The Balmer decrement of SDSS galaxies

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ABSTRACT

High resolution spectra are necessary to distinguish and correctly measure the Balmer emission lines due to the presence of strong metal and Balmer absorption features in the stellar continuum. This accurate measurement is necessary for use in emission line diagnostics, such as the Balmer decrement (i.e. Hα/Hβ), used to determine the attenuation of galaxies. Yet at high redshifts obtaining such spectra becomes costly. Balmer emission line equivalent widths are much easier to measure, requiring only low resolution spectra or even simple narrow band filters and therefore shorter observation times. However a correction for the stellar continuum is still needed for this equivalent width Balmer decrement. We present here a statistical analysis of the Sloan Digital Sky Survey Data Release 7 emission line galaxy sample, using the spectrally determined Balmer emission line fluxes and equivalent widths. Using the large numbers of galaxies available in the SDSS catalogue, we determined an equivalent width Balmer decrement including a statistically-based correction for the stellar continuum. Based on this formula, the attenuation of galaxies can now be obtained from low spectral resolution observations. In addition, this investigation also revealed an error in the Hβ line fluxes, within the SDSS DR7 MPA/JHU catalogue, with the equivalent widths underestimated by average ~0.35Å in the emission line galaxy sample. This error means that Balmer decrement determined attenuations are overestimated by a systematic 0.1 magnitudes in Aᵣ, and future analyses of this sample need to include this correction.

Key words: galaxies: starburst – galaxies: statistics – galaxies: active – dust, extinction

1 INTRODUCTION

The Balmer lines are the most well known and observed emission lines in astronomy, being both strong lines in the optical and ubiquitous as they arise from recombination to the n = 2 level of the most common element, hydrogen. As the atomic structure of hydrogen is so well understood, the strength of the emission line fluxes can be well determined if the radiation field ionizing the hydrogen gas is known, and, importantly, the relative fluxes of the resulting hydrogen emission lines are only weakly dependent on the local conditions. Given that the ratios are well determined, the difference between the measured ratios of the Balmer lines and the intrinsic values expected can be used to determine the reddening of galaxies, or, more accurately, the ionized regions within them. In association with a attenuation/reddening-law and selective-to-total attenuation, RV, the reddening can then give the total attenuation of a galaxy (e.g. the oft used work of Calzetti [2001]). This possibility of using the hydrogen emission lines, in particular the ratio of the two strongest Balmer lines Hα and Hβ, to measure the reddening and attenuation has been known and utilized for many years (e.g. Bernard 1936, was one of the first mentions of the Balmer decrement being affected by the path length through an absorbing medium).

However, the accurate measurement of the Balmer decrement (i.e. the ratio of Hα/Hβ) requires the measurement of both the relatively weak Hβ line and the continuum underneath it to distinguish the line. Separation of the underlying continuum from the Balmer emission lines is vital as the metal absorption lines and, especially, Balmer absorption lines present in the spectrum of later-type stars act to weaken or hide the relative flux of the emission lines in the integrated spectra of galaxies. Such was shown by Liang et al. [2004], where the Hβ line was not observed in a selection of galaxies in low-resolution spectra (R = 150) from the the Canada-France-Redshift Survey, but moderate resolution spectra (R > 600) revealed the weak Hβ lines hidden by both dust and absorption lines. The exposure times needed to obtain sufficient spectral resolution to distinguish the emission lines from the underlying stellar continuum of galaxies means that studies of the Balmer decrement tend to be biased to high emission-line equivalent widths objects. While this bias may not be a serious issue, with dustier, more attenuated objects tending to have higher specific star formation rates and thus higher emission line equivalent widths (see e.g. da Cunha et al. [2010]), such biases do tend to limit samples for...
investigations of dust and star formation. This is especially so at higher redshifts where the high spectral resolution needed to resolve both the line and continuum limits surveys to the brightest objects.

Even with moderate resolution spectra, the Balmer emission line fluxes are still sensitive to the way the stellar absorption is accounted for, and this can be a substantial source of error, particularly for weak emission line sources.

It is these issues that motivates us to examine the possibility of determining the decrement from the equivalent widths of the Balmer emission lines. Emission line equivalent widths do not require high resolution spectra to distinguish the line from the continuum, allowing the use of low resolution spectra, such as at R ~ 300, enabling fainter objects or more objects to be observed for the same exposure time as high resolution spectra. Even at only R ~ 100, as will be available with the multi-object NIRSpec on JWST, the influence of Hβ on the interpretation of galaxy spectra can still be significant (Pacifici et al., in prep), and the line is still detectable at a signal-to-noise ~ 5 for some of the brighter emission-line galaxies. The issue at these low resolutions (i.e. R < 300) becomes one of distinguishing individual lines, such as [NII]λ6584Å from Hα, not detecting the lines.

To examine how the Balmer line equivalent widths can be used to determine the Balmer decrement, we use the Sloan Digital Sky Survey (SDSS Abazajian et al. 2009), which contains a large spectroscopic sample of emission line galaxies covering a wide range of galaxy types and properties, including attenuations. With such a wide range of galaxies, and reasonably high resolution spectra (R ~ 1900), the SDSS provides the perfect sample for examining this issue. With the large number of SDSS galaxies it is possible to obtain a statistically representative estimate for the correction factors needed (i.e. the Balmer absorption lines) and thus determine the Balmer decrement from equivalent widths alone.

