Simple Pendulum Revisited

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Abstract

We describe a 8085 microprocessor interface developed to make reliable time period measurements. The time period of each oscillation of a simple pendulum was measured using this interface. The variation of the time period with increasing oscillation was studied for the simple harmonic motion (SHM) and for large angle initial displacements (non-SHM). The results underlines the importance of the precautions which the students are asked to take while performing the pendulum experiment.

1 Introduction

The simple pendulum is a very trivial experiment that physics students do in higher secondary. Yet students sometimes fail to appreciate why "the initial angular displacement of the pendulum must be small". Interesting letter to the Editor in Physics Education (UK) point that the pendulum’s time period increases by only 1% for pendulum’s oscillating through 30°. A 1% increase means the time period is 10msec more for than a pendulum undergoing SHM whose time period is 1sec, thus students do question the very need of the precaution that the pendulum should only be given small angle initial displacements. Variations as small as 10msec are very difficult to measure using stop-watches. Since computers (or microprocessors) can in principle make measurements in micro-seconds, we were tempted to study the simple pendulum using a micro-processor.

2 The interface

The microprocessor is essentially made up of digital devices, which communicate among itself in the language of binary, i.e. in ones (1) and zeros (0). That is voltage signals of preferred levels. To find the time interval of an oscillating pendulum, we keep an arrangement of laser source and light dependent resistor (LDR) such that the pendulum’s bob cuts the light path during its oscillation. As the bob cuts the path, the light is momentarily blocked. This produces a change in current generated in the LDR. For the microprocessor to communicate

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and understand this change (analog) in current, it has to be converted to TTL compatible
digital voltage. The conversion and subsequent wave shaping is done using the circuitry shown
in fig.1.

Figure 1: Circuit used to wave-shape and interface instant of pendulum’s cross-over in front
of the light dependent resistance (LDR).

The value of resistance of the LDR, as the name suggests, depends on whether light is falling
on it or not. The resistance of the LDR is usually inversely proportional to the intensity of
light falling on it. In our case when the laser light was falling on the LDR, it’s resistance
was 15KΩ, while on switching off the laser light it rose to 150KΩ. The voltage drop across
the 10KΩ resistance which forms a voltage divider with the LDR is 5v when the laser light is
‘ON’ (bright phase) and 0.75v when LDR is not exposed to laser (dark phase). This voltage
\( V_{o1} \) could have been directly fed into the microprocessor, however, the intensity of the laser
light strongly depends on the current supplied by it’s batteries. With time since the current
is likely to fall, the voltage \( V_{o1} \), would change as the experiment is being conducted. The
difference amplifier, amplifies the difference between the voltages \( V_{o1} \) and \( V_1 \). By selecting a
proper \( V_1 \), using the pot, the output \( V_{o2} \) varies from positive level for dark phase and negative
level for the bright phase. This inversion is bought about by the inverting amplifier (opamp).
The second opamp inverting action brings \( V_{o3} \) in phase with \( V_{o1} \). This opamp is essentially a
Schmitt’s trigger, which hard drives the output to +12v and -12v. The output of the second
opamp (\( V_{o3} \)) varies from -12v to +12v in accordance with the motion of the pendulum. Since,
the microprocessor can only understand zero or high state (∼ +5v), a 4.7v zener diode is used to protect the microprocessor (from +12v) by converting +12v level to +4.7v and to force -12v level to zero. The 1KΩ resistance kept between the opamp and zener diode is to control the current flowing into the zener diode.

Figure 2: Plot of pendulum length vs time period squared (\(T^2\)). The data point fall on a straight line with co-relation factor as good as 0.9995.

