Understanding destruction of \( n \)-th order quantum coherence in terms of multi-path interference

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The classic example of the destruction of interference fringes in a “which-way” experiment, caused by an environmental interaction, may be viewed as the destruction of first-order coherence as defined by Glauber many years ago [1]. However, the influence of an environment can also destroy the \( n \)-th order quantum coherence in a quantum system, where this high order coherence is captured. We refer to this phenomenon as the \( n \)-th order decoherence. In this paper we show that, just as the first-order coherence can be understood as the interference of the amplitudes for two distinct paths, the higher order coherence may be understood as the interference of multiple amplitudes corresponding to multiple paths. To see this, we introduce the concept of \( n \)-th order “multi-particle wave amplitude”. It turns out that the \( n \)-th order correlation function can be expressed as the square norm of some “multi-particle wave amplitude” for the closed system or as the sum of such square norms for the open system. We also examine, as a specific example, how an environment can destroy the second order coherence by eliminating the interference between various multiple paths.

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I. INTRODUCTION

A most profound concept in quantum physics is quantum coherence. The first order version of quantum coherence can be directly manifested by the superposition of two quantum states. This is quite similar to the optical coherence in Young’s double experiment. From the standpoint of the photon-detection theory by Glauber, this simple coherence can be mathematically depicted by the first order correlation function [1]. However, in quantum mechanics, this first order coherence phenomenon does not sound very marvelous since the same circumstance can also occur in a classical case, such as the optical interference in the above mentioned Young’s double experiment. In fact, by the first order correlation function only, it is impossible to distinguish the natures of a laser light field and a conventional light field with identical spectral properties. As an effective remedy, Glauber’s \( n \)-th order quantum correlation function, which accounts for various intrinsic ( \( n \)-th order) quantum coherence effects, such as the intensity-intensity correlation measurement in the Hanbury-Brown-Twiss experiment [2], was introduced [1]. Indeed, this function reflects the intrinsically quantum features of coherence beyond the classical analogue.

The quantum coherence embodies the wave nature in the world of microscopic particles. On the other hand, it is very fragile and can easily be destroyed by a “which-path(way)” experiment [2, 3], or by an environmental interaction. This phenomenon of destruction of coherence is usually referred to as quantum decoherence [2, 3].

The phenomenon of losing the coherence described by the first order correlation function is defined as the first order quantum decoherence. Motivated by the considerations in fundamental quantum measurement problem [2, 3], and also by the attempts to preserve quantum coherence of qubits in quantum computing [2, 4, 5], many recent experimental and theoretical investigations have been focused on revealing the physical mechanism of the decoherence problem, e.g., see [6]. According to these studies, this first order decoherence can be roughly understood through the quantum entanglement of the considered system with the environment or the measuring apparatus.

Obviously, this entanglement implies a “which-path(way)” detection [3] in the single particle picture. Precisely speaking, in an initial coherent superposition \( |\psi\rangle = \sum c_n |n\rangle \), each system state \( |n\rangle \) corresponds to a “path” and many “two path” interferences are reflected in the square norm of the spatial wave function \( \langle x|\psi_s\rangle \). Thus by considering \( \langle x|\psi_s\rangle \) the quantum coherence can be captured to some extent. After the interaction, each “path” is correlated with an environment state \( |e_n\rangle \) to form an entangling state \( |\psi_T\rangle = \sum c_n |n\rangle \otimes |e_n\rangle \). Here, the different states \( |e_n\rangle \) distinguish among the “paths” of different \( |n\rangle \) and thus record the information of each “path”. The interference terms in the spatial intensity \( I(x) = \text{Tr}(\langle x|\psi_T\rangle \langle \psi_T|x\rangle) \) will disappear when the environment states \( |e_n\rangle \) are completely distinct, i.e., \( \langle e_m|e_n\rangle = \delta_{mn} \). In that case each path is labeled by an environment state.

The above well-known explanation of the first order decoherence in terms of “which-path(way)” detection mechanism is simple but very profound. However, it is not yet clear whether this mechanism can be used to elucidate the \( n \)-th order quantum decoherence (n-QDC), the destruction of quantum coherence described by Glauber’s
nth order correlation function. The difficulty is we do not exactly know what are the “paths” and the corresponding “which-path” detection. Most recently, we have touched the second order quantum decoherence problem by-passing this difficulty. As a matter of fact, in our treatment, we did not define the concept of “path” directly. The concrete calculation in the ref. [13] motivated us to consider the “which-path” picture of the higher order quantum decoherence in general.

In our present investigation, it is crucial to show, we notice that, for a close system, in some cases Glauber’s nth-order correlation function is the square norm of the nth-order “multi-particle wave amplitude”, which will be defined later in this paper, while for an open system, it can become a sum of the square norms of the nth-order “multi-particle wave amplitude” over the states of an environment or an apparatus interacting with this open system. This observation is crucial in our present investigation. As an effective wave function, this multi-particle amplitude can be shown to be a supposition of many generalized “paths” (the multi-particle paths or simply multiple paths). Thus, the higher order coherence may be understood as the interference of multiple particle amplitudes. With this conception a generalized “which-path” detection may be established in terms of “multi-particle paths” as the physical mechanism of higher-order decoherence for some examples.

In section 2, we will briefly explain Glauber’s nth order quantum coherence in terms of the single and multi photon effective wave functions used in [14, 15]. As their generalizations, in section 3 the concepts of multi-particle path and multi-particle wave amplitude are introduced for both close and open bosonic systems. Especially, the nth order correlation functions of bosonic systems will be studied. In section 4, an interacavity model with two bosonic modes is used to demonstrate 2-QDC as a “which-path” detecting process. In section 5, the exact solution obtained in the appendix is utilized to show the dynamical process of 2-QDC, which is caused by the entanglement with the environment or an apparatus and indeed can be explained as a generalized “which-path” measurement for the explicitly -defined multi-particle paths.

