Intermittency of Fast MHD Modes and Regions of Anomalous Gradient Orientation in Low-β Plasmas

Ka Wai Ho and A. Lazarian

Department of Astronomy, University of Wisconsin-Madison, Madison, WI 53706, USA; kho33@wisc.edu, lazarian@astro.wisc.edu

Received 2020 September 5; revised 2021 February 13; accepted 2021 February 15; published 2021 April 15

Abstract

The strong alignment of small-scale turbulent Alfvénic motions with the direction of magnetic field that percolates the small-scale eddies and imprints the direction of the magnetic field is a property that follows from the MHD theory and the theory of turbulent reconnection. The Alfvénic eddies mix magnetic fields perpendicular to the direction of the local magnetic field, and this type of motion is used to trace magnetic fields with the velocity gradient technique (VGT). The other type of turbulent motion, fast modes, induces anisotropies orthogonal to Alfvénic eddies and interferes with the tracing of the magnetic field with the VGT. We report a new effect, i.e., in a magnetically dominated low-β subsonic medium, fast modes are very intermittent, and in a volume with a small filling factor the fast modes dominate other turbulent motions. We identify these localized regions as the cause of the occasional change of direction of gradients in our synthetic observations. We show that the new technique of measuring the gradients of gradient amplitudes suppresses the contribution from the fast-mode-dominated regions, improving the magnetic field tracing. In addition, we show that the distortion of the gradient measurements by fast modes is also applicable to the synchrotron intensity gradients, but the effect is reduced compared to the VGT.

Unified Astronomy Thesaurus concepts: Interstellar magnetic fields (845); Interstellar medium (847); Interstellar dynamics (839)

1. Introduction

Turbulence is ubiquitous in astrophysical environments. Magnetohydrodynamic (MHD) turbulence plays a crucial role in various astrophysical phenomena, including the formation of stars (see Mac Low & Klessen 2004; McKee & Ostriker 2007), the propagation and acceleration of cosmic rays (see Jokipii 1966; Yan & Lazarian 2008; Schleicher et al. 2010), the removal of angular momentum from accretion disks (see Krasnopolsky et al. 2012), and the regulation of heat and mass transfer between different ISM phases (see Draine 2009 for a list of the different ISM phases).

Over the decades, progress in the theory has improved our understanding of MHD turbulence (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999), and numerical studies (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2002, 2003) have helped to test the proposed theoretical ideas. A detailed discussion of the present understanding of MHD turbulence can be found in Beresnyak & Lazarian (2019). Based on the advancement of MHD theories, techniques such as correlation function analysis and the principal component analysis of anisotropies (Heyer et al. 2008) were proposed to study magnetic fields statistically using the theoretical understanding of anisotropic scaling (Lazarian et al. 2004; Esquivel & Lazarian 2005, 2011; Heyer et al. 2008; Burkhardt et al. 2014). They showed the ability to trace the magnetic field using anisotropy in the MHD simulations and observation.

The most promising way to trace magnetic field is related to a recently proposed technique, the velocity gradient technique (VGT). The technique makes use of the fact that magnetic field makes turbulence anisotropic, with turbulent eddies elongated along the magnetic field (see Brandenburg & Lazarian 2013 for a review). As a result, it induces fluid motion perpendicular to the local direction of the magnetic field and makes the gradients of velocity become perpendicular to that local direction. This property of magnetic turbulence was employed in González-Casanova & Lazarian (2017), in which an approach was proposed that used velocity centroid gradients (VCGs) to trace magnetic field orientations. With the sub-block averaging approach proposed in Yuen & Lazarian (2017b), the technique showed great potential.

The VGT has been numerically tested for a wide range of column densities from transparent diffuse gas to molecular self-absorbing dense gas. It was shown to be able to provide both the orientations of the magnetic field and a measure of the magnetization of the medium (see Lazarian & Yuen 2018a; Lazarian et al. 2018). A VGT survey (Hu et al. 2019) was conducted recently to study the morphology of five low-mass molecular clouds. The result is consistent with the Planck polarization measurement and successfully showed the reliability of the VGT technique.

Apart from tracing magnetic field orientation and the magnetization, the gradient technique can also trace the signature of important astrophysical processes imprinted in the fluid, for instance in the presence of gravity and shocks (see Yuen & Lazarian 2017a; Hu et al. 2020). The observational signature of this change in the cloud dynamics is that the VGT orientation flips by up to 90° to align parallel to the direction of the magnetic fields. We call this an orthogonal gradient. This opens a novel way for the community to study the region with strong gravitational collapse through VGT. However, a similar signature of an orthogonal gradient could also arise from the compressible nature of MHD turbulence (see Cho & Lazarian 2003; Lazarian & Yuen 2018a), in particular due to the fast modes.

In this paper, we would like to explore the relation between fundamental properties of the MHD turbulence and the orthogonal gradient that appears in the gradient technique. In what follows, we cover the theory in Section 2 and our numerical setup in Section 3. Then we discuss the relation
between MHD modes and properties of the gradient in Sections 4 and 5. We further study the impact of fast modes on both 3D and 2D observable measures in Section 6. We then introduce the new techniques to reduce the impact of fast modes in the 2D observable measures in Section 7. Finally, we discuss our work in Section 8 and summarize the paper in Section 9.

2. Properties of MHD Turbulence

The development of MHD turbulence theory has been boosted by the ability to perform high-resolution numerical simulations. The original studies of Alfvénic turbulence by Iroshnikov (1964) and Kraichnan (1965) were based on a hypothetical model isotropic MHD turbulence. Numerical studies, however, demonstrated the anisotropic nature of the MHD cascade.

2.1. Anisotropy in Incompressible MHD Turbulence

The predictive modern theory of incompressible MHD turbulence was formulated by Goldreich & Sridhar (1995, hereafter GS95). While the theory initially received a lukewarm response from the MHD turbulence community, later theoretical and numerical studies extended the theory and provided rigorous testing. In this process the theory was augmented with the essential concept of a local direction of magnetic field. This concept is most naturally derived from the properties of magnetic eddies undergoing fast turbulent reconnection (Lazarian & Vishniac 1999, hereafter LV99). Our present study is based on the modern understanding of the MHD turbulence cascade, and the statistical properties of MHD turbulence that are confirmed numerically.

