**Abstract**

In the framework of the littlest Higgs model with T parity, we study the $W_H/Z_H$ production in association with a T-odd (anti)quark of the first two generations at the CERN Large Hadron Collider up to the QCD next-to-leading order. The kinematic distributions of final decay products and the theoretical dependence of the cross section on the factorization/renormalization scale are discussed. We apply three schemes in considering the QCD NLO contributions and find that the QCD NLO corrections by adopting the (II) and (III) subtraction schemes can keep the convergence of the perturbative QCD description and reduce the scale uncertainty of the leading order cross section. By using these two subtraction schemes, the QCD NLO corrections to the $W_H(Z_H)q^-$ production process enhance the leading order cross section with a K-factor in the range of $1.00 \sim 1.43$.

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I. Introduction

The standard model (SM) \cite{1,2} provides a remarkably successful description of high energy physics at the energy scale up to $10^2$ GeV. Despite its tremendous success, the mechanism of electroweak symmetry breaking (EWSB) remains the most prominent mystery in current particle physics, and the smallness of the Higgs boson mass cannot be protected against perturbative quantum corrections. The instability of the Higgs boson mass leads to the so-called "hierarchy problem" between the electroweak scale, $m_H$, and the cutoff scale of the SM, $\Lambda \sim 10$ TeV \cite{3}.

However, the cutoff scale larger than 10 TeV will lead to a large radiative correction to the Higgs boson mass, which needs unnatural fine-tuning to give a proper EWSB scale. To solve the fine-tuning problem (or hierarchy problem) has become one of the main motivations to the construction of physics beyond the SM. Besides the supersymmetric and extra dimensions models, the little Higgs models \cite{4} are very attractive theories which offer an alternative approach to solve the "hierarchy problem" \cite{4,5}, and are proposed as one kind of EWSB models without fine-tuning in which the Higgs boson is naturally light as a result of nonlinearly realized symmetry \cite{5}-\cite{10}. The simplest version of the little Higgs models is the littlest Higgs (LH) model \cite{8}, which is based on an $SU(5)/SO(5)$ nonlinear $\sigma$ model \cite{7}. However, precision electroweak constraints \cite{11} require the LH model to characterize a large value of $f$, so the fine-tuning between the cutoff scale and the electroweak scale is again needed. This problem can be solved by the littlest Higgs model with T parity (LHT) \cite{12}-\cite{15}.

In the LHT, the SM particles are T-even and their T parity partners are T-odd. Then the SM gauge bosons cannot mix with the new gauge bosons, and the electroweak precision observables are not modified at tree-level. The loop suppression of the corrections to low energy electroweak observables allows the symmetry breaking scale $f$ to be significantly lower than 1 TeV \cite{14}. In order to cancel the quadratic divergence of the Higgs boson mass contributed by top loops, an additional T-even heavy top-quark $T_+$ is introduced. Then a T-odd partner $T_-$ is also required to implement the T parity. The CERN Large Hadron Collider (LHC) provides an opportunity for searching for the new particles predicted in the LHT. Many studies on the phenomenology of the LHT have been presented in detail \cite{13,17,18,19}. The phenomenology of these T-odd gauge
bosons and fermions in the LHT is very attractive. Due to the T parity conservation, the T-odd particles could be produced in pair at the LHC, such as (1) T-odd fermion pair (or gauge boson pair) production, (2) single T-odd fermion production associated with a T-odd gauge boson.

Of all the processes in the second type at the LHC, the $W_H q_-$ associated production, where $q_- = u_-, d_-, c_-, s_-, \bar{u}_-, \bar{d}_-, \bar{c}_-, \bar{s}_-$, has the largest cross section. Therefore, it deserves special attention. The production rate of the $pp \rightarrow Z_H q_- + X$ process is of the same order as, but quantitatively smaller than, that of the $W_H q_-$ production at the LHC. The $W_H q_-$ and $Z_H q_-$ production signals could be detected by their subsequent decays $q_- \rightarrow W_H q'$, $W_H \rightarrow A_H W$ and $Z_H \rightarrow A_H H$, where $q'$ represents $u, d, c, s, \bar{u}, \bar{d}, \bar{c}, \bar{s}$ and $A_H$ is the T parity partner of photon being undetectable as a candidate of dark matter [20]. The phenomenology of the T-odd $SU(2)$ doublet particle productions at the LHC has been already studied at the leading-order (LO) [17]. As we know, the LO predictions for the processes at hadron colliders are always sensitive to the factorization and renormalization scales ($\mu_f$ and $\mu_r$). For the LO $W_H(Z_H)q_-$ production at the LHC $\mu_f$ enters solely through the parton distribution functions (PDFs), while the parton level cross section does not depend on $\mu_r$ at this order. In general, the high order contributions can reduce the scale uncertainty of the LO cross section, and the QCD next-to-leading order (NLO) corrections enhance the LO cross section. Therefore, it is important to take into account the QCD NLO corrections to reduce the sensitivity to these scales.

In this paper, we focus on the $W_H/Z_H$ production in association with a T-odd (anti)quark of the first two generations at the LHC, $pp \rightarrow W_H(Z_H)q_- + X$ ($q_- = u_-, d_-, c_-, s_-, \bar{u}_-, \bar{d}_-, \bar{c}_-, \bar{s}_-$), up to the QCD NLO. The paper is organized as follows. In Sec.II, we briefly review the relevant masses and couplings in the LHT. The detailed strategies of calculation are given in Sec.III. The numerical results and discussions are presented in Sec.IV. Finally we present a short summary.

II. Related LHT theory

Based on an $SU(5)/SO(5)$ global symmetry breaking pattern, a subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ of the $SU(5)$ global symmetry is gauged in the LH model [15], and the gauge fields $W^a_{i\mu}$ and $B_{i\mu}$ ($a = 1, 2, 3$, $i = 1, 2$) are introduced. The kinetic terms for the gauge and
scalar fields can be written as

\[ \mathcal{L}_{G^S} = \sum_{j=1}^{2} \left[ -\frac{1}{2} \text{Tr} \left( W_{j\mu
u} W_{j}^{\mu\nu} \right) - \frac{1}{4} B_{j\mu\nu} B_{j}^{\mu\nu} \right] + \frac{f^2}{8} \text{Tr} \left[ \left( D_{\mu} \Sigma \right)^\dagger \left( D^{\mu} \Sigma \right) \right], \]  

(2.1)

where \( \Sigma \) is the nonlinear \( \sigma \) model field of the LH model and the covariant derivative \( D_{\mu} \Sigma \) is defined as

\[ D_{\mu} \Sigma = \partial_{\mu} \Sigma - i \sum_{j=1}^{2} \left[ g_j \left( W_{j\mu} \Sigma + \Sigma W_{j}^{T\mu} \right) + g_j' B_{j\mu} \left( Y_{j\mu} \Sigma + \Sigma Y_{j} \right) \right]. \]  

(2.2)

To implement T parity in the LHT, we make the following T parity assignment:

\[ W_{1\mu} \leftrightarrow W_{2\mu}, \quad B_{1\mu} \leftrightarrow B_{2\mu}, \]
\[ \Pi \rightarrow -\Omega \Pi \Omega, \quad \text{where} \quad \Omega = \text{diag}(1, 1, -1, 1, 1). \]  

(2.3)

The invariance of the above Lagrangian under T parity implies that the gauge couplings of the two \( SU(2) \times U(1) \) gauge groups have to be equal, i.e., \( g_1 = g_2 = \sqrt{2} g, \quad g_1' = g_2' = \sqrt{2} g' \).