This method should be seen as complementary to the stellar spectral synthesis continuum fitting methods widely used in the analysis of galaxy spectra. Codes such as PLATEFIT (Tremonti et al. 2004), used for the determination of the stellar properties in the SDSS MPA/JHU catalogue used here) and Lamarcille et al. 2006, applied this to lower resolution, higher redshift data in the VVDS sample), pPXF (Cappellari & Emsellem 2004), STARLIGHT (Cid Fernandes et al. 2005), STECKMAP (Ocvirk et al. 2006), and VESPA (Tojeiro et al. 2007), all use linear combinations of synthetic stellar population spectra (such as from Bruzual & Charlot (2003)) to fit the full observed spectra of galaxies using various optimized maximum likelihood approaches. These codes have been created to extract the maximum possible information from galaxy spectra given degeneracies and noise (see e.g. the discussion in Ocvirk et al. 2006). and thus are clearly the best approach when strong continuum is detected. Yet the amount of possible information to be extracted reduces with both decreasing signal-to-noise ratio and spectral resolution. In addition, these methods are limited by the available spectral libraries, which may not cover the full parameter range needed to match the observed galaxies, and may have intrinsic issues, as we demonstrate here in an issue we discovered in the course of this paper. Thus an empirical method as we explore here is fully complementary to the spectral synthesis methods used in most works.

In the following sections we introduce the Balmer lines, both in absorption and emission (22), the SDSS emission-line galaxy sample (33) provide a possible way to determine the Balmer decrement from the equivalent widths(44), and finally also point out an interesting problem with the fitting of the stellar continuum in the SDSS (33).

| $\lambda$ (Å) | 5000K | 10,000K | 20,000 K |
|-------------|--------|---------|---------|
| Hα         | 6562.80 | 3.04    | 2.86    | 2.75    |
| Hβ         | 4861.32 | 1.00    | 1.00    | 1.00    |
| Hγ         | 4340.46 | 0.458   | 0.468   | 0.475   |
| Hδ         | 4101.73 | 0.251   | 0.259   | 0.264   |

Table 1. Balmer lines, including rest-frame wavelengths (Å), and their ratios relative to Hβ for $n_e = 10^3$ cm$^{-3}$ and 3 different temperatures (values from Dopita & Sutherland (2003), based on data from Storev & Hummel (1995)).

2 THE BALMER LINES AND DECREMENT

2.1 Balmer emission lines

The Balmer emission lines in the interstellar medium arise predominantly from the recombination and subsequent cascade of electrons to the n = 2 level of hydrogen. While collisional excitation can also contribute to the Balmer line emission in hot media (see e.g. Ferland et al. 2009), photoionization and recombination are the predominant energetic processes in most galaxies.

As the atomic structure of hydrogen is so simple, it is possible to determine the exact electronic transition rates, and therefore the ratios of resulting emission lines from these transitions, as a function of physical conditions in the interstellar medium (see Menzel & Baker (1937), Baker & Menzel (1938) for the original theory, with updated work by Seaton (1959) and Storev & Hummel (1995), and treatments of this theory found in textbooks such as Osterbrock & Ferland (2006) or Dopita & Sutherland (2003)). In particular, two cases exist for which the Balmer decrement has been determined over a range of temperatures and densities: Case A and Case B. Case A assumes that a nebula is optically thin to all Lyman emission lines (i.e. lines emitted from transitions to n = 1 level of hydrogen), while Case B assumes that a nebula is optically thick to all Lyman lines greater than Lyα (i.e. transitions to n = 1 from levels n = 3 and above), meaning these photons are absorbed and re-emitted as a combination of Lyα and higher order lines, such as the Balmer lines. These two cases will lead to different intrinsic ratios for the Balmer lines, with variations of the same order as temperature effects (for other possible “Cases” of emission which may occur, see e.g. Ferland et al. 1999, Luridiana et al. 2009). While Case B is typically assumed for determining intrinsic ratios, in reality the ratio in typical H II regions lies between these two cases, and must be determined using radiative transfer codes such as MAPPINGS III (see e.g. Groves et al. 2004) or CLOUDY (Ferland et al. 1998).

In Table 1 we present the four strongest Balmer emission lines and their ratios relative to Hβ assuming Case B conditions. These ratios are only weakly sensitive to density, with the Hα/Hβ ratio at $T_e = 10^4$K equal to 2.86, 2.85, and 2.81 for the electron densities $n_e = 10^2$, $10^3$, and $10^4$ cm$^{-3}$ respectively, hence we only show the larger variation due to temperature here. For a full ratio description see Table B.7 in Dopita & Sutherland (2003), or Table 4.4 in Osterbrock & Ferland (2006). While these variations due to temperature and density are significant, they are still small relative to the effects of dust, as visible in the later sections, and hence strong diagnostics for the amount of reddening experienced by an emission line galaxy.
2.2 Balmer absorption lines

As discussed in the introduction, the fitting of the underlying stellar continuum is a vital step in determining line fluxes, especially for hydrogen (e.g. Balmer) and helium recombination lines, which have underlying corresponding absorption lines. The Balmer absorption lines arise from the absorption of light by hydrogen in the excited $n = 2$ level in the photospheres of stars. The strength of the absorption is dependent on both the effective temperature and gravity, as they require a significant fraction of hydrogen to be excited to the $n = 2$ level (see e.g., Table 4 in González Delgado & Leitherer 1999). The maximum equivalent widths occur around $T_{\text{eff}} \sim 9000K$, corresponding to early A-type stars.

For a simple, single-aged stellar population, the equivalent widths (EW) of the Balmer stellar absorption features depend on the age and, more weakly, on the metallicity, and vary from $\sim 2\,\AA$ to $\sim 15\,\AA$ with the maximum value occurring for stars aged around 500 Myr (i.e. dominated by the light from A & F stars). In Figure 1 we show the variation of the equivalent widths (EW) respectively for simple stellar populations as a function of age and metallicity (as labelled in the left hand of the figure in the H$\delta$ diagrams). We compare the four strongest lines; H$\alpha$, H$\beta$, H$\gamma$, and H$\delta$, using four models to determine the equivalent widths as labelled in the upper right; the Bruzual & Charlot (2003, BC03) stellar population synthesis code using the MILES Sánchez-Blázquez et al. (2006) and Stelib Le Borgne et al. (2003) stellar spectral libraries, and the 2008 version of the Charlot & Bruzual (in prep, CB08) code with the MILES library. Also shown by the dashed lines are the results from González Delgado & Leitherer (1999). The weak dependence on metallicity and the strong dependence on age, with the peak in EW at 0.5 – 1Gyr for all lines, are clearly seen for all models. The equivalent widths are reasonably similar between the dominant Balmer lines (i.e. H$\alpha$, H$\beta$, H$\gamma$, and H$\delta$) varying at most a factor of $\sim 2$ (Kauffmann et al. 2003c, González Delgado & Leitherer 1993, González Delgado et al. 1999).