In the MHD theory, the Alfvén mode cascade exhibits its anisotropic nature in the sub-Alfvénic regime, i.e., when the injection velocity \( V_L \) is less than the Alfvén velocity \( V_A \). That is, the Alfvén modes initially evolve along the so-called weak turbulent cascade, i.e., increasing the perpendicular wavenumber \( k_\perp \) while keeping the parallel wavenumber \( k_\parallel \) the same (see LV99; Galtier et al. 2005). However, at sufficiently small scales the nature of the cascade changes. The magnetic eddies that aligned with the local magnetic field surrounding them mix up this field and emit Alfvén waves with a period equal to the period of an eddy, which is

\[
L_\perp / V_\perp \approx l_\parallel / V_A
\]

(1)

where \( l_\parallel \) is the parallel scale of the eddy and \( l_\perp \) is the eddy size perpendicular to the magnetic field. In GS95, this condition comes with the critical balance, indicating the fact of assuming an incompressible nature, meaning zero velocity divergence throughout the entire space. As a result, the infall velocity gradient \( V_\perp / L_\perp \) should be equivalent to the propagating velocity gradients \( V_A / L_\parallel \) of the Alfvénic wave along the magnetic field line. Combining with velocity scaling \( V_\perp \sim L_\perp^{1/3} \), we could obtain the vital scaling relation between the parallel and perpendicular scales of the eddies,

\[
l_\parallel \sim l_\perp^{2/3}.
\]

The anisotropy scaling indicates that an increase in the perpendicular wavenumber makes the Alfvénic wavevectors more and more perpendicular to the magnetic field. The physical picture is that the turbulent eddies are getting more and more elongated along the direction of the magnetic field as we zoom into smaller scales.

At the same time, one can see that the velocity gradients follow the same anisotropy scaling, meaning the largest gradients correspond to the smallest and most elongated eddies. A later study (Lazarian et al. 2017) shows that the properties of the velocity gradient could also applied to the gradients of the turbulent magnetic field. So, any observable quantities containing information on the magnetic field and velocity could enable the tracing of the local magnetic field direction.

2.2. Compressible MHD Turbulence

In the case of compressible turbulence, three MHD modes arise, namely the incompressible Alfvén mode and the two compressible modes called fast and slow modes. We use the word “modes” rather than “waves” because, in strong MHD turbulence, the properties of motions may not be wave-like, while Alfvénic modes are essentially eddies, and they decay nonlinearly within one period. The first study of anisotropy of the three modes was proposed by Cho & Lazarian (2002, 2003) through the decomposition method. The corresponding equations determining the basis for the decomposition into modes are

\[
\hat{\xi}_A \propto \hat{k}_\parallel \times \hat{k}_\perp
\]

\[
\times \hat{\xi}_s \propto \left(1 + \frac{\beta}{2} - \sqrt{D}\right) k_\parallel \hat{k}_\perp + \left(-1 - \frac{\beta}{2} - \sqrt{D}\right) k_\perp \hat{k}_\parallel
\]

\[
\times \hat{\xi}_f \propto \left(1 + \frac{\beta}{2} + \sqrt{D}\right) k_\parallel \hat{k}_\perp + \left(-1 + \frac{\beta}{2} + \sqrt{D}\right) k_\perp \hat{k}_\parallel
\]

(3)

where \( D = (1 + \beta/2)^2 - 2\beta \cos^2 \theta, \quad \beta = 2M_A/M_S, \quad \cos \theta = k_\parallel \cdot \hat{B}, \) and \( \hat{\xi} \) is the displacement vector with the subscript indicating its mode (A: Alfvénic, f: fast, and s: slow).

Cho & Lazarian (2003) performed the numerical analysis for both gas-pressure-dominated (high-\( \beta \) regime) and magnetic-pressure-dominated (low-\( \beta \) regime) plasma. They showed for both regimes that the Alfvén and slow modes follow the GS95 scale-dependent anisotropy and the same Kolmogorov \( E(k) \sim k^{5/3} \) cascade, while the fast mode exhibits a \( k^{3/2} \) spectrum and isotropic scaling.

Apart from the scaling, Cho & Lazarian (2003) also showed that the cascade of Alfvén modes is almost independent of the slow and fast modes. The amount of the Alfvénic modes’ energy drained into compressible modes is negligible, which means that they have their own cascade and evolve independently.

In addition, Alfvén and slow modes together carry most of the energy of the turbulent cascade. In this case, tracing the gradient of the velocity/magnetic field of Alfvénic and slow modes could provide the local direction of the magnetic field. Fast modes, in many cases, are subdominant in terms of the energy cascade, although they play a very important role in a number of key astrophysical processes, e.g., cosmic-ray scattering (see Yan & Lazarian 2002, 2004). However, the study of fast modes is mainly statistical, while their spatial properties, such as the spatial energy distribution, have not been discussed or explored. The latter is performed in the current study.
We study the MHD modes and gradients using the numerical simulations obtained by two MHD codes. The first one is the 3D MHD compressible, single-fluid, operator-split, staggered-grid MHD Eulerian code Zeus-MP/3D (Hayes et al. 2006) to set up a three-dimensional, uniform turbulent medium. We use a range of Alfvénic Mach number $M_A = V_L/V_A$ and sonic Mach number $M_s = V_L/V_s$, where $V_L$ is the injection velocity; $V_A$ and $V_s$ are the Alfvén and sonic velocities respectively. The second one is the advanced MHD simulation code Athena++. Athena++ is a complete rewrite of the Athena MHD code (Stone et al. 2010) in C++. Compared to Zeus, the algorithm of Athena++ is based on directionally unsplit, higher-order Godunov methods, which not only are ideal for use with both static and adaptive mesh refinement, but also are superior for capturing shocks. We employ Zeus for normal low-$\beta$ simulation and Athena++ for extreme low-$\beta$ simulation. To study the properties of modes in a low-$\beta$ environment and prevent the density effect caused by supersonic turbulence, we selected the subsonic environment, with amounts of about 50% for Alfvénic modes, $\beta = 2(M_A/M_s)^2$. Where $M_A$ and $M_s$ are computed at the final snapshot for each simulation ($t = 2.0\tau_{cs}$ for H0S and H1S, $t = 3.0\tau_{cs}$ for H0SS). The mean $B$-field direction is toward the $z$-axis for all the simulations.

