Stability analysis for stochastic BAM nonlinear neural network with delays

Z W Lv\(^1\), H S Shu\(^2,3\), G L Wei\(^2\)
\(^1\)College of Applied Mathematics, Donghua University, Shanghai 200051, China.
\(^2\)School of Information Science and Technology, Donghua University, Shanghai 200051, China.
E-mail: hsshu@dhu.edu.cn

Abstract. In this paper, stochastic bidirectional associative memory neural networks with constant or time-varying delays is considered. Based on a Lyapunov-Krasovskii functional and the stochastic stability analysis theory, we derive several sufficient conditions in order to guarantee the global asymptotically stable in the mean square. Our investigation shows that the stochastic bidirectional associative memory neural networks are globally asymptotically stable in the mean square if there are solutions to some linear matrix inequalities (LMIs). Hence, the global asymptotic stability of the stochastic bidirectional associative memory neural networks can be easily checked by the Matlab LMI toolbox. A numerical example is given to demonstrate the usefulness of the proposed global asymptotic stability criteria.

1. Introduction

In [5]-[6], Kosko proposed a new class of networks called bidirectional associative memory (BAM) neural networks. This class of networks has been successfully applied to pattern recognition due to its generalization of the single-layer autoassociative Hebbian correlator to a two-layer pattern-matched heteroassociative circuit. Recently, the dynamics such as stability and periodicity of BAM neural networks have received much attention due to their potential application in associative memory, parallel computation and optimization problems. Some important results have been obtained in Refs. [1],[3]-[7],[12]-[13],[15],[23]. Most neural network models proposed and discussed in the literature are deterministic. As is well known, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random, as pointed out by Haykin [2] that in real nervous system synaptic transmission is a noisy process brought on by random fluctuation from the release of neurotransmitters and other probabilistic causes. Therefore, it is of prime importance and great interest to consider stochastic effects to the stability of neural networks. To date, some results on stability of stochastic cellular neural networks and stochastic Cohen-Grossberg neural networks have been reported (see, [8],[12]-[13],[17],[19]-[21],[22]). However, to the best of our knowledge, the stability of BAM neural networks with delays have been studied ([9],[10],[13]), but few authors study the stability of stochastic BAM neural networks with delays.

Motivated by the above discussion, in this paper, we analyze the stochastic BAM neural network models with constant or time-varying delays. By utilizing a Lyapunov-Krasovskii functional and conducting the stochastic analysis, we derive several sufficient conditions in order to guarantee the
global asymptotically stable in the mean square. Different from the commonly used matrix norm theories (such as the $M$-matrix method), a unified linear matrix inequality (LMI) approach is developed to establish sufficient conditions for the neural networks to be global asymptotically stable. Note that LMIs can be easily solved by using the Matlab LMI toolbox, and no tuning of parameters is required [18]. A numerical example is provided to show the usefulness of the proposed global asymptotic stability condition.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. For symmetric matrices $X$ and $Y$, the notation $X > Y$ (respectively, $X \geq Y$) means that $X - Y$ is positive definite (respectively, non-negative). $I$ denote the compatible dimension identity matrix. Denote by $\mathfrak{F}_0$ the family of all $\mathfrak{F}_0$-measurable $\mathbb{R}^n$-valued random variables $\xi = \{ \xi(\theta) : h \leq \theta \leq 0 \}$ such that $\sup_{h \leq \theta \leq 0} E|\xi(\theta)|^p < \infty$ where $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $P$. The shorthand $\text{diag} \{M_1, M_2, \ldots, M_N\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1, M_2, \ldots, M_N$. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

The BAM networks with constant delays described by the following differential equations ([3],[4],[7]):

$$
\begin{align*}
\dot{u}(t) &= -Au(t) + W^T g(v(t - \tau)) + I, \\
\dot{v}(t) &= -Bv(t) + V^T \tilde{g}(u(t - \delta)) + J,
\end{align*}
$$

