Non-Linear Effects in Resonant Tunneling; Bistabilities and Self-Sustained Oscillating Currents

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We study non-linear phenomena in double barrier heterostructures. Systems in 3D under the effect of an external magnetic field along the current and 1D systems are analyzed. Non-linearities are reflected in the I-V characteristic curve as bistabilities, instabilities and time dependent oscillations of the currents. The nature of the non-linear behavior depends upon the parameters that define the system.

I. INTRODUCTION

Resonant tunneling in double barrier heterostructures (DBH) have been found to exhibit a variety of interesting physical properties. Transport in a 3D DBH presents non-linear phenomena, which are reflected in the observation of multistabilities in the I-V characteristic curve. This property is produced by the rapid leakage of the electronic charge accumulated at the well between the barriers when the applied potential is just taking the device out of resonance. It is a result of the Coulomb charge interaction at the well and has been theoretically studied, in a 3D system, using a mean field approximation. In this work we have found a similar behavior in a 1D system constituted by a quantum dot connected to two leads. Although, the origin of the bistability is the non-linearity produced by the Coulomb interaction, as in the 3D system, what is dominant in a quantum dot is the spatial e-e correlation. It produces simultaneously the Coulomb blockade of the charges, that appears as periodic oscillations of the conductance as a function of the gate potential and the bistability of the current. For a 3D system instead, the Coulomb blockade is absent and the electronic interaction can be incorporated as a renormalization of the quasi-particle energies using a mean field approximation which gives rise to the non-linear effects.

Non-linear time dependent transport in double-barrier structures is another very interesting phenomena. In this work we report the presence of self-sustained current oscillations when a large magnetic field is applied along the direction of the current in a 3D DBH. The magnetic field produces a drastic change in the number of resonant electrons going through the well inducing a great enhancement of the non linearities of the system. The state of charge of the well controls the oscillatory behavior of the current. Our results show that the system bifurcates as the magnetic field is increased and may transit to chaos at large enough field. The oscillations are exponentially sensible to initial conditions or to infinitesimal changes in the parameter of the Hamiltonian, characterizing a chaotic behavior which has been studied in detail in the parameter space.

We obtain a similar result for the current going along the quantum dot connected to two leads. The produced self-sustained time dependent current oscillations is controlled in this case by the voltage applied to the metallic gate. In both cases the period of the oscillations depends upon the transit time of the electron going along the well, which is determined by the size of the barriers.

II. THE MODEL

Consider a 1D or a 3D double barrier structure (DBS) device. In order to study its time evolution under a bias we adopt a first-neighbors tight-binding model for the Hamiltonian. For the 3D situation we include an external magnetic field \( B \) in the growth direction. We assume a parabolic energy dispersion parallel to the interfaces while the field quantizes the motion of the electrons in the perpendicular plane giving rise to Landau levels with energies \( \epsilon_n = (n+\frac{1}{2})\hbar\omega_c \), where \( n = 0, 1, 2, ... \) is the Landau index and \( \omega_c = eB/m^*c \) is the cyclotron frequency. The probability amplitude \( b_j^{nk} \) for an electron in a time dependent state \( |kn> \) at plane \( j \) along \( z \) in the Landau level \( n \) is determined by the equation of motion.
\[ i\hbar \frac{db^{nk}}{dt} = \left( \epsilon_j + \epsilon_n + U \sum_{mk'} |b^{nk'}|^2 \right) b_j^{nk} + v \left( b_{j-1}^{nk} + b_{j+1}^{nk} - 2b_j^{nk} \right), \]  

(1)

where \( v \) is the hopping matrix elements between electrons in nearest-neighbor planes, and \( \epsilon_j \) represents the band contour and external bias. Here the sum over \((m, k)\) covers all occupied electron states of the system, within the various Landau levels \( m \) with energies below the Fermi level, incident on the DBS from the emitter side. In writing equation (1) we have adopted a Hartree model for the electron-electron interaction. We have neglected inter-Landau level transitions since they are of little importance at not too low magnetic fields, and averaged over allowed transitions between degenerate states within a Landau level, taking advantage of the localization induced by the Gaussian factor in the Landau basis.

