Mid-infrared variations of R Coronae Borealis stars

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Accepted 2014 December 23. Received 2014 December 23; in original form 2014 October 15

ABSTRACT
Mid-infrared (IR) photometry of R Coronae Borealis stars obtained from various satellites from Infrared Astronomical Satellite (IRAS) to Wide-field Infrared Survey Explorer (WISE) has been utilized in studying the variations of the circumstellar dust’s contributions to the spectral energy distribution of these stars. The variation of the fractional coverage (\(R\)) of dust clouds and their blackbody temperatures (\(T_b\)) have been used in trying to understand the dust cloud evolution over the three decades spanned by the satellite observations. In particular, it is shown that a prediction \(R \propto T_b^4\) developed in the paper is satisfied, especially by those stars for which a single collection of dust dominates the IR fluxes.

Key words: circumstellar matter – stars: individual: R CrB.

1 INTRODUCTION
R Coronae Borealis stars (hereafter RCBs) are a rare class of peculiar variable stars. Currently known stars number about 68 in the Galaxy, 19 in Large Magellanic Cloud (LMC) and three in the Small Magellanic Cloud (SMC; Clayton 2012). Two principal defining characteristics of RCBs are (i) a propensity to fade at unpredictable times by up to about 8 mag as a result of clouds of soot intercepting the line of sight to the star, and (ii) a supergiant-like atmosphere that is very H deficient and He and C rich.

RCB stars manufacture and disperse soot from a formation site near the star (Loreta 1935; O’Keefe 1939). The soot particles absorb starlight and re-emit the energy in the infrared (IR; e.g. Feast et al. 1997). Often, soot particles are sufficiently close to the star that they achieve an equilibrium temperature of several hundred degrees and so emit in the mid-IR bands. Such emission is measured with difficulty from the ground. Fortunately, several Earth-orbiting satellites built for IR photometric surveys have now reported measurements on RCBs. Available data are too sparse and inhomogeneous to define the variation of RCB dust emission on a daily, monthly or even annual time-scale but mean properties of the IR emission should be extractable to enable searches for correlations between dust shell properties derived from the IR emission and other RCB properties such as chemical compositions.

The Infrared Astronomical Satellite (IRAS) provided the first systematic photometry of these stars (Rao & Nandy 1986; Walker 1986) to complement limited ground-based photometry carried out earlier (e.g. Feast & Glass 1973; Kilkenny & Whittet 1984). IRAS confirmed not only that warm (300–900 K) dust was common around RCBs but also showed the presence of cold (i.e. distant from the star) dust for strong IR emitting stars (Rao & Nandy 1986). IRAS was followed by the Infrared Space Observatory (ISO). Although ISO provided higher spectral resolution mid-IR spectroscopy and spectral energy distributions (SEDs), it observed only a few bright RCBs (Lambert et al. 2001). Clayton et al. (2011) recently discovered using the Herschel satellite that the cold dust shell around R CrB radiates up to at least a wavelength of 1000 \(\mu\)m.

Major progress in IR photometry of RCBs could occur only after the launch of Spitzer satellite which provided high-quality low-resolution 8–40 \(\mu\)m spectra using Infrared Spectrograph (IRS) for a major sample of RCBs (García-Hernández, Kameswara Rao & Lambert 2011, hereafter Paper I). Spitzer spectra were combined with optical and near-IR photometry at maximum light corrected for interstellar reddening to define a star’s SED from which characteristics of the IR emitting dust shell were extracted (Paper I; García-Hernández, Kameswara Rao & Lambert 2013). Recently, it was realized that mid-IR band colours could distinguish RCB stars from other types of variable stars rather uniquely (Miller et al. 2012; Tisserand 2012; Tisserand et al. 2011, 2013).

Following Spitzer, the satellites AKARI and Wide-field Infrared Survey Explorer (WISE) have provided photometric surveys from which observations of RCBs may be extracted. The AKARI IR camera surveyed the sky at 9 and 18 \(\mu\)m between 2006 May 6 and 2007 August 28 (Ishihara et al. 2010). A few RCBs were also detected by AKARI at 65, 90 and 160 \(\mu\)m. WISE’s survey was conducted at 3.0, 4.6, 12 and 22 \(\mu\)m in 2010 (Wright et al. 2010). Tisserand (2012) used the available AKARI, WISE and other photometry to investigate the IR colours and the SEDs from the optical to the mid-IR and to characterize the circumstellar dust emission from a large sample of RCBs.
Here, we assemble and discuss the SEDs of our \textit{Spitzer} sample and their dust emission as measured in the mid-IR by the series of satellites from \textit{IRAS} to \textit{AKARI} and \textit{WISE} and by published ground-based photometry.

