Analytical consideration and computer simulation of DFWM

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Abstract. Degenerate four-wave mixing (DFWM) for co-propagating femtosecond laser pulses is considered in the axial-symmetric case for bulk medium with cubic nonlinear response, if pump-wave amplitudes are being equal. Computer simulation is based on the set of nonlinear Schrödinger equations describing this process. For the analytical consideration we use the framework of both plane wave approximation and long pulse duration approximation taking into account the phase matching. In opposite to widely used approach, based on the pump-waves non-depletion, the problem invariants are used for analytical solution developing. The solution demonstrates various DFWM modes existence and allows us to provide full analysis of the problem in dependence of its parameters. Analytical solution and derived pulse interaction modes can explain complicated regime of DFWM, which may appear at different intensities of interacting waves.

1. Introduction
As it is well-known, a problem of four-waves mixing (FWM) is permanently under investigation [1-19]. In particular, such kind of laser pulse interaction is widely used for both a substance spectroscopy and a medium diagnostic. Another very important application of FWM is caused by a possibility of FWM using for optical frequency conversion to any wavelength with a careful fiber design, and therefore it is a very attractive.

Obviously there are many physical factors which influence on FWM modes. To understand the FWM features one needs to make computer simulation of this problem or various analytical approaches developing. Obviously, the last is more preferable because it allows to see a visual dependence of a frequency conversion efficiency. Usually the analytical consideration is based on the pump-waves non-depletion approximation which is valid at the initial stage of the pulses interaction. This approach possesses the essential restrictions. For example, the interacting wave energy does not preserve. Its more significant limitation arises out of neglecting of self-modulation and cross-modulation of the pumping waves. As a result of this, the phase difference between pumping waves and other two waves becomes wrong and therefore an evolution of the interacting waves is wrong also. To revise this situation we develop new approach for analytical solution of the problem: we use conservation laws of the FWM and both plane wave approximation and long pulse duration approximation. In this case we accurate take into account the interacting waves phase changing. Hence, the analytical solution of the problem becomes more adequate to a physical situation if pulse dispersion and beam diffraction is weak at a medium length under consideration. In this paper we compare the problem analytical solution made in [20, 21] with computer simulation results obtained using the solution of four nonlinear Schrödinger equations describing FWM of axially-symmetry beams with taking into consideration a second order dispersion of interacting pulses, their self- and
cross–modulation and beam diffraction. Let us notice that early we have used with success the similar approach for analysis of frequency conversion efficiency when the laser pulses interact in a medium with quadratic nonlinear response [22-26]).

2. Problem statement

We consider the FWM of co-propagating laser pulses with current frequencies denoted as \( \omega_1, \omega_2, \omega_3, \omega_4 \) in a medium with cubic nonlinear response. In the framework of slowly varying envelope the considered process is governed by the set of dimensionless nonlinear Schrödinger equations with respect to complex amplitudes \( A_1, A_2, A_3, A_4 \) of interacting waves:

\[
L_j A_j = i \alpha_j (F_j + 2A_j A_{-j} e^{i \Lambda k z}), j = 1, 3, \quad L_j A_j = i \alpha_j (F_j + 2A_j A_{-j} e^{-i \Lambda k z}), j = 2, 4, \quad (1)
\]

\[
L_j = \frac{\partial}{\partial z} + i D_j \Lambda_j, \quad F_j = A_j (2(|A_j|^2 + |A_{-j}|^2 + |A_{+j}|^2 - |A_{-j}|^2)), 0 < z \leq L, \quad 0 < r < L, \quad 0 < t < L_t
\]

with the following initial and boundary conditions for complex amplitudes

\[
A_j(z = 0, r, t) = A_{j0}, 0 \leq r \leq L, \quad 0 \leq t \leq L_t, \quad A_j \bigg|_{t=0} = \frac{\partial A_j}{\partial r} \bigg|_{r=0} = A_{j0} \bigg|_{z=L} = 0, \quad j = 1-4. \quad (2)
\]

We assume that there is a group velocity matching. In Eqs. (1) the dimensionless parameters and functions can be expressed through the physical ones in the following manner:

\[
z = z'/l_{diff}, \quad t = t'/l_{t}, \quad r = r'/a_l, \quad l_{diff,j} = 2k_j a_l^2, \quad D_j = l_{diff,j}/l_{diff,j}, \quad \Lambda_j = l_{diff,j}/l_{self,j}, \quad (3)
\]

\[
l_{self,j} = \frac{2\tau_t^2}{d^2 k(\omega_j)/d\omega^2}, \quad l_{diff,j} = \frac{n(\omega_j) \cdot \Lambda_j}{3\pi^2 \chi^{(3)} A_{j0}}, \quad \Delta k = (k_j + k_{-j} - k_{+j} k_4). \quad (4)
\]

