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Self-similar Shape Mode of Optical Pulse Propagation in de-focusing Medium with Two-Photon Absorption

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Abstract. We investigate the self-similar shape mode of laser pulse propagation in medium with two-photon and multi-photon absorption under action of either cubic nonlinearity or resonant nonlinearity. The annalitical solution for laser pulse propagation under the detuning between carrier frequency of a wave packet and energy transition frequency for atom or molecule is developed. An existance of the self-similar shape mode for laser beam propagation in de-focusing medium is predicted if the nonlinear absorption takes place. This mode takes place along some distance of laser pulse propagation. We used as analytical approach for considered problem as well as computer simulation.

1. Introduction
Soliton or soliton-like profile of laser beam or shape of laser pulse plays a great role in modern nonlinear optics and attaches great attention of various scientist (see, for example [1-17]). In our opinion, an investigation of self-similar profile of laser beam, propagating in a medium with nonlinear absorption is actual problem because powerful laser system produces high intensive laser pulse which induces a formation of plasma under its propagation in a gas.

Below we consider the possibility of laser pulse propagation with the self-similar profile in the medium with TPA or multi-photon absorption including a presence of either cubic nonlinear response of a medium (self-focusing or de-focusing) or resonant nonlinearity. To achieve this aim we find the corresponding beam profile from the eigenvalue problem. Early we have applied this method for various problems of nonlinear optics [18-27].

It should be stressed that each problem requires unique iterative method for finding the soliton from eigenvalue problem because the corresponding equation is nonlinear. Therefore, a particular method should be developed for each type of nonlinear response of the medium. This is also true for the medium with nonlinear absorption but in this case, the finding of a soliton (self-similar profile of intensity) is a more difficult task.

It should be stress, that in [28] in contract to current paper we discussed considering problem for both homogeneous and layered medium with TPA only. We does not consider an influence of a cubic nonlinear response on an existence of soliton-like mode at the laser pulse propagation in a medium with the nonlinear response.
2. Problem statement

Below we consider three cases of laser pulse interaction with nonlinear medium. First of them corresponds to laser pulse propagation in a medium with both two-photon absorption (TPA) and cubic nonlinear response. The aim of this consideration is an illustration of possibility to existence a mode of laser pulse propagation with self-similar shape even if laser beam de-focusing takes place.

In the second case we analyze a laser pulse interaction with medium, possessing a cubic nonlinear response and three-photon absorption (ThPA). The aim of this consideration is an investigation of absorption type influencing on laser pulse shaping. The last case of consideration deals with resonant nonlinear response with taking into account wave packet carrier frequency detuning from the energy transition frequency for medium molecules or atoms. We write an analytical solution for simplified problem for the case under consideration.

All laser pulse interactions with medium are considered in the framework of 1D dimensionless Schrödinger equation, written for layered medium, because our previous results showed that they are valid also for layered medium. Consequently, we will analyze laser pulse propagation in three types of medium:

\[ \varepsilon(z) \frac{\partial A}{\partial t} + iD \frac{\partial^2 A}{\partial z^2} + \beta(i\varepsilon(z) + (\delta(z) + i\alpha(z)) |A|^2)A = 0, \quad 0 < z < L_z, \quad 0 < t \leq L_t \]  

- cubic nonlinear response with TPA,

\[ \varepsilon(z) \frac{\partial A}{\partial t} + iD \frac{\partial^2 A}{\partial z^2} + \beta(i\varepsilon(z) + \delta(z)) |A|^4 + i\alpha(z) |A|^2)A = 0, \quad 0 < z < L_z, \quad 0 < t \leq L_t \]  

- cubic nonlinear response with ThPA,

\[ \varepsilon(z) \frac{\partial A}{\partial t} + iD \frac{\partial^2 A}{\partial z^2} + \beta(i\varepsilon(z) + \delta(z)(1 + i\Delta) |A|^k)A = 0, \quad 0 < z < L_z, \quad 0 < t \leq L_t \]  

- resonant nonlinear response of medium.

