QUANTUM REFERENCE FRAMES, 
TIME AND MEASUREMENTS

S.N. MAYBurov

Lebedev Institute of Physics
Leninsky pr. 53, Moscow Russia, 117924

Abstract

We argue that correct account of the quantum properties of macroscopic objects which form reference frames (RF) demand the change of the standard space-time picture accepted in Quantum Mechanics. The presence of RF free quantum motion in the form of wave packet smearing results in formal nonapplicability of Galilean or Lorentz space-time transformations in this case. For the description of the particles states transformations between different quantum RF the special quantum space-time transformations are formulated. Consequently it results in corrections to Schrodinger or Klein-Gordon equations which depends on the RF mass. RF proper time becomes the operator depending of momentums spread in RF wave packet, from the point of view of other observer. The experiments with macroscopic coherent states are proposed in which this effects can be tested.

* E-mail Mayburov@sgi.lpi.msk.su
1 Introduction

In a modern Quantum Mechanics (QM) the particles states and other objects evolve in Minkowski space-time regarded as independently existing entity. Alternatively it’s the main method of the description of the surrounding world from the point of view of particular observer. In Classical Physics it corresponds to introduction of space coordinate axes associated with particular reference frame (RF) which is supposed to be some solid macroscopic object or the system of them. Despite that in QM the behavior of physical objects can be strikingly nonclassical it’s tacitly assumed describing RF properties in QM that any RF evolution is always exactly classical. Consequently all RF coordinate transformations in QM are supposed to be identical to classical - Galilean or Lorentzian ones. In our paper we argue that this assumption is in general incorrect and quantum features of RF should result in a special quantum space-time transformations. We must stress that this results are obtained without introducing new axioms or hypothesis, but staying in the framework of standard QM. It seems to us that current discussions of Quantum space-time should include the correct definition of quantum RF (Doplicher, 1995). The first consistent formulation of quantum RF problems was given in (Aharonov, 1984). Their algebraic and group properties were studied by Toller (Toller, 1997). The importance of RF quantum properties was noticed in Quantum Gravity study of distributed - dust or fluid RF (Rovelli, 1991). Yet the detailed analysis of Quantum Measurements problems concerned with quantum RF and quantum observers and the properties of their proper time wasn’t performed up to now.

In this paper only one quantum RF effect out of many possible ones will be analyzed in detail. In fact it’s the consequence of the existence of the wave packet for any free macroscopic object, regarded as RF which width gradually enlarges with the time. Despite that this effect scale in the standard laboratory conditions is quite small, it influences on the coordinate measurements in principle, and we can expect it can have important meaning both for Cosmology and for small distance Physics. It’ll be shown that in general quantum RF transformations corresponds to the additional quantum symmetry. We describe briefly the special experiment which can test this conclusions using modern experimental technic. Part of our results were published earlier in (Mayburov, 1997).

The formulated problem is closely related to the macroscopic quantum coherence topics which embrace different observations of the superpositions of the macroscopic objects states. The recent studies have shown that for low dissipation superconductor systems the superpositions of the macroscopic states can be observed (Legett, 1980).

Our paper is organised as follows: in the rest of this chapter we remind of the QM wave packet properties and formulate our model premises. In a chapter 2 the new canonical formalism of quantum RF states and their transformations developed and the analog of Schrodinger equation for finite mass observer is obtained. The possible experimental tests and their description by the Decoherence Model are considered in chapter 3. The relativistic equations for quantum RF and the resulting quantum space-time properties are regarded in chapter 4. In a final chapter the obtained
results and their interpretation are discussed.

Note that in QM framework the system defined as RF presumably should be able to measure the observables of quantum states i.e. to be observer i.e. to include measuring devices -detectors. At first sight it seems this problem can be solved only when the detailed microscopic model of state vector collapse will be developed. Despite the multiple proposals up to now well established theory of collapse which answer all difficult questions is absent (Mayburov,1995). Alternatively we’ll show that our problem premises doesn’t connected directly with the state vector collapse mechanism. To demonstrate it clearly we’ll perform in the framework of Decoherence model the calculation of the particular example of such measurement. As the result in place of its detailed description we can use two simple assumptions about the observer system properties, which are in the same time rather weak. The first one is that RF consists of finite number of atoms (usually rigidly connected) and have the finite mass. We don’t consider in our study the influence of detector recoil effects on the measurements results which can be made arbitrarily small (Aharonov,1984).

So in this paper we’ll assume that observer system (OS) or RF is the ensemble of particle detectors, meters and recording devices which perform the coordinate or other measurements on quantum objects. As the realistic example we can regard the photoemulsion plate or the diamond crystal which can measure microparticle position relative to its c.m. and simultaneously record it. We’ll regard measuring device to be in a pure state as usually done in Measurement Theory.

The experiments with the atomic and molecular beams confirm that the complex quantum system can be obtained in the delocalized state without change of their internal properties. We’ll consider the hypothetical situation when one free observer $F_1$ is described by some other macroscopic observer as a pure quantum state with large uncertainty of centre of mass coordinate $R_c = \sum \frac{m_i}{M} r_i$. The question which we had in mind preparing this work was: if the observer can be in such delocalised quantum state what will he see looking at the objects of our macroscopic world?

