Bearing Life Prediction With Informed Hyperprior Distribution: A Bayesian Hierarchical Approach

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ABSTRACT A Bayesian hierarchical model (BHM) is developed to predict bearing life using envelope acceleration data in combination with a degradation model and prior knowledge of the bearing rating life. The BHM enables the inference of individual bearings, groups of bearings, or bearings operating under certain conditions. The key benefit of the BHM approach is that the relationships between the bearing model parameters and their prior distributions can be expressed at different hierarchical levels. We begin our analysis using a bearing rating life calculation $L_{10h}$ and an estimate of its associated failure time distribution. Realistic variations to constrain our prior distribution of the failure time are then applied before measurements are available. When data become available, estimates more representative of our specific batch and operating conditions are inferred, both on the individual bearing level and the bearing group level. The proposed prognostics methodology can be used in situations with varying amounts of data. The presented BHM approach can also be used to predict the remaining useful life (RUL) of bearings both in situations in which the bearing is considered to be in a healthy state and in situations after a defect has been detected.

INDEX TERMS Bayesian hierarchical model, bearing life prediction, bearing life rating $L_{10h}$, probability distribution, prognostics, remaining useful life.

I. INTRODUCTION

ONE of the major interests in industry is the extension of the useful life of high-performance systems. Proper maintenance plays an important role by extending the useful life, reducing the life cycle costs and improving the reliability and availability. The reliability of a component or system is a measurement of its performance with respect to its intended function above a minimum standard for a specified period of time under defined circumstances. Prognostics and health management (PHM) is an engineering discipline that aims to maintain system behaviour and function while ensuring mission success, safety and effectiveness. Health management using proper predictive maintenance (EN13306 [1]) deployment is a worldwide-accepted strategy that has become popular in many industries in past decades. These techniques are relevant in environments where the prediction of a failure and the prevention and mitigation of its consequences increase the profit and safety of the facilities concerned. Prognosis is the most critical part of this process and is a key feature in this maintenance strategy since the estimation of the remaining useful life (RUL) is essential. The remaining useful life (RUL) can be defined as the length of time from the current time to the end of the useful life [2]. PHM can provide a state assessment of the future health of systems or components of interest, e.g., when a degraded state has been found. With this technology, one can estimate how long it will take before the equipment will reach a failure threshold under the future operating and environmental conditions. A major challenge in PHM is to accurately predict the RUL, which often depends on multiple parameters that are time and operation dependent. These relationships and models have to be derived from the physical understanding of the system or by measuring its degradation behaviour [3].
In the literature, prediction models are often classified according to different terms, see e.g., [4]–[8]. According to the type of model [9], they can be categorised into the following three groups: (i) physical degradation models, (ii) stochastic processes models and (iii) statistical models, seen in detail in Fig. 1, with a detailed explanation in reference [8], [9].

Bearings are normally considered to be non-repairable units; for example, in consumer cars, a failed wheel bearing is replaced and not restored. However, in some expensive and large bearing applications, the bearings can be restored. Whether a bearing is considered to be a repairable or non-repairable unit is dependent on the application and the economic aspects. The degradation rate of a bearing is driven by external factors, the system design and the operation (speed, load, etc.). For example, in a large wind turbine, whether a bearing can be restored depends on the failure mode [3].

In a wind turbine, the bearing loads and rotating speeds vary considerably due to changing winds. In this industry, the failure of a single roller or bearing can lead to a stoppage of power generation, with expensive consequences. Another example in which bearings play a vital role is in the paper industry. In a paper machine where pulp is transformed into paper, rotating components, such as bearing-mounted rollers, play an important role in driving the wire with the pulp through the process. In this type of serial layout, the failure of a single roller or bearing could lead to the stoppage of several production steps, with expensive consequences. In this case, a predictive maintenance strategy can be implemented to ensure and optimise the asset availability. Therefore, bearing life time prognosis plays a significant role in reducing plant down time [3].

