Thermal non-Gaussianity in holographic cosmology

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Abstract

Recently it has been shown that the thermal holographic fluctuations can give rise to an almost scale invariant spectrum of metric perturbations since in this scenario the energy is proportional to the area of the boundary rather than the volume. Here we calculate the non-Gaussianity of the spectrum of cosmological fluctuations in holographic phase, which can imprint on the radiation dominated universe by an abrupt transition. We find that if the matter is phantom-like, the non-Gaussianity $f_{NL}$ can reach $\mathcal{O}(1)$ or even be larger than $\mathcal{O}(1)$. Especially in the limit $\omega \to -5/3$, the non-Gaussianity is very large and negative. Furthermore, since the energy is proportional to the area, the thermal holographic non-Gaussianity depends linearly on $k$ if we neglect the variation in $T$ during the transition (fixed temperature).

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I. INTRODUCTION

Inflation is an extremely successful model in solving the horizon and flatness problems in the standard hot big bang cosmology \[1\]. Furthermore inflation shows us that the CMB anisotropy \[2, 3, 4\] and the observed structures in the universe \[5, 6, 7, 8, 9\] are results of the quantum fluctuations occurring in the very early stage of the universe. Its prediction of a nearly scale invariant spectrum has been confirmed very well in the experiments \[10\]. Nevertheless, the possibility that primordial thermal fluctuations might seed the structure of our universe is also an intriguing alternative to quantum fluctuations \[11, 12, 13, 14\]. Unfortunately the spectral index of thermal fluctuations is either too red \(n_s = 0\) or too blue \(n_s = 4\), failing to generate a nearly scale invariant spectrum \[15, 16, 17\]. Therefore, thermal scenarios call for new physics in order to produce nearly scale invariant fluctuations. For example, a thermal scenario in which the effect of new physics to change the equation of state of thermal matter, can produce nearly scale invariant spectrum. This happens in non-commutative inflation \[12, 18, 19\], and also in LQC \[16, 20\]. Another happens in the Hagedorn phase for certain types of string gas \[21\] or in holographic phase \[17\], in which the energy becomes strongly non-extensive, specifically proportional to the area. In addition, postulating a mildly sub-extensive contribution to the energy density in Near-Milne universe \[15\], the scale-invariance spectrum can also be obtained. In this paper, we focus on holographic thermal fluctuations and investigate the holographic thermal non-Gaussianity.

This scale invariance suggests that the primordial perturbation arising from a thermal holographic phase may be able to seed the structure of observable universe, and lead to the anisotropy observed in CMB as well. Thus it is interesting and significant to ask what is the distinct feature of this holographic primordial perturbations and which is testable in coming observations. There are at least two possible observables, accessible in the near future, that have the potential to rule out or support large classes of models: non-Gaussianity and primordial gravitational waves. In Ref. \[22\], Y. S. Piao has investigated the tensor perturbation. He finds that the tensor perturbation amplitude has a moderate ratio, which may be tested in coming observations. For comparison, we can also see Ref. \[23\] for tensor perturbation from string gas cosmology. Here we focus on the non-Gaussianity in holographic cosmology.

In the WMAP convention, the degree of non-Gaussianity is parameterized by \(f_{\text{NL}}\), which
can be written as

\[ \zeta = \zeta_g + \frac{3}{5} f_{NL} (\zeta_g^2 - \langle \zeta_g^2 \rangle) , \]

(1)

where \( \zeta \) is the primordial curvature perturbation and \( \zeta_g \) is its linear Gaussian part. The introduced factor \( 3/5 \) is convenient for cosmic microwave background comparisons \cite{4, 24}. However, this ansatz only describes the most generic form of non-Gaussianity which is local in real space. Theoretically, the non-Gaussianity estimator \( f_{NL} \) should be defined by the three-point function \( \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \) and has different shapes. We usually focus on two cases. One is the local, squeezed limit \( k_1 \ll k_2 \approx k_3 \), denoted by \( f_{NL}^{local} \). Another is the non-local, equilateral limit \( k_1 \approx k_2 \approx k_3 \), labeled by \( f_{NL}^{equil} \) \cite{25} (for brief, we can also refer to \cite{26}).

