Energy levels estimation on a quantum computer by evolution of a physical quantity

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Abstract

We show that the time dependence of mean value of a physical quantity is related with the transition energies of a quantum system. In the case when the operator of a physical quantity anticommutes with the Hamiltonian of a system, studies of the evolution of its mean value allow determining the energy levels of the system. On the basis of the result, we propose a method for determining energy levels of physical systems on a quantum computer. The method opens a possibility to achieve quantum supremacy in solving the problem of finding minimal or maximal energy of Ising model with spatially anisotropic interaction using multi-qubit quantum computers. We apply the method for spin systems (spin in magnetic field, spin chain,

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Ising model on squared lattice) and realize it on IBM’s quantum computers.

Key words: energy levels, spin systems, quantum supremacy, quantum computer.

1 Introduction

Calculation of energy spectrum of a Hamiltonian is an important problem in quantum mechanics. Recent development of quantum computers recognizes considering them as a powerful tool for solving this problem.

One of the algorithms allowing determination of eigenvalues for Hamiltonian is the quantum phase estimation (QPE) algorithm \[1, 2, 3, 4, 5, 6\]. It was originally proposed by Kitaev, Lloyd and Abrams \[1, 2, 3\]. This algorithm is based on finding eigenvalue \( \lambda = e^{i\phi} \) or phase \( \phi \) of a unitary operator. In the case when the unitary operator is the operator of evolution of a quantum system the phase \( \phi \) is related with eigenvalues of the Hamiltonian. Short review on this problem can be found in \[7\]. In \[8\] the method for estimation of the transition energies on the basis of robust phase estimation algorithm was proposed.

Also, hybrid classical-quantum algorithms that allows to examine energy levels are known. Among them are quantum approximate optimization algorithm (it recognizes to find ground state energy and is used to solve optimization problems \[9, 10, 11, 12\]), variational quantum eigensolver (it recognizes to obtain the transition energies \[13, 14, 15, 16\]).

In \[17\] the authors presented an efficient method for estimating the eigenvalues of a Hamiltonian from the time dependence of expectation values of the evolution operator. Originally this idea was suggested in \[18\]. In \[19\] qubit efficient circuit architecture was adopted for the variational quantum eigensolver and qubit efficient scheme to study ground-state properties of quantum many-body systems on a quantum computer was introduced. In \[20\] quantum algorithms (quantum Lanczos, quantum analogue of the minimally entangled typical thermal states, quantum analogue of the minimally entangled typical thermal states) that gives a possibility to detect ground, excited and thermal states on a quantum computer were described.

In this paper we show that studies of time dependence of mean value of a physical quantity allow to extract transition energies of a quantum system. In the case when the operator of the physical quantity anticommutes with the
Hamiltonian such a studies give a possibility to determine the energy levels of a system. On the basis of the results we propose method for detection of energy levels of physical systems on a quantum computer. Using the method, energy levels of spin systems (spin in magnetic field, spin chain, Ising model on squared lattice) are found performing calculations on IBM’s quantum computers.

The paper is organized as follows. In Section 2 we propose a method to estimate transition energies on the basis of studies of evolution of the mean value of physical quantity. In Section 3 we show that in particular case when the operator of a physical quantity anticommutes with the Hamiltonian of a physical system studies of evolution of the mean values of the operator gives a possibility to detect the energy levels of the physical system. In Section 4 we present results for energy levels of spin systems obtained on the basis of calculations on the IBM’s quantum computers. Conclusions are presented in Section 5.

2 Evolution of mean value of physical quantity and transition energies of a physical system

Let us consider a physical system with Hamiltonian \( H \). The evolution of a state vector of the system in time can be written as follows

\[
|\psi(t)\rangle = e^{-iHt/\hbar}|\psi_0\rangle = \sum_i c_i e^{-iE_i t/\hbar} |E_i\rangle,
\]

where we use notation \( |\psi_0\rangle \) for the state of the system at the initial moment of time \( t = 0 \) and expand it over the eigenstates of Hamiltonian \( |E_i\rangle \) namely,

\[
|\psi_0\rangle = \sum_i c_i |E_i\rangle.
\]

\( E_i \) are energy levels of the system.

