Generalized second law in phantom dominated universe

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Abstract

We study the conditions of validity of the generalized second law in phantom dominated era.
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1 Introduction

Astrophysical data show that the equation of state parameter \( \omega \) lies near \( \omega = -1 \) and possibly \( \omega < -1 \), leading to an accelerated universe [1]. Some present data seem to favor an evolving dark energy with \( \omega \) less than \(-1\) at present epoch from \( \omega > -1 \) in the near past [2]. So we may assume that the universe is dominated by a perfect fluid for which \( \omega < -1 \), dubbed phantom energy [3, 4]. This description of the universe may contain finite-time future singularity, accompanied with divergence of dark energy density, called Big Rip [3]. Depending on the phantom potential, this model may also lead to universes expanding for ever, e.g., universe tending to a de Sitter space time [5]. The effects of gravitational back-reactions can also counteract that of phantom energy and can become large enough to terminate the phantom dominated phase before the big rip [6].

Thermodynamics of the expanding universe has also been the subject of several studies [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Phantom thermodynamics looks leading to negative entropy of the universe [17] or to appearance of negative temperatures [18]. In accelerated expanding universe, besides the normal entropy, a cosmological horizon entropy can also be considered. One can investigate the conditions for which the generalized second law of thermodynamics (GSL) holds [10, 11]. In these cases GSL asserts that the sum of the horizon entropy, and the normal entropy of the fluid is an increasing function of time. In [10] the change in event-horizon area in cosmological models that depart slightly from de Sitter space was investigated, and it was shown that the area and consequently the (de Sitter) horizon entropy are non decreasing functions of time. In the presence of a viscous fluid, there was found that GSL was satisfied provided that the temperature of the fluid was equal to or lower than de Sitter horizon temperature, \( T_{dS} = H/(2\pi) \), where \( H \) is the Hubble parameter. In [13] the GSL

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for two specific examples of phantom dominated universe was discussed. In the first example it was shown that for a time independent parameter of state, the total entropy is a constant. In the second example using a power law potential and the slow climb approximation, it was shown that GSL was satisfied. In the absence of a well defined Hawking temperature for cosmological horizon, the temperature of the phantom fluid in [13] was assumed to be the same as the de Sitter temperature.

The phantom universe, has a future event horizon (cosmological horizon), $R_h$. In this model the Hubble parameter is an increasing function of time so we expect that the horizon entropy, in contrast to the model discussed in [10], decreases with time.

In this paper we consider the phantom dominated era, and prove that the future event horizon area is a non-increasing function of time. It is shown that one necessary condition for satisfying GSL is the positivity of the temperature. If the temperature is assumed to be proportional to the de Sitter temperature 

$$ T = \frac{bH}{2\pi}, $$

an inequality in terms of Hubble parameter, $H$, the future event horizon, $R_h$, and the parameter $b$, will be obtained. By solving this inequality one can determine the dynamics of $R_h$, $H$, and also the range of $b$. We show that GSL is satisfied for a wide range of models, provided that $0 < b \leq 1$. We also study the condition of validity of GSL in the transition from quintessence regime to the phantom dominated era. At the end, two specific examples are used to emphasize our general results.

We use the units $\hbar = c = G = k_B = 1$.

## 2 Phantom thermodynamics

The spatially flat FRW universe in comoving coordinates $(t, x, y, z)$, is described by the metric

$$ ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). $$

The relative expansion velocity of the universe, in terms of the scale factor $a(t)$, is given by the Hubble parameter $H = \dot{a}/a$. The over dot indicates derivative with respect to the comoving time $t$. We assume that the universe is filled with a perfect fluid with the energy density $\rho$, and the pressure $P$. The fluid state parameter $\omega$ is defined through the equation of state $P = \omega \rho$. Einstein equations are

$$ \frac{dH}{dt} = -4\pi(P + \rho), $$

$$ H^2 = \frac{8\pi}{3}\rho. $$

The above equations result

$$ \omega = -1 - \frac{2\dot{H}}{3H^2}. $$

For an accelerating universe we have $\dot{H} + H^2 > 0$, which, in terms of $\omega$, can be expressed as $\omega < -\frac{4}{3}$. The corresponding cosmological fluid is then consisted of
some sort of energy density which has a negative pressure known as dark energy. Matter with \( \omega < -1 \) is dubbed phantom energy. By considering \( 4 \), one can see that for phantom fluid: \( \dot{H} > 0 \), which using \( 3 \) results \( P + \rho < 0 \).

