Shrewd Selection Speeds Surfing: Use Smart EXP3!

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Abstract—In this paper, we explore the use of multi-armed bandit online learning techniques to solve distributed resource selection problems. As an example, we focus on the problem of network selection. Mobile devices often have several wireless networks at their disposal. While choosing the right network is vital for good performance, a decentralized solution remains a challenge. The impressive theoretical properties of multi-armed bandit algorithms, like EXP3, suggest that it should work well for this type of problem. Yet, its real-world performance lags far behind. The main reasons are the hidden cost of switching networks and its slow rate of convergence. We propose Smart EXP3, a novel bandit-style algorithm that (a) retains the good theoretical properties of EXP3, (b) bounds the number of switches, and (c) yields significantly better performance in practice. We evaluate Smart EXP3 using simulations, controlled experiments, and real-world experiments. Results show that it stabilizes at the optimal state, achieves fairness among devices and gracefully deals with transient behaviors. In real world experiments, it can achieve 18% faster download over alternate strategies. We conclude that multi-armed bandit algorithms can play an important role in distributed resource selection problems, when practical concerns, such as switching costs and convergence time, are addressed.

I. INTRODUCTION

Mobile devices often have several wireless networks at their disposal. Choosing the right network is vital for good performance. Yet, it is non-trivial. This is, in part, because network availability is transient and the quality of networks changes dynamically due to mobility of devices and environmental factors. The conventional wisdom is to choose WiFi over cellular, and to associate with a WiFi Access Point (AP) that has the highest signal strength—which is often suboptimal [6]. The challenge is for each device to make decentralized decisions, without any coordination, and yet achieve a fair allocation, where each device gets an equal share of the available bandwidth (to the extent that it is feasible). Given that the environment is dynamic, it is harder to achieve an optimal solution. Resource selection problems can be formulated as a congestion game, and multi-armed bandit problem relates to repeated multiple-player games. Furthermore, theoretical properties of multi-armed bandit algorithms suggest that they provide an excellent solution to this problem.

EXP3 (Exponential-weight algorithm for Exploration and Exploitation) [4], one of the leading bandit algorithms, is fully decentralized and Hannan-consistent, i.e. as time elapses, it performs nearly as well as always selecting the best action in hindsight. It has been proven to converge to (weakly stable) Nash equilibrium [22], [33] while guaranteeing good performance (i.e., minimizing regret). However, we observe (via simulation) that EXP3 tends to perform worse than even simple naive greedy solutions. The main reasons for the unexpectedly poor outcomes are (a) EXP3 does not capture switching cost, which is a non-trivial cost in network selection, and (b) it has a relatively slow convergence; in some of our simulations, it took the equivalent of over 14 days to stabilize. We do not want to treat switching cost as a “loss”, from the perspective of EXP3, as this will unfairly penalize networks with high data rates and high switching cost. Moreover, while the process of exploring networks is designed to minimize regret, it does not optimize for quick convergence to a Nash equilibrium. Both of these problems are exacerbated in dynamic wireless network settings.

We formulate the wireless network selection problem as a repeated congestion game (in each round, each device chooses a network and receives some reward, i.e. bandwidth), and model the behavior of devices using online learning in the adversarial bandit setting. We propose Smart EXP3, a novel bandit-style algorithm that retains the good properties of EXP3 while addressing the issues that prevent it from achieving good performance in practice. From a theoretical perspective, we focus on the static version of the problem; in our experiments, we explore dynamic settings. There are a few key insights underlying Smart EXP3. The first observation is that we can minimize the cost of switching networks by using adaptive blocking techniques. The second observation is that we can speed up the rate of reaching a “stable state” by carefully adding initial exploration and a greedy policy. The third observation is that once the system is stable, we want to remain in a good state; we rely on a switch-back mechanism. Finally, in a dynamic setting, a careful minimal reset mechanism is needed to ensure that the system adapts efficiently to changes.

To summarize, the following are our key contributions:

1) We formulate the problem as a repeated congestion game and model the behavior of devices using online learning in the adversarial bandit setting.
2) We show that EXP3 has relatively poor performance in a dynamic wireless network setting.
3) We propose Smart EXP3, an algorithm that has good theoretical and practical performance
4) We demonstrate, empirically using simulations, controlled experiments, and in-the-wild experiments that Smart EXP3 (a) gracefully deals with transient behaviors, (b) stabilize at the optimal state relatively fast, with reduced switching and without any coordination, and (c) achieves fairness among devices.
5) We give an upper bound on the expected number of network switches and prove that Smart EXP3 has the same convergence and regret properties as EXP3.

A major goal of this paper is to discover how to make bandit-style algorithms (like EXP3) more effective in practice, without compromising on theoretical properties, by focusing on important issues of switching cost, time to stabilize, and adaptation to transient behaviors.
II. WIRELESS NETWORK SELECTION

In this section, we describe the wireless network selection problem, and formulate it as a repeated congestion game.

A. Wireless network selection problem

We consider a collection of mobile devices operating in an environment with heterogeneous networks. For example, Figure 1 depicts mobile devices operating in three service areas (shaded areas A, B and C) with several wireless networks. The wireless networks are numbered from 1 to 5 and the dotted lines delimit their coverage. Different devices have access to different networks, e.g. devices at the food court will see the cellular network and WLANs 2 and 3. The goal is to connect each device to the best network, which may vary over time.

![Wireless network selection diagram](image)

Fig. 1: Service areas with heterogeneous wireless networks.

Three criteria are important when selecting a network: (a) the quality of the connection, which is influenced by the distance between a device and the AP, or the level of external interference; (b) the bandwidth of the network; and (c) the level of congestion, e.g. the number of devices sharing the network. While this information is not available to a device at the time of selection, the achievable data rates can be estimated by exploring the networks. Every time a device switches network, it incurs a cost, which we assume is measured in terms of delay, and sacrifices some available bandwidth.

B. Formulation of wireless network selection game

Since mobile devices operate in a dynamic environment, continuous exploration and adaptation are required. Wireless network selection can be formulated as a repeated resource selection game, a special type of congestion game [30].

We formally define the wireless network selection game as a tuple $\Gamma = (N, K, (S_j)_{j \in N}, (U_k^t)_{k \in K})$, where

1) $N = \{1 \cdots n\}$ is the finite set of $n$ active mobile devices indexed by $j$.
2) $K = \{1 \cdots k\}$ denotes the finite set of $k$ wireless networks available.
3) $S_j \subseteq K$ is the strategy set of mobile device $j$, i.e. the set of wireless networks available to it.
4) Gain (payoff or utility) $g_k(t)$ of mobile device $j$ refers to the bit rate observed by $j$ when selecting network $k$ at time $t$; it is given by a function of the number of devices $n_k(t)$ associated with $k$ as follows:

   $n_k(t) = \{|i \in N : x_i = k\}$

   where $x_j$ is the network selected by $j$ at time $t$.

   $g_k(t) = U_k^t(n_k(t))$

A device’s gain affects its strategy and, hence, ignores switching cost so that networks with high gain but high switching cost are not penalized.

5) Cumulative goodput of a device $j$ is given by

$$\sum_{t=1}^{T} U_k^t(n_x(t)) \cdot (\text{slot_duration} - \text{delay})$$

where delay is the switching cost, slot_duration (higher than delay) is the length of a time slot (assuming time is slotted), and $T$ is the time horizon.

6) A strategy profile is given by $S = S_1 \times \cdots \times S_n$. It is at Nash equilibrium if $g_x(S) \geq g_x(S_{-j}, S'_j)$ for every $S'_j$ and every $j \in N$, where $(S_{-j}, S'_j)$ implies that only device $j$ changes its strategy. Hence, no device wants to unilaterally change its strategy.

