The residual amplitude modulation (RAM) is the undesired, non-zero amplitude modulation that usually occurs when a phase modulation based on the electro-optic effect is imprinted on a laser beam. In this work, we show that electro-optic modulators (EOMs) that are used to generate the sidebands on the laser beam also generate a RAM in the optical setup. This result contradicts standard textbooks, which assume the amplitude remains unchanged in the process and should be considered as a fundamental RAM (RAM_F) for these devices. We present a classical model for the propagation of an infrared laser with frequency $\omega_1$ in a wedge-shaped crystal and an EOM with an RF modulating signal of frequency $\Omega$. Since $\Omega \ll \omega_0$, we solve Maxwell’s equations in a time-varying media via a WKB approximation and we write the electromagnetic fields in terms of quasi-plane waves. From the emerging fields of the setup, we compute the associated RAM_F and show that it depends on the phase-modulation depth $m$ and the quotient $\left( \frac{m}{\Omega} \right)$. We show that the genesis of the RAM_F is found in phenomena that occur at the level of the unit cell of the modulator crystal. The RAM_F values obtained for the EOMs used in gravitational wave detectors are presented. Both the detectability and the cancellation of RAM_F are then analyzed.

I. INTRODUCTION

The electro-optic modulators (EOMs) are devices designed to modulate a laser beam. Depending on the configuration adopted by the EOM, they can be used to change the polarization state, to modulate the phase or the amplitude of the laser [1]. It is also possible to simultaneously modulate the amplitude and phase of the beam [2].

The EOMs have multiple applications, for example, frequency modulation spectroscopy [3, 4], modulation transfer spectroscopy [5, 6], two-tone frequency modulation spectroscopy [7, 8], laser frequency stabilization and cavity length locking [9, 10]. Specifically, the EOMs are used in the Laser Interferometer Gravitational-Wave Observatory (LIGO) as well as in VIRGO where they play an important role. These observatories, which have recently achieved the first direct observation of gravitational waves emitted by black hole coalescence, are capable of detecting perturbations of the space-time on the order of $10^{-19}$ m [11–13].

To achieve these sensitivity levels it is necessary to accurately control the length of the two Fabry-Perot cavities, (each cavity 4 km in length) so that they are always in optical resonance. The length control system is done via a variation of the Pound-Drever-Hall technique to generate sidebands in the laser beams that go to the cavities. The sidebands are generated by EOM that produces a phase modulation in the laser beam [14]. To achieve phase modulation, the index of refraction of the crystal used in the EOM is modulated by a periodic, slowly varying external electric field. This external field is perpendicular to the direction of the laser wave and both are aligned with the principal axis of the crystal.

However, besides the required phase modulation, the experimental setup of the EOMs also produced an unwanted modulation in the amplitude of the transmitted wave. This residual amplitude modulation (RAM) could have pronounced effects on the optomechanical response of the interferometer, there is evidence that RAM affects the calibration of the Fabry-Perot cavities [15]. Thus, it is important to minimize RAM of advanced LIGO (aLIGO) when searching for much weaker sources of gravitational radiation like the coalescence of neutron stars.

RAM in the aLIGO setup was attributed to deficiencies in the phase modulation process. According to several authors, there are many sources that could contribute to RAM:

(i) Etalon effect caused by the multiple reflections on the crystal faces [16, 17].

(ii) Misalignment between the incident beam and the principal axis of the crystal [17].

(iii) Piezoelectric response of the crystal to the modulating frequency [18].

(iv) Deformation of the crystal with the local temperature [18].

(v) Non-uniformity of the modulating electric field [18].

(vi) Photorefractive effect on the crystal [19–21].

In order to avoid (i) and (ii), the crystal of the EOM is wedge-shaped so that the laser direction is no longer perpendicular to the opposite faces of the crystal. The aLIGO configuration uses a Rubidium Titanyl Phosphate (RTP) wedge-shaped crystal for each EOM [22].

Experimental studies performed in [23] analyzed the incidence of (iii) and (iv) for aLIGO laser power and showed that it is not relevant, given the current layout of aLIGO. In any case, since RAM is present even for low-intensity lasers, where it is expected that (iii) and (iv) reduce their incidence, it is clear that these would be second order effects.

Although RAM has been reduced to low levels (ranging from $10^{-5}$ to $10^{-6}$) it has not been possible to eliminate it from the setup.
It is important to note that RAM is always present in all technological designs which include EOM. In fact, experimental studies of RAM have been reported in a number of technological applications ([17], [24–28]).

In this work, we show that the same process that produces a phase modulation for the propagating electromagnetic wave in a time-dependent index of refraction medium will also give a modulating amplitude for the transmitted wave.

The theoretical model proposed in this work comes directly from Maxwell’s equations in a medium with a time-dependent index of refraction. Thus, it is not any of the proposed sources of RAM listed above. Directly from the field equations, one shows that RAM is an inherent feature of the physics of the problem and it is thus unavoidable. The mathematical expression of the transmitted electric field obeys all the parameteric dependence mentioned in [15] and should be regarded as a RAMF. It is worth mentioning that the present values of RAM for aLIGO are still two orders of magnitude above this predicted limit. Thus, there is still plenty of room for improvement until they reach the lower limit.

In Section (II) we provide a simple model to describe light propagation inside a crystal with an optical modulator. In Section (III) we obtain the transmitted wave for our model EOM and we present the principal result of this work, the expression for RAMF for our model of EOM. We compute the RAMF associated with the aLIGO setup. In section (IV) we analyze RAMF from the conservation of energy point of view and we present the microscopic model for the genesis of the RAMF effect. In section (V) we analyze the detectability and suppression of RAMF to different applications of EOMs. Finally, in section (VI) we summarize our results and analyze the physical validity of the approximations used to obtain this RAMF for the aLIGO laser and other different applications of EOM lasers.

II. LASER PROPAGATION IN A MEDIA WITH A TIME-DEPENDENT INDEX OF REFRACTION

The propagation of electromagnetic waves inside a crystal with a lossless and time-dependent media can be modeled by Maxwell’s equations,

\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \]  
\[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0, \]  
\[ \nabla \cdot \mathbf{B} = 0, \]  
\[ \nabla \cdot \mathbf{D} = 0, \]  

(1)  
(2)  

\[ \mathbf{B} = \mu_0 \mathbf{H}, \]

(3)

together with the constitutive relations:

\[ \mathbf{D} = \varepsilon(t) \mathbf{E}. \]  

(4)

The above relations show that the medium is non-magnetic and that the permittivity is time-dependent. Moreover, we assume that the electro-optical effect is linear.

We recall that the electro-optical effect is the change in the magnitude and direction of the refractive indices in the crystal due to the presence of an external electric field. If the field has a periodical time dependence (usually with a single frequency) we will denote it as a modulating electric field.

We thus write the dielectric permittivity as,

\[ \varepsilon(t) = \varepsilon_0 \, n^2(t), \]

where \( \varepsilon_0 \) is vacuum permittivity and \( n(t) \) is time-dependent index of refraction.

Remark: to observe an electro-optical effect the modulated electric field must vary slowly with time, i.e., its frequency must be much smaller than that of the laser beam propagating inside this media. Denoting by \( \Omega \) the frequency of the modulating field, by \( \omega_0 \) the frequency of the electromagnetic wave, we assume that,

\[ \frac{\Omega}{\omega_0} \ll 1. \]

Introducing the vector potential \( \mathbf{A} \) in the Coulomb gauge

\[ \nabla \cdot \mathbf{A} = 0, \]

we write the electric and magnetic fields of the laser wave as

\[ \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}, \]

\[ \mathbf{B} = \nabla \times \mathbf{A}, \]

where we have set the scalar potential \( \Phi = 0 \) since we are solving the source-free Maxwell equations. Outside the crystal, the scalar potential vanishes and since it satisfies an elliptic equation, by uniqueness, it vanishes everywhere.

