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TENSION FAILURE ASSESSMENT AT LUG HOLE EDGES

Daniela Scorza, Andrea Carpinteri, Sabrina Vantadori

Department of Engineering & Architecture, University of Parma, Parco Area delle Scienze 181/A, 43124 Parma, Italy

Corresponding Author: Daniela Scorza, email: daniela.scorza@unipr.it

ABSTRACT

The lug is the critical component of lug joints, and one of its failure mode is the tension failure at hole edges. In the present paper, the fatigue crack growth simulation of a semi-elliptical crack located in the lug net cross-section, at one of the two hole edges, is carried out by employing a theoretical model based on the Paris law. The lug sizes, loading conditions and crack configurations are assumed in accordance with those related to some experimental tests available in the literature, performed on lugs of PolyMethylMethAcrylate under pulsating tension.

KEYWORDS: Fatigue Crack Growth, Lug joint, PMMA, Stress-Intensity Factor, Semi-elliptical crack.

NOMENCLATURE
$a, b$  Semi-axes of the semi-elliptical crack

$a_0, b_0$  Initial values of the semi-elliptical crack semi-axes

$B_m$  Polynomial coefficients

$C,m$  Constants of the Paris law

$D$  Diameter of the lug hole

$K_{I(m)}, K_{I(m)}^*$  Stress intensity factor for a semi-elliptical crack in a finite-thickness plate under the $m$-th elementary stress distribution, and the corresponding dimensionless value

$K_T$  Stress-Concentration Factor

$m$  Order of the $m$-th elementary stress distribution

$N$  Number of loading cycles

$P$  Applied load

$R$  Stress ratio

$t$  Size of the lug net cross-section ($Y$-axis direction)

$T$  Lug thickness

$W$  Lug width

$w$  Local coordinate axis with origin in $O'$

$\alpha = a/b$  Crack aspect ratio

$\alpha_0 = a_0/b_0$  Initial value of the crack aspect ratio

$\eta = w/a$  Dimensionless local coordinate

$\xi = a/t$  Relative crack depth

$\xi_0 = a_0/t$  Initial value of the relative crack depth

$\sigma_{I(m)}$  $m$-th elementary stress distribution

$\sigma_{xr}$  Remote stress applied on the lug
1. INTRODUCTION

1.1 Lug joint fatigue behaviour

Fatigue failure in structures frequently occurs at joints, as is documented for various catastrophic accidents in the literature \[1\]. Therefore, Fatigue Crack Growth (FCG) analysis and damage control have to be performed \[2,3\].

The main function of a joint is to transmit loads to one element to another element of the structure. A lug joint consists of a fork structure, a lug and a pin (Figure 1). The load transmission between the fork, subjected to tension, and the lug occurs by means of the pin. The pin applies a distributed pressure to the upper half of the hole in the lug. Such a pressure is applied in the vicinity of the notch root, that is, in correspondence of the edges of the hole (Figure 1).

\[
\sigma_{xh}, \sigma_{xh}^* \quad \text{Stress distribution along X-direction in the lug net cross-section, and the corresponding dimensionless value}
\]

Acronyms

FCG\hspace{1em} Fatigue Crack Growth
PMMA\hspace{1em} PolyMethylMethAcrylate
SIF\hspace{1em} Stress Intensity Factor

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**Figure 1**
The lug is usually the critical component of such joints, since failure under cyclic loading is determined by both the stress concentration at hole edges and fretting due to the relative movement between pin and lug [4].

Different failure modes can affect the lug [5]:
(a) tension failure at the hole edges;
(b) bearing failure at the pin/lug interface;
(c) shear failure at the lug edge, that is, the pin tries to push out through the edge of the lug (fracture along two planes);
(d) tensile failure at the lug edge, that is, the pin tries to push out the edge of the lug (fracture on a single plane);
(e) out-of-plane buckling failure of the lug.

Lug joints are widely used in aerospace engineering: key examples of their use are in engine pylons for attachment of the engines, and in airplanes for connection of ailerons or flaps to the wing.

The review by Gran et al. [2] on significant United States Air Force aircraft structural failures reveals that bolt and rivet holes comprise over one third of the observed failure origins.

Over the past decades, several exhaustive experimental studies have been conducted to examine the fatigue behaviour of lug joints and the crack propagation under either constant or variable amplitude loads [3,6,7]. In particular, Schijve and Hoeymakers [3] performed fatigue tests on aluminium alloy lugs subjected to constant-amplitude loading, and recorded the FCG rate for both artificial corner crack started by a small saw cut, and natural cracks due to the fretting fatigue
between hole and pin. They observed that the local failure mode ranged from pure Mode I (near the hole boundary) to Mixed Mode (near the plate surface). The experimental work by Grandt et al. [7] was focused on the observation of the front evolution of initial embedded cracks (semi-elliptical crack, corner quarter-elliptical crack and through-the-thickness crack), located along a hole edge in both a wide plate and a lug made of PolyMethylMethAcrylate (PMMA). They monitored and recorded the crack size and shape evolution by using time lapse photography, and observed that both semi-elliptical and corner cracks quickly evolved into the through-the-thickness configuration.

On the basis of the experimental evidences, the initial crack shape in a lug can be usually approximated as either a semi-elliptical crack, a quarter-elliptical crack or a through-the-thickness crack [3,8-10] and, under cyclic loading, the first two types can quickly evolve in the last one [3,7].

