SWITCHING MECHANISM-BASED EVENT-TRIGGERED FUZZY ADAPTIVE CONTROL WITH PRESCRIBED PERFORMANCE FOR MIMO NONLINEAR SYSTEMS

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Abstract. This paper investigates the switching mechanism-based event-triggered fuzzy adaptive control issue of multi-input and multi-output (MIMO) nonlinear systems with prescribed performance (PP). Utilizing fuzzy logic systems (FLSs) to approximate unknown nonlinear functions. By using the switching threshold strategy, the system has more flexibility in strategy selection. The proposed control scheme can better solve the communication resource limitation. On account of the Lyapunov stability theory, the stability of the controlled system is proved. And all signals of the controlled system are bounded. Moreover, the tracking errors are controlled in a diminutive realm of the origin within the PP bounded. Simultaneously, the Zeno behavior is avoided. Finally, illustrate the effectiveness of the control scheme that has been proposed by demonstrating some simulation consequences.

1. Introduction. Several control schemes were developed in [1-4] for some unknown nonlinear SISO systems based on adaptive fuzzy approximation theory. In the real life, it is not univariate systems that are the majority, most systems are essentially multivariable. Each output of the multivariable systems are controlled and influenced by multiple inputs simultaneously. This phenomenon is called coupling or cross influence. Their control problems become sophisticated and challenging while multivariable systems are uncertain and nonlinear. Adaptive fuzzy control problems for unknown nonlinear MIMO systems were studied in [5-7].

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For the past several decades, the backstepping control scheme has become one of the most formidable techniques for nonlinear systems, meanwhile, it also has attracted substantial attention [8-9]. Since the nonlinear function is unknown in many practical applications, therefore, the proposed FLSs by some scholars in [10-12] and the proposed neural networks (NNs) in [12] are used to solve the approximation problem of the unknown functions.

In general, the most primitive methods are time-triggered control or periodic sampling control [13]. However, the time triggered control requires a conservative sampling period to ensure the control performance of system, which often gives rise to needless samplings, resulting in the dissipate of additional communication resources. Whereas the root cause of all this is that scholars do not consider the limitation of communication resources.

Nowadays, with the vigorous development of network technology, event-triggered control (ETC) has been attached more importance to the control field. It can be used to settle a matter of waste of communication resources. In [14], the authors have proposed the ETC scheme for nonlinear multi-agent systems (MASs) subjected to denial-of-service (DoS) attacks. It can be seen from [15], the authors have designed ETC scheme for non-affine systems with full state constraints. As mentioned in [16], the ETC approach of MIMO nonlinear systems with nonparametric uncertainties was presented. The authors in [17] have designed ETC scheme for MIMO nonlinear discrete-time systems. As appeared in [18], the ETC strategy is proposed, which is mainly handled the distributed tracking problem for MASs. However, none of the authors used a switching threshold strategy in the above description, which impels us continuously to study this paper.

Obviously, switching mechanism-based fuzzy adaptive ETC with PP is preferred for MIMO nonlinear systems from the above discussions. The problem of unknown functions are solved by FLSs. Combining the DSC technique with the backstepping technique, the problem of “complexity explosion” in the recursive design process of traditional adaptive backstepping has been overcome. Compared to the current works, the main feature of this article is that the proposed switching mechanism is applied in MIMO systems for the first time. The switching mechanism is the combination of fixed threshold strategy (FTS) and relative threshold strategy (RTS). Relatively speaking, the switching mechanism can be more flexible to select an appropriate mechanism according to system requirements, which can save communication resources more than the FTS or the RTS alone.

The remainder of this paper is distributed as follows. Section 2 describes the problem formulations and the preliminaries. Section 3 details the design procedure for the event-triggered controller and stability analysis. Section 4 shows the simulation results. Eventually, conclusions are described in Section 5.

2. Problem formulations and preliminaries.

2.1. Problem statement. Consider the following MIMO nonlinear systems as:

\[ \dot{x}_{i,j_i} = x_{i,j_i+1} + f_{i,j_i}(\bar{x}_{i,j_i}), \quad j_i = 1, 2, \cdots, m_i - 1 \]

\[ \dot{x}_{i,m_i} = f_{i,j_i}(X) + u_i \]

\[ y_i = x_{i,1}, \quad i = 1, 2, \cdots, n \] \hspace{1cm} (1)

where \( \bar{x}_{i,j_i} = [x_{i,1}, \cdots, x_{i,j_i}]^T \in \mathbb{R}^{j_i} (i = 1, 2, \cdots, n; j_i = 1, 2, \cdots, m_i) \) are the state of the \( i \)th subsystem, \( f_{i,j_i}(\cdot) \) is the unknown nonlinear functions with \( f_{i,j_i}(0) = 0 \). \( u_i \in \mathbb{R} \) and \( y_i \in \mathbb{R} \) describe the control input and output of the \( i \)th subsystem,
respectively. \( X = [x_{1,1}, \ldots, x_{1,m_1}, x_{2,1}, \ldots, x_{2,m_2}, x_{n,1}, \ldots, x_{n,m_n}]^T \) are the system states.

**Assumption 2.1.** [16] The smooth function \( y_{i,r} \) is referred to as the reference signal and \( y_{i,r}^{(j)}(i = 1, 2, \ldots, n; j_1 = 1, 2, \ldots, m_i) \) are bounded and continuous.

2.2. **Fuzzy logic systems.** In this paper, since the unknown nonlinear continuous functions \( f_{i,j,i}(\cdot) \) are contained in the system (1), thus, FLSs are used to dispose of the unknown functions.

**Lemma 2.2.** [19] On \( \Omega \) set, define a continuous function \( \Psi(x) \), there exists \( Y(x) = \vartheta^T \varphi(x) \), one has

\[
\sup_{x \in \Omega} |\Psi(x) - \vartheta^T \varphi(x)| \leq \delta
\]

where \( \delta \) is a given constant, \( \varphi(x) = [\psi_1(x), \ldots, \psi_N(x)]^T \), \( \vartheta = [\vartheta_1, \vartheta_2, \ldots, \vartheta_N]^T \) is the parameter vector and \( \vartheta_i \) is the inference result variable. \( \vartheta^* \) is the ideal parameter vector for FLS. Then, the optimal argument can be written as

\[
\vartheta^* = \arg \min_{\vartheta \in \mathbb{R}^n} \sup_{x \in \Omega} |\Psi(x) - \vartheta^T \varphi(x)| \leq \delta
\]

Since \( f_{i,j,i}(\bar{x}_{i,j,i}) \) are unknown. From Lemma 2.2 produces

\[
f_{i,j,i}(\bar{x}_{i,j,i})|\vartheta_{i,j,i}) = \vartheta_{i,j,i}^T \varphi_{i,j,i}(\bar{x}_{i,j,i})
\]

Take advantage of FLSs for MIMO systems, FLSs for MIMO systems, the approximation error of MIMO nonlinear function is

\[
\tilde{\delta}_{i,j,i} = f_{i,j,i}(\bar{x}_{i,j,i}) - f_{i,j,i}(\bar{x}_{i,j,i})|\vartheta_{i,j,i}^*), 1 \leq i \leq n
\]

where \( \tilde{\delta}_{i,j,i} \) satisfies \( |\tilde{\delta}_{i,j,i}| \leq \tilde{\delta}_{i,j,i}^* \), and \( \tilde{\delta}_{i,j,i}^* \) is an unknown constant.

