I. INTRODUCTION

In 1915 Albert Einstein [1] completed the general theory of relativity which then became an accepted theory of gravity. In general relativity the space-time geometry $g_{ik}$ (metric tensor) is the gravitational field described by the action

$$I_{GR} = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-g} g^{ik} R_{ik} - \int \rho \sqrt{g} g^{ik} \frac{dx^i}{dt} \frac{dx^k}{dt} d^4x,$$

where $G$ is the gravitational constant and $c$ is the speed of light. The second term in Eq. (1) describes interaction between gravitational field and matter with the rest mass density $\rho(t, r)$. Variation of (1) with respect to $g_{ik}$ yields Einstein equations

$$R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right),$$

where $R_{ik}$ is the Ricci tensor and $T_{ik}$ is the energy-momentum tensor of matter.

Einstein equations (2) are a consequence of the postulate that space-time geometry $g_{ik}$ is gravitational field, and thus we live in curved space-time. From our point of view this postulate is counterintuitive because the well-tested Standard Model of particle physics (which includes the electroweak theory and quantum chromodynamics) is a field theory in Minkowski space-time. We believe it is unlikely that the fourth fundamental interaction, gravity, should be described by a theory which such dramatically different from the Standard model as general relativity. One should mention that so far general relativity was tested only at weak gravitational field and thus it is not a theory fully confirmed experimentally.

General relativity also predicts existence of singularities such as black holes when a massive star collapses into a point with zero volume and infinite matter density. One can argue that general relativity becomes invalid in the vicinity of singularities and a quantum theory of gravity will remove them. In contrast, the present theory is free of such singularities at the classical level.

Motivated by the well-tested Standard model, here we propose a new classical theory of gravity which is a field theory in Minkowski space-time and based on the principle of equivalence. We postulate that space-time we live is flat Minkowski space-time and matter does not affect space-time geometry. Also we assume, similarly to electrodynamics, that gravitational field is a 4-vector $A_k$ which lives in Minkowski space-time.

Next we derive action for $A_k$. The principle of equivalence states that motion of test particles in the fixed gravitational field $A_k$ in Minkowski space-time is equivalent to motion in curved space-time with a metric $f_{ik}$ which is determined by the field $A_k$. Thus, the equivalent metric $f_{ik}$ must be a functional of the vector field $A_k$. In Appendix A we show that the principle of equivalence gives the following unique answer for $f_{ik}$

$$f_{ik} = \eta_{ik} e^{A^2} - \frac{2A_i A_k}{A^2} \sinh(A^2),$$

where $\eta_{ik} = \text{diag}(1, -1, -1, -1)$ is Minkowski metric tensor and

$$A^2 = \eta^{ik} A_i A_k.$$  

General covariance (invariance under general coordinate transformations) is a mathematical device used to implement the principle of equivalence [3]. Thus, action for gravitational field written in terms of the equivalent metric $f_{ik}$ must have general covariant form given by $I_{GR}$, where $I_{GR}$ is the action of general relativity (1) in which $g_{ik}$ is replaced by $f_{ik}$. As a result, we obtain the following expression for the action

$$I = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-f} f^{ik} R_{ik} - \int \rho \sqrt{f_{ik}} \frac{dx^i}{dt} \frac{dx^k}{dt} d^4x,$$
where $f = \det(f_{ik}) = -e^{2\phi^2}$ and $f_{ik}$ is the tensor inverse to $f_{ik}$ ($f_{im}f^{km} = \delta^k_i$). The first term in Eq. (5) is Einstein-Hilbert action in which the Ricci tensor $R_{ik}$ is formed from the field $f_{ik}$.

Action (5) is a functional of the gravitational field $A_k$. Variation of (5) with respect to $A_k$ yields four equations for gravitational field. In general relativity all components of $f_{ik}$ are treated as independent under variation of the action (5). In the present theory this is not the case due to constraint imposed by Eq. (3).

Action (5) is written in Minkowski metric in covariant form, has no free parameters and serves as a foundation of the present theory of gravity. Our derivation of the action (5) is unique and, hence, the vector theory of gravity which obeys the principle of equivalence is also unique.1

In Section VII we show that current theory passes all available tests. At strong field our theory substantially deviates from general relativity and yields no black holes. For cosmology the present theory gives essentially the same evolution of the Universe as general relativity.

A remarkable feature of our theory is that equations for gravitational field can be solved analytically for arbitrary static mass distribution (see Sec. IV). If point masses are located at $r_1, r_2, \ldots, r_N$ then exact solution is

$$f_{ik} = \begin{pmatrix}
e^{2\phi} & 0 & 0 & 0 \\
0 & -e^{-2\phi} & 0 & 0 \\
0 & 0 & -e^{-2\phi} & 0 \\
0 & 0 & 0 & -e^{-2\phi}
\end{pmatrix}, \quad (6)$$

where

$$\phi(r) = -\frac{m_1}{|r - r_1|} - \ldots - \frac{m_N}{|r - r_N|} \quad (7)$$

and $m_k (k = 1, \ldots, N)$ are constants determined by the value of masses.

Solution (6) is free of black holes. For a star of mass $M$ and radius $R$ Eq. (7) reduces to $\phi(r) = -GM/c^2r$ ($r \geq R$) and using Eq. (6) for energy conservation we obtain that escape velocity for a particle from the stellar surface is

$$v = c_s\sqrt{1 - e^{2\phi(R)}}, \quad (8)$$

where $c_s = e^{2\phi(R)}$ is the speed of light at the stellar surface (see Eq. (5)). Eq. (8) shows that escape velocity is always smaller then $c_s$ ($c_s \leq c$). In addition, solution (6) predicts that stars do not collapse into a point singularity but rather form stable compact objects with no event horizon and finite gravitational redshift [4].

In recent years, the evidence for the existence of ultracompact supermassive objects at centers of galaxies has become very strong. It is important to note that present solution (6) not only argues that such objects are not black holes, but also can explain quantitatively their observed properties (see Sec. VIII and Ref. [5]).

II. EQUATIONS FOR GRAVITATIONAL FIELD IN MINKOWSKI SPACE-TIME

Here we obtain equations for gravitational field from the action (5). In the rest part of the paper raising and lowering of indexes is carried out using Minkowski tensor $\eta_{ik} =$diag$(1, -1, -1, -1)$. It is convenient to introduce new independent functions, a scalar

$$\phi = -\frac{A^2}{2} \quad (9)$$

and replace $A_k$ by a unit vector which we also will denote as $A_k$

$$A_k \rightarrow \frac{A_k}{\sqrt{\eta^{km}A_lA_m}} \quad (10)$$

The new vector $A_k$ obeys the normalization constraint

$$A_kA^k = 1. \quad (11)$$

Because expression under the square root in Eq. (10) can be both positive or negative the unit vector $A_k$ is a real or pure imaginary vector. Observable quantities, of course, are always real.

