Exclusive $J/\Psi$ and $\Psi(2s)$ photo-production as a probe of QCD low $x$ evolution equations

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Abstract

We investigate photo-production of vector mesons $J/\Psi$ and $\Psi(2s)$, based on both HERA and LHC data, using 2 fits of unintegrated gluon distributions. The latter are subject to non-linear Balitsky-Kovchegov evolution (Kutak-Sapeta gluon; KS) and linear next-to-leading order Balitsky-Kuraev-Fadin-Lipatov evolution (Hentschinski-Sabio Vera-Salas; HSS gluon) respectively. Apart from extending previous studies to the case of radially excited charmonium $\Psi(2s)$, we further use an improved set of charmonium wave functions and provide an estimate of the uncertainties associated with the HSS gluon. While we observe that the difference between linear and non-linear evolution somehow diminishes and a clear distinction between both HSS and KS gluon is not possible using the currently available data set, we find that the differences between both gluon distributions are enhanced for the ratio of the photo-production cross-sections of $\Psi(2s)$ and $J/\Psi$ vector mesons.

1 Introduction

Due to its large center of mass energy, the Large Hadron Collider (LHC) provides a unique opportunity to explore the dynamics of strong interactions in the high energy or Regge limit. For a process with a hard scale, which renders the strong coupling constant $\alpha_s$ small, a study of the Regge limit is possible using perturbative Quantum Chromodynamics (QCD). The theoretical description is provided through the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution, which achieves a resummation of perturbative higher order corrections, which are enhanced by a large logarithm in $x$ to all orders in the strong coupling at leading (LL) [1–4] and next-to-leading (NLL) [5, 6] logarithmic accuracy. Here $x = M^2/s$ where $M$ denotes the characteristic hard scale of the process and $s$ the center of mass energy squared. The perturbative high energy limit is then defined as $x \to 0$ at $M =$fixed. BFKL evolution predicts a power-like rise of the proton structure function $F_2$ with $1/x$, which is driven by the gluon distribution. While this rise is seen in the data and can be described by BFKL evolution [7–10], it is known that it cannot continue down to arbitrary small values of $x$. Instead, BFKL evolution will eventually drive the proton into an over occupied system of gluons, which eventually leads to the saturation of gluon densities [11]. Finding convincing and substantial evidence for gluon saturation as well as for the transition into this region of
QCD phase space is still one of the open problems of QCD and at the core of the physics program of the future Electron Ion Collider [12].

A very useful observable to explore the gluon distribution at the LHC in this region of interest is provided by exclusive photo-production of vector mesons. The observable is somewhat complementary to the bulk of studies currently undertaken [13–21], which attempt to resolve the hadronic final state in order to explore the low \( x \) gluon. In contrast to those studies, exclusive photo-production of vector mesons allows for a direct observation of the energy dependence of the photo-production cross-section which directly translates into the \( x \)-dependence of the underlying gluon distribution. In particular, if both HERA and LHC data are combined, the probed region in \( x \) extends over several orders of magnitude of \( x \), down to smallest values of \( x = 4 \cdot 10^{-6} \). Photo-production of bound states of charm quarks, \( i.e. \) \( J/\Psi \) and \( \Psi(2s) \) vector mesons, are then attractive observables, since the charm mass provides a hard scale at the border between soft and hard physics; the observable is therefore expected to be particularly sensitive to the possible presence of a semi-hard scale associated with the transition to the saturation region, the so-called saturation scale.

