A New Class of Solutions to the Strong CP Problem with a Small Two-Loop $\theta$

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We present a new class of models which produce zero $\theta_{\text{QCD}}$ angle at the tree and one-loop level due to hermiticity of sub-blocks in the extended quark mass matrices. The structure can be maintained typically by non-abelian generation symmetry. Two examples are given for this class of solutions.

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Introduction  
While the Standard Model (SM) has been enjoying fantastic success, it does have many loose ends which are potentially our guidepost to the new physics of the future. Two of the most significant loose ends are strong CP problem and the fermion mass hierarchy. Within the SM, the Yukawa couplings give rise to the fermion masses of all three generations and their mixings including the CP violation. Indeed it was first observed by Kobayashi and Maskawa[1] (KM) that only two generations cannot support any CP violating phase. The fact that all three generations have to be involved to create a CP violating phenomena, makes KM model an extremely subtle and beautiful model for CP violation. It also makes CP violation tightly connected with flavor physics.

One of the weaknesses of the SM is that it does not address the issue of why fermion masses are so awkwardly different. The top quark mass is five orders of magnitude bigger than electron mass which is in turn bigger than the neutrino mass by another four to five orders. This is the fundamental issue in flavor physics. However, there is an even more serious problem if one looks into the CP issue in the KM mechanism. The mechanism allows a tree level CP violating parameters $\theta$ associated with strong interaction. The experimental limit on the neutron electric dipole moment (EDM) requires this parameters to be of order $10^{-26}$ or smaller. The KM mechanism does not address why $\theta$ is so small. There are two levels to this problem, the tree and the loop levels. Since CP violating phase in KM is a part of the dimension-four Yukawa couplings, just like the tree level $\theta$ parameter, there is no reason why $\theta$ can not be a bare parameter of the model with a natural value of order one. However, it should be noted that, by being tightly connected with the flavor physics as described above, the KM mechanism has already embedded in itself a natural mechanism to suppress the loop correction to $\theta$. It has been shown that if one heuristically set $\theta$ to zero at the tree level, the loop correction will not occur till three-loop level (with two weak and one strong loop)[2], and logarithmic divergent correction to $\theta$ will appear only at the 14th order of the electroweak coupling $g_2$ (or, at the 7-loop level)[3,4].

Even if one put in the Planck scale as the estimate for the cut-off, the divergent correction produces only a minute value for $\theta$ just like the 3-loop finite corrections.

This nice loop property is a direct result of the coupling between flavor physics and CP in the KM model. It indicates that the strong CP problem is in a sense only a tree level problem. All we need is to look for a mechanism extending SM to suppress the tree level $\theta$ parameter. After that, the loop correction will take care of itself. Such mechanism does not have to be a low energy phenomenon. It can be some features embedded in a high energy theory such as GUT or string theory. For example, a popular class of models of this type is the Nelson-Barr mechanism[5,6] in which a (softly broken or gauged) flavor symmetry and spontaneously broken CP symmetry are used at high energy in a GUT-like theory to suppress the tree level $\theta$. The phase of the KM model is generated by introducing additional heavy vectorial fermions which can have CP violating mixing with the ordinary fermions. In models such as this, one typically have a one-loop induced $\theta$ at the higher energy scale. Such contributions are typically not suppressed by the heavy scale and it is up to the adjustment of the model parameters to make such contributions small enough. While flavor symmetry was used in examples provided by Ref.[5,6,7], it can potentially be replaced by some other symmetry. Still another recent example, Ref.[8] involving even more direct use of flavor symmetry, adopts both flavor and CP symmetry to make the up (down) quark mass matrix lower (upper) triangular with real diagonal elements. This also guarantees that the tree level $\theta$ is zero while the model still has enough parameters to create the KM phase of any magnitude. Clearly such mechanism can be easily embedded in GUT or SUSY context.

While not all the proposed solutions to the strong CP problem are strongly associated with flavor physics (for example, the Peccei-Quinn solution[9] can be quite independent of flavor), it should be interesting to speculate that the solution to the strong CP problem may be the side product of high energy theory that addresses the issues in the flavor problem.

In this letter, we wish to propose a new class of solu-
tions to the strong CP problem that can also potentially address the flavor physics issue. The new class of models we propose involves additional heavy fermions in high energy and a special mass matrix pattern for this extended set of fermions such that it results in vanishing strong CP $\theta$ even at one-loop level. The small two-loop $\theta$ in the high energy theory naturally produces an effective KM model at low energy with small enough tree level effective $\theta$ that predicts a small but potentially measurable neutron EDM. The special mass pattern can be the result of some family symmetry which can serve to tie up the issue of strong CP problem with that of the flavor problem in the future.

