Topological Hall effect in the Shastry-Sutherland lattice

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We study the classical Heisenberg model on the geometrically frustrated Shastry-Sutherland (SS) lattice with additional Dzyaloshinskii-Moriya (DM) interaction in the presence of an external magnetic field. We show that several noncollinear and noncoplanar magnetic phases, such as the flux, all-in-all-out, 3-in-1-out, and canted-flux phases are stabilized over wide ranges of parameters in the presence of the DM interaction. We discuss the role of DM interaction in stabilizing these complex magnetic phases. Electron transport, when coupled to these noncoplanar magnetic phases results in a finite Berry phase, which manifests in the form of topological Hall effect, whereby a non-zero transverse conductivity is observed even in the absence of a magnetic field. We study this anomalous magneto-transport by calculating the electron band structure and transverse conductivity for a wide range of parameter values, and demonstrate the existence of topological Hall effect in the SS lattice.

I. INTRODUCTION

The interplay of charge and spin degrees of freedom manifests in novel phases in strongly correlated electron systems [1–4]. One of the basic models that describes this interplay is the Kondo lattice model or the double exchange (DE) model, in which localized magnetic moments are coupled to itinerant electrons [5–9]. In these systems the conduction electrons and localized spins affect each other in a self-consistent way. The mobile electrons mediate effective interactions between the localized spins, and dictate the magnetic behavior. On the other hand, the scattering of the mobile electrons from these localized moments decides the resulting electronic and transport properties of the system. This interplay becomes more interesting, when the localized moments are arranged on a geometrically frustrated lattice[10–13]. In these frustrated systems, the ground state has a large degeneracy, leaving them strongly susceptible to even small perturbations like longer-range exchange interactions mediated by conduction electrons coupled to the localized moments. In some cases, the resulting effect of the spin-charge coupling leads to unconventional magnetic phases[14–18].

Among these phases, some of the most interesting are those with noncoplanar spin orderings, with non-zero scalar spin chirality[19–21]. The chiral nature of these states break both the parity and time-reversal symmetries. When an electron moves through a background of noncoplanar spin texture, it picks up a Berry phase, which gives rise to many interesting transport phenomena such as the geometric or topological Hall effect (THE) and unconventional magnetoresistive behavior[14, 22–24]. In THE, a transverse Hall current is observed even in the absence of any external applied magnetic field – driven solely by the cumulative Berry phase acquired by the electrons. The acquired Berry phase is equivalent to the coupling of electron orbital moment to a fictitious magnetic field.

THE has been observed in the ferromagnetic pyrochlore compounds Pr2Ir2O7 and Nd3Mo6O17[14, 25–27]. The chiral spin ordering has been studied theoretically in the context of Kondo lattice model on frustrated lattices such as triangular[28, 29], kagome[21, 30–32], pyrochlore[33], face-centered cubic lattice[34], and checkerboard lattice[35]. Our plan is to extend this study to the geometrically frustrated SS lattice, which is a prototypical model of several materials like the rare-earth tetraborides [36–41]. These materials have rare-earth elements with large magnetic moments that can be treated as classical spins. This, in turn, renders the theoretical modeling of such systems more tractable. For classical spins, the Kondo lattice and double exchange models can be mapped onto one another, as the eigenstates corresponding to opposite signs of the Kondo coupling are related by a global gauge transformation.

The SS lattice has several competing interactions in play owing to its unique lattice symmetry. The competition between the axial and diagonal exchange interactions usually results in a collinear or coplanar ordered phase[42–44]. However, a rich variety of phases, including noncoplanar phases, are expected when the symmetry allowed DM interaction is taken into account. Further, the use of an external Zeeman field enhances the possibility of having noncoplanar phases significantly. Previously, we have shown that the Kondo lattice model on the SS lattice exhibits noncoplanar and non-collinear ground states over a wide ranges of parameters[45–47]. In this work, we aim to thoroughly study the effect of all the competing interactions in stabilizing the noncoplanar phases and investigate the transport properties of itinerant electrons on this lattice. The ability to realize multiple noncollinear and noncoplanar magnetic orderings by tuning different interactions for realistic values of model parameters make the SS lattice an ideal case for studying THE.