The gauge symmetry \([SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2\) breaks down to its diagonal subgroup \( SU(2) \times U(1) \), which is generated by the combinations \( \{(Q_1^a + Q_2^a)/\sqrt{2}, Y_1 + Y_2\} \), where \( \{Q_i^a, Y_i\} \) \((a = 1, 2, 3, \quad i = 1, 2)\) are the generators of the two \( SU(2) \times U(1) \) gauge groups respectively. This subgroup is identified with the SM electroweak gauge group \( SU(2)_L \times U(1)_Y \). The corresponding gauge fields of the residual gauge symmetry, \( SU(2)_L \times U(1)_Y \), are just the T-even eigenstates of the gauge sector:

\[ W_{L}^a = \frac{W_{1}^a + W_{2}^a}{\sqrt{2}} \quad \text{and} \quad B_{L} = \frac{B_{1} + B_{2}}{\sqrt{2}}. \]  

(2.4)

In addition, the other four orthogonal linear combinations of gauge fields,

\[ W_{H}^a = \frac{W_{1}^a - W_{2}^a}{\sqrt{2}} \quad \text{and} \quad B_{H} = \frac{B_{1} - B_{2}}{\sqrt{2}}, \]  

(2.5)

are odd under T parity.

The mass eigenstates of the gauge sector in the LHT are expressed as

\[ W_{L}^\pm = \frac{W_{L}^1 \mp i W_{L}^2}{\sqrt{2}}, \quad \left( \begin{array}{c} A_L \\ Z_L \end{array} \right) = \left( \begin{array}{cc} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{array} \right) \left( \begin{array}{c} B_L \\ W_{L}^3 \end{array} \right), \quad (T - \text{even}), \]
\[ W_{H}^\pm = \frac{W_{H}^1 \pm i W_{H}^2}{\sqrt{2}}, \quad \left( \begin{array}{c} A_H \\ Z_H \end{array} \right) = \left( \begin{array}{cc} \cos \theta_H & -\sin \theta_H \\ \sin \theta_H & \cos \theta_H \end{array} \right) \left( \begin{array}{c} B_H \\ W_{H}^3 \end{array} \right), \quad (T - \text{odd}), \]  

(2.6)
where the mixing angle $\theta_H$ at the $\mathcal{O}(v^2/f^2)$ is defined as

$$S_H = \sin \theta_H \simeq \frac{5g g' v^2}{4(5g^2 - g'^2) f^2}, \quad (2.7)$$

and $v \simeq 246$ GeV is the vacuum expectation value of the SM Higgs.

The T-even gauge bosons $A_L$, $Z_L$ and $W_L$ are identified with the SM photon, $Z$-boson and $W$-boson, respectively. The masses of the T-odd gauge bosons are given by

$$m_{A_H} \simeq \frac{1}{\sqrt{5}} g' f \left(1 - \frac{5}{8} \frac{v^2}{f^2}\right), \quad m_{W_H} \simeq g f \left(1 - \frac{1}{8} \frac{v^2}{f^2}\right), \quad m_{Z_H} \simeq m_{W_H}. \quad (2.8)$$

To implement T parity in the fermion sector (Here we only present the description of the quark sector as a representative.), we introduce two incomplete $SU(5)$ multiplets and an $SO(5)$ multiplet:

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ \psi_2 \end{pmatrix}, \quad \Psi_{HR} = \begin{pmatrix} \tilde{\psi}_{HR} \\ \chi_{HR} \\ \psi_{HR} \end{pmatrix},$$

where $\psi_i = -\tau^2 q_i = -\tau^2 (u_i, d_i)^T, \quad (i = 1, 2, HR)$, (2.9)

which transform under T parity as $\Psi_1 \rightarrow -\Sigma_0 \Psi_2, \quad \Psi_2 \rightarrow -\Sigma_0 \Psi_1$ and $\Psi_{HR} \rightarrow -\Psi_{HR}$, where $\tau^i (i = 1, 2, 3)$ are Pauli matrices and $\Sigma_0$ is a $5 \times 5$ symmetric tensor defined as

$$\Sigma_0 = \langle \Sigma \rangle = \begin{pmatrix} 1 \\ 1_{2 \times 2} \\ 1_{2 \times 2} \end{pmatrix}. \quad (2.10)$$

The transformations for $\Psi_1, \Psi_2$ and $\Psi_{HR}$ under $SU(5)$ are as $\Psi_1 \rightarrow V^* \Psi_1, \quad \Psi_2 \rightarrow V \Psi_2$ and $\Psi_{HR} \rightarrow U \Psi_{HR}$. There $V \in SU(5)$ and $U$ is an $SO(5)$ transformation in a nonlinear representation of $SU(5)$ defined as

$$\xi \rightarrow V \xi U^\dagger = U \xi \Sigma_0 V^T \Sigma_0 \quad \text{(under the } SU(5) \text{ transformation } V). \quad (2.11)$$

It tells us that $q_1, q_2$ and $q_{HR}$ are all $SU(2)$ doublets. Therefore, we obtain two T-odd $SU(2)$ doublets, $q_H = (q_1 + q_2)/\sqrt{2}$ and $q_{HR}$, which are left- and right-handed respectively, and a T-even left-handed $SU(2)$ doublet, $q_{SM} = (q_1 - q_2)/\sqrt{2}$.

Through the Lagrangian

$$\mathcal{L}_F = -\kappa f \left( \psi_2 \xi + \psi_1 \Sigma_0 \xi^\dagger \Omega \right) \Psi_{HR} + \text{h.c.}, \quad (2.12)$$
the T-odd Dirac fermion doublet \( q_- \), defined as \( (q_-)_L = q_H \) and \( (q_-)_R = q_{HR} \), gains a mass equal to \( \sqrt{2}\kappa f \) before EWSB. After EWSB, a small mass splitting between the T-odd up- and down-type quarks is induced, and the masses are given by \( m_u \sim \sqrt{2}\kappa f \left( 1 - \frac{1}{8}v_f^2 \right) \), \( m_d = \sqrt{2}\kappa f \). (2.13)

The T-even left-handed \( SU(2) \) doublet \( q_{SM} \) is identified with the left-handed SM fermion doublet. It can acquire Dirac masses \( m_u \) and \( m_d \) via Yukawa interactions with the T-even right-handed \( SU(2) \) singlets \( u_R \) and \( d_R \), respectively. As we know, the up-type quark is heavier than the down-type quark for each generation in the SM, while the partners of the SM quarks in a new physics model may exhibit an inverted mass hierarchy. For example, the bottom-squarks are considered to be heavier than the top-squarks in the MSSM, a minimal supersymmetric extension of the SM. As we expected, Eq.(2.13) indicates that the mass of the T-odd down-type quark is larger than that of the T-odd up-type quark in the LHT.

In order to cancel the large quadratic divergent corrections to the Higgs boson mass induced by the top quark, the Yukawa interaction for the top sector must be modified. The \( \Psi_1 \) and \( \Psi_2 \) multiplets for the top sector must be completed to representations of the \( SU(3)_1 \) and \( SU(3)_2 \) subgroups of \( SU(5) \) by introducing two additional left-handed \( SU(2) \) singlets \( U_{L1} \) and \( U_{L2} \). These multiplets are

\[
Q_1 = \begin{pmatrix} \psi_1 \\ U_{L1} \\ 0 \end{pmatrix} \quad \text{and} \quad Q_2 = \begin{pmatrix} 0 \\ U_{L2} \\ \psi_2 \end{pmatrix},
\]

which obey the same transformation laws under T parity and \( SU(5) \) as do \( \Psi_1 \) and \( \Psi_2 \). In addition to the T-even right-handed \( SU(2) \) singlet \( u_R \), the top sector contains two right-handed \( SU(2) \) singlets \( U_{R1} \) and \( U_{R2} \), which transform under T parity as \( U_{R1} \leftrightarrow -U_{R2} \). By using these new \( SU(2) \) singlets introduced in the top sector, we obtain four additional T parity eigenstates:

\[
U_{L\pm} = \frac{U_{L1} \mp U_{L2}}{\sqrt{2}} \quad \text{and} \quad U_{R\pm} = \frac{U_{R1} \mp U_{R2}}{\sqrt{2}}.
\]

Therefore, two new heavy partners with opposite T parity, \( T_+ \) and \( T_- \), should appear in the top sector.

