Baryon spectra and non-strange baryon strong decays in the chiral SU(3) quark model

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In the framework of the chiral SU(3) quark model, the baryon spectra within the band of $N \leq 2$ are studied, and the effect of the confining potential in different configurations, namely the $\Delta$-mode and $Y$-mode are compared. In the same way, the baryon spectra in the extended chiral SU(3) quark model, in which additional vector meson exchanges are introduced, are also calculated. It is shown that a reasonable baryon spectrum in the chiral SU(3) quark model can be achieved no matter whether the $\Delta$-mode or the $Y$-mode confining potential is employed. In the extended chiral SU(3) quark model, several energy levels are further improved. The resultant binding energies of excited baryon states in different confining modes deviate just by a few to several tens MeV, and it is hard to justify which confining mode is the dominant one. The non-strange baryon strong decay widths are further discussed in the point-like meson emission model by using the wave-function obtained in the spectrum calculation. The resultant widths can generally explain the experimental data but still cannot distinguish which confining mode is more important in this simple decay mode.

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I. INTRODUCTION

In the framework of the chiral SU(3) quark model \textsuperscript{2}, which is extrapolated from the SU(2) linear $\sigma$ model \textsuperscript{1}, a unified description of the experimental data on the masses of the baryon ground states, the binding energy of deuteron, and the baryon-baryon scattering has successfully been achieved \textsuperscript{2,3,4}. Later, this model has been applied to the study of multi-quark systems to predict new dibaryons and explain newly observed hadron states \textsuperscript{5,6}. Whether this model can also describe the baryon spectrum in a reasonable extent would be one more place to confirm the
reliability of the model in studying the hadron structure and the hadron-hadron scattering in the quark degrees of freedom.

In the chiral SU(3) quark model, the short range perturbative effect of QCD is generally characterized by the one-gluon-exchange (OGE) potential, the medium range non-perturbative effect of QCD is mainly described by one-Goldstone-bosons exchange (OBE) potentials, and the long distance non-perturbative effect is commonly depicted by a phenomenal confining potential, say a harmonic confinement potential.

In terms of the OGE quark model, we calculated the baryons spectra with different confining potential modes [7], and such potentials were derived from the flux tube model [8, 9] and later were confirmed by lattice QCD (LQCD) calculations [10]. For the baryon system, there exist two confining modes. In the first mode, called ∆-mode, the confining potential can approximately be described by a sum of two-body confining potentials. The second mode associates with a genuine three-body interaction, called Y-mode. In recent years, with the development of the fast computer, more and more LQCD calculations on three-quark potential have been carried out. Typical works on this aspect are done by Takahashi et al. [11] and Alexandrou et al. [14]. Takahashi et al. more accurately considered three quark Wilson loops and advocated that the Y-shape confining mode is the dominant confining mode in baryon. But, Alexandrou et al. believed that the ∆-mode is favored at least in the distances smaller than $1.2 \text{fm}$. Different from other works [12, 15, 16, 17, 18], we respectively calculated the contributions of these two confining potential modes directly, and found that by employing either the ∆-mode or Y-mode confining potential, one can achieve reasonable baryon spectra in OGE quark model [7]. In this paper we evaluate the spectra and decay properties of baryons with these two confining modes in the chiral SU(3) quark model, and hope that which confining mode is more important in baryon can be explored.

In the next section, the baryon spectrum in the chiral SU(3) quark model is discussed and the effect of different confining modes is compared. The baryon spectrum in the extended chiral SU(3) quark model is further examined in section III, and in section IV the decays of baryons are calculated in a point-like meson emission model. Finally, the conclusion is drawn in section V.
II. BARYON SPECTRUM IN CHIRAL SU(3) QUARK MODEL

A. Brief formulism

In the chiral SU(3) quark model \[2\], the interaction induced by the chiral field describes the non-perturbative QCD effect in the medium distance. The interacting Hamiltonian between the quark and the chiral field can be written as

\[ H^{ch}_I = g_{ch} F(q^2) \bar{\psi} \left( \sum_{a=0}^{8} \sigma_a \lambda_a + i \sum_{a=0}^{8} \pi_a \lambda_a \gamma_5 \right) \psi, \]

where \( g_{ch} \) is the coupling constant between the quark and the chiral-field, \( \lambda_0 \) is a unitary matrix, \( \lambda_1, \ldots, \lambda_8 \) are the Gell-Mann matrix of the flavor SU(3) group, \( \sigma_0, \ldots, \sigma_8 \) denote the scalar singlet and octet fields and \( \pi_0, \ldots, \pi_8 \) represent the pseudoscalar singlet and octet fields, respectively. \( F(q^2) \) is a form factor to describe the chiral-field structure \[19, 20\] and, as usual, is taken as

\[ F(q^2) = \left( \frac{\Lambda^2}{\Lambda^2 + q^2} \right)^{1/2}, \]

where \( \Lambda \) is the cutoff mass of the chiral field. It can be verified that \( H^{ch}_I \) is invariant under the infinitesimal chiral SU(3) transformation.

With \( H^{ch}_I \), the chiral-field-induced effective quark-quark potentials can be written as:

\[ V^{ch}_{ij} = \sum_{a=0}^{8} V_{\sigma_a}(r_{ij}) + \sum_{a=0}^{8} V_{\pi_a}(r_{ij}), \]

where

\[ V_{\sigma_a}(r_{ij}) = -C(g_{ch}, m_{\sigma_a}, \Lambda)X_1(m_{\sigma_a}, \Lambda, r_{ij})[\lambda_a(i)\lambda_a(j)] + V^{l,s}_{\sigma_a}(r_{ij}), \]

\[ V_{\pi_a}(r_{ij}) = C(g_{ch}, m_{\pi_a}, \Lambda)\frac{m^2_{\pi_a}}{12m_q_i m_q_j}X_2(m_{\pi_a}, \Lambda, r_{ij})(\sigma_i \cdot \sigma_j)[\lambda_a(i)\lambda_a(j)] + V^{ten}_{\pi_a}(r_{ij}), \]

