AdS$_5$ black strings in the stu model of FI-gauged $N = 2$ supergravity

Matteo Azzola, Dietmar Klemm, and Marco Rabbiosi

Dipartimento di Fisica, Università di Milano, and
INFN, Sezione di Milano,
Via Celoria 16, I-20133 Milano, Italy.
E-mail: matteo.azzola@mi.infn.it, dietmar.klemm@mi.infn.it, marco.rabbiosi@unimi.it

Abstract: We analytically construct asymptotically AdS$_5$ black string solutions starting from the four-dimensional domain wall black hole of [1]. It is shown that its uplift gives a black string in $d = 5$ minimal gauged supergravity, with momentum along the string. Applying instead the residual symmetries of $N = 2$, $d = 4$ Fayet-Iliopoulos-gauged supergravity discovered in [2] to the domain wall seed leads, after uplifting, to a dyonic black string that interpolates between AdS$_5$ and AdS$_3 \times \mathbb{H}^2$ at the horizon. A Kaluza-Klein reduction of the latter along an angular Killing direction $\phi$ followed by a duality transformation yields, after going back to five dimensions, a black string with both momentum along the string and rotation along $\phi$. This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed. These solutions all have constant scalar fields. As was shown in [3], the construction of supersymmetric static magnetic black strings in the FI-gauged stu model amounts to solving the SO(2, 1) spinning top equations, which descend from an inhomogeneous version of the Nahm equations. We are able to solve these in a particular case, which leads to a generalization of the Maldacena-Nuñez solution.

Keywords: Black Holes, Supergravity Models, Black Holes in String Theory, String Duality.
1 Introduction

Exact solutions to Einstein’s field equations and their supergravity generalizations have been playing, and continue to play, a crucial role in many important developments in general relativity, black hole physics, integrable systems, string theory and quantum gravity. Being highly nonlinear, coupled partial differential equations, these are notoriously difficult to solve, sometimes even in presence of a high degree of symmetry, for instance in supergravity where one has typically many other fields in addition to the metric. For this reason, solution generating techniques have become a very powerful tool to generate new solutions from a given seed. The basic idea is to reduce the action (if one has sufficiently many commuting Killing vector fields) to three dimensions, where all vector fields can be dualized to scalars, so that one ends up with a nonlinear sigma model coupled to gravity. The target space symmetries can then be used to obtain new solutions to the field equations by starting from a known one\(^1\). In the case of

\(^1\)Two of the many notable examples are the most general rotating black hole solution of five-dimensional \(N = 4\) superstring vacua constructed by Cvetič and Youm [4], or the black Saturn found in [5].
four-dimensional Einstein-Maxwell gravity, for instance, one gets a sigma model whose scalars parametrize the Bergmann space $SU(2,1)/S(U(1) \times U(1))$ [6, 7].

In gauged supergravity, supersymmetry requires a potential for the moduli (except for flat gaugings), that generically breaks the target space symmetries. For the simple example quoted above, the addition of a cosmological constant breaks three of the eight $SU(2,1)$ symmetries, corresponding to the generalized Ehlers and the two Harrison transformations. This leaves a semidirect product of a one-dimensional Heisenberg group and a translation group $\mathbb{R}^2$ as residual symmetry [8], that cannot be used to generate new solutions.

Recently however, elaborating on earlier work [9], the authors of [2] developed a solution generating technique for $N = 2, d = 4$ Fayet-Iliopoulos (FI)-gauged supergravity as well, which essentially involves the stabilization of the symplectic vector of gauge couplings (FI parameters) under the action of the U-duality symmetry of the ungauged theory. One of the main goals of the present paper is to provide an explicit application of the method introduced in [2]. Namely, we start from the supersymmetric domain wall black hole in the stu model of $N = 2, d = 4$ FI-gauged supergravity, with prepotential $F = -X^1X^2X^3/X^0$, constructed in [1]. We then act on it with one of the symmetry transformations of [2] and lift the resulting configuration to five dimensions. This leads to a dyonic black string in minimal $N = 2, d = 5$ gauged supergravity, with momentum along the string\(^2\). In the next step, one performs a Kaluza-Klein reduction along an angular Killing direction $\phi$ followed by another duality transformation. After going back to $d = 5$ one gets a black string with both momentum and rotation. This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed.

Black strings that interpolate between AdS\(_5\) at infinity and AdS\(_3 \times \Sigma\) near the horizon (where $\Sigma$ denotes a two-dimensional space of constant curvature), are interesting in their own right, since they provide a holographic realization of so-called RG flows across dimensions, from a four-dimensional CFT to a CFT\(_2\) in the IR. A first example for the gravity dual of such a scenario was given in [11], and subsequently Maldacena and Nuñez [12] found a string theory realization in terms of D3-branes wrapping holomorphic Riemann surfaces. Since then, many papers on this subject appeared, cf. [13–21] for an (incomplete) list of references.

The remainder of this paper is organized as follows: In the next section, we briefly introduce the stu model of $N = 2, d = 5$ FI-gauged supergravity and the $r$-map which relates it to the corresponding four-dimensional model. We also determine the residual

\(^2\)Notice that the idea of relating black strings in five-dimensional gauged supergravity to black holes in 4d gauged supergravity goes back to [10].
duality symmetries of the latter along the recipe of [2], and review the domain wall black hole seed solution constructed in [1]. In section 3, this configuration is lifted to \( d = 5 \), and it is shown that the result is a black string with momentum along the string. Subsequently, in 4, we apply a duality transformation to the seed of [1] to get, after uplifting, a dyonic black string. This is then Kaluza-Klein reduced along an angular Killing direction \( \phi \) in sec. 5, duality-transformed and lifted back to generate a black string with both momentum and rotation. Finally, 6 contains the construction of supersymmetric static magnetic black strings generalizing the Maldacena-Nuñez solution, by solving the SO(2,1) spinning top equations. Moreover, we make some comments on a possible inclusion of hypermultiplets. We conclude in section 7 with a discussion of our results. A summary of the supersymmetry variations of the five-dimensional theory is relegated to an appendix.

2 The stu model of FI-gauged \( N = 2 \) supergravity

The bosonic Lagrangian of \( N = 2, d = 5 \) FI-gauged supergravity coupled to \( n_v \) vector multiplets is given by [22]\(^3\)

\[
e^{-1} \mathcal{L}^{(5)} = \frac{R}{2} - \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - \frac{1}{4} G_{IJ} F^I_{\mu\nu} F^J_{\rho\sigma} + \frac{e^{-1}}{48} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F^I_{\mu\nu} F^J_{\rho\sigma} A^K_\lambda - g^2 V_5, \tag{2.1}
\]

where the scalar potential reads

\[
V_5 = V_I V_J \left( \frac{9}{2} G^{ij} \partial_i h^I \partial_j h^J - 6 h^I h^J \right). \tag{2.2}
\]

Here, \( V_I \) are FI constants, \( \partial_i \) denotes a partial derivative with respect to the real scalar field \( \phi^i \), and \( h^I = h^I(\phi^i) \) satisfy the condition

\[
\mathcal{V} \equiv \frac{1}{6} C_{IJK} h^I h^J h^K = 1. \tag{2.3}
\]

The stu model is defined by the symmetric tensor with only nontrivial component \( C_{123} = 1 \), up to permutations. In this case, the functions \( h^I = h^I(\phi^1, \phi^2) \) are given by

\[
h^1 = e^{-\phi^1 / \sqrt{6}} - e^{\phi^2 / \sqrt{2}}, \quad h^2 = e^{2\phi^1 / \sqrt{6}}, \quad h^3 = e^{-\phi^1 / \sqrt{6} + \phi^2 / \sqrt{2}}, \tag{2.4}
\]

and therefore the constraint (2.3) is satisfied.