As \( U_{L-} \) and \( U_{R-} \) do not mix with \( u_H \) and \( u_{HR} \), where \( u_H \) is the up component of the \( SU(2) \) doublet \( q_H \), \( T_- \) is simply given by \( (T_-)_L = U_{L-} \) and \( (T_-)_R = U_{R-} \). However, the T-even
eigenstates $U_L^+$ and $U_R^+$ mix with $u_{SM}$ and $u_R$ respectively, where $u_{SM}$ is the up component of the $SU(2)$ doublet $q_{SM}$, so that the mass eigenstates of the top quark $t$ and its heavy partner $T_+$ are given by

\[
\begin{align*}
\begin{pmatrix}
t_L \\
(T_+)_L
\end{pmatrix} &= \begin{pmatrix}
\cos \theta_L & -\sin \theta_L \\
\sin \theta_L & \cos \theta_L
\end{pmatrix} \begin{pmatrix}
u_{SM} \\
U_L^+
\end{pmatrix}, \\
\begin{pmatrix}
t_R \\
(T_+)_R
\end{pmatrix} &= \begin{pmatrix}
\cos \theta_R & -\sin \theta_R \\
\sin \theta_R & \cos \theta_R
\end{pmatrix} \begin{pmatrix}
u_R \\
U_R^+
\end{pmatrix},
\end{align*}
\tag{2.16}
\]

where the mixing angles $\theta_{L,R}$ and masses of $T_{\pm}$ are determined by the Yukawa interaction Lagrangian for the top sector.

The couplings of the T-odd $SU(2)$ doublet quarks and gauge bosons to the T-even SM particles used in our calculations are listed in Table 1 [13, 21], where $(V_{Hu})_{ij}$ and $(V_{Hd})_{ij}$ are the matrix elements of the CKM-like unitary mixing matrices $V_{Hu}$ and $V_{Hd}$, respectively. The two mixing matrices satisfy $V_{Hu}^\dagger V_{Hd} = V_{CKM}$ [21], therefore, they cannot simultaneously be set to the identity. In the following calculations we take $V_{Hu}$ to be a unit matrix, then we have $V_{Hd} = V_{CKM}$.

| Interaction | Feynman rule | Interaction | Feynman rule |
|-------------|--------------|-------------|--------------|
| $W_{H}^{\pm \mu} q^{-}_i \overline{u}^j_{-}$ | $i\frac{g}{\sqrt{2}}(V_{Hd})_{ij} \gamma^\mu P_L$ | $W_{H}^{-\mu} \overline{q}^i_{-} u_j^j$ | $i\frac{g}{\sqrt{2}}(V_{Hu})_{ij} \gamma^\mu P_L$ |
| $Z_{H}^\mu q^{-}_i \overline{u}^j_{-}$ | $i (\frac{g_2}{2}C_C - \frac{g_2}{10}S_{H})(V_{Hu})_{ij} \gamma^\mu P_L$ | $Z_{H}^\mu \overline{q}^i_{-} d_j^j$ | $i(-\frac{g_{C_2}}{2}C_C - \frac{g}{10}S_{H})(V_{Hd})_{ij} \gamma^\mu P_L$ |
| $\overline{q}^a_{i} q^a_{-} C_{\mu}$ | $i g_s(T^a)^{\alpha\beta} \gamma^\mu$ | $\overline{q}^a_{i} q^a_{-} C_{\mu}$ | $i g_s(T^a)^{\alpha\beta} \gamma^\mu$ |

Table 1: The related LHT Feynman rules used in this work, where $q_{-} = u_{-}, d_{-}, c_{-}, s_{-}, t_{-}, b_{-}$, $i$ and $j$ are the generation indices and $C_{H}^2 = 1 - S_{H}^2$.

III. Analytic calculations

III..1 LO cross sections

The partonic processes, which contribute to the parent process $pp \to W_H(Z_H)q_{-} + X$ at the LHC, are written as

\[
\begin{align*}
\begin{array}{c}
g(p_1) + q(p_2) \to V_{H}(p_3) + q_{-}(p_4), \\
(V_{H} = W_{H}, Z_{H}, \ q = u, d, c, s, \overline{u}, \overline{d}, \overline{c}, \overline{s}), \\
(q_{-} = u_{-}, d_{-}, c_{-}, s_{-}, \overline{u}_{-}, \overline{d}_{-}, \overline{c}_{-}, \overline{s}_{-}).
\end{array}
\end{align*}
\tag{3.1}
\]
Figure 1: The LO Feynman diagrams for the partonic process $gu \to W_H^+d_−$. 

There are two LO Feynman diagrams for each of the above partonic processes. We plot the LO Feynman diagrams for the partonic process $gu \to W_H^+d_−$ as a representative in Fig.1. The Fig.1(1) and Fig.1(2) diagrams are $s$- and $t$-channel Feynman diagrams respectively. The LO cross section for the partonic process $gq \to V_Hq_′−$ has the form as

$$\hat{\sigma}_{LO}(\hat{s}, gq \to V_Hq_′−) = \frac{(2\pi)^4}{4|\vec{p}_1|\sqrt{\hat{s}}} \int \sum |M_{LO}|^2 d\Phi_2, \ (q = u, d, c, s, \bar{u}, \bar{d}, \bar{c}, \bar{s}), \quad (3.2)$$

where $d\Phi_2$ is the two-body phase space element, and $\vec{p}_1$ is the momentum of the initial gluon in the center-of-mass system. The integration is performed over the two-body phase space of the final particles $V_H$ and $q_′−$. The summation is taken over the spins and colors of the initial and final states, and the bar over the summation indicates averaging over the intrinsic degrees of freedom of initial partons.

The LO total cross section for the parent process $pp \to V_Hq_− + X$ can be expressed as

$$\sigma_{LO}(pp \to V_Hq_− + X) = \sum_{\bar{u}, \bar{d}, \bar{c}, \bar{s}} \left\{ \int dx_A dx_B \left[ G_{g/A}(x_A, \mu_f)G_{q/B}(x_B, \mu_f)\hat{\sigma}_{LO}(gq \to V_Hq_′−, x_Ax_B, \mu_f, \mu_r) + (A \leftrightarrow B) \right] \right\}, \quad (3.3)$$

where $G_{i/P}$ ($i = g, q, \ P = A, B$) represents the PDF of parton $i$ in proton $P$, $x_P$ ($P = A, B$) is the momentum fraction of a parton (gluon or quark) in proton $P$, and $\mu_f$ and $\mu_r$ are the factorization and renormalization scales, respectively.
III..2 QCD NLO corrections

III..2.1 General description

The QCD NLO corrections to the $pp \rightarrow W^+ H(ZH)q^- + X$ process involve the following components:

(i) The QCD one-loop virtual corrections to the partonic processes $gq \rightarrow W^+ H(ZH)q'^-$. 

(ii) The contributions of the real gluon emission partonic processes $gq \rightarrow W^+ H(ZH)q'^- + g$. 

(iii) The contributions of the real light-quark emission partonic processes $gg \rightarrow W^+ H(ZH)q'^- + \bar{q}q''$, $q'' q'' \rightarrow W^+ H(ZH)q'^- + \bar{q} q''$. 