These equations are solved with zero-value boundary conditions

\[ A(t, 0) = A(t, L_z) = 0, \quad 0 \leq t \leq L_t \]  

and initial distribution of complex amplitude

\[ A(0, z) = A_o(z), \quad 0 \leq z \leq L_z. \]  

Above \( A(t, z) \) is dimensionless complex amplitude that is slowly varying in time; \( z \) is coordinate, along which the laser pulse propagates, \( L_z \) is dimensionless length of the nonlinear medium; \( t \) is a time, \( L_t \) is dimensionless time interval during which the propagation of the laser pulse is investigated. Function \( \varepsilon(z) \) describes a dielectric permittivity of medium. Parameter \( D \) characterizes a diffraction of laser beam and \( \beta \) relates with parameter \( D \). Signs of these two parameters must be the same and they depend on choosing of wave propagation direction for a derivation of the Schrödinger equation from Maxwell's equations. Equations (1)-(3) take into account a wave reflected from any boundaries of medium layers.
Function $\delta(z)$ characterizes a nonlinear absorption (or amplification) of a medium. Sign of this function depends on medium type: a medium with absorption $\delta(z)>0$ or amplification $\delta(z)<0$ of optical radiation. Parameter $k$ is equal to 2 for medium with TPA or 4 for medium with ThPA. Function $\alpha(z)$ describes the cubic nonlinear response of medium. In our notation its positive sign corresponds to compression of laser pulse at positive value of the diffraction coefficient. Opposite case corresponds to laser pulse decompression. It should be emphasized that the functions $\varepsilon(z)$, $\alpha(z)$ and $\delta(z)$ are equal to corresponding constants for the homogeneous medium ($\varepsilon(z)=\varepsilon=\text{const}$, $\alpha(z)=\alpha=\text{const}$, $\delta(z)=\delta=\text{const}$). Below we will consider a homogeneous medium.

It should be emphasized that the algorithm of finding the beam profile with the self-similar feature is valid for both homogeneous medium and layered one. However, for the layered medium the diffraction coefficient $D$ and parameter $\beta$ in our notation relate each with other in following way

$$D = -\frac{1}{4\pi\Omega}, \quad \beta = -\pi\Omega, \quad \Omega = \frac{\omega}{\omega_{\text{mr}}}.$$  \hspace{1cm} (6)

Here $\Omega$ is ratio of laser light frequency to frequency of periodic structure. The sign “minus” can be changed to sign “plus” if we follow the wave propagating in the direction which is opposite to the direction chosen by us above for the layered medium. Below if we consider a homogeneous medium, we choose a value of the diffraction coefficient $D$ and parameter $\beta$ as

$$D = 1, \beta = 0, 25.$$  \hspace{1cm} (7)

If we will consider other values of these parameters then we will specify their new values.

3. Homogeneous medium with resonant nonlinear response

To find the requiring profile of the beam we consider the nonlinear eigenfunction problem (see equation (3)). With this aim let us represent the complex amplitude in a way

$$A(t, z) = B(z)e^{-i\lambda t}.$$  \hspace{1cm} (8)

Substituting the function (6) into the Schrödinger equation (1), one can write the following eigenvalue and eigenfunction problem

$$\frac{D}{\varepsilon} \frac{d^2B}{dz^2} + \beta \left( 1 + \frac{\delta(\Delta - i\gamma)}{\varepsilon} \right) B = \lambda B, \quad B(0) = B(L_c) = 0.$$  \hspace{1cm} (9)

We can see that the equation (9) contains time-dependent factor $e^{2\text{Im}\lambda t}$. Generally speaking, in this case we need to solve the problem for various moments of time because the factor $\text{Im}\lambda \cdot t$ varies. It is obviously that we can consider the dependence $e^{2\text{Im}\lambda t}$ from time as the variation of the absorption coefficients because at the propagation of laser beam its intensity varies with accordance of this factor. Thus, different eigenfunctions corresponding to various values of the absorption coefficients will correspond to eigenfunction at various moments of time. Taking into account these arguments we solve the following eigenfunction problem.
\[ \frac{D}{\varepsilon} \frac{d^2 B}{dz^2} + \beta \left( 1 + \frac{\Delta (\Delta - i)}{\varepsilon} \right) B \left( \Delta B \right) \] \[ = \lambda B, \quad B(0) = B(L_\varepsilon) = 0 \] (10)

with changing the intensity maximum.

