Node-to-Set Disjoint Paths Problem in a Möbius Cube*

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SUMMARY  This paper proposes an algorithm that solves the node-to-set disjoint paths problem in an $n$-Möbius cube in polynomial-order time of $n$. It also gives a proof of correctness of the algorithm as well as estimating the time complexity, $O(n^4)$, and the maximum path length, $2n - 1$. A computer experiment is conducted for $n = 1, 2, \ldots, 31$ to measure the average performance of the algorithm. The results show that the average time complexity is gradually approaching to $O(n^4)$ and that the maximum path lengths cannot be attained easily over the range of $n$ in the experiment. key words: hypercube, multicomputer, interconnection network, parallel processing, dependable computing

1. Introduction

Recently, because the clock frequency has shown signs of leveling off, parallel processing systems, especially massively parallel systems are gathering much attention. Though the hypercube [21] was enthusiastically studied in 1980’s as a topology for parallel processing systems, it has been seen over the last two decades as obsolete. Recent massively parallel systems adopt the hierarchical topology instead of the conventional simple topologies such as a mesh, a torus, and so on to connect several tens of thousands of processors efficiently. Some massively parallel systems such as NASA Pleiades [19] and ACC Cyfronet AGH Zeus [2], [20] have adopted a hypercube as a higher-layer topology of their hierarchical topologies. Therefore, a hypercube and its variants have returned to the center of attention. A Möbius cube [6] is one such variant of a hypercube.

A Möbius cube can connect the same number of nodes as a hypercube while keeping its diameter about half of that of the hypercube. Hence, it has attracted much attention [8], [14], [23]−[25]. The unsolved problems in Möbius cubes include the node-to-set disjoint paths problem: given a source node $s$ and a set of destination nodes $D = \{d_1, d_2, \ldots, d_k\}$ in a $k$-connected graph $G = (V, E)$, find $k$ paths $s \leadsto d_i$ ($1 \leq i \leq k$) between $s$ and each element of $D$ that are node-disjoint except for $s$. Note that in this paper the notations $u \rightsquigarrow v$ and $u \to v$ for two nodes $u$ and $v$ represent a path from $u$ to $v$ and an edge from $u$ and $v$, respectively. The node-to-set disjoint paths problem is an important issue in parallel and distributed computation [5], [10], [15], [16] as well as the node-to-node disjoint paths problem [7], [12], [18], [22] and the set-to-set disjoint paths problem [3], [4], [9], [11].

In general, we can solve the node-to-set disjoint paths problem in polynomial-order time of $|V|$ by using the max-flow algorithm. However, the complexity of the algorithm is too large for an $n$-dimensional Möbius cube $M_n$ because it has $2^n$ nodes. For an $n$-dimensional hypercube, there is an algorithm that solves the node-to-set disjoint paths problem in $O(n^2)$ time [5]. The maximum length of the paths generated by the algorithm is $n + 1$. However, this algorithm is not applicable to Möbius cubes because they do not have some properties that hold in hypercubes. Therefore, it is necessary to invent an applicable algorithm by investigating properties of an $M_n$. In this paper, we propose an algorithm N2S (node-to-set) with polynomial-order time of $n$ instead of $2^n$. Algorithm N2S is comprised of two cases depending on the distribution of the source node and the destination nodes. The algorithm constructs $n$ disjoint paths from the source node to $n$ destination nodes where $n$ is equal to the connectivity of $M_n$’s. We also present the results of an average performance evaluation by a computer experiment.

The rest of this paper is organized as follows. A definition of a Möbius cube as well as other requisite definitions are introduced in Sect. 2. Section 3 explains our algorithm N2S in detail. Section 4 describes a proof of correctness and the theoretical complexities of N2S. Average performance of N2S is reported in Sect. 5. We conclude and give future works in Sect. 6.

2. Preliminaries

A definition of a Möbius cube and three lemmas are introduced in this section.

