Landau level mixing by full spin-orbit interactions

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We study a two-dimensional electron gas in a perpendicular magnetic field in the presence of both Rashba and Dresselhaus spin-orbit interactions. Using a Bogoliubov transformation we are able to write an approximate formula for the Landau levels, thanks to the simpler form of the resulting Hamiltonian. The exact numerical calculation of the energy levels, is also made simpler by our formulation. The approximate formula and the exact numerical results show excellent agreement for typical semiconductors, especially at high magnetic fields. We also show how effective Zeeman coupling is modified by spin-orbit interactions.

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I. INTRODUCTION

The manipulation of the spin of charge carriers in semiconductors, spintronics, has attracted increasing interest in recent years. In the paradigmatic Datta-Das spin transistor, the spin of the electron passing through the device is controlled by the Rashba spin-orbit (SO) interaction which in turn can be varied by the application of gate voltages. The Rashba interaction stems from the structural inversion asymmetry (SIA) introduced by a heterojunction or by surface or external fields. In semiconductors with narrower energy gap (InGaAs, AlGaAs), this effect is expected to be stronger. It has been shown experimentally that the Rashba spin-orbit interaction can be modified up to 50% by external gate voltages.

In addition to the Rashba coupling there is also a material-intrinsic Dresselhaus spin-orbit interaction. This originates from the bulk inversion asymmetry (BIA) of the crystal, and can be relatively large in semiconductors like InSb/InAlSb.

Both Rashba and Dresselhaus interactions contribute to the spin-dependent splitting of the band structure of the host material, leading to dramatic spin-dependent phenomena for electrons or holes in semiconductors: Effects on the charge and magnetic transport, spin relaxation, and spin-Hall conductivity have been recently studied. A drift-diffusive transistor, in contrast to the ballistic Datta-Das device, has been proposed to be more robust by Schliemann et al. for the case when the two SO interactions have the same strength. The experimental observation of the spin-galvanic effect and weak localization effects have increased the interest in understanding the interplay between different SO terms. Photocurrent measurements have been used to obtain the ratio of Rashba and Dresselhaus coefficients. Beautiful optical measurements of SO effects have also been performed recently in strained semiconductors.

It is well known that the determination of the eigenvalues and eigenstates of the system is crucial for the calculation of a number of important physical properties of the system. In the case of a two-dimensional electron gas (2DEG) in a perpendicular magnetic field in the presence of both SO interactions, few solvable cases have been analyzed in the literature (we devote the next section to review them). However, a comprehensive description of the more general case, in which both Rashba and Dresselhaus interactions are present with arbitrary strength, would be interesting.

In this work, we address the problem of SO coupling effects on the Landau level structure of a 2DEG in a strong perpendicular field. We find an excellent approximate expression for the spinor Landau levels in the most general case of different Rashba and Dresselhaus interactions. We compare our approach to numerical results, made easier and recursively exact by our formulation of the problem. We further study the effective Zeeman g-factor of the system, and spin-orbit coupling is found to enhance or suppress the Zeeman splitting, depending crucially on material parameters and gate voltages. This behavior may be found useful in the characterization of spin-filter and spin-polarized currents in two-dimensional systems.

In the next section we review the exactly solvable cases of SO coupling in a field, where either Rashba or Dresselhaus coupling is present. The third section is dedicated to the general case in which both terms are present. Using a Bogoliubov transformation we transform the Hamiltonian of the 2DEG in a perpendicular magnetic field, and in the presence of both Rashba and Dresselhaus terms, to the 2DEG with only an effective Rashba interacting term with modified strength. The specific form of the interacting term allows for the derivation of the numerical exact level structure of the model. Our approximate result is obtained and shown to be increasingly accurate for higher magnetic fields or weaker SO interaction. The last section contains typical results for different materials and discussion.

II. SOLVABLE CASES

The Hamiltonian of 2D electrons with effective mass $m$ and Zeeman coupling $g$ in a perpendicular magnetic
Here \( B \hat{z} \) is
\[
H_0 = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} - \frac{g\mu_B B}{2} \sigma^z,
\]
where \( \mu_B \) is the Bohr magneton, \( \sigma^z \) is the Pauli matrix, and \( P = \hat{p} + \gamma \hat{A} \) is the kinetic momentum.

In the absence of Zeeman coupling, the two spin-states are degenerate, but all states are shifted by a constant value \( \Delta E = -\hbar \omega_c (\alpha + \beta)^2 / 4 \). Note that there is no mixing between the two (rotated) spin states for different levels.

