The Strongest Experimental Constraints on
SU(5) x U(1) Supergravity Models

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ABSTRACT

We consider a class of well motivated string-inspired flipped SU(5) supergravity models which include four supersymmetry breaking scenarios: no-scale, strict no-scale, dilaton, and special dilaton, such that only three parameters are needed to describe all new phenomena (m_\text{t}, \tan \beta, m_{\tilde{g}}). We show that the LEP precise measurements of the electroweak parameters in the form of the \(\epsilon_1\) variable, and the CLEOII allowed range for \(B(b \rightarrow s\gamma)\) are at present the most important experimental constraints on this class of models. For \(m_\text{t} \gtrsim 155 (165)\) GeV, the \(\epsilon_1\) constraint (at 90(95)%CL) requires the presence of light charginos \((m_{\chi^\pm_1} \lesssim 50 - 100\) GeV depending on \(m_\text{t}\)). Since all sparticle masses are proportional to \(m_{\tilde{g}}\), \(m_{\chi^\pm_1} \lesssim 100\) GeV implies: \(m_{\chi^0_1} \lesssim 55\) GeV, \(m_{\chi^0_2} \lesssim 100\) GeV, \(m_{\tilde{g}} \lesssim 360\) GeV, \(m_{\tilde{q}} \lesssim 350 (365)\) GeV, \(m_{\tilde{e}_R} \lesssim 80 (125)\) GeV, \(m_{\tilde{e}_L} \lesssim 120 (155)\) GeV, and \(m_{\tilde{\nu}} \lesssim 100 (140)\) GeV in the no-scale (dilaton) flipped SU(5) supergravity model. The \(B(b \rightarrow s\gamma)\) constraint excludes a significant fraction of the otherwise allowed region in the \((m_{\chi^\pm_1}, \tan \beta)\) plane (irrespective of the magnitude of the chargino mass), while future experimental improvements will result in decisive tests of these models. In light of the \(\epsilon_1\) constraint, we conclude that the outlook for chargino and selectron detection at LEPII and at HERA is quite favorable in this class of models.
1 Introduction

Models of low-energy supersymmetry which have their genesis in physics at very high energies (i.e., supergravity or superstrings) embody the principal motivation for supersymmetry – the solution of the gauge hierarchy problem. These models can also be quite predictive since their parameters are constrained by the larger symmetries usually found at very high mass scales. In contrast, models of low-energy supersymmetry with no such fundamental basis (such as the minimal supersymmetric standard model (MSSM)) should properly be acknowledged as generic parametrizations of all possible supersymmetric beyond-the-standard-model possibilities, and as such not as real “theories”. Specifically, supergravity models with radiative electroweak symmetry breaking need only five parameters to describe all new phenomena: the top-quark mass ($m_t$), the ratio of Higgs vacuum expectation values ($\tan \beta$), and three universal soft-supersymmetry-breaking parameters ($m_{1/2}, m_0, A$). The MSSM on the other hand requires over twenty parameters to describe the same phenomena, and has led to the erroneous impression that supersymmetric models necessarily introduce numerous parameters which hamper the analysis of processes of interest. In order to ameliorate this situation, it is routinely assumed that these parameters are related among themselves in some arbitrary way. These ad-hoc simplifications can be misleading (e.g., implying “general” conclusions based on special cases) or simply insufficient (e.g., as in the analysis of branching fractions involving particles from all sectors of the model).

Perhaps one of the most interesting aspects of supergravity models is the radiative electroweak symmetry breaking mechanism [1, 2], by which the electroweak symmetry is broken by the Higgs mechanism driven dynamically by radiative corrections. This mechanism involves several otherwise unrelated physical inputs, such as the top-quark mass, the breaking of supersymmetry, the physics at the high-energy scale, and the running of the parameters from high to low energies. Needless to say, electroweak symmetry breaking has no explanation in the MSSM (i.e., the negative Higgs mass squared is put in by hand). In practice, this constraint is used to determine the magnitude of the Higgs mixing parameter $\mu$ [3], which is of electroweak size or larger, and implies: (i) a generic correlation among the lighter neutralino and chargino masses ($2m_{\chi_1^0} \sim m_{\chi_2^0} \sim m_{\chi_1^\pm}$) which has been observed in a variety of models [4, 5, 6], and (ii) the connection between the supersymmetric sector and the Higgs sector such that, as the supersymmetry breaking parameters are increased, the Higgs sector asymptotes quickly to a standard-model-like situation with one light Higgs scalar [7].

Experimental predictions in this class of models for processes at present and near future collider facilities can be obtained and have been determined for the various supergravity models we consider below (LEPI [8, 7], Tevatron [9], HERA [10], LEPII [11]). Unfortunately, the range of sparticle and Higgs masses is quite broad, even for the constrained models we consider. In fact, experimental exploration of all of the well motivated parameter space of these models would require the large hadron colliders (LHC/SSC) for the strongly interacting particles (gluino and squarks) and...
the next linear collider (NLC) for the weakly interacting particles (charginos and neutralinos). Of course, the present generation of experiments has probed and will continue to probe part of this allowed parameter space (i.e., the lighter end of the spectrum).

