Approaching optimal entangling collective measurements on quantum computing platforms

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Entanglement is a fundamental feature of quantum mechanics and holds great promise for enhancing metrology and communications. Much of the focus of quantum metrology to date has been on generating highly entangled quantum states which offer better sensitivity, per resource, than what can be achieved classically. However, to reach the ultimate limits in multi-parameter quantum metrology and quantum information processing tasks, collective measurements, which generate entanglement between multiple copies of the quantum state, are necessary. Here we experimentally demonstrate theoretically optimal single-copy and two-copy collective measurements for simultaneously estimating two non-commuting qubit rotations. This allows us to implement quantum-enhanced sensing, for which the metrological gain persists for high levels of decoherence, and to draw fundamental insights about the interpretation of the uncertainty principle. We implement our optimal measurements on superconducting, trapped-ion and photonic systems, providing an indication of how future quantum-enhanced sensing networks may look.

Quantum-enhanced single parameter estimation is an established capability, with non-classical probe states achieving precisions beyond what can be reached by the equivalent classical resources in photonic [1–3], trapped ion [4, 5], superconducting [6] and atomic [7, 8] systems. This has paved the way for quantum enhancements in practical sensing applications, from gravitational wave detection [9] to biological imaging [10]. For single-parameter estimation, entangled probe states are sufficient to reach the ultimate allowed precisions. However, for multi-parameter estimation, owing to the possible incompatibility of different observables, entangling resources are also required at the measurement stage. The ultimate attainable limits in quantum multi-parameter estimation are set by the Holevo Cramér-Rao bound (Holevo bound) [11, 12]. In most practical scenarios it is not feasible to reach the Holevo bound as this requires a collective measurement on infinitely many copies of the quantum state [13–16] (see Methods M1 for a rigorous definition of collective measurements). Nevertheless, it is important to develop techniques which will enable the Holevo bound to be approached, given that multi-parameter estimation is fundamentally connected to the uncertainty principle [17] and has many physically motivated applications, including simultaneously estimating phase and phase diffusion [18, 19], quantum super-resolution [20, 21], estimating the components of a 3D field [22, 23] and tracking chemical processes [24]. Furthermore, as we demonstrate, collective measurements offer an avenue to quantum-enhanced sensing even in the presence of large amounts of decoherence, unlike the use of entangled probe states [25, 26].

To-date, collective measurements for quantum multi-parameter metrology have been demonstrated exclusively on optical systems [27–32]. Contemporary approaches to collective measurements on optical systems are limited in their scalability, i.e. it is difficult to generalise present approaches to measuring many copies of a quantum state simultaneously. The limited gate set available can also make it harder to implement an arbitrary optimal measurement. Indeed, the collective measurements demonstrated so far have all been restricted to measuring two copies of the quantum state and, while quantum enhancement has been observed, have all failed to reach the ultimate theoretical limits on separable measurements [33, 34]. Thus, there is a pressing need for a more

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versatile and scalable approach to implementing collective measurements.

In this work we design and implement theoretically optimal collective measurement circuits on superconducting and trapped-ion platforms. The ease with which these devices can be reprogrammed, the universal gate set available, and the number of modes across which entanglement can be generated, ensure that they avoid many of the issues that current optical systems suffer from. Using recently developed error mitigation techniques [35] we estimate qubit rotations about the axes of the Bloch sphere with a greater precision than what is allowed by separable measurements on individual qubits. This approach allows us to investigate several interesting physical phenomena: i) We demonstrate both optimal single-copy and two-copy collective measurements reaching the theoretical limits [33, 34]. We also implement a three-copy collective measurement as a first step towards surpassing two-copy measurements. However, due to the circuit complexity, this measurement performs worse than single-copy measurements. ii) We investigate the connection between collective measurements and the uncertainty principle. Using two-copy collective measurements, we experimentally violate a metrological bound based on known, but restrictive uncertainty relations [36]. iii) Finally, we compare the metrological performance of quantum processors from different platforms, providing an indication of how future quantum metrology networks may look.