For the 1D system we can write a similar equation where the subindex \( n \) corresponding to the Landau level is eliminated. The electron-electron interaction restricted to the dot has to be treated in this case using a strong interaction limit approximation capable of including simultaneously the Coulomb blockade effect and the non-linear behavior. The electron-electron interaction is analyzed within the context of the Hubbard I approximation. In this case the equation of motion is written as,

\[ i\hbar \frac{db_j^{nk}}{dt} = \left( \epsilon_j + v_j + v_{j+1} b_{j+1} - v_{j-1} b_{j-1} \right) b_j^{nk} + v b_j^{nk} \]  

(2)

where \( \alpha = \pm: n^+ = n; n^- = 1 - n; \epsilon^+ = U; \epsilon^- = 0; v_j = v \forall j \neq 0, 1; v_j = 0 = 0, 1; n = \sum_k |b_k^{nk}|^2; b_0^k = \sum_{\alpha=1}^2 b_{\alpha}^k; V_p \) is the gate potential applied at the well and the sum over \( k' \) covers as before all occupied electron states of the system.

The time dependent equations are solved using a half-implicit numerical method which is second-order accurate and unitary. The approach taken here assumes that in the presence of a bias, the wave function at time \( t \) outside the structure is given by

\[ b_j^{nk} = I e^{i k r_j} + R_{jn} e^{-i k r_j} \quad r \leq -L \]

\[ b_j^{nk'} = T_{jn} e^{i k' r_j} \quad r \geq -L \]  

(3)

Here \( k \) and \( k' = \sqrt{2m^* (\epsilon_k + \epsilon_L)/\hbar} \) are the wave numbers of the incoming and outgoing states, respectively, with \( \epsilon_k = \epsilon_n - 4\pi n^2 (k^2) \) the energy of the incoming particle (for the 1D case \( \epsilon_n = 0 \) and \( \epsilon_L \) is the external bias. To model the interaction with the particle reservoir outside the structure the incident amplitude \( I \) is assumed to be a constant independent of the coordinates. The envelope function of the reflected and transmitted waves, \( R_{jn} \) and \( T_{jn} \), are allowed to vary with \( j \), however. Since far from the barriers these quantities are a weak function of the coordinate \( z_j \) we restrict ourselves to the linear corrections only. This approximations is appropriate provided the time step \( \delta t \) does not exceed a certain limiting value. For the results presented here, a maximum value of \( \delta t = 3 \times 10^{-17} \) s was found sufficient to eliminate spurious reflections at the boundary while maintaining numerical stability up to \( 20 \times 10^{-12} \) s.

In our numerical procedure the coefficients obtained for one bias are used as the starting point for the next bias step.

We study first the 3D DBH under the effect of an external magnetic field. The model is applied to an asymmetric double barrier structure. The second barrier is chosen phenomenologically so as to fit the experimental I-V characteristic for a GaAs device in the absence of an external magnetic field. For small values of the magnetic field a stationary solution is reached after a short transient. At large enough bias two stationary solutions emerge, obtained by either increasing or decreasing the applied voltage. This corresponds to the well known bistability of 3D DBH.

A completely novel feature starts up as the field is increased. At small values of the external bias a stationary solution is rapidly reached. As the bias is increased, however, a critical value arises beyond which no stationary solution exists, and the system begins to oscillate in a self-sustained way. After a range of voltages over which the oscillations persist a stationary solution is reached again.

In Fig.1 we show the current at the center of the well for \( B = 13T \) and a fixed bias \( V = 0.27V \). A transient period is followed by an oscillation that is never damped out. Although not perfectly periodic, the oscillation has two strong Fourier components at frequencies \( \nu \sim 0.3 \) and \( 0.8 \) THz.

