Possible formation of lowly luminous highly magnetized white dwarfs by accretion leading to SGRs/AXPs

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We sketch a possible evolutionary scenario by which a highly magnetized super-Chandrasekhar white dwarf could be formed by accretion on to a commonly observed magnetized white dwarf. This is an exploratory study, when the physics in cataclysmic variables (CVs) is very rich and complex. Based on this, we also explore the possibility that the white dwarf pulsar AR Sco acquired its high spin and magnetic field due to repeated episodes of accretion and spin-down. We show that strong magnetic field dramatically decreases luminosity of highly magnetized white dwarf (B-WD), letting them below the current detection limit. The repetition of this cycle can eventually lead to a B-WD, recently postulated to be the reason for over-luminous type Ia supernovae. A spinning B-WD could also be an ideal source for continuous gravitational radiation and soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs). SGRs/AXPs are generally believed to be highly magnetized, but observationally not confirmed yet, neutron stars. Invoking B-WDs does not require the magnetic field to be as high as for neutron star based model, however reproducing other observed properties intact.

Keywords: white dwarfs; strong magnetic fields; CVs; pulsars; SGRs/AXPs.

1. Introduction

Several independent observations repeatedly argued in recent past for the existence of highly magnetized white dwarfs (B-WDs). Examples are over-luminous type Ia supernovae\(^1\)\(^2\), white dwarf pulsars\(^3\)\(^4\), etc. Also soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) could be explained as B-WDs\(^5\)\(^–\)\(^7\), while they are generally believed to be highly magnetized neutron stars\(^8\) without however any direct detection of underlying required high surface field \(B_s \sim 10^{15}\) G. Interestingly, explaining SGR/AXP by a magnetized white dwarf requires a lower \(B_s \lesssim 10^{12}\) G, which may however correspond to central field \(\gtrsim 10^{14}\) G. Nevertheless, the origin of such fields in a white dwarf remains a question, when the observed confirmed surface field is \(\lesssim 10^{9}\) G.

Here we explore a possible evolution of a conventionally observed magnetized white dwarf to a B-WD by accretion, which may pass through a phase exhibiting currently observed AR Sco. This is an exploratory study, and the present venture is based more on an idealized situation, when the physics in accreting white dwarfs, i.e. cataclysmic variables (CVs), is very rich and complex. We also show, based on some assumption, that the thermal luminosity of such a B-WD could be very small, below their current detection limit. However, due to high field and rotation, their spin-down luminosity could be quite high. Hence, they could exhibit SGRs/AXPs.
2. Accretion induced evolution

The detailed investigation of the accretion induced evolution faces several difficulties including nova eruptions (hence nonsteady increase of mass) and the eruption and ejection of accumulated shells. Nevertheless, the discovery of AR Sco, which is a fast rotating magnetized white dwarf, argues for the possibility of episodic increase of mass in a CV. Hence, we sketch a tentative evolutionary scenario with repeated episodes of accretion phase leading to the high magnetic field via flux freezing and spin-power phase decreasing field. Eventually this mechanism can plausibly lead to a B-WD. Note that there are already observational evidences for transitions between spin-power and accretion-power phases in a binary millisecond pulsar. The conservation laws controlling the accretion-power phase around the stellar surface of radius \( R \) and mass \( M \), which could be inner edge of accretion disk, are given by

\[
\frac{l \Omega(t)}{(t)^2 R(t)^2} = \frac{GM(t)}{R(t)^2}, \quad I(t) \Omega(t) = \text{constant}, \quad B_s(t) R(t)^2 = \text{constant}, \quad (1)
\]

where \( l \) takes care of inequality due to dominance of gravitational force over the centrifugal force in general, \( I \) is the moment of inertia of star and \( \Omega \) the angular velocity of the star which includes the additional contribution acquired due to accretion as well. Solving the conservation laws given by equation (1) simultaneously, we obtain the time evolution of radius (or mass), magnetic field and angular velocity during accretion. Accretion stops when

\[
-\frac{GM}{R^2} = \frac{1}{\rho \, dr} \left( \frac{B_s^2}{8\pi} \right) |_{r=R} \sim -\frac{B_s^2}{8\pi R \rho},
\]

where \( \rho \) is the density of inner edge of disk.

