Heavy Quark Interactions and Quarkonium Binding

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Abstract

We consider heavy quark interactions in quenched and unquenched lattice QCD. In a region just above the deconfinement point, non-Abelian gluon polarization leads to a strong increase in the binding. Comparing quark-antiquark and quark-quark interaction, the dependence of the binding on the separation distance \( r \) is found to be the same for the colorless singlet \( \bar{Q}Q \) and the colored anti-triplet \( QQ \) state. In a potential model description of in-medium \( J/\psi \) behavior, this enhancement of the binding leads to a survival up to temperatures of \( 1.5 T_c \) or higher; it could also result in \( J/\psi \) flow.

1 Introduction

The interaction of a heavy quark-antiquark (\( Q\bar{Q} \)) pair in strongly interacting matter has been studied in finite temperature lattice QCD in the quenched approximation as well as for the cases of two light and two light plus one heavy quark species [1–4]. In all these studies, one obtains the difference \( F(r, T) \) between the color singlet free energy with and without the heavy quark pair, as function of the temperature \( T \) of the medium and the separation distance \( r \) of \( Q \) and \( \bar{Q} \). Schematically, we can write

\[
F_0(T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_0/T\}
\]

and

\[
F_Q(r, T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_Q(r)/T\}
\]

for the free energies in question; here the Hamiltonian \( \mathcal{H}_0 \) describes the interacting quark-gluon plasma alone, and \( \mathcal{H}_Q(r) \) that of the plasma containing the static \( Q\bar{Q} \) pair; the integral \( \int d\Gamma \) denotes the grand canonical phase space integration and summation. The color singlet free energy difference provided by lattice studies is then defined as

\[
F(r, T) = F_Q(r, T) - F_0(T). \tag{3}
\]

In Fig. 1, we show the \( r \)-dependence of \( F(r, T) \) at different temperatures above the deconfinement point, i.e., for \( T > T_c \). The vacuum form (for \( T = 0 \)) also shown here is the Cornell potential [5]

\[
V(r) = \sigma r - \frac{\alpha}{r}, \tag{4}
\]

defined in terms of the string tension \( \sigma \) and the Coulomb coupling \( \alpha \approx \pi/12 \). For very small \( r \ll T^{-1} \), the small color singlet pair is neither seen nor affected by the medium, so that the interaction is specified by the \( T \)-independent running coupling \( \alpha\) \((r)\), which for larger \( r \) saturates at the canonical string value \( \pi/12 \). On the other hand, at very high temperatures and comparatively large \( r \gg T^{-1} \), in the perturbative limit, we expect an \( r \)-independent running coupling \( \alpha(T) \).

In full QCD below \( T_c \), the presence of light quarks (\( q \)) leads to \( q\bar{q} \) pair production and string breaking; from quarkonium studies, the string breaking energy at \( T = 0 \) is estimated to be about 1.0 - 1.2 GeV.

In the quenched case below \( T_c \), the free energy diverges in the large distance limit, with a temperature-dependent string tension \( \sigma(T) \) which vanishes at \( T = T_c \). Temperatures above \( T_c \), however, as seen in Fig. 1, also lead to a finite large-distance limit, with a temperature-dependence which is very similar to that found in full QCD. Since here there are no light \( q\bar{q} \) pairs to provide string-breaking, the large...
F_{1}(R,T)/\sigma^{1/2}

Figure 1: Free energy difference for a color-singlet \( Q\bar{Q} \) pair as function of \( r \), (a) for quenched and (b) for two-flavor QCD [1].

The internal energy behavior is shown in Fig. 2 [3, 4]. From eqs. (1), (3) and (5), we obtain

\[
U(r,T) = \langle H_{Q}(r,T) \rangle - \langle H_{0}(T) \rangle,
\]

which for a static \( Q\bar{Q} \) pair, with no kinetic energy, measures the change in potential energy due to the introduction of the pair. It is seen in Fig. 2a that just above the deconfinement point, the potential is much stronger than at \( T = 0 \). In the following, we want to study the origin of this increase.