Another way of testing the predictions of supergravity models is via indirect experimental signatures, usually originating from virtual (e.g., one-loop) processes. In particular, one has the precise electroweak measurements at LEP (in the form of the $\epsilon_{1,2,3,6}$ parameters [12, 13, 14, 15]) and the rare radiative decay $b \to s\gamma$ [16, 17, 18, 19, 20], as observed by CLEOII Collaboration [21]. Both these probes have been investigated independently in the minimal $SU(5)$ and the no-scale flipped $SU(5)$ supergravity models in Ref. [22] and [19]. In this paper we expand these studies to a larger class of $SU(5) \times U(1)$ supergravity models and determine the allowed region of parameter space where both constraints are satisfied simultaneously.

These one-loop processes have the advantage of side-stepping the strong kinematical constraints which afflict the direct production channels, although they require high precision experiments which may not be accountable exclusively in terms of supersymmetric effects in the models we consider. Nevertheless, as we point out in this paper, these experimental constraints are at present the most stringent ones on the class of supergravity models we consider. It is also important to note that the knowledge acquired through the indirect tests will help sharpen the predictions for the direct detection processes, and thus focus the experimental efforts to detect these particles.

This paper is organized as follows. In Sec. 2 we describe briefly the class of $SU(5) \times U(1)$ supergravity models which we consider, as well as the four soft-supersymmetry-breaking scenarios of interest. In Sec. 3 we outline the calculational procedure followed to obtain $\epsilon_1$ and $B(b \to s\gamma)$. In Sec. 4 we present our results and discuss their phenomenological consequences. Finally in Sec. 5 we summarize our conclusions. The appendix contains the explicit expressions used to evaluate $B(b \to s\gamma)$.

2 The SU(5) x U(1) Supergravity Models

The $SU(5) \times U(1)$ supergravity models we consider include the standard model particles and their superpartners plus two Higgs doublets. These are supplemented by one additional vector-like quark doublet with mass $\sim 10^{12}$ GeV and one additional vector-like (charge $-1/3$) quark singlet with mass $\sim 10^6$ GeV [23], such that the unification scale is delayed until $\sim 10^{18}$ GeV as expected to occur in string models [24].

Besides the two parameters needed to describe the (third-generation) Yukawa sector of the models ($m_t$ and $\tan \beta$), one must also specify the values of the soft-supersymmetry-breaking parameters at the unification scale. The latter can be greatly simplified by assuming universality. That is, all three gaugino masses are taken to be degenerate ($M_1 = M_2 = M_3 = m_{1/2}$), all fifteen squark and slepton masses and the two Higgs scalar doublets are separately assumed to be universal.
$(M_{\tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}})_{i=1,2,3} = m_0$ and $M_{H_1,2} = m_0$), and the three trilinear scalar couplings are taken to be equal ($A_{t,b,\tau} = A$). Of these simplifying assumptions, only the one relating the (first- and second-generation) scalar masses is required experimentally to keep the flavor-changing-neutral-currents in the $K - \bar{K}$ system under control [25]. Nevertheless, these assumptions are seen to hold in large classes of supergravity models. The Higgs mixing parameter $\mu$ and its associated bilinear soft-supersymmetry-breaking mass $B$ do not need to be specified at the unification scale since they do not feed into the renormalization group equations for the other parameters. They are determined at the electroweak scale via the radiative electroweak symmetry breaking mechanism. In sum then, only five parameters are needed to describe the supergravity models we consider. In contrast, twenty-five parameters would be needed in the MSSM.

The minimization of the electroweak Higgs potential is performed at the one-loop level, and yields the values of $|\mu|$ and $B$ as well as the one-loop corrected Higgs boson masses [3]. The five-dimensional parameter space is restricted by several consistency conditions and experimental constraints on the unobserved sparticle masses (most importantly the chargino mass and the lightest Higgs boson mass [4]). Moreover, the cosmological constraint of a not-too-young universe (usually incorrectly referred to as “not overclosing the universe”) is also imposed [26].

Building on the above basic structure one is naturally led to the $SU(5) \times U(1)$ (or flipped $SU(5)$) gauge group since this model has a strong motivation in the context of string theory [27]. Moreover, the additional particles fit snugly into complete $10, \overline{10}$ $SU(5) \times U(1)$ representations. In this model proton decay and cosmological constraints are satisfied automatically [3, 4]. Concerning the soft-supersymmetry-breaking parameter space, one assumes two possible string-inspired supersymmetry breaking scenarios:

(i) the $SU(N,1)$ no-scale supergravity model [28, 2] which implies $m_0 = A = 0$, with a more constrained case called the strict no-scale scenario where $B(M_U) = 0$ is also assumed; and

(ii) the dilaton supersymmetry breaking scenario [29] where $m_0 = \frac{1}{\sqrt{3}} m_{1/2}$ and $A = -m_{1/2}$, including a special case where $B(M_U) = 2m_0$.

