Electron Heating and Saturation of Self-regulating Magnetorotational Instability in Protoplanetary Disks

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Abstract
Magnetorotational instability (MRI) has the potential to generate vigorous turbulence in protoplanetary disks, although its turbulence strength and accretion stress remain debatable because of the uncertainty of MRI with a low ionization fraction. We focus on the heating of electrons by strong electric fields, which amplifies nonideal magnetohydrodynamic effects. The heated electrons frequently collide with and stick to dust grains, which in turn decreases the ionization fraction and is expected to weaken the turbulent motion driven by MRI. In order to quantitatively investigate the nonlinear evolution of MRI, including the electron heating, we perform magnetohydrodynamical simulation with the unstratified shearing box. We introduce a simple analytic resistivity model depending on the current density by mimicking the resistivity given by the calculation of ionization. Our simulation confirms that the electron heating suppresses magnetic turbulence when the electron heating occurs with low current density. We find a clear correlation between magnetic stress and current density, which means that the magnetic stress is proportional to the squared current density. When the turbulent motion is completely suppressed, laminar accretion flow is caused by an ordered magnetic field. We give an analytical description of the laminar state using a solution of linear perturbation equations with resistivity. We also propose a formula that successfully predicts the accretion stress in the presence of the electron heating.

Key words: accretion, accretion disks – instabilities – magnetohydrodynamics (MHD) – methods: numerical – protoplanetary disks – turbulence

1. Introduction
Magnetorotational instability (MRI) has the potential to generate vigorous turbulence in protoplanetary disks. The turbulent viscosity made by MRI can explain the accretion rate suggested by observation (e.g., Hawley et al. 1995; Flock et al. 2011). That is why MRI has been expected to be a mechanism generating disk turbulence in most research of protoplanetary disks. Previous studies have investigated how MRI turbulence in the disks significantly affects planetesimal formation. For example, vigorous MRI turbulence causes the diffusion of the dust-condensed region (Carballido et al. 2005; Fromang & Papaloizou 2006; Fromang & Nelson 2009; Turner et al. 2010; Zhu et al. 2015) and the collisional fragmentation of grains (Carballido et al. 2010). The disk turbulence is important for both disk evolution and planetesimal formation.

However, MRI growth and the generation of vigorous magnetic turbulence need the disk to be sufficiently ionized. Decoupling between the gas and magnetic fields due to the low ionization fraction causes nonideal magnetohydrodynamic (MHD) effects, such as ohmic dissipation, the Hall effect, and ambipolar diffusion. The nonideal MHD effects can stabilize MRI (e.g., Fleming et al. 2000; Sano & Stone 2002; Bai & Stone 2011; Bai 2013; Kunz & Lesur 2013; Simon et al. 2015). The nonideal MHD effects strongly depend on the ionization fraction. Therefore, it is essential to understand the ionization state in the disk to determine the efficiency of MRI and the strength of the resulting turbulence.

Although a theoretical estimate of the turbulence strength in a disk still has large uncertainties, recent disk observations found indirect evidence of the turbulence strength. The disk around HL Tau, which is thought to be typical of protoplanetary disks surrounding T Tauri stars, has been observed by ALMA observatory with high spatial resolution (ALMA Partnership et al. 2015). The disk has many axisymmetric rings and gaps approximately within 100 au from the star. Pinte et al. (2016) reproduced a similar observational image with the radiative transfer simulation and obtained the dust and gas properties. According to their paper, such a clear gap requires the dust disk to be geometrically thin, which means weak turbulence of the Shakura–Sunyaev alpha parameter $\alpha \lesssim a few 10^{-4}$ (Shakura & Sunyaev 1973). Moreover, Flaherty et al. (2015) and Flaherty et al. (2017) observed a disk around an A-type star, HD 163296, and obtained the spectral map that limits the non-thermal gas velocity dispersion, which is mainly due to turbulent motion. Flaherty et al. (2017) constrained the velocity dispersion to less than ~0.04 times the sound speed, which corresponds to $\alpha \lesssim 10^{-3}$ around the midplane. The value is one order of magnitude less than that of typical $\alpha$ values of fully developed MRI turbulence $\alpha \sim 10^{-2}$. The direct imaging observation of HD 163296 by Isella et al. (2016), which observed multiple gaps, also suggested weak turbulence from the gap width and depth relation, assuming the presence of planets in the gaps. These observations provide a new question of why the disk turbulence is weak.

In this paper, we investigate the effect of electron heating on the MRI. Electron heating is one of the consequences of resistive MHD and has the potential to suppress MRI via changing ionization balance. MRI generates not only magnetic fields but also electric fields in the comoving frame of the gas. The electric fields induced by MRI heat charged particles, in particular, electrons, in the gas, due to collision with gas...
particles (Inutsuka & Sano 2005). The heated electrons are efficiently removed from the gas phase because they frequently collide with and stick to dust grains (Okuzumi & Inutsuka 2015, hereafter OI15). Therefore, the electron heating causes a decrease in the ionization fraction, which amplifies the nonideal MHD effects suppressing MRI. Since the electron heating takes place after MRI sufficiently grows, the nonideal MHD effects amplified by the electron heating can change the picture of MRI behavior, even in sufficiently ionized regions. Our previous study (Mori & Okuzumi 2016, hereafter MO16) investigated the region in protoplanetary disks where the electron heating influences MRI. We showed that this suppression mechanism becomes important even in outer regions of protoplanetary disks that retain abundant small dust grains. Since the MRI growth leads to suppressing the MRI itself in the presence of electron heating, the saturated turbulent motion could be weaker than that of fully developed MRI turbulence. MO16 also estimated the accretion stress of magnetic turbulence using a scaling relation between the magnetic stress and the current density, and proposed that the accretion stress suppressed by the electron heating could be reduced by more than an order of magnitude.

