Three-dimensional fracture Propagation model and simulation study of CBM wells

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Abstract: Hydraulic fracturing is an important measure and main method to increase storage, production, and effective development of coalbed gas wells. And correctly understanding the morphology and geometric dimensions of fractures is the key to hydraulic fracturing stimulation. In order to understand the formation, analyze and evaluate the quality of fracturing construction, a three-dimensional propagation model of vertical fracture is established by using fluid mechanics and linear elastic fracture mechanics. The model considers the multi-layered and heterogeneous characteristics of the coal seam, that is, multi-layered coal seam thickness, in-situ stress, and rock mechanics parameters such as Poisson's ratio, elastic modulus and fracture toughness. The model can simulate arbitrary multilayered stress distribution patterns and the propagation of the fracture before and after the layer. Research and develop three-dimensional extension simulation software for vertical cracks of CBM wells. Through the calculation of the example, the calculated value of the established three-dimensional fracture propagation model coincides with the fracture parameter interpretation result, which verifies the practicability and reliability of the model.

1. Introduction

Hydraulic fracturing is the main measure to increase production of coal reservoir, and the effect of hydraulic fracturing depends on the understanding of formation condition. Although hydraulic fracturing has made great progress in theory, equipment, and processes, there are still some problems. For example, there are no methods for directly measuring the hydraulic fracture parameters, the real parameters of underground fracture are unknown. Different methods only reflect the same problem from different perspectives[1~2]. This makes it difficult to understand the stratum and analyze and evaluate the fracturing operation quality.

With the development and application of hydraulic fracturing technology, post pressure analysis and evaluation technology has received extensive attention, and the corresponding testing methods and interpretation techniques have been further developed and improved. The well test analysis technique of vertical fractured well is an effective method to know and understand the formation, analyze and evaluate the quality of the fracturing. For the analysis and interpretation methods of vertical fracture wells, due to the traditional characteristic straight line method and the typical curve fitting method, the understanding of fracture geometry and the simulation of fracture are not accurate. Based on the knowledge of fluid mechanics, linear elastic fracture mechanics, heat transfer, computational mathematics and software engineering, propose a three-dimensional propagation model for coal seam
fracturing, develop a three-dimensional extension simulation software for vertical cracks of CBM wells. It has been applied to verify its practicality and reliability.

2. Establishment of three-dimensional fracture propagation model

During hydraulic fracturing, the fracture extend at the same time in three directions of length, height, and width. The actual coal seam conditions are various. Vertically, coal seams often have multiple and heterogeneous characteristics, that is, the thickness of each layer may be different, and the lithology is not necessarily the same, there are some or even significant differences in rock mechanical parameters and in-situ stress distribution. Therefore, the three-dimensional extension model of vertical cracks must be able to adapt to any multi-layered stress distribution pattern. The influence of mechanical parameters such as elastic modulus, Poisson's ratio and fracture toughness on the three-dimensional extension of fracture can be well considered. So, this paper establish a set of three-dimensional extension models for vertical fracture based on the study of vertical fracture propagation models at home and abroad. The model considers the multi-layered and heterogeneous characteristics of the coal seam, that is, the influence of the change of coal seam thickness, ground stress and rock mechanics parameters (such as Poisson's ratio, modulus of elasticity, fracture toughness). The model can simulate all kinds of stress distribution patterns of arbitrary multi-layer and the extension of fracture before and after penetration.

Fig.1 Fracture and coal seam longitudinal section

The fracture and coal seam longitudinal section as shown in Fig.1, and establish coordinate system. There is any M layer at the upper part of the fracture layer and any N layer at the lower part of the fractured layer, the thickness of coal seam, in-situ stress, Poisson's ratio, modulus of elasticity and fracture toughness of each layer can be different.

Take a unit of length $\Delta x$ along the length of the fracture for research, regardless of the compressibility of the fracturing fluid, the continuity equation is derived from the principle of volume balance. According to the method of Nolte's introduction of the pipe shape factor to the pressure drop equation of the fluid flow in the parallel plate, the pressure drop equation in the three-dimensional fracture can be determined. According to the England and Green formula to calculate the width at any coordinate $z$ on the width profile, the fracture width equation can be obtained. Calculate the height equation by calculating the stress intensity factors at both ends of the fracture in any vertical cross-section\textsuperscript{[3–8]}. The specific equation is

(1) Continuity equation
\[- \frac{\partial q(x,t)}{\partial x} = \frac{2h(x,t)C_i(x,t)}{\sqrt{t-\tau(x)}} + \frac{\partial A(x,t)}{\partial t}\]

(2) Pressure drop equation
\[\frac{\partial p(x,t)}{\partial x} = f_1[w(x,z,t), h(x,t), q(x,t)]\]

(3) Fracture width equation
\[w(x,z,t) = f_2[p(x,z,t), h(x,t)]\]

(4) Fracture height equation
\[p(x,t) = f_3[h_{up}(x,t), h_{low}(x,t)]\]

The above formula constitutes a set of equations about \(p(x,t)\), \(h_{up}(x,t)\), \(h_{low}(x,t)\) and \(w(x,z,t)\), implicit unknowns between equations. In order to further simplify the calculation and obtain the equation that is easy to solve, it needs to be sorted out. From the aforementioned fracture height equation, let \(p = p_f(x,t) - S_i\). The two ends are derived as the following equations:

\[
\begin{align*}
\frac{\partial h_{up}(x,t)}{\partial x} &= \Phi_1[h_{up}(x,t), h_{low}(x,t), q(x,t), x, t] \\
\frac{\partial h_{low}(x,t)}{\partial x} &= \Phi_2[h_{up}(x,t), h_{low}(x,t), q(x,t), x, t] \\
- \frac{\partial q(x,t)}{\partial x} &= \frac{2(h(x,t)C_i(x,t))}{\sqrt{t-\tau(x)}} + \frac{\partial A(x,t)}{\partial t}
\end{align*}
\]

