Effect of spin–orbit coupling on tunnelling escape of Bose–Einstein condensate

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Abstract

We theoretically investigate quantum tunnelling escape of a spin–orbit (SO)-coupled Bose–Einstein condensate (BEC) from a trapping well. The condensate is initially prepared in a quasi-one-dimensional harmonic trap. Depending on the system parameters, the ground state can fall in different phases—single minimum, separated or stripe. Then, suddenly the trapping well is opened at one side. The subsequent dynamics of the condensate is studied by solving nonlinear Schrödinger equations. We found that the diverse phases will greatly change the tunnelling escape behavior of SO-coupled BECs. In the single minimum and separated phases, the condensate escapes the trapping well continuously, while in the stripe phase it escapes the well as an array of pulses. We also found that SO coupling has a suppressing effect on the tunnelling escape of atoms. Especially, for BECs without inter-atom interaction, the tunnelling escape can be almost completely eliminated when the system is tuned near the transition point between the single minimum and stripe phases. Our work thus suggests that SO coupling may be a useful tool to control the tunnelling dynamics of BECs, and potentially be applied in the realization of atom lasers and matter wave switches.

Keywords: tunnelling, Bose–Einstein condensate, spin–orbit coupling

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum tunnelling, the phenomenon where a particle penetrates, and in most cases passes through, a potential barrier which it classically cannot surmount, is one of the most surprising effects in quantum mechanics. In the early days, it was mainly studied at the microscopic level [1, 2]. In pace with the discovery and development of various macroscopic quantum systems, research interest in macroscopic quantum tunnelling (for example, Josephson tunnelling [3]) has also begun to increase [2, 4]. Especially, Bose–Einstein condensates (BECs), a macroscopic matter wave with very good coherence and a manipulability, are very suitable for studying quantum tunnelling (and many other macroscopic quantum phenomena). Atomic Josephson junctions were realized a long time ago [5, 6]. Furthermore, the atomic analogy of superconducting quantum interference devices has also been experimentally demonstrated and proposed as a good compact rotation sensor [7]. The tunnelling escape of BECs from a trapping well has also been extensively studied [8–14] and recently has been experimentally observed, as well [15, 16].

Interference plays a key role in quantum physics. It can have a significant influence on quantum tunnelling. These influences have been reported in various fields. Fundamentally, it can lead to a breakdown of the exponential decay law [2, 17–19], and this phenomenon has already been observed in BEC systems [20]. It can also give rise to a modification of tunneling time [21]. In a nanomechanical system, it can cause a suppression of the tunneling between opposite magnetizations [22]. In atomic and molecular physics, recently it was reported to have a significant effect on the process of photoassociation [23] (which is also a quantum tunnelling-related phenomenon).

Spin–orbital (SO) coupling greatly enriches the quantum properties of BECs. Diverse ground state phases exist in which the matter wave properties are quite different [24–27].

In the zero momentum phase, the condensate stays in a state
Figure 1. Schematic diagram for the tunnelling escape of an SO-coupled BEC from a trapping well. As in reference [36], Zeeman levels 1 and 2 act as two spin states (the third Zeeman level 3 is far detuned and can be adiabatically eliminated, therefore corresponding lines are plotted in a lighter color). SO coupling is realized by the interaction between counter-propagating Raman beams L1, L2 and the atomic condensate. Initially, the condensate is prepared in the ground state of a harmonic trap described by equation (3) (green dashed line). At time \( t = 0 \) the trap is suddenly opened at the right side, i.e. changed to the form of equation (6) (violet solid line) with parameters \( V_0 = 1.57 \hbar \omega_0, x_0 = 1.5x_0, x_c = 2x_0, \delta = 0.87x_0 \). Then, atoms begin to escape the trapping well due to quantum tunnelling.

with zero momentum, while in a separated phase it possesses momentum of finite value either \( +p_0 \) or \( -p_0 \) and significantly in the stripe phase, its state is a superposition of the \( +p_0 \) and \( -p_0 \) states. The simultaneous existence of these two states will cause interference between them. Subsequently, the quantum tunneling phenomena of SO-coupled BECs show new features. In recent years, Josephson tunnelling of an SO-coupled BEC has attracted a large amount of research interest [28–32]. A new kind of Josephson effect—momentum space Josephson effect—has been predicted [33]. It is also reported that SO coupling will lead to rich dynamical phenomena of BEC Josephson vortices [34, 35].

