Extended Press–Schechter theory and the density profiles of dark matter haloes

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ABSTRACT
An inside–out model for the formation of haloes in a hierarchical clustering scenario is studied. The method combines the picture of the spherical infall model and a modification of the extended Press–Schechter theory. The mass accretion rate of a halo is defined to be the rate of its mass increase due to minor mergers. The accreted mass is deposited at the outer shells without changing the density profile of the halo inside its current virial radius. We applied the method to a flat Λ–cold dark matter universe. The resulting density profiles are compared with analytical models proposed in the literature, and a very good agreement is found. A trend is found of the inner density profile to become steeper for larger halo mass, which also results from recent N-body simulations. Additionally, present-day concentrations as well as their time evolution are derived and it is shown that they reproduce the results of large cosmological N-body simulations.

Key words: galaxies: formation – galaxies: haloes – galaxies: structure – cosmology: theory – dark matter.

1 INTRODUCTION
Numerical studies (Quinn, Salmon & Zurek 1986; Frenk et al. 1988; Hernquist 1990, hereafter H90; Dubinski & Calberg 1991; Crone, Evrard & Richstone 1994; Cole & Lacey 1996; Fukushima & Makino 1997; Navarro, Frenk & White 1997, hereafter NFW; Kravtsov et al. 1998; Moore et al. 1999, hereafter MGQSL; Huss et al. 1999; Jing & Suto 2000, hereafter JS; Klypin et al. 2001) show that the density profiles of dark matter haloes are fitted by models of the form

\[ \rho(r) = \frac{\rho_s}{(r/r_s)^\gamma [1 + (r/r_s)^\nu]^\beta}. \]  

More specifically NFW proposed a model with \( \lambda = 1, \mu = 1, \nu = 2, \) H90 proposed \( \lambda = 1, \mu = 1, \nu = 3, \) MGQSL \( \lambda = 1.5, \mu = 1.5, \nu = 1, \) while JS proposed \( \lambda = 1.5, \mu = 1, \nu = 1.5. \)

The logarithmic slope \( \gamma, \) of the density profile, defined by

\[ \gamma(r) \equiv -\frac{d \ln \rho(r)}{dr} = \lambda + \mu \nu \frac{(r/r_s)^\nu}{1 + (r/r_s)^\nu} \]  

is, in all the above models, a decreasing function of radius. It is smaller than 2 near the centre of the system and larger near its virial radius.

An interesting quantity that characterizes the shape of the density profile is the concentration. This is defined by \( c = R_{200}/r_s, \) where \( R_{200} \) is the virial radius of the system. In a hierarchical clustering model, the concentration is a decreasing function of the mass of the system. Thus, smaller systems, that formed earlier, have higher concentrations than larger ones. This reflects the high density of the Universe at the epoch of their formation. The concentration of the resulting structures is studied in detail by large N-body simulations. In particular, Bullock et al. (2001, BKPD hereafter), constructed a toy model that describes accurately the time evolution of the concentration in a way that is consistent with the results of their large cosmological simulations.

Although numerical experiments are the most powerful method of studying the formation of structures, the development of analytical or seminumerical methods is very important as well since they help to improve our understanding concerning the physical processes during formation. Density profiles of equilibrium cold dark matter haloes are studied by such methods (e.g. Avila-Reese, Firmani & Hernández 1998; Raig, González-Casado & Salvador-Solé 1998; Syer & White 1998; Henriksen & Widrow 1999; Kull 1999; Nusser & Sheth 1999; Lokas 2000; Hiotelis 2002).

Modifications of the extended Press–Schechter theory (PS) (Press & Schechter 1974; Bower 1991; Bond et al. 1991; Lacey & Cole 1993) based on the distinction between minor and major mergers (Manrique & Salvador-Solé 1996; Kitayama & Suto 1996; Salvador-Solé, Solanes & Manrique 1998; Cohn, Bagla & White 2001) are also used for studying the formation of structures. Recently, such a modified PS approximation was combined with a spherical infall model picture of formation by Manrique et al. (2003), MRRSS hereafter. Their results are in good agreement with those of N-body simulations.

In this paper we use the formalism of MRSSS, with justified modifications, and the same model parameters as in BKPD. We
compare the characteristics of the resulting structures with those in \( N \)-body results. In Section 2, we discuss the modified PS theory and its application to the calculation of the density profile. In Section 3, the characteristics of the resulting dark matter haloes are presented. A discussion is given in Section 4.

