Charged Particle Stopping Power Effects on Ignition:
Some Results from an Exact Calculation

Robert L. Singleton Jr.
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, USA

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Abstract

A completely rigorous first-principles calculation of the charged particle stopping power has recently been performed by Brown, Preston, and Singleton (BPS). This calculation is exact to leading and next-to-leading order in the plasma number density, including an exact treatment of two-body quantum scattering. The BPS calculation is therefore extremely accurate in the plasma regime realized during the ignition and burn of an inertial confinement fusion capsule. For deuterium-tritium fusion, the 3.5 MeV alpha particle range tends to be 20–30% longer than most models in the literature have predicted, and the energy deposition into the ions tends to be smaller. Preliminary numerical simulations indicate that this increases the $\rho R$ required to achieve ignition.
I. INTRODUCTION

The charged particle stopping power of a hot plasma plays a critical role in whether an inertial confinement fusion (ICF) capsule will undergo thermonuclear burn and ignition. For a robust experimental setup, in which the laser energy is sufficiently high to assure ignition and full burn, the fine details of the stopping power are not so relevant; however, in more marginal settings on the threshold of ignition, these details are likely to play a more important role. Conventional wisdom states that our current knowledge of the stopping power is probably “good enough”; however, I will present some analytic and numerical results to illustrate that the subleading order physics neglected or missed by most stopping power models, the terms of order unity inside the Coulomb logarithm, can in fact lead to a noticeable effect on ignition.

In this study, I would like to explore some implications of the stopping power calculation of Brown, Preston, and Singleton (BPS) \[1\]. This calculation of the charged particle stopping power, which includes the energy splitting of the projectile as it slows down and a rigorous treatment of the quantum to classical transition, is near-exact for the weakly coupled plasmas relevant for ICF ignition. I should also emphasize that the BPS stopping power is not a model of the Coulomb energy exchange, but a controlled first principles calculation of this process. For a detailed pedagogical explanation of the technique, please see Refs. \[2, 3, 4\]. We may therefore study the effects of stopping power on ignition and burn in a systematic fashion, and unlike utilizing a model, we know the range of validity of the BPS stopping power, along with an estimate of the error in any given plasma regime. Rather than launching straightway into the simulation of real ICF capsules, with their concomitant computational and physical complexities, I shall instead start more systematically and investigate the simpler problem of the deuterium-tritium (DT) microsphere studied by Fraley et al. \[5\]. We shall find that, for these so called Fraley spheres, the BPS stopping power increases the ignition threshold significantly compared to typical models of the stopping power found in the literature. For example, the value of $\rho R$ necessary to attain a self-sustaining burn in a Fraley sphere increases by 10% in comparison to the well-known state-of-the-art model of Li and Petrasso \[6\].

Specializing to the case of DT fusion, the dominant boot-strap heating mechanism necessary to achieve a self-sustained burn is $\alpha$ particle energy deposition in the background plasma. All things being equal, the larger the stopping power $dE/dx$, the greater the energy deposition of the supra-thermal 3.5 MeV $\alpha$ particles, and therefore the easier it becomes to achieve a self-sustained burn through $\alpha$ particle heating of the ions. In other words, a larger stopping power leads to a more efficient self-heating, and conversely, a smaller charged particle stopping power would tend to make ignition harder, since this would increase the range of $\alpha$ particles and lead to smaller energy deposition per unit volume. Other factors, such as
the relative amount of energy that the $\alpha$ particle apportions between plasma electrons and ions as it slows down, which is determined from $dE_e/dx$ for electrons and $dE_i/dx$ for ions, can also be quite important, with larger ion heating being more favorable for ignition.

Let us start by reviewing a study performed by N.M. Hoffman and C.L. Lee [7], in which they compared the stopping power model of ‘Corman-Spitzer’ (CS) [8] to that of C.K. Li and Petrasso [6]. A significant difference between these two models is that CS is valid only in a velocity window in which the projectile speed is much greater than the thermal ion velocity but much smaller than the thermal electron velocity, while Li and Petrasso (LIP) is more general and was constructed to remain valid for all non-relativistic velocities. The LIP stopping power tends to be smaller than that of CS, and consequently the $\alpha$ particle range for LIP is somewhat larger than for CS. By the aforementioned considerations, we would therefore expect the ignition threshold for LIP to be greater than that predicted from CS. Interestingly, however, Hoffman and Lee found that both models produce the same ignition threshold for the DT microspheres of Ref. [5], despite the longer range of the $\alpha$ particle predicted by LIP [7]. In addition to this, a preliminary study I have begun indicates that other models, such as that of Ref. [9], produce an ignition threshold almost identical to that of CS and LIP. As pointed out by Ref. [7], these observations can be explained by noting that the models of CS and LIP differ most near the low energy thermal regime of the $\alpha$ particle projectile, while their stopping powers are almost identical at higher energies near the 3.5 MeV threshold (at even higher energies, CS and LIP begin to diverge, but this is well above the production threshold). Consequently, the stopping power models of CS and LIP act to slow the $\alpha$ particle down in almost exactly the same way throughout most of its history, since they give equal values for $dE/dx$ throughout most of the projectile energy regime, and this is why the ignition profiles are virtually identical. In contrast, the stopping power of LIP and BPS differ significantly even in the high energy regime traversed by the $\alpha$ particle, and as we shall see, this produces a marked difference in the ignition profiles of LIP and BPS.

Hoffman and Lee also observed that certain diagnostics, those sensitive to small projectile energies of order the thermal background, are quite different between CS and LIP. For example, Hoffman and Lee found LIP to be in agreement with experiment for the spectrum of fast protons in a D$^3$He filled capsule implosion, while CS disagreed with the experimental data [7, 10]. The diagnostic signatures explored by Hoffman and Lee tend to be more sensitive to the low energy region of $dE/dx$, where CS and LIP have the largest discrepancy, and this explains the differences in diagnostics [7]. However, for diagnostics that are sensitive to the soft regime of the projectile, one must compute $dE/dx$ extremely accurately. This presents a potential problem for models that are inaccurate near the thermal regime, since their diagnostic signatures are sensitive to this regime. However, as explained in Section II
of Ref. [1], this criticism does not apply to the BPS stopping power, and consequently BPS should provide quite reliable diagnostics for such processes.

