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Published in:
IEEE Transactions on Network Science and Engineering

Published: 01/07/2024

Document Version:
Post-print, also known as Accepted Author Manuscript, Peer-reviewed or Author Final version

Publication record in CityU Scholars:
Go to record

Published version (DOI):
10.1109/TNSE.2024.3382367

Publication details:
Zhang, Y., Chen, B., Yu, L., & Ho, D. W. C. (2024). Distributed Estimation for Interconnected Systems with Arbitrary Coupling Structures. IEEE Transactions on Network Science and Engineering, 11(4), 3667-3677. https://doi.org/10.1109/TNSE.2024.3382367

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Zhang, Y., Chen, B., Yu, L., & Ho, D. W. C. (2024). Distributed Estimation for Interconnected Systems with Arbitrary Coupling Structures. *IEEE Transactions on Network Science and Engineering, 11*(4), 3667-3677. 
https://doi.org/10.1109/TNSE.2024.3382367
Distributed Estimation for Interconnected Systems with Arbitrary Coupling Structures

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Abstract—This paper is concerned with the problem of distributed estimation for time-varying interconnected systems with arbitrary coupling structures. A distributed stability condition requiring only the information from each subsystem and its neighbors is proposed to guarantee the stability of the designed distributed estimator. Then, a simplified condition without the real-time exchange of estimator gains is further proposed to reduce the communication burden. Under distributed stability constraints, the optimal estimator gain is designed by solving a convex optimization problem. Notice that the involved convex optimization problem can be easily solved by standard software packages because the constraints can be transformed into linear matrix inequalities. It is also shown that the designed distributed estimator is scalable for adding or subtracting subsystems. Finally, an illustrative example of a three-tank interconnected system is employed to show the effectiveness of the proposed methods.

Index Terms—Time-varying interconnected system; distributed stability condition; distributed estimation; optimal estimator; convex optimization.

I. INTRODUCTION

In recent decades, complex networks have received considerable attention in various fields, including physics, mathematics, engineering, biology, and sociology [1, 2]. One of the most essential focuses in complex networks is understanding, inferring, and intervening in complex dynamic systems’ behavior. For example, the controllability and observability for complex systems have been discussed in [3–5], and the pinning control for network synchronization has been investigated in [1, 6, 7]. Interconnected systems are high-dimensional complex dynamic systems composed of numerous dispersed subsystems that can be state-coupled with their neighboring subsystems [8]. The past few decades have witnessed a surge of research interest in interconnected systems due primarily to their extensive applications in power systems [9], multi-robot systems [10], epidemic networks [11], and gene regulatory networks [12].

Real-time state estimation is helpful to understand and infer interconnected systems’ behavior. However, the increased complexity of system topologies and dynamics has prevented traditional estimation approaches from achieving satisfactory performance [13]. The reason is mainly attributed to the poor scalability of a centralized structure in traditional approaches. The centralization structure will suffer from heavy computational and communication burdens as well as a high field deployment cost if the number and dimension of subsystems are increased. Different distributed estimation approaches have been developed to reduce computing and communication overhead [14–30]. Most distributed estimation approaches for interconnected systems are based on special coupling structures. For example, a distributed moving horizon estimator was designed in [20] for sparse banded interconnected systems by the Chebyshev approximation of a centralized solution. Moreover, a distributed Kalman filter [21] and a consensus-based decentralized estimator [22] were designed for sparse systems by decomposing them into interconnected subsystems with coupled and overlapping states. Meanwhile, a sub-optimal distributed Kalman filtering problem was addressed in [23] for a class of sequentially interconnected systems. As for general interconnected systems without any structure constraints, the problem is more complicated due to the coupled estimation error iterations. By a completely local design, distributed estimators with a decoupling strategy were designed in [24–26] to deal with coupled estimation errors, while a moving horizon estimator was proposed in [27] with the assumption of uncorrelated local estimation errors. The distributed estimators with a plug-and-play fashion were developed in [28, 29] by exploiting the small-gain condition of the one-dimensional error norm system. Besides, a distributed zonotopic estimator with a fusion strategy was proposed in [30] for interconnected systems with overlapping states. However, it is still challenging to design distributed estimators with stability guarantees for general interconnected systems.

As important as it is in network synchronization, the stability problem for interconnected systems is also fundamental and has received much attention since the 1960s [31–35]. The traditional stability analysis for interconnected systems can be roughly divided into three categories: 1) methods based on scalar or vector Lyapunov functions [36–38]; 2) methods based on the small-gain theorem [39–41]; 3) methods based on the dissipativity theory [42, 43]. The above methods usually derive a centralized stability condition from a local analysis. For example, the $M$-matrix condition [38] can be derived from local Lyapunov stability conditions, and the small
gain condition [41] can be obtained from local input-to-state stability conditions. The traditional analysis methods have two drawbacks that limit their application in distributed estimation and control. The first drawback is that the derived condition is more conservative than the centralized stability condition of the overall system. The second one is that these conditions are not scalable and require knowledge of overall systems’ dynamics and couplings. How to design distributed stability conditions for interconnected systems is still an open question. The promising idea is to decompose the centralized stability conditions of the overall system into distributed ones by a subsystem-level analysis. A centralized dissipativity condition was decomposed into distributed conditions for individual subsystems in [44]. Note that these conditions require a considerable communication burden to exchange message matrices among subsystems and cannot be generalized to time-varying interconnected systems.

It should be pointed out that the distributed estimator designed in [25] only provides stability conditions for specific coupling structures, which are further interpreted as the structure of a directed acyclic graph in [26]. As for interconnected systems with arbitrary coupling structures, the problem of distributed estimation with stability guarantees has not yet been fully solved. Motivated by the above analysis, we will address the problem of distributed estimation with stability guarantees has not yet been fully solved. Motivated by the above analysis, we will address the problem of distributed estimation with stability guarantees has not yet been fully solved.

