The systematic study of $B \to \pi$ form factors in pQCD approach and its reliability

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Abstract

The study of exclusive B decays in perturbative QCD are complicated by the endpoint problem. In order to perform the perturbative calculation, the Sudakov effects are introduced to regulate the endpoint singularity. We provide a systematic analysis with leading and next-to-leading twist corrections for $B \to \pi$ form factors in pQCD approach. The intrinsic transverse momentum dependence of hadronic wave function and threshold resummation effects are included in pQCD approach. There are two leading twist B meson distribution amplitudes (or generally wave functions) in general. The QCD equations of motion provide important constraints on B meson wave functions. The reliability of pQCD approach in $B \to \pi$ form factors is discussed. 70% of the result comes from the region $\alpha_s(t)/\pi < 0.2$ and 38% comes from the region where the momentum transfer $t \geq 1$GeV. The conceptual problems of pQCD approach are discussed in brief. Our conclusion is that pQCD approach in the present form cannot provide a precise prediction for $B \to \pi$ transition form factors.

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1 Introduction

The exclusive B decays provide an important test of the standard model of particle physics. With the running of BaBar and Belle B-factories and the proceeding of future B physics projects (BTeV and LHCb etc.), a large amount of B mesons will be accumulated to explore the origin of CP violation and determine the CKM parameters, such as the angles $\alpha$, $\beta$, and $\gamma$ in unitary triangle. In most cases, the limitation of our theoretical ability prevents the precise prediction for exclusive B decays so we have to refer to some phenomenological approaches.

The large B meson mass $m_B$ establishes a large scale so that perturbative QCD (pQCD) may be applicable in exclusive B decays. Recently, two different approaches based on perturbative QCD were proposed to calculate the exclusive B decays. One approach, usually say QCD factorization approach, states that two body hadronic B decays can factorize and the relevant amplitude can be written as the convolutions of non-perturbative quantities (the light-cone distribution amplitudes of mesons and semi-leptonic form factors) and perturbatively calculable hard scattering kernels [1]. The other approach is the modified pQCD, or say pQCD approach for simplicity. In pQCD approach, the semi-leptonic form factors for $B \to P$ transition (where P represents light meson) are claimed to be perturbatively calculable [2].

The central difference between these two approaches is whether $B \to P$ transition form factors are perturbatively calculable, or specifically, whether Sudakov effects can cure endpoint singularity. In $B \to P$ transition form factors, the endpoint singularity is generated even at leading twist. In pQCD approach, the transverse momentum are retained to regulate the endpoint singularity and Sudakov double logarithm corrections are included to suppress the long distance contributions from configuration of large transverse separation. In [3], the authors investigated the reliability of Sudakov effects in $B \to \pi$ form factors. Their conclusion is that Sudakov suppression are so weak that they can not be applied in B decays. If their criticism is right, it would bring the entire thought about Sudakov effects into crisis.

In our previous study [4], we investigated the Sudakov effects in QCD factorization approach. Our conclusion is contrary to the above criticism. Sudakov effects play an important role in exclusive B decays. It is well known that the standard approach or say BLER approach [5] is questionable at experimentally accessible energy scales, typically a few GeV region [6]. This question also occurs in the hard spectator scattering in $B \to P_1 P_2$ decays. The QCD factorization approach gives a finite result at leading twist. Carefully study shows that there are large contributions coming from the region where the momentum transfers are small. The introduction of Sudakov suppression enlarges
the application range of pQCD and makes it self-consistent [7]. Although the relevant numerical result is power suppressed, the Sudakov effects is potentially important. We disagree with the comment that the main purpose of introduction of Sudakov suppression is to improve the computation accuracy [3]. At twist-3 level, the chirally enhanced power correction is logarithmically divergent, so the QCD factorization approach can not apply. A phenomenological parametrization method is designed to regulate the endpoint singularity [8]. This method is not self-consistent and it introduces an arbitrary infrared cutoff. Sudakov suppression establish a natural infrared cutoff. Obviously, pQCD approach has the advantage of the parametrization method.

The disagreement of conclusions in [2, 4] and [3] motivates us to reconsider the reliability of pQCD approach in $B \rightarrow \pi$ form factors. In this paper, we present a systematic study of $B \rightarrow \pi$ form factors in pQCD approach and examine the reliability of pQCD calculation. Compare with the recent pQCD analysis of $B \rightarrow \pi$ form factors [9], the present analysis contains three new theoretical ingredients:

- The intrinsic transverse momentum dependence of wave functions for B and pion mesons are included. The importance of intrinsic transverse momentum dependence is first pointed out in pion form factor in [10]. Because Sudakov suppression is not severely strong for real $m_B$, the effect of intrinsic transverse momentum dependence of wave function can not be neglected.

- B meson contains two leading wave functions. The assumption of single B meson wave function in the previous pQCD analysis is not valid. Equations of motion in HQET provide important constraint on the choice of B meson wave functions.

- The threshold resummation effect for B meson is taken into account. The perturbative analysis depends on the endpoint behavior of B meson distribution amplitudes. The jet function obtained from threshold resummation suppress the endpoint contribution, so it modifies the power behavior of endpoint contribution. We expect that it can improve the perturbative result since one of the two B meson distribution amplitudes does not vanish at endpoint.

In addition, the chirally-enhanced power corrections (twist-3 contribution) are included in our analysis. It is noted that the contribution of chirally-enhanced power corrections is comparable with or even larger than leading twist contribution [9]. Our study confirm it. The numerical results of $B \rightarrow \pi$ form factors at large recoil region in pQCD approach are consistent with derivations of QCD sum rules [11].

The remainder of the paper is organized as follows. Sec. 2 introduces the theoretical ingredients of pQCD approach. In sec. 3, we present the formulas for $B \rightarrow \pi$ form factors
in pQCD approach, perform the numerical analysis, and examine the reliability of pQCD approach. Finally, in sec. 4, the conclusions and discussions are presented.

2 The $B \rightarrow \pi$ form factors in pQCD approach

$B \rightarrow \pi$ form factors are the basic non-perturbative parameters in semi-leptonic $B \rightarrow \pi l \nu$ decays and exclusive, nonleptonic decays such as $B \rightarrow \pi \pi, \pi K$ decays in QCD factorization approach. While in pQCD approach, it can be perturbatively calculated from the universal hadronic distribution amplitudes or wave functions. In this section, we will give a general discussion of $B \rightarrow \pi$ form factors and introduce the main ingredients of pQCD approach.

The form factors of $B \rightarrow \pi$ are defined by the following Lorentz decompositions of biquark current matrix elements:

$$\langle \pi | \bar{u} \gamma_{\mu} b | B \rangle = (P_B + P_\pi - m_B^2 - m_\pi^2 q^2/q^2) F^{B\pi}_{+}(q^2) + (q^2/m_B^2 - q^2/m_\pi^2) q_\mu F^{B\pi}_{0}(q^2),$$

where $q = P_B - P_\pi$ is the momentum carried by lepton pair in semi-leptonic $B \rightarrow \pi l \nu$ decays. At large recoil limit, $q^2 = 0$, the two form factors $F^{B\pi}_{+}, F^{B\pi}_{0}$ reduce to one parameter, $F^{B\pi}_{+}(0) = F^{B\pi}_{0}(0)$.

First, we give our conventions on kinematics. We work in the rest frame of B meson. The mass difference of b quark and B meson is negligible in the heavy quark limit and we take approximation $m_b = m_B$ in our calculation. The masses of light quarks $u$, $d$ and $\pi$ meson are also neglected. For discussion, the momentum is described in terms of light-cone variables. We take $k = (k^+, k_\perp)$ with $k^\pm = k^0 \pm k^3$ and $k_\perp = (k^1, k^2)$. The scalar product of two arbitrary vectors $A$ and $B$ is $A \cdot B = A_\mu B^\mu = A^{+B+} - A^{-B+}/2 - \vec{A}_\perp \cdot \vec{B}_\perp$.
The momentum of pion is chosen to be in the “$-$” direction. Under these conventions, $P_B = (m_B/\sqrt{2}, m_B/\sqrt{2}, 0), P_\pi = (0, m_\pi/\sqrt{2}, 0), \vec{P}_\pi = (0, m_\pi/\sqrt{2}, 0)$ with $\eta = 1 - \eta = q^2/m_B^2$. We define two light-like vectors $n_+ \equiv (\sqrt{2}, 0, 0)$ and $n_+ \equiv (0, \sqrt{2}, 0)$.

We denote $\xi$ as the momentum fraction of spectator anti-quark in B meson, and $x$ as the momentum fraction of the anti-quark in pion. As plotted in Fig.1.

