Necessary conditions for minimizing the impact of impulse noise on broadband systems for the transmission of discrete messages in infocommunicational systems of river water transport

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Abstract. The article is devoted to the issue of minimizing the impact of impulse noise on broadband transmission systems of discrete messages when operating AIS systems on inland waterways of the Russian Federation. Differences in the use of broadband and narrowband information transmission systems have been explored. Also, the stronger influence of impulse noise on broadband radio communication systems is mathematically proven.

1. Introduction
Application of Automatic Information Systems (AIS) for inland transport traffic control allows for efficient monitoring and automatic control of traffic flows. Data transfer in such systems shall conform to modern security and reliability requirements. Due to that, studying accuracy of reception under influence of various types of noise is one of the most important task.

AIS data transfer channels are influenced by not only noises, Delinger fading and narrow-band noises, but by impulse noises of natural and industrial origin as well. Presence of these noises has a significant influence over both noise immunity of AIS radio links, and their functional stability. Timeliness of the study the influence of the latter ones on the discrete message transfer systems due to introduction of various wideband data transmission systems into the channels that use signals of complex noise-like structure with the base, where \( F_r \) is the tentative frequency band taken by the \( r \)th implementation of the signal. Comparative evaluations of impulse noise influence onto such systems and widely employed traditional narrow-band systems with phase-subtracting modulation, ChT and AM were carried out previously [1-3]. In [1, 2] spectral representations were used to demonstrate that impulse noise has the same influence over systems of both classes when the latter are taken as optimal in assumption that only noise extends influence, but operate under conditions of simultaneous action of fluctuation noise and impulse noise. Thus, in such systems, unlike in the case of narrowband noise, extending the base of useful signal does not bring lower influence of the impulse noise. In [3], complex account of statistical and time-frequency parameters of impulse noise led to a more rigorous statement that influence of such noise onto wide- and narrowband systems is the same only for wideband signals with uniformly minimal peak-factor. In all the other cases, impulse noise has more impact onto the wideband systems than onto the narrowband ones.
Rigorous proof of this statement is given below, together with its organic link to a well-known Mandelshtam problem [4-5]. This link is widely used in practice by developers and researchers of discrete message transfer systems. A first-order requirement of uniform peak-factor is imposed onto such systems as a protective measure against impulse noise. To the knowledge of the authors, justification for such a requirement is based upon intuition and experience, while a rigorous proof of its necessity is not found in literature. On the other hand, a number of specialists hold an opinion that applying a time-frequency structure one may obtain additional attenuation of impulse noise in comparison with narrowband signals. Thus, the aim of this paper is also to show invalidity of such assumptions.

AIS data transfer channels are influenced by not only noises, Delinger fading, narrow-band noises, but also by impulse noises of natural and industrial origin. Presence of these noises has a significant influence over both noise immunity of AIS radio links, and their functional stability. Timeliness of studying the influence of the latter ones on the discrete message transfer systems due to introduction of various wideband data transmission systems into the channels that use signals of complex noise-like structure with the base, where \( F_r T < 1 \), where \( F_r \) is the tentative frequency band taken by the \( r \)th implementation of the signal. Comparative evaluations of impulse noise influence onto such systems and widely employed traditional narrow-band systems \((1 \leq F_r T < 10)\) with phase-subtracting modulation, ChT and AM were carried out previously [1-3]. In [1, 2] spectral representations were used to demonstrate that impulse noise has the same influence over systems of both classes when the latter are taken as optimal in assumption that only noise extends influence, but operate under conditions of simultaneous action of fluctuation noise and impulse noise. Thus, in such systems, unlike in the case of narrowband noise, extending the base of useful signal does not bring lower influence of the impulse noise. In [3], complex account of statistical and time-frequency parameters of impulse noise led to a more rigorous statement that influence of such noise onto wide- and narrowband systems is the same only for wideband signals with uniformly minimal peak-factor. In all the other cases, impulse noise has more impact onto the wideband systems than onto the narrowband ones.

