Tie Strength Distribution in Scientific Collaboration Networks

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Science is increasingly dominated by teams. Understanding patterns of scientific collaboration and their impacts on the productivity and evolution of disciplines is crucial to understand scientific processes. Electronic bibliography offers a unique opportunity to map and investigate the nature of scientific collaboration. Recent work have demonstrated a counter-intuitive organizational pattern of scientific collaboration networks: densely interconnected local clusters consist of weak ties, whereas strong ties play the role of connecting different clusters. This pattern contrasts itself from many other types of networks where strong ties form communities while weak ties connect different communities. Although there are many models for collaboration networks, no model reproduces this pattern. In this paper, we present an evolution model of collaboration networks, which reproduces many properties of real-world collaboration networks, including the organization of tie strengths, skewed degree and weight distribution, high clustering and assortative mixing.

I. INTRODUCTION

Teams are increasingly overshadowing solo authors in production of knowledge [1]. Examining patterns of scientific collaboration is therefore crucial to understand the scientific processes, knowledge production [1], research productivity [2], the evolution of disciplines [3], and scientific impact [4] [5], etc. Electronic bibliographic data and the development of network science make it possible to systematically investigate scientific collaboration at a large scale [6] [8]. A common approach to studying scientific collaboration is constructing a network of collaboration, where nodes represent authors and two authors are connected by co-authorship [9]. Various aspects of collaboration networks have been widely explored, including basic structural properties [9] [10], evolution [11], robustness [12] [13], assortative mixing [14], and rich-club ordering [15] [16]. Since coauthors usually know each other, collaboration networks have often been considered as proxies of social networks [9]. This viewpoint has been widely adopted, because collaboration networks can be systematically constructed without any subjective bias [9] and the size of these networks can be large.

However, recent studies have revealed that collaboration networks possess unique properties that are not presented in other proxies of real-world social networks such as mobile communication networks and online social networks. One example is the atypical distribution of weak and strong ties. Like most other networks, collaboration networks exhibit cohesive groups (‘communities’) [17] [20]. Since Granovetter pioneered the ideas of the relationship between network structure and tie strength, it has been assumed that strong ties tend to exist in the communities, while weak ties tend to connect these groups [17] [21]. Here we refer ‘communities’ in a purely structural point of view, ignoring weights and ‘weak’ and ‘strong’ ties refer the weight of edges. This organizational principle has been repeatedly confirmed in many networks [22] [27]. However, scientific collaboration networks exhibit the opposite pattern; weak ties constitute communities, while strong ties connect these communities [27] [28]. This counter-intuitive observation raises a question: How and why collaboration networks are shaped in this way?

Although there are many models of scientific collaboration networks or similar weighted networks [3] [11] [29] [35], the organization of tie strength and their roles on global connectivity have not been fully explored. Here we propose that the academic advising system, the patterns of academic career trajectory, and the active inter-group collaboration may provide an explanation. Our key notion is that weak ties are mainly formed from short-term collaborations between students and their advisors, while strong ties are formed through long-term collaborations between groups [28]. Built on this notion, our model reproduces the tie-strength distribution as well as other common properties, such as skewed degree and weight distribution, high clustering, and assortative mixing.

II. STRUCTURE AND LINK WEIGHT

To test the universality of the atypical tie-strength distributions in scientific collaboration reported in [27] [28], we analyze four scientific collaboration networks: Network Science, High-energy Physics, Astrophysics, and Condensed Matter. Link weights in these networks are defined by \( w_{ij} = \sum \frac{n_{ij}}{n_k} \), where \( n_k \) is the number of authors in paper \( p \) in which \( i \) and \( j \) participated [10] [36]. Although this particular definition of weight is not unique, it has been widely accepted as a standard metric (See Section III in [36] for a detail and thorough discussion about it). Table 1 lists basic statistics of these networks. As many studies demonstrated, both degree and link weights are broadly distributed [9] [10].
TABLE I. Structural statistics for weighted scientific collaboration networks include number of nodes \( N \), number of links \( M \), mean node degree \( \langle k \rangle \), mean link weight \( \langle w \rangle \), clustering coefficient \( c \) and assortativity coefficient \( r \) [14]. Net-sci is based on the coauthorship of scientists working on network science based on the papers posted on High-energy Physics E-Print Archive (http://arxiv.org/archive/hep-th), Astrophysics E-Print Archive (http://arxiv.org/archive/astro-ph) and Condensed Matter E-Print Archive (http://arxiv.org/archive/cond-mat), respectively [9]. For each network, we only consider the largest connected component. All the 4 networks are downloaded from http://www-personal.umich.edu/~mejn/netdata/.

