Evidence for superdiffusive shock acceleration at interplanetary shock waves

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Abstract. Recent analysis of time profiles of energetic particles accelerated at interplanetary shocks has shown evidence for superdiffusive transport upstream of the shock fronts, namely for a transport characterized by a particle mean square displacement that grows faster than linearly in time. While for normal, diffusive transport exponential particle time profiles are predicted, a large number of interplanetary shock events, included the termination shock of the solar wind, exhibits energetic particle time profiles that upstream decay as power laws. This power law trend has been derived in the framework of particle superdiffusion. The standard theory of diffusive shock acceleration has been further extended to the case of particle superdiffusive transport (superdiffusive shock acceleration), allowing for the derivation of harder energy spectral indices both for relativistic and non-relativistic particles. Here we test this theory for a couple of interplanetary shock waves that accelerate protons and that have been observed by the ACE spacecraft. We show that power law particle time profiles upstream of the shocks are common and clearly indicate superdiffusive transport. The particle energy spectra in some events are in a very good agreement with the superdiffusive shock acceleration prediction.

1. Introduction
Energetic particles are observed throughout the heliosphere and they originate from various sources: near the boundary of the solar system, the termination shock of the solar wind is a source of energetic particles [1, 2]; particles are observed to be accelerated in planetary magnetospheres [3, 4, 5], at interplanetary traveling shocks [6], and at the forward and reverse shock pairs of corotating interaction regions [7]; violent eruptive events in the solar atmosphere produce solar energetic particles (SEPs) [8, 9]. The mechanism of particle acceleration that has been invoked for more than 30 years is diffusive shock acceleration (DSA), which explained many observations. However, some observations cannot be explained in the classical framework of DSA: hard particle spectra inferred from radio emissions in supernova remnants cannot be reproduced by DSA [10]; particle time profiles at interplanetary shocks do not always follow the exponential profile upstream, in the case of a spatially constant diffusion coefficient $D$, and a flat profile downstream predicted by DSA [11, 12]; the recent Voyager 2 termination shock crossing has revealed that the spectral slope of energetic ions is compatible with the DSA prediction assuming a compression ratio (i.e., the ratio between the upstream and the downstream plasma speeds) $r = 3$, which is greater than the one observed, $r = 2$, from the spacecraft [2]. This might indicate that we need alternative physical mechanisms to explain particle acceleration. DSA is based on normal (Gaussian) spatial diffusion, but spacecraft observations suggest that the transport of SEPs can vary from normal to scatter-free (ballistic) [13] and the proton parallel...
mean free path at 1 AU is estimated to range from 0.1 AU [14] to more than 1 AU [15, 16]. Thus, besides normal diffusion, anomalous transport regimes can occur. Anomalous transport regimes are characterized by a mean square displacement for particles \( \langle \Delta x^2 \rangle = 2D_{\alpha} t^\alpha \) with \( \alpha \neq 1 \). \( D_{\alpha} \) is the anomalous diffusion coefficient having dimensions \([D_{\alpha}] = L^2/t^\alpha\). For processes slower than normal diffusion \( \alpha < 1 \) (subdiffusion), for processes faster than normal diffusion \( \alpha > 1 \) (superdiffusion). Systems exhibiting subdiffusion are characterized by trapping regions, while superdiffusive processes show random walk trajectories characterized by very long displacements. For ballistic transport, \( \alpha = 2 \). Recently, Refs. [17, 18] using non Gaussian, Lévy, statistics have derived a prediction for particle time profiles far upstream of a shock wave in the case of superdiffusive transport. At variance with DSA, a power law time profile \( j(t) \propto t^{-\beta} \) with \( \beta < 1 \) has been obtained. This has been tested for electrons accelerated at several corotating interaction region shocks [17, 18, 19], for ions accelerated at the termination shock of the solar wind [20], and in a case study of protons accelerated at a coronal mass ejection driven shock [21]. The power law decay fits much better the observed time profiles than the exponential decay predicted by DSA, indicating superdiffusive transport upstream of the shocks.

Further, Refs. [22, 23] have extended the first order Fermi acceleration process to include the case where particles accelerated at shocks propagate superdiffusively. They have developed a new theoretical framework called superdiffusive shock acceleration (SSA), which provides predictions for both the particle energy spectra and the acceleration times (see details in [22, 24]).

In this paper, we will first summarize the main aspects of SSA, then we will test the energy spectra predictions against a couple of interplanetary shocks observed by the Advanced Composition Explorer (ACE) spacecraft. In the last Section we will discuss the main results.