We shall illustrate the basic structure that characterizes this class of models and then provide two examples using two different kind of flavor group to produce the desired extended fermion mass pattern. Since it is not our purpose here to try to pin-point a particular realistic model at this point, we shall only present the simplest examples of such solution to the strong CP problem and leave the issues related to the fermion mass hierarchy to the future. This is reasonable because even if there is a common link to both problems, the strong CP problem seems to be more severe while the flavor problem seems to be more tedious and more tied to details of model building.

### Minimal Model of Hermitian Mass Matrix
The idea is simply to add vector-like fermions with the same charge as the up and down quarks so that the larger mass matrices are hermitian at tree level using flavor symmetry and spontaneously broken CP symmetry. This will guarantee that the tree level $\theta$ at high energy is zero. To make the radiative correction to $\theta$ small enough, one arrange additional symmetry to make one-loop contribution vanish in the high energy theory. This will result in a low energy effective theory whose CP violation is is KM in nature while effective tree level $\theta$ is naturally small.

Our simplest use a horizontal symmetry $SO(3)|_{\parallel}$ to achieve real determinants of quark mass matrices at both the tree and one-loop levels. In addition to the three existing generations, there are three new generations, which are individually $SU_L(2)$ singlets and vector-like; nonetheless, the hypercharges are chosen in a way that they have same electric charges as the known generations. Therefore they are labelled by

$$U_L, U_R, D_L, D_R,$$

in an analogous fashion with the known quarks,

$$q_{Li} \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri}.$$

Let us stress that their only difference is in $SU_L(2)$ assignments. A horizontal flavor symmetry $SO(3)|_{\parallel}$ transforms every Weyl fermion multiplet above in the $3$ representation, labelled by generation index $i = 1, 2, 3$. There are new horizontal neutral (invert to $SU(2)_L \times U(1)$) Higgs bosons, i.e. one quintet (symmetric traceless rank-2 tensor) CP-even $\phi_S$ and one triplet (antisymmetric rank-2 tensor) CP-odd $\phi_A$. The Yukawa couplings are

$$\bar{d}_R (\mu_d + g_{dS} \phi_S + i g_{dA} \phi_A) D_L$$

$$+ \bar{u}_R (\mu_u + g_{uS} \phi_S + i g_{uA} \phi_A) U_L$$

$$+ \bar{D}_R (\mu_D + g_{DS} \phi_S + i g_{DA} \phi_A) D_L$$

$$+ \bar{U}_R (\mu_U + g_{US} \phi_S + i g_{UA} \phi_A) U_L$$

$$(h_d \bar{d}_R + h_u \bar{U}_R) \hat{H}\dagger q_L + (h_u \bar{u}_R + h_d \bar{D}_R) \hat{H}\dagger q_L + H.c. (3)$$

with the usual $SU_L(2)$ Higgs doublet $H$ which couples to fermions flavor-blindly. We denote $\hat{H} \equiv i \tau_2 H^*$. CP symmetry is assumed to be a good symmetry before any symmetry breaking. Since $d_R$ and $D_R$ are identical triplet, one indeed can define $\bar{D}_R$ to be the one that couples with $q_L$ and there $h_d = 0$ by convention.

The horizontal symmetry is completely broken by VEV’s of $\phi_S$ and $\phi_A$, which together with $\langle H \rangle$ give rise to the following $6 \times 6$ down quark mass matrix term

$$\begin{pmatrix} \bar{d}_R \bar{D}_R \end{pmatrix} (M_6) \begin{pmatrix} d_L \\ D_L \end{pmatrix},$$

$$M_6 = \begin{pmatrix} 0 & \mu_d + g_{dS} \langle \phi_S \rangle + i g_{dA} \langle \phi_A \rangle \\ \mu_d + g_{dS} \langle \phi_S \rangle + i g_{dA} \langle \phi_A \rangle \end{pmatrix} (5)$$

and similarly for up quark mass matrix. The $6 \times 6$ matrix has real determinant when couplings are real, as required by the imposed CP symmetry.

