Nuclear matrix elements for neutrinoless $\beta\beta$ decays and spin-dipole giant resonances

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Nuclear matrix element (NME) for neutrinoless double beta decay (DBD) is required for studying neutrino physics beyond the standard model by using DBD. Experimental information on nuclear excitation and decay associated with DBD is crucial for theoretical calculations of the DBD-NME. The spin-dipole (SD) NME for DBD via the intermediate SD state is one of the major components of the DBD-NME. The experimental SD giant-resonance energy and the SD strength in the intermediate nucleus are shown for the first time to be closely related to the DBD-NME and are used for studying the spin-isospin correlation and the quenching of the axial-vector coupling, which are involved in the NME. So they are used to help the theoretical model calculation of the DBD-NME. Impact of the SD giant-resonance and the SD strength on the DBD study is discussed.

Neutrinoless double beta decay (DBD), which violates the lepton-number conservation law, is a sensitive and realistic probe for studying the neutrino ($\nu$) nature (Majorana or Dirac) relevant to the origin of matter in the universe, the absolute $\nu$-mass scale, and other $\nu$-properties beyond the standard model [1, 3]. The nuclear matrix element (NME) for the neutrinoless DBD is crucial to extract the effective $\nu$-mass and other $\nu$-properties of the particle physics interests from the decay rate. The NME is also needed to design DBD detectors [4–6]. Thus the NMEs are of great interest from astro-, particle- and nuclear physics view points. Theoretical works on DBD-NMEs by using various nuclear models are discussed in [7–12], and recent DBD experiments in [13–15].

Actually, accurate theoretical calculations for the DBD-NME are very hard since they are sensitive to nucleonic and non-nucleonic correlations and nuclear medium effects. Consequently, calculated DBD-NMEs, including the effective axial-vector coupling ($g_{A}$), scatter over an order of magnitude [3, 9], depending on the nuclear models, the interaction parameters and the effective coupling ($g_{A}^{\text{eff}}$) used in the models. Thus experimental inputs are useful to check the theoretical models and the nuclear parameters to be used for the models [1, 6].

The present letter aims to show for the first time that the experimental energy and the strength of the spin-dipole (SD) giant-resonance in the intermediate nucleus are closely related to the DBD-NME ($M^{\text{DBD}}$) based on the pnQRPA (proton-neutron Quasi-particle Random Phase Approximation) model, and reflect the nuclear structure and the quenching of the $g_{A}$, which are involved in the DBD-NME. Here the SD strength is given as $B^{-}(\text{SD})=|M^{-}(\text{SD})|^{2}$ with $M^{-}(\text{SD})$ being the SD NME. We note that $M^{-}(\text{SD})$ is associated with the SD component of the DBD-NME, which is one of the major components of the DBD-NME, and the quenching of $g_{A}$ is one of key parameters for the theoretical model calculation. Thus experimental information on the SD giant-resonance energy $E_{G}(\text{SD})$ and the SD strength $B^{-}(\text{SD})$ are used to help and check the theoretical model calculation of the DBD-NME.

Recently, the experimental $E_{G}(\text{SD})$ values were shown to depend on the isospin $z$-component $T_{z}=(N-Z)/2$ with $N$ and $Z$ being the neutron and proton numbers [6], and pnQRPA calculations for $M^{\text{DBD}}$ were performed by adjusting the particle-hole parameter to the $E_{G}(\text{SD})$ in [12], where $g_{A}^{\text{eff}}=1$ is assumed. Single-$\beta$ SD NMEs for the ground states in the medium-heavy nuclei (mostly non-DBD nuclei) were studied in the framework of the pnQRPA, and were found to be reduced by $g_{A}^{\text{eff}}/g_{A} \approx 0.5$, depending much on the individual states [10].

The present paper puts emphasis on studying and discussing universal features in the $E_{G}(\text{SD})$, the GT and SD strengths, and the $M^{\text{DBD}}$ as a function of the mass number $A$. For $M^{\text{DBD}}$, we use a common value for the $g_{A}^{\text{eff}}$ as derived by referring to the experimental summed GT strengths for the DBD nuclei and the SD NMEs for the ground state transitions in the medium-heavy nuclei. Then we discuss impact of the present findings on the DBD experiments.

