On division operation of any numbers: introducing a new technique

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Abstract. There are four basic mathematical operations. One of them is a division operation. A division is an inverse operation of multiplication. The symbol of division operation is: or /, thus we can write the division of any two numbers as \(x:y\) or \(x/y\). This operation is taught in a school from elementary to high level. Most students consider the mathematical operation of division is one of a complex operation. Many techniques are given such as direct technique by means of repeated subtraction, and ‘porogapit’ technique. Those techniques have been well-known and can be found in the students school text book or in the internet. In this paper, we develop a new technique namely a reminder-like theorem, and some other specific techniques for specific numbers. Those new techniques show the effectiveness in solving division problem, and it gains more efficiency in terms of algorithms.

1. Introduction
People always use mathematical operations in everyday life, even though they are not realised to use it, Harini [4]. When they go for shopping, especially, they uses at least four operations, namely addition, subtraction, multiplication and division, see Setyono [9]. Among of those operation, the division operation is considered to be the most difficult one to study by school students, see John [5], since when we do a division operation we also use a subtraction operation or even addition. In the learning process, the division operation will not only develop mathematical arithmetic skills, but it will impact on the rise of students’ critical thinking skills, refers to Dafik [2], Monalisa [6], Nazula [7], and Sumartiningsih [10]. Therefore, the basic concepts related to division operations in mathematics must be studied fundamentally. The division of any two numbers is defined as a repetition of subtraction, say \(6:2 \rightarrow 6-2=4, 4-2=2, 2-2=0\), thus \(6:2=3\). \(15:5 \rightarrow 15-5-5-5=0\), thus \(15:5=3\), see Ding [3].

Furthermore, understanding the concept, students need to master some basic facts of division. Some numbers are divisible by some numbers, otherwise un-divisible. For this case students should give a quick response to have an inefficient calculation process, Van de Walle [11]. Secondly, a number with specific properties is easy to deal but due to the lack of technique knowledge we are trapped by a long process of calculation, for instance we straightforwardly use a “porogapit”, even the problem is easy to solve by a fast technique. We need to develop a various technique in solving division operations for specific number involved in the operation. Dealing with division operation on one to two digit numbers divided by a one digit number is easy to use a simple way. However, if the operation is dealt with large numbers, say more than two digit numbers, it requires a special procedure. In the classroom, many teachers still use the ‘porogapit’ technique to solve division problems, even the students...
encounter many difficulties. The ‘porogapit’ technique gives students a certain complexity, since they should involve subtraction to solve the concept of division. In addition, Arima [1] claimed that they are also required to understand the position values of the digits in the multiplication process, and in this part, sometimes the students often make mistakes.

Furthermore, along with the development of this era, many researchers have developed many research results related to division techniques. This will make it easier for us to do the division calculation. Thus, in this paper we will provide the latest research results related to the efficient technique of division operations for any numbers, specific numbers, 2, 5, and 9 and its repetitions. We name this new technique as remaining technique as well. For this purpose we introduce a notation $[x]$ which role as a numeral storage of the division calculation.

2. Research Method
This study uses mathematical inductive-deductive processes, it is similar with the research carried out by Raharjo [8]. By means the notation $[x]$, we will solve the division problem. The meaning of brackets $[x]$ is a number that has not been decomposed from its place value, but the position of the place value has been clear. For example, we will illustrate as follows: $[45]$ means that the number contains the ones digit 5 and the tens digit 4, then $[786]$ means that the number contains the ones digit 6 and the tens digit 8, and hundreds is 7. Next, when we have $[45][28]$, this number cannot be read yet as a number, we will be able to read it if there is a concatenation process as follows $[45][28] = 4 \{5 + 2\} 8 = 478$. For $[79][23][48] = 7\{9 + 2\} 78 = 7\{11\} 78 = 81788$. Moreover $[78 ][29][47][98] = 7\{10\}[13 ][16] 8 = 81468$ and etc. To have the final result of the number representation in square brackets, we have to read them from behind.

Furthermore, the visualization of calculation are carried out under an analytical approach. The specific illustrations are given to go further for a purpose of making generalization. We use a simplest software library of Matlabor Excel to test the truth of the results. At the end, we establish an algorithm for each technique.

3. Result Findings
We will describe our results on two sections. The first section is related to the division of any two numbers, and the second one is related to the division of some specific numbers by using the reminder-like theorem.

