A Biased Review of Sociophysics

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Abstract. Various aspects of recent sociophysics research are shortly reviewed: Schelling model as an example for lack of interdisciplinary cooperation, opinion dynamics, combat, and citation statistics as an example for strong interdisciplinarity.

1 Introduction

The idea of applying physics methods to social phenomena goes back centuries ago, e.g. with the first (unsuccessful) attempt to establish mortality tables, involving astronomer Halley, the “sociology” of Auguste Comte who taught analysis and mechanics around 1840, or the 1869 book by Quetelet “Physique Sociale”. Majorana suggested to apply quantum physics in 1942 [5]. Some contemporary physicists [3, 4] have worked on the field since some decades. But it became a physics fashion about a dozen years ago, with opinion dynamics, applications of complex networks, etc. [6]. Presumably the best review is still the one from Italy during Berlusconi’s rule [1], which killed the present author’s chances to get a Nobel prize (in literature: science fiction) for his four articles in [2] on languages (p. 49), opinions (p. 56), retirement demography (p. 69) and Bonabeau hierarchies (p. 75). The field is now far too wide to be covered in a short review, and thus here only some biased selection is presented. Lecture notes of Fortunato [7] start with a nice introduction into the more ancient history of sociophysics, Galam wrote a recent book [8], with pages 75-77 on: The Soviet-Style Rewriting of the History of Sociophysics.

We start with a discussion why it may be useful to apply physics research style to human beings, then we bring the Schelling model as an example where sociophysics was lacking for decades. The following three sections review opinion dynamics, combat, and citations, followed (for readers outside statistical physics) by a critique of mean field theory. Econophysics is regarded here as outside of sociophysics, and also ignored because of recent reviews are languages [9] [10] [11], Penna ageing models [11] [12], networks [13] and traffic jams [14].

2 Does Sociophysics Make Any Sense?

People are not atoms. We may be able to understand quite accurately the structure of the hydrogen atom, but who really understands the own marriage partner. Nevertheless already Empedokles in Sicily more than two millennia ago found that people are like fluids: some people mix easily like water and wine (an ancient Greek crime against humanity), and some like water and oil do not mix. And a few months ago the German historian Imanuel Geiss died, who
described the decay of empires with Newton’s law of gravitational forces (but disliked simulations to explain diplomatic actions during the few weeks before World War I).

It is the law of big numbers which allows the application of statistical physics methods. If we throw one coin we cannot predict how it will fall, and if we look at one person we cannot predict how this person will vote, when it will die, etc. But if we throw thousand coins, we can predict that about 500 will fall on one side and the rest on the opposite side (except when we cheat, or the coin sticks in the mud of a sport arena). And when we ask thousand randomly selected people we may get a reasonable impression for an upcoming election. Half a millennium ago, insurance against loss of ships in the Mediterranean trade became possible, and life insurance relies on mortality and the Gompertz law of 1825 that the adult probability to die within the next year (better: next month) increases exponentially with age. Such insurance is possible because it relies on the large number of insured people: Some get money from the insurance and most don’t. My insurance got years ago most of my savings and now pays me a monthly pension until I die; the more the journal referees make troubles to my articles, the sooner I die and the less loss my insurance company will make with me. Only when all the banks and governments are coupled together by their debts, the law of large numbers no longer is valid since they all become one single unit (‘Maastricht’ rules on sovereign debts were broken by governments in Euroland since 1998, a decade before the Lehman crash.)

Outside physics the method of agent-based computer simulations is fashionable. It has nothing to do with 007 James Bond, but refers to methods simulating single persons etc instead of averaging over all of them. Physicists do that at least since 1953 with the Metropolis algorithm of statistical physics, also in most of the simulations listed here. This book by non-physicists, written about simultaneously and independently from one by physicists, covers similar fields and similar methods but barely overlaps in the references. Recent work from cognitive science and related disciplines is listed in.

