Remarks on Tachyon Condensation in Superstring Field Theory

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We generalize recent results on tachyon condensation in boundary string field theory to the superstring.
1. Introduction

The boundary String Field Theory (BSFT) of Witten and Shatashvili \cite{1, 2} is a version of open string field theory in which the classical configuration space is the space of two dimensional worldsheet theories on the disk which are conformal in the interior of the disk but have arbitrary boundary interactions. Solutions of the classical equations of motion correspond to conformal boundary theories. For early work on the closely related sigma model approach to string theory see e.g. \cite{3, 4, 5, 6, 7, 8}.

In a recent series of papers \cite{9, 10, 11} it has been shown that open string tachyon condensation on D-branes in bosonic string theory can be efficiently studied in BSFT. In particular:

(a) Condensation to the closed string vacuum and to lower dimensional branes involves excitations of only one mode of the string field – the tachyon.

(b) The exact tachyon potential can be computed in BSFT and its qualitative features agree with Sen’s conjecture \cite{12}.

(c) The exact tachyon profiles corresponding to decay of a higher brane into a lower one give rise to descent relations between the tensions of various branes which again agree with those expected from \cite{12}.

In contrast, in Witten’s cubic SFT tachyon condensation in general involves giving expectation values to an infinite number of components of the string field. As a consequence, one has to resort to level truncation \cite{13, 14} and only approximate results are available.

As explained in \cite{10}, the reason BSFT gives an efficient description of tachyon condensation in the bosonic string is that this process is easy to understand in the first quantized framework as a property of the worldsheet renormalization group \cite{15}. Thus, one would expect in general that BSFT would give rise to a useful description of all (classical) physical processes which correspond to solvable worldsheet RG problems. Exact results for tachyon condensation have also been obtained by introducing noncommutativity \cite{16, 17, 18, 19}; the recent results of \cite{20} might give a closer relation between this approach and BSFT.

In this note we will consider the generalization of the results of \cite{9, 10} to tachyon condensation on unstable brane configurations in the superstring \cite{21}. There are many interesting examples of such configurations, both in ten dimensions and in compactified theories with various degrees of supersymmetry. There are also some new issues, having to do with the presence of RR charges carried by some of the branes that participate in the process of condensation, and the corresponding spacetime supersymmetry structure. Some
aspects of tachyon condensation in this context have been studied \cite{22} by level truncating the superstring field theory of \cite{23}. We will discuss the simplest case – non-BPS branes and the $Dp - D\bar{p}$ system in flat ten dimensional spacetime. As explained in \cite{15}, the worldsheet description of condensation is again simple in this case, and one would expect the BSFT description to be useful.

2. The action in BSFT with worldsheet supersymmetry

The original papers on BSFT \cite{1,2} studied only the bosonic case, and as we will see there are some new elements that arise in the supersymmetric context. Therefore, before turning to the description of tachyon condensation, we start with a discussion of BSFT in the superstring.

Recall that in the bosonic open string, the BSFT action is constructed as follows. One studies a general worldsheet theory with boundary interactions, described by the action

$$S = S_0 + \int_0^{2\pi} \frac{d\tau}{2\pi} \mathcal{V},$$

where $S_0$ is a free action defining an open plus closed conformal background, and $\mathcal{V}$ is a general boundary perturbation, which can be parametrized by couplings $\lambda^i$:

$$\mathcal{V} = \sum_i \lambda^i \mathcal{V}_i.$$  \hspace{1cm} (2.2)

The couplings $\lambda^i$ correspond to fields in spacetime, and one is interested in constructing the spacetime action $S(\lambda^i)$. The proposal of \cite{1,2} is to take the classical spacetime action $S$ to be

$$S = (\beta^i \frac{\partial}{\partial \lambda^i} + 1)Z(\lambda),$$

where $Z(\lambda^i)$ is the disk partition sum of the worldsheet theory (2.1) and $\beta^i$ govern the worldsheet RG flow of the couplings $\lambda^i$ with distance scale $|x|$,

$$\frac{d\lambda^i}{d \log |x|} = -\beta^i(\lambda).$$  \hspace{1cm} (2.4)

