Form factors for the rare $\Lambda_b(\Lambda_b^0) \rightarrow N\ell^+\ell^-$ decays in light cone QCD sum rules

T. M. Aliev,1,* T. Barakat,2,3 and M. Savcı1,2

1Physics Department, Middle East Technical University, 06531 Ankara, Turkey
2Physics Department, King Saud University, Riyadh 11451, Saudi Arabia

(Received 11 July 2018; published 23 August 2018)

DOI: 10.1103/PhysRevD.98.035033

Form factors of the rare $\Lambda_b(\Lambda_b^0) \rightarrow N\ell^+\ell^-$ decays are calculated in the framework of the light cone QCD sum rules by taking into account the contributions from the negative parity baryons. Using the obtained results on the form factors, the branching ratios of the considered decays are estimated. The numerical survey for the branching ratios of the $\Lambda_b \rightarrow N\ell^+\ell^-$ and $\Lambda_b^0 \rightarrow N\ell^+\ell^-$ decays indicate that these transitions could be measurable in LHCb in near future. Comparison of our predictions on the form factors and branching ratios with those existing in the literature is also performed.

I. INTRODUCTION

Lately, impressive experimental progress has been made in investigation of the rare decays of heavy $\Lambda_b$ baryon. The CDF Collaboration [1] announced the first evidence of the rare $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ decay [2]. Very recently, the suppressed $\Lambda_b \rightarrow p\pi^-\mu^+\mu^-$ decay, excluding $J/\psi$ and $\psi(2S) \rightarrow \mu^+\mu^-$ resonance is observed [3]. This is the first observation of a $b \rightarrow d$ transition in a baryonic decay. The measured branching ratio is $B(\Lambda_b \rightarrow p\pi^-\mu^+\mu^-) = (6.9 \pm 1.9 \pm 1.1^{+1.3}_{-1.0}) \times 10^{-8}$. Next, the LHCb Collaboration performed a detailed analysis of differential branching ratio, forward–backward angular distributions, and asymmetries in the meson and hadronic systems [4]. This evidence stimulated the search of other similar decays, such as $\Lambda_b \rightarrow N\mu^+\mu^-$, which can in principle be discovered in the near future at LHCb.

The detailed study of $\Lambda_b$ baryon decays receives special attention for two reasons. The first reason is due to the fact that the $\Lambda_b$ baryon has spin one-half and therefore can give attention for two reasons. The first reason is due to the fact that the $\Lambda_b$ baryon has spin one-half and therefore can give

II. TRANSITION FORM FACTORS FOR THE $\Lambda_b(\Lambda_b^0) \rightarrow N\ell^+\ell^-$ DECAYS IN LIGHT CONE SUM RULES

At the quark level, $\Lambda_b(\Lambda_b^0) \rightarrow N\ell^+\ell^-$ decay is governed by the flavor changing neutral current $b \rightarrow d$ transition. The hadronic matrix elements responsible for $\Lambda_b(\Lambda_b^0) \rightarrow N$ transition are determined by considering the transition current between $\Lambda_b(\Lambda_b^0)$ and $N$ states. The relevant form factors of the vector, axial-vector, and tensor currents are defined as

\begin{equation}
\langle N(p) | \bar{b} \gamma_\mu (1-\gamma_5) d | N(p) \rangle \\
= \bar{u}_N(p-q) \left[ f_1(q^2) \gamma_\mu + i f_2(q^2) \frac{m_N}{m_b} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{m_b} q^\mu \right] u_N(p),
\end{equation}

and

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Funded by SCOAP3.

*taliev@metu.edu.tr
†tbarakat@ksu.edu.sa
‡savci@metu.edu.tr
\[
\langle \Lambda(p - q)|\bar{b}i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)d|N(p)\rangle = \bar{u}_\Lambda(p - q) \left[ \frac{f_1^\Lambda(q^2)}{m_{\Lambda_0}} (\gamma_\mu q^2 - q_\mu q^\mu) + if_2^\Lambda(q^2)\sigma_{\mu\nu}q^\nu \right. \\
\left. + \frac{g_f^\Lambda(q^2)}{m_{\Lambda_0}} (\gamma_\mu q^2 - q_\mu q^\mu)\gamma_5 + ig_f^\Lambda(q^2)\sigma_{\mu\nu}q^\nu \right] u_N(p),
\]

In order to determine the form factors \( f_i, f_f^\Lambda \) in Eqs. (1) and (2), we introduce the following correlation functions,

\[
\Pi_i^\Lambda(p, q) = i \int d^4x e^{i\eta(x)} \langle \Lambda_0 (0)|T(\eta_{\Lambda_0}(0)J_\alpha(x))|N(p)\rangle,
\]

where \( \eta_{\Lambda_0} \) is the interpolating current of \( \Lambda_0 \)-baryon, \( J_\alpha(x) \) is the heavy–light transition current, and \( \alpha \) index corresponds to the choice of form of interpolating current. For the decays at hand these currents are \( J_\alpha = \bar{b}\gamma_\mu(1 - \gamma_5)d \) or \( \bar{b}i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)d \).

The most general form of the interpolating current for the \( \Lambda_b \) baryon is given as

\[
\eta_{\Lambda_b} = \frac{1}{\sqrt{6}} e_{abc} \{2[(u^{aT}(x)Cd^b(x))\gamma_5 b^c(x) + \beta(u^{aT}(x)C\gamma_5 d^b(x))b^c(x)] + (u^{aT}(x)Cb^b(x))\gamma_5 d^c(x) \\
+ \beta(u^{aT}(x)C\gamma_5 b^b(x))d^c(x) + (b^{aT}(x)Cd^b(x))\gamma_5 u^c(x) + \beta(b^{aT}(x)C\gamma_5 d^b(x))u^c(x)\},
\]

where \( a, b, \) and \( c \) are the color indices, \( C \) is the charge conjugation operator, and \( \beta \) is an arbitrary parameter with \( \beta = -1 \) corresponding to the Ioffe current.