and

\[ V^{l,s}_{\sigma_a}(r_{ij}) = -C(g_{ch}, m_{\sigma_a}, \Lambda)\frac{m^2_{\sigma_a}}{4m_q_i m_q_j} \left\{ G(m_{\sigma_a}r_{ij}) - \left( \frac{\Lambda}{m_{\sigma_a}} \right)^3 G(\Lambda r_{ij}) \right\} \]

\[ \times [L \cdot (\sigma_i + \sigma_j)] [\lambda_a(i)\lambda_a(j)], \]

\[ V^{ten}_{\pi_a}(r_{ij}) = C(g_{ch}, m_{\pi_a}, \Lambda)\frac{m^2_{\pi_a}}{12m_q_i m_q_j} \left\{ H(m_{\pi_a}r_{ij}) - \left( \frac{\Lambda}{m_{\pi_a}} \right)^3 H(\Lambda r_{ij}) \right\} \]

\[ \times [3(\sigma_i \cdot \hat{r}_{ij})(\sigma_j \cdot \hat{r}_{ij}) - \sigma_i \cdot \sigma_j] [\lambda_a(i)\lambda_a(j)], \]

(1, 2, 3, 4, 5, 6, 7)
with
\[
C(g_{ch}, m, \Lambda) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{m^2 - m^2} m,
\]
(8)

\[
X_1(m, \Lambda, r) = Y(mr) - \frac{\Lambda}{m} Y(\Lambda r),
\]
(9)

\[
X_2(m, \Lambda, r) = Y(mr) - \left(\frac{\Lambda}{m}\right)^3 Y(\Lambda r),
\]
(10)

\[
Y(x) = \frac{1}{x} e^{-x},
\]
(11)

\[
G(x) = \frac{1}{x} \left(1 + \frac{1}{x}\right) Y(x),
\]
(12)

\[
H(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y(x),
\]
(13)

and \(m_{\sigma_a}\) and \(m_{\pi_a}\) being the masses of the scalar meson and the pseudoscalar meson, respectively.

The short-range interaction in the model is mainly governed by the one-gluon-exchange interaction
\[
V_{ij}^{OGE} = \frac{1}{4} g_i g_j \left(\lambda_i^c \cdot \lambda_j^c\right) \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \left(\frac{1}{m_{q_i}^2} + \frac{1}{m_{q_j}^2} + \frac{4}{3} \frac{1}{m_{q_i} m_{q_j}} (\sigma_i \cdot \sigma_j)\right) \right\} + V_{OGE}^{ls},
\]
(14)

with
\[
V_{OGE}^{ls} = -\frac{1}{16} g_i g_j \left(\lambda_i^c \cdot \lambda_j^c\right) \frac{3}{m_{q_i} m_{q_j} \rho_{ij}^3} \cdot (\sigma_i + \sigma_j).
\]
(15)

For the interaction in the long-distance range, confining potential dominates the behavior of the system. In this paper, two kinds of confining modes are considered. The \(\Delta\)-mode confining potential of a three quark system can generally be written as:
\[
V_{\text{conf}}^{\Delta} = \frac{1}{2} b \sum_{i<j} r_{ij} + C_\Delta,
\]
(16)

where \(b\) denotes the string tension, \(C_\Delta\) is an overall constant. The \(Y\)-mode confining potential is related to the energy of the flux tube connecting three valance quarks, and the total length of the flux tube should take the minimal value for stability. The general form of the \(Y\)-mode confining potential is written as
\[
V_{\text{conf}}^{Y} = b \sum_{i=1}^{3} | r_i - r_0 | + C_Y,
\]
(17)
where $C_Y$ is an overall constant, and $r_0$ is the coordinate of the junction point. The rule for finding the location of the junction point is the following: If all the inner angles of the triangle with three constituent quarks sitting at the apexes of the triangle are smaller than $2\pi/3$, the junction point is located inside the triangle and the angles spanned by two flux tubes are $2\pi/3$. If one of the inner angles of the triangle would take a value equal to or greater than $2\pi/3$, the junction point would be located at that apex. Let the lengths of the three sides of the triangle be $a$, $b$ and $c$, respectively. $L_{\text{min}}$ then can be expressed as

$$L_{\text{min}} = \begin{cases} \frac{1}{2}(a^2 + b^2 + c^2) + \frac{\sqrt{3}}{2}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)} & \text{if all the inner angles are smaller than } 2\pi/3, \\ a + b + c - \text{max}(a,b,c) & \text{if one of the inner angles is not smaller than } 2\pi/3. \end{cases}$$

(18)

For the baryon systems, the total Hamiltonian can be written as

$$H = \sum_{i=1}^{3} T_i - T_G + \sum_{i<j}^{3} V_{ij} + V^{\text{conf}},$$

(19)

with

$$V_{ij} = V_{ij}^{OGE} + V_{ij}^{ch}.$$  

(20)

Where $T_i$ and $T_G$ are the kinetic energy operators of the i-th quark and the center of mass, respectively. The kinetic energy operator in the semi-relativistic form is

$$T_i = \sum_{i=1}^{3} \sqrt{p_i^2 + m_i^2}. $$

(21)

As a physical baryon $B$, it has a certain spin $(J, J_Z)$ and a certain parity $(P)$, where $J$ and $J_Z$ represent the quantum number of the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and its magnetic quantum number, respectively, and $P = (-1)^L$. Therefore, the wave function of the baryon $B$ with quantum number $J^P$ should be constructed as

$$|J, J_Z, P, B> = \Phi_C \sum_{M=-L}^{L} \sum_{S_Z} (J, J_Z \mid L, M, S, S_Z) \phi_B^{SF}(S, S_Z, \xi, \Sigma_B^{SF}) \psi_{L,M}^{N}(\eta, \zeta, \Sigma_O^{O}),$$

(22)

where $(J, J_Z \mid L, M, S, S_Z)$ is the CG coefficient for L-S coupling, $\Phi_C$ denotes the total color wave function which should be totally antisymmetric, $\phi_B^{SF}(S, S_Z, \xi, \Sigma_B^{SF})$ represents the $SU_{SF}(6)$ spin and flavor wave functions, $\psi_{L,M}^{N}(\eta, \zeta, \Sigma_O^{O})$ is the spatial wave function with $N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda$.
being the principal quantum number, \( L \) and \( M \) being the quantum number of the total orbital angular momentum \( L = l_\rho + l_\lambda \) and its magnetic quantum number, respectively, \( \Sigma \) standing for the symmetry of the spatial wave function, \( \eta \) being the width parameter and \( \zeta \) representing the aggregate of the spatial variables, and \( \phi^S_F(S, S_Z, \xi, \Sigma^F) \) and \( \psi^N_{L,M}(\eta, \zeta, \Sigma^O) \) must be coupled to a symmetric wave function. The detailed forms of these wave functions can be found in Ref. [21, 22].