Recently, the WMAP 5-year data shows that at 95% confidence level, the primordial non-Gaussianity parameters for the local and equilateral models are in the region \( -9 < f_{NL}^{local} < 111 \) and \( -151 < f_{NL}^{equil} < 253 \), respectively \cite{10}. If this result is confirmed by the future experiments, such as the Planck satellite, then it will be a great challenge to many inflation models. For example, in the simplest single field slow roll inflation model, it has been found that \( |f_{NL}| < 1 \) \cite{27, 28}. Such a non-Gaussianity is too small to detect in the near future. However, there are by now a number of different ways of generating density perturbations from sources other than the fluctuations of slow roll inflation, and in these models it is possible to get larger and observable non-Gaussianity \cite{29, 30, 31, 32}.

In Ref. \cite{33}, B. Chen et al. find that thermal fluctuations can potentially produce large non-Gaussianity. They develop a method to calculate the non-Gaussianity of the fluctuations with thermal origin and apply it to study the non-Gaussianity in string gas cosmology \cite{34}. In this paper, we intend to investigate the thermal non-Gaussianity in the holographic cosmology.

The outline of this paper is the following. We first present a brief review on the holographic thermal fluctuations in section II. Then we use thermal correlation functions to calculate the power spectrum and the non-Gaussianity in section III. Finally, we discuss the conditions which may lead to a large non-Gaussianity in this scenario, in comparison with those in string gas cosmology in section IV.
II. BRIEF REVIEW ON THE HOLOGRAPHIC COSMOLOGY

A holographic version of thermal cosmology was originally proposed by J. Magueijo et al. (for more details, see Ref. [17]). They postulate that the very early universe underwent a phase transition from a high temperature, holographic phase (Phase I) to a usual low temperature phase of standard cosmology (Phase II). In holographic phase, the universe is disordered such that there is no classical metric [35], but its thermodynamics may be described by making use of the holographic principle [36]. The spacetime geometry emerged only during the phase transition such that in Phase II the universe may be described in terms of Einstein gravity theory. The basic idea in holographic cosmology is that owing to the existence of an early holographic phase, in which a specific heat scales as the area of the boundary, the thermal fluctuations which arise in holographic phase can finally imprint a scale invariant spectrum on Phase II through the phase transition [17].

More explicitly, in holographic phase it is conjectured that

$$\langle E \rangle = b M_{pl}^2 T \langle A \rangle,$$  \hfill (2)

where $b$ is some dimensionless constant, $M_{pl}$ is the Planck mass, $T$ is the temperature, and $\langle A \rangle$ is the expectation value of the area of the boundary. Thus, at fixed area, using the relation $c_A = \frac{\partial \langle E \rangle}{\partial T}(\langle A \rangle)$, the specific heat can be expressed as

$$c_A = \frac{b \langle A \rangle}{\hbar G},$$ \hfill (3)

where $\hbar$ is the Plank constant and $G$ is the Newton constant. Therefore, the specific heat at fixed area is proportional to the area which leads to scale-invariant spectrum.

When the temperature falls to a critical temperature $T_c$, the phase transition begins and classical metric emerges. As the concept of length is created, the area and volume can be expressed as $\langle A \rangle = A = 4\pi R^2$, and $V(R) = \frac{4}{3}\pi R^2$ as usual, where $R$ is the thermal correlation length. For more precisely depicting the phase transition process, the dependence of $R$ on $T$ was introduced [17],

$$\frac{R(T)}{l_0} = \left(\frac{T_c}{T_c - T}\right)^\gamma,$$ \hfill (4)

where the relation is valid for $T \leq T_c$, $l_0$ is the smallest scale but not zero, which is determined by quantum geometry, the critical exponent $\gamma$ is introduced to parameterize the speed of the phase transition.
From the above relation, we can see that when $T = T_c$, the correlation length $R$ is divergent and $T < T_c$, $R$ is created, the transition happens and enters into the usual radiation phase of standard cosmology.

In holographic cosmology, the holographic phase has a nearly divergent length, which is required to assure all interesting modes observed today are in causal contact before transition. This solves the horizon problem. In the next section, we will mainly discuss the holographic thermal non-Gaussianity.

III. THE HOLOGRAPHIC THERMAL NON-GAUSSIANITY

In this section, our purpose is to discuss the non-Gaussianity of thermal fluctuations in holographic cosmology. Before discussing the non-Gaussianity, a key question which has to be addressed is on what scale the initial conditions should be imposed. We note that in Ref. [33], they hold the thermal horizon $R$ as a parameter during the calculation. They find that if thermal horizon is smaller than Horizon scale at the horizon crossing, thermal fluctuations can lead to a large non-Gaussianity. In Ref. [19], the thermal correlation length $R = T^{-1}$ is adopted, which is a lower bound and so will lead to larger non-Gaussianity.

