Particle mixing as possible explanation of the dark energy conundrum

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Abstract.

The vacuum condensate due to neutrino and quark mixing behaves as a perfect fluid and, at the present epoch, as a cosmological constant. The very small breaking of the Lorentz invariance constrains today the value of the dark energy.

1. Introduction

Recent astrophysical data [1]-[7] represent observational evidence of the fact that the universe today observed is flat and it is undergoing an accelerated expansion. The hypothesis is that such an expansion is driven by a fluid called dark energy with negative pressure, nearly homogeneously distributed and making up to $\sim 70\%$ of the energy content of the universe.

Even if it seems clear how dark energy works, its nature remains an unsolved problem. Therefore, the understanding of the dark energy has become one of the main issue of modern physics. Its determination should provide the gravity vacuum state [8], should clarify the mechanisms which led from the early universe to the today observed large scale structures [9, 10], and to represent the theoretical solution to the observational data.

A lot of models have been proposed in order to explain the dark energy [11]-[19]. However, no comprehensive scheme is available, which may fit the observations and frame them into a fundamental theory.

In this paper we report recent results according to which the particle mixing phenomenon could explain the dark energy component of the universe [20]. Responsible of such a component is the non-perturbative vacuum structure associated with the particle mixing [21]-[23].

The estimated value of the cosmological constant is imposed by the small breaking of the Lorentz invariance of the flavor vacuum at the present epoch [20].

The paper is organized as follows. In Section II we present the QFT formalism for mixed fields [24]-[31] (for a detailed review see [29]). We introduce the particle mixing vacuum condensate and show that it is homogeneous and isotropic [20]. In Section III we exhibit the mixing contributions to the dark energy at the present epoch and Section IV is devoted to conclusions.

2. Particle mixing vacuum condensate

The mixing transformations among three generations of Dirac fields are: $\Psi_f(x) = U \Psi_m(x)$, where $U$ is the CKM matrix and $\Psi_m^T = (\psi_1, \psi_2, \psi_3)$ are the fields with definite masses $m_1 \neq m_2 \neq m_3$. The mixing relations can be written as $\psi_\sigma^\alpha(x) = G^{-1}_\theta(t) \psi_i^\alpha(x) G_\theta(t)$, where
\( (\sigma, i) = (A, 1), (B, 2), (C, 3); \ A, B, C \) denote lepton (\( e, \mu, \tau \)) or flavor (\( d, s, b \)) indices. \( G_\theta(t) \) is the mixing generator \([20, 28, 29]\). The flavor annihilators, at each time, are defined as:

\( \alpha_{k_i}^\dagger(t) \equiv G_\theta^{-1}(t) \alpha_{k_i}^\dagger G_\theta(t) \), and \( \beta_{k_i}^\dagger(t) \equiv G_\theta^{-1}(t) \beta_{k_i}^\dagger G_\theta(t) \). They annihilate the flavor vacuum \( |0(t)\rangle_f = G_\theta^{-1}(t) |0\rangle_m \), where \( |0\rangle_m \) is the vacuum annihilated by \( \alpha_{k_i}^\dagger \) and \( \beta_{k_i}^\dagger \), \( i = 1, 2, 3 \), \( r = 1, 2 \). In the infinite volume limit \( |0(t)\rangle_f \) turns out to be unitarily inequivalent to the vacuum \( |0\rangle_m \) \([24, 27]\). Moreover, \( |0(t)\rangle_f \) is a coherent condensate of particles. In the reference frame such that \( k = (0, 0, |k|) \), the numbers of condensed particles are:

\[
N^k_1 = f \langle 0(t)|N_{\alpha_1}^k|0(t)\rangle_f = f \langle 0(t)|N_{\beta_1}^k|0(t)\rangle_f = s_{12}^2 c_{13}^2 |V_{12}|^2 + s_{13}^2 |V_{13}|^2,
\]

and similar relations for \( N^k_2, N^k_3 \). In Eq. (1), \( N_{\alpha_1}^k = \alpha_{k_i}^\dagger \alpha_{k_i} \), \( N_{\beta_1}^k = \beta_{k_i}^\dagger \beta_{k_i} \) with \( i = 1, 2, 3 \), and \( V_{ij} \) are the Bogoliubov coefficients entering the mixing transformations (see Refs. [20, 28, 29]).

