Correlation between the Charged Current Interactions of Light and Heavy Majorana Neutrinos

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Abstract

The evidence for neutrino oscillations implies that three neutrino flavors ($\nu_e, \nu_\mu, \nu_\tau$) must have different mass states ($\nu_1, \nu_2, \nu_3$). The most popular idea of generating tiny masses of $\nu_i$ is to introduce three heavy Majorana neutrinos $N_i$ (for $i = 1, 2, 3$) into the standard model and implement the seesaw mechanism. In this approach the neutrino mixing matrix $V$ appearing in the charged current interactions of $\nu_i$ is not unitary, and the strength of unitarity violation of $V$ is associated with the matrix $R$ which describes the strength of charged current interactions of $N_i$. We present an explicit parametrization of the correlation between $V$ and $R$ in terms of nine rotation angles and nine phase angles, which can be measured or constrained in the precision neutrino oscillation experiments and by exploring possible signatures of $N_i$ at the LHC and ILC. Two special but viable scenarios, the Type-I seesaw model with two heavy Majorana neutrinos and the Type-II seesaw model with one heavy Majorana neutrino and one Higgs triplet, are taken into account to illustrate the simplified $V$-$R$ correlation. The implications of $R \neq 0$ on the low-energy neutrino phenomenology are also discussed. In particular, we demonstrate that the non-unitarity of $V$ is possible to give rise to an appreciable CP-violating asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations with short or medium baselines.

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Very robust evidence for non-zero neutrino masses and large lepton flavor mixing has recently been achieved from solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino oscillation experiments. This great breakthrough opens a new window to physics beyond the standard model (SM), because the SM itself only contains three massless neutrinos whose flavor states $\nu_\alpha$ (for $\alpha = e, \mu, \tau$) and mass states $\nu_i$ (for $i = 1, 2, 3$) are identical. The most natural and popular way to generate non-vanishing but tiny masses $m_i$ for $\nu_i$ is to extend the SM by introducing three right-handed neutrinos but preserving its $SU(2)_L \times U(1)_Y$ gauge symmetry. This class of neutrino mass models is commonly referred to as the Type-I seesaw models [5]. If one Higgs triplet $\Delta_L$ and three right-handed neutrinos are simultaneously introduced into the SM, the resultant $SU(2)_L \times U(1)_Y$ gauge models are usually classified as the Type-II seesaw models [6]. In either case the mass states of three right-handed neutrinos, denoted as $N_i$ (for $i = 1, 2, 3$), have the positive eigenvalues $M_i$ which can be much larger than the electroweak symmetry breaking scale $v \approx 174$ GeV. The smallness of $m_i$ is therefore attributed to the smallness of $v^2/M_i$ (and the smallness of the vacuum expectation value of $\Delta_L$ in the Type-II seesaw models). Both light and heavy neutrinos are the Majorana particles in such seesaw models, in which the lepton number ($L$) is not conserved. Associated with the seesaw mechanism, the leptogenesis mechanism [7] may naturally work to account for the cosmological matter-antimatter asymmetry via the CP-violating and out-of-equilibrium decays of $N_i$ and the $(B-L)$-conserving sphaleron processes [8].

To directly test the seesaw and leptogenesis mechanisms, it is desirable to experimentally discover the heavy Majorana neutrinos $N_i$ at high-energy $e^+e^-$ and (or) hadron colliders. Since such neutral and weakly interacting particles leave no trace in ordinary detectors, their possible collider signatures must involve charged leptons via the charged current interactions for some $\Delta L = 2$ processes (i.e., those processes with the lepton number being violated by two units) [9]. In the basis of mass states, the standard charged current interactions of $\nu_i$ and $N_i$ can be written as

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[ (e\quad \mu\quad \tau)_L V^\mu \nu_1 \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu + (e\quad \mu\quad \tau)_L R^\mu \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L W^-_\mu \right] + \text{h.c.} \ , \quad (1)$$

where $V$ is just the Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix [10] responsible for neutrino oscillations, and $R$ describes the strength of charged current interactions between $(e, \mu, \tau)$ and $(N_1, N_2, N_3)$. It has been noticed that $V$ is not exactly unitary in the seesaw models and its deviation from unitarity is essentially characterized by non-vanishing $R$ [11]. Indeed, the production and detection of $N_i$ at the Large Hadron Collider (LHC) or the International Linear Collider (ILC) require not only $M_i \lesssim \mathcal{O}(10)$ TeV but also appreciable sizes of the matrix elements of $R$. Hence the unitarity violation of $V$ might show up in the future long-baseline neutrino oscillation experiments, if the collider signatures of heavy Majorana neutrinos are really accessible. In this sense, the correlation between $V$ and $R$ signifies an important relationship between neutrino physics and collider physics.