| Name     | \( N \) | \( M \) | \( \langle k \rangle \) | \( \langle w \rangle \) | \( c \) | \( r \) | Time     |
|----------|--------|--------|----------------|----------------|-----|-----|---------|
| Net-sci  | 379    | 914    | 4.823          | 0.536          | 0.741 | -0.0817 | -       |
| Hep-th   | 5,835  | 13,815 | 4.74           | 0.990          | 0.506 | 0.185 | 1995–1999 |
| Astro-ph | 14,845 | 119,652 | 16.12          | 0.279          | 0.670 | 0.228 | 1995–1999 |
| Cond-mat | 36,458 | 171,735 | 9.42           | 0.506          | 0.657 | 0.177 | 1995–2005 |

Figure 1 shows the relationship between link weight \( w_{ij} \) and local clustering defined by the overlap measure

\[ O_{ij} = \frac{n_{ij}}{d_i - 1 + d_j - 1 - n_{ij}} \]

where \( n_{ij} \) is the number of common neighbors of node \( i \) and \( j \), and \( d_i \) (\( d_j \)) is the degree of node \( i \) (\( j \))[24]. \( O_{ij} \) quantifies the overlap between the neighbors of two end-points and measures embeddedness of an edge. For instance, \( O_{ij} = 0 \) indicates that nodes \( i \) and \( j \) have no common neighbors and the link is likely to connect communities. For a large portion of links, overlap decreases with weights. For a small portion of strongest links (20% links with \( w_{ij} > 0.640 \) for Net-sci, 4.3% links with \( w_{ij} > 3.327 \) for Hep-th, 7.8% links with \( w_{ij} > 0.765 \) for Astro-ph, and 14% links with \( w_{ij} > 0.869 \) for Cond-mat), overlap increases with weights. These results indicate that weak ties mainly constitute dense local clusters, whereas strong ties are connecting these clusters. In order to further confirm the universality of weight-topology coupling patterns in scientific collaboration networks, we examine network connectivity under link removal [23][25][28]. We remove links based on descending or ascending order of link weights and track the relative size of Largest Connected Component (LCC) \( R_{LCC} \) as a function of the fraction of removed links. Figure 2 shows that removing strong links breaks the networks into disconnected components faster than removing weak links, indicating that strong links are more important in maintaining global network connectivity. Strong links connect clusters (Fig. 3B), while weak links reside inside communities (Fig. 3C).

FIG. 1. The correlation between link overlap \( O_{ij} \) and link weight \( w_{ij} \) in scientific collaboration network of (A) Network Science, (B) High-energy Physics, (C) Astrophysics and (D) Condensed Matter. We use logarithmic binning for \( w_{ij} \). The error bars indicate the standard error of the mean \( O_{ij} \). For a large portion of links, overlap decreases with weight. For a small portion of strongest links, overlap increases with weight.

III. MODEL

Many models have been proposed to explain the known properties of scientific collaboration networks. However, these models either do not consider link weights or do not capture the role of strong ties in maintaining global connectivity. Some models focus on assortative mixing [29][30]. Some study the self-organizing evolution of collaboration networks as preferential attachment and “rich-get-richer” [11][31][32]. Others emphasize the evolution of disciplines [33] or social interaction of scientists [3]. The weak-tie hypothesis has often been considered as an evident truth about networks, and most models that produce community structures assume so [40][41].

By contrast, our model is based on the following observations: (i) scientific collaboration networks grow in time, as new papers and scientists join continuously; (ii) junior scientists become inactive with high probability; indeed, recent work on analysis of APS dataset reveals that 40% of authors only publish one paper in their entire career [28]; (iii) long-term collaboration usually occurs between senior scientists who have their own research groups [28].

Our model has two mechanisms of producing new papers: intra-group and inter-group collaboration. Starting with a research group of an advisor and a student, the collaboration network grows over time. At every time step:

- Each group publishes \( k \) papers by itself. Each paper is written by the advisor and \( l - 1 \) co-authors preferentially chosen from the same group based on the students’ scientific expertise \( e \). The probability to be chosen is proportional to \( e \). If a student joins a group at time \( \tau \), with initial expertise \( e(\tau) = 1 \), \( e \) increases linearly with time: \( e(t) = t - \tau + 1 \);
The relative size of Largest Connected Component (LCC) $R_{LCC} = N_{LCC}/N$ indicates that the removal of strong links leads to a faster breakdown of networks.

For simplicity, we assume the following parameters to be constants, namely $k = 3$, $l = 4$, $p = 1$, $G = 5$, and $f = 0.3$ in our analysis. Table II shows the meanings of the model parameters.