However, the tunnelling escape of an SO-coupled BEC from a trapping well has not been carefully examined yet. In this work, we will study this phenomenon, and try to find out what effect SO coupling will have on the system. Tunnelling escape dynamics of an SO-coupled BEC in different phases will be studied by solving nonlinear Schrödinger equations, and the results will be compared. The dependence of the tunnelling escaped atoms number on the SO coupling strength will examined. Possible applications of the system in the field of atom optics will also be discussed.

The contents of this article is organized as follows. In section 2, we introduce the model for studying the tunnelling escape of an SO-coupled BEC. Then in section 3, the tunnelling escape dynamics of the system are studied both analytically and numerically, and the results are discussed. Lastly, the work is summarized in section 4.

2. Model

We consider a system of SO-coupled BECs realized by the Raman coupling scheme [36], as schematically shown in figure 1. The condensate is assumed to be confined in a parabolic trap with frequencies \( \omega_0 \ll \omega_z \), so that the system can be reduced to one dimension [37], and the dynamic can be described by the following two-coupled nonlinear Schrödinger equations

\[
i\hbar \frac{\partial \psi_1}{\partial t} = \left( \frac{(p_x + p_y)^2}{2m} + U(x) \right) \psi_1 + \hbar \Omega \psi_1 + (g_0 |\psi_1|^2 + g_1 |\psi_1|^2) \psi_1,
\]

\[
i\hbar \frac{\partial \psi_2}{\partial t} = \left( \frac{(p_x - p_y)^2}{2m} + U(x) \right) \psi_2 + \hbar \Omega \psi_2 + (g_0 |\psi_1|^2 + g_1 |\psi_1|^2) \psi_2,
\]

where \( \psi_1 \) and \( \psi_2 \) are wave functions of the two spin components obeying the normalization condition \( \int (|\psi_1|^2 + |\psi_2|^2) dx = 1 \), \( \hbar \) is the reduced Planck’s constant, \( m \) is the mass of an atom, \( p_x = -i \hbar \frac{\partial}{\partial x} \) is the momentum operator in \( x \)-direction, \( p_x \) is the SO coupling strength determined by the momentum transfer of the Raman lasers, \( \Omega \) is the Rabi coupling strength accounting for the transition between the two spin states, \( g_0, g_1 \) are effective one-dimensional contact interaction strengths (their values are \( g_2 = 2N_0 a_0/\omega_z \), where \( a_0 \) and \( a_1 \) are s-wave scattering lengths for the collision of two atoms with same or different spins, \( N_0 \) is number of atoms in the condensate), and \( U(x) \) is the external trap potential. Experimentally, such a system is highly controllable, SO coupling can be tuned by manipulating the two Raman lasers [38, 39], and contact interaction can be tuned using the well-known Feshbach resonance technique [40].

Initially, the condensate is confined in a harmonic trap:

\[
V_0(x) = \frac{1}{2} m_0 \omega_0^2 x_0^2.
\]

Thus, the initial state is assumed to be the ground state \( \Psi_0 = (\tilde{\psi}_1, 0, \tilde{\psi}_2, 0)^T \) of this trapping potential. For vanishing of both SO coupling and inter-atom interactions \( p_x = g_0 = g_1 = 0 \), equations (1) and (2) are simplified to

\[
\mu \dot{\varphi}_{1,1} = \left( \frac{p_x^2}{2m} + V_0(x) \right) \varphi_{1,1} + \hbar \Omega \varphi_{1,1},
\]

with \( \psi_{1,1}(x, t) = \tilde{\varphi}_{1,1}(x) e^{-i \mu t/\hbar} \). Solving this time-independent Schrödinger equation, the ground state can be calculated precisely to be a Gaussian wave packet:

\[
\Phi_0^G = \frac{1}{\sqrt{2\pi(\varphi_0^G)^2}} \exp \left( -\frac{x^2}{2\varphi_0^G} \right).
\]

Otherwise, for no vanishing of SO coupling and inter-atom interaction, the initial state will be obtained by numerically evolving a trial wave function in an imaginary time dimension.

