Decay of Temperature Fluctuations in Homogeneous Turbulence Before the Final Period for the Case of Multi-point and Multi-time in a Rotating System in Presence of Dust Particle

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Abstract: Using Deissler’s approach the decay temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle is studied and have considered correlation between fluctuating quantities at two and three point. Two and three point correlation equations in a rotating system is obtained and the set of equations is made to determine by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier transforms. Finally integrating the energy spectrum over all wave numbers, the energy decay law of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle is obtained.

Keywords: Dusty fluid turbulence, temperature fluctuations, rotating system, multi-point and multi-time.

INTRODUCTION

Deissler\(^{(1,2)}\) developed a theory for homogeneous turbulence, which was valid for times before the final period. Following Deissler’s theory Loeffler and Deissler\(^{(3)}\) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their study, they presented the theory, which is valid during the period for which the quadruple correlation terms are neglected compared to the 2\(^{\text{nd}}\) and 3\(^{\text{rd}}\) -order correlation terms. Using Deissler’s same theory Kumar and Patel\(^{(4)}\) studied the first-order reactants in homogeneous turbulence before the final period for the case of multi-point and single-time. The problem\(^{(4)}\) is extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel\(^{(5)}\) and also the numerical result of\(^{(5)}\) carried out by Patel\(^{(6)}\). Following Deissler’s approach Sarker and Islam\(^{(7)}\) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. M. A. Islam and M. S. A. Sarke\(^{(8)}\) also studied the first –order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and L. Rahman\(^{(9)}\) studied the decay of temperature fluctuations in MHD turbulence before the final period. Sarker and Islamic\(^{(10)}\) also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time.

Azad and Sarker\(^{(11)}\) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Azad and Sarke\(^{(12)}\) also studied the decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Sarker and Azad\(^{(13)}\) studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system. Azad, Sarker and Mondol\(^{(14)}\) studied the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system.

In their approach, they used two and three point correlations and neglecting fourth- and higher-order correlation terms compared to the second-and third-order correlation terms.

In this paper the method of\(^{(1,2)}\) is used and we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle.

Basic Equations: For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written as

\[
\frac{\partial \tilde{T}}{\partial t} + \tilde{u}_i \frac{\partial \tilde{T}}{\partial x_i} = \frac{k}{\rho C_p} \frac{\partial^2 \tilde{T}}{\partial x_i \partial x_i} \tag{1}
\]
where \( \bar{T} \) and \( \bar{u} \) are instantaneous values of temperature and velocity; \( k \), thermal conductivity; \( \rho \), fluid density; \( C_p \), heat capacity at constant pressure; \( x \), space coordinate; \( t \), time and the repeated subscripts are summed from 1 to 3.

Breaking these instantaneous values into time average and fluctuating components as \( \bar{T} = \langle T \rangle + T' \) and \( \bar{u}_i = \langle u_i \rangle + u_i' \) allows equation (1) to be written as

\[
\frac{\partial}{\partial t}\langle T \rangle + \langle u \rangle \frac{\partial}{\partial x} \langle T \rangle = \gamma \frac{\partial^2}{\partial x^2} \langle T \rangle + T' + \langle u \rangle \frac{\partial}{\partial x} + \frac{\partial}{\partial x} \langle u \rangle \frac{\partial}{\partial x} + \langle u \rangle \frac{\partial}{\partial x} + u_i \frac{\partial}{\partial x} =
\]

\[
\gamma \left[ \frac{\partial^2}{\partial x^2} \langle T \rangle + \frac{\partial^2}{\partial x \partial x} \langle T \rangle \right]
\]

(2)

where \( \gamma = \frac{k}{\rho C_p} \). From the condition of homogeneity it follows that \( \frac{\partial \langle T \rangle}{\partial x} = 0 \), and in addition the usual assumption is made that \( \langle T \rangle \) is independent of time and that \( \langle u_i \rangle = 0 \). Thus equation (2) becomes

\[
\frac{\partial T}{\partial t} + u_i \frac{\partial}{\partial x_i} T = \left( \frac{\nu}{\rho} \right) \frac{\partial^2 T}{\partial x_i \partial x_i}
\]

(3)

where \( \frac{\nu}{\rho} \) Prandtl number; \( \nu \), kinematic viscosity.