For the homogeneous medium one can develop the analytical solution of this equation. With this aim let us consider the solution in the form

\[ B(z) = a \ ch^{-m}(cz) e^{if(z)} \] (11)
in which \( a \) and \( f(z) \) is a function, which describes the phase distribution along \( z \) coordinate, \( m \) is a power of hyperbolic cosine. Substituting the expression (11) in the equation (10) we get two equations:

\[ \frac{D}{\varepsilon} \left[ (m^2 + m) c^2 th^2 (cz) - m c^2 - (f'(z))^2 \right] + \beta \left[ 1 + \frac{\Delta \delta}{\varepsilon} a^k \right] = \text{Re} \ \lambda, \]

\[ \frac{D}{\varepsilon} \left[ f''(z) - 2 m c \cdot th (cz) f'(z) \right] - \frac{\beta \delta}{\varepsilon} a^k \] (12)

Two important conclusions follow from set of the equations (12). First, we see that in both equations a factor \( a \). Therefore, we can choose the parameter \( a \) be equal to 1. and below (as well as above in (11)) we omit this parameter. Changing in amplitude \( a \) corresponds to changing in the absorption coefficient \( \delta \) only. Second, for solution developing it is necessary to choose the parameter \( m \) depending from \( k \) in following way: \( km = 2 \). In this case, first equation of the set of equations (12) can be rewritten as

\[ th^2 (cz) \left[ \frac{D}{\varepsilon} (m^2 + m) c^2 - \frac{\Delta \beta}{\varepsilon} \right] - \frac{D}{\varepsilon} \left( f'(z) \right)^2 - m c^2 \frac{D}{\varepsilon} + \beta \frac{\Delta}{\varepsilon} = \text{Re} \ \lambda. \] (13)

Therefore, for the solution existence of this equation, the validity of the following set of equations is sufficient:

\[ f'(z) = \sqrt{\frac{2 c^2}{k} \left( \frac{1}{2} + 1 \right) - \frac{\Delta \beta}{D} \cdot th(c z)}, \quad \text{Re} \ \lambda = \beta \left[ 1 + \frac{\Delta}{\varepsilon} \right] - \frac{2 c^2}{\varepsilon k} D. \] (14)

Hence, the phase shift along the \( z \) - coordinate is described by

\[ f(z) = \frac{1}{c} \sqrt{\frac{2 c^2}{k} \left( \frac{1}{2} + 1 \right) - \frac{\Delta \beta}{D} \cdot \ln \left( c h(c z) \right)} + c_1, \] (15)

where \( c_1 \) is a constant, which defines the homogeneous phase shift and, obviously, does not influence on the intensity distribution of the laser beam.

Substitution of this relation in the second equation of (12) results in the following relation between coefficients of the problem
\[
\theta^2(\varepsilon) = D \left( \frac{2\beta \delta}{\varepsilon} \right) - \frac{\varepsilon}{D} \left[ \frac{2c^2}{k} \left( \frac{2}{k} + 1 \right) - \frac{\Delta \beta}{D} (1 + \frac{4}{k}) \right] + \frac{\varepsilon}{D} \left[ \frac{2c^2}{k} \left( \frac{2}{k} + 1 \right) - \frac{\Delta \beta}{D} \right] - \frac{\beta \delta}{\varepsilon} = \text{Im} \lambda. \quad (16)
\]

For validity of this equation for any value of \( z \) coordinate it is sufficient that the following equalities take place

\[
c \sqrt{\frac{2c^2}{k} \left( \frac{2}{k} + 1 \right) - \frac{\Delta \beta}{D} (1 + \frac{4}{k})} = \frac{\beta \delta}{\varepsilon} \text{Im} \lambda = -\frac{\beta \delta}{\varepsilon} \frac{4}{k + 4}. \quad (17)
\]

From first relation of equations (17) one follows that a sign of parameter \( c \) depends from a sign of a right part of this equation. It takes place if a sign of \( \delta \) changes to opposite sign, which corresponds to amplifying medium. The solution of this equation takes place for both signs of detuning \( \Delta \phi \). As a result, we can write the following relations:

\[
c = \pm k \sqrt{\frac{\Delta + \sqrt{\Delta^2 + \frac{8(k + 2)}{(k + 4)^2}}}{4(k + 2)}}. \quad (18)
\]

Choosing of a sign before square root is specified by a sign of the parameter \( \delta \).