(iv) The corresponding contributions of the PDF counterterms.

It should be noticed that for the $gg \rightarrow W^+ H(ZH)q'^- + \bar{q} q''$ and $q'' q'' \rightarrow W^+ H(ZH)q'^- + \bar{q} q''$ light-quark emission partonic processes there exists resonance effect due to the $q_-$ propagator. In Fig.2 we present the Feynman diagrams for these real light-quark emission partonic processes via intermediate on-shell T-odd quarks. To deal with the resonance effect in these partonic processes, we replace $m_{q^-}$ in the denominator of the $q_-$ propagator by $m_{q^-}^2 - i m_{q^-} \Gamma_{q^-}$. With the LHT parameter values used in this paper, the main decay channels of $q_-$ are $q_\rightarrow W^+ H q'$, $q_- \rightarrow Z_H q$ and $q_ \rightarrow A_H q$: 

$$\text{Br}(q_- \rightarrow W^+ H q') + \text{Br}(q_- \rightarrow Z_H q) + \text{Br}(q_- \rightarrow A_H q) \approx 100\%.$$ 

(3.4)

Therefore, the value of $\Gamma_{q_-}$ is obtained approximately by summing up the LO partial decay widths of these main decay channels. These QCD NLO contribution parts from the $gg \rightarrow W^+ H(ZH)q'^- + \bar{q}$ and $q'' q'' \rightarrow W^+ H(ZH)q'^- + \bar{q}$ partonic processes are quite large due to the high gluon luminosity and the $q_-$ resonance effect.

In this work, we apply three schemes in considering the QCD NLO corrections. In scheme (I) (denoted as "QCD NLO I") we include all the four components mentioned above in the QCD NLO corrections. With this scheme, the Feynman diagrams in Fig.2 could lead to large corrections to the Born $pp \rightarrow W^+ H(ZH)q^- + X$ process due to the $q_-$ resonance effect, and destroy the perturbative convergence. Furthermore, these Feynman diagrams are also counted towards
Figure 2: The Feynman diagrams for the real light-quark emission partonic processes via intermediate on-shell T-odd quarks.
production followed by an on-shell decay $q_- \to W_H(Z_H)q'$. Therefore, to avoid double counting and to keep the convergence of the perturbative QCD description of the $W_H(Z_H)q_-$ associated production channel, we should remove the intermediate on-shell $q_-$ contributions from the $W_H(Z_H)q_-$ associated production [23].

In scheme (II) (denoted as "QCD NLO II") we exclude the contributions of the partonic processes $gg \to W_H(Z_H)q_-' + q$ and $q''q'' \to W_H(Z_H)q_-' + q$ from the QCD NLO corrections. Since the corrections to the parent process $pp \to W_H(Z_H)q_- + X$ contributed by the partonic processes $gg \to W_H(Z_H)q_-' + q$ and $q''q'' \to W_H(Z_H)q_-' + q$ and their corresponding PDF counterterms are IR-safe, we could exclude them from the QCD NLO corrections to the parent process $pp \to W_H(Z_H)q_- + X$. With this subtraction scheme, the intermediate on-shell $q_-$ contributions to the $W_H(Z_H)q_-\text{ associated production}$ are removed and the perturbative convergence is kept. Since all the $gg$ and $q\bar{q}$ initiated contributions are excluded, this scheme subtracts some genuine QCD NLO contributions.

We adopt another subtraction strategy, the PROSPINO scheme [22, 23], which removes the on-shell T-odd quark pair production from the real light-quark emissions $gg \to W_H(Z_H)q_-' + q$ and $q''q'' \to W_H(Z_H)q_-' + q$, to avoid double counting and to not artificially ruin the convergence of the perturbative QCD description of the $pp \to W_H(Z_H)q_- + X$ process. This on-shell subtraction scheme can provide a reliable production rate since it only subtracts the squared on-shell amplitudes and does this point by point over the entire phase space. We call this subtraction scheme as the scheme (III) in this work, which is defined as a replacement of the Breit-Wigner propagator [23]

$$\frac{|M|^2(s_{VHq})}{(s_{VHq} - m_{q_-}^2)^2 + m_{q_-}^2\Gamma_{q_-}^2} \to \frac{|M|^2(s_{VHq})}{(s_{VHq} - m_{q_-}^2)^2 + m_{q_-}^2\Gamma_{q_-}^2} - \frac{|M|^2(m_{q_-}^2)}{(s_{VHq} - m_{q_-}^2)^2 + m_{q_-}^2\Gamma_{q_-}^2}\Theta(s - 4m_{q_-}^2)\Theta(m_{q_-} - m_{VH}),$$

where $s_{VHq}$ is the squared momentum flowing through the intermediate $q_-$ propagator. The results by adopting this scheme are denoted as "QCD NLO III".

To isolate the UV and IR singularities, we adopt the dimensional regularization method in $D = 4 - 2\epsilon$ dimensions. The collinear counterterm of the PDF, $\delta G_{i/P}(x, \mu_f)$ ($P =$ proton, $i = g, u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}$), is split into two parts: the collinear gluon emission part $\delta G_{i/P}^{(\text{gluon})}(x, \mu_f)$
and the collinear light-quark emission part $\delta G_{ij/P}(x, \mu_f)$,

$$
\delta G_{q(g)/P}(x, \mu_f) = \delta G_{q(g)/P}^{(\text{gluon})}(x, \mu_f) + \delta G_{q(g)/P}^{(\text{quark})}(x, \mu_f), \quad (q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}),
$$

where

$$
\begin{align*}
\delta G_{q(g)/P}^{(\text{gluon})}(x, \mu_f) &= \frac{1}{\epsilon} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_f^2}{\mu_f^2} \right)^\epsilon \right] \int_x^1 \frac{dz}{z} P_{qq(g)}(z) G_{q(g)/P}(x/z, \mu_f), \\
\delta G_{q(g)/P}^{(\text{quark})}(x, \mu_f) &= \frac{1}{\epsilon} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_f^2}{\mu_f^2} \right)^\epsilon \right] \int_x^1 \frac{dz}{z} P_{qq(q)}(z) G_{q(g)/P}(x/z, \mu_f), \\
\delta G_{g/P}^{(\text{quark})}(x, \mu_f) &= \frac{1}{\epsilon} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_f^2}{\mu_f^2} \right)^\epsilon \right] \sum_{q=u,\bar{u}} \int_x^1 \frac{dz}{z} P_{gq(q)}(z) G_{g/P}(x/z, \mu_f).
\end{align*}
$$

The explicit expressions for the splitting functions $P_{ij}(z)$ ($ij = qq, qg, gg$) in Eqs.(3.7) are available in Ref.[24].

**III.2.2 Virtual and real emission corrections to $gq \rightarrow W_H(Z_H)q_-$**

The one-loop level amplitudes for the partonic processes $gq \rightarrow W_H(Z_H)q_-$ in the LHT include the contributions of the self-energy, vertex and box graphs. In Fig.3 the box Feynman diagrams for the partonic process $gu \rightarrow W_H^+d_-$ are presented as a representative.

