accurate direct georeferencing, the accuracy of exterior orientation elements of images must be high enough, especially these angular elements. In the process of transformation from attitude data obtained via a position and orientation system (POS) to exterior orientation angular elements of images, there are two main error sources: boresight misalignment and transformation errors between different coordinate systems.\cite{5}

For the calibration of boresight misalignment, the most popular method is test field calibration approach. First, it is necessary to set up a calibration field and take a set of especial images using an aerial camera mounted POS over the calibration field. Second, the exterior orientation angular elements are calculated by using the traditional bundle block adjustment. At last, comparing the exterior orientation angular elements obtained via a POS with that computed by traditional aerial triangulation in the calibration field, the boresight misalignment can be computed directly.\cite{6-8} Moreover, a novel method of self-eliminating POS systematic errors in a POS-supported bundle block adjustment without the use of a special calibration field is proposed to eliminate error.\cite{9}

As is well known, data obtained via a POS are in WGS 84 global geocentric reference frame, while photogrammetric products are often required in national coordinate system. Therefore, data obtained via a POS must be transformed to the national coordinate system. There are three options: (1) Transformation of the final products: the restitution is performed in a suitable Cartesian reference frame first, and the resulting model of the scene is completely transformed to the national coordinate system. (2) Transformation of artificial GCPs: “Virtual” GCPs are acquired from partial scene first, and then, an aerial triangulation is run. According to the estimated parameters, the scene can be restituted in the national coordinate system. (3) Transformation of the exterior orientation elements: only the exterior orientation elements are transformed, and the scene restitution is carried out directly in the national coordinate system.\cite{10} Within three transformation methods, the third one is the most practical. However, the effects of the earth curvature and map projection on transformation have to be taken into consideration.\cite{11}

Attitude angles obtained directly by IMU in a POS are Euler angles of the IMU body coordinate system in the navigation coordinate system, including roll, pitch, and heading ($\Phi, \Theta, \Psi$). Angular elements required in photogrammetry are rotation angles from the object coordinate system to the image space coordinate system, including phi, omega, and kappa ($\varphi, \omega, \kappa$). The transformation from the object coordinate system to the image coordinate system can be performed directly by using the orthogonal matrix constituted by ($\Phi, \Theta, \Psi$). On the other hand, the transformation also can be performed indirectly by using ($\Phi, \Theta, \Psi$) through a series of transformation matrices: the object coordinate system $\rightarrow$ the earth centered earth fixed coordinate system $\rightarrow$ the navigation coordinate system $\rightarrow$ the IMU body coordinate system $\rightarrow$ the camera body coordinate system $\rightarrow$ the image space coordinate system.\cite{12} According to the equivalent relationship between the two transformations mentioned above, attitude angles obtained via a POS can be transformed to exterior orientation angular elements at the time of exposure without aerial triangulation. If the object coordinate system is a Cartesian rectangular coordinate system, all the coordinate systems are rigorous three-dimensional spatial rectangular coordinate systems. Transformation from one coordinate system to another can be performed through an orthogonal rotation matrix rigorously. However, the coordinate reference systems for topographic mapping in China are often Gauss-Kruger projection coordinate system. The effects of the earth curvature and the meridian deviation on angle transformation have to be taken into account, and an additional compensation is required.\cite{13}

The objective of this paper is to research a transformation method of exterior orientation angular elements obtained via a POS in Gauss-Kruger projection coordinate system. The main content is about the error compensation in the transformation of angular elements in Gauss-Kruger projection coordinate system. The effects of the earth curvature and meridian deviation on attitude angles are detailed in Section 1.
At the base of these analyses, a rigorous compensation matrix is deduced. Afterward, the transformation equations of attitude angles obtained via a POS under Gauss-Kruger projection coordinate system are summarized in Section 2. At last, two sets of actual images with POS data are selected and used to demonstrate the validity and practicability of compensation model deduced in this paper. The empirical results are analyzed and discussed in Section 3.

