Analysis of Assumptions in BIG Bell Test Experiments

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Recently, a group of experiments tested local realism with random choices prepared by humans. These various tests were subject to additional assumptions, which lead to loopholes in the interpretations of almost all of the experiments. Among these assumptions are fair sampling, no signaling, and faithful reproduction of a Bell-type quantum model. The data from 9 of 13 experiments were examined and occurring anomalies were analyzed in view of the above assumptions. It is concluded that further tests of local realism need better setup calibration to avoid apparent signaling or necessity of the complicated underlying quantum model.

1. Introduction

Rejection of local realism models is a theoretical and experimental challenge. The original argument for local realism by Einstein, Podolsky, and Rosen,[1] has later been turned into a form of testable inequalities governing measurement outcomes in such models by Bell and others.[2–4] The test assumes that two (or more) observers are separated and choose what to measure. Due to separation, a principle of special relativity applies that no information can be communicated outside the forward light cone. This is no-signaling and it cannot be deduced from the quantum field theory alone.[5,6] No-signaling is natural in large setups with relativistic scales. A weaker definition of no-signaling fitting also the non-relativistic setups would be the absence of any interactions between parties that could transfer information before the measurement is completed. The opposite, signaling, would mean that such transfer somehow occurred.

Local realism means the existence of a joint (positive) probability for all choices to have specific outcomes, under the assumption of locality and predetermination or local causality and free choice[7,8]. Here, locality implies that the outcome can depend only on the local choice (i.e., the choice and the measurement are made by the same party). Locality implies no-signaling but not vice versa. Correlations satisfy certain inequalities if local realism is assumed. Their violation indicates a violation of local realism or conflict with no-signaling assumption. Bell test is stronger than steering, where one assumes quantum representation of observables at one of the parties.[9–11]

The tests of local realism were realized in the past usually with photons.[8–11] However, no specific system is required; it only has to fit the simple quantum few-state approximation, which resulted in setups across nearly all branches of physics[14–17] (see the review in ref. [18]). Unfortunately, among various problems, the most significant appeared to be lack of sufficient distance (locality loophole), imperfect detection (detection loophole), e.g., a fraction of particles are lost,[19–21] and predetermined (often fixed) choices (random or free choice hypothesis). Loopholes allow for a local realistic model.[22,23]

In the recent Bell tests performed in Delft,[28] NIST,[29] Vienna,[30] and Munich,[31] claimed as loophole-free, violation of local realism is claimed with high confidence level (assuming local realism, the probability of the data is 4% in[28], ≈ 10−7 in ref. [29], ≈ 10−1 in ref. [30], and ≈ 10−9 in ref. [31]). However, all these experiments show also some moderate anomalies, that need either signaling or a complicated underlying quantum model to explain.[32] Moreover, the choice in these experiments was randomly generated by a machine, which cannot exclude some conspiracy models, where the choice is controlled by the other party. To rule out this possibility, one should use human-generated choices. In principle, to prevent locality loophole, the choices should be quicker than the time of communication between observers, which would need Moon–Earth distance.[31] A weaker challenge has been undertaken in the BIG Bell test (BBT),[34] where the human choices were collected all over the Earth, with a too long timescale to close this locality loophole in the strict sense. Nevertheless, trusting that the remote observers have no access to the other party’s choices, BBT can indeed test local realism. In the test, the stream of bits (either 0 or 1) was generated by Bellsters (self-selected people) and used to control choices of separate parties, which performed a test of local realism. In 13 various tests, local realism has been violated, but usually with additional assumptions. They have often assumed ideal quantum Bell states and measurement angles. The goal of the hereby analysis is to examine the data for these assumptions. We requested the data from all 13 experiments and obtained them from 9. Below we present the analysis of each of these nine experiments, discussing anomalies given the type of experiment and known technical features. Experiment 3 has been analyzed separately[35] but without any discussion of signaling and deviations from the underlying quantum model. The findings are later summarized in the discussion, with improvement recommendations for future tests of local realism.
2. Methodology and Main Tested Hypotheses

Local realism means existence of the joint positive probability $p(A_0, A_1, B_0, B_1)$ for $A_0$ being the outcome measured by the observer $A$ who chose $a$ (e.g., the measurement basis). The locality excludes dependence on the remote choice, that is, hidden random variables $A_{ab}$ are not allowed. It generalizes to three or more observers by adding $C_i$, etc., to the set of random variables. In an experimental test, one has to assume absence of communication, that is, information about the setting choice of the party $A$ cannot reach the party $B$ (and $C$, $D$, etc., in the multipartite case) before completion of its measurement. This assumption cannot be verified, only falsified. The measurable probability is a marginal of $p$ for two or three parties, that is,

$$p(AB|XY) = \sum_{A_b} p(A_bB_0|A_yX_bY_0)$$

$$p(A = A^*, B = B|Y_0, X_0)$$

\hspace{1cm} (1)

where $X, Y, Z = 0, 1$ are the chosen settings while $\tilde{X} = 1 - X$, etc., denote complementary (not measurable on the same state). Within the received data, we test the statistical independence of measurements by one observer from the choice of measurement basis by the other, that is, no-signaling, being the null hypothesis, that is,

$$p(a|B) = p(a|B|Y) = p(a|B|1Y)$$

$$p(A|X) = p(A|X|0) = p(A|X|1)$$

\hspace{1cm} (2)

where $\ast$ means ignoring (summing over) that outcome. In an ideal test, we could simply check the above inequality, as done, for example, in ref. [32]. Unfortunately, BBT experiments drop some events, and the probability to get outcome $0$ or $1$ is of finite efficiency $\eta$ not equal to $100\%$. It can depend on various factors, but the most important is setting dependence. We test the assumption that the measured probability $p$ is related to $\tilde{p}$ by an efficiency dependent only locally on the setting, that is,

$$p(AB|XY) = \tilde{p}(AB|XY)\eta_a(X)\eta_b(Y)$$

\hspace{1cm} (3)

with efficiencies $\eta < 1$. Note that additional possible efficiency dependent locally on the outcome is already included and does not alter the no-signaling hypothesis. However, a combined dependence of $\eta(A, X)$ is beyond the scope of this analysis as it gives too many free parameters. The only exception is when testing a particular quantum model, which we do for experiments 4 and 12.

