Set-up for observation thermal voltage noise and determination of absolute temperature and Boltzmann constant

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We describe a set-up for measurement of the absolute zero by Johnson-Nyquist thermal noise which can be performed within a week in every high-school or university. Necessary electronic components and technical guidelines for the construction of this noise thermometer are given. The operating temperature used is in the tea cup range from ice to boiling water and in this sense the set-up can be given in the hands of every high school and university physics student. The measurement requires a standard multi-meter with thermocouple and voltage probe and gives excellent for education purposes percent accuracy. The explanation is oriented to university level but due to the simplicity of the explanation motivated high-school students can follow the explanation derivation of the used formulas for determination of the absolute zero and the Boltzmann constant. As a by-product our set-up gives a new method for the determination of the spectral density of the voltage $e_N$ and current noise $i_N$ of operational amplifiers.

I. INTRODUCTION

On intuitive level a thermal motion of gas atoms was sensed even in the time of Democritus and Lucretius Carus but even now not in every university Maxwell distribution of the molecules is experimentally demonstrated. The same can be said about the thermal motion of Brownian particles which requires at least a microscope. One dimensional motion of electrons along a wire as a tram on a rail can be observed by electronic measurements but even in this case the standard equipment of the type of TeachSpin is affordable only for the elite faculties of physics. The purpose of the present article is to describe how a experimental simple set-up can be made in every university and even high-school within pocket money budget.

One of the purposes of our work is to suggest a new design of the set-up for measurement of absolute temperature, Boltzmann constant $k_B$ and electron charge $q_e$ which is hundred times cheaper than commercially available laboratory equipment. Hand made set-ups has one additional advantage – the understanding.

To whom is addressed this study, we describe the mixture of levels of the possible readers beginning with the senior one:

1. Starting with “XL” category of readers. they are our colleagues preparing every year lectures on disciplines Statistical or Thermal physics. Our manuscript interpolates between the Einstein, Schottky, Johnson and Nyquist papers and contemporary textbooks. Touching to the scientific archaeology following the style of McCombie in the excellent collection by Landsberg we give an example how the basic physical laws related to electric fluctuations can be illustrated using contemporary integrated circuits. This category of readers is typical for Eur. J. Phys. The theory corresponds to teaching of thermal physics as a theoretical discipline. Simultaneously, we explain all know-how details corresponding to the typical level of university teachers preparing new-set ups for experimental work. The set-up can be used for complimentary demonstration to the lectures on thermal physics which in many university has different names, for example thermodynamics and statistical physics, or statistical thermodynamics (as chemists call it). The multiplication of the set-up can be used for laboratory work for the initial years of university education.

2. The second “L” auditorium consists of young experimenters typically at assistant professor position which are designing set-ups for the physics laboratories. It is not supposed that these readers can follow the complete theoretical derivation of the used well-known formulae. Those formulae are their starting point of understanding how the set-up works. The detailed description how the set-up can be reproduced is addressed to this auditorium. We give all technical details of the used electronics in order to avoid additional reading of the textbooks on the art of electronics.

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We carefully cite the specification of the used integral circuits and in such a way we build a bridge between the theoretical physics and contemporary electronics; the authors of the present work have overlapping expertise. Using pocket money budget they can reproduce our set-up in a day and to compare its work with the same of the more sophisticated set-ups in terms of efficiency.

3. The third ("M") level are university students which are curious to reproduce something similar on the subject which they study, the mentioned above fundamental constants for example. There are a lot of examples available on Internet and academic journals can improve the standard of the presentation on this direction.

4. Some very motivated high-school students (category "S") which know that temperature is related to random thermal motions of the particles. Having access to a soldering station and an electronics store they can easily reproduce the set-up and measure 2 fundamental constant at home or at school. This low number category actually contains a many our future colleagues.

5. Last but not least our 5th column are high-school teachers which some time ago were curious and motivated students. Human curiosity like childhood never passes completely. We can send by conventional post free of charge a set-up for the Physics Teachers which indicate any interest and get in touch with us. Demonstration of Maxwell distribution of atoms requires expensive vacuum technique accessible only in high tech universities while demonstration of thermal voltage fluctuations becomes available in the whole world.

Study of electric fluctuations is for decades important laboratory work in good universities, for instance Refs. [2, 10–19], however all those set-ups are difficult to be reproduced or require expensive commercial electronics. The purpose of the present work is to describe and innovation of teaching of electrical noise with a set-up which can be easily reproduced within one week in every school. Moreover, all formulae of the used electronics are elementary and derived as a homework on algebra for undergraduates.

The paper is organized as follows: after the elementary theory given in the next section II we describe our set-up in Sec. III and finally we represent the typical experimental data in Sec. IV. In Sec. V we conclude what can be done in order slightly to improve high-school education related to thermal phenomena.

II. THEORY

A. Classical statistics

We start with recalling the Nyquist theorem [8] for the spectral density of the voltage noise \( (E^2)_f \) of an frequency dependent impedance \( Z(\omega) \) at temperature \( T' \)

\[
(E^2)_f = 4R(\omega)T, \quad T = k_BT' \gg \hbar\omega, \quad R = \Re(Z), \quad \omega = 2\pi f,
\]

where \( k_B \) is the Boltzmann constant and \( h \) is the Planck one. The temperature in energy units is denoted by \( T \), while in Kelvins by \( T' \), and \( f \) is frequency in Hz.

As a simple illustration of calculation mean squared voltage using this theorem let us analyze parallelly connected capacitor with capacitance \( C \) and a resistor with resistance \( R \). The total admittance is

\[
\frac{1}{Z} = \frac{1}{R} + j\omega C, \quad j = -i,
\]

where \( i \) is the imaginary unit which participate in the formula for the amplitude of plane electromagnetic waves \( \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t')] \). In electronics usually \( \exp[i(\omega t')] = \exp[i(-\omega t')] \). For the real part of the impedance Eq. (2) we have

\[
\Re(\omega) = \frac{R}{1 + (\omega T)^2}, \quad \tau = RC.
\]

The substitution in the Nyquists theorem Eq. (1) gives for the total mean square of the voltage

\[
\langle U^2 \rangle = \int_0^{\infty} (E^2)_f df = \frac{T}{C}, \quad \frac{1}{2} C\langle U^2 \rangle = \frac{1}{2} T,
\]

in agreement with the equipartition theorem. As the frequency independent white noise spectral density \( 4RT \) is the only one which leads to the equipartition theorem where the dissipation coefficient \( R \) does not participate one can conclude that Nyquist theorem is a consequence of the equipartition theorem. Albert Einstein [5] was the first who suggested that Boltzmann constant \( k_B \) can be measured using this relation if the thermally fluctuating voltage on the capacitor can be significantly amplified. However, our methodological experiment was made quite recently [20] and the set-up was given to the participant of the Experimental Physics Olympiad [3]. The drift of the zero was the
was given at the 6th Experimental Physics Olympiad \[4\]. No doubts this is methodologically instructive but the idea obtained contemporary development and for determination of the absolute temperature it is quite possible thermal noise to be accepted in the future as a new standard, for the development of the metrology, for example, the recent review Ref. \[17\] and references therein.

In order to illustrate the metrological idea to use Johnson \[7\]-Nyquist \[8\] noise as a thermometer let us analyze a simple consequence: the thermal noise of a resistor with resistance \(R_N\) is amplified \(Y\) times and then is applied to a Low-Pass Filter (LPF) with is actually a voltage divider with transmission coefficient (transmission function or transmittance)

\[
Υ_{\text{LPF}} = \frac{1/jωC_L}{R_L + 1/jωC_L} = \frac{1}{1 + jωτ_L}, \quad |Υ_{\text{LPF}}|^2 = \frac{1}{1 + (τ_L)^2}, \quad B \equiv \frac{R_L}{τ_L}.
\]

For the filtered voltage \(U_f\) we have in the frequency representation

\[
U_f = Υ_{\text{LPF}} U_a, \quad Υ_{\text{LPF}} = \frac{e^{jφ_L}}{\sqrt{1 + (ωτ_L)^2}}, \quad \tan φ_L = -ωτ_L.
\]

and for its mean square

\[
⟨U_f^2⟩ = 4R_N T \frac{|Y|^2}{4R_L C_L} = 4R_N TB, \quad B \equiv \frac{|Y|^2}{4R_L C_L}.
\]

This result differs from the initial one Eq. \[5\] only with the multiplier \(R_N / R_L\) and replacement of \(C\) with \(C_L\).

In the general case for arbitrary frequency, for the dependence of the amplification \(Υ_a(ω)\) of the pre-amplifier and the transmission function of the filter \(Υ(ω)\) we have to measure the averaged square of the amplified thermal voltage

\[
⟨U^2⟩ = 4R_N k_B T' B, \quad B \equiv \int_0^∞ |Υ_a| |Υ|^2 \frac{df}{df}, \quad \text{and for the total pass-band width to use the approximation of frequency independent amplification } B \approx Y^2 B_{\text{LPF}}.
\]

for the thermal noise thermometry. The calculation or measurement of the so defined pass bandwidth \(B\) is a routine problem of electronic engineering. For a high-frequency pre-amplifier it is often necessary to know only the bandwidth of the filter \(B_{\text{LPF}} \equiv \int_0^∞ |Υ_{\text{LPF}}|^2 \frac{df}{df}, \quad \text{and for the total pass-band width to use the approximation of frequency independent amplification } B \approx Y^2 B_{\text{LPF}}.\]

In any case for the measurement of averaged square of the filtered amplified voltage \(⟨U_f^2⟩\) we have to use analog multipliers for which the output voltage \(U_W\) is a product of the two input voltages \(U_X\) and \(U_Y\) divided to a specific for the product constant \(U_m\)

\[
U_W = \frac{U_X U_Y}{U_m}, \quad U_X = U_Y = U_f, \quad U_f = Υ_{\text{LPF}} U_a.
\]

These input voltages have to be equal to the filtered amplified by the pre-amplifier voltage \(U_f\). The technical details are described in the next section but before that next we describe in short the low temperature and high-frequency asymptotic.

\section{General case of thermal fluctuations}

The low frequency and high temperature approximation Eq. \[1\] for the spectral density of the voltage noise is applicable practically for all practical cases of electronics. But for some special cases, imagine mK temperatures and
simultaneously GHz frequencies, we have to take into account quantum effects. In this case the temperature in Eq. (1) has to be substituted by the thermal averaged energy of a quantum oscillator with inductance $L$ and capacitance $C$

$$T \rightarrow E_{\text{osc}} = \left( \frac{1}{2} \frac{L}{\hbar} \right)^2 + \frac{1}{2} \frac{Q^2}{C} = \frac{\hbar \omega_0 / 2}{\tanh(\hbar \omega_0 / 2T)}, \quad L \dot{Q} - Q \dot{L} = -i \hbar, \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad (12)$$

where $\hat{\cdot}$ denotes an operator, $\hat{Q}$ is the quantum operator of the charge and $L \dot{I}$ is the same of the canonical variable analogous to the momentum. For the proof it is necessary to apply the detailed balance principle to the quantum oscillator exchanging energy with a resistor with resistance $R$ immersed in environment with temperature $T'$. And we arrive at the general result

$$\langle E^2 \rangle_f = 4R(\omega_0) \frac{\hbar \omega_0 / 2}{\tanh(\hbar \omega_0 / 2T)} \quad (13)$$

The proof is very simple [23-25], for the total impedance of the sequential resonance circuit we have

$$Z = R + j\omega L + \frac{1}{j\omega C}, \quad |Z|^2 = R^2 + \left( \omega L - \frac{1}{j\omega C} \right)^2. \quad (14)$$

Then for the spectral densities for the current and charge on the capacitor we have

$$\langle I^2 \rangle_f = \frac{\langle E^2 \rangle_f}{|Z|^2}, \quad \langle Q^2 \rangle_f = \omega^2 \langle I^2 \rangle_f. \quad (15)$$

And finally elementary integration of the spectral densities of the energy gives the averaged energy of the oscillator Eq. (12)

$$E_{\text{osc}} = \int_0^\infty \left( \frac{1}{2} \frac{L}{\hbar} \right)^2 + \frac{1}{2} \frac{Q^2}{C} \right) df = \hbar \omega_0 \left( \frac{1}{\exp(\hbar \omega_0 / T) - 1} + \frac{1}{2} \right) \quad (16)$$

and in Eq. (13) the auxiliary index “0” from Eq. (13) can be omitted. The general formula for the spectral density of the voltage can be derived using the Einstein approach of the detailed balance principle applied to a quantum oscillator. Further applying casual principle one can derive the fluctuation-dissipation theorem as a consequence of Nyquist theorem [25].

