Holographic Dark Energy Model
with Modified Generalized Chaplygin Gas

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Abstract

We present a holographic dark energy model of the universe considering modified generalized Chaplygin gas (GCG). The modified GCG behaves as an ordinary barotropic fluid in the early epoch when the universe was tiny but behaves subsequently as a ΛCDM model at late epoch. An equivalent model with scalar field is obtained here by constructing the corresponding potential. The holographic dark energy is identified with the modified GCG and we determine the corresponding holographic dark energy field and its potential. The stability of the holographic dark energy in this case is also discussed.

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1 Introduction:

In the recent years a number of observational facts from high redshift surveys of type Ia Supernovae, WMAP, CMB etc. led us to believe that our universe is passing through an accelerating phase of expansion [1]. It is generally accepted that our universe might have also emerged from an accelerating phase in the past. Thus there might have two phases of acceleration of the universe (i) early inflation and (ii) late acceleration followed by a decelerating phase. It is known that perfect fluid assumption in the framework of Einstein general theory of Relativity (GTR) cannot fully explain the observational facts in the universe. It is known that early inflation may be realized in a semiclassical theory of gravity where matter is described by quantum fields [2]. Starobinsky obtained inflationary solution considering a curvature squared term in the Einstein-Hilbert action [3] long before the advent of inflation was known. However, the efficacy of the model is known only after the seminal work of Guth who first employed the phase transition mechanism to accommodate inflation. Thus inflation may be realized either modifying the matter sector or the gravitational sector of the Einstein’s field equation. A number of literature appeared in the last few years in which curvature squared terms [4] are added to the Einstein-Hilbert action to build early inflationary universe scenario.

Alternatively modification in the matter sector with an equation of state $p = \omega \rho$, permits inflation in the early universe if $\omega = -1$ but for the present acceleration one requires $\omega < -1$. The usual fields in the standard model of particle physics are not suitable to obtain the late accelerating phase of the universe. Thus it is a challenge to theoretical physics to formulate a framework to accommodate the observational facts. In order to describe the present accelerating phase of the universe, it may be useful to consider dark energy in the theory. The observational facts in the universe predict that dark energy content of the
universe is about 76% of the total energy budget of the universe. To accommodate such a huge energy various kinds of exotic matters in the theory are considered to identify possible candidate for the dark energy. Chaplygin gas is considered to be one such candidate of dark energy with an equation of state $p = -\frac{B}{\rho}$ [5], where $\rho$ and $p$ are the energy density and pressure respectively and $B$ is a constant. Subsequently, a modified form of the equation of state $p = -\frac{B}{\rho^\alpha}$ with $0 \leq \alpha \leq 1$ was considered to construct a viable cosmological model [6, 7], which is known as generalized Chaplygin gas (GCG) in cosmology. It has two free parameters. It behaves initially like dust but subsequently evolves to an asymptotic cosmological constant at late time when the universe is sufficiently large. The GCG behaves as a fluid obeying an equation of state $p = \omega \rho$ at a later epoch. To accommodate dark energy various kinds of matter with a modified equation of state are also considered in the literature. However, recently another form of equation of state for Chaplygin gas [8] is proposed similar to that considered in [9], which is given by

$$
p = A\rho - \frac{B}{\rho^\alpha} \quad \text{with} \quad 0 \leq \alpha \leq 1,
$$

where $A$ is an equation of state parameter and $B$ is a constant, known as modified GCG. This has three free parameters. In the early universe when the size of the universe $a(t)$ was small, the modified GCG gas corresponds to a barotropic fluid (if one considers $A = \frac{1}{3}$ it corresponds to radiation and $A = 0$ it corresponds to matter). So, the modified GCG at one extreme end behaves as an ordinary fluid and at the other extreme when the universe is sufficiently large it behaves as cosmological constant which can be fitted to a $\Lambda$CDM model. In a flat Friedmann model it is shown [6] that the modified generalized Chaplygin gas may be equivalently described in terms of a homogeneous minimally coupled scalar field $\phi$. Barrow [10] has outlined a method to fit Chaplygin gas in FRW universe. Gorini et al. [11] using the above scheme obtained the corresponding homogeneous scalar field $\phi(t)$.
in a potential $V(\phi)$ which can be used to obtain a viable cosmological model with modified Chaplygin gas.

