Spin-parities of the $P_c(4440)$ and $P_c(4457)$ in the One-Boson-Exchange Model

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(Dated: July 16, 2019)

The LHCb collaboration has recently observed three pentaquark peaks, the $P_c(3142)$, $P_c(4440)$ and $P_c(4457)$, and are very close to a pair of heavy baryon-meson thresholds, with the $P_c(3142)$ located 8.9 MeV below the $D^*\Sigma_c$ threshold, and the $P_c(4440)$ and $P_c(4457)$ located 21.8 and 4.8 MeV below the $D^*\Sigma_c$ one. The spin-parities of these three states have not been measured so far. In this work we assume that the $P_c(3142)$ is a $J^P=\frac{1}{2}^-$ $D^*\Sigma_c$ bound state, while the $P_c(4440)$ and $P_c(4457)$ are $D^*\Sigma_c$ bound states of unknown spin-parity, where we notice that the consistent description of the three pentaquarks in the one-boson-exchange model can indeed determine the spin and parities of the later, i.e. of the two $D^*\Sigma_c$ molecular candidates. For this determination we revisit first the one-boson-exchange model, which in its original formulation contains a short-range delta-like contribution in the spin-spin component of the potential. We argue that it is better to remove these delta-like contributions because, in this way, the one-boson-exchange potential will comply with the naive expectation that the form factors should not have a significant impact in the long-range part of the potential (in particular the one-pion-exchange part). Once this is done, we find that it is possible to consistently describe the three pentaquarks, to the point that the $P_c(4440)$ and $P_c(4457)$ can be predicted from the $P_c(3142)$ within a couple of MeV with respect to their experimental location. In addition the so-constructed one-boson-exchange model predicts the preferred quantum numbers of the $P_c(4440)$ and $P_c(4457)$ molecular pentaquarks to be $\frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively.

I. INTRODUCTION

The observation of three pentaquark-like resonances by the LHCb collaboration [1] — the $P_c(3142)$, $P_c(4440)$ and $P_c(4457)$ — provides three of the most robust candidates so far for a hadronic molecule, a type of exotic hadron conjectured four decades ago [2,3]. As a matter of fact molecular pentaquarks, i.e. bound states of a charmed antimeson and a charmed baryon, were predicted in a series of theoretical works [4–9]. Subsequent theoretical analyses after the experimental observation of the LHCb pentaquarks [1], if anything, further point out towards the hypothesis that they are molecular candidates. For this determination we revisit first the one-boson-exchange model, which in its original formulation contains a short-range delta-like contribution in the spin-spin component of the potential. We argue that it is better to remove these delta-like contributions because, in this way, the one-boson-exchange potential will comply with the naive expectation that the form factors should not have a significant impact in the long-range part of the potential (in particular the one-pion-exchange part). Once this is done, we find that it is possible to consistently describe the three pentaquarks, to the point that the $P_c(4440)$ and $P_c(4457)$ can be predicted from the $P_c(3142)$ within a couple of MeV with respect to their experimental location. In addition the so-constructed one-boson-exchange model predicts the preferred quantum numbers of the $P_c(4440)$ and $P_c(4457)$ molecular pentaquarks to be $\frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively.

But phenomenological models, even the most successful ones such as the OBE model, usually end up requiring a cer-
tarn amount of tweaking (see for instance Ref. [43] for a lucid exposition of a few of the quirks of the OBE model). For the OBE model as applied to nuclear physics, it was quickly realized that the correct description of the deuteron properties requires the cutoff to be \( \Lambda \approx 1.3 \) GeV for the pion contribution. The theoretical reason is the distortion of the long-range properties of the one-pion-exchange (OPE) potential by the form factors, which can be prevented if the cutoff is hard enough. The present manuscript indicates that this type of long-range distortion also happens for hadronic molecules, but proposes a different solution adapted to the particular circumstances of the application of the OBE model to the molecular pentaquarks.

The problem is as follows: if the \( P_c(4312) \) is indeed a \( J^P = \frac{1}{2}^- \) \( \bar{D}\Sigma \) molecule with a binding energy of 8.9 MeV, it can be described within the OBE model with a monopolar form factor and a cutoff \( \Lambda = 1119 \) MeV. If we use the most simple OBE model possible, i.e. we use the same form factor and cutoff for all the exchanged mesons, we can predict the \( J^P = \frac{1}{2}^- \) and \( J^P = \frac{3}{2}^- \) \(\bar{D}\Sigma \) binding energies from the cutoff that we already determined from the \( P_c(4312) \). In particular we arrive at

\[
B_E(\frac{1}{2}^-) \approx 74 \text{ MeV} \quad \text{and} \quad B_E(\frac{3}{2}^-) \approx 3 \text{ MeV},
\]

which are incompatible with the binding energies of the \( P_c(4440) \) and \( P_c(4457) \) as \(\bar{D}\Sigma \) bound states, \(B_E = 21.8\) and 4.8 MeV respectively. This happens regardless of which state we identify with the \( J^P = \frac{1}{2}^- \) and \( J^P = \frac{3}{2}^- \) quantum numbers. The failure of the naive OBE model to naturally explain the \( P_c(4312), P_c(4440) \) and \( P_c(4457) \) with the same cutoff can be traced back to a particular artifact generated by the form factors. The unregularized spin-spin piece of the OBE potential contains a contact-range and a finite-range piece, which we write schematically as

\[
V_S \propto \left[ -m^3 \delta^{(3)}(\vec{r}) + \frac{e^{-mr}}{4\pi mr} \right],
\]

with \( m \) the mass of the exchanged meson. It happens that the inclusion of a form factor regularizes the contact-range Dirac-delta piece of the OBE potential, making it finite range. The expectation is that the finite range of the regularized delta will be considerably shorter than the range of the Yukawa-like piece. However this condition is not actually fulfilled for a monopolar form factor and a cutoff \( \Lambda \sim 1 \) GeV. This is obvious in the OPE contribution of the OBE potential, which is in fact distorted at distances comparable with the Compton wavelength of the pion. In particular the excessive binding of the \( \frac{1}{2}^- \) \( \bar{D}\Sigma \) in Eq. (1) can be traced back to the regularized delta contribution stemming from the OPE potential: while the Yukawa-like piece of the OBE potential is repulsive in this system, the delta-like piece provides the system with a strong, probably unphysical, short-range attraction. If we remove the delta-like contribution to the OPE potential by hand, we end up with the set of predictions

\[
B_E(\frac{1}{2}^-) = 13.2 \text{ MeV} \quad \text{and} \quad B_E(\frac{3}{2}^-) = 11.6 \text{ MeV},
\]

which are much closer to the expected binding energies of the molecular pentaquarks. There are similar delta-like contributions in the spin-spin piece of the vector-meson-exchange potential. This piece of the OBE potential is of shorter range than the OPE piece. The removal of their delta-like contributions is not as crucial as in the OPE piece, yet it should better be done if we want the OBE model to be internally consistent, in which case we arrive at the predictions:

\[
B_E(\frac{1}{2}^-) = 4.2 \text{ MeV} \quad \text{and} \quad B_E(\frac{3}{2}^-) = 18.3 \text{ MeV},
\]

where the binding energies are in fact very close (within 1 – 3 MeV) to what we would expect from a molecular \( P_c(4440) \) and \( P_c(4457) \), namely 21.8 and 4.8 MeV. Owing to the compatibility of this set of predictions with the current experimental determination of the \( P_c(4440) \) and \( P_c(4457) \) masses, the removal of the Dirac-delta contributions could indeed be considered as the preferred solution to the form-factor problem. In this case the OBE model as applied to hadronic molecules ends up having the phenomenological success of its original nuclear physics version, modulo the larger experimental uncertainties associated with hadronic molecules. The seemingly ad-hoc removal of the Dirac-delta contributions, which has also been considered for instance in Ref. [44], finds a natural explanation within the renormalized OBE model of Ref. [43].

The manuscript is structured as follows: in Sect. II we briefly explain how the heavy hadron-hadron interaction is constrained by heavy-quark spin symmetry (HQSS), where we also advocate a notation based on the quark model for heavy-hadron interactions [45]. In Sect. III we explain the OBE model, while in Sect. IV we explain the regulator artifact within the OBE model. Then in Sect. V we show the predictions we arrive at after removing this artifact. Finally in Sect. VI we summarize our results.

