A resonant pair of warm giant planets revealed by TESS

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ABSTRACT
We present the discovery of a pair of transiting giant planets using four sectors of TESS photometry. TOI-216 is a 0.87M⊙ dwarf orbited by two transiters with radii of 8.2R⊕ and 11.3R⊕, and periods of 17.01d and 34.57d, respectively. Anti-correlated TTVs are clearly evident indicating that the transiters orbit the same star and interact via a near 2:1 mean motion resonance. By fitting the TTVs with a dynamical model, we infer masses of 26+24−11M⊕ and 190+220−80M⊕, establishing that the objects are planetary in nature and have sub-Kronian and Kronian densities. TOI-216 lies close to the southern ecliptic pole and thus will be observed by TESS throughout the first year, providing an opportunity for continuous dynamical monitoring and considerable refinement of the dynamical masses presented here. TOI-216 closely resembles Kepler-9 in architecture, and we hypothesize that in such systems these Saturn-analogs failed to fully open a gap and thus migrated far deeper into the system before becoming trapped into resonance, which would imply that future detections of new analogs may also have sub-Jupiter masses.

Key words: planets and satellites: detection — stars: individual (TIC 55652896)

1 INTRODUCTION
A small fraction of exoplanets in the cosmos have the correct orbital geometry to transit their star as seen from our home. These transiting planets have been a “royal road to success” in planet discovery (Russell 1948; Winn 2010) yielding thousands of discoveries in recent years (see the NASA Exoplanet Archive; Akeson et al. 2013), despite the fact that they represent just a sliver of the total population. An even more rarefied population is that of transiting planets exhibiting transit timing variations (TTVs; Agol et al. 2005; Holman & Murray 2005; Deck & Agol 2015). For these special worlds, not only can one infer the planetary size from the transit depths, but dynamical modeling of the TTVs can often provide planetary masses too - a fact heavily exploited by Kepler (Holman et al. 2010; Lithwick et al. 2012; Nesvorný et al. 2012). In such cases, it is therefore possible to confirm the planetary nature of a system almost exclusively from photometric observations (e.g. Ford et al. 2011; Steffen et al. 2013).

Kepler enjoyed many successes with this strategy, largely enabled by its patience to stare at the same stars for over four years continuously. With TESS, the full-sky nature of the survey means that most parts of the sky are observed for much shorter windows\(^1\), potentially posing a challenge to dynamical confirmation of planetary candidates. However, TESS does maintain a longer vigil on the ecliptic poles, observing these fields for up to a year continuously (Ricker et al. 2016).

In this work, we describe the discovery of two TESS planets near a 2:1 mean motion resonance (MMR) leading to highly significant TTVs. Thanks to the host star’s fortuitous location near the southern ecliptic pole, TESS can observe the target for most of the first year making TOI-216 an excellent target for monitoring planet–planet interactions. We describe the observations by TESS in Section 2 with attention to detrending, contamination and stellar properties. In Section 3 we regress light curve models and TTV models, demonstrating that the system is a pair of planet-mass objects gravitationally interacting with one another. Finally,

\(^1\) This situation could change if with an extended TESS mission (Bouma et al. 2017)
we discuss the possibilities opened up by this exciting new system in Section 4.

2 OBSERVATIONS

2.1 Identification

TIC 55652896 was observed by TESS in the first four sectors of year one, and indeed is scheduled to be observed in every sector of that year except for sector 10 (2019-Mar-26 to 2019-Apr-22). Falling on camera 4, the target is a relatively rare example of a transiting planetary system caught within the TESS continuous viewing zone (CVZ). With an ecliptic latitude of -82.476408°, we highlight that the target also lands close to JWST’s planned CVZ and would be observable for $\gtrsim 260$ days per year of the mission.

A TESS alert was issued on 2018-11-30 for two candidate transiting planets associated with TIC 55652896 using sectors 1 and 2, dubbed TOI-216.01 and TOI-216.02. With periods of $\sim 17.1$ days (TOI-216.02) and $\sim 34.5$ days (TOI-216.01), the outer candidate was only seen to transit twice during this time (once per sector).

Amongst the 300+ TOIs identified at the time of writing, this pair stood out as particularly interesting because the planetary candidates lie near a 2:1 period commensurability. If the objects were orbiting the same star, and gravitationally interacting, then it may be possible to confirm the planetary nature of the pair without any ground based follow-up (e.g. see Steffen et al. 2013). For this reason, we decided to further study this system.

