Numerical studies of the internal waves interaction with the Black Sea shelf topography at spring-summer stratification

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Abstract. This work investigates the regional specifics of the appearance and propagation of internal waves at the shelf of the Black Sea. The parameters of horizontal inhomogeneity associated with changes in the bottom relief and density of seawater that lead to instability and destruction of internal waves are studied. The numerical experiments for the spring and summer seasons on transects normal to the coast are conducted using data of temperature and salinity from the Bank of oceanographic data of the Marine Hydrophysical Institute of RAS. The article touches upon the actual theme of adaptation of satellite images obtained from open sources when solving oceanographic problems. The result of the processing of remote sensing data was information about the speeds and wavelengths for the observed phenomena. These values are used to recover the depth of the seasonal pycnocline within a two-layer approximation of the vertical density structure. For different bottom slopes and seasonal profiles of the buoyancy frequency, sites are determined where the transition from supercritical to critical or subcritical conditions is possible in the region of the Crimean coast.

1. Introduction
Internal waves determine the nature of many hydrophysical processes occurring in shelf areas of marine basins. In topographical boundary regions, changes in the sea depth lead to the generation of horizontally non-uniform waves, as well as to the appearance of boundary wave formations [1, 2]. The investigation of internal waves on the Black Sea shelf by contact methods has been carried out for a long time, and its results allowed exploring the processes underlying the generation of intense internal waves [3, 4].

In this paper, the study of the spatial-temporal parameters of internal waves and the physical mechanisms of their dissipation is carried out on the basis of a joint analysis of the simulation results, measurements and available satellite observations [5, 6].

2. Study area and datasets
Shelf irregularity of the coastline/bathymetry at the areas of the Crimean coast and the Rim Current passing near the shore create specific conditions for the internal wave generation. This study is focused on the Black Sea shelf to the North-West of the Heracles Peninsula (where the quasi-steady Sevastopol anticyclone is known to be formed) and to the North-East of it, in the shallow Kalamita Bay. The topography of study region with spatial resolution of 0.5’ obtained from the file of depth (www.gebco.net) is shown in figure 1.

The accessibility of satellite images and the presence of internal waves in the snapshots have become one of the decisive factors in the choice of research methods. Its horizontal structure was studied by processing the Sentinel-2 and Landsat-8 data for 2017 from open databases: glovis.usgs.gov,
scihub.copernicus.eu/dhus/#/home, earthexplorer.usgs.gov. Reading and processing of high resolution satellite images was performed in the SNAP Desktop environment. For each packet of internal waves, the direction of propagation, as well as 6 points describing its shape is noted. Perimeters with geographic reference of 6 points for each wave (3 front and 3 rear) were obtained. The parameters of the waves (front width, wavelength, number of waves in the packet, direction, date, location) were visualized with QGIS 2.18 and included in database with online access https://qgiscloud.com/April/WebAtlasWaves2.

![Figure 1. Map of the study region. (a) Bathymetric map of the slope/shelf areas with the superimposed sea surface image from satellite Sentinel-2 on 29 June, 2017. Snapshot represents the existence of internal waves. The transects location scheme (green lines) with its serial numbers is given. The red points indicate the presence of measurements in the Bank of oceanographic data. The color shows the location of study areas depending on shelf widths: #1 with wide shelf (magenta), #3 with narrow shelf (green) and #2 with narrower shelf (yellow). (b) The inset shows the location of Crimean coast and the blue box represents the study region shown in (a).]

As initial conditions for numerical experiments we used the seasonal water stratification profiles calculated on the basis of information from the Bank of oceanographic data of the Marine Hydrophysical Institute. The Bank’s data include the decadal temperature and salinity fields at standard levels in the 0-1500 m layer in the Black Sea. These fields were reconstructed using in-situ data and satellite measurements which have been collected from the year 1950 up to now [7]. 144 files with temperature and salinity from May to August were processed for transects shown at figure 1. The seawater density ($\rho$, kg m$^{-3}$) was calculated from the International Equation of State. Profiles of buoyancy frequency are defined as

$$N^2(z) = -\frac{g}{\rho(z)} \frac{\partial \rho(z)}{\partial z},$$  \hspace{1cm} (1)

where $\rho(z)$ is the density, $z$ is the vertical coordinate and $g$ is the force of gravity.