Consider first the underlying medium, a plasma of unbound (but interacting) gluons and (for full QCD) quarks at temperature \( T \). We had seen that gluon screening plays a decisive role in the medium, and when we speak of constituents, this should be kept in mind. Any interaction is mediated by gluons, and when the gluonic interaction range is reduced through color screening, this holds for quark interactions as well. We denote by \( n(T) \) the average density of constituents, so that \( d = n^{-1/3} \) is the average separation distance between adjacent color charges. The interaction range for the constituents is given by the correlation function,

\[
f(r) \sim \exp\{-r/\xi(T)\},
\]

which measures the interaction strength between constituents separated by a spatial distance \( r \). The correlation length \( \xi(T) \) is a basic property of the medium, and it will diverge for \( T \to T_c \) in case of a second order confinement/deconfinement transition. In the high temperature limit, we expect a non-interacting gas and hence \( \xi \to 0 \).

We now add a static \( Q\bar{Q} \) pair to our system. As long as the \( Q \) and the \( \bar{Q} \) are sufficiently far apart, their color charges are effectively screened and asymptotically, they do not interact with each other. Nevertheless, the \( Q \) as well as the \( \bar{Q} \) individually interact with the medium, and around each charge, this will lead to polarization effects in the region where its presence is felt. Hence in a sphere of average
radius $\xi(T)$ around the $Q$ (and similarly around the $\bar{Q}$), there will be an excess of color compared to the state of the medium before insertion of the static charges. The polarization screens each static charge, so that its strength is reduced the further away we measure it. This evidently is a non-Abelian effect, since an electric charge cannot be screened by photons, and in quenched QED the $Q\bar{Q}$ interaction would remain purely Coulombic even in the large distance limit. When the $Q\bar{Q}$ separation distance $r$ is reduced, eventually interaction between the charges becomes significant. This interaction initially has a Debye-screened Coulomb form, with $\alpha_{\text{eff}} \sim \alpha \exp\{-\mu r\}$ as the effective coupling, where $\mu$ denotes the screening mass. It has to be emphasized that such a picture makes sense only for $r \gg \xi$, so that we really have two well separated polarization spheres, each of which contains a static charge surrounded by many medium charges, with an overall excess of the opposite charge.

When the heavy quarks are brought still closer to each other, the polarization spheres begin to overlap [4]. The overlapping spheres continue to attract each other, but this interaction now has two components and can no longer be described by a pure Debye-screened potential. On one hand, there is the direct interaction between the $Q$ and the $\bar{Q}$, which in the limit of small $r$ becomes just the Coulombic vacuum form $\alpha/r$. On the other hand, the charged constituents inside the overlapping polarization spheres also attract each other directly, with a strength determined by the deviation of the region from the state without a $Q\bar{Q}$ pair. Once the spheres no longer overlap, this interaction becomes by Gauss’ law just the Debye-screened Coulomb form with a reduced effective charge.

In order to determine the behavior of $Q\bar{Q}$ binding in a deconfined medium, we shall first consider the large and small distance limits of the thermodynamics potentials.

## 2 The Large Distance Limit

In the limit of large $Q\bar{Q}$ separation, for $r \to \infty$, we have two fully screened and hence non-interacting charges. Nevertheless, the thermodynamic potential differences $F, U$ and $S$ do not vanish: they specify the effect of the interaction of each of the two independent charges with the medium. This is not related to the interaction of $Q$ and $\bar{Q}$, as is easily seen by considering the corresponding quantities for a pair of heavy quarks (a “diquark”). Lattice calculations [6, 7] have in fact shown that for $r \to \infty$ one obtains

$$F_{\text{singlet}}^{(1)}(T) = F_{\text{anti-triplet}}^{(3)}(T) = 2F_3(T),$$

where $F^{(1)}$ corresponds to the color singlet state of the $Q\bar{Q}$ and $F^{(3)}$ to the anti-triplet state of the diquark. Corresponding relations hold for $U(T)$ and $S(T)$. Hence $F_3(T)$ as defined above indeed specifies

![Figure 2: Internal energy difference for a color-singlet $Q\bar{Q}$ pair in two-flavor QCD, (a) as function of $r$ and (b) for $r \to \infty$, compared to the corresponding free energy (solid line) [4].](image-url)
the free energy associated to a single static (triplet or anti-triplet) color charge in a medium of temperature \( T \); \( U_3(T) \) and \( S_3(T) \) denote the corresponding internal energy and entropy. The relation
\[
U_3(T) = F_3(T) + T S_3(T)
\]  
(10)
can then be interpreted as the sum of the energy shift \( F_3(T) \) due to the interaction of the static \( Q \) with the constituents of the medium, and the energy shift \( T S_3(T) \) due to the interaction between the constituents of the cloud, with \( S_3(T) \) specifying their change in number and \( T \) their energy. It is clear that \( U_3(T) \) is not due to any interaction between the static \( Q \) and its partner \( \bar{Q} \), and it will thus not affect the binding of the heavy quark pair.