How much the electron heating suppresses MRI is still unclear, although the possibility of electron heating occurring in the disks has been investigated. The estimation of turbulence strength in MO16 is based on a scaling relation that has not been verified. In order to confirm the effectiveness for electron heating to suppress magnetic turbulence, accretion stress in the presence of the electron heating should be investigated quantitatively.

Our goal in this work is to quantify the effect of the electron heating on MRI with a numerical simulation. We perform MHD simulations where the suppression of the electric resistivity due to electron heating is modeled by a simple analytic function. Furthermore, we propose a formula that reproduces the Maxwell stress obtained from the simulation, which can be used to take into account the effect of the electron heating on the disk evolution. As a first step, we neglect ambipolar diffusion and the Hall effect, focusing on how the Ohmic resistivity increasing with the electric field strength affects the saturated state of MRI. In addition, although strong electric fields not only heat electrons but also ions, we also neglect the ion heating, which requires much higher electric field strengths than electron heating (OI15).

This paper is organized as follows. In Section 2, we present the numerical setup and procedure in our simulations. In Section 3, we then show some results and present the interpretations. In Section 4, we analytically derive a relation between current density and Maxwell stress. In Section 5, we summarize this paper and discuss implications for dust diffusion in protoplanetary disks.

2. Method

2.1. Numerical Method

We perform MHD simulations with a unstratified local shearing box, using Athena, an open source MHD code that uses Godunov’s scheme (Stone et al. 2008; Stone & Gardiner 2010). We adopt a local reference frame (x, y, z) corotating with the Keplerian flow at a fiducial distance $r_0$ from the central star. The coordinates x, y, and z refer to the radial, azimuthal, and vertical distances from the corotation point, respectively. Neglecting curvature and vertical gravity, the MHD equations in this local coordinate system can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -2\Omega \times \mathbf{v} + 3\Omega^2 x - \frac{1}{\rho} \nabla \left( P + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi \rho} (\mathbf{B} \cdot \nabla \mathbf{B}), \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\epsilon \nabla \times \mathbf{E}, \quad (3)$$

where $\mathbf{v}$ is the gas velocity, $\rho$ is the gas density, $P$ is the gas pressure, $\Omega$ is the angular velocity at radius $r_0$, $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the electric field, and $\epsilon$ is the speed of light. In this paper, we assume isothermal fluid and use the isothermal equation of state for an ideal gas,

$$P = \epsilon c_s^2 \rho, \quad (4)$$

where $c_s$ is the sound speed of isothermal gas and is constant. The electric field $\mathbf{E}$ in this reference frame is related to the electric field $\mathbf{E}'$ in the comoving frame of the gas,

$$\mathbf{E} = \mathbf{E}' - \frac{1}{c} \mathbf{v} \times \mathbf{B}, \quad (5)$$

by the Lorentz transformation in the limit of small velocity. To close the system of equations, we employ Ohm’s law,

$$\mathbf{J} = \frac{c^2}{4\pi \eta (E')} \mathbf{E}', \quad (6)$$

where $\mathbf{J} = (c/4\pi) \nabla \times \mathbf{B}$ is the current density. In this study, we assume that the electric resistivity $\eta$ depends on the amplitude of the electric field strength, $E' = |E'|$, which is the case when electron heating changes the ionization fraction.

The dependence of $\eta$ on $E'$ was investigated in (OI15). OI15 calculated the ionization fraction from the ionization equilibrium, including two important effects of plasma heating, i.e., the amplification of plasma adsorption onto dust grains and impact ionization by energetic plasma. The amplification of plasma adsorption decreases plasma abundance, while the impact ionization increases plasma abundance. They showed that the amplification of plasma adsorption occurs at lower $E'$ than impact ionization. In this work, we focus only on the amplification of plasma adsorption amplified by the electron heating and neglect impact ionization.

In this paper, we use an analytical resistivity model that mimics the behavior of $\eta$ as a function of $E'$ due to the electron adsorption, which is based on OI15. Figure 1 shows a schematic picture of our resistivity model. Effective electric resistivity is determined by the smaller of the electron and ion resistivity. The horizontal gray lines show electron and ion resistivity in the case without electron heating. The critical electric field strength $E_{\text{EH}}$ shows the threshold of electric field strength where electron heating occurs. For $E' > E_{\text{EH}}$, the resistivity increases with increase of $E'$ due to dust adsorption of heated electrons. When $E'$ is so small that electron heating does not work, i.e., $E' \ll E_{\text{EH}}$, electron resistivity is much smaller than ion resistivity. In that case, the effective resistivity is equal to electron resistivity without electron heating, which is
constant. On the other hand, the electron resistivity in $E' > E_{\text{EH}}$ increases with increases of $E'$ because electron abundance decreases due to electron heating. In this case, the effective resistivity also increases. At $E' \gg E_{\text{EH}}$, electron resistivity is larger than ion resistivity, therefore the effective resistivity is determined by ion resistivity and is constant.

In this work, we focus only on resistivity that increases by electron heating; we do not address an instability of electric fields caused by negative differential resistance, $dJ/dE < 0$ (see Section 6.1 in OI15). In this work, the gradient of $\eta$ to $E'$ is modified to be shallower than the resistivity given in OI15. In order to satisfy $dJ/dE = 1/(\alpha J/J d)$, the power-law index of $\eta$ to $J$ is taken to be larger than $-1$.

Imitating the $J$-$E'$ relation of OI15 including electron heating, we give the simple analytical resistivity model where the resistivity increases with an increase of $E'$ or $J$. In Figure 2, we show a schematic diagram of the $J$-$E'$ relation, including our resistivity model. The resistivity $\eta$ is written as

$$
\eta = \begin{cases} 
\eta_0, & J < J_{\text{EH}}, \\
\eta_0 \left( \frac{J}{J_{\text{EH}}} \right)^{1/\epsilon - 1}, & J_{\text{EH}} < J < 1000^{\epsilon/(1-\epsilon)} J_{\text{EH}}, \\
1000^{\epsilon/(1-\epsilon)} J_{\text{EH}} < J,
\end{cases}
$$

(7)

where $\eta_0$ is the initial resistivity, $\epsilon$ is a constant value sufficiently less than unity, and $J_{\text{EH}}$ is the current density at which the electron heating sets in. In this paper, we take $\epsilon$ to be 0.1, and $J_{\text{EH}}$ to be the arbitrary parameter. Here, we assume that the ion resistivity is higher than the electron resistivity by a factor of 1000.