2.3. **Prescribed performance.** Following the idea of [20], the PP function can be written as:

\[
-\ell_{i,\min} h_i(t) < \zeta_{i,1} < \ell_{i,\max} h_i(t), \quad \forall t \geq 0
\]

where \( \ell_{i,\min} > 0 \) and \( \ell_{i,\max} > 0 \) are design parameters, \( h_i(t) = (h_{i0} - h_{i\infty}) e^{-A_i t} + h_{i\infty} \) with positive design parameters \( h_{i\infty} \) and \( A_i \). \( h_{i0} = h_i(0) \), and \( h_{i0} \) is selected such that \( h_{i0} > h_{i\infty} \). \( \zeta_{i,1} = x_{i,1} - y_{i,r} \) is the tracking error, designed the appropriate parameters such that the inequality holds, \( -\ell_{i,\min} h_i(0) < \zeta_{i,1}(0) < \ell_{i,\max} h_i(0) \).

The control performance in (6) can be realized by the equivalent unconstrained signal obtained from the constraint tracking error transformation, so we define

\[
\zeta_{i,1}(t) = \Lambda_i(v_i(t)) h_i(t), \quad \forall t \geq 0
\]

where \( v_i \) is the transformed signal, \( \Lambda_i(v) \) is a strictly increasing, and

\[
\Lambda_i(v) = \frac{\ell_{i,\max} e^{v_i} - \ell_{i,\min} e^{-v_i}}{e^{v_i} + e^{-v_i}}
\]

From (7) and (8) produce

\[
v_i(t) = \Lambda_i^{-1}\left(\frac{\zeta_{i,1}(t)}{h_i(t)}\right) = \frac{1}{2} \ln \frac{\Lambda_i + \ell_{i,\min}}{\ell_{i,\max} - \Lambda_i}
\]
and
\[
\dot{\nu}_i(t) = \frac{1}{2} \ln \frac{\Lambda_i + \ell_{i, \text{min}}}{\ell_{i, \text{max}} - \Lambda_i} + \Gamma_i
\]
\[
= \frac{1}{2} \left( \frac{1}{\ell_{i, \text{min}} + \Lambda_i} + \frac{1}{\ell_{i, \text{max}} - \Lambda_i} \right) \dot{\gamma}_i
\]
\[
= \frac{1}{2} \left( \frac{1}{\ell_{i, \text{min}} + \Lambda_i} - \frac{1}{\Lambda_i - \ell_{i, \text{max}}} \right) \frac{\dot{\gamma}_i h_i - \dot{\gamma}_1 h_1}{h_i}
\]
\[
= \frac{1}{2h_i} \left( \frac{1}{\ell_{i, \text{min}} + \Lambda_i} - \frac{1}{\Lambda_i - \ell_{i, \text{max}}} \right) \dot{\gamma}_i (\dot{\gamma}_1 - \frac{\dot{\gamma}_1 h_i}{h_i})
\]
(10)

where \( \kappa_i = \frac{1}{2h_i} \left( \frac{1}{\ell_{i, \text{min}} + \Lambda_i} - \frac{1}{\Lambda_i - \ell_{i, \text{max}}} \right) \).

The state transformation is given by
\[
z_{i,1}(t) = v_i(t) - \frac{1}{2} \ln \frac{\ell_{i, \text{min}}}{\ell_{i, \text{max}}}
\]
(11)

Then we can obtain
\[
\dot{z}_{i,1}(t) = \kappa_i (\dot{\gamma}_i - \frac{\dot{\gamma}_1 h_i}{h_i})
\]
(12)

Similar to [20, 21], the PP as exhibited in (6) is satisfied while \( z_{i,1}(t) \) is bounded.

3. Control design and stability analysis. In this part, we present the design method of the adaptive fuzzy event-triggered controller by backstepping design procedure. Firstly, under the backstepping structure, there are coordinate transformations as follows
\[
\dot{\zeta}_{i,1} = x_{i,1} - y_{i,r}
\]
\[
z_{i,j} = x_{i,j} - \nu_{i,j}
\]
\[
\pi_{i,j} = \nu_{i,j} - \alpha_{i,j-1}, \quad j = 2, \cdots, m_i
\]
where \( \zeta_{i,1} \) is the tracking error, \( \pi_{i,j} \) is output error of the first-order filter, \( z_{i,j} \) is the virtual surface error, \( \nu_{i,j} \) is a state variable, \( \alpha_{i,j-1} \) is the virtual control signal. Define \( \tilde{\delta}_{i,j} = \dot{\theta}_{i,j} - \dot{\eta}_{i,j}, \quad \hat{\delta}_{i,j} \) is the estimation of \( \dot{\theta}_{i,j} \), where \( j = 1, \cdots, m_i \).

**Step 1.** It follows from (12) and (13) that
\[
\dot{z}_{i,1}(t) = \kappa_i (\dot{\gamma}_i - \frac{1}{h_i} \dot{\gamma}_1 h_i)
\]
\[
= \kappa_i (\dot{x}_{i,1} - \dot{y}_{i,r} - \frac{1}{h_i} \dot{\gamma}_1 h_i)
\]
\[
= \kappa_i (z_{i,2} + \pi_{i,1} + \alpha_{i,1} + f_{i,1} - \frac{1}{h_i} \dot{\gamma}_1 h_i - \dot{y}_{i,r})
\]
\[
= \kappa_i (z_{i,2} + \dot{\theta}_{i,1}^T \psi_{i,1} + \alpha_{i,1} + \pi_{i,2} - \hat{\delta}_{i,1} - \dot{y}_{i,r} - \frac{1}{h_i} \dot{\gamma}_1 h_i)
\]
(14)

The following Lyapunov function to take into account
\[
V_{i,1} = \frac{1}{2} z_{i,1}^2 + \frac{1}{2\eta_{i,1}} \dot{\theta}_{i,1}^2
\]
(15)

where \( \eta_{i,1} > 0 \) is design parameter.

It can be obtained from (14) and (15) that
\[
\dot{V}_{i,1} = z_{i,1} \dot{z}_{i,1} + \frac{1}{\eta_{i,1}} \dot{\theta}_{i,1} \dot{\theta}_{i,1}
\]
\[
= z_{i,1} \kappa_i (z_{i,2} + \dot{\theta}_{i,1}^T \psi_{i,1} + \pi_{i,2} + \alpha_{i,1} + \dot{\gamma}_1 h_i - \dot{y}_{i,r}) + \frac{1}{\eta_{i,1}} \dot{\theta}_{i,1} \dot{\theta}_{i,1}
\]
(16)

Based on Young’s inequality, we get
\[
\kappa_i z_{i,1}(z_{i,2} + \pi_{i,2}) \leq \kappa_i^2 z_{i,1}^2 + \frac{1}{2} z_{i,2}^2 + \frac{1}{2} \pi_{i,2}^2
\]
(17)
According to the Step 1, we design the virtual control signal \( \alpha_{i,1} \) and the parameter adaptive law of \( \vartheta_{i,1} \) as
\[
\alpha_{i,1} = -\frac{c_{i,1}}{\kappa_i} z_{i,1} - \frac{3}{2} \kappa_i z_{i,1} - \vartheta_{i,1} \psi_{i,1} + \dot{y}_{i,r} + \frac{1}{h_i} \dot{h}_i \zeta_{i,1}
\]
\[
\dot{\vartheta}_{i,1} = \kappa_i \eta_i \alpha_{i,1,1} z_{i,1} \psi_{i,1} - \vartheta_{i,1} \psi_{i,1}
\]
where \( c_{i,1} > 0 \) and \( \vartheta_{i,1} > 0 \) are design parameters.

By invoking (20) and (21) into (19) yields
\[
\dot{V}_{i,1} \leq -c_{i,1} z_{i,1}^2 + \frac{1}{2} z_{i,2}^2 + \frac{1}{2} \pi_{i,2}^2 + \frac{1}{2} \tilde{\vartheta}_{i,1}^2 + \frac{\dot{\vartheta}_{i,1}}{\eta_{i,1}} \dot{\vartheta}_{i,1} \vartheta_{i,1}
\]
To deal with the repeated differentiation of \( \alpha_{i,1} \), consider the first-order filter as follows
\[
\tau_{i,2} \dot{\psi}_{i,1} + \nu_{i,2} = \alpha_{i,1}
\]
where \( \tau_{i,2} \) is a given constant.

