Calibration of thermopile heat flux gauges using a physically-based equation

Oliver J Pountney, Mario Patinios, Hui Tang, Dario Luberti, Carl M Sangan, James A Scobie, J Michael Owen and Gary D Lock

Abstract
A thermopile, in which a number of thermocouple junctions are arranged on either side of a thin layer of insulation, is commonly used to determine the heat flux for steady-state measurements. Gauges using this method are available commercially and a new, generic calibration method is described here. For this purpose, an equation based on physical properties has been derived to determine the theoretical relationship between the measured voltage output of the gauge and the heat flux through it. An experimental rig has been built and used to calibrate gauges under steady-state conditions for heat fluxes between 0.5 and 8 kW/m². The gauge temperature was controlled between 30 and 110°C, and voltage-flux correlation – based on the theoretical relationship – was determined using maximum likelihood estimation (MLE). For tests with constant gauge temperature, there was a linear relationship between the voltage and heat flux; owing to the temperature dependency of the Seebeck constants of the thermoelectric materials, the voltage increased with increasing gauge temperature. In all cases, there was very good agreement between the measured and correlated values, and the overall uncertainty of the correlation was estimated to be less than 5% of the measured heat flux.

Keywords
Flux gauge calibration, heat flux measurement, conduction calibration, maximum likelihood estimation

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Introduction
The heat flux gauge described in this paper was chosen for use in a rotating-cavity rig, simulating an aero-engine compressor where the radial growth and operating-life of highly-stressed rotating discs depend critically on the metal temperatures. Childs et al. described a variety of methods that can be used to determine heat flux. Principally the physical methods are based on measurements of temperature differences, spectral emissions or the rate of change of surface temperature.

Transient methods
Turbomachinery applications of heat flux gauges are often employed under transient conditions. Thin-film resistance thermometers mounted directly on acrylic surfaces, or on either side of a polyimide insulation layer, have been used to measure the temperature history of models subjected to transient changes in temperature, usually in bespoke short-duration experiments. Heat flux is calculated from unsteady surface-temperature measurements using numerical solutions of the linear differential equations with controlled boundary conditions. Oldfield describes an impulse response processing method, subsequently applied to a gauge used by Guo et al. Piccini et al. discussed the development and calibration of a two-sided, direct-heat-flux-gauge using an insulating layer bonded to a metallic substrate. The heat fluxes measured were of the order of 25 kW/m², over temperatures ranges of less than 100°C. The experimental uncertainty of the calibration techniques (typically ±4–7%) was limited by the variation of the thermal properties of the materials used with temperature.

Nickol et al. used a double-sided gauge consisting of a Kapton (polyimide) insulating layer (51-μm thick) sandwiched between two nickel resistance thermometers. This was used to measure high-frequency response (>100 kHz) heat flux on the platform of a transient, cooled transonic turbine stage with
Transient temperatures up to 280°C. Time-resolved heat-flux was determined numerically using a one-dimensional transient heat transfer model of the Kapton layer with the temperature sensors providing boundary conditions. The method for data reduction and uncertainty analysis is described by Nickol et al.7 Siroka et al.8 described a calibration method for nano-fabricated gauges and compared well-established heat flux gauges developed for short-duration facilities to gauges designed to be used in continuous-duration facilities.

One dimensional transient conduction analytical solutions have been applied to turbomachinery flows at the University of Bath, with surface temperatures measured using thermochromic liquid crystals9 and infra-red sensors.10,11 Most experimenters use the solution of Fourier’s equation for a semi-infinite substrate with a step-change in the temperature of the fluid to determine the convective heat transfer coefficient; the adiabatic surface temperature can also be obtained, but this is an error-prone method suitable only for experiments with relatively large values of Bi, the Biot number. Pountney et al.9 showed that for Bi > 2, which covers most practical cases, more accurate results could be achieved using a composite substrate of two materials. Cho et al.10 described a new method to determine the heat transfer coefficient and adiabatic-surface temperature without having to measure the air temperature; here maximum likelihood estimation (MLE) was used in conjunction with Fourier’s equation, which also provided the 95% confidence intervals. Validation experiments were conducted in a small purpose-built wind tunnel, and there was good agreement with empirical correlations for turbulent flow over a flat plate. A further technique was presented by Tang et al.11 where transient air and surface temperatures were extrapolated to steady-state conditions, obtained using an MLE analysis. This technique was used to determine the adiabatic effectiveness on a rotating turbine disc rig.