The realibility of our microprocessor program (listed in the Appendix) was checked by finding the frequency of known square waves fed from audio function generators. The program is essentially a counter program which counts the time interval taken between two positive edges of a train of square waves. While counting, the program loops between instruction addresses C024H and C02DH for the square wave’s high state and between C02EH and C037H for the input square wave’s ground state. The latter part was to over come the requirement of IC555 monostable trigger hardware in our circuitry. For a count of ’N’, the time period is
given as

\[ T = \frac{40N - 3}{f} \]

\[ = \left( \frac{40N - 3}{3} \right) \mu s \]  

(1)

where \( f \) is the frequency of the microprocessor clock in MHz. The formula is obtained by calculating the \( T \) states or time taken for the microprocessor to execute each instruction. It should be noted, the microprocessor’s program takes 17\( \mu \)secs to identify an edge and write the count in a memory location. This 17\( \mu \)secs is a systematic error that would be present in the value of time period measured. From this exercise we realized the quartz crystal used as a clock for the microprocessor kit was 6.2MHz (Books [2] state it is 6MHz, however, it depends on local manufacturers). Thus, results of all time measurements (fig 2, 6, 8 and 11) have to be multiplied by 0.968 (=6.0/6.2) to accommodate this correction.

While the reliability of the software was established, to make sure the LDR (our transducer), as also our microprocessor interface was reliable, we measured the time period of oscillation for various pendulum lengths. Figure [2] shows a perfect linearity between the pendulum’s length and its time period squared (i.e. \( T^2 \)). This is expected and is in accordance to well established theory that we shall discuss below. It should be noted that on an average the response time of a LDR is in small milli-secs. Thus, the ability to resolve and measure any changes in time period with increasing oscillations would be in milli-secs. Also, the accuracy in terms of absolute value of ‘\( g \)’ calculated from experimental data would depend on how accurate the quartz crystals frequency has been reported.

3 Simple Harmonic Motion: small initial displacement

Before proceeding to discuss the results of our experiment it would be worthwhile to recapulate about the pendulum and under what conditions does it’s motion reduce to a simple harmonic motion. A pendulum is easily set up by suspending a point mass. Physically, this is achieved by suspending a bob which has an appreciable mass but whose radius is small as compared to the length of the string used to suspend the bob in consideration. The pendulum is set into to and fro motion by displacing it from its mean position. The forces acting on the displaced pendulum is shown in fig(3). The restoring force is given as

\[ F = - mgsin\Theta \]

where \( m \), \( \Theta \)’ and \( g \)’ are the mass of the bob, it’s angular displacement from the mean position, the acceleration due to gravity respectively. The above leads to the equation of motion

\[ mL \frac{d^2\Theta}{dt^2} = - mgsin\Theta \]
or

\[
\frac{d^2 \Theta}{dt^2} = - \frac{g}{L} \sin \Theta \tag{2}
\]

On considering the initial angular displacement \( i.e. \Theta \) to be small, \( \sin \Theta \) of eqn \( 2 \) reduces to \( \Theta \) and substituting \( \omega^2 = \frac{g}{L} \), we have

\[
\frac{d^2 \Theta}{dt^2} = - \omega^2 \Theta \tag{3}
\]

This second order differential equation describes the motion of the simple harmonic motion (SHM), whose analytical solution is easily derivable and given as

\[
\Theta(t) = A \sin(\omega t) + B \cos(\omega t)
\]

where A and B are constants. We can get the values of the constants by choosing suitable initial conditions. The time period of oscillation can be obtained from the relationship \( (\omega = \frac{2\pi}{T_0}) \)

\[
\omega = \sqrt{\frac{g}{L}}
\]
giving

\[ T_o = 2\pi \sqrt{\frac{L}{g}} \]  

(4)

The above equation shows the proportionality between \( T_o^2 \) and the pendulum’s length. To confirm the reliability of our time measuring device (interface and software etc), we confirmed this relationship, see fig(2). This relationship holds true for small angle displacements. Hence, the data for fig(2) were collected for various lengths of the pendulum with initial angular displacement being 5°, an universally accepted small angular displacement.

Students identify eqn(4) easily, since it is used by them to estimate the acceleration due to gravity. Also of interest is the fact that the above expression implies that the time taken to complete one oscillation is independent of the angular displacement (\( \Theta \)), abide subjected to the condition \( \sin \Theta \sim \Theta \). It’s here that the argument starts as to what would be the appropriate initial displacement that an experimentalist should give to attain the simple harmonic motion? As \( \Theta \) (in radians) increases, the disparity between itself and it’s sine (\( \sin \Theta \)) increases. This fact is seen in fig(3), where the increase in disparity is shown in terms of error \( \left( \frac{\Theta - \sin \Theta}{\Theta} \right) \), in % w.r.t. \( \Theta \).