It’s well known that the solution of Shrodinger equation for any free quantum system in a pure state consisting of $N$ constituents can be presented as the

$$\Psi(\vec{r}_1, ..., \vec{r}_n, t) = \sum c_l \Phi_l(\vec{R}_c, t) * \phi_l(\vec{r}_{ij}, t)$$

where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ are the relative or 'internal' coordinates of constituents (Schiff,1955). Here $\Phi_l$ describes the c.m. motion of the system. It demonstrates that in QM framework the functioning and evolution of the system in the absence of the external fields is separated into the external evolution as whole of the pointlike particle M and the internal evolution completely defined by $\phi_l(\vec{r}_{ij}, t)$ if the constituents interaction depends only on $\vec{r}_{ij}$ as usually have place. So the internal evolution is independent of whether the system is localized in the standard macroscopic ‘absolute’ reference frame (ARF) or not. Relativistic QM and Field Theory studies show that the factorization of c.m. and relative motion holds true even for nonpotential forces and variable $N$ in the secondarily quantized systems (Schweber,1961). Moreover this factorization expected to be correct for nonrelativistic systems where binding energy is much less then its mass $m_1$, which is characteristic for the most of real
detectors. For our problem it’s enough to assume that the factorization of the c.m. motion holds for the observer system only in the time interval $T$ from the system preparation procedure, until the act of measurement starts, i.e. when the measured particle collides with it. More exactly our second and last assumption about observer properties is that during period $T$ its state is described by the wave function generalizing (1) of the form

$$\Psi(R_c, q, t) = \sum c_l \Phi^*_c(R_c, t) \phi_l(q, t)$$

where $q$ denote all internal detector degrees of freedom which evolve during $T$ according to Schrödinger equation (or some field equation). To simplify our calculations we’ll take below all $c_l = 0$ except $c_1$ which wouldn’t influence our final results.

Any free object which was initially localized in the wave packet $\Phi(R, t_0)$ after it will gradually smear in space, which rate for the gaussian packet with initial dispersion $a_0$ described as (Schiff, 1955):

$$a(t) = (\int (R - \bar{R})^2 |\Phi(R, t)|^2 d^3R)^{1/2} = a_0(1 + \frac{t^2}{m^2a_0^4})^{1/2}$$

(Plank constant $\hbar$ and $c$ in our calculations is equal 1). The common conclusion is that to observe experimentally measurable smearing of macroscopic object demands too large time, but we’ll show that for some mesoscopic experiments it can be reasonably small to be tested in the laboratory conditions. In our work it’s permitted ad hoc preparation of any initial state vector described by the smooth function $\Psi(r_1, t_0)$ in agreement with QM postulates.

We don’t consider in our study the influence of detector recoil effects on the measurements results which can be made arbitrarily small (Aharonov, 1984). During this study we assume that RF and OS are always identical entities, but we’ll discuss in a final chapter their possible distinctions which in fact can weaken demands to RF formulated in this chapter.

## 2 Q-transformations Formalism

To explain the meaning of quantum RF transformations in QM we consider gedankenexperiment with the wave packet of observer system $F^1$. Assume that $F^1$ of mass $m_1$ is suspended in a vacuum chamber which is regarded as the classical absolute RF (ARF) with the mass $m_A \rightarrow \infty$ (gravitation field is absent). For the simplicity $F^1$ c.m. wave packet is supposed to smear significantly only along X axis described by $\psi_1(x)$.

$$\psi_1(r_1, t) = \psi_1(x_1, t)\delta(y_1)\delta(z_1)$$

The measured particle $n$ with mass $m_n = m_2$ belongs to a very narrow beam, so that its wave function $\psi_n(x_n)$ can be approximated by the delta-function $\delta(x_n - x_s)$. $F^1$ includes particle detector $D_0$ which can measure the distance between the particle and $F^1$ c.m. All states are taken at the fixed time $t = 0$ when $n$ collides with $F^1$ and due to it $t$ dependence in $\psi$ arguments omitted.
Due to the factorization of $F^1$ and $n$ states $n$ wave function in $F^1$ $\psi'$ can be extracted from the $F^1 + n$ system wave function by the canonical transformation

$$\psi(x_n, x_1) = \psi_1(x_1)\psi_n(x_n) = \Phi_c(X_c)\psi_n'(x_n - x_1) = \psi_1\left(\frac{m_1x_1 + m_nx_n}{m_1 + m_n}\right)\psi_1(x_n - x_1 - x_s)$$

Function $\Phi$ describes the state of this system as the whole and can't be found by no measurement on $n$ in $F^1$. Parameters of $\psi_n'$ in principle can be defined by the measurement of $n$ observables in $F^1$, which details wouldn't be discussed here. In this example $\psi_n'$ coincides with $\psi_1$ for $n$ being localized in ARF. It assumes that if in ARF $F^1$ wave function have the average $x$ dispersion $a_O$ then from the 'point of view' of observer $F^1$ any object localized in ARF is smeared with the same RMS $a_O$.

Considering the collapse in this two RF we note that $F^1$ and ARF observers will treat the same event unambiguously as $n$ detection (or it flight through $D_0$). In observer reference frame $F^1$ it reveals itself by the detection and amplification process in $D_0$ initiated by $n$ absorption. For ARF the collapse results from the nonobservation of neutron in a due time - so called negative result experiment. So the signal in $F^1$ will have the same relative probability as in ARF. Such kind of the measurement means obviously the reduction of $\psi_1$ in ARF. Because of it proceeding further we'll assume always that all our considerations are performed for the quantum ensemble of observers $F^1$ without additional referring to it. It means that each event is resulted from the interaction between the 'fresh' observer and also the particle, prepared both in the specified quantum states, alike the particle alone in the standard experiment. As we have no reason to assume that the transition from ARF to $F^1$ which we'll call Q-transformation can transfer pure states to mixed ones we must conclude that this distribution is defined by neutron wave function in $F^1$. It means that the result of measurement in $F^1$ is also described by QM Reduction postulate, i.e. that initial state during the measurement by RF detector evolve into the mixture of the measured observable eigenstates.