Steady-state methods

A thermopile, in which a number of thermocouple junctions are arranged on either side of a thin layer of insulation, is commonly used to determine the heat flux for steady-state measurements, and gauges using this method are available commercially. Pullins and Diller12 described the calibration of a thermopile heat flux gauge capable of operating at temperatures up to 1000°C. Gifford et al.13 developed a heat-flux gauge for high temperature (>1000°C) and high heat flux (130 kW/m²) conditions. The sensor used K-type thermocouple materials in a thermopile arrangement and a one-dimensional thermal-resistance model was used to determine the steady-state sensitivity.

Tang et al.14 recently developed new methods of steady-state analysis based on Bayesian statistics and this method will be applied to the conjugate problem of buoyancy-induced heat transfer in a new aero-engine compressor experimental facility at the University of Bath, where heat flux gauges would provide steady-state data within rotating cavities. During an aeroengine transient, the compressor discs can accelerate in a few seconds, but – owing to the buoyancy-induced flow inside the cavity15 – the temperature of the discs can take tens of minutes to reach a steady-state. As the transient disc heat transfer corresponds to quasi-steady flow, thermopile flux gauges could also be used for the slow transient that occurs inside these cavities.

This paper describes the calibration of thermopile heat flux gauges designed for such steady-state measurements, where the temperature difference (ΔT) across the thickness (Δr) of an insulating layer is used to determine the heat flux. Thermopile gauges manufactured by the RdF Corporation (model 27,160-C-L-A01) were selected for this application but, owing to the relatively low temperatures and small heat fluxes (around 100 °C and 8 kW/m²), they had to be calibrated individually. For this purpose, an equation based on physical properties has been derived to show the theoretical relationship between the measured voltage drop across the gauge and the heat flux through it. The relationship between voltage and flux is shown to depend on the number of thermocouple junctions, the Seebeck characteristics of the thermoelectric materials, and the thermal conductivity and thickness of the gauge material. Consequently, the voltage output of the gauge depends not only on the heat flux but also on the temperature of the gauge material. Using the derived equation, a correlation between the voltage and both the heat flux and the temperature of the gauge can be found using maximum likelihood estimation (MLE) applied to the experimental measurements.

The next section describes the derivation of the voltage-flux equation for a thermopile gauge based on the construction and physical characteristics of the gauge material. The Calibration method section describes the apparatus used for the calibration. The Calibration results section compares the experimental measurements with the correlated values, including a comprehensive uncertainty analysis, and the Conclusions section summarises the principal conclusions. Appendix A describes the finite element method used to confirm that the heat flux through the calibrated gauge was uniform, and Appendix B outlines the MLE method used in the correlation of the experimental data.

Construction, sensitivity and calibration equation for thermopile heat flux gauges

Construction of thermopile gauges

The schematic in Figure 1(a) shows the construction of a typical thermopile heat flux gauge. This type of
sensor measures the temperature difference (ΔT) across an internal layer of known conductivity (ki) and thickness (Δyi). The internal layer is sandwiched between protective layers. Assuming one-dimensional conduction through the internal layer, Fourier’s law can be applied to obtain the heat flux (q):

\[ q = -k_i \frac{\Delta T_i}{\Delta y_i} \]  

(1)

The measurement of ΔT is typically made using the thermopile, which comprises sets of thermo-element pairs connected in series to form n thermocouple junctions on the upper and lower surface of the internal layer – a thermopile comprising three pairs of thermo-elements is shown in Figure 1(a) by way of example. The difference in temperature between the thermocouple junctions on the upper surface and lower surface will generate a voltage output through the Seebeck effect. The voltage output V of the gauge is therefore a function of the heat flux passing through it, but it will also be affected by the surface temperature.