### 3. Numerical Simulations

To study the MHD modes and gradients using the numerical simulations obtained by two MHD codes. The first one is the 3D MHD compressible, single-fluid, operator-split, staggered-grid MHD Eulerian code Zeus-MP/3D (Hayes et al. 2006) to set up a three-dimensional, uniform turbulent medium. We use a range of Alfvénic Mach number $M_A = V_L/V_A$ and sonic Mach number $M_s = V_L/V_s$, where $V_L$ is the injection velocity; $V_A$ and $V_s$ are the Alfvén and sonic velocities respectively. The second one is the advanced MHD simulation code Athena++. Athena++ is a complete rewrite of the Athena MHD code (Stone et al. 2010) in C++. Compared to Zeus, the algorithm of Athena++ is based on directionally unsplit, higher-order Godunov methods, which not only are ideal for use with both static and adaptive mesh refinement, but also are superior for capturing shocks. We employ Zeus for normal low-$\beta$ simulation and Athena++ for extreme low-$\beta$ simulation. To study the properties of modes in a low-$\beta$ environment and prevent the density effect caused by supersonic turbulence, we selected the subsonic environment, with amounts of about 50% for Alfvénic modes, $\beta = 2(M_A/M_s)^2$. Where $M_A$ and $M_s$ are computed at the final snapshot for each simulation ($t = 2.0\tau_{cs}$ for H0S and H1S, $t = 3.0\tau_{cs}$ for H0SS). The mean $B$-field direction is toward the $z$-axis for all the simulations.

### 4. Anisotropy and Fast and Slow Mode Intermittency

To illustrate the properties of the MHD modes, we perform mode decomposition similarly to that in Cho & Lazarian (2003). For computing the specific component $b_i$ of each mode, the modes can be calculated as

$$b_{i,f,s,A} = \tilde{F}^{-1}\left(\tilde{b}(\xi_{f,s,A})\xi_{f,s,A}\right),$$

where $\tilde{F}$ is the Fourier transfer operator, $b$ is the computed quantity such as velocity or magnetic field in this case, and the subscript $i$ represents the direction such that $i \in \{x, y, z\}$. The upper panel of Figure 1 illustrates the decomposition procedure that takes place in Fourier space.

#### 4.1. Anisotropy of MHD Modes

Several studies have been done on the anisotropy of different MHD modes (Cho & Lazarian 2003; Lazarian & Yuen 2018a, 2018b). Here, we summarize the specific anisotropy of each mode briefly.

The middle panel of Figure 1 visualizes the anisotropy of the three modes and the bottom panel shows the slope of their energy spectra. We show the results of the decomposed velocity cube, which is projected along the $x$-axis (denoted as the LOS in this paper).

Alfvénic modes cascade on a scale of the order of one period, with the wavevector of the Alfvénic perturbations in strong turbulence being nearly perpendicular to the local direction of the magnetic field. As a result, the anisotropy and the isocontours of intensity correlation are both elongated parallel to the magnetic field. This feature is shown in Figure 1.

For slow modes, slow waves present perturbations that propagate along magnetic field lines. In the limit of an incompressible medium, slow waves are pure magnetic compression that propagates along magnetic field lines. Formally, the incompressible case corresponds to $\beta = \infty$, and in this limit the slow modes are frequently called pseudo-Alfvén modes. By contrast, for $\beta \ll 1$, the slow waves are density perturbations propagating along magnetic field lines. The anisotropy of those perturbations would be imprinted in the velocity as shown in Figure 1.

The properties of fast modes are rather different for the high-$\beta$ and low-$\beta$ regimes. For the high-$\beta$ case, the propagation of a fast wave is similar to that of a sound wave irrespective of its relation to the magnetic field, whereas in the low-$\beta$ case, the propagation of a fast wave corresponds to the magnetic field compression that propagates with Alfvén velocity. In this case, the anisotropy of the fast mode would be perpendicular to the magnetic field, which is also shown in Figure 1. Therefore, we would also expect velocity eddies to be perpendicular to the region that is dominated by the fast modes, which is the opposite relation to that of Alfvénic turbulence.

#### 4.2. Intermittency of Fast Modes

Numerical studies (Cho & Lazarian 2002, 2003; Lazarian & Yuen 2018a, 2018b) indicate that the fast modes are subdominant at least for the case of incompressible driving of turbulence (e.g., Lazarian & Yuen 2018a). For our simulations, we also found that this property holds on the global scale. We noticed that most of the energy is distributed through the Alfvén modes and slow modes in the case of a low-$\beta$ environment, with amounts of about 50% for Alfvén modes, 35% for slow modes, and only 15% for fast modes. This shows that the fast modes play a subdominant role on average.

If we talk about the local energy distribution of modes in 3D space, the situation is different in localized regions. Our simulations testify to a very intermittent spatial energy distribution of the fast mode. For each cell in our simulation cubes, we get the total kinetic energy by adding up the kinetic energy of the Alfvénic modes $E_A$, the slow modes $E_s$, and the fast modes $E_f$. We then divide the energy of the individual modes by the total energy to study the relative distribution of energy.
turbulent modes in space. Mathematically,

\[ E_{A,s,f} = \int V_{A,s,f}^2 \, dV \quad \text{and total energy} \]

\[ E_{\text{total}} = E_A + E_s + E_f. \]

In fact, we find that the energy of each mode is not distributed evenly across space. A strong concentration of fast modes is observed in localized regions. Figure 2 shows a visualization of this clustering effect from a slice of a map from 3D cubes. We define a region where the fast mode occupies more than 51% of the energy as a fast-mode-dominated region. As shown in the figure, fast modes play a minor role in most of the volume. However, in special localized regions, they would play the dominant role with up to 90% of the energy being in that form.

We notice that the fast-mode-dominated regions are not only clustering in the 2D slice but also continuous throughout the 3D space to form a 3D cluster. Figure 3 visualizes this 3D
Figure 2. Illustration of the intermittency of fast modes in a 2D slice. Maps of the fast-mode energy fraction in each location from a slice map of 3D cubes. The fast mode is dominant in red regions, it plays a minor role in blue regions, and white means the critical case. Color bar: The diverging color bar is white when the energy fraction is 0.5, red when it approaches 1, and blue when it approaches 0. Simulation used: H1S.

property using the simulation data. Those regions occupy about 10% of the total cube volume, but this varies from simulation to simulation. We also searched for relationships between the fast-mode-dominated regions and other MHD qualities, such as magnetic field strength and velocity amplitudes, but found no such relations. The location of those regions behaves as a random variable with no relation to the local physical variables.

4.3. Intermittency of Slow Modes

We noticed that intermittency is also found in the slow modes. Like fast modes, the energy of the slow modes is not evenly distributed in 3D space and clusters in local regions. About 30% of the total cube volume is dominated by the slow modes. The right panel of Figure 3 visualizes this 3D property of slow modes. The slow modes show a stronger clustering effect than fast modes and occupy more volume. However, most of them are concentrated in the component that is parallel to the magnetic field.