We treat spatial components $A_\alpha (\alpha = 1, 2, 3)$ as independent, while

$$A_0 = \sqrt{1 + A_1^2 + A_2^2 + A_3^2}. \quad (12)$$

In terms of new independent functions, $\phi$ and the unit vector $A_k$, the equivalent metric (5) has the form

$$f_{ik} = \eta_{ik}e^{-2\phi} + 2A_iA_k\sinh(2\phi), \quad \sqrt{-f} = e^{-2\phi} \quad (13)$$

and the inverse tensor is

$$\tilde{f}_{ik} = \eta_{ik}e^{2\phi} - 2A^iA^k\sinh(2\phi). \quad (14)$$

One can find variation of the action (5) using formulas

$$\delta \int d^4x\sqrt{-f}\tilde{f}^{ik}R_{ik} = \int d^4x\sqrt{-f} \left( R_{ik} - \frac{1}{2}f_{ik}R \right) \delta \tilde{f}_{ik},$$

$$\delta f_{ik} = -f_{im}f_{kj}\delta \tilde{f}^{mt}$$

1 One should mention that, similarly to the vector potential in electrodynamics, $A_k$ is not observable directly. Hence, $A_k$ can be any vector field which gives physically reasonable equivalent metric $f_{ik}$. Because $f_{ik}$ depends on $A_k$ quadratically the equivalent metric remains real also for pure imaginary $A_k$. If $A_k$ is real (pure imaginary) then always $f_{00} \leq 1$ ($f_{00} > 1$). We will allow both real and pure imaginary $A_k$. If gravitational field realizable in nature corresponds only to real $A_k$, then solutions of our equations for particular physical problems will automatically yield real $A_k$. This is, e.g., the case for static gravitational field produced by positive masses (see Eqs. (6) and (7) which yield $f_{00} \leq 1$).
and expressing variation of $\delta f_{ik}$ in terms of $\delta \phi$ and $\delta A^\alpha$. Variation of $\epsilon$ with respect to $\phi$ and $A^\alpha$ yields the following four equations in Minkowski space-time

$$R_{ik}A^k = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} f_{ik} T \right) A^k, \quad (15)$$

where the Ricci tensor $R_{ik}$ and Christoffel symbols $\Gamma_{ik}^l$ are formed from the equivalent metric $f_{ik}$

$$R_{ik} = \frac{\partial^{\prime} T_{ik}}{\partial x^l} + \Gamma_{ik}^m \Gamma_{ml}^l - \Gamma_{il}^m \Gamma_{km}^l, \quad (16)$$

$$\Gamma_{ik}^l = \frac{1}{2} f_{lm} \left( \frac{\partial f_{mi}}{\partial x^l} + \frac{\partial f_{mk}}{\partial x^l} - \frac{\partial f_{ik}}{\partial x^m} \right), \quad (17)$$

$T_{ik}$ is the energy-momentum tensor of matter

$$T_{ik} = \frac{\rho c^2}{\gamma} e^{2\phi} f_{ik} u^i u^k, \quad T = f_{ik} T_{ik} = \frac{\rho c^2}{\gamma} e^{2\phi}, \quad (18)$$

$\rho$ is the rest mass density, $u^i$ is the particle 4-velocity

$$u^i = \frac{\gamma dx^i}{c dt} \equiv \gamma \left( 1, \frac{V^\alpha}{c} \right), \quad (19)$$

$$V^\alpha = dx^\alpha/dt \quad (\alpha = 1, 2, 3) \text{ is three dimensional velocity of particle } (V^2 = V_1^2 + V_2^2 + V_3^2),$$

$$\gamma = \frac{1}{\sqrt{(c^2 - V^2)} e^{-2\phi} + 2 (A_0 c + A_\alpha V^\alpha)^2 \sinh(2\phi)} \quad (20)$$

is the $\gamma$–factor in the presence of gravitational field.

Covariant Eqs. (15) for the scalar $\phi$ and the unit vector $A_k$ are the main equations of the present theory of gravity. They are written in Minkowski metric which means that raising and lowering of indexes is carried out using Minkowski tensor. In our theory motion of particles in gravitational field is described by Eq. (15) identical to those in general relativity in metric $f_{ik}$. We obtain equations of particle motion in Appendix [13].

One should note that because $\phi$ and $A_k$ are not observable directly they may not be smooth functions and components of $A_k$ can have jumps at points where $\phi = 0$. However, equivalent metric $f_{ik}$ constructed from $\phi$ and $A_k$ must be continuous and smooth.

Present Eqs. (15) have exact analytical solutions in the class of functions for which only one component of $A_k$ (e.g. component $m$) is nonzero and $\phi$ is independent of $x^m$. In particular, solution $A_k = [1, 0, 0, 0], \ \phi = \phi(r)$ describes static field, while solution with $A_\alpha = i$ corresponds to a gravitational wave with polarization along the $\alpha$–axis. We discuss such solutions next.

### III. GRAVITATIONAL WAVES OF ARBITRARY AMPLITUDE

In general relativity there exist transverse gravitational waves of two polarizations [6]. Here we show that the same situation takes place in the present theory of gravity. Let us consider a plane gravitational wave in free space. Then gravitational field obeys equations

$$R_{ik}A^k = 0. \quad (21)$$

Exact solution of Eqs. (21) for arbitrary strength of gravitational field is given by

$$A_k = [0, 0, i, 0], \ \phi = \phi(t, x, z) \quad (22)$$

for a wave of $y$ polarization propagating in the $x - z$ plane and

$$A_k = [0, 0, 0, i], \ \phi = \phi(t, x, y) \quad (23)$$

for a wave of $z$ polarization propagating in the $x - y$ plane. For both polarizations, $\phi$ is a function satisfying the linear wave equation

$$\square \phi = 0, \quad \square \equiv \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}, \quad (24)$$

that is wave propagates with the speed of light $c$.

For example, for $y$ polarization (22) the equivalent metric (13) has the form

$$f_{ik} = \begin{pmatrix} e^{-2\phi} & 0 & 0 & 0 \\ 0 & -e^{-2\phi} & 0 & 0 \\ 0 & 0 & e^{-2\phi} & 0 \\ 0 & 0 & 0 & -e^{-2\phi} \end{pmatrix} \quad (25)$$

and the corresponding Ricci tensor is

$$R_{ik} = \square \phi \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -e^{4\phi} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - 2 \frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^k} \quad (26)$$

Obviously Eqs. (21) are satisfied provided $\phi(t, x, z)$ obeys the wave equation (24). Combining $y$ and $z$ polarizations one can find a general solution for arbitrary plane wave propagating along the $x$–axis:

$$A_k = i[0, 0, 0, 0], \ \phi = \phi(t, x), \ \alpha = \alpha(t, x) \quad (27)$$

The corresponding equivalent metric is given by

$$f_{ik} = e^{-2\phi} \eta_{ik} - \sinh(2\phi) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 \sin^2 \alpha \sin(2\alpha) & 0 \\ 0 & 0 & \sin(2\alpha) & 2 \cos^2 \alpha \end{pmatrix} \quad (28)$$

For the gravitational field (28) Eqs. (21) lead to the following equations for $\phi$ and $\alpha$

$$\square \phi + \sinh(4\phi) \left[ \frac{1}{c^2} \left( \frac{\partial \alpha}{\partial t} \right)^2 - \left( \frac{\partial \alpha}{\partial x} \right)^2 \right] = 0, \quad (29)$$

$$\sinh(2\phi) \square \alpha + 4 \cosh(2\phi) \left( \frac{\partial \alpha}{\partial x} \frac{\partial \phi}{\partial x} - \frac{1}{c^2} \frac{\partial \alpha}{\partial t} \frac{\partial \phi}{\partial t} \right) = 0 \quad (30)$$
which have exact analytical solution in the form

$$\phi = \phi(x - ct), \quad \alpha = \alpha(x - ct).$$

Eqs. (28) and (31) give a general solution for gravitational wave of arbitrary amplitude propagating along the positive $x$-direction.

