Options for cosmology at redshifts above one

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Abstract.
We show that detailed exploration of the $1 < z < 2$ redshift region can provide for definitive testing not only of the standard inflationary cosmological paradigm with its fine-tuned cosmological constant and its mysteriously late ($z < 1$) onset of cosmic acceleration, but also for the non fine-tuned, alternate conformal cosmological model, a cosmology which accelerates both above and below $z = 1$. In particular we confront both of these models with the currently available type Ia supernovae standard candle and extended FRII radio source standard yardstick data, with these latter data being particularly pertinent as they already include a sizeable number of points in the $1 < z < 2$ region. We find that both models are able to account for all available $0 < z < 2$ data equally well; and with the conformal model explicitly being able to fit the data while being an accelerating one in the $z > 1$ region, one is thus currently unable to ascertain whether the universe is accelerating or decelerating between $z = 1$ and $z = 2$. To be able to visualize the supernovae and radio galaxy data simultaneously, we present a representation of the radio galaxy data in terms of an equivalent apparent magnitude Hubble diagram. We discuss briefly some implications of the anisotropies in the cosmic microwave background for the conformal theory, and show that in that theory fluctuations which set in at around nucleosynthesis can readily generate the first peak in the anisotropy data.

1. INTRODUCTION

Through a detailed analysis of type Ia supernovae standard candles [1, 2], of the cosmic microwave background (CMB) [3, 4], and of clusters of galaxies [5], standard cosmology has homed in on a rather narrow range of allowed cosmological parameters, a range centered around $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ or so. While such allowed values are very encouraging for the standard flat inflationary universe model [6], they are, at the same time, equally deeply troubling for standard gravity, requiring a fine-tuning of the cosmological constant $\Lambda$ through as many as 60 to 120 orders of magnitude, with the fits to data being altogether disastrous if a value such as $10^{60}$ or $10^{120}$ for $\Omega_\Lambda(t_0)$ were to actually be used. The currently required value for the cosmological constant associated with an $\Omega_\Lambda(t_0)$ of order one thus poses an extremely severe challenge to the standard cosmological model which has so far stubbornly resisted resolution. However, even without any such resolution, it is nonetheless possible to directly test whether or not nature actually is governed by the fine-tuned value for $\Lambda$ suggested by the data analysis. Specifically, since the matter density $\rho_M(t)$ redshifts while the constant $\Lambda$ does not, as one looks back in redshift the relative strength of the contributions of these components to the cosmological expansion rate will vary. In particular, since on its own a normal matter density would lead to deceleration while by itself $\Lambda$ (if taken to be positive - a further ad hoc assumption of the standard paradigm) would lead to acceleration, their inferred current
era relative strengths are such that a net cosmic acceleration is to only be a very late \((z < 1)\) phenomenon, with the universe having to be decelerating at all higher redshifts. Study of cosmology above \(z = 1\) can thus serve as a major diagnostic for the standard paradigm. And moreover, as we shall show below, it can also provide for definitive testing of the fully covariant alternate conformal gravity theory whose cosmology was originally advocated \([7]\) precisely because it possessed an underlying symmetry, viz. conformal invariance, which was able to keep the cosmological constant under control, to thereby lead to a cosmological model \([8, 9, 10, 11]\) which was able to account \([12]\) for the accelerating universe supernovae data without any fine-tuning at all while being able to naturally accommodate a \(\Lambda\) as large as elementary particle physics suggests. In this paper then we therefore explore options for cosmology at redshifts greater than one.

With the acquisition of supernovae data at \(z > 1\) being quite difficult (currently there is only one \(z > 1\) supernova, SN 1997ff, for which both an apparent magnitude and redshift have been established \([13]\)), and with it being some time before the space based SNAP supernovae project will come on line, it is thus necessary to seek alternate techniques to explore \(z > 1\) cosmology. Since data for the very powerful extended FRII radio galaxies \([14]\) are already available out to \(z = 2\) or so (and are readily extendable to \(z = 3\)), we thus turn to the standard yardstick technique based on such radio galaxies which has been developed by Daly and coworkers \([15, 16, 17, 18, 19, 20]\), and explore its implications for cosmology. Even though the technique itself is based on completely conventional theoretical astrophysical ideas, it is nonetheless instructive to validate the technique purely by empirical means. Consequently, we shall first apply the technique to data below \(z = 1\), and show complete consistency between its cosmological expectations and those based on the \(z < 1\) supernovae data themselves. Thus armed, we shall then extend the predictions of the standard yardstick technique out to \(z = 2\) and explore its ensuing implications for cosmology. As such, the procedure that we are following here is is a well established one in astronomy, namely to check the validity of a candidate technique against an established one in a given kinematic region, and to then extend the candidate one into a region which the established one does not reach. Noting the complementary between the standard yardstick and standard candle techniques, our analysis thus nicely prepares the \(z > 1\) region for its eventual exploration via future supernovae data.

