Detection of Hertz Frequency Multiharmonic Field Line Resonances at Low-L ($L = 1.1–1.5$) During Van Allen Probe Perigee Passes

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Abstract We present new and previously unreported in situ observations of Hertz frequency multiharmonic mode field line resonances detected by the Electric Field and Waves instrument on-board the NASA Van Allen probes during low-L perigee passes. Spectral analysis of the spin-plane electric field data reveals the waves in numerous perigee passes, in sequential passes of probes A and B, and with harmonic frequency structures from $\sim 0.5$ to $3.5$ Hz which vary with L-shell, altitude, and from day-to-day. Comparing the observations to wave models using plasma mass density values along the field line given by empirical power laws and from the International Reference Ionosphere model, we conclude that the waves are standing Alfvén field line resonances and that only odd-mode harmonics are excited. The model eigenfrequencies are strongly controlled by the density close to the apex of the field line, suggesting a new diagnostic for equatorial ionospheric density dynamics.

1. Introduction

In the Earth's magnetosphere, standing Alfvén wave oscillations along the length of the field lines can occur in the form of harmonic field line resonances (FLRs) at a discrete set of eigenfrequencies. These eigenfrequencies depend on the length of the magnetic field line, as well as the magnetic field strength and plasma mass density along the field line (e.g., Chen & Hasegawa, 1974; Southwood, 1974). The standing waves can be excited internally by unstable energetic charged particle distributions (e.g., Ozeke & Mann, 2001; Southwood et al., 1969; Zhu et al., 2020) or externally by the interaction of the solar wind with the Earth's magnetic field (e.g., Allan et al., 1986; Kivelson & Pu, 1984; Mathie & Mann, 2000). These waves have also been shown to play an important role in the acceleration and transport of relativistic electrons in the outer radiation belt (e.g., Fälthammar, 1965; Ozeke et al., 2020).

Early work reported the existence of auroral zone discrete FLRs in ground-magnetometer data (e.g., Samson et al., 1971), with later work further characterizing the existence of discrete frequency FLRs associated with discrete frequency waveguide modes in ground-based magnetometer (e.g., Mathie et al., 1999), radar (e.g., Fenrich et al., 1995; Ruohoniemi et al., 1991), and in situ data (e.g., Rae et al., 2005). Plaschke et al. (2009) also present evidence that these discrete frequency FLRs may be associated with standing Alfvénic Kruskal–Schwarzschild surface modes occurring at discrete frequencies on the magnetopause. Mathie and Mann (2000) and Menk et al. (1994) used the cross-phase technique (e.g., Waters et al., 1995) applied to ground-based magnetometer data at high latitudes to clearly demonstrate that discrete frequency FLRs were local enhancements in the Alfvén continuum, with in situ satellite data also showing evidence of broad-band excitation of Alfvén waves in the continuum (cf., Anderson et al., 1990; Engebretson et al., 1986) consistent with the theory of Hasegawa et al. (1983).

Typically, in situ measurements of harmonic FLRs from the field instruments on satellites in high-altitude elliptical orbits are limited to higher L-shells (cf. Takahashi et al., 1990, for $L = 2–6$). Similarly, ground-based magnetometer data have shown Pc3–4 FLRs at midlatitudes (e.g., Mathie & Mann, 2000) and at lower L-shells (e.g., Menk et al., 1994; Ziesolleck et al., 1993; from around $L = 1.4–2.7$), with Green et al. (1993) reporting local FLR frequencies in ground-based magnetometer data at $L \gtrsim 1.5$ with frequencies from 66 to 84 mHz. In situ satellites in polar low-Earth orbit (LEO), such as those from the ESA Swarm constellation, have observed Hertz frequency waves at midlatitudes and high latitudes (e.g., Kim et al., 2018), but these are...
usually associated with a higher altitude driver such as the growth of electromagnetic ion cyclotron waves which are excited in the equatorial plane and then propagate along field lines to the ionosphere. Here, we report observations of Hertz frequency standing mode harmonic FLRs which are observed in the electric field dynamic power spectra during low inclination perigee passes of the NASA Van Allen probes. To our knowledge, such observations are new and previously unreported. By comparing the observed harmonic spectra to those from models, we conclude that only odd-mode standing wave harmonics are excited. There is also significant current interest in monitoring near-equatorial plasma dynamics, for example, in relation to plasma bubbles (e.g., Park et al., 2015, and references therein). We discuss the potential future utility of our in situ observations of these low-L FLRs as a new diagnostic for equatorial ionospheric plasma mass density dynamics.