One should note that in the weak field limit solutions obtained above reduce to those of general relativity. Indeed, in the weak field limit one can omit nonlinear terms in Eq. (20). The linearized $R_{ik}$ satisfies Einstein equations in free space $R_{ik} = 0$ and, thus, the present solutions are also solutions in general relativity. However for strong field this is not the case.

In Appendix C we show that in our theory radiation of weak gravitational waves is given by the same formula as in general relativity. However, the present vector theory of gravity is not equivalent to general relativity in the weak field limit and there are situations when two theories give different answers even for weak field. As an example, let us consider solution of Eqs. (29) and (30) in the weak field limit. The linearized $R_{ik}$ satisfies Einstein equations in free space $R_{ik} = 0$ and, thus, the present solutions are also solutions in general relativity. However for strong field this is not the case.

Indeed, for metric (28) with $\alpha = ckt$ and $2\phi = C \sin(2kx + \theta)$ the Ricci tensor in the first order in $\phi$ reads

$$R_{ik} = 4k^2 \phi \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$ (39)

Because $R_{00}$ and $R_{11}$ do not vanish our solution (36) disobey Einstein equations (38) even for weak field. However, present equations (21)

$$R_{22} \sin \alpha + R_{23} \cos \alpha = 0,$$

$$R_{33} \cos \alpha + R_{23} \sin \alpha = 0$$

are automatically satisfied because they do not involve $R_{00}$ and $R_{11}$.

Such an example demonstrates that our theory of gravity is not equivalent to general relativity in the weak field limit. This is expected because a vector theory can not be equivalent to a tensor theory. However, in the weak field regimes for which general relativity was tested (such as radiation of weak gravitational waves and weak static field) both theories give the same answer.

### IV. STATIC GRAVITATIONAL FIELD

Here we consider gravitational field produced by rest matter with density $\rho(r)$. In this case the energy-momentum tensor $\mathcal{T}$ has only one nonzero component $\mathcal{T}_{00} = \rho c^2 e^{\phi}$ and Eqs. (15) have the following exact analytical solution

$$A_k = [1, 0, 0, 0],$$

and $\phi(r)$ obeys the equation

$$\Delta \phi = \frac{4\pi G}{c^2} e^{\phi} \rho.$$ (41)

For solution (40) the equivalent metric is given by

$$f_{ik} = \begin{pmatrix} e^{2\phi(r)} & 0 & 0 & 0 \\ 0 & -e^{-2\phi(r)} & 0 & 0 \\ 0 & 0 & -e^{-2\phi(r)} & 0 \\ 0 & 0 & 0 & -e^{-2\phi(r)} \end{pmatrix}$$ (42)

and the corresponding Ricci tensor is

$$R_{ik} = \Delta \phi \begin{pmatrix} e^{4\phi} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - 2 \frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^k}.$$ (43)

Because $R_{00} = 0$ Eqs. (15) are automatically satisfied for $i = 1, 2, 3$. 

Indeed, for metric (28) with $\alpha = ckt$ and $2\phi = C \sin(2kx + \theta)$ the Ricci tensor in the first order in $\phi$ reads

$$R_{ik} = 4k^2 \phi \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$ (39)

Because $R_{00}$ and $R_{11}$ do not vanish our solution (36) disobey Einstein equations (38) even for weak field. However, present equations (21)
In Newtonian limit Eq. (11) reduces to $\Delta \phi = 4\pi G \rho / c^2$ and, thus, $c^2 \phi(r)$ has a meaning of gravitational potential.

Solution (42) is free of black holes for any mass distribution and field strength. For a point mass $M$ located at $r = 0$ Eq. (11) leads to $c^2 \Delta \phi = 4\pi G M \delta(r)$ and has a solution $\phi = -G M / c^2 r$. For $N$ point masses at $r_1, \ldots, r_N$ Eq. (11) yields

$$\Delta \phi = 4\pi \left[ m_1 \delta(r_1) + \ldots + m_N \delta(r_N) \right], \quad \text{(44)}$$

where $m_1, \ldots, m_N$ are positive constants. Solution of Eq. (44) is

$$\phi(r) = -\frac{m_1}{|r - r_1|} - \ldots - \frac{m_N}{|r - r_N|}, \quad \text{(45)}$$

Next we consider motion of a particle with rest mass $m$ in static gravitational field $\phi(r)$. Equation of particle motion in general case is obtained in Appendix B. Eq. (B5) for field (42) reduces to

$$\frac{d}{dt} \left( c^2 \phi \right) = 0, \quad \text{(46)}$$

where $\nabla \phi = \partial \phi / \partial r$, $r = x^\alpha, V = \partial r / \partial t$ is the particle velocity and

$$\gamma = \frac{e^{-\phi}}{\sqrt{1 - \frac{V^2}{c^2} e^{-2\phi}}}, \quad \text{(48)}$$

One can also find equation of particle motion (47) directly from Lagrange’s equation $\frac{d}{dt} \frac{\partial L}{\partial \dot{\chi}} = \frac{\partial L}{\partial \chi}$, where the Lagrangian (B7) for static gravitational field reads

$$L = -mc^2 \sqrt{e^{2\phi} - \frac{V^2}{c^2} e^{-2\phi}}, \quad \text{(49)}$$

Eq. (46) follows from Eq. (47) if multiply both sides of Eq. (47) by $e^{-\phi} V$ and make simple algebraic transformations.

Lagrangian (49) gives the following expression for the particle generalized momentum $p = \frac{\partial L}{\partial \dot{\chi}}$

$$p = e^{-\phi} m V, \quad \text{(50)}$$

and particle Hamiltonian $H = \dot{\chi} \frac{\partial L}{\partial \dot{\chi}} - L$

$$H = e^{2\phi} \gamma mc^2 = \sqrt{m^2 c^4 e^{2\phi} + p^2 c^2 e^{-4\phi}}. \quad \text{(51)}$$

Thus, Eq. (46) is the equation of energy conservation $E = \text{const}$, where

$$E = e^{2\phi} \gamma mc^2 = \frac{e^{\phi} mc^2}{\sqrt{1 - \frac{V^2}{c^2} e^{-4\phi}}} \quad \text{(52)}$$

is the particle energy and Eq. (47) is the equation for momentum.

For a massless particle one should use Eq. (B9) which for a static field reads

$$e^{-4\phi} \frac{\partial^2 \chi}{\partial t^2} - c^2 \Delta \chi = 0. \quad \text{(53)}$$

Eq. (53) describes propagation of a massless particle with speed

$$v = ce^{2\phi}. \quad \text{(54)}$$

One can see that speed of light depends on the gravitational field $\phi$ and $v \leq c$ if $\phi$ is given by Eq. (53) with positive masses. By proper rescaling of coordinates in Eq. (53) one can remove the factor $e^{-4\phi}$ at any given point. Let us fix $\phi = 0$ at infinite distance from masses. If an observer at infinity sends a light signal towards the Sun then near the solar surface $\phi < 0$ and light will propagate with a smaller speed. This is the explanation of Shapiro time delay in the present theory of gravity. In our theory the space-time geometry is fixed everywhere and given by Minkowski metric. Light signal traveling the same distance arrives with a delay if the light trajectory passes near the Sun. The delay occurs because the speed of light is smaller near the solar surface.