The wireless network selection problem is related to the adversarial bandit problem [4], in which a gambler must select a slot machine to play in a sequence of trials to maximize the cumulative reward. In our case, the aim of each device $j$ is to maximize its cumulative goodput by quickly identifying and connecting to the best network. The performance of a network degrades proportionally to the number of devices supported; other mobile devices accessing shared networks may be considered adversaries. We model the behavior of devices using online learning in the adversarial bandit setting, where EXP3 [4] is a standard algorithmic solution. The only information available to a device is its set of available networks.

III. SMART EXP3

In this section, we develop Smart EXP3, a distributed wireless network selection algorithm, by diligently modifying EXP3 [4] so as to retain its good properties while compensating for its shortcomings. It runs independently on each mobile device. Yet, it affects the choice of other devices that have a common set of available networks (by affecting their gains).

EXP3. We briefly explain how EXP3 [4] works. It maintains a weight for each network, which represents the confidence that the network is a good choice. Initially, a device assumes uniform weight over all networks. The weight of a network is affected by the gain (bit rate) the device observes by associating with it; a higher gain implies higher weight. EXP3 assumes that time is slotted. At each time slot, it selects a network randomly from a probability distribution, that mixes between using the weights and a uniform distribution; the latter ensures that EXP3 keeps exploring occasionally and discovers a better network that was previously “bad”. The best network will eventually gain higher weight and be selected most often.

Time and blocks. Each device partitions time into blocks, and selects a network to associate with for the entire block. Each block consists of a sequence of time slots of equal length. The duration of a time slot is long enough for a device to observe the gain. The block length used by a device grows over time and is given by $\lceil(1 + \beta)^x\rceil$, where $\beta \in (0, 1)$ and $x$ is the number of times the network has been selected by that device. This ensures that more time is spent in the optimal network, which is eventually selected more frequently. Every so often and upon significant decline in network quality, block lengths are reset for better adaptation. The use of blocks reduces
switching cost [3, 13, 23] and improves performance by de-synchronizing the selection time of devices.

**Algorithm description.** Algorithm 1 outlines the major steps in Smart EXP3, excluding the parts on reset and updates made when a change in the set of available networks is detected. See Table I for notations. We defer explanation on switch back.

Much like EXP3, Smart EXP3 assigns a weight to each network. At the beginning of a block, the probability distribution is updated based on the weights of the networks. The same multiplicative weight update and probability update rules as for EXP3 [4] are used. Smart EXP3 then selects a network \( i_b \) to associate with during the whole block. In the first \( k \) blocks, it explores the networks in random order, and \( \overline{p}(b) = \frac{1}{|\text{explore\_network}|} \). This improves the learning rate. From block \( k + 1 \) onward, it either selects randomly based on its probability distribution or considers the use of a greedy approach. In the prior case, \( \overline{p}(b) = \hat{p}_i(b) \). The mobile device observes a gain during the entire block, which is used to update the network’s weight at the end of the block. The estimated gain \( \hat{g}_i(b) \) in the weight update rule compensates for a potentially small probability of observing the gain.

**Algorithm 1: Smart EXP3**

Shows the major steps in the algorithm, leaving out the parts on (1) reset, and (2) updates made when a change in the set of available networks is detected. chooseGreedily() determines whether “greedy” selection can be leveraged; a device selects greedily with probability \( \frac{1}{2} \) at the beginning of an execution, or for some time after a reset.

Input : \( k \in \mathbb{Z}_{\geq 0} \), real \( \gamma \in (0, 1] \), real \( \beta \in (0, 1] \\
Initialze: \( w_i(1) \leftarrow 1 \) for \( i = 1, \ldots, k \), \( \text{explore\_network} \leftarrow \mathcal{K} \)
1. foreach block \( b = 1, 2, \cdots \) do
2. \( p_i(b) \leftarrow (1 - \gamma) \frac{w_i(b)}{\sum_{j=1}^{k} w_j(b)} + \frac{\gamma}{k} \) for \( i = 1, \ldots, k \)
3. if \( \text{explore\_network} \neq \emptyset \) then
4. \( i_b \leftarrow \text{random from } \text{explore\_network} \)
5. \( \text{explore\_network} \leftarrow \text{explore\_network} \setminus \{i_b\} \)
6. else if chooseGreedily() = True then
7. \( i_b \leftarrow \text{network with highest average gain} \)
8. else \( i_b \leftarrow \text{random according to } p(b) \)
9. \( l_{i_b} = \lceil (1 + \beta)x_{i_b} \rceil \)
\( \% \) execute block for \( l_{i_b} \) time slots. \( \% \) at the second time slot, switch back to the previous network if the current one is worse, and start a new block.
10. \( g_{i_b}(b) \leftarrow \text{gain observed, where } g_{i_b}(b) \in [0, l_{i_b}] \)
11. \( \hat{g}_{i_b}(b) \leftarrow \frac{g_{i_b}(b)}{\overline{p}(b)} \)
12. \( w_{i_b}(b + 1) \leftarrow w_{i_b}(b) \exp\left(\frac{\gamma\hat{g}_{i_b}(b)}{k}\right) \)

Greedy choices. At the beginning of an execution, or for some time after a reset, the mobile device flips an unbiased coin and decides (with equal probability) to use either a greedy or a random strategy. In the prior case, it selects the seemingly “best” network, i.e., the network from which the highest average gain has been observed. Then, \( \overline{p}(b) = \frac{1}{2} \). If the device decides to choose randomly, \( \overline{p}(b) = \hat{p}_i(b) \). An aggressive use of greedy selection generally leads to low efficiency in social welfare. However, allowing half the devices to choose greedily, at first, causes them to perturb the weight of their perceived “best” network and allows other devices to explore and adapt. Empirical results show that this drastically improves the rate of which the algorithm stabilizes.

Switching back. If a device switches network when the algorithm is at Nash equilibrium, it will observe a lower gain. Based on this intuition, if a device observes a worse performance during the first time slot of a block, it starts a special block at the next time slot. In that block, the mobile device simply associates to its previous network rather than executing lines 3 - 8 of Algorithm 1 Here, \( \overline{p}(b) = 1 \). Smart EXP3 does not allow a device to switch back in two consecutive blocks to prevent a ping-pong effect. The switch back mechanism reduces the time spent in a bad network (restricts it to a block of a single time slot), and prevents other devices from reacting. Empirical results show that this mechanism makes Smart EXP3 much more stable.

Resetting. Smart EXP3 resets every so often, and when it detects a significant drop in the network being selected for consecutive time slots. For instance, when the algorithm favors one particular network with sufficiently high probability and stays in that network for a long time, it becomes less adaptive to changes in the environment. It might take an unacceptable amount of time to discover new resources available when some devices leave the service area. Hence, Smart EXP3 periodically resets network block lengths, and details stored for use during greedy selection. It then forces exploration of available networks. As such, reset is minimal to allow the algorithm to adapt without forsaking everything it has learned. The duration between two resets is referred to as a reset period.

Change in set of networks. When a device discovers a new network, its weight is set to the maximum weight of the other networks or 1 if all networks are newly discovered; then, the algorithm resets. In addition, the algorithm resets when a network favored with significantly high probability is no longer available. These ensure that a newly discovered network is likely to be explored and the algorithm adapts quickly to the change. If the network to which the device was connected is no longer available, Smart EXP3 resets the block.

\[^{1}\text{It depends on the type of selection made, i.e. whether it was an initial exploration, a random choice, a greedy selection, or a switch back.}\]
IV. THEORETICAL ANALYSIS OF SMART EXP3

Due to the changes we have made to EXP3, it is not immediately apparent that Smart EXP3 has the same convergence and regret properties as EXP3. Here, we prove that it does and give an upper bound on its switching cost.

The duration of a time slot is denoted by \( t_d \) and a reset period by \( \tau \). For the purpose of the analysis, we assume that \( S_j = \mathcal{K} \) for every \( j \in \mathcal{N} \), i.e. all mobile devices have the same strategy set.

