Simulation of Wave Mixing of Surface Layers in the Sea Area

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Abstract. A problem of surface waves simulation is considered there in the context of combating oil spills. Being independent of waves penetrating into water, oil is disposed in sea as floating surface slicks and as drops suspended in water thickness. An oil surface slick incurs of transferring by wind and current actions, on the one hand, and of a variety of transforming processes, on the other hand. Gravitational spreading is one of the most influent transforming processes for oil on the sea surface. Among physical problems, there are dynamics of thin films of petroleum products on the sea surface and the need to consider moving on the waved sea-atmosphere border. The oil penetration into the water thickness occurs during the dispersion process --- of the transfer of oil from the water surface to the water phase as a result of wave collapse. The carried away oil splits into drops of different sizes which spread and diffuse into the water thickness. The factors such as droplet size, buoyancy, and turbulence affect dispersion stability. The main sources of dispersion energy are collapsing waves formed by wind on the surface of a water body. We suggest using a 2D surface wave model based on the dynamics of interacting particles. In contrast to complex hydrodynamic and spectral models, that makes it possible to estimate quite simply and effectively the proportion of dispersed oil pollution carried from the surface to the depth of the sea area.

1. Introduction
Surface waves are the main and essential factor for dispersing oil spills. Currents in the sea are determined by atmospheric wind action, and by sea-level fluctuations in the form of waves. There are a large number of models describing surface waves in marine areas. These include spectral models that use the boundary layer equation with the Hasselman force and the turbulent energy transfer equation for closure to parameterize the effects of sea surface waves [1]. The basis of such models is the concept of the description of the wave oscillations proposed by Hasselman [2]. Numerical models for calculating surface waves based on the Lagrangian approach for describing the free boundary use recently the turbulent energy transfer and turbulent dissipation equations for closing the system [3,4,5,6]. In these models, special attention is paid to the interaction of wave and shear currents and their influence on the formation of turbulent energy at the atmosphere - sea surface separation boundary. There are many papers devoted to experimental verification of the models used. The paper
[7] presents the results of field and laboratory studies of the mechanism of destruction of surfactant films by intense surface waves. The mechanism of film destruction in the gravitational waves collapse zone is modeled in a controlled laboratory experiment with artificial surface films. Based on the results of field observations, it is shown that in the areas of collapsing wave crests, the film is destroyed due to vertical mixing associated with the generation of turbulence during the collapse. The influence of surface waves on the formation of turbulence as one of the factors of inhomogeneity and destruction of waves is experimentally shown in works [8-10]. Semi-empirical models for evaluating the effect of natural dispersion on the vertical transport of pollutants into the water depth are widely used. The calculation of the vertical dispersion of oil in water, that is, the penetration of oil droplets into the surface layer of the sea due to the collapse of waves, resulting in the formation of an oil-in-water emulsion, can be performed, for example, using the method [11, 12].

\[ Q_{\text{disp}} = C_{\text{oil}} D^{0.57} f_S F_{\text{WC}} d_0^{6.7} \delta d \]  

(1)

where \( Q_{\text{disp}} \) – oil stream (kg/m); \( f_S \) – sea surface, covered by oil; \( d_0 \) – mean diameter of drops; \( \delta d \) – interval of drop diameters; \( C_{\text{oil}} \) – a parameter, depending on oil properties; \( D \) – dissipation of wave energy on square entity:

\[ D = 0.0034 \rho_w g H_{\text{rms}}^3, \quad H_{\text{rms}} = \frac{H_{\text{wave}}}{\sqrt{2}} \]  

(2)

where \( H_{\text{wave}} \) – height of wave; \( \rho_w \) – water density; \( F_{\text{WC}} \) – part of sea surface, covered by whitecapes, in time entity:

\[ F_{\text{WC}} = 0.032 \frac{(W - W_s)}{T_{\text{wave}}} \]

where \( W \) – wind speed; \( W_s \) – wind speed of whitecaps beginning (5 m/s); \( T_{\text{wave}} \) – wave period. It is believed that oil is driven into the water in the form of drops, but large drops immediately float to the surface. The threshold size of drops that remain in the water is estimated at 70 - 150 \( \mu \text{m} \) [12]. The parameter \( C_{\text{oil}} \) depends on the properties of the oil and is determined experimentally. In the work [11], experiments were carried out with three types of oil, as a result, the value of parameter a was estimated from 510 to 1800. It is noted that the parameter \( C_{\text{oil}} \) strongly depends on the viscosity of the oil [11,12]. The parameterization suggested in the model MOHID is possible [13]:

\[ C_{\text{oil}} = -312.25 \ln \left( \frac{\mu_{\text{oil}}}{\rho_{\text{oil}}} \right) + 2509.8 \]  

(3)

where \( \nu = \frac{\mu_{\text{oil}}}{\rho_{\text{oil}}} \) – is kinematic viscosity of oil. If \( C_{\text{oil}} \leq 0 \) it is supposed that \( C_{\text{oil}} = 0 \) and dispersion is absent.