1.2 State-of-the-art on Fatigue Crack Growth (FCG)

The state-of-the-art literature clearly shows that a significant effort has been made by the Scientific Community in order to develop numerical and analytical models able to evaluate the FCG rate with an acceptable degree of confidence [11-15]. Thanks to the pioneering work of Paris and Erdogan [16], the increment of fatigue crack propagation per stress cycle was empirically linked to the range of the Stress Intensity Factor (SIF) at the crack tip through the well-known Paris law. Then Elber [17] accounted for the crack closure
phenomenon through a modified version of the Paris law, by considering an effective SIF range. Numerous fatigue crack propagation models following the Elber’s idea were proposed, such as the semi-elliptical fatigue crack propagation model developed by Newman [18].

However, the crack tip closure model by Elber, employed in the aircraft industry, is not easy to be applied and requires some experimental calibrations. An empirical and relatively successful FCG model accounting for the mean stress effect was proposed by Walker [19] (two-parameter driving force model), where both the maximum value and the range of the SIF were used to determine the FCG. An analogous expression has been more recently formulated by Kujawski [20] including only the tensile part of the SIF range, whereas the mean stress effect has been accounted for by Noroozi et al. [21] analysing the elastic-plastic strain-stress history at the crack tip.

1.3 FCG in lug joint

In all the above empirical FCG equations, an accurate computation of the SIFs characterising the stress field near the crack tip/front is essential. Consequently, several methods have been developed to determine the SIF of structural components containing cracks, by applying both experimental and analytical/numerical approaches. Different crack configurations in both infinite and finite plates and lugs have been analysed [3, 6, 7, 22-26]. A single quarter-elliptical corner crack emanating from fastener holes was examined by Smith et al. [6] through a Finite Element Alternating Method (FEAM). Schijve
and Hoeymakers [3] developed a simple interpolation method, based on Newman-Raju formulation [22], to obtain the SIFs for corner cracks at holes. Then, they compared the computed SIFs with those experimentally determined by Grandt [7]. Hellen [23] applied the J-integral method to take into account the complex stress field in a lug under pin-loaded operation conditions. The Finite Element Method (FEM) was used by both Kathiresan and Brussat [24] and Gencoz et al. [25]: the first ones numerically simulated lugs with either through-the-thickness or corner cracks by modelling the contact between pin and lug through spring elements, whereas the latter ones reproduced the stresses observed in photo-elastic models of the lugs with particular stress distributions.

More recently, Kim et al. [9] have analysed a lug under random spectrum loading through experimental tests and numerical Boundary Element Method (BEM) analyses, in order to evaluate the SIFs of through-the-thickness cracks. By applying the weight function technique and the Unigrow Model, Mikheeviskiy et al. [10] have estimated the residual fatigue life of a lug subjected to cyclic loading. Since one of the drawbacks of FEM and BEM when determining the SIF values is the need of remeshing in order to reproduce the evolution of the crack front, additional numerical techniques based on meshless methods and Extended Finite Element Methods (XFEMs) [27-30] have been recently applied to examine FCG in lug joints [31].

An accurate determination of the SIF values at the front of 3D surface cracks is the main task in order to predict fatigue life of
lug joints and, consequently, a simple and computationally efficient procedure avoiding complex meshes or remeshing techniques can be an interesting tool for practical applications.

In the present paper, the numerical procedure proposed in Refs [32, 33] by the Authors is applied to analyse the FCG of a semi-elliptical crack in a lug made of PMMA. Such a procedure consists of five steps shown in Figure 2, as is discussed in the following, and is validated in Section 5 by comparing the numerical results (in terms of crack path, crack depth evolution, and crack front evolution) with the experimental ones provided by Grandt et al. in Ref. [7].

**Figure 2**

2. COMPONENT GEOMETRY AND MATERIAL DATA

The cracked lug under cyclic tension here analysed is shown in Figure 3. The geometric sizes and crack configurations examined are those related to the specimens experimentally tested by Grandt et al. [7]. Such specimen configurations are characterised by four different values of the width-to-diameter ratio $W/D$ (equal to 3.0, 2.5, 2.0 and 1.5, respectively), being $W$ the width of the lug and $D$ the diameter of the hole (Figure 3(a)). For each specimen configuration, two values of the maximum applied load $P$ (Figure 1) are analysed, as is listed in Table 1, being the loading ratio $R$ equal to zero. The specimen acronyms used in Table 1 agree with those employed in Ref. [7].
The reference coordinate frame XYZ is assumed to have its origin O located at one of the two hole edges (Figure 3(a)). The X-axis is taken parallel to the loading direction coincident with the lug longitudinal one, the Y-axis is oriented along the lug width W and, hence, the Z-axis completes the right-hand frame (Figure 3(b)).

A semi-elliptical crack is assumed to exist in correspondence to the lug net cross-section at one of the two hole edges (Figure 3(b)), and its initial semi-axes lengths (named $a_0$ and $b_0$ along Y- and Z-axes, respectively) are listed in Table 2. Two dimensionless variables are introduced to describe the crack front evolution: (i) the relative crack depth $\xi$, defined as the ratio between the crack depth $a$ and the lug net cross-section size $t$ along the Y-axis, and (ii) the crack aspect ratio $\alpha$, defined as the ratio between the crack semi-axis lengths, i.e. $\alpha=a/b$. The initial values of such variables, named $\xi_0$ and $a_0$, are listed in Table 2 for each sample.