We could first determine $\eta$ from the data and then test (3) but this is unnecessarily complicated. Instead, we keep parameters $\eta$ as unknown but constant so the no-signaling hypothesis becomes an independence hypothesis. Then, we can employ the standard Pearson’s $\chi^2$ test, with the independence being our null hypothesis. Let $O_{ab}$ be the number of measurements for which the two observers have each assigned one parameter $a$ and $b$, respectively, with two options ($a, b = 0, 1$). These parameters could be the outcome of one observer and the experimental setup of the other. The numbers for each of these measurements considered separately are $O_a = \sum_b O_{ab}$ and $O_b = \sum_a O_{ab}$; $N = \sum_{ab} O_{ab} = \sum_a O_a = \sum_b O_b$ is the total count of the registered outcomes. If the probability of the measurement is statistically independent with respect to $a$ and $b$, then $O_{ab}$ is expected to be equal to $E_{ab} = Np_a p_b$ with $p_a = O_a/N$ and $p_b = O_b/N$ being the probabilities of specific events, which is the statement of the null hypothesis. The $\chi^2$ test of (3) is $\chi^2 = \sum_{ab}(O_{ab} - E_{ab})^2/E_{ab}$ value is expected to be close to $0$ in order for the null hypothesis (the independence) to hold. More specifically, for $\chi^2$ from a given experiment, one could obtain the probability ($p$-value) as the upper tail region of $\chi^2$ distribution for one degree of freedom above a specific value $\chi^2_x$. For large $N$, it is equal $\text{erfc}(\sqrt{\chi^2}/2)$ by central limit theorem, and $p \approx \exp(-\chi^2/2)$ for large $\chi$ (see Figure 1 for the general behavior). In the case of no-signaling, we test independence of $p(A|XY) = p(A|\eta(X)\eta(Y)$ for a given $X$ and $2 \times 2$ table of $A$ and $Y$, and similarly for $B$. The obtained $p$-value has to be increased by Bonferroni correction, a.k.a. look-elsewhere-effect, that is, multiplied by the number of possible tests ($4$ in the two-party case).

3. BBT Experiments

The BBT is a group of 13 experiments. We asked for the data of all of them and received them for nine, that is, 1, 2, 4, 5, 6, 9, 10, 12, 13. Below, we present the analysis of each of these nine experiments, with a short description of the setup and aim, and then an explanation of the analyzed assumptions and found anomalies, according to the above methodology.

All of the experiments discussed here used photons and most of them suffer from loopholes. Most of the experimenters applied fair sampling in their analysis, that is, the instances in which all relevant photons that were measured are considered representative for all the instances produced, and the instances in which a photon was lost are discarded. The distances are too small to prevent all subluminal influence. Only experiment 13 is free from these loopholes.

3.1. 1: R.B. Patel et al. Quantum Steering using Human Randomness

The experiment tested steering on Bell state $|\psi\rangle = (|+ \pm \rangle - |\mp + \rangle)/\sqrt{2}$, measuring photons with random pairs of settings,
which are the same for both measuring parties. Essentially the measured quantity was \( \hat{A}_k = \mathbf{n}_k \cdot \hat{\sigma} \), with Pauli matrices \( \hat{\sigma}_1 = |+\rangle\langle-| + |-\rangle\langle+| \), \( \hat{\sigma}_2 = i |+\rangle\langle-| - i |-\rangle\langle+| \), \( \hat{\sigma}_3 = |+\rangle\langle+| - |\rangle\langle-| \). The 16 unit (|\mathbf{n}| = 1) directions \( \mathbf{n}_k \) are located at vertices and centers of sides of a dodecahedron (or centers of sides of a regular soccer ball) with \( \mathbf{n}_1 = (0, 0, 1) \) being one of the dodecahedron’s vertices (see Figure 2). The settings are controlled by a half-wave-plate and quarter-wave-plate. In general, Bell correlations read

\[
\langle \hat{A}_k \hat{B}_j \rangle = -\mathbf{n}_k \cdot \mathbf{n}_j = -\cos \phi_{kj} \tag{4}
\]

where \( \phi_{kj} \) is the angle between directions \( \mathbf{n}_k \) and \( \mathbf{n}_j \). Here, ideally the correlations should be \( \langle \hat{A}_k \hat{B}_j \rangle = -1 \) with \( A_k, B_k = \pm 1 \). Instead of fair sampling, applied usually to the fraction of coincidences in the set of all events (including \( A_k, B_k = 0 \)), this experiment has oversampling; that is, for a given setting, a large group of states is measured until the coincidence is registered. Since the settings are the same, we could not check signaling, that is, dependence of one party’s measured probability on the choice of the other. The only analysis we could do is the occurrence of a particular setting.

In Table 1, it is clear the choices 6 and 11 occur much often. According to authors in ref. [40], it can be attributed to anticorrelations from Bellsters. The bits provided by humans are usually not fully random, and this increased occurrence may be the result of combinations like 0101 and 1010 (binary

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**Figure 2.** The directions of the measured operators on the Bloch sphere.

**Table 1.** Numbers of occurrences of particular settings.

| Setting | Occurrence | Setting | Occurrence |
|---------|------------|---------|------------|
| 1       | 1549       | 9       | 922        |
| 2       | 1478       | 10      | 1529       |
| 3       | 1744       | 11      | 3575       |
| 4       | 716        | 12      | 900        |
| 5       | 1448       | 13      | 834        |
| 6       | 3947       | 14      | 1274       |
| 7       | 988        | 15      | 1239       |
| 8       | 736        | 16      | 1238       |
| all     | 24117      |         |            |
representations of 5 and 10, respectively). However, this should not cause any new loopholes.

3.2.2: M. Ringbauer and A. White, Quantum Correlations in Time

The experiment tested local realism translated into time correlations. Essentially, the setup consists of three observers A, B, C, each measuring 0 or 1 at respective choices X, Y, Z (in fact A measurement means a preparation of the state), made by an appropriate setting of halfwave plates (HWP) and quarterwave plates (QWP). The sequence of measurement allows the causal order depicted in Figure 3 and the violation of the classical correlation assumes this order. Due to finite detection efficiency, fair sampling is assumed.

