Using Randomness to decide among Locality, Realism and Ergodicity.

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The recently performed loophole-free tests have demonstrated that at least one of three properties is false in Nature when Bell’s inequalities are experimentally violated: Locality, Realism or (what is lesser known) Ergodicity. An experiment based on the observation of the time evolution of randomness is proposed to find which one is false. The results of such experiment would be important not only to the foundations of Quantum Mechanics; they would have direct practical consequences for quantum-based Random Number Generators and the security of Quantum Key Distribution.

Bell’s inequalities demonstrate the predictions of Quantum Mechanics (QM) to be incompatible with the intuitive properties of Locality and Realism (LR) [1]. The latter are considered valid not only in everyday life, but also in all scientific practice. Experiments have been performed for half a century to determine whether QM or LR is valid in Nature. Early observations reported a violation of Bell’s inequalities, hence disproving LR. This result was so disturbing, that mechanisms exploiting experimental imperfections (which were named, in general, loopholes) were claimed to be the actual cause of the observed violation. Experiments of increased sophistication were performed, successively closing each loophole [2-7]. In recent years, a bunch of experiments practically closed all the known loopholes simultaneously, reaching the loophole-free condition [8-11] (for a sort of critical review see [12]) and confirming the QM predictions.

In order to use Bell’s inequalities in an experiment, at least one property additional to LR is needed. Although this extra property was documented long ago, it was periodically forgotten and rediscovered with different names: ergodicity [13], homogeneous dynamics [14], uniform complexity [15], counter-factual stability [16], etc. In what follows, I use its earliest name “ergodicity”, which is also the easiest to understand. In short, it means that data recorded with different experimental settings (and that are, in consequence, unavoidably recorded at different times) can be combined into a single expression (that is, one of the Bell’s inequalities). As this property is lesser known than LR, I describe it briefly in the Appendix for the Reader’s convenience.

The loophole-free experiments demonstrate that theories able to describe the observed violation of Bell’s inequalities cannot have the properties of being Local, Real and Ergodic simultaneously. In Figure 1, sets corresponding to these properties are drawn in the space of theories dealing with Bell’s experiment. Theories inside the black “triangle” with red, blue and yellow sides, in the intersection of the three main sets, are refuted. The relevant question now is which one of these three properties is false in Nature when Bell’s inequalities are violated. Of course, more than one can be false. I consider here the cases where only one of them is false.

The key of the proposal is that a bounded, random evolution is ergodic. This appears true even if no unanimously accepted definition of “random” exists. The reasoning is as follows. An evolution is ergodic iff its time average is constant in phase space. In other words, the system spends the same time in each region of the same volume. If the evolution is not ergodic, then the system visits some regions more often than others. Hence, the probability of finding the system in some regions is larger than in others. Then: not ergodic

![Figure 1: Sets in the space of theories dealing with Bell’s experiment. The theories having the properties of being Real (red) and Local (blue) and Ergodic (yellow) are refuted by loophole-free experiments. The ones that are not-Local and not-random are refuted for they allow faster-than-light signaling. The set of refuted theories is painted in black. Not-Local (but random and Real) theories are painted in orange; they support “quantum certified” randomness. Not-Real (but Local and Ergodic) theories are painted in green (both light and dark) and correspond to the Copenhagen interpretation of QM. Not-Ergodic (but Real and Local) theories are painted in violet. Note they cannot be random.](image-url)
⇒ predictable (partially, at least) ⇒ not random, for all reasonable definitions of “random”. Inverting the logic implication, then: random ⇒ ergodic. Be aware that this is valid for bounded systems (finite phase space) only. Otherwise, the random walk (which is a not-ergodic evolution) is a counter-example.

In the Bell’s experiment, the sequence of measurement outcomes is the main observable of the system’s evolution. It provides a natural way to define the properties of a theory describing that experiment. Then, “random” theories are the ones that predict such sequence to be random. Drawing the set of these theories in Fig.1 leads to interesting conclusions. Not-Local and not-random theories allow faster-than-light signaling and must be rejected [17]. This is the black area additional to the above mentioned “triangle”. The theories in the orange painted set are not-Local, Real, Ergodic and random, and are the realm of quantum certified randomness. A lot of recent research activity is based on this possibility, so that deciding whether it is false, or not, is of utmost importance nowadays.

Not-Real theories correspond to the Copenhagen interpretation of QM and are included in the green areas. Strictly speaking, the Copenhagen interpretation says nothing about the randomness of a sequence of outcomes of successive measurements made on a set of identically prepared states. QM allows calculating probabilities, but not the features of the time sequence that underlies the actual measurement of such probabilities. The only explicit definition on this issue I know is von Neumann’s axiom, which states that quantum measurements violate Leibniz’s principle of sufficient reason. In other words: a quantum measurement produces one or another outcome without cause. A sequence of such outcomes is intuitively random, although this intuition is difficult to formalize [18]. Besides, von Neumann’s axiom can be understood in two ways, or strengths. Its “strong” form means that Leibniz’s principle is violated in Nature at the quantum scale. The “weak” form means that the axiom is part of a user’s guide or warning about what QM can or cannot predict, but not necessarily a property of Nature. Therefore, we don’t know if (always according to the Copenhagen interpretation, which denies Realism) the said sequences are random (light green area in Fig.1) or not (dark green area).