Did sociophysics have practical applications? When I got the work of Galam, Gefen and Shapir, I told a younger colleague that I liked it. But after reading it he disliked it and remarked to me that the paper helps management to control its workers better if a strike is possible. In the three decades since then I had read about many strikes but not about any being prevented by this paper. Two decades later Galam was criticised by other physicists for having helped terrorists with his percolation application to terror support. I am not aware that such percolation theory was applied in practice. However, a century ago physicist did not believe that nuclear energy can be used. Our own subsection “Retirement Demography” in ch. 6 of recommends immigration and higher retirement ages to balance the ageing of Europe; both aspects are highly controversial but I was not yet murdered. Helbing’s simulation that a column before a door improves the speed of evacuation during a panic seems to me very practical.
3 Schelling Model for Urban Ghettos

The formation of urban ghettos is well known in the USA and elsewhere. Harlem in Manhattan (New York) is the most famous “black” district since nearly one century, extending over dozens of blocks in north-south direction. Was it formed by conscious discrimination e.g. from real-estate agencies, or was it the self-organised result of the preferences of residents to have neighbours of the same group? Of course, the Warsaw Ghetto, famous for its 1943 uprising, was formed by Nazi Germany. Four decades ago, Schelling [21] showed by a simple Monte Carlo simulation (by hand, not by computer) of two groups A and B, that a slight preference of A people to have A neighbors, and of B people to have B neighbors, suffices to form clusters of predominantly A and predominantly B on a square lattice with some empty sites, out of an initially random distribution.

Statistical physicists of course would think of the standard Ising model on a square lattice to understand such a question, with A people corresponding to up spins and B people to down spins. Their ferromagnetic interaction gives a preference of A for A neighbours, and the same for B. The temperature introduces randomness. Simulations with Kawasaki dynamics (conserved magnetisation) have been made since decades. However, it took three decades before physicists took up the Schelling model; see [22, 23] for an early and some recent (physics) publications.

It seems quite trivial that the equilibrium distribution is no longer random if people select their residence with A/B preferences; but does it lead to “infinite” ghettos, i.e. to domains which increase in size to infinity if the studied lattice tends to infinity? This phase separation is well studied in the two-dimensional Ising model: for temperatures $T$ above a critical temperature $T_c$, only finite clusters are formed. For temperatures below this critical temperature, one large domain consisting mainly of group A, and another large domain consisting mainly of group B, are formed after a simulation time proportional to a power of the lattice size.

Schelling could not see that his model does not give large ghettos, only small clusters [21] as for $T > T_c$ in Ising models., but Jones [24] (from a sociology institute, publishing in a sociology journal) corrected that by introducing more randomness into the Schelling model; and Vinkovic (astrophysicist) and Kirman (economist) did it two decades after Jones. Then large ghettos are formed as for Ising models for $T < T_c$. Nevertheless, the Jones paper today is cited much more rarely than Schelling, and mostly by physicists, not by his sociology colleagues (see www.newisiknowledge.com for the Science Citation Index). Only now the physics and sociology communities show some cross-citations for the Schelling model.

Of course, instead of merely two groups A and B one could look at several. Empirical data for the preferred neighbours among four groups listed as White, Black, Hispanic and Asian in Los Angeles are given by [25]. Corresponding Potts generalisations of the Schelling-Ising version of [22] were published earlier [26]. For Schelling models on networks, one may break the links between nodes occupied by neighbors from different groups [27], as done before for other
networks [28].

In summary of this section, cooperation of physicists with sociologists could have pushed research progress by many years.

4 Opinion Dynamics

Much of the opinion simulations, [29] and ch. 6 in [11], is based on the voter or majority-vote models [30], the negotiators of Deffuant et al [31], the opportunists of Hegselmann and Krause [33], and the Sznajd missionaries [34], the latter three originating all near the year 2000. They check if originally randomly distributed opinions converge towards one (“consensus”), two (“polarisation”) or more (“fragmentation”) shared opinions. (Warning: In some fields “polarisation” means a non-centric consensus, like in ferroelectrics.) See also [4].

The voter or majority-vote models [30] are Ising-like: Opinions are +1 or −1, and at each iteration everybody follows one randomly selected neighbour or the majority of the neighbourhood, respectively, except that with a probability $q$ (which corresponds to thermal noise) it refuses to do so. Ref.[30] gave a recent application.