Thus, as the equivalent widths of the stellar absorption features are approximately constant with wavelength while the relative strength of the Balmer emission lines decrease rapidly with decreasing wavelength (i.e. for the higher order lines), stellar absorption affects strongly the measurement of the Balmer decrement i.e. H$\alpha$/H$\beta$, H$\gamma$/H$\delta$. This is especially so for weak emission line galaxies where the stellar absorption features are relatively stronger and only H$\alpha$ is seen in emission. This relative importance of the effect of the stellar Balmer absorption on the emission lines is important when considering the Balmer ratios, as discussed in later sections.

3 THE SDSS SAMPLE

Within this work we base our findings on the spectroscopic data from the seventh Data Release of the SDSS (DR7; Abazajian et al. 2009), though we also refer to the fourth Data Release (DR4; Adelman-McCarthy et al. 2006) as well when necessary. The SDSS used a pair of multi-fibre spectrographs with fibres of 3” diameter. In most galaxies the fibres were placed as close as possible to the centres of the target galaxies. The flux- and wavelength-calibrated spectra cover the range from 3800 to 9200Å, with a resolution of $R \sim 1900$.

We obtain our emission line fluxes from the MPA/JHU analy-sis of the SDSS spectroscopic sample. This database contains, in addition to the emission line fluxes, derived physical properties for all spectroscopically observed galaxies in the SDSS DR7. The procedure for emission line measurement, detailed in Tremonti et al. (2004), was to correct the line fluxes for stellar absorption, fitting a non-negative combination of stellar population synthesis models from Charlot & Bruzual (in prep., CB08) for the SDSS DR4 release and Bruzual & Charlot (2003, BC03) for the SDSS DR4 release. The best-fitting stellar population model also places constraints on the star formation history and metallicity of the galaxy (see e.g. Gallazzi et al. 2005), and has been used to estimate stellar masses and star-formation histories (Kauffmann et al. 2003).

The equivalent widths of the Balmer lines we use here, also available as part of the MPA/JHU database (listed as (line name)_reqe in the gaL line data file), are computed from straight integration over the continuum-subtracted bandpasses listed in table. Note that, by definition, emission lines have negative values of equivalent width but for clarity in the rest of the paper we assign all emission lines a positive value. The continuum in this case is estimated using a running median with a 200 pixel window and does not properly account for stellar absorption. This measurement is representative of the cases where a more accurate determination of the stellar continuum, as done with the MPA/JHU database, is not more accurate determination of the stellar continuum, as done with the MPA/JHU database, is not

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1 The data catalogues are available from http://www.mpa-garching.mpg.de/SDSS/

2 The model spectra used were from an early version of the models and differ from what will be eventually published. The differences from BC03 are primarily due to different treatment of TP-AGB stars and that the empirical stellar library used was the MILES library rather than STELIB.

3 The spectra are available as part of the GALAXEV package, which is can be obtained from http://www2.iap.fr/users/charlot/bc2003/index.html
Table 2. Equivalent width bandpass

| Line | Centre (Å) | Lower bound (Å) | Upper bound (Å) |
|------|------------|----------------|-----------------|
| Hδ  | 4101.73    | 4092.0         | 4111.0          |
| Hγ  | 4340.46    | 4330.0         | 4350.0          |
| Hβ  | 4861.32    | 4851.0         | 4871.0          |
| Hα  | 6562.80    | 6553.0         | 6573.0          |

possible, such as with low resolution or low S/N data, and helps characterize the effects of stellar absorption on the lines.

As we concentrate on emission line galaxies in this work, in particular galaxies with measurable Balmer emission lines, we have placed cuts on the signal-to-noise (S/N) of the Balmer emission lines using the uncertainties given by the MPA/JHU catalogue. As discussed on the website, the listed uncertainties are formal, and likely underestimates, thus we increase the uncertainty estimates on the emission lines to take into account continuum subtraction uncertainties by the factors listed on the web site determined by comparisons of duplicate observations within the SDSS sample. Specifically for the Balmer lines, we multiply the line flux uncertainty estimates by a factor of 1.882. From the full SDSS emission line galaxy sample we define three galaxy samples depending on the lines and ratios being examined: SN(Hα, Hβ), SN(Hα, Hβ, Hγ), and SN(Hα, Hβ, Hγ, Hδ), where we require a S/N> 3 in two or more of the four strongest Balmer lines (Hγ, Hβ, Hγ, & Hδ). The S/N cuts are dominated by the weakest line in each sample due to the strong decrease in relative flux for the higher order lines, thus the inclusion of each higher order line biases the samples to higher equivalent widths of the Hδ emission line. As shown in figure 2 beginning from the full galaxy sample of SDSS DR7 (∼928,000 galaxies), approximately half are emission line galaxies (∼510,000, as measured by the presence of Hα in emission), with a broad spread of equivalent widths peaking at ∼20 Å (as measured from the local continuum, not corrected for stellar absorption). As each higher order Balmer line is included, the sample rapidly decreases and is biased to higher equivalent widths, with the SN(Hγ, Hδ, Hγ, Hδ) sample limited to ∼120,000 galaxies with EW(Hα)> 10 Å, with a distribution peaking at 32 Å (SN(Hγ, Hδ) has ∼392,000 galaxies, while SN(Hγ, Hδ, Hγ) has ∼241,000 galaxies). As the emission line EW(Hα) in a galaxy spectrum can be considered a proxy for the specific star formation rate (the current star formation rate relative to the total stellar mass, sSFR=SFR/M) of a galaxy, the bias in EW(Hα) means a bias to more “starforming” galaxies, which means a bias to lower-mass, lower-metallicity, bluer galaxies as shown in previous works (Brinchmann et al. 2003, Tremonti et al. 2004). This bias needs to be kept in mind when considering the diagrams and analysis in this work.