II. CONTEXT

The basic physics responsible for charged particle stopping in a plasma is energy exchange through Coulomb interactions, a process that is quite similar to the Coulomb energy exchange that drives the temperature equilibration between plasma components at differing temperatures. Both of these processes, stopping power and temperature equilibration, are local, in that they do not involve energy transport in space. This should be contrasted with thermal conductivity, in which heat flow by electrons or ions is not only driven by Coulomb energy exchange, but also by the transport of energy from one spatial location to another. In this sense, the basic physics of the stopping power is identical to the physics of temperature equilibration between plasma components, both of which differ somewhat from the physics of thermal conductivity. I will have more to say about thermal conductivity in a future section, but for now, let us concentrate on the rate at which the charged particle loses energy as it traverses the plasma. For the Coulomb potential, a straightforward calculation of the energy exchange rate diverges logarithmically in both the short and long distance regimes [11]. To obtain a finite result, we must introduce \( ad hoc \) short and long distance cutoffs \( b_{\text{min}} \) and \( b_{\text{max}} \) as we integrate over the impact parameter \( b \), in which case the rate takes the form

\[
\frac{dE}{dt} = K \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{db}{b} = K \ln \left\{ \frac{b_{\text{max}}}{b_{\text{min}}} \right\}. \tag{2.1}
\]

The coefficient \( K \) is an exactly calculable analytic prefactor that depends critically on the physical process under consideration, while the logarithmic term is called the Coulomb logarithm. The exact values of the short and long distance cutoffs can only be estimated, and this is a dominant source of non-systematic error in model building. Choosing the values of \( b_{\text{min}} \) and \( b_{\text{max}} \) based on physical considerations (however well motivated they might be), rather than direct calculations from theory, is what I will call \textit{model building}. We may increase the sophistication of models such as Eq. (2.1) by including collective effects or improved short distance physics, as in Ref. [6], but I will still refer to this as \textit{modeling}. In many cases in plasma physics, modeling is essentially the only means by which to proceed, particularly in complicated cases like warm dense matter. In some situations, however, such as the weakly coupled plasma of a burning ICF capsule or the hot plasma at the center of the sun, we can avail ourselves to simple perturbative analytic techniques rather than model building, and this is the path taken by BPS.
Before turning to the rigorous calculation of the stopping power, let us investigate further the model building methodology and the physical arguments used to choose the short and long distance cutoffs. Debye screening sets the scale for the long distance cutoff, and we expect on physical grounds that $b_{\text{max}} = c_M \kappa^{-1}$ with the dimensionless coefficient $c_M \sim 1$, where $\kappa$ is the Debye wavenumber. The coefficient $c_M$ is by no means a constant, but rather, it is typically a function of the various plasma parameters (and the various projectile parameters in the case of stopping power), which I will write schematically as $c_M = c_M(m, T, n)$. Short of a rigorous calculation, there is no prescription for determining the value or functional form of $c_M$, and indeed, choosing the functional form of $c_M$ is part of the model construction (usually $c_M$ is set unity). There are also various choices for the Debye wavenumber $\kappa$; for example, should one use the electron screening length $\kappa^{-1}_e$ or the total screening length $\kappa^{-1}_D$ determined from the electrons and ions? The answer to this question is of course process dependent; for example, if the ions can be treated as static and screened by the electrons, then we should take $b_{\text{max}} = c_M \kappa^{-1}_e$. However, to be rigorous, such choices must come out of the calculation rather than being put into the calculation by hand. The situation for $b_{\text{min}}$ is even less clear. The short distance cutoff is set by scattering. In the extreme classical regime, we take $b_{\text{min}}^C = c_m^C \times \text{(distance of closest approach)}$, and in the extreme quantum regime we take $b_{\text{min}}^Q = c_m^Q \times \text{(momentum transfer)}/\hbar$. In either case, the functions $c_m^C$ and $c_m^Q$ are of order unity, but absent a rigorous calculation, this is all we can really say about them. To interpolate between the extreme quantum and classical regimes, one usually employs an *ad hoc* scheme such as $b_{\text{min}} = \left[ (b_{\text{min}}^C)^2 + (b_{\text{min}}^Q)^2 \right]^{1/2}$, although in Ref. [3] it was shown that this simple interpolation is missing an important logarithmic contribution. From these considerations, it is clear that the model building process is at best logarithmically accurate, that is to say, that the coefficient inside the Coulomb logarithm is known only to within a factor of order unity, and a complete and rigorous calculation is the only way to really settle the issue (this is why it is not surprising that the coefficient inside the logarithm varies by an order of magnitude within the literature, and with few exceptions, there is no reason to prefer one model over another\(^1\)).

Another point worth stressing is that the Coulomb logarithm itself is process dependent, despite the fact that the same basic physics of Coulomb energy exchange is at work in the stopping power and temperature equilibration. For example, if we consider the charged

\(^1\) There are *calculations* in the literature, such as that of Gould and DeWitt [12], that do obtain the correct coefficients inside the logarithm. However, to my knowledge, none of these calculations are systematic, in that they retain spurious higher-order terms and do not provide an error estimate. Therefore, without independent verification such as BPS, these calculations do not have the power to know just how accurate they are.
particle stopping power and the electron-ion temperature equilibration rate, we can write
\[ \frac{dE_p}{dx} = K_p \ln \left\{ \frac{b_{\text{max}}}{b_{\text{min}}} \right\} \]  
(2.2)
\[ \frac{dE_{ei}}{dt} = K_{ei} \ln \left\{ \frac{b_{\text{max}}}{b_{\text{min}}} \right\} \left( T_e - T_i \right). \]  
(2.3)

The first expression (2.2) represents the energy loss per unit distance of a projectile [the charged particle stopping power], while the second expression (2.3) is the energy exchange rate per unit volume between electrons at temperature \( T_e \) and ions at temperature \( T_i \) [this rate is proportional to the temperature difference, which I have explicitly indicated in Eq. (2.3)]. I have placed a bar over the short and long distance cutoffs in Eq. (2.3) to indicate that the Coulomb logarithm need not be the same as for other processes. Concentrating on the charged particle stopping power from here on, I will rewrite the generic stopping power model (2.2) in the form
\[ \frac{dE_{\text{model}}}{dx} = K_{\text{CS}} \ln \Lambda_{\text{model}}. \]  
(2.4)