Negotiators of Deffuant et al [31] each have an opinion which can be represented by a real number or by an integer. (Opinions on more than one subject are possible [32] but we first deal with one opinion only. Integer opinions are simplifications and often used in opinion polls when people are asked if they agree fully, partly, or not at all with an assertion.) Each agent interacts during one time step with a randomly selected other negotiator. If their two opinions differ, each opinion shifts partly to that of the other negotiator, by a fraction of the difference. If that fraction is 1/2, they agree which is less realistic than if they nearly agree (fraction $< 1/2$). But if the two opinions are too far apart, they don’t even start to negotiate and their opinions remain unchanged. Thus there are no periodic boundary conditions applied to opinions; in contrast to real politics, the extreme Left and the extreme Right do not cooperate. (Axelrod studied some aspects already earlier [35].)

The opportunists also talk only with people who are not too far away from their own opinion (a real number). Each person at each time step takes the average opinion of all the people in the system which do not differ too much of their own past opinion. Thus in contrast to the binary interactions of negotiators we have multi-agent interactions of opportunists. (Instead of opportunism one can also talk of compromise here but that word applies better to the negotiators of Deffuant et al.)

Finally, the missionaries of the Sznajd model [34] try to convince the neighbourhood of their own opinion. They succeed if and only if two neighbouring missionaries agree among themselves; then they force this opinion onto their neighbourhood (i.e. on six neighbours for a square lattice). For only two opinions on the square lattice, one has a phase transition depending on the initial fraction of randomly distributed opinions: The opinion which initially had a (small) majority attracts the whole population to its side. In one dimension
no such transition takes place \[34\], just as in the Ising model. For a review of Sznajd models see \[36\].

A review of both negotiators and opportunists was given by Lorenz \[29\]. More information on missionaries, negotiators and opportunists, also for more than one subject one has an opinion on \[32\], are given in \[11\].

A connection between the above opinion dynamics and econophysics are the modifications of the usual Ising model to simulate tax evasion. Spin up corresponds to honest tax payers, and spin down to people who cheat on their income tax declaration. (I am not an experimental physicist). For \( T > T_c \) without any modification, half of these Ising tax payers cheat. However, if every year with probability \( p \) the declarations are audited and fraud is detected, and if discovered tax cheaters then become honest for \( k \) consecutive years, the fraction of tax cheaters not surprisingly goes down towards zero for increasing \( p \) and/or \( k \) \[37, 38\], also on various networks. Journal of Economic Psychology even plans a special issue on tax evasion. According to www.taxjustice.net, July 2012, hundreds of Giga-Dollars of taxes due are not paid world-wide each year.

5 Wars and Lesser Evils

Reality is not as peaceful as a computer simulation, and World War II was presumably the most deadly of all wars, with World War I far behind.

The “guilt” for World War I was hotly debated for decades; in contrast to many books and articles, the Versailles peace treaty of 1919 did not blame only Germany for this war. Richardson \[39\] used simple differential equations to explain this and other wars as coming from the other side’s armaments and own dissatisfaction with the status quo, while the cost of armaments and war pushes for peace. Later work \[40\] used human beings to simulate ten leaders during the weeks before World War I; these simulators made more peaceful decisions than the politicians of July 1914. Another work simulated only the German emperor and the Russian tsar \[41\] while \[42\] for a more complex study used a supercomputer of that time. Historians criticised that work because it relied on outdated history books which explained the war more by accidents than by intentions. Except for Richardson \[39\] this early work is barely cited in physics journals.

The creation of the anti-Hitler coalition of World War II was simulated by Axelrod and Bennett \[35\]. It contains numerous parameters describing the properties of European countries and their interrelations and thus is difficult to reproduce. Galam \[43\] has more fundamental criticism of that work.

The decay of Yugoslavia led to the most murderous wars in Europe after World War II, particularly in Bosnia-Hercegovina 1992-1995 including genocide. The emergence of local fighting between three groups was simulated by Lim et al. \[44\] using a Potts model. Intergroup fighting is possible if the regions where one group dominates are neither too small not too large. Lim et al. apply their model to the Bosnia-Hercegovina war but neglect the outside
initiation and influence from Belgrade (Serbia) and Zagreb (Croatia) in that war \[45\]. Such influence was partially taken into account in a linguistic simulation later \[46\]. Fig.1 shows computer simulations of egoist, ethnocentric, altruistic and cosmopolitan behaviour in a population \[47\]. Often the Yugoslavia wars are described as ethnic. “Ethnic”, defined e.g. through language, religion \[48\], history, biology (race, “blood”, DNA) is now part of international law through UN resolutions since 1992 against “ethnic cleansing”. Ethnicity is often construed or even imposed on people \[49\], but the deaths and losses of homeland are real.