The first step in deriving the sum rules for the transition form factors is to insert the full set of beauty-baryon states between the interpolating current \( \eta_{\Lambda_b} \), and the transition current \( J_\alpha \), and then the ground state contributions are isolated. At this point we meet the following problem which is absent for the mesonic system. The interpolating current of the baryon has nonzero overlap not only with the ground state but also with the negative parity baryon. Calculations show that the mass difference between the negative and positive parity states of the heavy \( \Lambda_b \) baryon is about (250–300) MeV [15]. Therefore negative parity baryons can give considerable contribution to the sum rules. For this reason the standard quark-hadron duality approximation should be modified, which leads to the strong dependence of the sum rules predictions on the choice of interpolating current. Keeping these preliminary remarks in mind, the expression for the physical part of the correlation can be written as

\[
\Pi^\Lambda(p, q) = \sum_{i = +, -} \frac{\langle 0|\eta_{\Lambda_b}(0)|\Lambda_b(p - q, s)\rangle \langle \Lambda_b(p - q, s)|\bar{b}\gamma_\mu(1 - \gamma_5)d|N(p)\rangle}{m_i^2 - (p - q)^2},
\]

\[
\Pi^\Lambda(p, q) = \Pi^\Lambda(p, q)|J_\alpha = \bar{b}\gamma_\mu(1 - \gamma_5)d|\rightarrow i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)d|,
\]

where summation is performed over the positive and negative parity \( \Lambda_b \) baryons. The first matrix element in Eqs. (5) and (6) is defined in terms of the residues of \( \Lambda_b \) and \( \Lambda^*_b \) baryons as follows

\[
\langle 0|\eta_{\Lambda_b}|\Lambda_b(p - q)\rangle = \lambda_{\Lambda_b} u_{\Lambda_b}(p - q),
\]

\[
\langle 0|\eta_{\Lambda^*_b}|\Lambda_b(p - q)\rangle = \lambda_{\Lambda^*_b} \gamma_5 u_{\Lambda^*_b}(p - q).
\]

Using Eqs. (1), (2), (7), and (8), the hadronic part of the correlation function can be written as

\[
\Pi^\Lambda(p, q) = \frac{\lambda_{\Lambda_b}}{m_{\Lambda_b}^2 - (p - q)^2} \left\{ \frac{f_1^\Lambda(q^2)}{m_{\Lambda_0}} (2p_\mu - m_N\gamma_\mu - 2q_\mu + \gamma_5 q_\mu + m_{\Lambda_b} \gamma_5) \\
- \frac{f_2^\Lambda(q^2)}{m_{\Lambda_0}} (2p_\mu + \gamma_5 (m_{\Lambda_0}^2 - m_{\Lambda_b}^2) + (m_{\Lambda_b} + m_N)\gamma_5 q_\mu - (m_{\Lambda_b} + m_N)q_\mu - q_\mu q^\mu) \right\}.
\]
FORM FACTORS FOR THE RARE ...

\[ + f_3(q^2) \frac{q_\mu}{m_{\Lambda_b}} (m_{\Lambda_b} + m_N - \not{q}) - g_1(q^2)[2p_\mu\gamma_S + (m_{\Lambda_b} + m_N)\gamma_\mu\gamma_S - 2q_\mu\gamma_S \\
+ \gamma_\mu\not{q}\gamma_S] + g_2(q^2) \frac{q_\mu}{m_{\Lambda_b}} [2p_\mu\not{q}\gamma_S + \gamma_\mu\gamma_S(m_{\Lambda_b}^2 - m_N^2) + (m_{\Lambda_b} - m_N)\gamma_\mu\not{q}\gamma_S \\
- (m_{\Lambda_b} - m_N)q_\mu\gamma_S - q_\mu\not{q}\gamma_S] - g_3(q^2) \frac{q_\mu}{m_{\Lambda_b}} [(m_{\Lambda_b} - m_N)\gamma_\mu\gamma_S - \not{q}\gamma_S] \} \]

\[ + \frac{\lambda_{\Lambda_b}}{m_{\Lambda_b}^2 - (p-q)^2} \left\{ \tilde{f}_1(q^2)(-2p_\mu + m_N\gamma_\mu + 2q_\mu + \gamma_\mu\not{q} + \gamma_\mu\gamma_S) \right. \]

\[ + \tilde{f}_2(q^2) \frac{q_\mu}{m_{\Lambda_b}} (m_{\Lambda_b}^2 - m_N^2) - (m_{\Lambda_b}^2 - m_N^2)\gamma_\mu\not{q}\gamma_S + (m_{\Lambda_b}^2 - m_N^2)q_\mu - q_\mu\not{q}\gamma_S \]

\[ + \tilde{f}_3(q^2) \frac{q_\mu}{m_{\Lambda_b}} (m_{\Lambda_b} - m_N + \not{q}) - g_1(q^2)[-2p_\mu\gamma_S + (m_{\Lambda_b} - m_N)\gamma_\mu\gamma_S + 2q_\mu\gamma_S \\
- \gamma_\mu\not{q}\gamma_S] + \tilde{g}_2(q^2)[2p_\mu\not{q}\gamma_S - \gamma_\mu\gamma_S(m_{\Lambda_b}^2 - m_N^2) + (m_{\Lambda_b} + m_N)\gamma_\mu\not{q}\gamma_S \\
- (m_{\Lambda_b} + m_N)q_\mu\gamma_S + q_\mu\not{q}\gamma_S] - \tilde{g}_3(q^2) \frac{q_\mu}{m_{\Lambda_b}} [(m_{\Lambda_b} + m_N)\gamma_\mu\gamma_S + \not{q}\gamma_S] \} \right\}, \tag{9} \]

\[ \Pi^{\mu}_{ij}(p, q) = \frac{\lambda_{\Lambda_b}}{m_{\Lambda_b}^2 - (p-q)^2} \left\{ f_1^T(q^2) \frac{1}{m_{\Lambda_b}} \left[ ((m_{\Lambda_b} - m_N)\gamma_\mu\gamma_S + \gamma_\mu\not{q} + 2p_\mu)q^2 \right. \right. \]

\[ - ((m_{\Lambda_b} - m_N)\not{q} - (m_{\Lambda_b}^2 - m_N^2 - 2q^2)q_\mu) + f_2^T[(-2p_\mu + q_\mu)\not{q} \right. \]

\[ - (m_{\Lambda_b}^2 - m_N^2)\gamma_\mu - (m_{\Lambda_b} + m_N)\gamma_\mu\not{q}\gamma_S + (m_{\Lambda_b} + m_N)q_\mu \left. \right] \right. \]

\[ + \left[ g_1^T(q^2) \frac{1}{m_{\Lambda_b}} \left[ (m_{\Lambda_b} + m_N)\gamma_\mu\gamma_S + \gamma_\mu\not{q}\gamma_S + 2p_\mu\gamma_S)q^2 \right. \right. \]

\[ - ((m_{\Lambda_b} + m_N)\not{q}\gamma_S - (m_{\Lambda_b}^2 - m_N^2 - 2q^2)q_\mu) + g_2^T[(-2p_\mu - q_\mu)\not{q} \right. \]