As shown in Ref. [20], the vacuum condensate due to neutrino and quark mixing behaves as a perfect fluid. Indeed considering the Minkowski metric, the energy-momentum tensor density \( T_{\mu\nu}(x) \) for the fermion fields \( \psi_i, i = 1, 2, 3 \) is \([32]\):

\[
T_{\mu\nu}^\text{cond}(x) = f \langle 0(t)| \cdot T_{\mu\nu}(x) \cdot |0(t)\rangle_f ,
\]

for which the off-diagonal components are zero:

\[
\int d^3 x \ f \langle 0(t)|T_{0j}(x)|0(t)\rangle_f = \int d^3 x \ f \langle 0(t)|T_{j0}(x)|0(t)\rangle_f = 0 ,
\]

where \( j \neq 0 \), and \( j \neq l \), respectively. Thus \( T_{\mu\nu}^\text{cond} \) is homogeneous and isotropic and can be written as

\[
T_{\mu\nu}^\text{cond} = \text{diag}(T_{00}^\text{cond}, T_{11}^\text{cond}, T_{22}^\text{cond}, T_{33}^\text{cond}) ,
\]

where

\[
\int d^3 x \ T_{00}^\text{cond}(x) = \int d^3 x \ f \langle 0(t)| T_{00}(x) |0(t)\rangle_f = 4 \sum_i \int d^3 k \omega_{k,i} N_i^k ,
\]

\[
\int d^3 x \ T_{jj}^\text{cond}(x) = \int d^3 x \ f \langle 0(t)| T_{jj}(x) |0(t)\rangle_f = 4 \sum_i \int d^3 k \frac{k_j k_j}{\omega_{k,i}} N_i^k ,
\]

(no summation on \( j \) is intended) and \( N_i^k \) are the numbers of particles condensed in the vacuum (see Eq. (1)).

3. Particle mixing and dark energy

In Ref. [20] it has been shown that the energy density due to the vacuum condensate arising from particle mixing gives a contribution to the vacuum energy which evolves dynamically (see also Refs. [21, 22]). Moreover, it has been remarked that the negligible breaking of the Lorentz invariance of the vacuum is responsible of the very small value of the dark energy at the present epoch [20]. In this Section we report the main results obtained in Ref. [20].

Let us write the energy-momentum tensor density \( T_{\mu\nu}(x) \) for the fermion fields \( \psi_i, i = 1, 2, 3 \) as \([33]\):

\[
T_{\mu\nu}(x) = \Sigma_{\mu\nu}(x) + \mathcal{V}_{\mu\nu}(x) = \left\{ \frac{i}{2} \bar{\psi}(x) \gamma_\mu \frac{\partial}{\partial \nu} \psi(x) \right\} - \eta_{\mu\nu} \left[ \frac{i}{2} \bar{\psi}(x) \gamma^\alpha \frac{\partial}{\partial \alpha} \psi(x) \right] + \eta_{\mu\nu} \left[ \bar{\psi}(x) M_d \psi(x) \right] \right\} ,
\]
where

\[ \Sigma_{\mu\nu}(x) := \left\{ \frac{i}{2} \left( \bar{\Psi}_m(x) \gamma_{\mu} \gamma_{\nu} \Psi_m(x) \right) - \eta_{\mu\nu} \left[ \frac{i}{2} \bar{\Psi}_m(x) \gamma^\alpha \gamma_{\mu} \partial_{\alpha} \Psi_m(x) \right] \right\} , \]  

and

\[ \mathcal{V}_{\mu\nu}(x) := \eta_{\mu\nu} \left[ \bar{\Psi}_m(x) M_d \Psi_m(x) \right] , \]  

\[ M_d = \text{diag}(m_1, m_2, m_3) , \quad \Psi_m = (\psi_1, \psi_2, \psi_3)^T \]  

and \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \).

