Fracture Analysis of Griffith Interface Crack in Fine-Grained Piezoelectric Coating/Substrate under Thermal Loading

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Received 25 May 2020; Revised 31 August 2020; Accepted 2 September 2020; Published 15 September 2020

1. Introduction

Piezoelectric materials have played an important role in the production of intelligent structures. Due to strength demand, we often bond another piezoelectric material on the surface of the device to protect it or monitor the device. However, piezoelectric composites are prone to cracks at the interface during the process of fabricating defects and load conditions. This can even lead to structural failure. Therefore, problems of interface fracture of the piezoelectric composite under all kinds of loads have attracted wide concern [1–9]. Qin and Mai [10] analyzed the interface crack problem of a piezoelectric bimaterial subjected to combined thermal, mechanical, and electrical loads via Stroh’s formalism and the singular integral equation method. However, the influence of material size on the thermal intensity factor has not been considered in the numerical analysis. Ueda [11, 12] investigated the problem of a parallel crack in a piezoelectric strip under thermoelastic loading and the FGM material strip containing an embedded crack or an edge crack perpendicular to its boundaries. Wang and Noda [13] studied the problems of the piezoelectric material strip with a Griffith interface crack under thermal loading and discussed the effects of polarization direction, crack size, and location on the thermal strength factor. Kang et al. [14] considered the influences of piezoelectric laminated composite shells with interlaminar stresses under electrical, thermal, and mechanical loads. They concluded that the interlaminar shear stress of the laminated shell can be reduced by selecting the appropriate electric field value. Cook and Vel [15] examined a comprehensive multiscale analysis of laminated plates with integrated piezoelectric fiber composite actuators. They used the coupling method of microscale field variables and macroscale field variables to solve the three-dimensional macroscopic equilibrium equation of the piezoelectric laminated plate under arbitrary boundary conditions solved. Li et al. [16] presented a semianalytical technique to solve fracture behaviors of piezoelectric composites under thermal loading. This method could analytically represent the resulting stress and electric displacement distribution along the radial direction obtained. Kuutti and Virkkunen [17] justified the surface crack behavior under periodic thermal loads. The finite element method and weight function method could simulate the crack behavior over the entire load cycle. Mojahedin et al. [18] considered the mechanical behavior of a solid circular plate which is made of saturated...
and unsaturated porous material with piezoelectric actuators under thermal loading. They discussed the effect of porosity on the thermal and mechanical stability of the material. Herrmann and Loboda [19] performed fracture analysis on an electrically impermeable interface crack with contact zones in thermoelectric bimaterials. Using the admissible directions of the heat and the electrical fluxes, the dependencies of the electrical intensity factors on the intensities of the thermal and electrical fluxes were discussed.

Many researchers have studied the interface fracture problems of piezoelectric materials under loading [20–24]. However, all of polycrystalline materials studied above are composed of multidomain large grains, and such sizes cannot meet the requirements of the current intelligent devices, but fine-grained piezoelectric materials are gaining increasing interest. However, there are few studies on the mechanical properties of fine-grained piezoelectric materials.

As an important fracture parameter that can measure structural safety, the intensity factor plays a crucial role in checking the safety of structures. In this article, a Griffith interface crack in fine-grained piezoelectric coating/substrate under steady-state thermal loading is established. An integral transformation method and superposition principle of the solution of the equation are used to transform the thermal load problem into a singular integral equation to obtain the intensity factor conveniently. Furthermore, the interaction between the intensity factor and material parameters is investigated. The results indicate that the larger elastic modulus and thinner coating thickness improve the safety of the coating/substrate structure.

2. Problem Formulation

Figure 1 shows that a fine-grained ceramic powder coating and the substrate are bond together by plasma spraying, while the crack with a length of 2l is along the interface between the coating and the substrate. A fine-grained piezoelectric coating/substrate structure was obtained by the polarization treatment of the coating. The coating and substrate are polarized along the Y-axis, and both are transversely isotropic. The thicknesses of the coating and substrate are $h_1$ and $h_2$, respectively. We assumed that the crack faces remain thermally and electrically insulated, and $T_a$ and $T_b$ are the environment temperatures.

The constitutive equations for the elastic field are [12]

$$\sigma_{xx} = c_{11}^{(m)} \frac{\partial \omega}{\partial x} + c_{13}^{(m)} \frac{\partial u}{\partial y} + e_{31}^{(m)} \frac{\partial \phi}{\partial y} - \lambda_{11}^{(m)} T,$$  \(1\)

$$\sigma_{yy} = c_{13}^{(m)} \frac{\partial \omega}{\partial x} + c_{33}^{(m)} \frac{\partial u}{\partial y} + e_{33}^{(m)} \frac{\partial \phi}{\partial y} - \lambda_{33}^{(m)} T,$$  \(2\)

$$\sigma_{xy} = c_{44}^{(m)} \left( \frac{\partial \omega}{\partial y} + \frac{\partial u}{\partial x} \right) + e_{15}^{(m)} \frac{\partial \phi}{\partial x},$$  \(3\)

$$D_x = e_{15}^{(m)} \frac{\partial \omega}{\partial y} + \frac{\partial u}{\partial x} - e_{11}^{(m)} \frac{\partial \phi}{\partial x},$$  \(4\)

$$D_y = e_{31}^{(m)} \frac{\partial \omega}{\partial x} + e_{33}^{(m)} \frac{\partial u}{\partial y} - e_{33}^{(m)} \frac{\partial \phi}{\partial y} + p_{33}^{(m)} T.$$  \(5\)

In Equations (1), (2), (3), (4) and (5), the $\omega$ and $u$ are the displacement components; $\phi$ is the electric potential; $T$ is the temperature change; $\sigma_{xx}$, $\sigma_{yy}$, and $\sigma_{xy}$ are stress components; $D_x$ and $D_y$ are electric displacement components; $c_{11}^{(m)}$, $c_{13}^{(m)}$, and $c_{44}^{(m)}$ are elastic modulus; $e_{31}^{(m)}$, $e_{33}^{(m)}$, $e_{15}^{(m)}$ and $e_{33}^{(m)}$ are piezoelectric and dielectric constants, respectively. $\lambda_{11}^{(m)}$ and $\lambda_{33}^{(m)}$ are the stress-temperature coefficients; $p_{33}^{(m)}$ is the pyroelectric constant. The superscript $m = 1, 2$ stands for the fine-grained piezoelectric coating and piezoelectric substrate, respectively.

Assume that the temperature satisfies the Fourier heat conduction equation, as follows:

$$k_m \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,$$  \(6\)

where $k_m = \sqrt{k_x^{(m)} / k_y^{(m)}} k_i^{(m)}$, and $k_i^{(m)}$ are coefficients of thermal conductivity. $m = 1, 2$.

