Nonabelian Topological Mass Generation in 4 Dimensions

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Abstract

We study the topological mass generation in the 4 dimensional nonabelian gauge theory, which is the extension of the Allen et al.’s work in the abelian theory. It is crucial to introduce a one form auxiliary field in constructing the gauge invariant nonabelian action which contains both the one form vector gauge field $A$ and the two form antisymmetric tensor field $B$. As in the abelian case, the topological coupling $mB \wedge F$, where $F$ is the field strength of $A$, makes the transmutation among $A$ and $B$ possible, and consequently we see that the gauge field becomes massive. We find the BRST/anti-BRST transformation rule using the horizontality condition, and construct a BRST/anti-BRST invariant quantum action.

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I. Introduction

Allen, Bowick and Lahiri [1] found an interesting mechanism which they called topological mass generation in 4 dimensions. They studied the abelian gauge theory which contains the vector field $A$ and the second rank antisymmetric tensor field $B$, and incorporated the topological term $B \wedge F$ in the action as

$$S = \frac{1}{2} \int_{M_4} (H \wedge *H - F \wedge *F + mB \wedge F),$$  \hspace{1cm} (1)

where $H = dB$, $F = dA$, and $*$ is the Hodge star (duality) operator. The action (1) is invariant under the gauge transformation

$$A \to A + d\alpha, \quad B \to B + d\beta,$$  \hspace{1cm} (2)

and gives the equations of motion

$$d * H = mF, \quad d * F = mH.$$  \hspace{1cm} (3)

The coupled equations in (3), which can be considered as a generalization of the London equations, give rise to the massive Klein-Gordon equations

$$(\Box + m^2) F = 0, \quad (\Box + m^2) H = 0,$$  \hspace{1cm} (4)

which show that the fluctuations of $F$ and $H$ are massive.

The above mechanism of Allen et al. worked nicely in the abelian theory, and provided a new method of mass generation for the gauge field while preserving the gauge symmetry. So it is very tempting to extend this mechanism to the nonabelian theory. At first glance, it seems that we can do this extension by simply putting the trace operation in front of the action (1), and by covariantizing the gauge transformation (2). However, there is a difficulty in this naive approach. The $H \wedge *H$ term in the action, which is the kinetic term for the antisymmetric
tensor field, is not invariant under the gauge transformation. This fact causes a
difficulty already at the classical level, and is related to the geometrical aspect
that is intrinsic to the second rank antisymmetric tensor field (nonabelian Kalb-
Ramond field) [4]. This problem has been studied by many, and first solved by
Thierry-Mieg and Baulieu [3]. The BRST quantization with the inclusion of the
one form gauge field $A$ was also studied in Refs. [4] and [5]. These studies show
that in the theory with the nonabelian antisymmetric tensor field it is necessary to
introduce a one form auxiliary field in constructing a gauge invariant kinetic term
and a BRST invariant quantum action. The purpose of this paper is to apply the
above result to the nonabelian generalization of the topological mass generation
mechanism of Allen et al.

In section II, we obtain the BRST and anti-BRST transformation rule by ap-
plying the so-called horizontality condition. We also explain why and how the one
form auxiliary field is necessitated and solves the previously mentioned difficulty
[3]. In section III, we explain how the topological mass generation mechanism of
Allen et al. occurs in the nonabelian case. In section IV, we construct the qua-
tum action based on the BRST and anti-BRST symmetry of section II. Section V
constitutes the conclusion.

II. BRST and anti-BRST symmetry

One might expect that the nonabelian generalization of the action (1) could be
achieved by the following action $\tilde{S}$ which is obtained from (1) by simply replacing
$F = dA$ and $H = dB$ with $F = dA + AA$ and $H \equiv DB = dB + [A, B]$:

$$\tilde{S} = \frac{1}{2} \int_{M_4} \text{Tr}(H \ast H - F \ast F + mBF),$$

(5)
with the wedge($\wedge$) product between forms to be understood hereafter. We then should check whether the action $\bar{S}$ in (3) is invariant under the gauge transformation:

$$
\begin{align*}
\delta A &= D\epsilon_0 = d\epsilon_0 + [A, \epsilon_0], \\
\delta B &= -[\epsilon_0, B] + D\epsilon_1.
\end{align*}
$$

