Dissipative Properties and Isothermal Compressibility of Hot and Dense Hadron Gas using Non-extensive Statistics

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We evaluate the transport properties such as shear viscosity (η), bulk viscosity (ζ) and their ratios over entropy density (s) for hadronic matter using relativistic non-extensive Boltzmann transport equation (NBTE) in relaxation time approximation (RTA). In NBTE, we argue that the system far from equilibrium may not reach to an equilibrium described by extensive (Boltzmann-Gibbs (BG)) statistics but to a q-equilibrium defined by Tsallis non-extensive statistics after subsequent evolution, where q denotes the degree of non-extensivity. We observe that η/s and ζ/s initially decrease rapidly with the temperature (T) for various q-values while they become independent of q at higher T. As q increases, the magnitudes of η/s and ζ/s increase in the lower temperature region. We also show the upper mass cutoff dependence of these ratios for a particular q and find that they decrease with the increase in mass cutoff of hadrons. Further, we present the first estimation of isothermal compressibility (κT) using non-extensive Tsallis statistics in Boltzmann Transport equation (NBTE), where we employ the RTA for the collision term. The elliptic flow measurements at relativistic Heavy-Ion Collider (RHIC) experiment have found that the η/s is close to the KSS bound, which developed intense interest in this ratio of the strongly interacting matter described by quantum chromodynamics (QCD) [3, 4]. Also, a peak in Bulk viscosity to entropy density ratio (ζ/s) is expected near QCD critical temperature (Tc), where conformal symmetry breaking might be significant as expected by various effective models [5–7]. Other interesting thermodynamic properties are the isothermal compressibility (κT) and speed of sound (cs), which are used to define the equation of state of the system and quantify the softest point of the phase transition along with the location of the critical point [8].

Due to high multiplicities produced in high-energy collisions, the statistical models are more suitable to describe the particle production mechanism. Such a statistical description of transverse momentum (pT) of final state particles produced in high-energy collisions has been proposed to follow a thermalized Boltzmann-Gibbs (BG) distribution. But, a finite degree of deviation from the equilibrium statistical description of pT spectra has been observed by experiments at RHIC [9, 10] and Large Hadron Collider (LHC) [11–14]. Fortunately, for anomalous systems, where the usual ergodicity is violated such as the states in large systems involving long range forces between particles and metastable states in small systems where the number of particles are relatively smaller, a generalized BG entropy has been introduced by C. Tsallis [15, 16]. Recently, a growing attention has been paid towards the possible non-extensive effects in thermodynamics and statistical mechanics [17, 18] as well as to explain the particle spectra in high-energy hadronic and heavy-ion collisions [19–24]. The nuclear modification factor has also been successfully described by using Tsallis non-extensive statistics in Boltzmann Transport equation (BTE) with relaxation time approximation (RTA) [25, 26].

The aim of the present paper is to study various dissipative properties such as shear and bulk viscosities using the relativistic non-extensive Boltzmann transport equation (NBTE), where we employ the RTA for the collision term. In NBTE, we assume that a non-equilibrium system, which dissipates energy and produces entropy,
The collision integral can be approximated as a rate of change of the non-equilibrium force acting on a fermion while $-u_\mu$ for bosons.

The paper runs as follows. In section II, the dissipative properties such as shear and bulk viscosities are derived using the relaxation time approximation of the relativistic NBTE. The formulation of isothermal compressibility and squared speed of sound in non-extensive statistics are also given. In section III, the results and discussions are presented. Finally, we summarize with the findings of this work in section IV.

II. FORMULATION

We follow the approach mentioned in Ref. [27] to calculate the dissipative properties such as shear and bulk viscosity for a hadronic matter using non-extensive statistics. We start with the BTE given by

$$\frac{\partial f_p}{\partial t} + v_i^p \frac{\partial f_p}{\partial x^i} + F^p \frac{\partial f_p}{\partial p^i} = I(f_p),$$

where $v_i^p$ is the velocity of $i^{\text{th}}$ particle and $F^p$ is an external force acting on $i^{\text{th}}$ particle. $I(f_p)$ is the collision integral which gives the rate of change of the non-equilibrium distribution function $f_p$ when the system approaches $q$-equilibrium.