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2. Variants of constructing decision circuits

For studies, let us select binary \((r = 1; 2)\) systems of coherent and incoherent reception, which are optimal in channels with fluctuation noise only; such systems use opposite signals in coherent reception and orthogonal (under enhanced definition) signals in incoherent reception, both with an active pause. The variants of constructing receptive decision circuits in such multiplier- and matched filter-based systems are shown in Fig. 1 (coherent reception) and Fig. 2 (incoherent reception) [1], where \( \hat{z}(t) \) is the received sum of desired signal and noise;

\[ z_r(t) \] is a Hilbert-associated function with \( \hat{z}_r(t) \). Let

\[ \tau_0 \] much shorter than that of the useful element of the desired signal \( T \). Rigorous limits for the duration of the noise under consideration are determined by condition (11).
\[ z'(t) = \mu r(t, \psi) + \mu_{\Pi} z_{\Pi}(t) + \xi(t), 0 \leq t < T, \quad (1) \]

Where

\[ z_{\Pi}(t) = \begin{cases} 
\xi(t-t_0; \tau_0), & t_0 \leq t \leq t_0 + \tau_0 \\
0, & 0 \leq t \leq t_0, t_0 + \tau_0 < t < T, \end{cases} \quad (2) \]

is a function defining the structure of a single impulse noise on \([0, T]; \xi(t)\) and \(\mu_{\Pi}\) are, respectively, time of arrival and amplitude ratio of the impulse noise transmission determined by their distribution laws: \(z_{\xi}(t)\) is a finite function, integrated with a square;

\(\mu\) and \(\psi\) are, respectively, amplitude ratio of transmission and initial phase of high-frequency signal interpolation, determined by their distribution laws; \(\xi(t)\) is a fluctuation noise approximated with Gaussian white noise with a spectral density of \(V^2\).

**Figure 1.** Variants of constructing receptive decision circuits on the basis of multipliers and matched filters (coherent reception).
Figure 2. Variants of constructing receptive decision circuits on the basis of multipliers and matched filters (incoherent reception).

3. Assessing noise resistance

Noise resistance of the systems under consideration in case of received signal type (1), taking in account distribution of parameters $\mu$, $\Psi$, and $\mu\Gamma$, may, in a number of cases, be calculated following the method given in [6, 7], as it was previously done in [3], where the reader may find necessary details of such research; among them, two starting conditions are of importance for this work.
First, functional form of the element receipt error probability depends on the laws of distribution \( \mu \), \( \psi \), \( \mu_\Pi \), \( t_0 \); however, in all these cases there are certain arguments in these formulas: \( \bar{h}^2 \) is an average ratio of received desired signal energy to \( V^2 \) and

\[
\frac{g_r}{\mu_\Pi} = \frac{\sqrt{\mu^2 - \mu_\Pi^2}}{T} \int_0^T z_r(t) z_\Pi(t) dt + \frac{2}{T} \int_0^T \left( \frac{\mu^2 - \mu_\Pi^2}{T} z_r(t) z_\Pi(t) dt \right),
\]

is the average value of mutual difference coefficients in the time-frequency domain \( z_r(t) \) and \( z_\Pi(t) \), where \( \mu_\Pi \) and \( \mu_\Pi^2 \) are average values of \( \mu^2 \) and \( \mu_\Pi^2 \), \( z_\Pi(t) = \xi_\Pi(t - t_0; \tau_0) \); \( t_0 \leq t \leq t_0 + \tau_0 \); \( t_0 \in [0, T] \) is the average value of the moment of arrival of the impulse noise, \( r=1.2 \).

Second, the stated probabilities of error uniformly depend on changes in the values of coefficients \( \frac{g_r}{\mu_\Pi} \), getting better with lower values of the latter.

