Relaxation dynamics of aftershocks after large volatility shocks in the SSEC index

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Abstract. - The relaxation dynamics of aftershocks after large volatility shocks are investigated based on two high-frequency data sets of the Shanghai Stock Exchange Composite (SSEC) index. Compared with previous relevant work, we have defined main financial shocks based on large volatilities rather than large crashes. We find that the occurrence rate of aftershocks with the magnitude exceeding a given threshold for both daily volatility (constructed using 1-minute data) and minutely volatility (using intra-minute data) decays as a power law. The power-law relaxation exponent increases with the volatility threshold and is significantly greater than 1. Taking financial volatility as the counterpart of seismic activity, the power-law relaxation in financial volatility deviates remarkably from the Omori law in Geophysics.

Introduction. – Financial markets are complex systems, from which numerous empirical regularities have been documented in the Econophysics community [1–5]. A large proportion of these stylized facts deal with volatility, which is an important measure of risk of financial assets. The dynamics of asset volatility exhibit significant similarity in scaling compared to seismic activities such as the Gutenberg-Richter law [6] and Omori’s law [7]. We note that the Gutenberg-Richter law in Finance [8] has deep connection with the inverse cubic law in the right tail distribution of volatility [9]. Both scaling laws concern the dynamic behavior of volatility after stock market crashes.

Indeed, there are quite a few studies on the dynamic behavior of volatility after stock market crashes. Sornette and coworkers found that the implied variance of the Standard and Poor’s 500 (S&P 500) index after the infamous Black Monday (10/19/1987) decays as a power law decorated with log-periodic oscillations [10]. Lillo and Mantegna investigated 1-minute logarithmic changes of the S&P 500 index during 100 trading days after the Black Monday and found that the occurrence of events larger than some threshold exhibits power-law relaxation for different thresholds [11]. This Omori law was also found to hold after two other crashes on 10/27/1997 and 08/31/1998 [12]. There is more evidence from other stock indexes. Selçuk investigated daily index data from 10 emerging stock markets (Argentina, Brazil, Hong Kong, Indonesia, Korea, Mexico, Philippines, Singapore, Taiwan and Turkey) and observed Omori’s law after two largest crashes in each market [13]. Selçuk and Gençay utilized the 5-minute Dow Jones Industrial Average 30 index (DJIA) and identified Omori’s law after 10/08/1998 and 01/03/2001 [14].
In a recent paper, Weber et al. extended the procedure adopted in the aforementioned studies on two aspects [15]: (1) They have smoothed the volatility (absolute of return) a moving average over an aggregated time scale in order to remove insignificant fluctuations; and (2) They found that the Omori law holds not only after big crashes but also after intermediate shocks. These two issues are of essential importance, which enable us to discuss the Omori law in alternative manners. On one hand, the volatility can be defined by the average of absolute returns in a given time interval rather than the absolute of return over the same time scale. On the other hand, the main shocks in financial markets are not necessary to be defined by large crashes. Instead, we can investigate volatility shocks so that rallies are also included besides crashes.

Based on these considerations, we shall study in this work the volatility dynamics of the Shanghai Stock Exchange Composite (SSEC) index after large volatility shocks. The rest of this paper is organized as follows. We first present preliminary information about the composition of data sets, the definition of volatility, and the mathematical description of Omori’s law. We then provide an objective procedure for the identification of volatility shocks and investigate the aftershock dynamics of daily and minutely volatility after large shocks for different thresholds.

Preliminary information.

The data sets. We analyze two high-frequency data sets recording the SSEC index \(I(t)\) at two different sampling frequencies. The first data set contains the close prices at the end of every minute during the continuous double auction so that its sampling interval is one minute. This data set covers five and half years from February 2001 to August 2006. The size of the data exceeds 318,602.

The sampling frequency of the second data set fluctuates along time and is roughly 10 realizations per minute. This data set records every quotations during the continuous double auction (from 9:30 a.m. to 11:30 a.m. and from 13:00 p.m. to 15:00 p.m.) released by the Shanghai Stock Exchange and displayed on the terminals that investors can watch on. The data span from January 2004 to June 2006. The size of the data set is 1,253,440. It is worth noting that this data set is not ultra-high-frequency but high-frequency, since ultra-high-frequency data record all transactions [16].

