A comparative analysis of standard accretion discs spectra: an application to ultraluminous X-ray sources

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ABSTRACT

We compare standard models of accretion discs around black holes (BHs) that include the appropriate zero-torque inner boundary condition and relativistic effects on the emission and propagation of radiation. The comparison is performed adopting the multicolour disc blackbody model (MCD) as reference and looking for the parameter space in which it is in statistical agreement with ‘more physical’ accretion disc models. We find simple ‘recipes’ that can be used for adjusting the estimates of the physical inner radius of the disc, the BH mass and the accretion rate inferred using the parameters of the MCD fits. We applied these results to four ultraluminous X-ray sources for which MCD spectral fits of their X-ray soft spectral components have been published and find that, in three cases (NGC 1313 X-1, X-2 and M 81 X-9), the BH masses inferred for a standard disc around a Schwarzschild BH are in the interval $\sim 100-200\,M_\odot$. Only if the BH is maximally rotating are the masses comparable to the much larger values previously derived in the literature.

Key words: accretion, accretion discs – black hole physics – X-rays: binaries.

1 INTRODUCTION

Studying the spectra of X-ray binaries is one of the main tools for understanding the physics of these sources and determine the properties of the accretion disc and the compact object that they host. From an observational standpoint, Galactic black hole candidates (BHCs) show a number of spectral states, defined in terms of the components in their X-ray spectra and the flux level (e.g. McClintock & Remillard 2006; Remillard & McClintock 2006). In the so-called high state (Tananbaum et al. 1972; Oda 1977), the spectrum is dominated by a soft, thermal component up to 1–2 keV, with a power law emerging at higher energies. In the low state (Oda et al. 1971; Tananbaum et al. 1972) the soft component is not present and the spectrum is well fitted by a power law with a cut-off at a few tens of keV. Transitions to an intermediate state (Belloni et al. 1997; Méndez & van der Klis 1997) or a very high state (Miyamoto et al. 1991) are also observed in which both components are present and equally important in terms of the emitted flux. It is worth mentioning that this classification is based also on the timing properties of the sources (e.g. Méndez, Belloni & van der Klis 1998). It is largely accepted that the soft, thermal component originates from an accretion disc, while the power law is likely produced by Comptonization of the soft disc photons from a hotter phase of the accretion flow (usually referred to as corona). In these assumptions, in principle the properties of the disc and the physical parameters of the accreting black hole (BH) can be inferred from the study of the soft component.

Similar spectral states (e.g. Foschini et al. 2002; Feng & Kaaret 2005) and correlated spectral/flux variability (e.g. La Parola 2001; Mucciarelli et al. 2007) are also observed in ultraluminous X-ray sources (ULXs), very bright, point-like X-ray sources in the field of nearby galaxies (see e.g. Fabbiano 2006). In fact, several pieces of evidence indicate that the majority of ULXs are X-ray binaries, chiefly among them the detection of periodic modulations in the X-ray flux (e.g. Fabbiano et al. 2006; Kaaret, Simet & Lang 2006) and the identification of stellar optical counterparts (e.g. Liu, Bregman & Seitzer 2002, 2004; Kaaret, Ward & Zezas 2004; Mucciarelli et al. 2005). The spectra of several ULXs show a soft, thermal component, similar to that observed in BHCs, which has been interpreted as emission from an accretion disc, although for some ULXs with spectral curvature above 2–3 keV different spectral models have also been proposed (e.g. Gonalves & Soria 2006; Stobbart, Roberts & Wilms 2006; Mizuno et al. 2007).

To model the soft component in BHCs and ULXs the so-called multicolour disc blackbody model (MCD; Mitsuda et al. 1984) has been often adopted because it is easy to use and is efficiently implemented in the X-ray spectral fitting package XSPEC. The fitting parameters of the MCD model (the disc inner temperature and the normalization of the spectrum) depend on the BH mass and accretion rate, and can in principle be used to estimate the physical parameters of the accretion flow and of the BH. However, it is well known that this model represents an approximation of the standard, Shakura & Syunyaev (1973) disc and differs from it for the expression of the temperature profile, in which the effects of the
viscous torque at the inner boundary are neglected. Also, the locally emitted spectrum is assumed to be a blackbody, neglecting the vertical temperature structure and atmospheric radiative transfer effects. Although the effects of a zero-torque inner boundary in a pseudo-Newtonian potential (Paczynsky & Wiita 1980) were implemented in the disc model (Gierliński et al. 1999), temperature profile correction factors for the MCD model were also derived (e.g. Kubota et al. 1998; Makishima et al. 2000). At the same time, limited deviations from LTE caused by radiative transfer effects were accounted for using a hardening factor $f = T_{\text{col}}/T_{\text{eff}}$, where $T_{\text{col}}$ and $T_{\text{eff}}$ are the colour and effective temperatures (e.g. Shimura & Takahara 1995; Zampieri, Turolla & Szuszkiewicz 2001). Even adopting all these correction factors, deviations of the MCD with respect to a standard disc spectrum may be significant if the physical state of the disc changes or the coupling between the disc and the corona becomes important, which may occur especially at high accretion rates (Merloni, Fabian & Ross 2000).

Despite all these uncertainties, the parameters of the MCD fit have been used for estimating the mass of the BH and the accretion rate in X-ray binary systems. This has recently been done also for ULXs for which, at present, X-ray spectroscopy provides one of the few available methods to obtain zero-threshold estimates of the BH mass. Indeed, if the soft component observed in ULX spectra is fitted with a MCD model (e.g. Miller et al. 2003; Miller, Fabian & Miller 2004a), its characteristic temperature is typically $\sim 200$ eV, much lower than that observed in Galactic BHs. Both the low temperature and high normalization constant of the MCD model have been used to estimate the BH masses, typically obtaining values largely in excess of $100 M_\odot$. Modelling the soft component in these terms is clearly possible if ULXs are in an accretion stage similar to that of BHs in the soft (or nearly soft) state, so that a disc spectral component is present. Along with the very high (isotropic) luminosity of these sources, these 'spectroscopic estimates' have led to the suggestion that they may contain intermediate mass black holes (IMBHs).