At $E_{\text{EH}} < E' \lesssim 1000 E_{\text{EH}}$, current density is approximately equal to $J_{\text{EH}}$ in this model. Therefore, $J_{\text{EH}}$ also approximately corresponds to the saturated current density. The difference between the saturated current density and $J_{\text{EH}}$ is at most smaller than a factor of two.

### 2.2. Simulation Settings

We use a shearing box with a uniform shear flow with the background azimuthal velocity of $-1.5\Omega x$. The simulation box sizes in the radial, azimuthal, and vertical direction are $H, 2\pi H$, and $H$, respectively, where $H$ is the gas scale height, $c_s/\Omega$. We impose the shearing periodic boundary condition for $x$ and the periodic boundary condition for $y$ and $z$.

We take the computational units of length, time, and density to be, respectively, $H, \Omega^{-1}$, and the initial gas density $\rho_0$. Therefore, the unit of velocity is $c_s$, and the unit of pressure is the initial gas pressure $P_0 = \rho_0 c_s^2$. The unit of magnetic field strength is

$$
B_u = \sqrt{4\pi P_0}.
$$

(8)

We take the unit of current density to be

$$
J_u = \frac{c B_u}{4\pi H}.
$$

(9)

We nondimensionalize Ohm’s law $E = (4\pi \eta c^2)J$ as $E/E_u = (\eta/\eta_0)(J/J_u)$, where

$$
E_u = \frac{4\pi \eta_0 J_u}{c^2} = \frac{c_s}{c} B_u
$$

(10)

and

$$
\eta_u = H^2 \Omega = H c_s.
$$

(11)

The initial vertical magnetic field is uniform and its strength is

$$
B_z0 = \sqrt{2} \beta_0^{1/2} B_u,
$$

(12)

where

$$
\beta_0 = \frac{8\pi P_0}{B_{z0}^2}
$$

(13)

is the initial plasma beta. We consider the situation where MRI would be fully active if electron heating were absent. The activity of MRI is determined by the Elsasser number (e.g., Sano & Miyama 1999),

$$
\Lambda_z = \frac{\lambda \Lambda_{\zeta}}{\eta \Omega}
$$

(14)
where
\[ v_{A} = \frac{B}{\sqrt{4\pi \rho}} \]  
(15)
is the Alfvén velocity of the vertical magnetic field. MRI is fully active when \( \lambda \gg 1 \), while the resistivity suppresses the most unstable MRI mode when \( \lambda \ll 1 \). We choose the value of \( \eta_0 \) so that the Elsasser number in the initial state \( \Lambda_0 \) is equal to 10. \( \Lambda_0 \) is expressed as \( \Lambda_0 = \frac{v_0^2}{\eta_0 \Omega} \), where \( v_0 \) is the Alfvén velocity of initial state, \( v_0 = \frac{B_0}{\sqrt{4\pi \rho}} \). For this value of \( \Lambda_0 \), the Elsasser number in the final saturated state also satisfies \( \lambda \gg 1 \) as long as electron heating is neglected (\( \eta = \eta_0 \) for all \( E' \)), because we generally have \( v_{A} > v_{0} \). In order to investigate the dependence on the critical current density \( J_{\text{EH}} \), we take \( J_{\text{EH}} \) to be less than 10\( J_{\text{c}} \), which approximately corresponds to the maximum current density, at which current density is saturated in fully developed MRI turbulence (Munurashi et al. 2012). We give random perturbations of pressure \( \delta P \) and velocity \( \delta \mathbf{v} \) whose maximum amplitudes are \( \delta P/P_0 = 5 \times 10^{-3} \) and \( |\delta \mathbf{v}|/c_s = 2 \times 10^{-5} \), respectively. The amplitudes are taken to be so small that they never exceed the amplitudes of the perturbations left after electron heating suppresses MRI turbulence. We also take into account a small viscosity that is effective for damping initial perturbations.

The numerical resolutions are taken to be 64, 64/\pi, and 64 grids per \( H \) in the \( x, y, \) and \( z \) directions, respectively. In order to properly resolve the MRI turbulence, we take the vertical grid spacing \( \Delta z \) to be much smaller than the most unstable wavelength \( \lambda_{\text{MRI}} \) (Noble et al. 2010). In our fiducial model, \( \lambda_{\text{MRI}}/\Delta z \approx 20-120 \) in the final state. In order to resolve MRI, \( \lambda_{\text{MRI}}/\Delta z \gtrsim 6 \) is required (Sano et al. 2004). Our resolution satisfies this requirement. A Courant–Friedrichs–Lewy number of 0.4 is used.

### 2.3. Initial Conditions

We take \( \beta_0 = 10^4 \) and \( \Lambda_0 = 10 \) as the fiducial parameters. For this set of \( \beta_0 \) and \( \Lambda_0 \), we consider 10 different values of \( J_{\text{EH}} \): \( J_{\text{EH}}/J_0 = 1 \times 10^{-4}, 3 \times 10^{-3}, 1 \times 10^{-2}, 3 \times 10^{-2}, 1 \times 10^{-1}, 3 \times 10^{-1}, 1, 3, 10 \) and \( \infty \), where \( J_{\text{EH}}/J_0 = \infty \) corresponds to the case without electron heating. We also perform simulations with different values of \( \beta_0 \) and \( \Lambda_0 \) to see the dependence on these parameters. We take \( \beta_0 \) as \( \beta_0 = 10^3, 10^4, 10^5 \) and \( \Lambda_0 \) as \( \Lambda_0 = 30, 10, 0.3 \), with \( J_{\text{EH}} = 0.003, 0.03, 0.3, 3, \infty \) for each set of \( \beta_0 \) and \( \Lambda_0 \). We use these results for checking the accuracy of the analytic \( \alpha_{\text{EH}} \) relation presented in Section 4.