Below we will need the relationship between (2.1) and four-dimensional \( N = 2 \) FI-gauged supergravity, with bosonic Lagrangian

\[
e^{-1} \mathcal{L}^{(4)} = \frac{R}{2} - g_{IJ} \partial_\mu z^I \partial^\mu \bar{z}^J + \frac{1}{4} I_{\Lambda\Sigma} F_{\Lambda\mu\nu} F^{\Sigma}_{\mu\nu} + \frac{1}{4} R_{\Lambda\Sigma} F_{\Lambda\mu\nu} \star F^{\Sigma}_{\mu\nu} - V_4(z, \bar{z}), \tag{2.5}
\]

\(^3\)The indices \( I, J, \ldots \) range from 1 to \( n_v + 1 \), while \( i, j, \ldots = 1, \ldots, n_v \).
where \( R_{\Lambda\Sigma} = \text{Re} N_{\Lambda\Sigma} \), \( I_{\Lambda\Sigma} = \text{Im} N_{\Lambda\Sigma} \), and \( N \) is the period matrix that determines the couplings of the scalars to the vector fields. \( N \) is determined by a homogeneous function \( F \) of degree two, called the prepotential. The scalar potential can be written in the symplectically covariant form \[ V_4 = g^{IJ} D_I \mathcal{L} D_J \bar{\mathcal{L}} - 3 \mathcal{L} \bar{\mathcal{L}}, \] (2.6)

with \( \mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle \equiv \mathcal{G}'\Omega \mathcal{V} \), where \( \mathcal{V} \) denotes the covariantly holomorphic symplectic section, \( \mathcal{G} = (g^\Lambda, g_\Lambda)^t \) is the symplectic vector of FI parameters and \( D \) the Kähler-covariant derivative. More details can be found e. g. in \[2, 23–25\].

If one reduces the action (2.1) to four dimensions using the \( r \)-map \[26\]
\[ ds_5^2 = e^{\frac{\phi}{\sqrt{3}}} ds_4^2 + e^{-\frac{2}{\sqrt{3}} \phi} (dz + K_\mu dx^\mu)^2, \]
\[ A^I = B^I (dz + K_\mu dx^\mu) + C^I_\mu dx^\mu, \]
\[ x^I = B^I + ie^{\frac{\phi}{\sqrt{3}}} h^I, \]
\[ e^\kappa = \frac{1}{8} e^{\sqrt{3} \phi}, \]
\[ g_{IJ} = \frac{1}{2} e^{\frac{2}{\sqrt{3}}} G_{IJ}, \]
\[ F^\Lambda_{\mu\nu} = \frac{1}{\sqrt{2}} (K_{\mu\nu}, C^I_{\mu\nu}), \] (2.7)

one ends up with the model with cubic prepotential
\[ F = -\frac{X^1 X^2 X^3}{X^0}, \] (2.9)

and FI parameters
\[ \mathcal{G} = (0, 0, 0, 0, 0, g_1, g_2, g_3)^t. \] (2.10)

The theory (2.5) enjoys a residual symmetry that is determined by evaluating the stabilizer of the U-duality group acting in the symplectic representation on the vector \( \mathcal{G} \) of the couplings of the theory [2]. In the present case the embedding of \( \text{SL}(2, \mathbb{R})^3 \) in \( \text{Sp}(8, \mathbb{R}) \) can be found in [2] and the vector \( \mathcal{G} \) is given by (2.10). With a slight loss of generality we impose \( g_1 = g_2 = g_3 \equiv g \). Then the stabilizer algebra reads

\[ T(a_1, a_2) = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-a_1 - a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -a_1 - a_2 & a_1 + a_2 & 0 & 0 \\
0 & 0 & a_1 + a_2 & -a_2 & 0 & 0 & 0 & 0 \\
0 & a_1 + a_2 & 0 & -a_1 & 0 & 0 & 0 & 0 \\
0 & -a_2 & -a_1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \] (2.11)

\[ ^4 \text{We apologize for using the same greek indices } \mu, \nu, \ldots \text{ both in five and four dimensions, but the meaning should be clear from the context.} \]
a two-dimensional abelian nilpotent subalgebra of order three of \( \text{sp}(8, \mathbb{R}) \).

The solution generating technique of \([2]\) consists in the transformation of the seed configuration \((\mathcal{V}, \mathcal{G}, \mathcal{Q}) \rightarrow (S\mathcal{V}, \mathcal{G}, S\mathcal{Q})\), where \(S\) is an element of the stabilizer group \(\mathcal{S}_G\). Here, \(\mathcal{Q} = (p^A, q_A)^t\) is the symplectic vector of magnetic and electric charges that enters the field strengths as follows: For a static solution of the type that we shall consider below,

\[
ds^2 = -e^{2U} dt^2 + e^{-2U} \left( dY^2 + e^{2\psi}(d\theta^2 + \sinh^2 \theta d\phi^2) \right),
\]

the Maxwell equations are solved by

\[
F_{\Lambda Y} = \frac{1}{2} e^{2(U-\psi)} I^\Lambda_\Sigma (R_{\Sigma \Gamma} p^\Gamma - q_\Sigma), \quad F_{\theta \phi}^\Lambda = \frac{p^\Lambda}{2} \sinh \theta,
\]

and the corresponding dual field strengths

\[
F_{\Lambda \mu \nu} = R_{\Lambda \Sigma} F_{\mu \nu}^\Sigma - \frac{1}{2} I_{\Lambda \Sigma} \epsilon_{\mu \nu \rho \sigma} F_{\Sigma \rho \sigma}
\]

are

\[
F_{\Lambda Y} = \frac{1}{2} e^{2(U-\psi)} \left( I^\Lambda_\Sigma p^\Sigma + R_{\Lambda \Gamma} R^{\Gamma \Omega} R_{\Omega \Sigma} p^\Sigma - R_{\Lambda \Gamma} R^{\Gamma \Omega} q_\Omega \right), \quad F_{\Lambda \theta \phi} = \frac{q_\Lambda}{2} \sinh \theta.
\]

The charge vector \(\mathcal{Q}\) is therefore completely determined by the \((\theta \phi)\)-components of \(F^\Lambda\) and \(F_\Lambda\).