By calculating the gained tie strength within a group and between groups at each time step, we show that when $\alpha \approx 4.281$, removal of strong or weak links first will have the same effect on the network connectivity. We focus on stationary groups with $G$ students and with total expertise $G(G + 1)/2$. The professor $p$ in group $g$ writes $k$ papers with the group members. Each paper will add the weight of $1$ to the link between the professor and a chosen student. Let $p_i(e)$ be the probability that a student with expertise $e$ is chosen in an intra-group paper (see Appendix E for its calculation). Then the gained link weight between the professor and the student is

$$w_{p,i}^{(i)} = \frac{k}{l-1} p_i(e)^k.$$  

Let $p_i(e_1,e_2)$ be the probability that two students in the group $g$ with expertise $e_1$ and $e_2$ are chosen in an intra-group paper. Then the gained link weight between the two students is

$$w_{e_1,e_2}^{(i)} = \frac{k}{l-1} p_i(e_1,e_2)^k.$$  

Meanwhile, the professor $p$ in group $g$ writes $\alpha k$ papers with another group $g'$ ($p = 1$). The weight between the two professors increases by

$$w_{p,p'} = \frac{\alpha k}{l-1}.$$  

Let $p_0(e)$ be the probability that a student with expertise $e$ is chosen in an inter-group paper. Then

$$w_{p,i}^{(i)} = w_{p,i}^{(b)} = \frac{\alpha k}{l-1} p_0(e)^{\alpha k}.$$  

Let $p_0(e_1,e_2)$ be the probability that two students with expertise $e_1$ and $e_2$ are chosen in an inter-group paper. Then

$$w_{e_1,e_2}^{(i)} = w_{e_1,e_2}^{(b)} = \frac{\alpha k}{l-1} p_0(e_1,e_2)^{\alpha k}.$$  

### TABLE II. Model parameters and their explanations.

| Parameter | Meaning |
|-----------|---------|
| $k$       | Number of intra-group papers |
| $l$       | Number of authors in each paper |
| $p$       | Probability of preferred group to collaborate |
| $G$       | Expertise threshold for students to graduate |
| $f$       | Probability of graduates to form new groups |
| $\alpha$  | Ratio of inter-group to intra-group collaborations |
So the gained tie strength within a group at each time step is

\[ W^{(i)} = \sum_{e=1}^{G} w_{i,pe}^{(i)} + \sum_{e_1 \neq e_2} w_{e_1,e_2}^{(i)} + \sum_{e=1}^{G} \sum_{e_1 \neq e_2} w_{i,e_1,e_2}. \]  

(6)

\[ \Delta W = W^{(b)} - W^{(i)} = \frac{k}{l-1} \left( \alpha + \alpha \sum_{e=1}^{G} (p_b(e,e))^{\alpha k} - \sum_{e=1}^{G} (p_i(e))^k - \sum_{e_1 \neq e_2} (p_i(e_1,e_2))^k \right) \]

\[ W^{(b)} = w_{p,p'} + \sum_{e=1}^{G} w_{p,e} + \sum_{e_1=1}^{G} \sum_{e_2=1}^{G} w_{s_{e_1},e_2}. \]  

(7)

\[ \Delta W = W^{(b)} - W^{(i)} \]

The total gained tie strength between the two groups is

\[ \Delta W = \sum_{e=1}^{G} \sum_{e_1 \neq e_2} w_{e_1,e_2}. \]  

We next derive the number of groups \( n_g(t) \) and the number of students \( n_s(t) \) at \( t \). Let \( n^{(e)}(t) \) be the number of students at the expertise level \( e \) at time step \( t \). The expertise \( e \) increases in time and the students graduate when the expertise reaches \( G \). Graduates create their own group with the probability \( f \), namely

\[ n_g(t) = n_g(t-1) + fn_{s}^{(G-1)}(t-1), \]

(8)

with \( n_g(0) = \ldots = n_g(G-2) = 1 \). At each time step, there are the same number of new students as the number of groups

\[ n^{(1)}(t) = n_g(t). \]  

(9)

The number of students with expertise \( e \geq 2 \) is the same as the number of students with expertise \( e - 1 \) in the previous time step

\[ n^{(e)}(t) = \ldots = n^{(1)}(t - (e - 1)) = n_g(t - e + 1). \]  

(10)

Therefore, the number of groups is

\[ n_g(t) = n_g(t-1) + fn_g(t-G+1). \]  

(11)

The number of students is

\[ n_s(t) = \sum_{e=1}^{G} n^{(e)}(t) = \sum_{e=1}^{G} n_g(t-e+1). \]  

\[ (t \geq G-1) \]

(12)

with \( n_s(t) = t+1 \) for \( t \leq G-2 \). The increased number of nodes is the number of groups in the previous time step

\[ N(t) = N(t-1) + n_g(t-1) \]

(13)

with \( N(0) = 0 \) and \( N(t) = t+1 \) for \( 1 \leq t \leq G-2 \). These analytical results are in agreement with the numerical results, as shown in Fig. 5.

