Velocity profiles at non-stagnation point of viscous fluid passes a sliced sphere contains magnet

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Abstract. This paper will be presented to explain the velocity profile of viscous fluids at non-stagnation points when the fluid passes through a sliced sphere contains magnet. The design of this study was an experiment using computer simulations based on mathematical models built. Mathematical models and coordinate systems are built based on physical phenomena and are composed of continuity equations and momentum equations. The model was converted to a non-dimensional model and solved by the Keller Box numerical method. Numerical simulations are carried out by varying the incision angle and looking at the fluid velocity profile that occurs. The simulation results show that if magnetic parameter is not included then the velocity will be different at each point, the greater the observation angle the greater the velocity. If magnetic parameter is included then the velocity will tend to be the same. If an observation angle hold on any value then the greater the magnetic parameter the greater the velocity.

1. Introduction

Research in the field of hydrodynamics has often been done. Several studies have been conducted relating to hydrodynamics is hydrodynamics on spheres, stretching cylinders, circular cylinders, porous spheres, and sliced spheres [1-5]. These studies generally discuss the velocity profile around geometric objects. This velocity profile is useful for determining the effect of friction on objects [6].

Research has discussed the velocity profile at stagnation point. It concluded that at stagnation point, the greater the magnetic parameters the steeper the velocity profile [5]. This research still leaves a problem, how is the velocity profile at non-stagnation point. Future studies must answer this question.

This research will discuss hydrodynamics in the sliced sphere with a focus of research at the non-stagnation points. There are two areas that are non-stagnation points on the sliced sphere, namely the surface area of the sliced flat surface and the remaining surface of the sphere. This research will discuss the first non-stagnation points, namely the points that are in the flat surface area of the sliced result.

To solve the problem of the velocity profile at the non-stagnation point, a spherical coordinate system was modified with a slice. The coordinate system for this problem is shown in Figure 1. According to coordinate system of sliced sphere on Figure 1, mathematical models containing of continuity equations and momentum equations are presented below [1].

\[
\frac{\partial \tau \bar{u}}{\partial \bar{x}} + \frac{\partial \tau \bar{v}}{\partial \bar{y}} = 0
\]

\[
\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial p}{\partial \bar{x}} + \frac{\mu}{\rho} \left( \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) - \frac{\sigma}{\rho} B_0^2 \bar{u}
\]
\[
\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} \right) - \frac{\sigma}{\rho} B_0^2 \bar{v}
\]

\( t < 0 : \bar{u} = \bar{v} = 0, \text{ for any } \bar{x}, \bar{y}, \)

\( t \geq 0 : \bar{u} = \bar{v} = 0, \text{ at } \bar{y} = 0, \)

\( \bar{u} = \bar{u}_e(x), \text{ at } \bar{y} \to \infty. \)

Figure 1. Coordinate system of sliced sphere.

According to [6] we have \( \bar{u}_e = \frac{3}{2} U_\infty \sin \left( \frac{\bar{x}}{a} \right) \) and \( b = \frac{a \sin \theta_s}{\sin \alpha}. \) For calculation reasons, this model is changed to the non-dimensional form by using these equations.

\[
x = \bar{x}, \quad y = R_e \left( \frac{\bar{y}}{a} \right), \quad r(x) = \frac{\bar{r}(\bar{x})}{a}, \quad u = \frac{\bar{u}}{U_\infty}, \quad M = \frac{\sigma a B_0^2}{\rho U_\infty},
\]

\[
v = R_e \left( \frac{\bar{v}}{U_\infty} \right), \quad u_e = \frac{\bar{u}_e(x)}{U_\infty}, \quad p = \frac{\bar{p}}{\rho U_\infty^2}, \quad t = \frac{U_\infty \bar{t}}{a}, \quad b = \frac{\bar{b}}{a}.
\]

Substitution of these equations into mathematical models yields

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial x^2} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - Mu
\]

\[
\frac{1}{Re} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{1}{Re} \frac{\partial^2 v}{\partial x^2} + \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} - M v
\]

\( t < 0 : u = v = 0, \text{ for any } x, y, \)

\( t \geq 0 : u = v = 0, \text{ at } y = 0, \)

\( u = u_e(x), \text{ at } y \to \infty. \)

\( u_e = \frac{3}{2} \sin \left( \frac{\bar{x}}{\sin \theta_s} \right). \)

Velocity component in \( x \) and \( y \) direction can be expressed by using stream function. To expressed it, we use relations

\[
u = -\frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial \phi}.
\]

It is written as follows

\[
\frac{1}{r} \frac{\partial^2 \psi}{\partial \theta \partial t} + \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} \frac{\partial^2 \psi}{\partial \phi^2} - \frac{1}{r^3} \frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial \phi} \left( \frac{\partial \psi}{\partial \phi} \right)^2 - \frac{1}{r^3} \frac{\partial \psi}{\partial \phi} \frac{\partial^2 \psi}{\partial \theta^2} = u_e \frac{\partial u_e}{\partial x} + \frac{1}{r^3} \frac{\partial^3 \psi}{\partial \theta^3} + M \left( u_e - \frac{1}{r} \frac{\partial \psi}{\partial \theta} \right).
\]
\[ t < 0 : \psi = \frac{\partial \psi}{\partial y} = 0, \text{for any} \ x, y, \]
\[ t \geq 0 : \psi = \frac{\partial \psi}{\partial y} = 0, \text{at} \ y = 0, \]
\[ \frac{\partial \psi}{\partial y} = u_e(x)r(x), \text{at} \ y \to \infty. \]
\[ u_e = \frac{3}{2} \sin \left(\frac{x \sin x}{\sin \theta_s}\right). \]

