Inclusive and Exclusive Scatterings from Tensor Polarized Deuteron

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Abstract. The possibility of using a tensor polarized deuteron target in electroproduction reactions creates new opportunities for studying different phenomena related to the short-range hadronic and nuclear physics. The use of the tensor polarized deuteron allows us to isolate smaller than average inter-nucleon distances for the bound two-nucleon system. In this report we consider several high $Q^2$ reactions which are particularly sensitive to the short-range two-nucleon configurations in the deuteron. One is the relativistic dynamics of electron-bound-nucleon scattering, which can be studied in both inclusive and exclusive reactions, and the other is the strong final state interaction in close proximity of two nucleons that can be used as a sensitive probe for color-transparency phenomena.

1. Introduction
The deuteron is the simplest nuclear system with the wave function strongly dominated by the $pn$ component. Thus, it can be used for studies of many aspects of $pn$ strong interactions. One such aspect is the study of the $pn$ system at short distances where one hopes to gain access to many fundamental issues of nuclear dynamics, such as a relativistic description of nuclear structure, the dynamics of the $NN$ repulsive core, the role of the non-nucleonic degrees of freedom, and the hadron-quark transition at very short distances.

However one problem in realizing such a program of studies is that the deuteron is barely bound with the charge-rms radius of about 2.1 fm. In the momentum space, this fact is reflected in the very steep momentum distribution of the unpolarized deuteron wave function with the strength concentrated predominantly at the small relative momenta in the $pn$ system (see the curve labelled as $\rho^{\text{unp}}$ in Fig. 1).

However the fact that the deuteron has a D-wave which vanishes at small momenta indicates that isolating it in any given reaction with the deuteron will effectively suppress the small-momentum/long-range contributions.

This can be seen from the polarized density matrix of the deuteron [1]:

\[
\rho_5^2(k_1, k_2) = u(k_1)u(k_2) + \left[1 - \frac{3|k_2 \cdot a|^2}{k_2^2}\right] \frac{u(k_1)w(k_2)}{\sqrt{2}} + \left[1 - \frac{3|k_1 \cdot a|^2}{k_1^2}\right] \frac{u(k_2)w(k_1)}{\sqrt{2}} + \left(\frac{9}{2} \frac{(k_1 \cdot a)(k_2 \cdot a)^* (k_1 \cdot k_2)}{k_1^2 k_2^2} - \frac{3}{2} \frac{|k_1 \cdot a|^2}{k_1^2} - \frac{3}{2} \frac{|k_2 \cdot a|^2}{k_2^2} + \frac{1}{2}\right) w(k_1)w(k_2),
\]

(1)
where $u(k)$ and $w(k)$ represent the $S$ and $D$ partial waves respectively. The polarization vector $\vec{a}$ is defined through the deuteron spin wave functions

$$
\psi^{10} = i \cdot a_z, \quad \psi^{11} = -\frac{i}{\sqrt{2}}(a_x + ia_y), \quad \psi^{1-1} = \frac{i}{\sqrt{2}}(a_x - ia_y),
$$

where $\psi^{1\mu}$ is the projection of the deuteron’s spin in the $\mu$ direction. The unpolarized density matrix of the deuteron is defined as

$$
\rho^{\text{unp}}(k_1, k_2) = \frac{1}{3} \sum_a \rho_a^{d}(k_1, k_2).
$$

Since $\lim_{k \to 0} w(k) = 0$, it follows from Eq.(1) that any polarization combination of $\rho_a^{d}$, in which the $u^2$ term is canceled has an enhanced sensitivity to the larger internal momenta (smaller distances) of the deuteron as compared to the unpolarized case. It follows from Eq.(1) that the $u(k_1)u(k_2)$ term does not depend on the polarization vector $\vec{a}$, thus one can cancel this term summing any two polarization components of the density matrix and subtracting the doubled value of the third polarization component. One example is:

$$
\rho^{20}(k_1, k_2) \equiv \frac{1}{3}(\rho^{11} + \rho^{1-1} - 2\rho^{10}) = \left( \frac{3k_1^{2z}}{k_1^2} - 1 \right) \frac{u(k_2)w(k_1)}{\sqrt{2}} + \left( \frac{3k_2^{2z}}{k_2^2} - 1 \right) \frac{u(k_1)w(k_2)}{\sqrt{2}}
$$

$$
+ \left( \frac{3}{2} \left[ \frac{(k_1k_2)(k_1k_2 - 3k_1^{2z}k_2^{2z})}{k_1^2k_2^2} + \frac{k_1^{2z}}{k_1^2} + \frac{k_2^{2z}}{k_2^2} \right] \right) w(k_1)w(k_2).
$$

