Leptogenesis in a Hybrid Texture Neutrino Mass Model

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Abstract

We investigate the CP asymmetry for a hybrid texture of the neutrino mass matrix predicted by $Q_8$ family symmetry in the context of the type-I seesaw mechanism and examine its consequences for leptogenesis. We also calculate the resulting Baryon Asymmetry of the Universe (BAU) for this texture.

1 INTRODUCTION

In the Standard Model (SM), fermions acquire masses via spontaneous breakdown of SU(2) gauge symmetry. However, the values of fermion masses and the observed hierarchical fermion spectra are not understood within the SM. This results in thirteen free parameters in the SM which includes three charged lepton masses, six quark masses and the four parameters of the CKM matrix. The symmetries of the SM do not allow non-zero neutrino masses through renormalizable Yukawa couplings. However, non-zero neutrino masses can be introduced via non-renormalizable higher dimensional operators presumably having their origin in physics beyond the SM. Radiative and seesaw mechanisms often supplemented by additional inputs like texture zeros and flavor symmetries are widely discussed mechanisms for fermion mass generation. These mechanisms, most often, complement and reinforce each other. In the ongoing decade, significant advances have been made in understanding these mechanisms. In particular texture zeros and flavor symmetries have provided quantitative relationships between flavor mixing angles and the quark/lepton mass ratios. It has, now been realized that the “See-Saw GUT” scenario, on its own, cannot provide a complete understanding of the flavor structure of the quark and lepton mass matrices and new physics seems to be essential perhaps in the form of new symmetries mainly in the lepton sector. Moreover, a unified description of flavor physics and CP-violation in the quark and lepton sectors is absolutely necessary. This can be achieved by constructing a low energy effective theory with the SM and some discrete non-Abelian family symmetry and, subsequently, embedding this theory into Grand Unified Theory (GUT) models like SO(10). For this reason, the discrete symmetry

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will have to be a subgroup of SO(3) or SU(3)\[^2\]. The search for an adequate discrete
symmetry has mainly focused on the minimal subgroups of these groups with at least
one singlet and one doublet irreducible representation to accommodate the fermions
belonging to each generation. One such subgroup is the quaternion group \(Q_8\)[3] which
not only accommodates the three generations of fermions but also explains the rather
large difference between values of 2-3 mixings in the quark and lepton sectors.

Discrete quaternion groups have been extensively studied in the literature\[^3\]. \(Q_8\) is a
subgroup of \(SU(2)\). A two-dimensional representation of \(Q_8\) is provided by the eight
unimodular unitary matrices \((\pm 1, \pm i\sigma_1, \pm i\sigma_2, \pm i\sigma_3)\) where \(1\) is the \(2 \times 2\) unit matrix
and \((\sigma_1, \sigma_2, \sigma_3)\) are the Pauli Matrices. There are five irreducible representations
of \(Q_8\) corresponding to the five conjugacy classes in which the elements of
\(Q_8\) are divided and are conveniently denoted by \(1^{++}, 1^{+-}, 1^{-+}, 1^{--}\) and \(2\). The \(SU(2)\)
decomposition of these irreducible representations is given by \(1^{++}, 2^{++}\) and \(2^{+-}\) having the same group properties are equivalent and are, hence, interchangeable. The
basic tensor product rule for \(Q_8\) is \(2 \times 2 = 1^{++} \oplus 1^{+-} \oplus 1^{--} \oplus 1^{--} \oplus 1^{++}\). The quarks and
leptons can be differentiated by assigning them to different irreducible representations
of \(Q_8\) in the following manner:

\[
(u_\alpha, d_\alpha), u_c^\alpha, d_c^\alpha \in 1^{--}, 1^{-+}, 1^{++}, 1^{+-}, 1^{--}, 1^{++}, 1^{+-}, 1^{++}
\]

where \(\alpha\) is the flavor index.