(1)

in which $u = (u_1, \ldots, u_m)^T \in \mathbb{R}^m, v = (v_1, \ldots, v_n)^T \in \mathbb{R}^n, A = \text{diag}(a_1, \ldots, a_n) > 0, g = (g_1, \ldots, g_m)^T, B = \text{diag}(b_1, \ldots, b_m) > 0, \tilde{g} = (g_1, \ldots, g_n)^T, I = (I_1, \ldots, I_n)^T, J = (J_1, \ldots, J_m)^T, W = (w_i)_{m \times n}, V = (v_j)_{n \times m}, \tau \geq 0, \delta \geq 0$ are constants.

Throughout this paper, we assume that the activate functions $g_i$ possess the following properties:

(A1) $g_i$ are bounded on $R, i = 1, 2, \ldots, \max \{m, n\}$.

(A2) There exist real numbers $M_i > 0$ such that $|g_i(x) - g_i(y)| \leq M_i|x - y|$ for any $x, y \in R, i = 1, 2, \ldots, \max \{m, n\}$.

It is clear that under the assumption (A1) and (A2), system (1) has at least one equilibrium.

In order to simplify our proof, we shift the equilibrium point $u^* = (u_1^*, \ldots, u_m^*)^T \in \mathbb{R}^m, v^* = (v_1^*, \ldots, v_n^*)^T \in \mathbb{R}^n$ of system (1) to the origin. This transformation $x(t) = u(t) - u^*, y(t) = v(t) - v^*, \tilde{f}(x(t)) = g(u(t)) - g(u^*), f(y(t)) = g(v(t)) - g(v^*)$ put system (1) to system (2)

$$
\begin{align*}
\dot{x}(t) &= -Ax(t) + W^T f(y(t - \tau)), \\
\dot{y}(t) &= -By(t) + V^T \tilde{f}(x(t - \delta)),
\end{align*}
$$

(2)

where

$$
\begin{align*}
x(t) &= (x_1(t), \ldots, x_n(t))^T, y(t) = (y_1(t), \ldots, y_m(t))^T, f = (f_1, \ldots, f_m)^T, \tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_n)^T, M = \text{diag}(M_1, \ldots, M_m), \tilde{M} = \text{diag}(\tilde{M}_1, \ldots, \tilde{M}_n).
\end{align*}
$$
Obviously, the activate functions $f_i$ satisfy the following properties:

(H1) $f_i$ are bounded on $R, i = 1, 2, \ldots, \max \{m, n\}$.

(H2) There exist real numbers $M_i > 0$ such that $|f_i(x) - f_i(y)| \leq M_i |x - y|$ for any $x, y \in R, i = 1, 2, \ldots, \max \{m, n\}$.

(H3) $f_i(0) = 0, i = 1, 2, \ldots, \max \{m, n\}$.

It is often the case in practice that the network is disturbed by environmental noises that affect the stability of the equilibrium. In this paper, the stochastic BAM networks with constant delays described by the following differential equations:

$$
\begin{align*}
\dot{x}(t) &= [\sum_{i=1}^{m} A_{ii}x_i(t) + \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij}x_j(t - \tau)] \dot{\omega}(t), \\
\dot{y}(t) &= [\sum_{i=1}^{m} B_{ii}y_i(t) + \sum_{i=1}^{m} \sum_{j=1}^{n} B_{ij}y_j(t - \tau)] \dot{\sigma}(t),
\end{align*}
$$

where $\omega(t) = (\omega_1(t), \ldots, \omega_m(t))^T \in \mathbb{R}^m, \sigma(t) = (\sigma_1(t), \ldots, \sigma_n(t))^T \in \mathbb{R}^n$ are Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P})$, and assume that $\rho: \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n \times m}, \tilde{\rho}: \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{m \times n}$ are locally Lipschitz continuous and satisfy the linear growth condition. Moreover, $\rho, \tilde{\rho}$ satisfies

