Ion acoustic solitary wave in homogeneous magnetized electron–positron–ion plasmas

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Abstract. The nonlinear ion acoustic wave propagating obliquely with respect to an external magnetic field is studied in a homogeneous magnetized electron–positron–ion plasma. It is found that the amplitude of the solitary structure increases with the percentage presence of positrons, which is opposite behaviour to the previous study of these waves in an unmagnetized plasma. The speed of the obliquely propagating soliton in a magnetized plasma turns out to be subsonic, while it is supersonic in the unmagnetized case already studied.

Contents

1. Introduction 1
2. Nonlinear set of equations 2
3. Localized stationary solution 4
4. Analytical solution 6
5. Numerical solutions 8
6. Discussion 9
References 9

1. Introduction

The ion acoustic wave (IAW) is an ion timescale phenomenon and this mode does not exist in electron–positron (e–p) plasmas. The IAW in a two-component electron–ion (e–i) plasma has long been studied and both the linear [1]–[4] and nonlinear [5]–[8] dynamics associated with this wave have been investigated. This wave can couple with shear Alfvén waves and electrostatic
drift waves (in nonuniform plasmas) due to finite Larmor radius effects. Since the mode has low frequency \( (\omega < \omega_{ci} \text{ where } \omega_{ci} = eB_0/m_{ic} \text{ is the ion gyrofrequency}) \), electrons are assumed to follow the Boltzmann distribution and the electron pressure is balanced by the electrostatic field. The role of ion inertia is crucial for the existence of this wave. It can propagate in both unmagnetized and magnetized \((e-i)\) plasmas.

A few years ago, the IAW was studied in an unmagnetized three-component electron–positron–ion \((e-p-i)\) plasma [9]. The linear dispersion relation was obtained by assuming that both electrons and positrons are hot and that they obey the Boltzmann distribution. The nonlinear investigation showed that the amplitude of the electron density hump reduces due to the presence of positrons in the \(e-i\) plasma. The amplitude of the nonlinear structure depends upon the concentrations of the different species.

The \(e-p\) plasma has an important role in the understanding of the plasmas in the early universe [10, 11], in active galactic nuclei [12], in pulsar magnetospheres [14, 15] and in the solar atmosphere [13].

It is well known that when positrons are introduced into \(e-i\) plasma the response of the plasma changes significantly. The positrons can be used to probe particle transport in tokamaks and, since they have sufficient lifetime, the two-component \((e-i)\) plasma becomes a three-component \((e-i-p)\) plasma [16, 17].

Most of the astrophysical plasmas also contain ions in addition to electrons and positrons. It is important to study linear and nonlinear wave propagation in such plasmas. Although there is no concrete evidence, the magnetospheres of rotating neutron stars are believed to contain \(e-p\) plasmas produced in the cusp regions of the stars due to intense electromagnetic radiation. Since protons or other ions may exist in such environments, so the three-component \(e-p-i\) plasma can exist in nature.

During the last decade, \(e-p-i\) plasma has attracted the attention of several authors [18]–[26]. They have studied linear and nonlinear wave propagation in \(e-p-i\) plasmas using different models.

In this paper, we study the propagation of the ion acoustic solitary wave in a magnetized homogeneous \(e-p-i\) plasma. The electrostatic perturbations are assumed to be two-dimensional. Ion Larmor radius effects and the concentration of positrons modify the IAW dynamics both in the linear as well as in the nonlinear regime. In section 2, the nonlinear set of equations and the linear dispersion relation are presented. In section 3, the energy integral equation and the Sagdeev potential are obtained. In section 4, an analytical solution in the form of a solitary pulse in the small-amplitude limit is investigated. The nonlinear solitary structures are obtained numerically in section 5 corresponding to an arbitrary-amplitude perturbation using the Sagdeev potential approach. Finally in section 6, a discussion of the results is presented.

