A Sparsity-constraint Capon Beamforming Algorithm

Wenjing Wang¹, Hongyu Zhao² and Ying Zhang¹

¹School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China
²Luoyang electronic equipment test center, Luoyang, Henan, China

Abstract. In the presence of the steering vector of the signal of interest mismatches, nulls are formed in the direction of the signal of interest, which affects the performance of the beamforming. In this paper, a sparsity-constraint Capon beamforming based on iterative robust minimum variance beamforming is proposed. Using iterative robust minimum variance beamforming, the mismatched steering vector is corrected. To achieve sidelobe level and suppress interference, sparse constraints on beam response is added in the angel of sidelobe domain. Computer simulations have been conducted to verify validity and superiority of the proposed algorithm. It is shown that in the presence of mismatched steering vector, the proposed algorithm is more robust and has lower sidelobe level than the existing methods.

1. Introduction

Beamforming technology is one of the main research contents in array signal processing, which has been widely used in military and civilian fields such as radar, sonar, space communication, and biomedicine. The most critical issue in adaptive beamforming is the efficient reception of the signal of interest. There are two basic requirements for the effective reception. One is to form the main lobe in the spatial direction of the interested signal. The other is to suppress the noise and the interference signal, and improve the output Signal-to-Interference-plus-Noise Ratio (SINR) of the output signal. To meet these requirements, the excitation weights of each array element are adjusted online in beamforming. In this paper, a sparsity-constraint Capon beamforming based on iterative robust minimum variance beamforming[1] is proposed.

2. Problem Formulation

The output of a narrowband beamformer is

\[ y(n) = w^H x(n) = \sum_{m=1}^{N} w_m^* x_m(n) \]  

where \( n \) is the snapshot, \( N \) is the number of array elements, \( w=[w_1,...,w_N]^T \) is beamforming weight, \( x=[x_1,...,x_N]^T \) is the received snapshot vector given by

\[ x(n) = As(n) + v(n) + i(n) \]  

(2)
where $A = [a(\theta_0), a(\theta_1), \ldots, a(\theta_K)]$ is the direction matrix, $a(\theta_k)$ is the steering vector of the $k$th incoming signal, $s(n) = [s_1(n), s_2(n), \ldots, s_K(n)]^T$ is the desired signal vector, $v(n)$ is $N \times 1$ noise vector, $i(n)$ is $N \times 1$ interference vector.

According to the weight vector of the beamer, the pattern of the array is available:

$$ F(\theta) = \left| w^H a(\theta) \right| $$

The SINR is expressed by

$$\text{SINR} = \frac{\sigma_0^2 \left| w^H a(\theta_0) \right|^2}{w^H R_{in} w} $$  

where $\sigma_0^2$ is the desired signal power, $R_{in}$ is the interference-plus-noise covariance matrix, $a(\theta_0)$ is the steering vector of the desired signal.

When the real steering vector of the desired signal is not consistent with the ideal one, a null will appear in the direction of the desired signal, which affects the performance of the beamforming.

### 3. Proposed Sparsity-constraint Capon Beamforming Based on Iterative Robust Minimum Variance Beamforming

Assuming that the main lobe does not contain interference signals, the angle of side lobe is uniformly divided into equiangular meshes, and the corresponding steering vector matrix corresponding to the side lobe is

$$ \bar{A} = [a(\theta_1), a(\theta_2), \ldots, a(\theta_L)] $$

Where $\theta_1, \theta_2, \ldots, \theta_L$ correspond to the angular grid points within the side lobe.

Ideally, the gain in the side lobe of the normalized array pattern is much smaller than the gain in the main lobe. The gain in the side lobe can be approximately zero, and the main lobe width is small relative to the side lobes. Therefore, it has the sparse nature of the signal (most elements are zero).

In order to enhance the suppression of noise and interference, the norm $l_p$ is used to sparsely constrain the beam response of the side lobe, which is added as the penalty term of the cost function of minimum variance distortionless response (MVDR) [2]. The corrected steering vector of the desired signal $\hat{a}_0(\theta_0)$ is by iterative robust minimum variance beamforming. The optimization model is as follows:

$$ w_{\text{proposed}} = \arg \left\{ \min_w \left[ \frac{1}{2} w^H R_{\alpha} w + \lambda \left\| w^H \bar{A} \right\|_p^p \right] \right\} $$

s.t. $w^H \hat{a}_0(\theta_0) = 1$

where $R_{\alpha}$ is the covariance matrix of the received snapshot data $x$, $\left\| \cdot \right\|_p^p (0 \leq p \leq 1)$ is the $l_p$ norm. The smaller the value $p$, the thinner the beam response $w^H \bar{A}$ in the side lobe. Then, the side lobe level is suppressed. $\lambda$ is a regularization factor.

Using the Lagrangian multiplier method, the cost function is:
\[ J(w) = \frac{1}{2} w^H R_x w + \lambda \| w^H \tilde{A} \|^p_p + \gamma (w^H a(\theta_0) - 1) \]  
(7)

Let \( \tilde{A} = [\tilde{A}, \mu \cdot \hat{a}] \), \( d = [0, \mu \cdot 1] \), \( \mu = \gamma / \lambda \), equation (7) becomes

\[ J(w) = \frac{1}{2} w^H R_x w + \lambda \| \tilde{A} w^H - d^* \|^p_p \]  
(8)

