Abstract

In the Littlest Higgs model with T-parity, the heavy photon ($A_H$) is supposed to be a possible dark matter (DM) candidate. A direct proof of validity of the model is to produce the heavy photon at accelerator. In this paper, we study the production rate of $e^+e^- \rightarrow A_HA_H$ at the international $e^+e^-$ linear collider (ILC) in the Littlest Higgs model with T-parity and show the distributions of the transverse momenta of $A_H$. The numerical results indicate that the heavy photon production rate could reach the level of $10^{-1} \text{fb}$ at some parameter space, so it can be a good chance to observe the heavy photon via the pair production process with the high luminosity at the ILC ($500 \text{ fb}^{-1}$).

We know that DM is composed of weakly-interacting massive particles (WIMPs), so the interactions with standard model (SM) particles are weakness, how to detect heavy photon at a collider and distinguish it from other DM candidates are simply discussed in the final sector of the paper.

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1 Introduction

SM of particle physics, including the strong and electroweak interactions, is the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge model which has been extensively tested during the past 30 years. The great success of SM in so-far all the fields of particle physics manifests that it is obviously a valid theory. However, on other aspect, because so many parameters in the theory are free and need to be phenomenologically determined, one can believe it to be only an effective theory. Besides all the questions which would promote theorists to look for a more fundamental theory, there exists a famous problem, namely the hierarchy. This is because the Higgs of SM gets a mass through a “bare” mass term and interactions with other particles, but because of its scalar nature, the mass it gets through interactions is as large as the largest mass scale in the theory. In order to get the relative low Higgs mass, the bare mass must be fine-tuned to several tens of an expressive places so as to accurately cancel the very big interaction terms. Meanwhile, the astrophysical observations, for example, the rotation curves of galaxies [1] and the gravitational lensing effects [2] show that about 23% of the energy density of the universe is composed of DM. One of the most fundamental problems in cosmology and particle physics today is what the nature of DM is. A flood of discussions point out that the DM should have the following characters: non-luminous, non-baryonic, non-relativistic, and electrically neutral [3, 4, 5, 6, 7]. Extensive astronomical evidence indicates that DM is a stable heavy particle that interacts with SM particles weakly. That is to say, such a particle is a WIMP [8]. The microscopic composition of DM remains a mystery and SM does not offer an appropriate candidate to account for DM, however, many theories which extend SM contain new particles with the proper properties to play the role of DM.

As mentioned above, the new physical models are needed to solve the fine-tuning problem and provide the candidates of DM.

The best known example that can solve the above-mentioned problems is a supersymmetry (SUSY) model with R-parity [9]. The minimal supersymmetric standard model (MSSM) with R-parity [10, 11] is such a model. Supersymmetric partners are introduced, the contribution of each supersymmetric partner cancels off the contribution of each ordinary particle, so the fine-tuning doesn’t exist anymore. On the other hand, a discrete gauge symmetry named R-parity is included, so the lightest neutralino, a Majorana fermion ($\tilde{\chi}^0_1$) is supposed to be the lightest stable SUSY particle and stand as the DM component, where the neutralinos ($\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4$) are the lightest particles in the SUSY models, they are the fermionic SUSY partners of the neutral gauge and CP-even Higgs bosons.

A little Higgs model [12, 13] with T-parity [14, 15] provides a successful alternative to the SUSY with R-parity in solving the problems of the SM mentioned above. In the initial little Higgs model (LHM), Higgs originates as a pseudo-Nambu-Goldstone boson (PNGB) of a spontaneously broken
global symmetry. The global symmetry is definitely broken by the interactions of two sets, with each set preserving an unbroken subset of the global symmetry. The Higgs is an exact NGB when either set of couplings is no longer present. The hierarchy problem is explained by adopting new heavy particles at the TeV scale with the same spins as the corresponding SM particles. One-loop quadratic divergences of the Higgs boson mass are canceled by these new particles. Unfortunately, these initial models suffered from strict restrictions from the precision electroweak fits.

An ideal resolution to solve the problem is to apply a $Z_2$ discrete symmetry named T-parity (analogous to the R-parity in SUSY), by requiring SM particles to be even and the new heavy particles to be T-parity odd, forbidding all tree-level corrections to the electroweak precision tests. If the T-parity was conservation, the lightest T-odd particle (LTP) of a LHM would be stable and not decay, and it will play the role of the DM candidate.