As discussed in \cite{10}, the action (2.3) thus defined is nothing but the boundary entropy of \cite{24}. It coincides with the disk partition sum at RG fixed points, and decreases along RG flows.
Apriori, one might have expected that the boundary entropy [24] should be equal to the partition sum $Z(\lambda^i)$ throughout the RG flow. In the bosonic string, there are at least two (related) problems with this proposal. One is the requirement that the boundary entropy should have critical points whenever the boundary theory is conformal. This is not necessarily the case for the partition sum $Z(\lambda^i)$. Indeed,

$$\partial_i Z = -\int_0^{2\pi} d\tau \frac{d}{2\pi} \langle V_i \rangle$$ \hspace{1cm} (2.5)

which does not always vanish in a CFT. In particular, for operators $V_i$ of scaling dimension zero there is no apriori reason for the right hand side of (2.5) to vanish, and in general it does not. An example is the case where $V_i$ is taken to be the constant mode of the tachyon in the bosonic string.

The second problem is that the disk partition sum is linearly divergent in the bosonic string [8]:

$$Z = a_1 \Lambda |x| + a_2.$$ \hspace{1cm} (2.6)

$\Lambda$ is a UV cutoff (a large energy); $a_1$ and $a_2$ are finite coefficients. The origin of the divergence is the infinite volume of the Mobius group of the disk.

Both problems are avoided by the definition (2.3). Indeed, as shown in [1,2], $S$ can be alternatively defined by

$$\frac{\partial S}{\partial \lambda^i} = \beta^j G_{ij}(\lambda).$$ \hspace{1cm} (2.7)

where $G_{ij}$ is a non-singular metric. Therefore, $S$ is stationary at fixed points of the RG. Using the Callan-Symanzik equation for $Z$ one finds that (2.3) is equivalent to

$$S = Z - \frac{dZ}{d \log |x|}$$ \hspace{1cm} (2.8)

in which the divergent term in (2.4) precisely cancels.

In the superstring both of the above objections to thinking of the partition sum as the boundary entropy disappear. First, worldsheet SUSY implies that all boundary perturbations (2.2) are top components of worldsheet superfields,

$$V_i = \{ G_{-\frac{1}{2}}, W_i \}$$ \hspace{1cm} (2.9)

where $W_i$ are the bottom components of the corresponding superfields, and $G_{-\frac{1}{2}}$ is the worldsheet SUSY generator. The analog of eq. (2.5) involves in this case the correlator
\( \langle \{ G_{-\frac{1}{2}}, W_i \} \rangle \), which indeed vanishes at fixed points of the RG, since in that case \( G_{-\frac{1}{2}} \) annihilates both the incoming and outgoing vacua.

The second objection disappears as well since the linear divergence cancels due to a cancellation between bosons and fermions – the supersymmetrically regularized volume of the super-Mobius group of the disk is finite (2.4) [8,6]. Note that the above results do not require spacetime supersymmetry; they are valid in any vacuum of the fermionic string.

In view of the above observations, it is natural to propose that for the superstring, the BSFT action \( S \) is simply the disk partition sum,

\[
S(\lambda^i) = Z(\lambda^i). \tag{2.10}
\]

In the context of the low energy effective action for massless modes this was indeed proposed in [38]. We conjecture that this is the case for the full string field theory in the BSFT formalism, at least in backgrounds where ghosts and matter are decoupled.

Below, we will use this proposal to study tachyon condensation in the superstring. Our results can be viewed as further evidence for the validity of (2.10). Possible avenues for proving the conjecture (which we will not attempt here) are:

(a) The action (2.3) was obtained in [1] from a Batalin-Vilkovisky (BV) formalism applied to the space of worldsheet field theories. It would be interesting to generalize this formalism to the fermionic string and derive (2.10).

(b) A related conjecture is that the disk partition sum of a supersymmetric worldsheet field theory coincides with the boundary entropy [24]. \( Z \) satisfies two of the three properties associated with the boundary entropy: it is stationary at fixed points of the RG, and it takes the correct value there. If one can prove that it decreases along RG flows, it will be a strong candidate for the boundary entropy, and thus for the spacetime action in BSFT.