The sensitivity, S, of the gauge is given by the equation:

\[ S \overset{\text{def}}{=} \frac{V}{q} = \frac{\Delta y_i n S_{AB}}{k_i} \]  

(2)

where \( S_{AB} \) is the relative Seebeck coefficient for the combination of thermo-element materials A and B. Type-T thermo-element pairs (copper-constantan) are commonly used as they are readily available and have a large Seebeck coefficient. It is apparent from equation (2) that to maximise the sensitivity of the gauge it is necessary to manufacture the thermopile with as many thermo-element pairs (n) as practicable and to use a thick internal layer made from an insulator (i.e. large \( \Delta y_i \) and low \( k_i \)). While increasing the thickness and reducing the thermal conductivity of the internal layer helps to increase the sensitivity of the gauge, there will be an associated (and unwanted) increase in disturbance of the thermal field in the material in which the gauge is installed (unless that material happens to also be an insulator). In commercial gauges the internal layers are typically \( 0.05 < \Delta y_i < 0.2 \) mm and are manufactured from polyimide films, which have thermal conductivities in the range \( 0.1 \) W/mK < \( k_i < 0.35 \) W/mK.

The RdF heat flux gauges used here have an overall thickness of 0.18 mm and consist of 54 pairs of T-type thermocouples. The surface area of the gauge was \( 6.9 \times 10^{-2} \) m\(^2\) (690 mm\(^2\)), and the protective layers and the internal layer are made of polyimide.

**Gauge sensitivity**

The gauge is modelled as a three-layer composite substrate with perfect thermal contact between layers (i.e. no contact resistance is included). The steady-state temperature profile for this arrangement is shown in Figure 1(b) for the case where the lower surface temperature (\( T_{s1} \)) is hotter than the upper surface temperature (\( T_{s2} \)). It is assumed that the thermopile junction temperatures, denoted by \( T_{s1} \) and \( T_{s2} \) respectively in Figure 1(b), are located at the interfaces between the internal and protective layers. For convenience, it is assumed that the thicknesses of the lower and upper protective layers, \( \Delta y_{s1} \) and \( \Delta y_{s2} \) respectively, are equal (i.e. \( \Delta y_{s1} = \Delta y_{s2} \)) and that they are made from the same material as the internal layer, which provides a uniform thermal conductivity throughout the gauge (i.e. \( k_{s1} = k_{s2} = k_i = k \)).

Tabulated reference data\(^{16}\) are typically used to convert thermocouple voltage measurements to hot junction temperatures. The tabulated reference data are applicable to thermocouples with a cold junction temperature of 0°C. Childs\(^{17}\) provides a detailed discussion on the process of calculating a hot junction temperature for cases where the cold junction is not at 0°C. This process involves converting the cold junction temperature, which is measured using a calibrated peripheral device, such as a PRT, to a representative cold junction thermocouple voltage (\( V_C \)) using the tabulated reference data (note that by definition, \( V_C = 0 \) mV if the cold junction is at 0°C). The sum of the representative cold junction voltage and the measured thermocouple output

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**Figure 1.** Schematic of a typical thermopile heat flux gauge with three thermo-element pairs showing (a) construction and (b) thermal profile for one-dimensional conduction.
voltage \( V \) gives a representative hot junction voltage \( V_H \), i.e.
\[
V_H = V + V_C
\] (3)

Finally, \( V_H \) is used with the tabulated reference data to determine the hot junction temperature. The tabulated reference data is published in 1 °C increments. Where more resolute measurements are required, polynomial fits are used to convert between voltage and temperature. Ignoring higher order terms, \( V_H \) and \( V_C \) can thus be expressed as
\[
V_H = C_1 T_H + C_2 T_H^2
\] (4)
and
\[
V_C = C_1 T_C + C_2 T_C^2
\] (5)

where \( C_1 \) and \( C_2 \) are coefficients obtained from the fits to the tabulated reference data (these will differ between thermocouple types) and \( T_H \) and \( T_C \) are the hot and cold junction temperatures respectively.

