Supersymmetry and the Anomalous Anomalous Magnetic Moment of the Muon

Jonathan L. Feng\textsuperscript{a} and Konstantin T. Matchev\textsuperscript{b}

\textsuperscript{a} Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.

\textsuperscript{b} Theory Division, CERN, CH–1211 Geneva 23, Switzerland

The recently reported measurement of the muon’s anomalous magnetic moment differs from the standard model prediction by 2.6\sigma. We examine the implications of this discrepancy for supersymmetry. Deviations of the reported magnitude are generic in supersymmetric theories. Based on the new result, we derive model-independent upper bounds on the masses of observable supersymmetric particles. We also examine several model frameworks. The sign of the reported deviation is as predicted in many simple models, but disfavors anomaly-mediated supersymmetry breaking.

Measurements of spin magnetic dipole moments have a rich history as harbingers of profound progress in particle physics. In the leptonic sector, the electron’s gyromagnetic ratio $g_e \approx 2$ pointed the way toward Dirac’s theory of the electron. Later, the electron’s anomalous magnetic moment $a_e \equiv (g_e - 2)/2 \approx \alpha/2\pi$ played an important role in the development of quantum electrodynamics and renormalization. Since then, increasingly precise measurements have become sensitive both to very high order effects in quantum electrodynamics and to hadronic processes, and the consistency of experiment and theory has stringently tested these sectors of the standard model.

Very recently, the Muon ($g - 2$) Collaboration has reported a measurement of the muon’s anomalous magnetic moment, which, for the first time, is sensitive to contributions comparable to those of the weak interactions \cite{ref1}. (See Tables I and II.) The new Brookhaven E821 result is $a_\mu^{\exp} = 11.659 \pm 0.0006 \pm 0.0020 \times 10^{-10}$ (1.3 ppm), where the first uncertainty is statistical and the second systematic. Combining experimental and theoretical uncertainties in quadrature, the new world average differs from the standard model prediction by $2.6\sigma$ \cite{ref2}:

$$a_\mu^{\exp} - a_\mu^{SM} = (43 \pm 16) \times 10^{-10}. \tag{1}$$

Although of unprecedented precision, the new result is based on a well-tested method used in previous measurements. Polarized positive muons are circulated in a uniform magnetic field. They then decay to positrons, which are emitted preferentially in the direction of the muon’s spin. By analyzing the number of energetic positrons detected at positions around the storage ring, the muon’s spin precession frequency and anomalous magnetic moment are determined. The new result is based solely on 1999 data. Analysis of the 2000 data is underway, with an expected error of $\sim 7 \times 10^{-10}$ (0.6 ppm), and the final goal is an uncertainty of $4 \times 10^{-10}$ (0.35 ppm) \cite{ref3}.

The standard model prediction has been greatly refined in recent years. The current status is reviewed in Ref. \cite{ref4} and summarized in Table I. The uncertainty is dominated by hadronic vacuum polarization contributions that enter at 2-loops. This is expected to be reduced by recent measurements of $\sigma(e^+e^- \rightarrow \text{hadrons})$ at center-of-mass energies $\sqrt{s} \sim 1$ GeV. Thus, although the statistical significance of the present deviation leaves open the possibility of agreement between experiment and the standard model, the prospects for a definitive resolution are bright. If the current deviation remains after close scrutiny and the expected improvements, the anomalous value of $a_\mu$ will become unambiguous.

In this study, we consider the recent measurement of $a_\mu$ to be a signal of physics beyond the standard model. In particular, we consider its implications for supersymmetric theories. Supersymmetry is motivated by many independent considerations, ranging from the gauge hierarchy problem to gauge coupling unification to the necessity of non-baryonic dark matter, all of which require supersymmetric particles to have weak scale masses. Deviations in $a_\mu$ with the reported magnitude are therefore generic in supersymmetry. In addition, $a_\mu$ is both flavor- and CP-conserving. Thus, while the impact of supersymmetry on other low energy observables can be highly suppressed by scalar degeneracy or small CP-violating phases in simple models, supersymmetric contributions to $a_\mu$ cannot be. In this sense, $a_\mu$ is a uniquely robust probe of supersymmetry, and an anomaly in $a_\mu$ is a natural place for the effects of supersymmetry to appear.