Leaving a general derivation of (2.10) to future work, we now turn to the example of interest here, tachyon condensation on unstable D-branes in type II string theory.\(^1\) We follow closely the analysis of [1,10]. The worldsheet action is

\[
S = S_{\text{bulk}} + S_{\text{boundary}} \tag{2.11}
\]

with the standard NSR action in the bulk:

\[
S_{\text{bulk}} = \frac{1}{4\pi} \int d^2z \left( \partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial} \psi_\mu + \bar{\psi}^\mu \partial \bar{\psi}_\mu \right) \tag{2.12}
\]

\(^1\) i.e. Dp-branes with \( p \in 2\mathbb{Z} + 1 \) in IIA string theory, or \( p \in 2\mathbb{Z} \) in IIB.
The integral is over a disk of radius one. The conventions are those of [25], \( \alpha' = 2 \), and the signature of spacetime is Euclidean.

Supersymmetric boundary interactions corresponding to open string tachyon condensation are introduced following the discussion of [13]. The boundary is described by superspace coordinates \((\tau, \theta)\), with \(0 \leq \tau \leq 2\pi\) and \(\theta\) the boundary Grassman coordinate. The boundary superfields are \(\Gamma = \eta + \theta F\) and \(X = x + \theta \psi\). \(X\) is the restriction to the boundary of the standard worldsheet super-coordinate, while \(\Gamma\) is a quantum mechanical degree of freedom which lives on the boundary. Both \(\Gamma\) and \(X\) are real or, in the presence of Chan-Paton factors, Hermitian matrices. The boundary action (2.11) is given by:

\[
e^{-S_{\text{boundary}}} = \text{Tr} \exp \left[ \int \frac{d\tau}{2\pi} d\theta (D\Gamma + T(X)\Gamma) \right]
\]

where the trace is over the Chan-Paton indices and \(D = \partial_\theta + \theta \partial_\tau\). The fermions \(\eta, \psi\) are anti-periodic around the circle (as is appropriate to the NS sector).

Restricting for the moment to the case of one non-BPS brane (and hence a one-dimensional Chan-Paton space) and performing the integral over \(\theta\), the action (2.13) becomes

\[
S_{\text{boundary}} = -\int \frac{d\tau}{2\pi} \left( F^2 + \dot{\eta} \eta + T(X) F + \psi^\mu \eta \partial_\mu T \right)
\]

The boundary auxiliary fields are free and can be easily integrated out. This gives

\[
F = -\frac{1}{2} T
\]
\[
\eta = -\frac{1}{2} \frac{1}{\partial_\tau} (\psi^\mu \partial_\mu T)
\]
\[
= -\frac{1}{4} \int d\tau' \epsilon(\tau - \tau')(\psi^\mu \partial_\mu T)(\tau')
\]

where \(\epsilon(x) = +1\) for \(x > 0\) and \(= -1\) for \(x < 0\). The formula for \(\eta\) in (2.13) is non-local but well defined, since both \(\eta\) and \(\psi^\mu\) do not have zero modes in the NS sector. Plugging back into the action (2.14) one finds

\[
S_{\text{boundary}} = \frac{1}{4} \int \frac{d\tau}{2\pi} \left[ (T(X))^2 + (\psi^\mu \partial_\mu T) \frac{1}{\partial_\tau} (\psi^\nu \partial_\nu T) \right]
\]

3. Free field perturbations

One of the main points of [10] is that one can learn a lot about the physics of D-branes by studying solvable boundary perturbations. In the present context, boundary tachyon profiles of the form

\[
T(X) = a + u_\mu X^\mu
\]
give rise to free field theory on the worldsheet (see (2.14), (2.16)) and can be analyzed by using the results of [11,10]. Note that for any non-zero $u_\mu$, the interaction (3.1) gives a boundary mass to only one combination of the superfields $X^\mu$. This will play a role below.