Assuming no external force and proceeding with the RIA, the collision integral can be approximated as

$$I(f_p) \approx \frac{(f_p - f^0_p)}{\tau(E_p)},$$

where $\tau(E_p)$ is the relaxation time or collision time and $f^0_p$ is the Tsallis distribution function, which is given by

$$f^0_p = \left(\frac{E_p - p \cdot u - \mu}{T} \right) ^\frac{1}{1-q},$$

where $u$ is the fluid velocity and $+$ sign corresponds to fermions while $-$ sign stands for bosons. $T$ and $\mu$ are temperature and chemical potential, respectively. The function $e^{(x)}_q$ is defined as

$$e^{(x)}_q = \begin{cases} (1 + (q-1)x)^{\frac{1}{1-q}} & \text{if } x > 0, \\ (1 + (q-1)x)^{\frac{1}{1-q}} & \text{if } x \leq 0, \end{cases}$$

and in the limit $q \to 1$, it reduces to the standard exponential $e(x) = e(x)$.

Now, the stress-energy tensor ($T^\mu_\nu$) can be written as

$$T^\mu_\nu = T^\mu_\nu + T^\mu_{\text{dissi}},$$

where $T^\mu_{\text{dissi}}$ is the ideal part of the stress-energy tensor and $T^\mu_\nu$ is the dissipative part of the stress-energy tensor.

In the hydrodynamical description of QCD, shear and bulk viscosities enter in the dissipative part of the stress-energy tensor, which can be written (in the local Lorentz frame) as

$$T^{ij}_{\text{dissi}} = -\eta \left( \frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i} \right) - \left( \zeta - \frac{2}{3} \eta \right) \frac{\partial u^i}{\partial x^j} \delta^{ij},$$

In terms of distribution function, this can be expressed as

$$T^{ij}_{\text{dissi}} = \int \frac{d^3p}{(2\pi)^3} p^i p^j \delta f_p,$$

where $\delta f_p$ is the deviation of the distribution function from the $q$-equilibrium and is given by (from Eqs. 1 and 2),

$$\delta f_p = -\tau(E_p) \left( \frac{\partial f^0_p}{\partial t} + v_i^p \frac{\partial f^0_p}{\partial x^i} \right).$$

Assuming a steady flow of the form $u^i = (u_x(y), 0, 0)$ and space-time independent temperature, Eq. 6 simplifies to $T^{xy} = -\eta u_x \partial / \partial y$. Now, from Eqs. 7 and 8, we get (using $\mu = 0$),

$$T^{xy} = \frac{1}{T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau(E_p) p_x p_y}{E_p} f^0_p \frac{\partial u_x}{\partial y}.$$

Thus the coefficient of shear viscosity for a single component of hadronic matter can be expressed as

$$\eta = \frac{1}{15T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau(E_p)}{E_p} \frac{p_x^4}{f^0_p}.$$

The bulk viscosity ($\zeta$) is related to the dissipation when the system is uniformly compressed. From Eq. 6,

$$(T^{\text{dissi}})_{ij} = -3\zeta \frac{\partial u^i}{\partial x^j}$$
and using Eqs. 7 and 8 we get,

\[(T_{dssi})_i = - \int \frac{d^3p}{(2\pi)^3} \tau(E_p) \frac{p^2}{E_p} \left( \frac{\partial f^0_p}{\partial t} + v^i_p \frac{\partial f^0_p}{\partial x^i} \right). \quad (12)\]

Now, using the conservation law for energy-momentum i.e. \(\partial \mu \Gamma^{\mu\nu} = 0\), one can obtain [29]

\[\zeta = \frac{1}{T} \int \frac{d^3p}{(2\pi)^3} \tau(E_p) f^0_p \left( E_p c_a^2 - \frac{p^2}{3E_p} \right)^2, \quad (13)\]

where, \(c_a^2\) is the squared speed of sound at constant baryon density.

