Hadronic Light-by-Light scattering in the anomalous magnetic moment of the muon

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Status of \((g - 2)_\mu\) as a test of the Standard Model

Fig. from Jegerlehner 1705.00263

New experiments: \(\times 4\) improvement in accuracy \(\implies\) theory effort needed:

\[ a^\text{exp}_\mu - a^\text{SM}_\mu \approx 300 \cdot 10^{-11}; \delta a^\text{exp,\,future}_\mu \approx 16 \cdot 10^{-11}. \]

\[ \text{HVP (}=\text{O}(\alpha^2))\text{ target accuracy: } \lesssim 0.5\%. \]

\[ \text{HLbL (}=\text{O}(\alpha^3))\text{ target accuracy: } \lesssim 15\%. \]
Approaches to $\alpha_{\mu}^{\text{HLbL}}$

1. **Model calculations:** (the only approach until 2014)
   - based on pole- and loop-contributions of hadron resonances

2. **Dispersive representation:** Bern approach; Mainz approach; Schwinger sum rule.
   - identify and compute individual contributions
   - determine/constrain the required input (transition form factors, $\gamma^*\gamma^* \to \pi\pi$ amplitudes, …) dispersively

3. **Experimental program:** provide input for model & dispersive approach, e.g. $(\pi^0, \eta, \eta') \to \gamma \gamma^*$ at virtualities $Q^2 \lesssim 3\text{ GeV}^2$; currently active program at BES-III see talk by Y. Guo

4. **Lattice calculations:**
   - RBC-UKQCD T. Blum, N. Christ, T. Izubuchi, L. Jin, Ch. Lehner, …
   - Mainz N. Asmussen, A. Gérardin, J. Green, HM, A. Nyffeler, H. Wittig…

This talk: how do the findings from different approaches fit together?
Models for $a_{\mu}^{HLbL}$

A recently updated estimate: NB. much smaller axial-vector contribution

$$a_{\mu}^{HLbL} = (103 \pm 29) \times 10^{-11}$$

Jegerlehner 1809.07413
Wisdom gained from model calculations Prades, de Rafael, Vainshtein 0901.0306

- heavy (charm) quark loop makes a small contribution

\[
a_{\mu}^{HLbL} = \left(\frac{\alpha}{\pi}\right)^3 N_c Q_c^4 c_4 \frac{m_\mu^2}{m_c^2}, \quad c_4 \approx 0.62.
\]

- Light-quarks: (A) charged pion loop is negative & quadratically divergent:

\[
a_{\mu}^{HLbL} \xrightarrow{m_\pi \to 0} \left(\frac{\alpha}{\pi}\right)^3 c_2 \frac{m_\mu^2}{m_\pi^2}, \quad c_2 \approx -0.065.
\]

(B) The neutral-pion exchange is positive, \( \log^2(m_\pi^{-1}) \) divergent:

Knecht, Nyffeler, Perrottet, de Rafael PRL88 (2002) 071802

\[
a_{\mu}^{HLbL} = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{m_\mu^2}{48\pi^2(F_\pi^2/N_c)} \left[ \log^2 \frac{m_\rho}{m_\pi} + O\left(\log \frac{m_\rho}{m_\pi}\right) + O(1) \right].
\]

- For real-world quark masses: using form factors for the mesons is essential, and resonances up to 1.5 GeV can still be relevant ⇒ medium-energy QCD.

- Two closeby vector currents \( V_\mu(x)V_\nu(0) \overset{\text{OPE}}{\sim} \epsilon_{\mu\nu\rho\sigma} \frac{x^\rho}{(x^2)^2} A_\sigma + \ldots \) ‘look like’ an axial current from a distance: doubly-virtual transition form factors of \( 0^{-+} \) and \( 1^{++} \) mesons only fall like \( 1/Q^2 \); but, coupling of axial-vector meson to two real photons forbidden by Yang-Landau theorem.
Test of ‘model wisdom’ via exact dispersive sum rules

\[ m_\pi = 330 \text{ MeV}, \ 96 \cdot 48^3, \ a = 0.063 \text{ fm} \]