The anomalous magnetic moment of the muon is

$$a_\mu = \frac{e}{2m_\mu} \bar{\mu} \sigma^{mn} \mu F_{mn},$$

where $\sigma^{mn} = \frac{i}{2} [\gamma^m, \gamma^n]$. The supersymmetric contribution, $a_\mu^{\text{SUSY}}$, is dominated by well-known neutralino-smuon and chargino-sneutrino diagrams \cite{ref5}. In the absence of significant slepton flavor violation, these diagrams are completely determined by only seven supersymmetry parameters: $M_1$, $M_2$, $\mu$, $\tan \beta$, $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$, and $A_\mu$. The first four enter through the chargino and neutralino masses: $M_1$, $M_2$, and $\mu$ are the U(1) gaugino, SU(2) gaugino, and Higgsino mass parameters, respectively, and $\tan \beta = (H_u^0)/|H_d^0|$ governs gaugino-Higgsino mixing. The last five determine the slepton masses, where $m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_R}$ are the SU(2) doublet and sin-
glet slepton masses, respectively, and the combination \( m_{\tilde{\mu}}(A_{\mu} - \mu \tan \beta) \) mixes left- and right-handed smuons. In general, \( M_1, M_2, \mu, \) and \( A_{\mu} \) are complex. However, bounds from electric dipole moments typically require their phases to be very small. In addition, \( |a_{\mu}^{\text{SUSY}}| \) is typically maximized for real parameters. In deriving model-independent upper bounds on superparticle masses below, we assume real parameters, but consider all possible sign combinations; these results are therefore valid for arbitrary phases. Our sign conventions are as in Ref. [8].

The qualitative features of the supersymmetric contributions are most transparent in the mass insertion approximation. The structure of the magnetic dipole moment operator requires a left-right transition along the lepton-slepton line. In the interaction basis, this transition may occur through a mass insertion in an external muon line, at a Higgsino vertex, or through a left-right mass insertion in the smuon propagator. The last two contributions are proportional to the muon Yukawa coupling and so may be enhanced by \( \tan \beta \). For large and moderate \( \tan \beta \), it is not hard to show that the supersymmetric contributions in the mass insertion approximation are all of the form

\[
\frac{a_{\mu}^{\text{SUSY}}}{10^{\pi}} \sim m_{\tilde{\mu}}^2 \mu M_1 \tan \beta F, \tag{2}
\]

where \( i = 1, 2, \) and \( F \) is a function of superparticle masses, with \( F \propto M^{-1}_{\tilde{\mu}} \) in the large mass limit [9].

Equation (2) implies \( a_{\mu}^{\text{SUSY}}/a_{\mu} \sim m_{\tilde{\mu}}^2/m_\mu^2 \approx 4 \times 10^4 \); \( a_{\mu} \) is therefore far more sensitive to supersymmetric effects than \( a_{\mu} \), despite the fact that the latter is 350 times better measured. Also, for \( M_2/M_1 > 0 \), although the contributions of Eq. (3) may destructively interfere, typically \( \text{sign}(a_{\mu}^{\text{SUSY}}) = \text{sign}(\mu M_{1,2}) \); we have found exceptions only rarely in highly model-independent scans. Finally, the parameter \( \tan \beta \) is expected to be in the range \( 2.5 \lesssim \tan \beta \lesssim 50 \), where the lower limit is from Higgs boson searches, and the upper limit follows from requiring a perturbative bottom quark Yukawa coupling up to \( \sim 10^{16} \text{ GeV} \). Supersymmetric contributions may therefore be greatly enhanced by large \( \tan \beta \).

To determine the possible values of \( a_{\mu}^{\text{SUSY}} \) without model-dependent biases, we have calculated \( a_{\mu}^{\text{SUSY}} \) in a series of high statistics scans of parameter space. We use exact mass eigenstate expressions for \( a_{\mu}^{\text{SUSY}} \). Our calculations agree with Refs. [9,10] and cancel the corresponding standard model diagrams in the supersymmet-

\[
\frac{a_{\mu}^{\text{SUSY}}}{43 \times 10^{-10}} = \frac{\tan \beta}{50} \left( \frac{390 \text{ GeV}}{M_{\text{LOSP max}}} \right)^2. \tag{3}
\]

If \( a_{\mu}^{\text{SUSY}} \) is required to be within \( 1\sigma \) (2\sigma) of the measured
deviation, at least one observable superpartner must be lighter than 490 GeV (800 GeV).