In Sec. (2) we familiarize the reader with the standard yardstick radio galaxy method, while also presenting a procedure developed jointly with R. A. Daly which enables us to conveniently represent the standard yardstick technique predictions in the form of an equivalent apparent magnitude versus redshift plot, to thus make the method readily visualizable. In Sec. (3) we use the standard yardstick technique to test some candidate cosmological models, viz. the standard inflationary \((\Omega_M, \Omega_\Lambda)\) model, the quintessence model \([21]\), the rolling scalar field model \([22]\), and the cosmological model based on the alternate conformal gravitational theory. Through use of such a wide variety of models we are able to get the broadest possible reading on the interpretation of the data. Finally, in Sec. (4) we discuss some implications for conformal gravity of anisotropies in the CMB, another important testing ground for cosmology, and in work done jointly with K. Horne, show that in the conformal theory fluctuations which set in at around nucleosynthesis can readily generate the first peak in the anisotropy data.
2. THEORETICAL BACKGROUND

General use of a standard yardstick for cosmology requires comparing the measured size of some chosen system at a given redshift with an expected size for it at that same redshift. However, unlike the common intrinsic luminosity type Ia supernovae standard candle technique, there does not appear to be any common intrinsically sized family of astrophysical systems which could provide a purely empirical analog of the standard candle technique. One thus has to resort to theory to determine an expected size, with the radio galaxy method developed by Daly and coworkers relying on standard astrophysical theory which is independent of cosmology, and with the study of anisotropies in the CMB using models based on the cosmology itself to determine the requisite expected size. We discuss the radio galaxy method here and discuss the CMB technique below.

The primary advantage of the radio galaxy method is that in utilizing the properties of very powerful FRII classical double radio galaxy sources, one deals with systems that are luminous enough to permit observation out to $z = 2$ and beyond, to thus enable us to go beyond the region currently explored by type Ia supernovae. The FRII sources consist of an AGN that produces two oppositely directed supersonically propagating collimated jets which inject energy into two radio hot spots. The most powerful and least distorted of these radio sources (referred to as FRIIb by Daly and coworkers) form an unusually homogeneous population with the average distance between the radio hotspots, $<D>$, at a given redshift exhibiting a rather small dispersion. $<D>$ then serves as a requisite measured size, and with these sources subtending a small opening angle $\theta$ at the observer, we can set $<D> = \theta R(t) = \theta R(t_0) r / (1 + z)$ where $r$ is defined by the Robertson-Walker null geodesic relation $\int_0^{t_0} c dt / R(t) = \int_0^{r} dr / (1 - kr^2)^{1/2}$.

On assuming that the supersonic flow (of average rate of growth $v_L$) of the FRIIb sources can be described by strong shock physics, and assuming that the total lifetime of the source is related to the beam power, $L$, of the source according to the power law $t_* \propto L^{-\beta/3}$, Daly and coworkers show (see e.g. [20]) that the size $D_* = v_L t_*$ to which such an FRIIb system will eventually be expected to grow is related to $<D>$ as

$$<D> / D_* = k_0 y(z)^{(6\beta - 1)/7} [k_1 y(z)^{-4/7} + k_2]^{\beta/3 - 1}$$

where $y(z) = H_0 R(t_0) r / c$, and where $k_0$, $k_1$ and $k_2$ are specific (though rather complicated) functions of observables associated with the FRIIb systems which are given in [18]. In order to apply Eq. (1) to cosmology Daly and coworkers assume further that the expected $D_*$ will be universally proportional to the measured average size $<D>$ of all of the sources in the parent sample which are at the same redshift as the given source, i.e. that the ratio $<D> / D_*$ is a redshift independent constant $\kappa$. At the present time a parent population of 70 FRIIb radio galaxies has been identified, with 20 of the sources having been observed in detail. The full 70 source parent sample is thus used to determine $<D>$, while values for $D_*$ are determined from the well studied 20 sources. Even though none of the assumptions which go into this analysis is particularly contentious

1 The absolute normalization of $\kappa$ is unimportant. What matters for the technique is that $\kappa$ be independent of redshift, something which is borne out in the fits.
or unreasonable, nonetheless regardless of the validity of its theoretical underpinnings, the radio galaxy standard yardstick technique can be considered independently verified by the successful comparison between the $z < 1$ radio galaxy and supernovae data given below, a comparison which provides fitted values for the phenomenological $\beta$ and $\kappa$.

While the fits to be given in the tables below are based on the use of the full theoretical calculation outlined above, should energy losses due to inverse Compton cooling of relativistic electrons by CMB photons be negligible (something thought to be a good though not perfect approximation for the available $z < 2$ FRIIb sample), the parameter $k_2$ in Eq. (1) can then be neglected, with Eq. (1) then simplifying to [17]

$$\kappa = (R(t_0)r)g(\beta)Q$$

(2)

where $g(\beta) = 3/7 + 2\beta/3$ and where $Q = (H_0/c)^g(\beta)k_0k_1^{1/3-1}$ depends only on observed quantities. If we now define a quantity

$$m_{RG} = 5\log[(1+z)Q^{-1/g(\beta)}] + (5/g(\beta))\log \kappa + M + 5\log[H_0/c] ,$$

(3)

where $M = M_B - 5\log(H_0/c) + 25$, we can then introduce a convenient equivalent or effective apparent magnitude parameter $m_{RG}$ for radio galaxies, viz.

$$m_{RG} = M + 5\log[(1+z)H_0R(t_0)r/c] ,$$

(4)

an expression completely analogous to the B band Hubble diagram relation

$$m_B = M + 5\log[(1+z)H_0R(t_0)r/c] .$$

(5)