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A distributed stability condition is proposed by a subsystem-level analysis for a centralized stability condition. Compared with existing stability conditions for interconnected systems [36–43], the proposed distributed stability condition requires only the information from each subsystem and its neighbors.

A simplified distributed stability condition is proposed for a non-ideal communication environment. The proposed simplified condition avoids the real-time exchange of estimators’ gain information, which is different from the conditions in [44] that require a considerable communication burden to exchange message matrices.

An optimal and stable estimator is recursively designed for time-varying interconnected systems by solving an optimization problem with the proposed distributed stability constraints. Compared with the distributed estimator in [20–23], the designed estimator does not require any structural assumptions about the interconnected systems, such as sparse and sequential structures. The designed estimator also shows its advantages when interconnected systems cannot be decoupled by the approaches in [24–26].

Notation: The set of $n$-dimensional real vectors is denoted by $\mathbb{R}^n$. The identity matrix with appropriate dimensions is represented as $\mathbf{I}$. And the matrix with all zero elements is denoted by ‘0’. Define $\mathbb{N}_l := \{1, 2, ..., l\}$, where $l$ is a natural number excluding zero. Given sets $A$ and $B$, $A \setminus B$ represents the set of all elements of $A$ that are not in $B$, and $A \cap B$ is the intersection set of $A$ and $B$. The superscript ‘$T$’ represents the transpose, while the symmetric terms in a symmetric matrix are denoted by ‘*’. Meanwhile, the notations $A^{-1}$, $\mathcal{R}(A)$, $\|A\|_2$, and $\lambda_{\text{max}}(A)$ are the inverse, trace, 2-norm, and maximum eigenvalue of matrix $A$, respectively. The notation $X > 0$ ($< 0$) is a positive definite (negative definite) matrix, and $X \succeq 0$ ($\preceq 0$) is a positive semi-definite (negative semi-definite) matrix. Given two matrices $X, Y \in \mathbb{R}^{n \times n}$, the notation $X \preceq Y$ means every element of $X$ is not greater than the corresponding element of $Y$. The notation $\text{col}\{a_1, ..., a_n\}$ means a column vector whose elements are $a_1, ..., a_n$, while $\text{diag}\{\cdot\}$ stands for a block diagonal matrix. The $(i, j)$th element or block of a matrix $A$ is represented by $A_{i,j}$, and the mathematical expectation is denoted by $E\{\cdot\}$.

II. PROBLEM FORMULATION

A. Time-varying Interconnected Systems

Consider a time-varying interconnected system $S$ composed of $l$ subsystems. The state and measurement dynamics of the $i$th subsystem $S_i$ ($i \in \mathbb{N}_l$) are described as follows:

$\begin{align*}
&x_i(k+1) = A_i(k)x_i(k) + \Gamma_i(k)w_i(k) \\
&S_i : \quad y_i(k) = C_i(k)x_i(k) + D_i(k)v_i(k)
\end{align*}$

(1)

where the vectors $x_i(k) \in \mathbb{R}^{n_i}$, $y_i(k) \in \mathbb{R}^{m_i}$, $w_i(k) \in \mathbb{R}^{n_{wi}}$, and $v_i(k) \in \mathbb{R}^{n_{vi}}$ are the state, measurement, system noise, and measurement noise of the subsystem $S_i$, respectively. The bounded matrices $A_i(k), \Gamma_i(k), A_{i,i_{\text{in}}}(k), C_i(k)$, and $D_i(k)$ have appropriate dimensions. Meanwhile, the sets of $i$th subsystem’s in-neighbors, out-neighbors, and neighbors are denoted by $\overline{\mathcal{N}}_i, \mathcal{Q}_i$, and $\mathcal{O}_i$, respectively. The number of elements in these sets is $\overline{d}_i$, $\overline{q}_i$, and $\overline{\theta}_i$. In addition, let us denote upper bounds $\|A_i(k)\|_2 \leq \alpha_{i,i} = \alpha_i$ and $\|A_{i,i_{\text{in}}}(k)\|_2 \leq \alpha_{i,i_{\text{in}}}$, and set $\alpha_{i,j} = 0$ ($j \notin \mathcal{O}_i \cap \{i\}$).

Assumption 1. The noises $w_i(k)$ and $v_i(k)$ are uncorrelated Gaussian white noises satisfying

$\begin{align*}
&E\{w_i(k)w_j(k_1)\} = \delta_{i,j}\delta_{k,k_1}Q_w, \\
&E\{v_i(k)v_j(k_1)\} = \delta_{i,j}\delta_{k,k_1}Q_v, \\
&E\{w_i(k)v_j(k_1)\} = 0
\end{align*}$

(2)

where $Q_w$ and $Q_v$ are the known covariances of $w_i(k)$ and $v_i(k)$, respectively. The notation $k_0$ is the initial instant, while $\delta_{k,k_1} = 0$ if $k \neq k_1$ and $\delta_{k,k_1} = 1$ otherwise.

Assumption 2. Each subsystem knows only its own state and measurement dynamics and can communicate with its neighbors.

Remark 1. The considered interconnected systems with time-varying dynamics and couplings are more general and have wider applications. Taking blocked power systems as an example, the couplings among different blocks change with real-time power dispatching [9, 45]. Another example of heavy-duty vehicle systems with aerodynamic interconnections [46] can also be modeled as an interconnected system (1). Compared with the other models [20–23, 47], the interconnected...
The primary concern of the distributed estimation problem is to design a suitable gain matrix $K_i(k) \ (i \in \mathbb{N}_i)$ such that the estimation error system (4a-4b) is stable and the selected estimation performance index $J_i(k)$ about $P_i(k)$ is minimized. If the performance index for the local estimation is designed as $J_i(k) = \text{Tr} \{P_i(k)\}$, then the optimal estimator gain $K_i^{\text{opt}}(k)$ can be searched by solving an optimization problem:

$$K_i^{\text{opt}}(k) = \arg \min_{K_i(k) \in \mathcal{K}_i(k)} \text{Tr} \{P_i(k)\}$$

(5)

where $\mathcal{K}_i(k)$ is a subspace of stable estimator gains for subsystem $S_i$. It is usually difficult for subsystems to timely obtain the cross-covariance $P_{i,j}(k)$ used in (4c) by only local communication. Therefore, an upper bound of $\text{Tr} \{P_i(k)\}$ is required to improve the optimization problem (5). It is also known from (4a-4b) that the stability of each local estimator depends on the stability of its in-neighbors’ estimators due to the interconnected estimation error term $\tilde{x}_{i-1}(k-1)$. Hence, it is difficult to determine $\mathcal{K}_i(k)$ and design a stable estimator by a local analysis. On the other hand, a centralized analysis needs the overall system’s knowledge about its dynamics and couplings, which is not suitable for large-scale interconnected systems.