From (13), we can obtain \( \dot{\nu}_{i,2} = -\frac{1}{\tau_{i,2}} (\alpha_{i,1} - \nu_{i,2}) \) and
\[
\hat{\vartheta}_{i,2} = \dot{\vartheta}_{i,2} - \dot{\vartheta}_{i,1} = -\frac{\tau_{i,2}}{\tau_{i,2}} + Y_{i,2}(\cdot)
\]
where \( Y_{i,2}(\cdot) \) is a continuous bounded function, and
\[
\dot{Y}_{i,2}(\cdot) = -\dot{\vartheta}_{i,1} = \frac{d(\vartheta_{i,1})}{dt} + d(\vartheta_{i,1}) = \frac{\vartheta_{i,1}}{dt}
\]

**Step 2.** According to (13), we have
\[
\dot{z}_{i,2} = \dot{x}_{i,2} - \dot{\nu}_{i,2}
\]
\[
\dot{z}_{i,2} = \dot{x}_{i,3} + f_{i,2} - \dot{\nu}_{i,2}
\]
Opt to the Lyapunov function candidates
\[
\dot{V}_{i,2} = V_{i,1} + \frac{1}{2} z_{i,2}^2 + \frac{1}{2} \pi_{i,2}^2 + \frac{1}{2} \tilde{\vartheta}_{i,2}^2
\]
where \( \eta_{i,2} > 0 \) is design parameter.

In accordance with (13), (25) and (26), \( \dot{V}_{i,2} \) satisfies
\[
\dot{V}_{i,2} = \dot{V}_{i,1} + z_{i,2} \dot{z}_{i,2} + \dot{\pi}_{i,2} \pi_{i,2} + \frac{1}{\eta_{i,2}} \dot{\vartheta}_{i,2} \tilde{\vartheta}_{i,2}
\]
\[
\dot{V}_{i,2} = V_{i,1} + z_{i,2} \dot{z}_{i,2} + \dot{\pi}_{i,2} \pi_{i,2} + \dot{\vartheta}_{i,2} \tilde{\vartheta}_{i,2}
\]
Exploiting Young’s inequality gives
\[
z_{i,2}(z_{i,3} + \pi_{i,3}) \leq z_{i,2}^2 + \frac{1}{2} z_{i,3}^2 + \frac{1}{2} \pi_{i,3}^2
\]
where \( \eta \) is a given constant.

In the light of (13), (37) and (38), \( \dot{V}_{i,j} \) results in

\[
\dot{V}_{i,j} \leq -c_{i,j}z_{i,j}^2 + z_{i,j}(2z_{i,j} + \alpha_{i,j} + \psi_{i,j} - \dot{\psi}_{i,j})
\]

\[
+ \frac{1}{2}\tilde{\sigma}_{i,j}^2 + \frac{1}{2}\tilde{\sigma}_{i,j}^2 + \frac{\phi_{i,j}}{m_{i,j}}\tilde{\sigma}_{i,j}\psi_{i,j} + \frac{1}{2}\tilde{\sigma}_{i,j}^2 + \frac{1}{2}\tilde{\sigma}_{i,j}^2
\]

\[
+ \frac{1}{2}\tilde{\sigma}_{i,j}^2 + \tilde{\sigma}_{i,j}\tilde{\sigma}_{i,j} + \frac{\alpha_{i,j}}{m_{i,j}}(\eta_{i,j}z_{i,j} + \psi_{i,j} - \dot{\psi}_{i,j})
\]

Construct \( \alpha_{i,j} \) and the parameter adaptive law of \( \psi_{i,j} \) as

\[
\alpha_{i,j} = -(c_{i,j} + 2)z_{i,j} + \dot{\psi}_{i,j} - \psi_{i,j}
\]

\[
\dot{\psi}_{i,j} = \eta_{i,j}z_{i,j} + \dot{\psi}_{i,j} - \psi_{i,j}
\]

where \( c_{i,j} > 0 \) and \( \phi_{i,j} > 0 \) are design parameters.

Be identical with (19)-(22), substituting (31) and (32) into (30), one obtains

\[
\dot{V}_{i,j} \leq -\sum_{k=1}^2 c_{i,k}z_{i,k}^2 + \sum_{k=1}^2 \phi_{i,k}\tilde{\sigma}_{i,k}\psi_{i,k} + \sum_{k=1}^2 \frac{1}{2}\tilde{\sigma}_{i,k}^2 + \sum_{k=1}^2 \frac{1}{2}\tilde{\sigma}_{i,k}^2
\]

\[
+ \sum_{k=1}^2 \frac{1}{2}\tilde{\sigma}_{i,k}^2 + \tilde{\sigma}_{i,k}\tilde{\sigma}_{i,k} + \pi_{i,j}\tilde{\sigma}_{i,k}
\]

Similar to (23), one has

\[
\tau_{i,3}\dot{\psi}_{i,3} + \nu_{i,3} = \alpha_{i,2}
\]

\[
\nu_{i,3}(0) = \alpha_{i,2}(0)
\]

where \( \tau_{i,3} \) is a given constant.

From (34), we can get

\[
\dot{\nu}_{i,3} = \frac{1}{\tau_{i,3}}(\alpha_{i,2} - \nu_{i,3}) = \frac{\pi_{i,3}}{\tau_{i,3}}
\]

From (34) and (35), one can obtain

\[
\dot{\pi}_{i,3} = \dot{\nu}_{i,3} - \dot{\pi}_{i,3} = \frac{\pi_{i,3}}{\tau_{i,3}} + Y_{i,3}(\cdot)
\]

where \( Y_{i,3}(\cdot) = Y_{i,3}(z_{i,j}, \psi_{i,j}, \tau_{i,3}, \dot{\psi}_{i,j}, \dot{\psi}_{i,j}) \) is a continuous function.

**Step** \( j_i(3 \leq j_i \leq m_i - 1) \). From (1) and (13), we obtain

\[
\dot{z}_{i,j_i} = \dot{x}_{i,j_i} + \dot{f}_{i,j_i} - \dot{\psi}_{i,j_i}
\]

\[
= \dot{x}_{i,j_i} + \dot{f}_{i,j_i} + \dot{\psi}_{i,j_i}
\]

\[
= \dot{x}_{i,j_i} + \dot{f}_{i,j_i} + \dot{\psi}_{i,j_i} + \dot{\psi}_{i,j_i} + \dot{\psi}_{i,j_i} + \dot{\psi}_{i,j_i}
\]

Opt to the Lyapunov function as

\[
\dot{V}_{i,j_i} = V_{i,j_i} - \frac{1}{2}z_{i,j_i}^2 + \frac{1}{2}\pi_{i,j_i}^2 + \frac{1}{2}\eta_{i,j_i}^2
\]

where \( \eta_{i,j_i} > 0 \) is design parameter.