For a dipolar fixed field, \( \dot{\Omega} \propto \Omega^3 \), where over-dot implies time derivative. Generalizing for the present purpose it becomes \( \dot{\Omega} = k \Omega^n \) with \( k \) being constant. Therefore, during the phase of spin-power pulsar (when accretion inhibits), the time evolution of angular velocity and surface magnetic field may be given by

\[
\Omega = \left[ \Omega_0^{1-n} - k(1-n)(t-t_0) \right]^{1/n}, \quad B_s = \sqrt{\frac{5c^3 I k \Omega^n}{R^6 \sin^2 \alpha}},
\]

where \( \Omega_0 \) is the angular velocity when accretion just stops at the beginning of spin-power phase at time \( t = t_0 \), \( k \) is fixed to constrain \( B_s \) at the beginning of first spin-powered phase, which is determined from the field evolution in the preceding accretion-power phase, \( \alpha \) is the angle between magnetic and spin axes. Note that \( n = m = 3 \) corresponds to dipole field.

Figure 1 shows a couple of representative possible evolutions of angular velocity and magnetic field with mass (time). It is seen that initial larger \( \Omega \) with accretion drops significantly during spin-power phase (when accretion stops and hence no change of mass), followed by its increasing phase. Similar trend is seen in \( B_s \) profiles with a sharp increasing trend (with value \( \sim 10^{11} \text{ G} \)) at the last cycle leading to the increase of \( B_c \) as well, forming a B-WD. At the end of evolution, it could be left
out as a super-Chandrasekhar B-WD and/or a SGR/AXP candidate with a higher
spin frequency. Of course, in reality they may depend on many other factors and
the current picture does not match exactly with what is expected in AR Sco itself.

Fig. 1. Time evolution of (a) angular velocity in s$^{-1}$, (b) magnetic field in G, as functions
of mass in units of solar mass. The solid curves correspond to $n = 3$, $m = 2$, $\rho = 0.05$ gm
$cm^{-3}$, $l = 1.5$ and dotted ones to $n = 3$, $m = 2$, $\rho = 0.1$ gm $cm^{-3}$, $l = 2.5$; $k = 10^{-14}$ CGS,
$M = 10^{-8} M_\odot$ Yr$^{-1}$, $\alpha = 10^\circ$ and $R = 10^4$ km at $t = 0$. This is reproduced from a previous work.

3. Luminosity

With the increase of mass, the radius of white dwarfs, hence B-WDs, becomes very
small. Indeed, the increase of magnetic field is due to decreasing radius via
flux-freezing. Now due to smaller radius, UV-luminosity of B-WDs turns out to be
very small if the surface temperature is same as their nonmagnetic counterpart.
However more interestingly, from the conservation of energy, it is expected that
the presence of strong magnetic field enforces decreasing thermal energy and hence
lowering luminosity in stable equilibrium.

Combining the magnetostatic and photon diffusion equations in the presence of
magnetic field but ignoring tension, we obtain

$$\frac{d}{dT} (P + P_B) = \frac{4ac}{3} \frac{4\pi GM T^3}{L} \frac{\kappa}{\kappa},$$

which we solve to obtain the envelop properties. Here $P$ is the matter pressure, $P_B$
the magnetic pressure, $\kappa$ the opacity, $T$ the temperature, $a$ the radiation constant,
$M$ the mass of white dwarf within the core radius $r$, which is practically the whole
mass of white dwarf because the envelop is very thin, and $L$ is the luminosity.
For the strong field considered here, the radiative opacity variation with $B$ can be
modelled similarly to neutron stars as $\kappa = \kappa_B \approx 5.5 \times 10^{31} \rho T^{-1.5} B^{-2} \text{ cm}^2\text{g}^{-1}$.
We use a field profile proposed earlier for neutron stars to enumerate the field magnitude at a given density (radius), irrespective of other complicated effects, given by

\[ B \left( \frac{\rho}{\rho_0} \right) = B_s + B_0 \left[ 1 - \exp \left( -\eta \left( \frac{\rho}{\rho_0} \right)^\gamma \right) \right], \]

