Alternative scheme to generate a supersinglet state of three-level atoms

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In this paper we propose an alternative scheme to generate a supersinglet state of three-level atoms via a single-mode of a cavity QED based on the two-photon transitions described by the ‘full microscopical Hamiltonian approach’. In it, three three-level atoms prepared in suitable initial states are sequentially sent through the cavity originally prepared in its vacuum state. After an appropriate choice of the atom-cavity interaction times plus a field detection the state that describes the whole atom-field system is projected in the desired supersinglet state. The fidelity and success probability of the state as well as the practical feasibility of the scheme are discussed.

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INTRODUCTION

Entanglement of states is a fundamental resource for the quantum communication and quantum computation processes. To this end, there are some known entangled states useful for such works, namely: Einstein-Podolsky-Rosen (EPR) state [1], characterizing entangled qubits of two particles; Greenberger-Horne-Zeilinger (GHZ) [2] and W states [3], for qubits in a tripartite (or more) entanglement; Cluster states [4], corresponding to a class of four or more qubits in an entangled state; Werner states [5], a pure to mixed (or vice-versa) state controlled by a single parameter; etc. All these states violate the Bell’s inequality and are applied in quantum teleportation [6], quantum cryptography [7], one way quantum computer [8], etc.

Previously, three apparently unrelated problems without classical solution, namely, the “N-strangers”, “secret sharing”, and “liar detection”, were solved [9] via supersinglet entangled states \(|S^{(N)}\rangle\); the lower and upper indexes indicate the number of particles and the dimension in Hilbert space, respectively. Also, the liar detection problem was solved using the three-qubit triplet state \(|S^{(3)}\rangle\) [10] and the four-qubit singlet state \(|S^{(4)}\rangle\) [11]. Generally speaking, these states can be written in the form [9]

\[
|S^{(N)}\rangle = \frac{1}{\sqrt{N!}} \sum_{\text{permutations of } 01\ldots(N-1)} (-1)^{\tau[i,j,...,n]},
\]

where \(\tau\) is the number of transpositions of pairs of elements composed by those appearing in a canonical order, i.e., \(0, 1, 2, ..., N - 1\). As an example of Eq. (1), first consider the supersinglet \(|S^{(2)}_2\rangle\) with \(N = 2\) and the canonical order given by \([01]\). From the Eq. (1) one obtains \(|S^{(2)}_2\rangle = (|01\rangle - |10\rangle)/\sqrt{2}\). Another example is: for three three-level atoms the supersinglet \(|S^{(3)}_3\rangle\) reads (see Ref. [9] for more details)

\[
|S^{(3)}_3\rangle = \frac{1}{\sqrt{6}} \left[|gfe\rangle - |gef\rangle - |feg\rangle + |egf\rangle + |efg\rangle - |efg\rangle\right],
\]

where \(|g\rangle\), \(|f\rangle\), and \(|e\rangle\) (instead of 0, 1, and 2, respectively) represent the atomic levels configuration shown in Fig. 1.

Despite its relevance in the field of quantum information, as far as we know few experimental schemes have been proposed for the generation of the supersinglet states. Recently, a scheme for generation of the \(3 \times 3\) supersinglet states (2) was presented in the scenario of cavities [12]. It employs four three-level atoms, three cavities, and selective atomic detectors. In each cavity the atom-field interaction is governed by the Jaynes-Cummings model in which the atom works as two-level atom. However, in the present state of the art the manipulation of three cavities is missing yet. Then, inspired by the potential applications of the supersinglet states [9–11], in this paper we will propose an alternative scheme to generate the \(3 \times 3\) supersinglet state, as given in the Eq. (2). It uses only a single QED cavity, four three-level atoms in a ladder configuration, and selective atomic detectors. The atom-field interaction is described by the ‘full microscopical Hamiltonian approach’ that is a two-photon Jaynes-Cummings model. So, the use of a single cavity turns the present scheme more attractive in view of its experimental feasibility.

