Transitioning from equal-time to light-front quantization in $\phi^4_2$ theory

Sophia S. Chabysheva and John R. Hiller

Department of Physics and Astronomy
University of Minnesota-Duluth
Duluth, Minnesota 55812
(Dated: November 6, 2018)

Abstract

We implement the limiting procedure of Hornbostel for the quantization of two-dimensional $\phi^4$ theory in a sequence of coordinate systems that interpolate between equal-time and light-front coordinates. This allows computation of the vacuum state in the odd and even sectors of the theory and computation of massive states built on these vacua. Results are compared with those of the equal-time calculations of Rychkov and Vitale and those of standard light-front calculations.
I. INTRODUCTION

Recently, there has been a resurgence of interest in the spectrum of two-dimensional $\phi^4$ theory \[1, 6, 8, 9\], partly because of what appeared to be an inconsistency between results from equal-time quantization and light-front quantization. Although the apparent inconsistency has been resolved, as a difference in mass renormalizations \[8, 10, 11\], there remain various issues related to the structure of the vacuum. In light-front quantization \[12–14\], the vacuum is famously trivial \[2\] but in equal-time quantization, it is as complex as any of the other eigenstates.

In order to see more clearly what may be happening for the light-front vacuum, we apply the interpolation procedure championed by Hornbostel \[16\] and emphasized by Ji \[24\], in which the coordinates are chosen to be

$$x^\pm = \frac{1}{\sqrt{2}}[\sqrt{1\pm ct} \pm \sqrt{1\mp cz}],$$

\[(1.1)\]

with $x^+$ chosen as the time coordinate. The parameter $c$ ranges from 0 to 1, with 0 being the light-front limit \[26\] with $x^\pm = (t \pm z)/\sqrt{2}$, and 1 the equal-time limit, with $x^+ = t$ and $x^- = -z$. The minus sign for the equal-time spatial coordinate may seem incongruous, but it is a permissible choice that simplifies the notation.

The conjugate energy and momentum are

$$p^\pm = \frac{1}{\sqrt{2}}[\sqrt{1\pm cE} \mp \sqrt{1\mp cp_z}].$$

\[(1.2)\]

Dot products of the momentum and spatial two-vectors are then given by $p \cdot x = p_+ x^+ + p_- x^-$. The mass-shell condition becomes

$$\mu^2 = E^2 - p_z^2 = cp_+^2 - cp_-^2 + 2sp_+p_-,$$

\[(1.3)\]

with $s \equiv \sqrt{1-c^2}$. The positive root for $p_+$ yields

$$p_+ = \left[\sqrt{p_-^2 + c\mu^2} - sp_-\right]/c.$$

\[(1.4)\]

For the $c = 1$ and $c = 0$ limits, this expression becomes

$$p_+ \rightarrow \begin{cases} \sqrt{p_-^2 + \mu^2}, & c = 1 \\ \frac{\mu^2}{2p_-}, & c = 0, p_- > 0 \\ \frac{\sqrt{\mu^2}}{2cp_-}, & c \rightarrow 0, p_- = 0 \\ \frac{\sqrt{\mu^2}}{2cp_-}, & c \rightarrow 0, p_- < 0 \end{cases}.$$

\[(1.5)\]

Clearly, the zero modes ($p_- = 0$) and negative $p_-$ states have infinite light-front energy and are removed from the spectrum, as $c \rightarrow 0$.

1 For citations of older work, see \[8\].
2 See, however, the remarks by Collins \[15\] on nontrivial aspects.
3 There are earlier applications of interpolation, to two-dimensional QCD \[14\], the Dirac equation \[18\], and perturbation theory \[19\], as well as of quantizations close to the light-cone \[20, 21\] applied to two-dimensional QED and QCD \[22, 23\].
4 This coordinate transformation is not a Lorentz transformation, which makes the $c \rightarrow 0$ limit technically distinct from the infinite-momentum-frame limit \[25\].
5 Contrary to our usual convention but in keeping with an equally common choice, $x^\pm$ include a factor of $1/\sqrt{2}$. This matches Hornbostel’s construction \[16\] and simplifies some of the expressions.
However, these modes can contribute to light-front computations and, in particular, to vacuum expectation values [16, 24]. A standard illustration of this is in the spectrum and VEV of a free scalar field that has been shifted by a constant. The shift introduces to the Lagrangian a term that is linear in the field; on the light-front, this can only contribute via zero modes. This can be seen quite cleanly in the \( c \to 0 \) limit, where the Hamiltonian eigenvalue problem in an \( x^- \) box has an analytic solution for any \( c > 0 \). A numerical solution in a truncated Fock space works just as well. This is discussed in Sec. II.

