Simulation of acoustic wave equation in viscoelastic media by ONADM method

Rong Huang 1, * and Zhiliang Wang 2

1 Shool of Sciences of Southwest Petroleum University, Nanchong, China
2 Shool of Sciences of Southwest Petroleum University, Chengdu, China

*Corresponding author e-mail: 270511874@qq.com

Abstract. The propagation of acoustic wave in viscoelastic media is of extremely important in seismic exploration, seismology and so on. This paper propose a novel difference scheme for acoustic wave equation in viscoelastic media simulation, named Optimal approximate analytic discretization method (ONADM). This is a new method which can suppress numerical dispersion effectively in larger space step in recent years. The purpose of this paper is to study the propagation of acoustic wave in viscoelastic media by using ONADM method. Finally, The numerical results demonstrate that the ONAD method for acoustic wave equation in viscoelastic media has obvious advantages.

1. Introduction
In earlier studies, researchers suppose that the earth was an ideal elastomer which brought some uncertainty mistake for seismology interpretation, like amplitude decrease, phase error and video rate reduction. In order to represent the attenuation and dissipation of energy during acoustic wave propagation in underground media, the quality factor Q has been introduced to describe the attenuation and dissipation of energy during seismic wave propagation in inhomogeneous media with viscoelasticity.

In this years, many scholars have studied the the acoustic wave equation with the quality factor Q in viscoelastic media. McDonal et al. [1] found that the quality factor Q can be regarded as a frequency independent constant in the seismic exploration band. After that, Blanch et al. [2] researched the numerical method of constructing the constant Q model, which reduced the parameters describing the viscoelastic medium. At present, the Kelvin model is widely used model for describing viscoelastic media [3-8]. Based on Kelvin model, Yuan C F et al. [9] studied the transient response of the Kevin model under the small disturbance, and gave the analytic solution of the wave equation under pulse source condition.

Many numerical algorithms can be used to simulate seismic wave propagation in underground media, just like the finite difference method, the finite-element method, the pseudo-spectrum method, and so on. Compared to the pseudo-spectrum method and the finite-element method, the finite difference method have become the most popular and widely used numerical methods in computational geophysics and petroleum exploration for theirs smaller memory cost, fast computing speed, and high efficiency. However, when the conventional finite difference methods are used to simulation wave propagation with coarse grids, strong numerical dispersion will be aroused. The numerical dispersion is an important annoying shortcomings of numerical method.
Yang et. al. first proposed the so called NAD method and applied this method to the acoustic- and elastic- wave field simulation [10]. NAD method reconstruct the wavefield by combining the displacement and its gradient to approximating high-order spatial partial derivative. From the mathematical point of view, the gradient represented the tendency of waveform change. Therefore increased gradient information can effectively enhance the ability of numerical schemes to suppress the numerical dispersion. ONADM is one of the perfect of NAD-type algorithms[11], but it was not to used for qP-wave simulation. In this paper, we will expand the ONADM algorithm to viscoelastic media models.

2. Acoustic wave equation in viscoelastic media and ONADM method

2.1. Acoustic wave equation in viscoelastic media

The acoustic wave equation in viscoelastic media is

\[ \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{1}{\omega Q_p} \frac{\partial}{\partial t} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \]  

(1)

Where \( Q_p \) is the quality factor, \( \omega \) is the angular frequency and \( v \) is the acoustic velocity.

We can rewrite equation (1) as

\[ \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{1}{\omega Q_p} \left[ \frac{\partial^2 \left( \frac{\partial u}{\partial t} \right)}{\partial x^2} + \frac{\partial^2 \left( \frac{\partial u}{\partial t} \right)}{\partial z^2} \right] \]

(2)

According to the truncated Taylor expansion, we can get

\[ P = \frac{\partial^2 U}{\partial t^2} = \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} \right) + \frac{1}{\omega Q_p} \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} \left( \frac{\partial V}{\partial t} \right) \right] \]