2.2 Stellar properties

TOI-216 has an apparent magnitude of 11.5 in the TESS bandpass. From the TESS Input Catalog (TIC) version 7 (Stassun et al. 2018), catalog survey spectroscopy of the star constrains $T_{\text{eff}} = (5026 \pm 125)$ K, $[M/H] = 0.32 \pm 0.10$ and $\log(g) = 4.66 \pm 0.20$. These properties are used by TIC to infer $M_\star = (0.879 \pm 0.073)M_\odot$ and $R_\star = (0.715 \pm 0.166)R_\odot$.

We also queried the star within Gaia DR2 (Gaia Collaboration et al. 2018) and find a parallax measurement of $5.591 \pm 0.028$ mas (GAIA DR2 4664811297844004352). Rather than use the TIC-7 summary statistics for stellar mass and radius, we would prefer to work with posterior samples - as well as include the Gaia parallax - and so we elected to perform our own Bayesian isochrone fitting. To this end, we used the isochrones package by T. Morton using the previously listed constraints on $V$, $T_{\text{eff}}$, [M/H], logg and parallax with the Dartmouth stellar evolution models.

The inputs to our fits (the “star.ini” file) are given in the top panel of Table 1, and the derived parameters of interest in the lower panel. As expected, our results closely agree with those listed in the TIC.

2.3 Contamination

With a pixel size of 21 arcseconds, there is a greater chance of crowding with TESS than Kepler. The aperture used in each sector varies slightly but is approximately 4 by 3 pixels and thus sources out to 84 arcseconds can contaminate the aperture.

| parameter          | value          |
|--------------------|----------------|
| $V$                | 12.324 $\pm$ 0.069 |
| $T_{\text{eff}}$ K | 5026 $\pm$ 125   |
| $[M/H]$ [dex]      | 0.32 $\pm$ 0.10  |
| $\log(g)$ [dex]    | 4.66 $\pm$ 0.20  |
| parallax [mas]     | 5.591 $\pm$ 0.028 |
| $M_\star$ [$M_\odot$] | 0.874$^{+0.035}_{-0.034}$ |
| $R_\star$ [$R_\odot$] | 0.838$^{+0.043}_{-0.030}$ |
| $\rho_\star$ [kg m$^{-3}$] | 2090$^{+270}_{-300}$ |
| $d$ [pc]           | 178.84$^{+0.89}_{-0.88}$ |

Fortunately, there are no comparably bright stars that lie within this region. The nearest star listed in the TIC to our target is TIC 55652894, separated by 48.2 arcseconds but far fainter with an apparent TESS magnitude of 16.3 (1.2% the brightness level). Gaia DR2 (Gaia Collaboration et al. 2018) reports 46 stars within 84 arcseconds, with G-band magnitudes from 17.1 to 21.1 (TOI-216 is 12.2). Together, these sources could maximally dilute the target by 5.6% in $G$, although the true value will be less due to color correction to the redder TESS bandpass, location of the sources and the finite PSF widths.

These contaminating source cumulatively lead to a small amount of dilution of TOI-216, which is estimated within the TESS aperture to be 0.33%, 0.46%, 0.29% and 0.26% for sectors 1 to 4 respectively (values taken form the TESS light curve files) and these are included in our later light curve fits using the prescription of Kipping & Tinetti (2010).

We also highlight that an unresolved companion may reveal itself through centroid shifts during the moments of transit, but all centroid shifts for this star are below 1 $\sigma$ as reported by the TESS sector 1-3 cumulative Data Validation (DV) report.

2.4 Light curve detrending

We downloaded sectors 1 to 4 short-cadence (2 minute) data for TIC 55652896 and work with the PDC (Pre-search Data Conditioning) product in what follows (Jenkins et al. 2017). Any bad data flags were removed, and outliers filtered with 5 $\sigma$ clipping against a 20-point moving median. Using the ephemeris and duration for TOI-216.01 and TOI-216.02, we de-weight the in-transit points for the purposes of detrending. The PDC light curve for all four sectors is shown in Figure 1.

We next detrend each transit epoch of each planet independently, using four different algorithms following Teachey & Kipping (2018) - CoFiAM, PoLyAM, local polynomials and a Gaussian Process using a squared exponential kernel. The four light curves are then combined into a single time series - a method marginalized light curve - by taking the median at each time stamp and inflating the formal uncertainties.
by adding the inter-method standard deviation in quadrature. We direct the reader to Teachey & Kipping (2018) for a detailed description of the four algorithms as well as the method marginalization process. The resulting light curves from all four methods, as well as the method marginalized light curves, are made available at this URL.

We find that the inter-method standard deviation is many times smaller than the formal uncertainties, indicating a highly stable detrending. The median formal uncertainty is 2430 ppm but the median inter-method standard deviation is 16 times smaller at 150 ppm. After adding this extra component in quadrature to the formal uncertainties, the error increases by just 0.2%.