2 EXTENDED AND MODIFIED PRESS–SCHETCHER THEORY

One of the major goals of the spherical infall model is the PS approximation. It states that the comoving infall of haloes with mass in the range \( M, M + dM \) at time \( t \) is given by the relation

\[
N(M, t) dM = \frac{2}{\pi^2} \frac{\delta_i(t)}{\sigma(M)} \left( \frac{\sigma(M)}{M} \right)^2 \left( \frac{\rho_0}{m_H} \right) \left( \frac{H_0}{\sigma(M)} \right)^3 dM,
\]

where \( \sigma(M) \) is the present-day rms mass fluctuation on comoving scale containing mass \( M \) and is related to the power spectrum \( P \) by the following relation:

\[
\sigma^2(M) = \frac{2}{\pi^2} \int \tilde{W}^2(k) P(k) k^2 dk,
\]

where \( \tilde{W} \) is the Fourier transform of the window function used to smooth the overdensity field. The mass \( M \) and the radius \( R \) are related by the equation

\[
M = \frac{4}{3} \pi \rho_0 R^3 = \frac{\Omega_m H_0^2}{2G} R^3,
\]

where \( \rho_0 \) is the present-day value of the density of the unperturbed Universe, \( \Omega_m \) is the present-day value of the density parameter (defined at any scale factor \( a \)) and \( H_0 \) is the present-day value of Hubble’s constant, \( H \).

The only time-dependent term of equation (3) is \( \delta_i(t) \), which is the linear extrapolation up to the present epoch of the primordial density that collapses at time \( t \). It is calculated using the following arguments: in a model universe with cosmological constant \( \Lambda \), the radius \( r \) of a sphere having initial overdensity \( \Delta_i \), evolves according to the equation

\[
\frac{dr}{dt} = H_i g^{1/2}(s),
\]

where \( s \equiv r/r_i \) and \( r_i \) is the initial radius. \( H_i \) is the value of the Hubble constant at the initial time \( t_i \) and \( g \) is given by the relation

\[
g(s) = \Omega_m (1 + \Delta_i) (s^{-1} - 1) + \Omega_{\Lambda_i} (s^2 - 1) + \frac{2}{3} f_i \Delta_i
\]

and \( \Omega_{\Lambda_i} \) is the initial value of the quantity \( \Omega_{\Lambda}(a) = \Lambda / [3H^2(a)] \). Equation (7) is derived under the assumption that the initial velocity \( v_i \) of the shell is \( v_i = H_i r_i - v_{pec} \), where the initial peculiar velocity, \( v_{pec} \), is given according to the linear theory relation \( v_{pec} = \frac{1}{2} H_i r_i f_i \) (Peebles 1980). A very good approximation of the factor \( f_i \) is \( f_i = \Omega_m^0 + \frac{1}{3} [1 - \Omega_m(1 + \Omega_{\Lambda})] \) (Lahav et al. 1991). The radius of the maximum expansion is \( r_{ta} = r_{f1} \), where \( r_{f1} \) is the root of the equation \( g(s) = 0 \), which corresponds to zero velocity (\( ds/dt = 0 \)). The sphere reaches its turnaround radius at time

\[
ta = \left( \frac{2}{H_i} \right) \int_0^{r_{ta}} g^{1/2}(s) ds
\]

and then collapses at time \( t_c = 2ta \).

The scale factor \( a \) of the Universe obeys the equation

\[
\frac{da}{dt} = H_0 X^{1/2}(a),
\]

where

\[
X(a) = 1 + \Omega_m(a^{-1} - 1) + \Omega_{\Lambda}(a^2 - 1)
\]

and the subscript 0 denotes the present-day values of the parameters. However, the time and the scale factor \( a \) are related by the equation

\[
t = \left( \frac{1}{H_0} \right) \int_0^a X^{-1/2}(a) da.
\]

Setting \( t = t_c \) in the above equation and solving for \( a \) one finds the scale factor \( a_c \) at the epoch of collapse. If we call \( \delta_0(t) \) the initial overdensity required for the spherical region to collapse at time \( t \) and take into account the linear theory for the evolution of the matter density contrast \( \delta = \delta \rho / \rho \), we have

\[
\delta \propto \left( \frac{1}{H_0^2} \right) \int_0^a X^{-1/2}(a) da = D(a),
\]

(Peebles 1980), then \( \delta_i(t) \) is given by

\[
\delta_i(t) = \delta_i(t) \left( \frac{D(t_0)}{D(t)} \right)\left( \frac{D(t_0)}{D(t)} \right),
\]

where \( t_0 \) denotes the present epoch.

