Coulomb drag in intermediate magnetic fields

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(March 24, 2022)

We investigated theoretically the Coulomb drag effect in coupled 2D electron gases in a wide interval of magnetic field and temperature $1/\tau \ll \omega_c \ll E_F/\hbar$, $T \ll E_F$, $\tau$ being intralayer scattering time, $\omega_c$ being the cyclotron frequency. We show that the quantization of the electron spectrum leads to rich parametric dependences of drag transresistance on temperature and magnetic field. This is in contrast to usual resistance. New small energy scales are found to cut typical excitation energies to values lower than temperature. This may lead to a linear temperature dependence of transresistance even in a relatively weak magnetic field and can explain some recent experimental data.

We present a novel mechanism of Coulomb drag when the current in the active layer causes a magnetoplasmon wind and the magnetoplasmons are absorbed by the electrons of the passive layer providing a momentum transfer. We derived general relations that describe the drag as a result of resonant tunneling of magnetoplasmons.

PACS numbers:73.20.Mf, 73.61.-r, 73.23.Ps

I. INTRODUCTION

When two 2-dimensional electron systems are placed in close proximity, then even in the absence of electron tunneling between layers the current in one layer (the active layer) will cause the current in the other (the passive layer). This phenomenon is known as a frictional drag and is due to the interlayer Coulomb interaction which causes a momentum transfer from one layer to the other. If no current is allowed in the passive layer, the potential difference develops there to compensate the frictional interlayer force. The transresistance is measured as the ratio between a momentum transfer from one layer to the other. If no current is allowed in the passive layer, the potential difference develops there to compensate the frictional interlayer force. The transresistance is measured as the ratio between the electric field developed in the passive layer to the current density in the active layer. This is in contrast to usual resistance. New small energy scales are found to cut typical excitation energies to values lower than temperature. This may lead to a linear temperature dependence of transresistance even in a relatively weak magnetic field and can explain some recent experimental data.

Until recently, the experiments on frictional drag have been done in a zero magnetic field. In this regime, the theoretical description is well elaborated. The $T^2$ dependence of transresistance was explained by the phase-space arguments.

A number of experiments on Coulomb drag in magnetic field have been also reported. Most of them are done in Quantum Hall regime ($T \ll \hbar \omega_c \simeq E_F$, $\omega_c$ being temperature and cyclotron frequency respectively) when the screening of the interaction and the polarization function are determined by the states of the lowermost Landau level. The experiments have shown a strong filling factor dependence of drag resistance. It was strongly enhanced (compared to the $H = 0$ case) when the Fermi level lay within the Landau level and was strongly suppressed when the Fermi level was between the Landau levels. The recent theoretical work (which treats the case $T < \Delta \ll \hbar \omega_c \simeq E_F$) predicts a twin-peak structure of the transresistance as a function of the magnetic field. This is due to the interplay between the screened interlayer interaction and the phase-space available for the interlayer e-e scattering.

In the present work, we investigate the drag effect in intermediate magnetic fields $1/\tau \ll \omega_c \ll E_F/\hbar$. In contrast to Quantum Hall regime, the electrons occupy many Landau levels. Still these Landau levels remain well resolved since their width $\Delta \simeq \hbar \sqrt{\omega_c/\tau} \ll \hbar \omega_c$ is much smaller than the level spacing.

The electron density of states is strongly distorted in comparison with its zero-field value. That is why even in the "classical" regime $E_F \gg T \gg \hbar \omega_c$ and $R_c \gg d$ ($R_c = v_F/\omega_c$ being the cyclotron radius, $d$ being the distance between the layers) the polarization function $\text{Im}\chi(\omega \simeq T, q \simeq 1/d)$ which is responsible for the absorption of energy, differs strongly from its zero-field form. As a function of frequency it consists of a series of well resolved peaks at multiples of cyclotron frequency. We will see that this circumstance leads to rich parametric dependences of transresistance
on temperature and magnetic field. This is to be contrasted with the usual intralayer resistance that exhibits no anomalies except strongly suppressed Shubnikov-de Haas oscillations and does not manifest the electron density of states.

In the intermediate magnetic field at temperatures $T > \hbar \omega_c$, weakly damped boson excitations become important. Those are magnetoplasmons with energies close to multiples of cyclotron energy. These excitations provide a new mechanism of Coulomb drag in the system. The momentum transfer is provided by magnetoplasmons being excited in one layer and absorbed into the other. We have found general relations that allow to present the magnetoplasmon contribution to the drag resistance as the result of resonant tunneling of magnetoplasmons.

The problem we consider in this paper is interesting also because the experimental dependence of transresistance in the indicated parameter region has a universal form $\propto B^2 T$. This temperature dependence is of a highly nontrivial form because the common arguments based on consideration of the phase-space available for the scattering give the $T^2$ dependence. We show that new small energy scales (for example, $\Delta \ll T$) play the role of characteristic cut-off energies which leads to a linear temperature dependence of transresistance in a relatively weak magnetic field.

