Research Article

Designing a Repetitive Group Sampling Plan for Weibull Distributed Processes

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Acceptance sampling plans are useful tools to determine whether the submitted lots should be accepted or rejected. An efficient and economic sampling plan is very desirable for the high quality levels required by the production processes. The process capability index $C_L$ is an important quality parameter to measure the product quality. Utilizing the relationship between the $C_L$ index and the nonconforming rate, a repetitive group sampling (RGS) plan based on $C_L$ index is developed in this paper when the quality characteristic follows the Weibull distribution. The optimal plan parameters of the proposed RGS plan are determined by satisfying the commonly used producer’s risk and consumer’s risk at the same time by minimizing the average sample number (ASN) and then tabulated for different combinations of acceptance quality level (AQL) and limiting quality level (LQL). The results show that the proposed plan has better performance than the single sampling plan in terms of ASN. Finally, the proposed RGS plan is illustrated with an industrial example.

1. Introduction

In the manufacturing industries, acceptance sampling plan has been widely used for inspection purposes. It has played an important role in the inspection of raw materials, semifinished products, and finished products from product manufacture to marketing. Acceptance sampling plans provide the producer and the consumer with acceptance or nonacceptance criteria meeting both of their requirements for product quality, in which the decision is made on the sample information taken from the submitted lot. Because of human error and fatigue during the sampling inspection, there is a chance of making errors. The chance of rejecting a good lot is called the producer’s risk, and the chance of accepting a bad lot is called the consumer’s risk. One purpose of an acceptance sampling plan is to minimize the sample size so as to reduce the cost and time of the experiment while satisfying the producer’s risk as well as the consumer’s risk at the specified quality levels. So the use of an acceptance sampling plan earns good reputation of the organization and increases the profit. For more applications of the acceptance sampling plan can be found in Fernández [1], Wang [2], Yan et al. [3], Balamurali and Usha [4], and Wu et al. [5].

During the inspection of the products, the producers care about the inspection cost which is directly related to the sample size. So the researchers want to propose a more efficient sampling plan to lower the inspection cost, time, and efforts. A single sampling plan is very popular in the industrial engineering because of the simplicity, but the decision of lot sentencing based only on the single sample may undermine good relations between the producers and the consumers in some cases (Liu and Wu [6]). Recently, Sherman [7] proposed the attributes of repetitive group sampling (RGS) plan whose operational procedure is similar to that of the sequential sampling scheme. Balamurali and Jun [8] extended the concept of RGS to variables’ inspection and then showed that it is more efficient than single sampling and double sampling in terms of the average sample number (ASN) while providing the desired protection to producers and consumers. The RGS plan has been used widely in the industries when the inspection is costly and destructive. Aslam et al. [9] designed the repetitive sampling plan using the process loss function. Liu and Wu [6] designed the repetitive sampling plan for unilateral specification limit. Yen et al. [10] proposed a variable repetitive group sampling plan based on one-sided process capability indices. Wu et al. [11]
developed a variable repetitive group sampling plan based on the capability index $C_{pk}$. Aslam et al. [12] presented three repetitive types of sampling plans using the generalized process capability index $C_{pmk}$ for normally distributed processes was investigated by Lee et al. [13].

Process capability analysis is an effective method to measure the performance and potential capability of process. In the manufacturing and services industry, process capability indices (PCIs) are utilized to examine whether product quality meets the consumers’ required level. Recently, acceptance sampling plans based on process capability index have attracted many researchers. Examples include Pearn and Wu [14], Aslam et al. [15], and Wu [16]. All of the above PCIs have been developed or investigated under the assumption of normality. Nevertheless, the normality is very questionable in many processes including manufacturing process, service process, and business operation process [17]. The lifetime model of many products may generally follow a nonnormal distribution which includes Weibull, exponential, gamma, Rayleigh, and Burr XII or the other distributions.

The process capability index $C_L$ (or $C_{PL}$) proposed by Montgomery [18] is used to assess the lifetime performance of electronic components which have a larger-the-better type quality characteristic, where $L$ is the lower specification limit. Recently, for some well-known nonnormal lifetime distributions, statistical inferences for $C_L$ have been considered in the literature (see [19–22]).