The parameter $M$ introduced here is related to the intrinsic absolute magnitude $M_B$ of type Ia supernovae via the relation $M_B = M - 40 - 5 \log(3h^{-1})$, where $h$ is the Hubble constant as measured in units of 100 km/s/Mpc. For $h = 0.65$, $M_B = M - 43.32$, the typical measured value of $M = 23.95$ given below corresponds to the value $M_B = -19.37$ found in the supernovae data analyses themselves [1, 2]. By putting the radio galaxy data into the same format as the supernovae data we can thus plot both data sets on one and the same graph, and while this is only an approximate procedure, it nonetheless permits an easy visualization of the entire $0 < z < 2$ region.

Application of the theory to data requires the specification of a global cosmological model based on a theory of gravity. For standard gravity with a set of perfect fluid sources each with an equation of state $p_i(t) = w_i\rho_i(t)$ and with an $\Omega_i(t)$ parameter given by $\Omega_i(t) = 8\pi G\rho_i(t)/3c^2H^2(t)$, the coordinate distance is given in the canonical $k = 0$ universe case by the familiar

$$R(t_0)r = \frac{c}{H_0} \int_0^z \frac{dz}{[\sum_i\Omega_i(1+z)^{3+3w_i}]^{1/2}} , \quad \sum_i \Omega_i = 1 .$$

(6)

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2 Recently Daly [20] has generalized this formalism by using the full Eq. (1) to extract the dependence of $y(z)$ on $z$ directly without needing to make any approximation at all. Since $y(z) = (H_0/c)\delta L/(1+z)$ where $\delta L$ is the luminosity distance, a plot of $y(z)$ against $z$ is essentially a Hubble plot, with the plots given in [20] being found to exhibit the same general trends as those given in Figs. 1 and 2 below.
Eq. (6) encompasses not only a standard inflationary universe with a $w_M = 0$, $\Omega_M > 0$ matter fluid and a $w_\Lambda = -1$, $\Omega_\Lambda > 0$ cosmological constant, but also a quintessence model with a matter fluid and an $\Omega_Q > 0$ quintessence fluid whose $w_Q$ is negative. With slight adjustment Eq. (6) can also be applied to the rolling scalar field model [22], with its power law potential $V(\phi) \propto \phi^\alpha$ leading to a $w$ parameter which then depends on $\varepsilon$.

As well as study the standard theory, we shall also explore the fully covariant alternate conformal gravitational theory, a theory which sets out to solve some of the most troubling problems in astrophysics, viz. the cosmological constant and dark matter problems, by modifying gravity rather than by making ad hoc adjustments to the energy-momentum tensor. While the conformal theory is found to recover the results of standard gravity for solar system sized distances or less, its departure from the standard theory on larger distance scales has enabled it to naturally resolve the dark matter and dark energy problems without any fine-tuning at all, with its only known difficulty (a point we return to below) being an inability to nucleosynthesize sufficient primordial deuterium.

In the conformal theory it is found that even while the low energy limit of the theory is controlled by a dynamically induced but otherwise standard attractive Newton constant $G$, its cosmology is controlled by an entirely different induced gravitational constant $G_{\text{eff}}$, a repulsive rather than attractive coupling constant which is given as $G_{\text{eff}} = -3c^2/4\pi\hbar S_0^2$ where $S_0$ is the (very large) expectation value of a scalar urfield which is to spontaneously break the conformal symmetry cosmologically. Apart from this specific change the cosmological evolution equation is otherwise completely standard, taking (for a matter density which redshifts as $1/t^3$) the form [8, 9, 10, 11]

$$\ddot{R}^2(t) + kc^2 = \dot{R}^2(t)[\ddot{\Omega}_M(t) + \ddot{\Omega}_\Lambda(t)], \quad q(t) = (n/2 - 1)\ddot{\Omega}_M(t) - \ddot{\Omega}_\Lambda(t)$$

(7)

with $\Omega_i(t)$ having been replaced by $\tilde{\Omega}_i(t) = 8\pi G_{\text{eff}} \rho_i(t)/3c^2H^2(t)$. Moreover, in the conformal theory the sign of the parameter $\Lambda$ is explicitly known to necessarily be negative since it arises from elementary particle physics phase transitions which occur as the universe cools down, i.e. by transition from an unbroken symmetry phase with $\Lambda = 0$ to a broken one with a lower energy. Then, with $G_{\text{eff}}$ also being negative, it follows that $\ddot{\Omega}_\Lambda(t)$ itself must necessarily be positive. Moreover, with $\ddot{\Omega}_M(t)$ necessarily being negative precisely because $G_{\text{eff}}$ is negative, it follows that $q(t)$ is then always negative, with conformal cosmology thus automatically being an accelerating one in each and every epoch no matter how big or small $\Lambda$ might be. As regards the numerical value of $\ddot{\Omega}_\Lambda(t)$, we note further that the larger $S_0$ the smaller $G_{\text{eff}}$, and thus the smaller the amount by which the cosmological constant gravitates. Thus in the conformal theory it is not the cosmological constant which gets quenched but rather its effect on cosmic evolution, with the amount of gravity produced by a matter source being radically reduced from the amount generated by the same source in the standard theory. Moreover, this quenching is done by the theory itself without the need for any fine-tuning, leading to a theory in which no matter how huge $\Lambda$ might be, $\ddot{\Omega}_\Lambda(t)$ always [10, 11] has to lie between zero and one in all epochs except the very earliest (the only epoch where $\ddot{\Omega}_M(t)$ is of consequence even though $\rho_m(t)$ contains the completely standard amount of luminous material), with the late universe deceleration parameter being given as $q(t) = -\ddot{\Omega}_\Lambda(t)$, so that at late times $q(t)$ then has to automatically lie between zero and minus one no matter what. The theory thus gives a controlled amount of cosmic
acceleration in each and every late universe epoch without any fine-tuning at all, with it being the absence of any decelerating epoch above $z = 1$ which serves as a clear discriminator between it and the standard theory. Given Eq. (7), the conformal theory coordinate and luminosity distances are then found to be given by [11, 12]

