pp forward elastic scattering amplitudes at 7 and 8 TeV

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We analyse the recent LHC data at 7 and 8 TeV for pp elastic scattering with special attention for the structure of the real part, which is shown to be crucial to describe the differential cross section in the forward region. We determine accurately the position of the zero of the real amplitude, which corresponds to the zero of a theorem by André Martin.

Keywords: elastic scattering amplitudes; LHC; total cross section

I. INTRODUCTION

The elastic amplitude \( T(s,t) \) is a function of only two kinematical variables, controlled by principles of analyticity and unitarity, but no fundamental solution is known for its form, and representations of the differential cross section are given in terms of models, designed and applied for restricted ranges of \( s \) and \( t \). It is expected that at high energies the \( s \) dependence becomes relatively simple, but the enormous gaps and uncertainties in the data from CERN/ISR, Fermilab and CERN/LHC do not help in tracing the \( s \) dependence with reliability. On the other hand, for a given \( s \), the angular dependence has not been measured with uniformity in the full \( t \)-range, and the necessary disentanglement of the real and imaginary parts of the amplitude is a hard task, with unavoidable indetermination [1,2]. The forward \( t \) range has been measured more often. Recently Toten and Atlas groups at LHC measured \( d\sigma/dt \) in forward \( t \) ranges at \( \sqrt{s} = 7 \) and 8 TeV [3,4]. These data (Table I) offer an opportunity to study in detail several aspects of the very forward region, such as the magnitudes of the real and imaginary amplitudes, the position of the zero of the real part and the first derivatives of the amplitudes with respect to the variable \( t \) (slopes). The Coulomb-nuclear interference depends on the proton electromagnetic structure, and the relative phase requires specific assumptions for the forms of the nuclear amplitudes as described in detail in our recent work [5].

In the present work we propose independent parametrizations for the real and imaginary nuclear parts, writing

\[
T_R^N(t) = \frac{1}{(4\pi \hbar c)^2} \sigma(\rho - \mu_R t) e^{B_R t/2},
\]

and

\[
T_I^N(t) = \frac{1}{(4\pi \hbar c)^2} \sigma(1 - \mu_I t) e^{B_I t/2}.
\]

The parameter \( \sigma \) is the total cross section, \( \rho \) is the ratio of the real and imaginary parts at \(|t| = 0\), \( B_R \) and \( B_I \) are the local slopes of the amplitudes and the parameters \( \mu_R \) and \( \mu_I \) account for the existence of zeros in the amplitudes. The zero in the real part is crucial to explain the \( t \) dependence of \( d\sigma/dt \) for small \(|t|\).

We remark that parameters are determined fitting data in limited \(|t| \) ranges, at finite distance from the origin, so that the values obtained depend on the analytical forms [11,12] of the amplitudes. In particular, the slope parameter usually written in form \( d\sigma/dt = \sigma^2(\rho^2 + 1) \exp(Bt) \) does not agree with the expression for the differential cross section as sum of two independent squared magnitudes, each with its own slope. The assumption that \( B_R \) and \( B_I \) are equal is not justified. The average quantity \( B \) alone gives rough and unsatisfactory information. The importance of the different slopes in the analysis of pp elastic scattering has been investigated in the framework of the so called dispersion relations for slopes [6]. It is important to note that also the Coulomb-Nuclear phase \( \phi(t) \) depends essentially on the form of the nuclear amplitudes [7]. The zero of the real part is given by \(|t_R| = -\rho/\mu_R\). We understand that this quantity is the zero predicted in the theorem by André Martin [10].

Of course the parameters of the amplitudes are correlated, and in the present work we investigate the bounds of the correlations. We attempt to identify values of parameters that may be considered as common representatives for different measurements. We show that the differences between the two experimental collaborations may be restricted to quantities characterizing normalization. The question of normalization is essential, and our inputs are the values of \( d\sigma/dt \) given in the experimental papers [3,4].

The extraction of forward parameters in pp scattering has difficulties due to the small value of the \( \rho \) parameter, and consequently has suffered in many analyses from neglect of the properties of the real part. In our view the values of \( \sigma, \rho, B \) appearing in universal databases [11,12] as if they were direct experimental measurements should give room for critically controlled phenomenological determinations. A proper consideration for the properties
of the complex amplitude is necessary. We observe that the properties that \( B_R \neq B_I \) and of the presence of zeros are common to several models \[1, 2, 13\]. The determination of the amplitudes for all \(|t|\) in several models is coherent.

