Fragmentation of a spin-1 mixture in a magnetic field

Jie Zhang,¹,² Xue Hou,¹ Bin Chen,¹ and Yunbo Zhang³
¹College of Physics and optoelectronics, Taiyuan University of Technology, Taiyuan 030024, P. R. China
²Key Lab of Advanced Transducers and Intelligent Control System of Ministry of Education, Taiyuan University of Technology, Taiyuan 030024, P. R. China
³Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, P. R. China

We study the ground state quantum fragmentation in a mixture of a polar condensate and a ferromagnetic condensate when subject to an external magnetic field. We pay more attentions to the polar condensate and find that it will be less fragile in the mixture when perturbed by the magnetic field. Both atom numbers and the number fluctuations in the spin-0 component will keep in a high magnitude of order of N when the magnetization of the system is increased. The role of the ferromagnetic condensate is to provide a uniform and stable background which can delay the rapid shrink of the 0-component population and make it possible to capture the “super-fragmentation”. Our method has potential applications in measuring the inter-species spin-coupling interaction through adjusting the magnetic field.

PACS numbers: 03.75.Mn, 67.60.Bc, 67.85.Fg

I. INTRODUCTION

Recent experimental breakthroughs in spinor Bose-Einstein Condensate, such as the sub-Poissonian spin correlations generated by atomic four-wave spin mixing [1], the atomic squeezed states realized in the spin-1 ultracold atomic ensembles [2], and the antiferromagnetic spatial ordering observed in a quenched one-dimensional spin-1 gas [3], are all in connection with the vacuum fluctuations and recall attentions to the finite particle number effect beyond the mean-field treatment. The vacuum fluctuations become a significant subject in more and more experimental facts, e.g., atomic quantum waves, atomic spin squeezing and quantum information. As one of the active frontiers, the spin-1 ultracold atomic ensemble is often adopted. With the basic interaction form \( V(r) = (\alpha + \beta \mathbf{F} \cdot \mathbf{F}) \delta(r) \), the properties of such a three-component spinor condensate [4] have been demonstrated experimentally [5] and two different phases reflecting fundamental properties of spin correlation are identified: the so-called polar and ferromagnetic states for \( \beta > 0 \) (\(^{23}\text{Na}\)) and \( \beta < 0 \) (\(^{87}\text{Rb}\)) atomic condensates respectively. The mixture of two spinor condensate with the ferromagnetic and polar atoms, respectively, show more attractive quantum effects [6–13]. With the help of sympathetic cooling, the BEC mixtures of Na and Rb have been realized and it is interesting to observe the interspecies interaction induced immiscibility between the two condensates [12].

The ground state of the condensate with \( \beta > 0 \) has been predicted to be either polar (\( n_0 = N \)) or antiferromagnetic (\( n_1 = n_{-1} = N/2 \)) within the mean-field treatment, where the condensate is usually described by a coherent state. However, the many-body theory by Law, Pu and Bigelow [14] pointed out that the ground state of \( \beta > 0 \) atoms is a spin singlet with properties (\( n_1 = n_0 = n_{-1} = N/3 \)) drastically different from the results predicted by the mean field theory. Soon, Ho and Yip [15] show that this spin singlet state is a fragmented condensate with anomalously large number fluctuations and thus has fragile stability. The remarkable nature of this super-fragmentation is that the single particle reduced density matrix gives three macroscopic eigenvalues (\( N/3 \)) with large number fluctuations \( \Delta n_{1,0,-1} \sim N \). Similar considerations were also addressed by Koashi and Ueda [16–18]. The signature of fragmentation is then refer to the anomalously large fluctuations of the populations in the Zeeman levels. This is a super-Poissonian correlation character, and the large number fluctuations shrink rapidly as the experimentally adventitious perturbations exist, such as magnetic field or field gradient.