### III. EXPERIMENTAL SET-UP

In our set-up drawn schematically in Fig. 1 a multiplier AD633 [26] for which $U_m = 10 \text{V}$ is self-understanding that drawing the schematics the resistance of wires and paths in the printable circuit boards is much smaller than the resistance of the resistors and the capacity of the triaxial cable is much smaller than the capacity of the low-pass filter $\mathcal{C}_L$. Two of voltage inputs are grounded $X_2 = Y_2 = 0$ and the filtered amplified voltage

$$X_1 = Y_1 = U_t, \quad U_t = \mathcal{Y}_{\text{LPF}} U_a \quad (17)$$

is applied to the other two. During measurement $|U_a| < 1 \text{V}$ to avoid non-linear effects. Let us recall the general formula for this multiplier [26 Eq. 1, Fig. 12 and Fig. 17]

$$W = \frac{(X_1 - X_2)(Y_1 - Y_2)}{U_m} + Z, \quad Z = \frac{R_2}{R_1 + R_2} W, \quad W = \frac{(X_1 - X_2)(Y_1 - Y_2)}{U_m} R_1 + R_2 \quad (18)$$

Elementary substitution of these formulae from the specification gives

$$W = \frac{U_t^2}{U}, \quad \tilde{U} = \frac{R_1}{R_1 + R_2} U_m = 1 \text{V}, \quad R_1 = 2 \text{k}\Omega, \quad R_2 = 18 \text{k}\Omega. \quad (19)$$

The voltage $W$ directly proportional to the square of the filtered voltage $U_t$ is applied to another averaging LPF with large time constant

$$\tau_{av} = R_{av} C_{av} = 15 \text{s}, \quad R_{av} = 1.5 \text{M}\Omega, \quad C_{av} = 10 \mu\text{F}. \quad (19)$$
on the range of application. This DC voltmeter measures the noise created voltage
This averaged voltage is measured by a multimeter with an internal resistance
q
detail in Refs. [4, 22] this set-up can also be used for determination of the electron charge
Eq. (19) and Eq. (20). If additionally a shot noise current from a photo-diode is applied through
R
an oscilloscope marked with an eye is necessary to be connected. Tracing the path of the noise signal: the noise resistor
R
creates a thermal noise voltage
U
an averaging filter and a voltmeter. A repeater may be added before to the voltmeter, or else the internal resistance of the
voltmeter \( R_V \) has to be taken into account in this final voltage divider. For visual illustration of the amplified thermal noise
an oscilloscope marked with an eye is necessary to be connected. Tracing the path of the noise signal: the noise resistor
R
creates a thermal noise voltage \( U \) which is amplified by the pre-amplifier to \( U_a \) and then filtered by a Low Pass Filter (LPF)
to \( U_f \). This filtered voltage \( U_f \) is further applied to the 2 inputs \( X_1 \) and \( Y_1 \) of a multiplier as shown in Eq. 17. The output
voltage of the multiplier \( W \) is divided by the voltage divider with resistors \( R_1 \) and \( R_2 \). The divided voltage \( U_Z \) is applied to the
adding input of the multiplier; in Eq. \( 15 \) we use the simplified notations \( W \equiv U_W \) and analogously \( Z \equiv U_Z \). Finally the
voltmeter shows the averaged output voltage of the multiplier passed through a voltage divider \( U_N = (W) R_C / (R_V + R_a) \), see
Eq. \( 19 \) and Eq. \( 20 \). If additionally a shot noise current from a photo-diode is applied through \( R_N \), as it is described in great
detail in Refs. 3 22 this set-up can also be used for determination of the electron charge \( q_e \).

This averaged voltage is measured by a multimeter with an internal resistance \( R_V \) of the order of 1 or 10 M\( \Omega \) depending
on the range of application. This DC voltmeter measures the noise created voltage
\[
U_N = \frac{R_V \langle W \rangle}{R_V + R_a} = \frac{R_V}{R_V + R_a} \frac{\langle U_f^2 \rangle}{U^*} = \frac{\langle U_f^2 \rangle}{U^*}, \quad U^* \equiv \frac{R_V + R_a}{R_V} \frac{R_1 + R_2}{R_1 + R_2} U_m.
\] (20)

Here we take into account another voltage divider. After substitution of \( \langle U_f^2 \rangle \) from Eq. \( 9 \)
\[
U_N = \frac{B}{U^*} 4R_N k_n T',
\] (21)

where \( k_n T' \) is the interesting for us spectral density of the thermal voltage but it is not the whole truth. To this spectral density we have to add the voltage noise of the first operational amplifier \( e_N \approx 0.9 \text{nV}/\sqrt{\text{Hz}} \) [Table 1] 27 and the current noise \( i_N \approx 2.4 \text{pA}/\sqrt{\text{Hz}} \). In such a way the final measurable voltage reads
\[
U_N = \left[ \varepsilon_n^2 + 4R_N k_n T' + R_N^2 i_N^2 \right] / A, \quad 1/A = \frac{B}{U^*} = \frac{R_N}{R_V + R_a} \frac{(R_1 + R_2)}{R_1 U_m} B, \quad B = \frac{|Y|^2}{4R_L C_L} (1 - \varepsilon),
\] (22)

where the small correction
\[
\varepsilon \equiv 1 - \frac{B}{B'} = (1 - \varepsilon)B
\] (23)
takes into account more precise numerical integration for the exact bandwidth Eq. \( 10 \) of the pre-amplifier depicted
in Fig. 2. This pre-amplifier is a sequence of a buffer with a double Non Inverting Amplifiers (NIA) with amplification
\( \Upsilon_{\text{NIA}} \) with a virtual ground between them, followed by a difference amplifier (\( \Delta \)) with amplification \( \Upsilon_{\Delta} \) ending with an
Inverting Amplifier (1A) with amplification \( \Upsilon_{1A} \). In such a way the total amplification \( \Upsilon_a \) is a product of the sequence
\[
\Upsilon_a(\omega) = \Upsilon_{\text{NIA}}(\omega) \Upsilon_{\Delta}(\omega) \Upsilon_{1A}(\omega).
\] (24)
FIG. 2. Circuit representation of the used pre-amplifier in the experimental set-up schematically presented in Fig. 1. The pre-amplifier consists of coupled NIA or buffer with amplification $\Upsilon_{\text{NIA}}(\omega)$ to the left of the first dashed vertical, followed by a difference amplifier in the middle between the two verticals with amplification $\Upsilon_\Delta(\omega)$ and ending with an IA to the right with amplification $\Upsilon_{\text{IA}}(\omega)$. The whole amplification of the pre-amplifier $\Upsilon_a(\omega) = \Upsilon_{\text{NIA}}(\omega)\Upsilon_\Delta(\omega)\Upsilon_{\text{IA}}(\omega)$. The first couple ADA4898-2 [27] OpAmps labeled $B_1$ and $B_2$ are for the buffer or coupled NIA amplifiers, while the second couple are for the difference amplifier with label D and for the IA with label I. The first two amplifiers $\Upsilon_{\text{NIA}}(\omega)\Upsilon_\Delta(\omega)$ are actually one instrumentation amplifier. The voltage supplies for the OpAmps are omitted here for brevity and given in Fig. 3.

The amplification of these amplifiers is given by the well-known formulas described in many textbooks an specifications; see, for example, Ref. [28, Eq. 4, Eq. 7, Fig. 52 and Fig. 53]

$$\Upsilon_{\text{NIA}} = \frac{1}{1 + \frac{1}{Z_F} + \frac{1}{Z_G} + \tau_s}, \quad Z_G = \frac{r_G}{2}, \quad Z_F = \frac{1}{R_F} + sC_F,$$

$$\Upsilon_\Delta = \frac{Z_G}{R_F} \left( 1 + \frac{1}{Z_G} + \frac{1}{Z_F} + \tau_s \right), \quad Z_G = R_G + \frac{1}{sC_G}, \quad \tau = \frac{1}{2\pi f_c},$$

$$\Upsilon_{\text{IA}} = -\frac{1}{Z_F} \left( 1 + \frac{1}{Z_G} + \frac{1}{Z_F} + \tau_s \right), \quad Z_F = \frac{1}{R_F} + sC_F,$$

where $s \equiv j\omega$ is the purely imaginary at real frequencies $\omega$, and time-constant $\tau$ is parameterized by the crossover frequency $f_c$ of the OpAmps. For the used dual low noise OpAmp ADA4898-2 [27] $f_c = 97.73$ MHz [29] In short, the initial signal $U$ is amplified, then filtered, before being squared

$$U_f = \Upsilon_{\text{LPF}} U_a, \quad U_a = \Upsilon_a U = \Upsilon_{\text{IA}}\Upsilon_\Delta\Upsilon_{\text{NIA}} U.$$  

The numerical integration can be easily performed using software products of the type of Mathematica or Maple, or even by direct programming with languages such as Python or Fortran. It is simpler to use the complex $\Upsilon$ and to calculate corresponding modulus $|\Upsilon|^2$. This calculation of Eq. (10) gives $B = 55.44$ PHz and from Eq. (23) $\varepsilon = 8.7\%$.

In the broad frequency interval

$$\frac{1}{R_G C_G} \ll \omega \ll \frac{1}{R_F C_F}$$

(29)
the amplification is almost frequency independent
\[
Y_{\text{NIA}} = \frac{R_F}{r_G/2} + 1, \quad Y_\Delta = -Y_{1\text{A}} = \frac{R_F}{R_G}, \quad Y = Y_{\text{NIA}} Y_\Delta Y_{1\text{A}} = -1.01 \times 10^6.
\] 
(30)

The bandwidth of the filter must be in this interval
\[
\frac{1}{R_G C_G} \ll \frac{1}{R_I C_L} \ll \frac{1}{R_F C_F}.
\] 
(31)

We have also to give technical details related to the power supply of the circuit shown in Fig. 3. The large 10 \( \mu \)F capacitors [30, 31] stop the offset voltages of the operational amplifiers ADA4898-2, their floating of the zero, their low frequency \( 1/f \) noise and the low frequency AC voltages coming from the electric power supply in the lab. We use two sets of 9 V batteries for the buffer (B\(_1\)) and (B\(_2\)), see Fig. 2. While the right batteries give voltage supply for the OpAmp of the difference (D) and inverting amplifier (I) from Fig. 2. In parallel to each of the 2 batteries one electrolytic 10 \( \mu \)F and one ceramic 0.1 \( \mu \)F capacitor are soldered. The ground (common point) is connected also to the most external shield of the triaxial cable. This obligatory screening together with capacitors prevents the self-excitation (ringing) of the amplifier with amplification of 10\(^6\) or 60 dB in voltage. The use of two couple batteries makes the set-up much more stable against the ringing which always appears at big enough noise resistor \( R_N \). This self excitation can be easily observed by an oscilloscope, but physical conditions of the stability is an interesting question for further studies of similar set-ups.

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We wish to emphasize that the described set-up does not require specialized commercial electronic equipment like Butterworth filters, oscilloscopes, computers, preamplifiers and laboratory power supplies used in many educational noise thermometry experiments [10, 12, 18, 19, 32], see also the references therein. The electronic circuit is so stable that it does not even require screening metallic boxes and BNC cables between them. It is pedagogically instructive that all details are visible and the only grounding is the connection of the outer shield of the triaxial cable to the common point of the pre-amplifier. The used triaxial cable is Multicomp Pro RG403, practically a coaxial cable with bandwidth of the filter must be in this interval

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Summarizing, all values of the circuit elements of the set-up are given in Table VI.