In our work, we will adopt the Hubble scale $R = H^{-1}$, beyond which causality prohibits local causal interactions [37]. Therefore we will calculate at $R = H^{-1}$, i.e. $k = a/R = aH$. This means that when $T = T_c$, $R(T)$ is infinite, $H \approx 0$. Next, we will firstly calculate the 2-point correlations, 3-point correlations and the power spectrum. Then we give the non-Gaussianity of holographic thermal fluctuations.

Fluctuations in a thermal ensemble can be determined from the thermodynamic partition function

$$Z = \sum_r e^{-\beta E_r},$$

where the summation runs over all states, $E_r$ is the energy of the state, and $\beta = T^{-1}$. Let $U$ represents the total energy inside region $\mathcal{R}$. Then the average energy of the system is given by

$$\langle U \rangle = \langle E \rangle = \frac{\sum_r E_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}} = -\frac{d \log Z}{d \beta},$$

The 2-point correlation function for the energy fluctuations $\delta U \equiv U - \langle U \rangle$ is given by

$$\langle \delta U^2 \rangle = \frac{d^2 \log Z}{d \beta^2} = -\frac{dU}{d\beta} = T^2 c_{\langle A \rangle},$$

where $c_{\langle A \rangle}$ is the energy correlation function.
Similarly, the 3-point correlation function can be expressed as

\[
\langle \delta U^3 \rangle = -\frac{d^3 \log Z}{d^3 \beta} = \frac{d^2 U}{d^2 \beta} = T^3(2c\langle A \rangle + Tc'\langle A \rangle) .
\] (8)

where prime denotes the derivative with respect to the temperature \( T \).

Since the energy density perturbations \( \delta \rho = \delta E/V \). Using (3) and (7), the 2-point correlation functions of \( \delta \rho \) is given as

\[
\langle \delta \rho^2 \rangle = \frac{\langle \delta U^2 \rangle}{V^2} = \frac{T^2c\langle A \rangle}{R^6} = \frac{4\pi bT^2}{hGR^4} .
\] (9)

Now we calculate the 3-point correlation function of \( \delta \rho \). We note that \( c'\langle A \rangle = 0 \). Using (3) and (8), the 3-point correlation function of \( \delta \rho \) can be expressed as

\[
\langle \delta \rho^3 \rangle = \frac{\langle \delta U^3 \rangle}{V^3} = \frac{T^3(2c\langle A \rangle + Tc'\langle A \rangle)}{R^9} = \frac{8\pi bT^3}{hGR^7} .
\] (10)

Performing the fourier transformation, the density fluctuations \( \delta \rho_k \) in momentum space can be related the fluctuation \( \delta \rho \) in position space by

\[
\delta \rho_k = k^{-\frac{3}{2}} \delta \rho .
\] (11)

In longitudinal gauge (see [38, 39, 40]), and in the absence of anisotropic matter stress, the metric takes the form

\[
ds^2 = a^2(\eta)[-d\eta^2(1 - 2\Phi) + (1 + 2\Phi)dx^2] ,
\] (12)

where \( \Phi \) represents the fluctuations in the metric. Since during the phase transition, \( R(T) \) is infinite, \( H \approx 0 \), we can have \( k \gg H \), which means that the perturbations are deep in the horizon. Thus the Eq. (12) of metric perturbation may be reduced to the Poisson equation

\[
k^2\Phi_k = 4\pi Ga^2\delta \rho_k .
\] (13)

Using the horizon crossing condition \( k = a/R = aH \), the Poisson equation can be expressed as

\[
\Phi_{kL} = 4\pi G\delta \rho_k R^2 .
\] (14)

Using (9), (11) and (14), the 2-point correlation for \( \Phi_k \) is given

\[
\langle \Phi_k^2 \rangle = \frac{(4\pi)^3GbT^2}{\hbar}k^{-3} .
\] (15)
Similarly, the 3-point function for $\Phi_k$ is expressed as

$$\langle \Phi^3_k \rangle = \frac{2(4\pi)^4 G^2 b T^3}{h R} k^{-\frac{9}{2}}. \quad (16)$$

If we neglect the variation in $T$ during the transition (fixed temperature), the power spectrum for $\Phi_k$ can be written as

$$P_\Phi \equiv k^3 \langle \Phi_k^2 \rangle = \frac{32\pi b T^2}{T_{Pl}^2} \quad (17)$$

where $T_{Pl} = \sqrt{\frac{\hbar}{G}}$ is Planck temperature. Owing to the energy proportional to the area, that is a scale-invariant spectrum but not the white noise.