The two-photon transition in three-level atoms interacting with a single cavity-field mode was realized in Ref. [13]. As applications of this study we have proposed a teleportation of zero- and two-photon superposition [14], an entanglement swapping protocol [15], and a scheme for generation of the two-photon EPR and W states [16]. We have also investigated the entropy of the entanglement swapping [17] and the dynamics of a two-atom entanglement and the entanglement sudden death [18].

The paper is organized as follows: In the Sec. II we present an overview of the model; in Sec. III we show the scheme of generation of the supersinglet state; Sec. IV displays the numerical results and in the Sec. V we concludes the paper.
Consider a three-level atom that interacts with a single cavity-field mode via a two-photon Jaynes-Cummings model described in the interaction picture by the Hamiltonian [19]

\[
H_I = \hbar g_1 (a|e\rangle\langle f|e^{-i\delta t} + a^\dagger|f\rangle\langle e|e^{i\delta t}) + \hbar g_2 (a|f\rangle\langle g|e^{i\delta t} + a^\dagger|g\rangle\langle f|e^{-i\delta t})
\]

where \( g_1 \) and \( g_2 \) stand for the one-photon coupling constant with respect to the transitions \(|e\rangle \leftrightarrow |f\rangle\) and \(|f\rangle \leftrightarrow |g\rangle\), respectively. The detuning \( \delta \) is given by

\[
\delta = \Omega - (\omega_e - \omega_f) = (\omega_f - \omega_g) - \Omega,
\]

where \( \Omega \) is the cavity-field frequency and \( \omega_e, \omega_f, \) and \( \omega_g \) are the frequencies associated with the atomic levels \(|e\rangle\), \(|f\rangle\), and \(|g\rangle\), respectively. Fig. 1 shows a schematic representation of the atomic levels.

The state describing the combined atom-field system reads

\[
|\psi(t)\rangle = \sum_n [C_{e,n}(t)|e,n\rangle + C_{f,n}(t)|f,n\rangle + C_{g,n}(t)|g,n\rangle],
\]

where \(|k,n\rangle\), with \( k = e, f, g \), indicate the atom in the state \(|k\rangle\) and the field in the Fock state \(|n\rangle\). The coefficients \( C_{k,n}(t) \) stand for the corresponding probability amplitudes.

Inserting the Eqs. (3) and (5) into the time dependent Schrödinger equation one obtains the coupled first-order differential equations for the probability amplitudes

\[
\frac{dC_{e,n}(t)}{dt} = -ig_1 C_{f,n+1}(t)\sqrt{n+1}e^{-i\delta t},
\]

\[
\frac{dC_{f,n+1}(t)}{dt} = -ig_1 C_{e,n}(t)\sqrt{n+1}e^{i\delta t} - ig_2 C_{g,n+2}(t)\sqrt{n+2}e^{i\delta t},
\]

\[
\frac{dC_{g,n+2}(t)}{dt} = -ig_2 C_{f,n+1}(t)\sqrt{n+2}e^{-i\delta t}.
\]

As usually, we consider that the entire atom-field system is decoupled at the initial time \( t = 0 \),

\[
C_{e,n}(0) = C_e C_n(0),
\]

\[
C_{b,n+1}(0) = C_f C_{n+1}(0),
\]

\[
C_{c,n+2}(0) = C_g C_{n+2}(0),
\]

where the \( C_n(0) \) stand for the amplitudes of the arbitrary initial field state and the \( C_{e,n} \) are atomic amplitudes of the (normalized) initial atomic state

\[
|\chi\rangle = C_e |e\rangle + C_f |f\rangle + C_g |g\rangle.
\]