2. Data and Instrumentation

The primary data used in this study come from the two Van Allen probes orbiting at a highly elliptical, 10° inclination, geocentric orbit, originally with a perigee of ~600 km and an apogee of ~30,000 km (Kessel et al., 2013). However, toward the end of the Van Allen probe mission, the perigee was lowered to below ~300 km. The Electric Field and Waves (EFW) instrument suite used in this study consisted of four sensors in the spin plane and two sensors perpendicular to the spin plane on tubular extendable booms, resulting in three electric dipoles for differential potential measurements. The equipment was designed to detect quasistatic, low-frequency three-dimensional electric fields. The basic survey mode measures the electric field at 32 samples per second during the entire orbit. Experimental data show that the EFW instrument is sensitive to DC electric fields down to ~0.3 mV/m and <0.3 mV/m at higher frequencies (Wygant et al., 2013). The raw, spin-plane E-field used here was calculated from the potential difference between the sensors using \((V_2 - V_1)/X\) where \(V_1\) and \(V_2\) are the potentials from opposite sensors 1 and 2, respectively, and \(X\) is their separation length of around 100 m. Note that using sensors 3 and 4 resulted in similar electric fields. The probe’s spin axis points at the sun forming an angle of ±10.5°, with attitude maneuvers approximately every 21 days (Srinivasan et al., 2012).

The EFW data are publicly available in four different preprocessed formats (Level 1: raw to Level 4: fully processed) in “.cdf” files. The data presented here are from the EFW Level 2 16 samples per second electric field probe sensor potentials in day-long data sets. Note that the electric field due to the motion of the spacecraft through the background magnetic field has been removed in the Level 2 EFW data (Wygant et al., 2013). In the data presented here, a high-pass filter was applied to remove the slow-spin tone caused by the probes’ ~11.5 s spin period and very low-frequency elements. An Fast Fourier Transform (FFT) was used to produce dynamic spectrograms, which highlight the temporal evolution of the observed wave frequencies.

No clear and contemporaneous magnetic signatures of the waves were detected using the Electric and Magnetic Field Instrument Suite and Integrated Science magnetic search coil (MSC) or fluxgate magnetometer (MAG) data. This could be due to the reduced resolution of the MAGs near perigee and the frequency response of the MSC instruments, which are optimized for frequencies from 10 Hz to 12 kHz with a frequency response that drops off by a factor of 20 from 10 to 1 Hz (Kletzing et al., 2013). In addition, the Doppler effect of the fast-moving spacecraft close to perigee produces additional strong magnetic field signals making detection of the background waves difficult.

The electric field spectrograms during 72 and 75 perigee passes of probes A and B, respectively, occurring during January 2018 were manually analyzed. In the perigee pass data analyzed, Hertz frequency waves were a common occurrence both during and outside this interval. To illustrate this, some additional example spectrograms from later epochs after January 2018 are presented in the supporting material in Figures S2 and S4. Here, we select an example electric field spectrogram which occurred during the perigee pass of probe B from 16:22 to 17:22 UTC on 1 January 2018 for detailed analysis.

3. Observations

The low-frequency waves present in the spectrogram during the perigee pass of probe B on 1 January 2018, from 16:22 to 17:22 UTC, clearly illustrate evidence of multiple apparently harmonic wave frequencies. In order to examine if these wave frequencies result from different harmonic modes of standing FLRs, the
observed frequencies are compared with the eigenfrequencies from a standing wave mode, determined by solving the guided Alfvén wave equation in a dipole field.

Figure 1 shows the event in detail, presenting the spectrogram as a function of McIlwain L-shell and MLT in panel (a), and the altitude profile of probe B and the time series of the electric field derived from the potential difference between the electric field sensors in panel (b). The McIlwain L-shell (McIlwain, 1961) parameter was obtained from the probe’s ephemeris data and derived using the TS04D (Tsyganenko & Sitnov, 2005) and IGRF (Thebault et al., 2015) external and internal magnetic field models, for a locally mirroring particle. Geomagnetic activity remained quiet during January 2018 with Dst never dropping below −27 nT and Kp never exceeding 4.7. Wave signatures are present in the spectrogram during this entire 1-h-long data set where the altitude of probe B varied between 622.7 km and 5,937.3 km. Varying apparently harmonic wave frequencies can easily be distinguished during the in-bound perigee pass as the probe dropped below $L \lesssim 1.5$. As the probe moved inwards onto lower L-shells, the observed wave frequencies increased, reaching a maximum of $\sim 3.5$ Hz at $L = 1.2$. The frequency bands become less distinguishable during the outbound interval of the perigee pass, although there is evidence that they reappear again later in the outbound interval, the data suggesting that the frequencies in general decrease again as the probe moves toward higher $L$.

