Comparing the Cost of Using Single Policy for Periodic-Review Policy for a 2-Echelon Inventory Problem with Seasonal Demand

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Abstract

Mixed-integer programming models are used to investigate the difference between total costs of multiple and single ordering policies of an inventory system under seasonal demand. The system is a 2-echelon system with single warehouse and N retailers in which demand occurs only at retailers and unsatisfied demand is considered as demand loss. The multiple policies are policies which vary along the fluctuating demand and the single policies are policies which do not vary. It is found that the difference between total costs of single and multiple policies can be ranging from 0% to 100%. The economic-order-quantity-to-demand ratio, $\frac{EOQ}{D}$, can be used to decide whether to apply multiple policies or keep using single policy. With any size of demand, the system with $\frac{EOQ}{D}$ ratio smaller than 1, multiple policies are likely to perform better. The difference of total costs is at the highest when $\frac{EOQ}{D}$ ratio is zero and it decreases as $\frac{EOQ}{D}$ ratio gets larger. With the $\frac{EOQ}{D}$ ratio greater than 1, both types of policies tend to give the same total cost.

Keywords : Multi-echelon systems, Seasonal demand, Mixed-integer programming, Inventory system, Multiple policy
1. Introduction

This study focuses on an inventory system with single warehouse and multiple retailers under seasonal demand. The system is a 2-echelon inventory system in which demand only occurs at retailers. Demand that is not satisfied with on-hand inventory will be considered as demand loss. Retailers are supplied by the warehouse and the warehouse is supplied by external suppliers. In the system, items are stored at both warehouse and retailers. When the warehouse or any retailer orders items, they are replenished with known lead time. Demand is assumed to be seasonal which fluctuates in a cycle of a certain span of periods. The demand pattern repeats cycle after cycle and the total demand per cycle is assumed to be stationary. In this problem, demand is influenced by human decision, so its cycle is not only a year but could be a month or a week.

The objective of this paper is to investigate the difference between the total costs of policies which vary along the fluctuating demand and policies which do not vary and also investigate when to apply each of them.

The remainder of this paper is organized as follows. Section 2 reviews the literature related to multi-echelon system. Section 3 presents a problem description. Section 4 describes the methodology to determine ordering policies. Section 5 presents results and discussions. Finally, Section 6 concludes and suggests future research extensions.

2. Literature review

Multi-echelon model was first introduced as a serial multi-echelon system with stationary uncertain demand by Clark and Scarf [1] and it was further studied by many researchers. The system was extended from serial to divergent structure and was studied in many aspects. De Kok et al. [2] classified multi-echelon inventory research systematically with various dimensions such as structure, demand, and research goal.

There are papers considering multi-echelon inventory system with fluctuating deterministic demand. These studies were called the multi-echelon dynamic lot sizing problem [3]. They were mostly solved by mixed-integer programming models or algorithms such as Lagrangian relaxation [4] or decomposition strategy [5]. Tarim and Kingsman [6] developed an algorithm based on a mixed-integer programming model to solve lot-sizing problem with service-level constraints for single-item single location on multi-period. Then, the algorithm was applied to calculate the \((R, S)\) policies [7] and Tarim and Smith [8] improved the algorithm to solve within shorter time by using a constraint programming model.

With non-stationary uncertain demand, there are studies both in single-echelon and multi-echelon systems. Although, there are various methods to deal with non-stationary demand, many methods are based on the same concept. One of the concepts that is widely used is dividing the non-stationary demand into many phases of stationary demand and applying multiple policies on those phases. Reddy and Rajendran [9] developed a heuristic method to determine order-up-to policy for a 5-level serial supply chain with non-stationary demand at the lowest level. They proposed a dynamic order-up-to policy which the policy changed periodically and conducted a simulation study to evaluate the heuristics. Grewal, Enns, and Rogers [10] applied a simulation-optimization procedure to solve a single-echelon system with seasonal demand of two products. As demand had seasonal pattern which repeated cycle after cycle, each cycle could be divided into many phases with the same demand’s character as other cycles. Then, there were as many ordering policies as number of phases in a demand cycle. Therefore, reorder points and lot sizes varied along demand pattern regions. Ordering policy parameters were iteratively improved via process between simulation and optimization models.