There are various approaches to study the dark energy. One approach is introducing scalar fields. The behavior of \( \omega \) in the phantom regime, \( \omega < -1 \), can be related to the presence of a phantom scalar field, \( \phi \), with a wrong sign kinetic term \( 3 \). Depending on the form of the phantom scalar field potential, different solutions such as asymptotic de Sitter, big rip, and so on may be obtained \( 5 \). To describe \( \omega > -1 \), or quintessence regime, a normal scalar field, \( \sigma \), known as quintessence scalar field, can be used \( 19 \). The phase transitions in inflationary models can be investigated using hybrid models. These models, at least, are composed of two scalar fields \( 20 \), e.g., one of these models is the quintom model which in order to describe the transition from \( \omega > -1 \) to \( \omega < -1 \) regime, assumes that the cosmological fluid, besides the ordinary matter and radiation, is consisted of a quintessence and a phantom scalar field \( 21 \).

Another method to study the acceleration of the universe is to use a running cosmological constant based on the principles of quantum field theory (especially on the renormalization group) which can mimic the behavior expected for quintessence and phantom-like representations of the dark energy. In this method despite of the absence of scalar fields, an effective equation state like \( 4 \) can be obtained \( 22 \).

In this paper we will use the equations \( 3 \), \( 4 \), and assume that the universe is filled with a perfect fluid in phantom phase in the sense that \( \omega < -1 \), but our results will be independent of the scalar fields or other origins of the phantom energy.

Consider the era when phantom phase is dominated i.e, \( \dot{H} > 0 \). We assume that \( a(t) \rightarrow \infty \) when \( t \rightarrow t_s \), so the scale factor will diverge for a future value on the world time: \( t_s \). The radius of the observer’s (future) event horizon is

\[
R_h = a(t) \int_t^{t_s} \frac{dt'}{a(t')}, \quad \int_t^{t_s} \frac{dt'}{a(t')} < \infty. \tag{5}
\]

Using \( 5 \) one can see that \( R_h \) satisfies the following equations

\[
\dot{R}_h = HR_h - 1, \\
\ddot{R}_h = (\dot{H} + H^2)R_h - H. \tag{6}
\]

For de Sitter space time, \( t_s = \infty \), \( R_h = 1/H \), and the eq. \( 6 \) continues to hold.

**Theorem:** \( R_h \) is a non-increasing function of time, \( \dot{R}_h \leq 0 \).

**Proof.** We have \( \dot{H} \geq 0 \), therefore \( \ddot{a} \geq \dot{a}^2/a \). This inequality can be integrated from \( t \) to \( t_s \)

\[
\int_t^{t_s} \frac{\ddot{a}(t')}{a^2(t')} \, dt' \geq \int_t^{t_s} \frac{1}{a(t')} \, dt', \tag{7}
\]

resulting

\[
\frac{1}{\dot{a}(t)} - \frac{1}{\dot{a}(t_s)} \geq \int_t^{t_s} \frac{1}{a(t')} \, dt'. \tag{8}
\]

Suppose that \( \frac{1}{\dot{a}(t_s)} \) is bounded from below by a positive number \( \epsilon \). In our (super)accelerating model of universe, \( \ddot{a} > 0 \) yields

\[
\frac{1}{\dot{a}(t)} > \frac{1}{\dot{a}(t_s)} > \epsilon, \quad t < t_s. \tag{9}
\]
Therefore
\[ \int_a^\infty \frac{da}{aa} > \epsilon \int_a^\infty \frac{da}{a} = \infty. \quad (10) \]

But (10) conflicts with the assumption of having a future horizon (5). Therefore \( 1/\dot{a}(t_s) = 0 \), and using (8), we obtain \( \dot{H}R_h \leq 1 \). Using this result and (9), we arrive at \( \dot{R}_h \leq 0 \).