In this paper, we revisit and compare standard models of accretion discs around BHs that include the appropriate zero-torque inner boundary condition and/or relativistic effects on the emission and propagation of radiation. They are derived in the usual assumptions that: (1) the accretion disc is optically thick, geometrically thin and in a quasi-steady state; (2) the locally emitted spectrum from a small annulus of the disc is a blackbody ($L_\nu = B_\nu$, $i$), with a colour correction factor $f$ taking into account for atmospheric transfer effects. The comparison is performed adopting the MCD as reference model and looking for the parameter space in which it turns out to be in agreement with 'more physical' accretion disc models. The fits are performed keeping either the BH mass or the accretion rate fixed and allowing the inner radius of the MCD to vary. This gives us simple 'recipes' that can be used for adjusting the estimates of the physical inner radius of the disc, the BH mass and the accretion rate inferred using the parameters of the MCD fits.

The paper is organized as follows. Section 2 contains a short summary of the basic structure equations of the accretion disc models considered in this work. Section 3 presents the results of the comparison among the different spectral models. Section 4 is devoted to an application of the results of this comparison to some representative ULX spectra, while Section 5 summarizes our conclusions.

## 2 ACCRETION DISC SPECTRA

In this section, we summarize the basic equations of the accretion disc models that we have considered in our work. We start with recalling the main properties of the standard disc model (Shakura & Sunyaev 1973) and the MCD model (Mitsuda et al. 1984), that are usually adopted in fitting the spectra of X-ray binaries. Then, we shortly review the structure equations for a relativistic accretion disc orbiting around a Schwarzschild and a Kerr BH (Bardeen, Press & Teukolsky 1972; Novikov & Thorne 1973; Page & Thorne 1974). The emergent spectrum is calculated taking into account the relativistic effects on the emission and propagation of light (gravitational redshift, light bending, Doppler shift) using the formalism of the transfer function introduced by Cunningham (1975). More sophisticated treatments of photon propagation in curved spaces–times, both numerical (Laor 1991; Karas, Vokrouhlík & Polnarev 1992; Rauch & Blandford 1994; Bromley, Chen & Miller 1997; Founton et al. 1997; Wilms 1998; Beckwith & Done 2004; Muller & Camenzind 2004) and also analytical (Cadez & Gomboc 1996; Gomboc 2001; Cadez et al. 2003; Calvani 2005) are available. Such techniques adopt ray tracing of the photon geodesics from the disc plane to the observer and are particularly suited when dealing with the calculation of spectral line profiles. However, we are essentially interested in comparing the continuum spectral shape of the MCD disc with that of a standard and relativistic disc and, to this end, a treatment of the relativistic geometrical optics a là Cunningham (1975) appears acceptable.

### 2.1 Standard accretion disc and MCD

Let’s consider the Standard accretion disc model (Shakura & Sunyaev 1973) in the usual approximations (Newtonian, optically thick, geometrically thin and in a quasi-steady state). The expression of the temperature profile is obtained setting the energy rate of viscous dissipation equal to the total radiation flux (e.g. Frank, King & Raine 2002):

$$T(r) = T_\text{in} \left( \frac{r}{R_{\text{in}}} \right)^{-3/4} \left[ 1 - \left( \frac{R_{\text{in}}}{r} \right)^{1/2} \right]^{1/4},$$

where

$$T_\text{in} = \left( \frac{3GM^*}{8\pi\sigma R_{\text{in}}} \right)^{1/4},$$

$M$ is the BH mass, $M^*$ is the accretion rate and $R_{\text{in}}$ is the inner disc radius. The emitted spectrum is calculated assuming that the disc emits locally as a blackbody, so that:

$$L_{\nu} = 4\pi r_0^2 F_{\nu} = 8\pi^2 \cos i \int_{R_{\text{in}}}^{R_{\text{out}}} I_\nu(T) r \, dr,$$

where $i$ is the disc’s inclination angle with respect to the direction of the line of sight, $r_0$ is the distance to the observer, $R_{\text{out}}$ is the outer disc radius and $I_\nu(T)$ is the specific intensity. We assume that the gas is in LTE and hence $I_\nu(T) = B_\nu(T)$, where $B_\nu(T)$ is the Planck function.

The MCD model is an approximation of the standard disc and differs from it only for the expression of the temperature profile, in which the effects of the viscous torque at the inner boundary are neglected. The temperature profile is then given by (Mitsuda et al. 1984):

$$T(r) = T_\text{in} \left( \frac{r}{R_{\text{in},\text{BB}}} \right)^{-3/4},$$

where

$$T_\text{in} = \left( \frac{3GM^*}{8\pi\sigma R_{\text{in},\text{BB}}} \right)^{1/4},$$
and $R_{in, BB}$ is the inner radius of the MCD. The emitted spectrum can again be calculated from equation (3), with the temperature profile given by equation (4).

In the following, we will use the MCD model as reference for fitting the standard and relativistic disc spectra, because this model is frequently adopted for the spectral fits of X-ray binaries. The fitting parameters of the MCD model, the disc inner temperature $T_{in}$ and the spectral normalization factor $K_{BB} = \cos i \left( R_{in, BB} / r_0 \right)^2$, depend on $M$ and $M^*$.

Assuming that the inner radius $R_{in, BB} = b R_s (R_s = GM/c^2)$ is the gravitational radius), it is possible to write $M$ and $M^*$ as:

$$M = \frac{67.5}{b} \left( \frac{r_0}{1 \text{ Mpc}} \right) \left( \frac{K_{BB}}{\cos i} \right)^{1/2}$$

and

$$\frac{M}{M_{\odot}} = 0.1 b^2 \left( \frac{r_0}{1 \text{ Mpc}} \right) \left( \frac{K_{BB}}{\cos i} \right)^{1/2} \left( \frac{T_{in}}{1 \text{ keV}} \right)^4,$$

where $K_{BB} = [R_{in, BB} / 1 \text{ Km}]^2 (r_0 / 10 \text{ kpc})^{-2} \cdot \cos i]$

In these assumptions, if the distance $r_0$ is known and the inclination angle $i$ is fixed, the MCD parameters can be used for estimating $M$ and/or $M^*$.