Thus, the Ampere-Maxwell equation is the only non-trivial equation to be solved. Assuming that the incident wave is polarized in the "Z" direction and propagates in the "Y" direction we write,

\[ \mathbf{E} = E_z(y,t) \hat{e}_z, \]

\[ D_z(y,t) = \varepsilon(t) E_z(y,t). \]

It follows from the above conditions that the potential vector satisfies,

\[ \frac{\partial^2 A_z}{\partial y^2} - \mu_0 \frac{\partial}{\partial t} \left( \varepsilon \frac{\partial A_z}{\partial t} \right) = 0. \]  

(5)
To solve (5), we write the solution as the real part of,

$$ A_x = A_{0Z}(y, t) e^{i \phi(y, t)}. $$

This particular waveform will be useful when considering the case of interest to us, the so-called quasi-static solution where the phase $\phi(y, t)$ is a small deviation from the static case, and the amplitude $A_{0Z}(y, t)$ a slowly varying function of space and time. Replacing in (5), we obtain two differential equations, one for the real part:

$$ \frac{\partial^2 A_{0Z}}{\partial y^2} - \left( \frac{\partial \phi}{\partial y} \right)^2 A_{0Z} = \mu_0 \varepsilon \left[ \frac{1}{\varepsilon} \frac{\partial^2 A_{0Z}}{\partial t^2} + \frac{\partial^2 A_{0Z}}{\partial t^2} - \left( \frac{\partial \phi}{\partial t} \right)^2 A_{0Z} \right], $$

and the other for the imaginary part:

$$ \frac{\partial^2 \phi}{\partial y^2} A_{0Z} + 2 \frac{\partial \phi}{\partial y} \frac{\partial A_{0Z}}{\partial y} = \mu_0 \varepsilon \left( \frac{1}{\varepsilon} \frac{\partial \phi}{\partial t} A_{0Z} + \frac{\partial^2 \phi}{\partial t^2} A_{0Z} + 2 \frac{\partial \phi}{\partial t} \frac{\partial A_{0Z}}{\partial t} \right). $$

(6)

A. The WKB approximation

In order to solve for these variables, we first analyze the real part of the equation. If we assume that $A_{0Z}(y, t)$ varies slowly with position and time compared with $\phi(y, t)$ we can set

$$ \left( \frac{\partial \phi}{\partial y} \right)^2 A_{0Z} \gg \frac{\partial^2 A_{0Z}}{\partial y^2}, $$

$$ \left( \frac{\partial \phi}{\partial t} \right)^2 A_{0Z} \gg \frac{1}{\varepsilon} \frac{\partial^2 A_{0Z}}{\partial t^2} + \frac{\partial^2 A_{0Z}}{\partial t^2}. $$

In this way we obtain the eikonal equation for $\phi(y, t)$,

$$ \left( \frac{\partial \phi}{\partial y} \right)^2 - \frac{1}{v(t)^2} \left( \frac{\partial \phi}{\partial t} \right)^2 = 0, $$

(7)

where the following relation was used,

$$ \mu_0 \varepsilon(t) = \frac{1}{v(t)^2}. $$

B. The general solution and phase propagation

Eq. (7) can be solved by the characteristic method. We thus, rewrite this equation as

$$ \left( \frac{\partial \phi}{\partial t} \right) \pm v(t) \left( \frac{\partial \phi}{\partial y} \right) = 0. $$

The general solution can be written as

$$ \phi(y, t) = \phi(W), $$

with $\phi$ an arbitrary function of the argument

$$ W = W(y, t) = \pm y - \int_0^t v(\tau) d\tau. $$

(8)

Replacing this solution in the amplitude eq. (6) yields,

$$ \frac{\partial A_{0Z}}{\partial y} \pm v(t) \frac{\partial A_{0Z}}{\partial y} = \frac{1}{2} A_{0Z} \frac{\partial}{\partial y} \ln[v(t)]. $$

The solution can be obtained via the characteristic method giving

$$ A_{0Z}(y, t) = \sqrt{\frac{v(t)}{v_0}} A(W), $$

where $v_0$ is a constant with the dimension of velocity and $A(W)$ is an arbitrary function. For ease of notation, we have not distinguished between waves that propagate to increasing or decreasing values of $y$. This must be taken into account when solving for the propagation of the laser beam as light travels through different media.

Note also that both $\phi$ and $A$ depend on $W$. Thus one can write the solution as the real part of

$$ A_x = \sqrt{\frac{v(t)}{v_0}} A(W) e^{i \phi(W)}, $$

with $A(W)$ and $\phi(W)$ real functions and where the phase represents a traveling wave with a time-dependent velocity. It is also useful to rewrite the solution as

$$ A_x = \sqrt{\frac{v(t)}{v_0}} F(W) e^{ikW}, $$

(9)

with $F$ a real function and $k$ a constant with dimension of inverse of length (and can be taken as the wave-number). This last form of the solution is used to obtain the electric and magnetic fields in the different media.

III. MODEL OF EOM, THE TRANSMITTED FIELDS AND RAMF

The solution presented above can be used to obtain the propagation of light through media with different refractive indices. In particular, we are interested in describing an incoming laser in a media with constant index of refraction $n_0$ which then enters a region with a finite length and with a time-dependent index $n(t)$. After going through that region the laser then goes back to a region with constant index $n_0$. This step function model for the index of refraction describes the action of the EOM in a region of the crystal when the
border effects of the EOM are not taken into account. To obtain the electromagnetic field in the three regions one imposes matching conditions at the incoming boundary \(y = L_1\), at the outgoing boundary \(y = L_2\), and then solve for the amplitudes of the fields.

We first write the electric field and the magnetic field in the three different regions as,

\[
\mathbf{E}_j(y,t) = -\frac{\partial A_j}{\partial t} \hat{e}_z, \quad (10)
\]

\[
\mathbf{B}_j(y,t) = \frac{\partial A_j}{\partial y} \hat{e}_x, \quad (11)
\]

where the index \(j\) take the following values \(j = 1, 2, 3\) depending on the region of interest. In our case the index 1 identifies the incoming wave, 2 the region with a time-dependent index of refraction and 3 the outgoing region. Consequently we write the corresponding vector potential using (9) as,

\[
A_1(y,t) = F_1(W_+) e^{i k W_+} + F_1(W_-) e^{i k W_-},
\]

\[
A_2(y,t) = \sqrt{\frac{v(t)}{v_0}} [F_2(W_+) e^{i k W_+} + F_2(W_-) e^{i k W_-}],
\]

\[
A_3(y,t) = F_3(W_+) e^{i k W_+}.
\]

and imposing matching conditions for the electric and magnetic field at \(y = L_1\) and \(y = L_2\) to obtain the solution.

At this point we restrict ourselves to the discussion of laser light propagating through a crystal with an EOM whose dimensions are smaller than those of the crystal (see figure 2). For this case, the index of refraction of the crystal in the EOM region is given by:

\[
n(t) = n_0 \left[ 1 - \frac{\gamma}{2} \cos(\Omega t) \right], \quad (12)
\]

where \(n_0\) is the index of refraction of the crystal without the EOM and \(\gamma\) is a very small dimensionless parameter defined as,

\[
\gamma = n_0^2 r_{33} \frac{V_0}{d},
\]

where \(r_{33}\) is the electro-optic coefficient of the crystal, \(V_0\) is the amplitude of modulating voltage which generates electric field in the "Z" direction and \(d\) is the distance between the electrodes.

Thus, our EOM model to be solved is an incoming wave in a media with index \(n_0\) entering a region with index \(n(t)\) given by eq.(12) and then transmitted to a region with index \(n_0\) again.

Using the results presented in the Appendix A we obtain the transmitted vector potential. For clarity, we rewrite this expression in the following form

\[
A_3(y,t) = A_0 \exp[i \Phi(y,t)], \quad (13)
\]

where

\[
\Phi(y,t) = k y - \omega_0 t - m \cos \left\{ \frac{\Omega}{\nu_0} (t - t_0) \right\}, \quad (14)
\]

and \(m\) is the depth of the phase-modulation (modulation depth), defined as

\[
m = \frac{\omega_0 \gamma}{\Omega} \sin \left( \frac{\Omega L}{2 v_0} \right). \quad (15)
\]

In (14) \(t_0\) is a real constant given by

\[
t_0 = \frac{L - 2 L_2}{2 v_0}.
\]

Here \(L = L_2 - L_1\) represents the length of the electrode and \(v_0 = \frac{c}{n_0}\), where \(c\) is the speed of light in vacuum.

This formulation shows that the vector potential emerging from the EOM is simply a phase-modulated wave with modulation depth \(m\).

Since \(\left( \frac{\Omega L}{2 v_0} \right) \ll 1\), we can approximate the modulation depth (15) as

\[
m = \frac{\gamma \omega_0 L}{2 v_0} = \frac{\pi L}{\lambda_0} r_{33} \frac{V_0}{d}. \quad (16)
\]

This last expression agrees with the usual definition of modulation depth that appears in the standard textbooks [29].

We rename the electric and magnetic fields emerging from the modulator region as \(\mathbf{E}_{out}(t) = \mathbf{E}_3(L_2, t)\) and \(\mathbf{B}_{out} = \mathbf{B}_3(L_2, t)\), respectively.

Replacing the vector potential (13) in (10) we obtain

\[
\mathbf{E}_{out}(t) = i \mathbf{E}_0 \left[ 1 - \frac{m \Omega}{\omega_0} \sin (\Omega t) \right] \exp[i \Phi(L_2, t)] \hat{e}_x, \quad (17)
\]

with \(\mathbf{E}_0 = A_0 \omega_0\) and \(m\) is given by (16).

Similarly, using (11) the transmitted magnetic field results

\[
\mathbf{B}_{out}(t) = i \mathbf{B}_0 \left[ 1 - \frac{m \Omega}{\omega_0} \sin (\Omega t) \right] \exp[i \Phi(L_2, t)] \hat{e}_x. \quad (18)
\]

As one can see, the electric and magnetic fields emerging from the EOM have a RAM at frequency \(\Omega\). This is rather surprising since it is usually assumed that only the phase is affected by the EOM phase modulation. Indeed, expression (17) contradicts equations on EOM phase modulation from standard textbooks. (Please see page 333, eq. (12.19) of ref. [29] or also page 244, eq. (7.3-14) of ref. [30]). The textbook equation for the electric field emerging from an electro-optic modulator takes a form free from RAM. A direct consequence of this result is a modulation of the outgoing intensity of a laser beam after it goes through a region with an EOM.
Since the emerging beam of the crystal illuminates a photodiode, the resulting photocurrent \( i_c \) is given by
\[
i_c(t) \propto E_{\text{out}}(t) E_{\text{out}}^*(t), \tag{19}
\]
Replacing (17) in (19) we obtain
\[
i_c(t) \propto E_0^2 \left[ 1 - \frac{2m\Omega}{\omega_0} \sin (\Omega t) \right], \tag{20}
\]
where it is clear that there exists RAM in the intensity emerging of EOM and it is used to define \( \text{RAM}_F \).