The Higgs doublets have the following \(Q_8\) assignments:

\[
(\phi^o_1, \phi^-_1) \sim 1^{++},
\]

\[
(\phi^o_2, \phi^-_2) \sim 1^{+-}.
\]

Electroweak symmetry breaking generates the charged fermion mass terms \(M^{ij}_f\) and the neutrino mass terms \(M^{ij}_\nu\). The entries in the fermion mass matrices are
associated with \(Q_8\) assignments of the corresponding fermion bilinears leading to

\[
M_{\text{quark}} \sim \begin{pmatrix}
1^{++} & 1^{-+} & 1^{--} \\
1^{-+} & 1^{++} & 1^{--} \\
1^{--} & 1^{++} & 1^{++}
\end{pmatrix}
\]

and

\[
M_{\text{lepton}} \sim \begin{pmatrix}
1^{++} & 2 & 2 \\
2 & 1^{+-} + 1^{--} & 1^{++} + 1^{+-} \\
2 & 1^{--} + 1^{-+} & 1^{--} + 1^{--}
\end{pmatrix}.
\]

The \(1^{++}\) contribution in the 2-3 sector of the Majorana neutrino mass matrix is for-
bidden because of the symmetry of the mass matrix. Nonzero entries in the neutrino
mass matrix are induced by the vacuum expectation values (VEVs) of Higgs triplets
transforming in the corresponding \(Q_8\) irreducible representation. These triplet VEVs
the masses for charged fermions. At least four Higgs triplets are needed to correctly reproduce the current neutrino data. Nonzero mixing in the solar sector requires the VEVs of \((\xi_3, \xi_4) \in \mathbb{R}^2\). The other two Higgs triplets \(\xi_1\) and \(\xi_2\) must transform in two different 1-dimensional representations of \(Q_8\) for which four distinct choices are possible corresponding to \(Q_8\) assignments relative to \((\phi_o^1, \phi_{-1}^1) \in 1^{++}\) and \((\phi_o^2, \phi_{-2}^2) \in 1^{--}\) viz. (i) \(\xi_1 \in 1^{++}, \xi_2 \in 1^{+-}\) (ii) \(\xi_1 \in 1^{++}, \xi_2 \in 1^{-+}/1^{-+}\) (iii) \(\xi_1 \in 1^{+-}/1^{-+}, \xi_2 \in 1^{-+}\) (iv) \(\xi_1 \in 1^{+-}, \xi_2 \in 1^{--}\).

The resulting Majorana neutrino mass matrices\([3]\) corresponding to the above \(Q_8\) assignments are

\[
M_I^{\nu} = \begin{pmatrix}
 a & c & d \\
 c & 0 & b \\
 d & b & 0
\end{pmatrix},
M_{II}^{\nu} = \begin{pmatrix}
 0 & c & d \\
 c & a & 0 \\
 d & 0 & b
\end{pmatrix},
(7)
\]

\[
M_{III}^{\nu} = \begin{pmatrix}
 0 & c & d \\
 c & a & b \\
 d & b & a
\end{pmatrix},
M_{IV}^{\nu} = \begin{pmatrix}
 a & c & d \\
 c & b & 0 \\
 d & 0 & b
\end{pmatrix}
(8)
\]

in the charged lepton basis. Out of these four possible scenarios for the Majorana neutrino mass matrix, the scenario \(II\) and \(IV\) are excluded by the present experimental data on neutrino masses and mixings\([3, 5]\). In fact, scenario \(I\) and \(II\) correspond to two texture zero neutrino mass matrices studied extensively in the literature\([6, 7, 8, 9, 10, 11, 12, 13, 14]\) and leptogenesis in these two scenarios have been investigated\([15, 16]\). However, the hybrid scenario \(III\) is a new scenario predicted by the \(Q_8\) symmetry with a normal hierarchy of neutrino masses and nonzero 1-3 mixing angle\([5, 17]\). In this scenario, equalities between mass matrix elements can coexist with texture zeros. It is extremely important to investigate the connection between the textures of fermion mass matrices and the observables of flavor mixing. Discrete quaternion groups have been extensively applied to flavor physics\([18, 19, 20, 21, 22]\). It is, therefore, important to subject models based on \(Q_8\), in particular, to the test of a viable leptogenesis to explain the Baryon Asymmetry of the Universe (BAU). Leptogenesis is based on the \(CP\) asymmetry generated through out of equilibrium lepton number violating decays of heavy Majorana neutrinos resulting in a lepton asymmetry which is subsequently transformed into a baryon asymmetry via \((B + L)\) violating sphaleron processes and depends on the structure of Majorana neutrino mass matrix. In the present work, we study the \(CP\) asymmetry in the hybrid scenario \(III\) predicted by \(Q_8\) symmetry and examine its implications for leptogenesis and the resulting Baryon Asymmetry of the Universe (BAU).