These expressions depend on the value of $b$, the inner radius in units of $R_s$, and $T_{in}$. It is well known that these estimates of the BH mass and the disc accretion rate are uncertain, because the physical state of the disc may vary and, in addition, radiative transfer effects induced by scattering may cause departures of the BH from $B_\ast$. Several non-LTE accretion disc models, in which radiative transfer effects at the surface of the disc are computed, have been presented in the literature, for both hot discs around stellar mass BHs (e.g. Merloni et al. 2000; Davis, Blaes & Turner 2005) and cool discs around intermediate mass BHs (e.g. Hui, Krolik & Hubeny 2005). These effects are very important in predicting the local spectra emitted at a given disc annulus and can be accounted for using an hardening factor $f = T_{col} / T_{eff}$, where $T_{col}$ and $T_{eff}$ are the colour and effective temperature. It is easy to show that $K_{BB} \propto 1/f^4$ and, as a consequence, $M \propto f^2$ and $M / M_{\odot} \propto f^2$ (Kubota et al. 1998; Kubota & Makishima 2001). So, as a first approximation, we account for radiative transfer effects simply inserting $f^2$ into equations (6) and (7). Computations of $f$ for standard and relativistic disc gives values in the interval 1.4–2 (see Shimura & Takahara 1995; Zampieri et al. 2001; Davis et al. 2005; Hui et al. 2005).

### 2.2 Relativistic accretion disc

The structure equations and the emitted spectrum of the relativistic disc are obtained in the same assumptions stated above: (1) the accretion disc is optically thick, geometrically thin and in a quasi-steady state; (2) the local emitted spectrum from a small annulus of the disc is a blackbody ($I_i = B_i$).

In the following, we will refer to the radius of the innermost stable circular orbit around a BH with $r_{ms}$. Its expression is given by:

$$r_{ms} = M \left( 3 + Z_2 - [(3 - Z_1)(3 + Z_1) + 2Z_2]^{1/2} \right),$$

where $Z_1$ and $Z_2$ are functions of $M$ and $a$ (Novikov & Thorne 1973), and $a$ is the specific angular momentum of the BH.

#### 2.2.1 Temperature profile of the relativistic disc

A detailed derivation of the structure of a relativistic accretion disc around a BH is presented in Novikov & Thorne (1973) and Page & Thorne (1974) to which we refer for all the details. Here, we present only the relevant equations for the temperature profile and the locally emitted flux.

Following Novikov & Thorne (1973), the emitted radiation flux can be expressed as a function of the integrated $\phi - r$ component of the stress-energy tensor (evaluated in the comoving frame of the observer in geodetic circular motion) as:

$$F = \frac{3GM}{8\pi r^3} \frac{Q}{BC^{1/2}},$$

where $B$, $C$ and $Q$ depend on the radial coordinate $r$ and the specific angular momentum of the BH $a$ (Novikov & Thorne 1973). The function $Q$ depends also on the angular momentum per unit mass of a generic circular orbit and of the innermost stable circular orbit (Stoeger 1976). Assuming that the radiative transport of energy is dominant over the turbulent one, in LTE we can write

$$F = \sigma T^4,$$

Comparing equations (9) and (10) it is possible to obtain the surface temperature profile for a relativistic accretion disc:

$$T(r) = T_\odot \left( \frac{r}{R_\odot} \right)^{-3/4} B^{-1/4} C^{-1/8} Q^{1/4},$$

where $T_\odot$ is given by equation (2).

#### 2.2.2 Relativistic effects on the propagation of radiation

Following Bardeen et al. (1972) and Cunningham (1975) the spectrum of a relativistic accretion disc is calculated by means of a transfer function $f$, that accounts for all relativistic effects on the emission and propagation of radiation (Doppler shift, light bending and gravitational redshift). The expression for $f$ is (Cunningham 1975):

$$f \left( g^*, r_*, \theta_0 \right) dg^* dr_* = \frac{g}{\pi r_0} \left( g^* - g^{*2} \right)^{1/2} r_0^2 \cos \theta_0 d\Omega_0,$$

where $r_0$ is the radius of a generic emitting ring of the disc, $\theta_0$ is the polar angle between the line of sight of the distant observer and the disc’s polar axis, $d\Omega_0$ is the solid angle that a specific geodesics family subsumes to the observer at a given energy and for a given emission radius, $g = E_0 / E_* $ is the ratio of the observed photon energy and the emitted one, $g^*$, $\theta_{max}$ and $\theta_{min}$ are functions of $g$ (for their expression see Cunningham 1975).

The radiation flux seen by a distant observer is:

$$F_0 = \int I_0 \cos \theta_0 d\Omega_0,$$

where $I_0$ is the observed specific intensity. If $I_i$ is the emitted specific intensity, from the expression of the relativistic invariant $I / E^3$ and equation (13) we obtain the observed luminosity per unit energy

$$L_0 = 4\pi r_0^2 F_0 = 4\pi \int g^3 I_i r_0^2 \cos \theta_0 d\Omega_0.$$

Using equation (12) it is possible to rewrite $r_0^2 \cos \theta_0 d\Omega_0$ in terms of $f(g^*, r_*, \theta_0) dg^* dr_*$. Inserting the resulting expression into equation (14), we obtain:

$$L_0 = \int 2\pi f \left( g^2 - g^{*2} \right)^{-1/2} dg^* d\left( \pi r_0^2 \right).$$

Tabulated values of the transfer function for a Schwarzschild and a maximally rotating ($a = 0.9981$) Kerr BH were reported by Cunningham (1975). In the following, we will consider these two extreme cases as reference.
3 PARAMETER ESTIMATES FROM SPECTRAL FITS

We implemented the structure equations and spectra of the accretion disc models outlined above and produced fits of the standard and relativistic discs against the MCD model. This model–model comparison has been performed by means of a standard $\chi^2$ minimization procedure, that provides also an assessment of the statistical significance of the fit. A fixed reference error of $\sim 10$ per cent was assigned to the spectra. As mentioned above, we use the MCD model as reference for fitting all the other disc spectra, because this model is frequently adopted for the spectral fits of X-ray binaries within XSPEC. The values of the best-fitting MCD parameters are then compared with those of the other models.