A bird’s eye view of our results is provided in figure 1, where we present temperature and magnetic field exponents of the transresistance in five distinct parameter regions. We obtain detailed analytical results for the first three regions that correspond to $T \gg \hbar \omega_c$. In addition to slow $T$- and $B$-dependence the transresistance in the regions IV-VII ($T \ll \hbar \omega_c$) exhibits oscillatory dependence on inverse magnetic field (Shubnikov-de Haas oscillations). The oscillations can be hardly investigated analytically except simplest cases. In the present paper we only present analytical results for the regions VI and IV and give estimations of the transresistance in the regions V and VII for typical filling factors.

The outline of the paper is as follows. In section 2 we list the theoretical assumptions we made and present the method which is essentially the same as in Ref. 1. Details of polarization function and magnetoplasmon spectrum are presented in section 3. In section 4 we give a detailed description of magnetoplasmon mechanism of the drag. A phenomenological description of the resonant tunneling of magnetoplasmons is elaborated in section 5. We list our analytical results for high temperatures in section 6. Section 7 is devoted to evaluation of the transresistance at low temperatures.

II. METHOD

In the present paper, we cover parameter region $\Delta \ll \hbar \omega_c \ll E_0 = \hbar v_F / d$. Here $\Delta$ stands for the width of the Landau level. It determines the maximum one-particle density of states in a magnetic field. The ratio between $\hbar \omega_c$ and $T$ can be arbitrary.

We assume here that Landau levels acquire width due to scattering by impurities and, following 1, treat the effect in the selfconsistent Born approximation (SCBA). This approximation is known to lift the difficulties related to the high Landau level degeneracy. In this approach, $\Delta^2 = (2/\pi) \hbar \omega_c \hbar / \tau$. This expression for $\Delta$ is valid for a short range ($\delta$-correlated) random potential.

We also assume Coulomb mechanism of the drag 2, so that the d.c. drag current results from the rectification by the passive layer of the a.c. fluctuating electric field created by the active one. In diagrammatic language, the transconductance is given by a diagramm composed of three-body correlation functions, those are connected by Coulomb interaction lines (photons propagators). It was argued in 3 that under very general conditions the three-body correlation functions can be expressed in terms of electron polarization functions $\chi(\omega, q)_{1,2}$ in each layer.

This yields the following expression for the diagonal element of the transresistivity tensor (Eq. 28 of Ref. 4):

$$\rho_{12}^{xx} = -\frac{\hbar^2}{2e^2 n_1 n_2 T} \int \frac{d^2 q}{(2\pi)^2} q^2 \int_0^\infty \frac{d\omega}{2\pi} \frac{|V_{12}(q)|^2}{\mathcal{E}(\omega, q)} \frac{\text{Im} \chi_1(\omega, q) \text{Im} \chi_2(\omega, q)}{\sinh^2(\hbar \omega / 2T)}. \tag{1}$$

Here $n_1$, $n_2$ are electron concentrations in the layers, $V_{12}(q)$ is the Fourier component of interlayer Coulomb interaction and $\mathcal{E}(\omega, q)$ describes (dynamical) screening of this interaction. Electrostatics gives $V_{12} = V(q) \exp(-q l), V(q) = 2\pi e^2 / \epsilon q$ being the bulk dielectric constant, and

$$\mathcal{E}(\omega, q) = (1 + V(q) \chi_1)(1 + V(q) \chi_2) - V_{12}^2(q) \chi_1 \chi_2. \tag{2}$$

It has been argued 4 that Eq.1 is valid for arbitrary magnetic field provided $q^{-1} \ll l, R$, $l$ being mean free path. Our checks confirm that, so we use the Eq. 4 in our calculations in intermediate magnetic field regime.

As a reference, we give here the expression for transresistance in the absence of magnetic field for temperature $T \ll E_0$ 4.
\[ \rho^{12}_F = \frac{\zeta(3)}{64} \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^2} \frac{T^2 a_B^2}{\hbar^2 v_F v_F}. \]  

Here \( \zeta(3) \approx 1.202 \) is the Riemann zeta-function, \( v_{F1}, v_{F2} \) are the corresponding Fermi velocities and \( a_B \) is the Bohr radius.

To make use of Eq. [3] we shall evaluate the polarization function \( \chi(\omega, q; B) \). That we do in the following section.

We will assume that the layers are macroscopically identical. We keep the indices 1, 2 that label the layers in the formulas solely for the sake of physical clarity. The exception will be the discussion of Shubnikov-de Haas oscillations in the regions IV and VI. We allow there for different filling factors in the layers.

