The scalar sector of 3-3-1 models

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I. INTRODUCTION

The 3-3-1 models of electroweak interactions [1], based on the SU(3)$_L$⊗U(1)$_N$ gauge group, predict some new particles which lead to new physics and interesting phenomenology. In particular, these models predict no very high new mass scales, in such a way that they can be confirmed or ruled out in a near future [2,3]. They belong to a type of gauge models which has the SU(2)$_L$⊗U(1) electroweak doublet contained in a SU(3)$_L$⊗U(1) triplet. The SU(3)$_L$⊗U(1)$_N$ $\mapsto$ SU(2)$_L$⊗U(1)$_Y$ break scale is estimated by running sin$^2\theta_W$ towards large values, which give the bound $w \lesssim 3$ TeV [4].

This type of models, that includes the unified ones and chiral extensions of the standard model, is characterized by SU(2) doublets $(X^{\pm}, X^{\pm\pm})$ of vector and scalar gauge bosons which carry lepton number $L = 2$ (dileptons). The dileptons mediate processes which violate individual lepton numbers. The total lepton number ($L = L_e + L_\mu + L_\tau$) is conserved.

There are some aspects which turn the 3-3-1 models a very interesting extension of the standard model [1,4,5]. The main two of them are the prediction about the family number (which must be multiple of the color number) and the bound for the Weinberg angle ($\sin\theta_W < 1/4$). Experimentally an interesting feature is that the mass scale of the vector dileptons can be as low as the weak scale. Thus, the lower bound on these masses can be obtained from the available data of the weak processes such as muon decay, low energy neutral current experiments, Bhabha scattering, etc [3,6].

In this work we discuss the Higgs potential of the 3-3-1 models at the tree level. The phenomenology of the scalar sector of the 3-3-1 models is not studied yet. In the model of Refs. [1], the minimal number of Higgs multiplets required in order to generate all fermion masses is four: three triplets and one sextet. The latter is necessary in order to get the masses of the charged leptons. However, if we assume the presence of heavy leptons [7] the sextet is not necessary. Also, if a right-handed singlet neutrino is added, it is possible to generate the right mass spectrum of the charged lepton through radiative corrections without introducing the sextet of Higgs bosons [8]. Thus, we can consider the VEV of the
sextet rather small (a few GeV) relative to the VEV’s of the triplets which are \( \sim 100 \text{ GeV} \) (two) and \( \sim (1 - 3) \text{ TeV} \) (one). So, the three triplet models can be a good approximation for the scalar sector of the 3-3-1 models.

Here we study both, the three triplet and the three triplet and one sextet models. Each of them has neutral CP-even and CP-odd scalar bosons as well as singly charged and doubly charged ones. In this type of models there can be charged scalar masses of the same order (or smaller) than the neutral scalar ones. In the approximation which we are using here it is always possible to identify one of the Higgs bosons with the scalar of the minimal \( SU(2)_L \otimes U(1)_Y \) model.

In the next section we discuss the Higgs potentials of the models, presenting the masses and eigenstates for the two 3-3-1 models which are considered here. In Sec. 3 we summarize our results.

II. THE SCALAR POTENTIALS

The details of the 3-3-1 models were already pointed out in several works (see, for example, Refs. [1,4]). Therefore, we write below only the scalar fields. The scalar sectors which are considered here are the minimal ones. We also do not consider CP violation through the scalar exchange [1].

A. The Three Triplet Model

In this case the scalar sector is composed by the three \( SU(3) \) triplets

\[
\eta = \begin{pmatrix} \eta^0_0 \\ \eta^-_1 \\ \eta^+_2 \end{pmatrix} \sim (3, 0); \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (3, 1); \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (3, -1). \quad (1)
\]