We observe that the polynomial factors written in the exponent in some parameterizations of data \[5\] correspond to the linear and quadratic factors mentioned above, if the assumption is made that they are much smaller than 1 and can be converted into exponentials. However this substitution is not convenient, because it does not show explicitly the essential zeros, and it also gives unsatisfactory parameterization that cannot be extended even to nearby \(|t|\) values.

We thus have the framework necessary for the analysis of the data, with clear identification of the role of the free parameters. The quantities to be determined for each dataset are \( \sigma, \rho, B_I, B_R, \mu_I, \mu_R \).

II. DATA ANALYSIS AT \( \sqrt{s} = 7 \) AND 8 TeV

The analysed datasets and their \( t \) ranges are listed in Table I, where T7, T8, A7, A8 specify TOTEM (T) and Atlas (A) Collaborations and center-of-mass energies 7 and 8 TeV. In the measured ranges the Coulomb effects play important role and the relative Coulomb phase is properly taken into account \[1\].

In order to identify values for parameters valid for all measurements, we study four different conditions in the fits: I) all six parameters are free; II) fixing \( \rho \) at 0.14, as suggested by dispersion relations; III) fixing \( \mu_I \) from the expected positions of imaginary zero \[1, 2\] and dip in \( d\sigma/dt \); IV) fixing simultaneously \( \rho \) and \( \mu_I \) at the above values. A complete table with the results can be found in ref. \[7\]. In the present work we show the values obtained with Condition IV) in Table I. The first reason for this choice is the existence of a correlation between the parameter \( \mu_R \) and the parameter \( \mu_I \) of the imaginary amplitude, and since the dip structure is presented in pp elastic scattering for larger \( t \) values, and the position of the dip is intimately related with the parameter \( \mu_I \), the determination of \( \mu_I \) constrains the value of \( \mu_R \). The second reason is justified because the parameter \( \mu_R \) together with \( \rho \) construct the position of the zero of Martin, which is suggested to have an asymptotic form \(|t_R| \sim 1/\log^2 s \) \[2\]. As mentioned in \[7\] the data sets analysed (A7, A8, T7 and T8) do not cover the large \(|t|\) range where the dip structure is presented and also, the values of \( \rho \) are sensitive to the Coulomb phase, which depends on the structure of the nuclear amplitude, and is still an open question. In order to contour these difficulties we fix both \( \rho \) and \( \mu_I \) at their expected values and we obtain good modeling for all measurements, except for the total cross sections, that distinguish Atlas from TOTEM.

The regularity on the values of \( \mu_R \) is remarkable, and the position of the zero is stable in all measurements with \(|t_R| \simeq 0.037 \text{ GeV}^2\) within the statistical errors. The position of the zero, together with the magnitude of \( B_R \) determines the structure of the real amplitude. The zero of the real amplitude is responsible for the structure shown in Fig. 1 where the differential cross section was subtracted by a pure exponential form called \( ref = A \exp(Bt) \) and divided by this quantity. Roughly speaking the \( ref \) function has the similar structure and the same magnitude of the imaginary amplitude in the forward region, which means that the structure of a valley shown in l.h.s of Fig. 1 is due to the structure of the real part. The r.h.s of Fig. 1 is an alternative quantity that instead of working with the \( ref \) function we have the squared of the real and imaginary amplitudes. The advantage of this language is that the errors due to the normalization of the cross section are suppressed leading to a much narrower error band. It is also interesting to observe that on the r.h.s of Fig 1 at \(|t|\) near 0.01 GeV\(^{-2}\) the quantity \( T^2 / I \) has a zero due to the interference of the real and Coulomb amplitude, since for pp scattering the Coulomb amplitude is negative while the real nuclear is positive near the origin.

The zero of the imaginary part anticipates the dip in the differential cross section that occurs beyond the range of the available data under study.

Our analysis indicates that the real amplitude plays crucial role in the description of the differential cross section in the forward region. Interference with the Coulomb interaction is properly accounted for, and use is made of information from external sources, such as dispersion relations and predictions for the imaginary zero obtained in studies of full-\( t \) behaviour of the differential cross section \[1, 2\].

III. CONCLUSIONS

In this work we study the properties of the amplitudes in pp elastic scattering analysing experimental data at the LHC center-of-mass energies 7 and 8 TeV, based on a model for the complex amplitude, with explicit real and imaginary parts, each containing an exponential slope and a linear factor to account for the existence of a zero. The zero of the real part, close to the origin, corresponds to Martin’s Theorem, and the zero of the imaginary part anticipates the dip in the differential cross section that occurs beyond the range of the available data under study.