\[ + (m_{\Lambda_b}^2 - m_N^2)\gamma_\mu - (m_{\Lambda_b} - m_N)\gamma_\mu\not{q}\gamma_S + (m_{\Lambda_b} - m_N)q_\mu \left. \right] \right. \]

\[ + \left[ \tilde{g}_1^T(q^2) \frac{1}{m_{\Lambda_b}} \left[ ((m_{\Lambda_b} - m_N)\gamma_\mu\gamma_S - \gamma_\mu\not{q}\gamma_S - 2\gamma_Sp_\mu)q^2 \right. \right. \]

\[ - ((m_{\Lambda_b} - m_N)\not{q}\gamma_S + (m_{\Lambda_b}^2 - m_N^2 - 2q^2)q_\mu) + \tilde{g}_2^T[(-2p_\mu - q_\mu)\not{q} \right. \]

\[ + (m_{\Lambda_b}^2 - m_N^2)\gamma_\mu - (m_{\Lambda_b} - m_N)\gamma_\mu\not{q}\gamma_S + (m_{\Lambda_b} - m_N)q_\mu \left. \right] \right. \]

\[ + \left[ \tilde{g}_1^T(q^2) \frac{1}{m_{\Lambda_b}} \left[ ((m_{\Lambda_b} - m_N)\gamma_\mu\gamma_S - \gamma_\mu\not{q}\gamma_S - 2\gamma_Sp_\mu)q^2 \right. \right. \]

\[ - ((m_{\Lambda_b} - m_N)\not{q}\gamma_S + (m_{\Lambda_b}^2 - m_N^2 - 2q^2)q_\mu) + \tilde{g}_2^T[(-2p_\mu - q_\mu)\not{q} \right. \]

\[ + (m_{\Lambda_b}^2 - m_N^2)\gamma_\mu - (m_{\Lambda_b} - m_N)\gamma_\mu\not{q}\gamma_S + (m_{\Lambda_b} - m_N)q_\mu \left. \right] \right. \]

\[ - (m_{\Lambda_b}^2 + m_N)\gamma_\mu\gamma_S + 2\not{q}\gamma_Sp_\mu + (m_{\Lambda_b} + m_N)\gamma_\mu\not{q}\gamma_S - \not{q}\gamma_Sq_\mu \right\}. \tag{10} \]

We proceed now calculating the correlation functions [see Eq. (3)] for the \( \Lambda_b(\Lambda_b^*) \to N^e\bar{e}^- \) transitions from the QCD sides. Note that, in the rest of the study, the masses of the light quarks are neglected. Moreover the external momenta \((p - q)\) and \(q\) are both spacelike i.e., \((p - q) \ll m_{\Lambda_b}^2\) and \(q \ll m_{\Lambda_b}^2\), in order to justify the operator product expansion (OPE) near the light cone \(x^2 \approx 0\). The OPE is performed over the twist of the nonlocal operators and it includes the nucleon distribution amplitudes (DAs). Contracting the \(b\)-quark fields for the correlation functions, from the QCD side we get,
The heavy quark operator $S_Q(x)$ is obtained in [16], whose expression is given as

$$
S_Q(x) = \frac{m_Q^2}{4\pi} K_1(m_Q\sqrt{-x^2}) - i\frac{m_Q}{2\pi x} K_2(m_Q\sqrt{-x^2}) - i g_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[ \frac{k + m_Q}{(m_Q^2 - k^2)^2} G_{\mu\nu}(v x) \sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} v x \mu G_{\mu\nu} \right],
$$

where $K_i$ are the modified Bessel functions, and $G_{\mu\nu}$ is the gluon field strength tensor. It follows from Eq. (11) that in order to calculate the correlator function the matrix element $e^{abc}\langle 0|u^a_f(0)\sigma_\mu(0)\bar{d}^b_f(x)d^c_f(0)|N(p)\rangle$, of the three quark field operators between vacuum and nucleon near light cone $x^2 \to 0$ is needed. This matrix element is parametrized in terms of the nucleon DAs and is given in [6,7] (for more details about the nucleon DAs, see also [17]).

Substituting the parametrization of the matrix element $e^{abc}\langle 0|u^a_f(0)\sigma_\mu(0)\bar{d}^b_f(x)d^c_f(0)|N(p)\rangle$ in terms of the nucleon DAs and the heavy $b$-quark propagator, the correlation function from the QCD side can be calculated straightforwardly.

In order to suppress the higher states and continuum contributions, we perform the Borel transformation over $-(p-q)^2$ to the expressions of correlation function from the hadronic and the QCD sides, and matching the coefficients of the relevant structures, we get the following sum rules for the form factors:

(a) For the $\gamma_\mu$ current,

$$
2\lambda_{N_b} f_1(q^2) e^{-m_{N_b}^2/M^2} - 2\lambda_{N_b} \tilde{f}_1(q^2) e^{-\tilde{m}_{N_b}^2/M^2} = \Pi^{(1)B}_1(p, q),
$$

$$
-2\lambda_{N_b} f_2(q^2) e^{-m_{N_b}^2/M^2} + 2\lambda_{N_b} \tilde{f}_2(q^2) e^{-\tilde{m}_{N_b}^2/M^2} = \Pi^{(1)B}_2(p, q),
$$

$$
\lambda_{N_b} e^{-m_{N_b}^2/M^2} \left( m_{N_b} - m_N \right) \left( f_1(q^2) - \frac{f_2(q^2)}{m_{N_b}} (m_{N_b} + m_N) \right) + \lambda_{N_b} e^{-\tilde{m}_{N_b}^2/M^2} \left( m_{N_b} + m_N \right) \left( \tilde{f}_1(q^2) + \frac{\tilde{f}_2(q^2)}{m_{N_b}} (m_{N_b} - m_N) \right) = \Pi^{(1)B}_3(p, q),
$$

$$
\lambda_{N_b} e^{-\tilde{m}_{N_b}^2/M^2} \left( -2f_1(q^2) + \frac{f_2(q^2)}{m_{N_b}} (m_{N_b} + m_N) \right),
$$

$$
\lambda_{N_b} e^{-\tilde{m}_{N_b}^2/M^2} \left( 2f_1(q^2) + \frac{f_2(q^2)}{m_{N_b}} (m_{N_b} - m_N) \right),
$$

$$
\lambda_{N_b} e^{-\tilde{m}_{N_b}^2/M^2} \left( \tilde{f}_1(q^2) + f_3(q^2) \right) + \lambda_{N_b} e^{-\tilde{m}_{N_b}^2/M^2} \left( \tilde{f}_2(q^2) + \tilde{f}_3(q^2) \right) = \Pi^{(1)B}_4(p, q),
$$

$$
2\lambda_{N_b} \tilde{f}_1(q^2) e^{-m_{N_b}^2/M^2} - 2\lambda_{N_b} \tilde{f}_2(q^2) e^{-\tilde{m}_{N_b}^2/M^2} = \Pi^{(1)B}_5(p, q),
$$

$$
-2\lambda_{N_b} \tilde{f}_3(q^2) e^{-m_{N_b}^2/M^2} + 2\lambda_{N_b} \tilde{f}_3(q^2) e^{-\tilde{m}_{N_b}^2/M^2} = \Pi^{(1)B}_6(p, q).}

