A Critique of Core–Collapse Supernova Theory Circa 1997

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There has been a new infusion of ideas in the study of the mechanism and early character of core–collapse supernovae. However, despite recent conceptual and computational progress, fundamental questions remain. In this all-too-brief contribution, I summarize some of the interesting insights achieved over the last few years. In the process, I highlight as-yet unsolved aspects of supernova theory that continue to make it a fascinating and frustrating pursuit.

1 Introduction

It has recently been shown that neutrino–driven Rayleigh–Taylor instabilities between the stalled shock wave and the neutrinospheres (“Bethe” convection) are generic feature of core–collapse supernovae. Whatever their role in reigniting the stalled explosion, their existence and persistence have altered the way modelers approach their craft. Supernovae must explode aspherically, and this broken symmetry is stamped on the ejecta and character of the blast, as well as on its signatures. Consequences of asphericity include significant gravitational radiation, natal kicks to nascent neutron stars, induced rotation, mixing of iron–peak and r–process nucleosynthetic products, the generation and/or rearrangement of pulsar magnetic fields, and, in extreme cases, jetting of the debris.

However, there is no consensus yet on the centrality of overturn (or “convection”) to the mechanism of the explosion itself, with some deeming it either pivotal, potentially important, or diversionary. Nevertheless, all agree on the existence of convection in the gain region of the stalled protoneutron star, and this point must be stressed. A gain region is a prerequisite for the neutrino–driven mechanism. For heating to exceed cooling in steady–state accretion, the entropy gradient must be negative, and, hence, unstable. Therefore, a gain region is always convective. In order to achieve quantitative agreement with the variety of observational constraints (explosion energy, residual neutron star masses, Ni and “N = 50” peak yields, halo star element ratios, neutron star proper motions, etc.), the “final” calculations must be done multi–dimensionally. While if it can be shown that 1–D spherical models do explode after some delay, the true duration of that delay, the amount of fallback, and the energetics of the subsequent explosion must be influenced by the overturning motions that can not be captured in 1–D. Convection changes not only the character of the hydrodynamics, but the entropies in the gain region and
the “efficiency” of neutrino energy deposition that is the ultimate driver of the explosion. Furthermore, implicit in a focus on 1-D calculations is the assumption that multi–D effects could only help, that they do not thwart explosion. Hence, the belief that spherically–symmetric calculations are germane depends upon insights newly obtained from the multi–dimensional simulations. Nevertheless, it will be an important theoretical exercise to ascertain whether 1–D models with the best physics and numerics can explode, if only because such has been a goal for decades. The “viability” of 1–D models will be influenced in part by the transport algorithm employed (multi–group, flux–limited, full transport, diffusion), the microphysics (opacities and source terms at high and low densities), the effects of general relativity, the equation of state, convection in the inner core that can boost the driving neutrino luminosities, and the inner density structure.

2 Neutrino Transport

Though much of the recent excitement in supernova theory has concerned its multi–dimensional aspects, neutrino heating and transport are still central to the mechanism. The coupling between matter and radiation in the semi–transparent region between the stalled shock and the neutrinospheres determines the viability and characteristics of the explosion. Unfortunately, this is the most problematic regime. Diffusion algorithms and/or flux–limiters do not adequately reproduce the effects of variations in the Eddington factors and the spectrum as the neutrinos decouple. Hence, a multi–group full transport scheme is desirable.

To address the issues surrounding neutrino transport, we have recently created a neutrino transport code using the program Eddington developed by Eastman & Pinto. This code solves the full transport equation using the Feautrier approach, is multi–group, is good to order v/c, and does not employ flux limiters. The $\nu_e$, $\bar{\nu}_e$, and $\nu_\mu$ are handled separately and coupling to matter is facilitated with accelerated lambda iteration (ALI). By default, we employ 40 energy groups from 1 MeV to either 100 MeV ($\bar{\nu}_e$ and “$\nu_\mu$”) or 230 MeV ($\nu_e$) and from a few to 200 angular groups, depending on the number of tangent rays at the given radial zone, in the Feautrier manner. In this way, the neutrino angular distribution function and all the relevant angular moments (0’th through 3’rd) are calculated to high precision, for every energy group.

