Energy released by brush discharges from the fabric with conductive fibres

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Abstract. Fabrics containing regularly spread conductive fibres introduced into their structure may storage an electric charge, what results in a characteristic surface potential distribution. The brush, electrostatic discharges may occur when the grounded conductive object approaches to the charged fabric surface. A simplified, charged fabric-grounded object model is proposed. A metal sphere as the conductive object and a synthetic fabric with conductive yarn (forming regular cells) are considered. The analytical model allows to connect the energy $W_{rel}$, that can be released during the discharge, with geometry of the fabric-object system. The model has led to the power type relation between the energy $W_{rel}$ and the cell diameter of $a$.

1. Introduction
Most fabrics are made of the yarn containing synthetic fibres or their admixture. Their characteristic feature is their high resistivity. This leads to a long storage time of the electric charge, which may occur as a result of the electrification process. The electrified fabric is the source of an electric field in which energy can be stored [1, 2]. If the field intensity exceeds the dielectric strength of the air, electrostatic discharges (ESD) may occur. The energy accumulated in the electric field is released in a relatively short time ($10^{-9}$-$10^{-6}$ s) [3] and can be a cause of hazards. In order to assess the risk of a hazard, a comparison of the energy that can be released during the ESD-with the hazard threshold energy is necessary.

One of the methods of limiting the value of energy that can be released from the fabric during ESD is the insertion of conductive fibres into the fabric [4, 5]. Grounding of the fibres generally reduces the electric potential on the fabric surface. However, the conductive fibres practically do not affect the charge half-decay time of the fabric [6]. Metallic fibres have linear resistivity up to 500 $\Omega$/m, however conductive polymer yarns can have resistivity higher than 10 $k\Omega$/m [7]. In any case, the resistance of the conductive fibres, creating the conductive “border” of the particular cell, is significantly lower in comparison to the resistance (surface/volume) of the polymeric fabric within the cell. However, a finite resistance of conductive fibres may cause that the model will not work for very fast transient processes.

Usually, hazards due to the ESD are analysed for the case of plane-parallel objects with one grounded side [1]. It should be emphasised that energy stored in the electric field for an object with constant charge depends on its position. Energy released during ESD from such objects strongly depends on the distance between the object and the grounded surrounding.
2. Energy stored in the electric field
The energy $W$ stored in the electric field of the intensity $E$ can be determined from the general equation:

$$ W = \frac{1}{2} \varepsilon_0 \int \varepsilon |E|^2 \, dv, $$

(1)

where $\varepsilon_0=8.85 \times 10^{-12}$ F/m – free space permittivity, $\varepsilon$ – relative permittivity of a medium filling the element of volume $dv$. Application of eq. (1) is difficult, because it is necessary to know both, the field intensity $E(x,y,z)$ and the relative permittivity $\varepsilon(x,y,z)$ distributions in the volume $v$. Due to the lack of these distributions, a simplified analytical model for the grounded object-charged fabric system was proposed.

In a case of a flat, plane-parallel object, charged uniformly with density $q_o$, the charge stored on the elementary surface $\Delta S$ can be determined from the relation:

$$ Q = q_o \cdot \Delta S. $$

(2)

Assuming equipotentiality of the surface $\Delta S$ with a potential $V$ and its capacitance to the grounded surrounding $C_T$, the energy $\Delta W$ stored on this capacitance can be determined from the equation:

$$ \Delta W = \frac{1}{2} C_T V^2 = \frac{1}{2} Q^2 = \frac{1}{2} \frac{q_o^2 (\Delta S)^2}{C_T}. $$

(3)

3. Analytical and numerical calculation of energy released during ESD in a "grounded object-charged fabric" system
The model of the fabric (with a conductive fibres)-grounded object (i.e. a spherical electrode) system is shown in figure 1. To simplify the calculations, cylindrical symmetry in the geometry was assumed. A single cell was modelled not as a square with a side length of $a$, but as a circular dielectric disk with a thickness equal to the thickness of the fabric $d$, and with a radius $R=0.5a$. The relative permittivity of the fabric is equal to $\varepsilon_r$. The closest objects on the ground potential are conductive mesh (made by conductive fibres) and a spherical electrode with a radius $R_0$ (compliant with standard EN-13463-1), located in the axis of symmetry and distance $h$ from the surface of the fabric.

