Determining consignment inventory strategy for a vendor and multiple buyers using a hybrid metaheuristic algorithm

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This paper considers a centralized supply chain with consignment inventory (CI) system, where a vendor keeps a finished product and supplies it at the same price to multiple buyers. CI is a process where the buyer pays with delay for the goods supplied by the vendor. Considering the time value of money, this study compares two types of delay in payment in CI, which have been rarely considered (real use and order to order). Furthermore, two different metaheuristic algorithms, genetic algorithm (GA) and a hybrid algorithm, containing GA and particle swarm optimization algorithm, are used to maximize the supply chain net profit and calculate the proper values. Finally, a sensitivity analysis is performed to examine the effects of each parameter on the total net profit. Moreover, by applying a paired t-test, a comparison is made between two types of CI system and also between GA and the hybrid algorithm.

Keywords: inventory; nonlinear programming; delay in payment; hybrid metaheuristic algorithm; discounted cash flow approach (DCF)

1. Introduction

Vendor managed inventory (VMI), consignment inventory (CI) and a combination of both, so-called consignment vendor managed inventory (C&VMI), are supply chain sourcing agreements between a vendor and a buyer. In VMI, the vendor is responsible for determining the order size and replenishment time for himself as well as for his buyer based on shared information and inventory policies. In CI, the buyer will be invoiced with delay, but he still specifies his own time and quantity of orders. Under a C&VMI agreement, the vendor determines how much to store at the buyer’s premises, and he owns the goods stocked there.

The benefits of VMI and C&VMI to the buyers, where the vendor is authorized for the inventory management, include shifting the costs of placing orders to the vendor as well as increasing the inventory turns by a service level guarantee. Moreover, the vendor can benefit from the lessened demand uncertainty to reduce the manufacturing and transportation costs. Although, vendor managed system has been recognized as one of

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the most successful approaches in improving supply chain integration by some researchers (Darwish & Odah, 2010), they have been criticized by some others. (Copacino, 1993) discussed that a poorly designed VMI/ C&VMI agreement would harm a vendor which must transfer items frequently to satisfy the inventory turns needed at the buyer. (Saccomano, 1997) also disputed that VMI transfers the risks of inventory management from the buyers to the vendors. (Kim & Park, 2010) believed that VMI may be disadvantageous for the vendor as it increases the inventory management and information retaining costs. According to these viewpoints, the buyer would be also afraid of losing his information possession and shelf space management, so unwilling to share the information with the vendor.

On the other hand, CI contract is worthy to the buyers as they do not need to invest their capital on inventory. Besides, it is advantageous to the vendor in attracting more buyers and selling new and unproven products that the buyers may hesitate to buy. It can also act as a strategic tool for a weak vendor to satisfy the requirements of the strong buyers. (Gümüş, Jewkes, & Bookbinder, 2008) mentioned Automation and Drives division of Siemens as an example of CI in which springs and nuts can be consigned from the vendors, although the demand per annum is quite stable.

Based on the previous studies, CI is profitable for both the vendor and the buyer, while VMI and C&VMI would be more attractive to the buyer rather than the vendor. Therefore, we concentrate on CI in this study to provide a condition under which both supply chain members benefit.

Based on the time of transferring the ownership of goods, four types of CI could be categorized (Molamohamadi, Rezaeiahari, & Ismail, 2013; Piasecki, 2004): (1) paying as sold (real use), (2) paying as sold during a predefined period, (3) changing ownership after a predefined period, and (4) paying for an order when the next is placed (order to order consignment).

CI has been widely studied by many researchers, most of which have considered the third type of delay in payment called trade credit in literature. However, there are some studies related to the first type; (Gümüş et al. 2008) provide conditions under which CI benefits the vendor, the buyer, and the two parties by assuming that the ownership is exchanged when the goods are consumed by the buyer. (Sharifyazdi, Jafari, Molamohamadi, Rezaeiahari, & Arshizadeh, 2009) conduct a study on the real use of CI system to obtain proper values of replenishment interval at the warehouse, as the vendor, and retailers (buyers), and retail prices with particle swarm optimization (PSO) algorithm through an integrated inventory model. (Ru & Wang, 2010) build a game-theoretic model to compare the performance of the supply chain under two consignment contracts, CI and C&VMI. Considering retail competition, (Adida & Ratisoontorn, 2011) model a two echelon supply chain as a Stackelberg game, where the vendor is the leader and the buyers are his followers.