Definition of volatility. The return \(r(t)\) over time scale \(\delta t\) is defined as follows

\[
r(t) = \ln[I(t)] - \ln[I(t - \delta t)],
\]

where \(\delta t\) is time resolution for each data set (one minute for the first data set and one step in unit of event time for the second one). In the Econophysics literature including the aforementioned work on financial Omori law, the absolute of return \(|r(t)|\) is frequently adopted as a measure of volatility on the time interval \((t - \Delta t, t]\). Actually, there are various methods proposed for estimating daily volatility in financial markets utilizing intraday data [17–20]. We utilize the following definition for volatility [20–22]

\[
V(t) = \left[ \sum_{t-\Delta t < \tau \leq t} r^2(\tau) \right]^{1/2},
\]

which is the root of the sum of squared returns\(^1\). In this work, we consider minutely and daily volatilities. For daily volatility (\(\Delta t\) is one day), we adopt the first data set so that \(\delta t\) is one minute. For minutely volatility (\(\Delta t\) is one minute), we use the second data set so that \(\delta t\) equals to one step of event time.

Omori’s law vs. power-law relaxation. It is well-known that there is long memory and clustering in the volatility. Large shocks in the volatility are often followed by a series of aftershocks and the occurrence number of events with the volatility exceeding a given threshold decreases with time. Recently, the so-called Omori’s law was borrowed from Geophysics to describe the dynamics of financial aftershocks. Omori’s law states that the number of aftershocks decays with some power

\(^1\)The definition we adopt is different slightly from that of Ref. [20–22], in which the intraday returns are filtered with an MA(1) model and the variance \(V^2(t)\) is considered instead of \(V(t)\).
law of the time after large shocks: \( n(t) \propto t^{-p} \). In order to avoid divergence at \( t = 0 \), Omori’s law is often rewritten as
\[
n(t) = K(t + \tau)^{-p},
\]
where \( K \) and \( \tau \) are two positive constants, and \( n(t) \) is the occurrence rate of aftershocks during \( (0, t] \). Equivalently, the cumulative number of aftershocks after large volatility shocks can be expressed as follows
\[
N(t) = \begin{cases} 
K[(t + \tau)^{1-p} - \tau^{1-p}] / (1 - p), & p \neq 1 \\
K \ln(t/\tau + 1), & p = 1
\end{cases}
\]
(4)

In the empirical analysis, the parameters \( K, p \) and \( \tau \) are estimated using nonlinear least-squares regression.

**Empirical analysis.**

**Identifying large volatility shocks.** When crashes are concerned, one have to address the problem how to define a crash, on which a consensus is still lack. A quite feasible and unambiguous definition is based on large drawdowns [23–25]. An alternative option is to seek for large price drops within different time windows [26], which was essentially the same idea used by Selçuk [13]. A third method is to investigate those large price drops identified as crashes by academics and professionals. These three methods identify partially overlapping examples of crashes, that were used in the previous studies of the dynamics of volatility after crashes [11–15].

The situation is different in this study. We concern with large volatility shocks, which correspond not only to crashes but also to rallies. For the daily volatility, we first select the seven largest volatilities. If the time interval between two events are smaller than 30 trading days, the smaller one is excluded as a foreshock or aftershock. We then determine the duration of the impact by identifying the local minimum of volatility after the main shock within 60 trading days. This selection procedure is detailed in the following.

