Non-standard neutrino interactions in the Zee–Babu model

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We investigate non-standard neutrino interactions (NSIs) in the Zee–Babu model. The size of NSIs predicted by this model is obtained from a full scan over the parameter space, taking into account constraints from low-energy experiments such as searches for lepton flavor violation (LFV) and the requirement to obtain a viable neutrino mass matrix. The dependence on the scale of new physics as well as on the type of the neutrino mass hierarchy is discussed. We find that NSIs at the source of a future neutrino factory may be at an observable level in the $\nu_e \rightarrow \nu_\tau$ and/or $\nu_\mu \rightarrow \nu_\tau$ channels. In particular, if the doubly charged scalar of the model has a mass in reach of the LHC and if the neutrino mass hierarchy is inverted, a highly predictive scenario is obtained with observable signals at the LHC, in upcoming neutrino oscillation experiments, in LFV processes, and for NSIs at a neutrino factory.

I. INTRODUCTION

Experimental studies of neutrino oscillations have provided us with compelling evidence that neutrinos have masses and lepton flavors mix. Among many possible mechanisms to describe the origin of neutrino masses, radiative mass generation provides an attractive method to obtain small neutrino masses. In such a framework, neutrino masses are exactly vanishing at tree level, and are induced as finite radiative corrections. Typically, neutrino masses are suppressed by a loop factor and proportional to $m_\ell^2/M$, where $m_\ell$ are the charged lepton masses, $M$ is the mass scale of the new particles in the loop, and $\mu$ is the scale of lepton-number violation. Together with a modest suppression from Yukawa couplings, this allows sub-eV neutrino masses, while having new physics not too far from the electroweak scale, $M \sim 1$ TeV, opening the possibility of collider tests of the neutrino mass generation mechanism.

An economical way of radiative neutrino mass generation is to enlarge the scalar sector of the Standard Model [1, 2]. In the Zee model, neutrino masses are obtained at one-loop level by adding a singly charged scalar and a second $SU(2)_L$ doublet to the Standard Model [3]. While the simplest version of this model cannot accommodate current experimental data, since the predicted leptonic mixing angle $\theta_{12}$ is too large (close to $\pi/4$), a minor extension of the model remains a viable option [4]. Alternatively, in the Zee–Babu model [3, 5, 6, 7], two $SU(2)_L$ singlet scalars are introduced, one singly and one doubly charged, and neutrino masses are generated at two-loop level. Phenomenological studies of this model have been performed, e.g., in Refs. [8, 9, 10]. Through the exchange of heavy scalars, lepton flavor violating (LFV) processes such as $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ can be dramatically enhanced compared to the Standard Model. Furthermore, the new scalars could be accessible for the Large Hadron Collider (LHC). In particular, the doubly charged Higgs may induce very clean like-sign bi-lepton events.

Besides colliders, next generation neutrino oscillation experiments will also help us to unveil the underlying physics behind neutrino masses. A neutrino factory will be the ultimate facility to perform precision measurements of standard neutrino oscillations as well as to search for non-standard neutrino properties. We take this as a motivation to investigate non-standard neutrino interactions (NSIs) in realistic neutrino mass models. In a given model, NSIs are typically linked to LFV processes for charged leptons, yielding too tight bounds, see, e.g., Refs. [11, 12, 13]. For example, in the case of triplet scalar models (i.e. type-II seesaw), NSIs are always entangled with the interactions among four charged leptons, which suffer stringent constraints from LFV processes like $\mu \rightarrow 3e$. There are no sizable NSIs unless severe fine-tuning of Majorana phases is invoked [14]. In the case of the Zee–Babu model, the situation is more involved, since the masses of singly and doubly charged Higgs in principle can be well separated and a different set of Yukawa couplings controls charged lepton and neutrino interactions with the scalars. Non-trivial predictions for the NSI parameters emerge from the different combinations of Yukawa couplings responsible for the neutrino masses and mixing, and LFV processes. In this respect, we will investigate NSIs in the Zee–Babu model in detail.

The remaining parts of the work are organized as follows: In Sec. II, we sketch the Zee–Babu model and show how NSIs are induced. In Sec. III, experimental constraints from low-energy observables on the model pa-
rameters are measured, while in Sec. VI we present the results from our numerical study of NSI parameters within this model. Discussion and summary follow in Sec. VII.