In \([1]\), various BPS solutions to the theory \((2.5)\) were constructed, by using the recipe of \([27]\), where all timelike supersymmetric backgrounds of \(N = 2, d = 4\) FI-gauged supergravity are classified\(^5\). In particular, for the prepotential \(F = -X^1 X^2 X^3 / X^0\), the solution reads\(^6\)

\[
ds^2 = -4b^2 dt^2 + \frac{y dy^2}{b^2 \left( cy + 2gp \right)} + \frac{y^3}{b^2} (d\theta^2 + \sinh^2 \theta d\phi^2),
\]

where

\[
b^4 = \frac{8g_1 g_2 g_3 y^2}{H^0 (cy + 2gp)^{\frac{3}{2}}}, \quad H^0 = \frac{2g_0}{3g^2 p^2 y^2} (cy + 2gp)^{\frac{1}{2}} (cy - gp) + h^0,
\]

with field strengths and scalars

\[
F^0 = 4dt \wedge d(H^0)^{-1}, \quad F^I = \frac{p^I}{2} \sinh \theta d\theta \wedge d\phi,
\]

\[
z^I = i r^I = i \frac{\sqrt{g_1 g_2 g_3}}{\sqrt{2} g_1} \frac{\sqrt{H^0 y^3}}{(cy + 2gp)^{\frac{1}{2}}},
\]

\(^5\)For a classification of the null case cf. \([28]\).

\(^6\)To translate between \((2.5)\) and the conventions of \([1]\) take \(g_{\text{CK}} \rightarrow g/2\) and \(G_{\text{CK}} \rightarrow \frac{1}{16\pi}\).
and the magnetic charges are constrained by $g_I p^I \equiv gp = 2/3$ (no summation over $I$)\footnote{This follows from eq. (2.23) of [1] with $\kappa = -1$, by taking into account that $p^I_{\text{here}} = 8\pi p^I_{\text{CK}}$.} for $I = 1, 2, 3$. This field configuration preserves two real supercharges, while the near-horizon limit is a half-BPS attractor point $\text{AdS}_2 \times H^2$. The range of the parameters is $q_0 < 0$, $p^I > 0$, $c > 0$, $h^0 > 3|q_0|^{3/2}/2$ and $g_I > 0$. (2.15) has a horizon in $y = 0$ and asymptotes to a curved domain wall (whose worldvolume is an open Einstein static universe) for $y \to \infty$.

The $(\theta, \phi)$-components of the dual field strengths are given by

$$F_{\theta\phi} = \frac{q_0}{2}, \quad F_{\theta\phi} = 0.$$  \hspace{1cm} (2.18)

From this, together with (2.17), one can easily read off the charge vector $Q$.

## 3 Black string with momentum along $\partial_z$

The $r$-map (2.7), (2.8) can be used to uplift the field configuration (2.15), (2.17) to five dimensions. If we define $y =: r^2$, the resulting metric in $d = 5$ reads

$$\text{ds}^2 = 2 \left( \frac{1}{H^0 b} \right)^\frac{3}{4} \left( \frac{1}{b^2} \frac{4 r^4 dr^2}{cr^2 + 2gp} + \frac{r^6}{b^2} (d\theta^2 + \sinh^2 \theta d\phi^2) \right) + \frac{1}{4} (H^0 b)^\frac{1}{4} \left( dz^2 - \frac{8\sqrt{2}}{H^0} dt dz \right),$$  \hspace{1cm} (3.1)

with

$$b^4 = \frac{8 g_1 g_2 g_3 r^9}{H^0 (cr^2 + 2gp)^3}, \quad H^0 = \frac{2q_0}{3r^2 p r^3} (cr^2 + 2gp)^{1/2} (cr^2 - gp) + h^0.$$  \hspace{1cm} (3.2)

The fluxes and the scalars are given by

$$F^I = \frac{p^I}{\sqrt{2}} \sinh \theta d\theta \wedge d\phi, \quad h^I = \frac{(g_1 g_2 g_3)^{1/3}}{g_I}.$$  \hspace{1cm} (3.3)

The configuration (3.1), (3.3) satisfies the BPS equations (A.1) with Killing spinor

$$\epsilon = Y(r)(1 + i \Gamma^{32})(1 - \Gamma^1)\epsilon_0,$$  \hspace{1cm} (3.4)

where $\epsilon_0$ is a generic constant Dirac spinor. Since $\epsilon_0$ is subject to a double projection, the solution is one quarter BPS. It has a horizon in $r = 0$, where the spacetime becomes $\text{AdS}_3 \times H^2$, while asymptotically it approaches what is commonly termed magnetic [29] $\text{AdS}_5$. 


The solution (3.1), (3.3) was first constructed in [30], by using the results of [31], where all supersymmetric backgrounds of minimal gauged supergravity in five dimensions were classified. It describes a gravitational wave propagating along a magnetic black string, which can be seen by introducing the new coordinates

\[
\rho = \frac{l^2 (H^0)^{1/3} b^{1/3}}{\sqrt{2} r^3} , \quad u = \frac{8^{1/4} z}{\ell^{1/2} c^{3/4}} , \quad v = -\frac{32 t}{\ell^{1/2} c^{3/4} \sqrt{2} r^3} , \quad (3.5)
\]

where \( \ell \) is defined by

\[
\frac{2}{\ell^2} = (g_1 g_2 g_3)^{2/3} , \quad (3.6)
\]

such that the cosmological constant is \( \Lambda = -6 \ell^{-2} \). The metric (3.1) becomes

\[
d s^2 = \frac{\ell^2 d\rho^2}{\rho^2 h^2} + \frac{\ell^4}{\rho^2} (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{\ell^2}{\rho^2} h^{3/2} (H^0 du^2 + du dv) , \quad (3.7)
\]

with

\[
h = 1 - \frac{\rho^2}{3 \ell^2} . \quad (3.8)
\]

In the new radial coordinate \( \rho \), the wave profile reads

\[
H^0 = \frac{3 g_0 c^{3/2}}{2 h^{3/2}} \left( 1 - \frac{\rho^2}{2 \ell^2} \right) + h^0 . \quad (3.9)
\]

In the near-horizon limit \( \rho \to \sqrt{3} \ell \), (3.7) approaches

\[
d s^2 = (du + \hat{\rho} dv)^2 + \ell^2 \left( \frac{d\phi^2}{4 \hat{\rho}^2} - \hat{\rho}^2 dv^2 \right) + \ell^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) , \quad (3.10)
\]

where we eliminated the constant \( h^0 \) by shifting \( v \to v - h^0 u \), and subsequently rescaled

\[
u \to \frac{2u}{|g_0|^{1/2} c^{3/4}} , \quad v \to (|g_0|/2)^{1/2} (3c)^{3/4} \ell v . \quad (3.11)
\]

Moreover we defined

\[
\hat{\rho} = \left( \sqrt{3} - \frac{\rho}{\ell} \right)^{3/2} , \quad l = \frac{2 \ell}{3} . \quad (3.12)
\]

Note that the AdS_3 factor in (3.10) is written as a Hopf-like fibration over AdS_2, in which \( \partial_v \) is a null direction.