The \( \text{RAM}_F \) is a periodic modulation of the transmitted wave intensity with frequency \( \Omega \) for an ideal EOM with other sources of RAM being omitted. In the Appendix C we compute \( \text{RAM}_F \):
\[
\text{RAM}_F = \frac{2m\Omega}{\omega_0}. \tag{21}
\]

Table I gives the \( \text{RAM}_F \) levels for the EOM of aLIGO.

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
\( m \) & \( L \) [mm] & \( f_m \) [MHz] & \( \text{RAM} \) & \( \text{RAM}_F \) & \( \eta = \frac{2\Omega}{\omega_0} \) \\
\hline
0.15 & 15 & 45.3 & 6.2 \times 10^{-6} & 4.8 \times 10^{-8} & 3.2 \times 10^{-7} \\
0.09 & 7 & 24.0 & 1.0 \times 10^{-6} & 1.5 \times 10^{-8} & 1.7 \times 10^{-7} \\
\hline
\end{tabular}
\caption{\( \text{RAM}_F \) levels calculated from (21) for the EOM of aLIGO.}
\end{table}

Table I, the fourth column shows the experimental values of RAM for aLIGO at Livingston LIGO Observatory (LLO) \(^1\) and the sixth column shows \( \eta = \frac{\text{RAM}_F}{m} \), the ratio between the \( \text{RAM}_F \) and the modulation depth \( m \). Using (21) gives \( \eta = \frac{2\Omega}{\omega_0} \).

IV. A MICROSCOPIC MODEL OF \( \text{RAM}_F \) AND CONSERVATION OF ENERGY

In this section, we present several results that are relevant to understand the nature of \( \text{RAM}_F \).

We first present (in subsection IV A) a model of the radiation emitted by the crystal cell when the distribution of charges inside the cell is simultaneously affected by a high-frequency plane wave and by the external modulating field. It is shown that there exists a RAM effect at the level of each cell, which we call microscopic RAM (\( \text{RAM}_\mu \)) and the relation between \( \text{RAM}_\mu \) and \( \text{RAM}_F \) is obtained. This provides a classical interpretation on the genesis of the \( \text{RAM}_F \) effect.

In subsection (IV B) we show that \( \text{RAM}_F \) can also be obtained from conservation of energy if one introduces a more general version of the Poynting theorem when the electrical permittivity of the media is time-dependent. We also show that the same physical principles that yield the propagation of light inside a crystal can be used to describe \( \text{RAM}_F \).

Finally, a direct relation between \( \text{RAM}_F \) and the changes that the modulating field produces on the charge distribution inside the crystal cell is obtained.

A. A microscopic model for \( \text{RAM}_F \)

We present here a classical model of the RAM produced by the interaction of electromagnetic waves and an EOM with the crystal structure.

This model should not be understood as a theoretical model of the electro-optical effect based on the details of the molecular structure of matter, which has already been analyzed in the literature \([31]\).

In contrast, our simple model focuses on the \( \text{RAM}_F \) effect, showing that it is an inherent consequence of the electro-optical effect itself. Indeed, assuming the change of the refractive index due to the electro-optical effect and using, the Lorenz-Lorentz relation (which is essentially a bridge between macroscopic and microscopic quantities) and the elemental dispersion model of Abraham-Lorentz, we calculate the corresponding microscopic change of the dipole moment of the crystal cell. Our model shows that the change of the dipole moment (which is a consequence of the changes that the external field produces within the cell) generates, in addition to the electro-optical effect, a RAM effect at the microscopic level of the cell. This type of RAM has been named \( \text{RAM}_\mu \) and we have analyzed its relationship with \( \text{RAM}_F \).

We first discuss the propagation of a plane wave inside a dielectric media. The electric field of the wave generates displacements of the electric charges from its equilibrium position. The wavelength of the incoming wave (\( \lambda_0 = 1064 \text{ nm} \)) is much larger than the typical crystal cell (for an RTP crystal the lattice parameters are: \( a = 12.96 \text{ Å}, b = 10.56 \text{ Å} y c = 6.49 \text{ Å} \)). Thus, one can use the dipole approximation to describe the motion of the charges in each cell interacting with the total electric field (usually referred to effective field), i.e., the superposition of the plane wave field with the mean field generated by the other cells.

If one neglects dissipative effect in the crystal, the dynamic of the centers of charge in the dipole approximation are obtained using the \( H. A. \) Lorentz dispersion model. Since the plane wave and modulating field are aligned with the “Z” axis, the equation for this model is given by,
\[
\frac{d^2 z}{dt^2} + \omega_r^2 z = \frac{eE'}{m_0}, \tag{22}
\]
where \( \omega_r \) is the natural angular frequency of the dipole, \( e \) and \( m \) are the charge and mass of the electron respectively, and \( E' \)

\(^1\) https://dcc.ligo.org/public/0108/E1300758/001/E1300758-v1.pdf

\(^2\) http://www.redoptronics.com/RTP-crystal.html
is the effective field in each cell. Here $z(t)$ is the deviation of the center of charge inside the cell.

The dipole motion in each cell is used to derive the following expression for the atomic polarizability $\alpha$ ([32]),

$$\alpha_0 = \frac{e^2}{m_0 (\omega_0^2 - \omega^2)}, \quad (23)$$

with $\omega_0 = \frac{2\pi c}{\lambda_0}$ the angular frequency of the plane wave.

The Lorentz-Lorentz relation [32] yields a relationship between the atomic polarizability and the macroscopic index of refraction of the media:

$$\alpha_0 = 3\varepsilon_0 \frac{n_0^2 - 1}{N \ n_0^2 + 2}, \quad (24)$$

with $N$ the number of charges per unit volume ($N = 8.87 \times 10^{27}$ m$^{-3}$ for RTP crystal) and $n_0$ the index of refraction of the media at the frequency $\omega_0 (n_0 = 1.82)$.

Eqs. (23) and (24) yield a relation between $n_0$ and $\omega_r$. For RTP crystal we obtain $\omega_r = 4.97 \times 10^{15}$ Hz. Given that $\omega_0 = 1.77 \times 10^{15}$ Hz the system is not near the resonance zone. Thus, we can neglect absorption effects in the crystal, as we assume at the beginning.

Macroscopically, the electro-optical effect is described by a time-dependent index of refraction in the crystal. At a macroscopic level, the effect can be attributed to a time-dependent resonant frequency $\omega_{\mu}(t)$. More specifically, the modulating electric field affects the charge distribution inside each cell so that its natural oscillation frequency can be written as,

$$\omega_{\mu}(t) = \omega_r \left[ 1 + \frac{\delta}{2} f(t) \right], \quad (25)$$

where $\delta \ll 1$ is a dimensionless parameter representing the strength of the perturbation of $\omega_r$ and $f(t)$ represents the action of the modulating field on the relative position of the atomic nuclei in the crystal cell. The modulating field introduces vibrations in the crystal array, inducing a relative displacement on the nuclei in each cell. This modifies the electric potential for the center of charge motion, which in turn modifies the polarizability. Then in this model, the change in the polarizability is due to the change in the resonant frequency of the system. (In Appendix D we derive the specific values of $\delta$ and $f(t)$ when an EOM is present.)

It is worth mentioning that one of the first classical models of EOM was proposed in reference [33]. The authors showed that the addition of an anharmonic term to the interaction potential in the Lorentz equation implies a change in resonance frequency similar to that of (25). However, it is important to note that this model is only valid for static modulation fields.

The equation of motion for $z(t)$, the departure of the center of charge from its equilibrium position, is given by the generalization of (22) with a time-dependent $\omega_{\mu}$ given by (25),

$$\frac{d^2 z}{dt^2} + \omega_{\mu}^2 z = \frac{eE'}{m_0} \quad (26)$$

where $E'$ is the effective field on the cell, given by:

$$E' = \left( \frac{n_0^2 + 2}{6} \right) n_0 \left[ 1 - \frac{2\delta \omega_r^2 (n_0^2 + 2)}{3 (\omega_r^2 - \omega_0^2)} \cos (\Omega t) \right].$$

Analogously to the macroscopic RAM, we can define a microscopic RAM for the radiated intensity of each cell. Using (C1)-(C5), where $\delta_r$ is replacing by $\Delta_{\mu}$, we obtain

$$\text{RAM}_{\mu} = \frac{2\delta \omega_r^2 (n_0^2 + 2)}{3 (\omega_r^2 - \omega_0^2)}. \quad (29)$$

Moreover, we can establish a relationship between (21) and (29). Using (16), (D16), (23) and (24), yields the following result

$$\text{RAM}_{\mu} = \frac{\pi L (n_0^2 - 1)}{\lambda_m n_0} \text{RAM}_F, \quad (30)$$

where $\lambda_m = \frac{2\pi c}{\Omega m}$ is the wavelength of the modulating field, and $L$ is the length of the crystal.