### 3. Simulation Results

Table 1 summarizes the parameter sets explored in this study. We express the volume-averaged quantities as \( \langle \ldots \rangle \) and the time- and volume-averaged quantities as \( \langle \langle \ldots \rangle \rangle \). The volume averages are calculated over the entire simulation box, and the time averages are calculated from 100 to 150 in units of the orbital period \( 2\pi/\Omega_0 \). The range of time integration is taken so that the final saturated state dominates the average.

The most important quantity obtained from the simulations is the accretion stress, which controls the disk evolution. The accretion stress can be characterized in terms of the Shakura–Sunyaev alpha parameter \( \alpha \), which is defined as the time- and volume-averaged accretion stress divided by the time- and volume-averaged pressure, which is equal to \( P_0 \) for an isothermal gas,
\[ \alpha = \alpha_R + \alpha_M = \frac{\langle (\rho \delta v_{\perp}) \rangle}{P_0} + \frac{\langle (-B_x \delta B_y) \rangle}{4\pi P_0}, \]  
(16)
where we express \( \langle (\rho \delta v_{\perp}) \rangle/P_0 \) and \( \langle (-B_x \delta B_y) \rangle/(4\pi P_0) \) as, respectively, \( \alpha_R \) and \( \alpha_M \).

#### 3.1. The Fiducial Case

Figure 3 shows the saturated state \((t = 60 \) orbits\) observed in our fiducial simulations with \( J_{\text{EH}}/J_0 = 0.03 \). The saturated state for the case without electron heating \( J_{\text{EH}}/J_0 = \infty \) is also shown for comparison. We also show the crosscuts of the saturated state on the \( x-z \) and \( y-z \) planes for \( J_{\text{EH}} = 0.03, 0.3, 3, \) and \( \infty \) in Figure 4. We find that a laminar flow with an ordered magnetic field dominates the saturated state for \( J_{\text{EH}}/J_0 = 0.03 \), whereas the turbulent magnetic fields are generated in the case without electron heating. Comparing these two cases, we confirm that electron heating suppresses the turbulent motion that is characteristic of MRI. Moreover, the magnetic field strength \( |B| \) is also largely suppressed for the laminar case.

In Figure 4, we see that the azimuthal magnetic fields for \( J_{\text{EH}}/J_0 = 0.03 \) are sinusoidal in the vertical direction, with a wavelength as large as the vertical box size. In the presence of electron heating, the perturbations on small scales grow due to the increased resistivity, while perturbations on larger scales grow. For this reason, the magnetic field inside the box tends to be dominated by the component whose wavelength is equal to the box size. We also see that small structures of magnetic fields appear with increasing \( J_{\text{EH}} \). This too can be understood by the fact that the resistivity increased by the electron heating suppresses the perturbations on small scales.

In order to demonstrate that the resulting \( J \) and \( E' \) follow the given \( J-E' \) relation, in Figure 5 we show the evolutionary tracks of the volume-averaged current density \( (J) \) and electric field strength \( (E') \) in the \( J-E' \) plane. The current densities initially grow along the line of \( \Lambda_0 = 10 \) and then branch off the line after they reach \( J_{\text{EH}} \). We confirm that the resulting \( (J)-(E') \) tracks almost go along with the \( J-E' \) relation we give. We also find that, in the absence of electron heating cases, \( (J) \) and \( (E') \) are saturated near the line corresponding to \( \Lambda_0 = 0.1 \).

In Figure 6, we show the time evolution of the volume-averaged Maxwell stress for different values of \( J_{\text{EH}} \). MRI grows linearly in the first few orbits, then the Maxwell stress becomes saturated in \( \sim 30 \) orbits. We find that the Maxwell stress in the saturated state decreases with decreasing \( J_{\text{EH}} \), which means that MRI is stabilized by electron heating. We also find that the Maxwell stress in the saturated state is fluctuating when \( J_{\text{EH}}/J_0 > 0.3 \) and is highly stationary when \( J_{\text{EH}}/J_0 < 0.1 \). This suggests that electron heating completely suppresses turbulent motion caused by MRI when \( J_{\text{EH}} < 0.1J_0 \).

Here, we define the threshold current density as
\[ J_{\text{lam}} = 0.1J_0. \]  
(17)
At \( J \ll J_{\text{lam}} \), the saturated state is laminar.