Let us consider a physical quantity represented by operator \( \hat{A} \) Then evolution of mean value of the quantity reads

\[
A(t) = \langle \psi(t)|\hat{A}|\psi(t)\rangle = \sum_i \sum_j c_i^* c_j e^{i\omega_{ij}t} A_{ij} = \sum_i \sum_j g_{ij} e^{i\omega_{ij}t},
\]
where $\omega_{ij} = (E_i - E_j)/\hbar$ is frequency of transition between energy levels $E_i$ and $E_j$, $A_{ij} = \langle E_i|\hat{A}|E_j \rangle$ is matrix element of operator $\hat{A}$ and $g_{ij} = c_i^*c_jA_{ij}$ is hermitian matrix.

The goal of this paper is to extract from function $A(t)$ the frequencies $\omega_{ij}$. For this purpose we consider the following transformation of expectation value

$$A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt A(t)e^{-i\omega t}.$$  

(4)

Substituting (3) into (4) we find

$$A(\omega) = \sum_{i} \sum_{j} g_{ij} \delta(\omega - \omega_{ij}).$$

(5)

Thus function $A(\omega)$ has $\delta$ - peaks at $\omega = \omega_{ij}$. It allows knowing $A(\omega)$ to find the frequencies of transitions $\omega_{ij}$. Note that $A^*(\omega) = A(-\omega)$ and in general contains real and imaginary parts

$$A(\omega) = A_1(\omega) + iA_2(\omega),$$

(6)

where

$$A_1(\omega) = \int_{0}^{\infty} dt A(t) \cos(\omega t),$$

(7) $$A_2(\omega) = \int_{0}^{\infty} dt A(t) \sin(\omega t).$$

(8)

In order to apply this result for finding the frequencies of transitions on a quantum computer we have to take into account that on a quantum computer we can find mean values of physical quantity $A(t)$ at some fixed moments of time. Thus we write $t = \tau n$, were $n = -N, -N+1, ... ,N-1, N$ and $\tau$ is some fixed time interval. Then for $A(\omega)$ we have

$$A(\omega) = \sum_{i} \sum_{j} g_{ij} \frac{1}{2\pi} \sum_{n=-N}^{N} \tau e^{-i(\omega - \omega_{ij})\tau n} =$$

$$\sum_{i} \sum_{j} g_{ij} \frac{1}{2\pi} \tau \left( 1 + 2\cos((\omega - \omega_{ij})(T + \tau)/2) \frac{\sin((\omega - \omega_{ij})T/2)}{\sin((\omega - \omega_{ij})\tau/2)} \right),$$

(9)
where $T = N\tau$. At $\tau \to 0$ and fixed $T$ we find

$$A(\omega) = \sum_i \sum_j g_{ij} \frac{1}{\pi} \frac{\sin((\omega - \omega_{ij})T)}{(\omega - \omega_{ij})}.$$  

Each term in this expression takes maximal value at $\omega = \omega_{ij}$ which is $Tg_{ij}/\pi$.

Note that (10) corresponds to interval of integration in (4) from $-T$ to $T$. When in additional $T \to \infty$ then (10) tends to (5).

In general the idea to use these results for finding frequencies of transition between different energy levels is the following. We study evolution of quantum system and find time dependence of some physical quantity $A(t)$ on a quantum computer. Then we find Fourier transformation $A(\omega)$ of this function. Maximal values of $A(\omega)$ allow to detect the frequencies of transitions.

3 Detecting of energy levels of a physical system by evolution of mean value

In this section we show that in the particular case studying evolution of the mean value of operator $\hat{A}$ one can determine the directly energy levels of a physical system. Let us consider operator which anticommutes with Hamiltonian of a system

$$\{\hat{A}, H\} = 0.$$  

If such operator exists the energy spectrum of a system is symmetric with respect to $E \to -E$. In other worlds if $E$ is the energy level of a system, $-E$ is the energy level too. In order to show this let us consider stationary Schrödinger equation

$$H|\psi\rangle = E|\psi\rangle.$$  

Applying operator $\hat{A}$ anticommuting with $H$ to left and right hand side of the equation we have

