Construction of 6D supersymmetric field models in \( \mathcal{N} = (1,0) \) harmonic superspace

I.L. Buchbinder\textsuperscript{a}, N.G. Pletnev\textsuperscript{b}

\textsuperscript{a}Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk, 634061 Russia; joseph@tspu.edu.ru
and
National Research Tomsk State University, Tomsk, 634050 Russia

\textsuperscript{b}Department of Theoretical Physics, Sobolev Institute of Mathematics and National Research Novosibirsk State University, Novosibirsk, 630090 Russia; pletnev@math.nsc.ru

Abstract

We consider the six dimensional hypermultiplet, vector and tensor multiplet models in (1,0) harmonic superspace and discuss the corresponding superfield actions. The actions for free (2,0) tensor multiplet and for interacting vector/tensor multiplet system are constructed. Using the superfield formulation of the hypermultiplet coupled to the vector/tensor system we develop an approach to calculation of the one-loop superfield effective action and find its divergent structure.

1 Introduction

Construction of the non-Abelian (1,0) and (2,0) superconformal theories in 6D has attracted much attention for a long time (see e.g. [1], [2]). Such models can be considered as the candidates for dual gauge theories of the interacting multiple M5-branes [3] and can be related to near-horizon AdS\(_7\) geometries. A crucial ingredient of this construction is the non-Abelian tensor multiplet gauge fields [4]\(^1\). A solution to this problem has been found [6] in the framework of a tensor hierarchy [7] which, besides the Yang-Mills gauge field and the two-form gauge potentials of the tensor multiplet, contains the non-propagating three- and four-forms gauge potentials. Construction of the (2,0) models can be realized on the base of coupling the (1,0) non-Abelian tensor/vector models to the superconformal hypermultiplets [8]. Also we point out the work [9] where the Killing spinor equations of 6-dimensional (1,0) superconformal theories have been solved and the solutions for the configuration of the background fields preserving 1, 2, 4 and 8 supersymmetries have been found.

\(^1\)Recently, a number of papers devoted to constructing the class of non-Abelian superconformal (1,0) and (2,0) theories in six dimensions has appeared (see [5] and references therein). These works were inspired by papers [2], which explored the 3-algebra gauge structure and used a non-propagating vector field of negative scaling dimension which transforms nontrivially under the non-Abelian gauge symmetry.
Superfield formulation of the tensor hierarchy has been studied in the paper [10] where a set of constraints on the super-\((p + 1)\)-form field strengths of non-Abelian super-\(p\)-form potentials in \((1,0)\) \(D6\) superspace has been proposed. These constraints restrict the field content of the super-\(p\)-forms to the fields of the non-Abelian tensor hierarchy. The superfield formulation of the tensor hierarchy sheds light on a supersymmetric structure of the theory and can serve as a base for the various generalizations. They can be useful for searching the superfield action [11] and for studying the \((2,0)\) superconformal theory by superspace methods. However actually the superfield Lagrangian formulation of the theory under consideration has not been constructed so far.

In the given paper we develop the superfield methods for studying the open problems related to superfield formulating the vector/tensor system and calculating the quantum effective action. Our consideration is based on harmonic superspace technique formulated for four dimensions in [12], [13] and extended to six dimensions in [14], [15]. The superfield realization of the unitary representations of \((n, 0)\) superconformal algebras \(OSp(8^* / 2n)\) in six dimensions [16] has been found in [17] and it was shown that the \(D6, (1,0)\) and \((2,0)\) tensor multiplets are described by the analytic superfields in appropriately defined harmonic superspaces [18]. In this paper we demonstrate that a harmonic superspace formalism can be efficiently implemented for superfield Lagrangian construction of the tensor hierarchy models.

The paper is organized as follows. Section 2 is devoted to basic notations of \(6D\) harmonic superspace. In Section 3 we review the superfield formulations of \(6D (1,0)\) hypermultiplet [15], vector multiplet [14] and tensor multiplet [19], in harmonic superspace. We also discuss the structure of \((2,0)\) tensor multiplet. The material of Section 3 is used in the next Sections to formulate the new superfield models. Section 4 is devoted to superfield Lagrangian construction of the \((2,0)\) tensor multiplet in terms of \((1,0)\) hypermultiplet and \((1,0)\) tensor multiplet. In Section 5 we study the superfield Lagrangian formulation for non-Abelian vector/tensor system. We begin with a harmonic superspace reformulation of the results of paper [10], then we propose the superfield action for superconformal models of tensor hierarchy and, using the results of the previous part of Section 5, we derive the component structure of the superfield action and show that it coincides with component Lagrangian constructed in [6]. Section 6 demonstrates a power of superfield methods. It is devoted to studying the quantum effective action in \((1,0)\) hypermultiplet theory coupled to Abelian vector/tensor system. We develop the superfield proper time technique, allowing to calculate the effective action in manifestly supersymmetric and gauge invariant form, and calculate the divergent part of the effective action. It is proved that this divergent part contains a term, providing the charge renormalization in the vector/tensor action from Section 5, and a higher derivative action, found in [20]. In Conclusion we will summarize the results obtained. In Appendix A we describe the basic notations and conventions of \(6D\) supersymmetry. Appendix B contains some details of deriving the component action from superfield action of vector/tensor system.
2 6D, $\mathcal{N} = (1, 0)$ harmonic superspace

Harmonic superspace is a powerful formalism for off-shell construction of extended supersymmetric field theories in four and six dimensions [12], [13], [14], [15]. In this section we briefly describe the basic notations and conventions which are used in the paper (see the details of $D = 6$ superspace e.g. in [21]).

It is well known that in six dimensions there exists two independent supersymmetry generators, therefore the representations of the 6D superalgebra are defined by two integers $(p, q)$ [21]. The corresponding supersymmetries are in general denoted as $\mathcal{N} = (p, q)$ or simply $(p, q)$. In this paper we will construct the harmonic superfield models corresponding to $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (2, 0)$ supersymmetries.

The six-dimensional superspace is parameterized by the coordinates $z = \{x^{\alpha\beta} \equiv x^m \gamma_{m}^{\alpha\beta}, \theta_i^o\}$. Here the odd coordinates $\theta_i^o$ $(\alpha = 1, \ldots, 4)$ are the right-handed chiral spinors of the group $SU^\#(4) \sim SO(1, 5)$ (left-handed spinors are denoted $\psi_a$). The index $I$ is spinor one of the group $USp(2n)$ (below we use $n = 1, 2$) and $n$ corresponds to the $\mathcal{N} = (n, 0)$ supersymmetry. The properties of the matrices $\gamma_{m}^{\alpha\beta}$ are given in the Appendix A. The index $I$ is raised and lowered with the help of the $USp(2n)$ matrix $\Omega^{IJ}$ (see the properties of this matrix e.g. in [25]), $\psi_I = \Omega_{IJ} \psi^J$, $\psi^I = \Omega^{IJ} \psi_J$, $\Omega^{IJ} \Omega_{JK} = \delta^I_K$. The Grassmann coordinates obey the reality condition $\bar{\theta}^I = \theta^{0I} = \Omega^{IJ} \theta^J$. The basic spinor derivatives of the 6D, $\mathcal{N} = (n, 0)$ superspace are

$$D^I_\alpha = \frac{\partial}{\partial \theta^I_\alpha} - i \theta^I_\alpha \frac{\partial}{\theta^I_\alpha} \{D^I_\alpha, D^J_\beta\} = -2i \Omega^{IJ} \gamma_{m}^{\alpha\beta} \partial_m.$$

The symmetry group of the superspace involves $USp(2n)$ transformations of the R-symmetry.

The harmonic $D6$, $\mathcal{N} = (1, 0)$ superspace was introduced by [14], [15], [20] and it is parameterized by the coordinates $(x^m, \theta^\alpha, u^\pm)$, where harmonics $u^\pm (\tilde{u}_i^\pm = u^\pm, u^+ u^- = 1(i = 1, 2))$ live on the coset R-symmetry of the group $SU(2)/U(1)$. Besides the standard (or central) basis $(x^m, \theta^\alpha, u^\pm)$ one can introduce the analytical basis $(z^m_A, \theta^{+\alpha}, u^\pm, \theta^{-\alpha})$:

$$x^\alpha_A = x^\alpha + i \theta^- \gamma^\alpha \theta^+, \quad \theta^{+\alpha} = u^\pm \theta^{\alpha}.$$  

The important property of the coordinates $z^m_A, u^\pm$ is that they form a subspace closed under $\mathcal{N} = (1, 0)$ supersymmetry transformations. The covariant harmonic derivatives which form the Lie algebra of $SU(2)$ group ([$D^{++}, D^{--}] = D^0$) in the analytic basis have the form

$$D^{++} = u^+ \frac{\partial}{\partial u^{-i}} + i \theta^+ \frac{\partial}{\theta^{+\alpha}} + \theta^{+\alpha} \frac{\partial}{\theta^{-\alpha}}, \quad D^{--} = u^- \frac{\partial}{\partial u^{+i}} + i \theta^- \frac{\partial}{\theta^{+\alpha}} - \theta^{-\alpha} \frac{\partial}{\theta^{-\alpha}}, \quad D^0 = u^+ \frac{\partial}{\partial u^{+i}} - u^- \frac{\partial}{\partial u^{-i}} + \theta^{+\alpha} \frac{\partial}{\theta^{+\alpha}} - \theta^{-\alpha} \frac{\partial}{\theta^{-\alpha}}. $$

2 Harmonic superspace is closely related to projective superspace which is also successfully applied to off-shell formulations of extended supersymmetric theories (see e.g the recent papers [22], [23], [24]).
Using the analytic subspace one can define the analytical superfields, which do not depend on $\theta^{-\alpha}$, i.e. satisfy the condition of the Grassmann analyticity $D^+_\alpha \phi = 0$, where the spinor derivatives in the analytic basis have the form

$$
D^+_\alpha = \frac{\partial}{\partial \theta^{-\alpha}}, \quad D^-_\alpha = -\frac{\partial}{\partial \theta^{+\alpha}} - 2i\partial_{\alpha\beta}\theta^{-\beta}, \quad \{D^+_\alpha, D^-_\beta\} = 2i\partial_{\alpha\beta}.
$$

The possibility to formulate the theory in terms of G-analytic superfields is a crucial advantage of the harmonic superspace formalism \(^3\).

3 Harmonic superfields and their interactions

It is known that the massless conformal (1,0) and (2,0) superfields in six dimensions are divided into two classes: (i) the superfields whose first component carries any spin but it is an $USp(2n)$ singlet; (ii) the 'ultrashort' analytic superfields in harmonic superspace, their first component is a Lorentz scalar but it carries $USp(2n)$ indices \([17]\). All these superfields satisfy some superspace constrains. In this paper we consider the simplest superfields from above both classes, corresponding to the following three types of (1,0) 6D multiplets: the hypermultiplet, the vector multiplet and the tensor multiplet.

3.1 Hypermultiplet

The (1,0) and (2,0) hypermultiplets are described by the superfields $q^I(x, \theta)$ and their conjugate $\bar{q}_I(x, \theta)$, $\bar{q}_I = (q^I)^\dagger$, both in the fundamental representation of $USp(2n)$ group. Here $i = 1, 2$ for (1,0) case and $I = 1, \ldots, 4$ for (2,0) case. The corresponding constraint is

$$
D^{(I} q^{J)}(x, \theta) = 0 .
$$

In the case of $\mathcal{N} = (1,0)$ supersymmetry, the superfield $q^i(x, \theta)$ has a short expansion $q^i(z) = f^i(x) + \theta^{\alpha i} \psi_\alpha(x) + \ldots$. The doublet of scalars $f^i$ and the spinor $\psi_\alpha$ satisfy the equations $\Box f^i = 0$, $\partial^{\alpha \beta} \psi_\beta = 0$. As a result, the $\mathcal{N} = (1,0)$ hypermultiplet in six dimensions has 4 bosonic+4 fermionic real degrees of freedom.