Figure 2 also reveals the limitation of low-resolution spectroscopy in finding all emission line sources, and thus the limitation of applicability of the method we explore here. Approximately 5% of the SN(Hα) sample, and even 0.7% of the SN(Hα, Hβ) sample, actually have EW(Hα) less than zero (i.e. the emission line is lost in the stellar absorption feature). These sources would never be picked up as emission line galaxies in low-resolution spectra, and the use of an emission-line equivalent width Balmer decrement to determine the attenuation would return spurious results.

Note that we have not included any cuts on redshift or the type of emission-line galaxy as in previous works on SDSS emission line galaxies (e.g. Kewley et al. 2004). Such redshift cuts are necessary to make certain that aperture effects do not play a part in the derived galaxy properties (by sampling only a small, biased part of the galaxy, as discussed in Kewley et al. 2005) and to prevent luminosity biases at the higher redshifts. Yet as we care only for the derived Balmer emission line fluxes and stellar equivalent widths these issues do not strongly affect our findings.

Separating emission-line galaxies by class (i.e. star-forming or AGN) is necessary when examining the structure of the forbidden emission lines (e.g. [O iii] λ5007 Å) which depend strongly on the dominant ionization mechanism in the gas (see e.g. Kewley et al. 2006), and when comparing physical galaxy properties with emission line properties (see e.g. Kauffmann et al. 2003[6]). However, as discussed in the previous section, the relative strength of the Balmer lines depend only weakly on local conditions, and will vary little in their intrinsic ratios between being photoionized by AGN or by OB stars (i.e. Hα/Hβ should be ~ 2.86 in star-forming galaxies and ~ 3.1 in galaxies ionized purely by an AGN). Thus, while this difference is significant and will have some bearing on the work in this paper, it is secondary to the effects of dust attenuation. Only when collisional heating dominates the atomic gas and the excitation of hydrogen, such as in shocks or clouds in hot gas, can the intrinsic ratios be significantly different (see e.g. Ferland et al. 2003), but these processes are not expected to dominate most galaxies within our sample. An additional reason to include AGN is that, in accordance with our main aim, separating out the contribution of AGN from lower S/N samples may prove problematic as the weak diagnostic lines become lost within the noise.

Figure 2. Distribution of the Hα emission line equivalent widths for the samples considered in this work. All four histograms are normalized to their peak value, with the total number of galaxies in each sample listed in the key in the upper left. The solid histogram shows all emission line galaxies in SDSS DR7 (defined by the presence of Hα in emission), and the three dashed curves show the distributions of samples defined by S/N cuts in the Balmer lines (as labelled in the upper left) considered in this work.

4 THE EQUIVALENT WIDTH BALMER DECREMENT

The issue of simply using directly the equivalent widths of the Balmer lines as proxies for the line fluxes when calcu-
The Balmer decrement of SDSS galaxies

Figure 3. The variation of the equivalent width based Balmer decrement \((\log[EW(H_\alpha)/EW(H_\beta)])\) versus the stellar continuum-subtracted flux based Balmer decrement for the SDSS SN(H_\alpha,H_\beta) sample. All axes are normalized by the intrinsic Balmer ratio, \((H_\alpha/H_\beta)_0 = 2.86\). In the left diagram, the EW(H_\alpha/H_\beta) has not been corrected for the difference in continuum flux at the H_\alpha and H_\beta wavelengths \((F_\alpha(H_\alpha)/F_\alpha(H_\beta))\), while in the right this is included. The upper left hand corner shows the median uncertainties for the sample, and the straight line indicates a 1:1 relation. The colours and associated colourbars indicate the median H_\alpha abs and log[EW(H_\alpha)] respectively in each pixel. The top axes give the resulting E(B-V) from the balmer decrement assuming the O’Donnell (1994) Galactic extinction curve.

Figure 4. 2D histograms of the distribution of the SDSS SN(H_\alpha,H_\beta) sample galaxies’ equivalent-width based Balmer decrements \((\log[EW(H_\alpha)/EW(H_\beta)])\), including a constant correction for stellar Balmer absorption, versus the stellar continuum-subtracted flux based Balmer decrements. Each figure shows a different bin of H_\alpha equivalent width, as labelled in lower right of each plot (in Å). As in figure 3, both axes are normalized by the intrinsic Balmer ratio, \((H_\alpha/H_\beta)_0 = 2.86\). The colours indicate the log of the number density in each pixel, as labelled by the colour bar on the right, with pixels with less than 5 galaxies excluded. The error bars in the upper left indicate the median uncertainty for each sample, with the total number of galaxies listed in the lower right of each plot. Note that the y-axis has a different correction for stellar Balmer absorption, \(R\), for each plot as indicated in the lower left of each plot, and that galaxies with H_\beta still in absorption after correction (i.e. \(EW(H_\beta) + R < 0\)) have been excluded from the sample.

When uncorrected for the underlying continuum variation there is a clear systematic offset between the H_\alpha and H_\beta wavelengths. In the left diagram of figure 3 we show the distribution of \(\log(\text{EW}(H_\alpha)) / \text{EW}(H_\beta)\), uncorrected for the continuum flux variation, while on the right the more accurate form of the EW Balmer decrement is used: \(\log[\text{EW}(H_\alpha)] / \log[\text{EW}(H_\beta)]\), where \(F_\alpha(H_\lambda)\) is the continuum flux at H_\alpha, determined from a 200 pixel median smoothing of the emission-line subtracted continuum.