### III. POLARIZATION FUNCTION AND MAGNETOPLASMONS

We start with the following expression for the imaginary part of the polarization function:

\[ \text{Im} \chi(\omega, q) = \nu \omega c \sum_{n,m} J^2_{n-m}(qR_c) \int_{-\infty}^{+\infty} \frac{d\epsilon}{\pi} [n_F(\epsilon) - n_F(\epsilon + \omega)] \text{Im} G'_n(\epsilon) \text{Im} G'_m(\epsilon + \omega), \]  

where \( \nu = m_0/\pi \hbar^2 \) is the 2D thermodynamic density of states in the absence of magnetic field, \( n_F \) is the Fermi distribution function, and \( G'_n \) is the retarded Green’s function of the electrons in the nth Landau level. In the above formula, we have taken into account that under the conditions considered in this paper \( T, \hbar \omega_c \ll E_F \) only big Landau level numbers are important. Thus the bare vertex function is reduced to its quasiclassical form, which is Bessel function of the argument \( qR_c \). In the limit \( \Delta \to 0 \) (no disorder) this expression is equivalent to semiclassical approximations employed in Eq. [3]

The expression (4) disregards vertex corrections due to disorder. This is safe since we always assume that \( qv_F \gg 1/\tau \) and \( qR_c \gg 1 \). We will also disregard the rapidly oscillating part of Bessel function squares at \( qR_c \gg 1 \), that is, we assume that \( J^2_{n-m}(qR_c) \approx 1/\pi qR_c \). We discuss the relevance of this assumption in Appendix.

Using the SCBA expression for \( \text{Im} G_n \) in the limit of big \( n \),

\[ \text{Im} G'_n(\epsilon) = -\frac{2}{\Delta} \sqrt{1 - \left( \frac{\epsilon - \epsilon_n}{\Delta} \right)^2} \Theta \left( 1 - \left( \frac{\epsilon - \epsilon_n}{\Delta} \right)^2 \right), \]  

where \( \Theta \) is the step-function and \( \epsilon_n = (n + 1/2) \hbar \omega_c \), we obtain from Eq. [4] that

\[ \text{Im} \chi(\omega, q) = \nu \frac{4\omega_c}{\pi^2 \Delta} \frac{\omega_c}{qv_F} \sum_{n,m} [n_F(\epsilon_n) - n_F(\epsilon_n + \omega)] X_{im}(\frac{\epsilon_n - \epsilon_m + \hbar \omega}{2\Delta}). \]  

Here we define dimensionless function \( X_{im}(x) \equiv (4/3)[(1 + x^2)E(\sqrt{1 - x^2}) - 2x^2F(\sqrt{1 - x^2})] \) for \( |x| \leq 1 \) and \( X_{im}(x) = 0 \) otherwise. The functions \( F(x), E(x) \) are the complete elliptic integrals of the first and second kind, respectively.

Note that \( \int_{-1}^{+1} dx X_{im}(x) = \pi^2/8 \). The expression for \( \text{Im} \chi(\omega, q) \) assumes different forms depending on the temperature. In the most interesting case \( T \gg \hbar \omega_c \) and \( qv_F \gg \omega \) (the latter inequality is equivalent to \( T \ll E_0 \), because the characteristic \( q \approx 1/d \) and the frequency cannot be larger than the temperature) we get from Eq. [3]

\[ \text{Im} \chi(\omega, q) = \nu \frac{4\omega_c}{\pi^2 \Delta} \frac{\omega_c}{qv_F} \sum_{j=-\infty}^{+\infty} X_{im}(j) \frac{+\infty}{\pi} X_{im}(\frac{\omega - j \hbar \omega_c}{2\Delta}). \]

(8)

The functions \( X_{re}(x) \) and \( X_{im}(x) \) are plotted in Fig.2.

In the opposite limit of \( \Delta \ll T \ll \hbar \omega_c \) and \( \omega \ll 2\Delta; qR_c \gg 1 \) we obtain from Eq. [3]:

\[ \text{Im} \chi(\omega, q) = \nu \frac{4\omega_c}{\pi^2 T} \frac{\omega_c}{\Delta} \frac{1}{qR_c} X_{im}(\frac{\omega}{2\Delta}) f_n(1 - f_n), \]  

(9)
where \( f_n = 1/(1 + \exp(\epsilon_n - \mu)/T) \) being the filling factor of the \( n \)-th Landau level in the layer, \( \mu \) being chemical potential.

Finally, at \( T \ll \Delta \) we can set \( n = m \) and integrate expression (4) over \( \epsilon \) in close vicinity of \( \mu \). This gives

\[
\text{Im} \chi = \frac{4\nu \omega^2}{\pi^2 \Delta^2} \frac{\omega}{q v_F} (1 - \frac{(\mu - \epsilon_n)^2}{\Delta^2})
\]

(10)

Although the magnetoplasmon modes of a 2DEG have been extensively studied\(^9\), very little attention has been paid to their properties at high frequencies in short-wave limit. Since those are of interest for us, we investigated them in some detail. The dispersion curves of the magnetoplasmon modes in the case of the weak damping are determined by the equation \( \text{Re} \chi = 0 \), \( \chi \) being given by Eq. (4). From this equation we get \( (\text{Re} \chi/\nu) + z_{\pm} = 0 \), where \( z_{\pm} = qa_B/(2(1 \pm \exp(-q d))) \ll 1 \). Using Eq. (5) for \( \text{Re} \chi \), for each \( j \) we obtain two solutions corresponding to two (\( \pm \)) magnetoplasmon modes. First thing to note is that under our assumptions \( qa_B \ll 1 \) so that on a large frequency scale \( z_{\pm} \) can be safely omitted. The resulting equation \( \text{Re} \chi = 0 \) determines a series of Burstein-like magnetoplasmons\(^{10}\) with frequencies that in the interesting region of \( q \) are close to cyclotron harmonics \( j\omega_c \). (see Fig. 3) If \( q < q_0 \equiv -4jX_{ce}/(1\omega_c^2/\pi^2\Delta v_F \simeq 0.19j\omega_c/R_c \Delta \) the root of dispersion relation lies beyond the electron adsorption bands and the magnetoplasmon is not damped. These magnetoplasmons are of no interest for us since they do not talk to electrons and thus cannot participate in drag. If \( q \gg q_0 \) magnetoplasmons lie deep in the electron adsorption band and are strongly damped, so that their contribution to drag cannot be distinguished from the contribution of electron-electron scattering. This is why we concentrate now on a close vicinity of \( q_0 \) (right panel of Fig. 3).