(6)

Now, the action $\bar{S}$ is invariant only under the part of the transformation related with $\epsilon_0$. It is not invariant under the full transformation (3): the first term of the action $\bar{S}$, which is the kinetic term of the two form field $B$, is not invariant. This is because in contrast to the usual two form curvature which transforms as $\delta F = -[\epsilon_0, F]$, the corresponding curvature for the $B$ field, $H = DB$, does not transform as $\delta H = -[\epsilon_0, H]$ under the full transformation (3). Therefore we need some additional ingredient in order to have an invariant action for $B$. We may understand this situation from the existence of a constraint in the nonabelian theory which is induced from the equation of motion given by the action (3), that is, the constraint $DD^*H = [F, *H] = 0$ from the equation of motion $D^*H + mF = 0$. In order to untie this constraint Thierry-Mieg and Baulieu [3], and Thierry-Mieg and Ne’eman [4] introduced the 1-form auxiliary field $K$. This was done in the following manner: In order to implement the constraint, a Lagrange multiplier term should be added into the Lagrangian. However, simply adding a term like $K[F, *H]$ does not do the job because of a newly produced constraint. The correct expression for the kinetic term in the Lagrangian which does not produce further constraints is obtained by replacing $H = DB$ in (3) with a new $H'$ [4],

$$
H' \equiv DB - DDK.
$$

(7)

With the introduction of the one form auxiliary field $K$ and its accompanying ghost (classically a gauge parameter), the $H'$ in (4) transforms as $\delta H' = -[\epsilon_0, H']$
under the full transformation (8), as we shall see below. Thus we successfully get
the gauge invariant kinetic term for the $B$ field. On the other hand, the topological
term $mBF$ is gauge invariant by itself. Thierry-Mieg and Baulieu [3] studied the
quantization of antisymmetric tensor gauge theory whose action mainly consisted
of this topological term (modulo a term of auxiliary field), and found that it was
necessary to introduce a new auxiliary field at the quantum level which represents
an extra symmetry of the action. This auxiliary field is again related with the
constraint from the equation of motion in the nonabelian gauge theory, and in fact
it is rooted in the gauge symmetry of the previous classical auxiliary field $K$, and
is the ghost for the $K$ field. We shall see below how this prescription works nicely.

Now, we get into the BRST/anti-BRST transformation rule. Here we use the
so-called horizontality condition to get the BRST/anti-BRST transformation rule
[5, 7, 8, 9, 10, 11]. First we illustrate this in the usual Yang-Mills case. The
horizontality condition is in essence the Maurer-Cartan equation in the direction
of the gauge group of the principal fiber bundle with a doubled structure-group
$\mathcal{G} \otimes \mathcal{G}$,

$$\bar{F} \equiv \bar{d} \bar{A} + \bar{A} \bar{A} = F,$$

where

$$\bar{A} = A_\mu dx^\mu + A_N dy^N + A_{\bar{N}} d\bar{y}^{\bar{N}} \equiv A + \alpha + \bar{\alpha},$$

$$\bar{d} = d + s + \bar{s}, \quad d = dx^\mu \partial_\mu, \quad s = dy^N \partial_N, \quad \bar{s} = d\bar{y}^{\bar{N}} \partial_{\bar{N}},$$

$$F = dA + AA = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu.$$

Here $y$ and $\bar{y}$ denote the coordinates in the direction of gauge group of the principal
fiber bundle. Now (8) yields the BRST/anti-BRST transformation rule for the
Yang-Mills case:

$$(dx)^1 (dy)^1 : sA_\mu = D_\mu \alpha,$$
\[ (dx)^1(dy)^1 : \bar{s}A_\mu = D_\mu \bar{\alpha}, \]
\[ (dy)^2 : s\alpha = -\alpha \bar{\alpha}, \]  
\[ (dy)^2 : \bar{s}\bar{\alpha} = -\bar{\alpha} \bar{\alpha}, \]
\[ (dy)^1(d\bar{y})^1 : s\bar{\alpha} + \bar{s}\bar{\alpha} = -[\alpha, \bar{\alpha}]. \]  

However, in order to fix the transformation rule completely, we need to introduce an auxiliary field for the last equation of (9):
\[ s\bar{\alpha} \equiv t ; \quad \bar{s}\alpha = -t - [\alpha, \bar{\alpha}], \quad st = 0, \quad \bar{s}t = -[\bar{\alpha}, t]. \]  

This completes the BRST/anti-BRST transformation rule for the Yang-Mills case.