As can be seen, the error is below 10% for angles less than 45°. Would this limit be acceptable? Before answering this question, as to understand the boundary between SHM and non-SHM, we proceed to understand the modifications introduced in eqn(4), when the pendulum is set into motion with large angle displacements (non-SHM).

4 Pendulum with large initial displacement

The time period of oscillation of a pendulum oscillating with large angles can be found by solving eqn(2), i.e.

\[ \frac{d^2 \Theta}{dt^2} = - \omega^2 \sin \Theta \]  

(5)

However, that is easier said then done. Infact discussions on large amplitude oscillations are rarely carried out because there are no analytical solutions for the above differential equations. Infact, the solution is expressed interns of elliptical integrals [3, 4]

\[ T = \left( \frac{2}{\pi} \right) T_o \int_{0}^{\pi/2} \frac{d\Theta}{\sqrt{1 - \sin^2(\Theta_m/2)\sin^2 \Theta}} \]  

(6)

Hence, eqn(6) is either numerically solved or various approximations are used. Of these approximations, the most famous was given by Bernoulli in 1749 [4]

\[ T = T_o \left( 1 + \frac{\Theta_m^2}{16} \right) \]  

(7)
where $T_0$ is the time period had the SHM condition been satisfied and is given by eqn (4) and $\Theta_m$ is the maximum angular displacement given to the pendulum. Eqn (7) would suggest that take whatever initial displacement you want while doing the experiment to determine the acceleration due to gravity, all you have to do is to include the correction ($\Theta_m^2/16$) in the time period expression (eqn 2).

Figure 4: Plot of acceleration due to gravity with initial displacement. The solid line is essentially calculated from 'T' of eqn (7) while the constant line is 'g' evaluated using $T_0 = \frac{T}{1+\Theta_m^2/16}$ (i.e. time period after correcting for large angle displacement).

While the pendulum and it's time period of oscillation in itself is interesting, it is usually used to evaluate the acceleration due to gravity. Teachers insist that students do the experiment with small angular displacements. Students do this without appreciating "why" and as to "what is a small angle". Let us consider what kind of variation is expected theoretically if this precaution is not adhered to. Figure (4) shows the plot of the variation of acceleration due to gravity with the initial displacement done by numerically solving eq (5). The values of 'g' represented by the solid line is essentially calculated from 'T' of eqn (7) while the constant line is 'g' evaluated using $T_0 = \frac{T}{1+\Theta_m^2/16}$ (i.e. time period after correcting for large angle displacement). As the figure shows, with increasing angular displacement, the error in evaluated 'g' grows.

Eqn (7) suggests only a trivial consideration of including a correction factor is required if the small angle precaution is not followed, the question then arises "why fuss over small initial displacements?" Also, it is evident from fig (4) that the error in 'g' would be below 10% for initial displacements below 45° which is quite a large angle. This might be well within the limits of experimental error, induced by your measuring devices like scale and stop-watch. It should be noted that Nelson and Olsson have determined 'g' with an accuracy of $10^{-4}$ using a simple pendulum by including as many as 8-9 correction terms. Thus, the importance of maintaining small initial displacement while performing the experiment is still not convincing.
In the next section we report the measurements made by our microprocessor interface and try to address the questions we have asked above.

5 Results & Discussion

A bob of radius 2.5cm was suspended using a cotton thread of length one meter (the length of the pendulum thus is 1.025m). As the pendulum cut the lasers path to the LDR, an electric pulse is generated. From the point of the onset of the positive edge, the microprocessor counts the time elapsed till the bob cuts the light path again. In one complete oscillation, the bob cuts the light path thrice, say at instances $t_0$, $t_1$ and $t_2$ (see fig 5). The time period is given as $t_2 - t_0$. The program is however designed to store $t_1 - t_0$ and $t_2 - t_1$. This was done to make sure there is no error induced due to the inability to pin point the mean position of the pendulum. Data were collected for the pendulum oscillating with the initial displacements of $5^\circ$, $10^\circ$, $15^\circ$, $20^\circ$, $25^\circ$ and $30^\circ$.