From now on we refer to the different accretion disc models with the following expressions: Disc-stand for the standard accretion disc (Shakura & Syunyaev 1973), Disc-BB for the MCD (Mitsuda et al. 1984), Disc-rel or Disc-kerr for a relativistic disc around a Schwarzschild or a maximally rotating ($a = 0.981$) Kerr BH (Novikov & Thorne 1973), respectively. We will also use different indices to denote the mass $M$ and the accretion rate $\dot{M}$ of the different models ($M_{\text{BB}}$, $M_{\text{BBBB}}$ for the Disc-BB; $M_{\text{stand}}$, $M_{\text{stand}}$ for the Disc-stand; $M_{\text{rel}}$, $M_{\text{rel}}$ for the Disc-rel; $M_{\text{kerr}}$, $M_{\text{kerr}}$ for the Disc-kerr). The accretion rate is expressed in units of the Eddington accretion rate $\dot{M}_{\text{Edd}} = L_{\text{Edd}}/c^2$, where $L_{\text{Edd}}$ is the Eddington luminosity. For an accretion efficiency $\eta = 0.1$–0.4, the accretion luminosity reaches $L_{\text{Edd}}$ for $M = \dot{M}_{\text{Edd}}/\eta = 2.5 \times 10^3 \dot{M}_{\text{Edd}}$.

3.1 Disc-stand model

Fits of the standard disc with a MCD were performed in the literature in order to correct for factors in the inner disc radius, the BH mass and the accretion rate (e.g. Kubota et al. 1998; Makishima et al. 2000). We performed similar fits of the Disc-stand with the Disc-BB but keeping either the BH mass or the accretion rate fixed. Acceptable fits with different best-fitting values of $R_{\text{in BB}}$ were obtained (see Table 1). For the fit with fixed $M$, we obtain $R_{\text{in BB}} = 13R_g$, while for the fit with fixed $\dot{M}$, $R_{\text{in BB}} = 14.9R_g$. In the first case, the value of $M_{\text{BB}}$ returned by the fit is $\approx 20$ per cent smaller than $M_{\text{stand}}$ while, in the second case, the values of $M_{\text{BB}}$ derived from the fit is $\approx 12$ per cent smaller than the corresponding value of $M_{\text{stand}}$. Clearly, there are also fits with fixed $R_{\text{in BB}}$ in the interval $13 \leq R_{\text{in BB}} \leq 14.9$, that return intermediate values of $M_{\text{BB}}$ and $M_{\text{BB}}$ (see again Table 1). In Fig. 1, we show the results of one of these fits.

These results are in agreement with those of Kubota et al. (1998), who showed that the inner radius of the best-fitting MCD model does not correspond to the true inner radius of the standard disc, but

| $M_{\text{stand}}$ ($M_{\odot}$) | $M_{\text{stand}}$ ($M_{\odot}$) | $M_{\text{BB}}$ ($M_{\odot}$) | $M_{\text{BB}}$ ($M_{\text{Edd}}$) | $R_{\text{in BB}}$ ($R_g$) | $\chi^2_{\text{red}}$ |
|---------------------------------|---------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 100.0                          | 0.1                            | 100.0                       | 0.08 ± 0.01                 | 13.0 ± 1.5                  | 0.1                         |
| 100.0                          | 0.1                            | 94.0 ± 10.0                 | 0.09 ± 0.03                 | 13.9                        | 0.2                         |
| 100.0                          | 0.1                            | 88.0 ± 32.0                 | 0.1                         | 14.9 ± 1.3                  | 0.2                         |

Table 1. Values of the characteristic parameters of the Disc-stand model and those derived by the spectral fits with the Disc-BB model for $\cos i = 1$. The inner radius of the Disc-stand model is $6R_g$. In the first row $M_{\text{BB}}$ is set equal to $M_{\text{stand}}$; in the second row $R_{\text{in BB}}$ is fixed at an intermediate value between 13 and 14.9 $R_g$; in the last row $M_{\text{BB}}$ is set equal to $M_{\text{stand}}$. The results of the spectral fit at fixed radius are shown in Fig. 1.

In fact, in order for the spectral peaks of the Disc-BB and Disc-stand to be superimposed (see Fig. 1), the maximum temperature of the standard disc (0.487$T_c$; see equations 1 and 2) must be equal to the maximum temperature of the Disc-BB ($T_{\text{in}}$; see equation 5). From this, equation (16) easily follows. Therefore, assuming that the Disc-stand terminates at the ISCO (innermost stable circular orbit), $R_{\text{in}} = 6R_g$ and the inner radius of the best-fitting Disc-BB must be located at $\approx 15.6R_g$, in substantial agreement with the results of the model–model fits.

We note that all the results presented here hold for any inclination angle, as the Disc-BB and Disc-stand models have the same dependence on $i$.

3.2 Disc-rel model

Similar fits of the Disc-rel model with the Disc-BB were also performed. For sufficiently small inclination angles, also in this case the emitted spectrum can be well reproduced by the MCD. The fits were performed keeping either $M$ or $\dot{M}$ fixed. Results are shown in Table 2. The two fits return different values of the inner radius, $R_{\text{in BB}} = 19.2R_g$ for fixed $M$ and $R_{\text{in BB}} = 25R_g$ for fixed $M$. In that the two radii are related by the expression (see also Zimmerman et al. 2005)

$$R_{\text{in BB}} = 2.6 R_{\text{in}}. \quad (16)$$

Figure 1. Spectrum (top panel) and temperature profile (bottom panel) of the Disc-stand model (solid-green) for $M_{\text{stand}} = 100M_{\odot}$, $M_{\text{stand}} = 0.1M_{\text{Edd}}$, $R_{\text{in}} = 6R_g$, and the corresponding best-fitting Disc-BB model (dashed-red; $M_{\text{BB}} = 94M_{\odot}$, $M_{\text{BB}} = 0.09M_{\text{Edd}}$, $R_{\text{in BB}} = 13.9R_g$). These plots refer to the fitting results reported in Table 1, second row ($\cos i = 1$).
the first case the value of $\dot{M}_{BB}$ returned by the fit is $\approx 40$ per cent smaller than $\dot{M}_{rel}$ whereas, in the second case, the value of $\dot{M}_{BB}$ derived from the fit is $\approx 25$ per cent smaller than the corresponding value of $\dot{M}_{rel}$. From Table 2, we can see that good fits can be obtained also fixing $R_{in,BB}$ in the interval $19.2-25R_g$ and leaving $M_{BB}$ and $\dot{M}_{BB}$ free to vary. In Fig. 2, we plot the results of one of these fits ($R_{in,BB} \approx 21.6R_g$).

