Low-mass scalar production in $\gamma\gamma$ scattering

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Abstract

We estimate the $I = 0$ scalar meson $\sigma/f_0(600)$ $\gamma\gamma$ widths, from $\pi\pi$ and $\gamma\gamma$ scattering data below 700 MeV using an improved analytic K-matrix model.

1. Introduction. Preliminary remarks

- This is an attempt to get information on the nature of the low-mass scalar meson, the controversial $\sigma$ or $\epsilon$ or $f_0(600)$, from $\gamma + \gamma \to \pi + \pi$ at energies below $\sim 700$ MeV.
- Studies of this process go back to Lyth (1971) [3], Yndurain (1972) [4], ... [5] [6] [7], and recently Boglione-Pennington [8], Pennington [9], Achasov-Shestakov [10], Oller-Roca [11], Pennington et al. [12], Giacosa-Gutsche-Lyubovitskij [13] ... [18,19].
- Though not proved for composite particles (to my knowledge), I will assume "usual" analyticity properties in $s$ (the energy squared), with $\text{Left}$ cut from $t, u$ particle exchanges, and $\text{Right}$ cut above threshold, from physical channel.
- Working at lowest order in E.M., unitarity is linear, and involves STRONG amplitudes.
- From unitarity and analyticity, Muskheilishvili [14] have shown how to determine a set of Fundamental solutions, from which one can obtain the full family of solutions, which is determined up to Polynomial Ambiguity once we are given the Left singularities.
- In the 1-channel purely elastic case, the Fundamental solution reduces to the Omnes formula [15], where the phase shift must be chosen continuous, to get an analytic and invertible function.
- Analytic extrapolation is unstable if there are no bounds in all directions of the complex plane, and sensitive to even small but rapid variations. This is, presumably, one of the reason of the controversies on the low-mass scalar meson, which appear to be a very broad object. See a recent discussion by Yndurain et al. [16].

2. STRONG interaction parametrisation

Since we only study the low energy part, we assume elastic unitarity from threshold up to infinity, and unitarise only the S-waves.

Here we will neglect, in particular, the opening of the $K - \bar{K}$ threshold, the effect of the f2(1270) ...

We will use parametrisations, of generalized analytic K-matrix [17] type, which allow explicit expressions for the Fundamental solution, and explicit continuation on the Riemann sheets.

2.1. I=0, S-wave

- We use an $N/D$ representation for $T^0$:

$$T^0(s) = \frac{Gf_0(s)}{s_R - s - Gf_0(s)}$$

$$D = s_R - s - Gf_0(s);$$

with Fundamental solution

$$F^0(s) = 1/D;$$

where the shape function $f_0(s)$ has only Left singularities, while $f_0(s)$ has only Right singularities, with

$$3m_f_0(s) = \theta \rho(s)f_0(s)$$

and $\Re f_0$ is obtained by dispersion relation, with minimal subtraction at $s = 0$. For $G$ small, there would exist a bare pole at $s = s_R$.

2.1.1.

We choose a simple form for $f_0(s)$,

$$f_0(s) = \frac{s - s_{A_0}}{s + s_{D_0}}$$

which have an Adler zero and 1 pole to simulate the near Left singularities.

- The $I=0$, $S=0$ phase-shifts $\delta_0^0$ have been determined by several groups, in particular using ROY equations [18,19].

To determine the 4 parameters $(s_{A_0}, s_{D_0}, s_R, G)$, we fit the phase-shifts $\delta_0^0$ below 800 MeV, obtained by Caprini et al. [18].

For 26 points, total $\chi^2 = 0.55$, one obtains $s_{A_0} = 0.0167 \text{GeV}^2, s_{D_0} = 0.5013 \text{GeV}^2, s_R = 0.8232 \text{GeV}^2, G = 1.1839$.

The $T^0$ amplitude has 2 poles in the second sheet, P1 and P2, with energy $w$ and energy squared $s$ values

$$wP_1 = 0.422 - i 0.290 \text{ GeV}; sP_1 = 0.0936 - i 0.2447 \text{ GeV}^2$$

$$wP_2 = 1.043 - i 0.672 \text{ GeV}; sP_2 = 0.6360 - i 1.4027 \text{ GeV}^2$$

The first one, P1, is not far from the one : $0.441 - i 0.272 \text{ GeV}$. The second, P2, unexpected for the author, will certainly move a lot when taking into account what happens near $K\bar{K}$ threshold.