The rolling element bearing, which has several failure modes, is the central component in rotating machinery. Hence, there is a considerable need to monitor the health of a bearing and to detect faults early to increase the operating life. Vibrations, oil debris and temperature can be used to detect bearing degradation. Vibrations in a machine are due to the dynamic behaviour of the parts subjected to an exciting force. These vibrations can be measured, extracted, and analysed. The main objective in condition monitoring via vibration analysis, oil analysis, etc. is to detect the degradation of bearing health before failure occurs. This condition assessment can provide early warnings based on vibration analysis. This early-warning information is often used to plan maintenance, but it can also be used to change the operational conditions. Various methods are used to analyse vibration signals, including time-domain, frequency-domain and time-frequency analysis. Considering bearing monitoring and analysis, a wide range of vibration-based bearing fault detection methods have been examined in the literature [10]–[12].

Vibration signals are used for trend analysis of machinery and its components by tracking the changes in the amplitude of signal components. By assessing the changes in a monitored vibration signal, one can predict problems like spalling of the bearing raceways and plan maintenance activities. Vibration signals have been utilised in many applications to evaluate system degradation [13]. The RUL prediction of rolling element bearings is complex because several variables influence bearing behaviour. A failure in a single component of a bearing could lead to added cost, a higher risk to the people operating the machinery and a higher vulnerability of the surrounding environment.

PHM addresses the prediction of future conditions and how to manage the health of an asset. In complex industrial applications, fault propagation is often difficult to predict, especially when multiple dependencies and complex relations exist among various process parameters and asset health. In most cases, degradation data for the entire lives of the components are not available, and no explicit relationship has been established between damage and the measured health conditions. Therefore, there is a need for methods that can be used to classify the health status and to predict the RUL.

In a recently published article on bearing degradation and life prediction, the authors addressed the issue related to incomplete signals for RUL prediction of a nonlinear degradation system [14]. In another recently published article by Rodriguez [15], RUL prediction for multiple-component systems based on a systems-level performance indicator was studied, where a system-level performance indicator was computed based on the performance of each component.

The proposed method can provide a common prediction framework for combining all failure data to predict the EoL. Predictions based solely on the prior distributions and no measurement data can be achieved but have increased uncertainty. This method is also beneficial when sequential measurement and estimation is costly and when probability assessment and risk analysis prior to a mission is desired. We have referred to several works related to this topic As mentioned above our objective is to propose a hierarchical Bayesian estimation methodology and this methodology can be used in combination with different degradation models and processing techniques. The degradation model depends heavily on the data measured [3].

Several case studies and algorithm development processes were addressed in a recently published book [16].

This paper presents a Bayesian hierarchical model.
(BHM)-based RUL prognostic approach for bearing life. The model uses the bearing rating life \( L_{10} \) and its associated failure time distribution, in combination with run-to-failure measurements of 14 bearings and an exponential model of bearing degradation. The approach is illustrated using run-to-failure vibration measurements taken at constant load and speed.

The goal of the manuscript is not to present a method for improved detection time of bearing defects or fault diagnostics. The aim of the manuscript is to present a BHM prognostics approach for bearing prognostics that uses, e.g., a health indicator combined with other data sources. Other types of indicators can be used if desired. The description of the BHM approach is not dependent on the latest or most advanced health indicator.

Our objective is to propose a hierarchical prognostics approach. This approach is not dependent on a certain health indicator, degradation model or preprocessing technique.

The benefits of BHM are that we can obtain prognostics of bearing failure times in situations with little or no data relying on prior distributions governed by \( L_{10} \) and in situations with an abundance of failure time data that will adjust the prior distributions to reflect our specific batch of bearings [3].

II. DATA DESCRIPTION AND DEGRADATION MODELLING
In this section, the data collection and the degradation model are described. The vibration run-to-failure data used in this study were generated by a test rig at SKF. To reduce the complexity and clearly show the benefits of using the hierarchical prognostics approach, we present the hierarchical model using basic RMS-based health indicators and simple exponential degradation models. As mentioned earlier, more advanced preprocessing techniques, health indicators and degradation models can be included with added complexity. The degradation model was selected from the literature [7], [17].