Note that it follows from Eq. (30) that the macroscopic RAM$_F$ is proportional to RAM$_\mu$ with a factor that depends on the macroscopic characteristics of the crystal.

We conclude that RAM$_F$ has a microscopic nature. The same process in the crystal yields both RAM$_F$ and phase modulation of the field.

In table (II) we show the relationship between RAM$_F$ and RAM$_\mu$ for a LIGO optical layout.

| $m$ | $\gamma$ | $\delta$ | RAM$_\mu$ | RAM$_F$ |
|-----|-----|-----|-----|-----|
| 0.15 | $1.86 \times 10^{-6}$ | $1.31 \times 10^{-6}$ | $5.33 \times 10^{-6}$ | $4.82 \times 10^{-8}$ |
| 0.09 | $2.39 \times 10^{-6}$ | $1.69 \times 10^{-6}$ | $6.85 \times 10^{-6}$ | $1.53 \times 10^{-8}$ |

B. RAM$_F$ and generalized Poynting theorem

It follows from the Faraday and Ampere-Maxwell laws (1) and (2) that we can write down the electromagnetic
energy flux associated with a volume $V$ and its boundary $\partial V$ as:

$$
\oint_{\partial V} \mathbf{S} \cdot \mathbf{n} \, da = - \int_{V} \left( \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \, dV,
$$

(31)

where $\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$ is the Poynting vector, and with $V$ the EOM volume. Using the relations (3) and (4) and taking the time average of (31) in the adiabatic approximation on a given period of the incoming electromagnetic wave, $T = \frac{2\pi}{\omega_0}$, a straightforward calculation leads to

$$
\oint_{\partial V} \langle \mathbf{S} \cdot \mathbf{n} \rangle \, da = - \int_{V} \left( \langle \frac{\partial u_{\text{em}}}{\partial t} \rangle + \left\langle \frac{1}{2} \frac{\partial \varepsilon}{\partial t} \mathbf{E}^2 \right\rangle \right) \, dV,
$$

(32)

where

$$
u_{\text{em}} \equiv \frac{\varepsilon(t)}{2} \mathbf{E}^2 + \frac{1}{2\mu_0} \mathbf{B}^2,
$$

is the (generalized) energy density of the electromagnetic field and the symbol $\langle \bullet \rangle$ labels the time average over one cycle of the carrier.

The first term on the r.h.s. of (32) represents the average power of the electromagnetic field, and the second term on the r.h.s. of (32) depends on the variation rate of the electrical permittivity.

At this point we can address several issues:

- Note that when $\varepsilon$ does not depend on the time we recover the usual expression for the conservation of energy.

- It would appear that this first term on the r.h.s. gives a bigger contribution to the Poynting flux than the second term. However, each term gives an identical contribution to the equation. This follows from the fact that the magnetic contribution and the static part of the electric contribution to the energy density yield a vanishing flux. Thus, only the time-dependent part of the energy density matters and gives a similar contribution to the second term of the equation.

- If we compute the flux of the Poynting vector for our model we obtain

$$
\oint_{\partial V} \langle \mathbf{S} \cdot \mathbf{n} \rangle \, da = - \frac{E_0^2}{2\mu_0 \nu_0} l_x l_z \text{RAM}_F \sin(\Omega t),
$$

(33)

where $l_x$ and $l_z$ are the crystal dimensions perpendicular to the electromagnetic wave.

Therefore, from the macroscopic point of view, $\text{RAM}_F$ is related to an external flux of energy ingoing to the crystal. Since modulating field constant ($\Omega = 0$) implies null $\text{RAM}_F$, then $\text{RAM}_F$ is a dynamic effect which is a consequence of the work by the variable modulating field on the media.

It is interesting to analyze the electromagnetic energy in the crystal from a microscopic point of view.

The interaction energy between the plane wave representing the laser beam and each dipole generated in the crystal cell is $-\mathbf{E} \cdot \mathbf{p}$, where $\mathbf{E} = E_0 \cos(k_0 y - \omega_0 t)$ $\mathbf{e}_x$ with $k_0 = \frac{\omega_0}{c}$ and $\mathbf{p} \left( t - \frac{y}{c} \right)$ is the dipole moment of each cell given by (D17). Therefore, the interaction energy density per unit volume ($u_{\text{int}}$) will be

$$
u_{\text{int}} = -N (\mathbf{E} \cdot \mathbf{p}).
$$

Note that the field of the plane wave propagates in a vacuum and their interaction with the dipoles represents the macroscopic continuous medium. The average total power of interaction in the dielectric is obtained as follows

$$
\int_{V} \left\langle \frac{\partial u_{\text{int}}}{\partial t} \right\rangle \, dV = -N \int_{V} \left( \left\langle \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{p} \right\rangle + \left\langle \mathbf{E} \cdot \frac{\partial \mathbf{p}}{\partial t} \right\rangle \right) \, dV.
$$

(34)

The first term in r.h.s. vanish due to the temporal average, then

$$
\int_{V} \left\langle \frac{\partial u_{\text{int}}}{\partial t} \right\rangle \, dV = -N \int_{V} \left\langle \mathbf{E} \cdot \frac{\partial \mathbf{p}}{\partial t} \right\rangle \, dV,
$$

(35)

Replacing (D17) in this last expression and using (D16) and $\varepsilon_0 \nu_0^2 = (\mu_0 \nu_0)^{-1}$ we obtain

$$
\int_{V} \left\langle \frac{\partial u_{\text{int}}}{\partial t} \right\rangle \, dV = - \frac{E_0^2}{2\mu_0 \nu_0} l_x l_z \gamma \frac{c}{\sqrt{\Omega}} \times \left\{ \cos \left[ \Omega \left( t - \frac{L}{c} \right) \right] - \cos(\Omega t) \right\}.
$$

(36)

Since $\left( \frac{\Omega L}{c} \right) \ll 1$ and using (16) and (21) then (36) yields

$$
\int_{V} \left\langle \frac{\partial u_{\text{int}}}{\partial t} \right\rangle \, dV = - \frac{E_0^2}{2\mu_0 \nu_0} l_x l_z \text{RAM}_F \sin(\Omega t).
$$

(37)

Remarkably, the result of the last calculation agrees with (33). Thus, the average power that goes through the crystal boundary is equivalent to the average power of interaction between the modulating field and the variable dipoles inside the medium.

The last expression shows that $\text{RAM}_F$ is a direct consequence of the interaction between the plane wave field and all the non-constant dipole moments of the medium. Specifically, the expression (35) shows that $\text{RAM}_F$ arises from the rate change of the dipole moments in the presence of the field of the plane wave. Let us emphasize that the oscillating dipole moment is generated by the plane wave and the magnitude of this dipole changes due to the modulating external field. This can be seen from (D17), since the plane wave produces an oscillating dipole moment at frequency $\omega_0$ and constant magnitude, while the modulating field generates a change in the magnitude of the dipole moment at frequency $\Omega$. 

A physical interpretation of the rate of change of \( p \) in equation (35) is the following: the plane wave disturbs the electronic cloud of the cell generating a dipole moment while the modulating field produces lattice deformations (Please see Pag. 264 Section 7.6 of ref. [30]). The displacements of nuclei affect the dynamics of the electrons which in turns ultimately perturbs the dipole moment of the cell.

In terms of our classical model, the electronic perturbations arising from the nuclear movements are represented by the change in the natural frequency of the electronic dynamic in the H. A. Lorentz equation. From a quantum point of view, we can also give a simple explanation. The interaction between the crystal and the laser beam is produced by the absorption of photons that increases the energy of the atoms of the cell. Since the external field displaces the nuclei inside the cell, this modifies the energy levels which in turns changes the absorption rate of the laser photons. This would explain the amplitude variations of the emerging beam, giving rise to the RAM\(_F\) effect.

Summarizing, the genesis of RAM\(_F\) is found in the lattice deformation induced by the external modulating field. However, a deep understanding of this relationship involves a complete quantum model of the interaction between the laser wave, the modulating field, and the crystal. There are some works about this topic [34–36].

V. ANALYSIS OF CANCELLATION AND DETECTABILITY OF RAM\(_F\)

Since the RAM\(_F\) effect is fully predictable, it is natural to think about its total cancellation as an affordable goal.

It is worth noting that the intensity of the emerging field of the EOM can be affected by two types of RAM: RAM\(_S\) (systematic RAM) and RAM\(_N\) (noise RAM).

RAM\(_S\) represents all those RAM sources that are predictable. This type of sources includes RAM\(_F\) and, for example, the constant misalignment angles of the EOM optical configuration.

RAM\(_F\) is an absolutely foreseeable component of RAM which we have named it as fundamental because it is an unavoidable consequence of the phase modulation process itself. In other words, this component of RAM is called fundamental because even in the ideal case where all the other possible sources of RAM are eliminated, RAM\(_F\) would always be present.

