Isotropic and anisotropic behavior of SrPt$_3$P superconductor

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The orientation-specific response of SrPt$_3$P single crystals in normal and superconducting states was probed for different orientations between the crystallographic axes and the applied DC magnetic field. Upper critical field, $H_{c2}(T)$, was estimated using different criteria from four-probe resistivity, $\rho(T)$. London penetration depth, $\lambda(T)$, was evaluated from the magnetic susceptibility of the Meissner-London state measured using a tunnel-diode resonator. Whereas $H_{c2}(T)$ and normal-state $\rho(T)$ are nearly identical comparing $H \parallel c-$axis, and $H \parallel a-$axis measurement, the corresponding components of the London penetration depth are quite different for different directions of a magnetic flux penetration. Intriguingly, $\lambda_{ab}(T)$ is nearly exponentially attenuated upon cooling signaling a full gap in the $ab-$plane, but $\lambda_c(T) \sim T^2$ at low temperatures implying the nodal gap. Moreover, the anisotropy, $\gamma(T) = \lambda(T)/\lambda_{ab}(T)$ shows non-monotonic temperature dependence. Using recent theoretical analysis, we suggest that SrPt$_3$P is a single-gap superconductor with a fully gapped Fermi surface everywhere except for the two polar nodes along the $c-$axis. At the same time, in this multi-band material, band-specific dirty and clean scattering limits have nontrivial consequences for the measured quantities.

I. INTRODUCTION

Superconductivity in platinum-based phosphides, APT$_3$P (A = Sr,Ca,La), was discovered by Takayama et al. in 2012 [1]. These compounds have distorted antiperovskite structure, similar to non-centrosymmetric superconductors LaPt$_3$Si and CePt$_3$Si. They received much attention for the large variation of the electron-phonon (EP) coupling strength from weak to strong with the EP coupling constant values of, $\lambda_{EP} = 0.57, 0.86$ and $1.33$, for LaPt$_3$P, CaPt$_3$P and SrPt$_3$P, respectively [2,3].

SrPt$_3$P has $T_c \approx 8.4$ K, the highest among the 5d electron superconductors. From the heat capacity measurements, Takayama et al. concluded that SrPt$_3$P is an s-wave superconductor with strong coupling attributed to soft phonon modes, with the characteristic ratio, $2\Delta_0/k_BT_c \sim 5$. This scenario was further supported by the nuclear magnetic resonance (NMR) measurements of the Knight shift, though a Hebel-Slichter peak was not observed [4], which can also be due to the enhanced phonon damping. Nonlinear magnetic field dependence of the Hall resistivity was interpreted as coming from the multiple pockets of the Fermi surface, perhaps hinting at the multi-band superconductivity [4]. Hu et al. investigated the effect of Pd-doping in polycrystalline SrPt$_3$P and found that there is a complex interplay between electron correlations, electron-phonon coupling, and spin-orbit coupling [5].

To probe the pairing nature of SrPt$_3$P, London penetration depth has been measured in polycrystalline samples using transverse-field $\mu$SR [6]. Combined with the measurements of the critical fields the data led to a suggestion that SrPt$_3$P is a two-band superconductor with equal gaps, but different coherence lengths within different Fermi surface sheets.

On the theoretical side, from first principle calculations Chen et al. suggested unusual superconductivity in SrPt$_3$P caused by a charge density wave and strong spin-orbit coupling [7]. In contrast, Subedi et al. concluded that SrPt$_3$P is a conventional s-wave superconductor where spin-orbit coupling plays only “a marginal role” [2]. Many more theoretical works studying the electronic structure and phonon modes followed [8–13].

In this situation, it is important to not only establish the overall thermodynamic behavior of SrPt$_3$P, but also determine temperature-dependent anisotropy of the superconducting order parameter. This requires crystals with sufficiently large dimensions for all directions. The growth of SrPt$_3$P single crystals is non-trivial and requires high-pressure high-temperature synthesis, similar to MgB$_2$. Single crystals of SrPt$_3$P were synthesized in 2016 [14] and they are used in this study.