**Convergence.** We consider Smart EXP3 without reset and prove that it retains the convergence property of EXP3.

**Theorem 1:** When \( \gamma \) is arbitrarily small, the strategy profile of all devices using Smart EXP3 converges to a weakly stable Nash equilibrium \([22]\); almost everywhere weakly stable Nash equilibrium is a pure Nash equilibrium of the congestion game as defined in \([22], [33]\).

Hence, when all devices leverage Smart EXP3, they end up being optimally distributed across networks. No device will observe higher gain by unilaterally switching network. Although, it is not conveyed by the analysis, empirical results show that Smart EXP3 reaches a stable state (defined in section VI-A) 3.3x faster than EXP3 in some setting considered.

The formal proof is provided in appendix A.

**Bound on number of network switches.** We now bound the number of network switches.

**Theorem 2:** For any \( k > 0 \), \( \beta \in (0, 1] \), time slot duration \( t_d \in \mathbb{Z}_{\geq 0} \), reset period \( \tau > 0 \), and stopping time \( T > 0 \), the expected number of network switches over time \( T \) is upper bounded as:

\[
E[S(T)] < \frac{T}{\tau} \left(3 \frac{k \log(\frac{T}{\tau} + 1)}{\log(1 + \beta)}\right)
\]

The logarithmic bound implies that the number of switches reduces over time.

Assuming \( t_d = 1 \) and \( \tau = T \) (i.e. there is no reset),

\[
E[S(T)] < \frac{3k \log(T + 1)}{\log(1 + \beta)}
\]

It implies that longer time horizon \( T \), and higher number of wireless networks (to explore) increase the number of switches. Faster growth of block size (controlled by \( \beta \)) will reduce the number of switches.

Referring to Theorem 2, we also infer that a higher delay (switching cost) implies longer time slots, and hence, reduced number of switches. Longer reset periods will also reduce the number of switches, as the latter decreases over time in a reset period. Empirical results show a drastic reduction in the number of network switches compared to that of EXP3.

The formal proof is given in appendix B.

**Regret bounds.** We define weak regret as follows:

**Definition 1:** Weak regret. It refers to the difference between the cumulative goodput (capturing switching cost) achieved by always selecting the best network in hindsight and that of Smart EXP3.

We prove that Smart EXP3 retains the logarithmic weak regret property of EXP3. Let \( G_{\text{Smart EXP3}}(T) \) denote the cumulative gain of Smart EXP3 at \( T \), \( G_{\text{max}}(T) \) be the cumulative gain at \( T \) when always choosing the best network in hindsight, \( \mu_d \) be the mean delay observed, and \( \mu_g \) denote mean bit rate observed.

**Theorem 3:** For any \( k > 0 \), any fixed \( \gamma \in (0, 1] \), any \( \beta \in (0, 1] \), any assignment of rewards, stopping time \( T > 0 \), time slot duration \( t_d \in \mathbb{Z}_{\geq 0} \), reset period \( \tau > 0 \), the highest block length \( l \), mean delay \( \mu_d \geq 0 \), and mean gain \( \mu_g \geq 0 \), the expected weak regret is upper bounded as:

\[
E[R(T)] \leq \frac{T \cdot t_d}{\tau} \left((1 + \gamma l (e - 2)) G_{\text{max}}(\tau) + \frac{k \ln k}{\gamma}\right) + \frac{T \cdot \mu_d \cdot \mu_g}{\tau} \left(3 \frac{k \log(T + 1)}{\log(1 + \beta)}\right)
\]

Hence, Smart EXP3 is Hannan-consistent as its weak regret tends to zero. As time elapses, it performs nearly as well as always selecting the best network in hindsight.

Assuming \( t_d = 1 \) and \( \tau = T \) (i.e. there is no reset),

\[
E[R(T)] \leq (1 + \gamma l (e - 2)) G_{\text{max}}(T) + \frac{k \ln k}{\gamma} + \frac{T \cdot \mu_d \cdot \mu_g}{\tau} \left(3 \frac{k \log(T + 1)}{\log(1 + \beta)}\right)
\]

The first term implies that: (a) if the cumulative goodput achieved by always choosing the best network is high, the regret can be high (if the goodput of Smart EXP3 is low); in that case, having long blocks, increases the regret (which would imply Smart EXP3 is staying in a bad network for a long duration; but this is not seen in our evaluations), and (b) weak regret grows with an increase in number of networks (exploring sub-optimal networks). The second term implies that weak regret increases with a rise in (a) number of network switches, (b) mean delay observed, and (c) mean bit rate observed.

Referring to Theorem 3, we also infer that long time slot duration yields an increase in delay, as more time is spent in sub-optimal networks. Longer reset periods will reduce regret as the latter decreases over time in a reset period.

The formal proof is provided in appendix C.

V. IMPLEMENTATION DETAILS

We thoroughly evaluate Smart EXP3 and compare its performance against those of several other algorithms, through simulation and experiments. All algorithms are implemented in Python, using SimPy [31] for simulation. In this section, we discuss the implementation of Smart EXP3 focusing on the greedy, switch back and reset mechanisms. We discuss the parameter values chosen for the simulation and experiments. \( p \) denotes the probability distribution, \( i_{\text{max}} \) refers to the network with the highest probability, and \( i_{\text{max}} \) denotes the network selected for the highest number of time slots.

\[\text{This is true with very high probability when the parameters of the congestion game are chosen at random.}\]
Parameter choice. In our implementation, $\gamma = b^{\gamma}$, which tends to zero to ensure convergence [25]; $\beta = 0.1$ such that blocks are short during exploration; and the duration of one time slot is 15 seconds (simulated seconds for simulation).

Greedy choices. Smart EXP3 considers the use of greedy when: (a) $\max(p) - \min(p) \leq \frac{l_{i+}}{y}$, given that it starts with a uniform probability, or (b) $l_{i+} < y$, where $y$ is the value of $l_{i+}$ when condition (a) evaluates to false for the first time. The second condition allows for the use of greedy after a reset. Based on empirical results, these are good choices. When either of these conditions evaluates to true, the device selects greedily with probability $\frac{1}{2}$ (flipping an unbiased coin).

Switch back. A device switches back if (a) the gain from the current network is worse than the average gain observed in the preceding block or during its last time slot, or if more than 50% of the time, a higher gain was observed in the preceding block, and (b) the algorithm did not switch back at the beginning of the current block (to prevent a ping-pong effect). To ignore stale data, the decision is based on observations from only the last 8 time slots of the previous block.

Resetting. The algorithm resets when $p_{i+} \geq 0.75$ and $l_{i+} \geq 40$, i.e. the algorithm stays for a long duration in the network favored with sufficiently high probability. This allows for discovery of resources that have recently been freed. It also resets if a drop of at least 15% is observed in $p_{i+}$, where it starts with $p_{i+} = 1$. It assigns a weight to each network. At each time slot, it explores networks that have recently been freed. It also resets if a drop of at least 15% is observed in $p_{i+}$, where it starts with $p_{i+} = 1$. It assigns a weight to each network. At each time slot, it explores networks that have recently been freed.

VI. EVALUATION THROUGH SIMULATION

This section shows that EXP3 has poor performance in a dynamic wireless network setting. It then evaluates Smart EXP3, relying on simulations using synthetic data (Section VI-A), and trace-driven simulations (Section VI-B).

A. Simulation using synthetic data

In this section, we show that EXP3 incurs high switching cost, has slow convergence, and fails to adapt to changes in the environment. In contrast, Smart EXP3 (a) stabilizes at Nash equilibrium with reduced switching, (b) better utilizes available resources, (c) achieves fairness among devices, (d) scales with an increase in number of devices and networks, (e) adapts to changes in the environment, and (f) is robust against “greedy” devices. It outperforms alternative selection algorithms given in Table [11]. The performance of algorithms in Table [11] is also discussed to highlight benefits of key features of Smart EXP3.