Off-shell Lagrangian formulation of the hypermultiplet is based on use of the analytic superfields in harmonic superspace. In this formulation the hypermultiplet is described by an unconstrained analytic superfield $q^+_A(\zeta, u)$ satisfying the reality condition $(q^{+A}) \equiv q^+_A = \varepsilon_{AB} q^{+B}$

$$
D^+_\alpha q^+_A(\zeta, u) = 0 .
$$

\(^3\)In some cases it may be helpful to use an anti-analytic basis in which

$$
x^+ = x^a - i\theta^+ \gamma^a \theta^-, \quad D^- = -\frac{\partial}{\partial \theta^{+a}}, \quad D^+ = \frac{\partial}{\partial \theta^{-a}} - 2i\partial_{\alpha\beta}\theta^{+\beta}
$$

$$
D^{++} = u^{-i} \frac{\partial}{\partial u^{-i}} + \theta^{+\alpha} \frac{\partial}{\partial \theta^{-\alpha}} - i\theta^{+\alpha} \partial_{\theta^{-\alpha}}, \quad D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}} + \theta^{-\alpha} \frac{\partial}{\partial \theta^{+\alpha}} - i\theta^{-\alpha} \partial_{\theta^{+\alpha}}
$$

4
Here $A = 1, 2$ is a Pauli-Gürsey index lowered and raised by $\varepsilon_{AB}, \varepsilon^{AB}$. After expansion of $q^+ \theta^+$ and $u$ we obtain an infinite set of auxiliary fields which vanish on-shell due to the equations of motion
\[ D^{++} q^+(\zeta, u) = 0 . \] (3.3)

The equations of motion follows from the action
\[ S_q = -\frac{1}{2} \int d\zeta (-4) du q^+ A D^{++} q^+_A . \] (3.4)

Here $d\zeta (-4) = d^6xd^4\theta^+$. This formulation allows us to write down the most general self-couplings in the form of the arbitrary potential $\mathcal{L}^{(4)}(q^+, \bar{q}^+)$ [13]. The corresponding sigma models have the complex hyper-Kähler manifolds as their target manifolds [26].

### 3.2 Vector multiplet

The off-shell $(1,0)$ non-Abelian vector supermultiplet is realized in $6D$ conventional superspace as follows $^4$. As usual ones introduce the gauge-covariant derivatives
\[ \mathcal{D}_M = D_M + A_M, \quad [\mathcal{D}_M, \mathcal{D}_N] = T^L_{MN} \mathcal{D}_L + F_{MN}, \]

with $D_M = \{D_m, D^i_{\alpha}\}$ be the flat covariant derivatives obeying the anti-commutation relations (2.1), and $A_M$ be the gauge connection taking the values in the Lie algebra of the gauge group. The gauge-covariant derivatives under consideration obey the constraints $F_{\alpha\beta} = 0$ and
\[ \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -2i\varepsilon^{ij}\mathcal{D}_{\alpha\beta}, \quad [\mathcal{D}_\gamma, \mathcal{D}_{\alpha\beta}] = -2i\varepsilon_{\alpha\beta\gamma\delta} W^{i\delta} . \] (3.5)

Here $W^{i\alpha}$ is the superfield strength of the anti-Hermitian superfield gauge potential obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace. In this case, the integrability condition $\{\mathcal{D}_\alpha^+, \mathcal{D}_\beta^+\} = 0$ yields $\mathcal{D}_\alpha^+ = e^{-ib} D^+_{\alpha} e^{ib}$ with some Lie-algebra valued harmonic superfield $b(z, u)$ of zero harmonic U(1) charge. In the $\lambda$-frame, the spinor covariant derivatives $\mathcal{D}_\alpha^+$ coincide with the flat ones, $\mathcal{D}_\alpha^+ = D^+_\alpha = \frac{\partial}{\partial \theta^+} - \alpha$, while the harmonic covariant derivatives acquire the connection $V^{++}$,
\[ \mathcal{D}^{++} = D^{++} + V^{++} . \] (3.6)

Namely this connection is an unconstrained analytic potential of the theory. In the Wess-Zumino gauge, the component expansion of $V^{++}(\zeta, u)$
\[ V^{++}_{WZ} = \theta^{+\alpha} \theta^{+\beta} A_{\alpha\beta}(x_{A}) + (\theta^+)^3 \lambda^{-\alpha}(x_{A}) + 3(\theta^+)^4 Y^{--}(x_{A}) , \] (3.7)

involves the physical gauge fields and the auxiliary fields.

$^4$An incomplete list of the references includes [14], [15], [20], [21], [27], [28].
The other non-analytic harmonic connection \( V^{--}(z,u) \) is uniquely determined in terms of \( V^{++} \) as a solution of the zero-curvature condition \([27], [13]\)

\[
D^{++} V^{--} - D^{--} V^{++} + [V^{++}, V^{--}] = 0.
\] (3.8)

The connection \( V^{--} \) transforms as \( \delta V^{--} = -D^{--} \Lambda \) under gauge transformations. Here the gauge parameter \( \Lambda \) is an analytic anti-Hermitian superfield.

Using the connection \( V^{--} \) one can build the spinor and the vector superfield connections as follows

\[
A^{-}_{\alpha} = -D^{-}_{\alpha} V^{--}, \quad A_{\alpha\beta} = \frac{i}{2} D^{-}_{\alpha} D^{+}_{\beta} V^{--}.
\]

This yields

\[
W^{+\alpha}_{\lambda} = -\frac{1}{4} (D^{+})^{3\alpha} V^{--},
\] (3.9)

where \( W^{\alpha}_{\lambda} \) is the field strength in the \( \lambda \)-frame. The Bianchi identities lead to relations

\[
D^{-}_{\alpha} W^{+\alpha} = D^{+}_{\alpha} W^{+\alpha}, \quad D^{\pm}_{\alpha} F_{ab} = i D_{[a} (\gamma_{b])_{\alpha\beta} W^{\pm\beta}.
\] (3.10)

The vector superfield strength is defined as follows \( F^{\beta}_{\alpha} = (D^{-}_{\alpha} W^{+\beta} - D^{+}_{\alpha} W^{+\beta}) \). The other useful consequence of the Bianchi identities are

\[
D^{+}_{\beta} W^{+\alpha} = \frac{1}{4} \delta^{\alpha}_{\beta} Y^{++}, \quad Y^{++} = -(D^{+})^{4} V^{--}, \quad D^{++} Y^{++} = 0,
\]

\[
W^{-}_{\alpha} = D^{-\alpha} W^{+}_{\alpha}, \quad \frac{1}{2} D^{-\alpha} Y^{++} = D^{+}_{\alpha} W^{+\alpha},
\]

\[
D^{++} W^{+\alpha} = 0.
\] (3.11)

Pay attention to the fact that these relations serve a definition of important for our aims superfield \( Y^{++} \).

The superfield action of 6D SYM theory is written in the form

\[
S_{SYM} = \frac{1}{f^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{tr} \int d^{14} z d u_{1} \ldots d u_{n} \frac{V^{++}(z, u_{1}) \ldots V^{++}(z, u_{n})}{(u_{1}^{+} u_{2}^{+}) \ldots (u_{n}^{+} u_{1}^{+})}. \] (3.12)

Here \( f \) is the dimensional coupling constant \((|f| = -1)\). The corresponding equations of motion have the form \( Y^{++} = (D^{+})^{4} V^{--} = 0 \). The component fields of \( W^{+\alpha} \) and \( V^{++} \) are related to each other with help of the zero-curvature condition (3.8) and due to the definition (3.9).

It is known that the superfield action with dimensionless coupling constant [20] has the form

\[
S = \frac{1}{2 g^{2}} \text{tr} \int d \zeta (-4) d u \ (Y^{++})^{2}.
\] (3.13)

It possesses the superconformal symmetry and contains the higher derivatives.
To conclude this section, we give the decomposition of the superfield $V^-$ in terms of the component fields

$$V^- (x_A, \theta^-, \theta^+, u) = \theta^{-\alpha} \theta^{-\beta} v_{\alpha\beta} (x_A, \theta^+) + (\theta^-)^3 v^+ (x_A, \theta^+) + (\theta^-)^4 v^{++} (x_A, \theta^+), \quad (3.14)$$

$$v_{\alpha\beta} = A_{\alpha\beta} + \frac{1}{4} \varepsilon_{\alpha\beta\gamma\delta} \theta^{+\gamma} \lambda^{-\delta} - \frac{1}{4} \varepsilon_{\alpha\beta\gamma\delta} \theta^{+\gamma} \theta^{+\delta} Y^{--},$$

$$v^+ = -\frac{1}{2} \lambda^{+\alpha} + \theta^{+\alpha} Y^{+-} + \theta^{+\beta} i f^{\alpha\beta} + \theta^{+\gamma} \theta^{+\delta} \delta^{\alpha\beta}_{\gamma\delta} + (\theta^+)^3 \kappa^{(-2)\alpha} + (\theta^+)^4 \sigma^{(-3)\alpha},$$

$$\frac{1}{2} f^\alpha_{\beta} = (\gamma_{mn})^\alpha_{\beta} F_{mn}, \quad F_{mn} = \partial_m A_n - \partial_n A_m - i [A_m, A_n], \quad D_{\alpha\beta} = \partial_{\alpha\beta} - i [A_{\alpha\beta},],$$

$$3 \omega^\alpha = i D_{\alpha\beta} \lambda^{-\beta}, \quad \frac{1}{2} \varepsilon_{\alpha\beta\gamma\delta} \kappa^{(-2)\gamma\delta} = 2 i D_{\alpha\beta} Y^{--} + \frac{1}{4} \varepsilon_{\alpha\beta\gamma\delta} \{ \lambda^{+\gamma}, \lambda^{-\delta} \},$$

$$v^{++} = Y^{++} + \theta^{+\alpha} \chi^{+\alpha} + \theta^{+\alpha} \theta^{+\beta} \Omega_{\alpha\beta} + (\theta^+)^3 \rho^{-\alpha} + (\theta^+)^4 \pi^{-2},$$

$$\chi^+ = i D_{\alpha\beta} \lambda^{+\beta}, \quad \varepsilon^{\alpha\beta\gamma\delta} \Omega_{\gamma\delta} = -2 i D^{\alpha\beta} Y^{+-} - D^{[\alpha\gamma} f^{\beta]}_{\gamma} - \frac{1}{4} \{ \lambda^{+\alpha}, \lambda^{-\beta} \}, \quad D^{[\alpha\beta} f_{\gamma]} = 0,$$

$$\rho^{-\alpha} = 2 i D^{\alpha\beta} \chi^+_{\beta} + \frac{1}{2} [\lambda^{+\alpha}, Y^{--}] - [\lambda^{-\alpha}, Y^{+-}] + i [\lambda^{-\beta}, f^\beta_{\alpha}],$$

$$\pi^{(-2)} = D^{\alpha\beta} D_{\alpha\beta} Y^{--} - \frac{1}{2} \{ \lambda^{-\alpha}, D_{\alpha\beta} \lambda^{-\beta} \} + 3 [Y^{+-}, Y^{--}] .$$

Relation (3.14) defines the complete component structure of the superfield $V^-$ in terms of components of the superfield potential $V^{++}$. The component at $(\theta^-)^4(\theta^+)^4$ has been calculated earlier in [20] (at the different conventions). Further we will need all the components of $V^-(x_A, \theta^+, \theta^-, u)$.

### 3.3 Linear multiplet in harmonic superspace

In this subsection we briefly review a self-dual tensor multiple and its description in harmonic superspace [19].

As it is well known, the so-called self-dual tensor multiple contains a scalar $\phi$, a spinor $\psi_{\alpha}$ and an antisymmetric tensor $B_{ab}$ subject to the self-dual constraint

$$\partial_{[a} B_{bc]} - \frac{1}{6} \varepsilon_{abcdef} \partial^d B^{ef} = 0. \quad (3.15)$$

There two ways to describe the self-dual tensor multiplet in harmonic superspace.

Firstly, one introduces the superfield $\Phi(x, \theta)$ subject to the constraint

$$D^i_\alpha D^j_\beta \Phi = 0. \quad (3.16)$$

Such a superfield is also called a linear. This superfield has no external indices and can be obeyed the reality condition $\Phi = \bar{\Phi}$. In the case of $\mathcal{N} = (1, 0)$ supersymmetry, the component expansion of the superfield $\Phi(x, \theta)$ has the form

$$\Phi = \phi + \theta^i \psi^{i\alpha} + \theta^i \theta^j G_{(\alpha\beta)} + \ldots , \quad (3.17)$$
where the component fields satisfy the massless equations of motion. Note that the field $G_{(\alpha\beta)}$ is related to 3-form field $G_{abc}$: $G_{(\alpha\beta)}(x) = (\gamma^{abc})_{\alpha\beta}G_{abc}(x)$ and is self-dual by definition. We see that on shell the linear superfield contains all components of self-dual tensor multiplet.