Before measuring electric fluctuations, it is necessary to determine the “eigen” noise of the amplifier, terminating the triaxial cable with a short circuit, i.e. \( R_N = 0 \). Rewriting the final result Eq. (22) for experimental data processing is convenient to use the variable and its polynomial regressions

\[
S \equiv AU_N = \frac{U^2}{B} + 4R_N k_T T' + R_N^2 \tau_S^2, \quad A \equiv \frac{U^*}{B} = \frac{(R_N + R_{aw}) R_I U_m}{R_N (R_I + R_J) B} = 45 \text{aV/Hz}.
\] 
(32)
TABLE I. Table of the numerical values of the circuit elements from Fig. 1 and Fig. 2.

| Circuit element or parameter | Value |
|------------------------------|-------|
| $R_N$                        | 100 Ω |
| $R_L$                        | 100 Ω |
| $C_L$                        | 42 nF |
| $1/2\pi R_L C_L$             | 37.9 kHz |
| $C_{\text{cable}}$           | 96.5 pF/m |
| $r_{\text{cable}}$           | 0.8 Ω/m |
| $r_G$                        | 20 Ω |
| $R_F$                        | 1 kΩ |
| $C_F$                        | 10 pF |
| $C_G$                        | 10 μF |
| $R_G$                        | 100 Ω |
| $1/2\pi R_G C_G$             | 159 Hz |
| $R_F'$                       | 10 kΩ |
| $C_F'$                       | 10 pF |
| $R_1$                        | 2 kΩ |
| $R_2$                        | 18 Ω |
| $U_0$                        | 10 V  |
| $R_{\text{av}}$              | 1.5 MΩ |
| $C_{\text{av}}$              | 10 μF |
| $R_{\text{av}} C_{\text{av}}$| 15 s |
| $R_V$                        | 1 MΩ |
| $+V_S$                       | +9 V  |
| $-V_S$                       | -9 V  |
| $e_N$                        | 0.9 nV/√Hz [27] Table 1 |
| $i_N$                        | 2.4 pA/√Hz [27] Table 1 |

The variable $S$ has dimension $V^2/\text{Hz}$; this is actually the total low frequency spectral density of the voltage noise. The noise-meter calibration constant $A$ is the most important parameter of the device giving the connection between the measured voltage $U_N$ and the spectral density of the voltage noise $S$. The quantity $q_e A$ has dimension of action (energy times time as Planck constant $\hbar$). Our set-up is a device for measurement of the low frequency spectral density of the voltage noise. The constant voltage measured by a DC voltmeter $U_N$ is proportional to the low frequency spectral density of the noise $S$, and $A$ is a constant specific for the noise-meter which can be expressed by the parameters of the circuit.

Here we have to make a special clarification. Administrators related to education could say that spectral density is a sophisticated notion belonging to final courses of university education. However, spectral density in sense of energy per unit frequency is used in every high-school textbook in which black body radiation is described. Omitting units of the ordinate intensity of black-body radiation as function of frequency or wavelength is drawn in textbooks for a century. Let us introduce dimensionless frequency
\[ w = \frac{\hbar \omega}{T} = \frac{hf}{k_B T}, \quad \omega = 2\pi f, \quad h = 2\pi \hbar, \quad T = k_B T'. \] (33)

In space dimension $D$ the intensity of the black-body radiation is proportional to the function
\[ I(\omega; D) \propto \frac{w^D}{\exp(w) - 1}; \] (34)
theoretical explanation and experimental observation of this universal spectral density in $D = 3$ has been awarded by 4 Nobel prizes (Planck 1918, Penzias & Wilson 1978, Mather & Smooth 2006, Peebles 2019). The original consideration of thermal voltage fluctuations Eq. (1) by Nyquist [8] is based on the 1 dimensional black-body radiation and its low frequency limit
\[ I(\omega \ll 1; D = 1) \propto \frac{w}{\exp(w) - 1} \approx 1. \] (35)
This low frequency spectral density $S$ is measured by our set-up.

For short circuit in the input (front-end) we obtain a new method for determination of the voltage noise of the OpAmp

$$
e_N = \sqrt{S_0}, \quad S_0 \equiv S(R_N = 0).$$

(36)

At fixed temperature $T'$ we can perform measurement of $S_i$ for a set of noise resistors $R_N, i$ for $i = 1, 2, \ldots, N$. The parabolic fit of the data $S$ versus $R_N$

$$S = a_0 + a_1 R_N + a_2 R_N^2$$

(37)

gives a by product a new method for determination of the current noise and Boltzmann constant

$$e_N = \sqrt{S_0} = \sqrt{\alpha_0}, \quad a_1 = \frac{dS}{dR_N} \bigg|_{R_N=0}, \quad k_n = \frac{a_1}{4T'}, \quad a_2 = \frac{d^2S}{2dR_N^2} \bigg|_{R_N=0}, \quad i_N = \sqrt{\alpha_2}. \quad (38)$$

The derivatives here are related to the parabolic fit of the dependence $S(R_N)$. Here we hypocritically suppose that we know the absolute temperature. However, studying thermal noise it is most interesting to perform temperature measurements and to try to determine the absolute zero. If we use small noise resistor $R_N = 50 \Omega$ the quadratic term $\propto a_2$ has negligible influence. For a set of temperatures $t'_j$ (measured in Celsius degrees) we can measure $S_i$ and additionally the off-sets $S_{0,i}$ for short circuits $R_N = 0 \Omega$. Then we can perform the linear regression and to extrapolate the absolute zero temperature $t'_0$

$$S - S_0 = b_0 + b_1 t' = (t' - t'_0) b_1, \quad t'_0 = -b_0/b_1. \quad (39)$$

At this subtraction cancels the offset of the $U_N$ voltage which actually has to be written in the right side of Eq. (37). The temperature range between ice cold and boiling water is enough for a reliable measurement meaning that liquid nitrogen or dry ice are not required but their inclusion would of course dramatically improve the accuracy. Operating with tea cup temperatures $t' \in (0, 80)\degree C$, even for a teenager is safe to make the experiment in the kitchen. The higher temperature can be used is limited by the insulation of the triaxial cable $190\degree C$. But in principle using platinum 100 resistors much higher temperatures can be used in an appropriate laboratory. For low temperatures however there is no limitation but the accuracy becomes smaller. The accuracy is however not essential the purpose is understanding.

Here we have to mention the applicability of the linear regression analysis since Eq. (37) requires parabolic regression with respect to $R_N$. In order to be able to analyze linearly this equation, we need the quadratic term to be negligible leading to the condition $R_N \ll e_N/i_N = 375 \Omega$ according to the parameters from Table VI. On the other hand according to Eq. (1) the noise voltage of the ADA4898-2 operational amplifier, $e_N = 0.9 \mathrm{nV}$ corresponds to the thermal noise of a resistor of $e_N^2/4k_nT'_{\text{room}} = 53.8 \mathrm{K}$ at $T'_{\text{room}} = 273 + 25 = 297 \mathrm{K}$. The geometric mean ($\sqrt{53.8 \times 373} = 142$) $\Omega$ shows that a noise resistor $R_N = 100 \Omega$ is close to the optimal value if we wish to study temperature dependence if the Johnson noise with our set-up. As a resistor it is possible to use platinum 100 $\Omega$ resistance temperature detectors or similar realizations [33]. In this case temperature dependence of the noise resistor $R_N(t')$ can be used as a good reference thermometer well calibrated in 1 kK interval $t' \in (-200, 800)\degree C$. And simultaneously in the same interval we can observe thermal fluctuations of the voltage. In such a way contemporary technology of low noise operational amplifiers can be optimally used for educational purposes. More detailed analysis of the influence of the quadratic term comes from the input current noise $i_N$ is given in Sec. [34]

A. Additional determination of $k_n$ and $q_e$

The described device can be used also for determination of the electron charge $q_e$ by Schottky [6, 34] noise. In this case to the spectral density of the voltage and current we have to add the contribution by the Schottky shot noise created by the averaged photo-current

$$S_+ = R_N^2 q_e^2 + 2q_e I_{ph} R_N^2, \quad i_{N,+}^2 = 2q_e I_{ph}. \quad (40)$$

For a methodological derivation [22] and application for high-school students see Ref. [31] The electron charge can be determined as derivative of the noise spectral density $\propto U_N$ with respect of the averaged photo voltage measured after the amplification by the buffer

$$U_{ph} = \frac{Y_N I_{ph}}{2}$$

(41)
proportional to averaged photo-current $I_{ph}$ (see schematics in Fig. 2). The story is well-known, a young post-doc of Planck (Walter Schottky) attended in a lecture given by Enstein and impressed by the consideration of fluctuation he suggested that electron charge $q_e$ can be also determined by examining of current fluctuations. The electron charge can be expressed from the Schottky formula for the current noise $i_{ph}$ and averaged photo-current $I_{ph}$ Eq. (10). The excess spectral current density $i_{ph}^2$ have to be expressed by the excess voltage noise density $S$. According Eq. (32) the voltage spectral density $S$ can be expressed by the DC voltage of the voltmeter $U_N$. In this formula the device constant of the noise meter $A$ can be expressed by the pass-bandwidth $B$. The pass-bandwidth can be expressed from Eq. (22) by the maximal amplification $Y$ from Eq. (D1) and the bandwidth of the filter. For educational illustrations the small correction $\varepsilon$ defined by Eq. (23) can be neglected. Its evaluation requires numerical integration Eq. (10). Performing this chain of substitutions we finally obtain 22

$$q_e = \frac{d^2i_{ph}^2}{2dU_{ph}^2} = Y_{NIA}A \frac{dU_N}{dU_{ph}} = \frac{2G}{2RF + r_G} \left( \frac{R_G}{R_N} \right)^4 \frac{R_L C_L U_{m}}{1 - \varepsilon} R_1 + R_2 R_V + R_{av} \frac{dU_N}{dU_{ph}}. \quad (42)$$

cf. Ref. 35 Eq. 17. The photo voltage $U_{ph}$ has to be measured by a voltmeter switched after the buffer before the first capacitors in Fig. 2. Analogously to Eq. (38) derivative means the slope of the linear regression in the plot $U_N$ versus $U_{ph}$. In such a way the electron charge can be expressed by the parameters of the circuit and two voltages. The experiment consists of the linear regression in the plot $U_N$ versus $U_{ph}$ when the intensity of light is changed \[4, 22\]. Analogously to this formula for the electron charge we can rewrite the derived formula for the Boltzmann constant Eq. (38) as \[3, 20\]

$$k_B = A \frac{dU_N}{dR_N} \bigg|_{R_N=0} = \frac{1}{T} \left( \frac{r_G}{2RF + r_G} \right)^2 \left( \frac{R_G}{R_N} \right)^4 \frac{R_L C_L U_{m}}{1 - \varepsilon} R_1 + R_2 R_V + R_{av} \frac{dU_N}{dR_N} \bigg|_{R_N=0}, \quad (43)$$

cf. Ref. 20 Eq. 17, where we suppose that absolute temperature is already determined in the used units.

Starting from the fundamental physics and state of the art electronics we give comprehensive description of the set-up and the the theory of its work. This description is not addressed to the students but we give it only because it is new and cannot be found in the literature.

Let us make a comparison. Every electronic calculator is a sophisticated device. The description of all its element is understandable to professionals but not for the students which take the calculator only to calculate. The same is for our set-up for determination of the absolute zero. Imagine it is given to a high-school student. What should be the instruction which we have to give in order after some measurements of temperature and voltage by multimeter students to be able to determine the absolute temperature zero.

IV. DETERMINATION OF THE ABSOLUTE ZERO

Imagine that on the table we have several vessels with water in different temperatures. It they are of order of 1 liter the change of the temperature will be negligible during the time of the measurements. In the left, for example, we can put cold water with flowing ice $t' = 0^\circ$C; and on the right a thermos with almost boiling water $t' = 90^\circ$C. Between 1-3 vessels which interpolate different temperatures in the tea cup interval. On the table you have to have a towel as well.

Let us switch the first multi-meter as a thermometer and put the thermocouple sensor in the cold water. Put 4 batteries at the clips and carefully switch them on the set-up. Be careful not to inverse the polarity because the OpAmps will be burnt. Then switch on the second multi-meter as a voltmeter no measure to measure voltage output.