The momentum \( L_z \) along the string can be computed as a Komar integral associated to the Killing vector \( \xi = \partial_u \),

\[
L_z = \frac{1}{16 \pi G_5} \oint_{\partial V} dS^{\mu \nu} \nabla_\mu \xi_\nu , \quad (3.13)
\]

8To get (3.7) from (4.21) of [30], set \( k = -1 \) there (which implies \( P = 0 \) and \( H_2 = 1 \). Then the Heun equation (4.25) of [30] boils down to a simple differential equation that is solved by (3.9).
where $\partial V$ is the boundary of a spacelike hypersurface $V$ of constant $v$. $\partial V$ is defined by $\rho = \text{const.} \to 0$. The oriented measure on $\partial V$ reads

$$dS_{\mu\nu} = (\zeta^\mu n^\nu - n^\mu \zeta^\nu)\sqrt{\sigma} d\theta d\phi du,$$

(3.14)

where

$$n = \frac{\rho}{\ell h^{3/4} H^{01/2}} (\partial_u - 2H^0 \partial_v)$$

(3.15)

denotes the unit normal to $V$, while $\zeta = -\rho t^{-1} \partial_\rho$ and $\sigma$ is the determinant of the induced metric on $\partial V$,

$$\sqrt{\sigma} = \frac{\ell^5}{\rho^3} h^{3/4} H^{01/2} \sinh \theta.$$  

(3.16)

This leads to

$$L_z = \frac{L q_0 c^{3/2} (g - 1)}{16G_5}. $$

(3.17)

Here $g = 2, 3, \ldots$ denotes the genus of the Riemann surface to which $H^2$ is compactified, and $L$ is the period of $u$ in (3.10).

The Bekenstein-Hawking entropy of the solution (3.7) is given by

$$S_{\text{BH}} = \frac{A_{\text{hor}}}{4G_5} = \frac{L^2 \pi (g - 1)}{3G_5}. $$

(3.18)

It would be very interesting to generalize (3.7), for instance by allowing for a nontrivial dependence of the wave profile on the ‘angular’ coordinates $\theta, \phi$. According to the governing equations for the wave profile (cf. [4.22] and [4.26] of [30]) this is possible, and a Kaluza-Klein reduction of such a solution to four dimensions would lead to a static black hole with dipole- or higher multipole charges. Work in this direction is in progress.

### 4 Dyonic black string with momentum

As we said, the results of [2] can be used to generate a new configuration from the seed solution (2.15) and the respective fluxes and scalars (2.17). It would be interesting to fix the parameters of the symmetry transformation $T(a_1, a_2)$ in (2.11) to get a vanishing graviphoton and therefore a static metric in five dimensions. However, as the following calculation shows, this results to be impossible. Starting form the fluxes

$$Q = (0, p, p, p, -|q_0|, 0, 0, 0)^t,$$

(4.1)

---

9To compute the horizon area, we took a section of constant $\hat{\rho}$ and $v$ in (3.10).

10Note that $q_0 = -|q_0|$. Moreover, if we choose $g_1 = g_2 = g_3 = g$, the constraint $g_1 p^I = g p$ satisfied by the seed implies $p^1 = p^2 = p^3 = p$. 


the action of $U = e^{T(a_1,a_2)}$, with $T(a_1,a_2)$ given by (2.11), generates

$$Q' = \begin{pmatrix} 0 & p & 0 & -|q_0| + (a_1a_2 + a_1^2 + a_2^2)p \\ p & p & -a_1p & (a_1 + a_2)p \\ 0 & -a_2p & -a_1p & (a_1 + a_2)p \\ p & p & 0 & -a_1p \\ -|q_0| + (a_1a_2 + a_1^2 + a_2^2)p & (a_1 + a_2)p & 0 & -a_1p \\ -a_1p & -a_2p & -a_1p & 0 \\ (a_1 + a_2)p & (a_1 + a_2)p & (a_1 + a_2)p & 0 \end{pmatrix}. \quad (4.2)$$

The scalars acquire a constant axion,

$$z^1' = z^1 + a_1, \quad z^2' = z^2 + a_2, \quad z^3' = z^3 - a_1 - a_2. \quad (4.3)$$

Looking at (4.2) one may think that the graviphoton can be set to zero by choosing properly the parameters $a_1, a_2$. This is however not the case, as can be seen from the field strengths in presence of axions, that read

$$F^\Lambda = \frac{S^\Lambda b^2}{y^{5/2}(cy^2 + 2gp)^{1/2}} dt \wedge dy + \frac{p^\Lambda}{2} \sin \theta d\theta \wedge d\phi, \quad (4.4)$$

with

$$S^\Lambda = \frac{|q_0|}{r_1 r_2 r_3} (1, a_1, a_2, -a_1 - a_2)^t, \quad p^\Lambda = (0, p, p, p)^t. \quad (4.5)$$

The uplifted metric is still (3.1). The field strengths and scalars in five dimensions become respectively

$$F^I = \frac{2\sqrt{2}b^2 S^I}{r^4 (cr^2 + 2gp)^{1/2}} dt \wedge dr + \frac{p^I}{\sqrt{2}} \sin \theta d\theta \wedge d\phi, \quad h^I = 1, \quad (4.6)$$

where now

$$S^I = \frac{|q_0|}{r_1^2 r_2^2 r_3^3} (a_1, a_2, -a_1 - a_2)^t, \quad p^I = (p, p, p)^t, \quad (4.7)$$

so we have generated two additional electric charge parameters. Note that the metric remains untouched by the duality rotation. This solution describes again a flow between magnetic AdS$_5$ and AdS$_3 \times H^2$, and preserves the same amount of supersymmetry as before, as can be easily seen by using the Killing spinor equations following from (A.1).

5 Dyonic black string with both momentum and rotation

We now want to generate a rotating black string by applying the same technique as above. The starting point is the seed metric (3.1), with fluxes and scalars (4.6). After
a Kaluza-Klein reduction to four dimensions along the angular Killing direction $\partial_\phi$, one gets

$$ds^2 = \sinh \theta \left( \frac{\sqrt{2} (cr^2 + 2gp)^{1/2}}{g^3 r^2} dr^2 + \frac{(cr^2 + 2gp)^{3/2}}{2\sqrt{2}g^3} d\theta^2 + 2r^3 \left( \frac{H^0}{4\sqrt{2}} dz - dt \right) dz \right),$$

$$A^\Lambda = \left( 0, \frac{4q_I}{H^0} dt \right), \quad z^I = \frac{p^I}{\sqrt{2}} \cosh \theta + i \frac{(cr^2 + 2gp)^{1/2} \sinh \theta}{\sqrt{2} g_I}, \quad (5.1)$$

where $q_3 = -q_1 - q_2$. Applying the duality transformation (2.11) leads to a configuration with a nonvanishing graviphoton,

$$Q = (0, 0, 0, 0, q_1, q_2, -q_1 - q_2) \mapsto Q' = (0, 0, 0, 0, \omega, q_1, q_2, -q_1 - q_2)^t, \quad (5.2)$$

where $\omega = -a_1 (2q_1 + q_2) - a_2 (q_1 + 2q_2)$, and the scalar fields acquire a real constant part as in (4.3). Lifting the solution back to $d = 5$ gives

$$ds^2 = \frac{2dr^2}{g^2 r^2} + \frac{cr^2 + 2gp}{2g^2} d\Omega^2_{\text{univ}} + \frac{2\sqrt{2} gr^3}{(cr^2 + 2gp)^{1/2}} \left( \frac{H^0}{4\sqrt{2}} dz - dt \right) dz$$

$$+ 4\sqrt{2} \omega \sqrt{2} \frac{cr^2 + 2gp}{g^2 H^0} \sinh^2 \theta \left( d\phi + \frac{2\sqrt{2} \omega}{H^0} dt \right) dt, \quad (5.3)$$

$$A^I = \frac{p^I}{\sqrt{2}} \cosh \theta d\phi + \frac{4}{H^0} \left( q_I + \omega s^I + \frac{\omega p^I}{\sqrt{2}} \cosh \theta \right) dt, \quad h^I = 1, \quad (5.4)$$

with $s^I = (a_1, a_2, -a_1 - a_2)$ and $H^0$ was defined in (3.2).