Theoretical Results

In this work we implement theoretically optimal quantum circuits saturating the Nagaoka bound [33, 34], which sets an upper limit on the precision attainable with separable measurements. We consider the probe $|\psi\rangle = |0\rangle$, which experiences small rotations, $\theta_x$ and $\theta_y$, about the $x$ and $y$ axes of the Bloch sphere respectively before getting decohered (Fig. 1 (c) and (d)). For small rotations, the state becomes $\rho_1 \approx (1 - \epsilon)|0\rangle\langle 0| + \epsilon/2$, where $\epsilon$ is the decoherence strength. Such a noise model is relevant for quantum computing [37] and communic-
Figure 2: Surpassing single-copy limits through collective measurements. In all figures - - - and --- correspond to the mean squared error (MSE) attainable with separable measurements. MSEs below --- are forbidden by quantum mechanics. Error bars are obtained using the bootstrapping technique [52] and correspond to one standard deviation. All experimental points have error bars but some are smaller than the marker size. Each data point corresponds to the average of 400 individual experimental runs, each using 512 shots, as shown in the inset of (a) (see Methods M4). (a) and (c) show single-copy and two-copy estimates of $\theta$, respectively, both with and without error mitigation. Results for estimating $\theta_y$ are similar (Extended Data Fig. 1). (b) and (d) show the corresponding MSE. The distribution of MSE values follows the expected chi-squared distribution, shown in the inset of (d). The black circle in the inset corresponds to the mean MSE value. The results shown in (a)-(d) are for decoherence parameter $\epsilon = 0.5$ and are obtained on the F-IBM QS1 device. (e) Optimal single-, two- and three-copy measurements at different decoherence strengths, $\epsilon$. The pink, purple and blue markers correspond to experimental single-, two- and three-copy measurements respectively. For the superconducting devices all markers correspond to the precision after using error mitigation. The results of the AQTION trapped-ion processor for $\epsilon = 0.5$ are shown in the inset for clarity. (f) Bars are one minus the ratio of the Holevo bound to the $m$-copy Nagaoka bound, for $m$ up to and including 7, calculated theoretically at $\epsilon = 0.5$. Experimental points are one minus the ratio of the Holevo bound to the MSE obtained experimentally. $\blacklozenge$ corresponds to the precision which our three- and four-copy projective measurements can obtain in theory. The upper and lower $\blacklozenge$ are simulations based on a depolarising noise model with gate error rates of $5 \times 10^{-3}$ and $1 \times 10^{-3}$ respectively. Legend is the same as in (e).

The Nagaoka bound is given by

$$v_x + v_y \geq N_1 = \frac{4}{(1-\epsilon)^2},$$  

where $v_{x(y)}$ is the variance in the estimate of $\theta_{x(y)}$. This applies when the probe states are measured one by one (Fig. 1 (a)). We shall refer to measurements of this type as single-copy measurements. The two-copy Nagaoka bound is

$$v_x + v_y \geq N_2 = \frac{4 - 2\epsilon + \epsilon^2}{2(1-\epsilon)^2},$$  

which applies when we can perform a collective measurement on two copies of the probe, $\rho_2 = \rho_1 \otimes \rho_1$, which are entangled during the measurement (Fig. 1 (b)). These measurements are referred to as two-copy measurements. Finally, allowing for collective measurements on infinitely
many copies of the probe state, the Holevo bound is
\[ v_x + v_y \geq \mathcal{H} = \lim_{m \to \infty} m \times N_m = \frac{4 - 2\epsilon}{(1 - \epsilon)^2}. \] (3)

The hierarchy between the bounds is, $\mathcal{H} \leq 2N_2 \leq N_1$, with equality only for $\epsilon = 0$ or 1. Detail on the computation of the different bounds is given in Supplementary Note 1.