Finally, a solution of the problem (12) is

\[
m = \frac{2}{k}, \quad a = 1, \quad \lambda = \left\{ \begin{array}{ll}
\beta [1 + \frac{\Delta \beta}{\varepsilon}] - k \beta [\frac{\delta}{\varepsilon}] & 
\frac{\Delta + \sqrt{\Delta^2 + \frac{8(k + 2)}{(k + 4)^2}}}{2(k + 2)} \quad ;
\-
\frac{\beta \delta}{\varepsilon} \frac{4}{k + 4} \quad ,
\end{array} \right.
\]

\[
f(z) = \frac{1}{c} \sqrt{\frac{2c^2}{k} \left( \frac{2}{k} + 1 \right) - \frac{\Delta \beta}{D}} \cdot \ln \left( c h(\varepsilon z) \right) + c, \quad f'(z) = \frac{2c^2}{k} \left( \frac{2}{k} + 1 \right) - \frac{\Delta \beta}{D} \theta \left( \varepsilon z \right),
\]

\[
c = \pm k \sqrt{\frac{\Delta + \sqrt{\Delta^2 + \frac{8(k + 2)}{(k + 4)^2}}}{4(k + 2)}} \quad , \quad B(z) = \left( c h(\varepsilon z) \right)^{-\frac{3}{2}} e^{if(z)}.
\]

From the expression of the intensity distribution (19) one follows a value of \( z \) coordinate, at which the intensity of laser beam decreases to two times (\( |B(z)|^2 = 0.5 \)):

\[
|c| \cdot z_h = \text{arccosh}(2^{k/4}) = \ln(2^{k/4} + 1). \quad (20)
\]

Consequently, for the half-width of the intensity distribution we get the following ratio

\[
a_z = 2z_h = 2 \cdot \ln(2^{k/4} + 1)/|c|.
\]

From (19) and (21) a dependence of the half-width of the intensity distribution follows. One can see
that a positive value of detuning $\Delta$ leads to the half-width increasing. But its negative value results in the half-width decreasing.

4. Cubic nonlinearity and TPA
The influence of cubic nonlinear response on existence of the mode with self-similar beam profile propagation (see equation (1)) was investigated in [28] in detail. The main conclusion of this paper is an existence of such mode even for de-focusing medium. As example, a comparison of results of computer simulation with theoretical (analytical) consideration is shown in Tables 1, 2. In these Tables an influence of cubic nonlinear response on characteristics of laser beam propagation is depicted also.

Table 1. Dependence of the first eigenvalue $\lambda$ and of the half-width $a_z$ from the cubic nonlinearity $\alpha$ for the medium with TPA and $\epsilon(z) = 2.5, \delta = 10$.

| $\alpha$ | 2     | 5     | 10    | -2    | -2.5   |
|----------|-------|-------|-------|-------|--------|
| Re $\lambda$ | 0.16  | 0.36  | 0.66  | -0.14 | -0.18  |
| Theoretical | 0.16  | 0.36  | 0.66  | -0.14 | -0.19  |
| Im $\lambda$ | -0.67 | -0.67 | -0.67 | -0.67 | -0.67  |
| Theoretical | -0.67 | -0.67 | -0.67 | -0.67 | -0.67  |
| $a_z$    | 2.075 | 1.775 | 1.425 | 2.525 | 2.625  |
| Numerical | 2.07  | 1.78  | 1.44  | 2.55  | 2.57   |
| Theoretical | 2.07  | 1.78  | 1.44  | 2.55  | 2.57   |

Table 2. Dependence of the first eigenvalue $\lambda$ and of the half-width $a_z$ from the cubic nonlinearity $\alpha$ for the medium with TPA and $\epsilon(z) = 2.5, \delta = 5$.

| $\alpha$ | 2     | 5     | 10    | -2    | -3     |
|----------|-------|-------|-------|-------|--------|
| Re $\lambda$ | 0.27  | 0.45  | 0.72  | -0.028| -0.11  |
| Im $\lambda$ | -0.33 | -0.33 | -0.33 | -0.33 | -0.33  |
| $a_z$    | 2.625 | 2.025 | 1.525 | 3.975 | 4.375  |
| Numerical | 2.625 | 2.025 | 1.525 | 3.975 | 4.375  |

We see good coincides between analytical solution and computer simulation results. It should be noticed only that a convergence of our computer method disappears at $\alpha < -2.5$ if $\delta = 10$ and at $\alpha < -3$ if $\delta = 5$. Nevertheless, we see clear dependence of a beam width from a parameter $\alpha$ and the existence of a solution of the problem under consideration for de-focusing medium if the nonlinear absorption takes place.