Usually \( \delta_i \) is written in the form

\[
\delta_i(t) = \delta_i(t) \left( \frac{D(t_0)}{D(t)} \right)\left( \frac{D(t_0)}{D(t)} \right),
\]

where \( \delta_{crit}(t) \) is the linear extrapolation of the initial overdensity up to the time \( t \) of its collapse. In an Einstein–de Sitter universe (\( \Omega_m = 1, \Omega_{\Lambda} = 0 \)) this value is independent of the time of collapse and is \( \delta_{crit} \approx 1.686 \). In other cosmologies it has a weak dependence on the time of collapse (e.g. Eke, Cole & Frenk 1996). In a flat universe it can be approximated by the formula \( \delta_{crit}(t) \approx 1.686 \delta_0^2 M_{\nu}^{1/2} \delta_i(t) \).

The PS mass function agrees relatively well with the results of \( N \)-body simulations (e.g. Efstathiou, Frenk & White 1985; Efstathiou & Rees 1988; White, Efstathiou & Frenk 1993; Gelb & Bertschinger 1994; Lacey & Cole 1994; Bond & Myers 1996), while it deviates in detail at both high and low masses. Recent improvements (Sheth & Tormen 1999; Sheth, Mo & Tormen 2001; see also Jenkins et al. 2001) allow a better approximation involving some more parameters. The application of the above approximation to the model studied in this paper is a subject of future research.

Lacey & Cole (1993) extended the PS theory using the idea of a Brownian random walk, and were able to calculate analytically tractable expressions for the mass function, merger rate, and other properties. They show that the instantaneous transition rate \( r \) from haloes with mass \( M \) to haloes with mass between \( M', M' + dM' \) is given by

\[
r(M \rightarrow M', t) dM' = \left( \frac{2}{\pi} \right)^{1/2} \frac{\delta_i(t)}{\sigma(M)} \frac{1}{\sigma^2(M')} \frac{d\sigma(M')}{dM'} \left[ 1 - \frac{\sigma^2(M)}{\sigma^2(M')} \right]^{-3/2} \times \exp \left[ -\frac{\delta_{crit}^2(t)}{2} \left( \frac{1}{\sigma^2(M)} - \frac{1}{\sigma^2(M')} \right) \right] dM'.
\]

This provides the fraction of the total number of haloes with mass \( M \) at \( t \), which give rise per unit time to haloes with mass in the range \( M', M' + dM' \) through instantaneous mergers of any amplitude.

An interesting modification of the extended PS theory is the distinction between minor and major mergers (Kitayama & Suto 1996; Manrique & Salvador-Solé 1996; Salvador-Solé et al. 1998; Percival, Miller & Peacock 2000; Cohn et al. 2001).

Mergers that produce a fractional increase below a given threshold \( \Delta_m \) are regarded as minor. This kind of mergers corresponds to an accretion. Consequently, the rate at which haloes increase their mass
due to minor mergers is the instantaneous mass accretion rate and is given by the relation
\[ r^{in}(M, t) = \int_{M}^{M(1 + \Delta_{\text{vir}})} (M' - M) \rho(M' \rightarrow M', t) \, dM'. \] (16)

Thus the rate of increasing halo mass due to the accretion is
\[ \frac{dM(t)}{dt} = \rho_{b}(t) \Delta_{\text{vir}}(t), \] (17)

Before proceeding further with the model, it is useful to discuss briefly the cosmological considerations concerning the virial radius of a spherical system. Let \( \Delta_{\text{vir}}(a) \) be the ratio of the overdensity of a sphere, that has collapsed and virialized at scalefactor \( a \), to the background density. This can be expressed by the form
\[ \Delta_{\text{vir}}(a) = \frac{\rho(a)}{\rho_{b}(a)} = \frac{1}{s_{c}^{2} a^{3}} \left( \frac{a}{a_{c}} \right)^{3} (1 + \Delta), \] (18)

where \( c_{1} \) is the collapse factor of the sphere defined as the ratio of its final radius to its turnaround radius. Lahav et al. (1991) applied the virial theorem to the virialized final sphere assuming a flat overdensity and found the collapse factor to be \( c_{1} \approx (1 - n/2)/(2 - n/2) \), where \( n = (\Lambda r_{\text{vir}}^{3})/(3GM) \). For an Einstein–de Sitter universe \( \Delta_{\text{vir}}(a) \approx 18\pi^{2}t^{2} \) at any time. For flat models with cosmological constant, significantly good analytical approximations of \( \Delta_{\text{vir}} \) exist. Bryan & Norman (1998) proposed for \( \Delta_{\text{vir}} \) the following approximation:
\[ \Delta_{\text{vir}}(a) \approx (18\pi^{2} - 82x - 39x^{2})/\Omega_{m}(a), \] (19)

where \( x = 1 - \Omega_{m}(a) \).