### 2 Spectral Energy Distributions

In \textit{Paper I}, \textit{Spitzer} satellite spectra of RCBs in the wavelength range of 5–40 \(\mu m\) were paired with visual to near-IR photometry of the star to obtain a comprehensive SED of the star. Observed fluxes were corrected for interstellar reddening. For details, please see \textit{Paper I}. Comparisons were made with \textit{IRAS} 10 and 25 \(\mu m\) fluxes and, where available, with ground-based IR photometry. Here, we complement these \textit{Spitzer} and \textit{IRAS} observations with \textit{AKARI} (Ishihara et al. 2010) and \textit{WISE} (Wright et al. 2010) satellite band fluxes (Table 1). \textit{WISE} observations were done in four bands 3.0, 4.6, 12 and 22 \(\mu m\) whereas \textit{AKARI} observations in the two bands at 9.0 and 18.0 \(\mu m\) have been used. The flux calibration adopted was from Wright et al. (2010) for \textit{WISE} magnitudes and from Ishihara et al. (2010) for \textit{AKARI}. The epochs of these observations have been taken as 2006.0 and 2010.5 for \textit{AKARI} and \textit{WISE}, respectively. Epochs of the \textit{Spitzer} observations are given in \textit{Paper I}; a majority were taken in about mid-2008. To compare the flux densities of \textit{Spitzer} spectra with the other satellites bands we choose the same effective wavelengths of these bands 9, 12 and 18 \(\mu m\) and obtained monochromatic flux densities from the IRS spectra. We made a linear fit to the data points of three wavelengths on either side of these chosen band wavelengths and obtained the flux density listed in Table 1 at these central wavelengths. These flux densities are not band averaged flux densities as given by other satellites. Flux density uncertainties were taken from the following sources: \textit{IRAS} Point Source Catalogue 2.0 via SIMBAD; \textit{AKARI} and \textit{WISE} from http://irsa.ipac.caltech.edu/cgi-bin/Gator/nph-query; \textit{Spitzer} from the noise in the observed spectra. For the \textit{Spitzer} 9 \(\mu m\) flux density, the uncertainties are less than 0.01 \(Jy\) for all stars and are not indicated in Table 1. \textit{WISE} measurements at 3.0 and 4.6 \(\mu m\) are not tabulated but are shown in the appropriate figures which follow. Except for the \textit{IRAS} photometry, the ground-based and \textit{AKARI} and \textit{WISE} photometry have not been colour corrected. Apart from the 12 \(\mu m\) \textit{WISE} band, the other bands are sufficiently narrow not to require a significant colour correction for these spectra which span a narrow (500–900 K) temperature range.
Earlier, Tisserand (2012) used WISE as well as AKARI magnitudes to construct SEDs of RCBs. The recent release of the WISE catalogue contains more and fainter RCBs than dealt with by him and the magnitudes also were revised (Infrared Processing and Analysis Center, IPAC, website http://irsa.ipac.caltech.edu/cgi-bin/Gator/nph-query). It was mentioned by Tisserand (2012) that 4.6 μm band WISE magnitudes less than 4.0 have saturation problems. Some of the bright RCBs like R CrB, RX Sgr, etc. were too bright for accurate photometry. Table 1 gives the flux density measurements from the four satellites and, where available, ground-based mid-IR photometry. We make limited use of the WISE flux densities at 3.0 and 4.6 μm. These quantities are not tabulated in Table 1 but are provided by Tisserand (2012).

We have used the methodology described in Paper I for constructing the SED of the star and its dusty circumstellar shell. Interstellar reddening estimates listed in Paper I have been applied before constructing the SEDs; this correction for reddening primarily affects the stellar component of the combined SED. Blackbody fits have been made to the combined SEDs with the temperature of the stellar blackbody taken from Paper I.

The blackbody fits provide the dust temperature $T_d$ and the fraction of the stellar energy radiated by the cool circumstellar shell $R = f_{cool}/f_{star}$, where $f$ refers to the integrated flux density. In general, the dust component for RCBs can be well represented by a single blackbody. In a few cases a second and cooler blackbody is added to provide an adequate fit to the IR fluxes. With a more detailed coverage of the near-IR energy distribution, it might be possible to include a blackbody to represent emission by warmer dust near the star. The WISE observations at 3.0 and 4.6 μm and ground-based L and M photometry sample warmer dust, if present. Our focus here is on the dust emitting at mid-IR wavelengths.

3 DISCUSSION – THE PUFF MODEL

3.1 The concept

Our discussion of SEDs is framed in terms of the random dust-puff model for dust formation and ejection. In this scenario (Herbig 1949; Payne-Gaposchkin 1963; Feast 1979, 1986, 1996, 1997; Clayton et al. 1992; Feast et al. 1997), dust is ejected from points on or near the RCB surface in the form of puffs or clouds which then are expelled away from the stellar surface. Only those puffs formed on or very close to the Earth-facing hemisphere of the RCB will result in the characteristic fading. All puffs but those eclipsed by the star will contribute to the IR signal from the dust. In a given circumstellar envelope, there may be one, several or many puffs. Puffs may be created at preferred points of the stellar surface or uniformly over the surface. The rate of creation of fresh puffs will likely vary from star to star. Those RCBs prone to frequent formation of puffs will be expected to have nearly time-independent $R$ values.

The amplitude of IR variations will depend on several factors such as the number of puffs and their mean contribution to the mid-IR flux: a star with many weak puffs is likely to show smaller variations than a star with a few strong puffs. On the other hand, RCBs producing puffs infrequently are likely to have smaller mean $R$ values with larger variations resulting as a puff moves away from the star and subsequently a fresh puff is ejected. In this simple picture, puffs are formed and expand away from the star, i.e. there is no mechanism for storing dust (and gas) near the star as in a circumstellar disc. Chesneau et al. (2014) from interferometric observations of V854 Cen show that this star’s dust is concentrated in an elongated structure which, as the authors note, may reflect a bipolar wind off the star. Quasi-stable dusty discs have been found as circumbinary features for some binary stars. It has yet to be shown that V854 Cen is a binary star. Evidence for bipolar outflows has been presented earlier by Rao & Lambert (1993) for V854 Cen and Clayton et al. (1997) for R CrB.

Information about the formation and evolution of the circumstellar dust shell for RCBs is, thus, provided by the temporal evolution of the IR flux in terms of its spectral distribution and intensity which here are crudely represented by the covering factor $R$ and the blackbody temperature $T_d$. Unfortunately, available IR measurements are extremely sparse because the observations are difficult to obtain; ground-based mid-IR photometry is a rarely practiced art and IR-capable satellites are launched infrequently. An exceptional data base is the South African Astronomical Observatory (SAAO) series of JHKLM photometry of bright southern RCBs (Feast et al. 1997). And, in particular, Bogdanov, Taranova & Shenavrin (2010) mounted a beautiful long-term campaign of JHKLM observations of UV Cas.