5. Cubic nonlinear response and ThPA
The influence of cubic nonlinear response on existence of the mode with self-similar beam profile propagation in a medium with three-photon absorption (see equation (2) with $k=4$) is investigated numerically because an analytical solution is absent nowadays. Computer simulation results are shown in Tables 3-5. We see an existence of beam profile corresponding to the self-similar profile mode for de-focusing medium if a nonlinear absorption takes place (Tables 3, 4). As well as for the case of a resonant nonlinearity an imaginary part of the eigenvalue depends only from the absorption coefficient and does depend from a value of the parameter $\alpha$. Fig. 1 illustrates beam profile computed for for various relations between parameters characterizing a nonlinear response of medium. We see very good agreements between computed curve and approximating function.
Fig. 1. Intensity profile computed for $\varepsilon = 2.5; \delta = 10$ and various values of $\alpha$. Intensity profile computed for $\alpha = 4$ is approximated perfectly by the function $\text{ch}^{-1}(z / 1,1)$ (dashed line).

Table 3. Dependence of the first eigenvalue $\lambda$ and of the half-width $a_z$ from the cubic nonlinear response $\alpha$ for the medium with ThPA $\varepsilon(z) = 2.5, \delta = 10$.

| $\alpha$ | 0 | 0.1 | 0.5 | 1 | 2 | 4 | 6 | -0.1 | -0.5 | -1 |
|----------|---|-----|-----|---|---|---|---|------|------|----|
| Re $\lambda$ | -0.039 | -0.032 | -0.006 | 0.026 | 0.087 | 0.2 | 0.3 | -0.045 | -0.073 | -0.11 |
| Im $\lambda$ | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 |
| $a_z$ | 2.175 | 2.155 | 2.125 | 2.075 | 1.975 | 1.775 | 1.575 | 2.225 | 2.275 | 2.325 |

Table 4. Dependence of the first eigenvalue $\lambda$ and of the half-width $a_z$ from the cubic nonlinear response $\alpha$ for the medium with ThPA $\varepsilon(z) = 2.5, \delta = 5$.

| $\alpha$ | 0; 0.1 | 1 | 2 | 4 | 6 | -0.1 | -0.5 | -1 | -2 | -3 |
|----------|-------|---|---|---|---|------|------|----|----|----|
| Re $\lambda$ | 0.11 | 0.17 | 0.22 | 0.32 | 0.4 | 0.1 | 0.07 | 0.03 | -0.04 | -0.12 |
| Im $\lambda$ | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 |
| $a_z$ | 3.075 | 2.775 | 2.475 | 2.075 | 1.775 | 3.125 | 3.275 | 3.475 | 3.825 | 4.325 |

Table 5. Value of the first eigenvalue $\lambda$ and of the half-width $a_z$ for kerr solition propagating in transparent medium ($\delta = 0$) with $\varepsilon(z) = 2.5$. Im $\lambda$ is equal to 0 in this case.

| $\alpha$ | 0.1 | 1 | 2 | 4 | 6 |
|----------|----|---|---|---|---|
| Re $\lambda$ | 0.25 | 0.28 | 0.38 | 0.32 | 0.45 |
| $a_z$ | 8.975 | 4.525 | 2.275 | 2.075 | 1.875 |
6. Conclusion
We showed that the self-similar mode of laser beam propagation in a medium with nonlinear absorption takes place even for a presence of de-focusing of laser beam. We developed an analytical solution for the beam profile for laser pulse propagation in a medium with resonant nonlinear response for signs of detuning of a carrier frequency of a wave packet from energy transition frequency of atom or molecule.

