Monopoles, confinement and the photon propagator in QED$_3$

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We study the lattice gauge boson propagator of 3D compact QED in Landau gauge at zero and non-zero temperature. Non-perturbative effects are reflected by the generation of a mass $m$, by an anomalous dimension $\alpha$ and by the photon wave function renormalisation $Z$. These effects can be attributed to monopoles: they are absent in the propagator of the regular part of the gauge field. The rôle of Gribov copies is carefully investigated.

Three-dimensional compact electrodynamics (cQED$_3$) shares two essential features with QCD, confinement [1] and chiral symmetry breaking [2]. Confinement of electrically charged particles is caused by a plasma of monopoles which emerge due to the compactness of the gauge field. Recently we have interpreted the deconfinement phase transition in cQED$_{2+1}$ from the monopole point of view and have shown that the transition is independent of the strength of external fields [3]. In Ref. [4] we have demonstrated how the confinement property is manifest in the gauge fields [3]. In Ref. [1] we have shown that all nontrivial effects originate from the singular fields of the monopoles and disappear with the formation of dipoles.

Here we report on an extension of this study to the case of $T = 0$ and a careful investigation on the severity of the Gribov copy problem. At $T \neq 0$, it is important to realize that more structure functions are necessary for a full description. We found that one of the finite $T$ propagators is extremely gauge fixing sensitive. For our lattice study we have adopted the Wilson action, $S[\theta] = \beta \sum_\mu (1 - \cos \theta_\mu)$, where $\theta_\mu$ is the $U(1)$ field strength tensor corresponding to the compact link field $\theta_1$. The lattice coupling $\beta$ is related to the lattice spacing $a$ and the continuum coupling constant $g_3$ of the 3D theory, $\beta = 1/(a g_3^2)$. The Landau gauge is defined as the maximum of $\sum_i \cos \theta_i^2$ with respect to gauge transformations $G$. For the Monte Carlo algorithm (combining local and global updates) and the gauge fixing procedure see Ref. [5]. Simulations for $T = 0$ have been performed on a $32^3$ lattice, those for $T > 0$ on a $32^2 \times 8$ lattice.

We have studied the gauge boson propagator in Landau gauge in lattice momentum space $\vec{k}$ where it is defined as $D_{\mu\nu}(\vec{p}) = \langle A_{\vec{k},\mu}^\dagger A_{-\vec{k},\nu} \rangle$ in terms of the Fourier transformed gauge potential $A_{\vec{k},\mu}$ for $p_\mu = (2/a) \sin(2\pi k_\mu / L_\mu)$. The gauge field in lattice position space is taken as $A_{\vec{n}+\frac{1}{2},\mu} = \sin \theta_{\vec{n},\mu} / (g_3 a)$. The most general tensor structure of $D_{\mu\nu}$ at $T = 0$ is given by

$$ D_{\mu\nu}(\vec{p}) = P_{\mu\nu}(\vec{p}) D(p^2) + (p_\mu p_\nu) / p^2 F(p^2) / p^2 \quad (1) $$

with the 3-dimensional transverse projection operator $P_{\mu\nu}(\vec{p}) = \delta_{\mu\nu} - (p_\mu p_\nu) / p^2$. If the Landau gauge is fulfilled exactly, $F(p^2) \equiv p_\mu D_{\mu\nu} p_\nu = 0$. The transverse and longitudinal propagators, $D$ and $F$, are extracted by projection and are found approximately rotationally invariant or vanishing.

For $T > 0$ the propagator lacks $O(3)$ rotational

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gauge transforms of the original configuration. The monopole part of the transversal propagator \( D_{\text{mono}} \) reaches a maximum in the low momentum region before it drops towards \( p^2 = 0 \). The regular (photon) part is singular at \( p^2 \to 0 \) like \( D_{\text{phot}} \sim 1/p^2 \), while the full transversal propagator is not. \( p^2 D_{\text{phot}} \) is flat independently of \( \beta \). As expected, the longitudinal part \( F \) vanishes within errors.