The littlest Higgs model with T-parity (LHT) [16, 17] is such a type model, it is a modification of the original Littlest Higgs Model. One of the unique signatures of the theory is the existence of a neutral minimum weight heavy $U(1)$ gauge boson: heavy photon $A_H$ and some other special particles. Experimentally searching them would provide direct evidence for judging the validity of the theory. There are several ways to study DM, for example, one can use the relativistic mean field theory to determine the nuclear form factor for the detection of dark matter [18]. Astronomical observations and restrictions may provide another way. In our previous work, we have studied the heavy photon time-evolution in the LHT [19]. Finally, There is a good opportunity to combine theoretical models with the upcoming DM direct and indirect detection experiments, results from the present and future colliders in the TeV range. Because all the new particles are very heavy, they escaped the detection (if exist) in previous accelerators. By a general analysis, their masses, just like the particles predicted by other models, may be in TeV regions, so that one can expect to observe them at the high energy colliders. The production of heavy photon pair at the large hadron collider (LHC), $pp \rightarrow A_H A_H + X$ and the associated production of $Q_H A_H$ at the LHC have been discussed in Refs. [16] and [20] respectively, where $Q_H$ is the partner of the light quark. Because of the complex background, the LHC might be difficult to confirm a theory, so the ILC will be a proper tool to better understand the properties of the LHT. In this work, we analyze the production of the heavy photon pair at the ILC.

The paper is organized as follows. The numerical results of the production rate and the distributions of the transverse momenta are presented in Sec. II where all input model parameters are explicitly listed. Our discussions on various aspects and conclusions are made in the last section.
2 Theoretical formulation and numerical results

First, let us concisely recall the relevant characteristics of the LHT. The LHT is based on a non-linear $\sigma$-model describing a global $SU(5)/SO(5)$ symmetry breaking, which takes place at an energy scale $\Lambda \sim 4\pi f \sim 10$ TeV. The SM Higgs doublet is generally considered to be a subset of the Goldstone bosons associated with the breaking. The symmetry breaking also breaks the assumed embedded local gauge symmetry $[SU(2) \times U(1)]^2$ subgroup down to the diagonal subgroup $SU(2)_L \times U(1)_Y$, which is identified as the ordinary SM electroweak gauge group. The additional gauge structure leads to four extra gauge bosons at the TeV scale, $W^\pm_H$, $Z_H$ and $A_H$. The diagonal Goldstone bosons of the matrix $\Pi$ are “eaten” to become the longitudinal degrees of freedom of the heavy gauge bosons with odd T-parity at the scale $f$, the masses of them are

\[
M(Z_H) \approx M(W^\pm_H) \approx gf, \quad M(A_H) \approx \frac{g'}{\sqrt{5}} \approx 0.16f,
\]

where $g$ and $g'$ are the gauge couplings of SM $SU(2)_L$ and $U(1)_Y$. After electroweak symmetry breaking at the scale $v \ll f$, the masses of the new heavy gauge bosons as well as the SM gauge boson masses receive corrections of order $v^2/f^2$ and could be written as

\[
M(Z_H) = M(W^\pm_H) = gf(1 - \frac{v^2}{8f^2}) \approx 0.65f,
\]

\[
M(A_H) = \frac{fg'}{\sqrt{5}}(1 - \frac{5v^2}{8f^2}) \approx 0.16f,
\]

since the masses of the other T-odd particles are generically of level $f$, the $A_H$ can be assumed to be the LTP and be regarded as an ideal candidate of the WIMP cold DM.

The mirror fermions, acquire masses through a Yukawa-type interaction

\[
\kappa f (\bar{\Psi}_2 \xi \Psi' + \bar{\Psi}_1 \Sigma_0 \Omega \xi^\dagger \Omega \Psi')
\]

whereas $\Psi_1$, $\Psi_2$ are the fermion doublets and $\Psi'$ is a doublet under $SU(2)_2$.

One fermion doublet $\Psi_H = \frac{1}{\sqrt{2}}(\Psi_1 + \Psi_2)$ acquires a mass $\kappa f$, which is a free parameter, with the natural scale set by $f$. Specifically, the T-odd heavy partners of the SM leptons acquire the following masses $\sqrt{2}\kappa_l f$ [21], where the $\kappa_l$ is the flavor independent Yukawa coupling. The T-odd fermion mass for both lepton and quark partners will be assumed to exceed 300 GeV to avoid the colored T-odd particles from being detected in the squark searches at the Tevatron.