II. HEAVY-QUARK SPIN SYMMETRY

In this section we review a few basic consequences of HQSS for heavy antimeson-baryon molecules. As applied to molecular states, HQSS refers to the fact that interactions among heavy hadrons do not depend on the spin of the heavy quarks within the hadrons. This can automatically be taken into account by writing the heavy hadron interactions in a suitable notation. The standard notation for this purpose is to group heavy hadrons with the same light-quark structure in a single superfield, which we review in Sect. II A. Here we advocate instead for a simpler notation in terms of the light-quark degrees of freedom, which has been recently presented in Ref. [45] (though we note that it has been intermittently used in the literature for a long time [46]), which we explain in Sect. II B.

A. Heavy-Superfield Notation

The \( P \) and \( P^* \) heavy mesons are \( |Q\bar{q}\rangle \) states with total spin \( J = 0 \) and 1, respectively. They can be grouped into the single
non-relativistic superfield:

\[ H_Q = \frac{1}{\sqrt{2}} \left[ P + P^* \cdot \vec{\sigma} \right] , \]  

which has been adapted from its relativistic version \cite{47} and has good transformation properties with respect to heavy-quark spin rotations. In the formula above \( H_Q \) is a 2x2 matrix and \( \vec{\sigma} \) are the Pauli matrices. The \( \Sigma_Q \) and \( \Sigma_Q^* \) heavy baryons are \( |Qqq⟩ \) states with total spin \( J = \frac{3}{2} \) and \( \frac{1}{2} \). They can be written together as the following non-relativistic superfield \cite{48}

\[ \mathcal{S}_Q = \frac{1}{\sqrt{3}} \vec{\sigma} \Sigma_Q + \Sigma_Q^* , \]  

which corresponds to the relativistic heavy-baryon superfield written in Ref. \cite{49}. From these superfields, the simplest contact-range, no-derivative Lagrangian involving the heavy (anti-)meson and heavy baryon is \cite{33}

\[ \mathcal{L} = C_a \mathcal{S}_Q^i \cdot \mathcal{S}_Q^i \text{Tr} \left[ H_Q^{\dagger} H_Q \right] + C_b \sum_{i=1}^{3} \mathcal{S}_Q^i \cdot (J_i \mathcal{S}_Q^i) \text{Tr} \left[ H_Q^{\dagger} \sigma_i H_Q \right] , \]  

where \( J_i \) with \( i = 1, 2, 3 \) refers to the spin-1 angular momentum matrices and with \( C_a \) and \( C_b \) coupling constants. If we particularize for the \( D\Sigma_c \) family of molecules, we obtain the following potential:

\[ V(D\Sigma_c, J = \frac{1}{2}) = C_a , \]  

\[ V(D\Sigma_c^*, J = \frac{3}{2}) = C_a , \]  

\[ V(D\Sigma_c^*, J = \frac{1}{2}) = C_a - \frac{3}{2} C_b , \]  

\[ V(D\Sigma_c^*, J = \frac{3}{2}) = C_a + \frac{3}{2} C_b , \]  

\[ V(D\Sigma_c^*, J = \frac{1}{2}) = C_a - \frac{3}{2} C_b , \]  

\[ V(D\Sigma_c^*, J = \frac{3}{2}) = C_a + \frac{3}{2} C_b . \]  

We notice that, for simplicity, we have ignored isospin when writing the Lagrangian of Eq. \cite{7} and the potentials of Eqs. \cite{9,14}. Isospin can be trivially taken into account by adding a subindex indicating the isospin of the two-body state in thecouplings: \( C_{1a}, C_{1b} \) with \( I = \frac{1}{2}, \frac{3}{2} \).

**B. Light-Quark Notation**

Actually the heavy-quark symmetric interactions can be derived in an easier and more direct way if we consider that the heavy-quark acts as a spectator \cite{45}. Instead of building superfields for the \( P \) and \( P^* \) heavy mesons, we can simply express the interactions in terms of the light-quark subfield within the heavy mesons, \( q_L \). Equivalently, for the \( \Sigma_Q \) and \( \Sigma_Q^* \) heavy baryons we can use the light-diquark subfield within them: \( d_L \). After introducing these fields, the lowest-order contact-range Lagrangian can be simply written as

\[ \mathcal{L} = C_a (q_L^\dagger q_L) (d_L^\dagger d_L) + C_b (q_L^\dagger \vec{\sigma}_L q_L) \cdot (d_L^\dagger \vec{S}_L d_L) , \]  

where \( \vec{\sigma}_L \) and \( \vec{S}_L \) refer to the spin of the \( q_L \) and \( d_L \) subfields, respectively. This leads to the contact-range potential

\[ V_c(q_L d_L) = C_a + C_b \vec{\sigma}_L \cdot \vec{S}_L , \]  

where we have labeled the heavy meson and baryon with the light quark and light diquark inside them with the subscript 1 and 2. The contact-range potential is now written for the light-quark fields within the heavy hadrons. The translation into the heavy-hadron degrees of freedom can be encapsulated in a series of rules. In particular for the heavy mesons the light-quark spin operators are translated into

\[ \langle \vec{P} | \vec{\sigma}_L | \vec{P}^* \rangle = \vec{S}_L , \]  

where \( \vec{S}_L \) refers to the spin-1 matrices as applied to the heavy vector meson. For the heavy baryons the correspondence is

\[ \langle \Sigma_Q | \vec{J}_L | \Sigma_Q^* \rangle = \frac{2}{3} \vec{S}_L , \]  

\[ \langle \Sigma_Q^* | \vec{J}_L | \Sigma_Q \rangle = \frac{2}{3} \vec{S}_L \]  

We refer to the spin-1 matrices as applied to the heavy spin-\( \frac{1}{2} \) baryon and \( \vec{S}_L \) are the spin-\( \frac{3}{2} \) angular momentum matrices. With these substitutions it is easy to check that the contact-range potential of Eq. \cite{10} written in the light-quark field basis is indeed equivalent to the contact-range potential of Eqs. \cite{9,14} written in the particle basis. The notation in terms of the light-quark subfields is however much more compact and we will adopt it for the rest of this work.

**III. THE ONE-BOSON-EXCHANGE POTENTIAL**

In this section we present the OBE model that we use in this work. In the OBE model the potential between two hadrons is generated by the exchange of a series of light mesons, which includes the \( \pi \), the \( \sigma \), the \( \rho \) and the \( \omega \) (plus a few extra light mesons in its more sophisticated versions). This results in a description of the forces among hadrons that is both simple and physically compelling. Yet there are disadvantages in the OBE model, which usually include a large number of coupling constants and the requirement of form factors and a cut-off to remove the unphysical short-range components of the potential. Here the choice of coupling constants will be done in terms of experimentally known information or by recourse to phenomenological models. For the form factor we will choose a standard multipolar form, while the cutoff will be determined by the condition of reproducing a known molecular candidate, the \( P_c(4312) \) in this case. By determining the cutoff in this way we are partially renormalizing the OBE model,
i.e. removing cutoff ambiguities in terms of observable information. This concept is based on the fully renormalized OBE model of Ref. [43], which in turn helps to understand a few of the tweaks required in the original OBE model (e.g. the excessively large coupling to the \( \omega \) vector meson that is usually required in nuclear physics). We stress however that we have not implemented a renormalized OBE model in this work, but merely adapted a few of the ideas of Ref. [43].

