τ polarization and Randall-Sundrum scenario at $e^+e^-$ colliders

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Abstract

We study polarized cross-sections and forward-backward asymmetry for the process $e^+e^-\rightarrow\tau^+\tau^-$ in the stabilized Randall-Sundrum scenario. It is shown that there is substantial deviation from the Standard Model predictions, both in terms of the actual numerical values and angular distributions.

The hierarchy between the electroweak and the Planck scale remains to be a mystery till date. Many possible solutions have been proposed. The idea that the fundamental scale of gravity is not as large as the Planck scale, $\mathcal{O}(10^{-19} \text{ GeV})$, but could be as low as $\mathcal{O}(\text{TeV})$ has invited considerable attention. The basic idea behind such a proposal is the existence of $n$ compact extra spatial dimensions. The Standard Model (SM) fields all lie on a 3-brane while gravity is free to propagate in the entire bulk. The four dimensional Planck scale, $M_{Pl}$, gets related to the fundamental scale of gravity, $M_*$, and the volume, $V_n$, of the $n$ extra dimensions as

$$M_{Pl}^2 \sim V_nM_*^{n+2} \quad (1)$$

In this picture the bulk space-time is a direct product of the four-dimensioal Minkowski space and the extra dimensions. To attain the relevant numbers, the volume $V_n$ should be large and this in turn introduces a new hierarchy between the electroweak scale and the inverse of the size of these extra dimensions.

An alternative scenario due to Randall and Sundrum assumes the existence of two 3-branes which define the ends of the world in the context of a five dimensional bulk space-time. The bulk geometry in this case, in contrast to the previous one, is non-factorizable and the hierarchy is generated by the warp factor which is an exponential function of the inter-brane distance. The same warp factor relates the induced metrics on the two branes. The question of stability of this inter-brane distance still remains an important issue of concern. It was shown by Goldberger and Wise that the inclusion of a bulk scalar field minimally coupled to gravity does indeed admit a stable solution.

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In the present work, we assume that such a stabilizing mechanism is at play. The bulk space-time is a slice of $AdS_5$ and the metric is

$$ds^2 = e^{-2kr_c|y|} \eta_{\mu \nu} dx^\mu dx^\nu - r_c^2 dy^2 \quad (2)$$

where $x^\mu$ are the usual four dimensional coordinates and $y \in [-\pi, \pi]$ parametrizes an $S^1/Z_2$ orbifold. The quantity $r_c$ is the radius and $k^{-1}$ is the curvature of this $AdS_5$ space-time. The two branes lie at $y = 0$ and $y = \pi$ and have equal and opposite tensions.

The fluctuations about the flat metric, $\eta_{\mu \nu}$, contain both the four dimensional massless graviton and its massive Kaluza-Klein (KK) excitations. On the other hand quantum fluctuations of the inter-brane distance manifest themselves in the form of a scalar particle describing the modulus field. Inclusion of the bulk scalar field as in [3] provides a nontrivial potential for the modulus field, thereby stabilizing the inter-brane distance. It is quite clear that to an observer sitting on the $y = \pi$ brane, a field with fundamental mass scale $m_0$ will appear to have an effective mass $m = m_0 e^{-kr_c\pi}$. Therefore, with $kr_c \sim 12$, TeV scales are easily generated on the $y = \pi$ brane and no other hierarchy is introduced.

The couplings of SM fields with the massless and massive gravitons, and the modulus field can be easily found out [2, 3, 4, 5]. On the $y = \pi$ brane, we have after compactification, to the lowest order

$$L_{int} = -\frac{1}{M_{Pl}} T^{\alpha \beta}(x) h^{(0)}_{\alpha \beta} - \frac{1}{\Lambda_\pi} T^{\alpha \beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha \beta} - \frac{\varphi}{\langle \varphi \rangle} T^\alpha_\alpha \quad (3)$$

Here $h^{(n)}_{\alpha \beta}$ are the massive gravitons and $\varphi$ is the modulus field with coupling strengths $\Lambda_\pi$ and $\langle \varphi \rangle$ (both $\mathcal{O}(\text{TeV})$) respectively. $h^{(0)}_{\alpha \beta}$ is the massless four dimensional graviton and $T^{\alpha \beta}$ is the energy-momentum tensor for the SM fields. From this equation it is obvious to see that to the lowest order, the modulus field couples to the SM fields like the Higgs boson. In particular, the modulus field couples to the fermions via the fermion mass.

