A relativistic mixing-layer model for jets in low-luminosity radio galaxies

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ABSTRACT
We present an analytical model for jets in Fanaroff & Riley Class I (FR I) radio galaxies, in which an initially laminar, relativistic flow is surrounded by a shear layer. We apply the appropriate conservation laws to constrain the jet parameters, starting the model where the radio emission is observed to brighten abruptly. We assume that the laminar flow fills the jet there and that pressure balance with the surroundings is maintained from that point onwards. Entrainment continuously injects new material into the jet and forms a shear layer, which contains material from both the environment and the laminar core. The shear layer expands rapidly with distance until finally the core disappears, and all of the material is mixed into the shear layer. Beyond this point, the shear layer expands in a cone and decelerates smoothly. We apply our model to the well-observed FR I source 3C 31 and show that there is a self-consistent solution. We derive the jet power, together with the variations of mass flux and entrainment rate with distance from the nucleus. The predicted variation of bulk velocity with distance in the outer parts of the jets is in good agreement with model fits to VLA observations. Our prediction for the shape of the laminar core can be tested with higher-resolution imaging.

Key words: galaxies:active \textendash{} galaxies: jets \textendash{} galaxies: individual: 3C 31 \textendash{} galaxies: ISM

1 INTRODUCTION
Extragalactic radio sources were divided into two morphological classes by Fanaroff & Riley (1974). FR I sources have an edge-darkened structure, whereas FR II sources are edge-brightened with prominent outer hot-spots. This classification has proved to be extremely robust: the division between the classes depends primarily on radio luminosity (Fanaroff & Riley 1974), with FR II sources being more powerful, but also on the stellar luminosity of the host galaxy (Ledlow & Owen 1996). There are significant differences between the structures of the jets in the two classes: those in FR I sources often flare close to the nucleus and have large opening angles, whereas their equivalents in FR II sources are highly collimated out to the hot-spots (Bridle 1984). There is good evidence that FR I jets are initially relativistic, but decelerate on kiloparsec scales, whereas FR II jets remain relativistic until they terminate (e.g. Lainé 1993).

The process of deceleration in FR I jets appears to be complex, and may involve a transition to turbulent flow. In addition, the sources have a wide range of morphologies, ranging from well-defined lobes similar to those in FR II sources to extended plumes or tails (Parma, de Ruiter & Fanti 1996). For these reasons, attempts to construct global models of the evolution of FR I sources, linking observable quantities such as linear size and radio luminosity, have been less straightforward than the equivalents for FR II sources (e.g. Scheuer 1974; Kaiser & Alexander 1997; Kaiser, Dennett-Thorpe & Alexander 1997). This suggests that the jet flows are essentially laminar. Part of the motivation for the present study is therefore to construct a simple model of FR I jets for use as input to global models. We consider twin-jet sources, which make up at least one half of the FR I population (Parma et al. 1996), excluding wide-angle tails and fat-double sources (Owen & Laing 1984; Owen & White 1991), whose jet properties differ significantly.

Over the last few years, detailed modelling of deep VLA observations of jets in five FR I sources has allowed us to quantify their geometries, velocity distributions, magnetic fields and emissivity distributions in three
dimensions. We refer in detail to the analysis of 3C 31 by Laing & Bridle (2002a, hereafter LB02a); observations and models of a further four sources have subsequently been published (Canvin & Laing 2004; Canvin et al. 2005; Laing et al. 2006). A consistent picture of FR I jet deceleration on kiloparsec scales has emerged from these studies. The flow velocities are \( \beta = v/c \approx 0.8 - 0.9 \) where the jets first brighten abruptly, typically at \( \sim 1 \) kpc from the nucleus. The jets flare and then recollimate, decelerating rapidly to speeds of \( \beta \approx 0.1 - 0.4 \). The best-fitting transverse velocity profiles appear to be approximately self-similar. At least in the 4/5 cases where the jets appear to be propagating in contact with the interstellar medium of the host galaxy rather than inside radio lobes, they are roughly 30% faster on-axis than at their edges. Nevertheless, an evolution of the velocity profiles with distance from the nucleus is not excluded. In particular, the transverse velocity variations are poorly constrained where the jets first brighten abruptly and a top-hat profile would also be consistent with the observations in these regions of all five sources.

In order to decelerate, a jet must entrain matter, either from stars within its volume (Phinney 1983; Komissarov 1994) or by ingestion of the surrounding material at its boundary, as originally suggested by Baan (1984), De Young (1981) and Begelman (1982). In the latter case, the transverse velocity profile almost inevitably evolves with distance from the nucleus.

X-ray observations can be used to infer the temperature, density and pressure profiles of the hot gas associated with the host galaxies of FR I radio galaxies (e.g., Hardcastle et al. 2002; Worrall et al. 2003; Hardcastle et al. 2003). Together with the velocity distributions derived from modelling of the radio emission, these can be used in a conservation-law analysis (Bicknell 1994, hereafter B94) to derive jet energy fluxes and the variations of mass flux, pressure, internal density and entrainment rate with distance from the nucleus (Laing & Bridle 2002b, hereafter LB02b). Such an analysis is quasi-one-dimensional and therefore adopts values for the flow variables (in particular the velocity) averaged across the jet cross-section. This is reasonable if the velocity profiles have restricted ranges and do not evolve significantly with distance down the jets, as is consistent with the observations of 3C 31 (LB02b). If FR I jets are in pressure equilibrium with their surroundings after they recollimate, this analysis requires that a significant overpressure drives the initial flaring.

An alternative approach, which would also be consistent with the observations, is to postulate that the transverse velocity profiles evolve significantly as the jets interact with the external medium. The first approximation is then to assume pressure equilibrium between the jet and its surroundings and to take explicit account of the interaction between the jets and their surroundings using a simple mixing-layer model. This is the subject of the present paper. The key assumption is that there is a turbulent mixing layer between the jet and its environment, produced by the interaction of the two components. The mixing layer grows both into the jet and into the environment, and the initially laminar jet eventually becomes fully turbulent. As in the quasi-one-dimensional analysis of Laing & Bridle (2002b), we use the relativistic formulation of the laws of conservation of mass, momentum and energy given by B94.