Recently, holographic principle [12, 13] is incorporated in cosmology [14-17] to track the dark energy content of the universe following the work of Cohen et al. [18]. Holographic principle is a speculative conjecture about quantum gravity theories proposed by G’t Hooft. The idea is subsequently promoted by Fischler and Susskind [12] claiming that all the information contained in a spatial volume may be represented by a theory that lives on the boundary of that space. For a given finite region of space it may contain matter and energy within it. If this energy suppresses a critical density then the region collapses to a black hole. A black hole is known theoretically to have an entropy which is proportional to its surface area of its event horizon. A black hole event horizon encloses a volume, thus a more massive black hole have larger event horizon and encloses larger volume. The most massive black hole that can fit in a given region is the one whose event horizon corresponds exactly to the boundary of the given region under consideration. The maximal limit of entropy for an ordinary region of space is directly proportional to the surface area of the region and not to its volume. Thus, according to holographic principle, under suitable conditions all the information about a physical system inside a spatial region is encoded in the boundary. The basic idea of a holographic dark energy in cosmology is that the saturation of the entropy bound may be related to an unknown ultra-violet (UV) scale $\Lambda$ to some known cosmological scale in order to enable it to find a viable formula for the dark energy which may be quantum gravity in origin and it is characterized by $\Lambda$. The choice of UV-Infra Red (IR) connection from the covariant entropy bound leads to a universe dominated by blackhole states. According to Cohen et al. [18] for any state in the Hilbert space with energy $E$, the corresponding Schwarzschild radius $R_s \sim E$, may be less than the IR cut off value $L$ (where $L$ is a cosmological scale). It is possible to derive a relation between the
UV cutoff $\rho^{1/4}_\Lambda$ and the IR cutoff which eventually leads to a constraint \[ \left( \frac{8\pi G}{c^4} \right) L^3 \left( \frac{L}{\Lambda} \right) \leq L \] [19] where $\rho_\Lambda$ is the energy density corresponding to dark energy characterized by $\Lambda$, $G$ is Newton's gravitational constant and $c$ is a parameter in the theory. The holographic dark energy density is

\[ \rho_\Lambda = 3c^2 M_P^2 L^{-2}, \tag{2} \]

where $M_P^2 = 8\pi G$. It is known that the present acceleration may be described if $\omega_\Lambda = \frac{\rho_\Lambda}{\rho_\Lambda} < -\frac{1}{3}$. If one considers $L \sim \frac{1}{H}$ it gives $\omega_\Lambda = 0$. A holographic cosmological constant model based on Hubble scale as IR cut off does not permit accelerating universe. It is also examined [14] that the holographic dark energy model based on the particle horizon as the IR cutoff even does not work to get an accelerating universe. However, an alternative model of dark energy using particle horizon in closed model is also proposed [20]. However, Li [15] has obtained an accelerating universe considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Thus to have a model consistent with observed universe one should adopt the covariant entropy bound and choose $L$ to be event horizon.

The paper is organized as follows: in sec. 2, the relevant field equation with modified Chaplygin gas in FRW universe is presented; in sec. 3 we present an equivalent model with a scalar field by constructing the corresponding potential, in sec. 4, holographic dark energy fields in modified GCG is determined; in sec. 5, squared speed of sound for holographic dark energy is evaluated for a closed universe i.e., $k = 1$. Finally in sec. 6, a brief discussion.
2 Modified Chaplygin Gas in FRW universe:

The Einstein’s field equation is given by

\[ G_{\mu\nu} = \kappa^2 T_{\mu\nu} \]  \hspace{1cm} (3)

where \( \kappa^2 = 8\pi G \), and \( T_{\mu\nu} \) is the energy momentum tensor.

We consider a homogeneous and isotropic universe given by

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right] \]  \hspace{1cm} (4)

where \( a(t) \) is the scale factor of the universe, the matter is described by the energy momentum tensor \( T^\mu_\nu = (\rho, p, p, p) \) where \( \rho \) and \( p \) are energy density and pressure respectively.

