Chemical and spectrophotometric evolution of Low Surface Brightness galaxies

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ABSTRACT

Based on the results of recent surveys, we have constructed a relatively homogeneous set of observational data concerning the chemical and photometric properties of Low Surface Brightness galaxies (LSBs). We have compared the properties of this data set with the predictions of models of the chemical and spectrophotometric evolution of LSBs. The basic idea behind the models, i.e. that LSBs are similar to ‘classical’ High Surface Brightness spirals except for a larger angular momentum, is found to be consistent with the results of their comparison with these data. However, some observed properties of the LSBs (e.g. their colours, and specifically the existence of red LSBs) as well as the large scatter in these properties, cannot be reproduced by the simplest models with smoothly evolving star formation rates over time. We argue that the addition of bursts and/or truncations in the star formation rate histories can alleviate that discrepancy.

Key words: Galaxies: general - evolution - spiral - photometry - stellar content

1 INTRODUCTION

Galaxies with a central disc surface brightness (in the $B$ band) well below the Freeman value of $\mu_{B,0}=21.65$ mag arcsec$^{-2}$ - typical of the average previously catalogued “classical” High Surface Brightness galaxies or HSBs - are classified as LSBs. In the last decade, in particular, a considerable body of observational data has been collected on those objects (e.g. de Blok, van der Hulst & Bothun 1995; O’Neil, Bothun & Cornell 1997a; O’Neil et al. 1997b; Bories, de Blok & van der Hulst 1999; O’Neil, Bothun & Schombert 2000a; van den Hoek et al. 2000; Matthews, van Driel & Monnier Ragaigne 2001; Galaz et al. 2002). Although there is no unambiguous definition of an LSB galaxy, in the following we will adopt as the limit between HSBs and LSBs a central disc surface brightness value of $\mu_{B,0}=22$ mag arcsec$^{-2}$, a commonly used criterion.

LSB galaxies may turn out to be crucial for studies of galaxy formation and evolution and of the ‘cosmic’ chemical evolution if they constitute the majority of galaxies (as suggested by O’Neil & Bothun 2000). They are also of interest in the field of high redshift quasar absorbers: some Damped Lyman Alpha systems (DLAs) are identified with LSBs, and DLAs could be due as much to LSBs as to ‘classical’ spirals (e.g. Boissier, Péroux & Pettini 2003 and references therein).

Observations have shown a large variety in the properties of LSB galaxies, which range from dwarfs to massive systems (e.g. Bothun, Impey & McGaugh 1997) and from the “blue” to the “red” edge of galactic colours (O’Neil et al. 2000a). This variety suggests that they form a heterogeneous family, as argued by Bell et al. (2000).

Nevertheless, their potentially interesting implications for galactic astrophysics motivated various theoretical studies, aiming towards an understanding of their nature. For instance, Gerritsen & de Blok (1999) used N-body simulations to study their star formation, while van den Hoek et al. (2000) compiled observational characteristics of 24 LSBs and tried to derive their star formation histories by comparison with simplified chemical evolution models. On the other hand, Jimenez et al. (1998) modelled the chemical and spectro-photometric evolution of LSBs by assuming that they are discs with larger angular momentum than HSBs; this idea had been suggested through the study of the formation of disc galaxies by Dalcanton, Spergel & Sum-
mers (1997), who considered a gravitationally self-consistent model for the formation of both LSB and HSB discs.

In a recent series of papers (Boissier & Prantzos 2000, Prantzos and Boisier 2000, Boissier and Prantzos 2001, Boissier et al. 2001) the chemical and spectrophotometric evolution of spiral galaxies of various masses and spin parameters was computed and compared quite successfully to a large number of observational data: Tully-Fisher relation, colours, integrated spectra, star formation efficiency, gas fraction etc. Those works showed that for relatively large values of the spin parameter \( \lambda \) the properties of discs resembled closely those of LSBs (in particular the central surface brightness).

Following the ideas of Dalcanton et al. (1997) and Jimenez et al. (1998) in the present paper we extend the same models to larger values of the spin parameter, keeping the remaining galactic physics (stellar Initial Mass Function, prescriptions for Star Formation Rate and Infall rate etc.) the same. We compare then our results to an extended set of observed properties of LSBs (H\(_i\) mass, colours, abundances). Our main motivation is to answer the question: can LSB spirals and LSBs be modelled in the same framework, where LSBs merely have a larger angular momentum, or is it necessary to invoke specific assumptions in order to explain their properties?

In section 2 we present the compilation of data on LSBs properties that we adopted for our study. In section 3 we explain how the simplest (1st order) models of LSBs are constructed on the basis of our - quite successful - HSB models, by merely increasing the spin parameter. These simple models are then compared to the observations in section 4. In section 5 bursts and truncations are added to the smooth star formation history of the “simple models”, in order to obtain better fits to the ensemble of the observational data. Section 6 gives a summary of the obtained results.

2 LOW SURFACE BRIGHTNESS GALAXIES: THE DATA SAMPLE

Low Surface Brightness galaxies (LSBs) display a very broad range of sizes, masses and spectral line widths. Their typical observed 21 cm H\(_i\) line width is about 100 km/s, though it can be as high as 600 km/s. “Giant” LSBs have also very large scalelengths, the most striking case being Malin-1. In some sense, their large velocities and sizes make them similar to HSB spirals, as argued in Bothun et al. (1997).

In this paper, we explore possible links in the physical properties/histories between HSB and LSB discs. Since we have no a priori idea of which of the LSBs could actually be linked to HSBs, we consider observations for the whole range of LSBs, from dwarfs to giants. Our sample consists of the following data sets, available in the literature.

O’Neil et al. LSBs sample

The observational data on LSBs from O’Neil et al. (1997a,b) provides information on the surface brightness, scalelength, absolute magnitude of the disc and colours for 127 galaxies. 80% of them are well fitted by an exponential profile, which justifies the use of a disc model for studying LSBs. The data suggest that a large fraction of LSBs are redder than previously thought. H\(_i\) data for some of these galaxies are given in O’Neil, Bothun, & Schombert (2000) and Chung et al. (2002). Note that some of these objects are members of galaxy clusters (Pegasus and Cancer).

de Blok et al. LSBs sample

For 21 late-type field LSBs from the sample of Schombert & Bothun (1988) and the UGC, de Blok et al. (1995) published disc scalelengths and absolute magnitudes and colours; for subsets of that sample H\(_i\) data are given in de Blok et al. (1996), abundances in de Blok & van der Hulst (1998a), CO line upper limits in de Blok & van der Hulst (1998b) and He rotation curves in McGaugh, Rubin & de Blok (2001). The gas content of those galaxies is also available from the work of van den Hoek et al. (2000).

