Current carrying vortex crystals

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Abstract

Abrikosov vortices in a type-II superconducting film subjected to strong magnetic field $B$ with periodic array of nanoholes of the density $n_{\text{pin}}$ form sometimes a vortex crystal, even when they are driven by a transport current. It is shown numerically that the crystal melting and transition to the resistive state occurs as a coherent depinning of the single vortex dislocations. For a system with interstitial vortices, $f = B/\Phi_0 n_{\text{pin}} > 1$, the mechanism of depinning depends on the current direction with respect to the pinning array. It was found that slightly above the critical current trajectories of moving vortices are not straight, but rather acquire a snake-like shape enveloping the system of pins. In contrast to the matching field case, $f = 1$, the transition to a resistive state is not coherent and is developing through formation of the "snake-like" vortex trajectories. It is pointed out that the depinning is closely associated with the appearance of a strongly varying electric field. We calculated the electric fields accompanying vortex crystal melting and found the voltage-current characteristics. When the pinning array is made random, the critical current is reduced.
I. INTRODUCTION.

An important challenge in applications of type-II superconductors is achieving optimal critical currents $J_c(B)$ under an applied magnetic field $B$. Below this current the vortex matter is under sustainable stress (vortices being displaced) and can support a dissipationless electric current. The critical current decreases as magnetic induction increases and consequently the pinning efficiency should be optimized. Although intrinsic pinning always exists in bulk superconductors, it is rather inefficient at elevated fields. Recently a new possibility has been developed - pinning by artificially assembled well controlled periodic arrays of holes, blind holes and magnetic dots [1].

As the size of the pinning centers became smaller, it became comparable with the coherence length $\xi$ of a superconductor, while the range of the magnetic fields gradually increased[2]. The coherence length can also be significantly increased by tuning temperature to be just below critical $T_c$. The critical current is maximized when the nanoholes lattice is hexagonal at the matching field $B_0 = \Phi_0 n_{pin}$, where $\Phi_0$ is a unit of flux and $n_{pin}$ is the density of pins. This case was treated analytically in the framework of the Ginzburg-Landau (GL) theory [3]. When the number of vortices exceeds that of the pinning centers, namely when the filling factor $f = B/B_0 > 1$, additional, "interstitial" vortices appear. As a result the flux in this case can be clearly separated into two subsystems, a mobile, weakly pinned interstitial vortex subsystem (IVS) and almost rigidly pinned (PVS). Although not directly pinned by the pinning centers, at fields $B >> H_{c1}$ they strongly interact with PVS and with each other. In addition in many recent experiments penetration depth $\lambda$ is much larger than distances between the nanoholes, so that magnetic field is homogeneous and the vortex core physics (including strong electric field varying over the scale $\xi$) becomes of importance.

To accommodate these feature, namely strong interactions and, order parameter structure at constant magnetic field one considers the GL model[4]. Within the GL model the current carrying steady state for $f = 1$ (where there is no electric field) of the pinned flux line lattice was recently studied analytically using a simple variational method [3].

In the present paper we consider both the static (stationary) and the dynamic properties of the vortex matter in commensurate and random pinning arrays for several filling factors $f \geq 1$. The interstitial vortices reduce the critical current, since they are pinned just by a
repulsive potential created by the static PVS. It turns out that the critical current is very anisotropic with respect to orientation of the current relative to that of the primitive vector of the unit cell of the pinning array. When the array is made random pinning becomes more effective[2].

II. BASIC EQUATIONS

Superconducting slab subjected to a sufficiently high, homogeneous and time independent magnetic field perpendicular to the slab. Assuming, \( \kappa \equiv \lambda/\xi >> 1 \), the magnetization is small so that for magnetic fields several times larger than \( H_{c1} \), induction \( B \approx H \) and is practically homogeneous. It is therefore treated as a constant rather than a degree of freedom. The 2D time dependent effective GL equation in dimensionless variables (see for details Ref. [3]) has the form:

\[
\frac{\partial \psi}{\partial t} + i\phi \psi = -\frac{\partial}{\partial \psi^*} f_{GL} [\psi],
\]

where

\[
f_{GL} = \int d^2 r \left\{ \psi^* \left[ -\frac{D^2}{2} - \frac{1 - t}{2} + V_{pin} (r) \right] \psi + \frac{1}{2} (\psi^* \psi)^2 \right\},
\]

is the GL free energy. The covariant derivatives \( D \equiv \nabla - iA \) include the vector potential \( A = b \left( -\frac{1}{2} y, \frac{1}{2} x, 0 \right) \), describing the magnetic induction \( (b = B/H_{c2} (T) \) and \( t = T/T_c) \). In our gauge the electric field is \( E = -\nabla \phi \). Eq.(1) is supplemented by the charge conservation law, \( \nabla \cdot j = 0 \), where the dimensionless current density, including the normal component, is given in our gauge by

\[
j = \frac{i}{2} [\psi^* D \psi - \psi (D \psi)^*] - \nabla \phi.
\]