Solving these coupled differential equations with the initial conditions in (7) we get the time dependent coefficients as

\[
C_{e,n}(t) = \left[ \frac{g_1^2(n+1)}{\Lambda_n \alpha_n^2} \gamma_n(t) + 1 \right] \tan(\Lambda_n t)e^{-i\delta t} C_e C_{n+1} + \frac{g_1 g_2 \gamma_n(t)}{\Lambda_n \alpha_n^2} C_g C_{n+2},
\]
of the entire system is given by

\[ C_{f,n+1}(t) = -\frac{g_1 \sqrt{n+1}}{\Lambda_n} \sin(\Lambda_n t)e^{i\frac{\delta}{2} C_C} C_{n} + \left( \cos(\Lambda_n t) - \frac{i\delta}{2\Lambda_n} \sin(\Lambda_n t) \right) e^{i\frac{\delta}{2} C_f} C_{f,n+1} \]

\[ -i \frac{g_2 \sqrt{n+1}}{\Lambda_n} \sin(\Lambda_n t)e^{i\frac{\delta}{2} C_C} C_{n+2}, \]  

(10)

and

\[ C_{g,n+2}(t) = \frac{g_2 \sqrt{(n+1)(n+2)}}{\Lambda_n \alpha_n^2} \gamma_n(t) C_{n} - i \frac{g_2 \sqrt{n+1}}{\Lambda_n} \sin(\Lambda_n t)e^{-i\frac{\delta}{2} C_f} C_{f,n+1} \]

\[ + \left[ \frac{g_2^2 (n+2)}{\Lambda_n \alpha_n^2} \gamma_n(t) + 1 \right] C_{g} C_{n+2}, \]  

(11)

where

\[ \gamma_n(t) = \left[ \Lambda_n \cos(\Lambda_n t) + \frac{\delta}{2} \sin(\Lambda_n t) - \Lambda_n e^{i\frac{\delta}{2} t} \right] e^{-i\frac{\delta}{2} t}, \]

(12)

\[ \Lambda_n = \sqrt{\frac{\delta^2}{4} + \alpha_n^2}, \]

(13)

\[ \alpha_n = \sqrt{g_2^2 (n+1) + g_2^2 (n+2)}, \]  

(14)

\( \Lambda_n \) being the Rabi frequency. The substitutions \( n \to n - 1 \) in Eq. (10) and \( n \to n - 2 \) in Eq. (11) allow one to obtain the \( C_{f,n}(t) \) and \( C_{g,n}(t) \), respectively.

\section*{GENERATION OF SUPERSINGLET}

In this section, we consider three three-level atoms plus a single cavity field mode previously prepared in the vacuum state \( |\{0\}_C\rangle \). Firstly, we send the atom 1, in the excited state \( |e\rangle_1 \), to interact with the cavity field mode, leading the atom-field system to the state

\[ |\psi\rangle_{1C} = C_{c_0}^{(e)}(t_1)|e,0\rangle_{1C} + C_{f_1}^{(e)}(t_1)|f,1\rangle_{1C} + C_{g_2}^{(e)}(t_1)|g,2\rangle_{1C}, \]  

(15)

where the \( C_{m}^{(kl)} \), with atomic indexes \( m, k = e, f, g \) and cavity indexes \( n, l = 0, 1, 2, \ldots \), are the coefficients given by Eqs.(9-11).

In a second step the atom 2, previously prepared in the intermediate state \( |f\rangle_2 \), crosses the cavity in a way that the state of the atom-field system is written as

\[ |\psi\rangle_{2C} = C_{c_0}^{(e)}(t_1)|C_{f_0}^{(e)}(t_2)|e,f,0\rangle_{12C} + C_{g_1}^{(e)}(t_2)|e,g,1\rangle_{12C} + C_{f_1}^{(f)}(t_1)|C_{f_1}^{(f)}(t_2)|f,f,1\rangle_{12C} + C_{g_2}^{(f)}(t_2)|f,g,2\rangle_{12C} + C_{g_2}^{(e)}(t_1)|C_{f_2}^{(e)}(t_2)|f,g,2\rangle_{12C} + C_{g_3}^{(f)}(t_2)|g,g,3\rangle_{12C} + C_{g_3}^{(e)}(t_1)|g,g,3\rangle_{12C}. \]  

(16)