Note that here we used the graded commutator, that is, for instance \([\alpha, \bar{\alpha}] = \alpha \bar{\alpha} + \bar{\alpha} \alpha\) because of the anticommuting character of \(\alpha\)’s. The graded commutator is used throughout this paper.

For the two form antisymmetric tensor field \(B\), we first consider the horizontality condition for the modified field strength \(H'\) in (7):
\[ \widetilde{H}' \equiv \widetilde{D} \widetilde{B} - \widetilde{D} \widetilde{D} \widetilde{K} = DB - DDK \equiv H', \]  

where
\[ \widetilde{D} \equiv \widetilde{d} + [\widetilde{A}, \ ], \]
\[ \widetilde{B} \equiv \frac{1}{2} B_{\mu\nu} dx^\mu dx^\nu + B_{\mu N} dx^\mu dy^N + B_{\mu \bar{N}} dx^\mu d\bar{y}^{\bar{N}} \]
\[ + \frac{1}{2} B_{MN} dy^M dy^N + B_{M \bar{N}} dy^M d\bar{y}^{\bar{N}} + \frac{1}{2} B_{\bar{M} \bar{N}} d\bar{y}^{\bar{M}} d\bar{y}^{\bar{N}} \]
\[ \equiv B - \beta - \bar{\beta} + \phi + \rho + \bar{\phi}, \]
\[ \widetilde{K} \equiv K_\mu dx^\mu + K_N dy^N + K_{\bar{N}} d\bar{y}^{\bar{N}} \equiv K + \kappa + \bar{\kappa}, \]
\[ H \equiv DB \equiv \frac{1}{6} H_{\mu\nu\rho} dx^\mu dx^\nu dx^\rho, \]
\[ H' \equiv H - DDK \equiv \frac{1}{6} H'_{\mu\nu\rho} dx^\mu dx^\nu dx^\rho. \]
This horizontality condition for the $H'$ now yields the BRST/anti-BRST transformation rule for the $B$ and its ghosts:

\[
(dx)^2(dy)^1 : sB_{\mu \nu} = -[\alpha, B_{\mu \nu}] - D_{[\mu} \beta_{\nu]} - [\kappa, F_{\mu \nu}] ,
\]

\[
(dx)^2(d\bar{y})^1 : sB_{\mu \nu} = -[\bar{\alpha}, B_{\mu \nu}] - D_{[\mu} \bar{\beta}_{\nu]} - [\bar{\kappa}, F_{\mu \nu}] ,
\]

\[
(dx)^1(dy)^2 : s\beta_{\mu} = -[\alpha, \beta_{\mu}] + D_{\mu} \phi ,
\]

\[
(dx)^1(d\bar{y})^2 : s\bar{\beta}_{\mu} = -[\bar{\alpha}, \bar{\beta}_{\mu}] + D_{\mu} \bar{\phi} ,
\]

\[
(dx)^1(dy)^1 : s\bar{\beta}_{\mu} + \bar{s} \beta_{\mu} = -[\alpha, \bar{\beta}_{\mu}] - [\bar{\alpha}, \beta_{\mu}] + D_{\mu} \rho ,
\]

\[
(dy)^1 : s\phi = -[\alpha, \phi] ,
\]

\[
(d\bar{y})^1 : \bar{s} \phi = -[\bar{\alpha}, \bar{\phi}] ,
\]

\[
(dy)^2(d\bar{y})^1 : \bar{s} \phi + s \rho = -[\alpha, \rho] - [\bar{\alpha}, \phi] ,
\]

\[
(dy)^1(d\bar{y})^2 : s\bar{\phi} + \bar{s} \rho = -[\alpha, \bar{\phi}] - [\bar{\alpha}, \rho] ,
\]

where $D_{[\mu} \beta_{\nu]} \equiv D_{\mu} \beta_{\nu} - D_{\nu} \beta_{\mu}$, and $D_{\mu} \equiv \partial_{\mu} + [A_{\mu}, ]$. The horizontality condition (11) for the $B$ field implies

\[
\bar{B} - \bar{D} \bar{K} = B - DK .
\]