Bulge-dominated LSBs sample

Although most of the LSBs seem to be late-type objects without any apparent bulge, some of them clearly do have bulges, and we do not exclude these from our analysis, to avoid introducing a bias against the most evolved LSB discs. For this reason, we also use the data on surface brightness, scalelength, and colours of 20 galaxies from Beijersbergen et al. (1999). As our models concern only the disc component of LSBs, we used the “area-weighted” colours given by Beijersbergen et al., which should be representative of the discs.

Giant LSBs sample

We also use the data of Matthews et al. (2001), which provides surface photometry and H\(_i\) data for 16 giant LSB galaxies, with high luminosities (\(\sim 10^{10} L_\odot\)) and large scale-lengths (\(> 6 \) kpc). Though LSB Giants are relatively rare, they may play an important role in our investigation of the link between HSBs and LSBs because of their similar sizes/velocities (note that there are many more HSBs with large velocity widths than with small ones, like for LSBs).

Infra-red selected LSBs sample

A new sample of LSBs selected from the 2MASS near-infrared survey (Monnier Ragaigne et al., 2002, MR2002) is used as well. Though these 4,000 objects were selected on their low \( K_s \)-band central disc surface brightness (\( \mu_{K,s} < 18 \) mag arcsec\(^{-2}\)) they turned out to be bluer than expected; the \( B \)-band central disc surface brightnesses of the 20 objects for which we have optical surface photometry actually lie in the range of \( \sim 21-22.5 \) mag arcsec\(^{-2}\), straddling the 22 mag arcsec\(^{-2}\) \( B \)-band selection criterion for LSBs we adopted in the present paper. Here we use the subset of 20 galaxies for which we presently have available \( BVRI \) surface photometry, H\(_i\) spectra and \( JHK \) photometry. This sample allows the exploration of the link between optically HSB and LSB galaxies.

Homogenisation of the data

Because the data sets have different origins, they need homogenisation before a proper comparison with our disc models. We adopted the following procedure:

- All distance-dependent quantities are reduced to distances based on a Hubble constant of 65 km/s/Mpc.
- The models presented in the next section were computed under the hypothesis that the discs are observed face-on, and the data were corrected accordingly to an inclination angle \( i = 0^\circ \). This affects magnitudes and surface brightness by a quantity equal to \(-2.5C\log(\cos i)\), where \( C \) ranges from 0 for an optically thick disc to 1 for an optically thin one. Since LSBs are usually not much evolved chemically (they have low metallicities), it is likely that they are dust-free and we assume \( C = 1 \). This assumption is justified by the fact that
the colours of LSBs seem to be independent of inclination (e.g. O’Neil et al. 1997a).

- The absolute total disc magnitude used in this paper, $M_{B,d}$, is deduced from the fit with an exponential profile (e.g. O’Neil et al. 1997a), i.e. $M_{B,d} = \mu_0 - 2.5 \log(2\pi R_d^2)$, where $\mu_0$ and $R_d$ are the central surface brightness and the scalelength, in mag arcsec$^{-2}$ and arcsec, respectively.

- The total gas mass is obtained by correcting the HI mass for the helium fraction ($M_H/M_\odot=1.4$). The molecular component in LSBs is generally negligible, though a number of LSBs have now been detected in CO (Matthews & Gao 2001; O’Neil, Hofner & Schinnerer 2000, and references therein).

### 3 MODELS: FROM “CLASSICAL” HSB SPIRALS TO LSBs

Dalcanton et al. (1997) suggested that LSBs could be the large angular momentum equivalents of “classical” spirals. That idea was adopted in Jimenez et al. (1998) who constructed models relatively similar to those used in the present study; they showed that this assumption can lead to reasonably good results, at least for the limited observational sample they used (which did not include objects like the red LSBs of the O’Neil et al. sample).

In this paper, we have adopted the same assumption, i.e. that LSBs can be explained with models similar to those of HSB spirals, but with a larger angular momentum. The strength of this work is that it relies on detailed models of the chemical and spectro-photometric evolution of disc galaxies, already shown to be successful in reproducing the main properties of the Milky Way and of nearby LSB spirals: see Boissier & Prantzos 1999 (BP99) for the Milky Way model, Boissier & Prantzos 2000 (BP2000) for the extension to other spirals and Boissier et al. (2001) on the agreement of the models with the observed gas fraction, star formation efficiency and ages of nearby spirals.

Notice that Schombert, McGaugh & Eder (2001) determined the gas fraction of LSB dwarf galaxies, and argued that their data were compatible with the BP2000 models of discs with the largest angular momenta. As an alternative possibility, they also suggested that LSBs could be on average younger than HSBs. That conclusion is also in agreement with our models, since large angular momentum galaxies (LSBs) form the bulk of their stars later, on average, and thus they appear younger than HSBs. We shall note that their galaxies are dwarf-LSBs, corresponding to the lower masses found in the samples we collected (section 2).

Since the LSBs models presented here are a simple extrapolation of those presented previously in this series, we will recall here only briefly their main features (section 3.1); we discuss, in particular, the role of the spin parameter and of its distribution (section 3.2), since we assume that it is at the origin of the differences between HSBs and LSBs.