II. THE nTH-ORDER COHERENCE FOR QUANTIZED LIGHT FIELD AND MULTI-PHOTON WAVE AMPLITUDE

In quantum mechanics, a pure state is a superposition $|\psi\rangle = \sum_k c_k|k\rangle$ of many components $|k\rangle$, but the corresponding mixture $\rho = \sum_k |c_k|^2|k\rangle\langle k|$ can describe the same classical probability distribution $|c_k|^2$. However, $|\psi\rangle$ and $\rho$ represents different quantum realities. Usually it is said that the two components of a pure state is more coherent than those of a mixed one. This coherence property is obviously reflected by the intensity interference of two “paths” corresponding to the two components in the quantum state. For the system of one particle, only single particle property is relevant for this observation. In this sense, only the intensity interference experiment is essential for one particle system. For many particle system, however, there exist many experiments (such as the Hanbury-Brown-Twiss experiment and the intensity-intensity correlation measurement) to show the much richer nature of quantum coherence.

In order to study quantum coherence in many particle system, Glauber introduced the so-called $n$-th order quantum correlation function ($n$-QCF)

$$G^{(n)}[\alpha_1, t_1; \alpha_2, t_2; \cdots; \alpha_n, t_n] = Tr[\hat{\rho} E_{\alpha_1}^†(t_1) E_{\alpha_2}^†(t_2) \cdots E_{\alpha_n}^†(t_n) E_{\alpha_1}(t_1) \cdots E_{\alpha_2}(t_2) E_{\alpha_1}(t_1)],$$

for the electro-magnetic field $E_\alpha(t)$ in different mode $\alpha$. Here, $E_\alpha(t)$ is the annihilation operator of mode $\alpha$ at time $t$ in the Heisenberg picture, $E_\alpha^†(t)$ is the corresponding conjugate operator, and the density matrix $\hat{\rho}$ represents the initial state of many-mode electro-magnetic field. Of course, this formalism can also be used to study the coherence property for any quantum many-body system.

Furthermore, to describe the higher order coherence, Glauber also defined the nth-order coherence function in the form [16]

$$g_{\alpha\beta}^n = g[\alpha_1, \alpha_2, \cdots, \alpha_n; \beta_1, \beta_2, \cdots, \beta_n] = \frac{Tr[\hat{\rho} E_{\beta_1}^†(t_1) E_{\beta_2}^†(t_2) \cdots E_{\beta_n}^†(t_n) E_{\alpha_1}(t_1) \cdots E_{\alpha_2}(t_2) E_{\alpha_1}(t_1)]}{\sqrt{G[\alpha_1, t_1; \alpha_2, t_2; \cdots; \alpha_n, t_n]} \sqrt{G[\beta_1, t_1; \beta_2, t_2; \cdots; \beta_n, t_n]}}.$$
density matrix $g = (g_{\alpha\beta} : \alpha = (\alpha_1, \alpha_2, \cdots, \alpha_k), \beta = (\beta_1, \beta_2, \cdots, \beta_n))$ to its diagonal ones. It is easily observed from this definition that these off-diagonal elements represent the coherence effect, and each one $g_{\alpha\beta}$ correlates two diagonal ones $g_{\alpha\alpha}$ and $g_{\beta\beta}$. Thus, we can understand the coherence function as measuring the degree of coherence for the two diagonal elements, which correspond to two different “paths”. From this point of view, there is obviously no quantum coherence for a completely mixed state with vanishing off-diagonal elements. In the following discussion, we will give simple examples to illustrate this viewpoint.

Now let us turn to an instance [14] from which we can see clearly how the higher order quantum coherence effects given by 2-QCF is revealed in the multi-particle picture. For a single photon, the coherent superposition $|s\rangle = \frac{1}{\sqrt{2}}(|1_k\rangle + |1_{-k}\rangle)$ of two states with opposite wave vectors $k$ and $k'$ possesses the first order quantum coherence which can be described by the interference fringes

$$G^{(1)}(r, r, t) = \langle s | E^{-}(r, t) E^{+}(r, t) | s \rangle = |\langle 0 | E^{+}(r, t) | s \rangle|^2 \propto \cos^2(kr),$$

where

$$E^{+}(r, t) = \sum_{k} E_k a_k \exp(ikr - i\omega_k t)$$

is the photon field operator with positive frequency for the annihilation operator $a_k$. It should be noticed that the diagonal element $G^{(1)}(r, r, t)$ of the first order correlation function is just the square norm of the single photon wave-packet [13]

$$\langle 0 | E^{+}(r, t) | s \rangle = \frac{1}{\sqrt{2}} \left( \langle 0 | E^{+}(r, t) | 1_k \rangle + \langle 0 | E^{+}(r, t) | 1_{-k} \rangle \right)$$

$$= \frac{1}{\sqrt{2}} \left( E_k e^{ikr - i\omega_k t} + E_{-k} e^{-ikr - i\omega_{-k} t} \right).$$

Therefore, $G^{(1)}(r, r, t)$ represents the interference between the “two paths” $\langle 0 | E^{+}(r, t) | 1_k \rangle$ and $\langle 0 | E^{+}(r, t) | 1_{-k} \rangle$. This just develops the corresponding concept in Young’s double experiment for quantized light field. The above reformulation of the first order quantum coherence implies that “two paths” are necessary for the interference phenomenon.

