Collins Fragmentation and the Single Transverse Spin Asymmetry

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Abstract

We study the Collins mechanism for the single transverse spin asymmetry in the collinear factorization approach. The correspondent twist-three fragmentation function is identified. We show that the Collins function calculated in this approach is universal. We further examine its contribution to the single transverse spin asymmetry of semi-inclusive hadron production in deep inelastic scattering and demonstrate that the transverse momentum dependent and twist-three collinear approaches are consistent in the intermediate transverse momentum region where both apply.

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1. **Introduction.** Single-transverse spin asymmetries (SSA) in hadronic processes have attracted much attention from both experiment and theory sides, and great progress has been made in the last few years. They are defined as the spin asymmetries when we flip the transverse spin of one of the hadrons in the scattering processes: 

\[ A = \frac{d\sigma(S_\perp) - d\sigma(-S_\perp)}{d\sigma(S_\perp) + d\sigma(-S_\perp)} \]

where \( d\sigma \) is the differential cross section. These physics involve nontrivial nucleon structure and strong interaction QCD dynamics. In recent years, there has been great theoretical progress in exploring the underlying physics for the SSAs observed in various hadronic processes \([1, 2, 3, 4, 5, 6]\). Especially, it has been found that the final/initial state interactions play very important roles to leading to a nonzero SSA in the Bjorken limit \([1]\). These effects are closely related to the gauge properties in the definition of the associated transverse momentum dependent (TMD) parton distributions \([2, 3, 4]\) and the QCD factorizations for the relevant hadronic processes \([5, 6]\). Based on these developments, it has been shown \([7]\) that the two widely used approaches to study SSA physics: the transverse momentum dependent (TMD) approach \([8, 9]\) and the twist-three quark-gluon correlation in the collinear factorization approach \([10, 11, 12]\) are consistent in the intermediate transverse momentum region where both apply. These progresses have laid solid theoretical foundation to study QCD dynamics and the relevant nucleon structure from the SSA phenomena.

However, this consistency has only been studied for the SSA contributions coming from the polarized distributions of the incoming nucleon, where the so-called Sivers function \([8]\) in the TMD approach is equivalent to the Qiu-Sterman matrix element \([11]\) in the twist-three collinear factorization approach \([4, 7]\). It has been difficult to extend to the SSAs associated with the fragmentation functions, namely the Collins mechanism contribution to the SSA \([13]\). The transverse momentum dependent Collins fragmentation function describes the azimuthal hadron distribution correlated with the quark transverse polarization vector \([13]\). When combining with the quark transversity distribution, it will generate the SSAs in the semi-inclusive hadron production in deep inelastic scattering (SIDIS) \([13]\) and single inclusive hadron production in \(pp\) collisions \([14, 15]\). It also contributes to the azimuthal asymmetry in di-hadron production in \(e^+e^-\) annihilation process \([16]\). This contribution is very important not only because it is a significant contribution to the SSA observables in hadronic processes, but also because its contribution is crucial to extract the quark transversity distribution of nucleon, one of three leading twist quark distributions \([17]\) which is weakly
constrained \[18, 19\]. The experimental investigations of these physics have been recently very active from both SIDIS \[20\] and $e^+e^-$ processes \[21\].

Although they both belong to the naive-time-reversal-odd functions, the Collins fragmentation function and the Sivers distribution have different universality properties. For example, the Sivers TMD quark distributions have opposite signs in the SIDIS and Drell-Yan processes \[1, 2\]. However, the TMD fragmentation functions are found to be universal between different processes mentioned above \[6, 15, 22, 23, 24\]. Especially, the final/initial state interactions will not result into a sign change between different processes, although they are important to retain the gauge invariance for the TMD fragmentation functions \[15\].