The above (1,0) self-dual tensor multiplet (with the constraint (3.16)) is formulated in harmonic superspace \[19\] with the help of real superfields satisfying the constraints

$$D_{\alpha}^{+}D_{\beta}^{+}\Phi = 0, \quad D^{++}\Phi = 0. \tag{3.18}$$

The first of them means that $\Phi(x, \theta^+, \theta^-, u)$ is linear in $\theta^-:

$$\Phi = l(x_A, \theta^+, u) + \theta^{-\alpha}f_{\alpha}^{+}(x_A, \theta^+, u), \tag{3.19}$$

where the coefficient functions are the analytic superfields

$$l = \phi + \theta^+\psi^-, \quad f_{\alpha}^{+} = -\psi_{\alpha}^{+} - \theta^{+\beta}\partial\alpha\beta\phi + \theta^{+\beta}G_{(\alpha\beta)} - i\partial_{\alpha\beta}\theta^{+\beta}\theta^{+\gamma}\psi_{\gamma}^{-}. \tag{3.20}$$

The dynamical equations follow from the second constraint (3.18) which reduces the harmonic dependence of $l$ and $f^{+}$ to a polynomial and thus produces a finite supermultiplet. It leads to the two harmonic constraints

$$\hat{D}^{++}l + \theta^{-\alpha}f_{\alpha}^{+} = 0, \quad \hat{D}^{++}f_{\alpha}^{+} = 0, \tag{3.21}$$

from which we obtain the equations of motion for the components of the self-dual tensor multiplet

$$\partial^{\alpha\beta}\psi_{\beta}^{+} = 0, \quad \Box\phi = 0, \quad \partial^{\alpha\gamma}G_{\gamma\beta} = 0. \tag{3.22}$$

Note that all these components are the field strengths. Besides, the kinematical constraints (3.18) is solved \[19\] by introducing the prepotential

$$\Phi = (D^{+})^{-3}l. \tag{3.23}$$

Another way to formulate the tensor multiplet in superfield form is based on superfield $V^{\alpha i}$ subject the kinematical constraints \[19\]

$$D_{\beta}^{(i}V^{j)\alpha} - \frac{1}{4}\delta_{\beta}^{\alpha}D_{\gamma}^{(i}V^{j)\gamma} = 0. \tag{3.24}$$

In the harmonic superspace ones get the superfield $V^{+\alpha}(x, \theta^+, \theta^-, u)$ under the following constraints

$$D_{\alpha}^{+}V^{+\beta} - \frac{1}{4}\delta_{\alpha}^{\beta}D_{\gamma}^{+}V^{+\gamma} = 0, \quad D^{++}V^{+\alpha} = 0. \tag{3.25}$$

In the analytic basis we have

$$V^{+\alpha}(x_A, \theta^-, \theta^+, u) = v^{+\alpha}(\theta^+) + \theta^{-\alpha}v^{(+2)}(\theta^+), \tag{3.26}$$

here $v^{+\alpha}$ and $v^{(+2)}$ are the arbitrary analytic superfields. Then second dynamical constraint $D^{++}V^{+\alpha} = 0$ becomes

$$\hat{D}^{++}v^{(+2)} = 0, \quad \hat{D}^{++}v^{+\alpha} + \theta^{+\alpha}v^{(+2)} = 0. \tag{3.27}$$
The component expansions of these superfields are obtained from above relations in the form
\[ v^{(+2)} = f^{(+2)} + \theta^{+\alpha}k_{\alpha}^+ + (\theta^+)2\alpha_\beta a_{\alpha\beta} + (\theta^+)3\alpha\tau^{-\alpha} + (\theta^+)4C^{(-2)}, \]
\[ v^{+\alpha} = \rho^{+\alpha} + \theta^{+\beta}(B_{\beta}^\alpha + \delta_\beta^\alpha\Sigma) + (\theta^+)2\beta\gamma\omega^{-\alpha}_{\beta\gamma} + (\theta^+)3\beta E^{(-2)\beta\alpha} + (\theta^+)4\varphi^{(-3)\alpha}. \]

The kinematical constraints (3.24) is solved by introducing the prepotential [19]:
\[ \mathcal{V}^{+\alpha} = (D^+)3\alpha\mathcal{V}^{(-2)}, \tag{3.27} \]
where
\[ \mathcal{V}^{(-2)} = (\theta^-)3\alpha v^{+\alpha} + (\theta^-)4v^{(+2)}. \tag{3.28} \]

This prepotential is defined up to the Abelian gauge transformations
\[ \delta_\Lambda \mathcal{V}^{(-2)} = -D^{--}\Lambda. \tag{3.29} \]

If
\[ \Lambda \sim \ldots + \frac{1}{2}\theta^{-\alpha}\theta^{+\beta}i\Lambda_{\alpha\beta} + \varepsilon_{\alpha\beta\gamma\delta}\theta^{-\alpha}\theta^{-\beta}\theta^{+\gamma}\rho^{+\delta} + (\theta^-)3\theta^{+\alpha}f^{++} + \ldots, \]
then
\[ \delta \mathcal{V}^{+\alpha} \sim \theta^{+\beta}\partial^{\alpha\gamma}\Lambda_{\gamma\beta} - \rho^{+\alpha} - \theta^{-\alpha}f^{+2}, \quad \delta B^\alpha_{\beta} = \partial^{\alpha\gamma}\Lambda_{\gamma\beta} - \frac{1}{4}\delta^\beta_\gamma\partial^{\alpha\delta}\Lambda_{\delta\gamma}. \]

The fields \( f^{ij}(x), \rho^\alpha_i \) form a multiplet of gauge degrees of freedom, they can be excluded by appropriate gauge choice, i.e. one can use the Wess-Zumino gauge.

Substituting the quantities \( v^{+\alpha}, v^{(+2)} \) in (3.26) we find the general solution
\[ \tau^{i\alpha} = 2i\partial^{\alpha\beta}k^i_\beta, \quad \omega^{-\alpha}_{\beta\gamma} = -\frac{1}{2}\delta^{\alpha\beta}_{[\beta\gamma]}, \quad a^{\alpha\beta} = -\frac{i}{2}\partial^{[\alpha\gamma}B^\beta_{\gamma]} - i\partial^{\alpha\beta}\Sigma, \]
as well as the on-shell conditions
\[ \Box\Sigma = 0, \quad \partial^{(\alpha\gamma}B^\beta_{\gamma)} = 0, \quad \partial^{\alpha\beta}k^i_\beta = 0. \tag{3.30} \]

At the same time the fields \( C^{(-2)}, \varphi^{(-3)\alpha}, E^{(-2)\alpha\beta} \) are eliminated by the choice of gauge \( f^{ij} = 0, \rho^\alpha_i = 0 \).

The free action for the dynamical equations (3.26), (3.21) has been proposed in [19]
\[ S_{TM} = \int d^6xd^8\theta du\Phi^{(-3)}D^{++}\mathcal{V}^{+\alpha} = \int d^6xd^8\theta du\Phi D^{++}\mathcal{V}^{(-2)}. \tag{3.31} \]

This action is invariant under above gauge transformations of the \( \mathcal{V}^{(-2)} \) together with the gauge invariant condition for \( \Phi \) \( (\delta_\Lambda \Phi = 0) \),
\[ \delta S_{TM} = \int d^8\theta du\Phi D^{++}D^{--}\Lambda = \int d^8\theta duD^{++}\Phi D^{--}\Lambda = 0, \]
where the on-shell equation \( D^{++} \Phi = 0 \) has been used. Note also that all the constraints (3.23), (3.16) for the superfields \( V^{-\alpha} = D^{--} V^{+\alpha} \) and \( \Phi \) can be solved in the anti-analytic basis of the harmonic superspace, where \( D^-_{\alpha} = -\frac{\partial}{\partial \theta^+} \):

\[
V^{-\alpha}(x_A, \theta^-, \theta^+, u) = v^{-\alpha}(\theta^-) + \theta^{+\alpha} v(-2)(\theta^-) = (D^-)^{3\alpha} V^{+2}, \tag{3.32}
\]

and

\[
\Phi(x_A, \theta^-, \theta^+, u) = l(\theta^-) + \theta^{+\alpha} f^{-}(\theta^+) = (D^-)^{3\alpha} \Phi^{(+3)}. \tag{3.33}
\]

This BF-type action (3.31) describes two tensor multiplets one of which acts as a Lagrange multiplier for the equations of motion of the other multiplet:

\[
S = \int d^6 x \left( G^{++}_{\alpha\beta} \partial^{(\alpha\gamma} B^\beta_{\gamma)} + i \psi^+_\alpha \partial^{\alpha\beta} k^-_{\beta} + \phi \Box \Sigma \right). \tag{3.34}
\]

We see that the superfield \( \Phi^{(-3)} \) describes those degrees of freedom, which are killed in 3-form \( G_{abc} \) by the self-duality condition. According to the work [19] the self-dual fields do not exist off-shell on their own\(^5\).

In the analytic subspace of the harmonic superspace the analytic superfields

\[
G^{++} = D^+_{\alpha} \Phi V^{+\alpha} + \frac{1}{4} \Phi D^+ \Phi V^{+\alpha}, \quad D^+ \Phi V^{+\alpha} = 0, \tag{3.35}
\]

allow us to rewrite the action (3.31) in the form

\[
S = \int d\zeta(-4) du \left\{ D^+_{\alpha} \Phi D^{++} V^{+\alpha} + \frac{1}{4} \Phi D^{++} D^+ V^{+\alpha} \right\}. \tag{3.36}
\]

This expression completely corresponds to the standard recipe for constructing the superfield action in harmonic/projective superspace (see e.g. [13], [23], [24]) and will be used below for constructing the interacting superfield action of the vector/tensor system.

### 3.4 (2,0) Tensor multiplet

Field content of six dimensional (2,0) tensor multiplet consists of a self-dual 3-form curvature \( G_{(\alpha\beta)}(x) = (\gamma^{abc})_{\alpha\beta} G_{abc} \) with three on-shell degrees of freedom, four left-handed spinors \( \psi^I_{\alpha}(x) \) and five scalars \( \phi^{IJ}(x) = -\phi^{HI}(x) \) which satisfy the condition \( \Omega_{IJ} \phi^{IJ} = 0 \) [17]. All these component fields can be encoded into the \( \Omega \)-traceless scalar superfield \( L^{IJ}(x, \theta^I) \) (1,J=1,2,3,4; 5 of USp(4)), subject to the differential constraints

\[
D^K L^{IJ} - \frac{2}{5} D_{\alpha L} (\Omega^{KIJ} L^{JL} - \Omega^{KJI} L^{LI} + \frac{1}{2} \Omega^{LJK} L^{KL}) = 0. \tag{3.37}
\]

Also one imposes the reality condition \( \overline{L^{IJ}} = \Omega_{IK} \Omega_{JL} L^{KL} \). The constrains on the trace-free part of \( D^K L^{IJ} \) arise as a consistency condition on the embedding an M5-brane in

\(^5\)It would be interesting to quantize such a theory and study the effective action analogously to self-dual YM theory [32]. One can expect that these fields can be propagating due to quantum corrections.
eleven-dimensional superspace \[29\]. Using the spinor derivative algebra (2.1) it is not difficult to show that this superfield has the following \(\theta\) expansion

\[
L^{IJ} = \phi^{IJ} + \left(\theta^{a[I} \psi_{aJ]} + \frac{1}{2} \Omega^{IJ} \theta^{\alpha K} \psi_{aK} \right) + \left(\theta^{a[I} \theta^{\beta J]} + \frac{1}{2} \Omega^{IJ} \theta^{\alpha K} \theta^{\beta K} \right) \frac{1}{2} G_{(\alpha \beta)} + \ldots .
\]

The corresponding component fields satisfy the massless equations of motion

\[
\Box \phi^{IJ} = 0, \quad \partial^{\alpha \beta} \psi_{I} = 0, \quad \partial^{\alpha \gamma} G_{\gamma \beta} = 0.
\]

The latter equation implies that the 3-form \(G_{abc}\) is the curl of a 2-form \(G_{abc} = \partial_{[a} B_{bc]}\), or \(G_{\alpha \beta} = \partial_{(\alpha} B_{\beta)}\). The gauge transformations now take the form \(B_{\alpha}^{\beta} \rightarrow \partial^{\alpha \gamma} \Lambda_{\beta \gamma} - \frac{1}{4} \delta_{\alpha}^{\beta} \partial^{\gamma \delta} \Lambda_{\gamma \delta}\).

There are various complications in formulation of (2,0) interacting theories with non-abelian gauge group \[2\], \[3\], \[5\]. It is still unclear whether a superfield action for this multiplet actually exists.

4 \textbf{(2,0) tensor multiplet in D6, (1,0) harmonic superspace}

In this section we show that the (2,0) tensor multiplet can be formulated in (1,0) harmonic superspace in terms of (1.0) tensor multiplet and hypermultiplet.

It is easy to see that the total on-shell field contents of (1,0) hypermultiplet and (1,0) tensor multiplet exactly coincides with one of (2,0) tensor multiplet. Therefore it seems natural that the dynamical theory of (2,0) tensor multiplet can be constructed in (1,0) harmonic superspace in terms of (1,0) hypermultiplet and (1,0) tensor multiplet. Consider a sum of actions for hypermultiplet (3.4) and (1,0) tensor multiplet (3.26). We show that this total action possesses by extra hidden (1,0) supersymmetry. Taking into account the manifest (1,0) supersymmetry of the actions (3.4) and (3.26), one gets a (2,0) supersymmetry of the total action.

