Multi-objective optimization of energy-efficient buffer allocation problem for non-homogeneous unreliable production lines

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ABSTRACT The current context of rising ecological awareness and high competitiveness, reveals a strong necessity to integrate the sustainability paradigm into the design of production systems. The buffer allocation problem is of particular interest since buffers absorb disruptions in the production line. However, despite the rich literature addressing the BAP, there are no studies that use a multi-objective framework to deal with energetic considerations. In this study, the energy-efficient buffer allocation problem (EE-BAP) is studied through a multi-objective resolution approach. The multi-objective problem is solved to optimize two conflicting objectives: maximizing production throughput and minimizing its energy consumption, under a total storage capacity available. The weighted sum and epsilon-constraint methods as well as the elitist non-dominated sorting genetic algorithm (NSGA-II) are adapted and implemented to solve the EE-BAP. The obtained solutions are analyzed and compared using different performance metrics. Numerical experiments show that epsilon-constraint outperforms the NSGA-II when considering comparable computational time. The Pareto solutions obtained are trade-offs between the two objectives, enabling decision making that balances productivity maximization with energy economics in the design of production lines.

INDEX TERMS Buffer allocation problem, energy efficiency, multi-objective optimization, unreliable production lines, non-linear programming.

I. INTRODUCTION

The buffer allocation problem (BAP) is a design problem in production systems that aims to find the optimal allocation of storage space to achieve system efficiency. Allocating additional buffer sizes has a significant impact on system efficiency through the absorption of disruptions due to breakdowns or variations in processing times. However, it also generates a higher work-in-process inventory level and greater investment cost. Therefore, finding the optimal allocation of buffer capacities is a crucial research issue.

The BAP was firstly presented by [1]. It is an NP-hard combinatorial optimization problem [2] characterized by different variants depending on the objective functions and constraints of the model. The most studied versions are the dual BAP and the primal BAP [3]. The dual BAP focuses on throughput maximization with a total buffer capacity as a constraint (equation 1, where $K$ is the number of machines in the line, $N$ the buffer size vector, $N_{total}$ the total buffer space available in the system, and $\psi$ the throughput of the line). The primal BAP minimizes the total buffer capacity with a minimum desired throughput $\psi_d$ (equation 2). Both forms of the BAP have been widely addressed in the literature. Many other variants of the BAP exist, focusing on profit maximization [3], [4], average work-in-process [5], system availability [6], etc. Literature work also considered combining other design options within the BAP, such as system configuration, equipment selection or maintenance actions [7]–[9]. Comprehensive reviews on the BAP can be found in the literature [3], [10].

\[
\text{Find } N = (N_1, N_2, \ldots, N_{K-1}) \text{ so as to :} \\
\max \psi
\]
The novel variant of the BAP focusing on both energy efficiency and throughput optimization, introduced by [11], is rigorously studied and analyzed through a multi-objective framework. The goal is to find buffer configurations that respect the total buffer space allowed and provide maximum throughput with minimum energy consumption simultaneously for production systems.

- Two classical multi-objective optimization methods WS and ECM, as well as the evolutionary algorithm NGSA-II are implemented to search for Pareto-optimal solutions. Comparative studies highlight the out-performance of the ECM. The efficiency of the non-linear programming method used for solving the EE-BAP allows finding Pareto fronts with good spread and coverage in reasonable computational times.

- The simultaneous optimization of the conflicting objectives: throughput and energy consumption, offers a panel of alternatives for decision making. The complex trade-off between both performances is analyzed allowing to reach sustainability and energy efficiency in production systems design.

The article is presented as follows. Section 2 introduces basic concepts of multi-objective optimization and some popular resolution approaches. A literature review related to the multi-objective BAP is also presented in this section. Section 3 introduces the EE-BAP and its Multi-objective modeling based on Pareto optimization. In Section 4, numerical results from the multi-objective resolution of the EE-BAP and a comparative study using performance indicators are presented. Conclusions and future research directions are put forward in Section 5.

II. RELATED LITERATURE

A. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization has been a growing interest in the last two decades. Solving a problem with multiple contradicting objectives to be optimized simultaneously requires a set of optimal trade-offs between the conflicting objectives [12]. In the following, we describe basic theoretical concepts related to multi-objective optimization as well as some of the most popular resolution approaches.

1) Basic concepts

A general multi-objective optimization problem is given as follows:

\[
\begin{align*}
\text{minimize } & \quad F(x) = [F_1(x), F_2(x), ..., F_k(x)]^T \\
\text{subject to } & \quad g_j(x) \leq 0, \quad j = 1, 2, ..., m
\end{align*}
\]

\(k\) is the number of objective functions to be optimized and \(m\) is the number of inequality constraints, with \(m \geq 2\). \(x\) is a vector of design variables and \(F(x)\) is a vector of objective functions. \(g_j(x) \leq 0\) represents the constraints and \(\Omega\) contains all feasible \(x\).

Given two vectors \(x, x' \in \Omega\), the vector \(x\) is said to dominate \(x' (x > x')\) if \(x\) is not worse than \(x'\) in any
objective function \( x \) weakly dominates \( x' \) and it is strictly better in at least one objective function \([13], [14]\). If neither \( x \) dominates \( x' \), nor \( x' \) dominates \( x \), \( x \) and \( x' \) are said to be non-comparable.

The vector \( x \) is a Pareto-optimal solution if there is no other vector in \( \Omega \) that dominates it. The set of Pareto-optimal solutions is called the Pareto set. Its image is called the Pareto front.

In single-objective optimization, the set of optimal solutions is often composed of a singleton. In the multi-objective case, the Pareto front usually contains many elements (an infinity in continuous optimization and an exponential number in discrete optimization \([15]\)). To solve a multi-objective problem, the best discrete representation of the Pareto front is searched as it is, in general, not possible to enumerate all elements of the Pareto front. A set of decision vectors in the feasible set is called a Pareto set approximation if no element of this set is weakly dominated by any other. The image of this set in the objective space is called a Pareto front approximation \([16]\).