All in all, the energy of compressible modes is not evenly distributed across space but is concentrated in some local regions. The regions of fast-mode dominance distort tracing of the magnetic field using the gradient technique. Indeed, one may expect that some of the local regions may be dominated by fast modes, which, according to Lazarian & Yuen (2018a), can result in gradients that are orthogonal to the gradients arising from the dominant Alfvénic modes. The domination of slow modes for low $\beta$ is not expected to affect the magnetic field tracing.

4.4. Intermittency of Each Mode throughout the Evolution of Simulation

One may consider the properties of intermittency of each mode throughout the evolution of turbulence. We further study the change of intermittency using the simulation H0SS. For each snapshot ($\Delta t = 0.1 \tau_{cs}$), we compute the volume filling factor of regions dominated by each mode. We define dominated regions as those where the energy of a single mode comprises more than half of the total energy in that unit volume. We notice that the sum of the volume filling factors of regions dominated by the three modes will not be 1 because some of the unit volumes are not dominated by single modes. Figure 4 shows the result. One can see that the volume filling factor is about 0.3 for all modes at the beginning of the simulation at $t \approx 0.1 \tau_{cs}$. The volume filling factor of the Alfvénic mode then starts to grow steadily while the other two modes decrease. The system reaches a steady state at about $t \approx 1.7 \tau_{cs}$. The volume filling factors are then stable with a mild fluctuation throughout the simulation until its end at $t \approx 3 \tau_{cs}$. The result in the stable stage is consistent with our observation in the previous subsection for all of the modes, where half the cubes are occupied by the Alfvénic mode, about 25% by slow modes, and about 10% by fast modes.

5. Gradients for Different MHD Modes

5.1. Calculations of Gradient

As a tracer of magnetic field, the gradient technique aims to trace the local alignment of the eddies with the local magnetic field. Below we calculate gradients in the compressible mode and the slow and fast modes.

To calculate the gradient, we follow the procedure introduced in Yuen & Lazarian (2017a). We first compute the gradient field of the observable measures through the Sobel kernel. The whole gradient maps will then be divided into several sub-blocks. The number of sub-blocks depends on the resolution of the map and the block size. Then we adopt the sub-block averaging method to probe the peak in gradient orientation distributions in the sub-blocks of the gradient map. The peak orientation of the gradient orientation distribution represents the direction of the projected local magnetic field, and its dispersion would imprint the information of the local magnetization (see Lazarian et al. 2018).

To quantify how well the gradient field is aligned with the magnetic field, we employ the alignment measure that is introduced in analogy with the grain alignment studies (see Lazarian 2007):

$$AM = 2\langle \cos^2 \theta_f \rangle - 1,$$

with a range of $[-1, 1]$ and $\theta_f$ represents the angle between gradient orientation and the orientation of magnetic field/polarization. Perfect alignments will give $AM = 1$, whereas $AM = 0$ for random alignment and $AM = -1$ for orthogonal alignment. This measure was first used for gradients by González-Casanova & Lazarian (2017). Later it was adopted by other authors studying other statistics (see Soler et al. 2019).

5.2. Properties of Gradients for Different MHD Modes

Previous studies explored the characteristics of gradient in different observations, such as synchrotron intensity maps (Lazarian & Yuen 2018b) and spectroscopic channel maps (Lazarian & Yuen 2018a), through the employment of the mode decomposition method described in Cho & Lazarian (2002, 2003). Figure 5 illustrates the features of gradient in the separated MHD modes from one of the cubes using the Zeus simulation. It shows the results of the decomposed velocity cube, projected along the x-axis. The upper panel is a comparison of the rotated gradient vectors and magnetic fields, which demonstrates the dependence of gradient on magnetic field at the local scale. The bottom panel shows the orientation
histogram of rotated gradient vectors for each mode to visualize the global dependence. Consistent with the literature, the Alfvénic modes has strong alignment between the gradient vectors and the magnetic field and shows a very strong dependence for both local and global cases.

For slow modes, since they do not evolve on their own but are sheared by Alfvén modes, slow modes also show the alignment between magnetic field and its gradients. However, looking at the local alignment and orientation histogram in Figure 5, we see that the gradients arising from slow modes are less aligned than those from Alfvénic modes. Slow modes are less aligned with the magnetic field locally and show a stronger signature of alignment on the global scale. However, their distribution is more dispersed than that of Alfvén modes.

In contrast, the gradients in fast modes have a totally different anisotropy. The local gradient vectors show significant perpendicular features on the local scale while the orientation histogram also demonstrates a global tendency to be perpendicular to the magnetic field.

As a short summary, in terms of using gradients to trace magnetic fields, Alfvén and slow modes contribute to the expected alignment between the gradient and the magnetic field while fast modes play a disruptive role.

5.3. Regions with Perpendicular Gradient Alignment in Observations

We discussed the behavior of gradients arising from different MHD modes in the last subsection. In the real world, the environment of MHD turbulence is composed of a mixture of the three basic modes. We know that Alfvén modes play the most important role in MHD turbulence (Cho & Lazarian 2003). As discussed before, Alfvén modes and slow modes together dominate MHD turbulence, comprising more than 80% of the total energy. Therefore, one should expect a gradient aligned with the local magnetic field direction. However, the gradient in observations can be affected by fast-mode-dominated regions.

In Figure 6, we demonstrate the alignment between gradient and magnetic field from a cross-section map of 3D LOS velocity cubes. The map shows measurements of strong alignment with AM = 0.89, which indicates that the gradient is highly aligned with the magnetic field in most of the regions. However, one can notice that the gradient vectors are perpendicular to magnetic field in some of the local regions.

This tendency is also shown by the orientation histogram in Figure 6. The gradient orientation histogram shows the distribution of gradients rotated 90° perpendicular to the local mean field in that region. We called it an orthogonal alignment and called the perpendicular region ‘orthogonal regions’ (ORs). This property has not been noticed before in previous studies of gradients. In order to study magnetic field though gradient, it is very important to understand the underlying cause of the orthogonal regions and the way to identify them.
6. Fast Modes and the Orthogonal Regions

6.1. Orthogonal Regions in 3D Space

We further study the linkage between ORs and the intermittency of fast modes in this section. Figure 7 shows the decomposition of the orthogonal region and gradient orientation histogram we just showed in Figure 6. We also add the pre-decomposed data in the left panel of the figure as a reference. We notice that the morphology of the orthogonal region is very similar to that of its fast modes, with roundish structures located on the left; these are different from the Alfvénic modes, which have filamentary structure located in the bottom left corner of the figure. We have not shown the structure of slow modes of this orthogonal region but provide the corresponding discussion in the Appendix. The local orientation histogram shows the correct anisotropy with respect to different modes but with lower dispersion than the pre-decomposed data, which means higher statistically significant anisotropy. The results of decomposition and histogram together indicate that the orthogonal region is a superposition of two MHD modes, with the fast mode being dominant. We further check the distribution of energy in that region and find that about 90% of the energy corresponds to the fast modes and the remainder to the Alfvénic modes. To verify this hypothesis, we superimpose the Alfvénic modes and fast modes of that region and compare the orientation histogram and structure to the pre-decomposed data. We show the superimposed result in the right panel for reference. The superposition of fast and Alfvénic modes recovers well the pre-decomposed data. This result (see also Section 5.3) provides a description of the ORs in 3D space. Due to the high intermittency of fast modes, the clustering effect of the fast modes is dominant in
some local region, which makes the anisotropy of velocity gradients change direction by $90^\circ$.