MRSSS considered the following picture of the formation of a halo: at time \( t \), a halo of virial mass \( M_{i} \) and virial radius \( R_{i} \) is formed and at later times it accretes mass according to equation (17). Assuming that the accreted mass is deposited in an outer spherical shell without changing the density profile inside its current radius, then
\[ M_{i}(t) = M_{i} - \int_{R_{i}}^{R_{i}(t)} 4\pi r^{2} \rho(r) \, dr. \] (20)

The current radius \( R_{i}(t) \) contains a mass with mean density \( \Delta_{\text{vir}}(a) \) times the mean density of the Universe \( \rho_{b}(t) \). Therefore,
\[ R_{i}(t) = \left[ \frac{3M_{i}(t)}{4\pi \Delta_{\text{vir}}(a) \rho_{b}(t)} \right]^{1/3}. \] (21)

Differentiating with respect to \( t \)
\[ \rho(t) = \Delta_{\text{vir}}(t) \rho_{b}(t) \left[ 1 - \frac{M_{i}(t)}{M_{i}(t)} \frac{d[\ln(\rho_{b}(t)\Delta_{\text{vir}}(t))]}{dt} \right]^{-1}. \] (22)

Since one of the goals of this paper is the comparison of our results with the results of the \( N \)-body simulations of BKPD, we have used for \( \Delta_{\text{vir}}(t) \) the same approximation as BKPD did – that is equation (19) – and not the constant value of 200 that MRSSS used. It is convenient to express equation (22) in terms of the scale factor \( a \) instead of the time \( t \). Thus, equation (22) becomes:
\[ \rho(a) = \frac{3H^{2}(a)\Omega_{m}(a)}{8\pi G} \Delta_{\text{vir}}(a) \times \left[ \frac{1 + \frac{M_{i}(a)}{r^{\text{acc}}_{\infty}[M_{i}(a), a]} \left( \frac{3}{a} - \frac{d[\ln(\Delta_{\text{vir}}(a))]}{da} \right) \right]^{-1}, \] (23)

where we have used \( \rho_{b}(a) a^{3} = \text{constant} \) and
\[ r^{\text{acc}}_{\infty}[M_{i}(a), a] = \frac{dM_{i}}{da} = H_{0}^{-1} \chi^{-2/3}(a) r^{in}(M_{i}(t), t). \] (24)

Integrating equation (17) and using equations (21) and (23), we obtain the growth of virial mass and virial radius and, in a parametric form, the density profile of haloes.

### 3 Density Profiles of Dark Matter Haloes

The results described in this section are derived for a flat universe with \( \Omega_{m,0} = 0.3 \) and \( \Omega_{\Lambda,0} = 0.7 \). We have used two forms of power spectrum. The first one, named spect1, is that proposed by Efstathiou, Bond & White (1992). It is based on the results of the COBE Differential Microwave Radiometer (DMR) experiment and is given by the relation
\[ P_{\text{spect1}}(k) = \frac{Bk}{\left[ 1 + \left( ak + (bk)^{1/2} + (ck)^{1/2} \right)^{1/2} \right]^{2}}, \] (25)

where \( a = (6.4/\Gamma) h^{-1} \text{Mpc}, b = (3.0/\Gamma) h^{-1} \text{Mpc}, c = (1.7/\Gamma) h^{-1} \text{Mpc} \) and \( \nu = 1.13 \). Low-density cold dark matter (CDM) models in a spatially flat universe (i.e. \( \Lambda > 0 \)) are described for \( \Gamma = \Omega_{m,0} h \).

The second spectrum – called spect2 – is that proposed by Smith et al. (1998) and is given by
\[ P_{\text{spect2}}(k) = \frac{Ak^{n}}{\left( 1 + a_{1}k^{1/2} + a_{2}k + a_{3}k^{3/2} + a_{4}k^{2} \right)^{2}}. \] (26)

The values for the parameters are: \( n = 1, a_{1} = -1.5598, a_{2} = 47.986, a_{3} = 117.77, a_{4} = 321.92 \) and \( b = 1.8606 \).

We used the top–hat window function that has a Fourier transform given by
\[ \hat{W}(kR) = \frac{3[\sin(kR) - kR \cos(kR)]}{(kR)^{3}}. \] (27)

The constants of proportionality \( A \) and \( B \) are found using the procedure of normalization for \( \sigma_{8} \equiv \sigma(8 h^{-1} \text{Mpc}) = 1. \) In Fig. 1, the resulting rms fluctuations for both spectra are shown. It must be noted that we use a system of units with \( M_{\text{uni}} = 10^{12} h^{-1} M_{\odot} \), \( R_{\text{uni}} = h^{-1} \text{Mpc} \) and \( t_{\text{uni}} = 1.515 \times 10^{7} h^{-1} \text{yr} \). In this system of units \( H_{0}/H_{\text{uni}} = 1.5276 \).