Definition 1: An $n$-dimensional Möbius cube $M_n$ has $2^n$ nodes. A unique $n$-bit address is assigned to each node. Two nodes $\bar{u} = (u_1, u_2, \ldots, u_n)$ and $\bar{v}$ are connected if and only if one of the following conditions is satisfied:

$$\bar{v} = \begin{cases} \{ (u_1, u_2, \ldots, u_{i-1}, \bar{u}, u_{i+1}, \ldots, u_n) \} & (u_{i-1} = 0), \\ \{ (u_1, u_2, \ldots, u_{i-1}, \bar{u}, u_{i+1}, \ldots, u_n) \} & (u_{i-1} = 1). \end{cases}$$

where $\bar{u}$ represents a bit obtained by inverting $u$. Note that
Table 1: Comparison of a 0-M \(_n\) and a 1-M \(_n\) with other topologies.

| Topology | #nodes | degree | Diameter | Average Distance |
|----------|--------|--------|----------|------------------|
| 0-M \(_n\) | \(2^n\) | \(n\) | \([(n + 2)/2]\) | \(\dagger\) |
| 1-M \(_n\) | \(2^n\) | \(n\) | \([(n + 1)/2]\) | \(\dagger\) |
| \(H_n\) | \(2^n\) | \(n\) | \(n\) | \(n/2\) |
| \(T_n\) | \(2^n\) | \(n\) | \([(n + 1)/2]\) | \(\rightarrow 3n/8 (n \rightarrow \infty) \) [1] |

\(\dagger: \leq n/3 + [1 - (1/2]^d / 9 + 1 [6]\)

Fig. 1 Examples of a 0-M\(_n\) and a 1-M\(_n\).

Fig. 2 Disjoint paths between \(M^d_n\) and \(M^u_n\).

Regarding the average distance.

There is a shortest-path routing algorithm for an arbitrary pair of nodes in an \(M_n\) and it takes \(O(n)\) time [6]. In the rest of this paper, we refer the algorithm spr.

**Lemma 1:** In an \(M_n\), for an arbitrary node \(u \in M^j_n\) (\(j \in \{0, 1\}\)), there is exactly one edge \(u \rightarrow v_i(\in M^j_n)\).

(Proof) Assume \(u \in M^j_n\) (\(j \in \{0, 1\}\)). Then, \(u(i) \in M^j_n\) (2 \(\leq i \leq n\)). While \(u^{(1)} \in M^j_n\). Hence, there is exactly one edge \(u \rightarrow u^{(1)}(\in M^j_n)\).

**Lemma 2:** In an \(M_n\), for an arbitrary node \(u \in M^j_n\) (\(j \in \{0, 1\}\)), there are \(n\) paths of length at most 2 from \(u\) to nodes in the \(M^j_n\) that are disjoint except for \(u\).

(Proof) Let us consider \(n\) paths from \(u\) to nodes in the \(M^j_n\):

\[
U_i : \begin{cases} 
  u \rightarrow u^{(1)} \quad (i = 1), \\
  u \rightarrow u^{(1)} \rightarrow u^{(2)}(1) \quad (2 \leq i \leq n).
\end{cases}
\]

Then, from Lemma 1, \(u^{(1)} \neq u^{(1)}(2 \leq i \leq n)\). Hence, \(U_i\) is disjoint with other paths \(U_j\) (2 \(\leq i \neq j \leq n\)) except for \(u\). In addition, for two paths \(U_i\) and \(U_j\) (2 \(\leq i \neq j \leq n\)), from \(u^{(1)} \neq u^{(1)}\) and Lemma 1, these paths are also disjoint except for \(u\) (Fig. 2). From above discussion, the \(n\) paths \(U_i\) (1 \(\leq i \leq n\)) of length at most 2 are disjoint except for \(u\). 

**Lemma 3:** There is no cycle whose length is 3 in an \(M_n\).

(Proof) We prove this lemma by induction on \(n\). Clearly, an \(M_2\) does not have a such cycle. Then, we assume that \(n \geq 3\) and the lemma holds for an arbitrary \(M_{n-1}\). If an \(M_n\) has a cycle of C of length 3, C has at least two edges between the \(M^0_n\) and the \(M^1_n\). Then, from Lemma 1, the terminal nodes of these two edges are all different. Therefore, this fact contradicts that the length of \(C\) is 3.