Setup. We consider two settings of 20 devices and 3 networks, with an aggregate bandwidth of 33 Mbps. Setting 1 assumes non-uniform data rates 4, 7 and 22 Mbps, a factor close to the theoretical data rates of IEEE 802.11 standards [15] and cellular networks [18] that yields a unique Nash equilibrium. In Setting 2, the networks have a uniform data rate (11 Mbps each). Delay is modeled using Johnson’s SU distribution for WiFi and Student’s t-distribution for cellular, each identified as a best fit [14] to 500 delay values. We make the following assumptions (which are not pre-requirements for the algorithm)

as a best fit [14] to 500 delay values. We make the following assumptions (which are not pre-requirements for the algorithm)

\footnote{As a baseline, we include a Full information and a Centralized protocol; these assume the availability of global knowledge, even though it can not be implemented without coordination among devices or via a base station.}

Table II: Algorithms to which Smart EXP3 is compared

| Algorithm          | Description                                                                 |
|--------------------|-----------------------------------------------------------------------------|
| Full Information   | It assigns a weight to each network. At each time slot, it selects a network at random based on their weights. At the end of a time slot, the device receives feedback about the gain it could obtain from each network, and computes the loss of each of them. The weight of each network is updated based on their loss, using a multiplicative update rule [20]. |
| Greedy             | It starts by exploring each network in random order. Then, at each time slot, selects a network from which the highest average gain has been observed. |
| Centralized        | It is optimal (maintains Nash equilibrium) and assumes that a centralized entity allocates devices to the right network. |
| Fixed Random       | It picks a network at random and stays in that network. |

Table III: Algorithms highlighting features of Smart EXP3

| Algorithm                      | Description                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|
| Block EXP3                     | It is a version of EXP3 that selects a network for a block.                  |
| Hybrid Block EXP3              | It is a version of Block EXP3 that starts by selecting greedily with a probability $\frac{1}{2}$ as Smart EXP3. |
| Smart EXP3 w/o Reset           | It is a version of Smart EXP3 that never resets. |

in the simulation: (1) a network’s bandwidth is equally shared among its clients, and (2) clients are time-synchronized. Results involve data from 500 runs of 5 (simulated) hours each, i.e. 1200 time slots, unless specified otherwise.

Switching cost. Figure 2 shows that EXP3 and Full Information incur high number of network switches. Block-based algorithms experience around 80% lower switching cost, and lower variance, in both settings. The costs of Hybrid Block EXP3 and Smart EXP3 w/o Reset are lower than that of Block EXP3 as their greedy policy helps them become stable faster, as discussed later. Thus, block lengths increase faster. The cost of Smart EXP3 increases with resets, but is acceptable. As discussed later, reset promotes faster adaptation to changes in network conditions. Greedy may incur high cost in setting 2, where 8 devices switched networks more than 83.3% of time. Centralized and Fixed Random approaches have no cost.

![Fig. 2: Average number of network switches incurred by each algorithm in both settings (error bar shows standard deviation); Centralized and Fixed Random do not switch network.](image-url)

Stability and distance to Nash equilibrium. We define the notion of stable state to evaluate the algorithms’ performance.

Definition 2: Stable state An algorithm is said to have reached a stable state when all devices favor a network with sufficiently high probability (we assume $\geq 0.75$), and they
favor that same network until the end.

EXP3 and Full Information never reached a stable state in our simulation, due to frequent switching. Figure 3 shows that more than 40% of Block EXP3 runs stabilize, but rarely at Nash equilibrium. As given in Table IV, it takes very long to reach the stable state. The greedy policy in Hybrid Block EXP3 significantly improves the rate at which the algorithm stabilizes. The switch back mechanism retains Smart EXP3 w/o Reset in the optimal state, leading to 99.4% and 100% runs being stable at Nash equilibrium in settings 1 and 2, respectively, at a faster rate. As setting 2 has three Nash equilibria with an equal distribution of devices over networks, the algorithms perform better (their initial distribution is uniform).

![Fig. 3: Percentage run that reached a stable state and type of stable state (Nash equilibrium or some other state).](image)

**TABLE IV: Median no. of time slots taken to reach a stable state (whether Nash equilibrium or some other state).**

|             | Block EXP3 | Hybrid Block EXP3 | Smart EXP3 w/o Reset |
|-------------|------------|-------------------|---------------------|
| Setting 1   | 1026       | 583.5             | 339                 |
| Setting 2   | 810        | 366               | 244.5               |

Some algorithms cannot, by definition, be evaluated based on the notion of stable state, e.g., Greedy, Centralized, Fixed Random, and Smart EXP3 (due to resets). Thus, we define distance to Nash equilibrium as a common evaluation criterion.

**Definition 3: Distance to Nash equilibrium.** In line with the definition of \( \epsilon - \text{equilibrium} \), it refers to the maximum percentage higher gain a device can observe at Nash equilibrium, compared to its current gain.

As an example, we consider a setting with three mobile devices and two wireless networks. Assume that the three devices observe bit rates 1 Mbps, 1 Mbps, and 4 Mbps. At Nash equilibrium, they would each observe 2 Mbps. Compared to their current gains, two devices would observe 100% higher bit rate while the third one would observe a lower bit rate. The distance to Nash equilibrium is then considered to be 100%.

![Fig. 4: Average distance to Nash equilibrium (% higher gain a device can observe) across time slots (15 seconds each).](image)

(b) Setting 2 (legend is the same as that of Figure 4a). EXP3, Full information and Fixed Random maintains a distance close to 40%.

*Fig. 4: Average distance to Nash equilibrium (% higher gain a device would have observed, compared to its current gain, if the algorithm was at Nash equilibrium) — shaded region represents \( \epsilon \)-equilibrium, where \( \epsilon = 7.5 \).*

is vital for fast adaptation in a dynamic setting, as we shall see later. It also occasionally drifts away from the optimal state, shown as fluctuations, but is forced to return by the switch back mechanism. It spends 62.77% and 74.30% time at Nash equilibrium in settings 1 and 2, respectively, and is at \( \epsilon \)-equilibrium most of the time, when \( \epsilon = 7.5 \). Figure 4 shows that distances in setting 2 are lower, as expected.

**Unutilized resources.** In each setting, with aggregate bandwidth of 33 Mbps, the total bandwidth available over 1200 time slots (15 seconds each) is 74.25 GB. As Greedy starts by exploring available networks in a random order, it is highly likely that \( \frac{2}{3} \) of the devices will be associated with each network.
Cumulative download and fairness. The number of network switches and state at which an algorithm stabilizes affect its cumulative goodput. Table V shows that the block-based algorithms achieve higher cumulative gain, on average. Greedy has lower performance than Smart EXP3 in setting 1 but comparable performance in setting 2, as expected. Fixed Random also achieves comparable performance in setting 2.

We define the notion of fairness.

Definition 4: Fairness. An allocation is fair if each device gets an equal share of the available bandwidth (to the extent that it is feasible).

Figure 5 shows that EXP3, Smart EXP3 and Full Information are fairer among the algorithms. The standard deviations of Smart EXP3 are 80% and 55% less than that of Greedy in settings 1 and 2, respectively. While Nash equilibrium may not be fair, periodic reset can lead to fairness if devices converge to a different network after a reset. While Smart EXP3 incurs higher switches compared to Greedy, it is worth spending time exploring to achieve higher and fairer cumulative download.

Table V: (Mean) per run median cumulative download (GB).

| Algorithm                  | Setting 1 | Setting 2 |
|----------------------------|-----------|-----------|
| EXP3                       | 2.89      | 2.73      |
| Block EXP3                 | 3.54      | 3.65      |
| Hybrid Block EXP3          | 3.41      | 3.58      |
| Smart EXP3 w/o Reset       | 3.53      | 3.55      |
| Smart EXP3                 | 3.53      | 3.62      |
| Greedy                     | 3.12      | 3.62      |
| Full Information           | 2.92      | 2.71      |
| Centralized                | 3.54      | 3.54      |
| Fixed Random               | 2.56      | 3.43      |

Scalability. Scalability is evaluated in terms of the rate at which an algorithm reaches a stable state. Since Smart EXP3 cannot be evaluated based on this concept, Smart EXP3 w/o Reset is considered here. The algorithm was run 500 times, for 8640 time slots each, with different number of devices and networks. Figure 6 shows the median number of time slots taken to stabilize, in each setting. The rate increases linearly with an increase in number of networks and sub-linearly with an increase in number of devices. Furthermore, Smart EXP3 w/o Reset was stable at Nash equilibrium 100% (or nearly 100%) of times in each of the settings considered.

