A roadmap for searching cosmic rays correlated with the extraterrestrial neutrinos seen at IceCube

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We have built Sky maps showing the expected arrival directions of 120 EeV ultrahigh energy cosmic rays (UHECR) directionally correlated with the latest astrophysical neutrino tracks observed at IceCube, including the 4-year high-energy starting events (HESE) and the 2-year Northern tracks, taken as point sources. We have considered contributions to UHECR deflections from the galactic and the extragalactic magnetic field, and a UHECR composition compatible with the current expectations. We have used the Jansson-Farrar JF12 model for the Galactic magnetic field and an extragalactic magnetic field strength of 1nG and coherence length of 1Mpc. We observe that the regions outside of the Galactic plane are more strongly correlated with the neutrino tracks than those adjacent to or in it, where IceCube HESE events 37 and 47 and, from the 2-year Northern hemisphere sample, event N13 are good candidates to search for excesses, or anisotropies, in the UHECR flux. On the other hand, clustered Northern tracks around \((l, b) = (0^\circ, -30^\circ)\) are promising candidates for a stacked point source search. As an example, we have focused on the region of 150 EeV UHECR arrival directions correlated with IceCube HESE event 37 located at \((l, b) = (-137.1^\circ, 65.8^\circ)\) in the Northern Hemisphere, far away from the Galactic plane, obtaining an angular size \(\sim 5^\circ\), being \(\sim 3^\circ\) for 200 EeV, and \(\sim 8^\circ\) for 120 EeV.

I. INTRODUCTION

The discovery of extraterrestrial neutrinos made by the IceCube (IC) Neutrino Observatory [1–3] has boosted the multimessenger searches of point sources, which eventual results should lead us to understand the high-energy astrophysical phenomena. Within this context, it is believed that extraterrestrial neutrinos are created inside or outside the source, primarily through photopion production. The pion production is caused by the interaction of ultrahigh energy cosmic rays (UHECR) either with the cosmic microwave background (CMB) or with the extragalactic background light (EBL) [4], yielding neutrinos from their decay, with energies in the range of 10PeV-1EeV or in \(\mathcal{O}(PeV)\), respectively. Thus, given the connection between UHECR and neutrinos, some degree of correlation between their respective experimental observations is expected. This kind of study has been already conducted using the extraterrestrial neutrinos observed at IceCube and a combined UHECR data from the Pierre Auger Observatory (PAO) and Telescope Array (TA), with not a positive outcome yet [5]. There have also been other attempts to seek correlations between photons and neutrinos [6] or gravitational waves with neutrinos [7]. In the future, multimessenger searches, such as joint neutrino/gamma-ray transient sources, will be facilitated by AMON [8].

The correlation analysis between UHECRs and neutrinos, as has been done in [5], relies on the distribution of cosmic ray arrival directions. This paper uses another approach to this issue. In our case, we will focus on predicting the regions on the Sky where UHECRs correlated with neutrinos are expected to arrive, considering that the neutrino tracks are pointing to the sources. In this way, these regions will constitute a tool for searching UHECR excesses on the Sky. Besides, searches in these regions could be used as complementary test of the various hypothesis implied in their construction, among others, the magnetic field model, the UHECR composition and, at a more fundamental level, the expected associated production of UHECR and neutrinos.

