Model-independent Constraints on the Weak Phase $\alpha$ (or $\phi_2$) and QCD Penguin Pollution in $B \to \pi\pi$ Decays

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Abstract

We present an algebraic isospin approach towards a more straightforward and model-independent determination of the weak phase $\alpha$ (or $\phi_2$) and QCD penguin pollution in $B \to \pi\pi$ decays. The world averages of current experimental data allow us to impose some useful constraints on the isospin parameters of $B \to \pi\pi$ transitions. We find that the magnitude of $\alpha$ (or $\phi_2$) extracted from the indirect CP violation in $\pi^+\pi^-$ mode is in agreement with the standard-model expectation from other indirect measurements, but its four-fold discrete ambiguity has to be resolved in the near future.

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The major goal of KEK and SLAC $B$-meson factories is to test the Kobayashi-Maskawa mechanism of CP violation [1] within the standard model and to detect possible new sources of CP violation beyond the standard model. An elegant description of CP violation in $B$ physics is the unitarity triangle defined by the following orthogonality relation of six quark mixing matrix elements in the complex plane [2]:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$  \hfill (1)

Three inner angles of this triangle are denoted as $\alpha$, $\beta$ and $\gamma$ by the BaBar Collaboration, or equivalently $\phi_1$, $\phi_2$ and $\phi_3$ by the Belle Collaboration:

$$\alpha \equiv \phi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\beta \equiv \phi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

$$\gamma \equiv \phi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right).$$ \hfill (2)

So far $\beta$ has been rather precisely determined from the measurement of CP violation in $B_d^0$ vs $\overline{B}_d^0 \rightarrow J/\psi K_S$ transitions [3], and its value $\beta \approx 23^\circ$ is compatible very well with the standard-model expectation. The next experimental step is to measure $\alpha$ and $\gamma$ to a good degree of accuracy at $B$-meson factories, such that one may cross-check the Kobayashi-Maskawa picture of CP violation and probe possible new physics in the $B$-meson system.

It is well known that $\overline{B}_d^0 \rightarrow \pi^+\pi^-$, $\overline{B}_d^0 \rightarrow \pi^0\pi^0$ and $B_u^- \rightarrow \pi^0\pi^-$ decays can be used to extract the weak angle $\alpha$ in a model-independent way, because the isospin relation of their transition amplitudes allows us to remove the QCD penguin pollution [4]. Note, however, that the experimentally-reported branching fractions of $B \rightarrow \pi\pi$ decays are all charge-averaged:

$$\mathcal{B}_{+-} \equiv \frac{1}{2} \left[ \mathcal{B}(B_d^0 \rightarrow \pi^+\pi^-) + \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^+\pi^-) \right],$$

$$\mathcal{B}_{00} \equiv \frac{1}{2} \left[ \mathcal{B}(B_d^0 \rightarrow \pi^0\pi^0) + \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^0\pi^0) \right],$$

$$\mathcal{B}_{0\pm} \equiv \frac{1}{2} \left[ \mathcal{B}(B_u^+ \rightarrow \pi^0\pi^-) + \mathcal{B}(B_u^- \rightarrow \pi^0\pi^+) \right].$$ \hfill (3)

The world averages of current BaBar [5], Belle [6] and CLEO [7] data on $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$ are listed in Table 1 [8]. In addition, the direct CP-violating asymmetries between $\overline{B}_d^0 \rightarrow \pi^+\pi^-$, $\overline{B}_d^0 \rightarrow \pi^0\pi^0$, $B_u^- \rightarrow \pi^0\pi^-$ and their CP-conjugate decays can be defined as

$$C_{+-} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \pi^+\pi^-) - \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^+\pi^-)}{\mathcal{B}(B_d^0 \rightarrow \pi^+\pi^-) + \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^+\pi^-)},$$

$$C_{00} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \pi^0\pi^0) - \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^0\pi^0)}{\mathcal{B}(B_d^0 \rightarrow \pi^0\pi^0) + \mathcal{B}(\overline{B}_d^0 \rightarrow \pi^0\pi^0)},$$

$$A_{0\pm} \equiv \frac{\mathcal{B}(B_u^- \rightarrow \pi^0\pi^-) - \mathcal{B}(B_u^+ \rightarrow \pi^0\pi^+)}{\mathcal{B}(B_u^- \rightarrow \pi^0\pi^-) + \mathcal{B}(B_u^+ \rightarrow \pi^0\pi^+)}.$$