Immerse the resistor at the end of the triaxial cable in the same water cup. First measurement is using the short circuit and to write the voltmeter indication $U_0$. For brevity in this section we omit index V in the notation of the voltage measured by the DC voltmeter in Fig. 1. Switch the voltmeter in 200 mV range and work with mV. Then remove the short-circuit and measure again the voltage $U$ which is now influenced by the thermal voltage noise. The difference $\Delta U_N \equiv U - U_0$ can be ascribed to the thermal noise and it is proportional to the absolute temperature

$$\Delta U = DT', \quad T' = t' - t_0', \quad D \equiv \frac{4R_N k_B}{A} = k_B \left( \frac{2RF + r_G}{r_G} \right)^2 \left( \frac{R_G}{R_N} \right)^4 \frac{(1 - \varepsilon) R_N R_1 + R_2}{R_1} \frac{R_V}{R_V + R_{av}}. \quad (44)$$

Here the index $N$ of the voltages $U_N$ and $U_{N,0}$ is for brevity omitted. Introducing the reciprocal value $C \equiv 1/D$ we can express the absolute temperature by the measurable voltage difference $T' = C\Delta U$. The offset of the multiplier is canceled by the described procedure of subtraction from the measured signal the voltage at short circuit input. For the determination of the absolute zero the calculation of the coefficient of the proportionality $D$ is not necessary.
However, determining the slope of this proportionality after a linear regression, we can determine the Boltzmann constant at calculated parameter $\mathcal{A}$

$$
\mathcal{D} \equiv \frac{d(U - U_0)}{dt'}, \quad k_B = \frac{A}{4R_N} \mathcal{D} = \frac{1}{R_N} \left( \frac{r_G}{2R_T + r_G} \right)^2 \left( \frac{R_G}{R_T} \right)^4 \frac{R_L C_L U m}{(1 - \varepsilon) R_N} \frac{R_1 + R_2}{R_V + R_{av}} \frac{d(U - U_0)}{dt'},
$$

(45)

this is a new result of the present study. We use again the “≡” sign because the Eq. (45) is the experimental definition of $\mathcal{D}$ as a slope of a linear regression while Eq. (44) was the theoretical definition for the coefficient of the proportionality $\mathcal{D}$. We have to mention also that $dt' = dT'$ the temperature difference of Kelvin and Celsius degrees is equal. The difference is only in the initial of the scales.

It should be mentioned that described method is not the first simple estimate of the determination of the absolute zero temperature. From a long time simple demonstration is to use the ideal gas relationship for the pressure $P$ and volume $\tilde{PV} = \nu RT'$ in many cases [36], for the definition of the new Kelvin, for example. However, the purpose of the present study is not to design a thermometer but to demonstrate thermal fluctuations. To determine the Boltzmann constant $k_B$ it is necessary to study fluctuations or to count atoms (not moles $\nu$), i.e. Avogadro number $\tilde{R}/k_B$.

In such a way the Boltzmann constant can be determined by a measurable slope of the linear regression $\Delta U$ versus thermometer temperature $t'$ and the parameters of the set-up. We wish to emphasize that all formulas (except small correction $\varepsilon$) can be derived by derived by elementary calculation accessible for high-school students. Mathematician David Hilbert used a simple criterion for understanding a theorem – you understand the theorem if you consider that the proof is trivial. In this sense the theoretical physics is also trivial. The theoretician Lev Landau used to say something in the sense “I am the biggest trivialiser”. Following this line, our derivation of the proportionality coefficient $\mathcal{D}$ from Eq. (44) is also trivial. It contains several multipliers, for each of them we have simple algebraic derivation. The total number of letters necessary for re-derivation does not exceed the averaged high-school homework on math. Only a careful exercise on calligraphy. The corresponding numerical calculations are better to be done with a computer, the calculated parameters are given in Table II.

| Calculated or measured parameter | Value          |
|----------------------------------|----------------|
| $U^*$                            | 2.50 V         |
| $U$                              | 1.00 V         |
| $A$                              | 45 aV/Hz       |
| $B$                              | 60.72 PHz      |
| $\varepsilon$                    | 8.7%           |
| $\tau_L \equiv R_L C_L$          | 4.2 $\mu$s     |
| $|Y|$                             | $1.01 \times 10^6$ |
| $C$                              | 2.09 K/mV      |
| $\mathcal{D}$                    | 479 $\mu$V/K  |
| $A/4(T' = 296 K)$                | 38 $z$V/(Hz K) |
| $Y_{NIA}A/2R_N$                  | 22.73 aC       |
| $q_e A/\hbar$                    | 0.068          |
| $P$, peta                        | $10^{15}$      |
| $a$, atto                        | $10^{-18}$     |
| $z$, zepto                       | $10^{-21}$     |

Returning to the determination only the absolute temperature we describe the first row of Table III. Then we can pass to the next vessel and again determine $\Delta U_2$ and $t'_2$. The purpose of the experiment is to have several points $\Delta U_i$, $t'_i$, and to present this table in a plot $\Delta U$ [mV] versus $t'$ [°C]. Draw a line passing near all experimental points

| i | $U$ | $\Delta U$ | $t'$ |
|---|-----|------------|------|
| 1 | ... | ...        | ...  |
| 2 | ... | ...        | ...  |
in the ($\Delta U$ versus $t^\prime$) plot. This procedure is called a linear regression but you can use millimeter paper without knowing the corresponding math. An example of graphical representation of experimental data is given in Fig. 4.

Now you have to make the extrapolation. This extrapolation represents the accuracy of the measurement using this set-up. Extrapolating the line far from the experimental points in order to cross the new abscissa $\Delta U = 0$. Determine at which temperature $t^\prime_0$ is this crossing. Finally evaluate the accuracy in percents of our measurement

$$\epsilon = 100 \times \frac{|t^\prime_0 + 273|}{273}.$$  \hfill (46)

In order to demonstrate properties of the setup we make one experiment with laboratory conditions using thermal chamber and good voltmeter with very large $R_V$. The results are represented in Fig. 4. The interception of the line of linear regression $\Delta U \propto (t^\prime - t^\prime_0)$ with the abscissa gives $t^\prime_0 = -234^\circ C$ giving 14% accuracy which is quite acceptable for high-school and undergraduate result which is the purpose of our work, see for comparison the educational experiment in MIT [12]; cf. also unpublished guides [13–15]. By performing linear regression using a computer, we easily obtain the uncertainties of the coefficients. The evaluation of the experimental errors is as a rule tedious for the students, but our observation is that Jack-knife [37] method they easily accept and the corresponding programming can be easily performed.

With very careful measurements it is possible to reach percent accuracy, especially if we include liquid nitrogen to the tea cup temperatures. Theoretical formulae where only accessories how our set-up completes its precursors following the story of the physics in the last 115 years.

As a whole due to the careful choice of integral circuits, the accuracy of the described set-up is comparable with ones of university physics laboratories. As additional advance we have given state-of-the-art description of the electronics at the level of standard university courses and the budget of remake of the set-up is in pocket range order.

V. DISCUSSION

Let us summarize what the ingredients to make this oversimplified experiment which gives satisfactory accuracy and can be performed in high-schools are. First of all, the appearance of low noise OpAmps with $e_N < 1 \, \text{nV}/\sqrt{\text{Hz}}$ and simultaneously with crossover frequency $f_c > 10 \, \text{MHz}$ gives the opportunity to construct very cheap pre-amplifiers with huge maximal amplification $Y \gtrsim 10^6$. Only highly optimized device can reach such amplification even without screening boxes. The instrumental amplifier is less sensitive with respect of self-excitation (ringing), but here we have to emphasize the fundamental importance of the triaxial cable. The most dangerous parasitic feed-back is the electrostatic coupling of the amplified signal to the voltage input of the pre-amplifier. The triaxial cable substitutes the screening metallic boxes and BNC cables which connect the different amplifiers, filters, oscilloscopes. This simplification gives the possibility the experiment for thermal voltage fluctuations to gain a virulent mutation in order to infect the high schools physics labs from the university ones. The linear dependence Eq. (44) and Fig. 4 describes
The operational range is limited to the temperatures of use of the triaxial cable -40°C to 200°C. The cable, resistor and soldering of the resistor $R_N$ to cable wires can survive in immersing in liquid nitrogen, but when is cooled the cable must not be bent. Moreover, there is a contemporary tendency the new Kelvin to be performed by measurements of electric fluctuations and it is nice and useful high-school students to become familiar with what actually temperature is and the world in which they will live. From methodical point of view it will be nice together with electric demonstration of absolute temperature to be performed a demonstration with ideal gasses, see for example Ref. [36]. At fixed density $n$ of the particles the pressure $p$ of an ideal gas is proportional to the temperature $T = nT$. This is known for ages. However, thermal physics is incomprehensible. Many physical variables and phenomena, for instance Ref. [38], depend on the absolute temperature $T$ and in the simplest possible case of classical statistics the dependence is just proportionality. Relatively more recent example, only a half century ago is the observation by Timko [39] a fundamental property of the silicon transistors that: “if two identical transistors are operated at a constant ratio of collector current densities, $r$, then the difference in their base-emitter voltage is $(k_BT/q_e)(\ln r)$. Because both $k_B$ (Boltzman’s constant) and $q_e$ (the charge of an electron) are constant, the resulting voltage is directly Proportional To Absolute Temperature (PTAT).” This article by Timko is the basis for understanding the specification of a 2-Terminal Integral circuit (IC) Temperature Transducer. This IC has really excellent properties as an absolute thermometer: “Linear current output: 1 µA/K, wide temperature range: -55°C to +150°C. Probe-compatible ceramic sensor package 2-terminal device: voltage in/current out Laser trimmed to ±0.5°C calibration accuracy (AD590M). Excellent linearity: ±0.3°C over full range (AD590M). Wide power supply range: 4 V to 30 V. Sensor isolation from case.” If the current passes through 1 kΩ resistor the calibration gives $\Delta U_{\text{AD590}} = 1$ mV/K, a value comparable with our device. The quoted passages are taken from the AD590 specification [9]. However, using only an absolute temperature thermometer it is impossible to determine neither Boltzmann constant $k_B$ nor electron charge $q_e$ which are byproducts of study of the described noise-meter. The purpose of our work is not only to design an absolute thermometer but also to demonstrate fluctuations of the electric potential due to thermal or shot noise. Roughly speaking, we are making fluctoscopy of the moving electrons analyzing basic notion of the though in thermal physics.

Returning to the evolution, the ideas in physics of Langevin random forces are a convenient tool to study thermal fluctuations. But when an idea is born we can return to the Democritus and its popularization by Lucretius Carus. The thermal fluctuations of the voltage at the end of a resistor can be considered as Lucretius Carus exegium clinamen principiorum [1, Sec. 7.1].

Often a critique can be heard that high-school physics hardly accepts new ideas for innovation and modernization. The world condition in the next years will not be very favorable but at least we can try our best at the present times.

Perhaps not every physics teacher can make a cover version of the described set-up but definitely everyone of them can use it. Low price gives the unique possibility set-ups to be made in a town and send by post to the whole world. The present article is addressed to the teachers, but it can be the basis of the guide for the high-school students how to measure the absolute temperature zero. In short, the described set-up is a part of modernization of the teaching of fundamental physics, a modernization which became possible by the emergence of low price high-tech active electronic components. Upon request authors can provide the drawings of the PCB, the list of components and even the purchase orders.

We believe that our simple set-up and accompanying theoretical consideration will be interesting and accessible to reproduce by a diverse audience of physics students, educators and researchers. The level of our description is oriented to universities, but due to simplicity set-up can be reproduced and used even in high schools. The only device necessary to be added is an affordable multimeter which is easily acceptable within pocket money budget. And who knows maybe some of the ignited students can find career in the physics and even in the metrology.

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Appendix A: Simple derivation for the formulas for amplification

We use well-known formulas for the amplification of the used basic amplifiers but instead to cite textbooks, we give a simple re-derivation compatible with the used notations following Ref. [20]. Formally such material is part of the university education but simple algebra can be reproduced by a teenager. This supplementary material reveals the close relation between the basic physics and electronics and we are trying to build a bridge with unique education between the theoretical analysis of the thermal fluctuations and the contemporary electronics. As a rule students of thermal physics are not familiar with the necessary for our understanding electronics.

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1. Basic equation of operational amplifiers (OpAmp)

We recall the basic equation for the operational amplifiers (OpAmp) giving the relation between output voltage $U_o$ and the voltages at plus $U_+$ and minus $U_-$ inputs

$$U_o = (U_+ - U_-)G(\omega), \quad |G(\omega)| \gg 1. \quad (A1)$$

In short, an operational amplifier has two voltage inputs $U_+$ and minus $U_-$ with very small input current. And one voltage output $U_0$ which can give significant output current so the equation above to be satisfied in linear regime. In the schematics of the circuits voltage supply terminals for plus $+V_S$ and minus voltage supply $-V_S$. The used by us dual OpAmp ADA4898 has dual $(U_+, U_-, U_0)$ probes denoted at Fig. 2 of Ref. [27] as $+\text{IN1}$, $-\text{IN1}$, $\text{V_{OUT1}}$ and $+\text{IN2}$, $-\text{IN2}$, $\text{V_{OUT2}}$.

