Tunneling conductance and local density of states in time-reversal symmetry breaking superconductors under the influence of an external magnetic field

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We consider different effects that arise when time-reversal symmetry breaking superconductors are subjected to an external magnetic field, thus rendering the superconductor to be in the mixed state. We focus in particular on two time-reversal symmetry breaking order parameters which are believed to be realized in actual materials: $p + ip'$-$\Delta$ and $d + id'$-$\Delta$. The first order parameter is relevant for Sr$_2$RuO$_4$, while the latter order parameters have been suggested to exist near surfaces in some of the high-$T_c$ cuprates. We investigate the interplay between surface states and vortex states in the presence of an external magnetic field and their influence on both the tunneling conductance and the local density of states. Our findings may be helpful to experimentally identify the symmetry of unconventional time-reversal symmetry breaking superconducting states.

I. INTRODUCTION

Recently, considerable attention has been devoted to the chiral superconducting phase which is believed to be realized in the $p$-wave triplet superconductor Sr$_2$RuO$_4$. The chiral state of a $p$-wave superconductor corresponds to a non-zero projection $l_z = \pm 1$ of the Cooper pairs angular momentum $l$ along the $z$ axis, and thus breaks time-reversal symmetry. The spatially homogeneous triplet order parameter $\Delta = \Delta_0 (\hat{d} \cdot \hat{\tau}) i \hat{\sigma}_y$ is described by the vector $\hat{d}(p) = (0, 0, p_z + i \chi p_y)$, which depends on the direction of electron momentum $p$. Here $\Delta_0 \propto \cos(2\chi)$ is the bulk value of the order parameter, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y)$ is the vector of Pauli matrices of conventional spin operators, and $\chi = \pm 1$ corresponds to the two possible values of chirality. Also, chiral superconducting states can be associated with an admixture of two order parameters corresponding to different irreducible representations of a crystal point group. Naturally different order parameter components can coexist in the vicinity of interfaces between superconductors and surfaces due to broken symmetry of the crystal group. Among other possibilities of sub-dominant order parameter symmetry, there are states which break time reversal symmetry, as well as in the generation of spontaneous currents flowing along the surfaces in time reversal symmetry breaking cases.

Time-reversal symmetry breaking order parameters have been proposed to exist near surfaces in high-$T_c$ superconductors. This proposal stems from the observation of a split zero-bias conductance peak in the absence of any applied magnetic field. In this case, it has been suggested that the relevant order parameter is either $d + id'$ wave or $d + id$ wave. The gap may then be written as $\Delta = \Delta_0 g(\theta_p) + i \Delta_s$ or $\Delta = \Delta_0 g(\theta_p) + 1i \Delta_d g_1(\theta_p)$, respectively, where $\Delta_0$ is an amplitude of the main component and the admixture of another pairing symmetry is denoted by the amplitudes $\Delta_s$ and $\Delta_d$. Here, $\theta_p$ is a polar angle in momentum space $p = p(\cos \theta_p, \sin \theta_p)$, $g(\theta_p) = \cos(2\theta_p + \alpha)$ and $g_1(\theta_p) = \sin(2\theta_p + \alpha)$ where $\alpha/2$ is an angle measuring the misorientation of crystalline symmetry axes and coordinate axes. One obtains $d - d$-$\Delta$ symmetry of the main order parameter component for $\alpha = 0$ and $d - d$-$\Delta$ pairing for $\alpha = \pi/2$. While the experimental data so far clearly indicates an order parameter which breaks time-reversal symmetry, the question of whether the symmetry is $d + id'$ or $d + id$-$\Delta$ remains unsolved. Clearly, experimental signatures that may distinguish these two types of pairings would be highly desirable.

One of the important features of unconventional superconductors is the possibility for the existence of surface Andreev bound states. They occur in the vicinity of the scattering interface between a superconductor and an insulator, if the incident and reflected quasiparticles (QP) with different momentum directions see different phases of the order parameter. The consequence of the Andreev bound states formation is an increase of the local density of states at the surface resulting in zero-bias conductance peak anomaly observed in tunneling spectroscopy of high-$T_c$ cuprates with $d$-wave symmetry of superconducting pairing as well as in the $p$-wave triplet superconductor Sr$_2$RuO$_4$. Also, the Andreev bound states determine the anomalous low-temperature behaviour of the London penetration length and the Josephson critical current in $d$-wave and chiral superconductors.

Under the influence of an applied magnetic field, screening currents and vortices may be generated in a superconductor. As a result, the spectrum of surface states acquires a Doppler shift, leading to a splitting of the zero-bias conductance peak. Abrikosov vortices located near a superconducting surface generate an essentially inhomogeneous superfluid velocity field, which leads to a non-trivial electronic structure of the surface bound states. Also, it was recently proposed that the same Doppler shift effect should lead to a chirality selective influence of the magnetic field on the surface states in a $p$-wave chiral superconductor with broken time-reversal invariance. The quasiparticle density of states (DOS)
near a flat surface was shown to depend on the orientation of magnetic field with respect to the chirality as well as on the vorticity in case when the Abrikosov vortex is pinned near the surface of superconductor. Additionally, in superconductors featuring gap nodes, such as in the case in pure $d_{x^2-y^2}$-symmetric superconducting cuprates, a vanishing pair potential in nodal directions results in important ramifications for the physics of the system.\textsuperscript{11,23,24,25,26}

To understand the effect of an externally applied magnetic field on the surface DOS, let us consider a spectrum of Andreev bound states near a flat surface of a time-reversal symmetric superconducting cuprates, a vanishing pair potential in nodal directions results in important ramifications for the physics of the system.\textsuperscript{11,23,24,25,26}

The transformation of these spectra due to the Doppler shift effect is shown in Fig.\textsuperscript{11} To be definite we assume that $\Delta_s > 0$, $\Delta_d > 0$ and $\chi = 1$. Considering the DOS at Fermi level, $\nu = |d\varepsilon_a/dk_y|_{\varepsilon=0}$, in a chiral $p$-wave superconductor one can see that its dependence on the magnetic field is monotonic: it either increases or decreases for different field directions (see Fig.\textsuperscript{11}) as discussed in Ref.\textsuperscript{22}.