3 ANALYSIS

3.1 Light curve model

We initially built light curve models for the system which treated each planetary candidate as orbiting an independent star. It became immediately clear that the transitters displayed strong transit timing variations (TTVs), as can be seen by simple inspection of Figure 2. More detailed analysis of these light curves presented in Section 3.3 reveals strong evidence for anti-correlation - the hallmark of dynamically interacting planets (Steffen et al. 2013). This is only possible if the two transitters are orbiting the same primary, and thus in our final light curve modeling we decided to treat the objects as sharing a common host star.

This is particularly useful for modeling the inner candidate, TOI-216.02, whose light curve displays a V-shaped morphology consistent with a grazing geometry. Treated as an independent body, V-shaped transits display strong degeneracies between size, impact parameter, limb darkening coefficients and host star density (Carter et al. 2008). Since the anti-correlated TTVs imply a common host star, and the other transiter is non-grazing, the conditional relationship greatly aids in the inference of a unique light curve solution for TOI-216.02.

Our light curve model is that of the classic Mandel & Agol (2002) quadratic limb darkening code, which is over-sampled by a factor of 5 to correct for the slight distortion of finite integration time via the prescription of Kipping (2010). The quadratic limb darkening coefficients are re-parameterized to $q_1$ & $q_2$ following Kipping (2010).

Since TTVs are apparent in the light curve (Figure 2), we allow each transit epoch to have a unique time of transit minimum, $\tau$. In what follows, we also assume both transitters have nearly circular orbits ($e \approx 0$). If one (or both) were in fact eccentric, the derived stellar density would be erroneous (see Kipping 2010; Moorhead et al. 2011; Tingley et al. 2011; Dawson & Johnson 2012) by a factor of (marginalizing over argument of periastron, $\omega$):

$$\frac{<\rho'_\star>}{\rho_\star} = \frac{1}{2\pi} \int_{\omega=0}^{2\pi} \left(1 + e \sin \omega \right)^2 \frac{3}{(1-e^2)^{3/2}} d\omega = 1 + \frac{3}{8} e^2 \left(\frac{1}{1-e^2}\right)^{1/2}.$$  

Our later fits (see Section 3.2) reveal that our light curve derived density is measured to a precision of 6% and thus from Equation (1) one may show that $e < 0.14$ should be expected to lead to a less than 1σ systematic error in the inferred density. As an apparently fairly compact, multiple planet system, we consider this assumption is reasonable on the grounds of dynamical stability, and indeed our later TTV fits favor low eccentricities (see Section 3.3). Nevertheless, we choose not to use the light curve derived density in any attempt to refine the isochrone modeling from Section 2.2.

3.2 Light curve fits

Our light curve model has a total of 17 free parameters: two ratio-of-radii ($p_{\text{inner}}$ & $p_{\text{outer}}$), two impact parameters ($b_{\text{inner}}$ & $b_{\text{outer}}$), two limb darkening coefficients ($q_1$ & $q_2$), a mean stellar density ($\rho_\star$), six times of transit minimum for the inner transiter ($\tau_{\text{inner},i}$) and four times of transit minimum for the outer transiter ($\tau_{\text{outer},i}$). We assume uniform priors for all parameters except for $\rho_\star$ for which we adopt a broad log-uniform prior.

Fits were conducted using MULTINEST (Feroz & Hobson 2008; Feroz et al. 2009) with 4000 live points, in non-

![Figure 1. PDC light curve of TIC 55652896 as observed by TESS for sectors 1 to 4. We mark the location of the transits of TOI-216.01 in blue and TOI-216.02 in red. The penultimate transit of TOI-216.02 was found to be unusable due its proximity to a data gap.](image-url)
constant efficiency mode. The maximum a-posteriori light curve solution is plotted in Figure 2. We make the full posterior samples available at this URL but list the credible intervals on the 10 transit times in Table 2 and the other 7 global parameters in Table 3.

