Entanglement generation via phase-matched processes: different Bell states within the linewidth

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It is shown, theoretically and experimentally, that at any type-II spontaneous parametric down-conversion (SPDC) phase matching, the decoherence-free singlet Bell state is always present within the natural bandwidth and can be filtered out by a proper spectral selection. Instead of the frequency selection, one can perform time selection of the two-photon time amplitude at the output of a dispersive fibre. Applications to quantum communication are outlined.

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Type-II spontaneous parametric down-conversion (SPDC) provides the easiest way of generating polarization-entangled two-photon states, which find applications in quantum communication and quantum computation. When using polarization-entangled two-photon states in quantum communication protocols, the basic problem is their decoherence [1] due to the propagation of two-photon light through optical fibres or even through free space [2], if one takes into account the effect of the atmosphere. However, it has been shown [3, 4] that decoherence-free [3] propagation of polarization-entangled two-photon light can be achieved by using the singlet Bell state, $\Psi^-$, which is invariant to any polarization transformation.

Robustness against decoherence is not the only remarkable property of the singlet two-photon Bell state. Being antisymmetric with respect to the permutation of the two photons, it can be filtered out of the set of Bell states with the help of a 50% non-polarizing beamsplitter, due to the Hong-Ou-Mandel ‘dip’ effect [5]. Because of this property, it was the basic tool in the first quantum teleportation experiments [6]. This state is unpolarized not only in the usual sense (considering the intensity moments) but also with respect to all higher-order intensity moments [8]. Its ‘bright’ (high-photon-number) analogue is predicted to have the properties of ‘scalar light’, for which the fluctuations of all Stokes parameters are suppressed below the shot-noise limit [8].

There are two basic types of polarization-entangled singlet two-photon state [10]. Historically, the first method to efficiently prepare the singlet state was based on non-collinear type-II SPDC [11]. Later, it was suggested to produce the same state by using two type-I SPDC crystals under non-collinear phase matching [12]. In both these cases, the state $\Psi^-$ is prepared in two different spatial modes and can be called a polarization-wavevector entangled state. The second type of a singlet state is a polarization-frequency entangled state, that is, a polarization-entangled state in two different frequency modes. This can be done by using two type-I SPDC crystals under collinear frequency-nondegenerate phasematching [8, 13]. For quantum communication applications this type of the singlet state is preferable since both photons of an entangled pair can be easily transmitted through the same optical fibre and undergo the same polarization changes.

It is important to point out that to achieve polarization entanglement, the schemes based on type-II SPDC are always provided with a birefringent crystal compensating the e-o delay $\tau_0$ between the orthogonally polarized photons of a single pair. Alternatively, one can use interferometric schemes based on two type-I or two type-II crystals [14].

In this paper we show that the o-e delay $\tau_0$ between the signal and idler photons, which is caused by the nonlinear crystal and requires certain efforts to be eliminated, can be actually quite helpful. Namely, it turns out that the state $\Psi^-$ is always produced within the natural bandwidth of type-II frequency-degenerate SPDC, and it is the $\tau_0$ delay that is responsible for creating $\Psi^-$. This fact, beyond increasing the understanding of generation and manipulation of optical entangled states, can have important applications to quantum communication.

Consider collinear frequency-degenerate type-II SPDC from a continuous wave (cw) pump. In the low-gain regime the state of the two-photon light generated this way can be represented as a superposition of a vacuum state and a two-photon state given by the integral over the SPDC frequency spectrum,

$$\langle\Psi\rangle = \langle \text{vac}\rangle + \int d\Omega F(\Omega) |a_H^\dagger (\omega_0 + \Omega) a_V^\dagger (\omega_0 - \Omega) e^{i\Omega \tau_0}\rangle + a_V^\dagger (\omega_0 + \Omega) a_H^\dagger (\omega_0 - \Omega) e^{-i\Omega \tau_0}|\text{vac}\rangle,$$