Since Eq. (53) does not contain $t$ explicitly the photon frequency $\omega_\phi$ (measured in time $t$) remains the same during light propagation. However, physical processes occur with different rates at different $\phi$. Gravitational field (42) can be removed at a given point by rescaling in the factor $\sqrt{\frac{\omega_\phi}{c^2}} = e^\phi (t = \tau/e^\phi)$ and spatial coordinates by $\sqrt{-\frac{\partial^2}{\partial s^2}} = e^{-\phi}$. In such rescaled coordinates identical atoms emit light with equal frequencies $\omega \propto \partial \chi / \partial \tau = e^{-\phi} \partial \chi / \partial t$. Thus we obtain

$$\omega = \omega_\phi e^{-\phi}, \quad \text{(55)}$$

where $\omega_\phi$ is the photon frequency measured in time $t$.

Eq. (55) shows that if light emitted by an atom propagates into a region with larger gravitational potential then the detected light frequency is smaller then those an identical atom emits at the detection point. This phenomenon is known as gravitational redshift of light. Eq. (55) also shows that in our theory there are no black holes. Indeed for the gravitational field created by a point mass $M$: $\phi = -G M / c^2 r$. Therefore if a photon is emitted at a distance $r$ from the mass $M$ with frequency $\omega$ then an observer at infinity will detect the photon with the energy

$$\hbar \omega_0 = \hbar \omega e^{-GM/c^2 r}. \quad \text{(56)}$$

According to Eq. (55) no matter how close the photon is emitted to the mass $M$ the photon’s energy at infinity never becomes zero. This means that photon can escape from the mass $M$ from any distance. Such a conclusion is dramatically different from prediction of general relativity. In Einstein’s theory photons become trapped by
the mass $M$ if they are emitted from a distance smaller then the event horizon (that is point mass $M$ behaves as a black hole).

In Section [V] we show that our theory passes all tests of general relativity. In particular, Eqs. (49) and (57) explain correctly the precession of the perihelion of Mercury and Eq. (55) the gravitational redshift of light.

V. STATIONARY GRAVITATIONAL FIELD

Next we consider gravitational field produced by stationary mass currents, so that the energy-momentum tensor of matter (18) is independent of time. We assume that matter velocity $V$ is much smaller then the speed of light. However, the scalar potential $\phi$ is not necessarily small. In this case one can look for solution for the equivalent metric (13) in the form (in the Cartesian coordinate system)

$$f_{ik} = \begin{pmatrix}
e^{2\phi} & \tilde{A}_{1e^{2\phi}} & \tilde{A}_{2e^{2\phi}} & \tilde{A}_{3e^{2\phi}} \\
\tilde{A}_{1e^{2\phi}} & -e^{-2\phi} & 0 & 0 \\
\tilde{A}_{2e^{2\phi}} & 0 & -e^{-2\phi} & 0 \\
\tilde{A}_{3e^{2\phi}} & 0 & 0 & -e^{-2\phi}
\end{pmatrix}, \quad (57)$$

where the three dimensional vector $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$ is small, $\tilde{A} \ll 1$, and $\phi = \phi(r)$ is arbitrary. For metric (57) the Ricci tensor up to terms linear in $\tilde{A}$ reads

$$R_{00} = e^{4\phi} \Delta \phi,$$

$$R_{0\alpha} = e^{4\phi} \tilde{A}_\alpha \Delta \phi - \frac{1}{2} \left[ \text{curl}(e^{4\phi} \text{curl}\tilde{A}) \right]_\alpha,$$

$$R_{\alpha\beta} = -2 \frac{\partial \phi}{\partial x^\alpha} \frac{\partial \phi}{\partial x^\beta} + \Delta \phi \delta_{\alpha\beta}$$

and Eqs. (15) reduce to

$$\Delta \phi = \frac{4\pi G}{c^2} e^\phi \rho, \quad \nabla \phi \cdot \tilde{A} = 0, \quad (58)$$

$$\text{curl}(e^{4\phi} \text{curl}\tilde{A}) = \frac{16\pi G}{c^3} e^\phi \rho V, \quad (59)$$

where $V = (V^1, V^2, V^3)$ and $\rho$ are the matter velocity and density respectively. Eqs. (57), (58) and (59) are valid in the first order in $V/c$ and for $\tilde{A} = 0$ reduce to those for a static field.

For a point mass $M$ rotating around the $z$-axis with an angular momentum $L$ Eqs. (57), (58) and (59) give the following expression for $f_{ik}$ in spherical coordinates ($ct, r, \theta, \phi$)

$$f_{ik} = \begin{pmatrix}
e^{2\phi} & 0 & 0 & Ar \sin \theta \\
0 & -e^{-2\phi} & 0 & 0 \\
0 & 0 & -r^2 e^{-2\phi} & 0 \\
Ar \sin \theta & 0 & 0 & -r^2 \sin^2(\theta)e^{-2\phi}
\end{pmatrix}, \quad (60)$$

with

$$\phi = -\frac{GM}{c^2 r}, \quad A = \frac{3c^3 Lr \sin \theta}{16M^2 G^2} \left[ e^{-2\phi} - e^{2\phi} \left( 8\phi^2 - 4\phi + 1 \right) \right]. \quad (61)$$

In the weak field limit $GM \ll c^2 r$ Eq. (61) yields $A = 2G M \sin(\theta)/c^2 r^2$ which coincides with the answer obtained in general relativity.

One can find exact analytical solutions for gravitational field of any strength by making Lorentz transformation of Eq. (12). For example, gravitational field produced by a relativistic point particle moving with constant velocity $V$ along the $x-$axis is given by

$$A_k = \frac{1}{\sqrt{c^2 - V^2}} [c, -V, 0, 0], \quad (62)$$

and

$$\phi = \frac{GM}{c^2 \sqrt{(ix-Vx)^2 + y^2 + z^2}}, \quad (63)$$

where $M$ is the particle rest mass.

VI. TESTS OF THE THEORY OF GRAVITY

According to the present theory we live in Minkowski space-time in which there is vector gravitational field. However descriptions of particle motion in terms of the vector field or space-time geometry are equivalent. This is Einstein equivalence principle. One can consider interaction of particles with the field in two different, but equivalent ways. In the first approach the space-time is Minkowski space-time in which there is vector gravitational field and motion of a test particle is described by Eq. (65). In the second treatment we describe interaction of the particle with the gravitational field in a geometrical way. Namely we assume there is no vector field, but instead the space-time is curved with metric

$$g_{ik} = f_{ik}, \quad g^{ik} = \tilde{f}^{ik}, \quad (64)$$

and particles move along geodesic lines (6)

$$\frac{du^b}{ds} = -\Gamma^b_{ik} u^i u^k, \quad (65)$$

where $\Gamma^b_{ik}$ are Christoffel symbols, $ds = \sqrt{g_{ik} dx^i dx^k}$ and $u^k = dx^k/ds$ is the particle 4-velocity. Please note that Eq. (13) is written in Minkowski metric, while Eq. (65) is written in metric (64). However, mathematically equations (13) and (65) are identical and thus give the same answer.

