Lower and upper bound form of outage probability in one-way AF full-duplex relaying network under impact of direct link

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ABSTRACT

This paper proposed and investigated the one-way amplify-and-forward (AF) full-duplex relaying network under impact of direct link. For the system performance analysis, the exact and lower and upper bound form of the system outage probability (OP) are investigated and derived. In this system model, authors assume that the E uses the maximal ratio combining (MRC) technique. Finally, we can see that the analytical and the simulation values overlap to verify the analytical section using the Monte Carlo simulation. Also, we investigate the influence of the system primary parameters on the proposed system OP.

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1. INTRODUCTION

Radio frequency (RF) signals can carry both information and energy can be considered as the main electrical sources in communication network, called wireless powered networks (WPNs) [1]-[10]. In [11], authors studied the outage probability between some points based on the tradeoff fundamental, and [12] proposed and designed the practical receiver for energy and information transmission and its advantages for the communication network. Furthermore, the authors in [13] presented and demonstrated the practical energy harvesting communication network, and [14] proposed and investigated the continuous energy and power transmission in the cognitive relaying communication network. Moreover, the time switching and the power splitting protocols design for the communication network and the comparison between them are proposed and investigated in [15]-[18].

This paper proposed and investigated the one-way amplify-and-forward (AF) full-duplex relaying network under impact of direct link. For the system performance analysis, the exact and lower and upper bound form of the system outage probability (OP) are investigated and derived. In this system model, authors assume that the E uses the maximal ratio combining (MRC) technique. Finally, we can see that the analytical and the simulation values overlap to verify the analytical section using the Monte Carlo simulation. Also, we investigate the influence of the system primary parameters on the proposed system OP.
2. SISTEM MODEL

In Figure 1, we illustrated the system model of the proposed system and the energy harvesting (EH) and information processing (IT) phases are illustrated in Figure 2 as in [19]-[25]. In this model, all of the channels are Rayleigh fading. Then the CDF of the channel gains $|h_{SR}|^2$, $|h_{RD}|^2$ and $|h_{SD}|^2$ can be formulated as (1).

$$F_{|h_{SR}|^2}(x) = 1 - \exp(-\lambda_{SR}x),$$
$$F_{|h_{RD}|^2}(x) = 1 - \exp(-\lambda_{RD}x),$$
$$F_{|h_{SD}|^2}(x) = 1 - \exp(-\lambda_{SD}x)$$

Here, we assume (2)

$$\lambda_{SR} = (d_{SR})^\chi, \lambda_{RD} = (d_{RD})^\chi, \lambda_{SD} = (d_{SD})^\chi$$

then we have (3).

$$F_{|f|^2}(x) = 1 - \exp(-\lambda_{RR}x)$$

Finally, the PDFs of $|h_{SR}|^2$, $|h_{SD}|^2$ and $|f|^2$ can be given as the follows

$$f_{|h_{SR}|^2}(x) = \lambda_{SR} \exp(-\lambda_{SR}x),$$
$$f_{|h_{RD}|^2}(x) = \lambda_{RD} \exp(-\lambda_{RD}x),$$
$$f_{|h_{SD}|^2}(x) = \lambda_{SD} \exp(-\lambda_{SD}x),$$
$$f_{|f|^2}(x) = \lambda_{RR} \exp(-\lambda_{RR}x)$$

2.1. Energy harvesting and Information transmission

The received signal at the relay can be expressed as

$$y_{R} = h_{SR}x_s + f x_R + n_R$$

The average transmitted power at the relay can be computed as the following

$$P_R = \frac{E_s}{(1-\alpha)^2} = \frac{\eta\alpha}{1-\alpha} P_s |h_{SR}|^2 \quad \text{and} \quad \kappa P_s |h_{SR}|^2$$

where $\kappa = \frac{\eta\alpha}{1-\alpha}$. The received signal at the destination can be given by

$$y_D = h_{RD}x_R + n_D$$

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where \( n_d \) is the AWGN with variance \( N_0 \). The amplification factor are formulated as the (8).