The OpAmp consists of sophisticated electronic components: transistors, resistors and capacitors hidden in a commercially applicable black box shown in scale in Fig. 49 Ref. [27] as in scale 8-Lead Standard Small Outline Package with Exposed Pad. In the first approximation Op Amp is just a high gain amplifier $\alpha = 1/G \approx 0$ and we have to take into account only the first real and imaginary correction. The implementation of the described set-up become doable after recent appearance of cheap, low-noise and high-speed Op Amps. In such a way this was doable in Bell laboratories a century ago now can be in the hand in every student on physics in every school in the world. Last but not least existence minimum of electronics is important ingredient of teaching of physics. Young Richard Feynman used to repair nonworking radios in the beginning.

As open loop gain in modulus is a big number is useful to introduce its reciprocal value

$$\alpha(\omega) \equiv \frac{1}{G(\omega)} \approx \frac{1}{G_0} + \tau s = \frac{1}{G_0} + \frac{f}{f_c} \approx j\omega, \quad |\alpha(\omega)| \ll 1, \quad f_c \equiv \frac{1}{2\pi \tau}, \quad \omega = 2\pi f, \quad (A2)$$

which is small in modulus and linear function from frequency $\omega$. For an order of evaluation one can choose $G_0 \approx 10^6$ and $f_c \approx 100$ MHz. The last approximation is applicable in the interval

$$\frac{f_c}{G_0} \ll f \ll f_c, \quad (A3)$$

i.e, in kHz to MHz range which is just our case. Further we use the basic equation of Op amps in the form \[11\] \[43\]

$$\alpha U_o = U_+ - U_-, \quad \alpha \approx \tau s, \quad s = j\omega, \quad \tau = \frac{1}{2\pi f_c}. \quad (A4)$$

Our first problem is the amplification of noninverting amplifier.

2. Non-inverting amplifier (NIA)

The non-inverting amplifier is drawn in Fig. 5. One can see that input voltage of the amplifier is just the plus

\[ \text{FIG. 5. High impedance non inverting amplifier } U_i = U_+. \text{ The well-known result for the amplification is given by Eq. (A7).} \]
voltage of the OpAmp

\[ U_+ = U_i, \quad U_- = \frac{Z_G}{Z_G + Z_F} U_o, \]

(A5)

while minus voltage can be expressed as a voltage divider of output voltage and sequentially connected gain \( Z_G \) and feedback \( Z_F \) impedances. The substitution of Eq. (A5) in Eq. (A4) gives

\[ U_i - \frac{Z_G}{Z_G + Z_F} U_o = \alpha U_i, \]

(A6)

or

\[ \Upsilon_{NIA} \equiv \frac{U_o}{U_i} = \frac{1}{\frac{Z_F}{Z_G} + 1} + \frac{s}{2\pi f_c}, \]

(A7)

in agreement with the cited well-known formula which we used. The derivation of the amplification for inverting amplifier (IA) which we re-derive in the next section also requires elementary high-school algebra which is easy to trace.

The formula for the amplification of the multiplier Eq. (18) can also be understood considering a voltage divider.

3. Inverting amplifier (IA)

In the schematics of the Inverting Amplifier (IA) shown in Fig. 6 the plus input voltage is grounded

\[ U_+ = 0, \quad U_- = U_o + (U_i - U_o) \frac{Z'_F}{Z'_F + Z'_G}, \]

(A8)

while for consideration of minus input one can start from output voltage and to add the difference between the input voltage and output voltage which passes through a voltage divider of sequentially connected feedback and gain impedances. The substitution of these formulas for \( U_+ \) and \( U_- \) in the basic equation Eq. (A4) after some elementary calculation gives the well-known result we have cited

\[ \Upsilon_{IA} \equiv \frac{U_o}{U_i} = -\frac{1}{\frac{Z'_F}{Z'_F} + \left( \frac{Z'_F}{Z'_F} + 1 \right) \frac{s}{2\pi f_c}}. \]

(A9)

It is not necessary to repeat analogous calculations which gives almost the same result for the difference amplifier

\[ \Upsilon_{\Delta} = -\Upsilon_{IA}. \]

(A10)
4. The set-up as lock-in voltmeter

We derived the formula for the averaged square of the voltage Eq. [20] supposing that filtered signal is applied to both the input probes of the multiplier. However if amplified and filtered time dependent signal $U_f(t)$ is applied to the first input probe

$$X_1 = U_f(t), \quad Y_1 = U_b(t) \quad (A11)$$

and to other input is applied some basic time dependent signal $U_b(t)$ for the time averaged signal measured by the voltmeter $U_N$ instead of Eq. [20] now reads

$$\langle U_f U_b \rangle = U^* U_N. \quad (A12)$$

As a rule the basic signal is sinusoidal voltage

$$U_b(t) = U_b \cos(\omega t) \quad (A13)$$

which creates some subtle response

$$U(t) = U_0 \cos(\omega t + \theta) = \Re \{U_0 \exp[j(\omega t + \theta)]\} \quad (A14)$$

in the studied subject. This signal is then amplified and filtered

$$U_f(t) = U_f \cos(\omega t + \theta + \varphi + \varphi_L) = \Re (\mathcal{Y}_{\text{LPF}} T U_0 \exp(j(\omega t + \theta))) , \quad U_f = |\mathcal{Y}_{\text{LPF}}| \times |T| \times U_0. \quad (A15)$$

The substitution of those voltages in the main equation of the lock-in Eq. [A12] gives

$$U_N = \cos(\varphi + \varphi_L + \varphi_{\text{rot}} - \theta) |\mathcal{Y}_{\text{LPF}} T| \frac{U_b U_0}{2 U^*}, \quad (A16)$$

where $\varphi_{\text{rot}}$ is and additional tunable phase shift applied before basis signal to be applied to the lock-in. Let us take an example. For $f = \omega / 2\pi = 10$ kHz we have $|\mathcal{Y}_{\text{LPF}}| \approx 1, |T| \approx Y = 10^6$. Additionally for $R_V \gg R_{\text{rot}}$ the multiplication voltage constant is $U^* = 1$, and with tunable phase shifter we can get $\cos \approx 1$. In this case let we apply maximal possible for the AD633 multiplier voltage $U_b = 10$ V and to analyze can we measure a nanovolt signal of $U = 10$ nV.

The estimation obtained from the above formula gives additional output voltage $\Delta U_N \approx 50$ mV. This is comparable with the own noise of the device measures at short circuit input $U_N \approx 80$ mV. In such a way if we measure for one minute the output voltages obtained with switched on and off small signals the evaluated voltage difference can be measured by an affordable multi-meter. Actually even 5 mV one minute modulation can be also detected and in this sense the considered device is actually an affordable lock-in voltmeter which every body can do himself to measure nV signals. The sensitivity can be significantly improved by replacement of the low pass filter with a resonance one. The simplest idea is to substitute the $C_L$ with a frequency dependent negative resistor (D-element) [44–46], i.e. $1/(j\omega C_L) \rightarrow -1/(j\omega^2 D)$ and simultaneously to add for stability against ringing a big capacitor $R_L \rightarrow R_L + 1/(j\omega C_L)$. In the next subsection we will try to analyze some typical obstacles which prevents progress in the basic physics education.

5. Apology of the formulas

Let us emphasize again that noise thermometry, and determination of $k_b$ and $q_e$ are included in the university study education for decades. What is obligatory to all those experimental set-ups. First of all a good low-noise pre-amplifier. Many universities can allow to buy it without troubles. The problem is actually not completely financial. The commercial amplifier has a clearly written manual and within one our every student can use it.

What is our alternative – to make oneself the amplifier which requires one day soldering of electronic components on a printable circuit board. The corresponding theory is elementary and the total derivation does not exceed the volume of a college homework on algebra. The first crack is however obvious for many students and even for teachers electronics is a handicraft and on the other side the derivation of formulas is a tedious math similar to those required to pass corresponding exams to obtain diploma.

Actually some electronics is obligatory for all physics students but even in the textbook on the “art of electronics” one can read jocks like “the generators amplify and the amplifiers generate”. In any case a commercial amplifier is a box for the students which is opened only in the services. Our device is actually an open circuit at the screening is reduced only to the third most external shield of the triaxial cable.
In short our purpose is to provoke teachers and student to design his amplifier, to make necessary calculations and to perform the assembling. In short our purpose is to stimulate creativity.

Let us continue further. From the amplified signal $U_a(t)$ it is necessary to evaluate the mean square in some time period. The simplest way is to record the signal by a digital oscilloscope and further to calculate the corresponding mean values and fluctuations by a computer. No doubts physics students has to be able to use oscilloscopes and computers. What is the purpose of our article to stimulate student to design something which is working passing several times the non-existing boundary between physics an engineering. But for the derivation of the corresponding formulas is necessary to analyze how the numerical multiplication can be performed analogously by an integral circuit of analogous multiplier. The multiplication squaring and proportion are ingredients even in the initial school education. However the chain of elementary formulas looks for many students already as a tedious collection. A motivated student or teacher has to pass and assimilate some new notions. Perhaps the simplest one is the amplification $\mathcal{Y}$ of the amplifier followed by a filter which can be frequency dependent. The next already new notion is the pass-bandwidth $\mathcal{B}$. With the multiplication and analogous squaring of the signal is related the voltage multiplication constant of the device $U^*$.

Most aggressive resistance in our explanation meets the spectral density of the voltage $\mathcal{S}$ having dimension $V^2/$Hz.

Most of the university teachers consider that this is notion belonging of advanced electronics. This is because the change of the terminology. Spectral density in sense of intensity per unit frequency is accepted in the physics education since more that a century with explanation of the blackbody radiation. We can refer to Rayleigh, Jeans, Wien, Planck and Einstein and even before implicitly this notion was used by Hershel discovering infra red light. In conclusion nothing new if we use square voltages per unit frequency.

A motivated young man can use even new for him notions. In our case they are calibration constants for the noise-meter $\mathcal{A}$, and the thermometer $\mathcal{C}$ or $\mathcal{D}$. In order to explain our result and the ability of the described set-up we have to use some notions and notations. In such a way we have addressed to a a little bit philosophical problem: Whether accumulation of a many trivial ingredients lead to and non-trivial result and what is the crucial dose?

The people involved with administration of education are as a rule against every innovation. In some sense their duties are to conserve the status-quo and to stop the development. The places where music is learn are called Conservatory and as a jocular etymology the mission of the ministry of science is to conserve the present level of education and even to worsen it using the polite-correct word to alleviate it. Even the youngest coauthors of the present article express disagreement: Why the final result for the Boltzmann constant $k$ Eq. (15) and Eq. (16) and electron charge $q_e$. Eq. (12) are written in a doubled manner with short and long notations. The answer is related with the double purpose of the article. On one side we have tho express the final fundamental results using universal notions as amplification, bandwidth, or constant relating spectral density of the voltage fluctuations with the measured by voltmeter DC voltage. Those notions or their equivalents or synonyms indispensable will be used in the other implementation of the same idea to organize university lab-work to measure $T$, $k_\text{B}$ and $q_e$. On the other side for detailed description of every set-up it is obligatory the final result to be expressed by the parameters of the used components and the measurable physical variables. The relation between these two final goals is one of the main purposes of our methodological study.

Now let us consider what total dose of efforts is acceptable for a self-made set-up for a student lab. We made many experiments with smart white mice. The prototypes of the described set-up was used for illustrative experiments to the courses of statistical physics for more then then years. It was an alternative method students to pass the exam – to made from scratch the set-up for determination of a fundamental constant $k_\text{B}$ and $q_e$. It was an advantage that they used most recent integral circuits which was not used at those time in the commercial electronic devices as amplifiers and filters.

Developing some modification of the set-ups we attracted as co-collaborators even high-school students and as a medical experiment we can confirm that one week honest work is not a lethal dose for a motivated student. A by product our co-collaborators obtained additional initial speed in their professional career.

Performing experiments with hundreds high-school student we observed that most of them reading the tutorial to the set-up can further to perform the experiment and to determine $T$, $k_\text{B}$ and $q_e$. It was necessary to switch the batteries, integral circuits and changing resistors or light intensity to measure some voltages by DC voltmeters used in their high-schools. Some person of them was able even to reproduce the derivation of the used formulas and to understand completely the used electronics. The youngest one was able to make some simple test measurements with the set-up which was also a significant success.

Concerning the eldest potential readers of the present article which are in the position to design new set-ups for the physics education their peers are climbing to Mount Everest and even K2 and with this comparison to make the described set-up in one day is like to make a safety weekend trip to Mont Blanc.