The most general, renormalizable and gauge invariant \( SU(3)_L \otimes U(1)_Y \) Higgs potential which we can write with the three triplets of the Eqs. (1) is given by
\( V_T(\eta, \rho, \chi) = \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 + \\
+ (\eta^\dagger \eta) [\lambda_4 (\rho^\dagger \rho) + \lambda_5 (\chi^\dagger \chi)] + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_7 (\rho^\dagger \eta) (\eta^\dagger \rho) + \\
+ \lambda_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \chi) (\chi^\dagger \rho) + \left( \frac{f_1}{2} \varepsilon^{ijk} \eta_i \rho_j \chi_k + H.c. \right), \) \hspace{1cm} (2)

where the \( \mu \)'s, \( \lambda \)'s and \( f_1 \) are coupling constants, the latter with dimension of mass. Since the original model of Refs. [1] do not consider neutrino mass [9], the potential (2) is \( F \)-conserving [10], with \( F = L + B \), where \( L \) is the total lepton number and \( B \) is the baryon number.

Let the neutral scalars \( \eta^0, \rho^0, \) and \( \chi^0 \) acquire a vacuum expectation value \( v_\eta, v_\rho, \) and \( v_\chi, \) respectively, and define

\[ \varphi = v_\varphi + \xi_\varphi + i\zeta_\varphi, \] \hspace{1cm} (3)

with \( \varphi = \eta^0, \rho^0, \chi^0. \) We will use the notation \( v_\eta \equiv v, v_\rho \equiv u, \) and \( v_\chi \equiv w. \) The pattern of the symmetry breaking is

\[ \text{SU}(3)_L \otimes \text{U}(1)_N \overset{\langle \chi \rangle}{\longrightarrow} \text{SU}(2)_L \otimes \text{U}(1)_Y \overset{\langle \rho, \eta \rangle}{\longrightarrow} \text{U}(1)_{\text{em}}, \]

and the VEV’s are related with the standard model one \( (v_W) \) as \( v^2 + u^2 = v^2_W. \)

The \( \xi_\varphi (\zeta_\varphi) \) fields lead to three CP-even (one CP-odd) physical scalar bosons. The exact form of the CP-even mass and eigenstates are troublesome. Thus, we will give the masses and eigenstates of this sector in an approximate form and for clearing this point we will consider this approximation in some detail. The square mass matrix coming from the \( \xi_\varphi \) fields reads as

\[ M_\xi^2 = \frac{1}{2} \begin{pmatrix}
8\lambda_1 v^2 - f_1 uw/v & 4\lambda_4 vu + f_1 w & 4\lambda_5 vw + f_1 u \\
4\lambda_4 vu + f_1 w & 4\lambda_2 u^2 - f_1 vu/w & 4\lambda_6 uw + f_1 v \\
4\lambda_5 vw + f_1 u & 4\lambda_6 uw + f_1 v & 8\lambda_3 w^2 - f_1 vu/w
\end{pmatrix} \hspace{1cm} (4)\]

in the \( \xi_\eta, \xi_\rho, \xi_\chi \) basis. Here we have already imposed the constraints due to the non-linearity of the shifted potential in the \( \xi_\varphi \) and \( \zeta_\varphi \) fields and we have not considered phases in the VEV’s. A natural approximation is to impose \( |f_1| \sim w \) (this also avoids a new mass scale
beyond $w$ in the model) and to maintain only terms of higher order in $w$ in Eq. (3). But this results in one zero mass ($H_{0}^{1}$) and two massive physical states ($H_{0}^{2}$ and $H_{0}^{3}$) with masses

$$m_{02}^{2} \approx \frac{v^{2} + u^{2}}{2vu} w^{2}, \quad m_{03}^{2} \approx -4\lambda_{3} w^{2},$$

(5)

associated with the approximate eigenstates

$$\left( \begin{array}{c}
\xi_{\eta} \\
\xi_{\rho}
\end{array} \right) \approx \frac{1}{(v^{2} + u^{2})^{1/2}} \left( \begin{array}{cc}
v & u \\
u & -v
\end{array} \right) \left( \begin{array}{c}
H_{1}^{0} \\
H_{2}^{0}
\end{array} \right),$$

(6a)

$$\xi_{\chi} \approx H_{3}^{0}.$$  

(6b)