The equations of equilibrium are [12]

$$c_{11}^{(m)} \frac{\partial^2 \omega}{\partial x^2} + c_{44}^{(m)} \frac{\partial^2 \omega}{\partial y^2} + \left( c_{13}^{(m)} + c_{33}^{(m)} \right) \frac{\partial^2 u}{\partial x \partial y} + \left( c_{15}^{(m)} + c_{33}^{(m)} \right) \frac{\partial^2 \phi}{\partial x \partial y} = \lambda_{11}^{(m)} \frac{\partial T}{\partial x},$$  \(7\)

$$c_{44}^{(m)} \frac{\partial^2 \omega}{\partial x^2} + c_{33}^{(m)} \frac{\partial^2 \omega}{\partial y^2} + \left( c_{31}^{(m)} + c_{44}^{(m)} \right) \frac{\partial^2 u}{\partial x \partial y} + \left( c_{33}^{(m)} + c_{33}^{(m)} \right) \frac{\partial^2 \phi}{\partial x \partial y} = \lambda_{33}^{(m)} \frac{\partial T}{\partial y},$$  \(8\)

$$e_{15}^{(m)} \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \left( e_{33}^{(m)} + e_{33}^{(m)} \right) \frac{\partial^2 \phi}{\partial x \partial y} = - p_{33}^{(m)} \frac{\partial T}{\partial y},$$  \(9\)

Without loss of generality, the boundary conditions can be written as

$$\sigma_{yy}(x, 0^+) = \sigma_{yy}(x, 0^-) = 0, \quad |x| < l,$$  \(10a\)

$$u(x, 0^+) = u(x, 0^-), \quad |x| \geq l,$$  \(10b\)
where

\[ \begin{align*}
\sigma_{yx}(x, 0^+) &= \sigma_{yx}(x, 0^-) = 0, \quad |x| < l, \\
w(x, 0^+) &= w(x, 0^-), \quad |x| \geq l, \\
D_x(x, 0^+) &= D_x(x, 0^-) = 0, \quad |x| < l, \\
ϕ(x, 0^+) &= ϕ(x, 0^-), \quad |x| \geq l, \\
σ_{yy}(x, 0^+) &= σ_{yy}(x, 0^-), \quad |x| \geq l, \\
σ_{yx}(x, 0^+) &= σ_{yx}(x, 0^-), \quad |x| \geq l, \\
D_y(x, 0^+) &= D_y(x, 0^-), \quad |x| \geq l.
\end{align*} \]

(10a)

(10b)

(10c)

(10d)

(10e)

(10f)

(10g)

(10h)

(10i)

(10j)

The problem of the temperature field can be expressed as

\[ T(x, 0^+) = T(x, 0^-), \quad |x| \geq l, \]

(11a)

\[ T(x, h_1) = T_a(x), \quad T(x, h_2) = T_b(x), \quad |x| \geq l, \]

(11b)

\[ k_y^{(2)} \frac{∂T(x, 0^+)}{∂y} = k_y^{(2)} \frac{∂T(x, 0^-)}{∂y} = -Q_0(x), \quad |x| < l, \]

(11c)

\[ k_y^{(1)} \frac{∂T(x, 0^+)}{∂y} = k_y^{(2)} \frac{∂T(x, 0^-)}{∂y}, \quad |x| \geq l. \]

(11d)

Substituting formula (12) into Equations (11a), (11b), (11c) and (11d), we obtain

\[ A_{11}(s) + A_{21}(s) - A_{12}(s) - A_{22}(s) = \frac{j}{s} \int_{-l}^{l} G(t)e^{ist} dt, \]

(15)

\[ A_{11}(s)e^{-|s|k_y h_1} + A_{21}(s)e^{ik_y h_1} = \int_{-∞}^{∞} T_a(x)e^{i\alpha_1 x} d\alpha, \]

(17)

\[ A_{12}(s)e^{-|s|k_y h_2} + A_{22}(s)e^{ik_y h_2} = \int_{-∞}^{∞} T_b(x)e^{i\alpha_2 x} d\alpha. \]

(18)

Equations (16), (17) and (18) can be written in the form of a matrix as follows:

\[ A \cdot B = C, \]

(19)

where

\[ A = \begin{bmatrix}
-s|k_y k_x^{(2)}| & |s|k_y k_x^{(2)} & |s|k_y k_x^{(1)} & -|s|k_y k_x^{(2)} \\
1 & 1 & -1 & -1 \\
e^{-|s|k_y h_1} & e^{i|s|k_y h_1} & 0 & 0 \\
0 & 0 & e^{-|s|k_y h_2} & e^{i|s|k_y h_2}
\end{bmatrix}, \]

(20)

\[ B = [A_{11}(S), A_{21}(S), A_{12}(S), A_{22}(S)]^T, \]

\[ C = [0, I_0, I_1, I_2]^T, \]

\[ I_0 = \int_{-l}^{l} G(t)e^{ist} dt, \]

\[ I_1 = \int_{-l}^{l} T_a(x)e^{i\alpha_1 x} d\alpha, \]

\[ I_2 = \int_{-l}^{l} T_b(x)e^{i\alpha_2 x} d\alpha. \]

Solving the linear Equation (19), we get

\[ \begin{align*}
A_{11}(s) &= F_1 I_0 - F_2 I_1 + F_3 I_2, \\
A_{21}(s) &= F_1 I_1 - F_2 I_0 - F_3 I_2, \\
A_{12}(s) &= F_1 I_2 + F_2 I_0 - F_3 I_1, \\
A_{22}(s) &= F_1 I_1 - F_2 I_2 - F_3 I_0.
\end{align*} \]

(21)

The expressions of \( F_i \) and \( Δ, q = 1, 2, ... 12 \), are given in Appendix A. Obviously, the unknown functions \( A_{11}(s), A_{21}(s), A_{12}(s), \) and \( A_{22}(s) \) depend on \( I_0 \). Once \( I_0 \) is determined, the temperature field could be obtained.
By substituting Equation (12) into Equation (11c), we obtain

\[
\frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{s[k_{1y}k_{1y}^{(1)}(F_1 - F_5)]}{s} e^{i\tau x} ds \int_{-l}^{l} G(t) dt = Q_0(x) - d_0(x),
\]

where

\[
d_0(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |s[k_{1y}k_{1y}^{(1)}(F_1 - F_5)]| e^{i\tau x} ds.
\]

Furthermore, Equation (23) can be written as

\[
\frac{1}{\pi} \int_{0}^{\infty} M(s) \sin \left((t - x)s\right) ds \int_{-l}^{l} G(t) dt = Q_0(x) - d_0(x),
\]

where

\[
M(s) = \frac{|s[k_{1y}k_{1y}^{(1)}(F_1 - F_5)]|}{s}.
\]

Through changing the order of the integration on the cracks' surface, we obtain

\[
- \frac{1}{\pi} \int_{-l}^{l} M_0 \frac{\partial}{\partial x} \kappa_1(x, t) G(t) dt = Q_0(x) - d_0(x),
\]

where

\[
\lim_{s \to 0} M(s) = M_0,
\]

\[
M_0 = -\frac{k_1 k_2 k_{1y}^{(1)} k_{2y}^{(2)}}{k_1 k_{1y}^{(1)} + k_2 k_{2y}^{(2)}},
\]

\[
\kappa_1(x, t) = \int_{0}^{\infty} (M(s) - M_0) \sin \left((t - x)s\right) ds.
\]

Equation (26) is a singular integral equation with Cauchy kernel. The density function \(G(t)\) in Equation (26) has the square root-type singularity. Thus, we can express \(G(t)\) as follows:

\[
G(t) = \frac{1}{\sqrt{\tau^2 - t^2}} g_0(t),
\]

where \(g_0(t)\) is a continuous function defined in the interval \([-l, l]\).