Fig. 1 presents the examples of the density matrices for unpolarized, $((\rho^{11} + \rho^{1-1} + \rho^{10})/3$, transverse, $\rho^{10}$, and tensor polarized, $\rho^{20}$, deuteron targets as they enter in the impulse approximation term of the $d(e, e'p)n$ cross section (in this case $k_1 = k_2 = p$). As it can be seen from Eq.(3), the tensor polarized density matrix depends only on the terms proportional to $u(p)w(p)$ and $w(p)^2$.

This suggests ways for studying several issues of nuclear physics related to the short-range interactions using tensor polarized deuteron targets.

Figure 1. Momentum dependences of different combinations of the polarized density matrices. Solid, dashed and dotted curves correspond to unpolarized, tensor polarized, and transverse polarized distributions, respectively.
1.1. The D-wave Component of the Deuteron

The presence of the D-wave component in the deuteron follows from the existence of the finite quadruple momentum of the deuteron. It is directly related to the tensor part of the NN interaction. The modern NN interaction potentials which fit the existing NN phase-shifts with $\xi^2 \leq 1$ predict the overall probability of the D wave in the deuteron to be in the range of 4.87% (cdBonn [2]) to 5.76% (AV18 [3]). Although the difference seems rather small, these parameterizations predict substantially different high momentum strengths above 400 MeV/c, which is predominantly related to the D component. The need to understand the D-wave momentum distribution became more pressing recently, due to the observation of the strong dominance of pn short range correlations (SRCs) in nuclei as compared to the pp and nn SRCs [4, 5], which is related to the tensor component of NN interaction in the SRC [6, 7]. The dominance of the tensor interaction in the high momentum component of the nuclear wave function may play important role in the dynamics of the asymmetric nuclei [8, 9] with nontrivial implications for superdense nuclear matter and neutron stars [10, 11].

Very recently another interesting aspect of the role of the D-wave was revealed in the observation of $k^{-4}$ scaling of the momentum distributions in the deuteron and nuclei [12] in the momentum range of 300-600 MeV/c, which shows intriguing similarities with the contact observed in the two-component symmetric atomic systems of fermions.

As it follows from Eq.(3), the processes involving a tensor-polarized deuteron target are directly related to the structure of the D-partial wave component in the deuteron.

1.2. Relativistic Dynamics of the NN Bound System

One of the important issues in studying nuclear structure at short distances is the need for a relativistic description of the bound system. This is an important issue also in understanding the QCD medium effects with recent studies indicating that parton distribution modifications in nuclei are proportional to the high momentum component of nuclear wave function [13].

The deuteron is the simplest bound system and naturally any self-consistent attempt to understand the relativistic effects in the bound nuclear systems should start with the deuteron. The issue of the relativistic description of the deuteron has long history with extensive research started already in late 1970’s (see e.g. [14, 15, 16, 17]).

The experimental studies of the relativistic effects in the unpolarized deuteron up to now included the large $Q^2$ elastic $ed$ scatterings [18]. However, due to complexities in the reaction mechanism the relativistic effects were difficult to isolate.

The processes involving a tensor polarized deuteron target are expected to exhibit enhanced sensitivity to the relativistic effects due to higher average momentum of the bound nucleon entering in the polarized nuclear density matrix as compared to the unpolarized case. Thus one can discuss possible high momentum transfer reactions off tensor polarized deuterons and study their sensitivity to the relativistic effects of the bound nucleon motion.

1.3. Final State Hadronic Interactions

The fact that the tensor polarized deuteron is characterized by a larger average internal momenta indicates that the two nucleons are in close proximity and scattering from such target can yield large final state hadronic interactions in exclusive reactions. The latter can be used in studies of the hadronic properties of produced particles such as nucleons [1, 19, 20] or vector mesons [21, 22, 23].

2. Electroproduction Reactions Involving Tensor Polarized Deuterons

We will discuss several electroproduction reactions in which utilizing the unique features of the tensor polarized deuteron would allow us to study the above discussed properties of high energy processes.
2.1. High $Q^2$ Near-Threshold Inclusive Scattering

High $Q^2$ ($>1$ GeV$^2$) inclusive scattering at $x_{Bj}>1$ is known to probe large longitudinal momenta of bound nucleon interacting with the virtual photon. One of the important indications that these reactions indeed probe the high momentum component of the nuclear wave function is the observation of the scaling in the ratios of the $A(e,e'X)$ cross sections to that of deuteron [24, 25] or light nuclei [26, 27].

