Wrong-sign Kaons in B decays and New Physics

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Abstract

New physics can possibly emerge in the $B$ decays into wrong-sign kaons for which the standard model contributions are extremely suppressed. We analyze two-body decays of $\bar{B}^0$ and $B^-$ mesons involving the $b \to d\bar{s}$ ($\Delta S = -1$) and $b \to s\bar{s}d$ ($\Delta S = +2$) transitions in a model independent way, and examine various wrong-sign kaon signals which are expected to be observed in the future $B$ experiments. Our analysis shows that it would be possible to identify the origin of new physics through the combined analysis of several $B$ decay modes involving one or two wrong-sign $K^*$'s.

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I. INTRODUCTION

The standard processes of the $b$-quark decay to $s$-quark involve the $\Delta S = 0$ transition $b \to d \bar{s}$ coming from Penguin diagrams, and the $\Delta S = +1$ transitions $b \to s q q'$ ($q, q' = u, c$) induced from the tree-level $W^\pm$ exchange and $b \to sq \bar{q}$ ($q = u, c, d, s$) from Penguin diagrams. Regarding these processes as the “right-sign” $s$-quark (or kaon) decays of $B$ mesons, the so-called “wrong-sign” decays can appear through the $\Delta S = -1$ and $\Delta S = +2$ transitions,

$$b \to d \bar{d} s \quad (\Delta S = -1) \quad \text{or} \quad b \to s s \bar{d} \quad (\Delta S = +2).$$

In the standard model (SM), such processes come from box diagrams exchanging $W^\pm$ bosons in the loop inducing the effective Lagrangian:

$$L_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[ C_{dSM}^{dd} (\bar{d}_L \gamma_\mu b_L) (\bar{d}_L \gamma_\mu s_L) + C_{sSM}^{ss} (\bar{s}_L \gamma_\mu b_L) (\bar{s}_L \gamma_\mu d_L) \right]$$

where the SM coefficients are exceedingly small due to the strong GIM-suppression and the small CKM angles involved. Rough estimation gives $C_{dSM}^{dd} \sim \lambda^8 G_F m_W^2 / 2\sqrt{2}\pi^2$ and $C_{sSM}^{ss} \sim \lambda^7 G_F m_W^2 / 2\sqrt{2}\pi^2$ which make the inclusive branching ratios below $10^{-13}$ and $10^{-11}$, respectively. Such effects will be beyond the reach of any possible future experiments such as super-B factory, Tevatron or LHC which will produce about $10^{10} - 10^{12}$ $B \bar{B}$ mesons.

Given the suppressed SM contribution, certain new physics beyond the SM could give sizable contributions to the wrong-sign $s$-quark transitions and thus alter various observables of the $B$ decays predicted in the context of the SM. Typical examples of new physics such as two Higgs-doublet models and supersymmetric standard model with squark flavor mixing or R-parity violation have been considered in Ref.\cite{1}. In this paper, we investigate the effects of wrong-sign $s$-quark operators on various physical observables in the $B$ decays, and examine how to extract such effects in the future experiments, without resorting to specific models of new physics.

We start with introducing the most general scalar and vector current effective Lagrangian for the $\Delta S = -1$ transition;

$$L_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[ S_{LL}^{dd} (\bar{d}_R b_L) (\bar{d}_R s_L) + S_{RR}^{dd} (\bar{d}_L b_R) (\bar{d}_L s_R) \right. $$

$$+ S_{LL}^{sd} (\bar{d}_R b_L) (\bar{d}_R s_L) + S_{RR}^{sd} (\bar{d}_L b_R) (\bar{d}_L s_R) $$

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Let us first consider the two–body decays of the neutral and charged $B$ mesons into $\pi$ and $K$ mesons arising from $\Delta S = -1$ transition:

$\bar{B}^0 \to \pi^0 K^0$, $\pi^0 K^{*0}$ and $B^- \to \pi^- K^0$, $\pi^- K^{*0}$.

Within the factorization framework [6], we obtain the following amplitudes for the neutral $B$ meson decays:

$$A(\bar{B}^0 \to \pi^0 K^0) = \frac{i G_F}{2} f_K (m_B^2 - m_{\pi}^2) F_0^{B\to\pi} (m_{K^{*0}}^2)$$

II. $\Delta S = -1$ TRANSITION : $b \to dd\bar{s}$

where $\alpha$ and $\beta$ are color indices. Note that we omitted the scalar operators of the $LR$ and $RL$ types, the vector operators of the $LL$ and $RR$ types and tensor operators as they can be rewritten in terms of the above scalar and vector operators after Fierz transformations. For the $\Delta S = -2$ transition, one takes the exchange, $d \leftrightarrow s$. Among the typical examples of new physics, the largest possible coefficients may be obtained with R-parity violation which gives rise to $C'_{RL}$ and $C'_{LL}$ at the tree-level as follows:

$$C'_{LL} = \sum_n \frac{\sqrt{2}}{8} \lambda'_{n3l} \lambda'_{n12}, \quad C'_{RL} = \sum_n \frac{\sqrt{2}}{8} \lambda'_{n21} \lambda'_{n13},$$

where $m_{\tilde{\nu}_n}^2$ is the mass of the mediating sneutrino of the $n$–th generation. We define that the R-parity violating couplings are given in the superpotential as follows:

$$W_{\lambda'} = \lambda'_{ijk} (E_i V^\dagger_{jk} U_j D_k - L_i^0 D_j D_k^c),$$