The equation (3) was applied to the model in which the total capacitance of the system $C_T$ associated with its element $\Delta S$ can be determined from the simplified equation:

$$ C_T = C_1 + C_2 = \left( \frac{\varepsilon_r \Delta S}{\sqrt{r^2 + (h + R_0)^2 - R_0^2}} \right) + \left( \frac{\varepsilon_r \Delta S}{R - r} \right), $$

(4)

where: $C_1$ – the capacitance between the grounded spherical electrode and the element $\Delta S$, $C_2$ – the capacitance between the element $\Delta S$ and the grounded conductive fibre, $r$ – distance (radius) between the system’s symmetry axis and the element $\Delta S$.

Assuming that:

$$ \Delta S = 2\pi r \, dr, $$

(5)

the energy $W_{\text{stored}}$, stored in the system can be determined from the relation:

$$ W_{\text{stored}} = \frac{\pi}{\varepsilon_0} \int_{0}^{r} \frac{r q_o^2(r)}{\sqrt{r^2 + (h + R_0)^2 - R_0^2}} \frac{1}{R - r} + \frac{\varepsilon_r}{\varepsilon_0} \, dr. $$

(6)

Based on experimental tests and numerical modeling [6,8], it was assumed that after ESD a charge density crater $q_o(r)$, with a diameter between 50 and 90% of the single cell diameter, occurs. An average value of $0.7R$ was assumed (figure 2) for the crater radius. The "bottom" of the charge has a “depth” of 12.5 % of its initial density $q_{0b}$. 
The distribution of surface charge density $q_s(r)$ before and after ESD is determined from the following equations:

$$q_{s\,\text{before}}(r) = q_\infty,$$

$$q_{s\,\text{after}}(r) = \begin{cases} q_\infty \left[ 0.875 \sin \left( \frac{\pi r}{1.4R} \right) + 0.125 \right] & \text{for } 0 < r < 0.7R, \\ q_\infty & \text{for } 0.7R < r < R. \end{cases}$$

The total energy stored in the system before and after ESD, respectively $W_{\text{before}}$ and $W_{\text{after}}$, can be calculated by substitution relations (7) and (8) to equation (6). Finally, the energy $W_{\text{rel}}$ released in the charged fabric-grounded object system during ESD, is determined from the following equation:

$$W_{\text{rel}} = W_{\text{before}} - W_{\text{after}}.$$ 

Calculations of the values of integral (6), for charge distributions given by (7) and (8) were made numerically for the following data: the relative permittivity of the fabric $\varepsilon_r=1$, fabric thickness $d=0.6$ mm, radius of the spherical electrode $R_0=7.5$ mm, fabric-electrode distance is assumed to be constant, equal to $h=d$, surface charge $q_\infty=40$ $\mu$C/m² before ESD. It was the highest value measured experimentally for the fabric samples charged by DC corona [6]. The assumed charge density, may not occur in industrial condition (the worse case considered).

Numerical calculations of energy were carried out for the same geometry of the system and surface charge distributions using COMSOL Multiphysics software. Smooth sinus-type transition to 0 near the fibres was added to avoid numerical errors. Simulation details were presented in [6].

4. Results

The dependence between the energy $W_{\text{rel}}$ released during ESD, and the diameter $a$ of the single cell obtained from calculations (eq. (9)) and modelling is shown in figure 3. The $W_{\text{rel}}=f(a)$ characteristics can be approximated by a power-type function:

$$W_{\text{rel}} = Ka^n,$$

where: $W_{\text{rel}}$ – released energy in [$\mu$J], $a$ – single cell diameter in [mm], $K$ – constant equal approximately to $\sim 3 \times 10^{-4}$, $n$ – power function exponent, approximately to $\sim 3.3$.

Both, numerical and analytical models give similar results with the difference not exceeding of 36 %.
5. Limitation of the model
Experimental investigations have shown that for \( q_{s0} = 20-40 \, \mu \text{C/m}^2 \) [6] and \( a < a_{\text{min}} = 10 \, \text{mm} \) [8] no ESDs were observed. It can be explained using Paschen’s law [9]. When the spherical electrode is approaching the cell with \( a < 15 \, \text{mm} \), the potential difference between the electrode and the fabric is always under the Paschen curve – see figure 4. The curves (2), (3) and (4) were obtained using modelling for the system with uniform initial distribution of the charge \( q_s = 40 \, \mu \text{C/m}^2 \).

Results of experimental investigations carried out on samples with \( a > 50 \, \text{mm} \) [8] do not allowed to draw unequivocal conclusions due to non-uniform charge distributions obtained.

6. Conclusions
- The energy released during ESD from charged fabric can be described by a power type relation: \( K \cdot a^n \), where \( a \) – single cell diameter in [mm], \( K \) – constant, \( n \) – power function exponent.
- The presented model should be verified empirically by direct measurement of released energy for a wide range of the cell diameters.
- Future work should include developing a strict relation between crater shape and size and a single cell diameter.

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