where $\langle\Psi\rangle$ is the state of the two-photon light generated by the integral over the SPDC frequency spectrum.
where \( \omega_0 = \omega_p / 2 \), \( \omega_p \) is the pump frequency, \( a_H^\dagger \) and \( a_V^\dagger \) are the photon creation operators in the horizontal and vertical polarization modes (denoted by \( H, V \)). Since orthogonally polarized photons have different group velocities when propagating through the crystal, the phase factor \( e^{\pm i \theta / \Delta L} \) appears, where \( \tau_0 = DL / 2 \) is the mean temporal delay between orthogonally polarized photons, \( D \equiv 1 / u_H - 1 / u_V \) is the difference of the inverse group velocities and \( L \), the length of the crystal. The spectral amplitude of the state has the form

\[
F(\Omega) = \frac{\sin(\Omega \tau_0)}{\Omega \tau_0}.
\]

In existing experiments with frequency-degenerate SPDC and frequency selection, it is always the frequency corresponding to exact degeneracy, \( \Omega = 0 \), that is selected. The resulting two-photon state, a factorized one, \( a_H^\dagger(\omega_0)a_V^\dagger(\omega_0) \), can be turned into \( \Psi^+ \) by splitting the beam on a beamsplitter \[15\]. However, if one considers a small frequency shift from the exact degeneracy condition, \( \Omega = \pi / 2 \tau_0 \), then the two-photon part of the state \[14\] is

\[
\Psi^- \equiv F(\pi / 2 \tau_0)[a_H^\dagger(\omega_1)a_V^\dagger(\omega_2) - a_V^\dagger(\omega_1)a_H^\dagger(\omega_2)],
\]

where \( \omega_1 = \omega_0 - \pi / 2 \tau_0 \) and \( \omega_2 = \omega_0 + \pi / 2 \tau_0 \). The square modulus of the two-photon amplitude, which gives the total number of photon pairs, is in this case 0.41; it means that the singlet Bell state is produced with the efficiency almost twice higher than the \( \Psi^+ \) state is produced using a filter and a beamsplitter \[13\]. Therefore, the singlet Bell state is present within the natural bandwidth of any type-II SPDC spectrum and can be filtered out using a proper spectral selection.

In our experiment we demonstrate the generation of the \( \Psi^- \) state on the slopes of type-II SPDC spectral line by selecting the frequency of one of the photons using a monochromator. The setup is shown in Fig.1. Two-photon light was generated via spontaneous parametric down-conversion by pumping a type-II 0.5 mm \( \beta \)-barium borate crystal (BBO) with 0.45 Watt Ar\textsuperscript{+} cw laser beam at the wavelength 351 nm in the collinear frequency-degenerate regime. It is important that no birefringent material was inserted after the crystal to compensate for the e-o delay. The pump laser beam was eliminated by a highly reflecting UV mirror and the SPDC radiation was addressed to a 50/50 non-polarizing beamsplitter. To perform polarization selection, two Glan prisms were placed at the output ports of the beamsplitter. The spectral distribution of the coincidences was analyzed with a diffractive-grating monochromator with the resolution 0.8 nm placed in one of the output ports of the beamsplitter. Since the pump was cw, frequency selection in one output port automatically selected the frequency of the correlated photon. In order to reduce the contribution of accidental coincidences, a broadband interference filter centered around 702 nm was placed after the beamsplitter. Its transmission band (FWHM=40nm) was wider than the natural width of the SPDC spectrum under given experimental conditions (FWHM=12nm). Biphoton pairs were registered by two photodetection apparatuses, consisting of red-glass filters, pinholes, focusing lenses and avalanche photodiodes (PerkinElmer single-photon counting modules). The photocount pulses of the two detectors, after passing through delay lines, were sent to the START and STOP inputs of a Time-to-Amplitude Converter (TAC). The output of the TAC was finally addressed to a Multi-Channel Analyzer (MCA), and the number of coincidences of photocounts of the two detectors was observed at the MCA output.