After this qualitative example we'll regard the general situation for the system $S_N$ of $N$ objects $B_i$ which include $N_g$ pointlike 'particles' $G_i$ and $N_f$ frames $F^i$, which in principle can have also some internal degrees of freedom described by (1).

For the start we'll assume that RF and particles coordinates observables $\vec{r}_i$ are given in absolute (classical) ARF which is characterized by infinite mass $m_A$ and coordinate $\vec{r}_A = \vec{r}_0 = 0$ and $\vec{p}_A = \vec{p}_0 = 0$ in ARF, denoted also as $F^0$, but nonzero in some other ARF. At the later stage we can abandon this notion and consider only relations between quantum RF and observables defined in them. We should find two transformation operators - from ARF to quantum RF, and between two quantum RF, but we'll show that in most general approach they coincide. We'll start from the former case and use Jacoby canonical coordinates $\vec{u}_j$ ($l = 1, N_f$ denote corresponding RF) and conjugated momentums $\vec{p}_l$:

$$\begin{align*}
  \vec{u}_1 &= \vec{r}_N - \vec{r}_{N-1} , & \vec{u}_i &= \vec{r}_{N-i+1} - \vec{r}_{N-i}, \\
  \vec{u}_{N-1} &= \vec{r}_{N-1} - \vec{r}_1 , & \vec{u}_N &= \vec{R}_{cm} - \vec{r}_A
\end{align*}$$

(4)
where

\[ M^i_n = \sum_{j=i}^{n} m_j \]  

(If upper index i is omitted, it assumed that i = 1; also the vector sign omitted, where its use is obvious). The 'quantum' set \( u^k \) can be obtained from \( u^1 \) changing cyclically \( F^k \) and \( F^1 \) in the \( \vec{r}^i \) coordinates array so that \( r_1^i = r_k^i \), \( r_2^i = r_{k-1}^i \),..., \( r_k^i = r_1^i \). \( r_i^j \) is defined by the above formula in which \( r_j^i \) is substituted, and as the result \( u_{N-1}^k = r_2^s - r_k \), etc. For \( j \neq 1 \), \( r_j^i - r_1^i \) is the linear sum of several coordinates \( \vec{u}^i_1 \), so they don't commute, due to quantum movement of \( F^1 \). Conjugated to \( u^1_i \) (\( i = 1, N \)) canonical momentums are:

\[
\vec{\pi}^1_1 = \mu_i^1 (\vec{\tilde{p}}_{m_N}^{iN} - \vec{\tilde{p}}_{m_{N-1}^{-1}}^{iN})
\]

\[
\vec{\pi}^1_i = \mu_i^1 (\vec{\tilde{p}}_{m_{N-i+1}^{iN-i+1}}^{iN} - \vec{\tilde{p}}_{m_{N-1}^{-1}}^{iN})
\]

where \( \vec{\tilde{p}}^i = \sum_{j=i}^{N} \vec{p}_j \) The transformed free Hamiltonian of the system objects motion is:

\[
\hat{H}_e = \frac{(\vec{\pi}^1_N)^2}{2M_N} + \sum_{j=1}^{N-1} \frac{(\vec{\pi}^1_j)^2}{2\mu_i^j}
\]

(5)

where reduced mass \( \mu_i^{-1} = (M_N^{N-i+1})^{-1} + m_{N-i}^{-1} \).

To find the transformation between 2 quantum RF we start from the simplest case \( N_f = 2, N_g = 0 \). This is just the space reflection of \( F^1 \) coordinate \( u_2^1 = -u_1^1 \) performed by the parity operator \( \hat{P}_1 \). This discrete transformation is presented in the indirect form in all other cases. The next case \( N_f = 2, N_g = 1 \) is just the two \( u^1 \) coordinates linear transformation exchanging \( r_2, r_1 \):

\[ u_{1,2}^2 = \hat{U}_{2,1} u_1^1 \hat{U}_{2,1}^+ = a_{1,2} u_1^1 + b_{1,2} u_2^1 \]

The unitary operator \( \hat{U}_{2,1} \) in general can be decomposed as \( \hat{U} = \hat{C}_2 \hat{R} \hat{C}_1 \), where \( \hat{C}_{1,2} \) is conformal transformation of the kind \( u_i^1 = c_i u_i^1 \), such that \( c_1 c_2 = 1 \)

\[ c_1 = \left[ \frac{M_3 m_3 m_2}{m_1 (m_2 + m_1)^2} \right]^{\frac{1}{2}} \]

\( R \) is the rotation on \( u_{1,2}^0 \) intermediate coordinates hypersurface on the angle:

\[ \beta = \arccos \left( \frac{m_2 m_1}{(m_3 + m_2)(m_1 + m_3)} \right)^\frac{1}{2} \]

Then \( \hat{C}_2 \) results in \( u_2^2 = c'_i u_i^1 \). For the general case \( N > 3 \) it’s possible nonetheless to decompose the transformation from \( F^j \) to \( F^k \) as the product of such bilinear operators. Really if to denote as \( \hat{S}_{i+1,i} \) the operator exchanging \( F_i, F_{i+1} \) in \( u^1 \) set, which changes in fact only \( u_i^1, u_{i+1}^1 \) pair, values of other \( u_j^1 \) as easily seen conserved
under this exchange; and $\dot{U}_{2,1} = \dot{S}_{2,1}$ obviously. Then the transformation operator from $F^1$ to $F^k$ is:

$$\dot{U}_{k,1} = \dot{U}_{2,1} \dot{S}_{3,2} \cdots \dot{S}_{k,k-1}$$

It follows immediately that the transformation from $F^j$ to $F^k$ is $\dot{U}_{j,k} = \dot{U}_{k,1} \dot{U}_{j,1}^{-1}$.

