Kendall’s Shape Statistics as a Classical Realization of
Barbour-type Timeless Records Theory approach to Quantum Gravity

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Abstract

I already showed that Kendall’s shape geometry work was the geometrical description of Barbour’s relational mechanics’ reduced configuration spaces (alias shape spaces). I now describe the extent to which Kendall’s subsequent statistical application to such as the ‘standing stones problem’ realizes further ideas along the lines of Barbour-type timeless records theories, albeit just at the classical level.

Was an invited seminar at ‘Foundations of Physics, Munich 2013’.

Highlights: * Kendall’s shape geometry is Barbour’s mechanics’ reduced configuration space.
* Kendall’s shape statistics is a classical timeless records theory
* Records theory is a strategy for the Problem of Time

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1 Introduction

Julian Barbour proposed a scaled relational particle mechanics (RPM) in 1982 with Bruno Bertotti BB82 [1] and a pure-shape RPM in 2003 [2]. These implement both Temporal Relationalism and Spatial Relationalism [3, 4] in senses made precise in Sec 2 that are in accord with Leibniz and Mach’s critiques of Newtonian mechanics. Moreover, Spatial Relationalism is here implemented indirectly. A natural question then is what is the specific geometry of these theories’ reduced configuration spaces?

I began to study this from first principles in [5] but subsequently found that David Kendall’s theory of shape geometry (Sec 3) [6, 7, 8, 9] already covered this in far greater generality in the pure-shape case. Kendall’s work was done in a context entirely different from mechanics: the application he put it to is a statistical theory of shape. Nevertheless, his underlying notion of pure shape is the same as [10, 3] in Barbour’s 2003 RPM. Additionally, the cone over Kendall’s pure-shape geometry is the reduced configuration space for the scaled RPM [11, 3]. Overall, the result of applying the Jacobi–Synge procedure to construct a mechanics from a given metric geometry to Kendall’s geometry coincides (Sec 4) with the reduction of Barbour’s mechanics.

As further motivation, Barbour’s RPM’s are additionally useful as follows [12, 3]. 1) There is an analogy with (conformo)geometrodynamics including leading to new derivations of these forms of GR [13], and to various alternative theories/formulations for gravitational theory [14] and of gauge theory [15]. 2) They model a number of aspects of background independence [16] and of the subsequent Problem of Time in Quantum Gravity [17] and in Quantum Cosmology in particular [18], including various other closed-universe effects [3].

As well as working on RPM’s [1] and their analogy with geometrodynamics [19], Barbour additionally worked on timeless records theory [20] (Sec 5). The present paper shows that this can be tied to (Secs 6–8) Kendall’s own further application: shape statistics. Kendall used this e.g. to determine whether a set of standing stones at Land’s End contained more alignments than could be put down to random chance, which he approached by sampling in threes. Another early application of this method involved investigating whether quasars are aligned.

2 Barbour’s Mechanics in Indirect Form

Temporal Relationalism. This involves adopting ab initio Leibniz’s ‘there is no time for the universe as a whole’ principle [19, 3] as a desirable tenet of background-independence and of closed universes. This is mathematically implemented by 1) postulating geometrical Jacobi type actions [21, 1] that happen to be parametrization-irrelevant. 2) Additionally, such actions are not to contain any extraneous time (such as Newton’s) or time-like variables (such as the lapse of GR). Such actions are of the form

\[ S = \sqrt{2} \int ds \sqrt{W(Q)}. \] (1)

Here \( ds := ||dQ||_M \) the kinetic arc element (geometry on configuration space \( Q \) with metric \( M \)) and the potential factor is \( W = E - V \) for Mechanics or \( W = R - 2\Lambda \) for GR (Ricci 3-scalar \( R \))
and cosmological constant $\Lambda$), in which case $[1]$ is also integrated over space. A distinguished time then emerges as a simplifier to both the equation of motion and the change–momentum relation (relational analogue of velocity-momentum relation) $[19, 25]$. By its form,

$$t^{em} = \int ds/\sqrt{2W(Q)}$$

(2)

it clearly implements Mach’s ‘time is to be abstracted from change’ resolution of Leibniz’s conundrum, moreover doing so in such a way that all changes are given an opportunity to contribute.