Finally, we calculate the mean degree \( \langle d \rangle \) of model networks (see Appendix A). The analytical result is in agreement with the numerical result \( \langle d \rangle = 10.7 \).

### A. Model Results

We vary \( \alpha \) to investigate the impact of inter-group collaboration. Fig. 4 shows that when \( \alpha = 4 \), neither strong

\[ \text{FIG. 4. The robustness of model networks to link removal when } \alpha = 4. \text{ The result for } \alpha = 5 \text{ is shown in Figure 6B.} \]

\[ \text{FIG. 5. Comparison of calculation results with numerical results of (A) number of of groups } n_g(t) \text{ and (B) number of students } n_s(t). \text{ The numerical results are averaged over 100 repetitions.} \]
ties nor weak ties maintain global connectivity. As we increase $\alpha$ to 5, the network exhibits more similar weight organization with the real collaboration networks (Fig. 6B). These results indicate that our hypothesis can indeed explain the atypical tie-strength distributions in scientific collaboration networks.

Furthermore, our model reproduces other common properties of scientific collaboration networks: (i) skewed distribution of degree and link weights, as shown in Fig. 7 (ii) high average clustering coefficient: $c = 0.784$ ($\alpha = 5$); and (iii) assortative mixing: $r = 0.0796$ ($\alpha = 5$).

Having assumed that the number of authors $l$ in a paper and the probability $p$ for a preferred collaboration relationship between two groups are constants ($l = 4$ and $p = 1$), we now investigate the model’s sensitivity to the parameters $l$ and $p$. We draw $l$ from the range $[2,5]$ with equal probability for intra-group papers, and from the range $[3,6]$ with equal probability for inter-group papers. We also set $p$ to 0.9 and 0.8. These results are showed in Fig. 8.

First, we set $p$ to 0.9 and 0.8 while keeping other parameters to be constants ($\alpha = 5$ and $l = 4$). The results shown in Fig. 8A and 8B indicate that the proposed model is robust to variations of $p$. Strong ties still maintain global connectivity of model networks for different $p$. However, decreasing $p$ increases the randomness of model networks, as smaller $p$ indicates that each group has larger probability to collaborate with different groups. This is confirmed from different values of $R_{LCC}$. For $p = 1$ the model network begins to be disconnected when the fraction of removed links is 0.2 (Fig. 8B). But for $p = 0.8$ the model network is broken into components when the fraction is 0.5 (Fig. 8D).

Next, we set $l$ as a random variable while keeping keeping other parameters as constants ($p = 1$, $\alpha = 1$ or $\alpha = 5$). Fig. 8C and 8D show that the proposed model is robust for the variation of number of authors $l$ in a paper. $l$ being either fixed (Fig. 8B) or a random variable drawn from a given distribution (Fig. 8C and 8D) will not change the link removal results. For $\alpha = 1$, neither strong ties nor weak ties maintain global connectivity. When $\alpha = 5$ strong ties become important to connect different clusters.

IV. CONCLUSIONS

In this paper we explore the weight organization of scientific collaboration networks. We propose a model, which incorporates intra- and inter-group collaborations and reproduces many properties of real-world collaboration networks. We also provide detailed analysis of our model. Our work also raises further questions such as: How did the collaboration pattern change in time? How do scientific ideas flow through strong and weak ties? Are there any general coupling patterns (or classes) between structure and weights?
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Appendix A: Calculation of mean degree

In order to get the expected degree of a professor when graduation, we track it from time step $\tau$ when she joined the group with a indicator function (probability) that $a_i$ of the students with expertise $\alpha$ will be in the same group until the graduation, we track it from time step $\tau$ to $\tau + G - 1$ when graduation with expertise $\alpha$. Let $\bar{\nu}$ of different students collaborators in group $g$ at $\tau$. From $t = \tau + 1$ to $t = \tau + G - 2$, there are another $G - 2$ new students joining in the group. Let $I(a_1,a_2)$ (Pr($a_1,a_2$)) be the indicator function (probability) that $a_i$ have collaborated with $a_2$. The number of different students collaborators in the group $g$ for $a_i$ is

$$d_i = \sum_{a_e \in g} I(a_1,a_e) = \sum_{a_e \in g} 1 \times \text{Pr}(a_1,a_e)$$

$$= \sum_{a_e \in g} 1 - \text{Pr}(a_1,a_e)$$

$$= 2(G - 2) - \sum_{a_e \in g} \text{Pr}(a_1,a_e)$$

(A1)