A. The Lagrangian

First we write down the Lagrangians that encode the couplings between the heavy hadrons and the light mesons (\( \pi, \sigma, \rho, \omega \)). We use the light-quark notation introduced in Sect. ITB. For the light-quark field within the heavy mesons the Lagrangian reads as follows

\[
L_{q_1, q_2} = \frac{g_1}{\sqrt{2} f_\pi} q_L^\dagger \vec{\sigma}_L \cdot \nabla (\vec{\pi} \cdot \vec{\rho}) q_L, \quad (21)
\]

\[
L_{q_1, q_1, \sigma} = g_{\sigma 1} q_L^\dagger \sigma q_L, \quad (22)
\]

\[
L_{q_1, q_2, \rho} = g_{\rho 1} q_L^\dagger \rho q_L - \frac{f_{\rho 1}}{4 M_1} \sigma_{i j k} q_L^\dagger \partial_i \rho_j q_L - \frac{f_{\rho 1}}{4 M_1} \sigma_{i j k} \partial_i \rho_j q_L, \quad (23)
\]

\[
L_{q_2, q_2, \omega} = g_{\omega 1} q_L^\dagger \omega q_L - \frac{f_{\omega 1}}{4 M_1} \sigma_{i j k} q_L^\dagger \partial_i \omega_j q_L, \quad (24)
\]

where \( g_1 \) and \( g_{\sigma 1} \) are the couplings to the pion and the sigma meson respectively, while \( g_{\rho 1} \) and \( f_{\rho 1} \) with \( V = \rho, \omega \) are the electric- and magnetic-type couplings to the vector mesons; \( M_1 \) is a mass scale that we introduce for \( f_{\rho 1} \) to be dimensionless. For the light-diquark field within the heavy baryons we have

\[
L_{d_1, d_1} = \frac{g_2}{\sqrt{2} f_\pi} d_L^\dagger \vec{S}_L \cdot \nabla (\vec{T} \cdot \vec{\rho}) d_L, \quad (25)
\]

\[
L_{d_1, d_1, \sigma} = g_{\sigma 2} d_L^\dagger \sigma d_L, \quad (26)
\]

\[
L_{d_1, d_1, \rho} = g_{\rho 2} d_L^\dagger \vec{T} \cdot \vec{\rho} d_L - \frac{f_{\rho 2}}{4 M_2} \sigma_{i j k} d_L^\dagger \partial_i \rho_j d_L + \frac{h_{\rho 2}}{2 M_2} Q_{L, i j} \vec{T} \cdot \partial_i \rho_j d_L, \quad (27)
\]

\[
L_{d_1, d_1, \omega} = g_{\omega 2} d_L^\dagger \omega d_L - \frac{f_{\omega 2}}{4 M_2} \sigma_{i j k} d_L^\dagger \partial_i \omega_j d_L + \frac{h_{\omega 2}}{2 M_2} Q_{L, i j} \partial_i \omega_j d_L, \quad (28)
\]

The spin of the light-diquark field is \( S_L = 1 \), which means that there are three possible type of interactions with a vector field: electric-, magnetic- and quadrupole-type. They correspond to the \( g_{\rho 2}, f_{\rho 2} \) and \( h_{\rho 2} \) couplings. The mass \( M_2 \) is introduced to make the \( f_{\rho 2} \) and \( h_{\rho 2} \) couplings dimensionless. For the quadrupole-type term we have introduced the spin-2 tensor

\[
Q_{L, i j} = \frac{1}{2} \left[ S_{L, i} S_{L, j} + S_{L, j} S_{L, i} \right] - \frac{S^2}{3} \delta_{i j}. \quad (29)
\]

We expect the quadrupole-type term to be small though.

B. The OBE Potential

The OBE potential can be easily derived from the previous Lagrangians for the light-quark and light-diquark fields. We write the potential in the following from

\[
V_{\text{OBE}} = \zeta V_\pi + V_\sigma + V_\rho + \zeta V_\omega, \quad (30)
\]

where \( \zeta = \pm 1 \) is a sign, for which the convention is

\[
\zeta = +1 \quad \text{for } q_L d_L \quad (\text{e.g. } \vec{D}_\Sigma), \quad (31)
\]

\[
\zeta = -1 \quad \text{for } \bar{q}_L d_L \quad (\text{e.g. } D_\Sigma), \quad (32)
\]

that is, we take \( \zeta = +1 \) for the most representative type of molecule, the (hidden-charm) \( D_\Sigma \) in this case. In momentum space the different components of the OBE potential read

\[
V_\pi(q) = -\frac{g_{\pi 1} g_\pi}{2 f_\pi^2} \vec{T} \cdot \vec{T} \frac{\delta_{L, 1} \cdot \epsilon \vec{S}_{L, 2} \cdot \epsilon}{q^2 + m_\pi^2}, \quad (33)
\]

\[
V_\sigma(q) = -\frac{g_{\sigma 1} g_{\sigma 2}}{q^2 + m_\sigma^2}, \quad (34)
\]

\[
V_\rho(q) = \vec{T} \cdot \vec{T} \left[ -\frac{g_{\rho 1} g_{\rho 2}}{q^2 + m_\rho^2} \left( g_{\rho 2} \frac{h_{\rho 2}}{2 M_2} \frac{\epsilon \cdot (Q_{L, 2} \cdot \epsilon)}{q^2 + m_\rho^2} \right) - \frac{f_{\rho 2}}{2 M_2} \frac{\epsilon \cdot (Q_{L, 2} \cdot \epsilon)}{q^2 + m_\rho^2} \right], \quad (35)
\]

\[
V_\omega(q) = \frac{g_{\omega 1} g_{\omega 2}}{q^2 + m_\omega^2} \left( g_{\omega 2} - \frac{h_{\omega 2}}{2 M_2} \frac{\epsilon \cdot (Q_{L, 2} \cdot \epsilon)}{q^2 + m_\omega^2} \right), \quad (36)
\]
If we Fourier-transform the previous expressions to coordinate space we have
\[
V_{\rho}(\hat{r}) = +\hat{r}_1 \cdot T_2 \frac{g_{1g2}}{6f_2^2} \left[ -\vec{\sigma}_{L1} \cdot \vec{S}_{L2} \delta(\vec{r}) 
+ \frac{2}{3} \vec{\sigma}_{L1} \cdot \vec{S}_{L2} m_\rho^2 W_\rho(m_\rho r) 
+ \frac{1}{3} S_{L1L2}(\vec{r}) m_\rho^3 W_\rho(m_\rho r) \right],
\]

\[
V_{\sigma}(\hat{r}) = -g_{1g2} m_\sigma W_\sigma(m_\sigma r),
\]

\[
V_{\rho}(\hat{r}) = +\hat{r}_1 \cdot T_2 \left[ g_{1g2} m_\rho W_\rho(m_\rho r) 
+ \frac{2}{3} \vec{\sigma}_{L1} \cdot \vec{S}_{L2} m_\rho^2 W_\rho(m_\rho r) 
+ \frac{1}{3} S_{L1L2}(\vec{r}) m_\rho^3 W_\rho(m_\rho r) \right],
\]

where we have introduced the dimensionless functions
\[
W_\rho(x) = \frac{e^{-x}}{4\pi x},
\]

\[
W_\sigma(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{4\pi x},
\]

while \(S_{L1L2}\) represents the standard tensor operator
\[
S_{L1L2}(\vec{r}) = 3 \vec{\sigma}_{L1} \cdot \vec{r} \vec{S}_{L2} \cdot \vec{r} - \vec{\sigma}_{L1} \cdot \vec{S}_{L2}
\]

and \(Q_{L2}(\vec{r})\) is a second type of tensor operator
\[
Q_{L2}(\vec{r}) = \hat{r} \cdot (Q_{L2}\hat{r}) = Q_{L2,ij} \hat{r}_i \hat{r}_j,
\]

with \(Q_{L2,ij}\) defined in Eq. (29). This second type of tensor operator is theoretically interesting, but probably not particularly relevant as the \(h_{\omega2}\) coupling is expected to be small, see Sect. IIIID for a more detailed discussion.

### C. Form Factors

We have derived the previous OBE potential under the assumption that the interactions between heavy hadrons and light mesons are point-like. Hadrons have however a finite size, which can be taken into account by the introduction of a form factor for each vertex. In momentum space we will simply have
\[
V_M(q; \Lambda_1, \Lambda_2) = V_M(q) F_M(q; \Lambda_1) F_M(q; \Lambda_2).
\]

We will assume a monopolar form factor for vertices 1 and 2:
\[
F_M(q; \Lambda_i) = \frac{\Lambda_i^2 - m_M^2}{\Lambda_i^2 + q^2}.
\]

In principle we can use different cutoffs for different vertices to take into account the different internal structure of the heavy mesons and heavy baryon. Yet this is only necessary if we want to describe heavy meson-meson, heavy meson-baryon and heavy baryon-baryon molecules consistently. If we are only interested in the heavy meson-baryon system then we can simply assume a unique cutoff for both vertices 1 and 2.

If we now Fourier-transform the momentum space potential with a monopolar form factor into coordinate space, the outcome is that we simply have to make the following substitutions:
\[
\begin{align*}
\delta(\vec{x}) &\rightarrow m^3 d(x, \lambda), \\
W_\rho(x) &\rightarrow W_\rho(x, \lambda), \\
W_\sigma(x) &\rightarrow W_\sigma(x, \lambda),
\end{align*}
\]

where
\[
\begin{align*}
d(x, \lambda) &= \frac{(\lambda^2 - 1)^2}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \\
W_\rho(x, \lambda) &= W_\rho(x) - \lambda W_\rho(\lambda x) \\
&- \frac{(\lambda^2 - 1)}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \\
W_\sigma(x, \lambda) &= W_\sigma(x) - \lambda^2 W_\sigma(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \lambda^2 \left(1 + \frac{1}{\lambda x}\right) \frac{e^{-\lambda x}}{4\pi}.
\end{align*}
\]

The corresponding expressions for form factors of higher polarity (e.g. dipolar) can be consulted in the Appendix of Ref. [42].

