Decay of quantised vorticity by sound emission

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\textit{It is thought that in a quantum fluid sound generation is the ultimate sink of turbulent kinetic energy in the absence of any other dissipation mechanism near absolute zero. We show that a suitably trapped Bose-Einstein condensate provides a model system to study the sound emitted by accelerating vortices in a controlled way.}

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1. BACKGROUND

If quantised vorticity is created in helium II at temperatures so small that the normal fluid fraction (hence viscosity and friction) is negligible, it is found that it rapidly decay\textsuperscript{1}. It is thought that the mechanism which is responsible for kinetic energy dissipation is the creation of sound waves, or phonons\textsuperscript{2,3}. It was Nore \textit{et al.}\textsuperscript{4} who first found that during the evolution of a vortex tangle the total kinetic energy decreases and the total sound energy increases. By doing a more detailed analysis we revealed that a short sound pulse is emitted at each vortex reconnection\textsuperscript{5}. The importance of vortex reconnections was highlighted when it was discovered that\textsuperscript{6} the vortex cusp which is created at each reconnection event relaxes and triggers large amplitude Kelvin waves (helical displacements of the vortex core); these waves interact non-linearly and generate more waves of higher and higher wavenumber. This Kelvin wave cascade\textsuperscript{7,8}, which is analogous to the Kolmogorov cascade of classical turbulence, thus shifts the kinetic energy to wavenumbers which are large enough that a vortex line can efficiently radiate sound\textsuperscript{9}. Recently we demonstrated\textsuperscript{10} the simultaneous occurrence of both processes (reconnection pulses and sound radiation).
Unfortunately the study of quantised vorticity in helium II suffers from a lack of direct visualisation. Better detection techniques exist for trapped weakly-interacting atomic Bose-Einstein condensates (BECs). The key ingredients of the problem (quantised vorticity, sound waves, reconnections) are present in both systems. The aim of this paper is to show that a trapped Bose-Einstein condensate provides a model system to study sound radiation by vortices in a controlled way. Sound radiation in a BEC is thus not only interesting per se, but it can give insight into the more difficult problem of superfluid turbulence.

2. MODEL

In a recent paper we have suggested that sound emission by a quantised vortex can be studied in a controlled way by letting a vortex precess within a dimple embedded in a weaker harmonic potential which confines a quasi-two-dimensional (quasi-2D) atomic condensate, as illustrated in Fig. 1(a). If the dimple depth is less than the chemical potential the sound (which has an energy of the order of ) escapes, otherwise it remains in the region near the vortex and can be reabsorbed. This configuration thus allows us to control sound radiation in a sensitive way.

Our analysis is based on numerical simulation of the Gross-Pitaevskii (GP) equation in the quasi-2D limit. Fig. (a) shows the density profile of the trapped condensate in the presence of a vortex which is initially located near the axis of the trap. The vortex precesses around the axis, due to the
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Fig. 2. (a) Density plot of a corotating vortex-vortex pair in a homogeneous system. Quadrupolar sound emission, of amplitude $\sim 0.2\% n_0$, is generated from the corotating vortex cores (illustrated by plus signs). (b) Acceleration of the pair as a function of the radius $r$ (bottom axis), calculated from the time-dependent GP equation (black points with error bars), a time-independent numerical approach in the rotating frame (solid black line), and the analytic prediction corresponding to $a = \kappa^2/r^3$ (dashed line). The acceleration of a single vortex precessing in a harmonic trap $\omega = \sqrt{2} \times 10^{-1}(\mu/\hbar)$ is also shown (grey line), as a function of distance from the condensate edge (top axis) $R(\xi) = R_C - r(\xi)$, where the condensate radius $R_C = 10\xi$.

Magnus force: the density gradient due to the trapping potential causes a buoyancy force which induces tangential motion. What interests us here is the fact that the acceleration of the vortex induces emission of sound.