The near-horizon limit $r \to 0$ of (5.3) leads to the metric

$$ds^2 = \frac{|q_0|}{3p} (dz - \hat{\rho} dt)^2 + \frac{\hat{\rho}^2 d\Omega^2}{4 \hat{\rho}^2} - \frac{|q_0|}{3p} \rho^2 dt^2 + \frac{p}{g} \left[ \ell^2 d\theta^2 + \sinh^2 \theta (d\phi + 2\hat{\rho} \omega dt)^2 \right], \quad (5.5)$$

where

$$\hat{\rho} \equiv \frac{3\sqrt{gp}}{|q_0|} r^3. \quad (5.6)$$

(5.5) represents a deformation of (3.10), i.e., of $\text{AdS}_3 \times \text{H}^2$.

For $r \to \infty$ (5.3) approaches again magnetic $\text{AdS}_5$, as can be easily shown by using some simple coordinate transformations.

It would be interesting to check whether the solution (5.3), (5.4) is still BPS, or, more generally, if the solution-generating technique based on [2] preserves supersymmetry.
6 Solutions with running scalars

In the appendix of [3] the problem of finding one quarter magnetic BPS strings with running scalars is reduced to solve a system of three first-order differential equations. The metric is given by

\[ ds^2 = e^{2V}(-dt^2 + dz^2) + e^{2W}(du^2 + d\Omega_\kappa^2), \]  

(6.1)

with

\[ d\Omega_\kappa^2 = d\theta^2 + \frac{\sin^2 \sqrt{\kappa} \theta}{\kappa} d\phi^2, \quad \kappa = 0, \pm 1, \]

(6.2)

\[ e^{2V} = (x^1 x^2 x^3)^{-\frac{1}{3}} e^{-g \int (x^1 + x^2 + x^3) du}, \quad e^{2W} = (x^1 x^2 x^3)^{\frac{2}{3}}, \]

(6.3)

and the \( x^I(u) \) define the scalar fields according to

\[ h^I = x^I / (x^1 x^2 x^3)^{1/3}. \]  

They are determined by the system

\[ y^1' = y^2 y^3 + Q^1, \]

\[ y^2' = y^1 y^3 + Q^2, \]

\[ y^3' = y^1 y^2 + Q^3, \]

(6.4)

where a prime denotes a derivative w.r.t. the radial coordinate \( u \) and

\[ y^1 = x^1 + x^2 - x^3, \]

\[ y^2 = x^1 - x^2 - x^3, \]

\[ y^3 = x^1 - x^2 + x^3. \]

(6.5)

The fluxes are purely magnetic, i.e., \( F_{\theta \phi}^I = \sqrt{\kappa} q^I \sin \sqrt{\kappa} \theta \), and the parameters \( Q^I \) are defined by

\[ Q^1 = -\kappa (q^1 + q^2 - q^3), \]

\[ Q^2 = -\kappa (q^1 - q^2 - q^3), \]

\[ Q^3 = -\kappa (q^1 - q^2 + q^3). \]

(6.6)

(6.4) can be derived from an inhomogeneous version of the SU(2) Nahm equations [32–34]

\[ \frac{dT^I}{du} = \epsilon^{IJK} [T^J, T^K] + S^I \]

(6.7)

(where the \( T^I \) and \( S^I \) take values in the Lie algebra su(2)) by setting \( T^I = y^I \sigma^I \), \( S^I = Q^I \sigma^I \) (no summation over \( I \)), and the \( \sigma^I \) denote Pauli matrices. Notice that for

---

In this section we choose \( V_I = \frac{1}{3} \) in (2.2) without loss of generality.

In the following we set \( g = 1 \).
$Q^I = 0$ this leads to the Ercole-Sinha solution \cite{35}, which is given in terms of elliptic functions. (6.4) can be written as
\begin{equation}
y'' = C^I_{JK}y''y^K + Q^I, \quad (6.8)
\end{equation}
and thus its symmetries are determined by the transformations that leave the tensor $C^I_{JK}$ invariant, $T^{-1}CTT = C$. Unfortunately this implies $T = 1$. The discrete symmetry group of (6.8), which is easily shown to be $(\mathbb{Z}_2)^6 \times \mathbb{Z}_3$, is not very useful for generating new solutions from known ones.

The system (6.8) is equivalent to the SO(2,1) spinning top equations, which are given by
\begin{align}
I_1\omega'_1 &= (I_2 - I_3)\omega_2\omega_3 + M_1, \\
I_2\omega'_2 &= -(I_3 - I_1)\omega_3\omega_1 + M_2, \\
I_3\omega'_3 &= (I_1 - I_2)\omega_1\omega_2 + M_3, \quad (6.9)
\end{align}
where $I_K$ are the principal moments of inertia and $M_K$ represents an external torque. If we set
\begin{equation}
\omega_1 = \lambda_1 y^1, \quad \omega_2 = \lambda_2 y^2, \quad \omega_3 = \lambda_3 y^3, \quad (6.10)
\end{equation}
where
\begin{align}
\lambda_1^2 &= \frac{-I_2I_3}{(I_3 - I_1)(I_1 - I_2)}, \quad \lambda_2^2 = \frac{I_3I_1}{(I_1 - I_2)(I_2 - I_3)}, \quad \lambda_3^2 = \frac{-I_1I_2}{(I_2 - I_3)(I_3 - I_1)},
\end{align}
(6.9) reduces to (6.8). Here we assumed (without loss of generality) $I_1 > I_2 > I_3 > 0$. Then all $\lambda_K^2$ are positive. Note that, as compared to Euler’s equations, there is a minus sign on the rhs of the second of (6.9), which is the reason for the terminology SO(2,1). In fact, in the untorqued case $M_K = 0$, the eqns. (6.9) can be derived from the Hamiltonian
\begin{equation}
H = \frac{\ell_1^2}{2I_1} - \frac{\ell_2^2}{2I_2} + \frac{\ell_3^2}{2I_3}, \quad (6.11)
\end{equation}
by using the Poisson brackets
\begin{equation}
[\ell_I, \ell_J] = \epsilon_{IJK}\ell_K, \quad (6.12)
\end{equation}
as well as $\ell'_I = [\ell_I, H]$. In (6.12), $\epsilon_{123} = 1$ and the indices of the Levi-Civita tensor are raised with the Minkowski metric $\eta = \text{diag}(1,-1,1)$. W.r.t. the Euler top, (6.11) has one kinetic term that comes with the ‘wrong’ sign.