2. Nonlinear set of equations

Let us consider an ideal homogeneous magnetized three-component \((e-p-i)\) plasma. The external constant magnetic field is directed along the \(z\)-axis, i.e. \(B_0 = B_0 \hat{z}\). The electrons and positrons are assumed to be hot while the ions are treated as a cold fluid. The phase velocity of the IAW is assumed to be much larger than the ion thermal velocity and much less than the electron (positron) thermal velocities, i.e. \(v_t \ll \omega/k \ll v_{te}, v_{tp}\) (where \(v_{tj} = (T_j/m_j)^{1/2}\) is the thermal speed of the \(j\)th species while \(j = e, p, i\)). Under these conditions the nonlinear dynamics of the low-frequency IAW in the three-component plasma are governed by the following set of
equations: the ion continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v_i) = 0,$$

and the ion momentum equation

$$\frac{\partial v_i}{\partial t} + (v_i \cdot \nabla) v_i = -\frac{e}{m_i} \nabla \phi + \omega_{ci} v_i \times \hat{z}.$$  

(2)

The electrons and positrons in the electrostatic potential perturbation are assumed to follow the Boltzmann distributions, respectively, as

$$n_e = n_{e0} \exp \left( \frac{e\phi}{T_e} \right)$$  

(3)

and

$$n_p = n_{p0} \exp \left( -\frac{e\phi}{T_p} \right).$$  

(4)

We have assumed \( E = -\nabla \phi \) (where \( \phi \) is an electrostatic potential) and \( \omega_{ci} = eB_0/m_i c \) is the ion gyrofrequency, while \( n_\alpha \) is the perturbed density. Here \( n_{\alpha 0} \) is the unperturbed density of \( \alpha \) species (where \( \alpha = e, p, i \) stands for electrons, positrons and ions, respectively), \( v_i \) is the ion fluid velocity, \( e \) is the magnitude of the electron charge and \( m_i \) is the mass of the ion and \( T_e \) (\( T_p \)) denote the electron (positron) temperatures, respectively.

Equilibrium requires

$$n_{i(0)} + n_{p0} = n_{e0}.$$  

(5)

Let us consider the two-dimensional perturbation in the \( xz \)-plane. Then equations (1) and (2) can be written as

$$\partial_t n_i + \partial_x (n_i v_{ix}) + \partial_z (n_i v_{iz}) = 0$$  

(6)

$$\partial_t v_{ix} + (v_{ix} \partial_x + v_{iz} \partial_z) v_{ix} = -\frac{e}{m_i} \partial_x \phi + \omega_{ci} v_{iy}$$  

(7)

$$\partial_t v_{iy} + (v_{ix} \partial_x + v_{iz} \partial_z) v_{iy} = -\omega_{ci} v_{ix}$$  

(8)

and

$$\partial_t v_{iz} + (v_{ix} \partial_x + v_{iz} \partial_z) v_{iz} = -\frac{e}{m_i} \partial_z \phi.$$  

(9)

The quasi-neutrality is defined as

$$n_i + n_p \simeq n_e.$$  

(10)

The linear dispersion relation of the IAW in a magnetized e–p–i plasma is obtained by solving equations (3)–(10) algebraically, which yields

$$(1 - p)c_s^2 \left\{ \frac{k_x^2}{\omega^2} + \frac{k_z^2}{\omega^2} \right\} = (1 + \alpha p).$$  

(11)

In the limit of low-frequency waves (\( \omega \ll \omega_{ci} \)), the above linear dispersion relation reduces to

$$\omega^2 = \frac{c_s^2 k_z^2 (1 - p)}{[(1 - p)\rho_s k_x^2 + 1 + \alpha p]}.$$  

(12)
where \( \omega, k_x \) and \( k_z \) are frequency and wavenumbers along the \( x \)- and \( z \)-direction, respectively. Furthermore, \( c_s = (T_e/m_i)^{1/2} \) is the ion-sound speed in two-component e–i plasma, \( p = n_{po}/n_{eo} \) is the ratio of positron and electron unperturbed densities, \( \alpha = T_e/T_p \) is the ratio of electron and positron temperatures and \( \rho_s = c_i/\omega_{ci} \) is the ion Larmor radius at electron temperature.

The limiting case of two-component (e–i) plasma can be obtained from equation (12) by inserting \( n_{po} = 0 \) (i.e. in the absence of positrons), which gives

\[
\omega^2 = \frac{c_s^2 k_x^2}{(1 + \rho_s^2 k_z^2)}.
\]

(13)

This is the same linear dispersion relation of IAW in a homogeneous magnetized plasma with two components, electrons and ions, which has already been studied by Yu et al. [27]. The same dispersion relation has also been discussed by Mahmood and Saleem [28] in the presence of ion streaming in magnetized e–i plasma.