The effect of the full Feautrier scheme vis–a–vis previous calculations will soon be benchmarked and calibrated. However, we have already obtained several interesting results. Since the annihilation of $\nu-\bar{\nu}$ pairs into $e^+e^-$ pairs depends upon the 0’th, 1’st, and 2’nd angular moments of the neu-
trino angular distribution function, as well as upon the neutrino spectra, we can and have calculated the rate of energy deposition via this process exactly, though in the context of previous model runs (still ignoring general relativity). The $\nu_e + \bar{\nu}_e \to e^+ + e^-$ and $\nu_\mu + \bar{\nu}_\mu \to e^+ + e^-$ energy deposition rates in the shocked region are no more than 0.01 and 0.001, respectively, those of the dominant charged-current processes, $\nu_e + n \to e^- + p$ and $\bar{\nu}_e + p \to e^+ + n$. However, in the unshocked region ahead of the shock, depending upon the poorly-known $\nu$–nucleus absorption rates, the $\nu$–$\bar{\nu}$ annihilation rate can be competitive, though it is still irrelevant to the supernova. These calculations should put to rest the notion that $\nu$–$\bar{\nu}$ annihilation is important in igniting the supernova explosion.

It is thought that neutrino–electron scattering and inverse pair annihilation are the processes most responsible for the energy equilibration of the $\nu_\mu$'s and their emergent spectra. However, recent calculations imply that the inverse of nucleon–nucleon bremsstrahlung (e.g., $n + n \to n + n + \nu \bar{\nu}$) is also important in equilibrating the $\nu_\mu$'s. This process has not heretofore been incorporated in supernova simulations. Our preliminary estimates suggest that inverse bremsstrahlung softens the emergent $\nu_\mu$ spectrum, since the bremsstrahlung source spectrum is softer than that of pair annihilation. In addition, given the large $\nu_\mu$ scattering albedo, one must properly distinguish absorption from scattering, in ways not possible with a flux–limiter. Since the relevant inelastic neutral-current processes are stiff functions of neutrino energy, these transport issues bear directly upon the viability of neutrino nucleosynthesis (e.g., of $^{11}$B and $^{19}$F).

The new code allows us to calculate the difference between the flux spectrum ($h_\nu$) and the energy density spectrum ($j_\nu$). The latter couples to matter and drives the supernova in the neutrino mechanism, while the former, or some variant of it, is frequently substituted for the latter in diffusion codes. Since matter–neutrino cross sections are higher for higher–energy neutrinos, the energy density spectrum is always harder than the flux spectrum. This hardness boosts the neutrino heating rates in the semi-transparent region. To illustrate this effect, in Figure 1 the ratio $j_\nu / h_\nu$ is plotted versus neutrino energy at a time 30 milliseconds after bounce. The shock is then at 124 kilometers. It is clear that the ratio effect can be interesting. However, it is most pronounced in the cooling region below the gain region and tapers off as the shock is approached. Mezzacappa et al. in particular, have highlighted this correction, but self–consistent calculations from collapse to explosion, using the Fenuetier or Boltzmann techniques (in principle equivalent), are needed, given the notorious feedbacks in the supernova problem. The same effect may be important in driving the protoneutron star wind thought to be the site of
Figure 1: $j_\nu/h_\nu$ versus $\epsilon_\nu$ for electron neutrinos 30 milliseconds after the bounce of a $15 \, M_\odot$ core, using the code of Burrows & Pinto (1997)

Indeed, full transport calculations of r–process winds and the supernova, even in 1–D, will be illuminating.

3 Conclusions

In parallel with the ongoing evolution of the numerical tools being brought to bear on the supernova problem is the emerging realization that the systematics of the supernova phenomenon with progenitor is inching closer into view. As we unravel the mechanism, we simultaneously explore the origin of neutron stars and black holes, the birthplace of elements of which we are made, and the source of much of the energy of the ISM. As supernova modelers and the Jayhawks might say, ad astra per aspera.

Acknowledgments

Conversations with Willy Benz, Chris Fryer, Tony Mezzacappa, and Phil Pinto that materially altered the content of this squib are gratefully acknowledged, as is the support of the U.S. N.S.F. under grant #AST92-17322.

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