If we consider the two particle state $|1_k, 1_{-k}\rangle$ with a single component, an interesting situation arises where the first order quantum coherence does not appear, but we can see the second order effect through the second order quantum correlation function

$$G^{(2)}(r_1, r_2, t_1, t_2) = 2E_k^2 \left\{ 1 + \cos[(k-k')(r_1-r_2)] \right\}.$$}

Unlike the case of first order coherence, in this case there do not appear the obvious two or many “paths”. Nevertheless, the interference phenomenon can still be captured in a similar way with the introduction of generalized “path”. We understand the generalized “path” as described by The “two time” correlation function

$$G^{(2)}(r_1, r_2, t_1, t_2) = |\psi|^2$$

where $\psi$ is the “two-photon wave function”

$$\psi \equiv \psi(r_1, r_2, t_1, t_2) = \langle 0 | E^+(r_2, t_2) E^+(r_1, t_1) | 1_k, 1_{-k} \rangle$$

$$= E_k^2 e^{-2i\omega_k t} [e^{ikr_1 + ik'r_2} + e^{-ikr_1 + ik'r_2}].$$

$\psi$ was invoked as a two photon effective wave-function, and it was also called the biphoton wave packet for the photon field $E^+(r, t)$ [14]. Especially, we remark that the biphoton wave packet $\psi$ is a coherent superposition of two “probability amplitudes” corresponding to two “two-photon paths”

$$\langle 0 | E^+(r_2, t_2) | 0_k, 1_{-k} \rangle \langle 0_k, 1_{-k} | E^+(r_1, t_1) | 1_k, 1_{-k} \rangle$$

and

$$\langle 0 | E^+(r_2, t_2) | 1_k, 0_{-k} \rangle \langle 1_k, 0_{-k} | E^+(r_1, t_1) | 1_k, 1_{-k} \rangle$$

Starting from Glauber’s standpoint and proceeding along, we come to the conclusion that a set $\{G^{(n)}|\alpha_1, \alpha_2, \cdots, \alpha_n, t_n|n = 1, 2, \cdots\}$ of correlation functions, rather than a single one, is indispensable to describe comprehensively the wave-particle dual nature in the quantum world of many particle system. The above example of second order coherence shows that the conception of two-path interference in single photon picture still works with a proper generalization of the concept of “path”. Hence it is quite natural to seek a generalized “which-path(way)” measurement as the mechanism of higher-order quantum decoherence.

III. MULTI-PARTICLE AMPLITUDE FOR FREE BOSONS

In this section the conception of two photon effective wave-function will be generalized. It will be applied to the study of general quantum systems of identical particles. We first discuss the spatially-homogeneous case for the sake of simplicity.

We consider a homogeneous bosonic field with two modes $|V\rangle$ and $|H\rangle$. The generalized field operator in time-domain

$$\hat{\phi}(t) = c_V \hat{b}_V e^{-i\omega_V t} + c_H \hat{b}_H e^{-i\omega_H t}$$

$$\equiv c_V (t) \hat{b}_V + c_H (t) \hat{b}_H$$

is an annihilation operator with respect to the superposition state

$$|+\rangle = c_V^* |V\rangle + c_H^* |H\rangle.$$}

Here $b_H$ and $b_V$ are the annihilation operators of the boson system; $c_V$ and $c_H$ satisfy the normalization relation $|c_V|^2 + |c_H|^2 = 1$. Without loss of generality, we take $c_V = c_H = 1/\sqrt{2}$. This means we consider the measurement to detect the polarized boson along the 45° direction in the $V - H$ plane. We call $\phi$ a “measuring” operator.
Corresponding to $\hat{\phi}$, the generalized second order correlation function \[ G^{(2)} = \langle 1_V 1_H | \hat{\phi}^\dagger(t_1) \hat{\phi}^\dagger(t_2) \hat{\phi}(t_2) \hat{\phi}(t_1) | 1_V, 1_H \rangle \]

\[ = |\langle 0, 0 | \hat{\phi}(t_2) \hat{\phi}(t_1) | 1_V, 1_H \rangle|^2 \equiv |\Psi(t_1, t_2)|^2. \] (9)

The two time wave function

$$\Psi(t_1, t_2) = \langle 0, 0 | \hat{\phi}(t_2) \hat{\phi}(t_1) | 1_V, 1_H \rangle$$

can be understood in terms of the two “paths” picture from the initial state $|1_V, 1_H\rangle$ to the final state $|0, 0\rangle$:

$$|1_V, 1_H\rangle \xrightarrow{c_H(t_1)} |1_V, 0_H\rangle \xrightarrow{c_V(t_2)} |0, 0\rangle \xleftarrow{c_H(t_2)} |0_V, 1_H\rangle$$

The two “paths” are just associated with the two amplitudes forming a coherent superposition

$$\Psi(t_1, t_2) = c_{CV} H \exp(-i\omega t_2 - i\omega H) + c_H c_V \exp(-i\omega t_2 - i\omega v, t_1)$$ (10)

Correspondingly, the second order correlation function

$$G^{(2)} = 2 |c_{CV} c_H|^2 \{1 + \cos[(\omega V - \omega H)(t_2 - t_1)]\}$$ (11)

The above discussion for the second order quantum coherence is applicable to the higher order case. Our arguments in this paper are based on two novel observations: a. The generalized field operator $\hat{\phi} = \sum c_n \hat{b}_n$ is specified for a quantum measurement about a superposition single particle state $|\phi\rangle = \sum c_n |n\rangle$. b. For a certain initial single component state $|s_0\rangle$ of $N$ particles system, the $n$-th order quantum correlation function

$$G^{(n)}(r_1, r_2, \ldots, r_n, t_1, t_2, \ldots, t_n) = |\psi^{(n)}|^2$$ (12)

can be written as the norm square of an effective wave function $\psi^{(n)}$, which is just a superposition of many amplitudes.