In fact, the choice of the Galactic and extragalactic magnetic field model is one of the most important hypothesis in our work. These fields deviate the UHECRs from its path to the Earth, making their arrival directions to not coincide with the corresponding ones of the neutrinos. For the extragalactic magnetic field (EGMF), we will use a turbulent field of strengths \(\sim 1\mu G\) and coherence lengths \(\gtrsim 1\text{ Mpc}\) following the references [9–10]. For the Galactic magnetic field (GMF), we will use field strengths of \(\sim 1\mu G\). The GMF is divided into a regular and a turbulent component, the former described by models such as those in [11–12] and the latter in [13]. The GMF deflections are dominated by the regular components, to which is added, as a secondary effect, a smearing due to the turbulent component. Another premise in the calculation of the magnetic deviation is the UHECR mass composition, which is taken into account in this paper, as it is described in sections ahead. Currently, the PAO has yet to explore the mass composition above 50 EeV, although a trend towards a heavy composition above 10 EeV is apparent [14–15]. We select 13 muons tracks from the extraterrestrial neutrino data sample given by IceCube in the 79-string and 86-string configurations [13], which spans the deposited energy range 60TeV-1PeV. This is equivalent to four years of data taking and gives us an \(E_{\nu}^{-2.58}\) neutrino flux spectrum and a flux of \(2.2 \pm 0.7 \times 10^{-18}\) \(\text{GeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\) at 100 TeV. We choose 21 muon neutrino tracks from the two-year sample [14], consisting of tracks coming from the Northern Hemisphere, containing approximately 35000 muon tracks. A subsample of the 21 tracks with the highest energies was released and are likely to be of astrophysical origin.
The paper is divided as follows: in section II we describe the analysis ingredients, which are: the extragalactic magnetic field deflections with its corresponding treatment for UHECR propagation, the galactic magnetic field deflection and the definitions for signal and background. In section III we present our results and, finally, in section IV our conclusions.

II. ANALYSIS INGREDIENTS

We subdivided this section into three parts: the EGMF deflections and UHECR propagation, GMF deflections and the Signal and Background definitions.

A. EGMF Deflections and UHECR propagation

Typical deflections in a turbulent EGMF with a Kolmogorov spectrum are given by [10]

$$\delta_{\text{rms}} = 0.8^\circ Z \left( \frac{B_0}{E^{10^{-9} G}} \right) \sqrt{\frac{D}{10 \text{Mpc}}} \sqrt{\frac{L_c}{1 \text{Mpc}}},$$

where $B_0$ is the EGMF root-mean-square field strength, $E$ is the UHECR energy, $L_c$ the coherence length of the field and $D$ is the propagation distance, which starts from the UHECR. There is no general consensus on the values of $B_0$ and $L_c$ [9] [10], in particular, we are using the values of 1 nG and 1 Mpc, respectively. Due to the large propagation distances of order $> 10$ Mpc, energy losses are taken into account being obtained from forward tracking via Monte Carlo simulation using CRPropa 3 [17]. These energy loss processes include: cosmological expansion, photopion production and photodisintegration. For the latter two processes provide both the CMB and the infrared background light (IBL) described in [18].

We estimate the magnetic deflections through the injection of individual events from the following spectrum [19]

$$Q_Z(E_p) \propto \frac{E_p^{-\gamma}}{\cosh[E_p/(ZR_{\text{max}})],}$$

where $E_p$ stands for primary UHECR energy and $R_{\text{max}} = 20$ EV marks the rigidity cutoff, where rigidity is defined, in general, as $R = E/(Z e)$. The sources emit $p, \text{He}, \text{N}, \text{Si}, \text{Fe}$ nuclei [19] according to the following ratios

$$p, \text{He}: \text{N}: \text{Si}: \text{Fe} = 0.1 : 0.27 : 0.30 : 0.32 : 0.005$$

and we assume a homogenous distribution of identical sources. The composition in Equation (3) fits well the Auger data reasonably and gives us a maximum distance of $\sim 200$ Mpc from which UHECR above $100$ EeV may reach the Earth. The propagation distance $D$ decreases exponentially with the UHECR arrival energy $E$.

We then generate $10^6$ Monte Carlo events, calculating the magnetic deflection with Equation (1) for small steps $\Delta L_i$ due to the energy losses, adding them in quadrature. This amounts to the substitution

$$\frac{Z}{E^2} \sqrt{D_i} \rightarrow \sqrt{\sum_{i=1}^{N} \frac{Z^2(L)^i}{E^2(L)} \Delta L_i},$$

where $N \Delta L = D$. These magnetic deflections also increase the UHECR propagation length by $\Delta D = \sum_{i=1}^{N} \Delta r$ where

$$\Delta r \approx 0.195 \text{Mpc} \frac{Z^2}{(E/\text{EeV})^2} \frac{L_c}{1 \text{Mpc}} \left( \frac{\Delta L}{1 \text{Mpc}} \right)^2.$$ (4)

Therefore, we obtain the deflections using [1] and [4] for a distance $D$, we let the particle propagate an additional $\Delta D$, to see if the particle loses any extra energy. We use the latter approach, looking at extra energy losses, because $\Delta D \ll D$ and does not significantly contribute to the deflection.