## 2 Neutrino Mass Matrix

We consider the following neutrino mass matrix with a hybrid texture resulting from \(Q_8\) symmetry:

\[
M_{\nu} = \begin{pmatrix}
 0 & a & b \\
 a & c & d \\
 b & d & c
\end{pmatrix}
(9)
\]
where $a$, $b$, $c$ and $d$ are complex, in general. In the framework of type-I seesaw mechanism \cite{23, 21, 25, 26}, the effective light neutrino mass matrix $M_\nu$ is given by

$$M_\nu = M_D M_R^{-1} M_D^T \quad (10)$$

where $M_D$ is the Dirac neutrino mass matrix and $M_R$ is the right-handed Majorana neutrino mass matrix. The number of physical seesaw parameters contained in $M_D$ and $M_R$ on the right hand side is double the number of parameters in the low energy neutrino mass matrix $M_\nu$. As a result, the reconstruction of the seesaw is not possible solely from low energy neutrino physics and one requires additional observables. One such observable which can be used in reconstructing the seesaw is the observed Baryon Asymmetry of the Universe (BAU). The seesaw matrices $M_D$ and $M_R$ are totally unknown and even the light neutrino mass matrix $M_\nu$ is not completely known. In the absence of a complete knowledge of $M_\nu$ and $M_R$, there are an infinite number of possibilities for the seesaw Dirac neutrino mass matrix $M_D$ resulting in the so-called seesaw degeneracy \cite{27}. Therefore, to facilitate the reconstruction of the seesaw, one is constrained to make additional assumptions about the seesaw matrices motivated by some specific models. However, in general, in the absence of a specific model in hand, the best one can do is to parametrize one’s ignorance and that was the approach followed by Casas and Ibarra culminating in the so-called Casas-Ibarra (CI) parametrization \cite{28} given by

$$M_D = i V \sqrt{M^d_\nu} R \sqrt{M^d_R} \quad (11)$$

where $M^d_\nu = \text{diag}\{m_1, m_2, m_3\}$ ($m_i, i = 1, 2, 3$ are light neutrino masses), $M^d_R = \text{diag}\{M_1, M_2, M_3\}$ ($M_i, i = 1, 2, 3$ are right handed Majorana neutrino masses), $V$ neutrino mixing matrix and $R$ is a complex orthogonal matrix. The light neutrino mass matrix $M_\nu$ (from low energy phenomenology) can be parametrized as

$$M_\nu = m_0 \begin{pmatrix} 0 & \lambda & \lambda \\ \lambda & 1 & \epsilon - 1 \\ \lambda & \epsilon - 1 & 1 \end{pmatrix} \quad (12)$$

where the real parameters $\lambda$, $\epsilon$ and $m_0$ are given by $\lambda = 0.01$, $\epsilon = 0.12$ and $m_0 = \frac{\sqrt{\Delta m_{23}^2}}{2}$ with $\Delta m_{23}^2 = 2.37 \times 10^{-3} \text{ eV}^2$ \cite{29} and the apparent equalities between different elements of the neutrino mass matrix are to within the current precision $(\approx 10^{-2})$ of the oscillation parameters. One can diagonalize $M^d_\nu$ given by Eqn. (12) to calculate matrices $M^d_\nu$ and $V$:

$$M^d_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (13)$$

$$= m_0 \begin{pmatrix} \frac{1}{2} (\epsilon - \sqrt{\epsilon^2 + 8\lambda^2}) & 0 & 0 \\ 0 & \frac{1}{2} (\sqrt{\epsilon^2 + 8\lambda^2} + \epsilon) & 0 \\ 0 & 0 & 2 - \epsilon \end{pmatrix}, \quad (14)$$