2) Resolution approaches

One of the most popular methods used for solving multi-objective problems is to reduce it to a scalar problem. The WSM allows the multi-objective optimization problem to be cast as a single-objective mathematical optimization problem. The objective function therefore becomes an aggregated weighted sum of the normalized objectives. The weights \( \omega_k, k = 1, ..., m \) are chosen such that \( \sum_{k=1}^{m} \omega_k = 1 \) and \( \omega_k \geq 1 \). A convex combination of objectives is hence obtained. The well-known limitation of this method is its inability to generate non-convex portions of the Pareto front regardless of the weight combination used. Another traditional method from the field of multi-objective optimization is the ECM introduced in \([17]\). In this method, one of the objective functions is selected to be optimized while the other(s) are converted into additional constraints. By a systematic variation of the constraint bounds, different elements of the Pareto front can be obtained. One of the difficulties of this method is that the solution to the problem largely depends on the chosen epsilon vector requiring therefore problem knowledge. The WSM and ECM rely on the availability of a procedure to solve constrained single-objective problems. Multiple executions are required which may result in expensive computational time.

In addition to these classical methods, evolutionary algorithms are also used in multi-objective optimization. The first implementation of a multi-objective evolutionary algorithm dates back to the 1980’s with the Vector Evaluation Genetic Algorithm (VEGA) \([18]\). Considerable research effort has been devoted to the study of evolutionary multi-objective optimization. This is mainly due to their capacity to deal simultaneously with a set of possible solutions. Consequently, in a single run of the algorithm several members of the Pareto-optimal set are found, instead of having to perform multiple runs as in the case of the classical methods. Furthermore, these algorithms are less susceptible to the shape or continuity of the Pareto front and require very little knowledge about the problem.

The Non-dominated Sorting Genetic Algorithm (NSGA-II) \([19]\) is one of the popularly used evolutionary multi-objective algorithms. It is widely used for its diversity and faster convergence in solutions. At each generation, the off-spring population is created from the parent population of size \( N_{\text{pop}} \). The two populations are combined to form a new population of size \( 2N_{\text{pop}} \), which is then classified into different non-domination classes (fronts) to form a new population of size \( N_{\text{pop}} \) according to the non-domination ranking. The resting fronts are deleted. The crowding distance is used to keep members from the last front according to the remaining slots for more diversity.

Besides the NSGA-II method, other methods are also commonly used such as the Strength Pareto Evolutionary Algorithm (SPEA2) \([20]\), the Pareto envelope-based selection algorithm PESA \([21]\), etc. Additionally, there are other evolutionary algorithm-based methodologies, such as particle swarm or ant-based. For comprehensive literature reviews on multi-objective evolutionary algorithms, several survey papers are available \([22], [23]\). Lately, great progress has been made in the field of multi-objective optimization with many algorithms based on hybrid approaches. For instance, an effective modified multi-objective evolutionary algorithm with decomposition (MMOEA/D) is proposed in \([24]\) for the energy-efficient distributed job shop scheduling problem, minimizing makespan and energy consumption. A multi-objective Evolutionary Swarm Hybridization (MESH) algorithm is proposed in \([25]\) and applied to hydro-power plant modeling. In \([26]\), a novel algorithm called MOMPA multi-objective marine predator algorithm is introduced. This algorithm incorporates the non-dominated sorting approach and the reference point strategy to outperform the NSGA-II. Moreover, a novel hybrid swarm intelligence optimization algorithm, named PSO-Lévy-DFOA is developed in \([27]\) that integrates PSO search strategy and Lévy flight applied to WLAN planning.

B. THE MULTI-OBJECTIVE BAP

Although extensive efforts were devoted to the study of the BAP since decades as stated in the introduction, most of the research efforts in the literature only consider the single objective version of the BAP. Studies on multi-objective BAP mainly focus on productivity and buffer space optimization. \([28]\) studied the BAP for throughput maximization and total buffer size as well as costumer average waiting delay minimization for closed queuing networks. A multi-objective approach for the buffer allocation and throughput trade-off problem for single server general queuing networks was developed in \([29]\). The generalized expansion method was used as a performance evaluation tool along with a multi-objective genetic algorithm to minimize the total number of buffer sizes and maximize the throughput rate. \([30]\) proposed a multi-objective resolution approach based on an ant colony.
algorithm using the Lorenz dominance, for throughput and total buffer size optimization. Cross-entropy method was used in [31] to solve the BAP for throughput and buffer size optimization in unreliable serial lines. [32] studied buffer and preventive maintenance periods allocation problems. An integrated simulation and meta-heuristic algorithm method (genetic and particle swarm) was used to solve the problem for throughput maximization and total buffer size as well as total number of defective units minimization. In [33], a genetic algorithm combined to line search method was used to solve the multi-objective model for throughput and buffer size optimization. Later, [34] proposed a multi-objective mathematical formulation and a hybrid genetic algorithm to solve buffer sizing and machine allocation problems simultaneously for throughput maximization and total cost minimization. A hybrid multi-objective optimization algorithm based on an adaptation of the Pareto hill climbing and NSGA-II was proposed in [35]. Authors studied the line balancing, equipment selection, and buffer sizing problem for idle time and total unit costs minimization as well as throughput maximization. [36] developed a novel tabu-search NSGA-II hybrid method for throughput maximization and work in process (WIP) minimization. In [37], the BAP is studied for throughput, capital buffer installation costs, and inventory cost optimization in a series-parallel production line. This problem is solved using two evolutionary algorithms SIBEA (Simple Indicator-Based Evolutionary Algorithm) and SEMO (Simple Evolutionary Multiobjective Optimizer).

Recently, [38] focused on the multi-objective BAP for throughput and buffer size optimization. Discrete event simulation modeling was used as an evaluative tool for performance measure and the Pareto optimal set was derived using a hybrid approach combining NSGA-II and multi-objective simulated annealing. Moreover, in [39] the transfer line balancing problem and the BAP were studied simultaneously for throughput and total cost optimization. Simulation was used for evaluation, and the problem was solved using the NSGA-II and Multi-Objective Particle Swarm Optimization (MOPSO).

Although the literature related to multi-objective optimization algorithms is very rich with various evolutionary-based algorithms having interesting search and convergence performances, most recent multi-objective studies of the BAP use either the NSGA-II [39] or hybrid approaches combining NSGA-II with other evolutionary algorithms such as [35], [36], [38]. Although the developed hybrid algorithm TS-NSGA-II in [36] demonstrated good performance, the NSGA-II still shows equivalent overall performance with a better breadth of search and less computational time than the developed hybrid approach. Furthermore, in [39], authors compare the NSGA-II with PSO algorithm, and demonstrate the over performance of the NSGA-II when compared to PSO for the multi-objective BAP. Therefore, based on the literature relative to the BAP, using the NSGA-II as an evolutionary multi-objective algorithm could be a proper choice for solving this problem.