The thermal flow intensity factors are defined by [13].

\[
k^l_y = \lim_{x \to -l} 2\pi(-x - l) k_y \frac{\partial T_m(x, 0)}{\partial y},
\]

\[
k^l_y = \lim_{x \to l} 2\pi(x - l) k_y \frac{\partial T_m(x, 0)}{\partial y}.
\]

According to Equation (26), the singular part of \(k_y(\partial T_m(x, 0)/\partial y)\) is

\[
\lim_{x \to l} k_y \frac{\partial T_m(x, 0)}{\partial y} = M_0 \lim_{x \to l} \int_{-l}^{l} G(-l) \]  

\[
\lim_{x \to l} k_y \frac{\partial T_m(x, 0)}{\partial y} = M_0 \lim_{x \to l} \int_{-l}^{l} G(l).
\]

Substituting Equation (28) into Equations (31) and (32), we obtain

\[
k^l_y = M_0 \sqrt{\pi l} g_0(l),
\]

\[
k^l_y = -M_0 \sqrt{\pi l} g_0(-l).
\]

3.2. Thermal Stress and Electric Displacement Fields. Similar to the solution of linear equation, the solutions to Equations (7), (8) and (9) of equilibrium for displacements and electric potential consist of a homogeneous solution and a particular solution.

Equations (7), (8) and (9) can be written into homogeneous forms as follows:

\[
c_{11}^{(m)} \frac{\partial^2 w}{\partial x^2} + c_{14}^{(m)} \frac{\partial^2 w}{\partial y^2} + \left(c_{13}^{(m)} + c_{44}^{(m)}\right) \frac{\partial^2 u}{\partial x \partial y} + \left(e_{14}^{(m)} + e_{15}^{(m)}\right) \frac{\partial^2 \phi}{\partial x \partial y} = 0,
\]

\[
c_{44}^{(m)} \frac{\partial^2 u}{\partial x^2} + c_{13}^{(m)} \frac{\partial^2 w}{\partial y^2} + \left(c_{14}^{(m)} + c_{44}^{(m)}\right) \frac{\partial^2 w}{\partial x \partial y} + \left(e_{14}^{(m)} + e_{15}^{(m)}\right) \frac{\partial^2 \phi}{\partial x \partial y} = 0,
\]

\[
c_{15}^{(m)} \frac{\partial^2 u}{\partial x^2} + c_{13}^{(m)} \frac{\partial^2 u}{\partial y^2} + \left(c_{14}^{(m)} + c_{44}^{(m)}\right) \frac{\partial^2 \phi}{\partial x \partial y} - \left(e_{14}^{(m)} + e_{15}^{(m)}\right) \frac{\partial^2 \phi}{\partial x \partial y} = 0.
\]

By using the Fourier integral transform, the solutions of Equations (34), (35) and (36) can be expressed as [13]

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{1k}^{(m)}(s) e^{i(k\alpha + \beta)} f_k(s) e^{-i\nu x} ds,
\]

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{2k}^{(m)}(s) e^{i(k\alpha + \beta)} f_k(s) e^{-i\nu x} ds,
\]

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{3k}^{(m)}(s) e^{i(k\alpha + \beta)} f_k(s) e^{-i\nu x} ds,
\]

where \(B_{1k}^{(m)}, B_{2k}^{(m)}, B_{3k}^{(m)},\) and \(f_k(s), (m = 1, 2),\) are unknown functions to be determined.

The special solutions of Equations (7), (8) and (9) can be expressed as

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} C_{1k}^{(m)} A_{1m} e^{-i(k\alpha + \beta)} + C_{1b}^{(m)} A_{2m} e^{i(k\alpha + \beta)} e^{-i\nu x} ds,
\]

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} C_{2k}^{(m)} A_{1m} e^{-i(k\alpha + \beta)} + C_{1b}^{(m)} A_{2m} e^{i(k\alpha + \beta)} e^{-i\nu x} ds,
\]

\[
u_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} C_{3k}^{(m)} A_{1m} e^{-i(k\alpha + \beta)} + C_{1b}^{(m)} A_{2m} e^{i(k\alpha + \beta)} e^{-i\nu x} ds,
\]
where \( B_{1a}^{(m)}, B_{2a}^{(m)}, B_{3a}^{(m)}, B_{1b}^{(m)}, B_{2b}^{(m)}, \) and \( B_{3b}^{(m)} \) satisfy the expression as follows:

\[
\begin{align*}
(A_2 k_m^2 - A_1 k_m + A_0) B_{1a}^{(m)} &= \begin{bmatrix}
-\lambda_{11}^{(m)} \\
-\lambda_{12}^{(m)} \\
-\lambda_{13}^{(m)} \\
-\lambda_{14}^{(m)} \\
-\lambda_{15}^{(m)} \\
-\lambda_{21}^{(m)} \\
-\lambda_{22}^{(m)} \\
-\lambda_{23}^{(m)} \\
\end{bmatrix}, \\
A_2 k_m^2 + A_1 k_m + A_0) B_{2a}^{(m)} &= \begin{bmatrix}
\lambda_{11}^{(m)} \\
\lambda_{12}^{(m)} \\
\lambda_{13}^{(m)} \\
\lambda_{14}^{(m)} \\
\lambda_{15}^{(m)} \\
\lambda_{21}^{(m)} \\
\lambda_{22}^{(m)} \\
\lambda_{23}^{(m)} \\
\end{bmatrix}, \\
\end{align*}
\]

where

\[
\begin{align*}
A_0 &= \begin{bmatrix}
-\lambda_{11}^{(m)}^{(m)} & 0 & 0 \\
0 & -\lambda_{12}^{(m)} & -\lambda_{13}^{(m)} \\
0 & -\lambda_{14}^{(m)} & -\lambda_{15}^{(m)} \\
\end{bmatrix}, \\
A_1 &= \frac{-i\pi^2}{s} \begin{bmatrix}
\lambda_{11}^{(m)} & \lambda_{12}^{(m)} & \lambda_{13}^{(m)} \\
0 & \lambda_{14}^{(m)} & \lambda_{15}^{(m)} \\
0 & \lambda_{16}^{(m)} & \lambda_{17}^{(m)} \\
\end{bmatrix}, \\
A_2 &= \begin{bmatrix}
\lambda_{18}^{(m)} & \lambda_{19}^{(m)} & \lambda_{20}^{(m)} \\
0 & \lambda_{21}^{(m)} & \lambda_{22}^{(m)} \\
0 & \lambda_{23}^{(m)} & \lambda_{24}^{(m)} \\
\end{bmatrix}.
\end{align*}
\]

Substituting Equations (45), (46) and (47) into Equations (2), (3), and (5), the thermal stress and electric displacement fields are obtained:

\[
\begin{align*}
\sigma_{yy} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{1a}^{(m)} f_0 e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{1b}^{(m)} A_0 e^{-\text{i} k y} s, \\
\sigma_{yx} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{2a}^{(m)} f_0 e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{2b}^{(m)} A_0 e^{-\text{i} k y} s, \\
D_y &= \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{3a}^{(m)} f_0 e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{3b}^{(m)} A_0 e^{-\text{i} k y} s, \\
\end{align*}
\]

Therefore, the solution of Equations (7), (8) and (9) can be written as

\[
\begin{align*}
u(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{\infty} B_{1a}^{(m)}(s) e^{\text{i} k x} f_k(s) e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} s \int_{-\infty}^{\infty} s f_k(s) e^{-\text{i} k y} s, \\
\omega(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{\infty} B_{2a}^{(m)}(s) e^{\text{i} k x} f_k(s) e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} s \int_{-\infty}^{\infty} s f_k(s) e^{-\text{i} k y} s, \\
\phi(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{\infty} B_{3a}^{(m)}(s) e^{\text{i} k x} f_k(s) e^{-\text{i} k y} s + \frac{1}{2\pi} \int_{-\infty}^{\infty} s \int_{-\infty}^{\infty} s f_k(s) e^{-\text{i} k y} s.
\end{align*}
\]
In order to solve the thermal stress and electric displacement fields, the dislocation density functions are introduced:

\[ d_1(x) = \frac{d}{dx} [u(x, 0^+) - u(x, 0^-)], \]
\[ d_2(x) = \frac{d}{dx} [w(x, 0^+) - w(x, 0^-)], \]
\[ d_3(x) = \frac{d}{dx} [\phi(x, 0^+) - \phi(x, 0^-)]. \]