Next, we send the atom 3, previously prepared in the ground state \( |g\rangle_3 \), to interact with the cavity field. In this way, the state of the entire system is given by

\[ |\psi\rangle_{123C} = C_{c_0}^{(e)}(t_1)|C_{f_0}^{(e)}(t_2)|e,f,0\rangle_{123C} + C_{g_1}^{(e)}(t_2)|C_{g_1}^{(e)}(t_3)|e,g,1\rangle_{123C} + C_{f_1}^{(f)}(t_1)|C_{f_1}^{(f)}(t_2)|f,f,0\rangle_{123C} + C_{g_2}^{(f)}(t_2)|f,g,2\rangle_{123C} + C_{g_2}^{(g)}(t_3)|g,g,2\rangle_{123C} + C_{g_3}^{(f)}(t_1)|C_{f_2}^{(f)}(t_2)|g,g,2\rangle_{123C} + C_{g_3}^{(g)}(t_2)|C_{g_3}^{(g)}(t_3)|g,g,3\rangle_{123C} + C_{g_3}^{(e)}(t_3)|g,g,3\rangle_{123C} + C_{g_3}^{(e)}(t_1)|g,g,3\rangle_{123C}. \]  

(17)
FIG. 2: Plot of the fidelity versus the detuning. In (a) we consider the value of coupling constant as $t = 23 \mu s$, $t_2 = 1 \mu s$, and $t_3 = 45 \mu s$. In (b) we use $g = 17.5 \text{MHz}$ with $t_1 = 15 \mu s$, $t_2 = 38 \mu s$, and $t_3 = 95 \mu s$.

Now, we assume a cavity detection in the vacuum state. This can be realized by sending an auxiliary atom in its ground state to interact with the cavity field, and so after the atomic measurement projects the state of the cavity (see Appendix for details).

In this way the state given in Eq. (17) is reduced to

$$|\psi^{'}\rangle_{123} = |\mathcal{N}\{C^{(e_0)}_0(t_1)C^{(g_0)}_{f_0}(t_2)e,f,g\rangle_{123} + C^{(e_0)}_0(t_1)C^{(g_0)}_{g_0}(t_2)C^{(g_1)}_{f_0}(t_3)e,f,g\rangle_{123}
+ C^{(e_0)}_1(t_1)C^{(f_1)}_{j_1}(t_2)C^{(g_1)}_{f_0}(t_3)f,f,f\rangle_{123} + C^{(e_0)}_0(t_1)C^{(f_1)}_{e_0}(t_2)f,f,g\rangle_{123}
+ C^{(e_0)}_0(t_1)C^{(f_2)}_{e_0}(t_2)C^{(g_2)}_{e_0}(t_3)|g,f,e\rangle_{123}
+ C^{(e_0)}_0(t_1)C^{(f_1)}_{e_1}(t_2)C^{(g_1)}_{f_0}(t_3)|g,f,e\rangle_{123}\},$$

(18)

with a success probability given by

$$P_S = |\mathcal{N}|^{-2} = |C^{(e_0)}_0(t_1)C^{(f_0)}_{f_0}(t_2)|^2 + |C^{(e_0)}_0(t_1)C^{(g_0)}_{g_0}(t_2)C^{(g_1)}_{f_0}(t_3)|^2
+ |C^{(e_0)}_1(t_1)C^{(f_1)}_{j_1}(t_2)C^{(g_1)}_{f_0}(t_3)|^2 + |C^{(e_0)}_0(t_1)C^{(f_1)}_{e_0}(t_2)f,f,f\rangle_{123}^2
+ |C^{(e_0)}_0(t_1)C^{(f_2)}_{e_0}(t_2)C^{(g_2)}_{e_0}(t_3)|^2 + |C^{(e_0)}_0(t_1)C^{(f_1)}_{e_1}(t_2)C^{(g_1)}_{f_0}(t_3)|^2.$$

(19)