For a multi-component hadron gas at finite chemical potential, the shear and bulk viscosities can be written as,

\[\eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E^3_a} (\tau_a f^0_a + \tau_a f^0_a), \quad (14)\]

and,

\[\zeta = \frac{1}{T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E^3_a} \left[ T_a c^2_a + \left( - \frac{\partial P}{\partial n_B} \right)_c - \frac{p^2}{3E^2_a} \right]^2 \]

\[+ \frac{1}{T} \sum_a \int \frac{d^3p}{(2\pi)^3} \left[ T_a c^2_a - \left( - \frac{\partial P}{\partial n_B} \right)_c - \frac{p^2}{3E^2_a} \right]^2. \quad (15)\]

Here \(E_a^2 = p^2 + m_a^2\) and the barred quantities represent the antiparticles. \(f^0_a\) is the distribution function for \(a\)th particle. Now, the energy dependent relaxation time is given as,

\[\tau^{-1}(E_a) = \sum_{bcd} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p_c}{(2\pi)^3} \frac{d^3p_d}{(2\pi)^3} W(a, b \rightarrow c, d) f^0_b, \quad (16)\]

where \(W(a, b \rightarrow c, d)\) is the transition rate defined as,

\[W(a, b \rightarrow c, d) = \frac{2\pi^4 (p_a + p_b - p_c - p_d)}{2E_a 2E_c 2E_2 2E_d} \frac{|c|}{|M|^2}. \quad (17)\]

Here \(|M|\) is the transition amplitude.

In the center-of-mass frame, Eq. 16 can be simplified as,

\[\tau^{-1}(E_a) = \sum_b \int \frac{d^3p_b}{(2\pi)^3} \sigma_{ab} \sqrt{s - 4m_a^2} \frac{\sqrt{s}}{2E_a 2E_b} f^0_b, \quad \equiv \sum_b \int \frac{d^3p_b}{(2\pi)^3} \sigma_{ab} v_{ab} f^0_b, \quad (18)\]

where \(v_{ab}\) is the relative velocity and \(\sqrt{s}\) is the center-of-mass energy. \(\sigma_{ab}\) is the total scattering cross-section in the process \(a(p_a) + b(p_b) \rightarrow a(p_c) + b(p_d)\). For further simplification, \(\tau(E_a)\) can be approximated to averaged relaxation time \(\tilde{\tau} [30]\) and it can be obtained from Eq. 20 by averaging over \(f^0_a\) as,

\[\tilde{\tau}_a^{-1} = \int \frac{d^3p_a}{(2\pi)^3} \tau^{-1}(E_a) f^0_a = \frac{\sum b \int \frac{d^3p_a}{(2\pi)^3} \frac{d^3p_b}{(2\pi)^3} \sigma_{ab} v_{ab} f^0_b}{\int \frac{d^3p_a}{(2\pi)^3} f^0_a}, \quad (19)\]

Here \(n_b = \int \frac{d^3p_b}{(2\pi)^3} f^0_b\) is the number density of the \(b\)th hadronic species. \(f^0_b\) is the Tsallis distribution for \(b\)th particle which is given by Eq. 3.

The momentum space volume elements can be written as,

\[d^3p_a d^3p_b = 8\pi^2 p_a p_b dE_a dE_b d\cos \theta. \quad (20)\]

The numerator in Eq. 20 can be written as,

\[\sigma \int \frac{d^3p_a d^3p_b v_{ab} E_a/T E_B E_a/E_B}, \quad (21)\]

The denominator can be written as,

\[\int d^3p_a d^3p_b d\cos \theta E_a/E_B E_B E_a/E_B \cos \theta \tag{22}\]

and the denominator can be written as,

\[\int d^3p_a d^3p_b E_a/E_B E_B E_a/E_B \cos \theta \tag{23}\]

Now, using the generalized Tsallis non-extensive statistics for \(q\)-equilibrium, the \(\langle \sigma_{ab} v_{ab} \rangle\) can be written as,
\[
\langle \sigma_{ab} \nu_{ab} \rangle = \frac{\sigma}{\int 8\pi^2 p_a p_b dE_a dE_b d\cos \theta \ e^{-E_a/T} e^{-E_b/T} \sqrt{(E_a - p_a p_b \cos \theta)^2 - (m_a m_b)^2}} \int 8\pi^2 p_a p_b dE_a dE_b d\cos \theta \ e^{-E_a/T} e^{-E_b/T}. \tag{24}
\]

Here \( \sigma \) is the hadronic cross-sections used as a parameter in the calculations. Using Eqs. 19 and 24, the relaxation time can be calculated. Now, shear and bulk viscosities are calculated using Eqs. 14 and 15, respectively. The other thermodynamic quantities in non-extensive statistics are calculated as [33],

\[
n = g \int \frac{d^3 p}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}}. \tag{25}
\]

\[
\epsilon = g \int \frac{d^3 p}{(2\pi)^3} E \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}}. \tag{26}
\]

\[
P = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^2}{3E} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}}. \tag{27}
\]