A more interesting case is that of \( \phi^4 \) theory, where, as already mentioned, the vacuum needs to be better understood. In Sec. III we will explore what the solutions with \( c \neq 0 \) and the \( c \to 0 \) limit can tell us. The calculations are done numerically, in a Fock basis of discrete momentum states in an \( x^- \) box. A brief summary is given in Sec. IV.

II. SHIFTED FREE SCALAR

A free scalar field that is shifted by a constant provides an interesting example of the impact of zero modes on a light-front calculation. This can be seen explicitly in the \( c \to 0 \) limit, where a nonzero contribution is found for the vacuum energy and the VEV of the field. These analytic results [16, 24] can be replicated in a numerical calculation using a Fock basis of zero modes. We illustrate this here.

The Lagrangian of a free scalar field of mass \( \mu \) is

\[
L_0 = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \mu^2 \phi^2. \tag{2.1}
\]

In terms of the interpolating coordinates \([1,1]\) with arbitrary \( c \) in two dimensions, this becomes [16],

\[
L_0 = \frac{1}{2} c [ (\partial_+ \phi)^2 - (\partial_- \phi)^2 ] + s \partial_+ \phi \partial_- \phi - \frac{1}{2} \mu^2 \phi^2. \tag{2.2}
\]

The (free) Hamiltonian is

\[
P_+^0 = \int dx^- (\pi \partial_+ \phi - L_0), \tag{2.3}
\]

with \( \pi = c \partial_+ \phi + s \partial_- \phi \). The mode expansion for the field is

\[
\phi = \int_{-\infty}^{\infty} \frac{dp_-}{\sqrt{4\pi w_p}} [ a(p_-) e^{-ip^-x} + a^\dagger(p_-) e^{ip^-x} ], \tag{2.4}
\]

with \( w_p \equiv \sqrt{p^2 + c \mu^2} \). The nonzero commutation relation is

\[
[a(p_-), a^\dagger(p'_-) ] = \delta(p_- - p'_-). \tag{2.5}
\]

The normal-ordered free Hamiltonian can then be written as

\[
P_+^0 = \int_{-\infty}^{\infty} dp_- \frac{w_p - sp_-}{c} a^\dagger(p_-) a(p_-). \tag{2.6}
\]

Similarly, the momentum operator is

\[
P_- = \int_{-\infty}^{\infty} dp_- p_- a^\dagger(p_-) a(p_-). \tag{2.7}
\]
Discretization consistent with discrete light-cone quantization (DLCQ) is invoked by placing the system in a box $-L < x^- < L$ with periodic boundary conditions. The momentum is then discrete, $p_- = n\pi/L$, as set by the integer $n$; however, unlike DLCQ, $n$ ranges over all integers, not just the positive ones. As shown in (1.5), negative $p_-$ is removed from the spectrum only for $c = 0$. An energy cutoff is then required for a finite basis. We do still define a positive integer $K$ as the resolution, so that in the $c \to 0$ light-front limit, the total momentum is $P_- = K\pi/L$. The index $n$ for individual momentum then ranges from 1 to $K$ in the light-front limit, and momentum fractions $p_-/P_-$ are just $n/K$.

The discrete mode expansion for arbitrary $c$ is

$$\phi = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{4\pi w_n}} [a_n e^{-in\pi x^-/L} + a_n^\dagger e^{in\pi x^-/L}],$$

with $w_n \equiv \sqrt{n^2 + c\tilde{L}^2}$, $\tilde{L} \equiv \mu L/\pi$, and $[a_n, a_m^\dagger] = \delta_{nm}$. The free Hamiltonian becomes

$$P_+^0 = \frac{\mu}{L} \sum_{n=-\infty}^{\infty} \frac{w_n - sn}{c} a_n^\dagger a_n.$$

We now shift the field: $\phi \to \phi + v$. The new Lagrangian is

$$\mathcal{L} = \mathcal{L}_0 - \mu^2 v\phi - \frac{1}{2}\mu^2 v^2,$$

and the Hamiltonian, having dropped a constant, is

$$P_+ = P_+^0 + P_+^I,$$

with the interaction part

$$P_+^I = \int_{-L}^{L} dx^- \mu^2 v\phi = \mu v\sqrt{\frac{L\pi}{c^{1/4}}} [a_0 + a_0^\dagger].$$

In the light-front limit $c \to 0$ this interaction term completely disappears, as it does in a native light-front calculation where zero modes are neglected. Without this term, the shift in the field and the shift in the energy cannot be recovered. However, a calculation for arbitrary $c > 0$ succeeds, and the light-front limit can then be taken. This was discussed by Hornbostel, and we repeat the argument here.