III. PROBLEM STATEMENT
The BAP is an NP-hard combinatorial optimization problem [2] that aims to find the optimal allocation of storage space among buffer areas in a production line. As stated in the literature review section, the objectives to optimize are in most cases the equivalent throughput of the system (to be maximized) and the total buffer space allocated (to be minimized), either in a single or a multi-objective optimization procedure. In this study, based on the EE-BAP [11], a multi-objective procedure is conducted to optimize two conflicting objectives: the throughput of the line and its energy consumption. The total buffer space is considered here as a constraint.

The system studied in this paper is a serial production line composed of $K$ machines separated by $K-1$ buffer areas (see figure 1). Each machine $M_i \forall i \in \{1..K\}$ is characterized by a failure rate $\lambda_i$, a repair rate $\mu_i$, and a processing rate $\omega_i$. Machines are also characterized by a failure state energy consumption $E_{down,i}$, an idle state energy consumption $E_{idle,i}$, and an operating energy consumption composed of a constant part $E_{load,i}$ and a variable part $E_{op,i}$ per part processed. $\rho_i$ and $E_i$ are respectively the production rate and the energy consumption of each machine $M_i$. Finally, $\psi$ is the equivalent throughput of the line and $E$ its total energy consumption.

![FIGURE 1: Serial production line](image)

In addition, we consider commonly used assumptions in the literature. Unlimited supply before the first machine and unlimited storage capacity after the last machine are assumed. Therefore, the first machine cannot be starved and the last machine cannot be blocked. Operation dependent failures are considered. The failure and repair rates of the machines (respectively denoted $\lambda_i$ and $\mu_i$, $\forall i \in \{1..K\}$) are assumed to be exponentially distributed. Furthermore, each buffer $B_j$ $j \in \{1..K-1\}$ has a finite capacity to be determined denoted $N_j$ and cannot be down. Energy consumption of buffers is considered negligible in this study.

A. THE EE-BAP
The EE-BAP is a variant of the BAP that integrates energetic considerations. The objective is to find the optimal allocation of buffer spaces to maximize the equivalent throughput and minimize the energy consumption of the line, under a maximum total buffer space allowed. To solve the EE-BAP in [11], a standardized linear combination of the objectives is used to eventually implement the problem on a mathematical solver. Numerical results from conducted tests on literature instances illustrate a great potential of energy savings with low throughput deterioration when compared to the classic dual BAP for throughput maximization. A different buffer

\[\omega_i\]
profile from that of the dual BAP is suggested by the EE-BAP, that respects the same constraints, but allows a lower energy consumption with low or, in certain cases, no throughput deterioration.

The evaluation approach of the two crucial performances considered in the problem, i.e. throughput and energy consumption of the line, was developed in [40]. The throughput is evaluated using the Equivalent Machine Method [41]. In this analytical formulation, the different states of each buffer $B_j$ $j \in \{1...K-1\}$ are analyzed using birth-death Markov processes. Each stochastic process is defined by its processing rates $\alpha_j$. Based on the analysis of the simple system composed of two machines and one buffer, the probabilities of empty and full states $p_j^0$ and $p_j^N$ of each buffer $B_j$ are formulated. Thereafter, each original machine is replaced by an equivalent machine considering the probabilities of blockage and starvation. The effective production rates $\rho_i$ for each machine $M_i$ $i \in \{1...K\}$ are obtained. The throughput of the production line $\psi$ is defined as the bottleneck between the effective production rates of the equivalent machines. Due to its main approach that considers only full and empty buffer states, instead of all buffer states as in other methods from the literature, the state space cardinality of the Markov chain representation of the system is reduced drastically. Results from numerical experiments demonstrate a high accuracy with extensively reduced computational time when compared to other methods from the literature, such as the decomposition and aggregation methods [41], [42].

Corresponding formulations for: probabilities of empty and full buffer state $p_j^0$ and $p_j^N$ for each buffer $B_j$ $j \in \{1...K-1\}$, processing rates ratio $\alpha_j$ for each buffer $B_j$ $j \in \{1...K-1\}$, production rates $\rho_i$ for each machine $M_i$ $i \in \{1...K\}$, and equivalent throughput of the line $\psi$ are given in equations 4 - 8 respectively.

$$p_j^0 = \begin{cases} \frac{1}{1-\alpha_j} \frac{1}{\eta_j+1} & \text{if } \alpha_j \neq 1 \\ \frac{\eta_j}{1-\alpha_j} \frac{1}{\eta_j+1} & \text{if } \alpha_j = 1 \end{cases} \quad \forall j = \{1...K-1\}$$ (4)

$$p_j^N = \begin{cases} \frac{\eta_j (1-\alpha_j)}{1-\alpha_j} \frac{1}{\eta_j+1} & \text{if } \alpha_j \neq 1 \\ \frac{1}{1-\alpha_j} \frac{1}{\eta_j+1} & \text{if } \alpha_j = 1 \end{cases} \quad \forall j = \{1...K-1\}$$ (5)

$$\alpha_j = \frac{\min_{i=1...j} \rho_i}{\min_{i=j+1...K} \rho_i} \quad \forall j = \{1...K-1\}$$ (6)

$$\rho_i = \omega_i \times \frac{\mu_i \times \xi_i}{\mu_i + \xi_i \times \lambda_i} \quad \forall i = \{1...K\}$$ (7)

Where:

$$\begin{cases} \xi_1 = 1 - p_{1}^{N_1} \\ \xi_K = 1 - p_{K-1}^{N_{K-1}} \\ \xi_i = (1 - p_{j}^{N_{j-1}}) \times (1 - p_{j}^{N_j}) \quad \forall i = \{2...K-1\} \\ \psi = \min_{i=1...K} \{\rho_i\} \end{cases}$$ (8)

The second part of the method evaluates the energy consumption of the production line. The approach consists of an evaluation based on machine states. Each machine $M_i$ $i \in \{1...K\}$ consumes a specific amount of energy according to the state in which it could be. Five states are considered for each machine: operating, down, starved, blocked, and starved & blocked at the same time. These states are indicated by 1, 2, 3, 4, and 5 respectively. The first and the last machine are only characterized by three different states since the first machine cannot be starved and the last machine cannot be blocked. A Markov chain formulation is used to obtain transition and steady state probabilities $P_{l,i}$ $l \in \{1...5\}$ $i \in \{1...K\}$. These probabilities are obtained as a function of machine parameters as well as probabilities of empty and full buffer states ($p_j^0$ and $p_j^N$) derived from the throughput evaluation part. The transition probabilities matrix is given for each machine: the general case $M_i$ $\forall i = \{2...K-1\}$ (equation 9), the first machine (equation 10), and the last machine (equation 11).