Substituting Equations (52), (53) and (54) into Equations (10a), (10b), (10c), (10d), (10e), (10f), (10g), (10h), (10i), (10j), (10k) and (10l), we obtain

\[
\sum_{k=1}^{6} B_{1k}^{(1)} f_k = \frac{H_1 I_{1a} - H_3 I_{2a} - H_3 I_{3a} + a_{1a}}{H},
\]
\[
\sum_{k=1}^{6} B_{1k}^{(2)} f_k = \frac{H_1 I_{1a} - H_2 I_{2a} - H_3 I_{3a} + a_{2a}}{H},
\]
\[
\sum_{k=1}^{6} B_{2k}^{(1)} f_k = \frac{H_2 I_{1a} - H_2 I_{2a} + H_3 I_{3a} + a_{3a}}{H},
\]
\[
\sum_{k=1}^{6} B_{2k}^{(2)} f_k = \frac{H_1 I_{2a} - H_1 I_{2a} - H_2 I_{3a} + a_{4a}}{H},
\]
\[
\sum_{k=1}^{6} B_{3k}^{(1)} f_k = \frac{H_{13} I_{3a} - H_1 I_{2a} - H_3 I_{3a} + a_{5a}}{H},
\]
\[
\sum_{k=1}^{6} B_{3k}^{(2)} f_k = \frac{H_{26} I_{1a} - H_{26} I_{2a} - H_{26} I_{3a} + a_{6a}}{H},
\]

where

\[ I_{1a} = \frac{i}{s} \int_{-l}^{l} d_1(t) e^{ist} \, dt, \]
\[ I_{2a} = \frac{i}{s} \int_{-l}^{l} d_2(t) e^{ist} \, dt, \]
\[ I_{3a} = \frac{i}{s} \int_{-l}^{l} d_3(t) e^{ist} \, dt. \]

The expressions of \( H, a_{p*}, \) and \( H_{pq}, p = 1, 2, \cdots, 6, q = 1, 2, \cdots, 18, \) are given in Appendix B. Similar to the process of solving the temperature field, once \( I_{1a}, I_{2a}, \) and \( I_{3a} \) are determined, the thermal stress and electric displacement fields could be obtained.

Substituting Equations (55), (56), (57), (58), (59) and (60) into Equations (48), (49) and (50), we get

\[
\sigma_{xx}(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{1k}^{(1)} f_k \left[ \left( \lambda_2 c_{44} \right) H_{10} - \left( \lambda_4 c_{44} \right) H_{4} \right] e^{-ist} \, ds = 0,
\]
\[
\sigma_{xy}(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{1k}^{(2)} f_k \left[ \left( \lambda_2 c_{44} \right) H_{10} - \left( \lambda_4 c_{44} \right) H_{4} \right] e^{-ist} \, ds = 0,
\]
\[
\sigma_{yy}(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{6} B_{1k}^{(1)} f_k \left[ \left( \lambda_2 c_{44} \right) H_{10} - \left( \lambda_4 c_{44} \right) H_{4} \right] e^{-ist} \, ds = 0,
\]
\[
\sigma_{yy}(x, 0^-) = \sigma_{yy}(x, 0^-) + \sigma_{yy}(x, 0^-),
\]
\[
\sigma_{xy}(x, 0^-) = \sigma_{xy}(x, 0^-) + \sigma_{xy}(x, 0^-), \]
\[
\sigma_{yy}(x, 0^-) = \sigma_{yy}(x, 0^-) + \sigma_{yy}(x, 0^-),
\]
\[
D_{y}(x, 0^-) = D_{y}(x, 0^-) + D_{y}(x, 0^-).
\]
where

\[
\sigma_{y_1}(x, 0^-) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left[ \left( -i\epsilon_{13}^{(2)} H_{10} + \lambda e_{33}^{(2)} H_4 \right)
+ \lambda e_{33}^{(2)} H_{16} \right] I_{1\ast} + \left( -i\epsilon_{13}^{(2)} H_{11} - \lambda e_{33}^{(2)} H_5 \right) I_{2\ast} - \lambda e_{33}^{(2)} H_{17} I_{2\ast} + \left( i\epsilon_{13}^{(2)} H_{12} - \lambda e_{33}^{(2)} H_6 \right) I_{3\ast} \right] e^{-i\omega s} ds,
\]

\]

\[
\sigma_{\bar{y}_1}(x, 0^+) = \lim_{y \to 0^+} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left[ \left( \lambda e_{33}^{(2)} H_{10} - i\lambda e_{34}^{(2)} H_4 \right)
- i\epsilon_{15}^{(2)} H_{16} \right] I_{1\ast} + \left( i\epsilon_{14}^{(2)} H_5 - \lambda e_{33}^{(2)} H_6 \right) I_{2\ast} + \lambda e_{33}^{(2)} H_{17} I_{2\ast} + \left( i\epsilon_{14}^{(2)} H_6 - \lambda e_{33}^{(2)} H_6 \right) I_{3\ast} \right] e^{-i\omega s} ds,
\]

\]

\[
D_{y_1}(x, 0^+) = \lim_{y \to 0^+} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left[ \left( \lambda e_{33}^{(2)} H_{10} - i\lambda e_{34}^{(2)} H_4 \right)
- i\epsilon_{15}^{(2)} H_{16} \right] I_{1\ast} + \left( i\epsilon_{14}^{(2)} H_5 - \lambda e_{33}^{(2)} H_6 \right) I_{2\ast} + \lambda e_{33}^{(2)} H_{17} I_{2\ast} + \left( i\epsilon_{14}^{(2)} H_6 - \lambda e_{33}^{(2)} H_6 \right) I_{3\ast} \right] e^{-i\omega s} ds,
\]

\]

\[
\sigma_{y_2}(x, 0^+) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[ -i\epsilon_{13}^{(2)} \left( B_{3a} A_{12} + B_{2b} A_{22} \right) \right] I_{1\ast} + \frac{|s| k_{4}^{(2)} / s} \left( -B_{3a} A_{12} + B_{3b} A_{22} \right) \right] e^{-i\omega s} ds.
\]

\]

\[
D_{y_2}(x, 0^+) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[ -i\epsilon_{13}^{(2)} \left( B_{3a} A_{12} + B_{2b} A_{22} \right) \right] I_{1\ast} + \frac{|s| k_{4}^{(2)} / s} \left( -B_{3a} A_{12} + B_{3b} A_{22} \right) \right] e^{-i\omega s} ds.
\]

Obviously, it is easy to solve the solution of Equations (71), (72) and (73) according to Equations (39), (40), (41) and (42). Next, we need to transform Equations (68), (69) and (70) into singular integral equations of the first kind with Cauchy kernel.

In another form, integral Equations (68), (69) and (70) will be

\[
\sigma_{y_1}(x, 0^+) = \lim_{y \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left( Q_{1} I_{1\ast} + Q_{2} I_{2\ast} + Q_{3} I_{3\ast} \right) e^{-i\omega s} ds,
\]

\[
\sigma_{\bar{y}_1}(x, 0^+) = \lim_{y \to 0^+} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left( Q_{4} I_{1\ast} + Q_{5} I_{2\ast} + Q_{6} I_{3\ast} \right) e^{-i\omega s} ds,
\]

\[
D_{y_1}(x, 0^+) = \lim_{y \to 0^+} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{k=1}^{6} \frac{\delta e^{iky}}{H} \left( Q_{1} I_{1\ast} + Q_{4} I_{2\ast} + Q_{5} I_{3\ast} \right) e^{-i\omega s} ds.
\]

The expressions of $e^{iky}$ and $Q_{p}$, $p = 1, 2, \ldots, 9$, are given in Appendix C.