Thus, with an appropriate choice of the interaction times ($t_1$, $t_2$, and $t_3$) one obtains from (18) and (2) the fidelity, defined as $F_S = |\mathcal{N}|\langle S_3 |\psi^{'}\rangle_{123}|^2$, given by

$$F_S = \frac{|\mathcal{N}|^2}{6} - |C^{(e_0)}_0(t_1)C^{(f_0)}_{f_0}(t_2) + C^{(e_0)}_0(t_1)C^{(g_0)}_{g_0}(t_2)C^{(g_1)}_{f_0}(t_3)
+ C^{(e_0)}_1(t_1)C^{(f_1)}_{j_1}(t_2) + C^{(e_0)}_0(t_1)C^{(f_1)}_{e_0}(t_2)f,f,f\rangle_{123}
+ C^{(e_0)}_0(t_1)C^{(f_2)}_{e_0}(t_2)C^{(g_2)}_{e_0}(t_3) - C^{(e_0)}_0(t_1)C^{(f_1)}_{e_1}(t_2)C^{(g_1)}_{f_0}(t_3)|^2.$$

(20)

NUMERICAL RESULTS

In this section we present some numerical results. By choosing appropriate interaction times $t_1$, $t_2$, and $t_3$ we obtain larger values of the fidelity ($F_S$). However, we must also choose convenient values of the detuning ($\delta$) since it appears in the present configuration as shown in Fig. 1 (also in Ref. [13]). The control of the parameter $\delta$ can be done via the Stark-shift effect due to an external electric field [20]. In the present protocol our calculations show that the fidelity decreases when the detuning $\delta$ increases. Figs. 2a and 2b display the fidelity of the supersinglet state versus the detuning for $g = 1 \text{MHz}$ (with $t_1 = 23 \mu s$, $t_2 = 1 \mu s$, and $t_3 = 45 \mu s$) and $g = 17.5 \text{MHz}$ (with $t_1 = 15 \mu s$, $t_2 = 38 \mu s$, and $t_3 = 95 \mu s$), respectively.

In Tables I and II some values of the fidelity with the corresponding success probability are listed for different values of times $t_1$, $t_2$, and $t_3$, considering $g_1 = g_2 = g = 1 \text{MHz}$ with $\delta = 0$ and $\delta = 0.1 g$, respectively. Tables III and IV use the same convention of Tables I and II, except for $g_1 = g_2 = g = 17.5 \text{MHz}$.

For more details we have displayed the fidelity for a fixed interaction time $t_1 = 23 \mu s$ (considering $g = 1 \text{MHz}$ and $\delta = 0$) in Fig. 3a and $t_1 = 15 \mu s$ (considering $g = 17.5 \text{MHz}$ and $\delta = 0$) in Fig. 3b.
to interact with the cavity field, obeying the following possibilities:

\[ |g, 0 \rangle_{a,C} \rightarrow |g, 0 \rangle_{a,C}, \]  
\[ |g, 1 \rangle_{a,C} \rightarrow C_{g_1}^{(1)}(t')|g, 1 \rangle_{a,C} + C_{f_0}^{(1)}(t')|f, 0 \rangle_{a,C}, \]  
\[ |g, 2 \rangle_{a,C} \rightarrow C_{g_2}^{(2)}(t')|g, 2 \rangle_{a,C} + C_{f_1}^{(2)}(t')|f, 1 \rangle_{a,C} + C_{e_0}^{(2)}(t')|e, 0 \rangle_{a,C}, \]  
\[ |g, 3 \rangle_{a,C} \rightarrow C_{g_3}^{(3)}(t')|g, 3 \rangle_{a,C} + C_{f_2}^{(3)}(t')|f, 2 \rangle_{a,C} + C_{e_1}^{(3)}(t')|e, 1 \rangle_{a,C}. \]  

\( \delta = 0 \) in Fig. 3b. We note that the fidelity is more sensitive to the interaction time for larger values of the coupling constant. For example, this is shown by comparing Fig. 3b (\( g = 17.5 \text{MHz}, n \sim 50 \)), where the fidelity becomes more sensitive to fluctuations in the interaction times, and Fig. 3a (\( g = 1 \text{MHz}, n \sim 90 \)), where it suffers a little change.