\hfill (4)
The world averages of current BaBar [5] and Belle [6] data on $C_{+\to}$, $C_{00}$ and $A_{0\pm}$ are also shown in Table 1 [8]. These two collaborations have actually measured the time-dependent rates of $B_d^0 \to \pi^+\pi^-$ decays on the $\Upsilon(4S)$ resonance:

$$\Gamma[B_d^0(\Delta t) \to \pi^+\pi^-] = \frac{e^{-|\Delta t|/\tau_0}}{4\tau_0}[1 - S_+ \sin(\Delta m_d\Delta t) + C_+ \cos(\Delta m_d\Delta t)] ;$$

$$\Gamma[\overline{B}_d^0(\Delta t) \to \pi^+\pi^-] = \frac{e^{-|\Delta t|/\tau_0}}{4\tau_0}[1 + S_+ \sin(\Delta m_d\Delta t) - C_+ \cos(\Delta m_d\Delta t)] ,$$

(5)

where $\tau_0$ is the lifetime of neutral $B$ mesons, and $S_+$ signifies the indirect CP violation arising from the interplay between decay and $B_d^0\overline{B}_d^0$ mixing [9]. A similar time-dependent measurement can be done for $B_d^0$ vs $\overline{B}_d^0$ mixing, whose rates consist of $C_{00}$ and $S_{00}$ corresponding to $C_{++}$ and $S_{++}$ in Eq. (5). Only $S_{++}$ has been determined from the KEK and SLAC experiments, and its world average [8] is given in Table 1.

Although the relevant experimental data on direct CP violation remain quite preliminary, they can be used to do a quantitative analysis of $B \to \pi\pi$ decays. The main purpose of this paper is to recommend an algebraic isospin approach, which allows us to figure out the ranges of $B \to \pi\pi$ isospin parameters and to determine the weak phase $\alpha$ (or $\phi_2$) from $S_{++}$ and (or) $S_{00}$ in a more straightforward and model-independent way. We find that the allowed region of $\alpha$ is in agreement with the standard-model expectation from other indirect measurements, but its four-fold discrete ambiguity has to be resolved in the near future.

Under isospin symmetry and in the neglect of electroweak penguin contributions [10], the amplitudes of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$ and $B_d^+ \to \pi^0\pi^+$ decays (or their CP-conjugate processes) form a triangle in the complex plane:

$$A(B_d^0 \to \pi^+\pi^-) + \sqrt{2}A(B_d^0 \to \pi^0\pi^0) = \sqrt{2}A(B_d^+ \to \pi^0\pi^+) ;$$

$$A(\overline{B}_d^0 \to \pi^+\pi^-) + \sqrt{2}A(\overline{B}_d^0 \to \pi^0\pi^0) = \sqrt{2}A(\overline{B}_d^- \to \pi^0\pi^-) .$$

(6)

The magnitudes of $A(B_d^- \to \pi^0\pi^-)$ and $A(B_d^+ \to \pi^0\pi^+)$ are identical to each other in this safe approximation [4,11], hence the CP-violating asymmetry $A_{0\pm}$ vanishes. Table 1 indicates that the experimental data are in good agreement with the expectation of $A_{0\pm} \approx 0$. In terms of the charge-averaged branching fractions in Eq. (3) and the direct CP-violating asymmetries in Eq. (4), let us follow Ref. [12] to explicitly express the parameters

$$r = |r|e^{i\theta} \equiv \frac{A_0}{A_2} ,$$

$$\tau = |\tau|e^{i\varphi} \equiv \frac{A_0}{A_2} ,$$

(7)