RAM\(_N\) instead is an unpredictable parasitic component of RAM arising from several random sources such as temperature fluctuations, mechanical vibrations, etalon effects due to defects of the crystal, photorefractive effect, etc. In the usual experimental situations, RAM\(_N\) is much greater than RAM\(_F\).

Since until now the existence of RAM\(_F\) was unknown, efforts have been addressed to remove only RAM\(_S\). Thus, depending on the specific field of experimental application, different active feedback control schemes were proposed to remove or reduce RAM\(_N\) ([17], [26], [27]). In [17], for spectroscopy applications, the authors claim to reduce RAM\(_N\) up to shot noise level.

In optical-cavity-based frequency ultra-stabilization, different closed-loop control schemes were proposed ([26], [27]). In [26], the active control implemented in wave-guide EOM managed to reduce the RAM\(_N\) to limit it to maximum values of \( 5 \times 10^{-6} \) and the Allan deviation associated with RAM\(_N\) fluctuation was limited to \( 10^{-6} \) for average time between \( 1 - 1000 \) s. In [27], the authors claim to have reached a minimum value of the Allan deviation of RAM of \( 2 \times 10^{-7} \) for an average time of 2s. It should be noted that RAM\(_F\) does not affect the value of Allan deviation of RAM since it represents the constant term of the sideband amplitudes.

It is important to keep in mind that the different attempts to eliminate RAM are based on a model where the intensity of the emerging EOM laser is only affected by a random parasitic misalignment between the polarization of the laser and the main axes of the crystal. This misalignment causes two beams with orthogonal polarization, ordinary (o) and extraordinary (e). Due to the birefringence of the crystal, both beams travel with different speeds, and the superposition of both beams in the polarizer placed at the output of the EOM then generates RAM. This model does not include RAM\(_F\) since its existence was unknown. Therefore, we can conjecture that RAM, i.e. RAM\(_N\) and RAM\(_S\), should be able to be removed, provided that the feedback control system is based on an electromagnetic propagation model that includes both RAM sources. Thus, having the theoretical formula from de emerging beam of intensity that includes RAM\(_N\) and RAM\(_S\) will be helpful to understand the minimum experimental values of RAM\(_N\) obtained in [26] and [27], and to determine the optimal sensing and control schemes that will allow cancellation of the RAM.

On the other hand, the situation is different for the wedge-shaped crystal EOM analyzed in the present work. In fact, the wedge shape of the crystal separates the directions of the rays o and e when they emerge from the crystal, see fig. (1).

![FIG. 1. Separation of the polarizations in a wedge crystal.](image_url)

This avoids contributions to RAM arising from both the main etalon effect and the misalignment. For this reason, expression (21) of this work is valid for the wedge-shaped crystal EOM, which does not correspond to the experimental situations analyzed in [17], [26] and [27].

Finally, we want to make a comment about the detectability of RAM\(_F\). For this purpose, the technical features of the photodiode and the electronic processing of the photodiode signal are relevant. We analyze RAM\(_F\) detection for aLIGO photodiodes, namely PD1811. From the datasheet of the PD1811
we obtain the Noise Equivalent Power (NEP), Integrated Noise (IN), Conversion Gain (CG) and Continuous Wave - Saturation Power (CW-SP):

| Parameter          | Range                |
|--------------------|----------------------|
| \( \lambda \)     | 900 nm - 1700 nm     |
| NEP\(_1\)          | 2.5 pW/\( \sqrt{Hz} \) DC \( \leq f \leq 10 \) MHz |
| NEP\(_2\)          | 22.5 pW/\( \sqrt{Hz} \) 10 MHz \( \leq f \leq 130 \) MHz |
| IN                  | 246 nW\(_{RMS}\)     DC \( \leq f \leq 130 \) MHz |
| CW-SP               | 120 \( \mu \)W        at \( \lambda = 950 \) nm |
| CG                  | 2.4 \( \times 10^4 \) V/W |

Here \( \lambda \) is the wavelength of the radiation on the photodiode and \( f \) is the frequency of the photodiode signal.

In general, the detectability of the radiation incident on a photodiode depends on the comparison between the incident radiation power with the total noise power of the photodiode. As an example, taking into account the value of CW-SP, we choose an incident power of 100 \( \mu \)W. So, from table (III), the minimum RAM\(_F\) power incident on the photodiode will be \( P_{\text{RAM}_F} = 1.53 \) pW (for \( f_m = 24.0 \) MHz), with its RMS-value of \( P_{\text{RAM}_F} = 1.08 \) pW\(_{RMS}\). This value of power is far below the noise power of photodiode, which can be estimated as \( P_{\text{noise}} = \text{IN} = 246 \) nW\(_{RMS}\). The SNR of output signal of photodiode is then \( \text{SNR} = P_{\text{RAM}_F}/P_{\text{noise}} = 4.4 \times 10^{-6} \). Even in scenarios of such low SNR, lock-in amplifiers achieve the detection of the signal. The dynamic reserve (DR) of the lock-in amplifier necessary in this case can be calculated as \(^5\)

\[
\text{DR(dB)} = 20 \log_{10} \left( \frac{P_{\text{noise}}}{P_{\text{RAM}_F}} \right) = 107.1 \text{ dB}.
\]

Actually, modern lock-in amplifiers reach dynamic reserve values up to 120 dB.

Alternatively, the noise power of the photodiode can be drastically reduced if the signal of the photodiode first goes through a bandpass filter before the lock-in amplifier. As an example, we choose a value of 10 KHz for 3 dB bandwidth of the bandpass-filter, which means a quality factor \( Q = f_m/BW = 24.0 \) MHz/10 KHz = 2400. Now, the new value of the noise power for this reduced bandwidth is \(^6\)

\[
P_{\text{noise}} = \text{NEP}_2 \frac{R_{\text{max}}}{R(\lambda_0)} \sqrt{\text{BW}},
\]

where \( R_{\text{max}} \) is the maximum responsivity of the photodetector, \( R(\lambda_0) \) is the responsivity of the photodetector at laser carrier wavelength \( (\lambda_0 = 1064 \) nm\) and BW is the bandwidth of bandpass filter.

\(^4\) https://www.newport.com/p/1811-FC
\(^5\) https://www.ameteksi.com/-/media/ameteksi/download_links/  

documents/7210/tn1001_speifying_lock-in-amplifiers.pdf
\(^6\) https://www.thorlabs.com/images/TabImages/Noise_Equivalent_Power_White_Paper.pdf

In addition, a back of the envelope calculation can estimate the number of photons associated with RAM\(_F\), for a given power of the incident beam on the photodiode. In fact, in the case of the previous example, for an incident power of 100 \( \mu \)W, the number of photons per sec will be \( N_{\text{carriers}} = 5.35 \times 10^{14} \) s\(^{-1}\). Then, for a \( P_{\text{RAM}_F} = 1.53 \) pW (for \( f_m = 24.0 \) MHz) the number of photons per sec. of frequencies \( f_0 + f_m \) and \( f_0 - f_m \) (in equal quantity) corresponding to \( N_{\text{RAM}_F} = 8.21 \times 10^9 \) s\(^{-1}\). This calculation suggests that, despite the smallness of \( P_{\text{RAM}_F} \), the photodiode can detect RAM\(_F\), given the high number of photons associated with the RAM\(_F\) effect.

VI. CONCLUSIONS

In this work, we have established a lower limit for the RAM\(_F\) effect when using electro-optic modulators. We showed that when some region inside a crystal has a time-dependent index of refraction produced by EOM, the phase and amplitudes of the outgoing electromagnetic fields are modulated with the frequency of the EOM. This limit was obtained using an approximation to Maxwell’s equations and a particular model for the propagation media. We assume that the crystal permittivity is time-dependent inside the modulation zone (between the pair of electrodes) and is constant outside of it, so it is spatially piecewise homogeneous \( \epsilon(t) \). Furthermore, we assume that \( \epsilon(t) \) is a slowly varying function of time with its angular frequency \( \Omega \) much smaller than the corresponding frequency \( \omega_0 \) of the laser wave propagating in the crystal \((\frac{\omega}{\omega_0} \approx 1 \times 10^{-7})\). In this situation, we can apply the so-called WKB approximation and derive effective equations of motion for the propagating wave. The plane wavefront assumption simplifies the analysis but does not change the general result since the WKB equations are linear in the electric and magnetic fields and we are simply taking a Fourier decomposition.

We have not considered border effects on the EOM either, i.e., the electric field is assumed to be homogeneous even at both ends of the modulation electrodes and null outside of them. So the interfaces in which the laser enters and leaves the modulation zone to become discontinuity planes of the refractive index. Furthermore, we have restricted ourselves to consider wedge-shaped crystal EOM whose oblique faces lie outside of the modulation zone (see fig. 3). This does not represent a restrictive condition since it is usual in experimental layouts of EOM. For those reasons, the oblique interfaces separate two constant refractive index media (air/crystal without modulation field or vice versa). Consequently, when the laser goes through these interfaces no contribution to RAM\(_F\) can be produced neither on the reflected nor on the transmitted components of the laser beam. This means that RAM\(_F\) is strictly generated by modulation zone.
We could have taken border effect into account by substituting the step function model for \( n \) with a smooth function that takes into account the border effects of the EOM. We claim the effect is also present when a more general configuration is adopted since, as we elaborated in the previous section, it is a physical, global effect arising from the interaction of the dipole moments of the crystal with modulating field of EOM.