\( n, \epsilon \) and \( P \) are the number density, energy density and pressure of hadrons, respectively. The non-extensive entropy \( s \) can be calculated from the above expression as,

\[
s = \frac{\epsilon + P - \mu n}{T}. \tag{28}
\]

We use the basic expression for addition of entropies while calculating it for the multicomponent hadron gas, which is given by,

\[
s(A + B) = s(A) + s(B) + (q - 1)s(A)s(B), \tag{29}
\]

where \( s(A + B) \) is the total entropy of \( A \) and \( B \), \( s(A) \) and \( s(B) \) are the entropies of \( A \) and \( B \), respectively.

We have also calculated the isothermal compressibility for hadron gas using non-extensive statistics. The isothermal compressibility (\( \kappa_T \)) is defined as [34],

\[
\kappa_T = -\frac{1}{V} \frac{\partial V}{\partial P}. \tag{30}
\]

where \( V \) is the volume of the system. Again in terms of fluctuation and average number, isothermal compressibility can be expressed as [34, 35],

\[
\langle (N - \langle N \rangle)^2 \rangle = \text{var}(N) = \frac{T \langle N \rangle^2}{V} \kappa_T. \tag{31}
\]

Using the relation \( \langle (N - \langle N \rangle)^2 \rangle = V T \frac{\partial n}{\partial \mu} \), Eq. 31 can further be expressed in terms of number density and com-

pressibility,

\[
\frac{1}{\kappa_T} = \sum_a \frac{n_{aq}^2}{\frac{\partial n_{aq}}{\partial \mu}}, \tag{32}
\]

where \( \frac{\partial n_{aq}}{\partial \mu} \) is given as,

\[
\frac{\partial n_{aq}}{\partial \mu} = \frac{g q}{T} \int \frac{d^3 p}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{q+1}{q-1}}. \tag{33}
\]

In hydrodynamics, speed of sound plays an important role in understanding the Equation of State (EoS) and hence, the associated phase transition. This is because of the fact that speed of sound depends on the properties of the medium. The squared speed of sound \( (c_s^2) \) is defined as,

\[
c_s^2 = \frac{\partial P}{\partial \epsilon}. \tag{34}
\]

It is already well established that speed of sound must be calculated at a constant entropy to particle ratio. The \( c_s^2 \) at finite baryon chemical potential can be written as [36],

\[
c_s^2(T, \mu_B) = \frac{\frac{\partial P}{\partial T}}{\frac{\partial \epsilon}{\partial T}} + \frac{\frac{\partial \mu}{\partial \mu_B}}{\frac{\partial \mu}{\partial \mu_B}} \tag{35}
\]

Here we do not consider the strangeness neutrality condition for the sake of simplicity. The term \( \frac{\partial \mu}{\partial \mu_B} \) can be calculated using the constant entropy per particle \( (s/n) \) condition [36, 37], and given as,

\[
\frac{d\mu}{dT} = s \left( \frac{\partial n}{\partial T} \right) - n \left( \frac{\partial s}{\partial T} \right), \tag{36}
\]