#### 3.1 General features

In BP99, the disc of our Galaxy is simulated as an ensemble of concentric rings gradually built up by inflow of gas of primordial composition. The adopted Star Formation Rate (SFR), as suggested by the density wave theory, is

$$\psi = \alpha \Sigma_{2g}^{1.5} \frac{V(R)}{R}$$  \hspace{1cm} (1)$$

where $V(R)$ is the disc rotation velocity as function of radius, $\Sigma_{2g}$ the gas density (in $M_\odot$ pc$^{-2}$), $\psi$ the star formation rate (in $M_\odot$ pc$^{-2}$ Gyr$^{-1}$), and $\alpha$ is an efficiency parameter adjusted as to reproduce the properties of the Solar Neighborhood. That form of the SFR is in agreement with the observed current SFR profile of the Milky Way (BP1999) and of some nearby spiral galaxies (Boissier et al. 2003, in preparation). We adopt the Initial Mass Function (IMF) of Kroupa et al. (1993), characterised by a flattening at low masses, and consider it as universal, i.e. not evolving in time or space. The infall rate is declining exponentially, with a timescale $\tau = 7$ Gyr in the local disc in order to reproduce the observed G-dwarf metallicity distribution. Inner galactic zones are formed earlier than the outer disc (“inside-out” disc formation). The evolution is computed independently at each radius and the global properties (masses, luminosities, colours) are obtained by integration or by fitting the obtained profiles to recover scalelengths and central surface brightnesses. For further details on the ingredients and the model of the Milky Way, see BP99.

In BP2000, the mass and scalelength of the discs of HSB spirals are computed with similar models, using simple “scaling laws”, as suggested by Mo, Mao & White (1998) in the framework of Cold Dark Matter scenarios of galaxy formation. Within this simplified approach, discs are characterised by two parameters: their circular velocity $V_C$ (determined by the mass of the dark halo), and their spin parameter $\lambda = J/E^{1/2}G^{-1/2}M^{-1/2}$ where $J$, $E$, $M$ are, respectively, the angular momentum, total energy and mass of the dark halo, whereas $\lambda$ is a dimensionless quantity measuring the specific angular momentum of the dark halo. The disc scalelength can then be expressed as:

$$R_d = R_{d,MW} \times \frac{V_C}{V_{C,MW}} \times \frac{\lambda}{\lambda_{MW}}$$  \hspace{1cm} (2)$$

and the central surface density as

$$\Sigma_0 = \Sigma_{0,MW} \times \left(\frac{V_C}{V_{C,MW}}\right)^2 \left(\frac{\lambda}{\lambda_{MW}}\right)^{-2}$$  \hspace{1cm} (3)$$

where the index $MW$ refers to the corresponding value in the Milky Way (see BP2000 for details). The disc mass varies with $V_C^3$. From equation 3 it is obvious that the central surface density depends strongly on the spin parameter and that large values of it are likely to represent LSB galaxies. Actually, Dekel and Woo (2002) present some observational evidences (based on the Sloan Digital Sky Survey data) that the surface brightness decreases with the spin parameter $\lambda$, even if they consider $\lambda$ as a “second parameter” for the LSB class of galaxies, the first parameter being the stellar mass. This is not surprising since they include all dwarf galaxies in their study, and have then a much larger range of stellar mass than we consider. Note that our surface brightnesses also depends on the stellar mass (what can be deduced from figure 7).

On the other hand, since the same star formation rate prescription (equation 1) is applied for all galaxies, a lower star formation rate in LSBs will result naturally from the dependence of the SFR on radius and on gas surface density.

In BP2000 it was argued that observational data on nearby spirals suggest that the infall time-scale decreases...
with galaxy mass, i.e. that less massive discs were formed later, on average; this dependence of infall time-scales on the circular velocity $V_C$ is also included here. Infall rates depend also on the local surface density (in order to produce an inside-out formation of the galactic discs) and this results in a slower formation of LSB galaxies.

Because LSBs display a large range of H I line widths, we computed a new set of models with six values for the circular velocity: 40, 80, 150, 220, 290 and 360 km/s (only the first one was not considered in BP2000).

3.2 Spin parameter distribution

In BP2000, models of HSB spirals were obtained for $80 < V_C (\text{km/s}) < 360$ and 0.02 $\lesssim \lambda \lesssim 0.07$. It was noted that for $0.07 \lesssim \lambda \lesssim 0.09$, the surface densities were so low that the discs were in fact at the limit of LSB galaxies ($\mu_{1,0}$ between 22.5 and 23 mag arcsec$^{-2}$).

The models of BP2000 with $\lambda \gtrsim 0.06$ have a central surface brightness close to 22 mag arcsec$^{-2}$. In this work we consider larger values of the spin parameter $\lambda$ that produce "naturally" LSB galaxies.

Mo et al. (1998) provide a largely used spin parameter distribution (figure 1) which gives a reasonable fit to the results of numerical simulations of the formation of dark matter halos; it can be expressed as follows:

$$ P(\lambda)d\lambda = \frac{1}{\sqrt{2\pi}\sigma_\lambda} \exp \left[ -\frac{\ln^2(\lambda/\bar{\lambda})}{2\sigma_\lambda^2} \right] \frac{d\lambda}{\lambda} $$

(4)

where the average spin parameter $\bar{\lambda} = 0.05$ and $\sigma_\lambda = 0.5$.

Values of $\lambda$ larger than 0.06 will produce discs with a central blue surface brightness lower than 22 mag arcsec$^{-2}$. According to the distribution shown in figure 1, 35% of the galaxies will have spin parameters in the interval [0.06,0.21], while the 60% in the interval [0.02,0.06] are likely to be HSB spirals (see BP2000). Note that the limits of the intervals are a bit arbitrary since the surface brightness depends slightly on the velocity $V_C$ too. Especially, galaxies with the lower rotational velocities can be LSBs even with lower spin parameters. The limits adopted here represent however a conservative estimate for the number of LSB galaxies.

These figures are not fully consistent with the claim by O’Neil & Bothun (2000) that “the majority of the galaxies” are LSBs. However they suggest that a significant part of the population of LSB galaxies could indeed be discs of large spins. They are also at odds with the claim of Dekel and Woo (2002) that 95% of galaxies are LSBs, but this is mainly due to differences in definitions. Dekel and Woo (2002) include in their “LSB” class all dwarf galaxies, while we limit ourselves to discs with rotation velocities larger than 40 km/s. This choice excludes of course a large number of galaxies because of the shape of the luminosity function and is justified by the fact that we are indeed interested in the behaviours of (relatively) massive LSB disc galaxies, comparing them to similar HSBs.

The LSB disc models constructed for this paper were therefore computed for 4 values of $\lambda$: 0.07, 0.09, 0.15 and 0.21 (the first two values were already used in BP2000). All other ingredients of the models are identical to those used in the LSB disc models, as discussed in section 3.1.

