Structural Damage Assessment via Model Updating Using Augmented Grey Wolf Optimization Algorithm

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Abstract

Some civil engineering-based infrastructures are planned for the structural health monitoring (SHM) system based on their importance. Identification and detecting damage automatically at the right time are one of the major objectives this system faces. One of the methods to meet this objective is model updating with the use of optimization algorithms in structures. This paper is aimed at evaluating the location and severity of the damage combining two being-updated parameters of the flexibility matrix and the static strain energy of the structure using augmented grey wolf optimization (AGWO) and only with extracting the data of damaged structure, by applying 5 percent noise. The error between simulated and estimated results in average of ten runs and each damage scenario was less than 3 percent which proves the proper performance of this method in detection of the all damages of the 37-member three-dimensional frame and the 33-member two-dimensional truss. Moreover, they indicate that AGWO can provide a reliable tool to accurately identify the damage in compare with the particle swarm optimizer (PSO) and grey wolf optimizer (GWO).

1. INTRODUCTION

After long term utilization, the infrastructures should be evaluated in terms of safety and sustainability. Over time, a structure may lose its desired performance due to the factors such as earthquakes, floods, storms, etc. This may even leads to its collapse.

With the advent of advanced technologies including sensor networks, information and signal processing and managing systems [1–3], The SHM process has been able to enhance safety, sustainability, the development of infrastructure, measuring and management of cost of exploitation over time. The use of monitoring provides with the required information in building smart structures; such as, equipment needed to measure or record data before the structure gets damaged more.

The Prognosis of damage by using traditional methods of local inspection or testing due to an increase in the number and dimensions of structures and their deterioration is not feasible because inspection of such structures is time consuming, costly and along with human error. Therefore, to control the remaining useful life of large and more complex structures, new methods based on changes in the vibration properties of structures have been developed; that are commonly referred to as damage detection methods [4]. The basic idea is that the modal data of the structure, such as frequency and mode shapes are influenced by the physical properties of the structure, so changes in the physical properties of the structure lead to change in its modal properties. Consequently, by comparing the modal characteristics of the structure before and after the damage, the location and severity of damage to the structure can be detected [5].

First time Holland [6] investigated the problem of detecting damage based on natural frequency with genetic algorithm. One method that has attracted many researchers today is the numerical update model method. Detection of damage without the need for undamaged structural data is one of the advantages of this method.

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Defining the objective function and determining the being-updated parameters are among the most important factors affecting the success of these methods. In updating the numerical model using the inverse problem, the difference between the simulated and estimated results is minimized with the help of the optimizer algorithm.

A comprehensive review by Friswell and Mottershead [7] has been conducted on various methods of updating the model. Hajela and Soeiro [8] examined two optimization methods on a 15-member-dimensional truss, and obtained acceptable results. Boulkaibet et al. [9], used the Monte Carlo combining simulation, were able to provide a more precise method and introduce probabilities in a model update process for more sophisticated systems. Other researchers utilizing the sensitivity of the frequency response functions detected the severity of the damage [10, 11]. Ghodrati et al. [12, 13] used the flexibility matrix parameters and the modal residual forces and came up with strong and stable results.

Flexibility matrix and static strain energy are two being-updated parameters used in this method which are introduced in this paper.

Yan and Golinval [14] also use covariance-based subspace detection techniques to identify modal parameters. The stiffness matrix variations to detect damage is used as it is significantly altered due to major damage to the structure, but if the damage is small, this method is not very effective. Dynamic data and flexibility can be elicited out of dynamic experiments and structure frequency response measurements, respectively. One of the methods to detect vibration-based damage is to use statistical analysis [15, 16]. Tomaszewska [17] investigated the influence of statistical errors on damage detection methods and concentrated on the flexibility and mode shape curvature approaches as methodologies that use both natural frequencies and mode shapes. Damage detection from mode shape data requires measurements in many locations of a structure. Therefore, damage detection methods based on flexibility were utilized by researchers [18–20]. Li et al. [21] presented a generalized flexibility matrix for a definite reduction of natural frequencies of higher modes. The flexibility matrix by applying a unit force to values of degrees of freedom (DOFs) can be used as a modal displacement to calculate the strain variation of members. Accordingly, an efficient method was used to detect multiple damage to the truss system using strain-based flexibility index (SCBFI) [22] and another flexibility-based damage probability index (FBDPI) [23], simulation results showed high performance. Zare Hosseinzadeh et al. [24] employed an effective method based on the calculation of static displacement by a matrix of flexibility. The efficiency of the proposed method was verified by an experimental study of a five-story structure with shear frame. Kaveh and Zolghadr [25] used the object function of flexibility matrix and modal strain energy (MSE) method as a conducting tool in order to direct a beam and portal frame’s damage detection process.