According to Ref. [7], the specimen material is PMMA. The mechanical properties are listed in Table 3, whereas the fatigue properties in terms of Paris law constants are computed starting from the experimental data reported in Refs [7,34]: $C=6.0\cdot10^{-6}$ and $m=8.3$
(with the crack growth rate in \( \text{mm/cycle} \), and the Stress-Intensity Factor range in \( \text{MPa}\sqrt{\text{m}} \)).

Table 3

3. STEPS 1 AND 2: STRESS FIELD IN THE UNCRACKED LUG AND COEFFICIENT DETERMINATION

According to Step 1 of the procedure summarised in Figure 2, the stress field in the uncracked lug of the joint under a static tension loading \( P \) (Figure 1) is needed to be computed.

A two-dimensional FE analysis is initially performed, where the effect of the pin on the lug is simulated by using displacement radial constrains at the half hole, as is shown in Figure 4(a). The load \( P \) is applied directly to the lug through the uniform pressure \( \sigma_{xr} = P/(W \cdot T) \). Note that no friction is taken into account at the pin-lug interface. Such an assumption, that allows us to simplify the problem, is based on the numerical results provided by Naderi et al. [31]. They analysed the friction between the lug and the pin modelling the lug loaded by either a uniform or a cosine pressure along the half hole, and also examined the whole joint without friction. Their SIF evaluations along the front of the crack in the lug net cross-section show that the frictionless model provides safety results and, for such a reason, the friction effect is neglected in the numerical model here employed.
The FE analysis is linear elastic, performed by using about 1650 eight-node plane strain quadrilateral elements (minimum element size equal to about 0.01 mm) with reduced integration. As an example, the numerical model discretisation related to the lug characterised by $W/D=1.5$ is shown in Figure 4(a).

Figure 4

To determine the Stress-Concentration Factor $K_T$, the mesh is refined by increasing the FE number $n_e$. The results obtained are shown in Figure 5(a) for each lug geometric configuration examined. It can be noted that, independent of the $W/D$ value, $K_T$ tends to asymptotic values when $n_e$ is greater than about $20 \cdot (10)^3$. Such values are listed in Table 1.

Figure 5

Therefore, to evaluate the accuracy of the two-dimensional model previously employed to determine $K_T$, a 3D FE model of the whole joint is also implemented in ABAQUS/CAE Finite Element software (Simulia, Johnston, United States). As an example, the numerical model discretisation related to the lug characterised by $W/D=1.5$ is shown in Figure 4(b). The load $P$ is applied directly to the pin, whereas the lug is fixed. The FE analysis is linear elastic, performed by using
approximatively $75 \cdot (10)^4$ finite elements of the following type: 20-node quadratic brick elements (minimum element size equal to about 0.01mm) with reduced integration. The frictionless contact between lug and pin has been simulated as a normal “hard” contact with penalty method of resolution.

Independent of the $W/D$ value, by taking as a reference value the Stress-Concentration Factor obtained from the 3D analysis, the corresponding value derived through the 2D analysis differs less than 5% from the reference one. Therefore, only the results coming from the 2D analysis are used in the following.

The stress distribution at the net cross-section along $X$-direction, $\sigma_{xh}$ for $x=0$ and $0 \leq y \leq t$ (Figure 4(a)), is determined from the above 2D numerical model, and then normalised with respect to the remote stress, $\sigma_{xr}$ (Figure 4(a)), by obtaining $\sigma_{xh}^* = \sigma_{xh}/\sigma_{xr}$.

According to Step 2 of the procedure summarised in Figure 2, such normalised stress values can be approximated through a power series expansion by performing a 5th order polynomial fitting for the problem being examined:

$$\sigma_{xh}^*(\eta) \approx \sum_{m=0}^{M=5} B_m \frac{\eta^m}{\sigma_{xr}}$$

where $B_m$ are the polynomial coefficients, and $\eta = w/a$ is a normalised coordinate defined as the ratio between the local coordinate $w$ and
the crack depth \( a \) (Figure 3(b)). Note that the origin \( O' \) of the \( w \)-axis is located at \( y = a \).

For each lug configuration examined, the expression of the 5th order polynomial fitting curve is reported in the Appendix, together with the corresponding Pearson correlation coefficient which measures the correlation between the FE stress values and the approximated ones.

As an example, the normalised stress distribution for the lug geometric configuration characterised by \( W/D = 1.5 \) is plotted in Figure 5(b) (dot symbols) together with the corresponding 5th order polynomial fitting curve (continuous line). In such a case, the Pearson coefficient is equal to about 1.0, proving the accuracy of the approximation proposed.

4. STEPS 3 AND 4: SIF COMPUTATION

According to the procedure summarised in Figure 2, a simplified SIF computation for the cracked lug is herein described (Steps 3 and 4).