We checked if the causal order is satisfied in sense of no-signaling, that is, if the probability cannot depend on the choice not linked causally (see Figure 3). Let us denote the probability p(ABC|XYZ) of measuring A, B, and C for the choices X, Y, Z. Because of postselection of coincidences, we assumed also detection efficiency to depend only on the local choice but not on the outcome. It means in particular that

\[ p(ABC|XYZ) = \eta_A(X)\eta_B(Y)\eta_C(Z) \]

with efficiencies \( \eta < 1 \). Then, no-signaling implies \( \tilde{p}(**C0YZ) = p(**C1YZ) \), \( \tilde{p}(**B0|XY) = p(**B1|XY) \), \( \tilde{p}(A**|X**) \) and \( \tilde{p}(AB**|XY0) = \tilde{p}(AB**|XY1) \).

Here, \( * \) means ignoring/discarding outcome or every choice. The summed count for each combination of choices and outcomes is shown in Table 2.

No-signaling means that \( N(A**|XYZ) \) is independent of YZ. However, the differences (see Table 6) are so large (e.g., \( \chi^2 \approx 121 \) for \( XZ = 10 \) while comparing each Y and A) that without question they must originate from a systematic effect. According to the experiment authors in ref.\(^{[41]} \), this apparent signaling is due to the special Bob’s measurements which suppresses the rate of the measurement of A = 0 except the case X = Y = 1 when A = 1 is suppressed. The apparent signaling \( Y \rightarrow C \) is visible in \( N(**C1|Y1) \) with \( \chi^2 \approx 34 (p \approx 10^{-7}) \). It is allowed by the causality flow. However, this also may be a systematic effect revealed in \( YZ = 11 \) in the two last rows of Table 3. The other no-signaling tests are passed within the acceptable certainty level, but also the statistics are relatively small.

The dependence of the coincidences on the setting is unquestionable. The possible reason is angle-dependent deflection at HWP or QWP, which changes the cross section between the photon wave packet and the fiber. It could be also back-signaling due to small distances compared to the time photons need to pass their routes, but we cannot check it here. Nevertheless, the effect
Table 4. Counts of Table 2 with two outcomes ignored (marked by *) and total numbers of coincidences.

|       | 0** | 1** | 0* | 1* |
|-------|-----|-----|----|----|
| 00    | 159 | 328 | 000| 305| 182|
| 001   | 153 | 345 | 001| 312| 186|
| 010   | 169 | 268 | 010| 259| 178|
| 011   | 176 | 298 | 011| 298| 176|
| 100   | 160 | 318 | 100| 291| 187|
| 101   | 189 | 296 | 101| 295| 190|
| 110   | 336 | 151 | 110| 304| 183|
| 111   | 306 | 143 | 111| 278| 171|

Table 5. \( \chi^2 \) values calculated for causally linked choices and outcomes. It is expected to fail here. Each test is performed for other choices fixed; their values are given in the upper row in alphabetical order (XY, YZ, or XZ).

|       | 00 | 01 | 10 | 11 |
|-------|----|----|----|----|
| X and A | 0.07 | 7.3 | 85.4 | 88.9 |
| X and B | 0.3 | 0.3 | 0.9 | 0.08 |
| Y and B | 1.09 | 0.004 | 0.2 | 0.1 |
| Y and C | 0.9 | 20.1 | 1.2 | 34.7 |
| Z and C | 0.07 | 13.3 | 2.4 | 29.7 |

is so large that it should be possible to run a diagnostic test to confirm the cause in the future.

3.3.4: B. Liu et al., Violation of a Bell Inequality Using Entangled Photons and Human Random Numbers

The experiment tested standard CHSH inequality on the Bell state \(|\psi\rangle = \frac{(|HV\rangle - |VH\rangle)}{\sqrt{2}}\). Two experiments, one with human-generated random numbers (HRN1) and quantum random numbers (QRN1), the second with real-time human random numbers (HRN2) and the database of human random numbers (DB2), constitute four data sets to analyze. The random choices \(X = 0, 1\) and \(Y = 0, 1\) define bases of the respective photon measurements of \(A = 0, 1\) and \(B = 0, 1\). In particular, \(XA = 00, 01, 10, 11\) correspond to polarization angles 0°, 90°, 45°, and 135° respectively, while \(YB = 00, 01, 10, 11\) to \(-22.5°, -112.5°, -67.5°, 22.5°\). These angles translate to spin correlations (4) as \(n = (\cos2\phi, \sin2\phi, 0)\), so the angles in (4) are twice the photon polarization angle. The test assumes no signaling between parties \(XA\) and \(YB\) but the statistics are postselected on coincidences, so we checked it combined with the assumption of detection efficiency depending only on the local choice.

In the data, the total counts (Table 11, first column) depends on the settings. If (3) holds, then \(N(00)N(11) = N(01)N(10)\) while here, for example, 26933 = 5011 < 53764 = 29470 beyond statistical error (Pearson’s \( \chi^2 \) independence tests give \( \chi^2 > 240 \) so the \( p \)-value is < 10^{-10}). Nevertheless, even correcting for the setting-dependent efficiency, the Alice’s statistics in Table 9 depends on the Bob’s choice, (e.g., \(N(0\ast 01) = 29096\) is more than twice \(N(0 \ast 00) = 13775\)). The Bob’s statistics differences (Table 10) are smaller, for example, for HRN1 \(N(0\ast 00) = 13737\) and \(N(0\ast 01) = 15981\) give the difference 2244 at the variance \(\sqrt{29718}\) (over 13 times). At the same time, \(N(0 \ast 00) = 13196\) and \(N(1 \ast 10) = 13489\) have a difference only 293 (alternatively, \(\chi^2 \approx 58.6\) and \(p \approx 10^{-11}\)). Note even larger \(\chi^2\) for QRN1 and DB2. Trying to compensate the first difference by scaling the efficiencies will increase the second one. The large deviations are only slightly decreased by look-elsewhere-effect a.k.a. Bonferroni corrections \([37,38]\); that is, the \( p \)-value is increased four times (the number of testable combinations).

We see a violation of no-signaling if detection efficiency is only local-choice dependent. Vice-versa, assuming no-signaling, the detection rate must depend on the remote choice. Since only coincidences have been reported, the analysis of signaling could be repeated with the full data, including also single counts. Nevertheless, dropping fair sampling leads also to a reinterpretation of
Bell violations, which may disappear in agreement with classical realism based on no-signaling condition.