Figure 1(a) illustrates the evolution of daily volatility for the SSEC index constructed from minutely returns. The initial seven days are 10/23/2001, 11/16/2001, 05/21/2002, 06/24/2002, 02/02/2004, 10/29/2004 and 07/05/2006, respectively. We observe that 11/16/2001 is very close to 10/23/2001. Since the volatility of 10/23/2001 is larger than that of 11/16/2001, we treat the former as a main shock and the latter its aftershock. Similarly, the dates 05/21/2002 and 06/24/2002 are also close to each other and the volatility of the latter is much larger than that of the former. The latter is thus regarded as the main shock and the former is considered as its foreshock. Since 06/05/2006 is near the end of the sample period, it is excluded from investigation. We are thus left with four events on 10/23/2001, 06/24/2002, 02/02/2004 and 10/29/2004 for analysis, as indicated in Fig. 1(a) with arrows.
Figure 1(b) shows the locations of the four selected events in the price trajectory of the SSEC index. It is worthy of noting that four pieces of information took place on those four days. The Chinese stock market entered an antibubble regime since August 2001, which was triggered by the promulgation of the Tentative Administrative Measures for Raising Social Security Funds through the Sale of State-Owned Shares by the State Council of China on 06/24/2001 [27]. China’s Securities Regulatory Commission, however, suspended the fifth rule of the “Provisional Rules” in the evening of 10/22/2001 and the SSEC index rose up by 9.86%2. On 2002/06/24, the State Council of China announced the removal with immediate effect of the provisions of the Tentative Administrative Measures for Raising Social Security Funds through the Sale of State-Owned Shares. This caused an increase of 9.25% in the SSEC index. In the weekend right before 02/01/2004 (Monday), the State Council of China released Some Opinions of the State Council on Promoting the Reform, Opening and Steady Growth of Capital Markets [Effective] and the market fluctuated remarkably with a 2.08% rise. In the same year, the decision of the People’s Bank of China to raise the benchmark lending and deposit interest rates, lift the ceiling of RMB loan interest rates, and allow a downward movement of RMB deposit interest rates took effect from October 29. On the same day, the market swang a lot and closed with a -1.58% drop.

Volatility shocks with different magnitude may have distinct durations of impact. In order to investigate the relaxation behavior of volatility after a shock, it is crucial to determine this impact duration. The simplest but crude way would be to fix the impact duration for all shocks, say, 60 or 100 trading days. The shortcoming of this rule is obvious. In this work, we use a relatively objective way in which the local minimum of volatility after an identified shock is regarded as the end of its impact. Specifically, the local minimum is determined within a 60-trading-day window ensuing the shock. The main information of the four identified large volatility shocks are presented in Table 1. As expected, the impact duration of the main shock increases approximately with the relative magnitude $V_{\text{max}}/\sigma$, where $\sigma$ is the sample average of the volatilities. In addition, it is interesting to note that the four large volatility shocks are more relevant to rallies rather than crashes.

Table 1: Description of the four large volatility shocks for the SSEC index. The sample standard deviation of daily volatility $\sigma = 0.000421$. $t_0$ is the date of volatility shock and $t_1$ is the last date of shock impact. $T$ is the duration of shock impact from $t_0$ to $t_1$ in unit of trading day. $V_{\text{max}}$ and $V_{\text{min}}$ are the volatilities at time $t_0$ and $t_1$, respectively. $r(t_0)$ is the daily return on day $t_0$ referenced to the previous trading day.

| $t_0$     | $t_1$     | $T$ | $V_{\text{max}}/\sigma$ | $V_{\text{max}}$ | $V_{\text{min}}$ | $r(t_0)$ |
|-----------|-----------|-----|--------------------------|------------------|------------------|----------|
| 10/23/2001| 11/29/2001| 28  | 12.9                     | 0.00543          | 0.00019          | +9.86%   |
| 06/24/2002| 09/13/2002| 60  | 13.7                     | 0.00578          | 0.00015          | +9.25%   |
| 01/02/2004| 04/07/2004| 48  | 5.5                      | 0.00230          | 0.00022          | +2.08%   |
| 10/29/2004| 11/25/2004| 20  | 4.4                      | 0.00186          | 0.00023          | -1.58%   |

In order to investigate the relaxation behavior of large volatility shocks at different time scales, we also calculated minutely volatility and used the same shocks identified in daily volatility for comparison. Since the minutely time series is shorter, we have two large shocks left on 01/02/2004 and 10/29/2004. Compared with the large fluctuations in the Chinese stock market, these two shocks correspond to neither crashes nor rallies.