Considering the 7 global parameters, there are two noteworthy conclusions that can be drawn from the results. The first is that the impact parameter of the inner planet is un

Table 2. Medians and one-sigma uncertainties for the ten times of transit minimum in our light curve fit of TOI-218.01 and TOI-218.02.

| parameter | epoch | BJΔuUTC - 2,457,000 |
|-----------|-------|---------------------|
| τinner, 1 | 1     | 1325.3277 ± 0.0033  |
| τinner, 2 | 2     | 1342.4307 ± 0.0027  |
| τinner, 3 | 3     | 1359.5399 ± 0.0026  |
| τinner, 4 | 4     | 1376.6317 ± 0.0025  |
| τinner, 5 | 5     | 1393.7236 ± 0.0029  |
| τinner, 6 | 6     | 1410.8266 ± 0.0027  |
| τinner, 7 | 7     | 1427.9282 ± 0.0027  |
| τouter, 1 | 1     | 1331.2851 ± 0.0076  |
| τouter, 2 | 2     | 1365.8243 ± 0.00074 |
| τouter, 3 | 3     | 1400.3686 ± 0.00072 |
| τouter, 4 | 4     | 1434.9224 ± 0.00072 |

Our dynamical model has 14 parameters: mass ratios $M_{\text{inner}}/M_\star$ and $M_{\text{outer}}/M_\star$, orbital periods $P_{\text{inner}}$ and $P_{\text{outer}}$, eccentricities $e_{\text{inner}}$ and $e_{\text{outer}}$, longitudes of periapsis $\omega_{\text{inner}}$ and $\omega_{\text{outer}}$, impact parameters $b_{\text{inner}}$ and $b_{\text{outer}}$, difference in nodal longitudes $\Omega_{\text{outer}} - \Omega_{\text{inner}}$, stellar density $\rho_\star$, and time offsets $\tau_{\text{inner},0}$ and $\tau_{\text{outer},0}$ between a reference epoch and the first observed transit of each planet. All orbital elements are given at the reference epoch 2,458,325 BJDUTC.

We used uniform priors for all parameters except for $b_{\text{inner}}$, $b_{\text{outer}}$, and $\rho_\star$, since our earlier light curve fits provide strong constraints which can be leveraged here. Since MultiNest requires simple parametric forms of the priors for the purposes of inverse transform sampling, we approximated the marginal posteriors from our earlier fits such that $b_{\text{outer}}$ is uniform between 0 and 0.4, $b_{\text{inner}}$ is a Gaussian centered on 0.95 with 0.025 standard deviation and $\rho_\star$ is a Weibull prior with shape parameters 22.7 and 2425.1.

The fits converged to a unique solution and the joint posteriors are depicted in Figure 4 for reference. The maximum a-posteriori solution is plotted in Figure 3, which shows how the model is able to fully describe the observed deviations.

Combining the derived mass ratios with the stellar mass derived earlier (see Section 2.2) allows us to measure that $M_{\text{inner}} = 26^{+51}_{-14} M_J$ and $M_{\text{outer}} = 190^{+380}_{-80} M_J$, which establishes that the masses are far below the deuterium burning limit and these objects may be classified as “planets”.

3.4 Final parameters

To complete our analysis, we combine the fundamental stellar parameters derived earlier (see Section 2.2) with the relative radii (from Section 3.2) and relative masses (from Section 3.3) to calculate physical properties for both planets. Our final planet properties are listed in Table 3.

4 DISCUSSION

We have demonstrated that the TESS planetary candidates TOI-216.01 & 02 must orbit the same primary star given their anti-correlated TTVs. The light curve derived stellar density found by fitting both signals yields a value almost precisely equal to the target star’s density from an isochrone analysis, establishing that the objects indeed orbit the target rather than a contaminant. Finally, we have regressed an N-body dynamical model to the observed TTVs to demonstrate that the masses of each body are far below the deuterium burning limit making these bona fide “planets”.

The TOI-216 planetary system displays some close similarities to the Kepler-9 system (Holman et al. 2010), but is 1.6 magnitudes brighter in V. In both cases, one finds low-density gas giants in a 2:1 mean motion resonance orbiting a Sun-like star at similar periods (~20 d and 40 d). To a lesser degree, the system also resembles KOI-872 (Nesvorný et al. 2012). In both of these cases, the MMR pair of planets are
accompanied by a short-period super-Earth and thus it is natural to wonder if perhaps TOI-216 may also be accompanied by a small and currently unresolved terrestrial planet. We ran a box-least squares search (Kovács et al. 2002) for such a signal but find no significant peaks with the available TESS data.

The TTVs of TOI-216 are characterized by a super-period as the longitude of conjunctions circulates with a timescale of \( \sigma \) years. Although TOI-216 will be mon-

**Figure 2.** Left: The six available transits of TOI-216.02 observed by TESS in sectors 1 to 6 phase folded on the best-fitting linear ephemeris. We bin the data to 20-minute samples and overlay the maximum a-posteriori light curve model. Right: Same, but for TOI-216.01, where only four transit are available. In both cases, TTVs are clearly evident.