The purpose of this work is just to reveal how the charged current interactions of light and heavy Majorana neutrinos are correlated with each other. We shall show that the correlation between $V$ and $R$ can in general be parametrized in terms of nine rotation angles and nine phase angles. This parametrization is independent of any details of a Type-I or Type-II
seesaw model, and its parameters can be measured or constrained in the precision neutrino oscillation experiments and by exploring possible signatures of $N_i$ at the LHC and ILC. We shall take two special but viable examples, the Type-I seesaw model with two heavy Majorana neutrinos and the Type-II seesaw model with one heavy Majorana neutrino and one Higgs triplet, to illustrate the simplified form of $V-R$ correlation. The implications of $R \neq 0$ on the low-energy neutrino phenomenology will also be discussed. In particular, we shall demonstrate that the non-unitarity of $V$ is possible to give rise to an appreciable CP-violating asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations with short or medium baselines.

Without loss of generality, we choose the basis in which the flavor and mass states of three charged leptons are identical. In this basis, the neutrino mass terms generated from spontaneous $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ symmetry breaking can be written as

$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} \begin{pmatrix} \nu_L \ N_R^c \end{pmatrix} \begin{pmatrix} M_L & M_D & \nu_L^c \\ M_D^T & M_R & \end{pmatrix} \begin{pmatrix} N_R \\ \nu_R \end{pmatrix} + \text{h.c.},$$

where $\nu_L$ and $N_R^c$ are defined as $\nu_L^c \equiv C^{\nu_L}$ and $N_R \equiv C^{N_R}$ with $C$ being the charge conjugation matrix, $M_D = Y_\nu v$ and $M_L = Y_\Delta v_L$ result from the Yukawa interactions of Higgs doublet and triplet with $v \approx 174 \text{ GeV}$ and $v_L \approx 1 \text{ GeV}$ being the corresponding vacuum expectation values, and $M_R$ is the mass matrix of three right-handed Majorana neutrinos. The overall $6 \times 6$ neutrino mass matrix in $\mathcal{L}_{\text{mass}}$, denoted as $\mathcal{M}$, is symmetric and can be diagonalized by a unitary transformation $U^\dagger \mathcal{M} U = \hat{\mathcal{M}}$; or explicitly,

$$\begin{pmatrix} V & R \end{pmatrix} \begin{pmatrix} M_L & M_D & \nu_L^c \\ M_D^T & M_R & \end{pmatrix} \begin{pmatrix} V & R \end{pmatrix}^* = \begin{pmatrix} \hat{M}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix},$$

where $\hat{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ and $\hat{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ with $m_i$ and $M_i$ (for $i = 1, 2, 3$) being the light and heavy Majorana neutrino masses, respectively. After this diagonalization, one may express the neutrino flavor states $\nu_\alpha$ (for $\alpha = e, \mu, \tau$) in terms of the light and heavy neutrino mass states $\nu_i$ and $N_i$ (for $i = 1, 2, 3$):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L.$$  

Then the standard charged current interactions between $\nu_\alpha$ and $\alpha$ (for $\alpha = e, \mu, \tau$) in the basis of flavor states,

$$-\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} (e \mu \tau)_L \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L W^- \mu + \text{h.c.},$$

turn out to be of the form given in Eq. (1) in the basis of mass states. It is clear that $V$ and $R$ describe the charged current interactions of three light and heavy Majorana neutrinos, respectively. While $V$ can be measured from a variety of neutrino oscillation experiments, $R$ may be determined from possible collider signatures of $N_i$.  

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Note that \( V \) itself is not unitary. Indeed, \( VV^\dagger + RR^\dagger = 1 \) holds as a consequence of unitarity of the \( 6 \times 6 \) transformation matrix \( U \) in Eq. (3) [12]. The non-unitarity of the MNS matrix \( V \) is an intrinsic feature of the seesaw models, no matter whether they are of type I or of type II. Since \( V \) and \( R \) are two \( 3 \times 3 \) sub-matrices of \( U \), their elements must be correlated with each other. To find out the explicit correlation between \( V \) and \( R \), we may parametrize \( U \) in terms of 15 rotation angles and 15 phase angles [13]. Then the common parameters appearing in both \( V \) and \( R \) characterize their correlation. First of all, let us define the 2-dimensional \((1,2)\), \((1,3)\) and \((2,3)\) rotation matrices in a 6-dimensional complex space:

\[
O_{12} = \begin{pmatrix}
c_{12} & \hat{s}_{12}^* & 0 & 0 & 0 \\
-\hat{s}_{12} & c_{12} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]

\[
\begin{pmatrix}
c_{13} & 0 & \hat{s}_{13}^* & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
-\hat{s}_{13} & 0 & c_{13} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & c_{23} & \hat{s}_{23}^* & 0 & 0 \\
0 & -\hat{s}_{23} & c_{23} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]

where \( c_{ij} \equiv \cos \theta_{ij} \) and \( \hat{s} \equiv e^{i\delta_{ij}} \sin \theta_{ij} \) with \( \theta_{ij} \) and \( \delta_{ij} \) being the rotation angle and phase angle, respectively. Other 2-dimensional rotation matrices \( O_{ij} \) (for \( 1 \leq i < j \leq 6 \)) can be defined in an analogous way [14]. We parametrize the \( 6 \times 6 \) unitary matrix \( U \) as

\[
U = \begin{pmatrix} A & R \\ B & U \end{pmatrix} \begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix}
\]

(7)

with

\[
\begin{pmatrix} A & R \\ B & U \end{pmatrix} = O_{56}O_{46}O_{36}O_{26}O_{16}O_{45}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14},
\]

\[
\begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix} = O_{23}O_{13}O_{12}.
\]

(8)

Comparing between Eqs. (3) and (7), we get \( V = AV_0 \) and \( S = BV_0 \), in which

\[
V_0 = \begin{pmatrix}
c_{12}c_{13} & \hat{s}_{12}c_{13} & \hat{s}_{13}^* & c_{13}^* \\
-\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23} & c_{12}c_{23} - \hat{s}_{12}\hat{s}_{13}\hat{s}_{23} & \hat{s}_{23} & c_{13}\hat{s}_{23} \\
\hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}\hat{s}_{13}c_{23} & c_{13}\hat{s}_{23} & c_{13}c_{23}
\end{pmatrix}
\]

(9)
is just the standard parametrization of the unitary MNS matrix (up to some proper phase rearrangements) [15]. It is obvious that \( V \to V_0 \) in the limit of \( A \to 1 \) (or equivalently, \( R \to 0 \) and \( S \to 0 \)). Thus \( A \) signifies the unitarity violation of \( V \). After a lengthy but straightforward calculation, we obtain the explicit expressions of \( A \) and \( R \) as follows:

\[
A = \begin{pmatrix}
    c_{14}c_{15}c_{16} & 0 & 0 \\
    -c_{14}c_{15}\hat{s}_{16}\hat{s}_{24} - c_{14}\hat{s}_{15}\hat{s}_{25}c_{26} & c_{24}c_{25}c_{26} & 0 \\
    -\hat{s}_{14}\hat{s}_{24}\hat{s}_{25}c_{26} & -c_{24}c_{25}\hat{s}_{26}\hat{s}_{36} - c_{24}\hat{s}_{25}\hat{s}_{35}c_{36} & c_{34}c_{35}c_{36} \\
    c_{14}\hat{s}_{15}c_{16} & 0 & \hat{s}_{15}c_{16}
\end{pmatrix},
\]

\[
R = \begin{pmatrix}
    -\hat{s}_{14}\hat{s}_{15}\hat{s}_{16}\hat{s}_{26} - \hat{s}_{14}\hat{s}_{15}\hat{s}_{25}\hat{s}_{26}c_{26} & \hat{s}_{15}\hat{s}_{16}\hat{s}_{26} + c_{15}\hat{s}_{25}\hat{s}_{26}c_{26} & c_{16}\hat{s}_{26} \\
    +c_{14}\hat{s}_{24}\hat{s}_{25}c_{26} & -\hat{s}_{15}\hat{s}_{16}\hat{s}_{26} - c_{15}\hat{s}_{25}\hat{s}_{26}c_{26} & c_{16}\hat{s}_{26} \\
    -\hat{s}_{14}\hat{s}_{15}\hat{s}_{16}\hat{s}_{26} - \hat{s}_{14}\hat{s}_{15}\hat{s}_{25}\hat{s}_{26}c_{26} & -\hat{s}_{15}\hat{s}_{16}\hat{s}_{26} - c_{15}\hat{s}_{25}\hat{s}_{26}c_{26} & c_{16}\hat{s}_{26}
\end{pmatrix}.
\]

One can see that \( A \) and \( R \) involve the same parameters: nine rotation angles (\( \theta_{14}, \theta_{15} \) and \( \theta_{16} \) for \( i = 1, 2 \) and 3) and nine phase angles (\( \delta_{14}, \delta_{15} \) and \( \delta_{16} \) for \( i = 1, 2 \) and 3). If all of them are switched off, we shall be left with \( R = 0 \) and \( A = 1 \). Nontrivial \( A \) is therefore the bridge between \( V = AV_0 \) and \( R \).