Using the metric (4) and the energy momentum tensor, the Einstein’s field equation (3) can be written as

\[ H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\rho \]  \hspace{1cm} (5)

where we use \( M_P^2 = \kappa^2 \). The conservation equation for matter is given by

\[ \frac{d\rho}{dt} + 3H(\rho + p) = 0. \]  \hspace{1cm} (6)

For modified generalized Chaplygin gas given by eq. (1), the energy density is obtained from eq. (6), which is given by

\[ \rho = \left( \frac{B}{1 + A} + \frac{C}{a^n} \right)^{\frac{1}{1 + \alpha}} \]  \hspace{1cm} (7)

where \( B \) and \( C \) are arbitrary integration constants and we denote \( n = 3(1 + A)(1 + \alpha) \).

We now define the following

\[ \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \quad \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_k = \frac{k}{a^2H^2} \]  \hspace{1cm} (8)

where \( \rho_{cr} = 3M_P^2H^2 \), \( \Omega_\Lambda \), \( \Omega_m \) and \( \Omega_k \) represent density parameter corresponding to \( \Lambda \), matter and curvature respectively.
3 Modified GCG as a scalar field:

We assume here that the origin of dark energy is a scalar field. In this case, using Barrow’s scheme [10], we get the following

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \left( \frac{B}{A + 1} + \frac{C}{a^n} \right)^{\frac{1}{\alpha + 1}}, \]  
\[ (9) \]

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = -\frac{B}{A+1} + \frac{A C}{(B+1 + \frac{C}{a^n})^{\alpha + 1}}, \]  
\[ (10) \]

It is now simple to derive the scalar field potential and its kinetic energy term, which are given by

\[ V(\phi) = \frac{B}{A+1} + \frac{1-A}{2} \frac{B}{a^n} \]  
\[ \frac{1}{\left( B+1 + \frac{C}{a^n} \right)^{\alpha + 1}} \]  
\[ (11) \]

\[ \dot{\phi}^2 = \frac{(A+1)C}{\left( B+1 + \frac{C}{a^n} \right)^{\alpha + 1}} \]  
\[ (12) \]

We now describe two cases:

Case I: For a flat universe \( (k = 0) \), using eq. (5) one can integrate eq. (12) which yields

\[ \phi - \phi_o = \pm \frac{2M_P}{\sqrt{n(1 + \alpha)}} sinh^{-1} \left[ \sqrt{\frac{C(A+1)}{B}} a^{-\frac{2}{\alpha + 1}} \right] \]  
\[ (13) \]

and the corresponding potential is given by

\[ V(\phi) = \frac{B}{1+A} + \frac{B(1-A)}{2(1+A)} sinh^2 \left( \frac{\sqrt{n}}{2} \left( \phi - \phi_o \right) \right) \]  
\[ \frac{1}{\left( \frac{B}{1+A} \right)^{\alpha + 1} cosh^{\frac{2\alpha}{\alpha + 1}} \left( \frac{\sqrt{n}}{2} \left( \phi - \phi_o \right) \right) \}} \]  
\[ (14) \]

The potential behaves as a constant near \( \phi \to \phi_o \), otherwise it increases with increasing value of the field.

Case II: We now consider a non flat universe, in this case the evolution of the scalar field is obtained as

\[ \phi - \phi_o = \pm \int \frac{dz}{\sqrt{\frac{12 M_P^2 (A + 1)}{n^2} \left( \mu^2 + z^2 \right) - \frac{3M_P^2K}{C^{2/n}} z^{4/n}(\mu^2 + z^2)^{\frac{1}{\alpha + 1}}}} \]  
\[ (15) \]
where \( k = +1 \) for closed universe (\( k = -1 \) for open universe) and we denote \( \mu^2 = \frac{B}{A+1} \), \( z = \sqrt{\frac{C}{a^n}} \). The integral may be evaluated analytically for some special choice of the parameters. We choose the following:

- \( A = -\frac{1}{3}, \ B = 0 \), the scalar field evolves as

\[
\phi_\pm = \phi_o \pm \sqrt{\frac{2M_P^2}{n^2(1 - \frac{3M_P^2k}{c^{2/n}}})} \ln \left( \frac{C}{a^n} \right),
\]

the corresponding scalar field potential is given by

\[
V(\phi) = \frac{2}{3} \left( \frac{2M_P^2}{n^2(1 - \frac{3M_P^2k}{c^{2/n}})} \right)^{\frac{1}{2(\alpha+1)}} e^{\pm \frac{1}{\alpha+1} (\phi - \phi_o)}. \tag{16}
\]

The potential is exponential in nature, it is increasing or decreasing depending on the type of inflaton field one choose in this case. We also note that a positive potential is obtained for \( C > (3M_P^2k)^{n/2} \). In the case of closed universe the above inequality gives lower bound on the value of \( C \). But in the case of an open universe \( C \) can pick up negative values also for even integer values of \( n \).

