Theoretical investigation of induction heating of a two-layer conducting body by the example of an induction crucible furnace with a conductive crucible

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Abstract. Influence of the reflected electromagnetic wave on the electrical characteristics of an induction crucible furnace with a conductive crucible is considered. The review and comparison of the known methods of calculation of induction crucible furnaces with a conductive crucible have been carried out. A mathematical model of an induction crucible furnace with a ferromagnetic conductive crucible in the ELCUT package was developed. Electrical and power characteristics of the furnace were obtained depending on the frequency, temperature and current of the inductor.

1. Introduction

Induction crucible furnaces with a conductive crucible (hereinafter ICF with CC) (Figure 1) are used for melting copper and magnesium alloys, gold, as well as non-conductive materials [1, 2]. When the wall thickness of the crucible is half the depth of penetration of the electromagnetic wave (EMW) into the material of the crucible, the EMW penetrates into the charge, and energy is also released in it. In this way, it is possible to talk about induction heating of a two-layer body, which forms a crucible and a charge. Wherein each layer has its own value of the electrical resistivity \( \rho \) and the relative magnetic permeability. The questions of induction heating of a two-layer body with reference to the case of surface hardening of steel preforms are considered in the works A.M. Weinberg, A.E. Slukhotsky, V.B. and etc.

2. Calculation of the electrical characteristics of ICF with a nonmagnetic conductive crucible

Electric calculation of the furnace described above is carried out in a universal finite element program package ELCUT 6.0 (professional version) by solving the problem of the magnetic field of alternating currents.

Analysis of the magnetic field of alternating currents consists in calculating the electric and magnetic fields excited by the applied currents (sinusoidally varying in time) currents or an external variable field.

The full current in the conductor is considered as the sum of the external current caused by the externally applied voltage and the eddy current induced by the alternating magnetic field:

\[
\mathbf{j} = j_{\text{ext}} + j_{\text{eddy}}.
\]

The problem is formulated as a partial differential equation with respect to the complex amplitude of the vector magnetic potential \( A (B = \text{rot} A, B - \text{magnetic induction vector}) \). The magnetic induction vector is assumed to lie
in the plane of the $zr$ model (Figure 3), while the vector of the electric current density $j$ and the vector magnetic potential $A$ are orthogonal to it.

The equation for axisymmetric task will be:

$$\frac{\partial}{\partial r} \left( \frac{1}{r \mu_z} \frac{\partial A}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_z} \frac{\partial A}{\partial z} \right) - i \omega \gamma A = -j_{ext}.$$  

where a direct-current conductivity $\gamma$ and components of a tensor of magnetic conductivity $\mu_z$ and $\mu_r$ are constants within each block of the model.

Fig. 2 shows the calculated area divided by the finite elements. The number of grid nodes is about 350000. Grid irregularities can be achieved by setting different values of the sampling step for different sub-regions of the computational domain and directly at the nodes marked in Figure 3. Model allows one, by changing the properties of blocks, to perform calculations for the cases of a graphite crucible without metal and a crucible with different metal levels. The Neumann boundary condition $dH/dr = 0$ (the tangential component of the magnetic field strength) is given on the symmetry axis (boundary 1 in Fig. 3), at the outer boundaries (boundary 2) is the zero Dirichlet boundary condition $A = 0$ (the value of the magnetic Potential, i.e. the field is localized within the design area).

Experience shows that the radius of the computational domain (the distance along the $r$-axis in Figure 3.) should be set 2 - 2.5 times greater than the inner radius of the inductor of the furnace. And the distance of $z$ axis from the bottom of the crucible to the right to the border 2 and from the collar to the left to the boundary 2 – is approximately equal to the inner radius of the inductor.