### D. Couplings

| Light Meson | \(I^G (J^{PC})\) | M (MeV) |
|-------------|-----------------|--------|
| \(\pi\)     | 1+ (0++)        | 138    |
| \(\sigma\)  | 0+ (0++)        | 600    |
| \(\rho\)    | 1+ (1−−)        | 770    |
| \(\omega\)  | 0+ (1−−)        | 780    |

| Heavy Hadron | \(I^G (J^{PC})\) | M (MeV) |
|--------------|-----------------|--------|
| \(D\)       | \(\frac{1}{2}(0^+\uparrow)\) | 1867   |
| \(D^*\)     | \(\frac{1}{2}(1^+)\) | 2009   |
| \(\Sigma_c\)| \(1(4^+)\) | 2454   |
| \(\Sigma_c^*\)| \(1(4^+)\) | 2518   |

TABLE I. Masses and quantum numbers of the light mesons of the OBE model \((\pi, \sigma, \rho, \omega)\) and the heavy hadrons \((D, D^*, \Sigma_c, \Sigma_c^*)\). Notice that we work in the isospin-symmetric limit and take the isospin-averaged masses.
TABLE II. Couplings of the light mesons of the OBE model ($\pi$, $\sigma$, $\rho$, $\omega$) to the heavy-meson and heavy-baryon fields. For the magnetic-type coupling of the $\rho$ and $\omega$ vector mesons we have used the decomposition $f_V = \kappa_V g_V$, with $V = \rho$, $\omega$. $M$ refers to the mass scale (in MeV) involved in the magnetic-type couplings.

| Coupling      | Value for $P/P'$ | Value for $\Sigma_0/\Sigma_0$ |
|---------------|-------------------|-------------------------------|
| $g_1$         | 0.60              |                               |
| $g_{\rho1}$   | 3.4               |                               |
| $g_{\rho2}$   | 2.6               |                               |
| $g_{\omega1}$ | 2.6               |                               |
| $\kappa_{\rho}$ | 2.3              |                               |
| $\kappa_{\omega}$ | 2.3             |                               |
| $M_1$         | 940               |                               |

The choice of couplings for the $\rho$ and $\omega$ mesons is more laborious. First, from SU(3)-flavor symmetry and the OZI rule we expect that

$$g_{\rho1} = g_{\omega1}, \quad g_{\rho2} = g_{\omega2}, \quad (57)$$

$$f_{\rho1} = f_{\omega1}, \quad f_{\rho2} = f_{\omega2}, \quad (58)$$

$$h_{\rho2} = h_{\omega2}. \quad (59)$$

For the determination of the electric, magnetic and quadrupole couplings we will use the vector-meson dominance assumption. The original formulation of this idea states that hadrons do not couple directly to the electromagnetic field, but by means of the neutral vector meson fields, $\rho_3^0$ and $\omega^3$, where $\mu$ refers to the Lorentz indices of these fields, and the subindex $3$ indicates that we are dealing with the neutral rho meson. A practical way to apply this idea is to derive the electromagnetic Lagrangian from the substitutions

$$\rho_3^0 \rightarrow e \lambda_\rho A_\mu, \quad (60)$$

$$\omega_3^0 \rightarrow e \lambda_\omega A_\mu. \quad (61)$$

We can fix $\lambda_\rho$ and $\lambda_\omega$ from the nucleon case, in which case we obtain

$$\lambda_\rho = \frac{1}{2g_{\rho NN}} = \frac{1}{2g_p}, \quad (62)$$

$$\lambda_\omega = \frac{1}{2g_{\omega NN}} = \frac{1}{6g_\omega}. \quad (63)$$

where in the right-hand side we have written $g_{\rho NN}$ and $g_{\omega NN}$ in terms of the universal $\rho$ coupling (Sakurai’s universality [53])

$$g_p = \frac{m_o}{2g_p} \approx 2.9, \quad (64)$$

where we have also made use of the relation $g_{\omega NN} = 3g_{\rho NN}$, which is derived from SU(3)-flavor symmetry and the OZI rule. In can be trivially checked that this choice correctly reproduces that the proton and neutron charges are $e_p = +e$ and $e_n = 0$, respectively.

The application to the heavy hadrons requires a few modifications. For instance, vector-meson dominance is expected to reproduce the total charge of the light quarks only. It does not apply to the heavy quark, which we consider to couple directly to the electromagnetic field. Thus the application to the $D^0$ ($\bar{c}u$) charmed meson yields

$$g_{\rho 1} \left( \frac{1}{2g_\rho} + \frac{1}{6g_\rho} \right) = \frac{2}{3} e, \quad (65)$$

from which we deduce

$$g_{\rho 1} = g_p \approx 2.9. \quad (66)$$

For the magnetic moments we define the following quantity for sake of convenience

$$f_{\rho 1} = \kappa_\rho g_{\rho 1}, \quad (67)$$

which is related to the $D^0$ magnetic moment, $\mu(D^0)$, by the relation

$$\frac{2}{3} \kappa_\rho = \frac{2M_1}{e} \mu(D^0). \quad (68)$$

For the axial coupling between the $D$ and $D^*$ heavy mesons and the pion we take

$$g_1 = 0.60, \quad (53)$$

which is compatible with $g_1 = 0.59 \pm 0.01 \pm 0.07$ as extracted from the $D^* \rightarrow D\pi$ decay [50, 51]. For the $\Sigma_c$ and $\Sigma_c'$ heavy baryons the axial coupling is not experimentally available, but there is a lattice QCD calculation [52]

$$g_2 = 0.84 \pm 0.2. \quad (54)$$

which is the value we adopt here. We notice in passing that there are several conventions for the axial coupling to the heavy baryons in the literature and here we are effectively using the one in Ref. [52]. Other two popular conventions are the ones by Cho [49] and Yan [53], which are related to our convention by the relations $g_2 = -g_{2,\text{Cho}}$ and $g_2 = g_{1,\text{Yan}}$ (in Ref. [53] the axial coupling to the heavy baryons is denoted as $g_1$).

For the couplings to the $\sigma$ meson, in the case of the nucleon-nucleon interaction it can be determined from the linear sigma model [54] yielding

$$g_{\sigma NN} = \sqrt{2} \frac{M_N}{f_\pi} \approx 10.2. \quad (55)$$

For the case of the $D$, $D^*$ mesons and $\Sigma$, $\Sigma'$ baryons we can estimate the coupling to the $\sigma$ from the quark model. By assuming that the $\sigma$ only couples to the $u$ and $d$ quarks, we expect

$$g_{\sigma 1} = \frac{g_{\sigma 2}}{2} = \frac{g_{\sigma NN}}{3} \approx 3.4. \quad (56)$$

For the axial coupling from which we deduce

$$g_{\rho 1} = g_p \approx 2.9. \quad (66)$$

For the magnetic moments we define the following quantity for sake of convenience

$$f_{\rho 1} = \kappa_\rho g_{\rho 1}, \quad (67)$$

which is related to the $D^0$ magnetic moment, $\mu(D^0)$, by the relation

$$\frac{2}{3} \kappa_\rho = \frac{2M_1}{e} \mu(D^0). \quad (68)$$
If we set the scaling mass to be the nucleon mass $M_1 = M_N$, $\kappa_{\rho 1}$ simply coincides with $\mu(D^0)$ in units of the nuclear magneton. If we use the quark model $\mu(D^0) = \mu_u$, with $\mu_u = 1.85\mu_N$, we find

$$\kappa_{\rho 1}(M_1 = M_N) \approx 2.8.$$  

(69)

Notice that the definition of $\kappa_{\rho 1}$ is dependent on the mass scale $M_1$ in the Lagrangian. For $M_1 = m_\rho$, it happens that $\kappa_{\rho 1} \approx 5.5$. It should be noticed that the vector-meson dominance relation we have presented here can be further refined to obtain improved determinations of $g_{\rho 1}$ and $\kappa_{\rho 1}$. For instance, Ref. [56] applies a more sophisticated vector-meson dominance model to the weak decays of the charmed mesons, which translates into the couplings [41]

$$g_{\rho 1} \approx 2.6 \quad \text{and} \quad \kappa_{\rho 1}(M_1 = M_N) \approx 2.3 \pm 0.4.$$  

(70)

As can be appreciated this determination is compatible with the one in Eqs. (66) and (69) within errors. We will use the set derived from Ref. [56], i.e. the values in Eq. (70), to follow the same convention as in our previous works.