Substituting Equations (61), (62) and (63) into Equations (74), (75) and (76), we get

\[
\frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{1} e^{i(t-x)} ds \right) d_{1}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{4} e^{i(t-x)} ds \right) d_{2}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{5} e^{i(t-x)} ds \right) d_{3}(t) dt = \sigma_{1},
\]

\[
\frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{2} e^{i(t-x)} ds \right) d_{4}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{3} e^{i(t-x)} ds \right) d_{5}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{6} e^{i(t-x)} ds \right) d_{6}(t) dt = \sigma_{2},
\]

\[
\frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{7} e^{i(t-x)} ds \right) d_{7}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{8} e^{i(t-x)} ds \right) d_{8}(t) dt + \frac{1}{2\pi} \int_{l}^{l'} \left( \int_{-\infty}^{+\infty} \frac{\delta e^{iky}}{H} Q_{9} e^{i(t-x)} ds \right) d_{9}(t) dt = \sigma_{3}.
\]
where

\[\sigma_1 = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} -i\zeta_3^{(2)} \left( B_{3a}A_{12}e^{-i|k|y} + B_{3b}A_{22}e^{i|k|y} \right) \] 

\[- \zeta_1^{(2)} (A_{12} + A_{22}) + \frac{|s|k_3\zeta_3^{(2)}}{\pi} \left( -B_{3a}A_{12}e^{-i|k|y} \right) + B_{3b}A_{22}e^{i|k|y} + B_{1b}A_{22}e^{i|k|y}) e^{-\gamma x} ds,

\sigma_2 = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} -i\zeta_4^{(2)} (B_{1a}A_{12} + B_{1b}A_{22}) 

+ \frac{|s|k_4\zeta_4^{(2)}}{\pi} (-B_{3a}A_{12} + B_{3b}A_{22}) 

- i\zeta_5^{(2)} \left( -B_{3a}A_{12} + B_{3b}A_{22} \right) e^{-\gamma x} ds,

\sigma_3 = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} -i\zeta_6^{(2)} (B_{2a}A_{12}e^{-i|k|y} + B_{3a}A_{22}e^{i|k|y})

- \zeta_7^{(2)} (A_{12} + A_{22}) - \frac{|s|k_6\zeta_6^{(2)}}{\pi} \left( -B_{3a}A_{12}e^{-i|k|y} \right) + B_{3b}A_{22}e^{i|k|y} + B_{1b}A_{22}e^{i|k|y}) e^{-\gamma x} ds.

On the cracks’ surface,

\[\int_{-l}^{l} \left( \frac{Q_{1s}}{H_s t - x} + \kappa_2(x, t) \right) d_1(t) dt + \int_{-l}^{l} \left( \frac{Q_{2s}}{H_s t - x} + \kappa_3(x, t) \right) d_2(t) dt + \int_{-l}^{l} \left( \frac{Q_{3s}}{H_s t - x} + \kappa_4(x, t) \right) d_3(t) dt = \pi\sigma_1,

\int_{-l}^{l} \left( \frac{Q_{4s}}{H_s t - x} + \kappa_5(x, t) \right) d_1(t) dt + \int_{-l}^{l} \left( \frac{Q_{5s}}{H_s t - x} + \kappa_6(x, t) \right) d_2(t) dt + \int_{-l}^{l} \left( \frac{Q_{6s}}{H_s t - x} + \kappa_7(x, t) \right) d_3(t) dt = \pi\sigma_2,

\int_{-l}^{l} \left( \frac{Q_{7s}}{H_s t - x} + \kappa_8(x, t) \right) d_1(t) dt + \int_{-l}^{l} \left( \frac{Q_{8s}}{H_s t - x} + \kappa_9(x, t) \right) d_2(t) dt + \int_{-l}^{l} \left( \frac{Q_{9s}}{H_s t - x} + \kappa_{10}(x, t) \right) d_3(t) dt = \pi\sigma_3.

where

\[
\kappa_2(x, t) = \int_{-l}^{l} \left( \frac{Q_{1s}}{H_s t - x} \right) \sin (s(t-x)) ds,
\]

\[
\kappa_3(x, t) = \int_{-l}^{l} \left( \frac{Q_{2s}}{H_s t - x} \right) \sin (s(t-x)) ds,
\]

\[
\kappa_4(x, t) = \int_{-l}^{l} \left( \frac{Q_{3s}}{H_s t - x} \right) \sin (s(t-x)) ds,
\]

\[
\kappa_5(x, t) = \int_{-l}^{l} \left( \frac{Q_{4s}}{H_s t - x} \right) \sin (s(t-x)) ds,
\]

\[
\kappa_6(x, t) = \int_{-l}^{l} \left( \frac{Q_{5s}}{H_s t - x} \right) \sin (s(t-x)) ds,
\]

The expressions of \( H_s \) and \( Q_{ps} \), \( p = 1, 2, \cdots, 10 \), are given in Appendix C. Ultimately, Equations (68), (69) and (70) are reduced to coupling the first kind Cauchy singular integral equations.

In Equations (61), (62) and (63), the unknown function \( d_k(t), k = 1, 2, 3 \) with the following conditions:

\[\int_{-l}^{l} d_k(t) dt = 0, \quad k = 1, 2, 3.

Considering that the solution of the singular integral equation will use Chebyshev polynomials which are defined in the interval \([-1, 1]\), we introduce the following normalized quantities:

\[ r = \frac{x}{l}, \quad s = \frac{t}{l}, \]

\[ d_k(x) = \hat{d}_k(x), \]

\[ L_i(r, s) = \mathbf{l} \cdot \kappa_i(x, t), \]

\[ k = 1, 2, 3, i = 1, \cdots, 10. \]

Using Equations (84) and (85), integral Equations (79), (80) and (81) may be expressed as

\[ \int_{-l}^{l} \left( \frac{Q_{1s}}{H_s s - r} + L_2(s, r) \right) \hat{d}_1(s) ds + \int_{-l}^{l} \left( \frac{Q_{2s}}{H_s s - r} + L_3(s, r) \right) \hat{d}_2(s) ds + \int_{-l}^{l} \left( \frac{Q_{3s}}{H_s s - r} + L_4(s, r) \right) \hat{d}_3(s) ds = \pi\sigma_1, \]
\[ \int_{-1}^{1} \left( \frac{Q_s}{H_s} - \frac{1}{s^2} + L_5(s, r) \right) \tilde{d}_1(s) ds + \int_{-1}^{1} \left( \frac{Q_s}{H_s} - \frac{1}{s^2} + L_6(s, r) \right) \tilde{d}_2(s) ds + \int_{-1}^{1} \left( \frac{Q_s}{H_s} - \frac{1}{s^2} + L_7(s, r) \right) \tilde{d}_3(s) ds = \pi \sigma_2, \]

\[ \int_{-1}^{1} \left( \frac{Q_s}{H_s} + L_8(s, r) \right) \tilde{d}_1(s) ds + \int_{-1}^{1} \left( \frac{Q_s}{H_s} + L_9(s, r) \right) \tilde{d}_2(s) ds + \int_{-1}^{1} \left( \frac{Q_s}{H_s} + L_{10}(s, r) \right) \tilde{d}_3(s) ds = \pi \sigma_3, \]

\[ \sum_{k=0}^{N} X_k \left[ \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_1(s_k) \right] + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_2(s_k) + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_3(s_k) = \pi \sigma_1, \]

\[ \sum_{k=0}^{N} X_k \left[ \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_1(s_k) \right] + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_2(s_k) + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_3(s_k) = \pi \sigma_3, \]

The crack-tip behavior can be characterized by square-root singularity weighed down by the oscillation. However, because the zone of oscillation is very small [25], the unknown function \( \tilde{d}_k(s) \), \( k = 1, 2, 3 \), can be expressed as follows:

\[ \tilde{d}_k(s) = \frac{1}{\sqrt{1 - s^2}} g_k(s), \quad k = 1, 2, 3, \tag{87} \]

where \( g_k(s) \), \( k = 1, 2, 3 \), are continuous functions defined in the interval \([-1, 1]\).

