An Optimal Schedule for Urban Road Network Repair Based on the Greedy Algorithm

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Abstract

The schedule of urban road network recovery caused by rainstorms, snow, and other bad weather conditions, traffic incidents, and other daily events is essential. However, limited studies have been conducted to investigate this problem. We fill this research gap by proposing an optimal schedule for urban road network repair with limited repair resources based on the greedy algorithm. Critical links will be given priority in repair according to the basic concept of the greedy algorithm. In this study, the link whose restoration produces the ratio of the system-wide travel time of the current network to the worst network is the minimum. We define such a link as the critical link for the current network. We will re-evaluate the importance of damaged links after each repair process is completed. That is, the critical link ranking will be changed along with the repair process because of the interaction among links. We repair the most critical link for the specific network state based on the greedy algorithm to obtain the optimal schedule. The algorithm can still quickly obtain an optimal schedule even if the scale of the road network is large because the greedy algorithm can reduce computational complexity. We prove that the problem can obtain the optimal solution using the greedy algorithm in theory. The algorithm is also demonstrated in the Sioux Falls network. The problem discussed in this paper is highly significant in dealing with urban road network restoration.

Introduction

Research related to the road network reconstruction plan for earthquakes, floods, and other catastrophic events has noticeably increased over the past decade. Although these events are undeniably important, small daily life events should not be ignored. A wide variety of traffic accidents, car break down, road maintenance, storm-water ponding, road deterioration, and bad weather will cause partial or total reduction in capacity on a given link of urban road network. Traffic congestion, increase in road network travel cost, and even a gridlock can happen if these links are not repaired and their capacity are not restored in time. For example, during a heavy rain in Beijing in July 21, 2012, the capacity of the 95 road sections of the urban network became zero, and
the storm caused a traffic gridlock. This event still has profound effects on the urban road network despite it being relatively “minor” compared with catastrophes. On-time road network repair is urgent. However, the critical link should be identified when repair resources are limited. Accordingly, identifying which link or links will be given priority during repair becomes particularly important. An appropriate schedule should be prepared based on this information.

Critical links have varying definitions for different researchers or objectives. Corley and Sha [1] proposed that the most vital links in a weighted network would be those whose removal from the network would result in the greatest increase in the shortest distance between two specified nodes. Nardelli et al. [2] studied the difference between the length of the detour path after any link interrupted in the shortest path and that of the original shortest path to measure link importance. Scott et al. [3] defined the network robustness index (NRI) to identify critical links. The NRI is substantially equal to the change in the network-wide travel time when a given link is removed from the network. Oliveira et al. [4] pointed out that using congestion and vulnerability to acquire the importance ranking of road network links was appropriate. Rupi et al. [5] ranked network links according to their importance in maintaining proper connectivity among all origin–destination pairs. Hou and Jiang [6] proposed an indirect method to evaluate the relative importance of a link by using link reliability importance. Sohn [7] suggested that the accessibility index could be used to evaluate the significance of highway network links under flood damage. Current studies have identified the critical link mostly by considering the destruction or removal of a link. The link, which is vital for road network robustness, is not necessary for road network restoration. Therefore, we define critical link from the perspective of road network restoration. Our research focuses on how much the restoration of a link can contribute to road network performance in evaluating the critical link. Meanwhile, we also do not ignore the fact that a road network is dynamic. That is, evaluating the critical link is dynamic.