\[
\beta = \frac{x_R}{y_R} = \frac{P_R}{|h_{SR}|^2P_s + |f|^2P_R + N_0}
\]  

(8)

From (7) and combining with (5), we can obtain:

\[
y_D = h_{RD}\beta y_R + n_D = h_{RD}\beta [h_{SR}x_s + f x_R + n_R] + n_D
\]

After doing some algebra, the end-to-end signal to interference noise (SINR) can be obtained as (9),

\[
SINR_{AF} = \frac{E[|signal|^2]}{E[|interference|^2] + E[|noise|^2]} = \frac{P_s|h_{SR}|^2|h_{RD}|^2}{P_R|f|^2 + P_R|h_{RD}|^2 + N_0}
\]

(9)

where \( \Phi = \frac{P_s}{N_0} \). Next, the destination will also receive the information directly from the source. Therefore, the SINR in this phase can be obtained by (10),

\[
\gamma_{direct} = \Phi|h_{SD}|^2
\]

(10)

Finally, using the MRC technique at the receiver, the overall SINR of the system can be claimed as (11),

\[
\gamma_{MRC} = SINR_{AF} + \gamma_{direct} = \frac{\kappa\Phi|h_{SR}|^2|h_{RD}|^2}{\kappa^2\Phi|h_{SR}|^2|f|^2 + \kappa|f|^2 + 1} + \Phi|h_{SD}|^2 = X + Y
\]

(11)

where \( X = \frac{\kappa\Phi|h_{SR}|^2|h_{RD}|^2}{\kappa^2\Phi|h_{SR}|^2|f|^2 + \kappa|f|^2 + 1} \) and \( Y = \Phi|h_{SD}|^2 \).

3. SYSTEM PERFORMANCE ANALYSIS

3.1. Exact analysis

The System OP at the source destination can be defined as (12),

\[
OP = Pr(\gamma_{MRC} < \gamma_{th}) = Pr(X + Y < \gamma_{th}) = \int_0^{\gamma_{th}} F_X(y - \gamma_{th})f_Y(y)dy
\]

(12)

where \( \gamma_{th} \) is the predetermined threshold of the system. To find the probability in (12), we have to calculate the cumulative distribution function (CDF) of \( X \) and the probability density function (PDF) of \( Y \). So, the CDF of \( X \) can be found as (13),

\[
F_X(x) = Pr(X < x) = Pr\left(\frac{\kappa\Phi|h_{SR}|^2|h_{RD}|^2}{\kappa^2\Phi|h_{SR}|^2|f|^2 + \kappa|f|^2 + 1} < x\right)
\]

(13)

By denoting \( T = |h_{SR}|^2|h_{RD}|^2 \) and \( Z = |f|^2 \), the (13) can be reformulated by (14),

\[
F_X(x) = Pr\left(\frac{\kappa\Phi T}{\kappa^2\Phi T + \kappa Z + 1} < x\right)
\]

\[
= Pr[\kappa\Phi T < x(\kappa^2\Phi T + \kappa Z + 1)]
\]

\[
= Pr[T(\kappa\Phi - \kappa^2\Phi Zx) < x(\kappa Z + 1)]
\]
Therefore, the OP of the system in lower bound form can be given by

\[ \text{OP}_{LB} = Pr \left[ \min(X, Y) < \frac{\gamma_h}{2} \right] = 1 - Pr \left( X \geq \frac{\gamma_h}{2} \right) Pr \left( Y \geq \frac{\gamma_h}{2} \right) \]

3.2. Lower and upper bound analysis

From (10), the CDF of random variables (RVs) T can be computed by (15).