2.1 The physical picture of factorization in $B \rightarrow \pi$ form factors

The most important ingredient of pQCD is factorization, i.e., the separation of the long-distance dynamics from the short distance dynamics which is pertubatively calculable. Although rigorous proof of the factorization theorem is technically intricate, the physical picture is simple and intuitive.
First, we illustrate the factorization of pion electromagnetic form factor in order to introduce the basic idea of standard approach and the modified approach. A highly virtual photon collides on a quark in initial pion and makes it change its direction. In exclusive process $\gamma^* \pi \rightarrow \pi$, the simple power counting shows that at large momentum transfers, the valence quarks dominate the process and the higher Fock states and the intrinsic transverse momentum is power suppressed. The large momentum $Q$ means the high resolution at the distance of $1/Q$. In a small distance region, a parton only sees the other parton relative to it and the initial and final pion can be considered as a small-size color-singlet object during the hard interaction. What is important for small-size color-singlet pion is that soft gluon corrections cancel at the leading twist. This is the well-known phenomena, “color transparency”. The long-distance interactions only occur before and after the hard interaction, so their effects can be factorized into the initial and final pions respectively. Since the hard interaction is restricted in the short distance, we only need the probability for the $q\bar{q}$ pair in pion to be within the transverse distance of $1/Q$, in other words, the distribution amplitude of pion $\phi(x, Q^2)$. The scale $Q$ acts as factorization scale as well as renormalization scale. The above discussions can be grouped into the standard factorization formula for electromagnetic form factor of pion

$$F_z = \int dx dy \phi(x, Q^2)T(x, y, Q)\phi(y, Q^2). \quad (2)$$

The standard factorization is proved successfully in the asymptotic limit. The application of this standard approach at the experimentally accessible energy scales, typically a few $GeV$ region, is criticized in [6]. The authors pointed out that the perturbative calculations for electromagnetic form factor have a large amount of contributions coming from soft endpoint region $(x, y \rightarrow 0)$ where the perturbative analysis is invalid. This is the well-known “endpoint” problem even there is no endpoint singularity in the convolution of total amplitude. The Sudakov effects are introduced to modify the endpoint behavior and make pQCD applicable in few GeV region [7]. The basic idea of this modified approach, or say pQCD approach is using the mechanism of Sudakov suppression to suppress long-distance contributions of large transverse separations. Sudakov suppression establishes a factorization scale $1/b$ in addition to the momentum scale $Q$. The pion with small transverse separation has small color dipole moment so that QCD factorization is revised. In pQCD approach, Sudakov suppression plays a crucial role.

Second, We discuss the factorization in $B \rightarrow \pi$ form factors in the large $m_B$ limit. B meson is a heavy-light system. It has large size in longitudinal as well as transverse directions. If the soft B meson wave functions overlap with the wave functions of final meson, the separation of long-distance and short distance dynamics is impossible. From
this point, we conclude that $B \rightarrow D^{(*)}$ from factors in heavy quark limit are soft dominant and cannot be calculated by pQCD. For $B \rightarrow \pi$ form factors, the situation is different. The pion carries the energy of $\frac{m_B}{2}$ when $q^2 = 0$. When the fractional momentum of the antiquark in pion is far away from the endpoint region, i.e. $x \gg 0$, the pion is highly restricted in longitudinal direction because of Lorentz contraction. The soft gluons that attach to small size color-singlet pion decouple in the large $m_B$ limit. So factorization is applicable. Simply to say, the soft spectator anti-quark in B meson must undergo hard strong interaction to change into the fast moving parton in pion.

The above perturbative picture is destroyed by the endpoint contribution where antiquark in pion carries the momentum of order $\Lambda_{QCD}$. In the standard approach, $B \rightarrow \pi$ form factor is logarithmically divergent at leading twist (twist-2 for our case). The divergence becomes more serious for next-to-leading twist (twist-3) contribution and the linear divergence occurs. The origin of this divergence is that the distribution amplitude does not provide enough suppression at endpoint. In order to apply the perturbative picture, the soft endpoint contribution must be suppressed. As we have discussed for pion electromagnetic form factor, Sudakov effects can provide such suppression. With the help of Sudakov suppression, pQCD factorization can be applicable. In pQCD approach, the $B \rightarrow \pi$ form factors can be generally written as convolutions of wave functions and hard scattering kernels

$$F_{\pi}^{B} = \int d\xi \; dx \; d^2b_B \; d^2b_\pi \; \Psi_B(\xi, b_B, \mu) \; \Psi_\pi(x, b_\pi, \mu) \; T_{\pi,0}(\xi, x, b_B, b_\pi, \mu).$$

(3)

2.2 Pion wave functions

In the standard approach, the transverse momentum is assumed power suppressed and neglected. When the two valence quarks in pion carry large longitudinal momentum, i.e., the momentum fractions $x, \bar{x} \gg 0$, the transverse momentum is power suppressed compared to longitudinal momentum. In the endpoint region, the transverse momentum is important and cannot be neglected. In pQCD approach, transverse momentum is retained in hard scattering kernels and the non-perturbative parameters are hadronic wave functions in general.

The two valence quarks in pion have transverse momentum as well as the longitudinal momentum. Compared to the collinear limit, the momentum of the quarks in pion (with momentum $P_\pi$) changes to

$$k_1 = xp_\pi + k_\perp, \quad k_2 = \bar{x}p_\pi - k_\perp,$$

(4)

where $x$ and $\bar{x}$ denote the longitudinal momentum fractions of quark and anti-quark, respectively. For our case, the meson is on-shell and the partons are slightly off-shell.
The off-shellness of the parton is proportional to $k^2_\perp$ which is power suppressed compared to $m^2_B$.

As we have discussed above, the intrinsic transverse momentum dependence of wave functions of pion and B meson cannot be neglected because Sudakov suppression is not strong in the real $m_B$ energy. The distribution amplitude can be obtained from the integration of wave function over the transverse momentum

$$\phi(x) = \int_{|k_\perp|<\mu} d^2k_\perp \Psi(x,k_\perp),$$

(5)

where $\mu$ is the ultraviolet cutoff.

Similar to the definition of distribution amplitudes with leading and next-to-leading twist [12], the light-cone wave functions of pion are defined in terms of bilocal operator matrix element

$$\langle \pi(p) | \bar{q}_\beta(z) q_\alpha | 0 \rangle = \frac{i f_\pi}{4} \int_0^1 dx \int d^2k_\perp e^{i(xp - \vec{k}_\perp \cdot \vec{z}_\perp)}$$

$$\times \left\{ \gamma_5 \gamma_5 \Psi_\pi(x,k_\perp) - \mu_\pi \gamma_5 \left( \Psi_p(x,k_\perp) - \frac{1}{6} \sigma_{\mu\nu} p^\mu p^\nu \frac{1}{6} \Psi_\sigma(x,k_\perp) \lambda_{\mu\nu} \right) \right\}_{\alpha\beta},$$

(6)

where $f_\pi$ is the decay constant of pion. The parameter $\mu_\pi = m^2_\pi/(m_u + m_d)$ for charged pion. For neutral pion, we use the same parameter $\mu_\pi$ as the charged pion. $\Psi_\pi$, $\Psi_p$, and $\Psi_\sigma$ are the twist-2 and twist-3 wave functions, respectively. The twist-3 wave functions contribute power corrections. But at $m_B$ energy scale, the chirally enhanced parameter $r_\pi = \frac{\mu_\pi}{m_B} \sim O(1)$ is not small. So the twist-3 contribution should be considered in B decays.

In order to perform the analysis in pQCD approach, we need to transform the parameters in terms of coordinate variable in Eq.(6) into the momentum space configuration. We use the momentum projection given in [13]:

$$M^\pi_{\alpha\beta} = \frac{i f_\pi}{4} \left\{ \gamma_5 \Psi_\pi - \mu p \gamma_5 \left( \Psi_p - i \sigma_{\mu\nu} \frac{p^\mu p^\nu}{p \cdot \bar{p}} \frac{1}{6} \Psi_\sigma + i \sigma_{\mu\nu} \frac{p^\mu}{6} \frac{1}{6} \Psi_\sigma \frac{\partial}{\partial k_\perp \nu} \right) \right\}_{\alpha\beta},$$

(7)

where $\Psi'_{\sigma} = \frac{\partial \Psi_\sigma(x,k_\perp)}{\partial x}$. The wave functions $\Psi_\pi, \Psi_p, \Psi_\sigma$ may have different transverse momentum dependence, this will make the calculation difficult. In order to simplify discussions, we assume the same transverse momentum dependence for these wave functions.

In pQCD approach, the convolutions of wave functions and hard scattering kernel are presented in transverse configuration $b$-space. We need to define wave function in $b$-space through Fourier transformation

$$\Psi(x,b) = \int d^2k_\perp e^{-i\vec{k}_\perp \cdot \vec{b}} \Psi(x,k_\perp).$$

(8)
From Eq.(5) and (8), we can obtain

$$\phi(x) = \Psi(x, b = 0).$$

(9)

The impact parameter $b$ represents the transverse separation of quark and anti-quark in pion. Eq.(9) shows that the distribution amplitude is equal to the wave function at zero transverse separation.

The intrinsic transverse momentum dependence of wave function of pion is unknown from the first principle in QCD. We take a simple model in which the dependence of the wave function on the longitudinal and transverse momentum can be separated into two parts:

$$\Psi(x, k_\perp) = \phi(x) \times \Sigma(k_\perp),$$

(10)

where $\phi(x)$ is the pion distribution amplitude. $\phi(x)$ and $\Sigma(k_\perp)$ satisfy the normalization conditions

$$\int_0^1 \phi(x) = 1, \quad \int d^2 k_\perp \Sigma(k_\perp) = 1.$$  

(11)

The $k_\perp$ dependence of the wave function is contained in $\Sigma(k_\perp)$. In [10], $\Sigma(k_\perp)$ is assumed to be a Gaussian distribution,

$$\Sigma(k_\perp) = \frac{\beta^2}{\pi} \exp(-\beta^2 k_\perp^2),$$

(12)

Transforming it into the transverse configuration $b$-space, we obtain

$$\Sigma(b) = \int d^2 k_\perp e^{-i \vec{k}_\perp \cdot \vec{b} \Sigma(k_\perp) = \exp(-\frac{b^2}{4\beta^2}),$$

(13)

The oscillating parameter $\beta$ is fixed by requiring the root mean square transverse momentum (r.m.s.), $\langle k_\perp^2 \rangle^{1/2}$ is the order of $\Lambda_{QCD}$. Their relation can be obtained from

$$\langle k_\perp^2 \rangle = \frac{\int_0^1 dx \int d^2 k_\perp k_\perp^2 |\Psi(x, k_\perp)|^2}{\int_0^1 dx \int d^2 k_\perp |\Psi(x, k_\perp)|^2} = \frac{1}{2\beta^2},$$

(14)

Thus, $\langle k_\perp^2 \rangle^{1/2} = \frac{1}{\sqrt{2\beta}}$. If the root mean square transverse momentum $\langle k_\perp^2 \rangle^{1/2} = 0.35\text{GeV}$, then $\beta^2 = 4\text{GeV}^{-2}$. The detailed discussions about choice of the oscillation parameter $\beta$ can be found in [10].