3.1. Reminder-Like Theorem for Division of any Two Numbers
In this research, we have developed a new technique called a reminder-like theorem for any two numbers. We can do division for tens, hundreds, and thousands, etc. How does this technique works, we will give the following illustrations.

$$48 : 3 = 16$$

\[
\begin{array}{c}
\text{4} & \text{18} & \div & \text{3} \\
\hline
\text{1} & \text{6}
\end{array}
\]

Division of tens 48 divided by 3. It is the simplest one since the divisor is less than the tens unit.

$$688 : 8 = 86$$

\[
\begin{array}{c}
\text{6} & \text{8} & \text{48} & \div & \text{8} \\
\hline
\text{8} & \text{6}
\end{array}
\]

Division of hundreds by a divisor is greater than the hundreds unit.
5984 : 4 = 1496

Division of thousands 5984 divided by 4. It is the simplest one since the divisor is less than the thousands unit.

\[
\begin{array}{cccc}
5 & 19 & 38 & 24 \\
\hline
1 & 4 & 9 & 6
\end{array}
\]

54 : 4 = 13\frac{2}{4}

Division of tens 54 divided by 4. It is the simple, but it is not divisible by 4. It gives a reminder.

\[
\begin{array}{c}
54 \\
\hline
13\frac{2}{4}
\end{array}
\]

(reminder2)

\[
\begin{array}{c}
5 & 14 \\
\hline
1 & 3
\end{array}
\]

263 : 5 = 52\frac{3}{5}

Division of hundreds by divisor is greater than the first number of hundreds and it gives a reminder.

\[
\begin{array}{cccc}
2 & 6 & 13 \\
\hline
5 & 2
\end{array}
\]

(reminder3)

9539 : 7 = 2384\frac{3}{4}

Division of thousands by divisor is greater than the first number of hundreds and it gives a reminder.

\[
\begin{array}{cccc}
9 & 15 & 33 & 19 \\
\hline
2 & 3 & 8 & 4
\end{array}
\]

To have a brief steps, we can develop the following algorithm.

**Algorithm 1.** Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and divisor.

1. For \(i = 1, 2, \ldots, n\) do the following
   - Consider whether the divisor is less than the tens, hundreds, or thousands units
   - Divide each unit respectively, and store the reminder exactly in front of the following unit
   - Repeat the process till the last unit \(x_n\).
2. The result is the number under the line.
3. Done
3.2. Reminder-Like Theorem for Division of Some Specific Numbers

For the specific numbers, we will solve the division problem when a number is divided by 2, 5, twins 2 & twins 2 & triple 5, 5, 9, and twins 9 & triple 9. The specific numbers are very important to study since for certain case, any numbers are developed by a specific number. The followings, we give some illustration of the remainder-like theorem applied for this specific number.

First, we will show how any numbers divided by 2.

1. 438 : 2 = [20][15][40] = 219
2. 2876 : 2 = [10][40][35][30] = 1438
3. 75294 : 2 = [35][25][10][45][20] = 37647
4. 895738 : 2 = [40][45][25][35][15][40] = 447869

* first, multiply the numbers by 5

Algorithm 2. Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and divisor 2.

1. For \(i = 1,2, \ldots, n\) do the following
   - Multiply each digit of the number \(x_1x_2 \ldots x_n\) by 5
   - Write the results in right bracket format \([x]\)
   - Do the concatenation process of the \([x][y][z]\).
2. Obtain the result by obtaining the numbers from the right side
3. Done

Secondly, the divisor of the number is 5. To deal with this problem, the first step is multiplying each digit of the numbers by 2. Suppose we have a division operation of digits numbers \(x_1x_2 \ldots x_n\). By multiplying the numbers by 2, we \((x_1x_2), (x_2x_2), \ldots, (x_nx_2)\). By means of the sign \([x]\), we will develop the following. To have better understanding we give the following illustration.

1. 355 : 5 = [6][10][10] = 71
2. 4235 : 5 = [8][4][6][10] = 847
3. 85370 : 5 = [16][10][6][14][0] = 17674
4. 895735 : 5 = [16][18][10][14][6][10] = 179147

* first, multiply the numbers by 2

Algorithm 3. Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and divisor 5.

1. For \(i = 1,2, \ldots, n\) do the following
   - Multiply each digit of the number \(x_1x_2 \ldots x_n\) by 2.
   - Write the results in right bracket format \([x]\).
   - Do the concatenation process of the \([x][y][z]\).
2. Obtain the result by obtaining the numbers from the right side.
3. Done.