$$R(t_0)r = \frac{d_L}{(1+z)} = -\frac{c(1+z)}{H_0q_0} \left[ 1 - \left\{ 1 + q_0 - \frac{q_0}{(1+z)^2} \right\}^{1/2} \right],$$

(8)

to thus give a one parameter family of fits labelled by the current value of $q_0$, a value which, as we just noted, has to necessarily lie between zero and minus one. Given Eqs. (6) and (8) we turn now to the data.

3. OPTIONS FOR COSMOLOGY AT REDSHIFTS ABOVE ONE

For the supernovae data we follow the authors [2] and fit 38 of their 42 reported data points together with 16 of the 18 earlier lower $z$ points of [23], for a total sample of 54 $z < 1$ supernovae data points. (While we thus leave out 6 questionable supernovae data points for the fitting, nonetheless, for completeness we still include them in the displayed Fig. 1.) For the radio galaxies we use the 14 data points listed in [17] and the 6 listed in [18], for a total of 20 radio sources. Of the models for which we provide fits below, the implications for the radio galaxy data of three of them have already been well studied in the literature by Daly and coworkers, with the standard model radio galaxy predictions having been given in [18], the quintessence model predictions in [19] and the scalar field model predictions in [24]. Fits to the supernovae data using the standard model and the quintessence model abound in the literature, with supernovae fits using the scalar field model having been given in [25], and supernovae fits using conformal cosmology having been given in [12]. As far as all of those published fits are concerned what is new here is only in the way the fits are organized in the tables below, excepting that the conformal cosmology radio galaxy fits are new.3

For the $k = 0$ standard model fitting to the supernovae data we recover the results of [2], and obtain a minimum $\chi^2 = 56.76$ with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, with the 68% confidence region being given as $0.2 < \Omega_M < 0.35$, $0.8 > \Omega_\Lambda > 0.65$ (see Table 1, where our identification of 51 rather than 52 degrees of freedom is due to an internal aspect of the data extraction procedure used in [21]). Of the 20 radio sources 9 have $z < 0.9$, and a fit to them alone yields a best fit $\chi^2 = 11.65$. Combining now the two $z < 1$ data sets then yields a best fit $\chi^2 = 68.72$ for the 63 points with $z < 1$, where now the minimum is at $\Omega_M = 0.25$, $\Omega_\Lambda = 0.75$, with the 68% confidence region being given as $0.2 < \Omega_M < 0.35$, $0.8 > \Omega_\Lambda > 0.65$. Noting the complete overlap of the fitting parameters and noting that $56.76 + 11.65 = 68.41$ is extremely close to 68.72, we thus find complete compatibility between the cosmologies implied by the $z < 1$ supernovae and radio galaxy data. And

3 The fits themselves were prepared for the author by R. A. Daly and M. P. Mory using Daly’s master program which can generate radio galaxy fits for assigned cosmologies, and the author is altogether indebted to them for doing so.
with this very same concordance being found in the $z < 1$ analyses of all of the other cosmological models being considered here, we believe that one may therefore regard the standard yardstick technique as having been empirically confirmed.

Having established the credentials of the radio source technique, we now include the 11 radio galaxies with $z > 1$ and make an overall standard model fit to all 74 of the $z < 2$ data points. We find a best fit $\chi^2 = 74.41$ for the 74 points with $z < 2$, where now the minimum is at $\Omega_M = 0.25$, $\Omega_\Lambda = 0.75$, with the 68% confidence region being given as $0.2 < \Omega_M < 0.35$, $0.8 > \Omega_\Lambda > 0.65$. With a best fit $\chi^2 = 16.89$ being found for the 20 radio galaxy data points alone and with $56.76 + 16.89 = 73.65$ being within one standard deviation of 74.41, we again find complete compatibility between the standard yardstick and standard candle approaches.

In an examination of the fits it was found that a huge amount of the $\chi^2$ was contributed by just one radio galaxy, viz. 3C 427.1 at $z = .572$, an outlier which is more than 3 $\sigma$ away from the best fits. Consequently, we also investigated fits to the data with this potentially questionable source removed, with the resulting outcome for the standard inflationary cosmology being listed in Table 5 and displayed in Fig. 1 as an equivalent apparent magnitude fit and then in Fig. 2 as a residual equivalent apparent magnitude fit with respect to the convenient empty universe baseline.$^4$ With the plot of the altogether acceptable fitting of Eqs. (3) - (5) to the data being shown in Fig. 1, we believe that this figure can reasonably be interpreted as an early look at the $z < 2$ Hubble diagram.