$$
\begin{align*}
\text{trace} \left[ \sum_{i=1}^{m} A_{ii} x_i(t) y_i(t - \tau) \right] &\leq \sum_{i=1}^{m} x_i(t)^2 + \sum_{i=1}^{m} y_i(t - \tau)^2, \\
\text{trace} \left[ \sum_{i=1}^{m} B_{ii} y_i(t) x_i(t - \tau) \right] &\leq \sum_{i=1}^{m} y_i(t)^2 + \sum_{i=1}^{m} x_i(t - \tau)^2.
\end{align*}
$$

Respectively. where $\Sigma_1, \Sigma_2$ and $\tilde{\Sigma}_1, \tilde{\Sigma}_2$ are known constant diagonal matrices with appropriate dimensions.

Now, according to [14], it is obvious that system (3) has a unique global solution $x(t, \phi), y(t, \varphi)$ on $t \geq 0$ for any initial value $\phi \in L^{b}_{\mathcal{X}_\sigma}\left([-\tau, 0]; \mathbb{R}^n\right)$.

Furthermore, $\sigma(t, 0, 0) = 0, \tilde{\sigma}(t, 0, 0) = 0$ are required such that system (4) has a trivial solution $x(t, 0) = 0, y(t, 0) = 0$.

### 2. Definition and Lemmas

In this part, we will focus our attention on studying the stability of system (3). To obtain our results, we need introduce the following definition and lemmas.

**Definition 1.** For the neural network (3) and every $\xi \in L^{2}_{\mathcal{X}_\sigma}\left([-\tau, 0]; \mathbb{R}^n\right), \eta \in L^{2}_{\mathcal{X}_\sigma}\left([-\sigma, 0]; \mathbb{R}^m\right)$ the trivial solution (equilibrium point) is globally asymptotically stable in the mean square if the following holds:

$$
\lim_{t \to \infty} (E| x(t; \xi) |^2 + E| y(t; \eta) |^2) = 0
$$

**Lemma 1.** Let $x \in \mathbb{R}^n, y \in \mathbb{R}^m$ and $\varepsilon > 0$. Then we have

$$
x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y
$$

**Lemma 2.** For any positive definite matrix $M > 0$, scalar $\gamma > 0$, vector function $\omega: [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:

$$
\left( \int_{0}^{\gamma} \omega(s) ds \right)^T M \left( \int_{0}^{\gamma} \omega(s) ds \right) \leq \gamma \left( \int_{0}^{\gamma} \omega^T(s) M \omega(s) ds \right).
$$
Lemma 3. The LMI
\[
\begin{bmatrix}
Q(x) & S(x) \\
S^T(x) & R(x)
\end{bmatrix} > 0,
\]
where \( Q(x) = Q^T(x), R(x) = R^T(x) \) and \( S(x) \) depends affinely on \( x \), is equivalent to
\[
\begin{align*}
(i) & \quad Q(x) > 0, R(x) - S(x)^T Q(x)^{-1} S(x) > 0, \\
(ii) & \quad R(x) > 0, Q(x) - S(x) R(x)^{-1} S^T(x) > 0.
\end{align*}
\]

3. Main results

Theorem 1. If there exist positive scalars \( \rho > 0, \rho > 0, \epsilon_i > 0 \) \( (i = 1, 2) \), positive definite matrices \( P_1, Q_1 \), and positive diagonal matrices \( P_3, Q_3 \) with appropriate dimensions satisfying
\[
P_1 < \rho I \\
Q_1 < \rho I \\
\begin{bmatrix}
P_1 A + A P_1 - \epsilon_2 \tilde{M} \tilde{M} - \rho \tilde{\Sigma}_2^T \tilde{\Sigma}_2 - \delta \tilde{M} \tilde{Q}_1 \tilde{M} - \rho P W^T & P W^T \\
W P_1 & \epsilon_1
\end{bmatrix} > 0, \\
\begin{bmatrix}
Q_1 B + B Q_1 - \epsilon_3 \tilde{M} \tilde{M} - \rho \tilde{\Sigma}_2^T \tilde{\Sigma}_2 - \tau \tilde{M} P_2 \tilde{M} - \rho \tilde{\Sigma}_1^T \tilde{\Sigma}_1 & Q V^T \\
V Q_1 & \epsilon_2
\end{bmatrix} > 0.
\]
holds, then dynamics of the neural network (3) is globally asymptotically stable in the mean square.

