FSI and rare B decays: $B \rightarrow \pi\pi, \rho\rho$

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Abstract

The final state interactions (FSI) model, in which soft rescattering of low mass intermediate states dominate, is suggested. It explains why the strong interaction phases are large in the $B_d \rightarrow \pi\pi$ channel and are considerably smaller in the $B_d \rightarrow \rho\rho$ one.

Key words: B-mesons, rare decays, final state interactions

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1. Introduction

Understanding of final state interactions (FSI) in B decays is needed in order: 1. to predict/explain the ratios of branching ratios, $B \rightarrow \pi\pi, B \rightarrow \rho\rho$ is a very spectacular example; 2. to study strong interactions; 3. to understand DCPV: $C \sim \sin \alpha \sin \delta$, so: to understand values of $C$, $B \rightarrow K\pi$ is a very spectacular example.

$C$-averaged branching ratios of $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ decays are presented in the following Table [1]:

| Mode         | Br($10^{-6}$) | Mode         | Br($10^{-6}$) |
|--------------|---------------|--------------|---------------|
| $B_d \rightarrow \pi^+\pi^-$ | $5.2 \pm 0.2$ | $B_d \rightarrow \rho^+\rho^-$ | $24 \pm 3$ |
| $B_d \rightarrow \pi^0\pi^0$   | $1.5 \pm 0.2$ | $B_d \rightarrow \rho^0\rho^0$   | $0.74 \pm 0.29$ |
| $B_u \rightarrow \pi^+\pi^0$   | $5.6 \pm 0.4$ | $B_u \rightarrow \rho^+\rho^0$   | $18.2 \pm 3.0$ |

where we observe the absence of color suppression (naive factor $1/3^2/2 = 1/18$ in decay probability) of $\pi^0\pi^0$ mode.

Charmless strangeless $B$-decays are described by the sum of tree (T) and penguin (P) Feynman diagrams:
We work in the so-called “t-convention” for penguin amplitudes, when \((V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^*)f(m_c/M_W) = 0\) is subtracted from decay amplitudes. In this convention CKM phases difference of \(T\) and \(P\) amplitudes is \(\alpha \approx 90^\circ\) that is why they do not interfere in C-averaged decay probabilities.

2. Analysis of experimental data

Using isotopic invariance of strong interactions \(B \to \pi\pi\) decay amplitudes may be presented in the following form:

\[
M_{\bar{B}_d \to \pi^+\pi^-} = e^{-i\gamma} \frac{1}{2\sqrt{3}} A_2 e^{i\delta_2^0} + e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^0} + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} Pe^{i(\delta_2^p + \delta_2^0)} ,
\]

\[
M_{\bar{B}_d \to \pi^0\pi^0} = e^{-i\gamma} \frac{1}{\sqrt{3}} A_2 e^{i\delta_2^0} - e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^0} - \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} Pe^{i(\delta_2^p + \delta_2^0)} ,
\]

\[
M_{\bar{B}_u \to \pi^-\pi^0} = \frac{\sqrt{3}}{2\sqrt{2}} e^{-i\gamma} A_2 e^{i\delta_2^0} .
\]

Neglecting \(P\) from 3 branching ratios 3 parameters \(A_0\), \(A_2\) and \(|\delta_0^0 - \delta_2^0|\) can be extracted, and for phase difference of the amplitudes with \(I = 0\) and 2 we get:

\[
\cos(\delta_0^0 - \delta_2^0) = \frac{\sqrt{3}}{4} \frac{B_{++} - 2B_{00} + \frac{4}{3} \tau_+ B_{0+}}{\tau_+ B_{0+} \sqrt{B_{++} + B_{00} - \frac{2}{3} \tau_+ B_{0+}}} .
\]

Using experimentally measured branching ratios from the Table we obtain: \(|\delta_0^0 - \delta_2^0| = 55^\circ\).

\(P^2\) term we can extract from \(\text{Br}(K^0\pi^\pm)\): \(\text{Br}(B_d \to \pi^+\pi^-)_p \approx 0.59 \cdot 10^{-6}\); subtracting it from experimental data we see that penguin amplitude diminishes a bit phase difference: \(|\delta_0^p - \delta_2^p| = 47^\circ \pm 10^\circ\). Let us remind that in the case of \(D \to \pi\pi\) decays phase difference of isotopic amplitudes is twice times larger [2]: \(|\delta_0^D - \delta_2^D| = 86^\circ \pm 4^\circ\), which suggest approximate \(1/M\) scaling of FSI phases, where \(M\) is the mass of decaying meson.

\(\rho\)-mesons produced in \(B\)-decays are almost completely longitudinally polarized, so the analysis goes just like for \(\pi\)-mesons: \(|\delta_0^\rho - \delta_2^\rho| = 15^\circ + 5^\circ - 10^\circ\), small (unlike pion case).

This difference of FSI phases is responsible for different patterns of \(B \to \pi\pi\) and \(B \to \rho\rho\) decay probabilities.

We want to understand why FSI phases are large in \(B \to \pi\pi\) amplitudes but small in \(B \to \rho\rho\) amplitudes.
pQCD: PHASES ARRIVE FROM LOOPS, SO THEY ARE SMALL, which is correct for $B \to \rho\rho$-decays but it does not work for $B \to \pi\pi$.
SO: DYNAMICS at LARGE DISTANCES MATTER.