Finally looking only to the formulas which have to be re-derived in order to have comprehensive understanding of the described set-up in the level to design much better device is necessary to use several pieces A4 paper and one you. If we compare theoretical description of our set-up with complete description of the electronics theory of a Butterworth filter, commercial amplifier and an oscilloscope the criterion for simplicity will be obvious. It is very
instructive for the students to understand and feel all details of the set-up. For the beginning is even better to use self-made ugly set-up instead a good looking device. Professional use later will not escape.

Of course the present manuscript is not in the style of the papers written for carrier, but it is matter already to a social discussion

Appendix B: Bilinearity of the thermal noise \( \langle \Delta^2 \rangle_f = 4R_N T \)

The purpose of the present work is to demonstrate determination of the absolute temperature and to construct an absolute thermometer. But the linearity of the thermal noise with respect of the resistance is an important initial test for the work of the setup. The thermal noise however cannot be observed separately. To the measured noise the voltage noise of the first operational amplifier is added and for big enough noise resistors we have to take into account the current noise of the OpAmp as well. According to Eq. (32) and Eq. (37) referred to input is given by the quadratic polynomial

\[
U_N = \frac{BS}{U_*} = \frac{R_N}{(R_V + R_{av})} \frac{(R_1 + R_2)}{R_1 U_m} \frac{BS}{U_*}, \quad S = e_N^2 + 4k_BT_N + \tilde{i}_N^2 R_N^2. \tag{B1}
\]

Our device is actually a noisemeter of the spectral density, \( U_N \) is measured with an ordinary DC voltmeter. Experimental data is represented in Fig. 7 together with a polynomial fit and a linear extrapolation of the first 5 points, up to \( R = 1 \) kΩ including. As a byproduct of our consideration of fundamental physics we obtained a new method for measurement the spectral density of the voltage \( e_N \) and current noise \( i_N \) of operational amplifiers. Addressing

![FIG. 7. Resistance \( R_N \) dependence of of the voltage output \( U_N \) of the noise-meter which is proportional to the spectral density according Eq. (B1). The horizontal doted line describes the voltage noise of the OpAmp \( e_N \), the dashed line describe the linear extrapolation, its slope represents the thermal noise. The continuous line represents the parabolic fi which takes into account the current noise of the used OpAmp ADA4898-2. The fit with high correlation (\( > 99\% \)) gives \( \tilde{e}_N = 2.76 \) nV/\( \sqrt{2} \) Hz and \( \tilde{i}_N = 3.64 \) pA/\( \sqrt{2} \) Hz in excellent correspondence with Ref. \[27, Table 1\]. Tilde means our measurement and \( \sqrt{2} \) denotes that two operational amplifiers in the dual OpAmp in the buffer.

to the problem to make an absolute thermometer, we had to develop a new method for determination of current \( i_N \) and voltage \( e_N \) noise of the operational amplifiers. Our parameters agree with the numerical values given in the specification of the used op amp \[27, Table 1\] and in this sense our work can be considered as a research work in electronics which is however far beyond our methodical purposes. We give this appendix only to alleviate the work of colleagues constructing similar noise meters.

The analysis of Fig. 7 reveals that it is ignorant to use noise resistors bigger than 100 Ω. The instruction of the choice of resistors and OpAmps are described in Ref. \[47\] for instance. Actually this is well described by the manufacturers Analog Devices, Texas Instruments and Linear Technology. Op amps with low voltage noise have to be used only with small enough resistors. The voltage noise of ADA4898-2 corresponds to thermal noise of say 50 Ω resistor and that is why we recommend maximum 100 Ω for the described thermometer. The methodological woks are actually the most complicated scientific genre of research inspired student problems \[48\]. Before suggesting some innovation in the physics teaching a lot of engineering and scientific work has to be done.

Actually our set-up is a part of a device for measurement of the Bernoulli effect in superconductors \[49\] but it is another opera, as the saying goes in some languages.
Appendix C: Measurements with specialised equipment for education

We have also performed measurements with specialised educational equipment from Vernier. This equipment consists of LabQuest 3 data-collection platform with platinum 100 wide range temperature probe sensor and differential voltage probe sensor with $R_V = 20 \text{ M}\Omega$ therefore $U^* = 1.05 \text{ V}$. The experiment was performed on a relatively warm February evening with $R_N = 54 \text{ \Omega}$ with thermalization both in increasing and decreasing temperature. The resistor $R_N$ was put in a cigar tubos together with the temperature sensor. This tubos was put in a box and after the first measurement at ambient outside temperature, the box was filled with boiling water and a measurement per 15 s was recorded, this is Run 1, see additional data file. The second set of measurements Run 2 was performed after reach of maximal temperature on cooling back to outside temperature, a measurement per 1 minute. Occasionally, cold water was added to speed up a little bit the decrease rate to ambient temperature. The results from both sets of measurements are given in Table IV without taking into account the last measurement in Run 2, which was accidentally made upon switching to short circuit for which $U_0 = 130 \text{ mV}$.

| Parameter | Run 1 | Run 2 |
|-----------|-------|-------|
| $t'$ [°C] | -253.86 | -273.1 |
| corr. coeff. [%] | 92.0 | 76.97 |

If one looks carefully at the experimental data or reproduce the linear regression, will notice the “quantization” of the experimental data. This is no surprise since for our set-up $D = 479 \mu\text{V/K}$, while the best resolution at analog to digital conversion of the voltage probe sensor is 1.6 mV. This voltage step gives for the temperature quantum $\Delta t' = 3.34 \text{ K}$, revealing the meaning of the introduced parameters $C$ and $D$. They show the requirements for the temperature and voltage measurement apparatus for our experimental set-up. If at the output of the set-up a DC amplifier with amplification $\times 10$ is added, the temperature step will decrease to the more precise value of 0.34 K. Another solution is to extend the temperature interval between each successive measurement. Every technical device can be successfully upgraded.
Appendix D: Problem of the 9-th Experimental Physics Olympiad, Skopje, 8 May 2022
Kelvin’s Day

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This is the problem of the 9th International Experimental Physics Olympiad (EPO), Kelvin’s Day. The goal of the Olympiad is to measure the absolute zero with the given experimental set-up. If you have an idea how to do it, do it and send us the result; skip the reading of the detailed step by step instructions with increasing difficulties. We expect the participants to follow the suggested items – they are instructive for physics education in general. Only the reading of historical remarks given in the first section can be omitted during the Olympiad without loss of generality. All participants should try solving as much tasks as they can starting from the first one without paying attention to the age categories: give your best. The age categories are for ranking purpose only, work as far as you can.

Before to begin we wish to emphasize on of the purposes of EPO9, to measure the Boltzmann constant. After the Olympiad please repeat again the experimental tasks and you will see how simple it is to measure a fundamental constant. As a byproduct you have learned a lot of things related to the fundamental physics. Now you can read the tasks to the end to get to know them and you can start.

1. Description of the experimental set-up and the conditions for online participants

The set-up you received is represented in the Fig. 8. In the next sections different tasks with increasing difficulty are described in the sections corresponding to every age category. The jury will look at the experimental data, tables and graphics. It is not necessary (nor desirable) to write any humanitarian text between them, only mark the number of the corresponding task. We wish you success.

2. Tasks S. Getting to know the resistors set

1. Turn on the multimeter as a Direct Current Voltmeter (DCV) and measure the voltages of your four 9 V batteries with maximum accuracy and write them down.

2. Turn on the multimeter as an Ohm-meter (Ω). With maximum accuracy measure the resistance of every of crocodile cables. Connect them sequentially and measure again its resistance.

3. Again with maximum accuracy measure the resistance of all given resistors soldered on the pin connectors. Write on the label the value of each resistor. If you cannot write with so small numbers just write a character. and record the value corresponding of this character. Order the values of the resistors in a table like Table V, where i indicates the successive number of the resistor.

We recommend multimeter probes to be directly touched to the resistors. For the resistor wrapped with adhesive band you can use double pin connector shown in the upper right corner in Fig. 8.

If you prefer to use crocodile clips to connect resistors to the probes of the multimeter you have to subtract from the the value given by the multimeter the resistance of the crocodile cables. In such a way you will obtain the corrected values of the resistors.

The last column you will fill later.
FIG. 8. Details of the set-up given to the participants: Printed Circuit Board (PCB) with triaxial cable connected to it (left), two 9 V battery clip connectors soldered to 3× PCB female pin headers with orange labels on them (middle), 2 crocodile connecting cables (lower right), a small plastic bag containing: one 2× PCB male pin header (upper right), 1 short circuit connection on a 2× PCB female pin headers, and a set 6 resistors each soldered on 2× PCB female pin headers, one of them covered with black electrical tape.

\[
\begin{array}{ccc}
  i & R_i & U_i \\
  1 & \ldots & \ldots \\
  \ldots & \ldots & \ldots \\
\end{array}
\]

TABLE V. Table of the measured resistances of the given set of resistors and their measured voltage noise.

4. Calculate how many mV is the difference between \( \frac{1}{2} \) V and \( \frac{1}{3} \) V? Write the integer rounded value with accuracy of 1 mV.

5. Using the values in Table V, calculate and write down the values of the expressions

\[
Y_{NIA} = \frac{R_F}{r_G/2} + 1, \quad Y_\Delta = -Y_{IA} = \frac{R'_F}{R_G}, \quad Y = Y_{NIA}Y_\Delta Y_{IA}. \quad (D1)
\]

Those \( Y \)-values are actually the amplifications of different segments of the set-up amplifier, we analyze the theory of its work as an electronic device.
TABLE VI. Table of the numerical values of the circuit elements of the experimental set-up.

| Circuit element | Value     |
|-----------------|-----------|
| \( r_G \)       | 20 Ω      |
| \( R_F \)       | 1 kΩ      |
| \( C_F \)       | 10 pF     |
| \( C_G \)       | 10 \( \mu \) F |
| \( R_G \)       | 100 Ω     |
| \( R_F' \)      | 10 kΩ     |
| \( C_F' \)      | 10 pF     |
| \( R_L \)       | 100 Ω     |
| \( R_1 \)       | 2 kΩ      |
| \( R_2 \)       | 18 kΩ     |
| \( R_{av} \)    | 1.5 MΩ    |
| \( C_{av} \)    | 10 \( \mu \) F |

3. Tasks M. Determination of the Boltzmann constant

6. On every PCB at one of its corners there is a label with a preliminary measured capacity \( C \) written; it is close to 43 nF. Write down the value \( C \) for your set-up and from Fig. 9 determine \( \varepsilon \) for it both in % and absolute value. Then calculate \((1 - \varepsilon)\) for your set-up.

![FIG. 9. Error \( \varepsilon \) in percent in determination of \( k_B \) as a function of the capacitor \( C \).](image)

7. Take the value \( R_L \) from Table VI and calculate the quantity

\[ \tau_L = R_L C. \]  \hspace{1cm} (D2)

What is the dimensionality of this parameter? You can use standard mili (m) and micro (\( \mu \)) which are standard for the used SI units.
8. Calculate and write down the value

\[ B = \frac{|Y|^2}{4\tau_L} = \frac{|Y|^2}{4R_L C}. \]  

\[ \text{(D3)} \]

What is the dimensionality of this parameter? This is a very big number, you can rewrite your result using standard SI notations: k=Kilo=10^3, M=Mega=10^6, G=Giga=10^9, T=tera=10^{12}, P=Peta=10^{15}, E=Exa=10^{18}.

9. Now calculate and write down the corrected value

\[ B = (1 - \varepsilon)B \]  

\[ \text{(D4)} \]

in PHz. In electronics the parameter *B* is known as bandwidth from where the notation *B* is used. Knowing its value is crucial for our experimental task to measure the Boltzmann constant. Now we can go to our final task.

10. Switch the multimeter as voltmeter and using one crocodile connecting cable connect the common points of the voltmeter (usually marked as COM) with the cable from the PCB denoted with the usual triangular sign for ground $\triangle$.

11. Analogously, connect the other crocodile cable to the voltage output cable marked on the PCB with “OUT” to the voltage input of the multimeter often marked as VΩmA. You will work in the mV range of your multimeter, if you measure voltages larger than 200 mV, there is a problem either in connectivity or in the PCB itself. In such a case, before asking for assistance, check that the 2 female pins of the triaxial cable are connected by either a resistor or the short circuit.

12. Take the resistor with biggest resistance (and biggest number in your table) and connect it at the end to the of the three-axial cable of the set-up. This is the voltage input of the set-up. We actually measure the random voltage which thermal fluctuations induce at the end of the resistor. This random voltage is analogous to the Brownian motion of small particles immersed in a liquid. If our hearing were infinitely sharp, we would hear the drumming of air molecules on our eardrums. And today we measure the random motion and voltage of electrons moving along the length of a resistor.