where $L_i = (L_i^0, E_i)$ and $Q_i = (U_i, D_i)$ are the lepton and quark $SU(2)$ doublets, and $D_i^c$ is the $SU(2)$ singlet anti-quark superfields. Here $V_{ij}$ is the CKM matrix of quark fields. Let us note that the $C'_{LR,RL}$ couplings induced by R-parity violation can be as large as $0.1 - 0.01$ within the present experimental bounds [1, 5]. In the following, we will take the above new physics coefficients, $C'_{LR,RL}$, for specific illustrations.
\[
\left\{ \frac{1}{2} r_1 [(1 - \frac{1}{2} \xi) (S_{LL}^{dd} - S_{RR}^{dd}) + (\xi - \frac{1}{2}) (S_{LL}^{dd} - S_{RR}^{dd})] \\
+ [(1 + \xi) (C_{LL}^{dd} - C_{RR}^{dd}) + (r_1 \xi - 1) (C_{LR}^{dd} - C_{RL}^{dd}) + (r_1 - \xi) (C_{LR}^{dd} - C_{RL}^{dd})] \right\}.
\]

\[A(B^- \to \pi^- K^0) = \sqrt{2} A(\bar{B}^0 \to \pi^0 K^0),\]
\[A(\bar{B}^0 \to \pi^0 K^{*0}) = -G_F (e^* \cdot p_\pi) m_{K^*} \times \]
\[\left\{ \frac{m_{K^*}}{m_B + m_\pi} f_{K^*}^T F_{2\to\pi} (m_{K^*}^2) \frac{1}{2} [\xi (S_{LL}^{dd} + S_{RR}^{dd}) + (S_{LL}^{dd} + S_{RR}^{dd})] \\
- f_{K^*}^T F_{1\to\pi} (m_{K^*}^2) [(1 + \xi) (C_{LL}^{dd} + C_{RR}^{dd}) + (C_{LR}^{dd} + C_{RL}^{dd}) + \xi (C_{LR}^{dd} + C_{RL}^{dd})] \right\},\]
\[A(B^- \to \pi^- K^{*0}) = \sqrt{2} A(\bar{B}^0 \to \pi^0 K^{*0}),\]

where \(\xi \equiv 1/N_c\), \(r_1 \equiv 2m_{K^0}^2/(m_b - m_d)(m_s + m_d)\) and \(e^*\) is the polarization vector of the vector meson \(K^*\). The definitions of various form factors and their numerical values taken for our calculations are summarized in the Appendix.

For comparisons with the SM right-sign amplitudes, we define, for each decay mode, the ratio \(w_{\pi K}\) of the wrong-sign (WS) amplitude with \(\Delta S = -1\) to the corresponding SM one with \(\Delta S = +1\) (driven by the \(b \to s\bar{d}d\) transition) as:

\[w_{\pi K} \equiv \frac{A_{WS}(\Delta S = -1)}{A_{SM}(\Delta S = +1)}.\]

In Table I, we show the values of \(w_{\pi K}\) for each \(\Delta S = -1\) operator defined in Eq. (3), taking \(N_c = 3\) and \(S_{1J}^{(r)dd}, C_{1J}^{(r)dd} = |V_{tb} V_{ts}^* (a_4 - a_{10}/2)| \simeq 1.54 \times 10^{-3}\). For the numerical values used and definitions of the coefficients \(a_i\), etc., see Ref. [6, 7]. Table I clearly shows that there can be significant effects if \(S_{1J}^{(r)dd} \sim 10^{-3}\) or \(C_{1J}^{(r)dd} \sim 10^{-3}\) is allowed.

As pointed out in Ref. [1], the \(\bar{B} \to \pi K^{*0}\) mode will play major role for probing or constraining the \(\Delta S = -1\) transition. Here, \(\bar{B}\) denotes \(B^0\) or \(B^-\) and correspondingly \(\pi\) can be \(\pi^0\) or \(\pi^-\). Let us note that the ratio \(w_{\pi K^{*0}} = A_{WS}(\bar{B} \to \pi K^{*0})/A_{SM}(\bar{B} \to \pi K^{*0})\) can be determined by comparing two branching fractions of \(\Delta S = -1\) and \(\Delta S = +1\) transitions:

\[|w_{\pi K^{*0}}|^2 = \frac{\mathcal{B}(\bar{B} \to \pi K^{*0} \to \pi^{-} K^{+})}{\mathcal{B}(\bar{B} \to \pi K^{*0} \to \pi^{+} K^{-})}.\]

The current measurements at the \(B\) factories have started to constrain \(w_{\pi K^{*0}}\) which takes particularly simple form as

\[w_{\pi K^{*0}} = \left\{ (1 + \xi)[C_{LL}^{dd} + C_{RR}^{dd}] + [C_{LR}^{dd} + C_{RL}^{dd}] + \xi (C_{LR}^{dd} + C_{RL}^{dd}) \right\} \frac{1}{C_{SM}}\]
where $\xi = 1/N_c$ and $C_{SM} = V_{tb}V_{ts}^*(a_4 - a_{10}/2)$. Here we neglected the contributions from the scalar operators. Recently, accumulating about $(6 - 9) \times 10^7 BB$ pairs, BaBar Collaboration reported the measurements:

$$B(B^- \to \pi^-\pi^+K^-) = (59.1 \pm 3.8 \pm 3.2) \times 10^{-6} \quad [8],$$
$$B(B^- \to \pi^-K^{*0} \to \pi^-\pi^+K^-) = (10.3 \pm 1.2^{+1.0}_{-2.7}) \times 10^{-6} \quad [9].$$

(12)

With this, the measured upper bound of the 3-body wrong-sign branching ratio,

$$B(B^- \to \pi^-K^{*0} \to \pi^-\pi^+K^-) < 1.8 \times 10^{-6} \quad [8],$$

(13)

is translated to the bound,

$$B(B^- \to \pi^-K^{*0} \to \pi^-\pi^+K^-) < 3.1 \times 10^{-7}.$$

(14)

Therefore, from Eq. (10) one obtains the bound,

$$|w_{\pi^-K^{*0}}| < 0.17.$$

(15)

This implies that the coefficient $C_{LR,RL}'$ gets the constraint of $|C'| < 0.17N_c|C_{SM}| = 7.9 \times 10^{-4}(N_c/3)$. Applying this to the R-parity violation in Eq. (4), we obtain the following stringent bound:

$$|\lambda'_{n31}\lambda'_{n12}|, \ |\lambda'_{n21}\lambda'_{n13}| < 5.2 \times 10^{-4} \left(\frac{m_{\tilde{\nu}_n}}{100 \text{ GeV}}\right)^2.$$

(16)

Considering future experiments producing $10^{11} B$ mesons, it is expected to probe $|w_{\pi^-K^{*0}}|$ below the level of 1%, providing the limit on the coefficients $C's$ down to $3 \times 10^{-5}$.