Consider the student $a_2$ with expertise $e = 2$ at time $t = \tau$, $a_1$ and $a_2$ will be in the same group until the expertise of $a_2$ reaches $G - 1$. Let $\bar{p}^{(k)}(e_1,e_2)$ be the probability that two students with expertise $e_1$ and $e_2$ are not chosen in any of the $k$ papers

$$\bar{p}^{(k)}(e_1,e_2) = (1 - p_i(e_1,e_2))^k$$

(A2)

Then $\text{Pr}(a_1,a_2)$ is the probability that $a_1$ do not collaborate with $a_2$ from $t = \tau$ to $t = \tau + G - 3$

$$\text{Pr}(a_1,a_2) = \prod_{e=1}^{G-2} \bar{p}^{(k)}(e,e+1)$$

Similarly, for student $a_1, a_2, \ldots, a_{G-1}$ with expertise $e = 3, \ldots, G - 1$ at time $t = \tau$

$$\text{Pr}(a_1,a_3) = \prod_{e=1}^{G-3} \bar{p}^{(k)}(e,e+2)$$

$$\text{Pr}(a_1,a_{G-1}) = \prod_{e=1}^{G-1} \bar{p}^{(k)}(e,e+G-2)$$

From time step $t = \tau + 1$ to $t = \tau + G - 2$, the expertise of $a_1$ increases from $e = 2$ to $G - 1$. A new student joins the group at each time step

$$\text{Pr}(a_1,a_{G}) = \prod_{e=2}^{G-1} \bar{p}^{(k)}(e,e-1)$$

$$\text{Pr}(a_1,a_{2G-3}) = \prod_{e=G-1}^{G-1} \bar{p}^{(k)}(e,e-(G-2))$$

The intra-group degree for students collaborators (Eq. A1) now is

$$d_i = 2(G - 2) - \sum_{j=1}^{G-2} \prod_{e=1}^{G-1-j} \bar{p}^{(k)}(e,e+j) - \sum_{j=2}^{G-1} \prod_{e=j}^{G-1} \bar{p}^{(k)}(e,e-(j-1))$$

(A3)

Then, the number of student collaborators in another group $g'$ is

$$d_b = 2G - 3 - \sum_{j=0}^{G-2} \prod_{e=1}^{G-1-j} \bar{p}^{(ak)}(e,e+j) - \sum_{j=2}^{G-1} \prod_{e=j}^{G-1} \bar{p}^{(ak)}(e,e-(j-1))$$

(A5)

Considering two advisers, we get the degree of the student $a_1$ when graduation

$$d = d_i + d_b + 2$$

(A6)
After graduation, $a_1$ becomes a professor with probability $f$. The increased degree at each time step now is the probability of collaborating with the two newly joined students

$$\Delta d_p = (1 - (1 - p_1(1))^k) + (1 - (1 - p_b(1))^{\alpha k}) \quad (A7)$$

The total degree of professors at time step $t$ is

$$D_p(t) = \sum_{\tau=G-1}^{t-1} \left\{ n_s^{(G-1)}(\tau) \cdot f \cdot [d + (t - \tau + 1) \Delta d_p] \right\} \quad (A8)$$

Using the same idea, we can get the degree of a student with expertise $e$ ($e \leq G - 1$) at time step $t$. This is a general case of Equations [A3] and [A5].

$$d^{(e)}(t) = d^{(e)}_i(t) + d^{(e)}_b(t) + 2 \quad (A9)$$

So the mean degree $\langle d \rangle$ at time step $t$ is

$$\langle d \rangle(t) = \frac{1}{N(t)} \left[ D_p(t) + \sum_{e=1}^{G-1} n_s^{(e)}(t)d^{(e)}(t) \right] \quad (A10)$$

**Appendix B: Calculation of $p_i(e)$**

We describe how to calculate $p_i(e)$, $p_b(e)$, $p_i(e_1, e_2)$, and $p_b(e_1, e_2)$. Recall that $p_i(e)$ is the probability that a student with expertise $e$ is chosen for one intra-group paper. Its distribution is the sum of multiple multivariate Wallenius’ noncentral hypergeometric distributions [12] and can be calculated by using package `BiasedUrn` in R [43]. The calculations for $p_b(e)$, $p_i(e_1, e_2)$, and $p_b(e_1, e_2)$ are similar.

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