First, we studied the dependence of the coincidences counting rate on the wavelength selected by the monochromator. In the absence of the Glan prisms, we obtained the usual type-II SPDC spectrum with a FWHM of 12 nm. If two Glan prisms oriented at angles \( \theta_1, \theta_2 \) are inserted in the beamsplitter output ports, the coincidence counting rate \( R_c \) should depend on the selected frequency offset \( \Omega \) from exact degeneracy as \[17\]

\[
R_c = \frac{\sin^2(\Omega \tau_0)}{(\Omega \tau_0)^2}[\sin^2(\theta_1 + \theta_2)\cos^2(\Omega \tau_0) + \sin^2(\theta_1 - \theta_2)\sin^2(\Omega \tau_0)].
\]

Typical dependencies of \( R_c \) on \( \Omega \) at \( \theta_1 = \pi / 4 \) and different values of \( \theta_2 \) are shown in Fig.2. This behaviour demonstrates two-photon interference, which in this case manifests itself within the lineshape of SPDC frequency spectrum, as it was indeed predicted in Ref. \[17\].

The interference is observed most clearly for two orientations of the Glan prism in channel 2: at \( \theta_2 = 45^\circ \) and at \( \theta_2 = -45^\circ \). The experimental dependencies obtained for these cases are shown in Fig.3: in perfect agreement with formula \[14\], a maximum is observed at the center of the spectrum for the \( (45^\circ, 45^\circ) \) orientations of the Glan prisms and a minimum, for \( (45^\circ, -45^\circ) \) orientations.

From the theoretical calculation (Fig.2), as well as from the experimental spectra (Fig.3), one can see that with certain wavelengths selected, polarization interference with high visibility is observed under the rotation of the Glan prism in channel 2. This fact is well-known for the case where the selected wavelength is the central one. However, from Fig.3 we deduce that high-visibility polarization interference also takes place when the selected wavelength is 695.5 nm or 708.5 nm. Both cases correspond to the selection of the \( |\Psi^-\rangle \) state.

To confirm this fact experimentally, we have measured the polarization interference for the \(|\Psi^-\rangle\) state: the wavelength transmitted by the monochromator was fixed at 708.5 nm, and the coincidence counting rate was measured depending on the orientation of one of the polarizers, the other polarizer being oriented at \( \theta_1 = 45^\circ \). The
In order to prove the invariance of the produced $|\Psi^-(\tau)\rangle$ state under polarization transformation of the measurement basis we have placed quarter- and half-wave plates (QWP and HWP) at various orientations in front of the beamsplitter. In particular, if a HWP is placed in front of the beamsplitter and its orientation is changed, the dependencies similar to those shown in Fig. 2 transform as shown in Fig. 5(a,b), where both the theoretical curve and the experimental data are presented. As expected from the theoretical prediction, the experimental data prove that the coincidence counting rate corresponding to the wavelengths 695 nm and 708.5 nm (the $|\Psi^-(\tau)\rangle$ state) both at the positions of the Glan prisms ($45^\circ, 45^\circ$) and ($45^\circ, -45^\circ$) does not change depending on the HWP orientation, i.e., depending on the rotation of the birefringent polarization state before the beamsplitter. At all other wavelengths (including the central one, 702 nm), the coincidence counting rate clearly depends on the HWP orientation. Similar behaviour is observed for a QWP inserted before the beamsplitter.

The conclusion is that the frequency spectrum of collinear frequency-degenerate type-II SPDC contains, at about half-width from the center, the decoherence-free singlet Bell state $|\Psi^-(\tau)\rangle$. In order to use this state in quantum communications, one should select a relatively narrow frequency selection. This means a certain experimental difficulty, which, however, can be easily overcome if the produced state is to be transmitted through optical fibres.