Equation (3) is assumed to hold at the arbitrary point \( p \). For the point \( p' \) the corresponding equation can be written as

\[
\frac{\partial T'}{\partial t} + u_i' \frac{\partial}{\partial x_i} T' = \left( \frac{\nu}{\rho} \right) \frac{\partial^2 T'}{\partial x_i' \partial x_i'}
\]

(4)

Multiplying equation (3) by \( T' \), equation (4) by \( T \) and taking ensemble average, result in

\[
\frac{\partial \langle TT' \rangle}{\partial t} + \frac{\partial \langle TT'u_i \rangle}{\partial x_i} = \left( \frac{\nu}{\rho} \right) \frac{\partial^2 \langle TT' \rangle}{\partial x_i \partial x_i'}
\]

(5)

with the continuity equation

\[
\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_i} = 0
\]

(7)

Angular bracket \( \langle \cdots \rangle \) which is used to denote an ensemble average. Using the transformations

\[
\frac{\partial}{\partial x_i} = -\frac{\partial}{\partial r_i}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x_i} = \frac{\partial}{\partial r_i}
\]

into equations (5) and (6), one obtains

\[
\frac{\partial \langle TT' \rangle}{\partial t} - \frac{\partial \langle u_i TT' \rangle}{\partial r_i} (-r, -\Delta t, t + \Delta t) + \frac{\partial \langle TT'u_i' \rangle}{\partial t} \left( \delta, \Delta t, t \right) = 2 \left( \frac{\nu}{\rho} \right) \frac{\partial^2 \langle TT' \rangle}{\partial r_i \partial r_i}
\]

(8)

\[
\frac{\partial \langle TT' \rangle}{\partial r_i} + \frac{\partial \langle u_i TT' \rangle}{\partial r_i} (-\delta, -\Delta t, t + \Delta t) =
\]

\[
\left( \frac{\nu}{\rho} \right) \frac{\partial^2 \langle TT' \rangle}{\partial r_i \partial r_i}
\]

(9)

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms.

\[
\langle TT'(\delta, \Delta t, t) \rangle = \int_{\delta \to \infty} \left( \delta, \Delta t, t \right) \exp(i \delta \hat{K}) d\hat{K}
\]

(10)

\[
\langle u_i TT'(\delta, \Delta t, t) \rangle = \int_{\delta \to \infty} \left( \delta, \Delta t, t \right) \exp(i \delta \hat{K}) d\hat{K}
\]

(11)
and $\langle u'_i T'_j (\mathbf{p}, \Delta t, t) \rangle = \langle u'_i T'_j (\mathbf{-p}, -\Delta t, t + \Delta t) \rangle$

$$= \int \left\{ g \cdot \mathcal{S} \left( -\hat{K}, -\Delta t, t + \Delta t \right) \right\} \exp \left[ \hat{K} \cdot \mathbf{p} \right] d\hat{K} \quad (12)$$

(Interchange are made between the points $\mathbf{p}$ and $\mathbf{p}'$).

where $\hat{K}$ is known as a wave number vector and magnitude of $\hat{K}$ has the dimension $1/\text{length}$ and can be considered to be the reciprocal of an eddy size.

