Effect of dispersion coefficient on polymer injection studied with random walk particle tracking method

Rastlantısal parçacık hareket yöntemiyle incelenen dağılım katsayısının polimer enjeksiyonu üzerine etkisi

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Abstract

Polymer injection is a chemical EOR process, where the aim is to improve sweep efficiency of water flooding in an oil reservoir by increasing the water viscosity with the thickening effect of polymer injected with water. A random walk particle tracking model is developed to simulate the injection of polymer into an oil reservoir and it is integrated into an open source black-oil reservoir simulator (SINTEF’s MATLAB Reservoir Simulation Toolbox, MRST). The black-oil simulator solves the set of partial differential equations describing multiphase fluid flow in the porous medium, whereas the transport of the injected polymer due to advection and dispersion processes is obtained from the applied random walk method. The finite difference/finite volume discretizations of the continuity equations applied in MRST does not involve the physical dispersion processes. The dispersion observed on the results are called the numerical dispersion and are due to the numerical discretization methods applied to solve the continuity equations. Since dispersion in porous media is a scale dependent process, it is hard to quantify its coefficient by experiments or empirical equations in general. The random walk method to model the transport of injected polymer in the reservoir is independent of discretization. Hence, it does not involve numerical dispersion. Instead, the physical dispersion due to, for instance, the tortuosity of flow path or the adsorption of polymer in porous media can be included into the model. Therefore, we used the method to analyze the effect of dispersion coefficient. Increased dispersion coefficient causes the results to converge to actual MRST solution with increased uncertainties of polymer concentration. The dispersivity of a reservoir in simple one-dimensional problems can be determined with the method.

Keywords: Enhanced oil recovery, Polymer injection, Particle tracking method, Dispersion

1 Introduction

Polymer injection is one of the chemical enhanced oil recovery (EOR) methods that is used as a mobility control tool in petroleum reservoirs. The aim is to increase displacing phase viscosity and displacement efficiency [1]. Swollen polymer molecules in a good solvent (i.e. injection water) cause the polymer solution to have a higher flow resistance than regular waterflooding. Viscosity increase effect is the result of higher polymer concentration in solution [2]. Permeability reduction occurs due to polymer molecules being entrapped in small pore spaces and polymer molecules being adsorbed on rock surfaces during the polymer flooding [3]. Sheng (2011) states that the shear stress and interfacial viscosity between oil phase and injected polymer are important factors in polymer flooding as well [1]. These help polymer solutions to have higher push-pull force on oil droplets and oil droplets to move from dead-ends of porous medium.

The set of partial differential equations (PDE) describing multiphase fluid flow in the porous medium are used to develop a reservoir model and solve to forecast reservoir performances under different applications. However, although this approach needs minimum amount of simplifying assumptions [4], numerical dispersion errors are involved while solving the continuity equations in these simulators [5].

Increasing the grid number in a simulation to an acceptable number can reduce the numerical dispersion effect. However, it also increases the memory requirement of the simulations [6]. Moreover, increasing the grid block number is sufficient to some level only. Effect of increased number of grids on numerical dispersion in regular

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waterflooding and polymer flooding examples shows that the error does not disappear with finer grids \[7\].

Changes in reservoir flow velocity in porous medium adds the effect of extra and uneven mixing of chemicals. This added effect of mixing besides molecular diffusion is called dispersion. In a highly convective flow environment, dispersion coefficient and velocity are proportional. If a porous medium is assumed to have constant inhomogeneity factor, average grain size and diffusion coefficients, the dispersivity parameter can be defined as the ratio of dispersion coefficient and velocity. The highly scale dependent parameter is defined by experiments or empirical equations \[8\].

The random-walk particle tracking method is used to simulate flow in different studies. The method solves the advection and diffusion/dispersion of chemicals without numerical solutions of PDE’s, therefore is virtually free of numerical dispersion errors \[9\]. The method can be modified to inspect the effect of dispersivity in a reservoir since it does not involve the numerical dispersion problem. This study uses MATLAB Reservoir Simulation Toolbox (MRST), an open-source code developed by SINTEF Applied Mathematics in Oslo \[10\], to apply the random-walk particle tracking method. The method is applied in MRST’s polymer injection module to analyze the effect of dispersivity on the performance of polymer flooding.

2 Methodology

MRST is a tool for fast prototyping and demonstrating new reservoir modeling methods and concepts working with MATLAB. MRST involves several basic reservoir characterization modules and flow functions, and several add-on modules that can be applied for more specific cases and models \[10\]. Random walk method is applied in MRST to solve for polymer transport equation in the study. This section involves basic information on MRST and its working principal, random walk method and its equations, and the application of method to MRST.