For comparison purposes we have also included in Fig. 1 a plot of the apparent magnitude expectations associated with the illustrative ($\Omega_M = 1$, $\Omega_\Lambda = 0$, $\Omega_k = 0$), ($\Omega_M = 0$, $\Omega_\Lambda = 0$, $\Omega_k = 1$), and ($\Omega_M = 0$, $\Omega_\Lambda = 1$, $\Omega_k = 0$) models (as calculated from Eq. (5) with $M = 23.95$). The curvature dominated ($\Omega_M = 0$, $\Omega_\Lambda = 0$, $\Omega_k = 1$) empty universe is a coasting one with $q(t)$ being zero in all epochs, and is thus a particularly convenient baseline, a point we emphasize in the residual magnitude plot with respect to it given in Fig. 2. As we see, the ($\Omega_M = 0.25$, $\Omega_\Lambda = 0.75$) model apparent magnitude crosses this baseline at around $z = 1.6$, $^5$ a somewhat higher (rather than lower) value than the $z = 1.3$ value where an ($\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$) universe would cross, to thus indicate that no compelling case from the radio galaxy data can be made that the $z > 1$ region is any less cosmically repulsive than the $z < 1$ region. Additionally in the figures we have included the SN 1997ff data point at $z = 1.74^{+0.10}_{-0.15}$, and we see that the radio sources (and particularly the radio sources at the highest available redshifts) are not supporting its suggestion that cosmic repulsion is in fact weakening above $z = 1$. As regards SN 1997ff, we additionally recall that the authors of [13] had noted that this particular supernova just happened to be lensed by two foreground galaxies along the line of sight, so it might well be a lot dimmer than indicated in the figures, an effect which would then move it more toward the cosmically repulsive side of the empty universe baseline. From Fig. 2

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$^4$ While not included in the fit, for completeness we have still included 3C 427.1 in the figures. The horizontal error bars which are shown in the fits to the radio galaxy data points are obtained by combining the uncertainties in the data and the mean fitted deviations of the parameters $\beta$, $\kappa$ and $M$ as given in column 5 of Table 5. (Details of this error bar analysis are given in [24].)

$^5$ Because the dependencies on redshift of the deceleration parameter and the luminosity distance are quite different, for given assigned values of $\Omega_M(t_0)$ and $\Omega_\Lambda(t_0)$ the parameter $q(z)$ can change sign at a much lower redshift than the one at which $d_L(z)$ would cross the empty universe baseline.
we additionally infer that by extending the Hubble diagram out just a little bit further in \( z \), it should then rapidly become apparent whether the standard model \( z = 1.6 \) crossover is supported by higher \( z \) data. Moreover, even without this, filling in the \( z < 2 \) region with more data points could itself already sharply constrain cosmology.

We have also made fits to the data using a quintessence model, a scalar field model and a conformal cosmology model, and display their best fits in Tables 2, 3 and 4 (3C 427.1 included) and Tables 6, 7 and 8 (3C 427.1 excluded), and plot their best outlier excluded fits in Figs. 1 and 2.\(^6\) As we see, both the quintessence model fitting and the scalar field model fitting are every bit as acceptable as the standard \( (\Omega_M, \Omega_\Lambda) \) model fitting, with their best \( z > 1 \) fits also not being found to be any less cosmically repulsive than those for \( z < 1 \) (if anything both of the models go in the direction of making \( \Omega_M \) smaller). This suggestion of a potentially continuing cosmic repulsion above \( z = 1 \) is also shared by the conformal gravity fits, fits which are just as good as the standard model and quintessence fits while being strictly on the repulsive side of the empty universe baseline at all \( z \). The conformal gravity fitting to the 74 total data points which we present here is completely consistent not only with the earlier conformal gravity fitting to the 54 \( z < 1 \) supernovae data points given in [12], but also with the \( z > 1 \) predictions made in the same paper; with the very success of the conformal gravity fitting that we have presented here implying that it is not yet possible to ascertain whether the universe is actually accelerating or decelerating between \( z = 1 \) and \( z = 2 \),\(^7\) thus making any extension or filling in of the Hubble diagram potentially highly instructive. The conformal gravity fits are also significant in that at the present time they are in fact the only non fine-tuned fits to the accelerating universe data that have so far been presented in the literature, to thus at the very least show that it is in principle possible to fit the data without fine-tuning, with the currently available \( 0 < z < 2 \) data not at all rejecting the only predictive cosmological model presented so far in the literature in which the cosmological constant problem is naturally solved.

### 4. OPTIONS FOR COSMOLOGY AT RECOMBINATION

Other than the \( 0 < z < 2 \) region, the two other primary regions where cosmology can be tested are the nucleosynthesis era and the CMB recombination era, studies of which lead in the standard theory to remarkably successful fitting associated with an \( \Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7 \) universe. As we had indicated earlier such a universe can directly be tested in the \( 1 < z < 2 \) region. However, because it might be some time yet before a definitive answer to such testing is actually obtained, and because the cosmological constant problem associated with such an \( \Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7 \) universe is so very

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\(^6\) For fitting reasons we have constrained the quintessence \( w \) parameter to the range \(-3 \leq w \leq 0\), and the scalar potential parameter \( \alpha \) to the range \( 0 \leq \alpha \leq 8 \).