035033-4
where superscript $I$ represents transition current $\gamma_\mu$. Here, $\Pi_I^{(f)}(p,q)$, $\Pi_2^{(f)}(p,q)$, $\Pi_3^{(f)}(p,q)$, $\Pi_4^{(f)}(p,q)$, $\Pi_5^{(f)}(p,q)$, and $\Pi_6^{(f)}(p,q)$ are the invariant functions for the Lorentz structures, $p_\mu$, $p_\mu\bar{q}$, $\gamma_\mu\gamma_\mu\bar{q}$, $\gamma_\mu$, and $q_\nu\bar{q}$ structures, respectively.

The results for $\gamma_\mu\gamma_5$ current are obtained from Eq. (13) with the following replacements: $f_i \rightarrow g_i$, $\tilde{f}_i \rightarrow \tilde{g}_i$, $m_N \rightarrow -m_N$, and $\Pi_i^{(f)} \rightarrow \Pi_i^{(f)IB}$. For the sum rules for the form factors induced by the $i\sigma_{\mu\nu}q^2$ current, we get

$$-2\lambda_\Lambda e^{-m_N^2/M^2} f^T_1(q^2) + 2\lambda_\Lambda e^{-m_N^2/M^2} f^T_2(q^2) = \Pi_1^{(f)IB}(p,q)$$

$$\lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) + \lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) = \Pi_2^{(f)IB}(p,q)$$

$$\lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) + \lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) = \Pi_3^{(f)IB}(p,q)$$

$$\lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) + \lambda_\Lambda e^{-m_N^2/M^2} \left( f^T_1(q^2) m_\Lambda - m_N + f^T_2(q^2) \right) = \Pi_4^{(f)IB}(p,q)$$

where the denominator is given by $\Delta = m_b^2 - (xp + q)^2 = m_b^2 - \bar{x}q^2 + \bar{x}\bar{m}^2_N - x(p-q)^2$, and $\bar{x} = 1 - x$. In order to perform the Borel transformation, we rewrite the denominator in the form

$$\Delta = x(s(x) - (p-q)^2),$$

where $s(x) = (m_b^2 - \bar{x}q^2 + \bar{x}\bar{m}^2_N)/x$. Following this replacement, the Borel transformation and continuum subtraction are performed [6].
where $x_0$ is the solution of the equation

$$s_0 = \frac{m_b^2 - \bar{x}q^2 + x\bar{x}m_{\eta_b}^2}{x}.$$  

Solving Eqs. (13) and (14), we obtained the desired sum rules for the transition form factors $f_i, g_i, f_i^T, g_i^T, \tilde{f}_i, \tilde{g}_i, \tilde{f}_i^T$, and $\tilde{g}_i^T$.

One can easily see that the expressions of the form factors contain the residues $\lambda_{\Lambda_b}$ and $\bar{\lambda}_{\Lambda_b}$ of the $\Lambda_b$ and $\bar{\Lambda}_b$ baryons, respectively. These residues are determined from the analysis of the following two-point correlation function,

$$T = i \int d^4x e^{ix\xi} \langle 0 | \bar{\eta}_{\Lambda_b}(x) \eta_{\Lambda_b} \rangle (0) | 0 \rangle = T_1(q^2) \bar{T} + T_2(q^2).$$

Note that this correlator is used to calculate the residue of the $\Lambda_b$ baryon when for the pseudoscalar- and axial-vector currents are used [11]. We also use this correlator to calculate the residues of the $\Lambda_b(\bar{\Lambda}_b)$ baryons by using the most general form of the interpolating current of the $\Lambda_b$ baryon. Following the standard method, i.e., performing the Borel transformation and continuum subtraction procedures, we obtain

$$\lambda_{\Lambda_b} e^{-m_{\Lambda_b}^2/M^2} + \bar{\lambda}_{\Lambda_b} e^{-m_{\bar{\Lambda}_b}^2/M^2} = T_1^B,$$

$$\lambda_{\Lambda_b} m_{\Lambda_b} e^{-m_{\Lambda_b}^2/M^2} - \bar{\lambda}_{\Lambda_b} m_{\bar{\Lambda}_b} e^{-m_{\bar{\Lambda}_b}^2/M^2} = T_2^B.$$  

As has already been noted in further numerical analysis, the value of residues $\lambda_{\Lambda_b}(\bar{\lambda}_{\Lambda_b})$ of the $\Lambda_b(\bar{\Lambda}_b)$ baryons are needed. In this regard, the mass sum rules for the $\Lambda_b(\bar{\Lambda}_b)$ baryons for the most general form of the interpolating current contain three auxiliary parameters, namely, Borel mass parameter $M^2$, continuum threshold $s_0$, and the arbitrary parameter $\beta$. The working region of $M^2$ for the residue is determined by using the standard criteria; i.e., the power corrections and continuum contributions should be suppressed at the chosen values of $s_0$ and $\beta$. As a result of these requirements the working region of the Borel mass parameter is found to be $4 \text{GeV}^2 \leq M^2 \leq 6 \text{GeV}^2$. The continuum threshold can be obtained from the condition that the mass sum rules

where, $\lambda_{\Lambda_b}(\bar{\lambda}_{\Lambda_b})$, and $m_{\Lambda_b}(m_{\bar{\Lambda}_b})$ are the residues and masses of $\Lambda_b(\bar{\Lambda}_b)$ baryons. Solving these two equations for the residues $\lambda_{\Lambda_b}$ and $\bar{\lambda}_{\Lambda_b}$, we obtain,

$$\lambda_{\Lambda_b} = \frac{e^{-m_{\Lambda_b}^2/M^2}}{m_{\Lambda_b} + m_{\bar{\Lambda}_b}} (m_{\Lambda_b} T_1^B + T_2^B),$$

$$\bar{\lambda}_{\Lambda_b} = \frac{e^{-m_{\bar{\Lambda}_b}^2/M^2}}{m_{\Lambda_b} + m_{\bar{\Lambda}_b}} (m_{\bar{\Lambda}_b} T_1^B - T_2^B).$$

The expressions of the invariant functions $T_1^B$, and $T_2^B$ can be obtained from the results presented in [18].