We describe the geometry of the jet-layer model in Section 2. The relativistic conservation laws are introduced in Section 3. We derive and discuss the solutions for our model in Section 4. In Section 5 we apply our model to observations of 3C 31. We discuss the effects of varying model parameters in Section 6 and summarize our conclusions in Section 7.

2 STRUCTURE OF AN FR I JET

The basic structure of an FR I jet in our model is shown in Figure 1. Following the definition given by LB02a, we divide the jet into flaring and outer regions. Close to the nucleus in the flaring region, the outer isophotes have small, but increasing opening angles. Further out, they spread rapidly and then recollimate. In the outer region, the expansion is conical. The radio emission close to the base of the flaring region is usually faint and it is always possible to identify a distance from the nucleus where the jet brightens abruptly. We refer to this location as the brightening point.

We assume pressure equilibrium with the surroundings at all distances from the nucleus and adopt the simplest possible prescription for velocity variations following Canto & Raga (1991). Wherever possible, we approximate the velocity of a component of the flow by its spatially averaged value. We postulate that the flow close to the axis of the flaring region is laminar, with a constant relativistic bulk velocity \( v_j \), and that this occupies the full width of the jet at the brightening point, where interaction with the external medium becomes significant for the first time. As a result of entrainment of external material, a slower shear layer forms between the laminar jet and the environment. The bulk velocity of the shear layer should vary continuously in the radial \((r)\) direction from \( v_j \) at its boundary with the laminar core to 0 at its outer edge. The models derived by LB02a show, however, that the ratio of edge to centre velocity for the synchrotron-emitting material is \( \sim 0.7 \) throughout the flaring and outer regions, so the velocity range within the mixing layer is fairly narrow. Thus we assume that the steady-state flow in this layer has a constant bulk velocity \( v_s < v_j \). Material from both the environment and the laminar jet is continuously injected into the shear layer, the latter component supplying energy and momentum as well as mass. Integrated across the jet, the fraction of slower material then increases with distance from the nucleus; this would be interpreted as deceleration of the entire flow in fits to observations with poor transverse resolution.

The laminar jet in the centre eventually vanishes, so no more energy or momentum can be injected into the shear layer.

1 LB02a postulated the existence of an additional conical inner region in the faint inner jets of 3C 31, but observations of the better-resolved source NGC 315 by Canvin et al. (2005) are inconsistent with a constant expansion rate in the corresponding part of the brighter jet. A continuously increasing expansion rate is required in NGC 315 and is equally consistent with the observations of 3C31 and other sources. A two-zone model is adequate to describe the geometry in all cases.

2 This is also a change of terminology from LB02a, who refer to the flaring point, and is done to emphasise that the location marks a change in emissivity profile, not in geometry.
3 RELATIVISTIC CONSERVATION LAWS

We model the structure of FR I jets using relativistic fluid mechanics, applying the laws of conservation of mass, momentum and energy in the forms given by B94. As in that reference, we use the relativistic enthalpy \( \omega = \rho c^2 + \epsilon + p \) and the ratio \( \mathcal{R} = \rho c^2 / (\epsilon + p) \) of rest-mass energy to non-relativistic enthalpy. Here, \( \epsilon \) is the internal energy density and \( p \) is the pressure. In the laminar jet, we expect \( \mathcal{R}_j \propto p^{-1/4} \) (B94). For 3C31, the external pressure drops by a factor \( \approx 4 \) from the brightening point until the end of the flaring region. The approximation that \( \mathcal{R}_j \) is constant is therefore reasonable, and we adopt it in what follows.

For an ideal gas, \( \epsilon = p / (\Gamma - 1) \), so \( \mathcal{R} \) can be written as:

\[
\mathcal{R} = \frac{\Gamma - 1}{\Gamma} \frac{\rho c^2}{p} = \frac{\Gamma - 1}{\Gamma} \frac{\hat{n} c^2}{k_B T},
\]

where \( \Gamma \) is the adiabatic index, \( \hat{n} \) is the average particle mass and \( k_B \) is the Boltzmann constant. \( \mathcal{R}^{-1} \) is therefore a measure of the temperature. We make the approximation that the external medium around the jet is isothermal, so \( \mathcal{R}_e \) is constant. There is evidence for a temperature gradient on the relevant scales (Hardcastle et al. 2002), but the isothermal approximation has a very small effect on our results since the energy entrained from the external medium is negligible (B94, LB02b) and \( \mathcal{R}_e \gg 1 \) (Section 3.1.3).

3.1 Conservation laws for the flaring region

The main difference between our work and that of B94 and LB02b is that we divide the flaring region into two parts: the laminar jet and the shear layer. Thus our conservation equations include distinct terms associated with each of these components.

3.1.1 Conservation of rest mass

We use the following notations: \( r_j, r_s \) are the radii of the laminar jet and the shear layer respectively, \( \rho \) is the proper density, \( v \) is the bulk velocity, \( \beta = v/c \) and \( \gamma = (1 - \beta^2)^{-1/2} \) is the bulk Lorentz factor. The rest mass of the material...
passing through the total jet cross section $A(x) = \pi r_s(x)^2$ per unit time is equal to the rest mass of the material entering through the cross section 0 plus the total entrained mass from the environment. We express the mass fluxes $\dot{M}$, in the laminar jet and the shear layer at distance $x$ separately by:

$$
\dot{M}_j(x) = \gamma_j \rho_j(x) v_j \pi r_j(x)^2
$$

$$
\dot{M}_s(x) = \gamma_s \rho_s(x) v_s \pi [r_s(x)^2 - r_j(x)^2]
$$

From equation (9) of B94, we have

$$
\gamma_j \rho_j(x) v_j \pi r_j^2 + \int_{0}^{x} \rho_e(x') f(x') dx' = \dot{M}_j(x) + \dot{M}_s(x)
$$

The first term on the left of equation (1) is the rest mass of the material entering through cross section 0 per unit time. The second term on the left is the entrained mass flux. The terms on the right represent the rest masses of the material passing through the cross sections of the laminar jet and the shear layer per unit time at distance $x$. We assume that the laminar jet continuously supplies energy and momentum to the shear layer in such a way that $\beta_j$ and $\beta_s$ remain constant throughout the flaring region. The integral term $g(x) = \int_{0}^{x} \rho_e(x') f(x') dx'$ is the mass entrainment function, which was given in the form $g(x) = \int_{0}^{\infty} \rho(x) \cdot dS \cdot B94$ (where $\rho(x)$ is normal to the unit surface $dS$). $f(x)$ is therefore a function that expresses the combination of the perpendicular entrainment velocity and the shape of the jet boundary. The function $g(x)$ is a measure of the total mass entrained between the nucleus and distance $x$ per unit time.

