A novel reduced-order method for analysis of hyperspectral images

Yanming Zhang¹, ² and Lijun Jiang¹

¹Department of Electrical and Electronic Engineering, the University of Hong Kong, Pokfulam, Hong Kong
²Corresponding author’s e-mail: ymzhang@eee.hku.hk

Abstract. Hyperspectral imaging technology has been broadly applied in remote sensing because it collects echoed signals from across the electromagnetic (EM) spectrum and provides fruitfully helpful information. However, the processing or transformation of high-data-volume hyperspectral images, also viewed as snapshots varying with the EM spectrum, burdens the hardware resources, especially for the high spectral resolution and spatial resolution cases. To tackle this challenge, a novel reduced-order method based on the dynamic mode decomposition (DMD) algorithm is presented here to analyze hyperspectral images. The method decomposes the spatial-spectral hyperspectral images in terms of spatial dynamic modes and corresponding spectral patterns. Then, these spatial-spectral patterns are utilized to recover the raw hyperspectral images. Our proposed approach is benchmarked by the actual hyperspectral images measured at the Salinas scene. It is demonstrated that the proposed approach can represent the hyperspectral images with a low-rank model in spectral dimension. Our proposed approach could provide a useful tool for the model order reduction of hyperspectral images.

1. Introduction
Hyperspectral images refer to a set of images of the same observation scenario, each of which is measured at a specific EM spectrum. Moreover, it has been demonstrated that hyperspectral images have wide application in classifying objects, detecting targets, and extracting terrain’s features as it offering plenty of information not only in the spatial dimension but also the spectral dimension. Generally, the collected images form three dimensional hyperspectral data cube, (x, y, λ). Hereinto, the x and y denote the two spatial dimensions, and λ refers to the spectral dimension, which corresponds to the EM wavelength. However, while the development of hyperspectral images with high spectral resolution and spatial resolution can provide extremely rich and diverse spatial-spectral information, it also causes various issues. In particular, it is still challenging for hardware processor to deal with high spectral and spatial resolution hyperspectral images that are often very high-volume dataset. To address this challenge, several model order reduction (MOR) technologies have been developed. Hereinto, the principal components analysis (PCA) is explored to realize dimension reduction, and the principal components decomposed by PCA are evaluated to robustly detect the difficult targets in hyperspectral image datasets [1]. Afterwards various spatial dimensionality reduced methods have been developed and applied to MOR of hyperspectral images, such as graph embedding (GE) method [2], locally linear embedding (LLE) approach [3], and the low-rank learning [4]. It is worth mentioning that, with the recent development of machine learning and deep learning, the dimensionality reduction methods based on convolutional neural networks have been proposed.
However, in order to achieve great results, a large amount of hyperspectral data and consumption are required during the training step, which also puts a processing burden on the hardware. While the above-mentioned studies have achieved significant developments, it is still an open and challenging issue for the dimensionality reduction of hyperspectral imagery.

In this paper, we investigate the DMD approach to reduce the dimension of hyperspectral imagery. Through the DMD method, the spatial-spectral hyperspectral data cube is decomposed into the spatial dynamic modes and corresponding spectral patterns. These decomposed patterns can represent the raw hyperspectral data via the low-dimensional linear combination. The actual hyperspectral images measured at the Salinas scene are used to verify the proposed method. The advantages of this newly proposed approach are listed as followed. (1) The DMD-based method is a spectral dimension reduction and could be hybridized with traditional spatial dimension reduction methods. (2) Unlike supervised learning, this method does not demand a training process and then could be executed more economically and efficiently. (3) The output of this approach offers a spatial-spectral correlated subspace, which could be helpful for feature extraction and target classification. The remainder of the article is formed as follows. Section II presents the methodology of the proposed approach. The verification based on the actual hyperspectral imagery dataset is exhibited in Section III. Finally, the conclusions are drawn.

2. Methodology

2.1. Background

Often, the spatial-spectral hyperspectral images can be regarded as a three-dimensional data cube. To be specific, we assume that \( d(x_i, y_j, \lambda_k), i = 1, \ldots, O, j = 1, \ldots, P, k = 1, 2, \ldots, l, \ldots N \) denotes a pixel observed at a specific EM spectrum. If the vector \( r = (x, y) \) is defined to represent the spatial domain, one can obtain a set of snapshots varying with the EM spectrum, which is expressed as followed.

\[
D = \begin{bmatrix}
    d(r_1, \lambda_1) & \cdots & d(r_1, \lambda_N) \\
    \vdots & \ddots & \vdots \\
    d(r_M, \lambda_1) & \cdots & d(r_M, \lambda_N)
\end{bmatrix} \in \mathbb{R}^{M \times N} \tag{1}
\]

Where \( \mathbb{R} \) denotes real analytic space, \( M \) is the number of pixels in each image, and \( l \)th column of \( D \) means the observation sequence \( d_l \), also named as the \( l \)th snapshot. Then, the raw hyperspectral images can be expressed as: \( D = [d_1, d_2, \ldots, d_l, \ldots, d_M] \).