Firstly, the SIF along the front of a semi-elliptical surface crack in a finite-thickness plate (Figure 6(a)) is determined for elementary Mode I stress distributions \( \sigma_{I(m)} \) (\( I \) stands for Mode I, and \( m \) for the monomial order describing a given stress distribution). The thickness of the plate is equal to the lug net cross-section size in the \( Y \)-direction, and each stress distribution is characterised by zero value at point \( O' \) on the crack front and unit value in correspondence to the outer surface of the plate (Figure 6(b)): 

\[ \text{Equation} \]
\[ \sigma_{I(m)} = \sigma_{ref(m)} \left( \frac{w}{a} \right)^m = \sigma_{ref(m)} \cdot \eta^m \quad \text{with} \quad m = 0, \ldots, 5 \quad \text{and} \quad \sigma_{ref(m)} = 1 \]  \hspace{1cm} (2)

**Figure 6**

Such distributions are directly applied (one at a time) to the crack faces of the finite-thickness plate.

The SIFs along the crack front, \( K_{I(m)} \), are computed though a three-dimensional linear FE analysis by also applying the quarter-point displacement correlation technique [35]. The corresponding dimensionless SIFs, \( K_{I(m)}^* \), are defined as follows:

\[ K_{I(m)}^* = \frac{K_{I(m)}}{\sigma_{ref(m)} \sqrt{\pi a}} \quad \text{with} \quad m = 0, \ldots, 5 \]  \hspace{1cm} (3)

The crack configurations examined are characterised by two parameters: (i) the relative crack depth \( \xi = a/t \), ranging from 0.1 to 0.7, and (ii) the crack aspect ratio \( \alpha = a/b \), ranging from 0.1 to 1.2. Readers interested in details about the computed SIFs may look at Refs [32,33].

Then, by taking into account Eq.(2), the polynomial function reported in Eq.(1) can be formally rewritten as follows:

\[ \sigma_{xh}^*(\eta) \approx \sum_{m=0}^{M=5} \frac{B_m^*}{\sigma_{ref(m)}} \cdot \sigma_{I(m)} \]  \hspace{1cm} (4)

where \( B_m^* = B_m / \sigma_{xr} \).
Under linear elastic fracture mechanics assumption, the approximated
dimensionless SIFs along the crack front, \( K_{I(xh)}^* \), under the stress
distribution \( \sigma_{xh} \) may be determined through the superposition principle
(Step 4 in Figure 2):

\[
K_{I(xh)}^* = \sum_{m=0}^{M=5} B_m^* \cdot K_{I(m)}^*
\]

(5)

5. STEP 5: FATIGUE ANALYSIS OF THE LUG

The two-parameter theoretical model proposed by Carpinteri [36], based
on the well-known Paris law [16], is here applied to numerically
simulate the fatigue growth of a semi-elliptical crack in the lug net
cross-section, subjected to pulsating cyclic tension. Such a model
assumes that the surface crack under Mode I loading condition keeps a
semi-elliptical shape during the whole propagation. Details regarding
the application of the above model to metallic components with
different geometries, such as T-joint, double curvature shells and
round bars, can be found in Refs [33,34,37-41]. The dimensionless SIF
values, \( K_{I(xh)}^* \), employed in the Paris law, are computed according to
the procedure described in Section 4.

For each initial crack configuration listed in Table 2, the crack
aspect ratio \( \alpha \) is plotted against the relative crack depth \( \xi \) (Figure
Such results are compared with the experimental data provided in Ref. [7].

Note that, being the SIF values here computed for $0.1 \leq \xi \leq 0.7$ and $0.1 \leq \alpha \leq 1.2$, the fatigue crack paths can also be analysed in such ranges. Therefore, in the case of an experimental value $\xi_0$ out of the above range (see PT4, PT5 and PT6 in Table 2), $\xi_0$ is assumed to be equal to 0.1 in the theoretical model, whereas the corresponding value of both $\alpha_0$ and number of loading cycles, needed to the model, are estimated from the experimental data. Moreover, the FCG related to PT14 specimen (Table 2) has not been performed since the experimental value of $\alpha_0$ is out of the above range.

A quite satisfactory agreement, in terms of crack path, can be generally observed for all the examined specimens. The PT10 specimen shows an experimental FCG that is far from the numerical estimation, even if the proposed model is able to catch the experimental FCG of the PT11 specimen which presents geometry and loading condition very similar to those of PT10 specimen (see Table 1 and 2).

Moreover, for each initial crack configuration listed in Table 2, the relative crack depth $\xi$ is plotted against the number of loading cycles $N$ (Figure 8). Such results are compared with the experimental data in Ref. [7].
Figure 8

From Figure 8, we can observe a good agreement between the numerical results and the experimental data for all the geometric configurations except specimen PT10, as has been previously remarked.

Finally, the shape evolution of the crack front at some representative number of loading cycles is displayed in Figure 9, by comparing the numerical results with the experimental data presented in Ref. [7].

Figure 9

The quality of the numerical results can be evaluated through two error indexes computed at the examined number of loading cycles:

\[
I_\varepsilon = \left| \frac{\varepsilon_{\text{exp}} - \varepsilon_{\text{num}}}{\varepsilon_{\text{exp}}} \right| \cdot 100\% \quad (6a)
\]

\[
I_\alpha = \left| \frac{\alpha_{\text{exp}} - \alpha_{\text{num}}}{\alpha_{\text{exp}}} \right| \cdot 100\% \quad (6b)
\]

For each of the initial crack configuration examined, both the maximum values of \(I_\varepsilon\) and \(I_\alpha\) are reported in Figure 9. The error index related to the relative crack depth ranges from 1.2\% to 24.3\%, whereas that related to the crack aspect ratio ranges from 5.0\% to 14.2\%, with the
exception of the PT10 specimen for which such error indexes are quite greater ($I_{\varepsilon} = 65.5\%$ and $I_{\alpha} = 49.9\%$).