2.3 B meson wave functions

The intrinsic dynamic of B meson is different from the case of light pion meson. The momentum components of the spectator quark $l$ are of order $\Lambda_{QCD}$. The most convenient
tool to describe B meson is heavy quark effective theory (HQET). Since we have chosen $P_\pi$ in the "−" direction, the hard scattering amplitude does not dependent on $l_-$, the $l_-$ dependence of the wave functions can be integrated out. In HQET, the B meson wave functions are defined by the general Lorentz decomposition of the light-cone matrix element \[14, 3\]

\[
\langle 0|\bar{q}_\beta(z)b_\alpha(0)|\bar{B}(p_B)\rangle = -\frac{if_{B}}{4} \left\{ \frac{\hat{p}_B + m_B}{2} \left[ 2\bar{\Psi}_B^- (z^2, t) + \frac{\bar{\Psi}_B^+ (z^2, t)}{t} \right] \right\}_{\alpha\beta},
\]

(15)

with $v = \frac{p_B}{m_B}$ and $t = v \cdot z$. A path-ordered exponential is implicitly present in the gauge-independent matrix element.

The wave functions in momentum space can be obtained through the Fourier transformation:

\[
\Psi^\pm_B (l_+, l_-) = \frac{1}{2} \int \frac{d^2 \vec{z}_\perp}{(2\pi)^2} \frac{dz_-}{2\pi} e^{i(l_+z_-/2-l_-z_\perp)} \bar{\Psi}^\pm_B (-z^2_\perp, z_-/2),
\]

(16)

where $l = (l_+/\sqrt{2}, 0, \vec{l}_\perp)$ and $z = (0, z_-/\sqrt{2}, \vec{z}_\perp)$, so that $z^2 = -z^2_\perp$ and $t = z_-/2$.

A momentum projection for the matrix element of B meson is needed to simplify the calculation. In \[13\], the authors obtain a projection which is valid for distribution amplitudes. We extend their analysis to include the transverse momentum dependence of wave functions. We observe that their formulas need no modification except the replacement of distribution amplitudes by wave functions. The proof of this point is given in Appendix. So, the momentum-space projection operator for B meson is:

\[
M^B_{\alpha\beta} = -\frac{if_{B}}{4} \left\{ \frac{\hat{p}_B + m_B}{2} \left[ \Psi_B^+ \gamma_5 \frac{\partial}{\partial l_\perp^\mu} \Psi_B^+ - \Delta(l_+, l_-) \gamma_5 \frac{\partial}{\partial l_\perp^\mu} \right] \right\}_{\alpha\beta},
\]

(17)

where $\Delta(l_+, l_-) = \int_0^{l_+} dl (\Psi_B^- (l, l_-) - \Psi_B^+ (l, l_-))$.

The projection operator can be represented in the form which is helpful to compare with the results in the previous pQCD analysis \[9\]

\[
M^B_{\alpha\beta} = -\frac{if_{B}}{4} \left\{ (\hat{p}_B + m_B) \left[ \Psi_B^+ \frac{\hat{\Psi}_B^+}{2} - \frac{\hat{\Psi}_B^-}{2} \Psi_B^- - \frac{1}{2} \Delta(l_+, l_-) \gamma_5 \frac{\partial}{\partial l_\perp^\mu} \right] \right\}_{\alpha\beta},
\]

(18)

where $\Psi_B, \bar{\Psi}_B$ are defined by

\[
\Psi_B = \frac{\Psi_B^+ + \Psi_B^-}{2}, \quad \bar{\Psi}_B = \frac{\Psi_B^+ - \Psi_B^-}{2}.
\]

(19)

In \[1\], the authors consider a different projection operator for B meson

\[
M^B_{\alpha\beta} = -\frac{if_{B}}{4} \left\{ (\hat{p}_B + m_B) \gamma_5 [\Psi_B^1 + \hat{\Psi}_B^1] \right\}_{\alpha\beta},
\]

(20)
where $\Psi_{B1}, \Psi_{B2}$ are defined by

$$
\Psi_{B1} = \Psi_B^+, \quad \Psi_{B2} = \frac{\Psi_B^+ - \Psi_B^-}{2}.
$$

(21)

The above three projection operators are equivalent up to leading power in $1/m_b$. The projection given in Eq.(20) neglects the $\Delta$ term which is proportional to $l^+/m_b$ and thus power suppressed.

The $B$ meson distribution amplitudes are obtained from transverse momentum integral of the relevant wave functions or from wave functions at zero transverse separation. $\phi_B^+$ and $\phi_B, \bar{\phi}_B$ are distribution amplitudes relative to wave functions of $\Psi_B^\pm$ and $\Psi_B, \bar{\Psi}_B$ respectively and they are related by

$$
\phi_B = \frac{\phi_B^+ + \phi_B^-}{2}, \quad \bar{\phi}_B = \frac{\phi_B^+ - \phi_B^-}{2}.
$$

(22)

The equations of motion impose constraint on the wave functions. Using the equation of motion for the light quark of $B$ meson and neglecting the effects of three-parton and higher Fock states, we can obtain [13, 3]

$$
\frac{\partial \tilde{\Psi}_B^-}{\partial t} + \frac{\tilde{\Psi}_B^- - \tilde{\Psi}_B^+}{t} \bigg|_{z^2=0} = 0,
$$

(23)

$$
\frac{\partial \tilde{\Psi}_B^+}{\partial z^2} + \frac{1}{4} \frac{\partial^2 \tilde{\Psi}_B^-}{\partial l^2} \bigg|_{z^2=0} = 0.
$$

(24)

Similar to the discussion for pion wave function, we consider a model in which the dependence on the longitudinal and transverse momenta of $B$ meson is factorized:

$$
\Psi_B^\pm(\xi, l_\perp) = \phi_B^\pm(\xi) \times \Sigma_B^\pm(l_\perp),
$$

(25)

where $\xi = \frac{l_\perp}{m_B}$ is the longitudinal momentum fraction. The $\phi_B^\pm$ are the two distribution amplitudes of $B$ mesons. The normalization conditions are

$$
\int d\xi \phi_B^+(\xi) = 1, \quad \int d^2 l_\perp \Sigma_B^+(l_\perp) = 1.
$$

(26)

Under the above assumptions, the two constraints become

$$
\phi_B^+(\xi) = -\xi \frac{d\phi_B^-}{d\xi}(\xi),
$$

(27)

$$
\xi^2 m_B^2 \phi_B^+(\xi) = \omega_B^2 \phi_B^+(\xi),
$$

(28)

where $\omega_B^2 = \int d^2 l_\perp \Sigma_B^+(l_\perp)$.
The distribution amplitudes of $\phi^\pm_B$ can be solved analytically and the corresponding solution is:

$$\phi^-_B(\xi) = \sqrt{\frac{2}{\pi}} \frac{m_B}{\omega_B} \exp\left(-\frac{\xi^2 m_B^2}{2 \omega_B^2}\right), \quad \phi^+_B(\xi) = \sqrt{\frac{2}{\pi}} \frac{\xi^3 m_B^3}{\omega_B^3} \exp\left(-\frac{\xi^2 m_B^2}{2 \omega_B^2}\right). \quad (29)$$

The parameter $\omega_B$ is fixed by the value of root mean square transverse momentum of B meson. For the transverse momentum dependent function, we choose the Gaussian distribution:

$$\Sigma^+(l) = \frac{1}{2\pi \omega_B^2} \exp\left(-\frac{l^2}{2\omega_B^2}\right), \quad (30)$$

The parameters $\omega_B$ and $\omega'_B$ are not independent, they are related by $\omega_B = \sqrt{2}\omega'_B$. The root mean square transverse momentum $\langle l^2 \rangle = \frac{\omega_B}{\sqrt{2}} = \omega'_B$. In transverse configuration $b$-space, the transverse momentum dependent function is

$$\Sigma^+(b_B) = \exp\left(-\frac{\omega_B^2 b_B^2}{2}\right), \quad (31)$$

where $b_B$ is the conjugated variable of $l$. For $\Sigma^-_B(b_B)$ function, we assume it is the same as $\Sigma^+_B(b_B)$.

A more rigorous study of B meson wave functions in HQET is given recently in [15]. The authors find an analytic solution in the heavy quark limit using the QCD equations of motion. The transverse momentum distribution can also be derived without the assumption of Gauss behavior. The obtained B meson wave functions is model-independent.

### 2.4 Sudakov form factor

There are two types of resummation: Sudakov resummation (or say $b$-space resummation) and threshold resummation. These two resummation effects lead to suppression in different space: the region with large transverse separations $b$ for Sudakov resummation and the small longitudinal fractional momentum $x$ region for threshold resummation.

First, we discuss the Sudakov resummation in brief. At $\alpha_s$ order, the overlap of soft and collinear divergences produce double logarithms $-c \ln^2 Qb$. The transverse impact parameter $b$ is used to regulate the infrared divergence. In transverse configuration $b$ space, the Sudakov double logarithms are resumed up to next-to-leading-log approximation. A exponential factor $e^{-s(x,b,Q)}$ will be obtained from $b$-space resummation. We present the explicit expression of the exponent $s(x, b, Q)$ appearing in Sudakov form factor.