The allowed parameter spaces in these two cases have been determined in Refs. [5] and [6] respectively. It is found that there are no important constraints on $\tan \beta$ ($\tan \beta \lesssim 32, 46$ respectively) and the sparticle masses can be as light as their present experimental lower bounds. More importantly, the parameter spaces are three-dimensional ($m_t, \tan \beta, m_{1/2} \propto m_{\tilde{g}}$) and therefore the spectrum scales with $m_{\tilde{g}}$ [27]. (In what follows we consider only three representative values of $m_t = 130, 150, 170$ GeV.) In Table 1 we give a comparative listing of the most important properties of these models.

It is interesting to note that the special cases of these two supersymmetry breaking scenarios are quite predictive, since they allow to determine $\tan \beta$ in terms of $m_t$ and $m_{\tilde{g}}$. In the strict no-scale case one finds that the value of $m_t$ determines the sign of $\mu$ ($\mu > 0 : m_t \lesssim 135$ GeV, $\mu < 0 : m_t \gtrsim 140$ GeV) and whether the
lightest Higgs boson mass is above or below 100 GeV. In the special dilaton scenario, \( \tan \beta \approx 1.4 - 1.6 \) and \( m_t \lesssim 155 \text{ GeV} \), \( 61 \text{ GeV} \lesssim m_h \lesssim 91 \text{ GeV} \) follow. Thus, continuing Tevatron top-quark searches and LEPI,II Higgs searches could probe these restricted scenarios completely.

3 One-loop Constraints (\( \epsilon_1 \) and \( b \to s\gamma \))

The one-loop corrections to the \( W^\pm \) and \( Z \) boson self-energies (i.e., the “oblique” corrections) can be parametrized in terms of three variables \( \epsilon_{1,2,3} \) \( ^{[12]} \) which are constrained experimentally by the precise LEP measurements of the \( Z \) leptonic width (\( \Gamma_l \)), and the leptonic forward-backward asymmetries at the \( Z \)-pole (\( A_{FB} \)), as well as the \( M_W/M_Z \) ratio. A fourth observable is the \( Z \to b\bar{b} \) width which is described by the \( \epsilon_b \) parameter \( ^{[14]} \). Of these four variables, at present \( \epsilon_1 \) provides the strongest constraint in supersymmetric models at the 90%CL \( ^{[22, 15]} \). However, the \( \epsilon_b \) constraint is competitive with (although at present somewhat weaker than) the \( \epsilon_1 \) constraint \( ^{[15, 30]} \), and in fact may impose interesting constraints on supersymmetric models as the precision of the data increases.

The expression for \( \epsilon_1 \) is obtained from the following definition \( ^{[13]} \)

\[
\epsilon_1 = e_1 - e_5 - \frac{\delta G_{V,B}}{G} - 4\delta g_A, \tag{1}
\]

where \( e_{1,5} \) are the following combinations of vacuum polarization amplitudes

\[
e_1 = \frac{\alpha}{4\pi \sin^2 \theta_W M_H^2} [\Pi_{11}^{JJ}(0) - \Pi_{11}^{JJ}(0)], \tag{2}
\]

\[
e_5 = M_Z^2 F_{ZZ}(M_Z^2), \tag{3}
\]

and the \( q^2 \neq 0 \) contributions \( F_{ij}(q^2) \) are defined by

\[
\Pi_{ij}^{JJ}(q^2) = \Pi_{ij}^{JJ}(0) + q^2 F_{ij}(q^2). \tag{4}
\]

The \( \delta g_A \) in Eqn. (1) is the contribution to the axial-vector form factor at \( q^2 = M_Z^2 \) in the \( Z \to l^+l^- \) vertex from proper vertex diagrams and fermion self-energies, and \( \delta G_{V,B} \) comes from the one-loop box, vertex and fermion self-energy corrections to the \( \mu \)-decay amplitude at zero external momentum. These non-oblique SM corrections are non-negligible, and must be included in order to obtain an accurate SM prediction.

As is well known, the SM contribution to \( \epsilon_1 \) depends quadratically on \( m_t \) but only logarithmically on the SM Higgs boson mass (\( m_H \)). In this fashion upper bounds on \( m_t \) can be obtained which have a non-negligible \( m_H \) dependence: up to 20 GeV stronger when going from a heavy (\( \approx 1 \text{ TeV} \)) to a light (\( \approx 100 \text{ GeV} \)) Higgs boson. It is also known (in the MSSM) that the largest supersymmetric contributions to \( \epsilon_1 \) are expected to arise from the \( \tilde{t}-\tilde{b} \) sector, and in the limiting case of a very light stop, the contribution is comparable to that of the \( t-b \) sector. The remaining squark, slepton,
chargino, neutralino, and Higgs sectors all typically contribute considerably less. For increasing sparticle masses, the heavy sector of the theory decouples, and only SM effects with a light Higgs boson survive. (This entails stricter upper bounds on $m_t$ than in the SM, since there the Higgs boson does not need to be light.) However, for a light chargino ($m_{\chi^\pm} \rightarrow \frac{1}{2}M_Z$), a $Z$-wavefunction renormalization threshold effect can introduce a substantial $q^2$-dependence in the calculation, i.e., the presence of $e_5$ in Eq. (1) \[13\]. The complete vacuum polarization contributions from the Higgs sector, the supersymmetric chargino-neutralino and sfermion sectors, and also the corresponding contributions in the SM have been included in our calculations \[22\].