where \( B_0 \) (similar to central field) is a parameter with the dimension of \( B \), other parameters are set as \( \eta = 0.1, \gamma = 0.9, \rho_0 = 10^9 \text{ g cm}^{-3} \) for all the calculations. Further equating the electron pressure for the non-relativistic electrons on both sides of the core-envelop interface gives

\[ \rho_*(B_s) \approx 1.482 \times 10^{-12} T_s^{1/2} B_s \]

at interface. Now we solve equation (4) along with the photon diffusion equation

\[ \frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa (\rho + \rho_B)}{T^3} \frac{L}{4\pi r^2}, \]

with boundary conditions \( \rho(T_s) = 10^{-10} \text{ g cm}^{-3}, r(T_s) = R = 5000 \text{ km} \) and \( M = M_\odot \), where \( T_s \) is the surface temperature, and obtain \( \rho - T \) and \( r - T \) profiles. Further \( T_s \) and \( \rho_* \) can be obtained by solving for the \( \rho - T \) profile along with equation (6), as shown in Fig. 2, and knowing \( T_s \), we can obtain \( r_* \) from the \( r - T \) profile. We see that interface moves inwards (\( r_* \) decreases) with increasing \( B \) and \( L \). But \( \rho_* \) increases with increasing \( L \) and/or \( B \), as \( \rho_* \propto T_s^{1/2} B_s \).

![Fig. 2. Variation of density with temperature for \( B \equiv (B_s, B_0) = (10^{12} \text{ G, } 10^{14} \text{ G}) \) and \( L = 10^{-5} L_\odot \) (dashed line), \( 10^{-4} L_\odot \) (dotted line) and \( 10^{-3} L_\odot \) (dot-dashed line). The solid line represents equation (6). This is reproduced from a previous work.](image)

With above benchmarking, we now explore, based energy conservation, if the luminosity of a magnetized white dwarf or B-WD changes. For \( B = 0 \) and \( L = 10^{-5} L_\odot \), we have \( r_* = 0.9978R, \rho_* = 170.7 \text{ g cm}^{-3} \) and \( T_s = 2.332 \times 10^6 \text{ K} \). Using the same boundary condition as described above, we now solve equations (4) and (7) with \( B \neq 0 \), but vary \( L \) in order to fix \( r_* = 0.9978R \). We find interestingly that \( L \) decreases for \( B \neq 0 \), as shown in Table 1. Physically this corresponds to increasing \( B \), and thence magnetic energy, is compensated by decreasing thermal energy (decreasing \( T_s \)) and thence \( L \), when total energy is conserved. Similarly,
increasing $B$ may be compensated by decreasing gravitational energy (decreasing $r_*$). In either of the cases, $L$ decreases.

| $B/G = (B_0/G, B_0/G)$ | $L/L_\odot$ | $T_*/K$ | $\rho_*/g\text{ cm}^{-3}$ | $T_*/K$ |
|--------------------------|-------------|---------|-----------------|---------|
| (0, 0)                   | $1.00 \times 10^{-5}$ | $2.332 \times 10^8$ | $1.707 \times 10^2$ | $3.85 \times 10^2$ |
| $(10^{9}, 6 \times 10^{13})$ | $2.53 \times 10^{-7}$ | $4.901 \times 10^5$ | $1.037 \times 10^0$ | $1.53 \times 10^3$ |
| $(5 \times 10^{9}, 2 \times 10^{13})$ | $3.96 \times 10^{-8}$ | $3.262 \times 10^5$ | $4.232 \times 10^0$ | $9.65 \times 10^2$ |
| $(10^{10}, 10^{13})$ | $1.02 \times 10^{-6}$ | $7.189 \times 10^5$ | $1.257 \times 10^1$ | $2.17 \times 10^3$ |
| $(2 \times 10^{11}, 8 \times 10^{12})$ | $4.40 \times 10^{-9}$ | $2.063 \times 10^5$ | $1.346 \times 10^1$ | $5.57 \times 10^2$ |
| $(5 \times 10^{10}, 4 \times 10^{12})$ | $2.59 \times 10^{-8}$ | $3.185 \times 10^5$ | $4.182 \times 10^1$ | $8.68 \times 10^2$ |
| $(5 \times 10^{11}, 10^{12})$ | $2.93 \times 10^{-9}$ | $2.206 \times 10^5$ | $3.480 \times 10^2$ | $5.03 \times 10^2$ |