The future event horizon area is \( 4\pi R_h^2 \). We assume that this horizon area represents a true contribution to the total entropy. The corresponding entropy is proposed to be \( S_h = \pi R_h^2 \).

In this way \( \dot{S}_h = 2\pi R_h \dot{R}_h \) which is negative and shows that \( S_h \) decreases with time. This is in contrast to the assumption of (23), upon which it was argued that the holographic dark energy has no phantom-like behavior.

The entropy of the phantom fluid inside the cosmological horizon of a co-moving observer is related to the energy and the pressure via the first law of thermodynamics
\[ TdS = dE + PdV = (P + \rho)dV + Vd\rho. \quad (12) \]

In terms of \( H \) this law can be written as
\[ TdS = -\frac{1}{4\pi} \frac{dH}{dt}dV + Vd\rho. \quad (13) \]

Using \( V = \frac{4}{3} \pi R_h^3 \) and \( H^2 = (8\pi/3)\rho \), we obtain
\[ TdS = -\dot{H}R_h^2dR_h + \dot{H}R_h^2dH. \quad (14) \]

Then using (9), we arrive at
\[ T\dot{S} = \dot{H}R_h^2. \quad (15) \]

For phantom fluid \( \dot{H} > 0, \dot{S} > 0 \), provided we take \( T > 0 \). GSL states that the sum of the ordinary entropy and the horizon entropy cannot decrease with time
\[ \dot{S} + \dot{S}_h \geq 0. \quad (16) \]

In our model this law yields
\[ \frac{\dot{H}}{H}R_h + 2\pi \dot{R}_h \geq 0. \quad (17) \]

Note that in phantom regime, \( \dot{H} > 0 \) and \( \dot{R}_h < 0 \), so, a necessary condition that the GSL to be satisfied is positivity of the phantom fluid temperature \( T > 0 \). Therefore the GSL is violated in phantom models with positive entropy and negative temperature [18]. For a discussion about this subject see [23], [13].

The GSL asserts that the entropy of an isolated system cannot decrease, but does not determine the sign of the entropy. But if we assume that at the finite time \( t = t_s \) (i.e. when \( R_h = 0 \)) we have \( S = 0 \) [13], and the GSL is assumed to be satisfied in an era before \( t_s \), then the entropy will be negative in this epoch, which agrees to [17] and [24] in which a negative entropy for a specific model of phantom thermodynamics with positive temperature has been derived.
However in the above mentioned papers i.e., [17], [24] and [18], the role of the future event horizon has been ignored.

If the temperature is taken to be

\[ T = \frac{b H}{2\pi}, \]

GSL results

\[ b \leq -\frac{\dot{H} R_h}{H R_h}. \]

(18)

For a de Sitter space-time \( R_h = \frac{1}{H} \), which results \( b \leq 1 \), so we expect that in a phantom model which is small perturbed around de sitter space the fluid temperature is equal to or less than the de Sitter temperature.

If the total entropy is increasing in the universe expansion,

\[ \dot{S} + \dot{S}_h > 0, \]

(19)

equation [18] results

\[ (H^+R_h) > 0. \]

(20)

For \( b = 1 \), we must have \( (HR_h) > 0 \), or using [6]

\[ \ddot{R}_h > 0. \]

(21)

Here \( \dot{R}_h \) is an increasing negative function of time. When \( \dot{R}_h \to 0^- \), \( R_h \) tends to the hubble radius \( 1/H \) (see equation [6]).

Now let us assume that the expansion of the universe is a reversible adiabatic process such that

\[ \dot{S} + \dot{S}_h = 0. \]

(22)

From [18] we obtain \( HR_h^b = \lambda \) where \( \lambda \) is a constant.