3.2 Variability

Here, we compare mid-IR fluxes obtained with the satellites IRAS, AKARI, Spitzer and WISE. The epochs of these surveys are 1983 for IRAS, 2006 for AKARI, 2005–2008 for Spitzer and 2010 for WISE. Long-term – two decades approximately – variations are estimated from comparison between IRAS and the other three surveys. Shorter term – one to a few years – variations are estimated by comparisons among the fluxes from the three most recent satellites. Our present comparisons are an extension of the IRAS–Spitzer comparison discussed in Paper I where the Spitzer 12 and 25 μm fluxes for a large RCB sample were compared with their IRAS counterparts obtained about 25 yr ago.

IRAS and Spitzer 12 μm flux densities are compared in Fig. 1. When a few outliers are excluded, the mean relation $F(\text{IRAS}) = 1.24F(\text{Spitzer})$ provides a good fit to the data points and this linear relation is shown in the figure. A systematic offset of 24 per cent between Spitzer and IRAS observations may be due to the broad-band nature of the IRAS photometry and an imperfect colour correction. Given that there is a roughly 10 per cent uncertainty affecting IRAS photometry, it would appear that many RCBs had similar 12 μm flux densities in 1983 and 2005–2008 but this is not to suggest that they did not vary in the nearly 30-yr interval.

Outliers include RY Sgr, V854 Cen, UV Cas, Y Mus, RT Nor and WX CrA with an exceptionally large ratio of IRAS to Spitzer fluxes and FH Sct with an apparently small ratio of IRAS to Spitzer fluxes. The three stars – UV Cas, Y Mus and RT Nor – show a factor of 5 greater IRAS 12 μm flux than recorded by Spitzer. A striking and expected characteristic of the trio is that their covering factors $R$ as obtained from Spitzer were very small (0.01–0.03) and among the lowest of the sample. For their IRAS fluxes, the $R$ values were 0.07–0.28. In the puff model, the trio are most likely examples where very few puffs inhabit the circumstellar environment at any given time because ejection of fresh puffs is an infrequent event. Other outliers – RY Sgr, V854 Cen and WX CrA – show a less extreme difference between IRAS and Spitzer fluxes. This trio have larger $R$ covering factors but would appear to be still affected by a change in the dust in their circumstellar shell. FH Sct, another low $R$ star, is the only star in the sample which was fainter when observed by IRAS than by Spitzer.

A comparison of Spitzer and AKARI flux densities at 9 μm is shown in Fig. 2 where the relation $F(\text{AKARI}) = F(\text{Spitzer})$ is plotted. Similarly, WISE and Spitzer 12 μm fluxes are compared in Fig. 3.
Mid-IR variations of RCB stars

Figure 1. Flux densities \( F \) at 12 \( \mu \)m from Spitzer versus the values from IRAS. The solid line shows the mean relation \( F(\text{IRAS}) = 1.24F(\text{Spitzer}) \) defined by the bulk of the sample. Outliers are identified by shorthand labels: RY Sgr, V854 Cen, UX CrA, Y Mus, RT Nor and FH Sct. Red points refer to majority RCBs as defined by Lambert & Rao (1994), the green points refer to minority RCBs and two blue stars refer to two hot RCBs (DV Cen and MV Sgr) given in Table 1.

where the line corresponds to \( F(\text{WISE}) = F(\text{Spitzer}) \). In both figures, a few outliers are marked. In Fig. 3, RT Nor is the outstanding outlier with its WISE flux density raised by a fresh ejection of dust. V854 Cen is again brighter than expected from its Spitzer flux density and the mean for the sample. Two stars – VZ Sgr and S Aps – have faded noticeably between their observations by Spitzer and then by WISE. An overall impression from these comparisons is that the mid-IR fluxes of the majority of the sampled RCBs are not subject to major variations over the 1–25 yr time frame.

Qualitatively, the three extreme outliers in Fig. 1 – UV Cas, Y Mus and RT Nor – with the small \( R \) values from Spitzer may be identified as stars which at the time of the Spitzer observations had little warm dust – few dust puffs – in their circumstellar envelopes and, thus, large fractional changes in their mid-IR fluxes surely reflect fresh ejection of dust into the envelope and possibly a single ejection on or off the line of sight. We begin comparison of the SEDs constructed from IRAS, Spitzer and AKARI, and WISE observations with these three IRAS–Spitzer outliers – see Fig. 4 for UV Cas, Fig. 5 for Y Mus and Fig. 6 for RT Nor.

For UV Cas, a valuable series of \( JHKLM \) photometry by Bogdanov et al. (2010) from 1984 to 2009 (just following the IRAS observations) suggests that a major episode of dust ejection occurred prior to the IRAS observations.

This ejection did not cause a fading of the star. Following the ejection, UV Cas’s mid-IR flux decayed for about 2000 d with only two minor flux increases prior to 2009. Thus, the dust detected by AKARI, Spitzer and WISE was likely from the pre-IRAS ejection which had expanded away from the star in the intervening 25 yr; the drop in dust temperature from 800 to 510 K is consistent with this idea (Paper I). The dates of the AKARI and WISE observations generally bracket the Spitzer observations. Thus, it is not unexpected that for UV Cas that mid-IR fluxes from the three satellites are similar: Fig. 4 shows that the AKARI and Spitzer fluxes are essentially identical but the WISE fluxes are lower suggesting that the expansion of the ‘1980’ dust ejection appears to have continued and may have accelerated. A more quantitative discussion of the series of flux density measurements is given in Section 3.3. However, Clayton, Geballe & Zhang (2013) suggest that there might be some evidence for hundreds of km s\(^{-1}\) dust motions in the star.