2.1 MRST model

MRST model reads the ECLIPSE, Schlumberger’s industry-standard simulator data structures, and creates so called a simulation deck to define the model objects, which include the physical model, variables of the reservoir state, and simulation schedule of the problem at hand in the MATLAB environment. The grid system is generated using ECLIPSE’s form of input data. The governing equations (the system of partial differential equations describing multiphase flow in the porous media and the equations for imposed conditions including the well control equations and specified flux or pressure values at the boundaries of the porous media) are discretized following multidimensional Taylor’s series expansion scheme using Automatic Differentiation (AD) method. AD method linearizes the discretized equations with respect to unknown variables using the predefined analytical derivatives of the elementary arithmetic operations and the all the mathematical functions which appear in the discretized equations. The linearized system of equations is solved by Newton’s iteration method to obtain the primary unknown variables (the grid cell pressures and phase saturations, the well flow rates or pressures) \[2\].

2.2 Random walk particle tracking method

A statistical physics method can be used to find the solution to diffusion and dispersion problems that occur in porous medium, called random walk method \[5\]. The molecular diffusion of the molecules in a solution can be calculated with Equation (1) below, if the system is not injected with chemicals before \(t = 0\), and the injected chemical amount is \(N\) at origin \(x = 0\) in the system \[1\]:

\[
C(x,t) = \frac{N}{\sqrt{4\pi D_0 t}} \exp \left(\frac{-x^2}{4D_0 t}\right)
\]

(1)

\(C(x,t)\) is in mol/cm. \(D_0\) is the diffusion coefficient in cm\(^2\)/s. Chemical concentration is equal to 0 except at the origin at the beginning of injection. At the origin \(C \rightarrow \infty\). The equation of concentration distribution has similarities with Gaussian distribution function:

\[
n(x) = \frac{N}{\sqrt{2\pi}\sigma^2} \exp \left(\frac{-x^2}{2\sigma^2}\right)
\]

(2)

With the similarities between Equations (1) and (2), diffusion coefficient and variance can be related as:

\[
\sigma^2 = 2D_0 t
\]

(3)

Mean diffusion length of a chemical can be estimated as statistics states. Under Gaussian distribution, 68\% of the original mass travels the distance given in Equation (4) below \[1\]:

\[
\sigma = \sqrt{2D_0 t}
\]

(4)

Probable distance travelled by a particle with respect to diffusion in random walk method in a time step can be calculated from Equation (5) below \[11\], where, \(N(\xi)\) is a sample from a normal distribution with zero mean and unit variance:

\[
\Delta x_{spread} = N(\xi)\sqrt{2D_0 \Delta t}
\]

(5)

\[
N(\xi) = e^{-\frac{\xi^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}}
\]

(6)

Positions of the particles at each time step \(n\) is updated as:

\[
x_{i}^{n+1} = x_{i}^{n} + v_{w}^{n} \Delta t + \Delta x_{spread}
\]

(7)

where \(v_{w}^{n} \Delta t\) is the advection term for the particle and \(v_{w}\) is the water phase velocity obtained from the simulator since the chemicals are assumed to be travelling only in water phase.
2.3 Application in MRST

In random walk particle tracking method particles are initialized and moved, then the concentration profile of polymer is calculated and its effects to the system are added. MRST normally solves all continuity and polymer equations by itself and give related information at each time step.

For initialization of the particles, total number of particles to be injected is decided and the mass of each polymer particle is calculated from the simulation time and to be injected polymer mass data. All particles are initialized at the injection location at \( t = 0 \) and a determined number of particles are released to the system at each time step.

Movement of particles consists of advection and dispersion. MRST gives phase fluxes at each time step during the simulation at cell centers as averages. Water phase velocity is calculated from this flux information and for particles’ exact positions inside a grid, interpolated water phase velocity field is used. \( u \Delta t \) term in Equation (7) introduces the advection of particles. Dispersion is introduced within Equation (5) by replacing the diffusion coefficient with dispersion coefficient, \( D_C \). This value is obtained by multiplying the typical diffusion coefficient with the dispersivity of the porous medium. Dispersivity of the porous medium is a scale dependent parameter and calculated experimentally or empirically as stated before. [8] gives an empirical correlation in their study, which includes laboratory core flows and field data collected from different formation types. Equation (8) given below is used for the dispersivity in this study:

\[
\alpha_L = 0.044L^{1.13} \tag{8}
\]

The dispersion effect added calculations is the longitudinal dispersion, which occurs in the gross fluid flow direction. There is also the phenomena of transverse dispersion, however transverse dispersion and dispersivity are not studied much in the literature and the studies made show that their effects can be negligible compared to longitudinal dispersion and dispersivity [11].