We discuss double $\beta^{-}$ decays of the ground-state-to-ground-state transition of $0^{+} \rightarrow 0^{+}$. The neutrinoless DBD ($0\nu\beta\beta$) for $^{4}Z_{X_{1}} \rightarrow ^{4}Z_{X_{2}}+X_{1}$ is schematically shown in Fig. 1. The intermediate nucleus is $^{A}Z_{X_{2}}+X_{2}$. The $0\nu\beta\beta$ process to be discussed is the Majorana $\nu$-mass process of the current interest. Here, a light Majorana $\nu$ is virtually exchanged between two neutrons in the DBD nucleus. DBD nuclei discussed are medium-heavy ($A=76$-
where the transition operator $T_\alpha(k)$ with $\alpha=\text{GT}, T$ and $F$ are given by $T_{\text{GT}}(k) = t^\pm \sigma h_{\text{GT}}(r_{12}, E_k) t^\pm \sigma$, $T_T(k) = t^\pm h_T(r_{12}, E_k) S_{12} t^\pm$, and $T_F(k) = t^\pm h_F(r_{12}, E_k) t^\pm$. The operator includes the neutrino potential $h_{\alpha}(r_{12}, E_k)$ with $r_{12}$ being the distance between the two neutrons involved in the $\nu$ exchange and $E_k$ being the excitation energy of the intermediate state $k$. $S_{12}$ is the spin tensor operator and $t^\pm$ is the isospin operators for proton/anti-neutron.

Since the momentum of the virtual-$$\nu$$ is of the order of $1/r_{12}\approx 100$ MeV/c, the NME involves mainly intermediate states $k$ in the wide ranges of the spins of $J^\pi \approx 0^+ - 7^+$ and $E_k \approx 0$-30 MeV. Thus, the DBD-NME may reflect gross properties of the nuclear core.

The axial-vector SD ($J^\pi = 2^-$) component of the $M_{2\nu}^\alpha(\text{SD})$ in Eq. (2) is one of the major components since the orbital angular momentum matches the medium momentum of the virtual neutrino. The SD DBD-NME is associated with the product of the single $\tau^\pm$ SD NMEs of $M^+(\text{SD})$ and $M^-(\text{SD})$, as shown in Fig. 1. The single-$\beta$ SD NMEs are expressed as $M^\pm(\text{SD}) = <t^\pm|\sigma f(r)Y_1|\nu>$ with $Y_1$ being the spherical harmonics.

On the other hand, the two-neutrino DBD within the standard model is followed by the emission of the two real s-wave ($l=0$) neutrinos with low momentum, and the NME ($M_{2\nu}^{\beta\beta}$) involves exclusively the $M^\pm(\text{GT})$ for GT $1^+$ intermediate states at the low excitation-energy region. Thus $M_{2\nu}^{\beta\beta}$ is very sensitive to properties of the valence-nucleons at the proton neutron Fermi surface. So far, most theoretical DBD models use these valence-nucleon GT properties.
The SD giant-resonance energies are higher than the GT and SD 2QP states by \(\hbar \) increase. This is in accord with the GT and SD giant-resonance strengths in other nuclei [33]. The reduction is incorporated by the quenched axial-vector coupling of \(g_A^{\text{eff}}/g_A\) as shown in Fig. 3(b) [6]. These are consistent with GT strengths in other nuclei [33].

The giant-resonance energies depend on \(A\) and \(N – Z\), thus following the gross properties of nuclear core.