As we consider element-by-element reception, it is quite proper to substitute the impulse noise in (3) with a periodic function \( z_\Pi(t) = \xi_\Pi(t - t_0; \tau_0) = \xi_\Pi(t - t_0 + T; \tau_0) \), which, in its turn allows for representation with a trigonometric Fourier series over the element \( T \):

\[
z_\Pi(t) = \sum_{k=1}^{\infty} A_{k\Pi} \cos(k \omega_T t + \phi_{k\Pi}), 0 \leq t \leq T,
\]

Where

\[
A_{k\Pi} = \frac{2}{T} \int_0^T z_\Pi(t) \cos(k \omega_T t) dt + \frac{2}{T} \int_0^T z_\Pi(t) \sin(k \omega_T t) dt,
\]

\[
\phi_{k\Pi} = -\arctg \frac{b_{k\Pi}}{a_{k\Pi}}
\]

Then, using representation

\[
z_r(t) = \sum_{k=1}^{k_2} A_{k\Pi} \cos(k \omega_T t + \phi_{k\Pi}), 0 \leq t \leq T,
\]

Where \( k_{r2} - k_{r1} + 1 = F_r T \), we will get that, the mutual difference coefficients in (3) have the following form:

For opposite signals:
\[
\frac{g^2_1}{\bar{g}_1^2} = \frac{g^2_2}{\bar{g}_2^2} = \frac{\mu^2}{\mu^2} \left( \sum_{k=k_0}^{k_1} A_{k\pi} \cos(\varphi_{k\pi} - \varphi_{k\pi}) \right)^2 + \left( \sum_{k=k_0}^{k_1} A_{k\pi} \sin(\varphi_{k\pi} - \varphi_{k\pi}) \right)^2,
\]

(7)

For signals orthogonal under enhanced definition

\[
\frac{g^2_r}{\bar{g}_r^2} = \frac{\mu^2}{\mu^2} \left( \sum_{k=k_{r1}}^{k_{r2}} A_{k\pi} A_{k\pi} \cos(\varphi_{k\pi} - \varphi_{k\pi}) \right)^2 + \left( \sum_{k=k_{r1}}^{k_{r2}} A_{k\pi} A_{k\pi} \sin(\varphi_{k\pi} - \varphi_{k\pi}) \right)^2,
\]

(8)

In particular, from (7) and (8) it follows that the probability of error in the systems under consideration in the impulse noise conditions is determined only by the components of the noise that affects the band \( F_r \). In other words, the impact of the noise (4) is equivalent to the impact of

\[ z_{\pi k\pi} = \sum_{k=k_{r1}}^{k_{r2}} A_{k\pi} \cos(\omega_{k\pi} t + \varphi_{k\pi}). \]

Such transformation of (4) into (9) may be performed in most of the cases due to presence of selection filters with the bandwidth of at least \( F_r \) at the input of the receiver before its decision circuit.

From (7) and (8) we get, that for narrowband systems with \( F_r T = 1 \) and the coherent reception (for example, phase difference modulation), the mutual difference coefficients are equal to

\[
\frac{g^2_r}{\bar{g}_r^2} = \frac{\mu^2}{\mu^2} \frac{A_{r2}^2}{A_{r2}} = \frac{P_{r2}}{P_{r2}} = \frac{\overline{h}_{r2}^2}{h_{r2}^2},
\]

(9)

\[ \overline{h}_{r2}^2 = \frac{P_{r2} T}{\nu^2}; h_{r2}^2 = \frac{P_{r2} T}{\nu^2}; P_{r2} \] is the average value of the harmonic component of the impulse noise in the band \( \Delta \omega = 2\pi / T \) around the frequency \( k\omega_0 (k_{r1} \leq k \leq k_{r2}) \);

\[ P_{r2} \] is the average power of the received signal.

To study (7) and (8) in case of wideband systems, let us make a simplifying assumption: duration of the impulse noise is short enough, so that the following condition holds

\[ \tau_0 \leq 2\pi / \omega_0, k_2 = \max \{ k_{r2} \}. \]

(10)

Assumption (11), first, conforms with the fact that duration of actual impulse noises is usually \( \tau_0 \approx 10^{-5} \tau \) \( \tau \) or less frequently \( 10^{-8} \tau \), and second, (10) does not contradict (9). Then, we may hold that for \( A_{k\pi} \) and \( \varphi_{k\pi} \) from (5) we may get
\[
A_{k\Omega} = A_{\Omega} = \frac{2}{T} \int_0^T z_{\Omega}(t)dt; \quad \varphi_{k\Omega} = k\omega_0\overline{t_0}, 
\]

(11)