For \( b = 1 \), using [5], this results \( \dot{R}_h = \lambda - 1 \) and using the fact that \( R_h(t_s) = 0 \), for \( \lambda \neq 1 \), we obtain

\[ R_h = (1 - \lambda)(t_s - t). \]

(23)

Note that \( \dot{R}_h < 0 \), therefore \( \lambda < 1 \). \( \omega \) is given by

\[ \omega = -\frac{1}{3} - \frac{2}{3\lambda} < -1. \]

(24)

If we take \( t_s = \infty \), \( R_h \) becomes infinite as claimed in [9]. If \( \lambda = 1 \), the space-time is de Sitter, \( R_h = \frac{1}{H} \) and \( \omega = -1 \).

For \( b \neq 1 \), [22] and the first equation of [6] lead to the following equation

\[ \dot{R}_h - \lambda R_h^{1-b} + 1 = 0, \]

(25)

with solutions satisfying

\[ \frac{R_h \Phi(\lambda R_h^{1-b}, 1, 1/\pi)}{1-b} = t_s - t. \]

(26)

\( \Phi \), is the Lerchphi function. The condition \( \dot{R}_h < 0 \) yields \( \lambda < R_h^{b-1} \), but \( R_h(t_s) = 0 \), so to have \( \lambda \neq 0 \), \( b \) must satisfy the inequality \( b < 1 \), i.e, \( T < \frac{2\pi}{2\pi} \).

\( \dot{R}_h = 0 \) occurs at \( R_h = \lambda^{\pi-1} \), which is a branch point of \( \Phi \). At \( t = t_s \) we have \( R_h = 0 \) as expected. See fig. [11].

The above results are general and do not depend on the previous phase of the phantom universe. As it was mentioned in the introduction, based on recent
Figure 1: $t_s - t$ as a function of $R_h$, for $b = 0.9$, $\lambda = 1$ (continuous line) and $\lambda = 1.1$ (points).

astrophysical data, one can consider the possibility of the transition from $\omega > -1$ to $\omega < -1$. So it may be interesting to see how our results may be restricted, if the GSL still hold in the transition epoch. By transition epoch we mean a little neighborhood around the transition time: $t_t$. Like the phantom universe, for a quintessence universe we can consider the cosmological event horizon via (5). The relation (17) still valid. By considering that $\dot{H} < 0$, we deduce that a necessary condition for satisfying GSL in such universe is $\dot{R}_h > 0$. But in the phantom universe, $R_h$ is non-increasing. Therefore for a continuous $\dot{R}_h$ (which may be the result of continuity of $H$ and $R_h$, see (6)), we must have $\dot{R}_h(t_t) = 0$. Using (6), this results $HR_h = 1$ at $t_t$. At the transition time, we have also $\dot{H}(t_t) = 0$. In this way using (11) and (15), we obtain $\dot{S}(t_t) = \dot{S}_h(t_t) = 0$, therefore the total entropy is differentiable and thereby continuous at $t_t$. For $t < (>) t_t$, $S_h$ is an increasing (decreasing) function, while the fluid entropy is a decreasing (increasing) function of time. If the entropy is taken to be negative in the beginning of phantom dominated era (which is not a consequence of GSL which considers only the time derivative of the entropy) and the GSL is assumed to be valid in the transition epoch, then we must have a negative entropy in the quintessence era too.

3 Examples

To illustrate our general results, let us consider two examples. As a first example consider the phantom universe of pole-like type

$$a(t) = a_0(t_s - t)^{-n},$$

(27)

t_s \leq t_s$ and $n > 0$ is a positive real number. $a_0$ is a real positive constant. $\dot{H} > 0$, and $\omega = -1 - 2/(3n) < -1$ is a time independent constant. In this model $HR_h = n/(1+n)$, and $\dot{R}_h = -1/(n+1) < 0$. This is consistent with the theorem imposed after the eq. (6). Following the discussion after the eq. (22), and by taking $\lambda = n/(n+1)$, this model corresponds to a reversible adiabatic expansion...
of an universe with the phantom temperature \( T = H/(2\pi) \) in agreement with the results of [13].