Configurational Relationalism $[1, 3, 4, 3]$. This includes both Spatial Relationalism and Internal Relationalism in the sense of gauge theory. It involves regarding some group $G$ of transformations as physically irrelevant. An indirect implementation of this involves working on the principal bundle $P(Q, G)$; then taking into account the linear constraints ensuing from variation with respect to the auxiliary $G$-variables sends one to the relationally-desired quotient space $Q/G$. This use of linear constraints to eliminate auxiliary $G$-variables amounts to extremizing one’s action with respect to $G$, which process Barbour termed best matching. One is to use a cyclic differential form $dg$ for the auxiliary $G$-variables $[3]$ if this implementation of Configurational Relationalism is to be compatible with the above implementation of Temporal Relationalism.

For scaled RPM, $G = \text{Tr}(d) \mathbin{\#} \text{Rot}(d)$ for $\text{Tr}$ the translations, Rot the rotations and $\mathbin{\#}$ denoting semidirect product. The scaled RPM action is, for $V = V(||q^K - q^L||)$ alone,

$$S_{\text{scale}} = \sqrt{2} \int ds\sqrt{E-V} , \quad ds^2 := m_I\delta_{IJ}d_{A,B,C}q^Id_{A,B,C}q^J , \quad d_{A,B}q^I := dq^I - dA - dB \times q^I .$$

(3)

Here, $I$ runs over the particle labels and $p_I$ are momenta conjugate to $q^I$. From this,

$$\sum_{I=1}^N \delta_{IJ}p^I p^J/2m_I + V = E$$

(4)

follows as a primary constraint, and

(zero total momentum of the universe) , $P := \sum_{I=1}^N p_I = 0$

(5)

(zero total angular momentum of the universe) , $L := \sum_{I=1}^N q^I \times p_I = 0 .$

(6)

follow as secondary constraints from varying with respect to $A$ and $B$. For pure-shape RPM, $G = \text{Tr}(d) \mathbin{\#} \text{Rot}(d) \mathbin{\#} \text{Dil}$ for Dil the dilations. The action then is, for $V = V$(ratios of $||q^K - q^L||$ alone),

$$S_{ps} = \sqrt{2} \int ds\sqrt{V} , \quad ds^2 := m_I\delta_{IJ}d_{A,B,C}q^Id_{A,B,C}q^J , \quad d_{A,B,C}q^I := dq^I - dA - dB \times q^I + dCq^I .$$

(7)

This gives the same constraints as before and, from variation with respect to $C$,  

(zero total dilational momentum of the universe) , $D := \sum_{I=1}^N q^I \cdot p_I = 0 .$

(8)

One can instead attempt a direct implementation of Configurational Relationalism when one has the good fortune or skill; i.e. ‘using gauge-invariant quantities’.
3 Kendall’s shape geometry

\[ N \text{ points in dimension } d \text{ form the configuration space } Q(N, d) = \mathbb{R}^{Nd}. \] Kendall used the definition shape space \( S(N, d) := Q(N, d)/\text{Tr}(d) \otimes \text{Rot}(d) \otimes \text{Dil}; \) this coincides with Sec 2’s second quotient space. Moreover, Kendall demonstrated that some of these have highly tractable mathematics \([6, 8, 9]\); \( S^{N-2} \) spheres for 1-\( d \) and \( \mathbb{C}P^{N-2} \) complex projective spaces for 2-\( d \). I term these, respectively, \( N \)-stop metroland and \( N \)-a-gonland, the first nontrivial two of which I call triangleland and quadrilateralland \([22, 23]\). These results hold both topologically and metrically, with the natural spherical and Fubini–Study metrics respectively. The diagonal with \( N = d + 1 \) \([\text{Fig 1}]\) is also topologically simple but not known to be metrically simple. As a special case, \( \mathbb{C}P^1 = \mathbb{S}^2 \), rendering triangleland considerably simpler to study than any larger \( N \)-a-gonland. For this, one has not only tractable geometry and Methods of Mathematical Physics, but also Kendall’s spherical blackboard \([\text{Fig 2 a)}]\).