Here, I am emphasizing that the leading order coefficient \( K_{\text{CS}} \) is exactly known [8], while the coefficient \( \Lambda_{\text{model}} \) inside the logarithm is only known to an order of magnitude. The reason I do not call the BPS calculation a model is because Ref. [1] calculated the terms under the logarithm exactly from first principles, including the quantum to classical transition, along with a controlled estimate of the error, which, in contrast to Eq. (2.4), I will write as
\[ \frac{dE_{\text{BPS}}}{dx} = K_{\text{CS}} \ln \Lambda_{\text{BPS}} + \text{controlled error term small in weakly coupled plasma}. \]  
(2.5)

As I will discuss more fully in the next section, any thermodynamic quantity in a plasma can be expanded in integer powers of a dimensionless plasma coupling parameter \( g \) [13]. Measuring temperature in energy units and taking the electrostatic units to be rationalized Gaussian, the choices I will employ throughout this paper, the coupling takes the form
\[ g = \frac{e^2 \kappa}{4\pi} \frac{1}{T}, \]  
(2.6)
where \( \kappa \) is the Debye wavenumber. The \( g \)-expansion admits nonanalytic terms such as \( \ln g \), and indeed, such terms are essential in capturing the interplay of short and long distance

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2 In ordinary nonrationalized Gaussian units we would write \( g = e^2 \kappa / T \). There is an independent coupling parameter for each plasma component, and this must be taken into account for a real calculation; however, for the purposes of explanation, we may consider an isolated plasma component. Finally, the usual plasma coupling \( \Gamma \), defined in terms of the interparticle spacing, is related to the expansion parameter by \( g^2 \propto \Gamma^4 \). Integer expansions in \( g \) are therefore expansions in fractional powers of \( \Gamma \).
physics. As we shall see in the next section, the stopping power may be systematically expanded in the form

\[ \frac{dE_{\text{BPS}}}{dx} = - A g^2 \ln g \frac{\text{LO}}{\text{LO}} + B g^2 \frac{\text{NLO}}{\text{NLO}} + \mathcal{O}(g^3) , \]  

(2.7)

where I have indicated the leading order (LO) and the next-to-leading order (NLO) terms. To get a feel for the size of \( g \) in a plasma, at the center of the sun we find \( g = 0.04 \).

In a weakly coupled plasma in which \( g \ll 1 \), the error terms, which I have denoted by \( \mathcal{O}(g^3) \), are quite small and the expansion (2.7) will be near-exact, provided of course that we know the coefficients \( A \) and \( B \) exactly. The coefficient \( A = A(m,T,n) \) is well known, while \( B = B(m,T,n) \) was calculated by BPS [1], and as a matter of completeness, the final results of the BPS calculation are displayed in Appendix A. To make the connection with expression (2.5), I will write the leading order coefficient as \( K_{\text{CS}} = A g^2 \), and define the dimensionless coefficient \( C = \exp\{-B/A\} \). We can then express the rate (2.7) in the form

\[ \frac{dE_{\text{BPS}}}{dx} = K_{\text{CS}} \ln \Lambda_{\text{BPS}} + \mathcal{O}(g^3) , \quad \text{with} \quad \ln \Lambda_{\text{BPS}} = - \ln \{C g\} . \]  

(2.8)

This gives the exact Coulomb logarithm since we can calculate \( C = C(m,T,n) \). Incidentally, and this cannot be overemphasized, this perturbative methodology obviates the need for the cutoff parameters \( b_{\text{min}} \) and \( b_{\text{max}} \), and they can be dispensed with from here out. The long and short distant cutoffs are really only part of a heuristic device that, in my opinion, often introduces more confusion than it purports to settle.

As discussed at length in Ref. [1], to leading and next-to-leading order, and only to this order, can we decompose the stopping power into its contribution to the various plasma species,

\[ \frac{dE_{\text{BPS}}}{dx} = \sum_b \frac{dE_{\text{BPS}}^b}{dx} = \sum_b K_b \ln \Lambda_{\text{BPS}}^b + \mathcal{O}(g^3) , \]  

(2.9)

where we identify \( dE_{\text{BPS}}^b /dx \) as the stopping power contribution from species \( b \). The notion of dividing the energy into contributions uniquely associated with individual plasma species is valid only to order \( g^2 \), as three-body and higher collective effects render this division meaningless when working to order \( g^3 \) and higher. For the BPS stopping power, the quantity \( dE_{\text{BPS}}^b /dx \) depends not just upon parameters associated with the \( b \)-species, but upon the plasma parameters of all other species through the dielectric function. This should be contrasted with the corresponding quantity \( dE_{\text{LIP}}^b /dx \) for LIP (except for the Coulomb logarithm \( \ln \Lambda_{\text{LIP}}^b \)), which depends only upon the plasma conditions of species \( b \). Since we shall be comparing BPS with LIP, I will close this section by presenting the LIP stopping power in some detail.
Li and Petrasso [6] modeled $dE/dx$ by combining a generalization of the Fokker-Planck equation to account for short-distance collisions and a well-chosen term involving a step function to include long-distance collective effects. They define a Coulomb logarithm by using a minimum classical impact parameter that interpolates between the classical and quantum regimes, as described earlier. Using rationalized cgs units for the electric charge, the LIP stopping power for species $b$ can be written

$$
\frac{dE_{\text{LIP}}^b}{dx} = \sum_b \frac{dE_{\text{LIP}}^b}{dx} = \sum_b \frac{e_p^2}{4\pi} \frac{\kappa_b^2}{\beta_b m_b v_p^2} \left[ G \left( \frac{1}{2} \beta_b m_b v_p^2 \right) \ln \Lambda_{\text{LIP}}^b + H \left( \frac{1}{2} \beta_b m_b v_p^2 \right) \right],
$$

(2.10)

where the contribution to species $b$ is $dE_{\text{LIP}}^b/dx$. Here, the function multiplying the Coulomb logarithm is defined by

$$
G(y) = \left[ 1 - \frac{m_b}{m_p} \frac{d}{dy} \right] \mu(y),
$$

(2.11)

where

$$
\mu(y) = \frac{2}{\sqrt{\pi}} \int_0^y dz \frac{z^{1/2}}{e^{-z}},
$$

(2.12)

and

$$
H(y) = \frac{m_b}{m_p} \left[ 1 + \frac{d}{dy} \right] \mu(y) + \theta(y - 1) \ln \left( 2e^{-\gamma y^{1/2}} \right),
$$