Some researchers have also considered the second type of CI as the time of transferring ownership. In this type, a period is defined and whenever a product is sold, the bills must be paid to the vendor. The buyer can decide at the end of this period to buy the unsold goods or send them back to the vendor. (Chung, 1989) analyzes the optimal inventory policy in the presence of this type of trade credit by applying the discounted cash-flows (DCF) approach. DCF is a method which considers the time value of money by estimating the present values of the future cash flows. (Jaggi & Aggarwal, 1994) extend (Chung, 1989) to obtain the optimal order quantity of deteriorating items. (Chung & Liao, 2006) generalize (Jaggi & Aggarwal, 1994) by applying the credit term only for large order quantities. (Chung & Liao, 2009) generalize (Chung, 1989) by
considering the concepts of DCF and order quantity dependent trade credit. (Chung & Lin, 2011) disprove the approximations considered in (Jaggi & Aggarwal, 1994) for resulting in deterioration ignorance and obtain the optimal replenishment time by proposing a new algorithm.

Trade credit, the third type of CI, is referred to a contract in which the payment is done at the end of a specific period. This type of CI has already attracted many researchers’ attention. (Goyal, 1985) considers the lot-sizing problem of a buyer under trade credit, when the credit period is greater than/equal to, or smaller than the cycle time. (Dave, 1985) reformulates (Goyal, 1985)'s model by differentiating between selling and purchasing prices, but his standpoint did not receive any attention for around two decades. Considering the same issue as (Dave, 1985), (Teng, 2002) amends (Goyal, 1985)'s model and provides an analytical solution to solve it.

(Huang, 2003) extends (Goyal, 1985)'s model by determining the buyer’s optimal ordering policies under a two-level trade credit, where not only the vendor, but also the buyer would offer trade credit to the downstream level. (Huang, 2006) modifies (Huang, 2003) by including the buyer’s storage space limitation in the model. (Chung & Huang, 2007) develop (Huang, 2003)'s model for deteriorating items and limited storage space. (Chung & Huang, 2009) incorporate shortages to generalize (Goyal, 1985)'s model and prove convexities in the total annual variable cost function. (Kreng & Tan, 2010) develop optimal vendor’s replenishment decisions in a two-level order quantity dependent trade credit to extend (Huang, 2003)'s model. (Tsao, 2011) presents an inventory model with trade credit and logistic risk considerations. (Teng, Min, & Pan, 2012) introduce a linear time increasing demand function and develop an EOQ model under trade credit. By considering a two echelon supply chain, (Su, 2012) proposes an integrated inventory model, including trade credit, shortages, and defective items. (Thangam, 2012) considers advance payment scheme and two-level trade credit for developing an EOQ model with perishable items to find the optimal replenishment policy and the price discounting policy. Assuming a two warehouse inventory model and deteriorating items, (Guchhait, Maiti, & Maiti, 2013) applied a hybrid metaheuristic algorithm to evaluate the effect of order quantity dependent trade credit on buyer’s ordering policy under a two-level trade credit.

The fourth type of delay in payment, order to order consignment, has not been specifically discussed in literature, but there are some works in the second and third types of CI that consider the credit period less/greater than or equal to the cycle time (Chung, 1989; Chung & Huang, 2007; Goyal, 1985; Guchhait et al., 2013; Huang, 2003, 2006; Kreng & Tan, 2010; Su, 2012; Teng et al., 2012; Thangam, 2012; Tsao, 2011) (for a more comprehensive review of literature, please refer to [Chang, Teng, & Goyal, 2008; Molamohamadi et al., 2013]).

The literature review shows that there are research gaps in the first and fourth types of delay in payment, which have not been widely considered by the researchers. Therefore, this study focuses on modeling the inventory system for these two types, real use and order to order consignment.

In this paper, the vendor is a warehouse that transfers one type of product to the buyers 1,2, …, m at the same wholesale price. The buyers are retailers who then sell the product in independent markets at retail prices. Considering the time value of money, we propose the integrated formulation of supply chain net profit for two types of delay in payment, pay as sold and order to order consignment. These models are then solved by applying two metaheuristic algorithms; genetic algorithm (GA) and a hybrid algorithm. Afterwards, the results are compared by paired t-test to clarify the most
profitable delay in payment approach and the best algorithm. Moreover, the impacts of the parameters on the net profit will be evaluated by ANOVA test.

The rest of the paper is organized as follows. Section 2 describes the assumptions and notations to be used. After that, we formulate the net profits of the warehouse and the retailers for two types of delay in payment in Section 3 and represent the integrated models. The problem-solving procedure and algorithms used to solve the problem are then described in Section 4. In the penultimate section, results of the sensitivity analysis are given. Section 6 summarizes the paper and gives some further research directions.

2. Assumptions and notations

The symbols which will be used here are represented in Table 1.