In recent years, complex networks have been studied widely related to the properties and application of complex networks [8–10]. It will work well based on a good robustness for the network [11]. As to the complex road network, the studies on the network robustness mostly focus on dealing with disasters so far. Studies on dealing with disasters can be divided into two categories as follows: 1) enhancement of vital facilities to increase network robustness before a disaster happens and 2) quick response after a disaster. With regard to enhancing network robustness, the main research objective is to allocate limited resources to enhance vital facilities and reduce loss during a disaster. Protection and planning for recovering vital network segments are an efficient proactive approach to reduce the worst-case risk of service disruption because of budgetary limitations [12]. On the basis of such consideration, exploring the vulnerability of network nodes or arcs to disruption [13] and establishing the bi-level program model to protect the critical network segment to respond to attacks are the main research objectives [14, 15]. Most of the research background for network reconstruction and emergency rescue is disaster. The core of these studies is the effectiveness of limited resource allocation. Giving priority to the important edges which connected nodes with the largest populations is an effective repair strategy [16]. In addition, there are various measure indicators to help allocate resources. The effectiveness of limited resource allocation can be measured by minimizing system cost and maximizing system flow [17]; maximizing network accessibility [18]; minimizing user travel costs [19]; minimizing the rescue costs of primary and secondary disasters [20]; maximizing cumulative network accessibility and minimizing make span [21]; optimizing accessibility [22]; minimizing the travel time of travelers, total working time, and idle time between work troops [23]; minimizing combinatorial indicators [24]; maximizing the performance of emergency rehabilitation; minimizing the risk of rescuers and maximizing the saving of lives [25]; and minimizing unsatisfied demands for resources, time to delivery, and transportation costs [26] among others.
Protecting the critical network segment is vital before random or deliberate attacks. However, maintaining normal service is insufficient most of the time, which means that we should also quickly respond after network incidents occur. Moreover, we must recover its service on time. Most research objectives focus on disasters. Accordingly, the vehicle routing model is the core of these studies, and considerable constraints that should be solved optimally are involved in the model. In this study, we focus more attention on repairing damaged road networks resulting from minor events. We aim to minimize the cumulative whole network travel cost when we only have one repair crew (repair crew can be expanded). We propose the road network repair schedule-based greedy algorithm, which significantly improves computational efficiency, based on critical link identification. We can quickly obtain the optimal urban road network schedule even if the road network is extremely large. We prove that the greedy algorithm can obtain an optimal solution for our problem in theory. The test results show that an optimal schedule can be efficiently derived by our greedy algorithm.

The rest of this paper is organized as follows. Section 2 introduces the definition of the critical link and the optimal schedule for urban road network repair based on the greedy algorithm. Proof is also provided in this section. Section 3 tests the developed road network repair crew scheduling in the Sioux Falls network and presents the analysis results. Section 4 concludes the study.

Methodology
This study focuses on an optimal urban road network repair crew scheduling. The repair crew can only repair links when the capacity of some urban road network links is destroyed because of various reasons, and we only have one repair crew. Our research aims to minimize the cumulative whole road network travel cost along with damaged link restoration. A different repair order certainly results in a different effect in urban road network performance. The exhaustive search method requires a large calculation workload. Moreover, link restoration may worsen road network situations because of the Braess’ paradox. A greedy algorithm is an algorithm that applies the problem-solving heuristic of making a locally optimal choice at each stage with the aim of finding a global optimum. This algorithm performs efficiently for certain scheduling problems [27, 28]. We propose the optimal schedule for an urban road network repair based on the greedy algorithm because of its advantages. This algorithm aims to quickly obtain an optimal schedule, thereby ensuring that the effort of the repair crew will result in efficacious network improvement during the repair process. We also prove that the greedy algorithm is applicable to our problem in theory. Although our study is more theoretical rather than practical, it retains the basic characteristics of traffic. The result can still guide the repair of urban road networks in real life.

In early studies, scholars used to represent link damage with a 100% capacity reduction on the link. The most obvious problem resulting from such approach is the creation of isolated sub-networks. Moreover, a complete link from the network is not associated with reality. Several scholars have considered that using a high percentage-based link capacity reduction instead of 100% can be better. Sullivan et al. [29] extensively investigated this problem. The result showed that the most stable capacity disruption range for the ranking of critical link varied with network connectivity level. Consistent with the literature, the damaged links in our research indicate a high percentage-based link capacity reduction. The capacity reduction will be determined using road network connectivity.

Parameters
The critical link will be initially repaired in our greedy algorithm. Therefore, this part will introduce the definition of the critical link. In this study, we focus on the ratio of the travel cost in different network states, rather than on the specific travel cost, to facilitate comparison. The
link whose restoration produces the ratio of the system-wide travel time cost of the current network to the worst network is at minimum. We define such a link as the critical link for the current network. The notations listed in Table 1 have been adopted to facilitate description.