In [22] it has been found that an unintegrated gluon distribution subject to NLO BFKL evolution (the Hentschinski-Salas-Sabio Vera gluon; HSS) [7,8] is able to describe the energy dependence of the photo-production cross-section of \( J/\Psi \) and \( \Upsilon \) vector mesons. In [24], this study has been extended to the Kutak-Sapeta (KS) gluon [25], which is subject to non-linear Balitsky-Kovchegov (BK) evolution [26,27]. While both gluon distributions were able to describe the available data set, we found that a certain perturbative expansion, which underlies the linear HSS gluon, leads to an instability at highest values of the center of mass energy \( W \). While the instability can be removed through an improved scale setting, the growth of the stabilized gluon distribution with energy is too strong and linear evolution does no longer describe the data-set. This observation was then interpreted as a first indication for the transition towards saturated gluon densities. Note that in [28] is has been pointed out that this observation does not indicate saturation of gluon densities, but mainly the need for absorptive corrections (in the terminology of [28]). We agree in principle with this observation: the gluon does certainly not saturate at current values of the center of mass energy; one merely finds signs for the slow down of the power-like growth which points towards an increasing relevance of non-linear terms in low \( x \) QCD evolution equations. In other words, the cross-section is about to enter the so-called transition region, which separates the phase space region characterized by low and saturated gluon densities respectively. For a related study based on a different implementation of BFKL evolution, see [29], also [30].

In the present paper we extend the study of [24], to the case of radially excited charm-anti charm states, \( i.e. \) the \( \Psi(2s) \) vector meson. As for photo-production of \( J/\Psi \), the hard scale is provided by the charm mass, placing us at the boundary between soft and hard physics. On the other hand, the dependence of the light-cone wave function on the dipole size differs for \( \Psi(2s) \) and \( J/\Psi \). We therefore expect to test with \( \Psi(2s) \) photo-production a slightly different region in transverse momentum of the unintegrated gluon distribution. To increase the precision of our study we used instead of the previously implemented boosted Gaussian model for the vector meson wave function [31–33], a more refined description based on the numerical solution of the Schrödinger equation for the charm-anti charm state, provided in [34,35].
The outline of this paper is as follows: In Sec. 2 we provide the technical details of our theoretical description, in Sec. 3 we present the results of our numerical study and a comparison to data, while in Sec. 4 we summarize our results and draw our conclusions.

2 Theoretical setup of our study

In the following we describe the framework on which our study is based, see also Fig. 1. We study the process

\[ \gamma(q) + p(p) \rightarrow V(q') + p(p') \],

where \( V = J/\Psi, \psi(2S) \) while \( \gamma \) denotes a quasi-real photon with virtuality \( Q \rightarrow 0 \); \( W^2 = (q+p)^2 \) is the squared center-of-mass energy of the \( \gamma(q) + p(p) \) collision. With the momentum transfer \( t = (q-q')^2 \), the differential cross-section for the exclusive photo-production of a vector meson can be written in the following form

\[ \frac{d\sigma}{dt} (\gamma p \rightarrow Vp) = \frac{1}{16\pi} \left| A_T^{\gamma p \rightarrow Vp}(W^2, t) \right|^2. \]

where \( A_T(W^2, t) \) denotes the scattering amplitude for the reaction \( \gamma p \rightarrow Vp \) for color singlet exchange in the \( t \)-channel, with an overall factor \( W^2 \) already extracted. For a more detailed discussion see [22]. In the following we determine the total photo-production cross-section, based on an inclusive gluon distribution. This is possible following a two step procedure, frequently employed in the literature: First one determines the differential cross-section at zero momentum transfer \( t = 0 \) (which can be expressed in terms of the inclusive gluon distribution). In a second step the \( t \)-dependence is modeled, which then allows us to relate the differential cross-section at \( t = 0 \) to the integrated cross-section. In order to do so, we assume an exponential drop-off with \( |t| \) of the differential cross-section, \( \sigma \sim \exp \left[ -|t|B_D(W) \right] \)
with an energy dependent $t$ slope parameter $B_D$,
\[
B_D(W) = \left[ b_0 + 4\alpha' \ln \frac{W}{W_0} \right] \text{GeV}^{-2}.
\] (3)

The total cross-section for vector meson production is therefore obtained as
\[
\sigma^{\gamma p \rightarrow Vp}(W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt} \left. \left( \gamma p \rightarrow Vp \right) \right|_{t=0}.
\] (4)