Consequently, the problems to be solved in this paper are described as follows:

1) Derive distributed stability conditions for the proposed estimator (3) such that only subsystem-level knowledge of dynamics and couplings is required.
2) Design a distributed, stable, and optimal estimator for time-varying interconnected systems using the proposed distributed stability conditions.

**Remark 2.** To design a fully distributed estimator, both the iteration form and the stability conditions for the estimator need to achieve local communication, computation, and storage. Though the estimator (3) only uses the information of local measurement and neighbors’ estimates, the local estimation errors in (4b) are still interconnected. Therefore, the major difficulty of the distributed estimator design for general interconnected systems is calculating the optimal estimator gain and maintaining local stability for each subsystem without any global information about estimation errors.

### III. Main Results

In this section, two distributed stability conditions will be derived by a subsystem-level analysis for a centralized condition, and then a distributed, stable, and optimal estimator will be designed by optimization-based methods.

#### A. Centralized Stability Condition

To introduce the overall system dynamics and overall estimator iteration, let us define $x(k) := \text{col}\{x_1(k), ..., x_l(k)\}$, $y(k) := \text{col}\{y_1(k), ..., y_l(k)\}$, $w(k) := \text{col}\{w_1(k), ..., w_l(k)\}$, and $v(k) := \text{col}\{v_1(k), ..., v_l(k)\}$. Then, the dynamics of the overall system $\mathbf{S}$ can be obtained as

$$\begin{cases}
  x(k+1) = A(k)x(k) + \Gamma(k)w(k) \\
  y(k) = C(k)x(k) + D(k)v(k)
\end{cases}$$

(6)
Lemma 1. The overall estimator (8a) is mean-square UUB as

\[ P \] where \( \delta \) is uniformly bounded. Therefore, the boundedness of \( H(k) \) follows from (11) and the boundedness of system matrices. Therefore, \( \lim_{k \to \infty} \| P(k) \|_2 \) is bounded.

B. Distributed Stability Conditions

The centralized stability condition (11) needs the overall system’s knowledge, so it is hard to be implemented with a distributed communication structure. The following theorem provides a distributed condition to ensure stability for local estimators.

Theorem 1. The proposed distributed estimator (3) is mean-square UUB if the following distributed stability condition is satisfied \( \forall i \in \mathbb{N}_1 \) and \( \forall i_{\rho} \in \Omega_i : \)

\[ \| K_i(k)A_i(k-1) \|_2 < 1 \] (15a)

\[ \pi_{i,i_{\rho},i}N_i(k) - N_{i,i_{\rho}}(k)N_{i_{\rho}}^{-1}(k)N_{i,i_{\rho}}(k) < 0 \] (15b)

where parameter \( \pi_{i,i_{\rho}} \geq 0 \) satisfies \( \sum_{i_{\rho} \in \Omega_i} \pi_{i,i_{\rho}} = 1 \), and

\[ N_i(k) := \begin{bmatrix} -I & K_i(k)A_i(k-1) \\ * & -I \end{bmatrix} \]

\[ N_{i,i_{\rho}}(k) := \begin{bmatrix} A_{i_{\rho}}(k-1)K_i(k)^T \end{bmatrix} \]

\[ k \to \infty \quad \lim \| P(k) \|_2 \leq \delta \] (10)

Lemma 1. The overall estimator (8a) is mean-square UUB as (10) if the following centralized stability condition is satisfied:

\[ \| \tilde{K}(k)A(k-1) \|_2 < 1. \] (11)

Proof: It can be derived from (8c) that

\[ P(k) = \lim_{k \to \infty} \prod_{\mu=0}^{k-k_0-1} \tilde{K}(k-\mu) \]

\[ = \sum_{\tau=0}^{k-k_0-1} \left\{ \prod_{\mu=0}^{\tau-1} \tilde{K}(k-\mu) \right\} H(k-\mu) \]

\[ \times \left( \prod_{\mu=0}^{\tau-1} \tilde{K}(k-\mu) \right)^T \]

where \( \tilde{K}(k) := \tilde{K}(k)A(k-1) \). Then, one has that

\[ \| P(k) \|_2 \leq \left( \prod_{\mu=0}^{k-k_0-1} \| \tilde{K}(k-\mu) \|_2 \right) \| P(k_0) \|_2 \]

\[ + \sum_{\tau=0}^{k-k_0-1} \left\{ \prod_{\mu=0}^{\tau-1} \| \tilde{K}(k-\mu) \|_2 \right\} \| H(k-\mu) \|_2 \]. (13)

If the condition (11) is satisfied, it turns out that

\[ \lim_{k \to \infty} \prod_{\mu=0}^{k-k_0-1} \| \tilde{K}(k-\mu) \|_2 = 0 \] (14)

\[ \lim_{k \to \infty} \prod_{\mu=0}^{\tau-1} \| \tilde{K}(k-\mu) \|_2 = 0. \]

The boundedness of \( \| P(k) \|_2 \) follows from (11) and the boundedness of system matrices. Therefore, \( \lim_{k \to \infty} \| P(k) \|_2 \) is bounded.

\[ \| K_i(k) \|_2 \]

\[ \| H(k) \|_2 \]