The vacuum eigenstate is a coherent state of zero modes

$$|\text{vac}\rangle = e^{-\alpha(a_0^\dagger - a_0)}|0\rangle.$$

This works because the coherent state is, as always, an eigenstate of the annihilation operator

$$a_0|\text{vac}\rangle = -\alpha|\text{vac}\rangle$$

and, therefore,

$$P_+|\text{vac}\rangle = \left[ -\frac{\mu w_0}{Lc}\alpha a_0^\dagger + \frac{\mu v\sqrt{L\pi}}{c^{1/4}} a_0^\dagger - \frac{\mu v\sqrt{L\pi}}{c^{1/4}} \alpha \right]|\text{vac}\rangle.$$
Given \( w_0 = \tilde{L}\sqrt{c} \), we only need \( \alpha = v\sqrt{\tilde{L}\pi\sqrt{c}} \) to eliminate the \( a_0^\dagger \) terms and make this coherent state indeed an eigenstate of \( \mathcal{P}_+ \), with an eigenenergy of \( -\mu\sqrt{\frac{\tilde{L}}{c\pi}}\alpha = -\frac{1}{2}\mu^2v^2(2\tilde{L}) \). This restores the constant originally dropped from the Hamiltonian. In the light-front limit \( c \to 0 \), \( \alpha \) also becomes zero, and this state becomes the empty state \( |0 \rangle \), but the energy is independent of \( c \). All massive states are decoupled and remain as free.

The VEV of the field is given by
\[
\langle \text{vac} | \phi(0) | \text{vac} \rangle = \langle \text{vac} | \frac{1}{\sqrt{4\pi w_0}} [a_0 + a_0^\dagger] | \text{vac} \rangle,
\]
which reduces to
\[
\frac{1}{\sqrt{4\pi w_0}} (-\alpha - \alpha) = \frac{2}{\sqrt{4\pi \tilde{L}\sqrt{c}}} v\sqrt{\tilde{L}\pi\sqrt{c}} = -v.
\]
This, of course, reflects the original shift in the field. Obviously, this is independent of the value of \( c \). A non-zero result is obtained because the vanishing coefficients of zero mode contributions are compensated by the \( 1/c^{1/4} \) divergence in the zero-mode part of the field.

We need not rely on having an analytic solution to see this result for the vacuum state. A numerical solution in a finite basis of zero modes \( (a_0^\dagger)^n|0 \rangle \), truncated to \( n = 10 \), yields the spectrum shown in Fig. 1 as a function of \( c \). The lowest state’s energy is clearly independent of \( c \), with the energies of all higher states diverging as \( c \) approaches zero.

![FIG. 1. Spectrum for the shifted free scalar in a zero-mode basis, truncated to an occupation number of 10, as a function of the interpolating parameter \( c \). Equal-time quantization corresponds to \( c = 1 \) and light-front quantization to the limit \( c \to 0 \).](image)

This nontrivial light-front limit provides a connection with the known results for equal-time quantization. In the equal-time approach, the linear interaction term is not lost but makes a direct contribution to the Hamiltonian. The solution for the vacuum state then includes the consequences of the shift in the field, as can be seen here for \( c = 1 \). The light-front limit \( c \to 0 \) reproduces the results obtained from equal-time quantization.
To explore these connections further, we consider two-dimensional $\phi^4$ theory, where it is known that equal-time and light-front quantizations differ in the vacuum contributions to mass renormalization \cite{10}. Thus, the remainder of the paper is an analysis of $\phi^4$ theory in terms of the interpolating coordinates \cite{11}. The discrete form of the theory is constructed in the next subsection, and the results of the numerical solution are discussed in Sec. \textbf{III.B}.

