Elasto-plastic phase-field model for pullout tests of steel fiber embedded in high-performance concrete: numerical calibration and experimental validation

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In this paper, the pullout behavior of steel fiber embedded in high-performance concrete (HPC) is investigated. First, a constitutive framework of elasto-plastic phase-field model of fracture is constructed and applied to simulation of pullout tests for different embedded lengths of steel fibers. Thereby, the description of the mechanical behavior the Drucker-Prager plasticity model is used. The numerical calibration of the above mentioned model is performed by simulating pullout tests and validated by data obtained from experiments. Therefore, the mechanical behavior of high-performance concrete is studied in experiments.

1 Introduction

In the recent years new innovative concrete types such as high and ultra high-performance concrete (HPC & UHPC) has been investigated elaborately. However, HPC and UHPC fail in brittle manner due to its low resistance to cracking. Thus, to improve the strength of the concrete and its resistance to cracking were the main challenges in the last decades. The embedding of short steel fibers is common practice to increase the strength of the concrete as well as its resistance to cracking. These steel fibers influence the overall material behavior of HPCs with respect of formation and evolution of fracture. On this account the DFG priority program 2020 (SPP 2020) has been founded and aims the elaboration, understanding, description and prediction of degradation of HPCs, wherein the authors of this contribution have a joint project to work on the mentioned task. In the first step single steel fiber pullout tests are elaborated experimentally and numerically to set up a framework for the study of fiber’s influence on the overall material behavior. An elasto-plastic-phase-field model for fracture in concretes using the well known Drucker-Prager yield condition is implemented. Numerical calibration and validation of the presented model for experimental pullout tests has been performed.

2 Constitutive framework of elasto-plastic phase-field model

The phase-field parameter \( q \) representing the unbroken \((q = 0)\) and the fully broken \((q = 1)\) state of the material is introduced in [1]. Therein, the phase-field parameter \( q(x, t) \) is given by the minimization principle

\[
q(x, t) = \arg\{ \inf_{q \in W} \Gamma_1(q) \} \quad \text{with} \quad \Gamma_1(q) = \int_B \gamma(q, \nabla q) \, \text{d}x \quad \text{and} \quad \gamma(q, \nabla q) = \frac{1}{2} q^2 + \frac{1}{2} \|\nabla q\|^2, \tag{1}
\]

which is regularized by the crack functional \( \Gamma_1(q) \) using the crack surface density function per unit volume \( \gamma(q, \nabla q) \) of the solid. It is governed by the length scale parameter \( l \). Where \( l \to 0 \), achieves a sharp crack interface. Following [2], the free energy function \( \psi \), which depends on elastic strains \( \varepsilon^e \), the phase-field parameter \( q \) and its gradient \( \nabla q \) as well as the internal field variable \( \alpha \) describing equivalent plastic strains, reads

\[
\psi\left(\varepsilon^e, \alpha, q, \nabla q\right) = \psi^{ep}\left(\varepsilon^e, \alpha, q\right) + \psi^c - g(q, m) \psi^e + 2 \frac{\psi^c}{\zeta} \|\nabla q\| \quad \text{with} \quad \varepsilon^e := \varepsilon^e - \varepsilon^p. \tag{2}
\]

The specific critical fracture energy \( \psi^c \) is a material parameter, with \( \psi^c > 0 \), which controls the crack threshold. The parameter \( \zeta \) controls the post critical stress softening, especially its shape, due to fracture. The degradation function \( g(q, m) = (1 - q)^m \) is introduced in [3]. It provides the flexibility in controlling the speed of the fracture evolution by the newly introduced parameter \( m \) with its properties \( g(0, m) = 1 \), \( g(1, m) = 0 \) and \( g'(1, m) = 0 \). Potential of alternative degradation functions in the context of crack nucleation and propagation is discussed in [4].

The elasto-plastic energy function \( \psi^{ep} \) is additively decomposed into a reference elastic energy function \( \psi_0^{ep}(\varepsilon^e) \) and a reference plastic energy function \( \psi_0^{p}(\alpha) \) depending on the flow stress \( y_0 \), the equivalent plastic strain \( \alpha \) and hardening parameter \( h \), i.e.,

\[
\psi^{ep} = g(q, m) \left( \psi_0^{ep*}(\varepsilon^e) + \psi_0^{p*}(\alpha) \right) + \psi_0^{c*}(\varepsilon^e) \quad \text{with} \quad \psi_0^{ep}(\varepsilon^e) = \psi_0^{p*}(\varepsilon^e) + \psi_0^{c*}(\varepsilon^e) \quad \text{and} \quad \psi_0^{p*}(\alpha) = y_0 \alpha + \frac{1}{2} h \alpha^2. \tag{3}
\]