Now we apply the previous ideas to the $\Sigma_c$ and $\Sigma_c^*$ baryons. First we define the reduced couplings

$$f_{\rho 2} \equiv \kappa_{\rho 2} g_{\rho 2}, \quad h_{\rho 2} \equiv \eta_{\rho 2} g_{\rho 2}. $$  

(71)

We now apply vector-meson dominance to arrive at the relations

$$g_{\rho 2} = 2g_{\rho}, $$  

(72)  

$$\kappa_{\rho 2} = \frac{3}{4} \left( \frac{2M_2}{e} \right) \mu(\Sigma_{c c}^{++}), $$  

(73)  

$$\eta_{\rho 2} = \frac{3}{4} \left( \frac{M_2^2}{e^2} \right) Q(\Sigma_{c c}^{++}), $$  

(74)

where $\mu(\Sigma_{c c}^{++})$ and $Q(\Sigma_{c c}^{++})$ are the magnetic and quadrupole moment of the $\Sigma_{c c}^{++}$ baryon. From the quark model (and the assumption that the charm quark provides a minor contribution to the magnetic and quadrupole moments) we expect $\mu(\Sigma_{c c}^{++}) = 2\mu_q$ and $Q(\Sigma_{c c}^{++}) = 0$. We note that a non-vanishing quadrupole moment will require the light-diquark wavefunction to have a D-wave component, which is not the case in the naive quark model. Thus for $M_2 = M_N$ we arrive at

$$\kappa_{\rho 2} \approx 2.8, \quad \eta_{\rho 2} \approx 0.$$  

(75)

The fact that the quadrupole vector-meson coupling vanishes in the naive quark model probably indicates a relatively small contribution from this piece of the potential. This is actually good news in the sense that it simplifies the OBE potential. However the estimations from the quark model have been superseded by recent lattice QCD calculations, at least for the magnetic moment of the $\Sigma_{c c}^{++}$ baryon [57]. If we use the magnetic moment of the $\Sigma_{c c}^{++}$ to determine $\kappa_{\rho 2}$, we first note that the vector-meson dominance relation reads

$$\kappa_{\rho 2} = \frac{9}{8} \left( \frac{2M_2}{e} \right) \mu(\Sigma_{c c}^{++}).$$  

(76)

Ref. [57] obtains $\mu(\Sigma_{c c}^{++}) = 1.499(202)$, which leads to

$$\kappa_{\rho 2} \approx 1.7 \pm 0.2.$$  

(77)

This is the value we will adopt here. The charmed-antimeson and charmed-baryon masses that we use in this work, together with the couplings, can be consulted in Tables [III] and [III].

\section*{E. Wave Functions and Partial Wave Projection}

The wave function for a heavy meson-baryon system is

$$|\Psi\rangle = \Psi_{JM}(\vec{r}) |M_I\rangle,$$  

(78)

where $|M_I\rangle$ is the isospin wave function and $\Psi_{JM}$ the spin and spatial wave function, which can be written as a sum over partial waves

$$\Psi_{JM}(\vec{r}) = \sum_{LSJ} \eta_{LSJ}(r)^{(2S+1)L_J} \cdot \bar{Y}_{LM_J}(\hat{r}).$$  

(79)

We use the spectroscopic notation $^{2S+1}L_J$, which denotes a partial wave with total spin $S$, orbital angular momentum $L$ and total angular momentum $J$:

$$^{2S+1}L_J = \sum_{M_S,M_L} (LM_S,S_M) |S,M_S\rangle Y_{LM_J}(\hat{r}),$$  

(80)

where $\langle LM_S,S_M | J | M_J \rangle$ are the Clebsch-Gordan coefficients, $|S,M_S\rangle$ the spin wavefunction and $Y_{LM_J}(\hat{r})$ the spherical harmonics. For the $P^*\Sigma_Q$ and $P^*\Sigma_Q^*$ systems the spin wave functions are trivial

$$|S,M_S(P^*\Sigma_Q)\rangle = \frac{1}{2} |M_S\rangle,$$  

(81)

$$|S,M_S(P^*\Sigma_Q^*)\rangle = \frac{3}{2} |M_S\rangle,$$  

(82)

as they correspond to the spin wave functions of the heavy baryon (the heavy meson $P$ is a pseudoscalar). For the $P^*\Sigma_Q$ and $P^*\Sigma_Q^*$ systems,

$$|S,M_S(P^*\Sigma_Q)\rangle = \sum_{M_{S1},M_{S2}} \langle 1M_{S1}\rangle |1/2,M_{S2}\rangle |S,M_S\rangle$$

$$\quad \times |1,M_S\rangle, $$  

(83)  

$$|S,M_S(P^*\Sigma_Q^*)\rangle = \sum_{M_{S1},M_{S2}} \langle 1M_{S1}\rangle |3/2,M_{S2}\rangle |S,M_S\rangle$$

$$\quad \times |1,M_S\rangle, $$  

(84)

with $|1M_{S1}\rangle, |J_2M_{S2}\rangle$ the spin wavefunction of particles 1 and 2.

The partial-wave projection of the potential depends on the matrix elements of the spin-spin, tensor and quadrupole tensor operators, which are independent of $J$ and $M$,  

$$\langle S'J'LM'|O_{12}|SLJM\rangle = \delta_{JJ'}\delta_{MM'}O_{LS'LS}^{c}^{(c)}, $$  

(85)

with $O_{12} = C_{12}, S_{12}, Q_{12}$, which are in turn defined as

$$C_{12} = \vec{d}_1 \cdot \vec{d}_2, $$  

(86)

$$S_{12} = 3d_1 \cdot \hat{r} \cdot \vec{d}_2 - \vec{d}_1 \cdot \vec{d}_2, $$  

(87)

$$Q_{2ij} = \frac{1}{2} \left[ d_2 d_j d_i + a_2 d_i d_j \right] - \frac{a_2^2}{3} \delta_{ij}, $$  

(88)
with \( \hat{d}_1 \) (\( \hat{d}_2 \)) the corresponding spin operator for the \( \bar{D}, \bar{D}^* \) mesons (\( \Sigma_c, \Sigma_b^* \) baryons). In this work we are using the light-quark notation, which means that we have written the potentials in terms of the light-quark spin. The correspondence between the light-quark spin operators and \( C_{12}, S_{12} \) is given by

\[
\hat{d}_{12} \cdot \hat{S}_{12} = f_{12} C_{12}, \\
S_{12} = f_{12} S_{12}, \\
Q_{12} = f_{2} Q_{2},
\]

where \( f_{12} \) and \( f_{2} \) are factors related to the conversion from the light-quark to the hadron spin degrees of freedom (for all non-vanishing cases \( f_{12} = \frac{4}{5} \) and \( f_{2} = \frac{1}{2} \)). The specific matrix elements of the spin-spin, tensor and quadrupole-tensor operators can be consulted in Tables III, IV and V for all the molecular configurations that contain an S-wave (i.e. the ones that are more likely to bind).

**IV. THE CONSISTENT DESCRIPTION OF THE PENTAQUARK TRIO**

In this section we investigate whether the OBE model can describe the LHCb pentaquark trio consistently. We find that the removal of the short-range Dirac-delta contributions to the OBE potential is a necessary step for achieving this goal. We discuss the possible interpretations and justifications of this modification to the OBE model.

**A. Predictions of the \( P_c(4440) \) and \( P_c(4457) \)**

The main ambiguity of the OBE model is the choice of a cutoff. In this manuscript, following the ideas of Refs. [41, 42], we propose the determination of the cutoff from the condition of reproducing the mass of a known molecular candidate. As there are three hidden-charm pentaquarks, we are left with three possibilities: the \( P_c(4312) \) (as a \( D \Sigma_c \) bound state), the \( P_c(4440) \) and the \( P_c(4457) \) (as \( D \Sigma_c^* \) bound states). Owing to the aforementioned regulator artefact in the spin-spin piece of the OBE potential, the most suitable choice is the \( P_c(4312) \), which for the parameters of Table III is reproduced for

\[
\Lambda_1 = 1119 \text{ MeV}.
\]

In the naive OBE model, this cutoff leads to the predictions

\[
M(\frac{1}{2}^+) = 4388 \quad \text{and} \quad M(\frac{3}{2}^+ \Sigma_c) = 4459 \text{ MeV},
\]

which are not compatible with the experimental masses of the \( P_c(4440) \) and the \( P_c(4457) \), i.e.

\[
M_{P_c} = 4440.3 \pm 1.3^{+4.1}_{-4.6} \quad \text{and} \quad M_{P_{c'}} = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}.
\]

As already explained, the reason for this mismatch is the distortion of the OBE potential at relatively long distances owing to the delta-like contribution to the spin-spin interaction, which we will explain in what follows.

