MHD Jeffrey nanofluid past a stretching sheet with viscous dissipation effect

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Abstract. This study investigates the influence of viscous dissipation on magnetohydrodynamic (MHD) flow of Jeffrey nanofluid over a stretching sheet with convective boundary conditions. The nonlinear partial differential equations are reduced into the nonlinear ordinary differential equations by utilizing the similarity transformation variables. The Runge-Kutta Fehlberg method is used to solve the problem numerically. The numerical solutions obtained are presented graphically for several dimensionless parameters such as Brownian motion, Lewis number and Eckert number on the specified temperature and concentration profiles. It is noted that the temperature profile is accelerated due to increasing values of Brownian motion parameter and Eckert number. In contrast, both the Brownian motion parameter and Lewis number have caused the deceleration in the concentration profiles.

1. Introduction

Over the past few years, a better energy transfer rate has been a critical issue faced by our society. The use of new working fluid, which is functioned to transfer the energy from one position to another position may overcome the arising issue as it provides better thermal performance in comparison to the other working fluids, for example base fluids. This is because, the thermal conductivity of the base fluid is not suitable to achieve a better cooling rate in industry. In the current investigation, nanofluid is considered as new working fluid while Jeffrey fluid is assumed as base fluid where the presence of suspended nanoparticles can change the optical properties of the Jeffrey fluid as well as enhancing the thermal conductivity. Such improvement in the thermal conductivity is vital in various industrial and metallurgical applications such as micromanufacturing, transportation, power generation, ventilation, cooling, heating, air-conditioning etc.[1].

Sakiadis [2] is the first who studied the boundary-layer flow on continuous solid surfaces. The similar problem is extended by Erickson et al. [3], in which the effect of suction or injection are taken into account. Later, Crane [4] considered the boundary layer flow problem from a stretching sheet. It was then continued by Salleh et al. [5] on the boundary condition of Newtonian heating. In recent times, the non-Newtonian flow, i.e. Jeffrey fluid has gained worldwide attention due to growing importance in the processing industries such as metal and polymer sheet. For instance, Qasim [6] considered the effect of heat source or sink past a stretching sheet embedded in Jeffrey fluid. Soon after, Dalir [7] tackled the impact of entropy generation over a stretching sheet in Jeffrey fluid, for
which the effect of viscous dissipation is explained in details. The effect of viscous dissipation on the MHD Jeffrey fluid with surface slip and melting heat transfer was examined by Das et al. [8]. More recently, the Jeffrey nanofluid has been thoroughly investigated by Shehzad et al. [9], Abbasi et al. [10] and Ashraf et al. [11]. Their study has shown how the Brownian motion parameter, a fundamental factor that contribute to the performance of thermal conductivity has affected the fluid flow and heat transfer.

It is clear from the above investigations that the effect of viscous dissipation in Jeffrey nanofluid over a stretching sheet has never been examined. Therefore, the present study intends to examine the impact of viscous dissipation on MHD Jeffrey nanofluid over a stretching sheet. Based on the knowledge of authors, the present study is not yet published elsewhere, so the outcome of this study is considered as new.

2. Mathematical formulation

Following the Buongiorno model, a steady, two-dimensional and laminar flow of MHD Jeffrey nanofluid over a stretching sheet in the region \( y > 0 \) is considered and discussed with the influence of viscous dissipation, as illustrated in figure 1. The convective boundary condition is implemented. The stretched surface is in the plane \( y = 0 \), where \( y \) is measured perpendicular to the sheet while \( x \) is measured parallel to the sheet. The velocity of the stretching sheet changes linearly with the distance along the sheet, i.e. \( u_s(x) = ax \) where \( a > 0 \) is a constant. A constant magnetic field, \( B_0 \) is applied perpendicularly to the stretched sheet. The governing boundary layer equations are presented as follows [7, 12, 13],

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial x} \left[ \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^2 u}{\partial x^2 \partial y} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x \partial y^2} \right] + (1 - C_s) \frac{\rho f}{\rho_i} \frac{g \beta}{g \beta \rho_i} (T - T_\infty) - \frac{(\rho_p - \rho_f)}{\rho_f} g (C - C_s) - \frac{\sigma B_0^2 u}{\rho_f}
\]

Figure 1. Physical model of the coordinate system.

