RXJ1856.5-3754 and RXJ0720.4-3125 are P-Stars

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Abstract

P-stars are a new class of compact stars made of up and down quarks in \(\beta\)-equilibrium with electrons in a chromomagnetic condensate. P-stars are able to account for compact stars with \(R \lesssim 6 \text{Km}\), as well as stars comparable to canonical neutron stars. We show that P-stars once formed are absolutely stable, for they cannot decay into neutron or strange stars. We convincingly argue that the nearest isolated compact stars RXJ1856.5-3754 and RXJ0720.4-3125 could be interpreted as P-stars with \(M \simeq 0.8 M_{\odot}\) and \(R \simeq 5 \text{Km}\).

Keywords: Compact Star, Pulsar, Magnetic Field

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1. INTRODUCTION

Soon after the first radio pulsar were discovered, it becomes generally accepted that pulsars are rapidly rotating neutron stars endowed with a strong magnetic field. The exact mechanism by which a pulsar radiates the energy observed as radio pulses is still a subject of vigorous debate, nevertheless the accepted standard model based on the picture of a rotating magnetic dipole has been developed since long time.

Nowadays, no one doubts that pulsars are indeed neutron stars. However, quite recently we have proposed a new class of compact stars, named P-stars, which is challenging the two pillars of modern astrophysics, namely neutron stars and black holes. Indeed, in Ref. we showed that P-stars, which are compact stars made of up and down quarks in $\beta$-equilibrium with electrons in an abelian chromomagnetic condensate, are able to account for compact stars as well as stars comparable to canonical neutron stars. Moreover, for stars with mass $M \approx 1.4 M_\odot$ we found that the binding energy per nucleon is:

$$B \approx 0.386 \text{ GeV} \quad , \quad A \approx 2.9 \times 10^{57}$$

where $A$ is the baryon number of the star. This result should be compared with the binding energy per nucleon in the case of limiting mass neutron star, $B_{NS} \approx 100 \text{ MeV}$, showing that P-stars are gravitationally more bounded than neutron stars. Moreover, it turns out that P-stars are also more stable than strange stars. Indeed, it is known that there is extra energy of about $20 \text{ MeV}$ per baryon available from the conversion of matter to strange matter. So that, even the binding energy per nucleon for strange stars, $B_{SS} \approx 120 \text{ MeV}$, is much smaller than the value in Eq. (1). This shows that gravitational effects make two flavour quark matter in the chromomagnetic condensate globally more stable than nuclear matter or three flavour quark matter. However, one could wonder if locally, let’s say in a small volume of linear size $a \approx 1/m_\pi \approx 1 \text{ fermi}$, two flavor quark matter immersed in the chromomagnetic condensate could convert into neutrons or deconfined quarks of relevance for strange stars. In order to see how far this conversion will proceed, we need to estimate the appropriate transition rates. Let us first consider the decay into nuclear matter. In this case we must rearrange three quarks, which are in the lowest Landau levels, into configurations of quarks in nucleons. In doing this, we get a first suppression factor $\alpha$ due to color mismatch. In fact, the quarks into the lowest Landau levels are colorless due to presence of the chromomagnetic condensate. This suppression factor cannot be easily
evaluated. However, in general we have that $\alpha < \sim 1$. Another suppression factor arises from the mismatch of the quark wavefunctions in the directions transverse to the chromomagnetic field. This results in a factor $\left(\frac{m_{\pi}}{\sqrt{gH}}\right)^2$ for each quarks. Finally, to arrange three quarks into a nucleon we need to flip one quark spin, which increases the energy by a factor $\sqrt{gH}$ at least. So that we have for the transition rate:

$$P \sim \alpha \left(\frac{m_{\pi}}{\sqrt{gH}}\right)^6 e^{-\frac{\sqrt{gH}}{T}}, \quad (2)$$

where $T$ is the star core temperature. For a newly born compact star the core temperature is of order of several $MeV$ and it is rapidly decreasing. On the other hand, the typical values of $\sqrt{gH}$ is of order of several hundred of $MeV$. As a consequence we have:

$$P \sim 1.5 \times 10^{-46}, \quad (3)$$

which implies that the decays of two flavor quark matter immersed in the chromomagnetic condensate into nuclear matter is practically never realized. In addition, the conversion into three-flavor quarks is even more suppressed. Indeed, first we need to flip a large number of quark spin to fill the Fermi sphere, then the quarks at the top of the Fermi sphere are allowed to decay into strange quarks. We conclude, thus, that P-stars, once formed, are absolutely stable.