3. Algorithm N2S

In this section, for a source node \(s\) and a set of destination nodes \(D = \{d_1, d_2, \ldots, d_n\}\) in an \(M_n\) where \(s \notin D\), we show an algorithm N2S that finds \(n\) paths \(R_i : s \rightarrow d_i\) (1 \(\leq i \leq n\)) that are disjoint except for \(s\).
Since a 0-1 and a 1-1 are isomorphic and they consist of two nodes and an edge between them, the solution in a 0-1 or a 1-1 is trivially \( s \rightarrow d_1 \). Hence, in the rest of the paper, we assume that \( n \geq 2 \).

### 3.1 Procedure 1

In case that all destination nodes are included in an \( M^n \) (\( s \in M^n, D \subset M^n \)), we construct \( n \) paths from \( s \) to each destination node in \( D \) that are disjoint except for \( s \) by the following Procedure 1.

**Step 1** In the \( M^n \), apply Algorithm N2S recursively to construct \((n - 1)\) paths \( R_i: s \sim d_i \) (\( 1 \leq i \leq n - 1 \)) that are disjoint except for \( s \).

**Step 2** If the node \( d_n \) is included in one of the \((n - 1)\) disjoint paths constructed in Step 1, say \( R_h: s \sim d_h \) (Fig. 3), discard the subpath \( d_n \sim d_h \), and exchange the indices of \( d_h \) and \( d_n \). See Fig. 4.

**Step 3** Select edges \( s \sim s^{(1)} \) and \( d_n \sim d^{(1)}_n \).

**Step 4** In the \( M^n \), construct a path \( s^{(1)} \sim d^{(1)}_n \) by using Algorithm spr.

Finally, \( n \) paths \( P_i: s \sim d_i \) (\( 1 \leq i \leq n \)) that are disjoint except for \( s \) are constructed as follows:

\[
P_i: \begin{cases} \ s \sim d_i, & (1 \leq i \leq n - 1), \\ s \sim s^{(1)} \sim d^{(1)}_n \sim d_n, & (i = n). \end{cases}
\]

See Fig. 5.

### 3.2 Procedure 2

In case that some destination nodes are included in the \( M^n \) (\( s \in M^n, D \subset M^n \)), we construct \( n \) paths from \( s \) to each destination node in \( D \) that are disjoint except for \( s \) by the following Procedure 2.

**Step 1** Without loss of generality, we can assume that \( D \cap M^n = \{d_1, d_2, \ldots, d_k\} \) and \( D \cap M^n = \{d_{k+1}, d_{k+2}, \ldots, d_n\} \). For \( d_{k+1}, d_{k+2}, \ldots, d_n \), from Lemma 2 construct \((n - k)\) mutually disjoint paths \( Q_i: d_i \sim d_i' (k + 1 \leq i \leq n) \) of length at most 2 between \( M^n \) and \( M^n \) that do not include \( d_1, d_2, \ldots, d_k \). In particular, for each \( d_i \) of \( d_{k+1}, d_{k+2}, \ldots, d_n \) in this order, select a path \( Q_i: d_i \sim d_i' \) among \( n \) paths by Lemma 2 that does not include any node on the other paths \( Q_{k+1}, Q_{k+2}, \ldots, Q_{n-1} \) or the nodes \( d_1, d_2, \ldots, d_k \).

**Step 2** Select an edge \( s \sim s^{(1)} \).

**Step 3** In the \( M^n \), construct a path \( s^{(1)} \sim d_n \) by using Algorithm spr.

**Step 4** If the path constructed in Step 3 includes any nodes \( s \) that are disjoint except for \( Q_n \) constructed in Step 1, let \( d_{n+1} \) be the node on the path \( Q_n \) that is closest to \( s^{(1)} \), discard the subpath \( d_n \sim d_{n+1} \), and exchange the indices of \( d_n \) and \( d_{n+1} \). See Fig. 6.