which stand for the ratios of $I = 0$ and $I = 2$ isospin amplitudes in $B_d^0 \to \pi^+\pi^-$ (or $\pi^0\pi^0$) and $\overline{B}_d^0 \to \pi^+\pi^-$ (or $\pi^0\pi^0$) decays. The results are

$$|r| = \frac{\sqrt{3}B_{++}(1 + C_{++}) + 3B_{00}(1 + C_{00}) - 2\kappa B_{0\pm}}{\sqrt{\kappa B_{0\pm}}} ;$$

$$|\tau| = \frac{\sqrt{3}B_{++}(1 - C_{++}) + 3B_{00}(1 - C_{00}) - 2\kappa B_{0\pm}}{\sqrt{\kappa B_{0\pm}}} ;$$

(8)
where \( \kappa \equiv \tau_0 / \tau_\pm = 0.921 \pm 0.017 \) [2] denotes the lifetime ratio of neutral and charged \( B \) mesons. Eqs. (8) and (9) clearly show that \( \tau = r \) (i.e., \( |\tau| = |r| \) and \( \overline{\theta} = \theta \)) would hold, if \( C_{+} = C_{00} = 0 \) held. Hence the deviation of \( \tau \) from \( r \) is a measure of direct CP violation in \( B^0_d \) vs \( \overline{B}^0_d \rightarrow \pi^+\pi^- \) and \( \pi^0\pi^0 \) decays \(^1\).

The CP-violating parameters \( S_{+} \) and \( S_{00} \) in Eq. (5) are related to the weak angle \( \alpha \) as follows [12]:

\[
\begin{align*}
S_{+} &= (1 + C_{+})\Im \left[ \frac{q}{p} \frac{A(\overline{B}^0_d \rightarrow \pi^+\pi^-)}{A(B^0_d \rightarrow \pi^+\pi^-)} \right] = (1 + C_{+})|R| \sin[2(\alpha + \Theta)] , \\
S_{00} &= (1 + C_{00})\Im \left[ \frac{q}{p} \frac{A(\overline{B}^0_d \rightarrow \pi^0\pi^0)}{A(B^0_d \rightarrow \pi^0\pi^0)} \right] = (1 + C_{00})|\overline{R}| \sin[2(\alpha + \overline{\Theta})] ,
\end{align*}
\]

where \( q/p \approx (V_{td}V_{tb}^*)/(V_{td}V_{tb}) \) denotes the weak phase of \( B_d^0 \overline{B}_d^0 \) mixing [9], and

\[
\begin{align*}
R &= |R|e^{2i\Theta} = \frac{1 - \tau}{1 - r} , \\
\overline{R} &= |\overline{R}|e^{2i\overline{\Theta}} = \frac{2 + \tau}{2 + r} .
\end{align*}
\]

To be specific, we have

\[
\begin{align*}
|R| &= \sqrt{\frac{1 - 2|\tau| \cos \theta + |\tau|^2}{1 - 2|\tau| \cos \theta + |\tau|^2}} , \\
|\overline{R}| &= \sqrt{\frac{4 + 4|\tau| \cos \theta + |\tau|^2}{4 + 4|\tau| \cos \theta + |\tau|^2}} .
\end{align*}
\]

\(^1\)It is worth mentioning that the difference between \( \phi \equiv \arg[A(\overline{B}^0_d \rightarrow \pi^+\pi^-)/A(B^0_d \rightarrow \pi^0\pi^0)] \) and \( \varphi \equiv \arg[A(B^0_d \rightarrow \pi^+\pi^-)/A(\overline{B}^0_d \rightarrow \pi^0\pi^0)] \) measures the existence of direct CP violation too [12]. With the help of Eqs. (3), (4) and (6), one may obtain

\[
\begin{align*}
\cos \phi &= \frac{2\kappa B_{00} - B_{+-} (1 - C_{+-}) - 2B_{00} (1 - C_{00})}{2\sqrt{2}B_{+-}B_{00} (1 - C_{+-}) (1 - C_{00})} , \\
\cos \varphi &= \frac{2\kappa B_{00} - B_{+-} (1 + C_{+-}) - 2B_{00} (1 + C_{00})}{2\sqrt{2}B_{+-}B_{00} (1 + C_{+-}) (1 + C_{00})} .
\end{align*}
\]

