SUPER-CRITICAL GROWTH OF MASSIVE BLACK HOLES FROM STELLAR-MASS SEEDS

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ABSTRACT

We consider super-critical accretion with angular momentum onto stellar-mass black holes as a possible mechanism for growing billion-solar-mass black holes from light seeds at early times. We use the radiatively inefficient “slim disk” solution—advective, optically thick flows that generalize the standard geometrically thin disk model—to show how mildly super-Eddington intermittent accretion may significantly ease the problem of assembling the first massive black holes when the universe was less than 0.8 Gyr old. Because of the low radiative efficiencies of slim disks around non-rotating as well as rapidly rotating black holes, the mass e-folding timescale in this regime is nearly independent of the spin parameter. The conditions that may lead to super-critical growth in the early universe are briefly discussed.

Key words: accretion, accretion disks – black hole physics – cosmology: miscellaneous – galaxies: high-redshift

Online-only material: color figures

1. INTRODUCTION

The most distant quasar discovered to date, ULAS J1120+0641 at a redshift \( z = 7.084 \), is believed to host a black hole with a mass of \( 2.0^{+0.5}_{-0.7} \times 10^9 \, M_\odot \) that is shining 0.78 Gyr after the big bang (Mortlock et al. 2011). This object, together with the handful of bright Sloan Digital Sky Survey (SDSS) quasars at redshift \( z \gtrsim 6 \) (Fan 2006), sets some of the tightest constraints on models for the formation and growth of massive black holes (MBHs) at early epochs. The challenge provided by the existence of billion-solar-mass black holes at the end of the reionization epoch is easily described (see, e.g., Haiman 2013 for a review). If MBHs are assembled by the accretion of gas onto less massive “seed” holes at the rate \( \dot{m} \), and if in the process a fraction \( \epsilon \) of the rest-mass energy of the infalling material is released as radiation, then the growth of the black hole’s mass \( M \) is regulated by the equation

\[
\frac{dM}{dt} = (1 - \epsilon) \dot{m} = \left( \frac{1 - \epsilon}{\epsilon} \right) \left( \frac{L}{L_E} \right) \frac{M}{t_E}, \quad (1)
\]

Here \( L \) is the radiated luminosity, \( \epsilon \equiv L/\dot{m}c^2 \), \( L_E \equiv 4\pi G M \mu_e m_p c/\sigma_T \) is the Eddington limit when the continuum radiation force balances gravity, \( \sigma_T \) is the Thomson scattering cross-section, \( \mu_e \) is the mean molecular weight per electron, \( t_E \equiv M c^2/L_E = 0.44 \mu_e^{-1} \) Gyr is the Eddington timescale, and all other symbols have their usual meaning. The characteristic e-folding timescale \( t_{\text{acc}} \) for mass growth is then

\[
t_{\text{acc}} = \frac{1}{1 - \epsilon} \left( \frac{L}{L_E} \right) t_E \approx (4.3 \times 10^7 \, \text{yr}) \left( \frac{L}{L_E} \right), \quad (2)
\]

where the last equality assumes \( \mu_e = 1.15 \) (valid for primordial gas) and a radiative efficiency of \( \epsilon = 0.1 \). In a concordance cosmology with \( \Omega_M = 0.27 \), \( \Omega_k = 0.73 \), and \( H_0 = 70 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \), the time elapsed between \( z = 20 \) and \( z = 7 \) is 0.6 Gyr, corresponding to 14 e-foldings of Eddington-limited accretion (\( L = L_E \)) and a mass amplification factor of \( 10^6 \).

The growth of \( 2 \times 10^9 \, M_\odot \) MBHs at the Eddington rate from light black hole seeds of mass \( M_0 = 100 \, M_\odot \) requires \( \ln(2 \times 10^9/100) = 17 \) e-foldings at \( L = L_E \). Therefore, if the first seeds were \( \sim 100 \, M_\odot \) remnants of the first generation of massive stars (e.g., Madau & Rees 2001; Haiman & Loeb 2001; Heger et al. 2003; Volonteri et al. 2003), these could grow into billion-solar-mass black holes by \( z \sim 7 \) only if all the following conditions were fulfilled: (1) seeds were present early on, at \( z \gtrsim 20 \); (2) gas accretion continued more or less uninterrupted at the Eddington rate for \( z \gtrsim 0.6 \) Gyr; and (3) \( \epsilon \lesssim 0.1 \) (Tanaka & Haiman 2009). The second condition is hard to satisfy in the shallow potential wells of low-mass dark matter halos, as feedback effects resulting from the accretion process itself are expected to dramatically affect gas inflow and may result in sub-Eddington rates and negligible mass growth (Johnson & Bromm 2007; Pelupessy et al. 2007; Alvarez et al. 2009; Milosavljevic et al. 2009). The third condition requires radiative efficiencies that are below those expected for thin disk accretion onto rapidly spinning Kerr black holes (\( \epsilon \approx 0.3-0.4 \), Thorne 1974; Shapiro 2005), and approach the value, \( \epsilon = 0.057 \), characteristic of the Schwarzschild non-rotating solution.