The Nagaoka bounds, Eqs. (1) and (2), can be saturated by positive operator valued measures (POVMs) in 2- and 4-dimensional Hilbert spaces respectively (detailed in Supplementary Note 2). For single-copy measurements, it is possible to measure $\theta_x$ and $\theta_y$ separately, with two different POVMs, each using half of the total probe states without any loss in precision (Fig. 1 (c)). For the two-copy measurement this is not possible; both parameters have to be estimated simultaneously to take advantage of the collective measurement. In order to implement the optimal POVMs, we find a unitary matrix which diagonalises each POVM in the computational basis. Using standard techniques from quantum computing, we then convert these unitary matrices to quantum circuits [39], which can be executed experimentally (Fig. 1 (e) and (f)). We present three- and four-copy POVMs, and the corresponding quantum circuits, which theoretically surpass the two- and three-copy Nagaoka bounds respectively, in Supplementary Notes 3 and 4.

We also investigate the asymptotic attainability of the Holevo bound, examining how closely measurements on a finite number of copies of the probe state can approach it. In Fig. 2 (f), we compute the Nagaoka bound for performing collective measurements on up to 7 copies of the probe state simultaneously, corresponding to a 128-dimensional Hilbert space [40].

**Experimental Results**

In what follows we will describe the results of experiments conducted on multiple quantum platforms. The superconducting processors used were the Fraunhofer IBM Q System One (F-IBM QS1) processor, 11 cloud-accessible IBM Q processors and the Rigetti Aspen-9 processor. The trapped-ion (AQTON) processor is described in Ref. [41] and the Jena quantum photonic processor (JenQuant) is described in Methods M2. We implement the circuits corresponding to the optimal POVMs, shown in Fig. 1 (e) and (f), on the superconducting and trapped-ion processors. Additionally, we implement the single-copy measurements on JenQuant. The specific circuit parameters are provided in Supplementary Note 4. The outcomes of each run of a circuit are input to an estimator function to return the estimated values $\hat{\theta}_x$ and $\hat{\theta}_y$. This allows the mean squared error (MSE) to be determined.

Our first experiment investigates one possible application of error mitigation to quantum metrology. The details of the error mitigation used are found in Methods M4, but it is essentially a calibration process based on known angles as shown in Fig. 1 (g). For this experiment, conducted on the F-IBM QS1 processor, the decoherence parameter is fixed at $\epsilon = 0.5$ and we estimate a range of $\theta$ values. This verifies the unbiased-ness of the estimator after error mitigation. Fig. 2 (a) and (c) show the average estimate of $\theta_x$, $\theta_y$ both before and after error mitigation, with single- and two-copy measurements respectively. The improvement offered by error mitigation, evident in these figures, is quantified by the MSE in Fig. 2 (b) and (d). Error mitigation cannot reduce the MSE below what is theoretically allowed by the Nagaoka bound, but it does enable both the single-copy and two-copy measurements to reach the corresponding Nagaoka bounds. Crucially, Fig. 2 (d) shows the advantage of the two-copy measurement, achieving a precision beyond what is classically possible over the range of $\theta$ considered. This is the first experimental demonstration of a measurement saturating the two-copy Nagaoka bound. Averaged over the entire range of $\theta$, the two-copy measurements show a MSE $19 \pm 4\%$ below the theoretical single-copy measurement limit, which is only $6 \pm 4\%$ larger than the Holevo bound. In contrast, when restricted to single-copy measurements, the MSE is guaranteed to be at least $33\%$ larger than the Holevo bound. The ability to measure a range of angles is important for practical applications of quantum-enhanced metrology.

**Optimal Single-, Two- and Three-Copy Measurements**

We next fix the rotations to $\theta_x = \theta_y = 0$ and demonstrate a quantum enhancement over a range of $\epsilon$ values. Fig. 2 (e) shows the (scaled) MSE attained on different platforms. Using the F-IBM QS1 device we are able to demonstrate a clear quantum enhancement across a range of $\epsilon$ values. The two-copy measurement on the F-IBM QS1 device shows a maximum advantage over the theoretical single-copy limit of $21 \pm 4\%$. In contrast, the Rigetti Aspen-9 superconducting device does not approach the theoretical limits for any of the measurements, likely due to the higher gate and readout error rates. Notably, both JenQuant and the AQTON processor are able to reach the theoretical single-copy measurement limits without any error mitigation. The AQTON processor does not however, reach the theoretical two-copy limits. The demonstration of quantum-enhanced sensing with highly mixed states showcases that collective measurements may provide metrological gain in real-world sensing applications where decoherence is unavoidable.

