Plasma and thermal waves in a semiconducting cantilever photogenerated by a focused laser beam

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Abstract. The plasma wave (carrier-density wave) and thermal wave in an optically opaque semiconducting cantilever (3D geometry), photogenerated by a tightly focused and intensity-modulated laser beam, were analyzed. The theoretical model for the carrier-density and temperature distribution by using the Green function method and Hankel transformation was given. The amplitude and phase of the carrier-density and temperature space and frequency distribution in the cantilever are calculated, including the thermalization and surface and volume recombination heat processes. These investigations are important for many practical experimental situation (atomic force microscopy, thermal microscopy, thermoelastic microscopy, etc) and sensors and actuators based on cantilevers.

1. Introduction
The photoacoustic (PA) and photothermal (PT) techniques were recently established as diagnostic methods with good sensitivity to the thermal and electronic transport processes in semiconductors and microelectronic structures. Photogeneration of electron-hole pairs, i.e. the carrier-diffusion wave or plasma wave, generated by the absorbed intensity-modulated optical excitation can play the dominant role in the PA and PT experiments for most semiconductor materials and microelectronics structures. Depth-dependent plasma waves contribute to the generation of periodic heat, i.e. thermal waves. The theoretical analysis of the plasma and thermal effects in micromechanical structures consists in modeling a complex system by simultaneous analysis of the plasma and thermal wave equations. In previously published papers, the plasma, thermal and elastic fields in one-dimension (1D) micromechanical structures were theoretically and experimentally analyzed by Todorović et al. [1,2,3].

In this work the plasma and thermal waves, i.e. the carrier-density and temperature space and frequency distribution in a rectangular cantilever (3D geometry), photogenerated by a tightly focused and intensity-modulated laser beam, are analyzed. This theoretical treatment enables quantitative accounts of the carrier density field, \( n(r,t) \) and temperature field, \( T(r,t) \). In the case of periodical excitation, with angular modulating frequency of the incident beam \( \omega \), the excess carrier-density can be assumed as \( n(r,t) = \text{Re}[N(r,\omega) \exp(i\omega t)] \), and temperature \( T(r,t) = \text{Re}[T(r,\omega) \exp(i\omega t)] \), where \( n(r,\omega) \)
and \( T(r,\omega) \) are complex values which define the amplitude and phase of the carrier and temperature fields, respectively.

2. Carrier-density field

The plasma (carrier-density) field in an opaque semiconducting rectangular CL (3D geometry), where a tightly focused and intensity-modulated laser beam (an impulsive - Dirac harmonic source) centered at \((x_1, y_1)\) impinges at the surface plane \( z = 0 \), are taken in consideration.

A bipolar semiconductor model is considered with bulk and surface carrier recombination. The modulated light, absorbed in the semiconductor, excites electrons from the valence band. The laser beam excitation generates a depth-dependent plasma wave \( n(r,t) \), which propagates in the bulk of the semiconductor. The excitation energy, \( E \), is greater than the energy gap, \( E_G \), and the absorbed photons generate electron-hole pairs. The excess energy, \( \Delta E = E - E_G \), is converted by fast nonradiative processes into heat (the thermalization of the carriers). This electronic diffusion process is characterized by the diffusion coefficient \( D_E \). During this diffusion process the carriers can reach the sample surface, with a finite probability of recombination. The velocities \( s_1 \) and \( s_2 \) characterize the surface recombination processes at the illuminated surface \((z = 0)\) and rear side \((z = L_z)\) of the CL, respectively. On the other hand, the excess carriers recombine in the bulk of the CL, with a characteristic time \( \tau \) (the lifetime of photogenerated electron-hole pairs).

2.1. Green function method

The plasma (the carrier-density) field, when the plasma Green function \( G_c(r,\omega) \) is known, is given as \([4]\):

\[
N(r,\omega) = \int_{V_o} \int_\sigma Q_c(r_o,\omega) G(r|r_o,\omega) \, dV_o + \int_{S_o} \int_{S_o} F_L(r_o,\omega) G_c(r,\omega) \, dS_o, \tag{1}
\]

where \( Q_c(r_o,\omega) \) is the plasma volume source, \( F_L(r_o,\omega) \) is the plasma surface source, \( r_o \) is the position vector of the source, \( V_o \) and \( S_o \) are the volume and surrounding surface area of the particular spatial region within which the plasma wave problem and the associated Green function are defined. The first (volume) integral in Eq.(1) gives the contribution to the plasma wave field from all volume sources within \( V_o \). The second (surface) integral adds the contributions of all surface sources to the field over the entire surface surrounding and enclosing \( V_o \).