Proof: Consider the following Lyapunov-Krasovskii functional candidate
\[
V(t, x(t), y(t)) = x^T(t) P_1 x(t) + y^T(t) Q_1 y(t) + \int_{-\tau}^{0} f^T(\eta) P_1 f(\eta) d\eta + \int_{-\tau}^{0} \int_{-\gamma}^{0} \tilde{f}^T(\eta) Q_2 \tilde{f}(\eta) d\eta d\eta
\]
\[
+ \int_{\tau}^{0} \int_{\gamma}^{0} f^T(\eta) P_3 f(\eta) d\eta d\eta + \int_{-\tau}^{0} \int_{-\gamma}^{0} \tilde{f}^T(\eta) Q_3 \tilde{f}(\eta) d\eta d\eta
\]
where \( P_1, Q_1, P_3, Q_3 \) are the positive solution to (8), and \( P_2 \geq 0 \) is defined by
\[
P_2 = \epsilon_1 I + M^{-1} \rho \sum \tilde{\Sigma}_2 \tilde{M}^{-1}
\]
\( Q_2 \geq 0 \) is defined by
\[
Q_2 = \epsilon_2 I + \tilde{M}^{-1} \rho \sum \tilde{\Sigma}_2 \tilde{M}^{-1}
\]
Employing Itô’s differential rule, one can deduce that
\[
\ell V(t, x(t), y(t)) = x^T(t)(-P_1 A - A P_1)x(t) + y^T(t)(-Q_1 B - B Q_1)y(t) + 2x^T(t) P W^T f (y(t - \tau))
\]
\[
+ 2y^T(t) Q V^T \tilde{f}(x(t - \delta)) + \text{trace}(\sigma^T(t, x(t), y(t - \tau)) P \sigma(t, x(t), y(t - \tau))) + \text{trace}(\tilde{\sigma}^T(t, y(t), x(t - \delta)) Q \tilde{\sigma}(t, y(t), x(t - \delta))) + f^T(t) (P_2 + \tau P_3) f (y(t))
\]
\[
- f^T(t) (P_2 f (y(t - \tau)) + \tilde{f}^T (x(t))(Q_2 + \delta Q_3) \tilde{f}(x(t)) - \tilde{f}^T (x(t - \delta)) Q_2
\]
\[
\times \tilde{f}(x(t - \delta)) - \int_{-\tau}^{0} f^T(t) P_3 f (y(t)) d\xi - \int_{-\tau}^{0} \tilde{f}^T(t) Q_3 \tilde{f}(y(t)) d\eta d\eta
\]
next, it follows from (4) and (7) that
\[
\text{trace}\left(\sigma^T(t, x(t), y(t-t))P_1\sigma(t, x(t), y(t-t))\right) \leq \lambda_{\text{max}} (P_1) \text{trace}\left(\sigma^T\sigma\right) \\
\leq \lambda_{\text{max}} (P_1) \left[\|\Sigma_1 x(t)\|^2 + \|\Sigma_2 y(t-t)\|^2\right] \\
\leq \rho \left[\|x^T(t)\Sigma_1^T \Sigma_1 x(t) + \|y^T(t-t)\Sigma_1^T \Sigma_2\right] \times \|y(t-t)\] (13)
\]