Therefore, the previous studies on the consistency between the two approaches for the distribution contributions to the SSAs are not straightforward to extend to the fragmentation part, because the underlying physics and the roles played by the initial/final state interactions are totally different \[15, 22\]. In particular, the twist-three quark-gluon correlation function in the twist-three approach associated with the Collins contribution to the SSAs has not yet been identified. In this paper we will study this issue. There has been earlier attempt to construct the twist-three fragmentation function \[12, 25\] contributing to the SSA in hadronic processes. However, the function proposed there has been shown to vanish because of the universality arguments \[23, 24\] (see also the discussions below). In this paper, we will identify the twist-three fragmentation function corresponding to the Collins function. We will further calculate the large transverse momentum behavior of the Collins function from this twist-three fragmentation function, and will find that it is universal. These results will be presented in Sec. 2. In Sec. 3, we will study the Collins contribution to the SSA in SIDIS, and demonstrate that the TMD and collinear factorization approaches are consistent in the intermediate transverse momentum region. We conclude our paper in Sec. 4.

2. Collins Fragmentation at Large Transverse Momentum and Twist-three Fragmentation Function. For the TMD quark fragmentation function, we define the following matrix,

$${\mathcal{M}}_h(z, p_\perp) = \frac{n^+}{z} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{-i(k^+\xi^- - \vec{k}_\perp \cdot \vec{\xi}_\perp)}$$

\[1\]

\[\times \sum_X \left\{ \frac{1}{3} \sum_a \langle 0 | {\mathcal{L}}_0 \psi_b(0) | P_h X \rangle \right\} \times \langle P_h X | \langle \bar{\psi}_{aa}(\xi^-, \vec{\xi}_\perp) L^\dagger \xi | 0 \rangle , \]
where $a = 1, 2, 3$ is a color index, and $p_\perp$ is the transverse momentum of the final state hadron with momentum $P_h$ relative to the fragmenting quark $k$. The quark momentum $k$ is dominated by its plus component $k^+ = \frac{1}{\sqrt{2}}(k^0 + k^z)$, and we have $P_h^+ = zk^+$ and $\vec{k}_\perp = -\vec{p}_\perp/z$. For convenience, we have chosen a vector $n = (1^+, 0^-, 0_\perp)$ which is along the plus momentum direction. The gauge link $L_\xi$ is along the direction $v$ conjugate to $n$. In the case we need to regulate the light-cone singularity, we will choose an off-light-cone vector $v = (v^+, v^-, 0_\perp)$ with $v^- \gg v^+$ and further define $\hat{\zeta}^2 = (v \cdot P_h)^2/v^2$ [5]. The leading order expansion of the above matrix leads to two fragmentation functions for a scalar meson,

$$\mathcal{M}_h = \frac{1}{2} \left[ D(z, p_\perp) \not{\! P} + \frac{1}{M} H_1^\perp(z, p_\perp) \sigma^{\mu \nu} p_\mu n_\nu \right],$$

(2)

where $M$ is a mass scale chosen for convenience, and the second term defines the Collins function $H_1^\perp$. From the above equation, we can further define the transverse-momentum moment of the Collins function [9]:

$$\hat{H}(z) = \int d^2 p_\perp \frac{p_\perp^2}{2M} H_1^\perp(z, p_\perp).$$

By integrating out the transverse momentum, the fragmentation function will only depend on the longitudinal momentum fraction $z$ of the quark carried by the final state hadron. It is straightforward to show that this function can be written as a twist-three matrix element of the fragmentation function,

$$\hat{H}(z) = n^+ z^2 \int \frac{d\xi^-}{2\pi} e^{ik^+ \xi^-} \frac{1}{2} \left\{ \text{Tr} \sigma^\alpha \langle 0 \mid iD_\perp^\alpha + \int_{\xi^-}^{+\infty} d\zeta^- gF_{\alpha \beta}(\zeta^-) \right\} \psi(\xi^-) P_h X$$

$$\times \langle P_h X \mid \bar{\psi}(0) \rangle + h.c.,$$

(3)