The uncertainty introduced through the modeling of the $t$-dependence mainly affects the overall normalization of the cross-section with a mild logarithmic dependence on the energy. To determine the scattering amplitude, we first note that the dominant contribution is provided by its imaginary part. Corrections due to the real part of the scattering amplitude can be estimated using dispersion relations, in particular
\[
\frac{\text{Re} A(W^2, t)}{\text{Im} A(W^2, t)} = \tan \frac{\lambda \pi}{2}, \quad \text{with} \quad \lambda(x) = \frac{d\ln \text{Im} A(x, t)}{d\ln 1/x}. \] (5)

As noted in [22], the dependence of the slope parameter $\lambda$ on energy $W$ provides a sizable correction to the $W$ dependence of the complete cross-section. We therefore do not assume $\lambda = \text{const.}$, but instead determine the slope $\lambda$ directly from the $W$-dependent imaginary part of the scattering amplitude. To determine the latter, we go beyond the Gaussian model for the light-cone wave function of the vector mesons and use instead a refined description which includes relativistic spin-rotation effects. The imaginary part of the scattering amplitude is then in the forward limit obtained as [34–36]
\[
\text{Im} A_T(W^2, t = 0) = \int d^2r \left[ \sigma_{q\bar{q}} \left( \frac{M_V^2}{W^2}, r \right) \Sigma_T^{(1)}(r) + \frac{d\sigma_{q\bar{q}}}{dr} \left( \frac{M_V^2}{W^2}, r \right) \Sigma_T^{(2)}(r) \right],
\] (6)

with $r = |r|$. The functions $\Sigma_T^{(1,2)}$ describe the transition of a transverse polarized photon into a vector meson $V$ and are given by [35]
\[
\Sigma_T^{(i)}(r) = \hat{e}_f \sqrt{\frac{\alpha_{e.m.} N_c}{2\pi^2}} K_0(m_f r) \Xi^{(i)}(r), \quad i = 1, 2
\] (7)

where
\[
\Xi^{(1)}(r) = \int_0^1 dz \int \frac{d^2p}{2\pi} e^{ip\cdot r} \frac{m_T^2 + m_T m_L - 2p_T^2 z(1 - z)}{m_T + m_L} \Psi_V(z, |p|),
\]
\[
\Xi^{(2)}(r) = \int_0^1 dz \int \frac{d^2p}{2\pi} e^{ip\cdot r} |p| \frac{m_T^2 + m_T m_L - 2p^2 z(1 - z)}{2m_T(m_T + m_L)} \Psi_V(z, |p|),
\] (8)

and $\hat{e}_f = 2/3$ is the charge of the charm quark while $\alpha_{e.m.}$ the electromagnetic fine structure constant; $N_c = 3$ denotes the number of colors and $K_0$ is a Bessel function of the second kind. Finally, with $m_f$ the mass of the charm quark, we have
\[
m_T^2 = m_f^2 + p^2 \quad \quad m_L^2 = 4m_f^2 z(1 - z),
\] (9)
with $\Psi_V(z, |p|)$ the wave function of the vector meson. The latter has been obtained in [34,35] through the numerical solution of the Schrödinger equation for a given choice of the heavy quark interaction potential and provided in the boosted form as a table in both photon momentum fraction $z$ and transverse momentum $p$. The above form includes both effects due to the so-called Melosh spin rotation as well as a more realistic $r$-dependence of the photon-vector meson transition, with which we convolute the dipole cross-section $\sigma_{q\bar{q}}(x, r)$.