As many researchers chose to apply multiple ordering policies on multiple phases of non-stationary demand, the number of decision variables can grow rapidly if the system deals with many products or the demand changes frequently. Due to complexity, multiple policies are not usually practical in real-life situations. Tunc et al. [11] investigated that when demand followed a stable seasonal pattern with high uncertainty, a single policy could be reasonably used instead of the optimal multiple policies. The objective of this paper is to investigate the difference between the total costs of multiple policies and a single policy and when to apply each of them.

3. Problem statement

The problem is a 2-echelon inventory system having one warehouse and \(N\) retailers with seasonal demand. Each location
replenishes inventory in a fixed lead time. Retailers are supplied by the warehouse which is supplied by external suppliers and items can be stored at both warehouse and retailers. Demand that is not satisfied with on-hand inventory will be considered as demand loss. This demand loss must not exceed expected service level. The service level, in this problem, is fill rate - the proportion of demand served from on-hand inventory over period’s demand.

Demand is assumed to be seasonal without trend and fluctuating within a cycle of a certain span of periods. The demand pattern repeats cycle after cycle. For example, if each cycle consists of 4 periods, average demand of period 1 are equal to those of periods 5, 9, and 13 and so are periods 2, 6, 10, and 14.

Each location operates on a periodic review basis using reorder point and order-up-to points, \((R, s, S)\), or periodic review base-stock policy. The inventory level is reviewed every review period of time, \(R\), and if the level is at or lower than reorder point, \(s\), an order must be placed to raise the inventory level back to equal to or higher than order-up-to point, \(S\). The system applies echelon stock basis in which each location makes decision based on inventory levels of its own and of all locations downstream.

Demand is assumed to change period by period or each phase of demand is one period long while many researchers assumed that each phase of demand was a number of periods long. Since frequent change in demand causes the problem more complex, 2 types of ordering policies will be considered: (1) multiple policies - policies which vary along the fluctuating demand and (2) single policy - policy which does not vary. For example, if a cycle has 4 periods, for single policy, only one policy will be applied on each location, while, for multiple policies, 4 policies will be applied. This paper focuses on the difference between the total costs of a single policy and multiple policies and also when to apply single or multiple policies.

4. Methodology

The mixed-integer programming model is used to find two types of policies for the test instances. The first type applies a single policy and the other applies multiple policies. With a single policy, only one reorder point and one order-up-to point are determined for each location. On the other hand, with multiple policies, a number of reorder points and order-up-to points are determined for each location. Since the demand is assumed to repeat its pattern cycle after cycle, each location will have as many policies as the number of periods in a demand cycle. If there are 4 periods in a demand cycle, there will be 4 reorder points and 4 order-up-to points for each location. Each policy will be applied to each period in a cycle.

In this problem, review period, \(R\), is given to be one period long. Therefore, the model finds the policies which are reorder points, \(s\), and order-up-to points, \(S\), of each location as the demand used in the mixed-integer programming model is average demand of each period. Then, the total costs of these policies are compared.

In this paper, the mixed-integer programming model proposed by Sakulsom and Tharmmaphornphibal [12-13] is extended to find multiple policies. There is also a concept included in the models since the models will solve problems within finite-period horizon while real-life system lies within infinite-period horizon. While there is no expected demand after the planning horizon in the mixed-integer programming model, on-hand inventory level at the last period tends to be zero as it gives lower holding cost. Applying this type of solutions to an infinite-horizon problem will lead to shortage at the period beyond the planning horizon. Hence, there should be items left at the end of the last period for the following periods beyond the planning horizon. Therefore, there are constraints added to the mixed-integer programming model to force on-hand and on-order inventory at the end of horizon to be equal to those at the beginning. The model is as follows.