In this paper, we will firstly develop a repetitive group sampling plan based on the $C_L$ index for Weibull distributed processes with the lower specification limit for product acceptability determination using the close relationship between the index $C_L$ and the product nonconforming rate $p$. The plan aims to minimize the sample size required for inspection while controlling the nonconforming fraction or the number of nonconformities so as to meet the requirements of the producer and the consumer. The rest of this paper is organized as follows. In Section 2, the concept of the lifetime performance index $C_L$ is introduced briefly and the maximum likelihood estimation (MLE) of $C_L$ is also presented. The design and operating procedure of the proposed repetitive group sampling plan based on $C_L$ is presented; moreover the plan parameters are determined by solving the optimization problem and a detailed analysis is also discussed in Section 3. In Section 4, we will compare the efficiency of the proposed RGS plan with the single sampling plan in terms of OC curve and the average sample number (ASN). Section 5 gives an example for illustration. Finally, some concluding remarks are made in the last section.

2. The Lifetime Performance Index $C_L$

Montgomery [18] proposed a process capability index $C_L$ for evaluating the larger-the-better quality characteristic. The $C_L$ index can be defined as follows:

$$C_L = \frac{\mu - L}{\sigma},$$  

where $\mu$ and $\sigma$ are the process mean and the standard deviation, respectively, and $L$ is the lower specification limit.

The Weibull distribution is commonly used for the lifetime or durability of diverse types of manufactured items, such as ball bearings, automobile components, and electrical insulation. Suppose that the quality of interest $X$ follows a two-parameter Weibull distribution with the cumulative distribution function (cdf):

$$F(x; \lambda, \theta) = 1 - \exp \left( - \left( \frac{x}{\lambda} \right)^\theta \right), \quad x > 0,$$

where $\theta > 0$ is the known shape parameter and $\lambda > 0$ is an unknown scale parameter. As stated in Jun et al. [23] and Aslam and Jun [24], “the shape parameter can be assumed as known because engineering experience with a particular type of application makes such an assumption reasonable. We may use the estimated value from the past failure data even though it is not known.” Since the mean and the standard deviation of the Weibull distribution are given by $\mu = \lambda \Gamma(1 + 1/\theta)$ and $\sigma = \lambda A$, respectively, the index $C_L$ can be expressed as

$$C_L = \frac{\mu - L}{\sigma} = \frac{T(1 + 1/\theta) - L/\lambda}{A},$$

$$\lim_{-\infty} < C_L < \frac{\Gamma(1 + 1/\theta)}{A},$$

where $A = \sqrt{\Gamma(1 + 2/\theta) - \Gamma^2(1 + 1/\theta)}$ and $T(\cdot)$ is the complete gamma function.

The probability that an item will not meet the specification is called the fraction defective or nonconforming rate. Let $L$ denote the lower specification limit of an item from the Weibull distribution; then the nonconforming rate can be defined as

$$p = P(X < L) = 1 - \exp \left( - \left( \frac{L}{\lambda} \right)^\theta \right)$$

$$= 1 - \exp \left( - \left( T(1 + 1/\theta) - AC_L \right)^\theta \right),$$

$$\lim_{-\infty} < C_L < \frac{\Gamma(1 + 1/\theta)}{A}.$$
Wu and Kus [25] noticed that $2W/\lambda \theta$ has a chi-squared distribution with $2n$ degrees of freedom, that is, $2W/\lambda \theta \sim \chi^2_{2n}$. Denote the cumulative distribution function (cdf) of $\hat{C}_L$ as

$$P (\hat{C}_L \leq c) = P \left( \frac{\Gamma \left( 1 + \frac{1}{\theta} \right) - L \cdot (n/W) \cdot \theta}{A} \leq c \right)$$

$$= P \left( \frac{2W}{\lambda \theta} \leq 2n \left( \frac{L}{\lambda} \right) \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac \right) \right) \quad (6)$$

$$= F_{\chi^2_{2n}} \left( 2n \left( \frac{L}{\lambda} \right) \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac \right) \right),$$

where $F_{\chi^2_{2n}} (\cdot)$ is the cdf of the $\chi^2_{2n}$ distribution.

