Three-Body Scattering without Partial Waves

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Abstract. The Faddeev equation for three-body scattering at arbitrary energies is formulated in momentum space and directly solved in terms of momentum vectors without employing a partial wave decomposition. In its simplest form the Faddeev equation for identical bosons is a three-dimensional integral equation in five variables, magnitudes of relative momenta and angles. The elastic differential cross section, semi-exclusive d(N,N’) cross sections and total cross sections of both elastic and breakup processes in the intermediate energy range up to about 1 GeV are calculated based on a Malfliet-Tjon type potential, and the convergence of the multiple scattering series is investigated in every case. In general a truncation in the first or second order in the two-body t-matrix is quite insufficient.

Traditionally three-nucleon scattering calculations are carried out by solving Faddeev equations in a partial wave truncated basis. A partial wave decomposition replaces the continuous angle variables by discrete orbital angular momentum quantum numbers, and thus reduces the number of continuous variables, which have to be discretized in a numerical treatment. For low projectile energies the procedure of considering orbital angular momentum components appears physically justified due to arguments related to the centrifugal barrier. If one considers three-nucleon scattering at a few hundred MeV projectile energy, the number of partial waves needed to achieve convergence proliferates, and limitations with respect to computational feasibility and accuracy are reached. The amplitudes acquire stronger angular dependence, which is already visible in the two-nucleon amplitudes, and their formation by an increasing number of partial waves not only becomes more tedious but also less informative. The method of partial wave decomposition looses its physical transparency, and the direct use of angular variables becomes more appealing. It appears therefore natural to avoid a partial wave representation completely and work directly with vector variables.

Here we want to show that the full solution of the three-body scattering equation can be obtained in a straightforward manner, when employing vector variables, i.e. magnitudes of momenta and angles between the momentum vectors. As a simplification we neglect spin and isospin degrees of freedom and treat three-boson scattering. The interaction employed is of Malfliet-Tjon type, i.e. consists of a short range repulsive and intermediate range attractive Yukawa force. The parameters of the potential are adjusted so that a two-body bound state at $E_d = -2.23$ MeV is supported. Technical details of the calculation are given in [1, 2].

As first result we present in Fig. 1 the angular distribution for two energies, 0.2 and 1.0 GeV. In the upper panels the entire angle range is displayed using a logarithmic scale, whereas the lower panels focus on the forward angles.
Fig. 1: The elastic differential cross section at 0.2 and 1.0 GeV projectile energy as function of the laboratory scattering angle. The solid line represents the full solution of the Faddeev equation, whereas the other lines represent different orders in the multiple scattering series as indicated in the legends of the lower panels.

In addition to the exact Faddeev result, the cross sections are evaluated in first order in the two-body t-matrix, second order in t, third and 4th order in t and successively added up as indicated. This allows to study the convergence of the multiple scattering series. As expected, for the low energy, 0.2 GeV, rescattering terms of higher order are important, and even the 4th order is not yet close to the exact result. This is especially drastic for the large angles. At 1 GeV two rescattering terms (3rd order in t) are necessary to come into the vicinity of the final result in forward direction. For the large angles, the first rescattering correction has the biggest effect.

Fig. 2: The semi-exclusive cross section at 0.5 GeV laboratory incident energy and at 15° angle (left panels) and 33° angle of the emitted particle (right panels). In both cases the upper panel displays the high energy range of the emitted particle, whereas the lower panel shows the low energy range. The full solution of the Faddeev equation is given by the solid line in all panels. The contribution of the lowest orders of the multiple scattering series added up successively is given by the other curves as indicated in the legends.

The semi-exclusive cross section d(N,N') for scattering at 0.5 GeV is given in Fig. 2 for the emission angles 15° and 33°. The upper panels show the high energies and the lower panels the low energies of the emitted particle. Together with the full solution of the Faddeev equation (solid line) the sums of the lowest orders of the multiple scattering series are shown as indicated in the figure. The peak at the highest energy of the
emitted particle is the so called final state interaction (FSI) peak, which only develops if rescattering terms are taken into account. This peak is a general feature of semi-exclusive scattering and is present at all energies. The next peak is the so called quasi-free (QFS) peak, and one observes that at both angles one needs at least rescattering up to the 3rd order to come close to the full result. At both angles the very low energies of the emitted particle exhibit a strong peak in first order, which is considerably lowered by the first rescattering. Here the calculation up to 3rd order in the multiple scattering series seems already sufficient.

At 1 GeV the situation is similar, as shown in Fig. 3. It is interesting to observe that the strong QFS peak for the small angle (15°) only develops after two rescatterings (3rd order), and the final height of the peak requires orders higher than the ones shown. For the larger angle (33°) the third order calculation is already quite close to the exact solution. Again the final result for the peak at the very low energy of the emitted particle is reached with two rescattering contributions.

In conclusion we can say that the three-body Faddeev equation can be safely solved at intermediate energies using momentum vectors directly. Our present calculations are based on local forces, however this is not a restriction of our approach. To the best of our knowledge these are the first calculations of this kind. The key point is to neglect the partial wave decomposition generally used at low energies. Thus all partial waves are exactly included.

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REFERENCES

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