\[ F_T(t) = Pr(T < t) = Pr(|h_{SR}|^2 |h_{RD}|^2 < t) \]

\[ = \int_0^\infty F_{h_{SR}^2} \left( \frac{t}{|h_{RD}|^2} \right) f_{h_{RD}}(y) dy \]

\[ = 1 - \lambda_{RD} \int_0^\infty \exp \left( -\lambda_{RD} y - \frac{\lambda_{SR} t}{y} \right) dy \]  

(15)

Applying equation (3.324,1) of [23] as shown in (15) can be reformulated by (16),

\[ F_T(t) = 1 - 2 \times \sqrt{\lambda_{SR} \lambda_{RD} t} \times K_1(2 \sqrt{\lambda_{SR} \lambda_{RD} t}) \]

(16)

where \( K_1(\bullet) \) is the modified Bessel function of the second kind and \( v \) order. Applying (16) for (14), \( F_T(x) \) can be obtained by (17).

\[ F_T(x) = 1 - 2\lambda_{RR} \times \int_0^1 \frac{x}{t} \exp \left( -\lambda_{RR} z \right) \times \frac{\lambda_{SR} \lambda_{RD}(xz+1)}{\lambda_{SR} \lambda_{RD} \phi^2 xz} \times K_1(2 \sqrt{\frac{\lambda_{SR} \lambda_{RD}(xz+1)}{\lambda_{SR} \lambda_{RD} \phi^2 xz}}) \]  

(17)

Next, the CDF of Y can be found as (18).

\[ F_Y(y) = Pr(Y < y) = Pr(\Phi |h_{SD}|^2 < y) = Pr \left( |h_{SD}|^2 < \frac{y}{\phi} \right) \]

\[ = 1 - \exp \left( -\frac{\lambda_{SD} y}{\phi} \right) \]  

(18)

From (18), the PDF of Y can be obtained by (19).

\[ f_Y(y) = \frac{\partial F_Y(y)}{\partial y} = \frac{\lambda_{SD}}{\phi} \exp \left( -\frac{\lambda_{SD} y}{\phi} \right) \]

(19)

Substituting (17) and (19) into (12), finally, the OP in exact form can be claimed as (20),

\[ \text{OP} = \int_0^{\gamma_h} F_T(Y_{th} - y) f_Y(y) dy \]

\[ = 1 - \exp \left( -\frac{\lambda_{SD} y}{\phi} \right) \times \frac{2\lambda_{RR} \lambda_{SD}}{\phi} \int_0^{\gamma_h} \int_0^1 \frac{y}{\phi} \exp \left( -\lambda_{RR} z - \frac{\lambda_{SD} y}{\phi} \right) \times K_1(2 \sqrt{\frac{\lambda_{SR} \lambda_{RD}(yz+1)}{\lambda_{SR} \lambda_{RD} \phi^2 (yz+1)}}) \]  

(20)

where \( Y(y, z) = \sqrt{\frac{\lambda_{SR} \lambda_{RD}(yz+1)}{\lambda_{SR} \lambda_{RD} \phi^2 (yz+1)}} \).

3.2. Lower and upper bound analysis

From (11), we can compute as (21).

\[ 2 \min(X, Y) \leq X + Y \leq 2 \max(X, Y) \]

(21)

Therefore, the OP of the system in lower bound form can be given by (22).

\[ \text{OP}_{LB} = Pr \left[ \min(X, Y) < \frac{\gamma_h}{2} \right] = 1 - Pr \left( X \geq \frac{\gamma_h}{2} \right) Pr \left( Y \geq \frac{\gamma_h}{2} \right) \]

(22)
From (17), $P_1$ can be calculated as (23).

$$P_1 = 1 - Pr \left(X < \frac{\gamma_{th}}{2} \right) = 2\lambda_{RR} \times$$

$$\int_0^{\gamma_{th}} \exp(-\lambda_{RR}z) \times \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\phi - k^2\Phi \gamma_{th}}} \times K_1 \left(2 \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\phi - k^2\Phi \gamma_{th}}} \right) dz$$