The present work can not be considered as a proof that LSB galaxies are large-spin parameter discs since we did consider other possibilities as for instance that LSB galaxies are settling in halos with larger baryonic fractions or concentration parameters. We prefer to consider the large spin parameter hypothesis because discs with large spin parameters are expected to exist since the spin parameter distribution of figure 1 does present an important tail at large values, and that these discs with large spin parameter values were not examined in the grid of models previously published (BP2000).

4 MODEL RESULTS AND COMPARISON TO OBSERVATIONS

4.1 Rotation curves

In order to compute the Star Formation Rate (equation 1), we need to adopt a rotation curve $V(R)$ in our models. As in BP2000, it is computed as the sum of the contributions of the baryonic matter (exponential disc) and of the dark matter halo (non-singular isothermal sphere). The use of an exponential disc is justified by the fact that 80% of the galaxies of the O’Neil sample (the larger) are well fitted by an exponential disc (see also Bell et al., 2000). The shape of the halo used to compute the rotation curve is that of a non-singular isothermal sphere. We take the core radius to be a constant fraction of the scalelength of the disk obtained by equation 2. The CDM halo density profiles are known to be too “cuspy” with respect to observations (e.g. de Blok et al. 2001). It is thus difficult to decide which shape to use in the models. Our aim is not to include the most realistic profile of dark matter haloes, but to end up with rotation curves in rough agreement with the observed ones (this is actually all that matters for our simple models through the equation 1 used to compute the SFR). We make such a comparison in the following.
The shape and amplitude of the resulting curves depend on the spin parameter and the circular velocity, as a result of the scaling relations.

It is often argued that LSBs are dominated by dark matter (e.g. McGaugh et al. 2001, and references therein), and, indeed, in our models the contribution of the baryonic disc to the total rotation curve is lower for LSBs than for HSBs, as a result of the lower densities in the disc.

To check that our rotation curve are realistic enough for our purpose, we compare in figure 2 rotation curves computed with spin parameters larger than 0.07 to a number of typical rotation curves constructed by de Blok, McGaugh & Rubin (2001) for 26 LSB galaxies with available high quality Hα and Hβ data.

For most galaxies a reasonable agreement is obtained between model curves and observations. We note that in 2 out of the 26 objects (U6614 and U11748), a peak appears close to the centre, which may be due to a pronounced bulge component. Since our models concern discs only, we cannot reproduce such features and we clearly underestimate the rotation velocity (and hence the SFR) in the inner ∼10 kpc of these two objects. Still, these are large and massive galaxies and only their inner parts are affected.

We consider this comparison as satisfactory and we conclude that our simple rotation curves are realistic enough to compute the SFR with equation 1.

4.2 Model results

Figure 3 to 5 display some of our results concerning respectively the chemical and the spectro-photometric evolution of LSBs. On the left side of each figure, results are given for a typical value of the spin parameter λ=0.15 and for five values of the rotational velocity V_C of our grid of models. On the right side results are given for a typical value of the rotational velocity V_C=150 km/s and for three values of the spin parameter λ of our grid of LSB models.

In Figure 3, we present from top to bottom the SFR, the Star Formation Efficiency (SFE) and the gas fraction. The trends obtained in the case of HSBs (BP2000) are also reproduced here: more massive discs formed their stars earlier, had higher star formation efficiencies in the past and have smaller gas fractions than their lower mass counterparts; these features characterise also discs with low angular momentum. LSBs are however different from HSBs: the more massive galaxies (larger rotational velocities) are redder than low mass ones. This figure shows also that the baryon accretion in our models occurs later than the dark matter accretion. This might be due to feedback issues in low-mass galaxies (e.g. Dekel and Woo, 2002). In intermediate circular velocities, the mass history empirically established follows a similar trend as the one of the dark matter halo. For lower masses, we clearly observe that the baryon accretion in our models occurs later than the dark matter accretion. This might be due to feedback issues in low-mass galaxies (e.g. Dekel and Woo, 2002). In the simple models, no feedback is included explicitly, and the mass accretion rates we were conducted to adopt mimic its effect.

In Figure 4, the magnitude (top), B-K colour (middle) and central surface brightness (bottom) are presented. An important property is again similar to what was found in HSBs: the more massive galaxies (larger rotational velocities) are redder than low mass ones. This figure shows also that LSB galaxies were generally fainter in the past, and that their surface brightness has stayed low during their evolution, according to the models.

The mass accretion history of our models depends on the mass of the galaxy. This dependence was constrained by observations in spirals (e.g. Boissier et al., 2001): it is purely empirical and is not derived from a cosmological context. In figure 5, we compare the obtained histories with the predictions for dark matter halos in our hierarchical universe with the cosmology Ω_M=0.3, Ω_Λ=0.7, h=0.65). We assume for each disc that the cold dark matter halo is ten times as massive as the baryonic disc (at redshift 0), and that the halo follows the “universal mass accretion history” of van den Bosch (2002), depending mainly on the mass of the halo.

For intermediate circular velocities, the mass history empirically established follows a similar trend as the one of the dark matter halo. For lower masses, we clearly observe that the baryon accretion in our models occurs later than the dark matter accretion. This might be due to feedback issues in low-mass galaxies (e.g. Dekel and Woo, 2002). In the simple models, no feedback is included explicitly, and the mass accretion rates we were conducted to adopt mimic its effect.

When we move towards more massive galaxies, our baryonic discs build up more and more rapidly, and for the most massive galaxies even more rapidly than the dark matter halo. It has been already noticed that the properties of massive spirals (red colours, high metallicities, low gas fractions) are well reproduced in the simple models by assuming very high redshift formation. Some more realistic cosmological models might be useful to study this paradoxal situation,
Figure 3. Some properties of our models as a function of time. From top to bottom: Star formation rate, star formation efficiency, and gas fraction. Left: results for a mean value of the spin parameter $\lambda=0.15$ and for five values of the rotational velocity $V_C$ of our grid of models, as indicated in the top left panel. Right: results are given for a typical value of the rotational velocity $V_C=150$ km/s and for three values of the spin parameter $\lambda$ as indicated in the bottom right panel.

but it is beyond the scope of our paper. Note that very massive galaxies represent a small fraction of the galaxies we are studying in this paper.