In this work, we demonstrate that multiple noncollinear and noncoplanar magnetic ground state phases are stabilized in the SS lattice for different ranges of Hamiltonian parameters. The behavior of itinerant electrons is significantly modified by the coupling to the underlying spin textures. In particular, for noncoplanar magnetic orderings, this is manifested in the form of finite THE.

This paper is organized as follows. Following the introduction in section I, we discuss the models used in this study in section II. In section III we describe the method and the observables we calculate to characterize the magnetic and the
transport properties. We present the results of our work in section IV, followed by the summary in section V.

II. MODEL

We study the Hamiltonian,

$$\hat{H}^c = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\langle ij \rangle} D_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j - B \sum_i S_z^i \quad (1)$$
onumber

on the SS lattice, where, $\langle ij \rangle$ refers to nearest neighbor axial bonds on each plaquette, and next nearest neighbor diagonal bonds on alternate plaquettes. The first term represents the Heisenberg exchange interaction, with $J_{ij} = J(J')$ denoting the strength of antiferromagnetic exchange on the axial (diagonal) bonds. The second term is the antisymmetric DM interaction with $D_{ij}$ representing the DM vectors on SS bonds. The exact values and directions of these vectors are determined by the crystal structure, subject to the Moriya rules and the constraints imposed by the geometry of the lattice. In Fig. 1, the unit cell of the SS lattice together with the choice of all DM vectors on each bond is shown. We parameterize the DM vectors via their parallel ($D_{ij,s}$, $D_{ij,ns}$, $D'$) and perpendicular ($D_{ij}$) components. Further details of different components of the DM vectors are mentioned in the caption of Fig. 1. The last term is the Zeeman coupling between localized spins and an external applied magnetic field.

We treat the localized spins as classical vectors (true for $f$-electron systems with large magnetic moments) with unit length ($|S_i| = 1$). We use the spherical polar co-ordinates, $S_i = (\sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, \cos \theta_i)$ to denote the state of the localized spin. Henceforth, interactions on the diagonal bonds are represented with prime parameters while that on axial bonds with unprimed ones.

In order to study transport properties of itinerant electrons coupled to localized spin textures, we use the Kondo lattice model,

$$\hat{H}_c = -\sum_{\langle ij \rangle, \sigma} t_{ij} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) + J_K \sum_i S_i \cdot S_i \quad (2)$$

where $t_{ij}$ represents the hopping matrix elements of conduction electrons on the SS lattice bonds, and $J_K$ is the coupling strength of on-site Kondo term between classical spin, $S_i$, and the spin of itinerant electron, $s_i = c_{i,\sigma}^\dagger c_{i,\sigma}$. In the limit of $J_K \gg t$, the electron spins are fully aligned with the localized spins and Hamiltonian (2) reduces to an effective tight-binding model\cite{48}, given by,

$$\hat{H}_c = -\sum_{\langle ij \rangle, \sigma} \epsilon_{ij}^{\text{eff}} (d_i^{\sigma} d_j + \text{H.c.}) \quad (3)$$

where,

$$\epsilon_{ij}^{\text{eff}} = t_{ij} e^{i\theta_{ij}/2} \quad (4)$$

is the effective hopping matrix elements for the spin-parallel electrons between sites $i$ and $j$. The phase factor, related to spin chirality, is calculated as

$$\epsilon_{ij} = \arctan \frac{-\sin(\phi_i - \phi_j)}{\cos(\phi_i - \phi_j) + \cot \frac{\theta_i}{2} \cos \frac{\theta_j}{2}} \quad (5)$$

and $\theta_{ij}$ is the angle difference between the localized spins $S_i$ and $S_j$.

$$\cos \theta_{ij} = \cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j \cos(\phi_i - \phi_j). \quad (6)$$

III. METHOD AND OBSERVABLES

To investigate the model in (1), we use a Markov chain Monte Carlo (MC) to perform an importance sampling of the spin configurations, based on the Metropolis algorithm. The simulations are performed on lattices of dimension $L \times L$ with $L = 16 - 48$ over a wide range of Hamiltonian parameters. We use simulated annealing procedure to prevent the freezing of the localized moments which may happen at low temperatures. In this approach, we start the simulations with a random spin configuration at a high temperature ($T \approx J$), and equilibrate the system at this temperature. Next, we decrease the temperature by $\Delta T'$ and use the equilibrated spin configuration from previous $T$ as an initial configuration for equilibration at the new temperature. We repeat this process until we reach $T = 0.005J$, where measurements are made to calculate the thermal averages of the physical observables. 100,000 MC steps are used at each $T$ value as equilibration steps and