In this vicinity \( \chi(\omega, q) \) can be expanded in Taylor series in terms of \( w = \omega - j\omega_c - 2\Delta, \kappa = q - q_0 \), assuming that \( w \ll \Delta, \kappa \ll q_0 \),

\[
\chi/\nu = C_2 \frac{w}{2\Delta} + \frac{\kappa}{q_0} + iC_1 \left( \frac{w^2}{2\Delta} \right)^2 \Theta(-w)
\]

(11)

Here \( C_{1,2} \) are numerical constants characterizing behaviour of \( X_{im}, X_{re} \) near \( x = 1 \), \( C_1 \simeq 6.23, C_2 \simeq 2.19 \). This determines the dispersion law of magnetoplasmons

\[
w_{\pm} = -\frac{2\Delta}{C_2} (z_{\pm} + \kappa/q_0)
\]

(12)

and their damping

\[
\Gamma = \Theta(-w) \frac{C_1 w^2}{C_2 \Delta}
\]

(13)

where \( z \) is taken at \( q = q_0 \). Symmetric and asymmetric modes are split by

\[
\delta \omega = \Delta \frac{q_0 a_B}{C_2 \sinh q_0 d}
\]

(14)

We see that \( \Gamma \ll w \ll \Delta, \delta \omega \ll \Delta, \delta \omega \) can be comparable with \( \Gamma \).

**IV. MAGNETOPLASMON CONTRIBUTION.**

In this section we consider the magnetoplasmon mechanism of Coulomb drag.

In the absence of magnetic field there are two plasmon modes in double-layer system, the one with the electron densities in the two layers oscillating in phase (the optic mode), and the other one where the oscillations are out of phase (the acoustic mode). It was pointed out in\(^2\) that the drag effect can be greatly enhanced by dynamical "antiscreening" of the interlayer interaction due to coupled plasmon modes. Since the plasmon modes lie beyond the \( T = 0 \) particle-hole continuum, temperatures of the order of Fermi energy are required for a large plasmon enhancement of the drag effect. Only then the thermally excited electrons and holes with plasmon velocities provide sufficient damping of the plasmon modes and thus facilitate plasmon interaction with electrons.

In the revised situation of intermediate magnetic field, the magnetoplasmons have even better chances to enhance the drag. First, there are many modes and their typical energies are of the order of \( \hbar \omega_c \). Therefore these modes can be excited at temperatures much lower than Fermi energy. Second, the magnetoplasmons in our model acquire natural damping: due to finite Landau level width they may lay within the particle-hole continuum. The finite temperature
without disorder does not lead to magnetoplasmon damping and to the drag effect. This is in contrast to the situation without magnetic field, where Imχ at the plasmon frequency was calculated for collisionless plasmas. The magnetoplasmon mechanism of the Coulomb drag, when the current in the active layer causes a magnetoplasmon wind and the magnetoplasmons are absorbed by the electrons of the passive layer leading to transfer of the momentum, must be quite general one. In this section, we evaluate magnetoplasmon contribution with using Eq. 1.

5. RESONANT TUNNELING OF MAGNETOPLASMONS

We give here another derivation of this formula which clarifies its physical meaning. To describe resonant tunneling of plasmons between the layers, we introduce for each plasmon mode a density matrix

\[ \rho_{ij} = \frac{1}{2} (|i\rangle \langle i| + |j\rangle \langle j|) \]





For identical layers, diagonal elements of the Hamiltonian are equal to each other and dissappear from the equation. The non-diagonal element that is responsible for plasmon tunneling between layers can be readily express in terms of splitting \( \delta \omega \) between symmetric and assymmetric plasmon state: \( H_{12} = H_{21} = \delta \omega / 2 \).

