The Role of $M_W$ in Precision Studies of the Standard Model

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Abstract

Recent calculations have significantly decreased the scheme and residual scale dependence of basic radiative corrections of the Standard Electroweak Model. This leads to a theoretically accurate prediction of the $W$-boson mass $M_W$, as well as a reduced upper bound for the Higgs boson mass $M_H$. The implications of a precise $M_W$ measurement on the $M_H$ estimate are emphasized.
Two of the main objectives in current theoretical studies of the Standard Model (SM) are the improvement of the estimate of the Higgs boson mass $M_H$ and its upper bound, and the accurate prediction of the $W$ boson mass $M_W$. In this connection, theorists distinguish two types of errors: parametric ones, which in principle can be reduced by improving experimental inputs, and theoretical uncertainties derived from the truncation of the perturbative series. The latter are usually estimated by comparing different schemes of calculation that contain all the available theoretical information at a given order of accuracy. The difference between different approaches is referred to as the scheme-dependence.

Because of its accuracy and its sensitivity to $M_H$, the effective electroweak mixing parameter $\sin^2 \theta_{\text{eff}} \equiv s_{\text{eff}}^2$, determined at LEP and SLC, is of particular interest at present. Recent calculations of $s_{\text{eff}}^2$ and $M_W$ that incorporate reducible and irreducible contributions of $O(g^4 M_t^2/M_W^2)$ and use $\alpha$, $G_\mu$, and $M_Z$ as inputs [1, 2], examine three electroweak resummation approaches and two different ways of implementing the relevant QCD corrections. One of the approaches (MS) employs $\hat{\alpha}(M_Z)$ and $\sin^2 \hat{\theta}_W(M_Z) \equiv \hat{s}^2$, the MS QED and electroweak mixing parameters evaluated at the scale $\mu = M_Z$, while the other two (OSI and OSII) make use of the on-shell parameters $\alpha$ and $\sin^2 \theta_W \equiv s^2 \equiv 1 - M^2_W/M^2_Z$. As expected, the dependence on the electroweak scale cancels through $O(g^4 M_t^2/M_W^2)$. However, because complete $O(g^4)$ corrections have not been evaluated, the MS and OSI formulations contain a residual $O(g^4)$ scale dependence. On the other hand OSII is, by construction, strictly $\mu$-independent. It was shown in Ref.[2] that the incorporation of the irreducible $O(g^4 M_t^2/M_W^2)$ corrections sharply decreases the scheme dependence of the six calculations, to the level of $(4–5) \times 10^{-5}$ in $s_{\text{eff}}^2$ and 2–4 MeV in $M_W$, depending on $M_H$. It is worth pointing out that such variations are in rough accord with their expected order of magnitude. In fact, we have [3]

$$\frac{\delta s_{\text{eff}}^2}{s_{\text{eff}}^2} \approx \frac{\delta \hat{s}^2}{\hat{s}^2} \approx 1.53 \delta \hat{\Delta r}, \quad (1)$$

$$\frac{\delta M_W}{M_W} = -0.22 \delta \Delta r; \quad (2)$$

where $\delta s_{\text{eff}}^2$ and $\delta M_W$ are the variations induced by shifts $\delta \Delta \hat{r}$ and $\delta \Delta r$ of the basic radiative corrections $\Delta \hat{r}$ and $\Delta r$. As the two-loop corrections that have not been included are not enhanced by factors $(M_t^2/M_W^2)^n$ ($n = 1, 2$), they may be expected to be of $O(\hat{\alpha}/\pi \hat{s}^2)^2 \approx 10^{-4}$ in both $\Delta \hat{r}$ and $\Delta r$, implying $\delta s_{\text{eff}}^2 \approx 3.5 \times 10^{-5}$ and $\delta M_W \approx 2$ MeV. The same argument suggests that the incorporated $O(g^4 M_t^2/M_W^2)$ corrections may be larger by a factor 4–5, which is roughly what has been observed at low $M_H$ values [1, 2]. As illustrated in Fig.1, the irreducible $O(g^4 M_t^2/M_W^2)$ corrections also sharply reduce the residual electroweak scale dependence of the MS and OSI approaches.