Adaptability to changes in the environment. So far, we have seen that Greedy performs better compared to Full Information and Fixed Random. Thus, we only evaluate the performance of EXP3, Smart EXP3, Smart EXP3 w/o Reset and Greedy in 3 dynamic settings, with 20 devices each.

In settings 1 and 2, all devices see 3 networks with bandwidth 4, 7 and 22 Mbps. In setting 1, 9 devices join at the beginning of time slot 600 and leave at the end of time slot 800, while the others are always in the service area. Figure 7 shows that only Smart EXP3 and Smart EXP3 w/o Reset are able to adapt to these changes. Their distances increase when the 9 devices join and begin exploring, but they eventually move (at least very close) to the optimal state. In setting 2, 16 devices leave at the end of time slot 600, freeing resources. Figure 8 shows that only Smart EXP3 is able to discover the resources and adapt accordingly, highlighting the importance of reset.
Fig. 8: Average distance to Nash equilibrium (% higher gain a device would have observed, compared to its current gain, if the algorithm was at Nash equilibrium) — shaded region represents $\epsilon$-equilibrium, where $\epsilon = 7.5$; 16 devices leave at the end of $t = 600$.

Fig. 9: Average distance to Nash equilibrium (% higher gain a device would have observed, compared to its current gain, if the algorithm was at Nash equilibrium) — shaded region represents $\epsilon$-equilibrium, where $\epsilon = 7.5$; 8 devices moving.

Bus stop. 8 devices from the food court move to the study area at the beginning of $t = 401$ and eventually reach the bus stop at the start of $t = 801$. Figure 9 illustrates the performance of the algorithms for devices in each service area, and those moving across areas separately. Smart EXP3 outperforms all the other algorithms for each category of devices and moves to at least $\epsilon -\text{equilibrium}$, when $\epsilon = 7.5$.

Figure 10 shows that the number of network switches incurred by devices in a static and dynamic setting are comparable. Devices which are moving are likely to incur higher number of resets (median of 3 compared to median of 2 for temporarily stationary devices in our case), hence higher number of switches. This is because Smart EXP3 resets when it discovers new networks and when a device’s preferred network is no longer available.

Robustness against “greedy” devices. The performance of Smart EXP3 is evaluated in a setting, with 20 devices and 3 networks, where some devices use Greedy. In scenario 1, a single device uses Greedy while the others use Smart EXP3. In scenario 2, 10 devices employ each of the selection algorithms. And, in scenario 3 a single device uses Smart EXP3 while the others use Greedy. Figure 11 shows that, while Greedy is able to achieve good results in scenarios 1 and 2, it yields poor performance when the number of “greedy” users increase in scenario 3. On the other hand, Smart EXP3 performs well in all three scenarios.

B. Trace-driven simulation

Results from VI-A show that Greedy performs better among alternative approaches. Hence, we evaluate the performance of Smart EXP3, in comparison to that of Greedy only, based on network traces. We collected traces of a public WiFi network and a cellular network by downloading a file from a remote server [24] on both networks simultaneously and measuring their bit rates. We evaluate the algorithms on 4 network traces, of 25 minutes each. The bit rates fluctuate, especially for the cellular network, although cellular network is always better than WiFi in trace 2. Results presented, from 500 simulation runs, show that Smart EXP3 adapts to changing network conditions and achieves higher cumulative goodput.

Table VI gives the total download and switching cost incurred (in terms of download that could be achieved if there
TABLE VI: Median of cumulative download (MB) and total switching cost (MB) incurred by Smart EXP3 and greedy.

| Trace | Smart EXP3 Download (MB) | Cost (MB) | Greedy Download (MB) | Cost (MB) |
|-------|--------------------------|-----------|----------------------|-----------|
| Trace 1 | 764.16 | 39.74 | 671.07 | 3.05 |
| Trace 2 | 1188.56 | 32.48 | 1235.92 | 6.14 |
| Trace 3 | 657.81 | 44.11 | 428.47 | 2.96 |
| Trace 4 | 810.67 | 51.11 | 757.66 | 4.50 |

Figure 12 illustrates one run of Smart EXP3 on traces 1 and 3, showing how it adapts to changes in network conditions.

Fig. 12: Two pairs of simultaneous traces of a public WiFi and a cellular network, and an illustration of the network section process by Smart EXP3 at every time slot (shown as bit rate observed) - a random run with approximately the same cumulative download as the median cumulative download.

VII. EVALUATION THROUGH EXPERIMENTS

This section evaluates Smart EXP3 based on controlled experiments (Section VII-A) to see how it works in a real world setting where we still have some control over the network bandwidth and the number of devices, and experiments in the wild (Section VII-B), involving public networks. Since Greedy performs better among alternative approaches, we only consider Smart EXP3 and Greedy.

A. Controlled experiments

In this section, we that Smart EXP3 outperforms Greedy, in terms of cumulative goodput, resource utilization and adaptability to changes in network conditions in real world settings.

Setup. It consists of (a) 3 WiFi APs simulating 2.4GHz networks with bandwidth set to 4, 7 and 22 Mbps; (b) 2 laptops, each running a TCP server that continuously sends data to its clients (a request is sent to an alternate server when one fails to respond); (c) 14 raspberry pis that act as clients; and (d) a main AP that connects the servers and 3 WiFi APs through LAN cables. Devices run Smart EXP3 or Greedy and receive data from the server. They are synchronized, with drift of less than one second. Switching networks is implemented by closing and establishing new network and TCP connections. Gain is estimated based on the download during the time spent in a network. Results are based on 10 runs of 2 hours each, i.e. 480 time slots of 15 seconds.

Switching cost, download and resource utilization. As expected, Smart EXP3 incurs a higher number of switches (median of 73.5) compared to Greedy (median of 3). However, this enables it to explore and eventually achieve higher and fairer cumulative download, as shown in Table VII. Given the real world challenges, it also incurs a higher number of switches (median of 73.5 in 2 hours compared to 61 in 5 simulated hours) and resets (median of 5 in 2 hours compared to 2 in 5 simulated hours) than in simulation. Results show that it utilizes resources better than Greedy, which incurs a mean loss of 3.74% of the aggregate resources.

TABLE VII: Per run cumulative download (as a % of the estimated total download possible).

| Cumulative download (%) | (Average) median | (Average) standard deviation |
|-------------------------|-----------------|-----------------------------|
| Smart EXP3              | 6.89            | 1.55                        |
| Greedy                  | 6.29            | 2.87                        |

Distance from average bit rate available. Given that network bit rates fluctuate due to factors such as interference and packet loss. A device may not observe an equal share of a network’s bandwidth. As such, the notions of Nash equilibrium and stable state are hard to apply. Hence we define the notion of distance from average bit rate available.

Definition 5: Distance from average bit rate available. We compute the average bandwidth available for each device, based on bit rates observed by the devices. We then take its mean difference to lower bit rates observed by the devices. We normalize the result and refer to it as the distance from average bit rate available.

Figure 13 shows that the distance for Greedy gradually increases as the bit rates observed by some of the devices go down for some reason and the algorithm fails to adapt. The distance for Smart EXP3 eventually drops as the devices explore, learn and adapt, hence switching to a better network. However, noise in the real world perturbs the accuracy of the estimate of network quality and leads to a higher number of resets, preventing the distance from dropping any further.