Another behaviour of the DOS occurs in the case of a $d + i s$ wave superconductor. From Fig.\textsuperscript{11} it follows that for a certain field direction there are no states at the Fermi level $\varepsilon = 0$ (red dashed lines in Fig.\textsuperscript{11}). For the opposite field direction (blue dash-dotted lines in Fig.\textsuperscript{11}), intersections of spectral branches with the Fermi level appear when the superfluid velocity is large enough $|v_{sy}| > \Delta_s/p_F$ so that the value of momentum projection at the intersection point is smaller than the Fermi momentum $|k_s^0| < k_F$. Thus, one can expect that the DOS at the Fermi level should be zero when $H < H^*$, where $H^*$ is the magnetic field value providing the condition $|v_{sy}| = |\Delta_s|/p_F$ to be fulfilled.

On the contrary, in the $d + i d$ wave case the DOS at the Fermi level is non-zero even in the absence of a magnetic field. As can be seen from Fig.\textsuperscript{11}; (black solid lines) the spectral branches intersect the level $\varepsilon = 0$ at $k_s^0 = \pm k_F/\sqrt{2}$. The transformation of the spectrum due to the magnetic field of different directions is shown in Fig.\textsuperscript{11}; by red dashed lines ($H > 0$) and by blue dash-dotted lines ($H < 0$). Then, it can be easily seen that for $H > 0$ the coordinates of the intersection points $k_s^0$ shift towards $\pm k_F$ and for a certain value of the magnetic field $H > H^*$ the DOS at the Fermi level $\varepsilon = 0$ disappears.

In the presence of an Abrikosov vortex near the surface of chiral superconductor a non-trivial structure of the local density of states distribution appears which depends on the vortex orientation.\textsuperscript{22} Along with the Doppler shift effect, an important modification of the quasiparticle spectrum and the DOS can be obtained due to the overlapping of the surface states and the low-energyQP states localized within the vortex core, found in the pioneering work by Caroli- de Gennes and Matricon (CdGM).\textsuperscript{20} It was shown that QP states with energy lower than the bulk superconducting gap value $\Delta$ are localized within the vortex core at the characteristic scale of the order of coherence length $\xi$ and have a discrete spectrum $\varepsilon_\nu(\mu)$ as a function of the quantized (half–integer) angular momentum $\mu$. At small energies $|\varepsilon| \ll \Delta$ the spectrum for a vortex with vorticity $M$ is given by

$$\varepsilon_\nu(\mu) \approx -M\mu\omega,$$

where $k_F = p_F/\hbar$ and $\omega \sim \Delta_0/(k_F\xi)$. For the most of superconducting materials, including Sr$_2$RuO$_4$, the interlevel spacing $\omega$ is much less than the superconducting gap $\Delta$ since $(k_F\xi) \gg 1$. Therefore, the CdGM spectrum may be considered continuous as a function of the impact parameter of the
For certain QP trajectories the condition of resonance is obtained by applying Bohr-Sommerfeld quantization rule to the canonical variables $\mu = -k_F b$ and $\theta_p$. It should be noted that when the superconducting order parameter contains nodes, the quasiclassical expression (9) is invalid near the nodal directions since energy states near the vortex core are not truly localized, but rather “leak” out through the gap nodes. This is not the case for us since we consider superconducting order parameters which are gapped over the entire Fermi surface.

To study the interaction between vortex and surface states, let us consider an example of vortex positioned near a flat surface of chiral $p+ip$-wave superconductor. Comparing the energies of surface $\varepsilon_\alpha$ and vortex $\varepsilon_v$ states one can see that for certain QP trajectories the condition of resonance $\varepsilon_\alpha = \varepsilon_v$ is realized. Thus the spectrum transformation in such almost degenerate two-level system is given by a secular equation:

$$\left( \varepsilon - \varepsilon_\alpha \right) \left( \varepsilon - \varepsilon_v \right) = J^2.$$  (10)

Since we consider a low-energy spectrum $|\varepsilon| \ll \Delta_0$, the trajectories should pass close to the vortex center for the spectrum modification (10) to be effective. Then, the interaction of surface and vortex states is determined by the overlap integral $J \approx \Delta \exp(-\tilde{a}/\xi)$, where $\tilde{a} = a/\cos \theta_p$, and $a$ is the distance from the vortex to the surface. Taking a certain point at the surface (see point $A$ in Fig. 2) of the superconductor one can obtain a relation between the angles and impact parameters of trajectories passing through this point as follows $b = \tilde{a} \sin(\theta - \theta_p)$. Thus the energy of vortex core states can be written as $\varepsilon_v = M(\tilde{a}/\xi) \Delta_0 \sin(\theta - \theta_p)$. Then, from Eq. (10) we obtain the spectrum transformation shown qualitatively in Fig. 2 for the particular case of $\theta = 0$. It is easy to see that for equal values of vorticity and chirality (Fig. 2b) there appears a minigap in quasiparticle spectrum near the Fermi level and therefore the zero-energy DOS is suppressed. On the other hand, in the case of opposite vorticity and chirality (Fig. 2c) there is no minigap and the DOS is not suppressed.

In a $d+i\sigma$- and $d+i\tau$-wave superconductor, the interaction between vortex and surface states can also lead to noticeable effects, which will be discussed later in the present paper.