6. CONCLUSIONS

In the present paper, the fatigue behaviour of a lug joint made of PMMA, containing a semi-elliptical crack under pulsating cyclic tension, has been examined through a simplified procedure.

First of all, the uncracked lug has been modelled through a 2D FE mesh to determine the stress field in the lug net cross-section. Then such a stress field has been approximated through a power series expansion. The SIF values have been evaluated by applying the superposition principle to the dimensionless SIFs computed for a finite-thickness plate under some elementary stress distributions.

Finally, the obtained results in terms of crack path, crack depth evolution, and crack front evolution are compared with some experimental data available in the literature showing a good agreement. The proposed numerical procedure is simple and computationally efficient, avoiding 3D complex meshes or remeshing techniques. Such a procedure may be an interesting tool for practical applications, since it seems to be able to perform, with an acceptable degree of confidence, the fatigue assessment of a lug joint with a semi-elliptical surface defect.
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APPENDIX

Power series expansions approximating the normalised stress field $\sigma_{xh}^*$ through a polynomial fitting of 5th order for each lug geometric configuration:

(i) $W/D = 3.0$ (Pearson coefficient equal to 0.999):

$$\sigma_{xh}^*(\eta) \equiv \sum_{m=0}^{M=5} B_m \eta^m = \left[ 5.5047 - 0.6327 \cdot a + 4.4081 \cdot (10)^{-2} \cdot a^2 - 1.5512 \cdot (10)^{-3} \cdot a^3 + 2.5661 \cdot (10)^{-5} \cdot a^4 - 1.5963 \cdot (10)^{-7} \cdot a^5 \right] +$$

$$+ \left[ 0.6327 \cdot a - 8.8161 \cdot (10)^{-2} \cdot a^2 + 4.6536 \cdot (10)^{-3} \cdot a^3 - 1.0265 \cdot (10)^{-4} \cdot a^4 + 7.9814 \cdot (10)^{-7} \cdot a^5 \right] \eta +$$

$$+ \left[ 4.4081 \cdot (10)^{-2} \cdot a^2 - 4.6536 \cdot (10)^{-3} \cdot a^3 + 1.5397 \cdot (10)^{-4} \cdot a^4 - 1.5963 \cdot (10)^{-6} \cdot a^5 \right] \eta^2 +$$

$$+ \left[ 1.5512 \cdot (10)^{-3} \cdot a^3 - 1.0265 \cdot (10)^{-4} \cdot a^4 + 1.5963 \cdot (10)^{-6} \cdot a^5 \right] \eta^3 +$$

$$+ \left[ 2.5661 \cdot (10)^{-5} \cdot a^4 - 7.9814 \cdot (10)^{-7} \cdot a^5 \right] \eta^4 +$$

$$+ \left[ 1.5963 \cdot (10)^{-7} \cdot a^5 \right] \eta^5 \quad (A.1)$$

(ii) $W/D = 2.5$ (Pearson coefficient equal to 1.000):

$$\sigma_{xh}^*(\eta) \equiv \sum_{m=0}^{M=5} B_m \eta^m = \left[ 5.5137 - 0.7060 \cdot a + 6.2082 \cdot (10)^{-2} \cdot a^2 - 2.9278 \cdot (10)^{-3} \cdot a^3 + 6.6343 \cdot (10)^{-5} \cdot a^4 - 5.7292 \cdot (10)^{-7} \cdot a^5 \right] +$$

$$+ \left[ 0.7060 \cdot a - 0.1242 \cdot a^2 + 8.7834 \cdot (10)^{-3} \cdot a^3 - 2.6537 \cdot (10)^{-4} \cdot a^4 + 2.8646 \cdot (10)^{-6} \cdot a^5 \right] \eta +$$

$$+ \left[ 6.2082 \cdot (10)^{-2} \cdot a^2 - 8.7834 \cdot (10)^{-3} \cdot a^3 + 3.9806 \cdot (10)^{-4} \cdot a^4 - 5.7292 \cdot (10)^{-6} \cdot a^5 \right] \eta^2 +$$

$$+ \left[ 2.9278 \cdot (10)^{-3} \cdot a^3 - 2.6537 \cdot (10)^{-4} \cdot a^4 + 5.7292 \cdot (10)^{-6} \cdot a^5 \right] \eta^3 +$$

$$+ \left[ 6.6343 \cdot (10)^{-5} \cdot a^4 - 2.8646 \cdot (10)^{-7} \cdot a^5 \right] \eta^4 +$$

$$+ \left[ 5.7292 \cdot (10)^{-7} \cdot a^5 \right] \eta^5 \quad (A.2)$$

(iii) $W/D = 2.0$ (Pearson coefficient equal to 1.000):

$$\sigma_{xh}^*(\eta) \equiv \sum_{m=0}^{M=5} B_m \eta^m = \left[ 5.5967 - 0.8395 \cdot a + 9.2605 \cdot (10)^{-2} \cdot a^2 - 5.9004 \cdot (10)^{-3} \cdot a^3 + 1.8454 \cdot (10)^{-4} \cdot a^4 - 2.2365 \cdot (10)^{-6} \cdot a^5 \right] +$$