Now we must find the transformation operator from the classical ARF to $F^1$, where ARF 'classical' set $\vec{u}_i^A = \vec{r}_{N-i+1} - \vec{r}_A$. Here we must formally consider each object as RF to perform the intermediate transformations and include ARF in $F^i$ array as $F^{N+1}$, so $N_f' = N' = N + 1$. Note that the set $u^1$ can be rewritten as the Jacoby 'quantum' set for ARF if we formally add to it the $\vec{u}_i^{N+1} = \vec{r}_A - \vec{r}_E$, where $E$ is some other classical RF. Then it’s easy to observe that acting by $\hat{P}_1$ operator, resulting in $u^1_A = -u^A_N$ gives $u^1$ 'quantum' set of (1) for $m_{N+1} \to \infty$. We must add to it formally $u_{N+1}^1 = u_{N+1}^1$ and then the operator in question is equal to $\dot{U}_{A,1} = \dot{U}_{N+1,1} \hat{P}_1$ for infinite $m_{N+1}$.

Note that even settling $\vec{r}_1 = 0$, $\vec{p}_i = 0$ for $F^1$ we must account them as the operators in commutation relations, as was stressed in (Dirac,1956). Neglecting it result in so called 'Quantum Frames Paradox' (Aharonov,1984), which have no independent significance.

This problem become more intricate if we want to account the possible quantum rotation of our RF relative to ARF. We’ll consider here only stationary rotational states for 2-dimensional rotations. If $F^1$ is the solid object its orientation relative to ARF can be extracted from the relative (internal) coordinates of $F^1$ constituents (atoms). For the simplicity we assume that $F^1$ have the dipole structure and all its mass concentrated around 2 points $\vec{r}_{a_1}, \vec{r}_{b_1}$ so that this relative coordinate is $F^1$ independent degree of freedom $\vec{r}_{a_1} - \vec{r}_{b_1}$ or in polar coordinates $r_{1}^d, \theta_1$. Note that $r_{1}^d$ is observable which eigenvalue defined by $F^1$ constituents interaction. Thus after performing transformation $\dot{U}_{A,1}$ to $F^1$ c.m. we’ll rotate all the objects (including ARF) around it on the uncertain angle $\theta_1$, so the complete transformation is $U_{A,1}^T = \dot{U}_{A,1} \dot{U}_{A,1}^*$. In its turn this rotation operator can be decomposed as $\dot{U}_{A,1}^* = \dot{U}_{A,1}^c \dot{U}_{A,1}^d$, representing the rotation of objects c.m coordinates $u_i^1$ and $F^i$ constituents coordinates. This rotation introduces the quantum uncertainty of ARF orientation relative to $F^1$. The rotation of $F^1$ is performed by the operator $\dot{U}_{A,1}^d$, which action settles $\theta_1$ to zero and introduce in place of it the new observable $\theta_1'$ which corresponds to ARF angle in $F^1$.

The objects c.m. coordinates transformation operator is:

$$\dot{U}_{A,1}^c = e^{-i\theta_1 L_z}, \quad L_z = \sum_{i=1}^{N} l_{zi}, \quad l_{zi} = -id/d\alpha_i$$

where $\alpha_i$ is the polar angle coordinate of $u_i^1$. It results in:

$$u_{x_1}^{1r} = u_{x_1}^1 \cos \theta_1 + u_{y_1}^1 \sin \theta_1$$
$$u_{y_1}^{1r} = -u_{x_1}^1 \sin \theta_1 + u_{y_1}^1 \cos \theta_1$$

So the new polar angle is $\alpha_1' = \alpha_1 - \theta_1$. The transformation of the external objects
momentums is analogous:

\[ \pi_{xi}^{1r} = \pi_{xi} \cos \theta_1 + \pi_{yi} \sin \theta_1 \]
\[ \pi_{yi}^{1r} = -\pi_{xi} \sin \theta_1 + \pi_{yi} \cos \theta_1 \]

As easily seen Hamiltonian \( \hat{H}_c \) of (5) is invariant under this transformation. We must account also rotation of any \( F^i \) internal degrees of freedom described by formula (1). Assuming that any \( F^i \) have analogous to \( F^1 \) dipole form their 'internal' Hamiltonian in ARF is:

\[
\hat{H}_i = \sum_{j=1}^{N_f} \left[ \frac{1}{m_{d_j} \theta_j} \frac{\partial^2}{\partial \theta_j^2} + V_j + \hat{H}_j^f \right]
\]  

(7)

where \( m_{d_j} \) is the effective mass of the rotational moment which for the dipole is equal to its reduced mass, \( \hat{H}_j^f \) is the part of \( F^j \) constituents free Hamiltonian of their relative motion which is rotationally invariant. \( V_j \) is the potential of \( F^j \) constituents interaction and is obviously invariant of \( F^j \) rotation.