3 The Short Distance Limit

For the \( QQ \) system, we now have to distinguish between the attractive color singlet and the repulsive color octet state. In the case of the singlet, when \( r \) become sufficiently small compared to the average separation of charges in the medium, the \( QQ \) pair constitutes for the medium a color-neutral entity and is not “seen” by its constituents. Hence for \( r \ll T^{-1} \), the differences \( F^{(1)}_{QQ}(r, T) \) and \( U^{(1)}_{QQ}(r, T) \) measured in lattice studies are simply due to the perturbative interaction of the two static charges, with no medium effects. In the short distance regime we thus have the temperature-independent form
\[
F^{(1)}_{QQ}(r, T) = U^{(1)}_{QQ}(r, T) = -\frac{4}{3} \frac{\alpha(r)}{r},
\]  
(11)
where \( \alpha(r) \) is the \( r \)-dependent running coupling and \( 4/3 \) is the \( SU(3) \) color Casimir coefficient for \( 3 \times \bar{3} \rightarrow 1 \).

For the attractive anti-triplet case of the diquark state, the perturbative form of the direct \( QQ \) interaction gives \( 1/2 \) that of the singlet \( QQ \), as determined by color \( SU(3) \) Casimir coefficient \( 3 \times 3 \rightarrow 3 \). Now, however, the small overall system is still colored and hence is seen by the medium as a single point-like color charge. It will thus again lead to a polarization cloud, which for a triplet or anti-triplet just is \( F_3(T) \). As a result, we have in the short distance regime the free energy [6]
\[
F^{(3)}_{QQ}(r, T) \simeq -\frac{2}{3} \frac{\alpha(r)}{r} + F_3(T).
\]  
(12)
Comparing eqs. (11) and (12), we expect that
\[
F^{(1)}_{QQ}(r, T) \simeq 2[F^{(3)}_{QQ}(r, T) - F_3(T)]
\]  
(13)
should be satisfied in the small distance limit. Using eq. (9) we see that it also holds at large distance, and as seen in Fig. 3 it does so for \( T > T_c \) even at intermediate separation distances.

![Figure 3: Free energy of a singlet \( Q\bar{Q} \) and of an anti-triplet \( QQ \) [6], see eq. (13) [6].](image-url)
We thus find a general pattern of behavior for the attractive case (colorless singlet $Q\bar{Q}$ and colored antitriplet $QQ$). The only caveat is that while the singlet $Q\bar{Q}$ becomes simply Coulomb-like at short distance, for the attractive $QQ$ antitriplet state a polarization cloud remains even in the short distance limit. Once this is taken into account, the remaining interaction appears to be quite insensitive to whether we consider a $QQ$ or a $Q\bar{Q}$ system.

4 The Intermediate Separation Regime

To study the behavior at finite $r$, we consider the effective couplings $\alpha_F$ and $\alpha_U$ for the singlet $Q\bar{Q}$ state, with

$$\alpha_F(r, T) = \frac{3}{4} r^2 \left( \frac{\partial F(r, T)}{\partial r} \right),$$

(14)

and a corresponding relation for $U(r, T)$ in place of $F(r, T)$. For sufficiently small $r$, there are no medium effects and hence $\alpha_U = \alpha_F$. Since $S(r, T)$ becomes constant for large $r$, they also become equal in that limit. The behavior of $\alpha_F(r, T)$ for the singlet $Q\bar{Q}$ system, as found in lattice QCD, is illustrated in Fig. 4a for a range of temperatures $T > T_c$ [8]. It is seen that $\alpha_F(r, T)$ follows the vacuum form [4] up to the point where Debye screening begins to set in and eventually makes it vanish. In the range of $r$ values of interest here, this behavior disagrees strongly with a Yukawa-like Debye-screened form $F_1(r, T) \sim \alpha(T) r^{-1} \exp\{-r/r_D(T)\}$, with $r_D(T)$ as color-screening radius; the actual form has been studied in detail in [9]. As a result, models based on a Yukawa form [10] cannot correctly reproduce the free energy data in the crucial range of $r$ and $T$; either $\alpha$, or $m_D$, or both must depend on $r$ as well as on $T$. In Fig. 4b, we also show the corresponding behavior of $\alpha_U(r, T)$. As argued above, the two couplings agree at fixed $T$ in the small as well as in the large distance limits. In the intermediate $r$-range, the internal energy coupling considerably exceeds the vacuum Cornell form as well as that obtained from the free energy. The reason for this excess is that the internal energy difference contains not only the $Q\bar{Q}$ interaction, but also the effect of the interaction of the charges in the polarization clouds. With increasing temperature, the correlation length and hence the size of the polarization clouds decreases, so that the difference between the two couplings also decreases.