Shi et al. [26] proposed a damage detection-method using differences in the MSE for the simple two-story plain structure. The results were partially successful in quantification of the structural damage in spite of errors. Modal-strain-energy-based methods have generally shown promise for locating damage [27–31]. However, while it has numerous advantages over other methods; recent research has shown that its application to three-dimensional frame-type structures is limited [32]. Seyedpoor and Yazdanpanah [33] found in a study on a static strain energy-based damage index (SSEBI) that this method is more reliable under similar conditions than modal strain energy-based damage index (MSEBI). Cha and Buyukozturk [34] discovered a new method for detecting damage in three-dimensional steel structures using the hybrid multiobjective genetic. Their method well detects the small damages when there is no noise. Li et al. [35] developed an Improved Modal Strain Energy (IMSE) method for detecting damage in offshore platform structures and compared it with Stubbs index method. Their comparative studies showed that the IMSE index outperformed the Stubbs index and exhibited stronger robustness.

In this paper, detection of damage considered in five sections of the introduction, overview of the AGWO, structural damage detection approach based on taking advantage of the mentioned being-updated parameters with the strategy of choosing the best performance, numerical examples. Finally, the summary is outlined in conclusions. Figure 1 illustrates the flowchart of research methodology of present work.

2. AUGMENTED GREY WOLF OPTIMIZER

Algorithm AGWO modifies the global algorithm’s grey wolf optimization (GWO) by focusing on search parameter (A). This algorithm simulates the group behaviour of gray wolves in hunting, who have a leader called α. And secondary wolves with the name β, which help α in decision making (See Figure 2). Here α means estimated results to solve the problem in the research.

The hunting process is divided into four below categories.

2.1. Searching for Prey The exploration of the prey location can be achieved by the divergence of search agents, which can be achieved when |A| > 1, the main parameter responsible for exploration and exploitation is parameter A which mainly depends on parameter α as given in Equation (1).

\[ a = 2 - \cos(\text{rand}) \times \frac{t}{\text{Max}_\text{iter}} \] (1)
where \( \vec{X} \) is the position vector of grey wolf, \( \vec{X}_p \) is the position vector of the prey.

2. 3. Hunting

In the proposed AGWO algorithm, the hunting will depend only on \( \alpha \) and \( \beta \) as given in Equations (6)-(8).

\[
\vec{B}_\alpha = [\vec{C}_1 \cdot \vec{X}_{ai} - \vec{X}_i], \vec{B}_\beta = [\vec{C}_2 \cdot \vec{X}_{pi} - \vec{X}_i]
\]

\[
\vec{X}_1 = \vec{X}_{ai} - \vec{A}_1 \cdot \vec{B}_\alpha, \vec{X}_2 = \vec{X}_{pi} - \vec{A}_2 \cdot \vec{B}_\beta
\]

\[
\vec{X}_{i+1} = \vec{X}_1 + \vec{X}_2 / 2
\]

2. 4. Attacking the Prey

The exploitation of (attacking) the prey can be achieved by the convergence of search agents, which is investigated when \(|A| < 1\) [37].

3. PROPOSED METHOD

The free vibration equation of a structural in an undamped state is written as follows:

\[
[M]\ddot{X} + [K]X = 0
\]

where \([M]\) and \([K]\) are the matrices of the mass and stiffness of the structure, respectively. These matrices can be obtained from the direct stiffness method for the number of elements \((ne)\). Also \(\{\ddot{X}\}, \{X\}, [M_e]\) and \([k_e]\) are acceleration, displacement vectors, the matrices of the mass and stiffness of each element, respectively.

\[
[M] = \sum_{e=1}^{ne} [M_e]
\]

\[
[k] = \sum_{e=1}^{ne} [k_e]
\]

Damage to the structure reduces the stiffness of the damaged element, which is a function of the modulus of elasticity. Thus, by reducing the modulus of elasticity of the elements using Equation (12), the actual damage to the structure is simulated.

\[
E_d^e = \left(1 - \alpha_e\right)E_e
\]

where \(E_d^e\) the modulus of elasticity of the damaged element, \(\alpha_e\) the amount of damage to the element (a number between zero and one), where the zero indicates that there is no damage, the one indicates a damage of
100% of the element, and $E_e$ is the modulus of elasticity of the element in the undamaged state.