The baryon spectrum is solved by using the variational method. The matrix element of kinetic energy operator (21) is calculated in the momentum space [12]. According to the definitions of the coordinates

\[
R = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{m_1 + m_2 + m_3},
\]

\[
\rho = \frac{1}{\sqrt{2}}(r_1 - r_2),
\]

\[
\lambda = \sqrt{\frac{2}{3}} \left( \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - r_3 \right),
\]

the momenta in the rest frame are related to those in the center of mass frame by

\[
P_1 = \frac{1}{\sqrt{2}} p_\rho + \frac{1}{\sqrt{6}} p_\lambda,
\]

\[
P_2 = -\frac{1}{\sqrt{2}} p_\rho + \frac{1}{\sqrt{6}} p_\lambda,
\]

\[
P_3 = -\frac{2}{\sqrt{6}} p_\lambda.
\]

The detailed evaluation of the matrix elements can be found in Ref. [7] and references therein. In our numerical calculation, the spin-orbital interaction is dropped out due to the weakness of such an interaction showing in the experimental data of the baryon spectrum.

### B. Determination of parameters

There are four initial input parameters: the up (down) quark mass \( m_{u(d)} \), the strange quark mass \( m_s \), the harmonic oscillator frequency \( \omega \) and the confining strength \( b \). The up (down) quark mass \( m_{u(d)} \) and the strange quark mass \( m_s \) are taken to be commonly used values of 330\( MeV \) and 470\( MeV \), respectively. \( \omega \) is chosen to be 0.497 \( GeV \), which will produce a radius of about 0.5\( fm \) for a bare nucleon. The string tension is chosen, according to the LQCD result [11], to be 0.20\( GeV^{-1} \) for the Y-mode and an additional factor of 0.53 for the \( \Delta \)-mode. The other model parameters are fixed in the following way. The chiral coupling constant \( g_{ch} \) is fixed by

\[
\frac{g_{ch}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{NN\pi}^2}{4\pi} \frac{m_u^2}{M_N^2},
\]
with \( g_{NN\pi}^2/4\pi = 13.67 \) being the empirical value. The masses of exchanged mesons are adopted from the experimental data, except for the \( \sigma \) meson. In our calculation, the mass of \( \sigma \) is a free parameter and its value of 675 MeV is determined by the best fit to the experimental data available. The cutoff momentum \( \Lambda \) is taken to be close to the chiral symmetry breaking scale. After the parameters of chiral fields are fixed, the one gluon exchange coupling constants \( g_u \) and \( g_s \) can be determined by the mass splittings between \( N \) and \( \Delta \) and between \( \Sigma \) and \( \Lambda \), respectively.

The zero point energies \( C_{uu}, C_{us} \) and \( C_{ss} \) in the \( \Delta \)-mode confining potential are fixed by the masses of the ground state baryons \( N, \Lambda \) and the average masses of \( \Xi \) and \( \Omega \), respectively. And the zero point energies \( C^N, C^\Lambda = C^\Sigma, C^\Xi \) and \( C^\Omega \) in the \( Y \)-mode confining potential case are fixed by the masses of \( N, \Lambda, \Xi \) and \( \Omega \), respectively.

In the calculation, \( \eta \) and \( \eta' \) mesons are mixed by \( \eta_1 \) and \( \eta_8 \) with a mixing angle \( \theta^{PS} \) of \( -23^\circ \) as usual. The mixing angle \( \theta^S \) between \( \sigma_0 \) and \( \sigma_8 \) is still an open issue because the structure of \( \sigma \) meson is unclear and controversial. \( 35.264^\circ \) is adopted from Ref. which indicates that \( \sigma \) and \( \epsilon \) are ideally mixed by \( \sigma_0 \) and \( \sigma_8 \).

The resultant model parameters are tabulated in Table I.

C. Baryon spectrum

In the chiral SU(3) quark model, the baryon spectra for \( N \leq 2 \) bands with \( \Delta \)-mode and \( Y \)-mode confining potentials are calculated. The non-strange baryon spectra in \( N \leq 2 \) bands are plotted in Fig. and also tabulated in Table II. In this figure, the solid and dashed bars denote the results in the \( \Delta \)-confining mode and in the \( Y \)-confining mode, respectively. It is shown that no matter which confining mode is employed, most of resultant resonances are fairly well located within experimental errors. The differences of resultant resonances by using different confining modes are rather small and most of them are in several tens MeV to several MeV. For comparison, the results by using the so-called relativistic quark model, where the semi-relativistic corrections were considered and the OGE and confining potentials were employed only, are also given in Fig. Comparing with Fig. in our model, most resonances in positive-parity sector, except \( \Delta^*(3^+, 1600) \) which is 200 MeV higher than the experimental center value, can be fitted to the experimental data much better. The reason for worse fitting of \( \Delta^*(3^+, 1600) \) can be attributed to the fact that the contribution of the contact term for this state is much larger than that for the other states in this band (referring to the hyperfine matrix elements in the table in Ref.). To avoid the singular behavior of the wave function at the origin due to the \( \delta(r) \) function in the contact term in the
TABLE I: Model parameters. The meson masses and the cutoff masses are taken to be \( m_\rho' = 980 \text{ MeV}, \)
\( m_\kappa = 980 \text{ MeV}, m_\pi = 138 \text{ MeV}, m_K = 495 \text{ MeV}, m_\eta = 549 \text{ MeV}, m_\eta' = 957 \text{ MeV}, \)
\( \Lambda = 1100 \text{ MeV}. \)