### 6.2. Statistical Analysis of Observable Measures: Reduced Centroid

The situation of observable measures becomes complicated as a result of projecting different MHD turbulence components. The projection mixes up the contributions from three modes along the line of sight. The influence of the fast mode is subdominant in observable measures for most of the region as Alfvénic modes provide the major contribution along the line of sight.

We demonstrated the effect of different modes using reduced centroids (Lazarian & Yuen 2018a). The reduced centroid $C_R$ provides insight into the importance of the difference between the modes because the velocity can be treated as an average velocity along the line of sight and the fluctuations of the different modes are linearly separable:

$$
\delta C_R \propto \rho_0 \delta \langle v_{\text{LOS}} \rangle \\
= \rho_0 (\delta \langle v_{\text{Alfv}} \rangle + \delta \langle v_{\text{fast}} \rangle) \\
= \delta C_{R,\text{fast}} + \delta C_{R,\text{Alfv}} 
$$

where $\delta$ is the spatial differentiation operator of arbitrary direction and $\langle ... \rangle_{\text{LOS}}$ is the averaging operator along the line of sight. As we mentioned in Section 4, the fluctuation behavior of Alfvénic and fast modes is bimodal, with Alfvénic modes reaching their maximum fluctuation in the direction perpendicular to the local $B$-field lines and the fast modes parallel to $B$-field lines. Also, slow modes can treated as pseudo-Alfvénic modes and have the same properties. So, the amplitude of each mode is the crucial factor in deciding which mode takes a leading or dominant role and also the local anisotropic direction. In the language of the gradient in a reduced centroid map, the velocity fluctuation is equivalent to the gradient amplitude (GA). In a 2D observable measure like $C_R$, GA accounts for the absolute fluctuation in both parallel and perpendicular $B$-field directions. We can use the GA as an indicator to study the effect of mixing between the fluctuations of Alfvénic and fast modes. For a more precise mathematical description, we define the GA as the Euclidean norm of the gradient field in our computation procedure. (The reader may refer back to Section 5 for the calculation of gradient.)

To imitate the behavior of a fluctuation in the local fast-mode-dominated regime, we perform a synthetic test for the reduced centroid map by amplifying the fast mode energy. We amplify the fast mode velocity such that the kinetic energy ratio of Alfvénic to fast modes is one to one in the global environment. To study the relation between the fluctuation of each mode and the gradient orientation statistically, we perform a 2D histogram of the GA difference and gradient orientation. For each data point, we decomposed the contribution from fast and Alfvénic modes and computed the difference of their GAs, then we computed their gradient orientation without the decomposition. Figure 8 shows the result using simulation H1S.

In Figure 8, we define the GA difference as $\Delta \text{GA} = \text{GA}_{\text{fast}} - \text{GA}_{\text{Alfv}}$. $\Delta \text{GA}$ characterizes the mixture effect from modes in the 2D map, with a positive value.
indicating a regime dominated by fast mode fluctuation and a negative value representing domination by Alfvén mode fluctuation. As a reference, we show the direction of the global mean magnetic field in the figure. Figure 8 demonstrate a consistent picture with respect to properties of fast and Alfvénic modes. For the regime dominated by Alfvénic modes, a clear relation is found between the magnetic field and the GA. With greater amplitude difference on the negative side, meaning that Alfvénic modes dominate, the distribution of gradient orientation is more concentrated in the global mean field direction. In contrast, the fast-mode-dominated regime on the positive side shows the opposite relation: the distribution of gradient orientation is more concentrated in the direction perpendicular to the global mean field direction. For the region where $GA_{fast} \sim GA_{Alfven}$, the anisotropy from one mode would be canceled by another mode, so no preferred direction should be found. As a result, when $\Delta GA$ approaches 0, we see a broader dispersion of values, with gradient orientation randomly distributed in all directions.

6.3. Synchrotron Intensity

A similar argument could also apply to other observable measures such as a map of synchrotron intensity $I_S$. The physical environment of synchrotron radiation is hot and diffuse, so it is sub-Alfvénic and subsonic. Density fluctuation is moderate and the mean field is much stronger than its fluctuation. We can then write the fluctuation of the synchrotron intensity map $\delta I_S$ as

$$\delta I_S \propto \rho_0 \delta \left( B_{0,POS} + \delta B_{POS} \right)^2$$

$$= 2 \rho_0 \langle B_{0,POS} + \delta B_{POS} (\delta B_{A+POS} + \delta B_{f,POS}) \rangle$$

$$\approx 2 \rho_0 B_{0,POS} \langle \delta B_{A+POS} + \delta B_{f,POS} \rangle.$$  \hspace{1cm} \text{(7)}$$

where $B_{0,POS}$ is the mean field strength in the plane of the sky (POS), $\delta B_{POS}$ is the fluctuation of the magnetic field, which can be decomposed as Alfvénic modes $\delta B_{A,POS}$, slow modes $\delta B_{s,POS}$, and fast modes $\delta B_{f,POS}$. We group the Alfvénic and slow modes together since they share the same anisotropy. The expression comes with the same conclusion as the reduced centroid, which is that the anisotropy of the synchrotron intensity map depends on the amplitude of the fluctuations between the modes. Furthermore, as synchrotron intensity contains two components, the impact of the fast mode would be impaired and the gradient would be more aligned with the magnetic field. Studies of synchrotron intensity gradient (SIG) showed a better accuracy of magnetic field tracing (Lazarian et al. 2017).

To summarize, the anisotropy of observable measures depends on the comparison between the total fluctuations of each mode along the line of sight.