Figure 7 displays \( \alpha_M \) as a function of \( J_{\text{EH}} \). We confirm a positive correlation between \( \alpha_M \) and \( J_{\text{EH}} \). By fitting a quadratic function to the data, we obtain the empirical...
| Label   | $J_{EH}$ | $\beta$ | $\Lambda_0$ | $\langle R_0^2 \rangle/(8\pi \rho_0)$ | $\alpha_M$ | $\alpha_R$ | $(\langle J \rangle)/J_0$ |
|---------|----------|---------|-------------|-------------------------------------|-------------|-------------|--------------------------|
| EH0001  | 0.001    | $10^4$  | 10          | $1.00 \times 10^{-4}$               | $7.21 \times 10^{-9}$ | $9.00 \times 10^{-11}$ | $1.75 \times 10^{-3}$   |
| EH0003  | 0.003    | $10^6$  | 10          | $1.00 \times 10^{-4}$               | $6.49 \times 10^{-8}$ | $8.10 \times 10^{-10}$ | $5.25 \times 10^{-3}$   |
| EH001   | 0.001    | $10^4$  | 10          | $1.03 \times 10^{-4}$               | $7.21 \times 10^{-7}$ | $9.00 \times 10^{-9}$ | $1.75 \times 10^{-2}$   |
| EH003   | 0.03     | $10^4$  | 10          | $1.30 \times 10^{-4}$               | $6.49 \times 10^{-6}$ | $8.10 \times 10^{-8}$ | $5.25 \times 10^{-2}$   |
| EH01    | 0.1      | $10^4$  | 10          | $4.15 \times 10^{-4}$               | $6.47 \times 10^{-5}$ | $3.37 \times 10^{-4}$ | $1.71 \times 10^{-1}$   |
| EH03    | 0.3      | $10^4$  | 10          | $1.57 \times 10^{-3}$               | $3.08 \times 10^{-4}$ | $9.73 \times 10^{-4}$ | $4.62 \times 10^{-1}$   |
| EH1     | 1        | $10^4$  | 10          | $5.48 \times 10^{-3}$               | $1.53 \times 10^{-3}$ | $1.98 \times 10^{-3}$ | $1.39$                   |
| EH3     | 3        | $10^4$  | 10          | $1.17 \times 10^{-2}$               | $4.79 \times 10^{-3}$ | $2.95 \times 10^{-3}$ | $3.53$                   |
| EH10    | 10       | $10^4$  | 10          | $3.75 \times 10^{-2}$               | $1.69 \times 10^{-2}$ | $6.15 \times 10^{-3}$ | $8.23$                   |
| noEH    | $\infty$| $10^4$  | 10          | $7.15 \times 10^{-2}$               | $3.15 \times 10^{-2}$ | $9.67 \times 10^{-3}$ | $1.25 \times 10^1$      |
| B3-EH   | 0.003    | $10^3$  | 10          | $1.00 \times 10^{-3}$               | $1.41 \times 10^{-7}$ | $1.37 \times 10^{-8}$ | $4.60 \times 10^{-3}$   |
| B3-EH   | 0.03     | $10^3$  | 10          | $1.02 \times 10^{-3}$               | $1.41 \times 10^{-8}$ | $1.37 \times 10^{-6}$ | $4.60 \times 10^{-2}$   |
| B3-EH   | 0.3      | $10^3$  | 10          | $2.15 \times 10^{-3}$               | $5.99 \times 10^{-4}$ | $3.34 \times 10^{-4}$ | $4.22 \times 10^{-1}$   |
| B3-EH   | 3        | $10^3$  | 10          | $4.41 \times 10^{-2}$               | $2.52 \times 10^{-2}$ | $1.08 \times 10^{-2}$ | $3.56$                   |
| B3-noEH | $\infty$| $10^3$  | 10          | $2.22 \times 10^{-1}$               | $1.09 \times 10^{-1}$ | $2.79 \times 10^{-2}$ | $1.55 \times 10^1$      |
| B5-EH003| 0.003    | $10^4$  | 10          | $1.04 \times 10^{-5}$               | $2.70 \times 10^{-8}$ | $4.27 \times 10^{-11}$ | $5.98 \times 10^{-3}$   |
| B5-EH003| 0.03     | $10^4$  | 10          | $9.00 \times 10^{-5}$               | $2.69 \times 10^{-6}$ | $4.29 \times 10^{-9}$ | $5.97 \times 10^{-2}$   |
| B5-EH03 | 0.3      | $10^4$  | 10          | $1.66 \times 10^{-3}$               | $1.19 \times 10^{-4}$ | $2.91 \times 10^{-4}$ | $5.25 \times 10^{-1}$   |
| B5-EH3  | 3        | $10^4$  | 10          | $4.07 \times 10^{-3}$               | $1.66 \times 10^{-3}$ | $1.35 \times 10^{-3}$ | $3.49$                   |
| B5-noEH | $\infty$| $10^4$  | 10          | $2.56 \times 10^{-2}$               | $1.17 \times 10^{-2}$ | $4.09 \times 10^{-3}$ | $9.34$                   |
| L1-EH0003| 0.003    | $10^4$  | 1           | $1.00 \times 10^{-4}$               | $3.89 \times 10^{-8}$ | $4.86 \times 10^{-10}$ | $4.07 \times 10^{-3}$   |
| L1-EH003| 0.03     | $10^4$  | 1           | $1.18 \times 10^{-4}$               | $3.89 \times 10^{-8}$ | $4.86 \times 10^{-8}$ | $4.07 \times 10^{-2}$   |
| L1-EH03 | 0.3      | $10^4$  | 1           | $8.58 \times 10^{-4}$               | $2.56 \times 10^{-4}$ | $1.02 \times 10^{-3}$ | $3.50 \times 10^{-1}$   |
| L1-EH3  | 3        | $10^4$  | 1           | $1.08 \times 10^{-2}$               | $2.07 \times 10^{-3}$ | $3.12 \times 10^{-3}$ | $2.78$                   |
| L1-noEH | $\infty$| $10^4$  | 1           | $3.54 \times 10^{-2}$               | $1.67 \times 10^{-2}$ | $5.82 \times 10^{-3}$ | $8.20$                   |
| L30-EH0003| 0.003   | $10^4$  | 30          | $1.00 \times 10^{-4}$               | $8.28 \times 10^{-8}$ | $1.03 \times 10^{-9}$ | $5.94 \times 10^{-3}$   |
| L30-EH003| 0.03    | $10^4$  | 30          | $1.39 \times 10^{-4}$               | $8.28 \times 10^{-6}$ | $1.03 \times 10^{-7}$ | $5.94 \times 10^{-2}$   |
| L30-EH03 | 0.3     | $10^4$  | 30          | $1.84 \times 10^{-3}$               | $3.80 \times 10^{-4}$ | $1.32 \times 10^{-3}$ | $5.22 \times 10^{-1}$   |
| L30-EH3  | 3       | $10^4$  | 30          | $1.36 \times 10^{-2}$               | $5.68 \times 10^{-2}$ | $2.83 \times 10^{-3}$ | $3.93$                   |
| L30-noEH | $\infty$| $10^4$  | 30          | $7.28 \times 10^{-2}$               | $3.26 \times 10^{-2}$ | $1.01 \times 10^{-2}$ | $1.28 \times 10^1$      |

Figure 3. Snapshot of magnetic field strength $|B|/B_u$ at 60 orbits for $J_{EH}/J_0 = 0.03$ (left) and for the case without electron heating $J_{EH}/J_0 = \infty$ (right).