Considering the fact that the unitarity violation of \( V \) must be a small effect (at most at the 1% level as constrained by current neutrino oscillation data and precision electroweak data [16]), we may treat \( A \) as a perturbation to \( V_0 \). The smallness of \( \theta_{ij} \) (for \( i = 1, 2, 3 \) and \( j = 4, 5, 6 \)) allows us to make the excellent approximations

\[
A = 1 - \begin{pmatrix}
    \frac{1}{2}(s_{24}^2 + s_{25}^2 + s_{26}^2) & 0 & 0 \\
    \frac{1}{2}(s_{24}^2 + s_{25}^2 + s_{26}^2) & \frac{1}{2}(s_{24}^2 + s_{25}^2 + s_{26}^2) & 0 \\
    \frac{1}{2}(s_{24}^2 + s_{25}^2 + s_{26}^2) & \frac{1}{2}(s_{24}^2 + s_{25}^2 + s_{26}^2) & \frac{1}{2}(s_{34}^2 + s_{35}^2 + s_{36}^2)
\end{pmatrix} + O(s_{4j}^4),
\]

\[
R = 0 + \begin{pmatrix}
    \frac{s_{14}}{s_{16}} & \frac{s_{15}}{s_{16}} & \frac{s_{16}}{s_{16}} \\
    \frac{s_{24}}{s_{26}} & \frac{s_{25}}{s_{26}} & \frac{s_{26}}{s_{26}} \\
    \frac{s_{34}}{s_{36}} & \frac{s_{35}}{s_{36}} & \frac{s_{36}}{s_{36}}
\end{pmatrix} + O(s_{ij}^3),
\]

from which one can easily check the validity of \( V V^\dagger = AA^\dagger = 1 - RR^\dagger \) to a good degree of accuracy. Thus the nine mixing angles in Eq. (10) or (11) are all of \( O(10^{-1}) \) or smaller, such that the unitarity violation of \( V \) can maximally be of \( O(10^{-2}) \). The nine CP-violating phases of \( A \) or \( R \) are in general not suppressed, however. It is worth remarking that \( R \sim O(10^{-3}) \) to \( O(10^{-1}) \) may lead to appreciable collider signatures of lepton number violation, if the masses of heavy Majorana neutrinos \( M_i \) are of \( O(10^2) \) GeV to \( O(10) \) TeV. At the LHC, for instance, the promising lepton-number-violating processes mediated by \( N_i \) include \( pp \to W^\pm W^\pm \to \mu^\pm \mu^\pm jj \) and \( pp \to W^\pm \to \mu^\pm N \to \mu^\pm \mu^\pm jj \), where the \( \Delta L = 2 \) like-sign dilepton production can unambiguously signal the existence of \( N_i \) [9,17,18]. A preliminary analysis made in Ref. [9] has shown that it is possible to probe \( (RR^\dagger)_{\mu\mu} \approx s_{24}^2 + s_{25}^2 + s_{26}^2 \)
to $\mathcal{O}(10^{-4})$ for $M_i \sim 100$ GeV and to $\mathcal{O}(10^{-2})$ for $M_i \sim 400$ GeV at the LHC with an integrated luminosity $100$ fb$^{-1}$. The sensitivity will in general become worse for much larger values of the heavy Majorana neutrino masses.

Note that the triangular form of $A$ is a salient feature of our parametrization. Some straightforward consequences on $V = AV_0$ can be obtained from Eqs. (9) and (10).

- $V_{e3} = c_{14}c_{15}c_{16}\hat{s}_{13}$ holds. Given $\theta_{13} = 0$ for $V_0$, which might result from certain flavor symmetries imposed on $M_L$, $M_D$ and $M_R$ [12,18], $V_{e3}$ turns out to vanish.