**Step 5** Discard the subpaths of \( Q_n \) except for the subpath constructed in Steps 3 and 4.

**Step 6** In the \( M^n \), apply Algorithm N2S recursively to construct \((n - 1)\) paths \( R_i: s \sim d_i (2 \leq i \leq k) \) and \( R_i: s \sim d_i' (k + 1 \leq i \leq n - 1) \) that are disjoint except for \( s \).

Finally, \( n \) paths \( P_i: s \sim d_i \) (\( 1 \leq i \leq n \)) that are disjoint except for \( s \) are constructed as follows:

\[
P_i: \begin{cases} \ s \sim d_i, & (1 \leq i \leq k), \\ s \sim s^{(1)} \sim d_{n+1} \sim d_n, & (k + 1 \leq i \leq n - 1), \\ s \sim s^{(1)} \sim d_n, & (i = n). \end{cases}
\]

See Fig. 7.
4. Proof of Correctness and Estimation of Complexities

In this section, we prove the correctness of our algorithm and we give the estimates of the time complexity $T(n)$ and the maximum path length $L(n)$ for an $n$-dimensional Möbius cube $M_n$. Proofs are based on induction on $n$.

We assume that each node can be stored in a machine word, and construction of an edge by obtaining $u^{(i)}$ for any node $u$ requires $O(1)$ time. On the other hand, for any pair of nodes in an $M_n$, Algorithm spr takes $O(n)$ execution time to construct a shortest path between them whose length is at most $\lfloor (n+2)/2 \rfloor$ [6].

**Lemma 4:** In an $M_n$, the paths $P_i (1 \leq i \leq n)$ constructed by Procedure 1 are disjoint except for $s$. The time complexity of Procedure 1 is $T(n-1) + O(nL(n))$ and the maximum length of the paths constructed is $\max\{L(n-1), [n/2] + 3\}$.

(Proof) The paths $P_i(1 \leq i \leq n-1)$ constructed in Steps 1 and 2 are disjoint except for $s$ by hypothesis of induction. The path $P_n$ constructed in Steps 3 and 4 is outside of $M'_i$ except for $s$ and $d_n$. Hence, $P_i$ cannot share any common node with $P_j (1 \leq i \leq n-1)$ except for $s$, that is, $P_i$ is disjoint with $P_j (1 \leq i \leq n-1)$ except for $s$. Step 1 takes $T(n-1)$ time to construct $n-1$ paths and the maximum length of them is $L(n-1)$. Step 2 takes $O(nL(n-1))$ time to check whether $d_n$ is included in one of the paths constructed in Step 1. $P_n$ consists of two edges and a subpath by spr. Therefore, Steps 2 and 3 take $O(n)$ time to construct a path whose length is at most $2 + [(n+1)/2] = [n/2] + 3$. Hence, the time complexity of Procedure 1 is $T(n-1) + O(nL(n-1))$ and the maximum path length is $\max\{L(n-1), [n/2] + 3\}$.

**Lemma 5:** In an $M_n$, the paths $P_i (1 \leq i \leq n)$ constructed by Procedure 2 are disjoint except for $s$. The time complexity of Procedure 2 is $T(n-1) + O(n^3)$ and the maximum length of the paths constructed is $\max\{L(n-1)+2, [n/2]+2\}$.