According to previous studies (e.g., Sano & Stone 2002), $\Lambda_z$ expresses the MRI activity. When the Elsasser number is much higher than unity, MRI can make vigorous magnetic turbulence. Figure 8 shows the volume- and time-averaged Elsasser number $\langle \Lambda_z \rangle$ as a function of $J_{EH}$. For $1 < J_{EH}/J_0 < 10$, we see that although the Elsasser number is higher than unity, $\alpha_M$ gradually decreases with the decrease of $J_{EH}$, as we see in Figure 7. Because the increased resistivity can suppress magnetic fields by the small scale turbulent motion that forms strong current density, the electron heating takes place when

formula of the relation,

$$\alpha_M = 0.5 \left( \frac{J_{EH}}{10J_0} \right)^2.$$

(18)

The dependence on current density, $\alpha_M \propto J^2$, is consistent with a scaling relation obtained by MO16 (Equation (40) in their paper), although the magnitude in their equation is 50 times smaller than what is obtained here. This empirical fit can be used when $J_{EH}$ is less than $J_{\text{lam}}$. 
the MRI turbulence is generated. We also see that $\lambda_{\text{crit}}$ is constant at $J_{\text{EH}} < J_{\text{lam}}$. This is because $\eta$ is also constant for $J_{\text{EH}} < J_{\text{lam}}$, as we see below.

To see why the MRI is quenched in the laminar saturated state, we show the time- and volume-averaged critical wavelength $\lambda_{\text{crit}}$ in Figure 9. The critical wavelength $\lambda_{\text{crit}}$ is the shortest wavelength in an unstable MRI mode. This is obtained from the linearized equation system in Sano & Miyama (1999) by assuming a growth rate of zero. The critical wavelength in both resistive and ideal MHD is written as

$$\lambda_{\text{crit}} = \frac{2\pi}{\sqrt{3}} \frac{v_{A,0}}{\Omega} \left( 1 + \left( \frac{v_{A,0}^2}{\eta \Omega} \right)^{-2} \right)^{1/2},$$

where $v_{A,0} = B_0 / \sqrt{4\pi \rho_0}$. We see that the resulting critical wavelength is approximately equal to simulation box size $H$ for low $J_{\text{EH}}$. The MRI growth increases $\eta$, which in turn increases the critical wavelength $\lambda_{\text{crit}}$ when $\Lambda \lesssim 1$. For this reason, the shortest unstable wavelength increases until the wavelength reaches the box size, and eventually all MRI unstable modes die away. Note that the final state of this simulation depends on the vertical box size.

Figure 10 shows the time- and volume-averaged resistivity $\langle \langle \eta \rangle \rangle$ as a function of $J_{\text{EH}}$. In all simulations but with $J_{\text{EH}} = \infty$, the final resistivity is higher than the initial value $\eta_0$ (shown by the dotted line). We see that the saturated resistivity for low $J_{\text{EH}}$ is independent of $J_{\text{EH}}$. This value is given by $\lambda_{\text{crit}}(\eta) = H$ in the resistive MHD,

$$\frac{\eta_{\text{lam}}}{\eta_0} = \frac{2}{\sqrt{\beta_0 8\pi^2/3}} \approx 0.390 \times 10^{-2} \left( \frac{\beta_0}{10^4} \right)^{-1/2}.$$  \hspace{1cm} (20)

The resistivity cannot exceed this value because any higher resistivity would stabilize all unstable modes that can fit in the simulation box. The fact that $\langle \langle \eta \rangle \rangle$ reaches this critical value explains why the laminar saturated state is realized for $J_{\text{EH}} < 0.1 J_\text{in}$. We see in Figure 6 that the saturated state for the low $J_{\text{EH}}$ is steady. Although Figure 4 shows that the wavelength in the final state is equal to the vertical box size, the process leading to the saturated state has not been shown. How is the saturated laminar state determined? In the presence of electron heating, the resistivity also increases with growth of the unstable mode. When the increased resistivity reaches the critical resistivity Equation (20), MRI is stabilized since the unstable mode dies away. In this state, if perturbations of magnetic fields grow, then the resistivity is increased and in turn stabilizes the perturbations. On the other hand, if the perturbation is damped from the equilibrium state, then the resistivity becomes smaller and MRI grows again. In other words, the saturated laminar state is determined by the balance between the MRI growth by shear, and MRI decay by the increased resistivity. Therefore, the final state must settle into the stable equilibrium state.

Lastly, in order to see turbulent activity, we plot the root mean square of the vertical velocity $\langle \langle v_z^2 \rangle \rangle^{1/2}$ as a function of

![Figure 4. Slices in the x-z plane at y = 0 and in the y-z at x = 0 of the magnetic field strength |B|/B_0 (color) and direction of the magnetic field (arrows) at 60 orbits for J_{\text{EH}}/L = 0.03, 0.3, 3, and \infty, from top to bottom.](image)
In Figure 11. In particular, the vertical velocity of gas is important for the dynamics and spatial distribution of dust in protoplanetary disks. We see that the vertical velocity sharply drops at \( J_{\text{EH}} \), where the saturated state is laminar. Its implications for turbulent mixing of dust particles are discussed in Section 5.

4. Derivation of Current–Stress Relation

In this section, we derive a relation between \( \alpha \) and \( J_{\text{EH}} \) that reproduces our simulation results. Because \( J_{\text{EH}} \) can be calculated from disk parameters, this relation may provide a quantitative prediction for accretion stress without MHD simulations, when the saturated state is determined by electron heating. For example, this relation would be useful for simplified modeling with disk evolution using the \( \alpha \) parameter based on MHD simulations with electron heating. Here, we neglect the contribution of Reynolds stress to accretion stress. This is because Maxwell stress is generally larger than Reynolds stress according to Table 1. In addition, we regard the current density in the saturated state as \( J_{\text{EH}} \).