Since $(L/\lambda)^{\theta} = -\ln(1 - p)$, which can be obtained from (4), the cdf of $\hat{C}_L$ can be rewritten as

$$P (\hat{C}_L \leq c) = F_{\chi^2_{2n}} \left( -2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac \right) \right). \quad (7)$$

### 3. Design a Repetitive Group Sampling Plan Based on $C_L$

Suppose that the quality characteristic of interest follows a Weibull distribution and has a lower specification limit $L$. There is a one-to-one relationship between the $C_L$ index and the nonconforming rate $p$, so we can use $C_L$ as a quality benchmark for accepting a lot. Then the operating procedure of the repetitive group sampling plan based on $C_L$ is stated as follows.

**Step 1.** Choose the values of $(\rho_AQL, \rho_{LQL})$ at producer's risk $\alpha$ and consumer's risk $\beta$.

$$\pi (p) = P \left( \text{Accepting the lot} \mid p \right) = P_a (p) + R_p \cdot P_a (p) + R_p^2 \cdot P_a (p) + \cdots = \frac{P_a (p)}{1 - R_p}$$

$$= \frac{1 - F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta)}{1 - F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta) + F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta)}, \quad (11)$$

The average sample number (ASN) means the expected number of sampled units per lots for making decisions. Thus

$$\text{ASN} (p) = n \left( 1 - R_p \right) + 2nR \left( 1 - R_p \right) + 3nR^2 \left( 1 - R_p \right) + \cdots + mnR^{m-1} \left( 1 - R_p \right) + \cdots = \frac{n}{1 - R_p}$$

$$= \frac{1 - F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta) + F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta)}{1 - F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta) + F_{\chi^2_{2n}} (-2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta)}, \quad (12)$$

Step 2. Select a random sample ($X_1, X_2, \ldots, X_n$) from the lot; then compute $\hat{C}_L$ in (5).

Step 3. Accept the lot if $\hat{C}_L \geq k_a$; reject the lot if $\hat{C}_L < k_a$; if $k_a \leq \hat{C}_L < k_r$, repeat Step 2 by taking a new sample for further judgment, where $k_a$ and $k_r$ are acceptance constant and rejection constant, respectively.

There are three parameters $k_a$, $k_r$, and $n$ in the above proposed sampling plan. Note that the proposed RGS plan will reduce to the single sampling plan if $k_a = k_r$.

The probability of accepting the lot based on the single sampling can be expressed as

$$P_a (p) = P (\hat{C}_L \geq k_a)$$

$$= 1 - F_{\chi^2_{2n}} \left( -2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_a \right) \cdot \theta \right). \quad (8)$$

Similarly, the probability of rejecting the lot for the single sampling is given as

$$P_r (p) = P (\hat{C}_L < k_r)$$

$$= F_{\chi^2_{2n}} \left( -2n \ln (1 - p) \cdot \left( \Gamma \left( 1 + \frac{1}{\theta} \right) - Ac_r \right) \cdot \theta \right). \quad (9)$$

The probability required resampling is given by

$$R_p = P (k_r \leq \hat{C}_L < k_a \mid p) = 1 - P_a (p) - P_r (p). \quad (10)$$

Referring to Balamurali and Jun [8], the OC (operating characteristic) function of the RGS plan based on the $C_L$ index can be derived as follows:
Denote $p_{\text{AQL}}$ as the quality of the submitted lot at AQL (acceptable quality level) and $p_{\text{LQL}}$ as the quality of the submitted lot at LQL (limiting quality level). Yen and Chang [26] stated “a well-designed sampling plan must provide a probability of at least $1 - \alpha$ of accepting a lot if the nonconforming rate of the lot is at $p = p_{\text{AQL}}$ (in high quality), and a probability of no more than $\beta$ of accepting a lot if the nonconforming rate of the lot is at $p = p_{\text{LQL}}$ (in low quality).” For the specified values of $\alpha, \beta, p_{\text{AQL}}$, and $p_{\text{LQL}}$, the proposed RGS plan parameters must satisfy the following two inequalities:

\[
\pi(p_{\text{AQL}}) = P\{\text{Accepting the lot } | \ p = p_{\text{AQL}}\} \\
\geq 1 - \alpha,
\]