The main problem of such models in describing a surface wave remains the complexity of describing a free surface. Thus, one of the components of currents in the spill zone, namely surface oscillations, is described in insufficient detail by marine hydrodynamic models. This is especially important from the point of view of describing the process of dispersion of oil spills. One way to correct the situation may be to introduce additional artificial modeling of the distribution of oil...
components in the resulting emulsion. For this purpose, a wave model is constructed taking into account the dynamics of particles formed when oil is dispersed by solid sorbents. The resulting mixture is a dispersed system consisting of immiscible liquids. One of them, the polar liquid, is conventionally called the water phase, the other—the non-polar liquid, the oil phase. The continuous phase is called the dispersion medium, the phase that is in the form of individual droplets of small size is called the dispersed phase. The method of particle dynamics is used to describe the oscillations of a set of phases in the form of surface waves.

2. Simulation model

2.1. Simulation model statement

The simulation is performed in a two-dimensional vertical-longitudinal space. The method of particle dynamics used is essentially a description of interacting particle-oscillators [14]. Two media (sea water and oil) are represented as separate spherical elements. The nature of the interaction between elements is considered viscoelastic. The properties of media (density, modulus of elasticity, internal friction) are embedded in the primary properties of the individual elements (mass, diameter, stiffness and viscosity coefficients, interaction restriction radii). The motion of each element in space is described by Newton's second law, which results in the following system of equations:

\[
m_e \frac{d^2 x_i}{dt^2} = \sum_{j=1}^{N_e} \left\{ \begin{cases} \mu(d_e - r_y) \frac{x_i - x_j}{r_y}, & r_y < d_0 \\ 0, & r_y \geq d_0 \end{cases} \right\} + k(r_y - d_1)(v_{xi} - v_{yi})
\]

\[
m_e \frac{d^2 y_i}{dt^2} = -\sum_{j=1}^{N_e} \left\{ \begin{cases} \mu(d_e - r_y) \frac{y_i - y_j}{r_y}, & r_y < d_0 \\ 0, & r_y \geq d_0 \end{cases} \right\} + k(r_y - d_1)(v_{yi} - v_{yi})
\]

Thus, the wave model is a collection of interacting particles-elements of two phases [15]. Unlike the model [15], we assume that, firstly, the size of the elastic interaction region differs from the size of the dissipative interaction region, and secondly, we significantly change the mechanism of excitation of surface waves. In our model, we only perturb the upper layers of the liquid, which is more consistent with the idea of surface fluctuations in marine areas, where the perturbing force is provided by wind load.

The list of the variables and the parameters of the model as follows:

\( m_e = m_{oil}(m_{water}) \) — mass of elements oil-water;
\( d_e = d_{oil}(d_{water}) \) — diameter (size) of elements oil-water;
\( x_i, y_i \) — co-ordinates of elements for wave model;
\( t \) — the current time,
\( N_e \) — the number of elements in the model,
\( k \) — the stiffness coefficient (of the elastic interaction) in the model,
\( \mu \) — the viscosity coefficient (of the dissipative interaction) in the model,
\( d_0 \) — the dimension of the elastic interaction region,
\( d_1 \) — the dimension of the dissipative interaction region,
\( v_{xi}, v_{yi} \) — the velocity components of the \( i \)-th element,
\( r_{ij} \) — the distance between the centers of the \( i \)-th and \( j \)-th elements.
To create waves, the left and right borders of the modeling space of the original rectangular shape (of size $L_X \times L_Y$) move according to the law:

$$X_{Lg} = A \frac{Y}{h} \sin(2\pi ft), \quad X_{Rg} = L_X + X_{Lg};$$

(5)

$X_{Lg}$, $X_{Rg}$ define the motion of the left and right borders of the modeling space respectively,

- $f$ - the frequency of the wave oscillations,
- $A$ - amplitude of the horizontal oscillations of the water surface,
- $h$ - the depth of the modeling space.

This model can be considered as a superstructure (mesoscale model) for simulating surface oscillatory motion. The need to consider such a model is that it provides quantitative estimates of oil dispersion due to surface wave effects on the oil spill spot. It is noted \cite{c9} that at a wind speed of more than 12 m/s due to strong waves, the dispersion of "oil in water" occurs quickly enough and the emulsion of water in oil is almost not formed.

2.2. Numerical solution

Numerical integration of the equations of motion is performed by a modified Euler-Cauchy method, which for one of the spatial directions can be written as follows \cite{14}:

$$x_{i+1} = x_i + v_i \Delta t + a_i \frac{(\Delta t)^2}{2}$$

(6)

$$v_{i+1} = v_i + a_i \Delta t$$

(7)

A Fortran 95 program is written for the numerical integration.

The number of elements in the model $N$ is from 5000 to 50000, depending on problems to be solved. As an example there are some results for data as follows:

- $N = 5000$;
- $k = 5 \times 10^5$ kg/m*sec;
- $\mu = 10^{-3}$ kg*m/sec$^2$;
- $d_c = 10^{-3}$ m;
- $d_o = 1.1 \times 10^{-3}$ m;
- $d_1 = 10^{-3}$.