In the present work, we consider tau pair production at the $e^+e^-$ linear colliders. The presence of the extra dimension leads to additional contributions to the process $e^+e^- \rightarrow \tau^+\tau^-$ through the gravitational interactions induced by the gravitons and the modulus field. As can be seen from the interaction Lagrangian, the usual four dimensional massless graviton couples with the ordinary gravitational strength and thus its contribution is rather negligible. Also, the coupling of the modulus field with the fermions is proportional to fermion mass. Therefore, in the present case (with $m_e = 0$), the contribution due to the exchange of this modulus field is also neglected. We are thus left with an additional contribution from the massive graviton modes only. As is known [4, 5] that in Randall-Sundrum scenario, the mass spectrum of these graviton modes is discrete with the first massive graviton having a mass $\mathcal{O}(\text{TeV})$ and the successive modes are expected to be separated by $\mathcal{O}(\text{TeV})$. This leads to considering only the first massive graviton exchange contribution.

The process $e^+e^- \rightarrow \tau^+\tau^-$ receives contribution only from the s-channel diagram mediated by the massive spin-2 gravitons, apart from the SM contributions mediated by the photon and the Z-boson (Figure1.)
In SM, the scattering process takes place through the exchange of $\gamma/Z$, both of which are spin-1 particles. The situation becomes more interesting with the presence of a spin-2 graviton. The spin-2 nature of the graviton is responsible for significant deviation from the SM results. Moreover, SM being a renormalizable theory gives contributions that are well behaved at high energies. But the non-renormalizable gravitational interactions, in addition to SM interactions, lead to amplitudes not well behaved at higher energies. Therefore, any significant deviation from the SM predictions at high energies may serve as an indication of the possible existence of such interactions. The Next Linear Colliders, thus can be a good testing place for such theories.

For the process 
\[ e^+(p_2)e^-(p_1) \rightarrow \tau^+(k_2)\tau^-(k_1), \]
the scattering amplitude is
\[ M = M_\gamma + M_Z + M_G \]
where
\[ M_\gamma = \left( \frac{ie^2}{s} \right) \bar{u}(k_1)\gamma_\mu v(k_2) \bar{v}(p_2)\gamma^\mu u(p_1) \]
e is the electromagnetic charge and $(p_1 + p_2)^2 = s = (k_1 + k_2)^2$.

\[ M_Z = \left( \frac{ig^2}{16c_w} \right) \frac{1}{s - M_Z^2} \bar{u}(k_1)\gamma_\mu(a - \gamma_5)v(k_2) \bar{v}(p_2)\gamma^\mu(a - \gamma_5)u(p_1) \]
Here $g$ is the weak coupling, $c_w$ is the cosine of Weinberg angle and the parameter $a = -1 + 4\sin^2\theta_w$ is nothing but the vector coupling of the fermions to the Z-boson.

\[ M_G = \left( -\frac{i}{\Lambda^2} \right) \frac{1}{s - M_G^2} \left[ (k' \cdot p')\bar{u}(k_1)\gamma_\mu v(k_2) \bar{v}(p_2)\gamma^\mu u(p_1) \right. \]
\[ + \bar{u}(k_1) p' v(k_2) \bar{v}(p_2) k' u(p_1) \left. \right] \]
In the above expression
\[ p' = p_1 - p_2 \]
\[ k' = k_1 - k_2 \]