- The ratio $|V_{e2}/V_{e1}| = \tan \theta_{12}$ is completely irrelevant to the parameters appearing in $A$ or $R$. This result implies that the extraction of $\theta_{12}$ from the solar neutrino oscillation data is essentially independent of possible unitarity violation of $V$.

- $|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2 = c_{14}^2 c_{15}^2 c_{16}^2 \approx 1 - (s_{14}^2 + s_{15}^2 + s_{16}^2)$ holds. Hence non-vanishing $\theta_{14}$, $\theta_{15}$ and $\theta_{16}$ violate the normalization condition of three matrix elements in the first row of $V$. Current experimental data require $(s_{14}^2 + s_{15}^2 + s_{16}^2) \lesssim 1\%$ [16].

- $\langle m \rangle_{ee} = c_{14}^2 c_{15}^2 c_{16}^2 |m_1(c_{12}c_{13})|^2 + m_2(s_{12}^2 c_{13})^2 + m_3(s_{13}^2)^2$ holds for the effective mass of the neutrinoless double-beta decay. The smallness of $\theta_{14}$, $\theta_{15}$ and $\theta_{16}$ implies that their effects on $\langle m \rangle_{ee}$ are in practice negligible.

- $\langle m \rangle_e = c_{14}^2 c_{15}^2 c_{16}^2 \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2}$ holds for the effective mass term of the tritium beta decay. The smallness of $\theta_{14}$, $\theta_{15}$ and $\theta_{16}$ implies that their effects on $\langle m \rangle_e$ are also negligible.

Another consequence of the non-unitarity of $V$ is the loss of universality for the Jarlskog invariants of CP violation [19], $J_{\alpha\beta}^{ij} \equiv \text{Im}(V_{\alpha i}V_{\beta j}^*V_{\alpha j}^*V_{\beta i})$, where the Greek indices run over $(e, \mu, \tau)$ and the Latin indices run over $(1, 2, 3)$. The explicit expressions of $J_{\alpha\beta}^{ij}$ in terms of $\theta_{ij}$ and $\delta_{ij}$ are rather complicated and will be presented elsewhere. But we shall illustrate that the extra CP-violating phases of $V$ are possible to give rise to a significant asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations with short or medium baselines in the subsequent section.

For the sake of simplicity, here we only consider a special but interesting pattern of $V_0$: $\tan \theta_{12} = 1/\sqrt{2}$, $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (namely, $V_0$ takes the well-known tribimaximal mixing pattern [20]). Then the non-unitary neutrino mixing matrix $V = AV_0$ can approximate to

$$V \approx \begin{pmatrix}
\sqrt{\frac{1}{3}} e^{-i\delta_{12}} & \sqrt{\frac{1}{3}} (1 - X) e^{-i\delta_{23}} & \sqrt{\frac{1}{3}} e^{-i\delta_{23}} \\
-\sqrt{\frac{1}{6}} (1 + 2X) e^{i\delta_{12}} & \sqrt{\frac{1}{3}} (1 - X) & \sqrt{\frac{1}{2}} e^{-i\delta_{23}} \\
\sqrt{\frac{1}{6}} (1 - 2Y + Z) e^{i(\delta_{12} + \delta_{23})} & -\sqrt{\frac{1}{3}} (1 + Y + Z) e^{i\delta_{23}} & \sqrt{\frac{1}{2}} (1 - Z)
\end{pmatrix}, \quad (12)$$

where

$$X = (\hat{s}_{14}^* \hat{s}_{24}^* + \hat{s}_{15}^* \hat{s}_{25}^* + \hat{s}_{16}^* \hat{s}_{26}^*) e^{-i\delta_{12}},$$

$$Y = (\hat{s}_{14}^* \hat{s}_{34}^* + \hat{s}_{15}^* \hat{s}_{35}^* + \hat{s}_{16}^* \hat{s}_{36}^*) e^{-i(\delta_{12} + \delta_{23})},$$

$$Z = (\hat{s}_{24}^* \hat{s}_{34}^* + \hat{s}_{25}^* \hat{s}_{35}^* + \hat{s}_{26}^* \hat{s}_{36}^*) e^{-i\delta_{23}}.$$ 

(13)
The Jarlskog invariants $J_{ij}^{\alpha\beta}$ turn out to be $J_{e\mu}^{23} = J_{e\tau}^{23} = J_{\nu\mu}^{31} = J_{\nu\tau}^{31} = 0$, $J_{\nu\tau}^{12} \approx \text{Im} Y/3$, and

$$J_{\mu\tau}^{12} \approx (\text{Im} X + \text{Im} Y)/6, \quad J_{\mu\tau}^{23} \approx (\text{Im} X + \text{Im} Y + 2\text{Im} Z)/6, \quad J_{\mu\tau}^{31} \approx (\text{Im} X + \text{Im} Y - \text{Im} Z)/6.$$ (14)