The authors in ref. [42] claim that a similar apparent violation of the no-signaling assumption has already been observed in previous work by the same group, [43] also based on coincidence counts between Alice and Bob. Using single count rates (i.e., not post-selecting on a coincident outcome at the distant location) shows an agreement with no-signaling in that test. As described in the supporting information of that publication, the authors show that the effect results from the known efficiency differences of the detectors for outcomes 0 and 1, and they verified it with the help of a detailed quantum mechanical model for the experimental results. After correcting the data for the different detection efficiencies, the conditions for no-signaling were fulfilled for the coincidence counts as well. The same polarization analyzer modules, in particular, the same single photon detectors, as in ref. [43] have been used for the experiment conducted in the course of the BBT collaboration and discussed in the present paper.

We have checked if the inclusion of different outcome-dependent efficiency can explain one of the tests (Table 7), assuming the standard CHSH test conditions (ideal angles and input state), that is, \( \hat{p}(00|01) = \hat{p}(00|10) = \hat{p}(00|11) = \hat{p}(10|10) \). If \( p(AB|XY) = p(AB|XY)\eta_A(A)\eta_B(B) \), then \( N(00|01) = N(00|10) \). Even in the case of detection efficiency depending on both the choice and outcome, for example,

\[
p(AB|XY) = p(AB|XY)\eta_A(X, A)\eta_B(Y, B)
\]

Table 9. Counts \( XY \) (rows) for \( A = 0, 1 \) (columns) with Bob’s outcome ignored (marked by \( * \)) together with Pearson’s test of independence of \( A \) and \( Y \) with given \( X \) for each run of experiment 4.

| HRN1   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 13775    | 13158    | 63.6      |
| 01     | 29096    | 24668    | 63.6      |
| 10     | 16316    | 13154    | 63.6      |
| 11     | 27466    | 22545    | 63.6      |

| QRN1   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 32958    | 31505    | 0.07      |
| 01     | 40735    | 33980    | 159.9     |
| 10     | 45950    | 37298    | 92888     |
| 11     | 39812    | 32227    | 0.07      |

| HRN2   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 8409     | 7890     | 0.1       |
| 01     | 16650    | 14333    | 19.7      |
| 10     | 10333    | 8384     | 168.9     |
| 11     | 16069    | 12956    | 168.9     |

| DB2    | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 36403    | 34316    | 0.1       |
| 01     | 53112    | 45172    | 108.5     |
| 10     | 42026    | 33546    | 108.5     |
| 11     | 51384    | 41504    | 108.5     |

Table 10. Counts \( XY \) (rows) for \( B = 0, 1 \) (columns) with Alice’s outcome ignored (marked by \( * \)) together with Pearson’s test of independence of \( B \) and \( Y \) with given \( X \) for each run of experiment 4.

| HRN1   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 13737    | 13196    | 58.6      |
| 01     | 29108    | 24656    | 58.6      |
| 10     | 27724    | 22287    | 58.6      |
| 11     | 27466    | 22545    | 58.6      |

| QRN1   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 32938    | 31525    | 139.7     |
| 01     | 40668    | 34027    | 139.7     |
| 10     | 39823    | 32216    | 10.01     |
| 11     | 39812    | 32227    | 10.01     |

| HRN2   | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 8290     | 8009     | 34.3      |
| 01     | 16638    | 14345    | 34.3      |
| 10     | 16091    | 12934    | 34.3      |
| 11     | 16069    | 12956    | 34.3      |

| DB2    | \( 0^* \) | \( 1^* \) | \( \chi^2 \) |
|--------|----------|----------|-----------|
| 00     | 36028    | 34691    | 168.9     |
| 01     | 53108    | 45176    | 168.9     |
| 10     | 53112    | 45172    | 168.9     |
| 11     | 51384    | 41504    | 168.9     |

Table 11. Total counts \( XY \) (rows) for each run of experiment 4 together with Pearson’s test of independence of \( X \) and \( Y \).

| Total | HRN1 | QRN1 | HRN2 | DB2 |
|-------|------|------|------|------|
| 00    | 26933| 64643| 16299| 70719|
| 01    | 53764| 74965| 30983| 98284|
| 10    | 29470| 83248| 18717| 75572|
| 11    | 50011| 72039| 29025| 92888|

\( \chi^2 \approx \frac{n(\text{observed} - \text{expected})^2}{\text{expected}} \), beyond statistical error. It may indicate a drift of general constant non-ideal alignment of measurement angles.

3.4.5: L. Santodonato et al., Experimental Bilocality Violation with Human Randomness

The experiment tested local realism with two sources of entangled photons, one sending to \( A \) and \( B \), the other to \( B \) and \( C \). Essentially, the setup consists of three observers \( A, B, C \), each measuring 0 or 1 at respective choice \( X, Y, Z \). No signaling between each party is assumed. Due to finite detection efficiency, fair sampling is assumed.
We checked if the causal order is satisfied in a sense of no-signaling, that is, if the probability can depend on the choice not linked causally, assuming local-choice-dependent efficiencies in the sense of (5). Then, the test of (apparent) signaling is the same as in Section 3.2.

The summed count for each combination of choices and outcomes is shown in Table 12.

In the data, in the last of Table 14 that total counts still depend on the choices but not independently, that is, the assumption (5) implies \( N(010)N(101) = N(000)N(111) \) while 14103 ∗ 12172 ≠ 5577 ∗ 3842 beyond statistical error. Therefore, it may indicate the failure of the assumption (5), not no-signaling. In view of combined choice-dependent efficiency, we cannot directly check no-signaling (which apparently would be immediately violated) but rather no-signaling combined with independent efficiency. So, we test independence, for example, if \( p(00|0010)p(01|0110) \approx p(01|0010)p(10|0010) \) by Pearson’s \( \chi^2 \) test. Here, in Table 13 column 1 and 2, rows 3 and 7, \( \chi^2 = 48 \) giving \( p \)-value of the order 10\(^{-11} \). It is only slightly decreased by look-elsewhere-effect, that is, the \( p \)-value is increased about 100 times (the number of testable combinations). Another example is \( p(00|0001)p(00|1110) \approx p(01|0001)p(01|1110) \) giving (Table 16) \( \chi^2 ≈ 24 \) or \( p \)-values of the order 10\(^{-4} \). Nevertheless, most combination give much smaller \( \chi^2 \), often with \( p \approx 1 \).