Next, we will calculate the holographic thermal non-Gaussianity. Note that $\Phi$ perturbs CMB through the so-called Sachs-Wolfe effect [41]. However, it is useful to introduce a second variable, $\zeta$, which is the primordial curvature perturbation on comoving hypersurfaces [9, 42]. Then the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ can be calculated theoretically by

$$f_{NL}^{\text{equil}} = \frac{5}{18} k^{-\frac{4}{3}} \frac{\langle \zeta_k^3 \rangle}{\langle \zeta_k^2 \rangle^2}. \quad (18)$$

The variables $\Phi$ and $\zeta$ are related by

$$\zeta = \Phi - \frac{H}{\dot{H}}(\dot{\Phi} + H \Phi). \quad (19)$$

The variable $\zeta$ remaining nearly constant at super-horizon scales for adiabatic fluctuations but $\Phi$ not [38]. However, if the equation of state is constant, then $\Phi$ also remains constant at super-horizon. Therefore the relation (19) reduces to [43],

$$\zeta = \frac{5 + 3\omega}{3 + 3\omega} \Phi. \quad (20)$$

We also note that the primordial curvature variable $\zeta$ is independent of $\omega$, but the variable $\Phi$, which perturbs the CMB, changes as $\omega$ changes. For more detailed discussion on the variables $\zeta$ and $\Phi$, we can refer to [19, 38, 44].

Therefore, combining Eqs. (15), (16), (18) and (20), the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ can be expressed as

$$f_{NL}^{\text{equil}} = \frac{5(1 + \omega) h}{48(5 + 3\omega) \pi^2 b T_c R} = \frac{5(1 + \omega) h H}{48(5 + 3\omega) \pi^2 b T_c}. \quad (21)$$
Using the condition that modes exit the Hubble radius $k = a_c H$ and $k_0 = a_0 H_0$, and considering the relationship between scale factor and temperature $\frac{a}{a_0} = \frac{T}{T_c}$ in radiation-dominated era, we can estimate the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ to be

$$f_{NL}^{\text{equil}} = \frac{5(1 + \omega)\hbar}{48(5 + 3\omega)\pi^2 b} \frac{H_0 k}{T_0 k_0} = \frac{5(1 + \omega)\hbar}{48(5 + 3\omega)\pi^2 b} \times 10^{-30} \frac{k}{k_0},$$

where $H_0$, $T_0$, and $k_0$ represent today’s values, $a_c$ is the scale factor during the phase transition and we have neglected the variation in $a_c$.

From the above relation, we find that if $\omega \to -1$, the non-Gaussianity will be suppressed as in usual inflationary phase. However, if the matter is phantom-like, the non-Gaussianity $f_{NL}^{\text{equil}}$ can reach $\mathcal{O}(1)$ or larger than $\mathcal{O}(1)$ by fine tuning of the equation of state $\omega$. Especially in the limit $\omega \to -5/3$, the non-Gaussianity can be very large. Therefore, the thermal holographic non-Gaussianity depends on what kind of matter in holographic phase (Phase I), in which these fluctuations propagate to the primordial curvature fluctuation $\zeta$ or the potential $\Phi$ when the classical metric emerges in Phase II, so that the large thermal non-Gaussianity occurs in holographic phase imprints on Phase II. Also, we must point out that when $\omega < -1$, the non-Gaussianity estimator $f_{NL}^{\text{equil}} < 0$. Therefore the large non-Gaussianity in holographic cosmology will be negative.

Moreover, the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ depends linearly on the mode $k$. This is similar with string gas cosmology [34], where the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ also depends linearly on the mode $k$, but very different from what happens in inflationary models, where the non-Gaussianities are almost scale invariant. As pointed out in Ref. [34], depending linearly on the mode $k$ means that modes reentering the Hubble radius earlier have a larger non-Gaussianity. Therefore, if non-Gaussianity with an amplitude growing linearly with $k$ were to be detected, then thermal holographic cosmology with phantom-like matter should be desirable.