IRAS to WISE photometry shows that Y Mus behaved similarly to UV Cas (Fig. 5): the IRAS fluxes and the N flux from Kilkenny & Whittet (1984) are much greater than the fluxes from AKARI, Spitzer and WISE. Kilkenny & Whittet’s M and N observations were made in 1983 April during the year-long IRAS mission. Thus, the agreement between their N flux and the IRAS 10 \( \mu \)m flux is expected in the absence of rapid changes of the IR flux (see discussion of RT Nor below). Unfortunately, there is no long-term series of mid-IR photometry comparable to Bogdanov et al.’s (2010)
observations of UV Cas but we presume that the dust detected by IRAS moved away from Y Mus and had cooled substantially when detected by the other satellites. American Association of Variable Star Observers (AAVSO) records show Y Mus has not experienced a fading over the duration of the records (i.e. since early 1982) but these records do not sense puffs ejected off the line of sight to the star.

In two respects, different circumstances are found for RT Nor (Fig. 6). The AKARI and Spitzer mid-IR fluxes of RT Nor are in good agreement but the WISE fluxes are about an order of magnitude stronger and about twice the IRAS fluxes. The inference is that a fresh ejection of dust occurred after the 2005–2006 observations by Spitzer and AKARI and prior to the observation in 2010 by WISE. This sharp increase in mid-IR fluxes may be linked to a fading of the star: AAVSO records show that a minimum occurred between October 2008 when the star was at maximum and the next observation in mid-2009 when it was about 5 mag below maximum, the first decline since 1990. A second aspect setting RT Nor apart from Y Mus is that Kilkenny & Whittet’s (1984) M flux is considerably less than the IRAS measurement even though the M measurement was made during the year-long IRAS mission. We conjecture that between the M and IRAS observations fresh dust was added to the circumstellar shell. The AAVSO observations show no dimming in this period so that dust replenishment must have occurred off the line of sight to the star. These few observations suggest that RT Nor is more active than either UV Cas or Y Mus.

Frequency of ejection of dust is likely related in some way to the occurrence of visual fadings characteristic of RCBs but, as the examples of UV Cas and Y Mus illustrate, not every ejection of dust results in a dimming of the star. Jurcsik (1996) estimated interfade intervals for RCBs from reported visual magnitudes. Her two longest intervals were for Y Mus (15 300 d) and UV Cas (25 500 d) with typical values of 1000–2000 d reported for many RCBs. Thus, the low R values, the large amplitude mid-IR flux variations and the long interfade intervals are all consistent with the view that UV Cas and Y Mus are presently poor producers of dust. Jurcsik’s interfade interval for RT Nor is listed as 1950 d. In Paper I, we noted that the last minimum of RT Nor occurred about 15 yr before 2008 and intimated that this suggested that Jurcsik’s 1950 d estimate was an underestimate. However, recently two minima have been observed with onsets in about 2009 January and 2013 February. One may conjecture, as noted above, that the 2009 January fading was responsible for the large increase in mid-IR flux between the AKARI–Spitzer and WISE observations.

At the opposite end of the range of factors R to the above trio of outliers are RCBs with large R. Among this sample, there is a range of variation in the mid-IR fluxes from the four satellites. At one extreme is ES Aql (Fig. 7) where the mid-IR fluxes are very similar from all four satellites. The mean R is 0.67 ± 0.06. As noted in Paper I, the cool RCB ES Aql declines frequently with a
Mid-IR variations of RCB stars

3.3 The run of $R$ with $T_d$

Simple implementations of the puff model suggest a correlation between the covering factor $R$ and the blackbody dust temperature $T_d$. One may conjecture that in cases such as V3795 Sgr and VZ Sgr puffs form and/or dissipate on time-scales of a couple of years or less. This behaviour is in contrast to UV Cas, Y Mus and RT Nor where the time-scale appears to be a couple of decades.

Figure 5. The SED for Y Mus from IR observations at different epochs: left-hand panel – IRAS with M and N fluxes from Kilkenny & Whittet (1984); middle panel – AKARI and Spitzer; right-hand panel – WISE. In each panel, a two blackbody fit is shown with a 7200 K blackbody (green dashed curve) representing the stellar fluxes corrected for interstellar reddening and a cooler blackbody representing dust emission (655 K for IRAS, 420 K for AKARI and 395 K for Spitzer and 340 K for WISE).

Figure 6. The SED for RT Nor from IR observations at different epochs: left-hand panel – IRAS with M and N fluxes from Kilkenny & Whittet (1984); middle panel – AKARI and Spitzer; right-hand panel – WISE. In each panel, a two blackbody fit is shown with a 6700 K blackbody representing the stellar fluxes corrected for interstellar reddening and a cooler blackbody representing dust emission (500 K for IRAS, 400 K for AKARI and 850 K for WISE).

Other stars show modest variations in their mid-IR fluxes. One such example is SU Tau (Fig. 8) where the shape of the mid-IR spectrum is very similar from epoch to epoch but the flux level varies by about 20–30 per cent. SU Tau is a star which is frequently in decline: it ‘has been three or more magnitudes below maximum light for nearly half of the last 20 yr’ (Paper I).