$$A_1 = \frac{1}{2} \begin{bmatrix} \frac{\mu_1}{\lambda_1+\mu_1} (1 - p_{1}^{N_1}) & \frac{\lambda_1}{\lambda_1+\mu_1} & 0 & \frac{\mu_1}{\lambda_1+\mu_1} p_{1}^{N_1} \\ \frac{\mu_1}{\lambda_1+\mu_1} & \frac{\lambda_1}{\lambda_1+\mu_1} & 0 & \frac{\mu_1}{\lambda_1+\mu_1} p_{1}^{N_1} \\ \frac{1}{\lambda_1+\mu_1} & \frac{\lambda_1}{\lambda_1+\mu_1} & 0 & \frac{\mu_1}{\lambda_1+\mu_1} p_{1}^{N_1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (10)

$$A_K = \frac{1}{2} \begin{bmatrix} \frac{\mu_K}{\lambda_K+\mu_K} (1 - p_{K-1}^{0}) & \frac{\lambda_K}{\lambda_K+\mu_K} & 0 & \frac{\mu_K}{\lambda_K+\mu_K} p_{K-1}^{0} \\ \frac{\mu_K}{\lambda_K+\mu_K} & \frac{\lambda_K}{\lambda_K+\mu_K} & 0 & \frac{\mu_K}{\lambda_K+\mu_K} p_{K-1}^{0} \\ \frac{1}{\lambda_K+\mu_K} & \frac{\lambda_K}{\lambda_K+\mu_K} & 0 & \frac{\mu_K}{\lambda_K+\mu_K} p_{K-1}^{0} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (11)

Thereafter, energy consumption $E_i$ is formulated for each machine $M_i$ $i \in \{1...K\}$ and consequently for the production line, using steady state probabilities and specific state energy consumption. $E_{down,i}$ is the energy consumed in the failure state, $E_{load,i}$ the energy consumed while working without load (starved, blocked or both), $E_{cloud,i}$ the constant part of energy consumed in the operating state, and $c_{op,i}$ the variable part of energy consumed per part processed. Eventually, the energy consumption $E$ of a serial production line is defined as the sum of energy consumption of its $K$ machines. Corresponding formulations for steady state probabilities $P_{i,l}$ $l \in \{1...5\}$ $i \in \{1...K\}$ and energy consumption $E_i$ for each machine $M_i$ $i \in \{1...K\}$ as well as the total energy consumption of the line $E$ are given by equations 12-20 respectively. Further details can be found in [40].
The problem formulated as a mixed integer non-linear program is presented in algorithm 1.

\[ A_i = \begin{bmatrix}
\frac{\mu_i}{\lambda_i+\mu_i} (1-p_i^0) (1-p_i^{N_i}) & \frac{\lambda_i}{\lambda_i+\mu_i} p_i^{N_i} (1-p_i^{N_i}) & 0 \\
\frac{\mu_i}{\lambda_i+\mu_i} (1-p_i^0) (1-p_i^{N_i}) & 0 & \frac{\lambda_i}{\lambda_i+\mu_i} p_i^{N_i} (1-p_i^{N_i}) \\
\frac{\mu_i}{\lambda_i+\mu_i} (1-p_i^0) & 0 & 0 \\
(1-p_i^0) (1-p_i^{N_i}) & 0 & 0 \\
(1-p_i^0) (1-p_i^{N_i}) & 0 & 0 \\
\end{bmatrix} \]

**B. THE MULTI-OBJECTIVE EE-BAP**

The multi-objective EE-BAP is formulated as follows:

Find Pareto optimal set of \( N = (N_1, N_2, ..., N_{K-1}) \) so as to:

\[
\begin{aligned}
&\text{maximize } f_1(N) = \psi \\
&\text{minimize } f_2(N) = E \\
\text{s.t.} &
&\sum_{j=1}^{K-1} N_j \leq N_{total}, & j \in 1...K-1, \\
&N_j \in \mathbb{N}^*, & j \in 1...K-1.
\end{aligned}
\]

\( K \) is the number of machines in the production system, \( \psi \) the throughout of the line, and \( E \) the total energy consumption. Moreover, \( N \) is the buffer size vector and \( N_{total} \) the total buffer space available to be allocated among the \( K-1 \) buffer areas. \( N_j, \forall j = \{1...K-1 \} \) are non-negative integers denoting the capacity allocated for each buffer \( B_j \).

To solve this multi-objective problem, we adapt and implement two classical multi-objective methods namely the WSM and ECM, as well as the evolutionary multi-objective algorithm NSGA-II.

As stated in section II-B, the literature relative to the BAP shows a high interest in using the NSGA-II as an evolutionary multi-objective algorithm. Many studies demonstrate its good performance and suitability to the problem. Therefore, we will use this algorithm next to classical methods. The use of classical methods is motivated, from one hand, by the increasing trend for mathematical programming use to solve the BAP [5]. From the other hand, usually, the main criticism addressed to these methods is there computational expensiveness due to multiple executions of the single objective optimization algorithm. However, our mixed integer nonlinear programming (MINLP) method used for solving the single objective EE-BAP in [11] allows solving efficiently...
Algorithm 1: Energy-efficient BAP [11]

Input: $K, K - 1, N_{\text{total}}, \omega_i, \lambda_i, \mu_i, E_{\text{down},i}, E_{\text{no-load},i}, E_{\text{cloud},i}, e_{\text{op},i}$

Output: $(N_1, N_2, ..., N_{K-1}), \psi, E$

for each buffer $B_j$ do

for each machine $M_i$ do

$\text{Max } \left( \frac{\psi}{\frac{E}{E_{\text{max}}}} \right)$

$N_j \in N \quad N_j \neq 0$

$\sum_{j=1}^{K-1} N_j \leq N_{\text{total}}$

$\psi = \min_{i=1..K} \{ \rho_i \}$

$E = \sum_{i=1}^{K} E_i$

$E_i = P_{2,i} E_{\text{down},i} + (P_{3,i} + P_{4,i} + P_{5,i}) E_{\text{no-load},i} + P_{1,i} E_{\text{cloud},i} + e_{\text{op},i} \rho_i$

$\alpha_j = \frac{\min_{i=1..K} \rho_i}{\alpha_j + 1}$

if $\alpha_j = 1$ then

$p_j^0 = \frac{1 - \alpha_j}{N_j + 1}$

else

$p_j^0 = \frac{1 - \alpha_j}{N_j + 1}$

/end if

$P_{2,i} = \frac{\lambda_i}{\mu_i} P_{1,i}$

if $\rho_i = \omega_i \frac{\mu_i (1 - p_j^0 N_i)}{\mu_i + \lambda_i (1 - p_j^0 N_i)}$