Above \( z' \) is a physical longitudinal coordinate, along which the optical radiation propagates, \( z \) is measured in units of the diffraction length \( l_{diff,j} \) of the laser beam with the frequency \( \omega_j \) (the first wave). Parameters \( \tau_t, a_l \) denote the pulse duration and the beam radius of this wave at input section, correspondingly. Variable \( t' \) denotes the physical time. Time \( t \) is measured in unit of the first wave pulse duration. \( L_t \) is a dimensionless time interval, during which the interaction of waves is analyzed, \( L_j \) is a dimensionless length of nonlinear medium. Variable \( r \) is a transverse coordinate normalized by the radius \( a_l, \) \( L_r \) is the maximum value of this coordinate, \( \Lambda_j = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \) is the transverse Laplacian written in the axially symmetric case. Functions \( A_{j0}(r,t) \) describe the temporal and spatial distributions of a complex amplitude at input section of a medium. \( k_j \) is a wave number of \( j \)-wave. Parameter \( \Delta k \) is a phase mismatching on the first wave diffraction length, \( A_j \) are complex amplitudes of waves, normalized on the square root of the first wave intensity \( I_{10}^{0.5} \). \( A_{j0}(r,t) \) is a spatial-temporal distribution of the complex amplitude at the input section of a medium, \( A_{j0} \) are the dimensionless amplitudes of incident waves. We consider below that a wave with the frequency \( \omega_{l0} \) possesses zero-value amplitude \( (A_{l0} = 0), \) and the signal wave amplitude \( (A_{s0}) \) (the frequency \( \omega_2 \)) is not equal to zero, and amplitudes of the pump waves are equal each other \( (A_{l0} = A_{s0}) \) as well as the nonlinear
coefficients \( \alpha_i = \alpha_j \) because of an equality of the pump wave frequencies (\( \omega_1 = \omega_2 \)). Coefficient \( D_j \) characterizes the group velocity dispersion of waves, \( \tilde{D}_j \) is a diffraction coefficient. It should be stressed that a dependence of a cubic susceptibility (\( \chi^{(3)} \)) of a medium on the optical frequency is neglected. The similar assumption is valid for the refractive index of interacting waves. Parameters \( \alpha_j \) characterize self-action of the laser beams due to a cubic nonlinear response.

Below we will take into account the following relation for nonlinear coefficients of the problem:

\[
\alpha_1 + \alpha_3 = \alpha_2 + \alpha_4. \tag{3}
\]

Taking into account the equality of nonlinear coefficients for the pump waves, the equality (3) reduces to the next equality:

\[
2\alpha_1 = \alpha_2 + \alpha_4. \tag{4}
\]

3. Conservation laws (invariants) for the case of interacting waves phase matching (\( \Delta k = 0 \))

In this paper we compare computer simulation results with an analytical solution of the problem (1) in the case of a phase matching for the interacting waves. Therefore, we write invariants (conservation laws) for this case.

First invariant is energy one and it is written in the form:

\[
I_1 = \int_0^L \int_0^L \left( \frac{|A_1|^2}{\alpha_1} + \frac{|A_2|^2}{\alpha_2} + \frac{|A_3|^2}{\alpha_3} + \frac{|A_4|^2}{\alpha_4} \right) rdrdt. \tag{5}
\]

However, as it is well-known, there are other invariants which are similar to (5) and they are written as:

\[
I_{14} = I_{34} = \int_0^L \int_0^L \left( \frac{|A_1|^2}{\alpha_1} + \frac{|A_4|^2}{\alpha_4} \right) rdrdt = \int_0^L \int_0^L \left( \frac{|A_3|^2}{\alpha_3} + \frac{|A_4|^2}{\alpha_4} \right) rdrdr,
\]

\[
I_{24} = \int_0^L \int_0^L \left( \frac{|A_1|^2}{\alpha_2} - \frac{|A_4|^2}{\alpha_4} \right) rdrdt.
\]

The next invariant is Hamiltonian of the equation set (1) and it is seen as

\[
I_3 = \int_0^L \int_0^L \sum_{j=1}^4 \left( \frac{D_j}{\alpha_j} \frac{\partial A_j}{\partial t} + \tilde{D}_j \frac{\partial A_j}{\partial t} \right) + (4 \text{Re}(A_j A^*_j A_{j+1} A^*_{j+1})) + \frac{1}{2} (|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2) rdrdrdt.
\]

To obtain the analytical solution we essentially use these invariants. Moreover, they are used for a control of computer simulation results: we develop conservative finite-difference scheme for a numerical solution of the problem (1).