In the light of (13), (37) and (38), \( \dot{V}_{i,j_i} \) satisfies

\[
\dot{V}_{i,j_i} = \dot{V}_{i,j_i} - \frac{1}{2}z_{i,j_i}^2 + \frac{1}{2}\pi_{i,j_i}^2 + \frac{1}{2}\eta_{i,j_i}^2
\]

\[
= \dot{V}_{i,j_i} - \frac{1}{2}z_{i,j_i}^2 + \frac{1}{2}\pi_{i,j_i}^2 + \frac{1}{2}\eta_{i,j_i}^2
\]

\[
+ \dot{\psi}_{i,j_i}^2 + \dot{\psi}_{i,j_i} - \dot{\psi}_{i,j_i} + \frac{1}{m_{i,j}}\tilde{\sigma}_{i,j_i}^2
\]
By utilizing Young’s inequality, we obtain
\[ z_{i,j} (z_{i,j} + \pi_{i,j} + 1) \leq z_{i,j}^2 + \frac{1}{2} \pi_{i,j}^2 + \frac{1}{2} \eta_{i,j}^2 \] (40)
\[ z_{i,j} \tilde{z}_{i,j} \leq \frac{1}{2} z_{i,j}^2 + \frac{1}{2} \tilde{z}_{i,j}^2. \] (41)

Substituting (40) and (41) into (39) yields
\[ \dot{V}_{i,j} \leq \dot{V}_{i,j} - \frac{1}{2} z_{i,j}^2 + z_{i,j} (2z_{i,j} + \vartheta_{i,j} \psi_{i,j} - \hat{\vartheta}_{i,j}) \] (42)

Design \( \alpha_{i,j} \) and the parameter adaptive law of \( \hat{\vartheta}_{i,j} \) as
\[ \alpha_{i,j} = (c_{i,j} + 2) z_{i,j} + \hat{\vartheta}_{i,j} - \vartheta_{i,j} \psi_{i,j} \] (43)
\[ \dot{\hat{\vartheta}}_{i,j} = \eta_{i,j} z_{i,j} \psi_{i,j} - \vartheta_{i,j} \phi_{i,j} \] (44)
where \( c_{i,j} > 0 \) and \( \phi_{i,j} > 0 \) are design parameters.

In accordance with (43) and (44), it follows that
\[ \dot{V}_{i,j} \leq -\sum_{k=1}^{\bar{b}} c_{i,k} z_{i,k}^2 + \sum_{k=1}^{\bar{b}} \frac{\phi_{i,k}}{\nu_{i,k}} \dot{\vartheta}_{i,k} \tilde{\vartheta}_{i,k} + \frac{1}{2} \nu_{i,k} \tilde{\vartheta}_{i,k}^2 \] (45)
\[ + \sum_{k=2}^{\bar{b}} \frac{1}{2} \tilde{\vartheta}_{i,k}^2 + \sum_{k=2}^{\bar{b}} \nu_{i,k} \dot{\vartheta}_{i,k} + \frac{1}{2} \tilde{\vartheta}_{i,k}^2 \]

Then, be identical with (23), \( \alpha_{i,j} \) is given by
\[ \tau_{i,j} \nu_{i,j} + \nu_{i,j} + 1 = \alpha_{i,j} \]
\[ \nu_{i,j} + 1 = \alpha_{i,j} \] (46)

where \( \tau_{i,j} \) is a given constant.

From (46), one can obtain
\[ \nu_{i,j} + 1 = \frac{1}{\tau_{i,j} + 1} (\alpha_{i,j} - \nu_{i,j}) = -\frac{\pi_{i,j} + 1}{\tau_{i,j} + 1} \] (47)

From (46) and (47), we have
\[ \dot{\pi}_{i,j} + 1 = \nu_{i,j} + 1 = \frac{\eta_{i,j} + 1}{\tau_{i,j} + 1} + Y_{i,j} + 1 \] (48)
where \( Y_{i,j} = Y_{i,j} (z_{i,j}, \cdots, \pi_{i,j}, \cdots, \psi_{i,j}, \cdots, \phi_{i,j}, \cdots, \vartheta_{i,j}, y_{i,r}, \hat{y}_{i,r}) \) is a continuous function.

**Step** \( m_i \). In the light of (1) and (13), we get
\[ \dot{x}_{i,m_i} = \dot{x}_{i,m_i} - \dot{\nu}_{i,m_i} \]
\[ = u_i + f_{i,m_i} - \nu_{i,m_i} \]
\[ = u_i + \dot{\vartheta}_{i,m_i} \psi_{i,m_i} + \tilde{\vartheta}_{i,m_i} - \dot{\vartheta}_{i,m_i} \] (49)

Choose the Lyapunov function candidate as
\[ V_{i,m_i} = V_{i,m_i} + \frac{1}{2} \nu_{i,m_i} + \frac{1}{2} \tilde{\vartheta}_{i,m_i} - \dot{\vartheta}_{i,m_i} \] (50)

where \( \eta_{i,m_i} > 0 \) is design parameter.
According to (49) and (50) gives
\[
\dot{V}_{i,m_i} = \dot{V}_{i,m_i-1} + z_{i,m_i} \dot{z}_{i,m_i} + \pi_{i,m_i} \pi_{i,m_i} + \frac{\dot{\varphi}_{i,m_i} \dot{\varphi}_{i,m_i}}{\bar{m}_{i,m_i}} \\
= \dot{V}_{i,m_i-1} + z_{i,m_i}(u_i + \varphi^{*T}_{i,m_i} \psi_{i,m_i} + \tilde{\varphi}_{i,m_i}) + \dot{\varphi}_{i,m_i} \pi_{i,m_i} + \frac{\dot{\varphi}_{i,m_i} \dot{\varphi}_{i,m_i}}{\bar{m}_{i,m_i}}
\]
(51)
Utilizing Young’s inequality, one has
\[
z_{i,m_i} \tilde{\varphi}_{i,m_i} \leq \frac{1}{2} z_{i,m_i}^2 + \frac{1}{2} \tilde{\varphi}_{i,m_i}^2
\]
(52)
Substituting (45) and (52) into (51) yields
\[
\dot{V}_{i,m_i} \leq - \sum_{k=1}^{m_i-1} c_{i,k} z_{i,k}^2 + \sum_{k=1}^{m_i-1} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\varphi}_{i,k} \psi_{i,k} + \sum_{k=1}^{m_i-1} \frac{1}{2} \pi_{i,k}^2 + \sum_{k=1}^{m_i-1} \frac{1}{2} \tilde{\varphi}_{i,k}^2 \\
+ \sum_{k=2}^{m_i-1} \pi_{i,k} \dot{\varphi}_{i,k} + z_{i,m_i}(\dot{z}_{i,m_i} - \dot{\varphi}_{i,m_i} + u_i + \varphi_{i,m_i} \psi_{i,m_i}) \\
+ \dot{\varphi}_{i,m_i} \pi_{i,m_i} + \frac{1}{2} \tilde{\varphi}_{i,m_i} + \frac{\dot{\varphi}_{i,m_i}}{\bar{m}_{i,m_i}} (\eta_{i,m_i} z_{i,m_i} \psi_{i,m_i} - \dot{\psi}_{i,m_i})
\]
(53)
To make it easier to calculate, we design the auxiliary control signal \( \varphi \) and the parameter adaptive law of \( \dot{\varphi}_{i,m_i} \), as
\[
\varphi = -(c_{i,m_i} + 1) z_{i,m_i} + \dot{\varphi}_{i,m_i} - \psi_{i,m_i}
\]
(54)
\[
\dot{\psi}_{i,m_i} = \eta_{i,m_i} z_{i,m_i} \psi_{i,m_i} - \dot{\varphi}_{i,m_i}
\]
(55)
where \( c_{i,m_i} > 0 \) and \( \dot{\varphi}_{i,m_i} > 0 \) are design parameters.