As in [24], we calculate in the following the dipole cross-section from two underlying unintegrated gluon distributions $F(x, k^2)$, using the relation [37]

$$
\sigma_{q\bar{q}}(x, r) = \frac{4\pi}{N_c} \int \frac{d^2k}{k^2} \left(1 - e^{i|k-r|} \right) \alpha_s F(x, k^2).
$$

(10)

Our study is based on two different implementations, the KS and HSS unintegrated gluon densities respectively:

- the KS gluon has been obtained as a solution of the momentum space version of the BK equation with modifications according to the Kwieciński-Martin-Stasto (KMS) prescription [38]. This implies an implementation of a so-called kinematical constraint, leading to energy momentum conservation, as well as complete DGLAP splitting functions, including quarks. In the collinear limit, the underlying evolution equation reduces therefore to the conventional DGLAP evolution. The KS gluon distribution in the proton was fitted [25] to proton structure function data measured at the HERA experiments H1 and ZEUS [39]. For a more detailed discussion see [25,38]

- The HSS gluon is subject to NLO BFKL evolution, including a resummation of collinearly enhanced terms in the NLO BFKL kernel as well as a resummation of large running coupling corrections using the optimal scale setting procedure. The initial conditions have been fitted [7,8] to the same HERA data set as the KS gluon. For a more detailed discussion see [7,23].

While the HSS gluon provides a very good description of both $\Upsilon$ and $J/\Psi$ photo-production data [22, 24], it has been found in [24] that the perturbative expansion used for the solution of the NLO BFKL equation turns unstable at lowest values of $x$. In particular one finds at
the level of the dipole cross-section two terms
\[ \sigma_{q\bar{q}}^{(HSS)}(x, r, M^2) = \alpha_s \sigma_{q\bar{q}}^{(HSS)}(x, r), \quad \hat{\sigma}_{q\bar{q}}^{(HSS)}(x, r) = \hat{\sigma}_{q\bar{q}}^{(dom.)}(x, r) + \hat{\sigma}_{q\bar{q}}^{(corr.)}(x, r), \]

where
\[ \hat{\sigma}_{q\bar{q}}^{(dom.)}(x, r, M^2) = \int_{\frac{1}{2} - i \infty}^{\frac{1}{2} + i \infty} \frac{d\gamma}{2\pi i} \frac{4}{\pi^2 Q_0^2} \frac{\gamma}{\alpha_s(M^2)} f(\gamma, Q_0, \delta, r) \left( \frac{1}{x} \right)^{\chi(\gamma, M^2)} \]

\[ \hat{\sigma}_{q\bar{q}}^{(corr.)}(x, r, M^2) = \int_{\frac{1}{2} - i \infty}^{\frac{1}{2} + i \infty} \frac{d\gamma}{2\pi i} \frac{4}{\pi^2 Q_0^2} \frac{\gamma}{\alpha_s(M^2)} f(\gamma, Q_0, \delta, r) \left( \frac{1}{x} \right)^{\chi(\gamma, M^2)} \]
\[ \times \frac{\beta_0 \alpha_0(\gamma)}{8N_c} \log\left( \frac{1}{x} \right) \left[ -\psi(\delta - \gamma) + \log \frac{M^2 r^2}{4} - \frac{1}{1 - \gamma} - \psi(2 - \gamma) - \psi(\gamma) \right], \]

and
\[ f(\gamma, Q_0, \delta, r) = \frac{r^2 \cdot C \Gamma(\gamma) \Gamma(\delta - \gamma)}{N_c(1 - \gamma) \Gamma(2 - \gamma) \Gamma(\delta)}, \]