Here we investigate the anisotropy of the superconducting order parameter in SrPt$_3$P crystals by measuring different components of the London penetration depth, $\lambda(T)$, the upper critical fields, $H_{c2}(T)$, and temperature-dependent resistivity, $\rho(T)$, of single crystals of SrPt$_3$P in different orientations between the applied magnetic field and crystallographic axes. We developed a novel approach to estimate the components of London penetration depths from high-resolution tunnel diode resonator (TDR) measurements of the magnetic susceptibility and analyze the results in terms of temperature-dependent variations of $\lambda_i(T)$ ($i = a, b, c$) and anisotropy, $\gamma_i = \lambda_i(T)/\lambda_{ab}(T)$. Our analysis suggests that the Fermi surface of SrPt$_3$P is fully gapped everywhere, except for

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the poles that may have point-like nodes with the angular part of its order parameter, \( \Omega(\theta) \sim \sin \theta \), where \( \theta \) is the polar angle in spherical coordinates.

II. EXPERIMENTAL

Single crystals of SrPt\(_3\)P were grown under high pressure in a cubic anvil cell and mechanically separated from the flux as described in detail elsewhere [14]. As often is the case in high-pressure synthesis, the sample shape does not allow precision assignment of the facets to the crystallographical axes and the situation is further complicated considering nearly cubic crystal structure, \( a = b = 5.8 \ \text{Å}, c = 5.4 \ \text{Å}[14] \). Nevertheless, based on the x-ray Laue measurements we assign the short side of the sample to the \( c \)-axis and the longest side to a crystallographic \( a \)-axis. As shown below, the normal state transport is quite isotropic and, therefore, axes labeling is not important for the discussion of the superconducting state anisotropy and can be taken simply with respect to crystal facets. In other words, simple axes rotation cannot induce the anisotropy in otherwise isotropic crystal.

Crystals for the resistivity measurement were between 0.5 and 1 mm in the long side. For the electrical transport measurements, silver wires of 50 \( \mu \text{m} \) diameter were soldered to the sample using tin flux [15, 16], in a standard 4-probe resistivity configuration. The contacts had resistance in the m\( \Omega \) range. Resistivity measurements were performed in a Quantum Design PPMS down to 1.8 K in magnetic fields up to 9 T in configurations \( H \parallel a \)-axis, \( H \parallel b \)-axis and \( H \parallel c \)-axis. The upper critical field \( H_{c2} \) was determined from electrical resistivity measurements using different criteria as described in the text below.

The temperature variations of the components of the London penetration depth were extracted from the measurements of the normalized magnetic susceptibility as described in the Appendix. The susceptibility was inferred from the frequency shift of a self-oscillating tunnel diode resonator (TDR) when sample temperature varied down to 400 mK (~0.05 \( T_c \)) [17, 18]. The TDR circuit resonates at 14 MHz and the frequency shift is measured with one part per billion (ppb) precision. Its inductor coil generates ac magnetic field, \( H_{ac} < 20 \ \text{mOe} \), so that the sample is always in the Meissner state at the temperatures of interest. The size of the sample was 600 ± 5 \( \mu \text{m} \) (\( a \)-axis), 250 \( \mu \text{m} \) ± 5 \( \mu \text{m} \) (\( c \)-axis), 155 ± 5 \( \mu \text{m} \) (\( b \)-axis). The sample was mounted on a 1 mm diameter sapphire rod and inserted into a 2 mm in diameter inductor coil. The coil and the sample were in vacuum in a \( ^3 \text{He} \) cryostat with sample and TDR circuit temperatures actively stabilized by independent closed-loop Lakeshore controllers. The penetration depth was measured in two different configurations, \( H_{ac} \parallel c \)-axis and \( H_{ac} \parallel b \)-axis to minimize the difference in the demagnetization corrections. The resonant frequency of the circuit containing the sample depends on the mutual inductance of the sample in the coil and it is straightforward to show that the frequency shift compared to the empty resonator is proportional to the total magnetic susceptibility of a sample up to a calibration factor that depends on the sample dimensions, demagnetizing factor and parameters of the coil. The calibration constant is established for each experimental run by mechanically pulling the sample out of the coil at the base temperature of about 400 mK. Technical details of the technique are provided elsewhere [18–21]. Detailed derivation, estimates and step-by-step analysis leading to the individual components of the London penetration depth are discussed in the Appendix. This general method only requires magnetic susceptibility measured in different orientations. Indeed, precision of the measurements determines the precision of the final result. However, for estimates, even standard AC susceptometry can be used.