|                | \( \Delta\)-shape | Y-shape |
|----------------|---------------------|---------|
| \( g_u \)      | 0.900              | 0.900   |
| \( g_s \)      | 0.955              | 0.955   |
| \( b_{\Delta}^u(GeV/fm) \) | 1.01            | –       |
| \( b_{\Delta}^u(GeV/fm) \) | 1.01            | –       |
| \( C_{\Delta}^{uu}(MeV) \) | -1053          | –       |
| \( C_{\Delta}^{us}(MeV) \) | -936           | –       |
| \( C_{\Delta}^{ss}(MeV) \) | -751           | –       |
| \( b_{\Delta}^l(GeV/fm) \) | –               | 0.91    |
| \( b_{\Delta}^l(GeV/fm) \) | –               | 1.01    |
| \( b_{\Delta}^p(GeV/fm) \) | –               | 1.01    |
| \( b_{\Delta}^p(GeV/fm) \) | –               | 1.01    |
| \( C_Y^N(MeV) \) | –               | -1170   |
| \( C_Y^N(MeV) \) | –               | -1085   |
| \( C_Y^\Xi(MeV) \) | –               | -989    |
| \( C_Y^\Omega(MeV) \) | –               | -213    |

potential, the authors of Ref. \cite{12} employed a smearing factor and also included the relativistic corrections for the terms related to the quark mass. As a consequence, the mass of \( \Delta^*(\frac{3}{2}^+, 1600) \) in their calculation is smaller than ours but is still higher than the upper limit of experimental error bar. For the negative parity sector, our results are similar to Ref. \cite{12}, and our \( N^*(\frac{1}{2}^+, 1650) \) fits the experimental data much better. Our resultant mass of the Roper resonance \( (N^*(\frac{1}{2}^+, 1440)) \) is still higher than that of the first orbital resonance \( N^*(\frac{1}{2}^-, 1535) \), which is similar to the predictions in Ref. \cite{12}. In recent years, some one argued that the Roper resonance is not a pure baryon \cite{25}.

The baryon spectra with strange number \( S=-1 \) in \( N \leq 2 \) bands are given in Table III-IV and Fig. 3. Once again, the results of Ref. \cite{12} are pictured in Fig. 4. From Fig. 3 one sees that the fitting quality of spectra to the experimental data in this sector is not as good as that in the non-strange sector, but most of the resultant resonances are located within experimental error bars. In general, the fitting quality is similar to that in Ref. \cite{12}. The spectra with the \( \Delta\)-mode
FIG. 1: The non-strange baryon in \( N \leq 2 \) bands in the chiral SU(3) quark model. Boxes show the experimental regions of the resonances. The solid and dashed bars represent the results in the \( \Delta \)-mode case and in the \( Y \)-mode case, respectively.

FIG. 2: The non-strange baryon spectra in \( N \leq 2 \) bands in Ref. [12]. Boxes show the experimental regions of the resonances. The solid and dashed bars represent the results in the \( \Delta \)-mode case and in the \( Y \)-mode case, respectively.

The confining potential is close to those with the \( Y \)-mode, just as that in the non-strange sector. The evaluated mass of \( \Lambda^*_{1/2} \) (1405) is more than 100 MeV larger than the experimental value when the spin-orbit interaction, which causes the mass splitting between \( \Lambda^*_{1/2} \) (1405) and \( \Lambda^*_{3/2} \) (1520), is omitted in the calculation. Inclusion of such an interaction would result in a more reasonable spectrum for strange baryons Ref. [26].

The \( S= -2 \) and \(-3 \) baryon spectra in \( N \leq 2 \) bands are given in Fig. 5 and Table V. The deviations of the results with the \( Y \)-mode from those with \( \Delta \)-mode are similar to those in the former sectors. The results show that both the \( \Delta \)-mode and the \( Y \)-mode confining potentials can lead to reasonable baryon spectra.

D. Comparison of \( \Delta \)-mode and \( Y \)-mode

In order to study the effect of different confinement modes on the baryon spectrum, we now use a confining potential in which both \( \Delta \)-mode and the \( Y \)-mode are included

\[
V_{\text{conf}} = xV_{\Delta \text{conf}} + (1 - x)V_{Y \text{conf}},
\]

where \( x \) stands for the fraction of the \( \Delta \)-mode in the whole confining potential. The \( S=0 \) and \(-1 \) baryon spectra with \( x \) values of 0.2, 0.5 and 0.8 are plotted in Figs. 6 and 7 respectively. The
TABLE II: Non-strange baryon masses in $N \leq 2$ bands (in $MeV$). Experimental date are taken from [23].