Fig(6) shows the variation of the pendulum’s time period with oscillations. While for small initial angles, there is no or slight variation in time period, for large initial displacements, namely $20^\circ$, $25^\circ$ and $30^\circ$, the fall in time period with increasing oscillations is pronounced. The fall is better appreciated by plotting the normalized data ($T/T_1$). Deviation from the smooth variation in time period (scattering of data points) is seen for large angles. This primarily is due to the pendulum’s support not being perfectly stationary. For this reason we restrict our report to maximum angular displacement of $30^\circ$.

The fall in time period seems exponential. Since the experiment is not ideal, one can expect damping to play a role (the decrease in amplitude was visible to the naked eye with increasing
number of oscillations). In fact, damping is expected to attenuate the amplitude of oscillation exponentially with time. Thus, eqn(7) can be written as

$$T = T_o \left(1 + \frac{\Theta m e^{-2\beta t}}{16}\right)$$

Or, can be written as

$$T = a \left(1 + be^{-2\beta t}\right)$$

Our objective would be to fit the above the equation to the experimental data of fig(6). Table I lists the coefficient of eqn(9) obtained by curve fitting.

**Table I:** List of the coefficients obtained by fitting eqn(9) to the experimental data of fig(6). The last column lists the co-relation of fit with respective data points.

| S.No. | Θ (degrees) | a (sec) | b (radian)^2 | $\beta \times 10^{-2}$sec^-1 | $T(\beta = 0)$ (sec) | r  |
|-------|-------------|---------|--------------|-----------------------------|-----------------|----|
| 1.    | 5           | 2.00441 | 0.000333     | 0.34751                     | 2.00508         | 0.8138 |
| 2.    | 10          | 2.00676 | 0.001552     | 0.31740                     | 2.00988         | 0.9911 |
| 3.    | 15          | 2.01014 | 0.002072     | 0.38300                     | 2.01430         | 0.9968 |
| 4.    | 20          | 2.00181 | 0.006540     | 0.32090                     | 2.01490         | 0.9996 |
| 5.    | 25          | 2.00519 | 0.009755     | 0.41942                     | 2.01038         | 0.9921 |
| 6.    | 30          | 2.00961 | 0.011860     | 0.50847                     | 2.03344         | 0.9976 |
Figure 7: Plot of $T(\beta = 0)$ (or $a+ab$) vs angle. The data points fit well to Bernoulli’s approx (eqn 7) with co-relation factor as good as 0.985.

Instead of using coefficient ‘a’ as a variable for curve fitting, it should be taken as a constant ($T_o$, the SHM time period) defined by eqn(4). That is, for all six values of $\Theta$ (Table I), the value of ‘a’ should work out to be the same. However, this proved to be a difficult exercise where we were not able to achieve good co-relation between the experimental data points and the fitted curve. To overcome this, after obtaining the generalized coefficients (a, b) we calculated $T(\beta = 0)$ (=$a+ab$, listed in Table I), i.e. the time period of the pendulum’s oscillation through large angles without any damping (the variation with angle of oscillation is given by Bernoulli’s approximation eqn 7). Fig(7) shows the variation of $T(\beta = 0)$ with angle. The solid line shows the curve fit (eqn 7). The $T_o$ determined from our experimental data works out to be 2.00545sec. One can use this value in eqn(4) and determine the value of ‘$g$’, the acceleration due to gravity. We get the acceleration due to gravity as 9.963m/sec$^2$. This value is on the higher side. Even after accounting for the influences corrections terms discussed in reference [6], the value of ‘$g$’ would be on the higher side. The error, might be due to the lack of precise knowledge of the micro-processor’s clock frequency. Using eq(1) and eq(4), we get the maximum possible error as

$$dg = \left( \frac{8\pi^2 l}{T_o^2} \right) \frac{df}{f} = 6.54 \times 10^{-6}df$$

For an error of $+0.1 \times 10^6$MHz is the knowledge of the micro-processor’s clock frequency, the maximum error in ‘$g$’ would be $+0.65m/s^2$. Thus the error, as stated, is due to the imprecise knowledge of the micro-processor’s clock frequency.