The rare radiative flavor-changing-neutral-current (FCNC) $b \rightarrow s \gamma$ decay has been observed by the CLEOII Collaboration in the following 95% CL allowed range\[21\]

$$B(b \rightarrow s \gamma) = (0.6 - 5.4) \times 10^{-4}.$$ (5)

In Ref. \[19\] we have given the predictions for the branching ratio in the minimal $SU(5)$ supergravity model ($B(b \rightarrow s \gamma)_{\text{minimal}} = (2.3 - 3.6) \times 10^{-4}$) and in the no-scale flipped $SU(5)$ supergravity model. However, in that paper the experimental lower bound on $B(b \rightarrow s \gamma)$ was not available. Since a large suppression of $B(b \rightarrow s \gamma)$ (much below the SM value) can occur in the flipped $SU(5)$ models, such a bound can be quite restrictive. Below we give the predictions for $B(b \rightarrow s \gamma)$ in the two variants of the flipped $SU(5)$ model (and their special subcases) described in Sec. 4. The expressions used to compute the branching ratio $B(b \rightarrow s \gamma)$ are given in the Appendix for completeness.

4 Discussion

The results of our computations for $B(b \rightarrow s \gamma)$ and $\epsilon_1$ are shown in Figures 1,3,4,6 in the various models under consideration, for $m_t = 130, 150, 170$ GeV. (Smaller values of $m_t$ are not ruled out experimentally, although they appear ever more unlikely.) The LEP value for $\epsilon_1$ which we use in our analysis is $\epsilon_1 = (0.9 \pm 3.7) \times 10^{-3}$ \[31\], which implies $\epsilon_1 < 0.00517 (0.00632)$ at the 90(95)%CL. The experimentally allowed interval for $B(b \rightarrow s \gamma)$ is given in Eq. (5), although in what follows we will also consider a less conservative estimate of the lower bound, namely $B(b \rightarrow s \gamma) > 10^{-5}$. We now discuss the results and ensuing constraints on each model in turn.

4.1 No-scale flipped SU(5)

At the 90(95)%CL, for $m_t \lesssim 150 (165)$ GeV there are no restrictions on the model parameters from the $\epsilon_1$ constraint (see Fig. 1). For $m_t = 170$ GeV (see Fig. 1c) the $\epsilon_1$ constraint alone implies a strict upper bound on the chargino mass $\chi^\pm$ (i) for $\mu > 0$ there are no allowed points at 90%CL, while $m_{\chi^\pm} \lesssim 70$ GeV is required at

\[1\] An upper bound on the chargino mass implies upper bounds on all sparticle and Higgs masses, since they are all proportional to $m_S$, see Table 1.
95%CL; (ii) for $\mu < 0$ one obtains $m_{\tilde{\chi}_1^\pm} \lesssim 58$ (70) GeV at 90(95)%CL. Interestingly enough, for this range of chargino masses the $B(b \to s\gamma)$ constraint is also restrictive. Combining the $\epsilon_1$ and $B(b \to s\gamma)$ constraints we obtain: (i) for $\mu > 0$, $m_{\tilde{\chi}_1^\pm} \lesssim 67$ GeV at 95%CL and $\tan \beta \approx 8 - 10$; (ii) for $\mu < 0$, $m_{\tilde{\chi}_1^\pm} \lesssim 54$ (67) GeV at 90(95)%CL and $\tan \beta \lesssim 8$. No significant improvement is obtained by required $B(b \to s\gamma) > 10^{-5}$. Analogously, upper bounds on $m_t$ are only allowed for $\mu > 0$, $m_t > 130$ GeV is required $[5]$ could only be made consistent with LEP data if the chargino mass is very near its present experimental lower bound.

