FDTD simulation of millimeter-wave corrugated waveguides with cylindrical symmetry model

Y. Tamura¹, H. Nakamura², T. Okamura³, N. Kashima³, S. Fujiwara⁴ and S. Kubo²

¹ Department of Intelligence and Informatics, Konan University, Okamoto, Higashinada, Kobe, 658-8501, Japan
² Department of Helical Plasma Research, National Institute for Fusion Science, Oroshi, Toki, 509-5292, Japan
³ Department of Energy Engineering and Science, Nagoya University, Frocho, Chikusa, Nagoya, 464-8603, Japan
⁴ Department of Macromolecular Science and Engineering, Kyoto Institute of Technology, Matsugasaki, Sakyo, Kyoto, 606-8585, Japan
E-mail: tamura@konan-u.ac.jp

Abstract. The purpose of this study is to calculate the relationship between the circular shape of a corrugated waveguide and its electrical transfer characteristics by the finite-difference time-domain (FDTD) method. To improve the waveguide to transport with a lower loss of energy than the previous design, we optimize the waveguide structure by the time evolution of the electromagnetic fields in the waveguide. However, the calculation in a 3D model is difficult because of the large data size and the long calculation run time. Therefore, we used a cylindrical symmetry model. As a result of this simulation, a correlation of the electromagnetic field in the input source position and the output position has been calculated.

1. Introduction
In nuclear fusion, a high-density- and high-temperature environment is required. Therefore, it is necessary to provide the low-loss transmission of electromagnetic waves for the electron cyclotron heating (ECH) system to realize nuclear fusion. As one of the brilliant successes, it was found that corrugating the walls reduces the transmission loss further not only in straight runs but also in bends and that the corrugation makes the waveguide robust against small deformations [1]. However, the distance from the energy source to nuclear fusion reactor is generally very long because a nuclear fusion reactor and peripheral devices are so large that it is difficult to set the energy source, that is, the gyrotron, near the reactor. Therefore, small deformations in the waveguide can have a deleterious effect on energy transmission. In this study, we estimate the influences of structural errors on the corrugated circular waveguide by numerical simulation using the FDTD method [2].

2. Simulation Model
In the FDTD method, the 3D Maxwell equation is solved numerically, in the perfect matched layer (PML) [3] boundary condition, for the same situation as the experimental configuration. However, this equation is difficult to solve directly in the 3D environment because the data size
becomes big and the simulation time is quite long. To reduce these costs, we use a 2D model. Equations (1) - (6) show the Maxwell equations in the cylindrical symmetry model.

\[
\begin{align*}
\frac{1}{r} \frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z} &= \epsilon_0 \frac{\partial E_r}{\partial t}, \\
\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} &= \epsilon_0 \frac{\partial E_\theta}{\partial t}, \\
\frac{1}{r} \frac{\partial (r E_\theta)}{\partial r} - \frac{1}{r} \frac{\partial H_r}{\partial \theta} &= \epsilon_0 \frac{\partial E_z}{\partial t}, \\
\frac{1}{r} \frac{\partial E_z}{\partial r} - \frac{\partial E_\theta}{\partial \theta} &= -\mu_0 \frac{\partial H_r}{\partial t}, \\
\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} &= -\mu_0 \frac{\partial H_\theta}{\partial t}, \\
\frac{1}{r} \frac{\partial (r E_\theta)}{\partial r} - \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} &= -\mu_0 \frac{\partial H_z}{\partial t},
\end{align*}
\]

where \( E \) is the electric field and \( H \) is the magnetic field; the suffixes \( r, z \) and \( \theta \) denote each component in each direction; and \( \epsilon_0 \) and \( \mu_0 \) are permittivity and permeability under vacuum, respectively.

Figure 1 shows the corrugated waveguide geometry in this simulation. The walls of the waveguide are perfect conductors and the boundary at both sides of waveguide is the PML boundary. The actual material is aluminum; however, calculating with an aluminum wall is difficult because thin skin depth of the aluminum makes lattice size very small. The inside region of waveguide is under vacuum.