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\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_b \frac{\partial C}{\partial y} + D_x \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\nu}{C_f(1+\lambda)} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \lambda_1 \left( u \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) \right] \] (3)

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_x \frac{\partial^2 T}{\partial y^2}}{T_\infty} \] (4)

with the boundary conditions

\[ u = u_0(x) = ax, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_1(T_f - T), \quad -D_b \frac{\partial C}{\partial y} = h_2(C_f - C) \text{ at } y = 0 \]

\[ u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty \] (5)

where \( u \) and \( v \) are the respective components of velocity in the \( x \)– and \( y \)–directions, \( \nu \) is the kinematic viscosity, \( \lambda \) and \( \lambda_1 \) are the respective ratio of relaxation to retardation times and retardation time, \( \rho_f \) and \( \rho_p \) are the respective density for the base fluid and nanoparticles, \( \sigma \) is the electrically conductivity, \( T \) and \( C \) are the respective temperature and concentration of the fluid, \( \beta = \alpha \beta_k \) is the thermal expansion coefficients [14] and \( g \) is the gravitational acceleration. Also, \( \alpha = \frac{k}{\rho C_f} \) is the thermal diffusivity, \( \tau = \frac{(\rho c)_p}{(\rho c)_f} \) is the ratio of heat capacity where \( (\rho c)_p \) is the heat capacity of the nanoparticle while \( (\rho c)_f \) is the heat capacity of the fluid, \( T \) is the Brownian diffusion coefficient and \( D_x \) is the thermodiffusive diffusion coefficients, \( C_p \) is the specific heat at constant pressure, \( h_1 \) and \( h_2 \) are the respective heat and mass transfer coefficients and \( T_f = T_\infty + bx^2 \) and \( C_f = C_\infty + d(x^2) \) are the respective temperature and concentration of the hot fluid [15]. Notice that the subscripts \( \infty, p \) and \( f \) are the values which is far from the surface, the nanoparticles and the base fluid, respectively. Imposing the following similarity transformation variables,

\[ \eta = \sqrt{\frac{y}{v}}, \quad u = axf'(\eta), \quad v = -af'\sqrt{f(\eta)}, \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty} \] (6)

Substituting equation (6) into equations (2) to (4) and boundary conditions (5), the following nonlinear ordinary differential equations are obtained

\[ f'' + \lambda_2 \left( f'^2 - ff'' \right) + \left( 1 + \lambda \right) \left( ff'' - f'^2 - Mf'' + \gamma \theta + \gamma' \phi \right) = 0 \] (7)

\[ (1 + \lambda) \theta'' + (1 + \lambda) \Pr \left( f \theta' - 2f' \theta + Nb \theta \phi' + Nt \theta'^2 \right) + Ec \Pr \left( f'^2 + \lambda_2 f'' \left( f f'' - ff'' \right) \right) = 0 \] (8)

\[ \phi'' + Le \left( f \phi' - 2f' \phi \right) + \frac{Nt}{Nb} \theta'' = 0 \] (9)
\[ f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi_i (1 - \theta(0)), \quad \phi'(0) = -Bi_i (1 - \phi(0)) \]

\[ f'(\infty) \to 0, \quad f''(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0 \]

(10)

where \( \lambda_2 = \lambda_i a, \quad M = \frac{\sigma_i B_i^2}{\rho_f a}, \quad \gamma = (1 - C_m) \frac{\rho_f}{\rho_f} \frac{Gr_x}{Re_x^\gamma}, \quad Gr_x = \frac{g \beta (T_e - T_o) x^3}{\nu^2}, \quad \text{Re}_x = \frac{a x^2}{v}, \)

\[ \gamma' = (C_f - C_m) \frac{\rho_p - \rho_f}{\rho_f} \frac{g}{a^2 x}, \quad \Pr = \frac{\nu}{\alpha}, \quad Nb = \frac{\tau D_g (C_f - C_m)}{\nu}, \quad Nt = \frac{\tau D_g (T_f - T_o)}{\nu T_o}, \quad Le = \frac{v}{D_g}, \]