Finally, theories that violate Bell’s inequalities by being not-Ergodic are indicated with the violet area in Fig.1. Note that they must produce not-random sequences.

Consider now a typical Bell’s experiment, Figure 2. Assume the source S emits biphotons in the fully symmetrical Bell state $|\psi^+\rangle = 1/\sqrt{2}(|x_A x_B\rangle + |y_A y_B\rangle)$ in square pulses of duration about twice longer than $L/c$, where $L$ is the distance between stations A and B and $c$ is the speed of light. In each station, polarization analyzers are set at angles $\alpha, \beta$. The settings are changed “randomly” while the biphotons are in flight from the source to the stations, but they are left static during the pulse. If a detection occurs at the transmitted (reflected) port, a “0” (“1”) is written in a recording device together with the time it occurred (time stamping, or time tagging). After the experiment has ended, data processing eliminates detections that are not simultaneous with detections in the other station (that is, only coincidences are of interest). The pulses are assumed well separated in time. After many pulses, binary sequences of coincidences are recorded at each station. If $\alpha=\beta$ the sequences at both stations are identical, and anti-correlated if $\alpha \neq \beta$ (this is the basis of QKD). To keep low the rate of accidental coincidences the source intensity must be adjusted such that the probability $p$ to observe a detection per pulse is $p<<1$ [19]. F.ex., if $p=0.1$ and pulse repetition rate is 1 MHz, sequences 6 Mbits long are recorded at each station in a run lasting 300s. If the stations are separated 20m, then the pulse duration is $\approx 120$ ns and duty cycle $\approx 12\%$.

Assume now that the violation of Bell’s inequalities is observed to be constant along the pulse duration, as it was reported f.ex. in [5]. During the pulses’ first half the detections are spatially isolated and, if the detectors are efficient enough, the loophole-free condition is valid (Fig.3). Therefore, the violation is possible only because Locality, or Realism, or Ergodicity is false during this period. During the pulses’ second half instead, there has been enough time for the stations to communicate, the loophole-free condition is no longer valid and the violation is possible even if the three properties are all true. The recorded sequences can be separated in two: the ones recorded during the pulses’ first half, and the ones recorded during the second half. Of course, if sufficient statistics are available, the pulse can be sliced in more parts to get better time resolution.

If the violation of Bell’s inequalities observed during the first pulses’ half occurs because Locality (but not Realism and Ergodicity) is false, then we are in the condition of quantum certified randomness (orange area in Fig.1). The binary sequences recorded during this period must be “pure random”. Instead, the binary sequences recorded during the pulses’ second half may be or may be not random. If we had a “randometer” and plotted the variation of randomness in time, we would observe a maximal value during the pulses’ first half, and then some decay (Fig.3). Such decay is expected because the level of randomness

Figure 2: Sketch of a Bell’s experiment with a pulsed source of biphotons and remote stations. Detections produce binary time series at stations A and B.
Locality is false. But, once again, complexity can not be measured. It can only be estimated as a rate of compression, and the result depends on the compressor algorithm chosen. Another approach has been mentioned some lines before: to compute how many sequences (in a large set) are rejected by standard tests of randomness. This is because a sequence cannot be demonstrated random, but it can be demonstrated not-random. Pragmatically, a smaller rate of rejected sequences means a higher level of randomness. Therefore, if the rate of not-random sequences obtained in the first half of a pulse of duration ≈2L/c is larger (or smaller, or equal) than in the second half, then one gets some evidence that Ergodicity (or Locality, or Realism) is false, provided of course that the violation of Bell’s inequalities is observed all through the pulse.

Measuring the rate of rejected sequences suffices for a RNG user to get a device’s practical evaluation of reliability, but it is arguable as a proof in a fundamental discussion involving the falsity of Locality, Realism or Ergodicity. For this reason, I prefer to say here that the observation of a variation (or not) of the rate of rejected sequences may provide just some evidence of the falsity of one of the three properties. Nevertheless, the outcome of the proposed experiment would have an immediate practical impact, even if the foundational issues remained not fully decided. If the experiment shows that the rate of rejected sequences increases with time, then QKD is safer if pulses shorter than L/c are used to generate the key. If it shows that the said rate decreases instead, the final part (time > L/c) of long pulses should be preferred. And, if it shows that the said rate is constant, then both the pulse duration and the pulse’s section used are irrelevant. Analogous conclusions are valid for the best way to operate a quantum based RNG.