(2.13)

with $\theta(x)$ being the unit step function: $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x > 0$. Li and Petrasso define a Coulomb logarithm in terms of the combination of classical and quantum cutoffs as described above, namely

$$
\ln \Lambda_{\text{LIP}}^b = -\frac{1}{2} \ln \kappa_b^2 B_b^2,
$$

(2.14)

where the short distance cutoff is defined by

$$
B_b^2 = \left( \frac{\hbar}{2m_p u_b} \right)^2 + \left( \frac{e_p e_b}{4\pi m_p u_b^2} \right)^2,
$$

(2.15)

in which $m_{pb} = m_p m_b / (m_p + m_b)$ is the reduced mass of the projectile and species $b$, and

$$
u_b^2 = v_p^2 + \frac{2}{\beta_b m_b}
$$

(2.16)

defines an average of the squared projectile and thermal velocities. In rationalized units, the Debye wave number is

$$
k_b^2 = \frac{e_b^2 n_b}{T_b}.
$$

(2.17)

In future sections, we will also need the plasma frequency, and in rationalized units it takes the form

$$
\omega_b^2 = \frac{e_b^2 n_b}{m_b}.
$$

(2.18)
III. ANALYTIC CONSIDERATIONS: THE BPS CALCULATION

In this section I will review the salient features of the BPS calculation [1]. This calculation includes both hard (short distance) physics and dynamic collective (long distance) physics, joined together exactly and unambiguously, and systematized by a power series expansion in the plasma coupling constant g. Ref. [1] calculates both the charged particle stopping power and the electron-ion temperature equilibration rate, thereby providing an exact calculation of the coefficient inside the Coulomb logarithm for both processes in \( (2.2) \) and \( (2.3) \). An additional feature of the BPS calculation is that it also provides an exact interpolation between the extreme classical and extreme quantum regimes. The calculation exploits a procedure in quantum field theory known as dimensional regularization, or dimensional continuation as I will call it here. The basis idea is that divergent theories exhibit finite poles of the form \( 1/(\nu - 3) \) when analyzed in an arbitrary number of spatial dimensions \( \nu \). We can then manipulate finite quantities in such a way that we preserve the delicate relation between long and short distance physics. In a physical process, the divergent pole terms cancel and we can set the number of spatial dimensions to \( \nu = 3 \) at the end of the calculation, thereby giving a finite result with correct long and short distance physics.

To start the discussion, let \( x \) and \( v \) denote the \( \nu \)-dimensional position and velocity vectors of a particle. The Coulomb potential for two particles separated a distance \( r = |x - x'| \) is \( V_\nu(r) = C_\nu e^2/r^{\nu-2} \), where \( C_\nu = \Gamma(\nu/2 - 1)/4\pi^{\nu/2} \) is a spatially dependent geometric factor. This potential follows directly from a simple multidimensional generalization of Gauss’ Law. For every species \( b \), the single-particle distribution function \( f_b \) will be defined so that \( f_b(x, v, t) \, d\nu x d\nu v \) gives the number of particles of species \( b \) in a small hypervolume \( d\nu x \) about \( x \) and \( d\nu v \) about \( v \) at time \( t \). We shall take the plasma components to be Maxwell-Boltzmann, although with more work, the situation can be generalized to Fermi-Dirac statistics as well [3]. In the case of a projectile \( p \), the distribution function \( f_p \) will be peaked about a specific point in phase space, and the stopping power can then be calculated by

\[
\frac{dE_p}{dx} = \frac{1}{v_p} \frac{dE_p}{dt} = \frac{1}{v_p} \int d\nu' x d\nu' v \frac{p^2}{2m_p} \frac{\partial f_p}{\partial t} .
\]

(3.1)

Expression (3.1) is, of course, extremely problematic in three dimensions, but completely finite when \( \nu \neq 3 \). We can gain some additional insight into the divergence problem, however, by returning to three dimensions for a moment. In three dimensions, if we knew the exact form of \( f_p \) in the background of the other distribution functions \( f_b \) of the various plasma components, then Eq. (3.1) would be completely finite and well defined in three dimensions. Unfortunately, the requisite solution for \( f_p \) can only be obtained by solving the full BBGKY
hierarchy of kinetic equations, an impossible feat. The divergence problem in Eq. (3.1) arises only because we must use an approximation for \( f_p \), usually obtained by truncating the BBGKY equations to a first-order kinetic equation, such as the Boltzmann or Lenard-Balescu equation, and these approximates miss correlations that would otherwise render Eq. (3.1) finite in three spatial dimensions. Curiously, these divergences occur only for the Coulomb potential, and only then in three spatial dimensions!

These observations provide a path forward. Let us return to an arbitrary number of dimensions, where the Coulomb potential provides finite results. Let us also define multipoint correlation functions in a similar manner to the \( f \)'s, and in this way we can construct the full BBGKY hierarchy in an arbitrary number of dimensions. For simplicity I will drop the subscript on the distribution functions. In dimensions \( \nu > 3 \), the standard textbook derivation of the Boltzmann Equation (BE) goes through without the infrared divergent scattering kernel found in three dimensions, and I will denote this finite \( \nu \)-dimensional scattering kernel by the shorthand notation \( B_\nu[f] \). The important point to emphasize here is that the \( \nu \)-dimensional Coulomb potential \( V_\nu(r) \approx e^2/r^{\nu-2} \) emphasizes short-distance over long-distance physics when \( \nu > 3 \), and this means that the BBGKY hierarchy reduces to the Boltzmann equation to leading order in \( g \) in these dimensions:

\[
\text{BBGKY} \Rightarrow \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla x f = B_\nu[f] \quad \text{to LO in } g \text{ for } \nu > 3 .
\] (3.2)