In unsteady condition, we need to distinguish between small time \((t \leq t^*)\) and large time \((t > t^*)\). In order to obtain solution for small time and large time, we use similarity variable. It is presented below

\[ \psi = t^\frac{1}{2} u_e(x) r(x) f(x, \eta, t), \ \eta = y/t^{1/2} \text{for small time} \]
\[ \psi = u_e(x) r(x) f(x, Y, t), \ Y = y \text{for large time}. \]

By using it, we get an equation for small time as follows

\[ \frac{\partial^3 f}{\partial \eta^3} + \eta \frac{\partial^2 f}{\partial \eta^2} + \lambda t \left[ 1 - \left(\frac{\partial f}{\partial \eta}\right)^2 + f \frac{\partial^2 f}{\partial \eta^2} \right] + M t \left( 1 - \frac{\partial f}{\partial \eta} \right) = \frac{\partial^2 f}{\partial \eta \partial t} + tu_e \left[ \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2} \right] \]

\[ t < 0 : f = f' = 0, \text{for any} \ x, \eta, \]
\[ t \geq 0 : f = f' = 0, \text{at} \ \eta = 0, \]
\[ f' = 1, \text{at} \ \eta \to \infty, \]
\[ u_e = \frac{3}{2} \sin \left(\frac{x \sin x}{\sin \theta_s}\right). \]

We also get an equation for large time as follows

\[ \frac{\partial^3 F}{\partial Y^3} + \lambda \left[ 1 - \left(\frac{\partial F}{\partial Y}\right)^2 + F \frac{\partial^2 F}{\partial Y^2} \right] + M \left( 1 - \frac{\partial F}{\partial Y} \right) = \frac{\partial^2 F}{\partial Y \partial t} + tu_e \left[ \frac{\partial F}{\partial Y} \frac{\partial^2 F}{\partial Y \partial x} - \frac{\partial F}{\partial x} \frac{\partial^2 F}{\partial Y^2} \right] \]
\[ F = F' = 0, \text{at} \ Y = 0 \]
\[ F' = 1, \text{at} \ Y \to \infty. \]

If focus on stagnation point, we can cancel the \( tu_e \left[ \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2} \right] \) form [5]. In this case, we cannot cancel because \( u_e \) not equal to 0. Hence, we have the last equations. We will solve it numerically. The method used is the Keller box method as the other studies [8].

2. Method

This research uses an experimental design. Experiments carried out by computer simulations based on mathematical models. Simulation done by giving sliced angle variation, observation angle variation, and magnetic parameter variation. Simulation results are recorded to get the relationship between the observation angle, slicing angle, and magnetic parameters. Conclusions are built based on patterns formed from simulation results.

3. Result and Discussion

3.1. Velocity profile where magnetic parameter is not included

The velocity profile at the non-stagnation point which is on the flat surface of the sphere sliced can be described by giving variations of the observation angle and variations in the magnetic parameters. Velocity profile with variations of the observation angle can be seen in the following image.
Figure 2 shows that the flow velocity at a point near the stagnation point is lower than the point far from the stagnation point. We can conclude this based on the fact that the velocity profile at the nearest point $12^\circ$ is under the velocity profile at $24^\circ$. The velocity profile was also examined at the point of non-stagnation and there was a boundary layer separation [1]. However, this study found there was no boundary layer separation [10]. Based on Figure 2, at the stagnation point, velocity profile is steep [5]. This confirms that the mathematical model is suitable.

3.2. Velocity profile where magnetic parameter included
To show what happens if we have large magnetic parameter then the model is simulated by giving variations of the observation angle. Furthermore velocity profile is displayed in the following image.

Figure 3 shows that at magnetic parameter $M = 20$, velocity profile is looked equal in various observation angles ($0^\circ$, $12^\circ$, $24^\circ$, $36^\circ$, and $48^\circ$). The fluid velocity curves appear to overlap, showing the
fluid velocity at these points is relatively the same. In other words, there is no significant difference between fluid velocity at the nearest point and the farthest point. This result is in accordance with the study [5], on large magnetic parameters, the observation angle does not have a significant effect on the velocity profile.

3.3. Velocity profile with various magnetic parameter
To determine the effect of magnetic parameters, the model is simulated with variations in magnetic parameters. Velocity profiles with variations in magnetic parameters can be explained by the following picture

![Velocity profile](image)

Figure 4. Velocity profile at slicing angle $\theta_s = 60^\circ$ and observation angle $x = 30^\circ$.

Figure 4 shows that there are some differences velocity profile in various magnetic parameters (0, 1, 5, 10, and 15). Velocity profile where magnetic parameter $M = 15$ steeper than that magnetic parameter $M = 1$. We can conclude that the greater the magnetic parameter the greater the velocity profile. This result is in accordance with research [1], the greater the magnetic parameters, the steeper the velocity profile.

4. Conclusion
Velocity fluid will be different at each point where magnetic parameter is not included. The greater the observation angle the greater the velocity. Velocity will tend to be the same where magnetic parameter is included. If an observation angle hold on any value then the greater the magnetic parameter the greater the velocity. It also can be concluded that on the surface of the flat plane there is no boundary layer separation.

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