Let us note, that for an infinitely-small impulse \(z_{\Omega}(t) = A_{\Omega}(t - \overline{t_0})\), the equations (11) are exact. Taking (11) into account, ratios (7) and (8) take the following form:

\[
\overline{g_1}^2 = \overline{g_2}^2 = \frac{\mu_1^2}{\mu^2} \left[ \sum_{k=k_1}^{k_2} A_{k\Omega} A_{\Omega} \cos(k\omega_0\overline{t_0} + \varphi_{k\Omega}) \right]^2 + \left[ \sum_{k=k_1}^{k_2} A_{k\Omega} A_{\Omega} \sin(k\omega_0\overline{t_0} + \varphi_{k\Omega}) \right]^2
\]

(12)

Analysis of (12) leads us to the following conclusions. If on reception we know the moment of arrival of the impulse noise \(t_0\), then using wideband signals we may always ensure \(\overline{g_1}^2 = 0\), that is, maximum difference of the signals (orthogonality according to enhanced definition) with respect to single impeding impulses, which is impossible in narrowband systems. Actually, to that end it is necessary to satisfy the conditions

\[
\sum_{k=k_1}^{k_2} A_{k\Omega} \cos(k\omega_0\overline{t_0} + \varphi_{k\Omega}) = 0; \quad \sum_{k=k_1}^{k_2} A_{k\Omega} \sin(k\omega_0\overline{t_0} + \varphi_{k\Omega}) = 0. 
\]

(13)

For wideband signals with approximately uniform spectra, when

\[
A_{k\theta}^2 = \frac{2P_c}{\mu^2} F\tau \theta T = \text{const}(k), 
\]

(14)

ratios (13) are equivalent to conditions

\[
\sum_{k=k_1}^{k_2} \cos(k\omega_0\overline{t_0} + \varphi_{k\theta}) = 0; \quad \sum_{k=k_1}^{k_2} \sin(k\omega_0\overline{t_0} + \varphi_{k\theta}) = 0. 
\]

(15)

Such requirements are the easiest, if the implementation of noise-like signals have the value of \(\varphi_{k\theta}\) equal to 0 and \(\pi\) following a random law, or using solutions from [4, 5, 8] or otherwise, while at the same time we provide phase shift of the k-th component of the carrier signal that is formed in diagrams in Figure 1 by an angle \(\Delta \varphi_{k\theta} = -k\omega_0\overline{t_0}\). When there is no impulse noise, the shifts shall be removed.

More often than not, the moment of arrival of the impulse noise \(\overline{t_0}\) is not known at the reception. At that, \(\overline{t_0}\) may be anywhere within the range from 0 to T and it is common sense that none of the
values of $t_0$ has advantages over others. Then, the problem is reduced to finding such forms of signal for given $A_{Kr}$ and $k\omega_b$ by varying phases of $\phi_{Kr}$ that minimize the maximum possible values of $g_r^2$ for any $t_0 \in [0,T]$, that is, $\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} g_r^2$, is the target function of the impulse noise impact mitigation function, which according to (12) is

$$\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} \left\{ \left( \sum_{k=k_1}^{k_2} A_{Kr} \cos(k\omega_b t_0 + \phi_{Kr}) \right)^2 + \left( \sum_{k=k_1}^{k_2} A_{Kr} \sin(k\omega_b t_0 + \phi_{Kr}) \right)^2 \right\},$$

(16)

It is evident, that (16) is equivalent to the target function

$$\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} |\Delta|^2,$$

$$\Delta = \sum_{k=k_1}^{k_2} A_{Kr} \exp \left[ i(k\omega_b t_0 + \phi_{Kr}) \right],$$

Where $\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} |\Delta|^2$, and thus, the target function

$$\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} |\Delta|^2,$$

(17)

The Mandelshtam problem [4,5] with respect to radio signals is formulated in the following way:

$$\min_{\{\phi_{Kr}\} \in [0,2\pi]} \max_{\omega_b \in [0,T]} \left| \sum_{k=k_1}^{k_2} A_{Kr} \cos(k\omega_b t_0 + \phi_{Kr}) \right|, r = 1, 2.$$

(18)