4. Analytical solution f the problem in the frame-work of long pulse duration and plane wave approximation

Many special features of the FWM process can be clarified by using the long pulse and plane wave approximation. Taking into account the well-known representation of complex amplitude:

\[
A_j = a_j \exp(i\varphi_j), \quad j = 1 - 4,
\]

where an amplitude \( a_j \) and phase \( \varphi_j \) of the waves are real functions. In this case, the waves interacting can be described by the following equations:

\[
\frac{da_1}{dz} + 2\alpha_1 \cdot a_2 a_3 a_4 \sin \varphi = 0, \quad \frac{da_2}{dz} - 2\alpha_2 \cdot a_1 a_3 a_4 \sin \varphi = 0,
\]
\[
\frac{d\alpha_i}{dz} + 2\alpha_i \cdot a_{i2} \cdot a_{i4} \sin \varphi = 0, \quad \frac{d\alpha_4}{dz} - 2\alpha_4 \cdot a_{i2} \cdot a_{i3} \sin \varphi = 0,
\]

\[
\frac{d\varphi}{dz} + 2 \cos \varphi \cdot (\alpha_1 \cdot a_{12} \cdot a_{14} - \alpha_2 \cdot a_{13} \cdot a_{14} + \alpha_3 \cdot a_{14} - \alpha_4 \cdot a_{14}) - a_{1} \cdot a_{i1}^2 + a_{1} \cdot a_{i2}^2 - a_{1} \cdot a_{i3}^2 + a_{1} \cdot a_{i4}^2 = 0, \quad \varphi = \varphi_2 - \varphi_1 + \varphi_4 - \varphi_3
\]

with initial conditions

\[a_i|_{z=0} = a_{i0} = a_{i0}, \quad a_2|_{z=0} = a_{20}, \quad a_4|_{z=0} = a_{40} = 0, \quad \varphi|_{z=0} = \varphi_0.
\]

In new notations, the invariants can be rewritten in the next form:

\[I_i = a_i^2 + a_3^2 + a_4^2, \quad a_i^2 = a_i \cdot I_{14} - \frac{a_1}{a_4}, \quad a_2^2 = a_2 \cdot I_{24} + \frac{a_2}{a_4}, \quad a_3^2 = a_3 \cdot I_{34} - \frac{a_3}{a_4}, \quad a_4^2 = a_4 \cdot I_{44}, \quad \tilde{I}_4 = 4a_2 \cdot a_4 \cdot a_4 \cdot \cos \varphi + a_i^2 \tilde{\xi} + a_i^2 \eta = 0,
\]

where the invariant \( \tilde{I}_4 \) follows from the invariant \( I_4 \) and introduced parameters are defined as

\[
\tilde{\xi} = \frac{a_1}{a_4} \cdot (I_{14} - I_{24}) + \frac{a_3}{a_4} \cdot (I_{34} - I_{24}) - \frac{a_1}{a_4} \cdot (I_{14} + I_{34}) + \frac{a_2}{a_4} \cdot \frac{a_3}{a_4} \cdot I_{14} + \frac{a_2}{a_4} \cdot I_{24} + \frac{a_3}{a_4} \cdot I_{34},
\]

\[
\tilde{\eta} = \frac{a_2}{a_4} \cdot \frac{a_1}{a_4} - \frac{a_3}{a_4} \cdot \frac{a_3}{a_4} - \frac{a_2}{a_4} \cdot \frac{a_3}{a_4} + \frac{a_2}{a_4} \cdot \frac{a_3}{a_4} - \frac{a_2}{a_4} \cdot \frac{a_3}{a_4}.
\]