For interpretation of the observations we calculated wave properties and eigenvectors for the vertical velocity component of waves according to the linearized equations of motion under the Boussinesq approximation [8, 9]. We considered a rotating basin of variable depth filled with stratified water for the Coriolis frequency set at $f = 2 \Omega \sin \phi$ ( $\Omega = 7.292 \times 10^{-5}$ s$^{-1}$, $\phi$ is 44°N). The vertical velocity components for each of the three modes and the three areas near the Heraclès Peninsula were obtained by solving the boundary value problem simulated with MatLab application. As a result of this simulation supplementary information about wavelength, period and phase speed of internal waves was obtained and added to online database WebAtlasWaves2 mentioned above.

3. Problem and method description
In this section the problems of internal waves propagation along the coast with a shelf and their breaking mechanisms are described. The equations of common theory of fluid motion are also provided to describe the recovery procedure of the maximum depth of the seasonal pycnocline.
3.1. Formulation of the maximum depth of buoyancy frequency problem

The internal waves are waves that exist at the interface of liquids of various density. Usually in summer, the heated layer is located above the cold lower, and the pycnocline is approximated by a density jump. Therefore we can use two-layer approach in this investigation. We considered the oscillations of the interface between two layers in liquids arranged one above the other. As known for the irrotational fluid motion there is a potential for the speed \( \phi(x, z, t) \). Each of the functions \( \phi_1 \) and \( \phi_2 \) satisfies the Laplace equation inside its layer [10, 11]:

\[
\Delta \phi_1 = 0, \quad (-h < z < 0), \quad \Delta \phi_2 = 0, \quad (-H + h < z < -h),
\]

where \( h \) is the depth of the upper layer and \( \rho_1 \) is the density of the upper layer, \( H \) is the depth of the sea, and \( \rho_2 > \rho_1 \) is the density of the lower layer. On the solid horizontal walls the particles in contact with them remain in contact. At the interface between two fluids two boundary conditions are to be applied: (3) a kinematic condition which relates the motion of the free interface to the fluid velocities at the surface and (4) a dynamic condition which is connected with the force balance at the surface. The perturbed surface equation was written in form \( \zeta = a \cos(\kappa x) \exp(i \sigma t) \), the potential functions as \( \phi_1 = C_1 \text{ch}(k(y + h)) \cos(\kappa x) \exp(i \sigma t) \), \( \phi_2 = C_2 \text{ch}(k(y + H)) \cos(\kappa x) \exp(i \sigma t) \) and therefore we have

\[
\sigma = \frac{g(\rho_2 - \rho_1)}{k(\rho_1 \text{cth}(kh) + \rho_2 \text{cth}(k(H - h)))},
\]

Based on the available satellite images that allow observing a variation in wavelength we can calculate the depth of the Väisälä-Brunt frequency maximum using (5). Also for this calculation we need to have two datasets: the density difference by season and the phase velocity. First of these was taken from Bank's data and second was calculated by solving the boundary value problem numerically.

3.2. Internal wave breaking and dissipation conditions on the continental slope/shelf

The depth dump in the region of the Black Sea Crimean coast is a usual site of internal waves generation and propagation. One important breaking mechanism for internal waves is existence of critical slopes where the topographic steepness of slope is equal to the inclination of an internal wave beam [12]. The inertia gravity waves have frequencies \( f < \omega < N_a \), where \( \omega \) is the frequency of the internal wave, and \( N_a \) is the averaged maximum buoyancy frequency defined by (1). Internal wave beams have the slope of internal wave characteristics \( \alpha \) defined by

\[
\alpha^2 = \frac{\omega^2 - f^2}{N_a^2 - \omega^2}.
\]

In case if the slope of the seabed \( \gamma = \left[ \frac{\partial H(x, y)}{\partial x} \right] \) (where \( H(x, y) \) is the water depth, axis \( x \) is taken in the direction of propagation of the waves, \( y \) axis coincides with the coastal wall) is greater than \( \alpha \) (slope is supercritical), propagation of the internal wave upwards is not permitted, and the wave energy reflects offshore. The internal waves can propagate across the shelf break if \( \gamma < \alpha \) (slope is subcritical) and if \( \gamma = \alpha \) then the slope is critical and provides a site of dissipation [12, 13].
4. Results and discussion

4.1. The pycnocline depth obtained from two-layer approximation and internal waves observation at the Black Sea shelf

The analysis of numerical solutions of boundary value problem for the shelf of the Black Sea demonstrates that phase velocity of internal waves \( c \) for the first mode varies from 0.02 to 0.49 m/s. The available satellite images identify variations in wavelength \( \lambda \) from frontal to rear pairs of waves in the packet within the range of 72–1064 m. In the studied region for three areas ranged by shelf width (figure 1) we know diapasons of phase velocities, wavelengths and average seasonal density differences. This information is shown in table 1 and used to calculate the maximum depth of buoyancy frequency at the spring-summer stratification by equation (5). In the numerical experiments we suppose \( H = 75 \) m.