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Therein, the reference elastic energy function $\psi_0^0(\varepsilon^e)$ associated with the undamaged elastic solid is additively decomposed. This is represented by a positive part $\psi_0^{+}(\varepsilon^e)$ and a negative part $\psi_0^{-}(\varepsilon^e)$ of reference energy function as proposed in [5], i.e.,

$$\psi_0^{+}(\varepsilon^e) = \kappa \left( \langle \text{tr}[\varepsilon^e] \rangle^2 + 2 + \mu \| \text{dev} \varepsilon^e \| \right)^2$$

and

$$\psi_0^{-}(\varepsilon^e) = \kappa \left( \langle \text{tr}[\varepsilon^e] \rangle^2 / 2 \right),$$

where $\mu$ and $\kappa$ are the Lamé coefficients and Macaulay’s notation are used to describe the ramp function $(\bullet)_\pm = 1/2 (\bullet \pm |\bullet|)$. This formulation allows the crack to propagates only due to expansion of material. That means the spherical part of the reference energy function must not degrade due to negative volume change and whole reference energy function should degrade at positive volume change. We get the similar structure of the energy density function $\psi$, as introduced in [2],

$$\psi(\varepsilon, \alpha, q, \nabla q) = (1 - q)^m \left( \psi_0^{+}(\varepsilon^e) + \psi_0^{-}(\varepsilon^e) + \psi^{+}(\varepsilon^e) + \psi^{-}(\varepsilon^e) + 2 \psi^e \frac{1}{2} q^2 + \frac{l}{2} \| \nabla q \|^2 \right).$$

Here, the derivative of the energy function gives the stress tensor

$$\sigma := \partial_{\varepsilon} \psi = (1 - q)^m \left[ \kappa \langle \text{tr} \varepsilon^e \rangle, I + 2\mu \text{dev} \varepsilon^e \right] + \left[ \kappa \langle \text{tr} \varepsilon^e \rangle \cdot I \right].$$

The governing equation for the phase-field parameter is the Ginzburg-Landau and Allen-Cahn-type evolution equation, taking the value of parameter $m = 2$, according to the definitions given in [1],

$$q - l^2 \text{Div} [\nabla q] - (1 - q) H = 0.$$

To ensure irreversibility of the crack evolution the maximum value of a dimensionless crack driving state function is considered in accordance to [2], by

$$H := \max_{r \in [0, t]} H_0(x, s) \geq 0 \quad \text{with} \quad H_0 = \left( \frac{\psi_0^{+}(\varepsilon^e)}{\psi^e} + \frac{\psi_0^{-}(\alpha)}{\psi^e} - 1 \right).$$

For the description of non-linear behavior of concrete materials we considered the associative Drucker-Prager yield criteria [6]

$$\phi(I, I_D) = \sqrt{I_D + \beta_p I - \kappa_p} = 0 \quad \text{with} \quad I_D = \frac{1}{2} \| \text{dev} \sigma_0 \|^2, \quad I = \text{tr} \sigma_0 \quad \text{and} \quad \kappa_p := \partial_\alpha \psi_0^0 = y_0 + h \alpha,$$

where $I$ and $I_D$ are first and second invariant of effective stress tensor $\sigma_0 = \sigma_0^+ + \sigma_0^-$, see Eq. 6. $\beta_p$ is a material parameter and $\kappa_p$ is given by the linear isotropic hardening rule. The system of equations for numerical simulation of elasto-plastic fracture phase-field model contains the balance of linear momentum, governing equation for phase-field parameter, equation describing evolution of plastic strains $\varepsilon^p$ expressed in terms of the incremental plastic consistency parameter $\lambda^p$ and Kuhn-Tucker conditions, respectively, as

$$\begin{align*}
(1) : & \text{Div} \sigma = 0, \\
(2) : & q - l^2 \text{Div} [\nabla q] - (1 - q) H = 0, \\
(3) : & \varepsilon^p = \lambda^p \frac{\partial \phi(I, I_D)}{\partial \sigma} = 0, \\
(4) : & \phi \leq 0, \ \lambda \geq 0, \ \lambda \phi = 0,
\end{align*}$$

in the domain $B$ along with the boundary conditions $\sigma \cdot n = t$ on $\partial B$ and $\nabla q \cdot n = 0$ on $\partial B$. We implemented the above mentioned model in the framework of the Finite Element Method to analyze the fracture occurring in concrete. We solve the weak forms of the balance of linear momentum and the governing equation for phase-field parameter with incrementally decoupled updates using the staggered scheme documented in [1] and an adopted algorithm used in [7].

### 3 Experimental setup

To provide a feasible data basis for numerical simulation, pullout tests in direct tension on single steel fibers are conducted. All tests are performed using the HPC reference mixture used within the German priority program 2020. The concrete only consists of ordinary Portland cement, quartz sand, water and a small dosage of superplasticizer. To assess the properties of the concrete mixtures, multiple series of tests on cylindrical specimens have been investigated. All properties have been tested at a concrete age of 28 days. The cumulative results from different partner within SPP 2020 are given in Fig. 2a, see [8]. The experimental program presented here consists exclusively of typical single hook-end fibers of the type 3D 55/60 kindly provided by Bekaert, with a tensile strength of 1150 MPa, see Fig. 1a. To be able to determine bond strength and slip individually, straight fibers were generated by cropping the hook-ends of several fibers and deburring the cuts if necessary. In order to test the pullout behavior, the fibers were cast in the HPC prisms measuring 160 mm x 40 mm x 40 mm, each prism...
Fig. 1: (a) Single hook-end fibers of the type 3D 55/60 provided by Bekaert., (b) Embedded steel fibers with hooked ends before fiber pull-out test.