In the comparison with the Spitzer spectra obtained between the times of the AKARI and WISE observations, VZ Sgr (Fig. 10), a minority RCB, stands out; both the AKARI and the WISE fluxes are about a factor of 2 to three less than the Spitzer and IRAS fluxes. At the time of the Spitzer observation and as noted in Paper I, VZ Sgr was in a deep minimum and several magnitudes below maximum light, but, at the time of the AKARI and WISE observations, the star was at maximum light. The high mid-IR flux seen by Spitzer is likely attributable to the fresh dust contributed by the on-going minimum but, in contrast to many other RCBs, the dust dispersed quickly; a high-speed wind, as in V3795 Sgr, drove dust out to great distances?.

One may conjecture that in cases such as V3795 Sgr and VZ Sgr puffs form and/or dissipate on time-scales of a couple of years or less. This behaviour is in contrast to UV Cas, Y Mus and RT Nor where the time-scale appears to be a couple of decades.
The factor $R$ is defined to be the ratio of integrated dust emission to the stellar flux: $R = f_{\text{dust}}/f_{\text{star}}$, where corrections are applied for interstellar reddening. Consider the following scenario. Suppose that puffs occupy a volume $V(r)$ at a distance $r$ from the central star and the dust in the puffs is at an equilibrium temperature $T_d$. The thickness of a puff is assumed to be small relative to the radial distance $r$. The integrated emission from the dust will be proportional to the product of $V(r)$ and $T_d^4$. The dust temperature $T_d$ is taken to be the equilibrium temperature of a grey dust grain in an optically thin puff, i.e. $T_d \propto r^{-0.5}$ (Kwok 2007, p. 314, equation 10.32). In this simple picture, the ratio of star’s integrated dust emission from a given set of puffs moving out from distance $r_1$ to $r_2$ is $V(r_1)/V(r_2) = (T_d(r_1)/T_d(r_2))^4$. Thus, if the puffs are optically thin at IR wavelengths and their dust content are not evolving, $f_{\text{dust}}$ is expected to scale with $T_d^4$. Note that $T_d$ is determined from the wavelength dependence of the IR emission and $f_{\text{dust}}$ from the integrated IR emission and, thus, $T_d$ and $f_{\text{dust}}$ are different measures of the IR emission.

For the sample of RCBs, one expects that for a given $T_d$, there will be a spread in $f_{\text{dust}}$ because the size and number of puffs and their distribution with radial distance $r$ will vary from star to star and time to time for a particular star. The temperature $T_d$ distance $r$ relation will depend also on the stellar temperature $T_{\star}$; $T_d \propto T_{\star}$ (see Kwok 2007). Unfortunately, there is no observational way to determine the mean radial distance of the puffs at a given time or the distance travelled by puffs between two observations without assuming any expansion velocity for the dust. The covering factor $R = f_{\text{dust}}/f_{\text{star}}$ involves the stellar flux which is set by the luminosity $L$ or the product $R_{\text{star}}^2 f_{\text{star}}^2$.

Observational pursuit of the above idea may be most instructive when applied to those stars in Table 2 for which $R$ and $T_d$ both decrease with time. In such cases, the reasonable inference is that a set of puffs is expanding away from their formation site near the star and, thus, experiencing cooling as observations proceeded from IRAS to WISE. Six stars (Y Mus, UV Cas, V3795 Sgr, SV Sge, V1157 Sgr and RT Nor) meet our condition with an additional four satisfying the condition but for an increase in $R$ and $T_d$ with the observation by WISE which we interpret as the signal of the formation of a new puff or cloud of puffs. In Fig. 11, we show the $R$ versus $T_d$ relations for seven of the 10 stars and also for V854 Cen, a star with a near-constant $R$ and $T_d$. An evolutionary trend is indicated for each star. Note that for RT Nor and V CrA, the WISE result is disconnected from the indicated trend from earlier photometry because, we suppose, fresh puffs have appeared between the time of the Spitzer and WISE observations. Inspection of Fig. 11 shows that the evolutionary trends comprise a very diverse set. This diversity is likely to reflect several factors including the variation from star

\[ \text{Figure 7. SED of ES Aql. The fit of two cool blackbodies (750 and 140 K) is made to the Spitzer spectrum. IRAS, AKARI and WISE observations show no sensible departures from the Spitzer spectrum except that the WISE flux density at 6 \mu m exceeds the expectation from the Spitzer spectrum. The stellar blackbody corresponding to a temperature of 4500 K is shown by the green dashed curve.} \]

\[ \text{Figure 8. SED of SU Tau. The fit of a blackbody at 635 K is made to the Spitzer spectrum. IRAS, AKARI and WISE observations show modest variations around the Spitzer spectrum with larger variations according to AKARI and WISE at shorter wavelengths. The stellar blackbody for 6500 K is shown by the green dashed curve.} \]
Mid-IR variations of RCB stars

3.4 Dust and photospheric composition

Photospheric composition should be one of the key variables influencing the formation of the carbon soot, a presumed principal component of a puff’s dust. Chemical compositions of warm RCBs come mainly from Asplund et al. (2000) with results for V854 Cen from Asplund et al. (1998), V532 Oph from Rao et al. (2014) and V2552 Oph from Rao & Lambert (2003). Two introductory points about these compositions. First, the analysis by Asplund et al. (2000) using modern H-poor, He-rich model atmospheres (Asplund et al. 1997) revealed what was termed ‘the carbon problem’, i.e. the C abundance returned from the analysis of a spectrum was about 0.6 dex less than the C abundance adopted in the model atmosphere’s construction. This problem was discussed by Asplund et al. (2000) who suggested that an abundance ratio was
Dust parameters from mid-IR SEDs of RCBs at various epochs.