For the positive scalars 0 \varepsilon_1 > 0 it follows from Lemma 1 that
\[
2x^T(t)P_1W^Tf\left(y(t-t)\right) \leq \varepsilon_1 f^T(t)P_1W^Tf(t) + \epsilon_{-1}^{-1}x^T(t)P_1W^TWP_1x(t) \\
(14)
\]
Furthermore, it can be seen from Lemma 2 that
\[
\int_{t-\tau}^{t} f^T(t)P_1f(t)dt \geq \tau^{-1} \left(\int_{t-\tau}^{t} f(t)dt\right)^2P_1 \left(\int_{t-\tau}^{t} f(t)dt\right) (\tau) (15)
\]
Similarly, we can obtain
\[
\text{trace}\left(\sigma^T(t, y(t), x(t-t))Q_\sigma\sigma(t, x(t-t))\right) \leq \rho \left[\|y^T(t)\Sigma_1^T \Sigma_1 y(t) + \|x^T(t-t)\Sigma_1^T \Sigma_2\right] \times \|x(t-t)\] (16)
where 0 \varepsilon_2 > 0
\[
\int_{t-\delta}^{t} f^T(t)Q_2f(t)dt \geq \delta^{-1} \left(\int_{t-\delta}^{t} f(t)dt\right)^2Q_2 \left(\int_{t-\delta}^{t} f(t)dt\right) (\delta) (18)
\]
Using (13) to (18) and (H2), we obtain from (12) that
\[
\ell V(t, x(t), y(t)) \leq x^T(t)\left(-P_1A - AP_1 + \epsilon_{-1}^{-1}P_1W^TW + \rho \Sigma_1^T \Sigma_1 + \bar{M} (Q_2 + \delta Q_2) \bar{M}\right)x(t) + y^T(t) \\
\times \left(-Q_2B - BQ_2 + \epsilon_{-2}^{-1}Q_2V^TQ_2 + \rho \Sigma_1^T \Sigma_1 + \bar{M} (P_2 + \epsilon \bar{M})\right)y(t) + x^T(t-t)\bar{\rho} \\
\times \sum_1^T \sum_2^T - \bar{M} (Q_2 - \epsilon \bar{M}) \bar{M} x(t-t) + y^T(t-t)\left(\rho \sum_1^T \sum_2^T - \bar{M} (P_2 - \epsilon \bar{M})\right)y(t-t) \\
- \tau^{-1} \left(\int_{t-\tau}^{t} f(t)dt\right)^2P_1 \left(\int_{t-\tau}^{t} f(t)dt\right) \tau^{-1} \left(\int_{t-\tau}^{t} f(t)dt\right) (\tau) (19)
\]
Using (10) to (11) and by some manipulations, we obtain from (19) that
\[
\ell V(t, x(t), y(t)) \leq \alpha^T(t) \pi_1 \alpha(t) + \beta^T(t) \pi_2 \beta(t) \\
(20)
\]
where
\[
\pi_1 = \begin{bmatrix} -P_1A - AP_1 + \epsilon_{-1} \bar{M} \bar{M} + \rho \sum_1^T \sum_2 + \delta \bar{M} Q_2 \bar{M} + \epsilon_{-1} P_1 W^T W P_1 + \rho \sum_1^T \sum_1 \\
0 \end{bmatrix} - \tau^{-1} P_1 \\
\pi_2 = \begin{bmatrix} -Q_2B - BQ_2 + \epsilon_{-2} \bar{M} \bar{M} + \rho \sum_1^T \sum_2 + \tau MP_2 M + \epsilon_{-2} Q_2 V^T Q_2 + \rho \sum_1^T \sum_1 \\
0 \end{bmatrix} - \delta^{-1} Q_2
\]
In light of (8) and (2) we know that
\[-PA - AP_i + \varepsilon_i \dot{M}M + \rho \sum_i \sum_i + \delta \dot{M}Q_i \dot{M} + \varepsilon_i^2 P_i W^i WP_i + \rho \sum_i \sum_i < 0\]  
(21)

