**LETTER**

**Novel time-domain asymptotic-numerical solution for backward transient scattered magnetic field from a coated metal cylinder**

Toru Kawano¹, ² and Keiji Goto¹, ²

**Abstract** A novel time-domain (TD) asymptotic-numerical solution (novel TD-ANS) is proposed for a backward transient scattered magnetic field when an ultra-wideband (UWB) pulse wave is incident on a coated metal cylinder which is made of a metal cylinder covered with a homogenous medium layer. The TD-ANS is expressed by two kinds of notations. First is represented by a combination of a direct geometric optical ray (DGO) and a reflected GO (RGO). Second is given by a combination of a DGO and a RGO series. The TD-ANS is useful in understanding backward transient electromagnetic wave scattering because it can reproduce the wave packet of a reflected wave by superposing multiple RGO components. The accuracy and validity of the TD-ANS are confirmed by comparing with a reference solution.

**Keywords:** time-domain asymptotic-numerical solution (TD-ANS), backward transient electromagnetic wave scattering, coated metal cylinder, reflected geometric optical ray (RGO) series, arrival time estimation

**Classification:** Electromagnetic theory

1. Introduction

The studies on high-frequency (HF) electromagnetic (EM) (HF-EM) wave scattering in the frequency-domain (FD) from smooth convex bodies have been an important research subject for a variety of applications in the area of antennas and radar cross section [1, 2]. Canonical problems to analyze the HF-EM wave scattering in the FD from convex bodies with a complicated shape include a conducting cylinder [1, 2, 3, 4, 5, 6, 7, 8, 9] and a dielectric cylinder [10, 11, 12, 13]. With developments in materials science, various authors [10, 14, 15, 16, 17] discussed the problem of HF-EM wave scattering in the FD by a two-dimensional (2-D) cylinder whose surface is either coated or characterized by an impedance boundary condition.

On the other hand, the studies on HF-EM wave scattering in the time-domain (TD) from smooth convex bodies, which include a curved wedge, a coated cylinder, and a coated sphere, have recently received a lot of attention in the area of ultra-wideband (UWB) radars and their associated antennas for remote sensing [18, 19, 20, 21] and the nondestructive testing which investigates the degree of the corrosion of rebar in steel reinforced concrete structures [22, 23]. However, only a limited number of papers have been reported on the analysis and interpretation methods for backward transient HF-EM wave scattering problems [19, 21, 22, 23].

In our previous investigations, an FD uniform asymptotic solution (FD-UAS) [24, 25] and a TD asymptotic-numerical solution (TD-ANS) [26] have been derived for HF-EM wave scattering from a conducting cylinder covered with a thin lossy dielectric material. The FD-UAS in [25] is uniform in the sense that it remains valid within a transition region adjacent to a shadow boundary, and it smoothly connects a geometric optical ray (GO) solution and a geometrical theory of diffraction (GTD) solution exterior to the transition region, respectively.

Authors have also proposed an FD asymptotic solution [27, 28, 29, 30] and a TD-ANS [28, 31, 32] for HF-EM wave scattering from a coated conducting cylinder including the multiple reflection effect passing through a coating medium layer. The TD-ANS in [32] is proposed for a forward transient scattered magnetic field when a UWB pulse wave [20] is incident on a coated conducting cylinder. The TD-ANS in [32], which is represented by multiple reflected and surface diffracted ray (multiple RSD) series, provides the analysis and interpretation methods for forward transient HF-EM wave scattering.

The purpose of the present paper is to propose a novel TD-ANS for a backward transient scattered magnetic field [33, 34], which is developed from the conventional TD-ANS in [26, 28, 31], when a UWB pulse wave [20] is incident on a 2-D coated metal cylinder which is made of a metal cylinder covered with a homogenous medium layer. The TD-ANS is expressed by two kinds of notations. First is represented by a combination of a direct GO (DGO) and a reflected GO (RGO). Second is given by a combination of a DGO and a RGO series. The TD-ANS is useful in understanding backward transient HF-EM wave scattering because it can reproduce the wave packet of a reflected wave by superposing multiple RGO components. The accuracy and validity of the TD-ANS are confirmed by comparing with a reference solution.