Lattice: \[ \Pi_{\mu_1 \mu_2 \mu_3 \mu_4}^E(P_4; P_1, P_2) \equiv \int_{X_1, X_2, X_4} e^{-i \sum_a P_a \cdot X_a} \left\langle J_{\mu_1}(X_1) J_{\mu_2}(X_2) J_{\mu_3}(0) J_{\mu_4}(X_4) \right\rangle_E \]

\[ \mathcal{M}_{TT}(-Q_1^2, -Q_2^2, -Q_1 \cdot Q_2) = \frac{e^4}{4} R_{\mu_1 \mu_3}^E R_{\mu_2 \mu_4}^E \Pi_{\mu_1 \mu_3 \mu_4 \mu_2}^E (-Q_2; -Q_1, Q_1), \]

Dispersive sum rule in \[ \nu = \frac{1}{2}(s + Q_1^2 + Q_2^2) \]: [Pascalutsa, Pauk, Vanderhaeghen (2012)]

\[ \mathcal{M}_{TT}(q_1^2, q_2^2, \nu) - \mathcal{M}_{TT}(q_1^2, q_2^2, 0) = \frac{2\nu^2}{\pi} \int_0^\infty d\nu' \frac{\sqrt{\nu'^2 - q_1^2 q_2^2}}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} \left( \sigma_0 + \sigma_2 \right)(\nu') \sigma(\gamma^* \gamma^* \rightarrow \text{hadrons}) \]

J. Green et al. PRL115 222003 (2015); A. Gérardin et al. 1712.00421 (PRD).
Model for photon-photon fusion cross-section

Contribution of a narrow meson resonance to a $\gamma^*\gamma^* \rightarrow$ hadrons cross-section is

$$\propto \delta(s - M^2) \times \Gamma_{\gamma\gamma} \times \left[ \frac{F_{M\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F_{M\gamma^*\gamma^*}(0, 0)} \right]^2$$

- $\pi^0 \rightarrow \gamma^*\gamma^*$ transition form factor $F_{\pi^0\gamma^*\gamma^*}$ determined in dedicated Lat.QCD calculation
- seven other TFFs were parametrized by $1/(1 + Q^2/M^2)^k$ ($k = 1, 2$) and the parameters $M$ fitted.

Fitting all eight $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^*$ forward amplitudes:

|        | $M_{TT}$ | $M_{TT}^\tau$ | $M_{TT}^a$ | $M_{TL}$ | $M_{LT}$ | $M_{TL}^\tau$ | $M_{TL}^a$ | $M_{LL}$ |
|--------|--------|-------------|-------------|--------|--------|---------------|-------------|--------|
| Pseudoscalar | $\sigma_0/2$ | $-\sigma_0$ | $\sigma_0/2$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ |
| Scalar | $\sigma_0/2$ | $\sigma_0$ | $\sigma_0/2$ | $\times$ | $\times$ | $\tau_{TL}$ | $\tau_{TL}$ | $\sigma_{LL}$ |
| Axial | $\sigma_0/2$ | $-\sigma_0$ | $\sigma_0/2$ | $\sigma_{TL}$ | $\sigma_{LT}$ | $\tau_{TL}$ | $-\tau_{LT}$ | $\times$ |
| Tensor | $\sigma_0 + \sigma_2/2$ | $\sigma_0$ | $\sigma_0 - \sigma_2/2$ | $\sigma_{TL}$ | $\sigma_{LT}$ | $\tau_{TL}$ | $\tau_{TL}^a$ | $\sigma_{LL}$ |
| Scalar QED | $\sigma_{TT}$ | $\tau_{TT}$ | $\tau_{TT}^a$ | $\sigma_{TL}$ | $\sigma_{LT}$ | $\tau_{TL}$ | $\tau_{TL}^a$ | $\sigma_{LL}$ |
Forward LbL amplitudes: contributions of individual mesons

$N_f = 2$, $m_\pi = 193$ MeV, $128 \cdot 64^3$, $a = 0.063$ fm, fully connected diagram, in units of $10^{-6}$

Conclusion:

narrow resonances $+ \pi^+ \pi^-$ model for $\sigma(\gamma^*\gamma^* \rightarrow \text{hadrons})$

provides reasonable description of $\mathcal{M}_{\text{forward}}(\gamma^*\gamma^* \rightarrow \gamma^*\gamma^*)$ from Lat.QCD.
Quark-line contractions