\[-Q_i, B - BQ_i + \varepsilon_i MM + \rho \sum_i \sum_i + \tau MP_i M + \varepsilon_i^2 Q_i V^i VQ_i + \rho \sum_i \sum_i < 0\]

so we obtain \(\pi_1 < 0, \pi_2 < 0\). Then, there must exist scalar \(\gamma_1 > 0, \gamma_2 > 0\) such that

\[\pi_1 + \begin{bmatrix} \gamma_1 I & 0 \\ 0 & 0 \end{bmatrix} < 0, \pi_2 + \begin{bmatrix} \gamma_2 I & 0 \\ 0 & 0 \end{bmatrix} < 0\]  
(22)

Following from (22), one can obtain that

\[\ell V(t, x(t), y(t)) \leq -\gamma_1 \| \alpha(t) \|^2 - \gamma_2 \| \beta(t) \|^2\]  
(23)

Let \(\gamma = \min\{\gamma_1, \gamma_2\}\). Then we obtain from (23) that

\[\ell V(t, x(t), y(t)) \leq -\gamma \left( \| \alpha(t) \|^2 + \| \beta(t) \|^2 \right)\]

It implies that \(\ell V < 0\) for all \(\alpha(t) \neq 0, \beta(t) \neq 0\). According to Itô’s formula, system (3) is globally asymptotically stable in the mean square.

In the following, we will study the stability for stochastic BAM networks with time-varying delays. The model is described by the following differential equation:

\[
\begin{align*}
\dot{x}(t) &= \left[-Ax(t) + W^T f(y(t - \tau(t)))\right]dt + \sigma(t, x(t), y(t - \tau(t)))d\omega(t), \\
\dot{y}(t) &= \left[-By(t) + V^T \tilde{f}(x(t - \delta(t)))\right]dt + \tilde{\sigma}(t, y(t), x(t - \delta(t)))d\tilde{\omega}(t),
\end{align*}
\]

(24)

In this section, we always assume that \(\tau(t), \delta(t)\) are differentiable, nonnegative and bounded, \(0 \leq \tau(t) \leq \tau, 0 \leq \delta(t) \leq \delta\) and the derivative of \(\tau(t), \delta(t)\) are less than one, i.e., \(\dot{\tau}(t) \leq \zeta_1 < 1, \dot{\delta}(t) \leq \zeta_2 < 1\).

**Theorem 2.** If there exist positive scalars \(\rho > 0, \tilde{\rho} > 0, \varepsilon_i > 0 (i = 1, 2)\), positive definite matrices \(P_i, Q_i\), with appropriate dimensions satisfying

\[
P_i < \rho I, \quad Q_i < \tilde{\rho} I
\]

\[
\begin{bmatrix}
P_i A + AP_i - \kappa_i^{-1} \varepsilon_i M M - \kappa_i^{-1} \tilde{\rho} \sum_i \sum_i - \rho \sum_i \sum_i & P_i W^i WP_i \\

Q_i, B + BQ_i - \kappa_i^{-1} \varepsilon_i M M - \kappa_i^{-1} \tilde{\rho} \sum_i \sum_i & Q_i V^i VQ_i
\end{bmatrix} > 0,
\]

(26)

holds, then dynamics of the neural network (24) is globally asymptotically stable in the mean square.

Proof. Consider the following Lyapunov-Krasovskii functional candidate

\[
\begin{align*}
V(t, x(t), y(t)) &= x^T(t) P_1 x(t) + y^T(t) Q_1 y(t) + \int_{t-\tau(t)}^{t} f^T(\eta) P_1 f(\eta) d\eta \\
&+ \int_{t-\delta(t)}^{t} \tilde{f}^T(\eta) Q_2 \tilde{f}(\eta) d\eta
\end{align*}
\]