| Star         | Ground/other | IRAS (1983) | AKARI (2006) | Spitzer | WISE (2010.5) | Mean R |
|--------------|--------------|-------------|--------------|---------|---------------|--------|
|              | Epoch R Td   | R Td        | R Td         | Epoch R Td | R Td         | R av   |
| XX Cam       |              |             |              |          |               |        |
| SU Tau       | 0.46 600 0.09 850 2008.4 0.44 635 0.54 750 0.61 0.23 |
| UX Ant       | 0.14 650     |             |              |          |               |        |
| UW Cen       | 1984d 0.35 620 0.44 630 0.44 600 2008.7 0.46 636 0.44 650 0.38 0.10 |
| Y Mus        | 1984b 0.14 980 0.08 655 0.01 420 2008.4 0.01 395 0.01 340 0.06 0.06 |
| V854 Cen     | 1996.8c 0.94 1100 0.57 920 0.54 900 2007.8 0.30 900 0.54 880 0.58 0.22 |
| Z UMi        | 0.95 850 0.79 900 2008.9 0.41 695 0.79 900 0.74 0.20 |
| S Aps        | 0.42 750 0.22 710 2008.4 0.37 730 0.14 950 0.29 0.11 |
| R CrB        | 1998d 0.30 810 0.20 680 0.30 810 2004.6 0.30 950 0.30 810 0.28 0.04 |
| RT Nor       | 1984b 0.11 920 0.08 500 0.02 400 2005.4 0.01 365 0.46 850 0.14 0.17 |
| RZ Nor       | 1983.4c 0.72 720 0.72 720 0.54 650 2006.3 0.64 640 0.37 610 0.57 0.13 |
| V517 Oph     | 0.98 850 0.92 885 2008.4 0.92 885 0.83 880 0.91 0.05 |
| V2552 Oph    |              |             |              |          |               |        |
| V532 Ophf    |              |             |              |          |               |        |
| V1783 Sgr    | 0.53 770 0.65 790 2008.4 0.27 554 0.24 670 0.42 0.17 |
| WX CrA       | 0.46 670 0.39 740 2008.9 0.14 570 0.35 870 0.33 0.12 |
| V379 Sgr     | 0.98 900 0.71 700 2008.4 0.57 656 0.68 850 0.74 0.15 |
| V3795 Sgr    | 0.50 700 0.36 650 2008.4 0.29 600 0.17 580 0.33 0.12 |
| VZ Sgr       | 0.21 700 0.11 780 2008.4 0.21 692 0.07 770 0.15 0.06 |
| Rs Tel       | 1984b 0.26 790 0.28 700 0.36 850 2005.78 0.33 830 0.15 650 0.28 0.07 |
| MACHO135.27132.51 |              |             |              |          |               |        |
| GU Sgr       | 1977.6c 0.45 910 0.06 400 0.12 600 2008.4 0.27 510 0.27 0.18 |
| NSV 11154    | 0.63 720 0.92 800 2008.4 0.40 750 0.65 0.21 |
| FH Sct       | 0.04 393 0.17 740 2008.5 0.10 537 0.25 940 0.14 0.09 |
| V CrA        | 1984f 0.69 640 0.87 670 0.45 580 2005.4 0.38 550 0.65 700 0.61 0.18 |
| SV Sgr       | 0.17 730 0.11 600 2008.5 0.05 500 0.03 500 0.09 0.05 |
| V1157 Sgr    | 0.89 880 0.60 820 2008.5 0.44 753 0.34 680 0.57 0.21 |
| RX Sgr       | 1997.3c 0.38 820 0.76 870 0.38 820 2004.89 0.20 675 0.38 820 0.42 0.18 |
| ES Aql       | 0.70 830 0.70 830 2008.5 0.58 750 0.70 830 0.67 0.06 |
| V482 Cyg     | 0.09 650 0.14 850 2004.96 0.04 580 0.20 970 0.12 0.06 |
| U Aqr        | 0.37 560 0.37 560 2008.6 0.25 473 0.24 680 0.31 0.06 |
| UV Cas       | 0.26 830 0.03 507 2008.5 0.03 507 0.02 510 0.08 0.10 |

Notes. 
1. Goldsmith et al. (1990) also observed the star on 1985.5 from which R and Td of 0.16 and 540 K have been derived. They have been included in the average. 
2. The ground-based observations come from Kilkenny & Whittet (1984). Their observations were obtained mainly in 1983 April. 
3. V854 Cen was observed by ISO on 1996 September (Lambert et al. 2001). The R and Td values refer to ISO observations. The AKARI and WISE photometry needed a combination of two blackbodies to match the SED. 
4. R CrB was observed by ISO on 1998 January 15. The ISO SED could be fit by a combination of blackbodies 810, 750 and 550 K. The flux densities from AKARI and WISE match very well the ISO spectrum. All three observations have same value of R. 
5. The KW’s and IRAS observation together defined the SED from which R and Td have been estimated. 
6. The ground-based observations are from Glass (1978). 
7. RS Tel was observed by ISO on 1997 March 25. The ISO spectrum could be fit with a single blackbody of 870 K. The flux densities obtained with AKARI and WISE match the ISO spectrum very well. All three observations result in same R.