$P_{1,i} = \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i}} + \frac{p_j^0}{1 - p_j^0 N_i}$

$P_{4,i} = \frac{\lambda_i}{\lambda_i + \mu_i} P_{1,i} N_i$

else

if $i = K$ then

$\rho_i = \omega_i \frac{\mu_i (1 - p_j^0 N_i)}{\mu_i + \lambda_i (1 - p_j^0 N_i)}$

$P_{1,i} = \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i}} + \frac{p_j^0}{1 - p_j^0 N_i}$

$P_{3,i} = \frac{\mu_i}{\lambda_i + \mu_i} P_{1,i}$

else

$\rho_i = \omega_i \frac{\mu_i (1 - p_j^0 N_i)(1 - p_j^0 N_i)}{\mu_i + \lambda_i (1 - p_j^0 N_i)(1 - p_j^0 N_i)}$

$P_{1,i} = \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} + \frac{p_j^0}{1 - p_j^0 N_i}(1 - p_j^0 N_i) + \frac{p_j^0}{1 - p_j^0 N_i}(1 - p_j^0 N_i) + \frac{p_j^0}{1 - p_j^0 N_i}(1 - p_j^0 N_i)}$

$P_{3,i} = \frac{\mu_i}{\lambda_i + \mu_i} P_{1,i}$

$P_{4,i} = \frac{\mu_i}{\lambda_i + \mu_i} P_{1,i}$

$P_{5,i} = \frac{\mu_i}{\lambda_i + \mu_i} P_{1,i}$

/end if

/end if

/end if

/end if

/end if

/end for

/end for
the problem in much reduced computational times when compared to the literature. Therefore, since an execution of our MINLP using the mathematical solver is not computationally expensive, classical multi-objective methods are interesting to use. The WS and ECM are the most used classical methods in multi-objective optimization [22] able to find Pareto optimal solutions. In fact, our problem is bi-objective. Therefore, a pertinent choice of weights in the WS is simple to obtain. However, the WS being unable to find Pareto-optimal solutions that lie on the non-convex portion of the Pareto-optimal front, the ECM comes to alleviate this issue. For this method, the principal challenge of choosing a relevant epsilon vector is overcome. Again, the problem is bi-objective; therefore, we only have two forms with one constraint and epsilon value to choose for each. Moreover, the feasible space for \( \epsilon_\psi \) and \( \epsilon_E \) is easy to determine by solving the single objective MINLP.

Numerical tests and results are presented and compared in the following sections.

IV. COMPUTATIONAL STUDY

A. DEFINITION OF THE TEST PROTOCOL

For the numerical tests, a largely reported literature benchmark is used [3], [11], [43]–[46]. This benchmark is composed of 10 different instances of non homogeneous lines, with 4, 5, 6, 8, 9, and 10 machines with total buffer capacity varying from 10 to 315. The corresponding parameters of these instances are reported in table 1.

The energy parameters used are described in table 2 under the following assumptions:

- The energy consumption during the idle states (starved, blocked or both) is considered identical to the constant part of energy consumed while operating \( \left( E_{inload,i} = E_{need,i}, \forall i \in \{1...K\} \right) \). This assumption is in fact common in the literature [47], [48].
- The energy consumed in the failure state \( E_{down,i} \), being usually neglected in the literature, is considered in our study equal to 10% of the idle states energy.
- The fraction of the idle states energy and the operating energy is approximately equal to 50% for an isolated system. These energetic parameters are inspired from the literature [49] and considered without any loss of generality.

The multi-objective EE-BAP described in equation 21 is then solved using WSM, ECM, and NSGA-II. The classical methods WSM and ECM rely on the resolution of single objective EE-BAP using the MINLP in [11], whereas the NSGA-II requires the performance evaluation method developed in [40].

It is worth to mention that the buffer allocation problem belongs to the class of NP-hard combinatorial optimization problems [2]. As stated in [50], [51], this problem is difficult due to the lack of an algebraic relation between the throughput of the line (and also the energy consumption) and buffer sizes. Hence, an evolutive algorithm is necessary to calculate the throughput (and energy consumption) of the line, next to an optimization algorithm to search for the best buffer configuration. In addition, the BAP has a stochastic nature due to different processing times and/or random machine failures in the line. Furthermore, the problem is characterized by an inherent combinatorial optimization, as it is well established in the literature that the number of feasible solutions increases with the number of machines as well as with the storage capacity to be allocated. Indeed, for a production line with \( K \) machines and a total buffer capacity \( N_{total} \), a full enumeration solving approach is forced to consider \( K - 2 \) combinations among \( N_{total} + K - 2 \) possible configurations. The total number of possible buffer configurations can be calculated as follows:

\[
N_{solutions} = \binom{N_{total} + K - 2}{K - 2} = \frac{(N_{total} + 1)(N_{total} + 2) \cdots (N_{total} + K - 2)}{(K - 2)!}
\]

As it can be observed above, the total number of feasible solutions increases exponentially when \( N_{total} \) and \( K \) are large. For instance, there are 10626 possible ways to allocate a total storage capacity of 20 units to 5 buffers (a production line with 6 machines). If the production line involves 10 machines with a total buffer capacity of 100, then the total number of feasible buffer allocations becomes \( 3.52 \times 10^{11} \). This indicates the computational difficulty to search through the whole solution space by complete enumeration even for small sized problems. Therefore, finding optimal solutions by exact methods may require exponential computational time.

Using the NSGA-II, the algorithm has a computational complexity of \( O(MN^2) \), with \( M \) representing the number of objectives and \( N \) the size of the population. The fitness function uses the performance evaluation method in [40]. This method reduces drastically the computational complexity for evaluating throughput when compared to other evaluation methods from the literature such as decomposition or aggregation. This is due to the state space cardinality reduction when considering only full and empty buffer spates instead of all buffer states as in the other methods. Considering a production line of \( K \) machines and \( K - 1 \) buffers, the performance evaluation method requires solving a system of only \( 10K - 1 \) equations including \( 9K - 2 \) non-linear equations.

For solving the single objective BAP in WSM and ECM, we use an integrated optimization method formalizing the problem into a mathematical programming model, in which the performance evaluation method [40] is included. The obtained MINLP is solved using a mathematical solver, as in [11] where it demonstrated high accuracy with low computational time.
For WSM, the weights \( \epsilon \) were used a varying number of energy consumption with the throughput as a constraint.