To show the role of the parameter \( b \) in validity of GSL, as an another example, consider the model [15]

\[
a = a_0 \left( \frac{t}{t_s - t} \right)^n,
\]

(28)

where \( a_0 \) and \( n \) are two positive real constants. In this model

\[
H = \frac{n t_s}{t(t_s - t)},
\]

\[
\dot{H} = \frac{nt_s(2t - t_s)}{t^2(t_s - t)^2}.
\]

(29)

The condition \( \dot{H} > 0 \) is satisfied when \( t_s < 2t \). \( \omega \) is time dependent

\[
\omega = -1 - \frac{2(2t - t_s)}{3nt_s}.
\]

(30)

When \( 2t > t_s \), i.e., in phantom dominated era, we have \( \omega < -1 \).

In this model the future event horizon is

\[
R_h = t_s(x - 1)^{-n} \int_1^x (u - 1)^n u^{-2} du,
\]

(31)

where \( x := t_s/t, 1 < x < 2 \). We have

\[
\dot{R}_h = -1 + nx^2(x - 1)^{-n-1} \int_1^x (u - 1)^n u^{-2} du.
\]

(32)

Note that from the above equation one can show that \( \dot{R}_h < 0 \), in agreement with our previous results. This can be verified by considering that for \( x < 2 \) we have

\[
n \int_1^x (u - 1)^n u^{-2} du < \left( \int_1^x \frac{(u - 1)^n((n - 1)u + 2)}{u^3} du = \frac{(x - 1)^{n+1}}{x^2} \right),
\]

(33)

Which can be verified by noting that the integrands satisfy the above inequality for all \( u \) belonging to \([1,2]\).

If GSL is respected, we must have \( (H^\circ R_h) > 0 \), where \( c := 1/b \). This condition can be written as

\[
(x - 1)^{-n-1}(n - c)x^2 + 2cx \int_1^x (u - 1)^n u^{-2} du > 1.
\]

(34)

For \( 1 < x < 2, n > 0 \), and \( c > 0 \) (which follows from the discussion after the eq. (17)), we have \((n - c)x^2 + 2cx > 0\), therefore, (34) reduces to

\[
\int_1^x (u - 1)^n u^{-2} du > \frac{(x - 1)^{n+1}}{(n - c)x^2 + 2cx}.
\]

(35)

The right side of the above inequality can be written as a definite integral

\[
\int_1^x (u - 1)^n u^{-2} du > \int_1^x (u - 1)^n u^{-2} \left[ \frac{(1 - n)(c - n)u^2 + 2(n + cn - c)u + 2c}{((c - n)u - 2c)^2} \right] du,
\]

(36)
which can be rewritten as
\[
\int_1^x \frac{(u - 1)^n u^{-2}}{(c - n)u - 2c} qdu > 0. \tag{37}
\]

\( q \) is defined through
\[
q := Au^2 + 2Bu + D, \tag{38}
\]

\( A := (c - 1)(c - n), B := (n(c - 1) + c - 2c^2) \) and \( D := 4c^2 - 2c \). If for all \( u \) in the interval \( u \in [1, x] \subseteq [1, 2] \), \( q > 0 \), then the inequality \( \Psi_{n-1}(u) < 0 \) holds (does not hold). At \( u = 1 \), \( q(u = 1) = (c - 1)(n + c) \) and at \( u = 2 \), \( q \) is negative: \( q(u = 2) = -2c \). Therefore a necessary condition for validity of GSL is that \( q \) must have two roots such that at least one of them lies in \([1, 2]\). Otherwise \( q \) will be negative in this interval.

If \( n \) and \( c \) satisfy
\[
n^2 > \frac{c^2(1 - 2c)}{(c - 1)^2}, \tag{39}
\]
\( q \) has two roots.

For \( A > 0 \), \( q \) has a root \( 1 < r < 2 \) at \( r = (-B - \sqrt{B^2 - AD})/A \), provided that \( c > 1 \). The other root takes place at \( r' = (-B + \sqrt{B^2 - AD})/A < 2 \).

For \( A < 0 \), \( q \) has a root \( 1 < r = (-B - \sqrt{B^2 - AD})/A < 2 \), provided that \( c > 1 \). The other root takes place at \( r' = (-B + \sqrt{B^2 - AD})/A < 1 \).

