Singlet GB contributions in the chiral constituent quark model

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Abstract

The implications of the latest data pertaining to $\bar{u} - \bar{d}$ asymmetry and the spin polarization functions on the contributions of singlet Goldstone Boson $\eta'$ within $\chi$CQM with configuration mixing for explaining the "proton spin problem" have been investigated. It is found that the present data favors smaller values of the coupling of singlet Goldstone Boson $\eta'$ as compared to the corresponding contributions from $\pi$, $K$ and $\eta$ Goldstone bosons. It seems that a small non-zero value of the coupling of $\eta'$ ($\zeta \neq 0$) is preferred over $\zeta = 0$ phenomenologically.

The chiral constituent quark model ($\chi$CQM), as formulated by Manohar and Georgi \cite{1,2}, has recently got good deal of attention \cite{3,4,5,6} as it is successful in not only explaining the "proton spin crisis" \cite{7,8,9,10} through the emission of a Goldstone boson (GB) but is also able to account for the $\bar{u} - \bar{d}$ asymmetry \cite{11,12,13}, existence of significant strange quark content $\bar{s}$ in the nucleon, various quark flavor contributions to the proton spin \cite{3}, baryon magnetic moments \cite{3,4} and hyperon $\beta-$decay parameters etc..

Recently, it has been shown that configuration mixing generated by spin-spin forces \cite{14,15,16}, known to be compatible with the $\chi$CQM \cite{17,18,19},

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improves the predictions of $\chi$CQM regarding the quark distribution functions and the spin polarization functions \[20\]. Further, $\chi$CQM with configuration mixing (henceforth to be referred as $\chi$CQM$_{\text{config}}$) when coupled with the quark sea polarization and orbital angular omentum (Cheng-Li mechanism \[21\]) as well as “confinement effects” is able to give an excellent fit \[20\] to the octet magnetic moments and a perfect fit for the violation of Coleman Glashow sum rule \[22\].

Cheng and Li \[1\] realized that the key to understand the “proton spin problem” \[23\], within the Manohar and Georgi formalism of $\chi$CQM \[1\], lies in generating an appropriate quark sea in the proton through the chiral symmetry breaking mechanism. The basic process in the $\chi$CQM is the emission of a GB by a constituent quark which further splits into a $q\bar{q}$ pair, for example,

$$q_{\pm} \to \text{GB}^0 + q'_{\pm} \to (q\bar{q}') + q'_{\pm}, \quad (1)$$

where $q\bar{q}' + q'$ constitute the “quark sea” \[4, 5, 6\]. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can be expressed as

$$\mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{\eta'}{\sqrt{3}} q = g_8 \bar{q} \left( \Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) q, \quad (2)$$

where $\zeta = g_1 / g_8$, $g_1$ and $g_8$ are the coupling constants for the singlet and octet GBs, respectively, $I$ is the $3 \times 3$ identity matrix and

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \quad (3)$$

The GB field which includes the octet and the singlet GBs is written as

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha K^0 & -\beta \frac{2\eta'}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}. \quad (4)$$

SU(3) symmetry breaking is introduced by considering $M_s > M_{u,d}$ as well as by considering the masses of GBs to be nondegenerate ($M_{K,\eta} > M_{\pi}$) \[5, 6, 21\], whereas the axial U(1) breaking is introduced by $M_{\eta'} > M_{K,\eta}$ \[4, 5, 6, 21\]. The parameter $a(= |g_8|^2)$ denotes the transition probability of chiral fluctuation.
of the splittings $u(d) \rightarrow d(u) + \pi^+(-)$, whereas $\alpha^2a, \beta^2a$ and $\zeta^2a$ respectively denote the probabilities of transitions of $u(d) \rightarrow s+K^{-(0)}$, $u(d, s) \rightarrow u(d, s)+\eta$, and $u(d, s) \rightarrow u(d, s)+\eta'$. Recently, it has been pointed out that the new measurement of both the $\bar{u}/\bar{d}$ asymmetry as well as $\bar{u}-\bar{d}$ asymmetry by the NuSea Collaboration [12] may not require substantial contribution of $\eta'$ [6]. As the contribution of $\eta'$ not only has important implications for the $\chi$CQM but also has a deeper significance for axial U(1) anomaly as well as nonperturbative aspects of QCD including the effects of gluon anomaly on the spin polarizations [24], it therefore becomes interesting to understand the extent to which its contribution is needed in the $\chi$CQM to fit the data pertaining to the “proton spin problem”.