**Figure 1:** The distinct a) shape spaces and b) relational spaces at the topological level. Only the first 2 columns of each remain straightforward at the metric level (the ‘Casson diagonal’ \([9]\) does not).

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
N & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{N-a-gonlands} & S^0 & S^1 & S^1 & S^2 & S^2 \\
\text{N-stop metrolands} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
N & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{N-a-gonlands} & \mathbb{R} & \mathbb{R}^1 & \mathbb{R}^1 & \mathbb{R}^2 & \mathbb{R}^2 \\
\text{N-stop metrolands} & \mathbb{S}^1 & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} & \mathbb{S}^{d-1}/\mathbb{S}^{d-2} \\
\hline
\end{array}
\]

I then defined relational space \( R(N, d) := Q(N, d)/\text{Tr}(d) \otimes \text{Rot}(d); \) this is Sec 2’s first

**Figure 2:** a) Kendall’s Spherical Blackboard marked with with constituent triangles. This is the fundamental region (space of all unlabelled triangles) 1/3 of a hemisphere. b) Types of triangles are described as per b) in terms of anisoscelesness (left and right slanting departures from isosceles), ellipticity (sharp and flat departures from regularity) and area (maximal for the equilateral triangle \( E \) and minimal for any collinear configuration). Fig 3 further explains this terminology. Barbour calls sharp configurations ‘needles’, whilst Kendall referred collectively to considerably sharp and flat configurations as ‘splinters’. The double collision \( D \) is the sharpest triangle and the merger \( M \) is the flattest one. c) \([23]\) extends a) to quadrilateralland where each point is viewed as an axe (3 + 1 split) as depicted; \( T \) is the centre of mass of the triangular ‘blade’ subsystem.
quotient space. The geometry of this is, moreover, the cone over the above shape space \([11]\) (see \([24]\) for some earlier parallel uses in Celestial Mechanics and Molecular Physics). This further simplifies for the \(N\)-stop metroland case via \(C(S^{N-2}) = \mathbb{R}^{N-1}\) both topologically and metrically. There is no such simplification for \(N\)-a-gonlands except for triangleland, though that case is not flat (but is conformally flat). I note that 2-\(d\) suffices to have a very comprehensive analogy with spatially 3-\(d\) GR \([3]\); 3-\(d\) shape geometry is much harder \([9]\) and for entirely different reasons that GR’s own difficulties, by which 3-\(d\) RPM is not suitable as a productive toy model for GR \([3]\). (Moreover, triangleland itself does not distinguish between 2-\(d\) and 3-\(d\).) Finally, a consequence of the cone structure is that pure-shape problems occur as subproblems in models with scale. Thus considering Kendall’s pure-shape case turned out to be key to solving Barbour’s scaled RPM also.

4 Reducing Barbour = building a Mechanics on Kendall

Figure 3: Progression of coordinate systems for the triangle. a) are particle position coordinates relative to an absolute origin \(O\) and absolute axes \(A\). b) are relative Jacobi interparticle cluster separations; \(X\) denotes the centre of mass of particles 1 and 2; N.B. these coordinates still involve reference to \(A\). Then the configuration space radius \(\rho := \sqrt{\rho_1^2 + \rho_2^2}\). c) are scaled relational coordinates (ie no longer with respect to any absolute axes either). Pure-shape coordinates are then the relative angle \(\Phi\) and some function of the ratio \(\rho_2/\rho_1\); in particular, \(\Theta := 2\arctan(\rho_2/\rho_1)\). For \(n_i := \rho_i/\rho\), \(\text{aniso} = 2n_1 \cdot n_2\), \(\text{area} = 2\{n_1 \times n_2\}_3\), and \(\text{ellip} = n_2^2 - n_1^2\) (the regular configuration then corresponds to \(\text{ellip} = 0\), i.e. to equal partial moments of inertia for the constituent base and ‘median’ subsystems).