![Figure 4: Effective couplings from (a) free and (b) internal energy as function of $r$ [8].](image)

The difference between the values of $U(r, T)$ and $F(r, T)$ obtained in lattice studies thus becomes clear. The internal energy measures the binding potential between the two heavy quarks, that between each quark and the constituents of its polarization cloud, and that between the constituents of the overlapping polarization clouds. The free energy, on the other hand, measures only the potential between the “bare” heavy quarks, modified by the screening effects due to the polarization clouds. In the large distance limit, the heavy quark potential vanishes, so that $F$ approaches with increasing temperature the entropy change $-TS(T); since the entropy change becomes very weakly (logarithmic) temperature-dependent for
large $T$, this change grows almost linearly with $T$. For $r \to \infty$, $U(r,T)$ approaches the twice interaction energy between a heavy quark and its cloud constituents; if the correlation length and hence the cloud size vanish in this limit, $U(\infty, T)$ is also expected to vanish. In the short distance limit, the $Q\bar{Q}$ screens itself, so that both $F$ and $U$ measure only the direct $Q\bar{Q}$ interaction.

Stated in other words, the attractive interaction between two heavy quarks inside a deconfined medium has two distinct components: the direct Coulombic $Q\bar{Q}$ interaction, and the non-Abelian interaction between the constituents of the polarization clouds which the static quarks acquire in the medium. At large distances, this “dressing” simply reduces the effective charge through screening. But at intermediate distances, when the dressings overlap, they provide a strong additional attractive interaction. To corroborate this quantitatively, one would need to show that the color-averaged gluon-gluon interaction is attractive. For static quark sources of different quantum number, including adjoint octet quarks, such an attraction has been shown [7]. For gluons, it is not evident how similar studies could be carried out.

It should be emphasized that the non-perturbative interaction observed in the intermediate $r$ regime occurs (apart from the differences in the short distance limit) for a colored anti-triplet $QQ$ state in just the same functional form as it does for the colorless $Q\bar{Q}$ singlet.

### 5 Potential Model Studies

These considerations can throw some light on the problem of extracting from lattice results the temperature-dependent $Q\bar{Q}$ binding potential $V(r,T)$, to be used in a two-body Schrödinger equation,

$$\left\{2m_c - \frac{1}{m_c}\nabla^2 + V(r,T)\right\} \Phi_i(r) = M_i \Phi_i(r),$$

(15)

in order to study charmonium dissociation. Here $m_c$ denotes the charm quark mass, $M_i$ that of charmion state $i$. First studies had used the color-averaged free energy for $V(r,T)$ [13], but subsequently lattice results for the color-singlet free energy became available, leading to somewhat stronger binding. Eventually it was argued that the correct potential is given by the internal energy $U(r,T)$. The proposals of the last years now cover the whole range, with the general form $xU(r,T) + (1 - x)F(r,T)$, where $0 < x < 1$ [13–17]. As is evident from what was said here, the effective binding increases with $x$. The internal energy $U(r,T)$, as the expectation value of the difference of the Hamiltonians with and without the pair, includes the indirect “cloud-cloud” binding in addition to the direct $Q\bar{Q}$ interaction and hence provides a stronger binding. We believe that the indirect binding cannot be neglected, so that the correct form of the potential should indeed be $U(r,T)$.

To clarify the situation further, let us consider the semi-classical limit of eq. (15) applied to the case of $J/\psi$ dissociation,

$$2m_c + \frac{p^2}{m_c} + U(r,T) = 2m_c + \frac{c}{m_c r^2} + U(r,T) = M_{J/\psi}(r,T),$$

(16)

where the minimum of the energy

$$E(r,T) = \frac{c}{m_c r^2} + U(r,T)$$

(17)

as function of $r$ determines the ground state mass $M_{J/\psi}$ of the $J/\psi$. The constant $c$ arises from the uncertainty relation $p^2 r^2 \simeq c$ and depends on the form of the binding potential. We determine it by requiring the correct $J/\psi$ mass at $T = 0$. From

$$\frac{\partial E(r,T)}{\partial r} = 0$$

(18)

we obtain

$$c = \frac{m_r^3}{2} \left[ \sigma + \frac{\alpha}{r^2} \right]$$

(19)
which leads to
\[ M_{J/\psi}(r, T = 0) = 2m_c + \frac{3}{2} \sigma r - \frac{\alpha}{2r} = 3.1 \text{ GeV}. \] (20)