When uncorrected for the underlying continuum variation there is a clear systematic offset of the EW Balmer decrement from the 1:1 relation of \(-0.1\) dex. Correcting for the continuum variation removes this offset, yet a significant spread remains. This spread is due to the effect of the stellar Balmer absorption features. Without these, \(\text{EW}(H_\alpha) \times F_\alpha(H_\alpha)\) should be, by definition, the flux of the line. The colors in figure 3 indicate the median H_\alpha absorption index \((H_\alpha)_\text{abs}\), Worthey & Ottaviani (1997), Kauffmann et al. (2003) for the sample in each pixel. As the figure shows, while the absolute strength of the stellar Balmer absorption features does play a part in the observed offset of the SDSS galaxies’ EW Balmer decrements, the dominant mechanism for the offset and spread is the relative strength of the stellar absorption features to the emission lines. This can be seen by the distribution of the equivalent width of the H_\alpha emission line indicated by the colours in figure 3, where there is a clear gradient of decreasing EW(H_\alpha) with increasing offset from the line. As the emission lines become weaker overall, the stellar absorption features, which are <10 Å as discussed in section 2, obscure a greater fraction of H_\beta relative to H_\alpha and therefore lead to a larger offset.

As discussed in the introduction, the best way to compensate for the effect of the stellar absorption features on the emission lines is to fit the stellar continuum as done within the MPA/JHU SDSS database. However, when only poor quality spectra are available such as for high redshift galaxies, the determination of the Balmer absorption features may be unreliable. One possible approach when faced with low resolution spectra is to assume that the absorption equivalent width is constant for both Balmer lines across the whole sample. While figure 3 clearly shows that the absorption EW is not the same for all the Balmer lines, it provides a first step when information is sparse and uncertainties large. When a constant Balmer absorption correction \(R\) is assumed for both EW(H_\alpha) and EW(H_\beta), a correction factor of \(R = 4\) Å is determined when the offset of the SDSS galaxies’ EW Balmer decrements to the measured H_\alpha/H_\beta ratios is minimized using an error-based weighting. The inclusion of this simple correction factor improves the situation when compared to that shown in figure 3 but with a still significant scatter of \(\sigma \sim 0.1\) dex around the expected value and an extended tail of objects towards lower values. Both the scatter and the tail arise due to the assumption of a constant offset (i.e. Balmer absorption) for the whole sample. The value determined is biased towards high EW(H_\alpha) galaxies, as these galaxies both dominate the sample and have lower uncertainties, as discussed in section 3.

When split into bins of different EW(H_\alpha), more accurate fits with differing correction factors are obtained. In figure 4 we show the fits for the galaxies split into four bins; \(0.5 < \log(\text{EW}(H_\alpha)) < 1.0, 1.0 < \log(\text{EW}(H_\alpha)) < 1.5, 1.5 < \log(\text{EW}(H_\alpha)) < 2.0, \) and \(2.0 < \log(\text{EW}(H_\alpha)) < 2.5\). The number of galaxies in each bin is listed in the lower right of each figure, and the median uncertainties are shown by the error bars in the upper left. Note that galaxies with H_\beta still in absorption after the correction factor \(R\) is applied have been excluded, but that this is less than 0.2% of the sample in each
bin. As can be seen in the figure, the median uncertainties increase quickly with decreasing emission line equivalent width. It is for this reason that galaxies with log(EW(Hα)) < 0.5 have not been included here.

The y-axis for all four plots includes a correction for stellar absorption; log[(EW(Hα) + R)/[EW(Hβ) + R]] + log[F(Hα)/F(Hβ)]. As for the full sample, we determine the correction factor, R, for each binned sample by finding the value that leads to the minimum offset from the 1:1 relation with the uncertainties giving 1σ offsets from this relation. The values determined are R = 3.1, 3.5, 4.1, and 4.1Å for each increasing bin of EW respectively, as indicated in the lower-left of each plot in figure 4. The 1σ uncertainty around R for each bin is approximately 1.0 (slightly less for the EW(Hα) > 100Å bin). The lowest bin has an uncertainty of 1.5, though the probability distribution for R is slightly skewed to higher values, arising from the offset visible in [4], discussed below. The Balmer decrement determined from the EWs is significantly better than for the full sample, and especially so when no correction is included (figure [3], with dispersions of 1σ ~0.11, 0.06, 0.05, and 0.04 dex around the 1:1 line respectively for the 4 binned samples.

The lowest EW(Hα) (figure [3]) sample appears by eye to be slightly offset from the line. A reasonable hypothesis is that this offset arises due to our simple assumption of a constant correction R to both EW(Hα) and EW(Hβ), whereas it is clear from [4] that the absorption EW(Hα) is typically less than the absorption EW of Hβ by approximately a factor of 0.6 on average. However, changing the correction factor of EW(Hα) to 0.6R and redoing the fit does not remove this offset. On closer examination it is clear that this offset is due to a biasing of the fit to the high signal-to-noise data, which predominantly occur at high values of the Hα/Hβ ratio due to measurement biases (i.e. there is a clear gradient in EW(Hα) from top to bottom in figure [3]). Assuming uniform weighting for the fit (i.e. ignoring the errors in EWs) gives a value of R = 3.0, well within the large uncertainties for R. For the other figures, assuming an offset correction factor for EW(Hα) of 0.6R, results in R = 3.5, 3.7, and 3.7Å in terms of increasing EWs, with similar dispersion around the relations. The results are within the uncertainties for R when assuming a constant correction, but in all cases indicate the necessity for the correction of stellar absorption to the emission lines of a factor of ~ 4Å.

For figures [4], b, and d (0.5 < log(EW(Hα)) < 1.0, 1.0 < log(EW(Hα)) < 1.5, and 2.0 < log(EW(Hα)) < 2.5 respectively) the observed scatter is less than the median uncertainty. Only for figure [4] does there appear to be a significant, low number scatter above the line (note the log scale density in figure [3]). However, when examined, the scatter in this diagram, and also in figures [4], b, and d, is correlated with the uncertainty in the determined Hα and Hβ lines, with the median scatter in each pixel increasing significantly the further from the line. Thus the median uncertainty for the outliers is greater than the median uncertainty of the sample as a whole in figure [4].