The large deviations indicate that it can be either no-signaling or local choice dependent efficiency assumption to fail. Since only coincidences have been reported, the analysis of signaling could be repeated with the full data, including also single counts. According to authors,\(^{[44]} \) the apparent violation may be caused by the long runtime of the experiment when the efficiencies undergo systematic time-dependent bias.

### Table 12. Statistics of coincidences (columns A, B, C) at given choices (rows X, Y, Z) of experiment 5.

|        | 000 | 001 | 010 | 011 |
|--------|-----|-----|-----|-----|
| 000    | 586 | 376 | 353 | 628 |
| 001    | 716 | 357 | 436 | 748 |
| 010    | 1557| 794 | 846 | 1675|
| 011    | 302 | 516 | 724 | 233 |
| 100    | 741 | 548 | 551 | 844 |
| 101    | 1395| 939 |1060 |1375|
| 110    | 332 | 743 | 726 | 284 |
| 111    | 763 | 172 | 208 | 513 |

### Table 13. Counts of Table 12 with one of outcomes ignored (marked by *).

|        | 00* | 01* | 10* | 11* |
|--------|-----|-----|-----|-----|
| 000    | 535 | 1310| 1220| 569 |
| 001    | 676 | 1434| 1476| 611 |
| 010    | 1530| 3225|3018 |1458|
| 011    | 1344| 399 | 425 |1077|
| 100    | 704 | 1485| 1482| 659 |
| 101    | 1187| 2565|2586 |1065|
| 110    | 893 | 403 | 416 |1010|
| 111    | 290 | 741 | 892 |263 |

3.5. 6: K. Redeker, R. Garthoff, D. Burchardt, H. Weinfurter, W. Rosenfeld, Violation of Bell’s Inequality with a Single Atom and Single Photon Entangled Over a Distance of 400m

The experiment tested standard CHSH inequality on the entangled photon-atom Bell state \(|\psi=(|↓L⟩+|↑R⟩)/\sqrt{2}\) in the LR photon (A) basis and \(↑↓\) atom (B) basis. The measurement of the photon by avalanche photodiodes (APD) depended on the passive choice \(X = 0\), while the atom measurement (by ionization) depended on the choice \(Y = 0, 1\), realized by either a quantum random number generator (QRN) or human random number generator (HRN). The dichotomic outcomes of the respective measurements, \(A = 0, 1\) and \(B = 0, 1\) depend on the choices (\(X\) and \(Y\), respectively). In particular, \(XA = 00, 01, 10, 11\) correspond to polarization angles \(0°, 15°, 45°, 90°\), with \(YA = 0, 1\) corresponding to 22.5° in, 22.5° out, −22.5° in, −22.5° out. The angles enter the Bell correlation formula (4) multiplied by 2, as in Section 3.3. The test assumes no signaling between parties XA and YB but also fair sampling at the side of the photon (A).

We checked no-signaling assuming equal APD efficiencies, that is, if \(p(0|B|0Y)\approx p(0|B|1Y)\) and \(p(A|X0)=p(A|X1)\) for all runs. In the first case, one has to take into account...
Table 14. Counts of Table 12 with two outcomes ignored (marked by *) and total numbers of coincidences.

|       | **0** | **1** | **0** | **1** |
|-------|-------|-------|-------|-------|
|       | 000   | 001   | 010   | 011   |
| 000   | 1943  | 3634  | 000   | 2807  | 2770  |
| 001   | 2257  | 4197  | 010   | 3183  | 3271  |
| 010   | 4872  | 9231  | 011   | 7106  | 6997  |
| 011   | 1775  | 3245  | 011   | 2561  | 2459  |
| 100   | 2684  | 4330  | 101   | 3478  | 3536  |
| 101   | 4769  | 7403  | 110   | 6086  | 6086  |
| 110   | 2085  | 2722  | 110   | 2371  | 2436  |
| 111   | 1656  | 2186  | 111   | 1966  | 1876  |

Table 15. \( \chi^2 \) values calculated for each party. Each test is performed for other choices fixed; their values are given in the upper row in alphabetic order (XY, YZ, or XZ).

|       | 00   | 01   | 10   | 11   |
|-------|------|------|------|------|
|       |      |      |      |      |
| X and A | 15.69 | 31.81 | 120.15 | 55.007 |
| Y and B | 0.004 | 3.2  | 0.07  | 1.60  |
| Z and C | 9.97 | 60.48 | 4.44  | 39.55 |

Table 16. \( \chi^2 \) values calculated for independent choices and outcomes, as assumed. Each test is performed for other choices fixed; their values are given in the upper row in alphabetic order (XY, YZ, or XZ).

|       | 00   | 01   | 10   | 11   |
|-------|------|------|------|------|
|       |      |      |      |      |
| X and B | 0.69 | 0.78 | 1.61 | 0.02 |
| X and C | 2.04 | 0.001 | 0.003 | 0.11 |
| Y and A | 0.15 | 0.18 | 30.92 | 18.70 |
| Y and C | 1.54 | 22.80 | 0.13 | 27.78 |
| Z and A | 0.02 | 1.07 | 1.56 | 0.06  |
| Z and B | 1.22 | 0.58 | 0.30 | 2.91  |

Table 17. Counts XY (rows) for \( A = 0, 1 \) (columns) with Bob’s outcome ignored (marked by *), and the total counts, together with Pearson’s test of independence of \( A \) and \( Y \) with \( X \) given for each run of experiment 6.

|       | **0** | **1** | **0** | **1** | **0** | **1** |
|-------|-------|-------|-------|-------|-------|-------|
|       | 00    | 01    | 10    | 11    | 00    | 01    |
| HRN   | 2883  | 2462  | 5345  | 0.02  |
|       | 2635  | 2224  | 4888  | 0.02  |
|       | 2520  | 2041  | 4562  | 0.02  |
| QRN   | 2845  | 2403  | 5248  | 0.02  |
|       | 2773  | 2335  | 5108  | 0.02  |
|       | 2644  | 2137  | 4781  | 0.02  |
|       | 2658  | 2103  | 4761  | 0.02  |