Equations (3) to (5) relate the measured thermocouple voltage to the hot and cold junction temperatures as follows
\[
V = (C_1 T_H + C_2 T_H^2) - (C_1 T_C + C_2 T_C^2)
\] (6)

The voltage output from a thermoelectric pair with \( T_H = T_{i,1} \) and \( T_C = T_{i,2} \) can thus be written as
\[
V = -(C_1 + 2C_2 \bar{T}) \Delta T_i
\] (7)

where \( \Delta T_i \) and \( \bar{T} \) are the temperature difference across the internal layer and the mean temperature
\[
\Delta T_i \overset{\text{def}}{=} T_{i,2} - T_{i,1}
\] (8)
and
\[
\bar{T} \overset{\text{def}}{=} \frac{T_{i,1} + T_{i,2}}{2}
\] (9)

Note that the ratio between \( V \) and \( \Delta T_i \) is equivalent to the Seebeck coefficient for the thermocouple. Hence, dividing equation (7) through by \( \Delta T_i \) gives the following expression for the Seebeck coefficient
\[
S_{AB} = -\frac{V}{\Delta T_i} = C_1 + 2C_2 \bar{T}
\] (10)

For the T-type thermocouple used in the gauges here, values of \( C_1 = 38.64 \mu V/°C \) and \( C_2 = 0.0415 \mu V/°C^2 \) were obtained by fitting equation (4) to tabulated temperature-voltage reference data in \(^{16}\), using these coefficients, equation (10) provided good agreement with tabulated Seebeck coefficients obtained from the same data source (with a standard uncertainty of \( \pm 0.1 \mu V/°C \)).

The voltage output for a thermopile constructed from \( n \) thermoelectric pairs is found by multiplying equation (7) by \( n \), i.e.
\[
V = -n(C_1 + 2C_2 \bar{T}) \Delta T_i
\] (11)

For the gauges calibrated in this paper, \( n = 54 \).

Recall from equation (1) that for one-dimensional conduction
\[
q = -k \frac{\Delta T_i}{\Delta y_i}
\] (12)

Rearranging equation (11) and substituting into equation (12) gives
\[
q = \frac{k}{n(C_1 + 2C_2 \bar{T})} V
\] (13)

where \( \bar{k} \) is the ratio of thermal conductivity to thickness hence
\[
\bar{k} \overset{\text{def}}{=} \frac{k}{\Delta y_i}
\] (14)

It follows
\[
S = \frac{n(C_1 + 2C_2 \bar{T})}{k}
\] (15)

Note that equation (15) can also be found by substituting the Seebeck coefficient from equation (10) into the definition of sensitivity provided in equation (2).

The appendix describes the MLE method used to estimate \( \bar{k} \) from the measured data using equation (15), after which the relationship between \( S \) and \( \bar{T} \) can be obtained. Equation (13) can be used to correlate the experimental data when \( \bar{T} \) is known.

It is apparent from equation (13) that to measure heat flux using a thermopile gauge requires measurement of both \( V \) and \( \bar{T} \). Provided that the lateral surface temperature gradient is negligible – a prerequisite to the assumption of one-dimensional conduction necessary to use this type of gauge – then \( \bar{T} \) can be inferred from a single temperature measurement from a surface thermocouple installed laterally to the gauge. This approach is discussed below.

Consider the scenario where a gauge is attached to a test piece subjected to convective heat transfer, as shown in Figure 2. A thin foil thermocouple is bonded to the surface of the test piece laterally to the gauge (x-direction in the figure). It is assumed...
that the lateral temperature gradient in the test piece is negligible, i.e. \( \frac{\partial T}{\partial x} \approx 0 \). The gauge and thermocouple are encapsulated in a layer made from the same polyimide film as the gauge – this setup minimises any lateral disturbance resulting from the mismatch in thermal conductivities between the test piece and gauge.

