Geometrical Origin of Tricritical Points of various U(1) Lattice Models *†

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Abstract

We review the dual relationship between various compact U(1) lattice models and Abelian Higgs models, the latter being the disorder field theories of line-like topological excitations in the systems. We point out that the predicted first-order transitions in the Abelian Higgs models (Coleman-Weinberg mechanism) are, in three dimensions, in contradiction with direct numerical investigations in the compact U(1) formulation since these yield continuous transitions in the major part of the phase diagram. In four dimensions, there are indications from Monte Carlo data for a similar situation. Concentrating on the strong-coupling expansion in terms of geometrical objects, surfaces or lines, with certain statistical weights, we present semi-quantitative arguments explaining the observed cross-over from first-order to continuous transitions by the balance between the lowest two weights (“2:1 ratio”) of these geometrical objects.
1 Introduction

In many physical systems, even the qualitative features along a line of phase transitions may depend on the choice of the coupling constants which parametrize the bare interaction energies. Quite often one encounters the case where a continuous higher order transition changes into a discontinuous first-order one. In most cases, the cross-over behaviour is characterized by a tricritical point (TCP) \[1\]. In field theoretic language, the origin of a TCP is easy to understand on the basis of a Landau expansion of the effective potential in terms of an order- (or disorder-) parameter. One source for a TCP is the sign-change in the quartic term (assuming a stabilizing sextic term) in some range of the parameter space. A positive quartic term leads to a second-order transition, and a negative one to a first-order transition. A vanishing quartic term is associated with the TCP. Another important mechanism arises from the presence of a cubic term. Such a cubic term may be generated, in some range of the parameter space, by fluctuations of other fields. This always drives the transition first-order. While the analysis of the Landau expansion is very elementary, the non-trivial problem is, of course, its derivation from the underlying physical system with quite complicated microscopic forces (the most prominent example being the Gorkov derivation \[2\] of the effective Ginzburg-Landau theory of superconductivity). It is therefore desirable to find alternative properties of the system which allow at least for a semi-quantitative understanding of the appearance of TCP’s. In this lecture we exhibit such properties for compact U(1) lattice models, having in mind quite different physical systems (D=dimension):

1. D=4 Quantumelectrodynamics (U(1)-LGT)
2. D=3 Superfluid Helium (XY-model)
3. D=3 Defect models of melting

The common point of these systems is that they can be studied in

a) the weak-coupling or “defect” expansion. This is formulated in terms of lines with long-range Biot-Savart-like interactions. This is why all three systems can be described, alternatively, by Abelian Higgs models in which the complex fields account for the line-like disorder of the systems and the gauge fields for the long-range interactions \[3\].

b) The strong-coupling or “stress” expansion. This is also formulated in terms of geometrical objects (surfaces, lines), but with no long-range interactions. This simplifies their statistical behaviour and will provide for the desired alternative criterion for the existence of TCP’s.

For the specific example of U(1) lattice gauge theory the different representations, being dual to each other, and their corresponding field-theoretic descriptions are summarized in fig.1. The diagrams for the other systems are completely analogous.

\[1\]When the dimension is reduced by one, \(D \rightarrow D - 1\), “lines” have to be replaced by “points”. 
2 The “Defect” Representation

Let us first look at the weak-coupling expansion of the various U(1) lattice models. They all contain topological excitations with long-range interactions caused by the Nambu-Goldstone bosons of the U(1) symmetry. In a three-dimensional solid, the topological excitations are line-like defects, dislocations and disclinations [4], and the Nambu-Goldstone bosons are elastic distortions (phonons), leading to a stress-field around each defect and to Biot-Savart-like interactions between defect elements. In superfluid Helium in three dimensions, the “defects” are vortex-lines [5] and the long-range forces are caused by the superflow. In four-dimensional U(1)-LGT finally, the “defects” are world lines of magnetic monopoles [6] and the forces are due to electromagnetism. The upper right box in fig.1 symbolizes the equivalent disorder field description of these “defect” lines in terms of a complex field $\Psi$ interacting with a gauge field $A_\mu$. In superfluid Helium and U(1)-LGT, the disorder field theory of line-like defects turns out [7, 8] to be an Abelian Higgs model. In a solid, it is a more complicated field theory of a similar type, involving two disorder fields, one for dislocation and one for disclination lines [3].