If to denote \( \hat{P}^d_1 \) parity operator for \( \theta_1 \) \( l_j^d = -i \frac{\partial}{\partial \theta_j} \) then

\[
\hat{U}_{A,1}^d = \hat{P}^d_1 \exp(i \theta_1 L_d), \quad L_d = \sum_{i=2}^{N_f} l_i^d
\]

\[ \theta_j^r = \theta_j - \theta_1, \quad l_j^d = l_j^d \quad j \neq 1 \]
\[ \theta_1^r = \theta_{A}^r = -\theta_1, \quad l_1^d = l_A^d = -l_1^d - L_d \]

The new coordinates can be interpreted as corresponding to \( F^1 \) dipole rest frame, where its own angle \( \theta_1^r \) is fixed to zero but ARF angle in \( F^1 \) becomes uncertain and formally ARF acquires the orbital momentum \( l_A^d \).

The rotational part of Hamiltonian \( \hat{H}_i \) in this rest frame expressed through the new coordinates is:

\[
\hat{H}_i^r = \sum_{j=2}^{N_f} \frac{1}{m_{d_j} \theta_j} l_j^d \dot{\theta}_j^r + \left[ \frac{1}{m_d} \sum_{j=2}^{N_f} l_j^d \right]^2 + \frac{1}{m_1 \theta_1} l_1^d
\]

So we get the conclusive and noncontroversial description of \( G^i \) and \( F^i \) evolution in \( F^1 \) defined by Hamiltonian \( \hat{H}_c + \hat{H}_i \). Yet this transformation can result in the change of the objects \( G^i \) wave functions \( \Psi^1 \) in \( F^1 \) which will depend on \( F^1 \) orbital momentum which should be accounted performing the initial functions transformation.

Analogous considerations permit to find rotational transformation \( \hat{U}_{i,1}^R \) from \( F^1 \) to \( F^i \). We note that it just the additional rotation of all the objects on the angle \( \theta^r_i = \theta_i - \theta_1 \). So the form of \( \hat{U}_c \) part of rotational operator is unchanged and

\[
\hat{U}_{i,1}^d = \hat{P}^d_i e^{-i \sum_{j=1}^{N_f} l_j \dot{\theta}_j^r} e^{i \theta_1^r l_1^d}
\]

where \( \hat{P}^d_i \) is parity operator for \( \theta^r_i \).
For $d = 3$ the mathematical calculations are analogous, but more tedious, if to remind that any rotation in space can be decomposed as three consequent rotations in the specified orthogonal planes. So we omit the calculations for $d = 3$ here, and just explain what the new features appears. To describe this rotation $F^1$ should have the necessary structure, the simplest of which is the rigid triangle $abc$ with constituents masses concentrated in its vertexes. Then $Z'$ axe can be chosen to be orthogonal to the triangle plane and $X'$ directed along $ab$ side. Then the transformation which aligns ARF and $F^1$ axes can be performed rotating consequently ARF around $X', Y', Z'$ on the uncertain angles $\theta_x, \theta_y, \theta_z$. Each of three operators performing it is the analog of $\hat{U}^{R_{A1}}$ described above.

Now we regard the evolution equation in quantum RF taking the time $t$ as universal parameter independent of RF. Note that standard Schrödinger equation assumes mutely that observer for which the wave function is defined have infinite mass. For such observer ARF we define the system wave function $\psi_s(\vec{r}, t)$ in the standard picture. It satisfy to free Schrödinger equation for $N$ objects and with some initial conditions can be factorised into $\Phi_c(\vec{R}_c, t)\psi'_s(\vec{u}, t)$ as we discussed in chap. 1. Then the resulting equation for the relative motion of $N$ objects of $S_N$ can be obtained, if we remove system c.m. motion from $\hat{H}_c$ of (5). The resulting equation for $\psi'_s$ - wave function in $u^1$ coordinates is:

$$-\sum_{j=1}^{N-1} \frac{1}{2\mu'_j} \frac{\partial^2}{\partial u^2_j} \psi'_s(u^1, t) = i \frac{d\psi'_s}{dt}(u^1, t)$$

(8)

It is reduced to Schrödinger equation , if $m_1 \to \infty$ and is analogous to the equation for relative $e - p$ motion in Hydrogen atom (Schiff, 1955). Yet in distinction in $F^1$ rest frame ARF coordinate $\vec{r}_0 - \vec{r}_1$ becomes uncertain and to calculate its evolution through the coordinate $u^1_N$ in this equation we must use complete Hamiltonian $\hat{H}_c$. Then it’s easy to understand that if $m_0$ isn’t infinite, but just large we must change $M_N$ to $\mu'_N$ in the first $\hat{H}_c$ member in (5). We must notice the different form of $u^1_N, \vec{r}^1_N$ dependence on $\vec{r}_1$ in comparisons with other $\vec{u}_i, \vec{r}^1_i$. This can be regarded as ARF doesn’t belong to the studied system $S_N$, but is the 'outsider' object. This form of Jacoby coordinates can be extended in an obvious way for the description of the state of any other object of the universe. We conclude that equation (8), which can be modified if necessary to describe the evolution of 'outsider' objects in $F^1$ is the correct evolution equation which depends on observer $F^1$ mass.