Aftershock dynamics in daily volatility. We have calculated the cumulative number $N(t)$ of aftershocks larger than some fixed threshold after each main shock. The threshold $\theta$ is presented based on the sample standard deviation of daily volatility, which is $\sigma = 4.21 \times 10^{-4}$ within the time period concerned. Four thresholds are selected for each main shocks: $\theta/\sigma = 0.6, 0.7, 0.8, 0.9$ for 10/23/2001, 06/24/2002, and 10/29/2004 and $\theta/\sigma = 0.7, 0.8, 0.9, 1.0$ for 02/02/2004. The resulting functions $N(t)$ are plotted in Fig. 2. Note that the selection of thresholds is not arbitrary. If $\theta \ll \sigma$, all trading days are identified as aftershocks such that $N(t) = t$, which is illustrated in each panel of

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2There is a price limit of ±10% fluctuation compared with the closure price on the last trading day. A rise of 9.86% in the SSEC means that most of the stocks went up by 10%.
Fig. 2. If $\theta$ is too large, $N(t)$ becomes constant shortly after the main shock. This phenomena can be observed in Fig. 2.

Fig. 2: Cumulative number $N(t)$ of aftershocks larger than some fixed threshold after the main shock occurred on (a) 10/23/2001, (b) 06/24/2002, (c) 02/02/2004, and (d) 10/29/2004. The sample standard deviation of daily volatility is $\sigma = 4.21 \times 10^{-4}$. The thresholds are 0.6$\sigma$, 0.7$\sigma$, 0.8$\sigma$, and 0.9$\sigma$ for (a) 10/23/2001, (b) 06/24/2002, and (d) 10/29/2004 and 0.7$\sigma$, 0.8$\sigma$, 0.9$\sigma$, and 1.0$\sigma$ for (c) 02/02/2004. The solid lines are best fits to the data with Eq. (4). The dashed line is $N(t) = t$.

The power-law relaxation model (4) is calibrated for each empirical $N(t)$ function shown in Fig. 2. The estimated relaxation exponents $p$ and characteristic time scales $\tau$ are digested in Table 2. All the exponents $p$ are significantly larger than 1 except one case ($t_0 = 06/24/2002, \theta = 0.8\sigma, p = 0.99$). We notice that these exponents are much larger than the relaxation exponents of daily volatility for many other emerging markets [13]. In other words, the daily volatility after a main shock relaxes much faster in the Chinese stock market. In addition, the relaxation exponent $p$ increases with the threshold $\theta$ implying that larger aftershocks decay faster. This behavior is analogous to that of the intraday volatility of the S&P 500 index [12].

Table 2: Exponents and characteristic time scales of daily volatility relaxation after four large volatility shocks in Chinese stock market. The sample mean of daily volatility is $\sigma = 4.21 \times 10^{-4}$. The four thresholds for 02/02/2004 are 0.7$\sigma$, 0.8$\sigma$, 0.9$\sigma$, and 1.0$\sigma$, different from those shown in the table. The unit of $\tau$ is trading day.

| $t_0$    | 0.6$\sigma$ | 0.7$\sigma$ | 0.8$\sigma$ | 0.9$\sigma$ | 0.6$\sigma$ | 0.7$\sigma$ | 0.8$\sigma$ | 0.9$\sigma$ |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 10/23/2001 | 1.70        | 1.94        | 1.83        | 2.13        | 28.2        | 20.9        | 23.6        | 38.8        |
| 06/24/2002 | 1.86        | 1.86        | 0.99        | 2.14        | 28.1        | 14.8        | 2.1         | 7.9         |
| 02/02/2004 | 1.48        | 1.62        | 2.10        | 2.40        | 43.7        | 36.6        | 38.4        | 29.1        |
| 10/29/2004 | 1.79        | 1.80        | 1.91        | 2.31        | 23.2        | 23.7        | 18.4        | 12.7        |
Aftershock dynamics in minutely volatility. We have also calculated the cumulative number $N(t)$ of aftershocks larger than some fixed threshold after each main shock in minutely SSEC volatility. In this case, the sample standard deviation of minutely volatility is $\sigma = 1.49 \times 10^{-4}$ within the time period of the data. Three thresholds are selected for each main shocks: $\theta/\sigma = 1.5, 1.75, \text{and} 2.0$ for both shocks. Again, the selection of thresholds is not arbitrary. Smaller thresholds give straight lines $N(t) \approx t$, while too large thresholds produce constant $N(t)$ shortly after the main shock. The resulting functions $N(t)$ are plotted in Fig. 3. We observe that the three curves for 02/02/2004 on panel (a) of Fig. 3 exhibit nice power-law curvature. However, the three curves on panel (b) are not different remarkably from straight lines. Hence, the minutely SSEC volatility relaxation after big shocks behaves differently from that for the S&P 500 index, in which power-law behavior is unveiled after mediate shocks [15]. We submit that the difference between the construction of volatility in the two studies can not account for this discrepancy.