We assume that the jet is in pressure equilibrium with the external medium throughout the flaring and outer regions. Thus at fixed $x$, the pressures in the laminar jet, the shear layer, and the environment are all equal. Dividing by $p(x)$ on both sides of equation (1) and defining $F(x) = c_p g(x) / [\pi r(x)]$, we get:

$$
\frac{\mathcal{R}_j \Gamma_j}{\Gamma_j - 1} \gamma_j \beta_j \left[ \frac{p_0}{p(x)} r_j - r_j(x) \right]^2 = \frac{\mathcal{R}_s \Gamma_s}{\Gamma_s - 1} \gamma_s \beta_s \left[ r_s(x)^2 - r_j(x)^2 \right] - F_1(x).
$$

### 3.1.2 Conservation of momentum

The momentum flow through the cross section $A(x)$ per unit time should be equal to the momentum of the material coming out of the initial cross section 0 per unit time, modified by the effects of buoyancy and differences in pressure between the flow and its environment. We express the momentum flux, $\dot{P}$, by:

$$
\dot{P}_j(x) = \gamma_j \omega_j(x) v_j^2 + \Delta p_j(x) \pi r_j(x)^2,
$$

$$
\dot{P}_s(x) = \gamma_s \omega_s(x) v_s^2 + \Delta p_s(x) \pi [r_s(x)^2 - r_j(x)^2],
$$

where $\Delta p_j(x) = p_j(x) - p_j(0)$ and $\Delta p_s(x) = p_s(x) - p_s(0)$ are the pressure differences at distance $x$. In our pressure-matched case, they are equal to 0. We assume that material is entrained from the environment with a small bulk velocity and therefore that it contributes negligible momentum compared with that of the jet. We rewrite equation (16) in B94 for our case as:

$$
\gamma_j \omega_j(x) v_j^2 + \pi r_j(x)^2 = \dot{P}_j(x) + \dot{P}_s(x) + \phi(x),
$$

where $\phi(x) = \int_{0}^{x} \frac{dp_0}{p(x)} dx \left[ \frac{2p_0}{\rho_0} \int_{A} (1 - \frac{\rho}{\rho_0}) dS \right]$ is the buoyancy term and for $\rho_j \ll \rho_0$ in our model, we have $\phi(x) = \int_{0}^{x} \pi r_j(x)^2 dp_0$. The momentum equation can then be simplified to:

$$
\frac{(\mathcal{R}_j + 1) \Gamma_j}{\Gamma_j - 1} \gamma_j \beta_j \left[ \frac{p_0}{p(x)} r_j^2 - r_j(x)^2 \right] =
\frac{(\mathcal{R}_s(x) + 1) \Gamma_s}{\Gamma_s - 1} \gamma_s \beta_s \left[ r_s(x)^2 - r_j(x)^2 \right] + \frac{\phi(x)}{\pi p(x)}.
$$

### 3.1.3 Conservation of energy

The energy passing through the jet cross section must also be conserved. We express the energy flux (or jet power), $Q$, at distance $x$ for the two regions as:

$$
Q_j(x) = \gamma_j \omega_j(x) v_j r_j(x)^2,
$$

$$
Q_s(x) = \gamma_s \omega_s(x) v_s \pi [r_s(x)^2 - r_j(x)^2].
$$

B94 gives the relevant conservation law in his equation (26) and we rewrite this as:

$$
\gamma_j \omega_j(x) v_j \pi r_j^2 + \int_{0}^{x} \omega_e(x') f(x') dx' = Q_j(x) + Q_s(x).
$$

As the environment is dominated by the rest mass energy, so $\mathcal{R}_e$ is extremely large, and we can approximate $1 + 1/\mathcal{R}_e \approx 1$ at all position. Thus, $\int_{0}^{x} \omega_e(x') f(x') dx' \approx \int_{0}^{x} c^2 [1 + 1/\mathcal{R}_e(x')] f(x') dx' \approx c^2 g(x)$. Dividing both sides by $c p(x)$, we get:

$$
\frac{\mathcal{R}_s(x) + 1) \Gamma_s}{\Gamma_s - 1} \gamma_s \beta_s \left[ r_s(x)^2 - r_j(x)^2 \right] - F_1(x).
$$

### 3.2 Conservation laws for the outer region

For the outer region the conservation equations are similar, but without the laminar jet term. Another important difference is that the initial cross section is now at the end of the flaring region (point 1 in Fig. 1). The entrained mass and energy now denote the values integrated from point 1 (distance $x_1$) up to distance $x$. Finally, the velocity of the layer, $\beta_s$, is a function of distance $x$. The three equations analogous to equations (1), (8), and (12) are then given by:

$$
\gamma_1 \rho_1 v_1 \pi r_1^2 = \dot{M}_s(x) - g_0(x),
$$

$$
\gamma_1 \omega_1 v_1^2 \pi r_1^2 = \dot{P}_s(x) + \phi(x),
$$

$$
\gamma_1 \omega_1 v_1 \pi r_1^2 = Q_s(x) - \int_{x_1}^{x} \omega_e(x') f(x') dx'.
$$