$$H\hat{A}|\psi\rangle = -E\hat{A}|\psi\rangle.$$  

Thus $A|\psi\rangle$ is eigenstate with energy $-E$. Of course here we assume that state $A|\psi\rangle$ exists. In this case for mean value of the operator $\hat{A}$ we find

$$A(t) = \langle \psi_0 | e^{iHt/\hbar} \hat{A} e^{-iHt/\hbar} | \psi_0 \rangle = \langle \psi_0 | e^{2iHt/\hbar} \hat{A} | \psi_0 \rangle = \langle \psi_0 | \hat{A} e^{-2iHt/\hbar} | \psi_0 \rangle,$$  

where $\psi_0$ is initial state.
where we taking into account that $\hat{A} e^{-iHt/\hbar} = e^{iHt/\hbar} \hat{A}$.

Representing initial state in the form (2) the mean value reads

$$A(t) = \sum_j g_j e^{-i2\omega_j t}, \quad (16)$$

where $g_j = \sum c_i^* A_{ij} c_j$. The mean value $A(t) = A^*(t)$ is real, so we can also write

$$A(t) = \sum_j g_j^* e^{i2\omega_j t}. \quad (17)$$

Then $A(\omega)$ has a sharp pics at $\omega = 2\omega_i$

$$A(\omega) = \sum_j g_j^* \delta(\omega - 2\omega_i). \quad (18)$$

So, analysis of $A(\omega)$ allow to determine the energy spectrum of a physical system. In the next Section we apply this method for spin systems that are one of the most suitable systems for modeling them on a quantum computer.

4 Detecting energy levels of spin systems on IBM’s quantum computer

4.1 Spin in magnetic field

As the first example we consider a spin in magnetic field directed along $z$-axis with Hamiltonian

$$H = \hbar \omega_0 \sigma^z. \quad (19)$$

Let us apply the method proposed in the previous section and find the energy levels of the system using a quantum computer.

It is easy to write operator that anticommutes with the Hamiltonian (19). For example one can choose $\hat{A} = \sigma_x \{\sigma_x, \hbar \omega_0 \sigma^z\} = 0$ and study its mean value. We consider the initial state as

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle. \quad (20)$$
The state (20) is the eigenstate of operator $\sigma^x$ with eigenvalue 1 and is not the eigenstate of Hamiltonian (19). So, the mean value of $\hat{A}$ reads

$$A(t) = \langle \sigma^x(t) \rangle = \langle +|e^{i\omega_0\sigma^z t}\sigma^x e^{-i\omega_0\sigma^z t}|+ \rangle =$$

$$= \langle +|e^{2i\omega_0\sigma^z t}\sigma^x |+ \rangle = \frac{1}{2}(e^{2i\omega_0 t} + e^{-2i\omega_0 t}). \quad (21)$$

Then function

$$A(\omega) = \frac{1}{2}(\delta(\omega - 2\omega_0) + \delta(\omega + 2\omega_0)) \quad (22)$$

has peaks at $\omega = \pm 2\omega_0$ that are double eigenvalues of Hamiltonian (19) in the units of $\hbar$.

To study evolution of the mean value $\langle \sigma^x(t) \rangle$ given by (21) on a quantum computer we consider quantum protocol Fig. 1. Applying Hadamar gate $|0\rangle \to \sqrt{H}|0\rangle$ and $RZ(\phi)$ one prepares the state $\exp(-i\phi\sigma^z/2)|+\rangle$. To measure the mean value of the operator $\sigma^x$ in the state $\exp(-i\phi\sigma^z/2)|+\rangle$, we take into account that the operator $\sigma^x$ can be represented as $\sigma^x = \exp(-i\pi\sigma^y/4)\sigma^z \exp(i\pi\sigma^y/4)$. Therefore to find $\langle \sigma^x \rangle$ the Y-rotation gate $RY(\pi/2)$ is applied before measurement in the standard basis (see Fig. 1). The value $\langle \sigma^x \rangle$ can be calculated using the results of measurement as

$$\langle \sigma^x \rangle = \langle +|e^{i\phi\sigma^z/2}\sigma^x e^{-i\phi\sigma^z/2}|+ \rangle = \langle \tilde{\psi}|\sigma^x|\tilde{\psi} \rangle = |\langle \tilde{\psi}|0\rangle|^2 - |\langle \tilde{\psi}|1\rangle|^2, \quad (23)$$

where

$$|\tilde{\psi} \rangle = e^{i\pi\sigma^y/4}e^{-i\phi\sigma^z/2}|+\rangle. \quad (24)$$