Minimize \[
\frac{1}{2} \ (\text{ASN}(p_{\text{AQL}}) + \text{ASN}(p_{\text{LQL}}))
\]

Subject to \[
\frac{1 - F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{AQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta}}{1 - F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{AQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta})})} + F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{AQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta}}{1 - F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{AQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta})})} \geq 1 - \alpha
\]

\[
\frac{1 - F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{LQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta})} + F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{LQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta})}{1 - F_{\chi^2}(\frac{-2n \ln (1 - p_{\text{LQL}}) \cdot (\Gamma (1 + 1/\theta) - A_k)^{-\theta})})} \leq \beta.
\]

In order to determine the proposed RGS plan parameters, the Monte Carlo simulation using statistics software R is made to solve the above optimization problem. Tables 1–3 display the proposed plan parameters ($n, k_{\alpha}, k_{\beta}$, ASN) for the three values of the shape parameters ($\theta = 1, 1.97, 3$) of the Weibull distribution, respectively, with commonly used risks ($\alpha, \beta$) = (0.05, 0.10), (0.05, 0.05), and (0.10,0.10) and different quality levels ($p_{\text{AQL}}, p_{\text{LQL}}$). Thus, the practitioner can know the sample size required for inspection and the decisions whether accepting the submitted lot. For example, when the shape parameter $\theta = 1$, ($p_{\text{AQL}}, p_{\text{LQL}}$) = (0.05, 0.08) and ($\alpha, \beta$) = (0.05, 0.10), the plan parameters ($n, k_{\alpha}, k_{\beta}$, ASN) = (21, 0.9418, 0.9178, 31.143) from Table 1. It implies that, taking 21 inspected measurements randomly from the lot, the entire lot will be accepted if $C_L$ is larger than 0.9418, and the entire lot will be rejected if $C_L$ is smaller than 0.9178. Otherwise, take a new sample from the lot for further judgment if $0.9178 < C_L < 0.9418$. And the ASN required for making decisions on lot sentencing is 31.143.

Figure 1 shows the OC curves of the proposed RGS plan against the nonconforming rate $p$ for different sample sizes $n = 6, 10, 15$, and 25 under $\theta = 1.97, k_{\alpha} = 1.45$, and $k_{\beta} = 1.30$. It can be seen that the OC curve becomes more like the idealized OC curve shape (the slope is getting larger) as the sample size increases. It implies that the discriminatory power would become larger by increasing the sample size.

4. Comparative Analysis

In this section, we will use these two criteria, the OC curves and the sample size required for inspection, to demonstrate the advantages of the proposed RGS plan over the single plan based on the index $C_L$.

4.1. OC Curves. In order to examine the behaviour of the proposed RGS plan with different values of $\theta$, Figure 2 displays the OC curves of the RGS plan and the single sampling plan for two cases: (a) $\theta = 1$ and (b) $\theta = 3$, under ($p_{\text{AQL}}, p_{\text{LQL}}$) = (0.05, 0.08) and ($\alpha, \beta$) = (0.05, 0.05).

In Figure 2, we can see that the two curves of the sampling plans are very similar in case (a) or in case (b), but the sample size required by the RGS plan is much fewer. For example, the single plan requires $n = 47$ while the RGS plan requires ASN = 31.77 in case (a). In addition, all of the OC curves show that the probability of acceptance will become smaller as the nonconforming rate $p$ increases, which is as expected from the theory. Since the RGS plan requires fewer sample size to give the desired protection, the cost of inspection will greatly
Table 1: The proposed plan parameters when $\theta = 1$.