It is convenient to compare the present theory of gravity with observations based on the equivalent metric. According to Eq. (12) the equivalent metric for static gravitational field is

$$g_{ik} = \begin{pmatrix}
e^{2\phi} & 0 & 0 & 0 \\
0 & -e^{-2\phi} & 0 & 0 \\
0 & 0 & -e^{-2\phi} & 0 \\
0 & 0 & 0 & -e^{-2\phi}
\end{pmatrix}. \quad (66)$$
where for a point mass $M$: $\phi = -M/r$ (here we put $G = c = 1$). Metric \( ds^2 = h(r)(dx^0)^2 - g(r)((dx^1)^2 + (dx^2)^2 + (dx^3)^2) \), (67)

where

\[
 h(r) = \left(1 - \frac{M}{r} \right)^2, \quad g(r) = \left(1 + \frac{M}{2r} \right)^4.
\] (68)

For small $M/r$ both the Schwarzschild \( [68] \) and Yilmaz \( [66] \) metrics yield the same expansion

\[
 h(r) = 1 - \frac{2M}{r} + \frac{2M^2}{r^2} + \ldots, \quad g(r) = 1 + \frac{2M}{r} + \ldots \quad (69)
\]

which is known as Post-Newtonian approximation. The four classic tests of general relativity, namely the gravitational redshift of light, the deflection of light by the Sun, the precession of the perihelion of Mercury and time delay of a radar signal traveling near the Sun (Shapiro delay), have examined the metric in the Post-Newtonian approximation \( [69, 2] \). Because the equivalent metric \( [66] \) obtained in the present theory has correct Post-Newtonian limit \( [63] \), our theory of gravity also passes the four classic tests. For static field the present theory of gravity answers different from general relativity in the next correction beyond the Post-Newtonian approximation. So far, however, gravity have not been tested in this region.

In Appendix C we show that in our theory radiation of weak gravitational waves is described by the same formula as in general relativity. Such radiation was indirectly detected as energy loss by binary pulsars and serves as a quantitative test of Einstein equations for weak time-dependent field. The present theory also passes this test.

### VII. COSMOLOGY

In this section we apply our theory to evolution of the Universe. We assume that one can omit kinetic energy of matter compared to its rest energy. We also assume that matter is uniformly distributed in space with density $\rho$, where $\rho$ is independent of time. According to the present theory we live in Minkowski space-time and galaxies located in different parts of the Universe do not move relative to each other (apart from local random motion which we omit in this section). That’s why $\rho$ is constant. However gravitational field evolves with time. Light emitted by a distant source reaches an observer on Earth with a delay. As a result, at the moment of light detection the gravitational field is different from its value when light was emitted. This is the origin of cosmological redshift.

For spatially isotropic Universe gravitational field must have the form

\[
 A_k = [1, 0, 0, 0]
\] (70)

and, thus, equivalent metric \( [13] \) is given by

\[
 f_{ik} = \begin{pmatrix}
 1 & 0 & 0 & 0 \\
 0 & -a^2 & 0 & 0 \\
 0 & 0 & -a^2 & 0 \\
 0 & 0 & 0 & -a^2
\end{pmatrix}
\] (71)

where $a = e^{-\phi}$. As in general relativity, we add the cosmological constant term to energy-momentum tensor, $T_{ik} \rightarrow T_{ik} + \Lambda f_{ik}$, where $\Lambda$ is a constant. Then for gravitational field \( [70] \) Eqs. \( [15] \) read

\[
 R_{i0} = \frac{8\pi G}{c^4} \left( T_{i0} - \frac{1}{2} f_{00}T - \Lambda f_{i0} \right).
\] (72)

We also assume that the Universe is homogeneous on large spatial scales and, therefore, $a$ in Eq. \( [71] \) must be independent of $r$. Then for the equivalent metric \( [71] \) the Ricci tensor is

\[
 R_{00} = -\frac{3}{a^2} \frac{\partial}{\partial t} (a \dot{a}), \quad R_{\alpha0} = 0,
\] (73)

and

\[
 R_{\alpha\beta} = \delta_{\alpha\beta} \frac{\partial}{\partial t} (a^3 \dot{a}).
\] (74)

The energy-momentum tensor of matter \( [13] \) has only one nonzero component $T_{00} = \rho c^2/a^5$, $T = \rho c^2/a^3$, and therefore Eqs. \( [72] \) reduce to

\[
 -3 \frac{\partial}{\partial t} (a \dot{a}) = \frac{4\pi G}{c^4} \left( \frac{\rho c^2}{a^3} - 2\Lambda \right).
\] (75)

Multiplying both sides of Eq. \( [75] \) by $a \dot{a}$ and integrating over time we obtain

\[
 3a^2 = \frac{8\pi G}{c^4} \left( \frac{\rho c^2}{a^3} + \Lambda + \frac{C}{a^6} \right),
\] (76)

where $C$ is an integration constant. For $\Lambda = 0$ sign of $C$ determines whether a Universe is open or closed. In our theory the constant $C$ produces the same effect on $a(t)$ as the curvature of space in general relativity.

In the metric \( [71] \) Einstein equations \( [2] \) with the cosmological constant term give our Eq. \( [76] \) with $C = 0$. Indeed, present Eqs. \( [72] \) coincide with Einstein equations \( [4] \) with index $k = 0$. In addition, Einstein equations with $ik = \alpha \alpha$ yield

\[
 \frac{\partial}{\partial t} (a^3 \dot{a}) = \frac{8\pi G}{c^4} \left( \frac{\rho c^2}{2a} + \frac{\Lambda a^2}{2} \right).
\] (77)

Multiplying both sides of Eq. \( [77] \) by $a^3 \dot{a}$ and integrating over time we obtain

\[
 3a^2 = \frac{8\pi G}{c^4} \left( \frac{\rho c^2}{a^3} + \Lambda + \frac{C_1}{a^6} \right),
\] (78)
where $C_1$ is an integration constant. Eqs. (76) and (78) are compatible if $C_1 = C = 0$, and thus our Eq. (76) with $C = 0$ is also solution in general relativity. Such a solution agrees with available cosmological data.

By changing time coordinate into

$$\tau = \int^t \frac{dt}{a(t)}$$

(79)

Eq. (76) yields

$$\frac{3}{a^2} \left( \frac{\partial a}{\partial \tau} \right)^2 = \frac{8\pi G}{c^3} \left( \frac{\rho c^2}{a^3} + \Lambda + \frac{C}{a^2} \right)$$

(80)

which describes evolution of the Universe in the metric $ds^2 = c^2dt^2 - a^2(dx^2 + dy^2 + dz^2)$. Such form of metric is commonly used in cosmology for spatially flat Universe.

VIII. GALACTIC CENTERS AND DARK MATTER PROBLEM

In the present theory static gravitational field is described by the equivalent exponential metric (66). Metric (66) was also obtained in Refs. 2, 6, 8, 10. Exponential metric (66) predicts no black holes, but rather compact objects with no event horizon and very large, but finite, gravitational redshift.