The time convention $\exp(-i\omega t)$ is assumed and suppressed in this paper.

2. Formulation

Fig. 1 shows a 2-D coated metal cylinder with radius $\rho = a$ which is made of a metal cylinder with radius $\rho = b$ covered with a homogeneous medium 2 layer ($\varepsilon_{2}^{*}$, $\mu_{0}$) with thickness $h (= a - b)$, and coordinate systems $(x, y, z)$ and $(\rho, \phi)$. No-
3. TD-ANS for backward transient scattered magnetic field

3.1 TD-ANS by a combination of DGO and RGO
The $z$-component of a backward transient scattered magnetic field $y_z(p_0, \theta_0, p, \phi = \theta_0; t) = y(t)$ from a coated metal cylinder is given by the TD-ANS which is represented by a combination of a DGO and a RGO (is abbreviated to $y_{\text{DGO}}(t)$) as [26, 33]

\[
y(t) \sim y_{\text{TD-ANS}}(t) = y_{\text{DGO}}(t) + y_{\text{RGO}}(t)
\]

where $y_{\text{DGO}}(t)$ denotes a DGO solution in the TD and propagates along the path $Q \rightarrow P$ from the source point Q to the observation point P (see Fig. 2). While, $y_{\text{RGO}}(t)$ is a RGO solution in the TD and propagates along the path $Q \rightarrow Q_0 \rightarrow P$ which arrives at the point P after radiating from the point Q and reflected at the point $Q_0$ on a coated surface defined by radius $p = a$ (see Fig. 2).

In (1), $y_{\text{DGO}}(t)$ ($y_{\text{RGO}}(t)$) can be expressed by the inverse Fourier transform of the product of a DGO (RGO) solution $DGO(\omega)$ ($RGO(\omega)$) in the FD and the frequency spectrum $S(\omega)$ in (A-2) of a pulse source function $s(t)$ in (A-1) as [26, 33, 35]

\[
y_{\text{DGO}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} DGO(\omega)S(\omega) \exp(-i\omega t) d\omega
\]

\[
y_{\text{RGO}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} RGO(\omega)S(\omega) \exp(-i\omega t) d\omega.
\]

The readers can find the explicit representations for the DGO($\omega$) and the RGO($\omega$) in [25].

Pulse wave components $y_{\text{DGO}}(t)$ and $y_{\text{RGO}}(t)$ represented by integral form in (2) and (3) each are computable numerically by applying the fast Fourier transform (FFT) numerical code [36]. The TD-ANS for GO can be obtained by substituting the numerical results $y_{\text{DGO}}(t)$ and $y_{\text{RGO}}(t)$ into $y_{\text{GO}}(t)$ in (1). The response waveform of $y(t)$ can be obtained from the real part of $y(t)$, namely, $\text{Re}[y(t)]$. The response waveform of the TD-ANS in (1) can extract the DGO component $\text{Re}[y_{\text{DGO}}(t)]$ and the RGO component $\text{Re}[y_{\text{RGO}}(t)]$ from the $\text{Re}[y_{\text{GO}}(t)]$.

3.2 TD-ANS by a combination of DGO and RGO series
The TD-ANS by a combination of a DGO and a RGO series (is abbreviated to $y_{\text{GO series}}(t)$) for the $z$-component of a backward transient scattered magnetic field $y(t)$ from a coated metal cylinder is given by [28, 31, 33, 34]

\[
y(t) \sim y_{\text{TD-ANS}}(t) = y_{\text{DGO}}(t) + \sum_{p=0}^{M} y_{\text{RGO}_p}(t)
\]

where $y_{\text{DGO}}(t)$ is the DGO solution in the TD defined by (2). While, $y_{\text{RGO}_p}(t)$ represents a $p$ times RGO solution in the TD. The $y_{\text{RGO}_p}(t)$ propagates along the path $Q \rightarrow p(Q_0 \rightarrow R \rightarrow Q_0 \rightarrow P$ from the point Q to the point $P$ including multiple reflection effect $p(Q_0 \rightarrow R \rightarrow Q_0$ in a homogeneous medium 2 layer (see Fig. 2). Notation $p$ denotes the number of times of reflection on a metal cylinder.
ANS for GO series can be obtained by substituting the numerical RGO components depicted in Figs. 4(b) to 4(f) are not indicated by the same scales. We observe that the arrival time of the TD-ANSs in (1) and (4) because the computation time of each of the TD-ANSs is much shorter than that of the reference solution.