Using the invariant \( \tilde{I}_4 \), one can write the algebraic equation, containing amplitudes of the interacting waves and \( \cos \varphi \) instead the differential equation with respect to the phase difference \( \varphi \). This allows us to eliminate a term \( \sin \varphi \) in equation, concerning the interacting wave amplitudes, and write the fourth wave intensity \( p_4(z) = a_i^2(z) \) evolution in the form:

\[
a_4 \sqrt{\tilde{I}_4} = \frac{p_4}{\int f(x) dx}, \quad f = \text{sgn}(\lambda) \cdot x \left( x^3 + c_1 x^2 + c_2 x + c_3 \right).
\]

Above we introduced new notations:

\[
\lambda = 16 \frac{a_1 a_2 a_3}{a_4} - \eta^2, \quad c_1 = 16 \frac{a_1 a_2 a_3}{a_4} \cdot \left( I_{24} - I_{14} - I_{34} \right) - 2 \eta \tilde{\xi}) / \lambda,
\]

\[
c_2 = 16 \frac{a_1 a_2 a_3}{a_4} \cdot \left( I_{14} - I_{14} - I_{24} - I_{24} - I_{34} \right) - \tilde{\eta}^2 / \lambda, \quad c_3 = 16 a_1 a_2 a_3 \cdot I_{14} I_{24} I_{34} / \lambda.
\]

The intensity \( p_4(z) \) evolution is defined by roots of the equation \( f(x) = 0 \). We see from expression (9) and (10) that the root properties depend strongly on the problem parameters. One of the roots is always equal to 0 and others can be found from a cubic equation, which may possess either three real roots or one real root and two complex roots. Moreover, the fourth wave changing also depends on the sign of the parameter \( \lambda \) because of its influence on the radicand sign. Obviously, for the solution existence it is necessary a validity of the inequality:

\[
f(x) = \text{sgn}(\lambda) \cdot x \left( x^3 + c_1 x^2 + c_2 x + c_3 \right) \geq 0.
\]

In general case, the fourth wave intensity changing is described by an elliptical function. Thus, the FWM mode is governed by the relationship between the parameters \((a_{i0}, a_{20}, a_2)\). Below we write only two different solutions because in our previous papers we describe various solutions of the problem in detail.
If the equation \( f(x) = 0 \) has four real roots, satisfying the following relation: \( x_1 = 0 < x_2 < x_3 < x_4 \) and a sign of \( \lambda \) is negative, for zero-value of incident fourth wave intensity, the intensity \( p_4 \) belongs to the interval \([x_1 = 0, x_2]\) at the fourth wave propagation (this generation mode is named as low efficiency mode). In this case, the fourth wave intensity evolution is described by:

\[
p_4(z) = \left| u_4(z) \right|^2 = \frac{x_1(x_1-x_2)+x_2(x_1-x_3)\sin^2(0.5\omega_{\tau}\sqrt{4\sigma(x_2-x_3)(x_1-x_2)})}{x_1-x_2+(x_1-x_3)\sin^2(0.5\omega_{\tau}\sqrt{4\sigma(x_2-x_3)(x_1-x_2)})}, \quad m = \frac{(x_1-x_2)(x_4-x_1)}{(x_1-x_3)(x_4-x_2)},
\]

where \( \sin \) denotes the elliptical sinus.

If four real roots satisfy the following relation: \( x_1 < x_2 = 0 < x_3 < x_4 \) and a sign of \( \lambda \) is positive, then achievable value of the fourth wave intensity \( p_4 \) belongs to the interval \([x_2 = 0, x_1]\) if the incident intensity of the fourth wave satisfies the condition: \( p_4(z=0) = 0 \). The fourth wave intensity \( p_4 \) evolution along the medium is written in the form of elliptical sinus (sn):

\[
p_4(z) = \left| u_4(z) \right|^2 = \frac{x_2(x_1-x_2)-x_1(x_3-x_4)\sin^2(0.5\omega_{\tau}\sqrt{4\sigma(x_2-x_3)(x_1-x_2)})}{x_1-x_2-(x_3-x_4)\sin^2(0.5\omega_{\tau}\sqrt{4\sigma(x_2-x_3)(x_1-x_2)})}, \quad m = \frac{(x_1-x_2)(x_4-x_1)}{(x_1-x_3)(x_4-x_2)}.
\]

To clarify conditions of the used approach for the analytical solution developing we construct the conservative finite-difference scheme for the problem (1).

