Reliability-based Design with Size and Shape Optimization of Truss Structure Using Symbiotic Organisms Search

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Abstract. Studies on truss design optimization have been conducted extensively over the past decades. One of the significant current discussions is the reliability aspect of truss design in addition to optimal design. This problem has become more important, especially the sizing and shaping in the optimization of truss structures. Reliability-based design optimization is defined as finding the optimum structure while satisfying the given uncertainty and reliability criteria. This study aims to investigate the performance of metaheuristic algorithm in optimizing the truss structure design and satisfying the reliability constraints. Latin hypercube sampling method was used to model the presence of uncertainty. Symbiotic organisms search was also utilized as a metaheuristic algorithm to solve a modified 15-bar planar truss. The results indicated that reliability design gives a significant result in the shape and size of truss.

Keywords: metaheuristic algorithms, reliability-based design optimization, truss structure

1. Introduction

Structure optimization has become an important and challenging topic in civil engineering because it can increase the efficiency of a structure. Structure optimization is the act of designing and developing structures to achieve the maximum profit of available resources [1]. Many researchers are, therefore, interested in structure optimization to minimize cost and structure’s weight by optimizing the diameter size of steel pipe, the thickness of plate, or steel’s cross-section area [4]. Element’s number and constraint in a structure’s design cause complexity in structure optimization. Hence, metaheuristic has become more popular in solving structure optimization cases than gradient method [3]. In metaheuristic, the concept of randomness is useful to find the global solution of a case. Recently, Symbiotic Organisms Search (SOS) has been used by numerous researchers in optimization cases because its operations require no specific algorithm parameters [2].

Furthermore, uncertainty has also become an inevitable problem in structure optimization. In practice, truss structure is sensitive to uncertain design variables such as cross-section or uncertain parameters such as force and material’s modulus elasticity [5]. A structure’s strength and safety are also affected by changes in those variables and parameters; hence, uncertainty must be calculated in a design [5]. As a result, Reliability-Based Design Optimization (RBDO) has become an important matter in structure design. Some methods are needed to analyze the probability and reliability of a structure in order to solve RBDO problems. There are three methods available to analyze RBDO problem, i.e. the moment method, simulation method, and heuristic method [6].
This research aims to optimize a single variable which is the structure’s weight. In order to model the uncertainty of variables, random variables with certain mean and standard deviation were defined. The model was simulated using Latin Hypercube Sampling (LHS) method. Reliability of structure was analyzed by LHS method because it can achieve more reliable results compared with response surface method-based optimization [8]. To achieve the smallest reliable weight, SOS was employed as a metaheuristic algorithm and then some constraints were provided in the process with a probability of success not less than 99%.

2. Symbiotic Organisms Search

Symbiotic organisms search algorithm simulates interactive behaviors seen among organisms in nature [2]. This method is compatible with the nature of living organisms which cannot live alone and need interaction with others. Three kinds of interaction exist in SOS: mutualism phase, commensalism phase, and parasitism phase.

Mutualism phase occurs between two organisms that gain advantages from an interaction. If the new organism’s fitness is improved after the interaction, this organism is updated with a new one. The mathematic model of mutualism phase is defined by Cheng and Prayogo, as shown in equation 1, 2, and 3 [2].

\[ X_{i_{new}} = X_i + \text{rand}(0,1) \times (X_{best} - \text{Mutual}_{vector} \times BF_1) \]  
\[ X_{j_{new}} = X_j + \text{rand}(0,1) \times (X_{best} - \text{Mutual}_{vector} \times BF_2) \]  
\[ \text{Mutual}_{vector} = \frac{X_i + X_j}{2} \]

where \( X_i \) is an organism matched to the \( i \)-th member of the ecosystem, \( X_j \) is an organism that is selected randomly from the ecosystem, \( X_{i_{new}} \) is a new candidate from \( X_i \), \( X_{j_{new}} \) is the new candidate from \( X_j \), \( BF_1 \) and \( BF_2 \) are random numbers between one or two, and \( X_{best} \) is the global solution.

Commensalism phase is an interaction between two organisms where one of them gains an advantage while the other is not affected. If the new fitness value of the organism is better than the pre-interaction one, this organism is updated. Formula for \( X_{i_{new}} \) in this phase is:

\[ X_{i_{new}} = X_i + \text{rand}(-1,1) \times (X_{best} - X_j) \]  

Parasitism phase is an interaction where one organism benefits, and the other is harmed. \( X_i \) is given a role as the parasite named “Parasite_{Vector}”. Then, the fitness value of “Parasite_{Vector}” is compared with the fitness value of \( X_j \). If the fitness value of “Parasite_{Vector}” is better, the position of \( X_j \) is replaced with “Parasite_{Vector}”. After going through this phase, this algorithm is repeated until the criteria are satisfied.