Recently, it was pointed out that tunneling of quasiparticles into vortex core states leads to a resonant enhancement of subgap conductance of normal metal/superconductor (N/S) junction. In the case of chiral superconductors, such a tunneling effect can lead to either stimulation or suppression of conductance, depending on the direction of vorticity. We will show that if vortices are located far from the N/S interface, the conductance follows the behaviour expected from the Doppler shift approach. On the other hand, when the distance from the vortex to the interface becomes comparable with coherence length $\xi$ the tunneling into vortex core states comes into play, leading to the peculiar nonmonotonic conductance dependence on the vortex coordinate with respect to the super-
conducting surface.

This paper is organized as follows. In Sec. II we give an overview of the theoretical framework which is employed in this work, namely a Bogoliubov approach and a quasiclassical Eilenberger approach. In Sec. III we present our main results for the influence of magnetic field on bound surface states spectra and local density of states near the surface. We discuss the transformation of surface states in the Miessner state of superconductor as well as the effects of interplay between surface and vortex core states. We give our conclusions in Sec. IV.

II. THEORETICAL APPROACH

Our further considerations are based on the Bogoliubov–de Gennes (BdG) equations for particle– (u) and hole–like (v) parts of the wave function, which have the following form:

\[
\begin{align*}
-\frac{1}{2m} \left( \hat{p} - \frac{e}{c} \mathbf{A} \right)^2 u + \Delta v &= (\varepsilon + \varepsilon_F) u, \\
-\frac{1}{2m} \left( \hat{p} + \frac{e}{c} \mathbf{A} \right)^2 v + \Delta^\dagger u &= (\varepsilon - \varepsilon_F) v.
\end{align*}
\]

Here \(\Delta\) is a gap operator, \(\mathbf{A}\) is a vector potential, \(\hat{p} = -i(\partial/\partial \mathbf{x}, \partial/\partial \mathbf{y})\), and \(\mathbf{r} = (x, y)\) is a radius vector in the plane perpendicular to the anisotropy plane. Hereafter we assume the Fermi surface to be cylindrical along the \(z\)-axis and consider a motion of QPs only in \(xy\) plane.

In case of unconventional superconductors, the gap potential \(\Delta\) is a non-local operator, so the BdG system effectively becomes a very complicated integro-differential equation. Another complexity arises from the broken spatial invariance of the superconducting gap in presence of vortices near the N/S interface. A simplification can be obtained if one considers a quasiclassical approximation, assuming that the wavelength of quasiparticles is much smaller than the superconducting coherence length (see e.g. Ref[34]). Within such an approximation, QPs move along linear trajectories, i.e. straight lines along the direction of QP momentum \(\mathbf{n} = k_F k_F^{-1} = (\cos \theta_p, \sin \theta_p)\). Generally, the quasiclassical form of the wave function can be constructed as follows: \((u, v) = e^{i k_F \cdot \mathbf{r}} (U, V)\), where \((U(\mathbf{r}), V(\mathbf{r}))\) is a slowly varying envelope function. Then the system (11) reduces to a system of first-order differential equations along the linear trajectories defined by the direction of the QP momentum \(\mathbf{n} = k_F k_F^{-1} = (\cos \theta_p, \sin \theta_p)\). Introducing the coordinate along trajectory \(x' = (\mathbf{n} \cdot \mathbf{r}) = r \cos(\theta_p - \theta)\) we arrive at the following form of the quasiclassical equations:

\[
\begin{align*}
-i \hbar v_F \partial_{x'} + v_F \cdot \left( \frac{e}{c} \mathbf{A} \right) U + \Delta V &= \varepsilon U, \\
+i \hbar v_F \partial_{x'} + v_F \cdot \left( \frac{e}{c} \mathbf{A} \right) V + \Delta^\dagger U &= \varepsilon V,
\end{align*}
\]

where the Fermi velocity is \(v_F = n \hbar k_F / m\). The pairing potential in Eq. (12) may generally be written as

\[
\Delta(\mathbf{r}, \theta_p) = \Delta(\theta_p) \Psi(\mathbf{r}),
\]

where \(\Delta(\theta_p)\) describes the orbital symmetry of the superconducting order parameter in momentum space, while \(\Psi(\mathbf{r})\) describes its spatial dependence both magnitude- and phase-wise.

The local DOS (LDOS) can be expressed through the eigenfunctions of the BdG equation (11) in the following form:

\[
N(\varepsilon, \mathbf{r}) = \sum_n |u_n(\mathbf{r})|^2 \delta(\varepsilon - \varepsilon_n),
\]

where \(u_n(\mathbf{r})\) is electron component of quasiparticle eigen function corresponding to an energy level \(\varepsilon_n\). The eigenfunction has to be normalized: \(\int \int |u_n(\mathbf{r})|^2 + |v_n(\mathbf{r})|^2 \, d^2 r = 1\).

We will also later employ a quasiclassical Eilenberger approach to study the spatially resolved DOS. Let us here sketch the framework of the treatment which makes use of the Eilenberger equation, following the notation of Refs[35,36]. It is now convenient to solve the Eilenberger equation along trajectories along the Fermi momentum, and introducing a Ricatti-parametrization for the Green’s function[36]. In this way, one obtains:

\[
\begin{align*}
\hbar v_F \partial_{x'} a(x') + 2 \omega_n + \Delta^\dagger a(x') a(x') - \Delta &= 0, \\
\hbar v_F \partial_{x'} b(x') - 2 \omega_n + \Delta b(x') \bar{b}(x') + \Delta^\dagger &= 0,
\end{align*}
\]

where \(\omega_n = \omega_n + m v_F \cdot \mathbf{v}_s\) is a Doppler-shifted Matsubara frequency and

\[
\mathbf{v}_s = \frac{1}{2m} \left( \hbar \nabla \Phi - \frac{2e}{c} \mathbf{A} \right)
\]

is a gauge-invariant superfluid velocity where \(\Psi(\mathbf{r})\) is a gap function phase: \(\Psi(\mathbf{r}) = |\Psi|e^{i\Phi}\). The LDOS may be expressed through the scalar coherence functions \(a\) and \(b\) as follows:

\[
N(\varepsilon) = \int_0^{2\pi} \frac{d\theta}{2\pi} \text{Re} \left\{ \frac{1 - ab}{1 + ab} \right\} \omega_n - \varepsilon + i\delta,
\]

where \(\varepsilon\) is the quasiparticle energy measured from Fermi level and \(\delta\) is a scattering parameter which accounts for inelastic scattering.