\section{\textit{\boldmath $\phi^4$} Theory}

\textbf{A. Analysis}

The Lagrangian for $\phi^4$ theory is

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!}\phi^4.$$  \hfill (3.1)

We construct the (discrete) interaction Hamiltonian from the $\phi^4$ term as

$$P^I_+ = \int_{-L}^L dx \frac{\lambda}{4!} :\phi^4 :.$$  \hfill (3.2)

Substitution of the discrete mode expansion \cite{28}, with $L = \bar{L}\pi/\mu$, and evaluation of the now-trivial integrals, yields,

$$P^I_+ = \mu \frac{g\bar{L}}{4} \sum_{n_1...n_4} \frac{1}{\sqrt{w_{n_1}...w_{n_4}}} \left[ \frac{1}{12} (a_{n_1}^\dagger ... a_{n_4}^\dagger + a_{n_1}^\dagger ... a_{n_4}^\dagger) \delta_{n_1+...+n_4,0} \right. \hfill (3.3)$$

$$\left. + \frac{1}{3} (a_{n_1}^\dagger a_{n_2}^\dagger a_{n_3} a_{n_4}^\dagger a_{n_1}^\dagger a_{n_4}^\dagger a_{n_3} a_{n_2}^\dagger) \delta_{n_1,n_2+n_3+n_4} \right] \hfill (3.4)$$

$$+ \frac{1}{2} a_{n_1}^\dagger a_{n_2}^\dagger a_{n_3}^\dagger a_{n_4}^\dagger \delta_{n_1+n_2+n_3+n_4}, \hfill (3.5)$$

with $g \equiv \lambda/(4\pi\mu^2)$ the dimensionless coupling.

The Hamiltonian eigenstates are constructed as Fock-state expansions

$$|\psi\rangle = \sum_k \sum_{n_1...n_k} \psi_k(n_1 ... n_k) \frac{1}{\sqrt{k!}} \prod_{i=1}^k a_{n_i}^\dagger |0\rangle.$$  \hfill (3.6)

To take into account the symmetrization of states with $k$ identical bosons, we rewrite this sum as

$$|\psi\rangle = \sum_k \sum_{n_1\geq n_2\geq...\geq n_k} \frac{1}{\sqrt{N_{n_1}!...N_{n_k}!}} \phi_k(n_1 ... n_k) \prod_{i=1}^k a_{n_i}^\dagger |0\rangle,$$  \hfill (3.7)

where $N_{n_i}$ is the number of bosons with momentum index $n_i$ and the wave functions are related by

$$\phi_k = \sqrt{\frac{k!}{N_{n_1}!...N_{n_k}!}} \psi_k.$$  \hfill (3.8)

The normalization is

$$1 = \langle \psi | \psi \rangle = \sum_k \sum_{n_1...n_k} |\psi_k|^2 = \sum_k \sum_{n_1\geq n_2\geq...\geq n_k} |\phi_k|^2.$$  \hfill (3.9)
The probability $P_k$ for the Fock sector with $k$ bosons is then given by

$$P_k = \sum_{n_1 \ldots n_k} |\psi_k|^2 = \sum_{n_1 \geq n_2 \geq \ldots \geq n_k} |\phi_k|^2.$$  \hfill (3.10)

The eigenstates must satisfy $(P^0_+ + P^I_+)|\psi\rangle = E|\psi\rangle$. For simplicity, we look for eigenstates at rest, with total $P_-=0$, and either an odd or even number of constituents; the Hamiltonian changes particle number by only even amounts and therefore does not mix odd and even Fock states. The sums over the number of constituents $k$ are then limited to even or odd values. In particular, we have expansions in the form

$$|\text{even}\rangle = \psi_0|0\rangle + \sum_n \psi_2(n) \frac{1}{\sqrt{2}} a_n^\dagger a_{-n}|0\rangle + \cdots$$  \hfill (3.11)

$$|\text{odd}\rangle = \psi_1 a_0^\dagger|0\rangle + \sum_{n_1, n_2} \psi_3(n_1, n_2) \frac{1}{\sqrt{6}} a_{n_1}^\dagger a_{n_2}^\dagger a_{-n_1-n_2}|0\rangle + \cdots$$  \hfill (3.12)

For the purpose of having a finite numerical matrix calculation, the infinite Fock basis is truncated both in the sum over constituents and in energy. First, the number of constituents is limited to a maximum of $K$, so that the sum over $k$ in $|\psi\rangle$ is finite. Second, the total energy of each Fock state, as specified by the free Hamiltonian (2.9), is limited to be no more than a fixed energy, $E_{\text{max}}$.