III. RESULTS

A. Upper critical field

Figure 1 shows the temperature-dependent resistivity of SrPt\(_3\)P single crystal and compares it with the polycrystalline sample data reported by Takayama et al. [1] As shown in the upper-left inset, the single crystalline sample of the present study has much lower resistivity over the whole temperature range. The normalized resistivity in the main panel shows similar temperature dependence between two samples, but the residual resistivity ratio is higher in a single crystal by a factor of two, RRR \( \approx 10 \). The right-bottom inset zooms at the superconducting transition. Note that the transition temperature, \( T_c \), is also somewhat higher in the single crystalline sample. This may be due to a pair-breaking effect of the non-magnetic scattering expected for nodal superconductors.

The temperature-dependent resistivities, \( \rho(T) \), measured in the vicinity of the superconducting transition in magnetic fields applied in three different orientations, \( H \parallel a, b, c \)-axes, respectively, are plotted in Fig. 2. A very close to isotropic behavior is self-evident. Perhaps, only \( c \)-axis curves become more rounded than the other two orientations in fields above 2 T. Finally, Fig. 3 summarizes the temperature-dependent upper critical field, \( H_{c2}(T) \), estimated using four different criteria as shown in the inset in Fig. 3(b). Top panel, Fig. 3(a), shows \( H_{c2}(T) \) defined by the deviation, onset, and offset criteria. Expectedly, the onset criteria produces values close to the literature data on polycrystalline samples [1, 6], shown by open crossed symbols in both panels of Fig. 3. The commonly used mid-point transition data are plotted in Fig. 3(b) by different symbols and the same colors as defined in panel (a). We conclude that \( H_{c2}(T) \) is very similar between all three orientations regardless of the criterion used. Some difference is observed in the amount of smearing with the largest being for the magnetic field along the \( c \)-axis. We note that effective demagnetiza-
trical transport, as easily follows from the shape of the isotropic upper critical field suggests nearly isotropic electrical transport, as easily follows from the shape of the two quantities in two principal directions. The nearly flat behavior of the resistivity in the superconducting state is indicative of the two directions (solid line). In three orthogonal orientations, the resistivity is obtained on a polycrystalline sample by Takayama et al. (red squares) [1]. The upper inset shows the normalized data and the lower inset zooms at the superconducting transition.

We also note that often made assertion that the upward (positive) curvature of $H_{c2}(T)$ implies multi-band superconductivity as claimed, for example, in the cited above Ref. [6], is not supported by the theory [24–26]. Similar behavior, especially in the limited temperature interval, may be the result of complications of the Fermi surface topology, anisotropy of the order parameter, or nonmagnetic scattering even in a single-band material [25]. In fact, usually $H_{c2}(T)$ in multi-band superconductors shows “normal” concave behavior and is no different in shape from single-band result [24]. Similarly, temperature-dependent anisotropy, $\gamma(T) = \xi_a/\xi_{ab}$, does not imply multi-band superconductivity and is commonly found in single-band superconductors in different circumstances [26]. On the other hand, there may be consequences of multi-band character on the response to scattering as discussed below. The important conclusion from the resistivity measurements is that, as expected from the crystal structure, this material is practically isotropic. Of course, the electronic band structure is quite complicated, but for the analysis of transport and thermodynamic properties, we may approximate it by a Fermi sphere, an approach justified by the lack of anisotropy of the upper critical fields. We will use this fact for the analysis of the London penetration depth, which in the clean limit depends only on the normal state parameters and should become isotropic on a sphere in the dirty limit.

B. London penetration depth

We now examine the temperature dependence of the London penetration depth, $\lambda(T)$, which is linked directly to the structure of superconducting gap as it depends sensitively on the thermally excited quasiparticles. High-resolution magnetic susceptibility was measured at zero applied dc field in two orientations of small excitation ac field, $H_{ac} < 20$ mOe, along the $b-$ and along the $c-$axis as discussed in the Experimental section above. For this crystal structure we do not expect noticeable difference between $\lambda_b(T)$ and $\lambda_a(T)$, which reduces the analysis to only two components, in-plane $\lambda_{ab}(T)$, and out-of-plane $\lambda_c(T)$. The step-by-step procedure of the extraction of these components is given in the appendix. The Appendix describes in detail the evaluation of in general three components of $\lambda_i(T)$, $i = a,b,c$ from the magnetic susceptibility, $\chi_i(T)$, measured along three orthogonal directions $i$. The appendix contains all the illustrations required. Here we use the results of this data analysis.