Thus, the former eigenstate ($H_{0}^{1}$) must be a light Higgs boson, similar to that of the standard model. In order to confirm this assertion and to improve the approximation we take the eigenvalue equations with the exact mass matrix of Eq. (4) for the eigenvalue $m_{01}^{2}$, associated with $H_{1}^{0}$ field, with the correspondent approximate eigenstate and identify a new contribution to the initially null $m_{01}^{2}$ eigenstate. So, it is easy to see that the $H_{1}^{0}$ is a state associated with the mass

$$m_{01}^{2} \approx 4\lambda_{2} u^4 - \lambda_{1} v^4, \quad (v^2 - u^2)^{-1}.$$  

(7)

with the following relations among the coupling constants and VEV’s:

$$\lambda_{4} \approx 2\frac{\lambda_{2} u^2 - \lambda_{1} u^4}{v^2 - u^2}, \quad \lambda_{5} v^2 + 2\lambda_{6} u^2 \approx -\frac{vu}{2}.$$  

(8)

Notice that the $m_{01}^{2}$ mass of Eq. (4) is a function of $v$ and $u$ VEV’s only. So, this is the light scalar which we identify with the standard model Higgs boson. However, since the $\lambda$’s are still unbounded and the light Higgs mass is function of $(v^2 - u^2)^{-1}$, we have a large range for the mass scales, as in the standard model.

Concerning the imaginary part of the right-hand side of the Eq. (3) ($\zeta_{\varphi}$ fields) we have two Goldstone bosons and one massive physical state with exact mass

$$m_{h}^{2} = -\frac{f_{1} w}{vu} \left[ v^2 + u^2 + \left( \frac{vu}{w} \right)^2 \right],$$

(9)

where $f_{1} < 0$.  

5
The symmetry eigenstates in the CP-odd sector are related to the physical states as follows:

\[
\begin{pmatrix}
\zeta_n \\
\zeta_\rho
\end{pmatrix} \approx \frac{1}{(v^2 + u^2)^{1/2}} \begin{pmatrix} v & u \\ u & v \end{pmatrix} \begin{pmatrix} G_1^0 \\ G_2^0 \end{pmatrix},
\]

where the \( G^0 \)'s states are neutral Goldstone bosons and \( h^0 \) is the CP-odd massive neutral physical state.

In the charged scalar sector, we have obtained all masses and eigenstates exactly. In the singly charged sector we have two Goldstone bosons and two physical Higgs whose masses read

\[
m^2_{+1} = \frac{v^2 + u^2}{2vu} (f_1 w - 2\lambda_7 vu), \quad m^2_{+2} = \frac{v^2 + w^2}{2vw} (f_1 u - 2\lambda_8 vw),
\]

with mixings

\[
\begin{pmatrix}
\eta^+_1 \\
\rho^+
\end{pmatrix} = \frac{1}{(u^2 + w^2)^{1/2}} \begin{pmatrix} -v & u \\ u & v \end{pmatrix} \begin{pmatrix} G_1^+ \\ H_1^+ \end{pmatrix},
\]

\[
\begin{pmatrix}
\eta^+_2 \\
\chi^+
\end{pmatrix} = \frac{1}{(u^2 + w^2)^{1/2}} \begin{pmatrix} -v & w \\ w & v \end{pmatrix} \begin{pmatrix} G_2^+ \\ H_2^+ \end{pmatrix}.
\]

In the doubly charged sector there is only one doubly charged Goldstone and one physical Higgs boson with mass

\[
m^2_{++} = \frac{u^2 + w^2}{2uw} (f_1 v - 2\lambda_9 uw).
\]

The doubly charged mass eigenstates are given by

\[
\begin{pmatrix}
\rho^{++} \\
\chi^{++}
\end{pmatrix} = \frac{1}{(u^2 + w^2)^{1/2}} \begin{pmatrix} -u & w \\ w & u \end{pmatrix} \begin{pmatrix} G^{++} \\ H^{++} \end{pmatrix}.
\]