The following assumptions will also be considered in the formulation of the model:

1. Demand of each retailer is considered to be constant.

2. As integer-ratio policies are assumed, for each retailer \(i\), either \(g_i/g\) or \(g_i/g\) must be a positive integer. According to (Abdul-Jalbar, Segerstedt, Sicilia, & Nilsson, 2010), the warehouse does not have to hold inventory for the retailers with replenishment interval greater than \(g\); So, if \(g_i \geq g\), then each time retailer \(i\) places an order, so does the warehouse. However, when \(g_i \leq g\) the warehouse must hold inventory to satisfy retailer \(i\)'s demand. Figure 1 depicts a system consisting of one warehouse and two retailers with \(g_1 < g\) and \(g_2 > g\). In this example, the warehouse only holds inventory to fulfill the orders placed by retailer 1 and backorder is allowed only at the retailers.

3. Two types of delay in payment are considered in this paper: (i) Pay as sold (real use) and (ii) Order to order consignment.

4. Shortage is allowed at the retailers and would be completely backlogged. The retailers would send the backlogged orders at the time of receiving the items from the warehouse and would be paid simultaneously by end customers.

| Notation  | Definition |
|-----------|------------|
| Parameters \(A_{bi}\) | fixed replenishment cost per order of the product for retailer \(i\) ($/order) |
| \(A_w\) | fixed replenishment cost per order at the warehouse ($/order) |
| \(B_i\) | unit backorder cost of retailer \(i\) per time period ($/unit/time) |
| \(D_i\) | demand of the retailer \(i\) per unit time (unit/time) |
| \(h_{ri}\) | unit holding cost of the retailer per time period ($/unit/time) |
| \(h_w\) | unit holding cost of the warehouse per time period ($/unit/time) |
| \(p_i\) | retail price of retailer \(i\) ($/unit) |
| \(p_w\) | wholesale price of the product set by the manufacturer ($/unit) |
| \(r\) | interest rate |
| \(v_w\) | procurement price of the product for warehouse ($/unit) |
| \(W_w\) | allocated weight to the warehouse |
| \(W_r\) | allocated weight to the retailers |
| \(\phi_i\) | direct transportation cost for shipping one unit product from the warehouse to retailer \(i\) ($/unit) |
| Variables \(b_i\) | fraction of backlogging time in a cycle of retailer \(i\) |
| \(g\) | replenishment interval at warehouse |
| \(g_i\) | replenishment interval at retailer \(i\) |
(5) Lead time is assumed to be zero.
(6) For including the time value of money in the model, discounted cash flow (DCF) approach is considered where the interest rate compounds continuously.
(7) For assuring that backorder does not exist in all or most of the time in a cycle time, it is assumed that $g_i \geq 2b_i$.
(8) When there is power differential between a weak vendor and a stronger buyer, vendor can apply CI to meet his buyer’s wishes. Thus, it is assumed here that the warehouse (vendor) is weaker than the retailers (buyers) and uses CI as a strategy to sell his products. So two weight parameters with the following relations have been considered.

$$W_r + W_w = 1, \quad W_r > W_w$$

3. Problem formulation

3.1. Each retailer’s net profit

For real use consignment contract, because retailer $i$ must pay for the items at selling time, he pays for the backlogged items, which are equal to $D_i b_i g_i$, at the time of receiving them, when he sells them to the end customers. Therefore, he must pay $p_i D_i b_i g_i$ to
the warehouse for the backordered goods at the beginning of the period without delay, but delays paying off the rest until \( t, t \in [0, g_i(1 - b_i)] \), when they are sold. Hence, with regard to the continuously compound interest, the product procurement cost for the non-backlogged period in real use consignment would be \( P_{w1} = T \int_{0}^{g_i(1 - b_i)} D_t e^{-rt} dt \). Considering the fact that the payment is settled at the end of the period in order to order consignment, the procurement cost for this type of delay in payment would be \( P_{w} D_i e^{-rg_i} \). Moreover, the retailer spends \( D_i \left( \frac{1}{g_i} b_i g_i^2 \right) h_{sl} \) on holding cost, \( A_{bi} g_i \) on replenishment cost, \( \frac{D_i b_i g_i^2}{2g_i} B_i \) on backorder cost and \( D_i \varphi_i \) on transportation cost. Considering retailer \( i \)'s revenue \( (p_i D_i) \), his net profits per unit time (denoted by \( NP_{w1} \) for real use and \( NP_{w2} \) for order to order consignment) become:

\[
NP_{w1} = p_i D_i - (p_w D_i g_i + p_w \int_{0}^{g_i(1 - b_i)} D_t e^{-rt} dt)
- \frac{D_i(1 - b_i)^2 g_i^2}{2g_i} h_{sl} - \frac{A_{bi} g_i}{g_i} - \frac{D_i b_i g_i^2}{2g_i} B_i - D_i \varphi_i
\]

