Multiple gauge theories predict the presence of cosmic strings with different mass densities \( G \mu \). We derive an equation governing the perturbations of a rotating black hole pierced by a straight, infinitely long cosmic string along its axis of rotation and calculate the quasinormal-mode frequencies of such a black hole. We then carry out parameter estimation on the first detected gravitational-wave event, GW150914, by hypothesizing that there is a string piercing through the remnant, yielding a constraint of \( G \mu < 3.8 \times 10^{-3} \) at the 90% confidence interval with a comparable Bayes factor with a string-less analysis. In contrast to existing studies which focus on the mutual intersection of cosmic strings, or the cosmic string network, our work focuses on the intersection of a cosmic string with a black hole.

In this paper, we will show that cosmic string hair of a Kerr black hole would affect its ringdown waveforms, providing a way to search for cosmic strings by gravitational-wave detection and ringdown-spectroscopy analysis. The QNMFs of a Kerr-string black hole are first calculated by solving a modified Teukolsky equation that takes the effect of the cosmic string into account. Then, we will constrain the mass density of cosmic strings piercing through the remnant black hole formed in the merger event GW150914 detected by LIGO [1] with its ringdown signal, as well as discuss the degeneracies between different parameters in our analysis.

Throughout the remainder of this paper, we will adopt the \((+,-,-,-)\) convention and units where \( G = c = 1 \) will be used. However, to remain consistent with the literature, we will keep the \( G \) in front of \( \mu \).

### II. THE PERTURBATION EQUATION

The metric of a Kerr-string black hole, consisting of a black hole with mass \( M \) and specific angular momentum \( a \) pierced by a cosmic string of infinite length along the axis of rotation with dimensionless
mass density $G\mu \ll 1$, is given by \cite{50}

$$
\begin{aligned}
ds^2 &= \frac{\Delta \Sigma}{\Gamma} dt^2 - \frac{\Gamma \sin^2 \theta}{\Sigma} \left( b \frac{d\phi}{\Gamma} - \frac{2aMr}{\Gamma} \right)^2 \\
&\quad - \sum \left( \frac{d\Sigma^2}{\Delta} + d\theta^2 \right),
\end{aligned}
$$

(1)

where $b = 1 - 4G\mu$, $\Sigma = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 + a^2 - 2Mr$ and $\Gamma = (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta$. This metric can be obtained by introducing an azimuthal angular deficit of $8\pi G\mu$ on $\phi$ to the Kerr metric, and it remains a Petrov type D vacuum metric, suggesting that the Teukolsky formalism of treating black hole perturbation could be applied to the system.

Locally, Eq. (1) is equal to the Kerr metric with $\phi$ rescaled to $b \phi$, so the perturbation equation for $\Delta^m$ \cite{11, 52} with the following null tetrad:

$$
\begin{aligned}
l^\mu &= \frac{1}{\Delta} \left( r^2 + a^2, \Delta, 0, \frac{a}{b} \right), \\
n^\mu &= \frac{1}{2\Sigma} \left( r^2 + a^2, -\Delta, 0, \frac{a}{b} \right), \\
m^\mu &= \frac{1}{\sqrt{2(r^2 + i \Delta \cos \theta)}} \left( i a \sin \theta, 0, 1, \frac{i}{b \sin \theta} \right), \\
mn^\mu &= \frac{1}{\sqrt{2(r^2 - i \Delta \cos \theta)}} \left( -i a \sin \theta, 0, 1, -\frac{i}{b \sin \theta} \right).
\end{aligned}
$$

(2)