13. Connect the four (2 by 2) 9 V batteries to the 2 doubled connection clips by buckling the electrodes.

14. **Be careful! From this point on you can burn the operational amplifiers which will terminate your experiment.** The orange labels of the batteries voltage clip connectors should match with the orange labels of the two 3 × PCB male pin headers on the PCB these are voltage pins. Insert the 2 battery voltage clips in the 2 voltage pins, the orange labels on the clips must face the orange labels of the pins. Wait 5 minutes and measure the output voltage. Make 5 records with interval 1 minute. Your experimental data should be rewritten and ordered in a table similar to example Table VII.

| i | U_a | U_b | U_c | U_d | U_e |
|---|-----|-----|-----|-----|-----|
| 0 | . . . | . . . | . . . | . . . | . . . |
| 1 | . . . | . . . | . . . | . . . | . . . |
| . | . . . | . . . | . . . | . . . | . . . |

**TABLE VII.** Table of the measured voltages for every numbered resistor. For every resistor the median voltage should be underlined. Sequence of the measurements is opposite. The measurement starts with the resistor with biggest resistance and finishes with the short circuit numbered with zero and its median for the 5 measurements is denoted by \( U_0 \).

15. Then change the resistor, wait 2 minutes and make again 5 records every 1 minute. In such a way you will spend 7 minutes for every resistor. In these records take the median (the voltage biggest than 2 smallest in smaller that 2 biggest voltages); underline this value.

16. The short circuit should be the last in your measurements. Again wait 2 minutes, measure the voltage, wait 1 minute and measure again. This will take the last 6 minutes of this series of measurements; \((7\times6=42)+3=45\) and within one hour you can complete this essential part of the experiment. The median of the measurements for every resistor has to be written as 3rd.

17. For half an hour, within one hour you will have median averaged voltage for your set of resistors including the short circuit which you should denote with \( U_0 \). Write the median voltage as 3rd column in your Table V.
18. Using the multimeter probes measure the voltage of every 9 V battery supplying current to the setup. Disconnect the batteries from the clips and measure their voltage again. Present these 8 voltages in a table; before and after disconnecting. The voltage source should be preserved for later measurements. Then we can start the experimental data processing.

19. The results for the averaged voltage has to be graphically presented versus the resistance. The resistance \( R_i \) is in the abscissa the median voltages \( U_i \) on ordinate index 0 and corresponding voltage \( U_0 \) is for the short circuit. You have to use paper with squares, millimeter or even 5 mm from the notebook. Stop to think for a while, the choice of the intervals is most important for the accuracy of our further work. You will have several points on the \( U-R \) plot. Using a ruler draw a line which best approximate the experimental points; this is almost an art.

20. Choose 2 points “A” and “B” close to the opposite ends of the drawn line. Estimate the resistances \( R_A \) and \( R_B \) (the abscissas) and the voltage values \( U_A \) and \( U_B \) (the ordinates). Let us denote the voltage difference between these points as \( \Delta U \) and analogously difference in resistance by \( \Delta R \). Now we can calculate the most important for our measurement parameter

\[
i_o = \frac{\Delta U}{\Delta R} = \frac{U_B - U_A}{R_B - R_A},
\]

having dimension current \( V/\Omega = A \). This procedure for approximating of a cloud of experimental points by a line is called in mathematics linear regression.

21. Measure the room temperature with the thermocouple, calculate its absolute temperature value and write them down both. For example if the room temperature is 23 degree \( T' \approx 273 + 23 = 296 \text{ K} \).

22. Take from Table VI the necessary parameters \( R_1 \) and \( R_2 \) for the set-up. and use the value \( U_m = 10 \text{ V} \) which describes the work of the integral circuit which makes analogous multiplication. Calculate the Boltzmann constant, including its dimension, using the formula

\[
k_B = \frac{i_o U_m}{4BT'} \frac{R_1}{R_1 + R_2}.
\]

The detailed derivation of this formula is given in the scientific description of the used set-up in the main article above. Congratulation it is perhaps your first measurement of a fundamental constant!

23. Calculate the product \( T = k_B T' \) in Joules, or in SI design in zepto Joules; \( 1 \text{ zJ}=10^{-21} \text{ J} \). This very small energy determines the typical energy of thermal motion. For example averaged translational kinetic energy of one air molecule is \( E_{\text{kin}} = \frac{3}{2}T \).

4. **Tasks L.** Determine the absolute zero

This is an extremely difficult task, almost an adventure. We are using a pre-amplifier with a huge \( 10^6 \) voltage amplification. All precise electronics is in screened metal boxes but our set-up is not. That is why this experiment could be easily corrupted by every external source of noise: working electromotors, contact of room voltage supply, luminescent lamps etc. Try to minimize all suspicious source of noise close to your table or desk.

24. In the given set of the resistors one is with the electric insulation tape. Put more insulation tape again if it is necessary. Switch this resistor at the end of the triaxial cable.

25. Compare your obtained value of \( U_0 \) with your median voltage for this resistor. If \( U_0 \) is larger than the median value, your measurement of \( U_0 \) is not good. In this case, calculate \( U_0 \) from the linear regression using the formula

\[
U_0 = \frac{1}{2}(U_A + U_B) - \frac{i_o}{2}(R_A + R_B).
\]

You may perform this task even if your measurement of \( U_0 \) is good. Then compare both obtained values of \( U_0 \) and decide which one to use or use their average.

26. Using insulation tape wrap together the insulated resistor with the end of the of thermocouple of the temperature sensor. The resistor and the thermocouple have to be close each other. The thermocouple will measure the temperature around the resistor.
27. Using appropriate vessel heat the salt to around 100 °C. The temperature should be definitely below 200 °C.

28. Take a tea cup and return to the setup. Now you have to be very careful. Put the wrapped thermocouple in the center of the tea cup and carefully put the the hot salt around the resistor and thermocouple.

29. The most important part of the experiment begins. Each 2 °C write in a table with 4 columns: 1) number, 2) temperature, 3) the voltage and 4) reduced voltage as shown in Table VIII. When the temperature reaches 40 °C

| i | t_i | U'_i | \bar{U}_i = U'_i - U_0 |
|---|-----|------|------------------------|
| 0 | ... | ...  | ...                   |
| 1 | ... | ...  | ...                   |
| ... | ... | ...  | ...                   |

TABLE VIII. Table of the measured voltages as a function of temperature.

remove the batteries from the set-up as the experimental part is completed.

30. Fill the 4th row in the last table. The measured voltage minus $U_0$. The voltage $U_0$ is the median of the voltage of short circuit.

31. Present the experimental data graphically. In the abscissa (horizontally) the temperature $t$ in °C, in the ordinate (vertically) the reduced voltage $\bar{U}$.

32. Draw a line close to the experimental points, perform a linear regression just like in Task 20 using the temperature interval (40 °C, 80 °C). Calculate the slope

$$K \equiv \Delta \bar{U} \over \Delta t = {\bar{U}_B - \bar{U}_A \over t_B - t_A}. \quad (D8)$$

33. Now calculate the intercept, for which the abscissa value = 0 ($\bar{U} = D + Kt$)

$$D = {1 \over 2} (\bar{U}_A + \bar{U}_B) - {K \over 2} (t_A + t_B). \quad (D9)$$

34. Calculate

$$t_0 = {D \over K} = t_A - {\bar{U}_B - \bar{U}_A \over t_B - t_A} \bar{U}_A. \quad (D10)$$

This is actually your today evaluation of the absolute zero.

Do not be disappointed -100 °C and -400 °C are good results for your first determination of the absolute zero.

5. Tasks XL. Derivation of the used formulae

35. Derive Eq. (D7).

36. Derive Eq. (D10).

37. Derive

$$\bar{U}(t) = \bar{U}_A + {\bar{U}_B - \bar{U}_A \over t_B - t_A} (t - t_A). \quad (D11)$$

38. Using the spectral density of the thermal voltage noise of a resistor $4k_BT'R$ derive the formula for the determination of Boltzmann constant Eq. (D6). For tutorial use the main unabridged article above.
Underline your results, scan or photo all your pages and send the file in PDF format to EPO@bgphysics.eu. File name is your name and category you decide to participate, for example, “EnricoFermiXL.pdf”.

We will announce the results tomorrow.

With best regards,

EPO

6. Epilogue

EPO9 is held in a even in a more difficult for the whole world conditions than EPO8. Approximately half of the participants are on-line. The organizers of EPO9 would like to thank everyone who helped in the preparation of this wonderful competition especially the president of the Society of Physicists of Macedonia, Prof. Lambe Barandovski for ensuring on-spot participation in Skopje. We are waiting you at EPO10; next year same time.

7. Acknowledgments

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8. EPO – a historical perspective. Skip this section during the Olympiad

From its very beginning, the Experimental Physics Olympiad (EPO) is worldwide known; all Olympiad problems have been published in Internet [3, 4, 50–55] and from the very beginning there were 120 participants. In the last years high-school students from more than 10 countries participated and the distance between the most distant cities is more than 4 Mm.

Let us describe the main differences between EPO and other similar competitions.

- Each participant in EPO receives as a gift from the organizers the set-up, which one worked with. In such a way, after the Olympiad has finished, even bad performed participant is able to repeat the experiment and reach the level of the champion. In this way, the Olympiad directly affects the teaching level in the whole world. After the end of the school year, the set-up remains in the school, where the participant has studied.

- Each of the problems is original and is connected to fundamental physics or the understanding of the operation of a technical patent.

- The Olympic idea is realized in EPO in its initial from and everyone willing to participate from around the world can do that. There is no limit in the participants number. On the other hand, the similarity with other Olympiads is that the problems are direct illustration of the study material and alongside with other similar competitions mitigates the secondary education degradation, which is a world tendency.

- One and the same experimental set-up is given to all participants but the tasks are different for the different age groups, the same as the swimming pool pool water is equally wet for all age groups in a swimming competition.

- One of the most important goals of the Olympiad is the student to repeat the experiment at home and to analyze the theory necessary for the understanding. In this way any even badly performed motivated participant has the possibility to be introduced to the corresponding physics field, even though there is no physics classroom in his/her school, even though the physics education in his/her country to be deliberately destroyed.
We will briefly mention the problems of former 7 EPOs: 1) The setup of EPO1 [50] was actually a student version of the American patent for auto-zero and chopper stabilized direct current amplifiers. It was notable that many students were able to understand the operation of an American patent without special preparation [59]. 2) The problem of EPO2 [51] and EPO8 [55] was to measure Planck constant by diffraction of a LED light by a compact disk. 3) A contemporary realization of the assigned to NASA patent for the use of negative impedance converter for generation of voltage oscillations was the set-up of EPO3. [52] 4) EPO4 [53] was devoted to the fundamental physics – to determine the speed of light by measuring electric and magnetic forces. The innovative element was the application of the catastrophe theory in the analysis of the stability of a pendulum. 5) The topic of the EPO5 [3] was to measure the Boltzmann constant $k_B$ following the Einstein idea of study thermal fluctuations of electric voltage of a capacitor. 6) The EPO6 [4] problem can be considered as a continuation of the previous Olympiad. With a similar electronic circuit Schottky noise is measured and his idea for the determination of the electron charge is realized. 7) The EPO7 [54] problem was to measure a large inductance made by a general impedance converter by the Maxwell-Wien bridge. 8) The EPO8 problem was a modified repeat of EPO2 with first online participation in the complicated world global situation in 2021.

Each problem given at EPO can be considered as a dissertation in methodology of physics education.

In short, the established traditions is a balance between contemporary working technical inventions and fundamental physics.

The EPO problems are meant for high school and university students but are posed by teachers with co authorship with colleagues working in universities or scientific institutes. For colleagues interested in new author’s problems for the needs of the contemporary physics education we share our experience in the description of the experimental set-ups described at a university level. These are for instance:

1) The determination of the Planck constant without light but only with electronic processes study. [57] this set-up requires the usage of an oscilloscope but in some countries the oscilloscopes are available in in the high schools physics labs and the prices of the former is constantly going down.

2) The speed of the light without the usage of scales or high frequency equipment is another innovative set-up [58] for high school education. And the idea for this experiment is given by our teacher in electrodynamics Maxwell.

