Structural and torsional vibration analysis of a dry screw compressor

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Abstract. This paper investigates torsional vibration and pulsating noise in a dry screw compressor. The compressor is designed at Gardner Denver (GD) and is oil free and use for mounting on highway trucks. They are driven using a Power Take-Off (PTO) transmission and gear box on a truck. Torque peak fluctuation and noise measurements are done and their sources are investigated and reported in this work. To accurately predict the torsional response (frequency and relative angular deflection and torque amplitude), the Holzer method is used. It is shown that the first torsional frequency is manifested as sidebands in the gear train meshing frequencies and this can lead to noise that is the result of amplitude modulation. Sensitivity analysis of the drive train identifies the weakest link in the drive train that limits the first torsional frequency to a low value. Finally, the significance of higher mode shapes on inter-lobe clearance distribution of the rotors is investigated.

1. Introduction
This paper is a state of the art presentation of how screw machines mounted on trucks can be modelled torsionally. The developed models presented successfully predict the first torsional mode in the dry screw compressors developed by Gardner Denver (GD). The modelled dry screw compressors are labelled generically as A, B, and C. Based on sensitivity analysis performed the weakest links in the drive trains are identified. The paper proceeds to show how the excitation of the first torsional mode and higher modes can be prevented. Driving through the male and female rotors is investigated and compared in C (See figures 1 and 2). In both A and B, the main drive is through the female rotor. The compressors modelled are mounted on to trucks using brackets suspended from their chassis rail. The connection to the truck drive is done using an input shaft to the compressor and the PTO transmission system that includes gears. A detailed description of the drive train is presented in section 2.

The prediction and measurement of torsional modes in screw compressors have been reported by the authors in [1]. The reported work included the use of a 90kW 4 pole asynchronous electric motor as the main drive. It also gave the sources of pulsation and vibration noise in screw compressors and investigated and found out that torque peak fluctuation in screw machines is driven by torsional resonance and not structural resonance. Crucially, the authors determined that at critical compressor input speeds, the driving torque is negative and this can lead to torque reversal in the drive train and to gear backlash and noise. In such a case, the compressor is attempting to drive the electric motor instead of the motor driving the compressor. The dependence of the torque excitation amplitude on prop-shaft angle is also investigated and reported.

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In the current work, the torsional models developed are used for a truck drive and include the PTO transmission system and gears. After the successful prediction of the first torsional modes and validation with measurement, the second torsional mode is considered. Most importantly, the effect of this mode on the inter-lobe clearance distribution of the rotors is investigated. This is followed by the presentation of the explanation of why beating-like or pulsating noise is possible in screw compressors. It is observed that during torsional resonance the first mode frequency is manifested as sidebands in the entire speed range of the compressor at the gear meshing frequencies (GMFs) and their harmonics. It is concluded that pulsating noise in screw compressors is due to amplitude modulation and not frequency modulation [2, 3]. The excitation of the first torsional mode is responsible for the torque peak fluctuation measured in the drive train of screw compressors and partly responsible for the pulsating noise inside them. That is, the excitation of the first torsional mode is a necessary but not a sufficient condition for generating pulsating noise in screw compressors. If this frequency is shifted, the torque peak fluctuation will be avoided, but the torque excitation from the engine remains the same. It implies that the modulation due to input torque fluctuation will still exist at the gear mesh and this may still drive pulsating noise in the machine. A sufficient condition will be to also consider the micro-gear geometry optimization, including gear backlash and tip-relief. The latter will ensure that there is only mid-flank contact at the gears. Misalignment in the drive train and rotor deflection must also be avoided by using tighter clearances and bigger rotors. This will ensure that transmission error is eliminated or minimized in the drive train.

The paper begins with the presentation of the development of torsional models in section 2. In section 3, the results and discussions are reported and finally, the conclusions are given in section 4 and this is followed by the acknowledgment.

1.1. Torsional vibration theory

In general, rotating elements driven by conventional drives (Electric motors, engines, etc) are susceptible to torsional resonance if any of the the natural torsional frequencies of the system is excited. It is therefore critical to determine the torsional frequencies of such systems and ensure that they do not lie within the input speed range of the machine and do not coincide with any excitation frequency. If they do, stiffness or inertia tuning can be used to shift the torsional frequencies of the system to the left or the right.

Two general drive trains are encountered in practice. In one, the torque is transmitted from the drive through all the rotating elements without branching (See figure 1). In the other case, two or more rotating elements are connected by parallel shafts connected to the same gear stage (See figure 2). In such a case the torque is divided in the shafts at the branched point.