| $p_{AQL}$ | $p_{LQL}$ | $\alpha = 0.05, \beta = 0.05$ | ASN | | $\alpha = 0.05, \beta = 0.10$ | ASN | | $\alpha = 0.10, \beta = 0.05$ | ASN |
|-----------|-----------|-------------------------------|-----|---|-------------------------------|-----|---|-------------------------------|-----|
| 0.02      | 0.02      | 21.9                          | 13  | 9  | 0.9859                        | 50.94| 9  | 0.9885                        | 0.9800| 14.615 |
| 0.03      | 0.0857    | 6.8793                        | 5   | 3  | 0.9826                        | 35.06| 3  | 0.9874                        | 0.9662| 5.2036 |
| 0.04      | 0.9849    | 4.9917                        | 3   | 2  | 0.9807                        | 3.411| 2  | 0.9864                        | 0.9318| 4.4893 |
| 0.05      | 0.9819    | 4.2998                        | 3   | 2  | 0.9754                        | 3.787| 2  | 0.9831                        | 0.948 | 3.0405 |
| 0.06      | 0.9818    | 3.224                        | 2   | 1  | 0.9550                        | 2.239| 1  | 0.9611                        | 0.9484| 2.1883 |
| 0.07      | 0.9753    | 2.943                        | 2   | 1  | 0.9685                        | 2.489| 2  | 0.9726                        | 0.9440| 2.4449 |
| 0.08      | 0.9735    | 2.514                        | 2   | 1  | 0.9690                        | 2.333| 2  | 0.9666                        | 0.9439| 2.2715 |
| 0.09      | 0.9692    | 2.318                        | 2   | 1  | 0.9550                        | 2.239| 1  | 0.9611                        | 0.9484| 2.1883 |
| 0.10      | 0.9605    | 2.185                        | 2   | 1  | 0.9521                        | 2.177| 2  | 0.9625                        | 0.9593| 2.0244 |
| 0.02      | 0.03      | 18.02                        | 25  | 27 | 0.9791                        | 44.72| 27 | 0.9791                        | 0.9718| 41.903 |
| 0.03      | 0.0857    | 6.8793                        | 5   | 3  | 0.9826                        | 35.06| 3  | 0.9874                        | 0.9662| 5.2036 |
| 0.04      | 0.9849    | 4.9917                        | 3   | 2  | 0.9807                        | 3.411| 2  | 0.9864                        | 0.9318| 4.4893 |
| 0.05      | 0.9819    | 4.2998                        | 3   | 2  | 0.9754                        | 3.787| 2  | 0.9831                        | 0.948 | 3.0405 |
| 0.06      | 0.9818    | 3.224                        | 2   | 1  | 0.9550                        | 2.239| 1  | 0.9611                        | 0.9484| 2.1883 |
| 0.07      | 0.9753    | 2.943                        | 2   | 1  | 0.9685                        | 2.489| 2  | 0.9726                        | 0.9440| 2.4449 |
| 0.08      | 0.9735    | 2.514                        | 2   | 1  | 0.9690                        | 2.333| 2  | 0.9666                        | 0.9439| 2.2715 |
| 0.09      | 0.9692    | 2.318                        | 2   | 1  | 0.9550                        | 2.239| 1  | 0.9611                        | 0.9484| 2.1883 |
| 0.10      | 0.9605    | 2.185                        | 2   | 1  | 0.9521                        | 2.177| 2  | 0.9625                        | 0.9593| 2.0244 |

be reduced. Therefore, it is reasonable to conclude that the proposed RGS plan has a better performance.

4.2. Sample Sizes Required for Inspection. In order to compare the sample sizes required for inspection in the RGS plan and the single plan with different values of $p_{AQL}$ and $p_{LQL}$, the $p_{AQL}$ value is fixed at 0.05 and the $p_{LQL}$ value increases from 0.06 to 0.25 when the risks are $\alpha = 0.05$ and $\beta = 0.10$. The results are shown in Figure 3 ($\theta = 1$) and Figure 4 ($\theta = 3$). From Figures 3 and 4, we can note that the sample sizes required for both sampling plans decrease as the value of $p_{LQL}$ rises from 0.06 to 0.25. Clearly, the sample size required is larger as the value of $p_{LQL}$ is closer to the value of $p_{AQL}$.

Moreover, it is obvious that the proposed RGS plan requires smaller sample size for inspection than the single sampling plan when $p_{LQL}$ takes any value between 0.06 and 0.25. Therefore, the RGS sampling plan is a more cost-effective plan while the single plan is relatively uneconomical.