For $m_t = 130, 150$ GeV, the $\epsilon_1$ constraint is ineffective. However, for $\mu > 0$ the $B(b \to s\gamma)$ constraint is quite restrictive, as shown in Figs. 1a,1b. The various (dotted) curves correspond to different values of $\tan \beta$. For large values of $m_{\tilde{\chi}_1^\pm}$, these curves start off at values of $B(b \to s\gamma)$ which decrease with increasing $\tan \beta$, i.e., the largest value corresponds to $\tan \beta = 2$, and then $\tan \beta$ increases in steps of two. As the chargino mass decreases, these curves reach a minimum (i.e., zero) value and then increase again (except for $\tan \beta = 2$), and even exceed the upper bound on $B(b \to s\gamma)$ for large enough $\tan \beta$. To show better the excluded area, in Fig. 2 we have plotted those points in parameter space which survive the $B(b \to s\gamma)$ constraint, in the $(m_{\tilde{\chi}_1^\pm}, \tan \beta)$ plane. The swath along the diagonal is excluded because $B(b \to s\gamma)$ is too small. In fact, if we demand $B(b \to s\gamma) > 10^{-5}$, the points denoted by crosses would be excluded as well. The area to the left of the left group of points is excluded because $B(b \to s\gamma)$ is too large. Note that no matter what the actual value of $B(b \to s\gamma)$ ends up being, there will always be some allowed set of points, namely, a subset of both sets of presently allowed points. Another consequence of the $B(b \to s\gamma)$ constraint is an upper bound on $\tan \beta$: for $m_t = 130 (150)$ GeV, $\tan \beta \approx 26 (20)$ compared to the upper bound of $\tan \beta \approx 30 (26)$ which existed prior to the application of the $B(b \to s\gamma)$ constraint. Finally, note that $\tan \beta \gtrsim 20$ implies $m_{\tilde{\chi}_1^\pm} \gtrsim 100$ GeV.

4.2 Strict no-scale flipped SU(5)

In this variant of the model $\tan \beta$ is determined for given $m_t$ and $m_{\tilde{g}}$ values and is such that $m_t = 130$ GeV is only allowed for $\mu > 0$, whereas $m_t = 150,170$ GeV are only allowed for $\mu < 0$ (see Table 1 and Ref. 5). As above, the $\epsilon_1$ constraint is only effective for $m_{\tilde{\chi}_1^\pm} \gtrsim 150 (165)$ GeV at the 90(95)%CL. For $m_t = 170$ GeV there is an upper bound $m_{\tilde{\chi}_1^\pm} \lesssim 54 (62)$ GeV at 90(95)%CL, and there is no further constraint from $B(b \to s\gamma)$ (see Fig. 3). In fact, $B(b \to s\gamma)$ is only constraining for $m_t = 130$ GeV. Note that in this case there are two possible solutions for $\tan \beta$ which are most clearly seen in the $B(b \to s\gamma)$ plot (Fig. 3). The lower $\tan \beta$ solution (which asymptotes to the larger value of $B(b \to s\gamma)$) excludes chargino masses in the interval $54 \lesssim m_{\tilde{\chi}_1^\pm} \lesssim 175$ GeV, whereas the larger $\tan \beta$ solution excludes $m_{\tilde{\chi}_1^\pm} \gtrsim 150$ GeV.
4.3 Dilaton flipped SU(5)

The discussion in this model parallels closely that for the no-scale model given above, with some relevant differences. For \( m_t = 170 \) GeV, the \( \epsilon_1 \) constraint alone implies (see Fig. 4c): (i) for \( \mu > 0 \), \( m_{\chi^\pm_i} \lessapprox 53 (60) \) GeV at the 90(95)%CL and \( \tan \beta \lessapprox 10 \), and (ii) for \( \mu < 0 \), \( m_{\chi^\pm_i} \lessapprox 56 (70) \) GeV at the 90(95)%CL and \( \tan \beta \lessapprox 24 \). The further imposition of the \( B(b \rightarrow s\gamma) \) constraint is not significant.

For \( m_t = 130, 150 \) GeV, the \( B(b \rightarrow s\gamma) \) constraint is quite restrictive, and this time for both signs of \( \mu \) (see Figs. 4a,4b), although it is more important for \( \mu > 0 \).

The allowed regions of parameter space in the \((m_{\chi^\pm_i}, \tan \beta)\) space are shown in Fig. 5. The only qualitative difference with the results for the no-scale case is that for \( m_t = 150 \) GeV and \( \mu < 0 \), \( B(b \rightarrow s\gamma) \) can only be too small, and this excludes the area to the left of the set of allowed points. Finally, \( B(b \rightarrow s\gamma) \) does not impose any further constraints on \( \tan \beta \).

4.4 Special dilaton flipped SU(5)

In this case, only \( \mu < 0 \) is allowed and \( m_t \lessapprox 155 \) GeV is required. It also follows that \( \tan \beta < 2 \) (see Table [4] and Ref. [5]). At the moment \( B(b \rightarrow s\gamma) \) does not impose any constraints on the parameter space (see Fig. 6). The same is true for \( \epsilon_1 \), except for \( m_t = 155 \) GeV which at the 90%CL cannot be made acceptable since light chargino masses are not allowed (see Fig. 6); there are no constraints at the 95%CL.