The mean R – the quantity Rmean from Table 2 – is taken as a RCB’s propensity to shed puffs. The C abundances – see above remarks on the carbon problem – span a small range including both majority and minority RCBs and are uncorrelated with the mean R. The O abundances exhibit a range of about 1.5 dex. There appears to be a positive correlation between the O abundance and the mean R (Fig. 13) with barely a separation between majority and minority RCBs. The N abundances of majority RCB show a weak anticorrelation with the mean R with three of the four minority RCBs having a far lower N abundance (Fig. 13). Similarly, the Fe abundances of the majority are weakly anticorrelated with the mean R with, of course, the Fe abundances of minority RCBs falling below those of the majority RCBs. A least-squares solution of all the stars (both majority and minority) for O abundance with respect to the mean R suggests a slope of 1.24 ± 0.4 and a correlation coefficient of 0.52 which would increase to 1.33 ± 0.33 and a correlation coefficient of 0.70 if two outliers are dropped. For the N abundance with respect to the mean R, the slope is –0.5 ± 0.4 with a correlation coefficient of –0.3. For Fe abundance the majority RCBs suggest a slope of –0.4 ± 0.3 with a correlation coefficient of –0.37. For
minority RCBs the Fe abundances give the slope is $-0.15 \pm 0.7$ with a correlation coefficient of $-0.14$.

Correlations between abundance ratios and the mean $R$ have also been looked for. Using the spectroscopic C abundance, the O/C ratio also shows a weak correlation with mean $R$ without a distinction between majority and minority RCBs. A least-squares solution of the majority RCBs shows a slope of 1.54 and a correlation coefficient of 0.65. The slope becomes $1.24 \pm 0.4$ and correlation coefficient of 0.5 when majority and minority RCBs are considered together. The O/N and O/Fe ratios (Fig. 14) show a rough tendency to increase with increasing mean $R$ with two of the four minority RCBs falling amongst the majority RCBs; the exceptions among minority RCBs are VZ Sgr and V854 Cen with O/N and O/Fe ratios greater than those held by any majority RCB. The least-squares solution for O/N shows a slope of $1.7 \pm 0.4$ and correlation coefficient of 0.77 for majority and 0.55 for combined sample with the same slope. The O/Fe also suggests a slope of $1.67 \pm 0.67$ and a correlation coefficient of 0.60 for majority and combined sample has a slope of $2.17 \pm 0.7$. All three abundance ratios O/C, O/N and O/Fe seem to show a positive slope with respect to mean $R$.

Thus, it is possible that O abundance might have an influence in dust production in these stars. Woitke, Goeres & Sedlmayr's (1996) models for dust formation in RCBs suggest that the CO molecule is the most dominant cooling agent which determines whether the gas can reach condensation temperatures or not after passage of a pulsation shock. As long as the C abundance exceeds the O abundance, the CO abundance is likely to be influenced by the

### 3.5 Dust and luminosity

Is dust formation related to a star’s luminosity? In the absence of traditional (i.e. trigonometric) methods of estimating luminosity, we rely on the pulsation–period relation for RCBs. Pulsation is also of direct interest to the puzzle of dust formation in that a link, first suggested by Pugach (1977) and confirmed by Crause, Lawson & Henden (2007) ties onset of dust formation to a preferred phase in a RCBs radial pulsation. However, copious dust formation appears not to occur at every passage through the preferred phase. Also, it is unclear how dust formation is physically related to this phase. Although Woitke et al. (1996) developed a model for a shock-induced dust formation in a pulsating RCB, many details are still to be worked out, particularly in applying it to specific stars (however see Rao & Lambert 2010).

On the theoretical front, radial pulsations obey the relation $P \rho^{1/2} = Q$, where $P$ is the period, $\rho$ the density and $Q$ is a constant. Following Fadeyev (1996), $P$ in days can be written as follows:

$$ P = \frac{\left(\frac{T_{\text{eff}}}{T_{\text{eff}}^{\odot}}\right)^{3}}{\left(\frac{L}{L_{\odot}}\right)^{3/4}} \left(\frac{M}{M_{\odot}}\right)^{-1/2} Q, $$

where $L$ and $M$ are luminosity and mass, respectively.

Pulsation periods for many RCBs have been estimated from optical photometry with measurements available in the literature for the majority of stars in Table 1 (see Table 3 and Lawson & Kilkenny 1996). Photometric amplitudes are small in most cases: less than
several hundredths of a magnitude except for RY Sgr and a few others. Amplitudes may be variable and $P$ may vary too. With few exceptions, the period $P$ is in the range 30–50 d. Exceptions include U Aqr with $P = 81.3$ d (Lawson et al. 1990), $P = 114$ d for NSV 11154 (our unpublished estimate), $P = 120$ d for S Aps (Kilkenny & Flanagan 1983) and $P = 130$ d for Z UMi (Benson et al. 1994). Periods shorter than 30 d are found among the extreme helium stars, the hydrogen-deficient carbon stars and the hot RCBs.

The $T_{\text{eff}}$ versus $P$ plane is shown in Fig. 15. This plot is similar to Lawson et al.’s (1990) fig. 32. Effective temperatures for most RCBs are spectroscopic values from Asplund et al. (2000). However for the cooler RCBs, except for Z UMi, no spectroscopic $T_{\text{eff}}$s are available and $T_{\text{eff}}$ estimates are based on SED fits and photometry. Using Fadeyev’s formula, we find most of the RCB stars are distributed around the locus $\log L/M = 4.0$ for $M$ of 0.7 $M_\odot$ with the HdC stars located around $\log L/M = 3.5$.

Within the large sample with $P$ between 30 and 50 d, there is no correlation between the mean $R$ and $P$ or $T_{\text{eff}}$. Outside this period range, there are too few RCBs to search for connections with the mean $R$. However as seen from Table 2, generally cooler RCBs (presumably with longer periods) have larger mean $R$ values. At periods shorter than about 20 d, dust formation either does not occur or is an infrequent occurrence. As noted above, the stars in this category are the EHes warmer than RCBs, the HdC stars at the cool end and the hot RCBs at the hot end of the range for RCBs. Only one HdC star that showed mild IR excess, V4512 Sgr, is marked in Fig. 15.