3. Latin Hypercube Sampling

Latin Hypercube Sampling method was used to assure a good estimation of the statistical moments of response functions. In this method, sample points are well spread out when projected onto a subspace spanned by several coordinate axes. Latin Hypercube Sampling selects \( n \) different values of \( k \) variables \( X_1, \ldots, X_k \) where the range of each variable is divided into \( n \) nonoverlapping intervals on the basis of equal probability. It then selects a value randomly from each interval. The sampled cumulative probability can be written as:

\[ \text{Probi} = \left( \frac{1}{N} \right) r_n + \left( \frac{i-1}{N} \right) \]
where $r_n$ is uniformly distributed random number ranging from zero to one. Then, the probability of failure can be obtained from equation 6.

$$P_f \equiv \frac{N_H}{N}$$  \hspace{1cm} (6)

where $N_H$ is the number of failures, and $N$ is the number of simulations.

4. Problem Formulation

This study aims to minimize the weight of truss structure without violating any constraints. The constraints used in this study are static constraints and include element stress and reliability. The mathematical formulation of this problem optimization can be performed as follows:

Find, $X = \{A_1, A_2, \ldots, A_m, \xi_1, \xi_2, \ldots, \xi_n\}$

To minimize, $f(x) = \sum_{i=1}^{m} A_i \rho_i L_i$

Subjected to:

$g_1$: Check probability of success $\geq 99\%$

$g_2$: Stress constraints, $|\sigma_i| - |\sigma_i^{max}| \leq 0$

$g_3$: Shape constraints, $\xi_{lower} \leq \xi_j \leq \xi_{upper}$

where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. $A_i$, $\rho_i$, $L_i$, and $\sigma_i$ are cross-sectional area, weight density, length, and stress of element $(i)$, respectively.

5. Methodology

Reliability-Based Design Optimization was modeled by combining metaheuristic algorithm as optimization method and LHS to model the uncertainty. Metaheuristic was used to find the optimal cross-sectional area and shape of truss structure while Direct Stiffness Method (DSM) was used to analyze the structure. This paper also used DSM to obtain the displacement, axial force, and stress of each element. These outputs were utilized to detect the number of structures that failed. The structure’s probability of failure was then obtained from LHS. When the structure was not reliable, a penalty was given to the calculation of weight as the fitness value. Direct Stiffness Method as well as the metaheuristic algorithms were written using MATLAB R2018b. A flow chart of the truss optimization process is presented in Figure 1.

6. Test Problem and Results

In this paper, we compare the 15-bar planar truss structure problem, as shown in Figure 2, with a deterministic and non-deterministic variable. Each structure had its load cases. The goal was to minimize cross-sectional area so that the minimum weight could be obtained for the structure while meeting the strength, serviceability, and reliability requirements. Thirty experimental runs with 1000 iterations and 30 populations resulted in the same 120000 function evaluation. These two cases were simulated 100 times with modulus elasticity $(E) = 10^4$ ksi, weight density $(\rho) = 0.1$ lb/in.$^3$, and available cross-sectional areas $D = [0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180] (in.$^2$). Stress limits in tension or compression were 25 ksi. There were 23 design variables in this problem: 15 cross-section area variables and eight configuration variables. The configuration variables were the x- and y-coordinates of nodes 2, 3, 6, and 7 and the y-coordinates of nodes 4 and 8. However, nodes 6 and 7 were constrained to have the same x-coordinates of nodes 2 and 3. The side constraints for the configuration variables were 100 in. $\leq x_2 \leq 140$ in., 220 in. $\leq x_3 \leq 260$ in., 100 in. $\leq y_3 \leq 140$ in., 100 in. $\leq y_3 \leq 140$ in., 50 in. $\leq y_4 \leq 90$ in., -20 in. $\leq y_6 \leq 20$ in., -20 in. $\leq y_7 \leq 20$ in., and 20 in. $\leq y_8 \leq 60$ in.
Population, upper and lower bound, ground structure of truss, external load, iteration(max_iter), iter=0

Random initialisation

iter = iter + 1

(max_pop), pop=0

pop = pop + 1

LHS

(max_N), N=0

N = N + 1

Randomization of external load (P)

Calculation of displacement (D), internal force (N), dan element stress (s)

Number of simulation that exceed the constraint

MCS< max_MCS YES

NO

PENALTY FUNCTION

YES Exceed probability of failure constraint ?