is a function which collects both factors resulting from the proton impact factor and the transformation of the unintegrated gluon density to the dipole cross-section, see [7, 22] for details. The parameters \( Q_0 = 0.28 \text{ GeV}, C = 2.29 \) and \( \delta = 6.5 \) have been determined from fit to HERA data. Furthermore \( \bar{\alpha}_s = \alpha_s N_c / \pi \) with \( N_c \) the number of colors, and \( \chi(\gamma, M^2) \) is the next-to-leading logarithmic (NLL) BFKL kernel after collinear improvements a resummation of large terms proportional to the first coefficient of the QCD beta function, \( \beta_0 = 11 N_c / 3 - 2 n_f / 3 \) through the Brodsky-Lepage-Mackenzie (BLM) optimal scale setting scheme [40]. The NLL kernel with collinear improvements reads
\[ \chi(\gamma, M^2) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) - \frac{1}{2} \bar{\alpha}_s \chi_0^\prime(\gamma) \chi_0(\gamma) + \chi_{RG}(\bar{\alpha}_s, \gamma, \bar{b}) - \frac{\bar{\alpha}_s^2}{8N_c} \chi_0(\gamma) \log \frac{M^2}{M^2}. \]
those used in fits of the so-called IP-sat model \cite{41, 42}, through setting $M^2 = \frac{4}{r^2} + \mu_0^2$ with $\mu_0^2 = 1.51$ GeV$^2$. It is important to note that this change in the hard scale – even though well motivated – yields a dipole cross-section which does no longer fit the very precise HERA data; in particular the overall normalization requires an adjustment. The resulting dipole distribution provides an opportunity to explore stabilized perturbative NLO BFKL evolution for the description of exclusive vector meson photon production, while the parameters $Q_0$ and $\delta$ could in principle still be further adjusted. We will not make use of this possibility in this study. In the following we will distinguish the two possible implementation of the HSS dipole cross-section as ‘fixed’ and ‘dipole’ scale respectively.

3 Results

For our study we make use of two sets of vector meson wave functions provided by the authors of \cite{34, 35}. They are based on a numerical solution of the Schrödinger equation with harmonic oscillator (HO) and Buchmüller-Tye potential \cite{51} respectively, see \cite{34, 35} for a compact summary of the precise form of the underlying potentials. While the Buchmüller-Tye potential uses a charm mass of $m_f = 1.48$ GeV, the harmonic oscillator potential is associated with a charm mass of $m_f = 1.4$ GeV. For the parameters of the diffractive slope $B_D$, defined in Eq. (3), we use the following parameters determined in \cite{35} from a fit to HERA data:

\begin{align}
 b_0^{(J/\Psi)} &= 4.62, \\
 \alpha'_J(0) &= 1.71, \\
 b_0^{(J/\Psi)}(2s) &= 0.24, \\
 \alpha'_{\Psi(2s)}(0) &= \alpha'_J(0) - 0.02. \tag{15}
\end{align}

In agreement with the original fits of both KS and HSS gluon, the overall strong coupling constant which arises from Eq. (11) is evaluated at the charm mass. The results of our study for the $J/\Psi$ cross-section are shown in Fig. 3. Both KS and HSS gluons requires substantial $K$-factors to describe the data. At least partly these large $K$-factors can be traced back to the absence of the so-called skewness corrections, see \cite{52, 53}. As already pointed out in \cite{22}, we believe that it is not clear whether the approximations used in \cite{52, 53} are appropriate for the current setup based on high energy factorization, see also the related discussion in \cite{35}. We therefore do not include a skewness factor and correct instead for the off-set in normalization through a $K$-factor. The energy dependence of the photo-production cross-section is on the other hand almost identical for both $J/\Psi$ wave-functions. While the observations regarding the instability of the fixed scale solution to the HSS gluon made in \cite{24} holds, the uncertainty band associated with the dipole scale HSS gluon does no longer allow to clearly discard this solution through the data. Indeed, the non-linear KS gluon seems to slightly undershoot the data at highest $W$-values and therefore can be no longer identified as the preferred description. Regarding this observation there are two comments in order: first we suffer here the the consequences of our uncertainty in the overall normalization (which we fix through the central ZEUS data point at $W = 75$ GeV). It would have been of course possible to chose here a different data point which might place the three theory results slightly higher. In addition, similar to the case of the HSS gluon, one should also associate with the KS gluon an uncertainty band, which we estimate to be similar in magnitude or even larger than the one of the HSS gluon. At the same time it should be stressed that the error bars shown for the LHCb data at highest values of $W$ reflect only the error associated with the hadronic cross-sections.
Figure 3: Energy dependence of the $J/\Psi$ photo-production cross-section as provided by the KS and HSS gluon distribution (see text). The shaded regions correspond to a variation of the scale $M \rightarrow \{M/\sqrt{2}, M\sqrt{2}\}$. The upper/lower plot uses $J/\Psi$ wave functions based on the Buchm"uller-Tye and Harmonic Oscillator potential respectively. We further display photo-production data measured at HERA by ZEUS \cite{43,44} and H1 \cite{45,46} as well as LHC data obtained from ALICE \cite{47,48} and LHCb ($W^+$ solutions) \cite{49,50} collaborations.
and uncertainties due to the extraction of the photon-proton cross-section are not included. It is therefore likely that these error bars do not reflect the complete uncertainty associated with these data points. We therefore conclude that – even though the central values of both HSS gluon with dipole scale setting (which is subject to linear NLO BFKL evolution) and KS gluon (which is subject to a specific version of non-linear BK evolution) differ at highest \( W \)-values – it is not possible to clearly identify one of the two gluons as the appropriate description of the currently available data set.