In \cite{3} a particular solution to (6.4) is found. We will now show a different way to integrate these equations. Imposing $Q^1 = Q^2 = 0$, i.e., $q^2 = 0$ and $q^1 = q^3$, and defining $y_\pm = y^1 \pm y^2$ one finds that
\begin{equation}
y_\pm = k_\pm e^{\int y^3 du}, \quad y^3 = \frac{1}{4} \left(k_+^2 e^{2\int y^3 du} - k_-^2 e^{-2\int y^3 du}\right) + Q^3, \quad (6.13)
\end{equation}
where \( Q^3 = -2\kappa q^1 \) and \( k_\pm \) are integration constants. The Dirac-type charge quantization condition \( 3gV_I q^I = 1 \) of [3] implies \( 2gq^1 = 1 \), and thus \( Q^3 = -\kappa \). Introducing a new radial coordinate \( y = -\int y^3 du \), the last equation becomes

\[
y'' = \frac{1}{4} \left( k_-^2 e^{2y} - k_+^2 e^{-2y} \right) + \kappa, \tag{6.14}
\]

which can be integrated once to give\(^{13}\)

\[
y' = \frac{dy}{du} = -y^3 = -\sqrt{\frac{1}{4} (k_-^2 e^{2y} + k_+^2 e^{-2y}) + 2\kappa y}. \tag{6.15}
\]

This leads to the metric

\[
ds^2 = (x^1 x^2 x^3)^{-\frac{1}{2}} e^{\frac{x_1 x_2 + x_3}{4}} dy (-dt^2 + dz^2) + \left( x^1 x^2 x^3 \right)^{\frac{3}{2}} \left( \frac{dy^2}{y^3)^2 + d\Omega^2} \right), \tag{6.16}
\]

where

\[
x^1 = \frac{1}{4} \left( k_+ e^{-y} + k_- e^y + \sqrt{k_+^2 e^{-2y} + k_-^2 e^{2y} + 8\kappa y} \right),
\]

\[
x^2 = k_- e^y, \tag{6.17}
\]

\[
x^3 = \frac{1}{4} \left( -k_+ e^{-y} + k_- e^y + \sqrt{k_+^2 e^{-2y} + k_-^2 e^{2y} + 8\kappa y} \right).
\]

In what follows we assume \( k_- > 0 \). Then asymptotically for \( y \to \infty \) the geometry becomes (magnetic) AdS\(_5\),

\[
ds^2 = \frac{2e^{2y}}{k_-} (-dt^2 + dz^2) + dy^2 + \frac{k_-^2 e^{2y}}{4} d\Omega^2. \tag{6.18}
\]

For generic integration constants \( k_\pm \) the metric becomes singular at a certain point and the solution does not have a horizon. However, in the case \( \kappa = -1 \), if \( k_\pm \) are related by\(^{14}\)

\[
k_- = e^{-a} \sqrt{4a + 2}, \quad k_+ = -e^a \sqrt{4a - 2}, \tag{6.19}
\]

where \( a \) denotes an arbitrary parameter, there is a horizon for \( y = a \), where the solution approaches AdS\(_3 \times H^2\), as we will show below. In the cases \( \kappa = 0, 1 \) a similar reasoning cannot be done, and the metric has no event horizon.

\(^{13}\)The plus sign corresponds to an unphysical solution with negative scalars \( h^I \). A possible additive integration constant can be eliminated by shifting \( y \).

\(^{14}\)The other possibility \( k_\pm = e^{\pm a} \sqrt{4a + 2} \) is related to (6.19) by the \( \mathbb{Z}_2 \) symmetry \( x^1 \leftrightarrow x^3 \) and corresponds to negative \( h^I \).
(6.19) are real for $a \geq 1/2$. We note that, if (6.19) holds, one has

$$y^3(y) = \sqrt{2a \cosh(2(a - y)) - \sinh(2(a - y)) - 2y},$$

and thus

$$\frac{d(y^3)^2}{dy} = 2 \cosh(2(a - y)) - 4a \sinh(2(a - y)) - 2 \geq 2(e^{2(y-a)} - 1) \geq 0.$$  \hspace{1cm} (6.20)

$y^3$ is always well-defined for $y \geq a$ and becomes zero at the horizon, $y^3(a) = 0$. The scalar fields (6.17) are positive in the whole range $a \leq y < \infty$.

The value $a = 1/2$ is special since it corresponds to the limit in which $x^1 = x^3$, i.e., $\phi^2 = 0$. This truncation leads to the solution of [12] with two nonzero and equal fluxes. Indeed, one easily verifies that $\phi^1$ and the function $W$ appearing in the metric (6.1) satisfy the equation

$$e^{2W} + \frac{\phi^1}{\sqrt{6}} = e^{2W} - \frac{2\phi^1}{\sqrt{6}} + \frac{1}{4},$$

which is precisely equ. (17) of [12]. The black strings defined by (6.16), (6.17) represent thus generalizations of the Maldacena-Nuñez solution. Note that also the latter was not known analytically up to now.

The near-horizon limit $y \to a$ can be obtained from the expansion

$$\frac{x^1 + x^2 + x^3}{y^3} = \sqrt{\frac{1 + 2a}{2a}} \frac{1}{y - a} + O((y - a)^0),$$

$$\frac{(y^3)^3}{x^1 x^2 x^3} = 32a \sqrt{\frac{2a}{1 + 2a}} (y - a)^3 + O((y - a)^4),$$

$$x^1 x^2 x^3 = \sqrt{\frac{1 + 2a}{32}} + O((y - a)).$$

Introducing the new radial coordinate

$$\hat{u}^2 = (y - a)^{-\sqrt{\frac{1 + 2a}{2a}}},$$

the metric (6.16) becomes for $\hat{u} \to 0$

$$ds^2 = \frac{1}{\hat{u}^2} \left[ -dt^2 + dz^2 + \frac{d\hat{u}^2}{(2 + 4a)^{2/3}} \right] + \left(1 + \frac{2a}{32}\right)^{1/3} d\Omega_{-1}^2,$$

which is AdS$_3 \times H^2$. 

\hspace{1cm} (6.24)
The central charge of the two-dimensional SCFT dual to the near-horizon configuration is given by [12, 26]
\[ c = \frac{3R_{\text{AdS}_3}}{2G_5} = \frac{6\pi (g - 1)R_{\text{AdS}_3}R^2_{\Sigma_g}}{G_5}, \tag{6.26} \]
where \( g = 2, 3, \ldots \) is the genus of the Riemann surface \( \Sigma_g \) to which \( H^2 \) is compactified. The values of the curvature radii are
\[ R_{\text{AdS}_3} = \frac{1}{(2 + 4a)^{1/3}}, \quad R^2_{\Sigma_g} = \frac{(1 + 2a)^{1/3}}{2^{5/3}}, \tag{6.27} \]
with
\[ 2a = \sqrt{1 + \left( \frac{k_+ k_-}{2} \right)^2}. \tag{6.28} \]
For the truncation \( \phi^2 = 0 \), which means \( k_+ = 0 \), one has \( R_{\text{AdS}_3} = 2^{-2/3} \), i.e. the value found in [12].