The total energy of a Fock state is given by $\mu \frac{\tilde{L}}{L} \sum_n \sqrt{n^2 + \tilde{L}^2}$, where the sum extends over all bosons in the Fock state. For small $c$, the individual contributions behave as in (1.5):

$$\frac{w_n - sn}{c} \rightarrow \begin{cases} \frac{\tilde{L}^2}{2n}, & n > 0 \\ \frac{\tilde{L}^2}{c}, & n = 0 \\ \frac{\tilde{L}^2}{2n}, & n < 0. \end{cases}$$  \hfill (3.13)

Thus, for $n \leq 0$, the contributions diverge and Fock states with such constituent momenta will be removed by the energy cutoff as $c$ goes to zero. For eigenstates with total $P_-=0$, where the integers $n$ must sum to zero, there must be at least one constituent with $n \leq 0$. For such a state, a sufficiently small value of $c$ will cause the energy cutoff to remove all the Fock states, except the trivial empty state $|0\rangle$. However, this would be inconsistent with the analysis of the shifted free scalar, where the addition of a $c$-dependent energy cutoff would have removed the (infinite set of) Fock states needed to construct the coherent state for the vacuum eigenstate.

So, we instead keep the Fock basis unchanged as $c$ changes by imposing the energy cutoff at $c=1$ and then leaving the basis fixed when $c$ is decreasing. Therefore, for all $c$ values, the energy limit on Fock states is given by

$$\frac{\mu}{L} \sum_n \sqrt{n^2 + \tilde{L}^2} \leq E_{\text{max}}.$$  \hfill (3.14)

We do not study the dependence on these truncations, nor on the box size, in a systematic way, our purpose being a qualitative understanding of the light-front limit as the parameter $c$ goes to zero. In equal-time quantization there has been considerable work by Rychkov and collaborators [3] on the renormalization necessary to reduce the cutoff dependence and facilitate very accurate calculations with minimal basis sizes. Attempting this for arbitrary $c$ is certainly of some interest but is beyond the scope of the present work.
B. Results

As a check on the calculation, the even vacuum energy for equal-time quantization \((c = 1)\) is plotted in Fig. 2 as a function of the coupling \(g\). This is equivalent to the results of Rychkov and Vitale (RV) \([2]\), where \(g = 6g_{RV}/\pi\) and \(\tilde{L} = L_{RV}/(2\pi)\). We also plot the subtracted

\[
\Delta E \sim \frac{g}{w_0^2} \frac{1}{E_0 - 4w_0/c w_0^2},
\]

(3.15)

where the middle fraction is the contribution of the intermediate state with four bosons. The
FIG. 3. Subtracted equal-time spectrum $E_n - E_0$ computed with $c = 1$, $E_{\text{max}} = 20\, \mu$, $\tilde{L} = 1$, and up to 20 constituents. Here $E_n$ is refers to the nth level, with $n$ even (odd) for the even (odd) sector.

FIG. 4. Difference between even and odd vacuum states for decreasing values of $c$. The difference is larger for smaller $c$; at $g = 0$, the plotted ratio is just $1/\sqrt{c}$.

dimensionless individual zero-mode energy is $w_0 = \tilde{L}\sqrt{c}$. The shift then diverges as $c^{-3/2}$, which is consistent with the $c$ dependence shown in Fig. 5. Thus the $c \to 0$ limit includes the vacuum graphs that renormalize the boson mass differently from a purely light-front calculation, where zero-modes are not included. This explains the inconsistency between $c > 0$ calculations (which are consistent with the equal-time calculation) and $c = 0$, the
FIG. 5. Energy of the even vacuum state as a function of $1/c$ for different couplings $g$. For fixed $c$, the energy is more negative for larger $g$.

FIG. 6. Lowest order contribution to the vacuum energy.

pure light-front calculation.

IV. SUMMARY

We have shown that, for $\phi^4$ theory, the $c = 0$ light-front limit of the coordinates (1.1) is not equivalent to a native light-front calculation. The energy of the vacuum state is unbounded from below, and the value of the critical coupling remains consistent with the equal-time calculation. Thus, unlike the case of the shifted free scalar, the light-front limit of $\phi^4$ theory does not immediately provide a nontrivial light-front vacuum expectation value. The difference is primarily due to the way zero modes contribute; their contribution has a
finite limit for the shifted scalar but an infinite limit for $\phi^4$ theory.

This divergence means that one cannot include a nontrivial vacuum by grafting a $c \to 0$ limit onto a light-front ($c = 0$) calculation for $P^+ = P_- \neq 0$. Instead, one must consider $P_- \neq 0$ for a sequence of finite $c$ approaching zero and subtract the vacuum energy while taking the limit $c \to 0$. An alternative is to construct an effective Hamiltonian in the light-front limit [9, 13], one for which the direct correspondence with the equal-time approach is maintained. The construction is based on the evaluation of expectation values of $\phi^n$ with respect to the vacuum, something that should be readily calculable in the present formalism. We intend to pursue this construction.