A. DATA DESCRIPTION
The test rig was a dual rig with the possibility of measuring two bearings at the same time. The vibration data were collected for each bearing using a 3-axial accelerometer mounted on the bearing houses, and the measurement in the radial direction was stored. An SKF Multilog IMx and SKF @ptitude Observer software were used to collect the data and to extract the health indicator. The measurement set up for the bearing test rig is shown in Fig. 2. The SKF @ptitude was set to sample the acceleration signal with a sample rate of 2560 Hz. The health indicator was generated by selecting the inbuilt SKF @ptitude software option of calculating the RMS value of the enveloped signal. Measurements of 3.2 seconds were extracted with an interval of one hour until a first indication of a bearing fault was identified. The interval was then reduced to 5 minutes between each measurement. The tests were terminated when the health indicator reached the level of 3g of the enveloped signal, which is considered to be the failure threshold. A typical damaged raceway of a bearing after a test is shown in Fig. 3. During the 14 run-to-failure bearing tests, the speed was set to 1800 rpm, the bearing load was 18 kN, \( C/P = 2.16 \) (where \( C \) is the basic dynamic load rating in kN and \( P \) is the equivalent dynamic bearing load in kN).

During the test, different failure modes were achieved. The following failure modes were detected for the following bearings as shown in Fig. 6: Outer Race Bearing No. \( t_{i0}, t_{i1}, t_{i2}, t_{i5}, t_{i6}, t_{i7}, t_{i8}, t_{i11}, t_{i12}, t_{i13} \), Inner Race: Bearing

![FIGURE 2. Bearing test rig with the capacity of testing two bearings simultaneously with different speeds and loads. The upper image shows a view of the right bearing. The lower figure shows a view of the left bearing. The accelerometers (8) are mounted on top of the bearing house and measure vibrations in the radial direction (vertical direction).](image-url)
FIGURE 3. (a) & (b) Outer race spalling for faulty bearings.

TABLE 1. The operating condition parameters.

| Parameter       | Value       |
|-----------------|-------------|
| Load [kN]       | 18          |
| C/P             | 2.16        |
| Speed [rpm]     | 1800        |
| Lubrication     | Shell Turbo 100.0.201/m |
| Support bearing | 1309 ENT9   |
| Test bearings   | 21312E      |

No. t_{i_{10}}, Ball fault: Bearing No. t_{i_3}, t_{i_4}, t_{i_9}.

B. EXPONENTIAL DEGRADATION MODELLING

The exponential degradation model is designed to describe the degradation processes that follow the exponential-type degradation trend.

To predict the state of health behaviour, a degradation model that describes the relation between the bearing inputs, i.e., load and speed, and the degradation needs to be developed or selected. Bearing degradation behaviour is a complex phenomenon that is affected by several conditions (e.g., loading, speed, temperature, humidity and other operating conditions). Bearing degradation is stochastic in nature due to inherent randomness in manufacturing and during operation [17]. Using the BHM approach, we adopted the exponential degradation model as a signal model, which is one of the most widely used stochastic process models. This choice of model describes the central tendency of the observed data adequately (represented by the red solid line and blue curve, respectively, in Fig. 6) in combination with appropriate distributions to capture the necessary uncertainties (represented as the red area in Fig. 6) described in the next section. The exponential model is one of the most widely used stochastic process models. Many alternative and more complex models can easily be included in the same framework, and other choices will mainly affect model equation in Eq. (1-2) and Fig 4, leaving the model structure unchanged [3].

III. PROGNOSTICS METHOD

This paper proposes a BHM-based RUL estimation for bearing life. The hierarchical model uses the failure time distribution associated with the bearing rating life $L_{10h}$ as prior knowledge in combination with run-to-failure measurements and an exponential degradation model see (1). $L_{10h}$ is calculated using the bearing rating life described in the handbook of SKF, according to standard [13]. Using the $L_{10h}$ value, the associated distribution was estimated [18]. We relied on the Stan package [19] for modelling and the NUTS sampler to infer the posterior. Fig. 4 a three-level hierarchical bearing model. The graphs illustrate the distributions for each parameter and their dependencies in the hierarchical level. The first-level model contains the individual bearing model parameters, denoted as $\mu_j$, $\lambda_j$, $d_j$, and $\sigma_j$. The second-level model consists of prior distributions for the parameters in the first level. The distributions in third level are the hyper-prior distributions for the second-level parameters. The three levels of the hierarchical model structure are presented in the section below.