Using (6.27), the central charge can be written in the form
\[ c = \frac{6\pi (g - 1)}{4G_5} = 3N^2(g - 1), \tag{6.29} \]
where we used the AdS/CFT dictionary \( N^2 = \pi/(2G_5g^3) \) (with \( g = 1 \)). Near the conformal boundary the scalar fields behave like
\[ \sqrt{2} \phi^1 \sim 2Q^3y e^{-2y}, \quad \sqrt{2} \phi^2 \sim \frac{k_+}{k_-} e^{-2y}, \tag{6.30} \]
and thus are read in the dual SCFT as an insertion and an expectation value of an operator of scaling dimension \( \Delta = 2 \). The relevant deformation of the dual superpotential relative to \( \phi^1 \) is described in [12], while \( \phi^2 \) is a marginal deformation of two-dimensional \( N = (4, 4) \) SYM theory. Thus, the solution does not describe the gravity dual of 2d \( N = (2, 2)^* \) SYM [36]. The constant \( a \) represents the physical scale of the energy in the renormalization group flow at which the IR fixed point appears, but which being a CFT is independent of the energy scale.

### 6.1 Inclusion of hypermultiplets

It is worthwhile to note that with running hyperscalars the BPS equations, (5.17) of [26], can be simplified to a system for which the number of equations equals the number of the scalar fields in the model. The idea is basically the same as that of [3]: Introducing a new radial coordinate \( R \) by \( dR = e^{-\psi} dr \) and the rescaled scalars
\[ y^I = e^{\psi - 2T} h^I, \] where \( \psi(r) \) and \( T(r) \) are metric functions defined in equ. (3.1) of [26] and \( r \) denotes the radial coordinate used there, the system (5.17) of [26] boils down to

\[ \psi = \int 9 \kappa g^2 \mathcal{L}_y \, dR, \quad e^{3\psi - 6T} = \frac{1}{6} C_{IJK} y^I y^J y^K, \]

\[ y'^I - 9 g^2 \kappa (\mathcal{L}_y y^I - Q^x P^e_J G_{y^I}^J) - p^I = 0, \]

\[ q^\mu = -\frac{9}{2} \kappa g^2 h_{\tau \nu} \partial_\tau \mathcal{L}_y, \]

where

\[ \mathcal{L}_y = Q^x P^e_I y^I, \quad G_{y}^{IJ} = -C^{IJK} C_{KLM} y^L y^M + 2 y^I y^J, \quad C^{IJK} \equiv \delta^{IL} \delta^{JM} \delta^{KN} C_{LMN}. \]

Even if the complete integration of these equations in a particular model remains a hard task, this partial integration can be considered as a first step towards the solution. A numerical and asymptotic analysis of this type of models can be found in [37]\(^{15}\), where a particular truncation of \( N = 8, d = 5 \) gauged supergravity is studied.

### 7 Conclusions

In this paper, we used the residual symmetries of \( N = 2, d = 4 \) Fayet-Iliopoulos-gauged supergravity discovered in [2] to add an electric charge density and rotation to five-dimensional black strings that asymptote to \( \text{AdS}_5 \). This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed, and opens the possibility to construct in a similar way many solutions hitherto unknown, which are potentially interesting particularly in an \( \text{AdS}/\text{CFT} \) context.

The rotating string (5.3) interpolates between magnetic \( \text{AdS}_5 \) at infinity and a deformation of \( \text{AdS}_3 \times \mathbb{H}^2 \) near the horizon. This deformation implies that the \( \text{CFT}_2 \), to which the dual four-dimensional \( \text{CFT} \) flows in the IR, has less symmetry. We did not check explicitly how many supercharges are preserved by the near-horizon metric (5.5), but we would expect that the rotation breaks at least some of the original supersymmetries.

We also constructed static magnetic BPS black strings with running scalars in the FI-gauged \( \text{stu} \) model. It was shown that this amounts to solving the \( \text{SO}(2,1) \) spinning top equations, which descend from an inhomogeneous version of the Nahm equations. We were able to solve these in a particular case, which leads to a generalization of the Maldacena-Núñez solution. Moreover, we computed the central charge of the \( \text{CFT}_2 \) dual to the near-horizon configuration. From the behaviour of the two bulk scalar values.

\(^{15}\)Cf. also [38].
fields $\phi^1$ and $\phi^2$ near the conformal boundary we saw that they correspond in the dual SCFT to an insertion and an expectation value of an operator of scaling dimension $\Delta = 2$. The relevant deformation of the dual superpotential relative to $\phi^1$ is described in [12], while $\phi^2$ is a marginal deformation of two-dimensional $N = (4, 4)$ SYM theory. Thus, our solution does not describe the gravity dual of 2d $N = (2, 2)^*$ SYM [36].

We hope to come back in particular to further applications of the duality transformations of [2] in the near future.

Appendix

A Supersymmetry variations

The supersymmetry variations of the gravitino $\psi_\mu$ and the gauginos $\lambda_i$ in $N = 2, d = 5$ FI-gauged supergravity coupled to vector multiplets read [22]

$$\delta \psi_\mu = \left( D_\mu + \frac{i}{8} h_I (\Gamma_\mu^{\nu\rho} - 4 \delta_\mu^{\nu} \Gamma^\rho) F^I_{\nu\rho} + \frac{1}{2} g \Gamma_\mu h^I V_I \right) \epsilon,$$

$$\delta \lambda_i = \left( \frac{3}{8} \Gamma^{\mu\nu} F^I_{\mu\nu} \partial_i h_I - \frac{i}{2} G_{ij} \Gamma_\mu \partial_\mu \phi^j + \frac{3i}{2} g V_I \partial_i h^I \right) \epsilon,$$

where

$$D_\mu \epsilon = \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \Gamma_{ab} - \frac{3i}{2} g V_I A^I_\mu \right) \epsilon.$$  \hfill (A.2)

Acknowledgements

This work was supported partly by INFN. We would like to thank A. Amariti and N. Petri for useful discussions and C. Toldo and A. Tomasiello for valuable comments.

References

[1] S. L. Cacciatori and D. Klemm, “Supersymmetric AdS$_4$ black holes and attractors,” JHEP 1001 (2010) 085 [arXiv:0911.4926 [hep-th]].

[2] S. L. Cacciatori, D. Klemm and M. Rabbiosi, “Duality invariance in Fayet-Iliopoulos gauged supergravity,” JHEP 1609 (2016) 088 [arXiv:1606.05160 [hep-th]].