This we can see by operating $\bar{D}$ on the left hand side of (11):

\[
\bar{D} \bar{D} (\bar{B} - \bar{D} \bar{K}) = [\bar{F}, \bar{B} - \bar{D} \bar{K}] .
\]

Due to the previous horizontality condition (8), $\bar{F} = F$, one can write

\[
[\bar{F}, \bar{B} - \bar{D} \bar{K}] = [F, \bar{B} - \bar{D} \bar{K}] .
\]

However, the right hand side of the last equation should be purely horizontal, and we get the desired result (13) [3]. The condition (13) now yields the BRST/anti-BRST transformation rule for the $K$ field and its ghosts:

\[
(dx)^1(dy)^1 : sK_{\mu} = -[\alpha, K_{\mu}] + D_{\mu} \kappa - \beta_{\mu} ,
\]
Again, these BRST/anti-BRST equations from the two horizontality conditions, (11) and (13), do not fix the BRST/anti-BRST transformation rule completely, and we need extra auxiliary fields for $\beta$'s, $\phi$'s and $\rho$, and $\kappa$'s.

\begin{align*}
s \bar{\beta}_\mu &\equiv m_\mu; \quad \bar{s}\beta_\mu = -m_\mu - [\alpha, \bar{\beta}_\mu] - [\bar{\alpha}, \beta_\mu] + D_\mu \rho, \\
s \rho &\equiv n; \quad 
 \bar{s}\phi = -n - [\alpha, \rho] - [\bar{\alpha}, \phi], \\
s \bar{\varphi} &\equiv \bar{n}; \quad \bar{s}\rho = -\bar{n} - [\alpha, \bar{\phi}] - [\bar{\alpha}, \rho], \\
s \bar{\kappa} &\equiv u; \quad \bar{s}\kappa = -u - [\alpha, \bar{\kappa}] - [\bar{\alpha}, \kappa] + \rho.
\end{align*}

And the nilpotency of the $s$ and $\bar{s}$ operators fix the remainder.

\begin{align*}
sm_\mu &= sn = s\bar{n} = su = 0, \\
\bar{sm}_\mu &= -[\bar{\alpha}, m_\mu] - [D_\mu \alpha, \bar{\phi}] - D_\mu \bar{n} - [\bar{\beta}_\mu, t], \\
\bar{s}n &= -[\bar{\alpha}, n] - [\alpha \alpha, \bar{\phi}] - [\alpha, \bar{n}] - [\rho, t], \\
\bar{s}\bar{n} &= -[\bar{\alpha}, \bar{n}] - [\bar{\phi}, t], \\
\bar{s}u &= -[\bar{\alpha}, u] - [\bar{\kappa}, t] - \bar{n}.
\end{align*}

(12) and (14)-(16) constitute a complete set of the BRST/anti-BRST transformation rule. One can check that the above BRST/anti-BRST algebra is closed, that is, $s^2 = \bar{s}^2 = 0$, and the modified field strength for the $B$ field, $H'$ in (7), transforms like the usual field strength as we mentioned earlier,

\[ sH'_{\mu\nu\rho} = -[\alpha, H'_{\mu\nu\rho}] . \]
As we advertized earlier, the BRST/anti-BRST invariant classical action now can be written in terms with $H'$, the modified field strength for $B$, and the modified $B$ field, $B' = B - DK$:

$$S_o = \frac{1}{2} \int_{M_4} \text{Tr}(H' \ast H' - F \ast F + mB'F).$$

However, the last term in the above $S_o$ is different from $mBF$ only by a total derivative term, $-\text{Tr}[d(mKF)]$. Thus, for convenience, we shall use the term $mBF$ instead of $mB'F$ for our nonabelian action:

$$S = \frac{1}{2} \int_{M_4} \text{Tr}(H' \ast H' - F \ast F + mBF).$$ (17)

This action is invariant up to a total derivative under the BRST/anti-BRST transformation in (12) and (14)-(16).