Table 18. Counts XY (rows) for \( B = 0, 1 \) (columns) with Alice’s outcome ignored (marked by *), and the total counts, together with Pearson’s test of independence of \( B \) and \( X \) with \( Y \) given for each run of experiment 6.

|       | **0** | **1** | **0** | **1** | **0** | **1** |
|-------|-------|-------|-------|-------|-------|-------|
|       | 00    | 01    | 10    | 11    | 00    | 01    |
| HRN   | 2707  | 2638  | 0.9  |
|       | 2440  | 2481  | 1.16 |
|       | 2285  | 2603  | 51   |
|       | 2470  | 2092  | 51   |
| QRN   | 2649  | 2599  | 0.9  |
|       | 2368  | 2413  | 0.9  |
|       | 2387  | 2721  | 11   |
|       | 2648  | 2113  | 78   |

The different total counts can be explained by different APD efficiencies, but it should be confirmed by a diagnostic run. In the HRN test, there is human choice asymmetry, similar to experiment 13 (see later). On the other hand, the correlation

The authors confirmed that the efficiency was not equal. We have also checked if this effect could be explained by different efficiencies of APD, assuming correct input state and detectors’ angles. Suppose the APD has efficiency \( \eta_n \) for \( n = 1, 2, 3, 4 \). If the measurement axes are as in standard CHSH test, then the probabilities are as in Table 20. Then, still \( \chi^2 \) for QRN in the first two rows of Table 19 gives \( \chi^2 \approx 14 \) (\( p \approx 2 \times 10^{-4} \)) while the lower two rows (in case of alternative configuration) gives \( \chi^2 \approx 35 \) (\( p \approx 10^{-8} \)). Therefore, this explanation seems insufficient. Possible further reasons may include unspecified deviation from the ideal Bell state or measurement axes. The \( p \)-value is increased four times by the look-elsewhere-effect.
Table 19. Influence of asymmetric detection efficiencies on detection probabilities, with $4\Delta_n = 1 \pm 1/\sqrt{2}$ in the ideal Bell test in the case angles as in Section 3.3—upper two rows and with Alice’s angles additionally rotated by 45°—lower two rows.

| $\phi$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ |
|-------|----------------------------------|----------------------------------|
| 0     | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ |
| 1     | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ |
| 00    | $\eta_3 \Delta_1 + \eta_4 \Delta_2$ | $\eta_3 \Delta_1 + \eta_4 \Delta_2$ |
| 01    | $\eta_3 \Delta_1 + \eta_4 \Delta_2$ | $\eta_3 \Delta_1 + \eta_4 \Delta_2$ |

Table 20. In the case of ideal Bell angles, the outcome B should not depend on Y if either $X = 1$ or $X = 1$ for each run of experiment 6.

| HRN | $\phi$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ |
|-----|-------|----------------------------------|----------------------------------|
| 00  | 2707  | 2638                             |                                 |
| 01  | 2285  | 2603                             | 15.5                             |
| 10  | 2440  | 2481                             |                                 |
| 11  | 2470  | 2092                             | 19.7                             |

| QRN | $\phi$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ | $\eta_1 \Delta_1 + \eta_2 \Delta_2$ |
|-----|-------|----------------------------------|----------------------------------|
| 00  | 2649  | 2599                             |                                 |
| 01  | 2387  | 2721                             | 14.5                             |
| 10  | 2368  | 2413                             |                                 |
| 11  | 2648  | 2113                             | 35.5                             |

The experiment tested Clauser–Horne (CH) inequality on the entangled two-photon Bell state, one idler (Alice A) and signal (Bob B). The measurement can be translated into Alice–Bob 0, 1 numbers as follows: $A = 0, 1$ or $B = 0, 1$ corresponding to Alice’s and Bob’s +, − outcomes, while $X = 0, 1$ correspond to Alice’s choice $w$, $w'$ and $Y = 0, 1 − r$, $r'$. The measurement of the photon depended on the passive choice $X = 0, 1$ while the atom measurement depended on the choice $Y = 0, 1$. It assumes no signaling between parties $AX$ and $BY$ but also fair sampling, due to low detection efficiency, $\approx 5\%$. We checked no-signaling, assuming local choice dependent efficiency (3), that is, $|p(A* B|00Y) = p(A* B|11Y)$ and $p(A* X0) = p(A* X1)$ for all runs. We also checked if the rate of coincidences/trials depends on choices.

In the data, the total coincidence counts (Table 21) are roughly proportional to the number of trials but the latter depends on both settings in a correlated way (i.e., Alice and Bob do not choose the settings independently). No-signaling is violated as $\chi^2 \approx 20$ ($p \approx 10^{-5}$) for Table 21 taking the second and fourth row of $* 0$ and $* 1$ but it may be explained alternatively by correlation-dependent detection efficiency. The $p$-value is increased four times by the look-elsewhere effect.

The observed deviations put in question the local dependency of efficiency but may also indicate some form of signaling. In future experiments, these effects should be independently identified by diagnostic runs, preferably before the main test.

3.7.10: A. Lenhard, A. Seri, D. Rieländer, O. Jimenez, A. Mattar, D. Cavalcanti, M. Mazzera, A. Acín, and H. de Riedmatten, Violation of a Bell Inequality Using High-Dimensional Frequency-Bin Entangled Photons

The experiment tested Clauser–Horne (CH) inequality on the entangled two-photon Bell state, one idler (Alice A) and signal (Bob B). The measurement can be translated into Alice 0, 1, −1 (or nothing) by taking three different frequency modes. On the other hand, Bob always chooses 0 or nothing (missing photon due to low detection efficiency). Alice can choose $X = 0, 1$ (by changing modulation depth and phase) while Bob can choose $Y = nm$ where $n = 0, 1$ (modulation depth) while $m = 1, \ldots, 16$ (phase). The CH inequality is measured assuming fair sampling and symmetry of correlations. Bob’s labels 4 and 14 correspond to real phases 170 and 342, respectively, and were used to find Bell inequality violation.

Since Bob’s outcome was always 0 or nothing, the most appropriate test is one-sided signaling, that is, if Bob’s probabilities equality $p(* 0 |0 Y) = p(* 0 |1 Y)$ hold. We checked no-signaling, assuming local choice dependent efficiency (3), that is, $|p(A* B|00Y) = p(A* B|11Y)$.