Next, we will discuss the division of any number by twins 2. This technique is a combination technique of the division of any numbers by the specific number 2 and 5 mentioned above, and the division of any number by 11. The first step, we need to place down the beginning digit underneath the number, followed by subtracting the next digit, and so on until the last digit. The result of subtraction is divided by the last number, if the result is 0, it means that the division without remainder or the number is divisible by the divisor, and if it is not divisible, it means the division gives the remainder.

The next step is to perform the technique of dividing any number by number 11. For example the number is 990: 22, then 99: 11: 2 = x. The first step is to place down 9 underneath the number, subtract the following number 9 by 9 - 9 = 0, then the result 0 divided by the last number give 0.
Finally, we will deal with 90 by using the division technique by number 2. Follow the technique of dividing any number by number 2, we usually firstly multiply each digit in that number by 5, obtained \((9 \times 5 = 45)\) & \((0 \times 5 = 0)\). By means of the sign \([x]\), we will develop the following. To have better understanding we give the following illustration.

1. \(990 : 22 \rightarrow 990 : 11 : 2 = \)
   \[
   \begin{align*}
   \text{0/0} &= 0 \\
   990 &= 11 = 90 \\
   90 : 2 &= [45][0] \rightarrow 45 \\
   \end{align*}
   \]

2. \(1892 : 22 \rightarrow 1892 : 11 : 2 = \)
   \[
   \begin{align*}
   \text{2/2} &= 0 \\
   1892 &= 11 = 172 \\
   172 : 2 &= [5][35][10] \rightarrow 86 \\
   \end{align*}
   \]

3. \(4235 : 55 \rightarrow 4235 : 11 : 5 = \)
   \[
   \begin{align*}
   \text{5/5} &= 0 \\
   4235 &= 11 = 385 \\
   385 : 5 &= [6][16][10] \rightarrow 77 \\
   \end{align*}
   \]

4. \(3575 : 55 \rightarrow 3575 : 11 : 5 = \)
   \[
   \begin{align*}
   \text{5/5} &= 0 \\
   3575 &= 11 = 325 \\
   325 : 5 &= [6][4][10] \rightarrow 65 \\
   \end{align*}
   \]

*first, change the divisor with 22=11 x 2, then followed the following techniques.

**Algorithm 4.** Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and divisor of twin number.
1. For \(i = 1, 2, \ldots, n\) do the following.
   - Change the divisor by 22=11 x 2.
   - Place down the beginning digit underneath the number.
   - Subtracting the next digit by initial number.
   - Repeat until the last digit.
2. Obtain the result by obtaining the numbers from the right side.
3. Done.

Next we will present the division of any number with twin numbers 3. This technique is a combination technique of specific number techniques 2 and 5 and includes the division by number 111. The initial step is to divide the first three digits of the number 111 then do the consecutive subtraction from the fourth, fifth and so on until the last digit. If the result of subtracting the last digit is 0 it means division of the number is without remainder, but if the number is not evenly divided it means division by remainder. Then, the result of dividing the first triple number by 111 is divided by the number 2 or 5 using the technique described above. For example, an example of 8214: 222 is given. Like the previous step, we change it to 8214: 111: 2 = \(x\) the initial step 821: 11 = \(x\), multiply 111 by a number in which the result approaches 821, thus it is multiplied by 7, do subtraction (8-7), it gives the remainder of 1, combine with the next number (12-7) the remainder of 5, combine with the following numbers (51-7) the remainder of 44, combine with the last number, namely 4, thus 444: 111 = 4. Combine the result of division, it will give 74. The final step is 74: 2. We do it by multiplying the numbers with 5, \(7 \times 5 = [35]\) and \(4 \times 5 = [20]\). Combine these products to make 8214: 222= [35][20]= 370. By means of the sign \([x]\), it concludes the process. To have better understanding we give the following illustration.

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Algorit\[\text{m 4}$. Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and divisor of twin number.
1. For \(i = 1, 2, \ldots, n\) do the following.
   - Change the divisor by 22=11 x 2.
   - Place down the beginning digit underneath the number.
   - Subtracting the next digit by initial number.
   - Repeat until the last digit.
2. Obtain the result by obtaining the numbers from the right side.
3. Done.
1. \[ \frac{8214}{222} \div \frac{111}{2} = \begin{align*} 8-7 &= 1 \\
12-7 &= 5 \\
51-7 &= 44 \\
444 - 111 &= 333 \\
\text{Result: 333} \\
\end{align*} \]

2. \[ \frac{12432}{222} \div \frac{111}{2} = \begin{align*} 1-1 &= 0 \\
2-1 &= 1 \\
14-1 &= 13 \\
133 - 111 &= 22 \\
222 - 111 &= 111 \\
\text{Result: 111} \\
\end{align*} \]