4.2 Interpretation method of response waveform using the TD-ANS

In this section, we discuss the interpretation method of the response waveforms shown in Fig. 3.

Fig. 3(a) shows the response waveform of a transient scattered magnetic field from a metal cylinder with radius \( \rho = a \) without a medium 2 layer \( (\rho = a = b, h = 0.05a) \). We observe two wave packets in the \( \text{Re}[\text{Reference}(t)] \) (---). Referring to the \( \text{Re}[\text{YGO}(t)] \) in (1), we can recognize that two wave packets are \( \text{Re}[\text{YDGO}(t)] \) and \( \text{Re}[\text{YGO}(t)] \) in arrival order to the observation point P.

Fig. 3(b) shows the response waveform when the thickness of a homogeneous medium 2 layer is \( h = 0.5a_0 \). Two wave packets are observed from the \( \text{Re}[\text{Reference}(t)] \) (---). Referring to the \( \text{Re}[\text{YGO}(t)] \) (---), we also confirm that two wave packets are \( \text{Re}[\text{YDGO}(t)] \) and \( \text{Re}[\text{YGO}(t)] \) in arrival order to the point P. In comparison with the arrival time \( t = 35.103 \text{ ns} \) of the second wave packet \( \text{Re}[\text{YGO}(t)] \) in Fig. 3(a), the \( \text{Re}[\text{YGO}(t)] \) in Fig. 3(b) arrives at the point P by a time delay of 0.982 ns (\( = 36.085 \text{ ns} - 35.103 \text{ ns} \)). However, the TD-ANS in (1) cannot explain the time delay reasonably using the \( \text{Re}[\text{YGO}(t)] \).

Next, we discuss the time delay shown in Fig. 3(b) using the TD-ANS in (4). Fig. 4(a) shows the response waveform calculated from the \( \text{Re}[\text{YGO series}(t)] \) with \( M = 4 \), where numerical parameters used in Fig. 4(a) are the same as those given in the caption of Fig. 3 including \( h = 0.5a_0 \). The validity of TD-ANS in (4) is confirmed because the \( \text{Re}[\text{YGO series}(t)] \) (---) agrees excellently with the \( \text{Re}[\text{Reference}(t)] \) (---). We use the \( \text{Re}[\text{YGO series}(t)] \) as a new reference solution from now on.

Figs. 4(b) to 4(f) show the response waveforms (---) of multiple RGO components \( \text{Re}[\text{YGO}(t)], p = 0, 1, 2, 3, 4 \) respectively. Please note that the vertical axes of multiple RGO components depicted in Figs. 4(b) to 4(f) are not indicated by the same scales. We observe that the arrival time of...
time in (6), one obtains $\Delta t_h = 0.5 t_0 = 1.309$ ns. Figs. 4(c) to 4(f) indicate necessary times of multiple RGO components $p\Delta t_h = 0.5 t_0$, $p = 1, 2, 3, 4$, respectively. We can interpret that the time delay of 0.982 ns of the second wave packet $\text{Re}[\gamma_{\text{RGO}}(t)]$ in Fig. 3(b) is the result of interference of two RGO components $\text{Re}[\gamma_{\text{RGO}_{p=1}}(t)] = \text{Re}[\gamma_{\text{RGO}}(t)]$ and $\text{Re}[\gamma_{\text{RGO}_{p=2}}(t)] = \text{Re}[\gamma_{\text{RGO}}(t)]$ by referring to Figs. 4(a) to 4(f).

The above-mentioned discussions provide the usefulness of the interpretation method for a backward transient scattered magnetic field using the TD-ANS in (4) because it can reproduce the wave packet of a reflected wave by superposing multiple RGO components.

