Research Concerning the Dynamic Model of the Conventional Sucker Rod Pumping Units

GEORGETA TOMA*
Petroleum-Gas University of Ploiesti, 39 Bucuresti Blvd., 100680, Ploiesti, Romania

The study of the dynamic model of the conventional sucker rod pumping units requires first determining the variation on the cinematic cycle of the synthesis parameters (the reduced moment and the reduced mass moment of inertia) and then the variation of the angular speed of the cranks, in response to the dynamic and resistant actions on the component elements that appear during operation. The paper presents the way of determining the variation on the cinematic cycle of the synthesis parameters of the dynamic model corresponding to the conventional pumping unit mechanism and of the variation of the angular speed of its cranks. The experimental records have been processed with the Total Well Management program. The simulations have been performed with a computer program developed by the author using the Maple programming environment.

Keywords: conventional sucker rod pumping unit, dynamic model, synthesis parameters

Modeling the dynamics of the conventional sucker rod pumping installations represents an issue of study of prime importance for their optimal design [1-9]. In this scope the study of the dynamic model, namely determining the variation on the cinematic cycle of the synthesis parameters (the reduced moment and the reduced mass moment of inertia) and the variation of the angular speed of the cranks may lead to a better understanding of the influence of various functional or constructive parameters on the behavior of the sucker rod pumping installations in service [10-15].

In this paper is analyzed the dynamic model of the conventional sucker rod pumping installations. It is presented the way of determining the variation on the cinematic cycle of the synthesis parameters and of the angular speed of the cranks. Some significant results concerning the cinematic and dynamic analysis of the plane mechanisms that have strongly helped to the achievement of the research from this paper are presented in [16-23]. The simulations have been performed with a computer program developed by the author using the Maple programming environment [24] and the experimental records have been processed with the Total Well Management program [25].

Experimental part

It was used the program Total Well Management [25] for processing the experimental records in the case of a well serviced by a C-640D-305-120 pumping unit manufactured by Lufkin (fig. 1).

For establishing the variation of the synthesis parameters and of the angular speed of the cranks were used the variation of the force at the polished rod and the variation of the motor moment at the crankshaft during the stroke 65 represented in figure 2 and figure 3, respectively.

Establishing the variation on the cinematic cycle of the synthesis parameters and of the angular speed of the cranks

In figure 4 is represented a conventional pumping unit mechanism. \( C_1, C_2 \) and \( C_3 \) are the mass centers of the cranks, connecting rods and of the rocker, respectively; \( m_{CG} \) is the total mass of the balancing counterweights; \( m_{L1} \) is the total mass of the connecting bearings between

\*
* email: georgeta_tm@yahoo.com

Fig. 1. Data concerning the analyzed well

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the cranks and the connecting rods; \( m_1 \) is the mass of the spherical connecting bearing between the connecting rods and the rocker; \( m_2 \) is the mass of the equalizer traverse; \( m_3 \) is the mass of the rocker head considered to be concentrated in point \( D' \); \( M_m \) is the motor moment at the crankshaft; \( F \) is the force acting at the end of the polished rod.

The synthesis parameters of the dynamic model of a plane mechanism, namely the reduced mass moment of inertia \( J_{\text{red}} \) and the reduced moment \( M_{\text{red}} \) can be calculated from the following relations [22]:

\[
\frac{1}{2} J_{\text{red}} \cdot \omega_1^2 = \sum \left( \frac{1}{2} m_j \cdot \dot{v}_j^2 + \frac{1}{2} J_j \cdot \omega_j^2 \right)
\]  
\[
M_{\text{red}} \cdot \omega_1 = \sum (F_j \cdot \dot{v}_j + M_j \cdot \omega_j)
\]

where: \( m_j \) and \( J_j \) are the mass and the mass moment of inertia, respectively, of the component element \( j \) (\( J_j \) is calculated relative to its center of mass); \( \dot{v}_j \) and \( \omega_j \) are the speed of the mass center of the element \( j \) and its angular speed, respectively; \( F_j \) and \( M_j \) are the resultant force and the resultant moment, respectively, acting on the element \( j \) (\( M_j \) is calculated relative to its center of mass). It can be shown that the variation of the reduced mass moment of inertia \( J_{\text{red}} \) do not depend on the variation on the cinematic cycle of the angular speed \( \omega_1 \) [22], so this variation is the same regardless of the considered cinematic cycle.