7. Reducing the Impact of Fast Modes on the Gradient Technique

7.1. Amplitude Filtering (AF) Technique

In the last subsection, the net GA difference between the Alfvén modes and fast modes was shown in Figure 8. Even though we assumed equal global energy between two modes, we noticed that the amplitude of Alfvén modes is still a few times larger than that of fast modes. As a comparison shown in the figure, the maximum amplitude of the fast-mode-dominated data points is only $\sim 0.01$ while that for Alfvénic modes is $\sim 0.04$. This indicates that the pixels with a large value of GA in the centroid or synchrotron intensity map would essentially be dominated by the fluctuation of the Alfvén modes. So, we can reduce the impact of the fast mode by utilizing the effect of dominant Alfvén fluctuation at large GA.

To remove the pixels dominated by the fast mode, one can remove pixels with a low value of GA. Those pixels contain an equal mixture of fluctuations between the Alfvén and fast modes, which results in a random orientation distribution with no peak direction. The remaining pixels would then mainly come from the Alfvén modes and have a clear peak direction, which is the local mean direction of magnetic field or polarization.
In Figure 9, we compare the magnetic field tracing with centroid gradients using raw data and using the AF approach. For the latter, the filter suppressing regions of low amplitude gradient was applied. We observe an increase in the AM from 0.89 to 0.93.

We also noticed that this method would be incapable of removing the contribution of fast modes in the supersonic regime. As shocks are created in the supersonic regime, the density effect would play an important role in the shock region. The mass would accumulate at the shock front as the shock pushes across space. Taking an example of the radiative shock in a sub-Alfvénic environment, one that is usually valid in molecular clouds, the density before and after the shock would have a difference of ratio $M_s$. This would create a large GA with an anisotropy that is perpendicular to the mean field as the shock favors pushing in a direction parallel to the mean field.

7.2. Gradients of Gradient Amplitude (GGA)

Besides the GA filtering method, we can also use gradients of GA (hereafter GGA). GAs were first studied by Yuen & Lazarian (2020). There the GAs were used to obtain the sonic Mach number of turbulence, i.e., $M_s = V_L/V_s$, where $V_L$ is the turbulence injection velocity and $V_s$ is the sound velocity. Here we suggest a new way of using the GAs, i.e., to calculate their gradients and use the GGA to trace magnetic field.

Figure 10 shows a comparison of the SIG map and GGA map obtained with synchrotron intensities. To obtain the latter map, instead of using the raw observation maps as an input, we input the GA map for the calculation, taking the gradient direction from the gradient field of GA. The comparison of the two maps shows that GGA mitigates the influence of the fast modes and shows a better performance in magnetic field tracing than the SIG. For the GGA we used the standard procedure of sub-block averaging (Yuen & Lazarian 2017b).

Applying GGA to velocity centroids, we also found that the GGA technique demonstrated a better performance of tracing magnetic field for the reduced centroid. To compare the two techniques, we compute their alignment measure for different block sizes. Figure 11 shows the result. We see that the AM increases with block size for both techniques. However, GGA shows an earlier saturation of smaller block size with the size of $\sim 66^2$ pixels compared to the larger size of $\sim 200^2$ for the gradient technique. For both synthetic maps, the GGA performs better than the gradient with a performance gap of $\Delta AM \sim 0.4$ at the smallest block size. The performance gap then decreases gradually with increasing block size.

Nonetheless, we also notice that the new GGA technique is more sensitive to noise, which is an important factor in handling observational data. The performance of GGA drops significantly with increasing noise. This limits the use of GGAtO observational data with high signal-to-noise ratio only. We expect that the adoption of a smoothing filter, for instance smoothing with a Gaussian kernel, could restore the performance of GGA. We will study the new technique exhaustively in the future.

7.3. Using Velocity Channel Gradients and Combining GGA Approaches

As Lazarian & Yuen (2018a) showed, the decomposed channel map of the three modes shared the same anisotropy properties with centroid and synchrotron intensity maps. Unlike other observable measures imprinting the 3D information through projection, spectroscopic channel maps imprint the 3D information by reallocating it from real space to the LOS velocity space. The anisotropy property of a channel map depends on the channel width chosen. The contribution of the different modes in a channel map cannot be separated linearly even when constant density is assumed.

As demonstrated by Lazarian & Yuen (2018a), the gradient of a velocity channel map carries information on turbulence and its anisotropy. The thickness of the channel determines the information we trace, which is turbulent velocities in a thin channel and turbulent densities in a thick channel. For thin-channel maps, the gradient shows a better tracing performance of magnetic field direction than other techniques, such as correlation function anisotropy analysis, velocity centroid gradient, and reduced centroid gradient in environments with both subsonic and supersonic turbulence. In our study, we also notice that the impact of fast modes is being suppressed in the channel gradient using a thin channel. In Figure 12, we compare the magnetic field tracing with raw centroid gradient (without using the AF technique) and with thin-channel gradient. We observe the increase in AM from $\sim 0.88$ to $\sim 0.99$. Also, the locations of orthogonal gradients in the centroid map are being suppressed in the thin-channel gradient.

We also combined the GGA technique with channel maps for the further suppression of the fast mode. Compared with the raw channel gradients, channel gradients combined with GGA demonstrate a better performance of tracing magnetic field. However, the improvement of applying GGA to a channel is insignificant in contrast to synchrotron intensity or centroid. In Figure 13, we show the result of the improvement from...
combining channel and GGA together with various block sizes using the simulation H0S. A slight improvement of $\Delta AM \sim 0.03$ is observed at the smaller block size, and the performance gap narrows with increasing block size. The performance improvement become negligible when the block size is larger than 442. The result indicates that the hybrid approach is suitable for small block sizes.

8. Discussion

8.1. Intermittency of Fast Modes and Its Importance

The properties of fast modes were studied in a number of earlier papers (Cho & Lazarian 2002, 2003; Kowal & Lazarian 2010). In this paper we identified an important effect corresponding to the fast modes, namely, their intermittency in low-$\beta$ turbulence. The consequences of this property of fast modes should be explored further.

The fast mode plays a vital role in different astrophysical processes, especially in cosmic-ray (CR) scattering and acceleration. As turbulent energy is injected on large scales, the fast mode is the major source of CR scattering in the interstellar medium and intracluster medium (see Yan & Lazarian 2002, 2004). The earlier treatment of CR scattering typically assumes isotropic fast-mode scattering and a homogeneous distribution of fast-mode energy (see Yan & Lazarian 2008). The location of CR scattering usually includes the Galactic Halo and warm ionized medium, which is a low-$\beta$ and subsonic region. The numerical result in this study would suggest a physical picture of the current CR scattering model, i.e., localized CR scattering. The CR scattering may happen in some localized region where fast-mode energy is clustered.