Finally, Fig. 3(c) shows the response waveform when the thickness of the medium 2 layer is $h = 2.0 h_0$ and six wave packets are observed in the $\text{Re}[\gamma_{\text{RGO}}(t)]$ vs. time curve. Referring to the $\text{Re}[\gamma_{\text{RGO}_{p=1}}(t) \ldots]$ in (4), we can identify that six wave packets are $\text{Re}[\gamma_{\text{RGO}}(t)]$ and $\text{Re}[\gamma_{\text{RGO}_{p=1}}(t)]$, $p = 0, 1, 2, 3, 4$, in arrival order to the point $P$. In this case, the necessary time in (6) becomes $\Delta t_{h=2.0 h_0} = 5.236$ ns. We can also estimate the arrival time of each multiple RGO component $\text{Re}[\gamma_{\text{RGO}_{p=1}}(t)]$ along the path $Q \rightarrow P$, $P \rightarrow Q$, $P \rightarrow Q_0 \rightarrow Q$, and six wave packets are observed in the $\text{Re}[\gamma_{\text{RGO}}(t)]$ vs. time curve. The above-mentioned discussions provide the usefulness of the TD-ANS in (4).

5. Conclusion

We have proposed the novel TD-ANS for a backward transient scattered magnetic field when the UWB pulse wave is incident on a coated metal cylinder. We confirmed the accuracy and validity of the TD-ANS by comparing with the reference solution. We verified the usefulness of the TD-ANS because it can reproduce the wave packet of a reflected wave by superposing multiple RGO components. We also confirmed the validity of the TD-ANS because it can estimate the arrival times of pulse wave components such as the DGO, the RGO, and the multiple RGO solution in the TD.

References

[1] R.C. Hansen ed.: Geometric Theory of Diffraction (IEEE Press, New York, 1981) 230.
[2] G.L. James: Geometrical Theory of Diffraction for Electromagnetic Waves (Peter Peregrinus, London, 1986) 3rd ed. 7 (DOI: 10.1049/pb0001e).
[3] J.B. Keller: “Diffraction by a convex cylinder,” IRE Trans. Antennas Propag. 4 (1956) 312 (DOI: 10.1109/tap.1956.1144427).
[4] P.H. Pathak: “An asymptotic analysis of the scattering of plane waves by a smooth convex cylinder,” Radio Sci. 14 (1979) 419 (DOI: 10.1029/rs014i003p00419).
[5] P.H. Pathak, et al.: “A uniform GTD analysis of the diffraction of electromagnetic waves by a smooth convex surface,” IEEE Trans. Antennas Propag. 28 (1980) 631 (DOI: 10.1109/tap.1980.1142396).
[6] P. Hussar and R. Albus: “On the asymptotic frequency behavior of uniform GTD in the shadow region of a smooth convex surface,” IEEE Trans. Antennas Propag. 39 (1991) 1672 (DOI: 10.1109/8.121587).
[7] L.B. Felsen and N. Marcuvitz: Radiation and Scattering of Waves (IEEE Press, New York, 1994) 370 (DOI: 10.1109/9780470546307).
[8] T. Ishihara, et al.: “A modified UTD analysis of the diffracted electromagnetic field in the transition and shadow regions of a convex
conducting cylinder,” IIECE Trans. Electron. (Japanese Edition) J83-C (2000) 596.

[9] T. Ida and T. Ishihara: “Novel high-frequency uniform asymptotic solution for scattered field by a conducting cylinder,” IIECE Trans. Electron. (Japanese Edition) J87-C (2004) 754.

[10] T.B.A. Senior and J.L. Volakis: Approximate Boundary Conditions in Electromagnetics (IEE, London, 1995) 7 (DOI: 10.1049/ebw041e).

[11] T. Sasamori, et al.: “High frequency analysis of electromagnetic scattering due to a dielectric cylinder,” IIECE Trans. Electron. (Japanese Edition) J78-C-1 (1995) 9 (DOI: 10.1002/cejb.4420780405).

[12] T. Ida and T. Ishihara: “Novel high-frequency asymptotic solutions in the transition regions near geometrical boundaries and near caustics for scattering by a dielectric cylinder,” IIECE Trans. Electron. E87-C (2004) 1550.

[13] T. Ida, et al.: “Frequency-domain and time-domain novel uniform asymptotic solutions for scattered fields by an impedance cylinder and a dielectric cylinder,” IIECE Trans. Electron. E88-C (2005) 2124 (DOI: 10.1093/ietele/e88-c.11.2124).