Some comments on technical issues: in the form it is described, the proposed experiment is unattainable nowadays. Due to typical detectors’ efficiencies, Bell’s inequalities can be disproved with photons only by using Eberhardt’s states, which produce strongly biased sequences. Extraction methods are applied to get acceptable random sequences [24,25], but they may conceivably mask the phenomenon one wants to detect. Setups using entanglement swapping between photons and atoms do use Bell states, but produce a rate of detections too low to be practical. Besides, a true logical loophole may be lurking in these setups [12]. A simple solution at hand is to accept the fair sampling assumption [1] valid. The set of coincidences is assumed to be an unbiased statistical sample of the whole set of detected and not-detected biphotons, and hence Bell states and standard detectors can be used.

Another technical problem is achieving fast and random setting changes. In addition to the difficulty of fastness (flips must be completed usually in ≈10⁻⁸ s), there is the logical problem (a sort of infinite regress) of performing random setting changes. A solution to both difficulties is to assume that any hypothetical correlation between the stations vanishes when the biphotons’ source is turned off. An experimental basis supporting this assumption is that the curve of the S_CHS parameter, as the time coincidence window is increased, decays following exactly the curve predicted if the detections recorded outside the pulse are fully uncorrelated [5]. Note that non-correlation implies the curve but, of course, observing the curve does not necessarily imply the non-correlation. If the latter...

Figure 3: Up, left: pump pulse shape; for times t< L/c (or > L/c) the loophole-free condition is (is not) valid. Up, right: sketch of the time evolution of the reading R of a hypothetical randommeter applied to sequences produced in the setup in Fig.2 if Locality is false. Down, left (right): the same, if Ergodicity (Realism) is false.

If the violation of the Bell’s inequalities observed during the pulses’ first half occurs because Realism (but not Locality and Ergodicity) is false instead, then we are in the realm of the Copenhagen interpretation ofQM (green regions in Fig.1). The binary sequences may be random or not, but this is so regardless they are recorded in the first or the second half. In the average over many experimental runs, the randommeter would display a constant value (no matter if high or low).

Finally, if the violation of the Bell’s inequalities observed during the pulses’ first half occurs because Ergodicity (but not Realism and Locality) is false, then we are in the violet region in Fig.1. The binary sequences recorded during the pulses’ first half must be not-random. Instead, the sequences recorded during the second half may be or may be not random. The randommeter would plot a low value during the pulses’ first half, and then an increase (Fig.3). Such increase is expected for the same reason of the decay in the case Locality is false.

Unfortunately, randommeters do not exist. A possible approach is to calculate the sequences’ Kolmogorov complexity. But, once again, complexity cannot be properly measured. It can be only estimated as a rate of compression, and the result depends on the compressor algorithm chosen. Another approach has been mentioned some lines before: to compute how many sequences (in a large set) are rejected by standard tests of randomness. This is because a sequence cannot be...
implication is accepted true, then fast and random settings’ changes become unnecessary.

Under these two assumptions (“fair sampling” and, say, “uncorrelated when turned off”) the proposed experiment is achievable even with modest means. Although the results obtained in these conditions may be not decisive, they may give a clue about whether the effort to solve the problems of the complete experiment is worthy, or not. Besides, information of practical interest about the best use of quantum based RNG and the security of QKD would be immediately obtained.

In summary: by studying time evolution of randomness of sequences produced in a suitable Bell’s experiment, it is possible to get some evidence about which one of three fundamental properties (Locality, Realism or Ergodicity) is false in Nature. The result would also have practical impact on quantum based RNG and QKD.

Appendix: About Ergodicity.

Ergodicity is not necessary to derive Bell’s inequalities, but it is unavoidable to insert measured data into their mathematical expressions. Recall that in the derivation of (any of the many forms of) Bell’s inequalities, but it is unavoidable to insert measured data into their mathematical expressions. Recall that in the derivation of (any of the many forms of) Bell’s inequalities, the average over the space of hidden variables \( \lambda \) is done to get observable probabilities. F.ex. the probability of observing single photon detection in station A after a polarization analyzer oriented at angle \( \alpha \) is:

\[
P^+_{\lambda}(\alpha) = \int d\lambda \, \rho(\lambda). P^+_{\lambda}(\alpha, \lambda) = <P^+_{\lambda}>_{\lambda},
\]

(A1)

where \( \rho(\lambda) \) and \( P^+_{\lambda}(\alpha, \lambda) \) are well behaved density and probability functions. This is a weighted integral over the system’s space of states, or ensemble average. But, what is actually measured is:

\[
P^+_{\lambda}(\alpha) = (1/T) \int dt \, \rho(t). P^+_{\lambda}(\alpha, t) = <P^+_{\lambda}>_t,
\]

(A2)

which is an integral between values \( t \) and \( t+T \), a time average. The equality between these two averages is the ergodic property. Its validity allows experimental data to be meaningfully inserted into Bell’s inequalities. Strictly speaking the needed property is weaker, but the difference is unimportant here. The interested Reader may find details in, f.ex., [14].

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