where we have chosen the gauge link in Eq. (1) goes to $+\infty$, and $F^{\mu \nu}$ is the gluon field strength tensor and we have suppressed the gauge links between different fields and other indices for simplicity. Since the Collins function is the same under different gauge links [15, 22, 23, 24], we shall obtain the same result if we replace $+\infty$ by $-\infty$ in the above equation. This will immediately show that the matrix element used in [25] vanishes because of the universality property of the Collins fragmentation function [24]. From the above definition, we can see that $\hat{H}(z)$ involves derivative on the quark field and the filed strength tensor explicitly. Therefore, it belongs to more general twist-three fragmentation functions [26]. For example, extending the above definition, we can define a two-variable dependent twist-three fragmentation function as,

$$\hat{H}_D(z_1, z_2) = n^+ z_1 z_2 \int \frac{d\xi^- d\zeta^-}{(2\pi)^2} e^{ik^+ \xi^-} e^{ik^+ \zeta^-} \frac{1}{2} \left\{ \text{Tr} \sigma^\alpha \langle 0 \mid iD_\perp^\alpha(\zeta^-) \psi(\xi^-) \right\} P_h X$$

$$\times \langle P_h X \mid \bar{\psi}(0) \rangle + h.c.,$$

(4)
FIG. 1: Typical Feynman diagrams for the transverse momentum dependent Collins fragmentation function calculated in the collinear factorization approach: contributions from (a) $\partial_\perp$ and (b) $A_\perp$ associated operators in the twist-three quark-gluon correlation functions.

where $k_i^+ = P^+/z_i$ and $k_q^+ = k_i^+ - k_2^+$. Similarly, we can define a $F$-type fragmentation function by replacing $D_\alpha^\perp$ with $F^\perp\alpha$. However, the $F$ and $D$ types are related to each other by using the equation of motion [27]. For our case, it is easy to show that [12],

\[ \hat{H}_D(z_1, z_2) = PV \left( \frac{1}{z_1 - z_2} \right) \hat{H}_F(z_1, z_2) + \delta \left( \frac{1}{z_1} - \frac{1}{z_2} \right) \hat{H}(z_1), \]

where $PV$ stands for the principal value. Therefore, they are not independent. In the following calculations we will only keep $\hat{H}_F$ and $\hat{H}$ in the final results.

The above $\hat{H}_D$ function is different from the twist-three fragmentation function $\hat{E}(z_1, z_2)$ introduced in [26]. In particular, $\hat{H}_D$ is the imaginary part whereas $\hat{E}$ is the real part of the same matrix element involving twist-three quark fragmentation functions. Explicitly, if we replace $iD_\perp$ with $D_\perp$ in Eq. (5) we will obtain the definition of $\hat{E}$. If the time-reversal-invariance argument applies to the fragmentation functions the above $\hat{H}_D$ function would vanish. However, this argument does not apply here [13], such that the $\hat{H}_D$ function exists and contributes to the SSA in hadronic process. We emphasize that it is actually this function which corresponds to the Collins mechanism.

Therefore, the above defined $\hat{H}_D$ ($\hat{H}_F$) and $\hat{H}$ Eqs. (4,5) will be our starting point to formulate the Collins mechanism in the collinear factorization approach. First, we can calculate the transverse momentum dependence of the Collins function in the perturbative region from the twist-three fragmentation functions $\hat{H}_D$ and $\hat{H}$. To do this, we will have to not only calculate the perturbative diagrams with gluon radiation, but also to perform the twist expansion and take into account full contributions from the $\partial_\perp$ and $A_\perp$ operators in the...
definitions of $\hat{H}_D$ and $\hat{H}$ at this order \[27\]. We plot the typical Feynman diagrams in Fig. 1 for the Collins function calculation from these contributions, where a transversely polarized quark (with momentum $k = P_h/z_h + k_\perp$) fragments into a final state hadron $P_h$ by radiating a gluon with momentum $k_1$. For the contribution from Fig. 1(a), we do collinear expansion of the partonic scattering amplitude in terms of $k'_\perp$, the transverse momentum of the quark which couples to the final state hadron as shown in Fig. 1(a). Combining this collinear expansion with the hadron fragmentation matrix will form the $\partial_\perp \psi$ associated correlation function in Eqs. (4,5) \[26\]. Similarly, the contributions from Fig. 1(b) with transverse gluon field $A_\perp$ connecting the partonic part and the hadron fragmentation part will result into the $A_\perp$ associated correlation function. These contributions have to be sorted into gauge invariant functions such as $\hat{H}_F$ ($\hat{H}_D$) and $\hat{H}$, respectively. While the detailed derivations will be presented in a forthcoming publication, here we summarize the final result,