To investigate the transport properties of N/S junction, we employ an approach similar to what was used in work by Bardeen, Tinkham and Klapwijk[33]. The expression for the dimensionless zero- bias conductance of the N/S junction measured in terms of the conductance quantum \(e^2/(\pi h)\) can be written as follows:

\[
G = G_{sh} \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \left[ 1 - R_n(\theta_0) + R_n(\theta_0) \right] \cos \theta_0 \, d\theta_0,
\]

where \(R_n(\theta_0)\) and \(R_a(\theta_0)\) are the probabilities of normal and Andreev reflection respectively, \(\theta_0\) is the incident angle: \(k_F = k_F (\cos \theta_0, \sin \theta_0)\), characterizing the propagation direction of quasiparticles, coming from the normal metal region. The Sharvin conductance \(G_{sh} = k_F L_y / \pi\) equals the total number of propagating modes determined by the channel width \(L_y\).
The problem of quasiparticle scattering at the N/S interface is formulated within the BdG theory. An interfacial barrier separating the N and S regions can be modeled by repulsive delta function potential $W(x) = W_0 \delta(x)$, parameterized by a dimensionless barrier strength $Z = W_0/k_{F} F$. The boundary conditions at the N/S interface then read:

$$[f(0)] = 0, \quad [\partial_x f(0)] = (2k_{F} Z)f(0),$$

where $f = (u, v)$ and $[f(x)] = f(x + 0) - f(x - 0)$.

Considering a zero-bias problem we will have to analyze only zero-energy excitations with $\varepsilon = 0$. For wave functions decaying at the different ends of trajectory $(U, V) = e^{\xi} (e^{i(\eta + \Phi)/2}, e^{-i(\eta + \Phi)/2})$, where $\zeta = \zeta(s, b)$ and $\eta = \eta(s, b)$ are real-valued functions. Then, the quasiclassical equation can be written as follows:

$$\partial_x \eta + 2|\Delta| \cos \eta + 2\varepsilon_d = 0, \quad \partial_x \zeta + 2|\Delta| \sin \eta = 0.$$

The boundary conditions model the specularly reflecting N/S interface, coupling the waves with wave vectors $k_{F} = k_{F} (\cos \theta_0, \sin \theta_0)$, and $k_{F}' = k_{F} [\cos(\pi - \theta_0), \sin(\pi - \theta_0)]$. Therefore if the incident electron wave is $u_i = e^{ik_{F}\bar{r}}$, then the reflected electron $u_r$ and hole $v_r$ waves will have the form

$$u_r = U_r e^{i k_{F} \bar{r}}, \quad v_r = V_r e^{-i k_{F} \bar{r}},$$

where $U_r$ and $V_r$ are the envelope functions. Thus, each point $(0, y)$ at the N/S interface lies on the intersection of two quasiclassical trajectories, characterized by the angles $\theta_p = \theta_0$ and $\theta_p = \pi - \theta_0$. Let us denote the distribution of phases $\eta(x')$ along these trajectories as $\eta_+(x')$ and $\eta_-(x')$ correspondingly. Using the boundary conditions we obtain the following expression for the conductance:

$$G = \frac{N_0}{2} \int_{-L_y/2}^{L_y/2} \int_{-\pi/2}^{\pi/2} g(y, \theta_0) \cos \theta_0 d\theta_0 dy,$$

where $g(y, \theta_0)$ is given by

$$g(y, \theta_0) = \frac{2}{(Z_1^2 + Z_2^2)|1 - e^{i\theta}|^2 + 1},$$

with $Z_1 = Z / \cos \theta_0$ and $p(y, \theta_0) = \eta_+ - \eta_-$ is determined by a difference of phases $\eta_-(x')$ and $\eta_+(x')$ at the intersection point $(0, y)$. To evaluate the conductance, one needs to find the factor $e^{i\theta}$ in Eq. (22) and then the reflection probabilities by solving numerically Eq. (19) with the boundary conditions in Eq. (20).

III. RESULTS

To illustrate the basic effect of how the interplay between the Doppler-shift and the time-reversal symmetry breaking of the superconducting order parameter is manifested, we consider a situation where an external magnetic field is applied near the surface of the superconductor along the $z$-axis, thus inducing a vector potential $A$ in the superconductor which drives the shielding supercurrent. In order to proceed analytically, we make the simplifying assumption that the superfluid velocity field is nearly homogeneous and that the spatial variation of the superconducting order parameter near the interface is small. Choosing a real gauge, we then find that the Ricatti functions $a$ and $b$ in Eq. (15) may be written as

$$a(\theta) = s(\theta) \Delta(\theta), \quad b(\theta) = s(\theta) \Delta^*(\theta),$$

and $s(\theta) = 1/[\tilde{\omega}_n(\theta) + \sqrt{[\tilde{\omega}_n(\theta)]^2 + |\Delta(\theta)|^2}$, (23)

where $\tilde{\omega}_n$ depends on $\theta$ through the Doppler-shift. To evaluate the LDOS in Eq. (16) at the surface, we need to take into account proper boundary conditions at $x = 0$. Assuming an impenetrable surface with perfect reflection, these boundary conditions read:

$$a_{\text{surface}}(\theta) = a(\pi - \theta), \quad b_{\text{surface}}(\theta) = b(\theta).$$