To determine the system natural torsional frequencies, the structural dynamic equation

\[
[I] \{\dot{\theta} \} + [C] \{\dot{\theta} \} + [K] \{\theta \} = \{T(t)\}
\]

(1)

is used, in which \(T(t)\), the forcing torque or excitation, is set to zero, and \(C\), which is the damping coefficient, is usually negligible in torsional systems and therefore is set to zero. \(I\) is the moment of inertia and \(K\) is the torsional stiffness. Lumped mass models, in which inertias are concentrated at gears and rotors are developed, with the shafts inertias as well as the stiffness of the gears neglected. Gears are assumed to be infinitely stiff. Each inertia is treated as a node, \(N\), and the number of frequencies depends on \(N\). For \(N\) nodes, we have \(N-1\) natural torsional frequencies \(\omega_1, \omega_2, \ldots, \omega_{N-1}\), including rigid body modes. The corresponding relative angular deflection, \(\theta\), is a function of the angular frequencies. That is, \(\theta_N = f(\omega_N)\).

To determine actual angular deflections, actual torques and hence the shear stress in the shaft elements, forced torsional models are required. In this case, \(T(t) \neq 0\) and it is obtained by taking time series data of the torque from the drive and curve fitting it to obtain the analytical
function, which is of the form \( T(t) = T_0 \sin(\omega t + \alpha) + \bar{T} \), where \( T_0 \) is the torque amplitude, \( \bar{T} \) is the steady torque of the drive and \( \alpha \) is the phase shift.

2. Torsional model development

This paper looks at the application of the Holzer method in the development of torsional models for screw machines [3, 4]. Other applicable methods include the transfer matrix method and the eigenvalue method. The Holzer method and transfer matrix method are applicable to both branched and unbranched drive trains whereas the eigenvalue method works for only unbranched drive trains. For drive trains with branching this method is not able to predict accurately the relative angular deflection. This is true because at the branched point, boundary conditions in torque and deflection must be met as reported in [1]. In the previously reported work, the models are developed for an electric motor drive and include both free and forced torsional models. The unforced (excluding motor input torque excitation) models are presented and used to determine the natural frequencies and mode shapes of the drive trains of the compressors. The amplification factors, which are an indication of the severity of the torque in the drive train at resonance, are determined and reported. Sensitivity analysis of the stiffness and inertia is done to determine the most critical elements in the drive that can be used in tuning the drive. This is followed by the development of forced torsional models (with the excitation torque included in the dynamic equation) that are used to determine the torque amplitude [6]. The effect of prop-shaft angle on torque amplitude is also investigated. In each case, validation is done with measurement data.

In the work reported in this paper, free torsional models are developed to include the drive of a truck. The models developed are for machines with generic drive trains A, B and C. Sensitivity analysis is used to determine the weakest link in the drive train outside the compressor and this can be compared with that in the electric motor drive train. The developed models are implemented in MathCAD.

Illustrations of the drive trains A, B, and C using line diagrams are shown in figures 1 and 2 for unbranched and branched drive trains, respectively. In figure 1, N denotes the nodes (concentration of mass or inertia) with N1=truck engine, N2=truck gear box, N3=PTO gear box, N4=first compressor stage gear assembly, N5=intermediate compressor stage gear assembly, N6=female rotor, N7=synchronization gear stage assembly and N8=male rotor. The same is true in figures 2(a) and 2(b), but in these cases N6 denotes the synchronization gears and N7 and N8 the female and male rotors, respectively. The stiffness is denoted by \( k \) in these diagrams and \( G \) denotes the gears. In the equivalent models, I and \( k \) denote the reduced inertia and stiffness, respectively. The reduction is done to the shaft between nodes 1 and 2 by multiplying the raw inertia and stiffness by the square of the overall transmission ratio.

In generic machine B, there is no intermediate gear stage and so node 5 is missing. Figures 2(a) and 2(b) depict the line diagrams of the generic machine C with drives through the female and male rotors, respectively. In this way, the influence driving through the male and female rotors has on torsional response can be investigated.

Figure 2(c) presents the reduced or equivalent models of generic machines B and C. The male rotor is taken as node 8 for consistency and comparison of the torsional response in all the models. The models developed neglect the stiffness of the gears, damping, and shafts inertias.

In figure 3, the details of the drive train outside the compressor are shown. The truck inertia (II) is obtained from the manufacturer and the PTO transmission information is obtained by considering the sketch in figure 3, in which the PTO shafts and gear box are shown in addition to the truck gear box.