For a 1D system we study a model which consists of leads connected to a quantum dot represented by \( v_o = 0.1V \), \( U = 1.0V \) and a Fermi level \( E_f = 0.01eV \). The normalization of the wave function is taken so that each site could have up to a maximum of two electrons. We solve first equations (1) supposing that they possess stationary solutions. Since the electron density has rather long-range oscillations we made sure the sample was long enough to make finite size effects negligible. This is guaranteed taken a sample of \( N > 100 \) sites. Fig 2a shows the current voltage (I-V) characteristics for several a gate potential applied at the quantum dot \( V_p = 0.1V \). The system has a bistable behavior, which corresponds to the well known two solutions for the current; as the voltage is
increased when the dot is charged (solid line), and when the voltage is decreased the solution when the dot is without charge (dashed line). In either case the self-consistent solution converges to a stable fixed point after some finite number of iterations. Similarly to 3D tunneling devices in this 1D structure an increase of the Fermi level augments the charge content of the well and consequently the non-linearities and in particular the width of the hysteresis loop. When the gate potential is greater than a threshold value, which depends upon the parameters of the system, a completely novel feature starts to develop where two periods, two fixed points are encountered for certain voltages as shown in Fig 2b. This bubble-like solution is obtained as the voltage is increased outside the region of bistability showing that although both phenomena are derived from the non-linearities introduced by the local Coulomb repulsion, they corresponds to two different effects. Note that these new solutions are not reached, as the voltage is decreased (dashed line) as it corresponds to a discharged dot where the non-linearities are negligible. As the potential gate is raised still further, the area of the bistable bubble increases and it enters into de bistable region. At larger gate potentials the solution undergoes further bifurcations and finally a bifurcation cascade leading to a chaotic region. This behavior is in fact indicating the absence of stationary solution of the system.

Similarly to the 3D DBH under the effect of a magnetic field, self-sustained oscillations for the current are found in the case of a dot connected to two leads in the parameter region where there is no stationary solutions.

We interpret these results for both systems in the following way. In general, current flows through the heterostructure as long as a tunneling resonance lies within the emitter Fermi sea. We assume that at very low bias the resonance lies above the Fermi energy so that no current flows. As the bias increases it drops, reaching eventually the Fermi energy, thus opening a channel for electrons to tunnel through the double barrier. A current is thus established. The charge is trapped in the potential well, raising its bottom and pushing the resonance towards higher energies for the 3D system. For the 1D system the weight of the resonance level is proportional to $1 - n$ and as a consequence reduces when the charge increases. The current drops, some of the accumulated charge leaks out allowing the current to flow more easily once again, and a new cycle begins.

According to the picture drawn above the charge in the well lags the current, as exhibited in the inset of fig.1. Here the charge at the center of the well (dashed line) is plotted together with the current at the same point (full line), the former displaced a time $\tau \sim 1.4$ps to the left with respect to the latter. The relaxation time $\tau$ and the period of the oscillations are determined by the tunneling time for the electrons to leak out through the barriers and may be adjusted by modifying the barrier thickness.

### III. CONCLUSIONS

There is a wide region of accessible values of these parameters were bistability, instabilities and oscillations in the current are either present or absent. This opens up the interesting possibility of experimentally studying the transition from one behavior to the other as the parameters of both systems studied are varied. From the view point of possible applications these systems could operate as electromagnetic generators in the THz regime.

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1. L.L. Chang, L. Esaki and R. Tsu, Appl. Phys. Lett., 24, 593, 1974.
2. V. J. Goldman, D. C. Tsui and J. E. Cunningham, Phys. Rev. Lett., 58, 1256, 1987.
3. P. Pernas, F. Flores and E. V. Anda, J. Phy. C: Cond. Matter 3, 2346, 1992.
4. P. Orellana, E. Anda and F. Claro, Phy. Rev. Lett., 79, 1118, 1997.
5. D. Yoshida and P. A. Lee, Phys. Rev. B, 27, 4986, 1983.
6. J. Hubbard, Proc. Roy. Soc. A, 276, 238, 1963.
7. A. Zaslavsky, D. C. Tsui, M. Santos and M. Shayegan, Phys. Rev. B, 40, 9829, 1989.

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![Figure 1](image-url)  
**FIG. 1.** Current as a function of time at the center of the well for a 3D DBH under the effect of an applied magnetic field 13T. The inset shows the charge and the current shifted one relative to the other by 1.4ps.
FIG. 2. I-V characteristic of a quantum dot connected to 1D leads for:
a) $V_p = 0.1v$ and b) $V_p = 0.2v$