Using the tensor polarized deuteron in such reactions allows us to prepare the nucleus in the most compact state in which, due to the absence of the pure $(S$-wave)$^2$ contribution (Fig. 1), the system in average is sensitive to a higher momentum of the nucleon in the deuteron for given $x$ and $Q^2$. At large $Q^2 > 1$ GeV$^2$ kinematics, the probed longitudinal momenta of the bound nucleon is $p_z \approx m_N(1-x)$, or the light cone momentum fraction $\alpha \geq x$. Because of these kinematic conditions and the absence of the $(S$-wave)$^2$ contribution, one expects a measurable relativistic effects already at $x \sim 1.2$. Such an early onset of the relativistic effects indicates that one should be able to separate them from the uncertainty related to the choice of the NN potential.

The sensitivity to the relativistic effects is estimated using the theoretical calculations based on two very different approaches. The first approach describes the bound nucleon in the deuteron rest frame treating the interacting nucleon as being virtual (virtual nucleon approximation (VNA)) by taking the residue over the positive energy pole of the spectator nucleon. In this case the deuteron wave function satisfies the covariant equation of a two-nucleon bound system with the spectator being on mass-shell (see e.g. [28, 29]).

Another approach is based on the observation that high energy processes evolve along the light-cone. Therefore, it is natural to describe the reaction within the light-cone non-covariant framework [16, 17]. Negative energy states do not enter in this case, though one has to take into account so called instantaneous interactions. In the approximation when non-nucleonic degrees of freedom in the deuteron wave function can be neglected, one can relate the light-cone wave functions to those calculated in the lab frame by introducing the LC $pn$ relative three momentum $k = \sqrt{m^2+p_t^2/(\alpha(2-\alpha))} - m^2$. 

![Figure 2](image_url)
In Fig. 2 the predictions for VNA [28] and LC [24] approximations are given for the tensor asymmetry, $A_{zz} = \frac{T^{20}}{\sigma_{\text{unp}}}$, at the highest $Q^2$ kinematics for the recently proposed experiment at Jefferson Lab [30]. As the figure shows for $1.2 \leq x \leq 1.4$ the uncertainty due to the NN potentials used in the calculations is much smaller than the relativistic effects, while at high $x > 1.4$ we have substantial uncertainty due to the potentials. The calculations predict large $A_{zz}$ asymmetry in $x > 1$ region with significant potential of discrimination between both relativistic and NN potential effects.

2.2. High Momentum Transfer Exclusive Electrodisintegration of the Deuteron

The possibility of the extension of inclusive measurements to the exclusive electrodisintegration reactions in which struck nucleon carries almost all the momentum of the virtual nucleon gives direct access to the dynamics of the bound nucleon. Recently the first high $Q^2$ experiments were completed for $d(e, e'p)n$ reactions [31], in which it was observed that due to the onset of the eikonal regime in the final state interaction (FSI) of the stuck proton with the spectator neutron it is possible to isolate kinematic regions with minimal and maximal FSI effects. This observation is in agreement with the theoretical calculations which are based on high energy approximations in the description of FSI effects (see e.g. [1, 19, 20, 32, 28, 33]).

The pattern of FSI in $d(e, e'p)n$ reaction is best seen by considering the ratio of the cross section of the full calculation to that of the plane wave impulse approximation: $T = \frac{\sigma_{\text{full}}}{\sigma_{\text{PWIA}}}$. 

![Figure 3](image_url)

**Figure 3.** The dependence of the transparency $T$ on the angle $-\theta_{sq}$ and the momentum, $p_s$ of the recoil nucleon. The angle is defined relative to $\vec{q}$.