Equation (13) shows that the nonlinear wave steepening can be balanced by the dispersion of the IAW due to the ion Larmor radius effect. Such a compromise between the two effects can give rise to a stable and finite-amplitude solitary structure in a magnetized e–i plasma [27, 28].

In equation (12), the term of ion Larmor radius is multiplied by the factor \( p \), which is the ratio of unperturbed densities of positrons and electrons, so the wave dispersion is dependent upon the percentage presence of all the species. Hence, the positron density plays a significant role in the formation of a stable solitary structure in (e–p–i) plasma.

3. Localized stationary solution

We are interested in a localized stationary solution and therefore we define a moving coordinate as

\[
\xi = K_x x + K_z z - Mt
\]

(14)

where \( M \) is the speed of the localized nonlinear structure in the moving frame and \( K_x, K_z \) are the direction cosines along the \( x \)- and \( z \)-axis, respectively. Furthermore, \( K_x^2 + K_z^2 = 1 \).

Now, assuming that all the dependent variables are functions of \( \xi \), the ion continuity equation (6) can be written as

\[
v_{ix} \partial_x + v_{iz} \partial_z = M \left( 1 - \frac{1}{n_i} \right) \partial_\xi.
\]

(15)

Transforming equations (6)–(9) in terms of coordinate ‘\( \xi \)’ and using equation (15) in equations (7)–(9), we obtain the dimensionless nonlinear differential equation in terms of ion density \( n_i \) as

\[
\frac{d}{d\xi} \left[ \frac{1}{n_i} \left\{ \frac{d^2}{d\xi^2} \left( \frac{M_a^2}{2n_i^2} + \Phi \right) + 1 \right\} \right] = -\frac{K_z^2}{M_a^2} n_i \frac{\partial \Phi}{\partial \xi}
\]

(16)

where \( \xi = \xi/\rho_s \), \( M_a \) (Mach number) = \( M/c_s \), \( \Phi = e\phi/T_e \) and \( n_a = n_a/n_{ao} \) (where \( \alpha = e, i, p \)).

From the quasi-neutrality condition, we have

\[
n_i = \frac{1}{(1 - p)} (n_e - pn_e^{-\alpha})
\]

(17)
where \( \alpha = T_e/T_p \) and \( p = n_{p0}/n_{e0} \). Note that the above equation holds for \((0 < p < 1)\) in three-component \((e-p-i)\) plasmas.

Equation (3) becomes

\[
\Phi = \ln n_e.
\]  

(18)

Using equations (17) and (18) in (16) and then integrating, we obtain

\[
\frac{d^2}{d\xi^2} \left[ \frac{M_a^2(1 - p)^2}{2(n_e - pn_{e0}^{-a})^2} + \ln n_e \right] = -1 - \frac{(n_e - pn_{e0}^{-a})}{(1 - p)} \times \left[ \frac{K_z^2}{M_a^2} (1 - p) \left( n_e + \frac{p}{\alpha} n_{e0}^{-a} \right) - \frac{K_z^2}{M_a^2} \left( \frac{1 + \frac{p}{\alpha}}{1 - p} \right) - 1 \right]
\]

(19)

where we have used the boundary conditions, i.e. as \(|\xi| \to \pm \infty\), \(\partial^2 n_e/\partial \xi^2 \to 0\), \(\partial n_e/\partial \xi \to 0\) and \(n_e \to 1\).

Let us define

\[
u = \left[ \frac{M_a^2(1 - p)^2}{2(n_e - pn_{e0}^{-a})^2} + \ln n_e \right].
\]

Now multiplying by \(\partial \nu/\partial \xi\) on both sides of equation (19) and after integrating once, we obtain

\[
\frac{1}{2} \left( \frac{\partial n_e}{\partial \xi} \right)^2 + U(n_e, K_z, M_a, p) = 0
\]