In the presence of Zeeman coupling there is no exact solution of the energy spectrum. In a high magnetic field, where the Zeeman and spin-orbit splitting are small compared to the Landau level splitting, we can use perturbation theory near the \( \alpha = \beta \) point, and up to second order of perturbation we get \( n \geq 0 \)
\[
E_n^\lambda = \hbar \omega_c \left( n + \frac{1}{2} + \xi \right) + \frac{\alpha^2}{\lambda} \left( \frac{n+1}{2} - \frac{\beta^2}{\lambda} \right).
\]

When \( \xi = 0 \) we are left with the degenerate states with a constant shift of \(-\hbar \omega_c \beta^2\).

### III. GENERAL CASE

In what follows we consider the general case when both spin-orbit terms are present. Without loss of generality we assume that \( |\beta| < |\alpha| \). Adding the Rashba interaction to the 2DEG Landau level results in levels as in (12). In this case, keeping \( \alpha \) constant while increasing \( \beta \), it is expected that the level splitting will decrease and the system will return back to the levels of (11), when \( \beta \to \alpha \).

In the presence of both SO interaction terms, we use a Bogoliubov transformation, and introduce new operators \( c \) and \( c^\dagger \) by
\[
c = (\alpha a + i\beta a^\dagger) / \sqrt{\alpha^2 - \beta^2}
\]
for the case \( |\beta| < |\alpha| \). [For \( |\beta| > |\alpha| \) the definition of \( c \) and \( c^\dagger \) has to be interchanged.]

This transformation is just the rescaling of the spatial coordinates \( x \to x \sqrt{\alpha^2 - \beta^2} \) and \( y \to y \sqrt{\alpha^2 - \beta^2} \). The total Hamiltonian transforms to
\[
H = H_0(\omega) + H_{so} + V,
\]
In this formula \( H_0(\omega) \) is the Hamiltonian of a simple harmonic oscillator with modified energy \( \omega = \omega_c(\alpha^2 + \beta^2)/(\alpha^2 - \beta^2) \), which depends only on the ratio \( \beta / \alpha \).

The spin-orbit term \( H_{so} \) is
\[
H_{so} = \hbar \omega_c \sqrt{\alpha^2 - \beta^2} \begin{pmatrix} 0 & c^\dagger \\ c & 0 \end{pmatrix},
\]
which has the form of only an effective Rashba interaction term.

Finally the spin diagonal interacting term \( V \) is
\[
V = \hbar \omega_c (\alpha^2 - \beta^2) / 2,
\]
in which the perturbation parameter \( \lambda = 2\alpha / \alpha^2 + \beta^2 \) depends only on the ratio \( \beta / \alpha \). We treat (10) as a perturbation term. The advantage of using transformation (7)
lies in the simple form of (10) which makes it possible to achieve exact numerical solutions and most importantly a very accurate analytical ansatz for the spectrum.

The exact eigenvalues and eigenstates of $H_0(\omega) + H_{\text{so}}$, which we use as the basis for the perturbation expansion, are simply given by those in (4), but with properly scaled frequency $\omega$ and SO interaction term

$$
\psi_n^r = \left( \begin{array}{c} \cos \theta_n \phi_{n-1} \\ \sin \theta_n \phi_{n-1} \end{array} \right) \quad \psi_n^l = \left( \begin{array}{c} -\sin \theta_n \phi_{n-1} \\ \cos \theta_n \phi_{n-1} \end{array} \right),
$$

$$
E_{r/l} = \hbar \omega n + \frac{\delta}{2} \sqrt{1 + 4n(\alpha^2 - \beta^2)\hbar^2 \omega_c^2 / \delta^2}
$$

$$
\delta = \hbar \omega + 2\hbar \omega_c \xi
$$

$$
\tan 2\theta_n = 2 \sqrt{n(\alpha^2 - \beta^2)} \hbar \omega_c / \delta.
$$

(11)

Here $n \geq 1$ and the eigenstate $n = 0$ exists only for $\psi_n^i$ with $\theta_0 = 0$. Note how the mixing angle and level splitting are renormalized by the Dresselhaus interaction.

For $\beta \to 0$ we recover the results (4). On the other hand near the point $\alpha = \beta$ both the angle and level splitting approach zero, as expected.

Turning to the $V$ matrix elements, we obtain

$$
\lambda_{n_i n_j} = \langle \psi_{n_i}^p | V | \psi_{n_j}^p \rangle \sim \delta_{n_i, n_j, \pm 2}, \quad p_{i j} = r, l.
$$

(12)

Using the Brillouin-Wigner perturbation expansion, the basic formula of the wave-function is

$$
|\Psi_{n_i}^p \rangle = |\psi_{n_i}^p \rangle \sum_{n_j \neq n_i} \frac{\lambda_{n_i n_j}^p}{E_{n_i}^p - E_{n_j}^p} |\psi_{n_j}^p \rangle,
$$

(13)

in which the prime requires $n_i \neq n_j$ and $p_i \neq p_j$ and the exact energy $E_{n_i}^p$ has to be calculated from

$$
(E_{n_i}^p - E_{n_j}^p) \langle \psi_{n_j}^p | V | \psi_{n_j}^p \rangle = \langle \psi_{n_j}^p | V | \psi_{n_j}^p \rangle.
$$

(14)

The convenience of this perturbation scheme for $V$ is that the task of finding the wave functions can be avoided.