Carry out to completion the relevant set of moves in Fig 3 on the Jacobi arc elements \([3]\) and \([7]\) in 1- and 2-\(d\) for arbitrary \(N\) to obtain Barbour’s RPM in reduced form (the explicit end-product of the best-matching procedure) Apply the Jacobi–Synge approach \([21]\) to build a natural mechanics from a metric geometry to Kendall’s shape spaces and to the cone over these. These procedures coincide \([3]\); I term this the Direct = Best-Matched Theorem after the two types of implementation of Configurational Relationalism.

Thus indeed Kendall’s work (and its straightforward extension by coning) amounts to having already derived the detailed topological and metric structure of the reduced configuration spaces for Barbour’s RPM’s. This greatly strengthened the RPM program \([22, 23, 3]\).

Note that this coincidence of procedures was by no means guaranteed. E.g. \([9]\) attributes a distinct metric geometry on the space of shapes to Bookstein. I did not study mechanics for this geometry because it gives material significance to the plane figures themselves (a criterion by which a material is of not totally crushable), whereas relational mechanics considers the constellation of points itself to be the primary entity.
5 Records Theory

The Wheeler–DeWitt equation – the quantum wave equation for each whole-universe GR model – is stationary, i.e. frozen, i.e. timeless. One strategy for dealing with this unexpected result is to take it at face value and see how much Physics one can still do. Dynamics or history are now to be apparent notions to be constructed from the instant \[26\]. This amounts to supplanting ‘becoming’ with ‘being’ at the primary level. ‘Being at a time’ is simpler to deal with: replace this with correlations within a single instant between the configuration under study and another that constitutes the ‘hands of the clock’. Supplanting becoming \[27\] is more involved from a practical perspective.

Theoreticians have differed somewhat both in how to make the notion of record more precise, and in how they envisage the semblance of dynamics may come about. Thus there are in fact a number of Records approaches, Barbour’s included: \[28, 27, 20, 26, 29\]. The particular version I exposit is as follows at the classical level.

Records Postulate 1). Records are information-containing subconfigurations of a single instant that are localized both in space and in configuration space. This is partly so that they be controllable and accurately known and partly so that one can have more than one such to compare. For these reasons, Barbour’s own insistence on whole-universe configurations is dropped.

Records Postulate 2). Records are furthermore required to contain useful information. I take this to mean information along the lines of single-instant correlations. Information Theory, the study of complexity and of pattern recognition are thus relevant precursors to Records Theory.

Note 1) Notions of distance and notions of information as required by Records Postulates 1) and 2) are provided in \[3\].

Note 2) I do not adhere to a purely timeless interpretation myself (see the Conclusion); however timeless records do remain meaningful within further approaches to the PoT \[18\].

In order for this to additionally be a minimalistic timeless records theory, one has to be able to extract a semblance of dynamics or history from the same-instant correlation information in 2).

The combined study of the structural levels of Records Postulates 1) to 3) I term pre-records theory. In a nutshell, this concerns what questions can be asked about the presence of patterns in subconfigurations of a single instant.

[There is also a Records Postulate 3) \[3\] that records can be tied to atemporal propositions, which, amount to a suitable logic. However, this is classically straightforward and thus does not feature further in this article.]
6 Notions of correlation for RPM's

I build upon the most basic notions of this from Statistics. For random variables $X, Y$,

$$\textit{Pearson’s correlation coefficient } \rho_P := \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}}, \quad (9)$$

i.e. the normalization of the covariance by the square roots of the also-statistically significant variances. In this form it is invariant under exchange of dependent and independent status of the $X$ and $Y$. Moreover, one needs further notions of correlation as regards detecting correlations that take forms other than straight lines.