Substituting the equations (10)–(12) into equations (8) and (9) leads to the spectral equations.

$$\partial \langle \mathcal{S} \rangle / \partial t + 2 \left( \frac{\nu}{P_r} \right) k^2 \langle \mathcal{S} \rangle = iK_i \left[ \langle \mathcal{S} \rangle \mathcal{S} \left( \hat{K}, \Delta t, t \right) \right] - \langle \mathcal{S} \rangle \mathcal{S} \left( -\hat{K}, -\Delta t, t + \Delta t \right) \quad (13)$$

$$\partial \langle \mathcal{S} \rangle / \partial \Delta t + 2 \left( \frac{\nu}{P_r} \right) k^2 = -iK_i \left[ \mathcal{S} \left( -\hat{K}, -\Delta t, t + \Delta t \right) \right] \quad (14)$$

In equations (13)–(14) the quantity $\mathcal{S} (\hat{K})$ may be interpreted as a temperature fluctuation “energy” contribution of thermal eddies of size $1/k$. The time derivative of this “energy” as a function of the convective transfer to the wave numbers and the “dissipation” due to the action of the thermal conductivity. The term on the right hand side of equation (13) is also called transfer term while the second term on the left hand side is the “dissipation” term.

**Three-Point, Three-Time Correlation and Spectral Equations**: In order to obtain the three-point, three-time correlation and spectral equations, we write the Navier-Stokes equation for turbulent flow of dusty incompressible fluid in a rotating system at the point $P$, energy equations at the points $\mathbf{p}'$ and $\mathbf{p}$ separated by the vectors $\mathbf{p}'$ and $\mathbf{p}'$

$$\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x_i} \left( u_j u_i \right) = -\frac{1}{\rho} \frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 u_j}{\partial x_i \partial x_i} - 2 \left( \frac{\nu}{P_r} \right) \Omega \cdot u_j + f (u_j - \nu_j) \quad (15)$$

$$\frac{\partial T''}{\partial t} + u'_i \frac{\partial T''}{\partial x'_i} = \left( \frac{\nu}{P_r} \right) \frac{\partial^2 T''}{\partial x'_i \partial x'_i} \quad (16)$$

and

$$\frac{\partial T'''}{\partial t} + u'_i \frac{\partial T'''}{\partial x'_i} = \left( \frac{\nu}{P_r} \right) \frac{\partial^2 T'''}{\partial x'_i \partial x'_i} \quad (17)$$

Multiplying equations (15)–(17) by $T'' \cdot u'_T$ and $u_j T'$ respectively and then taking ensemble average, we obtained

$$\frac{\partial \langle u_j T'' u_i \rangle}{\partial t} + \frac{\partial \langle u_j T'' u_i \rangle}{\partial x_i} = -\frac{1}{\rho} \frac{\partial \langle F T'' \rangle}{\partial x_j} + f \left( \langle u_j T'' u_i \rangle - \langle u_j T' T'' \rangle \right) \quad (18)$$

$$\frac{\partial \langle T'' u_j T' \rangle}{\partial t} + \frac{\partial \langle T'' u_j T' \rangle}{\partial x'_i} = -\frac{1}{\rho} \frac{\partial \langle F T'' \rangle}{\partial x_j} + f \left( \langle u_j T'' u_i \rangle - \langle u_j T' T'' \rangle \right) \quad (19)$$

and

$$\left( \frac{\nu}{P_r} \right) \frac{\partial^2 \langle T'' u_j T' \rangle}{\partial x'_i \partial x'_i} \quad (20)$$

Using the transformations

$$\frac{\partial}{\partial x'_i} = -\left( \frac{\partial}{\partial x_i} + \frac{\partial}{\partial r'_i} \right), \frac{\partial}{\partial x'_i} = \frac{\partial}{\partial r'_i}, \frac{\partial}{\partial x'_i} = \frac{\partial}{\partial r'_i}$$

$$\left( \frac{\partial}{\partial t} \right)_{t'=t} = \left( \frac{\partial}{\partial t} \right)_{t=0} - \frac{\partial}{\partial t} \frac{\partial}{\partial t'} \frac{\partial}{\partial t'}$$

$$= -\frac{\partial}{\partial \Delta t} \frac{\partial}{\partial \Delta t'} \frac{\partial}{\partial \Delta t'} - \frac{\partial}{\partial \Delta t'}$$

into equations (18) – (20), we have
The six-dimensional Fourier transforms for quantities in the equations (21)-(23) may be defined as

\[
\langle u_{ij} T^{nm} \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = 
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_j \theta \theta' \rangle \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]  