Better estimates of both periods as well as the mode of pulsation, particularly for cool RCBs, are required for a proper assessment of the pulsation–dust connection.

### 3.6 Dust mass

It might be relevant to estimate the mass of dust that is hanging around these stars. The basic relation is

$$M_d = \left( \frac{F_\lambda}{\kappa_\lambda} \right) \left( \frac{D^2}{B_\lambda(T_d)} \right), \quad (2)$$

where $M_d$ is the dust mass, $F_\lambda$ is the excess flux over the stellar flux, $\kappa_\lambda$ is the absorption coefficient of the grain in cm$^2$ gm$^{-1}$, $D$ is the distance to the object and $B_\lambda(T_d)$ is the Plank function.

This quantity also depends on the estimate of the distance to the object. We use the $M_\text{v}$ versus $(V - I)$ colour established for LMC RCBs by Tisserand et al. (2009). As typical for F-type stars UV Cas and Y Mus, we assume the $M_\text{v}$ to be $-5$ and estimate the distance. Also assuming the dust to be amorphous carbon of BE type (Colangeli et al. 1995), we estimate the dust mass at the time of IRAS observations as $2.9 \times 10^{-9}$ m$_\odot$ for both UV Cas and Y Mus. A similar estimate for cool RCB Z UMi is obtained, by assuming $M_\text{v}$ of $-3.5$ as consistent with the observed $V - I$ colour, as $4.8 \times 10^{-9}$ m$_\odot$ at the time of IRAS observations.

Clayton et al. (2011) imaged R CrB and its environment during the prolonged deep minimum that started in 2007. They discovered several cometary knots around the star which are interpreted as...
past ejected dust puffs from the star. As they state ‘the puffs cause declines when they form directly in our line of sight and may be seen as cometary knots when they form to the side of or behind R CrB’. It is interesting to note that the dust mass estimated for these knots of $10^{-8}$ to $10^{-9}$ $M_\odot$ is about equal to dust mass estimated above for UV Cas, Y Mus and Z UMi from mid-IR excess at any given instant. It is of interest to note that Clayton et al. (2011) estimate total dust mass of $10^{-7}$ $M_\odot$ for the R CrB shell including the cold dust from their observations into far-IR with Herschel.

### 4 CONCLUDING REMARKS

Variations in the mid-IR flux densities measured by the different satellites reflect changes in the circumstellar shell’s mid-IR emission. Plots comparing flux densities from the four IR satellites are shown in Figs 1–3. It is of interest to compare such comparisons the intrinsic changes in mid-IR emission. Identification of intrinsic changes is certainly secure for the extreme outliers in the figures – most notably, UV Cas (Fig. 4), S Aps (Fig. 5) and RT Nor (Fig. 6). These must arise from the varying contribution of puffs in the circumstellar shell; the scale of these variations far exceeds the measurement uncertainties. As discussed above, UV Cas’s high $IRAS$ flux densities arise from puffs emitted somewhat prior to the $IRAS$ observation and, as these puffs evolved away from the star, cooled, lower flux densities were measured by the later satellites and in the ground-based important series of measurements by Bogdanov et al. (2010). Somewhat similarly, RT Nor owes its position as an outlier to the appearance of fresh puffs between the $Spitzer$ and $WISE$ observations and some fraction of these puffs caused the deep optical decline recorded by AAVSO.

Extreme outliers are valuable because they provide a measure of the contribution to the mid-IR emission from an individual puff or a collection of associated puffs; RT Nor experiences a 0.4 increase in $R$ between the $Spitzer$ and $WISE$ observations. Other RCBs with a circumstellar shell occupied by too many puffs will experience smaller variations in their mid-IR emission. One might suppose that variations will decrease in size as the number of puffs increases.

Stars where the mid-IR emission appears to be dominated by the slow evaporation of the circumstellar dust offer the opportunity to test theoretical ideas. In particular, movement of a given family of puffs away from the star is predicted to lead to the relation $R \propto T_d^2$. As shown in Section 3.3 and, in particular by Fig. 12, this prediction is
that a RCB’s propensity to form and eject dusty puffs evolves during this time. Some interesting insight that emerged from these mid-IR studies of RCBs are that the oxygen abundance has some influence on the dust production. It also appears that cool RCBs have more dust around them and higher luminosity at a given mass helps in dust production. Studies of pulsations, particularly in cool RCBs, would be a great help in future understanding of dust production in these stars.

Observational insights into the formation and evolution of dust around RCB stars are limited by the paucity of measurement at IR wavelengths. Campaigns extending Bogdanov et al.’s (2010) close scrutiny of UV Cas to other RCBs are to be encouraged. Such a campaign should extend to long wavelengths the valuable two decade long programme of *JHKL* photometry of a dozen southern RCBs by Feast et al. (1997; see also Feast 1997). Although, as shown here, the four IR satellites from *IRAS* to WISE have provided valuable insights into the evolution of circumstellar dust for a major sample of RCBs, it is most unlikely that a satellite will ever provide a thorough collection of IR photometry of these rare and fascinating stars.

**ACKNOWLEDGEMENTS**

We appreciate various comments made by the referee Geoff Clayton which improved the paper a great deal. We thank Aníbal García-Hernández for his assistance with the preparation and analysis of the observations of RCBs with the *Spitzer* satellite. We acknowledge with thanks the variable star observations from the AAVSO data base. This research has made use of the SIMBAD data base, operated at CDS, Strasbourg, France. This research has been supported in part by the grant F-634 from the Robert A. Welch Foundation of Houston, Texas.

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