Integrating out the exotic generation $D$, we have the effective mass matrix for the three light generations like CKM phenomenology. More explicitly, the effective masses of the known 3 generations can be understood as the amplitude given by the diagram below. It is a kind of see-saw mechanism in the quark sector. Similar diagrams occur for the up-type quarks.

In the limit of $\langle H \rangle = 0$, $d_L$ quarks decouple and we have the reduced mass matrix of a size $6 \times 3$ instead,

$$\begin{pmatrix} \bar{d}_R \bar{D}_R \end{pmatrix} \begin{pmatrix} M_d \\ M_D \\ M_L \end{pmatrix} \begin{pmatrix} D_L \\ d_L \end{pmatrix},$$

FIG. 1: A see-saw diagram for effective masses of the known 3 generations.
\[ M_d = \mu_d + g_{dS} \langle \phi_S \rangle + ig_{dA} \langle \phi_A \rangle, \]
\[ M_D = \mu_D + g_{DS} \langle \phi_S \rangle + ig_{DA} \langle \phi_A \rangle. \]  

(7)

First, we find a \( 6 \times 6 \) unitary matrix \( V \) to transform the above \( 6 \times 3 \) into one with zero entries in the upper \( 3 \times 3 \) block.

\[ V \left( \begin{array}{cc} M_d & 0 \\ M_D & M_D' \end{array} \right) \left( \begin{array}{c} 0 \\ M_D' \end{array} \right) = \left( \begin{array}{c} 0 \\ M_D' \end{array} \right), \]  

(8)

by choosing the top three row vectors of \( V \) perpendicular to the 3 column vectors in the mass matrix. This is possible because the 6 dimensional linear space is larger than the 3 dimensional space spanned by the three column vectors. Furthermore, by using bi-unitary transformation, we also rotate \( D_L \) into \( D_L' \) so that \( M_D' \) is diagonal. In this way, the massless states \( d_R \) and the massive states \( D'_R \) are generally mixed among the original \( d_R \) and \( D_R \). Nonetheless, the three generations of \( d_L \) remain massless and unmixed.

We include the effect \( (H) \) from the viewpoint of perturbation. The mass terms involving \( d_L \) are tabulated in the matrix form,

\[ (\bar{d}_R \, \tilde{D}_R) V \left( \begin{array}{ccc} 0 & & \\ h'_{d}(H^\dagger) & & \\ & & 1 \end{array} \right) d_L = (\bar{d}_R \, \tilde{D}_R) \left( \begin{array}{c} \bar{m}_d \\ \bar{m}'_d \end{array} \right) M_D' \left( \begin{array}{c} d_L \\ d_L' \end{array} \right). \]  

(9)

Including \( D'_L \), we have

\[ (\bar{d}'_R \, \tilde{D}'_R) \left( \begin{array}{ccc} \bar{m}_d & 0 & \\ 0 & M_D' & \\ \bar{m}'_d & 0 & M_D' \end{array} \right) \left( \begin{array}{c} d_L \\ d_L' \end{array} \right). \]  

(10)

As \( M_D' \gg \bar{m}_d \), masses of usual \( d \)-quarks are basically given by diagonalization of the complex mass matrix \( \bar{m}_d \). Similar procedures also apply to the \( u \)-quarks. Phenomenology of CKM mechanism follows.

**Strong CP at the one-loop level**  Since the hermiticity of each of the \( 3 \times 3 \) block of the \( M_6 \) mass matrix is a consequence of the horizontal flavor and CP symmetries, it is maintained even after \( M_6 \) receives radiative corrections from the induced higher dimensional operators. To see this, start with a simple case by working out an effective dim 5 operator of the form

\[ i\tilde{D}_L (f_{SA} \phi_S \phi_A + f_{AS} \phi_A \phi_S) d_R, \]  

(11)

which is induced at the loop level, like the figure below. The hermitian conjugate of above operator has the form

\[ -i\bar{d}_R (f'_{AS} \phi_S \phi_A + f'_{SA} \phi_A \phi_S) D_L. \]

However, there is a CP symmetry imposed in the beginning such that the operator is invariant under

\[ \phi_S \rightarrow \phi_S, \quad \phi_A \rightarrow -\phi_A, \quad \tilde{D}_L (\cdots) d_R \rightarrow \bar{d}_R (\cdots) D_L. \]  

(12)

This require \( f_{SA} = f'_{AS} \). This way, the induced effective mass matrix

\[ i(f_{SA} \langle \phi_S \rangle \langle \phi_A \rangle + f_{AS} \langle \phi_A \rangle \langle \phi_S \rangle) \]

remains hermitian. Generalization beyond this simple case is straightforward.