In the LHT model, the coupling term in the Lagrangian related to the heavy photon of our work is written as [16, 17]:

\[
A^\mu_L \bar{L}_i L_j : \frac{e}{10C_WS_W} (S_W - 5C_W(\frac{v}{f})^2 x_h) \gamma^\mu P_L \delta_{ij},
\]

where $\bar{L}$ is the heavy lepton of the LHT and $L$ is the SM lepton, and $e$ is the electromagnetic coupling constant, $x_h = \frac{5}{4g^2-g'^2}$, $v = \frac{2M_W S_W}{e}$, $f = 1000$, $P_L = \frac{1 - \gamma^5}{2}$. We identify $g$ and $g'$ with
the SM $SU(2)$ and $U(1)_Y$ gauge couplings, respectively. $S_W$ and $C_W$ are sine and cosine of the Weinberg angle, respectively.

The Feynman diagrams responsible for the process of $e^+e^- \rightarrow A_H A_H$ at the tree-level are shown in Fig.1.

![Feynman diagrams](image)

Figure 1: The Feynman diagrams of the process $e^+e^- \rightarrow A_H A_H$.

With the Lagrangian, the amplitude of the process can be written directly:

$$M = M_a + M_b$$

$$= -i\left(\frac{e}{10C_W S_W} (S_W - 5C_W \left(\frac{v^2}{f}\right) x_h)\right)^2 \bar{v}(P_1) \left(\frac{1}{P_{24}^2 - m_L^2}\right) \gamma^\mu \gamma^\nu P_{24\rho} + \frac{1}{P_{23}^2 - m_L^2} \gamma^\rho \gamma^\mu P_{23\rho} P_L u(P_2) \epsilon^*_\mu(P_3) \epsilon^*_\nu(P_4),$$

where $P_{23} = P_2 - P_3$, $P_{24} = P_2 - P_4$, and $\epsilon$ is the polarization vector for the heavy photon gauge boson.

The differential cross section is given by:

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|\vec{p}_1|^2} \frac{1}{\Sigma} |M|^2,$$

where $s$ and $t$ are the Mandelstam variables, $dt = 2 |\vec{p}_1| |\vec{p}_3| d\cos\theta$.

With the above production amplitudes and differential cross section, we can directly obtain the production cross section of the process. For the numerical computations of the cross section, we need the electroweak fine-structure constant $\alpha$ to satisfy $\alpha(M_Z) = \frac{e^2}{4\pi} = \frac{1}{128}$ [22].

There are three free parameters involved in the production amplitudes: the heavy photon mass $m_{AH}$, the mass of heavy lepton $m_L$, and the energy of the center-of-mass frame $\sqrt{s}$. Concretely, in order to expose possible dependence of the cross section on these parameters, we take two groups of values: $\sqrt{s} = 500, 1000$ GeV, $m_L = 300, 500, 700$ GeV respectively, the $m_{AH}$ takes 100 to 250 GeV in Fig.2 and 100 to 300 GeV in Fig.3. The numerical results of the cross sections are plotted in Fig.2 and Fig.3.
Figure 2: The dependence of the cross section of $e^+e^- \rightarrow A_H A_H$ on heavy photon mass $m_{A_H}$ (100~250 GeV) for $\sqrt{s}=500$ GeV and $m_{\tilde{L}}=300, 500, 700$ GeV at the ILC.

Figure 3: The dependence of the cross section of $e^+e^- \rightarrow A_H A_H$ on heavy photon mass $m_{A_H}$ (100~300 GeV) for $\sqrt{s}=1000$ GeV and $m_{\tilde{L}}=300, 500, 700$ GeV at the ILC.

Figure 4: The distributions of the transverse momenta of final state ($p_T$) for the process $e^+e^- \rightarrow$
$A_H A_H$ with $\sqrt{S}=500$ GeV.

Figure 5: The distributions of the transverse momenta of final state ($p_T$) for the process $e^+e^- \rightarrow A_H A_H$ with $\sqrt{S}=1000$ GeV.

From these figures, we observe that the production rate decreases sharply as $m_{A_H}$ increases due to the constraint from the phase space of the final state involving the heavy photon pair. The dependence of the cross section on $\sqrt{s}$ in the range of parameters that we use is apparent: when $\sqrt{s}$ becomes large the cross section increases obviously. The relations between the cross section and $m_{A_H}$ are the other way around, when $\sqrt{s}$ is fixed as 500 GeV, $m_{A_H}$ becomes large, the cross section decreases, but when $\sqrt{s}$ takes the value of 1000 GeV, the situations are more complicated, one could see the concrete relations from the figures.