1 Effect factors on angle transformation under Gauss-Kruger projection coordinate system

1.1 Effects of the curvature of the Earth on angle transformation

Generally, the planar coordinate system is under Gauss-Kruger projection, and the elevation system is based on the Yellow Sea elevation system in coordinate reference system for topographic mapping in China. Horizontal and vertical coordinate systems cannot compose a rigorous three-dimensional Cartesian rectangular coordinate system. When survey area is larger or accuracy demand is higher, the effects of the earth curvature must be properly considered.\(^{14}\) The ellipsoid can be replaced by a polyhedron of tangential planes, with each plane set up at the nadir point of the projection center. This together with the height related to the respective tangential plane for each image create a small individual Cartesian rectangular coordinate system. The correction needed for this replacement can remove the effect of ellipsoidal curvature.\(^{15}\) In the process of angle transformation from attitude angles \((\Phi, \Theta, \Psi)\) obtained via a POS to exterior orientation angular elements \((\phi, \omega, \kappa)\) needed in photogrammetry, the origin of the object coordinate system is often selected at one point \((B_0, L_0)\) in the survey area. Therefore, using the rigorous transformation between Cartesian rectangular coordinate systems, the tangential plane coordinate system at each projection center can be transformed to parallel with the tangential plane coordinate system at origin of coordinate system. In this way, the effects of earth curvature on angles can be removed.

The transformation between two points on ellipsoidal surface can be performed through the continuous rotations in the meridian and prime vertical (see Fig.1). According to the location relationship of points on ellipsoidal surface, from \(P_2(B, L)\) to \(P_1(0, 0)\), the tangential plane coordinate system is in the following rotation order: the first rotation: \(B\) around \(X_m\) axis; the second rotation: \(L\) around \(Y_n\) axis (after the first rotation). The combination of the rotations results in the following transformation matrix:

\[
R^{(0,0)}_{(B,L)} = R^{(0,0)}_{(B,L)} \cdot R^{(0,0)}_{(B,L)} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos L & -\sin L \\
0 & \sin L & \cos L
\end{bmatrix}
\begin{bmatrix}
\cos L & 0 & -\sin L \\
0 & 1 & 0 \\
\sin L & 0 & \cos L
\end{bmatrix}
\begin{bmatrix}
\cos L & 0 & -\sin L \\
-\sin L & \cos L & 0 \\
\cos L & \sin L & \cos \Delta L
\end{bmatrix}
\]

(1)

In order to express Eq.(2) conveniently, the following substitutions are made \(\Delta B = B - B_0, \Delta L = L - L_0\).

\[
R^{(0,0)}_{(B,L)} = R^{(0,0)}_{(B,L)} \cdot R^{(0,0)}_{(B,L)} =
\begin{bmatrix}
\cos L_0 & 0 & -\sin L_0 \\
-\sin L_0 & \cos L_0 & 0 \\
\cos L_0 & \sin L_0 & \cos L_0
\end{bmatrix}
\begin{bmatrix}
\cos L & 0 & - \sin L \\
- \sin L & \cos L & 0 \\
\cos L & \sin L & \cos L
\end{bmatrix}
\]

(2)
broad, $\Delta B$ can also be regarded as a small angle. Therefore, taking the small angle approximation \( \cos \Delta B = 1, \cos \Delta L = 1, \sin \Delta B = \Delta B, \sin \Delta L = \Delta L \), and neglecting the quadratic terms of $\Delta B, \Delta L$ in the expansion, the following linear form of Eq.(2) can be obtained:

$$
R_{(B, \Delta)}^{(L, B, \Delta)} = 
\begin{bmatrix}
1 & \Delta L \cdot \sin B & -\Delta L \cdot \cos B \\
-\Delta L \cdot \sin B & 1 & -\Delta B \\
\Delta L \cdot \cos B & \Delta B & 1 \\
\end{bmatrix}
$$

$$
= 
\begin{bmatrix}
1 & (L_0 - L) \cdot \sin B & -(L_0 - L) \cdot \cos B \\
-(L_0 - L) \cdot \sin B & 1 & (B - B_0) \\
(L_0 - L) \cdot \cos B & -(B - B_0) & 1 \\
\end{bmatrix}
$$

(3)