(2)

where \( \int_{0}^{g_i(1 - b_i)} D_t e^{-rt} dt = \frac{1}{r} D_i (1 - e^{-rg_i(1 - b_i)}) \) and

\[
NP_{w2} = p_i D_i - p_w D_i e^{-rg_i} - \frac{D_i(1 - b_i)^2 g_i^2}{2g_i} h_{sl} - \frac{A_{bi} g_i}{g_i} - \frac{D_i b_i g_i^2}{2g_i} B_i - D_i \varphi_i
\]

(3)

### 3.2. The warehouse’s net profit

Applying CI in a supply chain, the warehouse earns \( \sum_{i=1}^{m} (p_w D_i b_i + p_w \int_{0}^{g_i(1 - b_i)} D_t e^{-rt} dt) \) in real use consignment and \( \sum_{i=1}^{m} c_p D_i e^{-rg_i} \) in order to order consignment. The procurement cost at warehouse’s site, per unit time, is \( v_w \sum_{i=1}^{m} D_i \) and it spends \( \left( \frac{1}{g_i} \right) h_w \sum_{i, g_i > 1} D_i (g_i - g_i) g_i \) on holding cost and \( \frac{A_w}{g} \) on ordering cost. \( NP_{w1} \) and \( NP_{w2} \) represent the warehouse’s net profit per unit time for real use and order to order consignment, respectively.

\[
NP_{w1} = \sum_{i=1}^{m} \left[ p_w D_i b_i g_i + \frac{1}{r} p_w D_i \left( 1 - e^{-rg_i(1 - b_i)} \right) \right] - v_w \sum_{i=1}^{m} D_i - \left( \frac{1}{g_i} \right) h_w \sum_{i, g_i > 1} D_i (g_i - g_i) g_i - \frac{A_w}{g}
\]

(4)

\[
NP_{w2} = \sum_{i=1}^{m} p_w D_i e^{-rg_i} - v_w \sum_{i=1}^{m} D_i - \left( \frac{1}{g_i} \right) h_w \sum_{i, g_i > 1} D_i (g_i - g_i) g_i - \frac{A_w}{g}
\]

(5)

### 3.3. The integrated models

The objective here is to maximize the total net profit (\( NP_{T1} \) for real use and \( NP_{T2} \) for order to order consignment) with respect to the formulated net profits of warehouse and his retailers. The corresponding models become:
\[
\max NP_{T1} = W_r \sum_{i=1}^{m} NP_{Ri} + W_w NP_{W1} \\
= W_r \sum_{i=1}^{m} \left\{ p_i D_i - \left[ p_w D_i b_i g_i + \frac{1}{r} p_w D_i \left( 1 - e^{-rg_i(1-b_i)} \right) \right] \right\} \\
- D_i \left( 1 - b_i \right) g_i^2 h_{si} / 2 g_i - A_{bi} / g_i - D_i b_i g_i^2 / 2 g_i B_i - D_i \varphi_i \right\} + W_w \left\{ \sum_{i=1}^{m} \left[ p_w D_i b_i g_i + \frac{1}{r} p_w D_i \left( 1 - e^{-rg_i(1-b_i)} \right) \right] \right\} \\
- v_w \sum_{i=1}^{m} D_i - h_w \sum_{i, \frac{g_i}{g} > 1} D_i g / 2 - A_w / g \right\} 
\]
\[
\max NP_{T2} = W_r \sum_{i=1}^{m} NP_{R2} + W_w NP_{W2} \\
= W_r \left[ p_i D_i - p_w D_i e^{-rg_i} - D_i \left( 1 - b_i \right) g_i^2 h_{si} / 2 g_i - A_{bi} / g_i - D_i b_i g_i^2 / 2 g_i B_i - D_i \varphi_i \right] \\
+ W_w \left[ \sum_{i=1}^{m} p_w D_i e^{-rg_i} - v_w \sum_{i=1}^{m} D_i - h_w \sum_{i, \frac{g_i}{g} > 1} D_i g / 2 - A_w / g \right] 
\]
subject to \( g_i \geq 2b_i \)

subject to (7).

4. Solution procedure

Since the formulated optimization inventory model is a nonlinear programming problem, it is difficult to solve it analytically. Thus, we apply an evolutionary computation algorithm to solve the problem and find the optimum values. In this paper, a hybrid metaheuristic algorithm based on PSO and GA is proposed to find the retailers’ backorder time and replenishment cycle, as well as the warehouse’s cycle time. In the hybrid algorithm, offsprings are created not only by GA operators – crossover and mutation – but also by PSO.