In order to maintain a number of parameters were chosen equally spaced within an interval step size of 200.

The initial population is made to keep the individuals with the best Pareto front. If the two confronted members are of the same front, the decision is made according to the crowding distance criteria.

**Crossover:** simulated binary crossover (SBX) is adopted with the self-adapted procedure developed in [52]. This procedure allows updating the parameter responsible for the spread of offspring solution regarding that of the parent solutions. The crossover is performed with a probability \( p_c = 0.7 \).

**Mutation:** polynomial mutation is performed with a probability \( p_m = 1/n \) (where \( n \) is the number of decision variables).

**Stopping criteria:** A maximum number of iterations \( n_{gen} = 50 \) is fixed.

Tables 2-11 show the Pareto fronts obtained by WSM, ECM, and NSGA-II for the test instances given in table 1.

### C. PERFORMANCE COMPARATIVE STUDY

In the following, results of the implemented methods are compared using performance indicators.

1) **Performance Indicators**

A literature review on performance indicators used in multi-objective optimization can be found in [16]. Multi-objective performance indicators can be classified into: cardinality indicators that quantify the number of non-dominated points generated by an algorithm, convergence indicators that capture the degree of proximity between a Pareto front and its approximation, as well as distribution and spread indicators. In our comparative study we focus on cardinality, distribution, and spread indicators. In the following, a brief presentation of the used metrics.

| Instance | \( K \) | \( \sum_{j=1}^{K} N_j \) | \((\lambda_j, \mu_j, \omega_j)\) |
|----------|-------|----------------|-------------------------------|
| 1        | 4     | 10             | (0.07,0.17,3.7); (0.11,0.37,1.5); (0.49,0.78,1.1); (0.19,0.50,3.0) |
| 2        | 4     | 30             | (0.38,0.45,3.0); (0.30,0.55,1.0); (0.35,0.50,2.0); (0.43,0.40,3.6) |
| 3        | 5     | 10             | (0.10,0.30,1.2); (0.50,0.50,10); (0.50,0.20,3.0); (0.40,0.30,2.0); (0.20,0.10,1.8) |
| 4        | 5     | 15             | (0.30,0.64,2.8); (0.40,0.83,1.7); (0.45,0.75,2.5); (0.35,0.85,3.4); (0.10,0.74,1.9) |
| 5        | 5     | 115            | (0.08,0.40,2.6); (0.24,0.40,3.0); (0.20,0.60,3.4); (0.17,0.50,4.7); (0.10,0.30,1.5) |

| Table 1: Parameters of production lines reported in [3], [11], [43]–[46] |
|-------------------------|------------------|------------------|------------------|------------------|
| Instance | \( K \) | \( \sum_{j=1}^{K} N_j \) | \((\lambda_j, \mu_j, \omega_j)\) |
| 1        | 4     | 10             | (0.07,0.17,3.7); (0.11,0.37,1.5); (0.49,0.78,1.1); (0.19,0.50,3.0) |
| 2        | 4     | 30             | (0.38,0.45,3.0); (0.30,0.55,1.0); (0.35,0.50,2.0); (0.43,0.40,3.6) |
| 3        | 5     | 10             | (0.10,0.30,1.2); (0.50,0.50,10); (0.50,0.20,3.0); (0.40,0.30,2.0); (0.20,0.10,1.8) |
| 4        | 5     | 15             | (0.30,0.64,2.8); (0.40,0.83,1.7); (0.45,0.75,2.5); (0.35,0.85,3.4); (0.10,0.74,1.9) |
| 5        | 5     | 115            | (0.08,0.40,2.6); (0.24,0.40,3.0); (0.20,0.60,3.4); (0.17,0.50,4.7); (0.10,0.30,1.5) |

| Table 2: Energy parameters [11] |
|-------------------------|------------------|------------------|------------------|
| Instance | \( K \) | \( \sum_{j=1}^{K} N_j \) | \((\lambda_j, \mu_j, \omega_j)\) |
| 1        | 4     | 10             | (0.07,0.17,3.7); (0.11,0.37,1.5); (0.49,0.78,1.1); (0.19,0.50,3.0) |
| 2        | 4     | 30             | (0.38,0.45,3.0); (0.30,0.55,1.0); (0.35,0.50,2.0); (0.43,0.40,3.6) |
| 3        | 5     | 10             | (0.10,0.30,1.2); (0.50,0.50,10); (0.50,0.20,3.0); (0.40,0.30,2.0); (0.20,0.10,1.8) |
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### B. TEST RESULTS

For WSM, the weights \( \omega_1 \) and \( \omega_2 \) are varied with a constant step size of 0.05 considering \( \sum_{i=1}^{2} \omega_i = 1 \). Hence, we obtain 20 Mixed Integer Non Linear Programming (MINLP) problems to solve on Lingo solver 18.0. For estimating the Pareto front with ECM, the two versions of the problem are solved: ECM(v1) maximizing the throughput with the energy consumption as a constraint, and ECM(v2) minimizing the energy consumption with the throughput as a constraint. We used a varying number of \( \epsilon_0 \) and \( \epsilon_E \) values. These parameters were chosen equally spaced within an interval formed from the minimum and maximum values of these corresponding single-objective optimization problems. The step is calculated as a function of the feasible domain limits in order to maintain a number of 20 Mixed Integer Non Linear Programming (MINLP) problems to solve for fair comparison of all methods.

The NSGA-II described in section II-A is also implemented on Python to solve the EE-BAP. It is integrated with Lingo solver 18.0 for the evaluation of the objective functions. Several tests have been first performed in order to set efficiently the different parameters of the algorithm. Final values are determined as a compromise between the quality of the final solutions and the convergence time needed for fair comparison of the resolution approaches. The following parameters are considered:

- **Initial population:** the initial population is generated at random according to the number of defined chromosomes \( N_{pop} = 20 \).
- **Population sorting:** this sorting follows the method used by [19] using also the crowding distance.
- **Selection:** the selection is made by the tournament method, where the confrontation of two individuals of the initial population is made to keep the individuals with the best Pareto front. If the two confronted members are of the same front, the decision is made according to the crowding distance criteria.

**Crossover:** simulated binary crossover (SBX) is adopted with the self-adapted procedure developed in [52]. This procedure allows updating the parameter responsible for the spread of offspring solution regarding that of the parent solutions. The crossover is performed with a probability \( p_c = 0.7 \).

**Mutation:** polynomial mutation is performed with a probability \( p_m = 1/n \) (where \( n \) is the number of decision variables).