4. SGRs/AXPs as B-WDs

Paczynski\textsuperscript{5} and Usov\textsuperscript{6} independently proposed that SGRs and AXPs are moderately magnetized white dwarfs but following Chandrasekhar’s mass-radius relation\textsuperscript{14}. Many features of SGRs/AXPs are explained by their model at relatively lower magnetic fields, while the more popular magnetar model\textsuperscript{8} requires field $\gtrsim 10^{15}$ G, which is not observationally well established yet. Nevertheless, such a white dwarf based model suffers from a deep upper limit on the optical counterparts of some AXPs/SGRs, e.g. SGR 0418+5729, due to their larger moment of inertia.

Now B-WDs established here could be quite smaller in size and hence have smaller moment of inertia. Therefore, their optical counterparts, with very low UV-luminosities, are quite in accordance with observation. Hence, the idea of B-WD brings a new scope of explain SGRs/AXPs at smaller magnetic fields, which are observationally inerable, compared to highly magnetized magnetar model. For details see the work by Mukhopadhyay & Rao\textsuperscript{7}.

5. Continuous Gravitational Radiation

Due to smaller size compared to their regular counterpart, B-WDs rotate relatively faster. Now if the rotation and magnetic axes are misaligned, they serve as good candidates for continuous gravitational radiation due to their quadrupole moment, characterized by the amplitude\textsuperscript{15}

\[ h_+ (t) = \frac{h_0}{2} (1 + \cos^2 \alpha_0) \cos \Phi (t), \quad h_\times (t) = h_0 \cos \alpha_0 \sin \Phi (t), \quad h_0 = \frac{4\pi^2 GI_{zz} \epsilon}{c^2 P^2 D}, \quad (8) \]

where $\alpha_0$ is the inclination of the star’s rotation axis with respect to the observer, $\Phi (t)$ is the signal phase function, $\epsilon$ amounts the ellipticity of the star, $I_{zz}$ is the moment of inertial about $z$-axis, and $D$ is the distance between the star and detector.
A B-WD of mass $\sim 2M_{\odot}$, polar radius $\sim 700$ km, spin period $P_s \sim 1$ s\textsuperscript{16}, $\epsilon \sim 5 \times 10^{-4}$ and $D \sim 100$ pc would produce $h_0 \sim 10^{-22}$, which is within the sensitivity of the Einstein@Home search for early Laser Interferometer Gravitational Wave Observatory (LIGO) S5 data\textsuperscript{15}. However, DECIGO/BBO would give a firm confirmation of their gravitational wave because they are more sensitive in their frequency range. In fact, if the polar radius is $\sim 2000$ km with $P_s \sim 10$ s and other parameters intact, DECIGO/BBO can detect it with $h_0 \sim 10^{-23}$. Nevertheless, (highly) magnetized rotating white dwarfs approaching B-WDs are expected to be common and such white dwarfs of radius $\sim 7000$ km, $P_s \sim 20$ sec and $D \sim 10$ pc could produce $h_0 \gtrsim 10^{-22}$ which is detectable by LISA.

6. Summary

The idea of B-WD has been proposed early this decade, mainly to explain observed peculiar type Ia supernovae inferring super-Chandrasekhar progenitor mass. Lately it has been found with various other applications, e.g. SGRs/AXPs, white dwarf pulsars like AR Sco, continuous gravitational wave etc. Here we have attempted to sketch a plausible evolution scenario to explain the formation of such a highly magnetized, smaller size white dwarf. In our simplistic picture, ignoring many complicated CV features, we are able to show that a commonly observed magnetized white dwarf could be evolved to a B-WD via accretion. Hence, the existence of highly magnetized, rotating, smaller white dwarfs is quite plausible.

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