The main objective of this paper is to present simple analytic formulae that re-
produce to good accuracy the results of the new calculations, and to show how they lead to a useful estimate of $M_\mu$ and its upper bound, and to a theoretically precise prediction of $M_w$. The implications of an accurate $M_w$ measurement for the $M_\mu$ estimate are also emphasized. The analytic formulae are of the form:

$$s_{\text{eff}}^2 = (s_{\text{eff}}^2)_0 + c_1 A_1 + c_2 A_2 - c_3 A_3 + c_4 A_4, \quad (3)$$

$$M_w = M_w^0 - d_1 A_1 - d_5 A_1^2 - d_2 A_2 + d_3 A_3 - d_4 A_4, \quad (4)$$

where $A_1 \equiv \ln(M_\mu / 100 \text{ GeV})$, $A_2 \equiv [(\Delta \alpha)_h/0.0280 - 1]$, $A_3 \equiv [(M_t/175 \text{ GeV})^2 - 1]$, $A_4 \equiv [(\alpha_s(M_Z)/0.118) - 1]$, $(\Delta \alpha)_h$ is the five-flavor hadronic contribution to the QED vacuum-polarization function at $q^2 = M_Z^2$, and $(s_{\text{eff}}^2)_0$ and $M_w^0$ are the theoretical results at the reference point $(\Delta \alpha)_h = 0.0280$, $M_t = 175$ GeV, and $\alpha_s(M_Z) = 0.118$. The values of $(s_{\text{eff}}^2)_0$, $M_w^0$, $c_i$ ($i = 1 - 4$), and $d_i$ ($i = 1 - 5$) for the three electroweak schemes of Ref.[2] and $M_Z = 91.1863$ GeV are given in Tables 1 and 2. For brevity, we show the coefficients in the case of the $\mu_t$-parametrization, a procedure of implementing the QCD corrections in which the pole top-quark mass $M_t$ is expressed in terms of $\tilde{m}_t(\mu_t) = \mu_t$, the $\overline{\text{MS}}$-parameter, leading to sharply reduced QCD effects, and $\mu_t/M_t$ is evaluated by optimization methods. In Ref.[2] it was shown that in the three electroweak schemes this method of implementing the QCD corrections gives results very close to the direct use of $M_t$, an approach that is frequently employed in the literature. In the range 75 GeV $\leq M_\mu \leq 350$ GeV, with the other parameters within their $1 - \sigma$ errors, Eq. (3) approximates the detailed calculations of Ref.[2] with average absolute deviations of $\approx 4 \times 10^{-6}$ and maximum absolute deviations of $(1.1 - 1.3) \times 10^{-5}$, depending on the scheme; Eq. (4), which involves an additional parameter, shows average absolute deviations of approximately 0.2 MeV and maximum absolute deviations of $(0.8 - 0.9)$ MeV. Outside the above range, the deviations increase reaching $(2.6 - 2.8) \times 10^{-5}$ and $(3.1 - 3.3)$ MeV at $M_\mu = 600$ GeV.

We briefly discuss the estimation of $M_\mu$ from Eq. (3) and the prediction of $M_w$ from Eq. (4) using the direct experimental information on $s_{\text{eff}}^2$, $(\Delta \alpha)_h$, $M_t$, and $\alpha_s(M_Z)$. From Eq. (3) we have

$$A_1 = A_1^c \pm \sigma_1, \quad (5)$$

$$A_1^c = [(s_{\text{eff}}^2 - (s_{\text{eff}}^2)_0 - c_2 A_2 + c_3 A_3 - c_4 A_4)^c/c_1, \quad (6)$$

$$\sigma_1 = \left[\sigma_s^2 + c_2^2 \sigma_2^2 + c_3^2 \sigma_3^2 + c_4^2 \sigma_4^2\right]^{1/2}/c_1, \quad (7)$$