(24)

\[
\langle u_{ij} T^{nm} \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = 
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \theta \theta' \theta' \theta' \rangle \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]

(25)

\[
P^{nm}(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = 
\int \langle \alpha \theta \theta' \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]

(26)

By use of these facts we can write equations (21)-(23) give in the forms

\[
\frac{\partial}{\partial t} \langle u_{ij} T^{nm} \rangle = \langle u_{ij} T^{nm} \rangle
\]

(27)

\[
\int_{-\infty}^{\infty} \int \langle \beta_j \theta \theta' \rangle \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]

(28)

\[
\frac{\partial}{\partial t} \langle u_{ij} T^{nm} \rangle = 
\int_{-\infty}^{\infty} \int \langle \beta_j \theta \theta' \rangle \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]

(29)

\[
\frac{\partial}{\partial \Delta t} \langle u_{ij} T^{nm} \rangle = 
\int_{-\infty}^{\infty} \int \langle \beta_j \theta \theta' \rangle \exp[i (\hat{K} \hat{r} + \hat{K}' \hat{r'})] d\hat{K} d\hat{K}'
\]

(30)
If the derivative with respect to $x_j$ is taken of the momentum equation (15) for the point $p$, the equation multiplied by $\frac{x}{p}$ and taken the ensemble average, the resulting equation is

$$\frac{\partial^2 \langle u_{ij} T^n \rangle}{\partial x_i \partial x_j} = -\frac{1}{\rho} \frac{\partial^2 \langle pT^n \rangle}{\partial x_i \partial x_j} \tag{32}$$

Writing this equation in terms of the independent variables $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}'$

$$\left[ \frac{\partial^2}{\partial \hat{\mathbf{x}}_i \partial \hat{\mathbf{x}}_i} + \frac{\partial^2}{\partial \hat{\mathbf{x}}_j \partial \hat{\mathbf{x}}_j} + \frac{\partial^2}{\partial \hat{\mathbf{x}}_r \partial \hat{\mathbf{x}}_r} + \frac{\partial^2}{\partial \hat{\mathbf{x}}_j \partial \hat{\mathbf{x}}_j} \right] \langle u_{ij} T^n \rangle = -\frac{1}{\rho} \left[ \frac{\partial^2}{\partial \hat{\mathbf{x}}_j \partial \hat{\mathbf{x}}_j} + \frac{\partial^2}{\partial \hat{\mathbf{x}}_j \partial \hat{\mathbf{x}}_j} \right] \langle pT^n \rangle \tag{33}$$

Taking the Fourier transforms of equation (33).

$$\langle \alpha \beta \theta \theta' \rangle = -\frac{\rho}{k} \frac{k \cdot k' + k \cdot k' + k \cdot k' + k \cdot k'}{k \cdot k + k \cdot k' + k \cdot k'} \langle \beta \beta \theta \theta' \rangle \tag{34}$$

Equation (34) can be used to eliminate $\langle \alpha \beta \theta \theta' \rangle$ from equation (29).

**Solution for Times Before the Final Period:** To obtain the equation for times before the final period of decay, the three point correlations are considered and the quadruple correlation terms are neglected in comparison with the third-order correlation terms. Because the quadruple correlation terms decay faster than the lower-order correlation terms. If this assumption is made the equation (34) shows that the term $\langle \alpha \beta \theta \theta' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (29)-(31), we can write

$$\frac{\partial \langle \beta_j \theta \theta' \rangle (\hat{\mathbf{x}}, \hat{\mathbf{x}}', \Delta \mathbf{x}, \Delta \mathbf{x}', t)}{\partial \mathbf{t}} = 0 \tag{35}$$

$$\frac{\partial \langle \beta_j \theta \theta' \rangle (\hat{\mathbf{x}}, \hat{\mathbf{x}}', \Delta \mathbf{x}, \Delta \mathbf{x}', t)}{\partial \mathbf{t}} = 0 \tag{36}$$

$$\frac{\partial \langle \beta_j \theta \theta' \rangle (\hat{\mathbf{x}}, \hat{\mathbf{x}}', \Delta \mathbf{x}, \Delta \mathbf{x}', t)}{\partial \mathbf{t}} = 0 \tag{37}$$