Let us consider the third order situation as an example. Let the initial state be $|1_H, 2_V\rangle$. Then the generalized third order correlation function

$$G^{(3)} = \langle 2_V 1_H | \hat{\phi}^\dagger(t_1) \hat{\phi}^\dagger(t_2) \hat{\phi}^\dagger(t_3) \hat{\phi}(t_3) \hat{\phi}(t_2) \hat{\phi}(t_1) | 2_V, 1_H \rangle$$

$$= |\langle 0, 0 | \hat{\phi}(t_3) \hat{\phi}(t_2) \hat{\phi}(t_1) | 2_V, 1_H \rangle|^2$$

$$\equiv |\Psi(t_1, t_2, t_3)|^2$$ (13)

is a norm square of the two time wave function:

$$\Psi(t_1, t_2, t_3) = \langle 0, 0 | \hat{\phi}(t_3) \hat{\phi}(t_2) \hat{\phi}(t_1) | 2_V, 1_H \rangle$$

$$= \sqrt{2} c_{CV} c_H e^{-i\omega V(t_3 + t_2) - i\omega H t_1} + \sqrt{2} c_{CV} c_H e^{-i\omega V(t_2 - t_3) - i\omega H t_3} + \sqrt{2} c_{CV} c_H e^{-i\omega V(t_3 - t_2) - i\omega H t_2}$$ (14)

Each term in the above effective wave function is contributed by the corresponding one of the four “paths” from $|2_V, 1_H\rangle$ to $|0, 0\rangle$:

$$|2_V, 0_H\rangle \xrightarrow{c_V(t_3)} |1_V, 0_H\rangle \xrightarrow{c_H(t_2)} |0, 0\rangle \xleftarrow{c_H(t_2)} |0_V, 1_H\rangle$$

$$|2_V, 1_H\rangle \xrightarrow{c_V(t_3)} |1_V, 1_H\rangle \xrightarrow{c_H(t_2)} |0_V, 1_H\rangle$$

In terms of the effective 3-time-wave function defined above, the third order correlation function is explicitly written down:

$$G^{(3)} = 4 |c_{CV} c_H|^2 \left\{ \frac{3}{2} \cos[(\omega V - \omega H) t_2 - t_1] + \cos[(\omega V - \omega H) (t_3 - t_1)] + \cos[(\omega V - \omega H) (t_2 - t_3)] \right\}$$

It shows the quantum interference in the time-domain.

The above analysis is valid only for the case where the considered system is isolated from an environment and not measured by a detecting apparatus-a detector. For our purpose, we need to consider an open system $S$ interacting with an environment (reservoir) or a detector $E$, and we must extend the concepts of multi-particle(time)-wave functions and the corresponding many-particle paths defined above. To do the generalization, we first invoke the effective field operator

$$\hat{B}(t) = U(t, 0) \hat{\phi}(0) U(t, 0)$$

$$= \exp(i\hat{\gamma} t) \hat{\phi}(0) \exp(-i\hat{\gamma} t)$$ (15)

where $\hat{\phi}(0) = c_V \hat{b}_V + c_H \hat{b}_H$ has been given by Eq.(7). Then instead of the free time evolution governed by the free Hamiltonian $\hat{H}_0$, we use the evolution operator $U(t, 0)$ governed by the total Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{W} + \hat{H}_E \equiv \hat{H}_0 + \hat{V}$$

taking into account the role of the interaction between $E$ and $S$. Here, $\hat{H}_E$ is the free Hamiltonian for $E$. If we only consider an ideal quantum decoherence process without dissipation, $\hat{V}$ should possess the nature of quantum non-demolition: $[\hat{H}_0, \hat{W}] = 0$.
Let the states \( |n \rangle \equiv |n_V, n_H \rangle \) be the common eigenstates of \( H_0 \) and \( W \) corresponding to the eigenvalues \( E_n \) and \( V(n) = (n_V, n_H) \). If the initial state of the total system is

\[
|\psi(0) \rangle = |\phi_s \rangle \otimes |\phi_E \rangle
\]

where \( |\phi_s \rangle \) and \( |\phi_E \rangle \) are some specially-given initial states of \( S \) and \( E \) respectively, we can define the effective two-time state vector

\[
|\psi_B(t, t') \rangle = \hat{B}(t') \hat{B}(t) |\psi(0) \rangle
\]

(17)
as a reasonable generalization of the effective “two-time wave function” given above. Its norm is just the second order correlation function

\[
\langle \psi_B(t, t') | \psi_B(t, t') \rangle = Tr(\hat{\rho}(0) \hat{B}^\dagger(t) \hat{B}^\dagger(t') \hat{B}(t') \hat{B}(t)) = G^{(2)}[t, t', \hat{\rho}(0)],
\]

(18)
for the density matrix \( \hat{\rho}(0) = |\psi(0) \rangle \langle \psi(0) | \).