The exact tachyon potential is obtained by setting $u_\mu = 0$ in (3.1) and computing the path integral (2.16). This leads to

$$S(T) = V_0 e^{-\frac{1}{4}T^2}$$

(3.2)

where $V_0$ is a constant proportional to the volume of the unstable brane and to $1/g_{\text{string}}$. This constant can be normalized by requiring that the tension of the unstable D-brane, which corresponds to the vacuum at $T = 0$, comes out correctly. This follows from [26] and [15]. One could also perform the consistency check described in the bosonic case in [11], but we have not done this.

In any case, the potential term in the string field theory action on the unstable Dp-brane is proportional to

$$V(T) = e^{-\frac{1}{4}T^2}.$$  

(3.3)

This has all the features expected of the tachyon potential in superstring field theory: it is symmetric under $T \to -T$, and goes to zero at $T = \pm \infty$, which corresponds to the closed string vacuum. In contrast to the bosonic string, the potential is bounded from below in this case. In [10] it was proposed that this is related to the absence of a tachyon in the closed string sector.

To study condensation to lower dimensional branes we next turn to the case of non-zero $u_\mu$ in (3.1). By a Poincaré transformation we can shift away $a$ and take $u_\mu$ to point along a single coordinate direction, $X$. We are thus interested in evaluating the path integral (2.16) with $T(X) = uX$,

$$Z(u) = \int [DX][D\psi] e^{-S_{\text{bulk}} - S_{\text{boundary}}},$$

(3.4)

where

$$S_{\text{boundary}} = \frac{u^2}{4} \int_0^{2\pi} \frac{d\tau}{2\pi} \left( x^2 + \psi \frac{1}{\partial_\tau} \psi \right)$$

(3.5)

Differentiating with respect to the parameter

$$y := u^2$$

(3.6)
we have
\[
\frac{\partial}{\partial y} \log Z = -\frac{1}{8\pi} \int_0^{2\pi} d\tau (x^2 + \psi \frac{1}{\partial \tau} \psi)
\] (3.7)

As in the bosonic case, the correlator that appears in (3.7) needs to be regularized, since it involves products of fields evaluated at the same point. In [1] the correlator \( \langle x^2 \rangle \) was defined by point splitting,

\[
\langle x^2 \rangle = \lim_{\epsilon \to 0} [\langle x(\tau)x(\tau + \epsilon) \rangle - f(\epsilon)]
\] (3.8)

where \( f(\epsilon) \) is a function which has the same logarithmic singularity as the propagator, so that the limit (3.8) exists. Of course, this prescription is ambiguous by a \( u \)-independent constant (the physical import of this ambiguity is explained in footnote 2 below).

In the present case, worldsheet supersymmetry leads to a natural prescription for defining the right hand side of (3.7):

\[
\langle x^2 + \psi \frac{1}{\partial \tau} \psi \rangle = \lim_{\epsilon \to 0} \langle x(\tau)x(\tau + \epsilon) + \psi(\tau) \frac{1}{\partial \tau} \psi(\tau + \epsilon) \rangle
\] (3.9)

To see that this regularization preserves worldsheet supersymmetry note that by using (2.15) the right hand side of (3.9) is proportional to

\[
\int d\theta \langle X(\tau, \theta)\Gamma(\tau + \epsilon, \theta) \rangle.
\] (3.10)

To compute the right hand side of (3.9) we need the explicit form of the propagators of \( x \) and \( \psi \) in free massive boundary field theory. That of \( x \) was computed in [1]:

\[
G_B(\tau - \tau') := \langle x(\tau)x(\tau') \rangle = 2 \sum_{k \in \mathbb{Z}} \frac{1}{|k| + y} e^{ik(\tau-\tau')}
\] (3.11)

The propagator for fermions on the boundary in the NS sector is

\[
G_F(\tau - \tau') := \langle \psi(\tau)\psi(\tau') \rangle = 2i \sum_{k \in \mathbb{Z} + \frac{1}{2}} \frac{k}{|k| + y} e^{ik(\tau-\tau')}
\] (3.12)

To see that (3.12) is correct note the following facts. At \( y = 0 \), it reduces to the familiar result

\[
\langle \psi(\tau)\psi(\tau') \rangle|_{y=0} = -\frac{2}{\sin \frac{\tau-\tau'}{2}}
\] (3.13)