In addition, we must point out that the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ also depends sensitively on the parameter $b$. If the parameter $b$ is small, then the non-Gaussianity could be large. However, it will be confronted with the same problem as in string gas cosmology, in which if the string scale is decreased to the TeV scale, in order to obtain a power spectrum with reasonable amplitude, a fine-tuning on the temperature $T$ of the thermal string gas should be required [34]. In this one would also require a fine-tuning on the phase transition temperature $T_c$. 

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Now, in fact, if we depict more precisely the phase transition by the relation (4), the spectrum in phase II cannot be exactly scale-invariant and the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ cannot be exactly depends linearly on the mode $k$. In this case, the spectrum can be expressed as more precisely

$$P_\Phi = 32\pi b \frac{T_c^2}{T_{Pl}^2} \left[1 - \left(\frac{l_0}{R}\right)^2\right]^2 = 32\pi b \frac{T_c^2}{T_{Pl}^2} \left[1 - \left(\frac{l_0 T_c H_0 k}{T_0 k_0}\right)^2\right]^2. \quad (23)$$

Similarly, the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ takes the form

$$f_{NL}^{\text{equil}} = \frac{5(1 + \omega)h}{48(5 + 3\omega)\pi^2 b} \frac{H_0 k}{T_0 k_0} \left[1 - \left(\frac{l_0 T_c H_0 k}{T_0 k_0}\right)^{1/\gamma}\right]^{-1} \approx \frac{5(1 + \omega)h}{48(5 + 3\omega)\pi^2 b} \frac{H_0 k}{T_0 k_0} \left[1 + \left(\frac{l_0 T_c H_0 k}{T_0 k_0}\right)^{1/\gamma}\right]. \quad (24)$$

We can see that the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ depends on $k$ in a more complicated manner, which is very different from the string gas cosmology.

**IV. CONCLUSION AND DISCUSSION**

In this paper, we have calculated the non-Gaussianity parameter $f_{NL}^{\text{equil}}$ for thermal fluctuations in holographic cosmology with the use of the strategy developed in \[33\]. Since the energy is proportional to the area, the thermal holographic non-Gaussianity with an amplitude growing linearly with $k$ is obtained if we neglect the variation in $T$ during the transition (fixed temperature). Furthermore, if we assume the dependence of $R$ on $T$ as (4) during the transition, then we find that the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ depends on $k$ in a more complicated manner.

Furthermore, the thermal holographic non-Gaussianity depends on what kind of matter in Phase I. If $\omega \to -1$, the non-Gaussianity will be suppressed as in usual inflationary phase. However, if the matter is phantom-like, the non-Gaussianity $f_{NL}^{\text{equil}}$ can reach $O(1)$ or larger than $O(1)$ by fine tuning of the equation of state $\omega$. Especially in the limit $\omega \to -5/3$, the non-Gaussianity can be very large. In fact, from the expression (18) of the non-Gaussianity estimator $f_{NL}^{\text{equil}}$ and the relation (20), we can immediately see that when the $\omega \to -5/3$, $\zeta \to 0$, so the non-Gaussianity estimator $f_{NL}^{\text{equil}} \to \infty$. If $\omega \to -1$, $\zeta \to \infty$, the non-Gaussianity estimator $f_{NL}^{\text{equil}} \to 0$. In single field slow roll inflationary model, in order to obtain the scale-invariant spectrum, near deSitter spacetime ($\omega \to -1$) is required, then $\zeta \to \infty$, the non-Gaussianity estimator $f_{NL}^{\text{equil}} \to 0$. Therefore, the non-Gaussianity is suppressed in single field slow roll inflationary model. However, in holographic cosmology,
the scale-invariant spectrum is obtained by the specific heat scaling as area in holographic phase. Since the holographic phase transition is determined by geometry rather than the matter content \[17\], the large non-Gaussianity can be achieved by an appropriate choice of \(\omega\). We also note that the phase transition in string gas cosmology depends on the matter content (in the Hagedorn phase \(\omega = 0\) and in the radiation dominated universe \(\omega = 1/3\) \[21, 34\]) but not geometry. So the non-Gaussianity in string gas cosmology is small unless fine tuning of the string scale \[34\]. Therefore, in holographic phase, just as the geometry change abruptly, the large non-Gaussianity can be achieved. However, it is still an open question on what kind of matter will be dominant in early non-geometric phase. It will be very interesting to furthermore explore this question.

It is interesting that the large non-Gaussianity can be achieved by an appropriate choice of \(\omega\) in thermal holographic cosmology. Next, we will also consider the thermal non-Gaussianity in semiclassical loop cosmology and Near-Milne universe \[45\].

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