An experimental study of hydrodynamics and heat transfer at fluid filtration through a porous medium

R A Dekhtyar1,2* and V V Ovchinnikov1

1Kutateladze Institute of Thermophysics SB RAS, 630090 Novosibirsk, Russia.
2Novosibirsk State University, 630090 Novosibirsk, Russia.
E-mail: dekhtyar@itp.nsc.ru

Abstract. An experimental study of hydrodynamics and heat transfer under conditions of filtering of water and aqueous solution of glycerol through a porous insert in the channel was carried out. The values of the coefficient of hydraulic resistance and heat transfer for a chaotic backfill of balls with a diameter of 3.2 mm for filtering water and 47% glycerol solution have been determined. Three flow regimes were investigated: inertial, transient and turbulent. It has been found that, depending on the filtering mode through the porous insert, there are various laws of heat transfer. Comparison of the obtained results on heat transfer in ball fillings with the available literature data has been carried out. The obtained experimental data are in good agreement with the data of numerical calculations. For the thermal stabilization section at high values of the Reynolds number they are in good agreement with the data obtained by filtering air through a porous medium.

1. Introduction

In recent years, the study of heat transfer in granular media has intensified. This is due to their widespread use in the chemical and oil and gas industries, where interest in granular media is associated with the need to develop and improve the processes and devices of chemical technology, and to increase the efficiency of thermal methods for intensifying oil and gas production. Nuclear reactors are used as fixed-grain nuclear reactors in which the granular medium serves to cool the fuel elements immersed in it. And in traditional power engineering this medium is used in the design and improvement of compact and efficient heat exchangers. In this spectrum of tasks, the technological problems closely join an in-depth theoretical and practical analysis of the natural thermophysical processes in the earth's crust.

The above range of problems emphasizes the need for a fundamental study of the thermo-hydrodynamic characteristics of transfer processes in granular media. At the same time, the complexity of the object under study impedes the quick creation of a sufficiently rigorous theory for calculating the averaged and structural characteristics of the flows in such systems. In this connection, the role of systematic experimental studies and physical models is especially important to take into account the basic properties of the corresponding filtering modes and to explain and summarize the experimental data.

The pressure-drop-flow velocity relationship for very slow fluid flow through a porous structure can be described by the Hazen-Darcy equation:

\[
\frac{\Delta P}{L} = \frac{\mu}{\kappa} \cdot U, \quad \kappa = \frac{d^2}{B_k} \cdot \frac{\varepsilon^3}{(1-\varepsilon)},
\]  

(1)
where $\Delta P$ is the pressure difference across the length of the porous material in the flow direction, $L$ is the sample thickness in the same direction, $\kappa$ is the permeability, $\mu$ is the fluid viscosity and $U$ is the Darcy’s velocity, i.e. the volumetric flow rate divided by the cross-sectional flow area. For flow behavior obeying this case, the fluid is said to be flowing in the Darcy regime. The Blake-Kozeny equation, shown in equation (1) can be used to estimate the permeability $\kappa$ of a packed bed of spheres with diameter $d$ as a function of the porosity or void fraction $\varepsilon$ and the Blake-Kozeny constant $B_K$, which for spheres is 36 [1]. The Ergun relationship has been also adopted by researchers to describe the pressure-drop across porous materials [2, 3]. It has a similar form to the Forchheimer equation and was originally developed for packed columns of spherical particles [4]. The Ergun equation is presented in equation (2) where $\alpha$ and $\beta$ are the empirical constants (originally determined by experimentation to be 150 and 1.75) [5].

$$\frac{\Delta P}{L} = \alpha \cdot \frac{(1-\varepsilon)^2 \cdot \mu}{\varepsilon^3 \cdot d^2} \cdot u + \beta \cdot \frac{(1-\varepsilon) \cdot \rho}{\varepsilon^3 \cdot d} \cdot u^2,$$

Although Ergun [2] proposed $\alpha$ and $\beta$ values of 150 and 1.75, subsequent researchers have proposed a much broader range of values for these constants, due to variations in particle sphericity and roughness [4] and investigated structural morphology of the different packed beds. Although, $\alpha$ and $\beta$ values in the range of 160 to 180 and 1.8 to 4.36, respectively, have been measured even for spherical particles [6], there is little understanding of the direct influence of structural parameters on these variations.