In this paper we will report the influence of external magnetic field on the spinor condensate with \( \beta > 0 \), but on the premise of doping many ferromagnetic atoms in it. The interspecies spin coupling interaction arises and we propose a valid procedure to observe and control the fragmented states. If the ferromagnetic atoms in the mixture are condensed, the ground state favors all atoms aligned along the same direction and provides a uniform and stable background which can delay the rapidly shrinking of the number fluctuations when the inter-species coupling interaction is adjusted. The back action from polar atoms on to the more stable ferromagnetic atoms is negligible. Doping ferromagnetic atoms into spin-1 polar condensate can effectively influence the vacuum fluctuations and will have potential applications in quantum information and quantum-enhanced magnetometry.

II. HAMILTONIAN OF THE MIXTURE

We consider the mixture of two spinor condensates of \( N_1 \) ferromagnetic and \( N_2 \) polar atoms, respectively. The intra-condensate atomic spin-1 interaction takes the
standard interaction form \( V_k(r) = (\alpha_k + \beta_k \mathbf{F}_k \cdot \mathbf{F}_k) \delta(r) \) with \( k = 1, 2 \). The inter-condensate interaction between the ferromagnetic and polar atoms is \( V_{12}(r) = \frac{1}{2}(\alpha + \beta \mathbf{F}_1 \cdot \mathbf{F}_2 + \gamma \rho_0) \delta(r) \), which is more complicated because collision can occur in the total spin \( F_{\text{tot}} = 1 \) channel between different atoms \([3, 7]\). The parameters \( \alpha, \beta \), and \( \gamma \) are related to the \( s \)-wave scattering lengths in the three total spin channels and the reduced mass \( \mu \) for atoms in different species, and \( \rho_0 \) projects an interspecies pair into spin singlet state. Within the single spatial-mode approximation (SMA) \([14, 19, 21]\) for each of the two spinor condensates, the spin-dependent Hamiltonian for the mixture finally reads as

\[
\hat{H} = c_1 \beta_1 \hat{F}_1^2 + c_2 \beta_2 \hat{F}_2^2 + c_{12} \beta_1 \hat{F}_1 \cdot \hat{F}_2 + \frac{c_{12} \gamma}{3} \hat{\Theta}_{12} \hat{\Theta}_{12},
\]

where \( \hat{F}_1 = \hat{a}_1^\dagger \hat{F}_{1ij} \hat{a}_j \) (\( \hat{F}_2 = \hat{b}_1^\dagger \hat{F}_{2ij} \hat{b}_j \)) are defined in terms of the \( 3 \times 3 \) spin-1 matrices with \( i(j) = 1, 0, -1 \), and \( \hat{a}_1^\dagger (\hat{b}_1^\dagger) \) creates a ferromagnetic (polar) atom in the hyperfine state \( i \). The operator

\[
\hat{\Theta}_{12} = a_0^\dagger b_0^\dagger - a_1^\dagger b_{-1}^\dagger - a_{-1}^\dagger b_1^\dagger,
\]

creates a singlet pair with one atom each from the two species, similar to

\[
\hat{B}^\dagger = (\hat{b}_0^\dagger)^2 - 2\hat{b}_1^\dagger \hat{b}_{-1}^\dagger,
\]

for intra-species spin-singlet pair \([13, 16]\) when \( \beta_2 > 0 \).

The interaction parameters are \( c_1 = \frac{1}{2} \int dr |\Phi(r)|^4 \), \( c_2 = \frac{1}{2} \int dr |\Phi(r)|^4 \), and \( c_{12} = \int dr |\Psi(r)|^2 |\Phi(r)|^2 \), which can be tuned through the control of the frequency of the trapping potential \([3]\).

### III. Fragmentation in the Magnetic Field

#### A. Number distributions in a magnetic field

When the interspecies scattering parameters are calculated in the degenerate internal-state approximation (DIA) \([22, 23]\), the low-energy atomic interactions can be mostly attributed to the ground-state configurations of the two valence electrons, and the non-commutative term \( \hat{\Theta}_{12} \hat{\Theta}_{12} \) can be neglected \([3, 7, 9]\). The ground states are classified into four distinct phases: FF, MM-, MM+, and AA by three critical values of \( c_{12} = -\frac{(2N-1)\omega_2}{N} \), 0, and \( \frac{(2N-1)\omega_2}{N} \).