In Fig. 3 we repeat the above analysis, but for the case where the LSP decays visibly in collider detectors, as in models with low-scale supersymmetry breaking or \( R \)-parity violating interactions. In this case, the LOSP is the LSP. We relax the requirement of a neutral LSP, and require slepton masses above 95 GeV and neutralino masses above 99 GeV [13]. The results are given in Fig. 2. For this case, the solid envelope curve is parametrized by

\[
\frac{a_{\mu}^{\text{SUSY}}}{43 \times 10^{-16}} = \frac{\tan \beta}{50} \left[ \frac{300 \text{ GeV}}{M_{\text{LOSP}}^{\text{max}}} + \left( \frac{230 \text{ GeV}}{M_{\text{LOSP}}^{\text{max}}} \right)^{4} \right],
\]

and the 1σ (2σ) bound is \( M_{\text{LOSP}} < 410 \text{ GeV} \) (640 GeV).

These model-independent upper bounds have many implications. They improve the prospects for observation of weakly-interacting superpartners at the Tevatron and LHC. They also impact linear colliders, where the study of supersymmetry requires \( \sqrt{s} > 2M_{\text{LOSP}} \) (with the possible exception of associated neutralino production in stable LSP scenarios). Finally, these bounds provide fresh impetus for searches for lepton flavor violation, which is also mediated by sleptons and charginos/neutralinos.

We now turn to specific models. The supersymmetric contributions to \( a_{\mu} \) have been discussed in various supergravity theories [8,14], and more recently in models of gauge-mediated [10,14] and anomaly-mediated supersymmetry breaking [8,13].

We first consider the framework of minimal supergravity, in which the entire weak scale superparticle spectrum is fixed by four continuous parameters and one binary choice: \( m_{0} \), \( M_{1/2} \), \( A_{0} \), \( \tan \beta \), and sign (\( \mu \)), where the first three are the universal scalar, gaugino, and trilinear coupling masses at the grand unified theory (GUT) scale \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \). We relate these to weak scale parameters through two-loop renormalization group equations [16] with one-loop threshold corrections and calculate all superpartner masses to one-loop [17]. Electroweak symmetry is broken radiatively with a full one-loop analysis, which determines |\( \mu \)|.

In minimal supergravity, many potential low-energy effects are eliminated by scalar degeneracy. However, \( a_{\mu}^{\text{SUSY}} \) is not suppressed in this way and may be large. In this framework, \( \text{sign}(a_{\mu}^{\text{SUSY}}) = \text{sign}(\mu M_{1/2}) \). As is well-known, however, the sign of \( \mu \) also enters in the supersymmetric contributions to \( B \to X_{\gamma} \). Current constraints on \( B \to X_{\gamma} \) require \( \mu M_{1/2} > 0 \) if \( \tan \beta \) is large. In minimal supergravity, then, gaugino mass unification implies that a large discrepancy in \( a_{\mu} \) is only possible for \( a_{\mu}^{\text{SUSY}} > 0 \), in accord with the new measurement.

In Fig. 2 the 2σ allowed region for \( a_{\mu}^{\text{SUSY}} \) is plotted for \( \mu > 0 \). Several important constraints are also included: bounds on the neutralino relic density, the Higgs boson mass limit \( m_{h} > 113.5 \text{ GeV} \), and the 2σ constraint \( 2.18 \times 10^{-4} < B(B \to X_{\gamma} \gamma) < 4.10 \times 10^{-4} \).

For moderate \( \tan \beta \), the region preferred by \( a_{\mu}^{\text{SUSY}} \) is at low \( m_{0} \). Much of the favored region is excluded by the Higgs boson mass. However, the remaining region is consistent with the requirement of supersymmetric dark matter, and, intriguingly, is roughly that obtained in no-scale supergravity [18] and minimal gaugino-mediated [14] models. In contrast, for large \( \tan \beta \), there is a large allowed area that extends to large \( M_{1/2} \) and \( m_{0} \approx 1.5 \text{ TeV} \), and which also overlaps significantly with a region with desirable relic density. In focus point models with large and universal scalar masses [20], large \( \tan \beta \) is therefore favored. The cosmologically preferred regions of minimal supergravity are probed by many pre-LHC experiments [21]. Note, however, that the sign of \( \mu \) preferred by \( a_{\mu} \) implies destructive interference in the leptonic decays of the second lightest neutralino, and so the Tevatron search for trileptons is ineffective for 200 GeV < \( m_{0} < 400 \text{ GeV} \) [22].

We close by considering anomaly-mediated supersymmetry breaking [23]. One of the most robust and striking...
predictions of this framework is that the gaugino masses are proportional to the corresponding beta function coefficients, and so $M_{1,2}M_{3} < 0$. Consistency with the $B \to X_{s}\gamma$ constraint then implies that only negative $a_{\mu}^{\text{SUSY}}$ may have large magnitude, in contrast to the case of conventional supergravity theories [13].

