Abstract

A “fiducial” kinematical region for our calculations of instanton (I)-induced processes at HERA within I-perturbation theory is extracted from recent lattice simulations of QCD. Moreover, we present the finalized I-induced cross-sections exhibiting a strongly reduced residual dependence on the renormalization scale.

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1 Introduction

In this contribution, we briefly summarize some recent progress in our ongoing systematic study [1, 2, 3, 4, 5, 6, 7] of the discovery potential of DIS events induced by QCD instantons.

Instantons [8] are non-perturbative gauge field fluctuations. They describe tunnelling transitions between degenerate ground states (vacua) of different topology in non-abelian gauge theories like QCD. Correspondingly, (anti-)instantons carry an integer topological charge $|Q| = 1$, while the usual perturbation theory resides in the sector $Q = 0$. Unlike the latter, instantons violate chirality ($Q_5$) in (massless) QCD and the sum of baryon plus lepton number ($B + L$) in QFD, in accord [9] with the general ABJ-anomaly relation. An experimental discovery of instanton ($I$)-induced events would clearly be of basic significance.

The deep-inelastic regime is distinguished by the fact that here hard instanton-induced processes may both be calculated [10, 5] within instanton-perturbation theory and possibly detected experimentally [1, 2, 3, 7]. As a key feature we have recently shown [5], that in DIS the generic hard scale $Q$ cuts off instantons with large size $\rho \gg Q^{-1}$, over which we have no control theoretically.

There has been much recent activity in the lattice community to “measure” topological fluctuations in lattice simulations [11, 12] of QCD. Being independent of perturbation theory, such simulations provide “snapshots” of the QCD vacuum including all possible non-perturbative features like instantons. They also provide crucial support for important prerequisites of our calculations in DIS, like the validity of $I$-perturbation theory and the dilute $I$-gas approximation for small instantons of size $\rho \lesssim Q^{-1}$. As one main point of this paper (Sect. 2), these lattice constraints will be exploited and translated into a “fiducial” kinematical region for our predictions of the instanton-induced DIS cross-section based on $I$-perturbation theory. In Sect. 3 we display the finalized calculations of the various instanton-induced cross-sections [6]. The essential new aspect here is the strong reduction of the residual dependence on the renormalization scale $\mu_r$ resulting from a recalculation based on improved instanton densities [13], which are renormalization group (RG) invariant at the 2-loop level.
Figure 1: The leading instanton-induced process in the DIS regime of $e^\pm P$ scattering, violating chirality by $|\Delta Q_5| = 2n_f$.

2 Validity of Instanton Perturbation Theory in DIS

- Restrictions from Lattice-QCD Simulations

The leading instanton (I)-induced process in the DIS regime of $e^\pm P$ scattering is displayed in Fig. 1. The non-trivial topology of instantons is reflected in a violation of chirality by $|\Delta Q_5| = 2n_f$, in accord \[9\] with the general ABJ-anomaly relation (while in pQCD always $\Delta Q_5 \equiv 0$). The dashed box emphasizes the so-called instanton-subprocess with its own Bjorken variables,

$$Q'^2 = -q'^2 > 0; \quad 0 \leq x' = \frac{Q'^2}{2p \cdot q'} \leq 1. \tag{1}$$

The cross-section of interest may be shown \[1, 3, 6\] to exhibit a convolution-type structure, consisting of a smooth, calculable “flux factor” $P^{(I)}(x', \ldots)$ from the the $\gamma^* q\bar{q} t$ vertex, and the $I$-subprocess total cross-section $\sigma^{(I)}_{q\bar{g}}(Q', x')$, containing the essential instanton dynamics. We have evaluated the latter \[6\] by means of the optical theorem and the so-called $IT$-valley approximation \[14\] for the relevant $q'g \Rightarrow q'g$ forward elastic scattering amplitude.
in the $\mathcal{IT}$ background. This method resums the exponentiating final state gluons in form of the known valley action $S^{\mathcal{IT}}$ and reproduces standard $I$-perturbation theory at larger $\mathcal{IT}$ separation $\sqrt{R^2}$.

