Schwinger Model and String Percolation in Hadron–Hadron and Heavy Ion Collisions

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Abstract

In the framework of the Schwinger Model for percolating strings we establish a general relation between multiplicity and transverse momentum square distributions in hadron–hadron and heavy ion collisions. Some of our results agree with the Colour Glass Condensate model.

Experimental data from RHIC (Relativistic Heavy Ion Collider) show very interesting features concerning particle rapidity densities and transverse momentum, \( p_T \), distributions [1,2]. They exclude high particle densities, expected in naive multicollision models [3], as well as fast growing values for \( \langle p_T \rangle \) as a function of energy, expected in naive perturbative QCD models [4]. Physics seems to remain classical and, essentially, non-perturbative.

Multiparticle production is frequently described as resulting from multiple collisions at the parton level and, in the case of nucleus–nucleus collisions, also at nucleon level, with formation of colour strings stretched between the projectile and the target, which decay into other strings that subsequently hadronize into the observed hadrons [5]. There are
long strings in rapidity, valence strings, associated to valence quark (diquark) interactions, and short strings in rapidity, centrally produced (sea strings) associated to interactions of sea partons, mostly gluons. In a symmetrical AA collisions, with \( N_A \) participants from each nucleus, the number of valence strings equals the number of participants, as in the wounded nucleon model [6], while the number of sea strings behaves roughly as \( N_s \approx N_A^{4/3} \) [7], increasing with the energy.

We shall adopt for the mechanism of particle production the Schwinger model mechanism as developed in [8, 9]. In particular, the particle density and transverse momentum square will be considered proportional to the field (and the charge) carried by the string.

In multicollision models, many strings are produced, the number increasing with energy, atomic mass and centrality. If the strings are identical and independent, and approximately align with the collision axis, we have, for the rapidity particle density, \( dn/dy \), and for the average of the square of the transverse momentum, \( \langle p_T^2 \rangle \),

\[
\frac{dn}{dy} = N_s \bar{n}_1, 
\]

\[
\langle p_T^2 \rangle = \bar{p}_T^2, 
\]

where \( N_s \) is the number of strings, \( \bar{n}_1 \) is the single string particle density and \( \bar{p}_T^2 \) the average transverse momentum squared of the single string.

If the strings fuse in a rope [9], the colour randomly grows as \( \sqrt{N_s} \) and we have

\[
\frac{dn}{dy} = \frac{1}{\sqrt{N_s}} N_s \bar{n}_1, 
\]

\[
\langle p_T^2 \rangle = \bar{p}_T^2 \sqrt{N_s}. 
\]

In the situation of a hadron–hadron or nucleus–nucleus central collision, the strings overlap in the impact parameter plane and the problem becomes similar to a 2-dimensional continuum percolation problem [10]. If the strings are randomly distributed in the impact parameter plane then, in the thermodynamical approximation [11], the overlapping colour reducing factor is given by

\[
F(\eta) = \sqrt{1 - e^{-\eta}}, 
\]

where \( \eta \) is the transverse density percolation parameter,

\[
\eta \equiv \left( \frac{r_s}{R} \right)^2 N_s, 
\]

where \( \pi r_s^2 \) is the string transverse area and \( \pi R^2 \) the interaction transverse area. We thus have

\[
\frac{dn}{dy} = F(\eta) N_s \bar{n}_1, 
\]

\[
\langle p_T^2 \rangle = \frac{1}{F(\eta) \bar{p}_T^2}. 
\]
Equations similar to (7) and (8) were written in [11]. As with \( \eta \to 0 \) (low density limit) \( F(\eta) \to 0 \) and with \( \eta \to \infty \) (high density limit) \( F(\eta) \to 1/\sqrt{\eta} \), the behaviour of relations (1) and (2), and (3) and (4) is recovered from (7) and (8).

We shall now discuss the consequences of (7) and (8). Two straightforward results follow:

i) slow increase of particle density with energy and saturation of the normalised particle densities as \( N_s \) increases

As the number of strings, \( N_s \), increases with energy, at large energy \( \eta \) also increases and

\[
F(\eta) \approx \frac{1}{\sqrt{\eta}},
\]

which means, (7),

\[
\frac{dn}{dy} \approx \left( \frac{R}{r_s} \right) \sqrt{\frac{1}{N_s \bar{n}_1}}.
\]  

Instead of growing with \( N_s \), as one should have naively expected with independent strings, (1), the density grows more slowly, as \( N_s^{1/2} \).

