The nature of synchronization in power systems: a revelation from communication theory

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The large-scale integration of converter-interfaced resources in electrical power systems raises new stability threats which call for a new theoretic framework for modelling and analysis. Here we present the theory of power-communication isomorphism to solve this grand challenge. It is revealed that an intrinsic communication mechanism governs the synchronisation of all apparatus in power systems based on which a unified representation for heterogeneous apparatus and behaviours is established. We develop the mathematics to model the dynamic interaction within a power-communication isomorphic system which yield a simple stability criterion for complex systems that can be intuitively interpreted and thus conveniently applied in practice.

Driven by the trend of decarbonisation and clean growth, the primary energy of power systems is transforming from fossil fuels to renewable resources. The change of the primary energy is accompanied by the change of technologies for power generation and conversion. Renewable resources, mainly wind and solar energy, are interfaced to power systems by power electronic converters instead of conventional synchronous generators (SGs). The increasing penetration of converter-interfaced resources (CIRs) poses new threats to system stability. Converter-induced oscillations have been reported worldwide, many of which had major consequences. For example, the 2019 power outage in UK was triggered by the sub-synchronous oscillation of wind turbine converters in Hornsea wind farms according to report provided by National Grid UK [1]. Such converter-induced oscillation phenomena are beyond the prediction of state-of-the-art stability models and the underpinning mechanisms are not fully understood, which have drawn international attention.

The stability of a power system is defined as the ability to keep all apparatus in the system synchronised to a single frequency with expected power flows throughout the system [2], [3]. The classic stability theory for power systems is tailor-made for synchronous generators based on their self-synchronising capability which is governed by the physical law of the motion of rotors. However, converters are governed by control algorithms, which gives rise to almost infinite flexibility, and therefore complexity, in converter behaviours [4]. New synchronisation and power control schemes have been developed for converters to serve different design targets, such as reducing the sizing of power semiconductors and capacitors and improving harmonic distortions. The design targets are local oriented and the systemic impacts of these new schemes are unclear. It is extremely difficult to establish a system-compatible representation for the heterogeneous control-defined behaviours in a single model. Studies on converter-induced instabilities have been limited to specific cases and there have been no generic conclusions given nor consensus reached.

To solve this grand challenge, we look back to the nature of synchronisation in power systems to find the common principles for all apparatus and behaviours. We revealed that there is an intrinsic communication mechanism underlying power systems. The voltages and currents in power systems carry both energy and information, and the corresponding power flows and communication traffics are governed by the same principle, which we called power-communication isomorphism. Based on this isomorphism, we establish a unified phase locking mechanism for heterogeneous apparatus and behaviours so that they can be integrated into a systemic synchronisation model. We conceptualise the channel-frequency shift effect to illuminate the interactions of the base-band, carrier, and channel dynamics in the power-communication isomorphic system. This yield a surprisingly simple stability criterion for very complex power systems. The criterion is directly linked to the fundamental properties of apparatus and the graphical connectivity of networks, and thus provides a powerful tool to guide apparatus design and system operation.

**Power-Communication Isomorphism**

The concept of power-communication isomorphism is illustrated in Fig. 1. The voltages and currents in a power system are seen as communication signals carrying both energy and information. The power apparatus, including generators and converters, serve as modulators to create three-phase sinusoidal signals from the amplitude and angle of internal oscillators. The modulated signals are transmitted over a power network that serves as communication channels, and then demodulated by receiving apparatus via complex powers. Complex powers are associated with the angle differences of the transmitting and receiving apparatus, so power control has an equivalent effect of phase locking.

A three-phase sinusoidal signal in a three-phase balanced power system is represented as a complex number [5]

\[ A(\sin \theta + j \cos \theta) = Ae^{j\theta} \]  

(1)

where \( j = \sqrt{-1} \), and \( A \) and \( \theta \) are the amplitude and angle of the signal. The amplitude-angle can be jointly written as a complex angle \( \vartheta \)

\[ \vartheta = \ln A + j \theta, \quad e^{\vartheta} = Ae^{j\theta}. \]  

(2)

The time-derivative of the complex angle is called the complex frequency

\[ \omega = \dot{\vartheta} = A^{-1} \dot{A} + j\omega \]  

(3)
The real part $A^{-1} \dot{A}$ reflects amplitude variation, and the imaginary part $\omega = \hat{\theta}$ is the frequency in ordinary sense reflecting angle variation.