(27)

where \(P_1, Q_1\) are the positive solution to (26), and \(P_2 \geq 0\) is defined by

\[P_2 = \kappa_i^{-1} \varepsilon_i I + \kappa_i^{-1} \tilde{\rho} M^{-1} \sum_i \sum_i M^{-1}\]

(28)

\(Q_2 \geq 0\) is defined by

\[Q_2 = \kappa_i^{-1} \varepsilon_i I + \kappa_i^{-1} \tilde{\rho} M^{-1} \sum_i \sum_i \tilde{M}^{-1}\]

(29)
we denote \( \kappa_1 = \inf_{x \in \mathbb{R}} (1 - \hat{\tau}(t)) \), \( \kappa_2 = \inf_{x \in \mathbb{R}} (1 - \hat{\delta}(t)) \), and employ Itô’s differential rule, one can deduce that
\[
\ell V(t, x(t), y(t)) = x^T(t)(-P_iA - AP_i)x(t) + y^T(t)(-Q_iB - BQ_i)y(t) + 2x^T(t)P_iW^T f(y(t - \tau(t))) + 2 \times y^T(t)Q_iV^T \hat{f}(x(t - \delta(t))) + \text{trace}\left( \sigma^T(t, x(t), y(t - \tau))P_i\sigma(t, x(t), y(t - \tau)) \right) + \text{trace}\left( \hat{\sigma}(t, y(t), x(t - \delta(t)))Q_i\hat{\sigma}(t, y(t), x(t - \delta(t))) \right) + f^T(y(t))P_i f(y(t))
\]
\[
- f^T(y(t - \tau(t)))P_i f(y(t - \tau(t)))(1 - \hat{\tau}(t)) + \hat{f}^T(x(t))Q_i \hat{f}(x(t)) - \hat{f}^T(x(t - \delta(t))) \times Q_i \hat{f}(x(t - \delta(t)))(1 - \hat{\delta}(t))
\]
\[
\leq x^T(t)(-P_iA - AP_i + \tilde{M}_Q, \tilde{M}_M + \epsilon_i^2 P_iW^T W_i + \rho \Sigma_i^T \Sigma_i)x(t) + y^T(t)(-Q_iB - BQ_i + \rho \Sigma_i^T \Sigma_i + \epsilon_i^2 M_M)\]
\[
x(t - \tau(t)) + x^T(t - \delta(t))[-\inf_{x \in \mathbb{R}} (1 - \hat{\tau}(t))\tilde{M}_Q, \tilde{M}_M + \rho \Sigma_i^T \Sigma_i + \epsilon_i^2 M_M] x(t - \delta(t))
\]
\[
= x^T(t)(-P_iA - AP_i + \tilde{M}_Q, \tilde{M}_M + \epsilon_i^2 P_iW^T W_i + \rho \Sigma_i^T \Sigma_i)x(t) + y^T(t)(-Q_iB - BQ_i + \rho \Sigma_i^T \Sigma_i + \epsilon_i^2 M_M)\]
\[
x(t - \tau(t)) + x^T(t - \delta(t))[-\rho \Sigma_i^T \Sigma_i + \epsilon_i^2 M_M] x(t - \delta(t))
\]
\[
\sum_{i=1}^{\rho} \mathbb{E} \left[ \ell V(t, x(t), y(t)) \right] \leq -x^T(t) \hat{\pi}_1 x(t) - y^T(t) \hat{\pi}_2 y(t)
\]
where
\[
\hat{\pi}_1 = P_iA + AP_i - \epsilon_i^2 P_iW^T W_i - \kappa_2^2 \rho \Sigma_i^T \Sigma_i - \kappa_2^2 \epsilon_i^2 M_M - \varpi \rho \Sigma_i^T \Sigma_i - \rho \Sigma_i^T \Sigma_i
\]
\[
\hat{\pi}_2 = Q_iB + BQ_i - \epsilon_i^2 Q_iV^T V_i - \kappa_1^2 \rho \Sigma_i^T \Sigma_i - \kappa_1^2 \epsilon_i^2 M_M - \varpi \rho \Sigma_i^T \Sigma_i - \rho \Sigma_i^T \Sigma_i
\]
In light of (28) and (29) we obtain that
\[
\ell V(t, x(t), y(t)) \leq -x^T(t) \hat{\pi}_1 x(t) - y^T(t) \hat{\pi}_2 y(t)
\]
where
\[
\hat{\pi}_1 = P_iA + AP_i - \epsilon_i^2 P_iW^T W_i - \kappa_2^2 \rho \Sigma_i^T \Sigma_i - \kappa_2^2 \epsilon_i^2 M_M - \varpi \rho \Sigma_i^T \Sigma_i - \rho \Sigma_i^T \Sigma_i
\]
\[
\hat{\pi}_2 = Q_iB + BQ_i - \epsilon_i^2 Q_iV^T V_i - \kappa_1^2 \rho \Sigma_i^T \Sigma_i - \kappa_1^2 \epsilon_i^2 M_M - \varpi \rho \Sigma_i^T \Sigma_i - \rho \Sigma_i^T \Sigma_i
\]
In light of (26), we can see that \( \hat{\pi}_1 > 0, \hat{\pi}_2 > 0 \).