and

$$V = \begin{pmatrix} \sqrt{\lambda^2 + (\epsilon + \sqrt{\epsilon^2 + 8\lambda^2})^2} & \sqrt{\lambda^2 + (\epsilon - \sqrt{\epsilon^2 + 8\lambda^2})^2} & 0 \\ \frac{\epsilon + \sqrt{\epsilon^2 + 8\lambda^2}}{2\lambda} & \frac{\epsilon - \sqrt{\epsilon^2 + 8\lambda^2}}{2\lambda} & -1 \sqrt{2} \\ \frac{\sqrt{\epsilon^2 + 8\lambda^2} - \epsilon}{2\lambda} & \frac{\sqrt{\epsilon^2 + 8\lambda^2} + \epsilon}{2\lambda} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (15)$$
The complex orthogonal matrix $R$ can be parameterized as

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = T_{12}T_{13}T_{23},$$  \hspace{1cm} (16)

where $T_{ij}$ is the matrix of rotation by a complex angle $\xi_{ij} = \eta_{ij} + i\zeta_{ij}$ in the $ij$-plane with

$$T_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}; T_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}; T_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix},$$  \hspace{1cm} (17)

where $c_{ij} = \cos \xi_{ij}$, $s_{ij} = \sin \xi_{ij}$ for $i < j$. Using Eqns. (14-17) the Dirac neutrino mass matrix $M_D$ in the CI parameterization can be written as:

$$M_D = \begin{pmatrix} M_{11}^D & M_{12}^D & M_{13}^D \\ M_{21}^D & M_{22}^D & M_{23}^D \\ M_{31}^D & M_{32}^D & M_{33}^D \end{pmatrix},$$  \hspace{1cm} (18)

where elements of $M_D$ are given by

$$M_{11}^D = i\sqrt{M_1} \left( -A_2 c_{13} \sqrt{m_2} s_{12} - A_1 c_{12} c_{13} \sqrt{m_1} \right)$$

$$M_{12}^D = i\sqrt{M_2} \left( A_2 m_2 (c_{12} c_{23} + s_{12} s_{13} s_{23}) - A_1 \sqrt{m_1} (c_{23} s_{12} - c_{12} s_{13} s_{23}) \right)$$

$$M_{13}^D = i\sqrt{M_3} \left( A_2 m_2 (c_{12} s_{23} - c_{23} s_{12} s_{13}) - A_1 \sqrt{m_1} (c_{12} c_{23} s_{13} + s_{12} s_{23}) \right)$$

$$M_{21}^D = i\sqrt{M_1} \left( -\frac{2A_2 c_{13} \sqrt{m_2} s_{12}}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} + \frac{2A_1 c_{12} c_{13} \sqrt{m_1}}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} \right)$$

$$M_{22}^D = i\sqrt{M_2} \left( \frac{2A_2 m_2 (c_{12} c_{23} + s_{12} s_{13} s_{23})}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} + \frac{2A_1 m_2 (c_{12} s_{23} - s_{12} s_{13} s_{23})}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} \right)$$

$$M_{23}^D = i\sqrt{M_3} \left( -\frac{2A_2 c_{13} \sqrt{m_2} s_{12}}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} + \frac{2A_1 c_{12} c_{13} \sqrt{m_1}}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} \right)$$

$$M_{31}^D = i\sqrt{M_1} \left( -\frac{2A_2 c_{13} \sqrt{m_2} s_{12}}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} + \frac{2A_1 c_{12} c_{13} \sqrt{m_1}}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} \right)$$

$$M_{32}^D = i\sqrt{M_2} \left( \frac{2A_2 m_2 (c_{12} c_{23} + s_{12} s_{13} s_{23})}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} + \frac{2A_1 m_2 (c_{12} s_{23} - s_{12} s_{13} s_{23})}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} \right)$$

$$M_{33}^D = i\sqrt{M_3} \left( -\frac{2A_2 c_{13} \sqrt{m_2} s_{12}}{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}} + \frac{2A_1 c_{12} c_{13} \sqrt{m_1}}{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}} \right)$$  \hspace{1cm} (19)

and the quantities $A_1$ and $A_2$ are given by

$$A_1 = \frac{\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon}}{\sqrt{(\sqrt{\epsilon^2 + 8\lambda^2 + \epsilon})^2 + 8\lambda^2}}$$

$$A_2 = \frac{\sqrt{\epsilon^2 + 8\lambda^2 - \epsilon}}{\sqrt{(\epsilon - \sqrt{\epsilon^2 + 8\lambda^2})^2 + 8\lambda^2}}$$  \hspace{1cm} (20)