In a power system, the frequencies of signals are centred around the fundamental frequency $\omega_0$ (either 50 Hz or 60 Hz depending on regions) with minor deviations. The amplitudes of signals (voltages or currents) vary according to power flows. Depending on regions) with minor deviations. The amplitudes

Channel-Frequency Shift

The communication channels underlying power networks are linear systems. We first consider a first-order system represented by a first-order linear differential equation in time domain:

$$\frac{de^\theta}{dt} = ae^\theta + be^\theta$$  \hspace{1cm} (5)

where $e^\theta$ and $e^\hat{\theta}$ are the input and output of the channel, and $a$ and $b$ are channel parameters. We define the ratio of $e^\hat{\theta}$ and $e^\theta$ as the dynamic gain of the channel in time domain:

$$g = \frac{e^\hat{\theta}}{e^\theta} = e^{\hat{\theta} - \theta}.$$  \hspace{1cm} (6)

Combining (5) and (6) yields the differential equation for $g$

$$\dot{g} = (a - \omega)g + b.$$  \hspace{1cm} (7)

Thus we transform the dynamics of the signal to the dynamics of the channel gain. The channel gain is affected by the complex frequency $\omega$ of the signal being transmitted over the channel, which is named the channel-frequency shift effect.

Taking into account the channel gain, the complex power in (4) becomes

$$\hat{S}_{mn} = e^{\hat{\theta}_n}e^{\hat{\theta}_m} = g_{mn}e^{\theta_n}e^{\theta_m^*} = g_{mn}S_{mn}$$  \hspace{1cm} (8)

where $g_{mn}$ is the channel gain from the $n$-th node to the $m$-th node in the power network, $S_{mn}$ is the complex power transmitted from the $n$-th node to the $m$-th node over the channel, and $S_{mn}$ is the complex power without the channel effect. The total complex power received at $m$-th node is the sum of $\hat{S}_{mn}$ transmitted from all nodes

$$\hat{S}_m = \sum_n \hat{S}_{mn} = \sum_n g_{mn}S_{mn}.$$  \hspace{1cm} (9)

We now get the overall mathematical representation for the power-communication isomorphic system and summarise it in Fig. 2. For a power system containing $N$ active nodes, each node injects a signal $e^{\theta_n}$ into the power network which is de-modulated by another node $m$ to yield $S_{mn}$, where $n, m \in \{1, 2, \cdots, N\}$. $S_{mn}$ passes through the channel gain $g_{mn}$ to yield $\hat{S}_{mn}$. All traffics in the network share the channels according to the superposition principle, so the total complex power received at node $m$ is the summation of $\hat{S}_m = \sum_n \hat{S}_{mn}$. The complex power $\hat{S}_m$ is fed to a phase locking (power control) scheme to generate the internal complex frequency $\varpi_m$ and complex phase $\theta_m$. The complex phase $\theta_m$ is in turn modulated to signal $e^{i\theta_m}$. The complex frequency $\varpi_m$ determines the channel gain $g_{mn}$ due to channel-frequency shift.

The channel-frequency shift provides a convenient perspective to illuminate the interaction of base-band signals with the carrier and channels. We use the perturbation of complex angles and frequencies to represent base-band signals and find the corresponding perturbation of channel gain and complex power by linearising (7) and (8):

$$\Delta g_{mn} = \frac{g_{mn0}}{j\varpi_n - a} \cdot F \cdot \Delta \varpi_n$$

$$\Delta \dot{S}_{mn} = \dot{\hat{S}}_{mn0}(\Delta \varpi_m^* + F \cdot \Delta \theta_n)$$  \hspace{1cm} (10)
where the prefix $\Delta$ and subscript $0$ denote the perturbation and equilibrium of a dynamic variable, and $F$ is a low pass filter

$$F(s) = \frac{j\omega_0 - a}{s + j\omega_0 - a}. \quad (11)$$

The channel-frequency shift only applies on the transmitting end which induces asymmetry in base-band signal propagation, as is clear to see from (10). The angle perturbation $\Delta \varphi_m$ at the receiving end $m$ affects the complex power $\Delta S_{mn}$ instantaneously, whereas the angle perturbation $\Delta \varphi_n$ at the transmitting end $n$ passes through a low-pass filter $F$ before affecting $\Delta S_{mn}$. Thus the bandwidth of $F$ sets the limit speed of base-band traffics on the channel. The bandwidth of $F$ is determined by the carrier frequency $\omega_0$ as well as the channel parameter $a$.