### III. RESULTS AND DISCUSSIONS

In this section, we present the results and discussions of the dissipative and thermodynamic properties of hadronic matter calculated in the NBTE using relaxation time approximation, where non-extensive statistics is used as the \( q \)-equilibrium distribution function. We consider all hadrons and their resonances with a mass cutoff of 2.0 GeV. First, we estimate various dissipative properties of hadron gas such as shear viscosity to entropy density ratio (\( \eta/s \)) and bulk viscosity to entropy
density ratio ($\zeta/s$). We use Eq. 14 and 28 to estimate $\eta/s$ for zero baryon chemical potential. Here, we calculate the relaxation time of $\pi^0$ hadron by using Eq. 19, where a constant hadronic collision cross-section ($\sigma$) = 29 mb is taken [27]. The results of shear viscosity ($\eta$) of hadrons as a function of temperature ($T$) are presented in the top panel of Figure 1 for various $q$-values. We find that $\eta$ increases slightly with the temperature which is almost similar to that observed in Ref. [27], where Excluded-Volume Model with extensive statistics is used. But, for $q$-values greater than 1.11, $\eta$ changes its behaviour and decreases with $T$. $\eta$ has smaller values for lower $q$ and vice-versa, which goes in line with the observation that the transport coefficients become less as the system goes towards equilibrium and vice-versa. Middle panel presents the variations of shear viscosity to entropy density ratio ($\eta/s$) with respect to temperature at zero baryon chemical potential, where different lines are for different $q$-values. Here, solid horizontal line is the KSS bound. The general behaviour of $\eta/s$ is similar to that observed in Ref. [27]. This ratio decreases with $T$ at all $q$-values due to rapid increase in entropy density and gets closer to KSS bound at higher temperatures. As the degree of non-extensivity, $q$ increases, the magnitude of $\eta/s$ increases in the low temperature region. After $T \approx 0.110$ GeV, $\eta/s$ becomes $q$-independent which suggest that there is no effect of non-extensivity on this ratio above this temperature. This is a very important finding while understanding the dissipative properties, non-extensivity and the effect of temperature of the system. For comparison, we have also shown the calculation of $\eta/s$ for hadron gas at zero baryon chemical potential using Boltzmann-Gibbs distribution function shown by the grey solid line, which lies lowest among other results obtained using non-extensive statistics. This is due to the fact that the values of transport coefficients increase as we go towards non-equilibrium. $\eta/s$ as a function of temperature is presented in the lower panel of the figure for $q = 1.07$ for various upper mass cutoff of hadrons. We find that the magnitude of $\eta/s$ decreases with the increasing mass cutoff. This effect is less pronounced for higher mass cutoffs.

In Figure 2, we discuss the results of bulk viscosity ($\zeta$) and its ratio to entropy density ($s$) calculated by using Eqs. 15 and 28 at zero baryon chemical potential. Again, we take a constant value of hadronic cross-section ($\sigma$) equal to 29 mb while calculating the relaxation time ($\tau$). The upper panel represents $\zeta$ versus $T$ plot for different values of non-extensive parameter, $q$. We observe that $\zeta$ increases slightly as $T$ increases for lower $q$ but after $q = 1.11$, it changes its behaviour and decreases with the increasing $T$. This behaviour is similar to that observed for $\eta$. In the middle panel, we show the variations of bulk viscosity to entropy density ratio ($\zeta/s$) at $\mu_B = 0$ for various $q$-values. We find that $\zeta/s$ decreases rapidly with $T$ for all $q$ and becomes $q$-independent after $T \approx 0.140$ GeV. The value of $\zeta/s$ is higher for higher $q$-values. We also observe that $\zeta/s$ approaches an asymptotic value at a particular $T$, which is found to be higher for higher $q$-values. These findings suggest that the degree of approach towards an asymptotic value is sensitive to the non-extensive parameter, $q$. This goes inline with the expectations of dissipative properties of a system in approach to thermal equilibrium. In the lower panel, we show the upper mass cutoff dependence of $\zeta/s$ for $q = 1.07$. We observe similar effect of mass cutoff on this ratio as observed in the case of $\eta/s$ i.e. as we increase the upper mass cutoff of hadron resonance gas its magnitude decrease. This effect is most prominent at higher $T$ while significant change does not occur at lower temperature.

Next, we discuss one of the important thermodynamic observables i.e., isothermal compressibility ($\kappa_T$), which has drawn considerable interest in these days. It is very interesting to quantify this observable for a hot and dense hadron gas formed in hadronic and heavy-ion collisions. In [8], $\kappa_T$ is first time estimated for hadrons produced in heavy-ion collisions in extensive statistics using Hadron Resonance Gas (HRG) model. In this work, we have used non-extensive statistics for the calculation of $\kappa_T$. In Figure 3, results of $\kappa_T$ as a function of temperature at $\mu_B = 0$ are shown. It is observed that $\kappa_T$ initially decreases rapidly at low temperatures and becomes constant at higher temperatures for all the $q$-values. The temperature at which $\kappa_T$ becomes constant depends on the $q$-value. As $q$ increases the value of $\kappa_T$ decrease and the temperature at which it becomes constant shifts towards the lower values, which suggests that there is a strong dependence of $\kappa_T$ on non-extensivity. Figure 4 describes the dependence of isothermal compressibility on temperature at $\mu_B = 0.5$ GeV for various $q$. We find a similar behaviour of $\kappa_T$ in this case as observed for zero baryon chemical potential but comparatively has smaller values.