Obviously, \( \cos \varphi = \cos \phi \) would hold if both \( C_{+-} \) and \( C_{00} \) were vanishing. Note that the notations of direct CP-violating asymmetries in Ref. [12] are \( A_{+} = -C_{+-} \) and \( A_{00} = -C_{00} \).
\[
\Theta = \frac{1}{2} \arctan \left( \frac{|r| \sin \theta - |\bar{r}| \sin \bar{\theta} - |r| |\bar{r}| \sin(\theta - \bar{\theta})}{1 - |r| \cos \theta - |\bar{r}| \cos \bar{\theta} + |r| |\bar{r}| \cos(\theta - \bar{\theta})} \right),
\]
\[
\bar{\Theta} = -\frac{1}{2} \arctan \left( \frac{2|r| \sin \theta - 2|\bar{r}| \sin \bar{\theta} + |r| |\bar{r}| \sin(\theta - \bar{\theta})}{4 + 2|r| \cos \theta + 2|\bar{r}| \cos \bar{\theta} + |r| |\bar{r}| \cos(\theta - \bar{\theta})} \right).
\]

If there were no direct CP violation in \( B \to \pi \pi \) transitions (i.e., \( C_{+-} = C_{00} = 0 \) or \( r = \bar{r} \)), we would arrive at \( \bar{R} = R = 1 \) from Eqs. (11)–(13). In this case, Eq. (10) would be simplified to \( S_{+-} = S_{00} = \sin 2\alpha \). It is therefore necessary to pin down \( C_{+-} \) and \( C_{00} \) to a reasonable degree of accuracy, in order to extract the weak angle \( \alpha \) from \( S_{+-} \) and (or) \( S_{00} \).

Now we carry out a numerical analysis of \( B \to \pi \pi \) decays and CP violation by using the isospin formulas obtained above and current experimental data listed in Table 1. The 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) confidence regions of \( (|r|, |\bar{r}|), (\cos \theta, \cos \bar{\theta}), (|R|, |\bar{R}|), (\Theta, \bar{\Theta}) \) and \( (\alpha, S_{+-}) \) are shown in Figs. 1–3, while the central (best-fit) values of these parameters are given in Table 2. Some discussions are in order.

1. The moduli \( |r|, |\bar{r}|, |R| \) and \( |\bar{R}| \) can be determined without any discrete ambiguity (see Fig. 1 for illustration). The difference between \( |r| \) and \( |\bar{r}| \) is quite obvious, although it remains possible for \( |\bar{r}| = |r| \) to hold at the 3\( \sigma \) level. In contrast, it is likely to have \( |\bar{R}| = |R| \) at the 1\( \sigma \) level, but their best-fit values are different from each other. Direct CP violation is therefore expected to manifest itself in \( B \to \pi \pi \) transitions.

2. Although \( \cos \theta \) and \( \cos \bar{\theta} \) can be uniquely determined, \( \theta \) or \( \bar{\theta} \) involves two-fold ambiguity. The difference between \( \theta \) and \( \bar{\theta} \) is apparent, but \( \cos \bar{\theta} = \cos \theta \) remains possible at the 1\( \sigma \) level. More precise experimental data will allow us to fix \( r \) and \( \bar{r} \), both their moduli and their phases, to a better degree of accuracy.