From these expressions one can easily compute the polarized amplitudes for different possible combinations. The electrons are taken to be massless. Also, at the energy
range at which the linear collider is expected to operate, the τ’s can also be treated as massless. We follow the notation and convention as in [8] to compute these amplitudes. In the helicity basis, only the amplitudes with opposite (off-diagonal) helicity for both the pairs of external particles are non-vanishing. In what follows below, we label a right handed particle $\psi_R$ as $\psi^+(-)$ and similarly a left handed particle $\psi_L$ as $\psi^-(+)$ [9]. With these conventions, we have the following non-vanishing amplitude squares (as functions of invariants $s$ and $(p_1 - k_1)^2 = t = (k_2 - p_2)^2$):

$$|M^{(+--;+)}|^2 = 4(s + t)^2 \left[ \frac{4\pi \alpha_{em}}{s} + \left( \frac{g^2}{16\alpha^2_{em}} \right) \frac{(a-1)^2}{s - M_Z^2} + \frac{s + 4t}{\Lambda^2_\pi(s - M_G^2)} \right]^2$$

(8)

$$|M^{(+--;+)}|^2 = |M^{(-++;+)}|^2$$

(9)

$$= 4t^2 \left[ \frac{4\pi \alpha_{em}}{s} + \left( \frac{g^2}{16\alpha^2_{em}} \right) \frac{(a^2 - 1)}{s - M_Z^2} + \frac{3s + 4t}{\Lambda^2_\pi(s - M_G^2)} \right]^2$$

(10)

Let $\theta$ be the angle between the incoming $e^-$ and the outgoing $\tau^-$. The forward-backward asymmetry

$$A_{FB} = \frac{\int_{0}^{1} \frac{d\sigma}{d\cos \theta} d\cos \theta - \int_{-1}^{0} \frac{d\sigma}{d\cos \theta} d\cos \theta}{\int_{0}^{1} \frac{d\sigma}{d\cos \theta} d\cos \theta + \int_{-1}^{0} \frac{d\sigma}{d\cos \theta} d\cos \theta}$$

for different cases is summarised in the following table:

| $\sigma^{(+--;+)}$ | $\sqrt{s} = 500 \text{GeV}$ | SM=0.75 | SM+RS=-0.210615 |
|---------------------|-----------------------------|--------|------------------|
| $\sqrt{s} = 1000 \text{GeV}$ | SM=0.75 | SM+RS=0.119273 |
| $\sigma^{(+--;-)}$ | $\sqrt{s} = 500 \text{GeV}$ | SM=-0.75 | SM+RS=-0.149843 |
| $\sqrt{s} = 1000 \text{GeV}$ | SM=-0.75 | SM+RS=-0.0485942 |
| $\sigma^{(-++;-)}$ | $\sqrt{s} = 500 \text{GeV}$ | SM=0.75 | SM+RS=-0.201902 |
| $\sqrt{s} = 1000 \text{GeV}$ | SM=0.75 | SM+RS=0.109656 |

Table 1: The $A_{FB}$ values for $\sqrt{s} = 500, 1000 \text{ GeV}$ ($M_G = 600 \text{ GeV}, \Lambda_\pi = 1000 \text{ GeV}$). $SM$ and $SM + RS$ are the SM and combined results.

It is quite evident that there is a large deviation from the SM values when one has extra contribution from the massive graviton states, making such a measurement a possible testing ground for such theories.

Also, the spin-2 nature of the gravitons is responsible for significant change in the angular distribution of various differential cross-sections as compared to SM case where both the mediating particles are spin-1 objects. This is another feature that can be used

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1chirality equals helicity for massless particles while it is equal to the negative of helicity for massless antiparticles.
to establish the existence of such theories and put meaningful constraints on the parameters involved.