The decay into two pseudoscalar mesons is also useful to probe $\Delta S = -1$ though it is more difficult compared with the $\pi K^{*0}$ mode. First of all, the presence of the wrong–sign operators affects the experimental determination of the branching ratio of the mode $\bar{B} \to \pi\bar{K}^0$ from the measurements of $B(\bar{B} \to \pi K^{0}_{S,L})$. Both the $\Delta S = -1$ transition, $\bar{B} \to \pi K^0$, and the $\Delta S = +1$ one, $\bar{B} \to \pi\bar{K}^0$, contribute to the decays $\bar{B} \to \pi K^{0}_{S,L}$ through the $K--\bar{K}$ mixing. The amplitude of the $B$ decay into $\pi K_S$ or $\pi K_L$ is given by

$$\bar{A}_{\pi K^{0}_{S,L}} = p_K\bar{A}_{\pi K^0} \pm q_K\bar{A}_{\pi K^{*0}},$$

(17)

where $\bar{A}_{M_1M_2} \equiv A(B \to M_1 M_2)$. In Eq. (17), $p_K$ and $q_K$ are the coefficients relating the $K$ meson mass eigenstates with the flavor eigenstates;

$$|K_{S,L}\rangle = p_K|\bar{K}^0\rangle \pm q_K|K^{*0}\rangle.$$  

(18)
Recall that \( p_K, q_K = (1 \pm \bar{\epsilon})/\sqrt{2(1 + |\bar{\epsilon}|^2)} \) with \( |\bar{\epsilon}| \sim 10^{-3} \). Denoting the ratio of the wrong-sign amplitude to the SM one as \( w_{\pi K^0} = \bar{A}_{\pi K^0}/\bar{A}_{\pi K^0} \), we get

\[
2\mathcal{B}(\bar{B} \to \pi K^0_{S,L}) = \mathcal{B}(\bar{B} \to \pi \bar{K}^0)_{SM}\left|1 \pm \frac{p_K}{q_K} w_{\pi K^0}\right|^2,
\]

where \( p_K/q_K \simeq 1 \). The SM relation, \( \mathcal{B}(\bar{B} \to \pi \bar{K}^0) = 2\mathcal{B}(\bar{B} \to \pi K^0_{S,L}) \), can obviously be invalidated in the presence of the wrong-sign amplitudes, and thus it has to be checked experimentally. Current experiments at \( B \) factories only look for the modes \( \bar{B} \to \pi K^0_{S} \). The present world average of the \( \bar{B} \to \pi K^0_{S} \) branching ratios are [10]:

\[
2\mathcal{B}(\bar{B}^0 \to \pi^0 K^0_{S}) = (11.5 \pm 1.7) \times 10^{-6}, \\
2\mathcal{B}(B^- \to \pi^- K^0_{S}) = (20.6 \pm 1.4) \times 10^{-6}.
\]

This can be compared with the SM prediction: \( 2\mathcal{B}(\bar{B}^0 \to \pi^0 K^0_{S})_{SM} = 5.1 \times 10^{-6} \) and \( 2\mathcal{B}(B^- \to \pi^- K^0_{S})_{SM} = 15 \times 10^{-6} \), which are derived within the factorization scheme taking the standard values for the input parameters as specified in Appendix and \( \xi = 1/3 \). The apparent discrepancies between the experimental and theoretical values can be cured by the wrong-sign amplitude contribution as in Eq. (17). With the results of Table 1, the bound [13] can be translated to \( |w_{\pi^0 K^0}| < 0.13 \), and \( |w_{\pi^- K^0}| < 0.11 \) for the case of the new physics coupling \( C'_{LR,RL} \). Thus, the maximal contributions of the new physics (NP) to \( w_{\pi K} \) can give a better explanation of the data as we get \( 2\mathcal{B}(\bar{B}^0 \to \pi^0 K^0_{S})_{NP} = 6.5 \times 10^{-6} \) and \( 2\mathcal{B}(B^- \to \pi^- K^0_{S})_{NP} = 18 \times 10^{-6} \). However, it is premature to make any definite conclusion about the role of the wrong-sign amplitudes since the theoretical calculations have large uncertainties not only within the factorization scheme [9] but also in any other approaches [11, 12, 13, 14].

In relation to this, let us remark on the “wrong-sign” kaon contribution to the isospin violation [16] in the \( B \to \pi K \) modes:

\[
\bar{B}^0 \to \pi^0 K_S, \pi^- K^+; \quad B^- \to \pi^- K_S, \pi^0 K^-.
\]

As discussed, the \( \Delta S = -1 \) operators contribute only to \( \bar{B}^0 \to \pi^0 K_S \) and \( B^- \to \pi^- K_S \) as in Eq. (6). This shows that the experimental data [20], implying \( 2\mathcal{B}(\bar{B}^0 \to \pi^0 K^0_{S}) > \mathcal{B}(B^- \to \pi^- K^0_{S}) \), can be explained by an enhanced electro-weak penguin contribution coming from new physics as analyzed in a recent paper [17]. In fact, all the isospin violating relations could be a consequence of both the electro-weak penguin and the wrong-sign amplitude [18].
A direct way to probe the above wrong-sign amplitude, $w_{\pi K^0}$, is to reconstruct $K_L$ experimentally. This allows us to measure the following rate asymmetries [15] which are nearly vanishing in the SM:

$$\bar{A}_{\pi K^0} \equiv \frac{\Gamma(\bar{B} \to \pi^0 K^0_S) - \Gamma(\bar{B} \to \pi^0 K^0_L)}{\Gamma(\bar{B} \to \pi K^0_S) + \Gamma(\bar{B} \to \pi K^0_L)} = \frac{2\text{Re}(w_{\pi K^0})}{1 + |w_{\pi K^0}|^2}. \quad (21)$$