2.2. Dynamic mode decomposition

Traditionally, the DMD is used to decompose the spatial-temporal signal into the term of oscillation decay/growth rate and frequency [5]. This approach has been applied in many fields, such as EM vortex beam demultiplexing [6], EM radiation analysis [7], financial trading strategies [8], and ship wake detection [9]. If the spectral domain of hyperspectral images is analogous to the temporal domain, the spatial-spectral hyperspectral images can be analysed by this decomposition method. In particular, we first separate the data into two sets of snapshots as

\[
D_1 = [d_1, d_2, \ldots, d_{M-1}] \tag{2}
\]

\[
D_2 = [d_2, \ldots, d_{M-1}, d_M] \tag{3}
\]

We assume a local approximation: \( d_{l+1} = F(d_l) \), so the connection between equation (2) and equation (3) can be established, which is expressed as follows.

\[
D_2 = PD_1 \tag{4}
\]
Where $P$ means the DMD mapping matrix. DMD is to compute the eigenvectors and eigenvalues of the mapping matrix $P$. Hereinto, we adopt the singular value decomposition (SVD) based approach to realize the DMD algorithm. The steps are shown as follows. First, $D_1$ is represented by the decomposed results obtained by SVD.

$$D_1 = \Psi \Sigma V^*$$  \hspace{1cm} (5)

Where $*$ refers to the conjugate transpose. Then the mapping matrix $P$ can be expressed by

$$P = D_2 \Sigma^{-1} T^*$$  \hspace{1cm} (6)

Finally, through the eigen decomposition of $P$ in low-rank system that is calculated via computing proper orthogonal decomposition (POD) on the mapping matrix, one can obtain the approximate solution of the hyperspectral snapshots, which is given by:

$$d(\lambda) = \sum_{k=1}^{n} \psi_k e^{\omega_k \lambda} b_k = \Psi \exp(\Omega \lambda) b$$  \hspace{1cm} (7)

where $\psi_k$ and $\omega_k$ refer to the eigenvalue and eigenvector of the mapping matrix $P$, which correspond to the spatial dynamic mode and spectral pattern respectively, $b_k$ refers to the initial intensity of $k$th mode. The physical meaning of this decomposition is straightforward. The spatial-spectral hyperspectral images are represented by the combination of spatial dynamic mode and low-rank spectral pattern. Hence, MOR for high-volume hyperspectral datasets could be realized with this architecture.

3. Results

To verify the proposed method, we adopt the actual measured hyperspectral data at the Salinas scene. The hyperspectral images dataset is measured by the Airborne Visible Infrared Imaging Spectrometer (AVIRIS) sensor. Moreover, this hyperspectral observation is collected across 224 EM bands. Also, the spatial resolution of hyperspectral data measured at Salinas Valley is about 3.7 meter in each pixel. Figure 1 shows the hyperspectral images with different EM bands. It can be seen that the observed area has different geomorphic features. These geomorphic features can be distinguished well in some EM wavebands (such as number of band 100), but it is difficult to distinguish them in several EM wavebands (such as number of band 1). After obtaining the hyperspectral image data, we first reshape the two-dimensional spatial matrix into a spatial vector. Then, the three-dimensional hyperspectral data cub is arranged into a matrix according to the index of the EM spectrum. By inputting this narrow and tall deformation matrix into the DMD algorithm, we can obtain the spectral pattern and spatial modes. As shown in Figure 2, there are 11 eigenvalues in the decomposition. Since the hyperspectral data is in real analytic space, the extracted eigenvalues with non-zero imaginary part are all conjugate pairs to ensure that the reconstructed hyperspectral data is also in the real analytic space. It is worth noting that the extracted spectral pattern can be viewed as the dynamic characteristics of spectral direction.

Figure 3 plots the extracted spatial modes. We can see that all the geomorphic features are well represented in these modes. In different spatial modes, the intensity of the geomorphic features is also different, which provides a good foundation for classification problems in the hyperspectral images. Notably, unlike the classification is explored based on the original hyperspectral data, the classification in this scheme only use a streamlined amount of hyperspectral data, which is similar to the noise cancellation technology. In order to verify the effectiveness of the dimensionality reduction technology, we reconstruct the hyperspectral images according to the combination relationship shown in equation (7). Figure 4 shows the comparisons of the actual and reconstructed hyperspectral images under different EM bands. It is clear that the raw hyperspectral data is well represented in the reduced dimensionality obtained by the DMD approach. Also, the differences among the geomorphic features
are also well portrayed. That is to say, the hyperspectral data of 224 EM bands are compressed and well expressed through 11 the spectral pattern and spatial modes. Hence, the dimensionality of the hyperspectral images can be effectively reduced by the proposed DMD approach.

To further validate the proposed reduced-order approach, we quantitatively analyzed the error by calculating the normalized mean absolute error (NMAE) of the reconstructed results. Table 1 shows the NMAE versus the index of EM bands. It can be seen that NMAEs have small value at different EM bands, and all NMAE of the EM bands is less than 10%. This error is acceptable from the viewpoint of engineering. Performance of error analysis further demonstrates the effectiveness of the proposed approach.

Figure 1. The hyperspectral images at the Salinas scene with different EM bands.

Figure 2. The extracted spectral pattern.
Figure 3. The extracted spatial modes.

Figure 4. The comparisons of the actual and reconstructed hyperspectral images with different EM bands.
Table 1. NMAE versus the index of EM bands.

| Number of EM bands | NMAE   |
|--------------------|--------|
| #5                 | 0.0087 |
| #10                | 0.0158 |
| #100               | 0.0402 |
| #160               | 0.0581 |

4. Conclusion

In conclusion, we demonstrated the DMD-based approach to investigate and decompose the spatial-spectral hyperspectral images. The proposed data-driven approach deals with the hyperspectral image data as a two-dimensional correlated real signal. It decomposes hyperspectral image data into the DMD spatial modes and corresponding spectral patterns, which can be utilized to reconstruct the hyperspectral images. We verified the proposed based on the actual measured hyperspectral data at the Salinas scene. A two-dimensional spatial-spectral subspace can be obtained for feature classification through the extraction of these two relevant domains. Therefore, our paper offers a practical reduced-order technique for hyperspectral images.

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