**B. The One-Pion-Exchange Potential with a Monopolar Form Factor**

Now if we inspect the OPE contribution to the OBE potential, it contains a spin-spin and a tensor piece

\[
V_{\pi} = \hat{d}_{12} \cdot \hat{S}_{12} V_{\pi(S)} + S_{12} f(r) V_{\pi(T)}.
\]

The spin-spin piece reads

\[
V_{\pi(S)} = \frac{g_{\pi} f_{\Sigma_c}}{6f_{\pi}} f_1 \cdot f_2 \frac{m_{\pi}^3}{m^3} \left[ -d(m_{\pi} \tau_1, \frac{\Lambda}{m_{\pi}}) + W_f(m_{\pi} r, \frac{\Lambda}{m_{\pi}}) \right],
\]

where \( d \) and \( W_f \) are the regularized delta-like and Yukawa-like contributions defined in Eqs. (97) and (108).

As can be seen from Eq. (96) and Fig. 1 these two contributions have opposite sign: the delta-like contribution will generate a strong short-range attraction/repulsion that is unphysical. If the range of this unphysical contribution is short enough, it will have no observable effect in the predictions of the OBE model. However the problem is that this is not the case. If we compute the OPE potential contribution a monopolar cutoff \( \Lambda_1 = 1119 \text{ MeV} \), the OPE potential changes sign at \( r = 1.1 \text{ fm} \), which is comparable with the range of the OPE potential \( R_{\pi} = 1/m_{\pi} = 1.4 \text{ fm} \). This is unsettling to say the least: the modifications of the form factors to the OBE potential are expected to be short-ranged, but certainly not of the order of the pion range. This indicates that it is better to remove this contribution. If we remove the delta-like contributions of the pion and the vector mesons, we end up with the predictions

\[
M(\frac{1}{2}^+) = 4458.0 \quad \text{and} \quad M(\frac{1}{2}^\Sigma_c) = 4443.9 \text{ MeV},
\]

which are basically compatible with the experimental determination of the masses of the \( P_c(4440) \) and \( P_c(4457) \).
C. The OBE model from a modern perspective

The problems of the naive OBE model for reproducing the known hidden-charm pentaquarks are easy to understand from a modern perspective: they arise from the unknown short-range physics. The existence of short-range ambiguities in the OBE model is apparent from the fact that the unregularized OBE potential is singular, with the tensor contributions diverging as \(1/r^3\) for distances \(nr \ll 1\), with \(m\) the mass of the exchanged boson. This type of potentials require regularization if we want to have a unique solution of the Schrödinger equation [58] (for more modern treatments of singular interactions see Refs. [59, 62]). This is the reason that justifies the inclusion of form factors in the original OBE model.

Nowadays we know that the removal of short-range ambiguities requires not only regularization, but also renormalization. By this we mean the following: we expect to trade off the short-range ambiguities by observable information. In the original OBE model we simply regularize the potential by choosing a sensible form factor and cutoff. The renormalization process is more systematic: we explicitly include a contact-range potential to model the unknown short-range physics. By fitting the couplings in this contact-range potential to experimental information we are effectively absorbing the dependence on the form factor and the cutoff in these couplings. The price to pay is a reduction in the predictive power, as we have to include new parameters in the theory which have to be determined from experimental data.

Yet renormalization helps to understand in hindsight the success of phenomenological models. In the particular case of the OBE potential, choosing the form factor and the cutoff as to reproduce experimental information basically amounts to an implicit (but usually incomplete) renormalization process. This can be better understood by considering how a renormalized OBE potential in the line of Ref. [43] would look like. The idea would be to include a contact-range potential into the OBE potential, resulting in

\[ V = V_C + V_{OBE}, \]

with the \(V_C\) and \(V_{OBE}\) taken from Eqs. (16) and (30). This potential would be regularized by means of a form factor and a cutoff and the couplings \(C_{Ia}\) and \(C_{Ib}\) would be determined from known experimental information. It happens that this will require four experimental data in total, two per isospin channel. That is, in a properly renormalized OBE potential we will lose predictability (we will only be able to explore the \(I = 1/2\) sector) in favor of systematicity. This is not what we do in the present manuscript, which uses a phenomenological approach. Despite this, the present calculation can be understood as a particular case of the renormalized OBE potential of Eq. (58) for which

\[ C_a(\Lambda = \Lambda_1) = 0, \]

\[ C_b(\Lambda = \Lambda_1) = - \left[ C_{\sigma b}^\tau + C_{\rho b}^\rho + C_{\omega b}^\omega \right], \]

with \(\Lambda_1\) the cutoff for which the \(P_i(4312)\) is reproduced and where \(C_{\sigma b}^\tau, C_{\rho b}^\rho\) and \(C_{\omega b}^\omega\) are the Dirac-delta contributions to the

---

| Molecule | Partial Waves | \(J^P\) | \(\delta_{L1} \cdot \delta_{L2} = f_{L1} \times d_{L1} \cdot d_{L2}\) |
|----------|---------------|--------|--------------------------------------------------|
| \(\bar{D}_5\Sigma_i\) | \(^3S_{1/2}\) | 4+ | 0 \times (0 0) |
| \(\bar{D}_5\Sigma_i\) | \(^4S_{3/2} - ^4D_{3/2}\) | 4+ | 0 \times (0 0) |
| \(\bar{D}_5\Sigma_i\) | \(^2S_{1/2} - ^4D_{1/2}\) | 4+ | 0 \times (0 0) |
| \(\bar{D}_5\Sigma_i\) | \(^2D_{3/2} - ^4S_{3/2} - ^4D_{3/2}\) | 4+ | 0 \times (0 0) |
| \(\bar{D}_5\Sigma_i\) | \(^2S_{1/2} - ^4D_{1/2}\) | 4+ | 0 \times (0 0) |
| \(\bar{D}_5\Sigma_i\) | \(^2D_{3/2} - ^4S_{3/2} - ^4D_{3/2} - ^6G_{3/2}\) | 4+ | 0 \times (0 0) |

---

TABLE III. Matrix elements of the spin-spin operator for the partial waves we are considering in this work.
TABLE IV. Matrix elements of the tensor operator for the partial waves we are considering in this work.

| Molecule | Partial Waves | J' | $S_{413}(\hat{p}) = f_{41} \times S_{43}(\hat{p})$ |
|----------|---------------|----|----------------------------------|
| $\bar{D}\Sigma_c^+$ | $^3S_{1/2}$ | $\frac{1}{2}$ | $0 \times 0$ |
| $\bar{D}\Sigma_c^+$ | $^1S_{1/2} \bar{D}_{3/2}$ | $\frac{1}{2}$ | $0 \times \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ |
| $D^*\Sigma_c^+$ | $^3S_{1/2} \bar{D}_{1/2}$ | $\frac{1}{2}$ | $\frac{7}{3} \times \begin{pmatrix} 0 \\ \sqrt{2} \sqrt{-2} \end{pmatrix}$ |
| $D^*\Sigma_c^+$ | $^1S_{1/2} \bar{D}_{1/2} \bar{D}_{3/2}$ | $\frac{1}{2}$ | $\frac{7}{3} \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ |
| $D^*\Sigma_c^+$ | $^3S_{1/2} \bar{D}_{1/2} \bar{D}_{3/2}$ | $\frac{1}{2}$ | $\frac{7}{3} \times \begin{pmatrix} 0 \\ -\frac{7}{3} \sqrt{2} \\ -\frac{7}{3} \sqrt{2} \\ -\frac{7}{3} \sqrt{2} \end{pmatrix}$ |
| $D^*\Sigma_c^+$ | $^3S_{3/2} \bar{D}_{1/2} \bar{D}_{3/2} S_{1/2} \bar{D}_{3/2} G_{1/2}$ | $\frac{1}{2}$ | $\begin{pmatrix} 0 \\ -\frac{3}{2} \sqrt{10} \\ -\frac{3}{2} \sqrt{10} \end{pmatrix}$ |

spin-spin component of the OBE potential coming from $\pi$, $\rho$ and $\omega$ exchange:

$$C_b^\pi = -\frac{g_1 g_2}{2 f_\pi^2} \frac{1}{2} \cdot T_2,$$

$$C_b^\rho = -\frac{2 f_\rho}{3} \frac{f_\rho}{2 M_1} \frac{f_\rho}{2 M_2} \frac{1}{2} \cdot T_2,$$

$$C_b^{\omega} = \frac{2 f_\omega}{3} \frac{f_\omega}{2 M_1} \frac{f_\omega}{2 M_2}.$$

In this interpretation it can be clearly appreciated that the calculations presented here are not renormalized, but still can be identified with a renormalized calculation roughly reproducing the location of the $P_c(4440)$ and $P_c(4457)$ at a particular cutoff ($\Lambda = \Lambda_1$).