Define a permutation matrix as

\[ Q := \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & I_{n_{i_{\rho}}} & 0 \\ 0 & 0 & I_{n_{i_{\rho}}} \end{bmatrix}. \] (18)

By left and right multiplication of \( M_{i,i_{\rho}}(k) \) with \( Q \) and \( Q^T \), then the following inequality is derived:

\[ \tilde{M}_{i,i_{\rho}}(k) := \begin{bmatrix} U_{i,i_{\rho}} & V_{i,i_{\rho}}(k) \\ * & U_{i,i_{\rho}} \end{bmatrix} < 0 \] (19)

where

\[ U_{i,i_{\rho}} := \begin{bmatrix} -\pi_{i,i_{\rho}} & I \\ 0 & -\pi_{i,i_{\rho}} \end{bmatrix} \]

\[ V_{i,i_{\rho}}(k) := \begin{bmatrix} \pi_{i,i_{\rho}} & N_i(k) & N_{i,i_{\rho}}(k) & N_{i,i_{\rho}}(k) \end{bmatrix} \begin{bmatrix} \tilde{K}_i(k)A_i(k-1) \\ \cdot \cdot \cdot \tilde{K}_i(k)A_i(k-1) \end{bmatrix} \] (20)
Notice that the set \( \{ \tilde{M}_{i,i_s}(k) \mid i \in \Omega_i, i_s \in \Omega_i \} \) is equivalent to the set \( \{ \hat{M}_{i,i_p}(k) \mid i \in \Omega_i, i_p \in \Omega_i \setminus \Omega_i \} \), which can be augmented as
\[
\hat{M}(k) := \text{diag} \left( \hat{M}_{1,1}(k), \ldots, \hat{M}_{1,\xi_1}(k), \hat{M}_{2,2}(k), \ldots, \hat{M}_{2,\xi_2}(k), \ldots \right) < 0
\]
where \( \xi_i \) is the number of elements of \( \Omega_i \setminus \Omega_i \). Then, define the following matrix:
\[
\mathcal{R} := \text{row} \left\{ e_{1,1}, \ldots, e_{1,\xi_1}, e_{2,2}, \ldots, e_{2,\xi_2}, \ldots \right\}
\]
where \( e_{i,j} := \begin{bmatrix} e_i & e_j & 0 & 0 \\ 0 & 0 & e_i & e_j \end{bmatrix} \), and \( e_i \) is a matrix with dimension \( n \times n_i \) that contains all zero elements, but an identity matrix of dimension \( n_i \) at rows \( \sum_{j=1}^{i-1} n_j + 1 \) through \( \sum_{j=1}^{i} n_j \).
By the property of positive definite matrices, if \( \hat{M}(k) < 0 \), then the matrix by left and right multiplication of \( \hat{M}(k) \) with \( \mathcal{R} \) and \( \mathcal{R}^T \) is also negative definite, i.e.,
\[
\mathcal{R} \hat{M}(k) \mathcal{R}^T = \begin{bmatrix} -I & \hat{K}(k)A(k-1) \\ & -I \end{bmatrix} < 0.
\]
According to Schur complement lemma, one has that
\[
\left\| \hat{K}(k)A(k-1) \right\|_2 < 1.
\]
By Lemma 1, the inequality (24) is sufficient to ensure the ultimate boundedness of \( \| P(k) \|_2 \) and \( \| P_i(k) \|_2 \).

**Remark 3.** Intuitively, the stability of an independent subsystem is not affected by other subsystems, so it only needs to keep local mean-square UUB as in (15a). As for subsystems coupled with neighbors, the stability condition will be tighter due to the coupling terms. Therefore, the distributed stability condition in Theorem 1 contains two parts: the constraint (15a) ensures that the local estimation error system without interconnected terms is stable, while the constraint (15b) is an additional requirement for the stability of systems with coupling relationships.

**Remark 4.** The main idea of Theorem 1 is to derive the distributed stability condition (15) by a subsystem-level analysis for the centralized stability condition (11). It can be extended to derive other distributed stability conditions.

- For the time-varying interconnected system (1) without disturbance, the overall system dynamics can be represented as \( x(k + 1) = A(k)x(k) \). If there exists a sequence of diagonal positive definite matrices \( P(k) = \text{diag}\{ P_1(k), P_2(k), \ldots, P_l(k) \} \) such that \( A^T(k)P(k + 1)A(k) - P(k) < 0 \), then the system is Lyapunov stable [49, Th. 23-3]. This centralized Lyapunov stability condition leads to the following distributed condition:

\[
A^T(k)P_l(k + 1)A_i(k) - P_l(k) < 0 \quad (25a)
\]
\[
\pi_{i,i_p}\pi_{i_p,i}\hat{N}(k) - \hat{N}(k)\hat{N}^{-1}(k)\hat{N}^T(k) < 0 \quad (25b)
\]
where
\[
\begin{align*}
\hat{N}_i(k) & := \begin{bmatrix} -P_i(k + 1) & P_i(k + 1)A_i(k) \\ * & -P_i(k) \end{bmatrix} \\
\hat{N}_{i,i_p}(k) & := \begin{bmatrix} 0 & P_i(k + 1)A_{i,i_p}(k) \\ A^T_{i,i_p}P_i(k + 1) & 0 \end{bmatrix}
\end{align*}
\]
- For a time-invariant interconnected system with local inputs, the overall system dynamics can be represented as \( x(k + 1) = A(k)x(k) + Bu(k) \). If there exist diagonal matrices \( Q = Q^T = \text{diag}\{ Q_1, Q_2, \ldots, Q_l \} \), \( S = \text{diag}\{ S_1, S_2, \ldots, S_l \} \), \( R = R^T = \text{diag}\{ R_1, R_2, \ldots, R_l \} \), and \( P = \text{diag}\{ P_1, P_2, \ldots, P_l \} > 0 \) such that

\[
\begin{bmatrix}
-P & 0 \\
* & -R + B^TPB & B^TPA - S^T \\
* & * & -P - Q
\end{bmatrix} \leq 0 \quad (27)
\]
then the system is \( (Q, S, R) \)-dissipative [50]. This centralized dissipativity condition leads to the following distributed condition:

\[
\hat{N}_i(k) - \hat{N}_i(k)\hat{N}_i^{-1}\hat{N}_i^T(k) \leq 0 \quad (28a)
\]
\[
\pi_{i,i_p}\pi_{i_p,i}\hat{N}_i(k) - \hat{N}_i(k)\hat{N}_i^{-1}(k)\hat{N}_i^T(k) \leq 0 \quad (28b)
\]

where
\[
\begin{align*}
\hat{N}_i(k) & := \begin{bmatrix} -P_i & 0 \\ * & -R_i + B^TP_iB_i, B^TP_iA_i - S_i^T \\
* & * \end{bmatrix} \\
\hat{N}_{i,i_p}(k) & := \begin{bmatrix} 0 & 0 \\ 0 & B^TP_iA_{i,i_p} \\
A^T_{i,i_p}P_i & A^T_{i,i_p}P_iB_i \end{bmatrix}
\end{align*}
\]

**Remark 5.** The distributed stability condition (15) is slightly more conservative than the centralized stability condition (11). The conservatism is mainly attributed to the derivation step (23) from \( \hat{M}(k) < 0 \) to \( \mathcal{R} \hat{M}(k) \mathcal{R}^T < 0 \). However, the adjustable parameter \( \pi_{i,j} \) is introduced in (15) to reduce conservatism. By properly selecting these parameters, the conservatism caused by the above step can be mitigated. On the other hand, the conditions from traditional stability analysis of interconnected systems are more conservative than the centralized conditions of the overall system. For example, the \( M \)-matrix condition [38] and the small-gain condition [41] for a linear time-invariant system are more conservative than the centralized Lyapunov condition \( A^TPA - P < 0 \). In this sense, the subsystem-level analysis in Theorem 1 is not as conservative as the traditional analysis [36–43].

In Theorem 1, subsystem \( S_i \) needs to know the gain matrix \( K_{i,j} \) from subsystem \( S_{i_p} \) timely. However, it is usually difficult to exchange information among subsystems in real time if the communication environment is non-ideal, such as a wireless communication environment. In the following lemma, a distributed stability condition without synchronously knowing neighbors’ gain information is further proposed.
Lemma 2. The distributed estimator (3) is mean-square UUB if the following inequality is satisfied:
\[
\|\hat{X}_i(k)\|_2^2 \leq \beta_i \quad (\forall i \in \mathbb{N}_l) \tag{30}
\]
where \(\beta_i < \frac{1}{\alpha_i}\) is constrained by:
\[
\pi_{i,i}, \pi_{i,i'} (\alpha_i \beta_i - 1) (\alpha_i \beta_i - 1) > \alpha_i^2 \pi_{i,i'}^2 (\forall i \in \mathbb{N}_l). \tag{31}
\]

Proof: By (30), the following upper bounds can be derived:
\[
\|\hat{X}_i(k)A_i(k-1)\|_2 \leq \alpha_i \beta_i < 1 \tag{32a}
\]
\[
\|\hat{X}_i(k)A_{i,i'}(k-1)\|_2 \leq \alpha_i \pi_{i,i'} \beta_i. \tag{32b}
\]
According to the Schur complement lemma, the inequality (32a) can be converted into
\[
\begin{bmatrix}
-\alpha_i \beta_i I & \hat{X}_i(k)A_i(k-1) \\
-\alpha_i \beta_i I & \hat{X}_i(k)A_{i,i'}(k-1)
\end{bmatrix} \preceq 0. \tag{33}
\]
The inequality (33) means that \(N_i(k) + (1 - \alpha_i \beta_i)I \preceq 0\). Notice that the matrix \(N_i(k)\) is invertible, so it is found that
\[
\lambda_{\max}(N_i(k)) \leq \alpha_i \beta_i - 1 < 0 \tag{34a}
\]
\[
\lambda_{\min}(N_i^{-1}(k)) \geq \frac{1}{\alpha_i \beta_i - 1}. \tag{34b}
\]
It follows from (32b) that
\[
N_{i,i'}(k)N_{i,i'}^T(k) - \alpha_i^2 \pi_{i,i'}^2 \begin{bmatrix}
0 \\
\alpha_i^2 \pi_{i,i'}^2
\end{bmatrix} \preceq 0. \tag{35}
\]
Then, it can be concluded that
\[
\begin{align*}
\left( \pi_{i,i'}(\alpha_i \beta_i - 1)N_{i,i'} - \alpha_i^2 \pi_{i,i'}^2 \right) & \preceq 0, \\
\left( \pi_{i,i'}(\alpha_i \beta_i - 1) - \alpha_i^2 \pi_{i,i'}^2 \right) & \preceq 0. \tag{36}
\end{align*}
\]
Therefore, \((15a)\) follows from \(\beta_i < \frac{1}{\alpha_i}\), and \((15b)\) can be derived from (36) and the constraint (31). The property of stability follows from Theorem 1.

Remark 6. The distributed stability condition (15) is simplified into finding an appropriate time-invariant parameter \(\beta_i\) for each subsystem in Lemma 2. The determination of \(\beta_i\) in Algorithm 1 only needs subsystems to communicate with their neighbors to exchange the knowledge of \(\beta_i\) and \(\alpha_i\). Compared with the results in [44] that require a considerable communication burden to exchange message matrices among subsystems, the proposed condition (30) can be achieved offline with less communication overhead. Therefore, the stability condition (30) with less communication burden is more suitable for interconnected systems with time-varying couplings and dynamics.