Applying quantum protocol Fig. 1 with different values of parameter $\phi = 2\hbar\omega_0 t$ we measure dependence of the mean value of $\langle \sigma^x \rangle$ on time. Such studies were done on IBM’s quantum computer ibmq_manila. The structure of the quantum device is presented on Fig. 2.
We realized quantum protocol Fig. 1 on qubit $q[0]$ of ibmq_manila for parameter $\phi/2 = \hbar \omega_0 t$ changing from $-8\pi$ to $8\pi$ with the step $\pi/12$. On the basis of the obtained results taking into account (23) we find mean values $\langle \sigma^x(t) \rangle$ for $t = n\tau$, $\tau = \pi/12\omega_0$, $n = -N, -N + 1, \ldots, N - 1, N$, $N=96$ and calculate

$$A(\omega) = \sum_{n=-N}^{N} \langle \sigma^x(n\tau) \rangle e^{i\omega n\tau}$$  \hspace{1cm} (25)

The results of calculations are presented on Fig. 3. For convenience we put $\omega_0 = 1$. From Fig. 3 we see that the real part of $A(\omega)$ has peaks at $\omega = \pm 2$ that according to (18) correspond to energies $E = \pm \hbar \omega_0$. 

Figure 3: Result of detecting of the energy levels of a spin in magnetic field using studies of evolution of mean value $\langle \sigma^x \rangle$ on IBM’s quantum computer ibmq_manila. The real part of $A(\omega)$ has peaks at $\omega = \pm 2$ that correspond to energies $E = \pm \hbar \omega_0$. 

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**Figure 2:** Structure of IBM’s quantum computer ibmq_manila. Arrows link qubits between which the controlled NOT gate can be directly applied.
4.2 Spin chain

As the second example we consider a chain of three spins with the Ising interaction described by the following Hamiltonian

$$H = J \sigma^z_0 \sigma^z_1 + J \sigma^z_1 \sigma^z_2,$$

(26)

here $J$ is the interaction coupling. In this case we chose operator $\hat{A}$ to be $\sigma^x_1$.

Note that $\{\sigma^x_1, H\} = 0$. Quantum protocol for studies of $\langle \sigma^x_1(t) \rangle$ at different moments of time is presented on Fig. 4.

$$e^{-iHt/\hbar} |+\rangle$$

Figure 4: Quantum protocol for studies of mean value evolution $\langle \sigma^x(t) \rangle$ in the case of spin chain, $\phi = 2Jt/\hbar$

In the quantum protocol we take into account that operator $\exp(-iJ \sigma^z_i \sigma^z_j/\hbar)$ with exactness to total phase factor can be rewritten as $CX_{ij}RZ_j(2Jt/\hbar)CX_{ij}$, here $CX_{ij}$ is the controlled-NOT gate that acts on qubit $q[i]$ as a control and on qubit $q[j]$ as a target, Z-rotation gate $RZ_j(2Jt/\hbar)$ acts on qubit $q[i]$. So, with exactness to total phase factor the state $e^{-iHt/\hbar} |+\rangle$ is prepared in the result of action of gates $CX_{01}RZ_1(2Jt/\hbar)CX_{01}CX_{12}RZ_2(2Jt/\hbar)CX_{12}H_0H_1H_2$ on the state $|000\rangle$ (see Fig. 4). Then to detect the mean value $\langle \sigma^x_1(t) \rangle$ similarly as in the previous subsection we implement rotation of the state of qubit $q[1]$ around the $x$ axis by $\pi/2$ and perform measurement of the state in the standard basis (see Fig. 4).