For these relations to be consistent, we have

$$k_j \langle \beta_j \theta \theta' \rangle = f_j \exp \left\{ -\frac{\nu}{p_r} \left[ (1 + p_r)(k_j^2 + k_j'^2) \right] \right\} \tag{38}$$

and

$$k_j \langle \beta_j \theta \theta' \rangle = g_j \exp \left\{ -\frac{\nu}{p_r} k_j^2 \Delta t \right\} \tag{39}$$

$$k_j \langle \beta_j \theta \theta' \rangle = g_j \exp \left\{ -\frac{\nu}{p_r} k_j^2 \Delta t' \right\} \tag{40}$$

For these relations to be consistent, we have

$$k_j \langle \beta_j \theta \theta' \rangle = k_j \langle \beta_j \theta \theta' \rangle_0 \times \exp \left[ -\frac{\nu}{p_r} \left( (1 + p_r)(k_j^2 + k_j'^2) \right) (t - t_0) \right] + k_j^2 \Delta t + k_j'^2 \Delta t' + 2 p_r k_j k_j' \cos \theta (t - t_0)$$

$$+ \frac{p_r}{\nu} \left( 2 \varepsilon_{m \theta} \Omega_m - f_s \right) (t - t_0) \right] \tag{41}$$
where $\Theta$ is the angle between $k$ and $k'$ and $\langle \beta_j \theta \theta' \rangle_0$ is the value of $\langle \beta_j \theta \theta' \rangle$ at $t=t_0, \Delta t = \Delta t' = 0$. Letting $\hat{r}' = 0, \Delta t' = 0$ in the equation (24) and comparing the result with the equation (11) shows that

$$
\langle k_j \phi_j \Delta t, t \rangle = 
\int_{-\infty}^{\infty} \langle k_j \beta_j \theta' \theta''(\hat{k}, \hat{k'}, \Delta t, 0, t) \rangle \, d\hat{k}
(42)
$$

Substituting the equation (41) and (42) into the equation (13) we obtain

$$
\frac{\partial}{\partial t} \langle \tau' \rangle(\hat{k}, \Delta t, t) + 2 \nu k^2 \langle \tau' \rangle(\hat{k}, \Delta t, t) = 
\int_{-\infty}^{\infty} i k_j \left[ \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}, \Delta t, 0, t) \rangle - \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}, \Delta t, 0, t) \rangle \right] \, d\hat{k}'
(43)
$$

Now, $d\hat{k}'$ can be expressed in terms of $k'$ and $\Theta$ as

$$
\frac{\partial}{\partial t} \langle \tau' \rangle(\hat{k}, \Delta t, t) + 2 \nu k^2 \langle \tau' \rangle(\hat{k}, \Delta t, t) =
2 \pi i k_j \left[ \langle \beta_j \theta' \theta''(k, k') \rangle - \langle \beta_j \theta' \theta''(-k, -k') \rangle \right]_0
(43a)
$$

Hence, $d\hat{k}' = -2 \nu k^2 d \langle \cos \theta \rangle k' \hat{k}'$

In order to find the solution completely and following Loeffler and Deissler \cite{1} we assume that

$$
\int_{-\infty}^{\infty} i k_j \left[ \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}, \Delta t, 0, t) \rangle - \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}, \Delta t, 0, t) \rangle \right] \, d\hat{k}'
(45)
$$

where $\delta_0$ is a constant depending on the initial condition. The negative sign is placed in front of $\delta_0$ in order to make the transfer of energy from small to large wave numbers for positive value of $\delta_0$. The quantity $\left[ \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}) \rangle - \langle \beta_j \theta' \theta''(\hat{k}, \hat{k'}) \rangle \right]_0$

depends on the initial conditions of the turbulence.