The hybrid of GA with existing algorithms can always produce a better algorithm than either GA or the existing algorithms alone (Juang, 2004). Since GA and PSO are both population-based search algorithms, combining the searching abilities of both methods seems to be a good approach (Juang & Liou, 2004). Moreover, with a combination of GA and PSO, the premature convergence of PSO and memory loss of GA can be overcome and an efficient algorithm with diversity and information of all individuals will be made. In this section, we will introduce basic concepts of GA and PSO, followed by explaining how a combination of these two algorithms is used to solve the problem.
4.1. Basic concepts of GA

GA maintains a population of candidate solutions coded as individuals or chromosomes each of which consists of a linear list of genes. After implementing genetic operators, the fitness value of each chromosome will be evaluated iteratively to find its fitness value and generate the new population. The basic operators of GA used in producing the new generation are reproduction, crossover, and mutation. In reproduction, the most highly ranked chromosomes of the current generation would be reproduced in the new population. Crossover combines the information from two parents (current candidate solutions) to produce two offsprings as the new chromosomes. This operator creates offsprings by exchanging alternate pairs of randomly selected crossing sites of the parents. Mutation is the random changing of the values of an individual’s genes. Using these evolutionary inspired operators in GA, the best solutions are modified and passed on to the next generation. In this way, the population as a whole moves towards better solutions, ideally to the global optimum (Gen & Cheng, 1997).

4.2. Basic concepts of PSO

PSO is an algorithm for finding optimal regions of complex search spaces through the interaction of individuals in a population of particles which correspond to the chromosomes in GA. Initially, a population of particles is generated randomly. Every single particle is a representation of a potential solution and a swarm of particles moves through the problem search space. The particle movement is mainly affected by three factors: inertia, particle best position, and global best position. The inertia is the velocity of the particle in the latest iteration and can be controlled by inertia weight. Particle best position (pbest) is the best solution so far reached by the particle. Global best position (gbest) is the current best solution hitherto obtained by any particle in the population.

Considering $X_{kj}(t)$ and $V_{kj}(t)$ as the position and velocity of particle $k$ in dimension $j$ at time step $t$, a particle can update its velocity and position in each generation based on Equations (9) and (10):

$$X_{kj}(t+1) = X_{kj}(t) + V_{kj}(t+1)$$

$$V_{kj}(t+1) = wV_{kj}(t) + c_1 r_1(t) [y_{kj}(t) - X_{kj}(t)] + c_2 r_2(t) [\hat{y}_j(t) - X_{kj}(t)]$$

where $w$ is the inertia weight, $y_{kj}(t)$ is the best position found by particle $k$ in dimension $j$ at time step $t$, $\hat{y}_j(t)$ is the best position found by the whole swarm (gbest PSO) or by the particle $k$’s neighborhood (lbest PSO) in dimension $j$, $c_1$, and $c_2$ are positive acceleration constants used to scale the contribution of the cognitive and social components, respectively, and $r_1(t), r_2(t) \sim U(0,1)$ are random values introducing a stochastic element to the algorithm (Engelbrecht, 2007).

4.3. The proposed hybrid algorithm

Prior to the application of the hybrid algorithm, we need to design the genetic representation of the candidate solutions. The chromosome of the problem consists of $3m + 1$ genes with the second $m$ genes containing binary values to show whether the first $m$ genes represent $g_i/g_1$ or $g_i/g$:
These $2m$ genes are applied in a repairing strategy to guarantee that ratios of replenishment intervals are integer values. Based on this strategy, any non-integer values of $g/g_i$ or $g_i/g$ would be rounded up/down to the nearest integer values. The third $m$ genes are allocated to backorder fraction, and the last one is $g$.

The positive real number coding technique is used in the algorithm, and the initial population is generated randomly. In the proposed algorithm, the enlarged sampling space is applied, where parents and all offsprings (resulted from crossover, mutation, and PSO) have the same chance for being selected for the next generation through the elitist selection, by which the best individuals of a population move to another population. Besides, arithmetical operators, including affine crossover and dynamic mutation, are used as the genetic operators.

In order to improve the speed of convergence and the quality of solutions, velocity clamping is developed to the basic PSO. In the early applications of the basic PSO, it was found that the velocity quickly explodes to large values, especially for particles far from the neighborhood best and personal best positions. Consequently, particles would have large position updates, which result in leaving the boundaries of the search space. To control the global exploration of particles, velocities are clamped to stay within boundary constraints. If a particle’s velocity exceeds a specified maximum velocity, the particle’s velocity is set to the maximum velocity. Let $V_{\text{max},j}$ denote the maximum allowed velocity in dimension $j$; Particle velocity is then adjusted before the position updates, using Equation (12)

$$v_{kj}(t + 1) = \begin{cases} 
    v'_{kj}(t + 1) & \text{if } v'_{kj}(t + 1) < V_{\text{max},j} \\
    V_{\text{max},j} & \text{if } v'_{kj}(t + 1) \geq V_{\text{max},j} \\
    -V_{\text{max},j} & \text{if } v'_{kj}(t + 1) \leq -V_{\text{max},j}
\end{cases}$$

(12)

where $v'_{kj}$ is calculated using Equation (10).