2. Theoretical framework

Let an infinite planar shock be the source of accelerated particles, so that the geometry of the system is one dimensional and only variations along the coordinate perpendicular to the shock front (namely, \( x \)) are allowed. In such a system, it is possible to reconstruct a particle density profile [17, 18] as

\[
n(x, E, t) = \int P(x - x', t - t')S_{sh}(x', E, t')dx'dt',
\]

where the source term \( S_{sh}(x', E, t') = \Phi_0(E)\delta(x' - V_{sh}t') \) depends on the flux of particles in a given energy channel at the shock, \( \Phi_0 \), and \( V_{sh} \) is the shock speed in the observer frame. \( P(x - x', t - t') \) is the probability of finding a particle at the position \( x \) at time \( t \) if injected at \( x' \) at time \( t' \) and is called the propagator of the process. In the case of a Gaussian propagator, the classical upstream exponential decay predicted by DSA for the density of particles accelerated at a planar shock is recovered, i.e., \( n(x, E) \propto \exp(-V|x|/D) \), assuming a spatially constant diffusion coefficient \( D \). However, if we assume anomalous particle propagation, in particular superdiffusive propagation, the one dimensional propagator scales as [25]

\[
P(x - x', t - t') = A \frac{t - t'}{(x - x')^\mu},
\]

with \( 2 < \mu < 3 \). This form of the propagator can be derived in the limit of \( |x - x'| \gg |t - t'|^{1/\mu} \), that is far from the source. Refs. [17, 18] have explicitly derived the particle density under the hypothesis of superdiffusive transport for particles accelerated at a shock wave, that is

\[
n(x, E) \propto |x|^{-\beta},
\]

being the exponent \( \beta \) of the power law in Eq. (3) related to the exponent of the superdiffusive particle propagator, \( \beta = \mu - 2 \). Since superdiffusion is obtained when \( 2 < \mu < 3 \), this implies
0 < \beta < 1. Furthermore, a direct relation between \mu and the exponent of the mean square displacement \alpha can be obtained, that is \alpha = 4 - \mu \cite{25, 23}. Consequently, from the particle density profiles it is possible to estimate the increase of the mean square displacement using \alpha = 2 - \beta. As discussed above, the upstream decay of the particle time profiles has been studied in some shock events in interplanetary space (both for electrons and for protons), and for many of them the superdiffusive prediction reproduces the data much better than the diffusive exponential decay \cite{17, 18, 20, 19, 21}.

Figure 1. From top to bottom: 5 minute resolution ions fluxes displayed in a log-lin plot in five energy channels (see legend) during an interplanetary shock event detected by the ACE satellite on 1999/02/18; 10 min resolution solar wind bulk speed and proton density; magnetic field components; the azimuthal angle; total magnetic field normalized variance on 1 min time scale. The vertical dashed line indicates the shock time position. Data from http://cdaweb.gsfc.nasa.gov/

Refs. \cite{22, 23} have included superdiffusion within a first order Fermi acceleration process, giving rise to the SSA theory. In particular, they derived the particle density at the shock \( n_0 \) using Eq. (1) assuming isotropy and stationary conditions. They computed the probability of escaping from the acceleration region \( P_{\text{esc}} = \Phi_2 / \Phi_1 = n_2 V_2 / (n_0 v/4) \), that is the ratio between the particle flux downstream of the shock and the particle flux from upstream to downstream \cite{12} (\( \Phi_2 \) depends on the density and on the plasma speed downstream, \( \Phi_1 \) depends on the particle density at the shock and on the particle velocity). Hereafter, the index 1 indicates the unshocked, upstream region, while 2 refers to the shocked, downstream region. For relativistic particles \( P_{\text{esc}} \) is directly related to the exponent \( \hat{\gamma} \) of the integral energy spectrum (i.e., \( N(> E) = N_0(E/E_0)^{-\hat{\gamma}} \)), so that determining the escape probability in the superdiffusive
framework implies to derive the particle energy spectrum index corrected by the assumption of superdiffusive transport. In the case of non relativistic particles \( P_{\text{esc}} \) is related to the particle momentum spectrum, so that \( \gamma_p = P_{\text{esc}} / (\Delta p / p) \). Thus, the spectral indices for the differential density predicted by DSA and by SSA [22, 23] can be summarized, respectively, as

\[
\begin{align*}
\gamma_{p,\text{DSA}} &= \gamma_p + 1 = \frac{r + 2}{r - 1} \\
\gamma_{p,\text{SSA}} &= \gamma_p + 1 = \frac{2 - \alpha}{r - 1} + 1.
\end{align*}
\]

The DSA prediction depends on the compression ratio of the shock \( r = n_2 / n_1 \) only, while in the SSA framework the spectral index also depends on the exponent \( \alpha \). Notice that when \( \alpha \to 1 \) the classical \( \gamma_{p,\text{DSA}} \) is recovered. Since \( (dN/dE) dE = (dN/dp) dp \), it is straightforward to obtain \( \gamma_p = 2\gamma - 1 \), which relates the particle momentum spectral index to the particle energy spectral index.