Suppose we are given a constellation of $N$ points in 2-d – the physical content of what we term an $N$-a-gon in this Article. Is it relational to assess this for collinearity using Pearson’s correlation coefficient $\rho_P$? Immediately no\footnote{It is very well known that the upward-pointing line and the downward-pointing one have the opposite extremes of $\rho_P$. Thus $\rho_P$ cannot be entirely relational.} It is very well known that the upward-pointing line and the downward-pointing one have the opposite extremes of $\rho_P$. Thus $\rho_P$ cannot be entirely relational. In fact, variance and covariance are translation-invariant and Pearson’s normalization makes it scale-invariant also, but it indeed fails to be rotationally invariant. How can one remedy this for use in relational problems like Kendall’s standing stones problem or the study of snapshots from the $N$-a-gonland mechanics? One way follows from the covariance matrix

$$V = \begin{pmatrix} \text{Var}(X) & \text{Cov}(X,Y) \\ \text{Cov}(X,Y) & \text{Var}(Y) \end{pmatrix}, \quad (10)$$

being a 2-tensor under the corresponding $\text{Rot}(2) = \text{SO}(2)$ rotations. Thus consider the corresponding invariants: $\det V = \text{Var}(X)\text{Var}(Y) – \text{Cov}(X,Y)^2$ and $\text{tr} V = \text{Var}(X) + \text{Var}(Y)$. In particular, $\sqrt{\det V/\text{tr} V}$ is a scale-invariant ratio of translation-and-rotation invariant quantities and therefore a relationally invariant base-object for single-line linear correlations. This can be repackaged as

$$\rho_{\text{Rel}} := 2\sqrt{1 - \rho_P^2}/\{\omega + \omega^{-1}\}, \quad (11)$$

for $\omega := \sqrt{\text{Var}(X)/\text{Var}(Y)}$.

Note 1) Taking functions of this may allow for more flexibility as regards passing further hurdles from statistical theory.

Note 2) Since I found a basis of shape quantities for triangleland ($\text{aniso, ellip, area}$), (11) must be a function of these. What does it look like in terms of those? If the data is taken to represent the Jacobi vectors, $\sqrt{\det V} \propto I \text{ area}$, as one might anticipate from how area is attained by noncollinearity. Also, $\text{tr} V \propto I \{1 - \text{aniso}\}$. This forewarns us that normalizing by use of $I$ itself (c.f. Fig 3) is not natural in statistical investigations. All in all,

$$\rho_{\text{Rel}} \propto \text{area}/\{1 - \text{aniso}\}. \quad (12)$$

\footnote{The everyday rank correlation tests’ statistics also immediately fail to be rotationally relational, since any two data points can be rotated into a tie (and the procedures for these exclude ties).}
7 Kendall’s shape statistics

Moreover, seeking a single regression line for the best fit of data is not always appropriate, as is made clear by the study of the distribution of standing stones or quasars. One considers more sophisticated geometrical propositions about $N$-a-gon constellations such as sampling in threes: whether there is a disproportionate number of approximately-collinear triples of points. Investigating this relative angle information uses Kendall’s (and Barden, Carne, Le, W. Kendall and Small’s) techniques [7, 8, 30, 32, 9]. A suitable conceptual notion for approximately-collinear triples of points is Kendall’s notion of $\epsilon$-bluntness (see Fig 4.a); he also generalized this from triples to $M$-tuples in 2-$d$ [8].

Figure 4: Meanings in 2-$d$ space for a) Kendall’s [9] $\epsilon$-blunt notion of collinearity: angle at any vertex $\leq \epsilon$ is significant. b) My notion of $\epsilon$-collinear that is adapted to the dynamically useful Jacobi variables: angle $\Phi \leq \epsilon$ is significant.

Some relevant details of the statistical tests used are as follows. The standing stones problem was first posed as a problem to be addressed by some kind of geometrical statistics by Broadbent [31]. One approach to this by Kendall and collaborators involved the assumption that the standing stones lie within a compact convex polygon (‘the Cornish coastline’ for the Land’s End standing stones problem). Detail of the compact convex polygon then enters the test’s outcome. However, this restriction is made in order to cope with uniform independent identically-distributed distributions (much as quantum physicists often perform ‘normalization by boxing’). Using distributions that tail off, however, involvement of a ‘coastline’ ceases to be necessary.

N.B. I make a conceptual distinction between geometrical statistics and shape statistics as follows. The first is applying probability and statistics techniques to sample spaces that are differentiable manifolds [32, 9], involving suitable notions of $\sigma$-field, geometrical measure, change of variables formulae and taking isometries into account. The second concerns a differentiable manifold the points of which are, a fortiori, interpretable as ‘shapes in space’.