#### 3.1 Present-day structures

In the approximation used in this paper, for given models of the Universe and power spectrum there is only one free parameter, that is the value of the threshold \( \Delta_{m} \) (see equation 16). We found that

![Figure 1](https://example.com/image.png)

**Figure 1.** This plots shows the rms mass fluctuation as a function of mass. Solid line, spect1; dotted line, spect2.
the resulting density profiles are sensitive to the value of \( \Delta_m \). As an example, the density profiles of two systems with the same present-day mass \( 10^{12} h^{-1} \, M_\odot \) and different values of \( \Delta_m \) are plotted in Fig. 2. Both density profiles are derived from spect2. The solid line – shown in Fig. 2 – corresponds to the system derived for \( \Delta_m = 0.21 \), while the dotted line corresponds to the system for \( \Delta_m = 0.5 \). The density profile for smaller \( \Delta_m \) is steeper at both the inner and the outer regions. Additionally, for different \( \Delta_m \) the concentrations of the haloes are different too (a detailed description of the way the concentration is calculated is given below). The system that results for \( \Delta_m = 0.21 \) has \( c_{\text{vir}} = 15.2 \), while that for \( \Delta_m = 0.5 \) has \( c_{\text{vir}} = 8.7 \). In order to calculate the density profiles (that will be presented below), we used as a basic criterion the concentrations of the present-day structures. In fact, we have chosen the values of \( \Delta_m = 0.23 \) and 0.21 for spect1 and spect2, respectively, because the concentrations resulting from these values are close to the results of the toy model of BKPD. This model is constructed by BKPD to reproduce the results of their \( N \)-body simulations and it is able to give the concentration \( c_{\text{vir}} \) of a virial mass \( M_{\text{vir}} \) at any scale factor \( a \). First, the scale factor \( a \), at the epoch of collapse is calculated by solving the following equation:

\[
\sigma[M,(a)] = \sigma[F M_{\text{vir}}(a)], \tag{28}
\]

where \( F = 0.01 \) and \( M_\odot \) is the typical collapsing mass. Then, the concentration is calculated using the formula

\[
c_{\text{vir,BKPD}}[M_{\text{vir}}(a), a] = K \frac{a}{a_c}, \tag{29}
\]

where \( K = 4 \). We recall that the typical collapsing mass at scalefactor \( a \) satisfies \( \sigma[M,(a)] = 1.686 D(1)/D(a) \).

It is obvious that the above defined concentration depends only on the cosmology and the power spectrum used. Thus, for a given cosmology and halo mass, the concentration \( c_{\text{vir,BKPD}} \) is known a priori without taking into consideration any particular form of halo growth. We applied this toy model to find \( c_{\text{vir,BKPD}} \) for the present-day structures.

Another way to calculate the concentration is using \( c_{\text{vir}} = R_{\lambda}/r_\Sigma \), where \( r_\Sigma \) is the radius where the logarithmic slope of the density profile equals 2. This radius is found by the following procedure: first, the resulting density profiles are fitted by the general formula of equation (1). This is done by minimizing the sum

\[
S = \sum (\rho(r) - \rho_*(r))^2, \tag{30}
\]

where \( \rho_*, r_\Sigma, \lambda, \mu \) and \( v \) are fitting parameters. The minimization is performed using the unconstrained subroutine \texttt{znmfzd} of the IMSL mathematical library. Then, \( r_\Sigma \) is found by applying the following formula for \( n = 2 \):

\[
r_n = \left( \frac{n - \lambda}{\lambda + \mu v - n} \right)^{1/\mu} r_\Sigma. \tag{31}
\]

This formula gives the radius \( r_n \) at which the logarithmic slope equals \( n \). According to the model presented in this paper, haloes grow inside–out. Thus, the value of \( c_{\text{vir}} \) represents the rate of mass growth of the halo. In Fig. 3, the concentration is plotted as a function of the present-day virial mass. From the top of the figure, the first pair of curves (solid and dotted) correspond to spect1 and the second pair to spect2. Solid curves show our results while dotted curves depict the results of the toy model of BKPD. A very good agreement between the values of the concentration is shown. In particular, concentrations resulting from spect2 are in agreement with those obtained for the model of BKPD for the whole range of mass presented. On the other hand, small differences appear for very small and very large masses in the case of spect1.