Write the above total action in the form

\[
S^{(2,0)} = S_{q} + S_{T} = \int d^{6} x d^{8} \theta d u \Phi D^{++} \mathcal{V}^{(-2)} + \frac{1}{2} \int d^{3} \zeta d u q_{A}^{+} D^{++} q^{A}.
\]

Here \(A = 1, 2\) is the index of the Pauli-Gürsey rigid \(SU(2)\) symmetry. Accordingly two ways to interpret the action \(S_{T}\) we define two types of hidden supersymmetry transformations.

First, we treat the superfield \(\Phi\) in action \(S_{T}\) as Lagrangian multiplier and \(\mathcal{V}^{(-2)}\) as the basic superfield. Define the hidden supersymmetry transformations in the form

\[
\delta q_{A}^{+} = (D^{+})^{4} \epsilon_{A}^{\alpha} \Phi_{\alpha}^{(-3)}, \quad \delta \mathcal{V}^{(-2)} = -\epsilon_{A}^{\alpha} (\theta^{-})_{\alpha}^{3} q^{A}, \quad \delta \Phi = 0,
\]

where \(\epsilon_{A}^{\alpha}\) is the transformation parameter. Then, the variation of hypermultiplet action is

\[
\delta S_{q} = \int d^{6} x d^{8} \theta d u \epsilon_{A}^{\alpha} \Phi_{\alpha}^{(-3)} D^{++} q^{A}.
\]
The variation of the tensor multiplet action of the tensor multiplet looks like

\[ \delta S_T = \int d^6 x d^8 \theta d\Phi D^{++} \delta \mathcal{V}^{(-2)} = \int d^6 x d^8 \theta d\Phi^{-3} D^{++} (D^+)^{3\alpha} \delta \mathcal{V}^{(-2)} \quad (4.4) \]

\[ \quad = - \int d^6 x d^8 \theta d\epsilon^\alpha_A \Phi^{-3} D^{++} q^+ A . \]

We see that \( \delta S_q + \delta S_T = 0 \).

Second, we treat the superfield \( \mathcal{V}^{(-2)} \) in action \( S_T \) as the Lagrangian multiplier and the superfield \( \Phi \) as the basic superfield. Define the hidden supersymmetry transformations in the form

\[ \delta q^+_A = (D^+) A ^4 \epsilon^\alpha_A D^- \mathcal{V}^{(-2)}, \quad \delta \mathcal{V}^{(-2)} = 0, \quad \delta \Phi = -\epsilon^\alpha_A D^- q^+_A \quad (4.5) \]

Then

\[ \delta S_q = \int d^6 x d^8 \theta d\epsilon^\alpha_A D^- \mathcal{V}^{(-2)} D^{++} q^+_A, \quad (4.6) \]

and

\[ \delta S_T = - \int d^6 x d^8 \theta d\epsilon^\alpha_A q^+_A D^- \mathcal{V}^{(-2)} D^{++} q^+_A = \int d^6 x d^8 \theta d\epsilon^\alpha_A q^+_A D^- \mathcal{V}^{(-2)} . \quad (4.7) \]

We see again that \( \delta S_q + \delta S_T = 0 \).

As a result we have constructed the free action for (2,0) tensor multiplet in (1,0) harmonic superspace in terms of (1,0) hypermultiplet and (1,0) tensor multiplet. This action is invariant under the manifest (1,0) supersymmetry transformations and hidden (1,0) supersymmetry transformations.

It is interesting to study whether the supersymmetry algebra is closed. Begin with formulation on the base of superfields \( q^+ \) and \( \mathcal{V}^{(-2)} \). The transformation laws for these superfields are given by (4.2). Then it is not difficult to obtain that

\[ [\delta_2, \delta_1] \Phi = 2i \epsilon_1^{\alpha A} \epsilon_2^\alpha \partial_{\alpha \beta} \Phi. \quad (4.8) \]

Here we have used the identity \( (D^+_\alpha D^-_{\beta} + D^+_{\alpha} D^-_{\beta}) \Phi = 0 \) which follows from the constraints (3.16). For the hypermultiplet we have

\[ [\delta_2, \delta_1] q^+_A = 2i \epsilon_1^{\alpha B} \epsilon_2^\alpha \partial_{\alpha \beta} q^+_A . \quad (4.9) \]

Algebra of hidden supersymmetry transformations is closed. Analogous consideration can be carried out for the formulation with basic superfields \( q^+ \) and \( \Phi \). The corresponding algebra is also closed.

5 Interacting D6 (1,0) vector and tensor multiplets in harmonic superspace

5.1 Non-Abelian vector/tensor system

In this subsection we will briefly mention the general non-Abelian couplings of vectors and antisymmetric \( p \)-form fields in six dimensions following [6]. The (1,0) superconformal
6D field theory of [6] (vector/tensor system) describes a hierarchy of non-Abelian scalar, vector and tensor fields \{\phi^I, A^I_a, Y^{ij}, B^I_{ab}, C_{abc r}, C_{abcd \Lambda I}\} and their supersymmetric partners that label by indices \(r = 1, \ldots, n_V\) and \(I = 1, \ldots, n_T\). To label the \(C_{abc \ r}\) field a dual index \(r\) is used since the dynamically a vector fields dual to antisymmetric three-form tensors. The full non-Abelian field strengths of vector and two-form gauge potentials are given as

\[
F^r_{ab} = \partial_a A^r_b - f_{st}^r A^s_{a \ b} + h^r_I B^I_{ab},
\]

\[
H^I_{abc} = \frac{1}{2} \mathcal{D}_{[a} B_{bc]} + d^I_{rs} A^r_{[a} \partial_b A^s_{c]} - \frac{1}{3} f_{pq}^s d^I_{rs} A^r_{[a} A^p_{b\ c]} + g^{lr} C_{abc \ r}.
\]

Here \(f_{st}^r\) are the structure constants, \(d^I_{rs}\) are the \(d\)-symbols, defining the Chern-Simons couplings, and \(h^r_I, g^{lr}\) are the covariantly constant tensors, defining the general St"uckelberg-type couplings among forms of different degrees. Also the existence of the non-degenerate Lorenz-type metric \(\eta_{IJ}\), so that \(h^r_I = \eta_{IJ} g^{lr} \equiv g^{lr}, b_{Irs} = 2\eta_{IJ} d^I_{rs} \equiv d_{Irs}\) is assumed. The covariant derivatives are defined as \(D_m = \partial_m - A^I_m X_r\) with the gauge generators \(X_r\) acting on the different fields as follows \(X_r \cdot \Lambda^s \equiv -(X_r)^{s}_{t} \Lambda^t, X_r \cdot \Lambda^I \equiv -(X_r)^{I}_{J} \Lambda^J\). Covariance of the field strengths (5.1) requires that the gauge group generators in the various representations should have the form

\[
(X_r)^{s}_{t} = -f_{rs}^t + g^l_{rs} d^I_{rs}, \quad (X_r)^{I}_{J} = 2d^I_{rs} g^s_{r} - g^{ls} d_{Isr},
\]

in terms of the invariant tensors parameterizing the system (see the details in [6]). The field strengths (5.1) are defined such that they transform covariantly under the set of non-Abelian gauge transformations

\[
\delta A_m = D_m \Lambda^r - h^r_I \Lambda^I_m,
\]

\[
\delta B^I_{mn} = D_{[m} \Lambda^I_{n]} - 2d^I_{rs} (\Lambda^r F^s_{mn} - \frac{1}{2} A^r_{[m} \delta A^s_{n]})) - g^{lr} \Lambda_{mn \ r}.
\]

The superspace realization of the tensor hierarchy has been developed in the paper [10] in conventional 6D, (1,0) superspace. In the next subsection we consider the generalized Bianchi identities from [10] for the superfield vector/tensor system, reformulate them in the harmonic superspace and study the consistency conditions for the generalized Bianchi identities. For further use, it is convenient to introduce the generalized superfield strength

\[
\mathcal{W}^{s\alpha \ r} = W^{s\alpha \ r} + g^I_f \mathcal{V}^{s\alpha \ I},
\]

where the \(W^{s\alpha \ r}\) is the superfield strength of the super Yang-Mill theory (defined in subsection 3.2) and \(\mathcal{V}^{s\alpha \ I}\) is the superfield of tensor multiplet (defined in subsection 3.3), and write the generalized Bianchi identities in its terms. Then one can see that the conventional strength \(F_{mn}\) of the vector multiplet and the conventional strength \(B_{mn}\) of the tensor multiplet enter into \(\mathcal{W}^{s\alpha \ r}\) in the form \(F_{mn} + g B_{mn}\).

Using the generalized Bianchi identities and their consistency conditions we will formulate the superfield action for vector/tensor system and find its component form.
5.2 Harmonic superspace description of non-Abelian vector/tensor system

In this subsection we formulate the superfield version of non-Abelian vector/tensor system using the harmonic superspace technique. A complete set of the constraints on superfield strengths of the \( p \)-form potentials have been proposed in [10] in conventional 6D, (1,0) superspace. Our aim is to reformulate these constraints in harmonic superspace and study their consistency conditions.

First of all, the SYM constraint \( \mathcal{F}^{ij\ r}_{\alpha\beta} = 0 \) is not deformed, therefore we can use a harmonic superfield technique. Next, consider the dimension 2 component of the generalized Bianchi identities

\[
(\gamma^a)(aD^j_{\beta}W^{i\delta} r - 2\varepsilon^{ij}(\gamma^b)_{\alpha\beta}\mathcal{F}^{r}_{ab}) = \frac{1}{2}\varepsilon^{ij}\gamma_{\alpha\beta} \Phi^i g^r_f. \tag{5.4}
\]

This relation leads to the covariant derivatives of generalized superfield strength (5.3) in the form

\[
D^i_{\alpha}W^{j\beta\ r} = \delta^\beta_{\alpha}(\gamma_{ij} r + \frac{1}{2}\varepsilon^{ij}\Phi^i g^r_f) + \frac{1}{2}\varepsilon^{ij}(\gamma_{ab})_{\alpha\beta}\mathcal{F}^{r}_{ab}. \tag{5.5}
\]

Last equation is equivalent to the following set of relations

\[
D^i_{\alpha}W^{j\beta\ r} = \frac{1}{4}\delta^\beta_{\alpha}D^i_{\gamma}W^{j\gamma\ r}, \quad \gamma_{ij} = \frac{1}{8}D^i_{\alpha}W^{i\alpha\ r}, \quad \Phi^i g^r_f = \frac{1}{4}D^i_{\alpha}W^{i\alpha\ r}, \quad \mathcal{F}^{r}_{ab} = -\frac{1}{8}(\gamma_{ab})_{\alpha\beta}D^i_{\beta}W^{i\alpha}. \tag{5.6}
\]

Turn to harmonic superspace formulation. In terms of the harmonic superfields, the relations (5.6) take the form

\[
D^i_{\alpha}W^{j\beta\ r} = \delta^\beta_{\alpha}(\gamma_{ij} r + \frac{1}{2}\varepsilon^{ij}\Phi^i g^r_f) + \frac{1}{2}\varepsilon^{ij}(\gamma_{ab})_{\alpha\beta}\mathcal{F}^{r}_{ab}. \tag{5.7}
\]

Consider the dimension 5/2 component of the generalized Bianchi identities

\[
D^i_{\alpha}W^{j\beta\ r} + i(\gamma_{[a)}\alpha D^i_{b]}W^{n\delta\ r} = i(\gamma_{ab})_{\alpha\beta}\Psi^{ij} g^r_f. \tag{5.8}
\]

It yields \( 3\Psi^{ij} g^r_f = \frac{i}{10}D^i_{\alpha}D^j_{\beta} - \frac{1}{4}D^i_{\alpha}W^{i\beta\ r} + D^i_{\alpha\beta}W^{j\delta\ r} \). In addition, the above deformed identity determines the transformation law for the potential of 2-forms

\[
\delta B^i_{\alpha\beta} = i\varepsilon^{ij}_{\alpha\beta}\Psi^{ij} g^r_f. \tag{5.9}
\]

The selfconsistency conditions \{\( D^i_{\alpha}, D^j_{\beta} \}\) \( W^{k\delta} = -2i\varepsilon^{ij}D^i_{\alpha\beta}W^{k\delta} \) leads to

\[
D^i_{\alpha}\Phi^i = 2i\Psi^{ij}_{\alpha}, \quad iD^i_{\alpha\beta}W^{j\beta\ i} = \frac{1}{3}D^i_{\alpha}W^{ij\ r} + D^i_{\alpha}\Phi^i g^r_f, \tag{5.9}
\]

\[
D^i_{\alpha}\gamma^{ij\ r} = -i\varepsilon^{k}(D^i_{\alpha\beta}W^{\beta j\ r} - 2\Psi^{ij} g^r). \tag{5.10}
\]
By rewriting the above relations in terms of harmonic projection, one gets $D^+_\alpha Y^{++} r = 0$, $D^-_\alpha Y^{++} r = -2i(D_{\alpha \beta} W^{+\beta} r - 2\Psi^+_\alpha g^r_I)$, $D^+_\alpha Y^{-} r = \pm i(D_{\alpha \beta} W^{\pm\beta} r - 2\Psi^+_\alpha g^r_I)$. (5.11)