### 3 Hybrid Texture and Baryon Asymmetry

Baryon Asymmetry of the Universe (BAU) poses a puzzle for particle physics as well as cosmology. Even though the Standard Model (SM) has all the ingredients necessary for the dynamical generation of baryon asymmetry, it fails to explain the observed baryon asymmetry as the SM CP-violation is too small to gen-
(EWPT) is not strongly first order as required for successful baryogenesis. Baryogenesis, thus, requires new physics beyond the SM essentially in the form of new sources of CP-violation and must either provide a departure from thermal equilibrium in addition to the electroweak phase transition or modify the electroweak phase transition itself. Some of the possible new physics mechanisms are Affleck-Dine mechanism\cite{34, 35}, GUT baryogenesis\cite{36, 37, 38, 39, 40, 41, 42, 43, 44, 45} and baryogenesis via leptogenesis\cite{46, 47} etc. Out of these scenarios, the last one is particularly appealing since there is a plethora of reasons to believe that the SM is only a low energy effective theory and there are strong indications of new physics at a higher energy scale. The experimental evidence for massive neutrinos, the dark matter puzzle apart from the fine tuning problem of the Higgs mass and the gauge coupling unification are some of these reasons for invoking physics beyond the SM. It is, particularly interesting to note that the mechanism of baryogenesis via leptogenesis is motivated by some of the reasons listed above. In this mechanism, heavy singlet neutrinos are introduced via the seesaw mechanism whose Yukawa couplings provide new sources of CP-violation essential for a viable leptogenesis. The rates of these new Yukawa interactions can be slow enough to generate departure from thermal equilibrium. Majorana masses of heavy singlet neutrinos lead to necessary lepton number violation and the (B+L) violating SM sphaleron processes\cite{30} play a crucial role in partially converting the lepton asymmetry into a net baryon asymmetry. In this section we calculate the baryon asymmetry in the hybrid texture model (Eqn. (9)). The physical Majorana neutrino, $N_R$, decays in two modes:

\begin{align}
N_R &\to l_\alpha + \phi^+ \\
&\to \bar{l_\alpha} + \phi
\end{align}

where $l_\alpha$ is lepton and $\bar{l_\alpha}$ is antilepton. The CP asymmetry which is caused by the interference of tree level with one loop corrections for the decay of the lightest of heavy right handed Majorana neutrino $N_1$ is given by\cite{48, 49}

\begin{align}
\varepsilon_1^\alpha &= \frac{\Gamma - \Gamma}{\Gamma + \Gamma} = \frac{1}{8\pi v^2} \frac{1}{(M_D^\dagger M_D)_{ii}} \sum_{j\neq i} \Im \left[ (M_D^\dagger)_{i\alpha} (M_D)_{\alpha j} (M_D^\dagger M_D)_{ij} \right] f \left( \frac{M_2^2}{M_1^2} \right) + \\
&+ \frac{1}{8\pi v^2} \frac{1}{(M_D^\dagger M_D)_{ii}} \sum_{j\neq i} \Im \left[ (M_D^\dagger)_{i\alpha} (M_D)_{\alpha j} (M_D^\dagger M_D)_{ji} \right] \frac{1}{1 - M_2^2/M_1^2}
\end{align}

where $\Gamma = \Gamma(N_1 \to l_\alpha \phi^+)$ and $\Gamma = \Gamma(N_1 \to \bar{l_\alpha} \phi)$ are the decay rates and $v$ is the scale of the electroweak symmetry breaking, $v \simeq 174$ GeV. Within the SM the function $f(\varsigma)$ has the form

\begin{align}
f(\varsigma) &= \sqrt{\varsigma} \left[ \frac{1}{1 - \varsigma} + 1 - (1 + \varsigma) \ln \left( \frac{1 + \varsigma}{\varsigma} \right) \right],
\end{align}

where $\varsigma = \frac{M_2^2}{M_1^2}$.