In the current $B$ factories, only the direction of $K^0_L$ can be measured. Then, its momentum can be calculated from the $B$–mass constraint to reconstruct the mode $\bar{B} \to \pi^0 K^0_L$. This is the way to measure CP violation in the $\bar{B} \to J/\psi K^0_L$ mode. Contrary to the $J/\psi K^0_L$ case, the final state $\pi K^0_L$ suffers from a huge background which makes it hard to separate out the candidate events. In order to avoid it, one may have to fully reconstruct the other $B$, by which, however, we can only use about 0.1% of the produced $B\bar{B}$ pairs. Considering the branching ratio $\approx 10^{-5}$ of the $\pi K^0$ mode and the 0.1% detection efficiency, one can collect about 1000 $\pi K^0_L$ events from $10^{11} B\bar{B}$ pairs. Therefore, the $K_S$–$K_L$ asymmetry at the level of a few % could be seen in the future experiments.

The presence of the wrong-sign amplitude can appear also in the direct CP asymmetry of $B^\pm \to \pi^\pm K^0_S$. In the SM, the CP asymmetry

$$A_{CP} = \frac{\Gamma(B^+ \to \pi^+ K^0_S) - \Gamma(B^- \to \pi^- K^0_S)}{\Gamma(B^+ \to \pi^+ K^0_S) + \Gamma(B^- \to \pi^- K^0_S)} \quad (22)$$

is expected to be of order 1% arising from the interference of two penguin contributions (to $\bar{A}_{\pi K^0}$ in Eq. (17)) with the CKM-suppressed relative amplitude $\sim 0.02$. The wrong-sign amplitudes, $A_{\pi+ K^0}$ for $B^+$ and $\bar{A}_{\pi- K^0}$ for $B^-$ as in Eq. (17), can give rise to another interfering effect. Under the condition that the wrong-sign amplitudes dominates over the CKM-suppressed penguin amplitudes, we obtain

$$A_{CP} = \frac{2|w_{\pi- K^0}| \sin \Delta\phi \sin \Delta\delta}{1 + 2|w_{\pi- K^0}| \cos \Delta\phi \cos \Delta\delta + |w_{\pi- K^0}|^2}, \quad (23)$$

where $\Delta\phi$ ($\Delta\delta$) is the relative weak (strong) phase of the right and wrong sign amplitudes. At the moment, the above CP asymmetry is measured with the accuracy of 10-20% [10] which puts a constraint on $|w_{\pi- K^0}|$ (with $\Delta\phi, \Delta\delta \sim 1$ ) close to the bound, $|w_{\pi- K^0}| < 0.11$, coming from the branching ratio measurements discussed below Eq. (10). The CP asymmetry $A_{CP}$ is expected to be improved to the level of one percent in the future experiments, and thus could provide an indirect way to probe the wrong-sign amplitudes.
III. \( \Delta S = +2 \) TRANSITION : \( b \rightarrow ss\bar{d} \)

In this section, we discuss the two-body decays of the charged and neutral \( B \) mesons induced by the \( \Delta S = +2 \) operators:

**PP modes** : \( B^0 \rightarrow K^0 \bar{K}^0, \; B^- \rightarrow K^- \bar{K}^0 \),

**PV modes** : \( \bar{B}^0 \rightarrow \bar{K}^0 K^0, \; B^- \rightarrow K^- \bar{K}^* \), \( B^- \rightarrow K^{*-} \bar{K}^0 \),

**VV modes** : \( \bar{B}^0 \rightarrow \bar{K}^{*0} \bar{K}^0, \; B^- \rightarrow K^{*-} \bar{K}^* \).

The amplitudes for the decay modes into two pseudoscalar mesons (PP), a pseudoscalar and a vector mesons (PV), and two vector mesons (VV) are given as follows:

\[
A(\bar{B}^0 \rightarrow \bar{K}^0 \bar{K}^0) = i \sqrt{2} G_F f_K (m_B^2 - m_{K^0}^2) F_0^{B \rightarrow K}(m_{K^0}) \times \\
\left\{ \frac{r_2}{2} \{ (1 - \frac{1}{2}) \xi (S_{LL}^{ss} - S_{RR}^{ss}) + (\xi - \frac{1}{2}) (S_{LL}^{tss} - S_{RR}^{tss}) \} + \{ (1 + \xi) (C_{LL}^{ss} - C_{RR}^{ss}) + (r_2 \xi - 1) (C_{LR}^{ss} - C_{RL}^{ss}) + (r_2 - \xi) (C_{LR}^{ts} - C_{RL}^{ts}) \} \right\},
\]

\[
A(B^- \rightarrow K^- \bar{K}^0) = \frac{1}{2} A(\bar{B}^0 \rightarrow \bar{K}^0 \bar{K}^0),
\]

\[
A(\bar{B}^0 \rightarrow \bar{K}^{*0} \bar{K}^0) = -\sqrt{2} G_F (\epsilon^* \cdot p_B) m_{K^*} \times \\
\left\{ \frac{m_{K^*}}{m_B + m_{K^*}} f_{K^*}^T F_2^{B \rightarrow K}(m_{K^*}) \frac{1}{2} \{ (S_{LL}^{ss} + S_{RR}^{ss}) + (S_{LL}^{tss} + S_{RR}^{tss}) \} + \frac{r_3}{2} \{ (1 - \frac{1}{2}) \xi (S_{LL}^{ss} + S_{RR}^{ss}) + (\xi - \frac{1}{2}) (S_{LL}^{tss} + S_{RR}^{tss}) \} - f_{K^*} F_1^{B \rightarrow K}(m_{K^*}) \{ (1 + \xi) (C_{LL}^{ss} + C_{RR}^{ss}) + (C_{LR}^{ss} + C_{RL}^{ss}) + (C_{LR}^{ts} + C_{RL}^{ts}) \} + \frac{r_3}{2} \{ (1 + \xi) (C_{LL}^{ss} + C_{RR}^{ss}) + (C_{LR}^{ss} + C_{RL}^{ss}) \} \right\},
\]