| State | $\Delta$-mode | Y-mode | experimental data |
|-------|---------------|--------|------------------|
| $N^{\ast \frac{1}{2}^+}$ | 939 | 939 | 939 **** |
| | 1519 | 1548 | 1430-1470 **** |
| | 1700 | 1714 | 1680-1740 *** |
| | 1970 | 1961 | 1885-2125 * |
| | 2063 | 2056 | |
| $\Delta^{\ast \frac{1}{2}^+}$ | 1874 | 1853 | 1870-1920 **** |
| | 1949 | 1942 | |
| $N^{\ast \frac{3}{2}^+}$ | 1665 | 1669 | 1650-1750 **** |
| | 1910 | 1917 | 1879-1951 ** |
| | 1977 | 1970 | |
| | 2051 | 2048 | |
| | 2357 | 2366 | |
| $\Delta^{\ast \frac{3}{2}^+}$ | 1232 | 1232 | 1232 **** |
| | 1852 | 1870 | 1550-1700 *** |
| | 1961 | 1961 | 1900-1970 *** |
| | 2000 | 2024 | |
| $N^{\ast \frac{5}{2}^+}$ | 1675 | 1678 | 1675-1690 **** |
| | 2001 | 1990 | 1903-2025 ** |
| | 2356 | 2365 | |
| $\Delta^{\ast \frac{5}{2}^+}$ | 1957 | 1977 | 1870-1920 **** |
| | 2028 | 2020 | 1724-2200 ** |
| $N^{\ast \frac{7}{2}^+}$ | 2000 | 2007 | 1970-2080 ** |
| $\Delta^{\ast \frac{7}{2}^+}$ | 1961 | 1919 | 1940-1960 **** |
| $N^{\ast \frac{1}{2}^-}$ | 1454 | 1473 | 1520-1555 **** |
| | 1665 | 1685 | 1640-1680 **** |
| $\Delta^{\ast \frac{1}{2}^-}$ | 1586 | 1605 | 1615-1675 **** |
| $N^{\ast \frac{3}{2}^-}$ | 1459 | 1478 | 1515-1530 **** |
| | 1599 | 1619 | 1650-1750 *** |
| $\Delta^{\ast \frac{3}{2}^-}$ | 1586 | 1605 | 1670-1770 *** |
| $N^{\ast \frac{5}{2}^-}$ | 1632 | 1652 | 1670-1685 **** |

resultant baryon spectra with different x-values show very small differences.

From the structure of the flux-tube model, the difference between the Y-mode and the $\Delta$-mode potentials comes from different geometric shapes of flux-tubes in baryon. By denoting $L_Y$ and $L_\Delta$ as the lengths of the Y-mode and $\Delta$-mode flux-tubes, respectively, we have

$$\frac{L_\Delta}{2} \leq L_Y \leq \frac{2}{\sqrt{3}} \frac{L_\Delta}{2},$$

(27)

where the left equal sign means that the junction point is sitting at the apex and the right equal sign represents the fact that three quarks in baryon stay at apexes of an equilateral triangle. Further
TABLE III: Positive-parity $S=-1$ baryon masses in $N \leq 2$ bands (in $MeV$). Experimental data are taken from $[23]$.

| State | $\Delta$-mode | $Y$-mode | experimental data |
|-------|----------------|-----------|-------------------|
| $\Lambda^{\frac{1}{2}}_0$ | 1115 | 1115 | 1115.6 | **** |
| | 1690 | 1717 | 1560-1700 | *** |
| | 1801 | 1814 | 1750-1850 | *** |
| | 1844 | 1869 | | |
| | 2088 | 2079 | | |
| | 2127 | 2128 | | |
| | 2173 | 2239 | | |
| $\Lambda^{\frac{3}{2}}_0$ | 1727 | 1730 | 1850-1910 | **** |
| | 2039 | 2066 | | |
| | 2053 | 2072 | | |
| | 2098 | 2090 | | |
| | 2139 | 2196 | | |
| | 2230 | 2265 | | |
| | 2414 | 2323 | | |
| $\Lambda^{\frac{5}{2}}_0$ | 1733 | 1735 | 1815-1825 | **** |
| | 2092 | 2121 | 2090-2140 | *** |
| | 2133 | 2187 | | |
| | 2162 | 2207 | | |
| | 2414 | 2423 | | |
| $\Lambda^{\frac{3}{2}}_2$ | 2105 | 2115 | 2000-2140 | * |
| $\Sigma^{\frac{1}{2}}_2$ | 1192 | 1192 | 1192.6 | **** |
| | 1759 | 1784 | 1630-1690 | *** |
| | 1889 | 1918 | 1738-1790 | * |
| | 1992 | 1968 | 1826-1985 | ** |
| | 2064 | 2060 | | |
| | 2103 | 2092 | | |
| | 2139 | 2220 | | |
| $\Sigma^{\frac{3}{2}}_2$ | 1371 | 1370 | 1382 | **** |
| | 1859 | 1854 | 1800-1925 | * |
| | 1995 | 2003 | 2070-2140 | ** |
| | 2056 | 2060 | | |
| | 2079 | 2092 | | |
| | 2094 | 2096 | | |
| | 2103 | 2124 | | |
| | 2135 | 2212 | | |
| | 2200 | 2221 | | |
| $\Sigma^{\frac{5}{2}}_2$ | 1868 | 1859 | 1900-1930 | **** |
| | 2065 | 2069 | 2050-2080 | * |
| | 2099 | 2121 | | |
| | 2163 | 2217 | | |
| | 2199 | 2237 | | |
| $\Sigma^{\frac{7}{2}}_2$ | 2062 | 2023 | 2025-2040 | **** |
| | 2114 | 2204 | | |
considering a factor of $\frac{1}{2}$ in the string tension of the $\Delta$-mode confining potential, the ratio of the $Y$-mode and $\Delta$-mode confining potentials satisfies

$$1 \leq \frac{L_Y}{L_\Delta/2} = \frac{V_{Y}^{\text{conf}}}{V_{\Delta}^{\text{conf}}} \leq \frac{2}{\sqrt{3}} \simeq 1.15 .$$

This relation indicates that the maximal difference between the $Y$-mode and $\Delta$-mode confining potentials is 15%. Because the calculated baryon mass is closely related to the averaged values of potentials, one might not be able to distinguish the effects from two confining modes by studying the spectrum only. However, by carefully investigating the matrix element (ME) of the confining
TABLE IV: Negative-parity S=-1 baryon masses in N=1 bands (in MeV). Experimental data are taken from [23].