The logical consequence is that now we must admit that supernova explosions give rise to P-stars. In other words, we are lead to identify pulsars with P-stars instead of neutron or strange stars. Such a dramatic change in the standard paradigm of relativistic astrophysics has been already advanced in our previous paper [8] where we showed that, if we assume that pulsars are P-stars, then we may completely solve the supernova explosion problem. Indeed, the binding energy is the energy released when the core of an evolved massive star collapses. Actually, only about one percent of the energy appears as kinetic energy in the supernova explosion [9]. From Eq. (1) it follows that there is an extra gain in kinetic energy of about $1 - 10 \text{ foe}$ ($1 \text{ foe} = 10^{51} \text{ erg}$), which is enough to fire the supernova explosions. Moreover, further support to our point of view comes from cooling properties of P-stars. In fact, we found that cooling curves of P-stars compare rather well with available observational data. We are, however, aware that such a dramatic change in the standard paradigm of relativistic astrophysics which is based on neutron stars and black holes needs a careful comparison with the huge amount of observations collected so far for pulsar and black hole candidates.
In a forthcoming paper [10] we discuss the generation of magnetic field and the glitch mechanism in P-stars. In particular, we find that for P-star with canonical mass $M \simeq 1.4 M_\odot$ our theory allows dipolar surface magnetic fields up to about $B_S \simeq 10^{17}$ Gauss. Moreover, it turns out that the magnetic field is proportional to the square of the spin period:

$$B_S \simeq B_1 \left( \frac{P}{1 \text{ sec}} \right)^2 ,$$

where $B_1$ is the surface magnetic field for pulsars with period $P = 1 \text{ sec}$. Remarkably, assuming $B_1 \simeq 1.3 \times 10^{13}$ Gauss, we find the Eq. (4) accounts rather well the inferred magnetic field for pulsars ranging from millisecond pulsars up to anomalous X-ray pulsars and soft-gamma repeaters. As a consequence of Eq. (4), we see that the dipolar magnetic field is time dependent. In fact, it is easy to find:

$$B_S(t) \simeq B_0 \left( 1 + 2 \frac{\dot{P}}{P} t \right),$$

where $B_0$ indicates the magnetic field at the initial observation time. Note that Eq. (5) implies that the magnetic field varies on a time scale given by the characteristic age:

$$\tau_c = \frac{P}{2 \dot{P}} .$$

A remarkable consequence of Eq. (5) is that the effective braking index $n$ is time dependent. In particular, the braking index decreases with time such that:

$$-1 \lesssim n \lesssim 3 ,$$

the time scale variation being of order of $\tau_c/2$. However, it turns out that [10] the monotonic derive of the braking index is contrasted by the glitch activity. Indeed, in our theory the glitches originate from dissipative effects in the inner core of the star leading to a decrease of the strength of the dipolar magnetic field, but to an increase of the magnetic torque. Moreover, we find that the time variation of the dipolar magnetic field is the origin of pulsar timing noise.

Concerning black holes, we feel that the most interesting and intriguing aspect of our theory is that P-stars do not admit the existence of an upper limit to the mass of a completely degenerate configuration. In other words, our peculiar equation of state of degenerate up and down quarks in a chromomagnetic condensate allows the existence of finite equilibrium
states for stars of arbitrary mass. In a future publication we shall address the experimental evidence for massive P-star. In particular, we shall discuss the so-called galactic black hole candidates and SgrA*, the super massive compact object at the galactic center.