1.2 Effects of the meridian deviation on angle transformation

In Gauss-Kruger projection coordinate system, the central meridian of projection zone and the Equator are projected into perpendicular beelines, and they are regarded as $y$ axis and $x$ axis of the coordinate system, respectively. At the same time, all the other meridians in this projection zone become curves after projection, which distribute at $x$ axis symmetrically and are concave to $y$ axis. There will be angles between the central meridian and meridians of the projective points after projection, and they are meridian convergence of the projective points. In Fig.2, $p'$ is the projection of point $p$ on ellipsoidal surface; $p'N'$ is projection of meridian at point $p$; $p'n'$ is the tangent of $p'N'$ at point $p'$; and $p'\gamma'$ is the parallel of $y$ axis. The angle between $p'n'$ and $p'\gamma'$ is the meridian convergence of point $p'$, which is expressed by $\gamma$.\[16\]

$$
\gamma = \sin B \cdot l + \frac{1}{3} \sin B \cos^2 B \cdot (1 + 3\eta^2 + 2\eta^4) \cdot l^3 + \frac{1}{15} \sin B \cos^4 B \cdot (2 - \gamma') \cdot l^3 + O(l')
$$

(4)

where $t = \tan B, \eta' = e'' \cos^2 B, e''$ is the second eccentricity; $B$ is the latitude of projective point; and $l$ is the longitude difference between projective point and central meridian.

In Gauss-Kruger projection coordinate system, the scale factors of the horizontal and vertical directions at projective point are different because of the mapping length distortion. Moreover, the meridian deviation is affecting the orientation in respect to geographic orientation.\[13\] Gauss-Kruger projection is an orthomorphic projection, so there is no angle distortion in plane before and after projection. Nevertheless, there is a distortion of $\gamma$ in vertical direction due to the meridian deviation.\[10\] Therefore, in order to correct the effects of meridian deviation on angles, the coordinate system has to be rotated $\gamma$ around $Z_m$ axis.

The transformation matrix is as follows:

$$
R_\gamma = 
\begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(5)

In common topographic survey, meridian convergence can be computed approximately as follows:

$$
\gamma = l \cdot \sin B
$$

(6)

Meridian convergence $\gamma$ is a small angle. Taking the small angle approximation $\cos \gamma = 1, \sin \gamma = \gamma$, Eq.5 can be replaced by

$$
R_\gamma = 
\begin{bmatrix}
1 & -\gamma & 0 \\
\gamma & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} = 
\begin{bmatrix}
1 & -(L_{0g} - L) \sin B & 0 \\
(L_{0g} - L) \sin B & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(7)

where $L_{0g}$ is the central meridian longitude of projection zone; $L$ is longitude of projection point.

1.3 Compensation matrix of angle transformation

According to the analyses in Sections 1.1 and 1.2, the earth curvature and the meridian deviation can affect the angle transformation in Gauss-Kruger projection coordinate system. The earth curvature affects azimuth angle, while the meridian deviation affects zenith angle. Therefore, synthesizing these two effect factors, the correction of angle transformation obtained via a POS can be performed through Eqs.(2)
and (7). The compensation matrix of angle transformation in Gauss-Kruger projection coordinate system can be expressed as follows:

\[
R_m^m = R_{m_i}^{(b_i,c_i)} \cdot R_{c_i}^{(a_i)}
\]

\[
= \begin{bmatrix}
\cos L_0 & 0 & -\sin L_0 \\
-\sin B \sin L_0 & \cos B_0 & -\sin B_0 \cos L_0 \\
\cos B \sin L_0 & \sin B_0 & \cos B_0 \cos L_0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos L & -\sin B \sin L & \cos B \sin L \\
0 & \cos B & \sin B \\
-\sin L & -\sin B \cos L & \cos B \cos L
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -(1_{0gk} - L) \sin B & 0 \\
(1_{0gk} - L) \sin B & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(8)

If the range of survey area is not too large, Eq.(8) can be simplified according to the disposal principle of Eq.(2). The result of Eq.(8) is

\[
R_m^g = \begin{bmatrix}
1 & -(1_{0gk} - L) \sin B & 0 \\
(1_{0gk} - L) \sin B & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(9)

2 Transformation of angles obtained via a POS under Gauss-Kruger projection coordinate system

The transformation from attitude angles \((\Phi, \Theta, \Psi)\) obtained directly via a POS to exterior orientation angular elements \((\varphi, \omega, \kappa)\) required in photogrammetry can be performed indirectly through a series of transformation: the object coordinate system \((m)\) → the earth centered earth fixed coordinate system \((E)\) → the navigation coordinate system \((n)\) → the IMU body coordinate system \((b)\) → the camera body coordinate system \((c)\) → the image space coordinate system \((i)\).