| State          | Δ-mode | Y-mode | experimental data |
|----------------|--------|--------|-------------------|
| $\Lambda^{1/2} -$ | 1523   | 1540   | 1402-1410         |
|                | 1622   | 1639   | 1660-1680         |
|                | 1789   | 1808   | 1720-1850         |
| $\Lambda^{3/2} -$ | 1524   | 1541   | 1518-1520         |
|                | 1624   | 1641   | 1685-1695         |
|                | 1748   | 1767   | 2307-2372         |
| $\Lambda^{5/2} -$ | 1769   | 1788   | 1810-1830         |
| $\Sigma^{1/2} -$ | 1587   | 1604   | 1600-1640         |
|                | 1747   | 1763   | 1730-1800         |
|                | 1774   | 1791   | 1755-2004         |
| $\Sigma^{3/2} -$ | 1617   | 1635   | 1578-1584         |
|                | 1743   | 1760   | 1665-1685         |
|                | 1753   | 1769   | 1900-1950         |
| $\Sigma^{5/2} -$ | 1746   | 1762   | 1770-1780         |

FIG. 6: S=0 baryon spectra with mixed Δ- and Y-mode confining potentials in $N \leq 2$ bands. The solid, dashed and dotted bars correspond to the results with $x$ being 20%, 50% and 80%, respectively.

potential in different integrating intervals, one might see the different effects from different confining modes. In Fig. 8, we demonstrate the values of the matrix element of confining potentials in the Y-mode and Δ-mode with respect to the quark separation step by step in several states. In this figure, the matrix element at $r = \rho_{max} = \lambda_{max}$ with $\rho_{max}$ and $\lambda_{max}$ being the upper integrating limit of the $\rho$ and $\lambda$ integrations are normalized to 1 for an easy comparison. Evidently, at the short and medium distances ($0.2 fm - 0.8 fm$), the Δ-mode is dominant and at the large distances...
TABLE V: Ξ and Ω baryon masses in N≤2 bands in the chiral SU(3) quark model (in MeV). Experimental data are taken from [23].

| State | Model | Predicted masses (MeV) |
|-------|-------|------------------------|
| Ξ⁺¹/₂ | Δ-shape | 1327 1894 2016 2122 2190 2234 2263 |
|       | Y-shape | 1314 1907 2028 2085 2175 2212 2325 |
| Ξ⁺³/₂ | Δ-shape | 1506 1947 2122 2185 2204 2215 2234 2246 2361 |
|       | Y-shape | 1493 1927 2115 2176 2204 2212 2232 2312 2367 |
| Ξ⁺⁵/₂ | Δ-shape | 1952 2191 2218 2268 2361 |
|       | Y-shape | 1932 2177 2230 2327 2366 |
| Ξ⁺⁷/₂ | Δ-shape | 2184 2233 |
|       | Y-shape | 2134 2304 |
| Ω⁺¹/₂ | Δ-shape | 2262 2326 |
|       | Y-shape | 2516 2618 |
| Ω⁺³/₂ | Δ-shape | 1636 2241 2339 2347 |
|       | Y-shape | 1672 2623 2688 2770 |
| Ω⁺⁵/₂ | Δ-shape | 2333 2365 |
|       | Y-shape | 2674 2766 |
| Ω⁺⁷/₂ | Δ-shape | 2322 |
|       | Y-shape | 2636 |
| Ξ⁻¹/₂ | Δ-shape | 1756 1871 1890 |
|       | Y-shape | 1760 1875 1896 |
| Ξ⁻³/₂ | Δ-shape | 1756 1871 1896 |
|       | Y-shape | 1760 1875 1901 |
| Ξ⁻⁵/₂ | Δ-shape | 1892 |
|       | Y-shape | 1898 |
| Ω⁻¹/₂ | Δ-shape | 2104 |
|       | Y-shape | 2099 |
| Ω⁻³/₂ | Δ-shape | 2104 |
|       | Y-shape | 2099 |

the Y-mode provides more contributions. This result coincides with Alexandrou’s argument [14]. Moreover, in lower lying states, the increase rate of the matrix element of the Y-mode potential is much slower than that of the Δ-mode potential in the distances less than 0.6fm, but becomes much faster as r > 0.8fm, especially in higher level states.

III. THE EXTENDED CHIRAL SU(3) QUARK MODEL

Recently Zhang et al extended the chiral SU(3) quark model by including the exchanges of the singlet and octet vector meson fields between quarks. In this extended model, the vector meson
FIG. 7: S = -1 baryon spectra with mixed Δ- and Y-mode confining potentials in \( N \leq 2 \) bands. The solid, dashed and dotted bars correspond to the results with \( x \) being 20%, 50% and 80%, respectively.

FIG. 8: Contributions of the matrix elements of Δ-mode and Y-mode confining potentials with respect to the quark separation. (a) \( |2N(56,0^+)_1^2 > \) state, (b) \( |2N(70,1^-)_1^2 > \) state and (c) \( |2N(56,2^+)_1^2 > \) state with the notation \( |2J+1N(N_6, N_P)J_P > \), where \( J \), \( N_6 \), \( N \) and \( P \) denote the total angular momentum, the dimension of \( SU_{SF}(6) \) group, the principal quantum number and the parity of the state, respectively. The solid and dashed curves correspond to the Δ-mode confining potential and the Y-mode confining potential cases, respectively.

The induced quark-quark interaction is introduced through the interaction Lagrangian

\[
\mathcal{L}_I = -g_{chv} \bar{\psi} \gamma_\mu T^a A^\mu_\alpha \psi - \frac{f_{chv}}{2M_P} \bar{\psi} \sigma_{\mu\nu} T^a \partial^\nu A^\mu_\alpha \psi,
\]

where \( g_{chv} \) and \( f_{chv} \) are the coupling constants for the vector coupling and the tensor coupling, respectively. The form of the additional potential can be written as

\[
V_{ij}^V = \sum_{a=0}^{8} V_{0a}(r_{ij}),
\]
where $\rho_0, \ldots, \rho_8$ denote the singlet and octet vector fields, and

$$V_{\rho_8}(r_{ij}) = C(g_{chv}, m_{\rho_8}, \Lambda) \left\{ X_1(\rho_{\rho_8}, \Lambda, r_{ij}) + \frac{m_{\rho_8}^2}{6m_{q_i}m_{q_j}} \left( 1 + \frac{f_{chv}m_{q_i} + m_{q_j}}{g_{chv}} + \frac{f_{chv}^2}{g_{chv}^2} \right) \right\} X_2(\rho_{\rho_8}, \Lambda, r_{ij})(\sigma_i \cdot \sigma_j) \left[ \gamma_a(i)\gamma_a(j) \right] + V_{\rho_8}^{ls}(r_{ij}) + V_{\rho_8}^{ten}(r_{ij}), \quad (31)$$