For hierarchical right handed Majorana neutrino masses i.e. $M_1 \ll M_2$, $M_3$ the function $f(\varsigma) \simeq \frac{3}{2\sqrt{\varsigma}}$ and, in addition, the second term in Eqn. (23) is strongly suppressed, therefore, we will neglect this term in the following analysis. Also, the CP asymmetry given by Eqn. (23) when summed over the flavors $\alpha$ ($M_1 \geq 10^{12}$ GeV),
can be written as

\[ \varepsilon_1 = \sum_\alpha \varepsilon_1^\alpha \approx -\frac{3}{16\pi v^2} \left( 3 \left[ \frac{(M_D^\dagger M_D)_{12}^2}{M_1} M_2 + \frac{(M_D^\dagger M_D)_{13}^2}{M_2} M_3 \right] + \frac{3}{M_2} \right). \]  

(25)

It is evident from Eqn. (25) that for hierarchical right handed Majorana neutrinos, a non-vanishing decay asymmetry \( \varepsilon_1 \) depends on the imaginary part of \((1,2)\) and \((1,3)\) elements of \( M_D^\dagger M_D \) which in the Casas-Ibarra (CI) parameterization is given by

\[ M_D^\dagger M_D = \sqrt{M_R^d R^d_M R^d_R}. \]  

(26)

The elements \( (M_D^\dagger M_D)_{11}, (M_D^\dagger M_D)_{12} \) and \( (M_D^\dagger M_D)_{13} \) are given by

\[
(M_D^\dagger M_D)_{11} = \frac{1}{2} m_0 M_1 \left[ (R_{11}(R_{11})^*)(\epsilon - \sqrt{\epsilon^2 + 8\lambda^2}) + R_{21}(R_{21})^*(\sqrt{\epsilon^2 + 8\lambda^2} + \epsilon) \right]
- 2 R_{31}(R_{31})^*(\epsilon - 2)),
\]

(27)

\[
(M_D^\dagger M_D)_{12} = \frac{1}{2} m_0 \sqrt{M_1 M_2} \left[ (R_{12}(R_{11})^*)(\epsilon - \sqrt{\epsilon^2 + 8\lambda^2}) + R_{22}(R_{21})^*(\sqrt{\epsilon^2 + 8\lambda^2} + \epsilon) \right]
- 2 R_{32}(R_{31})^*(\epsilon - 2)),
\]

(28)

\[
(M_D^\dagger M_D)_{13} = \frac{1}{2} m_0 \sqrt{M_1 M_3} \left[ (R_{13}(R_{11})^*)(\epsilon - \sqrt{\epsilon^2 + 8\lambda^2}) + R_{23}(R_{21})^*(\sqrt{\epsilon^2 + 8\lambda^2} + \epsilon) \right]
- 2 R_{33}(R_{31})^*(\epsilon - 2)).
\]

(29)

The lepton asymmetry is related to the CP asymmetry through the relation

\[ Y_L = \frac{n_L - \bar{n}_L}{s} = \kappa \varepsilon_1 \frac{g_*}{g_s}, \]

(30)

where \( n_L \) and \( \bar{n}_L \) are number densities of leptons and antileptons, respectively, \( s \) is the entropy density, \( \kappa \) is the dilution factor which accounts for the washout processes such as inverse decay and lepton number violating scattering and \( g_* \) is the effective number of degrees of freedom, \( g_* = 106.75^{[1]} \). The possibility of generating an asymmetry between number of leptons and antileptons (lepton asymmetry) is due to a non-vanishing CP asymmetry, \( \varepsilon_1 \). The lepton asymmetry, thus, produced is converted into a net baryon asymmetry, \( Y_B \), through the sphaleron processes which is given by the relation\(^{[50,51]} \)

\[ Y_B = \frac{\zeta}{\zeta - 1} Y_L, \zeta = \frac{8N_f + 4N_H}{22N_f + 13N_H} \]

(31)

where \( N_f \) is number of fermion families and \( N_H \) is number of complex Higgs doublets. Taking \( N_f = 3 \) and \( N_H = 1 \), we get

\[ Y_B \approx \frac{28}{43} Y_L. \]