In Fig. 2 (e) and (f), we show the MSE of our three-copy measurement when implemented on the Rigetti Aspen-9 and F-IBM QS1 processors. In Supplementary
Note 6 we present further three-copy results for these and several other devices, all of which failed to reach the theoretical limit and display properties of a bad estimator. These experimental results are in qualitative agreement with simulations of three- and four-copy measurements based on the noise level expected for near-future quantum processors, also shown in Fig. 2 (f). From Fig. 2 (f), it is evident that for the problem we have considered, the benefit of three-copy measurements over two-copy measurements is marginal. This raises the question of whether measurements on many copies of a quantum state simultaneously are practically useful. In Supplementary Note 7 we present a similar problem, based on an amplitude damping noise model, where there is a sizeable gap between the two-copy Nagaoka and Holevo bounds, suggesting that collective measurements on many copies may be useful. With continually decreasing error rates, superconducting and trapped-ion devices may bridge this gap and approach the Holevo bound ever more closely. However, as the data from Fig. 2 (f) shows, there is a pertinent trade-off between what is gained by measuring more copies of the quantum state and what is lost by the increased experimental complexity.

Collective Measurements and the Uncertainty Principle

The uncertainty principle is one of the most fundamental features of quantum mechanics [17]. Recently, it has been observed that the original formulations of the uncertainty principle fail to hold in certain scenarios [42, 43], leading to the introduction of ‘universally valid’ uncertainty relations (UVUR) for operators [44–46]. In spite of the name, UVUR assume that measurements are carried out on single copies of the quantum state. This appears to be a natural assumption when considering how the measurement of one quantity disturbs any subsequent measurement of a second quantity. However, the same is not true when considering the precision with which two quantities can be jointly measured. Given this restriction, one might expect that UVUR can be violated through collective measurements.

Recently Lu and Wang extended the UVUR to quantum multi-parameter estimation [36], deriving a metrological bound on how well two parameters can be simultaneously estimated. We shall denote this as the LW uncertainty relation. For our problem this bound can be calculated as (see Supplementary Note 8)

$$\frac{1}{v_x} + \frac{1}{v_y} \leq (1 - \epsilon)^2,$$

(4)

which is saturated when $v_x = v_y = 2/(1 - \epsilon)^2$. The variances allowed by Eq. (4) coincide with our single-copy measurement variances. Indeed, our single-copy measurement variances, shown in pink in Fig. 3, verify the validity of UVUR in this scenario. However, our two-copy measurements implemented on the F-IBM QS1 processor were able to experimentally violate the LW uncertainty relation by more than 3 standard deviations as shown in purple in Fig. 3. The POVMs which give rise to the unbalanced variances are presented in Supplementary Note 9. To the best of our knowledge, this is the first time it has been observed that UVUR can be surpassed, either theoretically or experimentally. These results have significance for the manner in which the uncertainty principle is interpreted and suggest that tighter uncertainty relations are required when allowing for any measurement type. In Supplementary Note 10 we relate the violation of the LW uncertainty relation to the more common error-disturbance operator uncertainty relations.

Cross-platform Comparison

Our final experiment compares the performance of different platforms for estimating qubit rotations. This provides an indication of what resources may be used in a future quantum metrology network. For superconducting devices, we first perform simultaneous qubit rotation estimation using all non-neighbouring (pairs of) qubits,
to minimise cross-talk between qubits. The mean MSE and minimum MSE across all qubits is shown in Fig. 4 (a) and (b) for each device tested. Each MSE is averaged over estimating 5 angles in the range $\theta = -0.01$ to $0.01$, repeated 120 times for each angle. For the trapped-ion and photonic devices only one photon, ion or pair of ions was used, hence only the mean MSE is shown. We then repeat the experiment using only the best performing qubit(s), now applying error mitigation as shown in Fig. 4 (c) and (d). The benefits of error mitigation are most pronounced for the F-IBM QS1 processor as we had unrestricted access to this device. Having restricted access to a device means each experiment takes longer, hence the model for the device provided by error mitigation is likely to be less accurate by the end of the experiment.