In this paper we discuss the atom number distribution and fluctuation in an external magnetic field. The spin-dependent Hamiltonian in the magnetic field reads

\[
\hat{H} = c_1 \beta_1 \hat{F}_1^2 + c_2 \beta_2 \hat{F}_2^2 + c_{12} \beta_1 \hat{F}_1 \cdot \hat{F}_2 - c_{12} \rho_1 \hat{F}_{1z} - c_{22} \rho_2 \hat{F}_{2z},
\]

where only the linear Zeeman terms are considered. As the SU(2) symmetry is broken in a spinor mixture, one can not eliminate the linear Zeeman effect through a spin rotation \([20]\). Meanwhile the quadratic Zeeman energy, typically 2 orders of magnitude weaker than the linear terms, is negligible in the calculation of number distributions. Furthermore, we take \( p = c_1 \rho_1 = c_2 \rho_2 \) in the following discussion, which can easily realized through adjusting the trapping frequency.

We consider the direct product of the Fock states of the two species \( |n_1, n_0, n_{-1}\rangle \otimes |n_1, n_0, n_{-1}\rangle \) and do not restrict the model in the subspace with zero total magnetization \([4, 10]\). Instead, we consider the full space including all possible system magnetization \( m = n_1 + n_2 \). Using the full quantum approach of exact diagonalization, we study the response of the two species to the external magnetic field \( p \) for \( N_1 = N_2 = 40 \). The three critical points for the phase boundaries are approximately \( c_{12} = -4, 0, 4 \).

The field dependence of the population is shown in Fig. 1 for the MM+ phase at \( c_{12} = 2.5 \), where polar atoms are partly polarized in the opposite direction as the ferromagnetic atoms \([3]\). We notice that the ferromagnetic atoms (black dashed lines) are very sensitive to the magnetic field, i.e. atoms quickly redistribute in the \( n_1 \) component and the magnetization of ferromagnetic

![FIG. 1: (Color online) The dependence of atom numbers on \( p \), at fixed values of \( c_1 \beta_1 = -1, c_1 \beta_2 = 2, \) and \( c_{12} \beta = 2.5 \). The total numbers of the two species are \( N_1 = N_2 = 40 \), and we consider the full-space with total magnetization \( m \) a variable. Black dashed and red solid lines denote the number distributions in the ferromagnetic and polar condensate respectively. All interaction parameters are in the units of \( |c_1 \beta_1| \).](image-url)
condensate $m_1 = n_1 - n_{-1}$ saturates immediately. The ferromagnetic atoms actually form a stable condensate and provide a uniform magnetic background in the mixture. The polar atoms present a stepwise increase (decrease) in the atom number distribution $n_1(n_{-1})$ when the field increases. For small $p$ and positive $c_{12}\beta$, the system favors a negative magnetization ($m_2 = n_1 - n_{-1}$) of polar condensate, and $m_2$ will reverse and tend to saturate for large magnetic field. We notice that the super-fragmented state featured with $n_1 = n_0 = n_{-1} = \frac{N}{3}$ remarkably arises around a special value of $p = 100$.

The situation becomes more simple if the parameter $c_{12}\beta$ is negative, that is, in the FF phase (or MM phase), where polar atoms are fully (partly) polarized in the same direction as the ferromagnetic atoms. The enhanced ferromagnetic effect and the external magnetic field jointly suppress the atom number distribution $n_0$ and $n_{-1}$ of the polar condensate to zero, and at the same time saturate $n_1$ and the magnetization $m_2$ without magnetization reversal.