On the other hand, as

\[
N_s \approx N_A^{4/3}, \quad R \approx R_1 N_A^{1/3},
\]  

where \( R_1 \) is a quantity of the order of the nucleon radius,

\[
\frac{1}{N_A} \frac{dn}{dy} \approx \left( \frac{R_1}{r_s} \right) \sqrt{\frac{1}{N_A \bar{n}_1}}
\]

tends to saturate as \( N_A \) increase. Both behaviours (7) and (12) were confirmed by data [1].

The saturation, in our framework, is a consequence of string percolation [12]. At the level of QCD it can be seen as resulting from low-\( x \) parton saturation in the colliding nuclei [13].

ii) a universal relation between \( \frac{dn}{dy} \) and \( \langle p_T \rangle \)

For large density, Eqs. (7) and (8) become

\[
\frac{dn}{dy} = \left( \frac{R}{r_s} \right) N_s^{1/2} \bar{n}_1,
\]  

\[
\langle p_T^2 \rangle = \left( \frac{r_s}{R} \right) N_s^{1/2} \bar{p}_1^2,
\]

and, eliminating \( N_s^{1/2} \):

\[
\sqrt{\langle p_T^2 \rangle} = c \sqrt{\frac{1}{N_A^{2/3}}} \frac{dn}{dy},
\]

with

\[
c \equiv \left( \frac{r_s}{R_1} \right) \left( \frac{\bar{p}_1}{\bar{n}_1} \right)^{1/2}.
\]
A relation of this type,

\[
\sqrt{\langle p_T^2 \rangle} \approx \sqrt{\frac{1}{N_A^{2/3}}} \frac{dn}{dy}
\]

was obtained, in the framework of the Colour Glass Condensate (CGC) model [14], in [15]. Our formula (14) includes not only the functional dependence, but, as well, the proportionality factor \(c\).

We can make an order of magnitude estimate of the proportionality factor \(c\). In the dual string model \(r_s \approx 0.2 \text{ fm} \) [10, 16], \(R_1\) should be of the order of the proton radius (\(\approx 1 \text{ fm}\)) and for the string charged particle production parameters one has \(\bar{p}_1 \approx 0.3\) and \(\bar{n}_1 \approx 0.7\), as observed from low energy data [17], and \((\bar{p}_1^2/\bar{n}_1)^{1/2} \approx 0.35\). The proportionality factor is then \(\approx 0.07\) to be compared with 0.0348 for pions and 0.100 for kaons [15].

In the comparison with data we shall identify \(\sqrt{\langle p_T^2 \rangle}\) with \(\langle p_T \rangle\) and \(\sqrt{p_T^2}\) with \(\bar{p}_1\) (this overestimates the average values of \(\langle p_T \rangle\) and \(\bar{p}_1\)).

We have just considered the high \(\eta\) limit. In the low density end, which means low energy and peripheral collisions, we have just valence strings and \(\langle p_T \rangle \rightarrow \bar{p}_1 \approx 0.3 \text{ GeV}\). This is, in practice, the value of \(\langle p_T \rangle\) in pp collisions at low \((\sqrt{s} \lesssim 10 \text{ GeV})\) energies.

By putting these two limits together, we arrive at the formula obtained in [15], but now with all the parameters theoretically constrained:

\[
\langle p_T \rangle = \bar{p}_1 \{1 + \frac{r_s}{R_1 \bar{n}_1^{1/2}} \sqrt{\frac{1}{N_A^{2/3}}} \frac{dn}{dy}\}.
\]

In Fig. 1 we compare Eq. (18) with data. The agreement is not perfect, but there is an indication that some truth exists in CGC and string percolation models.

In the next step we make an attempt to generalise our results and to relate the (normalised) transverse momentum distribution \(f(p_T^2)\) to the multiplicity distribution \(P(n)\), in hadron–hadron and nucleus–nucleus collisions.

We work in the large \(\eta\) limit and start by changing the notation, and write

\[
N = \alpha N_s^{1/2},
\]

with

\[
\alpha \equiv R/r_s,
\]

such that \(N\) has the meaning of the number of effective strings (mostly sea strings or ropes). If \(n\) particles are produced

\[
n = Nn_1,
\]

see also Eq. (13). This effective number \(N\) takes into account percolation effects in the sum of colours of the \(N_s\) individual strings.

Let \(P(N)\) be the probability of producing \(N\) effective identical strings and \(p(n_i)\) the probability of producing \(n_i\) particles from the \(i\)-th string. We then have

\[
P(n) = \int P(N) \prod_{i=1}^N p(n_i)dn_i \delta \left(n - \sum_{i=1}^N n_i\right) dN
\]

(22)
$\langle p_T \rangle$ vs multiplicity density in $p\bar{p}$ collisions (where $N_A = 1$) at 1800 GeV [18] (open circles) and in central Au+Au collisions at 200 AGeV [19] (filled squares). Solid lines represent Eq. (18) with $\bar{p}_1$ adjusted separately to each species.