**CONCLUSION**

The ‘\( N \)-strangers’, the ‘secret sharing’, the ‘liar detection’, and the ‘Byzantine agreement’ are examples of unsolvable problems using the classical computation. On the other hand, they can be solved using quantum mechanics [9–11]. The supersinglet states are the key of this procedure. So motivated, we have presented here a feasible scheme for generation of the 3 \( \times \) 3 supersinglet state using three-level atoms. The present scheme sounds experimentally advantageous [20] in comparison with that in Ref. [12] since it uses only a single cavity. In our numerical simulations we have used two values for the coupling constant, given by \( g = 1 \text{MHz} \) [21] and \( g = 17.5 \text{MHz} \) [13], and Rydberg atoms with quantum number \( n \sim 90 \) and \( n \sim 50 \), respectively. We note that the fidelity of the wanted state increases for small values of the detuning. For example, for \( t_1 = 23 \mu s, t_2 = 1 \mu s, \) and \( t_3 = 45 \mu s (g = 1 \text{MHz} \text{ and } \delta = 0) \) the fidelity and success probability are 97.6\% and 62.9\%, respectively; for \( t_1 = 15 \mu s, t_2 = 38 \mu s, \) and \( t_3 = 95 \mu s (g = 17.5 \text{MHz} \text{ and } \delta = 0) \) the fidelity and success probability are 96.3\% and 32.0\%, respectively; etc. It is worth stressing that a nonideal fidelity does not forbid the application of this supersinglet to solve some protocols. For example, in the liar detection [9] a lot of supersinglet states are requested to provide a list of possible detections of the components. In this case, the occurrence of a few errors in the list due to imperfections in the state does not affect the main result. Also, the atomic decay and the control of the velocity distributions can be neglected regarding the fidelity of the scheme, since the lifetime of Rydberg atoms with \( n \sim 50 \) is about 30 ms, i.e., \( 10^7 \) times higher than the interaction times considered here and the velocity distribution of the atomic beam presents a small deviation, around 0.3\% [20]. In conclusion, taking into account the potential applications of this state and the feasibility of the scheme we believe that this supersinglet state can be experimentally implemented.

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**Appendix**

The detection of the cavity-field mode is discussed below. To this end, we consider an auxiliary atom in its ground state \(|g\rangle_a\) to interact with the cavity field, obeying the following possibilities:

\[ |g, 0 \rangle_{a,C} \rightarrow |g, 0 \rangle_{a,C}, \]  
\[ |g, 1 \rangle_{a,C} \rightarrow C_{g_1}^{(1)}(t')|g, 1 \rangle_{a,C} + C_{f_0}^{(1)}(t')|f, 0 \rangle_{a,C}, \]  
\[ |g, 2 \rangle_{a,C} \rightarrow C_{g_2}^{(2)}(t')|g, 2 \rangle_{a,C} + C_{f_1}^{(2)}(t')|f, 1 \rangle_{a,C} + C_{e_0}^{(2)}(t')|e, 0 \rangle_{a,C}, \]  
\[ |g, 3 \rangle_{a,C} \rightarrow C_{g_3}^{(3)}(t')|g, 3 \rangle_{a,C} + C_{f_2}^{(3)}(t')|f, 2 \rangle_{a,C} + C_{e_1}^{(3)}(t')|e, 1 \rangle_{a,C}. \]  

FIG. 3: Plot of the fidelity versus \( t_3 \) and \( t_2 \) for a fixed interaction time \( t_1 \). In (a) we consider the value of coupling constant as \( g = 1 \text{MHz} \) and detuning \( \delta = 0 \), as well as \( t_1 = 23 \mu s \). In (b) we use \( g = 17.5 \text{MHz} \) and \( \delta = 0 \) with \( t_1 = 15 \mu s \).
To ensure that the cavity is in its vacuum state, we set the interaction time \((t')\) appropriately to maximize the probability of photon absorption. As an example, considering the case with \(\delta = 0, g = 1 \text{MHz}\), and adjusting \(t' = 4.71 \mu\text{s}\), we obtain a maximum error of 0.0005\%, 1.8\%, or 1.7\% for the detection of the ground state in the cases with one-, two-, or three-photons, respectively (in Eqs. (21b-21d)). So, the selective atomic detection in the ground state guarantees the generation of the supersinglet state (2) with a success probability greater than 98.2\% using a single auxiliary atom. Note that by the support of more atoms (previously prepared in the ground state \(|g\rangle\), where the interaction time is tuned with the same value of \(t'\)) this error can be reduced even more, e.g., in the case of another auxiliary atom the maximum error in the absorption is about 0.03\% (success probability \(\geq 99.97\%\)).