3. Results

The results of calculations show that this model satisfactorily describes the evolution of a single wave crest. There is a qualitative similarity with the results of experimental studies conducted on the installation, the photo of which is given on Figure 1. The waves in the experiment are excited by rotating blades. The parameters of the wave are determined by its length, height from the wave sole to the top of the crest, and the angular speed of rotation of the blades. For this example, the wave height is 0.06 m and the rotation frequency is 5 Hz.

The results of calculations for the model (4) are presented on Figures 2 and 3.
Figure 1. Surface waves in the work part of the experimental setup.

Figure 2 shows an initial state, the oil slick particles are presented as black balls on the water surface, and stratification of water layers is represented by shades of gray.

Then, as a result of dispersion by wave motion, oil particles penetrate deep into the water, but the bottom layers of water remain undisturbed, as it can be seen on Figure 3. The black polyline in this drawing represents the boundary between floating oil particles on the surface and sunken ones.

Next, Figure 4 shows a diagram of the distribution of oil remaining on the surface (white columns of the diagram) and dispersed oil (black columns). The levels are counted up and down from the black polyline in the previous drawing. The extreme columns of the chart include the number of particles below or above this level, respectively. The ratio of the number of sunken oil particles to their total number is 0.247 (24.7%).

Figure 3. Distribution of oil slick particles in water depth and on the surface.

Figure 4. Diagram of oil particles distribution by levels.

To coordinate the parameters of the natural object and the experimental installation, the similarity conditions were accepted according to the Froude criterion. Thus, for a sea area with a depth of 10 m, the height of the waves corresponding to those shown in the installation photo will be approximately equal to 4.3 m.

4. Conclusion
Modeling of oil and petroleum products spills, as well as the use of models of their behavior, in environmental conditions is one of the important factors in making decisions on liquidation of pollution and forecasting of the state of the environment exposed to pollution from oil spills [16] – [17]. One of the factors affecting the removal of petroleum products from the surface of the water area is natural dispersion due to surface mixing.
The presented simulation model of surface waves is easy to implement and describes the characteristic features of surface vibrations quite adequately. At the same time, it should be noted that the model did not set the task of reproducing the real processes of sea wave oscillations: first, these waves are three-dimensional, and secondly, the hydrodynamic phenomena of surface oscillations in marine areas must be considered in conjunction with the dynamics of the atmosphere. All this significantly complicates the task of assessing the influence of surface waves on the processes of oil pollution dispersion. In addition, this model is part of the research program on ways to deal with the consequences of oil spills. Using this model, an attempt is made to quantify not only the natural dispersion process, but also the dispersing capacity of artificial dispersants. In this context, this model can be considered as an add-on to the macroscopic model for calculating the dynamics of oil spills. The basic, macroscopic model allows you to calculate the main parameters of the spill - the mass, area and thickness of the oil slick. Knowing these parameters and the dispersing capacity of the artificial dispersant, you can calculate its amount for effective use.

5. References
[1] Simon G, Dumas F and Dohaut T 2011 J. Ocean Dynamics 61 1887
[2] Hasselmann K 1970 Geophys. Astrophys. Fluid Dynamics 1(3) 463
[3] Kundu P K 1980 J. Phys Oceanogr. 10(2) 220
[4] Lazur P and Dumas F 2008 Adv Water Resour 31(2) 233
[5] Lazur P, Garnier V, Dumas F, Herry C, Chifflet M 2009 Cont. Shelf Resour 29(8) 985
[6] Ly L N and Garwood R 2000 J. Oceanography 56 473
[7] Ermakov S, Kapustin I and Shomina O 2015 Sovremennye problemy distantionnogo zondirovaniya Zemli iz Kosmosa 12(1) 72 (in Russian)
[8] Donelan M, Dobson F, Smith S and Anderson R 1995 J. Phys. Oceanogr. 25 1908
[9] Drennan W, Donelan M, Terray E and Katsaros K 1996 J. Phys. Oceanogr.26 808
[10] Kitaiigorodskii S. 1994 Tellus 46A 681
[11] Delvigne G and Sweeney C 1988 Oil Chem. Pollut. 4(4) 281
[12] Reed M and all 1999 Spill Sci. Technol. Bull. 5(1) 3
[13] Miranda R 2000 WIT Trans. Ecol. Environ. 40 1
[14] Gould H, Tobochnik J and Christian W 2005 An Introduction to Computer Simulation Methods (New York: Wiley–Interscience) p 819
[15] Kalach A V, Shcherbakov O V 2013 Vestnik Sankt-Peterburgskogo Universiteta Gocudarstvennoi Pozharnoi Sluzby MChS Rossii 4 88
[16] Kucherov O V, Shgoleva E D, Zanina M V, Nikolaeva A V 2015 Science and Technologies: Oil and Oil Products Pipeline Transportation 2(18) p 76
[17] Naumov A D, Nikolaeva A B 2004 Russian Meteorology and Hydrology 10 24