In Fig.4 and Fig.5 we present the transverse momentum distributions of heavy photon for $\sqrt{S}=500$ and 1000 GeV with $m_{A_H}$ equal to 100 GeV, respectively. From the figures, we could see that the differential cross section increases with the transverse momentum until the value reaches the maximum at the point that $p_T$ equals a certain value, then it begins to slide.

3 Discussions and conclusions

The advantages of observing the process of $e^+e^- \rightarrow A_H A_H$ at the ILC are obvious. First, astrophysical probes provide a way to study the characteristics of DM, however, astrophysical observations are unable to determine and examine the exact properties of a DM particle and it mostly remains for collider physics to reveal the nature of DM. Second, the LHC and the ILC may well turn out to be the DM factories and the ILC background is relatively clean, so this paper is a proper supplement to the Refs. [16, 20]. Finally, the heavy photon is the LTP and must be produced in pairs, constraint from the final state phase space of the above process is alleviated compared with other processes.
The combination of cosmology and high energy collider seems to be a good idea \cite{23, 24}, however, at the same time, we have to solve some problems.

The first problem is how to distinguish the $A_H$ pair of the LHT from other DM pair production at the ILC, for example, the lightest neutralino ($\tilde{\chi}^0_1$) of SUSY with R-parity \cite{10, 11}. Since different DM particles have different reaction channels and probabilities to be detected by the detectors of the ILC, which offer a way to distinguish the $A_H$ from others. In the MSSM with R-parity, tree-level production cross-sections of $e^+e^- \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_j$ have been studied systematically, readers who are interested in these processes and want to know more about them can see the paper \cite{25, 26}, so we don’t discuss the topic in detail here.

There is a more serious question we have to face, unlikely the production of heavy photons in $e^+e^-$ annihilation of six-dimensional SUSY QED \cite{27}, DM does not carry either color or electric charge, the direct detection of DM at a collider is very hard. In the collider experiments, DM would be like neutrinos and therein they would escape the detector without depositing energy in the experimental devices, causing an obvious imbalance of momentum and energy in collider events. DM would manifest itself as missing energy, however, DM isn’t the only source responsible for the missing energy. Limited calorimeter resolution, uninstrumented regions of the detector, and additional energy of cosmic rays must all be considered thoroughly in a collider experiment. All of these factors strongly complicate the investigation of DM in the collider experiments. For conservation of momentum, the sum of all momenta transverse to the beam direction must equal zero. Thus, we can ascertain the missing energy by measuring the energy deposited in each calorimeter cell of a detector. If all of the above uncertainties and the background caused by SM neutrinos were been are subtracted and the vector sum of all the transverse momenta is not equal to zero, we could claim that something invisible is produced, the undetected particle(s) may be the DM candidate(s) (such as a heavy photon).

There is a better way to observe the heavy photon pair production. They can be observed via radiative production $e^+e^- \rightarrow A_H A_H \gamma$. In this process, the signal is a single high energetic photon and missing energy, carried by the heavy photons. It would be our next work \cite{28}, we’ll discuss the problem later in detail.

Although it is very hard to detect the DM, $e^+e^- \rightarrow A_H A_H$ is the leading order process of the DM production in the Littlest Higgs model with T-parity at the ILC, so we need to predict the production rate. As a conclusion, we find that the production rate of $e^+e^- \rightarrow A_H A_H$ could reach the level of $10^{-2} fb$ at some small mass parameter space, which is a bit more larger than the process of $e^+e^- \rightarrow A_H Z_H$ \cite{29}, the heavy gauge bosons $A_H$ and $Z_H$ are produced with the cross section of $1.9 \ fb$ at the center of mass energy of 500 GeV and the large mass of heavy $Z_H$ boson(369 GeV) suppresses the final state phase space. The production of $A_H$ may be observable as the missing energy at the ILC thinks to its high energy and luminosity. However, for some uncertain reasons,
the missing energy related to the $A_H$ is very difficult to be accurately determined, so identification of the DM production at the ILC is very hard. We, so far, are not equipped with such expertise and ability to handle the complex analysis, but will definitely cooperate with our experimental colleagues and experts in this field to carry out a detailed analysis.

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