Quantum protocol Fig. 4 was realized on qubits $q[0], q[1], q[2]$ of ibmq_manila. Note that the quantum device has the chain structure (see Fig. 1). So, the CNOT gates in Fig. 4 can be applied directly to the respective qubits. Changing $\phi/2 = Jt/\hbar$ from $-8\pi$ to $8\pi$ with the step $\pi/12$, we detect the mean values $\langle \sigma^x(t) \rangle$ for $t = n\tau$, $\tau = \hbar \pi/J12$, $n = -N, -N + 1, \ldots, N - 1, N$, $N = 96$ and calculate $A(\omega)$ (25). The results of calculations are presented in Fig. 5. In Fig. 5 for convenience we put $\hbar/J = 1$. The real part of $A(\omega)$ has peaks at $\omega = 0$ and $\omega = \pm 4$ that correspond to the energy levels $E = 0$, $E = \pm 2J$, respectively.
Figure 5: Result of detecting of the energy levels of the spin chain by studies of evolution of mean value \( \langle \sigma^x_1 \rangle \) on IBM’s quantum computer ibmq_manila. Peaks of Re\( A(\omega) \) at \( \omega = 0 \) and \( \omega = \pm 4 \) correspond to energies \( E = 0, E = \pm 2J \).

### 4.3 Ising model on squared latice

Let us consider the Ising model

\[
H = \frac{1}{2} \sum_{i,j} J_{ij} \sigma^z_i \sigma^z_j. \tag{27}
\]

where \( J_{ij} \) are the interaction coupling.

We study square-lattices with isotropic and spatially anisotropic Ising interactions described, respectively, by the following Hamiltonians

\[
H_1 = J \sigma^z_0 \sigma^z_1 + J \sigma^z_1 \sigma^z_2 + J \sigma^z_2 \sigma^z_3 + J \sigma^z_3 \sigma^z_0, \tag{28}
\]
\[
H_2 = -J \sigma^z_0 \sigma^z_1 + J \sigma^z_1 \sigma^z_2 + J \sigma^z_2 \sigma^z_3 + J \sigma^z_3 \sigma^z_0, \tag{29}
\]

and detect the corresponding energy levels on IBM’s quantum computer ibmq_manila.

As the initial states we chose \( |\psi_0\rangle = |++++\rangle \). To detect the energy levels of the corresponding systems we examine evolution of the operator \( \hat{A} = \sigma^z_0 \sigma^z_2 \), \( \{\sigma^z_0 \sigma^z_2, H_1\} = \{\sigma^z_0 \sigma^z_2, H_2\} = 0 \). Quantum protocol for studies of \( \langle A(t) \rangle \) is presented on Fig. 6. To examine the case of spatially anisotropic
Figure 6: Quantum protocol for studies of mean value evolution \( \langle \sigma_0^x(t)\sigma_2^z(t) \rangle \) in the case of the Ising model on squared lattice with (28), \( \phi = 2Jt/\bar{\hbar} \).

Ising interaction (29) we change the sign of parameter \( \phi \) in \( RZ \) gate acting on the qubit \( q[1] \). The results for discrete Fourier transformation \( A(\omega) \) are presented on Fig. 7. Function \( \text{Re} A(\omega) \) (Fig. 7(a)) has peaks at \( \omega = 0 \) and \( \omega = \pm 8 \) that correspond to energies \( E = 0, E = \pm 4J \) of the Ising model on squared lattice (29). In the case of Ising model with spatially anisotropic Ising interaction the function \( \text{Re} A(\omega) \) has peaks at \( \omega = \pm 4 \) (Fig. 7(b)) corresponding to the energies \( E = \pm 2J \).

Figure 7: Results of detecting of the energy levels of Ising model on squared lattice with isotropic (28) (a) and spatially anisotropic Ising interactions (29) (b) by studies of evolution of mean value \( \langle \sigma_0^x\sigma_2^z \rangle \) on IBM’s quantum computer ibmq_manila. The peaks of \( \text{Re} A(\omega) \) (a) at \( \omega = 0 \) and \( \omega = \pm 8 \) correspond to energies \( E = 0, E = \pm 4J \). The peaks of \( \text{Re} A(\omega) \) (b) at \( \omega = \pm 4 \) correspond to energies \( E = \pm 2J \).
Finally, we also detect energy levels of a Ising model on squared lattice in the case of six spins (qubits). The calculations were made on 15-qubit quantum device ibmq.melbourne with structure given in Fig. 8 and on the quantum simulator ibmq.qasm.simulator.

