Spin wave driven domain wall motion in easy-plane ferromagnets: A particle perspective

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In easy-plane ferromagnets, we show that the interplay between a domain wall and a spin wave packet can be formulated as the collision of two massive particles with a gravitylike attraction. In the presence of magnetic dissipation, the domain wall mimics a particle subject to viscous friction, while the spin wave packet resembles a particle of variable mass. Due to the attractive nature of the interaction, the domain wall acquires a backward displacement as a spin wave packet penetrating the domain wall, even though there is no change in momentum of the wave packet before and after penetration.

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Introduction. Magnetic domain wall motion is widely used in manipulating the magnetic information for both storage and processing [1–3], and its understanding is crucial for both fundamental physics and technological applications [4–6]. Typical approaches to drive the domain wall motion include the magnetic field [7–9], current-induced spin-transfer torque [10–13] and spin-orbit torque [14–16], as well as spin waves [17–26]. Due to the intrinsic magnetic nature, the spin wave driven domain wall motion is of special interest toward purely magnetic computing [26–28].

Investigations on the interplay between spin waves and domain walls are complicated by the fast magnetization oscillations of spin waves in both time and space, as well as the inhomogeneous magnetization of domain walls. To overcome this complexity, a common and powerful approach is to make use of linear or angular momentum conservation [28], which focuses on global momentum transfer and avoids local interaction details.

In easy-axis ferromagnets, the spin wave can either drag or push the domain wall, depending on whether the spin wave is transmitted or reflected [17–19]. And when extending to antiferromagnetic and ferrimagnetic environments, the direction of the domain wall motion can be controlled by tuning the spin wave polarization [21–24] or frequency [25]. The easy-plane magnet is another important category of magnetic materials, and is a fertile ground for emerging physics including relativistic dynamics [29], magnon superfluids [30–32], magnetic vortices [33,34], and bimerons [35,36]. However, the domain wall in easy-plane ferromagnets is only studied in limited cases [37–40], and its motion driven by spin waves remains elusive.

In this Letter, we demonstrate that the domain wall in easy-plane ferromagnets is displaced toward the spin wave source as the wave passing through the wall, albeit the spin wave carries the same momentum before and after penetration. By developing a unified Lagrangian framework, we transform the interplay between a spin wave packet and a domain wall to an equivalent collision process between two massive particles subject to a gravitylike attraction, which greatly simplifies the wave-soliton interaction scenario and enables a classical yet intuitive understanding.

Basic model. We consider a one-dimensional ferromagnetic wire extending along the z axis as shown in Fig. 1(a), where the magnetization direction (red arrows) is denoted by a unit vector $\mathbf{m}(x)$. We assume that the ferromagnet has a strong hard-axis anisotropy favoring the z direction, and thus the x-y plane is the easy plane. In terms of the $z$ component of magnetization $m_z$, and the azimuthal angles $\phi$ with respect to the $z$ axis, the magnetization direction is explicitly denoted by $\mathbf{m} \equiv (\sqrt{1-m_z^2} \cos \phi, \sqrt{1-m_z^2} \sin \phi, m_z)$. The magnetic free energy is $\tilde{E} = (S/2) \int [K' m_z^2 + K \sin^2 \phi + A(\partial, \mathbf{m})^2] dx$, where $K'$ is a strong easy-plane anisotropy favoring the $x$-$y$ plane, $K \ll K'$ is a weak easy-axis anisotropy along the $x$ axis, and $A$ is the exchange coupling constant. Here, $S = \mu_0 M_s A$ is the magnetic flux, where $\mu_0$ is the vacuum permeability, $M_s$ is the saturation magnetization, and $A$ is the cross-section area of the magnetic wire. The main effect of the dipolar field is to renormalize the anisotropy constants in exchange regime, thus is not included explicitly in this Letter.

The Lagrangian of the magnetic system reads [8,39]

$$\mathcal{L} = \frac{S}{\gamma} \int m_z \phi \, dx - \tilde{E},$$

where the first term is the kinetic energy of magnetic system [8], and $\gamma$ is the gyromagnetic ratio. In addition, the

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and the Rayleigh function simplifies to

$$L_{\text{Rayleigh}} = \gamma \int m^2 dx,$$

where $\gamma$ is the Gilbert damping constant. The Euler-Lagrange variation of $L$ in Eq. (1) and the accompanying $R$ yields the Landau-Lifshitz-Gilbert (LLG) equation

$$m = -\gamma m \times H + \alpha m \times \dot{m},$$

where $H = -S^{-1}\delta E/\delta m$ is the effective magnetic field acting on magnetization $m$.