In these cases we have \( q(u = 1) > 0 \), hence \( q \) is positive for all values of \( u \) satisfying \( 1 < u < r \) and negative for all values of \( u \) in the interval \( r < u < 2 \). For \( c < 1 \), neither of the roots of \( q \) lies in the interval \([1, 2]\) and the sign of \( q \) is negative.

For \( A = 0 \) (in this case \( q(u) \) is a line), we have either \( c = 1 \) or \( c = n \). For \( c = 1 \), \( q(u = 1) = 0 \) and \( q \) is negative in \([1, 2]\). When \( c = n \), we have \( q(u = 1) = 2c(c - 1) \) and \( q(u = 2) = -2c \), so if \( c < 1 \), \( q \) will be negative in \([1, 2]\).

Using these results we conclude that \( q \) has a root in \([1, 2]\), provided that \( c > 1 \). Other values of \( c \) lead to negative values of \( q \) for all \( u \in [1, 2] \). Therefore \( T < H/(2\pi) \) is a necessary condition for validity of GSL.

Despite the above result, \( c > 1 \) is not an enough condition for validity of GSL, specially in the early period in which the phantom became dominated as is illustrated in fig.2. Using our arguments we can only claim that in the period \( 1 < x < r \), (in an interval before the big rip) the GSL holds. But note that for large \( c \) or large \( n \) we have \( r \approx 2 \), and GSL is applicable approximately in the whole region of the interval \([1, 2]\).

The transition from \( \dot{H} < 0 \) to \( \dot{H} > 0 \), occurs at \( x = 2 \), or \( t = \frac{\dot{t}}{\dot{r}} \), where \( \dot{H} = 0 \), which results \( \dot{S}(t_i) = 0 \). As we have discussed in the last part of the previous section, a necessary condition for satisfying GSL in \( t < \frac{\dot{t}}{\dot{r}} \) is \( \dot{R}(x = 2) = 0 \) which using \( \Psi_{n-1} \), implies
\[
4n \int_1^2 (u - 1)^n u^{-2} du = 1. \tag{40}
\]

The above integral can be written in terms of digamma functions, resulting
\[
4n \left(-\frac{n}{2(n + 1)} - \frac{1}{2(n + 1)} + \frac{1}{2} n \left(\Psi \left(\frac{1}{2} + \frac{1}{2} n\right) - \Psi \left(\frac{1}{2} n\right)\right)\right) = 1. \tag{41}
\]
Figure 2: $\int_1^x (u - 1)^n u^{-2} du - (x - 1)^{n+1}((n - c)x^2 + 2cx)^{-1}$, as a function of $x$ for $n = 1$ and $c = 2$.

Following our previous discussion, we expect that GSL remains valid in the region near $x = 2$ for large $n$. By asymptotic expanding of digamma functions, the above equation, (41), becomes

$$1 - \frac{1}{2n^2} + O\left(\frac{1}{n^4}\right) = 1$$

which is valid for large $n$. Therefore for large $n$, the GSL may be true in the transition epoch. The condition (42), in a short time before the transition: $t = t_t - \epsilon = t_s / 2 - \epsilon$, becomes

$$\int_1^2 \frac{(u - 1)^n u^{-2}}{((c - n)u - 2c)^2} qdu - \frac{c}{8n^2} \epsilon > 0,$$

which based on our previous discussion about the enough condition for validity of GSL, is correct for large $n$. Hence the GSL is respected in the transition epoch only for large values of $n$.