FIG. 1. (Color online) (a) The geometry of the SS lattice used in our study. Black lines represent the axial bonds while dotted black lines represent the diagonal bonds on alternate plaquette. The direction of arrow on these bonds indicates the order of cross product $S_i \times S_j$ in DM term for these bonds. The in-plane component of DM vector on axial bonds is divided into staggered, $D_{\text{ax}}$, and non-staggered, $D_{\text{ns}}$, components, and represented by red and purple arrows, respectively. The in-plane component of DM vector on diagonal bonds, $D'$ is indicated by blue arrows. The perpendicular component of DM vector, $D_{\perp}$ on axial bonds has out-of-plane and into-plane components. The directions of all these DM vectors are obtained using Moriya rules and crystal structure of SS lattice.
In order to identify the magnetic order of localized spins, we calculate the static spin structure factor given by the Fourier transform of spin-spin correlation function,

$$S(Q) = \frac{1}{N^2} \sum_{i,j} \langle S_i \cdot S_j \rangle \exp[iQ \cdot r_{ij}], \quad (7)$$

where \( r_{ij} = r_j - r_i \) denotes the position vector from the \( i \)-th to \( j \)-th site, and \( \langle \rangle \) represents the average over different MC configurations. Further, to distinguish between the coplanar and noncoplanar magnetic order, we calculate scalar spin chirality, as a measure of noncoplanarity of spin textures. On a triangular plaquette, the scalar spin chirality is defined as,

$$\chi_\alpha = S_i \cdot (S_j \times S_k). \quad (8)$$

The total chirality \( \chi \) is calculated by \( \chi = \frac{1}{N^2} \sum_\alpha \chi_\alpha \), where \( N_u \) is the number of SS unit cells. For collinear order (ferromagnetic and antiferromagnetic) and coplanar order (such as flux states), \( \chi = 0 \); whereas noncoplanar magnetic ordered phases such as canted-flux, all-in-all-out and 3-in-1-out phases are characterized by nonzero values of \( \chi \).

To study electronic transport on a magnetically ordered background, we diagonalize the Hamiltonian (3) to get the energy spectrum for itinerant electrons. We use the Kubo formalism to calculate the transverse conductivity in \( k \)-space which can be written as,

$$\sigma_{xy} = \frac{i e^2 \hbar}{N} \sum_{m,k} \sum_{nm} \left[ f(E_{mnk}) - f(E_{nmk}) \right] \times \frac{\langle mk | v_x | nk \rangle \langle nk | v_y | mk \rangle}{(E_{mnk} - E_{nmk})^2 + \eta^2}, \quad (9)$$

where \( m \) and \( n \) represent the band indices and \( f(E_{mnk}) \) is the Fermi-Dirac distribution function for energy \( E_{mnk} \); \( |mk \rangle \) and \( |nk \rangle \) are eigenstates in \( k \)-space corresponding to energies \( E_{mnk} \) and \( E_{nmk} \), respectively. \( N = L_x \times L_y \) represents the size of the sample and \( \eta \) is the scattering rate. \( v_x \) and \( v_y \) are the velocity operators in \( k_x \) and \( k_y \) directions and can be expressed as,

$$v_\mu = \frac{\partial H}{\partial k_\mu}, \quad \hat{\mu} = x, y \quad (10)$$

### IV. RESULTS

#### A. Magnetic properties

The Hamiltonian (1) has a rich ground state phase diagram owing to the large number of parameters. In the current work, we explore the role of different components of the DM interaction in stabilizing multiple noncollinear and noncoplanar ground states and the resulting effects on the electronic properties of the system. The exchange coupling along axial and diagonal bonds are set to \( J = 1.0 \) and \( J' = 0.8 \), respectively, which is motivated by experimental observation of nearly equal bond lengths in rare-earth compounds.
phase (flux state) with increasing \( D_\perp \).