The transformation is as follows:

\[
R_m^m(\varphi, \omega, \kappa) = R_m^n \cdot R_n^e \cdot R_c^e(\Psi, \Theta, \Phi) \cdot R_b^c \cdot R_i^c
\]

(10)

where \(R_i^c\) is the orthogonal transformation matrix from coordinate system \(i\) to coordinate system \(j\).

If the object coordinate system is a Cartesian rectangular coordinate system, Eq.(10) will be accurate rigorously. However, coordinate system for topographic mapping in China is under Gauss-Kruger projection coordinate system. For the above transformation, the effects of the earth curvature and the meridian deviation on angle transformation must be considered. According to the analyses in Section 1, Eq.(10) should be extended to the following one:

\[
R_m^n(\varphi, \omega, \kappa) = R_m^n \cdot R_n^e \cdot R_c^e(\Psi, \Theta, \Phi) \cdot R_b^c \cdot R_i^c
\]

(11)

where \(R_m^n\) is the transpose of matrix \(R_m^n\). The other parameters are the same as that in Eq.(10), the detailed models are shown in Reference [12].

Setting \(R_m^n(\omega, \varphi, \kappa) = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\) and adopting angular elements system of \(\varphi - \omega - \kappa\), the exterior orientation angular elements of each image required in photogrammetry can be given as follows, according to matrix \(R_m^n(\omega, \varphi, \kappa)\) in Eq.(11):

\[
\begin{align*}
\varphi &= -\arctg\left(\frac{a_1}{c_3}\right) \\
\omega &= -\arcsin(b_3) \\
\kappa &= \arctg\left(\frac{b_1}{b_2}\right)
\end{align*}
\]

(12)

3 Experiments and results analysis

3.1 Empirical test design

In this work, two sets of actual images taken from experimental projects that are different in terrain and photographic scale are selected and used for experiment. They were taken in November of 2004 and October of 2005, respectively. The main technical parameters of the empirical images are listed in Table 1.

| Table 1  | Technical data of the experimental images |
|----------|-----------------------------------------|
| Test 1   | Test 2                                  |
| Aerial camera | Leica RC-30 | Leica RC-30 |
| POS system    | POS AV 510 | POS AV 510 |
| Film         | Kodak 2442 | Kodak 2402 |
| Focal length  | 153.84 mm  | 153.53 mm  |
| Frame        | 23 cm × 23 cm | 23 cm × 23 cm |
| Photographic scale | 1:2500 | 1:60000 |
| Longitudinal overlap | 61%  | 64%   |
| Lateral overlap       | 32%  | 30%   |
| Number of strips      | 9    | 4     |
| Number of photos      | 255  | 48    |
| Number of ground control points | 73  | 29    |
| Maximum terrain undulation | 38.60 m | 107.50 m |
3.2 Transformation of exterior orientation angular elements obtained via a POS in WGS 84 Gauss-Kruger projection coordinate system

Attitude angles obtained directly via a POS are Euler angles from the IMU body coordinate system to the navigation coordinate system, which include roll, pitch, and heading \((\Phi, \Theta, \Psi)\). According to Eq.(10), exterior orientation angular elements without earth curvature and meridian deviation compensation can be obtained after a series of coordinate system transformation. These exterior orientation angular elements are in the coordinate system composed by WGS 84 ellipsoid after Gauss-Kruger projection (referred to as “WGS 84 Gauss-Kruger projection coordinate system” simply in this paper). Using WuCAPS system,[9] exterior orientation angular elements in WGS 84 Gauss-Kruger projection coordinate system can be computed by GPS-supported bundle block adjustment using four full GCPs in the four corners of the adjusted block. Comparing exterior orientation angular elements without compensation with the ones computed by WuCAPS system, the accuracies of exterior orientation angular elements without compensation are obtained (referred to as “Uncompensated” in Table 2). After that, errors of exterior orientation angular elements caused by the earth curvature and the meridian deviation can be corrected by Eq.(8). According to Eqs.(11) and (12), exterior orientation angular elements in WGS 84 Gauss-Kruger projection coordinate system after compensation can be computed. Comparing exterior orientation angular elements computed by ourselves with the ones computed by WuCAPS system, the accuracies of exterior orientation angular elements after compensation are obtained (referred to as “Compensated” in Table 2). On the other hand, comparing the exterior orientation elements computed by the POS with the ones computed by WuCAPS system, the accuracies of exterior orientation angular elements obtained via the POS are obtained (referred to as “POS” in Table 2). All the results are listed in Table 2.