[14] H.-T. Kim and N. Wang: “UTD solution for electromagnetic scattering by a circular cylinder with thin lossy coatings,” IEEE Trans. Antennas Propag. Antennas Propag. 37 (1989) 1463 (DOI: 10.1109/8.343566).

[15] H.H. Syed and J.L. Volakis: “High-frequency scattering by a smooth coated cylinder simulated with generalized impedance boundary conditions,” Radio Sci. 26 (1991) 1305 (DOI: 10.1029/91RS00999).

[16] P.E. Hussar: “A uniform GTD treatment of surface diffraction by impedance and coated cylinders,” IEEE Trans. Antennas Propag. 46 (1998) 998 (DOI: 10.1109/8.704801).

[17] H.C. Strifors and G.C. Gaunaudr: “Scattering of electromagnetic waves by a perfectly conducting cylinder with a thin lossy magnetic coating,” IEEE Trans. Antennas Propag. 48 (2000) 1526 (DOI: 10.1109/8.899669).

[18] P.R. Rousseau and P.H. Pathak, “Time-domain uniform geometrical theory of diffraction for a curved wedge,” IEEE Trans. Antennas Propag. 43 (1995) 1375 (DOI: 10.1109/8.475925).

[19] H.C. Strifors and G.C. Gaunaudr: “Scattering of electromagnetic pulses by simple-shaped targets with radar cross section modiﬁed by a dielectric coating,” IEEE Trans. Antennas Propag. Antennas Propag. 46 (1998) 1252 (DOI: 10.1109/8.719767).

[20] Federal Communications Commission ( FCC ) : “First report and order: in revision of part 15 of the commission’s rules regarding ultra-wideband transmission systems,” (2002) FCC 02-48.

[21] H. Vollmer and E.J. Rothwell, “Resonance series representation of the early-time field scattered by a coated cylinder,” IEEE Trans. Antennas Propag. 52 (2004) 2186 (DOI: 10.1109/TAP.2004.832331).

[22] G. Roqueta, et al.: “Analysis of the electromagnetic signature of reinforced concrete structures for nondestructive evaluation of corrosion damage,” IEEE Trans. Instrum. Meas. 61 (2012) 1090 (DOI: 10.1109/TIM.2011.2174106).

[23] M. Nishimoto and Y. Naka: “Analysis of transient scattering by a metal cylinder covered with inhomogeneous lossy material for nondestructive testing,” IIECE Trans. Electron. E101-C (2018) 44 (DOI: 10.1587/transele.E101.C.44).

[24] K. Sato and M. Hamano: “Asymptotic solutions for scattered field by a coated conducting cylinder with a thin lossy dielectric material,” IIECE Trans. Electron. E101-C (2018) 40 (DOI: 10.1587/transele.E101.C.40).

[25] K. Sato and L.H. Loc: “Asymptotic solutions for scattered field by a coated conducting cylinder with a thin lossy dielectric material,” IIECE Trans. Electron. E101-C (2018) 40 (DOI: 10.1587/transele.E101.C.40).

[26] K. Sato and T. Ida, et al.: “Frequency-domain uniform asymptotic solutions for scattered field by a coated cylinder with a thin lossy medium,” IIECE Trans. Electron. E99-C (2016) 18 (DOI: 10.1587/transiele.E99.C.18).

[27] K. Sato, et al.: “Time-domain asymptotic-numerical solution for transient scattered electric field by a coated conducting cylinder covered with a thin lossy dielectric material,” IIECE Trans. Electron. E99-C (2016) 18 (DOI: 10.1587/transiele.E99.C.18).

[28] K. Sato and T. Ida, et al.: “Study on transient scattered magnetic field by a coated conducting cylinder covered with a thick dielectric material,” The Papers of Technical Meeting on Electromagnetic Theory, IEE Japan, EMIT-20-25 (2016) 113.

[29] T. Kawano and K. Goto: “A geometric optical ray series solution for the high-frequency scattered magnetic field by a conducting cylinder covered with a thick dielectric material,” The Papers of Technical Meeting on Electromagnetic Theory, IEE Japan, EMIT-20-52 (2020) 8.