\[
A(B^- \rightarrow K^{*-} \bar{K}^0) = -\sqrt{2} G_F (\epsilon^* \cdot p_B) m_{K^*} \times \\
\left\{ \frac{m_{K^*}}{m_B + m_{K^*}} f_{K^*}^T F_2^{B \rightarrow K}(m_{K^*}) \frac{1}{2} \{ (S_{LL}^{ss} + S_{RR}^{ss}) + (S_{LL}^{tss} + S_{RR}^{tss}) \} - f_{K^*} F_1^{B \rightarrow K}(m_{K^*}) \{ (1 + \xi) (C_{LL}^{ss} + C_{RR}^{ss}) + (C_{LR}^{ss} + C_{RL}^{ss}) + (C_{LR}^{ts} + C_{RL}^{ts}) \} \right\},
\]

\[
A(B^- \rightarrow K^{*-} \bar{K}^0) = -\sqrt{2} G_F (\epsilon^* \cdot p_B) m_{K^*} \times \\
\left\{ \frac{r_3}{2} \{ (1 - \frac{1}{2}) \xi (S_{LL}^{ss} + S_{RR}^{ss}) + (\xi - \frac{1}{2}) (S_{LL}^{tss} + S_{RR}^{tss}) \} \right\}.
\]
\[ -[(1 + \xi)(C_{LL}^{ss} + C_{RR}^{ss}) - (1 + r_3 \xi)(C_{LR}^{ss} + C_{RL}^{ss}) - (\xi + r_3)(C_{LR}^{dss} + C_{RL}^{dss})] \right. \]

\[ A(B^0 \rightarrow K^* \bar{K}^0) = \]

\[ -\sqrt{2} G_F f_K T \left\{ \left[ (\epsilon_{l\mu \rho} \epsilon_1^{* \mu} \epsilon_2^{* \rho} p_1 p_2^\rho) T_{2}^{B \rightarrow K^*}(m_{K^*}^2) \right] \left[ \xi(S_{LL}^{ss} + S_{RR}^{ss}) + (S_{LL}^{dss} + S_{RR}^{dss}) \right] \right. \]

\[ - i(\epsilon_1^{*} \cdot p_2)(\epsilon_2^{*} \cdot p_1)(T_{2}^{B \rightarrow K^*}(m_{K^*}^2) + \frac{m_{K^*}}{m_B^2 - m_{K^*}^2} T_{3}^{B \rightarrow K^*}(m_{K^*}^2)) - \]

\[ \frac{1}{2}(m_B^2 - m_{K^*}^2) T_{2}^{B \rightarrow K^*}(m_{K^*}^2) \left[ \xi(S_{LL}^{ss} - S_{RR}^{ss}) + (S_{LL}^{dss} - S_{RR}^{dss}) \right] \left\} \right. \]

\[ \left[ (1 + \xi)(C_{LL}^{ss} + C_{RR}^{ss}) + (C_{LR}^{ss} + C_{RL}^{ss} + \xi(C_{LR}^{dss} + C_{RL}^{dss})) \right] \]

\[ \left[ (1 + \xi)(C_{LL}^{ss} - C_{RR}^{ss}) + (C_{LR}^{ss} - C_{RL}^{ss} + \xi(C_{LR}^{dss} - C_{RL}^{dss})) \right] \]

\[ A(B^- \rightarrow K^0 \bar{K}^0) = \frac{1}{2} A(B^0 \rightarrow K^* \bar{K}^0), \quad (30) \]

where \( r_2 \equiv 2m_{K^0}^2/(m_B - m_s)(m_s + m_d), \) \( r_3 \equiv 2m_{K^0}^2/(m_B + m_s)(m_s + m_d) \) and the index 1 or 2 labels each \( \bar{K}^0 \) in the \( \bar{B}^0 \rightarrow \bar{K}^0 \bar{K}^0 \) mode.

Again, Table I shows the ratio \( w_{KK} \) of the wrong-sign \( \Delta S = +2 \) amplitudes to the corresponding \( \Delta S = 0 \) SM ones (driven by the \( b \rightarrow d S \) transition) as:

\[ w_{KK} \equiv \frac{A_{WS}(\Delta S = +2)}{A_{WS}(\Delta S = 0)}, \quad (31) \]

counted by each \( \Delta S = +2 \) operator with the coefficient \( C_I^{(l)ss} \) or \( S_I^{(l)ss} \) where \( I, J = L, R \). The numerical values are taken with the choice of \( N_c = 3 \) and \( S_{II}^{(l)ss}, C_{II}^{(l)ss} = |V_{ub}^* V_{td}^2(a_4 - a_{10}/2)| = 2.91 \times 10^{-4} \). For the VV modes, we show the square-rooted ratio, \( |w_{K^* K^0}| \equiv \sqrt{\Gamma_{WS}(\Delta S = 2)/\Gamma_{SM}(\Delta S = 0)} \) since the direct comparison between amplitudes is not possible. In the following, we closely examine phenomenological implications of the wrong-sign \( \Delta S = +2 \) transition in each mode.