First two classes of diagrams thought to be dominant, with a cancellation between them:

| Weight factor of: | fully connected | $(2,2)$ topology |
|-------------------|-----------------|------------------|
| $SU(2)_f$:        |                 |                  |
| $m_s = \infty$    | isovector-meson exchange | $34/9 \approx 3.78$ | $-25/9 \approx -2.78$ |
|                   | isoscalar-meson exchange | $0$ | $1$ |

| SU(3)$_f$:         |                 |                  |
| $m_s = m_{ud}$     | octet-meson exchange | $3$ | $-2$ |
|                   | singlet-meson exchange | $0$ | $1$ |

Large-$N_c$ argument by J. Bijnens, 1608.01454; $SU(3)_f$ case in 1712.00421; Fig. by J. Green.
Contribution of $(2+2)$ disconnected diagrams to $\gamma^*\gamma^* \to \gamma^*\gamma^*$

$N_f = 2$, $m_\pi = 193$ MeV, $128 \cdot 64^3$, $a = 0.063$ fm, in units of $10^{-6}$

$Q_1^2 = 0.352$ GeV$^2$

- large-$N_c$ motivated prediction (no fit): $(\mathcal{M}_{TT}^{\tau,(2,2)}$ determined by $\sigma_{||} - \sigma_{\perp}$)
  $$\mathcal{M}_{TT}^{\tau,(2,2)} = -\frac{25}{9} \mathcal{M}_{TT}^{\tau,(2,2)\pi^0} + \mathcal{M}_{TT}^{\tau,(2,2)\eta'}$$

- agreement at $\sim 30\%$ level for $Q_i^2 \lesssim 1.2$ GeV$^2$. 
Dispersive methods: the Bern approach

Full HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma}^{(q_1, q_2, q_3)} = i^3 \int_{x,y,z} e^{-i(q_1 x + q_2 y + q_3 z)} \langle 0 | T \{ j_x^{\mu} j_y^{\nu} j_z^{\lambda} j_0^{\sigma} \} | 0 \rangle = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i,$$

e.g. $T_1^{\mu\nu\lambda\sigma} = \epsilon^{\mu\nu\alpha\beta} \epsilon^{\lambda\sigma\gamma\delta} q_1^{\alpha} q_2^{\beta} q_3^{\gamma} (q_1 + q_2 + q_3)^{\delta}$, where the 54 structures are really seven combined with crossing symmetry.

Computing $(g - 2)_\mu$ using the projection technique (directly at $q = 0$):

$$a_{\mu}^{\text{HLbL}} = -\epsilon^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \sum_{i=1}^{12} \hat{\Pi}_i(q_1, q_2; \rho) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)$$

with $\hat{\Pi}_i$ linear combinations of the $\Pi_i$.

Performing all “kinematic” integrals using Gegenbauer-polynomial technique after Wick rotation:

$$a_{\mu}^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1^4 \int_0^\infty dQ_2^4 \int_{-1}^1 d\tau \sqrt{1 - \tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \Pi_i(Q_1, Q_2, \tau)$$

Colangelo, Hoferichter, Procura, Stoffer (2015)
Dispersive methods (II)

▶ Charged-pion contributions: Colangelo et al. PRL118, 232001 (2017)

\[
a_{\mu}^{\pi \text{box}} + a_{\mu,J=0}^{\pi\pi,\pi-\text{poleLHC}} = -24(1) \cdot 10^{-11}
\]

▶ rescattering effects in $\pi^+\pi^-$ are being worked out for partial waves $\ell \leq 2$; first results for the $s$-wave (presented by Colangelo at $(g-2)$ theory workshop 2018).