The stiffness, gear ratios and inertias of the drive train inside the compressors are known while for the PTO drive, the overall gear ratio, \( i_{14} \) is measured and G3 and G4 are known and so \( i_{34} = \frac{G3}{G4} \) can be computed. It is therefore possible to compute \( i_{12} \) since \( i_{14} = i_{12} \times i_{34} \).
Similarly, for the PTO drive train, the transmission shaft is assumed to be infinitely stiff and so its stiffness is neglected. The PTO shaft (thin) is known and so the stiffness is computed using $k = \frac{G J}{L}$, where $G$ is the shear modulus, $J$ the polar moment of inertia about the principal axis ($= \frac{\pi D^4}{32}$ for a circular rotor and $= \frac{\pi (D_0^4 - D_i^4)}{32}$ with $D_0$ being the outer diameter and $D_i$ the
inner diameter, for a hollow rotor), and L is the rotor length. The PTO output shaft is also known and so its stiffness is computed. Unknown is the raw stiffness of the shaft between nodes 1 and 2 (engine shaft). The inertia of the flywheel is assumed to be included in that of the truck engine. To determine k12, an initial value equal to the raw or unreduced PTO shaft stiffness is assumed, and from measurements, the first torsional frequency measured is 10Hz on the truck (See figure 6). Because all other values in the model (Generic drive train A) are known, k12 is tuned till the first torsional frequency predicted by the torsional model is 10Hz. The k12 value obtained is used in the other torsional models (Generic drive trains B and C).

The rotors (male and female) are not uniform and so the method given above cannot be used to determine their stiffness. To do so, their equivalent diameters must first be determined and this is done using Finite Element Analysis (FEA) as shown in figure 4. From the maximum torque, T, applied and the maximum deflection, \( \varphi \), obtained from FEA, the rotor stiffness, \( k_r \), is determined. From this, the rotor equivalent diameter, \( D_{eq} \), can be determined using

\[
D_{eq} = \left( \frac{32k_r L}{\pi t^2} \right)^{1/4}
\]

This is especially true for the female rotor since the profile removes more material in it than in the male rotor for most applications. In the male, it might suffice to use the average of the tip diameter and the root diameter as the equivalent diameter.

2.1. Critical rotor speed determination

Conventionally, screw machines are designed to run below or above their critical speeds. This is done in order to prevent torsional resonance in the machine. The critical speed is determined by considering the sketch given in figure 5, in which the main drive is through the female rotor.
If $I_f$ denote the raw female rotor inertia, $I_m$ the raw male rotor inertia, $D$ the shafts diameters (with subscripts $f$ and $m$ denoting female and male, respectively), and $\tau_n$ the gear ratio of the synchronization gears, then the critical female rotor speed is given by:

$$\omega_{cf} = \frac{k_t (I_f + I_{mr})}{I_f I_{mr}} \frac{1}{2} \text{ in } \frac{rad}{s}$$

or $n_{cf} = \left(\omega_{cf} \cdot \frac{60}{\pi}\right)$ in rpm. Here, $I_{mr}$ is the reduced male rotor inertia and $k_t$ is the equivalent stiffness of the female and male rotors shafts.

One of the findings in this paper indicates that a screw machine can be designed to run below or above critical speed and still experience torsional resonance when it is connected to a truck drive via a PTO. However, modelling this behaviour is not common in the literature and this phenomenon is difficult to identify in the design stage. The measurement work by Feese et al. [7] shows that this type of torsional resonance is possible in the drive train of compressors mounted on trucks.

3. Results and discussions

The first torsional frequencies and mode shapes of the generic machines A, B, and C predicted by the torsional models are reported and validated with measurement data as shown in table 1 and figure 6.

| Generic machine type | Predicted first torsional freq. (Hz) | Measured first torsional freq. (Hz) |
|----------------------|-------------------------------------|-----------------------------------|
| A                    | 10.0                                | 10.0                              |
| B                    | 9.8                                 |                                   |
| C, female            | 9.15                                |                                   |
| C, male              | 8.97                                |                                   |

Torque is measured by mounting the torque flange onto the compressor input shaft. Brüel & Kjaer (B & K) microphones are used for the pulsed system in the torque measurement. In the truck test, the first torsional mode (10Hz or 600rpm) is occurring at $\frac{1}{2} \times$ critical input speed, whereas in the electric motor test, it was at $1 \times$ critical input speed at 22Hz. Using sensitivity analysis, the weakest element outside the compressor is identified. The first torsional mode is therefore limited by this element. The system can be tuned by increasing or decreasing the stiffness of this element. The weakest element in the link is acting as a torque limiter to protect the truck gearbox.

