Planck and reionization history: a model selection view

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9 July 2008

ABSTRACT
We use Bayesian model selection tools to forecast the Planck satellite’s ability to distinguish between different models for the reionization history of the Universe, using the large angular scale signal in the cosmic microwave background polarization spectrum. We find that Planck is not expected to be able to distinguish between an instantaneous reionization model and a two-parameter smooth reionization model, except for extreme values of the additional reionization parameter. If it cannot, then it will be unable to distinguish between different two-parameter models either. However, Bayesian model averaging will be needed to obtain unbiased estimates of the optical depth to reionization. We also generalize our results to a hypothetical future cosmic variance limited microwave anisotropy survey, where the outlook is more optimistic.

Key words: cosmology: theory, methods: data analysis, methods: statistical

1 INTRODUCTION

The five-year data from the Wilkinson Microwave Anisotropy Probe (WMAP; Hinshaw et al. 2008; Dunkley et al. 2008; Komatsu et al. 2008) have given reasonably tight constraints on the optical depth to Thomson scattering from the last-scattering surface, \( \tau = 0.09 \pm 0.02 \) with modest dependence on inclusion of additional datasets and changes to model assumptions. It has not however had the accuracy needed to go beyond this one-parameter description of the ionization history of the Universe, to give a more detailed view of how reionization took place and to distinguish between the various models in the literature (though combined with tentative indication of a change in Lyman-\( \alpha \) optical depth around redshift 7, it does give some indication that reionization is an extended process).

Theoretical studies suggest that the process of reionization can be quite complex (eg Barkana & Loeb 2001; Haiman & Holder 2003; Cen 2003; for reviews see Barkana & Loeb 2007a; Meiksin 2007). The Planck satellite may have the sensitivity to go beyond a one-parameter description of the formation process. For instance, Lewis, Weller & Battye (2006) considered three specific reionization histories (with other cosmological parameters held fixed), and assessed whether Planck would be able to distinguish amongst them, finding that it did indeed have some ability to do so.

However, the true data analysis problem is more complicated than in their study. Future experiments will not be trying to distinguish between a small set of specific reionization histories. Rather, there will be competing models for reionization each of which feature parameters that need to be determined from the data. That is, the problem is one of model selection (see Gregory 2005; Liddle, Mukherjee & Parkinson 2006a; Trotta 2008, and references therein). In this paper we use Bayesian model selection tools to forecast the ability of the Planck satellite, and a putative cosmic-variance-limited future survey, to distinguish between two reionization models, instantaneous reionization (parameterized solely by the optical depth to reionization \( \tau \) or equivalently the redshift of reionization), and a smooth transition to the ionized state, parameterized by a further parameter \( d_\eta \) which measures the rapidity of the transition (in conformal time \( \eta \)).

We consider only the large-scale bump in the cosmic microwave background (CMB) polarization spectrum generated by Thomson scattering of the CMB quadrupolar anisotropy during reionization. The detailed shape of the bump is related to the evolution of the globally-averaged ionized fraction during reionization (Kaplinghat et al. 2003; Hu & Holder 2003; Colombo et al. 2005). The power on scales smaller than the horizon size at reionization is uniformly damped by \( e^{-2\tau} \); this then cannot be used to constrain the details of reionization beyond \( \tau \), or even to constrain \( \tau \) itself which would be almost completely degenerate with the amplitude of perturbations. Other degeneracies are discussed in Martin et al. (2004) and Trotta & Hansen (2004). Reionization affects the CMB spectrum again on much smaller scales via secondary effects due to inhomogeneous or patchy reionization and the Ostriker–Vishniac effects (Ostriker & Vishniac 1986; Weller 1999; Hu 2000). We do not consider these effects which are beyond multipole \( \ell \sim 2000 \), modelling only uniform reionization. In the future, 21cm emission from neutral hydrogen is expected to provide a good tracer of the details of reionization (eg. see Barkana & Loeb 2007b), and there are experiments that will focus on mapping this emission.

2 THE MODELS

Our cosmological model is the usual spatially-flat \( \Lambda \)CDM cosmology, seeded by power-law adiabatic density perturbations. Its ad-
justable parameters are the dark matter and baryon densities $\Omega_c$ and $\Omega_b$, the Hubble parameter $h$, and the perturbation amplitude $A_s$ and spectral index $n_s$. These are fixed to WMAP3 best-fit values\(^{1}\) (Spergel et al. 2007) for $\Omega_b h^2$, $\Omega_c h^2$, the projected sound horizon $\theta$, $A_s \exp(-2\tau)$ and $n_s$. We then study the reionization signal from the TE and EE spectra out to $\ell$ of 100. It is possible to use such an analysis procedure because the non-reionization parameters are very well determined by the TT spectrum, and because the large-scale signal in CMB polarization is independent of the other parameters. A similar procedure has been followed in works including Kaplinghat et al. (2003), Holder et al. (2003), and Mortonson & Hu (2008a,b). The uncertainty on $\tau$ derived holding these parameters fixed is expected to be an underestimate by about 10\% (Mortonson & Hu 2008a).