(Proof) In Step 1, from Lemma 2, for each $d_i$ the nodes $d_{i+1}, d_{i+2}, \ldots, d_n$, $n$ paths can be constructed. Each of the nodes $d_1, d_2, \ldots, d_n$ is included at most one of the paths. In addition, from Lemma 3, each of other paths $Q_{k+1}, Q_{k+2}, \ldots, Q_{n-1}$ shares nodes with at most one of the paths given by Lemma 2 for $d_i$. Therefore, at least one of the $n$ paths given by Lemma 2 does not include $d_1, d_2, \ldots, d_n$ or any node on the paths $Q_{k+1}, Q_{k+2}, \ldots, Q_{n-1}$. Hence, in Step 1, $(n-k)$ disjoint paths $Q_i (1 \leq i \leq n)$ of lengths at most 2 can be constructed in $O(n^3)$ time from Lemma 2. The path $P_n$ constructed in Steps 2 to 5 is disjoint with other paths and the length is at most $L(n-1) + 2$. The time complexity for construction is $O(1) + O(n) + O(n^3) = O(n^3)$. The $(n-1)$ paths $R_j (1 \leq j \leq n-1)$ of lengths at most $L(n-1)$ can be constructed in $T(n-1)$ time in Step 6 and they are disjoint except for $s$ from induction hypothesis. Then, with above discussion, the paths $P_i (1 \leq i \leq n)$ are disjoint except for $s$. They can be constructed in $T(n) + O(n^3)$ time and their maximum length is $\max\{L(n-1)+2, [n/2]+2\}$.

**Theorem 1:** For a node $s$ and a set of $n$ nodes $D = \{d_1, d_2, \ldots, d_n\}$ in an $M_n$, Algorithm N2S finds $n$ paths from $s$ to $d_i (1 \leq i \leq n)$ that are disjoint except for $s$ in $O(n^4)$ time, and their maximum length is $2n-1$.

(Proof) From $L(1) = 1$ and Lemmas 4 and 5, the constructed paths are disjoint except for $s$ and $L(n) = 2n-1$. Then, $T(n) = O(n^4)$.

5. Performance Evaluation

We carried out a computer experiment to evaluate average performance of Algorithm N2S. In the experiment, we repeated following steps at least 10,000 times for random pairs of the source $s$ and the set of destination nodes $D = \{d_1, d_2, \ldots, d_n\}$ in a $0-M_n$ and a $1-M_n$ for each $n$ between 1 and 31.

1. Select a set of $n$ destination nodes $D = \{d_1, d_2, \ldots, d_n\}$ randomly.
2. Select a source node $s$ randomly other than $D$.
3. For $s$ and $D$, apply Algorithm N2S and measure the execution time and the maximum path length.

We implemented Algorithm N2S by using the programming language C++. The program was compiled with the GNU G++ compiler g++ with a -O option. The target machine is equipped with an Intel Core i5-3230M CPU 2.60 GHz and 4GB RAM. The program was running at Oracle VM VirtualBox with 1GB RAM.

Figures 8 and 9 show the average execution time to construct a node-to-set disjoint paths and their maximum lengths, respectively. Figure 8 shows that the average execution time is gradually approaching to $O(n^2)$ over the range of $n$ in the experiment. From Fig. 9, we can see that the theoretical maximum path length, $2n-1$, is not easily attainable.

Next, we compared difference of performance of Algorithm N2S in a $0-M_n$ and a $1-M_n$. We took the average execution times and the maximum path lengths in a $0-M_n$ and a $1-M_n$ for each $n$ shown in Figs. 8 and 9 as samples, and conducted a Wilcoxon rank-sum test. As a result, we could not find any statistically significant difference between them.
regarding both of the average execution time \( (W = 483, \rho = 0.98) \) and the maximum path length \( (W = 484.5, \rho = 0.96) \). From these results, we can conclude that Algorithm N2S is applicable to a 0-\( M_n \) and a 1-\( M_n \) equivalently.

6. Conclusions

In this paper, we proposed an algorithm that solves the node-to-set disjoint paths problem in \( n \)-Möbius cubes. Theoretical analysis has shown that its time complexity is \( O(n^4) \) and the maximum path length is \( 2n - 1 \). We also conducted a computer experiment and showed that the average execution time is gradually approaching to \( O(n^3) \) and the maximum path length \( 2n - 1 \) is not easily attainable over the range of \( n \) in the experiment.

Future works include theoretical analysis of average performance of the algorithm and improvement of the algorithm to construct shorter paths in smaller execution time.

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