- \( A = \frac{1}{3}, \ B \neq 0 \), in this case the scalar field evolves as

\[
\phi - \phi_o = \pm \sqrt{\frac{16M_P^2}{n^2(1 - \frac{3M_P^2k}{c^{2/n}}})} \sinh^{-1} \left( \frac{4C}{3B a^{n/2}} \right), \tag{17}
\]

and the corresponding potential is given by

\[
V(\phi) = \text{sech}^2 \left( \frac{n^2(1 - \frac{3M_P^2k}{c^{2/n}})}{16M_P^2} \right) \left( \phi - \phi_o \right) + \frac{1}{3} \text{tanh}^2 \left( \frac{n^2(1 - \frac{3M_P^2k}{c^{2/n}})}{16M_P^2} \right) (\phi - \phi_o). \tag{18}
\]

The potential is drawn in fig. 1. It is a new and interesting potential, it has a shape similar to that one obtains in the case of tachyonic field [21]. The difference is that there is an extra term to the potential which attains a constant value for large value of the inflaton field. This potential may be important for a viable cosmological model building. We also note that it permits an oscillatory scalar field for \( C < (3M_P^2k)^{n/2} \) with sinusoidal potential
Figure 1: shows the plot of $V$ versus $\phi$ with the parameter $n^2(1-\frac{3M_P^2k}{2n}) = 1$.

for any $n$ in closed universe but for open universe an even integer value of $n$ only gives such potential.

4 Holographic Dark Energy in Modified GCG:

In a FRW universe we now consider a non-flat universe with $k \neq 0$ and use the holographic dark energy density as given in (2) which is

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \quad (20)$$

where $L$ is the cosmological length scale for tracking the field corresponding to holographic dark energy in the universe. The parameter $L$ is defined as

$$L = ar(t). \quad (21)$$
where $a(t)$ is the scale factor of the universe and $r(t)$ is relevant to the future event horizon of the universe. Using Robertson-Walker metric one gets \[16\]

$$L = \frac{a(t)}{\sqrt{|k|}} \sin \left( \frac{\sqrt{|k|} R_h(t)}{a(t)} \right) \text{ for } k = 1,$$

$$= R_h \text{ for } k = 0,$$

$$= \frac{a(t)}{\sqrt{|k|}} \sinh \left( \frac{\sqrt{|k|} R_h(t)}{a(t)} \right) \text{ for } k = -1.$$  \hspace{1cm} (22)

where $R_h$ represents the event horizon which is given by

$$R_h = a(t) \int_t^{\infty} \frac{dt'}{a(t')} = a(t) \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}}.$$  \hspace{1cm} (23)

Here $R_h$ is measured in $r$ direction and $L$ represents the radius of the event horizon measured on the sphere of the horizon. Using the definition of $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2H^2$, one can derive \[17\]

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}}.$$  \hspace{1cm} (24)

Using eqs. (17)-(20), we determine the rate of change of $L$ with respect to $t$ which is

$$\dot{L} = \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} \cos \left( \frac{\sqrt{|k|} R_h(t)}{a(t)} \right) \text{ for } k = 1,$$

$$= \frac{c}{\sqrt{\Omega_\Lambda}} - 1 \text{ for } k = 0,$$

$$= \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} \cosh \left( \frac{\sqrt{|k|} R_h(t)}{a(t)} \right) \text{ for } k = -1.$$  \hspace{1cm} (25)