We assume standard recombination. If the recombination model eventually needs to be modified to account for two-photon decays (Dubrovich & Grachev 2005; Wong & Scott 2007; Chluba & Sunyaev 2008; Hirata 2008), this should not affect the model comparisons we present here because it would be common to all the models. A similar procedure has been followed in works including Kaplinghat et al. (2003), Holder et al. (2003), and Mortonson & Hu (2008a,b). The uncertainty on $\tau$ derived holding these parameters fixed is expected to be an underestimate by about 10\% (Mortonson & Hu 2008a).

We additionally force the ionization fraction to unity for $z < 6$, to avoid conflict with quasar absorption spectrum data, and to zero for $z > 30$ as no ionizing sources are expected so early. The optical depth $\tau$ is computed numerically for any such reionization model eventually to be modified to account for two-photon decays (Dubrovich & Grachev 2005; Wong & Scott 2007; Chluba & Sunyaev 2008; Hirata 2008), this should not affect the model comparisons we present here because it would be common to all the models. In addition, the spectrum changes on intermediate to small scales while we are using only the large scales here.

We mainly consider a two-parameter reionization model defined by the ionization fraction history

\[
x_e(\eta_i) = \left[1 - x_e(\eta_{i-1})\right] \tanh\left[\left(\frac{\eta_{zi}}{\eta_{zr}} - 1\right) d\eta + 1\right] + x_e(\eta_{i-1}).\]

1. Our calculations predated the WMAP five-year data release (Hinshaw et al. 2008), which however left the numbers almost unchanged.

where $x_e$ refers to the ionization fraction, $\eta_i$ and $\eta_{i-1}$ refer to consecutive time steps, $\eta$ to the conformal time at the $i$-th time step, $z_r$ is the redshift at which the ionization fraction is 0.5, $\eta_z$ is the conformal time corresponding to that redshift, and $d\eta$ gives the (inverse) width of the transition. Such a transition is implemented in CAMB (Lewis, Challinor & Lasenby 2000), and the commonly-used instantaneous reionization scenario corresponds to $d\eta$ having a large enough value, such as 50, that $z_r$ is effectively the redshift of instantaneous reionization\(^{2}\).

\(^{2}\) We use a version of CAMB prior to April 2008, that includes only hydrogen reionization and thus a final ionization fraction of unity. Including helium reionization the final ionization fraction would be $\sim 1.08$. See section XIII of CAMB notes, via a link from [http://camb.info/readme.html](http://camb.info/readme.html) for further details. The model selection results presented in this paper are not expected to change following this inclusion of helium reionization.
Reionization history forecasts for Planck

Through most of the following analysis we take a fiducial \(z_r-d_\eta\) model with \(d_\eta = 3\) and \(z_r = 8.9\), corresponding to \(\tau = 0.1\) as already well determined by WMAP5.

Flat priors are assumed on \(z_r\) and \(d_\eta\), over ranges of 6–30 and 0–10 respectively (the figures show that values of \(d_\eta\) larger than this result in scenarios very close to instantaneous). Figure 1 also motivates us to try a logarithmic prior on \(d_\eta\), for which we take the range 0.3–30. However a \(\tau < 0.3\) prior is also imposed, so that the one additional parameter as compared to instant reionization does not correspond to one additional degree of freedom. For this reason regular likelihood ratio tests, of the kind performed in Kaplinghat et al. (2003) and Holder et al. (2003), will not be valid. Figure 3 shows the induced prior on \(\tau\) resulting from linear priors on \(z_r\) and \(d_\eta\) (their own priors are not perfectly flat due to the extra imposition of the \(\tau < 0.3\) prior). The uncertainty on \(\tau\) from Planck is of order \(\Delta \tau = 0.01\), and over such widths the prior on \(\tau\) is roughly uniform; it is even more so around the fiducial value of \(\tau\). The same applies to the case of a log prior on \(d_\eta\).

Other models of reionization have been proposed (Haiman & Holder 2003; Cen 2003), for instance the double reionization scenario considered in Lewis et al. (2006), which would in general require a third parameter. From a model selection point of view (i.e. taking into account parameter uncertainties within each model) results in this paper indicate that discriminating such a model from a smooth transition model would be beyond the scope of Planck, though perhaps within the scope of a closer to cosmic variance limited experiment. Here our focus is mainly on clarifying what Planck can learn about reionization.