Table 2  Accuracy of exterior orientation angular elements in WGS 84 Gauss-Kruger projection coordinate system (arcsecond)

| Project | Uncompensated | Compensated | POS |
|---------|---------------|-------------|-----|
|         | \(\phi\)  | \(\omega\) | \(\kappa\) | \(\phi\) | \(\omega\) | \(\kappa\) | \(\phi\) | \(\omega\) | \(\kappa\) |
| Test 1  | 52.4 | 156.1 | 1434.1 | 31.4 | 36.9 | 46.6 | 31.6 | 36.8 | 45.6 |
| Test 2  | 535.1 | 324.5 | 702.3 | 35.5 | 34.6 | 65.2 | 47.5 | 35.3 | 65.2 |

Note: “RMS” in Table 2 is calculated from the error \(\Delta_i = (\phi, \omega, \kappa)\) between exterior orientation angular elements of \(n\) images computed by different methods, that is, \(\mu = \sqrt{\sum \Delta_i^2 / n}\) (same to the following Tables).

The following conclusions can be drawn from that in Table 2 and Fig. 3:

1) Transformation accuracy of exterior orientation angular elements uncompensated in WGS 84 Gauss-Kruger projection coordinate system is lower, containing obvious errors; but after the compensation matrix is used, transformation accuracy of exterior orientation angular elements improves evidently. These results indicate that the effects of earth curvature and meridian deviation on angle transformation have to be taken into account under Gauss-Kruger projection coordinate system, and it is necessary to adopt compensation matrix in the process of exterior orientation angular elements transformation obtained via a POS.

2) In WGS 84 Gauss-Kruger projection coordinate system, the accuracy of exterior orientation angular elements compensated is equal to that computed by
the POS, which means that the compensation matrix proposed in this paper is valid and feasible.

3) In WGS 84 Gauss-Kruger projection coordinate system, the accuracy of exterior orientation angular elements compensated cannot catch up with that computed by aerial triangulation. The differences between them change along with the different strips, but there is no obvious systematic error. The reason is that the whole survey area adopts one same compensation matrix, which cannot well reflect the change rule of errors in each strip.

3.3 Transformation of exterior orientation angular elements with different vertical axis in WGS 84 Gauss-Kruger projection coordinate system

In order to analyze the relationship between the transformation of exterior orientation angular elements and vertical axis of the coordinate system, two neighboring projection zones are selected on either side near the projection zones of the survey area (taking projection zone of 3° for an example). Central meridians of these five projection zones are used as the vertical axis of the Gauss-Kruger projection coordinate system, and basing on that, exterior orientation angular elements are transformed in WGS 84 coordinate system with different vertical axis, respectively. Comparing the transformation results with the exterior orientation angular elements computed by WuCAPS, the accuracies of exterior orientation angular elements with different central meridians can be computed. The results are listed in Table 3.

It can be seen in Table 3 that transformation accuracy of exterior orientation angular elements is relevant to the choice of central meridian; the larger the distance between test area and central meridian is, the lower the transformation accuracy of exterior orientation angular elements will be. The reason is that the meridian convergence is defined in each projection zone; and it is the angle between the meridian of projection point and the central meridian of projection zone the projection point locates after projection. Therefore, when the survey area spans more than one projection zone, the transformation of exterior orientation angular elements should choose the central meridian of the area as the vertical axis of the coordinate system in each projection zone.