Adaptability to changes in the environment. A dynamic setting is considered in which 9 devices leave at the end of time slot \( t = 240 \), i.e. after 1 hour. Figure 14 shows that both algorithms exhibit similar behaviors as in the static setting in the first 240 time slots. When the devices leave at \( t = 240 \), resources are freed. The distance of Smart EXP3 rises at that time slot. But, given that Smart EXP3 continuously explores its environment, it is able to eventually discover the new resources and adapt accordingly. On the other hand, Greedy fails to do so yielding a sudden increase in its distance.
Robustness against “greedy” devices. We consider a setting in which 7 devices use Smart EXP3 and 7 devices use Greedy. Figure 15 shows that, on average, those who leverage Smart EXP3 experience lower distance to the average bandwidth available, hence higher gain, given that it learns continuously and adapts to changes in its environment. On the other hand, Greedy may get stuck in the wrong network even if it experiences a drop in its gain, which may be different from the gain of other devices sharing the same network. While simulation shows that 50% of “greedy” devices in the environment succeed in performing well, it is not true in a real-world setting (as shown in the experiments).

B. Experiments in the wild

We evaluate the performance of Smart EXP3, in comparison to Greedy, through experiments in the wild and observe that Smart EXP3 achieves higher cumulative goodput (faster download). The number of devices and their selection approaches, and the bandwidth limit of available networks are unknown. The mobility of devices entering and leaving the service area are not controlled.

Smart EXP3 and Greedy were run sequentially on a laptop. A selection had to be made between 2 wireless networks, namely a public WiFi network and a cellular network. The laptop was equipped with a built-in WiFi interface and could connect to the cellular network through a tethered phone. The load of the 2 networks, monitored using Wireshark and by capturing the EcIo values from the mobile phone, varied during the experiments. The aim was to download a 500MB file, while connecting to the optimal network and optimizing on download time. Results from 12 runs of each algorithm show that Smart EXP3 could achieve 1.2x faster download, on average, compared to Greedy. Greedy took 15.67 minutes, on average, to download the file while Smart EXP3 took 12.90 minutes, on average, i.e. is about 18% faster.

VIII. OTHER RELATED WORK

We discuss state-of-art wireless network selection approaches and relevant work done on bandit algorithms.

A significant amount of work leverages the use of multiple wireless networks, such as Multinet, MPTCP, Coolspot, and others have been proposed to solve the wireless network selection problem. However, they are not scalable and are limited to managed networks. Several distributed solutions have been presented, but they all have some limitations. Some require coordination from APs or cooperation of peers. Others assume global knowledge, or availability of some information. In the problem is formulated as a continuous-time multi-armed bandit, but in a stochastic setting.

Multi-armed bandit algorithms were initially designed to solve a single-player problem. The adversarial bandit problem, where an adversary determines the payoff for each arm, can be easily related to a repeated game. While EXP3 ignores switching cost, the concept of updating in a block manner has been proposed to take into account switching cost. Multi-armed bandit techniques have also been applied to other resource selection problems, such as channel selection, selection of appropriate sensors to query in a sensor network, and selection of the appropriate replica server for content distribution networks. While switching network has a non-trivial cost, the notion of switching cost does not apply to the latter two problems. Channel selection approaches do not consider switching cost or require coordination of peers.

IX. CONCLUSION

We have presented Smart EXP3, a novel bandit-style algorithm that has good theoretical and practical performance. We prove that it has the same convergence and regret properties as EXP3. We evaluate its performance in dynamic wireless network settings. Empirical results show that it outperforms alternative selection approaches. It stabilizes at the optimal
state with reduced switching and without any coordination, gracefully deals with transient behaviors, and achieves fairness among devices. As future work, we intend to consider other selection criteria, such as application requirements, energy constraints and monetary cost, and evaluate the algorithm for other resource selection problems, e.g. WiFi channel selection.

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APPENDIX A

PROOF OF CONVERGENCE

We assume the version of Smart EXP3 without reset and show, following the steps in [33], that it retains the convergence property of EXP3.

From algorithm [1]

\[ w_i(b + 1) = w_i(b) \exp \left( \frac{\gamma g_i(b)}{k p(b)} \right) \]  

(1)

\[ p_i(b) = (1 - \gamma) \frac{w_i(b)}{k} + \frac{\gamma}{k} \sum_{j=1}^{k} w_j(b) \]  

(2)

From [2],

\[ (1 - \gamma)w_i(b) = \sum_{j=1}^{k} w_j(b) \left( p_i(b) - \frac{\gamma}{k} \right) \]  

(3)
Let $A_i = \exp\left(\frac{\gamma g_i(b)}{kP(b)}\right)$ (4).

We consider the effect of a client’s action $i_b$ on the probability of network $i$. We consider both cases when $i_b = i$ and $i_b \neq i$.

Let’s consider case 1: $i_b = i$.

Using (1), (2) and (4),

$$p_i(b) = \sum_{j=1}^{k} \frac{(1 - \gamma) w_j(b) A_i}{\sum_{j=1}^{k} w_j(b) - w_i(b) + w_i(b) A_i} + \frac{\gamma}{k}$$

Substituting (3) in (5).

$$p_i(b) = \sum_{j=1}^{k} \frac{w_j(b) (p_i(b) - \frac{\gamma}{k}) A_i}{(1 - \gamma) w_i(b) A_i} + \frac{\gamma}{k}$$

Given that

$$\frac{d}{dx} \left( e^{u(x)} \right) = e^{u(x)} \frac{d(u(x))}{dx},$$

$$\frac{dA_i}{d\gamma} = \frac{g_i(b)}{kP(b)} A_i$$

We obtain the continuous time process from the rate of change of $p_i$ with respect to $\gamma$ as $\gamma \rightarrow 0$ and dropping the discrete time script $t$.

$$\dot{p}_i = \lim_{\gamma \to 0} \frac{dp_i}{d\gamma}$$

$$= \lim_{\gamma \to 0} \frac{d}{d\gamma} \left( \frac{p_i(b) - \frac{\gamma}{k}}{1 + \frac{p_i(b) - \frac{\gamma}{k}}{(A_i - 1)} + \frac{\gamma}{k}} \right)$$

$$= \frac{p_i(b) g_i(b)}{kP(b)} (1 - p_i(b))$$

Taking expectation with respect to other clients’ actions

$$\xi_i = \frac{p_i}{k} \sum_{j \neq k - (i)} p_j (E[g_i] - E[g_j])$$

$$= \frac{p_i}{k} \sum_{j = 1}^{k} p_j (\overline{g_i} - \overline{g_j})$$

Given that this replicator dynamics is identical to the ones in [33] and [22], the rest of the proof follows from [22].

APPENDIX B

PROOF OF UPPER BOUND ON NUMBER OF NETWORK SWITCHES

Proof: As we seek to find an upper bound, we assume that reset periods are of equal lengths, a block length is given by $(1 + \beta)^x \leq \lceil (1 + \beta)^x \rceil$, and an equal number of time slots are spent in each network.
We start by identifying an upper bound on the number of network switches in one reset period. Let $\Delta$ be the number of switch backs (hence, $\Delta$ blocks of length one; aggregate of $\Delta$ time slots), and $f$ be the number of full blocks spent in each network.

Total number of time slots spent in each network

$$= (1 + \beta)^0 + \cdots + (1 + \beta)^{f-1}$$

Number of time slots in one reset period = $\frac{\tau}{t_d}$

This implies that

$$\left[(1 + \beta)^0 + \cdots + (1 + \beta)^{f-1}\right] \cdot k + k + \Delta \leq \frac{\tau}{t_d}$$

($k$ time slots for exploration; $\Delta$ time slots for switch backs; there might be a partial block at the end of the reset period, hence $\leq$).

Simplifying the equation and solving for $f$, we get

$$f \leq \log \left(\frac{\beta \tau}{k t_d} - \frac{\beta (\Delta + k)}{k} + 1\right) \log(1 + \beta)$$

Since we are looking for an upper bound, we can ignore the positive factor $\frac{\beta}{k}$ (which is $\leq 1$) of $\frac{\beta \tau}{k t_d}$ and eliminate the positive number $\frac{\beta (\Delta + k)}{k}$ being subtracted from $\frac{\beta \tau}{k t_d}$. Hence,

$$f \leq \frac{\log(\frac{\tau}{t_d} + 1)}{\log(1 + \beta)}$$

Number of blocks in one period $\leq k \cdot f + k + \Delta + 1$

(The one is to take care of a possible partial block at the end of the reset period).