To describe heat transfer in porous inserts, design models are based on traditional ideas about laminar and turbulent boundary layers, without due regard for the flow specificity in porous medium. This complicates generalization of calculation dependencies.

For the laminar sublayer, the expression for heat transfer, according to Poulikakos [7], can be represented as:

$$\text{Nu}_d = \text{const} \cdot \text{Re}^{0.2} \cdot \text{Pr},$$  \hspace{1cm} (3)

For the calculation of heat transfer from the wall to the liquid for Reynolds numbers less than 600 [9], the following expression for the Nusselt number is proposed:

$$\text{Nu}_d = \text{Nu}_0 + \text{const} \cdot \text{Re} \cdot \text{Pr},$$  \hspace{1cm} (4)

where $\text{Nu}_0$ is the component of heat transfer, independent of the fluid velocity, and the second term takes into account heat transfer in a moving fluid. In [6], based on the processing of the available experimental data in a turbulent filtering mode, the dependence

$$\text{Nu}_d = \text{const} \cdot \text{Re}^{0.3} \cdot \text{Pr}^a,$$  \hspace{1cm} (5)

For moderate Prandtl numbers, including air, in the literature the value $n = 0.4$ is usually used for heating, but there is still no particular clarity about the value of the index $n$ for Prandtl number.

However, there are few works simultaneously studying the heat transfer and hydrodynamics in a flowing medium and lacking methodological disadvantages associated with neglecting the influence of the initial part of heat transfer, the limited range of the mode under study and geometric parameters. For example, the authors of [11] managed to avoid such drawbacks. Therefore, systematic studies are necessary to clarify the basic laws of heat transfer depending on the quality characteristics of the flow.

The main purpose of the study is to show that there is the relationship between filtration modes through porous medium and the laws of heat transfer.

2. Experimental part

The experimental setup was made in the form of a closed circulation loop (Figure 1) providing a fluid flow rate from 0.007 to 0.4 kg/s. The fluid was fed from the tank 4 by the centrifugal pump 5 to the
After the working section, the liquid passed through the flow measuring section 2, cooled in the shell-and-tube heat exchanger 3 and returned to the tank 4. To maintain a constant flow rate through the porous medium at low filtration rates on the setup, a bypass line 6 was used.

The working area is made of a thin-walled (1.6 mm) copper pipe 1 with a length of 566 mm and an internal diameter of 52 mm (Figure 2). On the outer wall of the pipe there was a wound tape nichrome heater 3, isolated from the copper pipe with mica. A layer of heat-insulating material 5 was applied over the heater. The porous insert was formed of spherical glass balls 2. The insert at the top and bottom was limited to disc-shaped lattices 6 with holes evenly distributed over the area. As the granular layer was formed, its porosity ε was measured. Thermocouples 4 were installed on the pipe wall and in the granular layer. In the experiments, the differential pressure of the fluid in the working section was measured. The selection of static pressure was carried out through holes with a diameter of 0.8 mm in the pipe wall at the inlet and outlet from the backfilled area.

![Figure 1. Scheme of experimental setup](image1)

**Figure 1.** Scheme of experimental setup:
1 - pipe, 2 - flow meter section, 3 - heat exchanger, 4 - tank, 5 - centrifugal pump, 6 - bypass line

![Figure 2. Scheme of test section](image2)

**Figure 2.** Scheme of test section:
1 - pipe, 2 - porous insert, 3 - heater, 4 - thermocouples, 5 - heat insulating material, 6 - grid

During the experiments, we measured the temperature of the fluid at the inlet and outlet of the working section, the temperature of the heater and the air in the room. In special experiments, the calibration characteristics of pressure gauges, flow meters, thermocouples and heat losses through thermal insulation 5 into the environment were determined.

### 3. Results and Discussion

This work systematically studies the heat transfer coefficient $h$ on a stabilized heat exchange site with a constant heat flux on the wall ($q = \text{const}$) and hydraulic resistance when filtering water and an aqueous solution of glycerol in a circular pipe filled with monodisperse glass beads of various diameters.

Based on the analysis of [12], it is possible to distinguish the main modes of liquid filtration through a randomly packed granular layer: Darcy mode, transition from Darcy mode to inertia mode, inertial mode, transition from inertia mode to turbulent mode, and turbulent mode.