Over the last decade, a number of alternatives to the above picture have been proposed. If stellar seeds were present in large numbers at high redshifts, coalescing black hole binaries brought together by successive galaxy mergers may, in principle, help mass build-up and generate mass amplification factors as high as \( 10^4 \) (Yoo & Miralda-Escude 2004). More massive seeds, with \( M_0 \sim 10^4-10^5 \, M_\odot \), may form through the “direct collapse” of low angular momentum gas at high redshift (e.g., Loeb & Rasio 1994; Bromm & Loeb 2003; Kousshiappas et al. 2004; Lodato & Natarajan 2006; Spaans & Silk 2006; Regan & Haehnelt 2009; Mayer et al. 2010), likely via the intermediate stage of supermassive stars (Belgeman 2010), and therefore “jump start” the whole process. Questions remain about the idealized conditions needed in these models to avoid fragmentation, dissipate angular momentum, and drive gas toward the center of protogalaxies at extremely high rates.

In this Letter we discuss super-critical (i.e., super-Eddington) accretion with angular momentum onto stellar-mass seeds...
as a possible mechanism for bypassing some of the above difficulties. Evidence for near-Eddington or super-Eddington flows has been accumulating in recent years. Super-critical accretion onto stellar-mass black holes has been invoked to explain the nature of the ultraluminous X-ray sources (e.g., Gladstone et al. 2009; Middleton et al. 2013). A study of a large sample of active galactic nuclei (AGNs) suggests that many of them emit considerably more energy and have higher \( L/L_E \) ratios than previously assumed (Netzer & Trakhtenbrot 2014). Kormendy & Ho (2013) have recently argued that the normalization of the local black hole scaling relations should be increased by a factor of five to \( M_{\text{BH}} = 0.5\% M_{\text{edge}} \). This increases the local mass density in black holes by the same factor, decreases the required mean radiative efficiency to 1%–2%, and may be evidence for radiatively inefficient super-Eddington accretion (e.g., Soltan 1982; Novak 2013). At high redshifts, the very soft X-ray spectrum of ULAS J1120+0641 appears to suggest that this quasar is accreting at super-critical rates (Page et al. 2013). On the theoretical side, it is known that the standard, radiatively efficient thin disk solution (Shakura & Sunyaev 1973) can no longer be applied when mass is supplied to a black hole at high rates. In this regime, viscosity-generated heat does not have sufficient time to be radiated away, and is instead advected into the hole. The shorter mass e-folding timescales and the decreased radiative efficiencies that characterize these flows make them ideal for feeding and growing MBHs out of stellar-mass seeds at early times.

2. SUPER-CRITICAL ACCRETION: THE SLIM DISK SOLUTION

The Shakura–Sunyaev treatment of accretion onto a black hole via a thin disk posits a radiatively efficient flow where all the heat generated by viscosity at a given radius is immediately radiated away. It is a local model, described by algebraic equations, valid at any particular (radial) location in the disk, independently of the physical conditions at different radii. At high accretion rates, i.e., when \( \dot{m} \geq 0.3 \dot{m}_E \), this assumption is incorrect. Here, \( \dot{m}_E \equiv 16L_E/c^2 \) is the critical accretion rate that gives origin to about an Eddington luminosity in the case of a radiatively efficient thin disk around a non-rotating black hole.\(^5\) Optically thick, stationary “slim disks” offer a more general set of non-local solutions, obtained by numerical integration of the two-dimensional stationary Navier–Stokes equations with a critical point—the radius at which the gas velocity exceeds the local speed of sound (Abramowicz et al. 1988).