3. Model of FSI

Which intermediate states are important (here we follow papers [3], [4], see also [5])?
$b \to u\bar{d}d$ decay produce mostly 3 isotropically oriented jets of light mesons, each having about 1.5 GeV energy. In $e^+e^-$ annihilation at 3 GeV c.m. energy average charged particles (pions) multiplicity is about 4 - so, taking $\pi^0$'s into account in B-mesons decays to light quarks in average 9 “pions” are produced, flying in 3 widely separated directions (or almost isotropically, taken transverse momentum into account). Branching ratio of such decays is large, about $10^{-2}$. However such states NEVER rescatter into two pions or two $\rho$- mesons.

Which intermediate states will transform into two mesons final state we easily understand studying inverse process of two light mesons scattering at 5 GeV c.m. energy. In this process two jets of particles moving in the directions of initial particles are formed. Energy of each jet is $M_B/2$, while its invariant mass squared is not more than $M_B\Lambda_{QCD}$.

Following these arguments in the calculation of the imaginary parts of decay amplitudes we will take two particle intermediate states into account, to which branching ratios of $B$-mesons are maximal:

![Diagram](image)

It is convenient to transform integral over longitudinal (relatively to outgoing meson momentum) and time-like components of $k$ in the following way:

$$\int dk_0 dk_z = 1/(2 \cdot M_B^2) \int ds_X ds_Y .$$

Integrals over $s$ rapidly decrease when $s$ increase since only low mass clusters contribute to amplitude of 2 meson production. In this way we get:

$$M_{\pi\pi}^I = M_{XY}^{(0)I}(\delta_{\pi X} \delta_{\pi Y} + iT_{XY=0}^{I=0}).$$

Since Br$B \to \rho\rho$ is large it contributes a lot to FSI phase of $B \to \pi\pi$ decay; NOT VICE VERSA! $B \to \rho\rho \to \pi\pi$ chain can be calculated with the help of unitarity relation; for small $t$ we can trust elementary $\pi$-meson exchange in $t$- channel:

$$\text{Im}M(B \to \pi\pi) = \int \frac{d\cos \theta}{32\pi} M(\rho\rho \to \pi\pi) M^*(B \to \rho\rho) .$$
Introducing formfactor $exp(t/\mu^2)$ for $\mu^2 = 2m^2_\rho$ we obtain: $\delta_0^\pi(\rho\rho) = 15^0$, $\delta_2^\pi(\rho\rho) = -5^0$, $\delta_0^\pi(\rho\rho) - \delta_2^\pi(\rho\rho) = 20^0$ and half of experimentally observed phase difference is explained. Let us emphasize that $\delta_1^\pi(\rho\rho) \sim 1/M_B \to 0$.

It is remarkable that FSI phases generated by $B \to \pi\pi \to \rho\rho$ chain are damped by $\text{Br}(B \to \rho^+\rho^-,\rho^+\rho^0)/\text{Br}(B \to \pi^+,\pi^+\pi^0)$ ratios and are a few degrees: $\delta_0^\pi(\pi\pi) - \delta_2^\pi(\pi\pi) \approx 4^0$.

For $\pi\pi$ intermediate state we take Regge model expression for $T_{\pi\pi\to\pi\pi}$, which takes into account pomeron, $\rho$ and $f$ trajectories exchange. Pomeron exchange produces imaginary $T$ and does not contribute to phase shifts as far as it is critical, $\alpha_P(0) = 1$. However, for the amplitude of the supercritical pomeron exchange we have: $T \sim (s/s_0)^{\alpha_P(t)}(1 + \exp(-\pi\alpha_P(t)))/(-\sin(\pi\alpha_P(t))) = (s/s_0)^{(1+\Delta)}(i + \Delta\pi/2)$, where in the last expression $t = 0$ was substituted and the value of intercept $\alpha_P(0) = 1 + \Delta, \Delta \approx 0.1$ was used. So, $\delta_0^\pi(\pi\pi) = 5.0^0, \delta_2^\pi(\pi\pi) = 0^0$.

$\pi a_1$ intermediate state should also be taken into account. Large branching ratio of $B_d \to \pi^+ a_1^-\pi^-$-decay ($\text{Br}(B_d \to \pi^+ a_1^-) = (32 \pm 4) \times 10^{-6}$) is partially compensated by small $\rho\pi a_1$ coupling constant (it is 1/3 of $\rho\pi\pi$ one): $\delta_0^\pi(\pi a_1) = 4^0$, $\delta_2^\pi(\pi a_1) = -2^0$, where we assume that the sign of $\pi a_1$ contribution to phases difference is the same as that of the elastic channel.

Finally: $\delta_0^\pi(\pi\pi) = 23^0, \delta_2^\pi = -7^0, \delta_0^\pi - \delta_2^\pi = 30^0$, and the accuracy of this number is not high.

In conclusion the model of FSI in $B \to M_1M_2$ decays is suggested; it explains the absence of color suppression of $B \to \pi^0\pi^0$ decay. Relatively small $B \to \pi^+\pi^-$ branching ratio is the reason why $B \to \rho^0\rho^0$ mode remains small.

$B \to \pi\pi$: we cannot reproduce $C_{+-}$ value measured by Belle (-0.55(9)) while BABAR result (-0.25(8)) is much more acceptable and we predict almost maximal DCPV in $B(B) \to \pi^0\pi^0$ decays: $C_{00} \approx -0.60$.

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