From Eqs. (5), (7), (9), (11) and (13) we have the conditions for the potential to be below bounded:
\[ \frac{\lambda_1}{\lambda_2} \begin{cases} \approx u^4/v^4, & \text{if } v > u; \\ \approx u^4/v^4, & \text{if } v < u; \end{cases} \]  

\( v \neq u, \lambda_3 \lesssim 0, f_1 < 0; \) \[ \frac{f_1}{\lambda_7} < 2\frac{vw}{u}, \quad \frac{f_1}{\lambda_8} < 2\frac{vw}{u}, \quad \frac{f_1}{\lambda_9} < 2\frac{uw}{v}. \]  

(15a) (15b)

We recall that the masses of the CP-odd neutral scalar and the charged ones are exact.

The light Higgs boson \( H_1^0 \) can be of interest since it imitates the standard model one. The trilinear \((H_1^0Z^0Z^0)\) and quartic \((H_1^0H_1^0Z^0Z^0)\) interactions of the light neutral scalar boson are

\[ g^2\sqrt{3} \left[ u^2w^2 + 3(u^4 + 2v^2w^2 + 2w^4t^4) \right] + 6uw(v^2 + w^2t^2) \frac{6\sqrt{3}u^2w^2}{6\sqrt{3}u^2w^2}, \]  

(16a)

\[ g^2\sqrt{3} (u^2w^2 + 3v^4) + 3w \left[ (v^2 + t^2w^2) \left( 2u + \sqrt{3}t^2w \right) + 2\sqrt{3}u^2wt \right] \frac{12\sqrt{3}u^2w^2}{12\sqrt{3}u^2w^2}, \]  

(16b)

where \( t \equiv g'/g \) is the ratio among the coupling constants of the U(1) and the SU(3) groups, respectively. Since the coupling strengths are very different from the standard model ones (and other popular gauge models), the current results of the collider experiments on mass bounds are not directly applicable to 3-3-1 scalar bosons.

The scalar masses are functions of the unknown scalar potential parameters and of the VEV’s of the scalar neutral fields which are poorly bounded and so, the range for scalar mass scale is large.

As we comment in the Introduction the VEV of the sextet neutral field must be small in such a way that the three triplet case be a good approximation to the original model of Ref. [1]. If we also assume the condition \( w \sim -f_1 \) in the charged sector, the masses of the singly charged scalar [Eqs. (11)] and the doubly charged [Eq. (13)] scale with \( w^2 \), like the neutral scalar \( H_2^0 \) in Eqs. (3). In particular we have in this approximation \( m_{02} = m_{+2} \). On the other hand, \( m_{01} \) in Eqs. (7) can be large for some set of parameters. Therefore, it is possible in this model a charged Higgs mass be smaller than a neutral one. If the decay of the neutral Higgs in pairs of charged ones is kinematically allowed it is a good channel for Higgs searching [12]. The \( H_2^0 \) scalar boson is similar to the standard model one. The
similarity becomes exact if we put \( v(u) = 0 \) with \( v^2 + u^2 = (246 \text{ GeV})^2 \). In such cases the SU(2) scalar doublet is embedding in the \( \eta(\rho) \) 3-3-1 triplet of Eqs. (1).

A graphical analysis of \( m_{01} \) and the charged scalar masses is spoiled by our ignorance on the Higgs potential constants. However, we plot in the Figure 1 the curves for \( m_{02} \) and \( m_h \), which show that these masses can be relatively small for a large range of \( v \) VEV. There is practically no difference for \( 1 \text{ TeV} < w < 3 \text{ TeV} \).