3.1. **ETC design.** Consider the ETC scheme, then we design the event-triggered controller for \( i \)th subsystem is given as
\[
u_i(t) = \omega_i(t_{kk}), \quad \forall t \in [t_{kk}, t_{kk+1})
\]
(56)
We design the switching threshold strategy (STS) as
\[
t_{kk+1} = \begin{cases} 
\inf \{ t \in R, t > t_{kk} \mid |e_i(t)| \geq \lambda_i |u_i(t)| + \sigma_i \}, \ u_i(t) < M \\
\inf \{ t \in R, t > t_{kk} \mid |e_i(t)| \geq \sigma_{i1} \}, \ u_i(t) \geq M
\end{cases}
\]
(57)
where \( t_{kk}, \ k \in \mathbb{Z}^+ \), and \( \lambda_i \) meets \( 0 < \lambda_i < 1 \), it is the transition control law. Define measurement error as \( e_i(t) = \omega_i(t) - u_i(t) \). \( M \) is a design parameter, \( \sigma_i \) and \( \sigma_{i1} \) are parameters defined later. When (57) triggered, \( u_i(t) \) will be updated to \( \omega_i(t_{kk+1}) \).
Since the STS is a combination of the FTS and the RTS, we firstly give the following analysis.

A. **Relative threshold strategy**
In this case, the event-triggered controller is given by
\[
u_i(t) = \omega_i(t_{kk}), \quad \forall t \in [t_{kk}, t_{kk+1})
\]
(58)
We design the event-triggered mechanism (ETM) as
\[
t_{kk+1} = \inf \{ t \in R, t > t_{kk} \mid |e_i(t)| \geq \lambda_i |u_i(t)| + \sigma_i \}
\] 
\[
t_{k1} = 0
\]
(59)
where \( t_{kk}, \ k \in \mathbb{Z}^+ \), and \( \lambda_i \) meets \( 0 < \lambda_i < 1 \), it is the transition control law. \( \sigma_i \) are positive design parameters.
Form (59), we can get two cases. when \( u_i(t) > 0 \), on the basis of \( -\lambda_i |u_i(t)| - \sigma_i \leq \omega_i(t) - u_i(t) \leq \lambda_i |u_i(t)| + \sigma_i \), we have \( \omega_i(t) - u_i(t) = \rho_i(t)(\lambda_i u_i(t) + \sigma_i) \), where
SWITCHING MECHANISM-BASED EVENT-TRIGGERED FUZZY ADAPTIVE CONTROL

\( \rho(t) \in [-1, 1] \). When \( u_i(t) < 0 \), on the basis of \( \lambda_i |u_i(t)| - \sigma_i \leq \omega_i(t) - u_i(t) \leq -\lambda_i |u_i(t)| + \sigma_i \), we have \( \omega_i(t) - u_i(t) = \rho_i(t)(\lambda_i u_i(t) - \sigma_i) \).

In general, it holds that

\[
\omega_i(t) - u_i(t) = \rho_{i1}(t) \lambda_i u_i(t) + \rho_{i2} \sigma_i
\]

(60)

where \( \rho_{i1} = \rho_{i2} = \rho_i \), \( u_i(t) > 0 \). \( \rho_{i1} = \rho_1 \), \( \rho_{i2} = -\rho_i \), \( u_i(t) < 0 \).

Therefore, \( u_i(t) \) and \( \omega_i(t) \) can be finally given by

\[
u_i(t) = \frac{\omega_i(t)}{1 + \rho_1 \lambda_i} - \frac{\rho_{i2} \sigma_i}{1 + \rho_1 \lambda_i}
\]

(61)

\[
\omega_i(t) = -(1 + \lambda_i)[|\rho| \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right) + \bar{\sigma}_i \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right)]
\]

(62)

In accordance with the designed controller, we analyze the stability of the controlled system. Furthermore, it is also proved that the devised control scheme can avoid Zeno behavior [22]. These results are illustrated with the Theorem 3.1.

**Theorem 3.1.** Consider the MIMO nonlinear systems (1) consisting of the controllers (62), the devised virtual controllers (20), (31), (43) and (54), the parameter adaptive laws (21), (32), (44) and (55), the designed ETM (59), can pledge that the MIMO nonlinear systems all signals in the controlled system are bounded. The tracking error converges to a diminutive residual set with PP. And the Zeno behavior can be averted.

**Proof.** Invoking (54) and (55) into (53) produces

\[
\dot{V}_{i,m} \leq -\sum_{k=1}^{m_i} c_{i,k} z_{i,k}^2 + \sum_{k=1}^{m_i} \tilde{\phi}_{i,k} \tilde{\sigma}_i + \sum_{k=1}^{m_i-1} \frac{1}{2} \tilde{\pi}_{i,k+1}^2
\]

\[+ \sum_{k=1}^{m_i} \frac{1}{2} \tilde{\pi}_{i,k}^2 + \sum_{k=2}^{m_i} \tilde{\pi}_{i,k} \tilde{\pi}_{i,k} + z_{i,m}(u_i - \varphi)
\]

(63)

From (61) and (62), it follows that

\[
z_{i,m} u_i = z_{i,m}\left(\frac{\omega_i(t)}{1 + \rho_1 \lambda_i} - \frac{\rho_{i2} \sigma_i}{1 + \rho_1 \lambda_i}\right)
\]

\[
= z_{i,m}[-\frac{1 + \lambda_i}{1 + \rho_1 \lambda_i}(\varphi \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right) + \bar{\sigma}_i \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right)) - \frac{\rho_{i2} \sigma_i}{1 + \rho_1 \lambda_i}]
\]

(64)

Take into consideration \( \bar{\sigma}_i > \left|\frac{\sigma_i}{1 - \lambda_i}\right|, 0 < 1 + \rho_1 \lambda_i < 1 + \lambda_i \) and

\[
|\frac{z_{i,m,\sigma_i}}{1 - \lambda_i}|, \text{ since } 0 \leq |\bar{\sigma}| - |\tanh(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon})| \leq 0.2785 \varepsilon \text{ from } [23], \text{ (64) can be expressed as}
\]

\[
z_{i,m} u_i \leq -z_{i,m} \varphi \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right) - z_{i,m} \bar{\sigma}_i \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right) - z_{i,m} \frac{\rho_{i2} \sigma_i}{1 + \rho_1 \lambda_i}
\]

\[
\leq 0.2785 \varepsilon - z_{i,m} \bar{\sigma}_i \tanh\left(\frac{\bar{z}_{i,m,\bar{\sigma}}}{\varepsilon}\right) - \left|z_{i,m} \varphi\right| + \left|\frac{z_{i,m,\sigma_i}}{1 - \lambda_i}\right|
\]

(65)

Substituting (65) into (63), we can get

\[
\dot{V}_{i,m} \leq -\sum_{k=1}^{m_i} c_{i,k} z_{i,k}^2 + \sum_{k=1}^{m_i} \tilde{\phi}_{i,k} \tilde{\sigma}_i + \sum_{k=1}^{m_i-1} \frac{1}{2} \tilde{\pi}_{i,k+1}^2
\]

\[+ \sum_{k=1}^{m_i} \frac{1}{2} \tilde{\pi}_{i,k}^2 + \sum_{k=2}^{m_i} \tilde{\pi}_{i,k} \tilde{\pi}_{i,k} + 0.2785 \varepsilon
\]

(66)
We can obtain that
\[ \dot{\varpi}_{i,k} \varpi_{i,k} = \varpi_{i,k}(-\frac{\pi}{r_{i,k}} + Y_{i,k}(...)) \leq -\frac{\pi}{r_{i,k}} + \frac{1}{2}\pi_{i,k}^2 H_{i,k}^2 + \frac{1}{2}t \]
where \( Y_{i,k}(...) \) satisfies \( |Y_{i,k}(...)| \leq H_{i,k} \) with constant \( H_{i,k} > 0 \). \( t > 0 \) represents a constant.