C. Optimization-based Distributed Estimator

Then, an optimization-based distributed estimator will be designed. Let us define the following matrices:
\[
\begin{align*}
D_{P_i}(k) & := \text{col} \left\{ \sqrt{P_i(k)}, \ldots, \sqrt{P_i(k)} \right\}, \\
D_{\hat{P}_i}(k) & := \text{col} \left\{ \sqrt{\hat{P}_i(k)}, \ldots, \sqrt{\hat{P}_i(k)} \right\}, \tag{38}
\end{align*}
\]
where \([P_i(k)]_\tau\) and \([\hat{P}_i(k)]_\tau\) are the \(\tau\)th diagonal element of \(P_i(k)\) and \(\hat{P}_i(k)\), respectively. The optimal estimator gain is designed in the following theorem.

Theorem 2. For the distributed estimator (3), the optimal gain matrix \(K_i^{\text{opt}}(k)\) can be determined by solving the following optimization problem:
\[
\min_{K_i(k)} \text{Tr} \left\{ \tilde{G}_i(k) \right\} \quad \text{s.t.} \quad \left[ \begin{array}{ccc}
-\tilde{G}_i(k) & \hat{K}_i(k) & \hat{P}_i(k) \\
* & -\hat{P}_i(k) & 0 \\
* & * & -Q_{v_i}
\end{array} \right] \preceq 0 \tag{39}
\]
\[
\text{(15) or (30)} \tag{30}
\]
where \(\text{Tr} \left\{ \tilde{G}_i(k) \right\} \) is an upper bound of the trace of estimation error covariance, and (15) and (30) are the stability constraints for ideal and non-ideal communication environments, respectively. Let us denote \(\tilde{K}_i^{\text{opt}}(k-1) := I - K_i^{\text{opt}}(k-1)C_i(k-1)\), then the matrices \(\hat{P}_i(k)\) and \(\bar{P}_i(k)\) are recursively calculated by
\[
\hat{P}_i(k) = A_i(k-1)\hat{P}_i(k-1)A_i^T(k-1) \\
+ \Gamma_i(k-1)Q_{v_i}I_{\tau_i}^T(k-1) \\
+ \sum_{i' \in \mathbb{N}_l} A_i(k-1)D_{P_{i,i'}}(k-1)A_{i,i'}^T(k-1) \\
+ \sum_{i' \in \mathbb{N}_l} A_i(k-1)D_{\hat{P}_{i,i'}}(k-1)A_{i,i'}^T(k-1) \tag{40a}
\]
\[
\bar{P}_i(k-1) = \tilde{K}_i^{\text{opt}}(k-1)\hat{P}_i(k-2) \tilde{K}_i^{\text{opt}}(k-1)^T \\
+ K_i^{\text{opt}}(k-1)D_i(k-1)Q_{v_i}D_i^T(k-1) \tilde{K}_i^{\text{opt}}(k-1)^T \tag{40b}
\]
\[
A_i(k-1)D_{\hat{P}_{i,i'}}(k-1)A_{i,i'}^T(k-1) \tag{40a}
\]
\[
\left[ \begin{array}{ccc}
-\tilde{G}_i(k) & \hat{K}_i(k) & \hat{P}_i(k) \\
* & -\hat{P}_i(k) & 0 \\
* & * & -Q_{v_i}
\end{array} \right] \preceq 0 \tag{39}
\]
Proof: It is infeasible to calculate the one-step prediction error covariance $P_i^p(k)$ in (4c) directly because the cross-covariance $P_{i,j}(k)$ is hard to compute by local communication. Therefore, a matrix $P_i(k)$ satisfying $P_i(k) \leq P_i^p(k)$ is constructed for designing the estimator gain matrix. Let $\tilde{x}_{i}(k)_{r_1}$ be the $r_1$th component of $\tilde{x}_i(k)$ and $[\tilde{x}_j(k)]_{r_2}$ be the $r_2$th component of $\tilde{x}_j(k)$. By resorting to the Hölder inequality, one has that

$$
[P_{i,j}(k)_{r_1,r_2}] = E \left\{ [\tilde{x}_{i}(k)]_{r_1} [\tilde{x}_j(k)]_{r_2} \right\} \\
\leq E \left\{ \left[ [\tilde{x}_{i}(k)]_{r_1} [\tilde{x}_j(k)]_{r_2} \right]^2 \right\} \\
\leq \sqrt{E \left\{ [\tilde{x}_{i}(k)]^2_{r_1} \right\} \sqrt{E \left\{ [\tilde{x}_j(k)]^2_{r_2} \right\}} \\
= \sqrt{[P_i(k)]_{r_1}} \sqrt{[P_j(k)]_{r_2}}.
$$

It is known from (41) that

$$P_{i,j}(k) \leq D_{P_i}(k)D_{P_j}(k) \leq D_{P_i}(k)D_{P_j}(k).$$

By applying the inequality (42) to (4c), it is found that $P_i^p(k) \leq \bar{P}_i(k)$. Then, the matrix $P_i(k)$ can be constructed from (4d) as

$$P_i(k) = \bar{P}_i(k) := \tilde{K}_i(k)\tilde{P}_i(k)\tilde{K}_i^T(k) + K_i(k)D_i(k)Q_vD_i^T(k)K_i^T(k).$$

At the same time, it is proposed to construct a matrix $\tilde{G}_i(k)$ satisfying $\tilde{P}_i(k) - \tilde{G}_i(k) < 0$, so one has that

$$\text{Tr} \{P_i(k)\} \leq \text{Tr} \left\{ \tilde{P}_i(k) \right\} < \text{Tr} \left\{ \tilde{G}_i(k) \right\}.$$  

(44)

The optimal estimator gain is obtained by minimizing the upper bound $\text{Tr} \left\{ \tilde{G}_i(k) \right\}$, which turns out to be an optimization problem. The constraint $\tilde{P}_i(k) - \tilde{G}_i(k) < 0$ can be converted by the Schur complement lemma as follows:

$$[K_i(k)D_i(k)Q_vD_i^T(k)K_i^T(k) - \tilde{G}_i(k)\tilde{K}_i(k)\tilde{P}_i^p(k) - \tilde{P}_i^p(k)] < 0.$$  

(45)

Then, the first inequality constraint in (39) is further derived using the Schur complement lemma. By adding the distributed stability constraints in Theorem 1 or Lemma 2, the optimization problem (39) is formulated.

