INVESTIGATION OF A 2-COLOUR UNDULATOR FEL USING PUFFIN

L.T. Campbell¹², B.W.J. McNeil¹ and S. Reiche³
¹SUPA, Department of Physics, University of Strathclyde, Glasgow, UK,
²ASTeC, STFC Daresbury Laboratory and Cockcroft Institute, Warrington WA4 4AD, United Kingdom
³Paul Scherrer Institute, 5232 Villigen PSI, Switzerland

Abstract

Initial studies of a 2-colour FEL amplifier using one mono-energetic electron beam are presented. The interaction is modelled using the unaveraged, broadband FEL code Puffin. A series of undulator modules are tuned to generate two resonant frequencies along the FEL interaction and a self-consistent 2-colour FEL interaction at widely spaced non-harmonic wavelengths at 1nm and 2.4nm is demonstrated.

INTRODUCTION

With X-ray SASE FEL’s now in operation around the world, there is now user interest in the simultaneous delivery of two distinct wavelengths. This may be possible by preparing the beam before injection into the undulator, or by using two electron beams of different energy, known as a two-beam or two-stream FEL. Another method, the so-called 2-colour FEL, generates two radiation wavelengths simultaneously from a single mono-energetic electron beam by injecting it through alternative undulator modules that are tuned to the two different wavelengths. There may also be a need for more broadband broadband sources, which may be achievable using an energy chirped electron beam in a 2-colour FEL configuration, and matching together the spectra generated by each of the undulator modules.

Averaged FEL simulation codes such as [1 2 3 4] that make the Slowly VaryingEnvelope Approximation (SVEA) [5], can readily model 2-colour FEL interactions when the radiation wavelengths are about a fundamental wavelength λ₁ and e.g. its third harmonic λ₃ = λ₁/3. This requires the radiation to be described by two distinct computational fields as the field sampling at any resonant radiation wavelength λᵣ means the frequency range able to be sampled is limited by the Nyquist condition to ŵᵣ/2 < ω < 3ωᵣ/2. Furthermore, the accuracy of the model decreases the further away from the central resonant frequency ωᵣ. For a fundamental and say its third harmonic λ₃ = λ₁/3. This requires the radiation to be described by two distinct computational fields as the field sampling at any resonant radiation wavelength λᵣ means the frequency range able to be sampled is limited by the Nyquist condition to ŵᵣ/2 < ω < 3ωᵣ/2. Furthermore, the accuracy of the model decreases the further away from the central resonant frequency ωᵣ. For a fundamental and say its third harmonic λ₃ = λ₁/3.

MATHEMATICAL MODEL

Puffin uses a system of equations which utilizes scaled variables developed in [11]. These dimensionless variables are scaled with respect to the FEL parameter, defined as

\[ \rho = \frac{1}{\gamma k} \left( \frac{\bar{a}_w \omega_p}{4ck_w} \right)^{2/3}, \]

where \( \bar{a}_w \propto B_w \lambda_w \) is the usual rms undulator parameter, \( B_w \) is the rms undulator magnetic field strength, \( \lambda_w \) is the undulator period and \( k_w = 2\pi/\lambda_w \). In the cases simulated in this paper, the undulator tuning of \( \bar{a}_w \) is obtained by varying the rms undulator magnetic field alone. This tuning therefore also changes the FEL parameter and it is convenient to re-scale the equations in a general way for the different undulator module tunings by introducing the undulator module dependent parameter \( \alpha \equiv \bar{a}_w / \bar{a}_w 1 \), where the sub-script ‘1’ refers to the value of \( \bar{a}_w \) of the first module. By similarly re-defining

\[ \rho = \alpha^{2/3} \rho_1, \]
again where sub-script ‘1’ refers to the values of the first module, the parameter \( \alpha \) becomes an explicit parameter in the equations. A constant focussing channel, with strength expressed as a fraction of the natural undulator focussing which is a function of \( \tilde{a}_w \) \([6]\), maintains a constant electron beam radius for different modules by varying the focussing factor so that \( f = \alpha f_1 \). The net effect of these scalings maintains the exact form of the scaled equations of \([6]\) with the exception of two multiplicative factors of \( \alpha \) in the equations describing transverse and longitudinal motion of the electrons. The system of equations obtained is general, and \( \alpha \) can be made any function of distance through the interaction as required. It could, for example, be used to taper the undulator in Puffin, or to provide any number of differently tuned undulator modules. For the purposes of this paper, however, \( \alpha \) is varied between only two values corresponding to the two colours of radiation required. Further details of the scaling will be described elsewhere.

It is also noted here that in the 2-colour FEL, with only one resonant wavelength interaction occurring in each undulator module, radiation at the non-resonant wavelength will effectively propagate in free space. Two effects can be expected from this. Firstly, the non-resonant wavelength in an undulator module will undergo free-space diffraction (unless it is an harmonic when some guiding can occur \([4]\)), which will reduce the net coupling of the radiation to the electron beam and so increase the effective gain length of the interaction for both colours. Secondly, the addition of extra relative slippage between the radiation and the electrons in the non-resonant undulator modules may affect the spectrum by introducing modal effects \([12]\) and may also reduce the radiation bandwidth \([13][4][5]\).