**Figure 3.** Observed minus calculated (O-C) transit times of TOI-216.01 (blue) and TOI-216.02 (red). The TTVs are clearly anti-correlated indicating that the transiers orbit the same primary. We overlay the maximum a-posteriori dynamical model with solid lines, described in Section 3.3.
Figure 4. Corner plot of the joint posteriors from our dynamical fits to the TTVs for the TOI-216 system. Since the inferred masses are planetary, we dub the planets “b” and “c” here and in what follows. We omit the terms with informative priors from the earlier light curve fits.

Monitored throughout the first year of TESS observations, the super-period looks likely to exceed this baseline and thus continuous monitoring from the ground in 2020 would greatly benefit the determination of precise orbital elements. With the limited phase coverage available at the time of writing, the masses quoted in this work will surely be refined considerably in the future.

The resonance between the gas giants is consistent with dissipative processes in disk-planet interaction during their presumably inward migration from beyond the snow line (Crida et al. 2008; Havel et al. 2011; Cimerman et al. 2018). In the Grand Tack hypothesis of the Solar System (Hansen 2009; Walsh et al. 2011), Jupiter is thought to have opened up a gap, migrating slower (type II) than Saturn (type I), which likely failed to fully open a gap. This enabled Saturn to catch up to Jupiter, trapping the pair in resonance when Jupiter was at $\sim 1.5$ AU, which reversed subsequent migration. In the case of both Kepler-9 and TOI-216, the gas giants have Kronian masses, and thus may have failed to have opened up full gaps, causing them to type I migrate far deeper. As a larger sample of such systems is found in the future, it will be interesting to test if these giants tend to
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Table 3. Medians and one-sigma uncertainties for the system parameters of the planets TOI-216b \& c.

| parameter  | TOI-216b | TOI-216c |
|------------|----------|----------|
| $P$ [days] | 17.085\(^{+0.013}_{-0.013}\) | 17.089\(^{+0.012}_{-0.012}\) |
| $\delta$ [BJD$_{TDB}$ - 2,457,000] | 0.959\(^{+0.047}_{-0.047}\) | 0.16\(^{+0.11}_{-0.10}\) |
| $b$ | 0.080\(^{+0.032}_{-0.032}\) | 0.123\(^{+0.015}_{-0.014}\) |
| $\rho_\star$ [kg m$^{-3}$] | 2400\(^{+160}_{-140}\) | 2400\(^{+140}_{-140}\) |
| $i$ [$^\circ$] | 88.34\(^{+0.04}_{-0.04}\) | 89.83\(^{+0.11}_{-0.12}\) |
| $q_1$ | 0.48\(^{+0.25}_{-0.25}\) | 0.48\(^{+0.25}_{-0.25}\) |
| $q_2$ | 0.23\(^{+0.17}_{-0.17}\) | 0.23\(^{+0.17}_{-0.17}\) |
| $T_{14}$ [hours] | 2.047\(^{+0.082}_{-0.082}\) | 5.171\(^{+0.049}_{-0.049}\) |
| $T$ [hours] | 1.22\(^{+0.20}_{-0.20}\) | 4.89\(^{+0.054}_{-0.054}\) |
| $\tau_2$ [hours] | 0.39\(^{+0.19}_{-0.19}\) | 4.29\(^{+0.098}_{-0.098}\) |
| $R_p$ [R$_\star$] | 8.2\(^{+1.0}_{-1.2}\) | 11.2\(^{+0.58}_{-0.42}\) |
| $\epsilon$ | 0.105\(^{+0.15}_{-0.15}\) | 0.08\(^{+0.14}_{-0.14}\) |
| $\omega$ [$^\circ$] | 131\(^{+42}_{-42}\) | 65\(^{+22}_{-22}\) |
| $\Omega$ [$^\circ$] | 270 | 270\(^{+140}_{-140}\) |
| $M_p [M_\star]$ | 26\(^{+12}_{-12}\) | 189\(^{+94}_{-94}\) |
| $\rho_p$ [kg m$^{-3}$] | 240\(^{+160}_{-160}\) | 720\(^{+870}_{-870}\) |
| $a$ [AU] | 0.129\(^{+0.009}_{-0.009}\) | 0.206\(^{+0.010}_{-0.010}\) |
| $S [S_\star]$ | 25.9\(^{+0.11}_{-0.11}\) | 10.1\(^{+0.87}_{-0.87}\) |
| $T_{eq}$ [K] | 630\(^{+11}_{-11}\) | 497\(^{+10}_{-10}\) |

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Facilities: TESS

Software: MultiNest (Feroz & Hobson 2008; Feroz et al. 2009), isochrones (Morton 2015), Mercury (Chambers 1999), corner.py (Foreman-Mackey 2016)

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