II. THE ZEE–BABU MODEL AND NON-STANDARD NEUTRINO INTERACTIONS

In the minimal Zee–Babu model, two SU(2) L singlet scalar fields h + and k ++ are introduced with hypercharges 1 and 2, respectively. The corresponding Lagrangian is then given by

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{f_{\alpha \beta} L_{\alpha} C_i \sigma_2 L_{\beta} h^+ + g_{\alpha \beta} \overline{e}_\beta k^{++}}{m_h^2} - \mu h^- h^{++} + \text{h.c.} + V_H ,
\]

where \( L \) denote left-handed lepton doublets, \( e \) are the right-handed charged leptons, and the scalar potential \( V_H \) contains the couplings among scalar fields. The Yukawa couplings \( f \) and \( g \) are antisymmetric and symmetric, respectively. The trilinear \( \mu \) term breaks lepton number \( (L) \) in an explicit way, and hence, one can naturally expect the dimensionfull parameter \( \mu \) to be reasonably small, since the symmetry is enhanced in the limit \( \mu \to 0 \).

Light neutrino masses are generated via a two-loop diagram, which yields

\[
(m_\nu)_{ab} = 16 \mu f_{\alpha \beta} m_c g_{cd} I_{cd} m_d f_{bd} , \tag{2}
\]

where \( m_c \) are charged lepton masses and \( I_{cd} \) is a two-loop integral \( (12) \). Since the \( e^+ e^- \) collider LEP at CERN indicates that the masses of charged scalars are typically larger than \( O(100 \text{ GeV}) \), we can neglect the masses of charged leptons compared to them. In this case, one finds

\[
I_{cd} \approx I = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \left( \frac{m_k^2}{m_h^2} \right) , \tag{3}
\]

where \( M = \max(m_k, m_h) \) and \( I(r) \) is a dimensionless function of order unity \( (10) \). For our numerical calculations we use the expression given in Eq. (7) of Ref. \( (8) \). Using Eq. (3), the light neutrino mass matrix becomes

\[
m_\nu \approx \frac{1}{48\pi^2} \frac{\mu}{M^2} \int f D e g^T D e f^T , \tag{4}
\]

where the matrix \( D e = \mathrm{diag}(m_e, m_\mu, m_\tau) \) contains the charged-lepton masses. Light neutrino masses are suppressed by the heavy scalar masses and proportional to the lepton-number violating parameter \( \mu \). Due to the antisymmetric property of \( f \), we have \( \det m_\nu = 0 \), and therefore, one of the neutrinos is massless if higher-order corrections are not considered.

The heavy scalars will induce non-standard lepton interactions via tree-level diagrams as shown in Fig. 1. After integrating out the heavy scalars, the following dimension-6 operators are generated at tree level \( (12) \)

\[
\mathcal{L}_{\text{NSI}}^{d=6} = 4 \frac{f_{\alpha \beta} f_{\rho \sigma}}{m_h^2} \left( \overline{\nu}_\alpha P_L \nu_\beta \right) \left( \overline{\nu}_\sigma P_L \nu_\rho \right) - 2 \frac{f_{\alpha \beta} f_{\rho \sigma}}{m_h^2} \left( \overline{\nu}_\alpha P_L \nu_\beta \right) \left( \overline{\nu}_\rho P_L \nu_\sigma \right) , \tag{5}
\]

where a Fierz transformation has been applied for the second step, and

\[
\varepsilon_{\alpha \beta}^{\rho \sigma} = \frac{f_{\alpha \beta} f_{\rho \sigma}}{\sqrt{2} G_F m_h^2} \simeq 0.06 \frac{f_{\alpha \beta} f_{\rho \sigma}}{1 \text{ TeV}} \left( \frac{m_h}{\text{TeV}} \right)^{-2} \tag{6}
\]

are the canonical NSI parameters. For neutrinos propagating in normal matter, only the following NSIs are induced

\[
\varepsilon_{\alpha \beta}^{m} = \varepsilon_{\alpha \beta}^{ee} = \frac{f_{\alpha \beta} f_{\rho \sigma}}{\sqrt{2} G_F m_h^2} \tag{7}
\]