III. Nonabelian topological mass generation

In order to see how the topological mass generation phenomenon occurs in the nonabelian case, we first look into the equations of motion of our action (17).

$$D \ast H' = mF,$$
$$D \ast F = mH$$

$$+ \{-B \ast H' + \ast H'B + D(K \ast H' + \ast H'K)\},$$
$$DD \ast H' = 0.$$ (18)

From (18) we have

$$D \ast (D \ast F) = m^2F + mD \ast J$$
$$+ D \ast \{-B \ast H' + \ast H'B + D(K \ast H' + \ast H'K)\},$$
$$D \ast D \ast H' = m^2H$$ (19)
\[ DD \ast H' = 0, \]

where

\[ H' = DB - DDK = DB - [F, K], \]
\[ H = DB, \]
\[ J = [F, K] = \frac{1}{6} J_{\mu \nu \lambda} dx^\mu dx^\nu dx^\lambda. \]

After some work we have the following equations from (19).

\begin{align*}
(D^\mu D_\mu + m^2) & \frac{1}{2} F_{\alpha \beta} \\
+ (2F^\mu_\alpha F_{\beta \mu} - \frac{1}{6} m \varepsilon^{\mu \nu \lambda} \alpha D_\beta J_{\mu \nu \lambda}) \\
+ D_\alpha \{-[\frac{1}{2} B_{\mu \nu}, \frac{1}{6} H'_{\mu \nu \beta}] + D_\mu ([K_\nu, \frac{1}{6} H'_{\mu \nu \beta}]) \} = 0, \\
(D^\mu D_\mu + m^2) & \frac{1}{6} H_{\alpha \beta \gamma} \\
+ \frac{1}{2} \{D^\mu ([F_{\alpha \mu}, B_{\beta \gamma}] + [F_{\alpha \beta}, B_{\gamma \mu}] - D_\alpha J_{\mu \beta \gamma}) + [F^\mu, H'_{\mu \beta \gamma}] \} \\
+ m\{-[\frac{1}{2} B_{\alpha \beta}, \frac{1}{6} H'_{\mu \nu \lambda}] + D_\alpha ([K_\beta, \frac{1}{6} H'_{\mu \lambda \beta}]) \} \varepsilon^{\mu \nu \lambda} \gamma = 0, \\
[F^{\mu \nu}, H'_{\mu \nu \alpha}] = 0.
\end{align*}

The equations of motion for \( F \) and \( H \) are equivalent to the abelian counterpart (4) up to higher order terms which correspond to interaction terms. This will be apparent when we consider the propagators for these fields below. The last equation corresponds to our original constraint, and now this appears naturally as the equation of motion for the auxiliary field \( K \).

Now we write the Lagrangian including gauge fixing terms for the \( A \) and \( B \) fields to get the propagators for these fields.

\[ \mathcal{L} = \text{Tr} \left[ \frac{1}{12} H'_{\mu \nu \rho} H'_{\mu \nu \rho} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{m}{8} \epsilon_{\mu \nu \sigma \rho} B_{\mu \nu} F_{\rho \sigma} \\
- \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \frac{1}{2\xi} (D_\mu B^{\mu \nu})^2 \right], \]
where $H'_{\mu\nu\rho} = D_{[\mu}B_{\nu\rho]} - D_{[\mu}D_{\nu}K_{\rho]}$. The bare propagators for the $A$ and $B$ fields are given by

$$
\Delta^A_{\alpha\mu,\beta\nu} = -\frac{\delta_{\alpha\beta} \left[ g_{\mu\nu} - (1 - \zeta) p_{\mu} p_{\nu} / p^2 \right]}{p^2},
$$

$$
\Delta^B_{\alpha\mu\nu,\beta\eta\sigma} = \frac{\delta_{\alpha\beta}}{4p^2} \left[ g_{\mu[\eta} g_{\sigma]\nu} - \frac{4\xi - 1}{8\xi - 1} g_{\mu[\eta} p_{\sigma]} p_{\nu} / p^2 \right] + \frac{(4\xi - 1/8\xi - 1)}{8\xi - 1} g_{\nu[\eta} p_{\sigma]} p_{\mu} / p^2, \tag{22}
$$