To compare the counts they must be weighted by the number of trials (10s intervals, when the coincidences for a particular choices are collected) as shown in Table 23. The weighted trials $N$ are sums of $C/T$ while $e^2$ is the sum of $C/T^4$ for $C$ coincidences and $T$ trials. We adjusted $\chi^2$ test formula for varying trials with

$$\chi^2 = \frac{\sum \eta_{ij} \left( \frac{N_{00}N_{11} - N_{01}N_{10}}{N_{00} + N_{01} + N_{10} + N_{11}} \right)^2}{\sum \eta_{ij} e^{-2}}$$

The numbers of trials $T$ vary between 2 and 36. For the phases used in CH test, 170, and 342 but also 160, no-signaling is consistent with $\chi^2$ equal 2.97, 0.32, and 0.09, respectively. In the data,
there is an apparent violation of no-signaling or violation of local-dependent efficiency (3) at phase 267 of moderate significance where $\chi^2 \approx 14$ giving p-value $\approx 2 \times 10^{-4}$ increased 16 times (number of phases) by the look-elsewhere-effect. It may also indicate some form of signaling but also nonlocal-dependent efficiency. According to the authors of ref. [48], some problems could have occurred during the measurement for some point (e.g., laser temporarily out of a lock, leading to reduced coincidences for this setting and phase). The point at phase 267 is clearly below the curve in Figure S15 of the supporting information of BIG Bell test. [34] It might also be a small difference in fiber coupling for the different settings, which could lead to a difference in efficiency (locally for Alice and Bob) for the different settings. In future experiments, these effects should be independently identified by diagnostic runs, preferably before the main test.

3.8.12: J. Cariñe et al. Post-Selection Loophole-Free Energy-Time Bell Test Fed with Human-Generated Inputs

The authors report an optical experiment employing Franson "hug" configuration—a variation that avoids the post-selection of results present in basic Franson configuration. In the "hug" configuration, if the two emitted photons are detected by Alice and Bob, then it is guaranteed that they both traveled short ways (S) or long ways while obtaining phase shifts $\phi_A$ and $\phi_B$ (L). The results when single photons are measured by either Alice or Bob are discarded, due to low detection efficiency. Thus, fair sampling of a sort is inherent here. The remaining could be assumed to have been in a state $\ket{\psi} = \frac{1}{\sqrt{2}} (\ket{SS} + e^{i(\phi_A + \phi_B)} \ket{LL})$. CHSH inequality was tested, with varying phases set by human-generated input. We assume the fair sampling for the empty counts is also applied. As communicated, the closing of detection loophole was not the purpose of the experiment. [49] The setting at Alice and Bob are given by X and Y, respectively, equal 0 or 1, corresponding to $\phi_A = \pi/4$, $\phi_B = 0$ or $\phi_A = -\pi/4$, $\phi_B = \pi/2$. The outcomes A, B were also 0 or 1 (or nothing).

Based on the data provided and assuming local dependent efficiency (3), we assessed the number of empty counts and consequently the efficiency of the detectors. Further, we checked no-signaling, that is, if $p(\text{A} = 1 | \text{B} = 0, \text{Y}) = p(\text{A} = 1 | \text{B} = 1, \text{Y})$ and $p(\text{A} = 0 | \text{X} = 0) = p(\text{A} = 0 | \text{X} = 1)$, we also checked if the data are consistent with equal and independent detection efficiency, that is, if the probability $p(\text{A} | \text{B} | \text{X} | \text{Y})$ is consistent with the single efficiency $\eta$ and Bell state and angles $\phi(\theta)$. For arbitrary phases, the correlations can be written in the form similar to (4), $\langle AB \rangle = E_{\phi_B} = v \cos(\phi_A - \phi_B)$ with the visibility $v$. In the ideal Bell test, $E_{\phi_B} = E_{\phi_B'} = E_{\phi_B} = -E_{\phi_B'}$. The total number of coincidences depends on settings but independently, with $\chi^2 \approx 0.3$ for the last column of Table 25. Taking this effect into account, there is no signaling observed within the standard error. However, the correlations do not agree with the ideal Bell model, for example, $E_{\phi_B}$ is not equal $E_{\phi_B'}$. It can be quantified defining $M = NE$, where $N$ is the total number of coincidences. Here, $M$ is calculated using Table 24 while $N$ is in Table 25. The $\chi^2$ test for $M_{\phi_B}$, $M_{\phi_B'}$, $M_{\phi_B}$, and $M_{\phi_B'}$ gives $\chi^2 \approx 348$ ($p < 10^{-70}$). The difference can be however explained by a small long-term uncontrolled phase drift, confirmed by the authors. [49] Taking into account setting-dependent detection efficiency, no signaling signatures have been found. However, the strong deviation from the ideal Bell model should be confirmed in a diagnostic run, testing solely phase modulators.

| ph | X | Y |
|----|---|---|
| 140 | 7.83±0.56 | 6.17±0.54 |
| 151 | 10.67±0.74 | 8.75±0.83 |
| 160 | 7.00±0.92 | 6.96±0.79 |
| 170 | 8.98±0.73 | 6.78±0.75 |
| 183 | 9.00±0.75 | 7.03±0.64 |
| 197 | 8.91±0.83 | 7.45±0.96 |
| 215 | 8.82±0.85 | 9.48±1.34 |
| 232 | 8.17±0.61 | 6.99±0.76 |
| 250 | 5.79±0.66 | 8.24±0.76 |
| 267 | 7.04±0.78 | 7.37±0.87 |
| 284 | 7.14±0.77 | 7.28±0.75 |
| 298 | 6.70±0.79 | 7.05±0.87 |
| 321 | 7.82±0.76 | 5.43±0.76 |
| 342 | 6.01±0.65 | 5.71±0.77 |
| 352 | 7.07±0.67 | 6.99±0.82 |
| 358 | 7.91±0.63 | 4.56±0.58 |
| all | 124.87±2.95 | 112.25±3.27 |

Table 22. Pearson's test of independence of A and Y (left) and of B and X (right) for experiment 9.

| X | $\chi^2$ | Y | $\chi^2$ |
|---|---|---|---|
| 0 | 3.28 | 0 | 20.34 |
| 1 | 0.58 | 1 | 0.39 |

Table 23. Weighted counts C/T and errors $\epsilon^2 = C/T^2$ (at ± sign) at choices XY (columns) for B = 0 (columns) with Alice’s outcome ignored for special phases (ph – rows), or all summed in experiment 10 and $\chi^2$ in the last column.
The bias is independent (\( \chi \)) is also evident that there is a bias in the settings choice probabilities. Giving \( X \) outcome ignored in the middle and a total count on the right in experiment 12.