Modal parameters are obtained by the solution to this equation:
$$[K - \omega_i^2M] \phi_i = 0, \ i = 1, 2, \ldots, n$$
(13)

According to the Equation (13), the mode shapes and the square of the natural frequencies of the structure can be obtained for $n$ DOFs, respectively:
$$\omega_i^2 = \frac{\omega_{1i}^2 \cdots \omega_{ni}^2}{0 \cdots \omega_{ni}^2}$$
(14)
$$\phi_i = \begin{bmatrix} \phi_{1i} & \cdots & \phi_{ni} \end{bmatrix}$$
(15)

In case of noise, its effect in this section is applied to the damaged structure using the following equation:
$$[a_p] = [a] \times (1 + N \times \text{rand})$$
(16)
where $a_p$ is the output, $N$ represents the noise level which is 5% in this paper, and the rand vector [-1,1] is the random variable distributed by the software.

Thus, by using Equations (14) and (15) the flexibility matrix can be written as follows:
$$[F]_{n \times n} = [\phi]_{n \times nm}[\omega]_{nm \times nm}^{-1}[\phi]_{n \times nm}^T$$
(17)
in which, $nm$ is the number of the modes used. Now, the diagonal and anti-diagonal elements of the Equation (17) are used, respectively:
$$DF = [F_{1,1}, F_{2,2}, \ldots, F_{n,n}]^T$$
(18)
$$AdF = [F_{1,n}, F_{2,n-1}, \ldots, F_{n,1}]^T$$
(19)

Based on Equations (18) and (19), four vectors are defined in order to determine $C$. $DF^d$ and $AdF^d$ are the two vectors defined for the damaged structure, and $DF^m$ and $AdF^m$ are those of the model structure.

$$c_1 = \frac{|DF^d|}{(DF^d, DF^d)(DF^d, DF^d)}$$
(20)
$$c_2 = \frac{|AdF^d|}{(AdF^d, AdF^d)(AdF^d, AdF^d)}$$
(21)
$$C = (c_1 \times c_2)^2$$
(22)
Then, $F_1$ is obtained:
$$F_1 = \left( \cos^{-1}(C) \times \frac{\pi}{2} \right)^{1/2}$$
(23)
The static strain energy of the structure can be simulated by applying the following equation for the $nm$ mode used:
$$[K] \times [(U) - (P)] = 0$$
(24)

"If a static force, like the vector $[P]$, is applied to the free DOFs of the structure, the static displacements of these DOFs can be calculated by" [24]:
$$[U] = [K]^{-1} \times (P) = [F]_{n \times n} \times (P)_{n \times 1}$$
(25)
where $[K]^{-1}$ is the flexibility matrix, $[U]$ the vector of static nodal displacement and $[P]$ "A unique static load is as follows applied to the structure with $n$ DOFs" [24]:
$$[P] = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$$
(26)

Then, using the Equation (25), the static strain energy of each element can be calculated as follows [33]:
$$\Lambda_e = \frac{1}{2} \left( u_e \times K \times u_e \right)_e \ e = 1, 2, \ldots, ne$$
(27)
where $u_e$ is the vector of static nodal displacement of each element, and $\Lambda_e$ is the static strain energy of $e$-th element. For the function convergence, the static strain energy of the structure ($\Lambda$) gets normalized:
$$\Lambda_{\text{norm}} = \sqrt{\sum_{e=1}^{ne} (\Lambda_e)^2}$$
(28)
$$\Lambda_{ne} = \frac{\Lambda_e}{\Lambda_{\text{norm}}} \ e = 1, 2, \ldots, ne$$
(29)
where $\Lambda_{\text{norm}}$ is the static strain energy norm of the structure, and $\Lambda_{ne}$ is the unit static strain energy of $e$-th element. Thus, by defining two vectors of $\Lambda_{ne}^d$ and $\Lambda_{ne}^m$ for the damaged and model structures, $F_2$ is obtained as follows:
$$\Delta_j = \left| \log(\Lambda_{ne}^d) - \log(\Lambda_{ne}^m) \right| j = 1, 2, \ldots, ne$$
(30)
$$F_2 = \left( \max(\Delta_1, \Delta_2, \ldots, \Delta_{ne}) \right)^2$$
(31)
Therefore, the objective function is defined as follows:
$$f(a_1, a_2, \ldots, a_{ne}) = \min(F_1, F_2)$$
(32)

