Quark Recombination in Heavy Ion Collisions

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Data on high energy nuclear collisions collected at the Relativistic Heavy Ion Collider over the past decade have provided convincing evidence that hadronization is quite different in hot nuclear environments compared to \( p + p \) collisions. In particular, the data suggest that we see traces of quark degrees of freedom in elliptic flow, with the implication that collective flow is generated on the parton level and is transferred to hadrons through a simple recombination step. In this contribution we review the experimental evidence for quark recombination and discuss some recombination models which are used to describe these effects.

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1. Introduction

The experimental program at the Relativistic Heavy Ion Collider (RHIC) was born from the idea that a deconfined state of quarks and gluons, the quark gluon plasma (QGP), can be created and studied in collisions of heavy nuclei at the highest possible collision energies. Soon after turning on RHIC it became clear that we indeed see novel and unusual phenomena [1]. Some of those like strong jet quenching had been predicted qualitatively, others, like the quark number scaling of elliptic flow were surprising. Over the years a consensus seems to have emerged that quarks and gluons are indeed deconfined for a short amount of time in the fireball created at RHIC, and that this quark gluon plasma behaves like a very good liquid with small viscosity over entropy ratio $\eta/s$.

The key ingredient for this conclusion was the comparison of data with ideal (and later viscous) hydrodynamic calculations based on equations of state with partonic phases and small $\eta/s$ [1, 2]. However, there are plenty of reasons to check whether alternative explanations can be ruled out with certainty.

In this contribution we review the role of quark recombination models at RHIC [3, 4, 5, 6]. The main motivation for their emergence was the anomalous baryon enhancement and the observed quark number scaling of elliptic flow [7, 8, 9]. Such recombination models provide strong evidence that collective flow is partonic in origin. In particular, quarks and gluons seem to be the relevant degrees of freedom when elliptic flow is built up. This touches a topic that is of interest beyond heavy ion physics, namely the still not understood phenomenon of hadronization.

Hadronization, i.e. the color neutralization process that requires quarks and gluons to form hadrons, is a non-perturbative phenomenon that has largely defied a first-principle computation. There are 3 basic approaches to hadronization that are relevant in our context:

- Factorization: In some cases with large momentum transfer hadronization can be separated from the underlying (scattering) process in a rigorous way [10]. This works for single quarks and gluons fragmenting into jets in the vacuum at sufficiently large momentum and for exclusive processes at large momentum transfer. The fragmentation process for jets is universal and can be parameterized through fragmentation functions [11].

- Statistical and cluster emission concepts: They can do very well explaining certain bulk features of hadron production like hadron ratios, see e.g. [3].

- Microscopic models: They try to capture certain aspects of the underlying microscopic dynamics, though they are not comprehensive or derived from first principles. Examples are string fragmentation or quark recombination.

The idea that quarks coalesce into bound states similar to the coalescence of nucleons into light nuclei or plasma constituents into atoms has been around since the beginning of quark models and quantum chromodynamics (QCD) [12, 13, 14, 15]. The nature of QCD as a non-linear relativistic quantum field theory with a very complex non-perturbative sector limited the success of recombination models to particular situations. Generally those are characterized by the feature that a well-defined multi-quark state for hadronization can be identified. This is particularly true for the leading particle effect, in which a quark (heavy flavors such as charm and strange quarks are
experimentally accessible using identified particle tags) is produced in a collision in forward direction and coalesces with a quark from the beam remnant. One example is the $D^-/D^+$ asymmetry observed in the fixed target experiment E791 which used a $\pi^-$ beam on a nuclear target [16]. The asymmetry which grows to almost 100% at extreme forward direction ($F_{xF} \to 1$) can be explained by $c\bar{c}$ pair production with a preferential recombination of the $\bar{c}$ with the $d$ valence quark from the pion fragments, while the corresponding $c+\bar{d}$ combination does not involve a valence quark of the pion and is thus suppressed, see Fig. 1. The leading particle effect is probably the most convincing argument for the existence of a quark recombination mechanism outside of heavy ion physics [17].