Further, in order to see the effect of upper mass cutoff of hadrons on $\kappa_T$, we proceed as follows. We calculate $\kappa_T$ at $\mu_B = 0.5$ GeV for $q = 1.07$, which is shown in the Fig. 5 as a function of temperature. Again, a similar trend is obtained over all the temperatures as observed in the previous plots and as the upper mass cutoff increases, values of $\kappa_T$ decreases. This effect is more pronounced up to mass cutoff of 1 GeV and after that it gets weaker. $\kappa_T$ becomes independent of system temperature when higher resonances are added to the system. Hence a system is driven towards a critical behaviour, once we have resonances of higher masses as the ingredients of the system.

Another important thermodynamical observable is speed of sound. We use Tsallis statistics to calculate the squared speed of sound ($c_s^2$) for finite baryon chemical potential. In [38], the temperature and mass cutoff dependence of $c_s^2$ for various $q$-values have been studied in Tsallis statistics. It is noticed that the criticality in $c_s^2$, which is seen as the $q$-dependent peak in $c_s^2$, when studied as a function of temperature, disappears after $q = 1.13$. We have shown $c_s^2$ as a function of temperature at $\mu_B = 0.5$ GeV for different $q$-values in Figure 6. We
FIG. 1: (Color online) **Top panel** shows the variations of shear viscosity ($\eta$) as a function of temperature for different $q$-values. **Middle panel** shows $\eta/s$ versus temperature for different $q$-values. **Lower panel** presents the variations of $\eta/s$ with respect to temperature at $q = 1.07$ for various upper mass cutoff of hadron resonance gas.

FIG. 2: (Color online) **Top panel** represents bulk viscosity ($\zeta$) as a function of temperature for different $q$-values. **Middle panel** shows $\zeta/s$ versus temperature for different $q$-values. **Lower panel** presents the variations of $\zeta/s$ with respect to temperature at $q = 1.07$ for various upper mass cutoff of hadron resonance gas.
find that $c_s^2$ initially increases with $T$ and after reaching at a maximum value it starts decreasing. The maximum in this observable, which is related to the criticality shifts towards lower $T$ as $q$ increases and disappears after $q = 1.13$. These findings are same as observed for the case of $\mu_B = 0$ [38]. In Figure 7, the mass cutoff dependence of $c_s^2$ is shown for $q = 1.13$ and $\mu_B = 0.5$ GeV. The criticality in $c_s^2$ disappears when lower mass cutoff is reached, which advocates that criticality of the system depends on the number of hadrons present in the system. Also, we find that $c_s^2$ increases with the mass cutoff, which goes in line with the previous observations [38, 39].

IV. SUMMARY

In summary, we have studied the transport coefficients using the relativistic non-extensive Boltzmann transport equation (NBTE), where we employ the relaxation time approximation for the collision integral. Here, we have extended the approach using Tsallis non-extensive statistics as $q$-equilibrium distribution function and calculated shear and bulk viscosity to the entropy density ratios. We have also calculated isothermal compressibility and squared speed of sound using Tsallis statistics at finite baryon chemical potential for different $q$. The important findings of this work are summarized as follows:

1. We have found that the ratios of shear and bulk viscosity to entropy density have a strong dependence on non-extensivity. Their values increase with the
increasing $q$-parameter at lower temperatures while they become independent of $q$ at higher temperatures.

2. We have observed the effect of mass cutoff on these ratios and as mass cutoff increases the magnitudes of these ratios decrease. These effects are more pronounced at higher values of temperature.

3. We have first time studied the effect of non-extensive parameter, $q$ on isothermal compressibility. We have studied the temperature dependence of isothermal compressibility for various $q$-values and observed that it decreases rapidly with temperature and becomes constant at higher temperatures. The temperature at which the isothermal compressibility shows a saturation, may indicate an appearance of criticality in the system, which is dependent on $q$ and shifts towards lower values for higher $q$-values.

4. While studying the temperature dependence of isothermal compressibility for various $q$-values, it has been found that the values of $\kappa_T$ are smaller for higher $q$. It has also been found that, its values decrease at finite baryon chemical potential in comparison to that obtained at zero baryon chemical potential.

5. We have found that, isothermal compressibility strongly depends on the upper mass cutoff of hadron resonances and decreases with the increasing mass cutoffs. It becomes independent of system temperature when higher resonances are added to it. Hence a system is driven towards a critical behaviour, once we have resonances of higher masses.