ACKNOWLEDGMENTS

We dedicate this paper to the memory of Kent Hornbostel (1959-2018), thesis advisor and friend. This work was supported in part by the Minnesota Supercomputing Institute through grants of computing time and benefited from participation in the workshop on Hamiltonian methods in strongly coupled quantum field theory supported by the Simons Collaboration on the Nonperturbative Bootstrap. Interpretation of the results benefited from discussions with M. Burkardt.

[1] M. Hogervorst, S. Rychkov and B. C. van Rees, Phys. Rev. D 91, 025005 (2015).
[2] S. Rychkov and L.G. Vitale, Phys. Rev. D 91, 085011 (2015); Phys. Rev. D 93, 065014 (2016).
[3] J. Elias-Miro, M. Montull and M. Riembau, JHEP 1604, 144 (2016); J. Elias-Miro, S. Rychkov and L. G. Vitale, Phys. Rev. D 96, 065024 (2017); JHEP 1710, 213 (2017).
[4] A. Pelissetto and E. Vicari, Phys. Lett. B 751, 532 (2015).
[5] P. Bosetti, B. De Palma and M. Guagnelli, Phys. Rev. D 92, no. 3, 034509 (2015); B. De Palma and M. Guagnelli, PoS LATTICE 2016, 277 (2016).
[6] Z. Bajnok and M. Lajer, JHEP 1610, 050 (2016).
[7] N. Christensen, arXiv:1603.01273 [hep-ph].
[8] N. Anand, V. X. Genest, E. Katz, Z. U. Khandker and M. T. Walters, JHEP 1708, 056 (2017).
[9] A. L. Fitzpatrick, J. Kaplan, E. Katz, L. G. Vitale and M. T. Walters, JHEP 1808, 120 (2018).
[10] M. Burkardt, Phys. Rev. D 47, 4628 (1993).
[11] M. Burkardt, S.S. Chabyshева, and J.R. Hiller, Phys. Rev. D 94, 065006 (2016); S.S. Chabyshева and J.R. Hiller, Phys. Rev. D 95, 096016 (2017).
[12] S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, Phys. Rep. 301, 299 (1998).
[13] M. Burkardt, Adv. Nucl. Phys. 23, 1 (2002).
[14] J.R. Hiller, Prog. Part. Nucl. Phys. 90, 75 (2016).
[15] J. Collins, “The non-triviality of the vacuum in light-front quantization: An elementary treatment,” arXiv:1801.03960 [hep-ph].
[16] K. Hornbostel, Phys. Rev. D 45, 3781 (1992).
[17] Y. Frishman, C.T. Sachrajda, H. Abarbanel, and R. Blankenbecler, Phys. Rev. D 15, 2275 (1977).
[18] D.V. Ahluwalia, in the proceedings of HUGS at CEBAF, edited by W.W. Buck, 1990 (unpublished); D.V. Ahluwalia and D.J. Ernst, Report No. CTP-TAMU 91/90 (unpublished).

[19] M.S. Sawicki, Phys. Rev. D 44, 433 (1991); Phys. Lett. B 268, 327 (1991).

[20] T.W. Chen, Phys. Rev. D 3, 1989 (1971).

[21] E. Elizalde and J. Gomis, Nuovo Cim. A 35, 367 (1976).

[22] E.V. Prokhvatilov and V.A. Franke, Sov. J. Nucl. Phys. 49, 688 (1989).

[23] R. Lenz, M. Thies, S. Levit, and K. Yazaki, Ann. Phys. 208, 1 (1991).

[24] C.-R. Ji and C. Mitchell, Phys. Rev. D 64, 085013 (2001); C.-R. Ji and A. T. Suzuki, Phys. Rev. D 87, 065015 (2013).

[25] S. Weinberg, Phys. Rev. 150, 1212 (1966); S. J. Chang, R. G. Root and T. M. Yan, Phys. Rev. D 7, 1133 (1973); S. J. Chang and T. M. Yan, Phys. Rev. D 7, 1147 (1973); T. M. Yan, Phys. Rev. D 7, 1760 (1973); Phys. Rev. D 7, 1780 (1973).

[26] P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

[27] H.-C. Pauli and S.J. Brodsky, Phys. Rev. D 32, 1993 (1985); 32, 2001 (1985).

[28] S. Hellerman and J. Polchinski, Phys. Rev. D 59, 125002 (1999).