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References
[1] Akhmanov S A, Vysloukh V A and Chirkin A S 1992 Optics of femtosecond laser pulses (NY: American Institute of Physics)
[2] Shen Y R 1984 The Principles of Nonlinear Optics (NY: J. Wiley Press)
[3] Agrawal G P 1995 Nonlinear fiber optics (NY: Academic Press)
[4] Sukhorukov A P 1988 Nonlinear Wave Interactions in Optics and Radiophysics (Moscow: Nauka)
[5] Kivshar Y S and Agrawal G P 2003 Optical Solitons: From Fibers to Photonic Crystals (San Diego: Academic Press)
[6] Akhmediev N N and Ankiewicz A 1997 Solitons: Nonlinear Pulses and beams (London: Chapman and Hall Press)
[7] Brull L and Lange H 1986 Stationary, oscillatory and solitary wave type solution of singular nonlinear Schrödinger equations Math. Meth. in the Appl. Sci. 8 (4) 559–75
[8] Malomed B A 2006 Soliton management in periodic systems (NY: Springer)
[9] Berge L, Mezentsev V K, Rasmussen J J et al 2000 Self-guiding light in layered nonlinear media Opt. Lett. 25 (14) 1037–39
[10] Tower I and Malomed B A 2002 Stable (2+1)-dimensional solitons in a layered medium with sign-alternating Kerr nonlinearity JOSA B 19 (3) 537–43
[11] Yannopapas V, Modinos A and Stefonou N 2003 Anderson localization of light in inverted opals Phys. Rev. B 68 193205
[12] Goodman R H, Holmes Ph J and Weinstein M I 2004 Strong NLS soliton-defect interactions Physica D 192 215–42
[13] Driben R, Oz Y, Malomed B A et al. 2007 Mismatch management for optical and matter-wave quadratic solitons Phys. Rev. E 75 026612
[14] Serkin V N and Hasegawa A 2002 Exactly integrable nonlinear Schrodinger equation models with varying dispersion, nonlinearity and gain: application for soliton dispersion IEEE Journal of Selected Topics in Quantum Electronics 8 (3) 418
[15] Serkin V N, Hasegawa A and Belyaeva T L 2007 Nonautonomous solitons in external potentials Physics Review Letters 98 074102
[16] Baizakov B B, Malomed B A and Salerno M 2003 Multidimensional solitons in periodic potentials Europhysics Letters 63 (5) 642–48
[17] Al Khawaja U 2009 Integrability of a general Gross–Pitaevskii equation and exact solitonic solutions of a Bosel–Einstein condensate in a periodic potential Phys. Letters A 373 2710–16
[18] Trofimov V A and Varentsova S A 2005 Computational method for finding of soliton solutions of the nonlinear Schrödinger equation Lectures Notes in Mathematics ed L. Vulovk (Berlin: Springer-Verlag) 3401 pp 550–557
[19] Varentsova S A and Trofimov V A 2005 On a difference method for finding the eigenmodes of the nonlinear Schrödinger equation Moscow University Computational Mathematics and Cybernetics 3 118
[20] Matusevich O V and Trofimov V A 2008 Iterative method for finding the eigenfunctions of a
system of two Schrödinger equations with combined nonlinearity. Computational Mathematics and Mathematical Physics 48 (4) 677–87

[21] Matusevich O V and Trofimov V A 2009 Numerical method for finding 3D solitons of the nonlinear Schrödinger equation in the axially symmetric case Computational Mathematics and Mathematical Physics 49 (11) 1902–12

[22] Matusevich O V and Trofimov V A 2009 A numerical method for calculating solitons of the nonlinear Schrödinger equation in the axially symmetric case Moscow University Computational Mathematics and Cybernetics 33 (3) 117–26

[23] Trofimov V A and Matusevich O V 2008 Numerical method for 2D soliton solution at SHG in media with time-dependent combined nonlinearity Mathematical Modelling and Analysis 13 (1) 123–34

[24] Trofimov V A, Lysak T M and Matysevich O V 2010 Influence of transverse perturbation of soliton propagation direction on laser radiation evolution along the layered medium Photonic Crystal Fibers IV ed K Kalli and W Urbanczyk Proc. of SPIE 7714 77140K

[25] Trofimov V A, Lysak T M, Matysevich O V and Lan Sh 2010 Parameter control of optical soliton in one-dimensional photonic crystal Mathematical modeling and analysis 15 (4) 517–32

[26] Matusevich O V, Trofimov V A, Yudina E A and Malomed B A 2009 The evolution of two-frequency solitons in an optical fiber with a longitudinally nonuniform nonlinearity Optics and Spectroscopy 106 (1) 99–107

[27] Trofimov V A, Lysak T M and Matysevich O V 2010 Influence of transverse perturbation of soliton propagation direction on laser radiation evolution along the layered medium Proc. of SPIE 7714 77140K

[28] Trofimov V A, Matysevich O V and Smotrov D A 2011 Mode of propagation of optical radiation with self-similar pulse shape in layered medium with nonlinear absorption Proc. of SPIE 8095 80951K