Substituting (67) and (68) into (66), one has
\[
\dot{V}_{i,m} \leq -\sum_{k=1}^{m_1} c_{i,k} \varpi_{i,k}^2 - \frac{1}{2} \sum_{k=1}^{m_1} \delta_{i,k} \varpi_{i,k}^2 + \frac{1}{2} \sum_{k=1}^{m_1} \delta_{i,k} \varpi_{i,k}^2 + \frac{1}{2} \pi_{i,k}^2 + \sum_{k=2}^{m_1} (-\frac{1}{2} + \frac{1}{2} \pi_{i,k}^2 H_{i,k}^2 + \frac{1}{2}t) + 0.557\varepsilon
\]
where \( \Delta = 0.557\varepsilon + \frac{1}{2} \sum_{k=1}^{m_1} \delta_{i,k} \varpi_{i,k}^2 + \frac{1}{2} \sum_{k=1}^{m_1} \delta_{i,k} \varpi_{i,k}^2 + \frac{1}{2} (m_i - 1)t \).

Then, (70) can be represented as
\[
\dot{V}_{i,m} \leq -CV_{i,m} + \Delta
\]
where \( C = \min\left(2c_{i,1}, \ldots, 2c_{i,m_1}, \phi_{i,1}, \ldots, \phi_{i,m_1}, 2(\frac{1}{2} - \frac{1}{2} H_{i,2} - \frac{1}{2}), \ldots, 2(\frac{1}{2} - \frac{1}{2} H_{i,m_1} - \frac{1}{2})\right) \).

From (70), it can be seen clearly that all the signals of the controlled system are bounded. Integrating (70) over \([0,t]\), we have \( V_{i,m_1}(t) \leq V_{i,m_1}(0)e^{-Ct} + \frac{\Delta}{C}(1 - e^{-Ct}) \). We can obtain that \( \zeta_{i,1} \) satisfies \( \zeta_{i,1}^2 \leq 2V_{i,m_1}(t) \leq 2V_{i,m_1}(0)e^{-Ct} + 2\frac{\Delta}{C}(1 - e^{-Ct}) \), which will exponentially converge towards the compact set \( \bar{X} = \{\zeta_{i,1} | \zeta_{i,1}^2 \leq 2 \times \frac{\Delta}{C}\} \).

Afterwards, we prove that the adaptive ETC method can exclude Zeno behavior [22].

For \( \forall t \in [t_{kk}, t_{kk+1}] \) and the measurement error \( e_i(t) = \omega_i(t) - u_i(t) \) gives
\[
\frac{d|e_i(t)|}{dt} = \frac{d[|e_i(t)| e_i(t)]}{dt} \leq \text{sign}(e_i(t)) |\dot{e}_i(t)| \leq |\dot{\omega}_i(t)| \leq Y_i^* \]
where \( Y_i^* > 0 \) is a constant.

From (71), we can get
\[
|e_i(t)| = \int_{t_{kk}}^{t_{kk+1}} |\dot{e}_i(t)| dt \leq \int_{t_{kk}}^{t_{kk+1}} Y_i^* dt \leq Y_i^*(t_{kk+1} - t_{kk})
\]
In view of (59), one has \( \lim_{t \to t_{kk+1}} |e_i(t)| \geq \lambda_i |u_i(t)| + \sigma_i \). Then, we obtain
\[
t_{kk+1} - t_{kk} \geq \frac{\sigma_i}{Y_i^*} > 0
\]
All in all, it can be inferred from (73) that the Zeno behavior can be eliminated.
B. Fixed Threshold Strategy
In this part, the ETC strategy can be written as
\[ u_i(t) = \omega_i(t), \quad \forall t \in [t_{kk}, t_{kk+1}) \]  
\[ t_{kk+1} = \inf \{ t \in R, t > t_{kk} | \epsilon_i(t) | \geq \sigma_{i1} \} \]  
\[ t_1 = 0 \]  
where \( t_{kk}, \ k k \in \mathbb{Z}^+, \sigma_{i1} \) are positive design parameters. The measurement error is \( e_i(t) = \omega_i(t) - u_i(t) \). From (75), we have \( |\omega_i(t) - u_i(t)| \leq \sigma_{i1} \). Thus, we can obtain a continuous time-varying parameter \( \mu_i(t) \), satisfying \( \mu_i(t_{kk}) = 0, \mu_i(t_{kk+1}) = \pm 1 \) and \( |\mu_i(t)| \leq 1 \). \( \forall t \in [t_{kk}, t_{kk+1}) \), such that \( \omega_i(t) = u_i(t) + \mu_i(t)\sigma_{i1} \).
Therefore, \( u_i(t) \) and \( \omega_i(t) \) can be indicated as
\[ u_i(t) = \omega_i(t) - \mu_i\sigma_{i1} \]  
\[ \omega_i = \varphi - \sigma_{i1} \tanh \left( \frac{z_{i,m} \sigma_{i1}}{\varepsilon} \right) \]  
Similar to the RTS, we achieve the Theorem 3.2.

**Theorem 3.2.** Consider the MIMO nonlinear systems (1), under the controllers (76), by replacing the designed ETM in Theorem 3.1 with (75), the consequences still hold in Theorem 3.1.

**Proof.** Be identical with (63), and form (76) and (77), we have
\[ z_{i,m}, u_i = z_{i,m} (\omega_i(t) - \mu_i\sigma_{i1}) \]
\[ = z_{i,m} \left( \varphi - \sigma_{i1} \tanh \left( \frac{z_{i,m} \sigma_{i1}}{\varepsilon} \right) - \mu_i\sigma_{i1} \right) \]
\[ \leq z_{i,m} \varphi - z_{i,m} \sigma_{i1} \tanh \left( \frac{z_{i,m} \sigma_{i1}}{\varepsilon} \right) + |z_{i,m} \sigma_{i1}| \]
\[ \leq z_{i,m} \varphi + 0.2785 \varepsilon \]
where \( \sigma_{i1} \geq 0 \) are design parameters.
Substituting (78) into (63) gives
\[ \dot{V}_{i,m} = - \sum_{k=1}^{m_i} c_{i,k} \dot{z}_{i,k}^2 + \sum_{k=1}^{m_i} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\phi}_{i,k} \dot{\varphi}_{i,k} + \sum_{k=1}^{m_i} \left( \frac{1}{2} \pi_{i,k}^2 \right) \]
\[ + \sum_{k=1}^{m_i} \frac{\varphi_{i,k}}{\eta_{i,k}} \dot{\varphi}_{i,k} \dot{\phi}_{i,k} \sum_{k=1}^{m_i} \pi_{i,k} \varphi_{i,k} + 0.2785 \varepsilon \]  
(79)
Substituting (67) and (68) into (79), one has
\[ \dot{V}_{i,m} \leq - \sum_{k=1}^{m_i} c_{i,k} \dot{z}_{i,k}^2 - \frac{1}{2} \sum_{k=1}^{m_i} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\phi}_{i,k} \dot{\phi}_{i,k} + \frac{1}{2} \sum_{k=1}^{m_i} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\varphi}_{i,k} \dot{\varphi}_{i,k} + \sum_{k=1}^{m_i} \frac{1}{2} \pi_{i,k}^2 \]
\[ + \sum_{k=1}^{m_i} \left( \frac{\varphi_{i,k}}{\eta_{i,k}} \dot{\varphi}_{i,k} \dot{\varphi}_{i,k} \right) + \sum_{k=1}^{m_i} \left( \frac{\pi_{i,k} \varphi_{i,k}}{\eta_{i,k}} \right) + \frac{1}{2} \Delta \]
\[ \leq - \sum_{k=1}^{m_i} c_{i,k} \dot{z}_{i,k}^2 - \frac{1}{2} \sum_{k=1}^{m_i} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\phi}_{i,k} \dot{\phi}_{i,k} - \sum_{k=2}^{m_i} \left( \frac{1}{2} \pi_{i,k}^2 - \frac{1}{2} \pi_{i,k}^2 H_{i,k}^2 \right) + \Delta \]  
(80)
where \( \Delta = 0.2785 \varepsilon + \frac{1}{2} \sum_{k=1}^{m_i} \frac{\phi_{i,k}}{\eta_{i,k}} \dot{\phi}_{i,k} \dot{\phi}_{i,k} + \sum_{k=1}^{m_i} \frac{1}{2} \pi_{i,k}^2 + \frac{1}{2} (m_i - 1) \varepsilon \).
According to the same analysis as Theorem 3.1, all the controlled system signals still are bounded and \( \zeta_i, \sigma \) converges exponentially to a compact set \( \Xi = \{ \zeta_i, \sigma \} \leq 2 \times \frac{1}{\Delta} \) at a rate of \( C \). Finally, for \( \forall t \in [t_{kk}, t_{kk+1}) \) and the measurement error \( e_i(t) = \omega_i(t) - u_i(t) \), we have
\[ \frac{d|e_i(t)|}{dt} \leq \text{sign}[e_i(t)] |\dot{e}_i(t)| \leq |\dot{e}_i(t)| \leq \Upsilon_i, \dot{e}_i(t_{kk}) = 0 \]
and \( \lim_{t \to t_{kk+1}} |e_i(t)| = \sigma_{i1} \), from this we can get \( t_{kk+1} - t_{kk} \geq \frac{\sigma_{i1}}{\eta} > 0 \). By this point, we have proved that the Zeno behavior is avoided.