**B. The Three Triplet and One Sextet Model**

Here we need to add the sextet

\[
S = \begin{pmatrix}
\sigma_1^0 & s_2^+ & s_1^- \\
 s_2^+ & S_1^{++} & \sigma_2^0 \\
 s_1^- & \sigma_2^0 & S_2^{--}
\end{pmatrix} \sim (6^*, 0) \tag{17}
\]

of scalar fields to the three triplets of Eqs. (1). Thus we have additional terms in the Higgs potential of Eq. (2). The new potential is

\[
V_S(\eta, \rho, \chi, S) = V_T + \mu_4^2 \text{Tr} \left( S^\dagger S \right) + \lambda_{10} \text{Tr}^2 \left( S^\dagger S \right) + \lambda_{11} \text{Tr} \left[ \left( S^\dagger S \right)^2 \right] + \\
+ \left[ \lambda_{12} (\eta^\dagger \eta) + \lambda_{13} (\rho^\dagger \rho) + \lambda_{14} (\chi^\dagger \chi) \right] \text{Tr} \left( S^\dagger S \right) + \\
+ \frac{1}{2} \left( f_2 \rho_i \chi_j S^{ij} + \text{H.c.} \right), \tag{18}
\]

where \( V_T \) is given in Eq. (2). The VEV of the \( \sigma_2^0 \) field is \( v' \) and that for \( \sigma_1^0 \) vanishes, since we do not consider neutrino mass [1]. Eq. (3) have now an additional component given by \( \sigma_2^0 = v' + \xi' + i\zeta' \). The \( v' \) VEV contributes to the SU(2)\(_L\)\( \otimes \)U(1)\(_N\) \( \longrightarrow \) U(1)\(_{em}\) breaking in such a way that \( v^2 + u^2 + v'^2 = v_W^2 \).

Working with the same approximation as in the three triplet case (we are considering also \( |f_2| \sim w \)), we obtain the following square masses from the neutral CP-even sector:

\[
M_{01}^2 \approx -4\lambda \left( v^2 + u^2 + v'^2 \right), \tag{19a}
\]

\[
M_{02,03}^2 \approx \frac{w^2}{4 v u v'} \left\{ \left( s u^2 + r v v' \right) w \right. + \\
\]
This approximation implies

\[ w \approx \pm \left( \left( su^2 - rv \right)^2 w^2 - \frac{4uvu'v'}{w^2} \left[ f_1f_2u^2 - (f_1^2 + f_2^2) vv' \right] \right)^{1/2} \], \quad (19b)

\[ M_{04}^2 \approx -4\lambda_3 w^2. \] \quad (19c)

where we are defining

\[ r \equiv \frac{1}{w^2} (f_1v - vw') \approx -\frac{2}{uw} \left( \lambda_5 v^2 + \lambda_6 u^2 + 2\lambda_{14} v'^2 \right), \quad (20a) \]

\[ s \equiv \frac{1}{w^2} (f_1v' - vw). \] \quad (20b)

This approximation implies

\[ \lambda \equiv 4 \left( 2\lambda_{10} + \lambda_{11} \right) \approx \lambda_1 \approx \lambda_2 \approx \lambda_{12} \approx \lambda_{13} \approx \frac{\lambda_4}{2}, \] \quad (21)

In this case the standard model Higgs boson corresponds to \( H_1^0 \), in the sense that its mass has no dependence in the \( w \) parameter of the symmetry breaking [see Eq. (19b)].

Defining

\[ N_{\rho}^2 \equiv \frac{1}{w^4} \left\{ u \left( uw^2 - 2M_{04}^2 v' \right) + w^2 v'^2 \right\}^2 - vw \left[ uw^2 + 2M_{04}^2 v' + w^4 v'^2 \right] \}, \quad (22a) \]

\[ N_x^2 \equiv \frac{1}{w^{12}} \left\{ \left( v^2 + v'^2 \right) \left[ v \left( 2M_{04}^2 v' + uw^2 \right) + uw^3 \right] \right\}^2 w^4 + \]

\[ + \left[ u \left( uw^2 + 2M_{04}^2 v' \right) + \left( v^2 + vw \right) w^2 \right]^2 \times \]

\[ \times \left[ \left( u^2 + v'^2 \right) w^4 + 2vu \left[ v \left( 2M_{04}^2 v' + uw^2 \right) + uw^3 \right] \right] \}, \quad (22b) \]

\[ a_{21} \equiv \left[ u \left( uw^2 + 2M_{04}^2 v' \right) + v' w^2 \right] / w^2 N_{\rho}, \quad (22c) \]