The aim of the present paper is to discuss in detail the nearest isolated radio quiet compact stars. In the next Section we briefly review the theory of P-stars. After that, we discuss explicitly the isolated compact stars RXJ1856.5-3754 and RXJ0720.4-3125. From the emission spectrum we argue that the most realistic interpretation is that these objects are compact P-stars with $M \simeq 0.8 M_\odot$ and $R \simeq 5 \text{Km}$. Finally, our conclusions are drawn in Section 3.

2. RXJ1856.5-3754 AND RXJ0720.4-3125

In our previous paper we showed that P-stars are able to account for compact stars with $R \lesssim 6 \text{Km}$, as well as stars comparable to canonical neutron stars. In this Section, after reviewing P-stars, we will focus on the nearest isolated compact stars RXJ1856.5-3754 and RXJ0720.4-3125 (henceforth RXJ1856 and RXJ0720 respectively). We will convincingly argue that these compact stars are indeed P-stars.

P-stars are rather different from strange stars which are made entirely of deconfined u, d, s quarks. The possible existence of strange stars is a direct consequence of the so-called strange matter hypothesis, namely that u, d, s quarks in equilibrium with respect to weak interactions could be the true ground state of hadronic matter. Indeed, as discussed in Section 1, P-stars are more stable than neutron stars and strange stars whatever the value of the strength of the chromomagnetic condensate.

To investigate the structure of P-stars we need the equation of state appropriate for the description of deconfined quarks and gluons in an abelian chromomagnetic field. In general, the quark chemical potentials turn out to be smaller than the strength of the chromomagnetic field $\sqrt{gH}$. As a consequence up and down quarks are in the lowest Landau levels with energy $\varepsilon_{0,p_3} = |p_3|$, where $\vec{p}$ is the quark momentum. It is straightforward to determine the thermodynamic potential (at $T = 0$). The energy density is given by:

$$
\varepsilon = \frac{1}{4\pi^2} gH (\mu_u^2 + \mu_d^2) + \frac{\mu_e}{4\pi^2} + \frac{11}{32\pi^2} (gH)^2, \tag{8}
$$

where $\mu_f$ ( $f = d, u, e$) denotes the chemical potential. On the other hand, we find for the
pressure $P$:

$$P = \frac{1}{4\pi^2} gH (\mu_u^2 + \mu_d^2) + \frac{\mu_e^4}{12\pi^2} - \frac{11}{32\pi^2} (gH)^2. \quad (9)$$

Assuming that the system is in equilibrium with respect to weak $\beta$-decays we have the following constraint:

$$\mu_e + \mu_u = \mu_d. \quad (10)$$

Moreover from charge neutrality we also have:

$$\frac{2}{3} n_u - \frac{1}{3} n_d = n_e, \quad (11)$$

where

$$n_u = \frac{1}{2\pi^2} gH \mu_u, \quad n_d = \frac{1}{2\pi^2} gH \mu_d, \quad n_e = \frac{\mu_e^3}{3\pi^2}. \quad (12)$$

From previous equations one can easily write down the equation of state:

$$P = \varepsilon - \frac{\mu_e^4}{6\pi^2} - \frac{11}{16\pi^2} (gH)^2. \quad (13)$$

It is interesting to observe that the sound speed:

$$v_S^2 = \frac{dP}{d\varepsilon} = 1 - \frac{1}{\frac{49}{2} + \frac{18}{\pi^2} + \frac{15}{4\pi^2}}, \quad (14)$$

is quite close to the causal limit $v_S = 1$, for $\mu_e/\sqrt{gH} < 1$, leading to a very stiff equation of state.