**Discussion**

Superconducting and trapped-ion devices are natural platforms for attaining the maximal advantage of quantum metrology and quantum information tasks through collective measurements. By implementing collective measurements on pairs of quantum states, we have been able to perform quantum multi-parameter estimation with a precision that cannot be reached classically using the same resources. There are many scenarios where this work may prove beneficial, particularly when there is an intrinsic restriction on resources. One can envision an optical system connected to a quantum processor through optical-to-microwave converters [47]. With only a limited number of qubits, such a device could greatly enhance biomedical imaging or quantum communications, meaning these advantages may be leveraged with near-future technology. Furthermore, collect-
ive measurements can be beneficial for quantum tomography [48], entanglement distillation for quantum communication [49] and quantum illumination [50].

This work opens up a number of avenues for future investigation: a natural extension to using error mitigation for quantum metrology is error correction [51]. With the aid of the techniques presented here, it may be possible to demonstrate multi-parameter metrology which fully utilises quantum resources; benefiting from both entangled probe states and collective measurements. By simplifying our three-copy measurement circuit, the theoretical limits may be approachable with the present generation of quantum processors. It would also be pertinent to study further how gate error rates and circuit complexity need to scale to successfully implement many-copy collective measurements. Investigating further the connection between collective measurements and the uncertainty principle may reveal important aspects of fundamental physics and could lead to the development of tighter uncertainty relations which hold true for any measurement type. Finally, the ideal extension of our work is to demonstrate optimal collective measurements in a practical setting. We anticipate that our work brings this closer.

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AUTHOR CONTRIBUTIONS

PKL conceived the project. LC and SMA derived the theoretical results and designed the optimal quantum circuits. LC, DWB and SMA optimised the quantum circuits. TV ran the JenQuant experiment. CDM and IP ran the AQTION experiment. LC and FE ran the F-IBM QS1 experiment. LC performed the data analysis. LC, SY and SMA derived the bounds in Fig. 3. LC wrote the manuscript with contributions from all authors.

COMPETING INTERESTS

The authors declare no competing interests.