In recent years, the evidence for the existence of an ultra-compact concentration of dark mass at centers of galaxies has become very strong. However, a proof that such objects are black holes rather than compact objects without event horizon is lacking. If the present theory of gravity is correct then the compact supermassive objects at galactic centers can not be composed of baryonic matter.

Indeed, mass of a compact (neutron star like) baryonic object in the exponential metric (66) can not exceed about $12M_\odot$, but the objects at galactic centers possess masses up to a few $10^6M_\odot$. Hence, those objects must be made of dark matter of non baryonic origin. This fact gives us an opportunity to determine composition of dark matter based on observations of supermassive objects at galactic centers.

In the previous paper 8 we showed that properties of compact objects at galactic centers can be explained quantitatively assuming they are made of dark matter axions and the axion mass is about 0.6 meV. Analysis of Ref. 5 is based on the assumption that static gravitational field is described by the exponential metric (66) rather then by general relativity. A full time-dependent theory of gravity was unnecessary for calculations made in Ref. 8. The present paper provides such a theory and justifies our previous choice of the exponential metric.

Axions are one of the leading particle candidates for the cold dark matter in the Universe 11. Interaction of axions with QCD instantons generates the axion mass $m$ and periodic interaction potential 12

$$V(\varphi) = m^2F^2[1 - \cos(\varphi/F)],$$

(81)

where $\varphi$ is a real scalar axion field and $F$ is the Peccei-Quinn symmetry breaking scale. The interaction potential (81) has degenerate minima $V = 0$ at $\varphi = 2\pi nF$, where $n$ is an integer number. As a consequence, axions can form bubbles. Bubble mass is concentrated in a thin surface (interface between two degenerate vacuum states). In the exponential metric the potential energy of a spherical bubble with radius $R$ is given by [5]

$$U(R) = 4\pi\sigma R^2 \exp \left( \frac{M}{R} \right),$$

(82)

where $\sigma$ is the surface energy density and $M$ is the fixed bubble mass. $U(R)$ has a shape of a well. At $R \gg M$ one can omit gravity and $U(R) \simeq 4\pi\sigma R^2$ is just a surface energy (tension) which tends to contract the bubble. At $R \ll M$ gravity effectively produces large repulsive potential which forces the bubble to expand. As a result, the bubble radius $R(t)$ oscillates between two turning points.

In Ref. 5, based on quantitative analysis of available data, we argued that such oscillating axion bubbles, rather then supermassive black holes, could be present at galactic centers. Recent observations of near-infrared and X-ray flares from Sagittarius A*, which is believed to be a $3.6 \times 10^6M_\odot$ black hole at the Galactic center, show that the source exhibits about 20-minute periodic variability 13, 14, 15. An oscillating axion bubble can explain such variability. Known value of the bubble mass at the center of our Galaxy and its oscillation period yields the axion mass of about 0.6 meV. Size of the axion bubble at the center of the Milky Way oscillates between $R_{\text{min}} \approx 1R_\odot$ and $R_{\text{max}} \approx 1AU \approx 210R_\odot$.

Further, as shown in Ref. 5, the axion bubbles with no free parameters (if we fix $m = 0.6$ meV based on Sagittarius A* flare variability) quantitatively explain the upper limit (a few $10^6M_\odot$) on the supermassive “black hole” mass found in recent analysis of the measured mass distribution 16. Also, with no free parameters the bubble scenario explains observed lack of supermassive “black holes” with mass $M \lesssim 10^6M_\odot$ 17. For such low-mass bubbles the decay time $t \propto M^{9/2}$ becomes much shorter then the age of the Universe and, as a result, such objects are very rare.

Observation of the Galactic center with very long-baseline interferometry within the next few years will be capable to test theories of gravitation in the strong field limit. Such an observation will allow us to distinguish between the black hole (predicted by general relativity) and the oscillating axion bubble scenario. A defining characteristic of a black hole is the event horizon. To a distant observer, the event horizon casts a relatively large “shadow” over the background source with an apparent diameter of about $10GM/c^2 \approx 80R_\odot$ due to bending of light. The predicted size of this shadow for Sagittarius A* approaches the resolution of current radio-interferometers. Hence, there exists a realistic expectation of imaging the shadow of a black hole with VLBA within the next few years 18, 19, 20, 21, 22.
the axion bubble, rather then a black hole, is present at the Galactic center, the steady shadow will not be observed. Instead, the shadow will appear and disappear periodically with a period of about 20 min. Discovery of periodic appearance of the shadow from the Galactic center object will also be a strong evidence for the axion nature of dark matter and will lead to an accurate prediction of the axion mass.

One should mention that intrinsic size of Sagittarius A* at a wavelength of 1.3 mm was recently determined using VLBA. The intrinsic diameter of Sagittarius A* was found to be < 0.3 AU ≈ 65R⊙ which is less than the expected apparent size of the event horizon of the presumed black hole. Such observation might indicate lack of black holes, in agreement with the present theory.

IX. CONCLUSIONS

Here we propose a new classical theory of gravity which is based on the principle of equivalence and assumption that, similarly to electrodynamics, gravity is described by a vector field in Minkowski space-time. We show that present theory is the only possibility that can be obtained from these assumptions. Our theory fundamentally differs from general relativity which treats space-time geometry as gravitational field. In the present theory, similarly to the Standard Model, matter does not affect geometry of flat Minkowski space-time.

The current vector theory is not equivalent to general relativity even in the weak field limit. Nevertheless, our theory also passes all available tests and for static field the Post-Newtonian approximation gives the same answer as general relativity. Beyond the Post-Newtonian approximation the present theory gives different result and yields no singularities such as black holes. A defining characteristic of a black hole is the event horizon. So far there were no observations of the event horizon and, thus, a proof of black holes existence is lacking. For cosmology our theory predicts essentially the same evolution of the Universe as general relativity.

In the present theory gravitational field is described by four equations \[\text{Eq. (15)}\] which can be solved analytically for much greater number of problems then ten Einstein equations \[\text{Eq. (2)}.\] In particular, for arbitrary static mass distribution our Eqs. \[\text{Eq. (15)}\] have exact analytical solution \[\text{Eq. (45)}.\]

The present theory, if confirmed, can also lead to a break through in the problem of dark matter. Namely, the theory predicts that supermassive compact objects at galactic centers have non baryonic origin and, thus, yet undiscovered dark matter particle is a likely ingredient for their composition. As a result, observations of such objects can allow us to predict the nature of dark matter. In the previous paper \[\text{Ref. (5)}\] we showed that properties of compact objects at galactic centers can be explained quantitatively assuming they are made of dark matter axions and the axion mass is about 0.6 meV. Analysis of Ref. \[\text{Ref. (5)}\] is based on the present exponential metric \[\text{Ref. (66)}\] for static gravitational field rather then general relativity.

Our theory of gravity can be tested in several ways. For example, one can examine gravity beyond the Post-Newtonian approximation in the solar system by improving the accuracy of Shapiro time delay experiment (time delay of a radar signal traveling near the Sun). Another possibility is to resolve the supermassive object at the center of our Galaxy with VLBA. If general relativity is correct we must see a steady shadow from a black hole. If the present theory is right then likely the shadow will appear and disappear periodically with a period of about 20 min as we predicted in \[\text{Ref. (5)}\]. Observation of such oscillations will also provide evidence for dark matter axion with mass in meV range.