▶ Dispersive analysis of the $\pi^0 \rightarrow \gamma^*\gamma^*$ transition form factor leads to

\[
a_{\mu}^{\pi^0} = 62.6^{+3.0}_{-2.5} \cdot 10^{-11}
\]

Kubis et al. PRL121, 112002 (2018)

▶ Analysis of $\gamma^*\gamma^* \rightarrow \pi\pi$

Danilkin, Deineka & Vanderhaeghen, $(g-2)$ theory workshop, Mainz 2018
Lattice calculation of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

\[
M_{\mu\nu}(p, q_1) = i \int d^4x \, e^{iq_1 x} \langle \Omega | T \{ j_\mu(x) j_\nu(0) \} | \pi^0(p) \rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{\pi^*\gamma^*\gamma^*}(q_1^2, q_2^2),
\]

Double-virtual

Single-virtual

Contribution to the $(g - 2)_\mu$: using a conformal-mapping parametrization of $\mathcal{F}(Q_1^2, Q_2^2)$ in each virtuality, obtain

\[
a_{\mu}^{\text{HLbL}}|_{\pi^0} = (60.4 \pm 3.6) \cdot 10^{-11} \quad \text{(preliminary)}.
\]

Compatible (and competitive) with the dispersive result of Kubis et al. Gérardin et al 1607.08174 (PRD); $(g - 2)$ Theory workshop, Mainz 2018.
Direct lattice calculation of HLbL in \((g - 2)_\mu\)

At first, this was thought of as a QED+QCD calculation [pioneered in Hayakawa et al., hep-lat/0509016].

Today’s viewpoint: the calculation is considered a QCD four-point Green’s function, to be integrated over with a weighting kernel which contains all the QED parts.

**RBC-UKQCD:** calculation of \(d_{\mu}^{\text{HLbL}}\) using coordinate-space method in muon rest-frame; photon+muon propagators:
- either on the \(L \times L \times L\) torus \((\text{QED}_L)\) (1510.07100–present)
- or in infinite volume \((\text{QED}_\infty)\) (1705.01067–present).

T. Blum, N. Christ, T. Izubuchi, L. Jin, Ch. Lehner, . . .

**Mainz:**
- manifestly covariant \(\text{QED}_\infty\) coordinate-space approach, averaging over muon momentum using the Gegenbauer polynomial technique (1510.08384–present).

N. Asmussen, A. Gérardin, J. Green, HM, A. Nyffeler, . . .
Coordinate-space approach to $a_{\mu}^{HLbL}$, Mainz version

QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y)$

\[
a_{\mu}^{HLbL} = \frac{me^6}{3} \int d^4 y \left[ \int d^4 x \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y) \left( i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x, y) \right) \right].
\]

\[
i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x, y) = -\int d^4 z \, z_{\rho} \left\langle j_{\mu}(x) \, j_{\nu}(y) \, j_{\sigma}(z) \, j_{\lambda}(0) \right\rangle.
\]

- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects & only a 1d integral to sample the integrand in $|y|$.

[Asmussen, Gérardin, Green, HM, Nyffeler 1510.08384, 1609.08454]
What to expect: contribution of the $\pi^0$ to $a^{HLbL}_\mu$ (physical pion mass)

Even more freedom in choosing best lattice implementation than in HVP.

- The form of the $|y|$-integrand depends on the precise QED kernel used:
  - can perform subtractions (Blum et al. 1705.01067; $\mathcal{L} \to \mathcal{L}^{(2)}$), impose Bose symmetries on $\tilde{L}_{[\rho,\sigma];\mu\nu\lambda}(x, y)$ or add a longitudinal piece $\partial^{(x)}_{\mu} f_{\rho;\nu\lambda\sigma}(x, y)$. 

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HLbL in $(g - 2)_\mu$ from Lattice QCD
RBC-UKQCD: quark-connected diagram using QED$_L$

- $48^3 \times 96$, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV, $L = 5.47$ fm
- $a_\mu^{\text{HLbL}}(\text{connected}) = (116.0 \pm 9.6) \times 10^{-11}$

T. Blum et al, PRL118 (2017) no.2, 022005
RBC-UKQCD: (2,2)-disconnected diagram using QED$_L$

- $48^3 \times 96$, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV, $L = 5.47$ fm
- $a_{\mu}^{HLbL}(2,2) = (-62.5 \pm 8.0) \times 10^{-11}$
- together: $a_{\mu}^{HLbL} = (53.5 \pm 13.5) \cdot 10^{-11}$