3.4 Transformation of exterior orientation angular elements with different origin of coordinate system in WGS 84 Gauss-Kruger projection coordinate system

In order to analyze the effect of the origin of coordinate system on the transformation of exterior orientation angular elements, two points are selected on either side near the central point of survey area; the space is 1° in latitudinal direction. These five points are used as the origin of the Gauss-Kruger projection coordinate system. Then, exterior orientation angular elements are transformed with different origin, respectively. Comparing the transformation results with the exterior orientation angular elements computed by WuCAPS, the accuracies of exterior orientation angular elements with different origins of coordinate system can be computed. The results are listed in Table 4.

| Table 3  | Accuracy of exterior orientation angular elements with different central meridian (arcsecond) |
|---------|-----------------------------------------------------------------------------------------------|
| y-axis  | Central meridian | Central meridian −3° | Central meridian +3° | Central meridian −6° | Central meridian +6° |
|         | φ    | ω    | κ    | φ    | ω    | κ    | φ    | ω    | κ    | φ    | ω    | κ    |
| Test 1  | 31.4 | 36.9 | 46.6 | 91.6 | 54.0 | 6973.1 | 85.6 | 697 | 7030.7 | 172.2 | 96.8 | 13974.9 | 164.9 | 116.6 | 14032.4 |
| Test 2  | 35.5 | 34.6 | 65.2 | 130.5 | 68.6 | 6909.7 | 131.8 | 110.2 | 6916.8 | 253.7 | 151.7 | 13822.5 | 256.2 | 193.2 | 13829.6 |

| Table 4  | Accuracy of exterior orientation angular elements with different origin of coordinate system (arcsecond) |
|---------|-----------------------------------------------------------------------------------------------|
| Origin of coordinate system | Center point of survey area | Center point of survey area −1° | Center point of survey area +1° | Center point of survey area −2° | Center point of survey area +2° |
|         | φ    | ω    | κ    | φ    | ω    | κ    | φ    | ω    | κ    | φ    | ω    | κ    |
| Test 1  | 31.4 | 36.9 | 46.6 | 31.4 | 36.9 | 46.6 | 31.4 | 36.9 | 46.6 | 31.4 | 36.9 | 46.6 | 31.4 | 36.9 | 46.6 |
| Test 2  | 35.5 | 34.6 | 65.2 | 35.5 | 34.6 | 65.2 | 35.5 | 34.6 | 65.2 | 35.5 | 34.6 | 65.2 | 35.5 | 34.6 | 65.2 |
It can be seen in Table 4 that the choice of origin of coordinate system has no effect on the transformation of exterior orientation angular elements. The reason is that origin of coordinate system is chosen to transform all the tangential plane coordinate system at each projection center to a uniform tangential plane coordinate system; it is irrelevant to the location of the origin of coordinate system. Generally speaking, it is appropriate that the origin of coordinate system locates at the center point of the survey area.

3.5 Transformation of exterior orientation angular elements in national coordinate system

Using three parameters on angles in seven transformation parameters, the attitude angles of IMU body system in navigation coordinate system based on WGS 84 reference ellipse can be transformed to the reference ellipse that the national coordinate system will adopt. According to the Eqs. (11) and (12), after a series of coordinate transformation and angle compensation, the exterior orientation angular elements in the national coordinate system can be computed finally. Comparing the transformation results of exterior orientation angular elements with the exterior orientation angular elements in the national coordinate system computed by WuCAPS, the transformation accuracy of exterior orientation angular elements in the national coordinate system can be obtained (referred to as “Proposed by this paper” in Table 5).

Table 5 Accuracy of exterior orientation angular elements in national coordinate system (″)

| Transformation method | Proposed by this paper | Proposed by POS |
|-----------------------|------------------------|-----------------|
|                       | φ          | ω          | κ          | φ          | ω          | κ          |
| Test 1                | 34.0       | 33.7       | 38.4       | 42.6       | 53.9       | 31.6       |
| Test 2                | 34.6       | 39.9       | 60.8       | 50.7       | 35.1       | 65.8       |

It can be seen in Table 5 that compared with the exterior orientation angular elements computed by GPS-supported bundle block adjustment in WuCAPS, the total accuracy of exterior orientation angular elements computed based on the compensation matrix deduced in this paper is higher than that obtained via the POS.