\(^7\) A recently updated analysis of the lensing of SN 1997ff now indicates [26] that SN 1997ff was probably magnified even more than had previously been thought, to thus necessitate repositioning it even further toward the cosmically repulsive side of Fig. 2; with the authors of [26] noting that the conformal cosmology prediction for this supernova would then be brought well within the 2 \( \sigma \) level of acceptability.
severe, it is of value to ask whether the CMB anisotropy data could admit of any alternate explanation. As well as a being posed simply as a general question (namely, how much of the success of the CMB fitting is due to detailed features of a particular model and how much might be generic), one can also ask how well any candidate alternate theory might fare. While the cosmological fluctuation theory associated with the alternate conformal gravity theory being considered in this paper has yet to be fully developed, a first step in this regard has recently been taken by the author and K. Horne, one we now report on.

Basic to the CMB analysis is a determination of the true proper diameter \(d(\theta)\) of some candidate yardstick at coordinate \(r\) and redshift \(z = R(t_0)/R(t) - 1 = T(t)/T(t_0) - 1\) which subtends an angle \(\theta\) at an observer at \(r = 0, z = 0\), a proper diameter which for a general Robertson-Walker geometry is given by

\[
d(\theta) = 2R(t) \int_0^{\sin(\theta/2)} \frac{dr}{(1 - kr^2)^{1/2}},
\]

and which for small \(\theta\) reduces to the relation \(d(\theta) = \theta R(t) r\) used earlier for the radio galaxies. If the yardstick used for the CMB analysis is due to the growth of some cosmological fluctuation which started at some earlier fluctuation time \(t_F\), the proper distance \(D(t, t_F)\) of the fluctuation at the time \(t\) will be given by

\[
D(t, t_F) = R(t) \int_{t_F}^{t} \frac{dt}{R(t)},
\]

so that a comparison of the measured \(d(\theta)\) with a model choice for \(D(t, t_F)\) allows one to test and constrain the chosen model.

Since the treatment of the isotropy of the CMB in the conformal theory differs substantially from the discussion in the standard theory (the conformal cosmology CMB derived from Eq. (7) is already causally connected [9] even without any inflationary phase), it is instructive to first recall the standard model discussion. With the largest possible value for \(D(t, t_F)\) being given by a fluctuation which set out at \(t_F = 0\), for a standard \(k = 0\) cosmology which is radiation dominated \((R(t) = At^{1/2})\) until recombination, at recombination the maximum \(D(t_R, t_F)\) is then given as \(D(t_R, t_F) = 2t_R\). Similarly, for a \(k = 0\) standard cosmology which is matter dominated \((R(t) = Bt^{2/3})\) since recombination, \(d(\pi)\) is given as \(d(\pi) = 6t_R^{2/3} t_0^{1/3}\), with the ratio \(D(t_R, t_F = 0)/d(\pi) = (T(t_0)/T_R)^{1/2}/3\) thus being very much less than one. Thus despite the high isotropy found for the CMB, in a \(k = 0\) Robertson-Walker universe opposite points on the CMB sky would not be causally connected, with the angle \(\theta = 2(T(t_0)/T_R)^{1/2}/3\) subtended on the sky by \(D(t_R, t_F = 0)\) being of order only \(1^\circ\). In the standard theory this causality problem is solved by having an inflationary de Sitter phase occur very early in the history of the universe prior to the onset of the Robertson-Walker phase. This inflationary phase not only reconciles the causality conflict, it also converts what was a considerable difficulty into a potentially considerable triumph, since the very same \(D(t_R, t_F = 0)\) then no longer sets the scale for the isotropy of the CMB, but rather for its anisotropy as caused by fluctuations generated during the very same inflationary era. With the detection of an anisotropic peak in the CMB associated with precisely such a \(1^\circ\) scale, it is now taken as a given that the fluctuations which are seen in the CMB must indeed have originated in the very early universe,
with the very success of inflation in describing the CMB leading one to conclude that the spatial 3-curvature of the universe is having a negligible effect on current era cosmic expansion so that $\Omega_k(t_0) = -kc^2/R^2(t_0)$ is negligible. However, as we shall now show, in theories such as conformal cosmology whose Robertson-Walker phase already is causal, an altogether different option is possible.

With only the conformal matter density $\bar{\Omega}_M(t)$ contributing to the conformal evolution equation Eq. (7) at temperatures above the phase transition temperature $T_V$ at which the vacuum energy density $\Lambda$ is induced, and with $G_{\text{eff}}$ being negative, we see that Eq. (7) then only admits of solutions in which $k$ is negative, with the global topology of the universe thus being fixed once and for all in the conformal theory prior to the onset of any phase transition at all. Further, with $G_{\text{eff}}$ being negative there is no initial singularity in the theory, with the cosmology thus expanding from a finite minimum radius $R_{\text{min}}$ and a finite (though very large) maximum temperature $T_{\text{max}}$. In such a cosmology we find that at times which are not too early and not too late, the solution to Eq. (7) can be well approximated as $\Omega_k(t_0)$ is far from negligible.