The purpose of the present communication is to phenomenologically estimate the contribution of $\eta'$ GB by carrying out a fine grained analysis of “proton spin problem” within $\chi$CQM which also includes the implications of the latest E866 data. Further, it would be interesting to fine tune the contribution of $\eta'$, expressed through the parameter $\zeta$, by studying its implications on spin polarization functions and quark distribution functions.

The details of $\chi$CQM have already been discussed in Ref. [20], however to facilitate the discussion as well as for the sake of readability of the manuscript, some essential details of $\chi$CQM with configuration mixing have been presented in the sequel. As has already been discussed that spin-spin forces generate configuration mixing [14, 15, 16] which effectively leads to modification of the spin polarization functions [20]. The most general configuration mixing in the case of octet baryons [15, 16, 25] can be expressed as

$$|B\rangle = \left( |56, 0^+\rangle_{N=0} \cos \theta + |56, 0^+\rangle_{N=2} \sin \theta \right) \cos \phi$$

$$+ \left( |70, 0^+\rangle_{N=2} \cos \theta' + |70, 2^+\rangle_{N=2} \sin \theta' \right) \sin \phi, \quad (5)$$

where $\phi$ represents the $|56\rangle - |70\rangle$ mixing, $\theta$ and $\theta'$ respectively correspond to the mixing among $|56, 0^+\rangle_{N=0} - |56, 0^+\rangle_{N=2}$ states and $|70, 0^+\rangle_{N=2} - |70, 2^+\rangle_{N=2}$ states. For the present purpose, it is adequate [16, 20, 26] to consider the mixing only between $|56, 0^+\rangle_{N=0}$ and the $|70, 0^+\rangle_{N=2}$ states and the corresponding “mixed” octet of baryons is expressed as

$$|B\rangle \equiv \begin{pmatrix} 8, 1^+ \end{pmatrix} = \cos \phi |56, 0^+\rangle_{N=0} + \sin \phi |70, 0^+\rangle_{N=2}, \quad (6)$$

for details of the spin, isospin and spatial parts of the wavefunction, we refer the reader to reference [27].
To study the variation of the $\chi$CQM parameters and the role of $\zeta$ in obtaining the fit, one needs to formulate the experimentally measurable quantities having implications for these parameters as well as dependent on the unpolarized quark distribution functions and the spin polarization functions. We first calculate the spin polarizations and the related quantities which are affected by the “mixed” nucleon. The spin structure of a nucleon is defined as \[ B \equiv \langle B|N|B \rangle, \]

where $|B\rangle$ is the nucleon wavefunction defined in Eq. (6) and $N$ is the number operator given by

\[ N = n_{u^+} + n_{u^-} + n_{d^+} + n_{d^-} + n_{s^+} + n_{s^-}, \]

(8)

where $n_{q^\pm}$ are the number of $q^\pm$ quarks. The spin structure of the “mixed” nucleon, defined through the Eq. (6), is given by

\[ \langle 8, \frac{1}{2}^+ | N | 8, \frac{1}{2}^+ \rangle = \cos^2 \phi \langle 56, 0^+ | N | 56, 0^+ \rangle + \sin^2 \phi \langle 70, 0^+ | N | 70, 0^+ \rangle. \]