Using eqs. (15)-(20), it is possible to construct the required equation for the holographic energy density $\rho_\Lambda$, which is given by

$$\frac{d\rho_\Lambda}{dt} = -2H \left[ 1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} f(x) \right] \rho_\Lambda,$$  \hspace{1cm} (26)

where we use the notation, henceforth,

$$f(X) = \frac{1}{\sqrt{|k|}} \cos(n \sqrt{|k|} x) = \cos(X) \left[ 1, \cosh(X) \right] \text{ for } k = 1 \left[ 0, -1 \right].$$  \hspace{1cm} (27)
where $X = \frac{R_h}{a(t)}$. The energy conservation equation is

$$\frac{d\rho_\Lambda}{dt} + 3H(1 + \omega_\Lambda)\rho_\Lambda = 0$$

(28)

which is used to determine the the equation of state parameter

$$\omega_\Lambda = -\left(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X)\right).$$

(29)

Now we assume holographic dark energy density which is equivalent to the modified Chaplygin gas energy density. The corresponding energy density may be obtained using the equation of state given by (7). The equation of state parameter using (1) can also be re-written as

$$\omega = \frac{p}{\rho} = A - \frac{B}{\rho^{\alpha + 1}}.$$  

(30)

Let us now establish the correspondence between the holographic dark energy and modified Chaplygin gas energy density. In this case from eqs. (7) and (20), we get

$$C = a^n \left[(3c^2 M_P^2 L^{-2})^{1+\alpha} - \frac{B}{A + 1}\right].$$  

(31)

Thus using eqs. (29)-(30) in the above we determine the parameters as

$$B = (3c^2 M_P^2 L^{-2})^{\alpha + 1} \left[A + \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X)\right],$$  

(32)

$$C = (3c^2 M_P^2 L^{-2})^{\alpha + 1} a^n \left[1 - \frac{3A + 1}{3(A + 1)} - \frac{2\sqrt{\Omega_\Lambda}}{3(A + 1)c} f(X)\right].$$  

(33)

The scalar field potential becomes

$$V(\phi) = 2c^2 M_P^2 L^{-2} \left[1 + \frac{\sqrt{\Omega_\Lambda}}{2c} f(X)\right],$$  

(34)

and the corresponding kinetic energy of the field is given by

$$\dot{\phi}^2 = 2c^2 M_P^2 L^{-2} \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X)\right].$$  

(35)
Considering \( x = \ln a \), we transform the time derivative to the derivative with logarithm of the scale factor, which is the most useful function in this case. We get

\[
\phi' = M_P \sqrt{2 \Omega \left( 1 - \sqrt{\frac{\Omega}{c}} f(X) \right)} \quad (36)
\]

where \((\cdot)'\) prime represents derivative with respect to \( x \). Thus, the evolution of the scalar field is given by

\[
\phi(a) - \phi(a_o) = \sqrt{2} M_P \int_{\ln a_o}^{\ln a} \Omega \left( 1 - \sqrt{\frac{\Omega}{c}} f(X) \right) dx. \quad (37)
\]

\section{Squared speed for Holographic Dark Energy}

We consider a closed universe model \((k = 1)\) in this case. The dark energy equation of state parameter given by eq. (29) reduces to

\[
\omega = -\frac{1}{3} \left( 1 + \frac{2}{c} \sqrt{\Omega} \cos y \right) \quad (38)
\]

where \( y = \frac{R_H}{a(t)} \). The minimum value it can take is \( \omega_{min} = -\frac{1}{3} (1 + 2 \sqrt{\Omega}) \) and one obtains a lower bound \( \omega_{min} = -0.9154 \) for \( \Omega_A = 0.76 \) with \( c = 1 \). Taking variation of the state parameter with respect to \( x = \ln a \), we get \([17]\)

\[
\frac{\Omega'}{\Omega^2} = (1 - \Omega_A) \left( \frac{2}{c} \frac{1}{\Omega} \cos y + \frac{1}{1 - a \gamma} \frac{1}{\Omega_A} \right) \quad (39)
\]

and the variation of equation of state parameter becomes

\[
\omega' = -\frac{\sqrt{\Omega_A}}{3c} \left[ 1 - \frac{\Omega}{1 - \gamma a} + \frac{2\sqrt{\Omega_A}}{c} \left( 1 - \Omega_A \cos^2 y \right) \right], \quad (40)
\]

where \( \gamma = \frac{\Omega_m}{\Omega_m} \). We now introduce the squared speed of holographic dark energy fluid as

\[
v^2 = \frac{dp_A}{d\rho_A} = \frac{\dot{\rho}_A}{\rho_A} = \frac{\rho_A'}{\rho_A}, \quad (41)
\]
where variation of eq. (30) w.r.t. $x$ is given by

$$p'_\Lambda = \omega'_\Lambda \rho_\Lambda + \omega_\Lambda \rho'_\Lambda.$$  

(42)