[1] Kacprowicz, M., Demkowicz-Dobrzański, R., Wasilewski, W., Banaszek, K. & Walmsley, I. Experimental quantum-enhanced estimation of a lossy phase shift. Nat. Photonics 4, 357–360 (2010).
[2] Slussarenko, S. et al. Unconditional violation of the shot-noise limit in photonic quantum metrology. Nat. Photonics 11, 700–703 (2017).
[3] Guo, X. et al. Distributed quantum sensing in a continuous-variable entangled network. Nat. Phys. 16, 281–284 (2020).
[4] McCormick, K. C. et al. Quantum-enhanced sensing of a single-ion mechanical oscillator. Nature 572, 86–90 (2019).
[5] Leibfried, D. et al. Toward Heisenberg-limited spectroscopy with multiparticle entangled states. Science 304, 1476–1478 (2004).
[6] Wang, W. et al. Heisenberg-limited single-mode quantum metrology in a superconducting circuit. Nat. Commun. 10, 1–6 (2019).
[7] Muessel, W., Strobel, H., Linnemann, D., Hume, D. & Oberthaler, M. Scalable spin squeezing for quantum-enhanced magnetometry with Bose-Einstein condensates. Phys. Rev. Lett. 113, 103004 (2014).
[8] Gross, C., Zibold, T., Nicklas, E., Esteve, J. & Oberthaler, M. K. Nonlinear atom interferometer surpasses classical precision limit. Nature 464, 1165–1169 (2010).
[9] Aasi, J. et al. Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light. Nat. Photonics 7, 613–619 (2013).
[10] Casacio, C. A. et al. Quantum-enhanced nonlinear microscopy. Nature 594, 201–206 (2021).
[11] Holevo, A. S. Statistical decision theory for quantum systems. J. Multivar. Anal. 3, 337–394 (1973).
[12] Holevo, A. S. Probabilistic and statistical aspects of quantum theory, vol. 1 (Springer Science & Business Media, 2011).
[13] Kahn, J. & Guţă, M. Local asymptotic normality for finite dimensional quantum systems. Commun. Math. Phys. 289, 597–652 (2009).
[14] Yamagata, K., Fujiwara, A. & Gill, R. D. Quantum local asymptotic normality based on a new quantum likelihood ratio. Ann. Stat. 41, 2197–2217 (2013).
[15] Yang, Y., Chiribella, G. & Hayashi, M. Attaining the ultimate precision limit in quantum state estimation. Commun. Math. Phys. 368, 223–293 (2019).
[16] Czarnik, P., Arrasmith, A., Coles, P. J. & Cincio, L. Simple mitigation of global depolarizing errors in quantum simulations. Phys. Rev. E 104, 035309 (2021).
[17] Vatan, F. & Williams, C. Optimal quantum circuits for general two-qubit gates. Phys. Rev. A 69, 032315 (2004).
[18] Erhart, J. et al. Experimental demonstration of a universal error–disturbance uncertainty relation in spin measurements. Nat. Phys. 8, 185–189 (2012).
[19] Rozema, L. A. et al. Violation of Heisenberg’s measurement-disturbance relationship by weak measurements. Phys. Rev. Lett. 109, 100404 (2012).
[20] Ozawa, M. Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement. Phys. Rev. A 67, 042105 (2003).
[21] Branciard, C. Error-tradeoff and error-disturbance relations for incompatible quantum measurements. Proc. Natl. Acad. Sci. 110, 6742–6747 (2013).
[22] Higginbotham, A. P. et al. Harnessing electro-optic correlations in an efficient mechanical converter. Nat. Phys. 14, 1038–1042 (2018).
[23] Massar, S. & Popescu, S. Optimal extraction of information from finite quantum ensembles. Phys. Rev. Lett. 74, 1259–1263 (1995).
the two-dimensional material hexagonal boron nitride is based on a single photon emitting colour centre in single parameter estimation was considered.

Again in this work, the term multi-copy measurements, the multi-copy part referring to the fact that multiple (separable) measurement outcomes are used in making a final decision. Finally, Refs. [56, 57] examine multi-copy discrimination uses separable measurement thus simultaneously measures N copies of the quantum state simultaneously. An N-copy collective measurement of two quantum states is demonstrated. However, in principle the (0,2) and (1,2) schemes in Ref. [53] could be used for implementing collective measurements in the sense of our definition. Similarly, Ref. [54] refers to collective measurements as measurements of ensemble quantities of atoms, wholly unrelated to our terminology. In Ref. [55] multi-copy discrimination of two quantum states is demonstrated. However, this multi-copy discrimination uses separable measurements, the multi-copy part referring to the fact that multiple (separable) measurement outcomes are used in making a final decision. Finally, Refs. [56, 57] examine multi-copy metrology. Again in this work, the term multi-copy carries a different meaning compared to our work, as only single parameter estimation was considered.

M2. Photonic experiment

The Jena quantum photonic processor (JenQuant) is based on a single photon emitting colour centre in the two-dimensional material hexagonal boron nitride (hBN). The crystal defect introduces an effective two-level system into the bandgap that is excited optically. The emitter is fabricated by treating a multilayer hBN crystal with an oxygen plasma and subsequent rapid thermal annealing [58]. A suitable quantum emitter was therefore required to use a simple model of the form $\hat{\theta}_{x(y)} = \theta_{\text{noisy}, x(y)} + e_{x(y)}$, where $\theta_{\text{noisy}, x(y)}$ is the unmitigated $\theta_{x(y)}$ value predicted by the quantum processor.
and $c_{x(y)}$ is a constant. Detail on other possible models which were considered, but found to bias the estimator, is provided in Supplementary Note 5. We use 30 known $\theta$ values in the range $\theta \in [-0.2, 0.2]$ rad to determine a value for the model $c_{x(y)}$. An example of the model fitting is shown in Fig. 1 (g) for the F-IBM QS1 quantum processor. This model is then used to estimate some unknown angle $\theta = \theta_x = \theta_y$. Unless otherwise specified in the main text, the model is recalibrated after every 40 predictions of the unknown angle and the process is repeated to estimate each unknown angle 400 times. Our figure of merit is taken to be the average MSE over all 400 runs.