To model such super-critical flows—which are characterized by large radial velocities, non-Keplerian rotation, inner edges that are closer to the black hole than the innermost stable circular orbit, and low radiative efficiencies—we use the numerical solutions of the relativistic slim accretion disk equations obtained by Sadowski (2009) and available online at http://users.camk.edu.pl/as/slimdisk.html. Figure 1 shows how super-critical accretion is qualitatively different from the standard, sub-Eddington, thin disk solution. The right panel depicts the disk luminosity (in units of \( L_E \)) versus the accretion rate (in units of \( \dot{m}_E \)) for four values of the black hole spin parameter \( a = 0.983, 0.755, 0.505, \) and 0. The corresponding radiative efficiency \( \epsilon = L/\dot{m}c^2 \) is plotted in the left panel. Despite super-Eddington \( \dot{m} > \dot{m}_E \) accretion rates, slim disks remain only moderately luminous (\( L \gtrsim L_E \)), as a large fraction of the viscosity-generated heat is advected inward and released closer to the hole or not released at all. As a result of the increasing rate of advection, the efficiency of transforming gravitational energy into radiative flux decreases with increasing accretion rate. For ease of use and flexibility, we have fitted the two dimensional tabulated luminosity as

\[
L/L_E = A(a) \left( \frac{0.985}{\dot{m}_E/\dot{m} + B(a)} + \frac{0.015}{\dot{m}_E/\dot{m} + C(a)} \right),
\]

where the functions \( A, B, \) and \( C \) scale with the spin of the black hole as

\[
A(a) = (0.9663 - 0.9292a^{-0.5639}), \quad B(a) = (4.627 - 4.445a^{-0.5524}), \quad C(a) = (827.3 - 718.1a^{-0.7060}).
\]

Our fits to the emitted luminosity and ensuing radiation efficiency are compared in Figure 1 to the numerical results of Sadowski (2009). Over the range 0.001 < \( \dot{m}/\dot{m}_E \) < 500 and 0 < \( a < 0.998 \), fit residuals are typically below 7%. Since, in the case of photon-trapped super-critical accretion, the emitted luminosity is not linearly proportional to the accretion rate, it is convenient to rewrite Equation (2) as

\[
t_{\text{acc}} = \frac{t_E}{16(1 - \epsilon)} \left( \frac{\dot{m}_E}{\dot{m}} \right) \lesssim (8.4 \times 10^6 \text{ yr}) \left( \frac{3\dot{m}_E}{\dot{m}} \right),
\]

where the last inequality holds for modestly super-Eddington rates independently of the value of the black hole spin. Figure 2 shows how even small modifications to accretion rates and radiative efficiencies can have an exponential impact on the growth of seed black holes. In the left panel the cosmic assembly history of a seed hole of initial mass \( M_0 = 10^7 M_\odot \) accreting at \( \dot{m}/\dot{m}_E = 3 \) from redshifts 10 and 15 is compared to an Eddington-limited (\( \dot{m}/\dot{m}_E = 1 \)) growth that follows the classical thin disk solution. Two curves are shown, for (constant) spin parameter \( a = 0 \) and \( a = 0.99 \). Because of the low radiative efficiencies of slim disks around non-rotating as well as rapidly rotating holes, the mass e-folding timescale in this regime is nearly independent of the spin parameter. This is in contrast to the thin disk solution, where the mass of the growing hole is exponentially sensitive to its spin.

From the astrophysical standpoint, however, it seems unlikely that early growing black holes may be able to sustain uninterupted super-critical accretion rates for half a Gyr or so. The right panel of Figure 2 shows the illustrative growth histories of: (1) a seed non-rotating hole undergoing three major episodes of \( \dot{m}/\dot{m}_E = 3 \) accretion each lasting 50 Myr followed by a 100 Myr period of quiescence, i.e., a duty cycle of 0.5; and (2) a seed non-rotating hole undergoing five major episodes of \( \dot{m}/\dot{m}_E = 4 \) accretion each lasting 20 Myr followed by a 100 Myr period of quiescence, i.e., a duty cycle of 0.2. We have chosen a 100 Myr quiescence period since this is the mean time interval between major mergers (mass ratios \( \gtrsim 1:3 \)) at redshift 14 for a \( 10^{10} M_\odot \) descendant halo (Fakhouri et al. 2010). High duty cycles of 0.2–0.5 match those inferred from the observed clustering strength of bright quasars at redshift \( z = 3–4.5 \) (Shankar et al. 2010). Clearly, the shorter mass e-folding timescales of flows that are only mildly super-critical can significantly ease the problem of assembling MBHs out of

\(^5\) Note that many authors use a different definition of the critical accretion rate, i.e., \( \dot{m}_E \equiv L_E/c^2 \).
stellar-mass seeds at early times even in the case of intermittent accretion. We note here that while even shorter duty cycles may lead to the growth of MBHs if accretion was occurring at highly super-Eddington rates, \( \dot{m}/\dot{m}_E \gg 10 \), the slim disk solution is not directly applicable in such regimes. Indeed, general relativistic magnetohydrodynamic simulations of black hole accretion at rates in the range \( \dot{m}/\dot{m}_E = 20–200 \) have recently shown that these flows are actually efficient in terms of the total energy escaping from the system, which is mostly in the form of thermal and kinetic energy of outflowing gas and Poynting flux (Sadowski et al. 2014; McKinney et al. 2013). In highly super-critical flows, the magnitude of the outflow is found to be comparable with the inflow accretion rate (Sadowski et al. 2014).