\[ a_{22} \equiv -v \left( 2M_{04}^2 v' + uw^2 \right) / w^2 N_{\rho}, \quad (22d) \]

\[ a_{31} \equiv v' \left[ -v \left( 2M_{04}^2 v' + uw^2 \right) + uw^3 \right] / w^5 N_x, \quad (22e) \]

\[ a_{32} \equiv v' \left[ u \left( uw^2 + 2M_{04}^2 v' \right) + \left( v'^2 + vw \right) w^2 \right] / w^5 N_x, \quad (22f) \]

\[ a_{33} \equiv -\left( 2M_{04}^2 v' + uw^2 \right) \left( v^2 - u^2 \right) + u^2 v'^2 \right] / w^4 N_x \] \quad (22g)

we have the eigenstates

\[
\begin{pmatrix}
\xi_{\eta} \\
\xi_{\rho} \\
\xi_{\chi} \\
\xi'
\end{pmatrix}
\approx
\begin{pmatrix}
2\sqrt{|\lambda|} v/M_{01} & 2\sqrt{|\lambda|} u/M_{01} & 2\sqrt{|\lambda|} v'/M_{01} \\
a_{21} & a_{22} & vv'/N_{\rho} \\
av' & a_{31} & a_{32} & a_{33} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
H_{1}^0 \\
H_{2}^0 \\
H_{3}^0 \\
H_{4}^0
\end{pmatrix}, \quad (23a)
\]

\[ \xi' \approx H_{4}^0. \] \quad (23b)
In the CP-odd part we have two Goldstone and two physical Higgs bosons with masses:

\[
M^2_{J^1_1, J^2_2} \approx \frac{w^2}{4vuv'} \left\{ (su^2 + r v v')w + \right.
\pm \left[ (su^2 - r v v')^2 w^2 - \frac{4vuv'^2}{w^2} \left( f_1 f_2 u^2 + (f_1^2 - f_2^2) v v' \right) \right]^{1/2} \right\},
\] (24)

In the same way, for the CP-odd sector, the mass eigenstates read

\[
\begin{bmatrix}
\zeta_\eta \\
\zeta_\rho \\
\zeta_x \\
\zeta' \\
\end{bmatrix} \approx \begin{bmatrix}
2\sqrt{\lambda} v/M_0 & -2\sqrt{\lambda} u/M_0 & 2\sqrt{\lambda} v'/M_0 & 0 \\
b_{21} & b_{22} & b_{23} & 0 \\
u/(v^2 + u^2)^{1/2} & v/(v^2 + u^2)^{1/2} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
G^0_1 \\
G^0_2 \\
J^0_1 \\
\end{bmatrix},
\] (25a)

and

\[
\begin{bmatrix}
\eta^+ \\
\rho^+ \\
\eta^+_2 \\
\chi^+ \\
\end{bmatrix} \approx \begin{bmatrix}
2\sqrt{\lambda} v/M_0 & -2\sqrt{\lambda} u/M_0 & -2\sqrt{\lambda} v'/M_0 & 0 \\
2\sqrt{\lambda} w/M_0 & 2\sqrt{\lambda} v u/[M_0 (v^2 + u^2)] & a_{21} & 0 \\
0 & v'/ (v^2 + v'^2)^{1/2} & -u/ (u^2 + v'^2)^{1/2} & 0 \\
\end{bmatrix}
\begin{bmatrix}
G^+_1 \\
G^+_2 \\
H^+_1 \\
\end{bmatrix},
\] (25b)

where

\[
b_{21} \equiv \frac{2\sqrt{\lambda} v v'}{M_0 (v^2 + u^2)^{1/2}}, \quad b_{22} \equiv -\frac{2\sqrt{\lambda} uv'}{M_0 (v^2 + u^2)^{1/2}}, \quad b_{23} \equiv \frac{2\sqrt{\lambda} (v^2 + u^2)^{1/2}}{M_0}.
\] (26)