Inserting this into the expression for the LDOS, we obtain

$$N(\varepsilon) = 2\Re \left\{ \frac{1}{1 + a(\pi - \theta)b(\theta)} \right\}_{\omega_n \rightarrow -\delta} - 1.$$ (25)

The $\langle \ldots \rangle$ denotes angular averaging, which we restrict to angles $-\pi/2 \leq \theta \leq \pi/2$ due to the surface. It may be shown that for a chiral $p$-wave superconductor, the zero-energy DOS at the surface reads

$$N(0) = 1 + \frac{\hbar k_{F} v_{sy}}{\Delta_0} + \ldots,$$ (26)

while for pure $s$- or $d$-wave superconductors one finds

$$N(0) = C_1 + C_2 v_{sy}^2 + \ldots,$$ (27)

where $C_1$ and $C_2$ are arbitrary constants. From numerical investigations of Eq. (25) at $\varepsilon = 0$, we find that the zero-energy DOS may quite generally be written as

$$N(0) = C_1 + C_2 v_{sy} + \ldots,$$ (28)

whenever the superconducting order parameters $i)$ break time-reversal symmetry and $ii)$ support the presence of subgap surface-bound states. This is the case both for the $p_x + i p_y$-wave pairing which is believed to be realized in Sr$_2$RuO$_4$, as well as the $d_{x^2-y^2}$-wave and $d_{+1s}$-wave pairings that are relevant for the cuprates. In particular, tunneling spectroscopy measurements have indicated the presence of such a time-reversal symmetry breaking order parameter near surfaces by a split zero-bias conductance peak that was observed in the absence of an external field in several experiments.

In Ref. [5] it was pointed out that the neglect of the gradient term in the Eilenberger equation is expected to be a reasonable
approximation as long as the Doppler-shift energy \(mv_F \cdot v_s\)
is small compared to the local gap energy \(\Delta(\theta)\). Thisapproximation would then fail close to the vortex core or gap nodesof \(\Delta(\theta)\). Nevertheless, in the model case of spatially homoge-neous gap function and superfluid velocity field the gradientterms in Eilenberger equation can be neglected in the wholerang of Doppler shift energies. However considering a modelsituation the above discussion nevertheless serves to illustrateour main qualitative argument: namely, that chirality-sensitiveeffects should be expected in superconductors with order para-meters that \(i\) break time-reversal symmetry and \(ii\) supportthepresence of subgap surface-bound states. We now proceedtodiscuss the cases of \(p_x + ip_y\)-wave and \(d \pm is(d)\)-wavepairing in more detail, since these are relevant to actual materials.

A. Surface states in \(p + ip\) and \(d \pm is(d)\) superconductors
under the influence of magnetic field.

In Fig. we show numerical plots of the surface LDOSat the Fermi level given by Eq. \(25\) for the chiral \(p\)-wave(Fig. \(3b\)), \(d \pm is\)-wave (Fig. \(3c\)) and \(d \pm id\)-wave (Fig. \(3c\)) casesin a wide domain of superfluid velocities. The structure of gapfunctions is chosen in the form of Eqs. \(1\)–\(3\) and the param-eter characterizing inelastic scattering in Eq. \(16\) is chosen as\(\delta = 0.1\Delta_0\). We introduce the following notations for the dif-ferent critical velocities: \(v_c = \Delta_0/(\hbar k_F)\), \(v_{cs} = |\Delta_s|/(\hbar k_F)\)and \(v_{cd} = |\Delta_d|/(\hbar k_F)\).

As seen, the surface LDOS has sharp peaks at a certainvalue of the superfluid velocity in all cases. We will showbelow that peaked structure of LDOS is provided by boundsurface states. Another contribution to the LDOS comes fromthe delocalized states corresponding to the continuous part ofQP spectrum. A delocalized state with zero energy \(\varepsilon = 0\)exists provided (i) \(|v_{sy}| > v_c\) in case of chiral \(p\)-wave superconductor, (ii) \(|v_{sy}| > v_{cs}\) and (iii) \(|v_{sy}| > v_{cd}\) in case of\(d \pm is\)-wave and \(d \pm id\)-wave superconductors correspondingly. Condition (i) is unlikely to be realized because it means that the superfluid velocity is larger than the critical depair-ing value. Conditions (ii) and (iii) can be realized, because thevalues \(v_{cs}\) and \(v_{cd}\) can be well below the critical depairingvelocity if the amplitude of additional order parameter com-ponents is small enough.

To analyze the contribution to LDOS provided by the boundsurfaces we will consider the domain of low energies \(|\varepsilon| \ll \Delta_0\). By neglecting small deviations of the electron andhole momentum, the normalized wave function of QP local-ized near the boundary can be written as

\[
\begin{bmatrix}
 u \\
 v
\end{bmatrix} = \left( \frac{1}{i} \right) \sqrt{\frac{2}{\xi \cos \theta_p}} e^{i k_y y \sin(\xi x)} e^{-x/(\xi \cos \theta_p)},
\]

where \((k_x, k_y) = k_F(\cos \theta_p, \sin \theta_p)\). This wave function de-cay in the superconducting side \(x > 0\) at a characteristiclokalization scale \(\xi\) is given by \(\xi = \hbar v_F/\Delta_0\) for chiral \(p\)-wave and \(\xi = \hbar v_F/|\Delta_0 \sin(2\theta_p)|\) for \(d \pm is\)- and \(d \pm id\)- wave superconductor correspondingly with gap functions given byEqs. \(23\). The spectrum of the Andreev bound states, shiftedby the superfluid velocity, is given by

\[\varepsilon_a = \chi \Delta_0 k_y/k_F + \hbar v_{sy} k_y\]  
for the \(p\)-wave case,

\[\varepsilon_a = \Delta_s \sgn(k_y) + \hbar v_{sy} k_y\]  
for \(d \pm is\)-wave case and

\[\varepsilon_a = \Delta_d \sgn(k_y) \cos(2\theta_p) + \hbar v_{sy} k_y\]  
for \(d \pm id\)-wave case. Consequently, the contribution fromAndreev bound states to the zero-energy LDOS at the surfaceof a chiral \(p\)-wave superconductor is given by

\[N_a = N_0 \frac{1}{|v_{sy}/v_c + \chi|},\]

where \(N_0 = m/(2\pi \hbar^2)\) is the normal metal LDOS per one spin direction. For a chiral \(d \pm is\) superconductor, thebehaviour of the LDOS is more complicated. Assuming that\(\Delta_s > 0\), we obtain that the LDOS is zero for \(v_{sy} > -\Delta_s/\hbar k_F\). Otherwise, it is given by

\[N_a = 4N_0 \frac{v_{cs} v_{id}}{v_{sy}^2},\]