4. ANALYSIS AND RESULT

In this section the applicability of the presented method is demonstrated by studying two-dimensional truss and three-dimensional frame structures under different damage patterns. Moreover, applying the minimum modes number, noise and various scenarios, the efficiency of the object function and accuracy algorithm AGWO in comparison with to GWO and PSO are examined. It should be declared that all analyses have been made in the workspace of MATLAB software.

4.1. Two-Dimensional Truss

The finite element model of this structure consists of 33 elements as illustrated in Figure 3. Damage scenarios are given in Table 1, and its material properties are as follows: modulus of elasticity $E = 2 \times 10^6$ kg/cm², mass density
\[ \rho = 7.85 \text{gr/cm}^3 \] and area \( A = 36.2 \text{cm}^2 \), respectively. Also, parameters of optimization algorithm as follows: maximum number of iterations=1000, number of population of wolves=100, upper bound=1, lower bound=0.

The results of damage detection of the two-dimensional truss data with 0% noise, and 5% noise for the first, second and third scenarios are presented in Figures 4, 5 and 6, respectively.

Convergence curves for the AGWO in the third damage scenario of the two-dimensional truss data with: 0% noise and 5% noise are illustrated in Figure 7.

TABLE 1. Different damage scenarios for the two-dimensional truss

| Damage scenario 1 | Damage scenario 2 | Damage scenario 3 |
|-------------------|-------------------|-------------------|
| Element number    | Damage (%)        | Element number    | Damage (%)        | Element number    | Damage (%)        |
| 2                 | 10                | 9                 | 5                 | 1                 | 20                |
|                   |                   | 27                | 15                | 26                | 10                |
|                   |                   |                   |                   | 33                | 25                |

Figure 4. The results of damage detection in the first scenario of the two-dimensional truss data with: (a) 0% noise, (b) 5% noise

Figure 5. The results of damage detection in the second scenario of the two-dimensional truss data with: (a) 0% noise, (b) 5% noise
Figure 6. The results of damage detection in the third scenario of the two-dimensional truss data with: (a) 0% noise, (b) 5% noise

Figure 7. Convergence curves for the AGWO in the third damage scenario of the two-dimensional truss data with: (a) 0% noise, (b) 5% noise

4.2. Three-Dimensional Frame The three-dimensional frame model of this structure consists of 37 elements and 28 nodes which have six DOFs each as illustrated in Figure 8. Damaged scenarios are given in Table 2. For this structure, modules of elasticity are $E = 2 \times 10^6 \text{kg/cm}^2$, mass density, $\rho = 7.85 \text{g/cm}^3$, moment of horizontal inertia $I_h = 4162 \text{cm}^4$, moment of vertical inertia $I_v = 4162 \text{cm}^4$, shear modulus $G = 793000 \text{kg/cm}^2$, torsional constant $J = 2081 \text{cm}^4$, area $A = 64 \text{cm}^2$, the horizontal length $L_H = 500 \text{cm}$, the vertical length $L_V = 320 \text{cm}$. Also, parameters of optimization algorithm as follows: maximum number of iterations=1000, number of population of wolves=200, upper bound=1, lower bound=0.