The dissipation takes place independently in each of the layers. It contributes to the time derivative of the diagonal density matrix elements in the following way:

\[ \frac{\partial \rho_{ii}}{\partial t} = \Gamma_i (n_i^B - \rho_{ii}) \]

\[ \frac{\partial \rho_{ij}}{\partial t} = - \frac{(\Gamma_i + \Gamma_j)}{2} \rho_{ij} \]

The system of equations that incorporates both dissipation and resonant tunneling reads as follows:
\[
\frac{\partial \rho_{11}}{\partial t} = \Gamma_1 (n_1^B - \rho_{11}) + i \frac{\delta \omega}{2} (\rho_{12} - \rho_{21}) \\
\frac{\partial \rho_{22}}{\partial t} = \Gamma_2 (n_2^B - \rho_{22}) + i \frac{\delta \omega}{2} (\rho_{21} - \rho_{12}) \\
\frac{\partial \rho_{12}}{\partial t} = -\frac{\Gamma_1 + \Gamma_2}{2} \rho_{12} + i \frac{\delta \omega}{2} (\rho_{11} - \rho_{22})
\]
where \(\rho_{21} = \rho_{12}'\). The stationary solution takes the form
\[
\rho_{11} = n_1^B - \frac{\Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2 (n_1^B - n_2^B)}{\delta \omega^2 + \Gamma_1 \Gamma_2} \tag{21}
\]
Expression for \(\rho_{22}\) is obtained by reverting indices 1, 2.

We can now evaluate the drag force acting on electrons of each layer by equating it to a momentum flow between the layers. We sum over modes with all possible \(q\) and obtain
\[
F = -\sum_q \hbar q (\frac{\partial \rho_{11}}{\partial t})_{\text{diss}} = -\sum_q \hbar q \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2 (n_1^B - n_2^B)}{\delta \omega^2 + \Gamma_1 \Gamma_2} \tag{22}
\]
where \(\Gamma, n^B, \delta \omega\) may be \(q\)-dependent.

We assume that the current flows in the layer 2 and the drag force in the layer 1 is equilibrated by the electric field. The transresitivity is essentially the ratio of this field to that current. The effect of the current is that the equilibrium Bose distribution in the reference frame where the electrons of the second layer are in average at rest, rather than in the laboratory reference frame. So that
\[
\chi (\nu) = \frac{f_B (\nu) - \bar{h}(\nu_{\text{drift}} \nu)}{f_B (\nu) - \bar{h}(\nu_{\text{drift}} \nu)} \tag{23}
\]
\(\nu_{\text{drift}}\) being the drift velocity.

Substituting (23) in (22) we obtain
\[
F = \hbar^2 \nu_{\text{drift}} \int \frac{d^2 q}{8\pi^2} \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2}{\delta \omega^2 + \Gamma_1 \Gamma_2} \frac{\partial f_B (\nu)}{\partial \nu} \tag{24}
\]
The last things to note are that \(F = e n_1 E\) and \(I = e n_2 \nu_{\text{drift}}\). If we use this and set \(\Gamma_1 = \Gamma_2 = \Gamma\) we reproduce Eq. (17).

VI. RESULTS: HIGH TEMPERATURES

In this section we consider the drag resistance at temperatures \(T \gg \hbar \omega_c\), which are sufficiently high to excite magnetoplasmons and electrons in many Landau levels. As we can see in Fig. 1, at high temperature we encounter at least three distinct regions with different temperature and magnetic field exponents.

First of all, we shall explain why there are so many regions. If we compare imaginary part of polarization function with and without magnetic field, we see that their values averaged over frequency intervals bigger than \(\hbar \omega_c\) are the same \(<\langle \text{Im} \chi \rangle\rangle = \langle\langle \text{Im} \chi (H = 0) \rangle\rangle\). However, \(\text{Im} \chi = 0\) beyond narrow adsorption bands (Fig. 3). This means that within the bands \(\text{Im} \chi\) is significantly enhanced in comparison with its zero-field value, typically by a factor \(\omega_c/\Delta\). Surprisingly enough, the enhancement of \(\text{Im} \chi\) can lead both to enhancement and suppression of the drag.

If \(\text{Im} \chi \ll \nu\), \(\text{Re} \chi \approx \nu\). The denominator \(\mathcal{E}\) in Eq. (4) which is responsible for screening of interlayer potential is the same as without magnetic field. We refer to this situation as to normal screening regime. In this regime, the transresistance is in comparison to its value without magnetic field, since the effect is proportional to \(<\langle \text{Im} \chi \rangle^2\rangle >\langle\langle \text{Im} \chi (H = 0) \rangle^2\rangle\).

At further increase of \(\text{Im} \chi\), \(\text{Re} \chi\) develops as well so that both \(|\text{Re} \chi|, \text{Im} \chi\) become bigger than \(\nu\). The denominator \(\mathcal{E}\) strongly increases. That efficiently screens out the inter-layer interaction and leads to drastic descrease of the transresistance. That we will call overscreening.

However, \(\mathcal{E}\) can also decrease with increasing \(\text{Im} \chi\) and pass zero. Near this line, the inter-layer interaction is greatly increased. This is where the magnetoplasmon contribution dominates.

The actual value of the drag effect is thus determined by interplay of these three competing tendencies.
Let us first evaluate the magnetoplasmon contribution. Substituting expressions \([13], [14]\) to Eq.\([17]\) we notice that the integrand has a sharp extremum near \(\kappa \approx 0\) so we can formally integrate over \(\kappa\) in infinite limits. This yields the relation which is valid for all regions,

\[
\rho_{12}^{xx} = -0.00221 \frac{\hbar \Delta}{e^2 n_1 n_2 d^4 T} \frac{\alpha_B}{d} \sum_{j=1}^{\infty} \frac{(\hbar \omega_j/T)^4}{\sinh^2(\hbar \omega_j/2T)} \left( \frac{\alpha \hbar \omega_j/T}{\sinh(\alpha \hbar \omega_j/T)} \right)^{3/2},
\]

where \(\alpha \approx 0.19(T\omega_c/\Delta E_0)\).