With loop effects, the left upper and lower blocks begin to deviate from the ones proportional to a unity matrix. As the texture at the tree level disappears, the determinant is no longer real at the one-loop level, therefore there will be one-loop contribution to \( \theta \). Although this model neither falls into the Nelson–Barr class \([5, 6]\) nor the Glashow type of models \([8]\), they all suffer from the one-loop contribution to the strong CP problem, which in principle can be suppressed by some choices of hierarchy structure in parameters of the models through fine-tuning. However, in our model, there is an alternative and natural mechanism to remove the one-loop strong \( \theta \).

If we impose a discrete symmetry under which \( u, d \) and all \( \phi_{A,S} \), \( H \) fields are odd, while \( U \) and \( D \) are even, we have

\[ h_d = 0, \quad g_{DS} = 0, \quad g_{DA} = 0. \]

We keep the term \( \mu_d \) which only breaks this discrete symmetry softly. The mass matrix becomes

\[ M_6 = \left( \begin{array}{ccc} 0 & & \\ h'_{d}(H^\dagger) & & \\ & & 1 \end{array} \right) \left( \begin{array}{c} \mu_d + g_{dS} \langle \phi_S \rangle + ig_{dA} \langle \phi_A \rangle \\ \mu_d \end{array} \right). \]  

(13)

To see that the one-loop correction to \( \theta \) is zero in this case, one can consider the one-loop corrections to each \( 3 \times 3 \) block one by one. For example, when the left upper block deviates from \( 0 \) and develop a small hermitian component, one can show that the determinant maintains real. This property follows from the following lemma in the matrix algebra,

\[ \left| \begin{array}{cc} C & A \\ D & B \end{array} \right| = |CB - DA|, \quad \text{provided} \ CB = DC. \]  

(14)

Similarly when the other \( 3 \times 3 \) block develops a small hermitian correction, it is easy to show that the determinant remains. Therefore the strong CP \( \theta \) is zero at one-loop level.

**SU(3) horizontal symmetry**  Our second example uses \( SU(3) \) horizontal symmetry to achieve the same
goal. It is a modification of a model proposed by Masiero and Yanagida.[11]

The fermion spectrum is extended the same as the $SU(3)$ model earlier. The only difference is that each flavor is now transform as a triplet under a horizontal flavor symmetry $SU(3)_{\parallel}$, instead of $SO(3)_{\parallel}$. In the scalar sector, there are now horizontal neutral (insert into $SU(2)_L \times U(1)$) Higgs bosons, three octets $\phi^a_{\alpha}$ ($a = 1, \cdots, 8, \alpha = 1, 2, 3$). The Yukawa couplings in the hamiltonian (for down quarks) are

$$d_R(g_{d\alpha} \phi^a_{\alpha} \lambda^a + m_D)D_L + D_R(g_{D\alpha} \phi^a_{\alpha} \lambda^a + M_D)D_L + h_d D_R H^\dagger q_L + h'^d D_R H^\dagger q_L + H.c., \tag{15}$$

where $m_D$ and $M_D$ are bare masses, and similarly for up quarks. Three octets are needed in order that $SU(3)_{\parallel}$ is broken completely and that both symmetric and antisymmetric components of octet representation develop VEV. CP is again assumed to be a symmetry before any dimensional operator, $(1/M^n) d_R d_R H f_n(\phi)$, where $f_n$ is a function of $\phi$ or $\Phi$. However, since the fermion bilinear can only either be $SU(3)$ singlet or octet, $f_n$ has to be effective singlet or octet. Since the effective octet will still give hermitian form, the contribution to $\theta$ must come from the effective singlet $f_n$. Using the argument similar to that of the case of $SO(3)$ discussed earlier, it is easy to see that in this $SU(3)$ flavor model with discrete symmetry, the $\theta$ is nonzero starting at the two-loop level.

Conclusion We have shown that flavor symmetry properly arranged at high energy can have the powerful consequence of producing a low energy effective KM model with naturally small strong CP $\theta$. We argue that the success of the KM model with very small $\theta$ may be providing us the clue of an flavor symmetry in high energy. This letter provides two existence proofs of this idea, the next step is to tackle the harder question of actually using the flavor symmetry to explain the fermion mass problem.

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