Here, the \( \nu \)-dimensional spatial gradient has been denoted by \( \nabla x \). In dimensions \( \nu < 3 \), the standard textbook derivation of the Lenard-Balescu equation (LBE) goes through without the ultraviolet divergent scattering kernel found in three dimensions, and I will denote this finite \( \nu \)-dimensional scattering kernel by the shorthand notation \( L_\nu[f] \). In dimensions \( \nu < 3 \), the Coulomb potential \( V_\nu(r) \) emphasizes long-distance physics over short-distance effects, and consequently, to leading order in \( g \), the BBGKY hierarchy reduces to the Lenard-Balescu equation (LBE) in this spatial regime:

\[
\text{BBGKY} \Rightarrow \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla x f = L_\nu[f] \quad \text{to LO in } g \text{ for } \nu < 3 .
\] (3.3)

Space does not permit us to write down the exact forms of \( B_\nu[f] \) and \( L_\nu[f] \) here, but one may consult Ref. [1] for these expressions. These kinetic equations allow one to calculate the stopping power in \( \nu > 3 \) and \( \nu < 3 \), the results of which are presented in Sections 8 and 7 of Ref. [1], respectively. These calculations involve performing a series of momentum and wave number integrals in arbitrary dimensions \( \nu \), and they take form [1]

\[
\frac{dE^>_p}{dx} = \frac{1}{v_p} \int d\nu \frac{p^2}{2m_p} B_\nu[f] = H(\nu) \frac{g^2}{\nu - 3} + \mathcal{O}(\nu - 3) : \text{LO in } g \text{ when } \nu > 3 , \quad (3.4)
\]

\[
\frac{dE^<_p}{dx} = \frac{1}{v_p} \int d\nu \frac{p^2}{2m_p} L_\nu[f] = G(\nu) \frac{g^{\nu-1}}{3 - \nu} + \mathcal{O}(3 - \nu) : \text{LO in } g \text{ when } \nu < 3 . \quad (3.5)
\]
The analytic expressions for $H(\nu)$ and $G(\nu)$ are rather complicated, and space does not permit their reproduction here. In this paper, we are only interested in their analytic properties as a function of $\nu$. In particular, the coefficients $H(\nu)$ and $G(\nu)$ can be expanded in powers of $\epsilon = \nu - 3$, and we find

$$H(\nu) = -A + \epsilon H_1 + O(\epsilon^2) \quad \text{and} \quad G(\nu) = -A + \epsilon G_1 + O(\epsilon^2).$$

(3.6)

For our purposes, we do not require the exact forms of $H_1, G_1$, nor that of the leading term $A$. It is sufficient to note that the leading terms in Eq. (3.6) are equal, so that $H(\nu = 3) = G(\nu = 3)$. This is a fact that arises from the calculation itself, as it must, and it should be emphasized that this equality is not arbitrarily imposed by hand. It is a crucial point that the leading terms are identical, as this will allow the short- and long-distance poles to cancel, thereby giving a finite result.

Since the rates $dE_p^/> / dx$ of Eq. (3.4) and $dE_p^< / dx$ of Eq. (3.5) were calculated in mutually exclusive dimensional regimes, one might think that they cannot be compared. However, even though Eq. (3.5) was originally calculated in $\nu < 3$ for integer values of $\nu$, we can analytically continue the quantity $dE_p^< / dx$ to values $\nu > 3$ (for simplicity I have been omitting the functional dependence on $\nu$ from the stopping power, but it is there nonetheless). This process of analytic continuation to non-integer values of dimension is analogous to way in which the factorial function $n!$ on the positive integers can be generalized to the Gamma function $\Gamma(z)$ over the complex plane, including both the positive and negative real axes. We can then directly compare Eqs. (3.4) and (3.5). Upon writing the $g$-dependence of Eq. (3.5) as $g^{2+(\nu-3)}$, when $\nu > 3$ we see that Eq. (3.5) is indeed higher order in $g$ than Eq. (3.4):

$$\frac{dE_p^<}{dt} = -G(\nu) \frac{g^{2+(\nu-3)}}{\nu - 3} + O(\nu - 3) \quad \text{NLO in } g \text{ when } \nu > 3. \quad (3.7)$$

By power counting arguments, no powers of $g$ between $g^2$ and $g^{\nu-1}$ can occur in Eq. (3.4) for $\nu > 3$, and therefore Eq. (3.5) indeed provides the correct next-to-leading order term in $g$ when the dimension is analytically continued to $\nu > 3$. The individual pole-terms in Eqs. (3.4) and (3.7) will cancel giving a finite result when the leading and next-to-leading order terms are added. The resulting finite quantity will therefore be accurate to leading and next-to-leading order in $g$ as the $\nu \to 3$ limit is taken:

$$\frac{dE_p}{dx} = \lim_{\nu \to 3^+} \left[ \frac{dE_p^>}{dx}_{LO} + \frac{dE_p^<}{dx}_{NLO} \right] + O(g^3). \quad (3.8)$$

\[3\] For the related process of electron-ion temperature equilibration, in contrast, the expressions for $H(\nu)$ and $G(\nu)$ are quite simple.
FIG. 1: A comparison of the stopping power of BPS to Li and Petrasso (LIP) for a 3.5 MeV $\alpha$ particle in an equimolar DT plasma. The plasma temperature is $T = 3$ keV and the electron number density is $n_e = 1 \times 10^{25}$ cm$^{-3}$, giving a coupling $g = 0.01$. For BPS, the range and the energy deposited into ions and electrons are $R_{BPS}^\alpha = 30 \mu$m, $E_{iBPS}^\alpha = 0.38$ MeV, and $E_{eBPS}^\alpha = 3.16$ MeV, respectively. For LIP, these corresponding quantities are $R_{LIP}^\alpha = 25 \mu$m, $E_{iLIP}^\alpha = 0.45$ MeV, and $E_{eLIP}^\alpha = 3.09$ MeV. The percent differences between BPS and LIP are: $\Delta R_\alpha = +20\%$, $\Delta E_i = -16\%$, and $\Delta E_e = +2\%$, where $\Delta X \equiv (X_{BPS} - X_{LIP})/X_{LIP}$.

Note that this does not lead to any form of “double counting” since we are merely adding the next-to-leading order term (3.7) to the leading order term (3.4) at a common value of $\nu > 3$.