Within the channel bandwidth, the channel gain $g$ responds almost instantaneously to $\varpi$. In such a case, we can ignore the dynamic part of $g$ by letting $g = 0$ in (7) which yields

$$g \approx \frac{b}{\varpi - a}. \quad (12)$$

If a channel is a high-order system, we use a transfer function $G$ to represent its dynamics and factorise $G$ to a series of first-order systems

$$G(s) = \sum_k \frac{b_k}{s - a_k}. \quad (13)$$

Each of the factors induces a channel gain $g_k$ and the total gain is their summation

$$g = \sum_k g_k \approx \sum_k \frac{b_k}{\varpi - a_k} = G(\varpi) \quad (14)$$

which implies that the channel gain $g$ is approximated within the channel bandwidth by letting $s = \varpi$ in the transfer function $G(s)$.

**Unified Phase Locking Mechanism**

Phase locking plays a central role in the synchronisation of power systems. The power-communication isomorphism establishes the linkage between phase locking and power control and therefore allows for a unified view for different phase locking and power control schemes. Following (8) and (9) we rewrite $\dot{S}_m$ as

$$\dot{S}_m = g_{mm} e^{\theta_m + \bar{\theta}_m} + g_{mn} e^{\theta_n} \sum_{n \neq m} g_{mn} e^{\bar{\theta}_n}$$

$$= A_m^2 g_{mm} + A_m A_m e^{j(\bar{\theta}_m - \theta_m)} \quad (15)$$

where we define $A_m e^{j\theta_m} = \sum_{n \neq m} g_{mn} e^{\bar{\theta}_n}$ to represent the average amplitude and angle of signals transmitted from all other nodes to node $m$. The goal of phase locking is to generate $\bar{\theta}_m$ according to $\bar{\theta}_m$ so that $\theta_m$ is aligned with $\bar{\theta}_m$ with an expected angle difference $\delta_m = \theta_m - \bar{\theta}_m$ according to the expected power flow. $\dot{\bar{S}}_m$ is a complex number: $\bar{S}_m = P_m + jQ_m$, where $P_m$ and $Q_m$ are called real power and reactive power respectively. $\delta_m$ can be estimated by either $P_m$ and $Q_m$ but they have different sensitivities to $\delta_m$

$$\frac{\partial P_m}{\partial \delta_m} = -A_m A_m \sin \delta_m, \quad \frac{\partial Q_m}{\partial \delta_m} = A_m A_m \cos \delta_m. \quad (16)$$

$P_m$ is sensitive to $\delta_m$ around $\pi/2$ and $Q_m$ is sensitive to $\delta_m$ around $0$. This yields two types of phase locking schemes, as illustrated in Fig. 3 (a)-(b).

The first scheme compares the reference real power $P^*$ with the reference real power $P^*$ and uses the difference $P - P^*$ to control the acceleration or deceleration of $\omega$ [2]. The controller serves as the accumulator of real power and therefore is equivalent to a real inertia, which is embodied by energy storage components such as flywheels or capacitors. The real-power-based phase locking scheme is used in synchronous generators as well as virtual synchronous generators mimicked by converters.

The second scheme uses the reactive power difference $Q - Q^*$ to generate $\omega$ and the corresponding controller accumulates reactive power and is equivalent to reactive inertia. Since the reactive energy is imaginary, no real energy storage component is needed. The reactive-power-based phase locking scheme is used in current-controlled converters and is usually called a phase-locked loop (PLL) [6]. It is noted that there is a minor variant in the PLL used in practice that the imaginary part of the complex voltage signal, called the quadratic voltage $v_q$, is used instead of $Q$ for phase locking. $v_q$ is equivalent to $Q$ rescaled by the magnitude of current which can be counted into the reactive inertia.
The phase locking schemes based on real power and reactive power can be unified by defining hybrid power as below
\[ W = \text{Re}(e^{-j\varepsilon} \cdot \hat{S}) \] (17)
where \( \varepsilon \) is an displacement angle. The accumulator of hybrid power difference \( W - W^* \) is called hybrid inertia, as illustrated in Fig. 3 (c). If \( \varepsilon = 0 \), \( W = P \); if \( \varepsilon = \frac{\pi}{2} \), \( W = Q \). The displacement angle \( \varepsilon \) can be set to other values complementary to \( \delta \) so that \( W \) has a high sensitivity to \( \delta \). Thus we establish a unified phase locking mechanism that covers existing schemes and creates new schemes.

**Rethinking Power System Stability**

We now have all the ingredients ready to rethink power system stability from the perspective of power-communication isomorphism. The topology of a power system is visualised by a graph in Fig. 4 (a). A node \( m \) in the graph represents an apparatus in a power system and the weight \( H_m \) of a node is the hybrid inertia of the apparatus. There are two types of nodes, among which a voltage node transmits a voltage signal and a current node transmits a current signal and receives a voltage signal. A voltage node represents a synchronous-generator-like apparatus and creates new schemes.