First, we calculate \( c_0 \) and \( c_i \) as the base of all calculations. \( c_0 \) can be calculated as follows:

\[
c_0 = \sum_{j \in (E_{ni}, E_{ri})} t_j x_j,
\]

where \( t_j \) is the travel time across link \( j \), and \( x_j \) is the flow on link \( j \) in the initial network according to the user equilibrium assignment model [30]. User equilibrium assignment can be performed using TransCAD. The system-wide travel time cost \( c_i \) can be calculated as follows if the repair link \( e \) at the current situation after \( (i-1) \) links are repaired:

\[
c_i = \sum_{j \in (E_{ni}, E_{ri}, e)} t_j' x_j',
\]

where \( t_j' \) is the travel time across link \( j \), and \( x_j' \) is the flow on link \( j \) in the current network according to the user equilibrium assignment model [30]. The user equilibrium assignment model enables the travel time and flow in our study to be consistent with the realistic road network. The critical link can be obtained as follows:

\[
I_i^e = \frac{c_i}{c_0}.
\]

The value of \( I_i^e \) for the same road network state, of which the link is the smallest, is the critical link for the current network.

Algorithm

The objective of our research is to minimize the cumulative whole road network travel cost along with the restoration of the damaged link with only one repair crew. The following assumptions are made before constructing the model: (1) the travel time of the repair crew from one link to another is not considered; (2) damaged links only have two statuses: waiting for repair or return to normal after restoration; and (3) specific repair time for one damaged link is not considered. From these assumptions, for each repair step, our objective function and constraint set are formulated as follows:

\[
I_i^e = \frac{c_i}{c_0},
\]

Table 1. Notation description.

| Notation | Definition |
|----------|------------|
| \( E \)  | Set of all links in the road network          |
| \( E_{normal} \) | Set of normal links in the road network, abbrev \( E_n \) |
| \( E_{repair} \) | Set of abnormal links in the road network, abbrev \( E_r \) |
| \( E_{ni} \) | Set of normal links before repair the \( i \)-th link in the road network |
| \( E_{ri} \) | Set of abnormal links before repairing the \( i \)-th link in the road network |
| \( C_0 \)  | System-wide travel cost in the initial state |
| \( c_0 \)  | System-wide travel cost after repairing \( i \) links and link \( e \) is repaired in the last |
| \( I_i^e \) | Ratio of \( c_i \) to \( c_0 \), it represents the importance of a given link \( e \) |
| \( i \)    | \( i = 1, 2, 3, \ldots, m; m \) is equal to the number of links belonging to \( E_r \) |


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where \( I_e^i \) is denoted as follows:

\[
I_e^i = \begin{cases} 
1, & \text{link } e \text{ is repaired completely in the } i\text{th step} \\
0, & \text{otherwise}
\end{cases}
\]  

Eq 5 denotes that the repair crew can only repair one link at one step. Eq 6 denotes that any damaged link is rehabilitated at only one step.

Specifically, we hope each repair step of repair crew can reduce whole road network travel cost to the greatest extent. The final repair schedule derived by each step decision is also optimal. In other words, repairing crew make the best choice according to the current state at each step, and the each step best choice make the final global optimal choice as shown in Eq 8. The left side of Eq 8 which is our objectives indicates global optimal solution, repair order is optimization variables. The right side of Eq 8 shows the sum of each local optimal solution. We can achieve global optimal solution just through local optimal choice since Eq 8 is correct. Relevant proof will be given in the next section.

The exhaustive search algorithm is clearly feasible in resolving the aforementioned problem, but it will require a considerable amount of time. Therefore, we propose the greedy algorithm to solve the problem. We provide the critical link priority according to the greedy principle. The critical link determined by Eq 4. That means Eq 4 is the selection function, which determined which link to repair each step. We repair the critical link from the rest of \( E_r \) until all damage links are restored. However, the ranking of critical links cannot remain unchanged all the time because of the change in road network. Therefore, updating the ranking of critical links after a link restoration is necessary. Fig 1 shows the greedy algorithm. To make it more clear, Fig 2 indicates the greedy algorithm flowchart.

Proof

In the repair process, the repair crew repairs the critical link, whose \( I_e^i \) is the minimum. The result is a local optimal solution. We must prove that the greedy algorithm to our problem can...
obtain the global optimal solution through the local optimal solution, which confirms that the following equation is correct:

$$
\min \sum_{i=1}^{m} I_i = \sum_{i=1}^{m} \min(I_i).
$$

(8)

Fig 2. Greedy algorithm flowchart.