This partly justifies the rather ad-hoc removal of all the Dirac-delta contributions from a modern perspective. In the original OBE model this strong distortion of the pion contribution to the potential at long distances was avoided by the use of a large enough cutoff, usually $\Lambda_\pi > 1.3 \text{ GeV}$. Besides, the finite-range piece of the spin-spin piece of the OPE potential is attractive in the S-wave singlet and triplet partial waves, which in turn leads to a repulsive Dirac-delta contribution. But for the heavy antimeson-baryon system it is difficult to have a large enough cutoff that still reproduces the three

pentaquark poles\footnote{This will require making the $\omega$-meson contribution considerably more repulsive by breaking the SU(3) relation $g_\rho = g_\omega$, which is what happens in the OBE model as applied to the nucleon-nucleon system.}. From a modern perspective grounded on the ideas of renormalization, the removal of the deltas actually corresponds to the choice of the $C_b$ coupling that roughly reproduces the pentaquark poles at a certain cutoff. In any case it would be interesting to check whether the present identification of the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ will still be correct in a fully renormalized OBE model with a cutoff that is not fixed but floats within a natural range.

V. THE PENTAQUARK MULTIPLET

In this section we compute the predictions of the OBE model for the hidden-charm molecular pentaquarks. We determine the cutoff in the calculation from the condition of reproducing the $P_c(4312)$, as explained in Sect.\footnotemark[17] From this condition and the OBE potential we can simply determine the full hidden-charm molecular spectrum. We also explain how we estimate the uncertainties of the OBE model.
TABLE V. Matrix elements of the quadrupole-like tensor operator for the partial waves we are considering in this work.

| Molecule | Partial Waves | $J^P$ | $Q_{12}(\tilde{r}) = f_2 \times Q_{2}(\tilde{r})$ |
|----------|---------------|-------|---------------------------------|
| $\bar{D}\Sigma_c$ | $^3S_{1/2}$ | $\frac{1}{2}$ | $0 \times 0$ |
| $\bar{D}\Sigma'_c$ | $^4S_{1/2}^*D_{1/2}$ | $\frac{3}{2}$ | $\frac{1}{\sqrt{2}} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ |
| $D^*\Sigma_c$ | $^3S_{1/2}^*D_{1/2}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}} \times \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ |
| $D^*\Sigma'_c$ | $^3D_{3/2}^*S_{1/2}^*D_{1/2}$ | $\frac{3}{2}$ | $\frac{1}{3} \times \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ |
| $D^*\Sigma_c$ | $^3S_{1/2}^*D_{1/2}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}} \times \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ |
| $D^*\Sigma'_c$ | $^3D_{3/2}^*S_{1/2}^*D_{1/2}^*G_{3/2}$ | $\frac{3}{2}$ | $\frac{1}{3} \times \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ |
| $D^*\Sigma_c$ | $^3D_{3/2}^*S_{1/2}^*D_{1/2}^*G_{3/2}^*S_{1/2}^*D_{1/2}^*G_{3/2}$ | $\frac{5}{2}$ | $\frac{1}{3} \times \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ |

**A. Error Estimations**

The OBE model has a series of uncertainties, mostly stemming from the choice of the coupling constants. This error source can be dealt with by assigning a relative uncertainty to the OBE potential:

$$V(P_c) = V_{OBE}(1 \pm \delta_{OBE}),$$

where $V(P_c)$ is the molecular pentaquark potential in a given channel and $V_{OBE}$ is the central value of the OBE potential with the central value of the couplings, see Table III for details. We will assume the relative uncertainty to be $\delta_{OBE} = 30\%$, which is equivalent to assume that the average relative uncertainty of the coupling constants in Table III is $\delta_{coupling} = \delta_{OBE}/2 \sim 15\%$ (assuming that the uncertainty distribution in the couplings is Gaussian). With this uncertainty we can recalculate the cutoff $\Lambda_1$ by determining the location of the $P_c(4312)$, leading to

$$\Lambda_1 = 1.119^{+0.190}_{-0.094} \text{GeV}.$$  \hspace{1cm} (105)

The error in the binding energies is simply obtained by propagating the $(1 \pm \delta_{OBE})$ uncertainty in the OBE potential, with the condition of recalculating the cutoff as to reproduce the $P_c(4312)$. This condition implies the partial renormalization of the OBE model, which manifests in the fact that the errors derived from the overall uncertainty in the potential are rather small. For the particular case of the $D^*\Sigma_c$ bound states we arrive at

$$B_E(\frac{1}{2}^-) = 4.2^{+0.6}_{-0.7} \text{ and } B_E(\frac{3}{2}^-) = 18.3^{+0.6}_{-0.0} \text{ MeV},$$  \hspace{1cm} (106)

where the errors, besides being small, are also asymmetric.

There is a second error source: HQSS is not exact for finite heavy quark masses. The relative size of HQSS violations are expected to be of the order of $\delta_{HQSS} \sim \Lambda_{QCD}/m_Q$, with $\Lambda_{QCD} \sim (200–300) \text{ MeV}$ and $m_Q$ the mass of the heavy quark. This error manifests in random variations of the OBE potential around its expected HQSS limit

$$V(P_c) = V_{OBE}^{[HQSS]}(1 \pm \delta_{HQSS}),$$  \hspace{1cm} (107)

where in the charm sector we expect $\delta_{HQSS} \sim 15\%$. It is worth stressing the difference between the OBE error of Eq. (104) and the HQSS error of Eq. (107): the OBE error takes into account the error in the coupling constants but assumes that these couplings are identical for all the possible molecules, while the HQSS error considers that these couplings might be different for each of the molecular states. For the $D^*\Sigma_c$ bound
states the HQSS uncertainty is

\[ B_E(\frac{1}{2}^-) = 4.2^{+5.3}_{-3.3} \text{ and } B_E(\frac{3}{2}^-) = 18.3^{+11.6}_{-9.2} \text{ MeV,} \]

which is considerably larger than the OBE uncertainty. The reason why this happens is that the OBE uncertainty is renormalized away: changes in the couplings of the light mesons to the heavy hadrons are compensated by a change in the cutoff. On the contrary HQSS violations imply that the couplings are different for the ground and excited spin states of a heavy hadron, i.e. the couplings for the \( D \) and \( D^* \) (or \( \Sigma \) and \( \Sigma^* \)) are a bit different. This uncertainty is not absorbed by the cutoff variation and results in a larger error. Finally for the full error we will sum in quadrature the OBE and HQSS errors.

In addition to the binding energies of the molecular pentaquarks, we also compute the S-wave scattering lengths of the charmed antimeson-baryon systems. The reason is to identify molecular configurations in which the attraction is strong, but not strong enough to bind. The basis of this idea is a well-known relation between the two-body scattering length and binding energy, \( a_2 \) and \( B_2 \), that works in the limit in which the bound state is weakly bound

\[ a_2 = \frac{1}{\sqrt{2\mu B_2}} + O(\frac{\sqrt{2\mu B_2}}{m_\pi}), \]

with \( \mu \) the reduced mass of the system and \( m_\pi \) the pion mass. For a shallow bound state, i.e. \( m_\pi > \frac{\sqrt{2\mu B_2}}{3} > 0 \), the scattering length is positive and large \((m_\pi a_2 \gg 0)\). For \( B_2 \to 0 \) the scattering length diverges and for a system that almost binds, the scattering length is negative and large. We notice that we compute the scattering lengths under the assumption that the charmed antimeson and the charmed baryon are stable hadrons with respect to the strong interaction, which is not true in general. This is not important as we are actually using the scattering length as a tool to identify configurations that are close to binding.