6. It is observed that as the non-extensivity of the system increases the criticality shifts towards lower temperatures and disappears after $q = 1.13$. We have also noticed a strong dependence of criticality on mass cutoff. As upper mass cutoff decreases the criticality of the system disappears.

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[1] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).
[2] T. S. Biro and E. Molnar, Phys. Rev. C 85, 024905 (2012).
[3] P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007).
[4] T. Hirano and M. Gyulassy, Nucl. Phys. A 769, 71 (2006).
[5] A. Dobado and J. M. Torres-Rincon, Phys. Rev. D 86, 074021 (2012).
[6] C. Sasaki and K. Redlich, Phys. Rev. C 79, 055207 (2009).
[7] I. A. Shushpanov, J. I. Kapusta and P. J. Ellis, Phys. Rev. C 59, 2931 (1999).
[8] M. Mukherjee, S. Basu, A. Chatterjee, O. Chatterjee, S. P. Adhya, S. Thakur and T. K. Nayak, arXiv:1708.08692 [nucl-ex].
[9] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 75, 064901 (2007).
[10] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 83, 064903 (2011).
[11] K. Aamodt et al. [ALICE Collaboration], Eur. Phys. J. C 71, 1655 (2011).
[12] B. Abelev et al. [ALICE Collaboration], Phys. Lett. B 717, 162 (2012).
[13] B. Abelev et al. [ALICE Collaboration], Phys. Lett. B 712, 309 (2012).
[14] S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 72, 2164 (2012).
[15] C. Tsallis, J. Statist. Phys. 52, 479 (1988).
[16] C. Tsallis, Braz. J. Phys. 29, 1 (1999).
[17] C. Tsallis, Eur. Phys. J. A 40, 257 (2009).
[18] G. Kaniadakis, Eur. Phys. J. A 40, 275 (2009).
[19] D. Thakur, S. Tripathy, P. Garg, R. Sahoo and J. Cleymans, Adv. High Energy Phys. 2016, 4149352 (2016) and references therein.
[20] P. Sett and P. Shukla, Int. J. Mod. Phys. E 24, 1550046 (2015).
[21] T. Bhattacharyya, J. Cleymans, A. Khuntia, P. Pareek and R. Sahoo, Eur. Phys. J. A 52, 30 (2016).
[22] H. Zheng and L. Zhu, Adv. High Energy Phys. 2015, 180491 (2015).
[23] Z. Tang, Y. Xu, L. Ruan, G. van Buren, F. Wang and Z. Xu, Phys. Rev. C 79, 051901 (2009).
[24] B. De, Eur. Phys. J. A 50, 138 (2014).
[25] S. Tripathy, A. Khuntia, S. K. Tiwari and R. Sahoo, Eur. Phys. J. A 53, 99 (2017).
[26] S. Tripathy, T. Bhattacharyya, P. Garg, P. Kumar, R. Sahoo and J. Cleymans, Eur. Phys. J. A 52, 289 (2016).
[27] G. P. Kadam and H. Mishra, Phys. Rev. C 92, 035203 (2015).
[28] A. Khuntia, S. Tripathy, R. Sahoo and J. Cleymans, Eur. Phys. J. A 53, no. 5, 103 (2017).
[29] S. Gavin, Nucl. Phys. A 435, 826 (1985).
[30] O. Moroz, Ukr. J. Phys. 58, 1127 (2013).
[31] M. Cannoni, Phys. Rev. D 89, 103533 (2014).
[32] P. Gondolo and G. Gelmini, Nucl. Phys. B 360, 145 (1991).
[33] J. Cleymans and D. Worku, Eur. Phys. J. A 48, 160 (2012).
[34] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 78, 044902 (2008).
[35] S. Mrowczynski, Phys. Lett. B 430, 9 (1998).
[36] J. Cleymans and D. Worku, Mod. Phys. Lett. A 26, 1197 (2011).
[37] S. K. Tiwari, P. K. Srivastava and C. P. Singh, Phys. Rev. C 85, 014908 (2012).
[38] A. Khuntia, P. Sahoo, P. Garg, R. Sahoo and J. Cleymans, Eur. Phys. J. A 52, 292 (2016).
[39] P. Castorina, J. Cleymans, D. E. Miller and H. Satz, Eur. Phys. J. C 66, 207 (2010).