As at the initial stage of the technological process of melting of metal in ITP with PT loading of a crucible represents lumpy fusion mixture which separate pieces in the electric relation are almost not bound among themselves. Induced currents become isolated generally within each piece, and the equivalent resistance of such lumpy loading several times surpasses similar value at fluid copper. At this stage by existence in a crucible metal (in particular copper) furnace charges it is possible to neglect and use the results received for a crucible without metal. It was confirmed experimentally [12].
Calculating \( \rho \) the graphite was taken to be \( 7.14 \times 10^{-6} \text{ Om} \cdot \text{m} \) at 20\(^\circ\)C, \( 7.6 \times 10^{-6} \text{ Om} \cdot \text{m} \) –1100\(^\circ\)C.

Fig. 3 shows the calculated dependences of the electrical efficiency \( \eta \) and \( \cos \varphi \) of the “inductor-load” system of the ITP with the FET on the frequency of the inductor current in two modes of operation: 1 - cold mode at 20\(^\circ\)C, when the presence of a lump charge in the PC can be neglected in the calculation; 2 - hot mode at a temperature of 1100\(^\circ\)C, when there is liquid copper in the PT. Fig. 3 shows that while the crucible has a lump charge, the dependences of \( \eta \) and \( \cos \varphi \) correspond to the heating of a single-layer conductive charge. With increasing frequency, the electrical efficiency increases slightly and \( \cos \varphi \) decreases. Similarly, there are dependences of \( \eta \) and \( \cos \varphi \) at a temperature of 1100\(^\circ\)C, starting with a current frequency of about 32 kHz and up to 88 kHz. At these frequencies, the electromagnetic wave completely decays in the CC wall and the effect of the two-layered body does not manifest itself. The values of \( \eta \) and \( \cos \varphi \) are somewhat larger than at 20\(^\circ\)C, which is explained by the increase in the specific electrical resistance of the CC material during heating. The presence of maximum and minimum of the curve 1 (Fig. 3a) associated with the different character P1 and P21 changes from \( f \) at this temperature. Up to a frequency of about 32 - 33 kHz, the furnace load manifests itself as a two-layer conductive body. In this section, the value of the electrical efficiency increases with increasing frequency considerably, since the heat release in the crucible wall increases substantially, covering an insignificant decrease in the heat release in copper, and \( \cos \varphi \) increases in this case (Fig. 3b).

Fig. 4 shows the dependence of the total heat output in the ICF load from the CC on the value of the inductor current at different current frequencies: 10, 22, 44 and 88 kHz. Fig. 4 shows that at frequencies of 44 and 88 kHz the loading properties as a two-layer body do not appear, the values of the power in the hot mode slightly exceed the values of the power in the cold mode over the entire range of the inductor current variation. At frequencies of 10 and 22 kHz, the charge at 1100\(^\circ\)C behaves like a two-layer body, so the total heat release in the charge at this temperature is much less than the power of heat release in the cold regime.

It is established that for the considered furnace with PT, the charge has the properties of a two-layer body at frequencies no higher than 32-33 kHz, which corresponds to a ratio of the wall thickness of the crucible to a depth of penetration of about 1.3 at 1100\(^\circ\)C. If this condition is met, after melting the metal, the electrical efficiency and \( \cos \varphi \) will decrease. Thus, when melting in the ITP with FP, the control system should provide a change in frequency during the melting process in such a way as to prevent the appearance of the reflected wave effect.