Finally, once the particle enters the Galaxy with an arrival energy $E$, we assign a deflection $\delta_{\text{EG}}$ per energy bin of width $\Delta E$ via

$$\delta_{\text{EG}}(E) = \frac{\text{Average } \delta_{\text{rms}} \text{ in bin}},$$

where $E \in [E_i, E_i + \Delta E]$. In figure 1 we display the values of $\delta_{\text{EG}}$ as a function of E. A band $[\delta_{\text{EG}} - \omega, \delta_{\text{EG}} + \omega]$ is included, such that 68% of the events in the bin are enclosed in this interval. Where we start with deflections intervals ranging between $[3^\circ, 12^\circ]$ at 120 EeV until reach deflections intervals, as small as $[1^\circ, 3^\circ]$ at 200 EeV.

B. GMF deflections

Once the cosmic ray enters the Galaxy with an energy $E$, we can ignore energy loss processes due to the relatively small size of the Galaxy $\sim 40$ kpc and trajectory...
Starting from the energy field deflection, given by the von Mises-Fisher distribution with initial direction $P_c$, we propagate this particle from the Earth, through the GMF, until it leaves the Earth. We assume the final position of the particle. We also assume that the azimuthal distribution of this random deflection is flat. Contrary to the parametrization in [24], we were forced to extend the Rayleigh distribution to a von Mises-Fisher distribution in order to handle deflections that are not so small. This is caused by the low rigidity particles. The energy dependence in $\kappa$ is given by

$$\kappa(l_0, b_0, R) = A_1(l_0, b_0) R + A_2(l_0, b_0) R^2.$$

This approximation has been tested in the rigidity range $10 \, \text{EV} \leq R \leq 100 \, \text{EV}$. The parameters $A_1, A_2$ were obtained using HEALPix [22] to divide the sky into 3072 pixels of equal solid angle. We emphasize that in the vicinity of the Galactic plane, where large deflections are present provided by the high turbulent fields components, the parametrization given in Equation (8) is unreliable and we solve these cases numerically.

In the small deflection hypothesis (valid for $< 15^\circ$), where $\kappa \gg 1$, concentration parameter $\kappa$ is related to the root-mean-square deflection

$$\delta_{\text{Gal}} = \frac{1}{\sqrt{\kappa}}.$$

We assume an average rigidity $\langle R \rangle_E$ for all particles with a given energy $E$, which obeys the relation $\langle R \rangle_E \approx (E/10.5 \, \text{EeV}) \, \text{EV}$ according to our simulations.

The root-mean-square deflections at $R = 10 \, \text{EV}$ for different arrival directions are shown in figure 2. We see that trajectories close to the Galactic center and/or plane can be affected by high ($> 15^\circ$) non-coherent deflections which exceed the angular resolution of experiments ($\sim 2^\circ$) by an order of magnitude. As a reference, we have included the reconstructed arrival directions of the high-energy starting events (HESE) neutrino tracks, labeled according to their corresponding event numbers as presented in [3]. We also marked the respective arrival directions of 10 EV UHECR, considering the aforementioned tracks as point source and ignoring EGMFs and the JF12 random field components.