Corresponding to the symmetries of the theory, the instanton calculus introduces at the classical level certain (undetermined) “collective coordinates” like the $I$ ($\mathcal{T}$)-size parameters $\rho$ ($\overline{\rho}$) and the $\mathcal{IT}$ distance $\sqrt{R^2/\rho \overline{\rho}}$ (in units of the size). Observables like $\sigma^{(I)}_{q'q}(Q',x')$, must be independent thereof and thus involve integrations over all collective coordinates. Hence, denoting the density of $I$ ($\mathcal{T}$)’s by $D(\rho \overline{\rho})$ (see Eq. (3)), we have generically,

$$
\sigma^{(I)}_{q'q}(Q',x') = \int_0^\infty d\rho \int_0^\infty d\overline{\rho} \frac{D(\rho)D(\overline{\rho})}{\mathcal{IT} \text{-densities=Lattice!}} e^{-(\rho+\overline{\rho})Q'} \times \int_0^\infty d\xi M(\xi,x',Q',\ldots) e^{-\frac{4\pi}{\alpha_s}(S_{\mathcal{IT}}(\xi)-1)},
$$

where $\xi = R^2/\rho \overline{\rho} + \rho/\overline{\rho} + \rho/\rho$ is a convenient conformally invariant variable characterizing the $\mathcal{IT}$ distance. In Eq. (3), the crucial exponential cut-off $e^{-(\rho+\overline{\rho})Q'}$ is responsible for the finiteness of the $\rho, \overline{\rho}$ integrations. In addition, it causes the integrals (2) to be dominated by a single, calculable (saddle) point $(\rho^* = \overline{\rho}^* \sim Q'^{-1}, \xi^*(x',Q'))$, in one-to-one relation to the conjugate momentum variables $(x',Q')$. This effective one-to-one mapping of the conjugate $I$-variables allows for the following important strategy: We may determine quantitatively the range of validity of $I$-perturbation theory and the dilute $I$-gas approximation in the instanton collective coordinates $(\rho < \rho_{\text{max}}, R/\rho > (R/\rho)_{\min})$ from recent (non-perturbative) lattice simulations of QCD and translate the resulting constraints via the mentioned one-to-one relations into a “fiducial” kinematical region $(x' > x'_{\min}, Q' > Q'_{\min})$ at HERA! Experimentally, these cuts must be implemented via a $(x', Q')$ reconstruction from the final state topology [5], while theoretically, they are incorporated into our $I$-event generator [2] “QCDINS 1.6.0” and the resulting prediction of $\sigma^{(I)}_{\text{HERA}}(x' > x'_{\min}, Q' > Q'_{\min})$ (see Sect. 3).

In lattice simulations 4d-Euclidean space-time is made discrete, e.g. in case of the “data” from the UKQCD collaboration [12] which we shall use here,

- lattice spacing: $a = 0.055 - 0.1$ fm
- lattice volume: $V = l_{\text{space}}^3 \cdot l_{\text{time}} = [16^3 \cdot 48 - 32^3 \cdot 64] a^4$

In principle, such a lattice allows to study the properties of an ensemble of (anti-)instantons with sizes $a < \rho < V^{1/4}$. However, in order to make
instanton effects visible, a certain “cooling” procedure has to be applied first. It is designed to filter out (dominating) fluctuations of short wavelength $O(a)$, while affecting the instanton fluctuations of much longer wavelength $\rho >> a$ comparatively little. For a discussion of lattice-specific caveats, like possible lattice artefacts and the dependence of results on “cooling” etc., see Refs. [11, 12].

The first important quantity of interest, entering Eq. (2), is the $I$-density, $D(\rho)$ (tunnelling probability!). It has been worked out a long time ago in the framework of $I$-perturbation theory: (renormalization scale $\mu_r$)

$$D(\rho) \equiv \frac{dn}{d^4xd\rho} = d\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)\rho^b \rho^5. \tag{3}$$

Note the power law in the instanton size $\rho$ with the power $b$ given in Table 1, in terms of the QCD $\beta$-function coefficients: $\beta_0 = 11 - \frac{2}{3}n_f$; $\beta_1 = 102 - \frac{38}{3}n_f$.