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TABLE I: Fidelity and corresponding success probability as functions of $t_1$, $t_2$, and $t_3$ with $g = 1\text{MHz}$ and $\delta = 0$.

| $t_1$, $t_2$, $t_3$ ($\mu\text{s}$) | $P_s$    | $P_s(\%)$ |
|-------------------------------|---------|-----------|
| 1,1,45                        | 0.953017 | 70.9      |
| 5,1,1                         | 0.952057 | 42.0      |
| 5,1,46                        | 0.951075 | 50.5      |
| 12,1,1                        | 0.953527 | 51.3      |
| 12,1,2                        | 0.970373 | 73.9      |
| 12,1,20                       | 0.968870 | 75.5      |
| 12,1,27                       | 0.968235 | 76.3      |
| 12,1,34                       | 0.953858 | 49.3      |
| 12,1,45                       | 0.975297 | 67.0      |
| 12,1,46                       | 0.965878 | 59.8      |
| 23,1,1                        | 0.968455 | 46.8      |
| 23,1,2                        | 0.969310 | 69.7      |
| 23,1,20                       | 0.966943 | 71.4      |
| 23,1,27                       | 0.967875 | 72.0      |
| 23,1,34                       | 0.955247 | 45.8      |
| 23,1,45                       | 0.976124 | 62.9      |
| 23,1,46                       | 0.975231 | 55.3      |
| 34,1,1                        | 0.957197 | 42.7      |
| 34,1,45                       | 0.951170 | 58.9      |
| 34,1,46                       | 0.957395 | 51.2      |
| 41,1,2                        | 0.968149 | 74.6      |
| 41,1,20                       | 0.966810 | 76.2      |
| 41,1,27                       | 0.965816 | 77.1      |
| 41,1,34                       | 0.951813 | 49.8      |
| 41,1,45                       | 0.972791 | 67.7      |
| 41,1,46                       | 0.961744 | 60.7      |
TABLE II: Fidelity and corresponding success probability as functions of $t_1$, $t_2$, and $t_3$ with $g = 1$MHz and $\delta = 0.1g$.

| $t_1$, $t_2$, $t_3$ ($\mu$s) | $F_s$  | $P_s$ (%) |
|-------------------------------|--------|-----------|
| 1,1,1                         | 0.917148 | 55.9      |
| 1,1,2                         | 0.947847 | 77.6      |
| 1,1,9                         | 0.845914 | 54.4      |
| 1,1,13                        | 0.818697 | 50.7      |
| 1,32,5                        | 0.851816 | 78.4      |
| 2,30,3                        | 0.883008 | 3.5       |
| 5,1,1                         | 0.936816 | 42.6      |
| 5,1,2                         | 0.923425 | 65.1      |
| 5,1,8                         | 0.837938 | 44.4      |
| 5,1,9                         | 0.804010 | 45.0      |
| 5,1,15                        | 0.846834 | 26.4      |
| 5,32,5                        | 0.834717 | 66.1      |
| 5,32,10                       | 0.823751 | 33.1      |
| 6,30,1                        | 0.815561 | 44.6      |
| 6,30,2                        | 0.845026 | 62.7      |
| 8,1,2                         | 0.829526 | 83.7      |
| 8,1,9                         | 0.810296 | 54.5      |
| 12,1,1                        | 0.876137 | 52.8      |
| 12,1,2                        | 0.899566 | 74.7      |
| 12,1,8                        | 0.805662 | 55.9      |
| 12,1,9                        | 0.814788 | 52.4      |
| 12,1,13                       | 0.808298 | 48.5      |
| 12,32,5                       | 0.818571 | 75.5      |
| 50,1,1                        | 0.827253 | 54.7      |
| 50,1,2                        | 0.835046 | 76.5      |
| 50,1,48                       | 0.801231 | 50.7      |
TABLE III: Fidelity and corresponding success probability as functions of $t_1$, $t_2$, and $t_3$ with $g = 17.5$MHz and $\delta = 0$.