| Property           | Mean value | unit |
|--------------------|------------|------|
| Tensile strength $f_t$ | 5.7        | MPa  |
| Compressive strength $f_c$ | 112.0      | MPa  |
| Young’s modulus $E$      | 39.976     | GPa  |
| Poisson’s ratio $\mu$   | 0.192      | –    |

Fig. 2: (a) Mechanical properties of HPC, (b) testing setup.

containing three fibers, which were tested individually, see Fig. 1b. The fibers are tested for the embedded length of 20 mm and 30 mm for the straight fibers. The pullout tests are conducted in an electromechanical single column testing system, an Instron 5944 using pneumatic clamps for fixing the fibers. The prisms are fixed to the testing rig as shown in Fig. 2b. Two transducers (LVDT) attached to both sides of the prism are used to measure the slip between fiber and concrete matrix, i.e. the fiber pullout. The fibers are pulled out of the concrete at a constant displacement rate of three millimeters per minute. The influence of the embedded lengths on the pullout behaviour several tests were performed at a pullout rate of 0.3 mm/s using fibers with an embedded length of 20 mm and 30 mm. The curves generally show comparable characteristics, but some differences can be found, cf. Fig. 4a & 4b. Due to the often pronounced scatter of fiber pullout tests, especially at the softening, it is necessary to average the results. Since this averaging process also tends to smoothen the curves, pronounced points in the curve progression like the overall force-wise maximum can be affected disproportionally. Therefore, two different approaches for the statistical evaluation are used, as described in [8], intending to provide better possibilities to calibrate the numerical models.

4 Numerical example

The predictive capability of the presented model is checked by simulating the pullout behavior of a single embedded steel fibers. Fig. 3a shows the geometry and boundary conditions of a 2D test setup which is constructed according to actual experimental setup shown in Fig. 2b. There $l_f$ represents the embedded length of a fiber. In the simulation the loading is applied by a vertical displacement boundary condition $u_t$ at the top of the steel fiber. For the analysis the reaction forces of the constrained nodes at the fibers upper tip are computed and reported. The presented model is first calibrated for fibers with an embedded length of 20 mm to get similar behavior of test specimen as in experiments. Furthermore, the model is validated

| $E$ (GPa) | $\nu$ | $y_0$ (GPa) | $\psi^c$ | $h$ (Gpa) | $\beta_p$ | $l$ (mm) | $m$ | $\zeta$ |
|-----------|-------|-------------|-----------|-----------|-----------|----------|-----|--------|
| HPC       | 39.9  | 0.19 | 3e-3 | 2e-5 | 1e-5 | 2e-3 | 0.04 | 0.5 | 0.1 |
| Steel     | 210   | 0.3  | 0.45 | 1e-2 | 0.13 | 0 | 0.04 | 0.5 | 0.1 |

Fig. 3: Two dimensional boundary value problem for analysis of steel fiber pullout behavior. Analysis of steel fiber pullout behavior: (a) two dimensional boundary value problem, (b) material parameters.
for fibers with an embedded length of 30 mm. FE simulation and standard deviation of the averaged of the experimental data are compared in the load-displacement diagrams for $l_f = 20$ mm in Fig. 4a and for $l_f = 30$ mm in Fig. 4b. The calibrated material parameters are given in Fig. 3b. To consider the debonding and slipping in the steel-concrete interface the value of the specific critical fracture energy $\psi^{c^*}$ for elements surrounding the fiber is set to 50% of the value $\psi^{HPC}$ for HPC. In Fig. 5 the distribution of stresses in $x$-direction $\sigma_x$, equivalent plastic strains $\alpha$ and phase-field parameter $q$ at the peak load and at the displacement of $u_t = 0.5$ mm are shown.

![Load-displacement diagram](image1)

![Stress distribution](image2)

Fig. 4: Load-displacement diagram of experimental data and FEM simulation for (a) $l_f = 20$ mm and (b) $l_f = 30$ mm.

![Stress distribution](image3)

Fig. 5: Numerical simulation of steel fiber pullout with $l_f = 20$ mm: stresses $\sigma_x$, equivalent plastic strains $\alpha$ and phase-field parameter $q$ in (a),(c),(e) at peak load and (b),(d),(f) at the displacement of $u_t = 0.5$ mm, respectively.

5 Conclusion

In this contribution, an elasto-plastic phase-field model for fracture in high-performance concrete is presented which incorporates a diffusive crack interface approach. A Drucker-Prager plasticity model is formulated for crack propagation in high-performance concrete. The efficiency of the presented numerical model is tested by simulating the pullout behavior of a single embedded steel fibers using FEAP (UC, Berkeley). The experimental results were used to calibrate and validate the presented model for the numerical simulation of pull-out tests on fibers. From the results the good feasibility of the model to describe the damage behavior of concrete in a pullout test is shown.

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