Figure 4 shows low-temperature variation of the two components of London penetration depth, $\lambda_a(T)$ (filled green circles) and $\lambda_c(T)$ (blue line). This way of presenting was chosen to distinguish otherwise almost identical curves. As shown in the appendix, magnetic susceptibilities, $\chi_i(T)$, are quite different. This is because...
FIG. 3. (Color online) Upper critical field, $H_{c2}(T)$ of SrPt$_3$P single crystal measured with magnetic field applied along three principal directions, $H \parallel a$−axis (green symbols), $H \parallel b$−axis (red symbols) and $H \parallel c$−axis (blue symbols). The inset in panel (b) shows four different criteria used to define $H_{c2}$. Upper panel (a) shows deviation (circles), onset (squares) and offset (triangles) readings. Lower panel (b) shows commonly used midpoint-defined $H_{c2}$. The results are compared with the data on polycrystalline samples, from Takayama et al. (crossed circles) [1] and Khasanov et al. (crossed squares) [6].

We attempted fitting the data with a standard BCS expression, $\Delta\lambda(T) = \lambda(0) \sqrt{\pi \Delta(0)/2T} \exp(-\Delta(0)/T)$, but it did not produce satisfactory results. That is, the fit can be performed at least for one curve, $\lambda_{ab}(T)$, but the extracted gap, $\Delta(0)$, and penetration depth $\lambda(0)$ are unreasonable. London penetration depth depends on the integral of the projection of the Fermi velocity over the whole Fermi surface and having nodes or anisotropy in some parts of the order parameter has effect on all components of $\lambda$ [20, 21]. In addition, scattering may play a significant role and, depending on the order parameter structure, even non-magnetic (potential) scattering can be pair-breaking, which significantly affects the penetration depth and effectively turns exponential to a power-law behavior at low temperatures [20, 21]. We therefore use power-law fitting, $\lambda \sim T^n$ to compare the ease with which quasiparticles are created in different directions. The inset in Fig. 4 shows the large exponent, $n = 5.1$, for $\lambda_{ab}(T)$. Numerically, powers above $n = 5$ are indistinguishable from exponential. On the other hand, a close to $n = 2$ exponent in the temperature dependence of $\lambda_c(T)$, signifies either line-nodal gap in dirty limit, the multi-band sign-changing order parameter, such as $s^±$ in the dirty limit, or a point node [20, 21]. However, if the order parameter had a line node or was an $s^±$ type on a three dimensional Fermi surface, then both components of the penetration depth would show similar low power $n$. Our data seem to suggest a case with the point node along the $c$−axis.

FIG. 4. (color online) Two components of the London penetration depth, $\lambda_{ab}(T)$ (green circles) and $\lambda_c(T)$ (blue line), extracted from the measurements of magnetic susceptibility in two different orientations (every 5th point is plotted for clarity). Inset zooms at the low-temperature region of the same curves, green and blue symbols, respectively. The red dashed lines are the fits in the range of $T \leq T_c/3$, to the power-law: $\Delta\lambda \sim T^n$. 

when full temperature range is shown, the penetration depth increases and saturates at the value corresponding to the skin depth, which, as we saw from transport measurements, is the same in all orientations. The general treatment of temperature-dependent susceptibility and impedance taking into account normal quasiparticles is described elsewhere [27]. As mentioned above, in order to distinguish between different components of $\lambda$, high resolution measurements of magnetic susceptibility down to low temperatures are needed. The inset in Fig. 4 shows the same curves as in the main frame, but zoomed at the low-temperature range. Following the isotropic weak-coupling BCS asymptotic behavior, which is valid up to approximately $T_c/3$ we also limit our analysis to this range as shown schematically in the inset.
IV. DISCUSSION

Here we try to understand our quite constrained results. On the one hand, normal-state transport and $H_{c2}$ show practically isotropic behavior. On the other, London penetration depth shows very different low-temperature behavior along different directions. Moreover, if we plot the anisotropy of the penetration depth, $\gamma_\lambda(T) = \lambda_c(T)/\lambda_{ab}(T)$, it reveals a non-monotonic behavior as function of temperature as shown in Fig. 6. Recent theoretical analysis shows that temperature-dependent $\gamma_\lambda(T)$ does not automatically imply a multi-band pairing. Instead, some anisotropic single-band order parameters lead to a substantial temperature variation of the anisotropy [28].