4.3 Tully-Fisher relation

The Tully-Fisher relation, the linear relation between the absolute magnitude and the logarithm of the inclination-corrected H I line width, is one of the best-known properties of spiral galaxies. Observationally it is still difficult to establish if LSBs follow the same Tully-Fisher relation as HSB spirals or not. Some studies conclude that LSBs follow the same relation as classical spirals (Zwaan et al. 1995; Sprayberry et al. 1995; Chung et al. 2002), while in others LSB are found to be underluminous for a given line width (Persic & Salucci, 1991; Matthews et al. 1998). Chung et al. (2002) suggest that some of these differences may be due to selection effects.

Although O’Neil et al. (2000a) find that a few of their LSB galaxies show large departures from the standard Tully-Fisher relation, subsequent VLA H I imaging of four of them and a new analysis of the single-dish data (Chung et al. 2002) show that all (or most) of their original H I profiles may have been contaminated by H I from a nearby galaxy.

In figure 6 we plot the absolute blue magnitude as a function of the H I line-width, corrected for inclination, for the LSBs samples of De Blok et al. (crosses), O’Neil et al. (triangles), Giant LSBs (triangles) and the infrared-selected galaxies (filled dots). The data for the O’Neil et al. galaxies were taken from Chung et al. (2002), after cleaning from known contaminations. The combination of the different sample shows the existence of a Tully-Fisher relation, though with a considerable scatter. Our LSB models are overplotted, with each curve corresponding to a different spin parameter. Models with a lower spin parameter are also shown (stars) for reference. It is assumed that the circular velocity of our models, $V_c$, corresponds to half the inclination-corrected H I line-width, $0.5W_{50}/\sin(i)$.

The models predict the existence of a Tully-Fisher relation for both HSB and LSB disc galaxies. However, the adopted scaling relations (section 3.1) lead to a difference between the HSB and LSB TF relations: at a given rotational velocity the luminosity of LSBs is lower than for HSB.
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Figure 5. Total mass accreted on galaxies of various velocities as a function of redshift. The solid line indicates the mass of baryons in the disk of our models. The dotted line indicates the history of the dark matter halo ten times as massive as the disk at the end of the history, according to the universal mass accretion history of van den Bosch (2002). A redshift scale is indicated at the top.

Figure 6. The Tully-Fisher relation for different samples of LSBs, as indicated in the plot (see section 2 for details on the data). The curves correspond to models with various spin parameters indicated in labels. A fifth curve marked by stars shows a model with $\lambda = 0.05$, i.e. corresponding to HSBs.

Figure 7. $B$-band disc scalelength (top) and central surface brightness (bottom) deduced from fits to the surface brightness profiles as a function of the absolute blue magnitude of the disc component. Plotted are LSBs from the samples of O’Neil et al. (filled squares), de Blok et al. (crosses), Giant LSBs (triangles), Beijersbergen (open squares) and the infrared-selected galaxies (filled dots). Curves correspond to models with different spin parameters (as indicated in the bottom panel). The curve marked with stars correspond to HSBs ($\lambda = 0.05$), while the others correspond to LSBs. The grey lines join models with the same circular velocity (from left to right: 40, 150, and 360 km/s, respectively.

Spirals because of differences in their star formation history and gas fraction. The amplitude of the effect depending on the velocity $V_C$, the slope of the relation changes with the spin parameter.

Although the modeled and observed LSB Tully-Fisher relations show relatively similar slopes, the model values lie on average about 2 mag below the mean observed absolute magnitudes. This shift could be due to systematic differences between the measured and modeled circular velocities and to uncertainties in both of them.

4.4 Scalelength and central surface brightness

Figure 7 displays the disc scalelength and central surface brightness in the B-band as a function of $M_{B,d}$, the absolute magnitude of the disc component (as defined in section 2). From the adopted scaling relations (section 3.1), we expect that the scalelength will increase with luminosity, while the
central surface brightness will depend more strongly on the spin parameter. This is indeed the case, as seen in figure 7.

We note that the lack of observed galaxies in some parts of these diagrams (specifically, those with large luminosities but very faint central surface brightnesses, the so-called “Malin 1 cousins”) might result from selection effects. On the other hand, in view of the distribution of the spin parameter (figure 1) and of the circular velocity function (Gonzalez et al. 2000) it is expected that only a small number of LSB galaxies with large rotational velocities should exist. Therefore, one would expect such galaxies to be quite rare compared to LSBs of considerably smaller size or less extreme surface brightness.

Our models produce LSB disc galaxies similar in size and surface brightness to those of the presently available observational samples. However, within the framework of our models, giant discs with lower surface brightness could in principle be produced by systems with even larger $\lambda$ values than those adopted in our present calculations.

Finally, the scaling relationships indicate that LSBs should have larger scalelengths than HSBs for the same mass. The models for HSBs (stars) in Figure 7 are indeed located at smaller scalelengths than the models for LSBs. The same result is obtained in the observations since the MR2002 sample is characterised by smaller scalelengths since the other data, and correspondingly higher surfaces brightnesses.

4.5 Gas content

Our models predict a correlation between the absolute disc magnitude and the gaseous mass (bottom panel of figure 8). This results mainly from the mass-luminosity relation: more massive galaxies are more luminous and contain more gas (as expected if the gas-to-total mass ratio does not depend strongly on luminosity). The data follow the same trend, but the observational scatter is larger than what our models can account for, especially in the faint LSB range (where most of the data come from the O’Neil et al. sample).

Note that in a recent study, Galaz et al. (2002) presented a clear correlation between the HI mass and the Ks magnitude for a sample of galaxies they studied with near-infrared imagery, confirming the trend we observe in Figure 8. As a direct consequence of the scaling relationships, using the words of Galaz et al. (2002), “bigger galaxies have more of everything”.

The gas mass-to-light ratio ($M_{\text{gas}}/L_B$) is an interesting intensive quantity, more related to the “chemical state” of the galaxy than the absolute gas mass. Our models predict larger values of $M_{\text{gas}}/L_B$ for LSBs than for HSB spirals, as a result of the lower star formation efficiency in the former. In figure 8 it appears that our models predict a slightly declining trend with absolute magnitude; more massive galaxies have proportionally smaller gas fractions, due to the smaller time-scale of infall rates. The data show a steeper trend (arising mainly from the giant LSBs) with a broader dispersion than allowed by our models. For the LSBs data of O’Neil et al., two values of this ratio are shown, giving an indication of the corresponding uncertainty: one value is computed with a luminosity derived from the absolute magnitude $M_{B,d}$ of the disc only and one is based on the total absolute magnitude of the galaxy (the two filled square linked by a vertical line).