NO

Given penalty in the form of addition in fitness value

No penalty

Calculation of fitness value (weight of structure)

pop<max_pop YES

NO

Renew the fitness value and location using metaheuristic algorithm

iter < max_iter YES

NO

Optimal truss structure

Figure 1. Flow chart for truss optimization.
6.1. 15-bar planar truss structure with deterministic load

The structure model shown in Figure 2 [7] generated deterministic load $P = 10000$ lb on node 8. Table 1 shows that SOS has a better result than the reference. Figure 3 shows the iteration process of a 15-bar truss structure optimization. In terms of consistency, the convergence behavior of SOS is depicted in Figure 4.

Figure 3. Iteration of 15-bar truss structure with deterministic load: (a) iteration number 100; (b) iteration number 500; (c) iteration number 700; (d) iteration number 1000.
Table 1. Final design of size and shape for the 15-bar truss with deterministic load.

| Variable | Miguel et al. [7] | SOS |
|----------|-------------------|-----|
| A1 (in²) | 0.954             | 0.954 |
| A2 (in²) | 0.539             | 0.539 |
| A3 (in²) | 0.220             | 0.141 |
| A4 (in²) | 0.954             | 0.954 |
| A5 (in²) | 0.539             | 0.539 |
| A6 (in²) | 0.220             | 0.27  |
| A7 (in²) | 0.111             | 0.111 |
| A8 (in²) | 0.111             | 0.111 |
| A9 (in²) | 0.287             | 0.141 |
| A10 (in²)| 0.440             | 0.440 |
| A11 (in²)| 0.440             | 0.440 |
| A12 (in²)| 0.220             | 0.220 |
| A13 (in²)| 0.220             | 0.270 |

| Variable | Miguel et al. [7] | SOS |
|----------|-------------------|-----|
| A14 (in²)| 0.270             | 0.270 |
| A15 (in²)| 0.220             | 0.141 |
| X2 (in²) | 114.967           | 100.018 |
| X3 (in²) | 247.040           | 241.51 |
| Y2 (in²) | 125.919           | 135.727 |
| Y3 (in²) | 111.067           | 123.187 |
| Y4 (in²) | 58.298            | 57.189 |
| Y6 (in²) | -17.564           | -16.331 |
| Y7 (in²) | -5.821            | -8.822 |
| Y8 (in²) | 31.465            | 57.184 |

Best Weight (lb): 75.55
Average (lb): 82.64
Stddev: 2.96

Figure 4. Convergence behavior for the size and shape for 15-bar truss with deterministic load.

6.2. 15-bar planar truss structure with non-deterministic load

This case was given a non-deterministic load (P) using lognormal distribution with mean 10 kips and dispersion ± 5% on node 8. This random variable was modeled by LHS method. Figure 5 shows the iteration process of 15-bar truss structure optimization. In terms of consistency, the convergence behavior of SOS is shown in Figure 6.
Figure 5. Iteration of 15-bar truss structure with non-deterministic load: (a) iteration number 100; (b) iteration number 1000.

Table 2. Final design of size and shape for the 15-bar truss with non-deterministic load.

| Variable | SOS  |
|----------|------|
| A1 (in²) | 0.954|
| A2 (in²) | 0.954|
| A3 (in²) | 0.111|
| A4 (in²) | 1.333|
| A5 (in²) | 0.539|
| A6 (in²) | 0.44 |
| A7 (in²) | 0.111|
| A8 (in²) | 0.111|
| A9 (in²) | 0.111|
| A10 (in²)| 0.539|
| A11 (in²)| 0.111|
| A12 (in²)| 0.111|
| A13 (in²)| 0.539|

| Variable | SOS  |
|----------|------|
| A14 (in²)| 0.539|
| A15 (in²)| 0.111|
| X2 (in)  | 113.151|
| X3 (in)  | 221.644|
| Y2 (in)  | 114.121|
| Y3 (in)  | 132.255|
| Y4 (in)  | 53.554 |
| Y6 (in)  | 2.923 |
| Y7 (in)  | 2.582 |
| Y8 (in)  | 20.558|
| Best Weight (lb) | 87.314 |
| Average (lb)     | 92.2  |
| Stdev            | 4.018 |

Figure 6. Convergence behavior for the size and shape for 15-bar truss with non-deterministic load.

7. Conclusions
This paper compared the size and shape optimization result in a 15-bar planar truss structure with deterministic and non-deterministic load using SOS by reviewing two case studies. With the same number of function evaluation for each case, the result showed that uncertainty of load makes significant changes in size and shape optimization. A case with non-deterministic problem needs to be
designed with a larger size of steel which leads to increased weight. This paper shows that RBDO is important in structure design and cannot be neglected.

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