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The proof consists of two parts. First, the algorithm is proven to produce an urban road network repair schedule. Second, the urban road network repair schedule based on the algorithm is proven optimal. Let $T_2$ represent the repair schedule produced by the greedy algorithm. Evidently, the urban road network repair schedule problem must have a feasible solution. Accordingly, $T_2$ is a feasible solution. $T_2$ is clearly optimal if $E_0$ only contains one link.

The solution $T_1$ is produced assuming that an optimal algorithm to the problem of urban road network repair crew scheduling is available. $T_1$ is not equal to $T_2$, which indicates that the repair order of the two links is opposite, at least between $T_1$ and $T_2$. Assuming that $T_1$: $e_1 \rightarrow e_2 \rightarrow e_3 \rightarrow e_5 \rightarrow e_4 \rightarrow T_{11}$, then $T_2$: $e_1 \rightarrow e_2 \rightarrow e_3 \rightarrow e_4 \rightarrow e_5 \rightarrow T_{22}$, where $T_{11}$ and $T_{22}$ represent the repair order of the rest link of $E_0$, except for $e_1$, $e_2$, $e_3$, and $e_5$, which belong to $T_1$ and $T_2$, respectively. A solution $T_3$ for the problem of urban road network repair schedule is thus constructed. $T_3$ is nearly the same as $T_1$. The only difference is the repair order of $e_4$ and $e_5$, i.e., $T_3$: $e_1 \rightarrow e_2 \rightarrow e_3 \rightarrow e_4 \rightarrow e_5 \rightarrow T_{11}$. $T_3$ is partly the same as $T_2$. $T_3$ is clearly a feasible solution.

For $T_1$:

$$\min \sum_{i=1}^{m} I_i = I_{e_1}^1 + I_{e_2}^2 + I_{e_3}^3 + I_{e_4}^4 + I_{e_5}^5 + A_1;$$

(9)

For $T_3$:

$$\sum_{i=1}^{m} I_i = I_{e_1}^1 + I_{e_2}^2 + I_{e_3}^3 + I_{e_4}^4 + I_{e_5}^5 + A_2;$$

(10)

where $A_1 = \sum_{i=6}^{m} (I_i)$ for $T_1$ and $A_2 = \sum_{i=6}^{m} (I_i)$ for $T_3$.

For $T_1$:

$$I_{e_4}^1 = \frac{c_{e_4}}{c_0},$$

(11)

$$c_{e_4} = \sum_{j \in \{e_1 + e_2 + e_3 + e_5 \}} t_{j}^{e_4} a_{j}^{e_4},$$

(12)

$$E_{e_5} = E_{e_4} + e_1 + e_2 + e_3 + e_5.$$  

(13)

For $T_3$:

$$I_{e_5}^5 = \frac{c_{e_5}}{c_0},$$

(14)

$$c_{e_5} = \sum_{j \in \{e_1 + e_2 + e_3 + e_4 \}} t_{j}^{e_5} a_{j}^{e_5},$$

(15)

$$E_{e_5} = E_{e_4} + e_1 + e_2 + e_3 + e_4.$$  

(16)

Therefore, $I_{e_4}^1 = I_{e_5}^5$. Similarly, $A_1 = A_2$, $I_{e_4}^1$ and $I_{e_5}^5$ are the unique difference between $T_1$ and $T_3$ according to the user equilibrium assignment model. The greedy principle determines that
the repair order of e4 belongs to T3. Hence, $I_{e_4}^3 \geq I_{e_4}^1$, then
\[
\left( \min \sum_{i=1}^{m} I_i = I_1^1 + I_2^2 + I_3^3 + I_4^4 + I_5^5 + I_6^6 + I_7^7 + A_i \right) \geq \left( \sum_{i=1}^{m} I_i = I_1^1 + I_2^2 + I_3^3 + I_4^4 + I_5^5 + I_6^6 + I_7^7 + A_i \right)
\] (17)

T3 is actually closer to T2 than T1. T1 and T2 have made n different decisions. Similar to constructing T3, we can obtain T2 via finite transformation. The value of $\sum_{i=1}^{m} I_i$ is guaranteed to be no more than the value of $\min \sum_{i=1}^{m} I_i$ in translation. The solution of T2 is essentially $\sum_{i=1}^{m} \min I_i$.