3. RESULTS

If $V_0 \gg \mu$ (deep dimple) the vortex moves around the trap in a closed orbit, maintaining a mean distance from the axis. The sound emitted by the accelerating vortex re-interacts with it and is absorbed. The vortex energy does not decay but executes small oscillations which result from the beating of the vortex precession and the modes of the trapped condensate. If $V_0 \ll \mu$ (shallow dimple), the sound radiated by the accelerating vortex leaves the dimple, the vortex energy slowly decreases, and the vortex spirals outwards to lower densities. The sound waves are emitted in the direction perpendicular to the instantaneous direction of motion in the form of a dipolar pattern. The precessional motion of the vortex converts the dipolar emission into a spiral wave pattern—see Fig. 2(b). For a quasi-2D BEC, we find that the power radiated by the vortex is $P = \beta mN(a^2/\omega)$, where
Fig. 3. Sound burst produced by the close approach of three vortices. (a) The trajectory of the vortex-antivortex pair (solid line), initially located at \((0\xi, 20\xi)\) and \((5\xi, 20\xi)\), is deflected as it approaches a single vortex located at the origin. The density plot shows the final density distribution of the vortices (corresponding to dark spots). (b) The final density distribution, shown on a different density and length scale, shows a burst of sound which propagates radially outwards. (c) Acceleration experienced by the vortex in the pair nearest to the single vortex, for a vortex pair initially located at \((d, 20\xi)\) and \((d + 5\xi, 20\xi)\), where \(d/\xi = 0\) (solid line), 1 (dashed line), 2 (dotted line) and 4 (dot-dashed line). (d) Final radius of the pair following the interaction (rescaled by the initial radius of \(5\xi\)), as a function of distance \(d\).
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$a$ is the vortex acceleration, $\omega$ is the vortex angular velocity (induced by the trap), $N$ is the number of atoms in the BEC, and the dimensionless parameter $\beta \approx 6.3$ is determined by fitting. Our result is in fair agreement with calculations, based on the analogy to $(2+1)$-D electrodynamics\(^{(12)}\) (where vortices and phonons map onto charges and photons, respectively\(^{(13)}\) and classical acoustics\(^{(9)}\), which yield $\beta = \pi^2/2$, assuming perfect circular motion for a point vortex in a homogeneous system.

In Helium II, the relevant parameter is the decay of vortex line length $L$, typically given by\(^{(9)}\) $\frac{dL}{dt} = 4\pi P/[N\kappa^2 \ln(b/\xi)]$, where $\kappa = \hbar/m$, and $b$ is the average intervortex spacing in the tangle. Assuming a generalisation of our single vortex power radiation formula to a system of vortices with average separation $L^{-1/2}$, we can cast our result into Vinen’s form\(^{(9)}\) $\frac{dL}{dt} = -\alpha (\kappa/2\pi) L^2$, where $\alpha = \beta/[\pi \ln(\xi L^{1/2})]$. This generalisation of a numerically-computed expression based on the GP equation yields the correct power law decay of vortex line length. Using typical Helium II numbers, we obtain $\alpha \sim O(0.1)$ for the scenario considered here corresponding to a vortex decay driven by density inhomogeneity. This is larger than the value obtained for the ‘free’ decay arising from Kelvin wave excitations\(^{(10)}\) but still smaller than the experimentally obtained value for Helium II\(^{(9)}\). To establish a link between single vortices driven by trap inhomogeneity and superfluid turbulence we first consider the simple example of a vortex-vortex pair. As the vortices corotate about their central point, they radiate quadrupolar sound waves, which form a double-armed spiral wave pattern, as shown in figure Fig. 2(a). Fig. 2(b) shows the acceleration of a vortex-vortex pair in a homogeneous system and a single vortex in a harmonic trap. Although the acceleration is similar in both cases, the sound emission from the pair is less due to the quadrupolar character of the emission. This suggests that our value for $\alpha$ is an over-estimate for the case of a vortex tangle. The acceleration of the vortex-vortex pair tails off as the vortex cores merge ($r \to 0$), in contrast to the analytic prediction of Pismen\(^{(14)}\). Note that the acceleration of a vortex in a harmonic trap is strongly dependent on the trap frequency.

As an example of a three-vortex interaction, we consider the close approach of a vortex-antivortex pair towards a single vortex, as illustrated in Fig. 3(a). The single vortex and its nearest neighbour in the pair have the same polarity. During this interaction, the trajectory of the pair becomes deflected (black lines in Fig. 3(a)), while the single vortex makes small deviations about the origin (not shown). The sharp acceleration of the vortices (indicated in Fig. 3(c)) induces a sound burst, which propagates radially outwards. This is visible in the final density distribution shown in Fig. 3(b). This energy loss results in a reduction in the size of the vortex pair, shown in Fig 3(d). The magnitude of the loss decreases as the initial posi-
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tion of the pair is moved further away from the single vortex, in line with the decreasing acceleration experienced by the vortices (Fig.3c)). Note that the magnitude of the acceleration induced by the close approach is similar to the values studied in the dimple trap. Finally, if the single vortex and its nearest neighbour in the pair have opposing polarity, the interaction tends to involve a reconnection whereby these vortices form a pair, and leave behind the other vortex.

In summary, we have shown how sound emission can be controlled and quantified in experiments on dilute Bose-Einstein condensates. We discuss how the information gained can be mapped onto the understanding of the decay of superfluid turbulence in the limit of low temperature. However, further work is needed to exactly quantify the sound emission for the case of many vortices.

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