Substituting equation (45) into equation (44), we get

$$
\frac{\partial}{\partial t} \langle \tau' \rangle(\hat{k}, \Delta t, t), 2\pi + 2 \nu \frac{2 \pi k^2 \langle \tau' \rangle(\hat{k}, \Delta t, t)}{p_r}
(46)
$$

`Multiplying both sides of equation (46) by $k^2$, we get`

$$
\frac{\partial E}{\partial t} + 2 \frac{k^2 E}{p_r} = w
(47)
$$

where $E = 2 \pi k^2 \langle \tau \rangle$, the energy spectrum function and $w$ is the energy transfer term given by

$$
w = -2 \frac{\delta_0}{\nu} \left[ \langle k^2 k'^4 - k^4 k'^2 \rangle k^2 k'^2 \right]
(48)
$$
Integrating equation (46) with respect to $\theta$, we have

$$w = -\frac{\delta_0}{\nu(t-t_o)}\int_0^\infty \left( k^3 k'^5 - k^5 k'^3 \right) \exp\left\{ -\frac{\nu}{p_r} \left[ (1 + p_r) \left( k^2 + k'^2 \right) (t-t_o) + k^2 \Delta t \right. \right.$$ 

$$\left. -2 p_r k' (t-t_o) + \frac{p_r}{\nu} \left( 2 \epsilon_{m_{ij}} \Omega_m - f_s \right) (t-t_o) \right] \right\} d\theta'$$

$$+ \delta_0 \int_0^\infty \left( k^3 k'^5 - k^5 k'^3 \right) \right.$$ 

$$\times \left\{ \exp\left\{ -\frac{\nu}{p_r} \left[ (1 + p_r) \left( k^2 + k'^2 \right) (t-t_o) + k^2 \Delta t \right. \right.$$ 

$$\left. + 2 p_r k' (t-t_o) + \frac{p_r}{\nu} \left( 2 \epsilon_{m_{ij}} \Omega_m - f_s \right) (t-t_o) \right] \right\} d\theta'$$

(49)

Again, integrating equation (49) with respect to $k'$, we have

$$w = -\frac{\delta_0 \sqrt{\pi} p_r^{5/2}}{4 \nu^{3/2} (1 + p_r)^{3/2}} \int_0^\infty \exp\left\{-\frac{p_r}{\nu} (2 \epsilon_{m_{ij}} \Omega_m - f_s) \right.$$ 

$$\times (t-t_o) \right\} \times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} \right.$$ 

$$\times (t-t_o + (1 + 2 p_r) \Delta t) \right\} \times \left[ \frac{15 p_r k^4}{4 \nu^2 (t-t_o + \Delta t)^2 (1 + p_r)} \right.$$ 

$$+ \left( \frac{5 p_r^2}{(1 + p_r)^3} - \frac{3}{2} \right) \frac{k^6}{\nu (t-t_o + \Delta t)}$$

$$+ \left( \frac{p_r^3}{(1 + p_r)^3} - \frac{p_r}{1 + p_r} \right) k^8 \right\}$$

(50)

The series of equation (48) contains only even power of $k$ and start with $k'$. If we integrate equation (50) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty \omega dk = 0$$

(51)

which indicates that the expression for $w$ satisfies the condition of continuity and homogeneity. Physically it was to be expected, since $w$ is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (47) can be solved to give

$$E = \exp\left\{-\frac{2 \epsilon_{m_{ij}} \Omega_m - f_s}{p_r} \right.$$ 

$$\int_0^\infty \exp\left\{2 \epsilon_{m_{ij}} \Omega_m - f_s \right.$$ 

$$\times (t-t_o) \right\} \exp\left\{2 \epsilon_{m_{ij}} \Omega_m - f_s \right.$$ 

$$\times (t-t_o) \right\} \exp\left\{-\frac{2 \epsilon_{m_{ij}} \Omega_m - f_s}{p_r} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

(52)

where $J(k) = \frac{N k^2}{\pi}$ is a constant of integration.