This expression can be simplified further. The region I (\(\hbar \omega_c \ll T \ll E_0/\Delta \omega_c\)) corresponds to \(\alpha \ll 1\). Since \(\hbar \omega_c \ll T\) the characteristic values of \(j\) in the sum (25) are much bigger than unity. Therefore we can convert the sum over \(j\) into the integral. Since \(\int_0^\infty dx x^4 \sinh^{-2}(x/2) = 16 \pi^4/15\), we obtain

\[
\rho_{12}^{xx} \approx -0.0003 \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{(\hbar \omega_c)^3 T^4}{\Delta^3 E_0} \left( \frac{\alpha_B}{d} \right)^{3/2}.
\]

The region II is defined by inequalities \(T \gg E_0\Delta/\omega_c \gg \omega_c\). Here \(\alpha \gg 1\). As a result, the characteristic frequencies here \(\dot{\omega}_j \approx E_0\Delta/\omega_c \ll \omega_c\) (see Eq.\([27]\)). These frequencies, however, are still much larger than cyclotron frequency: \(\dot{\omega}_j \gg \omega_c\). This enables us again to introduce the continuous variable \(x = \alpha \hbar \omega_j/T\) and convert the sum into the integral:

\[
\rho_{12}^{xx} \approx -0.0095 \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{T}{E_0} \left( \frac{\alpha_B}{d} \right)^{3/2}.
\]

In region III (\(\omega_c \gg \sqrt{E_0\Delta}\)) the \(j = 1\) term dominates the sum. The corresponding expression for the plasmon drag resistance is exponentially small,

\[
\rho_{12}^{xx} \approx -2.7 \times 10^{-6} \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{(\hbar \omega_c)^3}{\Delta^9/2 E_0^{11/2}} \left( \frac{\alpha_B}{d} \right)^{3/2} \exp(-0.28(\hbar \omega_c)^2/\Delta E_0).
\]

This is due to the fact that the value of \(q\) which is needed to bring the magnetoplasmon pole to the vicinity of the Landau level is large compared to \(1/d\). Similar situation occurs in the region IV, with the exponential suppression being due to low temperature,

\[
\rho_{12}^{xx} \approx -1.12 \times 10^{-5} \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{(\hbar \omega_c)^8}{\Delta^4 E_0} \left( \frac{\alpha_B}{d} \right)^{3/2} \exp(-\hbar \omega_c/T).
\]

We will see below that even the exponentially suppressed magnetoplasmon contribution can efficiently compete with the quasiparticle one.

Now we will estimate quasiparticle contribution in all three regions.

Let us first consider region I (\(\omega_c \ll T \ll E_0\Delta/\omega_c\)). In this parameter interval the characteristic values of \(\hbar \omega_c \sim T\) and \(qd \approx 1\). It follows from Eqs.\([13, 14]\) that \(\Re \chi = \nu\) and \(\Im \chi \approx \nu\). Thus we are in the normal screening regime. Using the fact that \(V(q)\nu \gg 1\) (in other terms, \(qa_B \ll 1\)) we obtain from Eq.\([17]\)

\[
\rho_{12}^{xx} \approx -0.00725 \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{T^2 a_B^2}{\hbar^2 v_F^2} \frac{\omega_c}{\Delta}.
\]

This magnetotransresistance is bigger than the zero-field value (Eq.\([3]\)) by a factor of \(\omega_c/\Delta \gg 1\). This is due to the discreteness of the electron spectrum in the magnetic field when the density of states within the Landau level remarkably increases.

Region II (\(\omega_c \ll E_0\Delta/\omega_c \ll T\)). Though the main contribution to the drag in this region is due to the magnetoplasmon mechanism, it is instructive to give an estimation of the quasiparticle contribution. As in the region I, \(q \approx 1/d\). The characteristic frequency, however, is restricted by the value \(\omega_{cut} \sim E_0\Delta/\omega_c\) which is much smaller than the temperature (though still much larger than the cyclotron frequency). The reason is the overscreening. For estimations, we approximate \(\Im \chi\) near the cyclotron resonance by its value from Eq.\([7]\) at \(qd \approx 1\). That gives \(\Im \chi \approx \nu \omega/\omega_{cut}\). A good estimation for \(\Re \chi\) near the resonance is \(\nu \gg \Im \chi\) and \(\Im \chi\) otherwise. Thus the integrand in Eq.\([17]\), \((\Im \chi)^2/|E|^2 \infty \Im \chi)^2/(\Re \chi^2 + \Im \chi^2)^2\) achieves a maximum at \(\omega_{cut}\). Since the interval of integration over frequency is effectively smaller than the temperature, the transresistance exhibits a linear temperature dependence:

\[
\rho_{12}^{xx} \sim (\hbar/e^2)(1/n_1 n_2 d^4)(a_B^2/d^2)(T/E_0).
\]
Let us compare this with the $B = 0$ case. There, the linear temperature dependence starts at $T > E_0$. This is because the absorption of energy due to the Landau damping mechanism is possible only for frequencies smaller than $v_{f} / d$. Thermal frequencies are ineffective, because corresponding phase velocities are larger than $v_{f}$. In the presence of a magnetic field, the role of a cut-off energy is taken by a much smaller value $E_0 \Delta / \omega_c$. So that, the linear temperature dependence of the transresistance starts at much lower temperatures.