$$\text{MSE} = \frac{1}{400} \sum_{i=1}^{400} \left( (\theta_x - \hat{\theta}_{x,i})^2 + (\theta_y - \hat{\theta}_{y,i})^2 \right), \quad (5)$$

where $\hat{\theta}_{x(y),i}$ is the $i$th estimate of $\theta_{x(y)}$. To obtain each of the 400 estimates we average the results of 512 repetitions of the experiment for each of the single-copy circuits and for the two-copy circuit. For the three-copy circuit we average the results of 341 repetitions of the experiment to ensure equal resources are used in each experiment.

For the two-copy measurements in Fig. 3 with $v_x \neq v_y$, a slightly different error mitigation process was used. At the time this particular data was being taken it was not possible to recalibrate in between estimating the unknown angle. Hence, the calibration step was only performed once, immediately before estimating the unknown angle. To increase the utility of the error mitigation in this case, we used 30 known angles in the range $\theta \in [-0.05, 0.05]$ rad.

**DATA AVAILABILITY**

All data and codes are available at the following Github repository: https://github.com/LorcanConlon/Approaching-optimal-entangling-collective-measurements.

**CODE AVAILABILITY**

All data and codes are available at the following Github repository: https://github.com/LorcanConlon/Approaching-optimal-entangling-collective-measurements.

**EXTENDED DATA**

[53] Marciniak, Ch. D. et al. Optimal metrology with programmable quantum sensors. *Nature* **603**, 604–609 (2022).
[54] Bohnet, J. G. et al. Reduced spin measurement back-action for a phase sensitivity ten times beyond the standard quantum limit. *Nat. Photonics* **8**, 731–736 (2014).
[55] Jagannathan, A. et al. Demonstration of quantum-limited discrimination of multi-copy pure versus mixed states. *Phys. Rev. A* **105**, 032446 (2022).
[56] Tóth, G., Vértesi, T., Horodecki, P. & Horodecki, R. Activating hidden metrological usefulness. *Phys. Rev. Lett.* **125**, 020402 (2020).
[57] Trényi, R. et al. Multicopy metrology with many-particle quantum states. *arXiv preprint arXiv:2203.05538* (2022).
[58] Vogl, T., Campbell, G., Buchler, B. C., Lu, Y. & Lam, P. K. Fabrication and deterministic transfer of high-quality quantum emitters in hexagonal boron nitride. *ACS Photonics* **5**, 2305–2312 (2018).
[59] Vogl, T., Lecamwasam, R., Buchler, B. C., Lu, Y. & Lam, P. K. Compact cavity-enhanced single-photon generation with hexagonal boron nitride. *ACS Photonics* **6**, 1955–1962 (2019).
[60] Vogl, T., Knopf, H., Weissflog, M., Lam, P. K. & Eilenberger, F. Sensitive single-photon test of extended quantum theory with two-dimensional hexagonal boron nitride. *Phys. Rev. Res.* **3**, 013296 (2021).
[61] Jurcevic, P. et al. Demonstration of quantum volume 64 on a superconducting quantum computing system. *Quantum Sci. Technol.* **6**, 025020 (2021).
[62] Temme, K., Bravyi, S. & Gambetta, J. M. Error mitigation for short-depth quantum circuits. *Phys. Rev. Lett.* **119**, 180509 (2017).
Figure 5: Effect of error mitigation on estimation performance. Figs (a) to (d) show the estimated values of $\theta$, averaged over all 400 runs, before (blue squares) and after (red circles) applying error mitigation. Figs (a) and (b) ((c) and (d)) correspond to estimating $\theta_x$ and $\theta_y$ respectively with the optimal single(two)-copy measurement. Error bars are obtained using the bootstrapping technique [52] and correspond to one standard deviation. All results shown are for decoherence parameter $\epsilon = 0.5$ and are obtained on the F-IBM QS1 device.