Note that the HSBs models have lower gas contents than the LSBs models, what is in agreement with the observations: the MR2002 sample is characterized by smaller values of the gas to luminosity ratio than the other samples.

A way to obtain a stronger decrease of $M_{\text{gas}}/L_B$ with $M_{B,d}$ in our models would be to introduce a different parameterisation of the key physical ingredients in order to consume more gas in massive discs, either by either making the gas available earlier (more rapid infall) or by changing the prescription for star formation (enhancing the SFR efficiency). Still, given the large observational uncertainties and the crudeness of our “1st order” models, we consider that the simple “large-spin” models presented here reproduce satisfactorily the observed $M_{\text{gas}}/L_B$ values. We also note that departures from these values could be due to star formation events that occur in addition to the smooth history of the simple models, as will be discussed in section 5.

4.6 The luminosity-metallicity relation

The “luminosity-metallicity” relation is an important property of (HSB) spiral galaxies (e.g. Zaritsky, Kennicutt &
large scatter in the luminosity-metallicity relation (if there exists one), compared to the case of the HSBs.

In figure 9 we compare the data of McGaugh (1994) with the metallicities of our LSB models. Since the observations were done at various radii, for a comparison with the models we computed the distribution expected in the absolute magnitude - log(O/H) plane, by weighting the results with the distributions of the spin parameter (figure 1), circular velocity (we adopt the distribution of Gonzalez et al. 2000), and Star Formation Rate. The reason for introducing the latter is that abundances are measured in H ii regions, implying the existence of massive stars and, hence, important star formation activity.

As can be seen from figure 9, the abundances measured in LSBs are in satisfactory agreement with the values expected from our simple models, at least in a statistical sense. Galaz et al. (2002) argue that the J-Ks colour index they measured in their near-infrared galaxy sample is a good indicator of the metallicity. On this base, they deduce the existence of a luminosity-metallicity relationship, the redder colours corresponding to the galaxies with the largest stellar mass. From the variation of the J-Ks index between the less luminous and the more luminous galaxies, they conclude that the metallicity increases by a factor between 20 and 100 between them. In a logarithmic scale, this is 1.3 to 2 dex, what compares well with the slope we can estimate from Figure 9.

4.7 Colours

Three colour indices are presented as a function of absolute disc magnitude in figure 10. The models are characterized by blue colours and an extremely small colour dispersion, especially for low-mass discs. Both features stem from the fact that most of the model galaxies have formed the bulk of their stars toward the end of their history (see figure 3). The differences in their early star formation histories play a minor role (especially in low-mass discs) because the colours are always dominated by the youngest population of stars.

Although these model results are in broad agreement with the “blue” LSBs of the observational sample, they do not reproduce the colours observed in the large number of “red” LSBs. It is worth noting that the red LSBs are mainly found in the O’Neil et al. (2000) sample, part of which consists of cluster galaxies, though it should also be noted that O’Neil et al. (2000a) find no difference between the properties of their LSBs in clusters and in the field.

We also point out that colour uncertainties may be larger than usually quoted (≈ 0.1 mag), since O’Neil et al. (2000a) and Bell et al. (2000) find for the same object (P1-7) B−V ∼ 0.9 and B−V = 0.6±0.1, respectively. Notwithstanding this difference and the importance of the uncertainties in LSBs’ colours, the existence of “red” LSB discs is undisputable and that property is clearly not reproduced by the simple models presented here. In section 5, we suggest that another “ingredient” is necessary to reconcile the simple models with the existence of such red galaxies.

4.8 Summary of the “simple” model calculations

The results of our simulations can be summarised as follows:

Huchra (1994): massive galaxies contain proportionally more metals than low-mass ones. The metallicity is estimated from the oxygen abundance at a given radius in the disc of the galaxy (see Zaritsky et al. 1994 for details).

This relation is very important for studies of galactic evolution since it suggests that massive galaxies have processed more material into stars, or that low mass galaxies have lost part of their metals or a combination of both occurred. The former hypothesis was adopted in Boissier et al. (2001), on the basis of observed star formation efficiencies and gas fractions, for spirals with rotational velocities as low as 80 km/s. Recently, Garnett (2002) used a large data sample of spiral and irregular galaxies with rotational velocities as low as 5 km/s and convincingly argued that below ~120 km/s the effective yield of galaxies is reduced, most probably as a result of mass loss. We note that this effect, by itself, is not sufficient to explain the observed colour-luminosity relation of spirals, with the most massive ones being redder than their low mass counterparts; a younger age for the latter has to be assumed in order to account for the data (see Prantzos 2002 for a recent review).

Because our LSBs models are an extrapolation of the HSB ones, we expect them to produce a luminosity-metallicity relation in the case of LSBs also. However, McGaugh (1994) compiled metallicity measurements in LSBs and concluded that they do not display such a relation [his data are shown in figure 9 as a function of $M_{B,d}$. It should be noted that McGaugh (1994) excluded giant LSBs from his analysis, while we consider a continuous range of galaxies, from dwarfs to giants; if we add the giants to the sample, a small trend is observed (see figure 9). On the other hand, the abundance measurement in LSBs concern only a few H ii regions per object, located at various galactocentric radii. Therefore, it is impossible to determine an abundance at a “characteristic” radius and the situation is very different from the work of Zaritsky et al. (1994) concerning HSB spirals. This difficulty introduces a large scatter from Figure 9.

Figure 9. Luminosity-metallicity relation: the data correspond to the abundances of oxygen in H ii regions as a function of the absolute magnitude of the parent LSB galaxy (McGaugh 1994). The contours and greyscale levels show the expected model abundances, weighted by the velocity distribution, the spin distribution, and the star formation rate (see text).
Simple models (i.e. with smooth SF histories), differing by HSBs only by their larger spin parameters ($\lambda$), are in reasonable agreement with several observed properties of LSBs, e.g. the relations between absolute disc magnitude and central surface brightness, disc scalelength, metallicity, as well as total gas content. The models also predict a Tully-Fisher relation, in broad agreement with observations.