Following \[1\] we describe \( D \) by the function
\[
D(p^2) = (Z/\beta) m^{2\alpha} / [p_{\text{phot}}^{2(1+\alpha)} + m^{2(1+\alpha)}] + C. \tag{3}
\]

The photon part \( D_{\text{phot}} \) is fitted using \[3\] with \( \alpha = m = 0 \). At finite \( T \), the propagator data for \( D_L \) and \( D_T \) are analyzed for \( p_3 = 0 \), as a function of \( p^2 \) using the same fit function \[3\]. We have investigated the Gribov copy dependence of \( D \) and the finite \( T \) propagators \( D_L \) and \( D_T \). In this respect, the zero temperature \( D \) behaves similarly as \( D_L \). Some results are summarized in Fig. 2 at

![Figure 1](image1.png)

**Figure 1.** The propagator \( p^2 D \) and its contributions, as well as the (vanishing) propagator \( F \) vs. \( p^2 \) at \( \beta = 1.8 \). Only part of the data points averaged over the same \( p^2 \) are plotted.

In Fig. 1 we show for \( T = 0 \) at \( \beta = 1.8 \) the measured transverse propagator \( D \) and its different components in lattice momentum space. To create the best Landau gauge realization, we have evaluated \( N_G = 20 \) Gribov copies, independent maxima of \( F \) obtained from maximizing random

![Figure 2](image2.png)

**Figure 2.** Fitted \( \alpha \) and \( m \) vs. \( N_G \) for \( D_T \) and \( D_L \) at different \( \beta \)'s and updates. For \( D_L \) at \( \beta = 2.6 \) we have \( \alpha = 0 \) and \( m = 0 \).
\(\beta\) values below and above the phase transition. After a few Gribov copy attempts \(N_G\) all fit parameters of \(D_L\) are almost insensitive to \(N_G\). The results at large \(N_G\) are independent of whether the total Monte Carlo algorithm includes global updates or not (see the label “local ” for the latter case).

The parameters for \(D_T\) (and \(D_T(|p|, 0)\) itself), however, are strongly dependent on \(N_G\) if global updates are included. At \(N_G = 100\) a plateau is not yet reached. In the deconfinement phase, results are also sensitive to the presence of global update steps, in particular at high values of \(\beta\). Runs with only local updates produce propagators with significantly lower fit values of \(\alpha_T\) and \(m_T\), nearer to what is expected.

Realizing this, we decided to use in the final measurements at \(T > 0\), in the region \(\beta \geq 2.0\), only local updates and to use not less than \(N_G = 100\) Gribov copies per configuration. Deep in confinement (below \(\beta = 2.0\) for \(T \neq 0\), and generally for \(T = 0\)) our standard was \(N_G = 20\), with global updates included in the Monte Carlo. Thus, the results for \(D_T\) at larger \(\beta\) should be considered only as qualitative. The best fit parameters \(\alpha\) and \(m\) of the propagators \(D, D_L\) and \(D_T\) are presented in Fig. 3 as functions of \(\beta\). The anomalous dimensions \(\alpha \neq 0\) in the confinement region for all propagators are functions not only of the monopole density (which is monotonously decreasing with growing \(\beta\)). The cluster structure of the monopole configurations seems to play a significant role. The fit parameters \(\alpha_L\) and \(m_L\) of \(D_L\) vanish at \(\beta_c\), giving a clear signal of the finite temperature phase transition (1) caused by the formation of dipoles. This is not the case for \(\alpha_T\) and \(m_T\) of the \(D_T\) propagator. We associate the (so far) inconclusive behaviour of \(D_T\), at the transition and beyond, with the insufficient gauge fixing. The mass \(m\) for \(T = 0\) is in good agreement with the theoretical prediction (2) valid for a dilute monopole gas.

In conclusion, we have studied the gauge boson propagator in cQED\(_3\), both at zero and non–zero temperatures. We have found that the propagators in all cases under investigation can be fitted by (1) which is a sum of the massive propagator with an anomalous dimension plus a contact term. Similar to \(D_L\) in the confinement phase (1), \(D\) for \(T = 0\) has an anomalous dimension \(\alpha \neq 0\). The behavior of \(D_T\) at the phase transition is obscured by the extreme sensitivity with respect to Gribov copy effects, i.e. remaining wrapping Dirac strings.

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