In order to proceed with the analysis, we need to know $\lambda_i(0)$. If, as the zeroth approximation, we take the value of 134 nm obtained from the $\mu$SR measurements [6], the anisotropy shows substantial temperature dependence. As remarked before, this does not signify multi-band superconductivity. However, it means we are not in the dirty limit, because the dirty-limit values will be the same as our isotropic transport measurements. Approaching the transition, the anisotropy collapses to the isotropic value. This means that at $T = 0$ we may consider clean-limit $\lambda$, which depends only on the Fermi velocities and no superconducting parameters. But we know from transport that normal-state anisotropy is close to 1 at $T=0$. Considering all these results, we arrive to the following physical model.

- Both, the penetration depth and the coherence lengths are isotropic at $T = 0$ and at $T = T_c$, $\gamma(0) = \gamma(0) = \gamma(T_c) = \gamma(T_c) = 1$.
- At $T = 0$, the system is in clean limit with isotropic Fermi surface.
- At $T_c(H)$, the system is in dirty limit, which explains the isotropic $H_{c2}(T)$, and, as a result, $\gamma(0) = \gamma(T_c) = \gamma(T_c) = 1$. Here $a, b, c$ indicate the directions of the applied field with respect to the crystallographic axes. By definition, $H_{c2,ab} = \phi_0/(2\pi\xi_c\xi_{ab})$ and $H_{c2,ac} = \phi_0/(2\pi\xi^2_{ab})$, where $\phi_0$ is the magnetic flux quantum.
- Clean limit behavior (at low temperatures) and observed temperature-dependent $\lambda_i(T)$ is consistent with the point nodes along the $c-$axis.

Following [28], consider the simplest order parameter that satisfies these constraints. Using the usual ansatz of the separation of the temperature and angular parts of the order parameter, $\Delta(T, \vec{k}) = \Psi(T)\Omega(\vec{k})$, we take the angular part, we call an “apple”,

$$\Omega(\theta) = \sqrt{\frac{3}{2}} \sin \theta$$  \hspace{1cm} (1)
tering make this procedure quite unprecise. Yet, we are surprised to find such agreement for the simple order parameter and coarse assumption of the spherical Fermi surface.

Now we can plot the expected anisotropy of the London penetration depth as function of temperature. Surprisingly, the result, shown as thick solid line, is quite close to our experimental $\gamma(T)$ obtained with experimental $\lambda_i(0) = 134$ nm, see Fig. 6. Please note, there is no flexibility in calculated $\gamma(T)$, it varies from $1$ at $T = 0$ and grows to $\gamma(T_c) = \sqrt{2}$. For comparison, we show the calculations of $\gamma(T)$ using $\lambda_i(0) = 200$ nm obtained in working with Fig. 5. The result is not so good, but not too different either. It shows that the result is not too sensitive to the exact value of $\lambda(0)$ and we reproduce the main features. We conclude that the “apple” order parameter agrees reasonably well with our data, even considering substantial variation of the experimental $\lambda_i(0)$.