While figure [4] demonstrates that it is possible to determine the Balmer decrement to some accuracy from emission line equivalent widths, the determination of the correction is dependent upon the measurement of three quantities; EW(Hα), EW(Hβ), and the flux ratio, F(Hα)/F(Hβ). While the former two will be observable in strong emission line galaxies at high redshift, the flux ratio may prove problematic to measure from spectra. However this ratio is closely tied with the observed optical colours of the galaxy. In figure [5] we show the distribution of the continuum fluxes measured at the Hα and Hβ wavelengths (F(Hα)/F(Hβ)) against the restframe g − r colour as measured from the SDSS fibre spectrum within the full SN(Hα,Hβ) sample. A linear fit to the correlation in this figure returns

\[ \log (F(H(\alpha))/F(H(\beta))) = -0.26 + 0.39(g - r)_{\text{spec}}, \]

with a standard deviation of \( \sigma \sim 0.015 \) dex around this relation. We use the rest-frame g − r colour as a proxy for stellar continuum, but other colours such as r − i provide similar constraints. In the case of low-S/N spectra, the rest-frame colour could be from SED fitting to broadband magnitudes.

Thus, in summary, the Balmer decrement for a low-resolution, strong-emission-line spectrum can be measured from the emission line equivalent widths and colours alone with;

\[ \log(H/\beta) = \log \left( \frac{EW(H(\alpha)) + 4.1}{EW(H(\beta)) + 4.1} \right) + (-0.26 + 0.39(g - r)_{\text{rest}}), \]

with a scatter around this of \( \sigma \sim 0.05 \) dex, or \( \sim 0.3 \) mag in \( A_V \), assuming a Galactic extinction law (e.g. O'Donnell 1994). For weaker emission line galaxies (i.e. EW(Hα) < 30Å), a lower offset (R ~ 3.5) should be used, as shown in figure [4]. However, given the larger uncertainties and greater dispersion seen at lower EW(Hα), a correction factor of R ~ 4 can be used for the full sample with a 0.1 dex scatter (~ 0.7 mag in \( A_V \)) and an extension to lower values (i.e EW Balmer decrement underestimate) due to low EW systems.

One final note on this relation: as seen in figure [4] there is a strong bias in the sample of Balmer decrement with other galaxy properties, as discussed in detail in several other SDSS papers (see e.g. Kauffmann et al. 2003a; Garn & Best 2010). Thus, the relation shown above includes a combination of both galaxy type as well as variation in extinction. The only way to remove fully this effect is to match pairs of galaxies in as many property types excluding extinction, such as done in Wild et al. (2011b). Unfortunately when applied to the sample here, it was found that the range in extinction was not large enough to properly determine the relation. However, even given these uncertainties, this relation should still hold at several redshifts as high attenuations are on average associated with high gas masses, and thus high star formation rates and similar underlying continua at all redshifts.
The Balmer decrement of SDSS galaxies

5 STELLAR ABSORPTION EFFECTS ON THE EMISSION LINES

One issue with the previous section is that we assume throughout that the stellar-continuum subtracted emission line fluxes within the MPA/JHU database are correct. While the overall fits to the stellar continuum are impressively good with a median $\chi^2 = 1.01$ per pixel across the sample, there are appear to be remaining issues around the Balmer lines. In the following we concentrate on the SDSS DR7 Balmer emission-line fluxes corrected for the underlying stellar absorption features from the MPA/JHU database, and explore their uncertainties using the known intrinsic values and commonly used attenuation and extinction laws.

5.1 The issue with H$\beta$

When considered alone, the ratio of $H_\alpha/H\beta$ cannot indicate problems with the measurement of the lines involved unless it is significantly below the expected value of the unattenuated ratio. This is because the larger values of the emission line ratio can be caused by attenuation by intervening dust, with the intrinsic ratio dependent the emitting gas density and temperature (as discussed in section 2). However by examining several of the Balmer lines at once these dependencies can be accounted for.

Figure 6 shows the variation of the $H_\gamma/H\beta$ ratio against the $H_\alpha/H\beta$ ratio for the SN($H_\alpha$, $H_\beta$, $H_\gamma$) SDSS sample. Both ratios have been normalized to their Case B, $T = 10,000K$, $n_e = 100 \text{ cm}^{-3}$ intrinsic ratios ($H_\alpha/H\beta = 2.86$, $H_\gamma/H\beta = 0.468$). Immediately obvious in this figure is the offset of the sample from the zero point, indicating most SDSS galaxies undergo some attenuation (as seen in the previous plots), with the correlation between the two ratios as expected from the reddening laws applied to the intrinsic ratio.

The three different lines overplotted show the effect of three different attenuation laws commonly assumed in the analysis of galaxies. For all three lines, the symbols indicates steps of 0.5 in $A_V$, up to $A_V = 3$.

The O'Donnell (1994) law (O'D94) is an updated version of the Cardelli et al. (1989) fit to the average extinction law in the Galaxy, thus least representative of the integrated emission from the SDSS galaxies, which will suffer attenuation due to the mixture of emitting sources and absorbing medium. However, as discussed in Kennicutt et al. (2009) and can be seen in figure 6 the use of a foreground dust screen with galactic extinction is indistinguishable from the other laws, especially given the uncertainty within the SDSS sample. We assume a total to selective V-band extinction of $R_V = 3.1$, the average value in our galaxy.

The Calzetti et al. (2000) attenuation law (Cal00) was obtained from the continuum and Balmer decrement of local actively star-forming galaxies, thus matching the high EW($H\alpha$) galaxies in the sample. Note that as only ratios are analysed here, the difference between the colour excess ($E(B - V)$) of the stellar continuum and nebular lines noted by Calzetti et al. is effectively scaled out. The $R_V$ used here is 4.05, as given by Calzetti et al. (2000) from the comparison of the observed infrared flux to that predicted from the obscuration of the optical-ultraviolet light.