Figure 2. As Figure 1 but for models each with \(\tau\) fixed at 0.1. At fixed \(\tau\) the spectra, especially TT, have only a weak dependence on \(d_\eta\).

Figure 3. Uniform priors on \(z_r\) between 6 and 30, and on \(d_\eta\) between 0 and 10, result in a non-uniform prior on \(\tau\) (solid curves). We can work with such a prior on \(\tau\) because expected uncertainties from a Planck-like experiment are \(\Delta \tau = 0.01\) and over such a range the prior is fairly flat.

3 MODEL SELECTION FORECASTING METHODOLOGY

Model selection forecasting assesses a given experiment’s ability to distinguish between different cosmological models. This ability necessarily depends on the true model and on its parameter values (and of course on the overriding assumption that one of the models
Table 1. Analyzing TE and EE spectra of Planck specifications (first panel), with a fiducial model of $z_T = 8.9$ and $d_0 = 3$ (implying $\tau = 0.1$) using three test models. The second panel shows the same for a cosmic variance limited experiment. In Evidences are based on four estimates of the evidence for each model.

| Model, priors     | parameter estimates | ln Evidence | $\Delta \ln E$ |
|-------------------|---------------------|-------------|---------------|
| instantaneous reionization | $z_T = 12.9 \pm 0.5$ $\tau = 0.108 \pm 0.006$ | $-6.3 \pm 0.1$ | 0.0          |
| Planck satellite linear $d_q$ model | $z_T = 10.1 \pm 1.7$, $d_q = 4.4 \pm 1.9$ $\tau = 0.103 \pm 0.006$ | $-4.4 \pm 0.2$ | 1.9          |
| $\log d_q$ model | $z_T = 9.9 \pm 1.9$, $d_q = 4.1 \pm 1.9$ $\tau = 0.103 \pm 0.006$ | $-4.7 \pm 0.2$ | 1.6          |
| instantaneous reionization | $z_T = 12.7 \pm 0.2$ $\tau = 0.106 \pm 0.003$ | $-15.8 \pm 0.1$ | 0.0          |
| cosmic variance limited linear $d_q$ model | $z_T = 9.4 \pm 1.0$, $d_q = 3.3 \pm 0.5$ $\tau = 0.102 \pm 0.005$ | $-6.6 \pm 0.1$ | 9.2          |
| $\log d_q$ model | $z_T = 9.2 \pm 1.0$, $d_q = 3.2 \pm 0.4$ $\tau = 0.101 \pm 0.005$ | $-6.8 \pm 0.3$ | 9.0          |

4 RESULTS

4.1 The Planck satellite

We model Planck TE and EE data using just the 143 GHz polarization channel, following for its specifications the current Planck documentation$^3$. The full likelihood is constructed in the manner of Lewis (2005) and Pahud et al. (2006, 2007), without creating noisy data realizations. This ensures that the bias issues we discuss below are not a result of realization noise, but instead the forecast is equivalent to averaging over many data realizations and is thus itself effectively unbiased (see the appendix of Sahlén et al. 2008). We assume a sky coverage of 0.8, and take the likelihood up to a maximum multipole of $\ell = 100$.

The first entry in Table 1 shows that the result of analyzing Planck data based on this chosen fiducial model with a one-parameter instantaneous reionization model (i.e. with $d_q$ held fixed at 50, varying $z_T$ over the 6–30 prior range). The second and third entries show the data analyzed with the (correct) $z_T$-$d_q$ model. Linear and log priors on $d_q$ are assumed, to check for the dependence of results on such assumptions.

Our main result is the relative evidences of these models, where the instantaneous reionization model has a ln evidence which is less than 2 smaller than the two-parameter models. Accordingly, Planck will not be good enough to exclude the instantaneous reionization model, even though the true model appears to have a quite different ionization history. Put another way, Planck is not powerful enough to explore two-parameter models of reionization (at least unless the true model is even further from instantaneous reionization than our fiducial model).

Besides the evidence, one can also compute the Bayesian complexity of Planck data for the chosen fiducial model in the manner of Kunz, Trotta & Parkinson (2006). Using such an analysis Kunz et al. found that $\tau$ is already a required parameter with WMAP 3-yr data (and a similar analysis would likely show that another reionization parameter is required when the evidence supports it).