The term $g_0(x) = \int_{0}^{x} \rho(x) f(x') dx'$ is equal to the amount of entrained mass per unit time. With the same definitions of $F(x)$ and $R$ as given above, these three equations can be written in the following ways:
Table 1. Definitions of key parameters and functions. Columns 4 and 6 indicate whether the values are assumed a priori, inferred from fits of relativistic flow models to radio images ('Radio'), derived from X-ray observations of the surrounding hot gas ('X-ray') or calculated.

| Name | Physical meaning | Flaring region | Outer region |
|------|------------------|----------------|--------------|
|      |                  | value | origin | value | origin |
| \( \Gamma_j \) | adiabatic index of the laminar jet | 4/3, constant | assumed | - | - |
| \( \Gamma_s \) | adiabatic index of the shear layer | 4/3, constant | assumed | 4/3, constant | assumed |
| \( \Gamma_e \) | adiabatic index of the environment | 5/3, constant | assumed | 5/3, constant | assumed |
| \( \beta_j \) | bulk velocity of the laminar jet | constant | Radio | - | - |
| \( \beta_s \) | bulk velocity of the shear layer | constant | Radio | function of \( x \) | calculated |
| \( \beta_t \) | the bulk velocity at the beginning point of the outer region | - | constant | X-ray | calculated |
| \( \phi \) | ratio of rest mass energy to non-relativistic enthalpy for laminar jet | constant | Radio | function of \( x \) | calculated |
| \( \phi_s \) | ratio of rest mass energy to non-relativistic enthalpy for shear layer | function of \( x \) | calculated | function of \( x \) | calculated |
| \( \phi_1 \) | the value of \( \phi \) on the cross section 1 | - | constant | calculated |
| \( p \) | external pressure on cross section 1 | function of \( x \) | X-ray | function of \( x \) | X-ray |
| \( r_j \) | the radius of the laminar jet | function of \( x \) | calculated | - | - |
| \( r_s \) | the radius of the shear layer | function of \( x \) | Radio | function of \( x \) | Radio |
| \( r_0 \) | the jet radius at the brightening point | constant | Radio | - | - |
| \( r_1 \) | the shear layer radius at the beginning point of the outer region | - | constant | Radio | - |
| \( g_t \) | entrained mass per time from cross section 0 up to cross section 1 | function of \( x \) | calculated | - | - |
| \( g_0 \) | entrained mass per time from cross section 1 up to cross section 2 | - | function of \( x \) | calculated |

\[
\frac{\Gamma_s \gamma_1 \beta_1 p_1 r_1^2}{\Gamma_s - 1 \ p(x)} = \frac{\Gamma_s - 1 - \phi(x)}{\phi_1 + 1} \gamma_s(x) \beta_s(x) r_s(x)^2 - \frac{F_0(x)}{\phi_1 + 1}, \quad (17)
\]

\[
\frac{\Gamma_s \gamma_2 \beta_2 p_2 r_2^2}{\Gamma_s - 1 \ p(x)} = \frac{\Gamma_s - 1 - \phi(x)}{\phi_1 + 1} \gamma_s(x) \beta_s(x) r_s(x)^2 + \frac{1}{\phi_1 + 1} \pi p(x), \quad (18)
\]

\[
\frac{\Gamma_s \gamma_1 \beta_1 p_1 r_1^2}{\Gamma_s - 1 \ p(x)} = \frac{\Gamma_s - 1 - \phi(x)}{\phi_1 + 1} \gamma_s(x) \beta_s(x) r_s(x)^2 - \frac{F_0(x)}{\phi_1 + 1}. \quad (19)
\]

4 SOLUTIONS

In this section, we will solve equations \([5], [9], [13]\) for the flaring region, and equations \([17], [18], [19]\) for the outer region in terms of quantities which can be inferred either from fits of relativistic flow models to radio images (jet and layer velocities in the flaring region, together with the radius of the layer in both regions) or from X-ray observations of the surrounding hot gas (external density, temperature and pressure). We can then derive the shape of the laminar jet, \( r_j(x) \), the variation of velocity with distance in the outer region, \( \beta_s(x) \), the values of \( \phi \) in the various regions, the entrainment function and the velocity of entrainment.

We assume that the laminar jet has a relativistic equation of state with \( \Gamma_j = 4/3 \); the environment has \( \Gamma_e = 5/3 \). The shear layer contains mixed material but the energy density must still be dominated by relativistic particles (B94), and we therefore take \( \Gamma_s = 4/3 \).

4.1 Solutions for the flaring region

From equations \([5] \) and \([13] \), we have:
The pressure profile and the shape of the jet only depend on observable parameters together with the environment. Thus both \( \mathcal{R}_f \) and \( \mathcal{R}_l \) are expected to be functions of \( x \). We solve the equations numerically, using the following steps. Equations (17) and (18) give:

\[
\mathcal{R}_s(x) = \frac{\gamma_s}{\gamma_s - 1} \left[ \frac{\gamma_s (1 - \beta_s) - \gamma_s (x)}{\gamma_s (1 - \beta_s) - \gamma_s (x)} \right] - 1.
\]  

(29)

while equation (19) gives:

\[
\mathcal{R}_s(x) = \frac{\Gamma_s - 1}{\Gamma_s} \frac{\beta_s (x) - \beta_s (0)}{\beta_s (x) - \beta_s (0)} - 1.
\]  

(30)

Again, \( \mathcal{R}_s(x) \) occurs on the right-hand sides of equation (29) and (30), but \( M_s(x)/\mathcal{R}_s(x) \) and \( \dot{P}_s(x)/[\mathcal{R}_s(x) + 1] \) are just functions of \( \beta_s(x) \) and other observable parameters. Combining these two equations, we can solve numerically for \( \beta_s(x) \): the shape of the boundary, \( r_s(x) \), is constrained from observations, so the only unknown parameter is \( \beta_s(x) \), which in turn determines \( \gamma_s(x) \). Then, with the known value of \( \beta_s(x) \), we can express the entrainment function as follows:

\[
g_0(x) = \frac{M_s (\mathcal{R}_1 + 1) - \frac{M_s^2 (\mathcal{R}_s(x) + 1)}{\mathcal{R}_s(x) + 1}}{\gamma_s(x) - 1}.
\]  

(31)

Observations show that the radius of the shear layer in the outer region \( r_s(x) \) increases linearly with \( x \). We use this observed variation as the input function and predict the distributions of \( \beta_s(x) \), \( \mathcal{R}_s(x) \) and \( g_0(x) \).