On the other hand, it would appear that we have neglected the etalon effect without justification. However, this is not so. As it is well known, this effect is generated by the contribution of the successive internal reflections to the emerging laser. Two types of interfaces generate internal reflections: the oblique ones (air/crystal without modulating field or vice versa) and the vertical ones (crystal without modulating field/crystal with the modulating field, or vice versa). The reflections at the oblique faces do not contribute to the outgoing beams because they are deliberately deflect away from the path of the main beam. The reflections at the two vertical interfaces are crucial for the calculation of the emerging fields of the crystal. However, it is important to note the jump of the refractive index through both surfaces is of the first order in \( \gamma \), with \( \gamma \ll 1 \) (see(12)). So from the second reflection onwards, the amplitudes of the reflective fields are \( \gamma^l \)-order with \( l \geq 2 \), and they are out of our approximation order.

Based on the above considerations, we claim the results presented in this work set a \( \text{RAM}_F \) to the laser beam. In other words, it is impossible to obtain a pure phase modulation without an associated \( \text{RAM}_F \) effect. The order of magnitude of \( \text{RAM}_F \) obtained ranges from \( 10^{-7} \) to \( 10^{-8} \), depending on the modulation frequency \( \Omega \), carrier laser frequency \( \omega_0 \) and the phase modulation depth \( m \), as it is shown in table I. Since one of the main goals of this work is to understand the \( \text{RAM} \) effect on the laser beam of aLIGO, it is instructive to compare our results to the experimental values of \( \text{RAM} \) in aLIGO, which range from \( 10^{-5} \) to \( 10^{-6} \). Although this work shows that it is impossible to eliminate the \( \text{RAM} \) effect in the beam intensity, a meticulous search of other sources may help to reduce the present level of \( \text{RAM} \). It is worth mentioning that the \( \text{RAM} \) level predicted in this work is also present in more general configurations. As we said before, a wedge-shaped crystal avoids the etalon effect so the transmitted laser beam in the crystal is not subject to multiple reflections on the wedge boundary. As a consequence of this shape of the crystal, it is clear to see that in all experimental EOM layouts, every single EOM can be modeled as we did in section III. For these reasons \( \text{RAM}_F \) obtained in this work will be presented in all experimental configurations of EOM. As an example, the aLIGO EOM layout is a wedge-shaped RTP crystal over which three consecutively coupled of an electrode are placed. Each pair of electrodes is fed with AC voltage needed to generate three modulating fields of different frequencies. Therefore, the emerging laser beam of this series of modulators undergoes three-phase modulation processes, in three different frequencies. The value of each frequency is chosen to control the length of three different cavities. Our result for this case implies that, regardless of the particular position inside the series, each modulator generates a \( \text{RAM}_F \) in laser, at different frequency, which can be calculated using (21). It is also important to note that our result implies \( \text{RAM}_F \) will be present in all experimental applications that use EOMs, for example, frequency-modulation spectroscopy and optical-cavity-based frequency ultra-stabilization. Moreover, we conjecture that \( \text{RAM}_F \) will exist in any phase modulation process where an external agent (the modulating electric field for the crystal-based EOM) produces changes in the refractive index of the medium where the laser travels. As an example, we can mention the acoustic optical modulators and the waveguide based EOM. Indeed, \( \text{RAM}_F \) will be a consequence of the work done by the external agent on the medium of laser propagation, in the presence of the electromagnetic fields of the laser beam.

We want to emphasize that, although \( \text{RAM}_F \) will always be present in phase modulation processes with EOMs, in general, the value of \( \text{RAM}_F \) depends on the specific optical layout of the EOM.

Finally, we have analyzed the detection and suppression of \( \text{RAM}_F \). In spite of the small magnitude of \( \text{RAM}_F \), the appropriate choice of the bandpass filter and the lock-in amplifier allow detecting \( \text{RAM}_F \) in the photodiode signal. In addition, we conjecture that all types of \( \text{RAM} \), including \( \text{RAM}_F \), should be able to be removed, provided that the feedback control system is based on an electromagnetic propagation model that includes both \( \text{RAM} \) sources.

Acknowledgements: This research has been supported by the Faculty of Engineering of Instituto Universitario Aeronáutico and by CONICET. The authors wish to thank the LIGO Scientific Collaboration for sharing relevant information concerning \( \text{RAM} \) in LIGO.

Appendix A: The solutions to the WKB equations in a crystal of the EOM

Following the steps outlined in section III, we write down the solutions to the WKB equations in the three regions of interest. There are in principle four unknown functions of \( W \) that should be fixed by the matching conditions, i.e., they provide 4 equations for 4 unknowns functions of time. Since the idea is to solve them for a quasi-static configuration, it is worth noting that it should be similar to the static situation, \( v = \text{const} \). We thus obtain first the solution for this familiar case and then generalize to the quasi-static model.

1. The static situation

The three media have refractive indices given by \( n_1 = n_0, n_2 = n_0(1 - \frac{\gamma}{2}), n_3 = n_0 \), with interfaces at \( y = L_1 \) and \( y = L_2 \). In this case we have the known result:

\[
\begin{align*}
A_1(y,t) &= \left[ A_{1+} e^{i(k_1 y - \omega_0 t)} + A_{1-} e^{-i(k_1 y + \omega_0 t)} \right] \hat{e}_z, \\
A_2(y,t) &= \left[ A_{2+} e^{i(k_2 y - \omega_0 t)} + A_{2-} e^{-i(k_2 y + \omega_0 t)} \right] \hat{e}_z, \\
A_3(y,t) &= A_{3+} e^{i(k_3 y - \omega_0 t)} \hat{e}_z,
\end{align*}
\]
where $A_{j=1,2,3}$ are constants that are fixed from the matching conditions. The wavenumber is given by $k_j = \frac{n_j \omega_0}{v_j}$, and $j = 1, 2, 3$ labels the three refractive indices.

After solving the matching conditions one obtains,

\[
A_1(y, t) = A_0 \exp[i (ky - \omega_0 t)] \hat{e}_x + \frac{\gamma}{4} A_0 \exp[-i (ky + \omega_0 t - 2kL_1)] \hat{e}_x
\]

\[- \frac{\gamma}{4} A_0 \exp\{-i [ky + \omega_0 t - 2kL_2 + k\gamma L]\} \hat{e}_x,
\]

(A4)

\[
A_2(y, t) = A_0 \left(1 + \frac{\gamma}{4}\right) \exp\left\{i \left[ky - \omega_0 t - \frac{k\gamma}{2} (y - L_1)\right]\right\} \hat{e}_x
\]

\[- \frac{\gamma}{4} A_0 \exp\{-i [ky + \omega_0 t - \frac{k\gamma}{2} (y + L_1) + kL_2 (\gamma - 2)]\} \hat{e}_x,
\]

(A5)

\[
A_3(y, t) = A_0 \exp\left\{i \left[ky - \omega_0 t - \frac{k\gamma L}{2}\right]\right\} \hat{e}_x,
\]

where $k = \frac{\omega}{v_0}$ and $L = L_2 - L_1$.

2. The non-static case

Solving the matching condition for the non-static case yields the following results:

\[
F_1(W_+) e^{ikW_+} = A_0 \exp[i (ky - \omega_0 t)],
\]

(A7)

\[
F_1(W_-) e^{ikW_-} = \frac{\gamma}{4} A_0 \cos \left(\frac{\Omega}{2} \frac{y - L_1}{v_0}\right)
\]

\times \exp\{-i [ky + \omega_0 t - 2kL_1]\}

\[- \frac{\gamma}{4} A_0 \cos \left(\frac{\Omega}{2} \frac{y - L_2}{v_0}\right)
\]

\times \exp\{-i [ky + \omega_0 t - 2kL_2 + k\gamma L]\},

(A8)

\[
\sqrt{\frac{v(t)}{v_0}} F_2(W_+) e^{ikW_+} = A_0 \left(1 + \frac{\gamma}{4} \cos(\Omega t)\right)
\]

\times \exp\{i \left[ky - \omega_0 t - \frac{\omega\gamma}{\Omega} \sin \left(\frac{\Omega (y - L_1)}{2v_0}\right)\right]

\times \cos \left(\Omega \left(t - \frac{y - L_1}{2v_0}\right)\right)\},

(A9)

As one can check, the solutions $A_j(y, t)$, satisfy the matching conditions and converge to the static case when the limit $\Omega \rightarrow 0$ is taken.

Appendix B: An alternative method: following the wave

In this Appendix we reobtain the solution presented in Appendix A corresponding to the three media with indices of refraction: $n_1 = n_0$, $n_2(t) = n_0 \left[1 - \frac{2}{\gamma} \cos(\Omega t)\right]$ and $n_3 = n_0$ using a different ansatz. This method is based on ray tracing and considers the successive transmissions and reflections of the original incoming wave as it goes through the two interfaces between the three media.