In fact, due to the non-demolition interaction not resulting in dissipation, the basic dynamic properties of the open system do not change even in the presence of \( E \). If we choose \( |\phi_s \rangle = |1_H, 1_V \rangle \), there are only two “paths” from the initial state \( |1_V, 1_H \rangle \) to the final state \( |0, 0 \rangle \) for \( S \), and \( |0, 0 \rangle \) is the unique state which can be reached by the action of \( \hat{B}(t') \hat{B}(t) \). Then,

\[
\langle \psi_B(t, t') | \psi_B(t, t') \rangle = \sum_{n, \beta} \langle \psi_B(t, t') | n, \beta \rangle \langle n, \beta | \psi_B(t, t') \rangle = \sum_{\beta} |\langle 0, \beta | \psi_B(t, t') \rangle|^2
\]

and

\[
= \sum_{\beta} |\langle 0, \beta | \hat{B}(t') \hat{B}(t) | 1_H, 1_V, \phi_E \rangle|^2 = \sum_{\beta} |\Psi_\beta(t_1, t_2)|^2
\]

(19)

Here, the summation ranges over the complete set of states \( |\beta \rangle \) of \( E \), and each term in the sum is a norm square of the effective two particle wave function

\[
\Psi_\beta(t_1, t_2) = \langle 0, \beta | \hat{B}(t') \hat{B}(t) | 1_H, 1_V, \phi_E \rangle
\]

(20)
for the open system.

From the above calculations for the second and third order quantum decoherence, we observe that for a specially-given initial state, a higher order correlation function may be explicitly written down as the norm square (or its sum) of the multi-time wave function, which is a coherent superposition of several complex components associated with the generalized many-particle paths. It is pointed out that this kind of many-particle path is not a simple product of single-particle paths, but it can be determined by the specially designed measurement.

IV. GENERALIZED WHICH-PATH DETECTION IN AN INTRACA VITY MODEL

In this section, an intracavity model is presented to demonstrate in multi-particle picture the “which-path” detection associated with higher-order quantum decoherence.

In a recent paper [13], we have studied the problem of 2-QDC for a cavity-QED system. The concrete calculation in the ref. [13] shows that the 2-QDC effects can indeed be observed in the proposed experiment. But it involves dissipation effect losing energy. However, it is well known that quantum decoherence can still occur for an energy conserving system. So in principle dissipation is not indispensable for the discussion about decoherence effect. For this reason it is natural and interesting to consider pure decoherence process without dissipation. The pure decoherence can be well understood through the quantum entanglement of the considered system with the environment or the measuring apparatus. For a model with pure decoherence process, many concepts (such as multi-particle path and the corresponding which-path detection) can be made much clearer. Unlike the approximately-solvable model treated in the ref. [13], which loses its energy and coherence simultaneously, the model proposed in this section, a bosonic system of two modes interacting with an external system of many harmonic oscillators, does not dissipate its energy. This property makes the model exactly solvable, and as a result the problem of higher-order quantum decoherence can be studied in a straightforward way.

By taking \( \hbar = 1 \), the model Hamiltonian \( \hat{H} = \hat{H}_0 + \hat{V} \) is defined by

\[
\hat{H}_0 = \omega_V \hat{b}_V^\dagger \hat{b}_V, \quad \hat{V} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{ij} [d_V(\omega_j) \hat{b}_V^\dagger \hat{b}_V + d_H(\omega_j) \hat{b}_H^\dagger \hat{b}_H + \hat{a}_j^\dagger (\hat{a}_j)],
\]

(21)

(22)
where \( \hat{H}_0 \) is the free Hamiltonian of the system, \( \hat{V} \) the free Hamiltonian \( \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j \) of the reservoir (or a detector) plus a non-demolition interaction between the system and the reservoir; and \( \hat{b}_V^\dagger (\hat{b}_V), \hat{b}_H^\dagger (\hat{b}_H) \) the creation (annihilation) operators for the two modes with frequencies \( \omega_V \neq 0 \) and \( \omega_H = 0 \). The operators \( \hat{a}_j^\dagger (\hat{a}_j) \) are
creation (annihilation) operators of the reservoir modes of
the frequencies $\omega_j$. The frequency-dependent constant $d_H(\omega_j)\langle \hat{d}_V(\omega_j) \rangle$ measures the coupling constant between $H(V)$ mode and the $j$ mode of the reservoir.

The most important feature of the model is the non-demolition condition $[H_0, \hat{V}] = 0$. It means that the system does not dissipate energy to the reservoir. On the other hand, the system can leave imprint on the reservoir since, for different number states $|n_V, n_H\rangle$, there are different interactions

$$\sum_j [n_V d_V(\omega_j) + n_H d_H(\omega_j)](\hat{a}_j^\dagger + \hat{a}_j)$$

acting on the oscillator reservoir with different driving forces $\sim n_V d_V(\omega_j) + n_H d_H(\omega_j)$. When there is only one mode in the external system (reservoir) the whole system can physically be described by an intracavity model: Two mode cavity field interact with a moving wall of the cavity, which is attached to a spring and can be regarded as a harmonic oscillator with a small mass [4,8]. The fields are coupled to the cavity wall (a moving mirror) by the radiation pressure forces in proportion to the photon numbers $\hat{b}_H^\dagger \hat{b}_H$ and $\hat{b}_V^\dagger \hat{b}_V$.

For the above introduced model, we can discuss the higher order decoherence problem in the Heisenberg picture by explicitly defining the many-particle “which-path” measurement. The second order coherence is directly determined by the second order correlation function.

$$G[t, t', \hat{\rho}(0)] = Tr(\hat{\rho}(0)\hat{B}(t)\hat{B}(t')\hat{B}(t')\hat{B}(t))$$

which is defined as a functional of the density operator $\hat{\rho}(0)$ of the whole system at a given time 0. Here, the bosonic field (measuring) operator

$$\hat{B}(t) = \exp(i\hat{V}t)(c_H \hat{b}_H + c_V \hat{b}_V \exp(-i\omega_V t)) \exp(-i\hat{V}t)$$

is defined for the interacting system. Like the operator defined for the non-interacting system, it also describes a specific destructive quantum measurement [4] with respect to the polarized states

$$|+\rangle = c_H^* |H\rangle + c_V^* |V\rangle$$

where $c_H$ and $c_V$ satisfy the normalization relation $|c_H|^2 + |c_V|^2 = 1$. Without loss of the generality, we take $c_H = c_V = 1/\sqrt{2}$, considering a specific measurement.