Then we have \( \ell V < 0 \) for all \( x(t) \neq 0, y(t) \neq 0 \). According to Itô’s formula, system (24) is globally asymptotically stable in the mean square.

4. An illustrative example
Consider a two-neuron stochastic BAM networks with constant delays (3), where
\[
A = \begin{pmatrix} 1.8 & 0 \\ 0 & 2.5 \end{pmatrix}, \quad B = \begin{pmatrix} 1.0 & 0 \\ 0 & 1.5 \end{pmatrix}, \quad W = \begin{pmatrix} 0.4 & 0.5 \\ 0.8 & 0.5 \end{pmatrix}, \quad V = \begin{pmatrix} 0.5 & 0.3 \\ 0.3 & 1.0 \end{pmatrix}, \quad \\
\Sigma_2 = \begin{pmatrix} 0.6 & 0 \\ 0 & 1.7 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad \Sigma_0 = \begin{pmatrix} 0.21 & 0 \\ 0 & 0.4 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 0.12 & 0 \\ 0 & 0.3 \end{pmatrix}. \]

We have \( \tilde{M}_i = M_i = 1 \) \( \forall i = 1, 2 \), \( \tilde{M}_j = M_j = 1 \) \( \forall j = 1, 2 \), we can easily obtain \( \tilde{M} = M = I \), the constant delays can be taken as \( \delta = 0.5 \), \( \tau = 0.5 \), thus by using the Matlab LMI toolbox, we solve the LMIs (7),(8) for \( \rho > 0, \tilde{\rho} > 0, \epsilon_i > 0, \rho_i > 0, P_i > 0, Q_i > 0, Q_i > 0 \) and obtain
\[
P_1 = \begin{pmatrix} 77.5289 & 4.9878 \\ 4.9878 & 142.7478 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} 118.4040 & -2.3203 \\ -2.3203 & 88.5336 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 73.8815 & 0 \\ 0 & 95.9217 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 86.8113 & 0 \\ 0 & 79.7554 \end{pmatrix}, \quad \\
\rho = 214.2029, \tilde{\rho} = 157.0210, \epsilon_1 = 129.9591, \epsilon_2 = 57.0586.
\]
Therefore, it follows from Theorem 1 that the two-neuron stochastic BAM networks with constant delays (3) is globally asymptotically stable in the mean square.

5. Conclusions
In this paper, stochastic bidirectional associative memory networks with delays is studied. By constructing a Lyapunov-Krasovskii functional and using the stochastic stability analysis theory, we derive several sufficient conditions of globally asymptotically stable in the mean square. A LMI approach has been developed to solve the problem addressed. A simple example has been used to demonstrate the usefulness of the main results.

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