Due to the strong easy-plane anisotropy $K'$, the magnetization lies predominatingly in the $x$-$y$ plane with $m_z \approx 0$, as illustrated in Fig. 1(a). Hence, the Lagrangian simplifies to

$$L = \frac{S}{2} \int \left[ \frac{\dot{\phi}^2}{\gamma^2 K'} - A(\dot{\phi})^2 + K \sin^2 \phi \right] dx,$$

and the Rayleigh function simplifies to $R = (\alpha S/2 \gamma) \int \dot{\phi}^2 dx$. Correspondingly, the magnetic dynamics reduces to a damped sine-Gordon equation on the azimuthal angle $\phi$ [29,37–40],

$$\frac{1}{c^2} \phi - \frac{\alpha}{2} \phi = -\partial_t^2 \phi + \frac{\sin 2\phi}{2W^2},$$

where $c = \sqrt{\alpha K'}$ is the effective “speed of light” of the magnetic system, $W = \sqrt{A/K}$ is the characteristic magnetic length (or the domain wall width), and $\beta = \alpha \gamma K$ is the dissipation coefficient.

**Domain wall and spin wave packet.** The dynamics of the azimuthal angle $\phi(x, t)$ in Eq. (3) can be naturally divided into the slow dynamics due to the domain wall motion and the fast dynamics due to the spin wave excitation, i.e., $\phi = \phi_0 + \phi'$. The clear separation of slow and fast dynamics is ensured by the high frequency of the exchange-type spin wave under investigation here, regardless of the hard or soft anisotropy materials. In the following, we first investigate the dynamics of $\phi_0$ and $\phi'$ separately, and then their interaction caused by the nonlinearity embedded in the $\sin 2\phi$ term of Eq. (3).

It is well known that the static domain wall solution hosted by Eq. (3) has the following soliton form [41,42],

$$\phi_0(x, t) = 2 \arctan \left[ \frac{\exp \left( -\frac{x - X(t)}{W} \right)}{1} \right],$$

where $X$ is the domain wall central position. The domain wall profile in Eq. (4) corresponds to the magnetization rotating steadily from $m = -\hat{x}$ ($\phi_0 = \pi$) at $x \ll X$ to $m = +\hat{x}$ ($\phi_0 = 0$) at $x \gg X$, as illustrated in Fig. 1(a). Due to the invariance of the topological charge [41,42], $Q = (1/\pi) \int d\phi = -1$, the moving domain wall in Eq. (4) maintains a relatively fixed shape, hence its evolution is mainly determined by the variation of its central position $X(t)$: $\phi_0(t) \equiv \phi_0[X(t)]$.

In the meantime, we consider a spin wave packet in the Gaussian form,

$$\phi'(x, t) = \frac{1}{k_0 W} \sqrt{\frac{2nh\omega_0}{\gamma \pi s K}} \exp \left( -\frac{(x - \chi(t))^2}{2a^2} \right) \cos \{k_0[x - \chi(t)]\},$$

where $n$ is the magnon number representing the spin wave intensity, $\sigma$ is the typical width, $\chi$ is the central position, and $k_0$ and $\omega_0$ are the central wave vector and frequency of the wave packet. For the exchange-type spin wave under investigation in this work, we may assume $k_0 W \gg 1$, then the spin wave packet is narrow in both spatial and wave-vector spaces, i.e., $\sigma \ll W$ and $1/\sigma \ll k_0$ [43,44]. Hence, the evolution of the spin wave packet can also be described by the variation of its central position $\chi(t)$: $\phi'(t) \equiv \phi'[\chi(t)]$.