Define the variables,

$$\hat{q} \equiv \ln\left(\frac{xQ}{\sqrt{2}\Lambda_{QCD}}\right), \quad \hat{b} \equiv \ln\left(\frac{1}{b\Lambda_{QCD}}\right). \quad (32)$$
According to [16], the exponent \( s(x, b, Q) \) is presented up to next-to-leading-log approximation

\[
s(x, b, Q) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln \left( \frac{\hat{q}}{b} \right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b}) + \frac{A^{(2)}}{4\beta_1^2} (\frac{\hat{q}}{b} - 1) - \left[ \frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln \left( \frac{e^{2\gamma_E} - 1}{2} \right) \right] \ln \left( \frac{\hat{q}}{b} \right) \\
+ \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[ \ln(2\hat{q}) + 1 - \frac{\ln(2\hat{b}) + 1}{b} \right] + \frac{A^{(1)}\beta_2}{8\beta_1^3} [\ln^2(2\hat{q}) - \ln^2(2\hat{b})] \\
+ \frac{A^{(1)}\beta_2}{8\beta_1^3} \ln \left( \frac{e^{2\gamma_E} - 1}{2} \right) \left[ \ln(2\hat{q}) + 1 - \frac{\ln(2\hat{b}) + 1}{b} \right] - \frac{A^{(2)}\beta_2}{16\beta_1^4} \left[ 2\ln(2\hat{q}) + 3 - 2\ln(2\hat{b}) + 3 \right] \\
- \frac{A^{(2)}\beta_2}{16\beta_1^4} \hat{q} - \frac{\hat{b}}{b^2} \left[ 2\ln(2\hat{b}) + 1 \right] + \frac{A^{(2)}\beta_2}{432\beta_1^6} \frac{\hat{q} - b}{\hat{b}^3} \left[ 9\ln^2(2\hat{b}) + 6\ln(2\hat{b}) + 2 \right] \\
+ \frac{A^{(2)}\beta_2}{1728\beta_1^6} \left[ 18\ln^2(2\hat{q}) + 30\ln(2\hat{q}) + 19 - 18\ln^2(2\hat{b}) + 30\ln(2\hat{b}) + 19 \right] b^2, \tag{33}\]

where the coefficients \( \beta_i \) and \( A^{(i)} \) are

\[
\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \\
A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3} \beta_1 \ln \left( \frac{e^{\gamma_E}}{2} \right). \tag{34}\]

with \( \gamma_E \) the Euler constant.

The running coupling constant \( \alpha_s \) up to next-to-leading-log is written as

\[
\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_1 \ln(\mu^2/\Lambda^2)} - \frac{\beta_2 \ln \ln(\mu^2/\Lambda^2)}{\beta_1^3 \ln^2(\mu^2/\Lambda^2)}. \tag{35}\]

The exponent \( s(x, b, Q) \) is obtained under the condition that \( xQ/\sqrt{2} > 1/b \), i.e. the longitudinal momentum should be larger than the transverse degree. So \( s(x, b, Q) \) is defined for \( \hat{q} > \hat{b} \), and set to zero for \( \hat{q} < \hat{b} \). The previous formulas [17] about the exponent \( s(x, b, Q) \) picks up the most important first six terms in the first and second lines of the expression of \( s \). Note that the sign of the fifth and sixth terms are different from those in [17].

The Sudakov form factor factor \( e^{-s(x,b,Q)} \) falls off quickly in large \( b \) region and vanishes as \( b > 1/\Lambda_{QCD} \). Therefore it suppresses the long-distance contribution, which is called Sudakov suppression. In axial-gauge, the Sudakov form factor is included in each hadronic wave function. So we can define the pion and B hadronic wave functions with Sudakov corrections as

\[
\Psi_{\pi}(x, b_\pi; t) = \exp(-S_{\pi})\Psi_{\pi}^0(x, b_\pi, t), \quad \Psi_B(\xi, b_B; t) = \exp(-S_B)\Psi_B^0(\xi, b_B, t), \tag{36}\]
where \( t \) is the factorization scale in the hard scattering kernel. The \( \Psi^0_{\pi}(x, b_\pi, t) \) and \( \Psi^0_B(\xi, b_B, t) \) are wave functions without Sudakov corrections.

Combining with the evolution of wave functions and hard scattering kernel, a complete factor \( e^{-S_{\pi,B}} \) for pion and B mesons can be given as

\[
S_{\pi} = s(x, b_\pi, m_B) + s(\bar{x}, b_\pi, m_B) - \frac{1}{\beta_1} \ln \frac{\ln(t/\Lambda_{QCD})}{\ln(1/(b_\pi\Lambda_{QCD}))},
\]

\[
S_B = s(\xi, b_B, m_B) - \frac{1}{\beta_1} \ln \frac{\ln(t/\Lambda_{QCD})}{\ln(1/(b_B\Lambda_{QCD}))}.
\]

(37)

About the Sudakov form factor \( e^{-s(x,b,m_B)} \) for B and pion meson, some comments are in order:

- The Sudakov factor for B meson is only associated with the light quark since there is no collinear divergence associated with the heavy \( b \) quark. Due to the fact that the momentum of light spectator quark is concentrated at the order of \( \Lambda_{QCD} \), we may expect the Sudakov effects is small because of the suppression of B meson wave function. Our numerical analysis shows that its effect is at one percent level.

- Notice that the effect of the evolution term in the last of eq.(37) gives slight enhancement. If Sudakov suppression takes place, the enhancement will be masked (for example, the case of pion). On the other hand, if Sudakov suppression is not effective (for example, the case of B meson), this enhancement will emerge.

- In the endpoint region, the exchanged gluon carries small longitudinal momentum. The transverse degree cannot be neglected. The mechanism of Sudakov suppression begin to take into effect, and the long-distance dynamics from large transverse separations are suppressed by Sudakov effects. If the momentum fraction of one quark \( x \) is small, the Sudakov form factor associated with it is 1, no suppression takes place. However the Sudakov form factor associated with the other quark will provide strong suppression because the momentum fraction of this quark \( \bar{x} \) is large now.

2.5 Threshold resummation

Now, we discuss the threshold resummation effects. Double logarithms \( \alpha_s \ln^2 x \) can be produced by the higher order loop corrections. It diverges at the endpoint. If the endpoint contribution is important, the double logarithms \( \alpha_s \ln^2 x \) needs to be resumed to all orders. We review the basic idea of threshold resummation. Our discussion underlies on the analysis given in [9, 18]. The vertex corrections at \( \alpha_s \) order produce the double
logarithms \(-\frac{\alpha_s}{4\pi}C_F \ln^2 x\) where \(C_F = 4/3\) is the color factor. This collinear divergences can be factorized into a quark jet function \(S_t(x)\). In order to resume the double logarithms to all orders, it is necessary to introduce the moment (N) space. In N space, the Sudakov factor has the exponential form up to the accuracy of leading-log approximation (LL),

\[
S_t^{(LL)}(N) = \exp \left[ -\frac{1}{4} \gamma^{(LL)}_K \ln^2 N \right],
\]

where the anomalous dimension \(\gamma^{(LL)}_K = \frac{\alpha_s}{4\pi}C_F/\pi\).

The jet function \(S_t(x)\) can be obtained from \(S_t(N)\) through the transformation

\[
S_t(x) = \int_{a-i\infty}^{a+i\infty} \frac{dN}{2\pi i} (1-x)^{-N} S_t(N) S_t^{(0)}(N),
\]

where \(a\) is an arbitrary real constant larger than all the real parts of poles involved in the integrand. The \(S_t^{(0)}(N)\) comes from Mellin transformation of the initial condition \(S_t^{(0)}(x) = 1\),

\[
S_t^{(0)}(N) = \int_0^1 dx (1-x)^{N-1} S_t^{(0)}(x) = \frac{1}{N}.
\]

The upper index “0” means that there is no QCD corrections.

The contour integral in Eq. (39) can be transformed to

\[
S_t^{(LL)}(x) = -\exp \left( \frac{\pi}{4\alpha_s C_F} \int_{-\infty}^{\infty} \frac{dt}{\pi} (1-x)^{\exp(t)} \sin \left( \frac{1}{2} \alpha_s C_F t \right) \exp \left( \frac{\alpha_s}{4\pi} C_F t^2 \right) \right).
\]

The above analysis can be directly extended to next-to-leading logarithms. The jet function \(S_t(x)\) satisfies the normalization condition \(\int_0^1 S_t(x) = 1\). It vanishes at the end points \(x \to 0\) and \(x \to 1\). The most important property of jet function \(S_t(x)\) is that it damps faster than any power of \(x\).

Since the resumed factor \(S_t(x)\) suppresses small \(x\) contribution, it may play crucial role in B decays. The B meson distribution amplitude \(\phi_{B}(x)\) and the twist-3 distribution amplitudes \(\phi_{P}(x)\) do not vanish at \(x = 0\) in general. Although the transverse momentum can regulate the endpoint singularity, there is still a substantial contribution coming from endpoint region. The factor \(S_t(x)\) can suppress endpoint contribution and make pQCD more applicable.

In [19], the authors discuss another resummation whose formula is similar to Sudakov resummation. The obtained Sudakov form factor suppresses small \(x\) contribution more rapidly than the factor \(S_t(x)\). It shows the importance of double-log corrections from another point.

The factor \(S_t(x)\) presented in Eq.(41) involves one parameter integration. In order to simplify the numerical calculation, a simple parametrization for \(S_t(x)\) is proposed [9]

\[
S_t(x) = \frac{2^{1+2c}}{\sqrt{\pi} \Gamma(1+c)} \left[ x(1-x) \right]^c,
\]

(42)
where the parameter $c$ is determined around 0.3. The factor $S_t(x)$ with the above simple parameterization form vanishes at $x = 0, 1$. But it does not damp faster than any power of $x$. Thus the above parametrization is proposed only for phenomenological application. The rigorous treatment should retain the integral.