The GT 1+ strength \(B^-(\text{GT}) = |M^-(\text{GT})|^2\) and the SD 2– strength \(B^-(\text{SD}) = |M^-(\text{SD})|^2\) are mostly concentrated, respectively, in the GT and SD giant-resonances at the high excitation-energy region in all nuclei (see Fig. 2). Several sharp peaks in the low (0-6 MeV) energy region are two-quasiparticle states (2QP). The giant-resonance energies in units of MeV are found to be expressed as

\[ E_G(\text{GT}) \approx 0.06A + 7.0, \quad E_G(\text{SD}) \approx 0.06A + 14.5, \quad (4) \]

as shown in Fig. 3 A. They are given also as \(E_G(\text{GT}) \approx 0.4T_z + 9\) MeV and \(E_G(\text{SD}) \approx 0.4T_z + 16.5\) MeV [6]. Note that \(A \approx 4(N – Z) + 28\) for the present nuclei. The \(E_G(\text{GT})\) and \(E_G(\text{SD})\) increase gradually as \(A\) and \(N – Z\) increase. This is in accord with the GT and SD giant-resonances in other nuclei [33].

The GT 1+ strength \(B^-(\text{GT})\) increase as \(A\) and \(N – Z\) increase, and they are around 0.55 of the nucleon-based sum-rule limit of \(B^-(\text{GT})\) as shown in Fig. 3(b) [6]. These are consistent with GT strengths in other nuclei [33].

The GT and SD cross sections for the \((^3\text{He},t)\) reactions show the maximum at the \(s\) and \(p\)-wave \(t\) scattering-angles of \(\theta = 0\) deg., and around 2 deg., respectively. The GT and SD cross sections there are the same within 10%. The SD cross section for the 2QP states below the SD giant resonance is also nearly same as the GT one below the GT giant resonance. The SD cross sections for the 2QP region are found to be also of the order of 10-5 % of the total SD cross sections. Thus the SD strengths at

FIG. 3. (a) Experimental GT 1+ and SD 2– giant-resonance energies of \(E_G(\text{GT})\) and \(E_G(\text{SD})\). Solid lines: Fits by Eq. (4). Squares: The observed energies. Inverse triangles: The energies from the experimental \(E_G(\text{GT}) \approx 0.4T_z + 9\) MeV and \(E_G(\text{SD}) \approx 0.4T_z + 16.5\) MeV [6] as used in the pnQRPA. (b): The GT strengths in logarithmic scale. GT 1+ strength \(B^-(\text{GT})\). S: \(B_S^{-}(\text{GT})\) for the nucleon-based sum-rule limit. T: \(B_T^{-}(\text{GT})\) for the observed total strength up to \(E_G(\text{GT}) + 10\) MeV. The strength beyond \(E_G\) is corrected for the small contribution from the quasi-free scattering. 2QP: \(B_{2\text{QP}}^{-}(\text{GT})\) for the sum of the observed 2QP strengths up to 6 MeV. Straight lines are fits to the data.
the giant resonance and the 2QP region show the similar trends as the GT strengths there. The 2QP GT and 2QP SD strengths for the very low excitation-energy (a few MeV) region depend on individual nuclei, reflecting the valence nucleon configurations containing single particle states near the proton and neutron Fermi surfaces of each nucleus.

Note that for $^{106}$Mo the $0g$-shell effect on the $M^{0\nu}$ is huge, but looks little on the $M^{2\nu}$. Therefore, we focus on the $M^{2\nu}$, which increase as $SD$ and GT giant-resonance energies and the strengths, $\chi_{A}$, nuclear core, as shown in Fig. 4. This is in contrast to the $M^{0\nu}$, which decreases as $N$ and $Z$ increase, the susceptibility $\chi$ being the $\sigma$ susceptibility. Likewise the DBD-NME $M^{0\nu}$ may be doubly reduced by the factor $(1/(1+\chi))^2$. Then, the NME may be expressed as $M^{0\nu} \approx M^{0\nu}_0/(1+\chi)^2$ with $M^{0\nu}_0 \approx 6.5$ and $\chi \approx 0.025$ for the present DBD nuclei. As $(N-Z)$ and $A$ increase, the susceptibility $\chi$ increases, and $M^{0\nu}$ decreases. The value for $\chi$ is around 0.4 for DBD nuclei with $N-Z$ around 16, and $M^{0\nu} \approx 0.5 M^{0\nu}_0$.

