Mathematical Modeling of Surface Roughness in the Forming of Innovative Materials

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Abstract. Vacuum moulding (VM) process has many potential engineering applications. Not much work hitherto has been reported for modeling the surface roughness (SR) in VM process. In the present study, outcome of Taguchi model has been used for developing a mathematical model for SR; using Buckingham’s $\pi$-theorem. Three input parameters namely type of metal/ pouring temperature; shape factor and vacuum pressure were selected to give output in form of SR. This study provides main effect of these variables on SR and shed light on the mechanism of SR in VM process.

1. Introduction
VM process has the ability to obtain high dimensional accuracy and surface finish as compared to other casting processes [1]. Today, this process is used for the casting of pressure pipe fittings, thin-sectioned curtain walls, pulleys, bathtubs, railroad bolsters, machine tools, and engine parts [2]. The size of the castings is ranging from a few ounces up to 10 metric tons. Aluminum alloys, copper alloys, cast iron, carbon steel, manganese and stainless steels are some of common metals, which are currently used in VM process [3-5]. This is one of the casting processes, which is distinctly different from other sand casting processes as this process requires no binders for holding the sand grains together in the mould [6, 7]. The vacuum inside the mould results in a net pressure pushing in, holding the sand rigidly in the shape of the pattern, even after the pattern is removed. The process uses a specially designed, strong, highly flexible polymer film to seal the open ends of the sand mould and form the mould cavity [8]. The literature review reveals that lot of work has been reported on optimization of VM process [9-12]. Various process parameters like vacuum imposed, vibration frequency, pouring temperature and plastic film thickness for the sound casting produced by VM process have been reported. But hitherto no work has been reported for modeling the SR in VM. So, the present investigation has been focused to develop mathematical model for SR in VM. For VM process, an approach to model SR was proposed and applied by some researchers [13]. This model was an attempt for predicting the SR as macro model in VM and is based upon robust design concept of Taguchi technique. The model was mechanistic in sense that parameters can be observed experimentally from a few experiments for a particular material and then used in prediction of SR over a wide range of process parameters. This was demonstrated for VM, where very good predictions were obtained using an estimate of multi parameters at a time. In that study, effects of three process...
parameters (type of metal/ pouring temperature; vacuum pressure; and shape factor/ geometrical shape) were revealed. Table 1 shows various input and output parameters used in experimental study.

Table 1. Various input and output parameters.

| Input Process parameters                  | Levels       | Output process parameter |
|------------------------------------------|--------------|--------------------------|
| Type of metal/pouring temperature        | Pb/375°C     | Al/700°C                 |
| Vacuum pressure (KPa)                    | 43           | 48                       | 53 |
| Geometrical shape                        | Cylinder     | Hemi-sphere              | Cuboid |

The relationships were studied by considering interaction between these variables. Table 2 shows control log of experimentation (based upon Taguchi L9 OA) and experimental observations for SR.

Table 2. Control log of experimentation and experimental observations for SR.

| S. No. | Material’s pouring Temp. (°C) | Geometrical Shape | Vacuum Pressure (KPa) | SR (µm) | Observations |
|--------|------------------------------|-------------------|-----------------------|---------|--------------|
|        |                              |                   |                       | R1      | R2           | R3           |
| 1      | 375 (Pb)                     | Cylinder          | 43                    | 3.71    | 3.73         | 3.76         |
| 2      | 375 (Pb)                     | Hemisphere        | 48                    | 3.72    | 3.75         | 3.79         |
| 3      | 375 (Pb)                     | Cuboid            | 53                    | 3.39    | 3.41         | 3.43         |
| 4      | 510 (Zn)                     | Cylinder          | 48                    | 3.18    | 3.19         | 3.22         |
| 5      | 510 (Zn)                     | Hemisphere        | 53                    | 3.31    | 3.32         | 3.34         |
| 6      | 510 (Zn)                     | Cuboid            | 43                    | 3.42    | 3.43         | 3.44         |
| 7      | 700 (Al)                     | Cylinder          | 53                    | 3.09    | 3.12         | 3.14         |
| 8      | 700 (Al)                     | Hemisphere        | 43                    | 3.19    | 3.21         | 3.24         |
| 9      | 700 (Al)                     | Cuboid            | 48                    | 3.14    | 3.15         | 3.16         |

Note: R1, R2 and R3 are the three repetitions of the experiments to reduce the experimental error.

Based on this model, Pandher and Singh (2011) studied the relationships between SR and controllable process parameters [13]. These relationships agree well with the trends observed by experimental observations made otherwise [3-7].

1.1 Description of the VM process

Figure 1 shows the setup of VM process. VM process can be limited to as few as five items (namely: vacuum system, Film heater, vibrating table, pattern carrier and flasks) for automation or for higher production [9].

Figure 1. VM process [13].

Figure 2. Cause and effect diagram of SR in VM.
There is no need for heavy, noisy jolt squeeze equipment, ramming of slingers. Any shape or size can be produced in a VM from thin walls to thick sections, or from castings weighing ounces to several tons. Fine surface finish and excellent dimensional accuracy, no moisture related defects, no cost for binders, excellent sand permeability, and no toxic fumes from burning the binders are key advantages of VM. The major VM process variables affecting SR are shown as cause and effect diagram (see figure 2).