Taking into account the antisymmetric property of \( f \), one can find that the only relevant NSI parameters in matter are \( \varepsilon_{\mu \tau}^{\mu \mu} \), \( \varepsilon_{\mu \tau}^{\mu \nu} \), and \( \varepsilon_{\tau \nu}^{\mu \nu} \). Furthermore, NSIs may show up at neutrino production in a neutrino factory, related to the processes \( \mu \to e \nu_\mu \nu_\alpha \) due to \( \varepsilon_{\mu \alpha}^{\mu \mu} \). To be consistent with the notation in the literature, e.g., Ref. \( (10) \), we define

\[
\varepsilon_{\mu \tau}^{\mu \mu} = \varepsilon_{\mu \tau}^{\mu \nu} = \frac{f_{\mu \nu} f_{\tau \nu}}{\sqrt{2} G_F m_h^2} , \tag{8}
\]

which correspond to the source effects in the \( \nu_\mu \to \nu_\tau \) and \( \nu_\tau \to \nu_\tau \) channels, respectively. By definition, the relation

\[
\varepsilon_{\mu \tau}^{m} = -\varepsilon_{\mu \tau}^{s} \tag{9}
\]

holds, since both the NSI parameters \( \varepsilon_{\mu \tau}^{m} \) and \( \varepsilon_{\mu \tau}^{s} \) are related to the Yukawa couplings \( f_{\mu \nu} \) and \( f_{\tau \nu} \). Let us mention that Eq. (9) holds in a rather general class of models, where NSIs are generated by dimension-6 operators \( (17) \).

The light neutrino mass matrix can be diagonalized by means of a unitary transformation as

\[
m_\nu = U D U^T \tag{10}
\]
where \( D = \text{diag}(m_1, m_2, m_3) \) and \( U \) can be parametrized

\[
U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{23}s_{13}c^{\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}c^{\delta} \\
-s_{12}c_{13} - c_{12}s_{23}s_{13}c^{\delta} & s_{12}c_{23}s_{13}c^{\delta} & s_{12}c_{23}s_{13}c^{\delta} \\
s_{12}c_{13} - c_{12}s_{23}s_{13}c^{\delta} & -c_{12}c_{23}s_{13}c^{\delta} & c_{12}c_{23}s_{13}c^{\delta}
\end{pmatrix} \begin{pmatrix}
f_1 \\
e^{i\sigma} \\
1
\end{pmatrix},
\]

(11)

where \( c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} \) (for \( ij = 12, 13, 23 \)), and \( \delta \) is the Dirac CP-violating phase. Here only one Majorana phase \( \sigma \) is involved, since one light neutrino is exactly massless. As a result of \( \det U = 0 \), there is an eigenvector \( v_0 = (f_{\mu\tau}, -f_{\tau\mu}, f_{\mu\mu}) \) which corresponds to the zero eigenvalue \( f v_0 = 0 [8] \). Note that \( v_0 \) is also an eigenvector of \( m_\nu \), and therefore, we have

\[
DV^T v_0 = 0,
\]

(12)

for both normal mass hierarchy \((m_1 \ll m_2 \ll m_3, \text{NH})\) and inverted mass hierarchy \((m_2 > m_1 \gg m_3, \text{IH})\).

Equation (12) provides us with three equations, among which one is trivially satisfied, since one element of \( D \) is zero. The other two equations lead to relations between \( f \) and the lepton mixing parameters. In the NH case, we have

\[
\begin{align*}
  f_{\tau\tau} &= \frac{s_{12}c_{23}}{c_{12}c_{13}} + \frac{s_{13}s_{23}}{c_{13}} e^{-i\delta}, \\
f_{\mu\tau} &= \frac{s_{13}}{c_{12}c_{13}} e^{-i\delta}.
\end{align*}
\]

(13)

According to the current global fit of neutrino oscillation experiments, the second terms in the right-hand sides of Eqs. (13) and (14) can be neglected, since they are suppressed by the small mixing angle \( \theta_{13} \). Then, we obtain the approximate relation \( f_{\mu\tau} \approx f_{\tau\tau} \approx f_{\tau\tau}/2 \). Taking into account the experimental constraints from Eqs. (18) and (19) below, we can roughly estimate that \( |f_{\mu\tau}| \approx |f_{\tau\tau}| \lesssim 0.05 \) (\( m_{\text{V}}/\text{TeV} \)) and \( |f_{\tau\tau}| \lesssim 0.1 \) (\( m_{\text{V}}/\text{TeV} \)). Compared with Eqs. (7) and (8), the possibly important NSI parameter is \( \epsilon_{\tau\tau}^{e} \), which mainly affects the \( \nu_e \rightarrow \nu_\tau \) channel. The CP-violating phases of \( f_{\alpha\beta} \) are suppressed by \( \theta_{13} \), and thus, NSIs cannot induce very distinctive CP-violating effects in the NH case.