2.2. Plasma source

It is responsible for the appearance of a surface plasma source, in the Si sample. For example, for Si cantilever optically excited with laser beam at wavelength \( \lambda = 532 \text{ nm} \), where the optical absorption coefficient \( \alpha \approx 10^4 \text{ cm}^{-1} \), the optical absorption length is \( L_{op} \approx 1 \mu \text{m} \). The process of photogeneration is practically closed to the sample surface. Approximately all carriers will be generated at the distance \( L_{gen} \approx L_{op} \). In this case, the plasma source is situated approximately at the same region where the optical energy (the photons) are absorbed, i.e. it is situated in a thin surface layer \((L_{gen} << L_z)\). Then, the plasma source can be given with an ac surface source, i.e., with an ac plasma flux, \( F_L(x,y,\omega) \). The ac photogenerated carrier flux at the surface \( z = 0 \) can be approximated by a two-dimensional impulsive (Dirac) \( \delta \) - function:

\[
F_L(x,y,\omega) = F_{ca}(\omega) \delta(x-x_1') \delta(y-y_1'), \quad F_{ca}(\omega) = \frac{\eta_c (1 - R_s) P_s(\omega)}{2E}, \tag{2}
\]

where \( \eta_c \) is the quantum efficiency for non-radiative photogeneration of the carriers, \( P_s(\omega) \) is the laser power of the incident optical excitation and \( R_s \) is the reflectivity of the sample surface. If it is supposed that only the surface carrier flux exist at surface \( z = 0 \) (the volume sources are zero, \( Q_c(x,y,z,\omega) \approx 0 \)), the carrier-density field is
where $F_{co}$ is the ac surface carrier flux [carrier / (m²s)] and $G_c(x,y,z | x_1, y_1, 0; \omega)$ is the appropriate carrier Green function.

3. Thermal waves

3.1. Thermal sources

There are three main mechanisms in thermal wave generation, i.e. there are three thermal sources in semiconductors: the thermalization heat source, $Q^T(r, \omega)$ [W/m³]; the bulk recombination source, $Q^{BR}(r, \omega)$ [W/m³] and two different heat sources at the cantilever surface $z = 0$, $Q^{1SR}$ [W/m²] and $Q^{2SR}$ at the cantilever surfaces $z = L_z$.

The periodic temperature distribution, $T(x,y,z;\omega)$ in the cantilever can be given as a sum of three components:

$$T(x,y,z;\omega)=T^T(x,y,z;\omega)+T^{BR}(x,y,z;\omega)+T^{SR}(x,y,z;\omega),$$

where $T^T(x,y,z;\omega)$, $T^{BR}(x,y,z;\omega)$ and $T^{SR}(x,y,z;\omega)$ are the thermalization, bulk and surface recombination components of the temperature distribution, respectively.

3.2. Thermalization component of the temperature field

The thermalization heat source can be approximated with an ac surface heat source, i.e. with an ac TW flux, $Q^T(r, \omega) \approx F_{th}(x,y;\omega)$. For tightly focused laser beam, the thermal source is a point source, located approximately at the same position as the spot of the laser beam. The ac TW flux $F_{th}(x,y;\omega)$ [W/m²], can be approximated by a two-dimensional impulsive (Dirac) $\delta$ – function.

If it is supposed that only the surface thermal flux at surface $z = 0$ (the volume sources are zero) exists, the thermalization component of the TW field is:

$$T^T(x,y,z;\omega)=\frac{D_L}{K}\int_0^{L_z}dx_1\int_0^{L_z}dy_1 F_{th}(x_1, y_1;\omega)G_{th}(x,y,z|x_1, y_1, 0;\omega)=\frac{D_L}{K}P_{th}G_{th}(x,y,z|x_1, y_1, 0;\omega),$$

where $G_{th}(x,y,z | x_1, y_1, 0; \omega)$ is thermal Green functions, $P_{th}(\omega)$ is the power at excited surface, $D_T$ is the thermal diffusivity and $K$ is the thermal conductivity.