### 3 Calculations

#### 3.1 The formulas of $B \to \pi$ form factors in pQCD approach

We have defined $B \to \pi$ matrix element in terms of form factors $F_{B \pi}^{+,0}$ in Eq. (1). The $B \to \pi$ matrix element can also be expressed in another form

$$\langle \pi \mid \bar{u} \gamma_{\mu} b \mid B \rangle = f_1(q^2) P_{B\mu} + f_2(q^2) P_{\pi\mu}. \quad (43)$$

From Eq. (1) and Eq. (43), we can obtain $F_{+}^{B\pi}$ from $f_{1,2}$

$$\begin{align*}
F_{+}^{B\pi} &= \frac{1}{2}(f_1 + f_2), \\
F_{0}^{B\pi} &= \frac{1}{2}(f_1 + f_2) + \frac{1}{2} \eta(f_1 - f_2). \quad (44)
\end{align*}$$

with $\eta = \frac{q^2}{m_B^2}$.

In the large recoil region, the $B \to \pi$ transition is dominated by the single gluon exchange in the lowest order as depicted in Fig. 1. The formulas for the amplitude of Fig. 1(a) and (b) are

$$\begin{align*}
A &= \frac{\pi C_F}{N_c} f_\pi f_B m_B^2 \int d\xi dx \int d^2 \mathbf{l}_\perp \int d^2 \mathbf{k}_\perp \alpha_s \int \frac{1}{(\xi \eta m_B^2 + \mathbf{k}_\perp^2)(\xi \eta m_B^2 + (\mathbf{l}_\perp - \mathbf{k}_\perp)^2)} \\
& \times 2 \left\{ \Psi \left[ (x\eta + 1)\Psi_B + (x\eta - 1)\Psi_{\bar{B}} + \frac{\Delta(\xi, l_\perp)}{m_B} \frac{\mathbf{k}_\perp \cdot (l_\perp - \mathbf{k}_\perp)}{\xi \eta m_B^2 + (l_\perp - \mathbf{k}_\perp)^2} \right] P_{\pi\mu} \\
& + \frac{\mu_\pi}{m_B} \Psi_p \left[ (\Psi_B - \bar{\Psi}_B) P_{B\mu} + 2\left( \frac{1}{\eta} \bar{\Psi}_B - x\Psi_B \right) P_{\pi\mu} \right] \right. \\
& - \frac{\mu_\pi}{m_B} \Psi_\sigma \left[ (\Psi_B - \bar{\Psi}_B) P_{\bar{B}\mu} - 2\left( \frac{1}{\eta} - x \right) \Psi_B P_{\pi\mu} \right] \right\} \quad (45)
\end{align*}$$

and

$$\begin{align*}
B &= \frac{\pi C_F}{N_c} f_\pi f_B m_B^2 \int d\xi dx \int d^2 \mathbf{l}_\perp \int d^2 \mathbf{k}_\perp \alpha_s \int \frac{1}{(\xi \eta m_B^2 + \mathbf{k}_\perp^2)(\xi \eta m_B^2 + (\mathbf{l}_\perp - \mathbf{k}_\perp)^2)} \\
& \times 2 \left\{ \Psi \left[ (x\eta + 1)\Psi_B + (x\eta - 1)\Psi_{\bar{B}} + \frac{\Delta(\xi, l_\perp)}{m_B} \frac{\mathbf{k}_\perp \cdot (l_\perp - \mathbf{k}_\perp)}{\xi \eta m_B^2 + (l_\perp - \mathbf{k}_\perp)^2} \right] P_{\pi\mu} \\
& + \frac{\mu_\pi}{m_B} \Psi_p \left[ (\Psi_B - \bar{\Psi}_B) P_{B\mu} + 2\left( \frac{1}{\eta} \bar{\Psi}_B - x\Psi_B \right) P_{\pi\mu} \right] \right. \\
& - \frac{\mu_\pi}{m_B} \Psi_\sigma \left[ (\Psi_B - \bar{\Psi}_B) P_{\bar{B}\mu} - 2\left( \frac{1}{\eta} - x \right) \Psi_B P_{\pi\mu} \right] \right\}.
\end{align*}$$
Figure 1: Lowest order hard-scattering kernel for $B\pi$ form factor

\[
\times 2 \left\{ \Psi_{\pi} \left[ \xi\eta(\Psi_B + \bar{\Psi}_B)P_{B\mu} - \xi(\Psi_B + \bar{\Psi}_B)P_{\pi\mu} \right] + \frac{\Delta(\xi, l_{\perp})}{m_B} \left( 1 - \frac{\vec{l}_{\perp}^2}{\xi\eta m_B^2 + \vec{t}_{\perp}^2} - \frac{\vec{t}_{\perp}^2 - \vec{k}_{\perp}^2 \cdot \vec{t}_{\perp}}{\xi \eta m_B^2 + (\vec{t}_{\perp} - \vec{k}_{\perp})^2} \right) P_{\pi\mu} \right\} \\
+ 2 \frac{\mu_{\pi}}{m_B} \Psi_p \left[ \left( \xi(\bar{\Psi}_B - \Psi_B) + \frac{\Delta(\xi, l_{\perp})}{m_B} \left( 1 - \frac{\vec{l}_{\perp}^2}{\xi\eta m_B^2 + \vec{t}_{\perp}^2} - \frac{\vec{t}_{\perp}^2 - \vec{k}_{\perp}^2 \cdot \vec{t}_{\perp}}{\xi \eta m_B^2 + (\vec{t}_{\perp} - \vec{k}_{\perp})^2} \right) P_{B\mu} \right) \\
+ \left( 1 - \frac{2}{\eta} \xi \bar{\Psi}_B + \Psi_B \right) P_{\pi\mu} \right\}.
\]

In hard scattering kernels, transverse momentum $k_{\perp}$ in the denominators are retained to regulate the endpoint singularity. The $k_{\perp}^2$ in the numerator are power suppressed compared to $m_B^2$ and they must be dropped in order to ensure the gauge invariance required by factorization theorem. Transform the formulas from the momentum space into transverse configuration $b$-space and include Sudakov factors, we obtain the final
formulas for $F_{+\pi}$ and $F_{0\pi}$ in pQCD approach

\[
F_{+\pi} = \frac{\pi C_F}{N_c} \int d\xi d\eta \int b_B db_B b_\pi db_\pi \alpha_s(t) \int f_\pi f_B m_B^2 \int d\xi dB \left[ \frac{\Psi_\pi(x, b_\pi)}{\eta} \left( (x\eta + 1) \Psi_B(\xi, b_B) + (x\eta - 1) \bar{\Psi}_B(\xi, b_B) \right) + \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (1 - 2\xi) \Psi_B(\xi, b_B) + \frac{2}{\eta} \bar{\Psi}_B(\xi, b_B) \right) - \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (1 + 2\xi - \frac{2}{\eta}) \Psi_B(\xi, b_B) - \bar{\Psi}_B(\xi, b_B) \right) + \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (1 - 2\xi) \Psi_B(\xi, b_B) + (1 + \xi - \frac{2}{\eta}) \bar{\Psi}_B(\xi, b_B) \right) \right] h_1
\]

and

\[
F_{0\pi} = \frac{\pi C_F}{N_c} \int d\xi d\eta \int b_B db_B b_\pi db_\pi \alpha_s(t) \int f_\pi f_B m_B^2 \int d\xi dB \left[ \frac{\Psi_\pi(x, b_\pi)}{\eta} \left( (x\eta + 1) \Psi_B(\xi, b_B) + (x\eta - 1) \bar{\Psi}_B(\xi, b_B) \right) + \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (2 - \eta - 2\xi) \Psi_B(\xi, b_B) + \eta \bar{\Psi}_B(\xi, b_B) \right) - \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (2\xi - 1) \Psi_B(\xi, b_B) - (2 - \eta) \bar{\Psi}_B(\xi, b_B) \right) + \frac{\mu_\pi}{m_B} \Psi_p(x, b_\pi) \left( (2\xi) \Psi_B(\xi, b_B) + (2 - \eta) \bar{\Psi}_B(\xi, b_B) \right) \right] h_2
\]

where

\[
h_1 = K_0(\sqrt{x\eta} m_B b_B) \left[ \theta(b_B - b_\pi) I_0(\sqrt{x\eta} m_B b_\pi) K_0(\sqrt{x\eta} m_B b_B) + \theta(b_\pi - b_B) I_0(\sqrt{x\eta} m_B b_B) K_0(\sqrt{x\eta} m_B b_\pi) \right],
\]

\[
h_2 = K_0(\sqrt{x\eta} m_B b_\pi) \left[ \theta(b_\pi - b_B) I_0(\sqrt{x\eta} m_B b_B) K_0(\sqrt{x\eta} m_B b_\pi) + \theta(b_B - b_\pi) I_0(\sqrt{x\eta} m_B b_\pi) K_0(\sqrt{x\eta} m_B b_B) \right].
\]
h_3 = K_{-1}(\sqrt{\xi x \eta m_B b_B}) \left[ \theta(b_B - b_\pi) I_0(\sqrt{\xi x \eta m_B b_B}) K_0(\sqrt{\xi x \eta m_B b_B}) + \theta(b_\pi - b_B) I_0(\sqrt{\xi x \eta m_B b_B}) K_0(\sqrt{\xi x \eta m_B b_B}) \right]. \tag{51}

The function \( K_i \) and \( I_i \) are modified Bessel functions with \( i \) their orders.