We are now in a position to evaluate the limit in Eq. (3.8). Defining $\epsilon = \nu - 3$ as before, note that $g^\epsilon = \exp\{\epsilon \ln g\} = 1 + \epsilon \ln g + \mathcal{O}(\epsilon^2)$, which gives the relation

$$
\frac{g^\epsilon}{\epsilon} = \frac{1}{\epsilon} + \ln g + \mathcal{O}(\epsilon) .
$$

(3.9)

Substituting Eq. (3.9) into Eq. (3.7), adding this result to Eq. (3.4), and then taking the limit gives

$$
\frac{dE_p}{dx} = -A g^2 \ln g + B g^2 + \mathcal{O}(g^3) ,
$$

(3.10)

with $B = H_1 - G_1$, in agreement with Eq. (2.7). In this way, BPS has calculated the charged particle stopping power accurate to leading order and next-to-leading order in $g$.

For completeness, the full BPS stopping power is presented in Appendix A.

As alluded to earlier, the BPS calculation predicts the range of the 3.5 MeV $\alpha$ particle in a hot DT plasma to be 20–30% longer than typical plasma models in the literature with a smaller energy deposition into the ions. For LIP and BPS, this is illustrated Figs. 1 and 2. As we shall see in the next section, the longer $\alpha$ particle range and less efficient ion heating tend to make ignition more difficult to achieve.
FIG. 2: As in Fig. 1, BPS vs. LIP for a 3.5 MeV α particle, in a DT plasma at $T = 30$ keV and $n_e = 1 \times 10^{27}$ cm$^{-3}$, giving a plasma coupling is $g = 0.003$; $R_{α}^{BPS} = 3.7 \mu$m, $E_{I}^{BPS} = 1.51$ MeV, and $E_{e}^{BPS} = 1.51$ MeV; $R_{α}^{LIP} = 2.9 \mu$m, $E_{I}^{LIP} = 2.15$ MeV, and $E_{e}^{LIP} = 1.36$ MeV; $ΔR_{α} = 28\%$, $ΔE_{I} = 7\%$, and $ΔE_{e} = 11\%$.

IV. NUMERICAL CONSIDERATIONS: EFFECTS OF STOPPING POWER ON IGNITION

Since this is a preliminary study, which I would like to keep as clean and simple as possible, I will not model a real ICF capsule here. Instead, I would like to look at the essential features of the stopping power without the complications of additional processes like hydrodynamic instabilities and thermal conductivity. I have already mentioned the latter problem, but I would like to make a few more comments. In a real ICF capsule, consistency requires that we model or calculate the thermal conductivity just as accurately as the stopping power. However, calculating the heat flow is harder than calculating the stopping power: one must not only contend with the short and long distance physics of the instantaneous Coulomb interactions, but one must also consider the rate of energy transport from one spatial location to another. It is not clear to me whether the Coulomb logarithm of the thermal conductivity can be calculated with the BPS methodology, since one must invert a heat kernel in addition to performing the appropriate multidimensional integrals. One must also consider the possibility of α particles emerging from the hot spot into the surrounding colder and more strongly coupled plasma, a region where the BPS stopping power may not apply. Including these effects is of course essential, but at this stage, they are somewhat premature.

For the purposes of this study, it suffices to concentrate only on the time period after maximum compression and minimum volume, near the onset of thermonuclear burn. Stopping power plays less of a role in the implosion process, which is set in motion by an assembly of high intensity lasers or pulsed-power diodes that generate a radiation field that ablates a
FIG. 3: The yield in MJ as a function of initial $\rho R$ in g/cm$^2$ for a DT microsphere of Ref. [5]. The initial uniform temperature is taken to be $T_0 = 3$ keV and the initial mass of the sphere is $m_0 = 100 \mu g$. The upper axis gives the initial density $\rho$ in g/cm$^3$. The dashed curve is the yield profile for the stopping power of Li and Petrasso (LIP), while the solid curve corresponds to that of Brown, Preston, and Singleton (BPS). The points labeled by (a) and (b) correspond to full yield 1% maximum yield, respectively. The points labeled by (c) and (c') correspond to 10% maximum yield, the first for LIP and the second for BPS. The BPS stopping power has a longer $\alpha$ particle range than LIP, and it delivers less $\alpha$ particle energy to the ions. This has the effect of increasing the ignition threshold for BPS relative to LIP.

Material shell (such as beryllium or plastic) encapsulating the DT gas. In fact, I will ignore the presence of a surrounding shell completely, and instead look at the idealized case of Fraley et al. [5]. These authors consider an initially static microsphere of compressed DT at uniform temperature $T_0$ and density $\rho_0$. Because of the non-zero pressure in the microsphere, the outermost surface of the spherical assembly acquires an outward velocity (comparable to the speed of sound in the DT plasma), which produces an incoming rarefaction wave. The net effect is that the center of the sphere remains slightly hotter than the surface, and consequently thermonuclear burn (if it occurs) will start at the center of the sphere and burn outward. The $\alpha$-particle from the fusion reaction $D + T \rightarrow \alpha + n$ will provide the bootstrap heating that will initiate ignition and maintain thermonuclear burn.

For definiteness, we will take the initial temperature of these so called Fraley spheres to be $T_0 = 3$ keV, with an initial mass $m_0 = 100 \mu g$. We shall vary the initial density $\rho_0$ and measure the final yield $Y$ produced in the simulation. To find the final state of the system, we numerically integrate the Navier-Stokes equations using a two-dimensional Lagrangian finite-
difference scheme. Radiative energy exchange is included via a multi-frequency diffusion approximation. Other transport processes are included as well, and plasma electrons and ions are each assumed to be characterized by separate Maxwellian thermal distributions, and exchange energy at a rate characterized by Spitzer theory. The results of these simulations are illustrated in Fig. 3, where we plot the yield in MJ as a function of initial $\rho R$ in g/cm$^2$. The dashed curve is the yield profile predicted by the LIP stopping power, while the solid curve corresponds to that of BPS. The points labeled by (a) and (b) correspond to full yield and 1% of full yield, respectively. When full burn is assured, as in case (a), note that the yield is relatively independent of the stopping power. The points labeled by (c) and (c') correspond to the 10% maximum yield threshold for LIP and BPS respectively. As we see, the effect of BPS relative to LIP is to push the 10% threshold from $(\rho R)_{LIP} = 2.2$ g/cm$^2$ to $(\rho R)_{BPS} = 2.4$ g/cm$^2$, an increase in $\rho R$ of approximately 10%.