(20)

where the Sagdeev potential is defined as

\[
U(n_e, K_z, M_a, p) = \left[ 1 - \frac{M_a^2(1 - p)^2(1 + \alpha n_{e0}^{-a-1})^2}{(n_e - pn_{e0}^{-a})^3} \right]^{-2} \left[ \frac{M_a^2}{2} + \frac{K_z^2(1 + \frac{p}{\alpha})^2}{2M_a^2(1 - p)^2} + \frac{(1 + \frac{p}{\alpha})}{(1 - p)} \right]
\]

\[+ (1 - K_z^2) \ln n_e + \frac{K_z^2(n_e + \frac{p}{\alpha} n_{e0}^{-a})^2}{2M_a^2(1 - p)^2} + \frac{M_a^2(1 - p)^2}{2(n_e - pn_{e0}^{-a})^2} \]

\[- \left\{ \frac{1}{(1 - p)} + \frac{K_z^2(1 + \frac{p}{\alpha})}{M_a^2(1 - p)^2} \right\} \left( n_e + \frac{p}{\alpha} n_{e0}^{-a} \right) \]

\[- \frac{M_a^2}{(n_e - pn_{e0}^{-a})} \left\{ \frac{K_z^2(1 + \frac{p}{\alpha})}{M_a^2} + (1 - p) \right\} + \frac{K_z^2(n_e + \frac{p}{\alpha} n_{e0}^{-a})}{(n_e - pn_{e0}^{-a})} \]

(21)

and we have used boundary conditions, i.e. \(\partial n_e/\partial \xi \to 0\) and \(n_e \to 1\) as \(|\xi| \to \pm \infty\), to obtain equation (20). Equation (20) is a well known equation in the form of the ‘energy integral’ of an oscillating particle of unit mass, with velocity \((\partial n_e/\partial \xi)\) and position \(n_e\) in a potential well \(U(n_e, K_z, M_a, p)\).

The limiting case of two-component \((e-i)\) magnetized plasma can be obtained by substituting \(n_{p0} = 0\) in equation (20), which then reduces to the case of Yu et al [27] for the ion acoustic solitary wave in a homogeneous magnetized \((e-i)\) plasma.

The conditions for the existence of a localized solution of equation (20) require that (i) \(U(1) = U(N) = \partial U/\partial n_e|_{n_e=1} = 0\) and (ii) \(\partial^2 U/\partial \xi^2|_{n_e=1} < 0\) (where the fixed point at \(n_e = 1\) is unstable). Here \(N\) is the point where the curve crosses the axis as shown in figure 1(a) and it represents the maximum amplitude of the solitary wave. From the second condition, it is seen that solitary structures are formed only in the subsonic region, i.e. with Mach number \((M_a < 1)\).
4. Analytical solution

In this section, an analytical solution of equation (20) in the form of a KdV (Korteweg–de Vries) equation in the small-amplitude limit is obtained assuming the electron and positron temperatures to be equal, i.e. $T_e = T_p$ or ($\alpha = 1$). We investigate the behaviour of the Sagdeev potential analytically in this case and show that the soliton solution can exist under the localized boundary conditions.

For the localized solution, we have one of the boundary conditions as $U(N) = 0$ at $n_e = N$, where $N$ is the maximum amplitude of the nonlinear structure.

The above boundary condition gives the nonlinear dispersion relation as follows:

$$\frac{M_a^2}{2} + \frac{K_z^2}{2M_a^2} \frac{(1 + p)^2}{(1 - p)^2} + \frac{(1 + p)}{(1 - p)} + (1 - K_z^2) \ln N + \frac{K_z^2}{2M_a^2} \frac{(N^2 + p)^2}{N(1 - p)^2} \ln N + \frac{M_a^2 N^2}{2} \frac{(N^2 - p)^2}{(N^2 - p)^2} \left\{ \frac{(N^2 + p)}{N(1 - p)^2} + \frac{M_a^2 N}{(N^2 - p)} \right\} Q + \frac{K_z^2(N^2 + p)}{(N^2 - p)^2} = 0 \quad (22)$$

where $Q = (1 - p) + K_z^2 M_a^2 / (1 + p)$.