### 4.3 Summary of the solutions

To get solutions for both the flaring region and the outer region, we adopt the shape function \( r_s(x) \) from model-fitting to radio images, together with the velocities \( \beta_f \) and \( \beta_l \) for the flaring region. We also adopt the pressure profiles from X-ray observations. This leaves three functions which need to be evaluated at each distance \( x \): \( r_s(x), F(x) \) and \( \mathcal{R}_s(x) \) for the flaring region, and \( \beta_s(x), \mathcal{R}_s(x) \) and \( F(x) \) for the outer region.

The three equations from the conservation laws thus form a closed system. The input and derived parameters are listed in Table 1.

## 5 APPLICATION TO 3C31

Having established a system of equations which describe the structure and kinematics of an FR I jet, we now compare the results with observational data and models for the well-observed source 3C31. Geometrical (projection factor and radius) and velocity information are inferred from the relativistic-flow models of LB02a. Fits to the density, temperature and pressure of the hot gas surrounding the jets are as given by Hardcastle et al. (2002) and used in the quasi-one-dimensional conservation-law analysis of LB02b. As in these references, we adopt a concordance cosmology with Hubble constant, \( H_0 = 70 \mathrm{km} \mathrm{s}^{-1} \mathrm{Mpc}^{-1} \), \( \Omega_M = 0.7 \) and \( \Omega_{\Lambda} = 0.3 \). At the redshift of the host galaxy of 3C31, \( z = 0.0169 \), this gives a scale of 0.344 kpc arcsec\(^{-1}\).

### 5.1 Inferences from observation

The parameters defining the edge of the shear layer projected on the sky are determined by fitting to the total-intensity distribution. The angle to the line of sight required to correct for projection (52\(^\circ\) for 3C31) is derived from the relativistic-flow model. In LB02a, the shape of the shear layer in the flaring region is described by the polynomial \( r_s(x) = a + bx + cx^2 + dx^3 \) with \( r_0 = 0.125 \mathrm{kpc} \) at 1.1 kpc and \( r_1 = 0.815 \mathrm{kpc} \) at 3.5 kpc. The shear layer initially expands slowly, then goes through a phase of faster expansion before recollimating at the end of the flaring region. In the
outer region, the shear layer expands conically with an intrinsic half-angle of 13.1°. At the beginning of the flaring region, we assume that there is no shear layer, so we use the on-axis bulk velocity inferred by LB02a to characterize the jet, \(v_j = 0.77c\). We suppose that the shear layer makes up essentially all of the flow at the end of the flaring region. LB02a infer a variation of velocity across the flow from 0.37\(c\)–0.55\(c\) at this distance so we take a representative value of \(v_s = 0.45c\).

Hardcastle et al. (2002) have estimated the external density and pressure profiles for 3C 31 from X-ray observations. The density profile is given by:

\[
\rho_e(x) = \rho_{e,0}(x/x_0)^{-\alpha_1},
\]

(32)

where \(\rho_{e,0} = 2.16 \times 10^{-22}\) kg m\(^{-3}\) and \(\rho_0 = 1.93 \times 10^{-11}\) Pa are the density and pressure at \(x_0\), respectively. The values \(\alpha_1 = 1.5\) and \(\alpha_2 = 1.1\) give good approximations to the profiles, and we adopt them in the following calculation. The corresponding density and the pressure profiles are compared with those from Hardcastle et al. (2002) in Figure 2. Although we use an isothermal approximation in the development of our model (Section 3), the assumed pressure profile includes the effects of the temperature gradient.

The numerical values of the parameters are: \(n_e = 1.8 \times 10^5\) m\(^{-3}\), \(n_g = 1.9 \times 10^3\) m\(^{-3}\), \(\beta_e = 0.73\), \(\beta_g = 0.38\), \(x_c = 1.2\) kpc, \(x_g = 52\) kpc. The temperatures estimated by Hardcastle et al. (2002) range from 4.9\(\times\)10\(^6\) K to 1.7\(\times\)10\(^7\) K, corresponding to \(R_e = 5\times\)10\(^5\) to 1.5\(\times\)10\(^5\). Thus the approximation \(1 + 1/R_e \approx 1\) (Section 3.1.3) is valid to high accuracy.

The pressure is given by Birkshaw & Worrall (1993):

\[
p(x) = kT(x)n_e(x)/(\mu\chi_H),
\]

(34)

where \(\mu = 0.6\) is the mass per particle. For simplicity, we approximate the pressure and density distributions using power-law forms:

\[
\rho_e(x) = \rho_{e,0}(x/x_0)^{-\alpha_1},
\]

(35)

\[
p(x) = p_0(x/x_0)^{-\alpha_2},
\]

(36)

where \(x_0\) is the position of the brightening point. \(\rho_{e,0} = 2.16 \times 10^{-22}\) kg m\(^{-3}\) and \(p_0 = 1.93 \times 10^{-11}\) Pa are the density and pressure at \(x_0\), respectively. The values \(\alpha_1 = 1.5\) and \(\alpha_2 = 1.1\) give good approximations to the profiles, and we adopt them in the following calculation. The corresponding density and the pressure profiles are compared with those from Hardcastle et al. (2002) in Figure 2. Although we use an isothermal approximation in the development of our model (Section 3), the assumed pressure profile includes the effects of the temperature gradient.

Figure 3. Results from our model for the flaring region of 3C 31. (a) Geometry. The outer edge of the flow and the boundary between the laminar core and shear layer are shown. (b) Mass flux at distance \(x\). The full and dashed lines indicate the total mass flux and the contribution from entrainment, respectively. (c) Profile of \(R_s(x)\). (d) The entrainment velocity perpendicular to the outer boundary at distance \(x\).
5.2 Results from the model

5.2.1 Flaring region

With the parameters given in Section 5.1, we obtain $R_j = 13.4$ in the flaring region. The profiles of $R_s(x)$ and the total mass flux passing through a given cross section, $M$, are plotted in Figure 4. In the same figure, we also plot $v_{\text{ent}}$, the normal component of the entrainment velocity across the surface of the jet. This is related to the entrainment function by $v_{\text{ent}} = (1/\rho_e) d\rho / ds$.