- **PP modes**

  When there exist the wrong-sign amplitude of the process \( \bar{B}^0 \rightarrow \bar{K}^0 \bar{K}^0 \) as well as the SM amplitude of the process \( \bar{B}^0 \rightarrow \bar{K}^0 \bar{K}^0 \), the final states \( |K_A; K_B \rangle \) with \( A, B = S, L \) can be written as

\[ |K_{S,L}; K_{S,L} \rangle = +q_k^2|\bar{K}^0; \bar{K}^0 \rangle \pm p_k q_k \left( |K^0; \bar{K}^0 \rangle + |\bar{K}^0; K^0 \rangle \right), \]
\[ |K_{S,L}; K_{L,S} \rangle = -q_K^2 |K^0; \bar{K}^0 \rangle \mp p_K q_K \left( |K^0; \bar{K}^0 \rangle - |\bar{K}^0; K^0 \rangle \right), \tag{32} \]

where the first and second \( K \)'s are to be labeled by its momentum \( \vec{k} \) and \(-\vec{k}\), respectively, in the \( \bar{B}^0 \) rest frame. Note that we have neglected the state \( |K^0; \bar{K}^0 \rangle \) as it is irrelevant for our discussion. Rotation invariance implies that the antisymmetric combination of two different \( K \)'s vanishes in the amplitude. Thus, we obtain the following amplitudes with symmetrized final states:

\[
\hat{A}_{K_SK_S} = q_K^2 \hat{A}_{K^0\bar{K}^0} + \sqrt{2} p_K q_K \hat{A}_{K^0\bar{K}^0}, \\
\hat{A}_{K_LK_L} = q_K^2 \hat{A}_{K^0\bar{K}^0} - \sqrt{2} p_K q_K \hat{A}_{K^0\bar{K}^0}, \\
\hat{A}_{K_SK_L} = -\sqrt{2} q_K^2 \hat{A}_{K^0\bar{K}^0}, \tag{33} \]

for the \( \bar{B}^0 \) decays to \( K_SK_S \), \( K_LK_L \) and \( K_SK_L \), respectively. We see that the SM predictions, \( \Gamma(K_SK_S) = \Gamma(K_LK_L) \) and \( \Gamma(K_SK_L) = 0 \), are modified as

\[
\Gamma(\bar{B}^0 \to K_SK_S) = \Gamma(\bar{B}^0 \to K_SK_S)_{SM} \left| 1 + \frac{1}{2} \frac{q_K}{p_K} w_{\bar{K}^0K^0} \right|^2, \\
\Gamma(\bar{B}^0 \to K_LK_L) = \Gamma(\bar{B}^0 \to K_SK_S)_{SM} \left| 1 - \frac{1}{2} \frac{q_K}{p_K} w_{\bar{K}^0K^0} \right|^2, \tag{34} \\
\Gamma(\bar{B}^0 \to K_SK_L) = \Gamma(\bar{B}^0 \to K_SK_S)_{SM} \left( \frac{q_K}{p_K} w_{\bar{K}^0K^0} \right)^2, \]

where \( w_{\bar{K}^0K^0} = \bar{A}_{K^0\bar{K}^0}/A_{K^0\bar{K}^0} \).

The best way to observe \( \Delta S = +2 \) transition is to measure the following observables by reconstructing \( K_L \) experimentally:

\[
\hat{R}_{SL} \equiv \frac{\Gamma(\bar{B}^0 \to K_SK_L)}{\Gamma(\bar{B}^0 \to K_SK_S)} = \frac{|w_{\bar{K}^0K^0}|^2}{\left| 1 + \frac{1}{\sqrt{2}} |w_{\bar{K}^0K^0}| \right|^2}, \\
\hat{A}^K_{SL} \equiv \frac{\Gamma(\bar{B}^0 \to K_SK_S) - \Gamma(\bar{B}^0 \to K_LK_L)}{\Gamma(\bar{B}^0 \to K_SK_S) + \Gamma(\bar{B}^0 \to K_LK_L)} = \frac{2 \sqrt{2} \text{Re}(w_{\bar{K}^0K^0})}{2 + |w_{\bar{K}^0K^0}|^2}. \tag{35} \]

The observable \( \hat{R}_{SL} \) is of a particular interest as it measures the absolute value of the wrong-sign amplitude. Following the similar argument below Eq. (21), we find that \( \hat{R}_{SL} \) could be measured up to the level of 10 \% with \( 10^{11} B \bar{B} \) mesons taking the branching ratio \( B(\bar{B}^0 \to K^0\bar{K}^0)_{SM} \sim 10^{-6} \). Note that the current experimental results give the bound; \( 2B(\bar{B}^0 \to K_SK_S) < (1.6 - 3.2) \times 10^{-6} \) \([10]\). However, it will be almost impossible to measure \( \hat{A}^K_{SL} \) involving two \( K_L \) final state. We expect that precision measurements of the above observables can be made if future \( B \) experiments are equipped with a hadronic calorimetry with a significant ability of reconstructing \( K_L \) which is not anticipated in the present plans.
For the decays $B^- \to K^- K_{S,L}$, we can get the similar expressions as in Eqs. (21) and (23) by replacing $\pi^-$ with $K^-$ and $(p_K/q_K) w_{\pi^K}$ with $(q_K/p_K) w_{K^-K^0}$. Note that the CP asymmetry $A_{CP}$ of $B^\pm \to K^\pm K_S$ is known to be about 20% and thus the wrong-sign contribution has to be fairly large to see a new physics effect.

- **PV modes**:

As discussed in the previous section, the production of $K^{*0}$ or $\bar{K}^{*0}$ in the $B$ decays provides a straightforward way to identify the right-sign or wrong-sign signals. Let us first consider the $\bar{B}^0$ decays. As shown in Table I, the wrong-sign amplitude for $\bar{B}^0 \to K^0 \bar{K}^{*0}$ can be compared with two right-sign amplitudes for $\bar{B}^0 \to K^0 K^{*0}$ and $K^0 \bar{K}^{*0}$. Here, it is amusing to note that the latter right-sign amplitude exhibits a cancellation among SM contributions thus one predicts $B(K_S K^{*0})_{SM}/B(K_S K^{*0})_\text{SM} \lesssim 0.1$ [6, 11, 12, 13, 14]. Now that the wrong-sign amplitude contributes only to the $K_S \bar{K}^{*0}$ mode, it can alter the above SM prediction. Namely, the observation of

$$\frac{B(\bar{B}^0 \to K_S \bar{K}^{*0})}{B(\bar{B}^0 \to K_S K^{*0})} > 0.1$$

will clearly be a signal for new physics inducing the wrong-sign amplitude.