### III. Numerical Analysis

In this section, we present the numerical results of the form factors of the rare $\Lambda_b(\bar{\Lambda}_b) \rightarrow N\ell^+\ell^-$ decays, and their total decay rates and branching ratios. The main input parameters used in the numerical calculations are, $m_N=0.938\text{GeV}$, $m_{\Lambda_b}=5.620\text{GeV}$, and $m_{\bar{\Lambda}_b}=5.920\text{GeV}$ [19]. For the mass of the $b$-quark, we take its MS mass value $m_b=(4.16\pm0.03)\text{GeV}$ [19], $\langle \bar{u}u \rangle (1\text{GeV})=\langle \bar{d}d \rangle (1\text{GeV})=-(246\pm10)\text{MeV})$.

The input parameters entering the DAs of the nucleon are taken from [5,6], whose values are

$$f_N = (5.0 \pm 0.5) \times 10^{-3} \text{ GeV}^2,$$

$$\lambda_2 = (54 \pm 19) \times 10^{-3} \text{ GeV}^2,$$

$$V_d^d = 0.30,$$

$$f_{f_1} = 0.09,$$

$$\lambda_1 = -(27 \pm 9) \times 10^{-3} \text{ GeV}^2,$$

$$A_1 = 10.13,$$

$$f_{f_1} = 0.33,$$

$$f_{d_2} = 0.25.$$  

reproduce the lowest baryon mass with an accuracy of 10%, for a given value of $\beta$. The numerical analysis performed in this regard, determines the working region of the continuum threshold to be $s_0 = (40 \pm 1) \text{GeV}^2$. Finally, in order to find the working region for the parameter $\beta$, we studied the dependence of $\Lambda_b(\bar{\Lambda}_b)$ on $\cos \theta$, where $\tan \theta = \beta$. We observe that the residue demonstrates a good stability to the variation of $\cos \theta$ in the domain $-1 \leq \cos \theta \leq -0.5$, within an uncertainty of less than 4%. Therefore, in the proceeding analysis we shall use $\beta = -1$. Taking into account the working regions of the aforementioned parameters $M^2$, $s_0$ and $\beta$, the values of the residues $\lambda_{\Lambda_b}$ and $\bar{\lambda}_{\Lambda_b}$ which we shall use our analysis are found to be,
TABLE I. Form factors of the $\Lambda_b \to \ell^+\ell^-$ decay.

| $f_i(0)$ | $a_0$        | $a_1$        | $a_2$        |
|----------|--------------|--------------|--------------|
| $f_1$    | $-0.075 \pm 0.005$ | $0.17 \pm 0.03$ | $-1.56 \pm 0.09$ | $2.46 \pm 1.00$ |
| $f_2$    | $0.11 \pm 0.01$ | $0.79 \pm 0.05$ | $-3.15 \pm 1.30$ | $3.35 \pm 1.40$ |
| $f_3$    | $0.08 \pm 0.00$ | $0.78 \pm 0.06$ | $-3.5 \pm 1.3$ | $4.3 \pm 1.4$ |
| $g_1$    | $-0.090 \pm 0.002$ | $0.24 \pm 0.04$ | $-1.96 \pm 0.90$ | $3.18 \pm 1.30$ |
| $g_2$    | $0.08 \pm 0.003$ | $0.80 \pm 0.06$ | $-3.5 \pm 1.3$ | $4.1 \pm 1.3$ |
| $g_3$    | $0.14 \pm 0.04$ | $1.10 \pm 0.03$ | $-4.6 \pm 1.4$ | $5.0 \pm 1.6$ |
| $f_1^\prime$ | $-0.13 \pm 0.03$ | $11 \pm 3$ | $-66 \pm 15$ | $100 \pm 25$ |
| $f_2^\prime$ | $0.12 \pm 0.03$ | $2.5 \pm 0.9$ | $-13 \pm 3$ | $17 \pm 6$ |
| $g_1^\prime$ | $-1.8 \pm 0.03$ | $-1.0 \pm 0.2$ | $-0.60 \pm 0.14$ | $0.10 \pm 0.02$ |

$\lambda_{\Lambda_b} = (6.5 \pm 1.5) \times 10^{-2}$ GeV$^3$, $\lambda_{\Lambda_b^\prime} = (7.5 \pm 2.0) \times 10^{-2}$ GeV$^3$. (24)

We now turn our attention to the calculation of the $\Lambda_b(\Lambda_b^\prime) \to N$ transition form factors. The working region of $M^2$ for these form factors is determined in accordance with the aforementioned requirement, i.e., sufficient suppression of the power correction and continuum contributions. Our analysis shows that these conditions are simultaneously satisfied when $M^2$ lies in the domain $15 \text{ GeV}^2 \leq M^2 \leq 25 \text{ GeV}^2$. For the continuum threshold and the arbitrary parameter $\beta$, we use $s_0 = (40 \pm 1) \text{ GeV}^2$ and $\beta = -1$, respectively.

The LCSR predictions, unfortunately, do not work in the entire physical region. The prediction of LCSR for the form factors is reliable up to $q^2 = (10-11) \text{ GeV}^2$. Since at large $q^2$ the contributions of higher twists become sizable and convergence of OPE is questionable. In order to extend the LCSR prediction for the form factors to the entire physical domain $q^2 = (m_{\Lambda_b} - m_N)^2 \text{ GeV}^2$, we use the $z$-series parametrization suggested in [20], where

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

with $t_0 = q^2_{\text{max}} = (m_{\Lambda_b} - m_N)^2$, $t_+ = (m_B + m_a)^2$.