Polymer effects are added by calculating the concentration profile in the system. The particles present in each grid are counted and the mobile mass is divided by water inside the grid to calculate polymer concentration. Concentration calculation equation is given below:

\[
c = \frac{m_p}{V \Phi S_w} \tag{9}
\]

where:
- \( c \) = Concentration, kg/m\(^3\)
- \( m_p \) = Mobile polymer mass in the cell, kg
- \( V \) = Volume of the cell, m\(^3\)
- \( \Phi \) = Porosity
- \( S_w \) = Water saturation

After the concentration is calculated, the amount of polymer that can be adsorbed by the rock surface is calculated at each time step. The input file includes the adsorption capacity of the rock for the porous medium as a table. The adsorption effect is added by reducing an equal mass with respect to adsorbed polymer amount from each particle in the cell. Adsorption procedure is assumed irreversible in the model. Equations used for adsorption calculations are given below:

\[
m_{ads}^n = M \rho_p V (1 - \Phi (1 - S_{dpp})) \tag{10}
\]

\[
m_{red} = \frac{m_{ads}^n - m_{ads}^{n-1}}{N} \tag{11}
\]

where:
- \( m_{ads}^n \) = Mass of polymer that can be adsorbed by the cell at the current time step, kg
- \( M \) = Mass of polymer that can be adsorbed by unit mass of rock, kg
- \( \rho_{rock} \) = Density of rock, kg/m\(^3\)
- \( S_{dpp} \) = Dead pore volume
- \( m_{red} \) = Particle mass to be reduced due to adsorption, kg
- \( N \) = Number of particles in the cell

3 Description of test case

In our study, a three-dimensional MRST dataset obtained from MRST EOR module polymer examples is modified to have a one-dimensional problem. The first and last grid cells of the system has the injection well and the production well respectively in the scenario. Complete mixing of polymer with water is assumed. The model domain dimensions are 200 m in the \( x \)-direction, 20 m in the \( y \)-direction, and 4 m in the \( z \)-direction and has no flow boundary conditions. Reservoir porosity and permeability are taken constant and are 30% and 100 mD respectively. Dead pore volume and rock density are given as 5% and 2000 kg/m\(^3\). Residual resistance factor (RRF), which is a rock mineral and polymer chemistry dependent parameter, is the ratio between water relative permeability before, and after polymer flood, is taken 1.1. RRF is used in the calculations of permeability reduction effect. Water injection rate and the concentration of polymer injected with it are continuous during the simulation time and taken 10 m\(^3\)/day and 0.8 kg/m\(^3\) respectively. Typical diffusion coefficient is taken as \( 4 \times 10^{-10} \) m\(^2\)/s [11]. Total simulation duration is 500 days and each time step is taken 0.5 days.

Polymer viscosity multiplier and polymer adsorption curves are given as a table in the input file. Their plots are given in Figure 1 and 2 respectively.

Number of blocks can be increased in order to reduce the effect of numerical dispersion in reservoir simulations. After solving the problem described above with MRST model with different number of grids, the optimum number of grids is decided to be 400 for this case since the numerical dispersion does not disappear and the computation time increases if the grid number is increased further [7].

100 particles per time step is released to the system during the simulation time. Each particle released has the same polymer mass data, then their mass is reduced at each time step according to the adsorption capacity of the rock with respect to the grid they are present. Calculated polymer concentration profile is used at the next time step for polymer effect calculations.
4 Results and discussion

The problem described is solved with MRST and random walk particle tracking method, the results are compared for concentration and saturation profile movements, and production curves. Two saturation fronts are observed in the polymer injection scenarios, the first front is caused by the increased water saturation in the upstream with injection, which is denoted with green in the first saturation profile figure, and the second front following at the connection point of polymer water and connate water, which is denoted with orange in the same figure, as expected [12].

Figure 3.a and 3.b shows the concentration and saturation profile movements at different time steps during the simulation. Location of the particles are also scattered on the concentration profile graphs. However, only one fifth of total number of particles are plotted for easy visualization. Most of the particles lose all of their masses due to adsorption. Therefore, the particles ahead of the concentration front are not plotted.

The fluctuations seen on both profiles are caused by the random nature of the method. However, injection concentration of both MRST and random walk profiles agree with each other. The concentration and saturation profiles of the MRST model moves faster with the effect of numerical dispersion than particle model. Numerical dispersion causes the MRST profile to have a curved nature and simulates polymer reaching earlier to the production well, while particle model has a steep front. When four different realizations are run and compared with the MRST production profile, all realizations give almost the same result as seen in Figure 3.
sample from a normal distribution with zero mean and unit variance, and polymer concentration is calculated from the mobile mass inside each grid, we do not see a smooth concentration profile throughout the system. Increasing the total number of particles injected to the system can reduce the oscillations but they do not disappear. Moreover, increasing the number of particles also result in a longer computational time. 20 particles in a cell is stated as the optimum number to calculate the profile properly by [5]. Using small number of particles increase the uncertainty since the masses of each particle is too much and there are not enough particles in the cells during the simulation. Figure 4.a and 4.b shows the effect of total number of particles to the system.