**SIMULATION RESULTS**

Simulation results using this modified version of Puffin are now presented that model a 2-colour FEL interaction using a single electron beam in sequential undulators tuned to two different wavelengths. The first example is seeded from some temporally coherent external laser source at both frequencies - this results in a simpler (cleaner) electron beam phase space. The second example starts up from the electron beam shot-noise i.e. 2-color SASE. Radiation output spectra for differently tuned 2-colour FELs are then summarised.

In a 2-color FEL, there will be a small drift section between each undulator module that may require radiation/electron beam phase matching using small tuning chicanes. Although Puffin is capable of modelling the drift sections none are modelled here and an instantaneous change in undulator parameter is applied. Puffin is also used here in the 1D limit as described in \([6]\), so that simulations cost considerably less computational effort and the 2-colour interaction can be observed in its simplest form. More detailed effects including diffraction, that can be expected to alter the optimum set-up, and will be investigated in future work.

![Figure 1: Modular undulator layout for 2-colour operation.](image)

Table 1: Table of parameters for 2-colour simulations

| SASE | SASE scan |
|------|-----------|
| \( \rho \) | 0.005 | 0.005 | 0.001 |
| \( \gamma \) | 6300 | 6300 | 6900 |
| \( \sigma_\phi/\gamma \) | \( 10^{-4} \) | \( 10^{-4} \) | \( 10^{-4} \) |
| \( k_2 \) | \( 1.12 \times 10^{-2} \) | \( 1.12 \times 10^{-2} \) | \( 1.12 \times 10^{-2} \) |
| \( N_m \) | 6 | 13 | 13 |
| \( N_w \) | 25 | 25 | 100 |
| \( \alpha_1 \) | 1 | 1 | 1 |
| \( \alpha_2 \) | 1.946 | 1.946 | scan |

**Seeded Example**

Equation (2) defines how \( \rho \) changes with different undulator tunings. At longer wavelengths (larger \( \tilde{a}_w \)) a shorter gain length (at least in the 1D limit) results in saturation in a shorter interaction length. The simplest FEL configuration will have undulator modules with a fixed number of periods, limiting somewhat the options for driving both wavelengths to the same intensity. To compensate here for the difference in gain length of the two colours, the \( N_m = 6 \) undulator modules are alternated in the configuration of Fig. 1. This allows both wavelengths to receive gain as the interaction progresses unlike other options that amplify each wavelength in two consecutive interactions. The method used here stops the longer wavelength radiation dominating the interaction at the initial stages by inducing electron energy spread that inhibits amplification of the shorter wavelength, longer gain length interaction. It can be expected from the larger \( \rho \) value for the longer wavelength, and the equal total interaction length for both wavelengths, that the longer wavelength will have the larger saturation intensity \([11]\). It is preferable, therefore, that the shorter wavelength interaction, which induces a smaller electron energy spread than that at the longer wavelength, saturates first, thereby allowing the longer wavelength to extract more energy in the final undulator modules. For this reason it is preferable to end the interaction using undulator modules resonant at the longer wavelength.

The parameters for the first seeded case are shown in Table 1. A small seed field is injected at each resonant frequency, and the electron beam has a “flat-top” current profile. In the scaled \( \tilde{z} \) frame, the resonant wavenumbers are \( k_1 = 100 \) and \( k_2 \approx 41.7 \). A bunching parameter for each wavelength can be defined as:

\[
b_{1,2} = \frac{1}{L_{1,2}} \sum_{k=1}^{N_k} \tilde{\chi}_k e^{i k_{1,2} z_{2k}}
\]

where \( N_k \) is the number of macroparticles, and \( \tilde{\chi}_k \) is the...
Figure 2: Electron parameters at output of 3rd undulator module: Detail of electron phase space (top); modulus of bunching parameter at $\lambda_1$ (middle); modulus of bunching parameter at $\lambda_2$ (bottom).

Figure 3: Scaled radiation intensity (top) and spectrum (bottom) at output of 3rd undulator module.

The 2-colour FEL interaction is now modelled starting from noise via SASE. A similar, but extended, undulator beamline as Fig. 1 is used with a 1, 1, 2, 1, 1, 2, ..., 1, 1, 2, undulator module configuration, to allow for the build-up from noise. Otherwise all parameters are identical to the seeded case. Note the extra final undulator module resonant at the longer wavelength, as discussed above, that allows extraction of further longer wavelength energy from the beam that is saturated to further shorter wavelength emission. Plots similar to the seeded case of above are shown at the output of the $N_m = 13$ module beamline in figure 6. As the system now starts from noise the resultant beam phase space is more complicated but maintains similar character-
istics to the seeded case. The system is close to saturation and the beam is seen to have bunching components at both wavelengths simultaneously.