Usually, $V_{\text{max},j}$ values are selected to be a fraction of the domain of each search space dimension, that is,

$$V_{\text{max},j} = \delta(x_{\text{max},j} - x_{\text{min},j})$$

(13)

where $x_{\text{max},j}$ and $x_{\text{min},j}$ are the maximum and minimum values of the domain of $x$ in dimension $j$, respectively, and $\delta \in (0, 1]$.

The sensitivity of PSO to the value of $\delta$ in Equation (13) can be reduced by constraining velocities with the hyperbolic tangent function, i.e.

$$v_{kj}(t + 1) = V_{\text{max},j} \tanh\left(\frac{v'_{kj}(t + 1)}{V_{\text{max},j}}\right)$$

(14)

Figure 2 is the flow chart showing how the proposed hybrid algorithm works.

The flexibility of PSO in controlling the balance between local and global exploration of the search space assists in overcoming GA's uncertainty in finding the global optimum. Moreover, using stochastic operators in GA helps PSO particles to be dispersed through the whole problem space and search it thoroughly.

Matlab is used in this study, and 500 independent runs have been implemented to test the performance of the algorithms. Population size, probability of mutation, and probability of crossover are equal to 150, 0.02, and 0.8, respectively. For the purpose of
comparing GA and hybrid algorithm, the swarm size that will be used for the hybrid algorithm is as the same as the population size. The other parameters of PSO algorithm used in the hybrid one, including $\delta$, inertia weight, swarm initial velocity, and penalty value are set to 0.09, 0.1, 0, and $10^{10}$, respectively. In order to satisfy the limitation on $g_i$ and $h_i$, penalty method is used to penalize infeasible solutions and transform the constrained problem into an unconstrained one. This approach lets some points outside the feasible region be generated, and thus yields more rapid optimization and better final results, especially in highly restricted problems (Gen & Cheng, 1997).

Here, a penalty function would be deducted from the objective function as follows, whenever infeasible solutions are generated.

$$F(x) = f(x) - p(x), \quad \begin{cases} p(x) = 0 & \text{if } x \text{ is feasible} \\ p(x) > 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (15)

where $x$ is a solution to the problem and $p(x)$ is the variable penalty term consisting of a constant penalty amount and a variable penalty ratio. The constant penalty value is set to $10^{10}$ in this paper and the variable penalty ratio is adjusted based on the iteration number of the proposed hybrid algorithm. This is referred to a dynamic penalty approach, where the penalty pressure increases as the evolutionary process grows.

The algorithm continues till the termination criterion is satisfied, it is when the number of iterations reaches 100.
5. Sensitivity analysis

Here, we first study the sensitivity of the parameters and then compare the results of the two types of delay in payment and the performance of the hybrid algorithm with GA. Considering the results of the hybrid algorithm, we evaluate the effects of the parameters on the model by ANOVA with 0.05 as the significance level. For each parameter, an interval is given which is then divided into five subintervals. The beginning value of each subinterval is chosen as the representative of the interval. The parameters’ intervals of the example are based on given parameters by (Yu, Chu, & Chen, 2009) as: \( D \in [500, 1000], A_{b1} \in [50, 150], A_{b2} \in [2, 10], \varphi_1 \in [5, 15], \varphi_2 \in [5, 15], v_\omega \in [60, 80], h_w \in [1, 3], A_w \in [15, 50], B_1 \in [300, 700], r \in [0.05, 0.15], p_1 \in [600, 1000], p_w \in [200, 400] \), and the number of retailers is assumed to be two. Then, 500 random samples of all parameters are produced based on the given values with an equal probability being assigned to each. Hybrid algorithm and GA are executed based on these samples.

Table 2 shows \( p \)-values obtained from ANOVA test. In this table, values less than 0.05 indicate parameters, which are significantly effective on the output variables, i.e. \( D_1, p_1, \) and \( p_w \) considerably affect both net profits.

As two examples, \( D_1 \) and \( c_p \) are selected from Table 2 and the effects of \( D_1 \) on the net profit of real use CI and the influence of \( p_w \) on order to order consignment net profit are shown in Figure 3. The first figure (a) shows that \( D_1 \) has a direct effect on \( NP_{\text{Real use}} \) and demand increase would result in net profit growth. Moreover, according to (b), any increase in \( p_w \) from 200 to 350 leads \( NP_{\text{Order to order}} \) to decrease, but after that, with the increase of \( p_w \), the net profit increases gently.