In Fig. 3 we investigate how large a positive $a_{\mu}^{\text{SUSY}}$ may be in the minimal anomaly-mediated model. This model is parametrized by $M_{\text{aux}}, m_{0}, \tan \beta,$ and $\text{sign}(\mu)$, where $M_{\text{aux}}$ determines the scale of the anomaly-mediated soft terms, and $m_{0}$ is a universal scalar mass introduced to remove tachyonic sleptons. To get $a_{\mu}^{\text{SUSY}} > 0$, we choose $\mu M_{1,2} > 0$. We see, however, that the constraint from $B \to X_{s}\gamma$ is severe, as this sign of $\mu$ implies a constructive contribution from charginos to $B \to X_{s}\gamma$ in anomaly mediation. Even allowing a 1σ deviation in $a_{\mu}$, we have checked that for all $\tan \beta$, it is barely possible to obtain $2\sigma$ consistency with the $B \to X_{s}\gamma$ constraint. Minimal anomaly mediation is therefore disfavored. The dependence of this argument on the characteristic gaugino mass mechanism is therefore disfavored. The dashed (dotted) contour is excluded by $B(B \to X_{s}\gamma) < 4.10 \times 10^{-4}$, and the LSP is a stau to the left of the dashed line.

In conclusion, the recently reported deviation in $a_{\mu}$ is easily accommodated in supersymmetric models. Its value provides model-independent upper bounds on masses of observable superpartners and already discriminates between well-motivated models. We await the expected improved measurements with great anticipation.

Acknowledgments — This work was supported in part by the U. S. Department of Energy under cooperative research agreement DF–FC02–94ER40818.

[1] H. N. Brown et al. [Muon $(g - 2)$ Collaboration], hep-ex/0102017.
[2] J. Bailey et al. [CERN-Mainz-Daresbury Collaboration], Nucl. Phys. B150, 1 (1979).
[3] R. M. Carey et al., Phys. Rev. Lett. 82, 1632 (1999).
[4] H. N. Brown et al. [Muon $(g - 2)$ Collaboration], Phys. Rev. D 62, 091101 (2000) [hep-ex/0009022].
[5] A. Czarnecki and W. J. Marciano, hep-ph/0010194.
[6] R. M. Carey, talk at ICHEP00, Osaka, 1 August 2000; R. Frigl [Muon $(g - 2)$ Collaboration], hep-ex/0101014.
[7] For references to the early literature, see Ref. [1].
[8] J. L. Feng and T. Moroi, Phys. Rev. D 61, 095004 (2000).
[9] T. Moroi, Phys. Rev. D 53, 6595 (1996) [hep-ph/9512390]; Erratum, D 56, 4424 (1997).
[10] M. Carena, G. F. Giudice and C. E. Wagner, Phys. Lett. B390, 234 (1997) [hep-ph/9610233].
[11] T. Blazek, hep-ph/9912106.
[12] S. Ferrara and E. Remiddi, Phys. Lett. B53, 347 (1974).
[13] See, e.g., LEPFest, CERN, 10-11 October 2000, http://delphiiwww.cern.ch/~offline/physics_links/lepc.html.
[14] E. Gabrielli and U. Sarid, Phys. Rev. Lett. 79, 4752 (1997) [hep-ph/9707544]; K. T. Mahanthappa and S. Oh, Phys. Rev. D 62, 015012 (2000) [hep-ph/9908531].
[15] U. Chattopadhyay, D. K. Ghosh and S. Roy, Phys. Rev. D 62, 115001 (2000) [hep-ph/0006040].
[16] I. Jack et al., Phys. Rev. D50, 5481 (1994).
[17] D. M. Pierce, J. A. Bagger, K. Matchev and R. Zhang, Nucl. Phys. B491, 3 (1997) [hep-ph/9606211].
[18] A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. 145, 1 (1987).
[19] M. Schmaltz and W. Skiba, Phys. Rev. D 62, 095005 (2000) [hep-ph/0001172].
[20] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. 84, 2322 (2000) [hep-ph/9908309]; Phys. Rev. D 61, 075005 (2000) [hep-ph/9909334].
[21] J. L. Feng, K. T. Matchev and F. Wilczek, Phys. Lett. B 482, 388 (2000) [hep-ph/0004043]; Phys. Rev. D 63, 045024 (2001) [astro-ph/0008115].
[22] K. T. Matchev and D. M. Pierce, Phys. Rev. D 60, 075004 (1999) [hep-ph/9904282].
[23] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [hep-th/9810155].