To examine whether the macroscopic feature of the reservoir causes the second order decoherence or not, we consider the whole system in an initial state

$$|\psi(0)\rangle = |1_H, 1_V\rangle \otimes |\{0_j\}\rangle,$$

where $|\{0_j\}\rangle$ is the vacuum state of the reservoir. Here, we have denoted the general Fock states of the many mode field by $|\{n_j\}\rangle \equiv |n_1, n_2, ...$. Because the present discussion concerns the external system interacting the considered system, the conceptions presented in last section must be alternated. Actually, in stead of the effective “two-time wave function”, we use the effective two-time state vector

$$|\psi_B(t, t')\rangle = \hat{B}(t')\hat{B}(t)|\psi(0)\rangle.$$  

Then we can re-write the second order correlation function as

$$G[t, t', \hat{\rho}(0)] = \langle \psi_B(t, t') | \psi_B(t, t') \rangle$$

It is interesting that the effective state vector can be evaluated as the superposition

$$|\psi_B(t, t')\rangle = \frac{1}{2} e^{i\hat{V}(0,0)t'}[\exp(-i\omega_V t')e^{-i(1,0)t'}e^{i(1,0)t}+\exp(-i\omega_V t)e^{i(0,0)t'}e^{-i(0,1)t'}e^{i(0,1)t}e^{-i(1,1)t}]|\{0_j\}\rangle \otimes |0_H, 0_V\rangle$$

of two components for the two paths from the initial two particle state $|1_H, 1_V\rangle$ to the two particle vacuum $|0_H, 0_V\rangle$. It should be noticed that the effective actions of the reservoir

$$\hat{V}(m, n) = \sum_j \hat{V}_j(m, n) = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j (d_V(\omega_j)m + d_H(\omega_j)n)(\hat{a}_j^\dagger + \hat{a}_j)$$

can label the different paths and record the path informa-

tion in the reservoir. Thus, this generalized “which-path
measurement \(^{\text{\textsuperscript{\textdagger}}\text{\textdagger}}\) leads to the second order quantum decoherence.

The above result clearly demonstrates that, with the presence of the reservoir, the different probability amplitudes \((\sim \exp(-i\omega_j t')\) and \(\exp(-i\omega_j t))\) from \(|1_H,1_V\rangle\) to \(|0_H,0_V\rangle\) entangle with the different states
\[
\frac{1}{2}e^{i\hat{V}(0,0)t'}e^{-i\hat{V}(1,0)t'}e^{i\hat{V}(1,0)t}e^{-i\hat{V}(1,1)t}|\{0_j\}\rangle
\]
and
\[
\frac{1}{2}e^{i\hat{V}(0,0)t'}e^{-i\hat{V}(0,1)t'}e^{i\hat{V}(0,1)t}e^{-i\hat{V}(1,1)t}|\{0_j\}\rangle
\]
of the reservoir. This is just the physical cause of the second order quantum decoherence. In the following section an explicit calculation of the second order correlation function will be given to illustrate this crucial observation.

V. DYNAMIC DECOHERENCE IN HIGHER ORDER CASE

After a straightforward calculation the second order correlation function can be expressed in a factorization form \([9]\):
\[
G[t,t',\hat{\rho}(0)] = \frac{1}{2}[1 + \text{Im}(e^{i\omega_V(t-t')}\prod_j F_j)]
\]
where each factor
\[
F_j = \langle 0_j|e^{i\hat{V}_j(1,1)t}e^{-i\hat{V}_j(1,0)t'}e^{i\hat{V}_j(0,1)t'}e^{-i\hat{V}_j(1,0)t}e^{-i\hat{V}_j(1,1)t}|0_j\rangle \equiv \langle 0_j|\hat{u}_j^6(t_0)|0_j\rangle
\]
is a two-time transition amplitude of the \(j\)'th mode of the reservoir. Obviously, the term \(\prod_j F_j\) measures the extent of coherence and decoherence in the second order case. It plays the same role as the decoherence factor of the first order decoherence \([6]\). So it is also called the decoherence factor.

In the following, to give the factor \(F_j\) explicitly, we adopt the Wei-Norman method \([17,18]\) to calculate the effective time evolution defined by \(\hat{u}_j(t)\). It can be imagined as an evolution governed by a discrete time-dependent Hamiltonian \(H(t)\) dominated by \(\hat{V}_j(1,1), -\hat{V}_j(1,0), \hat{V}_j(1,0), -\hat{V}_j(0,1), \hat{V}_j(0,1)\) and \(-\hat{V}_j(1,1)\) in six time-intervals \([t_0 = 0, t_1 = t_1, t_2 = 2t, t_2, t_3 = 2t + t'], [t_3, t_4 = 2t + 2t'], [t_4, t_5 = 3t + 2t'], [t_5, t_6 = 4t + 2t']\) respectively. In the \(k\)-th step of calculation, we take the final state of \((k - 1)\)-th step as its initial state. Therefore, we obtain \(\hat{u}_j^6(t_0)\) as the sixth order evolution
\[
\hat{u}_j^6(t_0) = e^{g_j^6(t_0)\hat{a}_j^\dagger}e^{g_j^6(t_0)\hat{a}_j}e^{g_j^6(t_0)\hat{a}_j}e^{g_j^6(t_0)\hat{a}_j}
\]
Here, \(g_j^k(t_0)(k = 1, 2, 3, 4)\) are the coefficients that can be explicitly obtained. But for the calculation of the \(j\)-th component
\[
F_j = \exp[g_j^6(t_0)]
\]
of the decoherence factor, we only need to know \(g_j^k\). The detailed discussion in the appendix gives