We now investigate the remaining coefficients ‘b’ and $\beta$. The coefficient ‘b’ is proportional
The solid line shows that the experimental data points fall on a parabola. Second plot shows how the damping coefficient of the pendulum varies with initial angular displacement.

to $\Theta^2$ (compare eqn 8 and 9). This is evident from fig 8 which shows the data points to fall nearly perfectly (co-relation factor is 0.984) on a parabola. The proportionality constant by eqn 8 should be $1/16$ (if $\Theta$ is in degrees), or $1.9 \times 10^{-5}$ rad$^2$. Our result gives the proportionality constant as $1.4 \times 10^{-5}$ rad$^2$ (or 1/21). Eqn 8 is only an approximation hence, we can confidently say that our data follows the solution given by Bernoulli (eqn 7).

The second graph of Fig 8 shows the plot between the damping coefficient and the angular displacement. The damping factor is nearly constant for small angular displacements which shows a rapid linear increase for angular displacements above $20^\circ$. As can be seen the variation is similar to that of the IV characteristics of a diode and as in it’s case, we can extend the linear region to cut the 'X' axis and look for the limiting initial angular displacement which does not show sharp exponential fall in oscillation time period. This works out to be $11 - 12^\circ$. Beyond this limit, the damping coefficient is large and a pronounced exponential fall is seen in the oscillation’s time period (fig 6). Kleppner and Kolenkow [7] have discussed the nature of $\beta$ and have stated that it depends on the shape of the mass and the medium through which the mass moves. The amount of frictional force depends on the instantaneous velocity ($d\Theta/dt$) of the pendulum ($\beta$ being the proportionality constant). However, this nature of the frictional force ($F = -\beta d\Theta/dt$) is restricted for motion where velocity is not large enough to cause turbulence. Beyond angles of displacement of $11 - 12^\circ$, the frictional drag might not be following the linear relationship with velocity. This however needs further investigation.

Before summarizing the results of our experiment, it would be of use to understand how the pendulum experiment is done in the undergraduate lab. The student records the time taken to complete 40-50 oscillation oscillations from which the time period is calculated by
Figure 9: Plot of $T(\beta = 0)$ and $T_{av}$ vs angle. The solid line is the curve fit of $T(\beta = 0)$ to eqn(7) while the dash line is for visual aid to show variation of $T_{av}$.

dividing the total time taken by the number of oscillations measured in that time. We call this as $T_{av}$ (average). This term obviously does not take into consideration the influence of damping which is pronounced in large angle oscillations. This obviously leads to erroneous results. Figure (9) compares $T_{av}$ and $T(\beta = 0)$, the time period after accounting for damping with the pendulum’s displacement. The figure clearly depicts the increasing disparity with large displacements.

Thus, if a student performs the pendulum experiment without taking the necessary precaution of small angular displacement to get simple harmonic motion and in turn $T_o$, he or she would have to filter out the large angle correction and the damping coefficient. If no correction is made and 'g' is calculated using $T_{av}$ (listed in Table II), the variation in 'g' with angle $\Theta$, would increase (fig(10) iii). The fall in time period with successive oscillations is evident in this experiment, since a micro-processor measures the time period. This would not have been evident in ordinary circumstances. Thus, the experimenter would not have been obvious of this and would only inco-operate large angle corrections, with no corrections regarding damping. The resulting variation is seen as the falling value of 'g' with angle in fig(10) ii). The true constant nature of 'g' (fig 10 ii) is only obtained when both corrections are inco-operated.