The physical quantities should not depend on the choice of the scale parameter \( t \equiv \mu \) if the calculation can be performed up to infinite orders. Therefore, the scale parameter can be chosen as any value in principle. However, in practice the calculation can only be made perturbatively. To make the perturbative expansion meaningful, the scale parameter should be chosen in such a way that can make the higher order corrections small. The natural choice is \( t = \sqrt{\xi x \eta m_B} \) in the standard approach. If \( \xi, x \to 0 \), \( \alpha_s(t) \) will be divergent at \( t \leq \Lambda_{\text{QCD}} \). When including the transverse degree of freedom, \( \alpha_s(\mu) \ln \sqrt{\xi x \eta m_B}/\mu, \alpha_s(\mu) \ln b_B \mu \) and \( \alpha_s(\mu) \ln b_\pi \mu \) will appear in higher order corrections. Therefore we take \( t = \max(\sqrt{\xi x \eta m_B}, 1/b_B, 1/b_\pi) \). The scale \( t \equiv \mu \) must be larger than \( \Lambda_{\text{QCD}} \) thus avoids the divergence of coupling constant.

The wave functions include Sudakov corrections coming from Sudakov and threshold resummations

\[
\Psi_\pi(x, b_\pi) = S_t(x) \exp(-S_\pi) \phi_\pi(x) \Sigma_\pi(b_\pi), \tag{52}
\]
\[
\Psi_B(\xi, b_B) = S_t(\xi) \exp(-S_B) \phi_B(\xi) \Sigma_B(b_B).
\]

The similar expressions are given for \( \Psi_\sigma, \Psi'_\sigma, \bar{\Psi}_B \). The jet function \( S_t \) comes from threshold resummation. The simplified parametrization form in Eq.(42) is taken to estimate threshold resummation effects in this paper. The complete Sudakov factors \( S_{\pi, B} \) are given in Eq.(37). \( \Sigma_{\pi, B} \) are intrinsic transverse momentum dependence of pion and B meson wave functions.

We compare our formulas with the results in [3, 9]. In [3], only the leading twist of pion is discussed. Set the twist-3 terms to zero, the two formulas of ours and [3] are the same except the definition of \( h_1 \). The difference comes from the Fourier transform of hard part. In [9], the single B meson wave function \( \Psi_B \) is assumed and the terms of \( \bar{\Psi}_B \) and \( \Delta \) are neglected. The twist-3 power correction is included. The momentum projector in [9] for pion meson is slightly different from our projector in Eq.(7). Except for these differences, the formulas in [9] are consistent with ours.

### 3.2 The assumption of single distribution amplitude

Before we perform analysis of \( B \to \pi \) form factors in pQCD approach, we discuss an assumption of using single B meson distribution amplitude at first. To our knowledge,
the use of B meson distribution amplitude in B decays firstly appeared in [20]. The authors suggest the simplest momentum projection for B meson which contains two terms

\[ M_B = -\frac{i f_B}{4} [\not{p} + m_B g(\xi)] \gamma_5 \phi_B(\xi). \]  

(53)

The function \( g(\xi) \) is assumed to be at the order of 1. Setting \( g(\xi) = 1 \), the B meson momentum projection reduces to

\[ M_B = -\frac{i f_B}{4} [\not{p} + m_B] \gamma_5 \phi_B(\xi). \]  

(54)

This is the widely used formula in previous pQCD analysis which contains single B meson distribution amplitude. Here, we discuss the most familiar model used in the previous pQCD analysis

\[ \phi_B(\xi) = N_B \xi^2 (1 - \xi)^2 \exp \left( -\frac{\xi^2 m_B^2}{2 \omega_B^2} \right), \]  

(55)

where \( N_B \) is the renormalization constant makes \( \int_0^1 \phi_B(\xi) d\xi = 1 \). In this model \( \phi_B(\xi) \) has a peak at \( \xi = \bar{\Lambda}/m_B \) where \( \bar{\Lambda} = m_B - m_b \). This model of B meson distribution amplitude has been used to fit different channels of B decays.

B meson distribution amplitudes are important ingredients in pQCD approach. We should be careful about the choice of B meson distribution amplitude. In HQET, the definition of B meson distribution amplitude contains two terms \( \phi_B^+, \phi_B^- \). Compare Eq.(55) and Eq.(29), the difference lies in \( 1/m_B \) effect and can be neglected. The choice of single distribution amplitude amounts to taking \( \phi_B = \phi_{B1} \) and neglects \( \phi_{B2} \). So, the validity of the assumption of single distribution amplitude depends on whether the contribution of \( \phi_{B2} \) is small or not. To test this assumption, we make approximation that \( \phi_B \approx \phi_{B1} = \phi_B^+ \) where \( \phi_B^+ \) is given in Eq.(29). This approximation is reasonable because the calculated form factor \( F^{B\pi}_B(0) \) is: 0.239 using \( \phi_B \) in Eq.(55) and 0.227 using \( \phi_B^+ \) in Eq.(29).

Table 1 shows the numerical result for the contributions from \( \phi_{B1}, \phi_{B2} \) and \( \Delta_B \). If considering only \( \phi_{B1} \) contribution, \( F^{B\pi}_B(0) \) is 0.227. This result is consistent with the one using QCD sum rule within the theoretical errors. The \( \phi_{B1} \) contribution is dominant. For \( \phi_{B2} \) term, its contribution vanish at twist-3 but cannot be neglected at twist-2. The power suppressed term of \( \Delta_B \) is about 20% in the total numerical result. From a general point, there should have two leading B meson distribution amplitudes (or generally wave functions) in pQCD framework. But the single B meson distribution amplitude \( \phi_{B1} \) is the dominant contribution. It should be noted that \( \phi_{B2} \) contribution vanishes in the hard spectator scattering and weak annihilation diagrams in \( B \rightarrow \pi\pi \) decays.
Table 1: The $B \to \pi$ form factors $F_{B\pi}^{B\pi} = F_{B\pi}^{B\pi}(0)$ with $\phi_{B1}$, $\phi_{B2}$ and $\Delta_B$. The column “$\phi_{B1}$” represents $\phi_{B1}$ contribution only. The meanings of columns “$\phi_{B2}$” and “$\Delta_B$” are similar. The column “sum” represents the summation of all these contributions.

|      | $\phi_{B1}$ | $\phi_{B2}$ | $\Delta_B$ | sum    |
|------|-------------|-------------|-------------|--------|
| twist-2 | 0.042      | 0.016      | 0.014      | 0.072  |
| twist-3 | 0.185      | 0          | 0.048      | 0.233  |
| total   | 0.227      | 0.016      | 0.062      | 0.305  |

3.3 The $B \to \pi$ form factors for different models of distribution amplitudes

The distribution amplitudes for pion depends on the renormalization scale. This dependence is controlled by the evolution equation. For the scale related to our discussion, evolution effect should be important but precise estimate of this effect depends on the unknown input parameters. In this paper, what we concern most is B meson wave functions and the reliability of pQCD framework. We will not consider the evolution effects of pion distribution amplitudes. Thus, the pion distribution amplitudes for both the twist-2 and twist-3 are taken as their asymptotic form for simplicity, i.e.,

$$\phi_\pi = 6x\bar{x}, \quad \phi_p = 1, \quad \phi_\sigma = 6x\bar{x}. \quad (56)$$

For B meson distribution amplitudes (wave functions), three models exist in literatures:

- Model I
  $$\phi_B^-(\xi) = \sqrt{\frac{2m_B}{\pi \omega_B}} \exp\left(-\frac{\xi^2 m_B^2}{2\omega_B^2}\right), \quad \phi_B^+(\xi) = \sqrt{\frac{2}{\pi}} \frac{\xi^2 m_B^3}{\omega_B^3} \exp\left(-\frac{\xi^2 m_B^2}{2\omega_B^2}\right), \quad (57)$$

- Model II
  $$\phi_B^-(\xi) = \frac{m_B}{\omega_0} \exp\left(-\frac{\xi m_B}{\omega_0}\right), \quad \phi_B^+(\xi) = \frac{\xi m_B^2}{\omega_0^2} \exp\left(-\frac{\xi m_B}{\omega_0}\right), \quad (58)$$

where $\omega_0 = \frac{2}{\sqrt{3}} \bar{\Lambda}$ and $\bar{\Lambda} = m_B - m_b$ in this model.

- Model III
  $$\Psi_B^-(\xi, k_\perp) = \frac{2\bar{\xi} - \xi}{2\pi \xi^2} \theta(2\bar{\xi} - \xi) \delta(k_\perp^2 - m_B^2 \xi(2\bar{\xi} - \xi)), \quad \Psi_B^+(\xi, k_\perp) = \frac{\xi}{2\pi \xi^2} \theta(2\bar{\xi} - \xi) \delta(k_\perp^2 - m_B^2 \xi(2\bar{\xi} - \xi)), \quad (59)$$

with $\bar{\xi} = \frac{\bar{\Lambda}}{m_B}$. 

20
Model I is proposed in [3]. It uses the equations of motion for light spectator quark in HQET. The $\omega_B$ is at the order of $\Lambda_{\text{QCD}}$. The possible range is $0.2\text{GeV} - 0.5\text{GeV}$. Model II is based on a QCD Sum rule inspired analysis [14]. The above two models are both Gaussian type and there is a peak near $\bar{\Lambda}/m_B$. Model III uses the equations of motion for both light spectator quark and heavy b quark in HQET [15]. The distribution amplitudes have a cutoff at $\xi = 2\xi$ because it takes the approximation $m_B = \infty^2$. The distributions are linear functions of $\xi$. For model I, the transverse momentum dependent function $\Sigma_B(b_B)$ is given in Eq.(31). For model II, the transverse momentum dependence part is unknown, we take the same transverse momentum dependence function $\Sigma_B(b_B)$ as model I.