2. Experimental Evidence

A factor 5 suppression of high momentum hadrons was found soon after RHIC was turned on. This jet quenching phenomenon was expected from the predictions of parton energy loss. The suppression was roughly consistent for pions and kaons with a few GeV/$c$ transverse momentum $P_T$. However, the discovery that protons and $\Lambda$ baryons show little or no suppression was a surprise. It also threatened the partonic interpretation of energy loss since jet hadronization, even if altered by the presence of a medium was thought to basically transfer quenching equally to all hadrons. These findings, that are now the cornerstones of experimental evidence for quark recombination, were first known as the baryon anomaly at RHIC, since they defied our expectations for the intermediate $P_T$ range. In that $P_T$ range (roughly between 1 and 5 GeV/$c$) we find [1, 2].

- anomalously large baryon-to-meson ratios which were up to a factor 4 larger than expected from $e^+ + e^-$ or $p + p$ collisions, see Fig. 2. The proton-to-pion ratio can be one.

- systematically larger suppression (shown by smaller nuclear modification factors $R_{AA}$ and $R_{CP}$) for mesons than for baryons.
Figure 2: Data from STAR and PHENIX on $p/\pi$ and $K/\Lambda$ ratios in central Au+Au collisions at RHIC together with model calculations in the GKL and FMNB formalisms [4].

- systematically larger elliptic flow for baryons than for mesons with peak values roughly 50% larger. This was later recognized to follow the simple scaling law (see Fig. 3)

$$\frac{1}{3} v_2^B (3p_T) = \frac{1}{2} v_2^M (2p_T)$$

where the factors 3 and 2 refer to the number of valence quarks in baryons $B$ and mesons $M$ respectively [18].

Prior to RHIC this intermediate $p_T$ regime was expected to be dominated by the physics of jets and hard processes, but the experimental data seemed to be telling otherwise. Attempts to treat this as a transition region between bulk hydrodynamics below 1 to 2 GeV/c and pure jet quenching and fragmentation at larger $p_T$ failed to capture crucial details, e.g. the fact that the $\phi$ meson does not behave similar to the almost equally heavy proton, but rather than the much lighter pion [19]. This is evidence that at intermediate $p_T$ the number of valence quarks, and hence hadronization, is more important than collectivity in the hadronic phase which in a hydrodynamic picture depends solely on the mass of the hadron. We conclude that the intermediate $p_T$ region in heavy ion collisions shows features which are neither hydrodynamic nor consequences of jet fragmentation.

The basic experimental findings of the early RHIC years have stood the test of time. In more recent years it has been found that a quark number-scaling of $v_2$ using kinetic energy instead of transverse momentum improves the scaling at low momentum (or kinetic energy), see Fig. 3. But as we will argue below this has no direct implications for quark recombination.

3. Modeling Quark Recombination

As in the case of the leading particle effect, recombination models for heavy ion collisions are based on the notion of a well-defined distribution of quarks just before hadronization. This is
thought to be a thermalized plasma characterized by a temperature $T = T_c + \epsilon$ with some modest deviations from equilibrium allowed. Usually it is never clearly specified what the precise assumptions are, but the following properties seem to be important in most models:

- Gluons are frozen as degrees of freedom and quarks have already acquired effective constituent-like masses.

- The effective quarks are close to the mass shell such that the formation of additional quark-antiquark pairs is suppressed.

In such a scenario one can compute the projection of the density matrix $\rho$ of effective quarks onto hadron states

$$N_h = \int \frac{d^3 P}{(2\pi)^3} \langle h; P | \rho | h; P \rangle$$

where mesons and baryons are represented by their valence quarks. This approach is called the instantaneous quark recombination formalism since it happens suddenly, and it only conserves 3 out of 4 components of the energy-momentum vector. From the projection formula one can derive a straight-forward overlap integral for the spectrum of mesons (baryons are analogous) coalescing from partons $a, b$ [20, 4]

$$\frac{dN_M}{d^3 P} = \sum_{a,b} \int \frac{d^3 R}{(2\pi)^3} \int \frac{d^3 q d^3 r}{(2\pi)^3} W_{ab} \left( R - \frac{r}{2} , -q; P + \frac{P}{2} + q \right) \Phi_M(r, q)$$