To determine the \( y \) dependence consider the path integral (see (3.3))

\[
G_F(\tau_1 - \tau_2; y) = \langle \psi(\tau_1)\psi(\tau_2) e^{-\frac{y}{2\pi} \int_0^{2\pi} d\tau \psi \frac{1}{\partial \tau} \psi} \rangle
\] (3.14)
Differentiating with respect to $y$ we find that
\[ \partial_y G_F(\tau_1 - \tau_2; y) = -\frac{1}{8\pi} \int_0^{2\pi} d\tau \langle \psi(\tau_1) \psi(\tau_2) \psi(\tau) \frac{1}{\partial_\tau} \psi(\tau) \rangle \] (3.15)

Computing the right hand side using the propagator $G_F$ leads to the differential equation
\[ \partial_y G_F(\tau_1 - \tau_2; y) = \frac{1}{4\pi} \int_0^{2\pi} d\tau G_F(\tau_1 - \tau; y) \frac{1}{\partial_\tau} G_F(\tau_2 - \tau; y) \] (3.16)

which together with the “boundary condition” (3.13) uniquely determines the propagator to be (3.12).

We are now ready to compute the regularized correlator (3.9). Define
\[ \tilde{G}_F(\epsilon; y) := \langle \psi(\tau) \frac{1}{\partial_\tau} \psi(\tau + \epsilon) \rangle. \] (3.17)

Comparing to (3.12) we see that
\[ \tilde{G}_F(\epsilon; y) = -2 \sum_{k \in \mathbb{Z} + \frac{1}{2}} \frac{1}{|k| + y} e^{ik\epsilon}. \] (3.18)

$\tilde{G}_F(\epsilon)$ has a very similar form to $G_B(\epsilon)$ (3.11), with the only differences being the overall sign and range of the index $k$. This is natural, since if the fermions $\psi$ were periodic, their contribution would precisely cancel that of the bosons, so the full partition sum would be trivial \[6\] – a consequence of unbroken worldsheet supersymmetry.

By writing the index $k$ in (3.18) as one half of an odd number, and rewriting the sum over odd integers as a difference of a sum over all integers and that over even integers, one finds the relation
\[ \tilde{G}_F(\epsilon; y) = G_B(\epsilon; y) - 2G_B(\frac{\epsilon}{2}; 2y). \] (3.19)

Substituting in (3.9), we find:
\[ \langle x^2 + \psi \frac{1}{\partial_\tau} \psi \rangle = \lim_{\epsilon \to 0} \left[ G_B(\epsilon; y) + \tilde{G}_F(\epsilon; y) \right] = \lim_{\epsilon \to 0} \left[ 2G_B(\epsilon; y) - 2G_B(\frac{\epsilon}{2}; 2y) \right]. \] (3.20)

To evaluate this limit it is convenient to use the fact that $[11]
\[ G_B(\epsilon; y) = -2 \log(1 - e^{i\epsilon}) - 2 \log(1 - e^{-i\epsilon}) + \frac{2}{y} - 2y \sum_{k=1}^{\infty} \frac{1}{k(k+y)} (e^{ik\epsilon} + e^{-ik\epsilon}). \] (3.21)

Using this form we find that
\[ \lim_{\epsilon \to 0} [G_B(\epsilon, y) - G_B(\epsilon/2, 2y)] = -4 \log 2 + f(y) - f(2y), \] (3.22)
where

\[ f(y) = \frac{2}{y} - 4y \sum_{k=1}^{\infty} \frac{1}{k(k + y)} \]  

(3.23)

One can now proceed as in [1] and integrate the differential equation (3.7). This leads to

\[ Z(y) = 4y \frac{Z_1(y)^2}{Z_1(2y)} \]  

(3.24)

where \( Z_1 \) is a function appearing in the bosonic case [1],

\[ Z_1(y) = \sqrt{y} e^{\gamma y} \Gamma(y) \]  

(3.25)

and \( \gamma \) is the Euler number. In integrating the differential equation (3.7) one also picks up an integration constant, \( Z' \), so the partition sum is in fact

\[ Z = Z' Z(y). \]  