Then, the master equation for black holes pierced by cosmic strings is derived to be

$$
\begin{aligned}
\frac{d^2 \Psi}{dt^2} + \frac{4Ma r}{\Delta} \frac{d^2 \Psi}{d\phi^2} + \frac{1}{b^2} \left[ \frac{a^2}{\Delta} - \frac{1}{\sin^2 \theta} \right] \frac{d^2 \Psi}{d\phi^2} \\
- \frac{\partial}{\partial r} \left( \frac{\Delta}{\Delta^2} \frac{d \Psi}{dr} \right) - \frac{1}{\sin \theta \cos \theta} \left( \sin \theta \frac{d \Psi}{d\theta} \right) - 2s \frac{a(r - M)}{\Delta} + \frac{i \cos \theta}{\sin^2 \theta} \frac{d \Psi}{d\phi} \\
- 2s \left( \frac{M(r^2 - a^2)}{\Delta} - r - i a \cos \theta \right) \frac{d \Psi}{dt} + \left( s^2 \cot^2 \theta - s \right) \Psi = 4\pi \Sigma T.
\end{aligned}
$$

(6)

This is equivalent to the original Teukolsky equation with $\phi$ replaced by $b \phi$.

III. QUASINORMAL MODE FREQUENCIES

By considering a vacuum perturbation (i.e., $T = 0$), we can compute the QNMFs using Leaver’s method of continued fraction \cite{11, 52}. Analogous to Teukolsky’s computations \cite{51}, by putting $\Psi = e^{i(m\phi - \omega t)} R(r) S(\theta)$, Eq. (6) is separable into a radial equation and an angular equation:

$$
\begin{aligned}
\Delta - s \frac{d}{dr} \left( \frac{\Delta^{s+1} dR}{dr} \right) + \frac{K^2 - 2i s(r - M)K}{\Delta} + 4is \omega r - \lambda \right) R = 0, \\
\frac{1}{\sin \theta \cos \theta} \left( \frac{d \sin \theta ds}{dr} \right) + \left( a \omega \cos \theta - s \right)^2 \\
- \frac{m/b - s \cos \theta}{\sin \theta} \left( \frac{m/b - s \cos \theta}{\sin \theta} \right) - s^2 + s - A \right) S = 0,
\end{aligned}
$$

(7)

where $K = (r^2 + a^2) \omega - am/b$, $\lambda = A + a^2 \omega^2 - 2am \omega/b$ and $A$ is a separation constant. Effectively, the inclusion of a cosmic string amounts to changing $m$ to $m/b$ in Eq. (7) and Eq. (8), the QNMFs can hence be computed by changing the value of $m$ in Leaver’s algorithm.

Fig. 1 shows the dependence of the $(n, l, m) = (0, 2, 2)$ overtone frequency on $G\mu$ at various values of the spin parameter $a/M$. A bluer hue represents a larger value of $G\mu$ and a redder hue represents a larger value of $a$. Both the frequency and the decay rate of gravitational wave emissions increase with the mass density of the cosmic string. Notably, an increasing spin and an increasing $G\mu$ drive the $(0, 2, 2)$ overtone QNMFs in different directions. This suggests that parameter estimation through ringdown analysis is possible. Moreover, none of the common alternative gravitational theories that would affect the QNMFs increase both $|\tilde{\omega}|^m$ and $|\tilde{\omega}|^n$ \cite{59, 57}, so parameter estimation on $G\mu$ done with the QNMFs will not be degenerate with these theories.

Interesting trends are found in the QNMFs of black holes with cosmic strings. In the Schwarzschild case ($a = 0$), the separation constant is given by $A_{lm} = l(l + 1) - s(s + 1)$ \cite{51}. With the effect of a cosmic string included, for modes with $n = 0$ and $l = m = 2, 3, 4$, it is noticed that the separation constant is modified to $A_{lm} = (l/b)(l/b + 1) - s(s + 1)$.
FIG. 1. The real and imaginary parts of the \((n, l, m) = (0, 2, 2)\) overtone QNMFs for \(0 \leq G \mu \leq 0.1\) and \(0 \leq a/M \leq 0.8\). Such a relation is not present in modes with \(l \neq m\).