[3] S. L. Cacciatori, D. Klemm and W. A. Sabra, “Supersymmetric domain walls and strings in $d = 5$ gauged supergravity coupled to vector multiplets,” JHEP 0303 (2003) 023 [hep-th/0302218].
[4] M. Cvetiˇ c and D. Youm, “General rotating five-dimensional black holes of toroidally compactified heterotic string,” Nucl. Phys. B 476 (1996) 118 [hep-th/9603100].

[5] H. Elvang and P. Figueras, “Black Saturn,” JHEP 0705 (2007) 050 [hep-th/0701035].

[6] P. Breitenlohner, D. Maison and G. W. Gibbons, “Four-dimensional black holes from Kaluza-Klein theories,” Commun. Math. Phys. 120 (1988) 295

[7] P. Breitenlohner and D. Maison, “On nonlinear sigma models arising in (super-)gravity,” Commun. Math. Phys. 209 (2000) 785 [gr-qc/9806002].

[8] D. Klemm, M. Nozawa and M. Rabbiosi, “On the integrability of Einstein–Maxwell–(A)dS gravity in the presence of Killing vectors,” Class. Quant. Grav. 32 (2015) no.20, 205008 [arXiv:1506.09017 [hep-th]].

[9] N. Halmagyi and T. Vanel, “AdS black holes from duality in gauged supergravity,” JHEP 1404 (2014) 130 [arXiv:1312.5430 [hep-th]].

[10] K. Hristov, “Dimensional reduction of BPS attractors in AdS gauged supergravities,” JHEP 1412 (2014) 066 [arXiv:1409.8504 [hep-th]].

[11] D. Klemm and W. A. Sabra, “Supersymmetry of black strings in \( d = 5 \) gauged supergravities,” Phys. Rev. D 62 (2000) 024003 [hep-th/0001131].

[12] J. M. Maldacena and C. Nuñez, “Supergravity description of field theories on curved manifolds and a no go theorem,” Int. J. Mod. Phys. A 16 (2001) 822 [hep-th/0007018].

[13] N. Kim, “AdS\(_3\) solutions of IIB supergravity from D3-branes,” JHEP 0601 (2006) 094 [hep-th/0511029].

[14] A. Kapustin, “Holomorphic reduction of \( N = 2 \) gauge theories, Wilson-´t Hooft operators, and S-duality,” hep-th/0612119.

[15] A. Donos, J. P. Gauntlett and N. Kim, “AdS solutions through transgression,” JHEP 0809 (2008) 021 [arXiv:0807.4375 [hep-th]].

[16] F. Benini and N. Bobev, “Two-dimensional SCFTs from wrapped branes and \( c \)-extremization,” JHEP 1306 (2013) 005 [arXiv:1302.4451 [hep-th]].

[17] P. Karndumri and E. Ó Colgáin, “Supergravity dual of \( c \)-extremization,” Phys. Rev. D 87 (2013) no.10, 101902 [arXiv:1302.6532 [hep-th]].

[18] A. Donos and J. P. Gauntlett, “Flowing from AdS\(_5\) to AdS\(_3\) with \( T^{1,1} \),” JHEP 1408 (2014) 006 [arXiv:1404.7133 [hep-th]].

[19] F. Benini, N. Bobev and P. M. Crichigno, “Two-dimensional SCFTs from D3-branes,” JHEP 1607 (2016) 020 [arXiv:1511.09462 [hep-th]].

[20] A. Amariti and C. Toldo, “Betti multiplets, flows across dimensions and \( c \)-extremization,” JHEP 1707 (2017) 040 [arXiv:1610.08858 [hep-th]].
[21] A. Amariti, L. Cassia and S. Penati, “Surveying 4d SCFTs twisted on Riemann surfaces,” JHEP 1706 (2017) 056 [arXiv:1703.08201 [hep-th]].

[22] M. G"unaydin, G. Sierra and P. K. Townsend, “Gauging the $d = 5$ Maxwell-Einstein supergravity theories: More on Jordan algebras,” Nucl. Phys. B 253 (1985) 573.

[23] G. Dall’Agata and A. Gnechi, “Flow equations and attractors for black holes in $N = 2$ $U(1)$ gauged supergravity,” JHEP 1103 (2011) 037 [arXiv:1012.3756 [hep-th]].

[24] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fré and T. Magri, “$N = 2$ supergravity and $N = 2$ super-Yang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map,” J. Geom. Phys. 23 (1997) 111 [hep-th/9605032].

[25] D. Z. Freedman and A. Van Proeyen, “Supergravity,” Cambridge University Press (2012).

[26] D. Klemm, N. Petri and M. Rabbiosi, “Black string first order flow in $N = 2$, $d = 5$ abelian gauged supergravity,” JHEP 1701 (2017) 106 [arXiv:1610.07367 [hep-th]].

[27] S. L. Cacciatori, D. Klemm, D. S. Mansi and E. Zorzan, “All timelike supersymmetric solutions of $N = 2$, $d = 4$ gauged supergravity coupled to abelian vector multiplets,” JHEP 0805 (2008) 097 [arXiv:0804.0009 [hep-th]].

[28] D. Klemm and E. Zorzan, “All null supersymmetric backgrounds of $N = 2$, $d = 4$ gauged supergravity coupled to abelian vector multiplets,” Class. Quant. Grav. 26 (2009) 145018 [arXiv:0902.4186 [hep-th]].

[29] K. Hristov, C. Toldo and S. Vandoren, “On BPS bounds in $d = 4$, $N = 2$ gauged supergravity,” JHEP 1112 (2011) 014 [arXiv:1110.2688 [hep-th]].

[30] A. Bernamonti, M. M. Caldarelli, D. Klemm, R. Olea, C. Sieg and E. Zorzan, “Black strings in $\text{AdS}_5$,” JHEP 0801 (2008) 061 [arXiv:0708.2402 [hep-th]].

[31] J. P. Gauntlett and J. B. Gutowski, “All supersymmetric solutions of minimal gauged supergravity in five dimensions,” Phys. Rev. D 68 (2003) 105009 Erratum: [Phys. Rev. D 70 (2004) 089901] [hep-th/0304064].

[32] W. Nahm, “The algebraic geometry of multi-monopoles,” Lect. Notes Phys. 180 (1983) 456.

[33] N. J. Hitchin, “On the construction of monopoles,” Commun. Math. Phys. 89 (1983) 145.

[34] S. K. Donaldson, “Instantons and geometric invariant theory,” Commun. Math. Phys. 93 (1984) 453.

[35] N. Ercolani and A. Sinha, “Monopoles and Baker functions,” Commun. Math. Phys. 125 (1989) 385.
[36] J. Nian, “Gravity dual of two-dimensional $N = (2, 2)^*$ supersymmetric Yang-Mills theory and nonlinear Schrödinger equation,” arXiv:1706.09016 [hep-th].

[37] N. Bobev, K. Pilch and O. Vasilakis, “(0, 2) SCFTs from the Leigh-Strassler fixed point,” JHEP 1406 (2014) 094 [arXiv:1403.7131 [hep-th]].

[38] M. Suh, “Magnetically charged supersymmetric flows of gauged $N = 8$ supergravity in five dimensions,” JHEP 1808 (2018) 005 [arXiv:1804.06443 [hep-th]].