(3.26)

This is our final result for the partition sum as a function of \( y \). The integration constant \( Z' \) can be fixed as discussed above (after eq. (3.2)). One can check that \( Z(y) \) (3.24) is a monotonically decreasing function of \( y \) which approaches the value

\[ \lim_{y \to \infty} Z(y) = \sqrt{2\pi}. \]  

(3.27)

The result (3.24) admits several straightforward generalizations. For example, nontrivial Chan-Paton factors are easily included by taking the boundary action to be

\[ \text{Tr} P \exp \left[ -\frac{1}{8\pi} \int d\tau \left( (T(x))^2 + (\psi^\mu \partial_\mu T) \frac{1}{\partial_\tau} (\psi^\nu \partial_\nu T) \right) \right] \]  

(3.28)

Therefore the potential is proportional to

\[ \text{Tr} e^{-\frac{1}{4} \tau^2} \]  

(3.29)

Following [18,20] we can introduce a \( B \) field and take the \( B \to \infty \) limit, in which case the action becomes (3.29) with all products now being \( \ast \)-products.

Moreover, when there is a nontrivial Chan-Paton space it is possible to generalize the free-field ansatz in (3.1) in an interesting way, which describes condensation into codimension \( M \) branes with \( M > 1 \). Consider a tachyon profile of the form

\[ T(x) = \sum_{i=1}^{n} u_i x^i \gamma_i \]  

(3.30)
where $\gamma_i$ are Hermitian matrices. Suppose the matrices $\gamma_i$ form a Clifford algebra, $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. Since the $\gamma$ matrices are $2^{[n/2]}$ dimensional, the starting point of the discussion is thus a system of $2^{[n/2]}$ unstable D-branes.

Substituting (3.30) into (3.28) and using the symmetry of $x^ix^j$ and the Clifford relations we find that the interaction is proportional to the identity matrix, and has the form (3.5) for the $n$ dimensions (1, 2, $\cdots$, $n$). The fact that the interaction is proportional to an identity matrix in Chan-Paton space means that the $2^{[n/2]}$ unstable D-branes condense in this case to one (stable or unstable, depending on the parity of $n$) codimension $n$ brane. Roughly speaking, the interaction (3.30) acts only on the center of mass of the $2^{[n/2]}$ unstable branes. The partition sum (3.28) is simply equal to

$$2^{[n/2]}Z'' \prod_{i=1}^{n} Z(y_i)$$

where $y_i = u_i^2$, $Z(y)$ is the function in (3.24), and $Z''$ is a constant analogous to $Z'$ above.

The tachyon profile (3.30) is of course the standard Atiyah-Bott-Shapiro configuration [27] which played a central role in [28, 29, 30]. Restricted to a sphere it has unit winding number [27], which is another way to see that the $2^{[n/2]}$ unstable branes are condensing to a single brane. It is curious that the ABS construction arises in the present context in a somewhat novel way as a configuration preserving the free-field subspace.

4. Condensation to lower dimensional D-branes

We can now follow the discussion of [10] to study lower dimensional D-branes as solitons in BSFT on a higher dimensional brane. As a first step, let us determine the spacetime action (2.10) restricted to the tachyon field in the two-derivative approximation. The potential for the tachyon has already been determined in (3.3). By considering the partition sum $Z$ for general slowly varying tachyon fields $T(X)$ it is clear that the kinetic term will similarly have the form $\exp(-T^2/4)\partial_\mu T \partial^\mu T$. To fix the coefficient we should compare the spacetime action evaluated with a tachyon profile (3.1) to the result (3.24). As in [10], this involves regularizing the volume divergences, as we review next.