In the case of $B^-$ decays, there are two ways of identifying the wrong-sign amplitude. One is to look for $B^- \to K^- \bar{K}^{*0} \to K^- K^- \pi^+$ which is almost absent in the SM. Another way is to observe $B^- \to K_S K^{*-}$. As in the $\bar{B}^0$ case, the SM amplitude of $B^- \to K^0 K^{*-}$ is similarly suppressed and thus the wrong-sign amplitude may have a larger contribution. As a result,

$$\frac{2B(B^- \to K_S K^{*-})}{B(B^- \to K^- K^{*0})} > 0.1$$

may arise together with Eq. (36).

If the branching ratios of $B^- \to K^- \bar{K}^{*0}$, $K_S K^{*-}$ and $\bar{B}^0 \to K_S \bar{K}^{*0}$ are measured above the SM predictions, it will be possible to identify which operators in Eq. (3) contribute to the wrong-sign amplitude. To get an idea, let us compare the branching ratios assuming one type of the coefficients, $C(\cdots)$ and $S(\cdots)$ exists:

$$\frac{B(B^- \to K^- K^{*0})}{B(B^- \to K_S K^{*-})} = 3.8 (C_{II}), \ 2.4 (C_{IJ}), \ 0.35 (C'_{IJ}), \ 0.024 (S_{II}), \ 0.54 (S'_{II}),$$

$$\frac{B(\bar{B}^0 \to K_S \bar{K}^{*0})}{B(B^- \to K_S K^{*-})} = 5.7 (C_{II}), \ 0.01 (C_{IJ}), \ 0.34 (C'_{IJ}), \ 1.2 (S_{II}), \ 0.4 (S'_{II}),$$

where $I, J = L, R$ and $I \neq J$. This shows, for instance, that we should find the ratio $B(K_S K^{*-}) : B(K^- \bar{K}^{*0}) : B(K_S \bar{K}^{*0}) \simeq 3:1:1$ if the R-parity violation is the source of the
wrong-sign amplitude as in Eq. (4).

Considering again the $K_L$ measurement, $\Gamma(K_S\bar{K}^*) \neq \Gamma(K_L\bar{K}^*)$ can arise due to the interference between the right and wrong sign amplitudes, whose ratio $q_K\bar{A}_{K^0\bar{K}^*0}/p_K\bar{A}_{K^0\bar{K}^*0}$ can be separated out by measuring the $K_S-K_L$ asymmetry in the decay $\bar{B}^0 \to \bar{K}^*0K_{S,L}$. This has to be contrasted with the modes, $\bar{B}^0 \to K_{S,L}K^*$, in which no wrong-sign amplitude can interfere, and therefore, the SM prediction $\Gamma(K_SK^*) = \Gamma(K_LK^*)$ persists.

- **VV modes**: Having two $K^*$s in the final states, these modes also provide a clean way to identify the wrong-sign signals [1]. In the $K^*K^*$ modes, the standard right-sign processes contain two opposite-sign $K$’s: $\bar{B}^0 \to \bar{K}^0K^0 \to (K^+\pi^-)(K^-\pi^+)$ and $B^- \to K^-K^*0 \to (K^-\pi^0)(K^+\pi^-)$. On the other hand, the wrong-sign processes give rise to two same-sign $K$’s in the final states: $\bar{B}^0 \to \bar{K}^*0\bar{K}^*0 \to (K^-\pi^+)(K^-\pi^+)$ and $B^- \to K^-\bar{K}^*0 \to (K^-\pi^0)(K^-\pi^+)$. It is worthwhile to look into the ratio of branching ratios of the decay modes $\bar{B}^0 \to \bar{K}^*0\bar{K}^*0$ (or $B^- \to K^-\bar{K}^*0$) and $B^- \to K^-K^*0$, which are almost forbidden in the SM framework. We observe that the contributions to $B(B^- \to K^-\bar{K}^*0)$ from the scalar operators are suppressed by the factor $\sim (m_{K^*}/2m_B)^2$ comparing with those from the vector operators. On the other hand, the contributions to $B(B^- \to \bar{K}^*0\bar{K}^*0)$ from the scalar operators are not much different from those from the vector operators. Specifically we find

$$\frac{B(\bar{B}^0 \to \bar{K}^*0\bar{K}^*0)}{B(B^- \to K^-\bar{K}^*0)} = 340 \left( S_{IJ}^{(l)} \right), \quad 1.9 \left( C_{IJ}^{(l)} \right),$$

where $I, J = L, R$. This ratio can be served as a clear discriminant to identify whether the wrong-sign operators are purely scalar type or not.

**IV. CONCLUSION**

In the future $B$ experiments which can examine rare $B$ decays with the branching ratio down to $10^{-10}$, it is worthwhile to look for the $\Delta S = -1$ and $+2$ processes which have extremely small standard model background. In this regards, we analyzed exclusive two-body decay modes containing wrong-sign kaons in the final states, signaling new physics effect. As is well-known, the observation of $B^- \to \pi^-K^*$ or $K^-\bar{K}^*$ provides a clean signal for the existence of the wrong-sign amplitude. However, with a reasonable efficiency
in measuring $K_L$, the wrong-sign amplitudes can be also probed in the $K_S-K_L$ asymmetry of $B \to \pi(K^{(*)})K_{S,L}$ and $K_SK_L$, etc, or in the CP asymmetry of $B^\pm \to \pi^\pm(K^{(*)})K_S$.