Then, the computation procedures of the distributed estimation for time-varying interconnected systems with ideal and non-ideal communication environments can be summarized by Algorithm 2. The shared information among subsystems includes $\hat{x}_i(k)$ and $P_i(k)$ for the estimator iteration and $K_i(k)$ for the stability guarantee in an ideal communication environment.

Algorithm 2 Distributed Estimation for Time-varying Interconnected Systems

1: Offline Setup: If the communication environment is non-ideal, then offline allocate $\beta_i$ by Algorithm 1;  
2: for $i := 1$ to $l$ do  
3: Subsystem $S_i$ collects the local measurement $y_i(k)$ and the estimation results from its neighbors, including $\hat{x}_i(k - 1), P_i(k - 1), \text{and} K_i(k)$ ($i = \omega \in \mathbb{I}_l$);  
4: Subsystem $S_i$ calculates $P_i^p(k)$ by (40a);  
5: Subsystem $S_i$ determines the estimator gain $K_i(k)$ by solving the optimization problem (39), constrained by (30) if the communication environment is non-ideal, and (15) if the communication environment is ideal;  
6: Subsystem $S_i$ calculates $\tilde{P}_i(k)$ by (40b);  
7: Subsystem $S_i$ obtains the estimate $\hat{x}_i(k)$ by (3);  
8: Subsystem $S_i$ sends the calculated $\hat{x}_i(k), \tilde{P}_i(k), \text{and} K_i(k)$ to its out-neighbors;  
9: end for  
10: Return to Step 2 and implement Steps 2-9 for calculating $\hat{x}_i(k + 1)$ ($i = 1, 2, \ldots, l$).

Remark 7. The inequality constraints (15) and (30) can be rewritten as linear matrix inequalities, so the optimization problem in Theorems 2 can be directly solved by the function “mincx” of the MATLAB LMI toolbox [48]. Notice that the first inequality constraint in (39) is converted from $\tilde{P}_i(k) - \tilde{G}_i(k) < 0$ for constructing the matrix $\tilde{G}_i(k)$ and is a feasible constraint. Therefore, the solvability of the optimization problem (39) can be guaranteed by allocating the parameters in the stability constraints (15) and (30) in advance according to subsystems’ dynamics and couplings. These adjustable parameters make the derivation of stability conditions less conservative than the derivation in [44], and one possible parameter allocation can be found in (37) and Algorithm 1. On the other hand, the computational complexity of (40) is $O(\hat{P}_i^3n_{i,\text{max}}^3)$, where $n_{i,\text{max}} := \max\{n_i, n_{i_1}, \ldots, n_{\hat{i}}\}$. The computational complexity of (39) is also mainly determined by $\beta_i$ and $n_{i,\text{max}}$. For a large-scale interconnected system with a sparse coupling structure, the computational burden of (39) in the proposed distributed estimator will be much less than that of a centralized estimator.

Remark 8. The design methodology in Theorem 2 takes the optimality and stability of the distributed estimator into account by constructing an optimization problem. It can overcome the difficulty in stability assurance of completely local design methods, which means the proposed distributed estimator is more appealing when compared with the work in [25, 26] that only provides stability conditions for specific coupling structures. Moreover, the designed estimator enables plug-and-play operations due to the flexibility of the proposed distributed stability conditions. If one subsystem is removed, then the stability conditions and the estimator design for other subsystems do not require any changes. If a new subsystem is added, then only the stability condition and estimator design for the added subsystem need to be redesigned. The plug-and-play property is helpful for the large-scale deployment of distributed estimators for interconnected systems.

IV. NUMERICAL EXAMPLES

To illustrate the effectiveness of the proposed distributed estimators, we shall report the numerical study results for a three-tank system in this section. The three-tank system consists of three cylindrical tanks, T1, T2, and T3, with an equivalent cross-section $S_A$. These tanks are connected serially.
by cylindrical pipes with the cross-section $S_n$, and the layout is shown in Fig. 2. There are two pumps configured for T1 and T2 to supply water, and one outflow valve with the cross-section $S_n$ is located at the bottom of T2. The pump flow rates $Q_1$ and $Q_2$ are viewed as the input signals for the system, while the flow rates $Q_{13}, Q_{32},$ and $Q_{20}$ can be determined with outflow coefficients $\mu_{13}(t), \mu_{32}(t),$ and $\mu_{20}(t)$ by the general Torricelli rule. Let the levels of three tanks be the system state $x := [h_1 \ h_3 \ h_2]^T$ and assume they satisfy $h_1 > h_3 > h_2$ during the whole experiment. Then, the nonlinear continuous-time model [52] obtained from the balance equation can be linearized at the equilibrium point $x_* := [h_1^* \ h_3^* \ h_2^*]^T$ as:

$$\dot{x}(t) = A_c(t)x(t) + B_cu(t)$$  \hspace{1cm}(47)$$

where $u := [Q_1 \ Q_2]^T$ is the input signal, and the system matrices are defined in (46). The continuous-time model (47) should be discretized into a discrete-time model: $x(k+1) = A_d(k)x(k) + B_d(k)u(k)$. To maintain $A_d(k)$ the same coupling structure as $A_c(t)$, the block-wise discretization [53] with a sampling period $T_s$ is applied. Thus, the parameters of the discrete-time model can be obtained by $A_d(k) = I + F(T_s)A_c(kT_s)$, $B_d(k) = F(T_s)B_c$, and $F(T_s) := \text{diag} \left\{ \int_0^{T_s} e^{A_c(kT_s)} \, dt_1, \ldots, \int_0^{T_s} e^{A_c(kT_s)} \, dt_3 \right\}$. Then, the overall system is decomposed into three subsystems $S_i$ ($i = 1, 2, 3$) with the coupling structure in Fig. 3, where