$$+ \left[ 0.8395 \cdot a - 0.1852 \cdot a^2 + 1.7701 \cdot (10)^{-2} \cdot a^3 - 7.3916 \cdot (10)^{-4} \cdot a^4 + 1.1183 \cdot (10)^{-5} \cdot a^5 \right] \eta +$$

$$+ \left[ 9.2605 \cdot (10)^{-2} \cdot a^2 - 1.7703 \cdot (10)^{-3} \cdot a^3 + 1.1072 \cdot (10)^{-4} \cdot a^4 - 2.2365 \cdot (10)^{-5} \cdot a^5 \right] \eta^2 +$$

$$+ \left[ 5.9004 \cdot (10)^{-3} \cdot a^3 - 7.3816 \cdot (10)^{-4} \cdot a^4 + 2.2365 \cdot (10)^{-5} \cdot a^5 \right] \eta^3 +$$

$$+ \left[ 1.8454 \cdot (10)^{-4} \cdot a^4 - 1.1183 \cdot (10)^{-5} \cdot a^5 \right] \eta^4 +$$

$$+ \left[ 2.2365 \cdot (10)^{-6} \cdot a^5 \right] \eta^5 \quad (A.3)$$
(iv) \( W/D = 1.5 \) (Pearson coefficient equal to 1.000):

\[
\sigma_{st}(\eta) \equiv \sum_{m=0}^{M=5} B_m \cdot \eta^m = \left[ 6.5217 - 1.2174 \cdot a + 0.1841 \cdot a^2 - 1.6142 \cdot (10)^{-2} \cdot a^3 + 6.2243 \cdot (10)^{-4} \cdot a^4 - 7.8813 \cdot (10)^{-6} \cdot a^5 \right] +
\]
\[
+ \left[ 1.2174 \cdot a - 0.3683 \cdot a^2 + 4.8427 \cdot (10)^{-2} \cdot a^3 - 2.4897 \cdot (10)^{-3} \cdot a^4 + 3.9406 \cdot (10)^{-5} \cdot a^5 \right] \eta +
\]
\[
+ \left[ 0.1841 \cdot a^2 - 4.8427 \cdot (10)^{-2} \cdot a^3 + 3.7346 \cdot (10)^{-3} \cdot a^4 - 7.8813 \cdot (10)^{-5} \cdot a^5 \right] \eta^2 +
\]
\[
+ \left[ 1.6142 \cdot (10)^{-2} \cdot a^3 - 2.4897 \cdot (10)^{-3} \cdot a^4 + 7.8813 \cdot (10)^{-5} \cdot a^5 \right] \eta^3 +
\]
\[
+ \left[ 6.2243 \cdot (10)^{-4} \cdot a^4 - 3.9406 \cdot (10)^{-5} \cdot a^5 \right] \eta^4 +
\]
\[
+ \left[ 7.8813 \cdot (10)^{-6} \cdot a^5 \right] \eta^5
\]