We first derive an analytical expression of the Maxwell stress in the laminar state, \( \alpha_{\text{M, lam}} \). To express \( \alpha_{\text{M, lam}} = \langle (B_x B_y) \rangle / (4\pi P_0) \) as a function of \( J_{\text{EH}} \) we estimate \(-B_x B_y/4\pi P_0\) using Ampère’s equation, \( J = c/(4\pi) \nabla \times B \). We take \( \nabla \) to be the typical wavenumber \( \kappa \). Here, we consider
the vertical sinusoidal wave we see in Figure 4, and therefore $k = k_x e_x$ is assumed. The $x$-direction component of the current density is described as $J_x \approx -ck_x B_y/4\pi$, and thereby $B_y$ is written as

$$B_y \approx -\frac{4\pi}{ck_x} J_x. \quad (21)$$

According to Figure 9, the critical wavelength in the laminar case is the vertical box size, $H$. Thus, here we assume that the vertical wavenumber in the saturated state is

$$k_{z,\text{crit}} = \frac{2\pi}{H}. \quad (22)$$

Using Equations (21) and (22), we express $-B_x B_y/4\pi P_0$ as

$$\frac{B_x B_y}{4\pi P_0} \approx -\frac{100}{4\pi^2} \left( \frac{B_x}{B_y} \right)^2 \left( \frac{J}{10J_u} \right)^2, \quad (23)$$

where the current densities are normalized by the typical current density of fully developed turbulence, $\approx 10J_u$, and we assume that $J_x \approx J$ because $J_e$ dominates the total current $J$.

The relationship between $B_x$ and $B_y$ is given from the linearized equation system, Equations (10) and (12) in Sano & Miyama (1999),

$$B_x = \frac{2v_{\text{eq}}}{\eta_{\text{lam} \Omega}} B_y, \quad (24)$$

where $\eta_{\text{lam}}$ is the resistivity in the laminar case, and we use the fact that the saturated state is steady and resistivity is spatially uniform. Thus, we give $B_y/B_x$ in the laminar state as

$$\frac{B_x}{B_y} = -\frac{4}{\beta_0 \eta_{\text{lam}}} \eta_{\text{lam}}. \quad (25)$$

The saturated resistivity $\eta_{\text{lam}}$ is given by Equation (20).

Using Equations (25) and (20) for Equation (23) in the saturated state, we obtain $\alpha_{M, \text{lam}}$ as

$$\alpha_{M, \text{lam}} = 0.26 \left( \frac{\beta_0}{10^4} \right)^{-1/2} \left( \frac{J_{\text{EH}}}{10J_u} \right)^2, \quad (26)$$

where we assume $J$ to be equal to $J_{\text{EH}}$. Equation (26) approximately equals the fit in Figure 7. The difference in the coefficients between Equation (26) and the fit comes from the difference between the saturated current density and $J_{\text{EH}}$. Although Equation (26) approximately reproduces the Maxwell stress in the laminar state, it is not available for the turbulent state.

On the other hand, the fully developed turbulent state is empirically given from the data without electron heating. We find an empirical formula of $\alpha_{M, \text{turb}}$ from Table 1,

$$\alpha_{M, \text{turb}} \approx 0.036 \left( \frac{\beta_0}{10^4} \right)^{0.56}, \quad (27)$$

which can reproduce $\alpha_{M}$ in the case without electron heating in the calculations of this paper.

To reproduce simulation results, we make a function that approaches $\alpha_{M, \text{turb}}$ and $\alpha_{M, \text{lam}}$ with a high $J_{\text{EH}}$ limit and low...
$J_{\text{EH}}$ limit, respectively,
\[ \alpha_M = (\alpha_{M, \text{turb}} + \alpha_{M, \text{lam}})^{-3}. \] (28)

To verify these equations, we compare them to the results with different $\beta_0$ ($\beta_0 = 10^3, 10^4$, and $10^5$) and $\Lambda_0$ ($\Lambda_0 = 30, 10$, and 1). Figure 12 shows the $\alpha_M - J_{\text{EH}}$ relation, with varying $\beta_0$ and $\Lambda_0$, respectively. We see that Equation (28) reproduces the resulting $\alpha_M$.

We note that these results are based just on the simple analytic $J-E'$ relation. In general, the saturated current density might not be equal to $J_{\text{EH}}$. In that case, the saturated current density would need to be modified instead of $J_{\text{EH}}$. Moreover, the $J-E'$ relation including the electron heating can be a multivalued function of $J$ (see Figure 4 in MO15). The electric fields may jump to the other branch at $dJ/dE < 0$ because the electric field can vary with a much shorter timescale than the current density (see more details in OI15). In that situation, the current density may not converge on a value at the final state. This issue needs to be addressed in future calculations.

5. Summary and Discussion

We have investigated the effect of electron heating on MRI, which has the potential to stabilize MRI (OI15). In this paper, we have performed an MHD simulation that includes the effect of damping resistivity with electron heating, to numerically show the possibility and efficiency of electron heating. We have clearly found that electron heating suppresses the generation of the magnetic turbulence. In particular, when electron heating effectively operates, the ordered magnetic fields create a laminar flow. The accretion stress caused by the magnetic fields is much less than the conventional turbulent stress of magnetic turbulence. We also find a clear relation between the Maxwell stress and current density. As the saturated current density is suppressed at lower and lower levels by electron heating, the Maxwell stress becomes small. Additionally, we have shown the analytical expression of the laminar flow, which allows us to predict the Maxwell stress in the presence of electron heating.