3. \[ \frac{7215}{555} \div \frac{111}{5} = \begin{align*} 7-6 &= 1 \\
12-6 &= 6 \\
61-6 &= 55 \\
555 - 111 &= 444 \\
\text{Result: 444} \\
\end{align*} \]

4. \[ \frac{23865}{555} \div \frac{111}{2} = \begin{align*} 2-2 &= 0 \\
3-2 &= 1 \\
18-2 &= 16 \\
166 - 111 &= 55 \\
555 - 111 &= 444 \\
\text{Result: 444} \\
\end{align*} \]

*first, change the divisor with 222=111 x 2, then followed the following techniques.

**Algorithm 5.** Given a number of tens, hundreds, and thousands, \( x_1x_2 ... x_n \) and divisor of triple number.

1. For \( i = 1, 2, ..., n \) do the following.
2. Change the divisor by 222=111 \times 2.
3. Divide the first three digits of the number 111 then,
   - Do the consecutive subtraction from the fourth, fifth and until the last digit.
   - Divided by the number 2 or 5 using the technique described above.
   - Do subtraction, and combine the reminder to the next digit.
   - Store the individual result and use \([x] \) to have final result.
4. Done.

Next, we will present the division of two-digit, three-digit numbers and so on with the number 9. The beginning step is starting from the first digit, then adding the derived numbers with the numbers behind them in sequence. If the sum results in two numbers, the first number is stored above the beginning number and so on until the last digit. Next, divide the final sum by the number 9. Finally add up all the obtained numbers. For example, \( 333:9 \), then the first step is to lower down the number 3 at the front, then add the number 3 with the 3 behind it and the result is 6, then add the number 6 with the number 3 with the last number 9 written below it. The final step is the number 9 divided by the number 9 is 1 then place it above the number 9. Add up the results of the numbers that have been lowered down to determine the final result. By means of the sign \([x] \) we will develop the following.

To have better understanding we give the following illustration.

1. \[ \begin{align*} 3 & \ 3 \ 3 : 9 = \\
1 & \ 9/9=1 \\
3 & \ 6 \ 9 \\
3 & \ 7,0 \\
\end{align*} \]

4. \[ \begin{align*} 4 & \ 7 \ 6 \ 4 \ 1 : 9 = \\
1 & \ 1/127 \\
7 & \ 3 \ 7 \ 8 \\
8 & \ 4 \ 9,0 \\
\end{align*} \]
2. \[405 : 9 = \frac{45}{9} = 5\]
3. \[4923 : 9 = \frac{1112}{9} \approx 89\]
4. \[4236 : 9 = \frac{4278}{9} \approx 475\]
5. \[8856 : 9 = \frac{2723}{9} \approx 302\]
6. \[23256 : 9 = \frac{25728}{9} \approx 286\]

Algorithm 6. Given a number of tens, hundreds, and thousands, \(x_1x_2 \ldots x_n\) and the divisor 9.
1. For \(i = 1, 2, \ldots, n\) do the following
   - Place down the beginning digit underneath the number.
   - Add the next digit by initial number.
   - Repeat until the last digit.
2. Divide the last result by number 9.
3. Obtain the result by obtaining the numbers from the right side.
4. Done.

Next, we will present the division of more than two digit numbers by 99. The first step to take is to place down the beginning number and add the numbers that are placed down with the numbers behind them. The second step is to combine the numbers derived with the sum and delimited by a comma. If the number is divided by 3 digits then the number is derived by 1 digit, if the number is divided by 4 digits then the number is derived 2 digits and so on. For example, 339: 99, then the first step is to place down the number 3 then add the number with the 39 behind it which results number 42, the last step is to combine the number 3 with 42 by placing comma sign between them, namely 3.42. By means of the sign \([x]\), we will develop the following. To have better understanding we give the following illustration.

1. \[339 : 99 = \frac{39 + 3 = 42}{3,42}\]
2. \[425 : 99 = \frac{25 + 4 = 29}{4,29}\]
3. \[4236 : 99 = \frac{36 + 42 = 78}{42,78}\]
4. \[5763 : 99 = \frac{63 + 57 = 120 - 99 = 21}{57,21}\]
5. \[7435 : 99 = \frac{35 + 74 = 109 - 99 = 10}{74,10}\]
6. \[75236 : 99 = \frac{75 + 23 = 98}{75,98}\]

\[\frac{98 + 61 = 159}{75,9959}\]
If the addition gives 3 numbers, then the addition is aligned with the last digit, if the addition gives 2 numbers then only add the number behind the answer.