How to win a war is another question. Kress \[50\] reviewed modern simulation methods of war and other armed conflicts. Mongin \[51\] applied game theory to a military decision made by Napoléon before the battle of Waterloo, two centuries ago. Mongin concludes that the decision was correct; nevertheless Napoléon lost and ABBA won.

Revolutions may lead to war; the ones of 2011 in Tunisia and Egypt did not and inspired an Ising model for revolutions \[52\]. Ising spins point up for people
wanting change, and down for staying with the government. They are influenced by an up-field proportional to the number of up spins [53], and by a random local down field measuring the conservative tendency of each individual “spin”. Initially all spins are down, and they flip irreversibly up by heat bath kinetics. After some time, spontaneously through thermal fluctuations and without the help of initial revolutionaries like Lenin and Trotsky, enough revolutionary opinions have developed to flip the magnetisation from negative to positive values. This time obeys an Arrhenius law proportional to \( \exp(\text{const}/T) \), Fig. 2.

The US War of Independence is the reason that the British game of football is called soccer in the USA. [54] confirmed that single stars in the team are not enough; one needs multi-player team coordination. And [55] found that goals are not achieved randomly in time; scoring one goal increases the chances for the scoring team to make another goal soon thereafter. (See [56] for lesser games.)

The sad state of football in the author’s home town, mentioned by the New York Times, prohibits further discussion.

Figure 2: Time until revolution versus \( T_c/T \). A straight line here means an Arrhenius law; the line in the figure corresponds to \( L = 1001 \), the plus signs to \( L = 301 \); also \( L = 3001 \) (for higher temperatures only) gave about the same flip times. From [52].
6 Citations

The Science Citation Index (www.newisiknowledge.com) is expensive but useful. One can find which later journal articles cited a given paper or book, provided the company of the Institute of Scientific Information subscribes to that journal. Since the end of the 1960s I check my citations on it. But one should be aware of the fact that cited books are listed not at all or only under the name of the first author, while cited journal articles are also listed under the name of the further authors. And citing books are ignored completely. For example, the most cited work of the late B.B. Mandelbrot is his 1982 book: The Fractal Geometry of Nature. The nearly 8000 citations can be found on the Web of Science under “Cited Reference Search”, but if after “Search” and “Create Citation Record” one gets his whole list of publications, ranked by the number of citations, the book is missing there and a journal article with much less citations heads the ranked list.

Thus it is dangerous to determine scientific quality by the number of citations or by the Hirsch index (h-index with “Create Citation Record”) [57] as long as books are ignored. An author who knows the own books and their first authors can include the cited books on “Cited Reference Search”, but automated citation counts like the h-index ignore them. If scholars get jobs or grants according to their h-index of other book-ignoring citation counts, this quality criterion will discourage them to write books, and push them to publish in Science or Nature. This is less dangerous for physics than for historiography, but seldomly mentioned in the literature.

(To determine the h-index, the ranked list of journal articles produced by “Create Citation Record” starts with the most-cited paper with \( n_1 \) citations, then comes the second-most cited one with \( n_2 \) citations, and in general the \( r \)-ranked article with \( n_r \) citations. Thus, \( n_r \) decreases with increasing rank \( r \). The h-index is that value of \( r \) for which \( n_r = r \).

Physicists have systematically analysed citation counts (instead of merely counting their own and those of their main enemies) at least since Redner 1998 [58]. His work was cited more than 500 times, about half as much as the later h-index [57]. Recent papers by physicists deal with universality in citation statistics [59], the tails of the citation distribution [60], co-author ranking [61], allocation among coauthors [62], and clustering within citation networks [63].

Other criticism of quality measures via citations are better known [64]. One should not forget, however, that experienced scientists often have to grade works of their students; are these evaluations more fair than citation counts? And what about peer review? My own referee reports are infallible, but those for my papers are nearly always utterly unfair. Measuring quality is difficult, but the one who evaluates others has to accept that (s)he is also evaluated by others. As US president and peace Nobel laureate Jimmy Carter said three decades ago: “Life is unfair.”