Consider, for concreteness, a single non-BPS D9-brane in the IIA theory. We would like to evaluate the spacetime action for a profile $T(X^\mu) = uX^1$. It is convenient to regularize the volume divergence of the remaining coordinates as in [10] by periodic identification

$$X^\mu \sim X^\mu + R^\mu, \quad \mu = 2, \cdots, 10.$$  (4.1)
To determine the correct normalization of the \(X\) zero mode, we notice that, for tachyon profiles of the form \(T(X) = uX\),

\[
\int_{-\infty}^{\infty} \frac{dX}{\sqrt{2\pi}} e^{-\frac{u}{4} \int_0^{2\pi} (T(X))^2} = \sqrt{\frac{2}{y}},
\]

which reproduces the leading term in the expansion of \(Z(y)\) around \(y = 0\):

\[
Z(y) = \sqrt{\frac{2}{y}} + 2\sqrt{2} \log 2\sqrt{y} + \cdots.
\]

Therefore, the normalization of the \(X\) zero mode is \(1/\sqrt{2\pi}\). The total string field action evaluated for the boundary perturbation \(T(X) = uX\) is then given by:

\[
S(y) = S_0 4^y \frac{Z_1(y)^2}{Z_1(2y)} \prod_{\mu=2}^{10} \left( \frac{R^\mu}{\sqrt{2\pi}} \right).
\]

\(S_0\) is an overall constant related to \(V_0, Z'\) above.

We can now obtain the exact string field action up to two-derivatives. It has the form,

\[
S = T_9 \int d^{10}x \left[ 2 \log 2 e^{-T^2/4} \partial\mu T \partial_{\mu} T + e^{-T^2/4} \right],
\]

where the tension of the non-BPS D9-brane is given by

\[
T_9 = \frac{S_0}{(2\pi)^5}.
\]

As we have discussed above, the value of \(S_0\) can now be fixed by comparing (4.6) to the standard tension of a non-BPS D9-brane. The expression (4.3) is obtained by putting \(T(X) = uX\) and comparing the kinetic energy with the second term in the expansion (4.3), and the potential energy with the first term.

The action (4.5) has been recently proposed in [31] as a toy model describing the tachyon dynamics on a non-BPS D-brane in superstring theory, and conjectured to be a

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2 The coefficient of the kinetic term raises an interesting technical issue. In the bosonic string case, the choice of renormalization prescription for \(X^2(\tau)\) (see (3.8)) renders ambiguous the term in \(S\) appearing at order \(\sqrt{u}\) in an expansion at small \(u\). This has the effect of rendering ambiguous the coefficient of the two-derivative term for \(T\) in the spacetime action. This ambiguity does not influence any physical observables, such as the mass of the tachyon and the tensions of D-branes viewed as solitons. In the present case, the principle of worldsheet supersymmetry removes the ambiguity (see section 3).
two-derivative truncation of BSFT. Notice that the tachyon field configuration \( T(X^1) = cX^1 \), with \( c = 1/(2 \log 2)^{1/2} \) is a kink solution of the equations of motion following from this action:

\[
8 \log 2 \partial_\mu \partial^\mu T - 2 \log 2 T \partial^\mu T \partial_\mu T + T = 0. \tag{4.7}
\]

As explained in [21], this kink describes a D8-brane of type IIA theory and one can compute the tension ratio \( T_8/T_9 \) by plugging the kink profile in (4.3). Since this configuration is not a solution of the equations of motion of the full string field action (which includes an infinite number of higher derivative terms), one only gets in this way an approximate value for the tension of the D8-brane.

However, as in the bosonic case [10], we know that the exact profile of the soliton in the exact string field theory will also be of the form \( T(X) = uX \) for some value of \( u \). The reason is that this particular tachyon mode corresponds to a free field theory on the worldsheet, and does not mix with the rest of the modes. Therefore, to obtain the exact profile we just have to take \( u \) to correspond to the infrared attractive fixed point of the RG flow [15]. This is the value of \( u \) that minimizes the string field action (4.4). Since (3.24) is monotonically decreasing, the minimum is achieved at \( y_* = \infty \), as expected from the RG flow picture. At this infrared fixed point, the exact value of the action is:

\[
S(y_*) = S_0 \sqrt{2\pi} \prod_{\mu=2}^{10} \left( \frac{R^\mu}{\sqrt{2\pi}} \right). \tag{4.8}
\]

We can now determine the ratio of the D-brane tensions. From the spacetime point of view, the kink describes a D8-brane, therefore we have \( S(y_*) = T_8 \prod_\mu R^\mu \). After restoring units, taking into account the fact that \( \alpha' = 2 \), we conclude that

\[
T_8 = (2\pi \sqrt{\alpha'}) \frac{T_9}{\sqrt{2}}, \tag{4.9}
\]

which is the expected value\(^3\) the tensions of BPS Dp-branes are proportional to \( (2\pi \sqrt{\alpha'})^{3-p}/\kappa_{10} \) [25]. The tensions of the non-BPS branes are larger by a factor of \( \sqrt{2} \).