If we increase the inclination angle, the relativistic effects (especially the gravitational focusing) tend to increase the contribution of radiation coming from the inner parts of the disc, making the spectrum of the $\text{Disc-rel}$ intrinsically harder than that of the $\text{Disc-BB}$. In fact, for high inclination angles the $\text{Disc-BB}$ is only in rough agreement (within a $\sim 10$ per cent fractional error) with the $\text{Disc-rel}$ spectrum, and no satisfactory fit is obtained ($\chi^2_{red} \geq 1.3$). Similar results for $\cos i < 1$ were obtained by Ebisawa & Kazuhisa (1991). For illustrative purposes alone, in Table 2 we report also the values of the fits for $\cos i = 0.5$.

### 3.3 $\text{Disc-kerr}$ model

The last comparison concerns the $\text{Disc-kerr}$/$\text{Disc-BB}$ models. If the inclination angle is close to $i = 0$ (face-on disc), the spectral fits of the $\text{Disc-kerr}$ with the $\text{Disc-BB}$ at constant $M$ or $\dot{M}$ are statistically acceptable and return different values of the inner radius, $R_{in,BB} = 7.9R_g$ and $R_{in,BB} = 9.2R_g$, respectively (see Table 3). The values of $\dot{M}_{BB}$ and $M_{BB}$ are larger than the corresponding values of the $\text{Disc-rel}$ model (as high as $\sim 20$ per cent for $M$ and $\sim 25$ per cent for $\dot{M}$). Also in this case, there are other fits with fixed $R_{in,BB}$ in the range $7.9-9.2R_g$ for which the $\text{Disc-BB}$ and $\text{Disc-kerr}$ spectra are in good agreement (see again Table 3). In Fig. 3, we show the results of one of these fits obtained for $R_{in,BB} \approx 8.9R_g$. We have also tried fits varying the inclination angle $i$. For a maximally rotating Kerr BH, the relativistic effects on the emitted spectrum are significantly stronger than for a Schwarzschild BH. The spectrum of the $\text{Disc-kerr}$ model becomes intrinsically different and much harder than the $\text{Disc-BB}$ one as $i$ increases, so that it is no longer possible to obtain a satisfactory fit ($\chi^2 > 2$). For illustrative purposes alone, in Table 3 we report the values of the fits for $\cos i = 0.5$.

### 3.4 Comparison with fitting results from XSPEC

In order to test our approach, we simulated disc spectra for both a Schwarzschild BH and a maximally rotating Kerr BH using the $\text{kerrBB}$ model in XSPEC (v. 12.3.0). This model implements the relativistic structure equations and the relativistic effects on the emission and propagation of radiation (Li et al. 2005). We fitted the $\text{kerrBB}$ spectra with a $\text{discBB}$ model (the xspec implementation of

### Table 2. Values of the characteristic parameters of the $\text{Disc-rel}$ model and those derived by the spectral fits with the $\text{Disc-BB}$ model for $\cos i = 1$ and $0.5$. The inner radius of the $\text{Disc-rel}$ is $R_{in,rel} = 6R_g$. In the first row $M_{BB}$ is set equal to $M_{rel}$; in the second row $R_{in,BB}$ is fixed at an intermediate value between $19.2$ and $25R_g$; in the last row $\dot{M}_{BB}$ is set equal to $\dot{M}_{rel}$ (for each value of $\cos i$). The results of the spectral fit at fixed radius are shown in Fig. 2.

| $\cos i$ | $\dot{M}_{rel}$ | $M_{rel}$ | $M_{BB}$ | $\dot{M}_{BB}$ | $R_{in,BB}$ | $\chi^2_{red}$ |
|----------|-----------------|-----------|----------|----------------|-------------|----------------|
| $1$      | 50.0            | 0.1       | 50.0     | 0.06 $\pm$ 0.03 | 19.2 $\pm$ 11.6 | 0.2        |
|          | 50.0            | 0.1       | 43.5 $\pm$ 6.7 | 0.07 $\pm$ 0.02 | 21.6        | 0.1        |
|          | 50.0            | 0.1       | 37.3 $\pm$ 10.2 | 0.1       | 25.0 $\pm$ 7.1 | 0.1        |
| $1$      | 100.0           | 0.1       | 100.0    | 0.06 $\pm$ 0.03 | 19.2 $\pm$ 3.5 | 0.2        |
|          | 100.0           | 0.1       | 87.0 $\pm$ 31.0 | 0.07 $\pm$ 0.02 | 21.6        | 0.1        |
|          | 100.0           | 0.1       | 75.0 $\pm$ 34.0 | 0.1       | 25.0 $\pm$ 6.7 | 0.1        |
| $0.5$    | 100.0           | 0.1       | 100.0    | 0.03 $\pm$ 0.05 | 11.2 $\pm$ 3.4 | 1.3        |
|          | 100.0           | 0.1       | 50.0 $\pm$ 32.0 | 0.1       | 21.5 $\pm$ 12.3 | 1.8        |

### Table 3. Values of the characteristic parameters of the $\text{Disc-kerr}$ model and those derived by the spectral fits with the $\text{Disc-BB}$ model for $\cos i = 1$ and $0.5$. The inner radius of the $\text{Disc-kerr}$ model has been calculated using equation (8) (for $a = 0.9981$, $R_{in,kerr} \approx 1.36R_g$). In the first row $M_{BB}$ is set equal to $M_{Kerr}$; in the second row $R_{in,BB}$ is fixed at an intermediate value between $7.9$ and $9.2R_g$; in the last row $M_{BB}$ is set equal to $M_{Kerr}$ (for each value of $\cos i$). The results of the spectral fit at fixed radius are shown in Fig. 3.