On the other side, we also list the sample sizes required for the single sampling plan and RGS plan in Table 4 with commonly used values of $p_{AQL}$ and $p_{LQL}$ when $(\alpha, \beta) = (0.05, 0.10)$, $(0.10, 0.05)$, and $(0.10, 0.10)$ assuming that $\theta = 1.97$. From Table 4, it is obvious that the sample size required by the RGS plan is fewer than required by the single sampling plan for all cases. For example, when $p_{AQL} = 0.02$, $p_{LQL} = 0.03$, and $(\alpha, \beta) = (0.10, 0.05)$, the sample size of
Table 2: The proposed plan parameters when $\theta = 1.97$.

| $p_{AQL}$ | $p_{LQL}$ | $\alpha = 0.05, \beta = 0.05$ | $\alpha = 0.05, \beta = 0.10$ | $\alpha = 0.10, \beta = 0.05$ |
|-----------|-----------|-----------------------------|-----------------------------|-----------------------------|
| $\alpha$  | $\beta$   | $n$  | $k_a$ | $k_r$ | ASN  | $n$  | $k_a$ | $k_r$ | ASN  | $n$  | $k_a$ | $k_r$ | ASN  |
| 0.01      |           | 0.02 | 0.1663 | 1.5908 | 16.458 | 11  | 1.6607 | 1.5964 | 16.959 | 8  | 1.6725 | 1.59 | 14.2 |
|           |           | 0.05 | 0.5553 | 1.6224 | 31.244 | 2  | 1.5435 | 1.5928 | 16.959 | 2  | 1.5435 | 1.59 | 14.2 |
|           |           | 0.06 | 0.5553 | 1.6224 | 31.244 | 3  | 1.5435 | 1.5928 | 16.959 | 3  | 1.5435 | 1.59 | 14.2 |
| 0.02      |           | 0.02 | 0.1663 | 1.5908 | 16.458 | 11  | 1.6607 | 1.5964 | 16.959 | 8  | 1.6725 | 1.59 | 14.2 |
|           |           | 0.05 | 0.5553 | 1.6224 | 31.244 | 2  | 1.5435 | 1.5928 | 16.959 | 2  | 1.5435 | 1.59 | 14.2 |
|           |           | 0.06 | 0.5553 | 1.6224 | 31.244 | 3  | 1.5435 | 1.5928 | 16.959 | 3  | 1.5435 | 1.59 | 14.2 |
| 0.05      |           | 0.02 | 0.1663 | 1.5908 | 16.458 | 11  | 1.6607 | 1.5964 | 16.959 | 8  | 1.6725 | 1.59 | 14.2 |
|           |           | 0.05 | 0.5553 | 1.6224 | 31.244 | 2  | 1.5435 | 1.5928 | 16.959 | 2  | 1.5435 | 1.59 | 14.2 |
|           |           | 0.06 | 0.5553 | 1.6224 | 31.244 | 3  | 1.5435 | 1.5928 | 16.959 | 3  | 1.5435 | 1.59 | 14.2 |
| 0.10      |           | 0.02 | 0.1663 | 1.5908 | 16.458 | 11  | 1.6607 | 1.5964 | 16.959 | 8  | 1.6725 | 1.59 | 14.2 |
|           |           | 0.05 | 0.5553 | 1.6224 | 31.244 | 2  | 1.5435 | 1.5928 | 16.959 | 2  | 1.5435 | 1.59 | 14.2 |
|           |           | 0.06 | 0.5553 | 1.6224 | 31.244 | 3  | 1.5435 | 1.5928 | 16.959 | 3  | 1.5435 | 1.59 | 14.2 |

The RGS plan is 38.264, while the single plan is 51. Therefore, the proposed sampling plan will give the desired protection with the less required sample size so that the RGS plan is economically superior to the single sampling plan.

5. An Industrial Example

In this example, we use the data about the failure times of 25 ball bearings in endurance test, which have been discussed by Lee [20]. The following observations are the number of million revolutions before failure for each of 25 ball bearings (Lawless [27]):

| Observation | 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.48, 51.84, 51.96, 54.12, 55.56, 67.80, 67.80, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.40. |
Based on these observations, the estimation of the $C_L$ is calculated as follows:

$$C_L^{\hat{}} = \frac{\Gamma(1 + 1/\theta) - L \cdot (n/W)^{1/\theta}}{A} = 1.3864, \quad (15)$$

where $W = \sum_{i=1}^{25} X_i^\theta = 139525.2$ and $A = \sqrt{\Gamma(1 + 2/\theta) - \Gamma^2(1 + 1/\theta)} = 0.46974$.