Wave signatures similar to those shown in the spectrogram in Figure 1 can be identified during almost all perigee passes from both probes during January 2018, as shown in Figure 2 (see also Figure S1). Figure 2 consists of six different spectrogram plots of 1-h-long perigee-centered windows. The data are from perigee passes on 1, 3, and 6 January 2018, with additional spectrograms during perigee passes on 2, 6, 19, 14, 18, 22, 26, and 30 January shown in Figure S1. The time domain waveform data were detrended by a 0.4-Hz high-pass filter, illustrated in Figure S5, to remove the strong spin tone and its harmonics generated by the probes’ $\sim 11.5$ s spin period. The rise and fall in the wave frequencies detected during each perigee pass is similar for both probes A and B. However, more intense frequency band structures are detected on probe B making identification of the different frequencies clearer along the perigee passes of probe B. Note that...
EFW sensor 1 on probe A deteriorated in October 2015 which may be the reason why less intense frequency band structures were measured on probe A.

The spectrograms were calculated using a FFT algorithm with 10-s (160 samples)-long Hanning windows (Harris, 1978). No geomagnetic storms were observed during the period of 6 days spanned by the perigee passes in Figure 2, as indicated in Figure 2g.

Wave spectrograms with similar frequency profiles to those shown in Figure 1, as well as those shown in Figure S1, are commonly detected during perigee passes of the Van Allen probes. For example, on January 2018, out of the 72 perigee passes which occurred, probe A detected 69 events with clear signatures of low-frequency waves manually identified in the spectrograms. Similarly, during all of the 75 perigee passes of probe B, clear wave signatures were identified. While most of the wave signatures detected during the perigee passes have the same rising and falling frequency profile, some events have frequency profiles which do not fit this behavior, such as the double humped and asymmetric frequency profiles shown in the spectrogram presented in Figure S2 in the supporting material. During the end of the Van Allen probe mission, the perigee of the probes was lowered to altitudes <300 km. Figures S3 and S4 illustrate that the wave signatures can be identified in the spectrograms even during this time interval where the perigee of the probes dropped below ∼300 km. Analysis of these events with more complex frequency–time profiles is beyond the scope of the current paper and will be examined in future work. However, most likely, these are the signatures of changing profiles of ionospheric mass density as a result of dynamical changes to the low-L ionosphere and magnetosphere.

Figure 2. (a)–(f) The electric field spectrograms measured along perigee passes of probes A and B from 1 to 6 January 2018, in a similar format to that shown in Figure 1. (g) The time evolution of geomagnetic indices Kp and Dst from 1 to 6 January 2018.
4. Plasma Mass Density Model

In order to investigate whether the observed perturbations in the electric field are consistent with those expected from standing field line resonant oscillations of the magnetic field lines, the standing field-aligned Alfvén eigenfrequencies have been determined. The wave eigenfrequencies have been estimated using the numerical solutions to the guided toroidal Alfvén wave equation presented in Ozeke et al. (2005) (see also, Allan & Knox, 1979; Ozeke & Mann, 2004). These guided toroidal Alfvén wave equations assume a dipole magnetic field and a thin sheet ionosphere. In order to determine the eigenfrequencies of the standing waves, the height of the ionosphere and the Pedersen conductance need to be specified, as well as the plasma mass density along the length of the field lines (Allan & Knox, 1979; Ozeke et al., 2005). Here, we assume a thin sheet ionosphere with a height of 100 km and a Pedersen conductance of 10 S. The assumption of a thin sheet ionosphere is typically used for solving the guided Alfvén wave equation on higher L-shells ($L > 2$) where only a negligible fraction of the total field line length passes through the ionosphere.

In order to estimate what impact an ionosphere with a finite thickness may have on the eigenfrequencies, the height of the thin sheet ionosphere was increased from 100 km to 150 km, representing reflection of the guided Alfvén waves from either the bottom or the top of an ionosphere with a finite width of 50 km.