Our study shows that the real amplitude plays crucial role in the description of the differential cross section in the forward region. Interference with the Coulomb interaction is properly accounted for, and use is made of information from external sources, such as dispersion relations and predictions for the imaginary zero obtained in studies of full-\( t \) behaviour of the differential cross section \[1, 2\]. We organize the analysis under four conditions, according to the specifications of the parameters with values fixed in each case. Comparison is made of the results obtained for the four experimental measurements.
FIG. 1. The left plot shows the non-exponential behaviour of the differential cross section for T8. The figure is obtained subtracting from the best fit of the differential cross section a reference function which is \(d\sigma/dt = A \exp(Bt)\) and dividing the subtraction by this reference function. The dashed lines show the normalization error band in \(d\sigma/dt\), that is quite large. The plot in the RHS shows the ratio \(T_8^2/T_I^2\) which exhibits information of a non-exponential behaviour with advantages compared with the first plot, since \(\sigma\) is cancelled, and with it most of the normalization systematic error.

| \(\sqrt{s}\) (GeV) | dataset | \(\Delta|t|\) range (GeV\(^2\)) | N points | Ref. | \(\sigma\) (mb) | \(B_I\) (GeV\(^{-2}\)) | \(\rho\) |
|---------------------|---------|----------------------------------|---------|-------|----------------|--------------------|------|
| 7                   | T7      | 0.0052-0.3709                    | 87      | 1     | 98.6±2.2      | 19.90±0.30         | 0.14 (fix)\(^a\) |
| 8                   | T8      | 0.0007-0.1948                    | 60      | 3     | 103.0±2.3     | 19.56±0.13         | 0.14 (fix)\(^b\) |
| 8                   | A8      | 0.0105-0.3635                    | 39      | 4     | 96.1±0.2      | 19.74±0.05         | 0.136 (fix)\(^d\) |

TABLE I. Values of parameters at \(\sqrt{s} = 7\) and 8 TeV determined by Totem and Atlas Collaborations at LHC \(^3\)\(^\#\)\(^3\). Values for \(\rho^{[a]}\) and \(\rho^{[a]}\) taken from COMPETE Collaboration \(^11\); \(\rho^{[c]}\) obtained by the authors with a forward SET-I and kept fixed in a complete SET-II; \(\rho^{[d]}\) is taken from \(^12\).

We obtain the results shown in Table \(\|\) that we believe to be a good representation of the experimental data of Table \(\|\).

The quantity \(\rho_R\) is related with the scaling variable \(\tau = t \log^2 s\) introduced by J. Dias de Deus \(^14\) connecting \(s\) and \(t\) dependences at high energies and small \(|t|\). A. Martin \(^15\) uses the same idea of a scaling variable, writing an equation for the real part \(\rho(s,t)\) using crossing symmetric scattering amplitudes of a complex variable, valid in a forward range. The proposed ratio is

\[
\rho(s,t) \simeq \frac{\pi}{\log s} \left(1 + \frac{\tau (df(\tau)/d\tau)}{f(\tau)}\right),
\]

where \(f(\tau)\) is a damping function, with the implicit existence of a real zero. The form of \(f(\tau)\) determines the properties of the real zero \(^14\), that is found in the analysis of the data. This may be a clue for the introduction of explicit crossing symmetry and analyticity in our phenomenological treatment of the data.

Other models \(^13\) also deal with the position of the real zero, discussing different analytical forms for the amplitudes, and it would be interesting to investigate their predictions for the amplitudes in the forward range.

In non-perturbative QCD, in several instances, the proton appears as a structure with expanding size as the energy increases \(^17\), with varied mechanisms, as distribution of valence quarks in a cloud around a core, modifications in QCD vacuum in the region of the colliding particles, and so on. Together with the evolution of the proton hadronic size, its electromagnetic properties, as they appear in high energy collisions, may change also. A linear increase in \(\log s\) is a usual assumption for the effective proton radius, and the form factor parameter \(\Lambda^2\) would then be reduced by about 1/2, corresponding to increase of about 40 \% in proton radius. In Appendix A of reference \(^7\) we calculate the interference phase with this example.

We expect that future data in pp elastic scattering at 14 TeV will have high quality covering a wide \(t\) range...
to allow determination of the properties of the real and imaginary amplitudes in pp elastic scattering, including studies of the amplitudes up to the perturbative tail for large $|t|$. Hopefully the experimental groups will receive the necessary support and encouragement for this effort.