5 Conclusions

We have considered a class of well motivated string-inspired supergravity models based on the gauge group \( SU(5) \times U(1) \). The various constraints on the models allow one to predict all new low-energy phenomena in terms of just three parameters \((m_t, \tan \beta, m_{\tilde{g}})\) and as such these models are highly predictive. We have shown that one-loop processes which do not create real sparticles can nonetheless be used to constrain these models in interesting ways. The LEP \( \epsilon_1 \) constraint and the CLEOII \( B(b \rightarrow s\gamma) \) allowed range are perhaps surprisingly restrictive in the models which we have considered. The \( \epsilon_1 \) constraint requires the presence of light charginos if the top-quark mass exceeds the moderate value of \( \approx 155 \) GeV. In fact, \( m_{\chi^\pm_i} \lessapprox 100 \) GeV is the weakest possible constraint in this case. Since all sparticle masses are proportional to \( m_{\tilde{g}} \), \( \tan \beta \)-dependent upper bounds on all of them also follow. The weakest possible upper bounds \((i.e., \tan \beta\text{-independent})\) which follow for \( m_t \gtrsim 155 (165) \) GeV at the 90(95)%CL are:

\[
\begin{align*}
m_{\chi^0_i} & \lessapprox 55 \text{ GeV,} \\
m_{\chi^\pm_i} & \lessapprox 100 \text{ GeV,} \\
m_{\tilde{g}} & \lessapprox 360 \text{ GeV,} \\
m_{\tilde{q}} & \lessapprox 350 (365) \text{ GeV,} \\
m_{\tilde{e}_R} & \lessapprox 80 (125) \text{ GeV,} \\
m_{\tilde{e}_L} & \lessapprox 120 (155) \text{ GeV,} \\
m_{\tilde{\nu}} & \lessapprox 100 (140) \text{ GeV}
\end{align*}
\]
in the no-scale (dilaton) flipped $SU(5)$ supergravity model.

The $B(b \to s\gamma)$ constraint can probe the models irrespective of the mass scales involved because of a large suppression of the amplitude for a wide range of chargino masses ($50\,\text{GeV} \lesssim m_{\tilde{\chi}^\pm} \lesssim 280\,\text{GeV}$ depending on the value of $\tan\beta$). It should be noted that most of the qualitative aspects of our discussions should apply quite generally to the class of supergravity models with radiative electroweak symmetry breaking, where the parameters $m_0$ and $A$ are not necessarily related to $m_{1/2}$ as discussed here.

We conclude that future refinements of the allowed $B(b \to s\gamma)$ range are likely to result in decisive tests of this class of models. Moreover, if the top-quark mass is too heavy to be easily detectable at the Tevatron, then the $\epsilon_1$ constraint would require light charginos. These could nevertheless escape detection at the Tevatron, since the characteristic trilepton events are generally suppressed for light charginos \cite{9}. However, the outlook for chargino and selectron detection at LEPII \cite{11} and at HERA \cite{10} will remain quite favorable in this class of models.

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**A Expression for $B(b \to s\gamma)$**

The expression used for $B(b \to s\gamma)$ is given by \cite{18}

$$B(b \to s\gamma) = \frac{6\alpha}{\pi} \left[ \eta^{16/23} A_{\gamma} + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) A_{g} + C \right]^2,$$

where $\eta = \alpha_s(M_Z)/\alpha_s(m_b)$, $I$ is the phase-space factor $I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$, and $f(m_c/m_b) = 2.41$ the QCD correction factor for the semileptonic decay. In our computations we have used: $\alpha_s(M_Z) = 0.118$, $B(b \to ce\bar{\nu}) = 10.7\%$, $m_b = 4.8\,\text{GeV}$, and $m_c/m_b = 0.3$. The $A_{\gamma}, A_g$ are the coefficients of the effective $bs\gamma$ and $bsg$ penguin operators evaluated at the scale $M_Z$. Their simplified expressions are given below, in the justifiable limit of negligible gluino and neutralino contributions \cite{16} and degenerate squarks, except for the $\tilde{t}_{1,2}$ which are significantly split by $m_t$.

The contributions to $A_{\gamma,g}$ from the $W - t$ loop, the $H^\pm - t$ loop, and the $\chi^\pm_i - \tilde{t}_k$ loop are given by

$$W: \quad A_{\gamma,g} = \frac{3}{2} \frac{m_t^2}{m_W^2} f^{(1)}_{\gamma,g} \left( \frac{m_t^2}{m_W^2} \right)$$

$$H^\pm: \quad A_{\gamma,g} = \frac{1}{2} \frac{m_t^2}{m_{H^\pm}^2} \left[ \frac{1}{\tan^2 \beta} f^{(1)}_{\gamma,g} \left( \frac{m_t^2}{m_{H^\pm}^2} \right) + f^{(2)}_{\gamma,g} \left( \frac{m_t^2}{m_{H^\pm}^2} \right) \right]$$