![Figure 8: Structure of IBM’s quantum computer ibmq.melbourne. Arrows link qubits between which the controlled NOT gate can be directly applied.](image)

Taking into account the structure of the quantum computer ibmq.melbourne we study a system described by the following Hamiltonian

\[ H = J \sigma^z_0 \sigma^z_1 + J \sigma^z_1 \sigma^z_2 + J \sigma^z_2 \sigma^z_{12} + J \sigma^z_{13} \sigma^z_{14} + J \sigma^z_1 \sigma^z_{13} + J \sigma^z_0 \sigma^z_{14}, \]

(30)

We choose the initial state to be \(|++++++++\rangle\) and study the evolution of mean value of operator \(\hat{A} = \sigma^z_0 \sigma^z_2 \sigma^z_{13}\). Note that \(\hat{A}\) anticommutes with Hamiltonian (30). For this purpose quantum protocol Fig. 9 was implemented for different values of \(\phi = 2Jt/\hbar\) on qubits \(q[0]\), \(q[1]\), \(q[2]\), \(q[12]\), \(q[13]\), \(q[14]\) of ibmq.melbourne.

![Figure 9: Quantum protocol for studies of mean value evolution \(\langle \sigma^z_0(t) \sigma^z_2(t) \sigma^z_{13}(t) \rangle\) in the case of the Ising model on squared lattice with (30), \(\phi = 2Jt/\hbar\).](image)
We change the parameter $\phi/2 = Jt/h$ from $-8\pi$ to $8\pi$ with step $\pi/24$ and detect the mean values $\langle \sigma_0^x(t)\sigma_2^x(t)\sigma_{13}^x(t) \rangle$ for $t = n\tau, \tau = h\pi/J24, n = -N, -N+1, \ldots N-1, N, N=192$ on quantum computer ibmq_melbourne and on quantum simulator ibmq_qasm_simulator. On the basis of the obtained results we find $A(\omega)$ (25) (see Fig. 10). The peaks of the real part of the function $A(\omega)$ at $\omega = 0$, $\omega = \pm 6$, $\omega = \pm 14$ correspond to energies $E = 0$, $E = \pm 3J$, $E = \pm 7J$ in the system (30). Note that the peaks obtained on the basis of calculations on the quantum device (see Fig. 10 (b)) are not so clear as that obtained on quantum simulator ibmq_qasm_simulator and in the case of spin in magnetic field, spin chain, Ising model on squared lattice (28), (29). This is because the quantum protocol Fig. 9 contains more gates and measurements than that considered in the previous examples (Fig. 4, (Fig. 5) that leads to accumulation of errors. Nevertheless even in this case the method of detecting of the energy levels by studies of evolution of mean value gives as possibility to detect the energy levels.

![Figure 10](image)

Figure 10: Results of detecting of the energy levels of Ising model on squared lattice (30) on ibmq_qasm_simulator (a) and ibmq_melbourne (b) by studies of evolution of mean value $\langle \sigma_0^x\sigma_2^x\sigma_{13}^x \rangle$. The peaks of $\text{Re}A(\omega)$ at $\omega = 0$, $\omega = \pm 6$, $\omega = \pm 14$ correspond to energies $E = 0$, $E = \pm 3J$, $E = \pm 7J$.

Note that it is not a trivial problem to find minimal or maximal eigenvalue of Ising model with spatial anisotropy. The proposed algorithm allows to solve this problem on quantum computer and we hope that using quantum
computers with largest numbers qubits it will be possible to achieve quantum supremacy.

5 Conclusions

The method for detecting transition energies by studies of evolution of mean value of physical quantity on quantum computer has been proposed. In the case when the operator of a physical quantity anticommutes with the Hamiltonian of a physical system, the proposed method gives a possibility to detect energy levels of the system. A spin in magnetic field, spin chain with Ising interaction, Ising model on squared lattice with isotropic and spatially anisotropic Ising interaction have been studied. We have examined evolution of mean values of operators anticommuting with Hamiltonians of the systems on IBM’s quantum computers ibmq_manila, ibmq_melbourne using quantum protocols presented on Figs. 1, 4, 6, 9 and detect energy levels of the corresponding systems (see Figs. 3, 5, 7, 10).