The laminar flow formed by electron heating could have impacts on planetesimal formation. As we see in Figure 11, the vertical velocity dispersion drops when the electron heating completely suppresses the turbulence. In the laminar flow, the turbulent diffusion in the vertical direction is no longer effective. Under classical planetesimal formation theories, the dust sedimentation forms a dusty layer on the midplane that might be gravitationally unstable (Safronov 1972; Goldreich & Ward 1973). The dust layer might cause the gravitational instability that forms planetesimals. This model has been focused in terms of avoiding the meter-size barrier. However, vigorous disk turbulence easily stirs up the dust layer and diffuses it. The dust layer with weak turbulence may also provide a possible place for secular gravitational instability that produces multiple ring-like structures and resulting planetesimals (Takahashi & Inutsuka 2014, 2016; Tominaga et al. 2017). Therefore, weak disk turbulence may help the planetesimal formation. Such dust sedimentation on the midplane also might help cause the streaming instability, which requires a high dust-to-gas mass ratio (Youdin & Goodman 2005; Johansen & Youdin 2007; Bai & Stone 2010; Carrera et al. 2015). Therefore, efficient electron heating may help the formation of a dust layer and planetesimal formation. Moreover, such a weak turbulent disk might explain observed disks that have been suggested to have weak turbulence (e.g., Pinte et al. 2016; Flaherty et al. 2017).

In this paper, we neglect the stratified structure, non-Ohmic resistivities, and the negative slope in the $J-E'$ relation predicted by the ionization calculation. The stratified structure could affect the structure of the magnetic field in the saturated state. Non-Ohmic resistivities, such as the Hall effect and ambipolar diffusion, could affect the final structure (e.g., Bai & Stone 2011; Kunz & Lesur 2013; Lesur et al. 2014; Béthune et al. 2016; Bai 2017). Therefore, the importance of electron heating should be investigated for all resistivities. Moreover, the change of the ionization balance by electron heating could affect also non-Ohmic resistivities. Although the simple analytic $J-E'$ relation could not address how much the current density would be saturated in reality, this work has shown that current density is suppressed by electron heating and there is a relation between Maxwell stress and current density. We will
address the saturated current density with the more detailed \(J-E\) relation in future work.

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References
ALMA Partnership, Brogan, C. L., Pérez, L. M., et al. 2015, ApJL, 808, L3
Bai, X.-N. 2013, ApJ, 772, 96
Bai, X.-N. 2017, ApJ, 845, 75
Bai, X.-N., & Stone, J. M. 2010, ApJ, 722, 1437
Bai, X.-N., & Stone, J. M. 2011, ApJ, 736, 144
Béthune, W., Lesur, G., & Ferreira, J. 2016, A&A, 589, A87
Carballido, A., Cuzzi, J. N., & Hogan, R. C. 2010, MNRAS, 405, 2339
Carballido, A., Stone, J. M., & Pringle, J. E. 2005, MNRAS, 358, 1055
Carrera, D., Johansen, A., & Davies, M. B. 2015, A&A, 579, A43
Flaherty, K. M., Hughes, A. M., Rose, S. C., et al. 2017, ApJ, 843, 150
Flaherty, K. M., Hughes, A. M., Rosenfeld, K. A., et al. 2015, ApJ, 813, 99
Fleming, T. P., Stone, J. M., & Hawley, J. F. 2000, ApJ, 530, 464
Flock, M., Dzyurkevich, N., Klahr, H., Turner, N. J., & Henning, T. 2011, ApJ, 735, 122
Fromang, S., & Nelson, R. P. 2009, A&A, 496, 597
Fromang, S., & Papaloizou, J. 2006, A&A, 452, 751
Goldreich, P., & Ward, W. R. 1973, ApJ, 183, 1051
Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1995, ApJ, 440, 742
Inutsuka, S., & Sano, T. 2005, ApJL, 628, L155
Isella, A., Guidi, G., Testi, L., et al. 2016, Phil. Trans. R. Soc. A, 374, 20150164
Johansen, A., & Youdin, A. 2007, ApJ, 662, 627
Kunz, M. W., & Lesur, G. 2013, MNRAS, 434, 2295
Lesur, G., Kunz, M. W., & Fromang, S. 2014, A&A, 566, A56
Mori, S., & Okuzumi, S. 2016, ApJ, 817, 52
Muranushi, T., Okuzumi, S., & Inutsuka, S. 2012, ApJ, 760, 56
Noble, S. C., Krolik, J. H., & Hawley, J. F. 2010, ApJ, 711, 959
Okuzumi, S., & Inutsuka, S. 2015, ApJ, 800, 47
Pinte, C., Dent, W. R. F., Méndez, F., et al. 2016, ApJ, 816, 25
Safronov, V. S. 1972, Evolution of the Protoplanetary Cloud and Formation of the Earth and Planets (Jerusalem: Keter Publishing House)
Sano, T., Inutsuka, S., Turner, N. J., & Stone, J. M. 2004, ApJ, 605, 321
Sano, T., & Miyama, S. M. 1999, ApJ, 515, 776
Sano, T., & Stone, J. M. 2002, ApJ, 577, 534
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Simon, J. B., Lesur, G., Kunz, M. W., & Armitage, P. J. 2015, MNRAS, 454, 1117
Stone, J. M., & Gardiner, T. A. 2010, ApJS, 189, 142
Stone, J. M., Gardiner, T. A., Teuben, P., Hawley, J. F., & Simon, J. B. 2008, ApJS, 178, 137
Takahashi, S. Z., & Inutsuka, S. 2014, ApJ, 786, 55
Takahashi, S. Z., & Inutsuka, S. 2016, AJ, 152, 184
Tomimaga, R. K., Inutsuka, S., & Takahashi, S. Z. 2017, PASJ, submitted
Turner, N. J., Carballido, A., & Sano, T. 2010, ApJ, 708, 188
Youdin, A. N., & Goodman, J. 2005, ApJ, 620, 459
Zhu, Z., Stone, J. M., & Bai, X.-N. 2015, ApJ, 801, 81