where $f^{(1)}_{\gamma,g}(x)$ and $f^{(2)}_{\gamma,g}(x)$ are expressions that depend on $x = m_t^2/m_{H^\pm}^2$.
\[ \chi^\pm_i : \quad A_{\gamma, g} = \sum_{j=1}^{2} \left\{ \begin{array}{l}
\frac{m_W^2}{m_{\chi^\pm_i}^2} \left[ |V_{j1}|^2 f^{(1)}_{\gamma, g} \left( \frac{m_Q^2}{m_{\chi^\pm_i}^2} \right) 
- \sum_{k=1}^{2} V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} m_W \sin \beta} f^{(1)}_{\gamma, g} \left( \frac{m_{\tilde{t}_k}^2}{m_{\chi^\pm_i}^2} \right) \right] \\
- \frac{U_{j2}}{\sqrt{2} \cos \beta} \frac{m_W}{m_{\chi^\pm_i}^2} \left[ V_{j1} f^{(3)}_{\gamma, g} \left( \frac{m_Q^2}{m_{\tilde{t}_k}^2} \right) 
- \sum_{k=1}^{2} \left( V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} m_W \sin \beta} f^{(3)}_{\gamma, g} \left( \frac{m_{\tilde{t}_k}^2}{m_{\chi^\pm_i}^2} \right) \right) \right] \end{array} \right. \right\}, \quad (12) \]

with

\[
\begin{align*}
 f^{(1)}_{\gamma}(x) &= \frac{(7 - 5x - 8x^2)}{36(x - 1)^3} + \frac{x(3x - 2)}{6(x - 1)^4} \log x & (13) \\
 f^{(2)}_{\gamma}(x) &= \frac{(3 - 5x)}{6(x - 1)^2} + \frac{(3x - 2)}{3(x - 1)^3} \log x & (14) \\
 f^{(3)}_{\gamma}(x) &= (1 - x) f^{(1)}_{\gamma}(x) - \frac{x}{2} f^{(2)}_{\gamma}(x) - \frac{23}{36} & (15) \\
 f^{(1)}_{g}(x) &= \frac{(2 + 5x - x^2)}{12(x - 1)^3} - \frac{x}{2(x - 1)^4} \log x & (16) \\
 f^{(2)}_{g}(x) &= \frac{(3 - x)}{2(x - 1)^2} - \frac{1}{(x - 1)^3} \log x & (17) \\
 f^{(3)}_{g}(x) &= (1 - x) f^{(1)}_{g}(x) - \frac{x}{2} f^{(2)}_{g}(x) - \frac{1}{3}. & (18) \\
\end{align*}
\]

In these formulas \( U_{ij}, V_{ij} \) are the elements of the matrices which diagonalize the chargino mass matrix, \( m_{\chi^\pm_i} \) are the chargino masses, \( m_{\tilde{t}_k} \) are the stop mass eigenvalues, \( T_{ij} \) are the elements of the matrix which diagonalizes the \( 2 \times 2 \) stop mass matrix.
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Table 1: Major features of the class of \( SU(5) \times U(1) \) supergravity models, and a comparison of the supersymmetry breaking scenarios considered. (All masses in GeV).

| SU(5) × U(1) |  |
| --- | --- |
| • Easily string-derivable, several known examples  
• Symmetry breaking to Standard Model due to vevs of \( 10, \overline{10} \) and tied to onset of supersymmetry breaking  
• Natural doublet-triplet splitting mechanism  
• Proton decay: \( d = 5 \) operators very small  
• Baryon asymmetry through lepton number asymmetry (induced by the decay of flipped neutrinos) as recycled by non-perturbative electroweak interactions |  |

| No-Scale | Dilaton |
| --- | --- |
| • Parameters 3: \( m_{1/2}, \tan \beta, m_t \)  
• Universal soft-supersymmetry-breaking automatic  
• \( m_0 = 0, \ A = 0 \)  
• Dark matter: \( \Omega h_0^2 < 0.25 \)  
• \( m_{1/2} < 475 \) GeV, \( \tan \beta < 32 \)  
• \( m_\tilde{g} > 245 \) GeV, \( m_\tilde{q} > 240 \) GeV  
• \( m_\tilde{q} \approx 0.97m_\tilde{g} \)  
• \( m_{\tilde{t}} > 155 \) GeV  
• \( m_{\tilde{e}_R} \approx 0.18m_\tilde{g}, m_{\tilde{e}_L} \approx 0.30m_\tilde{g} \)  
• \( m_{\tilde{e}_R}/m_{\tilde{e}_L} \approx 0.61 \)  
• \( 60 \) GeV < \( m_h \) < \( 125 \) GeV  
• \( 2m_{\chi_1^0} \approx m_{\chi_2^0} \approx m_{\chi_1^\pm} \approx 0.28m_\tilde{g} \lesssim 290 \)  
• \( m_{\chi_3^0} \sim m_{\chi_4^0} \sim m_{\chi_2^\pm} \sim |\mu| \)  
• Spectrum easily accessible soon | • Parameters 3: \( m_{1/2}, \tan \beta, m_t \)  
• Universal soft-supersymmetry-breaking automatic  
• \( m_0 = \frac{1}{\sqrt{3}} m_{1/2}, \ A = -m_{1/2} \)  
• Dark matter: \( \Omega h_0^2 < 0.90 \)  
• \( m_{1/2} < 465 \) GeV, \( \tan \beta < 46 \)  
• \( m_\tilde{g} > 195 \) GeV, \( m_\tilde{q} > 195 \) GeV  
• \( m_\tilde{q} \approx 1.01m_\tilde{g} \)  
• \( m_{\tilde{t}} > 90 \) GeV  
• \( m_{\tilde{e}_R} \approx 0.33m_\tilde{g}, m_{\tilde{e}_L} \approx 0.41m_\tilde{g} \)  
• \( m_{\tilde{e}_R}/m_{\tilde{e}_L} \approx 0.81 \)  
• \( 60 \) GeV < \( m_h \) < \( 125 \) GeV  
• \( 2m_{\chi_1^0} \approx m_{\chi_2^0} \approx m_{\chi_1^\pm} \approx 0.28m_\tilde{g} \lesssim 290 \)  
• \( m_{\chi_3^0} \sim m_{\chi_4^0} \sim m_{\chi_2^\pm} \sim |\mu| \)  
• Spectrum accessible soon |