Indices

\(n\) = number of retailers

\(m\) = number of periods in horizon

\(q\) = number of periods in a cycle (in a single policy, \(q = 1\))

\(I_r\) = \(\{1, 2, \ldots, n\}\), a set of retailers

\(I_{rw}\) = \(\{0, 1, \ldots, n\}\), a set of stock points including a warehouse and retailers (the warehouse is referred as \(i = 0\))

\(J\) = \(\{1, 2, \ldots, m\}\), a set of periods in the planning horizon

\(P\) = \(\{0, 1, \ldots, q - 1\}\), a set of periods in a demand cycle (in a single policy, \(P = \{0\}\))
Parameters

$D_{ij}$ = demand of retailer $i$ during period $j$ (unit)

$K_i$ = ordering cost of stock point $i$ ($) 

$h_i$ = holding cost of stock point $i$ ($$/unit/period)$

$T_i$ = lead time of stock point $i$ (period)

$servlevel_i$ = expected service level of retailer $i$

$\gamma_{ij}$ = 1 if a stock point $i$ reviews its inventory in period $j$; 0 otherwise

$M$ = a large positive number that is greater than inventory level

Decision variables

$I_{ij}$ = on-hand inventory level of stock point $i$ at the end of period $j$ (unit)

$O_{ij}$ = on-order amount of stock point $i$ at the beginning of period $j$ (unit)

$L_{ij}$ = demand loss of retailer $i$ during period $j$ (unit)

$I_{ij}$ = 1 if on-hand inventory of retailer $i$ in period $j$ is not sufficient to cover period’s demand, 0 otherwise

$Z_{ij}$ = 1 if an order at stock point $i$ in period $j$ is placed; 0 otherwise

$S_{ip}$ = reorder point of stock point $i$ on period $p$ of a cycle (unit)

$S_{ip}$ = order-up-to point of stock point $i$ on period $p$ of a cycle (unit)

Objective function

Minimize

$$\sum \sum Z_{ij} \times K_i + \sum \sum I_{ij} \times h_i$$  (1)

Subject to

$$I_{ij-1} + O_{ij-T_i} + L_{ij-1} = D_{ij} + I_{ij} \quad \forall i \in I_r, \forall j \in J$$  (2)

$$O_{ij} - S_{0j} = \sum O_{ij} + I_{ij} \quad \forall j \in J$$  (3)

$$D_{ij} \leq (I_{ij-1} + O_{ij-T_i}) \leq L_{ij} \times M \quad \forall i \in I_r, \forall j \in J$$  (4)

$$L_{ij} \leq L_{ij} \times M \quad \forall i \in I_r, \forall j \in J$$  (5)

$$I_{ij} \leq (1 - L_{ij}) \times M \quad \forall i \in I_r, \forall j \in J$$  (6)

$$Z_{ij} \times r_{ij} \times M \geq O_{ij} \quad \forall i \in I_w, \forall j \in J$$  (7)

$$I_{ij-1} + \sum_{k=j-T_i}^{j-1} O_{ik} + (1 - Z_{ij}) \times M \geq s(i mod q) \quad \forall i \in I_r, \forall j \in J$$  (8)

$$I_{ij-1} + \sum_{k=j-T_i}^{j-1} O_{ik} \leq s(i mod q) + (1 - Z_{ij}) \times M \quad \forall i \in I_r, \forall j \in J$$  (9)

The objective function (1) is to minimize total cost of the system due to ordering and holding costs. Constraints (2) and (3) are inventory levels and product flows in and out (and also loss) in each period at each retailer and at the warehouse. Since it is assumed that the warehouse must always have sufficient items for retailers’ orders, there will be no demand loss at the warehouse. Constraints (4) to (7) force decision variables $L_{ij}$ and $I_{ij}$. The models have both variables $L_{ij}$ and $I_{ij}$ because of lost-sale assumption. Since unsatisfied demand is considered as demand loss, $I_{ij}$ will never be negative (it can be negative with backordered assumption). However, $L_{ij}$ and $I_{ij}$ will not be positive on the same period. If available inventory at the retailer is sufficient to serve period’s demand, $L_{ij}$ will be zero and $L_{ij}$ must be zero. Otherwise, $L_{ij}$ will be 1 and $I_{ij}$ must be zero. Constraints (8) define that whenever a retailer or the warehouse orders, ordering cost occurs.