| $\cos i$ | $M_{Kerr}$ | $M_{BB}$ | $\dot{M}_{BB}$ | $R_{in,BB}$ | $\chi^2_{red}$ |
|----------|------------|----------|----------------|-------------|----------------|
| $1$      | 100.0      | 100.0    | 0.075 $\pm$ 0.015 | 7.9 $\pm$ 0.9 | 0.2           |
|          | 100.0      | 88.0 $\pm$ 18.0 | 0.095 $\pm$ 0.02 | 8.9        | 0.2           |
|          | 100.0      | 85.0 $\pm$ 20.0 | 0.1       | 9.2 $\pm$ 1.3 | 0.2           |
| $0.5$    | 100.0      | 100.0    | 0.04 $\pm$ 0.015 | 2.4 $\pm$ 1.7 | 2.3           |
|          | 100.0      | 60.0 $\pm$ 5.0 | 0.1       | 4.0 $\pm$ 0.4 | 2.2           |

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the MCD) and used the parameters of the fit to estimate the mass
and accretion rate using equations (6) and (7).

Spectra were simulated using the response matrices of the EPIC-
pron camera onboard XMM–Newton. For the sake of comparison,
in the kerrBB model we adopted a zero torque inner boundary
condition and switched off the effects of self-irradiation and limb-
darkening. The disc was assumed to be face-on \((i = 0)\) and the
colour correction factor \(f\) was set equal to 1. A fixed distance of
5 Mpc was assumed. For the mass of the BH and the accretion
rate we chose 100 M\(_{\odot}\) and 0.485 M\(_{\text{Edd}}\), respectively, while the BH
angular momentum \(a\) was set equal to 0 (Schwarzschild) or 0.9981
(Kerr).

For a kerrBB spectrum with \(a = 0\), the best-fitting MCD parame-
ters are: \(T_{\text{in}} = 0.122 \pm 0.002\) keV and \(K = 39 \pm 6\) \((2\sigma\) errors). From these parameters, assuming \(b = 19.2 \pm 3.5\) \((\text{see Table 2})\),
we obtain \(M_{\text{BB}} = (110 \pm 29)\) M\(_{\odot}\) \((\text{equation 6})\), in good agreement
with the assumed value of the BH mass. Similarly, assuming \(b = 25 \pm 6.7\), it is \(M_{\text{BB}} = (0.4 \pm 0.28)M_{\text{Edd}}\) \((\text{equation 7})\), which is also
in agreement with the assumed value for the accretion rate. For a
kerrBB spectrum with \(a = 0.9981\), the best-fitting MCD parameters are:
\(T_{\text{in}} = 0.248 \pm 0.003\) keV and \(K = 6 \pm 0.4\) \((2\sigma\) errors). From equations (6) and (7) and the values of \(b\) reported in Table 3, we ob-
tain \(M_{\text{BB}} = (105 \pm 15)\) M\(_{\odot}\) and \(M_{\text{BB}} = (0.4 \pm 0.14)M_{\text{Edd}}\), again in
satisfactory agreement with the parameters assumed in simulating
the spectrum.

4 ESTIMATES OF \(M\) AND \(\dot{M}\) FROM THE X-RAY
SOFT COMPONENT OF ULXs

A certain number of ULXs show two component X-ray spectra
that can be adequately modelled with a soft thermal component
plus a power law. If the soft component is fitted with a MCD, the
fitting parameters (in particular the normalization \(K_{\text{in}}\)) return very
large values for the BH mass, often in excess of several hundreds
M\(_{\odot}\) \((\text{e.g. Miller et al. 2003; Miller et al. 2004a})\). These masses
are calculated from equation (6) assuming specific values for the
inner radius \(R_k\) (in units of \(R_g\)) and the colour correction factor \(f\).
Miller et al. (2003) and Miller et al. (2004a) adopted \(b = 9.5\) and
\(f = 1.7\).

In the previous section, we performed fits of the standard and
relativistic disc with a MCD keeping the BH mass fixed and deter-
mined the (range of) values of \(b\) for which the spectra are in
statistical agreement. These values are reported in Tables 1–3 and
differ from what assumed by Miller et al. (2003, 2004a). For the
standard disc, there appears to be more agreement with what re-
ported by Kubota et al. (1998) and Makishima et al. (2000). We
are then in the position to revise previous estimates of the BH mass
(and accretion rate) of ULXs based on MCD spectral fits of the soft
component, adopting the appropriate value of \(b\) for both a standard
and a relativistic disc.

For illustrative purposes, we use the results of the MCD spec-
tral fits of NGC 1313 X-1, NGC 1313 X-2, M 81 X-9 and NGC
4559 X-7 obtained by Miller et al. (2003, 2004a), Zampieri et al.
(2004) and Cropper et al. (2004). From equations (6) and (7) of
Section 2.1, we calculate the values of \(M\) and \(\dot{M}\) for different values
of \(b\) and assuming \(f = 1.7\), and report them in Tables 4 and 5. The
values that correspond to \(b = 9.5\) are those reported by Miller et al.
(2003, 2004a), whereas the values for \(b = 7.9, 13, 19.2\) correspond
to those expected for a standard disc around a maximally rotating
Kerr, Newtonian or Schwarzschild BH, respectively. As can be seen
from the tables, the values of the mass are systematically lower (up
to a factor 3) than those of a MCD with \(R_{\text{in BB}}\) at the ISCO \((b = 6)\).
This is clearly a consequence of the larger fitting radius required
by the Newtonian and relativistic discs. Comparing the masses of
the different ‘physical’ models, it is possible to see that the largest
value is obtained for a disc around a maximally rotating Kerr BH