V. ELECTRON-ION TEMPERATURE EQUILIBRATION

The final question I shall address is the effect of electron-ion temperature equilibration on the ignition threshold. The rate at which electrons and ions come into equilibrium,

$$\frac{\dot{E}_{ei}}{dt} = -C_{ei} (T_e - T_i),$$

was also calculated in Ref. [1] to the same level of rigor as the stopping power, and this is presented in Appendix B in full generality. In the high temperature limit that applies to ignition, the rate coefficient takes the simple form

$$C_{ei} = \frac{\omega_i^2}{2\pi \kappa_e} \frac{m_e}{2\pi T_e} \ln \Lambda_{BPS}, \quad \text{with} \quad \ln \Lambda_{BPS} = \frac{1}{2} \ln \left( \frac{8T_e^2}{\hbar^2 \omega_e^2} \right) - \gamma - 1,$$

where $\gamma = 0.57721 \cdots$ is the Euler constant, $\kappa_e$ and $\omega_e$ are the electron Debye wave number and plasma frequency, and $\omega_i^2 = \sum_i \omega_i^2$ is sum of the squares of the ion plasma frequencies.$^5$ Comparing Eqs. (5.2) and (2.3), we again see that there is no need to reference the heuristic scales $b_{min}$ and $b_{max}$. The BPS calculation predicts a smaller rate than typical model calculations in the literature. This is ostensibly advantageous for ignition, since the ion temperature is more weakly coupled to the electron temperature and can run away more easily;

---

$^4$ The BPS electron-ion coupling rate calculated in Ref. [1] was also used, and little difference between this near-exact result and Spitzer.

$^5$ Equation (5.2) corresponds to Eqs. (3.61) and (12.12) of Ref. [1], where I have taken this opportunity to correct a small transcription error: when passing from Eq. (12.43) to Eq. (12.44) in Ref. [1], a factor of $1/2$ was dropped. Restoring this factor of $1/2$ changes the additive constant outside the logarithm from the $-\gamma - 2$ that appears in Eq. (12.12) of Ref. [1] to the constant $-\gamma - 1$ in Eq. (5.2) of this paper.
however, initial simulations indicate that the electron-ion equilibration has little effect upon
ICF yield. Nonetheless, I would expect this rate to have some effect upon burn diagnostics,
and I will explore this possibility in the future.

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APPENDIX A: THE BPS CHARGED PARTICLE STOPPING POWER

As a matter of completeness, I will present the full form of the BPS stopping power
 calculated in Ref. [1]. Consider a projectile of mass $m_p$ and charge $e_p$, moving through a multi
component plasma at speed $v_p$. Each plasma component is assumed to be in equilibrium
with itself but not necessarily with the other components, and will be labeled by an index
$b$ (including the electron component). The mass of a component is $m_b$, the number density
is $n_b$, and the temperature is $T_b$. Temperature will be measured in energy units, and I will
use the notation $\beta_b = 1/T_b$ for the inverse temperature. To second order in the plasma
coupling $g$, the order to which we are working, the stopping power $dE/dx$ of the projectile
can meaningfully be broken into the contributions $dE_b/dx$ from the separate components, so
that $dE/dx = \sum_b dE_b/dx$. Each contribution can be expressed as a sum of three terms,

$$
\frac{dE_b}{dx} = \left( \frac{dE_{b,S}^C}{dx} + \frac{dE_{b,R}^C}{dx} \right) + \frac{dE_{b,Q}^C}{dx},
$$

(A1)

where the first two arise from classical short and long distance physics, and the latter term
from short distance two-body quantum diffraction. The stopping power $dE_b/dx$ is a function
of the parameters of the projectile and the plasma background. For convenience, however,
we shall emphasize the functional dependence only upon the projectile energy $E_p = \frac{1}{2}m_p v_p^2$. 
The terms in Eq. (A1) are given by Eqs. (3.4), (3.3) and (3.19), respectively, in BPS [1]:

\[
\frac{dE^C_{b,s}(E_p)}{dx} = \frac{e_p^2}{4\pi} \frac{\kappa_b^2}{m_p v_p} \left( \frac{m_b}{2\pi \beta_b} \right)^{1/2} \int_0^1 du \frac{u^{1/2} \exp \left\{ -\frac{1}{2} \beta_b m_b v_p^2 u \right\}}{1/u} \left[ -\ln \left\{ \frac{\beta_b e_p e_b K m_b}{4\pi m_{pb}} \frac{u}{1-u} \right\} + 2 - 2\gamma \left( \beta_b M_{pb} v_p^2 \right) + \frac{2}{u} \right]
\]

(A2)

\[
\frac{dE^C_{b,R}(E_p)}{dx} = \frac{e_p^2 i}{2\pi} \int d\cos \theta \frac{\rho_b(v_p \cos \theta)}{\rho_{total}(v_p \cos \theta)} F(v_p \cos \theta) \ln \left\{ \frac{F(v_p \cos \theta)}{K^2} \right\} + \frac{e_p^2}{2\pi} \frac{\rho_b(v_p)}{\rho_{total}(v_p)} \left[ F(v_p) \ln \left\{ \frac{F(v_p)}{K^2} \right\} - F^*(v_p) \ln \left\{ \frac{F^*(v_p)}{K^2} \right\} \right]
\]

(A3)

\[
\frac{dE^Q_{b}(E_p)}{dx} = \frac{e_p^2}{4\pi} \frac{\kappa_b^2}{2\beta_b m_p v_p^2} \int d v_{pb} \left[ 2\psi(1 + i\eta_{pb}) - \ln \eta_{pb}^2 \right]
\]

\[
\left[ 1 + \frac{M_{pb} v_p}{m_b v_{pb}} \left( \frac{1}{\beta_b m_b v_p v_{pb}} - 1 \right) \right] \exp \left\{ -\frac{1}{2} \beta_b m_b (v_p - v_{pb})^2 \right\} + \left[ 1 + \frac{M_{pb} v_p}{m_b v_{pb}} \left( \frac{1}{\beta_b m_b v_p v_{pb}} + 1 \right) \right] \exp \left\{ -\frac{1}{2} \beta_b m_b (v_p + v_{pb})^2 \right\}
\]

(A4)