FIG. 3: Plot of the normalized zero-energy LDOS \(N(\varepsilon)\) for (a) \(p\)-wave superconductor with \(\chi = 1\), (b) \(d \pm is\)-wave case with \(\Delta_s = 0.4\Delta_0\) and (c) \(d \pm id\)-wave case with \(\Delta_d = 0.4\Delta_0\). Dashed lines areguides for eyes: the vertical ones denote positions of LDOS peaksand horizontal ones correspond to the level of normal metal DOS \(N_0\).
On the contrary, for the $d + id$ case the LDOS is zero if $v_{sy} > \Delta_d/(\hbar k_F)$ (for $\Delta_d > 0$) and otherwise it is given by

$$N_a = N_0 \frac{\Delta_0}{|\Delta_d|} \left(1 + \frac{v_{cd}}{v_{cs} + v_{cd}^2} \right).$$

It can be seen that these contributions to LDOS have peaks at $v_{sy} = v_c$ for $p$-wave case. For $d + is$-wave and $d + id$-wave superconductors the peaks are positioned at $v_{sy} = -sgn(\Delta_s)v_{cs}$ and $v_{sy} = sgn(\Delta_d)v_{cd}$ correspondingly. Even though the position of the peaks are different, the dependencies of the surface LDOS on the superfluid velocity (and consequently on magnetic field) are very similar for $d + is$ and $d + id$-wave superconductors. Therefore, it might be difficult to distinguish which case is realized experimentally.

On the other hand the considered model with a spatially homogeneous gap function $\Psi(r) = 1$ is adequate only when the applied magnetic field is not too large. When the magnetic field is large enough, it breaks the Meissner state and generates vortices near the surface of superconductor. Therefore, we investigate the influence of vortices on the LDOS distribution near a superconducting surface as well as on the conductance of normal metal/superconducting junctions. We will show that vortices have different effect on the conductance in $d + is$ and $d + id$ cases.

### B. Interplay of vortex and surface states in chiral superconductors.

A chirality sensitive LDOS transformation due to vortices situated near the surface of a chiral $p$-wave superconductor was considered in Ref. 22. It was shown that depending on the chirality and vorticity value, the surface LDOS near is either enhanced or suppressed upon decreasing the distance from the vortex to the surface. In case of $d + is (d)$ superconductors the transformation of LDOS profile is also sensitive to the value of vorticity. Similar behaviour is expected for a conductance of normal metal/chiral superconductor junction in the presence of vortices.

To investigate the influence of a single vortex on the LDOS profile and conductance, we assume that at $x > 0$ (superconducting region) the coordinate dependence of the order parameter may be written as follows:

$$\Psi(r) = e^{i\Phi}.$$  \hspace{1cm} (32)

Here, we consider a model situation where the magnitude of the order parameter is constant. The phase distribution $\Phi(r)$ consists of a singular part $\Phi_s(r) = \arg(r - r_v)$ and a regular part $\Phi_r(r)$, determined by the particular metastable vortex lattice configuration realizing near the boundary. We assume that the regular part of the phase distribution is $\Phi_r(r) = -\arg(r - r_{av})$ corresponding to the image vortex situated at the point $r_{av} = (-2a, 0, 0)$ behind the N/S interface.

In Fig 4 we show the LDOS profile near the surface of a chiral $p$-wave superconductor in the presence of a single vortex, positioned at some distance $a$ from the surface. When the vortex is positioned far from the surface $a \geq 2\xi$ the LDOS profile follows the behaviour, expected from the picture of local Doppler shift. Depending on the relative value of vorticity and chirality, the surface LDOS is either increased (Fig 4a) or decreased (Fig 4b). An analytical estimate with the help of spectrum Eq. (33) yields a following estimation of the amplitude of LDOS peak in Fig 4: \(\Delta N/N_0 = (1 + M \chi a)^{-1}\). At smaller distances $a \leq 2\xi$, the behaviour of LDOS changes drastically. In the case of opposite vorticity and chirality, the surface LDOS grows at $a \leq 2\xi$, obviously due to the overlapping with the peak of vortex core states. In case of equal vorticity and chirality the same overlapping occurs, but on the contrary it leads to reduction of DOS, as it was discussed in Sec. II. The peak of the LDOS at the surface discussed in Ref 22 transforms into a dip-and-peak structure as the vortex comes close to the surface.

![FIG. 4: (Color online) Plot of the normalized zero-energy LDOS](image)

**FIG. 4: (Color online) Plot of the normalized zero-energy LDOS**

$N(0)$ in the presence of a vortex near the surface of a chiral $p$-wave superconductor. (a) and (c) correspond to equal vorticity and chirality, (b) and (d) correspond to opposite vorticity and chirality. The distance from vortex to the surface is $a = 2\xi$ for (a) and (b) and $a = \xi$ for (c) and (d).