8 Application to RPM Records Theory

I first note that the preceding Sec concerns relative angle information. However, not all shape information is relative angle information; some is, rather, ‘ratios of relative separations’ information, i.e. ‘clumping’. Astrophysical examples of clumping include tight binary stars, globular clusters, galaxies and voids. In fact, in 2-$d$ relative angles and ratios of relative separations occur in 1 : 1 proportion, whereas in 1-$d$ there are only ratios of relative separation. Thus the 1-$d$ case permits the simpler study of clumping in isolation from the more complicated relative angle issues. Moreover, clumping considerations are the complementary half to Kendall’s work as regards assessing shape information in 2-$d$. Being simpler, it is unsurprising that statistical techniques for clumping precede Kendall’s work in the literature. E.g. Roach [33] provided a discrete study of this (which is then interpretable in terms of coarse-grainings of RPM configurations).
Figure 5: a) clumping in 1-d and b) a discrete model of it as per [33]. c) Shape data in 2-d. This could consist of e.g. standing stones or of the particle positions in an RPM. d) Is the number of almost-collinear triangles present accountable for by coincidence or is it statistically significant (i.e. a pattern)?

**Clumping–Kendall paradigm for RPM Records Theory.** Roach’s clumping and Kendall’s alignment in threes cover the detection of *discernible patterns* in the ratios of relative separations and relative angle types of shape information in the $N$-a-gon configurations of RPM. Furthermore, it is discernible patterns that distinguish certain subconfigurations of an instant as pre-record, so these statistical techniques constitute quantitative means of determining whether one is in possession of pre-records rather than of purely random subconfigurations. This pioneers the use of a wider variety of notions of complexity and pattern recognition so as to render any physical theory’s Records Approach quantitatively meaningful.

Note 1) Kendall’s ‘collinearity in threes’ method is based on probing with the 2-d model’s minimal fully relational subsystems: the constituent triangles.

Note 2) This method involves the probability measure on the triangle and shape sphere, so it is indeed a geometrical statistics.

Note 3) Fitting within a convex polygon is not per se a restriction for RPM instants, since one can always rescale these so they fit. However, detail of the convex shape in question entering the conclusion would now constitute an unwanted vestige of the absolute. Thus the use of distributions that tail off is welcome as a freeing from such a background-independent imprint.

Note 4) The pre-records found peak about the $\mathbb{R}P^{N-2}$ ‘equator’ of collinearity that each $N$-a-gonland shape space $\mathbb{C}P^{N-2}$ possesses [3].

Note 5) Since RPM’s hitherto studied have involved small particle numbers [22, 23, 3], I point out that 3 points are now too few, as they correspond to the statistically-meaningless sample size of 1 when sampling with triangles. Nontriviality in this sense starts with quadrilateral, which allows for sampling with up to 4 triangles, though one needs a constellation containing somewhat more points in order to attain a statistically significant sample size.

Note 6) Shape Geometry furnishes the notion of distance needed for Records Postulate 1) and shape statistics furnishes the notions of useful information and correlation needed for Records Postulate 2) as a corollary of the ‘Direct = Best-Matched’ Theorem. Thus we have a working theory of classical pre-records.

8
Note 7) How about obtaining a semblance of history? Now significant results for different values of $\epsilon$ carry different implications \[31\]. Were they laid out skillfully by the epoch’s standards for e.g. astronomical or religious reasons ($\epsilon \leq 10$ minutes), or were they just the markers of routes or plots of land ($\epsilon \leq 1$ degree)?