$$V_{\rho_8}^{ls}(r_{ij}) = -C(g_{chv}, m_{\rho_8}, \Lambda) \frac{3m_{\rho_8}^2}{4m_{q_i}m_{q_j}} \left( 1 + \frac{f_{chv}2(m_{q_i} + m_{q_j})}{g_{chv}} \right) \right\} G(m_{\rho_8}, r_{ij}) \left[ L \cdot (\sigma_1 + \sigma_3) \right] [\gamma_a(i)\gamma_a(j)], \quad (32)$$

$$V_{\rho_8}^{ten}(r_{ij}) = -C(g_{chv}, m_{\rho_8}, \Lambda) \frac{m_{\rho_8}^2}{12m_q m_{q_i}} \left( 1 + \frac{f_{chv}m_{q_i} + m_{q_j}}{g_{chv}} + \frac{f_{chv}^2}{g_{chv}^2} \right) \right\} H(m_{\rho_8}, r_{ij}) \left[ L \cdot (\sigma_1 + \sigma_3) \right] S_{ij} [\gamma_a(i)\gamma_a(j)], \quad (33)$$

with $m_{\rho_8}$ being the mass of the vector meson.

In the extended model, some additional parameters are introduced \[3\]. Based on $g_{NN\rho}$ value in the phenomenological $NN$ interaction model, the coupling constants $g_{chv}$ and $f_{chv}$ can be estimated as $g_{chv} = 2.351$ and $f_{chv} = 2/3g_{chv}$, respectively. The mixing angle between $\omega_1$ and $\omega_8$ is taken to be $\theta^V = 35.26^\circ$, which indicates an ideal mixture. The model parameters are summarized in Table VI.

In the chiral SU(3) quark model, we find that the baryon spectra in the $\Delta$-mode and the $Y$-mode confining potential cases are rather close, and that one cannot definitely distinguish which confining mode is better through the spectra only because the mass of baryon is closely related to the averaged value of the confining potential operator. Therefore, in order to see whether a reasonable description for baryon spectra can be obtained in the extended chiral SU(3) quark model also, we calculate the baryon spectra with the $\Delta$-mode only. The baryon spectra for $S = 0$, -1, -2, -3 in the extended chiral SU(3) quark model are plotted in Figs 9-11. In these figures, the solid and dashed bars denote the results with and without the tensor coupling in the vector meson potential, respectively. The fitting quality in the extended model is close to that in the chiral SU(3) quark model except for a few states, and the deviations between the states with vector coupling of the vector meson potential only and those with both vector and tensor couplings of the vector meson potential are very small. The resultant mass of Roper resonance ($N(1/2^+, 1440)$) is 25 MeV smaller than that in the chiral SU(3) quark model. It is even closer to the experimental value, but
TABLE VI: Parameters of extended chiral SU(3) quark model. The vector meson masses $m_\rho = 770 \text{MeV}$, $m_{K^*} = 892 \text{MeV}$, $m_{\omega} = 782 \text{MeV}$, $m_{K^*} = 1020 \text{MeV}$ and the cutoff momentum $\Lambda = 1100 \text{MeV}$. (1) $f_{chv} = 0$, (2) $f_{chv} \neq 0$.

| Extended chiral SU(3) quark model | (1) | (2) |
|-----------------------------------|-----|-----|
| $\omega (\text{MeV})$            | 497.6 | 497.6 |
| $g_{ch}$                          | 2.621 | 2.621 |
| $g_{chv}$                         | 2.351 | 1.972 |
| $f_{chv}/g_{chv}$                 | – | 2/3 |
| $m_{\sigma} (\text{MeV})$        | 535 | 547 |
| $g_u$                             | 0.521 | 0.598 |
| $g_s$                             | 0.545 | 0.614 |
| $b_{uu} (\text{GeV}/\text{fm})$  | 0.57 | 0.58 |
| $b_{us} (\text{GeV}/\text{fm})$  | 1.03 | 0.99 |
| $b_{ss} (\text{GeV}/\text{fm})$  | 2.01 | 2.04 |
| $c_{uu} (\text{MeV})$            | -566 | -524 |
| $c_{us} (\text{MeV})$            | -855 | -769 |
| $c_{ss} (\text{MeV})$            | -1285 | -1224 |

is still larger than the first orbital resonance ($N(\frac{1}{2}^-, 1535)$). Moreover, in the extended model, the level intervals among $\Lambda(\frac{1}{2}^-)$ states become larger.

It should be mentioned that because the effect of the vector meson exchange can partially replace the role of the one gluon exchange, introducing vector meson exchange potential can reduce the strong coupling constant $\alpha_s$ to some extend, which is desirable by QCD expectation.

**IV. NONSTRANGE BARYON STRONG DECAY**

A successful hadron model should be able to explain as much data as possible such as the spectrum, magnetic moments, decay width and etc.. The chiral SU(3) quark model is quite successful in reproducing the experimental data of the hadron-hadron scattering and baryon spectrum. Now, we would check if this model can give a reasonable description of the strong decay width of non-strange baryon resonances.