The strong coupling constant, the masses and wave functions of the relevant colored particles in the LHT are renormalized to remove the UV divergences of the virtual corrections. In the QCD NLO calculations of the $pp \rightarrow W_H(Z_H)q_+ + X$ process, the following renormalization constants are introduced:

$$
\psi_{q(q-)}^{0,L,R} = \left( 1 + \frac{1}{2} \delta Z_{q(q-)}^{L,R} \right) \psi_{q(q-)}^{L,R}, \quad m_{q-}^0 = m_{q-} + \delta m_{q-},
$$

$$
G_{\mu}^0 = \left( 1 + \frac{1}{2} \delta Z_g \right) G_{\mu}, \quad g_s^0 = g_s + \delta g_s.
$$

(3.8)
where $g_s$ denotes the strong coupling constant, $m_{q_-}$ is the T-odd quark mass, $q_{q(q_-)}^{L,R}$ and $\mu$ denote the fields of the SM quark, T-odd heavy quark and gluon, respectively. The masses and wave functions of the colored fields are renormalized by adopting the on-shell scheme, then the relevant renormalization constants are expressed as

$$
\begin{align}
\delta Z_q^{L,R} &\equiv \delta Z_q = -\frac{\alpha_s(\mu_r)}{3\pi} \left[ \Delta_{UV} - \Delta_{IR} \right], \\
\delta Z_{q_-}^{L,R} &\equiv \delta Z_{q_-} = -\frac{\alpha_s(\mu_r)}{3\pi} \left[ \Delta_{UV} + 2\Delta_{IR} + 4 + 3 \ln \left( \frac{\mu_r^2}{m_{q_-}^2} \right) \right], \\
\frac{\delta m_{q_-}}{m_{q_-}} &\equiv -\frac{\alpha_s(\mu_r)}{3\pi} \left\{ 3 \left[ \Delta_{UV} + \ln \left( \frac{\mu_r^2}{m_{q_-}^2} \right) \right] + 4 \right\}, \\
\delta Z_g &\equiv -\frac{\alpha_s(\mu_r)}{2\pi} \left\{ \frac{3}{2} \Delta_{UV} + \frac{5}{6} \Delta_{IR} + \frac{1}{3} \ln \left( \frac{\mu_r^2}{m_{q_-}^2} \right) + \frac{1}{3} \sum_{T=T_+}^{T_-} \ln \left( \frac{\mu_r^2}{m_T^2} \right) + \frac{1}{3} \sum_{q_-} \ln \frac{\mu_r^2}{m_{q_-}^2} \right\},
\end{align}
$$

where $\Delta_{UV} = 1/\epsilon_{UV} - \gamma_E + \ln(4\pi)$ and $\Delta_{IR} = 1/\epsilon_{IR} - \gamma_E + \ln(4\pi)$.

For the renormalization of the strong coupling constant $g_s$, we adopt the $\overline{MS}$ scheme at the renormalization scale $\mu_r$, except that the divergences associated with the massive top-quark, T-odd $SU(2)$ doublet quarks ($u_-, d_-, c_-, s_-, t_-, b_-$) and $T_\pm$ loops are subtracted at zero momentum [25]. Then the renormalization constant of the strong coupling constant can be obtained as

$$
\frac{\delta g_s}{g_s} = -\frac{\alpha_s(\mu_r)}{4\pi} \left[ \frac{3}{2} \Delta_{UV} + \frac{1}{3} \ln \frac{m_{q_-}^2}{\mu_r^2} + \frac{1}{3} \sum_{T=T_+}^{T_-} \ln \frac{m_T^2}{\mu_r^2} + \frac{1}{3} \sum_{q_-} \ln \frac{m_{q_-}^2}{\mu_r^2} \right],
$$

where $\Delta_{UV} = 1/\epsilon_{UV} - \gamma_E + \ln(4\pi)$ and $\Delta_{IR} = 1/\epsilon_{IR} - \gamma_E + \ln(4\pi)$.

The LO amplitude for $gq \to W_H(Z_H)q'_-$ can be expressed as

$$
\mathcal{M}_{LO} = \mathcal{M}_s + \mathcal{M}_t,
$$

where $\mathcal{M}_s$ and $\mathcal{M}_t$ are the amplitudes for the $s$- and $t$-channel Feynman diagrams, respectively. Then the QCD NLO counterterm amplitude can be written as

$$
\mathcal{M}_{CT} = \left( \frac{\delta g_s}{g_s} + \frac{1}{2} \delta Z_g + \frac{1}{2} \delta Z_{q} + \frac{1}{2} \delta Z_{q_-} \right) \mathcal{M}_{LO} + \delta m_{q_-} \mathcal{M}_t \bigg|_{\left( q'_- - m_{q_-} \right)^2},
$$

where $\mathcal{M}_t \bigg|_{\left( q'_- - m_{q_-} \right)^2}$ represents the amplitude obtained from the $t$-channel amplitude $\mathcal{M}_t$ by doing the replacement of $\left( q'_- - m_{q_-} \right)^2$ to $\left( q'_- - m_{q_-} \right)^2$, and $p_{q_-}$ is the four-momentum of the
T-odd quark in the $t$-channel propagator. From Eqs.\,(3.12) and \,(3.13) we can see that the terms of $\sum_{T=T_+}^{T_0} \ln \frac{m_T^2}{\mu_f^2}$ are exactly canceled in Eq.\,(3.15), therefore, the values of $m_{T\pm}$ are unnecessary in our numerical calculations.

We use our developed in-house programs to isolate analytically the IR singularities of loop integrals and calculate numerically one-loop integrals based on the LoopTools-2.4 package [26, 27], where the analytical expressions for the IR-singular parts of loop integrals are adopted from Ref.\,[28], and the numerical evaluations of IR-safe $N$-point ($N \leq 4$) integrals are implemented by using the formulas in Refs.\,[29, 30, 31].

We employ the two cutoff phase space slicing (TCPSS) method [24] to calculate the corrections from the real gluon/light-quark emission partonic processes. An arbitrary soft cutoff $\delta_s$ separates the real gluon emission phase space into two regions, the soft gluon region and the hard gluon region. Another cutoff $\delta_c$ decomposes the real hard gluon/light-quark emission phase space region into the hard collinear ($HC$) region and the hard noncollinear ($\overline{HC}$) region. Then the soft and collinear IR singularities are isolated from the IR-safe region. The integration over the $\overline{HC}$ region of phase space is performed in the four-dimensions by using the Monte Carlo integrator [32]. Finally, the total cross section for the real emission process can be expressed as

$$\Delta \sigma_R = \Delta \sigma_S + \Delta \sigma_H = \Delta \sigma_S + \Delta \sigma_{HC} + \Delta \sigma_{\overline{HC}}. \quad (3.16)$$

The UV singularities of the loop corrections are canceled by those of the related counterterms contributed by the renormalization constants in Eqs.\,(3.9)-(3.13). Therefore, the renormalized virtual corrections (loop corrections combined with the related counterterms) are UV-finite. Furthermore, the renormalized virtual corrections also contain soft and collinear IR singularities. These IR singularities exactly vanish after combining the renormalized virtual corrections with the contributions of the real gluon/light-quark emission processes and the PDF counterterms. These cancelations have been verified analytically and numerically in our calculations.
IV. Numerical results and discussions

IV..1 Input parameters

In the study of the dependence of the QCD NLO corrected cross section on the factorization and renormalization scales, we set the two unphysical scales equal to a common value \((\mu_f = \mu_r = \mu)\) and do not vary them in an independent way for simplicity. This setting of scales may render the results more stable than they actually are due to the logarithmic term \(\ln \frac{\mu_r^2}{\mu_f^2}\) in the QCD NLO contributions. For example, the PDF counterterms in Eqs. (3.7) have the form as
\[
\frac{\alpha_s}{2\pi} \left( \frac{1}{\epsilon} - \gamma_E + \ln 4\pi + \ln \frac{\mu_r^2}{\mu_f^2} \right)(P \otimes G),
\]
where \(P \otimes G\) represents the convolution of the splitting function \(P\) with the PDF \(G\). When we set \(\mu_f = \mu_r\), we obtain \(\ln \frac{\mu_r^2}{\mu_f^2} = 0\) and the factorization/renormalization scale dependence of these PDF counterterms is underestimated.