The QNMFs for each \(nlm\) overtone were fitted to the form

\[
\tilde{\omega}_{nlm}(a, G \mu) = \tilde{\omega}_{nlm}(a, 0) + \tilde{\omega}_{nlm}(0, 0)[\hat{c}_1(a)G \mu + \hat{c}_2(a)(G \mu)^2].
\]  

(9)

The real and imaginary parts of \(\hat{c}_1(a)\) and \(\hat{c}_2(a)\) were then fitted to the form \(k_1 - k_2(1 - a/M)^k_3\) as motivated by Ref. [58]. For the \((0, 2, 2)\) mode, which is the dominant mode in GW150914, the maximum error of the fit over the range \(0 \leq G \mu \leq 0.1, 0 \leq a/M \leq 0.95\) in the real and imaginary parts of the QNMFs is about 0.8% and 1.4% respectively. Table I shows the numerical values of the coefficients for this mode.

| \(f(a)\) | \(k_1\) | \(k_2\) | \(k_3\) |
|----------|---------|---------|---------|
| \(\text{Re}\{\hat{c}_1(a)\}\) | 10.1 | 5.73 | 0.223 |
| \(\text{Im}\{\hat{c}_1(a)\}\) | 2.34 | 1.43 | 0.289 |
| \(\text{Re}\{\hat{c}_2(a)\}\) | 108 | 77.7 | 0.180 |
| \(\text{Im}\{\hat{c}_2(a)\}\) | 60.7 | 53.4 | 0.0503 |

TABLE I. Fitted coefficients for \(f(a) = k_1 - k_2(1 - a/M)^k_3\) for different \(f(a)\) of the \((0, 2, 2)\) mode, where \(f(a)\) are the real and imaginary parts of coefficients appearing in a quadratic of the QNMFs to \(G \mu\).

Fig. 2 plots the ringdown waveforms of a Kerr-string black holes of different \(G \mu\). The waveforms are assumed to be generated from a black hole merger event where the \((n, l, m) = (0, 2, 2)\) and \((1, 2, 2)\) modes are dominant in the ringdown signals, and the mass of the final black hole \(M_f\) and final spin parameter \(a_f\) are the same for all the waveforms. Using two or more modes instead of one, we can ensure that our waveform has four degrees of freedom in its QNMFs, so it can carry information about all three parameters \(M, a\) and \(G \mu\). Altering \(G \mu\) would induce a change in the shape of the waveform, and the higher the value of \(G \mu\), the more the waveform deviates from that from a system without a cosmic string.

Before making use of the QNMF fits to constrain \(G \mu\) with real data, it would be insightful to look into possible degeneracies between \(M_f\) or \(a_f\) with \(G \mu\). We do this by considering the match between two waveforms, one with fixed parameters and \(G \mu = 0\), the other with non-zero \(G \mu\) and varying \(M_f\) or \(a_f\). The match between waveforms is defined by

\[
\text{match}[h_1, h_2] = \frac{(h_1|h_2)}{\sqrt{(h_1|h_1)(h_2|h_2)}}.
\]  

(10)

with

\[
h_1 \equiv h_1(M_{f1}, a_{f1}, G \mu_1; t),
\]  

(11)

\[
h_2 \equiv h_2(M_{f2}, a_{f2}, G \mu_2; t),
\]  

(12)

and

\[
\langle h_1|h_2 \rangle = \int_{t_1}^{t_f} h_1^*(M_{f1}, a_{f1}, G \mu_1; t)h_2(M_{f2}, a_{f2}, G \mu_2; t)dt,
\]  

(13)
where \( h_1(M_{f1}, a_{f1}, G\mu_1; t) \) and \( h_2(M_{f2}, a_{f2}, G\mu_2; t) \) correspond to ringdown waveforms in the time domain with different parameters \( M_f, a_f \) and \( G\mu \), and * denotes complex conjugation. The starting and ending time of the ringdown signals are set to be \( t_i = 0 \) s and \( t_f = 0.030 \) s for matching.