$$H_1^+(z_h, p_\perp) = \frac{\alpha_s}{2\pi^2}(p_\perp^2)^2 \int \frac{dz}{z} \left[ A + \delta(\hat{\xi} - 1)\hat{H}(z)C_F \ln \frac{\hat{\xi}^2}{p_\perp^2} \right],$$

where $\hat{\xi} = z_h/z$ and the function $A$ is defined as

$$A = C_F \left[ z^3 \frac{\partial}{\partial z} \hat{H}(z) \right] \left( -2\hat{\xi} \right) + \hat{H}(z) \frac{2\hat{\xi}^2}{1 - \hat{\xi}} + \int \frac{dz_1}{z_1^2} PV \left( \frac{1}{z - z_1} \right) \hat{H}_F(z, z_1)$$

$$\times \left[ -C_F \frac{2z_h}{z} \left( 1 + \frac{z_h}{z_1} - \frac{z_h}{z} \right) - C_A \frac{2z_h z_1 (z + z_1)}{2} \frac{z}{z(z - z_1)(z_1 - z_h)} \right].$$

In the above calculations, we have adopted an off-light-cone gauge link in Eq. (1) to regulate the light-cone singularity and $\hat{\xi}^2$ has been defined above.

An important check of the above result is its universality property. Indeed, we find that our calculations are free of the gauge link direction used in Eq. (1), i.e., the Collins function in Eq. (6) is universal. In particular, in the calculations we find that the gauge link does not contribute to a pole in the Feynman diagrams of Fig. 1. Therefore, the gauge links going to $+\infty$ and $-\infty$ lead to the same results. This is consistent with the universality argument for the Collins fragmentation function \[6, 15\]. We have checked that the contribution from the twist-three function $\hat{E}_F$ introduced in \[25\] is also consistent with the universality property as those calculated above.

3. Collins Effect in Semi-inclusive DIS. In this section, we extend to calculate the the Collins contribution to the SSA in semi-inclusive DIS, $ep_1 \rightarrow e'\pi X$, and show that the TMD and collinear factorization approaches are consistent in the intermediate transverse
momentum region $\Lambda_{QCD} \ll P_{h,\perp} \ll Q$, where $\Lambda_{QCD}$ is the typical nonperturbative scale and $P_{h,\perp}$ is the transverse momentum of the final state hadron. Again, the above defined $\hat{H}_D$ and $\hat{H}$ will be our starting basis to calculate this contribution in the collinear factorization approach.

In the SIDIS process $e p \uparrow \rightarrow e' \pi X$ where the incoming nucleon is transversely polarized, the transverse spin dependent differential cross section can be formulated as

$$\frac{d\sigma(S_{\perp})}{d x_B dy dz_h d^2\vec{P}_{h,\perp}} = \frac{2\pi\alpha_{em}^2}{Q^2} L^{\mu\nu}(\ell, q) W_{\mu\nu}(P_A, S_{\perp}, q, P_h),$$

where $\alpha_{em}$ is the electromagnetic coupling, $\ell$ and $P_A$ are incoming momenta for the lepton and nucleon, $S_{\perp}$ the polarization vector for nucleon, $q$ the momentum for the exchanged virtual photon with $Q^2 = -q \cdot q$, $P_h$ is the momentum for the final state hadron. The kinematic variables are defined as $x_B = \frac{Q^2}{2P_A \cdot q}$, $z_h = \frac{P_A \cdot P_h}{P_A \cdot q}$, $y = \frac{P_A \cdot q}{P_A \cdot \ell}$. $L^{\mu\nu}$ and $W^{\mu\nu}$ are leptonic and hadronic tensors, respectively. To calculate the above differential cross section, it is convenient to decompose the hadronic tensor into several terms: $W^{\mu\nu} = \sum_i W_i V_i^{\mu\nu}$, where $V_i$ follow the definitions of [28].