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APPENDIX A: DERIVATION OF EQUIVALENT METRIC

Here we obtain expression for the metric \( f_{ik} \) which is equivalent to the vector field \( A_k \) using the principle of equivalence. For small \( A_k \) one can expand \( f_{ik} \) in powers of \( A_k \). In the leading order there are two possible terms \( A_i A_k \) and \( A_i^2 \eta_{ik} \), where \( \eta_{ik} = \text{diag}(1, -1, -1, -1) \) is Minkowski metric tensor and \( A^2 = A_k A^k \) is square of vector \( A_k \) \( (\text{Eq. (A1)} \). To obey available experimental tests these terms must enter \( f_{ik} \) in the combination \( A^2 \eta_{ik} - 2A_i A_k \) and, therefore, \( f_{ik} \) has the form

\[
\begin{align*}
\alpha f_{ik} & \approx \eta_{ik} + A^2 \eta_{ik} - 2A_i A_k. \\
\end{align*}
\]

One can write Eq. \[\text{Eq. (A1)}\] as

\[
\begin{align*}
\alpha f_{ik} & \approx \eta_{ik} + \eta_{im} \phi_k^m, \\
\end{align*}
\]

where

\[
\begin{align*}
\phi_k^m & = A^2 \delta^m_k - 2A^m A_k. \\
\end{align*}
\]

In curved space-time with metric \( g_{ik} \) Eq. \[\text{Eq. (A2)}\] reads

\[
\begin{align*}
\alpha f_{ik} & \approx g_{ik} + g_{im} \phi_k^m, \\
\end{align*}
\]

where \( \phi_k^m \) is written in metric \( g_{ik} \).

Let us now assume that gravitational field is not small. According to the equivalence principle one can describe motion of particles as if there is no field, but instead the space-time is curved with an equivalent metric \( g_{ik} \). If we change \( \phi_k \) by a small amount \( \phi_{ik} \rightarrow \phi_{ik} + \alpha \phi_{ik} \), where \( \alpha \) is a small number, then we can write (\( \phi \equiv \phi_{ik} \))

\[
\begin{align*}
f_{ik}(\phi + \alpha \phi) & \approx f_{ik}(\phi) + \partial f_{ik}(1 + \alpha \phi) \biggl|_{\alpha=0} \\
\end{align*}
\]

From the other hand Eq. \[\text{Eq. (A1)}\] yields

\[
\begin{align*}
f_{ik}(\phi + \alpha \phi) & \approx f_{ik}(\phi) + f_{im}(\phi) \alpha \phi_k^m. \\
\end{align*}
\]
Comparing Eqs. (A5) and (A6) we obtain that for any \( \phi_{ik} \)

\[
\frac{\partial f_{ik}(1 + \alpha) \phi}{\partial \alpha} \bigg|_{\alpha=0} = f_{im}(\phi)\phi^m_{ik}. \tag{A7}
\]

Because Eq. (A7) must be satisfied for any \( \phi_{ik} \) it is essentially a differential equation, rather than a condition at one point \( (\alpha = 0) \). Solution of Eq. (A7) yields the following unique expression for the metric equivalent to the gravitational field \( \phi_{ik} \)

\[
f_{ik} = \eta_{im} \exp(\phi^m_{ik}), \tag{A8}
\]

where the exponent of a tensor is defined as

\[
\exp(\phi^m_{ik}) = \delta^m_{k} + \frac{1}{2!} \phi^m_{ik} \phi^l_{ik} + \frac{1}{3!} \phi^m_{ik} \phi^l_{ik} \phi^p_{ik} + \ldots \tag{A9}
\]

Substituting Eq. (A3) into Eq. (A8) and taking into account that

\[
\exp(-2A^iA_k) = \delta^l_k - \frac{A^iA_k}{A^2} \left(1 - e^{-2A^2}\right) \tag{A10}
\]

we obtain

\[
f_{ik} = \eta_{im} \exp(A^2 \delta^m - 2A^m A_k) = e^{A^2} \eta_{im} \exp(-2A^m A_k)
\]

\[
= e^{A^2} \eta_{ik} - \frac{2\eta_{im} A^m A_k}{A^2} \sinh(A^2). \tag{A11}
\]

In Eq. (A11) \( A_k \) is the gravitational field in metric \( f_{ik} \). Therefore, \( A_i = f_{ik}A^k \). Taking into account Eq. (A11) we find

\[
A_i = e^{-A^2} \eta_{ik} A^k, \quad A^i = e^{A^2} \eta^{ik} A_k, \tag{A12}
\]

and \( A^2 = A_k A^k \) is determined from the equation

\[
A^2 e^{-A^2} = \eta^{ik} A_i A_k. \tag{A13}
\]

As a result one can write \( f_{ik} \) as

\[
f_{ik} = e^{A^2} \eta_{ik} - \frac{A^iA_k}{A^2} (e^{2A^2} - 1). \tag{A14}
\]

It is convenient to rewrite this equation in a different form introducing \( \tilde{A}_k = A_k e^{A^2/2} \). Then

\[
f_{ik} = e^{A^2} \eta_{ik} - 2 \tilde{A}_k \tilde{A}_k e^{A^2/2} \sinh(A^2), \tag{A15}
\]

where

\[
\tilde{A}^2 = \eta^{ik} \tilde{A}_i \tilde{A}_k. \tag{A16}
\]

### APPENDIX B: MOTION OF PARTICLES IN GRAVITATIONAL FIELD

Here we find how a test particle with rest mass \( m \) moves in an external gravitational field \( f_{ik} \). Interaction of the particle with the field is described by the action

\[
I_{\text{field-matter}} = -mc \int \sqrt{f_{ik}dx^i dx^k}, \tag{B1}
\]

where the integral is taken along the particle trajectory. One can find equation of particle motion varying the action (B1) at fixed \( f_{ik} \)

\[
\delta I_{\text{field-matter}} = -\frac{mc}{2} \int \left[ \left( \frac{\partial f_{ik}}{\partial x^i} - \frac{\partial f_{ik}}{\partial x^k} \right) u^i u^k - 2f_{ik} \frac{du^k}{ds} \right] ds \delta x^j.
\]

Principle of least action \( \delta I_{\text{field-matter}} = 0 \) yields the following equation

\[
f_{ik} \frac{du^k}{ds} = \frac{1}{2} \left( \frac{\partial f_{ik}}{\partial x^i} - \frac{\partial f_{ik}}{\partial x^k} \right) u^i u^k - 2f_{ik} \frac{du^k}{ds} \tag{B4}
\]

Multiplying both sides of Eq. (B4) by tensor inverse to \( f_{ik} \) we find

\[
\frac{du^b}{ds} = \frac{1}{2} \int 2 \delta^{ib} \left( \frac{\partial f_{ik}}{\partial x^i} - \frac{\partial f_{ik}}{\partial x^k} \right) u^i u^k, \tag{B5}
\]

or

\[
\frac{du^b}{ds} = -\Gamma^b_{ik} u^i u^k. \tag{B6}
\]

This is equation of motion of a particle in gravitational field \( f_{ik} \).