| $t_1$, $t_2$, $t_3$(µs) | $F_s$  | $P_s(\%)$ |
|-------------------------|--------|---------|
| 15,38,19                | 0.955450 | 34.1   |
| 15,38,47                | 0.955040 | 29.6   |
| 15,38,53                | 0.956239 | 39.3   |
| 15,38,61                | 0.953247 | 31.6   |
| 15,38,89                | 0.956397 | 42.0   |
| 15,38,95                | 0.963001 | 32.0   |
| 32,38,19                | 0.951438 | 32.9   |
| 32,38,47                | 0.954557 | 28.5   |
| 32,38,53                | 0.951940 | 38.1   |
| 32,38,61                | 0.951898 | 30.4   |
| 32,38,89                | 0.951164 | 40.7   |
| 32,38,95                | 0.961062 | 30.8   |
| 38,38,89                | 0.951349 | 47.6   |
| 49,38,95                | 0.956297 | 29.7   |
| 55,38,19                | 0.952102 | 38.1   |
| 55,38,25                | 0.950950 | 54.3   |
| 55,38,53                | 0.952674 | 43.7   |
| 55,38,89                | 0.955947 | 46.3   |
| 55,38,95                | 0.952517 | 36.1   |
| 72,38,19                | 0.955415 | 36.9   |
| 72,38,25                | 0.952313 | 52.9   |
| 72,38,53                | 0.956310 | 42.4   |
| 72,38,89                | 0.958697 | 45.0   |
| 72,38,95                | 0.957923 | 34.9   |
| 89,38,25                | 0.951564 | 51.6   |
| 89,38,95                | 0.961521 | 33.7   |
TABLE IV: Fidelity and corresponding success probability as functions of $t_1$, $t_2$ and $t_3$ with $g = 17.5$ MHz and $\delta = 0.1g$.

| $t_1$, $t_2$, $t_3$ ($\mu$s) | $F_s$  | $P_s$ (%) |
|-----------------------------|--------|----------|
| 10,30,17                    | 0.842295 | 58.4     |
| 10,30,21                    | 0.837639 | 46.8     |
| 10,30,43                    | 0.803492 | 62.4     |
| 10,30,46                    | 0.855158 | 87.6     |
| 13,30,9                     | 0.816954 | 49.0     |
| 13,30,17                    | 0.854786 | 58.3     |
| 13,30,21                    | 0.819474 | 46.6     |
| 13,30,34                    | 0.806463 | 68.8     |
| 13,30,46                    | 0.848566 | 86.5     |
| 13,30,49                    | 0.825522 | 68.2     |
| 15,27,50                    | 0.809714 | 18.8     |
| 18,27,11                    | 0.844388 | 53.4     |
| 18,27,14                    | 0.832976 | 23.3     |
| 18,27,36                    | 0.870633 | 27.4     |
| 18,27,40                    | 0.830723 | 24.5     |
| 18,27,50                    | 0.921186 | 20.4     |
| 21,27,36                    | 0.802649 | 30.3     |
| 21,27,50                    | 0.868293 | 23.1     |
| 39,30,17                    | 0.805561 | 57.6     |
| 39,30,21                    | 0.828222 | 45.7     |
| 39,30,43                    | 0.811022 | 62.6     |
| 39,30,46                    | 0.862737 | 83.8     |
| 42,30,9                     | 0.861602 | 46.4     |
| 42,30,17                    | 0.831920 | 56.9     |
| 42,30,21                    | 0.835240 | 45.0     |
| 42,30,34                    | 0.836005 | 64.8     |