**B. Retrieving the super-fragmented state**

The super-fragmented state, with equal population $n_{1,0,-1} = N/3$ and zero magnetization $m_2 = 0$, has been predicted in the pure spin-1 polar condensate and described by a spin singlet state in the form $\propto (\hat{B}^\dagger)^N/2 |0\rangle$ [15]. $\hat{B}^\dagger (B)$ is invariant under spin rotations and commutes with $\hat{F}_2$ and $\hat{F}_{2z}$. In the ground state of the system subject to an external magnetic field $(\hat{b}_1)^{m_2} (\hat{B}^\dagger)^{N/2 - m_2} |0\rangle$, one can see a rapid shrink of the spin-0 component distribution $n_0$ and an unbalanced population $m_2 > 0$. The super-fragmented state then reduces to a much more generic fragmented state: a two component number state with essentially zero fluctuations. Such state with polar interaction was not likely realized in typical experiments due to its fragility towards any perturbation breaking spin rotational symmetry.

For the spin-1 polar condensate doped with many ferromagnetic atoms, we can retrieve this super-fragmented state in the presence of an external field. For some special values of the magnetic field, both the spin-0 component population and number fluctuations would not shrink but revive to macroscopic orders of the super-fragmented state. In Fig. 2, we illustrate the revival points for three inter-species coupling parameters $c_{12}\beta$. These revival points are found to move towards larger value of $p$ as $c_{12}\beta$ increases.

As learned from previous studies [21], the mean-field treatment is efficient for atomic interaction of the ferromagnetic type. The much more stable ferromagnetic condensate in the mixture can be formulated in the mean field treatment as a boson-enhanced effective magnetic
field. This simplifies the Hamiltonian (4) as
\[ H = c_1 \beta_1 (\hat{F}_{z1}^1) + c_2 \beta_2 \hat{F}_{z2}^2 + c_{12} \beta (\hat{F}_1) \cdot \hat{F}_2 \] 
\[ -c_1 p_1 \langle \hat{F}_{z2} \rangle - c_2 p_2 \hat{F}_{z2} \] 
\[ = c_2 \beta_2 \hat{F}_{z2}^2 + A \hat{F}_{z2} + C \] 

where \( \langle \hat{F}_{z1} \rangle = N_1, A = c_1 \beta_1 N_1 - c_2 p_2, C = c_1 \beta_1 N_1(N_1+1) - c_1 p_1 N_1. \) The criterion for the emergence of super-fragmented state is \( p = c_1 \beta_1 N_1, \) where the magnetic field \( p, \) the optical trapping frequency \( (c_{12}), \) and the number of the doped ferromagnetic atoms \( (N_1) \) are all adjustable. When the magnetic field matches the condition that \( c_1 \beta_1 N_1 \) and \( c_2 p_2 \) cancel each other, we may achieve the super-fragmented state in a magnetic field. The three critical points in Fig. 2 are found to agree with the numerical results exactly.

Next, we turn to the situation with population imbalance in the two species. Fig. 3 illustrates the location of the critical point when the inter-species coupling parameter \( c_{12} \beta \) is fixed to be 2.5 and the atom numbers for the two species are \( N_1 = 20 \) and \( N_2 = 100. \) As the mean-field picture works well for the ferromagnetic atoms, we still get the crucial point \( p = 2.5 \times N_1 \) in Fig. 3. When equal population \( n_1 = n_0 = n_{-1} = N/3 \) occurs for the polar condensate, the number fluctuations also instantaneously reach to the macroscopic levels. Our numerical results for the fluctuation relation \( \langle \Delta n_0 = 2 \Delta n_{\pm 1} \rangle \) agree exactly with the algebraic results in [15] for pure polar condensate. With the emergence of equal population \( N/3 \) regarded as a sign of anti-ferromagnetic spin interaction, the inter-species spin coupling interaction \( c_{12} \beta \) can be estimated by the location of the critical magnetic field.