![Contour plots of match between waveforms with different parameters to test the degeneracy of \( G\mu \) with \( M_f \) and \( a_f \).](image)

(a) Match between a fixed string-less waveform with \( M_f \) fixed at 100\(M_\odot\) and one with variable \( G\mu \) and \( M_f \) (both with \( a_f/M_f \) fixed at 0.67). Degeneracy is observed for \( \log(G\mu) > -3 \), where a string-less waveform will look like a string-carrying waveform with a higher mass. (b) Match between a fixed string-less waveform with \( a_f/M_f = 0.4 \) and one with variable \( G\mu \) and \( a_f \) (both with \( M_f \) fixed at 100\(M_\odot\)). Similar to (a), there is degeneracy observed for \( \log(G\mu) > -3 \), but with a string-less waveform looking like a string-carrying waveform with less spin.

The match between the waveforms \( h(M_f, 0.67M_f, G\mu) \) and \( h(100M_\odot, 67M_\odot, 0) \) is plotted in Fig. 3a (calculated with the PyCBC software [59]). By varying \( M_f \) and \( G\mu \), it is found that the two parameters are degenerate in the \( \log(G\mu) > -3 \) regime. Nonetheless, for sufficiently high values of the match, the majority of the contour area is still centered at \( M_f = 100M_\odot \) and terminates at some high value of \( \log(G\mu) \), meaning that it is still possible to constrain \( G\mu \) by matched filtering without affecting too significantly the parameter estimation of \( M_f \). Similarly, Fig. 3b test the degeneracy of \( G\mu \) with \( a_f \). Again, there is degeneracy in the high \( \log(G\mu) \) regime, which will cause a string-less waveform to look like a string-carrying waveform with less spin. This will cause the constrain on \( G\mu \) to be less tight.

## IV. RESULTS

With the QNMFs of the Kerr-string black hole calculated, we will put a constraint on \( G\mu \) of the GW150914 system by assuming that its final black hole is a Kerr-string black hole. Parameter estimation is carried out with the PyCBC pipeline introduced in Ref. [60].

If GW150914 were a merger of primordial black holes (as suggested by [61, 62]), it might be possible for at least one of the black holes in the binary to hold cosmic strings. When this Kerr-string black hole merges with the other black hole, the remnant will also be a Kerr-string black hole and it will emit a ringdown signal characterized by the parameters \( M_f, a_f \) and \( G\mu \). Thus, we can hypothesize that the ringdown signal of GW150914 comes from a Kerr-string black hole and constrain the mass density of the hypothetical string piercing through it.

As mentioned earlier, we will have to use at least two modes to estimate the three parameters \( M, a \) and \( G\mu \). We might have chosen to use the \( (0, 2, 2) \) and \( (0, 2, 1) \) modes in theory due to them being the two most excited modes of GW150914 [64], but the large \( G\mu \) \( (0, 2, 2) \) QNMFs have values too close to those of low \( G\mu \) \( (0, 2, 2) \), which will cause the pipeline to falsely recognize high \( G\mu \) waveforms in the data. Therefore, the \( (0, 2, 2) \) and \( (1, 2, 2) \) ringdown modes are used. The prior distribution for \( \log(G\mu) \) and start time are respectively set to be a uniform distribution within \([-10, -1]\) and \([10, 20]M_\odot \), where \( M = 68M_\odot \) is the reported median value of the final mass in Ref. [60]. The prior distributions of other parameters as well as the treatment of noise of the LIGO data are the same as those in Ref. [60].

Figure 3a shows the contour plot of the final
FIG. 4. The posterior distribution of $\log(G\mu)$ for the GW150914 event. The shape of the distribution resembles that of a step function. There is a sharp drop near $\log(G\mu) = -2$. We obtain a constraint of $G\mu < 3.8 \times 10^{-3}$ at the 90% confidence level. The prior bounds are marked with dotted lines.

spin parameter $a_f/M_f$ and final mass $M_f$, with the median values estimated by a full IMR analysis marked by two black dotted lines. When compared to the same graph for an analysis without the parameter estimation of $G\mu$ [Fig. 3(b)], it can be seen that the plots have generally the same shape. However, the distribution in Fig. 3(a) leans more to the larger $M_f$ side, while the degeneracy in $a_f$ is not clearly seen. Fig. 3(a) also shows a slightly wider spread in $M_f$, but the spread in $a_f$ is reduced. Fig. 4 plots explicitly the three dimensional posterior of $M_f$, $a_f$ and $G\mu$. It can be clearly seen that the higher $G\mu$ points lie to the higher $M_f$ and smaller $a_f$ side, agreeing with our waveform degeneracy analysis in Sec. III.