In Constraints (9) through (16), reorder points and order-up-to points for the stock points are defined. The number of reorder points and order-up-to points depends on which model is used - single policy or multiple policies. With single policy each location will have one reorder point and order-up-to point. With multiple policies, the number of these points for each location depends on the number of periods in a demand cycle.
For example, if a cycle of demand consists of 4 periods there are 4 reorder points and order-up-to points. Each reorder point and order-up-to point will be applied to each period of demand as they are indicated. The modulus function, \((j \mod q)\), is used to find the remainder after divided period’s number, \(j\), by the number of periods in a cycle, \(q\). For example, on period 13, if \(q = 4\) and \((j \mod q) = 1\), the policy \((R, S_{1i}, S_{ii})\) is applied.

Constraints (9) to (12) are applied to the retailer, while Constraints (13) to (16) are applied to the warehouse. At every location, an order is placed when on-hand inventory at the end of period, \(I_{ij}\), is realized. With 1-period lead time, for instance, an order placed at the end of period \(j - 1\) will arrive at the end of period \(j\) and it will be ready to use on period \(j + 1\). Therefore, the models illustrate the system that an order placed by considering inventory level at the end of period \(j - 1\) will appear on period \(j\) as on-order inventory at the beginning of period \(j\), \(O_{ij}\), and will be added to on-hand inventory on period \(j + 1\). Constraints (9) and (10) force the retailers to place an order when their inventory positions (the total of on-hand and on-order inventory levels) are less than or equal to the reorder points and there must be no order placed if inventory positions are higher than the reorder points. In Constraints (9), if the inventory positions are equal to or lower than reorder points and \(r_{ij} = 1\), \(Z_{ij}\) must be 1. If \(r_{ij} = 0\), \(Z_{ij}\) can be either 1 or 0 where it tends towards 0 due to the objective function. There is a -0.5 term on the left-hand side because, without this term, when the inventory position is equal to reorder points, \(Z_{ij}\) can be either 0 or 1 which means that it might be no order placed.

In Constraints (10), on the other hand, if the inventory positions are higher than reorder points, \(Z_{ij}\) must be 0. Since Constraints (9) and (10) consider about whether to order at the beginning of period \(j\) or variables \(Z_{ij}\) which relate to \(O_{ij}\), the inventory positions are the total amount of on-hand and on-order until the end of period \(j - 1\). Constraints (11) and (12) force the inventory positions after placing orders to be equal to the order-up-to points. Constraints (13) through (16) are similar to Constraints (9) to (12) but they are applied to the warehouse. The major difference between the warehouse and retailers is that, at the warehouse, an echelon stock concept is applied so the inventory level is the summation of on-hand and on-order inventory of the warehouse and all retailers (the warehouse and all locations downstream). According to the echelon stock

concept, the warehouse can realize when retailers are about to place orders and can manage to fill its inventory just before those orders are placed. The total on-order of all locations up to period \(j - 1\), \(\sum_{k=0}^{n} \sum_{i=1}^{r_j} O_{ik}\), is considered as a part of inventory position of the system because it is items in the system which is being transported and not stored at any location yet. If the models do not take this inventory into account, the warehouse will order too early since it sees that the system has low inventory position although there are on-order items in transit to retailers.

Constraints (17) guarantee service level for every retailer. Constraints (18) and (19) force on-hand inventory and on-order inventory at the end of horizon to be equal to those at the beginning. Constraints (20) and (21) force all decision variables to be either positive value or binary.