3. The two-fold ambiguity of \( \theta \) or \( \bar{\theta} \) leads to the four-fold ambiguity of \( \Theta \) or \( \bar{\Theta} \), as illustrated in Fig. 2 and Table 2. An interesting feature of our results is that the signs of \( \Theta \) and \( \bar{\Theta} \) are essentially opposite. Although it is possible to have \( \bar{\Theta} = \Theta \) at the 2\( \sigma \) level for case (A) or (D), we find that \( \bar{\Theta} \neq \Theta \) holds even at the 3\( \sigma \) level for case (B) or (C). In particular, the possibility of \( \Theta = 0 \) and (or) \( \bar{\Theta} = 0 \) is strongly disfavored, implying the presence of QCD penguin pollution or direct CP violation in \( B \to \pi \pi \) decay modes. The typical values of \( \Theta \) and \( \bar{\Theta} \) are \( \Theta \sim \pm 19^\circ \) and \( \bar{\Theta} \sim \mp 26^\circ \), as shown in Table 2.

4. Because of the four-fold ambiguity associated with \( \Theta \) or \( \bar{\Theta} \), the result of \( \alpha \) determined from \( S_{+-} \) involves the four-fold discrete ambiguity too, as illustrated in Fig. 3. Table 2 tells us that the central values of \( \alpha \) can be \( \alpha \sim 122^\circ \) (A), \( 135^\circ \) (B), \( 86^\circ \) (C) or \( 95^\circ \) (D). This result is certainly in agreement with the standard-model expectation of \( \alpha \) (i.e., \( \alpha \sim 90^\circ \)) from other indirect measurements [2]. Once the CP-violating asymmetry \( S_{00} \) is also measured, it will be possible to completely or partly remove the discrete ambiguity of \( \alpha \) [4]. Then one may constrain the weak phase \( \alpha \) and QCD penguin pollution in \( B \to \pi \pi \) decays at a much better confidence level.

It is worth remarking that the validity of our isospin analysis relies on the assumption of negligible electroweak penguin effects. The electroweak penguin contribution to \( B \to \pi \pi \) decays is in general expected to be insignificant [10]. This expectation would be problematic or incorrect, if \( A_{0, \pm} \neq 0 \) were experimentally established [12]. Note also that final-state
interactions in $B \to \pi\pi$ transitions consist of both elastic $\pi\pi \leftrightarrow \pi\pi$ rescattering and some possible inelastic rescattering effects. Whether the latter is negligibly small or not remains an open question. To answer this question requires more precise measurements of both branching fractions and CP-violating asymmetries of $B \to \pi\pi$ decays.

We have presented an algebraic isospin analysis of rare $B \to \pi\pi$ decays by taking account of the fact that the experimentally-reported branching fractions are charge-averaged and large direct CP violation may exist in them. This approach is more straightforward than the originally-proposed geometric approach, from which the weak phase $\alpha$ (or $\phi_2$) and QCD penguin pollution are determined through the reconstruction of two isospin triangles. Therefore, our method is expected to be very useful to analyze the future experimental data on $B \to \pi\pi$ transitions and CP violation in a model-independent way.

Although the present experimental data (in particular, those on direct and indirect CP violation in $B \to \pi\pi$ decays) are not sufficiently precise, they can impose some instructive constraints on the parameter space of QCD penguin effects. Furthermore, we find that the allowed region of $\alpha$ (or $\phi_2$) is actually in agreement with the standard-model expectation from other indirect measurements. To resolve the four-fold discrete ambiguity associated with the magnitude of $\alpha$ (or $\phi_2$) determined from the indirect CP-violating asymmetry in $\pi^+\pi^-$ mode, a measurement of the similar CP-violating asymmetry in $\pi^0\pi^0$ mode is necessary. We expect that more accurate measurements of such charmless $B$ decays will help us to test the consistency of the Kobayashi-Maskawa mechanism of CP violation and to probe possible new physics beyond the standard model.

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2The transition amplitudes of $B^0_d \to \pi^+\pi^-$, $B^0_d \to \pi^0\pi^0$ and $B^+_u \to \pi^0\pi^+$ decay modes (or their CP-conjugate processes) may still form an isospin triangle in the complex plane, even if the inelastic $\pi\pi \leftrightarrow D\bar{D}$ rescattering effects are taken into account. In this complicated case, however, a model-independent determination of $\alpha$ from $S_{+-}$ and $S_{00}$ would be rather difficult.