In the long distance contribution (A3), we define \( \theta \) as the angle between the vectors \( k \) and \( v_p \), while \( F \) is equivalent to the plasma dielectric function,

\[
k^2 \epsilon(k, \omega = k \cdot v_p) = k^2 + F(v_p \cos \theta) \quad \text{in which} \quad F(v) = \int_{-\infty}^{\infty} du \frac{\rho_{total}(u)}{v - u + i\eta}, \quad \text{in which} \quad \rho_{total}(v) = \sum_b \rho_b(v) \quad \text{and} \quad \rho_b(v) = \kappa_b^2 \sqrt{\frac{\beta_b m_b}{2\pi}} v \exp \left\{ -\frac{1}{2} \beta_b m_b v^2 \right\}.
\]

(A5)

In Eq. (A4) the integration variable is

\[
v_{pb} = |v_p - v_b|
\]

(A7)

and the dimensionless quantum parameter is

\[
\eta_{pb} = \frac{e_p e_b}{4\pi \hbar v_{pb}}.
\]

(A8)

The Diagamma function is defined by \( \psi = \Gamma^{-1} d\Gamma/dz \), so that

\[
\text{Re} \psi(1 + i\eta) = \sum_{k=1}^{\infty} \frac{1}{k} \frac{\eta^2}{k^2 + \eta^2} - \gamma,\quad \text{in which} \quad \psi(1 + i\eta) = \gamma + \frac{1}{2} \ln(2\pi\eta) + \sum_{k=1}^{\infty} \frac{\psi^2}{\eta^2 + \psi^2}.
\]

(A9)
where $\gamma = 0.5572 \cdots$ is the Euler constant. The total mass and the reduced mass are

$$M_{pb} = m_p + m_b \quad \frac{1}{m_{pb}} = \frac{1}{m_p} + \frac{1}{m_b}.$$ \hspace{1cm} (A10)

The sum of terms (A3) and (A2) form the classical contribution, and the factor $K$ is an arbitrary wave number that cancels in the sum of Eqs. (A3) and (A2). It is convenient to set $K = \kappa_e$.

**APPENDIX B: THE BPS TEMPERATURE EQUILIBRATION RATE**

For the multi component plasma described in Appendix A, we assumed that the various components were in thermal equilibrium with themselves but not with each other. In practice, however, components exchange energy via Coulomb interactions, and will equilibrate according to a common temperature according to

$$\frac{dE_{ab}}{dt} = - C_{ab} (T_a - T_b). \hspace{1cm} (B1)$$

As with the stopping power, the rate coefficient can be written as a sum of three terms,

$$C_{ab} = \left(C_{ab,s}^C + C_{ab,R}^C \right) + C_{ab}^Q,$$ \hspace{1cm} (B2)

given by (12.31), (12.25), and (12.50) respectively in BPS [1]:

$$C_{ab,s}^C = -\kappa_a^2 \kappa_b^2 \left(\frac{\beta_a m_a \beta_b m_b}{\beta_a m_a + \beta_b m_b}\right)^{1/2} \left(\frac{1}{2\pi}\right)^{3/2} \left[ \ln \left\{ \frac{e_a e_b}{4\pi} \frac{K}{4 m_{ab} V_{ab}^2} \right\} + 2\gamma \right]$$ \hspace{1cm} (B3)

$$C_{ab,R}^C = \frac{\kappa_a^2 \kappa_b^2}{2\pi} \left(\frac{\beta_a m_a}{2\pi}\right)^{1/2} \left(\frac{\beta_b m_b}{2\pi}\right)^{1/2} \int_{-\infty}^{\infty} dv v^2 e^{-\frac{1}{4}(\beta_a m_a + \beta_b m_b)v^2} \frac{i}{2\pi} \frac{F(v)}{\rho_{total}(v)} \ln \left\{ \frac{F(v)}{K^2} \right\}$$ \hspace{1cm} (B4)

$$C_{ab}^Q = -\frac{1}{2} \kappa_a^2 \kappa_b^2 \left(\frac{\beta_a m_a \beta_b m_b}{\beta_a m_a + \beta_b m_b}\right)^{1/2} \left(\frac{1}{2\pi}\right)^{3/2} \int_0^{\infty} d\zeta e^{-\zeta/2} \left[ \text{Re} \psi \left(1 + i \frac{\bar{\eta}_{ab} \zeta}{1/2}\right) - \ln \left\{ \frac{\bar{\eta}_{ab} \zeta}{\zeta^{1/2}} \right\} \right].$$ \hspace{1cm} (B5)

The function $F(v)$ in Eq. (B4) is defined by Eqs. (A5) and (A6), and the strength of the quantum effects associated with the scattering of two plasma species $a$ and $b$ is characterized by the dimensionless parameter

$$\bar{\eta}_{ab} = \frac{e_a e_b}{4\pi \hbar V_{ab}},$$ \hspace{1cm} (B6)

where the square of the thermal velocity in this expression is defined by

$$V_{ab}^2 = \frac{T_a}{m_a} + \frac{T_b}{m_b}. \hspace{1cm} (B7)$$
In the limit $\beta_e m_e \ll \beta_i m_i$ we have

$$V_{ei}^2 = \frac{1}{\beta_e m_e}.$$  \hspace{1cm} (B8)

Upon taking the sum over ions, $C_{ei} = \sum_i C_{ei}$, and using the inequality $\beta_e m_e \ll \beta_i m_i$, we can express the rate coefficient as

$$C_{ei} = \kappa_e^2 \left( \frac{\beta_e m_e}{2\pi} \right)^{1/2} \frac{1}{2} \sum_i \omega_i^2 \left[ \ln \left\{ \frac{8T_i^2}{\hbar^2 \omega_e^2} \right\} - \gamma - 1 - \Delta_i(\bar{\eta}_{ei}) \right].$$  \hspace{1cm} (B9)

with

$$\Delta_i(\bar{\eta}_{ei}) = \int_0^\infty d\zeta e^{-\zeta/2} \left[ \text{Re} \psi \left( 1 + i \frac{\bar{\eta}_{ei}}{\zeta^{1/2}} \right) + \gamma \right].$$  \hspace{1cm} (B10)

In the extreme quantum limit the term $\Delta_i$ in Eq. (B10) vanishes, and we obtain Eq. (5.2).

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