$$A_c = \begin{bmatrix}
-\mu_{13}(t)gS_n & 0 & -\mu_{20}(t)gS_n \\
\frac{\mu_{13}(t)gS_n}{S_A\sqrt{2g(h_1^2-h_3^2)}} & 0 & -\frac{\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_2^2-h_3^2)}} \\
\frac{\mu_{20}(t)gS_n}{S_A\sqrt{2g(h_1^2-h_3^2)}} & \frac{\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_3^2-h_2^2)}} & 0
\end{bmatrix} \quad B_c = \frac{1}{S_A} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm}(46)$$

the state dynamics of these subsystems with system noises and leakage faults can be described as follows:

$$\begin{align*}
x_1(k+1) &= A_1(k)x_1(k) + A_{13}(k)x_3(k) + B_1u_1(k) + f_1(k) + w_1(k) \\
x_2(k+1) &= A_2(k)x_2(k) + A_{23}(k)x_3(k) + B_2u_2(k) + f_2(k) + w_2(k) \\
x_3(k+1) &= A_3(k)x_3(k) + A_{31}(k)x_1(k) + A_{32}(k)x_2(k) + f_3(k) + w_3(k)
\end{align*}$$  \hspace{1cm}(48)$$

where $x_i := h_i$, $u_i := Q_i$, $f_i$, and $w_i$ are the state, the input, the possible leakage fault, and the system noise of each subsystem, respectively. The matrices $A_i(k), A_{ij}(k),$ and $B_i$ are the corresponding submatrices (scalars for this example) in $A_d(k)$ and $B_d$. Moreover, the liquid level of the $i$th tank is monitored by a sensor with the measurement dynamics $y_i(k) = x_i(k) + v_i(k)$.

In the simulation, the system parameters are set as follows: $T_s = 1s$, $S_n = 0.0154 \text{ m}^2$, $S_m = 5 \times 10^{-5} \text{ m}^2$, $g = 9.8 \text{ m/s}^2$, $\mu_{13}$ is randomly selected from 0.42 to 0.50 at each instant, $\mu_{32}$ is randomly selected from 0.44 to 0.52 at each instant, $\mu_{20} = 0.58$, and $x_* = [0.4890 \ 0.2332 \ 0.3611]^T$. The leakage faults only happen over a certain period of time, as $f_1(k) = -2 \times 10^{-3}$ (100 $\leq k \leq 110$) and $f_2(k) = -2 \times 10^{-3}$ (50 $\leq k \leq 60$). The system noises $w_i(k)$ and the measurement noises $v_i(k)$ are uncorrelated Gaussian noises with covariances $Q_i = 0.0001$ and $R_i = 0.0005$. To control the liquid levels of T1 and T2 at the equilibrium point, two PID controllers with the same parameters, $K_P = 0.02$, $K_I = 5 \times 10^{-6}$, and $P_D = 2 \times 10^{-6}$, are employed in subsystems. Our primary concern in this numerical study is
to estimate the real states of each subsystem using only local communication. The proposed distributed estimators have been successfully applied to this interconnected three-tank system. In a non-ideal communication environment, the liquid levels of three tanks and their estimated values by Theorem 2 are plotted in Fig. 4. It is found that the leakage faults in T1 and T2 also cause a sudden drop in the liquid level of T3. At the same time, the estimators can track these real liquid levels well based on only local information. The result also shows that the estimation error in each subsystem can converge to a relatively small interval. To better reflect the estimation results, a Monte Carlo simulation with $N = 100$ runs has been performed by randomly varying the realization of process and measurement noises. The mean square error (MSE) is introduced to evaluate estimators’ performance, where

$$\text{MSE}(k) = \frac{1}{N} \sum_{s=1}^{N} \| e_s(k) \|^2$$

(49)

with $e_s(k)$ being the state estimation error at the instant $k$ in the $s$th simulation. The MSE comparison for the decentralized estimator [51] and the distributed estimators in Algorithm 2 using (15) and (30) is plotted in Fig. 5. It is shown in Fig. 5 that the MSE performance of the proposed distributed estimators is much better than the decentralized estimator. Moreover, the average MSE (AMSE) calculated from the mean of MSEs at all instants is also introduced to show the accuracy of estimators. The AMSE comparison under different measurement noise realizations for the distributed estimators in Algorithm 2 using (15) and (30), the distributed estimator with the assumption of uncorrelated errors in [27], and the centralized estimator is plotted in Fig. 6. It can be seen from Fig. 6 that the designed distributed estimators show their performance superiority over the distributed estimator that completely ignores the estimation errors correlations. It means that the distributed estimation method by constructing an upper bound of the trace of estimation error covariance is better than the estimation by ignoring the estimation error correlations. In addition, the performance of the distributed estimators does not degrade too much when compared with the centralized estimator.

V. CONCLUSION

In this paper, we presented new results of distributed estimator design for time-varying interconnected systems with arbitrary coupling structures. A distributed stability condition was proposed in Theorem 1 by a subsystem-level analysis for a centralized stability condition. The proposed distributed condition can keep the distributed estimator mean-square UUB without requiring the overall system’s knowledge of dynamics and couplings. By further considering a non-ideal communication environment, a simplified distributed condition was proposed to avoid the real-time exchange of local estimator gains. These distributed conditions enable plug-and-play operations due to their distributed nature. Moreover, the distributed estimators were designed by applying distributed conditions to an optimization-based estimator design approach. The designed estimators are fully distributed because the estimator form and the stability conditions only use information from each subsystem and its neighbors. Finally, an illustrative example of a three-tank interconnected system was employed to show the effectiveness of the proposed methods.

Several topics for future research are left open. It will be important to extend the presented distributed stability conditions to nonlinear interconnected systems and apply them to distributed control problems. Another interesting extension is the development of secure distributed estimators for interconnected systems. Notice that interconnected systems are more vulnerable to various attacks due to the distributed communication structure. If one subsystem is compromised, then the security risk may propagate to other subsystems. How to design anti-eavesdropping mechanisms to prevent more sophisticated attacks and how to design attack detection mechanisms through subsystem cooperation are the two important problems to be solved.

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