Comparing (17) and (18), one may easily see that problem (18) is a modified form of the Mandelshtam problem applied to signal envelope. As for such signals

$$(k_r - k_{r1} + 1) \omega_b \cup \frac{1}{2} \left( k_r + k_{r1} \right) \omega_b$$

is always true, solution of (18) is at the same time a solution of (18) [5]. Of course, formal substitution of $t$ with $t_0$ has no crucial significance. Thus, set $\{\phi_{Kr}\}$, found from (18) for minimization of complex signal peak factor and determining the shape of such signal ensures minimization of $g_r^2$ coefficients as well. Subsequently, signals with a uniform minimal peak-factor on $t_0 \in [0,T]$ ensure also minimal possible impact of impulse noise compared to any other form of wideband signals.

4. Evaluating the impact of impulse noise

Let us give upper and lower estimates of the impact of impulse noise on wideband signals (6). The upper bound estimate may be based on partial solutions that have important practical applications, when minimization of peak-factor in (18) is achieved by a proper selection of signs of the components, corresponding to $\{\phi_{Kr}\}$, selected from a set of values equal to 0 or $\pi$ [4, 5, 8]. For instance, let us estimate the value $\min g_r^2$ for one of possible solutions (18), when $\phi_{Kr}$ are selected from $\{\phi_{Kr}\}$ in accordance with the law from [4],

$$\phi_{Kr} = \left[ \frac{k}{p} + 1 \right] \frac{\pi}{2},$$
\( \left( \frac{k}{p} \right) \) is the Legendre symbol [9], \( p = F_T + 1 \) is a prime number. In this case, when conditions (14) hold, we have

\[
\min_{\{\varphi_k\} \in [0,2\pi]} \max_{t \in [0,T]} |\Delta|^2 = \frac{P_c}{\mu^2 F_T} \sum_{k=k_{i1}}^{k_{i2}} \exp \left[ i \left[ k \omega_k t_0 + \frac{\pi}{2} \left( \frac{k}{p} + 1 \right) \right] \right]^2 = \frac{P_c}{\mu^2 F_T} p c, F_T \geq 4
\]

Then, for \( F_T \geq 1 \) we have

\[
\min_{\{\varphi_k\} \in [0,2\pi]} \max_{t \in [0,T]} g_r^2 = \frac{\mu_1^2 A_{h1}^2}{2} p c = \frac{h_{r1}^2}{h^2} \frac{(F_T + 1) c}{2}.
\]

For \( F_T \geq 1 \) we have

\[
\min_{\{\varphi_k\} \in [0,2\pi]} \max_{t \in [0,T]} g_r^2 \approx \frac{h_{r1}^2}{h^2} \frac{c}{2},
\]

which is the same as (9) or a little larger. Or course, we may hope to get a better estimation of (20) if we use more accurate solutions of the Mandelshtam problem, when \( \{\varphi_k\} \) is selected out of random values in \([0,2\pi]\). However, application of lower bound estimate [5-8] for this case

\[
\min_{\{\varphi_k\} \in [0,2\pi]} \max_{t \in [0,T]} |\Delta| \geq \left( \sum_{k=k_{i1}}^{k_{i2}} A_{h1} \right)^{\frac{1}{2}}
\]

for conditions (14) gives us

\[
\min_{\{\varphi_k\} \in [0,2\pi]} \max_{t \in [0,T]} g_r^2 \geq \frac{h_{r1}^2}{h^2},
\]

which for the best result is the same as (9) again.

5. Conclusion

Thus, when constructing wideband systems one shall not hope that it is possible to obtain some additional noise reduction due to time-frequency structure of the signals (6) compared to narrowband systems. Moreover, provision of uniformly minimal peak factor of the wideband signals, according to solution of the problem (18) is a necessary condition for non-increased impact of such noise. Sufficient conditions for efficient noise attenuation are in the application domain of special, previously known protection measures (limitation, compensation, etc.), as well as in search for optimal operational algorithms of coherent and incoherent reception systems under condition of simultaneous impact of a whole complex of noises (fluctuation and impulse ones, or fluctuation, impulse and narrowband ones). The latter, however, shall be subject for a separate discussion and goes beyond the scope of this research.

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