Using the Lobatto-Chebyshev collocation method, one can transform Equations (79), (80), (81) and (83) into a system of algebraic equations:

\[ \sum_{k=0}^{N} X_k \left[ \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_1(s_k) \right] + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_2(s_k) + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_3(s_k) = \pi \sigma_1, \]

\[ \sum_{k=0}^{N} X_k \left[ \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_1(s_k) \right] + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_2(s_k) + \left( \frac{Q_s}{H_s} + \frac{1}{s^2} - r_q \right) \tilde{d}_3(s_k) = \pi \sigma_3, \]

\[ N \text{ is the node number of quadrature formula; } s_k \text{ and } r_q \]

are the zero points of the first and second kinds of Chebyshev polynomials. According to Theocaris and Ioakimidis [26] as well as Lu [27], the convergence rate of the Lobatto-Chebyshev collocation method is first order (i.e., \( o(h) \), where \( h \) stands for step size).

The intensity factors are defined by [13]

\[ K_{\alpha} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} \sigma_{\alpha x}(x, 0), \tag{90} \]

\[ K_{\beta} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} \sigma_{\beta y}(x, 0), \tag{91} \]

\[ K_{\gamma} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} \sigma_{\gamma y}(x, 0), \tag{92} \]

\[ K_{\delta} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} D_y(x, 0), \tag{93} \]

\[ K_{\varepsilon} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} D_x(x, 0), \tag{94} \]

\[ K_{\zeta} = \lim_{x \rightarrow -l} \sqrt{2\pi(-l-x)} D_x(x, 0). \tag{95} \]

Substituting Equation (87) into Equations (90), (91), (92), (93), (94) and (95), we obtain
where superscript 2 represents the substrate material. Assume that the coefficients of thermal conductivity are $k_m = \sqrt{K^{(m)}}, K^{(m)} = 1, m = 1, 2$.

4.1. Verification. We chose a special case to verify the foregoing derivation. We first assumed that the fine-grained piezoelectric coating and substrate are the same material. The problem in Figure 1 then becomes that of a Griffith crack in a single material plate with thermal loading.

4.2. Thermal Intensity Factors of Fine-Grained Piezoelectric Coating/Substrate Structure

In what follows, we assume that the piezoelectric substrate is cadmium selenide piezoelectric ceramic. Its parameters are given as [24]

\[
\begin{align*}
\epsilon_{11}^{(2)} &= 7.41 \times 10^{10} \text{ N/m}^2, \\
\epsilon_{15}^{(2)} &= -0.138 \text{ C/m}^2, \\
\epsilon_{33}^{(2)} &= 3.93 \times 10^{10} \text{ N/m}^2, \\
\epsilon_{44}^{(2)} &= 0.825 \times 10^{-10} \text{ C/Vm}, \\
\epsilon_{44}^{(2)} &= 1.32 \times 10^{10} \text{ N/m}^2, \\
\epsilon_{66}^{(2)} &= 0.903 \times 10^{-10} \text{ C/Vm}, \\
\lambda_{11}^{(2)} &= 0.16 \text{ C/m}^2, \\
\lambda_{33}^{(2)} &= 0.347 \text{ C/m}^2, \\
\lambda_{33}^{(2)} &= 0.551 \times 10^{6} \text{ N/m}^2 \text{K}, \\
\end{align*}
\]

Therefore, in Equation (26), $M_0 = -k_1 k_2 k_y^{(1)} k_y^{(2)}/(k_1 k_y^{(1)} + k_2 k_y^{(2)})$ becomes

\[
k_1 = k_3 = k = \sqrt{\frac{k_x}{k_y}}
\]

\[
k_y^{(1)} = k_y^{(2)} = k_y
\]

\[
M_0 \rightarrow M_* = -\sqrt{\frac{k_x}{2}}.
\]

Thus, the thermal flow intensity factors (29) and (30) can be written as follows:

\[
k_0^L = -\sqrt{\frac{k_x k_y}{2}} \sqrt{\sigma_0 (-l)},
\]

\[
k_0^R = \sqrt{\frac{k_x k_y}{2}} \sqrt{\sigma_0 (l)}.
\]

Because the crack is symmetrical about the $Y$-axis, we get

\[
k_0 = k_0^L = k_0^R.
\]
4.2.1. Studies of the Effect of Elastic Modulus on Thermal Intensity Factor. Here, we select material 1, which is a fine-grained coating made of cadmium selenide piezoelectric ceramic; material 2 is normal cadmium selenide piezoelectric ceramic (where 1 and 2 represent the fine-grained coating and substrate material, respectively). This means that material 1 and material 2 have approximately the same piezoelectric and dielectric constants under suitable conditions, and the elastic modulus of material 1 is larger than that of material 2.

Figures 3–5 show the effects of the elastic modulus on the thermal intensity factor for different $c^{(1)}_{44}/c^{(2)}_{44}$.

**Figure 3:** Effects of $l/h_1$ on mode I thermal stress intensity factor for different $c^{(1)}_{44}/c^{(2)}_{44}$.

**Figure 4:** Effects of $l/h_1$ on mode II thermal stress intensity factor for different $c^{(1)}_{44}/c^{(2)}_{44}$.

**Figure 5:** Effects of $l/h_1$ on electric displacement intensity factor for different $c^{(1)}_{44}/c^{(2)}_{44}$.

**Figure 6:** Effects of $l/h_2$ on mode I thermal stress intensity factor for different $h_1/h_2$.

400, $h_1 = 5$ mm, $K_I = \lim_{l/h_1 \to 0} K_{I0}$, $K_{II} = \lim_{l/h_1 \to 0} K_{II0}$, and $K_D = \lim_{l/h_1 \to 0} K_D$. These data indicate that with an increase in the crack length, the thermal stress intensity factor and the electric displacement intensity factor increase, and with the increase of the ratio of the elastic modulus between the fine-grained piezoelectric coating and substrate. The peaks of the thermal stress intensity factor and the electric displacement intensity factor increase correspondingly with the increase in the ratio of the elastic modulus between the fine-grained piezoelectric coating and substrate. Concurrently, when $l/h_1$ increases from 2, the growth rates of $K_I$ and $K_{II}$ are greater than that of $K_D$, and the interface fracture will eventually occur with a continuous increase in $l/h_1$. 

100, $h_1 = 5$ mm, $K_I = \lim_{l/h_1 \to 0} K_{I0}$, $K_{II} = \lim_{l/h_1 \to 0} K_{II0}$, and $K_D = \lim_{l/h_1 \to 0} K_D$. These data indicate that with an increase in the crack length, the thermal stress intensity factor and the electric displacement intensity factor increase, and with the increase of the ratio of the elastic modulus between the fine-grained piezoelectric coating and substrate. The peaks of the thermal stress intensity factor and the electric displacement intensity factor increase correspondingly with the increase in the ratio of the elastic modulus between the fine-grained piezoelectric coating and substrate. Concurrently, when $l/h_1$ increases from 2, the growth rates of $K_I$ and $K_{II}$ are greater than that of $K_D$, and the interface fracture will eventually occur with a continuous increase in $l/h_1$. 