Let us start from the second order correction to $E_{n_i}^p$:

$$
\Delta^{(2)}(E_{n_i}^p) = \lambda^2 \sum_{n_j \neq n_i} \frac{|\psi_{n_i}^{p_j}|^2}{E_{n_i}^p - E_{n_j}^p},
$$

(15)

in which $E_{n_i, n_j}^p = E_{n_i}^p - E_{n_j}^p$. For the fourth order correction we first calculate the irreducible term

$$
\Delta^{(4)}_{\text{ir}}(E_{n_i}^p) = \lambda^4 \sum_{n_j, n_k, n_l} \frac{|\psi_{n_i}^{p_j p_k p_l}|^2}{E_{n_i, n_j}^p E_{n_i, n_k}^p E_{n_i, n_l}^p}.
$$

(16)

By irreducible we mean that the intermediate states are different from the original state $\psi_{n_i}^p$. There is also another contribution to the fourth order energy correction due to the reducible term

$$
\Delta^{(4)}_{\text{re}}(E_{n_i}^p) = -\lambda^4 \sum_{n_j, n_k \neq n_i} \frac{|\psi_{n_i}^{p_j p_k}|^2}{(E_{n_i, n_j}^p)^2} \frac{|\psi_{n_i}^{p_k}|^2}{E_{n_i, n_k}^p}.
$$

(17)

This reducible term is the product of two irreducible terms with complicated coefficients in general. The simple form of the interaction term $V$ allows us to derive (17) just by replacing the bare energy $E_{n_i}^p$ by the renormalized energy $E_{n_i}^p + \Delta^{(2)}$ inside (10) and expanding up to $\lambda^4$ terms:

$$
\Delta^{(2)}(E_{n_i}^p + \Delta^{(2)}) = \Delta^{(2)}(E_{n_i}^p) + \Delta^{(4)}_{\text{ir}}(E_{n_i}^p) + O(\lambda^6).
$$

(18)

It can be shown in general that all reducible terms of a given order can be derived from the irreducible terms of the lower orders if we use the renormalized energy in them. That means that we need to calculate only the irreducible terms. Once calculated, we renormalize the energy and iterate until we get the desired accuracy. Moreover, the calculation of the irreducible terms is simplified significantly because the transitions to intermediate states are highly restricted, due to the peculiar form of $\lambda$. For a given level $n$ the energy can be calculated with accuracy $\lambda^{(N-n)/2}$, where $N$ is the total number of levels included ($\lambda < \beta/\alpha < 1$).

Knowing the limits for $\beta = 0$ (4) and $\alpha = \beta$ (9) of the spectrum, we can also propose an approximate formula for the energy levels, increasingly accurate at high magnetic field or weak SO interactions. Both $\omega$ and $\lambda$ depend on $\beta/\alpha$ and only $H_{\text{so}}$ in (3) depends on the values of $\alpha$ and $\beta$ as well. Rescaling back (7), we notice that the effect of $V$ is to transform $H_0(\omega) + V$ into the simple harmonic oscillator, with modified energy $\omega \sqrt{1 - \lambda^2} = \omega_c$, over the whole range of $0 \leq \lambda \leq 1$. The main effect of $V$ is to rescale the energy in (3), but this consideration misses the constant shift of the energy levels near the point $\beta = \alpha$. To compensate, we add an ad hoc energy shift which gives the correct level spectrum at $\alpha = \beta$ and at the same time does not affect the correct limit of the formula (4). Thus we obtain

$$
E_n^l = \hbar \omega_c n + \frac{1}{2} \sqrt{\delta^2 + 4n(\alpha^2 - \beta^2)\hbar^2 \omega_c^2}
$$

$$
+ \hbar \omega_c \beta^2 \left( \frac{n + 1}{n + 2 \xi} \right) - \frac{n + 1}{n - 2 \xi} - \frac{n + 1}{n + 2 \xi}
$$

(19)

in which $\delta = \hbar \omega_c (1 + 2 \xi)$, $n \geq 0$ for $E_n^l$ and $n \geq 1$ for $E_n^r$. As we will see in the examples, the agreement of (19) with the exact numerical results is excellent, especially at strong magnetic fields, when $\alpha$ and $\beta$ are smaller.