Now we shall try to understand the crossover to dirty limit approaching $T_c$. A similar crossover was suggested from the analysis of microwave surface impedance measurements [30] where the authors compare temperature-dependent coherence length, $\xi(T)$, and the mean-free path, $\ell$. Theoretically however, the “dirtyness” of the superconductor is determined by comparing the mean-free path and temperature-independent BCS coherence length, $\xi_0 = h v_F / \pi \Delta_0$ [31]. Therefore, their ratio does not depend much on temperature since in most materials resistivity saturates to residual impurity scattering at low temperatures. This saturation is obvious in our case of SrPt$_3$P from Fig.2. Let us estimate the relevant parameters for SrPt$_3$P. There is no published average Fermi velocity, but we know that the density of states at the Fermi energy is $N(E_F) \approx 3.95$ states/eV per unit cell [9]. Unit cell volume is $V_i \approx 180.29 \AA^3$ [14] and the effective mass varies from zero to $T_c$, $m = (2.25 - 2.46)m_0$ [9]. Therefore, the crude estimate from the isotropic Fermi gas formula, $v_F = (\pi^2 h^3)/(m^2 V_i) N(E_F)$ gives, $v_F \approx 4 \times 10^7$ m/s. Using the experimental [6, 14] and theoretical [10] gap amplitude, $\Delta_0 \approx 1.6$ meV, the BCS coherence length is $\xi_0 = h v_F / \pi \Delta_0 \approx 50$ nm. It is hard to reliably estimate the upper critical field, but even at $5$ T, there is a sign of superconductivity below $2$ K, see Fig.2. Considering scenarios of possible saturation at low temperatures and a direct linear extrapolation coarse estimate is, $H_{c2}(0) \approx 4 - 6$ T. The “clean” limit upper critical field is, $H_{c2} = \phi_0/(2\pi \xi_0^2) \approx 1.5$ T, where magnetic flux quantum, $\phi_0 = 2.07 \times 10^{-15}$ Wb. We can now estimate the mean free path as, $\ell = \xi_0 H_{c2}^{clean}/H_{c2}^{dirty}$, where $H_{c2}^{dirty} = \phi_0/(2\pi \xi_0 \ell)$ and we obtain, $\ell = 11 - 17$ nm. The actual coherence length, $\xi(0) = \sqrt{\phi_0/2\pi H_{c2}(0)} \approx 7 - 9$ nm. Finally, we can estimate the mean free path from a simple isotropic Drude approach using measured residual resistivity, $\rho(0) = 8.6 \mu\Omega\cdot\text{cm}$, which gives the coherence length, $\ell = 3\pi^2 h^3 (m v_F)^{-1} / \rho_0^2 \approx 14$ nm, which falls exactly in the middle of the independent estimate from $H_{c2}$ above. To summarize, we see that our sample appears to be in the moderate dirty limit if treated as an isotropic single-band metal.

However, temperature-dependent anisotropy, Fig.6, and the overall good agreement with the clean-limit calculations of the superfluid density, Fig.5 imply that it is still possible that multi-band character plays a role. A similar conclusion of the necessity to consider multi-band structure for SrPt$_3$P (but with different physics) was drawn in the field-dependent $\mu$SR studies [6]. In our hypothetical scenario, consider two bands with their own densities of states and Fermi velocities. Each band is characterized by its own BCS length corresponding to the size of the Cooper pair, $\xi_{0,i} = h v_{F,i} / \pi \Delta_{0,i}$. Importantly, there is still only one physical length scale determining the spatial variation of the order parameter and measurable physical coherence length, because, indeed, there is only one upper critical field [32, 33]. However, two bands may respond differently to scattering because as we discuss above, the criterion of “dirtiness” involves these “band” lengths. Due to strong inter-band interaction (coupling), the system is likely to have one single gap, but still have different $\xi_{0,i}$. Since our single-band estimates gave $\xi_0 \approx 50$ nm and $\ell \approx 10$ nm, not in the extreme difference, it is possible that $\xi_0 < \ell \approx \xi_{0,i}$, so that one band is in the clean limit and another in the dirty limit.

What should we expect in such situation? In zero magnetic field, the “clean” band shunts the “dirty” band. The superfluid density has two additive terms coming from each band [34], therefore the “clean” band contribution will still have significant anisotropy, but the second “dirty” band will contribute only isotropic background. For the upper critical field the situation is opposite - all we can measure is the higher $H_{c2}$, but this will be the one determined by the “dirty” band ($H_{c2}$ increase with scattering [25].)

V. CONCLUSIONS

Single crystals of SrPt$_3$P superconductor were studied by measuring the orientation-dependent resistivity and London penetration depth. A peculiar situation is encountered when the normal state resistivity and upper critical field are isotropic, but the London penetration depth has different temperature dependence in different directions. To evaluate the components of the London penetration depth we developed a general approach that uses precision measurements of anisotropic magnetic susceptibility, described in the Appendix. The temperature dependence of the London penetration depth anisotropy strongly suggests unconventional pairing with the order parameter having nodes along the polar directions, $\Omega(\theta) \propto \sin \theta$. Furthermore, to explain the difference between transport and London anisotropies, we suggest multi-band (but still a single effective gap) effects where one band is in the “dirty” limit and another is in the
FIG. 6. (color online) Anisotropy of the London penetration depth, $\gamma \lambda(T) = \lambda_c(T)/\lambda_{ab}(T)$, calculated using the experimental $\lambda_i(0) = 134$ nm [6] (blue circles), and $\lambda_i(0) = 200$ nm (green squares) which corresponds to the best agreement with the superfluid density, Fig. 5. Solid line is the numerical solution for the clean “apple” order parameter, Eq.(1).