Considering the same significance level (0.05), a left-tailed paired \( t \)-test is conducted and zero \( p \)-values showed that the hybrid algorithm performs better than GA. However,

| Par./NP | \( D_1 \) | \( D_2 \) | \( A_{b1} \) | \( A_{b2} \) | \( h_{11} \) | \( h_{21} \) | \( \varphi_1 \) | \( \varphi_2 \) | \( B_1 \) | \( B_2 \) | \( r \) | \( p_1 \) | \( p_2 \) | \( v_\omega \) | \( h_w \) | \( A_w \) | \( p_w \) |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( NP_{T1} \) | 0 | 0 | 0.85 | 0.23 | 0.9 | 0.98 | 0.8 | 0.28 | 0.41 | 0.5 | 0.76 | 0 | 0 | 0.75 | 0.5 | 0.42 | 0.04 |
| \( NP_{T21} \) | 0 | 0 | 0.7 | 0.22 | 0.76 | 0.24 | 0.84 | 0.36 | 0.36 | 0.61 | 0.23 | 0 | 0 | 0.65 | 0.39 | 0.49 | 0 |
less time is consumed in GA. On an average, GA spends 2.23 s per run, while the hybrid algorithm takes 36.30 s for each run.

Moreover, the similar t-test is done to assess the differences between real use consignment and order to order consignment. The calculated p-value is zero and small enough to reject the null hypothesis and deduce that real use policy is more profitable than order to order one.

It is, however, necessary to mention that although the real use consignment shows more profitability, its results for the proportion of replenishment times of retailers and warehouse are virtually unrealistic and a great, impractical difference exists between $g$ and $g_t$. Hence, the results of order to order consignment can be more acceptable and sensible.

6. Conclusion

Considering two types of delay in payment, a centralized CI system is studied in this paper. Generally, in CI, the vendor places some inventory in the buyer’s warehouse and allows him to sell or consume directly from his stock and purchase it with delay. The vendor here is a warehouse that stores one type of product and supplies it at the same wholesale price to multiple retailers, assumed as buyers, who then sell it independently.

Our main aim is to design and compare two types of CI system (real use and order to order consignment) which generates maximum net profit for the supply chain, when the time value of money is considered. A hybrid algorithm of PSO and GA is developed to calculate the optimum values, and its performance is compared with the GA. Moreover, a sensitivity analysis test is performed to examine the effects of parameters on the supply chain net profit.

The model discussed here can be further improved by considering a three echelon supply chain or a multi-product system. Moreover, the shortage can be determined as lost sale or partially backlogged and lead time can be included in the model. There is also one more extension of the present model that deserves further investigation. We can also consider price- and time-dependent demand and compare the results with this paper.

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References

Abdul-Jalbar, B., Segerstedt, A., Sicilia, J., & Nilsson, A. (2010). A new heuristic to solve the one-warehouse N-retailer problem. Computers & Operations Research, 37, 265–272. doi:10.1016/j.cor.2009.04.012

Adida, E., & Ratisoontorn, N. (2011). Consignment contracts with retail competition. European Journal of Operational Research, 215, 136–148. doi:10.1016/j.ejor.2011.05.059

Chang, C.-T., Teng, J.-T., & Goyal, S. K. (2008). Inventory lot-size models under trade credits: A review. Asia-Pacific Journal of Operational Research, 25, 89–112. doi:10.1142/S0217595908001651

Chung, K. H. (1989). Inventory control and trade credit revisited. Journal of the Operational Research Society, 40, 495–498.

Chung, K. J., & Huang, T. S. (2007). The optimal retailer’s ordering policies for deteriorating items with limited storage capacity under trade credit financing. International Journal of Production Economics, 106, 127–145. doi:10.1016/j.ijpe.2006.05.008
Chung, K. J., & Huang, C. K. (2009). An ordering policy with allowable shortage and permissible delay in payments. *Applied Mathematical Modelling, 33*, 2518–2525. doi:10.1016/j.apm.2008.07.016

Chung, K. J., & Liao, J. J. (2006). The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity. *International Journal of Production Economics, 100*, 116–130. doi:10.1016/j.ijpe.2004.10.011

Chung, K. J., & Liao, J.-J. (2009). The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. *European Journal of Operational Research, 196*, 563–568. doi:10.1016/j.ejor.2008.04.018

Chung, K. J., & Lin, S. D. (2011). The inventory model for trade credit in economic ordering policies of deteriorating items in a supply chain system. *Applied Mathematical Modelling, 35*, 3111–3115. doi:10.1016/j.apm.2010.12.001

Copacino, W. C. (1993). How to get with the program. *Traffic Management, 32*, 23–24.