In this approximation two of the masses of the singly charged scalar bosons coincide with the CP-odd ones \([M_{+1} (M_{+2}) = M_{J^1_1} (M_{J^2_2})]\). The other two are degenerated:

\[
M^2_{+3} = M^2_{+4} \approx f_2 \frac{w}{2v'},
\] (27)

where \(f_2 > 0\). The associated eigenstates are

\[
\begin{bmatrix}
\eta^+_1 \\
\rho^+ \\
\eta^+_2 \\
\chi^+ \\
\end{bmatrix} \approx \begin{bmatrix}
2\sqrt{\lambda} v/M_0 & -2\sqrt{\lambda} u/M_0 & -2\sqrt{\lambda} v'/M_0 & 0 \\
2\sqrt{\lambda} w/M_0 & 2\sqrt{\lambda} v u/[M_0 (v^2 + u^2)] & a_{21} & 0 \\
0 & v'/ (v^2 + v'^2)^{1/2} & -u/ (u^2 + v'^2)^{1/2} & 0 \\
\end{bmatrix}
\begin{bmatrix}
G^+_1 \\
G^+_2 \\
H^+_1 \\
\end{bmatrix},
\] (28a)

and

\[
\begin{bmatrix}
\eta_1^+ \\
\rho^+ \\
\eta_2^+ \\
\chi^+ \\
\end{bmatrix} \approx \begin{bmatrix}
\frac{u^2}{4v'} \left\{ u - 2\lambda_9 v' \pm \left[ (2\lambda_9 v' + u)^2 + 4v'^2 \right]^{1/2} \right\} \\
\end{bmatrix},
\] (29a)

\[
M^2_{++3} \approx f_2 \frac{w}{2v'},
\] (29b)

Notice that \(M^2_{3+} = M^2_{4+} = M^2_{3+4}\) in this approximation. Here we have the eigenstates
\[
\begin{pmatrix}
\rho^{++} \\
\chi^{++}
\end{pmatrix} \approx \frac{1}{(u^2 + v'^2)} \begin{pmatrix}
u \\ v' - u
\end{pmatrix} \begin{pmatrix}
G^{++} \\
\mathcal{H}_1^{++}
\end{pmatrix}, \tag{30a}
\]

\[
S_1^{++} \approx \mathcal{H}_2^{++}, \quad S_2^{++} \approx \mathcal{H}_3^{++}. \tag{30b}
\]

We show the behavior of the \(M_{02}\) and \(M_{03}\) masses as functions of \(v\) and \(u\) VEV’s in surface graphics in Figure 2. The physical values for \(M_{03}\) constraint the \(u\) VEV in range \(178 \text{ GeV} \lesssim u < 246 \text{ GeV}\) (Figure 2b). This mass increases indefinitely as \(v \rightarrow 0\) and \(u \rightarrow 246 \text{ GeV}\).

The behavior of the CP-odd neutral masses and singly charged ones are given in surface graphics in the Figure 3. We can see that the \(v\) VEV is constrained in these figures by \(0 < v \lesssim 37 \text{ GeV}\). The behavior of the \(M_{02} (M_{03})\) is similar to \(M_{J1} (M_{J2})\). We study the doubly charged scalar masses as functions of the ratio \(u/v'\) and \(\lambda_9\). \(M_{++1}\) is small when \(u/v'\) and \(\lambda_9\) constant are near zero and increases when these variables leave from the origin. We do not plot the correspondent graphic. \(M_{++2}\) has a more interesting behavior. In Figure 4 we plot the curve for \(M_{++2}\) as function of \(u/v'\) for \(\lambda_9 = -0.5\) and \(-1\). These curves show that \(M_{++2}\) has a limiting value as \(u/v'\) increases. This upper value increases as \(\lambda_9\) decreases.