Acting on the relation $D^i_\alpha \Phi^I = 2i\Psi^I_\alpha$ by the spinor derivative, one obtains

$$D_\alpha \Phi^I = \frac{1}{4} D_{\alpha \tau} \gamma^\alpha_\tau \Psi^I_\beta .$$

The dimension 3 component of the generalized Bianchi identities is

$$D_\alpha \Phi^I = i \frac{1}{4} D_{\alpha \tau} \gamma^\alpha_\tau \Psi^I_\beta .$$

In the spinor notations it has the form

$$\frac{1}{2} D^{(i\delta \Phi^\delta)}_\beta r = \frac{1}{3} H^{(\delta)}_\alpha \gamma_\alpha_\delta \Phi^I r, \quad \frac{1}{2} D^{(i\delta \Phi^\delta)}_\beta r = \frac{1}{3} H^{(\delta)}_\alpha \gamma_\alpha_\delta \Phi^I r .$$

Besides, from (5.8) ones obtain the equations for the symmetric in $(i, j)$ parts

$$D^{(i\alpha (ab)}_\delta \Psi^j_\beta = 2id_{rs} W^{\gamma i} r^{a}_{\gamma (\alpha} r^{b}_{\beta) \gamma} W^{\gamma i} s , \quad \frac{1}{2} D^{(i\alpha (ab)}_\delta \Psi^j_\beta = 2i W^{i} r^{\gamma_\alpha} W^{j} s d_{rs} .$$

For antisymmetric in $(ij)$ parts we have

$$H^{(+)}_\alpha = \frac{i}{4} d_{rs} W^{\alpha} r^{\gamma_\alpha} W^{\gamma i} s , \quad H^{(-)}_\alpha = \frac{1}{8} D_{\alpha \tau} \gamma_\alpha_\tau \Psi^I_\beta .$$

The above equations for symmetric and antisymmetric parts together imply

$$D^{(i\alpha (ab)}_\delta \Psi^j_\beta = -\frac{1}{2} \epsilon^{ij} D_{\alpha \beta} \Phi^I - \frac{1}{12} \epsilon^{ij} \gamma_\alpha_\delta H^{(-)I}_\beta \gamma_\beta_\delta W^{\gamma i} r^{\gamma} W^{\gamma i} s d_{rs} .$$

In terms of the harmonic superfields these relations take the form

$$D^+_\alpha \Psi^I_\beta = \mp \frac{1}{2} D_{\alpha \beta} \Phi^I + \frac{1}{12} \gamma_{\alpha \beta} H^{(-)I}_\delta + i \varepsilon_{\alpha \beta \gamma \delta} W^{-\gamma} r^{\gamma} W^{r\delta} s d_{rs} ,$$

$$D^\pm_\alpha D^\pm_\beta \Phi^I = -2 \varepsilon_{\alpha \beta \gamma \delta} W^{\pm \gamma} r^{\gamma} W^{\pm \delta} r d_{sr} ,$$

$$D^-_\alpha D^-_\beta Y^{++} r = -i(D_{\alpha \beta} \Phi^I g^r_I + \frac{1}{3} \gamma^{ij}_\alpha H^{(-)I}_\delta - D_{\beta \gamma} F^{\gamma}_\alpha r - 2 D_{\alpha \beta} Y^{++} r) .$$

The spinor derivative of the 3-rank tensor superfield $H^{I}_{abc}$ is

$$D^i_\alpha H^{I}_{abc} = i \gamma^a_{\alpha \beta} W^{\beta i} r^{F^{bc}} s d_{sr} + \frac{i}{2} D^{(a} (\epsilon^{bc)}_\beta \Psi^I_\alpha - i \gamma^{abc} W^{\beta i} s F^I d_{sr} g^r_I .$$

\footnote{An important feature of these equations is that the anti-self-dual part of the field strength $H$ is fixed in terms of the dynamical vector multiplet.}
This relation also determines the transformation law of the 3-form potential

$$\delta C_{abc} r = -i \epsilon_i \gamma_{abc} \mathcal{W}^i s \Phi^J d_{Jrs}.$$  

The corresponding degrees of freedom are not dynamic since the generalized 4-form field strength satisfies the duality conditions

$$-\frac{1}{4!} \epsilon^{abcd} \mathcal{H}_{abcd} r = (\mathcal{F}^e f s \Phi^r + i \mathcal{W}^i s \gamma_{ef} \Psi^I_i) d_I r s.$$  

(5.19)

As shown in the paper [10], all other relations among the main superfield strengths and the equations of motion can be derived from the following relations

$$\left( \gamma^{ij} s \Phi^r - 2i \mathcal{W}^i s \Psi^I_i \right) d_I r s = 0,$$  

(5.20)

d_{Irs} \{ \Phi^l D_\alpha \mathcal{W}^i _{rs} + \frac{1}{12} \mathcal{H}^{(r)}_{\alpha \beta} \mathcal{W}^i _{rs} + \frac{1}{2} D_\alpha \Phi^r \mathcal{W}^i _{rs} + \frac{1}{2} \mathcal{F}_\alpha \gamma_{ij} \Psi^I_i + \gamma^{ij} \Phi^s r j \} =$$

$$+ \frac{1}{2} \Phi^I_{rt} \Phi^r (4g^r d_{Irs} - g^s d_{Irs}) + \frac{2i}{3} \gamma_{\alpha \beta \gamma} \mathcal{W}^r_{\beta} \mathcal{W}^r_{\gamma} \mathcal{W}^a v d_I r s d_{Iav},$$

$$D_e (\Phi^I \mathcal{F}^a r s + i \mathcal{W}^i s \gamma_{ea} \Psi^I_i) d_{Irs} + \frac{1}{6} \epsilon^{abcd} \mathcal{F}^e \mathcal{H}^r_{def} d_{Irs}$$

$$+ (\frac{1}{4} \Phi^I D^a \Phi^J + \frac{1}{2} \Psi^{ij} \tilde{\gamma}^a \Psi^I_i) X_{rlj} - \frac{1}{2} \Phi^I \mathcal{W}^{is} \gamma^a \mathcal{W}^i_s (X_r)_{\alpha}^u d_I t I u = 0.$$  

They lead to Dirac equation for the fermions of tensor multiplet

$$D^\alpha \Psi^I_\beta = -\gamma^{ij} \mathcal{W}^r _{ij} \mathcal{W}^a r s d_I r s + \frac{1}{2} \mathcal{W}^{ia} r \Phi^r (4d_{Irs} g^r s - g^s d_{Irs}) - \frac{1}{2} \mathcal{W}^{ij} \mathcal{F}^a r s d_I r s ,$$  

(5.21)

to scalar superfield equation of motion

$$\Box \Phi^I = d_I r s (\mathcal{F}^a r s \mathcal{F}^a r b s - \gamma^{ij} \mathcal{W}^a r s - i \mathcal{W}^{ia} r D_\alpha \mathcal{W}^i s) + \frac{3}{2} \Phi^J g^r \Phi^K g_K r d_I r s$$

$$-i \mathcal{W}^{ia} r \Psi^J_i (4g^r d_I r s - g^s d_I r s),$$  

(5.22)

and to the second order equations for the 3-form field strength $\mathcal{H}^I_{abc}$

$$D^c \mathcal{H}^I_{abc} = -\frac{1}{4} \epsilon^{abcd} \mathcal{F}^c d_I r s + \mathcal{F}_a r s \mathcal{F}_b r d_I r s g^r I + i \mathcal{W}^i r \gamma_{abc} D^r \mathcal{W}^{is} d_I r s - i \mathcal{W}^{i} r \gamma_{abc} \Psi^I_i g^s d_I r s.$$

These equations of motion allow us to construct a component action of the theory in the form

$$S = \int d^8 x \{ \frac{1}{2} D^a \Phi^I D^a \Phi^I + \Phi^I d_I r s (-Y_{ij} \mathcal{W}^r s + \mathcal{F}^r s \mathcal{F}_{rs} - i \mathcal{W}^i r D_\alpha \mathcal{W}^i s) + \frac{1}{2} \Phi^I \mathcal{F}^a r s \mathcal{F}^a r s d_I r s$$

$$+ \Phi^I i \mathcal{W}^a r \Psi^I_i r (4g^r d_I r s - g^s d_I r s) + i \mathcal{W}^i r \mathcal{F}_a r d_I r s - i \mathcal{W}^i r \mathcal{F}_a r d_I r s$$

$$+ i \mathcal{W}^i r \mathcal{F}_a r d_I r s + i \mathcal{W}^i r \mathcal{F}_a r d_I r s \}.$$
Here we assume the existence of the non-degenerate symmetric metric \( \eta_{IJ} = g_{Ir}g^{r}_J \). Also the first components of the superfields are denoted the same way as the corresponding superfields, e.g. \( \mathcal{W}^{\alpha i}|_{\theta=0} = W^{\alpha i} \), \( Y^{ij}|_{\theta=0} = Y^{ij}, \ldots \). The action (5.27) coincides (up to field redefinition) with the component action of the superconformal vector/tensor system constructed in the papers [6], [30].

The other set of equations given in [10] includes supersymmetrizations of the Hodge-duality relations between the 3-form potential and the non-Abelian vectors and scalar-4-forms relations. All the relations of this subsection are used in the next subsection for finding the component form of superfield action of vector/tensor system.

### 5.3 Superfield Lagrangian formulation of vector/tensor system

In this subsection we propose the superfield action for non-Abelian vector/tensor system in harmonic superspace and find its component form.

Let us introduce the superfield

\[
\Upsilon^I = \Phi^I + \frac{1}{2} d_{rs}(D^+_{\alpha}V^{-r}\mathcal{W}^{\alpha s} + 2V^{-r}\mathcal{Y}^{++s}) ,
\]

where

\[
\mathcal{Y}^{++s} = \mathcal{Y}^{++s} + \frac{1}{4} D^+_{\alpha} \mathcal{Y}^{++s} .
\]

Remind that the \( \mathcal{Y}^{++} \) is defined in subsection 3.2 and the \( \mathcal{Y}^{++s} \) is defined in subsection 3.3. The expression \( \Upsilon^I \) (5.24) is the only extension of \( \Phi \) preserving linearity, \( D^+_{\alpha} D^+_{\beta} \Upsilon = 0 \), i.e the \( \Upsilon^I \) is a linear superfield. Using the superfield (5.24), one defines the superfield action in harmonic superspace as follows

\[
S = \int d\zeta_4 d\mu \{ \Upsilon^I D^{++} \Upsilon^{++} + D^+_{\alpha} \Upsilon^I D^{++} \mathcal{W}^{++\alpha} \} .
\]

The invariant tensor \( g_{Ir} \) has been defined in [6]. The integrand of expression (5.26) is the only (up to common coefficient) analytic superfield constructed from \( \Upsilon^I, \mathcal{Y}^{++}, \mathcal{W}^{++} \) and containing no higher derivatives of \( D^{++}, D^+_{\alpha} \). The action (5.26) depends both on superfields of vector multiplet and superfield \( \Phi \) responsible for tensor multiplet. If \( \Phi \) is a constant \( 1/f^2 \), this action takes the form (3.12) of SYM action

\[
S \sim \frac{1}{f^2} \int d^6x d\theta d\bar{\theta} V^{++} V^{-} .
\]

Besides, the proposed action possesses supersymmetry and gauge symmetries of vector/tensor system.

The action (5.26) is the natural generalization of the free action (3.36). Indeed if we put in (5.24) \( \Upsilon = \Phi \) and use the relations (5.25), (5.3) and the identities \( D^{++} \mathcal{Y}^{++} = 0, D^{++} \mathcal{W}^{++\alpha} = 0 \), one gets the action (3.36). Thus, the action (5.26) is the only possible superfield action for non-Abelian vector/tensor system which has the free action (3.36) in the Abelian limit.
Now we derive the component form of the action (5.26). For simplicity we consider only Abelian case. Integrating over the anticommuting coordinates, one gets

\[ S = \frac{1}{8} \int d^6x d\theta \left\{ \mathcal{Y}^I T^{++} + 2 \mathcal{W}^{I+} \right\} |_{\theta=0}. \]  

(5.27)

Further we act by the derivatives \( D^{-} \) and put all the theta’s equal to zero. Then act by the harmonic derivative \( \partial^{++} \). After the cumbersome enough calculations\(^7\), we obtain all the functional structures which present in the component action of the vector/tensor system (5.23)\(^8\).