In Fig. 4, we present the density profiles of the resulting structures with present-day masses in the range of \( 0.2 \times 10^{11} \) to \( 8 \times 10^{14} h^{-1} M_\odot \). The left-hand side figures (a1, b1, c1, d1, e1) have been produced using spect1, while the right-hand ones used spect2. Figs (a1) and (a2) correspond to mass \( 0.2 \times 10^{11} h^{-1} M_\odot \), (b1) and (b2) have mass \( 10^{12} h^{-1} M_\odot \), (c1) and (c2) to mass \( 10^{13} h^{-1} M_\odot \), (d1) and (d2) to mass \( 10^{14} h^{-1} M_\odot \) and (e1) and (e2) correspond to mass \( 8 \times 10^{14} h^{-1} M_\odot \). Solid lines represent the resulting density profiles, while dotted lines are the fits using the general formula of equation (1). It is shown that the fits using the general formula of equation (1) are exact. We also fit every halo density profile using the analytical models that have been proposed in the literature (H90, NFW, MGQSL, JS) and are described in Section 1. The best fit of these models to our resulting profiles is shown in Fig. 4 (circles). This best fit is found by the minimizing procedure described above, for \( \lambda, \mu \) and \( v \) constants and equal to the proposed values, while \( \rho_* \) and \( r_\Sigma \) are the only fitting parameters. The best fit for the resulting density profile in (a1) is the H90 model, in (a2) and (b2) the NFW model, in (b1), (c1) and (c2) the MGQSL model and in (d1), (d2), (e1) and (e2) the JS model. Additionally, haloes of different mass are fitted well by different analytical models. This is due to the
different inner and outer slopes of the density profiles. The inner slope (defined as that at a radial distance \( r = 10^{-2} R_{\text{vir}} \)), is an increasing function of the virial mass of the halo. For example, in the case of spect2 the inner slope varies from 1.43 for \( M = 10^{12} h^{-1} M_\odot \) to 1.65 for \( M = 8 \times 10^{14} h^{-1} M_\odot \). Additionally, the outer slope (at \( r = R_{\text{vir}} \)) is a decreasing function of the virial mass and it varies from 3.67 to 2.64 for the above range of masses.

Although density profiles resulting in simulations seem to be similar, systematic trends that relate them to the power spectrum have been reported. For example, Subramanian, Cen & Ostriker (2000) found in the results of their N-body simulations the following: for power spectra of the form \( P(k) \propto k^n \) the density profiles have steeper cores for larger \( n \). Therefore, a dependence of the density profile on the power spectrum is expected. This dependence is shown in our results comparing the profiles of haloes with the same present-day mass. It should be noted that the method studied in this manuscript is applicable for the era of slow accretion when the infalling matter is in the form of small haloes that have a mass of less than \( \Delta m \) times the mass of the parent halo. This kind of accretion occurs at the late stages of formation and thus determines the profile of the outer regions of the halo under study. However, the values of the inner slopes may be questionable. Real haloes have followed different mass growth histories and thus their properties show a significant scatter about a mean value. Unfortunately, the method studied in this manuscript results in one profile for a halo of given mass. Thus, its purpose is just to approximate the mean density profile of a large number of mass growth histories. Since the mass growth history resulting from the method is in good agreement with the mean growth history resulting from the N-body simulation – as will be shown below – then the values of the inner slopes could be close to those of N-body simulations. A Monte Carlo analytical approach based on the construction of a large number of mass accretion histories is under study. This study could solve some of the above problems.

In Fig. 5 the exponent \( \lambda \) is plotted, which gives the asymptotic slope at \( R \to 0 \), derived by the general fit as a function of present-day virial mass for both power spectra. It is shown that the exponent \( \lambda \) is an increasing function of the virial mass. This trend of the inner density profile is also found in the results of recent N-body simulations (Ricotti 2002).

### 3.2 Time evolution

In Fig. 6 we plot mass growth curves. The curves show \( M_{\text{vir}}(a) \) as a function of \( a \) on a logarithmic slope. The solid lines show our resulting structures and the dotted lines show the mass growth curves of the model proposed by Wechsler et al. (2002). The curves of the left-hand panel correspond to spect1, while those of the right-hand panel correspond to spect2. From the top to the bottom, the curves correspond to masses of \( 2 \times 10^{11}, 10^{12}, 10^{13}, 10^{14} \) and \( 8 \times 10^{14} h^{-1} M_\odot \), respectively. It is obvious that massive haloes show a substantial increase of their mass up to late times, while the growth curves of less massive haloes tend to flatten out earlier. This behaviour of mass growth curves characterizes the hierarchical clustering scenario where small haloes are formed earlier than more massive ones. Additionally, it helps to define the term ‘formation time’ in a measurable way. Wechsler et al. (2002) define the formation scalefactor \( \bar{a}_c \) as the scale factor when the logarithmic slope of the mass growth, \( \langle \ln M(a)/\ln a \rangle \), falls below some specified value, \( S \). They use the value \( S = 2 \). It should be noted that this definition of formation scalefactor differs from \( a_c \), defined by BKPD, since \( a_c \) is the value of the scale factor at the epoch the typical collapsing mass is \( F \) times the virial mass of the halo. We found that the values of \( \bar{a}_c \) and \( a_c \) for \( F = 0.01 \) and \( S = 2 \) are different. This is also noticed in Wechsler et al. (2002) since they state that \( \bar{a}_c \) and \( a_c \) have similar values for \( S = 2 \) but for \( F = 0.015 \). However, the use of the value \( F = 0.015 \) in the toy model of BKPD changes the resulting concentrations and so our basic criterion for the choice of the threshold \( \Delta m \) is not satisfied. Therefore, it is preferable to choose a different value of \( S \) for the definition of \( \bar{a}_c \), that of \( S = 1.5 \). In Fig. 6, the dotted lines