The coherence length \( \xi \) will be used as a unit of length \( r \rightarrow r/\xi \). The dimensionless order parameter \( \psi \) while \( \phi \) is the scalar potential [3]. The potential \( V_{pin} \) describes a pinning array, \( V_{pin} (r) = \sum V (r - r_a) \), where \( r_a \) denote the locations of the nanoholes centers. In a geometry considered here the applied DC current will be always oriented along the \( y \) axis, Fig1. The system of equations should be complemented by the following boundary conditions, \(-\nabla_y \phi = j^\text{ext}|_{y=0,L_y}; \nabla_x \phi = 0|_{y=0,L_y}, \) where \( L_y \) is the length of the sample in the current direction \( y \). Periodic boundary conditions in the perpendicular \( x \) direction are assumed. The order parameter is therefore subject to "metallic electrodes" boundary conditions, \( \psi = 0|_{y=0,L_y}, \) and periodic under magnetic translations in the \( x \) direction.
III. ELECTRIC FIELD AND THE VOLTAGE-CURRENT CHARACTERISTICS.

The above equations were treated numerically using Wilson’s discretization [3] for $f = 3$. Below the critical current vortices are displaced in the direction perpendicular to that of the persistent current, while the electric field vanishes. The critical current, $j_c = 0.0073$, is determined by the stability of the static distorted Abrikosov lattice carrying a net supercurrent. Above the critical current the electric field becomes nonzero and vortices start to move. Some of the interstitial vortices in this case are confined inside the channels formed by the line of alternating strongly pinned and interstitial vortices and are weakly pinned. As current increases the system undergoes a transition from the static pinned state to the moving incoherent (snake) phase, followed by the coherent motion.

Just above the critical current, at $j_\ast = 0.0075$, the snake-like vortex phase inside the channels appears[3]. The transition to a resistive state is therefore due to the snake-like instabilities although interstitial vortices trapped inside the channel walls are eventually also involved. At large currents all of the vortices are completely depinned. In this case both the vortices of the former channels and those that formed PVS participate in coherent laminar stream motion. The voltage-current characteristic in this case demonstrate extremely small critical currents. The electric field becomes strongly oscillating for $j_\ast$.

We also considering the structure and dynamics of the current carrying state for disordered array of nanoholes. In this case the transition to a resistive state is accompanied by
the appearance of weak electric fields that becomes homogeneous in accordance with the analytical estimate of electric field coherence length. The I-V curve, Fig.2, at large current still approaches the Bardeen - Stephen law and the structure is very similar to that of the time dependent GL model with renormalized viscosity, namely, the friction due to artificial pinning centers is accounted for by the increase of the effective inverse diffusion constant in the GL equation[4]. However at small transport currents the I-V curve becomes powerwise. This means that, while hexagonal array can arrest the stiff Abrikosov lattice commensurate with it, the random array of the same filling factor is unable to hold the moving fluxons. This is different from results of experiments[2] and simulations of the London model[5] in which the critical current increased as the array was made more random. There is no contradiction since in the later case the vortex lattice is so soft (close to $T_c (H)$ elastic modulii decrease rapidly) that vortices are pinned as a soft manifold or individually.

To summarize, we have studied numerically dynamics of pinned vortex matter in superconducting film under strong magnetic fields for $f > 1$ using the time dependent Ginzburg - Landau equations. If the pinning array is periodic, the depinning transition due to interstitial vortices is characterized by strong electric fields and creation of "snake - like" channels. For random pinning the vortex lattice is stiff enough to avoid pinning and the critical current decreases dramatically.

The voltage current characteristics demonstrates the universal behavior $V \sim J^n$, where $n$ depends on the correlation function of the disorder. The critical current in this case is very
close to zero.

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