Figure 4.a. Polymer concentration profile with different number of particles

Figure 4.b. Polymer concentration profile with different number of particles

In an advection dominant flow, the movement of the flood front is expected to be piston like. However, if the heterogeneity of the porous medium is high, flow becomes dispersion dominant. Since dispersion phenomena is not solved in the MRST system and it only comes from numerical errors. In order to see the effects of higher dispersion coefficients, the default dispersion coefficient used in the particle model is modified and polymer concentration profiles are compared on 200th day of simulation in Figure 5:

The uncertainties of polymer concentration increase with increasing dispersion coefficient. We also see some particles moving too fast and creating a small polymer concentration ahead of the flood front. When the production curves are plotted, it is seen that with increasing dispersion coefficient, the profiles converge more to MRST profile. The fluctuations on the production curve before breakthrough also increases but the general profile is easily distinguishable.

5 Conclusions

Random walk particle tracking method is implemented in MRST model to simulate polymer injection and analyze the effect of dispersion coefficient in this study.

A three-dimensional polymer injection scenario dataset is modified to have a one-dimensional problem and both MRST and particle models are run on the same problem for comparison. Random walk particle tracking model solves the polymer advection and diffusion/dispersion equation to add the effect of polymer to the system, and removes the effect of numerical dispersion on polymer continuity equation.
Figure. 5. Effect of dispersivity coefficient
The conclusions drawn can be listed as:

1. Random walk particle tracking method allows user to see effects of dispersion in the porous medium. Increasing dispersivity of the porous medium increases the uncertainties of the results, however flow and production profiles agree with MRST model.

2. True dispersivity of a simple one-dimensional system can be decided through history matching using random walk particle tracking method.

3. The uncertainties caused by the random nature of the method can be lowered by increasing the number of particles injected, using finer grids, or using shorter time steps. However, each of these solutions does not improve the results much after some point, and computation time of the simulation increases. The oscillations can also be reduced by using smoothing methods. On the other hand, smoothing methods would add some degree of numerical errors.

**Conflict of interest**

The authors declare that there is no conflict of interest.

**Similarity rate (Turnitin):** 14%

**References**

[1] J. Sheng. Modern Chemical Enhanced Oil Recovery: Theory and Practice. Gulf Professional Pub, 2011.

[2] K. Bao et al., Fully implicit simulation of polymer flooding with MRST. 15th Eur. Conf. Math. Oil Recover. ECMOR 2016, 42 (3), 545–63, 2016.

[3] W. B. Gogarty, Mobility control with polymer solutions. SPE J., 7 (02), 161–73, 1967. https://doi.org/10.2118/1566-B.

[4] T. Ertekin, J. H. Abou-Kassem, and G. R. King, Basic Applied Reservoir Simulation. SPE, Richardson, Texas, 2001.

[5] W. Kinzelbach, Simulation of pollutant transport in groundwater with the random walk method. Dresden Symposium, pp. 265–79, 1990.

[6] L. W. Lake, J. R. Johnston, and G. L. Stegemeier, Simulation and performance prediction of a large-scale surfactant/polymer project. Soc. Pet. Eng. J., 21 (06), 731–9, 1981. https://doi.org/10.2118/7471-PA.

[7] G. Mamak, Random walk particle modelling of polymer injection using matlab reservoir simulation toolbox. M.Sc Thesis, Middle East Technical University, Turkey, 2017.

[8] A. Arya, T. A. Hewett, R. G. Larson, and L. W. Lake, Dispersion and reservoir heterogeneity. SPE Reserv. Eng., 3 (01), 139–48, 1988.

[9] B. C. Zheng and P. P. Wang, MT3DMS: A Modular Three - Dimensional Multispecies Transport Model for Simulation of Advection, Dispersion, and Chemical Reactions of Contaminants in Groundwater Systems; Documentation and User’s Guide. Tuscaloosa, Alabama, 1999.

[10] Sintef, MRST - MATLAB Reservoir Simulation Toolbox, Accessed 13 July 2017. http://www.sintef.no/projectweb/mrst/

[11] S. R. Cranmer, Monte Carlo Solutions to Diffusion-Like Equations: A Practical Application of the Ito Calculus. Harvard-Smithsonian Cent. Astrophys. Cambridge, 2003.

[12] G. A. Pope, The application of fractional flow theory to enhanced oil recovery. Soc. Pet. Eng. J., 20 (03), 191–205, 1980. https://doi.org/10.2118/7660-PA.