In [22], as the colour percolation effects were absorbed in $N$, we treated the effective strings as independent (see [20]).

Regarding transverse momentum distributions, the natural generalisation for (14) is to write

$$p_T^2 = \frac{N}{\alpha^2} \bar{p}_1^2,$$

and for the distribution itself

$$F(p_T^2) = \int P(N) f(p_1^2) \delta \left( p_T^2 - \frac{N}{\alpha^2} \bar{p}_1^2 \right) d\bar{p}_1^2 dN.$$

In this case, for a given $F(p_T^2)$ contribute all effective strings with $f(p_1^2)$, such that $\bar{p}_1^2$ satisfies (23). As all strings are assumed equal, $f(p_1^2)$ is representative of any string.

In order to construct $P(n)$ and $F(p_T^2)$, one of course needs the elementary string distributions $p(n_1)$ and $f(p_1^2)$ and the distribution $P(N)$ of effective strings. Concerning the $p(n_1)$ distribution, it should be Poisson or close to Poisson type (as seen in $e^+e^-$ at low energy [21]). The $p_T^2$ distribution in the Schwinger model is an exponential in $-p_T^2$. The $P(N)$ distribution contains the nucleonic and the partonic structure of the colliding particles and the combinatorial factors of Glauber-Gribov calculus; its shape is investigated in [22].

Our objective here is not to solve Eqs. [22] and [24], but simply to try to relate $P(n)$ to $F(p_T^2)$. In view of that, let us proceed by calculating the $\langle n^q \rangle$ and $\langle p_T^{2q} \rangle$ moments of the distributions [22] and [24], respectively. The calculations are straightforward, but
lengthy in the case of multiplicities (see, for example, \( [20] \)). In this case, to simplify, we shall assume

\[
p(n_1) = \delta(n_1 - \bar{n}_1).
\]  

(25)

It has been shown, sometime ago, that this approximation in hadron–hadron and nucleus–nucleus collisions is very reasonable \([20]\).

We have then for the moments:

\[
\langle n^2 \rangle = \langle N^2 \rangle \bar{n}_1^q.
\]  

(26)

It is clear, because of (25), that all fluctuations come from fluctuations in the number of effective strings. For \( p_T \) distribution

\[
\langle p_T^{2q} \rangle = \frac{\langle N^q \rangle}{\alpha^q} \bar{p}_1^{2q}.
\]  

(27)

Eqs. (26) and (27) are the natural generalisation of (13) and (14). As before, the moments of the effective string distribution can be eliminated by dividing (27) by (26) and a relation between \( \langle n^q \rangle \) and \( \langle p_T^{2q} \rangle \) established. But one can now do better and eliminate the strongly model-dependent parameter \( \alpha = R/r_s \), eq. (20). If one writes the KNO moments

\[
C_q^n \equiv \frac{\langle X^q \rangle}{\langle X \rangle^q}, \quad q = 1, 2, \ldots,
\]  

(28)

the parameter \( \alpha \) disappears. By using a capital \( C \) for final distributions KNO moments and a small \( c \) for single string distributions KNO moments, our final result can be written as

\[
\frac{C_q^n}{c^n_q} = \frac{C_q^{p_T^{2q}}}{c^{p_T^{2q}}_q}.
\]  

(29)

This equation, as mentioned before, is strictly correct only for \( c^n_q = 1 \).

It is not easy to check Eq. (29) accurately, as most experiments can only measure \( p_T > \langle p_T \rangle \), but one can nonetheless attempt a somewhat rough comparison. In the Schwinger model the \( p_T \) distribution is Gaussian, which means \( c^{p_T^q} = q! \). If the final \( p_T \) distribution is also a Gaussian, then one obtains \( C_q^n = 1 \), which is not a good approximation. If the final \( p_T \) distribution is an exponential, which is closer to reality \([23]\), then \( C_q^{p_T^q} = (2q+1)!/(3!)^2 \), and we obtain, for instance, \( C_2^n = 5!/(3!)^2! \approx 1.66 \). This is to be compared with the experimental value \( C_2^n \approx 1.3 \) at \( \sqrt{s} = 200 \text{ GeV} \) \([24]\).

Finally, the main point we want to make with (29) is that multiplicity and transverse momentum distributions are deeply related: one should remember that, in general, from the \( C_q^x \) moments one can construct the distribution in KNO form, \( \langle x \rangle P(x/\langle x \rangle) \).

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