To study the properties of the star we consider it to be a spherical symmetric object, corresponding to a non rotating star. The stability of the star is governed by the general-relativistic equation of hydrostatic equilibrium for a spherical configuration of quark matter, which is the Tolman-Oppenheimer-Volkov equation [11]:

$$\frac{dP}{dr} = - \frac{GM(r)\varepsilon(r)}{r^2} \left[1 + \frac{4\pi r^3 P(r)}{M(r)}\right] \frac{1 + \frac{P(r)}{\varepsilon(r)}}{1 - \frac{2GM(r)}{r}}, \quad (15)$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r). \quad (16)$$

These equations can be solved numerically for a given central density $\varepsilon_c$. In this way we obtain $\varepsilon(r)$ and $P(r)$. The radius of the star is determined by:

$$P(r = R) = 0, \quad (17)$$
FIG. 1: Gravitational mass $M$ plotted versus stellar radius $R$ for P-stars for different values of $\sqrt{gH}$. Thick solid and dashed lines correspond to M-R curve for RXJ1856 obtained solving Eq. (21) with $R^\infty = 4.4 \text{ Km}$ and $R^\infty = 6.6 \text{ Km}$ respectively, and assuming that $R \geq R_\gamma = 3GM$. While the total mass by:

$$M = M(r = R).$$

(18)

In Figure 1 we display the gravitational mass $M$ versus the star radius $R$ for different values of $\sqrt{gH}$. At fixed value of the chromomagnetic condensate the mass and radius of the star can be thought of as function of the central energy density $\varepsilon_c$. We see that, as in strange stars, the mass first increases with $\varepsilon_c$ until it reaches a maximum such that $\frac{dM(\varepsilon_c)}{d\varepsilon_c} = 0$. The further increase of $\varepsilon_c$ leads in a region where $\frac{dM(\varepsilon_c)}{d\varepsilon_c} < 0$ and the system becomes unstable. Moreover, we see that there is no lower limit for the radius $R$. Indeed, for small mass we find:

$$R \sim M^{\frac{2}{3}}.$$  

(19)

Interestingly enough, Fig. 1 shows that there are stable P-stars with $M \lesssim M_\odot$ and $R \lesssim 6 \text{ Km}$. This region of the $M - R$ plane could be relevant for the recently observed
compact star \textit{RXJ1856} \cite{17,18}.

\textit{RXJ1856} is the nearest and brightest of a class of isolated radio-quiet compact stars (for a recent review see \cite{19,20}). We shall also consider \textit{RXJ0720}, the second brightest isolated compact star. \textit{RXJ1856} has been observed with Chandra and XMM-Newton \cite{21}, showing that the X-ray spectrum is accurately fitted by a blackbody law. Assuming that the X-ray thermal emission is due to the surface of the star, the authors of Ref. \cite{21} found for the effective radius and surface temperature:

\begin{equation}
R_\infty \approx 4.4 \frac{d}{120 \text{ pc}} \text{Km} \ , \ T_\infty \approx 63 \text{ eV} , \tag{20}
\end{equation}

where

\begin{equation}
R_\infty = \frac{R}{\sqrt{1 - \frac{2GM}{R}}} , \ T_\infty = T \sqrt{1 - \frac{2GM}{R}} . \tag{21}
\end{equation}

Assuming \( R_\infty = 4.4 \text{ Km} \) (i.e. \( d = 120 \text{ pc} \)) we can solve Eq. (21) for the true radius \( R \). We also impose that \( R \geq R_\gamma = 3GM \), for it is well known that the last circular photon orbit for the Schwarzschild geometry is at \( R_\gamma \). Remarkably, Fig. 1 shows that there are stable P-star configurations which agree with observation data. However, it should be stressed that in the observed spectrum there is also an optical emission in excess over the extrapolated X-ray blackbody. By interpreting the optical emission as a Rayleigh-Jeans tail of a thermal blackbody emission, one finds that the optical data are also fitted by the blackbody model yielding an effective radius \( R_\infty > 16 \text{ Km} \frac{d}{120 \text{ pc}} \) \cite{21}. In our previous paper \cite{8}, however, we suggested that the seven data points in the optical range could be interpreted as synchrotron radiation \cite{22} emitted by electrons with energy spectrum:

\begin{equation}
N(E) = \kappa E^{-\eta} , \ 2 \times 10^{-3} \text{KeV} \leq E \leq 7 \times 10^{-3} \text{KeV} . \tag{22}
\end{equation}