Then, the gradient of \( w \) is

\[ \nabla_w J(w) = R_x w + \lambda^* \tilde{A} \Pi(w)(\tilde{A}^H w - d^*) \]  
(9)

where \( \lambda^* = \lambda^p \), \( \Pi(w) \) is

\[ \Pi(w) = \begin{bmatrix} |(\tilde{A}^H w - d^*)|^{p-2} & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & |(\tilde{A}^H w - d^*)|^{p-2} \end{bmatrix} \]  
(10)

where \((\tilde{A}^H w - d^*)\) denotes the \( i \)th element of \((\tilde{A}^H w - d^*)\), \( |\cdot|^{p-2} \) is the \( p - 2 \) norm. With the iterate \( w(k) \) given at the \( k^{th} \) iteration, the \( w(k+1) \) is obtained by letting equation (9) equal to zero:

\[ w(k+1) = \lambda^* \left( R_x + \lambda^* \Pi(w(k)) \tilde{A}^H \right)^{-1} \tilde{A} \Pi(w(k)) d^* \]  
(11)

Where \( P \) and \( \lambda \) jointly determines the sparse of the beam response to the side lobe. The iteration summary of the proposed beamforming algorithm as follows:

a) Select the regularization factor \( \lambda \), the number of iterations Loop;
b) Get \( \hat{a}_0(\theta_0) \) by iterative robust minimum variance beamforming;
c) Set the side lobe region \( \Theta \) and form a sparse steering vector matrix \( \tilde{A} \) on it;
d) Get \( \Pi(w) \) from (10);
e) Get \( w(k+1) \) from (11);
f) Terminate the algorithm when \( k \) reaches Loop, otherwise, repeat c)-e).

4. Computer Simulations

The proposed algorithm is tested on the quarter of 8×8 -element cylinder array. The array has a radius of 0.2 m, a height of 0.3 m, and a fan angle of 90°. Assuming that the array element is omidirectional, the desired signal, interference and noise are mutually uncorrected. The noise is spatially White Gaussian with unit variance.

To verify validity and superiority of the proposed method, the fixed diagonal loading (FDL) [3], the Completely auto diagonal loading (GDL) [4], and the NESUS [5] are added into simulation.

In the first simulation, the desired signal is present in the array snapshots with direction and SNR of \( [\theta_0 = 0^\circ, \varphi_0 = 0^\circ, \text{SNR} = 10 \text{ dB}] \). There are 2 interferences with directions and interference-to-noise ratios (INRs) of \( [\theta_0 = 30^\circ, \varphi_0 = 45^\circ, \text{INRs} = 30 \text{ dB}] \) and \( [\theta_0 = 60^\circ, \varphi_0 = -30^\circ, \text{INRs} = 30 \text{ dB}] \),...
respectively. The center frequency of signal is \( f_0 = 4 \text{ GHz} \), the number of snapshots is 500. Assume that the elevation angles error and the azimuth error of the desired signal are \( \Delta \theta = 1^\circ \) and \( \Delta \phi = 1^\circ \), respectively. The diagonal loading of FDL takes 10 times the variance of the noise, and the uncertainty set size of NESUS is \( \varepsilon_1 = \| \mathbf{a}_0 - \overline{\mathbf{a}} \|_2 \). In the proposed algorithm, the uncertainty is \( \varepsilon_2 = 0.1, \delta = 0.01 \), the side lobe region \( \Theta \) contains \( \theta \in [-80^\circ, -5^\circ) \cup (5^\circ, 80^\circ] \), \( \phi \in [-80^\circ, -5^\circ) \cup (5^\circ, 80^\circ] \), angle sampling interval is \( 5^\circ \). And \( \rho = 1, \alpha = 50, \lambda = 0.2 \). The three-dimensional pattern and the contour line profile of the proposed method are shown in Figures 1 and Figure 2, respectively.

Figure 3 shows that FDL forms a null in the direction of the desired signal, and the sidelobe level is also higher than the other three algorithms; the main lobe of GDL and NESUS are offset, and the main lobe offset of NEUS is less than GDL and maintain a good beam shape. The main lobe of the proposed algorithm still points to the true incoming wave direction, and the sidelobe level is much lower than the other three algorithms.

![Figure 1. The three-dimensional pattern](image-url)
In the second simulation, the desired signal and interference is same to the first simulations. Considering that the elevation angles error and the azimuth error of the desired signal are $\Delta \theta = 3^\circ$ and $\Delta \phi = 2^\circ$. The relationship between SINR of the proposed method and the number of snapshots $N$ is shown in figure 4.

**Figure 2.** The contour line profile

**Figure 3.** The section of pattern at $\phi = 0^\circ$
Figure 4. SINR and snapshot number curve

Figure 4 shows the relationship between the SINR and the number of snapshots N after 100 independent simulation experiments. In Figure 4, it is found that the SINR of the proposed algorithm is higher than that of other algorithms under the small snapshot, and still has a higher SINR. The SINR of the FDL is not maintained under different snapshots. The MVDR has the lowest SINR because the MVDR guarantees that the desired signal is not distorted and the system requires the minimum output power. However, due to the mismatched steering vector of the desired signal, the beam main lobe shifts and the true signal direction is suppressed as an interference signal.

5. References
[1] Nai S E, Ser W, Yu Z L and Chen H W 2011 Iterative robust minimum variance beamforming IEEE Trans. Signal Process. 59 1601-11
[2] Ferguson G B 1998 Minimum variance distortionless response beamforming of acoustic array data J. Acoust. Soc. Am. 104 947
[3] Wang Z, Wang Y and Li Z 2015 An improved robust adaptive beamforming algorithm based on the conformal array 2nd Int. Conf. on Electrical Computer Engineering and Electronic (Jinan: Atlantis Press) pp 899–9044
[4] Du L, Li J and Stoica P 2010 Fully automatic computation of diagonal loading levels for robust adaptive beamforming IEEE Trans. Aerosp. Electron. Syst. 46 449-58
[5] Stoica L P and Wang Z 2003 On robust Capon beamforming and diagonal loading IEEE Trans. Signal Process 51 1702-15