The other input parameters are as follows: decay constants for pion and B meson $f_\pi = 0.13\text{GeV}$, $f_B = 0.19\text{GeV}$; $\Lambda_{\text{QCD}} = 0.25\text{GeV}$; $\bar{\Lambda} = m_B - m_b = 0.5\text{GeV}$; the oscillator parameter in the transverse momentum distribution of pion wave function $\beta^2 = 4\text{GeV}^{-2}$ [10]; parameter in B meson wave function $\omega_B = 0.35\text{GeV}$; pion twist-3 coefficient $\mu_\pi = 2.2\text{GeV}$ [8].

We present the result of form factors $F^{B\pi}_{+,0}$ at large recoil $q^2 = 0$. Using B and pion wave functions and the input parameters presented above, predictions for the $F^{B\pi}_{+,0}(0)$ with the three models for B meson distribution amplitudes are listed in Table 2.

| Model | $\phi_B$ | $\bar{\phi}_B$ | $\Delta_B$ | total |
|-------|----------|----------------|------------|-------|
| I     | 0.564    | -0.321         | 0.062      | 0.305 |
| II    | 0.641    | -0.338         | 0.060      | 0.363 |
| III   | 0.540    | -0.335         | 0.055      | 0.260 |

The predicted $B \to \pi$ form factors $F^{B\pi}_{+,0}(0)$ in model I, II and model III are around 0.3 which is favored by experiment and consistent with the prediction by other methods, such as QCD sum rule [11], BSW model [21]. The advantage of model III is that it only uses the QCD equations of motion. In reality, it is not a model. There is only one universal phenomenological parameter $\bar{\Lambda}$ in wave functions.

At the end of this subsection, we would like to discuss the power corrections. Table 3 shows that the chirally enhanced twist-3 contribution is numerically larger than leading twist contribution. The large twist-3 contribution is not consistent with the assumption.

\footnote{We thank J. Kodaira for the discussions about this topic.}
of twist expansion. The study of power corrections should be studied in a more careful and more systematic way which is beyond the scope of this paper.

Table 3: The twist-2 and twist-3 contributions of pion in $B \rightarrow \pi$ form factor $F_{B\pi} = F_{B\pi}^{+0}(0)$ with different $\omega_B$.

| $\omega_B$(GeV) | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 |
|-----------------|------|-----|------|-----|------|-----|
| twist-2         | 0.086| 0.078| 0.072| 0.066| 0.061| 0.056|
| twist-3         | 0.345| 0.280| 0.233| 0.198| 0.171| 0.150|
| total           | 0.431| 0.358| 0.305| 0.264| 0.232| 0.206|

3.4 The reliability of pQCD approach in $B \rightarrow \pi$ form factors

Now, we examine the reliability of pQCD analysis in $B \rightarrow \pi$ form factors. We choose model I of the $B$ meson distribution amplitudes for illustration. The conclusions of model II and III are similar.

The basic idea of pQCD approach is to use Sudakov effects to suppress the long-distance contribution with large transverse separations. A reliable pQCD analysis of $B \rightarrow \pi$ form factors should satisfy that most of the result comes from the region where impact parameters $b_\pi$, $b_B$ are both small. In order to study the impact parameter $b$ dependence of $B \rightarrow \pi$ form factors, we introduce a cut-off $b^c$ in impact parameters $b_\pi$ and $b_B$ in the integrals of Eq. (47) and (48) by $\int_{b^c_\pi}^{b_\pi} \int_{b^c_B}^{b_B} db_\pi db_B$. In Fig. 2, we show the dependence of $B \rightarrow \pi$ form factors $F^{B\pi} = F^{B\pi}_{+0}(0)$ on $b^c_\pi$ and $b^c_B$. Fig. 2(a) plots the $b^c_\pi$ dependence with $\omega_B = 0.35$GeV and $\mu_\pi$ varied. Fig. 2(b) plots the $b^c_\pi$ dependence with $\mu_\pi = 2.2$GeV and $\omega_B$ varied. One can see that varying these two important input parameters $\mu_\pi$ and $\omega_B$ does not change the behavior of the $F^{B\pi}$ dependence on $b^c$. Similarly, the $b^c_B$ dependence of $F^{B\pi}$ is depicted in Fig. 2(c) and (d). From Fig. 2, 50% of $F^{B\pi}$ comes from the $b^c_\pi < 1.5$GeV$^{-1}$ for impact parameter $b_\pi$ and $b^c_B < 2.0$GeV$^{-1}$ for impact parameter $b_B$. The contributions from regions with large impact parameters $b_\pi$, $b_B > 2$GeV$^{-1}$ are substantial: 31% for impact parameter $b_\pi$ and 50% for $b_B$. The calculation of pQCD approach may still include large long-distance contributions. Therefore, the reliability of leading order calculations in $\alpha_s$ expansion should be checked carefully. The more direct criteria is to check the distribution of the coupling constant $\alpha_s(t)$.

The standard to judge the reliability of perturbative analysis is that most of the contribution comes from the region where the coupling constant $\alpha_s(t)$ is small. Fig. 3 plots the form factors $F^{B\pi}$ coming from the region where $a_1 \leq \alpha_s(t)/\pi \leq a_2$. The last bar is the contribution of $\alpha_s(t)/\pi > 0.9$. In our calculation, 70% of the result comes from the
The non-perturbative contribution constitutes the intrinsic systematic error in pQCD approach.

One may also wonder if the integration over $b_B$ may not converge because the form factor $F^{B\pi}_+(0)$, as shown in Fig. 2(c) and (d), does not saturate at any value of $b_B$. Therefore, it is necessary to study the property of the dependence of $F^{B\pi}_+(0)$ on $b_B$ in the region $b_B > 1/\Lambda_{QCD}$. However, since the complete factor $e^{-S_B}$ in B meson wave function is divergent at $\Lambda_{QCD}$, we drop this term and perform the calculations again. We find that

region where $\alpha_s(t)/\pi < 0.2$ and the contribution for $t \geq 1\text{GeV}$ is 38%. If we consider the energy 1\text{GeV} is the point that the perturbation theory is broken, the non-perturbative contribution is about 60%. If we consider a weaker criterion that $\alpha_s(t)/\pi = 0.2$ is the broken point, one can see the non-perturbative contribution is 30%. No matter which criterion is chosen, the non-perturbative contribution is comparable to the perturbative part. So, the prediction of $B \to \pi$ form factor in pQCD approach can not be precise.

The non-perturbative contribution constitutes the intrinsic systematic error in pQCD approach.

Figure 2: $b_\pi$ and $b_B$ dependence of $F^{B\pi}$.
3.5 Comparison with other works

Whether $B \rightarrow \pi$ form factors are hard dominant or soft dominant is a highly controversial topic in B physics. There are many theoretical investigations using various approaches. In this paper, we have studied the perturbative method. The comparison of our result with other recent works in pQCD approach is helpful to interpret the perturbative method.

In [9], the authors calculate the $B \rightarrow \pi$ form factors in pQCD approach which includes pion twist-3 power corrections. The intrinsic transverse momentum dependence of B meson wave function is considered and of pion is neglected. Neglecting the intrinsic transverse momentum effect in pion is unjustified. The physical reason is that Sudakov suppression is not so strong for realistic B decays and the large $b$ contribution is very sensitive to the non-perturbative dynamics in hadrons. We have discussed that the model of B meson wave function chosen in [9] is nearly same as the $\phi_B^+$ in model I. So, we find an explanation of this model in HQET. The reliability of $B \rightarrow \pi$ form factor in pQCD is given in [2] by Y. Keum, H. Li and A.I. Sanda. They observed that 97% of the contribution comes from the region with $\alpha_s(t)/\pi < 0.3$. This criterion of $\alpha_s(t)/\pi < 0.3$ is not strong enough to guarantee the reliability of leading order result in $\alpha_s$ expansion series.

The authors in [3] address some critical questions about Sudakov effects in $B \rightarrow \pi$ form
The property of $F^{B\pi}$ vs. $b_B$ in $B$ meson. The dotted curve is the result with Sudakov and evolution effects in $B$ meson, the dashed one is for the case without Sudakov effect, i.e., without the contribution of $s(\xi, b_B, m_B)$ in $S_B$, and the solid curve is the result without both Sudakov and evolution effects in $B$ meson, i.e., without the total contribution of $S_B$.

Figure 4: The property of $F^{B\pi}$ vs. $b_B$ in $B$ meson. The dotted curve is the result with Sudakov and evolution effects in $B$ meson, the dashed one is for the case without Sudakov effect, i.e., without the contribution of $s(\xi, b_B, m_B)$ in $S_B$, and the solid curve is the result without both Sudakov and evolution effects in $B$ meson, i.e., without the total contribution of $S_B$.

The first question concerns theoretical issue on Sudakov form factor. For pion meson, it is pointed out that Sudakov form factor with next-to-leading-log (NLL) approximation is gauge dependent. This dependence can be cancelled by a soft function $U(b)$ which resummed the single soft logarithms [22]. A complete analysis of $B \rightarrow \pi$ form factors should include the next-to-leading-order (NLO) calculation in order to ensure gauge invariance. In this paper, we take the NLL Sudakov form factor and hard kernel to leading order, so the gauge dependence exists in our analysis. Whether this gauge dependence will lead to large numerical difference is not clear at present. The NLO calculation in pQCD approach must include the transverse degrees of freedom, therefore the calculation is difficult in technique.

The second question is about numerical dependence on the uncertainties in meson’s distribution amplitude. This is an important and inevitable problem in perturbative method. We find that the three models of $B$ meson wave functions which satisfy the QCD equations of motion give the consistent numerical results. More importantly, the $B$ meson wave functions derived in [15] is model-independent. So, the theoretical errors caused by the $B$ meson wave functions can be reduced largely.