All the three layers in the gauge are made of a common material with conductivity, \( k \). Equating the heat flux through the entire gauge to that through the internal layer gives

\[
\frac{\Delta T_i}{\Delta y_i} = \frac{\Delta T}{\Delta y} = \frac{T_{s,2} - T_{s,1}}{\Delta y_{s,2}} = \frac{T_{s,1} - T_{s,1}}{\Delta y_{s,1}}
\]

which

\[
\Delta T \overset{\text{def}}{=} T_{s,2} - T_{s,1}
\]

It is convenient at this point to define the ratio of the internal layer thickness to overall thickness of the gauge as

\[
R \overset{\text{def}}{=} \frac{\Delta y_i}{\Delta y}
\]

For a gauge with \( \Delta y_{s,1} = \Delta y_{s,2} \), equations (16) to (18) can be used to rewrite equation (9) as

\[
\tilde{T} = \frac{T_{s,2} + T_{s,1}}{2} = T_{s,1} + \frac{\Delta T_i}{2R}
\]

Replacing \( \Delta T_{s,2} \) using the equation above gives

\[
\tilde{T} = T_{s,1} + \frac{\sqrt{(T_{s,1} + C_1) - \frac{V}{nR} - (T_{s,1} + C_2)}}{2}
\]

Finally, equation (20) can be substituted into equation (15) to give the new calibration equation for the gauge

\[
S = \frac{V}{q} = \frac{\bar{q}(C_1 + 2C_2T_{s,1}) + \frac{V}{nR}[(C_1 + 2C_2T_{s,1})^2 - 4C_2V]}{k}
\]

After \( \bar{k} \) is determined from the calibration of the sensitivity, and if \( R \) is known, the gauge sensitivity \( S \) can be obtained from equation (21) using the measured values of \( V \) and \( T_{s,1} \). If \( R \) is unknown – which may be the case if a proprietary commercial gauge is used – then its value needs to be determined. A method for doing this is described in the Calibration results section.

**Calibration method**

Calibration measurements were simultaneously made for two RdF thermopile heat flux gauges of the same type (model 27,160-C-L-A01) using the experimental arrangement shown in Figure 3. Heat was transferred to, through and from the gauges exclusively via conduction, rendering this setup a ‘conduction calibration’, analogous to the guarded hot plate systems discussed in reference.18 The setup enabled \( q \) and \( T \) to be independently controlled and measured alongside \( V \) and \( T_{s,1} \), providing data to validate the theory proposed in the Construction, sensitivity and calibration equation for thermopile heat flux gauges section. Associated results are presented in the Calibration results section.

The calibration configuration comprised a low thermal conductivity Rohacell block (\( k = 0.03 \) W/mK) in which the two heat flux gauges and a 50 mm x 50 mm thin film resistance heater were installed. The gauges were located laterally to one another and were sandwiched between two copper plates of thickness 10 mm. K-type thermocouples were embedded...
within the copper plates to measure $T_{s,1}$ and $T_{s,2}$. The thermocouples were calibrated in a water bath against the output of a platinum resistance thermometer (PRT) pre-calibrated to within $\pm 0.1^\circ$C. A National Instruments CompactDaq system fitted with an NI9213 module (24-bit ADC with $<0.02$ °C measurement sensitivity in high-resolution mode) was used to acquire the voltage output of the heat flux gauges ($V$) and thermocouples. The acquisition rate for all signals was $5$ Hz. The top-surface of the upper copper plate was exposed to the surroundings through a cut-out in the Rohacell; the bottom-surface of the lower copper block was placed in contact with the thin film resistance heater. Silicone grease was used to minimise the contact resistance between the gauges, copper plates and the heater (which were neglected in the analysis in the Construction, sensitivity and calibration equation for thermopile heat flux gauges section).