The best parametrization of the form factors, with which the predictions of the LCSR are reproduced with a high accuracy in the $q^2 \leq 11 \text{ GeV}^2$ region, is given as

$$f(q^2) = \frac{1}{1 - q^2 / (m_{\text{pole}}^2)^2} [a_0^f + a_1^f z(q^2, t_0) + a_2^f [z(q^2, t_0)]^2].$$

For the pole masses, we use

$$m_{\text{pole}} = \begin{cases} 
  m_{B^+} = 5.325 \text{ GeV} & \text{for the form factors } f_1, f_2, f_1^\prime, f_2^\prime; \bar{g}_1, \bar{g}_2, \bar{g}_1^\prime, \bar{g}_2^\prime \\
  m_{B_1} = 5.723 \text{ GeV} & \text{for the form factors } g_1, g_2, g_1^\prime, g_2^\prime, \bar{f}_1, \bar{f}_2, \bar{f}_1^\prime, \bar{f}_2^\prime \\
  m_{B_0} = 5.749 \text{ GeV} & \text{for the form factors } f_3; \bar{g}_3 \\
  m_{B} = 5.280 \text{ GeV} & \text{for the form factors } g_3; \bar{f}_3 
\end{cases}$$

(27)

Our analysis predicts the following values of fit parameters $a_0^f$, $a_1^f$, and $a_2^f$ for the $\Lambda_b \to N\ell^+\ell^-$ and $\Lambda_b^\prime \to N\ell^+\ell^-$ form factors, respectively, which are presented in Tables I and II.

TABLE II. The same as Table I, but for the $\Lambda_b^\prime \to \ell^+\ell^-$ decay.

| $f_i(0)$ | $a_0$        | $a_1$        | $a_2$        |
|----------|--------------|--------------|--------------|
| $f_1$    | $-0.002 \pm 0.001$ | $0.60 \pm 0.20$ | $-2.71 \pm 0.5$ | $3.04 \pm 0.70$ |
| $f_2$    | $-0.040 \pm 0.001$ | $-0.36 \pm 0.10$ | $1.19 \pm 0.24$ | $-1.0 \pm 0.2$ |
| $f_3$    | $-0.052 \pm 0.002$ | $0.0085 \pm 0.0002$ | $-0.53 \pm 0.12$ | $1.1 \pm 0.23$ |
| $\bar{g}_1$ | $-0.30 \pm 0.006$ | $-0.11 \pm 0.02$ | $0.28 \pm 0.06$ | $-0.230 \pm 0.045$ |
| $\bar{g}_2$ | $-0.044 \pm 0.002$ | $-0.11 \pm 0.02$ | $0.11 \pm 0.02$ | $0.19 \pm 0.03$ |
| $\bar{g}_3$ | $-0.020 \pm -0.004$ | $-0.085 \pm 0.002$ | $0.24 \pm 0.05$ | $0.170 \pm 0.032$ |
| $\bar{f}_1$ | $0.070 \pm 0.003$ | $-11 \pm$ | $52 \pm 1$ | $-65 \pm 15$ |
| $\bar{f}_2$ | $-0.030 \pm 0.004$ | $-2.7 \pm 0.6$ | $12.5 \pm 2.5$ | $-15 \pm 3$ |
| $\bar{g}_1$ | $-0.080 \pm 0.002$ | $-3.0 \pm 0.7$ | $-13.5 \pm 3.5$ | $-15.3 \pm 3.5$ |
| $\bar{g}_2$ | $-0.060 \pm 0.002$ | $0.040 \pm 0.008$ | $-0.90 \pm 0.05$ | $-1.8 \pm 0.4$ |

PHYS. REV. D 98, 035033 (2018)

035033-7
\[
H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{td}^\ast \left\{ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + V_{ub}V_{ud}^\ast \sum_{i=1}^{2} C_i(\mu) - O_i^\prime \right\},
\] (28)

where, \(C_i\) are the Wilson coefficients and set \(O_i\) represent local operators. The matrix element responsible for the \(\Lambda_b \to N\ell^+\ell^-\) transition can be obtained from the effective Hamilton by sandwiching it between \(\Lambda_b\) and \(N\) states, after which the matrix element responsible for \(\Lambda_b \to N\ell^+\ell^-\) decay takes the following form,

\[
\mathcal{M} = \frac{G_F\alpha}{\sqrt{2}\pi} |V_{tb}V_{td}^\ast| \left\{ \bar{u}_N(p) \right\} \frac{i\sigma_{\mu\nu}\epsilon}{m_N} F_2 - \frac{q_\mu}{m_N} F_3 - \gamma_\mu \gamma_5 G_1 + \frac{i\sigma_{\mu\nu}\epsilon}{m_N} G_2 \\
- \frac{q_\mu}{m_N} \gamma_5 G_3 \left( p + q \right) (\bar{\epsilon}\gamma^\mu\epsilon) + \bar{u}_N(p) \gamma_\mu F_4 + \frac{i\sigma_{\mu\nu}\epsilon}{m_N} F_5 - \frac{q_\mu}{m_N} F_6 - \gamma_\mu \gamma_5 G_4 \\
+ \frac{i\sigma_{\mu\nu}\epsilon}{m_N} \gamma_5 G_5 - \frac{q_\mu}{m_N} \gamma_5 G_6 \left( p + q \right) (\bar{\epsilon}\gamma^\mu\gamma_5\epsilon) \},
\] (29)

where

\[
F_1 = c_9 f_1 - \frac{2m_b}{m_N} c_7 f_1^T,
F_2 = c_9 f_2 + \frac{2m_b}{q^2} m_N f_2^T,
F_3 = c_9 f_3 - \frac{2m_b}{q^2} c_7 (m_N - m_N) f_3^T,
G_1 = c_9 g_1 - \frac{2m_b}{m_N} c_7 g_1^T,
G_2 = c_9 g_2 + \frac{2m_b}{q^2} m_N g_2^T,
G_3 = c_9 g_3 - \frac{2m_b}{q^2} c_7 (m_N + m_N) g_3^T,
F_4 = c_{10} f_1,
F_5 = c_{10} f_2,
F_6 = c_{10} f_3,
G_4 = c_{10} g_1,
G_5 = c_{10} g_2,
G_6 = c_{10} g_3.
\] (30)

The matrix element for the \(\Lambda_b^* \to N\ell^+\ell^-\) transition can be obtained from the matrix element for the \(\Lambda_b \to N\ell^+\ell^-\) by first making the following replacements, \(F_i \to \tilde{G}_i\), \(G_i \to \tilde{F}_i\), \(m_N \to -m_N\), \(m_N \to m_N^{-}\) and further making the following ones, \(f_i \to \tilde{f}_i\), \(f_i^T \to \tilde{f}_i^T\), \(g_i \to \tilde{g}_i\), \(g_i^T \to \tilde{g}_i^T\).