This is illustrated in Fig 5(a), where we plot the LDOS at the surface point $(0, 0)$, which is the nearest point to the vortex in Fig 4. At large distances $a \gg \xi$ the LDOS is a monotonic function of $a$, either increasing or decreasing depending on the relation between vorticity and chirality. At smaller distances $a \leq 2\xi$, the extremum of LDOS appears. In the case of opposite vorticity and chirality [lower curve in Fig 5(a)], the surface LDOS grows at $a \leq 2\xi$, due to the overlapping with the peak of vortex core states. In case of equal vorticity and chirality [upper curve in Fig 5(a)] the same overlapping leads to reduction of LDOS.

To investigate the influence of vortices on the transport
properties of normal metal/chiral \( p \)-wave superconductor junction we solve the generic problem of the influence of a single vortex near the N/S surface on the zero-bias conductance of the junction. A numerical plot of the conductance \( G \) as a function of a distance of vortex to the junction interface is shown by the solid lines in Fig. 5(b) for equal (upper curve) and opposite (lower curve) values of chirality and vorticity. The conductance is normalized to the value of Sharvin conductance \( G_{sh} = k_F L_y / \pi \).

At large distances \( a \gg \xi \) an analytical estimation of conductance can be obtained by using a local Doppler shift approximation on the quasiparticle spectrum. Indeed, the modification of the surface states energy due to a supercurrent flowing along the boundary of superconductor can be written as

\[
\varepsilon_n \approx (\chi \Delta_0 + \hbar v_{sy} k_F) k_y / k_F, \tag{33}
\]

where \( k_y \) is a quasiparticle momentum along the surface, \( \chi = \pm 1 \) is a chirality value and \( v_{sy} = (M \hbar / m) a / (y^2 + a^2) \) is a projection on the surface plane of superfluid velocity generated by the vortex and image antivortex, \( M \) is vorticity value and \( m \) is the electron mass. It follows from Eq.(33) that the Doppler shift effect leads to a change in the slope of anomalous branch. It is easy to obtain that in this case the function \( g(y, \theta_0) \) in expression Eq. (22) takes the following form:

\[
g(y, \theta_0) = \frac{2}{4(Z^2 + Z^2)(\varepsilon_n/\Delta_0)^2 + 1}. \tag{34}
\]

The straightforward integration in Eq.(21) yields \( G = G_0 + \delta G \), where \( G_0 = G_{sh} (\pi / Z^2) \) is the conductance without vortex and

\[
\delta G / G_{sh} = \pm \frac{2 \pi \xi}{Z^2 L_y} \arctan(L_y / 2a) \tag{35}
\]

is a vortex-induced conductance shift, where the upper (lower) sign corresponds to equal (opposite) vorticity and chirality.

At distances smaller than \( 2\xi \), an extremum of the conductance appears. Upon placing the vortex closer to the surface, an opposite effect occurs: one obtains a conductance suppression instead of enhancement and vice versa. The origin of the conductance extremum is a tunneling of quasiparticles into the vortex core states, or in other words, the overlapping of vortex and surface bound states. Comparing Figs. 5(a) and 5(b) one can see that the conductance in general follows the behaviour of the surface DOS.

2. \( d + is \) and \( d + id \) wave.

In chiral \( d + is \) and \( d + id \) superconductors the LDOS transformation appears to also be vorticity sensitive. In the Fig. 6 we show the profile of zero-energy LDOS in the case when the vortex is placed at a distance of \( a = 2\xi \) from a flat boundary of a \( d + is \)-wave superconductor characterized by a gap function in momentum space given by Eq.(2). In this section, we use the notation \( \xi = \hbar v_F / \Delta_0 \).

One can see that for one sign of vorticity the surface LDOS shows two peaks which are symmetric with respect to the vortex position. As we have shown above, the large peaks in surface LDOS appear when the energy coincides with the position of bound state level. For a different sign of the vorticity, there are no surface states at the Fermi level and the LDOS along the surface is a flat function. A non-zero level of LDOS in this case is provided by inelastic scattering which leads to the smearing the QP energy levels. Applying a local Doppler shift approach, which holds if the distance from vortex to surface is rather large (\( a \gg \xi \)) one can interpret the results shown in Fig. 6.

The coordinates \( y^* \) of surface LDOS peaks can be estimated from the relation \( v_{sy} = \Delta_s / p_F \), where \( v_{sy} = \)}
(M/\hbar)\alpha/(y^2 + a^2) is a projection on the surface plane of superfluid velocity generated by the vortex with vorticity \(\mathcal{M}\) and image antivortex. It can be seen that for \(a > \xi(\Delta_0/|\Delta_s|)\) the peak is situated at \(y^* = 0\) i.e. at the surface point nearest to the vortex. Otherwise, we obtain \(y^* = \pm a\sqrt{1 - (a/\xi)(|\Delta_s|/|\Delta_0|)}\). Comparing this estimation with the numerical results in Fig.6 one observes a minor difference. For example, it follows from the estimation that the LDOS peaks should be positioned at \(y^* = \pm 2.4\xi\) for \(\Delta_s = 0.2\Delta_0\), but in Fig.6 they are located at \(y^* = \pm 2.0\xi\). This discrepancy can be attributed to the complex shift of the energy \(\varepsilon - \varepsilon + id\) due to the effective scattering parameter \(\delta = 0.1\Delta_0\) which was used in the numerical calculations. If we increase the distance from vortex to surface \(a\), the LDOS peaks will merge when \(a > \xi(\Delta_0/|\Delta_s|)\) (see Fig.6).

In Fig.7 we show the LDOS profile modulated by a vortex placed at a distance of \(a = 2\xi\) from a flat boundary of \(d + id\) superconductor. The structure of the gap function was chosen in the form \(\Delta_{id}\). Applying the approach based on the local Doppler shift we obtain the similar expression for the coordinates of the peaks of surface LDOS: \(y^* = \pm a\sqrt{1 - (a/\xi)(|\Delta_d|/|\Delta_0|)}\). For the particular values of parameters \(\Delta_d = 0.2\Delta_0\) and \(a = 2\xi\) this estimation yields \(y^* = \pm 2.4\xi\), which is much less than obtained from numerical plot in Fig.7. \(y^* \approx \pm 4\xi\). This discrepancy can also be attributed to the effect of inelastic scattering, which appears to have a larger effect in \(d + id\)-wave case than in discussed above \(d + is\)-wave case.