One factor that is present in the 2-colour SASE, that does not exist in the seeded case, is the potential for phase mismatching between the radiation and the electron beam due to the extra induced slippage in the alternate non-resonant undulator modules. This may be mitigated by using modules that are less than one gain length ensuring relative slippages are less than a cooperation length, so maintaining beam/radiation phase matching.

Similar 2-colour SASE simulations were also carried out for different wavelengths. This was achieved by using the extended configuration of Fig. 1 above, but for different values of $\alpha_2$. The FEL $\rho$ parameter has been reduced to $\rho = 0.001$ for this scan, and the number of undulator periods per module has been increased to $N_w = 100$ to allow for the longer gain length. The electron beam has similar parameters to a shorter wavelength FEL such as that on the Athos FEL proposed at SwissFEL [16]. The spectra of the scaled output intensities are plotted in Fig. 8 for a range of non-harmonic values of $\lambda_2$.

It is seen that the principle of operation of the 2-colour FEL applies to across a wide range of wavelengths. No attempt was made to optimise the output, so the output at a given wavelength can be expected to be improved upon. The statistical nature of the output is also unknown from these single-shot simulations.

**CONCLUSION**

The principle of a 2-colour FEL amplifier interaction using a single electron beam in an alternating series of differently tuned undulator modules was simulated for widely spaced, non-harmonic wavelengths. This modeling is only practical using non-averaged, broad bandwidth FEL simulation codes such as Puffin. Puffin was modified from that described in [6], to allow variation of the undulator magnetic field strength to change the resonant FEL wavelength. This allows Puffin to model not just a 2-color FEL, but any general variation in undulator field strength across a wide
range of resonant wavelengths as the interaction proceeds. Both seeded and SASE 2-colour operation were demonstrated to saturation and gave similar peak intensity spectra at both the resonant wavelengths. This simultaneous FEL lasing to saturation by one electron beam at two distinct, harmonically uncoupled, wavelengths is perhaps not an immediately intuitive result. However, the electron phase-space evolution has a rich structure that clearly demonstrates simultaneous electron bunching and emission at the two distinct wavelengths.

This result may open up other avenues for further research, for example in the generation of higher-modal radiation emission, or perhaps frequency mixing process from the FEL.

In the case of the 2-color SASE simulations, only a limited small of simulations were performed for each case so that the statistical nature of the 2-colour processes were not explored.

Other factors must be taken into account in future 3D simulations. Diffraction will clearly have an effect during ‘free’ propagation of the radiation in the non-resonant undulator modules. Another factor is the transverse beam matching to different undulator modules. This could become more problematic for larger differences in $\bar{n}_w$ between undulator modules and may limit the range of the 2-colour operation. Clearly, further examination in 3D is needed to identify and perhaps find solutions to such issues.

REFERENCES

[1] H.P. Freund, Phys. Rev. E, 52, 5401 (1995).
[2] S. Reiche, Nucl. Instr. and Meth. in Phys. Res. A 429, 243-248 (1999).
[3] E.L. Saldin et al, Nucl. Instr. and Meth. in Phys. Res. A 429, 233-237 (1999).
[4] W.M. Fawley, Proceedings of FEL 2006, BESSY, Berlin, Germany, 218-221 (2006).
[5] F.T. Arecchi and R. Bonifacio, IEEE J. Quantum Electron. QE-1, 169 (1965).
[6] L.T. Campbell and B.W.J. McNeil, Physics of Plasmas 19, 093119 (2012)
[7] N. Piovella, Phys. Plasmas 6, 3358 (1999)
[8] B.W.J. McNeil, G.R.M. Robb and D.A. Jaroszynski, Optics Comm. 165, 65 (1999)
[9] S.I. Bajlekov, S.M. Hooker and R. Bartolini, Proc. FEL 2009 Liverpool, MOPC42 (2009)
[10] C. Maroli, V. Petrillo and M. Ferrario, Phys. Rev. ST Accel. Beams 14, 070703 (2011)
[11] R. Bonifacio, C. Pellegrini and L.M. Narducci, Opt. Comm. 50, 373 (1984).
[12] N.R. Thomson and B.W.J. McNeil, Phys. Rev. Lett. 100, 203901 (2008)
[13] N.R. Thompson, D.J. Dunning and B.W.J. McNeil, Proceedings of IPAC10, Kyoto, Japan, TUPE050 (2010)
[14] Dao Xiang, Yuantao Ding, Zhirong Huang, and Haixiao Deng, Phys. Rev. ST Accel. Beams 16, 010703 (2013)
[15] B.W.J. McNeil, N.R. Thompson and D.J. Dunning, Phys. Rev. Lett., 110, 134802 (2013)
[16] SwissFEL Conceptual Design Report, PSI Bericht Nr. 10-04, (Ed. Romain Ganter), 2012