3 Measurements aspects

Now we’ll discuss briefly the feasibility of the experiments with RF wave packets. Its principal scheme is analogous to described above gedankenexperiment where in the vacuum chamber the solid state detector-recorder initially rigidly fixed is released and suspended at $t = -T_L$. After some time period the detector performs the coordinate measurement of the particle $n$ from the very narrow beam, which permit to find the detector quantum displacement. As the detector-recorder system in fact can be used any detector with the memory like the photoemulsion which
have coordinate accuracy of the order .1 micron. Especially attractive seems to be the plastic or crystal track detectors which under the electron microscopic scanning can in principle define the position of dislocation induced by particle track with the accuracy up to several interatomic distances. The same order will have the initial packet smearing $a_0$, because it’s defined by the surface effects between the detector and fixator surfaces which extended to the interatomic scale. If we suppose the mass of the detector to be $10^{-10}$ gram (the mass of emulsion grain which acts as the elementary individual detector) and $a_0$ value $10^{-2}mk$ we get the average centre mass deflection of the order .1 mk for the exposition time $10^6$ sec i.e. about one week. Despite that the performance of such experiments will be technically extremely difficult it’s important nonetheless that no principal prohibitions for them exist.

Now we’ll consider the analogous experiment description in the framework of Decoherence model (DM) of quantum measurements (Zurek, 1982). It’s reasonable to take that the result of the particle-grain interaction is roughly dichotomic: or $n$ passes by without changing grain D initial state $|U_-\rangle$, or passes through and darken it corresponding to $|U_+\rangle$, i.e. to assume that D have the single dichotomic degree of freedom (DF). If the consequent interaction of D with the environment E results into the state collapse as DM assumes, then the crude measurement of the distance $\delta r$ between $n$ and D c.m. occurs. Let’s rewrite $n$ state vector in the discrete form

$$|\psi^n_G\rangle = a|+\rangle + b|−\rangle$$

where $|+, −\rangle$ are state components inside and outside $r_G$ vicinity. We’ll assume that $n$-D interaction, which Hamiltonian is given below turns in at $t = −T$ and turns off at $t = 0$, so that $T$ roughly corresponds ton time of flight through the grain.

$$H^{n-D} = g_0 (1 + \sigma^n_z)\sigma_u^x = g_0 |+\rangle \langle + | (|U_+\rangle \langle U_-| + |U_-\rangle \langle U_+|)$$

We choose $T$ to be equal to:

$$T = \frac{\pi \hbar}{2g_0}$$

Then at $t > 0$ D starts to interact with E having very large number $N_E$ of DF. In the regarded set-up its realization means that the vacuum chamber is open and some gas which molecules have single dichotomic DF $|±_i\rangle$ filled it. We assume that in $n$-D and D-E Hamiltonians free parts for $n, D$ in the evolution equations can be neglected. Their account doesn’t change results principally, but makes the calculations more complicated (Bell, 1975). D-E interaction Hamiltonian is additive relative to E molecules, and the interaction between the molecules and their free motion neglected:

$$H^{D-E}_i = \sum_{i=1}^{N_E} g_i \sigma_u^y \sigma_i^y$$

In Zurek model $g_i$ - E coupling constants and initial E molecules state vectors $\alpha_i, \beta_i$ values are distributed at random independently of each other, so that $g_{min} < g_i < g_{max}$. Solving Shrodinger equation we find $\Psi(t)$ from which the density matrix $\rho(t)$ of $n$-D system can be obtained tracing over $E$ DF, which assumedly are unobservable.

$$\rho(t) = |a|^2|s_+\rangle \langle s_+| + |b|^2|s_-\rangle \langle s_-| + iz(t)ab^*|s_+\rangle \langle s_-| - iz^*(t)ab|s_-\rangle \langle s_+|$$
where $|s^\pm\rangle = |\pm\rangle|U^\pm\rangle$,

$$z(t) = N_E \prod_{k=1}^{N_E} [\cos 2g_k t + i(|\alpha_k|^2 - |\beta_k|^2) \sin 2g_k t]$$

Zurek showed that for large $N$ $z(t)$ soon become very small and the corresponding difference between $\rho(t)$ and $\rho_M$ - matrix of completely mixed state also becomes very small. So we conclude that that the state of $n$-D system during its evolutions approximates to the collapsed mixed state, corresponding to the measurement of $n$-D relative distance. So it seems that the state evolution in this delocalized measurements mainly follows the collapse postulate. This results supports our point that it’s correct to restrict ourself to the two assumptions formulated in chap.1 in place of using any detailed collapse model.

4 Relativistic Equations

Obviously the most important aim for this model development is to give relativistically covariant quantum RF description, which will describe also the quantum time properties. Here we’ll argue that they are mainly analogous to the space coordinates properties found in the former chapter. We’ll apply the same approach concerned with the relativistic wave packets of solid objects -regarded as quantum RF. We’ll suppose that this observer can be described as the zero-spin boson , because normally all its constituents spins are roughly compensated. For the simplicity we’ll consider only 1-dimensional situations which permits us to neglect the quantum rotations.

In nonrelativistic mechanics time $t$ is universal and 'external' relative to any system and so is independent of any observer. In relativistic case each observer in principle has its own proper time $\tau$ measured by his clocks, which must be presented in relativistic evolution equation in his RF.