This is, however, not quite the end of the story. The parameter distributions given from the nested sampling algorithm are shown in Figure 2. It is apparent from this that the estimated $\tau$ (and $z_T$) are held fixed. We will be considering is, if not the actual true model, at least representative of its predictions for the experiment under consideration). We call this true model and its parameter values the fiducial model. Trotta (2007) introduced an approach to model selection forecasting, PPOD, which averaged the model selection forecast over the current knowledge of model parameters, so as to give the probability of the future experiment giving different model selection outcomes. Mukherjee et al. (2006b) adopted a different approach to model selection, first implemented for cosmological applications in Mukherjee, Parkinson & Liddle (2006), computes the Bayesian evidence for any given model, as well as providing parameter estimates within that model. The evidence is the probability of the data given the model, hence can be used to determine how likely each model is to have given rise to the data. A difference of 2.5 in log evidence can be taken to be significant, and 5 decisive, evidence in favour of the model with larger evidence (Jeffreys 1961).
biased high in the instantaneous reionization model. The bias goes away in the two-parameter model, as it should since that model can describe the true behaviour of the data. Accordingly, to avoid a possible bias in measuring $\tau$ one should consider both the one-parameter and two-parameter models, and Bayesian model average as in Liddle et al. (2006b) to obtain constraints on $\tau$ (the cost being a slightly increased uncertainty in $\tau$). Similar conclusions have been reported in other papers (eg. Kaplinghat et al. 2003; Holder et al. 2003).

We have made some assumptions about the true (fiducial) model that we are not certain about. In practice we don’t know the fiducial $z_{\text{max}}$ when reionization started. This has been assumed to be 30 in the fiducial model. A different $z_{\text{max}}$ corresponds to a different reionization history, hence $z_{\text{max}}$ could be treated as an additional reionization parameter, but we don’t go into a third reionization parameter here. Instead we ask what outcome arises if we analyze data so simulated (with a $z_{\text{max}}$ of 30) with a $z_r-d_\eta$ model with $z_{\text{max}} = 20$. Such an ‘incorrect’ model would not be distinguishable from the true model by Planck. Further, if our incorrect model was not a smooth transition model but one involving a step function, again with only two parameters, corresponding to $z_{\text{max}}$ (prior range 7–30), with reionization ending at redshift 6, and with a constant reionization fraction in between these two redshifts of $x_e$ (prior range 0–1), such a model would again not be distinguishable from the assumed true model by Planck. These results are borne out of numbers presented in the next subsection for a cosmic variance limited hypothetical experiment.

4.2 Cosmic variance limited case

For a cosmic variance limited hypothetical experiment, the corresponding results are shown in the lower panel of Table 1 and in Figure 5. Again only TE and EE spectra are considered out to a maximum multipole of 100. This time the evidence favours the smooth and gradual transition $z_r-d_\eta$ model decisively over the instantaneous reionization model.

These results also show that, as before, a simpler model leads to a biased $\tau$, a bias that disappears upon using a complicated enough model for reionization. The choice of prior on $d_\eta$ (log or linear) doesn’t make much difference.
Table 2 shows an incorrect assumption regarding $z_{\text{max}}$ of the $z_{\text{e}}$-$d_\eta$ model does not significantly affect the evidence, i.e. $z_{\text{max}} = 20$ is indistinguishable from $z_{\text{max}} = 30$. $d_\eta$ is underestimated to make up for the difference in $z_{\text{max}}$, and $\tau$ is not misestimated. It also shows that the smooth and gradual transition model is favoured strongly, almost decisively, over the incorrect step model based on the difference in log evidence, while $\tau$ is biased low under this incorrect model assumption. Both incorrect models are clearly distinguishable from (and favoured over) the instantaneous reionization model. However the fact that different choices of $z_{\text{max}}$ are not distinguishable indicates that even cosmic variance limited experiments cannot probe very fine details of the reionization history.

## 5 CONCLUSIONS

We find that Planck is not expected to be able to distinguish significantly between a single-parameter and a two-parameter model of reionization in the model comparison sense, though it will mildly favour the two-parameter model for our chosen fiducial values. If the parameter values of the two-parameter true model were more extreme, then Planck might favour it significantly. However Bayesian model averaging the parameters over the two models will eliminate any bias in the optical depth to rescattering.

A cosmic variance limited hypothetical experiment will be able to decisively distinguish between the one- and two-parameter models, to distinguish between some two-parameter models, and may be able to go onto a third parameter.

The model comparison approach advocated here should if possible be applied to models parameterized by describing physically relevant quantities for the reionization history model; the phenomenological quantities employed here are an intermediate step

## ACKNOWLEDGMENTS

We thank Richard Battye, Jochen Weller, and Yun Wang for discussions.

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