“clean” limit. The former is probed by the upper critical field and the latter by the London penetration depth. While our interpretation is only one possible scenario, we hope it will motivate further detailed electronic band-structure calculations and estimate band-specific parameters related to the superconductivity of this material.

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Appendix: Meissner-London state of anisotropic cuboidal samples

1. Anisotropic magnetic susceptibility

The total magnetic moment of arbitrary shaped magnetic sample is given by [35]:

$$m = \int (\mu_0^{-1} B(r) - H_0) \, dV$$  \hspace{1cm} (A.1)

where $H_0$ is the applied magnetic field and the integration is carried over volume that completely encloses the sample. This equation is trivial in the infinite geometries (without demagnetization effects), but for arbitrary samples requires a rigorous derivation from the general equation for the magnetic moment via the integration of shielding currents, $m = \int [r \times j(r)] \, dV/2$ [35]. In fact, Eq.(A.1) includes the demagnetization effects of and can be used to define the effective demagnetization factors [35]. Initial magnetic flux penetration into a superconductor is linear in $\lambda/R$ if this ratio is much smaller than one. When penetrating flux sweeps about 1/3 of the sample dimension in the direction of penetration, highly non-linear (hyperbolic) functions take over so that diverging $\lambda/R$ results in zero magnetic susceptibility. For example, for an infinite slab of width $2w$, $\chi = (\lambda/w) \tanh(w/\lambda) - 1$.

However, for small $\lambda/R$, the penetration is linear and can be quite generally written as [36]:

$$\chi (1 - N) = D \frac{\lambda}{R} - 1$$

where $D$ is the dimensionality of the flux penetration and $N$ is the demagnetizing factor in the direction of the applied magnetic field. In particular, $D = 1$ for an infinite slab in a parallel magnetic field, $D = 2$ for an infinite cylinder in a parallel field and $D = 3$ for a sphere). Moreover, linear approximation means that we can simply calculate magnetic susceptibility as the ratio of shielded volume to the total volume. Owing to the exponential attenuation of the magnetic field from the surface, one can roughly assume a complete field penetration into the layer of depth $\lambda$ and no magnetic field at all beyond that. Then, the volume penetrated by the magnetic flux is determined by the corresponding components of the London penetration depth as shown in Fig.7.

FIG. 7. Definitions of symbols and directions in a cuboidal sample with different components of the London penetration depths.

Magnetic susceptibilities can be measured in three principal directions, so that, for example, $\chi_a$ is magnetic susceptibility measured with a magnetic field applied along the $a-$ axis. These definitions of magnetic
susceptibility includes demagnetization correction. This means that for any sample, components of the magnetic susceptibility are normalized so that, for example, $\chi_a = -1$ when $\lambda = 0$. For example, if total magnetic moment, $m(\lambda(T))$, is measured, then the normalized magnetic susceptibility is given by $\chi(T) = m(\lambda(T))/(m(\lambda = 0))$ where the denominator is the theoretical magnetic moment of a perfect diamagnetic sample of the same shape, $m(0) = -VH_b/(1 - N)$. Therefore, for any measurement, we need to know the value of the measured quantity for this ideal case of perfect screening.

For solid bulk superconductors showing nearly perfect diamagnetism at low temperatures, $\lambda_1(T_{min})/R_i \ll 1$ one can simply normalize the measured magnetization by the total variation of magnetization from zero screening to (almost) perfect screening. The details of the procedure depend on the measurement technique.

In case of DC magnetometry one can simply take the value of the magnetic moment at the lowest temperature and use it for normalization. However, the sensitivity and dynamic range are the main issues here, because it should be possible to detect the change of a magnetic moment when magnetic field penetrates into a depth of $\lambda$, which is about three orders of magnitude less than the total signal. To resolve the temperature variation of $\lambda(T)$ one needs to resolve the fractions of that signal. Moreover, in order to estimate London penetration depths, the sample must be in London-Meissner state, in a magnetic field much smaller that $H_{c1}$. Therefore, while theoretical possible, direct measurements of $\lambda(T)$ from DC magnetization are not practical at least with the commercial magnetometers.