3.3. Surface Recombination Components of the temperature field

Heat sources, due to the carrier recombination at the surfaces, $Q^{iSR}$ ($i = 1, 2$), can be given:

$$Q^{iSR}(x,y,z;\omega)|_{z=0} = s_i E_G N(x,y,0;\omega), \quad Q^{iSR}(x,y,z;\omega)|_{z=L_z} = s_i E_G N(x,y,L_z;\omega)$$

where $N(x,y,0;\omega)$ and $N(x,y,L_z;\omega)$ are the appropriate carrier density distribution at surfaces $z = 0, L_z$, respectively.

The heat sources $Q^{iSR}$ ($i = 1, 2$), due to the carrier recombination at the surfaces of the semiconductor $z = 0, L_z$, give the periodic temperature distribution $T^{SR}(x,y,z;\omega)$ in the CL as a sum of two components:

$$T^{SR}(x,y,z;\omega)=T^1_{SR}(x,y,z;\omega)+T^2_{SR}(x,y,z;\omega),$$

where the surface recombination component $T^1_{SR}(x,y,z;\omega)$ has the form:

$$T^1_{SR}(x,y,z;\omega)=\frac{D_L}{K}F_{co}s_1 E_G\int_0^{L_z}dx_1\int_0^{L_z}dy_1 G_c(x_1, y_1, 0|x_1, y_1, 0;\omega)G^{SR}(x,y,z|x_1, y_1, 0;\omega),$$

where $G_c$ is the appropriate plasma Green function and $G^{SR}$ is the thermal Green functions.

3.4. Bulk recombination component of the temperature field

The bulk recombination heat source $Q^{BR}(r, \omega)$, can be defined:
where $\eta_R$ is the coefficient of bulk recombination [$\eta_R = (\text{phonons} / \text{carrier}) \approx 1$] and $N(x,y,z;\omega)$ is the carrier density distribution in the semiconducting CL.

The bulk recombination component of the temperature field is:

$$T^{\text{BR}}(x,y,z;\omega) = \frac{D_T}{K} \frac{E_g}{\tau} \left( \frac{E_g}{\tau} \right) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} G(x_1,y_1,z_1|x,y,z;\omega) G^{\text{BR}}(x,y,z;\omega) dx_1 dy_1 dz_1.$$

4. Analysis of carrier-density and temperature fields

The theoretical model, derived in this work, enables to calculate the 3D carrier-density and temperature fields. The carrier-density and temperature fields were calculated for typical parameters of Si and dimensions: length $L_x = 3000 \, \mu\text{m}$, width $L_y = 400 \, \mu\text{m}$, and thickness $L_z = 150 \, \mu\text{m}$. Fig.1 shows a typical example of the 3D carrier-density distribution at the surface ($z = 0$) of the rectangular Si cantilever, excited with tightly focused beam centered at point $(x_1 = 100 \, \mu\text{m}, y_1 = L_y/2 = 200 \, \mu\text{m})$ on the CL surface.

![Figure 1](image1.png)

**Figure 1.** Amplitude of carrier-density distribution at the surface ($z = 0$) in the rectangular cantilever excited with tightly focused beam, impinges at point $x_1 = 100 \, \mu\text{m}, y_1 = L_y/2 = 200 \, \mu\text{m}$.

Fig.2 shows a typical example of the 3D temperature distribution (the amplitude of the surface recombination component) at the surface ($z = 0$) of the same Si cantilever, (the other parameters are the same as for example in Fig.1).
Figure 2. Amplitude of carrier-density distribution at the surface (z = 0) in the rectangular cantilever excited with tightly focused beam, impinges at point $x_1 = 100 \mu m$, $y_1 = 200 \mu m$.

Conclusion
The theoretical model for 3D carrier-density field and three components of temperature field (the thermalization, surface and bulk recombination) were derived. The carrier-density and temperature space and frequency distribution in a rectangular cantilever, photogenerated by a tightly focused and intensity-modulated laser beam, were analyzed.

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