The results of damage detection of the three-dimensional frame and the first seven modes for the first, second and third scenarios are presented in Figures 9, 10 and 11, respectively. Because of the random nature of the heuristic optimization algorithms, the average results of 10 damage detection independent runs of studied optimization algorithms investigated and shown in Figure 12.
### Table 2. Different damage scenarios for the three-dimensional frame

| Damage scenario 1 | Damage scenario 2 | Damage scenario 3 |
|-------------------|-------------------|-------------------|
| Element number    | Damage (%)        | Element number    | Damage (%)        | Element number    | Damage (%)        |
| 10                | 10                | 7                 | 5                 | 1                 | 10                |
|                   |                   | 24                | 15                | 15                | 20                |
|                   |                   |                   | 35                | 25                |                   |

**Figure 9.** The results of damage detection in the first scenario of the three-dimensional frame for the first seven modes

**Figure 10.** The results of damage detection in the second scenario of the three-dimensional frame for the first seven modes

**Figure 11.** The results of damage detection in the third scenario of the three-dimensional frame for the first seven modes

To compare the reliability and efficiency of optimization algorithms, the best, worst and, the standard deviation (SD) of the results among the 10 independent runs are presented in Table 3. As shown in Figure 13 from the left to right, the convergence curves for the third damage scenario of the three-dimensional frame data with 0% noise and 5% noise are illustrated for the AGWO, GWO and PSO, respectively.
Figure 12. The average results damage detection of ten independent runs for the PSO, GWO and AGWO in the third damage scenario of the three-dimensional frame data with: (a) 0% noise, (b) 5% noise.

TABLE 3. The best, worst, average and the standard deviation of the results among at ten independent runs for the optimization algorithms in the third damage scenario of the three-dimensional frame

| Algorithm | AGWO ($f_{\text{min}}$) | GWO ($f_{\text{min}}$) | PSO ($f_{\text{min}}$) |
|-----------|-----------------|----------------|-------------------|
| Noise     | 0 (%) | 5 (%) | 0 (%) | 5 (%) | 0 (%) | 5 (%) |
| Run Number 1 | 0.00000 0.00066 | 0.00000 0.00015 | 0.0001 | 0.000610 |
| Run Number 2 | 0.00000 0.00017 | 0.00031 0.00016 | 0.000109 0.000432 |
| Run Number 3 | 0.00000 0.00076 | 0.00000 0.00011 | 0.0001945 0.000772 |
| Run Number 4 | 0.00000 0.00009 | 0.00000 0.00013 | 0.0002431 0.000593 |
| Run Number 5 | 0.000568 0.00276 | 0.00000 0.00031 | 0.0004679 0.00311 |
| Run Number 6 | 0.00067 0.00021 | 0.00000 0.00025 | 0.0004165 0.000210 |
| Run Number 7 | 0.00000 0.00062 | 0.00000 0.00016 | 0.0001465 0.000210 |
| Run Number 8 | 0.00000 0.00064 | 0.00000 0.00008 | 0.0000476 |
| Run Number 9 | 0.00000 0.00323 | 0.00000 0.00026 | 0.000639 0.000217 |
| Run Number 10 | 0.00058 0.00010 | 0.00000 0.00027 | 0.000054 |
| Maximum    | 0.00068 0.00323 | 0.00031 0.00031 | 0.00468 0.00772 |
| Minimum    | 0.00000 0.00009 | 0.00000 0.00008 | 0.000028 |
| Average    | 0.00069 0.00092 | 0.00003 0.00019 | 0.00140 0.000370 |
| Sd         | 0.00177 0.00113 | 0.00010 0.00008 | 0.00182 0.00248 |

Figure 13. Convergence curves shown left to right respectively for the AGWO, GWO and PSO in the third damage scenario of the three-dimensional frame data with: (a) 0% noise, (b) 5% noise.
5. CONCLUSION

In this study, an updating-based-model strategy is presented in which by combining two being-updated parameters of the flexibility matrix and the static strain energy of the structure along with the use of optimization, structural damage assessment is achieved.

Despite the limitation in process of damage assessment in two-dimensional and three-dimensional structures with high DOFs along with applying multiple damages in different parts of the structure, using noise. It is assumed that by using structural static strain energy advantages to improve the performance of the proposed objective function and reduce the weakness of small and general damage detection in flexibility-matrix-based methods and using the first few modes of the structure, damages are evaluated very precisely.

Moreover, by comparing different studies in section 4 including average results of the 10 runs, statistical results and convergence with other evolutionary optimization algorithms of PSO and GWO, the stability of the AGWO algorithm is evaluated.

The error between simulated and estimated results in average of ten runs and each damage scenario was less than 3 percent which proves the proper performance of this method in detecting the damage of the 37-member frame and the 33-member truss. Investigation on the experimental model, combining other being-updated parameters and using other new heuristic and multi objective algorithms in the method is recommended.

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