In our opinion there is a strong and deep analogy between irreversible wave function collapse in the measurement and clock hands motion+measurement which can be regarded as the system self-measurement (Horwitz,88). It was argued recently that physical time can originate from some non-Hamiltonian dynamics, because any Hamiltonian Dynamics automatically guarantees reversibility (Rovelly,90). Meanwhile without choosing one or other mechanism , it’s possible to assume that as in the case of the position measurement this internal processes can be disentangled from the clocks c.m. motion. Then clocks + observer i.e. RF $F^2$ wave packet evolution can be described by the relativistic equation for their c.m. motion. In this packet different momentums and consequently velocities relative to external observer $F^1$ are presented. It makes impossible to connect external time $\tau_1$ and $F^2$ proper time
τ_2 by any Lorentz transformation, which corresponds to the unique definite Lorentz factor γ(\vec{v})

To illustrate the main idea we remind the well-known situation with the relativistic lifetime dilation of unstable particles or metastable atoms. Imagine that the prepared beam of them is the superposition of two or more momentums eigenstates having different Lorentz factors \gamma_i. Then detecting their decay products we’ll find the superposition of several lifetime exponents, resulting from the fact that for each beam component Lorentz time boost has its own value. If in some sense this unstable state can be regarded as elementary clock when their time rate for the external observer is defined by the superposition of Lorentz boosts responding to this momentums.

From this arguments we can assume that the proper time of any quantum RF being the parameter in his rest frame simultaneously will be the operator for other quantum RF. If this is the case the proper time \tau_2 of F^2 in F^1 can be the parameter depending operator, where parameter is \tau_1.

$$\hat{\tau}_2 = \hat{F}(\tau_1) = \hat{B}_{12}\tau_1$$

,where \hat{B}_{12} can be called Lorentz boost operator, which can be the function of F^1, F^2 relative momentum. To define its form it’s necessary first to find Hamiltonian of free particle G^2 in F^1. Obviously in relativistic case Hamiltonian of relative motion of very heavy RF and light particle should approximate Klein-Gordon square root Hamiltonian, but in general it can differ from it (Schweber, 1961). The main idea how to find it is the same as in nonrelativistic case: to separate the system c. m. motion and the relative motion of the system parts, but in relativistic case this is much more complicated problem (Coester, 1965). For the simplicity we’ll consider first the evolution of RF F^1 and the particle G^2 which observables are defined in classic ARF. In Classical Relativity the objects relative motion is characterized by their invariant mass square \(s^m\), which is equal to system total energy in its c.m.s., equivalent of nonrelativistic c.m. kinetic energy \(E_k\). In our case it’s equal to:

$$s^m_{12} = (m_1^2 + \vec{q}_{12}^2)^\frac{1}{2} + (m_2^2 + \vec{q}_{12}^2)^\frac{1}{2}$$

,where \vec{q}_{12} is G^2 momentum in c.m.s. . We also define G^2 momentum in F^1 rest frame which corresponds to Klein-Gordon momentum operator:

$$\vec{p}_{12} = \frac{s^m_{12}\vec{q}_{12}}{m_1} = \frac{E_1\vec{p}_2 - E_2\vec{p}_1}{m_1}$$

where \(E_i, \vec{p}_i\) are total energies and momentums in ARF. We can expect that Hamiltonian of free particle in F^1 will correspond to 4-th component of \(\vec{p}_{12}\) 3-vector. Really if to transform F^1, G^2 total momentum in c.m.s., which 4-th component is \(s^m_{12}\) to F^1 rest frame one obtains:

$$E_1 = [(s_{12}^m)^2 + \vec{p}_{12}^2]^\frac{1}{2} = m_1 + (m_2^2 + \vec{p}_{12}^2)^\frac{1}{2}$$

Following this classical analysis in accordance with Correspondence Principle we can regard \(E_1\) as possible Hamiltonian \(\hat{H}^1\) in F^1 rest frame and the evolution equation
for $G_2$ for corresponding proper time $\tau_1$ is:

$$\hat{H}^1 \psi^1(\vec{p}_{12}, \tau_1) = -i \frac{d\psi^1(\vec{p}_{12}, \tau_1)}{d\tau_1}$$  \hspace{1cm} (9)$$

It's easy to note that $\hat{H}^1$ depends only of relative motion observables and in particular can be rewritten as function of $\vec{q}_{12}$. This equation coincides with Klein-Gordon one, where it's possible to consider $m_1$ as arbitrary constant added to total energy. Consequently we can use in $F^1$ the same momentum eigenstates spectral decomposition (Schweber, 1961).

Space coordinates in $F^1$ is difficult to define unambiguously, as usual in relativistic problems, so we choose the general form of Newton-Wigner ansatz (Wigner, 1986).

$$x_{12} = i \frac{d}{dp_{12}} + F_x(\vec{p}_{12})$$

If we consider the evolution of RF $F^2$ in place of $G^2$ described by the same equation, then its proper time operator $\tau_2$ in $F^1$ can be defined from the correspondence with the classical Lorentz time boost as:

$$\hat{\tau}_2 = \hat{B}_{12}\tau_1 = [(s_{12}^m)^2 - m_1^2 - m_2^2]^{-1}m_1m_2\tau_1$$

Note that this form is completely symmetrical and the same operator relates the time $\tau_1$ in $F^1$ and $F^2$ proper time - parameter $\tau_2$. Despite this novelty no qualitatively new physical effects for the individual observer $F^2$ in addition to described in the previous chapters appears. By himself (or itself) $F^2$ can’t find any consequences of time arrow superpositions registrated by external $F^1$, for $F^2$ exists only unique proper time $\tau_2$. The only new effect will be found when $F^1$ and $F^2$ will compare their initially synchronized clocks. If this experiment will be repeated identically several times (to perform quantum ensemble) they find not only the standard Lorentz proper times difference, but also the statistical spread having quantum origin and proportional to the time interval and $F^2$ momentum spread.