|  |  |
|  |  |
| • Strict no-scale: \( B(M_U) = 0 \)  
\( \tan \beta = \tan \beta(m_t, m_\tilde{g}) \)  
\( m_t \lesssim 135 \) GeV \( \Rightarrow \mu > 0, m_h \lesssim 100 \) GeV  
\( m_t \gtrsim 140 \) GeV \( \Rightarrow \mu < 0, m_h \gtrsim 100 \) GeV | • Special dilaton: \( B(M_U) = 2m_0 \)  
\( \tan \beta = \tan \beta(m_t, m_\tilde{g}) \)  
\( \tan \beta \approx 1.4 - 1.6, m_t < 155 \) GeV  
\( m_h \approx 61 - 91 \) GeV |
Figure Captions

Figure 1: The values of $B(b \to s\gamma)$ (top row) and the $\epsilon_1$ parameter (bottom row) versus the chargino mass in no-scale flipped $SU(5)$ supergravity model for (a) $m_t = 130$ GeV, (b) $m_t = 150$ GeV, and (c) $m_t = 170$ GeV. The 95% CL CLEOII limit on $B(b \to s\gamma)$ and the 90% CL LEP upper limit on $\epsilon_1$ are indicated.

Figure 2: The region in $(m_{\chi_1^\pm}, \tan \beta)$ space which is allowed by the CLEOII limit on $B(b \to s\gamma)$ in the no-scale flipped $SU(5)$ supergravity model, for $\mu > 0$ and $m_t = 130, 150$ GeV. The value of $B(b \to s\gamma)$ is too large to the left of the left group of points, and too small in between the two groups of points. (No such constraints exist for $\mu < 0$.) The points denoted by crosses would become excluded if the lower bound on $B(b \to s\gamma)$ is strengthened to $B(b \to s\gamma) > 10^{-5}$.

Figure 3: The values of $B(b \to s\gamma)$ (top row) and the $\epsilon_1$ parameter (bottom row) versus the chargino mass in the strict no-scale flipped $SU(5)$ supergravity model, for $m_t = 130$ GeV ($\mu > 0$) and $m_t = 150, 170$ GeV ($\mu < 0$). The 95% CL CLEOII limit on $B(b \to s\gamma)$ and the 90% CL LEP upper limit on $\epsilon_1$ are indicated. For $m_t = 130$ GeV ($\mu > 0$) two $\tan \beta$ solutions exist which are clearly visible in the $B(b \to s\gamma)$ plot.

Figure 4: The values of $B(b \to s\gamma)$ (top row) and the $\epsilon_1$ parameter (bottom row) versus the chargino mass in the dilaton flipped $SU(5)$ supergravity model, for (a) $m_t = 130$ GeV, (b) $m_t = 150$ GeV, and (c) $m_t = 170$ GeV. The 95% CL CLEOII limit on $B(b \to s\gamma)$ and the 90% CL LEP upper limit on $\epsilon_1$ are indicated.

Figure 5: The region in $(m_{\chi_1^\pm}, \tan \beta)$ space which is allowed by the CLEOII limit on $B(b \to s\gamma)$ in the dilaton flipped $SU(5)$ supergravity model, for $m_t = 130, 150$ GeV. The value of $B(b \to s\gamma)$ is too large to the left of the left group of points, and too small in between the two groups of points (except for $m_t = 150$ GeV ($\mu < 0$), where it is too small to the left of the single group of points). The points denoted by crosses would become excluded if the lower bound on $B(b \to s\gamma)$ is strengthened to $B(b \to s\gamma) > 10^{-5}$.

Figure 6: The values of $B(b \to s\gamma)$ (top) and the $\epsilon_1$ parameter (bottom) versus the chargino mass in the special dilaton $SU(5)$ supergravity model (only allowed for $\mu < 0$) for $m_t = 130, 150, 155$ GeV. (In this model $m_t \leq 155$ GeV is required.) The 95% CL CLEOII limit on $B(b \to s\gamma)$ and the 90% CL LEP upper limit on $\epsilon_1$ are indicated.