Table 25. Counts at choices XY (rows) for A = 0, 1 (columns) with Bob’s outcome ignored (marked by *) on the left and for B = 0, 1 with Alice’s outcome ignored in the middle and a total count on the right in experiment 12.

|     | 0* | 1* | 0 | 1 | Total |
|-----|----|----|---|---|-------|
| 00  | 2968 | 2895 | 2943 | 2920 | 5863  |
| 01  | 2513 | 2543 | 2558 | 2498 | 5056  |
| 10  | 2614 | 2662 | 2656 | 2620 | 5276  |
| 11  | 2214 | 2267 | 2226 | 2255 | 4481  |

Table 26. Counts at choices XY (columns) with either Bob’s or Alice’s outcome ignored (rows AB with the mark * in experiment 13).

|     | 00 | 10 | 01 | 11 |
|-----|----|----|----|----|
| 0  | 2195 | 7185 | 1935 | 6088 |
| 1  | 2233 | 7695 | 1944 | 6675 |
| 2  | 2119 | 7672 | 2007 | 6739 |
| 3  | 2136 | 7652 | 1997 | 6841 |
| 4  | 2310 | 7692 | 1967 | 6708 |
| 5  | 2228 | 7765 | 1934 | 6798 |
| 6  | 2205 | 7690 | 1956 | 6908 |
| 7  | 2206 | 7927 | 1921 | 6931 |
| 8  | 2195 | 7953 | 1941 | 6824 |
| 9  | 2199 | 7995 | 1905 | 6788 |
| 10 | 2320 | 7900 | 1869 | 7016 |
| 11 | 2245 | 7886 | 1931 | 6911 |
| 12 | 2241 | 7755 | 2093 | 7124 |
| 13 | 2234 | 7831 | 2013 | 6921 |
| 14 | 2169 | 7856 | 2009 | 6958 |
| 15 | 11636736 | 10510154 | 10237027 | 9240316 |

Table 24. Total counts at choices XY (rows) for each configuration AB (columns).

|     | 00 | 10 | 01 | 11 |
|-----|----|----|----|----|
| 00  | 2296 | 647 | 672 | 2248 |
| 01  | 1796 | 762 | 717 | 1781 |
| 10  | 2332 | 324 | 282 | 2338 |
| 11  | 318  | 1908| 1896| 359  |

3.9.13: L.K. Shalm et al., Using Human-Generated Randomness to Violate a Bell Inequality without Detection or Locality Loopholes

The experiment is the standard two-party (Alice and Bob) Bell test, with two entangled (not maximally) photons detected at optimal polarization angles chosen by X = 0, 1 by Alice and Y = 0, 1 by Bob. Due to imperfect detection efficiency, the configuration differs from the ideal CHSH model. To increase statistics, each of the observers can detect a photon in one of the 16 time bins. Nevertheless, it allows to test if the correlations violate local realism, that is, existence of the joint probability of outcomes depending on the local choice. The influence of the remote choice is excluded by relativity, that is, the time between the choice and the end of the measurement is shorter than the light-speed signal.

We checked no-signaling, that is, if \( p(\ast B|0Y) = p(\ast B|1Y) \) and \( p(\ast |X0) = p(\ast |Y0) \) where A, B = 0, ..., 16 are the photon detections in the appropriate time bin, with 0 standing for no detection. We have taken into account the fact noted by the authors that X, Y = 0 are chosen about 5% more often than 1. As a stopping criterion, we took the last XY = 11 event.

Taking the last column \( \ast 0 \) or \( 0 \ast \) as a reference, we find in the independence test \( \chi^2 \approx 8 \) giving \( p \approx 0.005 \) for A = 11 and X = 0. Correcting by the look-elsewhere effect, this is increased to 32 giving \( p \approx 0.16 \), consistent with no-signaling assumption. It is also clear that there is a bias in the settings choice probabilities. The bias is independent \( (\chi^2 \approx 1) \) but different for the two parties (> 60 variances). It is also evident (but less significant) with the original authors’ stopping criterion giving \( N_{\chi^2} = 10125716 \) and \( N_{\eta} = 10105777 \approx 4.43 \) variances) for the human test. Within the available data, no signaling signatures have been found. However, the setting choice bias is asymmetric.

4. Discussion

In the data received from BBT experiments, there are observed disagreements with various simple assumptions. First, the human choice is not perfectly random but biased with more 0s than 1s (experiments 6 and 13). It is also highly anticorrelated, with very likely sequences 0101 and 1010 (experiment 1). The data from other experiments depend on too many parameters to confirm these observations. Nevertheless, a biased human choice can be simply incorporated into \( \eta \) in (3), so it does not affect our analysis based on independence tests. Second, the photon detection efficiency is often different for different detectors (experiment 6, communicated by the authors, also mentioned as possible by the authors of experiment 10\(^{48}\)) and dependent on the state of
the other part of the setup (experiment 2). Third, the actual quantum state and measurement axes are different from the ideal Bell case (experiment 6 and 12), probably by some phase drifts (as suggested in the case of 12 by the authors) or misalignments. Ignoring these effects could lead to apparent signaling, also in experiments 4, 5, 9, and 10.

These observations suggest for future tests of local realism: a) to narrow the problem of varying detection efficiency, for example, by its better control; b) to block known communication between detectors; c) to collect sufficient data to check no-signaling, preferably with closed loopholes; d) to focus more attention on diagnostic runs checking the input state, operation flow, and detectors. The last suggestion follows also from the general expectation from quantum tests of local realism—they are not only performed to violate local realism but also to confirm quantum predictions. In the future, one can also test if the same time-tagged stream of choices give the same statistical results for different experiments in separate locations. Due to the lack of global synchronization between experiments, we were unable to test it.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

Bell test, local realism, no-signaling

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