We take one-loop and two-loop running \(\alpha_s\) in the LO and QCD NLO calculations, respectively \[33\]. The central value of the factorization/renormalization scale \(\mu\) is chosen as \(\mu_0 = (m_{W_H} + m_{d_\perp})/2\). We adopt the CTEQ6L1 and CTEQ6M parton densities with five flavors in the LO and NLO calculations, respectively \[34\]. The strong coupling constant \(\alpha_s(\mu)\) is determined by the QCD parameter \(\Lambda_{5\text{LO}}^5 = 165\) MeV for the CTEQ6L1 at the LO and \(\Lambda_{5\text{MS}}^5 = 226\) MeV for the CTEQ6M at the NLO \[33\]. We ignore the masses of \(u\)-, \(d\)-, \(c\)-, \(s\)-, \(b\)-quarks, and take \(\alpha_{ew}(m_Z^2)^{-1}|_{\text{MS}} = 127.925\), \(m_W = 80.399\) GeV, \(m_Z = 91.1876\) GeV, \(m_t = 171.2\) GeV and \(\sin^2 \theta_W = 1 - \left(\frac{m_W}{m_Z}\right)^2 = 0.222646\).

The colliding energy in the proton-proton center-of-mass system is taken as \(\sqrt{s} = 7\) TeV for the early LHC and \(\sqrt{s} = 14\) TeV for the later running at the LHC. The Cabibbo-Kobayashi-Maskawa (CKM) matrix elements are taken as
\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
0.97418 & 0.22577 & 0 \\
-0.22577 & 0.97418 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (4.1)
\]

IV..2 Independence on two cutoffs

In order to demonstrate the independence of the total QCD NLO correction to the \(pp \rightarrow ug \rightarrow W^+_Hd^- + X\) process on the two cutoffs, \(\delta_s\) and \(\delta_c\), we present the QCD NLO correction parts as the functions of the cutoffs in Fig. 4(a), where we take \(\delta_c = \delta_s/100\), \(f = 600\) GeV and \(\kappa = 1\). From Eqs. (2.8) and (2.13), we obtain the related masses of the T-odd particles as
$m_{W_H} = 398.57$ GeV, $m_{u_-} = m_{c_-} = 830.70$ GeV and $m_{d_-} = m_{s_-} = 848.53$ GeV. In this figure we take $\mu = \mu_0 \equiv (m_{W_H} + m_{d_-})/2 = 623.55$ GeV. Although the decay width of $q_-$ is less than 1% of $m_{q_-}$, we take $\Gamma_{d_-} = 0.1 m_{d_-}$ to suppress the resonance effect, which makes the cutoff independence more clear. The amplified curve for $\Delta \sigma_{tot}$ of Fig 4(a) is shown in Fig 4(b). The figures demonstrate that the total QCD NLO correction $\Delta \sigma_{tot}$ which is the summation of the two-body and three-body corrections, is independent of the two cutoffs within the statistical errors, even though the two-body correction ($\Delta \sigma^{(2)}$) and three-body correction ($\Delta \sigma^{(3)}$) are strongly influenced by the cutoffs $\delta_s$ and $\delta_c$. As we know, the independence of the total QCD NLO correction to the $pp \to ug \to W^+_H d_- + X$ process on the cutoffs $\delta_s$ and $\delta_c$ is a necessary condition that must be fulfilled for the correctness of our calculations. In the further numerical calculations, we fix $\delta_s = 1 \times 10^{-4}$ and $\delta_c = \delta_s/100$.

Figure 4: (a) The dependence of the QCD NLO corrections to the $pp \to ug \to W^+_H d_- + X$ process on the cutoffs $\delta_s$ and $\delta_c$ at the LHC, where $f = 600$ GeV, $\kappa = 1$, $\delta_c = \delta_s/100$ and $\mu = \mu_0 = (m_{W_H} + m_{d_-})/2 = 623.55$ GeV. (b) The amplified curve for $\Delta \sigma_{tot}$ of Fig 4(a).

IV.3 Dependence on factorization/renormalization scale

In Figs 5(a,b,c) and Figs 6(a,b,c) we present the LO, QCD NLO corrected cross sections and the corresponding K-factors for the $pp \to W_H q_- + X$ and $pp \to Z_H q_- + X$ processes as the functions of the factorization/renormalization scale at the LHC with $\sqrt{s} = 7$ TeV and 14 TeV, respectively. In Figs 5(a,b) and Figs 6(a,b) the LHT input parameters are taken as $f = 500$ GeV and $\kappa = 1$, while in Figs 5(c) and Figs 6(c) we take $f = 1$ TeV and $\kappa = 1$. The masses of $W_H$, $Z_H$ and $q_-$ ($q = u, d, c, s$) corresponding to these LHT parameters are presented in Table 2. In these
IV.4 Dependence on LHT parameters

We depict the LO, QCD NLO corrected cross sections and the corresponding K-factors for the $pp \to W_H q_- + X$ and $pp \to Z_H q_- + X$ processes as the functions of $f$, the $SU(5)$ global symmetry breaking scale of the LHT, at the LHC with $\sqrt{s} = 7\,TeV$ and $14\,TeV$ in Figs.7(a,b,c) and Figs.8(a,b,c), respectively. In Figs.7(a,b) and Figs.8(a,b) the parameter $\kappa$ is set to be 1, while in Fig.7(c) and Fig.8(c) we take $\kappa = 3$. The curves labeled by ”NLO I”, ”NLO II” and ”NLO III” are for the QCD NLO corrected cross sections using the (I), (II) and (III) schemes, respectively. One can conclude from these figures that the cross section for the $pp \to W_H(Z_H)q_- + X$ process decreases quickly with the increment of $f$, because the two final T-odd particles become heavier with the increment of $f$. However, in the plotted range of $f$ we could have observable production.
Figure 5: The dependence of the cross sections and the corresponding K-factors for the $pp \rightarrow W_H q^- + X$ process on the factorization/renormalization scale $\mu$ at the LHC. (a) $f = 500 \text{ GeV}$, $\kappa = 1$ and $\sqrt{s} = 7 \text{ TeV}$. (b) $f = 500 \text{ GeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. (c) $f = 1 \text{ TeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. 
Figure 6: The dependence of the cross sections and the corresponding K-factors for the $pp \to Z_H q^- + X$ process on the factorization/renormalization scale $\mu$ at the LHC. (a) $f = 500 \text{ GeV}$, $\kappa = 1$ and $\sqrt{s} = 7 \text{ TeV}$. (b) $f = 500 \text{ GeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. (c) $f = 1 \text{ TeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. 

rates for the $pp \rightarrow WHq_− + X$ and $pp \rightarrow ZHq_− + X$ processes, especially when $\kappa = 1$.

![Figure 7: The cross sections and the corresponding K-factors for the $pp \rightarrow WHq_− + X$ process as the functions of the LHT parameter $f$ at the LHC. The corresponding $m_{Hq}$ and $m_d$ values are also scaled on the x-axis.](image)

(a) $\kappa = 1$ and $\sqrt{s} = 7$ TeV. (b) $\kappa = 1$ and $\sqrt{s} = 14$ TeV. (c) $\kappa = 3$ and $\sqrt{s} = 14$ TeV.