From Eq. (B1) we obtain the following Lagrangian of the particle

\[
L = -mc \sqrt{f_{ik} \frac{dx^i}{dt} \frac{dx^k}{dt}}. \tag{B7}
\]

Action (B1) and Eq. (B5) are invalid for massless particles. Let us consider a massless scalar field \( \chi \). In the gravitational field \( f_{ik} \) the action for \( \chi \) reads (in Minkowski metric)

\[
I = \frac{1}{8\pi} \int d^4x \sqrt{-f} f^{\mu\nu} \frac{\partial \chi^*}{\partial x^\mu} \frac{\partial \chi}{\partial x^\nu}. \tag{B8}
\]
Variation of Eq. (B8) yields the following equation of motion for the field $\chi$

$$\frac{\partial}{\partial x^\mu} \left( \sqrt{-f} f^{\mu\nu} \frac{\partial \chi}{\partial x^\nu} \right) = 0. \quad (B9)$$

For geometrical optics one can write $\chi$ as $\chi = |\chi|e^{i\psi}$, where $\psi$ (eikonal) has a large value. Substituting this into Eq. (B9) and keeping only the leading term we obtain eikonal equation in gravitational field

$$f^{\mu\nu} \frac{\partial \psi}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} = 0. \quad (B10)$$

Appendix C: Radiation of Gravitational Waves

Here we consider radiation of weak gravitational waves. In general relativity the metric for weak plane gravitational waves propagating along the $x-$axis in a properly chosen coordinate system reads [6]

$$g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{22}(t,x) & h_{23}(t,x) & 0 \\ 0 & h_{23}(t,x) & -h_{22}(t,x) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (C1)$$

where $h_{23}$ and $h_{22}$ are small perturbations. In linear approximation the corresponding Ricci tensor is

$$R_{ik} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \left( T_{22} - \eta_{22} T \right), \quad (C2)$$

To find energy radiated in the $x-$direction one should find functions $h_{23}$ and $h_{22}$ in the wave zone. They are determined from Einstein equations [2]

$$\frac{1}{2} \eta_{22} = \frac{8\pi G}{c^4} \left( T_{22} - \frac{1}{2} \eta_{22} T \right), \quad (C3)$$

$$\frac{1}{2} \eta_{23} = \frac{8\pi G}{c^4} T_{23}, \quad (C4)$$

$$-\frac{1}{2} \eta_{22} = \frac{8\pi G}{c^4} \left( T_{33} - \frac{1}{2g_{33} T} \right). \quad (C5)$$

In the right hand side of Eqs. (C3)–(C5) one can replace $g_{ik}$ with Minkowski metric $\eta_{ik}$.

In the present theory of gravity for weak gravitational waves ($|\phi| \ll 1$) propagating along the $x-$axis

$$A_k = i\eta_{k} \sin \alpha \cos \alpha \quad (C6)$$

and the equivalent metric [13] has the form

$$f_{ik} = \eta_{ik} + 2\phi \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & \cos(2\alpha) & 0 & -\cos(2\alpha) \\ 0 & -\sin(2\alpha) & \cos(2\alpha) & 0 \end{pmatrix} \quad (C7)$$

where $\phi = \phi(x - ct)$ and $\alpha = \alpha(x - ct)$. Eqs. (C6) and (C7) are equations in Minkowski space time. Let us make the following change of coordinates

$$x \rightarrow x - \int_{0}^{x-ct} \phi(q) dq. \quad (C8)$$

Taking into account that under coordinate transformation $x^i \rightarrow x^i + \xi^i$, where $\xi^i$ are small, tensor $f_{ik}$ transforms as $f_{ik} \rightarrow f_{ik} - \partial \xi/\partial x^k - \partial \xi/\partial x^i$, we obtain that in new coordinates

$$f_{ik} = \eta_{ik} + 2\phi \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & -\sin(2\alpha) & \cos(2\alpha) & 0 \end{pmatrix} \quad (C9)$$

which is precisely Eq. (C1) with $h_{22} = 2\phi \cos(2\alpha)$ and $h_{23} = -2\phi \sin(2\alpha)$. As a result, for weak gravitational waves our equivalent metric $f_{ik}$ in the wave zone is equal to $g_{ik}$, where $g_{ik}$ is solution of Einstein equations (C3)–(C5). Indeed, if $f_{ik}$ obeys Eqs. (C3)–(C5) then it automatically satisfies present equations

$$R_{ik} A^k = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} f_{ik} T \right) A^k \quad (C10)$$

which are linear combinations of Eqs. (C3)–(C5) with coefficients $A^k$.

We obtain that general relativity and our theory yield the same answer for the equivalent metric of emitted weak gravitational waves. Therefore, in both theories the energy loss due to emission of weak gravitational waves is given by the same formula.

Appendix D: Weak Gravitational Field

Here we obtain equations for weak gravitational field. In this case $|\phi| \ll 1$, however, the unit vector $A_k$ can be arbitrary. For weak field the Ricci tensor (16) reduces to

$$R_{ik} \approx \frac{\partial T_{ik}}{\partial x^k} - \frac{\partial T_{ik}}{\partial x^k}. \quad (D1)$$

Taking into account Eq. (17) for $\Gamma_{ik}^l$ and formula $\Gamma_{ik}^l = \partial \ln \sqrt{-f}/\partial x^l = -2\phi/\partial x^l$, we find that Eqs. (15) in the weak field limit yield

$$2A^k \left[ \frac{\partial^2}{\partial x^l \partial x^k} \left( A_i A^l \phi \right) + \frac{\partial^2}{\partial x^l \partial x^k} \left( A_k A^l \phi \right) + \Box(A_i A_k \phi) \right] -$$

$$- A_i \Box \phi \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} \eta_{ik} T \right) A^k, \quad (D2)$$
where
\[ \Box \equiv \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}, \]
\( T_{ik} \) is the energy momentum tensor for free particles
\[ T_{ik} = \sqrt{1 - \frac{V^2}{c^2}} \rho c^2 u_i u_k, \quad T = \sqrt{1 - \frac{V^2}{c^2}} \rho c^2, \quad (D3) \]
\( \rho \) is the rest mass density and \( u_k \) is the particle 4-velocity
\[ u_k = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \left( 1, \frac{V_k}{c} \right). \quad (D4) \]
The weak field equations (D2) are Lorentz covariant and nonlinear in \( A_k \). Eqs. (D2) differ from the weak field limit of Einstein equations [6]. This is the case because the present vector theory of gravity is not equivalent to general relativity (tensor theory) even for weak field.

If \( A_\alpha \) are also small (\( |A_\alpha| \ll 1 \) and \( A_0 \approx 1 \)) one can rewrite Eqs. (D2) in a form similar to Maxwell’s equations (for consistency we assume that \( V \ll c \) and omit second order time derivatives)
\[ \text{div} \mathbf{E} = 4\pi \frac{G \rho}{c^2}, \quad (D5) \]
\[ \text{curl} \mathbf{B} = 4\pi \frac{G \rho V}{c^2} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (D6) \]
where \( \mathbf{E} = \nabla \phi, \quad \mathbf{B} = \text{curl}(\mathbf{A} \phi) \),
\( \mathbf{A} = (A^1, A^2, A^3) \) and \( \mathbf{V} = (V^1, V^2, V^3) \) is the three dimensional velocity of particle.

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