Indeed, we find a quite good fit with \( \eta \approx 0.71 \). If this is the case the radiation in the optical range should display a rather large linear polarization \cite{22}:

\begin{equation}
\Pi = \frac{\eta + 1}{\eta + 7/3} \approx 0.56 . \tag{23}
\end{equation}

Interestingly enough, quite recently the distance measurement of \textit{RXJ1856} has been reassessed and it is now estimated to be at 180 pc instead of 120 pc \cite{23}. In this case, from Eq. (20) we obtain \( R_\infty = 6.6 \text{ Km} \). Assuming this value for \( R_\infty \) we can solve Eq. (21). The result, displayed in Fig. \ref{fig:fig1} indicates that:

\begin{equation}
R \approx 5.0 \text{ Km} , \ M \approx 0.8 M_\oplus . \tag{24}
\end{equation}
It is worthwhile to stress that such a value for radius cannot be accounted for within the neutron star theory. However, it must be stressed that unusually small mass and radii are also obtainable within the strange star theory (see, for instance Ref. [24, 25, 26]).

Even more, the new determination of the distance of RXJ1856 rules out the two blackbody interpretation of the spectrum. Indeed, this model leads now to an effective radius $R^\infty > 24$ Km, which is too large for a neutron star. Thus, the new determination of the distance of RXJ1856 strongly supports our interpretation of the optical emission. Moreover, we feel that alternative interpretations recently proposed are problematic. Indeed, a different possibility is to assume that the surface of the compact star is composed of a solid matter [27, 28, 29, 30]. Such a situation may occur at low temperatures and high magnetic fields [31, 32]. Even though there are no reliable calculations for this model, it has been suggested that it will give an optical flux even lower than the blackbody model [20]. We are lead, thus, to conclude that the most realistic interpretation of the spectrum of RXJ1856 is a thermal blackbody emission in X-ray and an optical electron synchrotron radiation emission.

Let us consider the second nearest isolated compact star RXJ0720, which has been detected by ROSAT [33, 34] and observed with XMM-Newton [35, 36]. Remarkably, it turns out that the spectrum of RXJ0720 is almost identical to that of RXJ1856. Indeed, RXJ0720 exhibits a blackbody X-ray spectrum with surface temperature $T^\infty \simeq 80$ eV, a large X-ray to optical flux ratio, a low X-ray luminosity, and an optical emission in excess over the extrapolated X-ray blackbody. A recent analysis [37] of the optical, ultraviolet, and X-ray data showed that the optical spectrum of RXJ0720 is not well fitted by a Rayleigh-Jeans tail, but it is best fitted by a non thermal power law. We find, indeed, that the optical spectrum can be interpreted as synchrotron radiation emitted by electrons with energy spectrum given by Eq. [22] in the relevant energy range. Moreover we obtain $\eta \simeq 0.63$ which is remarkably close to the value for RXJ1856. It should be stressed, however, that in the fitting procedure we do not consider the wavelength dependence of the interstellar extinction [38, 39]. It turns out that interstellar extinction leads to somewhat larger values of $\eta$, leading to a linear polarization which may reach about 90 %. On the other hand, the authors of Ref. [40, 41] found that the best fit to the X-ray spectrum on RXJ0720 is provided by a blackbody model with the effective radius and surface temperature:

$$R^\infty \simeq (2.1 \pm 0.1) \frac{d}{100 \, pc} \, Km \quad , \quad T^\infty \simeq 81 \pm 1 \, eV \, .$$

(25)
The assumed distance to RXJ0720 is \( d \approx 300 \text{ pc} \) \(^{37}\). So that, from Eq. (25) we infer \( R^\infty \approx 6.3 \text{ km} \), almost identical to the RXJ1856 effective radius.

We are led to conclude that the most realistic interpretation of the emission spectra of RXJ1856 and RXJ0720 indicates that these stars are rather compact with radius \( R \approx 5.0 \text{ km} \) and mass \( M \approx 0.8 M_\odot \).