Its boundary conditions and initial conditions are:
\[
\begin{align*}
h_{up}(x,t) \bigg|_{x=L_j} &= 0 \\
h_{low}(x,t) \bigg|_{x=L_j} &= 0 \\
q(x,t) \bigg|_{x=L_j} &= 0
\end{align*}
\]

3. Method for solving fracture three-dimensional model
Since the above differential equations and their boundary conditions and initial conditions contain unknown variables \(L_j\). Therefore, the two-dimensional extensional mathematical model formed by them is not complete. There are also additional conditions:
\[q(x,t) \bigg|_{x=0} = \frac{Q(t)}{2}\]

The method of solving the model is as follows: If the length of the fracture is \(L\), the above differential equations and their boundary conditions and initial conditions can be solved, and the flow rate \(q(x,t) \bigg|_{x=0}\) at the wellbore can be calculated. It is clear that the calculated flow of the wellbore should be related to the length of the fracture \(L\), that is \(q(x,t;L) \bigg|_{x=0}\). So, we construct the objective function:

\[f(L) = \int_0^T \left[ q(x,t;L) \bigg|_{x=0} - \frac{Q(t)}{2} \right]^2 dt \]

Thus, the problem of finding the length of fracture translates into the following optimization problems:
\[f(L_j) = \min \{ f(L) \}\]

Since the objective function is a single down-peak function, the above-mentioned optimization problem of finding fracture lengths is not difficult to solve with a general optimization solution.
Where, \( q(x,t) \) is the flow at the x in the fracture at time t, m\(^2\)/s; \( h(x,t) \) is the fracture height, m; \( C_r(x,t) \) is the comprehensive loss coefficient of X in the t time slot, m/s\(^{0.5}\); \( A(x,t) \) is the cross-sectional area at x in the seam at time t; \( \tau(x) \) is the time of the fracturing fluid to reach the point x of inside of the fracture, s; \( t \) is the time fracturing construction, s; \( p(x,t) \) is the pressure, MPa; \( w(x,z,t) \) is the fracture width the cross section at the x position in the slit, m; \( h_{up} \) is the height of the upper fracture, m; \( h_{low} \) is the height of the bottom fracture, m; \( Q \) is the construction displacement, m\(^3\)/s; \( L_f \) is the half length of fracture, m.

4. Simulation and calculation of three-dimensional fracture model

Based on the above three-dimensional fracture extension model, combined with the numerical model of wellbore temperature field and temperature field\(^{[9]}\), proppant migration and distribution model\(^{[10]}\), and fracturing parameter optimization design method, a three-dimensional extension simulation software for vertical cracks in CBM wells was developed. According to a coal bed gas well fracturing basic parameters, the depth of the coal seam is 890m, coal thickness is 11.4m, permeability is 0.02mD, porosity is 3013%, elastic Modulus is 6309, poisson's ratio is 0.22, the minimum principal stress is 19.25MPa, the maximum principal stress is 26.84MPa, the vertical stress is 22.74, bursting pressure is 24.14MPa. The the upper cover of elastic modulus is 37500MPa, porosity is 0.36, the minimum principal stress is 22.55MPa, the maximum principal stress is 27.95MPa, the vertical stress is 23.65, bursting pressure is 35.91MPa. The the lower layer of elastic modulus is 36570MPa, poisson's ratio is 0.38, the minimum principal stress is 28.73MPa, the maximum principal stress is 28.37MPa, the vertical stress is 26.78, bursting pressure is 36.98MPa. Based on the above basic parameters of a coal seam gas well, simulations and calculations were performed. And given a three-dimensional dynamic fracture propagation in the coal seam(Fig.2.) and the corresponding proppant transport distribution(Fig.3.). According to the calculation of the well, the half length of fracture is 181.6m, the fracture height is 26.84m and the fracture width is 11.63mm(Table.1), the calculated results are basically consistent with the actual interpretation of the fracture. It also verifies the correctness and practicality of this model.

| Table.1 Comparison of fracture parameter interpretation results with model calculation results |
|-----------------------------------------------|--------|--------|--------|
| Fracture geometry                            | Half length of fracture (m) | Fracture height (m) | Fracture width (mm) |
| Model calculation results                     | 181.6  | 26.84  | 11.63  |
| Actual interpretation result                 | 186.2  | 28.65  | 10.46  |
5. Conclusion
(1) Based on comprehensive knowledge of fluid mechanics, linear elastic fracture mechanics, heat transfer theory, computational mathematics and software engineering, propose a three-dimensional propagation model of fractures in coal seams.

(2) Considering the influence of multilayer and heterogeneity of coal seam, that is, coal seam thickness, ground stress and rock mechanics parameters, establish a set of three-dimensional propagation model of vertical fracture.

(3) Develop a three-dimensional propagation simulation software for vertical fracture of coalbed gas wells. The calculated values of the three-dimensional fracture propagation model are in good agreement with the results of fracture parameter interpretation, which verifies the practicability and reliability of the model.

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