In the case of the conventional sucker rod pumping units mechanism, by applying the relations (1) and (2), the reduced mass moment of inertia \( J_{\text{red}} \) and the reduced moment \( M_{\text{red}} \) may be calculated from the following relations (eq. (3,4)):
where: $m_1$, $m_2$, and $m_3$ are the masses of the cranks, of the connecting rods and of the rocker, respectively, and $J_{C1}$, $J_{C2}$ and $J_r$ are their mass moments of inertia.

The manner of establishing the variation on the cinematic cycle of the speed of different points situated on the pumping units mechanism is presented in [9]. In [9] is also presented the manner of calculating of the crank angles $\varphi_{1d}$ and $\varphi_{1a}$, corresponding to the beginning of the upward and downward movements of the sucker rod column.

The variation on the cinematic cycle of the angular speed of the cranks $\omega_1$ can be obtained by integrating the movement equation of the pumping unit mechanism with the following relation [22]:

$$\omega_1(\varphi_1) = \sqrt{\frac{J_{red,0} \omega_{1,0}^2}{J_{red}} + 2 \int_{\varphi_{1,0}}^{\varphi_1} \frac{M_{red}(\varphi_1)}{J_{red}} d\varphi_1}$$

where: $\omega_{1,0}$ and $J_{red,0}$ are the values of $\omega_{1,0}$ and $J_{red,0}$ respectively, at the beginning of the cinematic cycle corresponding to the cranks angle $\varphi_{1,0}$.

Results and discussions

The simulations have been performed in the case of a C-640D-305-120 pumping unit produced by Lufkin [26] with a computer program developed by the author using Maple programming environment [24]. The component elements of the pumping unit mechanism (fig. 4) have the following dimensions: $OA = 30$ in. (0.762 m); $AB = 133.5$ in. (3.3909 m); $BC = 111.09$ in. (2.8217 m); $CD = 155$ in. (3.937 m). The coordinates of the point C (fig. 4) are: $x_C = 2.8194$ m and $y_C = 3.5052$ m. The values of the crank angle $\varphi_1$ corresponding to the beginning of the upward and downward movements of the sucker rod column are: $\varphi_{1d} = 88.976^\circ$ and $\varphi_{1a} = 266.929^\circ$, respectively.

The other parameters involved in calculations have the following values: $OA' = 46$ in. (1.1684 m); $OA'' = 95$ in. (2.413 m); $CD' = 140$ in. (3.556 m); $m_1 = 88$ kg; $m_2 = 169$ kg; $m_r = 580$ kg; $m_{cr} = 840$ kg; $m_{cg} = 5200$ kg; $q_1 = 722$ kg/m; $q_2 = 34$ kg/m; $q_3 = 300$ kg/m ($q_1$, $q_2$, and $q_3$ are the linear masses of the cranks, connecting rods and of the rocker, respectively). The nominal angular speed of the cranks is of 6.667 rot/min.

In figure 5 is represented the variation of the reduced moment $M_{red}$ during the stroke 65 and in figure 6 is presented the variation of the reduced mass moment of inertia $J_{red}$ during a cinematic cycle. In both cases the variation is considered beginning with the angle $\varphi_{1d}$.

A way to validate the variation on a cinematic cycle of the reduced moment $M_{red}$ is to verify if $\int_{\varphi_{1d}}^{\varphi_{1a}} M_{red}(\varphi_1) d\varphi_1 = 0$ [22]. In figure 7 is represented the variation during the stroke 65 of the kinetic energy [22]:

$$\Delta E_k(\varphi_1) = \int_{\varphi_{1d}}^{\varphi_{1a}} \frac{1}{2} M_{red}(\varphi_1) d\varphi_1$$

where $\varphi_1 = [\varphi_{1d}, \varphi_{1d} + 2\pi]$. It can be observed that $\Delta E_k(\varphi_1 + 2\pi) = \int_0^{2\pi} M_{red}(\varphi_1) d\varphi_1$ is not exactly equal to zero, but is very close to zero. This little difference occurs especially due to the errors in the recording of the values of the force at the polished rod and of the motor moment at the crankshaft.

In figure 8 is represented the variation during the stroke 65 of the angular speed of the cranks expressed in rot/min. In applying the relation (5) has been considered that $\omega_{1,0}$ is equal to the nominal angular speed of the cranks.
Conclusions
In this paper has been analyzed the dynamic model of the conventional sucker rod pumping units. It has been presented the way of determining the variation on the cinematic cycle of the synthesis parameters of the dynamic model and of the angular speed of the cranks. Some results concerning a way to validate the variation on a cinematic cycle of the reduced moment have been also presented.

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