Even during the lowest perigee pass of Van Allen probe B shown in Figure 1, where the apex of the magnetic field line intersecting the location of the probe is at an altitude of $\sim 1,000$ km, changing the height of the ionosphere from 100 km to 150 km increased the eigenfrequencies by only a small amount <14%. In addition, the eigenfrequencies are only weakly dependent on the value of the Pedersen conductance, with similar frequencies produced for Pedersen conductance values >0.1 S. The eigenfrequencies are however strongly dependent on the plasma mass density, especially near the field line apex and/or in regions of low Alfvén speed (see e.g., Ozeke et al., 2005). The geographic latitude, longitude, and altitude along the length of the magnetic field lines which the Van Allen probe crossed during the perigee pass have been determined using the IGRF field and the plasma mass density along the length of the magnetic field line has been determined using the empirical International Reference Ionosphere (IRI) model. The IRI model estimates the average monthly plasma mass density as a function of geographic latitude, longitude, and altitude up to a height of $\sim 1,500$ km (Bilitza, 1990) for magnetically quiet conditions outside the auroral zone. The electric field spectrograms shown in Figure 2 (see also Figure S1) appear similar during each of the perigee passes, suggesting that during January 2018 the plasma mass density along the field lines remained relatively stable. Consequently, since the mass plasma density appears stable and the geomagnetic conditions were quiet as indicated by the Kp and Dst indices shown in Figure 2, it is likely that the IRI model may give a reasonable estimate for the density along the field lines during these perigee passes.

To make progress with modeling the Alfvén eigenfrequencies along the entire field line, the plasma mass density profiles along the magnetic field lines as a function of latitude have been fitted to the function presented in Equation 1.

$$ y = a \left[ \sin \left( \lambda - b \right) \right]^{n} + c $$

Here, $y$ represents the log$_{10}$ of the plasma mass density, $\lambda$ is the geographic latitude of a point along the field line. The constants $a$, $b$, $c$ and the integer constant $n$ are selected to give the best least squares fit to the values of $y$ and $\lambda$. The constant $c$ gives the equatorial plasma mass density (at the apex of the field line) and $b$ gives the geographic latitude of the field line apex (the equatorial midpoint of the field line). Using the fitting function shown in Equation 1, the plasma mass density was extrapolated to points along the field line beyond the $\sim 1,500$ km altitude limit of the IRI model. An example of the plasma mass density along the IGRF model-derived field lines crossed during the perigee pass of Van Allen probe B from 16:22 to 17:22 UTC on January 1, 2018 is presented in Figure 3. In Figure 3, panel (a) shows the altitude versus geographic latitude profile of the field lines crossed by the probe based on the IGRF model. Panel (b) shows the plasma mass density values along the same field lines derived from the IRI model up to an altitude of 1,500 km and the density values derived using the fitting method given by Equation 1 extrapolated beyond an altitude of 1,500 km.

The standing Alfvén wave equation in a dipole field is typically solved assuming the plasma mass density along the field line varies as given by Equation 2 (see Allan & Knox, 1979; Cummings et al., 1969; Ozeke et al., 2005).
Here, \( \rho_{\text{eq}} \) is the equatorial plasma mass density, \( L \) is the L-shell of the field line, \( R_e \) is the radius of the Earth, and \( r \) is the distance from the center of Earth to a point along the field line. Analytic solutions for the eigenfrequencies of standing toroidal Alfvén waves are only possible when the plasma density along the field varies as given by Equation 2 with \( p = 6 \) (see, Allan & Knox, 1979). Here, we examine how the model wave eigenfrequencies change when the plasma mass density along the field line varies as given by the IRI model using the fits presented in Equation 1 (see also Figure 3b), and alternatively when the plasma mass density is assumed to vary more gradually along the field line with a field-aligned power law density profile as given by Equation 2 with \( p = 6 \), as shown in Figure 3c.