3For example, the relationship $\sin \alpha / \sin \beta = |V_{cd}/V_{ud}|/|V_{ub}/V_{cb}|$ [16], which holds as a straightforward result of the unitarity-triangle defined in Eq. (1), can be numerically tested with more accurate data of $\alpha$ and $|V_{ub}/V_{cb}|$. 

6
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TABLES

TABLE I. The world averages of current experimental data on the charge-averaged branching fractions ($B_{+-}$, $B_{00}$, $B_{0\pm}$), direct CP-violating asymmetries ($C_{+-}$, $C_{00}$, $A_{0\pm}$) and indirect CP-violating asymmetries ($S_{+-}$, $S_{00}$) of $B \to \pi\pi$ decays [8].

|       | World average          |
|-------|------------------------|
| $B_{+-}$ | $(4.6 \pm 0.4) \times 10^{-6}$ |
| $B_{00}$  | $(1.51 \pm 0.28) \times 10^{-6}$ |
| $B_{0\pm}$ | $(5.5 \pm 0.6) \times 10^{-6}$ |
| $C_{+-}$  | $-0.37 \pm 0.11$ |
| $C_{00}$  | $-0.28 \pm 0.39$ |
| $A_{0\pm}$ | $-0.02 \pm 0.07$ |
| $S_{+-}$  | $-0.61 \pm 0.14$ |
| $S_{00}$  | —                     |

TABLE II. The central values of eight isospin parameters ($|r|$, $|\bar{r}|$; $\theta$, $\bar{\theta}$; $|R|$, $|\bar{R}|$; $\Theta$, $\bar{\Theta}$) and the weak phase $\alpha$ constrained from the world averages of current BaBar and Belle data on $B \to \pi\pi$ decays [8], where we have taken into account the two-fold ambiguity associated with $\theta$ and $\bar{\theta}$ as well as the four-fold ambiguity associated with $\Theta$, $\bar{\Theta}$ and $\alpha$.

|       | Case (A) | Case (B) | Case (C) | Case (D) |
|-------|----------|----------|----------|----------|
| $|r| $  | 0.6      | 0.6      | 0.6      | 0.6      |
| $|\bar{r}|$ | 1.7      | 1.7      | 1.7      | 1.7      |
| $\theta$ | $+180^\circ$ | $-180^\circ$ | $+180^\circ$ | $-180^\circ$ |
| $\bar{\theta}$ | $+120^\circ$ | $+120^\circ$ | $-120^\circ$ | $-120^\circ$ |
| $|R| $  | 1.5      | 1.5      | 1.5      | 1.5      |
| $|\bar{R}|$ | 1.3      | 1.3      | 1.3      | 1.3      |
| $\Theta$ | $-18^\circ$ | $-20^\circ$ | $+20^\circ$ | $+18^\circ$ |
| $\bar{\Theta}$ | $+25^\circ$ | $+27^\circ$ | $-27^\circ$ | $-25^\circ$ |
| $\alpha$ | $122^\circ$ | $135^\circ$ | $86^\circ$ | $95^\circ$ |
FIG. 1. The $1\sigma$, $2\sigma$ and $3\sigma$ confidence regions of $(|r|, |\tau|)$, $(\cos \theta, \cos \bar{\theta})$ and $(|R|, |\bar{R}|)$ parameters, constrained by the isospin relations and current experimental data.
FIG. 2. The 1σ, 2σ and 3σ confidence regions of $\Theta$ and $\bar{\Theta}$ with four-fold discrete ambiguity, obtained from the isospin analysis of current experimental data.
FIG. 3. The $1\sigma$, $2\sigma$ and $3\sigma$ confidence regions of $\alpha$ with four-fold discrete ambiguity, extracted from current experimental data on $S_{+-}$. 