The situation is even less clear if we turn to the \( \Psi(2s) \) photo-production cross-section Fig. 4. While the dipole scale HSS gluon and the KS gluon both provide a very good description of the energy dependence with essentially identical result for the wave function based on Buchmüller-Tyle and Harmonic Oscillator potential, the fixed scale HSS gluon fails completely to describe the data. This is not only evident at the highest values of \( W \), but also in the HERA region where the energy dependence is too flat. A satisfactory explanation of this observed failure is somehow difficult: while the flat energy dependence hints at a photon-\( \Psi(2s) \) transition which is dominated by a rather soft scale, the breakdown at large values of \( W \) hints at an enhancement of the region of small dipole separations \( r \), which leads to a negative enhanced logarithmic contribution, see the discussion of Eqs. (12) and (13). In this context we note that the functions \( \Sigma^{(2)}(r) \) increase in magnitude with respect to the function \( \Sigma^{(1)}(r) \) for the \( \Psi(2s) \), see Fig. 2. As a consequence the weight of the \( d\sigma_{q\bar{q}}/dr \) term in Eq. (6) increases, which in turn might give rise to the observed instability.

While both the \( J/\Psi \) and \( \Psi(2s) \) photo-production cross-section can currently not distinguish between linear and non-linear QCD evolution, we make an interesting observation if we consider instead the ratio of both cross-section, Fig. 5. While we discard the HSS solution based on a fixed renormalization scale due to the found instabilities, we find that the KS gluon, subject to non-linear BK evolution, and the dipole scale gluon, subject to linear NLO evolution, predict a different energy dependence for the ratio of \( \Psi(2s) \) and \( J/\Psi \) photo-production cross-sections. While linear NLO BFKL evolution predicts a ratio which slightly decreases with energy, non-linear KS evolution predicts an increase with energy of the cross-section ratio. We believe that this observation can be useful for two reasons: First of all it is well known that uncertainties are generally reduced for such cross-section ratios. This refers both to the aforementioned skewness factor as well as the extraction of the photo-production cross-section from hadronic data, which requires to control the so-called rapidity gap survival probability. Second, while the differences between linear and non-linear evolution are in general not large at current center-of-mass energies, they seem to follow a different tendency, \( i.e. \) the cross-section ratio increases for non-linear evolution and decreases for linear evolution. As far as data are concerned, we find that H1 data seem to prefer a rise of the ratio with energy. Nevertheless, due to the relative large error bars as well as their limitation to the region \( W = 50 - 110 \) GeV, the H1 data set is in complete agreement with both linear and non-linear evolution. LHCb data, which would cover the region of large energies \( W \), are currently only provided for \( J/\Psi \) and \( \Psi(2s) \) photo-production cross-sections separately. While it is in principle possible to take ratios of these results, the published \( W \)-bins of \( J/\Psi \) and \( \Psi(2s) \) cross-sections differ, which complicates a proper extraction of the cross-section ratio. We however believe that it would be very interesting to compare in the future our predictions to
Figure 4: Energy dependence of the $\Psi(2s)$ photo-production cross-section as provided by the KS and HSS gluon distribution (see text). The shaded regions correspond to a variation of the scale $M \rightarrow \{M/\sqrt{2}, \sqrt{2}M\}$. The upper/lower plot uses the wave function based on the Buchmüller-Tyle and Harmonic Oscillator potential respectively. We further display photo-production data measured at HERA by the H1 [54,55] as well as LHC data obtained from the LHCb collaboration ($W^+$ and $W^-$ solutions) [50].
Figure 5: Energy dependence of the ratio of $\Psi(2s)$ vs. $J/\Psi$ photo-production cross-section as provided by the KS and HSS gluon distributions for both Buchmüller-Tye (BT) and Harmonic Oscillator (HO) vector meson wave functions. The shaded regions correspond to a variation of the scale $M \rightarrow \{M/\sqrt{2}, M\sqrt{2}\}$; the normalization has been fixed through $K$-factors as shown in Figs. 3, 4. We further display photo-production data measured at HERA by the H1 collaboration [55].