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**Figure 4.** Density profiles as a function of radius. Solid curves: resulting density profiles. Dotted curves: fits of the resulting density profiles using the formula of equation (1). Circles: best bit to our results using models proposed in the literature (H90, NFW, MGQSL, JS). Left-hand side: spect1. Right-hand side: spect2.

**Figure 5.** Exponent \( \lambda \) as a function of present-day virial mass for both power spectra. It is shown that \( \lambda \) is an increasing function of the virial mass. Solid curve: spect1. Dashed curve: spect2.
show the mass growth curves of the model proposed by Wechsler et al. (2002). In this model the mass growth is calculated using the relation

\[ M_{\text{vir}}(a) = M_{\text{vir},0} \exp[-\tilde{a}_c S(1/a - 1)], \]

(32)

where \( M_{\text{vir},0} \) is the present-day virial mass and the formation scale-factor \( \tilde{a}_c \) is defined by the condition \( \frac{d \ln M(a)}{d \ln a} = S \) with \( S = 1.5 \). In Fig. 6, a very satisfactory agreement is shown, particularly for the less massive haloes.

We have to note that our model haloes grow inside-out. Therefore, at early enough times—when the slope of the density is smaller that 2 all the way from the centre up to the current radius—it is meaningless to define \( c_{\text{vir}} \). Once the building of the halo has proceeded beyond the point with slope 2, the evolution of \( c_{\text{vir}} \) is due to the growth of the virial radius and is given by

\[ c_{\text{vir}}(M(a), a) = c_{\text{vir}}(M_0) \frac{R_{\text{vir}}(M(a))}{R_{\text{vir}}(M_0)}, \]

(33)

where \( c_{\text{vir}}(M_0) \) denotes the present-day concentration and \( R_{\text{vir}}(M(a)) \) and \( R_{\text{vir}}(M_0) \) are the values of the virial radius at scalefactor \( a \) and at the present day, respectively. In Fig. 7 the time evolution of concentrations is plotted. Solid lines describe \( c_{\text{vir}} \) while dotted lines denote \( c_{\text{vir,BKPD}} \). More massive haloes have lower concentrations that evolve slower, while the concentrations of less massive haloes are higher and evolve more rapidly.

4 CONCLUSIONS

Since the formation of structures in a hierarchical clustering scenario is a complicated process, any attempt to construct analytical models requires a number of crucial assumptions.

The model studied in this paper was proposed by MRSSS and assumes that:

(i) the rate of mass accretion is defined by the rate of minor mergers;

(ii) haloes grow inside-out. The accreted mass is deposited at the outer shells without changing the density profile of the halo inside its current virial radius.

The first assumption indicates that structures presented in this paper formed by a gentle accretion of mass. The physical process implied by the second assumption is that the infalling matter does not
penetrate the current virial radius. This process requires an amount of non-radial motion. This amount has to be large enough so that the pericentre of the accreted mass is larger than the current virial radius. It should be noted that a density profile that results from a radial collapse has an inner slope steeper than 2. It is the presence of non-radial motion during the collapse that leads to inner slopes shallower than 2 (e.g. Subramanian et al. 2000; Nusser 2001; Hiotelis 2002). Non-radial motions are always present in the structures formed in N-body simulations.

Despite the above assumptions, the results of the model studied in this paper are in good agreement with the results of N-body simulations. The summary of these results is as follows.

(i) Density profiles of haloes are close to the analytical models proposed in the literature as good fits to the results of N-body simulations. A trend of the inner slope of the density profile to be an increasing function of the mass of the halo is also found, in agreement with recent results of N-body simulations.

(ii) Concentration is a decreasing function of virial mass. Its values are in agreement with the results of numerical methods.

(iii) Massive haloes increase their mass substantially up to late times. Growth curves of less massive haloes tend to flatten out earlier. The concentrations of less massive haloes evolve more rapidly, while those of more massive haloes evolve slowly.