\[
g_j^6(t_0) = -\frac{2}{\omega_j^2}[d_H(j) - d_V(j)]^2 \sin^2\left[\frac{1}{2}\omega_j(t' - t)\right] + \frac{i}{\omega_j^2}[d_V^2(j) - d_H^2(j)]
\]
\[
[\omega_j(t' - t) + 2(1 - \cos(\omega_j|t' - t|)) \sin \omega_j t] + (1 - 2 \cos \omega_j t) \sin(\omega_j|t' - t|)].
\]

It is noticed that the real part
\[-R_j(t - t') = -\frac{2}{\omega_j^2}[d_H(j) - d_V(j)]^2 \sin^2\left[\frac{1}{2}\omega_j(t' - t)\right]
\]
only depends on the time interval \(t' - t\), but the imaginary part \(\Omega_j(t, t')\) depends on both \(t\) and \(t'\) as a two time function.
of the decoherence factor

\[ F = \prod_{j=1}^{N} F_j = \prod_{j=1}^{N} e^{i\Omega_j(t,t')} \cdot \prod_{j=1}^{N} \exp\left(-R_j(t-t')\right) \]

\[ = |F| \exp[i\Omega(t,t')] \]

implies the vanishment of the second order correlation in the macroscopic limit that the number \( N \) of particles making up the reservoir approaches the infinity. This is because

\[ |F| = \exp\left[-\sum_{j=1}^{N} R_j(t,t')\right] \rightarrow 0 \]

as \( N \rightarrow \infty \) since \( \sum_{j=1}^{N} R_j(t,t') \) is a diverging series or a monotonously-increasing function of \( t - t' \) under some reasonable conditions.

In order to demonstrate the above conclusion quantitatively, we give the numerical results for the second order decoherence for different numbers \( N \) of the quantum oscillators. As \( N \) increase, these results are illustrated in Fig.1. In the numerical calculation, the coupling constants \( \{d_V(j)\} \) take random values in the domain \([0.8, 1.0]\), the coupling constants \( \{d_H(j)\} \) in \([0.2, 0.4]\), and the frequencies \( \{\omega_j\} \) in \([0.5, 1.5]\). The other parameters are given in the caption of the figure.

From both the above graphic illustration given by the numerical calculation and the analytic result given by Eqs.(30,33,34), we see clearly that the second order correlation function depends not only on the time interval \( t' - t \), but also on \( t \) or \( t' \). In fact it follows from the analytic result

\[ \text{Im}(e^{i\omega_V(t-t')} \prod_j F_j) = \frac{1}{2} |F| \cos[\omega_V(t - t') + \Omega(t,t')] \]

that the amplitude of the second order correlation function is mainly determined by the time interval \( t' - t \), but the phase is determined by the two time parameters. What is more important is that as the number of the quantum oscillators increases, the second order coherence vanishes faster and faster, and the amplitude for quantum revival becomes smaller and smaller. With a reasonable extrapolation, it can be predicted that, when the number of the quantum oscillators approaches the infinity in the macroscopic limit, the second order coherence will vanish in a very short time and thus no quantum revival phenomenon can be observed.

VI. CONCLUDING REMARKS

In this paper we first depict the high order quantum coherence of a boson system by introducing the concept of multi-particle wave amplitude. For some cases with the specifically given initial state, we show that the
norm square of multi-particle wave amplitude (or a sum of the norm squares for an open system) gives the high order correlation function. As an effective multi-time wave function, this amplitude can be shown to be a superposition of several “multi-particle paths”. When the environment or an apparatus entangles with them to form a generalized “which-path(way)” measurement, the high order quantum decoherence happens dynamically. Some explicit and general illustrations are presented in this paper to construe our observation. But to prove our conjecture that any high order correlation function indicating the existence of high order quantum coherence can be expressed as the norm square of a properly-defined effective wave amplitude for many particle system, there is still a long way to go. If this conjecture is true under certain general conditions, then there still exist the problem of clarifying these conditions. Moreover, experimental proposals that can be implemented at least in principle are still unavailable.

Our present investigations shed a new light on the understanding of quantum coherence. According to arguments in this paper the high order quantum coherence can also be sculptured as the generalized interference phenomenon by two “multi-particle paths”, and the intrinsically quantum features (higher order ones) of coherence beyond the classical analogue reflected by the spatial interference of two paths in classical electromagnetic field can be theoretically unified in a framework to embody the wave - particle duality in the quantum world. Therefore, as the essential element, the “which-path(way)” detection in both the original and the extended versions, naturally provides a complete decoherence mechanism in understanding quantum measurement and the transition from quantum to classical mechanics.

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APPENDIXES

In this appendix, the Wei-Norman method \[17, 18\] is adopted to calculate the second order decoherence factor \(F_j\). The calculation is completed in six steps.