where $\alpha, \beta$ are group indices, and $\mu, \nu, \eta, \sigma$ are space-time indices. The $A - B$ vertex is given by

$$
V^\mu_{\alpha, \beta, \nu, \lambda} = -im\epsilon^{\mu\nu\rho\lambda} p^\rho \delta_{\alpha\beta}, \tag{23}
$$

where $\mu\nu$ are polarization tensor indices for the $B$ field, and $\lambda$ is that of the $A$ field. Because of this two point $A - B$ vertex, we have to take this exchange effect, which we call “transmutation”, into account when we consider $A$ or $B$ propagator. With the suppression of group indices which are easy to put into, we find that the combined propagator for the $A$ field, $\tilde{\Delta}^A$, is given by

$$
\tilde{\Delta}^A_{\mu\nu}(p) = \Delta^A_{\mu\nu} + \Delta^A_{\mu\mu'} V^{\lambda\sigma,\mu'} \Delta^B_{\lambda\sigma, \chi\nu'} V^{\chi\nu', \nu'} \Delta^A_{\nu\nu'} + \cdots. \tag{24}
$$

As in Ref. [1], we note that

$$
V^{\lambda\sigma,\mu'} \Delta^B_{\lambda\sigma, \chi\nu'} V^{\chi\nu', \nu'} = -m^2 \left( g^{\mu'\nu'} - p^{\mu'} p^{\nu'} / p^2 \right) \equiv \theta^{\mu'\nu'}, \tag{25}
$$

and obtain the same result for the combined $A$ propagator except for the group indices which we suppress here:

$$
\tilde{\Delta}^A_{\mu\nu} = \Delta^A_{\mu\nu} + \Delta^A_{\mu\mu'} \theta^{\mu'\nu'} \Delta^A_{\nu\nu'} + \Delta^A_{\mu\nu} \theta^{\mu'\nu'} \Delta^A_{\nu\nu'} + \cdots
$$

$$
= -\frac{g_{\mu\nu} - p_{\mu} p_{\nu} / p^2}{p^2 - m^2} + \frac{\zeta}{p^4} p_{\mu} p_{\nu}. \tag{26}
$$
By choosing \( \zeta = 0 \), which corresponds to the Landau gauge for the \( A \) field, we can immediately see that the combined \( A \) propagator has a pole at \( p^2 = m^2 \). This feature is the same topological mass generation phenomenon found in Ref. [1], but now for the nonabelian case. As in the abelian case, the \( A \) field has two physical degrees of freedom and the \( B \) field has one for each gauge group index [12] when the topological \( B \wedge F \) term is absent. Introduction of this topological term makes the exchange between \( A \) and \( B \) fields possible. Thus as we saw above, it appears that the \( A \) field absorbs the \( B \) field and becomes massive, and vice versa. This phenomenon is somewhat similar to the Higgs mechanism as explained in Ref. [1].

IV. Quantum action

We write the BRST and anti-BRST invariant quantum Lagrangian as [3, 9]

\[
\mathcal{L}_Q = \mathcal{L}_C + \mathcal{L}_Q' \tag{27}
\]

with

\[
\mathcal{L}_Q' = \text{Tr}[s\bar{s}(-\frac{1}{2}A_\mu A^\mu + a_1 \bar{\alpha}\alpha + \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + a_2 \bar{\beta}_\mu \beta^\mu + a_2' \bar{\phi}\phi + a_2'' \frac{1}{2}\rho^2)], \tag{28}
\]

and \( \mathcal{L}_C \) given by (17). It is useful in the following calculation to note that

\[
\text{Tr}[s\bar{s}(\cdots)] = s \text{Tr}[\bar{s}(\cdots)] = s\bar{s} \text{Tr}[\cdots]. \tag{29}
\]