**Particle collision model.** As seen from Eqs. (4) and (5), both the soliton-like domain wall and the spin wave packet are reduced to particle-like objects characterized by their positions: $X(t)$ for the domain wall and $\chi(t)$ for the spin wave packet. In terms of these two degrees of freedom, the Lagrangian is recast from Eq. (2) to

$$L = \frac{M}{2} \dot{X}^2 + \frac{m_c^2}{2} \chi^2 - U,$$

where the effective masses of the domain wall and spin wave packet are defined by their static energies as $M c^2 = 2SKW$ and $mc^2 = n\hbar\omega_0$, respectively. The interaction energy $U$ in Eq. (6), originated from the sin $2\phi$ term in the sine-Gordon equation, takes the following attractive gravitylike form,

$$U = -GMm \sech^2 \left( \frac{\chi - X}{W} \right) \frac{2W^2}{c^4},$$

where $G = c^4/(SAk_0^2)$ is the effective gravitational constant. This interaction energy $U$ is collaboratively caused by the inhomogeneous domain wall magnetization and the reduction of its magnitude by the spin wave [44]. It is noteworthy that the static energies of the domain wall and spin wave packet $M c^2$ and $mc^2$ are omitted in the Lagrangian of Eq. (6) since they are both constants. Equations (6) and (7) indicate that the interplay between a domain wall and a spin wave packet in
easy-plane ferromagnets can be viewed as the collision of two particles with mass $m$ and $M$ subject to a gravitational-like attraction of energy $U$, as illustrated in Fig. 1(b).

Similar to the transformed Lagrangian in Eq. (6), the Rayleigh dissipation function is transformed to $\mathcal{R} = \beta MX^2/2 + \beta m\dot{X}^2/2$, where the dissipation of the domain wall and spin wave packet share the same coefficient $\beta$. Because of the topological protection, the domain wall profile in Eq. (4) maintains the same form, regardless of dissipation. However, the intensity of the spin wave packet in Eq. (5) is expected to decay due to dissipation. As a result, the domain wall mass $M$ is a constant of time, but the mass $m$ of the spin wave packet reduces in the magnetic background, just like an icy ball dissolving in water.

From the Lagrangian in Eq. (6) and the accompanying Rayleigh dissipation function, the dynamics of the domain wall and spin wave packet are then governed by

$$\frac{dP}{dt} : \dot{M}X = -\beta U - \beta MX, \quad (8a)$$
$$\frac{dp}{dt} : \left\{ \begin{array}{l} m\dot{X} = -\frac{\alpha}{\gamma}, \\ m = -\beta m, \end{array} \right. \quad (8b)$$

where Eqs. (8a) and (8b) describe the evolution of the domain wall momentum $P \equiv MX$ and the momentum of the spin wave packet $p \equiv m\dot{X}$, respectively. In the absence of dissipation ($\beta = 0$) we have $d(P + p)/dt = 0$ in this isolated two-body system, i.e. the total momentum is conserved or the momenta of the domain wall and spin wave are exchanged to one another via mutual potential $U$. The dissipation has different effects on the domain wall and the spin wave packet: The domain wall experiences a viscous force and slows down due to dissipation, but the spin wave packet loses its mass by dissolving into the background while maintaining its speed. The above viscosity and dissolution scenarios represent two opposite limits of particle dynamics in a fluid, with the former and latter denoting the full resistance and compliance of mutual deformation, respectively [45]. With Eq. (8), we successfully transformed the highly nontrivial interaction between an inhomogeneous magnetic domain wall and a fast-oscillating spin wave packet into a simple collision scenario between a particle of constant mass $M$ and a particle with variable mass $m$.

**Domain wall motion driven by spin wave packet.** We now consider the simplest case without dissipation ($\beta = 0$), for which Eq. (8) becomes $\dot{M}X = -m\dot{X} = -\beta U$. Due to the attractive nature of $U$, as the spin wave packet passes through the domain wall, the wave packet and the domain wall experience a forward and backward jump, respectively, as denoted by $\Delta X$ and $\Delta \chi$ in Fig. 1(c).

More explicitly, a spin wave packet is typically much lighter than the domain wall, $m \ll M$, therefore we may regard the domain wall as more or less static when the packet passes through the domain wall. The wave packet velocity at position $\chi$ can be approximately evaluated via the energy conservation $m\dot{X}^2/2 - mv_0^2/2 = -U(\chi - X)$, which yields

$$v_\chi - v_0 \simeq -\frac{U(\chi - X)}{m v_0} > 0, \quad (9)$$

where $v_0 = v_\chi \rightarrow \pm \infty$ is the initial and final velocity of the packet before and after penetration with $U = 0$. Because of this velocity enhancement, the spin wave packet gains a velocity inside the domain wall, thus leading to a forward jump in comparison to the case without the domain wall:

$$\Delta \chi = \int (v_\chi - v_0)dt \simeq -\int \frac{U}{mv_0^2}d\chi = \frac{GM}{v_0^2}. \quad (10)$$

In turn, the domain wall acquires a backward jump due to momentum conservation:

$$\Delta X = -\frac{m}{M} \Delta \chi = -\frac{Gm}{v_0^2}. \quad (11)$$

Here, we focus on the displacement of the domain wall, because the domain wall stops once the wave packet leaves the domain wall behind.