Figure 5 shows a corner plot for $M_f$, $a_f$ and $\log(G\mu)$. As evident from the absence of clear trend lines in the contour plots of $\log(G\mu)$ against $M_f$ or $a_f$, the degeneracy between the effects of $G\mu$ and those of $M_f$ or $a_f$ are not too significant.

This three-parameter analysis gives a Bayes factor of $\log B = 59.3$. When compared with $\log B = 59.1$ for an analysis with only $M_f$ and $a_f$, this suggests that the two models are similar in plausibility.

V. DISCUSSION AND CONCLUSIONS

A. Assumptions Made

For the analysis using QNMFs obtained from Eq. (6) to be valid, three conditions need to be satisfied:

1. The spins of the black holes during the inspiral phase are aligned with the orbital angular momentum vector.

2. The string is infinitely long and straight after the merger.

3. The string lies on the axis of rotation of the final black hole.
\[ \log(G\mu) \]

FIG. 6. Posterior plot of all three parameters \( M_f, a_f \) and \( G\mu \). The yellow hue corresponds to points with higher \( G\mu \), and they are located more towards the high \( M_f \) and low \( a_f \) side, agreeing with our waveform degeneracy analysis in Sec. III.

FIG. 7. A corner plot showing the posterior distribution of \( M_f, a_f \) and \( \log(G\mu) \). Unlike the contour plot for \( a_f \) and \( M_f \), no clear trend lines can be observed in the contour plots of \( \log(G\mu) \) versus \( a_f \) and \( M_f \).

(1) is necessary for (2) and (3): If (1) is not true, the spin of the Kerr-string black hole will precess, so the string will likely curl around itself, violating (2). Moreover, if the spins of the binary black holes are misaligned, the spin of the final black hole might also be misaligned with the string, violating (3). A rotating black hole pierced by a cosmic string would gradually approach an equilibrium position where the string aligns with the rotational axis of the black hole [15]. We assumed that the time scale for the approach to this equilibrium is much shorter than the time scale for merger and ringdown so that the ringdown signal is not contaminated by the signal from this stabilization.

In reality, as disturbances travel at a finite speed on the string, even if the spins of the binary black holes and the total angular momentum of the black hole merger are aligned perfectly, the string will spiral with the black holes during the inspiral phase. However, as the orbital radius of the black holes decreases, the string will steadily approach its final configuration, in which it passes through the center of mass of the system (i.e., the center of the final black hole) and aligns with the system’s angular momentum, so it could stabilize soon after the merger. In that case, assumptions (2) and (3) can be satisfied locally and approximately.

In the future, work may be done to relax these assumptions and consider more general configurations of the Kerr-string black hole. In particular, numerical simulation of the disturbances to the Kerr-string black hole during the inspiral and merger phase will allow for a more comprehensive analysis of the ringdown signals emitted by such a system.

B. Comparison with Existing Results

Unlike our constraints, methods based on analysis of the cosmic microwave background or the stochastic gravitational wave background apply to entire cosmic string networks. The effects of the cosmic string network on the background spectrum are model-dependent since one must take the spatial arrangement and temporal evolution of the cosmic strings into account. Hence, the constraint so obtained will depend on the precise model used to simulate the cosmic string network (see, for example, Ref. [15]).

Our methods are more similar to searches for gravitational wave bursts or signs of lensing from cosmic strings in that these are methods of direct search. The constraint obtained only applies to a single event and a global constraint can only be obtained given models of event rates and arrangements of cosmic strings.

C. Conclusion

In conclusion, we showed that it is possible to constrain the mass density of cosmic strings piercing rotating black holes by analyzing its ringdown signal. Although the constraint on the GW150914 event is
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