The model predicts that the laminar jet initially expands at the beginning of the flaring region and then starts to collapse $\approx 1.7$ kpc away from the brightening point. Meanwhile, the value of $R_s(x)$ drops a little at the beginning of the flaring region and then reaches an asymptotic value of $\approx 6.7$. The initial decrease of $R_j(x)$ occurs because the small amount of entrained material at the beginning of the flaring region can easily be heated by the the laminar jet. The functional forms of $R_s(x)$ and $v_{\text{ent}}(x)$ are constrained by the parameters inferred for 3C31 and may differ in other sources. For example, if the shear layer initially expands faster, $R_s(x)$ will be higher and $v_{\text{ent}}$ lower throughout the flaring region.

5.2.2 Outer region

In the outer region, our model predicts that the bulk velocity $\beta_s$ should decrease smoothly with $x$. $\beta_s = 0.45$ at 3.5 kpc, where it is normalized to the mean value of the distribution derived by LB02a, decreasing to 0.22 at 12 kpc. This is reasonably consistent with the velocity range derived by LB02a ($\beta = 0.15 - 0.22$ at the same distance). The value of $R_s$ increases with $x$ in our solution, reflecting the increasing dominance of the mass by entrained material. We plot $R_s(x)$ and $\beta_s(x)$ together with profiles of mass flux and velocity in Figure 4.

5.2.3 Estimate of jet power

Using the calculated and observed parameters given above, we can estimate the power of the jets in 3C31. The relevant parameter for comparison with estimates by other methods (e.g. Birzan et al. 2008) is $\Phi$ (LB02b), the energy flux of the jet with the rest-mass contribution subtracted. $\Phi = Q - Mc^2$ in the notation of the present paper. Applying equation (10) at the brightening point, we get values of $Q = 3.4 \times 10^{37}$ W and $\Phi = 1.6 \times 10^{37}$ W. 3C31 is a fairly powerful FRI source,
with a monochromatic luminosity of $10^{24.5}$ W at 1.4 GHz, approximately a factor of 10 below the FR I/FRII dividing line plotted by Ledlow & Owen (1996) given the absolute magnitude of the host galaxy (Owen & Laing 1989). A total power of $\Phi = 1.6 \times 10^{37}$ W for the twin jets of 3C 31 is well within the range derived from observations of cavities in the X-ray gas surrounding other radio galaxies of comparable monochromatic luminosity (Birzan et al. 2008).

### 5.2.4 Mass input from stellar mass loss

It has been argued that the deceleration in the flaring region could be caused by the entrainment of stellar wind material from stars located inside the jet (Komissarov 1994). In order to test this idea, we adopt the estimate of mass input from LB02b, who used a deprojection of R-band surface photometry for 3C 31 (Owen & Laing 1989), together with the same assumptions on conversion between stellar luminosity and mass loss as in Komissarov (1994) and Bowman et al. (1996). The corresponding entrainment per unit length (the derivative of the entrainment function defined above) can be written as

$$dg_s/dx = 2.4 \times 10^{28} \pi r_s(x)^2 x^{-2.65} \text{ kg kpc}^{-1} \text{ yr}^{-1},$$

where $r_s(x)$ and $x$ are in units of kpc. In Fig. 5 we compare the entrainment function from our model and its derivative with those estimated for stellar mass loss. At the beginning of the flaring region, the stellar mass input rate is remarkably close to that required, given the crudity of the assumptions. At larger distances, however, it falls well below the level required to decelerate the jet. In the outer region, the entrainment rate per unit length required by our model continues to increase, whereas that from stellar mass loss decreases. Thus, although stellar mass loss may be important in initiating the jet deceleration at the start of the flaring region, boundary-layer entrainment, as described by our model, is clearly required on larger scales. Mass input distributed throughout the jet volume, as would be expected from stellar mass loss, is a potential complication to our analysis.
The values of $r_0$, $r_1$, $\beta_j$ and $\alpha_s$ are fixed at the values determined for 3C 31. The plus sign indicates the value of $R_j$ for 3C 31.

5.3 Comparison with LB02b

It is of interest to compare the results of the present model with the conservation-law analysis of LB02b. The treatments are very similar in many respects, both relying on quasi-one-dimensional approximations and using conservation of mass, momentum and energy in a realistic external environment. The formulation of the conservation laws is identical in the two treatments. The principal differences in the assumptions are as follows.

(i) The analysis of LB02b explicitly assumed that there are no variations in physical parameters across the jets, as in our treatment of the outer region. We split the flaring region into laminar jet and shear layer components.

(ii) The jets in LB02b’s analysis are assumed to come into approximate pressure equilibrium with their surroundings only after they recollimate. This then requires that they are over-pressured at the start of the flaring region. In contrast, we assume that the jets are everywhere in pressure equilibrium with the external medium. In this picture, the initial expansion is caused by transfer of momentum from the laminar core to the shear layer rather than a pressure-driven expansion.

(iii) The models are constrained in slightly different ways. Both specify the radius of the jet as a function of distance from the nucleus. In LB02b, the velocity is given everywhere and the best average match to pressure equilibrium is found for the outer region. In the present model, velocities are specified only in the flaring region, but pressure equilibrium is enforced along the entire length of the jet.

(iv) In the solutions preferred by LB02b, momentum flux $= \Phi/c$ initially. This is required for the jets to decelerate from highly-relativistic velocities on parsec scales, as in unified models of BL Lac objects and FR I radio galaxies. It is not an explicit constraint in the present models, where the momentum flux is relatively higher (corresponding to the solutions in section 3.3.6 of LB02b).

(v) We use power-law, isothermal approximations for the external density and pressure distributions, whereas LB02b use a double-beta-model with varying temperature. The differences are minor (Fig. 2).