In the presence of dissipation ($\beta \neq 0$), the dynamics of packet position $\chi$ is unaltered in Eq. (8b), hence the packet forward jump $\Delta \chi$ in Eq. (10) remains the same. However, the domain wall velocity $V$ and the wave packet mass $m$ are subjected to a similar form of dissipation in $-\beta V$ and $-\beta m$, which means that both of them decrease by a common factor of $e^{-\beta t}$. Consequently, the domain wall displacement in Eq. (11) is modified to

$$\Delta X = -\frac{Gm_0}{v_0^2} \exp \left( \frac{\beta}{v_0} (\chi_0 - X_0) \right), \quad (12)$$

where $m_0$ and $\chi_0$ are the initial mass and position of the spin wave packet, and $X_0$ is the initial position of the domain wall.

As shown in Fig. 2, the domain wall displacements $\Delta X$ in Eqs. (11) and (12) formulated in terms of particle collision are confirmed numerically using the micromagnetic...
in the domain wall.

In the case of finite dissipation $\beta \neq 0$, if the spin wave density is $\rho_0$ at the source at $x = X_0$, then the spin wave density at the domain wall center $X(t)$ attenuates exponentially due to dissipation, $\rho = \rho_0 \exp[-\beta |X_0 - X(t)|/v_0]$, hence the spin wave density experienced by a moving domain wall satisfies $\dot{\rho}/\rho = -\beta V/v_0$. In conjunction with Eq. (13), one has $V/V = \dot{\rho}/\rho = -\beta V/v_0$, i.e., the domain wall effectively experiences a drag force toward the spin wave source with drag coefficient $\beta/v_0$. Therefore, the evolution of domain wall velocity is explicitly described by

$$V = -\frac{|V_0|}{1 - \beta V/v_0} t,$$  

where $V_0 = -G\rho_0 \exp[-\beta (X_0 - X_b)/v_0]$ is the initial velocity at the moment that the spin wave touches the domain wall.

The validity of the domain wall velocity given by Eqs. (13) and (14) is confirmed by micromagnetic simulations [47] as shown in Fig. 3. Irrespective of dissipation, the domain wall stops immediately once the spin wave leaves the domain wall behind, indicating that the domain wall is only temporarily driven by the spin wave during its penetration. In Fig. 3(b), the evolution of domain wall velocity with dissipation adopts the reciprocal form in Eq. (14) instead of an exponential growth form, endorsing the unconventional role of dissipation in shaping the interplay between the domain wall and spin wave.

**Discussions.** The easy-plane ferromagnet is distinct from its easy-axis counterpart in two aspects: The spin wave is linearly polarized, and thus does not carry any angular momentum; the domain wall is inertial, and tends to maintain its original velocity [29,49,50]. In addition, the equal spin wave amplitudes and velocities before and after penetration indicate that the domain wall motion in this work is not induced by the permanent transfer of angular momentum [17] or linear momentum [18,19]. The elevated spin wave velocity inside the domain wall, on the other hand, indicates a temporary borrowing and a later return of linear momentum between the spin wave and the domain wall. The above temporary momentum transfer scenario naturally extends to antiferromagnets and ferrimagnets [29], where the magnetic dynamics can also be mapped to Lorentz-invariant sine-Gordon equation (3).

The spin wave passes through the domain wall perfectly in this work, and therefore also shares many common features with the Balazs thought experiment on light passing through a block of transparent medium [51,52]. With a noticeably lower “speed of light” in this magnetic environment, more insights on the Abraham-Minkowski dilemma are accessible [52].

**Conclusions.** In conclusion, based on a particle collision scenario, we show that the domain wall is dragged backward during penetration of a spin wave in an easy-plane ferromagnet. The particle-based viewpoint established in this Letter provides a simple yet powerful tool to analyze the interplay between soliton and fluctuation waves in various nonlinear systems [41,42].

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