![Fig. 3](image-url) Cumulative number $N(t)$ of aftershocks larger than some fixed threshold after the main shock occurred on (a) 02/02/2004 and (b) 10/29/2004. The sample standard deviation of daily volatility is $\sigma = 1.49 \times 10^{-4}$. The thresholds are $1.5\sigma, 1.75\sigma$, and $2.0\sigma$. The solid lines are best fits to the data with Eq. (4).

We fitted the empirical $N(t)$ functions to Eq. (4), as shown in Fig. 3 with smooth curves. The estimated relaxation exponents $p$ and characteristic time scales $\tau$ are digested in Table 3. All the exponents $p$ except for $p = 0.97$ are significantly larger than 1. Similar to the daily volatility case, the relaxation exponent $p$ for minutely volatility increases with the threshold $\theta$. This observation is different from those in other stock markets [11, 12, 14, 15]. It is interesting to note that the characteristic timescale $\tau$ for 10/29/2004 is quite large. Applying Taylor’s expansion, we have $(1 + t/\tau)^{1-p} = 1 + (1-p)(t/\tau) + o(t/\tau)$ and $\ln(t/\tau + 1) = t/\tau + o(t/\tau)$ when $t \ll \tau$. Instituting these two linear approximations into Eq. (4) gives

$$N(t) = K\tau^{-p}t$$

This simple algebraic derivation explains the almost-linear behavior of $N(t)$ for 10/29/2004 shown in Fig. 3(b). In this case, $p$ is a parameter controlling the slope of the linear $N(t)$. Therefore, the observation that $p$ increases with $\theta$ and the fact that the slope of $N(t)$ decreases with $\theta$ by definition are consistent with each other.

Concluding remarks. - In summary, we have investigated the dynamic behavior of the Chinese SSEC volatility after large volatility shocks. Daily and minutely volatilities are considered, which are calculated based on high-frequency data at finer scales. We stress that large volatility shocks are adopted as financial earthquake, rather than the crashes. These large volatility shocks are selected objectively, together with the durations of shock impact. In this way, no main shock is qualified as a crash in our analysis. Instead, two rallies are selected, which have large volatility. The selection of the volatility threshold $\theta$ is no arbitrary. Too small or too large $\theta$ gives $N(t) \sim t$ or $N(t) = \text{const.}$, respectively.
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Table 3: Exponents and characteristic time scales of minutely volatility relaxation after two large volatility shocks in the Chinese stock market. The sample mean of minutely volatility is $\sigma = 1.49 \times 10^{-4}$. The unit of $\tau$ is trading day.

| $t_0$          | $p_{1.5\sigma}$ | $p_{1.75\sigma}$ | $p_{2.0\sigma}$ | $\tau_{1.5\sigma}$ | $\tau_{1.75\sigma}$ | $\tau_{2.0\sigma}$ |
|----------------|------------------|------------------|------------------|---------------------|---------------------|---------------------|
| 02/02/2004     | 0.97             | 1.58             | 1.72             | 9.4                 | 16.3                | 18.0                |
| 10/29/2004     | 1.54             | 1.74             | 1.76             | 72.7                | 104.0               | 43.7                |

We have found that the cumulative number $N(t)$ of aftershocks with magnitude exceeding a given threshold $\theta$ increases as a power-law to the time distance $t$ to the main shock. The power-law exponent $p$ value is an increasing function of the volatility threshold $\theta$. The aftershock dynamics is very different for the Chinese SSEC index volatility in the sense that most of the values of $p$ are significantly larger than 1. Hence, the power-law relaxation behavior is different from the Omori law where $p$ is close to 1. This study thus adds a new ingredient to the effort in searching for idiosyncratic behaviors in the Chinese stock markets [27–29].

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