Combination of all the observations will be useful to investigate new physics beyond the standard model. Thus, we consider it desirable to improve the detection efficiency for the identification of $K_L$ in the future $B$ experiments. Observing the wrong-sign kaons in the $B$ decays to one or two vector mesons can lead us to study the origin of the wrong-sign amplitude. For the $B$ decays driven by $\Delta S = +2$ transitions, the type of the wrong-sign operators can be identified if anomalously high branching ratios are measured for the modes $B^- \to K^-\bar K^*0$, $K_SK^*$ and $\bar B^0 \to K_S\bar K^*0$, or if the observation of the modes, $B^- \to K^-\bar K^*0$ and $\bar B^0 \to K^*0\bar K^*0$, is made.

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**APPENDIX A: FORM FACTORS**

The form factors for the $B$ decays used in our calculation are defined as follows.

- **Meson decay amplitudes:**

\[
\langle \pi^- (q) | \bar{d}\gamma_\mu (1 \pm \gamma_5) u | 0 \rangle = \sqrt{2} \langle \pi^0 (q) | \bar{u}\gamma_\mu (1 \pm \gamma_5) u | 0 \rangle = - \sqrt{2} \langle \pi^0 (q) | \bar{d}\gamma_\mu (1 \pm \gamma_5) d | 0 \rangle = \mp i f_\pi q_\mu ,
\]

\[
\langle K^- (q) | \bar{s}\gamma_\mu (1 \pm \gamma_5) u | 0 \rangle = - \langle K^0 (q) | \bar{d}\gamma_\mu (1 \pm \gamma_5) s | 0 \rangle = - \langle \bar{K}^0 (q) | \bar{s}\gamma_\mu (1 \pm \gamma_5) d | 0 \rangle = \mp i f_K q_\mu , \tag{A1}
\]

\[
\langle K^{*-} (q, \epsilon) | \bar{s}\gamma_\mu u | 0 \rangle = - \langle K^{0*} (q, \epsilon) | \bar{d}\gamma_\mu s | 0 \rangle = - \langle K^{0*} (q, \epsilon) | \bar{s}\gamma_\mu d | 0 \rangle = f_{K^*} m_{K^*} \epsilon_\mu^*, \tag{A2}
\]

\[
\langle K^{*-} (q, \epsilon) | \bar{s}\sigma_{\mu\nu} u | 0 \rangle = - \langle K^{0*} (q, \epsilon) | \bar{d}\sigma_{\mu\nu} s | 0 \rangle = - \langle K^{0*} (q, \epsilon) | \bar{s}\sigma_{\mu\nu} d | 0 \rangle = - \langle K^{0*} (q, \epsilon) | \bar{s}\sigma_{\mu\nu} d | 0 \rangle = - i f_{K^*T} (\epsilon_{\mu}^* q_\nu - \epsilon_{\nu}^* q_\mu) .
\]

For the tensor form factors, it is useful to remember $\sigma_{\mu\nu} \gamma_5 = - \frac{i}{2} \sigma_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}$ where $\epsilon_{0123} = +1$.

- **$B$ meson transition amplitudes:**

\[
\langle \pi^- | \bar{d} \Gamma b | B^- \rangle = \sqrt{2} \langle \pi^0 | \bar{d} \Gamma b | \bar{B}^0 \rangle ,
\]

\[
\langle K^- | \bar{s} \Gamma b | B^- \rangle = \langle K^0 | \bar{s} \Gamma b | \bar{B}^0 \rangle . \tag{A3}
\]
For the tensor form factors, we adopt the results of light cone sum rule calculation [19, 20].

\[ \langle P(p)|q\gamma_{\mu}(1 \pm \gamma_5)b|B(p_B)\rangle = \left( (p_B + p)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu \right) F_{1 B \to P}^B(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_{0 B \to P}^B(q^2), \] (A5)

\[ \langle V(p, \epsilon)|(V \pm A)_\mu|B(p_B)\rangle = \pm i\epsilon^*(m_B + m_V) A_1^V(q^2) \]
\[ \pm i(p_B + p)_\mu (\epsilon^* \cdot q) \frac{2m_V}{q^2} \left[ A_3^V(q^2) - A_0^V(q^2) \right] \]
\[ + \epsilon_{\mu\alpha\beta} \epsilon^{*\nu} q^\alpha p^\beta \frac{2V(q^2)}{m_B + m_V}, \] (A6)

where \( q = p_B - p \). Here we have the relation;

\[ A_3^V(q^2) = \frac{m_B + m_V}{2m_V} A_1^V(q^2) - \frac{m_B - m_V}{2m_V} A_2^V(q^2). \]

For the tensor form factors, we adopt the results of light cone sum rule calculation [19, 20], given by

\[ f_{K^*}^T = 185 \text{ MeV}, \]
\[ F_{2B \to \pi}^B = 0.296, \quad F_{2B \to K}^B = 0.374, \]
\[ T_{1B \to K^*}^B = T_{2B \to K^*}^B = 0.379, \quad T_{3B \to K^*}^B = 0.260. \] (A9)
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TABLE I: The amplitude ratios of the wrong-sign (WS) and the right-sign standard model (SM) processes; $w = A_{WS}/A_{SM}$ defined in Eqs. (29) and (31). The numerical values are obtained with each non-vanishing new physics coefficient $C^{(t)dd}_I$ or $S^{(t)dd}_I$ shown in the first row, which is normalized as $C^{(t)dd} = |V_{tb}V_{ts}^{*}(a_4-a_{10}/2)| = 1.54 \times 10^{-3}$ or $C^{(t)ss} = |V_{tb}V_{td}^{*}(a_4-a_{10}/2)| = 2.91 \times 10^{-4}$. In the last two lines, $|w|$ is shown to denote the ratio of $\sqrt{\Gamma_{WS}/\Gamma_{SM}}$ for each decay modes. Here $\xi \equiv 1/N_c$ and $N_c = 3$ is taken.