where the superscript $c$ means that the central experimental values in $s_{\text{eff}}^2$ and the $A_i$ ($i = 1 - 4$) are to be taken, and $\sigma_s$ and $\sigma_i$ are the corresponding standard deviations. The predicted central value of $M_\mu$ is obtained by inserting $A_1^c$ in the r.h.s. of Eq. (4). Because $A_1$ is correlated with $A_i$ (cf. Eq. (5)) and Eq. (4) contains a quadratic term in $A_1$, the error analysis is slightly more involved in the $M_w$ case. Defining
\[ \sigma^2_{M_W} \equiv (M_W - \hat{M}_W)^2 \] and taking into account the correlations one finds

\[ \sigma^2_{M_W} = \sigma^2_1 \left[ \hat{d}_1^2 + 3 d^2_5 \sigma^2_1 \right] + \sum_{i=2}^4 d_i \sigma^2_i [d_i - 2 \hat{d}_1 c_i / c_1], \] (8)

where \( \hat{d}_1 = d_1 + 2 A_1^c 5^c \). In the linear approximation \( (d_5 = 0) \) Eq. (8) reduces to the simpler expression:

\[ \sigma^2_{M_W} = (d_1 \sigma_s / c_1)^2 + \sum_{i=2}^4 \sigma^2_i [d_i - d_1 c_i / c_1]^2. \] (9)

We illustrate the application of these expressions using the first row coefficients in Tables 1 and 2 (\( \overline{\text{MS}} \) approach). Inserting the current world averages \( s^2_{\text{eff}} = 0.23152 \pm 0.00023 \) [6], \( M_t = 175.6 \pm 5.5 \text{ GeV} \) [6], \( \alpha_s(M_Z) = 0.118 \pm 0.005 \) [7], and the evaluation \( (\Delta \alpha)_{h} = 0.02804 \pm 0.00065 \) [8] we find from Eqs. (5-7) [9]:

\[ \ln \left( M_H / 100 \right) = 0.029 \pm 0.709; \quad \ln \left( M_H / 100 \right) < 1.195, \] (10)

or, equivalently,

\[ M_H = 103^{+106}_{-52} \text{ GeV}; \quad M_H < 330 \text{ GeV}, \] (11)

where henceforth the inequalities represent 95\% C.L. upper bounds. From Eqs. (4,8) and (10) one obtains the prediction

\[ M_W = 80.384 \pm 0.033 \text{ GeV}. \] (12)

We repeat this analysis for the other two schemes, with the results listed in Table 3. We see that the three approaches give close values. One way of combining them is to average the central values of \( \ln(M_H/100) \) and \( M_W \) and expand the error to cover the range of the three calculations. This gives

\[ \ln \left( M_H / 100 \right) = 0.000^{+0.738}_{-0.729}; \quad \ln \left( M_H / 100 \right) < 1.214, \] (13)

\[ M_H = 100^{+109}_{-52} \text{ GeV}; \quad M_H < 337 \text{ GeV}, \] (14)

\[ M_W = 80.384 \pm 0.034 \text{ GeV}. \] (15)

The dominant QCD contribution in these calculations is \( \delta_{QCD} \), the relevant correction in the evaluation of the electroweak parameter \( \Delta \rho \). For \( M_t = 175 \text{ GeV} \), its theoretical error has been estimated as \( \pm 5.2 \times 10^{-3} \) [10]. This induces errors of \( \pm 1.8 \times 10^{-5} \) in \( s^2_{\text{eff}} \) and \( \pm 3.1 \text{ MeV} \) in \( M_W \), which are of the same magnitude although somewhat larger than the differences between the \( \mu_t \) and \( m_t \) parametrizations found in Ref.[4]. As there are additional QCD contributions, we may enlarge the QCD theoretical error to \( \pm 3 \times 10^{-5} \) in \( s^2_{\text{eff}} \) and \( \pm 5 \text{ MeV} \) in \( M_W \). An incremental uncertainty of \( 3 \times 10^{-5} \) in \( s^2_{\text{eff}} \) shifts the \( M_H \) bounds by \( \approx 6\% \), and Eqs. (14,15) become

\[ M_H = 100^{+122}_{-54} \text{ GeV}; \quad M_H < 357 \text{ GeV}, \] (16)
\[ M_w = 80.384 \pm 0.039 \text{ GeV.} \quad (17) \]