Figure 3 presents the measurement data of pressure loss $\Delta P$ on a granular layer of length $L$ for balls of diameter $d = 3.2$ mm. It can be seen that our data for porosity of 0.364 and balls $d = 3.2$ mm coincide with the numerical calculations of [4] for various values of porosity and diameter of balls $d = 3$ mm.
The Ergun equation (2) is shown in the same figure (solid black line). It can be seen that this equation describes our data well with an accuracy of no worse than 3%.

Figure 3. Plots of pressure drop per unit flow length against filtering velocity

Figure 4 shows the results of experimental data processing in the form of dependence \( \frac{\text{Nu}_d}{\text{Pr}^{0.4}} = f(\text{Re}_d) \) for a monodisperse granular medium, formed by glass beads \( d = 3.2 \) mm for filtering water (\( \text{Pr} = 7 \)) and 47% glycerol solution (\( \text{Pr} = 48 \)). It can be seen that in the studied range of operating parameters, three basic laws of heat transfer can be distinguished: \( \frac{\text{Nu}_d}{\text{Pr}^{0.4}} \sim \text{Re}_d^{1/2} \), \( \frac{\text{Nu}_d}{\text{Pr}^{0.4}} \sim \text{Re}_d \), \( \frac{\text{Nu}_d}{\text{Pr}^{0.4}} \sim \text{Re}_d^{2/3} \). In the figure, transition zones are shaded: zone 1 - transition from \( \text{Nu} \sim \text{Re}^{1/2} \) to \( \text{Nu} \sim \text{Re} \), and zone 2 - transition from \( \text{Nu} \sim \text{Re} \) to \( \text{Nu} \sim \text{Re}^{2/3} \). The obtained results are compared with the experimental data of other authors. The most reliable are the data obtained for air and filtration rates corresponding to the turbulent regime or to high Reynolds numbers. Some of these data [9] for \( \text{Pr} = 0.7 \) are presented in Figure 4 in comparison with our data. For turbulent filtration, data for air, water, and glycerin solution are well summarized by the dependence \( \frac{\text{Nu}_d}{\text{Pr}^{0.4}} = 0.4 \cdot \text{Re}_d^{2/3} \cdot \text{Pr}^{0.4} \).

Figure 5 shows changes in the hydraulic resistance of the granular medium under study with \( d = 3.2 \) mm, performed simultaneously with the determination of heat transfer coefficients when filtering water and 47% glycerol solution. It can be seen from the figure that there are similar, as in [12], laws of hydraulic resistance to filtration rate and transitional filtration zones, which are highlighted in the figure (zone 1 and zone 2). Transition to a turbulent flow regime when water is filtered through a porous medium occurs at Reynolds number \( \text{Re}_d \sim 200 \).

Analyzing the experimental data on heat transfer when filtering a fluid through a porous medium and data obtained for the hydraulic characteristics of the filtration flow, it may be noted that the areas shaded in Figures 4 and 5 correspond exactly to the ranges of \( \text{Re}_d \) number where the heat transfer laws change in accordance with the flow regime change: inertial, transient and turbulent.
Figure 4. Nusselt vs. Reynolds number for various liquids in porous medium

Figure 5. Hydraulic resistance of the granular layer when filtering water and glycerol solution

In the initial sector of heat exchange, there is also a variety of heat transfer laws. On the graph, the experimental data obtained in the initial section when filtering a solution of 47% glycerol are compared with the dependence

\[ \text{Nu}_d \left( \frac{x}{D} \right) = \text{Nu}_{\infty} \left[ 1 + \left( \frac{x}{D} \right)^{-2/3} \right] \]

It can be seen that for Re_d > 45 and Re_d < 23 for x/D < 5, this dependence well describes the experimental data. But outside this range (Re_d ~ 33) describing heat transfer data in this area is difficult due to an assumption of different filtering modes occurrence in different sections along the channel.
Conclusion
An experimental study of hydrodynamics and heat transfer was carried out in the channel with a porous insert in the form of a random filling of glass beads with a diameter of 3.2 mm with the flow of water and 47% glycerin solution. As a result of research, three filtration modes have been distinguished: inertial, transient and turbulent. It is shown that, depending on the filtration mode, there are various laws of heat transfer, and the zones of changing heat transfer regimes coincide with those with a change in flow regimes. A comparison of the obtained results with the data available in the literature has shown that in the turbulent mode at the thermal stabilization section, the data on filtration of water (Pr = 7) and glycerol solution (Pr = 48) are in good agreement with data for air (Pr = 0.7); data on the pressure drop in the porous insert are consistent with the data of numerical calculations.

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