The emergence of a nucleosynthesis scale for the onset of fluctuations is not only intriguing, it may also prove to be of help for the conformal theory. Specifically, as noted earlier, one of the outstanding challenges for conformal cosmology is that while it can readily synthesize the requisite amounts of primordial helium and lithium [27], it fails to yield the needed amount of deuterium. Specifically, because the cosmology expands at a much slower rate than the standard theory, there is abundant time to destroy...
any deuterium which is generated during nucleosynthesis, to thus leave the cosmology deuterium deficient. However, as noted by the authors of [27], once helium is nucleosynthesized, the setting in of inhomogeneities would lead to helium rich and helium deficient regions whose spallation would then generate deuterium, with deuterium production then occurring between nucleosynthesis and recombination. The emergence of inhomogeneities toward the end of the nucleosynthesis era might then serve to alleviate the conformal gravity deuterium problem. Development of a full conformal cosmological fluctuation theory would allow one to address this question while also making predictions for the CMB in a way which might then prove definitive for the conformal theory. The author is indebted to Dr. R. A. Daly, Dr. K. Horne and M. P. Mory for their helpful comments and for their active collaboration in this work. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

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11 In compensation though it should be noted that because of this very same slow expansion, and contrary to the situation in the standard theory, conformal nucleosynthesis is able to not only get passed the \( Z = 8 \) bottleneck of no stable nuclei with \( Z = 8 \), but to then actually produce the measured amount of \(^9\)Be [27].
### TABLE 1. Standard Cosmology with $k = 0$

|        | 54 SN | 9 RG | 54 SN + 9 RG | 20 RG | 54 SN + 20 RG |
|--------|-------|------|--------------|-------|---------------|
| $\Omega_M$ | $0.30 \pm 0.05$ | $0.10 \pm 0.40$ | $0.25 \pm 0.10$ | $0.05 \pm 0.25$ | $0.25 \pm 0.10$ |
| w | $23.95 \pm 0.03$ | $23.93 \pm 0.03$ | $23.95 \pm 0.03$ | $23.93 \pm 0.03$ |
| $\kappa$ | $8.93 \pm 0.07$ | $8.82 \pm 0.07$ | $8.97 \pm 0.05$ | $8.83 \pm 0.05$ |
| $\beta$ | $1.75 \pm 0.10$ | $1.8 \pm 0.10$ | $1.65 \pm 0.05$ | $1.85 \pm 0.04$ |
| $\chi^2$ | $56.76$ | $11.65$ | $68.72$ | $16.89$ | $74.41$ |
| DOF | 51 | 6 | 58 | 17 | 69 |

### TABLE 2. Quintessence with $k = 0$

|        | 54 SN | 9 RG | 54 SN + 9 RG | 20 RG | 54 SN + 20 RG |
|--------|-------|------|--------------|-------|---------------|
| $\Omega_M$ | $0.48 \pm 0.10$ | $0.00 \pm 0.48$ | $0.38 \pm 0.17$ | $0.00 \pm 0.24$ | $0.00 \pm 0.45$ |
| w | $-2.08 \pm 1.39$ | $-0.75 \pm 1.28$ | $-1.36 \pm 1.64$ | $-0.73 \pm 0.58$ | $-0.62 \pm 1.18$ |
| M | $23.91 \pm 0.03$ | $23.93 \pm 0.03$ | $23.95 \pm 0.03$ | $23.93 \pm 0.03$ |
| $\kappa$ | $8.88 \pm 0.07$ | $8.81 \pm 0.07$ | $8.88 \pm 0.05$ | $8.81 \pm 0.05$ |
| $\beta$ | $1.75 \pm 0.10$ | $1.80 \pm 0.10$ | $1.70 \pm 0.04$ | $1.75 \pm 0.04$ |
| $\chi^2$ | $56.18$ | $11.43$ | $68.56$ | $16.53$ | $74.09$ |
| DOF | 50 | 5 | 57 | 16 | 68 |

### TABLE 3. Scalar Field Model with $k = 0$

|        | 54 SN | 9 RG | 54 SN + 9 RG | 20 RG | 54 SN + 20 RG |
|--------|-------|------|--------------|-------|---------------|
| $\Omega_M$ | $0.29 \pm 0.08$ | $0.05 \pm 0.45$ | $0.28 \pm 0.08$ | $0.05 \pm 0.24$ | $0.05 \pm 0.29$ |
| $\alpha$ | $0.00 \pm 0.00$ | $1.15 \pm 1.23$ | $0.00 \pm 1.23$ | $0.90 \pm 3.35$ | $3.35 \pm 3.35$ |
| M | $23.94 \pm 0.03$ | $23.94 \pm 0.03$ | $23.95 \pm 0.03$ | $23.95 \pm 0.03$ |
| $\kappa$ | $8.89 \pm 0.07$ | $8.81 \pm 0.07$ | $8.90 \pm 0.05$ | $8.81 \pm 0.05$ |
| $\beta$ | $1.75 \pm 0.10$ | $1.80 \pm 0.10$ | $1.70 \pm 0.04$ | $1.80 \pm 0.03$ |
| $\chi^2$ | $56.72$ | $11.54$ | $68.63$ | $16.73$ | $74.14$ |
| DOF | 50 | 5 | 57 | 16 | 68 |