In contrast to the bosonic case, the D8-brane obtained in this way is stable, and cannot further decay. This is consistent with the fact that the tachyonic profile which gives a free theory on the worldsheet is \textit{linear} in \( X \): as we remarked above, by a Poincaré

\[^3\text{Using the exact action up to two derivatives (1.3), one finds a ratio of tensions which is 1.151 times the expected value (see also [31]).}\]
transformation we can always take it to be of the form $uX^1$, and therefore it always describes a codimension one kink. At first sight this appears to be a problem for describing tachyon condensation to lower dimensional branes since one can only condense a single direction in spacetime. However, as discussed above, adding Chan-Paton factors and taking a tachyon profile as in (3.30) leads to condensation of $2^{[n/2]}$ D9-branes to a single $D(9-n)$-brane. Repeating the considerations of this section for the partition sum (3.31) we conclude that BSFT gives rise to the descent relation

$$\frac{T_{9-n}}{2^{[n/2]}T_9} = (\pi \sqrt{2\alpha'})^n,$$

(4.10)

or, equivalently

$$\frac{T_{9-n}}{T_9} = \begin{cases} 
(2\pi \sqrt{\alpha'})^n & \text{n even} \\
\frac{1}{\sqrt{2}} (2\pi \sqrt{\alpha'})^n & \text{n odd}
\end{cases}$$

(4.11)

which is indeed the correct answer.

One of the most important attributes of type II D-branes is that they carry RR charge \[25\]. Since BSFT is so closely allied to worldsheet techniques this property is easily checked in the present formalism using standard techniques. We follow a computation described in \[28\]. To compute the RR charge we compute the one-point function on the disk of the RR vertex operator in the \((-3/2, -1/2)\) picture,

$$V = C_{\hat{a}\hat{b}} S^\hat{a}(z) \tilde{S}^{\hat{b}}(\bar{z}) e^{-3/2\phi(z) - 1/2\tilde{\phi}(\bar{z})} + \cdots. \quad (4.12)$$

Here $\hat{a}, \hat{b}$ are chiral spinor indices for $SO(10)$, $C$ is the RR potential, $S^\hat{a}$ are spin operators, and $\phi, \tilde{\phi}$ are bosonized superconformal ghosts. When such a vertex operator is inserted into the disk, $\eta, \psi$ become periodic. In particular, we must soak up the $\eta, \psi$ zero-modes. The tachyon vertex operator is

$$V_T = \int \frac{d\tau}{2\pi} d\theta \Gamma T(X) = \int \frac{d\tau}{2\pi} (FT(x) + \eta \psi^\mu \partial_\mu T(x)).$$

(4.13)

The resulting spacetime interaction is proportional to $\int C \wedge e^{-\frac{1}{4}T^2} dT$, and, in a solitonic background $T = uX$, the charge is proportional to $u/|u|$. Note that the charge density is distributed equally on both sides of the D8 brane in accord with \[32\].

---

4 Note that the above discussion does not imply that a single unstable D9-brane cannot decay to Dp-branes with $p < 8$. As explained in \[15\] it can, but not within the free field subspace of configuration space which we are focusing on here.
Finally, the above discussion has focused on the case of an unstable D-brane, but it can be extended to $D\bar{D}$ systems by considering tachyon configurations of the form $\begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix}$.

In this paper we have made some preliminary remarks on the formulation of BSFT in the superstring case. Our results indicate that this approach might be a useful alternate route to superstring field theory. In addition to the open problems listed in [10], the supersymmetric case raises many further interesting open questions. Further development of BSFT for the superstring appears to be a worthwhile enterprise.

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