Let us briefly recall the physical content of the dual equivalence. From Feynman’s path-integral representation of quantum mechanics, we know that the statistical mechanics of one fluctuating line (“orbit”) corresponds to the quantum mechanics of one particle. It is then easy to see that the grand-canonical ensemble of fluctuating lines corresponds to the quantum mechanics of a many-particle system. This in turn is described most conveniently by a second quantized field theory. The long-range Biot-Savart-like nature of the interactions between line-elements is what permits their description by an Abelian gauge-field $A_\mu$. The minimal coupling of $A_\mu$ to the disorder field $\Psi$ leads then immediately to an Abelian Higgs model (scalar electro-
dynamics). Indeed, in the lattice formulation of interacting defect lines, these steps can be carried out rigorously [7, 8], leading to the Higgs action

\[ A = \int d^Dx \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |(\partial_\mu - ieA_\mu)\Psi|^2 + \frac{1}{2} m^2 |\Psi|^2 + \frac{1}{4} g |\Psi|^4 + \ldots \right] \]  

(1)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and \( e, m^2, g > 0 \) can in principle be calculated from the couplings of the original compact U(1) lattice model, indicated by the circle in the center of fig.1. According to the famous Coleman-Weinberg argument [9, 10], this Abelian Higgs model should always undergo a first-order transition when the mass parameter turns negative. The argument goes as follows: Assuming \( \Psi = \) const., the gauge fields can be integrated out, yielding a one-loop correction to (1)

\[ \text{tr} \log (k^2 + e^2 |\Psi|^2) \]  

(2)

This amounts to additional \( \Psi \) interactions of the form

\[ - |\Psi|^3 \quad \text{in D = 3} \]  

(3)

and

\[ |\Psi|^4 \log |\Psi| \quad \text{in D = 4} \]  

(4)

In both cases, the additional term should drive the transition first-order.

In three dimensions, this conclusion is in clear contradiction to the well established fact that the original D=3 XY model has a continuous phase transition [11, 12]. The crucial point in the whole argument is, of course, the assumption of an almost constant \( \Psi \)-field. If this assumption breaks down, then also the generation of a cubic term (in D=3) is no longer reliable and the transition may stay continuous even with \( A_\mu \)-field fluctuations. The decisive parameter is the ratio of the length-scales of \( A_\mu \)- and \( \Psi \)-fluctuations,

\[ \kappa = \frac{1}{\sqrt{2}} \frac{\text{penetration depth}(A_\mu)}{\text{coherence length}(\Psi)} \]  

(5)

In three dimensions, it was possible to show that there exists a tricritical value of \( \kappa \approx 1/\sqrt{2} \), i.e. near the separation line between type-I and type-II superconductivity [7]. For small \( \kappa \), the Coleman-Weinberg mechanism is valid, leading to a first-order transition, whereas for large \( \kappa \), the transition stays second order.

This led to the suggestion that also four-dimensional scalar electrodynamics could have a TCP [13]. By the above duality arguments, this would also hold for U(1)-LGT. Indeed, the first-order nature predicted by the Coleman-Weinberg mechanism was at odds with early Monte Carlo investigations of the dual U(1)-LGT which all claimed evidence for a continuous transition and even reported estimates for critical indices [14]. More recent work favors the existence of both transition regions, first-order and second-order, in agreement with ref. [13]. Only one paper claims the validity of the Coleman-Weinberg mechanism everywhere. The present status will be reviewed in more detail in the next section.
The goal of this lecture is to present semi-quantitative explanations for these TCP’s, working within the dual strong-coupling expansions of the various U(1) models. This has the advantage that, since the geometrical objects appearing in this expansion have no long-range interactions, subtleties of the type (3)–(4) are absent.

3 The “Stress” Representation

Let us start by recalling the definitions of the various U(1) models mentioned in the introduction. The partition functions

$$Z = \sum_{\{\gamma_{- \text{conf.}}\}} (\prod B)$$

are products of local Boltzmann factors which may be chosen as

$$B = \begin{cases} 
  e^{\beta \cos \Theta} & \text{Wilson} \\
  e^{\beta \cos \Theta + \gamma \cos 2\Theta} & \text{Mixed} \\
  \sum_n e^{-\frac{\beta V}{2}(\Theta - 2\pi n)^2} & \text{Villain} 
\end{cases}$$

with $\Theta$ standing short for

$$\Theta = \begin{cases} 
  \nabla_i \gamma_j - \nabla_j \gamma_i \equiv (\nabla_i \gamma_j)^A & \text{Gauge} \\
  \nabla_i \gamma_j & \text{XY} \\
  \nabla_i \gamma_j + \nabla j \gamma_i \equiv (\nabla_i \gamma_j)^S & \text{Melting} 
\end{cases}$$