(ii) Next, for a representative \( D_\perp (> 0.62) \), the evolution of the magnetic ordering in the presence of an external magnetic field, \( B \), is investigated. Introduction of magnetic field leads to the canting of localized spins along the direction of \( B \)-field, which results in an extra peak in \( S(Q) \) at \( Q = (0, 0) \). This is shown in Fig. 3(a), where the magnitude of the peak at \( Q = (0, 0) \) is plotted as a function of \( D_\perp \) and \( B \) with the help of colorbar. Fig. 3(b) shows \( S(Q) \) in the \( Q \)-space for \( B/J = 0.5 \) and \( T/J = 0.005 \) calculated on a \( 48 \times 48 \) SS lattice. It shows three equal-magnitude peaks at \( Q = (0, 0), (0, \pi) \) and \( (\pi, 0) \) demonstrating a 3\( Q \) magnetic ordering in the ground state.

(iii) The introduction of the parallel components of DM vector either on axial or on diagonal bonds also results in the canting of localized spins. Fig. 4 shows the effect of parallel component of DM vector on flux state. There is an additional peak in \( S(Q) \) at \( (\pi, \pi) \) and its weight increases with the increase of strength of parallel component. Qualitatively, the effect is same for all three parallel components of DM vectors namely \( D_{\parallel, x}, D_{\parallel, y} \) and \( D' \). The ground state has 3\( Q \) magnetic ordering with equal-magnitude peaks in \( S(Q) \) at \( Q = (0, \pi), (\pi, 0) \) and \( (\pi, \pi) \) as shown in Fig. 4(d). This phase corresponds to an all-in-all-out state, where four neighboring localized spins placed on four corners of a tetrahedron point radially inwards/outwards from the center of tetrahedron.

(iv) Next, we apply magnetic field in the presence of both in-plane and perpendicular components of DM vector. In the absence of magnetic field as mentioned in the previous paragraph the magnetic ordering is that of an all-in-all-out type. With the introduction of \( B \)-field the localized spins reorient in the direction of field and we get an additional out-of-plane canting of these moments. The peak in spin structure factor at \( Q = (0, 0) \) grows with the increase of strength of magnetic field as shown in Fig. 4(a) as a colorbar. The ground state now has 4\( Q \) ordering with peaks in \( S(Q) \) located at \( Q = (0, 0), (0, \pi), (\pi, 0) \) and \( (\pi, \pi) \) as shown in Fig. 4(b). This is a 3-in-1-out state with three spins pointing in and one spin pointing out from the center of tetrahedron. Further increase in magnetic field results in all localized spins pointing in the direction of \( B \)-field, a fully polarized ferromagnetic state.

**Spin chirality**: As seen above, multiple magnetic ordered phases are stabilized in the current model due to the interplay of the antiferromagnetic exchange interaction, the DM interaction and the external magnetic field. To quantify the noncoplanarity of these spin textures we look into the scalar spin-chirality Eq. (8). Our calculation of \( \chi \) gives the following results (see Fig. 6).

The chirality for flux state is zero as it is a 2\( Q \) noncollinear, but coplanar state. With increasing magnetic field on this flux state, the canting of the local moments in the direction of \( B \)-field increases continuously until the local moments are fully polarized. The chirality for canted flux state is non-zero as it is a noncoplanar state with 3\( Q \) magnetic ordering. The chirality increases monotonically up to an intermediate value of the applied field and then decreases continuously to zero at saturation.

Introduction of any of the parallel components of DM vectors causes the flux state to have an out-of-plane canting of the localized spins. For such states, \( S(Q) \) shows additional peaks at \( Q = (\pi, \pi) \). The weight of this peak increases with the increase of any of the in-plane component of DM vectors. This 3\( Q \) state is an all-in-all-out state with a very small value of chirality. Applying the magnetic field changes the all-in-all-out state to a 3-in-1-out state. For this state, \( S(Q) \) shows one more peak at \( Q = (0, 0) \). The enlarged out-of-plane component of the spins contribute to an increase in noncoplanarity of the ground state. The magnitude of the spin chirality increases with increasing magnetic field strength. The 3-in-1-out state is a 4\( Q \) state with a non-zero chirality as shown in Fig. 6(b).
FIG. 4. (Color online) The color plot of peak in $S(Q)$ at $Q = (\pi, \pi)$ as a function of $D_\perp$ and parallel components of DM vectors (a) $D_{D_{\parallel n}}$, (b) $D_{D_{\parallel s}}$, and (c) $D'_{D_{\parallel s}}$. (d) The spin structure factor in $Q$-space for a particular value of $D_{D_{\parallel n}} = 0.775$ which exhibits $3Q$ ordering with peaks at $Q = (0, \pi), (\pi, 0)$ and $(\pi, \pi)$. The results are shown for $L = 48$ lattice size at $J'/J = 0.80$ and $T/J = 0.005$.