We present the numerical results for the $pp \rightarrow WHq_− + X$ and $pp \rightarrow ZHq_− + X$ processes at the LHC for some typical values of the LHT parameters in Table 3 and Table 4 respectively. In the two tables we list the values of the proton-proton colliding energy $\sqrt{s}$, the LHT parameters $\kappa$ and $f$, the LO cross section $\sigma_{LO}$, the QCD NLO corrected cross sections using the (I), (II) and (III) schemes, and the corresponding K-factors. From both the tables and the $f$-dependence figures, we can see that the QCD NLO corrected cross sections for the $pp \rightarrow WHq_− + X$ and $pp \rightarrow ZHq_− + X$ processes at the LHC by using the subtraction scheme (II) are almost the same as those by adopting the subtraction scheme (III) in the LHT parameter space considered in this paper. At the early LHC, the QCD NLO corrected cross section for the $pp \rightarrow WHq_− + X$
Figure 8: The cross sections and the corresponding K-factors for the $pp \rightarrow Z_{H}q_{-} + X$ process as the functions of the LHT parameter $f$ at the LHC. The corresponding $m_{W_{L}}$ and $m_{d_{-}}$ values are also scaled on the x-axis. (a) $\kappa = 1$ and $\sqrt{s} = 7 \text{ TeV}$. (b) $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. (c) $\kappa = 3$ and $\sqrt{s} = 14 \text{ TeV}$.
Table 3: The numerical results for the $pp \to W_H q^- + X$ process at the LHC for some typical values of the LHT parameters.

| $\sqrt{s}$ (TeV) | $\kappa$ | $f$ (GeV) | $\sigma_{LO}$ (fb) | $\sigma_{NLO}^{(I)}$ (fb) | $K^{(I)}$ | $\sigma_{NLO}^{(II)}$ (fb) | $K^{(II)}$ | $\sigma_{NLO}^{(III)}$ (fb) | $K^{(III)}$ |
|------------------|----------|-----------|-------------------|---------------------|---------|---------------------|---------|---------------------|---------|
| 7                | 1        | 500       | 253.53(1)         | 544.6(8)            | 2.15    | 296.6(8)            | 1.17    | 295.3(8)            | 1.16    |
|                  |          | 700       | 21.72(1)          | 40.52(7)            | 1.87    | 27.87(7)            | 1.28    | 27.73(7)            | 1.28    |
|                  |          | 900       | 2.5287(1)         | 4.352(9)            | 1.72    | 3.514(9)            | 1.39    | 3.503(9)            | 1.39    |
| 14               | 1        | 500       | 2830.7(1)         | 7648(3)             | 2.70    | 2823(2)            | 1.00    | 2832(3)            | 1.00    |
|                  |          | 700       | 432.10(2)         | 1011.9(4)           | 2.34    | 467.9(3)           | 1.08    | 465.8(3)           | 1.08    |
|                  |          | 900       | 93.171(3)         | 197.66(8)           | 2.12    | 106.86(7)         | 1.14    | 106.45(8)         | 1.14    |
|                  |          | 1100      | 24.460(1)         | 47.82(2)            | 1.96    | 29.32(2)           | 1.20    | 29.08(2)           | 1.20    |
|                  |          | 1300      | 7.2751(3)         | 13.349(6)           | 1.83    | 9.053(6)          | 1.24    | 8.995(7)          | 1.24    |
| 14               | 3        | 500       | 2.6543(1)         | 46.14(6)            | 1.74    | 31.42(5)          | 1.18    | 31.42(6)          | 1.18    |
|                  |          | 700       | 1.5903(1)         | 2.320(6)            | 1.46    | 2.072(6)          | 1.30    | 2.067(6)          | 1.30    |
|                  |          | 900       | 0.12741(1)        | 0.1841(5)           | 1.44    | 0.1810(5)         | 1.42    | 0.1806(5)         | 1.42    |

The $pp \to Z_H q^- + X$ process using the subtraction scheme (III) can reach 295.3 fb (158.2 fb) and the corresponding K-factor is 1.16 (1.20) when $f = 500$ GeV and $\kappa = 1$. While at the 14 TeV LHC, the QCD NLO corrected cross section for the $pp \to W_H q^- + X$ process using the subtraction scheme (III) can reach 2832 fb (1491 fb) and the corresponding K-factor is 1.00 (1.03) when $f = 500$ GeV and $\kappa = 1$. We can also find that the QCD NLO K-factor increases with the increment of the LHT parameters $\kappa$ and $f$. By adopting the subtraction scheme (III), the K-factor can reach 1.42 and 1.43 when $\kappa = 3$ and $f = 900$ GeV for the $pp \to W_H q^- + X$ and $pp \to Z_H q^- + X$ processes at the 14 TeV LHC, respectively.

### IV..5 Transverse momentum and rapidity distributions of final particles

In this subsection we inspect the characteristics of the transverse momentum and rapidity distributions of the final decay products. The $W_H q^-_\text{associated production}$ at the LHC can be followed by the subsequent decays $W_H \to A_H W, q^- \to W_H q' \to A_H Wq'$ and $W^\pm \to l^\pm (\nu)$. Therefore, the $W_H q^-_\text{production signal}$ can be found by detecting the final states $l^+ l^- + \text{jet} + E_{T,\text{missing}}$. Similarly, the $Z_H q^-_\text{production}$ can be detected through the decays $Z_H \to A_H H, H \to b\bar{b}$ and $q^- \to W_H q' \to A_H Wq' \to A_H l\nu q'$, with the final states as $l^\pm b\bar{b} + \text{jet} + E_{T,\text{missing}}$. The results in subsections [IV..3] and [IV..4] show that the QCD NLO corrections using the
Table 4: The numerical results for the $pp \rightarrow Z_H q_- + X$ process at the LHC for some typical values of the LHT parameters.

| $\sqrt{s}$ (TeV) | $\kappa$ | $f$ (GeV) | $\sigma_{LO}$ (fb) | $\sigma_{NLO}^{(I)}$ (fb) | $K^{(I)}$ | $\sigma_{NLO}^{(II)}$ (fb) | $\sigma_{NLO}^{(III)}$ (fb) | $K^{(II)}$ | $K^{(III)}$ |
|------------------|-----------|-----------|-------------------|-------------------|--------|-------------------|-------------------|--------|--------|
| 7                | 1         | 500       | 132.15(1)         | 282.7(6)          | 2.14   | 158.9(6)          | 1.20              | 158.2(7) | 1.20   |
|                  |           | 700       | 11.205(1)         | 20.94(5)          | 1.87   | 14.54(5)          | 1.30              | 14.54(6) | 1.30   |
|                  |           | 900       | 1.2977(1)         | 2.232(6)          | 1.72   | 1.813(6)          | 1.40              | 1.808(6) | 1.39   |
| 14               | 1         | 500       | 1446.5(1)         | 3898(6)           | 2.69   | 1497(6)           | 1.03              | 1491(6)  | 1.03   |
|                  |           | 700       | 219.36(2)         | 513.6(9)          | 2.34   | 241.6(9)          | 1.10              | 240.6(9) | 1.10   |
|                  |           | 900       | 47.141(4)         | 99.9(2)           | 2.12   | 54.6(2)           | 1.16              | 54.3(2)  | 1.15   |
|                  |           | 1100      | 12.351(1)         | 24.22(6)          | 1.96   | 14.89(6)          | 1.21              | 14.83(6) | 1.20   |
|                  |           | 1300      | 3.6685(3)         | 6.758(9)          | 1.84   | 4.598(9)          | 1.25              | 4.582(9) | 1.25   |
| 14               | 3         | 500       | 14.125(1)         | 24.54(7)          | 1.74   | 17.02(6)          | 1.21              | 17.18(7) | 1.22   |
|                  |           | 700       | 0.83522(7)        | 1.227(4)          | 1.47   | 1.095(5)          | 1.31              | 1.102(4) | 1.32   |
|                  |           | 900       | 0.066474(5)       | 0.0969(3)         | 1.46   | 0.0947(3)         | 1.42              | 0.0952(3) | 1.43   |