Table 4. Values of the BH mass \(M\) and accretion rate \(\dot{M}\) for NGC 1313
X-1 and X-2, obtained using the parameters of the MCD that best-fits
their soft components \((K_{\text{BB}} = 28.3^\circ, T_{\text{in}} = 0.22\) keV for NGC 1313 X-1; \(K_{\text{BB}} =
6.66_{-16}^{+16}, T_{\text{in}} = 0.25\) keV for NGC 1313 X-2) and different values of \(b\) that
correspond to the different accretion disc models (see Tables 2, 3 and section
4; Miller et al. 2003 and Miller et al. 2004a adopted \(b = 9.5\)). The assumed
distance of NGC 1313 is 3.7 Mpc (Tully 1988) and the colour correction
factor is \(f = 1.7\) (Shimura & Takahara 1995; Zampieri et al. 2001; Hui et al.
2005).

| \(R_{\text{in BB}}(R_g)\) | NGC 1313 X-1 | NGC 1313 X-2 |
|----------------------|--------------|--------------|
| \(M (M_{\odot})\) | \(M (M_{\text{Edd}})\) | \(M (M_{\odot})\) | \(M (M_{\text{Edd}})\) |
| 6                  | 630_{-250}^{+500} | 0.5_{-0.1}^{+0.01} | 310_{-120}^{+260} | 0.4_{-0.2}^{+0.01} |
| 7.9                | 480_{-200}^{+200} | –               | 240_{-120}^{+240} | –               |
| 9.2                | 11_{-0.1}^{+1.0} | –               | –               | –               |
| 9.5                | 400_{-200}^{+200} | 1.2_{-0.1}^{+0.1} | 200_{-100}^{+200} | 1.0_{-0.2}^{+0.01} |
| 13                 | 290_{-200}^{+200} | –               | 140_{-120}^{+240} | –               |
| 14.9               | 3_{-0.2}^{+0.2} | –               | –               | 2.4_{-0.8}^{+0.8} |
| 19.2               | 200_{-200}^{+200} | –               | 100_{-100}^{+100} | –               |
| 25                 | –               | 8.3_{-0.8}^{+0.8} | –               | 6.7_{-2.0}^{+2.0} |
(Disc-kerr; see Tables 4 and 5). This is consistent with the fact that the (Boyer–Lindquist) radial coordinate of the ISCO for a maximally rotating, Kerr BH is $\approx 1/6$ that of a Schwarzschild BH of the same mass (cmp. Hui & Krolik 2008). Indeed, for a fixed value of $R_{\text{BH}}$ (in cm) as returned by the MCD fit of an observed spectrum, a lower value of $b$ necessarily implies a larger mass as $b = R_{\text{BH}}/R_g \propto 1/M$. We need to fill the same physical radius $R_{\text{BH}}$ with a bigger BH in order to reduce the value of $b$. Ultimately, the smaller value of $b$ returned by the fit of the Disc-kerr model is a consequence of the fact that the Kerr spectrum is significantly harder than those of the other ‘physical’ disc models computed with the same value of the parameters (see Fig. 4).

It is also interesting to note that, apart from NGC 4559 X-7, the BH masses for a Schwarzschild disc are no longer significantly in excess of $\sim 100 M_\odot$, with a distribution clustering in the interval 100–200 $M_\odot$ (see again Tables 4 and 5). This is true also for M 81 X-9, for which the second observation returns somewhat larger values of the mass. However, the limits inferred from the two observations of this source must be consistent within the errors, the mass of the BH must be in the interval 150–220 $M_\odot$ (smaller/larger values would not be consistent with the second/first observation; see the last row in Table 5). All the mass estimates obtained for the Disc-rel model are then significantly lower than those reported by Miller et al. (2003, 2004a). The only exception is NGC 4559 X-7 for which, however, there are hints of timing features in the power density spectrum and it has been proposed that it may contain a BH of several hundreds solar masses (e.g. Cropper et al. 2004). The inferred BH masses can reach the values estimated by Miller et al. (2004a) only for a disc around a maximally rotating, Kerr BH.

### Table 5. Values of the BH mass $M$ and accretion rate $\dot{M}$ for M81 X-9 and NGC 4559 X-7, obtained using the parameters of the MCD that best-fits their soft components ($K_{\text{BB}} = 20^{+40}_{-20}$; $T_{\text{in}} = 0.26 \text{ keV}$ for M81 X-9 (obs. 1); $K_{\text{BB}} = 60^{+70}_{-40}$; $T_{\text{in}} = 0.21 \text{ keV}$ for M81 X-9 (obs. 2)); $K_{\text{BB}} = 158^{+340}_{-107}$; $T_{\text{in}} = 0.148 \text{ keV}$ for NGC 4559 X-7) and different values of $b$ that correspond to the different accretion disc models (see Tables 2, 3 and section 4; Miller et al. 2003; Miller et al. 2004a adopted $b = 9.5$). The assumed distance is 9.69 Mpc for NGC 4559 X-7 (Sanders et al. 2003) and 3.4 Mpc for M81 X-9 (Georgiev et al. 1991; Hill et al. 1993). The colour correction factor is $f = 1.7$ (Shimura & Takahara 1995; Zampieri et al. 2001; Hui et al. 2005).

| $R_{\text{BH}}(R_g)$ | M 81 X-9 (obs. 1) | $\dot{M}(\dot{M}_{\text{Edd}})$ | M 81 X-9 (obs. 2) | $\dot{M}(\dot{M}_{\text{Edd}})$ | NGC 4559 X-7 | $\dot{M}(\dot{M}_{\text{Edd}})$ |
|----------------------|------------------|-------------------|------------------|-------------------|----------------|-------------------|
| 6                    | $490_{-140}^{+210}$ | $0.7_{-0.2}^{+0.3}$ | $860_{-370}^{+400}$ | $0.6_{-0.2}^{+0.3}$ | $4000_{-100}^{+300}$ | $0.6_{-0.3}^{+0.5}$ |
| 7.9                  | $380_{-110}^{+140}$ | $1.3_{-0.5}^{+0.7}$ | $650_{-280}^{+300}$ | $1.4_{-0.6}^{+0.7}$ | $3000_{-130}^{+120}$ | $1.4_{-0.6}^{+1.1}$ |
| 9.2                  | $310_{-90}^{+130}$ | $1.8_{-0.5}^{+0.8}$ | $540_{-230}^{+260}$ | $1.4_{-0.6}^{+0.7}$ | $2500_{-110}^{+190}$ | $1.5_{-0.6}^{+1.2}$ |
| 13                   | $230_{-70}^{+90}$ | $2.9_{-1.3}^{+1.8}$ | $400_{-170}^{+180}$ | $3.3_{-1.4}^{+1.5}$ | $1800_{-750}^{+1400}$ | $3.8_{-1.6}^{+2.9}$ |
| 14.9                 | $160_{-50}^{+60}$ | $3.5_{-1.8}^{+2.5}$ | $270_{-120}^{+120}$ | $9.3_{-3.9}^{+4.3}$ | $1200_{-500}^{+1000}$ | $10.1_{-4.6}^{+8.2}$ |