**Stopping criteria:** A maximum number of iterations \( n_{gen} = 50 \) is fixed.

Figures 2-11 show the Pareto fronts obtained by WSM, ECM, and NSGA-II for the test instances given in table 1.

### C. PERFORMANCE COMPARATIVE STUDY

In the following, results of the implemented methods are compared using performance indicators.

1) **Performance Indicators**

A literature review on performance indicators used in multi-objective optimization can be found in [16]. Multi-objective performance indicators can be classified into: cardinality indicators that quantify the number of non-dominated points generated by an algorithm, convergence indicators that capture the degree of proximity between a Pareto front and its approximation, as well as distribution and spread indicators. In our comparative study we focus on cardinality, distribution, and spread indicators. In the following, a brief presentation of the used metrics.
C-metric [54]

We consider two Pareto Front approximations $Y_N^1$ and $Y_N^2$. This metric captures the proportion of points in a Pareto front approximation $Y_N^2$ weakly dominated by the Pareto front approximation $Y_N^1$.

Spacing (SP) [55]

The SP indicator captures the variation of the distance between elements of a Pareto front approximation (equation 23).

$$SP(Y_N) = \sqrt{\frac{1}{|Y_N| - 1} \sum_{j=1}^{|Y_N|} (d - d^1(y^j, Y_N\backslash\{y^j\}))^2} \quad (23)$$

where $d^1(y^j, Y_N\backslash\{y^j\}) = \min_{y \in Y_N\backslash\{y^j\}} ||y - y^j||_1$ is the $l_1$ distance of $y_j \in Y_N$ to the set $Y_N\backslash\{y^j\}$ and $d$ is the mean of all $d^1(y^j, Y_N\backslash\{y^j\})$ for $j = 1...|Y_N|$.

Since this indicator cannot account for holes in the Pareto front approximation, we complete the study with the Hole relative size (HRS) indicator.
Hole relative size (HRS) [56]

This indicator identifies the largest hole in a Pareto front approximation for a bi-objective problem (equation 24).

\[
HRS(Y_N) = \frac{1}{\bar{d}} \max_{j=1,\ldots,|Y_N|} d_j \tag{24}
\]

where \(Y_N\) is a Pareto front approximation. The elements of \(Y_N\) have to be sorted in ascendant order according to the first objective, \(d_j = ||y^j - y^{j+1}||_2\) is the \(L_2\) distance between the two adjacent objective vectors \(y_j \in Y_N\) and \(y_{j+1} \in Y_N\), and \(\bar{d}\) the mean of all \(d_j\) for \(j = 1,\ldots,|Y_N| - 1\).

Hyper-volume metric (HV)

The hyper-volume indicator is described as the volume of the space in the objective space dominated by the Pareto front approximation \(Y_N\) and delimited from above by a reference objective vector \(r \in \mathbb{R}^m\) such that for all \(y \in Y_N\), \(y \leq r\). A higher value of the HV metric is desirable.

2) Analysis

Using the performance indicators, we conduct a comparative study on Pareto fronts generated by WSM, ECM and NSGA-II. Table 3 gives the ONVG, SP, HRS, and HV metrics of...
the generated Pareto fronts for each instance as well as the average values and standard deviation. Computational time is also given in table 3. Table 4 gives c-metric results and table 5 a summary of worst, average, and best c-metric values for each compared couple combination.

In addition, figures 12 -14 present a comparison between the used methods using boxplots of SP, HRS, and HV metrics respectively, while figure 15 illustrates the average coverage performance of the methods.

For cardinality, although the ONVG is not a pertinent measure on its own, it can be noticed from figures 2-11 and table 3 that ECM and NSGA-II generate more solutions in the Pareto front approximation. In contrast, the Pareto fronts generated by WSM appear to have only few solutions.

As of the distribution and spread, table 3 and figures 12-13 show that, in average, ECM outperforms WSM and NSGA-II in SP and HRS metrics. The NSGA-II obtains good values for these indicators as well. At the opposite, SP and HRS values for WSM are relatively high. From figures 2-11, it is easy to notice the existence of noticeable holes and the non-uniformity of the solutions spread for WSM Pareto fronts. As of the HV metric, all methods obtain close average values, with ECM outperforming here as well with the higher values (see table 3 and figure 14).

For the sets coverage comparison, using tables 4-5 and figure 15, we can see that ECM outperforms WSM in average (10% to 17% of ECM solutions are dominated by WSM,
TABLE 3: Performance metrics results

| Instance | K  | N_{total} | ONVG | SP | HRS | HV | CPU(s) | ONVG | SP | HRS | HV | CPU(s) |
|----------|----|-----------|------|----|-----|----|--------|------|----|-----|----|--------|
| 1        | 4  | 10        | 5    | 1.020 | 2.827 | 0.201 | 7 | 13 | 0.515 | 2.692 | 0.212 | 14 |
| 2        | 4  | 30        | 5    | 2.982 | 3.837 | 0.189 | 7 | 15 | 0.244 | 2.514 | 0.214 | 14 |
| 3        | 5  | 10        | 4    | 1.102 | 2.033 | 0.168 | 36 | 14 | 0.149 | 2.588 | 0.170 | 984 |
| 4        | 5  | 15        | 5    | 0.993 | 1.748 | 0.223 | 57 | 16 | 0.560 | 2.302 | 0.230 | 847 |
| 5        | 5  | 115       | 6    | 3.651 | 3.129 | 0.305 | 36 | 11 | 0.659 | 2.065 | 0.317 | 1101 |
| 6        | 6  | 130       | 17   | 0.347 | 6.898 | 0.203 | 92 | 17 | 0.080 | 2.667 | 0.207 | 134 |
| 7        | 8  | 125       | 11   | 1.964 | 4.326 | 0.270 | 447 | 12 | 0.441 | 2.202 | 0.276 | 1140 |
| 8        | 9  | 200       | 11   | 6.178 | 3.956 | 0.279 | 1447 | 15 | 0.249 | 1.250 | 0.294 | 6129 |
| 9        | 10 | 310       | 14   | 3.177 | 7.616 | 0.232 | 3204 | 16 | 1.138 | 1.206 | 0.258 | 5720 |
| 10       | 10 | 315       | 9    | 1.197 | 5.409 | 0.218 | 1071 | 10 | 0.132 | 1.337 | 0.223 | 3542 |