Fig. 3 presents the expectations for $T$ as a function of the recoil nucleon angle $\theta_{sq}$ relative to $\vec{q}$ for different values of recoil nucleon momentum. The figure demonstrates the distinctive angular dependence of the ratio $T$. At recoil nucleon momenta $p_s \leq 300\text{MeV/c}$, $T \leq 1$ and has a minimum, while at $p_s > 300\text{MeV/c}$, $T > 1$ and has a distinctive maximum. It can be seen from this picture that the FSI is small at kinematics in which recoil momenta in the reaction is parallel or anti-parallel to $q$ (referred to as collinear kinematics). The FSI dominates in the kinematics where $\theta_{pq} \approx 90^0$, more precisely the maximal re-scattering corresponds to the kinematics in which $\alpha \equiv \frac{E_{pq}}{m} = 1$ (referred to as transverse kinematics). The analysis of Fig. 3 shows that one can indeed isolate the kinematic domains where PWIA term is dominant from the domain in which FSI plays a major role. The ability to identify these two kinematics is an important advantage of high $Q^2$ $d(e, e'p)n$ reactions. It allows us to concentrate on the
different aspects of the dynamics of $d(e,e'p)n$ reaction with less background effects. Namely the
collinear kinematics are best suited for studies of bound nucleon dynamics, while in transverse
kinematics one can concentrate on the physics of hadronic re-interaction.

![Figure 4](image-url)

Figure 4. The $p_n$ dependence of the $d(e,e'p)n$ tensor polarization asymmetry at $\theta_{sq} = 180^0$. Solid and dashed lines are PWIA predictions of the LC and VNA approximations. The marked curves present the same (LC, VNA) calculations including the FSI effects.

To quantify these statements, in Fig. 4 we present the calculation of $A_d$ (which is similar to $A_{zz}$ but for exclusive processes) in collinear kinematics ($\theta_{sq} = 180^0$) within virtual nucleon and light-cone approximations described above. These estimates, similar to the inclusive scattering case, show significant relativistic effects already at moderate momenta of 250 – 300MeV/c for the spectator nucleon. It is interesting that the FSI effects further increase the difference between VNA and LC predictions, making it even easier to discriminate between VNA and LC approximations.

2.3. Dynamics of the Final State Interactions
As it follows from Fig. 3, studying the $d(e,e'p)n$ reactions at transverse kinematics allows us to enhance the effects due final state interaction of the struck nucleon off the spectator nucleon. One expects that FSI effects will be further enhanced for tensor-polarized deuterons since it corresponds, on average, to a smaller configuration than in the case of the unpolarized target.

If one assumes that FSI are determined by diffractive small angle $pn$ rescatterings, one can estimate the effect of the FSI on $A_d$, which is presented in Fig. 5.

The comparison of Fig. 5(a) and (b) shows the very strong sensitivity of the asymmetry $A_d$ to the final state interaction at the transverse kinematics: the FSI significantly diminishes the asymmetry starting at $p_n = 300$ MeV/c and $\theta_{sq} \sim 90^0$ (or $\alpha_n \approx 1$). This indicates that the quantity $A_d$ can be used as a very sensitive observable for studying the FSI dynamics; for example, the onset of the Color Transparency (CT) phenomena (for details of CT phenomena see Ref. [34]) at large $Q^2$ ($\geq 4 \text{ GeV}^2$).

For numerical estimates, we consider the $Q^2$ dependence of the asymmetry $A_d$ for fixed and transverse momenta of the spectator neutron. This dependence for $p_t = 300$ MeV/c, is presented in Fig. 6. One can see from this figure that CT effects can change $A_d$ by as much as factor of two for $Q^2 \sim 10 \text{ GeV}^2$. It is worth noting that the same models predict only 10-15% effect for $(e,e'p)$ reactions on unpolarized nuclear targets.
Figure 5. The dependence of tensor asymmetry $A_d$ on the angle and momentum of the spectator neutron in $d(e, e'p)n$ reaction. (a) PWIA approximation (b) Full calculation that includes FSI.

Figure 6. The $Q^2$ dependence of $A_d$ for $\alpha_n = 1$. Solid line - Full calculation without CT effects, dashed prediction of Quantum Diffusion Model [35], dashed-dotted - prediction of three state resonance model model [36], dotted -PWIA.

3. Conclusion and Outlook

The use of the tensor polarized deuteron targets in high energy electro-production reactions would provide a unique possibility for studying the NN strong interaction at short space-time separations. We demonstrate that such reactions allow a direct test of the relativistic effects in the short range NN interaction. They also allow us to discriminate between different NN potentials.

Another advantage of the tensor polarized deuteron as a compact two-nucleon system is the possibility to significantly enhance the final state hadronic interaction in the exclusive $d(e, e'p)n$ reactions. This provides a very sensitive tool for studying color transparency phenomena, which will be manifested in the suppression of the FSI effects at large $Q^2$ kinematics.

All these studies are a part of the broad program dedicated to the investigation of the structure of deeply bound nucleons as well as the physics of color transparency [37]. This program could benefit tremendously from the advances of building polarized deuteron targets that can be
operated with high current electron beams.

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