For the localized solution, the Sagdeev potential must be negative between two zeros, i.e. at $n_e = 1$ and $N$. Using Taylor’s expansion, the Sagdeev potential ‘$U$’ near $n_e = 1$ can be expressed
as

\[ U(n_e, K_z, M_a, p) = (A + BC)(n_e - 1)^2 + (D + EF + GH)(n_e - 1)^3 \]  

(23)

where

\[ A = \frac{1}{R^2} \left[ (1 - p^2)(K_z^2 - 1) + M_a^2(3 + 8p + p^2) + \frac{K_z^2}{M_a^2}(1 + 3p^2) \right. \]
\[ \left. + \frac{1}{(1 - p)} \{ 4K_z^2p(3 + p) - 2S[M_a^2(1 + p) + p(1 - p)] \} \right] \]
\[ B = \frac{2(1 - p)^2}{R^4} \left[ M_a^2 + (1 + K_z^2) + \frac{K_z^2}{M_a^2} - \frac{1}{(1 - p)^2}ST \right] \]
\[ C = 2R \left[ 3M_a^2p(4 + p) + (6M_a^2 - 1)(1 - p) + 3[M_a^2(5 + 3p) - (1 - p)^2] \right] \]
\[ D = \frac{1}{R^2} \left[ 2(1 - p)^2(1 - K_z^2) + 6p \left( S - \frac{2K_z^2p}{M_a^2} \right) - \frac{12M_a^2(1 + 5p + 2p^2)}{(1 - p)} \right. \]
\[ \left. + \frac{1}{(1 - p)^2} \{ 6M_a^2(1 - p)(1 + 6p + p^2) + K_z^2(1 + p)(1 - 36p) \} \right] \]
\[ E = \frac{6}{R^3} \left[ 1 - \frac{2M_a^2p}{(1 - p)} + \frac{3M_a^2(1 + p)^2}{(1 - p)^2} \right] \]
\[ F = 4K_z^2p(3 + p) - (1 - K_z^2)(1 - p)^3 + 2S[p(1 - p) - M_a^2(1 + 3p)] \]
\[ + (1 - p) \left[ M_a^2(3 + 8p + p^2) + \frac{K_z^2}{M_a^2}(1 + 3p) \right] \]
\[ G = \frac{2}{R^2} \left[ \frac{ST}{(1 - p)^2} - M_a^2 - (K_z^2 + 1) \frac{(1 + p)}{(1 - p)} - \frac{2K_z^2(1 + p)^2}{M_a^2(1 - p)^2} \right] \]
\[ H = \frac{3}{(1 - p)} \left[ 4W^3 + 6RW \times (6M_a^2(1 + p)^3 + 12M_a^2p(1 - p)^2 - (1 - p)^3) + 10M_a^2(1 + p)^4 \right. \]
\[ \left. + 6M_a^2p \times \{ 4(1 + p)(1 - p)^2 + (2 + 3p)(1 - p)^2 \} - 4M_a^2p(1 - p)^3 - (1 - p)^4 \right] \]

and

\[ R = (1 - p) - M_a^2(1 + p) \]
\[ S = (1 - p) + \frac{K_z^2}{M_a^2}(1 + p) \]
\[ T = (1 + p) + M_a^2(1 - p) \]
\[ W = 3M_a^2(1 + p)^2 + 2M_a^2p(1 - p) - (1 - p)^2. \]

Similarly, expanding the Sagdeev potential ‘\(U\)’ near \(n_e = N\), we obtain

\[ U(n_e, K_z, M_a, p) = \frac{(N^2 - p)^3}{NX^3} \left[ \frac{(N - 1)X^2Y}{M_a^2(1 - p)^2} \right. \]
\[ \left. + \{ (N^2 - p)^2N^2(M_a^2N^2(1 - p)^2(3N^4 + 8N^2p + p^2) - (N^2 - p)^4) \} \times \left\{ 2(K_z^2 - 1)\ln N - M_a^2 - \frac{M_a^2N^2(1 - p)^2}{(N^2 - p)^2} \right\} \right] \]
density structure in the presence of positrons, i.e. $p\delta N$ of the pulse is has been shown in figure 1(a) for due to the presence of positrons in a magnetized e–i plasma. Popel et al [9] have studied the nonlinear IAW in unmagnetized e–p–i plasma and they have reported the formation of solitary structure in the absence as well as in the presence of ion streaming in magnetized e–i plasma. The same values of $K_z$ and $Ma$ have been taken from Yu et al [27] where they were used for two-component e–i magnetized plasma. The soliton solution, under the limit of small- but finite-amplitude perturbation, can be obtained by using the expanded form of the Sagdeev potential ‘U’ as given in equation (23), and then integrating equation (21) we have

$$\delta n_e = \delta N \sec h^2(\lambda \xi)$$

(25)

where $n_e = 1 + \delta n_e$ and $\delta n_e \ll 1$ have been used. Here $\lambda^2 = -(A + BC)/2$ and $\delta N = -(A + BC)/(D + EF + GH)$ where $(A + BC) < 0$. Therefore, the maximum amplitude of the pulse is $\delta N$ and the width of the structure is of the order of $[-(A + BC)]^{-1/2}$.