LB02b discussed the effects of varying the assumptions of their analysis. This led to a spread of values around those for their reference model which we quote here. Table 2 compares values of key parameters for our model jet and that from LB02b’s reference model at the brightening point and at 12 kpc from the nucleus.

The energy fluxes of the two model jets are quite similar, despite the differences in starting assumptions. In terms of the available energy flux $\Phi$ (with the rest-mass component subtracted, as in Section 5.2.3 and LB02b), we find $\Phi = 1.6 \times 10^{37}$ W, compared with $\Phi = 1.1 \times 10^{37}$ W for LB02b. This is because the geometries of the two jets are identical; in the outer region their velocities are very similar and they are both close to pressure equilibrium with the surroundings. The main difference is in the mass flux, which is a factor of 1.5 times larger at 12 kpc from the nucleus in the present model.

There is a larger difference between the initial conditions for the two models at the brightening point. The model jet of LB02b has an initial density roughly 5 times lower than that described here, but is also overpressured: its energy density is dominated by the internal energy of relativistic particle rather than by bulk kinetic energy, as can be seen from the differences in the value of $\alpha_s$ at the brightening point (Table 2). The very low initial density in LB02b’s reference model is derived from the requirement for FR I jets to be able to decelerate from bulk Lorentz factors $\sim 5$ on parsec scales. If this requirement is relaxed, as in the high-momentum solutions described in section 3.2.6 of that paper, results closer to those in the present paper are obtained. The entrainment rate at the beginning of the flaring region in both models is very low and could be provided by mass input from stars (Section 3.2.4). Both models require an additional source of mass at larger distances from the nucleus, however.

Table 2. Comparison between derived parameters for 3C 31 in this paper and LB02b. Following B94 and LB02b, we quote the relativistic Mach number, $\mathcal{M} = \gamma_2/c_s$, where $c_s$ is the sound speed and $\gamma_2 = [1 - (c_s/c)^2]^{-1/2}$.

| Quantity                        | This paper | LB02b |
|--------------------------------|------------|-------|
| Energy flux $(10^{37}$ W)       | 1.6        | 1.1   |
| (excluding rest mass)           |            |       |
| Initial momentum flux $(10^{28}$ kg m s$^{-2}$) | 7.7        | 3.7   |
| Density at brightening point $(10^{-27}$ kg m$^{-3}$) | 12         | 2.5   |
| Mass flux at brightening point $(10^{27}$ kg yr$^{-1}$) | 6.2        | 1.0   |
| Mass flux at 12 kpc $(10^{27}$ kg yr$^{-1}$) | 47         | 32    |
| Pressure at brightening point $(10^{-11}$ Pa) | 1.9        | 15    |
| $\mathcal{M}$ at brightening point | 13.4 (jet) | 0.4   |
| 7.7 (layer)                     | 0.4        |       |
| Mach number at brightening point | 7.7 (jet)  | 1.5   |
| 2.5 (layer)                     | 1.5        |       |
6 EXPLORING PARAMETER SPACE FOR 
THE MODEL

Our model uses several parameters derived from observations of 3C31 to calculate the key physical properties of this object. For other FR I sources, these parameters will be inappropriate and in this section, we discuss the effects of altering them.

6.1 Flaring region

The parameters affecting the solution in the flaring region are the value of $R_e$, the polynomial coefficients for the outer boundary, the jet and layer velocities and the gradient of the external pressure. We have argued that $R_e$, which is always very large, cannot affect our solutions significantly. The shape of the outer boundary plays an important role in determining the buoyancy term and varies from source to source. As the shape function has four free parameters, we will not discuss this point in detail here but we note that faster expansion of the shear layer will lead to larger values of $R_s(x)$ and smaller entrainment velocities. We can vary the remaining three parameters, $\beta_1$, $\beta_2$, and $\alpha_2$, individually to determine their effect on our solutions and we plot them against $R_j$ below. The distributions of $R_s$, mass flux and $v_{ent}$ are closely related to that of $R_j$.

Given that the laminar jet is assumed to be in pressure equilibrium with its surroundings at the brightening point, its internal energy is determined. If $\beta_1$ and the form of the pressure profile are also fixed, then the energy flux minus the rest mass term, $\Phi$ (defined by its value at $x_1$) is also unchanged. Since $\Phi$ is a conserved quantity, this is also true for the laminar jet at $x_0$. A faster jet with the same internal energy must therefore have smaller density and $R_j$ (Fig. 6).

Moreover, if we have a faster shear layer at $x_1$, which means that $\Phi$ is higher, but $\beta_j$ remains constant, then the density of the laminar core at $x_0$ must increase, since the internal energy is fixed there by the pressure balance condition. $R_j$ therefore increases with $\beta_j$ (Fig. 7). The shapes of the distributions of $g(x)$, $R_s(x)$ and $v_{ent}(x)$ remain the same but their normalizations change if the jet or layer velocities are varied. For a faster laminar jet or a slower shear layer, $R_s$ and $v_{ent}(x)$ both become smaller, indicating that the shear layer is less dense.

The value of $R_j$ also depends on the pressure profile, quantified here by the exponent $\alpha_2$ of a power-law distribution. If the pressure decreases more slowly with distance, then the assumption of pressure equilibrium requires the internal energy of the layer to be higher at the end of the flaring region, increasing the energy flux. If the velocity of the laminar core is fixed at the brightening point, as is its internal energy, then we need a denser laminar jet and therefore a higher value of $R_j$ (Fig. 8).

6.2 Outer region

For the outer region, the situation is much simpler. As $R_1$, $\beta_1$ and $v_1$ are determined by continuity at the boundary with the flaring region, the only additional parameters inferred from the observations are the half opening angle $\theta$ and the power-law exponent of the external pressure profile, $\alpha_2$. Two factors influence the opening angle: the decrease of external pressure and the expansion associated with entrainment. Of the two, the latter is more important for 3C31: if we set $v_{ent} = 0$ to remove the entrainment terms, the predicted jet opening angle is around $3^\circ$ (compared with the observed value of $13^\circ$), suggesting that entrainment dominates the expansion.