T. Blum et al, PRL118 (2017) no.2, 022005

Comments [1712.00421]:
- Total is about a factor 2 lower than model estimates.
- This method has $O(1/L^2)$ finite-size effects. Or model missing something?
- Based on the model and large-$N_c$-based argument, one would expect $a_{\mu}^{HLbL}(2,2) \approx -150 \cdot 10^{-11}$, dominated by $(\pi^0, \eta, \eta')$ exchange.
Update by RBC-UKQCD: continuum, infinite-volume extrapolation.

\[ F_2(a, L) = F_2 \left( 1 - \frac{c_1}{(m\mu L)^2} \right) \left( 1 - c_2 a^2 \right) \]

My comment: the central value is much more in line with model expectation; uncertainty still large.
RBC-UKQCD first results for (3,1) diagram topology

$24^3 \times 64, \ m_\pi = 141 \text{ MeV}, \ a^{-1} = 1.015 \text{ GeV}$

- calculation on coarse lattice strongly suggests the (3,1) topology is negligible.

L. Jin, private communication
Mainz: integrand of $a_{\mu}^{cHLbL}$ with $\mathcal{L}^{(2)}$, $m_\pi = 340$ MeV, $a = 0.064$ fm, $96 \cdot 48^3$

- fully connected diagram only
- the $\pi^0$ exchange with VMD form factor provides a decent approximation to the full QCD computation.
Mainz: pion mass dependence of $\alpha_{\mu}^{cHLbL}$

$\alpha = 0.064 \text{ [fm]}$

- $m_\pi = 340 \text{ MeV}$
- $m_\pi = 285 \text{ MeV}$
- $m_\pi = 200 \text{ MeV}$

bands $= \pi^0$ contributions (band-width is difference between factor 3 and 34/9)

- upward trend for decreasing pion mass? needs more statistics.
Mainz: investigating systematic effects at $m_\pi = 285$ MeV

Check of finite-size effects

Check of discretization effects

- finite size and discretisation effects appear to be under control.

Mainz, A. Gérardin et al.
Conclusion

- Model approach to hadronic light-by-light scattering in $(g - 2)_\mu$ is gradually getting superseded by lattice and dispersive approach.

- Significant progress in the Bern dispersive framework.

- Lattice QCD now has a well-established method to handle $a_{\mu}^{HLbL}$.

- So far, lattice results (the Mainz forward scattering amplitudes and RBC-UKQCD $a_{\mu}^{HLbL}$ results extrapolated to infinite volume) are in line with model expectations.

- Could $a_{\mu}^{HLbL}$ explain the tension between the SM prediction and the experimental value of $a_{\mu}$? It does not look like it, but the effort to reduce uncertainties is worthwhile.
Backup Slides
Continuum tests: contribution of the $\pi^0$ and lepton loop to $\alpha^\text{HLbL}_\mu$

Integrand of the pion-pole contribution with VMD transition form factor.

Integrand of the lepton-loop contribution.

> Even more freedom in choosing best lattice implementation than in HVP.

> The form of the $|y|$-integrand depends on the precise QED kernel used: can perform subtractions (Blum et al. 1705.01067; $\mathcal{L} \to \mathcal{L}^{(2)}$), impose Bose symmetries on $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y)$ or add a longitudinal piece $\partial^{(x)}_{\mu} f_{\rho;\nu\lambda\sigma}(x, y)$. 

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HLbL in $(g - 2)_\mu$ from Lattice QCD
Hadronic vacuum polarization in $x$-space

$HM \ 1706.01139$

QED kernel $H_{\mu\nu}(x)$

$$a_{\mu}^{\text{hvp}} = \int d^4x \ H_{\mu\nu}(x) \left\langle j_\mu(x) j_\nu(0) \right\rangle_{\text{QCD}},$$

$$j_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s + \ldots; \quad H_{\mu\nu}(x) = -\delta_{\mu\nu} \mathcal{H}_1(|x|) + \frac{x_\mu x_\nu}{x^2} \mathcal{H}_2(|x|)$$