The Charlot & Fall (2000) attenuation law (CF00) is a more simple, empirical law put forward to allow for the different colour excesses and attenuations observed by Calzetti et al. (2000) between the nebular emission lines and stellar continuum. It breaks the attenuation into two components; the ‘diffuse ISM’ component that describes the effective obscuration of all stars in a galaxy by the diffuse dust, and the ‘birth cloud’ component that describes the additional extinction suffered by the $H\alpha$ regions from which the nebular emission lines arise, giving

$$
\frac{A_\lambda}{A_V} = \mu(\lambda/A_V)^{-0.7} + (1 - \mu)(\lambda/A_V)^{-1.3},
$$

where $A_V = 5500\AA$. The exponent of $-0.7$ for the diffuse ISM was empirically derived by Charlot & Fall (2000) with a comparison of nearby galaxies, while the $-1.3$ exponent for the birth clouds was chosen to match the extinction within our own Galaxy. The parameter $\mu$ indicates the fraction of the attenuation suffered by the nebular lines by each component. We assume $\mu = 0.3$ here, as used by Wild et al. (2007) and Wild et al. (2011a) in their analyses of SDSS galaxies.

There are three things of note to take from figure 6. One, given enough precision in the data, the SDSS galaxies should be able to distinguish between these three different attenuation laws, and provide an answer on which law is best (or least bad) to apply to an ensemble of galaxies. Such work has been done before for comparing galactic extinction laws using planetary nebulae (Phillips 2007). Note that the different $R_V$ between the Cal00 and O’D94 laws is what causes the difference in expected $H_\alpha/H\beta$ for the same $A_V$, while the CF00 and Cal00 have similar $A_V$ as determined from $H_\alpha/H\beta$, but not from $H_\gamma/H\beta$. Two, that the scatter of the SDSS galaxies is large around the three laws, preventing this possibility, though this scatter is not significant when compared to the median uncertainty as shown by the error bars. Third, and most importantly, while the slope of SDSS galaxies matches that given by the attenuation laws, there is a systematic offset of the SDSS galaxies when compared to all three attenuation laws. This offset is significant, and cannot be explained by assuming more extreme (and therefore
less likely) attenuation laws or by assuming large values of $R_V$, as the zero point itself appears to be offset.

Neither can this situation be remedied by assuming different values for the unattenuated Balmer ratios. As discussed in section 3, the intrinsic Balmer emission-line ratios are sensitive to the temperature of the ionized gas from which they arise, and, more weakly, to the density of the gas as well. The inset in figure 6 shows a close up of the zero point of figure 5, i.e. galaxies with little or no attenuation. Over this are plotted 3 symbols indicating the position where the zero point would be for different average H_n region temperatures: 5,000K, 10,000K (assumed within the figure), and 20,000K. All assume Case B ratios, and a typical H_n gas density of $n_e = 10^2$ cm$^{-3}$. These 3 temperatures encompass the range of temperatures expected, with typical solar metallicity H_n regions having $T_e \sim 8,000$K. What this figure demonstrates is that the variation in intrinsic Balmer ratios is small relative to both the effects of attenuation and the observed offset, and that the variation in intrinsic ratios is in the same sense as that due to the effects of attenuation (as shown by the Calzetti et al. (2000) law), thus the intrinsic ratio cannot be the cause of the offset.

A possible cause for this offset can be found when the MPA/JHU DR4 catalogue is examined instead. Using the same SN cuts on H$\alpha$, H$\beta$, and H$\gamma$, figure 7 shows the same plot as figure 6 for the DR4 sample. In most ways this figure is exactly the same (as it should be), except in two respects: the DR4 SN(H$\alpha$,H$\beta$,H$\gamma$) sample only has ~160,000 galaxies in the sample compared to the ~240,000 galaxies in the DR7 sample, visible in both the number density (colours) and in the scatter, and the DR4 sample does not have the offset with respect to the attenuation laws seen in figure 6.

As mentioned in section 3, the major difference between the DR4 and DR7 line fluxes from the MPA/JHU catalogues is the version of the GALAXEV models used for the continuum fits. In some respects the difference seen in the figures is surprising, as the median $\chi^2$ of the fits to the continuum in DR7 is reduced compared to the DR4 fits, from $\chi^2 = 1.5$ for DR4 to $\chi^2 = 1$ for DR7, suggesting a much better fit to the spectra.

The issue most likely lies within the H$\beta$ line region, as an offset in the H$\beta$ line flux would also explain the increasing offset at higher (H$\alpha$/H$\beta$) between the SDSS galaxies and the attenuation laws, and this region has been observed to be mismatched between models and the spectra of some globular clusters (see e.g. Walcher et al. 2009, Poole et al. 2010). This difference in slope is more clearly seen in figure 8a where we have rotated and scaled figure 6 to the $A_V$ plane using the attenuation law of Calzetti et al. (2000), where the offset from the $A_V$-axis is more clearly seen. Similar results are seen if another attenuation law is used. In this frame, the $A_V$ is the offset from the expected (H$\beta$/H$\gamma$) ratio based on the $A_V$, as determined by the (H$\alpha$/H$\beta$) ratio.

By “correcting” for the incorrectly subtracted stellar H$\beta$ equivalent width we can fix figure 8a. This is what we have done in figure 8b, where we have added 0.35Å to the emission line equivalent width (i.e. “True” H$\beta$=H$\beta$+0.35F(H$\gamma$)). The correction of 0.35Å was determined by minimizing the offset from the Calzetti law (this value depends only weakly on the choice of attenuation law). This means that the stellar absorption equivalent width of H$\beta$ is systematically underestimated by 0.35Å in the CB08 continuum fits to DR7 SDSS spectra.