Figure 9 shows how the jet properties change as functions of the exponent of the external density and pressure distributions, $\alpha_2$. For a jet with a fixed opening angle, a larger value of $\alpha_2$ (a faster decrease of pressure) reduces the amount of material entrained from the environment into the jet and leads to a slower entrainment velocity. As the buoyancy force can accelerate the material in the jet, a larger value of $\alpha_2$ can also lead to a slower deceleration in the outer region. The outer region cools due to continuous entrainment of thermal matter from the environment into the shear layer, so $R_s$ increases with distance at a rate dependent on the entrainment velocity.

If we keep $\alpha_2 = 1.1$ and alter the opening angle, $\theta$, the jet properties vary as shown in Figure 10. We find that when the opening angle is small, the jet hardly entrains any material from the environment, and so decelerates more slowly. In extreme cases, the jet could even be accelerated slightly by the pressure gradient. It is interesting to note that the other four sources which have been modelled in detail all have outer region opening angles $< 5^\circ$ (Canvin & Laing 2004; Canvin et al. 2005; Laing et al. 2006) and show little evidence for deceleration on these scales. Compared with 3C31, their external environments are significantly less dense and it may be that entrainment is relatively less important at large distances from the nucleus.

7 CONCLUSIONS AND FURTHER WORK

We have constructed an analytical mixing-layer model for jets in FR I radio sources that satisfies the relativistic mass, momentum and energy conservation laws. FR I jets are observed to expand rapidly and then recollimate into conical outflows, and we divide them into flaring and outer regions based on this morphological distinction. We assume that the jet is in pressure equilibrium with its surroundings throughout both regions and divide the flaring region into two parts: a laminar jet with very high bulk velocity, and a slower shear layer. We prescribe the shape of the shear layer and the (constant) velocities of the laminar jet $v_1$ and shear layer $v_s$ in the flaring region. We can then derive the jet power $Q$ and the ratio of rest mass energy to non-relativistic enthalpy for the laminar jet, $R_e$. We calculate profiles along the jet of the mass flux $M(x)$, the entrainment velocity $v_{ent}(x)$ and the ratio of rest mass energy to non-relativistic enthalpy for the shear layer, $R_s(x)$. Finally, we predict the variation of the bulk velocity of the shear layer, $v_s(x)$, with distance from the nucleus in the outer region and the radius of the laminar core $r_1(x)$ in the flaring region.

We have applied the model to the well-observed FR I radio source 3C31, and find self-consistent solutions for the jet properties. In the flaring region, we take the shape of...
the shear layer \( r_s(x) \) and the bulk velocities of \( v_j = 0.77c \) and \( v_s = 0.45c \) from fits to VLA observations (LB02a). In the outer region, our model predicts that the bulk velocity should decrease smoothly to \( 0.22c \) at 12 kpc, which is consistent with the values derived by LB02a. The corresponding energy flux is \( Q = 3.4 \times 10^{37} \) W, equivalent to \( \Phi = 1.6 \times 10^{37} \) W if the rest-mass contribution is subtracted.

We find that \( \beta_s(x) \) and \( R_s(x) \) in the shear layer decreases from \( \approx 7 \) at the beginning of the flaring region to 6.7 and then stays almost constant until the jet collimates. In the outer region, \( R_s(x) \) increases from 6.7 to 15.7 at 12 kpc, indicating that the temperature of the material in the outer region is decreasing with distance. The velocity of entrainment into the jet varies with distance, but has a characteristic value of a few hundred ms\(^{-1}\).

Our model gives a somewhat larger energy flux for 3C 31 than that of LB02b, who find \( \Phi = 1.1 \times 10^{37} \) W assuming that there are no transverse velocity variations in the jets. The two models are quite similar in the outer region, but differ more significantly at the start of the flaring region: our analysis assumes pressure equilibrium whereas LB02b require a significant over-pressure and consequently find a lower initial density. Both models require entrainment rates which are consistent with estimates of mass input from stars at the base of the flaring region, but not at larger distances.

We plan to apply a slightly generalized version of our analysis to the other FR I jets for which velocity models and adequate X-ray data are available (Canvin & Laing 2004; Canvin et al. 2003; Laing et al. 2006). Complex, non-axisymmetric structures are observed at the start of the flaring regions of these jets, as they are in 3C 31 (LB02a). It is plausible that these are shocks in the supersonic flow required in the core, although the detailed morphology of the best-resolved example, NGC 315, suggests otherwise (Laing et al. 2006). Our model requires that there should be a clear demarcation in velocity between the core and the shear layer in FR I jets and predicts the shape of the former. This can in principle be tested using the techniques developed by LB02a, but existing observations are limited by insufficient resolution or sensitivity in regions of rapid deceleration close to the nucleus.

Figure 9. The jet properties in the outer region for different values of \( \alpha_2 \), the exponent in the external pressure distribution. The solid line is the value estimated for 3C 31, \( \alpha_2 = 1.1 \). The dotted line, dash dot line and dashed line are for \( \alpha_2 = 0.5 \), \( \alpha_2 = 1.5 \) and \( \alpha_2 = 2 \), respectively. (a) Velocity profile, \( \beta_s(x) \). (b) The entrainment function \( g_o(x) \). This is the entrained mass flux between the start of the outer region \( (x = x_1) \) and distance \( x \). (c) Profile of \( R_s(x) \). (d) The entrainment velocity perpendicular to the shear layer surface. Irregularities in the profile are numerical artefacts.

\[^4\] Transverse velocity gradients are clearly detected, but they are
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Figure 10. The jet properties in the outer region for different values of the opening angle, $\theta$. The solid line is the default value for 3C 31 with $\theta = 13.1^\circ$. The dotted line, dash dot line and dashed line are for $\theta = 3.5^\circ$, $\theta = 8^\circ$ and $\theta = 20^\circ$ respectively.

should be able to image the flaring regions in detail and to resolve a core/shear-layer structure if one is present.

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well characterized only at larger distances from the nucleus, where the shear layer makes up much or all of the flow in our picture.

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