From (12) and (14)-(16), we get each term in (28) as

\[
\text{Tr}[s\bar{s}(\frac{1}{2}A_\mu A^\mu)] = \text{Tr}[s(A_\mu(\partial^\mu\bar{\alpha}))]
\]

\[
= \text{Tr}[A_\mu(\partial^\mu t) - (\partial^\mu\bar{\alpha})(D^\mu\alpha)], \tag{30}
\]

\[
\text{Tr}[s\bar{s}(\bar{\alpha}\alpha)] = \text{Tr}[tt + t[\alpha, \bar{\alpha}] - \alpha\bar{\alpha}\bar{\alpha}], \tag{31}
\]

12
\[ \text{Tr}[s \bar{s} (\frac{1}{2} B_{\mu \nu} B^{\mu \nu})] = - \text{Tr}[s (B_{\mu \nu} (D[\mu \bar{\beta}^\nu] + [\bar{\kappa}, F^{\mu \nu}]))] = \text{Tr}[B_{\mu \nu} \times (D[\mu m^\nu] + [D \alpha]^{\mu, \bar{\beta}^\nu] + [u, F^{\mu \nu}] + [\bar{\kappa}, [\alpha, F^{\mu \nu}]]]) \times (D[\mu \bar{\beta}^\nu] + [\alpha, B^{\mu \nu}] + [\kappa, F^{\mu \nu}])], \]

\[ \text{Tr}[s \bar{s}(\bar{\beta}_\mu \beta^\mu)] = \text{Tr}[s(\bar{\beta}_\mu m^\mu + (D_\mu \bar{\phi}) \beta^\mu - \bar{\beta}_\mu (D^\mu \rho) + [\bar{\beta}_\mu, \beta^\mu] \alpha)] = \text{Tr}[m_\mu m^\mu - m_\mu (D^\mu \rho - 2[\alpha, \bar{\beta}^\mu]) + (D_\mu \bar{n}) \beta^\mu + \bar{\beta}_\mu (D^\mu n) + [(D_\mu \alpha), \bar{\phi}] \beta^\mu + \bar{\beta}_\mu [(D^\mu \alpha), \rho] - [\bar{\beta}_\mu, \beta^\mu] \alpha \alpha + (D_\mu \bar{\phi}) (D^\mu \phi - [\alpha, \beta^\mu])], \]

and

\[ \text{Tr}[s \bar{s}(\bar{\phi} \phi)] = - \text{Tr}[s \bar{s}(\frac{1}{2} \rho^2)] = \text{Tr}[\bar{n} n - \bar{n} [\alpha, \rho] + n [\alpha, \bar{\phi}] + \alpha \alpha [\rho, \bar{\phi}]]. \]

We plug (30)-(34) into (28), and integrate out over \( t, m_\mu, n, \) and \( \bar{n}, \) which is equivalent to setting

\[ t = - \frac{1}{2} \left( \frac{1}{a_1} \partial_\mu A^\mu + [\alpha, \bar{\alpha}] \right), \]

\[ m^\nu = \frac{1}{2} \left( - \frac{1}{a_2} D_\mu B^{\mu \nu} + D^\nu \rho - 2[\alpha, \bar{\beta}^\nu] \right) \]

\[ n = - \frac{a_2}{a_3} D_\mu \beta^\mu - [\alpha, \rho] \]

\[ \bar{n} = - \frac{a_2}{a_3} D_\mu \bar{\beta}^\mu - [\alpha, \bar{\phi}], \]