Region III ($\sqrt{E_0 \Delta} \ll h \omega_c \ll T$). Here $\omega_{cut}$ becomes smaller than $\omega_c$. There is overscreening at $qd \simeq 1$ for all cyclotron resonances. The contribution of resonances to integral Eq. (3) is of the order of $\rho_{12}^{xx} \sim (h / e^2)(1 / n_1 n_2 d^4)(R_{c1}^2 a_B^2 / d^4)((T \Delta^3 / \omega_c^2)^2).

The main contribution is determined by the frequencies $\omega \sim \Delta \ll \omega_c$. So that the quasiparticles are created within the same Landau level. For these frequencies we always have the normal screening situation. We can set $\text{Re} \chi = \nu$. The $\text{Im} \chi$ is determined by the $j = 0$ term of Eq. (2). We obtain

$$\rho_{12}^{xx} = -\frac{\hbar}{e^2} \frac{2}{\pi^4} \frac{1}{n_1 n_2} \int \frac{d^2 q d^2 q'}{(2\pi)^2} \frac{1}{\sin^2(qd)} \frac{T a_B^2}{\Delta^2 R_{c1} R_{c2}} \int_0^{2\Delta} \frac{d\omega}{2\pi} \frac{\omega^2}{X_{im}} \left( \frac{\omega}{2\Delta} \right) \simeq -0.0011 \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^4} \frac{\omega_c^2 T a_B^2}{E_{F1} E_{F2}} \left( \frac{\omega}{2\Delta} \right)$$

(31)

It is interesting to note that this magnetic and temperature dependence coincides precisely with the observed one if we assume that $\Delta$ does not depend on magnetic field. Indeed, the magnetic dependence of $\Delta$ is rather weak. These experiments were performed in rather strong magnetic field where only few Landau levels were occupied so that one should not expect quantitative agreement with our calculations. From the other hand, the linear $T$-dependence is rather remarkable. We believe that in any case it indicates a reduction of typical excitation energies to values much smaller than $T$, possibly due to overscreening at energies of the order of $T$.

Now we are in position to compare quasiparticle and magneto-plasmon contribution and thus to set the borders of the gray-shaded regions in Fig. 1. The magneto-plasmon contribution dominates throughout the region II. In the region I we compare expressions (23) and (31). Magneto-plasmon contribution dominates provided $T \simeq E_0 \Delta / h \omega_c (a_B / d)^{1/2}$. Since experimentally $d \simeq a_B$ this happens in fact close to the border between the regions I and II $T \simeq E_0 \Delta / h \omega_c$. The exponentially small magneto-plasmon contribution given by Eq. (23) in the region III and dominates provided $0.28 \omega_c^2 / (E_{F0} \Delta) < \ln(\omega_c^2 / E_0 \Delta)^{1/2} (d / a_B)^{1/2}$. It also dominates in the region IV if $T / h \omega_c > 1 / \ln(\omega_c^2 T d^{1/2} / (E_0 \Delta d B / a_B^{1/2}))$. The latter condition is obtained by comparing expressions (24) and (32).

VII. RESULTS FOR LOW TEMPERATURES

In this section we present our results for low temperatures $T \ll h \omega_c$. Owing to energy limitations, only the states of the upper partially filled Landau level are involved into the drag. This makes the transresistance sensitive to concrete value of this filling factor. In addition to slow dependence on magnetic field, the drag effect exhibits oscillatory dependence on inverse magnetic field related to the filling factor. The detailed study of these Shubnikov-de Haas oscillations is beyond the limits of the present work and will be presented elsewhere. This is why we provide here analytical results for the regions IV and VI only. As to the regions V and VII, we present below estimations of the transresistance for typical filling factors rather than detailed analytical results.

In the region IV ($\Delta \ll T \ll h \omega_c \ll \sqrt{E_0}$) the situation is the most straightforward one since it follows from Eq. (3) that $\text{Im} \chi \ll \nu$. Thus we encounter here the normal screening and we may set $\text{Re} \chi = \nu$. We obtain for the transresistance:

$$\rho_{12}^{xx} = -0.0011 \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^4} \frac{a_B^2}{v_{F1} v_{F2}} \frac{\omega_c^4}{T \Delta} f_{n_1}(1 - f_{n_1}) f_{n_2}(1 - f_{n_2})$$

(32)

$f_{n_1,2}$ being filling factors in the layers.