Substituting the values of $w$ from (50) and $J(k)$ into the equation (52) and integrating with respect to $t_o$ we get

$$E = \exp\left\{-\frac{2 \epsilon_{m_{ij}} \Omega_m - f_s}{p_r} \right.$$ 

$$\int_0^\infty \exp\left\{2 \epsilon_{m_{ij}} \Omega_m - f_s \right.$$ 

$$\times (t-t_o) \right\} \exp\left\{2 \epsilon_{m_{ij}} \Omega_m - f_s \right.$$ 

$$\times (t-t_o) \right\} \exp\left\{-\frac{2 \epsilon_{m_{ij}} \Omega_m - f_s}{p_r} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

$$\times \exp\left\{-\frac{k^2 v (1 + 2 p_r)}{p_r (1 + p_r)} (t-t_o + \Delta t) \right.$$ 

(52)

where $J(k) = \frac{N k^2}{\pi}$ is a constant of integration.
where, $F(\eta) = \eta \int_0^\eta e^\eta d\eta$

\[ \eta = k \sqrt{n(t-t_0)} \frac{\sqrt[3]{v(t-t_0+\Delta t)}}{p_r(1+p_r)} \]

or \[ \eta = k \sqrt{\frac{\sqrt[3]{v(t-t_0+\Delta t)}}{p_r(1+p_r)}} \]

By letting $\eta = 0$ in equation (10) and use is made of the definition of $E$, the result is

\[ \langle TT' \rangle \frac{9}{2} = \frac{T^2}{2} = \int \frac{1}{2} \frac{E dk}{1+2p_r} \] 

Substituting equation (53) into equation (54) and integrating with respect to $k$, gives

\[ \langle TT' \rangle = \frac{N_e P_r^{3/2}}{2} (T + \Delta T)^{-3/2} \]

\[ \frac{9}{16T^{5/2}} (T + \frac{1+2p_r \Delta T}{1+2p_r}) \]

\[ \frac{5p_r(7p_r - 6)}{16(1+2p_r)(T + \Delta T)^{3/2}} \]

\[ \frac{5p_r(7p_r - 6)}{16(1+2p_r)T^{3/2}} (T + \frac{1+2p_r \Delta T}{1+2p_r}) \]

\[ \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1+2p_r)(T+\Delta T)^{1/2}} \]

\[ \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1+2p_r)T^{1/2}} (T + \frac{1+2p_r \Delta T}{1+2p_r})^{9/2} \]

\[ \frac{35p_r(3p_r^2 - 2p_r + 3)}{3.2^{3/2} (1+2p_r)^{11/2}} \]

\[ \frac{(T^{(2n+4)}/2)}{(T+\Delta T/2)^{(2n+1)}/2} + \frac{(T+\Delta T)^{(2n+3)/2}}{(T+\Delta T/2)^{(2n+1)}/2} \]

where $T=t-t_0$.

Equation (55) is the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle.

Conclusions: In equation (55) we obtained the decay law of temperature fluctuations in homogeneous turbulence before the final period in a rotating system in presence of dust particle by neglecting the quadruple correlation terms in comparison with the third-order terms for the case of multi-point and multi-time. If the fluid is clean and the system is non rotating then $f=0$, and $\Omega_m = 0$ the equation (55) becomes
\( \frac{\langle T^2 \rangle}{2} = \frac{N_p \rho_0^2 (T + \Delta T)^{3/2}}{8 \sqrt{2} \pi^{3/2} \sigma^{3/2}} + \frac{\pi \delta_p \rho_0^4}{4 \sqrt{2} \pi (1 + 2 p_r) (1 + 2 p_r)^{3/2}} \)