In the light of [24] and combined with the above analysis, the STS adopted in this paper combines the FTS and the RTS. Clearly, strategies can be flexibly selected to meet the system requirements and maintain certain performance of the system. Thus, we can obtain

\[
\bar{e}_i = \sup |e_i(t)| \leq \max \{ \lambda_i |M| + \sigma_i, \sigma_{i1} \}, \quad \forall t \in [t_{kk}, t_{kk+1})
\]

(81)

Using the STS and (81), we can obtain the Theorem 3.3.

**Theorem 3.3.** With the controller (56) and the STS (57) applied to the MIMO nonlinear systems (1), the results in Theorem 3.1 and Theorem 3.2 still hold with \( \bar{e}_i \leq \sigma_{i1} \).

**Proof.** Since the control law of the STS is identical to the RTS, so, the tracking error bound \( \Omega \) also is identical with Theorem 3.1. In the same way, we can get \( t_{kk+1} - t_{kk} \geq \frac{\max \{ \sigma_i, \sigma_{i1} \}}{\eta} > 0 \), the Zeno behavior also can be exclude. \( \square \)

4. Simulation results. In this section, we provide MATLAB simulation consequences to verify the reasonableness and the effectiveness of the presented fuzzy adaptive ETC scheme. Take into account the system as follows

\[
\begin{align*}
\dot{x}_{1,1} &= x_{1,2} + f_{1,1}(x_{1,1}) \\
\dot{x}_{1,2} &= f_{1,2}(x) + u_1 \\
y_1 &= x_{1,1} \\
\dot{x}_{2,1} &= x_{2,2} + f_{2,1}(x_{2,1}) \\
\dot{x}_{2,2} &= f_{2,2}(x) + u_2 \\
y_2 &= x_{2,1}
\end{align*}
\]

(82)

where

\[
f_{1,1}(x_{1,1}) = x_{1,1}e^{-0.5x_{1,1}^2}, \quad f_{1,2}(x) = -\cos \left( \frac{1}{1+x_{1,1}^2} \right)x_{1,2}^2, \quad f_{2,1}(x_{2,1}) = -0.1x_{2,1}^2, \quad f_{2,2}(x) = 2x_{2,2}x_{1,1}\sin(x_{1,2}),
\]

\( y_{1,r} = \sin(t) \) and \( y_{2,r} = \sin(t) \) are the reference signals.

The fuzzy membership functions are selected as

\[
\begin{align*}
\mu_N_{i,j_1} &= \exp\left[-\frac{(x_{i,j_1} - 0)^2}{2} \right], \quad \mu_S_{i,j_1} = \exp\left[-\frac{(x_{i,j_1} - 2)^2}{2} \right], \quad \mu_T_{i,j_1} = \exp\left[-\frac{(x_{i,j_1} + 2)^2}{2} \right], \quad i = 1, 2, j_1 = 1, 2.
\end{align*}
\]

Then, fuzzy basis functions are written as

\[
\psi_{i,j_1,l}(x_{i,j_1}) = \frac{2}{\sum_{l=1}^{5} \prod_{j=1}^{2} \exp\left[-\frac{(x_{i,j_1} + 2l - 6)^2}{2} \right]} \prod_{j=1}^{2} \exp\left[-\frac{(x_{i,j_1} + 2l - 6)^2}{2} \right]
\]

where \( l = 1, \cdots, 5, \quad i = 1, 2, \quad j_1 = 1, 2. \)

Opt to the devise parameters in controller (56), virtual controller (20) and adaptive laws (21) and (32) as \( c_{1,1} = 30, c_{1,2} = 100, c_{2,1} = 30, c_{2,2} = 100, \quad \tau_2 = 0.1, \quad \lambda_1 = 0.5, \quad \sigma_1 = 5, \quad \varepsilon = 30, \quad \eta_{1,1} = 10, \quad \eta_{1,2} = 20, \quad \eta_{2,1} = 10, \quad \eta_{2,2} = 20, \quad \phi_{1,1} = 0.02, \quad \phi_{1,2} = 0.3, \quad \phi_{2,1} = 0.5, \quad \phi_{2,2} = 0.5, \quad \ell_{1,\text{min}} = 2.8, \quad \ell_{1,\text{max}} = 2.4, \quad h_{10} = 2.1, \quad h_{1\infty} = 0.9, \quad A_1 = 0.5. \)

The initial conditions are chosen as \( x_{1,1} = 0.3, x_{2,1} = 0.8, x_{2,2} = 0.3, x_{2,2} = 0.2, \quad \psi_{1,1} = [0, 0, 0, 0, 0], \quad \psi_{1,2} = [0, 0, 0, 0, 0], \quad \psi_{2,1} = [0, 0, 0, 0, 0], \quad \psi_{2,2} = [0, 0, 0, 0, 0]. \)

Then, FIGURES. 1-9 exhibit the obtained MATLAB simulation consequences, from the simulation results of FIGURE. 1 and FIGURE. 5, we can see that the
MIMO system’s tracking performances. FIGURE. 2 and FIGURE. 6 picture the curves of tracking errors. As shown in FIGURE.3 and FIGURE.7, the ETC signals \( u_1(t) \), \( u_2(t) \) and the updated signal \( \omega(t) \), it clear that the ETC scheme can save certain communication bandwidth. The triggering time intervals \( t_{kk+1} - t_k \) are given by FIGURE. 4 and FIGURE. 8. On the basic of the PP function, the tracking error and the prescribed upper bound and lower bound are illustrated in FIGURE. 9.

Compared with the existing methods, the proposed switching mechanism-based fuzzy adaptive ETC method in this paper further saves the communication resources, and choice of the ETM is also more flexible. Obviously, the MATLAB simulation results show that the presented control scheme is efficient.