Eq. (17) is in good agreement with the current world average, \( M_{w}^{\text{exp}} = (80.43 \pm 0.08) \) GeV \( [8] \). Dividing Eq. (3) and Eq. (4) by \((s_{\text{eff}}^2)_0 \) and \( M_0^W \), respectively, we see that, at equal levels of relative experimental accuracy (which is, in fact, the current situation), \( s_{\text{eff}}^2 \) is more sensitive than \( M_w \) by factors \( \approx 2.7 \) in \( \ln(M_H/100) \), 6.6 in \( (\Delta \alpha)_h \), 1.8 in \( M_t \), 1.8 in \( \alpha_s \). Thus, at present, one of the main implications of Eq. (17) and \( M_{w}^{\text{exp}} \) is to provide a sharp test of the SM, at the 0.1% level, with very small theoretical uncertainty. There is, however, an important caveat in these considerations. The current estimates of \( M_H \) depend crucially on the world average \( s_{\text{eff}}^2 = 0.23152 \pm 0.00023 \), and this follows from a combination of experimental results that are not in good harmony. For example, the current \( \chi^2/d.o.f. \) in the determination of \( s_{\text{eff}}^2 \) from LEP + SLC asymmetries is 12.5/6, with a C.L. of 5.2%. As a rough and perhaps extreme illustration of this situation, we note that employing only the LEP average \( s_{\text{eff}}^2 = 0.23196 \pm 0.00028 \) one obtains a 95% C.L. upper bound for \( M_H \) larger than 800 GeV, while using the SLD value \( s_{\text{eff}}^2 = 0.23055 \pm 0.00041 \) alone the corresponding upper bound is \( \approx 80 \) GeV. It is clear that the \( M_H \) upper bound depends in a very sensitive manner on the precise central value and error of \( \sin^2 \theta_{\text{lept}} \): using the world average one obtains an interesting constraint; however, the LEP value alone leads to a very loose constraint and the SLD value alone is barely compatible with the lower limit on \( M_H \) from direct searches! If the \( s_{\text{eff}}^2 \) error is increased by a scaling factor \( S = [\chi^2/(N - 1)]^{1/2} \) according to Particle data group prescription \( [7] \), we have \( s_{\text{eff}}^2 = 0.23152 \pm 0.00033 \). Combining the results of the three schemes as before one finds

\[ M_H = 100^{+153}_{-60} \text{ GeV}; \quad M_H < 443 \text{ GeV,} \quad (18) \]

where we have included the QCD uncertainty. Although this scaling method is not generally employed in current analyses of the electroweak data (an exception is Ref. [11]), it provides a more conservative and perhaps more realistic estimate of \( M_H \).

This state of affairs strongly suggests the desirability of obtaining constraints on \( M_H \) derived from future precise measurements of \( M_w \). Using \( M_W \) as an input, we see from Eq. (4) that \( y = d_1 A_1 + d_5 A_1^2 \) is normally distributed about

\[ y^* = M_W^0 - M_W^c - d_2 A_2 + d_3 A_3 - d_4 A_4, \quad (19) \]

with standard deviation

\[ \sigma_y = \left[\sigma_{M_W}^2 + \sum_{i=2}^4 d_i^2 \sigma_i^2 \right]^{1/2}. \quad (20) \]

As an illustration, we assume future measurements of \( M_W \) and \( M_t \) with \( \sigma_{M_W} = 35 \) MeV, \( \sigma_{M_t} = 3 \) GeV, without changes in \( (\Delta \alpha)_h, \alpha_s(M_Z) \) and \( M_t^c \). To compare the sensitivity of such a measurement with the current \( s_{\text{eff}}^2 \) determination, we assume \( M_W^c = 80.384 \) GeV, i.e. the value predicted in Eq. (12) and Eq. (15). Using the \( \overline{\text{MS}} \)
scheme (first row of Table 2), Eqs. (13,20) lead to \( y = 0.0017 \pm 0.0417, y < 0.0703, \) which correspond to

\[
M_H = 103^{+95 \neg 57} \text{ GeV}; \quad M_H < 288 \text{ GeV}. \tag{21}
\]

The QCD uncertainty \( \Delta M_W = \pm 5 \) MeV increases the 95% C.L. upper bound to 308 GeV.