(3)

\[ \frac{\partial P}{\partial t} = \frac{\partial}{\partial t} \left( \frac{\partial^2 U}{\partial t^2} \right) = \frac{\partial^3 V}{\partial t^3} \]

\[ = \frac{1}{v^2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial^2 U}{\partial x^2} \right) + \frac{\partial}{\partial z} \left( \frac{\partial^2 U}{\partial z^2} \right) \right] + \frac{v^2}{\omega Q_p} \left[ \frac{\partial}{\partial x} \left( \frac{\partial^2 V}{\partial x^2} \right) + \frac{\partial}{\partial z} \left( \frac{\partial^2 V}{\partial z^2} \right) \right] \]

\[ = \frac{1}{v^2} \left( \frac{\partial^3 V}{\partial x^2 \partial z^2} + \frac{\partial^3 V}{\partial z^2 \partial x^2} \right) + \frac{v^2}{\omega Q_p} \left[ \frac{\partial^3 V}{\partial x^4} + \frac{\partial^3 V}{\partial z^4} \left( \frac{\partial V}{\partial t} \right) \right] \]

\[ = \frac{1}{v^2} \left( \frac{\partial^3 V}{\partial x^2 \partial z^2} + \frac{\partial^3 V}{\partial z^2 \partial x^2} \right) + \frac{v^2}{\omega Q_p} \left[ \frac{\partial^3 U}{\partial x^4} + \frac{\partial^3 U}{\partial z^4} + \frac{\partial^3 U}{\partial x^2 \partial z^2} \right] \]

(4)
Where \( U = \left( u \ \frac{\partial u}{\partial x} \ \frac{\partial u}{\partial z} \right)^T \), \( P = \frac{\partial^2 U}{\partial t^2} = \left( \frac{\partial^2 u}{\partial t^2} \ \frac{\partial^2 u}{\partial t \partial x} \ \frac{\partial^2 u}{\partial t \partial z} \right)^T \), 
\( V = \frac{\partial U}{\partial t} = \left( \frac{\partial u}{\partial t} \ \frac{\partial u}{\partial t \partial x} \ \frac{\partial u}{\partial t \partial z} \right)^T \), 
the required expressions of high-order spatial partial derivatives can be obtained by using the truncated Taylor-series expansion of \( P \) and \( R \) at grid points \( A_{i\pm 1,j\pm 1,m} (l \in \{-1,0,1\}) \) \( (m \in \{-1,0,1\}) \).

### 2.2. ONADM method

According to ONAD method:

\[
\left( \frac{\partial^2 U}{\partial x^2} \right)^k_{i,j} = \frac{2}{(\Delta x)^2} \left( U^k_{i+1,j} - 2U^k_{i,j} + U^k_{i-1,j} \right) - \frac{1}{2\Delta x} \left[ \left( \frac{\partial U}{\partial x} \right)^k_{i+1,j} - \left( \frac{\partial U}{\partial x} \right)^k_{i-1,j} \right] 
\]

(5)

\[
\left( \frac{\partial^2 U}{\partial z^2} \right)^k_{i,j} = \frac{2}{(\Delta z)^2} \left( U^k_{i,j+1} - 2U^k_{i,j} + U^k_{i,j-1} \right) - \frac{1}{2\Delta z} \left[ \left( \frac{\partial U}{\partial z} \right)^k_{i,j+1} - \left( \frac{\partial U}{\partial z} \right)^k_{i,j-1} \right] 
\]

(6)

\[
\left( \frac{\partial^4 U}{\partial x^4} \right)^k_{i,j} = \frac{6}{(\Delta x)^4} \left[ \left( \frac{\partial U}{\partial x} \right)^k_{i+1,j} - \left( \frac{\partial U}{\partial x} \right)^k_{i-1,j} \right] - \frac{12}{(\Delta x)^2} \left[ U^k_{i+1,j} - 2U^k_{i,j} + U^k_{i-1,j} \right] 
\]

(7)

\[
\left( \frac{\partial^4 U}{\partial z^4} \right)^k_{i,j} = \frac{6}{(\Delta z)^4} \left[ \left( \frac{\partial U}{\partial z} \right)^k_{i,j+1} - \left( \frac{\partial U}{\partial z} \right)^k_{i,j-1} \right] - \frac{12}{(\Delta z)^2} \left[ U^k_{i,j+1} - 2U^k_{i,j} + U^k_{i,j-1} \right] 
\]

(8)

\[
\left( \frac{\partial^4 U}{\partial x^2 \partial z^2} \right)^k_{i,j} = \frac{3}{(\Delta x)^2 (\Delta z)^2} \left[ U^k_{i+1,j+1} + U^k_{i+1,j-1} + U^k_{i-1,j+1} + U^k_{i-1,j-1} + 4U^k_{i,j} - 2U^k_{i+1,j} - 2U^k_{i,j+1} - 2U^k_{i,j-1} - 2U^k_{i-1,j} \right] 
\]

(9)

Based on equation (5)–(9), we can solve equation (4).