where $W_{ab}$ is the Wigner function of partons $a$ and $b$ in the fireball and $\Phi_M$ is the Wigner function of meson $M$, and $r$ and $q$ are the relative position and momentum of the two quarks.
From this equation different implementations have emerged, ranging from full phase space overlap integrals (e.g. the model by Greco, Ko and Levai [GKL] [21, 22]) to simplified schemes using 1-dimensional momentum integrals as in collinear factorization (e.g. the models by Fries, Müller, Nonaka and Bass [FMNB] [23, 20, 24, 25], and Hwa and Yang [HY] [26, 27]). The quark Wigner function is usually approximated by the $n$-quark phase space distribution. The hadron Wigner functions are not known a priori and are usually modeled according to simple guiding principles (e.g. exclusive wave functions in the case of FMNB) with a few simple parameters to fit. Indeed it turns out that applying this formalism to phase space densities of thermalized quarks at large momenta, makes the results very insensitive to details of the hadron Wigner function, since

$$W_{ab} \sim e^{-\frac{P}{2} - q T} e^{-\frac{P}{2} + q T} = e^{-\frac{q}{T}}.$$  \hspace{1cm} (3.3)

independent of the relative momentum $q$ in the hadron Wigner function.

Despite their differences in detail all instantaneous recombination models share common benefits and shortcomings [4]:

- They violate energy conservation on the level of $M/P_T$ and $k_T/P_T$ where $M$ and $k_T$ are hadron masses and intrinsic transverse momenta of quarks. Hence we can only expect them to provide reasonable results for large enough $P_T$, at least 1-2 GeV/c.

- None of these models enforce quark number conservation. Rather quarks at lower momentum are seen as a fixed background. Since the description should be limited to a small part of phase space there is no problem of entropy conservation in these instantaneous models.

- Recombination does not make any a priori predictions about the quark or quark gluon plasma phase itself. However, if hadron spectra are experimentally measured one can fit quark distributions before hadronization as input for recombination models.

- This immediately leads to the most stringent test for recombination model: the slew of different hadrons measured should be fit by only one quark distribution as input. All recombination models do this remarkably well, including describing hadrons that clearly break from hydrodynamic behavior like the $\phi$ meson.

Instantaneous recombination models give access to fundamental parameters of the quark phase at intermediate $P_T$ which is modeled as being close to thermal equilibrium so that the concepts of temperature and collective flow apply. The temperature, flow profile and the volume of the fireball (or more precisely the hadronization hypersurface) are fit parameters. Typical away-from-equilibrium deviations needed to fit the data are modifications to the flow that make the elliptic flow saturate at intermediate $P_T$ [20]. This saturation in the data — in Fig. 3 seen for $1 \text{ GeV/c} < P_T/n < 2 \text{ GeV/c}$ is a clear indication that thermalization is no longer perfect at intermediate $P_T$. Using these assumptions models do well describing the differences in suppression between baryons and mesons, and describing hadron ratios at intermediate $P_T$, including the $\phi$ meson. The available data is not sufficient to completely constrain the space-time dependence of the flow profile. A simple factorized ansatz is usually used in which an asymmetry in momentum space is imprinted on the quark phase. Note that this is different from a blastwave where there is a strong correlation in
direction and magnitude between the position vector $\mathbf{r}$ of a fluid cell in the transverse plane, and the local flow vector $\mathbf{v}$.

With the factorized ansatz it is easy to show that

$$v_2^h(P_T) = n_h v_2^q \left( \frac{P_T}{n_h} \right)$$

(3.4)

where $n_h$ is the number of valence quarks in hadron $h$. This leads naturally to the experimentally observed scaling law and is seen as a direct observation of quark degrees of freedom with collective flow. However, one needs to be cautious about the assumptions used for the factorization ansatz for the quark flow field. Indeed, using blastwave-like flow profiles as they naturally emerge from hydrodynamics lead to modifications of the scaling law which can not be reconciled with experimental data [28, 29]. This remains an unsatisfactory situation to this date. Recombination models lead to quark number scaling using flow profiles which are not consistent with hydrodynamic concepts. On the other hand, quark number scaling in data is extremely robust and holds to a surprisingly large accuracy, and other attempts to explain the scaling have mostly failed. E.g. hadronic transport models can get similar scaling but the overall size of the elliptic flow is too small. Recent attempts in viscous hydrodynamics are more promising but need fine-tuning of viscous freeze-out distributions which is not under control [30].