### III. SUMMARY

Summarizing, we have studied the scalar sector of the 3-3-1 models in the natural condition \(v, u, v' \ll w\). It is showed that the 3-3-1 models have neutral and charged scalar masses which can be small for a large range of the parameters, turning the scalar sector of these models interesting for searching of Higgs bosons in available and future colliders. In 3-3-1 models is hopeful to find neutral and charged scalar Higgs bosons with masses which are not heavier than a few TeVs. One of the doubly charged Higgs bosons can be relatively very light and so, as in two doublets models \[^{13}\], it can be used as a probe of the 3-3-1 models. It is interesting to observe here two aspects: firstly, in our approach the scalar masses do not depend on many free parameters and secondly, in the model with sextet, we are able to obtain natural constraints on the two VEV’s \((0 < v \lesssim 40 \text{ GeV} \text{ and } 180 \text{ GeV} \lesssim u < 246 \text{ GeV})\) independently of the experimental results.
Notice that the model with sextet is more predictive. Our analysis in this case would be improved if we had constraints for small $v$ VEV. This might be obtained through experimental data. We cannot put bound on the 3-3-1 parameters by requiring the validity of the perturbative regime, as it is done in two Higgs doublets models [14]. In our case the quark–Higgs coupling structure and the presence of new quarks [1] with unknown masses spoil these bounds.

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REFERENCES

[1] F. Pisano and V. Pleitez, Phys. Rev. D 46 (1992) 410; P. H. Frampton, Phys. Rev. Lett. 62 (1992) 2889; R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47 (1993) 4158.

[2] P. H. Frampton and D. Ng, Phys. Rev. D 45, (1992) 4240; H. Fuji, Y. Mimura, K. Sasaki and T. Sasaki, Phys. Rev. D 46, (1994) 559.

[3] J. Agrawal, P. H. Frampton and D. Ng, Nucl. Phys. B 386 (1992) 267.

[4] D. Ng, Phys. Rev. D 49, (1994) 4805.

[5] F. Pisano, V. Pleitez and M. D. Tonasse, Flavor Chiral Extensions of the Standard Model (in preparation).

[6] E. D. Carlson and P. H. Frampton, Phys. Lett. B 283 (1992) 123.

[7] V. Pleitez e M. D. Tonasse, Phys. Rev. D 48 (1993) 2353.

[8] F. Pisano, V. Pleitez e M. D. Tonasse, Radiatively Induced Lepton Masses, preprint IFTP.059/93 (not published).

[9] On the subject of neutrino mass in 3-3-1 models see P. H. Frampton, P. I. Krastev and J. T. Liu, Mod. Phys. Lett. A 9 (1994) 761.

[10] V. Pleitez and M. D. Tonasse, Phys. Rev. D 48 (1993) 5274.

[11] J. T. Liu and D. Ng, Phys Rev. D 50 (1994) 548; L. Epele, H. Fanchiotti, C. García Canal and D. G. Dumm, Phys. Lett. B 243 (1995) 291.

[12] J. Donoghue and L. -F. Li, Phys. Rev. D 19 (1979) 945.

[13] K. Huitu and J. Maalampi, Phys. Lett. B 344 (1995) 217.

[14] V. Barger, L. Hewett and R. J. N. Phillips, Phys. Rev. D 41 (1990) 3421; Y. Grossman, Nucl. Phys. B 426 (1994) 355.
FIGURE CAPTIONS

Figure 1. Graphics for the neutral Higgs masses $m_{02}$ (filled line) and for $m_{h}$ (dashed line), with $w = 3$ TeV, in the tree triplet Higgs model. $m/w$ stands for $m_{02}/w$ or $m_{h}/w$.

Figure 2. A surface graphic shows the range for physical neutral $M_{02}/w$ (a) and $M_{03}/w$ (b) masses as functions of the VEV’s $v$ and $u$ in the three triplet and one sextet model.

Figure 3. The same as in the figure 2, but $M/w$ in the third coordinate is $M_{\gamma 1}/w$ or $M_{+1}/w$ (a) and $M_{\gamma 2}/w$ or $M_{+2}/w$ (b).

Figure 4. The behavior of the $M_{++2}/w$ as function of the ratio $u/v'$ of the VEV’s for $\lambda_{9} = -0.5$ (dashed line) and $\lambda_{9} = -1$ (filled line).
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