Efficiency study of aeroelastic energy harvester via analytical approach

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Abstract. In the paper we examined the impact of structure parameters of aeroelastic energy harvester on the efficiency it provides. Study of system was preceded by deriving dimensionless analytical equations describing the power generated by the device. For this purpose, the Harmonic Balance Method method was used while adopting a simplifying assumption about the sinusoidal motion of the device. By analysing the obtained solutions, the characteristic quantities of system has been defined. By further calculations, impact of device parameters on operational flow velocity range was studied.

1. Introduction

The essence of aeroelastic energy harvesting is to convert mechanical energy of vibrations caused by flow on electrical energy. One of mechanisms of vibration excitation induced by constant flow is galloping – phenomena caused by occurrence of negative aerodynamic damping in the system. Mathematical model of this effect was first derived by Den Hartog in [1] and in the following years significantly extended i.a. in [2]. According to these works, we consider a body with one degree of freedom (translation parallel to axis Z), mounted on a damped spring system, subjected to flow in a direction parallel to the X axis (see Figure 1). The dynamics of the system is described by the equation:

\[ m \ddot{z}(\bar{t}) + c \dot{z}(\bar{t}) + s(z(\bar{t})) = F_z(\alpha) = \frac{1}{2} h \bar{\rho} U^2 C_z(\alpha) \]  

(1)

where: \( m \) – mass of the body [kg], \( c \) – damping coefficient [kg/s], \( s(\bar{z}(\bar{t})) \) – restoring force [N], \( z(\bar{t}) \) – displacement in Z direction [m], \( \dot{\bar{z}} \) and \( \ddot{\bar{z}} \) – first and second differential with respect to time, \( C_z(\alpha) \) – coefficient of aerodynamic force acting in the Z direction at \( \alpha \) angular orientation of the body [], \( F_z \) – aerodynamic force component acting in the Z direction [N], \( h \) – characteristic length of the body [m], \( \bar{\rho} \) – fluid density [kg/m²], \( U \) – flow velocity [m/s].
As reported in [3] and [4] the approximation of the $C_x(\alpha)$ function with a third order polynomial is sufficient for energy harvesting purposes. In addition, we assume that the flowed body has a symmetry axis in the direction normal to the flow. In this case, the even coefficients $a_n$ will equal to zero:

\begin{align}
C_x(\alpha) &= a_1 \alpha + a_3 \alpha^3 \\
a_1 &= \frac{dC_x}{d\alpha} + C_p \\
a_3 &= \frac{1}{6} \left( \frac{d^3C_x}{d\alpha^3} + \frac{d^2C_p}{d\alpha^2} \right)
\end{align} \tag{2}

Conversion of mechanical energy into electricity can be carried out using a variety of transducers, however, the most commonly used are electrostatic, electromagnetic and piezoelectric. Electrostatic may be the cheapest solution for large scale production [5], [6]. Electromagnetic [7], [8], due to their complicated construction turn out to be the most expensive but at the same time characterized by high efficiency. The multitude of possible transducer designs of this type increases their versatility, allowing their use in both small devices [9] and massive hydro or wind power plants. Prototypes, however, most often are consist of piezoelectric transducers [10-12], which is justified by great simplicity in their implementation while maintaining high efficiency. It will also be used in this work. Assuming that there is no structural damping ($c = 0$) in the system, the piezoelectric vibration energy harvester (PVEH) mathematical model takes the form [13]:

\begin{align}
m \ddot{\tau} + s (\tau) + \theta v (\tau) &= \frac{1}{2} \rho U^2 h \left( a_1 \frac{\dot{\tau}}{U} + a_3 \left( \frac{\dot{\tau}}{U} \right)^3 \right) \\
C_p \dot{v} (\tau) + \frac{v (\tau)}{R} - \theta \dot{\tau} &= 0
\end{align} \tag{3}