Number of network switches in one reset period

$$\leq k \cdot \frac{\log(\frac{\tau}{t_d} + 1)}{\log(1 + \beta)} + k + \Delta$$

$$\leq 3 k \cdot \frac{\log(\frac{\tau}{t_d} + 1)}{\log(1 + \beta)}$$

Thus, the expected number of network switches over $T$ is upper bounded as

$$T \cdot \frac{3 k \cdot \log(\frac{\tau}{t_d} + 1)}{\log(1 + \beta)}$$

which concludes the proof.

APPENDIX C

PROOF OF UPPER BOUND ON WEAK REGRET

Proof: We assume that reset periods are of equal lengths and $B$ is the number of blocks in one reset period.

We start by identifying an upper bound on weak regret for one reset period. We also assume that the algorithm spends the following fractions of time for each type of action: $\omega$ for exploration, $\delta$ for switch back, $\lambda$ for random selection, and $\alpha$ to flip a coin and following which it selects greedily with probability $\frac{1}{2}$ (hence $\omega + \delta + \lambda + \alpha = 1$). The proof closely relates to that of EXP3 [1] and leverages the following 4 simple facts derived from definitions:

$$g_{i_b}(b) = \frac{g_{i_b}(b)}{\bar{p}(b)}$$

$$\bar{p}(b) = \frac{\omega}{|\text{explore_network}|} + \delta + \lambda \cdot p_{i_b}(b) + \frac{\alpha}{2} \cdot p_{i_b}(b) + \frac{\alpha}{2}$$

$$> p_{i_b}(b) \cdot \left(\frac{\lambda + \delta}{2}\right)$$

$$> p_{i_b}(b) \cdot \psi, \text{ where } \psi < 1; \psi = (\lambda + \delta)$$

Hence,

$$g_{i_b}(b) < \frac{g_{i_b}(b)}{\psi} \cdot p_{i_b}(b) \quad (1)$$

Given that $\hat{g}_i(b) = 0$ for all actions $i$ except $i_b$,

$$\sum_{i=1}^k p_{i_b}(b) \hat{g}_i(b) = p_{i_b}(b) \hat{g}_{i_b}(b)$$

$$< p_{i_b}(b) \cdot \frac{g_{i_b}(b)}{\psi} \cdot p_{i_b}(b) \quad \text{from } (1)$$

$$< \frac{g_{i_b}(b)}{\psi}$$

$$g_{i_b}(b) \in [0, l_{i_b}]$$

$$\sum_{i=1}^k p_{i_b}(b) (\hat{g}_i(b))^2 < \frac{g_{i_b}(b)}{\psi} \cdot \hat{g}_{i_b}(b) \quad \text{from } (2)$$

$$< \frac{1}{\psi} \cdot \sum_{i=1}^k l_i \hat{g}_i(b)$$

By definition,

$$E[\hat{g}_i(b)|i_1, \cdots, i_b] = \sum_{i=1}^k \hat{g}_i(b) \cdot \bar{p}(b)$$

$$= g_{i_b}(b) \cdot \bar{p}(b), \text{ given that } \hat{g}_i(b) = 0 \text{ if } i \neq i_b$$

$$= g_{i_b}(b) \quad (4)$$

We now proceed with the proof. Let $W_{i_b} = w_1(b) + \cdots + w_k(b)$. The proof involves trying to find a bound on the ratio of weights from one round to the next, i.e. $\frac{W_{i_b+1}}{W_{i_b}}$.

$$\frac{W_{i_b+1}}{W_{i_b}} = \sum_{i=1}^k \frac{w_i(b+1)}{W_b}, \text{ given that } W_{i_b+1} = \sum_{i=1}^k w_i(b+1)$$

$$= \sum_{i=1}^k \frac{w_i(b)}{W_b} \exp \left(\frac{\tau \cdot \hat{g}_i(b)}{k}\right), \quad (5)$$

using the weight update rule in algorithm [4].
Given the probability update rule, we solve for \( \frac{w_i(b)}{W_b} \):

\[
p_i(b) = (1 - \gamma) \frac{w_i(b)}{\sum_{j=1}^{k} w_j(b)} + \frac{\gamma}{k}
\]

\[
= (1 - \gamma) \frac{w_i(b)}{W_b} + \frac{\gamma}{k}
\]

Thus,

\[
\frac{w_i(b)}{W_b} = \frac{p_i(b) - \frac{\gamma}{k}}{1 - \gamma}
\]

Combining this with (5), we get

\[
\frac{W_{b+1}}{W_b} = \sum_{i=1}^{k} p_i(b) - \frac{\gamma}{k} \exp \left( \frac{\gamma \tilde{g}_i(b)}{k} \right)
\]

From Taylor series,

\[
e^x \leq 1 + x + \frac{1}{2} x^2
\]

\[
\leq 1 + x + (e - 2)x^2
\]

In our case \( x = \frac{\gamma \tilde{g}_i(b)}{k} \). Combining this with (6), we get

\[
\frac{W_{b+1}}{W_b} \leq \sum_{i=1}^{k} p_i(b) - \frac{\gamma}{k} \left[ 1 + \frac{\gamma \tilde{g}_i(b)}{k} + (e - 2) \left( \frac{\gamma \tilde{g}_i(b)}{k} \right)^2 \right]
\]

\[
\leq \sum_{i=1}^{k} p_i(b) - \frac{\gamma}{k} + \frac{\gamma}{1 - \gamma} \sum_{i=1}^{k} \tilde{g}_i(b) \left( p_i(b) - \frac{\gamma}{k} \right)
\]

\[
+ \frac{(\frac{\gamma}{k})^2(e - 2)}{1 - \gamma} \sum_{i=1}^{k} (\tilde{g}_i(b))^2 \left( p_i(b) - \frac{\gamma}{k} \right)
\]

We solve each of the 3 terms in (7) individually. Solving the first term, we get

\[
\sum_{i=1}^{k} p_i(b) - \frac{\gamma}{k} = \frac{1}{1 - \gamma} \left( \sum_{i=1}^{k} p_i(b) - \sum_{i=1}^{k} \frac{\gamma}{k} \right)
\]

\[
= \frac{1}{1 - \gamma}(1 - \gamma)
\]

\[
= 1
\]

We now solve the second term. As we seek to find an upper bound, we can eliminate the positive number \( \frac{\gamma}{k} \) being subtracted from \( p_i(b) \).

\[
\sum_{i=1}^{k} \tilde{g}_i(b) (p_i(b) - \frac{\gamma}{k}) < \sum_{i=1}^{k} \tilde{g}_i(b) p_i(b)
\]

\[
< \frac{\gamma}{1 - \gamma} \frac{g_{\text{is}}(b)}{\psi} \text{ from (2)}
\]

\[
< \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \frac{1}{\psi(1 - \gamma)}
\]

We solve the third term, again ignoring the positive number \( \frac{\gamma}{k} \) being subtracted from \( p_i(b) \).

\[
(\frac{\gamma}{k})^2(e - 2) \sum_{i=1}^{k} (\tilde{g}_i(b))^2 \left( p_i(b) - \frac{\gamma}{k} \right)
\]

\[
< \left( \frac{\gamma}{k} \right)^2(e - 2) \sum_{i=1}^{k} \tilde{g}_i(b) p_i(b)
\]

\[
< \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{i=1}^{k} l_i \tilde{g}_i(b) \text{ from (3)}
\]

Combining (5), (9) and (10) in (7), we get

\[
\frac{W_{b+1}}{W_b} \leq 1 + \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \frac{1}{\psi(1 - \gamma)} + \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{i=1}^{k} l_i \tilde{g}_i(b)
\]