The product in (3) runs over the plaquettes or links of the lattice, and $\nabla_i$ are the usual lattice derivatives ($\nabla_i f(\vec{x}) = f(\vec{x} + \vec{i}) - f(\vec{x})$). The $\gamma$-configurations are, in U(1)-LGT, the euclidean electromagnetic fields, in the case of superfluid Helium, the phase angles of the condensate, and in the melting model, the atomic displacements. The common important property of these models is the U(1) invariance

$$\gamma \to \gamma + 2\pi$$

suggesting a Fourier (“character”) expansion of the Boltzmann factors

$$B(\Theta) = \sum_{b=-\infty}^{\infty} W_b e^{ib\Theta}$$

with Fourier coefficients

$$W_b = \int_{-\pi}^{\pi} \frac{d\Theta}{2\pi} B(\Theta) e^{-ib\Theta}$$

Inserting (10) into the partition function and summing over the $\gamma$-configurations, one obtains the strong-coupling expansion ($\bar{W}_b \equiv W_b/W_0$)

$$Z = (\prod W_0) \sum_{\{b-\text{conf.}\}} (\prod \bar{W}_b)$$
The algebraic structure of $\Theta$ in eq.(8) leads to conservation laws which constraint the admissible $b$-configurations. It is most transparent to characterize them in terms of geometrical objects:

$$\{b-\text{conf.}\} = \{\text{surfaces} \quad \text{Gauge} \}
\begin{cases} 
\text{lines} & \text{XY} \\
\text{complicated lines} & \text{Melting}
\end{cases}$$

In the U(1)-LGT, the $b$-variables correspond to the electromagnetic field strengths, in the superfluid to the superfluid currents, and in the melting model to the physical stress. This is why we call the expansion (12) generically the “stress”-representation. The geometrical objects in (13) are subject only to short-range contact interactions. Notice that the geometrical characterization of these “stress”-graph configurations does not depend on the dimension $D$. This is in contrast to the “defect”-representation where the dimensionality of the geometrical objects depends on the dimension in which the duality transformation is performed.

4 Analytical and Numerical Results

Let us now analyze the “stress” representation (12) in some detail. To each choice of action in (7) corresponds a special “natural” set of weights $\tilde{W}_b$ in (12). Their relative importance with increasing strength $b = \pm 1, \pm 2, \ldots$ can be studied conveniently by simulating each model with different actions and comparing their thermodynamic quantities such as their internal energies.

We start with the comparison between Wilson’s and Villain’s action for which the weights are $\tilde{W}_b^W = I_b(\beta)/I_0(\beta)$ ($I_b$ : modified Bessel function) and $\tilde{W}_b^V = \exp(-b^2/(2\beta_V))$, respectively. The two actions are made as similar as possible by equating the $b=\pm 1$ weights which amounts to relating the Villain parameter $\beta_V$ to the Wilson parameter $\beta$ as follows (see fig.2)

$$\beta_V(\beta) = -\frac{1}{2\log[I_1(\beta)/I_0(\beta)]}$$

This relation was first written down by Villain [15] when analyzing the Wilson type of action of the XY model in terms of the discrete Gaussian model (Villain approximation). If furthermore the overall normalizations of the partition functions are adjusted, then the difference between the two actions lies all in the higher weights, $\tilde{W}_b^W(\beta) = I_b(\beta)/I_0(\beta) \neq [I_1(\beta)/I_0(\beta)]^b = \exp(-b^2/(2\beta_V)) = \tilde{W}_b^V$ for $b \geq 2$. In order to measure their importance, we have performed Monte Carlo simulations in the representation (6) with both Wilson’s and Villain’s action [16]. As a typical example, we compare in fig.3 our results for the internal energy of the U(1) lattice gauge model. The excellent agreement for low $\beta$ up to the phase transition around $\beta \approx 1$ demonstrates that, in this range, the systems are dominated by $b = \pm 1$ excitations. It is therefore not surprising that also the transition temperatures of the Villain action (with $\beta_V$ as a free parameter) are mapped by (14) very precisely onto the corresponding ones of the Wilson action (see table 1). Furthermore, from fig.3 we
Figure 2: The parameter $\beta_V$ of the Villain approximation versus $\beta$ of the corresponding Wilson action. The curve results from the requirement of equal weights for graphs of strength-1 in the “stress” representation for both actions.