If the division is 3 digits, then the division is one digit, whereas if the division is 4 or 5 digits it will be decreased by 2 numbers.

Algorithm 7. Given a number of tens, hundreds, and thousands, \( x_1 x_2 \ldots x_n \) and divisor 99.
1. For \( i = 1, 2, \ldots, n \) do the following.
2. Subtract the left side number by the number to be divided, subtract by 1 digit if division is 3 digits and 2 numbers if division is 4 or 5 digits.
3. Add the numbers that are derived with the numbers behind them.
4. Combine the derived number with their sum then put comma between them to determine the final result.
5. Done.

Finally, we will present the division of a number that is more than two digits by the number 999. The initial step taken is to place down the three digits of the first left side, then add the two digits of the derived numbers with the two numbers behind them. The second step is to combine the numbers derived with the sum result, delimited by a comma between them. For example, 3762: 999, then the first step is to place down 376, the second step is to add up the two digits in front, namely 37 with the last two digits, which gives 2. Since it gives 2, we need to add 0 to 2, so that \( 37 + 20 = 57 \). The final step is to combine the number 376 with the number 57 behind it, delimited by a comma, the result is 3.7657. By means of the sign \( [x] \), we will develop the following. To have better understanding we give the following illustration.

\[
\begin{align*}
3762 : 999 &= \quad 38265 : 999 \\
&= (37 + 20=57) \quad (38 + 65=103) \\
&= 3,7657 \quad 382 \\
&= 103 \\
&= 38,303 \quad 385 \\
7246 : 999 &= \quad 78569 : 999 \\
&= (72 + 60 = 132) \quad (78 + 69 = 147) \\
&= 724 \quad 785 \\
&= 147 \\
&= 78,647 \\
\end{align*}
\]

If the addition gives 3 digits, then the addition is aligned with the last digit, if the addition gives 2 digits then only add the number behind the result.

If the division is 4 digits then add the number after it by adding the number 0.

Algorithm 8. Given a number of tens, hundreds, and thousands, \( x_1 x_2 \ldots x_n \) and divisor 999.
1. For \( i = 1, 2, \ldots, n \) do the following.
2. Take down the 3 leftmost digits of the number to be divided.
3. Add 2 digits derived with 2 digits after it, if the division is 4 digits then the addition with the numbers behind it is added by 0.
4. Combine the derived number with the sum then delimit with a comma to determine the final result.
5. Done.

4. Discussion
Based on the results of this study, these new techniques have many advantages, but also disadvantages. Using a reminder like-theorem for arbitrary numbers makes it easier for us to do division by minimizing the errors that often occur in students. When dividing using "porogapit", the error often occurs in placing the results of the division with the multiplication results which are often confused. Then division with specific numbers 2 & 5, twins 2 & twigs 3, and 9, 99 & 999 makes it easier for us to calculate division, this technique only uses multiplication and addition counting operations. The division says specific 2 & 5, we use the multiplication operation in its operation, so that students do not fail to do the division. While the division of twins 22, 222, 55 and 555 uses a combination technique between the methods such as division 2 & 5 and division of numbers 11. Meanwhile, division of number 9 uses more sum operation of the numbers. This technique is believed to be very useful for students to improve their mathematical generalization thinking skills, see Monalisa, et. al. [6]. Apart from making it easy to do multiplication, these quick techniques help us to improve the three types of mathematical operations at once namely addition, multiplication and division. In addition, the advantages of this quick technique, apart from being the most practical technique and the easiest technique to apply, we can also use for the twin division of all numbers. The division process by mineralizing the number to be divided by the divisor 11 and the number itself. As a result, this implies that calculating the multiplication with this technique will be faster and easier than the "porogapit" division technique, even compared to other techniques as well.

5. Conclusion
This research has succeeded in developing a new technique of division of any number by any number as divisors, and also developed division of any number with a special number as divisor. Previously, this technique of division had never been found, we have developed since the division operation is the hardest operation in mathematics for school students. We can conclude that the results of developing this technique have many advantages, since it can make it easier for students to solve division problems quickly and accurately. However, there are still many specific problems in division, so some new techniques need to be developed, thus that students have many choices in solving division problems.

Acknowledgment
Herewith, I deeply acknowledge to CGANT-RG, the University of Jember Indonesia, for the consistent support of this research activity for the year 2021.

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