Since $C_L^{\hat{}} = 1.3864 > k_a$, the consumer should accept the entire lot.
6. Conclusions

Acceptance sampling plans can make a lot sentencing decision for the producer and the consumer a general rule in order to meet the desired quality requirement and protection.

This paper presents a RGS plan based on the process capability index $C_L$ for accepting a lot whose quality characteristic follows a Weibull distribution. The optimal plan parameters of the proposed RGS plan are determined by minimizing the ASN function with two constraints required by the producer.
Table 4: The comparison of sample sizes for two sampling plans based on $C_L$ ($\theta = 1.97$).

| $P_{AQL}$ | $P_{LQL}$ | ($0.05, 0.05$) | ($0.05, 0.10$) | ($0.10, 0.05$) |
|-----------|------------|----------------|----------------|----------------|
|           | $n$ | $ASN_R$ | $n$ | $ASN_R$ | $n$ | $ASN_R$ |
| 0.01      | 0.06 | 4.9532 | 4 | 3.0521 | 3 | 2.4443 |
|           | 0.07 | 2.5903 | 3 | 2.5397 | 3 | 2.2039 |
|           | 0.08 | 2.4721 | 3 | 2.2746 | 3 | 2.0847 |
|           | 0.09 | 2.2566 | 3 | 2.1822 | 2 | 2.0516 |
|           | 0.10 | 2.1392 | 3 | 2.0756 | 2 | 2.0123 |
| 0.02      | 0.03 | 65 | 53 | 40.844 | 51 | 38.264 |
|           | 0.04 | 23 | 19 | 14.232 | 18 | 13.192 |
|           | 0.05 | 13 | 11 | 9.4282 | 10 | 7.3731 |
|           | 0.06 | 10 | 8 | 6.9586 | 7 | 5.3262 |
| 0.05      | 0.07 | 8 | 6 | 4.7171 | 6 | 4.0192 |
|           | 0.08 | 6 | 5 | 4.2030 | 5 | 3.5767 |
|           | 0.09 | 5 | 4 | 3.6085 | 4 | 3.0002 |
|           | 0.10 | 5 | 4 | 3.1886 | 4 | 2.6436 |
|           | 0.15 | 5 | 3 | 2.3361 | 3 | 2.0693 |
| 0.10      | 0.06 | 309 | 246 | 175.95 | 242 | 182.68 |
|           | 0.07 | 91 | 73 | 54.051 | 71 | 52.267 |
|           | 0.08 | 47 | 38 | 28.966 | 36 | 26.693 |
|           | 0.09 | 30 | 25 | 18.281 | 23 | 17.328 |
|           | 0.10 | 22 | 18 | 13.692 | 17 | 12.001 |
|           | 0.15 | 5 | 4 | 2.9261 | 3 | 2.4444 |
| 0.15      | 0.15 | 59 | 47 | 35.526 | 46 | 31.445 |
|           | 0.20 | 20 | 17 | 12.333 | 16 | 10.695 |
|           | 0.25 | 12 | 10 | 7.161 | 9 | 6.1821 |
| 0.20      | 0.30 | 8 | 7 | 5.1473 | 6 | 4.3262 |
|           | 0.35 | 6 | 5 | 4.1959 | 5 | 3.3826 |
|           | 0.40 | 5 | 4 | 3.3677 | 4 | 2.7629 |
|           | 0.50 | 4 | 3 | 2.5199 | 3 | 2.2246 |

Note: $n$ and $ASN_R$ denote the sample size of the single sampling plan and VRGS plan, respectively.

and the consumer. Then we use two criteria (the required sample size and OC curve) to compare the efficiency of the proposed RGS plan with the single plan proposed. The results imply that our proposed RGS plan requires the smaller ASN but provides the desired protection at the same time. So the industrialists can save the inspection cost if they use the proposed RGS plan. Finally, an example is also given to show the application of the proposed plan in various industries. For future research, it might be interesting to consider other sampling schemes, such as multiple dependent states (MDS) sampling based on the index $C_L$.

Competing Interests

The authors declare no competing financial interests.

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