5. Results and discussion

The mixed-integer programming model is used to solve problem instances with period’s average demand as they are used to determine initial policies. The results are used to analyze how the solutions behave when parameters such as ordering cost and size of demand change. A preliminary study on a simple problem with single warehouse and 2 identical retailers is conducted. Each demand cycle has 4 periods in which demand changes period by period and each instance is solved within 24-period time horizon. The review period, \(R\), is given to be one period long. 45 problem instances are tested with multiple policies and single policy. The parameters such as cycle demand and ordering cost vary among the instances.

There are 3 values set of total demand per cycle; 1,200, 1,600, and 4,000. Demand pattern repeats as proportion of each period demand over cycle demand. The patterns of demands are shown in Table 1.

Since the multiple policy model is more flexible, it is likely to give better solutions. However, it is found that, in some settings, the single policy model can give solutions which are as good as solutions from the multiple policy model. The result shows that there is relationship between values of parameters in the system and the difference between the objective values of the multiple and single policy models.
Firstly, the very basic parameters of interest are $K$ and $h$ which are ordering cost and holding cost. Generally, at any holding cost, a higher ordering cost leads to a greater size of order and less frequent order placed. From the instances, as the proportion of ordering cost to holding cost increases, the difference between objective values of the single and multiple policy models decreases. The single policy model can give total costs that are 100% higher than those from the multiple policy model. With small $\frac{K}{h}$ ratio, the system orders more frequent and total holding cost is the major cost. In this case, multiple policies can adjust order size on each period while the single policy cannot. Consequently, on-hand inventory of the system with multiple policies is lower than the system with single policy which means lower total cost. Furthermore, as the $\frac{K}{h}$ ratio gets smaller, the difference between total costs become larger since the holding cost becomes more important. However, this relationship is affected by the size of demand that the greater size of demand leads to greater difference. Hence, taking demand into account could display a clear vision of whether to apply single or multiple policies. The relationship is shown in Fig. 1.

When it comes to ordering policy, the relationship among ordering cost, holding cost and demand could be written as economic order quantity, $EOQ$. As it is assumed that the average demand of each cycle is stationary $EOQ$ could be used to find preliminary order quantity. The $EOQ$ of any location can be calculated as $\frac{\sqrt{2KD}}{h}$ where $K$ is ordering cost, $D$ is average demand per cycle and $h$ is holding cost per unit per cycle. Therefore this $\frac{EOQ}{D}$ ratio can be used to analyze further as shown in Fig. 2.

The $\frac{EOQ}{D}$ shown in Fig. 2 ranges from 0 to 2.83. To use $\frac{EOQ}{D}$ in the analysis, demand rate and holding cost used to calculate $EOQ$ must be the total demand per cycle. It is shown in Fig. 2 that, with any size of demand, when $\frac{EOQ}{D}$ ratio is smaller than 1, multiple policies are likely to perform better. However, as the ratio becomes greater, the difference between objective values becomes smaller and the single policy model can give as good objective values as the multiple policy model when the ratio equals to 1 or greater.
which an order should be placed every 2 periods (or half of a cycle), the single policy model can find policies which correspond to that frequency with slightly different order sizes from those of multiple policies. On the other hand, at $\frac{COQ}{D}$ of 0.63 to 1, the multiple policy model finds policies which place orders every 4 periods, which is the number of periods in a demand cycle. Therefore, the single can find the policy with the same ordering timing which leads to the same total costs.

6. Conclusion

A 2-echelon inventory system having one warehouse and N retailers with seasonal demand is studied. The system operates on periodic review basis with $(R,s,S)$ policy using reorder points and order-up-to points. The echelon stock concept is also applied. The mixed-integer programming model is proposed to determine two types of policies. One is to determine single ordering policy and the other is to determine multiple ordering policies. The mixed-integer programming model is used to investigate the different between total costs of multiple and single ordering policies. It is found that the difference can be ranging from 0% to 100%. The ratio can be useful for the inventory system managers to decide to use multiple or single policy based on their system parameters. The numerical results shown in this paper only considered initial ordering policies. Safety stock levels should be studied further how the difference between multiple and single policies will behave with safety stock.

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