The study presented in this paper is based on a previously published macro model based on Taguchi robust design [13]. The parameters like AFS No. 70 /grain size (0.210mm), frequency of vibration (30Hz) and casting volume (57697.5mm³) was kept constant for present study.

2. Mathematical modelling of ‘SR’

As per Taguchi design SR in VM was significantly dependent on type of metal and vacuum pressure. Table 3 and 4 respectively shows percentage contribution of input parameters and geometric model for SR [13]. The case study under consideration deals primarily with obtaining optimum system configuration in terms of response parameters with minimum expenditure of experimental resources. The best settings of control factors have been determined through experiments.

| Parameters                | Sum of square | Percentage contribution |
|---------------------------|---------------|-------------------------|
| Metal type/Pouring Temp.  | 2.235101      | 77.1295                 |
| Geom. Shape               | 0.107156      | 3.697764                |
| Vacuum pressure           | 0.29454       | 10.16406                |
| Error                     | 0.261058      | 9.008678                |

Table 3. Percentage contribution for SR.

| Optimized SR conditions |
|-------------------------|
| Metal type/ pouring temperature | Pb/ 375°C |
| Vacuum pressure          | 43 KPa    |
| Geometrical Shape        | Hemisphere |

Table 4. Geometric model of SR.

The Buckingham’s \( \pi \)-theorem proves that, in a physical problem including “n” quantities in which there are “m” dimensions, the quantities can be arranged in to “n-m” independent dimensionless parameters. In this approach dimensional analysis is used for developing the relations [14, 15].

Since SR, ‘Ra’ depends upon type of metal; cast metal density has been selected to represent this input parameter. Therefore by selecting basic dimensions: M (mass); L (length); T (time); and \( \theta \) (temperature) the dimensions of foregoing quantities would then be:-

1. Surface finish “Ra” (\( \mu \)m) \( L \)
2. Grain fineness number “N” (\( \mu \)m) \( L \)
3. Cast metal density “\( p \)” (kg/mm³) \( ML^{-3} \)
4. Shape factor “F” (dimensionless) \( M^0 L^0 T^0 \)
5. Pouring Temperature “\( \theta \)” (°C) \( \theta \)
6. Vacuum pressure “\( P \)” (kgf/mm²) \( ML^{-1} T^{-2} \)
7. Frequency of vibration “\( \gamma \)” (1/sec) \( T^{-1} \)

Now, \( Ra = f (N, p, F, \theta, P, \gamma) \) (1)

In this case \( n \) is 7 and \( m \) is 4. So, we can have \((n-m = 3) \pi_1, \pi_2, \) and \( \pi_3 \) three dimensionless groups. After some lengthy calculations, we finally obtain

\[ Ra = K1 \{ F, p, N3, \gamma 2 / P \} \] (2)
Here ‘K1’ represents vacuum moulding correction factor.

Now by keeping \( F \cdot \rho \cdot N^3 \cdot \gamma^2 \) fixed, experiments were performed for different values of \( P \), to find out ‘Ra’ and ‘K1’ in equation (2). The actual experimental data for three different types of metals/ metal density have been collected and plotted in figure 3.

\[
Ra = K1 \left( F \cdot \rho \cdot N^3 \cdot \gamma^2 / P \right)
\]

For metal Al (\( \rho = 2.7 \text{g/cm}^3 \)), pouring temperature 700°C

\[
Ra = (-0.01P + 3.62) \cdot F \cdot N^3 \cdot \gamma^2
\]  

For metal Zn (\( \rho = 7.14 \text{g/cm}^3 \)), pouring temperature 510°C

\[
Ra = (0.0074P^2 - 0.7214P + 20.758) \cdot F \cdot N^3 \cdot \gamma^2
\]  

For metal Pb (\( \rho = 11.34 \text{g/cm}^3 \)), pouring temperature 375°C

\[
Ra = (-0.0068P^2 + 0.6208P - 10.411) \cdot F \cdot N^3 \cdot \gamma^2
\]

It has been experimentally observed that dependence of SR in form of \( Ra \) is of the form given in equation 3-5. Since metal type/ pouring temperature is coming as most contributing factor, three different equations (3-5) have been separately deduced for expressing three different conditions of metal type. So, this final form of equations with any consistent system of units for the quantities may be used to get the main effect of variables on SR.

3. Conclusions

The Buckingham’s \( \pi \)-theorem has been used for mathematical modeling of SR in VM process. The contribution of input parameters to SR of casting is: Metal type/pouring temperature 77.12%, vacuum pressure 10.1%, and geometrical shape 3.6%. The optimized settings for minimum SR is: pouring temperature 700°C, geometrical shape (Cuboid) and vacuum pressure 53(KPa). The mathematical equation developed here sufficiently express all significant input parameters (see equation3-6) with coefficient of co-relation \( \approx 1 \). The results of the study presented in this paper pave the way to relevant future research on the optimal fabrication of innovative engineering materials and structures [16-54], taking advantage of the high dimensional accuracy and surface finish of vacuum molding.

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