Therefore, Eq 8 is correct, and T2 is optimal. This result implies that T2 is the optimal urban road network repair schedule.

**Numerical Results**

We propose the optimal schedule for urban road network repair based on the greedy algorithm on the well-known Sioux Falls network (Fig 3), which contains 24 nodes, 76 links, and 576 origin–destination (OD) movements. The Sioux Falls network is abstracted by Chen and Tzeng according to the Northridge earthquake in America [23]. It is a classic experimental network in transport research. The mean OD demand (Table 2), free-flow travel time (Table 3), and network capacity (Table 3) are the same as those used in the research of Li and Ma [31].

The link capacity reduction range between 80% and 75% is the most appropriate for the test network according to the research of Sullivan et al. [29] and the connectivity of the Sioux Falls network. The two experiments in the test are as follows. The first experiment supposes that eight links are damaged in the Sioux Falls network. We pay attention to the variety of ranking of the critical link. We illustrate our greedy algorithm clearly through the first experiment. The second experiment supposes that four links are damaged in the Sioux Falls network. We provide all 24 repair schedules for comparison. The second experiment proves the correctness of the greedy algorithm with respect to our research objective. The damaged links are random without losing generality.

**The First Experiment**

Suppose that links e9, e19, e29, e40, e46, e53, e60, and e74 of the Sioux Falls network (Fig 3) are damaged. The capacity reduction is 80%. We obtain $E_s = \{e9, e19, e29, e40, e46, e53, e60, e74\}$, and then calculate the value of $I_i$ for every damaged link that belongs to $E_s$. Table 4 shows that under the circumstances, repair link e40 will enable the repair work gain maximum benefit. After repair link e40, the network state also changes because of the interaction among links. Therefore, we cannot repair link e74 after repairing link e40. We must re-evaluate the relative importance of the damaged links after link e40 restoration. That is, we should calculate the value of $I_i$ for every damaged link that belongs to $E_{s2}$, and then decide which link to repair. In this case, the link for repair happens to be e74, which is the optimal choice. From this analogy, we can finally obtain the optimal schedule as e40 → e74 → e53 → e46 → e29 → e19 → e9 → e60.

The rank of the critical link changes with the road network change are shown in Table 4. Table 4 shows that the rank of the critical link has almost nearly changed after link restoration. The links in the road network are affected by each other one another. We pay attention to focus on the situation after link e40 restoration. The value of $I_i$ is 0.8731. However, the values of $I_{e_9}^1$, $I_{e_9}^2$, and $I_{e_9}^3$ will be 0.8838, 0.8818, and 0.9240, respectively, if we repair links e29, e53, or e60 subsequently. These values are all greater than 0.8731, indicating that the effect of repairing two links is less than that of one key link. The occurrence of this situation is attributed to the...
Braess’ paradox. The situation considerably wastes limited repair resources, which should be strongly avoided. In our research, we can predict which link will cause a significantly higher whole network travel cost. With regard to the urban road network repair schedule based on the greedy algorithm, we guarantee that limited repair resources will play the biggest role in each repair stage. The repair schedule is optimal for the current situation, but also the best for the global situation. Our schedule considers link interaction. Therefore, the optimal schedule is e40→e74→e53→e46→e29→e19→e9→e60 if we have only one crew. We can obtain the optimal schedule of e40, e74→e53, e46→e29, e19→e9, e60, rather than recalculate, if we have two crews. In the same manner, we can also directly obtain the optimal schedule if we have three or more crews. That is, our optimal schedule based on only one crew can be expanded.

The Second Experiment

Suppose that links e29, e40, e53, and e60 in the Sioux Falls network are damaged (Fig 3), the capacity reduction is 80%. According to the greedy algorithm, our optimal schedule is e53→e40→e29→e60 (Table 5). We also obtain all 24 repair crew schedules using the exhaustion method for comparison.

Fig 3. Sioux Falls network.