The ground state properties of an SO-coupled BEC have been carefully studied in reference [26]. Here we give a brief review. Omitting the nonlinear and trapping terms, at the same time assuming a plane wave solution \( \tilde{\psi}_{1,1}(x, t) = \psi_{1,1} \exp \left[ i \left( p_x x - E_t \right) / \hbar \right] \), the dispersion relation of an SO-coupled matter wave can be obtained by diagonalizing...
equations (1) and (2), which reads \( E_\pm(p_s) = (p_s^2 + p_\xi^2) / (2m) \pm \sqrt{\hbar^2 \Omega^2 + p_\xi^2 V_s^2 / m^2} \). This dispersion relation splits into two branches—\( E_+(p_s) \) and \( E_-(p_s) \). The higher energy branch \( E_+(p_s) \) is quite normal. It always has a single minimum at \( p_s = 0 \), and will have little influence on the ground state property of the system. Interestingly, the lower energy branch \( E_-(p_s) \) behaves very differently for different strengths of SO and Rabi coupling. When \( \hbar \Omega > p_s^2 / m \), it also has only one minimum at \( p_s = 0 \). Therefore, the lowest energy state will carry zero momentum, and the corresponding phase is called the ‘zero momentum phase’. While in the case of \( \hbar \Omega < p_s^2 / m \), the lower energy branch dispersion curve has two equal minima at \( p_s \) and \( p_0 \), with \( p_0 = \sqrt{p_s^2 - (m \hbar \Omega / p_s)^2} \). As a result, the lowest energy state may carry a separated momentum \(-p_0 \) or \(+p_0 \) (separated phase), or a superposition of them (due to the interference between these two states, there exist interference stripes, thus the corresponding phase is called the ‘stripe phase’), and these states are degenerate. This degeneracy may be lifted by the external potential or interaction between atoms. When there is only the external harmonic trap potential, the system prefers firstly approach the separated phase, before reaching the stripe phase. In the following content, these conclusions will be used directly without more explanation.

At time \( t = 0 \) trapping potential (3) is suddenly opened at the right side. Mathematically speaking, the trapping potential is suddenly changed to the following form (see figure 1):

\[
V(x) = \begin{cases} 
\frac{m \omega_0^2 x^2}{2}, & x < x_b, \\
V_0 \exp \left[ -\left( \frac{x - x_e}{b} \right)^2 \right], & x > x_b.
\end{cases}
\]

(6)

Here parameters \( V_0, x_e, \delta \) are determined by matching the potential function and its first derivative at point \( x = x_b \). They represent the height, location and width of the barrier respectively, and their values are set to \( V_0 = 1.57 E_{\text{ho}}, x_b = 1.5 x_0, x_e = 2 x_0, \delta = 0.87 x_0 \), with \( E_{\text{ho}} = \hbar \omega_0 \) and \( x_0 = \sqrt{\hbar / (m \omega_0)} \) being the harmonic oscillator (with frequency \( \omega_0 \)) energy and length units. Consistently, frequency, time and momentum will be measured in units \( \omega_0, t_0 = 1 / \omega_0 \) and \( \hbar / \omega_0 \).

After the trapping potential well is opened, i.e. changed to the form of equation (6), the atoms begin to escape from the trapping well due to quantum tunnelling. The dynamic is studied by solving the time dependent nonlinear Schrödinger equations (1) and (2) with a Crank-Nicolson scheme.

To characterize the tunnelling escape, we define the escaped spin-down/up atoms number as

\[
N_{\text{out}}^{\text{tot}} = \int_{x_f}^{x_b} |\psi_{\text{1,}}(x)|^2 dx, \tag{7}
\]

where \( x_f = 3 x_0 \) is the demarcation point between the potential area and free space, and the total escaped atoms number is the sum of the two spin components \( N_{\text{out}} = N_{\text{out}}^{\text{up}} + N_{\text{out}}^{\text{down}} \). It should be noted, the total wave function is normalized to 1 (as previously mentioned), therefore the escaped atoms number defined here is indeed a normalized atoms number or in other words—fraction to the total atoms number.