Instantaneous recombination models are often supported by calculations of jet energy loss and fragmentation at large $P_T$. This opens the possibility to introduce a jet-like component in the quark phase (either leading partons or full jet showers) and to allow for recombination of quarks coming partly from the bulk and partly from jets. This is known as soft-hard or thermal-shower recombination. Such cross terms can be seen as a step toward describing the modified hadronization of jets piercing a fireball. They also introduce jet-like correlations at intermediate $P_T$ although the absolute yields are dominated by hadrons from coalescence of bulk fireball quarks. The farthest reaching approach is the HY model which introduces a quark distribution at hadronization [31, 32]

$$f(P_T) = f_{\text{soft}}(P_T) + f_{\text{shower}}(P_T)$$

(3.5)

and applies quark recombination to all quarks equally such that mesons formed from two jet-shower quarks reproduce jet fragmentation, mesons from two soft quarks represent the usual recombination from the fireball, and the cross-terms describe the modifications to jet fragmentation in the medium.

Fig. 4 shows spectra for four different hadrons calculated in the FMNB model together with data from RHIC. The FMNB models uses quark recombination at intermediate $P_T$ supplemented with a jet energy loss and fragmentation calculation at high $P_T$. One can clearly see those two domains with crossovers around 4 GeV/$c$ for mesons and around 6 GeV/$c$ for baryons. At small $P_T$ deviations from data start to occur due to missing energy conservation and due to the mass mismatch for Goldstone bosons.

We want to conclude this section about recombination models by exploring ideas to implement quark recombination for the bulk fireball at low transverse momenta. Such a model clearly needs to enforce energy and momentum conservation and should allow for the preservation of thermal and chemical equilibrium. One approach is the resonance recombination model by Ravagli and Rapp [33, 34]. It couples mesons as resonances to a fixed background of quarks which resembles the
fireball just before hadronization. Formation of mesons is governed by a Boltzmann equation with gain ($q + \bar{q} \rightarrow M$) and loss ($M \rightarrow q + \bar{q}$) terms. To be precise

$$p^\mu \partial_\mu F_M(t, x, p) = -m \Gamma F_M(t, x, p) + p^0 \beta(x, p),$$

(3.6)

where $F_M(t, x, p)$ denotes the phase space density of the meson, and the gain term is given by

$$\beta(x, p) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} f_q(x, p) f_{\bar{q}}(x, p) \sigma(s) \nu_{rel}(p_1, p_2) \delta^3(p - p_1 - p_2),$$

(3.7)

with the resonant cross section modeled by a relativistic Breit-Wigner form:

$$\sigma(s) = g^2 \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2 + \Gamma m)^2}.$$  

(3.8)

The resonance recombination (RRM) approach has been criticized for treating the quark phase as a static background, but note that the instantaneous recombination formalism as well does not treat the quark phase explicitly. In other words both formalisms do not address the question of confinement and could as well be applied to a theory with bound states but without confinement. The big advantage of the RRM formalism is the conservation of momentum and energy, and the

Figure 4: Spectra for neutral pions, kaons, protons and $\Lambda$ particles calculated in the FMNB model including quark recombination and jet fragmentation compared to data from RHIC [20].
enforcement of detailed balance. Hence one can show that in the long-time limit a meson distribution recombining from a quark phase in local equilibrium is also in local equilibrium with the same temperature and collective flow velocity as the quark phase \[^{[35]}\]. This can be seen in Fig. 5 which shows spectra and elliptic flow of hadrons obtained once directly from blastwave models with a temperature equal to the critical temperature, and once from resonance recombination of quarks where the quark phase space distributions have been determined from the same blast wave at the critical temperature.