**Figure 2:** Differential cross-section for the process $e_R^- e_L^+ \rightarrow \tau_R^- \tau_L^+ \rightarrow e_R^- e_L^+$ with respect to the scattering angle of $\tau^-$ relative to $e^-$ at $\sqrt{s} = 1000$ GeV, $M_G = 600$ GeV and $\Lambda_\pi = 1000$ GeV.

**Figure 3:** Differential cross-section for the processes $e_R^- e_L^+ \rightarrow \tau_R^+ \tau_L^- \rightarrow e_R^- e_L^+$ with respect to the scattering angle of $\tau^-$ relative to $e^-$ at $\sqrt{s} = 1000$ GeV, $M_G = 600$ GeV and $\Lambda_\pi = 1000$ GeV.
Figure 4: Differential cross-section for the process $e_L e_R^+ \rightarrow \tau_L \tau_R^+$ with respect to the scattering angle of $\tau^-$ relative to $e^-$ at $\sqrt{s} = 1000$ GeV, $M_G = 600$ GeV and $\Lambda_\pi = 1000$ GeV.

In the Figs. (2)-(4), the pure SM contribution has been scaled by three orders of magnitude and is denoted by $SM \times 10^3$ while $SM + RS$ denotes the combined contribution of SM and gravitons. The later is far above the former and also the distribution is quite different as expected.

In whatever has been discussed above, we have assumed 100% polarization of the initial beams. But in practice this may not be the case. It is worth emphasizing that the same expressions with obvious modifications can be used to accommodate the partial/zero degree of polarization of the initial beams.

Further, we can calculate the final state polarization asymmetry $P^r$

$$P^r = \frac{\sigma_{\tau^-}^+ - \sigma_{\tau^-}^-}{\sigma_{\tau^-}^+ + \sigma_{\tau^-}^-}$$

Again, we expect it to be different from the SM value. In the case at hand, it is the graviton and Z-boson cross contribution that gives the additional contribution to the asymmetry. The results are presented in the following table:

| $\sigma(\tau^-) = \sigma(\tau^-)$ | $\sqrt{s} = 500$ GeV | SM = 0.666 | SM + RS = 0.038 |
|---------------------------------|----------------------|-------------|-----------------|
| $\sigma(\tau^-) = \sigma(\tau^-)$ | $\sqrt{s} = 1000$ GeV | SM = 0.654 | SM + RS = 0.0052 |

| $\sigma(\tau^-) = \sigma(\tau^-)$ | $\sqrt{s} = 500$ GeV | SM = 0.6183 | SM + RS = 0.0311 |
|---------------------------------|----------------------|-------------|-----------------|
| $\sigma(\tau^-) = \sigma(\tau^-)$ | $\sqrt{s} = 1000$ GeV | SM = 0.6056 | SM + RS = 0.0419 |

Table 2: The final state polarization asymmetry for $\sqrt{s} = 500$, 1000 GeV ($M_G = 600$ GeV, $\Lambda_\pi = 1000$ GeV).

The $\tau$ polarization measurement (through the study of correlations between the decay products) can provide further useful hints about the existence of physics beyond
SM. It is then straightforward to use the above relations and study correlations between the decay products of $\tau^+$ and $\tau^-$. For example in the present case, $\pi^-$, from the decay channel $\tau^- \rightarrow \pi^- \nu\tau$, will be emitted in the $\tau$-spin direction in the rest frame of $\tau^-$ while $\pi^+$ from $\tau^+ \rightarrow \pi^+ \bar{\nu}\tau$ emitted in the direction opposite to the $\tau^+$ spin. Thus, an appropriate Lorentz transformation to the Lab frame can indicate at the expected angular distribution of $\pi^\pm$ and provide information about deviation from SM.

In conclusion, we can say that $\tau$ polarization can give useful hints on the possible existence of Randall-Sundrum type scenarios and can be used to constrain the parameters of such theories.

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\(^2\)one expects similar results in the case of the scenario proposed by Arkani-Hamed et al. where instead of a single graviton exchange, one has a tower of massive gravitons till the effective Planck/string scale, $M_*$.