| Areas | \( \lambda \) (m) | \( c \) (m s\(^{-1}\)) | \( \Delta \rho \) (kg m\(^{-3}\)) | May | June | July | August |
|-------|-----------------|-----------------|-----------------|-----|-----|------|--------|
| 1     | \( 130 < \lambda \leq 1064 \) | 0.173 < \( c \leq 0.494 \) |    | 1.5 | 2.6 | 3.4 | 3.9  |
| 2     | \( 72 < \lambda < 335 \)  | 0.227 < \( c < 0.461 \)  |    | 1.6 | 2.6 | 3.6 | 3.9  |
| 3     | \( 139 < \lambda < 563 \)  | 0.227 < \( c < 0.462 \)  |    | 1.9 | 2.9 | 4.1 | 4.0  |

Figure 2 presents the calculation data of maximum depth of the seasonal pycnocline \( h \), m) for the spring and summer stratification obtained for each of the 18 transects located on the shelf.

**Figure 2.** The calculation results of the maximum depth of buoyancy frequency for the shelf of the Black Sea. The contours of \( h \) are superimposed in the form of lines of colour corresponding to the colour of three studied areas presented in figure 1. The values of \( h \) are the results of calculation based on values of the phase velocity and differences of density given in table 1.

(a) Spring stratification and (b) summer stratification.

The results shown in figure 2 serve to recognize waves on the satellite snapshots corresponding to the appearance of internal waves. These waves have the wavelengths that for permitted values of the
phase velocity provide the density jump at a depth of 20 m. In each of three study areas for wavelengths greater than 180 m it is possible to determine the phase velocity for which the calculated value of $h$ is nearly constant and close to 20 m. Note that the less is the phase velocity the higher a contour of $h$ is in the plot in figure 2.

The horizontal structure of internal waves was estimated by processing the Sentinel-2 and Landsat-8 data for 2017. For a few wave packets, quasi-synchronous images were available (two satellites passing within a time range 15–30 minutes), which allowed us to estimate phase velocity for these wave packets. The data were used for comparison of the calculated maximum depth of buoyancy frequency with the known summer stratification. For wave with wavelength of 379 m, and phase velocity of 0.34 m/s the calculated value of $h$ is 3.4 m. For wave with wavelength of 901 m, and phase velocity of 0.35 m/s the calculated value of $h$ is 3.9 m. For wave with wavelength of 532 m, and phase velocity of 0.28 m/s the calculated value of $h$ is 2.4 m. These results demonstrate an agreement with calculated values of $h$ shown as lines plotted in figure 2. It can be seen at panel (b) of figure 2 that for $\lambda > 250$ m values of $h$ exceed 20 m for all phase velocities.

4.2. Numerical investigation of internal waves instability on the Black Sea slope

As the characteristic slope of an internal wave depends on the vertical stratification (6), the value of critical slope $\alpha$ changes with season. For given topographic slope, a sufficiently large difference in $N(z)$ can switch the slope from supercritical to critical or subcritical. Supercritical bathymetry becomes subcritical, allowing the shoreward propagation of the internal wave energy which was previously reflected.

To establish the impact of shelf topography on internal wave propagation, the spring and summer water stratification profiles across the study area are used for the analysis. The impact of shelf topography on internal wave propagation was made using the frequency of the first normal mode of internal waves which was obtained as a result of a numerical solution of boundary value problem. In these numerical experiments we used the initial conditions from the seasonal water stratification profile. Table 2 provides the values of $\alpha$ and $\gamma$ for spring and summer seasons at the 18 transects on the shelf of the Black Sea.