C. Signal and Background

We work in a similar scenario such as described in [5], where a sample of $N_{\text{CR}}$ UHECR and $N_{\nu}$ neutrinos is given. The neutrinos are considered as point sources, while the $N_{\text{CR}}$ cosmic rays are a combination of signal and background events. We define $S_i^j$ as the probability density (pdf) that the $i$th cosmic ray came from the direction of the $j$th neutrino event.

$$S_i^j = \frac{\kappa_i}{2\pi(e^{\kappa_i} - e^{-\kappa_i})} \exp(\kappa_i x_i \cdot x_j).$$

FIG. 2. RMS deflection of cosmic rays with rigidity 10 EV. The white circles correspond to the reconstructed directions of HESE neutrino tracks and the white squares mark the expected arrival direction assuming only the JF12 coherent field. The white lines joining both are to match UHECRs with their corresponding track, they do not show the actual trajectory taken by the particle. Tracks are labeled by event numbers given in [3].
where \( \kappa_i = 1/\sigma_i^2 \) and \( \sigma_i \) accounts for the overall smearing of the ith cosmic ray. In the limit of small smearings \( S^i \) reduces to a two-dimensional Gaussian. For single source searches, when \( N_\nu = 1 \), the signal pdf is \( S_i \equiv S^i \) and is used in the unbinned likelihood analysis like the mentioned in [23]. For the so-called stacked source searches, when \( N_\nu > 1 \), we add up the contributions from multiple faint sources and the signal pdf is modified to

\[
S_i = \frac{1}{N_\nu} \sum_{j=1}^{N_\nu} S^i_j. \tag{11}
\]

In order to use any of these formulas, we substitute the arrival direction \( x_i \) of the UHECR by its backtracked direction \( x'_i \), assuming that the only magnetic field involved is the regular JF12 component and that the particle’s rigidity is given by \( \langle R \rangle_E \). We then determine the values of \( \kappa_i \) and \( \delta_{\text{Gal}} \) via Equations \( 8 \), that parametrizes the smearing effect of the non-regular component, and \( \delta_{\text{EG}} \), respectively, which are functions of the UHECR energy and its arrival direction. The EGMF deflections and angular resolution effects are incorporated by making the substitution

\[
\kappa_i \rightarrow \frac{1}{\sqrt{\delta_{\text{Gal}}^2 + \delta_{\text{EG}}^2 + \delta_{\text{res}}^2}}, \tag{12}
\]

with \( \delta_{\text{Gal}} = \langle (R) \rangle_E \), \( \delta_{\text{EG}} = \delta_{\text{EG}}(E) \) and \( \delta_{\text{res}} \) is the angular resolution of the experiment. We will assume an energy independent deflection \( \delta_{\text{res}} = 2^\circ \) as a characteristic angular resolution for IceCube and UHECR ground array experiments.

For the large GMF deflections present at the Galactic plane, where Equation \( 8 \) is not valid, \( S_i \) is determined entirely via Monte Carlo. We also define \( B_i \) as the probability density that the CR is a background event. Typically \( B_i = 1/4\pi \) or, in the case of the analysis in [5], the normalized exposure of the experiment.

### III. RESULTS

Now, we quantify how likely an observed UHECR, with a given arrival direction \( x_i \) and energy \( E_i \), could have the same origin as the neutrino track, which we treat as the UHECR point source, through the following ratio:

\[
W_i = \frac{\int S_i d\Omega}{\int_{B_i} d\Omega}, \tag{13}
\]

where \( d\Omega \) is integrated in a region of 1° in the Sky, centered around the neutrino track.

In figure 3, we show the effect of \( W_i \) in a single point source search, located at IC event 37 \((l,b) = (-137.1^\circ, 65.8^\circ)\) in the Northern hemisphere, labeled according to the numbering in Reference [5], in two plots. One is a two-dimensional map in the coordinates \((l,b)\), where \( P_c \) marks the expected UHECR arrival direction, when considering only the JF12 coherent component. The other one is its corresponding one-dimensional projection in angular distance centered in \( P_c \), where the solid line is the average value of \( W_i \) and the shaded region covers the whole set of values given by the points at the angular distance contour. We have selected IC event 37 because it belongs to a region where the GMF random component deflections are very small, as shown in figure 2 being the random EGMF responsible for most of the smearing around \( P_c \), displayed in the two-dimensional plot. These characteristics turn IC event 37 into a good candidate for searches of UHECR excesses around it. In both plots it is clear that \( P_c \) gives the highest ratio, since, by construction, it is here where the signal pdf is maximized. Ellipsoidal (vertical) solid/dashed lines are shown for the two (one) dimensional plot. The solid line represents the size of the typical or average angular search region \( \delta_s = \sqrt{\delta_{\text{Gal}}^2 + \delta_{\text{EG}}^2 + \delta_{\text{res}}^2} \), measured from \( P_c \), while the dashed lines include the effects of the spread in \( \delta_{\text{EG}} \) shown in figure 1. Our results show that UHECRs confined within the region of size \( \delta_s \), that have the appropriate energy and arrival direction, would have very high values of \( W_i \). In the one-dimensional plot, a band enclosing the \( W_i \) average is also displayed, and is caused by the anisotropy of the random deflections, or, equivalently, the B field itself. Otherwise, if the random deflections were isotropic, there would be no band, which means that all the values would converge to a single one. For small angular distances, the variation, or width of the band, of \( W_i \), is small because the parameter \( \kappa_i \) is essentially constant, and as we move away from \( P_c \) this variation is significant.