A. THE HIERARCHICAL MODEL STRUCTURE

In this paper, a three-level BHM is considered, as shown in Fig. 4, which illustrates the complete hierarchical approach, a standard technique for showing the dependence in the hierarchy and between parameters [20]. The method implementation follows standard references in Bayesian hierarchical modelling (see, e.g., [20]–[22]). The first-level model describes the $j$th individual bearing and its model parameters denoted by $\mu_j$, $\lambda_j$, $d_j$, and $\sigma_j$. The bearing exponential model is noted as $h$.

The second-level model contains the prior distributions for the parameters in the first level and is parameterised by the hyper-parameters $\mu_\mu$, $\sigma_\mu$, $\mu_\lambda$, $\sigma_\lambda$, $\alpha_d$, $\sigma_d$, and $\alpha_\sigma$, $\beta_\sigma$. The third layer defines the hyper-prior distributions that are vague and non-committal [20] except for the failure time distribution that is based on $L_{10h}$ calculations. The hierarchical model describes the behaviour of the group of bearings under a particular operating condition. With measurements from our batch, we use the proposed hierarchical model to estimate more specific prior distributions for our particular conditions. The three levels of the model are described in detail below.

Note that when using Bayesian statistics, all probabilities are conditional probabilities. To simplify notations, we follow the notations in [20] and do not repeat the parameters we condition on to the left of the equal signs in (1) and in (3–6).
1) Level 1: The Individual Bearing

Here $y_{ij}$ represents the $i$th time sample of the vibration envelop signal (RMS) of the $j$th bearing, with the associated time vector $t_{ij}$ for that particular bearing. The exponential degradation modelling used in the first level is expressed as follows:

$$y_{ij} \sim \ln N(h(\mu_j, \lambda_j, d_j, t_{ij}), \sigma_j), \quad (1)$$

where

$$h(\mu_j, \lambda_j, d_j, t_{ij}) = \mu_j + e^{\lambda_j(t_{ij} - d_j)} \quad (2)$$

is the signal model (in log scale) for the $j$th bearing, parameterised by $\mu_j$, $\lambda_j$, and $d_j$. The uncertainty $\sigma_j$ reflects both modelling errors and measurement noise. Note that all parameters are random variables [20]–[22] and associated with prior distributions explained in detail in Level 2 below.

Many alternative and more complex models can easily be included in the same framework, and other choices will mainly affect equation $h()$ in (1–2) and Fig. 4, leaving the model structure unchanged.

We use a log-normal distribution to describe the measurements, as the RMS values are strictly positive. Note that for log-normally distributed data, the expected value (in original scale) is given by $\exp(h(\mu_j, \lambda_j, d_j, t_{ij})) + 0.5\sigma_j^2$, and $h(\mu_j, \lambda_j, d_j, t_{ij})$ will instead represent the expected value of $\log(y_{ij})$ in log scale.

2) Level 2: The Group of Bearings

The second layer contains the distributions for the parameters in the first layer and describe the variations associated with our particular batch of bearings ($j = 0, \ldots, 14$) and in our operating conditions:

$$\mu_j \sim N(\mu_\mu, \sigma_\mu), \quad (3)$$
$$\lambda_j \sim N(\mu_\lambda, \sigma_\lambda), \quad (4)$$
$$d_j \sim \text{Weibull}(\alpha_d, \sigma_d), \quad (5)$$
$$\sigma_j \sim \text{Gamma}(\alpha_\sigma, \beta_\sigma). \quad (6)$$

We consider standard distributions for these parameters [20]–[22], and these distributions capture the similarity in the behaviour of our group. If they behave similarly, then these prior distributions will be sharp; if they deviate, then the prior distributions are wider, to capture the dissimilarity among them. We know the distribution of L10 of Bearing life. If the probability distribution of the parameter is identical or close to the general distributions, then the hypothesis is adopted according to that distribution. We have proceed with the mathematical calculations to estimate the parameter.

Here $\mu_j$ in (3) is normally distributed, as it can take negative values due to our choice of using log-normally distributed measurements in (1). The exponential increase in $\lambda_j$ is log-normally distributed, as it is non-negative. The failure times are Weibull distributed [18], and finally...
the standard deviation parameter $\sigma_j$ is gamma distributed following [20]–[22].