Our final goal is to calculate the differential width of the \(\Lambda_b^* \to N\ell^+\ell^-\) decay, whose expression is given as

\[
\frac{d\Gamma(s)}{ds} = \frac{G^2\alpha^2 m_N}{4096\pi^5} |V_{tb}V_{td}^\ast|^2 v\sqrt{\lambda(1,r,s)} \left[ \Gamma_1(s) + \frac{1}{3} \Gamma_2(s) \right],
\] (31)

where \(\alpha\) is the fine structural constant, \(v = \sqrt{1 - 4m_\ell^2/q^2}\) is the lepton velocity, \(\lambda(1,r,s) = 1 + r^2 + s^2 + 2r - 2s - 2rs\), \(s = q^2/m_\Lambda^2\), and \(r = m_\ell^2/m_\Lambda^2\), the expressions of \(\Gamma_1(s)\) and \(\Gamma_2(s)\) are given in the Appendix.

We also calculate the differential decay width for the \(\Lambda_b^* \to N\ell^+\ell^-\) transition whose expression can easily be obtained from the differential decay width of the \(\Lambda_b \to N\ell^+\ell^-\) transition with the help of appropriate replacements.

The differential branching ratios for the \(\Lambda_b \to N\mu^+\mu^-\), \(\Lambda_b \to N\tau^+\tau^-\), \(\Lambda_b^* \to N\mu^+\mu^-\), and \(\Lambda_b^* \to N\tau^+\tau^-\) decays at \(s_0 = 40\text{ GeV}^2\) and \(M^2 = 25\text{ GeV}^2\) are presented in Figs. 1–4, respectively.

In order to calculate the branching ratios of the \(\Lambda_b(\Lambda_b^*) \to N\ell^+\ell^-\) transitions, the differential decay width of the respective decays should be integrated over \(s\) in the

![FIG. 1. The dependence of the differential branching ratio for the \(\Lambda_b \to N\mu^+\mu^-\) transition on \(s\), at \(s_0 = 40\text{ GeV}^2\) and \(M^2 = 25\text{ GeV}^2\).](image-url)
As the result of these considerations, the branching ratios for the \( \Lambda_b \to N \ell^+ \ell^- (e, \mu, \tau) \) transitions are calculated to have the values \( \text{Br}(\Lambda_b \to N e^+ e^-) = (8 \pm 2) \times 10^{-8} \), \( \text{Br}(\Lambda_b \to N \mu^+ \mu^-) = (7 \pm 2) \times 10^{-8} \), and \( \text{Br}(\Lambda_b \to N \tau^+ \tau^-) = (2 \pm 0.4) \times 10^{-8} \), respectively.

In determining the branching ratios of the \( \Lambda_b^* \to N \ell^+ \ell^- (e, \mu, \tau) \) decay, the lifetime of the \( \Lambda_b^* \) is needed, which approximately has the same value as that of the \( \Lambda_b \) baryon. So multiplying the branching ratios of \( \Lambda_b \) or \( \Lambda_b^* \) with the factor \( \tau(\Lambda_b)/\tau(\Lambda_b^*) \) will yield more precise values for these transitions.

At the end of this section we would like to make few comments on the results of the \( \Lambda_b \to N \) transition form factors with the ones existing in the literature. The form factors of \( \Lambda_b \to N \ell \nu \) transition has already been calculated in framework of the LCSR in \[17\], by taking into account the contributions of the \( \Lambda_b \) and \( \Lambda_b^* \) decays. Our results for the form factors \( f_1, f_2, g_1, \) and \( g_2 \) are different compared to the ones presented in \[17\]. This can be attributed to the fact that, in the present work, we have used different form of the interpolating current than that used in \[17\]. As we have already noted, the results for the \( \Lambda_b^* \) baryon are very sensitive to the choice of the interpolating current. We have also checked that if interpolating current presented in \[17\] were used, our results on the form factors coincide with the predictions of the work \[17\]. The \( \Lambda_b \to N \) transition is studied in \[21\] in framework of the LCSR, but without taking into account the contributions of \( \Lambda_b^* \) baryons. And also the continuum subtraction procedure is performed rather in an inconsistent manner. For this reason, our predictions for the branching ratios are a bit off compared to the ones presented in \[21\], due to the considerable differences on the predictions of the form factors. Moreover, the \( \Lambda_b \to N \ell^+ \ell^- \) decay is studied within the relativistic quark-diquark picture in \[22\]. When compared, our predictions on the relevant form factors are different than those predicted in \[22\], where the results for the branching ratios are, approximately, 2 times smaller compared to our predictions.

The sum rules for the form factors can further be improved by taking into account the \( \alpha_s \) corrections to the DAs, and improvements on the input parameters present in these DAs. The results we obtain for branching ratios of the CKM suppressed \( \Lambda_b \to N \ell^+ \ell^- \) and \( \Lambda_b^* \to N \ell^+ \ell^- \) decays governed by the \( b \to d \ell^+ \ell^- \) transition, give confidence that these decays can be discovered at LHCb at near future.

**IV. CONCLUSION**

In the present work, we calculate the transition form factors of \( \Lambda_b \to N \ell^+ \ell^- \) decay in the framework of the LCSR. We take into account the contribution of negative parity \( \Lambda_b^* \) baryon to the sum rules. Using the obtained results for the form factors we estimate the branching ratios of \( \Lambda_b \to N \ell^+ \ell^- \), and \( \Lambda_b^* \to N \ell^+ \ell^- \) decays. We also compare our predictions on the form factors and branching...
ratios with the ones existing in the literature. From these results we can conclude that these decays can be observed at the near future at LHCb.

ACKNOWLEDGMENTS

One of the authors (T. Barakat) extends his appreciation to the International Scientific Partnership Program ISPP at King Saud University for funding this research work through ISPP No. 0038.

APPENDIX: DIFFERENTIAL DECAY WIDTHS FOR THE $\Lambda_b \rightarrow N\ell^+\ell^-$ AND $\Lambda_b^* \rightarrow N\ell^+\ell^-$ TRANSITIONS

In this Appendix, we present the differential decay widths for the $\Lambda_b \rightarrow N\ell^+\ell^-$ and $\Lambda_b^* \rightarrow N\ell^+\ell^-$ transitions.