For the IH case, the two non-trivial equations are

\[
\begin{align*}
  f_{\tau\tau} &= \frac{s_{23}c_{13}}{s_{13}} e^{-i\delta}, \\
f_{\mu\tau} &= \frac{c_{13}c_{23}}{s_{13}} e^{-i\delta}.
\end{align*}
\]

(15)

(16)

Indeed, it is obvious that \( |f_{\mu\tau}| \approx |f_{\tau\tau}| \) and \( |f_{\mu\tau}| \approx |f_{\tau\tau}|s_{13}/s_{23} \) hold. Thus, the potentially sizable NSI parameters are \( \epsilon_{\mu\tau}^{e}, \epsilon_{\mu\tau}^{\mu}, \epsilon_{\mu\tau}^{\tau} \) for neutrino propagation in matter, and \( \epsilon_{\mu\tau}^{e} \) for source effects in the \( \nu_\mu \rightarrow \nu_\tau \) channel at a neutrino factory. Equations (15) and (16) imply that \( \epsilon_{\mu\tau}^{e} \) and \( \epsilon_{\mu\tau}^{\mu} \) are real, whereas the phase of \( \epsilon_{\mu\tau}^{\tau} \) is given by \( \delta \). This may lead to an interesting correlation of CP-violation in standard oscillations and \( \epsilon_{\mu\tau}^{e} \)-induced CP-violating effects [18,19].

### III. EXPERIMENTAL CONSTRAINTS

At low-energy scales, stringent constraints from LFV processes mediated by the heavy scalars have to be included when we confront the model with experimental data. In the following, we compile the bounds given in Ref. [10].

- \( \ell_{e}^+ \rightarrow \ell_{e}^- \ell_{e}^- \ell_{e}^- \). As shown in Fig. [4], these rare lepton decays are mediated by \( k^{++} \) at tree level, and set very stringent constraints on the corresponding Yukawa coupling \( g \) and the mass of \( k^{++} \). The bounds at 90 \% C.L. read

\[
\begin{align*}
  |g_{\mu\mu}g_{e\tau}^*| &< 2.3 \times 10^{-5} \ (m_{k}/\text{TeV})^2, \\
  |g_{\mu\tau}g_{e\mu}^*| &< 0.010 \ (m_{k}/\text{TeV})^2, \\
  |g_{\mu\tau}g_{e\tau}^*| &< 0.006 \ (m_{k}/\text{TeV})^2, \\
  |g_{\mu\tau}g_{\mu\mu}^*| &< 0.008 \ (m_{k}/\text{TeV})^2, \\
  |g_{\mu\tau}g_{e\mu}^*| &< 0.008 \ (m_{k}/\text{TeV})^2, \\
  |g_{\mu\tau}g_{\mu\mu}^*| &< 0.010 \ (m_{k}/\text{TeV})^2.
\end{align*}
\]

(17)

Note that NSIs are induced by exchanging the singly charged Higgs \( h^{+} \), and one can in principle tune the mass of \( k^{++} \) or the scale of \( g \) in order to suppress its contributions to the LFV decays. However, since all the parameters \( f, g, m_{h}, \) and \( m_{k} \) enter in the expression for the neutrino mass matrix, c.f. Eq. (3), the constraints from Eqs. (17) have to be included also in an analysis of NSIs, in order to obtain the correct parameter space available for the model.