3) In the physics curriculum in all countries it is mentioned that the temperature is a measure of average kinetic energy of the gas molecules but the Boltzmann constant $k_B$ that gives the relation between energy and temperature is not measured in high school and even rarely in the best universities. The experimental set-up for $k_B$ by the method proposed by Einstein (the EPO5 problem) is described [20] as a set-up for university school lab exercise in a impact factor journal. But what larger impact an experimental set-up that is used by high school students from Kazakhstan and Macedonia and the surrounding countries can have. More than 100 set-ups were distributed around the world.

4) Similar thoughts can be expressed for the electron charge $q_e$. This fundamental constant is also mentioned in the high school education as a humanitarian incantation but is not measured. We broke this tradition and described an experimental set-up (from EPO6) in the European Journal of Physics [22]. This set-up can be built for a week in every high school. The Schottky idea for determination of the electron charge by measuring voltage fluctuations is used. From the idea to the realization more than 100 years have passed and one of the reasons is that in many countries the largest enemy of the education is the ministry of education.

The development of some set-ups required additional study of circuits with operational amplifiers (OpAmps). This led to introduction of the master equation of OpAmps applied to the stability of circuits with Negative Impedance Converter (NIC) [42], Generalized Impedance Converter [59] and study of Probability Distribution Function [29] of the crossover frequency of OpAmps. Continuing this stile, the scientific description of the set-up of the present Olympiad (EPO9) is given at university education level [60].

The mission of the physics teachers in the worldwide progress is evident – precisely our science reshaped the world in the last century. Successful innovative EPO set-ups after some update can be manufactured by companies specialized in production of educational equipment like TeachSpin[3] and PHYWE[4] for instance.
Appendix E: Solutions to the EPO9 Problem by the absolute champion Kristijan Shishkoski

| 1 | 2 | 3 |
|---|---|---|
| 54.6 | 155.1 | 44.8 |
| 22.2 | 126.2 | 22.2 |
| 105.1 | 115.5 | 154.8 |
| 154.8 | 125.0 | 145.0 |

\[ \frac{V}{2} - \frac{V}{2} = \frac{10^7 \cdot 10^7}{2} - \frac{10^7 \cdot 2 \cdot 10^4}{2} = \frac{10^7}{6} = 166.67 \text{ mV} \]

\[ V_a = \frac{8 \mu}{20 \mu} + 1 = \frac{10^7 \cdot 10^7}{20 \cdot 10^7} + 1 = 101.5 \]

\[ Y = V_{in} \cdot V_{out} = 101 \cdot 100 (-100) = 10100000 \]
\[ C = 40,4 \text{ nF} \]
\[ E \approx 9,37\% = 9,37 \cdot 10^{-2} \]
\[ (1 - E) \approx 90,63\% = 0,9063 \]
\[ \tau_c = R \cdot C = 100 \Omega \cdot 40,4 \text{ nF} = 100 \Omega \cdot 40,4 \cdot 10^{-9} \text{ F} = \\
\tau_c = 4,04 \cdot 10^{-6} \text{ s} = 4,04 \mu\text{s} \]

Задача 1: Единиця мерки на \( \tau_c \):

Од формулираща за еднуциям на кокео:

\[ F = \frac{E}{\tau_c} \]

Од формулираща за бремствуване на кокео:

\[ C = \varepsilon_0 \frac{S}{d} \Rightarrow 1F = 1 \frac{C^2}{N \cdot m} \]

\[ : 1F = 1 \frac{C^2}{N \cdot m} \]

Од Уил-Ленчов Закон и Оомб Закон:

\[ Q = UI t; U = IR \Rightarrow Q = I^2 R t \Rightarrow [ J = 1A; 1S = 1 \text{ A} \cdot 1S = 1 \text{ W} \cdot 1 \text{ m}] \]

\[ \Rightarrow [ J = 1A = \frac{1}{1S} = 1 \text{ W} \cdot 1 \text{ m}] \]

\[ \Rightarrow J = 1A = 1 \frac{N \cdot m \cdot s}{C^2} \]

\[ : 1A \cdot 1F = 1 \frac{C^2}{N \cdot m} \Rightarrow 1A \cdot 1F = 1S \]

\[ B = \frac{1V}{1 \cdot 10^{-12} \Omega} = 6,3125 \cdot 10^{16} \text{ Hz} = 63,125 \text{ Гц} \]

Задача 2: Единиця мерки на \( B \):

Од Фреку формулираща за кокео и Уил-Ленчов Закон се заключува дека:

\[ [B] = 1 \frac{\text{A}}{\text{m}} \]

\[ = \frac{1}{\mu} \text{H/m} \]

B \[ [B] = \left[ \frac{1}{\mu} \right] = \left[ \frac{1}{\text{H/m}} \right] \]
$B = (1 - 2) B = 0,9063 \cdot 63,125 \text{ PHz}$

$B = 57,2101875 \text{ PHz}$

| $i$ | $U_a$ | $U_b$ | $U_c$ | $U_d$ | $U_e$ |
|-----|-------|-------|-------|-------|-------|
| 0   | 109.2 | 106.6 | 106.7 | 105.9 | 106.6 |
| 1   | 158.6 | 158.9 | 158.1 | 157.1 | 158.6 |
| 2   | 149.7 | 147.8 | 149.9 | 151.1 | 149.9 |
| 3   | 127.2 | 126.3 | 125.4 | 125.5 | 126.2 |
| 4   | 115.3 | 114.9 | 115.4 | 115.0 | 115.3 |
| 5   | 142.5 | 141.6 | 141.8 | 142.1 | 141.3 |
| $*$ | 148.2 | 143.1 | 137.3 | 145.0 | 148.3 |

- Првомерење на $U_0$, се добијање многих свих сокоови, од 106 mV до 180 mV до некога секунда, та добија вредноста $U_0$. Се нациртише. За време $U_0$, се мерење говедећац. Узима се свих сокоови се добијање и прво мерење на дугуше наживе, на торењко.

$R_a = 17.5 \Omega$  $U_{a0} = 122 \text{ mV}$

$R_b = 52.5 \Omega$  $U_{b0} = 155.5 \text{ mV}$

$\delta = \frac{\Delta U}{\Delta U} = \frac{U_a - U_{a0}}{R_a - R_a} = \frac{155.5 - 122}{52.5 - 17.5} = 9,51423 \cdot 10^{-4}$

$T' = 22^\circ C = 295 K$

$R_b = \frac{4U_0}{4T' + R_1 + R_2} = \frac{9,51423 \cdot 10^{-4} \cdot 10 \text{ V}}{4 \cdot 295 \cdot 57,2101875 \cdot 10^{15} \text{ Hz}} (2 \pm 10) \text{ k} \alpha = \frac{4 \cdot 10^{23}}{K}$

$T = 1,409 \cdot 10^{-23} J \cdot 295 K \approx 4,15655 \cdot 10^{-21} J = 4,15655 \text{ eJ}$

$E_{kin} = \frac{1}{2} T = \frac{3}{2} \cdot 4,15655 \text{ eJ} = 6,234825 \text{ eJ}$
1) $\tilde{U}_A = 32,4 \text{ mV}$  \hspace{0.5cm} t_A = 46 ^\circ C$

$\tilde{U}_B = 35,15 \text{ mV}$  \hspace{0.5cm} t_B = 74 ^\circ C$

$$K \equiv \frac{\tilde{U}}{\Delta t} = \frac{\tilde{U}_B - \tilde{U}_A}{t_B - t_A} = 0,0382 \text{ mV} / ^\circ C$$

2) $\tilde{U} = D + Kt$

$$D = \frac{1}{2} (\tilde{U}_A + \tilde{U}_B) - \frac{K}{2} (t_A + t_B)$$

$$D = \frac{1}{2} (32,4 + 35,15) \text{ mV} - \frac{9,0982 \text{ mV}}{2} \frac{46 + 74}{2} ^\circ C \approx 27,883 \text{ mV}$$

3) $t_0 = - \frac{D}{K} = t_A - \frac{\tilde{U}_B - \tilde{U}_A}{t_B - t_A}$

$$t_0 = - \frac{27,883 \text{ mV} \cdot ^\circ C}{0,0382 \text{ mV}} \approx -283,94 ^\circ C$$

Moja dobija vrijednosti je $-283,94 ^\circ C$

Pogleda vrijednosti je $-273,15 ^\circ C$

$\Delta t = | -283,94 - (-273,15) | ^\circ C = 10,79 ^\circ C$

$E_{\text{st}} = 0,04$

U ovom slučaju zavisnosti funkcija na vrijednosti (linearna regresija, frekvencija), može se rešiti ovisno otemenje na oko 4% od toga dužina vrijednosti.

4) (7) $U_0 = \frac{1}{2} (U_A + U_B) - \frac{I_0}{2} (R_A + R_B)$

Og izraza od (8):

$U = U_0 + I_0 R$ \hspace{1cm} $I_0$ je koeficijent na izrazu $I$.

$U_0 = U - I_0 R$ \hspace{1cm} $U = \frac{U_A + U_B}{2}$, \hspace{0.5cm} $R = \frac{R_A + R_B}{2}$ \hspace{0.5cm} $U_0$ je srednja vrijednost

$U_0 = \frac{U_A + U_B}{2} - I_0 \frac{R_A + R_B}{2} = \frac{1}{2} (U_A + U_B) - \frac{I_0}{2} (R_A + R_B)$
\[ t_o = - \frac{D}{K} = t_a - \frac{\tilde{U}_a - \tilde{U}_b}{t_b - t_a} \]

From the graph and Eq. (37)
\[ \tilde{U} = U_0 + K t \]

Eq. (37): \( D = U_0 \)
\[ \tilde{U} = D + K t \]

At \( t = t_o \) and \( \tilde{U} = 0 \)
\[ O = D + K t_o \]
\[ t = - \frac{D}{K} \]

\( K \) is the coefficient of the graph.
\[ K = \frac{\tilde{U}_b - \tilde{U}_a}{t_b - t_a} \]

Analogous to Eq. (37), considering the graph
\[ D = \frac{\tilde{U}_a + \tilde{U}_b}{2} - \frac{K}{2} (t_a + t_b) = \frac{1}{2} (\tilde{U}_a + \tilde{U}_b - K (t_a + t_b)) \]

\[ t_o = - \frac{1}{2} \frac{(\tilde{U}_a + \tilde{U}_b - K (t_a + t_b)) (t_b - t_a)}{\tilde{U}_b - \tilde{U}_a} \]

\[ = - \frac{1}{2} \frac{(\tilde{U}_a + \tilde{U}_b - (\tilde{U}_b - \tilde{U}_a) \frac{t_b - t_a}{t_b - t_a}) (t_b - t_a)}{\tilde{U}_b - \tilde{U}_a} \]

\[ = - \frac{1}{2} \frac{(\tilde{U}_a + \tilde{U}_b) (t_b - t_a) - (\tilde{U}_b - \tilde{U}_a) (t_a + t_b)}{\tilde{U}_b - \tilde{U}_a} \]

\[ = \frac{1}{K} \frac{2 \tilde{U}_a t_b - 2 \tilde{U}_b t_a}{\tilde{U}_b - \tilde{U}_a} = \frac{\tilde{U}_b t_a - \tilde{U}_a t_b}{\tilde{U}_b - \tilde{U}_a} \]

\[ t_o = t_a - \frac{\tilde{U}_b - \tilde{U}_a}{t_a - t_b} \tilde{U}_a \]
\( \ddot{U}(t) = \ddot{U}_B + \frac{\ddot{U}_A - \ddot{U}_B}{b_b - b_A} (b - b_A) \) - ?

\[ \begin{align*}
0 \quad \text{condition on } \alpha_0 \\
\ddot{U}(t) &= \ddot{U}_0 + Kt \\
\ddot{U}_A &= \ddot{U}_0 + Kt_A \\
\ddot{U}(t) &= \ddot{U}_0 + Kt_A + U_0 + Kt - \ddot{U}_A \\
\ddot{U}(t) &= \ddot{U}_A + K(t - t_A)
\end{align*} \]

\[ \begin{align*}
\text{Problem 3:} \quad &4k_b T_R' \\
\text{Suggested equation for the problem:} \quad 4k_b T_R'
\end{align*} \]

\[ k_B = \frac{\varepsilon_0 U_m}{4 \pi' R} \frac{R_1}{R_1 + R_2} \] - ?

\[ \text{S.G. for} \\
\text{Problem 4:} \quad 4k_b T_R' \Rightarrow \quad k_b = \frac{A}{4 \pi' R} \]