### 3. Numerical experiments

The numerical experiment in this section is one inhomogeneous media with viscoelasticity with domain \( 8km \times 8km \). The quality factor is \( Q_p = 150 \). The wave velocity is \( v = 4.99 km/s \). The spatial and time increments are \( \Delta x = \Delta z = 20m \) and \( \Delta t = 2ms \) respectively. The courant number is \( \alpha = \Delta t \frac{v}{h} = 0.002 \times \frac{4.99}{0.02} = 0.499 \), which satisfies the maximum courant number given by us. The time variation of the source function is

\[
f(t) = -5.76f_0^2(1-16(0.6f_0\tau - 1)^2)e^{-8(0.6f_0\tau - 1)^2} 
\]

(10)

where the frequency is \( f_0 = 15Hz \).
From Fig.1, we can observe a clear and non-dispersive wave field snapshot, which shows that the stability condition of our calculation is effective.

Fig.1 give the wavefield snapshots of acoustic wave propagation in the viscosity of medium. The snapshot at 0.6s is satisfying the maximum stability condition, where the spatial step is 20m, the time steps is 2ms,and the wave velocity is 4.99km/s. From the wavefields, the snapshot generated by ONADM is very clear without any visible numerical dispersion.

4. Conclusion
An ONADM for 2D acoustic wave propagation simulation in inhomogeneous media with viscoelasticity is proposed in this paper. ONADM can suppress the numerical dispersion successfully when coarse grids are used or strong velocity contrast is encountered. Compared with the traditional high order finite difference scheme, ONADM’s numerical results is satisfactory.

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References
[1] McDonal F J, Angona F A, Mills R L, et al. Attenuation of shear and compressional waves in Pierre shale[J]. Geophysics, 1958, 23(3): 421-439.
[2] Blanch J O, Robertson J O A, Symes W W. Modeling of a constant Q: Methodology and algorithm for an efficient and optimally inexpensive viscoelastic technique[J]. Geophysics, 1995, 60(1): 176-184.
[3] Bohlen T. Parallel 3-D viscoelastic finite difference seismic modelling[J]. Computers & Geosciences, 2002, 28(8): 887-899.
[4] Saenger E H, Bohlen T. Finite-difference modeling of viscoelastic and anisotropic wave propagation using the rotated staggered grid[J]. Geophysics, 2004, 69(2): 583-591.
[5] Treysse F. Three-dimensional modeling of elastic guided waves excited by arbitrary sources in viscoelastic multilayered plates[J]. Wave Motion, 2015, 52: 33-53.
[6] Colombaro I, Giusti A, Mainardi F. On transient waves in linear viscoelasticity[J]. Wave Motion, 2017. 74:191-212.
[7] Zhu T. Numerical simulation of seismic wave propagation in viscoelastic-anisotropic media using frequency-independent Q wave equation[J]. Geophysics, 2017, 82(4): WA1-WA10.

[8] Sorokin S, Darula R. On attenuation of free and forced waves in an infinitely long visco-elastic layer of a constant thickness[J]. Wave Motion, 2017, 68:114-127.

[9] Yuan C F, Peng S P, Zhang Z J, Liu Z K. Seismic wave propagating in Kelvin-Voigt homogeneous viscoelastic medium [J]. Chinese Science (series D: Earth Sciences) (in Chinese), 2005,35 (10): 957-962.

[10] Yang D, J Teng, Z Zhang and E Liu, A nearly analytic discrete method for acoustic and elastic wave equations in anisotropic media: Bull. Seis. Soc. Amer., 2003, 93, 882–890.

[11] Yang, D, G Song and M Lu, Optimally accurate nearly analytic discrete scheme for wave-field simulation in 3D anisotropic media: Bull. Seis. Soc. Amer., 2007, 97, 1557 – 1569.