### TABLE 4. Conformal Cosmology

|        | 54 SN | 9 RG | 54 SN + 9 RG | 20 RG | 54 SN + 20 RG |
|--------|-------|------|--------------|-------|---------------|
| $q_0$ | $-0.38 \pm 0.38$ | $-0.53 \pm 0.53$ | $-0.38 \pm 0.38$ | $-0.50 \pm 0.50$ | $-0.38 \pm 0.38$ |
| M | $23.95 \pm 0.03$ | $23.95 \pm 0.03$ | $23.95 \pm 0.03$ | $23.95 \pm 0.03$ |
| $\kappa$ | $8.85 \pm 0.07$ | $8.81 \pm 0.07$ | $8.85 \pm 0.05$ | $8.81 \pm 0.05$ |
| $\beta$ | $1.75 \pm 0.10$ | $1.8 \pm 0.10$ | $1.70 \pm 0.04$ | $1.70 \pm 0.03$ |
| $\chi^2$ | $57.62$ | $11.41$ | $69.05$ | $16.46$ | $74.11$ |
| DOF | 51 | 6 | 58 | 17 | 69 |
### Table 5. Standard Cosmology with $k = 0$ and Outlier 3C 427.1 Removed

|        | 54 SN + 8 RG | 19 RG | 54 SN + 19 RG |
|--------|--------------|-------|---------------|
| $\Omega_M$ | 0.30 ± 0.05 - 0.10 | 0.30 ± 0.05 - 0.10 | 0.30 ± 0.05 - 0.10 |
| M | 23.95 ± 0.03 | 23.95 ± 0.03 | 23.95 ± 0.03 |
| $\kappa$ | 8.80 ± 0.07 | 8.70 ± 0.07 | 8.86 ± 0.05 |
| $\beta$ | 1.55 ± 0.11 | 1.55 ± 0.11 | 1.60 ± 0.05 |
| $\chi^2$ | 56.76 | 2.20 | 59.06 |
| DOF | 51 | 57 | 16 |

### Table 6. Quintessence with $k = 0$ and Outlier 3C 427.1 Removed

|        | 54 SN + 8 RG | 19 RG | 54 SN + 19 RG |
|--------|--------------|-------|---------------|
| $\Omega_M$ | 0.48 ± 0.10 - 0.38 | 0.45 ± 0.12 - 0.23 | 0.45 ± 0.12 - 0.23 |
| $w$ | -2.08 ± 0.92 | -1.80 ± 1.23 | -0.69 ± 0.32 |
| M | 23.91 ± 0.03 | 23.91 ± 0.03 | 23.91 ± 0.03 |
| $\kappa$ | 8.76 ± 0.07 | 8.72 ± 0.07 | 8.78 ± 0.05 |
| $\beta$ | 1.55 ± 0.11 | 1.55 ± 0.10 | 1.60 ± 0.05 |
| $\chi^2$ | 56.18 | 2.14 | 58.66 |
| DOF | 50 | 4 | 56 |

### Table 7. Scalar Field Model with $k = 0$ and Outlier 3C 427.1 Removed

|        | 54 SN + 8 RG | 19 RG | 54 SN + 19 RG |
|--------|--------------|-------|---------------|
| $\Omega_M$ | 0.29 ± 0.08 - 0.24 | 0.28 ± 0.09 - 0.23 | 0.28 ± 0.09 - 0.23 |
| $\alpha$ | 0.00 ± 0.00 | 0.00 ± 0.00 | 0.00 ± 0.00 |
| M | 23.94 ± 0.03 | 23.94 ± 0.03 | 23.94 ± 0.03 |
| $\kappa$ | 8.76 ± 0.07 | 8.71 ± 0.07 | 8.80 ± 0.05 |
| $\beta$ | 1.55 ± 0.11 | 1.55 ± 0.11 | 1.60 ± 0.05 |
| $\chi^2$ | 56.72 | 2.16 | 58.99 |
| DOF | 50 | 4 | 56 |

### Table 8. Conformal Cosmology with Outlier 3C 427.1 Removed

|        | 54 SN + 8 RG | 19 RG | 54 SN + 19 RG |
|--------|--------------|-------|---------------|
| $q_0$ | -0.38 ± 0.38 - 0.17 | -0.38 ± 0.38 - 0.17 | -0.38 ± 0.38 - 0.17 |
| M | 23.95 ± 0.03 | 23.95 ± 0.03 | 23.95 ± 0.03 |
| $\kappa$ | 8.74 ± 0.07 | 8.72 ± 0.07 | 8.73 ± 0.05 |
| $\beta$ | 1.55 ± 0.11 | 1.55 ± 0.11 | 1.60 ± 0.04 |
| $\chi^2$ | 57.62 | 2.13 | 59.76 |
| DOF | 51 | 57 | 16 |
FIGURE 1. Equivalent Hubble plot for supernovae (open circles) and radio galaxies (closed circles).
FIGURE 2. Equivalent residual apparent magnitudes with respect to an empty universe.