4 Summary

In this paper we discussed the constraints and conditions imposed on cosmological future horizon $R_h$, Hubble parameter $H$, and the temperature $T$, in a phantom dominated universe in order to satisfy the GSL. We proved that the future event horizon is a non-increasing function (see the theorem after eq. (6)) and thereby the corresponding horizon entropy is also a non-increasing function of comoving time. To obtain this result we assumed that one can relate an entropy to the future cosmological horizon, in the same way as black-hole event horizons, despite their different structures [10, 11, 13]. Using the assumption that the (super)accelerated universe is filled with a perfect fluid satisfying the first law of thermodynamics (eq. (12)), we obtained the total entropy and found that the positivity of the temperature is a necessary condition for GSL
We obtained also an inequality in terms of \( R_h \), \( H \), and \( T \) (see eqs. (15), (20) and (21)). This inequality becomes an equation for the adiabatic reversible expansion of the (super) accelerated universe. We solved this equation for \( R_h \), for de Sitter and non-de Sitter temperatures (see eqs. (23) and (26)). We showed that in various cases \( T \) must be less than or equal to de Sitter temperature. By studying the influence of the transition from the quintessence to phantom dominated universe on the GSL, we deduced that the time derivative of the future event horizon and the entropy must be zero at the transition time. However if we adopt that the GSL is valid in the transition epoch and also assume that the entropy is negative in the beginning of the phantom era (which is not a consequence of GSL), a negative entropy in the quintessence regime will be deduced, which is not expected.

**References**

[1] S. Hannestad and E. Mortsell, Phys. Rev. D 66, 063508 (2002); H. Jassal, J. Bagla and T. Padmanabhan, Phys.Rev. D 72, 103503 (2005); A. G. Riess et al., Astrophys. J. 607, 665 (2004); U. Seljak et al., Phys. Rev. D 71, 103515 (2005); D. Huterer and A. Cooray, Phys. Rev. D 71, 023506 (2005); Y. Wang and M. Tegmark, Phys. Rev. D 71, 103513 (2005); U. Alam, V. Sahni and A. A. Starobinsky, JCAP 0406, 008 (2004).

[2] B. Feng, X. Wang and X. Zhang, Phys.Lett. B 607, 35 (2005).

[3] R. R. Caldwell, Phys. Lett. B 545, 23 (2002).

[4] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett 91, 071301 (2003).

[5] V. Faraoni, Class. Quant. Grav. 22, 3235 (2005).

[6] P. Wu and H. Yu, Nucl.Phys. B 727, 355 (2005).

[7] R. Brustein, Phys. Rev. Lett. 84, 2072 (2000).

[8] M. Li, Phys. lett. B 603, 1 (2004).

[9] P. F. Gonzalez-Diaz, [hep-th/0411070](http://arxiv.org/abs/hep-th/0411070).

[10] P. C. W. Davies, Class. Quant. Grav 4, L225 (1987).

[11] P. C. W. Davies, Class. Quant. Grav 5, 1349 (1988).

[12] R. Bousso, Phys. Rev. D 71, 064024 (2005).

[13] G. Izquierdo and D. Pavon, [astro-ph/0505601](http://arxiv.org/abs/astro-ph/0505601), to be appeared in Phys. Lett. B.

[14] G. Izquierdo and D. Pavon, Phys. Rev. D 70, 127505 (2004).

[15] S. Nojiri and S. D. Odinstov, [hep-th/0506212](http://arxiv.org/abs/hep-th/0506212).

[16] Q. Huang and M. Li, JCAP 0408, 013 (2004).
[17] I. Brevik, S. Nojiri, S.D. Odintsov and L. Vanzo, Phys. Rev. D70, 043520(2004).

[18] P. F. Gonzalez-Diaz and C. L. Siguenza, Nucl. Phys. B 697, 363 (2004); E. Babichev, V. Dokuchaev and Y. Eroshenko, Phys. Rev. Lett. 93, 021102 (2004).

[19] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D 59, 063505 (1999).

[20] A. D. Linde, Phys. Rev. D 49, 748 (1994) astro-ph/9307002; A. D. Linde, Phys. Lett. B 259, 38 (1991); H. Wei and R. Cai, astro-ph/0512018.

[21] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005); Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608, 177 (2005); X. F. Zhang, H. Li, Y. S. Piao and X. M. Zhang, astro-ph/0501652.

[22] J. Sola and H. Stafinc, astro-ph/0507110 Phys. Lett. B 624 (2005) 147.

[23] Y. Gong, B. Wang and Y. Zhang, Phys. Rev. D 72, 043510 (2005).

[24] S. Nojiri, S. D.Odintsov, Phys. Rev. D 70, 103522 (2004).