With \( m_c = 1.3 \text{ GeV}, \sigma = 0.2 \text{ GeV}^2, \) and \( \alpha = \pi/12, \) this results in \( c = 1.56 \) and for the radius of the \( J/\psi \) the value \( r(T = 0) \approx 0.42 \text{ fm}, \) which agrees very well with that obtained in the corresponding quantum-mechanical study [18]. The minimization requirement [18] for finite \( T \) leads to
\[ \frac{2c}{m_c r} = r^2 \frac{\partial U}{\partial r} = \frac{4}{3} \alpha U(r, T), \] (21)

where \( \alpha U(r, T) \) is the effective coupling determined above (see Fig. 4). In Fig. 5 we solve this equation graphically. It is seen that up to some temperature value \( T_{\text{dis}} \approx 1.5 T_c, \) \( \alpha U(r, T) \) attains a peak large enough to intersect the kinetic term \( c/m_c r, \) so that there is a minimum in the energy and hence a bound state. For \( T > T_{\text{dis}}, \) this is no longer the case, the energy decreases monotonically with \( r \) and there is no more bound state. The radius of the surviving \( J/\psi \) is seen to vary very little with temperature; it remains in the range \( 0.3 - 0.45 \text{ fm} \) up to \( T_{\text{diss}}, \) where the bound state disappears.

![Figure 5: Semi-classical picture of \( J/\psi \) binding and dissociation ranges](image)

The potential \( U(r, T) \) used in the Schrödinger equation has the correct \( T \)-independent form for small \( r, \) and enhanced attraction in the intermediate \( r \) range up to \( T_{\text{diss}}. \) At large \( r, \) it leads to the \( r \)-independent value \( 2U_3(T), \) which, as we saw above, has little effect on the strength of the binding. Its value does, however, specify the binding energy as the gap between bound and free heavy quarks. With
\[ \Delta E(T) = 2[m_c + U_3(T)] - M_{J/\psi}(T) = 2U_3(T) - \frac{c}{mr_0^2} - U(r_0, T) \] (22)
as binding energy, with \( r_0 \) determined by the minimization condition (21), we obtain at the dissociation point a value of \( \Delta E(T_{\text{diss}}) \approx 0.2 \text{ GeV}. \)

For \( T < T_{\text{dis}}, \) the kinetic curve in Fig. 5 crosses the potential curve at two \( r \) values: the first crossing specifies the minimum of the energy and hence the \( J/\psi \) bound state radius, while the difference between the second and the first provides a measure of the thickness \( \Delta r \) of the potential wall. As \( \Delta r \rightarrow 0, \) quantum effects (tunneling) will become more and more likely and thus provide a quantum-mechanical possibility of the dissociation even below \( T_{\text{diss}}, \) as well as a finite binding probability even above \( T_{\text{diss}}. \)

6 Charmonium Flow

Our arguments suggest that charmonium binding in a hot quark-gluon plasma is to a large extent due to a binding between the clouds of the medium surrounding the heavy quarks. If this medium is experiencing an overall flow, the motion of the clouds will be transmitted to the \( Q\bar{Q} \) core and cause a drag leading to charmonium flow. Similarly, the isolated heavy quarks (for \( r \rightarrow \infty \)) will experience the drag of their
polarization clouds, and at hadronization lead to flow of open charm mesons. An observation of $J/\psi$ flow in heavy ion experiments should therefore not be interpreted as evidence for primary $J/\psi$ dissociation, followed by regeneration due to binding of charm constituents from different collisions. The cloud-cloud binding discussed here can transfer any medium motion also to a primary $Q\bar{Q}$ pair.

7 Outlook

Understanding the relation between lattice and potential studies of quarkonium binding is obviously an essential step in solving the in-medium behavior of quarkonia. What we have presented here is certainly not a final answer. It is only meant to recall

- that the heavy quark interaction in a quark-gluon plasma is not a simple two-component problem, but that on the other hand,
- this does not rule out a description in terms of a suitably formulated potential theory.

Recent analytical work [19–22] may provide a key to further developments in this direction.

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