\[
\Gamma_1(s) = 8m_{\Lambda_b}^3\{[(1 - 2\sqrt{r} + r - s)[4m_r^2 + m_{\Lambda_b}^2(1 + 2\sqrt{r} + r + s)]|F_1|^2 - [4m_r^2(1 - 6\sqrt{r} + r - s) - m_{\Lambda_b}^2((1 - r)^2 - 4\sqrt{r}s - s^2)]|F_4|^2 + (1 - 2\sqrt{r} + r - s)[4m_r^2(1 + \sqrt{r})^2 + m_{\Lambda_b}^2s(1 + 2\sqrt{r} + r + s)]|F_2|^2 + m_{\Lambda_b}^2s((-1 + r)^2 - 4\sqrt{r}s - s^2)v^2|F_4|^2 + 4m_r^2(1 + 2\sqrt{r} + r - s)s|F_6|^2 + (1 + 2\sqrt{r} + r - s)[4m_r^2 + m_{\Lambda_b}^2(1 - 2\sqrt{r} + r + s)]|G_1|^2 - [4m_r^2(1 + 6\sqrt{r} + r - s) - m_{\Lambda_b}^2((1 - r)^2 + 4\sqrt{r}s - s^2)]|G_4|^2 + (1 + 2\sqrt{r} + r - s)[4m_r^2(1 - \sqrt{r})^2 + m_{\Lambda_b}^2s(1 - 2\sqrt{r} + r + s)]|G_2|^2 + m_{\Lambda_b}^2s((-1 + r)^2 + 4\sqrt{r}s - s^2)v^2|G_4|^2 + 4m_r^2(1 + 2\sqrt{r} + r - s)s|G_6|^2 + 4(1 + \sqrt{r})(1 - 2\sqrt{r} + r - s)[2m_r^2 + m_{\Lambda_b}^2s]\text{Re}[F_1^*F_2] - 4m_{\Lambda_b}^2(1 + \sqrt{r})(1 - 2\sqrt{r} + r - s)s v^2\text{Re}[F_1^*F_5] - 8m_r^2(1 - \sqrt{r})(1 + 2\sqrt{r} + r - s)\text{Re}[F_4^*F_6] - 4(1 - \sqrt{r})(1 + 2\sqrt{r} + r - s)[2m_r^2 + m_{\Lambda_b}^2s]\text{Re}[G_1^*G_2] - 4m_{\Lambda_b}^2(1 - \sqrt{r})(1 + 2\sqrt{r} + r - s)s v^2\text{Re}[G_4^*G_6] + 8m_r^2(1 + \sqrt{r})(1 - 2\sqrt{r} + r - s)\text{Re}[G_4^*G_6] \},
\]

\[
\Gamma_2(s) = -8m_{\Lambda_b}^4 v^2\lambda(1, r, s)[|F_1|^2 + |F_4|^2 + |G_1|^2 + |G_4|^2 - s(|F_2|^2 + |F_5|^2 + |G_2|^2 + |G_5|^2)].
\]

After lengthy but straightforward calculations, for the differential decay rate of the $\Lambda_b \rightarrow N\ell^+\ell^-$, we get

\[
\frac{d\Gamma(s)}{ds} = \frac{G_s^2\alpha_{em} m_{\Lambda_b}}{4096\pi^5}\left| V_{tb} V_{td}^* \right|^2 v^2 \sqrt{\lambda(1, r, s)} \left[ \Gamma_1(s) + \frac{1}{3} \Gamma_2(s) \right],
\]

where $s = q^2/m_{\Lambda_b}^2$, $r = m_N^2/m_{\Lambda_b}^2$, $G_F = 1.17 \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant, $v = \sqrt{1 - 4m_r^2/q^2}$ is the lepton velocity, and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the usual triangle function. For the element of the CKM matrix $|V_{tb} V_{td}^*| = (8.2 \pm 0.6) \times 10^{-3}$ has been used [19]. The functions $\Gamma_1(s)$ and $\Gamma_2(s)$ are given as

The differential decay width for the $\Lambda_b^* \rightarrow N\ell^+\ell^-$ transition can be obtained from the differential decay width for the $\Lambda_b \rightarrow N\ell^+\ell^-$ by making the following replacements: $F_i \rightarrow G_i$, $G_i \rightarrow F_i$, $m_N \rightarrow -m_N$, and by changing the sign in front of the terms $\text{Re}[F_4^*F_5]$, $\text{Re}[F_4^*F_6]$, and $\text{Re}[G_4^*G_6]$, as well as $m_{\Lambda_b} \rightarrow m_{\Lambda_b}^*$, $s \rightarrow s' = q^2/m_{\Lambda_b}^*$, and $r \rightarrow r' = m_N^2/m_{\Lambda_b}^*$.
[1] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 107, 201802 (2011).
[2] R. Aaij et al. (LHCb Collaboration), Phys. Lett. B 725, 25 (2013).
[3] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 04 (2017) 029.
[4] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 06 (2015) 115.
[5] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. B312, 509 (1989); V. L. Chernyany and I. R. Zhitnitsky, Nucl. Phys. B345, 137 (1990).
[6] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. B589, 381 (2000); B607, 433(E) (2001); V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, Phys. Rev. D 65, 074011 (2002).
[7] T. M. Aliev, K. Azizi, A. Ozpineci, and M. Savci, Phys. Rev. D 77, 114014 (2008).
[8] Y. L. Liu and M. Q. Huang, Phys. Rev. D 79, 114031 (2009); Nucl. Phys. A821, 80 (2009).
[9] Y.-M. Wang and Y.-L. Shen, J. High Energy Phys. 02 (2016) 179.
[10] T. M. Aliev, K. Azizi, and M. Savci, Phys. Rev. D 81, 056006 (2010).
[11] R. N. Faustov and V. O. Galkin, Phys. Rev. D 96, 053006 (2017).
[12] W. Detmold and S. Meinel, Phys. Rev. D 93, 074501 (2016); 87, 074502 (2013).
[13] L. F. Gan, Y. L. Liu, W. B. Chen, and M. Q. Huang, Commun. Theor. Phys. 58, 872 (2012).
[14] T. Feldmann and M. W. Yip, Phys. Rev. D 85, 014035 (2012).
[15] C. H. Chen and C. Q. Geng, Phys. Rev. D 64, 074001 (2001).
[16] Y.-M. Wang, Y. Li, and C.-D. Lu, Eur. Phys. J. C 59, 861 (2009).
[17] A. Khodjamirian, Ch. Klein, Th. Mannel, and Y. M. Wang, J. High Energy Phys. 09 (2011) 106.
[18] T. M. Aliev, K. Azizi, T. Barakat, and M. Savci, Phys. Rev. D 92, 036004 (2015).
[19] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[20] C. Bourrely, I. Caprini, and L. Lellouch, Phys. Rev. D 79, 013008 (2009).
[21] K. Azizi, M. Bayar, H. Sundu, and Y. Saraç, J. Phys. G 37, 115007 (2010).
[22] R. N. Faustov and V. O. Galkin, Mod. Phys. Lett. A 32, 1750125 (2017).