Adhesive friction based on finite element study and n-point asperity model

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Abstract. The present work considers analysis of adhesive friction of rough surfaces using n-point asperity concept for statistical definition of surface roughness features, and accurate finite element analysis of elastic-plastic deformation of single asperity contact. Well defined adhesion index and plasticity index are used to study the prospective contact situations arising out of variation in material properties and surface roughness features. From the present results it is possible to locate the combinations of adhesion index and plasticity index that may yield very low coefficient of friction. Thus suitable choice of surface and material parameters for the contact of two rough surfaces can be made in order to minimize friction typically at low load and micro scale roughness situations.

1. Introduction
Friction is defined as resistance to movement of one body over the other. For smooth surfaces carrying nano to micro scale size of roughness features, adhesion at contact spots is the main cause of friction. Bowden and Tabor [1] in their classical theory, described the adhesive friction as a tangential force required to shear off adhesive bonds formed at the tip of contacting spots due to local plastic deformation. Tabor [2] emphasized that three basic elements are involved in dry friction of solids viz. the true area of contact, the nature and strength of the interfacial bonds formed at the region of contact, and the way in which the material around the contact regions is sheared during sliding. Chang et al. [3] developed a static friction model incorporating these basic elements. The model relates the start of slip to material properties and treats the first yielding of a single material point as a criterion for sliding inception. But, Kogut and Etsion [4] proved that such assumption underestimates the permissible tangential force as the first failed point remains surrounded by considerable volume of elastic material which can support additional tangential load. Roy Chowdhury and Ghosh [5] (henceforth called RG model) used slip and yield as two separate limiting criteria to get total tangential force. In their analysis, RG assumed purely elastic and plastic deformations of contact spots with normally applied load but they ignored the intermediate elastic-plastic deformation. Chang, Etsion, and Bogy [6] (henceforth called as CEB approach) used conservation of tip volume concept to account for behaviour of material under intermediate elastic-plastic deformation. But it is observed that the CEB analytical model suffers from a discontinuity in the contact load as well as in the first derivatives of contact load and contact area at elastic to elastic-plastic transition point. Literature shows some work [7-9] intended to remove the discrepancies found in CEB’s model. But these works were either mere mathematical exercises without much relevance to physical contact phenomenon [7-8] or they lacked the accuracy [9]. Kogut and Etsion [10] (henceforth called as KE approach), based on their accurate FEA (Finite
Element Analysis) results of single contact spot, developed analytical expressions for transition zone behaviour of load and contact area.

Surface roughness (asperity) plays significant role in the study of adhesion and friction. In conventional models the asperities are defined as peaks having some regular form at the tip. Also all such asperities on the surface are assumed to have identical tip radius with only variation in peak heights. Such definition of surface roughness features is far away from reality. The n-point asperity concept introduced by Hariri, Zu, and Ben Mrad [11] assumes that at particular level of separation between an interfering plane and a rough surface, there exist asperities of different sizes (i.e. different radii of curvatures and different heights). Also each asperity doesn’t exist as a separate entity throughout the progression of contact but the earlier asperity gets merged into a new asperity with contact progression. Thus scope exists to study adhesive frictional contact by extending the RG model to cover intermediate transition zones of deformation and using n-point asperity model. To the best of authors knowledge no such study is found in the existing literature. In earlier two studies [12, 13], the present authors studied the frictional contact in n-point asperity frame work. The first study [12] considers adhesive friction without any consideration for transition type deformation; while the second study [13] though uses KE considerations for transition zone, it is for non-adhesive contact situation.

2. Modeling adhesive friction in n-point asperity frame work

Contact between two rough surfaces can be analysed as a contact between an equivalent rough surface and a rigid smooth plane [3-6, 11-14]. In present study, the equivalent rough surface is assumed to be carrying equivalent height ordinates, and existence of n-point asperities is defined statistically by using such height ordinates [11]. In modeling adhesive friction, first formulations are obtained for normally applied load and tangentially applied load (friction load) on a single n-point asperity and then by incorporating this into statistical multi asperity contact model, the total effect is quantified for the whole surface. As mentioned before, in present study the RG model of adhesive friction is extended to cover the transition zones of deformation also by incorporating CEB and KE expressions.