Figure 3: Effects of $l/h_1$ on mode I thermal stress intensity factor for different $c^{(1)}_{44}/c^{(2)}_{44}$.
5. Conclusions

The Griffith crack problem of fine-grained piezoelectric coating/substrates under steady-state thermal loading is studied. Based on the principle of integral transformation and superposition of equation solutions, the thermomechanical problem is transformed into a solution of thermal stress and electric displacement intensity factors. We then analyzed the effects of crack geometry, coating thickness, and material modulus of elasticity on the fracture behavior of the crack under thermal load. The results show that a larger elastic modulus and thinner coating thickness improve the safety of the coating/substrate structure. This reflects the advantages of fine-grained structure materials.

Appendix A.

\[ F_1 = \frac{k_2 k_y^{(2)}}{\Delta} e^{2h_1 k_y |x|} \left( e^{2h_2 k_y |x|} + 1 \right), \]

\[ F_2 = \frac{\phi_l k_y^{(2)}}{\Delta} \left[ k_2 k_y^{(2)} - k_1 k_y^{(1)} + \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_2 k_y |x|} \right], \]

\[ F_3 = \frac{2k_2 k_y^{(2)}}{\Delta} e^{2h_1 k_y |x|} \phi_l k_y^{(2)}, \]

\[ F_4 = \phi_l k_y^{(1)} \left[ k_1 k_y^{(1)} - k_2 k_y^{(2)} - \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_2 k_y |x|} \right], \]

\[ F_5 = \frac{k_2 k_y^{(2)}}{\Delta} e^{2h_1 k_y |x|}, F_6 = \frac{k_2 k_y^{(2)}}{\Delta} e^{2h_1 k_y |x|}, \]

\[ F_7 = \frac{\phi_l k_y^{(2)}}{\Delta} \left[ k_1 k_y^{(1)} - k_2 k_y^{(2)} + \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_1 k_y |x|} \right], \]

\[ F_8 = \frac{k_1 k_y^{(1)}}{\Delta} e^{2h_1 k_y |x|} \left( e^{2h_1 k_y |x|} + 1 \right), F_9 = \frac{k_2 k_y^{(1)}}{\Delta} e^{2h_1 k_y |x|}, \]

\[ F_{10} = \frac{2k_2 k_y^{(1)}}{\Delta} e^{2h_1 k_y |x|}, \]

\[ F_{11} = \frac{\phi_l k_y^{(2)}}{\Delta} \left[ k_2 k_y^{(2)} + \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_1 k_y |x|} \right], \]

\[ F_{12} = \frac{k_2 k_y^{(1)}}{\Delta} \left( e^{2h_1 k_y |x|} + 1 \right), \]

\[ \Delta = k_1 k_y^{(1)} - k_2 k_y^{(2)} + \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_1 k_y |x|} \cdot e^{2h_2 k_y |x|} - \left( k_1 k_y^{(1)} + k_2 k_y^{(2)} \right) e^{2h_1 k_y |x|} e^{2h_2 k_y |x|}. \]

(A.1)
\[ a_1 = \frac{1}{\delta} \left( B_{1a}^{(1)} A_{11} + B_{1b}^{(1)} A_{21} - B_{2a}^{(2)} A_{12} + B_{2b}^{(2)} A_{22} \right), \]
\[ a_2 = \frac{1}{\delta} \left( B_{3a}^{(1)} A_{11} + B_{3b}^{(1)} A_{21} - B_{3a}^{(2)} A_{12} + B_{3b}^{(2)} A_{22} \right), \]
\[ a_3 = \frac{1}{\delta} \left( B_{4a}^{(1)} A_{11} + B_{4b}^{(1)} A_{21} - B_{4a}^{(2)} A_{12} + B_{4b}^{(2)} A_{22} \right), \]
\[ a_4 = -i c_{13}^{(1)} \left( B_{2a}^{(1)} A_{11} e^{-i|k| h_1} + B_{2b}^{(1)} A_{21} e^{i|k| h_1} \right), \]
\[ a_5 = \frac{k c_{13}^{(1)}}{\delta} \left[ -B_{1a}^{(1)} A_{11} e^{-i|k| h_1} + B_{1b}^{(1)} A_{21} e^{i|k| h_1} \right. \]
\[ - \lambda_{11}^{(1)} \left( A_{11} e^{-i|k| h_1} + A_{21} e^{i|k| h_1} \right)], \]
\[ a_6 = -i c_{13}^{(1)} \left( B_{3a}^{(1)} A_{11} e^{i|k| h_1} + B_{3b}^{(1)} A_{21} e^{-i|k| h_1} \right), \]
\[ a_7 = \frac{-k c_{33}^{(2)}}{\delta} \left[ -B_{1a}^{(2)} A_{11} e^{i|k| h_1} + B_{1b}^{(2)} A_{21} e^{-i|k| h_1} \right. \]
\[ - \lambda_{11}^{(2)} \left( A_{11} e^{i|k| h_1} + A_{21} e^{-i|k| h_1} \right)], \]
\[ a_8 = \left[ \frac{k c_{44}^{(1)}}{\delta} \left( -B_{2a}^{(1)} A_{11} + B_{2b}^{(1)} A_{21} \right) - i c_{44}^{(1)} \right. \]
\[ \cdot \left( B_{1a}^{(1)} A_{11} + B_{1b}^{(1)} A_{21} \right) - i c_{15}^{(1)} \left( B_{3a}^{(1)} A_{11} + B_{3b}^{(1)} A_{21} \right)], \]
\[ a_9 = \left[ \frac{k c_{44}^{(2)}}{\delta} \left( -B_{2a}^{(2)} A_{11} + B_{2b}^{(2)} A_{21} \right) - i c_{44}^{(2)} \right. \]
\[ \cdot \left( B_{1a}^{(2)} A_{11} + B_{1b}^{(2)} A_{21} \right) - i c_{15}^{(2)} \left( B_{3a}^{(2)} A_{11} + B_{3b}^{(2)} A_{21} \right)], \]
\[ a_{1*} = \left[ \frac{c_{14}^{(1)}}{c_{15}^{(1)}} - \left( c_{33}^{(1)} \sum_{k=1}^{6} \lambda_k \right)^2 + 1 \right] \left( c_{44}^{(2)} e_{15}^{(2)} - c_{44}^{(1)} e_{15}^{(1)} \right) a_1 \]
\[ + 6 \left( \sum_{k=1}^{6} \lambda_k a_2 + e_{15}^{(1)} a_3 + \sum_{k=1}^{6} \lambda_k \left( c_{44}^{(2)} e_{15}^{(1)} \right. \right. \]
\[ - c_{44}^{(1)} e_{15}^{(2)} \left. \right) (a_6 + a_7) - i c_{15}^{(2)} a_8 + i c_{15}^{(1)} a_9, \]
\[ a_{2*} = \frac{c_{14}^{(1)}}{c_{15}^{(1)}} a_1 + \frac{c_{14}^{(2)}}{c_{15}^{(2)}} \sum_{k=1}^{6} \lambda_k a_2 + c_{15}^{(1)} a_3 \]
\[ + \sum_{k=1}^{6} \lambda_k \left( c_{44}^{(2)} e_{15}^{(1)} - c_{44}^{(1)} e_{15}^{(2)} \right) (a_6 + a_7) - i c_{15}^{(2)} a_8 + i c_{15}^{(1)} a_9, \]
\[ a_{3*} = \frac{c_{14}^{(1)}}{c_{15}^{(1)}} \left( \sum_{k=1}^{6} \lambda_k a_1 + \sum_{k=1}^{6} \lambda_k \right) \left( c_{44}^{(2)} e_{15}^{(1)} \right. \]
\[ + \frac{c_{14}^{(2)}}{c_{15}^{(2)}} \sum_{k=1}^{6} \lambda_k a_3 - i c_{15}^{(1)} e_{15}^{(1)} a_3 - i c_{15}^{(2)} (a_6 + a_7) - i c_{15}^{(1)} a_8 + i c_{15}^{(1)} a_9, \]
\[ a_{4*} = \frac{c_{14}^{(1)}}{c_{15}^{(1)}} \left( \sum_{k=1}^{6} \lambda_k a_1 + \sum_{k=1}^{6} \lambda_k a_3 - i c_{15}^{(1)} e_{15}^{(1)} a_3 \right. \]
\[ - i c_{15}^{(1)} e_{15}^{(2)} a_9 - \left( c_{44}^{(2)} e_{15}^{(2)} - c_{44}^{(1)} e_{15}^{(1)} \right) (a_6 + a_7) - c_{15}^{(2)} (a_6 + a_7) - c_{15}^{(1)} \left. \sum_{k=1}^{6} \lambda_k a_9, \right] \]
\[ a_{5*} = -i c_{14}^{(2)} + i c_{14}^{(1)} a_1 + i c_{44}^{(1)} \right. \]
\[ \left. \left( \sum_{k=1}^{6} \lambda_k a_1 + \sum_{k=1}^{6} \lambda_k a_3 - i c_{15}^{(1)} e_{15}^{(1)} a_3 \right) \right. \]
\[ - i c_{44}^{(1)} e_{15}^{(2)} a_9 - \left( c_{44}^{(2)} e_{15}^{(2)} - c_{44}^{(1)} e_{15}^{(1)} \right) (a_6 + a_7) - c_{15}^{(2)} (a_6 + a_7) - c_{15}^{(1)} \left. \sum_{k=1}^{6} \lambda_k a_9, \right] \]
\[ a_{6*} = -i c_{14}^{(2)} + i c_{14}^{(1)} a_1 + i c_{44}^{(1)} \right. \]
\[ \left. \left( \sum_{k=1}^{6} \lambda_k a_1 + \sum_{k=1}^{6} \lambda_k a_3 - i c_{15}^{(1)} e_{15}^{(1)} a_3 \right) \right. \]
\[ - i c_{44}^{(1)} e_{15}^{(2)} a_9 - \left( c_{44}^{(2)} e_{15}^{(2)} - c_{44}^{(1)} e_{15}^{(1)} \right) (a_6 + a_7) - c_{15}^{(2)} (a_6 + a_7) - c_{15}^{(1)} \left. \sum_{k=1}^{6} \lambda_k a_9, \right] \]
\[ H_1 = \left[ \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{i \lambda h_1} + 1 \right] \left( c_{44}^{(2)} e_{15}^{(2)} - c_{44}^{(1)} e_{15}^{(1)} \right. \]
\[ - i c_{44}^{(2)} e_{15}^{(2)} \right], \]
\[ H_2 = i c_{44}^{(1)} \left( \sum_{k=1}^{6} \lambda_k e^{i \lambda h_1} \right), \]
\[ H_3 = c_{15}^{(1)} e_{15}^{(1)}, \]
\[ H_4 = -i c_{44}^{(1)} e_{15}^{(2)}, \]
\[ H_5 = i c_{44}^{(1)} \left( \sum_{k=1}^{6} \lambda_k e^{i \lambda h_1} \right), \]
\[ H_6 = c_{15}^{(1)} e_{15}^{(1)}. \]
\[ H_7 = -i c^{(1)}_{33} \sum_{k=1}^{6} \lambda_k \left[ \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda h_1 h_2} + 1 \right] \left( c^{(2)}_{44} \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda(h_1 + h_2)} \] 