The contribution to the proton spin in $\chi$CQM$_{\text{config}}$, given by the spin polarizations defined as $\Delta q = q^+ - q^-$, can be written as

\[ \Delta u = \cos^2 \phi \left[ \frac{4}{3} - \frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2\right)\right] \]
\[ + \sin^2 \phi \left[ \frac{2}{3} - \frac{a}{3} \left(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2\right)\right], \]

(10)

\[ \Delta d = \cos^2 \phi \left[ -\frac{1}{3} - \frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2\right)\right] \]
\[ + \sin^2 \phi \left[ \frac{1}{3} - \frac{a}{3} \left(4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2\right)\right], \]

(11)

\[ \Delta s = -a\alpha^2. \]

(12)

After having formulated the spin polarizations of various quarks, we consider several measured quantities which are expressed in terms of the above mentioned spin polarization functions. The quantities usually calculated in the $\chi$CQM are the flavor non-singlet components $\Delta_3$ and $\Delta_8$, obtained from the neutron $\beta$–decay and the weak decays of hyperons respectively. These can be related to Bjorken sum rule 28 and the Ellis-Jaffe sum rule 29 as

\[ \text{BSR} : \quad \Delta_3 = \Delta u - \Delta d, \]
\[ \text{EJSR} : \quad \Delta_8 = \Delta u + \Delta d - 2\Delta s. \]
Another quantity which is usually evaluated is the flavor singlet component of the total quark spin content defined as

\[ 2\Delta \Sigma = \Delta_0 = \Delta_u + \Delta_d + \Delta_s. \] (15)

Apart from the above mentioned spin polarization we have also considered the quark distribution functions which have implications for \( \zeta \) as well as for other \( \chi \text{CQM} \) parameters. For example, the antiquark flavor contents of the “quark sea” can be expressed as \[ 1, 5, 6 \]

\[ \bar{u} = \frac{1}{12}[(2\zeta + \beta + 1)^2 + 20]a, \quad \bar{d} = \frac{1}{12}[(2\zeta + \beta - 1)^2 + 32]a, \quad \bar{s} = \frac{1}{3}[(\zeta - \beta)^2 + 9\alpha^2]a, \] and

\[ u - \bar{u} = 2, \quad d - \bar{d} = 1, \quad s - \bar{s} = 0. \] (16)

The deviation of Gottfried sum rule \[ 13 \] is expressed as

\[ I_G = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)]dx = 0.254 \pm 0.005. \] (18)

In terms of the symmetry breaking parameters \( a, \beta \) and \( \zeta \), this deviation is given as

\[ \left[ I_G - \frac{1}{3} \right] = \frac{2}{3} \left[ \frac{a}{3}(2\zeta + \beta - 3) \right]. \] (19)

Similarly, \( \bar{u}/\bar{d} \) \[ 12, 30 \] measured through the ratio of muon pair production cross sections \( \sigma_{pp} \) and \( \sigma_{pn} \), is expressed in the present case as follows

\[ \bar{u}/\bar{d} = \frac{(2\zeta + \beta + 1)^2 + 20}{(2\zeta + \beta - 1)^2 + 32}. \] (20)

Some of the important quantities depending on the quark distribution functions which are usually discussed in the literature are as follows

\[ f_q = \frac{q + \bar{q}}{\sum_q (q + \bar{q})}, \quad f_3 = f_u - f_d, \quad f_8 = f_u + f_d - 2f_s. \] (21)