For a wider perspective, we are showing in figure 4 the dependence of \( \delta_s \) with UHECR arrival energy. It follows the same behaviour of \( \delta_{\text{EG}} \) (see figure 1) since \( \delta_s \) in the case of IC event 37 is dominated by the extragalactic contribution.

It is important to mention that in case the current or future ground array experiments were able to identify the UHECR mass, we would have a sensible improvement in our analysis since the uncertainties in the rigidity (i.e. \( \delta_{\text{EG}} \)) are going to be small and inside the Galaxy we would know with greater precision the backtrack trajectory because \( R \) is constant within the Galaxy. The analysis is also dependent on the UHECR-neutrino sources being within the GZK sphere, such that we may observe the cosmic rays.

The sky map of \( W_i \) for a stacked search of cosmic ray events by using the IceCube tracks as the assumed point sources is presented in figure 3 which assumes incoming 120EeV cosmic rays, which is our lower energy limit in our analysis, and unknown mass number. This figure contains two Sky maps, one for the IC HESE event sample [5] and another for the combined sample of HESE events and the second one using both the 4-year HESE tracks and the 2-year IceCube Northern Sky tracks [10] in a combined search. As before, the circles represent the positions of the neutrino tracks. In the HESE Sky map, we also included squares indicating their respec-
IV. SUMMARY AND CONCLUSIONS

We have built two Sky maps showing different regions where 120 EeV UHECR excesses with respect to the isotropic background should appear: one for the IC 4-year HESE tracks and another for a combined search using the 4-year HESE events and the 2-year Northern Sky tracks. These excesses are inferred from the measurement of the correlation between a given UHECR arrival direction with the IceCube neutrino tracks, which are taken as point sources. The GMF and EGMF deflections have been calculated, using, correspondingly, the JF12 model and EGMF of strength $\sim 1\text{ nG}$ and coherence lengths $\gtrsim 1\text{ Mpc}$. We note that the out-of-plane
regions concentrate higher correlation values, quantified by the probability ratio $W_i$, being more promising than the ones near the Galactic plane for revealing excesses. Some of these regions can be correlated clearly with a single neutrino track, for instance in events 37, 47 and N13. These events are candidates to include in a point source search. For the stacked source search, good candidates include the tracks that contribute to the region in $(l, b) = (0^\circ, \sim 30^\circ)$.

In particular, we take a closer look into event 37, where the GMF random component is negligible, considering an energy of 150 EeV and getting a region, where most of the UHECR excess should be located, of angular size $\sim 5^\circ$. If the UHECRs had energies of 120 EeV or 200 EeV the angular size would be $\sim 8^\circ$ or $\sim 3^\circ$, respectively. This similar tendency can be extrapolated to the Sky map where we expect regions with a much smaller angular spread, as long as we increase the UHECR energy. It is clear that the Sky map presented relies on the current statistic of the neutrino tracks and as new data is released, we may obtain more favourable search regions. Naturally, the possibility to find UHECR in these regions intrinsically depends on these sources being located within the GZK sphere. Finally, we must say that the results of this analysis should improve if the ground array experiments identify the UHECR mass.

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