Note that $J_{ij}^{\alpha\beta} = J_{ji}^{\beta\alpha} = -J_{ij}^{\alpha\beta}$ holds as a direct consequence of the definition of $J_{ij}^{\alpha\beta}$. It becomes clear that different $J_{ij}^{\alpha\beta}$ may in general have different values in the presence of unitarity violation, which can result in some new CP-violating effects in neutrino oscillations via the phase parameters $\delta_{kl}$ (for $k = 1, 2, 3$ and $l = 4, 5, 6$) hidden in $X$, $Y$ and $Z$.

Taking the steps outlined in Ref. [16], one may easily derive the probabilities of $\nu_{\alpha} \rightarrow \nu_{\beta}$ oscillations in vacuum. The result is

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \frac{\sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2 + 2 \sum_{i<j} \text{Re} \left( V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \right) \cos \Delta_{ij} - 2 \sum_{i<j} J_{ij}^{\alpha\beta} \sin \Delta_{ij}}{\left( V V^\dagger \right)_{\alpha\alpha} \left( V V^\dagger \right)_{\beta\beta}}, \quad (15)$$

where $\Delta_{ij} \equiv \Delta m_{ij}^2 L/(2E)$ with $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, $E$ being the neutrino beam energy and $L$ being the baseline length. It is straightforward to write out the expression of $P(\nu_\alpha \rightarrow \nu_\beta)$ from Eq. (15) by making the replacement $V \rightarrow V^*$ or equivalently $J_{ij}^{\alpha\beta} \rightarrow -J_{ij}^{\beta\alpha}$. If $V$ is exactly unitary (i.e., $A = 1$ and $V = V_0$), the denominator of Eq. (15) will become unity and the conventional formula of $P(\nu_\alpha \rightarrow \nu_\beta)$ will be reproduced. It has been observed in Ref. [21] that $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations may serve as an excellent tool to probe possible signatures of CP violation induced by the non-unitarity of $V$. To see this point more clearly, we consider a short- or medium-baseline neutrino oscillation experiment with $|\sin \Delta_{13}| \sim |\sin \Delta_{23}| \gg |\sin \Delta_{12}|$, in which the terrestrial matter effects are expected to be insignificant or negligibly small. Then the dominant CP-conserving and CP-violating terms of $P(\nu_\mu \rightarrow \nu_\tau)$ and $P(\nu_\mu \rightarrow \nu_\tau)$ can simply be obtained from Eq. (15) $^2$:

$$P(\nu_\mu \rightarrow \nu_\tau) \approx \sin^2 2\theta_{23} \sin^2 \frac{\Delta_{23}}{2} - 2 \left( J_{\mu\tau}^{23} + J_{\mu\tau}^{13} \right) \sin \Delta_{23},$$

$$P(\nu_\mu \rightarrow \nu_\tau) \approx \sin^2 2\theta_{23} \sin^2 \frac{\Delta_{23}}{2} + 2 \left( J_{\mu\tau}^{23} + J_{\mu\tau}^{13} \right) \sin \Delta_{23}, \quad (16)$$

where the good approximation $\Delta_{13} \approx \Delta_{23}$ has been used in view of the experimental fact $|\Delta m_{13}^2| \approx |\Delta m_{23}^2| \gg |\Delta m_{12}^2|$ [1–4], and the sub-leading and CP-conserving “zero-distance”

$^1$Note that the signs of three CP-violating terms in our Eq. (15) are opposite to those in Eq. (18) of Ref. [16]. The latter might result from a misprint.

$^2$Note that the CP-violating term shown in our Eq. (16) is apparently different from that given in Eq. (12) of Ref. [21], where a very different parametrization of the unitarity violation of $V$ has been adopted.
No matter whether a neutrino mass model is of Type-I seesaw or Type-II seesaw, nine
mass matrix with one heavy Majorana neutrino and one Higgs triplet [24]. In either case, we can arrive
with two heavy Majorana neutrinos [23] and the other is the minimal Type-II seesaw model
phenomenologically viable scenarios of this nature: one is the minimal Type-I seesaw model
may consider the “unbalanced” seesaw scenarios in which the number of heavy Majorana
ν (see Ref. [21] for some detailed discussions).