The \( \chi \text{CQM}_{\text{config}} \) involves five parameters: \( a, \alpha, \beta, \zeta \) and \( \phi \). Before carrying out the detailed analysis involving quantities which are dependent on \( \zeta \), to begin with we have fixed some of the \( \chi \text{CQM} \) parameters. The mixing angle \( \phi \) is fixed from the consideration of neutron charge radius \[ 16, 25, 31 \]. It has
been shown [4, 6] that to fix the violation of Gottfried sum rule [13], we have to consider the relation
\[ \bar{u} - \bar{d} = \frac{a}{3}(2\zeta + \beta - 3), \] (22)
which constrains the parameters \( a, \zeta \) and \( \beta \) when the data pertaining to \( \bar{u} - \bar{d} \) asymmetry [12] is used. The parameters \( \alpha \) and \( \beta \) suppress the emission of \( K^- \) and \( \eta \) as compared to that of pions as these strange quark carrying GBs are more massive than the pions. However, because of the very small mass difference between them, the suppression factors \( \alpha \) and \( \beta \) are taken to be equal. In Table 1, we summarize the input parameters and their values.

In Table 2, we have presented the various spin dependent phenomenological quantities which are affected by the variation of the symmetry breaking parameters. In the table, to highlight the particular values of \( a \) and \( \zeta \), we have presented the results for their different values. A general look at the table shows that the results of all the quantities affected by the inclusion of \( \zeta \) get improved in the right direction for lower values of \( \zeta \). In fact, for the case of \( a = 0.13 \) and \( \zeta = -0.10 \), we are able to get a perfect fit for \( \Delta_3 \) and \( \Delta_8 \).

Further, the results corresponding to quark distribution functions having implications for the symmetry breaking parameters have been presented in Table 3. In general both for \( \zeta = 0 \) and \( \zeta = -0.10 \), we are able to obtain an excellent fit, however in the case of \( \bar{u} - \bar{d}, \bar{u}/\bar{d} \) and \( f_3/f_8 \), the non-zero (small) value of \( \zeta \) gives a better fit than \( \zeta = 0 \).

A closer scrutiny of the table reveals several interesting points. \( \Delta_3 \) and \( \Delta_8 \) from Table 2 as well as \( f_3/f_8 \) from Table 3 perhaps suggest that a small non-zero value of \( \zeta \) gives a better fit than the zero value of \( \zeta \). In the case of \( \Delta \Sigma \) (Table 2), it seems that \( \zeta = 0 \) is a preferred value. However, as has been discussed earlier in \( \chi \)CQM [32] that the flavor singlet component of the spin of proton \( \Delta \Sigma \) receives contributions from various sources such as gluon polarization and gluon angular momentum, therefore, we cannot conclude that \( \zeta = 0 \) is preferred over \( \zeta \neq 0 \). In this context, we would like to mention that the above contribution of \( \eta' \) is in agreement with the experimental value of \( \Delta \Sigma \) in case we consider the contribution of the effects of gluon polarization and gluon angular momentum through gluon anomaly [32].

The results corresponding to small values of \( \zeta \) including \( \zeta = 0 \) clearly show better overlap with the data after the latest \( \bar{u} - \bar{d} \) asymmetry measurement [12]. In the \( \chi \)CQM, it is difficult to think of a mechanism wherein the contribution of \( \eta' \) or the ninth GB becomes zero. However, a small value of \( \zeta \) looks to be in
order from phenomenological considerations pertaining to the different GBs. For example, in case we consider the coupling of the GB corresponding to the pion, $K$, $\eta$ and $\eta'$ mesons being inversely proportional to the square of their respective masses, we find that their couplings are of the order $a\alpha^2 \sim 0.02$, $a\beta^2 \sim 0.02$ and $a\zeta^2 \sim 0.001$ for $a \sim 0.13$ which strangely agrees with our values obtained through the fit. These findings are also in agreement with the suggestions of Cheng and Li [4] who have advocated that the $\eta'$ contribution corresponds to the non-planar contributions in the $1/N_c$ expansion.

To summarize, we have investigated in detail the implications of the latest data pertaining to $\bar{u} - \bar{d}$ asymmetry and the spin polarization functions on the singlet Goldstone Boson $\eta'$ within $\chi$CQM with configuration mixing for explaining the “proton spin problem”. We find that the lower values of $\zeta$ are preferred over the higher values. Specifically, in the case of $\Delta_3$, $\Delta_8$, $\bar{u} - \bar{d}$, $\bar{u}/\bar{d}$ and $f_3/f_8$, it seems that the small non-zero value of $\zeta$ is preferred over $\zeta = 0$.