To complete our analysis we must check if the effective surface temperature given by Eqs. (20) and (25) are compatible with the P-star cooling curves. As discussed in Ref. [8], we assume stars of uniform density and isothermal. The equation which determines the thermal history of a P-star is:

\[
C_V \frac{dT}{dt} = -(L_\nu + L_\gamma), \tag{26}
\]

where \( L_\nu \) is the neutrino luminosity, \( L_\gamma \) is the photon luminosity and \( C_V \) is the specific heat. Assuming blackbody photon emission from the surface at an effective surface temperature \( T_S \) we have:

\[
L_\gamma = 4 \pi R^2 \sigma_{SB} T^4_S, \tag{27}
\]

where \( \sigma_{SB} \) is the Stefan–Boltzmann constant. In Ref. [8] we assumed that the surface and interior temperature were related by:

\[
\frac{T_S}{T} = 10^{-2} a , \quad 0.1 \lesssim a \lesssim 1.0. \tag{28}
\]

Equation (28) is relevant for a P-star which is not bare, namely for P-stars which are endowed with a thin crust. It turns out that, like strange stars \(^{42,43}\) (see also the discussion in Ref. [11], pag. 428), P-stars have a sharp edge of thickness of the order of about 1 fermi. On the other hand, electrons which are bound by the coulomb attraction, extend several hundred fermis beyond the edge. As a consequence, on the surface of the star there is a positively charged layer which is able to support a thin crust of ordinary matter (most probably, atomic hydrogen). Thus we see that the vacuum gap between the core and the crust, which is of order of hundred fermis, leads to a strong suppression of the surface temperature with respect to the core temperature. The precise relation between \( T_S \) and \( T \) could be obtained by a careful study of the crust and core thermal interaction. In the case of neutron stars this study has been performed in Ref. [44]. In any case, our phenomenological relation Eq. (28) allows a wide variation of \( T_S \), which encompasses the neutron star relation of Ref. [44]. Moreover, our cooling curves display a rather weak dependence on the parameter
a in Eq. (28) only for stellar age up to $\tau \sim 10^3$ years (see Fig. 2).

The neutrino luminosity $L_\nu$ in Eq. (26) is given by the direct $\beta$-decay quark reactions [45, 46], the dominant cooling processes by neutrino emission. It turns out that the neutrino luminosity is:

$$L_\nu \simeq 3.18 \times 10^{36} \frac{erg}{s} T_9^8 \frac{M}{M_\odot} \frac{\varepsilon_0}{\varepsilon} \frac{\sqrt{gH}}{1 GeV},$$

(29)

where $T_9$ is the temperature in units of $10^9$ $^o$K, and $\varepsilon_0 = 2.51 \times 10^{14} gr/cm^3$ is the nuclear density. Note that the neutrino luminosity $L_\nu$ has the same temperature dependence as the neutrino luminosity by modified URCA reactions in neutron stars (see, for instance Ref. [47]), but it is more than two order of magnitude smaller. The specific heat is given by:

$$C_V \simeq 0.92 \times 10^{55} T_9 \frac{M}{M_\odot} \frac{\varepsilon_0}{\varepsilon} \left(\frac{\sqrt{gH}}{1 GeV}\right)^2,$$

(30)

which in physical units reads:

$$C_V \simeq 1.27 \times 10^{39} \frac{erg}{\varepsilon K} T_9 \frac{M}{M_\odot} \frac{\varepsilon_0}{\varepsilon} \left(\frac{\sqrt{gH}}{1 GeV}\right)^2.$$

(31)
From Eq. (31) we see that the P-star specific heat is of the same order of the neutron star specific heat [47]. In Figure 2 we report our cooling curves, obtained by integrating Eq. (26) with $T^i_0 = 1.4$. It is worthwhile to note that the effective surface temperature almost does not depend on the star mass. Indeed, even though the luminosity and the energy density vary by about an order of magnitude when the stellar mass $M/M_\odot$ ranges from 0.6 to 2.7, the derivative of the temperature depends only on the ratio of the luminosity to the specific heat. It turns out that this ratio varies by less than a factor two in the above range of masses.