3. Results

Firstly, for comparison purposes and a most basic understanding of the system, we examine the no SO coupling case. Without any SO coupling \( p_s = 0 \), the symmetry between spin-down and spin-up components suggests the following form of wave function:

\[
\Psi(x, t) = \psi(x, t)(1 - i)^T e^{i\omega t}.
\]

(8)

Substituting it into nonlinear Schrödinger equations (1) and (2), the dynamics of the system can be simplified to a single governing equation:

\[
\frac{i \hbar}{\partial t} \psi(x, t) = \left( \frac{p_s^2}{2m} + U(x) \right) \psi(x, t)
+ \frac{g_0 + g_1}{2} |\psi(x, t)|^2 \psi(x, t),
\]

(9)

which has no dependence on the Rabi frequency \( \Omega \). This is to say that without the presence of SO coupling, Rabi coupling will not solely affect the dynamics of the system. This fact is also confirmed by our numerical results. In the first row (a1–d1) of figure 2, we set the SO coupling parameter \( p_s = 0 \), and each of the spin components show the initial wave packet, evolution of atomic density, wave packet at time \( t = 20 t_0 \) and the escaped atoms number during the evolution. We see that for all these quantities the spin-up and spin-down components have the same values.

As SO coupling is added to the system, symmetry between spin-down and spin-up components is broken, see figures 2(a2)–(d4). For single minimum (figures a2–d2) and stripe phases (figures a3–d3), although the two spin components have the same initial wave packets, they gain an evident difference during the evolution. And for separated phase (figures a4–d4), both the initial wave packets and their subsequent evolutions for the two spin components are very different. Interaction is essential in achieving the separated phase [26], and the repulsive interaction can considerably reinforce the escape of the condensate [11, 15], resulting in a much larger value regarding the escaped atoms number and atomic density compared to the non-interacting cases.

In figure 2, we also noticed that an initial single minimum or separated phase wave packet escapes from the trapping well continuously; while because of interference between the \( +p_0 \) and \(-p_0 \) momentum components, an initial stripe phase wave packet escapes from the trapping well as a series of pulses (for a better view, see figure 3 in which (b3,1), (b3,2), (c3) of figure 2 are enlarged to show details of the pulsive property). This indicates the tunneling escape of an SO-coupled BEC from a trapping well may be useful in realizing an atom laser which can be operated in both continuous and pulsed modes.

From figure 2, one can also somewhat see that SO coupling will affect the escaping speed of the condensate. This
fact can be more clearly demonstrated by figure 4 where we plot the escaped atoms number \( N_{\text{out}} \) as a function of time for different SO coupling strengths in the case of no inter-atom interaction. From the figure, we see the line for \( p_c = 0 \) (no SO coupling) locates above all the other ones, i.e. without SO coupling there are the most escaped atoms, that is to say SO coupling has a suppressing effect on the tunnelling escape of atoms. In the figure, we also noticed that in the single minimum phase (\( p_c = 0.0, 1.0, 1.5, 3.0 h k_0 \)), a stronger SO coupling tends to reduce the tunnelling escaped atoms number more; while in the stripe phase (\( p_c = 3.5, 4.0, 4.5, 8.0 h k_0 \)), it is quite the opposite—as the coupling gets stronger, it tends to reduce the tunneling escaped atoms number less. Around the phase transition point \( p_c = 3 h k_0 \), the escaped atoms number reaches a minimum with a value of almost zero. Using this feature, an efficient matter wave switch may be realized. In the case with inter-atom interaction, a similar suppression phenomenon can be observed as well (see figure 5). In the single minimum phase (\( p_c = 0.0, 1.0, 2.0, 2.5 h k_0 \)), the escaped atoms number also decreases with the increasing of coupling strength. As the SO coupling getting stronger (\( p_c = 3.0, 3.5, 4.5, 5.0 h k_0 \)), the system goes into the separated phase. In the separated phase, the escaped atoms number increases with the increasing of coupling strength. However, due to the escape reinforcement caused by repulsive interaction, even near the phase transition point a sufficiently large amount of atoms will escape the trapping well.

In a strong SO coupling limit, according to dynamical equations (1) and (2), the spin-down/up components tend to carry a large gauge momentum in the negative/positive direction. Thus, we can assume a wave function approximately having the following form:

\[
\Psi(x, t) = \frac{1}{\sqrt{2}} \phi(x, t)(1) e^{-i \psi x / \hbar} - \frac{1}{\sqrt{2}} \phi(x, t)(0) e^{i \psi x / \hbar},
\]

for the stripe phase, and

\[
\Psi(x, t) = \phi(x, t)(1) e^{-i \psi x / \hbar},
\]

for the separated phase, with \( \phi(x, t) \) being a spatially slowly varying wave function. Inserting them into nonlinear coupled Schrödinger equations (1) and (2), we found that for the stripe phase wave packet, \( \phi(x, t) \) obeys the same equation as
non-coupling equation (9); while for the separated phase wave packet, $\phi(x, t)$ obeys a slightly different equation:

$$i\hbar \frac{\partial}{\partial t} \phi(x, t) = \left[ \frac{p^2}{2m} + U(x) \right] \phi(x, t) + g_0 |\phi(x, t)|^2 \phi(x, t).$$