The thin film resistance heater supplied the heat input to the system, with fluxes of up to $8$ kW/m$^2$ generated through the gauges. Power to the heater was provided by a DC supply with an uncertainty of $0.06\%$ and $0.2\%$ for the voltage ($V_h$) and current ($I$) respectively. Given the low thermal conductivity of the surrounding Rohacell block, most of the provided heat was conducted through the copper plates and heat flux gauges, before being transferred via convection to an air jet impinging on the free surface of the upper copper block. The velocity of the jet was regulated so that the convective heat transfer coefficient could be varied on the exposed copper surface. This enabled the gauge temperature $T$, which is determined from averaging $T_{s,1}$ and $T_{s,2}$, to be controlled independently of $q$ between ambient and $110$ °C.

The large thermal conductivity of the copper blocks minimised their internal temperature gradients. This was particularly important in the upper block as the jet would have produced a lateral variation in heat flux over the impinged surface. This variation had the potential to propagate through the block and disturb the uniformity of the heat flux profile at the gauges. A simplified finite element model (FEM), details and results of which are provided in Appendix A, showed that for the strongest jet and highest heat flux case, the lateral variation of the component of heat flux normal to the gauge was insignificant (approximately $\pm 0.25\%$ of the laterally-averaged heat flux).

Some of the heat generated by the thin film heater was lost through the base of the Rohacell block. The level of heat loss was estimated in a series of experiments where the cut-out in the Rohacell block was filled with a Rohacell plug. With the plug in place, the thin film heater was used to heat the copper plates to a series of discrete values of $T$. At each value of $T$ the input power to the thin film heater was measured, indicating the total rate of heat loss from the system, $\dot{Q}_{l,tot}$, as shown in Figure 4. The measured heat loss was correlated by a linear fit using Maximum Likelihood Estimation (MLE), so that

$$\dot{Q}_{l,tot} = 0.0197(\bar{T} - T_\infty)$$

where $T_\infty$ is the temperature of the ambient air which is measured using the pre-calibrated PRT.

The voltage output from a single flux gauge was measured during one of the heat loss experiments to estimate the proportion of the heat loss transferred through the gauge layer. The difference between this loss and the total loss is the heat loss below the gauge layer $\dot{Q}_l$. It was calculated that $\dot{Q}_l$ was approximately $56\%$ of the total heat loss, so that

$$\dot{Q}_l = 0.56\dot{Q}_{l,tot}$$
The uncertainty of \( \dot{Q}_l \) is discussed in the Uncertainty analysis for sensitivity calibration section.

The total rate of heat transfer through the gauge layer during the calibrations was thus determined from

\[
\dot{Q} = P - \dot{Q}_l \tag{24}
\]

where \( P \) was the electrical power supplied to the thin film heater (\( P = V_hI \)) and \( \dot{Q}_l \) was determined from the measurement of \( T \) using equations (22) and (23).

Finally, the heat flux through the gauges, which was calculated from

\[
q = \frac{\dot{Q}}{A_e} \tag{25}
\]

where \( A_e = 0.00248 \text{ m}^2 \) is the effective area of the copper plates.

**Calibration results**

Eighty-five calibration steady-state tests were conducted for a range of surface temperatures (30°C < \( T_{s,1} < 110°C \)) and heat fluxes (0.5 < \( q < 8 \text{ kW/m}^2 \)).

**Gauge sensitivity**

As shown in equation (2), the gauge sensitivity \( S \) can be determined from the ratio of the measured gauge voltage \( V \) to the heat flux through the gauge \( q \) determined from equation (25). The 85 values of the measured sensitivity at different gauge temperatures \( T \) are shown in Figure 5. The sensitivity is seen to vary linearly with \( T \), and there is an approximate 15% increase in \( S \) as \( T \) increases from 30°C to 100°C.