Darwish, M. A., & Odah, O. M. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research, 204*, 473–484. doi:10.1016/j.ejor.2009.11.023

Dave, U. (1985). Letters and viewpoints on “Economic Order Quantity under Conditions of Permissible Delay in Payments” by goyal. *Journal of the Operational Research Society, 36*, 1069.

Engelbrecht, A. P. (2007). Particle swarm optimization. In *Computational intelligence: An introduction* (pp. 289–358). West Sussex: John Wiley & Sons.

Gen, M., & Cheng, R. (1997). *Genetic algorithms and engineering design*. New York, NY: John Wiley.

Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society, 36*, 335–338.

Guchhait, P., Maiti, M. K., & Maiti, M. (2013). Two storage inventory model of a deteriorating item with variable demand under partial credit period. *Applied Soft Computing, 13*, 428–448. doi:10.1016/j.asoc.2012.07.028

Gümüş, M., Jewkes, E. M., & Bookbinder, J. H. (2008). Impact of consignment inventory and vendor-managed inventory for a two-party supply chain. *International Journal of Production Economics, 113*, 502–517. doi:10.1016/j.ijpe.2007.10.019

Huang, Y.-F. (2003). Optimal retailer’s ordering policies in the EOQ model under trade credit financing. *Journal of the Operational Research Society, 54*, 1011–1015. doi:10.1057/palgrave.jors.2601588

Huang, Y.-F. (2006). An inventory model under two levels of trade credit and limited storage space derived without derivatives. *Applied Mathematical Modelling, 30*, 418–436. doi:10.1016/j.apm.2005.05.009

Jaggi, C. K., & Aggarwal, S. P. (1994). Credit financing in economic ordering policies of deteriorating items. *International Journal of Production Economics, 34*, 151–155.

Juang, C.-F. (2004). A hybrid of genetic algorithm and particle swarm optimization for recurrent network design. *EEE Transactions on systems, Man, and Cybernetics. Part B, Cybernetics: A Publication of the IEEE Systems, Man, and Cybernetics Society, 34*, 997–1006.

Juang, C.-F., & Liou, Y.-C. (2004). TSK-type recurrent fuzzy network design by the hybrid of genetic algorithm and particle swarm optimization. *IEEE International Conference on Systems, Man and Cybernetics (IEEE Cat. No.04CH37583)*, 3, 2314–2318. doi:10.1109/ICSMC.2004.1400674

Kim, B., & Park, C. (2010). Coordinating decisions by supply chain partners in a vendor-managed inventory relationship. *Journal of Manufacturing Systems, 29*, 71–80. doi:10.1016/j.jmsy.2010.09.002

Kreng, V. B., & Tan, S.-J. (2010). The optimal replenishment decisions under two levels of trade credit policy depending on the order quantity. *Expert Systems with Applications, 37*, 5514–5522. doi:10.1016/j.eswa.2009.12.014

Molamohamadi, Z., Rezaeiahari, M., & Ismail, N. (2013). Consignment inventory: A literature review and critique. *Journal of Basic and Applied Scientific Research, 3*, 707–714.

Piasecki, D. (2004). Consignment inventory: What is it and when does it make sense to use it? White Paper. *Inventory Operations Consulting LLC*. 
Ru, J., & Wang, Y. (2010). Consignment contracting: Who should control inventory in the supply chain? European Journal of Operational Research, 201, 760–769.
Saccomano, A. (1997). Risky business. Traffic World, 250, 48.
Sharifyazdi, M., Jafari, A., Molamohamadi, Z., Rezaeiazhari, M., & Arshizadeh, R. (2009). Particle swarm optimization approach in a consignment inventory system. In Numerical Analysis and Applied Mathematics: International Conference on Numerical Analysis and Applied Mathematics, pp. 240–243.
Su, C.-H. (2012). Optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. International Journal of Production Economics, 139, 247–256.
Teng, J. (2002). On the economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society, 53, 915–918.
Teng, J., Min, J., & Pan, Q. (2012). Economic order quantity model with trade credit financing for non-decreasing demand. Omega, 40, 328–335.
Thangam, A. (2012). Optimal price discounting and lot-sizing policies for perishable items in a supply chain under advance payment scheme and two-echelon trade credits. International Journal of Production Economics, 139, 459–472.
Tsao, Y.-C. (2011). Replenishment policies considering trade credit and logistics risk. Scientia Iranica, 18, 753–758.
Yu, Y., Chu, F., & Chen, H. (2009). A Stackelberg game and its improvement in a VMI system with a manufacturing vendor. European Journal of Operational Research, 192, 929–948. doi:10.1016/j.ejor.2007.10.016