Avg. - - - 2.261 4.178 0.229 - - 0.417 2.082 0.240 -

Std. - - - 1.674 1.854 0.041 - - 0.306 0.569 0.042 -

| Instance | K  | N_{total} | ONVG | SP | HRS | HV | CPU(s) | ONVG | SP | HRS | HV | CPU(s) |
|----------|----|-----------|------|----|-----|----|--------|------|----|-----|----|--------|
| 1        | 4  | 10        | 16   | 0.610 | 3.933 | 0.211 | 8 | 20 | 0.493 | 4.121 | 0.212 | 55 |
| 2        | 4  | 30        | 18   | 0.267 | 3.461 | 0.212 | 14 | 20 | 0.282 | 3.226 | 0.214 | 623 |
| 3        | 5  | 10        | 12   | 0.242 | 2.833 | 0.169 | 35 | 20 | 0.157 | 2.602 | 0.174 | 1163 |
| 4        | 5  | 15        | 15   | 0.582 | 2.550 | 0.228 | 70 | 20 | 0.525 | 2.876 | 0.230 | 873 |
| 5        | 5  | 115       | 16   | 0.406 | 2.372 | 0.317 | 30 | 20 | 0.661 | 2.568 | 0.318 | 850 |
| 6        | 6  | 130       | 16   | 0.133 | 2.769 | 0.202 | 83 | 20 | 0.149 | 1.671 | 0.209 | 977 |
| 7        | 8  | 125       | 16   | 0.395 | 2.153 | 0.276 | 349 | 20 | 0.502 | 3.574 | 0.224 | 769 |
| 8        | 9  | 200       | 17   | 1.059 | 3.031 | 0.285 | 1109 | 20 | 1.691 | 2.558 | 0.253 | 3321 |
| 9        | 10 | 310       | 19   | 1.073 | 1.981 | 0.254 | 1236 | 20 | 1.261 | 3.022 | 0.225 | 2271 |
| 10       | 10 | 315       | 14   | 0.381 | 3.443 | 0.216 | 1814 | 20 | 1.056 | 3.644 | 0.191 | 4381 |

Avg. - - - 0.515 2.853 0.237 - - 0.756 3.095 0.225 -

Std. - - - 0.308 0.591 0.042 - - 0.613 0.661 0.037 -

FIGURE 14: The boxplots of HV metric

FIGURE 15: Average coverage c-metric

against 27% to 33% of WSM solutions dominated by ECM). The NSGA-II also covers up to 22% of solutions generated by the classical methods. However, the Pareto fronts generated with the NSGA-II are always composed of solutions...
TABLE 4: C-metric results

| Instance | WSM  | ECM(v1) | ECM(v2) | NSGA-II |
|----------|------|---------|---------|---------|
| Instance 1 | 0.00 | 0.31   | 0.19    | 0.20    |
| Instance 2 | 0.00 | 0.27   | 0.06    | 0.10    |
| Instance 3 | 0.00 | 0.14   | 0.00    | 0.00    |
| Instance 4 | 0.00 | 0.25   | 0.13    | 0.20    |
| Instance 5 | 0.00 | 0.09   | 0.19    | 0.05    |

TABLE 5: C-metric results summary

| C-metric | WSM | ECM(v1) | ECM(v2) | NSGA-II |
|----------|-----|---------|---------|---------|
| Best     | -   | -       | -       | -       |
| Avg.     | 0.308 | 0.171 | 0.063 | 0.188 |
| Worst    | 0.100 | 0.000 | 0.000 | 1.000 |
| Avg.     | 0.393 | 0.000 | 0.000 | 0.000 |

weakly dominated by the classical methods. In average, 40% up to 60% of the solutions obtained by NSGA-II are dominated by at least one solution of the classical methods. Additionally, all Pareto front solutions generated by the NSGA-II are dominated by those obtained with WSM and ECM(v1) for instance 7 and instance 10. In fact, the NSGA-II performs well for small sized instances. However, when the problem complexity increases, it requires greater computational time to generate a Pareto front at least as good as the classical methods. This can be noticed from instance 7 to instance 10. In addition to the quality of the solution, the computational time of the NSGA-II is important when compared to the classical methods. This is mainly due to the efficiency of the non-linear programming method used to solve the single objective EE-BAP. Indeed, solving the single objective EE-BAP for each execution (20 predefined executions) of the classical methods takes at most few minutes each, whereas the NSGA-II requires various evaluations (population size × number of generations) using the mathematical solver which results in greater computational time. Although the evaluation method used demonstrates extensively reduced calculation time when compared to other popular methods such as decomposition or aggregation, running several execution of the evaluation function results in important computational time.

V. CONCLUSION

This paper presented a study on the multi-objective EE-BAP. The goal was to find the optimal allocation of storage capacities in order to simultaneously optimize two conflicting objectives: the throughput of the production line and its energy consumption, under a total buffer space constraint. Two classical methods, namely WSM and ECM, as well as the well known evolutionary algorithm NSGA-II, were adapted and implemented to solve the problem. Performance indicators were used to compare and evaluate the performance of the adopted methods.

The computational results show the advantage of ECM over WSM based on the adopted indicators. Indeed, ECM generates rich Pareto fronts with good spread, distribution, and coverage. On another hand, NSGA-II requires large computational time to generate solutions at least as good as the
classical methods. Although it performs well when it comes to cardinality, spread, and distribution, coverage performance is weak when considering comparable computational time next to the classical methods. A considerable proportion of solutions in Pareto fronts generated by the NSGA-II are dominated by the solutions from the classical methods, particularly for large sized instances. This is mainly due to the necessity for the NSGA-II to perform a considerable amount of evaluations using the mathematical solver, whereas the classical methods only need to run a specific number of executions. Usually, solving a single run of the BAP requires high computational time. However, the adopted non-linear programming method for solving the single objective EE-BAP reduces drastically the computational time. As a result, running the classical methods requires a reasonable time, which is not sufficient for the NSGA-II to generate an efficient Pareto front.

In conclusion, the Pareto solutions obtained are trade-offs between the two objectives, enabling decision making that balances productivity maximization with energy economics in the design of production lines. Moreover, ECM is found to be a very performing method to solve the multi-objective EE-BAP. By virtue of the non-linear programming method used to solve the EE-BAP, running the classical methods requires extensively reduced calculation time.

As a future perspective, we suggest investigating the potential of hybrid approaches in multi-objective optimization combining classical methods for their convergence properties, and evolutionary algorithms for their search properties that enable finding good spread solutions. In addition, the generalization of the approach to other production systems configurations could provide interesting insights. Further multi-objective resolution methods and evaluation approaches could be used to solve the EE-BAP as well.

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