A numerical plot of the N/S junction conductance as a function of distance from vortex to surface is shown in Fig.8 for the \(d + is\) and \(d + id\) cases. The conductance is normalized to the Sharvin conductance \(G_{sh} = k_F L_y/\pi\). Comparing Fig.8 and Fig.8 one can see that the conductance behaviour is qualitatively different for \(s\) and \(d\)-wave symmetry of the additional gap function component. For \(d + is\)-wave, the conductance has a sharp peak for one vortex orientation (upper curve in Fig.8) and it is a flat function of \(a\) for another vortex orientation (lower curve in Fig.8). The origin of the conductance enhancement is a formation of Andreev bound states at the Fermi level which are localized near the superconducting surface. As we discussed in the Introduction (see the Fig.1b), the zero-energy Andreev bound states can appear only for a certain direction of superfluid velocity flowing along the superconducting surface and if the value of the superfluid velocity is greater than a critical value \(|\nu_{sh}| > |\Delta_d|/(\hbar k_F)\). For a high interface barrier \(Z \gg 1\) applying an approximate analytical expression \(\langle 35\rangle\) we find that a sharp increase of conductance can be attributed to the effect of inelastic scattering, which appears due to the interaction of vortex and surface states.

\[
G/G_{sh} = \frac{16\pi}{3Z^2} \frac{\Delta_0}{L_y|\Delta_s|} \left[ 1 - \frac{a}{a^*} \right]^{3/2} + \lambda Z^{-4},
\]

where \(a^* = \xi(\Delta_0/|\Delta_s|)\) and \(\lambda \sim 1\). Otherwise, if \(a > a^*\) the conductance is much smaller since \(Z \gg 1\):

\[
G/G_{sh} \approx \left( \frac{\Delta_0}{\Delta_s} \right)^2 \frac{4}{3Z^4}.
\]

When the distance \(a\) is decreased further, the conductance is suppressed (see Fig.8, upper curve). The decrease of conductance can be attributed to the gap at the Fermi level which appears due to the interaction of vortex and surface states in a similar way as for the \(p + ip\)-wave case discussed in the previous section.

In a \(d + id\)-wave superconductor, zero-energy Andreev bound states may exist even in the absence of vortex. An asymptotic value of the conductance \(G_0\) at \(a \gg \xi\) can be obtained using the expression \(\langle 35\rangle\) as follows:

\[
G_0/G_{sh} = \left( \frac{\Delta_0}{\Delta_d} \right) \frac{\pi}{2\sqrt{2}Z^2}.
\]

When the vortex approaches the superconducting surface, the conductance is either suppressed (lower curve in Fig.8) or slightly enhanced (upper curve in Fig.8). This behaviour can be understood by again using the Eqs.\(\langle 35\rangle\) with the Doppler shifted spectrum of Andreev bound states \(\langle 6\rangle\). The decrease (increase) of conductance corresponds to the transformation of spectrum shown qualitatively in Fig.1 and it follows from the transformation of spectrum shown qualitatively in Fig.1 that \(\langle 35\rangle\).

\[
\delta G/G_{sh} = \pm \frac{\pi}{2\sqrt{2}Z^2} \left( \frac{\Delta_0}{\Delta_d} \right)^2 \frac{\xi}{L_y} \arctan \left( \frac{L_0}{2a} \right),
\]

where the upper and lower sign corresponds to the different vortex orientations. As the vortex approaches the surface further, there appears an extremum of the conductance. Such behaviour can be explained by a conductance enhancement due to the tunneling of QP into the vortex core states, discussed in Ref.\(\langle 40\rangle\). A sharp decrease of the upper curve in Fig.8 can be attributed to the opening of an energy gap at the Fermi level due to the interaction of vortex and surface states.
IV. SUMMARY

In summary, we have investigated how the tunneling conductance and the local density of states (LDOS) in superconductors are affected by the influence of an external magnetic field when the superconducting order parameter (OP) breaks time-reversal symmetry (TRS). This is directly relevant for both Sr$_2$RuO$_4$, where chiral $p+ip$-wave pairing is believed to be realized, and for the high-$T_c$ cuprates, where a $d+is$- or $d+id$-wave OP has been suggested to exist near surfaces. In addition to breaking TRS, all of these OPs feature surface bound zero-energy states at surfaces under appropriate circumstances (e.g. a dominant $d$-wave OP in the $d+is$-wave case).

We have shown how the Doppler-shift conspires with an interaction of vortex and surface states to produce a considerable qualitative modification of both the tunneling conductance and the LDOS. When the vortex is located at distances well above a coherence length $\xi$ from the surface, the Doppler-shift produces an enhancement or suppression of the LDOS depending on the relative sign of the vorticity and the chirality of the superconducting OP. This effect may be directly probed by first applying an external magnetic field in a direction while measuring the LDOS and then reversing the field direction and measuring again. When the vortex is located very close to the surface (a distance of order $\xi$ or smaller), there is an overlap between the vortex and surface states which effectively cause a dramatic change in the tunneling conductance and LDOS. This effect is also sensitive to the relative sign of the vorticity and the chirality of the superconducting OP. The overlap between these two sets of states results in either a strongly enhanced or suppressed tunneling conductance/LDOS at zero bias voltage/zero energy.

We have demonstrated the aforementioned effects both qualitatively and quantitatively for $p+ip$, $d+is$, and $d+id$-wave symmetries. Experimentally, the distance from the surface to the closest vortex can be altered by modifying the field strength. All of our predictions should be possible to test experimentally with present-day techniques.

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