- \( \mu^{+}e^{-} \rightarrow \mu^{-}e^{+} \). The muonium to antimuonium conversion through the exchange of \( k^{+} \) are well bounded experimentally. For the similar reason mentioned above, these constraints are mainly on \( m_{k} \) and \( g \), and the current bound at 90 \% C.L. is \( |g_{e\tau}g_{\mu\mu}^*| < 0.2 \ (m_{k}/\text{TeV})^2 \).
\begin{itemize}
  \item Universality in $\ell_e \to \ell_\mu \nu$ decays. The Fermi coupling constant measured in muon and tau decays obtains corrections from the exchange of $h^+$, which sets strong constraints on the Yukawa coupling $f$:

\[
|f_{\mu\mu}|^2 < 0.015 \left( \frac{m_h}{\text{TeV}} \right)^2,
\]

\[
|f_{\tau\mu}|^2 - |f_{\tau\mu}|^2 < 0.05 \left( \frac{m_h}{\text{TeV}} \right)^2,
\]

\[
|f_{\tau\tau}|^2 - |f_{\mu\mu}|^2 < 0.05 \left( \frac{m_h}{\text{TeV}} \right)^2,
\]

\[
|f_{\mu\tau}|^2 - |f_{\mu\tau}|^2 < 0.06 \left( \frac{m_h}{\text{TeV}} \right)^2.
\]

\end{itemize}

\[\text{(18)}\]

- Rare lepton decays $\ell_e \to \ell_\mu \gamma$. Both $h^+$ and $k^+$ contribute to LFV photon interactions at one-loop level, and the most stringent bound comes from $\mu \to e\gamma$. Neglecting the contributions from the doubly charged Higgs, we obtain experimental bounds at 90% C.L.

\[
|f_{\tau\tau}f_{\mu\tau}|^2 < 3.4 \times 10^{-5} \left( \frac{m_h}{\text{TeV}} \right)^4,
\]

\[
|f_{\mu\mu}f_{\mu\tau}|^2 < 1.7 \left( \frac{m_h}{\text{TeV}} \right)^4,
\]

\[
|f_{\tau\mu}f_{\mu\tau}|^2 < 0.7 \left( \frac{m_h}{\text{TeV}} \right)^4.
\]

\[\text{(19)}\]

\[
\text{Note that in our numerical analysis, contributions from both singly and doubly charged Higgs are included, see, e.g., Ref. [10].}
\]

Besides the bounds above, there also exist other constraints, like the $\mu-e$ conversion in nuclei and the anomalous magnetic moments of the muon, which are relatively loose, and hence will not be considered in this work. The perturbativity of the model imposes limits on the Yukawa couplings $f$ and $g$, in particular for very massive charged scalars. Similarly, the stability of the vacuum requires $\mu \ll 4\pi \min(m_k, m_h)$.}

\IV. NUMERICAL RESULTS FOR NSI IN THE ZEE–BABU MODEL

We have performed a full scan of the parameter space of the model in order to obtain predictions for NSIs. Following Refs. [8, 11], we take as independent parameters the lepton mixing angles, the Dirac and Majorana phases $\delta$ and $\sigma$, the Yukawa couplings $g_{e_\ell}, g_{\mu_\ell}, g_{\tau_\ell}$ and one of the three $f_{\alpha\beta}$'s, and the scalar masses $m_h, m_k$, as well as the trilinear coupling $\mu$. (Neutrino mass-squared differences are fixed to their best-fit values, since their uncertainties are comparably small.) The remaining Yukawa couplings $f_{\alpha\beta}$ and $g_{\alpha\beta}$ are then fixed by Eqs. (4), (13), and (14) for NH or Eqs. (4), (13), and (14) for IH. For each set of these parameters, we compare the model predictions to the experimental data with a $\chi^2$ function

\[
\chi^2 = \sum_i \frac{\left( \rho_i - \rho_i^0 \right)^2}{\sigma_i^2},
\]

where $\rho_i^0$ represents the data of the $i$th experimental observable, $\sigma_i$ the corresponding $1\sigma$ absolute error, and $\rho_i$ the prediction of the model. The experimental observables are the neutrino mixing angles (taken from Ref. [20]), and the constraints from LFV and universality tests given in Eqs. (17)–(19).