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| Parameter | Data | \(\chi_{\text{CQM}_\text{config}}\) |
|-----------|------|-------------------------------|
| \(\Delta u\) | 0.85 ± 0.05 | 0.95 \(a = 0.1\) 0.91 \(a = 0.14\) 0.91 \(a = 0.13\) |
| \(\Delta d\) | -0.41 ± 0.05 | -0.31 \(\zeta = -0.65\) -0.35 \(\zeta = 0\) -0.36 \(\zeta = -0.10\) |
| \(\Delta s\) | -0.07 ± 0.05 | -0.02 \(\zeta = -0.65\) -0.02 \(\zeta = 0\) -0.02 \(\zeta = -0.10\) |
| \(\Delta_3\) | 1.267 ± 0.0035 | 1.27 \(\zeta = -0.65\) 1.26 \(\zeta = 0\) 1.27 \(\zeta = -0.10\) |
| \(\Delta_8\) | 0.58 ± 0.025 | 0.67 \(\zeta = -0.65\) 0.60 \(\zeta = 0\) 0.59 \(\zeta = -0.10\) |
| \(\Delta\Sigma\) | 0.19 ± 0.025 | 0.31 \(\zeta = -0.65\) 0.27 \(\zeta = 0\) 0.28 \(\zeta = -0.10\) |

Table 1: Input parameters and their values used in the analysis.

| Parameter | Data | \(\chi_{\text{CQM}}\) |
|-----------|------|-----------------|
| \(\bar{u}\) | - | 0.168 \(a = 0.1\) 0.25 \(a = 0.14\) 0.23 \(a = 0.13\) |
| \(\bar{d}\) | - | 0.288 \(\zeta = -0.65\) 0.366 \(\zeta = 0\) 0.35 \(\zeta = -0.10\) |
| \(\bar{s}\) | - | 0.108 \(\zeta = -0.65\) 0.07 \(\zeta = 0\) 0.07 \(\zeta = -0.10\) |
| \(\bar{u} - \bar{d}\) | -0.118±0.015 | -0.108 \(\zeta = -0.65\) -0.116 \(\zeta = 0\) -0.117 \(\zeta = -0.10\) |
| \(\bar{u} / \bar{d}\) | 0.67 ± 0.06 | 0.58 \(a = 0.1\) 0.68 \(a = 0.14\) 0.67 \(a = 0.13\) |
| \(I_G\) | 0.254 ± 0.005 | 0.253 \(\zeta = -0.65\) 0.255 \(\zeta = 0\) 0.255 \(\zeta = -0.10\) |
| \(f_u\) | - | 0.655 \(a = 0.1\) 0.677 \(a = 0.14\) 0.675 \(a = 0.13\) |
| \(f_d\) | - | 0.442 \(\zeta = -0.65\) 0.470 \(\zeta = 0\) 0.466 \(\zeta = -0.10\) |
| \(f_s\) | 0.10 ± 0.06 | 0.061 \(\zeta = -0.65\) 0.039 \(\zeta = 0\) 0.039 \(\zeta = -0.10\) |
| \(f_3\) | - | 0.213 \(a = 0.1\) 0.207 \(a = 0.14\) 0.209 \(a = 0.13\) |
| \(f_8\) | - | 0.975 \(\zeta = -0.65\) 1.07 \(\zeta = 0\) 1.06 \(\zeta = -0.10\) |
| \(f_3 / f_8\) | 0.21 ± 0.05 | 0.22 \(a = 0.1\) 0.19 \(a = 0.14\) 0.20 \(a = 0.13\) |

Table 2: The phenomenological values of the spin polarizations and dependent parameters.

Table 3: The quark flavor distribution functions and dependent parameters.