In any epochs the contributions of the particle mixing to the vacuum energy density \( \rho_{\text{mix}} \) and to the vacuum pressure \( p_{\text{mix}} \) are given respectively by the \((0,0)\) and the \((j,j)\) components of the energy momentum tensor density of the vacuum condensate:

\[ \rho_{\text{mix}} \equiv \frac{1}{V} \eta^{00} \int d^3 x \, T_{00}^{\text{cond}}(x) , \]  

\[ p_{\text{mix}} \equiv -\frac{1}{V} \eta^{jj} \int d^3 x \, T_{jj}^{\text{cond}}(x) , \]  

where no summation on the index \( j \) is intended. By using Eqs. (5) and (6) we have:

\[ \rho_{\text{mix}} = \frac{2}{\pi} \sum_i \int dk \, k^2 \omega_{k,i} N^k_i , \]  

\[ p_{\text{mix}} = \frac{2}{3\pi} \sum_i \int dk \, k^2 \frac{m_i^2}{\omega_{k,i}} N^k_i . \]  

We note that the use of the identity \( \omega_{k,i} = \frac{k^2}{\omega_{k,i}} + \frac{m_i^2}{\omega_{k,i}} \) enables to write \( \rho_{\text{mix}} \) as

\[ \rho_{\text{mix}} = \Sigma_{\text{mix}} + \mathcal{V}_{\text{mix}} \]  

where the kinetic term \( \Sigma_{\text{mix}} \) and the potential term \( \mathcal{V}_{\text{mix}} \) are given by

\[ \Sigma_{\text{mix}} = \frac{2}{\pi} \sum_i \int_0^K dk \, k^2 \frac{m_i^2}{\omega_{k,i}} N^k_i , \quad \mathcal{V}_{\text{mix}} = \frac{2}{3\pi} \sum_i \int_0^K dk \, k^2 \frac{m_i^2}{\omega_{k,i}} N^k_i . \]

However, in the present epoch, the very small breaking of the Lorentz invariance [7], imposes that the vacuum expectation values of \( T_{\mu\nu}(x) \) are space-time independent. This implies that the kinematical part \( \Sigma_{\text{cond}}^{\mu\nu} \) of \( T_{\text{cond}}^{\mu\nu}(x) \) is today very small [20]:

\[ \Sigma_{\text{cond}}^{\mu\nu} = \int \langle 0| \Sigma_{\mu\nu}(x) : |0(0)\rangle_f \simeq 0 \]  

and \( T_{\text{cond}}^{\mu\nu} \) is given by:

\[ T_{\text{cond}}^{\mu\nu} \simeq \int \langle 0| \mathcal{V}_{\mu\nu}(x) : |0(0)\rangle_f = \eta_{\mu\nu} \int \langle 0| : \bar{\Psi}_m(x) M_d \Psi_m(x) : |0(0)\rangle_f . \]  

By using Eqs. (4) and (17), at the present epoch, we obtain:

\[ \frac{1}{V} \int d^3 x \, T_{\mu\nu}^{\text{cond}}(x) = \text{diag}(\rho_{\text{mix}}, p_{\text{mix}}, p_{\text{mix}}, p_{\text{mix}}) \]  

\[ = \eta_{\mu\nu} \sum_i m_i \int \frac{d^3 x}{(2\pi)^3} \int \langle 0| : \bar{\psi}_i(x) \psi_i(x) : |0(0)\rangle_f . \]  

The today state equation is then given by: \( \rho_{\text{mix}} \simeq -p_{\text{mix}} \), that is, the adiabatic index is

\[ w_{\text{mix}} = p_{\text{mix}}/\rho_{\text{mix}} \simeq -1 . \]
This means that the vacuum condensate coming from particle mixing, today, mimics the behavior of the cosmological constant [21], [22]. From Eq.(18), introducing the cut-off on the momenta $K$, we derive $\rho_{\text{mix}}$ [20]:

$$\rho_{\text{mix}} \approx \frac{2}{\pi} \sum_i \int_0^K dk k^2 \frac{m_i^2}{\omega_{k,i}} N^i_k.$$  

(20)

The integral (20) diverges in $K$ as $m_i^4 \log(2K/m_j)$, with $i,j = 1,2,3$ [22]. However, the value close to $-1$ of $w_{\text{mix}}$ at the present epoch constrains the value of $K$ and consequently the value of $\rho_{\text{mix}}$ [20]. Indeed, by comparing Eqs.(12) and (20), we note that the condition (16) for the $(0,0)$ component of $T_{\mu\nu}$ imposes $\Sigma_{\text{mix}} \ll \nu_{\text{mix}}$, where $\Sigma_{\text{mix}}$ and $\nu_{\text{mix}}$ are given in Eq.(15).