5. Numerical solutions

The numerical solution of equation (20) is presented under the required conditions for the existence of the localized solution. It is assumed that the electron temperature is equal to the positron temperature, i.e. $T_e = T_p$ or $(\alpha = 1)$. The plot of the Sagdeev potential ‘U’ versus ‘$n_e$’ has been shown in figure 1(a) for $K_z = 0.8$ and $Ma = \sqrt{0.75}$. The nonlinear density humps for electrons have been shown in figure 1(b). The solid curve shows the normalized electron density structure in the absence of positrons, i.e. $p = 0$ and the dashed curve represents the density structure in the presence of positrons, i.e. $p = 0.1$ for $K_z = 0.8$ and $Ma = \sqrt{0.75}$. The values of $K_z$ and $Ma$ have been taken from Yu et al [27] where they were used for two-component e–i magnetized plasma. The same values of $K_z$ and $Ma$ were also used by Mahmood and Saleem [28] in the absence as well as in the presence of ion streaming in magnetized e–i plasma. It can be seen from figure 1(b) that the amplitude of the electron density hump increases due to the presence of positrons in a magnetized e–i plasma. Popel et al [9] have studied the nonlinear IAW in unmagnetized e–p–i plasma and they have reported the formation of solitary structure in the supersonic region. They have also found that the amplitude of the density hump decreases with the percentage presence of positrons in e–p–i unmagnetized plasmas. We have found that in a magnetized e–p–i plasma solitary structures are formed in the subsonic region.

The amplitude of the solitary structure is also dependent upon the propagation direction of the solitary wave. The profile of the electron density against the direction of propagation has been shown in figure 1(c) for $K_z = 0.8$ (dashed curve) and $K_z = 0.7$ (solid curve) for the same concentration of positrons, i.e. $p = 0.1$ while $Ma = \sqrt{0.75}$.
Figure 2. For the same propagation direction $K_z = 0.8$ the Mach number changes with the change in concentration of positrons. Here $M_a = \sqrt{0.75}$ for $p = 0.1$ (solid curve) and $M_a = \sqrt{0.4}$ for $p = 0.4$ (dashed curve).

In figure 2, the dependence of positron density on Mach number has been shown for constant value of $K_z = 0.8$. It can be seen that with the increase in concentration of positrons, the Mach number decreases and the amplitude of the density hump increases in a magnetized e–p–i plasma.

6. Discussion

The nonlinear dynamics of the IAW in the three-component e–p–i magnetized plasma has been investigated. It has been found that the amplitude of the solitary structure depends upon several of the parameters involved: the concentration of positrons in e–i plasma, the propagation direction and the Mach number. It is interesting to note that the amplitude increases due to the presence of positrons in the magnetized plasma while in the unmagnetized plasma it decreases, as was reported in [9]. Furthermore, the formed solitary structures are subsonic ($M_a < 1$) in the magnetized case while they are supersonic ($M_a > 1$) in the unmagnetized plasma. This result is similar to the case of two-component e–i plasma, as has already been stated in the literature [27].

We have adopted the Sagdeev potential approach to obtain solitary ion acoustic density humps in magnetized e–p–i plasmas. The comparison of our result with an earlier study for the unmagnetized case [9] shows that the main features of the structures in (e–p–i) plasma and (e–i) plasmas are similar. The magnetic field has almost the same role in both cases. Since the electrostatic IAW is a fundamental mode of the plasma, such a comparative study seems to be important academically.

In section 4, we have also deduced an analytical solution of equation (20) in the form of a KdV equation under a small-amplitude limit. The numerical solutions of equation (20) corresponding to arbitrary amplitude perturbation have been presented in section 5. We think that these results will be helpful in understanding the nonlinear propagation of electrostatic perturbation in magnetized e–p–i plasmas which are believed to exist in the early universe, active galactic nuclei and the pulsar magnetospheres.

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