B. Electronic properties

**Band structure**: Coupling to the local moments modifies the transport properties of itinerant electrons dramatically. For simplicity, we consider a single band of $s$-electrons interacting with the magnetic ordering via a Kondo coupling between the electron spin and the local moments, as given by the Hamiltonian (3). The dynamics of the electrons is fast compared to that of the localized classical spins. Consequently, at short time scales, the electrons effectively move in a static, but spatially varying magnetic field. Each local moment, $S_i$, acts as a local magnetic field whose action on the spin magnetic moment of the itinerant electrons $s_i$ is described by a Kondo-like interaction $J_K S_i \cdot s_i$. In comparison, the Zeeman energy due the external magnetic field is small and shall be neglected. In the following, the hopping amplitude along the axial bonds $t$ is chosen to be unity ($t = 1.0$). For diagonal bonds the hopping matrix element is fixed at $t'/t = 0.8$.

In the absence of an external field, the electron band structure of SS lattice consists of 4 bands with 2-fold spin degeneracy as the SS lattice has 4-site unit cell. One of the bands is flat along the diagonal of the Brillouin zone (BZ) which gives rise to strong Van Hove singularity, where any interaction effects are maximized. A coupling to the spin texture increases the size of the unit cell in accordance with the periodicity of the magnetic ordering. The BZ is proportionately reduced and the bands are folded into the first BZ. A non-zero $J_K$ lifts the spin degeneracy and the energy bands for electrons with spins parallel and anti-parallel to the local moments are shifted downwards and upwards respectively. For sufficiently strong Kondo-coupling, i.e., $J_K \gg t$, the spin parallel and anti-parallel bands are completely separated by a gap $2J_K$, and we end up with an effective tight-binding model as discussed in section II. In this limit, the effective magnetic field produced by the spin texture couples directly to the charge degrees of freedom of the itinerant electrons, analogous to Quantum Hall systems. The electron energy bands are modified depending on the nature of the underlying magnetic order. Here we discuss the band structure features corresponding to the four complex magnetic phases which are stabilized in the SS lattice.

We observe following key features. (i) **Flux state**: The magnetic unit cell of the SS lattice remains as four sites for flux type ordering of the localized spins. The band structure consists of eight bands and for large $J_K$ these split into four bands each for spin parallel and antiparallel alignment of itinerant electrons with the localized moments. We show the dispersion of itinerant electrons when they move on the background of flux phase in Fig. 7(a) along a high symmetry path in the 1st BZ. The high-symmetry points of BZ taken in the calculations are $\Gamma = (0, 0)$, $M = (\pi/2, 0)$ and $K = (\pi/2, \pi/2)$. The four spin parallel bands are doubly degenerate and touch each other at diagonal point of BZ. (ii) **Canted flux state**: For this magnetic state, the dispersion
FIG. 5. (Color online) (a) The magnitude of the peak in spin structure factor at \( Q = (0, 0) \) as a function of one of the parallel component of DM vector \( D_{∥,ns} \) and \( B \)-field. (b) The plot of \( S(Q) \) in terms of \( Q_x \) and \( Q_y \) indicating 4 dominants peaks at \( Q = (0, 0), (0, \pi), (\pi, 0) \) and \( (\pi, \pi) \). The calculations are done at \( J'/J = 0.8 \), \( D_{∥,ns}/J = 0.775 \), \( B/J = 0.8 \) and \( T/J = 0.005 \) on a 48 \( \times \) 48 SS lattice of conduction electrons are plotted in Fig. 7(b). It consists of four bands with degeneracy of the bands is partially lifted. There is a direct gap opening between upper and lower pair of bands. The non coplanar canted flux state not only lifts the degeneracy, but also opens up a gap at the \( K \)-point.