Taking into account the number of assumptions and approximations used to build the model presented in this paper, we can conclude that the agreement with the results of N-body simulations is very good. Consequently, this model provides a very promising method to deal with the process of structure formation. Further improvements to this model could help give a better understanding of the physical picture during this process.

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REFERENCES

Avila-Reese V., Firmani C., Hernández X., 1998, ApJ, 505, 37
Bond J.R., Myers S., 1996, ApJS, 103, 41
Bond J.R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
Bower R.J., 1991, MNRAS, 248, 332
Bryan G., Norman M., 1998, ApJ, 495, 80
Bullock J.S., Kolatt T., Primack J.R., Dekel A., 2001, MNRAS, 321, 559 (BKPD)
Cohn J.D., Bagla J.S., White M., 2001, MNRAS, 325, 1053
Cole S., Lacey C., 1996, MNRAS, 281, 716
Crone M.M., Evrard A.E., Richstone D.O., 1994, ApJ, 434, 402
Dubinski J., Calberg R., 1991, ApJ, 378, 496
Efstathiou G., Rees M., 1988, MNRAS, 230, 5
Efstathiou G., Frenk C.S., White S.D.M., 1985, ApJ, 292, 371
Efstathiou G., Bond J.R., White S.D.M., 1992, MNRAS, 258, 1
Eke V.R., Cole S., Frenk C.S., 1996, MNRAS, 282, 263
Frenk C.S., White S.D.M., Davis M., Efstathiou G., 1988, ApJ, 327, 507
Fukushige T., Makino J., 1997, ApJ, 477, L9
Gelb J., Bertschinger E., 1994, ApJ, 436, 467
Henriksen R.N., Widrow L.M., 1999, MNRAS, 302, 321
Hernquist L., 1990, ApJ, 356, 539 (H90)
Hiotelis N., 2002, A&A, 382, 84
Huss A., Jain B., Steinmetz M., 1999, MNRAS, 308, 1011
Jenkins A., Frenk C.S., White S.D.M., Colberg J.M., Cole S., Evrard A.E., Couchman H.M.P., Yoshida N., 2001, MNRAS, 321, 372
Jing Y.P., Suto Y., 2000, ApJ, 529, L69 (JS)
Kitayama T., Suto Y., 1996, MNRAS, 280, 638
Klypin A.A., Kravtsov A.V., Bullock J.S., Primack J.R., 2001, ApJ, 554, 903
Kravtsov A.V., Klypin A.A., Bullock J.S., Primack J.R., 1998, ApJ, 502, 48
Kull A., 1999, ApJ, 516, L5
Lacey C., Cole S., 1993, MNRAS, 262, 627
Lacey C., Cole S., 1994, MNRAS, 271, 676
Lahav O., Lilje P.B., Primack J.R., Rees M.J., 1991, MNRAS, 251, 128
Lokas E.L., 2000, MNRAS, 311, 423
Manrique A., Salvador-Solé E., 1996, ApJ, 467, 504
Manrique A., Raig A., Salvador-Solé E., Sanchis T., Solanes J.M., 2003, ApJ, 593, 26 (MRSSS)
Moore B., Governato F., Quinn T., Stadel J., Lake G., 1998, ApJ, 499, L5 (MQSL)
Navarro J.F., Frenk C.S., White S.D.M., 1997, ApJ, 490, 493 (NFW)
Nusser A., 2001, MNRAS, 325, 1397
Nusser A., Sheth R., 1999, MNRAS, 303, 685
Peebles P.J.E., 1980, The Large-Scale Structure of the Universe. Princeton Univ. Press, Princeton
Percival W.J., Miller L., Peacock J.A., 2000, MNRAS, 318, 273
Press W.H., Schechter P., 1974, ApJ, 187, 425
Quinn P.J., Salmon J.K., Zurek W.H., 1986, Nat, 322, 329
Raig A., González-Casado G., Salvador-Solé E., 1998, ApJ, 508, L129
Ricotti M., 2002, MNRAS, in press (astro-ph/0212146)
Salvador-Solé E., Solanes J.M., Manrique A., 1998, ApJ, 499, 542
Sheth R.K., Tormen G., 1999, MNRAS, 308, 119
Sheth R.K., Mo H.J., Tormen G., 2001, MNRAS, 323, 1
Smith C.C., Klypin A., Gross M.A.K., Primack J.R., Holtzman J., 1998, ApJ, 508, L129
Subramanian K., Cen R.Y., Ostriker J.P., 2000, ApJ, 538, 528
Syer D., White S.D.M., 1988, MNRAS, 239, 337
Wechsler R.H., Bullock J.S., Primack J.R., Kratsov A.V., Dekel A., 2002, ApJ, 568, 52
White S.D.M., Efstathiou G., Frenk C., 1993, MNRAS, 262, 1023

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