| Fixed Quantities : $\rho = 0.14$ , $\mu_I = -2.16$ GeV$^{-2}$ (8 TeV) [2], $\mu_I = -2.14$ GeV$^{-2}$ (7 TeV) [1] |
|---------------------------------------------------------------|
| $N$ | $\sigma$ (mb) | $B_l$ (GeV$^{-2}$) | $B_R$ (GeV$^{-2}$) | $\mu_R$ (GeV$^{-2}$) | $-t_R$ (GeV$^2$) | $\chi^2$/ndf |
|-----------------------------------------------|----------------|----------------|----------------|----------------|----------------|---------------|
| T8 | 60 | 102.40±0.15 | 15.27±0.39 | 21.15±0.39 | -3.69±0.15 | 0.038±0.002 | 69.2/56 |
| A8 | 39 | 96.82±0.11 | 15.26±0.06 | 21.65±0.24 | -3.69±0.12 | 0.038±0.001 | 30.0/35 |
| T7 | 87 | 99.86±0.21 | 15.71±0.14 | 24.26±0.47 | -4.24±0.31 | 0.033±0.002 | 95.1/83 |
| T7 | 87+17 | 99.44±0.14 | 15.44±0.07 | 22.62±0.19 | -3.49±0.13 | 0.040±0.002 | 203.5/100 |
| A7 | 40 | 95.75±0.16 | 15.23±0.11 | 21.86±0.44 | -3.99±0.22 | 0.035±0.002 | 27.3/36 |

TABLE II. Proposed values of parameters for the four datasets. The T7 data are also shown with inclusion of points at higher $|t|$ (0.005149 < $|t|$ < 2.443 GeV$^2$) that are important for confirmation of the value of $\mu_I$ [2].

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[1] A. Kendi Kohara, E. Ferreira, and T. Kodama, Eur. Phys. J. C 73, 2326 (2013).
[2] A. K. Kohara, E. Ferreira, and T. Kodama, Eur. Phys. J. C 74, 3175 (2014).
[3] G. Antchev et al. (TOTEM Coll.), Eurphys. Lett. 101, 21002 (2013).
[4] G. Aad et al. (ATLAS Collaboration), Nucl. Phys. B 889, 486 (2014).
[5] G. Antchev et al. (TOTEM Coll.), Eur. Phys. J. C. 16, 661 (2016); Nucl. Phys. B 899, 527 (2015).
[6] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B 761, 158 (2016).
[7] A. K. Kohara, E. Ferreira, T. Kodama, and M. Rangel, Eur. Phys. J. C 77, 877 (2017).
[8] E. Ferreira, Int. Jour. Mod. Phys. E 16, 2893 (2007).
[9] V. Kundrát and M. Lokajícek, Phys. Lett. B 611, 102 (2005); V.Kundrát, M.Lokajícek and I. Vrooc, Phys. Lett. B 656, 182 (2007).
[10] A. Martin, Phys. Lett. B 404, 137 (1997).
[11] J. R. Cudell et al. (COMPETE Collaboration), Phys. Rev. Lett. 89, 201801 (2002).
[12] C. Patrign et al. (Particle Data Group), Chinese Physics C 40, 100001 (2016).
[13] C. Bourrely, J. Soffer, and T. T. Wu, Nucl. Phys. B 247, 15 (1984); Phys. Rev. Lett. 54, 757 (1985); Phys. Lett. B 196, 237 (1987); A. K. Kohara, E. Ferreira, and T. Kodama, Phys. Rev. D 87, 054024 (2013); V. A. Petrov, E. Predazzi and A. V. Prokudin, Eur. Phys. J. C 28, 525 (2003); O. V. Selyugin, Phys. Rev. D 60, 074028 (1999); M. M. Islam, R. J. Luddy, and A. V. Prokudin, Mod. Phys. Lett. A 18, 743 (2003); M.M. Islam and R.J. Luddy, Acta Phys. Pol. B Proc. Sup., B 4 (2015).
[14] J. D. Deus, Nucl. Phys. B 59, 231 (1974); Phys. Lett. B 718, 181 (2013).
[15] A. Martin, Lett. Nuovo Cim. 7, 811 (1973).
[16] I. M. Dremin, arXiv:1204.1914 [hep-ph].
[17] J. Dias de Deus and P. Kroll, Nuovo Cimento A 37, 67 (1977); Acta Phys. Pol. B 9, 157 (1978); J. Phys. G 9, L81 (1983); P. Kroll, Z. Phys. C 15, 67 (1982); T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968); Phys. Rev. D 19, 3268 (1979); Phys. Lett. B 128, 457 (1983); Phys. Lett. B 244, 113 (1990); B. Povh and J. Hüfner, Phys. Rev. Lett. 58, 1612 (1987); Phys. Lett. B 215, 722 (1988); Phys. Lett. B 245, 653 (1990); Phys. Rev. D 46, 990 (1992); Z. Phys. C 63, 631 (1994); E. Ferreira and F. Pereira, Phys. Rev. D 55, 130 (1997); Phys. Rev. D 56, 179 (1997).