Comparison of Eqs. (14) and (21) shows that an \( M_W \) determination of \( M_H \) with \( \sigma_{M_W} = 0.035 \) GeV and \( \sigma_{M_t} = 3 \) GeV would be somewhat more restrictive than the current \( s_{eff}^2 \) estimate. This scenario is consistent with the expectation \( \sigma_{M_W} \approx 40 \neg 50 \) MeV, \( \sigma_{M_t} \approx 3 \) GeV in Tevatron Run 2 and \( \sigma_{M_W} \approx 20 \neg 30 \) MeV, \( \sigma_{M_t} \approx 1 \neg 1.5 \) GeV in Run 3 [14]. One also expects increased accuracy in the \( M_W \) measurements at LEPII. An important element in the \( M_H \) estimate based on \( M_W \) is the insensitivity to \( (\Delta \alpha)_h \), a parameter whose future accuracy is not clear at present. On the contrary, in the \( M_H \) estimate based on \( s_{eff}^2 \), \( (\Delta \alpha)_h \) is responsible for one of the largest uncertainties. It is also worth pointing out that the current world average central value \( M_W = 80.43 \) GeV favors smaller \( M_H \) values than Eqs. (14,18). For example, assuming a future measurement \( M_W = 80.430 \pm 0.035 \) GeV and \( \sigma_{M_t} = 3 \) GeV we would get in the \( \overline{\text{MS}} \) scheme \( M_H < 149 \) GeV (95% C.L.) which would be a very interesting constraint.

Next, we consider the simultaneous use of \( (s_{eff}^2)_{exp} \) and \( M_W^{exp} \) in the \( M_H \) estimate. Because of the correlations and quadratic form of Eq. (4), this is most easily done with a numerical \( \chi^2 \)-analysis employing the theoretical expressions of Eqs. (3,4). Using the \( \overline{\text{MS}} \) scheme, Table 4 gives the \( M_H \) values and the 95% C.L. upper bounds for the current experimental inputs and for a future scenario with \( \sigma_{M_W} = 35 \) MeV and \( \sigma_{M_t} = 3 \) GeV. In both cases we employ the conventional and scaled versions of the \( s_{eff}^2 \) uncertainty. We see that the constraints in the future scenarios are somewhat less restrictive than when we consider \( M_W \) alone. This is due to the fact that the present \( M_W^{exp} \) leads to a lower \( M_H \) value than the one derived from \( (s_{eff}^2)^c \).

Finally, it is important to note that the incorporation of the \( O(g^4 M_t^2/M_W^2) \) terms, as implemented in Refs. [1, 4] leads, for equal inputs, to significantly lower \( M_H \) estimates than those obtained in conventional calculations which do not include such contributions. As an illustration, we consider a recent fit to the electroweak data and \( M_t^{exp} \) [4] which yields \( \sin^2 \theta_{eff}^{lept} = 0.23152 \pm 0.00022 \), \( M_t = 173.1 \pm 5.4 \) GeV, \( \alpha(M_Z)^{-1} = 128.898 \pm 0.090 \) (corresponding to \( (\Delta \alpha)_h = 0.02803 \pm 0.00065 \)), \( \alpha_s(M_Z) = 0.120 \pm 0.003 \), and an indirect determination \( M_H = 115^{+116}_{-66} \) GeV. For the same input data for \( \sin^2 \theta_{eff}^{lept} \), \( M_t \), \( (\Delta \alpha)_h \), \( \alpha_s(M_Z) \), Eq. (3) in the \( \overline{\text{MS}} \) scheme leads to \( M_H = 88^{+87}_{-44} \) GeV (the main difference with Eq. (10) is due to the fact that the fit to the electroweak data lowers the value of \( M_t \)). We see that the central value and 1\( \sigma \) upper bound estimated in the analysis of the fit are about 30% larger than the value derived in the \( \overline{\text{MS}} \) scheme from Eq. (3). If we use the same inputs in the OSI and OSII schemes, average the \( \ln(M_H/100) \) results as before and include the estimate of
the QCD error, we find $M_H = 85^{+100}_{-46}$ GeV, $M_H < 295$ GeV (95% C.L.). The fact that the latter is significantly smaller than the 95% C.L. upper bound 420 GeV reported in Ref. [6], which includes an estimate of all theoretical errors, is due not only to the $\approx 30\%$ effect explained before, but also to the fact that the scheme dependence is considerably larger when the $O(g^4 M_t^2/M_w^2)$ terms are not included.