3) Level 3: The Distributions for the Hyper-Parameters
For the hyper-parameters $\mu_{\mu}, \sigma_{\mu}, \mu_{\lambda}, \sigma_{\lambda}$ and $\alpha_{\sigma}, \beta_{\sigma}$ at the second level, we assign hyper-prior distributions that are vague and non-committal

$$\mu_{\mu} \sim N(0, 10), \quad (7)$$
$$\sigma_{\mu} \sim \text{Gamma}(0.01, 0.01), \quad (8)$$
$$\mu_{\lambda} \sim N(0, 10), \quad (9)$$
$$\sigma_{\lambda} \sim \text{Gamma}(0.01, 0.01), \quad (10)$$
$$\alpha_{\sigma} \sim \text{Exp}(10), \quad (11)$$
$$\beta_{\sigma} \sim \text{Exp}(10), \quad (12)$$

following [20]–[22]. However, for the parameters $\alpha_d$ and $\sigma_d$ governing the failure time distribution in (5), we use an informed hyper-prior distribution based on $L_{10h}$ calculations and a factor of one third the standard deviation around $L_{10h}$:

$$\alpha_d' \sim \ln N(\ln(0.7), 0.33), \quad (13)$$
$$\sigma_d \sim \ln N(\ln(344), 0.33), \quad (14)$$

where $\alpha_d = \alpha_d' + 1$ in the Weibull distribution in (5). These choices will constrain the prior distribution of failure time around $L_{10h}$ in the absence of measurements. A standard deviation of one third (0.33) allows the prior parameters $\alpha_d$ and $\sigma_d$ in the prior distribution (5) to vary around its median values of 1.7 and 344, respectively, obtained from the $L_{10h}$ calculations described in the next paragraph. The $\alpha_d' = \alpha_d + 1$ re-parametrisation in the Weibull distribution will restrict the parameter $\alpha_d$ to always exceed one and at the same time return a median value of 1.7 instead of 0.7, following the procedure proposed in [20]. This will force the Weibull distribution to zero at time zero. The set of parameter values (1.7 and 344) will constrain the prior distribution of failure times around $L_{10h}$, and the standard deviation (0.33) will allow it to adjust to our specific batch and operation conditions. Decreasing or increasing the standard deviation means that $L_{10h}$ will have more or less influence, respectively, on the prior for failure times. If the standard deviation is zero, then the prior distribution will be governed completely by $L_{10h}$, and we assume that $L_{10h}$ is perfectly representative of our batch of bearings. If we increase the standard deviation to a very large value, we assume that $L_{10h}$ has very little or no impact on the prior distribution for failure times for our batch, as the hyper-prior will be very non-informative [3].

As we shall see in Section IV and Fig. 5, our specific choice of (0.33) will give more than enough flexibility for the prior distribution to capture dissimilarities from $L_{10h}$ for our batch while at the same time providing necessary information prior to any measurements.

**TABLE 2. Life adjustment factor $a_1$ [SKF handbook].**

| Reliability | Failure probability | SKF rating life $L_{nm}$ | Factor $a_1$ |
|-------------|---------------------|--------------------------|-------------|
| %           | %                   | Million revolutions     |             |
| 90          | 10                  | $L_{10m}$               | 1           |
| 95          | 5                   | $L_{5m}$                | 0.64        |
| 96          | 4                   | $L_{4m}$                | 0.55        |
| 97          | 3                   | $L_{3m}$                | 0.48        |
| 98          | 3                   | $L_{2m}$                | 0.37        |
| 99          | 1                   | $L_{1m}$                | 0.25        |

**TABLE 3. Weibull distribution and $\alpha_d$ values.**

| (%)    | 10  | 5   | 4   | 3   | 2   | 1   |
|--------|-----|-----|-----|-----|-----|-----|
|        | 18  | 1.61| 1.59| 1.64| 1.66| 1.67|
| 5      | 151 | 1.51| 1.69| 1.70| 1.73|
| 4      | 18  | 1.87| 1.77| 1.78|
| 3      | 171 | 1.71| 1.76|
| 2      | 179 | 1.79|