9 Conclusion

Barbour’s relational particle mechanics (RPM) are of considerable interest as Leibniz–Mach-compliant theories of mechanics and as toy models of many aspects of GR-as-geometrodynamics. Direct Jacobi–Synge construction of the mechanics corresponding to Kendall’s shape space geometry and to the cone thereover produces the same mechanics as reducing Barbour’s pure-shape and scaled RPM’s respectively, which he formulated indirectly using ‘best matching’. (This is a procedure that bears much conceptual similarity to the Procrustes procedure of Shape Geometry \[9\].) Subsequent application of shape statistics to Theoretical Physics is new. In the present article, I have shown that this is ready-built for use in the case of the 1- and 2-d RPM’s – the RPM’s that are productive toy models of GR-as-geometrodynamics. This can be used to quantify whether given subconfigurations are records, a matter of interest in timeless (and histories — see below) approaches to Quantum Gravity. Quadrilateraland and N-a-gonaland analogues of the present article’s triangleland set of shape quantities aniso, ellip and area are known \[3, 23\]. I leave expressing the shape statistics quantifiers in terms of these for a future occasion.

Instead, I now pose a difficult and interesting question. In GR-as-geometrodynamics, the redundant configuration space $Q = \text{Riem}(\Sigma)$: the space of Riemannian metrics on a 3-space of fixed topology $\Sigma$, and choices of $G$ include Diff($\Sigma$): the 3-diffeomorphisms, Diff($\Sigma$) $\circ$ Conf($\Sigma$) for Conf($\Sigma$) the conformal transformations, and Diff($\Sigma$) $\circ$ VPConf($\Sigma$) for VPConf($\Sigma$) the global volume-preserving conformal transformations. The corresponding quotients are Wheeler’s superspace($\Sigma$), and York’s conformal superspace CS($\Sigma$) and {CS + V}($\Sigma$) respectively. What are the corresponding notions of (scale and) shape statistics, which one would use to quantitatively detect records within geometrodynamical subconfigurations?

Some stepping-stones toward this are as follows (see \[31\] for details and references). Minisuperspace (homogeneous GR), 2 + 1-d GR and inhomogeneous perturbations about homogeneous GR are all simpler models. Diagonal minisuperspace has a flat shape space (space of anisotropies), so it is too simple for nontrivial application of geometrical statistics. Full GR’s shape space — CS($\Sigma$) — is infinite-d and a stratified manifold. The 2 + 1 GR shape space geometry is considerably simpler than for 3 + 1 and has been studied under the name of Teichmüller space. The inhomogeneous perturbation model is also considerably simpler than the full 3 + 1 GR model, and can be seen as a replacement of the point particles of RPM’s by small inhomogeneous lumps within a GR framework. By that, and by CS($\Sigma$) being a space of shapes, the name ‘shape statistics’ continues to be merited for this example by the argument given in Sec 7. This line of enquiry could lead to new methods and insights as regards the analysis of the detailed data recently gathered by the Planck satellite.

Finally, of course, one’s ultimate interest as regards Quantum Gravity and the Records Theory approach to this is, additionally, quantum-mechanical. Barbour suggested \[20\] the formation of
tracks by $\alpha$-particles in bubble chambers as a paradigm for timeless records formation from which a sense of dynamics or history could be extracted. However, it may well be far more typical for decoherence to leave most to all information in an irretrievable state, as suggested e.g. by the Joos–Zeh paradigm of a dust particle decohering due to the microwave background photons [34]. Barbour also conjectured that quantum probability density function ‘mist’ might peak in some geometrically distinguished region of configuration space, whose configurations happen to be meaningful records. Unfortunately for this conjecture, the concrete examples of small classical and quantum RPM’s that I have worked out so far do not support it [3]. N.B. that Records Theory is useful not only in purely timeless approaches but also in approaches which assume a sense of history [26] [18]. This further adds to the present research’s value.

Acknowledgements I thank the Conference Organizers for inviting me to speak at the ‘Colloquium in Julian Barbour’s honour’ part of this Conference, and I thank Julian Barbour also for many discussions over the years. I also thank Christopher Small, Huiling Le and many people at the ‘Conformal Nature of the Universe’ Conference at the Perimeter Institute in 2012 for discussions, and Grant FQXi-RFP3-1101 from the Foundational Questions Institute (FQXi) Fund, administered by Silicon Valley Community Foundation, Theiss Research and the CNRS, hosted with Marc Lachieze-Rey at APC.

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