As we have seen in the previous section, the WKB approximation can be used to obtain an explicit form of the vector potential $A_z(W) = A_{0z}(W) e^{i\phi(W)}$, with $W$ given in eq. (8). Assuming the phase $\phi(W)$ to be an increasing function of $W$, we define

\[ k(y, t) \equiv \left(\frac{\partial \phi}{\partial y}\right)_t, \quad \text{and} \quad \omega(y, t) \equiv -\left(\frac{\partial \phi}{\partial t}\right)_y, \]

(B1)

(B2)

as the generalized wavenumber and frequency respectively. Inserting the definitions (B1) and (B2) in (7) yields

\[ k(y, t) = \pm \frac{\omega(y, t)}{v(t)}. \]

In the above equation $\pm$ means a wave propagating to increasing or decreasing values of $y$.

In principle, imposing the matching conditions for $E$ and $B$ at $y = L_1$ and $y = L_2$ yields an infinite set of waves going back and forth which contribute to the final form of the fields.
in the three regions. However, as one follows the original incoming ray and assumes $\gamma \ll 1$, it is clear that the second and third reflection inside the media $n_2(t)$ will be proportional to the second and third power of $\gamma$ respectively. We thus assume that only the rays that contribute to linear order in $\gamma$ are relevant for the calculation (see figure 2 and figure 3).

Motivated by the above example we introduce the following ansatz: "the transmitted and reflected waves at a given interface, that are generated by the same incident wave, possess the same angular frequency as the latter". The condition imposed on the angular frequencies at a given interface yields a matching condition for the phase of the incoming and outgoing fields. It follows from (B2), that these conditions fix the phase $\phi$ up to a constant.

When $E_1$ reaches $y = L_1$ a transmitted $E_t$ and reflected $E_r$ fields are produced. The matching conditions for the angular frequencies are:

$$\omega_i(L_1, t) = \omega_t(L_1, t) = \omega_r(L_1, t) = \omega_0,$$

with the corresponding conditions for the amplitude of the fields:

$$E_i(L_1, t) + E_r(L_1, t) - E_t(L_1, t) = 0,$$

$$B_i(L_1, t) + B_r(L_1, t) - B_t(L_1, t) = 0.$$

The above equations are sufficient to determine $A_r$ and $A_t$ from $A_i$.

A similar set of equations are obtained for $E_t$, the transmitted $E_{tt}$ and reflected $E_{tr}$ fields at $y = L_2$. The matching conditions for the angular frequencies at this interface are:

$$\omega_i(L_2, t) = \omega_{tt}(L_2, t) = \omega_{tr}(L_2, t),$$

with the corresponding conditions for the fields:

$$E_t(L_2, t) + E_{tr}(L_2, t) - E_{tt}(L_2, t) = 0,$$

$$B_t(L_2, t) + B_{tr}(L_2, t) - B_{tt}(L_2, t) = 0.$$

Taking $A_t$ as the incident data, the above equations yield $A_{tt}$ and $A_{tr}$.

Finally, when the wave $E_{tr}$ reaches $y = L_1$ a transmitted field $E_{tt}$ that goes back to the initial media $n_1$ is produced. The reflected wave is order $\gamma^2$ and thus, it is discarded.

The resulting matching condition for the angular frequency is given by:

$$\omega_{tr}(L_1, t) = \omega_{ttt}(L_1, t),$$

whereas the matching condition for the field is:

$$E_{tt}(L_1, t) - E_{tr}(L_1, t) = 0,$$

and the last vector potential $A_{tt}$ is obtained.

The final set of vector potentials that satisfy the angular frequency and amplitude matching conditions is given by:

$$A_i(y, t) = A_0 \exp[i(ky - \omega_0 t)] \hat{e}_x,$$

$$A_t(y, t) = A_0 \left[ 1 + \frac{\gamma}{4} \cos(\Omega t) \right]$$
$$\times \exp \left[ i \left( ky - \omega_0 t - \frac{\omega_0 \gamma}{\Omega} \sin \left[ \frac{\Omega (y - L_1)}{2v_0} \right] \right) \right] \hat{e}_x.$$

These propagating fields are: $A_i$ the incident vector potential in media $n_1$, $A_r$ the reflected potential at $L_1$ propagating in media $n_1$, $A_t$ the transmitted vector potential to the media $n_2$, $A_{tr}$ the reflected potential at $L_2$ propagating in media $n_2$, $A_{tt}$ the transmitted wave to the media $n_1$ after it was reflected at $L_2$. Finally, $A_{tt}$ is the transmitted potential propagating in $n_3$. Note that the reflected wave in $n_3$ is formed by the addition of two rays that are linear in $\gamma$: $A_r$ and $A_{rt}$.

In the static case, we assume the angular frequency is the same for all the waves propagating in the three media. However, in the quasi-static case, the angular frequencies differ from $\omega_0$. After the calculation is done we must check that in the static limit, $(\Omega \to 0)$, all the angular frequencies converge to $\omega_0$.

As an example, we consider the different fields that are traveling back in media $n_1$. $A_r$ has a constant angular frequency since it does not enter the time-dependent region, whereas $A_{tr}$ has an angular frequency $\omega_{tr}(y, t)$ since it contains information of the transit time in the modulated media.
Finally, similarly, provide a relation between the amplitude of the electric and magnetic planes waves. Indeed, Fresnel’s equations (B3)-(B8) cannot be obtained using the static solutions (A4)-(A6) when the limit $\Omega \to 0$ is taken. Likewise, $A_4 + A_r + A_{tr}$ coincides exactly with $A_1$ in (A7) and (A8), as expected. Similarly, $A_s + A_{tr}$ coincides with $A_2$ in (A9) and (A10). Finally, $A_{tt}$ is equal to $A_3$ given in (A11).

It is important to note that (B3)-(B8) cannot be obtained using the usual Fresnel’s equations. Indeed, Fresnel’s equations provide a relation between the amplitude of the electric and magnetic fields at both interfaces $(y = L_1$ and $y = L_2$ in our cases).

In the static case, these let resolve the problem because the amplitudes of the fields are constants and their phases are determined for wavenumber of the medium and the angular frequency of the incident wave, $\omega_0$ (see (A1)-(A3) and (10),(11)).

However, in the dynamic case, the amplitudes and phases of the fields satisfy a coupled partial differential equations system and the matching conditions give relations between the boundaries values of the amplitudes in both interfaces. Indeed, in this case, both the wavenumbers and the angular frequencies of the fields depend on the spatial coordinate and the time.

### Appendix C: Computing the RAM$_F$

In this appendix we compute the RAM$_F$. In general, given a signal emerging of the EOM, we can define RAM as

$$\text{RAM} = \frac{\text{AC}}{\text{DC}}, \quad \text{(C1)}$$

where AC is the alternating component and DC the continuous component of the photocurrent generated by the emerging beam from EOM.

DC is the first coefficient in the Fourier expansion,

$$\text{DC} = \frac{\Omega}{2\pi} \int_{-\frac{\pi}{\Omega}}^{\frac{\pi}{\Omega}} i_c (\tau) \, d\tau, \quad \text{(C2)}$$

where $i_c$ represents the photocurrent. The alternating component AC is the magnitude of the modulation components $I$ and $Q$ of $i_c$, given by

$$AC = \sqrt{I^2 + Q^2}, \quad \text{(C3)}$$

where

$$I = \frac{\Omega}{\pi} \int_{-\frac{\pi}{\Omega}}^{\frac{\pi}{\Omega}} i_c (\tau) \cos (\Omega \tau) \, d\tau, \quad \text{(C4)}$$

$$Q = \frac{\Omega}{\pi} \int_{-\frac{\pi}{\Omega}}^{\frac{\pi}{\Omega}} i_c (\tau) \sin (\Omega \tau) \, d\tau. \quad \text{(C5)}$$

Now we compute the RAM for an ideal EOM that is to say RAM$_F$. Replacing (20) in (C2)-(C5) we obtain the DC and AC components. Finally, from (C1) we obtain:

$$\text{RAM}_F = \frac{2m\Omega}{\omega_0}. \quad \text{(C6)}$$

### Appendix D: Computing the total intensity radiated by the crystalline cell when an EOM is present

In this appendix, we derive the total intensity radiated away by a crystal cell arising from a redistribution of the internal charges when an EOM is present.

We assume the nuclear charges in each cell of the crystal remain fixed whereas the electronic cloud can move around the equilibrium position. We also assume the total dipole moment vanishes at the equilibrium position. If an electromagnetic plane wave with angular frequency $\omega_0$ travels inside the crystal, each cell reacts to an effective electric field given by the superposition of the wave field and the total field of the remaining cells in the crystal. This effective field is given by:

$$E' = E + \frac{P}{3\varepsilon_0}, \quad \text{(D1)}$$
where \( \mathbf{E}' \) is the effective field, \( \mathbf{E} \) is the electromagnetic wave and \( \mathbf{P} \) is the polarization produced by the remaining cells. Note that the effective and the wave field have the same frequency since each cell in the crystal is forced to oscillate at the frequency of the incident wave field.