The particle-hole interaction ($g_{ph}$) pushes up the giant resonances in energy, shifting the spin-isospin strengths from the low-lying states to the giant resonances, and reduces the DBD-NME $M^{0\nu}$. The susceptibility $\chi$ in (iv) is proportional to $g_{ph}$ [6]. Thus $\chi$ increases as $g_{ph}$ increases and $M^{0\nu}$ decreases as $g_{ph}$ increases, 10% increase of $g_{ph}$ leading to around 5% decrease of $M^{0\nu}$.

Table I. The pnQRPA NMEs. $M^{0\nu}_A = (g_A/g_A)^2 M^{0\nu}_A$ and $M^{2\nu}_A = (g_A/g_A)^2 (M^{0\nu}_A + M^{2\nu}_A)$ and $M^{0\nu}$ (See Eq. (2)). $a$ and $b$ are the NMEs with $g_{ph}/g_A=0.74$ and 0.55, respectively. Note that for $^{106}$Mo the $0g$-shell effect on the $M^{2\nu}$ is huge, but looks little on the $M^{0\nu}$.

| $A$  | $M^{0\nu}_V$ | $M^{0\nu}_A$ | $M^{2\nu}_A$ | $M^{0\nu}$ | $M^{2\nu}$ |
|------|--------------|--------------|--------------|-----------|-----------|
| $^{16}$Ge | -1.16 | 2.59 | 2.02 | 3.75 | 3.18 |
| $^{96}$Zr | -1.03 | 2.12 | 1.29 | 3.14 | 2.31 |
| $^{100}$Mo | -1.51 | 2.11 | 1.81 | 3.62 | 3.32 |
| $^{116}$Cd | -1.01 | 2.03 | 1.43 | 3.03 | 2.44 |
| $^{128}$Te | -0.95 | 1.88 | 1.29 | 2.82 | 2.24 |
| $^{130}$Te | -0.81 | 1.57 | 1.08 | 2.37 | 1.89 |
| $^{136}$Xe | -0.63 | 1.57 | 1.05 | 2.19 | 1.68 |

FIG. 4. $M^{0\nu}$ (blue squares) and $M^{2\nu}$ (light blue square) with $g_{ph}/g_A=0.74$ are plotted against $A$ (panel (a)) and $N-Z$ (panel (b)). Solid lines are fits. See text.

(iii). The pnQRPA NMEs of $M^{0\nu}=(g_A^2/g_A)^2 [M^{0\nu}_G + M^{0\nu}_T]$ and $M^{2\nu}$ for $g_{ph}/g_A=0.74$ (see Eq. (2)) are shown in Table 1. They are found to decrease gradually as $A$ and $N-Z$ increases, reflecting the gross properties of the nuclear core, as shown in Fig. 4. This is in contrast to the SD and GT giant-resonance energies and the strengths, which increase as $A$ and $N-Z$ (see Fig. 3). The DBD-NMEs are found to behave like the 2QP GT and the 2QP SD strengths of $B_{QP}^{-1}$ (GT) and $B_{QP}^{-1}$ (SD) (see Fig. 3 B). $M^{2\nu}$, which is associated with the GT NMEs for low-lying 2QP $1^+$ states, changes more than a factor 10 among the DBD nuclei, depending on the valence nucleon configurations of states near the Fermi surfaces [6,14,36].

(iv). The repulsive $\sigma\tau$ interaction pushes up the GT and SD strengths to the GT and SD giant resonances, and reduces the low-lying GT and SD NMEs with respect to the 2QP NMEs due to the $\sigma\tau$ nuclear polarization effects [4,37,38]. The reduction rate is given by $1/(1+\chi)$ with $\chi$ being the $\sigma\tau$ susceptibility. Likewise the DBD-NME $M^{0\nu}$ may be doubly reduced by the factor $(1/(1+\chi))^2$. Then, the NME may be expressed as $M^{0\nu} \approx M^{0\nu}_0/(1+\chi)^2$ with $M^{0\nu}_0 \approx 6.5$ and $\chi \approx 0.025$ for the present DBD nuclei. As $(N-Z)$ and $A$ increase, the susceptibility $\chi$ increases, and $M^{0\nu}$ decreases. The value for $\chi$ is around 0.4 for DBD nuclei with $N-Z$ around 16, and $M^{0\nu} \approx 0.5 M^{0\nu}_0$.