a transverse tensor known analytically in terms of Meijer’s functions,

$$\mathcal{H}_i(|x|) = \frac{8\alpha^2}{3m^2} f_i(m_\mu |x|) \quad \text{and}$$

$$f_2(z) = \frac{G_{2,4}^{3,2} \left( \begin{array}{c} z^2 \\ 4,5,1,1 \end{array} \right) - G_{2,4}^{3,2} \left( \begin{array}{c} z^2 \\ 4,5,0,2 \end{array} \right)}{8\sqrt{\pi} z^4},$$

$$f_1(z) = f_2(z) - \frac{3}{16\sqrt{\pi}} \left[ G_{3,5}^{2,3} \left( \begin{array}{c} z^2 \\ 2,3,-2,0,0 \end{array} \right) - G_{3,5}^{2,3} \left( \begin{array}{c} z^2 \\ 2,3,-1,-1,0 \end{array} \right) \right].$$
Explicit form of the QED kernel

\[ \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) = \sum_{A=I,II,III} G^{A}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda} T^{(A)}_{\alpha\beta\delta}(x,y), \]

with e.g.

\[ G^{I}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda} \equiv \frac{1}{8} \text{Tr}\{ \left( \gamma_{\delta} [\gamma_{\rho}, \gamma_{\sigma}] + 2(\delta_{\delta\sigma} \gamma_{\rho} - \delta_{\delta\rho} \gamma_{\sigma}) \right) \gamma_{\mu} \gamma_{\alpha} \gamma_{\nu} \gamma_{\beta} \gamma_{\lambda} \}, \]

\[ T^{(I)}_{\alpha\beta\delta}(x,y) = \partial_{\alpha}^{(x)} (\partial_{\beta}^{(x)} + \partial_{\beta}^{(y)}) V_{\delta}(x,y), \]

\[ T^{(II)}_{\alpha\beta\delta}(x,y) = m \partial_{\alpha}^{(x)} \left( T_{\beta\delta}(x,y) + \frac{1}{4} \delta_{\beta\delta} S(x,y) \right) \]

\[ T^{(III)}_{\alpha\beta\delta}(x,y) = m (\partial_{\beta}^{(x)} + \partial_{\beta}^{(y)}) \left( T_{\alpha\delta}(x,y) + \frac{1}{4} \delta_{\alpha\delta} S(x,y) \right), \]

\[ S(x,y) = \int_{u} G_{m\gamma}(u-y) \left\langle J(\hat{\epsilon},u)J(\hat{\epsilon},x-u) \right\rangle_{\hat{\epsilon}}, \quad J(\hat{\epsilon},y) \equiv \int_{u} G_{0}(y-u) e^{m\hat{\epsilon} \cdot u} G_{m}(u) \]

\[ V_{\delta}(x,y) = x_{\delta} \bar{g}^{(1)}(|x|, \hat{x} \cdot \hat{y}, |y|) + y_{\delta} \bar{g}^{(2)}(|x|, \hat{x} \cdot \hat{y}, |y|), \]

\[ T_{\alpha\beta}(x,y) = (x_{\alpha} x_{\beta} - \frac{x^{2}}{4} \delta_{\alpha\beta}) \bar{t}^{(1)} + (y_{\alpha} y_{\beta} - \frac{y^{2}}{4} \delta_{\alpha\beta}) \bar{t}^{(2)} + (x_{\alpha} y_{\beta} + y_{\alpha} x_{\beta} - \frac{x \cdot y}{2} \delta_{\alpha\beta}) \bar{t}^{(3)}. \]

The QED kernel \( \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \) is parametrized by six weight functions.
\((g - 2)_\mu\): a reminder

\[
\mu = g \mu_B s, \quad \mu_B = \frac{e}{2m_\mu}
\]

- \(g = 2\) in Dirac’s theory

- \(a_\mu \equiv (g - 2)/2 = F_2(0) = \frac{\alpha}{2\pi}\) (Schwinger 1948)

- direct measurement (BNL): \(a_\mu = (11659208.9 \pm 6.3) \cdot 10^{-10}\)

- Standard Model prediction \(a_\mu = (11659182.8 \pm 4.9) \cdot 10^{-10}\).

- \(a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (26.1 \pm 8.0) \cdot 10^{-10}\).

Numbers from 1105.3149 Hagiwara et al.