(iii) All-in-all-out state: For the all-out state, the magnetic unit cell is still four sites, and the band structure plot shown in Fig. 7(c) comprises of four bands. The degeneracy of the bands is lifted and we observe an indirect gap between upper and lower pair of bands at the \( K \)-point. Interestingly, the middle two bands touch each other at a point close to the \( K \)-point.

(iv) 3-in-1-out state: The size of the SS lattice unit cell remains unchanged for this magnetic ordering. The degeneracy of all four bands is fully lifted and we observe direct as well as indirect gaps between the bands as shown in Fig. 7(d).

Hall conductivity: The coupling to local moments modifies the transport properties of itinerant electrons significantly in metallic magnets. The effect is most dramatic in the transverse conductivity, especially when the underlying spin arrangement is noncoplanar. In a magnetic metal, the Hall resistivity consists of three contributions,

\[
\rho_{xy} = \rho_{NHE}^{xy} + \rho_{AHE}^{xy} + \rho_{THE}^{xy},
\]

where NHE, AHE and THE refer to Normal, Anomalous and Topological Hall effects, respectively. The AHE appears in metals with a net magnetization due to spin-orbit coupling. On the other hand, THE arises due to the Berry phase acquired by an electron moving in a noncoplanar spin texture. The phenomenon is best understood within the framework of the effective Hamiltonian (2) in the strong coupling limit (\( J_K \gg t \)). In this limit, the Berry phase acquired by an electron moving around a closed plaquette results in an effective flux threading each such plaquette that acts as a fictitious magnetic field and drives a Hall effect, whose origin is purely geometrical. In this work, we focus on isolating the contribution of THE to the transverse conductivity in different phases.

(i) Flux state: The zero temperature Hall conductivity for this magnetic phase is plotted in Fig. 8(a) by varying the chemical potential for spin parallel itinerant electrons in the strong coupling limit. The value of transverse conductivity remains zero throughout the range of Fermi energy. As identified earlier flux state is a coplanar state with zero chirality which explains why it does not exhibit any THE.

FIG. 6. (Color online) The scalar spin chirality as a function an external magnetic field, and (a) \( D_{⊥,ns} \) and (b) \( D_{∥,ns} \). The colorbars adjacent to the plots indicate the magnitude of spin chirality.
we observed band opening at the $K$-point for this state. The chirality associated with this state is non-zero which contributes to THE. The transverse conductivity as a function of Fermi energy for canted flux state is shown in Fig. 8(b). The quantized Hall conductivity has integer value $−1$ (in unit of $e^2/h$) for the width of the band gap for this noncoplanar state.

(iii) **All-in-all-out state**: TheHall conductivity for this phase is shown in Fig. 8(c) by changing the Fermi energy. The chirality of this state is very small as compared to canted flux state. There is a non-zero value of Hall conductivity for small range of Fermi energy which is attributed to small noncoplanarity of this state. The value of the conductivity is not integer as there is no direct band gap between the energy bands.

(iv) **3-in-1-out state**: The most interesting outcome of our work is observed for the 3-in-1-out magnetic state. This magnetic phase is noncoplanar with non-zero value of chirality. We also observed that degeneracy of all the bands is fully lifted and there are direct and indirect gaps between the bands. The Hall conductivity for this phase is shown in Fig. 8(d) and it remains non-zero for a large window of Fermi-energy lying between $−2t$ and $2t$. Again the noncoplanarity of this phase manifests itself through non-zero value of THE. When the gap between the energy bands is direct, the quantized Hall conductivity remains $−1$ (in unit of $e^2/h$) for the width of band gap. This is a signature of integer THE similar to integer quantum Hall effect observed in quantum Hall systems.

**V. SUMMARY**

We have shown that noncollinear and noncoplanar magnetic phases of localized spins are stabilized on the SS lattice in the presence of antiferromagnetic exchange coupling, DM interaction and an external magnetic field. We have discussed the role of in-plane, and out-of-plane components of DM vectors, and external magnetic field in the stabilization of these exotic ground states of localized moments. The electronic properties are modified in the presence of these complex noncoplanar magnetic phases. We have studied topological Hall effect on the SS lattice in the strong Kondo-coupling limit between localized spins and itinerant electrons. The itinerant electrons hopping on the background of these magnetic textures of localized spins are shown to exhibit THE.

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