For the dimensionful parameters $m_h, m_k$, and $\mu$, we adopt first two representative choices, namely $m_k = m_k = \mu = 10$ TeV or $m_k = m_k = \mu = 1$ TeV. The latter case might be just in reach for the LHC. For a luminosity of 300 fb$^{-1}$ and under optimistic assumptions, this may lead to order 10 four-lepton events from the pair-production of the doubly charged scalars $pp \to k^++k^-=\ell^+\ell^+\ell^-\ell^-$. We do not consider much lower scalar masses, since already at 1 TeV many of the experimental bounds are saturated. At 10 TeV, the constraints are less tight, which leaves more freedom in choosing the parameters of the model, with the obvious disadvantage of not being testable at LHC. Choosing $\mu$ of the same order as the scalar masses is conservative in the following sense. For fixed scalar masses, neutrino masses are proportional to $\mu$. Hence, decreasing $\mu$ would require to increase the Yukawa couplings, see Eq. (4). This would make the constraints from Sec. [11] more severe and the parameter space would be more constrained. Therefore, we decided to take $m_h = m_k = \mu$ in order to keep $\mu$ relatively large, but still ensure the stability of the vacuum [8]. Towards the end of this section we also investigate the case $m_h \neq m_k$.

In Fig. 2 we present the allowed regions of NSI parameters at 1$\sigma$, 2$\sigma$, and 3$\sigma$ C.L., defined as contours in $\Delta \chi^2$ for two degrees of freedom with respect to the $\chi^2$ minimum. The left (right) panels correspond to scalar masses of 10 TeV (1 TeV). The upper plots in Fig. 2 show the NSI parameters $|\varepsilon_{\mu\mu}|$ and $|\varepsilon_{\tau\tau}|$ for neutrino propagation. We find that values of order $\varepsilon \sim 10^{-3}$ can be obtained only in the IH case, whereas in the NH case they are typically one order of magnitude smaller. These flavor diagonal propagation NSI parameters induce effectively a non-standard matter effect and it has been shown that present long-baseline experiments are not very sensitive to these NSI effects [21]. Even a two-baseline neutrino factory would only be sensitive to such NSI parameters at the level of $\varepsilon \gtrsim 10^{-2}$ [22]. The lower plots in Fig. 2 indicate that the NSI parameter $|\varepsilon_{\mu\tau}|$ relevant at the source of a neutrino factory may reach values up to $10^{-3}$ for both hierarchies. In the IH case, the parameters $|\varepsilon_{\mu\tau}| = |\varepsilon_{\mu\tau}|$ may also be as large as few $\times 10^{-3}$. For a scalar mass scale of 1 TeV, a non-trivial lower bound on the NSI parameters of order $10^{-4}$ is found in the right-hand column of Fig. 2. Indeed, for scalar masses in the TeV range, the model is rather constrained and the requirement of a correct neutrino mass matrix pushes the Yukawa couplings close to the bounds from Sec. [11, 8, 10], which in turn implies “large” NSIs. The extended region seen in the figure comes mainly from the freedom to adjust the Dirac CP-violating phase $\delta$.

Source NSIs related to the muon decay in a neutrino factory are probed efficiently with a near detector, see e.g., Ref. [23], since they lead to the appearance of
“wrong” neutrino flavors even at “zero distance” \cite{24}. For the cases of interest in the Zee–Babu model, \( |\varepsilon_{e\tau}^{\ast}| \) and \( |\varepsilon_{\mu\tau}^{\ast}| \), obviously a tau detector at the near site would be useful \cite{14, 15, 23, 24, 27, 28}. The authors of Ref. \cite{26} consider as an example a 2 kt OPERA-like near detector and find sensitivities for \( |\varepsilon_{e\tau}^{\ast}|, |\varepsilon_{\mu\tau}^{\ast}| \gtrsim 7 \times 10^{-4} \). Note that in order to disentangle the effect of \( \varepsilon_{e\tau}^{\ast} \) and \( \varepsilon_{\mu\tau}^{\ast} \), the ability to identify the charge of the tau lepton would be required.