4 Conclusion

Coordinate reference systems for topographic mapping in China are often Gauss-Kruger projection coordinate system. The effects of the earth curvature and meridian deviation on angle transformation have to be taken into account in the process of computing exterior orientation angular elements by a POS. On the basis of effect theories of earth curvature and meridian deviation on exterior orientation angular elements, the method using a compensation matrix to correct the transformation errors from attitude angles obtained via a POS to exterior orientation angular elements of images is proposed in this paper. Moreover, the rigorous equation of the compensation matrix is deduced. Two sets of actual data obtained via a POS AV 510 that are different in scale and terrain are selected and used to perform experiments. The empirical results not only indicate that the compensation matrix proposed in this paper is correct and practical but also show that transformation accuracy of exterior orientation angular elements obtained via a POS based on compensation matrix is closely related with the choice of vertical axis of coordinate system; proper vertical axis should be the Gauss-Kruger projection of the central meridian of projection zone in which survey area locates. However, the accuracy of exterior orientation angular elements is irrelevant to the choice of origin of coordinate system. It is appropriate that the origin of coordinate system locates at the center point of the survey area. Moreover, the transformation accuracy of exterior orientation angular elements computed based on the compensation matrix deduced in this paper is higher than that obtained via the POS. Because of the limitations in experiments, transformation method of exterior orientation angular elements obtained via a POS still requires further researches and experiments.

Acknowledgement

The empirical data acquisition was supported by
the Institute of Remote Sensing Applications in Chinese Academy of Sciences, Zhongfei General Aviation Company, Liaoning Jingwei Surveying & Mapping Technology INC, Siwei Aviation Remote Sensing Co. Ltd., and others. This support is gratefully acknowledged.

References

[1] Li Xueyou (2005) Principle, method and practice of IMU/DGPS-based photogrammetry [D]. Zhengzhou: Information Engineering University (in Chinese)

[2] Mostafa M, Hutton J (2001) Airborne remote sensing without ground control [C]. Proceedings of IGARSS’01, Sydney

[3] Müller R, Lehner M, Müller R, et al. (2002) A program for direct georeferencing of airborne and spaceborne line scanner images [J]. International Archives of Photogrammetry and Remote Sensing, 34(A1): 148-153

[4] Cramer M, Stallmann D, Haala N (2000) Direct georeferencing using GPS/Inertial exterior orientations for photogrammetric applications [J]. International Archives of Photogrammetry and Remote Sensing, 33(B3): 198-205

[5] Yastikli N, Jacobsen K (2005) Influence of system calibration on direct sensor orientation [J]. Photogrammetric Engineering and Remote Sensing, 71(5): 629-633

[6] Jacobsen K, Wegmann H (2002) Dependencies and problems of direct sensor orientation [C]. Proceedings of OEEPE Workshop on Integrated Sensor Orientation, Hanover

[7] Cramer M, Stallman D (2002) System calibration for direct georeferencing [J]. International Archives of Photogrammetry and Remote Sensing, 34(A3):79-84

[8] Jacobsen K (2002) Calibration aspects in direct georeferencing of frame imagery [J]. International Archives of Photogrammetry and Remote Sensing, 34(A1): 82-89

[9] Yuan Xiuxiao (2008) A novel method of systematic error compensation for a position and orientation system [J]. Progress in Natural Science, 18(8): 953-963

[10] Legat K (2006) Approximate direct georeferencing in national coordinates [J]. ISPRS Journal of Photogrammetry and Remote Sensing, 60(4): 239-255

[11] Skaloud J, Legat K (2008) Theory and reality of direct georeferencing in national coordinates [J]. ISPRS Journal of Photogrammetry and Remote Sensing, 63(2): 272-282

[12] Liu Jun, Zhang Yongsheng, Wang Dondhong, et al. (2004) Computing method of exterior Orientation elements of POSAV510-DG system [J]. Geomatics Technology and Equipment, 6(4): 43-47 (in Chinese)

[13] Bäumker M, Heimes F J (2002) New calibration and computing method for direct georeferencing of image and scanner data using the position and angular data of an hybrid inertial navigation system. [C]. Proceedings of OEEPE Workshop on Integrated Sensor Orientation, Hanover

[14] Gray I D (1997) Effects of the earth’s curvature on radar tracking system [J]. Radar System, 449: 606-608

[15] Ressl C (2002) The impact of conformal map projections on direct georeferencing [J]. International Archives of Photogrammetry and Remote Sensing, 34(3A): 283-288

[16] Kong Xiangyuan, Mei Shiyi (1996) Controlling surveying [M]. Wuhan: Wuhan University Press (in Chinese)