(32)
In order to calculate the baryon asymmetry we need the dilution factor $\kappa$ which involves integration over the full set of Boltzmann equations\cite{52,53}. The approximate value of the dilution factor which is sufficient for our purpose is given by\cite{54,55,56}

$$\kappa \simeq \frac{2 \left(1 - e^{-\frac{1}{2} K_R Z_B(K_R)}\right)}{K_R Z_B(K_R)}, \quad (33)$$

with

$$Z_B(K_R) \simeq 4 e^{-\frac{2}{3} R K_R^{0.13}} + 2, \quad (34)$$

where $K_R$ is the ratio of the thermal average of the $N_1$ decay rate and Hubble parameter and is given by

$$K_R = \frac{M_P}{1.7 \times 8 \pi v^2 \sqrt{g_*}} \left(\frac{M_D^{\dagger} M_D}{M_1}\right)_{11} \quad (35)$$

where $M_P \simeq 1.22 \times 10^{19}$ GeV is the Planck mass. The baryon asymmetry $Y_B$ given by Eqn. (32) contain three right handed Majorana neutrino masses $M_1$, $M_2$, $M_3$ and complex phases $\xi_{ij}$. Assuming hierarchical mass spectrum of $M_1$, $M_2$ and $M_3$ with $M_3 = M_0$, $M_2 = 10^{-2} M_0$, $M_1 = 10^{-5} M_0$ and giving full variation to phases $\xi_{ij}$, we have shown in Fig. (1) $Y_B$ as a function of the right handed Majorana neutrino mass scale, $M_0$. Fig. (1) has been plotted for three randomly picked representative sets of the complex phases $\xi_{ij}$ or equivalently three sets of the complex orthogonal matrix $R$ to show the dependence of $Y_B$ on $M_0$ which otherwise will not be clear. We have, also, shown the observed baryon asymmetry as grey region between horizontal lines. It is clear from the figure that $Y_B$ has a linear dependence on the heavy neutrino mass scale $M_0$. Also, for hybrid texture (Eqn. (9)) of the light neutrino mass matrix, the observed baryon asymmetry predicts that the mass scale of the right handed Majorana neutrino, $M_0$ lies in the range $(0.3 \times 10^{16} \leq M_0 \leq 1.4 \times 10^{16})$ GeV for set 1 and $(0.8 \times 10^{16} \leq M_0 \leq 3.4 \times 10^{16})$ GeV and $(1.8 \times 10^{16} \leq M_0 \leq 7.0 \times 10^{16})$ GeV for sets 2 and 3.

4 Conclusions

In conclusion, we examined the implications of neutrino mass matrix with hybrid texture resulting from $Q_8$ symmetry for CP asymmetry and Baryon Asymmetry of the Universe (BAU). We considered the Dirac neutrino mass matrix $M_D$ in the Casas-Ibarra (CI) parameterization (Eqn. (11)) alongwith the neutrino mass matrix $M_\nu$ with the hybrid texture obtained from $Q_8$ symmetry. With the parameterized form of the light neutrino mass matrix $M_\nu$ (Eqn. (12)) we find the form of the Dirac neutrino mass matrix $M_D$ corresponding to this hybrid texture. Assuming a normal hierarchy of the right handed Majorana neutrino masses, $M_1 \ll M_2$, $M_3$, we calculate the baryon asymmetry, $Y_B$, as a function of the right handed Majorana neutrino mass scale, $M_0$. It is found that hybrid texture of the kind considered here gives consistent value of the baryon asymmetry, $Y_B$, for neutrino mass scale, $M_0$, of the order of $10^{16}$ GeV which is several orders of magnitude higher than the other viable $Q_8$ scenario with two texture zeros\cite{15,16}. In particular, we have shown in Fig. (1) $Y_B$ for three different sets of complex orthogonal matrices $R$. 
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Figure 1: Baryon Asymmetry of the Universe (BAU) $|Y_B|$ (in the units of $10^{-10}$), as a function of right handed Majorana neutrino mass scale $M_0$ (in the units of $10^{18}$ GeV). The region between the two horizontal lines is the observed $|Y_B|$. 