In our previous paper [8] we compared the cooling curves with available pulsar data and found a quite satisfying agreement. In Figure 2 we report the effective surface temperatures of RXJ1856 and RXJ0720. The ages of the two compact stars have been fixed by matching with the cooling curves. In this way we obtain:

$$\tau \simeq 10^5 \text{ years for } RXJ1856, \quad \tau \simeq 3.2 \times 10^4 \text{ years for } RXJ0720. \quad (32)$$

In absence of a measured period and period derivative our age estimate for RXJ1856 is a challenge to future observations. On the other hand, recent timing analysis performed in Ref. [48, 49] allowed to determine the period and period derivative of RXJ0720:

$$P \simeq 8.391 \text{ s }, \quad \dot{P} \simeq 5.41 \times 10^{-14} \frac{S}{s}, \quad (33)$$

where for definiteness we use solution (1) of Table 2 in Ref. [49]. From the period and period derivative we may estimate the age of the star. Indeed, assuming star slowdown by dipolar magnetic braking, one obtains the age and the surface magnetic field:

$$\tau = \frac{P}{2 \dot{P}} \left[ 1 - \left( \frac{P_0}{P} \right)^2 \right], \quad (34)$$

$$B_S \simeq \sqrt{\frac{3 I P \dot{P}}{8 \pi^2 R^6}}, \quad (35)$$

where $P_0$ is the initial period, $I$ is the moment of inertia. Assuming that the pulsar started out life rotating much faster than present, then $P_0 \ll P$ and Eq. (34) reduces to the characteristic age $\tau_c$, Eq. (6). Using Eq. (33) we get the characteristic age for RXJ0720:

$$\tau_c \simeq 2.5 \times 10^6 \text{ years}, \quad (36)$$
which is about a factor $10^2$ greater than our estimate in Eq. (32). Note, however, that the characteristic age gives only an upper limit to the true age $\tau$, Eq. (34), which can be significantly smaller than $\tau_c$ if the initial period $P_0$ is close to $P$. As a matter of fact, within the neutron star theory it is believed that pulsars are generally born as rapid rotators, so that they generate a sizeable magnetic field by the dinamo mechanism. On the other hand, the magnetic field of RXJ0720 is quite large. Indeed, using Eq. (35) together with the mass, radius, period and period derivative values given in Eq. (24) and Eq. (33) respectively, we get:

$$B_S \simeq 9.4 \times 10^{19} \sqrt{P \dot{P}} \text{ Gauss} \simeq 6.3 \times 10^{13} \text{ Gauss} .$$  \hspace{1cm} (37)$$

In Ref. [10], where we discuss the generation of magnetic field in P-stars, we argue that huge magnetic field requires that P-stars must born rotating slowly. So that we see that, in the case of RXJ0720, our estimate of the magnetic field Eq. (37) implies a rather large value of $P_0$, in fact comparable to $P$. It is easy to see that, if

$$P_0 \simeq 8.336 \text{ sec} ,$$  \hspace{1cm} (38)$$

then Eq. (34) gives $\tau \simeq 3.2 \times 10^4 \text{ years}$, in perfect agreement with Eq. (32). Moreover, we shall show in Ref. [10] that in this way we may solve completely the puzzling discrepancy between the characteristic age and the true age inferred from the associated supernova remnants in the case of anomalous X-ray pulsars 1E 2259+586 and 1E 1841-045.

Interestingly enough, our estimate of the magnetic field agrees with a recent analysis of the spectrum of RXJ0720 [50]. The authors of Ref. [50] performed a spectral analysis of four XMM-Newton observations of RXJ0720. They find deviations in the spectra from a planckian shape which are interpreted as an absorption line. From the pulse-phase averaged spectra they derive for a gaussian-shaped line:

$$E \simeq 271 \text{ eV} , \sigma \simeq 64 \text{ eV} ,$$  \hspace{1cm} (39)$$

and an equivalent width of $-40 \text{ eV}$. It is natural to assume that cyclotron resonance absorption are likely the origin for the absorption feature seen in the spectra. If we restrict to electrons and protons as the origin of this absorption, we get:

$$E_{B_e} \simeq 11.6 \ B_{12} \ K eV , \ E_{B_p} \simeq 6.3 \ B_{12} \ eV .$$  \hspace{1cm} (40)$$
From the magnetic field Eq. (37), it is clear that electrons are excluded as the origin of the cyclotron line. On the other hand, in the case of protons, using
\[ \sqrt{1 - \frac{2GM}{R}} \simeq 0.726 \] (41)
we obtain the proton cyclotron line at:
\[ E_p \simeq 288 \text{ eV} \] (42)
in striking agreement with Eq. (39).