As discussed in the Gauge sensitivity section under the Construction, sensitivity and calibration equation for thermopile heat flux gauges section, the sensitivity can be correlated using equation (15), and MLE was used to estimate the parameter \( \tilde{k} \) and the randomness of \( S, \sigma_r(S) \), which represents the repeatability of measurements. More details about MLE are given in Appendix B, and the correlated curves of the sensitivity using the estimated \( \tilde{k} \) are plotted in Figure 5. There is good agreement between the measured and correlated sensitivity for both gauges. The estimated values of \( \tilde{k} \) for both gauges are given in Table 1. The major source of the uncertainty of the measured sensitivity was from the estimation of the heat losses. The outliers at 80°C were measured at low heat fluxes and high gauge temperatures, hence the relative effect of the heat loss is maximised. The estimated uncertainty of the heat loss was able to capture this effect, hence the outliers lied within the uncertainty bound.

**Uncertainty analysis for sensitivity calibration**

The sensitivity is determined from the measured gauge voltage \( (V) \) and the heat flux through the gauge \( (q) \), which is in turn determined by the voltage \( (V_h) \) and current \( (I) \) of the heater, the heat loss below the gauge \( (\dot{Q}_l) \) and the area of the copper block \( (A_e) \). Each quantity used to determine the gauge sensitivity

![Figure 4. Variation of total heat loss with temperature difference between copper block and ambient air.](image)

![Figure 5. Calibration curves for gauge sensitivity (Symbols denote experimental data; solid line denotes theoretical fit based on equation (15) using MLE; broken line denotes 95% uncertainty of S).](image)
Table 1. Gauge properties.

|                | Gauge a | Gauge b |
|----------------|---------|---------|
| \( \Delta y \) (mm) | 0.18    |         |
| \( n \)         | 54      |         |
| \( C_1 \) (\( \mu V/°C \)) | 38.6    |         |
| \( C_2 \) (\( \mu V/°C^2 \)) | 0.0415  |         |
| \( k \) (W/mK)  | 0.21    | 0.21    |
| \( u(k) \) (W/mK) | 0.013   | 0.013   |
| \( k \) (W/m²K) | \( 2.2 \times 10^3 \) | \( 2.2 \times 10^3 \) |
| \( u(k)/W/m²K \) | \( 0.048 \times 10^3 \) | \( 0.049 \times 10^3 \) |
| \( R \)         | 0.53    | 0.52    |
| \( u(R) \)      | 0.038   | 0.038   |
| \( \Delta y \) (mm) | 0.095   | 0.094   |

The calculated thermal conductivity with its 95% uncertainty was \( 0.21 \pm 0.026 \) W/(mK). This value is consistent with those given in the literature.\(^{20-22}\) The approach in the Uncertainty analysis for sensitivity calibration section was also used to estimate the standard uncertainty of the thermal conductivity, \( u(k) \), and the values for both gauges are given in Table 1.

From the estimated values of \( \tilde{k} \), the thickness ratio for both gauges, \( R \), can be calculated from equation (21). The MLE process was used to determine the \( R \) values for both gauges. These values, which are listed in Table 1, are close to 0.5, which is consistent with the information on the gauge manufacturing process provided by RdF. The standard uncertainties of \( R \), \( u(R) \), are also listed in Table 1.

**Comparison between heat flux measurements and correlations**

As shown in the Gauge sensitivity section under the Calibration results section, the gauge sensitivity is a function of \( T \); hence, for a fixed \( T \), the voltage generated from the gauge increases linearly with the heat flux through the gauge. Figure 6 shows the comparison between the measured and correlated variation of voltage with heat flux for tests in which the gauge was maintained at a constant temperature. Note that it was not possible to obtain measurements for \( q > 4 \) kW/m\(^2\) at \( T = 30^°C \) owing to limitations in the obtainable velocity of the impinging air jet. It can be seen from the figure that the variation is linear and that there is very good agreement between the
measured and correlated values for both gauges. It is clear that the effect of $T$ on the voltage output of the gauges is significant, with 10% increase in $V$ as $T$ was raised from 30°C to 80°C.

It can be seen from Figure 7 that the heat fluxes correlated by equation (21) are in excellent agreement with the experimental data, which supports the theory proposed in the Construction, sensitivity and calibration equation for thermopile heat flux gauges section.