During the time period \([t_{k-1}, t_k] (k = 1, 2, \ldots, 6)\), Let \(W_j^k(t, t')\) be an time evolution dominated by the single particle Hamiltonian

\[
\hat{H}_j^k = \alpha_j^k \hat{a}_j \hat{a}_j^\dagger + \beta_j^k \hat{a}_j \hat{a}_j^\dagger + \gamma_j^k \hat{a}_j, \quad (k = 1, 2, \ldots, 6). \tag{36}
\]

The coefficients \(\alpha_j^k, \beta_j^k, \gamma_j^k\) and the time intervals \(T_k = t_k - t_{k-1}\) take different values in the six different steps:

\[
\begin{align*}
\alpha_j^1 &= \omega_j, \beta_j^1 = \gamma_j^1 = d_V(\omega_j) + d_H(\omega_j), T_1 = t, \\
\alpha_j^2 &= -\omega_j, \beta_j^2 = \gamma_j^2 = d_V(\omega_j), T_2 = t, \\
\alpha_j^3 &= \omega_j, \beta_j^3 = \gamma_j^3 = d_V(\omega_j), T_3 = t', \\
\alpha_j^4 &= -\omega_j, \beta_j^4 = \gamma_j^4 = -d_H(\omega_j), T_4 = t', \\
\alpha_j^5 &= \omega_j, \beta_j^5 = \gamma_j^5 = d_H(\omega_j), T_5 = t, \\
\alpha_j^6 &= \omega_j, \beta_j^6 = \gamma_j^6 = -d_V(\omega_j) - d_H(\omega_j), T_6 = t \tag{37}
\end{align*}
\]

Because of the fact that the four operators \(n_j = \hat{a}_j^\dagger \hat{a}_j, \hat{a}_j^\dagger, \hat{a}_j, \hat{a}_j^\dagger \) and \(1\) form a closed algebra - the Heisenberg-Weyl algebra, the unitary time evolution operator at each step takes the following form (Wei-Norman theorem)

\[
\hat{u}_j^k(T) = e^{\alpha_j^k(T) \hat{a}_j \hat{a}_j^\dagger} e^{\beta_j^k(T) \hat{a}_j^\dagger \hat{a}_j} e^{\gamma_j^k(T) \hat{a}_j^\dagger \hat{a}_j}. \tag{38}
\]

for \(T \in [t_{k-1}, t_k]\) in a special sequence. Here coefficients \(g_j^k(T) (s = 1, 2, 3, 4)\) are functions of \(T\) to be determined. The benefit of the above form is that only the coefficient \(g_j^k(T)\) is needed in the calculation of the average value at the vacuum state. So we can largely reduce the complexity of our calculation as we need to pay attention only to things concerning \(g_j^k(T)\). Substituting \(\hat{u}_j^k(T)\) into the Schrödinger equation

\[
i \frac{d}{dT} \hat{a}_j^k = \hat{H}_j^k \hat{a}_j^k,
\]

we find the coefficients \(g_j^k(T)(s = 1, 2, 3, 4)\) satisfy the following system of equations:

\[
\begin{align*}
\frac{d}{dT} g_{2j}^k &= -i \alpha_j^k, \\
\frac{d}{dT} g_{1j}^k - g_{1j}^k \frac{d}{dT} g_{2j}^k &= -i \beta_j^k, \\
\frac{d}{dT} g_{3j}^k &= -i \gamma_j^k, \\
\frac{d}{dT} g_{4j}^k - g_{4j}^k \frac{d}{dT} g_{3j}^k &= 0
\end{align*}
\] \tag{39}

Using the results

\[
\frac{d}{dT} g_{1j}^k = -i \alpha_j^k g_{1j}^k - i \beta_j^k, \quad \frac{d}{dT} g_{4j}^k = -i \gamma_j^k g_{4j}^k \tag{40}
\]

obtained by simplifying the above system of equations, we get the solution

\[
\begin{align*}
g_{1j}^k(T) &= (g_{1j}^k(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k}) e^{-i \alpha_j^k(T - t_{k-1})} - \frac{\beta_j^k}{\alpha_j^k}, \\
g_{2j}^k(T) &= g_{2j}^k(t_{k-1}) + \frac{\gamma_j^k}{\alpha_j^k} (g_{1j}^k(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k}) \\
&\quad (e^{-i \alpha_j^k(T - t_{k-1})} - 1) + i \frac{\beta_j^k}{\alpha_j^k} (T - t_{k-1}) \tag{41}
\end{align*}
\]
Notice that, to obtain the above result we have used the initial conditions

\[
g_{ij}^k(t_{k-1}) = g_{ij}^{k-1}(t_{k-1}), \quad (42)\\
g_{ij}^k(t_{k-1}) = g_{ij}^{k-1}(t_{k-1}) = g_{ij}^0(t_0) = 0 \quad \text{for the first step. Then, we obtain a set of iteration equations}
\]

\[
g_{ij}^k(t_k) = (g_{ij}^{k-1}(t_{k-1}) + \frac{\beta_{ij}^k}{\alpha_{ij}^k}) e^{-i\alpha_{ij}^k T_k} - \frac{\rho_{ij}^k}{\alpha_{ij}^k},
\]

for each step and the initial conditions \(g_{ij}^0(t_0) = 0\) for the first step. Then, we obtain a set of iteration equations

\[
g_{ij}^k(t_k) = g_{ij}^{k-1}(t_{k-1}) + \frac{\gamma_{ij}^k}{\alpha_{ij}^k} (g_{ij}^{k-1}(t_{k-1}) + \frac{\beta_{ij}^k}{\alpha_{ij}^k}) (e^{-i\alpha_{ij}^k T_k} - 1) + i \frac{\beta_{ij}^k}{\alpha_{ij}^k} \gamma_{ij}^k T_k. \quad (44)
\]

Iterating six times with different initial conditions and coefficients, the final result of \(g_{ij}^k(t_0)\) is obtained as Eq. (44).

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