**TABLE 4. Weibull distribution and $\sigma_d$ values.**

| (%)    | 10  | 5   | 4   | 3   | 2   | 1   |
|--------|-----|-----|-----|-----|-----|-----|
|        | 367 | 386 | 367 | 362 | 352|
| 5      | 429 | 347 | 334 | 331|
| 4      | 286 | 311 | 310|
| 3      | 335 | 320|
| 2      | 309|

4) $L_{10h}$ to Weibull Calculations
The $L_{10h}$ ($L_{10h}$ with hour as the unit) for the bearings used in this study was calculated using the formula in [18]:

$$L_{10h} = 10^9 \cdot 60 \cdot n \left( \frac{c}{p} \right)^3$$

(15)

where $c/p = 2.16$ and the speed $n = 1800$ rpm, resulting in $L_{10h} = 93.312h$. Hence, 10% of a bearing population will have failed after approximately 93 hours. In this paper, it is assumed that $L_{10h}$ follows a Weibull distribution. To estimate the Weibull parameters (to use in the hyper-prior distribution described earlier), the failure probability function of a Weibull distribution

$$F(t) = 1 - \exp\left(-\frac{t}{\sigma_d}\right)^{\alpha_d},$$

(16)

was used together with relations between different values of failure probability, as shown in Table 2.

An estimate of the values was obtained by solving a system of equations using two sets of this equation with different failure probabilities in percentage (%) (1, 2, 3, 4, 5, 10) and with related scale factors $a_1$. A scale factor of 1 represents the $L_{10h}$ failure time. This was done for all combinations in Table 2, resulting in 15 sets of parameter values; see Tables 3 and 4. The mean value of these estimates ($\alpha + 1 = 1.7$ and $\sigma = 344$) was used to describe the initial failure time Weibull distribution of the bearings. A graphical representation of all 15 estimated Weibull distributions, including the mean distribution, can be seen in Fig. 5.
Measurements in which the enveloped signal deviated more areas illustrate the 2.5% and 97.5% confidence intervals. The exponential model fitted to the data. The light red j envelope signals. The bearing number is denoted by j estimate the parameter $\theta_j$. In Fig. 7, the posterior distribution of the model parameter $d_j$ is plotted for each bearing. The parameter represents the distributions in time at which the argument in the exponential part of the signal model (2) is zero, which is considered the RUL. Signals with a larger deviation compared to the exponential model behaviour, resulting in the same behaviour as in Fig. 6, present a wider distribution for the predicted RUL. Fig. 8 shows the posterior distributions for mean values of the RMS signal prior to failure. For bearing no 7 ($j = 7$), a wider distribution is seen due to the increased uncertainty also visible in Fig. 6.

The exact failure initiation time is not known due to the nature of the experiment and may only be inferred in this approach through the failure time (Fig. 7) in combination with the exponential decay model (Fig. 6) if desired.

In Fig. 9, the red graph represents the prior Weibull distribution of bearing life in (5) without access to any measurements, using the informed hyper-prior distribution associated with $L_{10h}$ and a 33% standard deviation of the Weibull parameters. The bold red line represents the prior distribution in (5) using the median hyper-parameter values, $\alpha_d = 1.7$ and $\sigma_d = 344$, i.e., the median of the hyper-prior distributions in (13). The thin red lines represent 100 credible hyper-parameters drawn from the hyper-prior distribution in (13) and depicts the possible variations of the prior distribution controlled by the hyper-prior distributions. The blue graph represents the updated prior distribution for our particular conditions when we have access to our 14 run-to-failure measurements. The thin blue lines represent 100 credible prior distributions given the data and depict the possible variations combining both the uncertainty in the data and the allowed variations defined by the hyper-prior distributions. The blue dots represent the mean value of the posterior failure times $d_j$ for our $j = 0, \ldots, 13$ bearings, shown in Fig. 7.

The initially calculated $L_{10h}$ bearing rating life of approximately 93 h and its associated Weibull distribution (see Fig. 9 thick red curve) over-estimate the life of the bearings. Using the BHM approach, which combines the prior knowledge and the measurement data, this estimate can be changed to represent the real lifetimes (blue thick curve). The initial failure distribution based on the designed $L_{10h}$ value can also be updated based on changes in the input parameters of the bearing life ($L_{10h}$) rating calculation e.g., a changing load $p$ during operation.