### 3. Data analysis

We analyze a couple of interplanetary shock events reported in [6] and detected by ACE. Figure 1 shows the 1999/02/18 shock (compression ratio \( r \sim 2.1 \)). Notice in particular from top to bottom: the 5 minute resolution proton time profiles, which tend to peak at the shock front (indicated by a vertical dashed line) in five energy channels, the 10 minute averaged proton bulk speed exhibiting a rapid increases from upstream to downstream, the 10 minutes averaged proton density, the magnetic field components in Geocentric Solar Ecliptic coordinates (notice that the magnetic field tends to fluctuate more in the shocked region), the latitudinal angle that identifies magnetic sectors, and the normalized variance of the total magnetic field at one minute time scale. The variance has been computed as \( \sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \), where \( \sigma_i^2 = \sum_{j=1}^N (B_j - B_0)^2 / N \) (the index \( i \) runs over the magnetic field components) is evaluated over time windows of \( N \) points and \( B_0 \) is the mean magnetic field within each window. Those time windows have a one minute length. This value is comparable with the time scale corresponding to the Larmor radius.

**Figure 2.** A log-log plot of ion time profiles as a function of the time (distance) from the shock. All the energy channels exhibit a power law trend for more than one decade. The time profiles tend to change in the vicinity of the shock. The best power law fits are indicated by the thick black solid lines. The power law slopes for each energy channel are displayed as well.
of the energetic protons shown in Figure 1 (i.e., \( \tau = 1/f_{\rho p} = 2\pi \rho p/V_{sw} \equiv 2\pi m_p v/(q'(B_1)V_{sw}) \),
where \( \langle B_1 \rangle \) is an average magnetic field value upstream). This corresponds to the resonant scale
for particles interacting with magnetic field turbulent fluctuations. As addressed in Ref. [26],
the normalized variance does not vary significantly upstream as the distance from the shock
increases; this is crucial since a constant level of magnetic field variance implies a spatially
constant diffusion coefficient which implies an exponential decay of the particle time profiles in
the DSA framework.

Figure 3. Average differential fluxes as a function of the particle kinetic energy computed
upstream of the 1999/02/18 shock from the ion fluxes shown in Figure (1) (filled circles). The
best power law fit is indicated by the black solid line. The black dashed line refers to the DSA
prediction, while the two red dashed lines indicate the power laws foreseen by the SSA theory
for the maximum and minimum values of the particle mean square exponent \( \alpha \) derived from the
profiles in Figure 2.

Following the data analysis procedure described in Ref. [17], we have plotted in Figure 2 the
upstream proton time profiles on log-log axes as a function of the time distance from the shock
front (in hours). At about 0.3 hours from the shock the time profiles decay as power laws for
all the energy channels considered. Higher energy particle fluxes have not been analyzed since
they tend to reach the background level. The best power law fits are shown in Figure 2 as solid
black lines and the slopes are also indicated for each energy channel. The power law slope is
\( \beta < 1 \) (see Eq. (3), where \(|x| = V_{sh} \Delta t| \) from single spacecraft observations), thus \( \alpha = 2 - \beta > 1 \)
indicating superdiffusive transport for energetic protons. It is important to remark that close
to the shock front (\(|\Delta t| < 0.3 \) hr) the particle profiles vary and become flatter: this is related to
the fact that the propagator form in Eq. (2) does not hold in the vicinity of the source (where
a modified Gaussian is obtained for the propagator) [25].

In order to test the SSA prediction for the particle differential spectrum, we have calculated
a differential flux \( \langle dj/dE \rangle \) averaging the proton time profiles displayed in the upper panel of
Figure 1 within the portion of the upstream region from day 48.6 to day 49.0. Notice that the
differential flux for non-relativistic particles can be written as \( dj/dE = vdN/dE = vE^{-\gamma} \sim
E^{-(\gamma - 1/2)} \equiv E^{-\gamma_j}. \) Thus, we easily get that \( \gamma_j = \gamma - (1/2) = \gamma_p/2, \) so that obtaining \( \gamma_j \)
from the differential flux analysis implies obtaining a \( \gamma_p \) value to be compared with Eq.s (4). Figure
2 reports the observed \( \langle dj/dE \rangle \) (filled circles) as a function of the kinetic energy corresponding
to the ‘central’ value of each energy channel. The DSA prediction, based on the observed
compression ratio, (see Eq. (4)) is displayed by the black dashed line, while the two red dashed
lines indicate the SSA predictions for the maximum and the minimum \( \beta \) (i.e., \( \alpha \)) reported in Figure 2. The latter agree better with the data values. The same analysis on differential fluxes have been reported in Ref. [24] for relativistic electrons accelerated at a corotating interaction region event; the result obtained is in good agreement with the SSA model.