3. DISCUSSION

The assembly of pregalactic MBHs following a phase of super-critical quasi-spherical accretion in metal-free halos has been explored by Volonteri & Rees (2005). Highly super-critical rates at early times have been recently advocated by Volonteri & Silk (2014). Here, we have extended and updated these works by focusing on the “slim disk” solution—advective, optically thick disks that generalize the standard model of radiatively efficient thin flows to moderately super-Eddington accretion rates. Under the assumption that a gas supply rate of a few \( \times \dot{m}_E \approx 0.01 M_\odot \, \text{yr}^{-1} \) (where \( M_\odot \) is the mass of the hole in units of \( 10^5 M_\odot \)) is indeed able to reach the central MBH for a period of 20 Myr or so, we have shown how a few episodes of super-critical accretion may turn a handful of light seed holes into the population of rare bright quasars observed at \( z \gtrsim 6 \) by the SDSS. Such high rates may both be determined by local physics within the accretion flow as well as by the large-scale cosmological environment in which MBHs and their hosts are growing. An example of small-scale physics is the presence of density inhomogeneities induced by radiative-hydrodynamic instabilities that reduce the effective opacity and allow super-Eddington fluxes from slim, porous disks (Dotan & Shaviv 2011). On large scales, gas-rich major mergers between massive protogalaxies have been shown to drive large gas flows toward the center of the merger remnant, owing to continuous losses of angular momentum by torques and shocks over a wide range of spatial scale (e.g., Mayer et al. 2010). Contrary to the formation of MBH seeds by rapid “direct collapse,” this gas supply is not
limited by the Kelvin–Helmholtz contraction timescale of the central supermassive proto-star (see, e.g., Shang et al. 2010), but can add to the central black hole mass over the longer accretion timescale of Equation (7). While these high-accretion episodes may generate AGN outbursts, drive hot, high-velocity outflows from the host galaxy, and occasionally clear the nuclear region of most of its gas, additional gas inflows may quickly rebuild the nuclear reservoir as AGN feedback appears to couple only weakly to a galaxy disk (Gabor & Bournaud 2014).

On intermediate scales, models of circumnuclear disks supported by the turbulent pressure caused by supernova explosions predict high mass supply rates to the central 1 pc lasting for 10^6 yr (Kawakatu & Wada 2008). Even in the absence of star formation, thick, turbulent pressure-supported disks are often seen in simulations of early protogalaxies (Wise et al. 2008). Viscous torques generated by the dissipation of supersonic turbulent motions generate a mass accretions rate (Pringle 1981)

$$m = 2\pi v\Sigma_0 \left| \frac{d \ln \Omega}{d \ln r} \right|,$$

where $\Sigma_0$ is the central surface density of the disk, $v \approx \sigma h$ is the viscous parameter (Wada & Norman 2002), $\sigma$ is the gas turbulent velocity, $h$ is the disk scale height, $h/r \sim \sigma/V_{\text{rot}}$, and $V_{\text{rot}}$ is the disk rotational velocity. Observations show that, at a fixed stellar mass, galaxies systematically increase in disordered motions and decrease in rotation velocity with increasing redshift and limited by the Kelvin–Helmholtz contraction timescale of the central supermassive proto-star (Shapiro, S. L. 2005, ApJ, 620, 59).

The estimate above shows that large mass supply rates in the dense environments of high redshift, dispersion-dominated, massive protogalaxies are theoretically plausible. It is consistent with the mass accretion rates measured at 100 pc from the center of simulated atomic cooling halos at $z > 10$ (Prieto et al. 2013), and is comparable to the mass accretion rates, $\sim c^3/G \propto T^{3/2}/G$ expected in self-gravitating, collapsing $T \sim 10^4$ K gas clouds. Rapid MBH growth may occur in rare special environments where gas can flow toward the center at super-Eddington rates relatively unaffected by feedback processes. Indeed it would seem fortuitous if the mass supply rate was always regulated exactly at $m_E$ during the early growth of MBHs, as it is commonly assumed. Equally contrived, of course, are gas fueling rates that remain at $3-4 \times m_E$ for $20-50$ Myr while the central MBHs are growing by many orders of magnitude in mass. Perhaps a more plausible scenario is one where $m \gg m_E$ early on (when black holes are still of intermediate mass), but only a fraction of this external mass supply reaches the horizon, and where $m$ “dwindles” to a few times Eddington during the last few e-foldings when the MBH is already above $10^8 M_\odot$.

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