Several types of prognostic technologies have been reported in the literature. However, the applicability of such technologies in industry remains a challenge, mainly due to the time-consuming calculations (which are dependent on the requirements and the applications) and the lack of required data and health status models directly related to the physical parameters. Unfortunately, an analytical expression of the joint posterior using BHM with realistic distributions

**FIGURE 5. Estimated Weibull distributions and the distribution based on the mean $\alpha + 1$ and $\sigma$ (thick red curve).**

**IV. RESULTS AND DISCUSSION**

The model uses the bearing rating life $L_{10h}$ and its associated failure time distributions as prior knowledge in combination with run-to-failure measurements of 14 bearings and an exponential model (see 1) of the bearing degradation. In this paper, we start our analysis using $L_{10h}$ and its associated failure time distribution together with realistic variations around the distribution parameters to constrain our prior beliefs of the failure time distribution prior to our measurements. When data become available, estimates of failure times representative of our specific batch and conditions can be inferred. The complete hierarchical model is updated as soon as more information (measurements) becomes available. When no measurements are available, the prognostics are based only on prior distributions as explained later in this section. Ideally, the hierarchical model updates sequentially, but this sequential updating procedure may be computationally demanding, depending on the sample rates of the health indicators and the number of bearings in the model. For large models with many individuals and at high sampling rates, other sampling procedures are more appropriate, such as the particle filter approach [23]. The paper proceede with the mathematical calculations to estimate the parameter [3].

Fig. 6 shows the posterior predictive bearing vibration envelope signals. The bearing number is denoted by $j$, where $j = 0, 1, \ldots, 13$. The blue curve represents the envelope measurements $y_{ij}$ (gE). The red solid line shows the exponential model fitted to the data. The light red areas illustrate the 2.5% and 97.5% confidence intervals. As shown in Fig. 6, the confidence interval is larger for measurements in which the enveloped signal deviated more (e.g. bearing no. $j = 7$) compared to the exponential behaviour of the model than for measurements with smaller deviations (e.g., bearing no. 9). In Fig 7, the posterior distribution of the model parameter $d_j$ is plotted for each bearing. The parameter represents the distributions in time at which the argument in the exponential part of the signal model (2) is zero, which is considered the RUL. Signals with a larger deviation compared to the exponential model behaviour, resulting in the same behaviour as in Fig. 6, present a wider distribution for the predicted RUL. Fig. 8 shows the posterior distributions for mean values of the RMS signal prior to failure. For bearing no 7 ($j = 7$), a wider distribution is seen due to the increased uncertainty also visible in Fig. 6.

The exact failure initiation time is not known due to the nature of the experiment and may only be inferred in this approach through the failure time (Fig. 7) in combination with the exponential decay model (Fig. 6) if desired.
is seldom achievable. To infer both the central tendency and the dispersion of the posterior, we must rely on time-consuming sampling techniques [19], [20].

Investigating the uncertainties and estimates in comparison with the measurements in Fig. 6 and the prior distributions and estimates in Figs. 7–9, we have no reason to believe the choices of distributions are insufficient or not flexible enough to describe the observed data.

This paper presented a BHM for bearing life prediction. The BHM can use data from different groups of bearings to characterise the similarities and group dependencies of the bearings (see, i.e., [24]–[26]). The BHM approach can be further improved by applying the approach to other types of health indicators, like the inner race, outer race, cage and ball indicators, which represent different failure modes. These indicators can be associated with different degradation models and prior distributions for the failure times. However, in this study, a general health indicator is considered to clearly present the BHM approach. The BHM approach can be further used to investigate similarities and group-level relationships [3].

The bearings can be grouped based on material hardening, location in a plant, and operation or maintenance actions. The data from the different groups are then used to estimate individual model parameters. The method can also provide RUL predictions based on prior distributions, such as the Weibull distribution associated with the bearing rating life ($L_{10h}$). This method is beneficial when little or no measurement data are available for a particular bearing. A typical situation is RUL prediction for a newly installed bearing or for bearings with little data. Hierarchical estimates of the group-level parameters in the model can be obtained from prior calculations for different bearing groups or for individual bearings under different conditions. The parameter values can be fixed during operation but can also be changed if required. Fixing the parameters in the model limits the number of parameters that need to be sequentially estimated. The Bayesian approach is commonly applied to account for the effects of parameter or data uncertainty in condition monitoring and to obtain more precise RUL predictions [27]. In these cases, the RUL prediction is based on stochastic degradation processes and performance degradation data modelling, where the distribution of the RUL is inferred, making it easy to quantify the uncertainty in the prediction results [28]. The BHM is a method for generating a probabilistic forecast of parameters associated with a degradation model and other input values.