### B. Predictions

With the OBE model regularized without the delta-like contributions, we can now predict the seven possible S-wave \( \bar{D}^* \Sigma^* \) molecules. The results are summarized in Table VI. For the isodoublet \( (I = \frac{1}{2}) \) molecular pentaquarks, the states predicted in the OBE model are indeed very similar to the ones obtained in scenario B of the contact-range effective field theory of Ref. [42]. Here we note that within the contact-range EFT description of Ref. [42] there are two coupling constants whose values have to be determined from experimental information. Thus two scenarios were considered: scenario A, in which the \( P_c(4440) \) and \( P_c(4457) \) are the \( J = \frac{1}{2} \) and \( J = \frac{3}{2} \) \( D^* \Sigma \) molecules, and scenario B for the opposite identification. Our OBE model naturally selects scenario B.

For the isosinglet \( (I = \frac{1}{2}) \) molecular pentaquarks, we find that the \( J = \frac{1}{2} \) \( D^* \Sigma \) and the \( J = \frac{1}{2} \) \( D^* \Sigma \) bind. Yet this conclusion is not particularly strong: these two molecular pentaquarks are weakly bound and once we consider the error in the binding energies the outcome is that there is a fair likelihood that they will not bind. The isosinglet \( J = \frac{1}{2} \) \( D^* \Sigma \) molecule is close to binding, as can be inferred from the large negative scattering length. Conversely, the uncertainties in the OBE model mean that this molecular pentaquark might be able to bind. The other isosinglet molecules display mild attraction, a conclusion which can be deduced from the negative (but natural) values of the scattering length shown in Table VI.

### VI. SUMMARY

In this manuscript we have investigated the spectroscopy of the hidden-charm pentaquarks from the point of view of the OBE model. In particular we considered the impact of the short-range delta-like contributions in the OBE potentials. The removal of these contributions, in combination with the condition of reproducing the mass of the \( P_c(4312) \) pentaquark as a \( D^* \Sigma \) bound state, leads to the following predictions for the \( D^* \Sigma \) molecules:

\[
M(\frac{1}{2}^-) = 4458.0^{+3.4}_{-5.3} \text{ and } M(\frac{3}{2}^-) = 4443.9^{+9.2}_{-11.6} \text{ MeV,}
\]

which are close to the experimental masses of the \( P_c(4440) \) and \( P_c(4457) \) pentaquarks. This suggests the identification of the \( P_c(4440) \) with the \( J = \frac{1}{2} \) \( D^* \Sigma \) bound state and the \( P_c(4457) \) with the \( J = \frac{1}{2} \) one. In fact the expectation from
OPE alone is that the \( J = \frac{3}{2} \) molecule should be more bound than the \( J = \frac{5}{2} \) one \([7]\), as a consequence of \( \langle \text{spin-spin component of OPE} \rangle \) being attractive (repulsive) in the \( J = \frac{3}{2} \) \((\frac{1}{2})\) channel. The combination of OPE with short-range physics, as in Ref. [63] (which uses the hidden-gauge approach to which it adds pion-exchange diagrams), leads to the same conclusion. The recent work of Ref. [4] also explains the molecular pentaquark spectrum on the basis of OPE and proposes the same spin-parity identification as here, but suggest that the reason why the \( J = \frac{3}{2} \) molecule is more bound is the tensor component of the OPE potential (instead of the spin-spin component, as in Ref. [7]). Be it as it may, we warn that theoretical predictions in the OBE model have significant uncertainties and that these uncertainties cannot be systematically estimated, as we are dealing with a model (instead of an effective field theory).

Besides proposing a possible identification for the quantum numbers of the three hidden-charm pentaquarks, we predict the existence of other four molecular pentaquarks with \( I = \frac{1}{2} \). This prediction indeed confirms the conclusion of Ref. [13], which used a contact-range effective field theory to describe the molecular pentaquarks, and of Ref. [65], which used the hidden-gauge formalism (constrained by HQSS) instead. Among the predicted states there is the \( J = \frac{3}{2} D^0 \Sigma_c \) molecule, which was conjectured in Refs. [6, 33] and recently reproduced in a few recent theoretical works \([13, 14, 63, 66]\). Finally, in the isoquartet sector \( (I = \frac{3}{2}) \) there might be two or three molecular pentaquarks that bind. We note that this will impact the size of the proposed isospin-breaking decay \( \Gamma(P_c \rightarrow J/\Psi K^0) / \Gamma(P_c \rightarrow J/\Psi p) \) calculated in Ref. [18] (in a similar way, for instance, as the presence of a bound or virtual state in the \( D \bar{D} \) system will affect the decay of the \( X(3872) \) to \( D^0 \bar{D}^0 \pi^0 \) \([67]\)). Conversely, the experimental measurement of the isospin-breaking decay ratio proposed in Ref. [18] might provide important clues regarding the existence of isoquartet molecular pentaquarks.

**ACKNOWLEDGMENTS**

We would like to thank Atsushi Hosaka for useful discussions, Eulogio Oset for suggesting a few interesting references and Ruprecht Machleidt for his clarifications and explanations regarding the OBE model. This work is partly supported by the National Natural Science Foundation of China under Grant No.11735003, the fundamental Research Funds for the Central Universities, the Youth Innovation Promotion Association CAS (No. 2016367) and the Thousand Talents Plan for Young Professionals.

**Appendix A: Lagrangians for the Magnetic and Quadrupole Moments**

In this appendix we discuss the magnetic and quadrupole couplings of a heavy hadron to the electromagnetic field. This is useful for the derivation of the magnetic- and quadrupole-like couplings to the vector mesons in the vector-meson dominance model. In particular we write

\[
L_\mu = \mu(h) h^{\mu} \left[ \frac{1}{2} \epsilon_{ijk} S_i \partial_j A_k \right] h, \quad (A1)
\]

\[
L_Q = Q(h) h^{\mu} \left[ \frac{1}{2} \epsilon_{ijk} Q_{ij} \partial_k \partial_0 A_0 \right] h, \quad (A2)
\]

for the magnetic-dipole and electric-quadrupole coupling of a heavy hadron field \( h \). In the magnetic term, \( \mu(h) \) is the magnetic-dipole moment of the heavy hadron, \( S \) represents the spin operator of this heavy hadron, which we assume to be spin-S (with \( S \geq \frac{1}{2} \) if we want the magnetic moment to be non-vanishing). In the quadrupole term, \( Q(h) \) is the electric-quadrupole moment of the heavy hadron and \( Q_{ij} \) is a spin-2 tensor that can be constructed from the spin operator \( S \):

\[
Q_{ij} = \frac{1}{2} \left[ S_i S_j + S_j S_i \right] - \frac{1}{3} S (S + 1) \delta_{ij}, \quad (A3)
\]

which requires \( S \geq 1 \) to be non-vanishing and with \( Q_{33} = \frac{1}{2} S (2S - 1) \). These definitions ensure that

\[
\langle S S | \hat{\mu}_3 S S \rangle = \mu(h), \quad (A4)
\]

\[
\langle S S | \hat{Q}_{33} S S \rangle = Q(h), \quad (A5)
\]

where \( |SS\rangle \) represents a spin state of the heavy hadron \( h \) where the third component is \( S_3 = +S \), while \( \hat{\mu}_3 \) and \( \hat{Q}_{33} \) are the \( i = 3 \) and \( ij = 33 \) components of the magnetic and tensor operators, which can be identified with

\[
\hat{\mu}_i = \mu(h) \frac{1}{|S_3|} S_i, \quad (A6)
\]

\[
\hat{Q}_{ij} = Q(h) \frac{1}{|Q_{33}|} Q_{ij}. \quad (A7)
\]

Conversely, the moments of order \( n \) can be defined analogously as

\[
M_{i_1...i_n}^{(n)}(h) = M^{(n)}(h) \frac{1}{|T_{i_1...i_n}^{(n)}|} T_{i_1...i_n}^{(n)}, \quad (A8)
\]

with \( M^{(n)}(h) \) the \( n \)-pole moment of hadron \( h \) and \( T^{(n)} \) a spin-\( n \) tensor constructed from the hadron spin operator \( S \).

For the heavy-baryon sextet, assuming that the multipole moments are dominated by the light-quarks, only the magnetic-dipole and electric-quadrupole moments will be relevant; as discussed, the quadrupole moment is expected to be small (it requires either HQSS breaking or a sizable D-wave component for the light quark pair).
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