The LO and QCD NLO corrected transverse momentum distributions of the final $W^-$-boson, jet, lepton $l^-$ and missing energy ($A_H A_H \nu \bar{\nu}$) for the $pp \rightarrow W_H q_- + X$ process and the corresponding K-factors are plotted in Figs.9(a,b,c,d), respectively. The results in these figures are obtained by taking $f = 1$ TeV, $\kappa = 1$ and $\sqrt{s} = 14$ TeV. With these parameters we obtain $m_{A_H} = 153.0$ GeV from Eq.(2.8). It should be declared that in Fig.9(b) the $p_T$ distribution labeled by ”NLO III” is for the leading jet ($j_1$ is called as leading jet if $E_{j_1} > E_{j_2}$), if there exist two jets in one event. From these figures we find that the distributions $\frac{d\sigma}{dp_{T,W^-}}$, $\frac{d\sigma}{dp_{T,jet}}$ and $\frac{d\sigma}{dp_{T,missing}}$ increase with the increment of $p_T$ in the low $p_T$ region, and reach their maxima at $p_{T,W^-} \sim 200$ GeV, $p_{T,jet} \sim 500$ GeV and $p_{T,missing} \sim 400$ GeV, respectively. The transverse momentum distribution of $l^-$ is quite different from the former ones. It decreases rapidly with the increment of $p_{T,l^-}$ for experimentally acceptable lepton.

The LO and QCD NLO corrected transverse momentum distributions of the final $W$-boson,
Figure 9: The LO and QCD NLO corrected $p_T$ distributions of final particles for the $pp \rightarrow W_H q\bar{q} + X$ process at the LHC by taking $f = 1 \, TeV$, $\kappa = 1$ and $\sqrt{s} = 14 \, TeV$. (a) $W^-$-boson, (b) jet, (c) lepton $l^-$, (d) missing $p_T$. 
jet, lepton and missing energy \( A_H A_H \, (\nu) \) for the \( pp \to Z_H q_+ X \) process and the corresponding K-factors are depicted in Figs.10(a,b,c,d), respectively. We do not distinguish the electric charge of \( W^\pm \) and \( l^\pm \) in the \( p_T \) distributions of the final \( W \)-boson and lepton. In these figures we set the input parameters as \( f = 1 \) TeV, \( \kappa = 1 \) and \( \sqrt{s} = 14 \) TeV. Again the \( p_T \) distribution labeled by ”NLO III” in Fig.10(b) is for the leading jet in a two-jet event. These figures show that the transverse momentum distributions of \( W^\pm \), jet, \( l^\pm \) and missing energy for the \( pp \to Z_H q_+ X \) process are quite similar to those of \( W^- \), jet, \( l^- \) and missing energy for the \( pp \to W_H q_- + X \) process, respectively, while the production rate of the \( pp \to W_H q_- + X \) process is almost twice larger than that of the \( pp \to Z_H q_+ + X \) process.

Figure 10: The LO and QCD NLO corrected \( p_T \) distributions of final particles for the \( pp \to Z_H q_- + X \) process at the LHC by taking \( f = 1 \) TeV, \( \kappa = 1 \) and \( \sqrt{s} = 14 \) TeV. (a) \( W \)-boson, (b) jet, (c) lepton, (d) missing \( p_T \).

We present the LO and QCD NLO corrected rapidity distributions of final particles and the
corresponding K-factors for the $pp \rightarrow W_{Hq^-} + X$ and $pp \rightarrow W_{Hq^-} + X$ processes in Figs 11(a,b,c) and Figs 12(a,b,c), respectively. Because $\frac{d\sigma}{dy}\bigg|_{-y} = \frac{d\sigma}{dy}\bigg|_{y}$ for the processes at the LHC, we plot only the distributions $\frac{d\sigma}{dy}\bigg|_{-y}$ and $\frac{d\sigma}{dy}\bigg|_{y}$ for the $pp \rightarrow W_{Hq^-} + X$ process. The values of the input parameters are taken the same as those used in Figs 9 and Figs 10. For the $Z_{Hq^-}$ production, we do not distinguish the electric charge of $W^\pm$ and $l^\pm$ in the rapidity distributions of the final $W$-boson and lepton. We can see from these figures that the rapidity distributions $\frac{d\sigma}{dy}\bigg|_{-y}$ for the $pp \rightarrow W_{Hq^-} + X$ process are quite similar to the distributions $\frac{d\sigma}{dy}\bigg|_{-y}$ and $\frac{d\sigma}{dy}\bigg|_{y}$ for the $pp \rightarrow Z_{Hq^-} + X$ process, respectively. All these differential cross sections decrease with the increment of $|y|$. That means all the final products, including $W$-boson, jet and lepton, prefer producing transversely.

Figure 11: The LO and QCD NLO corrected rapidity distributions of final particles for the $pp \rightarrow W_{Hq^-} + X$ process at the LHC by taking $f = 1 \text{ TeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. (a) $W^-$-boson, (b) jet, (c) lepton $l^-$. 
Figure 12: The LO and QCD NLO corrected rapidity distributions of final particles for the $pp \rightarrow Z_{Hq}^{-} + X$ process at the LHC by taking $f = 1 \text{ TeV}$, $\kappa = 1$ and $\sqrt{s} = 14 \text{ TeV}$. (a) $W$-boson, (b) jet, (c) lepton.
V. Summary

In this paper, we calculate the $W_H(Z_H)q_-$ ($q_- = u_-, d_-, c_-, s_-, \bar{u}_-, \bar{d}_-, \bar{c}_-, \bar{s}_-$) associated production at the CERN LHC up to the QCD NLO. We investigate theoretically the dependence of the cross section on the factorization/renormalization scale, and present the transverse momentum and rapidity distributions of the final decay products.

The real light-quark emission partonic processes $gg \rightarrow W_H(Z_H)q'_- + \bar{q}$ and $q''\bar{q}'' \rightarrow W_H(Z_H)q'_- + \bar{q}$ are at the same order of the on-shell T-odd quark pair production with the subsequent decay $q_- \rightarrow W_H(Z_H) + q$. Including these partonic processes will give a large contribution to the NLO QCD corrected rate for the associated $W_H(Z_H)q_-$ production. In order to show how to provide a reliable production rate of the $pp \rightarrow W_H(Z_H)q_- + X$ process at the LHC, we adopt three schemes in considering the QCD NLO corrections for comparison. Our calculations demonstrate that by using the scheme (I) the perturbative convergence could be destroyed due to the double counting showing up, while we can keep the convergence of the perturbative QCD description and obtain moderate QCD NLO corrections to the production rate with a strongly reduced scale uncertainty by adopting the (II) and (III) subtraction schemes. The smallness of the discrepancy between the numerical results using the (II) and (III) subtraction schemes indicates that the contributions of the partonic processes $gg \rightarrow W_H(Z_H)q'_- + \bar{q}$ and $q''\bar{q}'' \rightarrow W_H(Z_H)q'_- + \bar{q}$ are dominated by their $q_-$ resonance effects. The QCD NLO corrections by adopting these two subtraction schemes enhance the LO cross section with a K-factor in the range of $1.00 \sim 1.43$. We conclude that for associated production processes like the $pp \rightarrow W_Hq_- + X$ and $pp \rightarrow Z_Hq_- + X$ processes investigated in this paper, it is crucial to implement a consistent and reliable on-shell subtraction scheme separating associated production from QCD mediated pair production properly. The scheme (II) subtracts some genuine QCD NLO contributions, since all the $gg$ and $q\bar{q}$ initiated contributions are excluded from the QCD NLO corrections. Therefore, the PROSPINO scheme is more consistent that the subtraction scheme (II).

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