(a) shows typical raw measurement of the frequency shift variation upon sweeping temperature. The calibrated susceptibility, needed for our analysis is shown in panel (b). Basically, London penetration depth dictates the depth of magnetic field penetration below $T_c$, whereas normal-metal skin depth, $\delta$, takes over above $T_c$. In order to determine the total screening, one can estimate the normal state screening from the independent resistivity measurements through $T_c$ and known frequency. This is not the most precise approach. Alternatively, the measurement device must allow for the extraction of the sample from the sensing coil in situ. This requires substantial modification of the experimental setup, but once built gives the ability to calibrate every measured sample. This technique was used in the present work. Details of the calibration of tunnel-diode resonator are given elsewhere [19].

2. London penetration depth

Assuming that we now have all three measurements of the normalized magnetic susceptibility in small magnetic fields, represented by a vector,

$$X = (1 + \chi_a, 1 + \chi_b, 1 + \chi_c)$$

it is straightforward to show that counting penetrated volume in each direction with the designations of Fig.7 and keeping only linear in $\lambda$ terms we readily obtain:

$$X = L \cdot \Lambda$$

where vector $\Lambda = (\frac{\lambda_a}{a}, \frac{\lambda_b}{b}, \frac{\lambda_c}{c})$ and the coupling London matrix elements are given by $L_{ij} = 1 - \delta_{ij}$ where $\delta_{ij}$ is Kronecker delta,

$$L = \begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{pmatrix}$$

The solution follows,

$$\Lambda = L^{-1} \cdot X$$

where the inverse of the London matrix is given by $L_{ij}^{-1} = 1/2 - \delta_{ij}$,

$$L^{-1} = \begin{pmatrix}
-1/2 & 1/2 & 1/2 \\
1/2 & -1/2 & 1/2 \\
1/2 & 1/2 & -1/2
\end{pmatrix}$$

The resulting solution is:

$$\begin{cases}
\frac{2\Delta a}{\lambda} = 1 - \chi_a + \chi_b + \chi_c \\
\frac{2\Delta b}{\lambda} = 1 + \chi_a - \chi_b + \chi_c \\
\frac{2\Delta c}{\lambda} = 1 + \chi_a + \chi_b - \chi_c
\end{cases}$$

FIG. 8. Measured signal, resonator frequency shift in this case, is used to calculate normalized magnetic susceptibility. See details in the text.

In the case of frequency-domain AC susceptibility measurements, such as tunnel - diode resonator used in this work, microwave cavity perturbation or even amplitude-domain AC susceptibility, sufficient sensitivity and the dynamic range can be achieved (although this requires significant effort) for precision measurements of $\lambda(T)$. However, one can no longer take the signal difference from above $T_c$ to the lowest temperature, because of the screening of the applied AC fields by the induced eddy currents. This situation is shown in Fig.8 where panel
This allows evaluating three principal components of London penetration depths from three independent measurements of the magnetic susceptibility. Indeed, it is hard find perfect samples with ideal geometry and, therefore errors in the amplitudes are inevitable and this is only an approximation. However, as we demonstrate in this work, this procedure allows finding non-trivial temperature dependencies of the penetration depth, especially when they are supported by independent measurements of, for example, anisotropic normal state resistivity and upper critical fields that are tied together by thermodynamic Rutgers relations.

3. Simplification for tetragonal crystals

A typical case of tetragonal system or close to tetragonal that is often used to describe majority of iron pnictides and high-\textit{Tc} cuprates, there are two principal values of the London penetration depth, in plane, $\lambda_a = \lambda_b = \lambda_{ab}$, and out of plane, $\lambda_c$. Note, however, that sample still has three different dimensions, $a, b, c$. This means that all three components of magnetic susceptibility will be different. In this case the above general solutions are simplified as:

$$
\begin{align*}
\lambda_{ab} &= \frac{ab}{a+b} (1 + \chi_c) \\
\lambda_c &= c (1 + \chi_a) - \frac{ac}{a+b} (1 + \chi_c) \\
\ldots &= c (1 + \chi_b) - \frac{bc}{a+b} (1 + \chi_c)
\end{align*}
$$

These are very useful formulas as they show that in order to evaluate in-plane London penetration depth one need to measure only $\chi_c$ which is what most experiments do. To obtain $c$–axis penetration depth one needs to measure perpendicular components $\chi_a$ and/or $\chi_b$. Having both will improve the accuracy of the estimate. An important quantity in determining the microscopic mechanisms behind unconventional superconductors is temperature dependence of penetration depth anisotropy and described procedure enables just that. Main text shows the step by step application of the described procedure in the case of SrPt$_3$P superconductor.
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