**Table II:** Listed is the average time period $T_{av}$ that a student would measure manually along with the time period if he bothers to correct it for large angle oscillations. Also listed are the values of acceleration due to gravity he would have got with his time periods.
Figure 10: Plot shows how the calculated acceleration due to gravity, ‘g’, varies depending on what corrections have been done and its variation with initial angular displacement.

| S.No. | Θ (degrees) | T_{av} (sec) | g_{av} (m/sec^2) | \frac{T_{av}}{1+\frac{\Theta^2}{16}} (sec) | g_{cor} (m/sec^2) |
|-------|-------------|--------------|------------------|---------------------------------|------------------|
| 1.    | 5           | 2.00477      | 9.970            | 2.00382                         | 9.979            |
| 2.    | 10          | 2.00866      | 9.931            | 2.00486                         | 9.969            |
| 3.    | 15          | 2.01246      | 9.893            | 2.00389                         | 9.978            |
| 4.    | 20          | 2.00980      | 9.920            | 1.99466                         | 10.071           |
| 5.    | 25          | 2.01563      | 9.862            | 1.99197                         | 10.098           |
| 6.    | 30          | 2.02093      | 9.811            | 1.98696                         | 10.149           |

**Conclusion**

By doing the pendulum experiment with large angle displacements, calculations become complicated. As much as two informations have to be filtered out, the effect of large angle displacement and the damping factor. The damping coefficient is related to the initial displacement itself. These informations can only be processed if the time period of each oscillation is measured. This is quite impossible manually and only a micro-processor interface is capable of highlighting these features.
Acknowledgement

The authors would like to express their gratitude to the lab technicians of the Department of Physics and Electronics, S.G.T.B. Khalsa College for the help rendered in carrying out the experiments.
The microprocessor program required for measuring the time period of eighty oscillations is listed below.

| Address | Instruction | Hex-code |
|---------|-------------|----------|
| C000    | MVI E       | 1E       |
| C001    | 160D        | A0       |
| C002    | LXI H       | 21       |
| C003    | 00          | 00       |
| C004    | C1          | C1       |
| C005    | MVI A       | 3E       |
| C006    | 00          | 00       |
| C007    | OUT         | D3       |
| C008    | 08          | 08       |
| C009    | IN          | DB       |
| C00A    | 09          | 09       |
| C00B    | ANI         | E6       |
| C00C    | 01          | 01       |
| C00D    | CPI         | FE       |
| C00E    | 00          | 00       |
| C00F    | JZ          | CA       |
| C010    | 15          | 15       |
| C011    | C0          | C0       |
| C012    | JMP         | C3       |
| C013    | 09          | 09       |
| C014    | C0          | C0       |
| C015    | IN          | DB       |
| C016    | 09          | 09       |
| C017    | ANI         | E6       |
| C018    | 01          | 01       |
| C019    | CPI         | FE       |
| C01A    | 01          | 01       |
| C01B    | JZ          | CA       |
| C01C    | 21          | 21       |
| C01D    | C0          | C0       |
| C01E    | JMP         | C3       |
| C01F    | 15          | 15       |
| Address | Instruction | Hex-code |
|---------|-------------|----------|
| C020    | C0          | C0       |
| C021    | LXI B       | 01       |
| C022    | 00          | 00       |
| C023    | 00          | 00       |
| C024    | INX B       | 03       |
| C025    | IN          | DB       |
| C026    | 09          | 09       |
| C027    | ANI         | E6       |
| C028    | 01          | 01       |
| C029    | CPI         | FE       |
| C02A    | 01          | 01       |
| C02B    | JZ          | CA       |
| C02C    | 24          | 24       |
| C02D    | C0          | C0       |
| C02E    | INX B       | 03       |
| C02F    | IN          | DB       |
| C030    | 09          | 09       |
| C031    | ANI         | E6       |
| C032    | 01          | 01       |
| C033    | CPI         | FE       |
| C034    | 00          | 00       |
| C035    | JZ          | CA       |
| C036    | 2E          | 2E       |
| C037    | C0          | C0       |
| C038    | MOV M, B    | 70       |
| C039    | INX H       | 23       |
| C03A    | MOV M, C    | 71       |
| C03B    | INX H       | 23       |
| C03C    | DCR E       | 1D       |
| C03D    | MOV A, E    | 7B       |
| C03E    | CPI         | FE       |
| C03F    | 00          | 00       |
| C040    | JNZ         | C2       |
| C041    | 21          | 21       |
| C042    | C0          | C0       |
| C043    | HLT         | 76       |
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