Analoguously to Classical Relativity average time boost depends on whether $F^1$ measures $F^2$ observables, as we considered or vice versa. To perform this measurement we must have at least two synchronized objects $F^1_a$ and $F^1_b$, which make two $F^1$ and $F^2$ nonequivalent.

If the number of objects $N > 2$ we’ll use multilevel ‘clasterization’ formalism, which will be described here for the case $N = 3$ (Coester, 1965). In its framework Hamiltonian in $F^1$ and describing the two particles $G^2, G^3$ state evolution for proper time $\tau_1$ is equal to:

$$\hat{H}^1 = m_1 + [(s_{23}^m)^2 + p_{1,2+3}^2]^{\frac{1}{2}}$$

,where $s_{23}^m$ is two particles $G^2, G^3$ invariant mass given above. In this formalism at first level we consider their relative motion defined by $\vec{q}_{23}$. At second level we regard them as the single quasiparticle - cluster with mass $s_{23}^m$. Then $\vec{p}_{1,2+3}$ is the cluster momentum in $F^1$. So at any level we regard the relative motion of two objects only. Despite that this Hamiltonian isn’t factorized between this two levels some
of its feature is analogous to the nonrelativistic one of (5). We can extract small Hilbert space $H_{12}^l$, which basis is $|\vec{p}_{12}\rangle$ from the total space $H_s^l$ (Coester, 1965). In this small space the evolution of $m_2$ state $\psi'(\vec{p}_{12}, \tau_1)$ defined in $F^1_l$ can be described completely. The evolution of $m_3$ depends in its turn on $s_{12}^m$ and it means in fact one-way factorization of the cluster momentums states. This procedure can be extended in the obvious inductive way to incorporate an arbitrary number of the objects.

Now we’ll regard the simple and crude model of the quantum clocks and RF in which $F^1_l$ includes some ensemble (for example the crystal) of $\beta$-radioactive atoms. Their nucleus can radiate neutrino $\nu$ (together with the electron partner) which due to its very small cross-section practically can’t be reflected by any mirror and reabsorbed by this nucleus to restore the initial state. Then for our purposes this decay can be regarded as the irreversible stochastic process. Taking the trace over $\nu$ degrees of freedom, the final nucleus state can be described by the density matrix of mixed state $\rho_N(t)$ and the proper time of this clocks of $F^1_l$ can be defined as:

$$\tau_1 = -T_d \ln(1 - \frac{N_d}{N_0})$$

where $N_0$ is the initial number of this atoms $N_d$ - the number of decays, $T_d$ is the nucleus lifetime . It’s easy to understand from the previous discussion how the superposition of Lorentz boosts can be applied to such system state, if it has momentum spread .

We consider in fact infrared limit for macroscopic object, so the role of negative energy states, which is important for the standard relativistic problems must be small. Despite the locality paradox found for the relativistic wave packets we can expect this solutions to be valid at least in the infrared limit and beyond , when the packet size is much larger then Compton length (Hegerfeldt, 1976).

5 Concluding Remarks

We’ve shown that the extrapolation of QM laws on the macroscopic objects demands to change the approach to the space-time coordinate frames which was taken copiously from Classical Physics. It seems that QM permits the existence of RF manifold, the transformations between which principally can’t be reduced to Galilean or Lorentz transformations. This new global symmetry means that observer can’t measure its own spread in space, so as follows from Mach Principle it doesn’t exist. The physical meaning have only the spread of relative coordinates of RF and some external object which can be measured by this RF or other observer.

Historically QM formulation started from defining the wave functions on Euclidian 3-space $R^3$, which constitute Hilbert space $H_s$. In the alternative approach developed here we can regard $H_s$ as primordial states manifold. Introducing particular Hamiltonian results in the relative assymmetry of $H_s$ vectors which permit to define $R^3$ as a spectrum of the continuous observable $\hat{r}$ which eigenstates are $|\vec{r}_i\rangle$. But as we’ve shown for several quantum objects one of which is RF this definition become ambiguous and have several alternative solutions defining $R^3$ on $H_s$. In the
relativistic case the situation is more complicated, yet as we’ve shown it results in ambiguous Minkovsky space-time definition.

At first sight Q-transformation will violate Locality principle, but it’s easy to see that it holds for each particular RF, despite that the coordinate point in one RF doesn’t transforms into the point in other RF. This is easy to see for the nonrelativistic potential $V(r_2-r_1)$, but we can expect it true also in relativistic Field Theory. So we can suppose the generalisation of locality principle for Q-transformations, which yet must be formulated in a closed form.

In our work we demanded strictly that each RF must be quantum observer i.e. to be able to measure state vector parameters. But we should understand whether this ability is main property characterising RF? In classical Physics this ability doesn’t influence the system principal dynamical properties. In QM at first sight we can’t claim it true or false finally because we don’t have the established theory of collapse. But it can be seen from our analysis that collapse is needed in any RF only to measure the wave functions parameters at some $t$. Alternatevely this parameters at any RF can be calculated given the initial experimental conditions without performing the additional measurements. It’s quite reasonable to take that quantum states have objective meaning and exist independently of their measurability by the particular observer, so this ability probably can’t be decisive for this problem. It means that we can connect RF with the system which doesn’t include detectors, which can weaken and simplify our assumptions about RF. We can assume that more important for the object to be regarded as RF is the ability to reproduce space and time points ordering and to record it, as the solid states like the crystalls and the atomic clocks can do.

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