IV. RESULTS AND DISCUSSION

In Fig. 1 the ground state and the first two excited states of a 2DEG in InSb are plotted. The magnetic field is chosen ($B = 2T$) such that the effect of spin-orbit splitting is visible and at the same time let the reader appreciate the accuracy of the approximate result (19). In
the left part of this figure, first the parameter $\alpha_0$ varies from zero to maximum value of $\alpha_0^{\text{max}} = 250 \text{ meV} \, \AA$ while the Dresselhaus parameter is zero. Here the exact spectrum (1) is plotted versus the Rashba interaction, akin to an applied gate voltage. Then keeping $\alpha_0$ constant, on the right panels of the figure, we increase the value of $\beta_0$ from zero to $\beta_0 = \alpha_0$. Here the circles show the exact numerical result, while the solid lines correspond to the approximate solution (19). Note that as $\beta$ can not affect the ground state energy directly, the correction to this level starts as $\alpha^2 \beta^2$, while for all other levels, the correction is of order $\beta^2$ and higher.

In Fig. 2 we first switch on the Dresselhaus term (left panels) and then keeping $\beta$ constant, we increase the value of the Rashba interaction (right panels). This time it is the correction to the first excited level that is of fourth order $\alpha^2 \beta^2$.

Figure 3 shows how the SO interaction modifies the effective Zeeman coupling. Here this parameter is defined by $g_{\text{eff}}/g = 2(E_0^0 - E_1^0)/(\mu_B B g)$ where $E_0^0$ and $E_1^0$ are the first excited state and the ground state respectively, i.e., the lowest spin-split level pairs. In all cases, the Rashba term increases $g_{\text{eff}}$ while the Dresselhaus coupling decreases it. The reason is simple as we note that the $\phi_{0\uparrow}$ state is coupled to $\phi_{1\downarrow}$ through Rashba interaction and pushed further down, resulting in increasing spin splitting of each Landau level. Contrary to this the Dresselhaus interaction reduces the energy gap between the two spin states of each Landau level. Although not realized for the parameters chosen here, when the Zeeman splitting is small enough (as in low magnetic field), the Dresselhaus term can dominate the Zeeman coupling and interchange the ground state $\phi_{0\uparrow}$ with $\phi_{0\downarrow}$. This is also found in the spectrum of quantum dots (20).

In general the effective Zeeman coupling depends on the strength of the spin-orbit coupling, bare Zeeman coupling and magnetic field. In the upper panel of Fig. 4 we have plotted this quantity for different semiconductors. The value of the magnetic fields is fixed at $B = 2T$ for all cases. The bare value of the Zeeman splitting and the maximum value of the spin-orbit term is different and chosen as in the literature (21,22). The larger the spin-orbit interaction, the larger is the change in the $g_{\text{eff}}$. Notice that this modulation of $g_{\text{eff}}$ via the spin orbit effect is in principle controllable, via external gate voltage (to control $\alpha_0$). This behavior may be useful in experiments with spins (17). In the lower panel of Fig. 4 we plot $g_{\text{eff}}$ as a function of the magnetic field when the value of the Rashba and Dresselhaus interactions is fixed at $\alpha_0^{\text{max}}$ and $\beta_0^{\text{max}}$. 

FIG. 1: (Color online) Ground state and first two excited states for InSb as a function of $\alpha_0/\alpha_0^{\text{max}}$ at $\beta_0 = 0$ (left panels) and as a function of $\beta_0/\alpha_0$ at $\alpha_0 = \alpha_0^{\text{max}}$ (right panels). The fixed parameters are $\alpha_0^{\text{max}} = 250 \text{ meV} \, \AA$, $g/g_0 = -51$, $m/m_e = 0.014$, $B = 2T$. Energy is in units of $\hbar \omega_c = 16.5 \text{ meV}$.

FIG. 2: (Color online) Ground state and first two excited states in InSb as a function of $\beta_0/\beta_0^{\text{max}}$ at $\alpha_0 = 0$ (left panels) and as a function of $\alpha_0/\beta_0$ at $\beta_0 = \beta_0^{\text{max}}$ (right panels). The fixed parameters are $\beta_0^{\text{max}} = 250 \text{meV} \, \AA$, $g/g_0 = -51$, $m/m_e = 0.014$, $B = 2T$. Energy is in units of $\hbar \omega_c = 16.5 \text{ meV}$.

FIG. 3: (Color online) The effective Zeeman coupling of the lowest spin-split levels in InSb, corresponding to Fig. 4, $B = 2T$. 

$\text{InSb, } B = 2T$

$\text{InSb, } B = 2T$
\( \beta_0 = \alpha_0^{\text{max}}/4 \), respectively. By increasing the magnetic field the effective Zeeman coupling decreases because the ratio of the spin-orbit splitting to the Landau level separation decreases.

Once the energy is known the wave function can be calculated via \((13)\) which in turn can be used to calculate transport parameters like charge and spin conductivity.\(^{11,23,24}\) That work is in progress.

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