(A.4)
REFERENCES

[1] Schijve J. Fatigue of Structures and Materials. Springer Netherlands; 2009.
[2] Gran RJ, Orazio FD, Paris PC, Irwin GR, Hertzberg R. Investigation and analysis development of early life aircraft structural failures. Air Force Flight Dynamic Laboratory, Ohio 45433, AFFDL-TR-70-149, 1971.
[3] Schijve J, Hoeymakers AHW. Fatigue crack growth in lugs. Fatigue Fract Eng Mater Struct 1979;1:185-201.
[4] Frost NE, Marsh KJ, Pook LP. Metal Fatigue. Dover Pubns; 2003.
[5] Rama kishan CV, Mahaboob Basha A. Design analysis of lug joint in an airframe structure using Finite Element Method. IRJET 2017; 4:2282-2287.
[6] Smith CW, Jolles M, Peters WH. Stress intensities for cracks emanating from pin-loaded holes. ASTM STP 631 1977, 190-201.
[7] Grandt AF Jr, Harter JA, Tritisch DD. Semielliptical cracks along holes in plates and lugs. AFWAL-TR-83-3043. Air Force Wright Aeronautical Laboratories, OH 1982, 1-125.
[8] Geier W. Strength behavior of fatigue cracked lugs. Royal Aircraft Establishment, Dissertation Technical University, Munich, 1980.
[9] Kim JH, Lee SB, Hong SG. Fatigue crack growth behavior of Al7050-T7451 attachment lugs under flight spectrum variation. Theor Appl Fract Mec 2003;40:135-44.
[10] Mikheevskiy S, Glinka G, Algera D. Analysis of fatigue crack growth in an attachment lug based on the weight function technique and the UniGrow fatigue crack growth model. Int J Fatigue 2012;42:88-94.
[11] Carpinteri A. Handbook of Fatigue Crack Propagation in Metallic Structures. 1st ed. Elsevier Science; 1994.
[12] Schijve J. Fatigue of structures and materials in the 20th century and the state of the art. Int J Fatigue 2003;25:679-702.
[13] Krupp U. Fatigue Crack Propagation in Metals and Alloys: Microstructural Aspects and Modelling Concepts. WILEY-VCH; Darmstadt 2007.
[14] Palmert F, Moverare J, Gustafsson D, Busse C. Fatigue crack growth behaviour of an alternative single crystal nickel base superalloy. Int J Fatigue 2018;109:166-81.
[15] Chai M, Zhang Z, Duan Q, Song Y. Assessment of fatigue crack growth in 316LN stainless steel based on acoustic emission entropy. Int J Fatigue 2018;109:145-56.
[16] Paris PC, Erdogan F. A critical analysis of crack propagation laws. J Basic Eng 1963;85:528-34.
[17] Elber W. The significance of fatigue crack closure. ASTM STP 486 1971 230-42.
[18] Newman JC. Fracture analysis of surface-and through cracked sheets and plates. Eng Fract Mech 1973;5:667-89.
[19] Walker EK. The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7076-T6 aluminium, in: Effect on environment and complex load history on fatigue life. ASTM STR 462,1970, p. 1-4.
[20] Kujawski D. A fatigue crack driving force parameter with load ratio effects. Int J Fatigue 2001;23:S239-46.
[21] Noroozi AH, Glinka G, Lambert S. A two parameter driving force for fatigue crack growth analysis. Int J Fatigue 2005;27:1277-96.
[22] Newman JC Jr, Raju IS. Stress intensity factor equations for cracks in three-dimensional finite bodies subjected to tension and bending loading. In: NASA technical memorandum 85793, 1986.
[23] Hellen TK. On the method of virtual crack extension. Int J Numer Meth Eng 1975;9:187-207.
[24] Kathiresan K, Brussat TR. Advanced life analysis methods. AFWAL-TR-84-3080, OH; 1984.
[25] Gencoz O, Goransonu G, Merrill R. Application of finite element analysis techniques for predicting crack propagation in lugs. Int J Fatigue 1980;2(3):121-9.
[26] Barter S, Burchill M, Jones M. Measured fatigue crack growth increments versus predictions for small cracks in 7XXX aluminium alloys. Int J Fatigue 2017;105:144-59.
[27] Belytschko T, Black T. Elastic crack growth in finite elements with minimal remeshing. Int J Numer Meth Eng 1999;45(5):601-20.
[28] Sukumar N, Moës N, Moran B, Belytschko T. Extended finite element method for three-dimensional crack modelling. Int J Numer Meth Eng 2000;48:1549-70.
[29] Bergara A, Dorado JI, Martin-Meizoso A, Martínez-Esnaola JM. Fatigue crack propagation in complex stress fields: Experiments and numerical simulations using the Extended Finite Element Method (XFEM). Int J Fatigue 2017;103:112-21.
[30] Dirik H, Yalçinkaya T. Crack path and life prediction under mixed mode cyclic variable amplitude loading through XFEM. Int J Fatigue 2018; 114:34-50.

[31] Naderi M, Iyyer N. Fatigue life prediction of cracked attachment lugs using XFEM. Int J Fatigue 2015; 77:186-93.

[32] Vantadori S, Carpinteri A, Scorza D. Simplified analysis of fracture behaviour of a Francis hydraulic turbine runner blade. Fatigue Fract Eng Mater Struct 2013; 36:679-88.

[33] Carpinteri A, Ronchei C, Scorza D, Vantadori S. Fracture mechanics based approach to fatigue analysis of welded joints. Eng Fail Anal 2015; 49:67-78.

[34] Grandt AF Jr, Hinnerichs TD. Stress intensity factor measurements for flawed fastener holes. AMMRC MS 74-8, 1974.

[35] Lim IL, Johnston IW, Choi SK. On stress intensity factor computation from the quarter-point element displacements. Comm Appl Numer Meth 1992; 8:291-300.

[36] Carpinteri A. Shape change of surface cracks in round bars under cyclic axial loading. Int J Fatigue 1993; 15:21-6.

[37] Carpinteri A, Brighenti R, Vantadori S. Circumferentially notched pipe with an external surface crack under complex loading. Int J Mech Sci 2003; 45(12):1929-47.

[38] Carpinteri A, Brighenti R, Huth HJ, Vantadori S. Fatigue growth of a surface crack in a welded T-joint. Int J Fatigue 2005; 27(1): 59-69.

[39] Carpinteri A, Brighenti R, Vantadori S. Notched shells with surface cracks under complex loading. Int J Mech Sci 2006; 48(6): 638-49.

[40] Carpinteri A, Brighenti R, Vantadori S. Notched double-curvature shells with cracks under pulsating internal pressure. Int J Pres Ves Pip 2009; 86(7): 443-53.

[41] Carpinteri A, Vantadori S. Sickle-shaped surface crack in a notched round bar under cyclic tension and bending. Fatigue Fract Eng M 2009; 32(3): 223-32.
TENSION FAILURE ASSESSMENT AT LUG HOLE EDGES

Daniela Scorza, Andrea Carpinteri, Sabrina Vantadori

Department of Engineering & Architecture, University of Parma, Parco Area delle Scienze 181/A,
43124 Parma, Italy

Corresponding Author:
Daniela Scorza, email: daniela.scorza@unipr.it

LIST OF FIGURES AND TABLES CAPTIONS

Figure 1. Lug joint under tension.
Figure 2. Framework of the procedure employed for the FCG analysis.
Figure 3. Lug: (a) geometric configuration; (b) semi-elliptical crack at the lug net cross-section.
Table 1. Maximum applied load, geometric sizes, and Stress-Concentration Factor.
Table 2. Initial semi-axes lengths of the semi-elliptical cracks and net cross-section size along Y-direction.
Table 3. Mechanical properties of the PolyMethylMethAcrylate [7].
Figure 4. FE discretisation of the lug characterised by $W/D=1.5$: (a) 2D model; (b) 3D model.