As well known, the theory of a baryon strongly decay into hadrons is far away from establishment. In this section, we use a simple model, called the point-like meson emission model, to estimate the decay width of the $N^* \rightarrow NM$ process. In this model, the baryon has its own quark structure, and the point-like meson is emitted from one of the quarks in baryon (see Fig. 12).
The amplitude responsible for the emission of a pseudoscalar meson is usually assumed as

\[ <NM \mid H_s \mid N^* > = \sum_{i=1}^{3} < Ne^{-ik \cdot r_i} \mid (xk \cdot \sigma_i + yp_i \cdot \sigma_i)X_{i,M} \mid N^* > = 3 < Ne^{-ik \cdot r_1} \mid (xk \cdot \sigma_1 + yp_1 \cdot \sigma_1)X_{1,M} \mid N^* >, \]  

(34)

where the factor 3 is due to the symmetry of the wave function of three identical quarks, \( k \) is the
momentum of the emitted meson, $r_i$, $p_i$, $\frac{1}{2}\sigma_i$ and $X_{i,M}$ are the coordinate, the momentum, the spin and the flavor matrix, that describe the quark transition process $q_i \rightarrow q'_i + M$, of the $i$-th quark, respectively, and $x$ and $y$ are phenomenological constants. For neutral-pion emission, $X_M$ can be written as

$$X_{\pi^0} = \lambda_3,$$

(35)

where $\lambda_3$ is the Gell-Mann matrix. The wave function of the $| N^* >$ state is obtained by the configuration mixing in calculating the spectrum in the chiral SU(3) quark model.

In the center of mass frame of the non-strange baryon, the width of a non-strange baryon decaying into a ground state nucleon and a $\pi$ meson can be written as

$$\Gamma_{N\pi} = \frac{1}{\pi} \frac{|<f | \mathcal{H}_s | i>|^2 kE_N}{2J_R + 1} \frac{m_R}{m_N} <I_N, I_{3N}, I_\pi, I_{3\pi}|I_{N^*}, I_{3N^*}>^{-2},$$

(36)

where $J$, $I$ and $I_3$ represent the total angular momentum, the isospin and the third component of isospin, respectively.

In Table VII, we show the calculated decay width $\Gamma_{N\pi}^{1/2}(MeV^{1/2})$ of a four-star or a three-star non-strange resonance decaying into a nucleon and a pion. For comparison, the results in [28] are also listed in the table VII. The parameters $x$ and $y$ are obtained by a $\chi^2$ fit. From the table, one sees that most of the decay widths in our model agree with the experimental data and are comparable to those in Ref. [28]. The decay widths of $N(\frac{1}{2}^+, 1440)$ and $N(\frac{1}{2}^-, 1535$) deviate from the data greatly, because the order of the masses of these two resonances in the model prediction are opposite down. Moreover, the corresponding decay widths in two confining mode cases are also very close because the corresponding wave functions are very close.
TABLE VII: Square root of the non-strange baryon decay widths, $\Gamma_{N\pi}^{1/2}$, in MeV$^{1/2}$, together with the parameter $x$ and $y$. Column 3 are the results from [28]. Column 3 and 4 correspond to $\Delta$-shape and Y-shape confining potential and the last column are experimental data from [23].

| Resonance | Isgur | $\Delta$ - shape | Y-shape | Expt. |
|-----------|-------|-----------------|---------|-------|
| $N_{1}^{\frac{3}{2}}$(1535) | 5.3 | 13.2 | 13.3 | 8.2 $\pm$ 2.3 |
| $N_{1}^{\frac{1}{2}}$(1650) | 8.7 | 8.6 | 8.2 | 10.4 $\pm$ 2.0 |
| $N_{2}^{\frac{3}{2}}$(1520) | 9.2 | 11.5 | 10.9 | 8.1 $\pm$ 0.8 |
| $N_{2}^{\frac{1}{2}}$(1700) | 3.6 | 2.9 | 2.9 | 3.2 $\pm$ 1.6 |
| $N_{3}^{\frac{3}{2}}$(1675) | 5.5 | 6.7 | 6.4 | 8.2 $\pm$ 1.0 |
| $N_{3}^{\frac{1}{2}}$(1440) | 6.8 | 6.6 | 6.1 | 15.1 $\pm$ 2.8 |
| $N_{4}^{\frac{3}{2}}$(1710) | 6.7 | 3.1 | 3.2 | 3.9 $\pm$ 2.4 |
| $N_{4}^{\frac{1}{2}}$(1720) | 6.5 | 3.4 | 3.8 | 4.7 $\pm$ 1.6 |
| $N_{5}^{\frac{3}{2}}$(1680) | 7.1 | 2.4 | 2.6 | 9.2 $\pm$ 0.7 |
| $N_{5}^{\frac{1}{2}}$(1620) | 3.3 | 5.7 | 5.7 | 6.1 $\pm$ 1.2 |
| $N_{5}^{\frac{1}{2}}$(1700) | 4.9 | 6.4 | 6.9 | 6.7 $\pm$ 2.2 |
| $N_{7}^{\frac{3}{2}}$(1910) | 5.3 | 5.7 | 4.9 | 7.5 $\pm$ 1.8 |
| $N_{7}^{\frac{1}{2}}$(1232) | 10.2 | 10.3 | 10.7 | 10.9 $\pm$ 0.2 |
| $N_{7}^{\frac{1}{2}}$(1905) | 4.0 | 5.5 | 5.7 | 5.9 $\pm$ 2.2 |
| $N_{7}^{\frac{1}{2}}$(1950) | 7.5 | 10.6 | 10.4 | 10.6 $\pm$ 0.9 |

V. CONCLUSIONS

The baryon spectra with both Y-mode and $\Delta$-mode confining potentials in $N \leq 2$ bands are studied in the chiral $SU(3)$ quark model. The results show that, no matter which type of confining potential, say the $\Delta$-mode or the Y-mode or even the mixed mode, is employed, the experimental baryon spectra can be well-explained. The resultant baryon spectra with the Y-mode and $\Delta$-mode confining potentials are very close to each other. For most of the states, the corresponding mass difference with different confining modes is less than 20 MeV. Moreover, the $\Delta$-mode confinement is more effective in the short and medium distances and the Y-mode confinement provides more contributions in the long distance. It is also shown that the effect of the different confining mode does not distinctly show up in the spectrum. The strong decay widths of the non-strange baryon resonances are calculated in the point-like meson emission model with the wave-functions obtained in the baryon spectrum calculation. The resultant decay widths are generally in agreement with the experimental data. However, this mode is too simple. The detailed decay information of baryons should be extracted from the investigation in the implicit quark-gluon degrees of freedom.
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