### 6 One-loop effective action in the hypermultiplet theory

In this section we will consider a calculation of superfield quantum effective action in the hypermultiplet theory coupled to the external field of vector/tensor system. We will show that the \((1,0)\) super Yang-Mills action (3.12), the vector/tensor multiplet action (3.36) or (5.27) and higher derivative vector multiplet action \([20]\) are generated as the divergent parts of the effective action. For simplicity we assume that the background is Abelian.

The classical conformal invariant action for a massless hypermultiplet of canonical dimension 2 coupled to a background 6D \( \mathcal{N} = (1,0) \) vector/tensor system is written as

\[ S = -\frac{1}{2} \int d\zeta \mathcal{D}^{++} q^A \mathcal{D}^{++} \tilde{q}_A = -\int d\zeta \mathcal{D}^{(-4)} \tilde{q}^A \mathcal{D}^{++} q^A, \]

(6.1)

with \( \mathcal{D}^{++} = D^{++} + gV^{++} \) the analyticity-preserving covariant derivative and \( V^{++} \) the analytic potential. We want to emphasize that the superfield \( V^{++} \) here is not one for pure vector multiplet, the superfield strengths, involving the superfield \( V^{++} \), obey the Bianchi identities which contain the superfield \( \Phi \) related to tensor multiplet (see subsection 5.2).

As a result the action (6.1) describes interaction of hypermultiplet with vector/tensor system. The dynamical variable \( q^A \) is a covariantly analytic superfield and \( \tilde{q}_A \) is the conjugate of \( q^A \) with respect to the analyticity preserving conjugation \([13]\) \( q^A = \epsilon^{AB} q^B = (q^+_A), \quad q^+_A = (q^+, -\tilde{q}^+) \).

The hypermultiplet effective action \( \Gamma \) is defined by

\[ e^{i\Gamma[V^{++}]} = \int \mathcal{D} q^A \mathcal{D} \tilde{q}^A \exp(-i \int d\zeta \mathcal{D}^{(-4)} \tilde{q}^A \mathcal{D}^{++} q^A). \]

(6.2)

The expression (6.2) yields

\[ \Gamma[V^{++}] = i \text{Tr} \ln \mathcal{D}^{++} = -i \text{Tr} \ln G^{(1,1)}. \]

\(^7\)The intermediate calculations are given in Appendix B with use of the relations from the subsection 5.2.

\(^8\)Derivation of component action in non-Abelian case demands an additional study.
Here $G^{(1,1)}(\zeta_1, u_1|\zeta_2, u_2) = \langle \hat{q}^+(\zeta_1, u_1)q^+(\zeta_2, u_2) \rangle$ is the superfield Green function in the $\tau$-frame. This Green function is analytic with respect to both arguments and satisfies the equation
\[ D_{\tau}^{++} G^{(1,1)}(\tau|2) = \delta^{(3,1)}(1|2) . \] (6.4)

Here $\delta^{(3,1)}(1|2)$ is the appropriate covariantly analytic delta-function
\[ \delta^{(3,4-q)}(\tau_{\pm}) = (D^{++}_\tau)^4 \delta^{14}(z_1 - z_2) \delta^{(q-q)}(u_1, u_2) = (D^{++}_\tau)^4 \delta^{14}(z_1 - z_2) \delta^{(4-q,4-q)}(u_1, u_2). \] (6.5)

The formal solution to this equation can be found analogously to four-dimensional case [33], [34], [35] and looks 9 like
\[ G^{(1,1)}(\tau|2) = -\frac{1}{4} \Box (D^{++}_\tau)^4 (D^{++}_2)^4 \delta^{14}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} , \] (6.6)
where $1/(u_1^+ u_2^+)^3$ a special harmonic distribution. In Eq. (6.6) the $\Box$ is the covariantly analytic d’Alembertian ($[D_\alpha, \Box] = 0$) which arises when $(D^{++})^4(D^{--})^2$ acts on the analytical superfield and has the form
\[ \Box = -\frac{1}{8} (D^{++})^4(D^{--})^2 = D_\alpha D^\alpha + W^{+\alpha}D^{-\alpha} + Y^{++}D^{--} - \frac{1}{4} (D^{-\alpha}W^{+\alpha}) - \frac{1}{2} \Phi . \] (6.7)

Pay attention to the fact that the term $\Phi$ in above relation is responsible for the tensor multiplet contribution. Like in four- and five-dimensional cases [35] one can obtain the useful identity
\[ (D^{++}_1)^4(D^{++}_2)^4 \frac{1}{(u_1^+ u_2^+)^3} = (D^{++}_1)^4((u_1^+ u_2^+)(D^{++}_1)^4 - (u_1^- u_2^-)\Delta^{+-} - 4 \Box \frac{(u_1^- u_2^-)^2}{(u_1^+ u_2^+)} . \] (6.8)

Here
\[ \Delta^{+-} = iD^{\alpha\beta}D^{\alpha}_\beta - 4W^{+\alpha}D^{--} - (D^{-\alpha}W^{+\alpha}) . \]

This identity is used latter for computing the effective action.

The definition (6.3) of the one-loop effective action is purely formal. The actual evaluation of the effective action can be done in the various ways (see e.g. [34], [35]). Further we will follow [35] and use the relation
\[ \Gamma(V) = \Gamma_{y=0} + \int_0^1 dy \partial_y \Gamma(y V) = -i \text{Tr} \int_0^1 dy (V^{++} G^{(1,1)}(y)) , \] (6.9)

\[ \text{As well as in the work [34] we will act by the operator } (D^{++}_1)^2 \text{ on both sides of (6.4)} \]
\[ (D^{++}_1)^2 G^{(1,1)}(1|2) = (D^{++}_1)^2 \delta^{(3,1)}(1|2) = D^{++}_1 + 2(D^{++}_2)^4 \delta^{14}(z_1 - z_2) \]
\[ \frac{1}{(u_1^+ u_2^+)^3} . \]

Now, since the equation $D^{++} f^{-[q]} = 0$ has only the trivial solution $f^{-[q]} = 0$, after the action of the operator $(D^{++}_1)^4$ we obtain:
\[ (D^{++}_1)^4(D^{++}_1)^2 G^{(1,1)}(1|2) = -8 \Box G^{(1,1)}(1|2) = 2(D^{++}_1)^4(D^{++}_2)^4 \delta^{14}(z_1 - z_2) \]
\[ \frac{1}{(u_1^+ u_2^+)^3} . \]
where
\[ \text{Tr} (V^{++} G^{(1,1)}) = \int d\zeta_1 d\zeta_2 (-4) V^{++} (1) G^{(1,1)} (1|2) |_{1=2}. \] (6.10)

Here \( G^{(1,1)} (yV) \) means the Green function depending on superfield \( yV^{++} \).

The effective action in local approximation is represented as a series in powers of the background fields and their derivatives. Further we will consider the calculation of the effective action on the base of superfield proper-time technique.

It is obvious that the leading non-vanishing contribution on the diagonal \((z_1, u_1) = (z_2, u_2)\) of the two-point function
\[ -\frac{1}{4} \int d\zeta_1 d\zeta_2 (-4) V^{++} (1) \frac{1}{(D_1^{+})^4 (D_2^{+})^4} \delta^{14} (z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} |_{1=2}, \] (6.11)
arises when \( D^{-} \) from \( \Box_1 \) hits on \((u_1^+ u_2^+)|_{u_1 = u_2}\) and in addition at least eight spinor derivatives acting on the Grassmann delta-function are required to produce a non-vanishing result, \( (D^{-})^4 (D^{+})^4 \delta^{8} (\theta_1 - \theta_2)|_{\theta_1 = \theta_2} = 1 \). On the right hand side of (6.8) the third term contains a harmonic distribution which is singular at coincident points. However, this singular terms does not contribute to \( \Gamma^{(1)} \) in the leading approximation since there is no necessary degree of \( D^{-} \).

In the framework of the proper-time technique, the inverse operator \( \frac{1}{\Box_1} \) is defined as follows
\[ -\frac{1}{\Box_1} = \int_0^\infty d(is) e^{is\Box_1}. \] (6.12)

To avoid the divergences on the intermediate steps it is necessary to introduce a regularization. We will use a variant of dimensional regularization (so called \( \omega \)-regularization) accommodative for regularization of proper-time integral (see e.g. [36]). The \( \omega \)-regularized version of the relation (6.12) is
\[ -\frac{1}{\Box_1}_{\text{reg}} = \int_0^\infty d(is) (is\mu^2)\omega e^{is\Box_1}, \] (6.13)

where \( \omega \) tends to zero after renormalization and \( \mu \) is an arbitrary parameter of mass dimension. Taking into account the (6.13) and the relations (6.6), (6.9) ones get for effective action
\[ \frac{1}{4} \int d\zeta_1 d\zeta_2 (-4) V^{++} (1) \int_0^\infty d(is) (is\mu^2)\omega e^{is\Box_1} (D_1^{+})^4 (D_2^{+})^4 \frac{1}{(u_1^+ u_2^+)^3} \delta^{14} (z_1 - z_2) |_{1=2}. \] (6.14)

Here \( \delta^{14} (z_1 - z_2) = \delta^{6} (x_1 - x_2) \delta^{4} (\theta_1^+ - \theta_2^+) \delta^{4} (\theta_1^- - \theta_2^-) \). Now one uses the representation of the delta function
\[ \delta^{14} (z_1 - z_2) = \int \frac{d^6 p}{(2\pi)^6} e^{ip_\alpha \rho_\alpha} \delta^8 (\rho_\alpha), \]
where
\[ \rho_\alpha = (x_1 - x_2)^\alpha - 2i(\theta_1^- - \theta_2^-)\gamma^\alpha \theta^-, \quad \rho^{\alpha i} = (\theta_1 - \theta_2)^{\alpha i}, \]
and \( i = +, - \). In the expression (6.14) we commute the exponent \( \exp ip_a \rho^a \) through all the operator factors to the left and then use the coincidence limit. It yields to \( e^{is\Box_1^0(X)} \cdot \delta^8(\theta_1 - \theta_2) \) where \( X_a = D_a + ip_a \), \( X^-_a = D^-_a + 2p_{a\beta} \rho^-_{\beta} \). To get expansion of effective action in background fields and their derivatives we should expand \( e^{is\Box_1^0(X)} \) and calculate the momentum integrals. All these integrals have the standard Gauss form.

Further we will concentrate on calculating the divergent part of effective action. In the regularization scheme under consideration the divergences mean the pole terms of the form \( \frac{1}{\omega} \). Expanding the \( e^{is\Box_1^0(X)} \) in the (6.14) and leaving only the terms relating to divergences one gets

\[
e^{is\Box_1^0(u^+_1 u^+_2)(D^+_1)^4(D^-_1)^4\delta^8(\theta_1 - \theta_2)|_{1=2} = - \int_0^\infty \frac{d(is)}{(is)^3}(is\mu^2)^\omega e^{-ism^2}\{isY^{++} + \frac{(is)^2}{2} [\Box, Y^{++}]\}. \tag{6.15}
\]

Here \( m^2 = \Phi \). By calculating the proper-time integral and extracting the pole terms ones get for the right hand side of the above expression

\[
\frac{1}{\omega} m^2 Y^{++} - \frac{1}{2\omega} \Box Y^{++} - \frac{1}{2\omega} W^{+\alpha} D^-_\alpha Y^{++}. \tag{6.16}
\]

Using the conditions (5.11), we obtain finally the divergent part of the effective action in the form

\[
\frac{1}{4(4\pi)^3\omega} \int dud\zeta (-4)V^{++}(D^+_\alpha \Phi W^{+\alpha} + \Phi Y^{++}) - \frac{1}{2(8\pi)^3\omega} S_{ISZ}. \tag{6.17}
\]

In principle there is also the term \( \frac{1}{\omega} \int dud\zeta (-4)V^{++}\Box Y^{++} = \frac{1}{8} \int d^6x d^8\theta V^{++}(D^{--})^2 Y^{++} = \frac{1}{8} \int d^6x d^8\theta V^{--}[D^{++}, D^{--}] Y^{++}. \) However it is cancelled out with the corresponding term \(-i D^{+\alpha\beta} D^-_{\alpha} D^-_{\beta} (is)^2 W^{+\gamma} D^-_{\gamma} W^{+\delta} D^-_{\delta} \) from the second order expansion \( \Delta^{--} e^{is\Box} \).