The other tuning option is based on sensitivity analysis performed inside the compressor. In this case, the most sensitive element, based on inertia, is identified. To tune the system, the
The rotors used in the generic machines A and B have center distance (CD) of 93mm and an L/D ratio of 1.6, while the CD of generic machine C is 105mm and it has an L/D ratio of 1.6. The ratio of the discharge volume in machine C to that in A and B is 1.2. Similarly, the ratio of the male rotor speed in A and B to C is 1.3. The low speed in the latter (generic machine C) has noise advantage. Table 1 and figure 7 indicate that generic machine C has a lower first torsional frequency and response when compared to A and B. The slope, \(\tan(\beta)\), of the line connecting two nodal points is an indication of the severity of the torque in the shaft element [8]. Thus, the shaft element between nodes 2 and 3 has the largest slope and is therefore the most sensitive. This coincides with the position of the weakest element in the drive train as expected. Figure 7 also shows that the entire compressor is behaving as a rigid body and moving out of phase (anti-phase motion) and relative to the truck engine. The torsional response in generic machine C when we drive through the male rotor is better than when we drive through the female rotor.
Based on the outcome of the torsional response in figure 7, it is possible to determine the optimum position of the synchronization gears in the drive train. It ensures a stiffer and an optimum distribution of the torque in both the male and female rotors in the drive train.

3.1. Significance of higher mode shapes on inter-lobe clearance distribution

The effect of the first and second mode shapes on inter-lobe clearance is investigated and reported for generic machine A. It is found that in the first mode, there is likely to be no issues with clearance because in this case, there is no anti-phase motion (torsional wind-up) between the rotors (See figure 7). In the second mode and between nodes 6 and 7 and 7 and 8 (Figure 8), an anti-phase motion between the male and female rotors is present at the second modal frequency of 314Hz. In some applications, this frequency can be very close to the lobe-passing frequency of the rotors. If true, this mode can be excited, and this can lead to noise generation in the machine. Also, the relative clearance between the rotors will reduce making rotor-to-rotor contact possible (See figure 9). Using sensitivity analysis, the element that must be tuned in the drive train in order to shift this mode to the right is determined.

3.2. Pulsating noise

Pulsating noise is usually associated with screw compressors experiencing torsional resonance. This is especially true when there is torque reversal at the critical input speeds and when the input torque fluctuation is high and the gears are not optimized at the micro level. Transmission error in the drive train can also contribute to this effect. When there is torsional resonance, vibration signature (Fast Fourier Transform or FFT of time series vibration data) at the gears shows that this frequency is usually manifested as sidebands in the gear meshing frequencies and their harmonics of screw machines as depicted in figure 10. The noise shown in figure 11 is independent of the input speed. That is, irrespective of this speed, the 10Hz sideband is always present in the noise and vibration data at the gear meshing frequencies. The pulsating noise is therefore due to amplitude modulation and not frequency modulation [2, 3]. The noise data plot (Figure 11) shows that the amplitude is increasing and decreasing, but the 10Hz modulation

Figure 8. Second mode shape of generic machine A
The frequency is the same. The torque excitation coming from the engine is always present but the amplitude can be large or small depending on the prop-shaft angle in the drive train. The larger this angle, the larger the torque excitation. Tuning the drive train can prevent torsional
resonance and torque reversal at this critical speed from occurring, but it may not be sufficient to eliminate the pulsating noise. A sufficient condition will be to optimize the micro-gear geometry in addition. Specifically, the tip-relief can be optimized in order to ensure mid-flank contact at the gears. An optimized backlash will prevent the fluctuating input torque from adversely affecting the movement of the gears and ensure that they move only in the drive direction. Fluctuation in torque will not lead to backlash at the gears in this case.

4. Conclusions
This paper presents the successful development of torsional models of screw compressors when they are driven by trucks. The models developed are an extension to those developed for an electric motor drive. It explains the effect of higher order modes on inter-lobe clearance between rotors and presents the source of the pulsating noise experienced in screw compressors. The following conclusions can be drawn: (i) Excitation of the first torsional mode can lead to torque peak fluctuation in screw compressors and to pulsating noise. (ii) Driving through the male rotor leads to lower torsional response. (iii) When the second torsional mode in screw compressors is excited it can lead to a reduction in the inter-lobe clearance and to rotor-to-rotor contact. (iv) Pulsating noise in screw compressors is due to amplitude modulation.

The following work is planned for the future: (a) Torque measurements in machines B and C and using the torsional frequency obtained to tune k12 and comparing it to the stiffness k12 obtained when torque measurement data of machine A is used as presented in this paper and (b) Presentation of the methodology that is used in the detailed calculation of the inter-lobe clearance distribution between rotors.

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