The BHM approach enables inference for both individual bearings and groups of bearings. Estimates of the hierarchical model parameters and the individual bearing parameters are presented, although the investigated data set was obtained from a single group of bearings. The proposed method can produce new predictions of the RUL distribution for each new bearing measurement, enabling real-time prediction. Referring to the bearing failure measurement, by adding different load conditions or other changes in the model parameters, the BHM approach can update the prediction in real time. As the prognostic results are highly data dependent, it is not common to compare different methods (see e.g., [25], [29], [30]).

Bearing life is normally calculated in the design stage using the method of bearing rating life, given a failure time $L_{10h}$. In the presented BHM approach, this prediction is the driving force in the first phase of the bearing life, where all data sources indicate a healthy bearing. In the second phase, when the data indicate a degraded bearing, the measurement data become important in the calculation, resulting in an inferred prediction distribution. Historical degradation behaviour and failure times can also be used to train the model to obtain a better prediction system. By including more information on temperature, load, operation and maintenance conditions, etc., more precise (narrower) priors can be achieved using group-level regressions. In a real operational environment, with an accumulated number of measurements of failed bearings for different batches and conditions, the hierarchical model can estimate more specific prior distributions. The system will hence learn and improve over time [3].
FIGURE 6. Posterior predictive bearing vibration envelope signals. The bearing number is denoted by \( j \). The blue curve represents the envelope measurements \( y_{ij} \) (gE). The red solid line shows the exponential model fitted to the data. The light red areas illustrate the 2.5% and 97.5% confidence intervals.

FIGURE 7. The posterior distribution of the model parameter \( d_j \) is plotted for each bearing. The parameter represents the distributions in time at which the argument in the exponential part of the signal model (2) is zero, which is considered the RUL.
FIGURE 8. Posterior distributions for mean values of RMS signal prior to failure. For bearing no 7 ($j = 7$), a wider distribution is seen due to increased uncertainty.

FIGURE 9. The red graph represents the prior Weibull distribution of bearing life in (5) without access to any measurements, using the informed hyper-prior distribution with $L_{10h}$ and the allowed standard deviation of 33% around $L_{10h}$. The bold red line represents the prior distribution in (5) using the median hyper-parameter values, $\alpha_d = 1.7$ and $\sigma_d = 344$, i.e., the median of the hyper-prior distributions in (13). The thin red lines represent 100 credible hyper-parameters drawn from the hyper-prior distribution in (13) and depicts the possible variations of the prior distribution controlled by the hyper-prior distributions. The blue graph represents the updated prior distribution for our particular conditions, when we have access to our 14 run-to-failure measurements. The thin blue lines represent 100 credible prior distributions given the data and depict the possible variations combining both the uncertainty in the data and the allowed variations defined by the hyper-prior distributions. The blue dots represent the mean value of the posterior failure times $d_j$ for our $j = 0, \ldots, 13$ bearings, shown in Fig. 7. (The dots are randomly scattered vertically to see their identity.)
V. CONCLUSIONS

1) The BHM approach enables the relationships between the bearing model parameters and their prior distributions to be expressed in different hierarchical levels and to be used to predict the RUL of bearings.

2) The probabilistic results of the BHM approach, together with the economic consequences of failure, can be used to estimate the business risk of the bearing application and to assess different operation and maintenance strategies by simulating different future load conditions and maintenance actions.

3) Bearing rating life calculations of failure times and their distribution can be used to constrain our prior distribution of the failure time before measurements are available. As data become available, estimates more representative of our specific batch and operating conditions can be inferred.

4) The initial bearing rating life calculation and distribution can be replaced or combined with measures of previous failure times for similar batches and conditions to obtain a more specific prior distribution.

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