[30] T. Kawano and K. Goto: “Geometric optical ray series solution for high-frequency scattered electric field by a conducting cylinder covered with a thick dielectric material,” The Papers of Technical Meeting on Electromagnetic Theory, IEE Japan, EMIT-20-68 (2020) 47.

[31] K. Hagiwara, et al.: “Novel time-domain asymptotic-numerical solutions for transient scattered electric field from a coated cylinder covered with a thick dielectric medium,” IIECE Electron. Express 14 (2017) 20170085 (DOI: 10.1587/ex.14.20170085).

[32] T. Kawano and K. Goto: “Novel time-domain asymptotic-numerical solution for forward transient scattered magnetic field from a coated metal cylinder,” IIECE Electron. Express 17 (2020) 20200246 (DOI: 10.1587/elex.17.20200246).

[33] K. Goto, et al.: "Backward transient scattered magnetic field by a conducting cylinder covered with a homogeneous medium layer," The Papers of Technical Meeting on Electromagnetic Theory, IEE Japan, EMIT-20-27 (2020) 1.

[34] T. Kawano, et al.: "Study on backward transient scattered electric fields from a coated metal cylinder," submitted to ISAP (2020).

[35] L.B. Felsen ed.: Transient Electromagnetic Fields (Springer-Verlag, New York, 1976) 54 (DOI: 10.1007/3-540-07553-4).

[36] E.O. Brigham: The Fast Fourier Transform (Prentice-Hall, New Jersey, 1974) 148.

[37] M. Abramowitz and I.A. Stegun eds.: Handbook of Mathematical Functions (Dover, New York, 1972) 10th ed. 297.

[38] K. Goto, et al.: “Time-domain asymptotic-numerical solution for transient scattered field from a cylindrically curved conducting open sheet excited by UWB pulse wave,” IIECE Electron. Express 13 (2016) 20151041 (DOI: 10.1587/elex.13.20151041).

Appendix: UWB pulse source function and reference solution

We assume a truncated Gaussian-type modulated pulse source function \( s(t) \) defined by

\[
s(t) = \begin{cases} 
\left(-\alpha_0^2(t-t_0)^2 + \frac{(t-t_0)^2}{(2d)^2}\right) & \text{for } 0 \leq t \leq 2t_0 \\
0 & \text{for } t \leq 0, \ 2t_0 < t < 2t_0 
\end{cases} 
\]

where \( \alpha_0 \) denotes a central angular frequency, and \( t_0 \) and \( d \) are constant parameters. The frequency spectrum \( S(\omega) \) of \( s(t) \) is given by

\[
S(\omega) = \frac{2 \sqrt{\pi}}{\alpha_0} \text{Re} [\text{erf} (\alpha_0 \omega)] \left(1 - d^2(\omega - \alpha_0)^2\right) \quad (A.1)
\]

\[
\beta(\omega) = \frac{t_0}{2d} - i d(\omega - \alpha_0) \quad (A.2)
\]

where the error function \( \text{erf} z \) is defined as

\[
\text{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt. \quad (A.3)
\]

The \( s(t) \) in (A.1) becomes a Gaussian-type modulated UWB pulse source \[20\] with fractional bandwidth (FB) \( 0.50 \leq 0.25 \) when numerical parameters \( \alpha_0, t_0, \) and \( d \) given in the caption of Fig. 3 are applied. After substituting the FD exact solution \( H_\tau^2(\omega) \) obtained from an eigenfunction expansion \[14, 29\] and the \( S(\omega) \) in (A.2) into the z-component of a backward transient scattered magnetic field integral \( \gamma_z(\rho, \phi, \rho, \phi = \phi; t) = \gamma(t) \) given by \[32, 33, 35\]
$y(t) = y_{\text{reference}}(t)$

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} H^*_\varepsilon(\omega)S(\omega) \exp(-i\omega t) d\omega, \quad (A\cdot 5)
\]

by applying the fast Fourier transform (FFT) numerical code [36] to the \(y(t)\), one obtains a reference solution \(y_{\text{reference}}(t)\) numerically.

In Section 4, the \(\text{Re}[y_{\text{reference}}(t)]\) is used to confirm the accuracy, validity, and usefulness of the TD-ANS presented in Section 3.