The fact that this 0.35Å underestimate is systematic is interesting, as we would expect that any error would depend on the strength of the Balmer features, either measured through the H$\alpha$$_{Abs}$ or the emission line equivalent widths, both of which do correlate with H$\alpha$/H$\beta$ as seen in figure 5, yet no correlation is observed. What the underlying cause of this systematic underestimation is unknown, yet it must be taken into consideration when determin-
ing the $A_V$ from the Balmer decrement in the SDSS DR7 MPA/JHU catalogue. It is possible that this issue arises due to the misclassification of the spectral resolution of the MILES library (as discussed in Falcón-Barroso et al. (2011)) in the implementation in the CB08 code, but this issue is now known and currently under investigation. This investigation goes beyond the scope of the work presented here but we note that the models used here are early versions of the CB08 library and these issues are expected to be solved within the to-be-published models. When the new $A_V$ is calculated from the $H_\alpha/H_\beta$ ratio including the systematic 0.35 Å offset (using e.g. Calzetti et al. (2000) law), a mean difference of ~0.07 magnitudes is found with the uncorrected $A_V$ estimates, increasing slightly at higher $A_V$. This suggests that previous DR7 $A_V$ estimates, such as in Garn & Best (2010), are overestimated by this value.

Similarly this 0.35 Å correction must be included in our EW Balmer decrement (equation 2) leading to a new equation,

$$\log(H_\alpha/H_\beta) = \log \left( \frac{\text{EW}(H_\alpha) + 4.1}{\text{EW}(H_\beta) + 4.4} \right) + (-0.26 + 0.39(g-r)_{\text{corr}}),$$

(4)

which more closely matches our expectations of different stellar absorption equivalent widths between $H_\alpha$ and $H_\beta$. For weaker emission line galaxies, the offset would be smaller than 4.1, as shown in figure 4 and discussed at the end of section 6.

5.2 The issue with $H_\delta$

While examining the issue in $H_\beta$, a similar issue was found for $H_\delta$ that we present here as a curiosity. When $H_\delta/H_\alpha$ versus $H_\gamma/H_\delta$ is plotted, using the SN($H_\alpha$,$H_\beta$,$H_\gamma$,$H_\delta$) sample of SDSS galaxies and avoiding the problematic $H_\delta$ line, the tight correlation between these two ratios, matching closely that expected from the attenuation by dust. However, upon closer examination, a systematic offset is observed between the galaxies and attenuation laws. As in the previous subsection on $H_\beta$, the offset is clearer when rotated to the $A_V$ plane, which is what we show in figure 9, where the x-axis is the $A_V$ determined from $H_\gamma/H_\delta$ using the Calzetti et al. (2000) law, with the y-axis the offset from this $A_V$ when determined from the $H_\delta/H_\alpha$ ratio. The median offset is ~ -0.05 for most of the $A_V$ range shown here, meaning that the $A_V$ determined from $H_\delta/H_\alpha$ is lower than that determined from $H_\gamma/H_\delta$. The offset is seen for all attenuation laws considered here.

More importantly, the offset is also observed in the DR4 sample, though with greater uncertainty due to low number statistics. As with the $H_\delta$ line there appears to be no correlation of the offset with emission line equivalent widths, or stellar age determinants like $H_\delta$, or the $D_{4000}$ index (see Kauffmann et al. (2003) for definitions of these indices). Neither does it appear to be correlated with $H_\delta/H_\beta$ or other emission line or attenuation tracers. Thus, due to the lack of difference between DR4 and DR7, the strong EW bias of the sample (as shown in figure 2), and the fact that the uncertainty is dominated by the $H_\delta$ line, it is still not known what exactly causes this offset. It is most likely an issue due to the underlying continuum, but an investigation into the stellar models goes beyond the scope of this work. Thus we present this issue for now as a curiosity and a cautionary note of the level of systematic uncertainties in determining weak line fluxes from the SDSS sample.

6 CONCLUSION

We have examined the possibility of using equivalent widths of the Balmer emission lines to determine the Balmer decrement, and hence attenuation, of a galaxy. Using the Sloan Digital Sky Survey we were able to determine a statistically representative relation between the continuum-subtracted Balmer emission line flux ratio and the equivalent widths (EW) of the Balmer emission-lines combined with a rest-frame colour, correcting for the effects of the stellar absorption features:

$$\log(H_\alpha/H_\beta) = \log \left( \frac{\text{EW}(H_\alpha) + 4.1}{\text{EW}(H_\beta) + 4.4} \right) + (-0.26 + 0.39(g-r)_{\text{corr}}),$$

(5)

for galaxies with $\text{EW}(H_\alpha) \geq 30$ Å, with a scatter of 1σ ~ 0.06 dex, or 0.4 mag in $A_V$, indicating the possible variation for individual objects. For galaxies with $\text{EW}(H_\alpha) < 30$ Å smaller correction factors (3.5 for $\text{EW}(H_\alpha)$, 3.8 for $\text{EW}(H_\beta)$) should be used. However, given the scatter at low EW values, the equation above can be used above for all galaxies allowing for a much greater uncertainty in the final Balmer ratio or $A_V$ determined.

In addition, by comparing the Balmer decrement ($H_\alpha/H_\beta$ versus $H_\gamma/H_\delta$) we discovered that the $H_\gamma$ emission line equivalent width (and hence flux) is underestimated by ~0.35 Å in the JHU/MPA DR7 SDSS database, due to an issue in the $H_\delta$ region of the 2008 version of the Charlot & Bruzual stellar population synthesis code GALEXEV. This leads to an overestimation of the attenuation of the SDSS galaxies of 0.07 magnitudes in $A_V$ assuming the Calzetti et al. (2000) attenuation law.

Finally, we also discovered a strange offset in the $H_\delta$ emission line fluxes observable in both the DR4 and DR7 releases of the MPA/JHU database which we present both as a curiosity and as a warning on the underlying issues in interpreting weak-line emission lines in a statistical sample.
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