\[ H_9 = c^{(1)}_{33} c^{(1)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{10} = -c^{(2)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{11} = c^{(1)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{12} = c^{(2)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{13} = -i c^{(1)}_{44}, \]

\[ H_{14} = i c^{(1)}_{33} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{15} = i c^{(1)}_{44} e^{\lambda h}, \]

\[ H_{16} = -i c^{(1)}_{44}, \]

\[ H_{17} = i c^{(2)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H_{18} = i c^{(1)}_{44} \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ H = \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda(h_1 + h_2)} + 1 \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda h_1 h_2}. \] (B.1)

C.

\[ Q_1(s) = \sum_{k=1}^{6} \left[ i c^{(1)}_{33} H_{10} + \lambda_k c^{(2)}_{33} H_4 \right] + \lambda_k e^{\lambda h} H_{16}, \]

\[ Q_2(s) = \sum_{k=1}^{6} \left[ -i c^{(1)}_{33} H_{11} - \lambda_k c^{(2)}_{33} H_5 \right] - \lambda_k e^{\lambda h} H_{17}, \]

\[ Q_3(s) = \sum_{k=1}^{6} \left( c^{(1)}_{33} H_{12} - \lambda_k c^{(2)}_{33} H_6 \right) - \lambda_k e^{\lambda h} H_{18}, \]

\[ Q_4 = \sum_{k=1}^{6} \left( \lambda_k c^{(2)}_{44} H_{10} - i \lambda_k c^{(2)}_{44} H_4 \right) - i e^{\lambda h} H_{16}, \]

\[ Q_5(s) = \sum_{k=1}^{6} \left( i c^{(2)}_{44} H_5 - \lambda_k c^{(2)}_{44} H_{11} \right) + i e^{\lambda h} H_{17}, \]

\[ Q_6(s) = \sum_{k=1}^{6} \left( i c^{(2)}_{44} H_6 - \lambda_k c^{(2)}_{44} H_{12} \right) + i e^{\lambda h} H_{18}, \]

\[ Q_7(s) = \sum_{k=1}^{6} \left( \lambda_k c^{(2)}_{33} H_4 - i e^{\lambda h} H_{10} \right) - \lambda_k e^{\lambda h} H_{16}, \]

\[ Q_8(s) = \sum_{k=1}^{6} \left( \lambda_k c^{(2)}_{33} H_5 - i e^{\lambda h} H_{11} \right) - \lambda_k e^{\lambda h} H_{17}, \]

\[ Q_9(s) = \sum_{k=1}^{6} \left( \lambda_k c^{(2)}_{33} H_6 - i e^{\lambda h} H_{12} \right) + \lambda_k e^{\lambda h} H_{18}, \]

\[ Q_{10} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{11} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{12} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{13} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{14} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{15} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{16} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{17} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{18} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{19} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{20} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{21} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{22} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{23} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{24} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{25} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{26} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{27} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{28} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{29} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{30} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{31} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ Q_{32} = \sum_{k=1}^{6} \lambda_k e^{\lambda h}, \]

\[ Q_{33} = \sum_{k=1}^{6} e^{\lambda h}, \]

\[ H = \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda(h_1 + h_2)} + 1 \left( \sum_{k=1}^{6} \lambda_k \right)^2 e^{\lambda h_1 h_2}. \] (C.1)
Data Availability

The program codes that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no competing interests.

Acknowledgments

This study was funded by the National Natural Science Foundation of China (11972019).

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