C. AA phase in a magnetic field

AA phase is another super-fragmented state which have been predicted in the absence of magnetic fields [8]. It is a many-body spin singlet, which requires exactly the same atoms number of the two species \( (N = N_1 = N_2), \) and total spins from different species polarized in opposite directions. In the notation of the angular momentum representation

\[ |F_1, F_2, F, m\rangle = \sum C_{F_1, m_1; F_2, m_2} |F_1, m_1\rangle |F_2, m_2\rangle, \] 

AA phase is denoted as \( |\phi_{AA}\rangle = |N, N, 0, 0\rangle \) with \( F_1, F_2 \) and \( F \) the total spin quantum numbers of the ferromagnetic atoms, polar atoms, and the mixture and \( m_1, m_2 \) and \( m \) the corresponding \( z \)-components. The intra-species angular momentum states involved in the AA phase, \( |N, m_1\rangle \) and \( |N, m_2\rangle, \) should obey the constraint \( m_1 + m_2 = 0. \) The interesting feature of AA phase is the equal distribution of atoms in the six components \( (N/3) \) and the large number fluctuations. Using the full quantum approach of exact diagonalization, and considering the full space including all possible system magnetization \( m = m_1 + m_2, \) we study the responses of the AA phase to the external magnetic field with \( N_1 = N_2 = 40, \) and compare the results with the super-fragmented state in the pure condensate [12].

In Fig. 4, we compare these two typical fragmented ground states: the inter-species entangled singlet \( |\phi_{AA}\rangle \) and the pure polar singlet \( (B^1)^{\pm} |0\rangle, \) which belong to two special phases characterized by typical values of the interaction parameter: \( c_{12} \beta = 4.5, \) and \( c_{12} \beta = 0. \) We find that the responses of the \( n_0 \) component to the magnetic field are quite different. Fig. 4a shows the influence of a magnetic field on a pure polar condensate. As \( p \) increases, the \( 0 \)-component distribution \( n_0 \) (red dashed line) shrink rapidly, which agree with the algebraic results in [13] \( n_0 = (N_2 - m_2)/(2m_2 + 3). \) Fig. 4b shows the influence of a magnetic field on the polar atoms in the AA phase. We find that \( n_0 \) does not shrink rapidly in the beginning, instead, it increases first and remains in a high value for a certain range of \( p. \) Compared with the pure polar spin-singlet state, the inter-species entangled singlet seems to be more difficult to be magnetized as \( p \) increases. When all the polar atoms are forced to arrange in the opposite direction of the ferromagnetic ones, the bose-enhanced magnetic background formed by ferromagnetic condensate will be canceled. The system spontaneously breaks into a high symmetry state when the interaction parameter \( c_{12} \beta \) goes across the phase transition point \( (2N-1)c_{12} \beta_{N+1}. \) The magnetic behaviors of the two species are then identical (except for the atom mass), and...
both ferromagnetic atoms and polar atoms are equally to be magnetized when the field intensity is increased. Compared with the pure singlet in Fig. 4a, the involved total atoms are doubled, and at least a field strength $p \sim 400$ is needed to saturate the magnetization. The external magnetic field can be used to characterize these two spin-singlets through tracing the atoms numbers of $n_0$ component of the polar atoms.

D. The inter-species $P_b$ effect

If we refer to more general case beyond the DIA approximation, the $\gamma$ term of the Hamiltonian (1) should be considered. We notice that

$$\left[\hat{F}_{12}^\dagger, \hat{\Theta}_{12}^\dagger \hat{\Theta}_{12}\right] \neq 0, \quad \left[\hat{F}^2, \hat{\Theta}_{12}^\dagger \hat{\Theta}_{12}\right] = 0, \quad (7)$$

which means in general they do not belong to a set of commutative operators. However, we can numerically study the phase transition through the order parameter $\langle \hat{\Theta}_{12}^\dagger \hat{\Theta}_{12}\rangle$ [10]. To see more clearly the role played by the parameter $c_{12}\gamma$ on the fragmentation, we numerically diagonalize the Hamiltonian (1) with $N_1 = N_2 = 40$.