where: $v(t)$ – generated voltage $[V]$, $\theta$ – electromechanical coupling $[N/V]$, $R$ – circuit resistance $[\Omega]$, $C_p$ – circuit equivalent capacity $[F]$. Mathematical model (3) can be rewritten in dimensionless form by introducing dimensionless parameters:

\begin{align}
\ddot{x}(t) + S(x(t)) + \kappa n(t) &= a_1 \rho u \dot{x}(t) + a_3 \rho \frac{\dot{x}(t)^3}{u^3} \\
\dot{n}(t) + \frac{n(t)}{r} - \dot{x}(t) &= 0
\end{align} \tag{4}
where: \( x(t) = \frac{z(t)}{h} \), \( n(t) = \frac{v(t)}{\theta Cp} h \), \( \kappa = \frac{\theta^2}{Cp m \omega_n^2} \), \( \rho = \frac{h^2 \tilde{p}}{2m} \), \( r = Cp \omega_n R \), \( u = \frac{v}{h \omega_n} \), \( t = \omega_n \tilde{t} \) and \( S(x(t)) \) – dimensionless restoring force.

The efficiency of the device is largely determined by the geometry of the flowing body, which in the model (3) is represented by coefficients \( a_1 \) and \( a_3 \). In [14], elliptical cross-sections with different ratios between the length of the semi-minor axis and the semi-major axis were examined. A substantial set of aerodynamic coefficients of various typical sections is included in [15]. The maximum efficiency of PVEH depending on the shape of the flowing body was analyzed in [16]. The work [17] is devoted to the analysis of the impact of trapezoid arm inclination on its aerodynamic coefficients. Another factor strongly affecting the performance of PVEH is its mechanical structure. The typical one degree of freedom beam devices [18], [19] seem to give way in this respect to more complex multi-degree of freedom systems [20-22]. It is worth noting that devices showing also torsional vibrations should not be modeled using the Den Hartog’s hypothesis – for torsional vibrations the quasi-stationarity condition is never satisfied. The nonstationary flow model was used, among others in work [23], [24].

2. Analytical solution

The energy generated by the device depends on the flow velocity to which it is exposed. In order to determine this relationship, the mathematical model of the device (eq. 3) will be solved by utilizing the Harmonic Balance Method, adopting a linear model of restoring force \( S(x) = k x(t) \). Based on the analysis of numerical solutions, it was determined that it would be sufficient to set up solutions in the form:

\[
\begin{align*}
    x(t) &= A_x \cos(\omega t) \\
    n(t) &= A_N \cos(\omega t + \varphi)
\end{align*}
\]

where \( A_x \) – dimensionless amplitude of vibration, \( A_N \) – dimensionless voltage amplitude, \( \omega \) – dimensionless frequency and \( \varphi \) – phase shift are unknown quantities. Substituting the assumed solutions (5), (6) to mathematical model (4) leads to system of equations:

\[
\begin{align*}
    A_n \left( \omega \sin(\omega t + \varphi) - \frac{\cos(\omega t + \varphi)}{r} \right) - A_x \omega \sin(\omega t) &= 0 \\
    A_x \left( (\omega^2 - k) \cos(\omega t) - \frac{r \kappa \omega \cos(\omega t + \varphi)}{\sqrt{1 + r^2 \omega^2}} - up \omega \sin(\omega t) a_1 - A_x^2 \rho \omega^3 \sin(\omega t)^3 a_3 \right) &= 0
\end{align*}
\]

by collecting and balancing expressions with identical trigonometric functions, the following system of equations can be obtained:

\[
\begin{align*}
    A_N \omega \sin(\varphi) - A_N \frac{\cos(\varphi)}{r} &= 0 \\
    A_N \omega \cos(\varphi) + \frac{A_N \sin(\varphi)}{r} - \omega A_x &= 0 \\
    a_3 \rho \omega^3 A_x^3 &= 0 \\
    \omega^2 - k - \frac{r^2 \kappa \omega^2}{1 + r^2 \omega^2} &= 0 \\
    \frac{r \kappa}{1 + r^2 \omega^2} - a_1 up - \frac{3a_3 \rho \omega^2 A_x^2}{4u} &= 0
\end{align*}
\]