Table 1: The power $b$ in Eq. (3) from Ref. [9] and Ref. [13], making the $I$-density $D(\rho)$ RG-group invariant at the 1-loop and 2-loop level, respectively.

| $b$                      | $\frac{1}{b} \frac{dD}{d\mu_r}$ | Ref.          |
|-------------------------|----------------------------------|---------------|
| $\beta_0$               | $O(\alpha_s)$                   | ’t Hooft [1]  |
| $\beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}(\beta_1 - 12\beta_0)$ | $O(\alpha_s^2)$ | Morris, Ross, & Sachrajda [13] |

This power law $D(\rho)|_{n_f=0} \propto \rho^6$ of $I$-perturbation theory is confronted in Fig. 2 (top) with recent lattice “data”, which strongly suggests $I$-perturbation theory to be valid for $\rho \lesssim \rho_{\text{max}} = 0.3$ fm. Next, consider the square of the total topological charge, $Q^2 = (n \cdot (+1) + \bar{n} \cdot (-1))^2$ along with the total number of charges $N_{\text{tot}} = n + \bar{n}$. For a dilute gas, the number fluctuations are poissonian and correlations among the $n$ and $\bar{n}$ distributions absent, implying $\langle (n - \bar{n})^2 \rangle = \langle n + \bar{n} \rangle$, or $\langle Q^2_{\text{tot}} \rangle = 1$. From Fig. 2 (bottom), it is apparent that this relation characterizing the validity of the dilute $I$-gas approximation, is well satisfied for sufficiently small instantons! Again, we find $\rho_{\text{max}} \simeq 0.3$ fm, quite independent of the number of cooling sweeps. For increasing $\rho_{\text{max}} \gtrsim 0.3$ fm, the ratio $\langle Q^2_{\text{tot}} \rangle$ rapidly and strongly deviates from one. Crucial information about a second instanton parameter of interest, the average $IT$ distance $\langle R \rangle$, may be obtained as well from the lattice [11, 12]. Actually, the ratio $\left[\frac{\langle R \rangle}{\rho}\right] \simeq 0.83$ has good stability against “cooling”, from
Figure 2: Support for the validity of $I$-perturbation theory for the $I$-density $D(\rho)$ (top) and the dilute $I$-gas approximation (bottom) for $\rho \leq \rho_{\text{max}} \simeq 0.3$ fm from recent lattice data [12].
which we shall take $R/\rho \gtrsim 1$ as a reasonable lower limit for our $I$-perturbative DIS calculations.

Finally, the “fiducial” kinematical region for our cross-section predictions in DIS is found from lattice constraints and the discussed saddle-point translation as

$$\rho / R \gtrsim 0.3 \text{ fm}; \quad \frac{R}{\rho} \gtrsim 1 \quad \Rightarrow \quad \left\{ \begin{array}{l}
Q'/\Lambda \\
50.0 \quad \text{1-loop} \\
70.0 \quad \text{2-loop}
\end{array} \right\} \Rightarrow \left\{ \begin{array}{l}
Q' > Q'_{\text{min}} \simeq 8 \text{ GeV} \\
x' > x'_{\text{min}}(Q'_{\text{min}}) \simeq 0.35.
\end{array} \right.$$ 

\hspace{1cm} (4)

\section{3 $I$-Induced Cross-Sections for HERA}

We have achieved great progress in stability by using the 2-loop RG invariant form of the $I$-density $D(\rho)$ from Eq. (3) and Table 1 in a recalculation of the $I$-subprocess cross-sections [6]: The residual dependence on the renormalization scale $\mu_r$ turns out to be strongly reduced (Fig. 3). As “best” scheme we use $\mu_r = 0.15 \, Q'$ throughout, for which $\partial \sigma^{(I)}_{qg}/\partial \mu_r \simeq 0$. The quantitative calculations of $\sigma^{(I)}_{qg}$ (Fig. 4) nicely illustrate the qualitative arguments from Sect. 2, that the $Q'$ dependence probes the effective instanton size $\rho$ (top),
while the $x'$ dependence maps the $\IT$ distance $R$ in units of the $I$-size $\rho$ (bottom).

Figure 4: Instanton-induced cross-sections

Fig. 5 displays the finalized $I$-induced cross-section at HERA, as function of the cuts $x_{\text{min}}'$ and $Q_{\text{min}}'$ in leading semi-classical approximation, as obtained with the new release “QCDINS 1.6.0” of our $I$-event generator. For the minimal cuts (4) extracted from lattice simulations, we specifically obtain

$$\sigma_{\text{HERA}}^{(I)}(x' \geq 0.35, Q' \geq 8 \text{ GeV}) \approx 126 \text{ pb}; \ x_{\text{Bj}} \geq 10^{-3}; \ 0.9 \geq y_{\text{Bj}} \geq 0.1.$$

(5)
In view of the fact that the cross-section varies strongly as a function of the $(x', Q')$ cuts, the constraints from lattice simulations are extremely valuable for making concrete predictions.

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