FIG. 1 illustrates the CP-violating asymmetry between \( \nu_\mu \rightarrow \nu_\tau \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_\tau \) oscillations, where \( \theta_{23} \sim \pi/4 \), \( \Delta m^2_{23} \sim 2.5 \times 10^{-3} \text{ eV}^2 \) and \( (J^2_{\mu\tau} + J^3_{\mu\tau}) \sim 0.01 \) have typically been input. One can see that it is possible to measure this asymmetry in the range \( L/E \sim (100 \cdots 400) \text{ km/GeV} \), if the experimental sensitivity is \( \lesssim 1\% \). A short- or medium-baseline neutrino factory with the beam energy \( E \) being above \( m_\tau \approx 1.78 \text{ GeV} \) is expected to have a good chance to probe or constrain the effect of CP violation induced by the non-unitarity of \( V \) (see Ref. [21] for some detailed discussions).

4 No matter whether a neutrino mass model is of Type-I seesaw or Type-II seesaw, nine
rotation angles and nine phase angles are in general needed to parametrize the correlation
between the charged current interactions of three heavy Majorana neutrinos \( (N_1, N_2, N_3) \) and their light counterparts \( (\nu_1, \nu_2, \nu_3) \). To reduce the number of free parameters, one
may consider the “unbalanced” seesaw scenarios in which the number of heavy Majorana
neutrinos is smaller than that of lighg Majorana neutrinos [22]. There are at least two
phenomenologically viable scenarios of this nature: one is the minimal Type-I seesaw model
with two heavy Majorana neutrinos [23] and the other is the minimal Type-II seesaw model
with one heavy Majorana neutrino and one Higgs triplet [24]. In either case, we can arrive
at the simplified correlation between \( V \) and \( R \).

(1) The minimal Type-I seesaw model with two heavy Majorana neutrinos. In this case,
\( M_R \) is \( 2 \times 2 \), \( M_D \) is \( 3 \times 2 \), and \( M_L = 0 \) holds in the overall \( 5 \times 5 \) neutrino mass matrix \( M \). Switching off the rotation matrices \( O_{i6} \) (for \( i = 1, \cdots, 5 \)) in Eq. (8), we are able to fully parametrize the \( 3 \times 3 \) matrix \( A \) and the \( 3 \times 2 \) matrix \( R \) in terms of six rotation angles \( (\theta_{i4} \text{ and } \theta_{i5} \text{ for } i = 1, 2, 3) \)
and six phase angles \( (\delta_{i4} \text{ and } \delta_{i5} \text{ for } i = 1, 2, 3) \):

\[
A = \begin{pmatrix}
\frac{c_{14}c_{15}}{s_{15}c_{15}} & 0 & 0 \\
-c_{14}c_{15} - s_{14}c_{24}c_{25} & c_{24}c_{25} & 0 \\
-c_{14}c_{15} - s_{14}c_{24}c_{25} + s_{14}c_{14} & s_{15} & -c_{14}c_{15} - s_{14}c_{24}c_{25} & c_{24}c_{25} & 0 \\
\end{pmatrix}, \\
R = \begin{pmatrix}
\frac{s_{14}c_{15}}{s_{15}} \\
\frac{s_{14}c_{15}}{s_{15}} \\
-s_{14}c_{15} + c_{14}c_{24}c_{25} & c_{15}c_{25} \\
\end{pmatrix}.
\]

(18)

It is therefore straightforward to obtain the correlation between \( V = AV_0 \) and \( R \).

(2) The minimal Type-II seesaw model with one heavy Majorana neutrino and one Higgs
triplet. In this case, \( M_R \) is \( 1 \times 1 \), \( M_D \) is \( 3 \times 1 \), and \( M_L \) is \( 3 \times 3 \) in the overall \( 4 \times 4 \) neutrino
mass matrix \( M \). Switching off the rotation matrices \( O_{i5} \) (for \( i < 5 \)) and \( O_{i6} \) (for \( i < 6 \)) in
Eq. (8), we can parametrize the \( 3 \times 3 \) matrix \( A \) and the \( 3 \times 1 \) matrix \( R \) in terms of three
rotation angles \( (\theta_{i4} \text{ for } i = 1, 2, 3) \) and three phase angles \( (\delta_{i4} \text{ for } i = 1, 2, 3) \) as follows:
\[ A = \begin{pmatrix} c_{14} & 0 & 0 \\ -\hat{s}_{14}\hat{s}_{24} & c_{24} & 0 \\ -\hat{s}_{14}\hat{c}_{24}\hat{s}_{34} & -\hat{s}_{24}\hat{s}_{34} & c_{34} \end{pmatrix}, \]
\[ R = \begin{pmatrix} \hat{s}_{14} \\ c_{14}\hat{s}_{24} \\ c_{14}\hat{c}_{24}\hat{s}_{34} \end{pmatrix}. \] (19)