Figure 5. (a) Stress concentration factor $K_f$ against $n_e$ (No. of FE) by varying $W/D$; (b) normalised stress distribution $\sigma_{xh}^*$ against the coordinate $y$, determined by 2D FE analysis and corresponding polynomial fitting, related to the lug characterised by $W/D=1.5$.

Figure 6. (a) Semi-elliptical surface crack in a finite-thickness plate; (b) elementary stress distributions directly applied to the crack faces.

Figure 7. Crack aspect ratio against relative crack depth for specimens: (a) PT4 and PT5; (b) PT6 and PT7; (c) PT8; (d) PT10 and PT11. Experimental results related to such specimens [7] are also plotted.

Figure 8. Number of loading cycle against relative crack depth for specimens: (a) PT4 and PT5; (b) PT6 and PT7; (c) PT8; (d) PT10 and PT11. Experimental results related to such specimens [7] are also plotted.

Figure 9. Both numerical and experimental crack front evolutions for specimens: (a) PT4; (b) PT5; (c) PT6; (d) PT7; (e) PT8; (f) PT10; (g) PT11.
Figure 1.
Determination of the stress distribution $\sigma_{xh}$ by means 2D FE analysis

Set the material parameters $E, \nu, C, m$

Set the lug geometric sizes $W, D, t, T$

Set the loading condition $p_{max}$

STEP 1

STEP 2

STEP 3

STEP 4

STEP 5

SIFs evaluation for a semi-elliptical crack in a finite-thickness plate under $\sigma_{t(m)}$ (Eq.(3))

Set the crack configuration $\alpha_0, \xi_0$

SIF evaluation for a semi-elliptical crack in the lug (Eq.(5))

FCG analysis through the theoretical model proposed by Carpinteri (Ref.[36])

STOP

Figure 2.
Figure 3.
### Table 1.

| Specimen No. | $P_{max}$ [N] | $W/D$ [-] | $W$ [mm] | $D$ [mm] | $T$ [mm] | $K_T$ [-] |
|--------------|--------------|-----------|----------|---------|---------|---------|
| PT4          | 4893         | 3.0       | 171.45   | 57.15   | 18.12   | 5.50    |
| PT5          | 5338         |           |          |         |         |         |
| PT6          | 4893         | 2.5       | 142.88   | 57.15   | 18.10   | 5.51    |
| PT7          | 4448         |           |          |         |         |         |
| PT8          | 3336         | 2.0       | 114.30   | 57.15   | 18.14   | 5.60    |
| PT14         | 4003         |           |          |         |         |         |
| PT10         | 2224         | 1.5       | 85.73    | 57.15   | 18.22   | 6.52    |
| PT11         | 2224         |           |          |         |         |         |

### Table 2.

| Specimen No. | $a_0$ [mm] | $b_0$ [mm] | $t$ [mm] | $\xi_0 = a_0/t$ [-] | $\alpha_0 = a_0/b_0$ [-] |
|--------------|------------|------------|----------|---------------------|------------------------|
| PT4          | 2.691      | 6.650      | 57.150   | 0.05                | 0.40                   |
| PT5          | 3.395      | 5.159      | 57.150   | 0.06                | 0.66                   |
| PT6          | 1.576      | 5.061      | 42.863   | 0.04                | 0.31                   |
| PT7          | 4.211      | 4.898      | 42.863   | 0.10                | 0.86                   |
| PT8          | 3.821      | 4.799      | 28.575   | 0.13                | 0.80                   |
| PT14         | 12.246     | 5.414      | 28.575   | 0.43                | 2.26                   |
| PT10         | 3.979      | 5.530      | 14.288   | 0.28                | 0.72                   |
| PT11         | 3.549      | 5.315      | 14.288   | 0.25                | 0.67                   |

### Table 3.

|                              |                 |
|------------------------------|-----------------|
| Young modulus, $E$           | 1.8-3.1 GPa     |
| Poisson ratio, $\nu$        | 0.35-0.4        |
| Ultimate tensile strength, $R_m$ | 48-76 MPa      |
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
| No. CYCLES | $I_{\xi, max} = 5.6\%$ | $I_{\alpha, max} = 8.6\%$ |
|------------|------------------|------------------|
| 18000      |                  |                  |
| 16000      |                  |                  |

| No. CYCLES | $I_{\xi, max} = 10.2\%$ | $I_{\alpha, max} = 7.7\%$ |
|------------|------------------|------------------|
| 26000      |                  |                  |
| 25000      |                  |                  |
| 24000      |                  |                  |

| No. CYCLES | $I_{\xi, max} = 8.8\%$ | $I_{\alpha, max} = 7.0\%$ |
|------------|------------------|------------------|
| 21000      |                  |                  |
| 20000      |                  |                  |
| 19000      |                  |                  |
$I_{\xi, \text{max}} = 1.2\%$
$\alpha_{\text{max}} = 9.0\%$

Figure 9.