As mentioned above, we will calculate the transverse spin dependent differential cross section from the Collins mechanism, in particular the contributions from the twist-three fragmentation functions $\hat{H}_D$ and $\hat{H}$ defined in Sec.2. We follow the same procedure as that for the large transverse momentum Collins fragmentation function calculated in the last section, and we will take into account the contributions from both $\partial_{\perp}$ and $A_{\perp}$ associated fragmentation matrix elements in Eqs. (4,5). The relevant Feynman diagrams can be drawn accordingly, and we show two examples in Fig. 2. Similarly, their contributions can be
summarized into the terms associated with the gauge invariant functions $\hat{H}_F$ and $\hat{H}$. Furthermore, we are interested in the differential cross section in the intermediate transverse momentum region $\Lambda_{QCD} \ll P_{h\perp} \ll Q$. In the limit of $P_{h\perp} \ll Q$, we find that only $V_4$ and $V_9$ in the hadronic tensor decomposition contribute in the leading power of $P_{h\perp}/Q$. These two terms contribute the same to the differential cross sections except the azimuthal angular dependence: the contribution from $V_4$ is proportional to $\cos(2\phi_h)\sin(\phi_s - \phi_h)$ whereas that from $V_9$ is proportional to $\sin(2\phi_h)\cos(\phi_s - \phi_h)$, where $\phi_h$ and $\phi_s$ are the azimuthal angles of the transverse momentum $P_{h\perp}$ and the polarization vector $S_\perp$ relative to the lepton scattering plane. The total contributions from these two terms will be proportional to $\sin(\phi_h + \phi_s)$,

$$\left.\frac{d\sigma(S_\perp)}{dx_Bdydz_{\perp}d^2P_{h\perp}}\right|_{P_{h\perp} \ll Q} = \left.\frac{4\pi\alpha_s^2}{Q^4}x_B(1 - y)\sin(\phi_h + \phi_s)\frac{1}{z_{\perp}^2}\frac{1}{2\pi^2|q_{\perp}|^3}\int \frac{dx dz}{xz} h_1(x) \left\{ A\delta(\xi - 1) + B\delta(\hat{\xi} - 1) \right\}, \quad (9)$$

in the limit of $P_{h\perp} \ll Q$, where $\xi = x_B/x$ and $\hat{\xi} = z_{\perp}/z$ and $A$ function has been given in Eq. (7) and $B$ is defined as

$$B = C_F\hat{H}(z_h) \left[ \frac{2\xi}{(1 - \xi)_+} + 2\delta(\xi - 1)\ln\frac{Q^2}{|q_{\perp}|^2} \right]. \quad (10)$$

Following the same procedure as that in [7] for the Sivers effects, we will find that the above single transverse spin dependent differential cross section calculated from the twist-three fragmentation functions $\hat{H}_D$ and $\hat{H}$ in the collinear factorization approach can be reproduced by the TMD factorization for the same observable [6] by using the large transverse momentum Collins fragmentation function calculated in Sec.2, and the known results for the quark transversity distribution and the soft factor [7]. This clearly demonstrates that in the intermediate transverse momentum region, the twist-three collinear factorization approach and the TMD factorization approach provide a unique picture for the Collins contribution to the SSA in the semi-inclusive DIS.

4. Conclusion. In this paper, we have studied the Collins mechanism contribution to the single spin asymmetry in semi-inclusive hadron production in DIS process. We have identified the corresponding twist-three fragmentation function, and shown that the transverse momentum dependent and collinear factorization approaches are consistent in the intermediate transverse momentum region. Especially, we have also demonstrated that the Collins
fragmentation function calculated is universal, and free of the gauge link direction. It will be important to extend our calculations to the Collins contributions to the SSAs in other processes, such as in hadron production in polarized $pp$ scattering and di-hadron correlation in $e^+e^-$ annihilation. We will address these issues in a future publication, together with a detailed derivation of this paper.

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