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Table 2. Traffic demand of Sioux Falls network (vehicle/h).

| From/To | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1       | 0  | 100| 100| 500| 200| 300| 500| 800| 500| 1300|500  |200 | 300| 500| 500| 500 |300 | 100| 300 |100 | 400| 300 |100 |
| 2       | 100| 0  | 100| 200| 100| 300| 200| 400| 200| 600 |200  |100 | 300| 100| 100| 400 |200 | 0  | 100 |100  |0   | 100 |0   |
| 3       | 100| 100| 0  |200 | 100| 300| 200| 100| 300| 300 |300  |200 | 100| 100| 100| 200 |100 | 0  | 0   |0   | 0   | 100 |100 |
| 4       | 500| 200| 200| 0  | 500 |400 |400 |700 |700 |1200 |1400 |600 | 600| 500 |500 | 800 |500 | 100| 200 |300 | 200 |400 |500 |
| 5       | 200| 100| 100| 500| 0  | 200 |200 |500 |800 |1000 |500 |200 | 200| 100| 200 |500 |200 | 0  |100 | 100| 100| 200 |0  |
| 6       | 300| 400| 300| 400 |200 | 0  |400 |800 |400 |800  |400 |200 | 200| 100| 200 |900 |200 | 0  |100 | 100| 100| 200 |0  |
| 7       | 500| 200| 100| 400| 200| 400 |0  |1000|600 |1900 |500 |700 | 400| 200| 500 |1400|1000| 200| 400| 500 |500 | 200| 100|
| 8       | 800| 400| 200| 700 |500 | 800| 1000|0  |800 |1600 |800 |600 | 600| 400| 600 |2200|1400| 300| 700 |900 | 400| 500 |300|
| 9       | 500| 200| 100| 700 |800 | 400| 600 |800 |0  |2800 |1400|600 | 600| 600 |900 |1400| 900| 200| 400| 600 |300| 700 |500|
| 10      | 1300|600 |300 |1200|1000|800 |1900|1600|2800|0  |4000 |2000|1900|2100|4000|3900|700 |1800|2500|1200|2600|1800|800|
| 11      | 500| 200| 300|1500|500 |400 |500 |800 |1400|3900|0  |1400|1000|1600|1400|1400|1000|400 |600 |400 |1100|1300|600|
| 12      | 200| 100| 200| 600 |200 | 200| 700 |600 |600 |2000|1400|0  |1300| 700| 700 |700 |700 |600 |200 |300 |400 |300 |700 |500|
| 13      | 500| 300| 100| 600 |200 | 200| 400 |600 |600 |1900|1000|1300|0  |600 |700 |600 |500 |100 |300 |600 |600 |1300|800 |800|
| 14      | 300| 100| 100| 500 |100 | 100| 200 |400 |600 |2100|1600|700 |600 |0  |1300| 700| 700 |100 |300 |500 |400 |1200|1100|400|
| 15      | 500| 100| 100| 500 |200 | 200| 500 |600 |1000|4000|1400|700 |700 |1300|0  |1200|1500|200 |800 |1100|800 |2600|1000|400|
| 16      | 500| 400| 200| 800 |500 | 900|1400|2200|1400|4400|1400|700 |600 |700 |1200|0  |2800|500 |1300|1600|600 |1200|500 |300|
| 17      | 400| 200| 100| 500 |200 | 500|1000|1400|900 |3900|1000|600 |500 |700 |1500|2800|0  |600 |1700|1700|600 |1700|600 |300|
| 18      | 100| 0  | 0   |100 | 0  | 100| 200|300 |200 |700 |200 |200 |100 |100 |200 |500 |600 |0  |300 |400 |100 |300 |0  |
| 19      | 300| 100| 0  | 200 |100 | 200| 400 |700 |400 |1800|400 |300 |300 |300 |800 |1300|1700|300 |0   |1200|400 |1200|300 |100|
| 20      | 300| 100| 0  | 300 |100 | 300| 500 |900 |600 |2500|600 |500 |600 |500 |1100|1600|1700|400 |1200|0   |1200|2400|700 |400|
| 21      | 100| 0  | 0   |200 | 100| 100| 200|400 |300 |1200|400 |300 |600 |400 |800 |600 |600 |100 |400 |1200|0   |1800|700 |500|
| 22      | 400| 100| 100| 400 |200 | 200| 500 |500 |700 |2600|1100|700 |1300|1200|2600|1200|1700|300 |1200|2400|1800|0   |2100|1100|
| 23      | 300| 0  | 100| 500 |100 | 100| 200|300 |500 |1800|1300|700 |800 |1100|1000|500 |600 |100 |300 |700 |700 |2100|0  |700 |
| 24      | 100| 0  | 0   |200 | 0  | 100| 100|200 |200 |800 |600 |500 |700 |400 |400 |300 |300 |0  |100 |400 |500 |1100|700 |0  |