(23)

Next, $P_2$ can be found as (24).

$$P_2 = 1 - Pr \left(Y < \frac{\gamma_{th}}{2} \right) = \exp \left(-\frac{\lambda S D Y_{th}}{2\Phi} \right)$$

(24)

Substituting (23) and (24) into (22), we have:

$$OP_{LB} = 1 - 2\lambda_{RR} \times \exp \left(-\frac{\lambda S D Y_{th}}{2\Phi} \right) \times \int_0^{\gamma_{th}} \exp(-\lambda_{RR}z) \times \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\Phi - k^2\Phi \gamma_{th}}} \times K_1 \left(2 \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\Phi - k^2\Phi \gamma_{th}}} \right) dz$$

(25)

Similar to the above, the upper bound OP of the system can be computed as (26).

$$OP_{UB} = Pr \left[ \max(X,Y) < \frac{\gamma_{th}}{2} \right] = Pr \left(X < \frac{\gamma_{th}}{2} \right) Pr \left(Y < \frac{\gamma_{th}}{2} \right)$$

$$= \left\{ 1 - 2\lambda_{RR} \times \int_0^{\gamma_{th}} \exp(-\lambda_{RR}z) \times \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\Phi - k^2\Phi \gamma_{th}}} \times K_1 \left(2 \sqrt{\frac{\lambda S R \lambda D Y_{th}(kz+1)}{2k\Phi - k^2\Phi \gamma_{th}}} \right) dz \right\} \times \left\{ 1 - \exp \left(-\frac{\lambda S D Y_{th}}{2\Phi} \right) \right\}$$

(26)

4. NUMERICAL RESULTS AND DISCUSSION

The system OP versus $\alpha$ is shown in Figure 3 with $\eta=1$, $\gamma_{th}=1$, and $\Phi=7$ dB. The results show that the OP of the model system has a massive decrease with the rising of $\alpha$ from 0 to 0.45 and has a considerable increase when $\alpha$ rises to 1 in three cases with exact, lower, and upper bound analysis. The maximal value of the system OP can be obtained with $\alpha=0.45$. Furthermore, the OP is considered as the function of $\gamma_{th}$, as shown in Figure 4. Here we set $\eta=0.8$, $\alpha=0.25$, and $\Phi=5$ dB. Here, $\gamma_{th}$ increases from 0 to 6, as shown in Figure 4. As shown in Figure 4, the system OP increases significantly when $\beta$ rises in three cases with exact, lower, and upper bound analysis. From Figures 4 and 5, the analytical and the simulation curves overlap each others as shown in the analytical section.

![Figure 3. OP versus $\alpha$](image1)

![Figure 4. OP versus $\gamma_{th}$](image2)
Furthermore, the system OP versus $\eta$ and $\Phi$ are investigated in Figures 5 and 6, respectively. In Figure 5, the main system parameters are set as $\alpha=0.75$, $\gamma_{th}=1$ and $\Phi=5$ dB, and in Figure 6, we set $\alpha=0.5$, $\gamma_{th}=1$, and $\eta=1$ respectively. From Figures 5 and 6, it can be observed that the system OP has a slight increase with rising $\eta$ from 0 to 1 and has a massive decrease when $\Phi$ varies from -5 to 15 dB, respectively. Also, the simulation and analytical values agree to justify the analytical section.

Figure 5. OP versus $\eta$

Figure 6. OP versus $\Phi$

5. CONCLUSION

This paper proposed and investigated the one-way AF full-duplex relaying network under impact of direct link. For the system performance analysis, the exact and lower and upper bound form of the system outage probability (OP) are investigated and derived. In this system model, authors assume that the E uses the MRC (maximal ratio combining) technique. Finally, we can see that the analytical and the simulation values overlap to verify the analytical section using the Monte Carlo Simulation. Also, we investigate the influence of the system primary parameters on the proposed system OP.

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