Note that for the case when different $J_{ij}$ takes different values with different signs there is not trivial problem to find minimal or maximal eigenvalue of Ising model (27) one has a combinatorial optimization problem. Therefore the proposed in this paper method of finding the energy levels on a quantum computer opens a possibility to achieve quantum supremacy with development of quantum computers with largest number of qubits.

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References

[1] D. S. Abrams and S. Lloyd, Simulation of many-body fermi systems on a universal quantum computer Phys. Rev. Lett. 79, 2586 (1997).

[2] D. S. Abrams and S. Lloyd, Quantum algorithm providing exponential speed increase for finding eigenvalues and eigenvectors, Phys. Rev. Lett. 83, 5162 (1999).
[3] A. Y. Kitaev, Quantum computations: algorithms and error correction, Russian Math. Surveys 52 1191 (1997).

[4] M. Dobšíček, G. Johansson, V. Shumeiko, G. Wendin, Arbitrary accuracy iterative quantum phase estimation algorithm using a single ancillary qubit: A two-qubit benchmark, Phys. Rev. A 76, 030306 (2007).

[5] S. Paesani, A. A. Gentile, R. Santagati et al Experimental Bayesian Quantum Phase Estimation on a Silicon Photonic Chip, Phys. Rev. Lett. 118, 100503 (2017).

[6] J. B. Parker, I. Joseph, Quantum phase estimation for a class of generalized eigenvalue problems Phys. Rev A 102, 022422 (2020).

[7] P. M. Q. Cruz, G. Catarina, R. Gautier, J. Fernández-Rossier, Optimizing quantum phase estimation for the simulation of Hamiltonian eigenstates, Quantum Sci. Technol. 5 044005 (2020).

[8] A. E. Russo, K. M. Rudinger, B. C. A. Morrison, A. D. Baczewski, Evaluating energy differences on a quantum computer with robust phase estimation, Phys. Rev. Lett. 126, 210501 (2021).

[9] E. Farhi, J. Goldstone, S. Gutmann, A Quantum Approximate Optimization Algorithm [arXiv:1411.4028] (2014).

[10] E. Farhi, J. Goldstone, S. Gutmann, A Quantum Approximate Optimization Algorithm Applied to a Bounded Occurrence Constraint Problem, [arXiv:1412.6062] (2014).

[11] N. Moll, P. Barkoutsos, L. S. Bishop et al. Quantum optimization using variational algorithms on near-term quantum devices. Quantum Sci. Technol. 3, 030503 (2018).

[12] F. G. Fuchs, H. O. Kolden, N. H. Aase, G. Sartor Efficient Encoding of the Weighted MAX k-CUT on a Quantum Computer Using QAOA SN Computer Science 2, 89 (2021).

[13] A. Peruzzo, J. McClean, P. Shadbolt et al A variational eigenvalue solver on a photonic quantum processor Nat. Commun. 5, 4213 (2014).
[14] J. R. McClean, J. Romero, R. Babbush, A. Aspuru-Guzik, The theory of variational hybrid quantum-classical algorithms New J. Phys. 18, 023023 (2016).

[15] P. J. J. O’Malley, R. Babbush, I. D. Kivlichan, Scalable Quantum Simulation of Molecular Energies Phys. Rev. X 6, 031007 (2016).

[16] R. M. Parrish, E. G. Hohenstein, P. L. McMahon, Quantum Computation of Electronic Transitions Using a Variational Quantum Eigensolver Phys. Rev. Lett. 122, 230401 (2019).

[17] R. D. Somma, Quantum eigenvalue estimation via time series analysis, New J. Phys. 21 123025 (2019).

[18] R. Somma, G. Ortiz, J. E. Gubernatis, E. Knill, R. Laflamme, Simulating physical phenomena by quantum networks, Phys. Rev. A, 65, 042323 (2002)

[19] Jin-Guo Liu, Yi-Hong Zhang, Yuan Wan, Lei Wang, Variational quantum eigensolver with fewer qubits Phys. Rev. Research 1, 023025 (2019).

[20] M. Motta, Ch. Sun, A. T. K. Tan et al, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution, Nature Physics 16, 205 (2020).