Because of its properties of energy conservation and detailed balance resonance recombination is a first logical step to a comprehensive modeling of quark recombination for the bulk fireball at low \(P_T\). One of the more impressive features is that it can produce negative values of \(v_2\) (which in blastwave and hydrodynamic models can occur at low \(P_T\) for very heavy particles) using quarks with strictly positive \(v_2\). For instance this can happen in \(c\bar{c} \rightarrow J/\psi\) coalescence. This also indicates that there is no simple scaling law for elliptic flow at low momenta, neither for quark number nor for kinetic energy.

Resonance recombination suggests that one can access the quark phase space distributions at hadronization also as low \(P_T\). Note that the fitting of quark phase space distributions to describe hadron data with instantaneous coalescence at intermediate \(P_T\) works since hadronic rescattering for hadrons with such large momentum is rather scarce and the measured distribution resembles the spectrum at hadronization. This is not true for the bulk fireball at low \(P_T\) for which rescattering in the hadronic phase is believed to be very important. However, one can analyze bulk hadrons which are known to have very small cross sections and for which we have indication from data that they freeze-out just below the critical temperature. Multi-strange hadrons (\(\phi\), \(\Xi\), \(\Omega\)) are such hadrons. We can use them to fit quark distributions at low \(P_T\) using resonance recombination, which has been done in \[^{[35]}\].

**Figure 5:** Left panel: Spectrum of \(\phi\) mesons calculated from a blastwave model, and from resonance recombination using quark phase space distributions from the same blastwave. Right panel: Elliptic flow \(v_2\) for three different hadrons calculated from a blastwave and from resonance recombination using the same blastwave \[^{[35]}\].
4. Open Questions and Outlook

We have established quark recombination models to successfully explain key features of hadron production in heavy ion collisions. We note that there is no other natural mechanism to explain both the large yields of baryons, and the quark number scaling law for elliptic flow at momenta of several GeV. However, recombination should not be seen as irreconcilable with hydrodynamics and jet fragmentation. In fact we have proof of principle that quark recombination can reproduce both fragmentation functions and local thermal equilibrium hadrons. If both regions joined smoothly at some intermediate $P_t$ in the data recombination as a hadronization mechanism would not have come to our attention since fragmentation functions and a phase transition in the equation of state in hydrodynamics would have taken care of hadronization. However, at RHIC there seems to be a sufficiently large range of momenta in which matter is not in local thermal equilibrium (note e.g. the saturation of elliptic flow) but not at all in a “dilute” jet fragmentation regime (note e.g. the large baryon/meson ratio). In this regime neither the hydrodynamic nor the fragmentation concept are available and we have to resort to some microscopic model which quark recombination supplies. Our discussion above about successfully establishing either fragmentation or equilibrium distributions through quark recombination shows that both at low and high momenta recombination is compatible with the concepts available in those respective regions.

This gives recombination models a valuable place in heavy ion phenomenology. The question whether quark recombination gives a direct glimpse of the parton phase, and, among other things, proves that collective flow is first carried by partons needs further study. The fact that quark-number scaling laws have only been proved for unrealistic space-momentum correlations in the flow field is of concern. On the other hand, kinetic energy scaling which has sometimes been advertised as not compatible with quark recombination, is actually not at all related to it. It is rather an effect of equilibrium and hydrodynamic flow. In Ref. [35] it has been shown that it can be reproduced in the resonance recombination model with some simple assumptions about freeze-out times. Kinetic energy scaling seems to be somewhat accidental at RHIC and we should not be surprised if it is not manifest at other collision energies. On the other hand, quark-number scaling is a true test for quark recombination and we should see it hold at larger collision energies, as long as we can neglect rescattering in the hadronic phase.

We also need to find a way to incorporate confinement and chiral symmetry breaking, i.e. hadron mass generation into recombination models. This would allow fully exclusive simulations that track the evolution of all quarks in a sector of the fireball and hadronize them into hadrons obeying all conservation laws and symmetries of QCD.

5. Summary

Quark recombination is an effective microscopic model for hadronization which works very well to explain certain key features of the underlying dynamics. The basic concept is simple and has withstood dramatic improvements. However, it has been shown that recombination can pass as a microscopic model for both equilibrium hadronization and fragmentation. Recombination models do not make predictions for the quark phase, but they can be used to extrapolate measured hadron data back to the time just before hadronization.
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