The sensitivity to \( |\varepsilon_{\mu\tau}^{m}| \) for neutrino propagation has been discussed for atmospheric neutrinos \cite{24, 30}, the OPERA long-baseline experiment \cite{31}, superbeam experiments \cite{32}, and a neutrino factory, e.g. in Refs. \cite{22, 27, 33, 34}. Typically, the reach is at a few \( \times 10^{-2} \) or worse, which is not sufficient to probe the parameter range predicted by the Zee–Babu model. However, there are two reasons why in our case we may expect better prospects to observe NSI effects in this channel. First, in many of the above mentioned studies the complex phase of \( \varepsilon_{\mu\tau}^{m} \) has been marginalized over, whereas in the Zee–Babu model \( \varepsilon_{\mu\tau}^{m} \) is predicted to be real, see the discussion after Eqs. \cite{13, 14}. Second, Eq. \cite{19} relates source and propagation NSIs in this channel. The relevance of the phases can be understood from Eq. \cite{35} of Ref. \cite{32}, which shows that the relevant leading terms in the survival probability \( P_{\mu\mu} \) are proportional to \( |\varepsilon_{\mu\tau}^{s}| \sin(\phi^{s}) \) and \( |\varepsilon_{\mu\tau}^{m}| \cos(\phi^{m}) \), where \( \phi^{s,m} \equiv \arg(\varepsilon_{\mu\tau}^{s,m}) \). Hence, these terms can be set to zero if \( \phi^{s} \) and \( \phi^{m} \) can be chosen independently, but if they are coupled by \( \phi^{s} = \pi - \phi^{m} \) following from Eq. \cite{19}, at least one of them will always be non-zero. Under these special conditions, we estimate from the results of Refs. \cite{24, 27} sensitivities of a neu-

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**FIG. 2:** The allowed region of NSI parameters at 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \) C.L. in the Zee–Babu model. We take \( m_h = m_k = \mu = 1 \text{ TeV} \) for the figures in the left-hand side column and \( m_h = m_k = \mu = 10 \text{ TeV} \) for the figures in the right-hand side column.
trino factory for $\varepsilon_{\mu\tau}^m$ in the range of $10^{-3}$, even without a near detector. In the presence of a near tau-detector, a sensitivity for $|\varepsilon_{\mu\tau}^m| = |\varepsilon_{\mu\tau}^s| > 6 \times 10^{-4}$ is reached.

In Fig. 3 we show the correlations between NSI parameters and the mixing angle $\theta_{13}$. In the IH case and scalar masses at the TeV scale, one obtains a quite strong prediction for the mixing angle $\theta_{13}$. From Eqs. (15) and (10) follows that $|f_{\mu\tau}|$ is suppressed by $s_{13}$, whereas a correct neutrino mass matrix requires $f_{\mu\tau}$ to be of the same order as $f_{e\mu}$ and $f_{e\tau}$. Figure 3 indeed shows that for the IH case and scalar masses in the TeV range, values of $\theta_{13}$ close to its present bound are predicted, with a lower bound of $\sin^2\theta_{13} \gtrsim 10^{-2}$. Such a sizable lower bound is of particular interest, since it would guarantee a discovery at the forthcoming reactor [35, 36] or long-baseline [37] experiments in the near future [38].

In the NH case, no lower bound on $\theta_{13}$ is obtained. In this case, the presence of NSIs may have an impact on the search for $\theta_{13}$ at a future neutrino factory, especially if $\theta_{13}$ is relatively small. In particular, $\varepsilon_{\mu\tau}^m$ may lead to $\nu_{\tau}$ at the source, which will oscillate to $\nu_{\mu}$ and lead to so-called “wrong-sign” muons in the far detector, which might be confused with the effect of a tiny $\theta_{13}$.

Finally, let us relax the assumption $m_h = m_k$ and investigate the dependence on the masses of the scalars. In Fig. 4 we show the size of NSI parameters by fixing one of the two scalars at 1 TeV and varying the mass of the other one. In order to guarantee the stability of the vacuum [8], while keeping neutrino masses as large as possible, we use for the scale of lepton number violation...
$\mu = \min(m_h, m_k)$. The qualitative behaviour of these results can be understood from the expression for the neutrino mass matrix in Eq. (4) and the fact that the constraints on the Yukawa couplings $f$ and $g$ from Sec. III scale with $m_h$ and $m_k$, respectively.

First, the lower bound on the scalar masses follows from the fact that decreasing either $m_h$ or $m_k$ decreases the neutrino mass because of $\mu = \min(m_h, m_k)$. At the same time, the bounds on the Yukawas become more severe and it is impossible to obtain sufficiently large neutrino masses. Second, if we increase $m_k$, while keeping $m_h$ at 1 TeV (right panel of Fig. 4), neutrino masses decrease with $m_k^{-2}$ because of $M = \max(m_h, m_k)$. Since the constraints on $g$ increase only with $m_k$, it is not possible to compensate for the $m_k^{-2}$ suppression by increasing $g$. Hence, if the singly charged scalar is at the TeV scale also the doubly charged one has to be in that range. However, the opposite statement is not true and relatively large $m_h$ is possible for $m_k = 1$ TeV. If $m_h$ is increased, again neutrino masses decrease as $m_h^{-2}$, but in this case there are two factors of $f$ entering Eq. (4), and since the constraints on them increase as $m_h$, it is possible to keep neutrino masses constant by increasing $f$. However, note that this does not lead to larger NSIs, since $\varepsilon \propto f^2/m_h^2$, see Eq. (3), which is constant, in agreement with the left panel of Fig. 4.