\[
x\left[ \frac{9}{16 T^{5/2} (T + 1 + p_r) \Delta T^{5/2}} \right.
\]
\[
+ \frac{9}{16 (T + \Delta T)^{3/2} (T + 1 + p_r) \Delta T^{3/2}} \right.
\]
\[
+ \frac{5 p_r (7 p_r - 6)}{16 (1 + 2 p_r) (T + \Delta T)^{3/2} (T + 1 + p_r) \Delta T^{3/2}} \right.
\]
\[
+ \frac{5 p_r (7 p_r - 6)}{16 (1 + 2 p_r) (T + \Delta T)^{3/2} (T + 1 + p_r) \Delta T^{3/2}} \right.
\]
\[
+ \frac{35 p_r (3 p_r^2 - 2 p_r + 3)}{8 (1 + 2 p_r) T^{3/2} (T + 1 + p_r) \Delta T^{9/2}} \right.
\]
\[
+ \frac{35 p_r (3 p_r^2 - 2 p_r + 3)}{8 (1 + 2 p_r) (T + \Delta T)^{3/2} (T + 1 + p_r) \Delta T^{9/2}} \right.
\]
\[
+ \frac{8 p_r (3 p_r^2 - 2 p_r + 3) (1 + 2 p_r) (1 + 2 p_r)^{3/2}}{3.2^{23/2} (1 + p_r)^{11/2}} \right.
\]
\[
\times \sum_{n=0}^{\infty} \frac{1.35 \ldots \ldots (2n + 9)}{n! (2n + 1) 2^{2n} (1 + p_r)^n} \times \left( \frac{T^{2n+1/2}}{(T + \Delta T)^{1/2}} \right)^{(2n+1)/2} \right.
\]
\[
+ \frac{(T + \Delta T)^{(2n+1)/2}}{(T + \Delta T / 2)^{(n+1)/2}} \right] \]

which was obtained earlier by Sarker and Islam \((10)\).

If we put \( \Delta T = 0 \), we can easily find out

\[
\langle T^2 \rangle = \frac{N_p \rho_0^2 T^{3/2}}{8 \sqrt{2} \pi^{3/2} \sigma^{3/2}} + \frac{\pi \delta_p \rho_0^4 T^{5/2}}{4 \sqrt{2} \pi (1 + 2 p_r) (1 + 2 p_r)^{3/2}} \]

\[
x\left[ \frac{9}{16} + \frac{5 p_r (7 p_r - 6)}{16 (1 + 2 p_r) (1 + 2 p_r)^{3/2}} \right.
\]
\[
+ \frac{35 p_r (3 p_r^2 - 2 p_r + 3)}{8 (1 + 2 p_r)^2} \right]
\]
\[
= A T^{-3/2} + BT^{-5}
\]
\[
= A(t - t_0)^{-3/2} + B(t - t_0)^{-5} \quad (57)
\]

where
\[
A = \frac{N_p \rho_0^{3/2}}{8 \sqrt{2} \pi^{3/2} \sigma^{3/2}} \quad \text{and}
\]
\[
B = \frac{\pi \delta_p \rho_0^4}{2 \sqrt{2} \pi (1 + 2 p_r) (1 + 2 p_r)^{3/2}} \left[ \frac{9}{16} + \frac{5 p_r (7 p_r - 6)}{16 (1 + 2 p_r) (1 + 2 p_r)^{3/2}} \right.
\]
\[
+ \frac{35 p_r (3 p_r^2 - 2 p_r + 3)}{8 (1 + 2 p_r)^2} \right]
\]

which was obtained earlier by Sarker and Deissler \((11)\).

In this problem, due to rotation (of the fluid) in presence of dust particle, the temperature energy fluctuations decays more rapidly than the energy for non-rotating clean fluid for times before the final period.

If higher order correlation equations are considered in the analysis it appears that more terms of higher power of time would be added to the equation (55). For large times, the second term in the equation becomes negligible leaving the -3/2 power decay law for the final period.

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