3. Effect of a reflected wave in induction heating of a two-layer conducting body

In the course of experimental studies carried out by the authors on a laboratory furnace with a non-magnetic graphite crucible for melting copper, it was found that after the molten metal forms in the crucible, there is a significant reduction in the heat release power in the charge (from 1300 W to 1100 W) and electric efficiency with the same inductor current and frequency. Parameters of the experimental setup are: power – 2 KW, inductor current – 180 A, frequency – 22 KHz, inside diameter of inductor – 94 mm, outer diameter of the crucible – 70 mm, wall thickness of the crucible – 10 mm, loading diameter – 50 mm, diameter of the inductor tube – 8 mm. ICF was powered by a transistor inverter. This is explained by the fact that the depth of penetration of the EMW into the wall of a graphite crucible 10 mm thick at the current frequency of the 22 kHz inductor is about 13 mm, and therefore the integral...
value of the induced current in copper is comparable to the total current in the crucible (about 400 A and 780 A, respectively). This leads to the appearance of an inverse EMV reflected from the media interface with different values of $\rho$, and the distribution of the magnetic field strength $H$ along the thickness of the crucible wall changes (Fig. 5a). In this connection, the specific volume capacity of heat release $p_0V$ at each point along the wall thickness of the crucible decreases (Fig. 5b), which leads to a significant decrease in power in the crucible. In this case, the active power in the inner layer is insignificant (about 50 W) due to relatively small values of $\rho$ of the inner layer. In Fig. 5, the area corresponding to the wall is bounded by two vertical lines.

![Figure 5](image.png)

**Figure 5.** The distribution of $H$ (a) and $p_0V$ (b) along the radius $r$ for an empty crucible (1) and a crucible with liquid copper (2)
4. Calculation of electrical characteristics of ITP with ferromagnetic conductive crucible

Fig. 6 shows a sketch of the "inductor-charge" system of a laboratory furnace for melting magnesium (power 2.5 kW) with dimensions. The ICF loading with the conductive crucible is formed by a steel crucible and magnesium. The main difficulty in calculating the furnace is that the electrical characteristics of the charge material vary with the temperature \( t \) and the magnetic field strength \( H \) [1]. So \( \mu = \xi_1(H, t), \rho = \xi_2(t) \), where \( \mu \) is the relative magnetic permeability, \( \rho \) is the electrical resistivity, \( \xi_1, \xi_2 \) are the functional dependencies. At a loading temperature above the melting point and \( \delta_t/\Delta_2 \leq 1.3 \), where \( \Delta_2 \) is the penetration depth of the electromagnetic wave into the crucible wall with a thickness \( \delta_t \), the load can be considered as a two-layer body. The ratio \( \delta_t/\Delta_2 \leq 1.3 \) in the considered frequency range \((10 - 100 \text{ kHz})\) is performed only at a temperature above the Curie point (about 723 °C). Up to the melting point, the charge is a single-layered body, since magnesium in the crucible is in the form of a lumpy charge with a high electrical resistivity, and the crucible itself has magnetic properties. It is difficult to solve analytically the problem, therefore it is necessary to solve it using numerical methods [2].

As an example, fig. 7 shows the dependences of the active resistance \( R \) and the inductance \( L \) of the "inductor-charge" system on \( t \) at different frequencies \( f \) obtained in the ELCUT package with the inductor current \( I_1 = 100 \text{ A} \) and the average value of \( H \) on the outer surface of the crucible \( H = 7.7 \text{ kA/m} \). These and other dependencies are needed to reconcile the ICF and the power supply - a transistor or thyristor inverter.

![Figure 6. System sketch "inductor-charge": 1 - inductor; 2 - crucible; 3 - liquid metal; 4 - thermal insulation](image)

![Figure 7. Dependences of the active resistance and inductance of the system "inductor – charge" on the temperature at different frequencies: 1 - f = 22 kHz; 2 - f = 44 kHz; 3 - f = 66 kHz](image)

A mathematical model of the "inductor-charge" system is developed in the ELCUT package. At a constant frequency \( f \), the active resistance \( R \) increases with increasing temperature to the Curie point. At the Curie temperature, the resistance decreases sharply - 2.5 times for \( f = 22 \text{ kHz} \). At a constant frequency \( f \), the inductance \( L \) increases with increasing temperature up to the Curie point. At the Curie temperature, the inductance decreases sharply.
5. Conclusion
1. A method for determining the electrical characteristics of ITPs with PTs has been developed and corresponding studies have been carried out.
2. It is established that in the case when the PT is partially transparent to the electromagnetic wave, the total heat release in the crucible with the molten metal becomes less than the heat release power in the PT without metal with the same inductor current and frequency.

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