Figure 3: The internal energy of D=4 U(1) lattice gauge model with Wilson action in comparison with the Villain approximation.
Table 1: Transition temperatures of U(1) lattice gauge model for D=4, and of XY model for D=2,3 compared with the values obtained from the Villain mapping \( \beta_V = -\frac{[2 \log(I_1(\beta)/I_0(\beta))]^{-1}}{2}. \) The values are taken from: a) Jersák et al. [22], b) Gupta et al. [25], c) Dasgupta and Halperin [12], d) Furer et al. [11], e) Shugard et al. [17], f) Tobochnik and Chester [18], g) Samuel and Yee [19], h) Harten and Suranyi [20].

| Model | \( \beta_{Vc} \) | \( \beta_c \) (from VA) | \( \beta_c \) |
|-------|-----------------|-------------------|-------------|
| 4D-U(1) | 0.643 \(^a\) | 1.04 | 1.0111 \(^b\) |
| 3D-XY | 0.33 \(^c\) | 0.45 | 0.4539 \(^d\) |
| 2D-XY | 0.73 \(^e\) | 1.18 | 1.12 \(^f\), 1.18 \(^g\), 1.14 \(^h\) |

Table 2: The “2:1 ratio” near the transition points of the D=3 XY and D=4 U(1) lattice gauge models. It shows that the “stress” graphs of strength-2 are much more important in the Wilson than in the Villain action.

| \( W_2/W_1 \) | XY (\( \beta = 0.45 \)) | Gauge (\( \beta = 1.0 \)) |
|----------------|-----------------|-----------------|
| Wilson | 0.11 | 0.24 |
| Villain | 0.01 | 0.10 |

read off that at and above the phase transition the graphs of higher strengths proliferate much stronger using the Wilson action. Among these we expect the graphs of strength-2 to be most important. This suggests that the weight ratio \( W_2/W_1 \) should be the relevant distinguishing feature of the different actions. In the sequel, it will be called the “2:1 ratio”. In the Wilson action, the “2:1 ratio” is much larger than in the Villain action as is shown in table 2.

It is easy to convince ourselves that the higher ratios \( W_3/W_1, \ldots \) are practically irrelevant near the phase transition. For this purpose we compare the Villain action with further Monte Carlo simulations of the mixed \( \beta - \gamma \) action (see eq.(14)), which is chosen in such a way that the “2:1 ratio” coincides with that of the Villain action. The coincidence is achieved by requiring

\[
\frac{I_1(\beta, \gamma)}{I_0(\beta, \gamma)} = \exp\left(-\frac{1}{2\beta_V}\right) \quad (15)
\]

\[
\frac{I_2(\beta, \gamma)}{I_0(\beta, \gamma)} = \exp\left(-\frac{4}{2\beta_V}\right) \quad (16)
\]

where \( I_b(\beta, \gamma) \) are generalized modified Bessel functions

\[
I_b(\beta, \gamma) = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \cos(b\phi) e^{\beta \cos(\phi) + \gamma \cos(2\phi)} \quad (17)
\]

Here, it is convenient to solve eqs.(15,16) in the form \( \beta(\beta_V), \gamma(\beta_V) \) which we shall call the “Villain locus” in the \( \beta - \gamma \) plane. The comparison of our Monte Carlo data
for the internal energy of U(1)-LGT in fig.4 clearly demonstrates that the differences due to graphs of higher strength $b = \pm 3, \ldots$ (which still have different weights) are completely negligible - the data for the Villain action and the mixed $\beta - \gamma$ action (along the “Villain locus”) fall almost perfectly on top of each other.

As a conclusion, the differences between the Wilson and the Villain action near the phase transition are completely explained by the different weights of strength-2 “stress” graphs, whose proliferation is much more pronounced in the Wilson case.

That an admixture of strength-2 graphs with large enough weight can in principle drive the transition first-order, can be demonstrated easily in the disorder field theory of “stress” lines (i.e. in the mean-field formulation of the D=3 XY model with mixed action). This is described by an effective action [31]

$$\mathcal{A} = a|\phi_1|^2 + b|\phi_1|^4 + c|\phi_2|^2 - \frac{1}{2}[\phi_1^2\phi_2^* + c.c.] + \text{gradients} + \ldots$$  \hspace{1cm} (18)

where the complex fields $\phi_1, \phi_2$ represent strength-1 and strength-2 “stress” lines, respectively. The coupling $\phi_1\phi_1^\dagger$ corresponds to the merging of two lines of strength-1 into one line of strength-2. The complex conjugate coupling describes the reversed process. Integrating out the field $\phi_2$, one obtains an additional quartic term in $\phi_1$, $-\frac{1}{4c}|\phi_1|^4$, such that the total quartic term in $\phi_1$ may change sign, signalizing a first-order transition.