In Fig. 5, we illustrate the influence of a small $c_{12}\gamma \neq 0$ to the population $n_0$ and $n_1$ of super-fragmented state which has been retrieved in MM+ phase in the presence of the magnetic field. We find that the crucial point is still located at $p = 2.5 \times N_1$, but a tiny $c_{12}\gamma = 0.15$ will elevate the $n_0$ component to a dominated value, meanwhile suppress the $n_1$ and $n_{-1}$ components to lower level. The high occupation on $n_0$ component is an evidence of the nematic order [2], but the signature of fragmentation is still obvious. Away from for the critical point where $n_{+1}$ are suppressed to be nearly zero, the $n_1 \ (n_{-1})$ component is linearly increased (decreased) to meet the conditions that the system magnetization should be increased. For $c_{12}\gamma = -0.15$, both $n_0$ and $\Delta n_0$ shrink, the system reduced to a much more generic fragmented state with the system magnetization $m_2 = n_1 - n_{-1}$ which increases linearly with $p$. This situation is more like the pure polar condensate in a weak magnetic field.

The negative $\gamma$ term encourages pairing two different types of atoms into singlets [11]. In Fig. 6, the influences of negative singlet pairing coefficient $c_{12}\gamma$ on the numbers and quantum fluctuations of the two species are illustrated. We notice that the typical $N/3$ number distributions arise both in the $c_{12}\beta < 0$ and $c_{12}\beta > 0$ regions when $c_{12}\gamma$ reaches to $-20$. The number fluctuation $\Delta n_0$ gives two steady values, which represent two typical fragmented ground state: the inter-species entangled fragmented state for $c_{12}\beta > 2$, and the inter-species independent fragmented state for $c_{12}\beta < 0$. The fluctuations...
for these two states

\[ \Delta n_0 = \frac{\sqrt{N^2 + 9N}}{3\sqrt{5}}, \quad c_{12} \beta > 2, \]

\[ \Delta n_0 = \frac{2\sqrt{N^2 + 2N}}{3\sqrt{5}}, \quad c_{12} \beta < 0, \]

are found to match the numerical results in Fig. 6.

IV. CONCLUSION

To conclude, we studied the ground state properties of a binary mixture of ferromagnetic and polar spinor condensates in a magnetic field. Using the full quantum approach of exact diagonalization, we can study the competition between magnetic linear Zeeman effect and the inter-species spin coupling interaction \( c_{12} \beta \). The large vacuum fluctuation of number distributions on the three zeeman levels inside the polar condensate is worthy of investigation. We point out that the fragmentation properties of polar condensate can be adjusted through the magnetic field \( p \), trapping frequency \( c_{12} \), and the number of doped ferromagnetic atoms \( N_1 \). The ferromagnetic condensate is involved to provide a uniform and stable background which can delay the rapidly shrinking of the large number fluctuations. We illustrated the influences of the magnetic parameter \( p \), and identified two typical fragmented state with total spin \( \langle F^2 \rangle = 0 \). The positive inter-species spin coupling interaction \( c_{12} \beta > 0 \) can effectively entangle the different species, while for \( c_{12} \beta < 0 \) the different species on their \( F = 1 \) manifold are essentially independent. We propose a possible mechanism to effectively measure the inter-species spin coupling interactions through applying a magnetic field, as well as discriminate the two types of many-body spin singlets. Our work highlights the significant promises for experimental work on sodium and rubidium atomic condensate mixtures and provide some useful information for the study of photo-association of heteronuclear molecules.

This work is supported by the NSF of China under Grant Nos. 11204204, 11347181, the NSF of Shanxi Province under Grant Nos. 2012021003-2, 2014021011-1 and the fund of Taiyuan University of Technology for young teachers. YZ is also supported by the National Basic Research Program of China (973 Program) under Grant No. 2011CB921601, Program for Changjiang Scholars and Innovative Research Team in University (PCSIRT)(No. IRT13076).