Using the eqs. (41) and (42) we get

$$v^2_\Lambda = \omega'_\Lambda \rho_\Lambda \rho'_\Lambda + \omega_\Lambda$$

which now becomes

$$v^2_\Lambda = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda \cos y} + \frac{1}{6c} \sqrt{\Omega_\Lambda} \left[ \frac{1-\Omega_\Lambda}{1-\gamma a} + \frac{2}{\sqrt{\Omega_\Lambda}} (1 - \Omega_\Lambda \cos^2 y) \right].$$  

(43)

The variation of $v^2_\Lambda$ with $\Omega_\Lambda$ is shown in fig. 2 for different $y$ values. It is found that for a given value of $c$, $a$, $\gamma$, the model admits a positive squared speed for $\Omega_\Lambda > 0$. However, $\Omega_\Lambda$ is bounded below otherwise instability develops. We note also that for $\frac{(2n+1)\pi}{2} < y < \frac{(2n+3)\pi}{2}$, (where $n$ is an integer) no instability develops. We plot the case for $n = 0$ in fig. 2, it is evident that for $y \leq \frac{\pi}{2}$ and $y \geq \frac{3\pi}{2}$, the squared speed for holographic dark energy becomes negative which led to instability. But for the region $\frac{\pi}{2} < y < \frac{3\pi}{2}$ with $n = 0$ no such instability develops. It is also found that for $y = 0$ i.e., in flat case the holographic dark energy model is always unstable [22].

6 Discussions:

In this paper we explored the holographic dark energy model in FRW universe with a scalar field which describes the modified generalized Chaplygin gas (GCG). We determine the equivalent scalar field potential of the modified GCG in flat universe which is different from that obtained in Ref. 6. In the non-flat case although it is not so simple to obtain an analytic function of the potential in terms of the field we discuss two special cases in which potentials are shown as a function of field $\phi$. For $A = \frac{1}{3}$, $B \neq 0$, we obtain
Figure 2: shows the plot of $v^2_{\Lambda}$ versus $\Omega_{\Lambda}$ for different values of $y$ with $c = 1$, $\gamma = 1/3$ and $a = 1$, in the first array the figures are for $y = \frac{\pi}{3}$ and $y = \frac{\pi}{2}$, in the second array for $y = \frac{1.5 \pi}{2}$, $y = \pi$ and in the third array for $y = \frac{2.5 \pi}{2}$, $y = \frac{3\pi}{2}$. 
a new scalar field potential similar to the potential of a rolling tachyon obtained earlier [21] but with an extra piece in it. We also obtain the evolution of the holographic dark energy field and the corresponding potential in the framework of modified GCG in a non flat universe. Although recent observational evidence supports a flat universe, it is not yet decided that our universe is perfectly flat. Thus it is important to study a closed or open universe to account for the observational facts. We recover the equation of state considered by Setare [21] when $A = 0$ and $\alpha = 1$ to derive the fields of dark energy. We note that in the closed model the holographic dark energy is stable for a restricted domain of the values of $\Omega_\Lambda$. It is also observed that inclusion of a barotropic fluid in addition to Chaplygin gas (which is modified GCG) does not alter the form of potential and evolution of the holographic dark energy field but the parameter $B$ in the equation of state becomes proportional to $a^n$ with $n = 3(1 + A)(1 + \alpha)$. Thus the contribution of the holographic dark energy is more if ($A \neq 0$) compared to the case when one considers $A = 0$ as was done in Ref. [23]. Thus it is noted that although the form of the potential does not have impact on the addition of a barotropic fluid it changes the overall holographic dark energy density. It is found that the holographic dark energy is stable for a restricted domain of the values of $\Omega_\Lambda$ in a closed model of the universe.

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