5. **Conservative finite-difference scheme**

The conservative finite-difference scheme for problem (1-2) is constructed as follows. In the domain \( \Omega = \Omega_z \times \Omega_r \times \Omega_t = [0 \leq z \leq L_z] \times [0 \leq r \leq L_r] \times [0 \leq t \leq L_t] \) we introduce the uniform grid \( \omega = \omega_z \times \omega_r \times \omega_t : \omega_z = \{z_n = nh_z, n = 0, N_z, h_z = L_z / N_z\}, \omega_r = \{r_k = (k+0.5)h_r, k = 0, N_r, h_r = L_r / (N_r+0.5)\}, \omega_t = \{t_I = l\tau, l = 0, N_t, \tau = L_t / N_t\}\).

The grid functions \( A_j = A_{jk} = A_{jk}(z_n, r_k, t_I) \) are defined on the grid \( \omega \). For brevity, we introduce the index-free notations:

\[
\hat{A}_j = A_{jk}(z_{n+1}, r_k, t_I), \quad \hat{A}_j = 0.5(A_j + \hat{A}_j), \quad A_j \|^2 = 0.5( | A_j |^2 + | \hat{A}_j |^2 ), \quad j = 1 \sim 4.
\]

The Laplacian with respect to \( \tau \) and \( r \) coordinates is approximated as

\[
w_{\hat{\omega}} = \frac{w(z_n, r_k, t_{I+1}) - 2w(z_n, r_k, t_I) + w(z_n, r_{k+1}, t_I)}{\varepsilon^2} A_j = \frac{1}{r_k(r_{k+1} - r_k)} \left[ \frac{w(z_n, r_{k+1}, t_I) - w(z_n, r_k, t_I)}{r_{k+1} - r_k} - \frac{w(z_n, r_k, t_I) - w(z_n, r_{k-1}, t_I)}{r_k - r_{k-1}} \right], \quad j = 1 \sim 4.
\]

at the internal point \((z_n, r_{k}, t_{I})\) of the grid \( \omega \). There a function \( w \) means one of the grid functions \( A_{jk} \).

Using the notations introduced above, the finite-difference scheme for the problem (1) in the case of the phase matching can be written in the form:

\[
\bar{L}_u = \frac{\hat{u} - u}{\varepsilon^2} + \frac{\partial}{\partial z} A_{j} 0.5 \left( | A_j |^2 + | \hat{A}_j |^2 \right), \quad \bar{L}_u = 0.5 \left( \frac{\partial}{\partial z} A_j 2 \left( | A_j |^2 + | \frac{\partial}{\partial z} A_j |^2 + | \frac{\partial}{\partial r} A_j |^2 + | \frac{\partial}{\partial t} A_j |^2 \right) \right) - \frac{\partial}{\partial z} A_j 0.5 \left( | A_j |^2 + | \hat{A}_j |^2 \right),
\]

\[
\bar{L}_A_j = i\alpha_j (\bar{F}_j + A_j A_{j-1}) \quad A_j = 0.5 \left( \frac{\partial}{\partial z} A_j 2 \left( | A_j |^2 + | \frac{\partial}{\partial z} A_j |^2 + | \frac{\partial}{\partial r} A_j |^2 + | \frac{\partial}{\partial t} A_j |^2 \right) \right) - \frac{\partial}{\partial z} A_j 0.5 \left( | A_j |^2 + | \hat{A}_j |^2 \right), \quad j = 1,3,
\]

\[
\bar{L}_A_j = i\alpha_j (\bar{F}_{j+1} + A_j A_{j-1}) \quad A_j = 0.5 \left( \frac{\partial}{\partial z} A_j 2 \left( | A_j |^2 + | \frac{\partial}{\partial z} A_j |^2 + | \frac{\partial}{\partial r} A_j |^2 + | \frac{\partial}{\partial t} A_j |^2 \right) \right) - \frac{\partial}{\partial z} A_j 0.5 \left( | A_j |^2 + | \hat{A}_j |^2 \right), \quad j = 2,4
\]
with corresponding approximation of the boundary conditions and initial definition of the mesh functions.