The polarization vector \( \mathbf{P} \) in (D1) is given by:

\[
\mathbf{P} = \varepsilon_0 \chi(t) \mathbf{E}, \tag{D2}
\]

where \( \chi(t) \) is the crystal susceptibility when the electro-optic effect is present, i.e.,

\[
\chi(t) = (n_0^2 - 1) - n_0^2 \gamma \cos(\Omega t). \tag{D3}
\]

Using (D1), (D2) and (D3) we compute \( \mathbf{E}' = \mathbf{E}' \hat{e}_x \), giving,

\[
\mathbf{E}' = \frac{E_0 (n_0^2 + 2)}{6} \left[ 1 - \frac{n_0^2 \gamma}{n_0^2 + 2} \cos(\Omega t) \right] e^{-i\omega_0 t} + c.c. \tag{D4}
\]

This effective field will move the center of charge in each cell away from its equilibrium position. The motion of each center of charge will then produce dipole radiation that we wish to compute.

To do that we first analyze the motion of the center of charge position \( z(t) \) in each cell. The simplest model to describe a dispersive media is the Abraham-Lorentz [37] model. In this model \( z(t) \) satisfies,

\[
\frac{d^2 z}{dt^2} + \sigma \frac{dz}{dt} + \omega_p^2 z = \frac{eE'}{m_0}, \tag{D5}
\]

where \( \omega_p = \omega_r [1 + \frac{\delta}{N} \cos(\Omega t)] \). The time dependence of \( \omega_p \) follows from the action of the modulating field on the cell, as was shown in section IV A. In the above equation \( \sigma \) represents a dissipative term, and it is only relevant when the propagating wave frequency (\( \omega_0 \)) is close to the resonant frequency (\( \omega_r \)). Since in our case \( \omega_0 = 1.77 \times 10^{15} \) Hz and \( \omega_r = 4.97 \times 10^{15} \) Hz, we can neglect dissipative terms. Thus, \( z(t) \) satisfies the H. A. Lorentz equation,

\[
\frac{d^2 z}{dt^2} + \omega_p^2 z = \frac{eE'}{m_0}, \tag{D6}
\]

We propose the following solution for the forced oscillation,

\[
z(t) = \frac{1}{2} Z(t) e^{-i\omega_0 t} + c.c. \tag{D7}
\]

where \( Z(t) \) is a real amplitude which is assumed to vary much slower than the wave frequency. \( Z(t) \) can be written up to linear order as \( Z = Z_0 + \delta Z_1(t) \), where \( Z_0 \) is the constant solution in absence of a modulating field and \( Z_1(t) \) is a slowly varying function of time. Thus, in this approximation we neglect first and second derivatives of \( Z \) since \( \dot{Z} \ll Z \omega_0^2 \). Then, the first order deviation from the static case is remarkably simple, i.e.,

\[
(\omega_p^2 - \omega_0^2) z = \frac{eE'}{m_0}, \tag{D8}
\]

form which we obtain an algebraic relationship between \( z(t) \) and \( \mathbf{E}' \), namely,

\[
z = \frac{e}{m_0 (\omega_p^2 - \omega_0^2)} \mathbf{E}' . \tag{D9}
\]

This expression is consistent with the definition of the polarizability in terms of the microscopic parameters [32],

\[
\mathbf{p} = \alpha \mathbf{E}', \tag{D10}
\]

where

\[
\mathbf{p} = ez \hat{e}_x, \tag{D11}
\]

represents the cell dipole moment. Combining the above expressions we immediately obtain

\[
\alpha = \frac{e^2}{m_0 (\omega_p^2 - \omega_0^2)}, \tag{D12}
\]

Expanding the above expression up to linear terms in \( \delta \) yields

\[
\alpha(t) = \frac{e^2}{m_0 (\omega_p^2 - \omega_0^2)} - \delta \frac{e^2 \omega_r^2}{m_0 (\omega_r^2 - \omega_0^2)^2} f(t) . \tag{D13}
\]

On the other hand, for the static case, the polarizability can also be expressed in terms of macroscopic parameters (24),

\[
\alpha_0 = \frac{3\varepsilon_0 n_0^2 - 1}{N\left(n_0^2 + 2\right)}, \tag{D14}
\]

where \( n_0 \) is the constant index of refraction for the RTP crystal at the angular frequency \( \omega_0 \). If in the above formula we replace \( n_0 \rightarrow n(t) = n_0 \left[1 - \frac{\delta}{N} \cos(\Omega t)\right] \) and \( \alpha_0 \rightarrow \alpha(t) \), we obtain a generalized Lorentz-Lorentz (24) relation when an electro-optic effect is present. Up to linear terms in \( \gamma \) we have,

\[
\alpha(t) = \frac{3\varepsilon_0 (n_0^2 - 1)}{N(n_0^2 + 2)} - \gamma \frac{9\varepsilon_0 n_0^2}{N(n_0^2 + 2)^2} \cos(\Omega t) . \tag{D15}
\]

Combining equations (D13) and (D14) yields

\[
f(t) = \cos(\Omega t) \tag{D16}
\]

\[
\delta = \frac{9\varepsilon_0 m_0 (\omega_p^2 - \omega_0^2)^2}{Ne^2 \omega_r^4} \left(\frac{n_0^2 \gamma}{n_0^2 + 2}\right)^2 . \tag{D16}
\]
Using $\omega_c = 4.97 \times 10^{15}$ Hz, $\omega_0 = 1.77 \times 10^{15}$ Hz, $N = 8.87 \times 10^{27}$ m$^{-3}$ and $n_0 = 1.82$, we obtain $\delta = 0.71 \gamma$. This shows that $\delta$ and $\gamma$ have the same order of magnitude.

Computing $p$ from (D10) or (D11) yields,

$$p = e^2 E_0 \left( \frac{n_0^2 + 2}{3m_0} \right) \frac{\delta}{c} \frac{\omega_0^2}{3 (\omega_0^2 - \omega_{0b}^2)} \cos(\omega_0 t) \hat{e}_z. \tag{D17}$$

The electric and magnetic fields generated by this dipole in the radiation zone ($\frac{\omega_0^2}{c} \gg 1$) are given by [32],

$$E_\mu = \frac{\mu_0}{4\pi} \frac{[\hat{p}] \sin(\theta)}{r} \hat{e}_\theta, \tag{D18}$$

$$B_\mu = \frac{\mu_0}{4\pi c} \frac{[\hat{p}] \sin(\theta)}{r} \hat{e}_\phi, \tag{D19}$$

where $p = \sqrt{\hat{p} \cdot \hat{p}}$, and the subindex $\mu$ labels the microscopic origin of the fields. $\hat{e}_\theta$ and $\hat{e}_\phi$ are respectively the polar and azimuthal unit vectors in a spherical coordinate system, with the dipole $p$ aligned with the "Z" axis.

The corresponding Poynting vector is given by,

$$S_\mu = \frac{E_\mu \times B_\mu}{\mu_0} = \frac{\mu_0}{16\pi^2 c} \frac{[\hat{p}]^2 \sin^2(\theta)}{r^2} \hat{e}_r, \tag{D20}$$

where $\hat{e}_r$ is the radial unit vector of the coordinate system.

The rate of energy radiated by the dipole is obtained by integration of the Poynting vector on a large sphere centered at the dipole. The result is given by

$$\frac{dU}{dt} = \oint S_\mu \cdot \hat{n} \, da = \frac{\mu_0}{6\pi c} [\hat{p}]^2, \tag{D21}$$

where $U$ is the total energy radiated by the dipole.

The intensity emitted by the dipole is

$$I_\mu = \frac{dU}{dt} = \frac{\omega_0}{2\pi} \int_t^{t+\frac{2\pi}{\omega_0}} \frac{dU}{dt} \, dt, \tag{D22}$$

where the symbol $\langle \bullet \rangle$ indicates a time average over a cycle on the incident wave.

Replacing (D17) (D21) on (D22) yields,

$$I_\mu = \frac{\mu_0}{6\pi c} \left[ e^2 E_0 \left( \frac{n_0^2 + 2}{3m_0} \right) \frac{\omega_0^2}{3 (\omega_0^2 - \omega_{0b}^2)} \right] \times \left\{ \frac{1}{2} - \frac{\delta}{3} \frac{\omega_0^2}{(\omega_0^2 - \omega_{0b}^2)} \cos \left( \frac{\omega_0}{c} \left( t - \frac{R}{c} \right) \right) \right\}, \tag{D23}$$

where $R$ is the distance between observation point and dipole. The contribution of a single dipole inside the crystal to the intensity of signal emerging from the EOM is evaluated taking $R \leq L$ in (D23). Since $\left( \frac{\omega_0}{c} \right) \ll 1$, we neglect the correction by the retarded time in (D23):

$$I_\mu = \frac{\mu_0 \alpha_0^2 (n_0^2 + 2)^2}{108 \pi c} \frac{E_0^4 \omega_0^4}{\omega_{0b}^4} \left[ 1 - \frac{2\delta \omega_0^2 (n_0^2 + 2)}{3 (\omega_0^2 - \omega_{0b}^2)} \cos \left( \frac{\omega_0}{c} \left( t - \frac{R}{c} \right) \right) \right]. \tag{D24}$$

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