3. Results

Table 1 shows the parameters used in this simulation. For estimating the influence of structural error, the shape error described in eq. (7) is appended.

\[
d_{\text{error}} = d + a \sin\left(\frac{2\pi}{p_{\text{error}}} z\right),
\]

where \( a \) is the amplitude of the error and \( p_{\text{error}} \) is the period of the error. In the simulation, we used several values of \( a, p_{\text{error}}, \) and \( d, \) and inputted the transverse electric mode wave (TM01 mode, electromagnetic field is constant in the \( \theta \)-direction) at position \( E^{in} \) as shown in fig. 2.
Table 1. Parameters in FDTD simulation of the corrugated waveguide

| Element                        | Parameter                                      |
|--------------------------------|-----------------------------------------------|
| Input source frequency         | 84 GHz                                        |
| Wave length                    | 3.57 mm                                       |
| Inside radius                  | 15.9 mm                                       |
| Depth of groove                | 0.5 mm, 0.75 mm, 1.0 mm                       |
| Width of groove                | 1.0 mm                                        |
| Amplitude of error             | 0.1 mm, 0.3 mm, 0.5 mm, no error              |
| Pitch of error                 | 10 mm, 50 mm, 100 mm                          |
| Time step                      | 8.3e-14 sec                                   |
| Total simulation time          | 16.7 nsec                                     |
| Lattice size (r-direction and z-direction) | 0.05 mm                                      |
| PML thickness                  | 1.0 mm                                        |

Figure 2. Simulation results in which \( d \) is 0.75 mm, \( a \) is 0.3 mm, and \( p_{error} \) is 10 mm. The length of the z-direction in the simulation model is 200 mm. The "input" and "output" show the lines for calculating the correlation factor.

Figure 2 shows one example of simulation results. Plane waves are transmitted; however small turbulence can occur in local areas.

For estimating simulation results and turbulence, we introduce the correlation factor eq. (8).

\[
C = \frac{\sum_{i=1}^{n}(E_{in}^i - E_{in})(E_{out}^i - E_{out}^i)}{\sqrt{\sum_{i=1}^{n}(E_{in}^i - E_{out}^i)^2} \sqrt{\sum_{i=1}^{n}(E_{in}^i - E_{out}^i)^2}},
\]  

(8)

where \( C \) is the correlation factor, \( n \) is the number of data points (inside radius/lattice size) in the r-direction, and \( E_{in} \) and \( E_{out} \) are the values of the electric field in the input and output positions, respectively. The output position is apart from the edge of the simulation box to remove the influence of PML. The distance between input and output position is 125 mm, and 35 waves are included in this area. Therefore, the phase is the same for both the input and output positions, and therefore, eq. (8) indicates to calculates the auto correlation factor.

Figure 3 shows the values obtained for the dependency of depth without error, and indicates that \( d = 0.75 \) mm is the best parameter. This depth parameter is used in later estimations. The results obtained for structural error (Figures 4 and 5) show that the worse the error is, the smaller is the correlation factor. These results can be understood intuitively. However, this estimation is just compared with the error and shape change of the electric field, and therefore, a comparison with experimental results is necessary.
4. Conclusion

We applied cylindrical FDTD simulation to a corrugated waveguide using structural error parameters. As the results of this simulation, we established the relationship between structural variance and electromagnetic waveform shape as a correlation factor. In this simulation, we do not show transmission loss directly. In an actual nuclear fusion device, electromagnetic waves are emitted to the nuclear fusion reactor. For an estimate of the validity of this simulation, it is necessary to compare the simulated results with experimental results from the actual system. Moreover, the input source is a TM01 mode wave; however, the actual input source is the HE11 mode. It is difficult to apply this mode wave in a cylindrical symmetry model. Therefore, it will be necessary to simulate it in a 3D model and compare it with a 2D model.

Acknowledgement

This work is supported by KAKENHI (22500114).

References

[1] J. L. Doane 2008 Fusion Science Technology 53 159
[2] K. Kunz and R. Luebbers 1993 The Finite Difference Time Domain Method for Electromagnetics Florida: CRC Press
[3] J. P. Berenger 1994 Journal of Computational Physics 114 2