Indeed, as it was shown in Refs. [16, 17], when a two-photon state is transmitted through an optical fibre, due to the group-velocity dispersion (GVD) the shape of the spectral amplitude is transferred into the shape of the two-photon time amplitude and hence, into the distribution of the time interval between the arrivals of two photons of a pair. The frequency argument of the spectral two-photon amplitude will then be transformed into the time argument of the time two-photon amplitude as

$$\Omega \rightarrow \tau = \frac{1}{2} k'' z \Omega,$$

where $k''$ is the second derivative of the dispersion law and $z$ is the fibre length. Therefore, the frequency selection of the $|\Psi^-(\tau)\rangle$ state can be performed through the time selection of the delay between registering two photons. This can be done by selecting events corresponding to certain MCA channels. For instance, if a 1 km of single-mode fibre with $k'' = 3.2 \times 10^{-28} s^2/cm$ is inserted before the beamsplitter, selection of the $|\Psi^-(\tau)\rangle$ state is provided by choosing coincidence events with the delay between the signal and idler photons being $\pm 3 \text{ ns}$.

This fact can be used in quantum communications whenever propagation of polarization-entangled photons through optical fibres is involved. On the one hand, the receiver can always post-select the singlet state by picking only those coincidence events for which the signal and idler photons come with a fixed nonzero delay $\tau = \pi k'' z/\tau_0$. On the other hand, polarization drift introduced by the fibre will never influence the entangled state $|\Psi^-(\tau)\rangle$.

Finally, we would like to notice that in order to implement any protocol of information transmission [1], one should be able to generate another entangled state at the same frequencies as the singlet state $|\Psi^-(\tau)\rangle$. In particular, the state $|\Psi^+(\tau)\rangle$ can be easily created at the same frequencies by introducing after the crystal a birefringent material twice increasing the $\tau_0$ delay. This can be done by inserting a quartz plate with the same thickness as it is necessary to compensate for the $\tau_0$ delay, but with the optic axis oriented orthogonally [18].

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FIG. 1: Experimental setup for observing two-photon polarization interference within the lineshape of the two-photon spectral amplitude. A type-II BBO crystal cut for collinear frequency-degenerate phasematching is pumped by cw $\text{Ar}^+$ laser at 351 nm; NPBS is a 50/50 nonpolarizing beamsplitter; P1 and P2 are linear polarization filters (Glan prisms); D1, D2 are single-photon counting modules with outputs connected to the coincidence counter. A diffraction-grating monochromator is placed in one arm for the frequency selection; retardation plates (QWP and HWP) are used to study the invariance of the Bell state $|\Psi^-\rangle$ under polarization transformations.

FIG. 2: Theoretical dependence of the coincidences counting rate on the parameter $\Omega \tau_0$ for various orientations $\theta_2$ of the Glan prism in channel 2: $(0^\circ, \pm 22.5^\circ, \pm 45^\circ)$. The Glan prism in channel 1 is fixed at $\theta_1 = 45^\circ$.

[15] Because in half of the cases, both photons go to the same output, the state is produced with 50% loss.
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[18] On the other hand, $\phi^\pm$ could be generated by applying a polarization rotation on one arm after beam splitting.
FIG. 3: Experimental dependence of the coincidence counting rate on the wavelength selected by the monochromator for two cases: \( \theta_1 = \theta_2 = 45^\circ \) (squares, solid line) and \( \theta_1 = 45^\circ, \theta_2 = -45^\circ \) (triangles, dashed line). The lines represent the theoretical fit to the experimental data.

FIG. 4: Polarization interference fringes for the singlet Bell state \( |\Psi^-\rangle \) (the selected wavelength is \( \lambda = 708.5\text{nm} \)). Solid line represents the theoretical fit to the experimental data.

FIG. 5: Experimental dependence of the coincidence counting rate on the wavelength selected by the monochromator for the following orientations of the HWP placed after the crystal: \( 7^\circ \) (circles, dotted line); \( 17^\circ \) (squares, dashed line); \( 22.5^\circ \) (triangles, solid line). The orientations of the Glan prisms being \( 45^\circ, 45^\circ \) (a) and \( 45^\circ, -45^\circ \) (b). The high visibility of polarization interference corresponding to the wavelengths \( \lambda = 695.5\text{nm} \) and \( \lambda = 708.5\text{nm} \) (highlighted with vertical dashed bars) confirms the invariance of the \( |\Psi^-\rangle \) state to polarization rotation. The lines represent the theoretical fit to the experimental data.