The divergent part of the effective action contains two contributions. The first of them is the part of the Abelian action (5.26) of the vector/tensor system proposed in Section 5. If we consider a sum of action (5.26) and first term in (6.17), we will see that this term from (6.17) determines a renormalization of coupling constant in the (6.16)\(^{10}\) The second contribution is the Ivanov-Smilga-Zupnik Abelian higher derivative action of the vector multiplet [20]\(^{11}\)

\[
S_{ISZ} = \frac{1}{2} \int dud\zeta (-4)V^{++}Y^{++}. \tag{6.18}
\]

One should emphasize once more that we have considered only the divergent parts of the effective action. Of course, the effective action contains the finite part, the calculation of which is extremely interesting but is a more difficult and delicate problem.

\(^{10}\)Coupling constant \( g \) is defined through the covariant derivative \( D^{++} = D^{++} + gV^{++} \).

\(^{11}\)The action (6.18) can be written in the different superfield forms

\[
S_{ISZ} = \int dud\zeta (-4)V^{++}\Box Y^{++} = -\frac{1}{8} \int d^6x d^8\theta V^{++}(D^{--})^2 Y^{++} = -\frac{1}{8} \int d^6x d^8\theta V^{--}[D^{++}, D^{--}] Y^{++}.
\]
The divergent part of the effective action has been calculated within the $\omega$-regularization. If we use the other regularization schemes, we can expect some extra terms in the divergent part of the effective action. For example, application of the cut-off regularization leads the same two terms as in (6.17) with replacement $1/\omega$ by $\sim \log L^2$ where $L^2$ is the cut-off on the lower limit of the proper-time integral (see the details e.g. in [36]). However within this regularization we will get the extra contribution to divergent part of the effective action in the form

$$S_{L^2} \sim L^2 \frac{1}{4(4\pi)^3} \int d\zeta^{-4} du V^{++} Y^{++}.$$  \hspace{1cm} (6.19)

This term is generated from (6.15) when we take only the $Y^{++}$ in the integrand, put $m^2 = 0$, $\omega = 0$ and cut the integral on lower limit by $L^2$. It is easy to see that this result is (up to a coefficient) the Abelian (1,0) vector multiplet action (3.12).

As a result we see that the classical actions (3.12) and (5.26) of the Abelian theory are generated in quantum theory as the one-loop counterterms. The Abelian higher derivative action introduced in [20] is also generated as the one-loop counterterm. We emphasize once more that coupling to tensor multiplet is stipulated by the superfield $\Phi$, at $\Phi = 0$ such a coupling vanishes and (6.17) gives us the divergent part of effective action for hypermultiplet in pure vector multiplet background. It is worth to pointing out that the superfield calculation of divergent part of effective action is simple enough in comparison with component calculation and demonstrates a power of superfield methods.

7 Conclusion

We have considered the superfield formulations of a class of six dimensional supersymmetric models related to low-energy dynamics of $M5$ branes. These models possess the $\mathcal{N} = (1,0)$ supersymmetry and describe the hierarchy of interacting scalar, vector and tensor fields and their superpartners [6]. Our main aim was a construction of the superfield actions for the above models. It has been shown that this aim has been achieved in framework of six dimensional harmonic superspace. As a demonstration of power of harmonic superspace approach we have considered a problem of effective action in the hypermultiplet model coupled to background field of the vector/tensor system.

We have constructed the harmonic superfield Lagrangian formulation of free $\mathcal{N} = (2,0)$ tensor multiplet in $\mathcal{N} = (1,0)$ superspace. The system of (1,0) hypermultiplet and (1,0) tensor multiplet was considered. The corresponding action is a sum of actions for the corresponding (1,0) harmonic superfields. We have found the hidden (1,0) supersymmetry transformation which mixes the hypermultiplet and tensor multiplet and shown that this transformation leaves the action invariant.

We have proposed the superfield Lagrangian formulation of the non-Abelian tensor hierarchy in (1,0) harmonic superspace. The superfield action is formulated in terms of harmonic superfields of vector and tensor multiplets. We reformulated the constraints on superstrengths [10] in terms of harmonic superspace and using these constraints computed
the component action corresponding to proposed superfield action. It was shown that in Abelian case this component action is analogous to the action of tensor hierarchy [6].

To demonstrate a power of superfield methods we have considered a problem of quantum effective action in Abelian hypermultiplet theory coupled to background fields of vector/tensor system. Such an effective action is generated by hypermultiplet loop and depends on vector and tensor multiplet superfields. We have constructed the second order differential operator with coefficients, depending on background superfields, which acts on harmonic superfields and defines a form of effective action. The superfield proper-time technique for evaluating the effective action is developed. Such a technique allows to compute the effective action in manifestly supersymmetric and gauge invariant manner. We calculated the divergent part of the effective action and showed that it has a structure analogous to one of vector/tensor multiplet superfield action and defines a renormalization of coupling constant. Also it was shown that the actions of vector and tensor multiplet are generated as the parts of divergences of the effective action.

There are the various ways to generalize and apply the results obtained. We will point out two of them. Firstly, in Section 4 we constructed the action of free (2,0) tensor multiplet in terms of (1,0) hypermultiplet and tensor multiplets. The main element of this construction was an existence of hidden (1,0) supersymmetry transformations. We hope that such transformations can also be found in non-Abelian case which allows us to construct non-Abelian superfield action for (2,0) tensor multiplet. Secondly, in Section 6 we began to study the effective action of (1,0) hypermultiplet coupled to Abelian background field of vector/tensor system. We developed the superfield proper-time technique for evaluating the effective action and calculated the divergent part of the effective action. It would be extremely interesting to find the finite part of this effective action since it can be new 6D superconformal functional written in terms of harmonic superspace. Also, it would be interesting to study the effective action for hypermultiplet in non-Abelian vector/tensor background. We hope to consider the above problems in the forthcoming papers.

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8 Appendix A. Notations and conventions

In six dimensions (1,0) and (0,1) Weyl spinors belong to the fundamental representation of $SU^*(4) \sim SO(1,5)$ group and to the transpose representation, respectively. The $8 \times 8$
Dirac matrices $\Gamma^a$ (where $a = 0, 1, \ldots, 5$) satisfy the Clifford algebra

$$\Gamma^a \Gamma^b + \Gamma^b \Gamma^c = 2\eta^{ab}. \quad (A.1)$$

The Dirac matrices for even dimensions can be chosen in the form

$$\Gamma^a = \left( \begin{array}{cc} 0 & (\gamma^a)_{\alpha\beta} \\ (\gamma^a)^{\alpha\beta} & 0 \end{array} \right),$$

with $\alpha = 1, \ldots, 4$.

Our notation and conventions follow to [10]. We use the metric $\eta^{ab} = \text{diag}(+,-,-,-,-,-)$ as well as $\varepsilon_{abcdef}\varepsilon^{a_1a_2a_3} = -6\delta_{[a}^{a_1} \delta_{b}^{a_2} \delta_{c]}^{a_3}$. Everywhere the antisymmetrization with the weight 1 is used. We chose the antisymmetric representation of the 6D Weyl $4 \times 4$ $\gamma$-matrices $\gamma^a_{\alpha\beta} = -\gamma^a_{\beta\alpha}$ and

$$\bar{\gamma}_{\alpha} = -\gamma_{\alpha} = \frac{1}{2} \varepsilon^{\alpha\beta\delta} (\gamma_{\alpha})_{\beta\delta}, \quad \gamma^a_{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\delta} (\bar{\gamma}_{\alpha})_{\beta\delta}, \quad (A.2)$$

where the $SU^*(4)$ invariant $\varepsilon^{\alpha\beta\delta}$ is the totally antisymmetric symbol ($\varepsilon_{1234} = \varepsilon^{1234} = 1$). The matrices $\gamma^a_{\alpha\beta}$ obey the relation

$$(\gamma^a \bar{\gamma}^b + \gamma^b \bar{\gamma}^a)_{\alpha\beta} = 2\eta^{ab} \delta_{\alpha\beta}^\gamma, \quad [\gamma_{ab}, \gamma_c] = 2\eta_{bc} \bar{\gamma}_a. \quad (A.3)$$

The six-dimensional Pauli-type matrices $\{\gamma^a\}$ and $\{\bar{\gamma}^a\}$ are two separate bases of $4 \times 4$ antisymmetric matrices so that

$$\gamma^a_{\alpha\beta} \bar{\gamma}_{\beta\gamma} = -2\delta_{[a}^{\sigma} \delta_{b]}^{\delta}, \quad \gamma^a_{\alpha\beta} \bar{\gamma}_{\beta\sigma} = -2\delta_{[a}^{\alpha} \delta_{b]}^{\sigma}, \quad \bar{\gamma}_{\alpha} \bar{\gamma}_{\beta} = -2\varepsilon_{\alpha\beta\sigma\delta}. \quad (A.4)$$

The normalized antisymmetrized product of Pauli-type matrices

$$\gamma^{ab} = \frac{1}{2}(\gamma^a \bar{\gamma}^b - \gamma^b \bar{\gamma}^a), \quad (\bar{\gamma}^{ab})_{\alpha\beta} = -(\gamma^{ab})_{\alpha\beta}, \quad (A.5)$$

$$\gamma_{abc} = \frac{1}{3!} \gamma_{[a \beta\gamma c]} = \gamma_{a} \gamma_{b} \gamma_{c} - \gamma_{a} \gamma_{[b} \gamma_{c]} - \gamma_{a} \gamma_{b} \gamma_{c} - \gamma_{a} \gamma_{b} \gamma_{c} - \eta_{abc} = \gamma_{a} \gamma_{b} \gamma_{c} - \eta_{abc} - \eta_{a[b} \gamma_{c]} = \gamma_{a} \gamma_{b} \gamma_{c} - \eta_{abc} - \eta_{[a[b} \gamma_{c]}$$

$$(\gamma_{abc})_{\alpha\beta} = (\gamma^{abc})_{\beta\alpha} = \frac{1}{3!} \varepsilon_{abcdef}(\gamma^{def}_{\alpha\beta})_{\alpha\beta}, \quad (\bar{\gamma}_{abc})_{\alpha\beta} = (\bar{\gamma}^{abc})_{\beta\alpha} = \frac{1}{2} \varepsilon_{abcdef}(\bar{\gamma}^{def}_{\alpha\beta})_{\alpha\beta}$$

form the basis of general $4 \times 4$ matrices with the completeness relation

$$(\gamma^{ab})_{\sigma} = 2\delta_{[a}^{\alpha} \delta_{b]}^{\beta \delta} - 8\delta_{[a}^{\alpha} \delta_{b]}^{\delta \beta}, \quad (\bar{\gamma}_{abc} \bar{\gamma}^{\delta\beta} = -24\delta^{(\sigma}_{a} \delta\beta), \quad (\gamma_{abc} \gamma^{\delta\beta} = 0, \quad (A.6)$$

$$(\gamma_{a}^{b}_{\alpha})_{\beta} = 2\delta_{[a}^{\beta} \delta_{b]}^{\alpha \delta}, \quad (\gamma_{a}^{b}_{\alpha})_{\beta} = 2\delta_{[a}^{\beta} \delta_{b]}^{\alpha \delta} - 2\delta_{\delta}^{\alpha \beta} \gamma_{a}^{\beta} = 2\delta_{[a}^{\beta} \delta_{b]}^{\alpha \delta} - 2\delta_{\delta}^{\alpha \beta} \gamma_{a}^{\beta} - 8\delta_{[a}^{\alpha} \delta_{b]}^{\delta \beta}, \quad (A.7)$$
The supersymmetric covariant derivatives in the central basis of the \((1,0)\) D6 harmonic
Minkowski six-vector can be written as the bi-spinor:

\[
\gamma_{\alpha\beta}^{a} (\gamma_{abc})_{\gamma} = 2 \varepsilon_{\alpha \beta \gamma} (\gamma_{bc})_{\delta} - \gamma_{[a}^{b} (\gamma_{bc})_{\delta]}
\]
\[
\gamma_{\alpha\beta}^{a} (\gamma_{abc})_{\delta} = -2 \delta_{[a}^{\gamma} (\gamma_{bc})_{\delta]} - \gamma_{[a \beta}^{b \gamma} (\gamma_{c]}_{\delta})
\]
\[
(\gamma_{abc})_{\alpha\beta}^{a} (\gamma_{ab})_{\beta} = 20 \tilde{\gamma}_{c}^{\alpha \beta}
\]
\[
\gamma_{\alpha\beta}^{a} (\gamma_{abc})_{\delta} = 4 (\gamma_{bc})_{\alpha}
\]
\[
(\gamma_{abc})_{\alpha\beta}^{a} (\gamma_{ab})_{\gamma} = 4 \delta_{\alpha}^{\delta} \gamma_{\beta \gamma} + 4 \delta_{\beta}^{\delta} \gamma_{\alpha \gamma}
\]