When the inter- and intra-components interaction strengths are close to each other $g_0 \approx g_1$, equation (12) is approximately the same as non-coupling equation (9). So, for strong SO coupling, the tunnelling escape dynamic will go back to the non-coupling case. This conclusion is also demonstrated by the fact that in figure 4 lines for $p_c = 0.8\hbar k_0$ and in

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**Figure 3.** Details of figure 2 (b3.1, b3.2) and (c3), the pulsive tunnelling escape feature of a stripe phase wave packet. The left two panels are an enlargement of figure 2 (b3.1) and (b3.2) in the range of $x \in (30x_0, 200x_0)$. The two lines in figure 2 (c3) are enlarged as the right two panels—top panel for $|\psi(x, t = 20t_0)|^2$, while bottom panel for $|\psi(x, t = 20t_0)|^2$.

**Figure 4.** Escaped atoms number for different SO coupling strengths in the non-interacting case. The solid lines represent $p_c = 0.0, 1.0, 1.5, 3.0/\hbar k_0$ (single minimum phase), while the dashed lines represent $p_c = 3.5, 4.0, 4.5, 8.0/\hbar k_0$ (stripe phase), respectively. For all the lines, Rabi coupling and interaction strength are $\Omega = 8\omega_0$, $g_0 = g_1 = 0$.

**Figure 5.** Escaped atoms number for different SO coupling strengths in the interacting case. The solid lines represent $p_c = 0.0, 1.0, 2.0, 2.5/\hbar k_0$ (single minimum phase), the dashed lines represent $p_c = 3.0, 3.5, 4.5, 5.0/\hbar k_0$ (separated phase). The Rabi coupling and interaction strength are $\Omega = 8\omega_0$ and $g_0 = 10E_0\omega_0$, $g_1 = 9E_0\omega_0$ for all the lines.
During the evolution for the initial separated phase than in the stripe phase. This can be seen from escape the trapping well for a wave packet in the separated phase with energy $E = -11.91 E_0$. The initial stripe phase state is a nearly degenerate state with a slightly higher energy $E = -12.03 E_0$. The initial ground state is in the separated phase with energy $E = -12.03 E_0$. Under such parameters, the initial ground state is in the separated phase with energy $E = -12.03 E_0$. The initial stripe phase state is a nearly degenerate state with a slightly higher energy $E = -11.91 E_0$. figure 5 lines for $p_x = 0.5h k_0$ are nearly overlapping with each other.

For the inter-/intra-component interaction parameters $g_0 > g_1$, as repulsive interaction will reinforce the tunnelling escape, equations (9) and (12) suggest that more atoms will escape the trapping well for a wave packet in the separated phase than in the stripe phase. This can be seen from figure 6, where we compared the escaped atoms number during the evolution for the initial separated (violet solid line) and stripe (green dashed line) phase wave packets under parameters $p_x = 5 h k_0$, $\omega = 8 \omega_0$, $g_0 = 10 E_0 k_0$, $g_1 = 9 E_0 k_0$. Here, it should be pointed out that under these parameters, the ground state really falls in the separated phase with energy $E = -12.03 E_0$. But since the system is nearly degenerate, a stripe phase wave packet can also be obtained with a slightly higher energy $E = -11.91 E_0$.

4. Summary

In summary, we have analyzed the tunnelling escape of an SO-coupled BEC wave packet from a trapping well. Our results show that the dynamics of the system are quite different in different phases: the single minimum or separated phase wave packet escape the trapping well continuously, while the stripe phase one escapes in a pulsed manner. This feature may be used in realizing an atom laser which can be operated both in continuous and pulsed modes. We also found that SO coupling can suppress the tunnelling escape of atoms. Especially, in the non-interacting case, by tuning the SO coupling strength to the transition point between the single minimum and the stripe phase, the tunnelling escape can be almost totally suppressed, thus a matter wave switch may potentially be realized. In the strong SO coupling limit, the dynamics go back to the non-coupling case except for a phase factor.

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