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DBD studies. On the bases of the above discussions, we use $g_{A}^{el}/g_{A} \approx 0.65 \pm 0.1$. $M^{0\nu}$ with this $g_{A}^{el}/g_{A}$ region corresponds approximately to the $\pm 10\%$ region around $M^{0\nu} \approx 5.2 - 0.023A$, $M^{0\nu} \approx 4.2 - 0.08(N-Z)$.

The DBD NMEs for $A=76-136$ are considered to be around 3-2. Since they do not depend much on individual nuclei, selection of the DBD nucleus for the experiment may be made from experimental requirements such as the availability of ton-scale DBD isotopes, the large $Q$ value, the low-background and the high energy-resolution.

The present analyses show that the pnQRPA $M^{0\nu}$ is closely related to the SD giant resonance and the distribution of the strength $B^{-}(K) = |M^{-}(K)|^{2}$ with $K=GT$, SD in the intermediate nucleus. Various nuclear models are being used for calculating $M^{0\nu}$. Then, it is interesting to compare the calculated strength distribution of $B^{-}(K)$ in $\frac{A}{2}+1X_{2}$ by using the model wave function for $\frac{A}{2}X_{1}$ with the observed giant resonance and the strength. Actually, the SD $(J^{\pi}=2^{-})$ giant resonance is accompanied at the higher energy side by the resonances with $J^{\pi}=1^{-}, 0^{-}$ and the quasi-free scattering. Further experimental studies for their strengths and also for higher multipole strengths with $J^{\pi}=3^{-}$ and $J^{\pi}=4^{-}$ are interesting. The low and high multipole giant resonances are studied in the pnQRPA [14].

OMC(ordinary muon capture) of $(\mu, \nu_{\mu})$ is used to study $M^{+}(K)$ [6] [15]. The large strength is found in the giant resonances [15] [16]. The quenching of $g_{A}$ in the muon strengths is under discussions [17] [18]. Actually, $\tau^{\pm}$ strengths of $B^{\pm}(K)$ are also studied by using various nuclear reactions [6] [14] [15].

The $\tau^{-}$ strengths in the intermediate nucleus $\frac{A}{2}+1X_{2}$ are mostly in the giant resonances, and little strengths at the 2QP and ground states. Likewise, the DBD $(\tau^{-}\tau^{-})$ strengths are considered to be mostly in the double giant resonances, and little DBD strengths at the 2QP and ground states (Fig. 1). Double charge-exchange reactions on $\frac{A}{2}X_{1}$ are used to study the DBD strengths in $\frac{A}{2}+2X_{3}$ [6] [19]. The double GT and SD giant-resonance energies measured from the ground state of $\frac{A}{2}X_{1}$ are around $E_{G}(DGT)=26-32$ MeV and $E_{G}(SD)=42-48$ MeV for DBD nuclei with $A=76-136$. The RCNP [11B,11Li] data [20] show that most of the strengths are at the high excitation-energy region. The double GT strengths for the ground states are shown to have a positive correlation with the DGT-state DBD-NMEs [51].

The delta isobar ($\Delta$) is strongly excited by the quark $\sigma\tau$ flip in a nucleon to form the $N^{-1}\Delta$ giant resonance. Then, the axial-vector $M^{\pm}(K)$ and the axial-vector $M^{0\nu}$ with $\sigma\tau$-GT and T are reduced with respect to the model NMEs without the $\Delta$ isobar effect, which is incorporated by using the effective coupling of $g_{A}^{eff}$ [6] [52] [55]. Studies of the $N^{-1}\Delta$ effects are interesting.

In conclusion, the present work shows for the first time (i) a clear relation between the experimental SD strength distribution in the intermediate nucleus and the DBD NME in the pnQRPA formalism based on the SD data, and (ii) a simple expression of $M^{0\nu}$ as a function of $A$ on the basis of the pnQRPA with the $g_{A}^{el}/g_{A}$ derived experimentally. Thus such experimental inputs are useful for pinning down the values for the DBD NMEs.

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