**Acknowledgments**

The authors are indebted to G. Altarelli, W.A. Bardeen, S. Fanchiotti, and W.J. Marciano for very useful discussions. The work of M.P. and A.S. was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76-CH00016 and by the NSF Grant PHY-9722083, respectively.

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Table 1: Values of \((s_{eff}^2)_0\) and \(c_i\) \((i = 1 - 4)\) in Eq. (3) for three electroweak schemes that incorporate \(O(g^4M_t^2/M_W^2)\) corrections in the \(\mu_t\)-parametrization of QCD corrections [2].

| Scheme | \((s_{eff}^2)_0\) | \(10^2c_1\) | \(10^3c_2\) | \(10^3c_3\) | \(10^4c_4\) |
|--------|----------------|-----------|-----------|-----------|-----------|
| MS     | 0.231510       | 5.23      | 9.86      | 2.78      | 4.5       |
| OSI    | 0.231524       | 5.19      | 9.86      | 2.77      | 4.5       |
| OSII   | 0.231540       | 5.26      | 9.86      | 2.68      | 4.4       |

Table 2: Values of \(M_W^0\) and \(d_i\) \((i = 1 - 5)\) in Eq. (4), in GeV, for the same electroweak schemes as in Table 1.

| Scheme | \(M_W^0\) | \(10^2d_1\) | \(10^2d_2\) | \(10^2d_3\) | \(10^3d_4\) | \(10^3d_5\) |
|--------|----------|-----------|-----------|-----------|-----------|-----------|
| MS     | 80.3827  | 5.79      | 5.17      | 5.43      | 8.5       | 8.0       |
| OSI    | 80.3807  | 5.73      | 5.18      | 5.41      | 8.5       | 8.0       |
| OSII   | 80.3805  | 5.81      | 5.18      | 5.37      | 8.5       | 7.8       |

Table 3: Values of \(M_H\) and \(M_W\) obtained from the current world averages of \(s_{eff}^2\), \(M_t\), \(\hat{\alpha}_s\), and the evaluation of \((\Delta \alpha)_h\) [8], on the basis of Eqs. (3,4) and Tables 1,2.
Table 4: Values and 95% C.L. upper bounds of $M_H$ obtained in the $\overline{\text{MS}}$ scheme combining $s_{\text{eff}}^2$ and $M_W$ data and expressed in GeV. The current scenario involves the present experimental values, the future scenario assumes $\sigma_{M_W} = 35 \text{MeV}$ and $\sigma_{M_t} = 3 \text{GeV}$ with unchanged central values. The error on $s_{\text{eff}}^2$ is taken to be $2.3 \times 10^{-4}$ (conventional) or $3.3 \times 10^{-4}$ (scaled). The theoretical uncertainty due to QCD corrections is included as a systematic error.

| scenario       | $M_H$ (GeV) | upper bound |
|---------------|------------|-------------|
| current conv. | $96^{+103}_{-49}$ | 305         |
| current scaled| $90^{+116}_{-51}$ | 332         |
| future conv.  | $76^{+60}_{-35}$  | 188         |
| future scaled | $68^{+51}_{-34}$  | 182         |

Figure 1: Dependence of the $M_W$ prediction on the electroweak scale $\mu$ in the $\overline{\text{MS}}$ scheme for $M_t = 175 \text{ GeV}$, $M_H = 100 \text{ GeV}$, including only the leading $O(g^4M_t^2/M_W^4)$ correction (dotted curve) or incorporating also the irreducible $O(g^4M_t^2/M_W^2)$ contribution (solid curve). QCD corrections are not included.