5. **Conclusion.** In this paper, the fuzzy adaptive ETC scheme has been developed for MIMO nonlinear systems with PP. By combining the backstepping technique and the DSC technique, the computational problems have been enormously simplified. The presented STS is more flexible than the single FTS or the RTS, and it has absolute advantages to economize the communication bandwidth. In addition, the devised control programme can ensure all the signals of the controlled system are bounded and the tracking errors are always limited to the specified range. The switching threshold strategy in this paper can be extended to all kinds of systems, such as multi-agent systems[25]. And it can further solve the problem of resource waste in digital communication channel transmission[26]. However, we also have to consider the robustness of the system, such as dead-zone, saturation and time-delay[27-29]. Furthermore, the future work will focus on the application of event-triggered control in practical systems[30], and it can maximize that the value of the ETC embodies in Unmanned-Aerial-Vehicle formation or vehicle formation control.
Figure 2. System tracking error $y_1 - y_{1,r}$.

Figure 3. Control signal.
Figure 4. The time intervals $t_{k+1} - t_k$ of triggering events.

Figure 5. System output $y_2$ and the reference signal $y_{2,r}$. 
Figure 6. System tracking error $y_2 - y_{2,r}$.

Figure 7. Control signal.
Figure 8. The time intervals $t_{k+1} - t_k$ of triggering events.

Figure 9. Tracking performance.
REFERENCES

[1] W. W. Bai, T. S. Li and S. C. Tong, NN reinforcement learning adaptive control for a class of nonstrict-feedback discrete-time systems, *IEEE Transactions on Cybernetics*, 50 (2020), 4573–4584.

[2] C. P. Bechlioulis and G. A. Rovithakis, A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems, *Automatica*, 50 (2014), 1217–1226.

[3] C. Deng, C. Wen, J. Huang, X. M. Zhang and Y. Zou, Distributed observer-based cooperative control approach for uncertain nonlinear MASs under event-triggered communication, *IEEE Transactions on Automatic Control*, (2021), 1–1.

[4] K. W. Li and Y. M. Li, Adaptive fuzzy finite-time dynamic surface control for high-order nonlinear system with output constraints, *International Journal of Control, Automation and Systems*, 19 (2021), 112–123.

[5] T. S. Li, W. W. Bai, Q. Liu, Y. Long and C. L. Philip Chen, Distributed fault-tolerant containment control protocols for the discrete-time multi-agent systems via reinforcement learning method, *IEEE Transactions on Neural Networks and Learning Systems*, (2021), 1–13.

[6] X. D. Li and P. Li, Stability of time-delay systems with impulsive control involving stabilizing delays, *Automatica*, 124 (2021), 109336.

[7] X. D. Li and X. Y. Yang, Lyapunov stability analysis for nonlinear systems with state-dependent state delay, *Automatica*, 112 (2020), 108674.

[8] Y. M. Li, K. W. Li and S. C. Tong, Finite-time adaptive fuzzy output feedback dynamic surface control for MIMO non-strict feedback systems, *IEEE Transactions on Fuzzy Systems*, 27 (2019), 96–110.

[9] Y. M. Li, Y. J. Liu and S. C. Tong, Observer-based neuro-adaptive optimized control for a class of strict-feedback nonlinear systems with state constraints, *IEEE Transactions on Neural Networks and Learning Systems*, (2021), 1–15.

[10] Y. M. Li and S. C. Tong, Fuzzy adaptive control design strategy of nonlinear switched large-scale systems, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 48 (2018), 2209–2218.

[11] L. Liu, D. Wang, Z. H. Peng and Q. L. Han, Distributed path following of multiple under-actuated autonomous surface vehicles based on data-driven neural predictors via integral concurrent learning, *IEEE Transactions on Neural Networks and Learning Systems*, 32 (2021), 5334–5344.

[12] M. Liu, L. X. Zhang, P. Shi and Y. X. Zhao, Fault estimation sliding-mode observer with digital communication constraints, *IEEE Trans. Automat. Control*, 63 (2018), 3434–3441.

[13] Z. Liu, F. Wang, Y. Zhang, X. Chen and C. L. P. Chen, Adaptive tracking control for a class of nonlinear systems with a fuzzy dead-zone input, *IEEE Transactions on Fuzzy Systems*, 23 (2015), 193–204.

[14] W. Qian, W. W. Xing and S. M. Fei, $H_{\infty}$ state estimation for neural networks with general activation function and mixed time-varying delays, *IEEE Trans. Automat. Control*, 32 (2021), 3909–3918.

[15] J. Qiu, K. Sun, T. Wang and H. Gao, Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance, *IEEE Transactions on Fuzzy Systems*, 27 (2019), 2152–2162.

[16] Z. W. Ruan, Q. M. Yang, S. Z. S. Ge and Y. X. Sun, Adaptive fuzzy fault tolerant control of uncertain MIMO nonlinear systems with output constraints and unknown control directions, *IEEE Transactions on Fuzzy Systems*, (2021), 1–1.

[17] X. F. Shao and D. Ye, Fuzzy adaptive event-triggered secure control for stochastic nonlinear high-order MASs subject to DoS attacks and actuator faults, *IEEE Transactions on Fuzzy Systems*, 29 (2021), 3812–3821.

[18] W. Sun, S. F. Su, J. W. Xia and Y. Q. Wu, Adaptive tracking control of wheeled inverted pendulums with periodic disturbances, *IEEE Transactions on Cybernetics*, 50 (2020), 1867–1876.

[19] S. C. Tong, X. Min and Y. X. Li, Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions, *IEEE Transactions on Cybernetics*, 50 (2020), 3903–3913.
[20] S. C. Tong, K. K. Sun and S. Sui, Observer-based adaptive fuzzy decentralized optimal control design for strict-feedback nonlinear large-scale systems, IEEE Transactions on Fuzzy Systems, 26 (2018), 569–584.

[21] J. H. Wang, Z. Liu, C. L. Philip Chen and Y. Zhang, Event-triggered neural adaptive failure compensation control for stochastic systems with dead-zone output, Nonlinear Dynamics, 96 (2019), 2179–2196.

[22] T. Wang, Y. F. Zhang, J. B. Qiu and H. J. Gao, Adaptive fuzzy backstepping control for a class of nonlinear systems with sampled and delayed measurements, IEEE Transactions on Fuzzy Systems, 23 (2015), 302–312.

[23] W. Wang and S. Tong, Observer-based adaptive fuzzy containment control for multiple uncertain nonlinear systems, IEEE Transactions on Fuzzy Systems, 27 (2019), 2079–2089.

[24] L. B. Wu, J. H. Park, X. P. Xie, C. Gao and N. N. Zhao, Fuzzy adaptive event-triggered control for a class of uncertain nonaffine nonlinear systems with full state constraints, IEEE Transactions on Fuzzy Systems, 29 (2021), 904–916.

[25] L. Xing, C. Wen, Z. Liu, H. Su and J. Cai, Adaptive compensation for actuator failures with event-triggered input, Automatica, 85 (2017), 129–136.

[26] L. Xing, C. Wen, Z. Liu, H. Su and J. Cai, Event-triggered adaptive control for a class of uncertain nonlinear systems, IEEE Trans. Automat. Control, 62 (2017), 2071–2076.

[27] W. Q. Xu, X. P. Liu, H. Q. Wang and Y. C. Zhou, Event-triggered adaptive NN tracking control for MIMO nonlinear discrete-time systems, IEEE Transactions on Neural Networks and Learning Systems, (2021), 1–11.

[28] J. P. Yu, P. Shi and L. Zhao, Finite-time command filtered backstepping control for a class of nonlinear systems, Automatica, 92 (2018), 173–180.

[29] S. Zeghlache, L. Benyettou, A. Djeriou and M. Z. Ghellab, Twin rotor MIMO system experimental validation of robust adaptive fuzzy control against wind effects, IEEE Systems Journal, (2020), 1–11.

[30] Y. Zhang, X. H. Su, Z. Liu and C. L. P. Chen, Event-triggered adaptive fuzzy tracking control with guaranteed transient performance for MIMO nonlinear uncertain systems, IEEE Transactions on Cybernetics, 51 (2021), 736–749.

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