[1] Eva M. Bookjans, Christopher D. Hamley, and Michael S. Chapman, Phys. Rev. Lett. 107, 210406 (2011).
[2] C.D. Hamley, C.S. Gerving, T.M. Hoang, E.M. Bookjans and M.S. Chapman, Nature Physics, Vol. 8, No. 4. (2012); T.M. Hoang, C. S. Gerving, B. J. Land, M. Anquez, C. D. Hamley, and M. S. Chapman, Phys. Rev. Lett. 111, 090403 (2013).
[3] A. Vinit, E. M. Bookjans, C. A. R. Sá de Melo, and C. Raman, Phys. Rev. Lett. 110, 163501 (2013).
[4] T.-L. Ho, Phys. Rev. Lett. 81, 742 (1998); T. Ohmi and K. Machida, J. Phys. Soc. Jpn. 67, 1822 (1998).
[5] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur and W. Ketterle, Nature (London) 396, 345 (1998); L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, Nature (London) 443, 312 (2006).
[6] M. Luo, Z. Li, and C. Bao, Phys. Rev. A 75, 043609 (2007).
[7] Z. F. Xu, Y. Zhang, and L. You, Phys. Rev. A 79, 023613 (2009).
[8] Z. F. Xu, Z. F. Xu, J. Zhang, Y. Zhang, and L. You, Phys. Rev. A 81, 033603 (2010).
[9] J. Zhang, Z. F. Xu, L. You, and Y. Zhang, Phys. Rev. A 82, 013625 (2010).
[10] J. Zhang, T.T. Li, and Y. Zhang Phys. Rev. A 83, 023614 (2011).
[11] Yu Shi, Phys. Rev. A 82, 023603 (2010); Yu Shi and Ge Li, Phys. Rev. A 83, 013616 (2011); Yu Shi and Ge Li, Int. J. Mod. Phys. B 26, 1250002 (2012); Li Ge, Fei Zhou, and Yu Shi, Phys. Rev. A 89, 043623 (2014).
[12] F.D. Wang, D.Z. Xiong, X.K. Li, D.J. Wang, E. Tiemann, Phys. Rev. A 87, 080702(R) (2013).
[13] Z.F. Xu, D.J. Wang and L. You, Phys. Rev. A 86, 013632 (2012).
[14] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. 81, 5257 (1998).
[15] T.-L. Ho and S.-K. Yip, Phys. Rev. Lett. 84, 4031 (2000).
[16] Masato Koshi and Masahito Ueda, Phys. Rev. Lett. 84, 1066 (2000).
[17] M. Ueda and M. Koashi, Phys. Rev. A 65, 063602 (2002).
[18] Erich J. Mueller, Tin-Lun Ho, Masahito Ueda, and Gordon Baym, Phys. Rev. A 74, 033612 (2006).
[19] H. Pu, C. K. Law, S. Raghavan, J. H. Eberly, and N. P. Bigelow, Phys. Rev. A 60, 1463 (1999).
[20] M.-S. Chang, Q. S. Qin, W. X. Zhang, L. You, and M. S. Chapman, Nature Physics 1, 111 (2005).
[21] S. Yi, Öz. E. Müstecaplıoğlu, C. P. Sun, and L. You, Phys. Rev. A 66, 011601(R) (2002).
[22] H. T. C. Stoof, J. M. V. A. Koelman, and B. J. Verhaar, Phys. Rev. B 72, 4688 (1998).
[23] A. C. Maan, E. Tiesinga, H. T. C. Stoof, and B. J. Verhaar, Physica B. 165, 17 (1990).
[24] S. Wei, M. Bhattacharya, and N. P. Bigelow, Phys. Rev. A 68, 024708 (2003); ibid. 69, 049903(E) (2004).
[25] A. Pashov, O. Docenko, M. Tamanis, R. Ferber, H. Knoeckel, and E. Tiemann, Phys. Rev. A 72, 062505 (2005).
[26] W. X. Zhang, D. L. Zhou, M.-S. Chang, M. S. Chapman, and L. You, Phys. Rev. A 72, 013602 (2005).