Of course, the correlation between \( V = AV_0 \) and \( R \) is more obvious in this scenario. Taking the Jarlskog invariant \( J_{e\mu}^{23} \) for example, we find

\[ J_{e\mu}^{23} = s_{12}c_{13}s_{13}c_{14}\hat{c}_{24}[c_{12}c_{13}c_{23}s_{24}\sin(\delta_{13} - \delta_{12} - \delta_{23})]
- c_{12}s_{13}s_{14}c_{23}s_{24}\sin(\delta_{14} - \delta_{12} - \delta_{24}) + s_{12}s_{14}s_{23}s_{24}\sin(\delta_{14} - \delta_{13} + \delta_{23} - \delta_{24}) \] , (20)
in which the first term is essentially governed by the phase parameters of \( V_0 \), but the second and third terms result from the unitarity-violating effects. One may similarly calculate the other Jarlskog invariants of CP violation. As for \( \nu_\mu \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_\tau \) oscillations with short or medium baselines, the approximate probabilities obtained in Eq. (16) are generally applicable and the CP-violating quantity \( (J_{\mu\tau}^{23} + J_{\mu\tau}^{13}) \) may unambiguously signify the non-unitarity of \( V \) or the existence of certain non-standard neutrino interactions.

In summary, we have investigated how the charged current interactions of light and heavy Majorana neutrinos are correlated with each other in the Type-I and Type-II seesaw models. It is the first time that an explicit parametrization of this correlation, which is independent of any details of the seesaw models, has been presented to bridge the gap between neutrino physics and collider physics. The rotation angles and phase angles in such a parametrization are expected to be measured or constrained in the precision neutrino oscillation experiments and by exploring possible signatures of heavy Majorana neutrinos at the LHC and ILC. We have taken two special but viable examples, the minimal Type-I seesaw model with two heavy Majorana neutrinos and the minimal Type-II seesaw model with one heavy Majorana neutrino and one Higgs triplet, to illustrate the simplified \( V-R \) correlation. The implications of \( R \neq 0 \) on the low-energy neutrino phenomenology, such as the neutrinoless double-beta decay, the tritium beta decay and CP violation in neutrino oscillations, have also been discussed. In particular, we have demonstrated that the non-unitarity of \( V \) is possible to give rise to an appreciable CP-violating asymmetry between \( \nu_\mu \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_\tau \) oscillations with short or medium baselines. Our generic results remain valid even if the Type-I or Type-II seesaw mechanism is embedded in the supersymmetric standard model and some other extensions of the standard model.

How to naturally realize an appreciable correlation between \( V \) and \( R \) in a TeV-scale seesaw model is actually a real challenge to model builders. The reason is simply that the main textures of \( M_D \) and \( M_R \) in the Type-I seesaw scenarios or those of \( M_D^L \) and \( M_R^L \) in the Type-II seesaw scenarios, which are relevant to possibly observable collider signatures, are difficult to imprint on those sub-leading effects (due to explicit perturbations or radiative corrections) responsible for the tiny masses of light Majorana neutrinos [12,18]. Much more efforts are certainly needed to build viable and natural seesaw models at the TeV scale. One may even speculate the possibility to naturally achieve the TeV-scale leptogenesis and to experimentally test it at the LHC and ILC.
Let us stress that testing the unitarity of the light Majorana neutrino mixing matrix in neutrino oscillations and searching for the signatures of heavy Majorana neutrinos at TeV-scale colliders can be complementary to each other, both qualitatively and quantitatively, in order to deeply understand the intrinsic properties of Majorana particles. Any experimental breakthrough in this aspect will pave the way towards the true theory of neutrino mass generation and flavor mixing.

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FIG. 1. Illustration of the CP-violating asymmetry, which is induced by the non-unitarity of $V$, between $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations with short or medium baselines. Here $\theta_{23} \sim \pi/4$, $\Delta m^2_{23} \sim 2.5 \times 10^{-3}$ eV$^2$ and $(J_{\mu\tau}^{23} + J_{\mu\tau}^{13}) \sim 0.01$ have typically been input.