5. Data-Model Comparison Results and Discussion

In Figure 4, the wave frequencies detected by probe B during the perigee pass from 16:22 to 17:22 UTC on 1 January 2018 are compared with the eigenfrequencies of standing FLRs determined by solving the guided toroidal Alfvén wave equation. The spectrogram of the waves’ electric field perturbations clearly illustrates that a discrete set of rising and falling wave frequencies occurred as the probe moved inward onto lower L-shells and then outward onto higher L-shells during this perigee pass, as shown in Figure 4a (see also Figure 1). Figure 4b illustrates the same spectrogram as that shown in panel (a) with the addition of the
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Figure 4. (a) The electric field spectrogram during the perigee pass of probe B from 16:22 to 17:22 UTC on 1 January 2018, in a similar format to that shown in Figure 1. (b) The same spectrogram from (a) overplotted with the standing wave odd-mode harmonic eigenfrequencies of the field lines crossed by probe B derived with the plasma mass density along the field line using the method discussed in Section 4. (c) The same as (b) except the eigenfrequencies are obtained using the same equatorial plasma density as in (b) but the plasma density profile along the field lines is the same as that given in Equation 2 with \( p = 6 \). (d) The equatorial plasma mass density of the magnetic field lines crossed during the perigee pass of probe B. The blue and red regions indicate plasma density values obtained from the IRI model with and without extrapolation, respectively. IRI, International Reference Ionosphere.

odd harmonic mode standing wave eigenfrequencies of the magnetic field lines crossed by the probe during the perigee pass overplotted. In Figure 4b, the plasma mass density along the length of magnetic field lines used to determine the eigenfrequencies is obtained using the IRI model and the fitting function discussed in Section 4, with the mass density having the form illustrated in Figure 3b.

Figure 4b shows that the odd harmonic mode eigenfrequencies of these standing waves are in close agreement with the discrete set of rising and falling wave frequencies detected by EFW during the perigee pass of the probe, indicating the observed wave frequencies are likely caused by odd harmonic mode standing FLRs excited on L-shells down to \( L \sim 1.1 \). Figure 4c is similar to Figure 4b except that the eigenfrequencies have been determined from analytic solutions of the guided toroidal Alfvén wave equation with a density variation along the field lines as given by Equation 2 with \( p = 6 \), where the equatorial plasma mass densities are as illustrated in Figure 3c. Note that the model eigenfrequencies shown in both panels (b) and (c) of Figure 4 are determined from the guided toroidal Alfvén wave equation using the same values for the equatorial plasma mass density at the apex of the field lines given by the IRI model. Interestingly, these standing wave eigenfrequencies illustrated in panels (b) and (c) are almost identical, indicating that the eigenfrequencies are more dependent on the plasma mass density near the apex of the field lines compared with how the density varies along the length of the field lines closer to the ionosphere.

The temporal variation in the equatorial plasma mass density used to determine the standing eigenfrequencies shown in panels (b) and (c) is illustrated in Figure 4d. The region in red indicates points along the
perigee pass where the apex of the field lines crossed by the probe were at altitudes below 1,500 km. At those points, the density can be determined entirely from the IRI model without requiring any extrapolation of the IRI densities to higher altitudes using the fitting function given by Equation 1.

6. Conclusions
The results presented in this study show that electric field perturbations at a set of discrete wave frequencies are regularly detected by the EFW instruments at \( L \lesssim 1.5 \) during perigee passes of the Van Allen probes. These waves occur at frequencies ranging from \( \sim 0.5 \text{ Hz} \) to \( \sim 3.5 \text{ Hz} \), with the wave frequencies typically reaching a peak value on the lowest \( L \)-shell. The observed discrete wave frequencies are consistent with the excitation of only odd harmonic mode standing FLRs, as validated by modeling which solves the guided toroidal Alfvén wave equation in a dipole field with plasma mass density along the field lines constrained by the IRI model. These waves appear similar to the odd harmonic mode standing guided toroidal waves detected by Van Allen probe B, recently reported on by Takahashi et al. (2020). However, those waves occurred at much higher \( L \)-shells between dipole \( L \)-values of 4.2 and 6.1, as is commonly observed. To our knowledge, this is the first report of in situ harmonic FLRs with Hertz frequencies at low-\( L \) near the equator.

Our results clearly show that the eigenfrequencies of these low \( L \)-shell standing guided toroidal waves are strongly controlled by the equatorial plasma mass density at the apex of the magnetic field lines. Consequently, the regular occurrence of these standing waves offers a potentially new application of magnetoseismology for examining the dynamics of plasma mass densities in the coupled magnetosphere–ionosphere system at equatorial latitudes on very low \( L \)-shells. Inversion of the wave frequencies into density estimates could also be used for assessing the accuracy of existing plasmasphere and ionosphere models during both geomagnetically quiet and active time intervals.

Data Availability Statement
The EFW data are available from http://www.space.umn.edu/rbsefw-data/. The work by the EFW team was conducted under JHU/APL contract 922613 (RBSF-EFW). All data used in the paper are publically available from the links above. Supporting material is publically available on the zenodo data repository from https://doi.org/10.5281/zenodo.4287559.

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