Table 2. The bottom steepness ($\gamma \cdot 10^3$) and its ratio ($\gamma/\alpha$) to the inclination of the characteristic paths of the internal wave propagation for 18 transects.

| Transects | $\gamma \cdot 10^3$ | $\gamma/\alpha$<br>Spring | $\gamma/\alpha$<br>Summer | Transects | $\gamma \cdot 10^3$ | $\gamma/\alpha$<br>Spring | $\gamma/\alpha$<br>Summer |
|-----------|---------------------|--------------------------|--------------------------|-----------|---------------------|--------------------------|--------------------------|
| 1         | 2.07                | 0.01                     | 0.09                     | 10        | 3.85                | 0.03                     | 0.45                     |
| 2         | 1.71                | 0.01                     | 0.71                     | 11        | 6.30                | 3.36                     | 1.77                     |
| 3         | 1.85                | 0.02                     | 0.06                     | 12        | 2.51                | 1.09                     | 0.93                     |
| 4         | 2.33                | 0.02                     | 0.09                     | 13        | 4.30                | 1.53                     | 1.20                     |
| 5         | 2.70                | 0.03                     | 0.05                     | 14        | 5.67                | 0.15                     | 0.95                     |
| 6         | 2.82                | 0.03                     | 0.10                     | 15        | 6.63                | 0.18                     | 0.26                     |
| 7         | 2.98                | 0.02                     | 0.16                     | 16        | 5.84                | 0.06                     | 0.25                     |
| 8         | 3.81                | 0.03                     | 0.22                     | 17        | 5.45                | 0.10                     | 0.23                     |
| 9         | 3.81                | 0.03                     | 0.22                     | 18        | 4.96                | 0.06                     | 0.21                     |

For each of the 18 transects one of the regimes of the internal waves generation is detected (table 2). If $\gamma/\alpha < 1$ (subcritical slopes), waves are reflected upslope, if $\gamma/\alpha \to 1$ (critical slope) the wavelengths tend to zero and if $\gamma/\alpha > 1$ (supercritical slope) waves are reflected down and back toward deeper water. Seasonal increase in stratification leads to changes in critical slope $\alpha$. For example at the 14 transect a change in $N(z)$ between spring and summer switches the slope from subcritical to critical and the supercritical bathymetry at 12 transect becomes subcritical.

These results are confirmed by the internal waves observation at the shelf near the Crimean coast. Most frequently the wave packets at a depth of 50-100 m were observed between Sevastopol and
Yevpatoria at the shallow Kalamita bay shown in figure 1 (1–10 transects). These waves were detected in 47 cases from total number of satellite images in 2017. At the areas with critical slopes (11–14 transects) representative images of the wave packets were obtained in 3 cases, and in the rest of the study area (15–18 transects) in 19 cases of total number of available satellite observations.

**Conclusion**

A numerical study of the internal waves and their characteristics using realistic stratifications and bathymetry gains an insight into the conditions of their appearance, propagation, and dissipation. The available satellite images allowed us to observe a variation in wavelength, identify the phase velocity of internal waves and calculate the maximum depth of buoyancy frequency based on two-layer approximations. The diapasons of the phase velocity and wavelengths have been obtained for which the recovered depth of the seasonal pycnocline meets with measured values for the shelf of the Black Sea. For realistic topographic slopes the sites where difference between spring and summer stratification can switch the slope from supercritical to critical or subcritical have been obtained.

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**References**

[1] Mysak L A 1980 *Rev. Geophys. Space Phys.* **18** (1) 211–41
[2] Munk W H, Snodgrass F E, Wimbush M 1970 *Geophys. Fluid Dyn.* **1** (1) 161–235
[3] Serebryany A N, Ivanov V A 2013 *Fundamental’naya i Prikladnaya Gidrofizika* **6** (3) 34–45
[4] Vlasenko V I, Ivanov V A, Krasin I G, Lisichenok A D 1997 *Physical Oceanography* **3** 3–16
[5] Lavrova O Yu, Mityagina M I 2017 *Seas Remote Sensing* **9** (9) 892–918
[6] Dulov V A, Yurovskaya M V, Kozlov I E 2015 *Physical Oceanography* **6** 39–54
[7] Polonsky A B, Shokurova I G, Belokopytov V N 2013 *Physical Oceanography* **6** 27–41
[8] Yankovsky A E 2009 *J. Geophys. Res.* **114** 1–13
[9] Ke Z, Yankovsky A E 2010 *J. Phys. Oceanogr.* **40** 2757–67
[10] Lamb G 1947 *Hydrodynamics* (M.-L: GITTL) p 928
[11] Cherkesov L V 1984 *Basics of incompressible fluid dynamics* (Kiev: Naukova dumka) p 166
[12] Lamb K G 2014 *Annu. Rev. Fluid Mech.* **46** 231–54
[13] Stephenson G R Jr, Hopkins J E, Mattias Green J A, Inall M E, Palmer M R 2015 *Geophys. Res. Lett.* **42** 1826–33