properly extracted cross-section ratios. In particular, regardless of still size-able theoretical uncertainties, it would be interesting to see whether experimental data indicate a rising or falling ratio with energy.

4 Conclusion

In this paper we extended previous studies, dedicated to the study of the energy dependence of the exclusive $J/\Psi$ photo-production cross-section to $\Psi(2s)$ vector mesons. We furthermore used a more accurate description of the photon to vector meson transition, as provided by [34, 35], as well as a refined discussion of the theoretical uncertainties of the energy dependence of the linear HSS gluon. Reconsidering $J/\Psi$ photo-production including the above mentioned improvements, we find that we cannot completely confirm the claim made in [24]. Linear, stabilized HSS evolution, based on the dipole scale setting and non-linear KS evolution differ for largest scattering energies $W$, but the difference is not big enough such that current LHC data can unambiguously distinguish between one of the two QCD evolution equations, in particular once uncertainties of the HSS gluon are included. While the difference between HSS gluon with dipole scale setting and KS gluon is even less pronounced for $\Psi(2s)$ photo-production, we find somehow unexpectedly that the HSS gluon with fixed scale cannot describe the energy dependence of the $\Psi(2s)$ photo-production cross-section in the HERA region, while it completely breaks down for the highest center of mass energies of the
photon-proton reaction. On the other hand, we find it encouraging that the ratio of $J/\Psi$ and $\Psi(2s)$ photo-production cross-section shows a different energy behavior for linear and non-linear evolution. In this context we would like to stress that a similar observation has been already made in [35]: with the gluon modeled through the phenomenological KST dipole cross-section [56], an increase of the ratio with energy has been found. At the same time, an almost constant ratio has been found for the ratio of $\Upsilon(2s)$ and $\Upsilon(1s)$ photo-production cross-section which are both placed well in the perturbative region due to the hard scale provided by the bottom quark mass. The current study goes beyond this observation, since our gluon distributions are obtained as the solution to low $x$ QCD evolution equations. In particular we find that linear evolution leads even to slight decrease of the $\Psi(2s)$ and $J/\Psi$ ratio.

While from the theory side it is necessary to further increase the accuracy of predictions for photo-production cross-sections, we believe that a precise extraction of the ratio of $\Psi(2s)$ and $J/\Psi$ photo-production cross-sections could be very useful to distinguish in the future between linear and non-linear QCD evolution. We believe that this applies both to photon-proton scattering at highest center of mass energies as measured at LHC, as well as for photo-production cross-sections obtained in electron-ion scattering at the future Electron Ion Collider. While in the latter case, center-of-mass energies will be naturally lower, nuclear effects will likely enhance gluon densities and therefore the possible relevance of non-linear QCD evolution.

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