And as the citation lists for Redner [58] and Hirsch [57] show, this problem is truely interdisciplinary.
7 Critique of Mean Field Theories

This section explains mean field theory for readers from outside statistical physics, as well as its dangers.

If you want to get answers by paper and pencil, you can use the mean field approximation (also called molecular field approximation), which in economics corresponds to the approximation by a representative agent. Let us take the Ising model on an $L \times L$ square lattice, with spins (magnetic moments, binary variables, Republicans or Democrats) $S_i = \pm 1$ and an energy

$$E = -J \sum_{<ij>} S_i S_j - H \sum_i S_i$$

where the first sum goes over all ordered pairs of neighbor sites $i$ and $j$. Thus the “bond” between sites $A$ and $B$ appears only once in this sum, and not twice. The second sum proportional to the magnetic up field $H$ runs over all sites of the system. We approximate in the first sum the $S_j$ by its average value, which is just the normalised magnetisation

$$m = M/L^2 = \frac{1}{L^2} \sum_i S_i$$

Then the energy is

$$E = -J \sum_{<ij>} S_i m - H \sum_i S_i = -H_{eff} \sum_i S_i$$

with the effective field

$$H_{eff} = H + J \sum_j m = H + J q m$$

where the sum runs over the $q$ neighbours only and is proportional to the magnetization $m$. Thus the energy $E_i$ of spin $i$ no longer is coupled to other spins $j$ and equals $\pm H_{eff}$. The probabilities $p$ for up and down orientations are now

$$p(S_i = +1) = \frac{1}{Z} \exp(H_{eff}/T); \quad p(S_i = -1) = \frac{1}{Z} \exp(-H_{eff}/T)$$

with

$$Z = \exp(H_{eff}/T) + \exp(-H_{eff}/T)$$

and thus

$$m = p(S_i = +1) - p(S_i = -1) = \tanh(H_{eff}/T) = \tanh[(H + J q m)/T]$$

with the function $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$. This implicit equation can be solved graphically; for small $m$ and $H/T$, $\tanh(x) = x - x^3/3 + \ldots$ gives

$$H/T = (1 - T_c/T)m + \frac{1}{3} m^3 + \ldots; \quad T_c = q J$$
related to Lev Davidovich Landau’s theory of 1937 for critical phenomena \((T \text{ near } T_c, m \text{ and } H/T \text{ small})\) near phase transitions.

All this looks very nice except that it is wrong: In the one-dimensional Ising model, \(T_c\) is zero instead of the mean field approximation \(T_c = qJ\). The larger the number of neighbours and the dimensionality of the lattice is, the more accurate is the mean field approximation. Basically, the approximation to replace \(S_i S_j\) by an average \(S_i m\) takes into account the influence of \(S_j\) on \(S_i\) but not the fact that this \(S_j\) again influences \(S_i\) creating a feedback.

Thus, instead of using mean field approximations, one should treat each spin (each human being, …) individually and not as an average. Outside of physics such simulations of many individuals are often called “agent based” \([17]\), presumably the first one was the Metropolis algorithm published in 1953 by the group of Edward Teller, who is historically known from the US hydrogen bomb and Strategic Defense Initiative (Star Wars, SDI).

Of course, physicists are not the only ones who noticed the pitfalls of mean field approximations. For example, a historian \([65]\) years ago criticised political psychology and social sciences: “There is no collective individual” or “generalised individual”. And common sense tells us that no German woman gave birth to 1.4 babies, even though this is the average since about four decades. A medical application is screening for prostate cancer. Committees in USA, Germany and France in recent months recommended against routine screening for PSA (prostate-specific antigen) in male blood, since this simple test is neither a sufficient nor a necessary indication for cancer. However, I am not average, and when PSA concentration doubles each semester while tissue tests called biopsies fail to see cancer, then relying on PSA warnings is better than relying on averages.

8 Conclusion

Inspite of much talk since decades about the need for interdisciplinary research, the bibliographies on the same subject by authors from different disciplines do not show as much overlap as they should (and as they do in citation analysis). Perhaps the present (literature) review helps to improve this situation.

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