However, if one just take the limit $G \to 0$, the heavy pole P2 goes to the bare pole ($wP_2 \to \sqrt{s_R}$), while P1 goes to unphysical negative $s$ value, on the Left cut ($wP_1 > \sqrt{(s_R - \sigma_{D_0})}$).
2.1.2. For a choice of \( f_0(s) \) with a cut instead of a pole,
\[
f_0(s) = \lambda + G \frac{1}{s - s_D} \log(1 + \frac{s - s_D}{mu})
\]  
and corresponding \( \tilde{f}_0(s) \), fitting again the phase-shifts \[15\], one obtains with \( \chi^2 = 0.66 \ s_D = 0.0778, mu = 0.3462, G = -0.4673, \lambda = 1.477. \)

When scaling both \( \lambda \) and \( G \) to zero, the lowest pole \( P1 \) disappear before reaching Left cut (and before couplings vanish), while \( P2 \) keep the same behaviour, going to the bare pole (\( w P2^2 > \sqrt{s_R} \)).

Though this does not correspond to a true QCD limit, it could indicate that the existence of the \( P1 \) pole is related to possibility of physical decay.

2.2. \( I=2 \), S-wave

- We take for \( T^2 \):
\[
T^2(s) = \frac{\Lambda f_2(s)}{1 - \Lambda f_2(s)}
\] (7)
where
\[
f_2(s) = \frac{s - s A2}{(s + \sigma D1)(s + \sigma D2)}
\] (8)
is more convergent to avoid an unwanted bound state pole, with
\[
\Im m f_2(s) = \theta |s| f_2(s)
\] (9)

3. EM interaction

3.1. Pion exchange

The charged \( \pi^+\pi^- \) production in \( \gamma\gamma \) scattering, is dominated by the Pion exchange, but which does not contribute to the \( \pi^0\pi^0 \). However once produced, the charged pions can rescatter also into neutral ones.

Let \( \alpha f^B \) be the S-wave projection of total Born Pion exchange. Then
\[
T^0 = \sqrt{2/3} \alpha f^B \text{ and } T^2 = \sqrt{1/3} \alpha f^B
\] (10)
are the corresponding isospin \( I=0 \) and 2 amplitudes.

3.1.1.

Let us define \( \tilde{f}_0^B \) analytic on the Left, with
\[
\Im m \tilde{f}_0^B = \theta |s| f_0(s) \ f^B(s)
\] (11)
subtracted at \( s=0 \) to satisfy Thompson limit , in a minimal way.

Then
\[
T^0 = \sqrt{2/3} \alpha (f^B + G \tilde{f}_0^B) + \alpha P(s) F^0
\] (12)
is analytic, unitary for any polynomial \( P(s) \). The \( \tilde{f}_0^B \) term is naturally interpreted as the rescattering contribution, and the \( P(s) F^0 \) as a direct one. This interpretation is not completely unambiguous, since over-subtracting (or renormalising differently the unitarisation s-bubbles) can lead to a different definition, with same total amplitude. Here also, not to violate Thompson limit, one must restrict to \( P(s) \) vanishing at \( s = 0 \), and limit its degree to avoid too divergent partial-wave.

Thus we will choose \( P(s) = \sqrt{2} F_\gamma s \).
3.2. Results

One uses the $f_0$ in Eq(5). Fit to MARK II data \cite{21} for $\pi^+\pi^-$, and to CRYSTAL BALL \cite{20} for $\pi^0\pi^0$ below 0.7 GeV gives $F_\gamma \sim -0.08$

This corresponds to residues at the P1 pole $\text{resc} = (0.091,0.116)$ for rescattering, $\text{direct} = (0.007,0.031)$ for the direct contribution, $\text{tot} = (0.098,0.151)$ for the total photon-photon width, which can be translated into (using full hadronic width=580.0 MeV) into partial widths $\Gamma_{\text{resc}}=2.805$, $\Gamma_{\text{direct}}=0.126$, $\Gamma_{\text{tot}}=4.0$ keV.

Stephan Narison will speak on consequences for the nature of the particles associated to the poles.

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