The last point which we would like to comment is the general feature of the emission spectrum which seems to be consistent with a soft X-ray thermal blackbody emission from the surface and synchrotron (or, more generally, power law) optical and UV radiation. As it is well known, in general pulsar emission is powered by the rotational energy:
\[ E_R = \frac{1}{2} I \omega^2 \] (43)
Thus, the spin-down power output is given by:
\[ -\dot{E}_R = -I \omega \dot{\omega} = 4\pi^2 I \frac{\dot{P}}{P^3} \] (44)
On the other hand, an important source of energy is provided by the magnetic field. Indeed, the classical energy stored into the magnetic field is:
\[ E_B = \int_{r \geq R} \frac{1}{8\pi} B^2(r) \] (45)
Assuming a dipolar magnetic field:
\[ B(r) = B_S \left(\frac{R}{r}\right)^3 \text{ for } r \geq R \] (46)
Eq. (45) leads to:
\[ E_B = \frac{1}{6} B_S^2 R^3 \] (47)
As discussed in Section 1, the surface magnetic field turns out to be time dependent. So that, using Eq. (3) the magnetic power output is given by:
\[ \dot{E}_B = \frac{2}{3} B_0^2 R^3 \frac{\dot{P}}{P} \] (48)
For rotation-powered pulsars it turns out that \( \dot{E}_B \ll -\dot{E}_R \). However, if the dipolar magnetic field is strong enough, then the magnetic power can be of the order, or even
greater than the spin-down power. In Ref. [10] we will show that this fact leads to the pulsar death line, which is the line that in the $P - \dot{P}$ plane separates rotation-powered pulsars from magnetic-powered pulsars.

In the case of $RXJ0720$, by using Eqs. (24) and (37), we get:

$$-\dot{E}_R \simeq 0.9 \times 10^{44} \text{ erg } \frac{\dot{P}}{P},$$

(49)

and

$$\dot{E}_E \simeq 0.3 \times 10^{44} \text{ erg } \frac{\dot{P}}{P}.$$

(50)

We see that, within our uncertainty, $\dot{E}_E + \dot{E}_R \simeq 0$, namely almost all the rotation energy is stored into the increasing magnetic field. As a consequence, the emission from the star consists in thermal blackbody radiation form the surface. In addition, it could eventually also be present a faint synchrotron emission superimposed to the thermal radiation. We believe that $RXJ1856$ is exactly in this state. On the other hand, the energy stored into the magnetic field can be released if the star undergoes a glitch. Indeed, as thoroughly discussed in Ref. [10], glitches originate from dissipative effects in the inner core of the star leading to a decrease of the strength of the dipolar magnetic field. So that, soon after the glitch there is a release of magnetic energy. It is remarkable that this picture is consistent with the recently detected long-term variability in the X-ray emission of $RXJ0720$ [51]. As a consequence we predict that a similar emission could also be detected from $RXJ1856$ when and if this compact star will suffer a glitch.

3. CONCLUSIONS

In this paper we have discussed in details the nearest isolated radio quiet pulsar $RXJ1856$ and $RXJ0720$. Our results show that these stars are most likely P-stars with $M \simeq 0.8 M_\odot$ and $R \simeq 5 Km$. In a forthcoming paper we will discuss the generation of magnetic field and the glitch mechanism in P-stars. Moreover, we will show that our results compare rather well with available pulsar data.

In conclusion, we feel that the ability of our P-star theory for accounting in a coherent fashion several observational features of pulsars should suggest that it warrants serious
consideration.

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