8.2. Relation to Earlier Studies

We noticed that earlier studies have also found the intermittency effect of the MHD compressible mode in the subsonic environment (Lehmann et al. 2016; Park & Ryu 2019; Makwana & Yan 2020). They have explored the spatial distributions of fast modes through various statistics. In this paper, we further explore the relationship between magnetic field direction, gradient orientation distribution, and spatial distributions of fast modes. We also study how the intermittency effect influences the anisotropy of observational measures such as centroid and synchrotron intensity.

The gradient technique recently introduced a new way of tracing magnetic field. Its foundation is the theory of MHD turbulence, particularly the properties of Alfvenic modes.

Velocity gradients as well as their counterparts, e.g., synchrotron intensity and synchrotron polarization gradients, have shown good alignment with the underlying magnetic field, but the alignment was not perfect. The fast modes were suspected to cause the deviations, but it was surprising to see occasional significant deviations of gradients in particular directions. Indeed, in the simulations that were employed, the fast modes were subdominant compared to Alfven modes.

Through a careful analysis of the low-$\beta$ turbulence simulations in this paper we have identified that the reason for this behavior is that fast modes are very intermittent.
Therefore, in spite of general subdominance, the fast modes can dominate the signal along given lines of sight.

8.3. Comparison of Orthogonal Alignment between Gravitating Systems, Shocks, and Fast-mode-dominated Systems

The gravitationally dominated system, the shock region, and the fast-mode-dominated system all share the same gradient signature, an orthogonal gradient distribution in the gradient orientation histogram.

Hu et al. (2020) and Yuen & Lazarian (2017a) studied the velocity gradient in the presence of gravity. When gravity is absent, consider the GS95 picture and we would see that the maximum change in velocity gradient is in the direction perpendicular to the local magnetic field. In the case of strong self-gravity, gravitational force is expected to modify the properties of flows in the vicinity of centers of gravitational collapse. Mass flows in the gravitational center and causes the negative divergence of velocity. This gravitational pull produces the most significant acceleration of the plasma in the direction parallel to the magnetic field, and the velocity gradients are parallel to the magnetic field. As a result, the velocity field deviates from the normal magnetized turbulent fluid. The orthogonal gradient occurs in the gravitational center. This is often closely related to a high column density as mass is accumulated in the dense region. Additionally, Crutcher et al. (2010) showed that there is a power-law relationship between the value of the LOS component of magnetic field strength inferred from Zeeman splitting and the column density. So, orthogonal alignment, density, and magnetic field strength are bound tightly together in gravitationally dominated systems.

In addition, orthogonal alignment in a fast-mode-dominated region is not necessarily related to the density or magnetic field strength. It depends on the local energy ratio in the observable measures. The effect of components and the angle between mean field and LOS play a vital role here. Observable measures containing more than one component could include more weight of slow and Alfvénic modes and balance the clustering effect of the fast mode. We would expect a slight influence on synchrotron intensity and a stronger influence on the centroid. Although the nature of intermittency provides a clustering effect throughout the 3D space, this effect is not noticeable and smooths out during the projection because it is not related to the density and magnetic field strength.

On the other hand, a study found the existence of parallel velocity gradients in an ideal, compressible MHD simulation (Beattie et al. 2020). As shown in Section 4 of Beattie et al. (2020), the authors demonstrated that a large velocity gradient appears in the shock region and its direction is along the mean

![Figure 13](image-url). The variation of AM with block size of channel gradient and GGA using simulation H0S. Block size covered: [11, 18, 22, 33, 36, 44, 66, 72, 99, 132, 198, 396].
magnetic field. As a result, an orthogonal gradient could be found in those regions. We noticed that this phenomenon is closely related to the turbulent system that is both sub-Alfvénic and supersonic. The parallel gradient usually appears in and around the shock regions. Also, it is closely related to the density because a shock accumulates mass. These two properties differentiate the orthogonal gradient that occurs between the shock region and a fast-mode-dominated system. As mentioned in the previous paragraph, we have not found a relationship between the density and fast-mode-dominated region but a parallel gradient occurs in a shock region usually associated with high density and large velocity GA. Those regions usually can be visualized as a density structure with thin and long filamentary shape, which makes them easier to identify than the fast-mode-dominated system. (See Yuen & Lazarian 2017b; Lazarian & Yuen 2018a; Yuen & Lazarian 2020 for the identification of the shock in the regime of high sonic Mach number.)

9. Summary

This paper further studies the properties of fast modes and their impact in both 2D and 3D space using low-$\beta$ MHD simulations through the numerical method introduced by Cho & Lazarian (2003). Our main discoveries are:

1. The fast modes in low-$\beta$ MHD turbulence have less energy than other modes, but they are very intermittent and are dominant in localized subregions.

2. In those localized regions, the physical properties would change; for instance, the velocity anisotropy could be perpendicular to the local magnetic field.

3. We also studied the impact of fast modes on typical observable measures such as centroid and synchrotron intensity maps. Even though the Alfvén modes dominated along the line of sight, we found that the mean fluctuation caused by fast modes could have a noticeable influence on the 2D observable measures. For a local region where the fluctuation is dominated by the fast mode, the anisotropy would be perpendicular to the plane-of-sky magnetic field, therefore resulting in an orthogonal gradient.

4. Based on the development of the VGT, we further developed the amplitude filtering technique and GGA technique to suppress the impact of the slow mode. We applied the new techniques to synthetic centroid and synchrotron maps and showed that they could reduce the impact of the fast mode and therefore improve the tracing power of the gradient technique.

5. We compared the orthogonal gradients that occur in a collapsing self-gravitating region and a fast-mode-dominated region. The two are distinguishable because gravitational centers are often closely related to high column density, strong magnetic field, and high GA.

6. We discussed that the intermittency of fast modes could be very important for many astrophysical applications, e.g., cosmic-ray propagation and acceleration.

Appendix

Component Effect of MHD Modes

Our paper shows that the spatial distribution of fast mode energy is not uniform but exhibits a clustering effect. In this section, we also discuss the energy distribution of each mode across different components as their features of propagation determine their energy distribution.

Taking Alfvénic waves as a starting point, in a simple physical picture an Alfvénic wave propagates transversely along the magnetic field lines. As a result, fluctuation is only allowed in the direction perpendicular to the local magnetic field lines. In other words, energy would only be distributed in the $k_\perp$ direction. This feature would be preserved in the turbulent environment because the structure function shows the fluctuation of decay in the $k_\perp$ direction in the $5/3$ power law as opposed to the $k_\parallel$ direction.