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Fig 4 provides the value $\sum_{i=1}^{4} l_i$ of 24 repair schedules. The column indicates the $\sum_{i=1}^{4} l_i$ value of every repair schedule, the row indicates schedule number. The column clearly shows that the value of $\sum_{i=1}^{4} l_i$ in the 9th repair schedule is the minimum. The 9th repair schedule is the same as the repair schedule based on the greedy algorithm. The result indicates the correctness of Eq 8. The value of $\sum_{i=1}^{4} l_i$ in the 24 repair schedules appears to be less different. However, this

| Link | Link capacity(vehicle/h) | Free-flow travel time (h) |
|------|--------------------------|--------------------------|
| 1 and 3 | 15000 | 6 |
| 2 and 5 | 10000 | 2 |
| 4 and 14 | 10000 | 1.5 |
| 6 and 8 | 10000 | 2 |
| 9 and 11 | 12500 | 3.5 |
| 12 and 15 | 15000 | 3 |
| 7 and 35 | 10000 | 4 |
| 10 and 31 | 12500 | 3.5 |
| 13 and 23 | 10000 | 1.5 |
| 25 and 26 | 10000 | 1.5 |
| 21 and 24 | 15000 | 2.5 |
| 16 and 19 | 10000 | 1 |
| 22 and 47 | 15000 | 1.5 |
| 17 and 20 | 15000 | 2.5 |
| 18 and 54 | 15000 | 1.5 |
| 33 and 36 | 10000 | 2 |
| 27 and 32 | 15000 | 3 |
| 29 and 48 | 15000 | 2.5 |
| 50 and 55 | 15000 | 2.5 |
| 37 and 38 | 10000 | 10 |
| 34 and 40 | 10000 | 4.5 |
| 42 and 71 | 10000 | 2.5 |
| 73 and 76 | 10000 | 3.5 |
| 41 and 44 | 15000 | 3 |
| 70 and 72 | 15000 | 3 |
| 28 and 43 | 15000 | 4 |
| 46 and 67 | 15000 | 2 |
| 65 and 69 | 15000 | 3 |
| 30 and 51 | 15000 | 3.5 |
| 45 and 57 | 15000 | 2.5 |
| 63 and 68 | 15000 | 4.5 |
| 49 and 52 | 15000 | 2 |
| 53 and 58 | 15000 | 2 |
| 59 and 61 | 15000 | 5.5 |
| 56 and 60 | 15000 | 10 |
| 39 and 74 | 10000 | 2 |
| 66 and 75 | 10000 | 3.5 |
| 62 and 64 | 15000 | 3 |

Table 3. Link Parameters.

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value is only the ratio. The difference of the restoration effect will be large if multiplied by the whole road network travel cost for different repair schedules. As shown in Fig 4, the difference between the best and worst schedules remains significant. Therefore, quickly and efficiently obtaining the optimal repair schedule is significant in road network restoration.

### Conclusion

Certain incidents in urban road networks can cause the decline of the capacity of some links, which will lead to traffic congestion or even gridlock, and increase the travel cost of the whole network. Although such events are not as serious as disasters, they happen more frequently, and thus, are more relevant to our life. Only a few studies are related to this topic. We intend to conduct basic research regarding this problem. The core of the problem is how to allocate limited resources to achieve similar goals to those of disaster research. How limited resources can be allocated to minimize the cumulative whole road network travel cost along with the restoration of damaged link is the objective of our research. We define the critical link for our objective, which considers link interaction. Moreover, the link is dynamic. We repair the critical link to quickly achieve our objective based on the greedy algorithm, which aims to obtain the global optimal solution using the local optimal solution. The repair order of the damaged links is the optimal schedule. We prove that the greedy algorithm is applicable to our objective in theory and through a case study.