KE model, based upon the critical value of interference between flat plane and an asperity, $\delta_{(y)h}^*$ (which marks the yield inception), demarcates the whole range of deformation into four distinct zones. Total non dimensional applied load for all asperities having specific ‘n’ value will be

$$P_n^* = P_{(e)_n}^* + P_{(ep-1)_n}^* + P_{(ep-II)_n}^* + P_{(p)_n}^*$$  \hspace{1cm} (1)

Where the non-dimensional applied load values in elastic, elastic-plastic stage-I, elastic-plastic stage-II, and plastic zones of deformation are calculated as:

$$P_{(e)_n}^* = M_{(e)_n}^* E[\Delta P_{(e)_n}^*] \text{ for } 0 < \delta_n^* < \delta_{(y)h}^*$$ \hspace{1cm} (2)

$$P_{(ep-I)_n}^* = M_{(ep-I)_n}^* E[\Delta P_{(ep-I)_n}^*] \text{ for } \delta_{(y)h}^* \leq \delta_n^* \leq 6\delta_{(y)h}^*$$ \hspace{1cm} (3)

$$P_{(ep-II)_n}^* = M_{(ep-II)_n}^* E[\Delta P_{(ep-II)_n}^*] \text{ for } 6\delta_{(y)h}^* < \delta_n^* \leq 110\delta_{(y)h}^*$$ \hspace{1cm} (4)

$$P_{(p)_n}^* = M_{(p)_n}^* E[\Delta P_{(p)_n}^*] \text{ for } \delta_n^* > 110\delta_{(y)h}^*$$ \hspace{1cm} (5)

Here parameter ‘n’ indicates the number of height ordinates of which an asperity is comprised; presence of 'n' at subscript or sub-subscript position in a parameter indicates that the value of the parameter is for an asperity which is comprised of specific ‘n’ number of ordinates. $M$ values are the number of n-point asperities lying in respective zones of deformation, $E[]$ gives expected values, and \( \Delta \) terms are the values of single n-point asperity. $\delta_n^*$ is the non dimensional interference of an n-point asperity while $\delta_{(y)h}^*$ marks non dimensional critical interference corresponding to yield inception. For brevity, \( \Delta \) term for only elastic zone of deformation is provided here as below.

$$P_{(e)_n}^* = M_{(e)_n}^* E[\Delta P_{(e)_n}^*] \text{ for } 0 < \delta_n^* < \delta_{(y)h}^*$$ \hspace{1cm} (6)

$$P_{(ep-I)_n}^* = M_{(ep-I)_n}^* E[\Delta P_{(ep-I)_n}^*] \text{ for } \delta_{(y)h}^* \leq \delta_n^* \leq 6\delta_{(y)h}^*$$ \hspace{1cm} (7)

$$P_{(ep-II)_n}^* = M_{(ep-II)_n}^* E[\Delta P_{(ep-II)_n}^*] \text{ for } 6\delta_{(y)h}^* < \delta_n^* \leq 110\delta_{(y)h}^*$$ \hspace{1cm} (8)

$$P_{(p)_n}^* = M_{(p)_n}^* E[\Delta P_{(p)_n}^*] \text{ for } \delta_n^* > 110\delta_{(y)h}^*$$ \hspace{1cm} (9)
\[ \Delta P^*_{(e)} = \frac{(n-1)}{(-\ln \rho)} \left\{ \frac{0.67\delta^{*3/2}}{\psi \epsilon_n^{1/2}} - 1.77(n-1)^{1/2}(-\ln \rho)^{1/2} \phi^{1/2} \delta^{*1/4} \right\} \quad \text{for } 0 \leq \delta^* \leq \delta_{(e)}^*, \tag{6} \]

Where, \( \epsilon_n \) is the curvature coefficient of an n-point asperity; \( \phi = (\gamma \sigma^2)/(E \sigma^2) \) is the adhesion index [14]; and \( \psi = (H \beta^*)/(E \sigma^2) \) is the plasticity index [11]. The terms \( \sigma, \gamma, E \) and \( H \) used in these indices are standard deviation of height ordinates (root mean square roughness), work of adhesion, composite elastic modulus of equivalent rough surface and hardness value of soft surface respectively.

Equation (1) as given above when solved for all values of \( n \), viz. \( n = 3 \to \infty \) (minimum 3 height ordinates are required to define an n-point asperity), will give total applied load on the rough surface.