where \( a_3 \equiv a_2' - a_2^n. \) Then we obtain

\[ \mathcal{L}_\mathcal{Q}' = \text{Tr} \left[ - \frac{1}{4a_1} (\partial_\mu A^\mu + a_1 [\alpha, \bar{\alpha}])^2 \right] - \frac{1}{4a_2} \left( (D_\mu B^{\mu \nu} + 2a_2 [\alpha, \bar{\beta}^\nu])^2 - 2(D_\mu B^{\mu \nu} + 2a_2 [\alpha, \bar{\beta}^\nu])(D_\nu \rho) \right) \]
\[ + \frac{1}{a_3} (a_2 D_\mu \beta^\mu + a_3 [\alpha, \bar{\phi}])(a_2 D_\nu \beta^\nu + a_3 [\alpha, \rho]) \\
+ (\partial_\mu \bar{\alpha})(D^\mu \alpha) - a_1 \alpha \alpha \bar{\alpha} + a_3 \alpha \alpha [\rho, \bar{\alpha}] \tag{36} \\
- \frac{1}{2} \left( (D_{[\mu} \beta_{\nu]} + [\bar{\kappa}, F_{\mu\nu}]) (D^{[\mu} \beta^{\nu]} + [\alpha, B^{\mu\nu}] + [\kappa, F^{\mu\nu}]) \\
+ B_{\mu\nu}([D_{\mu} \beta_{\nu}], [D_{\mu} \beta_{\nu}], [\bar{\kappa}, F_{\mu\nu}] + [\alpha, F^{\mu\nu}]) \right) \\
+ a_2 \left( [(D_\mu \alpha), \bar{\phi}] \beta^\mu + \bar{\beta}_\mu [(D^\mu \alpha), \rho] - [\bar{\beta}_\mu, \beta^\mu] \alpha \alpha \\
+ (D_\mu \bar{\phi})(D^\mu \phi) - \frac{1}{4} (D_\mu \rho)(D^\mu \rho) - (D_\mu \bar{\phi})[\alpha, \beta^\mu] \right]. \]

The first three terms of (36) correspond to gauge fixing terms for the gauge field \( A \), for the antisymmetric tensor field \( B \), and for the ghost-antighost fields \( \beta, \bar{\beta} \) of the \( B \) field, respectively. Note that the first generation ghost-antighost fields \( \beta, \bar{\beta} \) of the second rank tensor field \( B \) need only one gauge fixing condition, because they behave like conjugate fields as we can see from their kinetic term located in the fifth term of (36), and they have \( \rho \) as their common ghost (one of the second generation ghosts for \( B \)). We further notice that the gauge fixing terms for the \( A \) and \( B \) fields in our quantum action are the same type as we used in section III, and the gauge parameters \( \zeta \) and \( \xi \) in (21) correspond to \( 2a_1 \) and \( 2a_2 \) in (36), respectively.

In the quantum action (36), there are no kinetic terms for the auxiliary fields, one form field \( K \) and its ghosts etc., as we expected. Also, if we count the propagating degrees of freedom in the quantum action (36), we can see that two physical degrees of freedom remain for the \( A \) field, and only one physical degree of freedom for the \( B \) field remains, but there is no physical degree of freedom for the one form classical auxiliary field \( K \). This is consistent with our propagator analysis in the previous section.

V. Conclusion
By incorporating a one form auxiliary field, we have been able to extend the abelian action for topological mass generation in 4 dimensions to the nonabelian case. This one form auxiliary field was essential in constructing the gauge invariant kinetic action for the antisymmetric tensor field, and also in finding a consistent BRST/anti-BRST symmetry for the theory which contains one form gauge and two form antisymmetric tensor fields. Basically, this auxiliary field is equivalent to a Lagrange multiplier though we need a special combination for the Lagrange multiplier term in the action, and it resolves the constraint which appears in the naively extended nonabelian action from the abelian one. In fact, the inclusion of the auxiliary field was the key to the successful construction of the nonabelian version of the topological mass generation in 4 dimensions. The mechanism of topological mass generation was the same as in the abelian case: the topological coupling term $mB \wedge F$ makes the transmutation between the vector field and the antisymmetric tensor field possible, and consequently the vector gauge field becomes massive, or vice versa. The counting of physical degrees of freedom in the BRST/anti-BRST invariant quantum action, two from the vector field and one from the antisymmetric tensor field for each gauge group index, also confirms this transmutation. In finding the BRST/anti-BRST symmetry, we used the geometrical “horizontality condition” scheme and obtained a consistent set of the BRST/anti-BRST transformation rule. In constructing the quantum action, we followed the Baulieu and Thierry-Mieg’s method [3, 9] for constructing BRST/anti-BRST invariant quantum action of the Yang-Mills theory, and the method worked nicely also for our system which is composed of vector and antisymmetric tensor fields. We hope that the realization of nonabelian topological mass generation in this paper will raise further interests in this mechanism.
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