\[ Ec = \frac{u^2_w}{C_p (T_e - T_o)^2}, \quad Bi_i = \frac{h_i}{k} \left( \frac{v}{a} \right), \quad Bi_i = \frac{h_i}{D_g} \left( \frac{v}{a} \right) \]

are the respective Deborah number, magnetic parameter, local thermal buoyancy parameter, local thermal Grashof number, local Reynolds number, local nanoparticle buoyancy parameter, Prandtl number, Brownian motion parameter, thermophoresis parameter, Lewis number, Eckert number, heat transfer of Biot number and mass transfer of Biot number. According to Hayat et al. [16], the exact solution of equation (7) when \( M = \gamma = \gamma' = 0 \) is,

\[ f(\eta) = \frac{1 - e^{-m\eta}}{m}, \quad m = \left( 1 + \frac{\lambda}{1 + \lambda_2} \right)^{1/2} \]

(11)

The second derivative of equation (11) with its dimensionless velocity gradient at the sheet surface are

\[ f''(\eta) = -me^{-m\eta} \quad \text{and} \quad f''(0) = -m \]

(12)

3. Results and discussion

The numerical solutions of equations (7) to (9) together with the boundary conditions (10) are obtained using the Runge-Kutta Fehlberg method, which is programmed in MAPLE software. Three parameters have been selected for analyses, i.e. Brownian motion, \( Nb \) Eckert number, \( Ec \) and Lewis number, \( Le \). The fixed parameters used in the simulations are \( \lambda = 0.1, \quad Pr = 0.72 \) (air), \( \gamma = 4, \quad \gamma' = 2, \quad Le = 1, \quad M = 0.5, \quad \lambda_2 = Ec = Bi_i = 0.2, \quad \text{and} \quad Nb = Nt = Bi_i = 0.4, \quad \text{except otherwise stated}. \)

Table 1 shows the comparison of the present study with the exact solution of skin friction coefficient \( f''(0) \) from equation (12) and the previous published results obtained by Dalir [7]. The results show a great agreement; thus, the present codes are now confident to be used.

| \( \lambda_2 \) | Exact solution of equation (12) | Dalir [7] | Present |
|-----------------|-------------------------------|-----------|---------|
| 0.0             | -1.09544512                   | -1.09641580 | -1.09544512 |
| 0.4             | -0.92582010                   | -0.92724220 | -0.92582010 |
| 0.8             | -0.81649658                   | -0.81808091 | -0.81649659 |
| 1.2             | -0.73854895                   | -0.74010502 | -0.73854899 |
| 1.6             | -0.67936622                   | -0.68074654 | -0.67936634 |
| 2.0             | -0.63245553                   | -0.63352833 | -0.63245579 |
The effect of Brownian motion parameter, $Nb$ on the temperature and concentration profiles are illustrated in figures 2 and 3. It is detected that an increase in the Brownian motion parameter has caused the increment in the temperature profile while the concentration profile is decelerated. This is because, increasing Brownian motion parameter will increase the collision between the fluid particles and their random motions, accordingly more heat is produced. Therefore, the profiles for temperature and concentration are accelerated and decelerated, respectively.

Figure 4 shows the effect of Eckert number, $Ec$ in which both the temperature profile and thermal boundary layer thickness are improved for increasing $Ec$. A growth in $Ec$ implies that more heat energy is stored resulting from the frictional forces, thereby increasing the temperature profile. It is clear from figure 5 that the concentration profile is strongly reduced due to increasing Lewis number, $Le$. In fact, smaller $Le$ signifies stronger Brownian diffusion coefficient whereas larger $Le$ signifies
weaker Brownian diffusion coefficient. As such, the reduction in nanoparticle concentration profile is expected, however depending on the Brownian diffusion coefficient.

4. Conclusions
In this paper, the study on the influence of viscous dissipation effect on MHD Jeffrey nanofluid past a stretching sheet is conducted. It is found that the temperature profile is growing function for $Nb$ and $Ec$ while the concentration profile is declining function for $Nh$ and $Le$.

Acknowledgement
The authors are thankful to the Universiti Malaysia Pahang (UMP) for the funding through grants PGRS1703100, RDU170358 and RDU141306.

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