Even if $T \gg \Delta$, we encounter overscreening in the region V ($\sqrt{E_0} \ll h \omega_c \ll E_0$). For estimations, we set $\text{Im} \chi \sim \text{Re} \chi \gg \nu$ and obtain

$$\rho_{12}^{xx} \sim (h / e^2)(1 / n_1 n_2 d^4)(a_B^2 R_{c1} R_{c2} / d^4)((T^3 / \omega_c^2 \Delta))$$

(33)

As a consequence of the overscreening, the transresistance rapidly decreases with increasing magnetic field.

At low temperature $T \ll \Delta$ the integral in Eq. 1 is contributed by $\omega \simeq T$. This gives rise to featureless $T^2$ temperature dependence of Eq. 3 with the coefficient depending on magnetic field. The region VI ($T \ll \Delta$, $\omega_c \ll \sqrt{E_0} \Delta$) again corresponds to normal screening. We take $\text{Im}$ from Eq. (31) and set $\text{Re} \chi = \nu$. This yields
\[ \rho_{12}^{xx} = -0.0031 \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^2} \frac{a_B^2}{v_{F1} v_{F2}} T^2 \omega_c^4 \frac{\Delta^4}{\Delta^2} \left( 1 - \frac{(\mu_1 - \epsilon_n)^2}{\Delta^2} \right) \left( 1 - \frac{(\mu_2 - \epsilon_n)^2}{\Delta^2} \right) \]  

where \( \mu_{1,2} \) are chemical potentials in the layers. They are related to filling factors \( f_{n_1,n_2} \) by means of

\[ f_n = 1/2 + (1/\pi) \left[ 1 - \frac{(\mu_1 - \epsilon_n)^2}{\Delta^2} \right] \sqrt{1 - \left( 1 - \frac{(\mu_1 - \epsilon_n)^2}{\Delta^2} \right)^2 + \arcsin \left( 1 - \frac{(\mu_1 - \epsilon_n)^2}{\Delta^2} \right) } \].  

For a typical filling factor, the effect is bigger than zero filed transresistance by a factor of \( (\omega_c/\Delta)^4 \).

ACKNOWLEDGMENTS

This work is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie (FOM)", which is financially supported by the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)". We thank G. E. W. Bauer and L. I. Glazman for useful discussions, A.V. Khaetskii acknowledges NWO for its support of his stay in Delft. He is also grateful to F. Hekking for collaboration at the earlier stage of the work, J.T. Nicholls and D.E. Khmelnitskii for useful discussions and M. Pepper for hospitality and financial support at Cavendish Laboratory where a part of this work has been done.

APPENDIX:

In this work we disregard rapidly oscillating parts of polarization function given by Eq. 4. This means we approximate Bessel function squares in Eq. 4 in the limit of \( qR \) small enough, which is the mathematically correct expression. Let us explain why.

First let us note that if we take these oscillating parts into account it would significantly alter our results. In the normal screening regime it would give an extra factor of \( \sqrt{\omega_c} \). The answer would change even more drastically in the overscreening regime. The point is that oscillating terms would set the polarization function in coordinate representation, \( \chi(x,x') \) has a sharp edge at \( |x - x'| = 2R_c \). This gives rise to Fourier components \( \sim \cos(2qR_c) \).

If the edge is not sharp, the oscillating part is exponentially suppressed. The suppression is of the order of \( \exp(-q^2(\delta R)^2) \), \( \delta R \) being a typical rounding of the edge. By virtue \( \delta R \) is the typical uncertainty of the coordinate of the electron that makes a half of Larmor circle. Such an uncertainty can be of quantum-mechanical origin. In this case we estimate \( \delta R \approx \sqrt{R_c/k_F} \ll R_c \). Another cause of uncertainty may be small-angle scattering by smooth potential fluctuations in the heterostructure. For this case we estimate \( \delta R \approx \sqrt{R_c^3/\ell_{sa}} \), where transport mean free path \( \ell_{sa} \gg R_c \). It is interesting to note that scattering on point-like defects does not contribute to \( \delta R \) provided \( \omega_c \tau \gg 1 \).

Now we note that typical \( q \) contributing to the drag resistance are of the order of \( 1/d \). We conclude that the oscillating part is exponentially suppressed provided \( d < \sqrt{R_c/k_F} \) or \( d < R_c \sqrt{R_c/\ell_{sa}} \). We assume that at least one of these conditions is fulfilled.
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FIG. 1. Seven regions of different analytical behaviour of the transresistance in the intermediate magnetic field regime. Note log-log scale. First number in each region corresponds to temperature exponent, second number indicates magnetic field exponent. Vertical line corresponds to the condition $(\hbar \omega_c)^2 \simeq E_0 \Delta$.

FIG. 2. The functions $X_m(x)$ and $X_r e(x)$ determine the shape of $\text{Im} \chi$, $\text{Re} \chi$ in the vicinity of each cyclotron resonance.

FIG. 3. Electron adsorption in $\omega, q$ plane. Left plane: electron adsorption occurs i. in narrow strips around $\omega = 0$ and cyclotron resonances (particle-hole continuum) ii. on the magnetoplasmon dispersion curves. Right plane: intersection of the magnetoplasmon dispersion curve and the edge of the strip at a smaller scale. We illustrate splitting of the magnetoplasmons and the level width $\Gamma$ they acquire inside the strip.