Another interplanetary shock detected by ACE is displayed in Figure 4. It occurred on 2002/04/23 and it has a compression ratio \( r \sim 4.0 \) due to a large amplitude jump in the proton density from upstream to downstream (see Section 4). Also in this case the proton time profiles analyzed in four energy channels exhibit a power law decay for one decade upstream of the shock front (see Figure 5). The slopes are \( \beta < 1 \), which agree with superdiffusive transport for energetic protons. In Figure 6 the differential flux (obtained averaging the particle fluxes in each channel from day 112.6 to day 113.0) as a function of the kinetic energy shows again a good agreement with the SSA predictions. Notice that the ‘scattering’ of the filled circles in Figure 6 is due to the high irregularity in the proton fluxes (see the upper panel in Figure 4), indeed upstream in three energy channels the profiles tend to overlap.

4. Discussions and Conclusions
We have studied the particle differential fluxes upstream of two interplanetary shocks observed at 1 AU by the ACE satellite. The two events indicate a superdiffusive transport for protons in several energy channels, as inferred from the analysis of particle time profiles upstream of the shock front. For the analysis of the spectra we have estimated the compression ratios of the shocks, which have been derived from the proton density measurements upstream and downstream from the front. Since the plasma parameters upstream and downstream can vary
Figure 5. Ion time profiles in log-log axes for the event displayed in Figure 4 as a function of the time distance from the shock. The best power law fits are indicated by the black solid lines and extend for one decade. The power law slopes for each energy channel are reported.

Figure 6. Average differential fluxes as a function of the particle kinetic energy computed upstream of the 2002/04/23 shock (filled circles). The best power law fit is indicated by the black solid line and show a very good agreement with the two power laws predicted by the SSA (red dashed lines). The black dashed line refers to the DSA prediction.

very rapidly, thus giving rise to a high uncertainty in the estimation of the plasma mean values, we have followed the data analysis procedure described in Ref. [24]. More specifically, we considered the plasma data in time intervals in the shocked and in the unshocked regions after a couple of hours from the time of the shock. Thus, we computed plasma density and speed average values within running windows of about 1 hr. The running window can slide over a time period of almost 3 hrs in the two regions, within portions of the signal not affected by large amplitude variations. This procedure tends to minimize the errors associated with the estimation of the plasma mean values, however a certain degree of variation in the plasma measurements upstream and downstream of the shocks gives rise to unavoidable (and sometimes
large) errors in their evaluation (see also the discussion in Ref. [27, 6]). Because of both the uncertainty in the derivation of the compression ratio, and the non-regular behavior of the proton time profiles, which in some cases tend to almost overlap in different energy channel (see upper panels of Figures 1 and 4 in the upstream side), the differential fluxes as a function of kinetic energy are not always regular power laws. However, in the two events considered the best power law fit of $\langle dj/dE \rangle$ vs $E$ is in a much better agreement with the power laws predicted by SSA than with the ones from DSA, considering the $\alpha = 2 - \beta$ values deduced from the fits in Figures 2 and 5. It is worth stressing that superdiffusive transport has been found in all the events analyzed from the particle time profiles upstream of the shocks and it has not directly been tested downstream. However, here we assume that particle superdiffusion holds within the downstream region, too. This assumption can be supported by the time trend of the normalized magnetic field variance shown in the bottom panels in Figs. 1 and 4. The variance $\sigma^2/B_0^2$ tends indeed to remain constant (or even decrease a bit in the 1999/02/18 event), suggesting that the particle scattering downstream does not increase owing to an increased power in the magnetic field fluctuations.

The analysis reported here, together with the spectrum of relativistic electrons accelerated at a corotating interaction region observed by Ulysses and presented in Ref. [24], is a first step in testing the SSA theory developed in Refs [22, 23] and more differential spectra need to be analyzed. In addition, an evaluation of the acceleration time, as presented in Eq. (19) in Ref. [22], directly from spacecraft observations will be presented in a forthcoming paper.

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