The situation becomes more complicated for the other two modes, the fast and slow modes, because of their compressible nature. Here, we try to discuss their features in the low-$\beta$ regime. We can start by looking at the presentation of slow modes in unit vector form in Cho & Lazarian (2003):

$$\hat{\xi}_s \propto \left(1 - \frac{\beta}{2} - \sqrt{D}\right)k_\perp \hat{k}_\perp + \left(-1 - \frac{\beta}{2} + \sqrt{D}\right)k_\parallel \hat{k}_\parallel \quad (A1)$$

By considering $\beta \ll 1$ and taking the Taylor expansion, the expression can be simplified as

$$\hat{\xi}_{s,\beta<1} \propto \frac{\beta}{2} \left(2\cos^2 \theta - \frac{\beta}{2}\right)k_\perp \hat{k}_\perp + \left(-2 + \frac{\beta}{2} \left(2\cos^2 \theta - \frac{\beta}{2}\right)k_\parallel \hat{k}_\parallel \quad (A2)$$

We see that slow modes depend on $\beta$ and the angle between the LOS and the mean field lines. Another point to make here is that apparently the $k_\perp$ component dominates over the $k_\parallel$ direction because of the $\beta/2$ factor. Comparing the $k_\parallel$ and $k_\perp$ components, the most important consequence is that the slow mode behaves oppositely to the Alfvénic modes in the low-$\beta$ environment: almost no slow mode energy is distributed in the direction perpendicular to the magnetic field lines.

On the other hand, the ratio between the $k_\parallel$ and $k_\perp$ components of the fast mode plays a minor role here. The ratio does not change much and remains close to unity. This indicates that the energy of the fast mode is distributed uniformly in all directions. This matches the feature of an acoustic-type cascade for fast modes because its propagation is like a sound wave, marginally concerned with the magnetic field direction.

As observational variables may contain only one or two vector components, this component effect makes a huge impact on them. A typical example would be the centroid. This could be shown by the reduced centroid map, in which the contribution of velocity could be decomposed as a superposition of different modes:

$$C(R) = \rho_0 \int v_{1,LOS}(R, s) ds = \rho_0 \int v_{A,LOS} + v_{s,LOS} + v_{r,LOS} ds = C_A(R) + C_s(R) + C_{f}(R), \quad (A3)$$

where $C_A$, $C_s$, and $C_f$ are the separate contributions from each mode and $\rho_0$ is the constant density. As we have just shown, the angle $\theta$ between the mean field vector and the line of sight determined the contribution of each mode, especially for slow and Alfvénic modes. For $\theta \sim \pi/2$, Alfvénic and fast modes contribute most of the anisotropy while the slow mode plays a minor role on the centroid. In this case, the fast modes play a more important role than slow modes. This effect would
become moderate on the synchrotron intensity map since it contains two components, which average out the contribution of each mode.

References

Beattie, J. R., Federrath, C., & Seta, A. 2020, MNRAS, 498, 2
Beresnyak, A., & Lazarian, A. (ed.) 2019, Turbulence in Magnetohydrodynamics (Berlin: De Gruyter)
Brandenburg, A., & Lazarian, A. 2013, SSRv., 178, 163
Burkhart, B., Lazarian, A., Leão, I. C., de Medeiros, J. R., & Esquivel, A. 2014, ApJ, 790, 130
Cho, J., & Lazarian, A. 2002, PhRvL, 88, 245001
Cho, J., & Lazarian, A. 2003, MNRAS, 345, 325
Cho, J., & Vishniac, E. T. 2000, ApJ, 539, 273
Crutcher, R. M., Wandelt, B., Heiles, C., et al. 2010, ApJ, 725, 1
Draine, B. T. 2009, SSRv., 143, 333
Esquivel, A., & Lazarian, A. 2005, ApJ, 631, 320
Esquivel, A., & Lazarian, A. 2011, ApJ, 740, 117
Galtier, S., Pouquet, A., & Mangeney, A. 2005, PhPl, 12, 092310
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
González-Casanova, D. F., & Lazarian, A. 2017, ApJ, 835, 41
Hayes, J. C., Norman, M. L., Fiedler, R. A., et al. 2006, ApJS, 165, 188
Heyer, M., Gong, H., Ostriker, E., & Brunt, C. 2008, ApJ, 680, 420
Hu, Y., Yuen, K. H., & Lazarian, A. 2020, ApJ, 897, 2
Hu, Y., Yuen, K. H., Lazarian, V., et al. 2019, NatAs, 3, 776
Iroshnikov, P. S. 1964, SvA, 7, 566
Jokipii, J. R. 1966, ApJ, 146, 480
Kowal, G., & Lazarian, A. 2010, ApJ, 720, 742
Kraichnan, R. H. 1965, PhFl, 8, 1385
Krasnopolovy, R., Li, Z.-Y., Shang, H., & Zhao, B. 2012, ApJ, 757, 77
Lazarian, A. 2007, JQSRT, 106, 225
Lazarian, A., Vishniac, E. T., & Cho, J. 2004, ApJ, 603, 180
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Lazarian, A., & Yuen, K. H. 2018a, ApJ, 853, 96
Lazarian, A., & Yuen, K. H. 2018b, ApJ, 865, 59
Lazarian, A., Yuen, K. H., Ho, K. W., et al. 2018, ApJ, 865, 46
Lazarian, A., Yuen, K. H., Lee, H., & Cho, J. 2017, ApJ, 842, 30
Lehmann, A., Federrath, C., & Wardle, M. 2016, MNRAS, 463, 1026
Mac Low, M.-M., & Klessen, R. S. 2004, RvMP, 76, 125
Makwana, K. D., & Yan, H. 2020, PhRvX, 10, 031021
Maron, J., & Goldreich, P. 2001, ApJ, 554, 1175
McKee, C. F., & Ostriker, E. C. 2007, ARA&A, 45, 565
Park, J., & Ryu, D. 2019, ApJ, 875, 2
Schleicher, D. R. G., Banerjee, R., Sur, S., et al. 2010, A&A, 522, A115
Soler, J. D., Beuther, H., & Rugel, M. 2019, A&A, 622, A166
Stone, J. M., Gardiner, A. T., Teuben, P., et al. 2010, ApJS, 178, 1378
Yan, H., & Lazarian, A. 2002, PhRvL, 89, 281102
Yan, H., & Lazarian, A. 2004, ApJ, 614, 757
Yan, H., & Lazarian, A. 2008, ApJ, 673, 942
Yuen, K. H., & Lazarian, A. 2017a, arXiv:1703.03026
Yuen, K. H., & Lazarian, A. 2017b, ApJL, 837, L24
Yuen, K. H., & Lazarian, A. 2020, ApJ, 898, 1