### Figure 4. Emitted spectra (top panel) and temperature profiles (bottom panel) of the Disc-BB, Disc-stand, Disc-rel and Disc-kerr models, obtained for the same characteristic parameters: $M = 100 M_\odot$, $\dot{M} = 0.1 \dot{M}_{\text{Edd}}$, $R_{\text{BH}} = R_{\text{in}} = 6R_g$, while $R_{\text{in,kerr}}$ has been obtained using equation (8). For all models $\cos i = 1$.

This is in substantial agreement with the approximate value ($b = 15.6$) that can be derived using analytic arguments.

Also in the case of a relativistic disc around a Schwarzschild BH, for sufficiently small inclination angles the emitted spectrum can be well reproduced by a MCD model (see also Ebisawa & Kazuhisa 1991 and Kubota et al. 2005). For small angles, in fact, one can try to guess the disc and BH parameters...
using the best-fitting MCD model but, as for the standard disc, a suitable assumption must be made on the inner disc radius. We found that there exist precise values of $R_{in, BB} \approx 19.2 R_g$ and $\geq 25 R_g$ for which the values of $M$ and $M$ inferred from the MCD are the same as those provided by the relativistic disc. For large angles, owing to genuine relativistic effects, no satisfactory MCD spectral fit of the relativistic disc spectrum can be obtained (e.g. Ebisawa & Kazuhisa 1991).

The spectrum of an accretion disc around a fast rotating Kerr BH, in general more strongly affected by relativistic effects compared with those of a disc around a Schwarzschild black hole. As a consequence, it can be well reproduced by a MCD model, only in the case in which the disc is essentially face-on. In this assumption, there exist values of $R_{in, BB}$ for which the MCD and Kerr discs return similar values of $M$ and $M(R_{in, BB} \sim 7.9 R_g$ and $R_{in, BB} \sim 9.2 R_g$, respectively). Also in this case, for large inclination angles, the spectrum of the Disc-kerr model becomes intrinsically different and much harder than the Disc-BB one, so that it is no longer possible to obtain a satisfactory fit with the MCD model.

The model–model comparison approach adopted in this paper was prompted by the idea to determine the parameter space in which the MCD is in agreement with ‘more physical’ standard accretion disc models and provide ‘recipes’ for adjusting the estimates of the disc inner radius, the BH mass and the accretion rate. In fact, despite the uncertainties related to radiative transfer and to the physical state of the inner disc, the parameters of the MCD fit are often used for inferring the mass and accretion rate in X-ray binaries, in particular in ULXs. The results of the comparison that we performed can be used to revise estimates of $M$ and $M$ obtained in this way. This has also been tested by applying our ‘recipe procedure’ to the results from a Disc-BB fit to simulated Kerr-BB spectra performed directly in XSPEC.

We considered the case of a few ULXs (NGC 1313 X-1, X-2, M81 X-9 and NGC 4559 X-7) for which MCD spectral fits of their X-ray soft spectral components have been published and/or values of the BH mass estimated (Miller et al. 2003, 2004a; Cropper et al. 2004; Zampieri et al. 2004). From the parameters of the fit with the Disc-BB ($T_{in}$ and $K_{BB}$) we found that, assuming that the inner disc boundary of the MCD is at or close to $6 R_g$, the values of $M$ and $M$ can be severely overestimated. Adopting the appropriate value of the inner radius $b$ (in units of $R_g$) for the Disc-rel model, we obtain that the BH masses are in the range $100–200 M_\odot$ for NGC 1313 X-1, NGC 1313 X-2, M81 X-9 and 700–2000 $M_\odot$ for NGC 4559 X-7 (see Tables 4 and 5). The relativistic disc models have BH masses systematically lower (up to a factor 3) than those of a MCD with $R_{in, BB}$ at the ISCO.

As already mentioned, sophisticated relativistic accretion disc models (e.g. Hanawa 1989; Hui et al. 2005; Li et al. 2005) that implement the relativistic structure equations and the relativistic effects on the emission and propagation of radiation, are available in XSPEC and can be clearly used to perform accurate fits of observed spectra, in particular for large inclination angles when the relativistic effects produce significant differences. An analysis of this type has recently been performed by Winter, Mushotzky & Reynolds (2007) and Hui & Krolik (2008). It is interesting to note that the BH masses for a sample of disc-dominated ULXs obtained by Hui & Krolik (2008) are below $100 M_\odot$. Even if their best fits require large values of the BH spin, BHs of several hundreds solar masses are not needed, in substantial agreement with our findings.

Clearly, it is possible that at least some ULXs for which curvature above $\sim 2–3$ keV is present in the X-ray spectrum are in a different accretion regime, in which the disc is in a diverse physical state (accreting at or above the Eddington rate; see e.g. Roberts 2007; Soria 2007). In this case, the assumptions of a standard accretion disc break down and different spectral models should be adopted (e.g. Stobbart et al. 2006; Mizuno et al. 2007). However, in the assumption that they are in a standard accretion regime (below Eddington), we found that, in three cases (NGC 1313 X-1, X-2 and M 81 X-9), the BH masses inferred for a standard disc around a Schwarzschild BH are in the interval $\sim 100–200 M_\odot$. Only if the BH is maximally rotating are the masses comparable to the much larger values previously derived in the literature.

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