Such a condition is satisfied when $K$ in Eq.(15) is $K \ll \sqrt{m_1 m_2 m_3}$. For this reason here we do not need to consider the regularization problem of the ultraviolet divergence of the integral (20).

We now show that particular values of $w_{\text{mix}}$, for neutrinos and for quarks, lead to contributions of the dark energy that are in agreement with its estimated upper bound. To do that we derive an expression of $w_{\text{mix}}$ as function of the cut-off $K$ [20].

Let us consider the adiabatic expansion of a sphere of volume $V$. Since the pressure $p$, at which the sphere expands, does work, the total energy, $E = \rho V$, varies: $dE = -pdV$. That is $ho dV + V \rho d\rho = -pdV$, that can be expressed as $d[(\rho + p)V] = 0$, and then

$$\rho + p = \frac{\text{const}}{V}.$$  

(21)

Thus if the volume is very large ($V \to \infty$), we have $\rho \simeq -p$ and $w = p/\rho \simeq -1$.

Being $\rho = \Sigma + \nu$ where $\Sigma$ and $\nu$ are the kinetic and the potential terms respectively, and taking into account Eq.(21), for a volume $V \to \infty$, we obtain $\rho = \Sigma + \nu \simeq -p$. Taking into account the condition $\Sigma \ll \nu$ imposed by the negligible breaking of the Lorentz invariance at the present epoch, we have $\rho \simeq \nu \simeq -p$. Then, from the equality: $\rho = \frac{\text{const}}{V} - p = \Sigma + \nu$, we have $\Sigma \simeq \frac{\text{const}}{V} \simeq 0$, for $V \to \infty$. This implies that, at the present epoch, $\rho$ can be written as $\rho = \Sigma - p \simeq -p$. Then, in the case of the flavor vacuum condensate, $w_{\text{mix}}$ is given by:

$$w_{\text{mix}} = \frac{p_{\text{mix}}}{\Sigma_{\text{mix}} - p_{\text{mix}}}.$$  

(22)

Since today $\Sigma_{\text{mix}} \sim 0$, then $w_{\text{mix}} \sim -1$. Eq.(22) gives an expression of $w_{\text{mix}}$ as function of $K$ (since $\Sigma_{\text{mix}}$ and $p_{\text{mix}}$ are function of $K$).

By using different values of $w_{\text{mix}}$ close to $-1$, we compute the contributions given to the dark energy by the particle mixing condensates, at the present epoch. We found the following results [20]:

**Neutrino mixing condensate contribution:**

$$\rho_{\text{mix}}^\nu \sim 10^{-47}\text{GeV}^4$$ for $-0.98 \leq w_{\text{mix}}^\nu \leq -0.97$. Such contributions are compatible with the estimated upper bound of the dark energy and $w_{\text{mix}}^\nu$ is in agreement with the constraint on the dark energy state equation [17].

For $w_{\text{mix}}^\nu < -0.98$ we have negligible contributions of $\rho_{\text{mix}}^\nu$. The results we found are dependent on the neutrino mass values one uses. (We have considered values of the neutrino masses such that the experimental values of squared mass difference [34] are satisfied, as for example: $m_1 = 4.6 \times 10^{-3}\text{eV}$, $m_2 = 1 \times 10^{-2}\text{eV}$, $m_3 = 5 \times 10^{-2}\text{eV}$).

**Quark mixing condensate contribution:**

A contribution compatible with the estimated upper bound of the dark energy: $\rho_{\text{mix}}^q = 1.5 \times 10^{-47}\text{GeV}^4$ is found for $w_{\text{mix}}^q = -1$ [20]. Very small deviations from the value $w_{\text{mix}}^q = -1$ give rise to contributions of $\rho_{\text{mix}}^q$ that are beyond the accepted upper bound of the dark energy.
4. Conclusions and discussion
In this report we have shown that the vacuum condensate from particle mixing provides a contribution to the dark energy which is compatible with the estimated value of the cosmological constant. Such value is imposed by the small breaking of the Lorenz of the flavor vacuum at the present epoch.

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