Since the equations (15) are nonlinear ones, then we apply an iteration process:

\[
\overline{L}_{j} A_{j}^{s+1} = i\alpha_{j} (\overline{F}_{j} + A_{2} A_{1} A_{s-j}), \quad j = 1, 3, \quad \overline{L}_{j} A_{j} = i\alpha_{j} (\overline{F}_{j} + A_{1} A_{3} A_{s-j}), \quad j = 2, 4. \tag{16}
\]

At boundaries points, the mesh functions are taken from \( s+1 \) iteration. At zero iteration \( (s = 0) \) they are taken as

\[
A_{j}^{s} = A_{j}, \quad j = 1-4
\]

and the iteration process is terminated if the following inequality:

\[
\max_{k,l} |A_{j}^{s} - A_{j}^{s+1}| \leq \varepsilon \max_{k,l} |A_{j}^{s}| + \delta, \quad j = 1-4
\]

is valid. Here \( \varepsilon, \delta \) are positive parameters which defines the iteration process accuracy.

6. Comparison of computer simulation results with analytical solution
In figure 1 the analytical solution (12) and the corresponding computer simulation results are depicted. We see very good coincidence of both dependencies. In figure 2 the comparison is shown for different values of diffraction and dispersion coefficients: \( \tilde{D}_{j} \) and \( D_{j} \). We see that with increasing

![Figure 1](image1.png)

**Figure 1.** Analytical solution (a) and computer simulation result (b) corresponding to parameters \( \alpha_{1} = \alpha_{2} = \alpha_{3} = 1, \quad a_{00} = A_{00} = a_{30} = A_{30} = 1, \quad a_{20} = A_{20} = 0.2, \quad a_{02} = A_{02} = 0, \quad D_{14} = 10^{-12}, \quad \tilde{D}_{0} = 10^{-12}. \)
Figure 2. Analytical solution (dash line) and computer simulation result (solid line) corresponding to parameters: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$, $a_{10} = A_{10} = a_{30} = A_{30} = 1$, $a_{20} = A_{20} = 0.2$, $a_{40} = A_{40} = 0$ and $D_j = \tilde{D}_{\eta} = 10^{-6}$ (a), $D_j = \tilde{D}_{\eta} = 10^{-5}$ (b), $D_j = \tilde{D}_{\eta} = 10^{-5}$ (c), $D_j = \tilde{D}_{\eta} = 10^{-5}$ (d), $D_j = \tilde{D}_{\eta} = 10^{-4}$ (e).

Figure 3. Analytical solution (dash line) and computer simulation result (solid line) corresponding to parameters: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$, $a_{10} = A_{10} = a_{30} = A_{30} = 1$, $a_{20} = A_{20} = 0.2$, $a_{40} = A_{40} = 0$ and $\tilde{D}_{\eta} = 10^{-12}$, $D_j = 10^{-6}$ (a), $10^{-5}$ (b), $5 \cdot 10^{-5}$ (c), $10^{-4}$ (d).

values of diffraction of a laser beam and dispersion of a laser pulse the analytical solution deviation from the computer simulation result increases. Nevertheless, coinciding of both curvatures takes place along big distance of the optical beam propagation. To clarify a playing role of a pulse dispersion and
a beam diffraction in breaking of the long pulse duration approximation and plane wave approximation we make computer simulation with unchangeable value of the diffraction coefficient with the dispersion coefficient changing and vice versa. The corresponding comparison is shown in figures 3, 4. We see that a second order dispersion influence results in the pulse compression and consequently, the maximal intensity of the pulse increases and this leads to the intensity oscillation period decreasing (see formula (12)).

The beam diffraction also leads to the maximal intensity enhancing. However, this enhancing is not pronounced along a pulse propagation distance under consideration.

![Graphs showing intensity vs Z for different parameters]

**Figure 4.** Analytical solution (dash line) and computer simulation result (solid line) corresponding to parameters: $a_1 = a_2 = a_3 = a_4 = 1$, $a_{40} = A_{10} = a_0 = A_0 = 1$, $a_{20} = A_{20} = 0.2$, $a_{40} = A_{40} = 0$ and $D_j = 10^{-12}$, $\tilde{D}_j = 10^5$ (a), $10^2$ (b), $5 \cdot 10^{-5}$ (c), $10^{-4}$ (d).

7. Conclusions

Thus, in this paper we made a comparison of the analytical solution, obtained in the frame-work of long pulse and plane wave approximation for FWM problem, with computer simulation results, obtained by using the conservative finite-difference scheme proposed for this problem solution.

We consider a case of equality of the pump wave intensities at co-propagating FWM under phase matching condition. We showed that along big distance of a laser pulse propagation both solutions coincide. Main reason of a deviation of the solutions is an influence of second order dispersion. Beam diffraction influences less than pulse dispersion till propagation distance under analysis.

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