Conclusions

An equation for the voltage output of a thermopile heat flux gauge has been derived using the physical properties of the gauge materials. It is shown that the voltage-flux relationship depends on the number of thermocouple junctions and on the thermal conductivity and thickness of the insulating material separating the junctions. Importantly, the relationship also depends on the temperature-dependent Seebeck constants of the thermoelectric materials. The two RdF flux gauges that were calibrated each had 54 junctions made from copper-constantan pairs, and the Seebeck constants were based on published values. The thickness and thermal conductivity of the polyimide insulating film were determined for each gauge using a maximum likelihood estimate (MLE) based on the experimental measurements.

An experimental rig was used to calibrate the gauges, which were sandwiched between two horizontal copper blocks insulated with Rohacell foam. The lower block was heated by an electric element, and the upper one was cooled by an air jet; this allowed the temperature and heat flux to be independently controlled. A two-dimensional finite-element analysis of the calibration rig confirmed that the heat flux through the copper blocks was uniform. The heat loss below the lower copper block was used to correct the heat flux measured in the calibration tests. For the steady-state tests reported here, the corrected heat flux was between 0.5 and 8 kW/m² for gauge temperatures between 30°C and 110°C.

Voltage-flux correlations for the gauges were obtained using MLE based on the theoretical equations. For tests with constant gauge temperature, there was a linear relationship between the voltage and heat flux; owing to the temperature dependency of the Seebeck constants, the voltage increased with increasing gauge temperature. In all cases, there was very good agreement between the measured and correlated values, and a detailed uncertainty analysis

Figure 6. Variation of voltage output with heat flux for constant $T$. (Symbols denote experimental data; solid lines denote correlation based on equation (13)).

Figure 7. Comparison of correlated and measured values of heat flux.
showed that the overall uncertainty of the correlation was less than 5% of the measured heat flux.

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Data access: due to confidentiality agreements with research collaborators, supporting data can only be made available to bona fide researchers subject to a nondisclosure agreement. Details of how to request access are available at the University of Bath data archive website (http://dx.doi.org/10.15125/BATH-00116).

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ORCID iDs
Oliver J Pountney https://orcid.org/0000-0003-0244-6295
Mario Patinios https://orcid.org/0000-0001-9256-0432
Hui Tang https://orcid.org/0000-0002-6389-7658
James A Scobie https://orcid.org/0000-0002-1827-3635

Supplemental Material
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Appendix

Notation

\[ A_e \] effective area of the copper block
\[ C_1, C_2, C_i \] coefficients for T type thermocouples
\[ C_j \] coefficient for power series expansion
The likelihood function for correlating $S$ index for likelihood functions

index for power series expansion

thermal conductivity

thermal conductivities of the internal layer

thermal conductivities of the upper and lower protective layers

ratio of thermal conductivity to thickness of the internal layer

maximum index for power series expansion

number of data points

number of thermocouple pairs in flux gauges

normal distribution

power supplied to the heater

heat flux through flux gauge

heat flow through flux gauge

heat loss below flux gauge

total heat loss through the Rohacell block

ratio of the thickness of the internal layer to that of the gauge

gauge sensitivity ($= V/q$)

relative Seebeck coefficient for the combination of thermo-element materials A and B

data set of $S$

temperatures of the hot and cold junctions of a thermocouple

lower and upper surface temperatures of the internal layer

lower and upper surface temperatures of the gauge

temperature of ambient air

average temperature of the internal layer

standard uncertainty

95% uncertainty

voltage output of flux gauges

voltage supplied to thin film heater

lateral direction

normal direction

normal distance to the top-surface of the upper copper plate

overall thickness of thermopile gauges

thickness of the internal layer

thicknesses of the upper and lower protective layers

temperature difference across the entire gauge ($= T_{s,2} - T_{s,1}$)

temperature difference across the internal layer ($= T_{l,2} - T_{l,1}$)