The third question is about the reliability of pQCD approach in $B \rightarrow \pi$ form factors. The conclusion given in [3] is that Sudakov suppression is so weak that pQCD approach
cannot be applied in B physics. After including the intrinsic transverse momentum effects and threshold resummation, the perturbation behavior of the numerical result in [3] is similar to ours. Whether the perturbative calculation is reliable or not should be checked by higher order calculations in $\alpha_s$ expansion series. Our present analysis shows that one can not guarantee higher order terms will be small. More works are needed before a completely reliable method is set up to control the higher order contributions.

4 Conclusions and discussions

In this paper, we have presented a systematic analysis of $B \rightarrow \pi$ form factors in pQCD approach. The intrinsic transverse momentum effects and threshold resummation are important ingredients in the pQCD framework in B decays. One important finding in this paper is that all the three models of B meson wave functions predict $F^{B\pi}$ about 0.3 in pQCD approach. This result should not be accidental. We find that the numerical results are not so sensitive to the choice of B meson wave functions in HQET. The B meson wave functions which satisfy the QCD equations of motion can reduce the theoretical errors largely. The consistent numerical results give us confidence that our knowledge about the B meson wave functions is not so poor as expected before. HQET provides a reasonable framework to interpret the intrinsic dynamics in B meson.

Although the prediction of $B \rightarrow \pi$ form factors in pQCD approach seems favorable in experiment, we cannot avoid many serious conceptual problems of pQCD approach. The calculation in our paper is at the leading order (LO). There is no strong evidence that NLO result is small especially in our case where most of contribution comes from the momentum region of $t < 1\text{GeV}$. A critical problem in NLO calculation is how to deal with the dependence on the choice of renormalization scale $\mu$. In this paper, the scale $\mu$ is taken as the largest momentum carried by the exchanged gluon. Whether this choice is best or not depends on the NLO calculation. However, as we have discussed, the NLO calculation in $B \rightarrow \pi$ form factors is technically difficult. Another problem comes from power corrections. We only discuss the chirally enhanced twist-3 correction. There are many other power corrections, such as twist-4, higher Fock states etc. Although one can argue that they are power suppressed in the heavy quark limit, we don’t know their numerical effects in the realistic case. In principle, all the above issues should be treated in a more systematic way than the present form. The seeming correspondence of the prediction of pQCD approach in $B \rightarrow \pi$ form factors with other theoretical expectations is based on many assumptions and one must be careful to obtain a conclusion. To some extent, we think it is difficult to estimate the theoretical errors in the present pQCD
approach in B decays.

Although there are many crucial problems in pQCD approach, completely rejecting it should be with caution. There are at least 30% hard contribution coming from the momentum $t > 1 \text{GeV}$ in $B \to \pi$ form factors. This contribution cannot be calculated reliably in any other present theoretical approaches. We cannot throw one method before we get the final answers.

There is no universal, satisfactory criteria of QCD because we don’t know how to quantitatively calculate the non-perturbative dynamics especially in exclusive, non-leptonic B-meson decays at present. The complicated strong interactions in exclusive B decays where both perturbaive and non-perturbative dynamics contribute is a challenging task for the present non-perturbative methods, such as QCD sum rules, lattice QCD etc.

In conclusion, $B \to \pi$ form factors provide an interesting places to study the perturbative and non-perturbative strong interactions. The success of B meson wave functions in HQET is helpful to understand the exclusive B decays. The perturbative method provides a simple phenomenological method to estimate the perturbative aspects of strong dynamics in B decays.

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Appendix

In this appendix we will derive the projector for $B$ meson in the momentum space with including transverse momentum. We begin with the generalized Lorentz decomposition of the light-cone matrix element [14, 3]

$$M(z)_{\alpha\beta} \equiv \langle 0 | \bar{q}_s(z) b_\alpha(0) | \bar{B}(p_B) \rangle = -i F_B \left\{ \frac{\not{p}_B + m_B}{2} \left[ 2 \bar{\Psi}_B^+(z^2, t) + \frac{\bar{\Psi}_B^-(z^2, t) - \Psi_B^+(z^2, t)}{t} \right] \gamma_5 \right\}_{\alpha\beta}, \quad (60)$$
with \( v = \frac{p_B}{m_B} \) and \( t = v \cdot z \). In light-cone coordinate \( z = (\frac{z^+}{\sqrt{2}}, \frac{z^−}{\sqrt{2}}, z_⊥) \), with \( z_± = z_0 ± z_3 \).

To obtain the projector in the momentum space we consider the amplitude of one process, \( T = \int \frac{d^4z}{(2\pi)^4} M(z)A(z) \),

where \( A(z) \) is the hard scattering kernel of the relevant process. Then

\[
T = \int \frac{d^4z}{(2\pi)^4} M(z) \int d^4l e^{-it \cdot z} A(l) = \frac{1}{4} \int dz_+ dz_- d^2z⊥ M(z) \int dl_+ dl_− d^2l⊥ e^{-i\frac{\vec{l}_+ \cdot \vec{l}_-}{2} - \vec{l}_⊥ \cdot \vec{z}_⊥} A(l_+, l_−, l⊥). \tag{62}
\]

For the case that \( A(l) \) is independent of \( l_− \), we have \( A(l) = A(l_+, l⊥) \). Then the integration over \( l_− \) in the above equation can be accomplished to get a delta function \( \delta(z_+/2) \).

Therefore we get

\[
T = \frac{1}{2} \int \frac{dz_- d^2z⊥}{(2\pi)^3} M(z)|_{z_+=0} \int dl_+ d^2l⊥ e^{-i\frac{\vec{l}_+ \cdot \vec{l}_-}{2} - \vec{l}_⊥ \cdot \vec{z}_⊥} A(l_+, l⊥). \tag{63}
\]

So we just need to consider the case \( z_+ = 0 \). For convenience the above equation can be written in the form

\[
T = \int dl_+ d^2l⊥ \frac{1}{2} \int \frac{dz_- d^2z⊥}{(2\pi)^3} e^{-i\frac{\vec{l}_+ \cdot \vec{l}_-}{2} - \vec{l}_⊥ \cdot \vec{z}_⊥} M(z)|_{z_+=0} A(l_+, l⊥). \tag{64}
\]

Introduce Fourier transformation:

\[
\Psi_B^\pm(l_+, l⊥) = \frac{1}{2} \int \frac{dz_- d^2z⊥}{(2\pi)^3} e^{-i\frac{\vec{l}_+ \cdot \vec{l}_-}{2} - \vec{l}_⊥ \cdot \vec{z}_⊥} \tilde{\Psi}_B^\pm(\vec{z}_⊥^2, z_-/2), \tag{65}
\]

and substitute eq.(60) into eq.(64), we can obtain

\[
T = \int dl_+ d^2l⊥ \frac{-i f_B}{4} \left[ \frac{\not p_B + m_B}{2} \left\{ 2\Psi_B^+(l_+, l⊥) + \frac{1}{2} \int \frac{dz_- d^2z⊥}{(2\pi)^3} e^{-i\frac{\vec{l}_+ \cdot \vec{l}_-}{2} - \vec{l}_⊥ \cdot \vec{z}_⊥} \tilde{\Psi}_B^+(\vec{z}_⊥^2, z_-/2) \right\} \frac{\not \gamma_5}{\not z_-/2} \right] A(l_+, l⊥). \tag{66}
\]

By defining \( \Delta(l_+, l⊥) = \int_0^{l^+} dl (\Psi_B^+(l, l⊥) - \Psi_B^+(l, l⊥)) \) and making the integrations by part, we can finally get

\[
T = \int dl_+ d^2l⊥ \frac{-i f_B}{4} \left[ \frac{\not p_B + m_B}{2} \left\{ 2\Psi_B^+(l_+, l⊥) - \Delta(l_+, l⊥) \gamma_\mu \frac{\partial}{\partial l^\mu} \right\} \frac{\not \gamma_5}{\not z_-/2} \right] A(l_+, l⊥). \tag{67}
\]

To deal with \( \frac{\partial}{\partial l^\mu} \), we need to re-express \( l^\mu \) as

\[
l^\mu = \frac{l^+}{2} n^\mu_+ + \frac{l}{2} n^\mu_- + l^\mu⊥,
\]

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therefore we can get
\[
\frac{\partial}{\partial l^\mu} = n_\mu^\nu \frac{\partial}{\partial l_+^\nu} + n_\mu^\nu \frac{\partial}{\partial l_-^\nu} + \frac{\partial}{\partial l_\perp^\mu}.
\]
(68)

Substitute eq.(68) into eq.(67) and do the integration by part and drop the surface term, we get
\[
T = \int dl_+d^2l_\perp \frac{-if_B}{4} \left[ \frac{\not{p}_B + m_B}{2} \left\{ \Psi_B^+(l_+, l_\perp)\gamma_+ + \Psi_B^-(l_+, l_\perp)\gamma_- - \Delta(l_+, l_\perp)\gamma^\mu \frac{\partial}{\partial l_\perp^\mu} \right\} \gamma_5 \right] \cdot A(l_+, \vec{l}_\perp).
\]
(69)

So the projector for $B$ meson in the momentum space is
\[
M_{\alpha\beta} = -\frac{if_B}{4} \left[ \frac{\not{p}_B + m_B}{2} \left\{ \Psi_B^+(l_+, l_\perp)\gamma_+ + \Psi_B^-(l_+, l_\perp)\gamma_- - \Delta(l_+, l_\perp)\gamma^\mu \frac{\partial}{\partial l_\perp^\mu} \right\} \gamma_5 \right]_{\alpha\beta}.
\]
(70)

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