The trace relation are
\[
\text{tr} (\gamma^{a} \gamma^{b}) = 4 \eta^{ab}, \quad \text{tr} (\gamma_{a} \gamma_{c}) = -4 \delta_{[a}^{\gamma} \delta_{b]}^{\delta}, \quad \text{tr} (\gamma_{abc} \gamma_{def}) = -4 \varepsilon_{abc} \varepsilon_{def} - 4 \delta_{[a}^{\delta} \delta_{[e}^{\gamma} \delta_{f]}^{\delta}
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{\beta} = 4 \delta_{[a}^{\gamma} (\gamma_{bc})_{\delta]} - \gamma_{[a \beta}^{b \gamma} (\gamma_{c]}_{\delta})
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = 0
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = -4 \gamma^{b}
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = 2 \gamma^{a}
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = 0
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = -2 \gamma^{a}
\]

A Minkowski six-vector can be written as the bi-spinor:

\[
x_{\alpha \beta} = \gamma_{\alpha \beta}^{a} x_{a}, \quad x^{\alpha \beta} = \gamma_{\alpha \beta}^{a} x_{a}, \quad x^{2} = \frac{1}{4} \varepsilon_{a b c} x_{a} x_{b c}, \quad x_{a} = \frac{1}{4} \text{tr} \left( \gamma_{a} x \right) \equiv \frac{1}{4} \gamma_{a}^{\alpha \beta} x_{\beta \alpha}
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = -2 \delta_{[a}^{\gamma} \delta_{b]}^{\delta}
\]
\[
(\gamma_{abc})^{a} (\gamma_{ab})_{a} = -2 \varepsilon_{a \beta \gamma}
\]

The supersymmetric covariant derivatives in the central basis of the \((1,0)\) D6 harmonic superspace have the form

\[
D_{\alpha}^{+} = \frac{\partial}{\partial \theta_{- \alpha}} - i \partial_{\alpha} \theta^{+} \beta, \quad D_{\alpha}^{-} = - \frac{\partial}{\partial \theta_{+ \alpha}} - i \partial_{\alpha} \theta^{-} \beta
\]
\[
\{D_{\alpha}^{+}, D_{\beta}^{-}\} = 2 i \partial_{\alpha} \beta
\]

The definition of the vector superfield strength looks like

\[
[D_{a}, D_{b}] = F_{ab}, \quad \{D_{\alpha}^{i}, D_{\beta}^{j}\} = 0
\]
\[
F_{ab} = - \frac{1}{8} (\gamma_{ab})_{\alpha}^{\beta} F_{\alpha}^{\beta}, \quad F_{\alpha}^{\beta} = (\gamma_{ab})_{\alpha}^{\beta} F_{ab}
\]

The field strength of the 2-form potential can be decomposed as follows

\[
H_{abc}^{\pm} = \frac{1}{2} (H_{abc} \pm *H_{abc}), \quad *H_{abc} = \frac{1}{6} \varepsilon_{abcde} H_{def},
\]
where the spinor representation the (anti-)self-dual parts of a 3-form \(H\) satisfy the relations

\[
H_{\alpha \beta}^{-} = H_{abc} (\gamma_{abc})_{\alpha \beta}, \quad H_{\alpha \beta}^{(+)} = H_{abc} (\gamma_{abc})^{\alpha \beta}
\]

We also use the following notations and conventions

\[
(D^{\pm})^{4} = - \frac{1}{4!} \varepsilon^{\alpha \beta \rho \sigma} D_{\alpha}^{\pm} D_{\beta}^{\pm} D_{\rho}^{\pm} D_{\sigma}^{\pm}
\]
\[
(D^{+})^{3 \alpha} = - \frac{1}{6} \varepsilon^{\alpha \beta \gamma \delta} D_{\beta}^{+} D_{\gamma}^{+} D_{\delta}^{+}
\]
\[
D_{\alpha}^{+} D_{\beta}^{+} D_{\rho}^{+} = \varepsilon_{\alpha \beta \rho} (D^{+})^{3 \gamma}
\]
\[
D_{\alpha}^{+} D_{\beta}^{+} D_{\gamma}^{+} D_{\delta}^{+} = \varepsilon_{\alpha \beta \gamma \delta} (D^{+})^{4}
\]
\[
D_{\alpha}^{+} (D^{+})^{3 \beta} = \delta_{\alpha}^{\beta} (D^{+})^{4}
\]
\[
\theta^{\pm \alpha} \theta^{\pm \beta} \theta^{\pm \gamma} = - \varepsilon^{\alpha \beta \gamma \delta} (\theta^{\pm})^{3}, \quad (\theta^{\pm})^{3} = \frac{1}{6} \varepsilon_{\alpha \beta \gamma} \theta^{\pm \beta} \theta^{\pm \gamma} \theta^{\pm \delta},
\]
\[
(\theta^{\pm})^{3} = - \delta_{\beta}^{\gamma} (\theta^{\pm})^{4}
\]
\[
(D^{+})^{3 \alpha} (\theta^{-})^{3} = \delta_{\beta}^{\alpha}, \quad (D^{+})^{4} (\theta^{-})^{4} = 1, \quad (D^{+})^{3 \alpha} (\theta^{-})^{4} = \theta^{- \alpha}
\]
\[
(D^{+})^{4} (\theta^{-})^{3} = 1
\]
Appendix B. Component expansion of the action (5.26)

We will consider now the main steps to derive the component decomposition of the action $S \sim \int d\zeta (-4) du L^{(+4)}$ in Abelian case. Here $L^{(+4)}$ is given by (5.26). To do that we should integrate over harmonics and over all anticommuting coordinates.

Let us begin with the integration rule over anticommuting coordinates

$$\int d\zeta (-4) du = \int d^3x du (-\frac{1}{4!})\varepsilon^{\alpha\beta\gamma\delta} D_\alpha^- D_\beta^- D_\gamma^- D_\delta^-.$$ (B.1)

First, we act by spinor derivatives and kill all the theta's and then integration over harmonics $u^\pm_i$. It is obvious that $D^-_\alpha$ does not act on $\theta^{-\alpha}$, therefore the dependence on them in (5.26) can be omitted from the very beginning. Using the rules $[D^{++}, D^-_\alpha] = D^+_\alpha$ in the expression $D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta L^{(+4)}$ we get a number of the terms which are conveniently grouped into

$$-\Phi(D^+_\alpha D^-_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^+_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^-_\beta D^+_\gamma D^-_\delta Y^{++})$$ (B.2)

$$+4D^-_\alpha \Phi D^{++} D^-_\beta D^-_\gamma D^-_\delta Y^{++} - 4D^-_\alpha \Phi(D^+_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\beta D^+_\gamma D^-_\delta Y^{++})$$ (B.3)

$$+D^-_\alpha \Phi D^{++} D^-_\beta D^-_\gamma D^-_\delta Y^{++} - D^\rho \Phi(D^+_\beta D^-_\gamma D^-_\delta + D^-_\beta D^+_\gamma D^-_\delta + D^-_\beta D^-_\gamma D^+_\delta)W^{+\rho}$$

$$\{+6D^-_\alpha D^-_\beta \Phi D^{++} D^-_\gamma D^-_\delta Y^{++} - 6D^-_\alpha D^-_\beta \Phi D^+_\gamma D^-_\delta Y^{++}\} + 4D^-_\alpha D^-_\beta \Phi D^-_\gamma D^-_\delta Y^{++}$$ (B.4)

$$+4D^-_\alpha D^-_\beta \Phi(D^+_\gamma D^-_\delta + D^-_\gamma D^+_\delta)W^{+\rho} - 4D^-_\alpha D^\rho \Phi D^{++} D^-_\beta D^-_\gamma D^-_\delta W^{+\rho}$$

$$\{+6D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta \Phi D^+_\gamma D^+_\delta Y^{++} + 6D^-_\alpha D^-_\beta D^\rho \Phi D^{++} D^+_\gamma D^+_\delta W^{+\rho} - 6D^-_\alpha D^-_\beta D^\rho \Phi D^+_\gamma D^+_\delta W^{+\rho}\}$$

$$-4D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta \Phi D^{++} D^-_\rho W^{+\rho} + 4D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta \Phi Y^{++}$$ (B.5)

$$+\Phi D^{++} D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta Y^{++} + D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta \Phi D^{++} Y^{++}$$

$$+D^\rho \Phi D^{++} D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta W^{+\rho} + D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta D^\rho \Phi D^{++} W^{+\rho} + 3D^-_\alpha D^-_\beta D^-_\gamma D^-_\delta \Phi D^{++} D^-_\delta Y^{++}.$$  

Further we will investigate each of these expression separately.

First, note that the set of terms (B.5) obviously vanishes if we recall the properties of harmonic superfields. To transform the expression (B.2) to the component form we commute the spinor derivative $D^+_\alpha$ to the right, use the fact that $D^+_\alpha Y^{++} = 0$ and take account that $[D^-_\gamma, D^-_\alpha] = -2i\varepsilon_{\alpha\beta\gamma\delta} W^{-\delta}$. As a result one gets

$$\frac{1}{4!}\varepsilon^{\alpha\beta\gamma\delta} \Phi(12iD^-_\alpha D^-_\gamma D^-_\delta + 32\varepsilon_{\alpha\beta\gamma\delta} W^{-\rho} D^-_\delta + 12\varepsilon_{\beta\gamma\delta\rho}(D^-_\alpha W^{-\rho}))Y^{++}.$$  

The equations (5.11), (5.17) of the section 5.2 allow us to rewrite it as follows

$$\Phi(D^{\alpha\beta} D_{\alpha\beta} - 16iW^{-\alpha} D_{\alpha\beta} W^{+\beta} - 12Y^{-} Y^{++} + 32iW^{-\alpha} \Psi^+_\alpha - \frac{1}{2} F^\alpha_\beta F^{\alpha}_{\beta} - 2D^{\alpha\beta} D_{\alpha\beta} Y^{-}).$$  

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The last term of the above relation vanishes after integration over the harmonic variables $\varepsilon_{ij}Y_{ij} \equiv 0$. The obvious transformations of the other terms under the integral $\int duu_+^iu_j^-u_k^-u_l^- = \frac{1}{6}\varepsilon_{ik}\varepsilon_{jl}$ give

$$\int d^6x\{4D^a\Phi D_a\Phi + \Phi(4F^{ab}F_{ab} - 4Y_{ij}Y_{ij} - 8iW_+^aD_\alpha^b W^{ij\beta} + 16iW_i^a\Psi_i^a)\}. \quad (B.6)$$

The other terms in the expressions (B.3) are considered analogously. As a result, one gets

$$-24i\Psi_+^aW^{+a}Y^{--} + 16i\Psi_+^aW^{-a}(Y^{++} + \frac{1}{2}\Phi) + 16i\Psi_+^aD^{a\beta}\Psi_{-\beta} - 16i\Psi_-^aD^{a\beta}\Psi_{+\beta}$$

$$- 8i\Psi_+^aD^{a\beta}D_{\beta\gamma}W^{-\gamma} - 4i\Psi_-^aD^{a\beta}D_{\beta\rho}W^{-\rho} + 8i\Psi_-^aD^{a\beta}D_{\beta\gamma}W^{+\gamma}.$$

Integrating over harmonics leads to

$$+ 4i\Psi_+^aW^{+a}\Phi + 16i\Psi_+^aD^{a\beta}\Psi_{+\beta} + 4i\Psi_+^aW^{+a}\Phi D^{a\beta}\Psi_{+\beta} - \frac{32}{3}i\Psi_+^aW^{+a}Y_{ij}. \quad \text{(B.7)}$$

The expression (B.4) has a complicated structure, but keeping in mind that, in the Abelian case, we have the properties $W^{+a}W^{-b} = W^{-b}W^{+a}$. This allows us to make cancellations of the potentially admissible of the terms $\varepsilon_{a\beta\gamma\delta}W^{-a}W^{+b}W^{-\gamma}W^{+\delta}$, $D_{a\beta}\Phi W^{-a}W^{+b}$. As a result, we have

$$- \frac{1}{12}\mathcal{H}_{a\beta}^{(--)D^{(a\delta}F_{-\delta)}} - \frac{8i}{3}\mathcal{H}_{a\beta}^{(-)W^{-a}W^{+b}} + D^{a\beta}\Phi D_{\beta\rho}F^{a\rho} - D_{a\beta}\Phi D^{a\delta}F^{a\delta},$$

where the last two terms disappear. Finally, after using the identity (5.14) this expression takes the form

$$\int d^6x\{-\frac{1}{18}\mathcal{H}_{a\beta}^{(-)H^{(+)\alpha\delta}} - \frac{4i}{3}\mathcal{H}_{a\beta}^{(--)}W_i^aW^{i\beta}\}. \quad \text{(B.8)}$$

Thus we see that all functional structures of the component action (5.23) are obtained from the superfield action (5.26).

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