Some important discrepancies exist between the observations and the “simple” models: the scatter in many of the observed relations (Tully-Fisher, gas mass vs. luminosity, colours) is much larger than predicted by the models. In particular, the “red” LSB galaxies are not reproduced by any of our models, since “by construction” they are dominated by the young stellar population.

The discrepancies between models and observations are particularly important. In fact, the models of BP2000 (on which this work relies) were developed mainly for field galaxies with smooth star formation histories. If the environment of LSB is different or if they are more sensitive to perturbations (because of their lower densities), some of their properties may be due to departures from the ideal smooth star formation histories. This idea is tested in the next section, where we try to reconcile the large-spin models and the discordant observations by adding “events” (bursts and truncations) on otherwise smooth star formation histories. Indeed, O’Neil et al. (2000a) suggested that the properties of both blue and red LSB galaxies can be reproduced by population synthesis models, assuming “on” and “off” states of the star formation for different stellar populations. Van den Hock et al. (2000) already suggested that small amplitude bursts may play a role in determining the colours of LSBs. In their N-body simulations, Gerritsen & de Blok (1999) argued that Star Formation Rate fluctuations are responsible for the large range of colours observed in LSBs and predicted that less than 20% of gas-rich LSBs should be “red”.

5 STARBURSTS AND TRUNCATED STAR FORMATION HISTORIES

In this section, we investigate a way to produce red LSB galaxies while introducing only small changes to the “simple” models presented in section 3. We will refer to the star formation histories derived with the simple models as “standard” histories, which represent a smoothed average of the true evolution of the LSBs, to which we will now add star formation “events”.

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First, we study the effect of a starburst added to the “standard” SF history of one of our models, and then we consider the effect of a truncation of the “standard” star formation at a recent epoch.

There are reasons to believe that the star formation rate behaves in such a way in LSB galaxies.

Martin and Kennicutt (2001) show that in nearby spirals, beyond the “threshold radius”, where the density of gas becomes smaller than the critical density (according to the Toomre instability criteria) a few HII regions (and then recently formed stars) are still found. They argue that local perturbations and density waves can temporarily and locally enhance the density to values high enough to allow star formation. This way of forming stars is obviously less smooth than the one described in the simple averaged models. Despite a large scatter, we can derive from the figure 9 of Kennicutt (1989) that the critical gas density at which the Toomre instability criteria are satisfied, beyond the “threshold radius”, where the density of gas becomes smaller than the critical density (according to the Toomre instability criteria) a few HII regions (and then recently formed stars) are still found. They argue that local perturbations and density waves can temporarily and locally enhance the density to values high enough to allow star formation. This way of forming stars is obviously less smooth than the one described in the simple averaged models. Despite a large scatter, we can derive from the figure 9 of Kennicutt (1989) that the critical gas density at which the Toomre instability criteria are satisfied.

Obviously, the gas surface density has been lower than (or close to) the threshold value observed in spiral galaxies, in most of our model galaxies and most of the time. Thus, the star formation is more likely to proceed through a succession of quiescent phases (when the density is lower than the threshold), and relatively active phases (when enough gas has been accumulated by infall and condensed by local perturbations or spiral modes to reach the threshold). This scenario is indeed suggested by Galaz et al. (2002) in order to explain the scatter in optical data by a sporadic past star formation history.

Another possibility is that the star formation history is affected by feedback from supernovae. Feedback effects may be crucial to the history of LSB galaxies (see for instance the study of Dekel and Woo, 2002). We can speculate that the effect of supernovae could be similar to the empirical threshold discussed above: stopping the star formation by blowing the gas away and delaying the birth of the next generation of stars until enough gas has settled again in the disk.

It is beyond the capacities of the simple models to test the details of these ideas concerning the way the star formation rate is affected in low density environment by threshold effects, spiral modes, perturbation, feedback. This task could be undertaken only with much more sophisticated hydrodynamical models (e.g. Samland and Gerhard, 2003). We compute instead two simple examples for illustrative purpose.

Of course, the star formation history of a galaxy may consist of a series of “bursts” and “drops” in its star formation rate. However, the “chemical” properties of a 13 Gyr old galaxy with such a jittery SFR would be virtually identical to those resulting from an averaged star formation history, while its colours may be affected mainly by the last of such events. Therefore, our adopted “smooth plus last event” model, while simplistic, may be a good enough approximation for the present study of LSBs.

5.1 Adding a star burst

Figure 12 shows the $B-I$ colour and the gas mass-to-light ratio in the $B$-band for the galaxies from the various samples and the grid of “standard” models.

It would be difficult to get a clear insight by adding bursts of various ages, intensities, metallicities, etc. to all of the “simple” models. Instead, we prefer to study one illustrative example, by adding a single burst of star formation to the “standard” star formation history of one of our models ($\lambda=0.07$, $V_C=80$ km/s) is shown by the arrow labelled “B” (at an age of 0.005 Gyr). The curve towards the tip of the arrow corresponds to increasing burst ages along the curve. The effect of the truncation on the star formation history of the same model one Gyr before the present epoch, is shown with the arrow labelled with a “T”.

![Figure 12. Effects of star bursts and truncations of the star formation on the gas mass-to-light ratio in the $B$-band (top panel) and $B-I$ colour (bottom panel). The data and the model grids are identical to those in figure 10 and 8 (except that we do not reproduce the HSB models). The effect of a star burst in addition to a “simple” model ($\lambda=0.07$, $V_C=80$ km/s) is shown by the arrow labelled “B” (at an age of 0.005 Gyr). The curve towards the tip of the arrow corresponds to increasing burst ages along the curve. The effect of the truncation on the star formation history of the same model one Gyr before the present epoch, is shown with the arrow labelled with a “T”.](image-url)
For very recent bursts, it seems likely that the brightest part of the galaxy would be the burst itself, in which case the galaxy would probably not be classified as an LSB, but rather as a “blue compact” (if the burst is centrally located). Legrand et al. (2000) proposed such a scenario for Zw 18, a blue compact galaxy of very low metallicity, which could result from a star burst occurring in an LSB galaxy after a long period of low and constant star formation rate.