In RG model, asperities deforming in purely elastic zone are considered to contribute to friction force. But in present study while using KE to demarcate different zones of deformation, contribution to friction force \( (T^*) \) is obtained from the asperities deforming in first two zones (viz. purely elastic and elastic-plastic stage-I) by applying RG’s slip-yield theory. The method adopted to obtain slip and yield parts of tangential resistance is given in author’s earlier works [12].

Finally the same contact situation is formulated with CEB’s considerations of transition zone deformation using n-point asperity surface roughness features.

3. Results and discussion
In order to study adhesive friction with two different approaches viz., CEB and KE, the corresponding formulations modeled for total applied load and total frictional force are solved by using MATLAB code. The applied load and tangential load formulations consists of different surface roughness and material properties related parameters like number of height ordinates \( n \), correlation coefficient \( \rho \), adhesion index \( \phi \) and plasticity index \( \psi \). The different conditions of contact with respect to variation in material properties and surface roughness features is obtained by varying the indices, \( \phi \) and \( \psi \) while variation in normally applied and frictional load is obtained by varying the mean surface separation \( (h^* = h/\sigma) \) between contacting surfaces in the range of 0 to +3\( \sigma \). For standard normal distribution of height ordinates, this range of mean separation covers 99.7% asperities on the surface under consideration. Physical significance of the different parameters and the related data used in present study is the same as discussed in author’s earlier study [12]. In present study three different values of adhesion index (viz., \( \phi = 0.1, 0.5 \) and 0.9) and three different values of plasticity index (viz., \( \psi = 2.0, 0.9 \) and 0.3) are used. Care is taken in selection of these indices for study so that we get significantly different zones with respect to adhesion effect and plasticity effect. Increase in adhesion index value from 0.1 to 0.9 represents increased adhesion effect while decrease in plasticity index from 2.0 to 0.3 represents increased plasticity effect. Physically, increase in adhesion effect is characteristic of smoother surfaces and/or surfaces possessing higher surface energy, while increase in plasticity effect is the characteristic of rough and/or soft surfaces. Figure 1 shows non-dimensional friction force against non-dimensional applied load. The general trend of behaviour observed is that the friction force is almost proportional to applied load. For the same applied load surfaces with higher \( \phi \) value carry higher friction force. In other words, the smoother surfaces with higher tendency for adhesion yield higher resistance to tangential movement. Also it is observed that plots with combination of higher \( \psi \) and lower \( \phi \) show small non linearity. Physically it means surfaces which have higher tendency to deform elastically and are more rough show non-linear behaviour of friction force with normally applied load. Figure 2 shows coefficient of friction versus non-dimensional applied load plots in high adhesion effect \( (\phi = 0.9) \) zone. From the plots it is observed that plots with \( \psi = 0.3 \) and 0.9 have almost zero slope while that with \( \psi = 2.0 \) have some slope. Physically it means surfaces undergoing predominantly plastic type of deformation and having moderate to higher adhesion have constant coefficient of friction. Also it is observed that at a particular value of applied
load, for smaller value of $\psi$, coefficient of friction values are high. Physically it means higher the tendency to deform plastically higher will be coefficient of friction. It is to be noted here that the rough surfaces have higher tendency to undergo plastic type of deformation. Comparison of results from the KE and CEB approaches show that there is qualitative agreement between the results but from quantitative agreement point of view, CEB approach formulation overestimates the results in predominantly elasto-plastic and plastic zones of deformations. From the present results it is possible to locate the combinations of adhesion index and plasticity index that may yield very low coefficient of friction. Thus suitable choice of surface and material parameters for the contact of two rough surfaces can be made in order to minimize friction typically at low load and micro scale roughness situations.

Figure 1 Typical variation of friction force with applied load

Figure 2 Typical variation of coefficient of friction with applied load

4. Conclusion
Use of n-point asperities to define surface roughness features represent more realistic picture than any other conventional method. Also incorporation of KE approach formulation for intermediate transition state of deformation gives complete solution. Qualitative trend of the results found is same as that observed in literature with conventional asperities. It is possible to locate the combinations of adhesion index and plasticity index that may yield very low coefficient of friction. Thus suitable choice of surface and material parameters for the contact of two rough surfaces can be made in order to minimize friction typically at low load and micro scale roughness situations.

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