whose can be solved in terms of unknown quantities:
The average electrical power is defined as:

\[ P = \frac{1}{T} \int_0^T \frac{n(t)^2}{r} \, dt = \frac{A_N^2}{2r} \quad (10) \]

where \( T = \frac{2\pi}{\omega} \) stands for vibration period. By referring power \( P \) to the total flow power \( P_t = u^3 \rho \) together with the application of identities (9) allows the formal definition of device efficiency \( \eta \) as a function of flow velocity:

\[ \eta = \frac{P}{P_t} = \frac{2\kappa a_1 u \rho (1 + r^2 \omega^2) - r \kappa}{3a_3 u^2 \rho^2 (1 + r^2 \omega^2)^2} \quad (11) \]

3. Results

Figure 2 represents the function (11) for various system parameters. Three characteristic quantities can be observed on each of these curves. The first is critical velocity \( u_{cr} \), i.e. the minimum flow velocity at which the system generates energy. It is given by the expression:

\[ u_{cr} = \frac{1 + r^2 (k + \kappa) - \sqrt{k^2 r^4 + (r^2 \kappa - 1)^2 + 2k (r^2 + r^4 \kappa)}}{2r \rho a_1} \quad (12) \]
Another value that can be distinguished on the efficiency chart is the flow velocity for which the system achieves maximum efficiency – resonance velocity $u_R$. Interestingly, regardless of the selected set of system parameters, the identity $u_R = 2u_{cr}$ holds. The last parameter – peak efficiency $\eta_p$, also turns out to be constant for each configuration and takes the value:

$$\eta_p = \eta(u_R) = -\frac{a_1^2}{6a_3}$$

(13)

It is worth noting that the above formula shows that the peak efficiency $\eta_p$ of the device depends only on the resonator geometry. Another feature of the system that should be considered is the fact that attempts to select the device parameters in such a way that it has a low critical velocity also inevitably lead to a significant narrowing of the high efficiency bandwidth. This phenomenon can be traced by analyzing the impact of parameters on the vibration frequency $\omega$ and resonance frequency $\omega_R$. Utilizing the expression for $A_x$ from system of equations (9) and examining the condition $\frac{\partial A_x}{\partial \omega} = 0$, one can get an expression for the resonance frequency $\omega_R$:

$$\omega_R = \sqrt{\left(\frac{r^5K}{r^5K(r\kappa - u_p a_t) - r^2}\right)^{-1}}$$

(14)

Figure 3 presents the $\omega$ and $\omega_R$ graphs for parameter sets providing different critical velocities $u_{cr}$.

![Figure 3. resonance frequency $\omega_R$ and vibration frequency $\omega$ as a functions of flow velocity for different critical velocities $u_{cr}$](image)

As it results from the above graphs, with the decreasing $u_{cr}$ the vibration frequency is moving away from the horizontal $\omega_R$ asymptote, which due to the hyperbolic course of this function leads to faster receding values of both functions with progressive flow velocity. This effect is highly undemand, because devices designed for recovering energy from slow flows would only work effectively in a very narrow velocity band and even a slight deviation from the assumed operation conditions would cause a dramatic decrease in the efficiency of the device. Solutions to this problem can be found in multistable systems.

4. Conclusions

The paper presents the essence of operation aeroelastic energy harvester. On basis of this analysis a nonlinear mathematical model describing its motion has been proposed. This model was solved using the Harmonic balance Method, which allowed deriving the analytical expression describing the energy efficiency of the system. Analysis of the solution allowed to conclude that the flow velocity for which the device shows the highest efficiency, regardless of the system parameters, is twice the critical velocity. The key conclusion, however, is the fact that the maximum efficiency of the device depends only on the geometry of the resonator. It was also pointed out that effect of narrowing of the high efficiency band as critical velocity decreases is inevitable.
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