For burst ages under 0.5 Gyr, the colours of the galaxy are bluer than without the burst; beyond that age the old stars of the burst dominate the stellar populations of the galaxy, which becomes redder than in the “standard” model. However, even for a very old burst, we do not obtain a very red galaxy. In fact, after 12.5 Gyrs the colours of the galaxy are quite close to those of the smooth “standard” model.

Assuming that all the gas taking part in the burst is removed from the galaxy, we can also compute a total gas mass-to-light ratio. For young bursts, M$_{gas}$/L$_{B}$ is lower than without the burst (because the luminosity increases significantly and the gas amount is reduced). For older bursts, M$_{gas}$/L$_{B}$ proceeds towards “standard” values, as the luminosity of the burst (which dominates the stellar population) decreases with age.

The results obtained here suggest that adding star bursts to the smooth SF history of the “standard” model can increase the scatter in the gas mass-to-light ratio (see figure 12); it will also increase the scatter around the Tully-Fisher relation, especially for luminous LSBs with a relatively low gas mass-to-light ratio.

### 5.2 Truncating the star formation

To illustrate the effect of truncations of the star formation history, we added to the same model ($\lambda$=0.07, $V_c$=80 km/s) the condition that 1 Gyr before the present epoch the star formation stopped abruptly. As a result, the stellar population reddens passively, since no young and blue stars are created. The resulting stellar population is different from that of a “normal” galaxy, without the need to invoke a special IMF.

The effect of truncating the star formation history in this way is shown in figures 12 and 13 by arrows labelled with a “T”. The luminosity decreases by about 1.5 magnitudes due to the lack of newly formed stars, while the colours become quite red. This model corresponds rather well to the faint red LSB galaxies of the O’Neil et al. sample.

Because of the decrease in luminosity, galaxies with larger than average gas mass-to-light ratio are obtained. Notice, however, that the scatter in the $M_{gas}$/L$_{B}$ vs $M_{B}$ relation that is obtained by adding truncations to a smooth SF history is smaller than that due to the diversity in the values of the spin parameter $\lambda$. Another consequence is that truncations of the SF will increase the scatter in the Tully-Fisher relation, especially for low-luminosity systems (as is the case with starbursts).

### 5.3 The effects of bursts and truncations

The two simple examples presented in the previous sections illustrate that a starburst, or a truncation of the star formation, may strongly affect the colours, luminosity, and gas mass-to-light ratio of LSB galaxies, with respect to the “simple” models with smooth histories discussed in section 3.

If the simple models reproduce the average history of LSB galaxies, then the occurrence of star bursts of various ages, masses and metallicities, and of truncations of star formation at various galactic ages, may explain the existence of the “red” LSBs as well as the large scatter found in several observational properties of LSBs, like their Tully-Fisher relation, gas mass-to-light ratio, and colours.

If some LSBs have had recent truncations in their star formation history their colours may be a poor indicator of their true nature; indeed, a massive LSB disc (having an old stellar population according to our models) may have the same $B-I$ colour as a less massive one with a truncated star formation history, evolving passively for e.g. a Gyr. This is in agreement with the finding of Bell et al. (2000) that “red” LSB galaxies form an heterogeneous group.

Finally, we note that the events we invoked (bursts and truncations) should affect more strongly the optical than the infra-red properties of LSBs, as the latter are more sensitive to the old, average, stellar population. We present in figure 13 the equivalent of figure 12 for the near infra-red $K$-band, showing the effects of the burst and the truncation. A comparison between figures 12 and 13 shows that infra-red...
properties are indeed less affected, and that we obtain less extreme colours in the infra-red, indicating that adding star formation events will introduce a relatively much smaller scatter in the colours, Tully-Fisher relation and gas mass-to-light ratio in the infra-red than in the optical.

In their recent study of a near-infrared sample of galaxies, Galaz et al. (2002) found that the near-infrared data and colours are less scattered than the optical ones. This is in full agreement with our previous deduction. They also found a decrease of the HI mass to light ratio with the Ks magnitude, together with an increase of the value of the J-K colour index, both corresponding qualitatively to the results of figure 13.

6 CONCLUSIONS

We have compiled data from various observational samples of Low Surface Brightness galaxies (LSBs) in order to cover the whole presently known range of their properties, from “dwarfs” to “giants”, from “blue” to “red” galaxies and from those at the limits of detectability in central disc surface brightness to objects straddling the adopted boundary between LSBs and HSBs (High Surface Brightness galaxies). We have not included in our study very faint dwarfs, but only gas-rich LSB discs, with rotational velocities larger than 40 km/s or so. We compared them to the results of models of their chemical and photometric evolution, obtained under the assumption that these LSBs are similar to HSBs except for a larger angular momentum. This was done on the base of successful models of HSBs that were presented in a recent series of papers.

The comparison is relatively satisfactory: observed disc scalelengths, central surface brightnesses and total gas masses are compatible with the model predictions. In their recent work, Schombert et al. (2001) also found that the gas fraction of their LSB dwarfs is compatible with our main assumption. However, the star formation history of some of the LSBs appears to be different from this simple picture, as indicated by the large scatter found in several observational relations (Tully-Fisher, total gas mass-to-luminosity) as well as by the existence of red galaxies, which cannot be not explained within the framework of the simplest models (i.e. large angular momentum discs with a smooth star formation history).

In order to account for the discrepancies between the simple models and the observations we suggest that star formation “events”, bursts and truncations, should be included to the simple model. We show that these ingredients may easily reconcile the observations with the models, since they reproduce the observed large scatter in certain relations; besides, truncations in the SF history may account for the existence of the red LSBs found by O’Neil et al. (1997b). Such “events” may result from the fact that the gas surface density in most LSBs is probably lower than the threshold for star formation found by Kennicutt (1989), or from supernovae feedback (Dekel and Woo, 2002). Such events are however unlikely to alter as much the near infrared data as the optical ones, quite in agreement with the observations of Galaz et al. (2002).

We conclude that LSBs may be the equivalent of HSBs with larger angular momentum (spin parameter \( \lambda \)), but that their observed properties could be largely affected by recent events in their star formation history; in that respect, they present similarities to the case of blue compact galaxies.

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