This explains at least qualitatively why in careful Monte Carlo studies with the Wilson action a first-order transition was reported while with the Villain action
Figure 5: The parameters $\beta$, $\gamma$ of the mixed action $\beta \cos \Theta + \gamma \cos 2\Theta$ which can be studied by means of the improved Villain approximation (“Villain locus”). The fat and dashed lines show an interpolation to the critical points found by Jersák et al.\cite{22}. The dotted line was estimated by those authors to be the locus of the Villain model in a different way from ours (which we believe to be less accurate).

Evidence for a continuous transition was claimed. Moreover, this picture is consistent with Monte Carlo studies of the mixed $\beta - \gamma$ action by Jersák et al.\cite{22} who located a TCP for slightly negative $\gamma$. Looking at fig.5, we see that our “Villain locus” indeed crosses the phase transition line in the range of second-order transitions. It would be interesting to investigate whether the TCP is connected with a universal tricritical “2:1 ratio” $\bar{W}_2/\bar{W}_1$.

The most recent Monte Carlo renormalization group (MCRG) studies are inconsistent with this nice picture. They are interpreted as evidence for a first-order transition, with both the Wilson and the Villain action. A historic summary is compiled in table 3. The latest study\cite{30} even speculates that the transition is always first-order, down to $\gamma = -\infty$. The last result would imply that in the whole parameter range of the D=4 Abelian Higgs model which is covered by the dual $\beta - \gamma$ action, the Coleman-Weinberg mechanism is indeed working. We feel, however, that even in view of this impressive list of work in table 3, the answer has not yet settled and much more high precision studies are necessary to provide the final answer.

While in D=4 the situation is still quite controversial, in D=3 dimensions the numerical evidence is clearly against the analog of the Coleman-Weinberg mechanism, as advanced by Halperin, Lubensky and Ma\cite{10}. Here, it was confirmed many times
Table 3: Summary of work on the order of the phase transition in the D=4 U(1) lattice gauge model with Wilson and Villain action and on the location of the tricritical point in the parameter space of the mixed $\beta - \gamma$ action.

| Author          | $\gamma_{TCP}$ | Wilson | Villain |
|-----------------|-----------------|--------|---------|
| Bhanot (1982)   | $> 0$           | 2      | 2       |
| Jersák et al. (1985) | $-0.11(5)$     | 1      | 2       |
| Grösch et al. (1985)  | metastability  |        |         |
| Morikawa et al. (1985) | $> 0$          | 2      | 2       |
| Gupta et al. (1986)  | $> 0$           | 2      | 2       |
| Burkitt (1986)  | $> 0$           | 2      | 2       |
| Lang (1986/87)   | $> 0$           | 2      | -       |
| Lang and Rebbi (1987) | -              | -      | 2       |
| Decker et al. (1988) | -              | 1      | -       |
| A. Hasenfratz (1988) | no             | 1      | 1       |

by different methods that both Wilson’s and Villain’s action undergo continuous transitions. The question was then whether there exists at all a parameter range in the mixed $\beta - \gamma$ action which shows first-order transitions. According to our above analysis, a good candidate was only the range $\gamma \geq 0$. We therefore concentrated on this range and found \[31\], first in a mean-field (MF) treatment, first-order transitions for $0.166 \leq \gamma \leq 0.375$. The full MF phase diagram is displayed in fig.6 (use the labelings on the right and top axis). Since already in the pure XY model ($\beta = 0$ or $\gamma = 0$ axis), the MF transition temperatures are off by a factor $0.45/0.33 = 1.36$, we have rescaled the MF curves by this factor when comparing with our Monte Carlo simulations which we run in the range where the rescaled MF results show the largest entropy jump, $\gamma \approx 0.35...0.40$. The numerical results confirmed the first-order nature of the transition. As a typical signal, we show in fig.7 the double peak structure in the internal energy histogram, corresponding to tunnelings between two metastable states. Since the observed first-order transition is very weak and the $\gamma$-range is probably very small, we were not able to locate the two tricritical points with reasonable accuracy. We can therefore only claim evidence for a short line of first-order transitions in the range $\gamma \approx 0.35...0.40$.

In defect models of melting, no TCP seems to exist. The reason is the very large activation energy of the graphs in the stress expansion so that there are no pretransitional excitations up to the point where the free energy intercepts the defect expansion. This is responsible for a jump in the slope of the free energy, i.e. for first-order transitions \[3\].
Figure 6: Phase diagram of the D=3 XY model with mixed action as obtained by mean-field methods and Monte Carlo simulations.

Figure 7: Energy histogram near the first-order transition line of the D=3 XY model with mixed action.
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