We conclude that from the point of view of NSIs, the crucial mass parameter is the one of the doubly charged scalar. Note also that this one has the most striking signature at colliders, namely the decay into two like-sign leptons. Apparently, vastly separated masses for the singly and doubly charged scalars is either not phenomenologically viable or does not affect the prediction for NSIs. Therefore, the NSI results obtained for $m_h = m_k$ are generic.

V. DISCUSSION AND SUMMARY

We have studied NSIs in the Zee–Babu two-loop neutrino mass model, which are mediated by the singly charged scalar of the model. We have shown that non-standard neutrino matter interactions relevant for the propagation of neutrinos may be induced. The relevant parameters $\varepsilon_{\mu\tau}^m$, $\varepsilon_{\mu\tau}^m$, and $\varepsilon_{\mu\tau}^m$ may reach values of order $10^{-3}$. While flavor diagonal NSIs of this size are too small to be observable, the off-diagonal term $\varepsilon_{\mu\tau}^m$ may be within the reach of a future neutrino factory. In addition to these matter NSIs, NSIs affecting the muon decay at the source of a neutrino factory may be induced in the Zee–Babu model, in both the $\nu_e \rightarrow \nu_\tau$ $(\varepsilon_{e\tau}^m)$ and $\nu_\mu \rightarrow \nu_\tau$ $(\varepsilon_{\mu\tau}^m)$ channels. The possible size of NSI parameters depends on the scale of new physics (i.e., the masses of the singly and doubly charged scalars, $m_h$ and $m_k$, respectively, and the scale of the lepton-number violating parameter $\mu$) and on the type of the neutrino mass hierarchy, NH or IH.

The most constrained situation is obtained for IH and scalar masses at 1 TeV. In this case, the NSI parameters $\varepsilon_{e\tau}^m$ and $\varepsilon_{\mu\tau}^m$ are predicted to be in the range $10^{-4} - 10^{-3}$, probably in reach of a near tau-detector at a future neutrino factory. Thanks to the fact that $\varepsilon_{\mu\tau}^m$ is real
in this model combined with the relation $\varepsilon_{\mu\tau} = -\varepsilon_{\mu\tau}^m$, even a standard neutrino factory without a near tau-detector may be sensitive to the NSI values predicted in this case. Furthermore, this configuration predicts a value of $\theta_{13}$ in reach of the upcoming oscillation experiments [38], as well as signals in LFV processes close to the present bounds, with good prospects for a signal in $\mu \rightarrow e\gamma$ [2, 10].

If kinematically accessible, the singly and doubly charged scalars of the model could be directly produced through the s-channel processes at the Tevatron and the LHC. There is no severe suppression of the cross section for the production of doubly charged scalar $k^{++}$, and if $2m_{k^+} > m_{k^{++}}$, it will predominantly decay into like-sign charged-lepton pairs with a very striking experimental signature. However, note that doubly charged scalars occur in a variety of models (e.g., the triplet scalar model for neutrino mass), and therefore, complementary signatures are required to identify the model. If the singly charged and doubly charged scalars are found at LHC, then—besides signals in LFV searches—the Zee-Babu model predicts a rather large value of $\theta_{13}$ and signals for NSI at a neutrino factory at a level of $10^{-3}$ if the mass hierarchy is inverted. In case of NH, $\theta_{13}$ as well as NSIs may still be in reach of future experiments, but no signal is guaranteed in either case. A similar situation emerges if the scale of new physics is beyond the reach of the LHC, for example at 10 TeV. In this case, observable signals may still arise for NSIs at a neutrino factory (as well as for $\theta_{13}$), but no relevant lower bound is obtained.

In conclusion, we have shown that, in the case of the Zee-Babu model for radiative neutrino masses, the interplay of the phenomenology at colliders, searches for LFV, and NSI effects at a neutrino factory could play a complementary role towards the goal of identifying the true mechanism of neutrino mass generation.

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