![Fig. 3. Unified phase locking mechanism. (a)-(c): Phase locking by real, reactive and hybrid power. The power-angle sensitivity is visualised by the pink shades (representing angle perturbation) projecting on the \( P \), \( Q \) and \( W \) axes (representing the associated power perturbation).](image)

![Fig. 4. Interconnection structure of a power system: (a) graphical topology and (b) synchronisation dynamics.](image)

**Synchronisation Dynamics**

The synchronisation dynamics in Fig. 4 (b) contains two loops and the total loop gain combining Loop 1-2 is
\[ [T_{L,12}] = [\Gamma_H]\left(\frac{1}{T} [I] + \frac{1}{\delta} [\xi]\right)^{-1} \] (20)
where \([I]\) is the unit matrix, \([\xi]\) and \([\Phi]\) are the eigenvalue and eigenvector matrices of \([K_H]\), and \([\Gamma_H]\). The synchronisation model so \([\Delta \theta] = j[\Delta \theta]\) and \([\Delta \omega] = j[\Delta \omega]\). The angle \([\Delta \theta]\) and frequency \([\Delta \omega]\) affect the hybrid power \([\Delta W]\) due to modulation-demodulation and channel-frequency shift, which yield the matrices \([K] \) and \([\Gamma]\) in the model. \([\Delta W]\) is fed to the inertias \([H]\) to generate \([\Delta \omega]\) via a transfer function \( T \) representing the inertia dynamics, as per the unified phase locking mechanism.

\([K]\) and \([\Gamma]\) are given by linearising (9), (14), and (17) around the equilibrium
\[ K_{mn} = \begin{cases} -|S_{mn}| |G_{mn}(j\omega_0)\sin \kappa_{mn}, & \text{for } n \neq m \\ - \sum_{l \neq m} K_{ml}, & \text{for } n = m \end{cases} \]
\[ \Gamma_{mn} = -|S_{mn}| |G'_{mn}(j\omega_0)| \sin \gamma_{mn} \] (18)

where
\[ \kappa_{mn} = \varepsilon_m - \angle S_{mn0} - \angle G_{mn}(j\omega_0), & \text{for } n \neq m \]
\[ \gamma_{mn} = \varepsilon_m - \angle S'_{mn0} - \angle G'_{mn}(j\omega_0), & \text{for } n \neq m \] (19)

where the prefix \( \angle \) donates the phase angle of a variable, and \( G'(s) \) denotes the s-derivative of \( G(s) \). It is noted that \( K_{mn} \) equals the static channel gain \( |G_{mn}(j\omega_0)| \) multiplied by the power flow over the channel, and therefore \([K]\) is defined as the loaded-channel matrix. We further define \([K_H] = [H]^{-1}[K]\) as the inertia-channel matrix to represent the channel gain scaled by inertias.
is the inertia-scaled channel-frequency shift matrix in the coordinates of $[\Phi]$. We give the following sufficient stability criterion by invoking the small-gain theorem [7]:

$$
\zeta_m = \inf_{\text{Re}(s)>0} \left| \frac{\xi_m + s/T(s)}{s} \right| > \sigma_{\text{max}} \quad (21)
$$

for all $m \in \{1, 2, \cdots, N\}$, where $\xi_m$ is the $m$-th eigenvalue of $[K_H]$ and $\sigma_{\text{max}}$ is the maximum singular value of $[\Gamma_H\Phi]$. $\mathcal{F} = \{-s/T(s) \mid \text{Re}(s) > 0\}$ is call the forbidden region and $\zeta_m$ is interpreted as the distance between $\xi_m$ and $\mathcal{F}$ scaled by $s$. The maximum of $\zeta_m$ is called $\zeta_{\text{max}}$, and $\zeta_{\text{max}} - \sigma_{\text{max}}$ is defined as the stability margin for the whole system.

The criterion (21) has important implications. $\xi_m$ indicates the graphical connectivity mode of channels under power flows and inertia scaling. $\zeta_m$ indicates the interaction of channel connectivity with inertia dynamics. $\sigma_{\text{max}}$ quantifies the potential destabilising effect caused by channel-frequency shift. All channel connectivity modes should be strong enough and the inertia dynamics should be properly damped to keep $\zeta_m$ well above $\sigma_{\text{max}}$ to counteract the channel-frequency shift for sufficient stability margin.

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