Our concern is road network restoration. Therefore, the critical links we define are highly suitable for road network repair instead of road network robustness. The link, whose restoration is best for the current road network, will be the critical link. The ranking of the critical link obviously changes because of the interaction among links after a link is repaired. The case study clearly demonstrates this situation. That is, the evaluation of the critical link must be dynamic. The case study also shows that the effect of repairing two links is not always better than the effect of repairing one link because of the Braess’ paradox. If the wrong link is selected for repair, the road network condition will worsen rather than improve. Our research can completely avoid the aforementioned poor decision. The evaluation of the critical link before

### Table 4. Rank of critical link under different road network states.

| link | e9   | e19  | e29  | e53  | e40  | e46  | e60  | e74  | Ranking of critical link |
|------|------|------|------|------|------|------|------|------|--------------------------|
| I<sub>1</sub> | 0.9663 | 0.9510 | 0.9594 | 0.9216 | 0.8731 | 0.9626 | 0.9540 | 0.9096 | e40, e74, e53, e19, e60, e29, e46, e9 |
| I<sub>2</sub> | 0.8538 | 0.8601 | 0.8838 | 0.8818 | ✓ | 0.8552 | 0.9240 | 0.8375 | e74, e9, e46, e19, e53, e29, e60 |
| I<sub>3</sub> | 0.8286 | 0.8262 | 0.8044 | 0.7760 | ✓ | 0.8163 | 0.8172 | ✓ | e53, e29, e46, e60, e19, e9 |
| I<sub>4</sub> | 0.7590 | 0.7665 | 0.7690 | ✓ | ✓ | 0.7481 | 0.7618 | ✓ | e46, e9, e60, e19, e29 |
| I<sub>5</sub> | 0.7298 | 0.7372 | 0.7251 | ✓ | ✓ | ✓ | 0.7466 | ✓ | e29, e9, e19, e60 |
| I<sub>6</sub> | 0.7161 | 0.7092 | ✓ | ✓ | ✓ | ✓ | ✓ | 0.7124 | e19, e60, e9 |
| I<sub>7</sub> | 0.6981 | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 0.7054 | e9, e60 |
| I<sub>8</sub> | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 0.6880 | e60 |

Note: ✓ represents the link has been repaired.

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### Table 5. Greedy algorithm results.

| min(I<sub>1</sub>) | min(I<sub>2</sub>) | min(I<sub>3</sub>) | min(I<sub>4</sub>) | min(I<sub>5</sub>) | \( \sum_{i=1}^{n} \min(I_{i}) \) |
|-------------------|-------------------|-------------------|-------------------|-------------------|-----------------|
| value             | 0.917             | 0.8619            | 0.8242            | 0.8123            | 3.4154          |
| link              | e53               | e40               | e29               | e60               | —               |

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each repair step fully utilizes the limited resources. Although our optimal schedule assumes that we can only repair one link for every step, the operation can be expanded to repair two or more links for every step rather than recalculate. For example, the optimal schedule is e40 → e74 → e53 → e46 → e29 → e19 → e9 → e60 because the case shows that we have only one crew. The optimal schedule is e40, e74 → e53, e46 → e29, e19 → e9, e60 if we have two crews, and so on. Varying solutions are available for the road network repair schedule. The greedy algorithm we apply can obtain the global optimal schedule through the local optimal schedule, which considerably reduces computational complexity and improves computational efficiency. The algorithm is highly efficient even if the road network is extremely large. In addition, it is significant and can be used as a guide in real-life applications.

Actually, greedy algorithm can obtain the global optimal solution through the local optimal solution thus reduces computational complexity and improves computational efficiency. However, not all problems can obtain global optimal solution through greedy algorithm. Therefore, we have proved that theoretically in section 2. The second experiment also proved it. Our optimal schedule has some limitations. The specific repair time of different damaged links and the time the crew travels from one damaged link to another are not considered. However, these issues are essential in real life. Consequently, the optimal schedule obtained using our proposed technique cannot be directly applied to real-life situations. These issues require further investigation. Combining the current research results with practical issues can be a worthwhile direction for future research.

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