Taking logarithms on both sides,

\[
\ln \frac{W_{b+1}}{W_b} \leq \ln \left( 1 + \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \frac{1}{\psi(1 - \gamma)} + \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{i=1}^{k} l_i \tilde{g}_i(b) \right)
\]

\[
1 + a \leq e^a, \text{ when } a > 1
\]

In our case, \( a = \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \frac{1}{\psi(1 - \gamma)} + \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{i=1}^{k} l_i \tilde{g}_i(b) \)

Hence, from (11)

\[
\ln W_{b+1} - \ln W_b \leq \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \frac{1}{\psi(1 - \gamma)} + \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{i=1}^{k} l_i \tilde{g}_i(b)
\]

Summing over \( B \)

\[
\sum_{b=1}^{B} \left( \ln W_{b+1} - \ln W_b \right) \leq \left( \frac{\gamma}{k} \right) g_{\text{is}}(b) \sum_{b=1}^{B} l_b \tilde{g}_i(b)
\]

\[
+ \left( \frac{\gamma}{k} \right)^2(e - 2) \psi(1 - \gamma) \sum_{b=1}^{B} \sum_{i=1}^{k} l_i \tilde{g}_i(b)
\]

\[
W_{B+1} \geq w_j(B + 1)
\]

\[
w_j(B + 1) = w_j(B) \exp \left( \frac{\gamma \tilde{g}_j(B)}{k} \right)
\]

\[
= w_j(B - 1) \exp \left( \frac{\gamma \tilde{g}_j(B - 1)}{k} \right) \exp \left( \frac{\gamma \tilde{g}_j(B)}{k} \right)
\]

\[
= \prod_{b=1}^{B} \exp \left( \frac{\gamma \tilde{g}_j(b)}{k} \right)
\]

\[
= \exp \left( \frac{\gamma}{k} \sum_{b=1}^{B} \tilde{g}_j(b) \right)
\]

\[
W_{B+1} \geq \exp \left( \frac{\gamma}{k} \sum_{b=1}^{B} \tilde{g}_j(b) \right)
\]
Taking logarithms on both sides

\[
\ln W_{B+1} \geq \frac{\gamma}{k} \sum_{b=1}^{B} \hat{g}_j(b) \tag{13}
\]

Simplifying the left-hand side of (12), which is a telescoping sum, and combining (13), we have

\[
\sum_{b=1}^{B} (\ln W_{b+1} - \ln W_b) = \ln W_{B+1} - \ln W_1 \geq \frac{\gamma}{k} \sum_{b=1}^{B} \hat{g}_j(b) - \ln k \tag{14}
\]

\(g_{i_b}(b)\) refers to gain from Smart EXP3 in block \(b\). Summing \(g_{i_b}(b)\) over \(B\) gives the total gain of the algorithm, \(G_{\text{Smart EXP3}}(B)\). We combine this and (14) with (12).

\[
\frac{\gamma}{k} \sum_{b=1}^{B} \hat{g}_j(b) - \ln k \leq \frac{\gamma}{\psi(1-\gamma)} G_{\text{Smart EXP3}}(B) + \left(\frac{\gamma^2}{\psi(1-\gamma)}\right)(e-2) \sum_{b=1}^{B} \sum_{i=1}^{k} l_i \hat{g}_i(b)
\]

Multiplying both sides by \(\frac{\psi(1-\gamma)}{k}\) and simultaneously solving for \(G_{\text{Smart EXP3}}(B)\),

\[
G_{\text{Smart EXP3}}(B) \geq \psi(1-\gamma) \sum_{b=1}^{B} \hat{g}_j(b) - \psi(1-\gamma) \frac{\ln k}{k} 
- \frac{\gamma}{k} (e-2) \sum_{b=1}^{B} \sum_{i=1}^{k} l_i \hat{g}_i(b)
\]

Taking expectation on both sides

\[
E[G_{\text{Smart EXP3}}(B)] \geq \psi(1-\gamma) \sum_{b=1}^{B} E[\hat{g}_j(b)] - \psi(1-\gamma) \frac{\ln k}{k} 
- \frac{\gamma}{k} (e-2) \cdot l \sum_{b=1}^{B} \sum_{i=1}^{k} E[\hat{g}_i(b)] \tag{15}
\]

where \(l\) is the largest block length.

Using (4), \(\sum_{b=1}^{B} E[\hat{g}_j(b)] = G_{\text{max}}(B)\) if we pick the best action \(j\).

Combining this with (15)

\[
E[G_{\text{Smart EXP3}}(B)] \geq \psi(1-\gamma)G_{\text{max}}(B) - \psi(1-\gamma) - \frac{\gamma}{k} (e-2) \cdot l \sum_{b=1}^{B} \sum_{i=1}^{k} E[\hat{g}_i(b)] \tag{16}
\]

\[
\sum_{b=1}^{B} \sum_{i=1}^{k} E[\hat{g}_i(b)] = \sum_{i=1}^{k} \sum_{b=1}^{B} g_i(b) \text{ from (4)}
\]

\[
= \sum_{i=1}^{k} \sum_{b=1}^{B} g_i(b) \text{ switching } b \text{ and } i \text{ sums}
\]

\[
\sum_{i=1}^{k} \sum_{b=1}^{B} g_i(b) \leq G_{\text{max}}(B) \text{ if } i \text{ is fixed; } i \text{ is at most the best action}
\]

Combining this with (16),

\[
E[G_{\text{Smart EXP3}}(B)] \geq \psi(1-\gamma)G_{\text{max}}(B) - \psi(1-\gamma) \frac{\ln k}{k} 
- \gamma \cdot (e-2) \cdot l G_{\text{max}}(B) 
\geq \psi(1-\gamma) \cdot \gamma \cdot (e-2) G_{\text{max}}(B) 
- \psi(1-\gamma) \frac{\ln k}{k} \tag{17}
\]

Subtracting \(G_{\text{max}}\) from both sides,

\[
E[G_{\text{Smart EXP3}}(B)] - G_{\text{max}}(B) 
\geq \psi(1-\gamma) \cdot \gamma \cdot (e-2) G_{\text{max}}(B) 
- \psi(1-\gamma) \frac{\ln k}{k} 
\]

Flipping the inequality,

\[
G_{\text{max}}(B) - E[G_{\text{Smart EXP3}}(B)] 
\leq (1+\gamma \cdot (e-2) - \psi(1-\gamma)) G_{\text{max}}(B) 
+ \psi(1-\gamma) \frac{\ln k}{k} 
\leq (1+\gamma \cdot (e-2)) G_{\text{max}}(B) + \frac{k}{\gamma} \ln k 
\]

\[
G_{\text{max}}(\tau) - E[G_{\text{Smart EXP3}}(\tau)] 
\leq (1+\gamma \cdot (e-2)) G_{\text{max}}(\tau) + \frac{k}{\gamma} \ln k 
\]

Given that there are \(\frac{T}{\tau}\) reset periods,

\[
G_{\text{max}}(T) - E[G_{\text{Smart EXP3}}(T)] 
\leq \frac{T}{\tau} \left((1+\gamma \cdot (e-2)) G_{\text{max}}(\tau) + \frac{k}{\gamma} \ln k \right) 
\]

Since gain ignores switching cost,

\[
E[R(T)] = (G_{\text{max}}(T) - E[G_{\text{Smart EXP3}}(T)]) \cdot t_d 
+ E[S(T)] \cdot \mu_d \cdot \mu_g 
\tag{17}
\]
\[ E[R(T)] \leq \frac{T \cdot t_d}{ \tau } \left( (1 + \gamma l (e - 2)) G_{\text{max}}(\tau) + \frac{k \ln k}{\gamma} \right) \]

\[ + \frac{T \cdot \mu_d \cdot \mu_g}{ \tau } \left( \frac{3 k \log(\frac{T}{t_d} + 1)}{\log(1 + \beta)} \right) \]

(18)

which concludes the proof.