The reach of a future Linear Collider
after the g-2 result

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Abstract
Combining the cosmological requirement on dark matter with the recent BNL g-2 measurement it is argued that, within the mSUGRA framework, the preferred region for SUSY mass parameters falls well inside the area covered by the future linear colliders under consideration for right handed sleptons and of the 2 lightest neutralinos. The coverage for the lightest chargino and left handed sleptons is also favoured but with smaller confidence.

The main uncertainty on the physics potential of future $e^+e^-$ colliders comes from our ignorance of the SUSY mass spectrum. As recently pointed out in [1], if the mass spectrum of gauginos (charginos, neutralinos) and sleptons is light enough to be observable with a 500 GeV LC, then signals should appear before LHC starts(2006) either in the precise measurement of g-2 under way at BNL, or in the observation of primordial neutralinos in the high sensitivity experiments under construction (CDMS, EDELWEISS, CRESST).

The recent indication [2] reported from the BNL experiment although not yet conclusive (2.6 s.d. effect), is encouraging since it is compatible with the expected contributions of a light SUSY spectrum as will be discussed in more detail below.

As already mentioned, there is no definite SUSY symmetry breaking, SSB, mechanism which can allow a precise prediction of the SUSY mass spectrum. In the most general approach this spectrum has more than 100 free parameters but there are various experimental constraints (in particular the requirement to avoid FCNC) which impose to reduce considerably this number.

In the so-called gravity-mediated SSB, mSUGRA, there are 2 mass parameters, $m_0$ and $m_{1/2}$, one related to the mass of the scalar superpartners and the other related to the mass of the fermionic superpartners, and 3 mixing parameters $\tan\beta$, $\mu$ and $A$. These parameters enter in the definition of the SUSY particles, the Higgs bosons and the Z masses. The latter provides one constraint which allows to determine $\mu^2$, leaving unknown sign($\mu$).
Four ingredients play an important part in defining the allowed domain for 
\( m_0, m_{1/2} \) and \( \tan\beta \) (\( A \) plays a less important role):

- **The Higgs mass** which, according to LEP, should be above 114 GeV (and probably at 115 GeV) which imposes \( m_{1/2} \) above 350 GeV (this lower bound can be reduced to \( \sim 200 \) GeV taking into account various uncertainties) unless \( m_0 \) is very large (unlikely given the observation on g-2 as discussed below).

- **The recent measurement of g-2 for the muons** at BNL with a 2.6 sd excess above the SM value which, interpreted in terms of SUSY, crudely speaking says \[ \tilde{m} \] at the 1.5 sd level that \( \tilde{m} \) is below \( 65\sqrt{\tan\beta} \) GeV, where \( \tilde{m} \) is of order \( M_{\text{chargino}}+M_{\text{sneutrino}} \). From LEP2 we can tell that this quantity is above 200 GeV, implying that \( \tan\beta \) should be larger than \( \sim 10 \). The sign of g-2 also implies that \( \mu \) should be positive.

- **The cosmological solution** which favours small values of \( m_0 \) to generate the proper amount of dark matter in the universe (\( \Omega h^2 \sim 0.3 \)). To insure that the LSP is neutral (neutralino) and not charged (stau lepton) \( m_0 \) is limited from below. The co-annihilation process stau+neutralino, active when both particles have the same mass, favours solutions which are close to this limit.

- **The branching ratio** \( b\to s\gamma \) implies that for large \( \tan\beta \), \( m_{1/2} \) and \( m_0 \) should be large to be compatible with the data. This constraint is only very restrictive for negative values of \( \mu \) while for positive values, compatible with the g-2 result, it simply implies that \( \tan\beta \) cannot be above \( \sim 30 \).

Putting together the 4 requirements, one can easily derive a valid SUSY spectrum. Typical mass spectra for 3 relevant values of \( \tan\beta \) are given in table 1. Note that one can adjust the lightest Higgs mass to 115 GeV with the A parameter.

As often discussed in the literature, the dark matter constraint suggesting light neutralinos and sleptons can be evaded in several ways. Without discussing the details one may simply state that the g-2 result prevent most of these scenarios. One can also predict whether the direct neutralino search is likely to produce a signal when the detectors under construction will reach a mass of 10-100kg of Ge (in the table the rate per day with 100kg of Ge is indicated). Provided that the background can be controlled according to expectations.

To compute the SUSY contribution to g-2=2\( \delta a_\mu \), I have used the detailed formulae given in [6]. The solution with \( \tan\beta=10 \) is clearly disfavored. The reason for this is simply that the Higgs mass constraint from LEP2 gives large mass values for the sneutrino and the chargino which therefore forces large values of \( \tan\beta \) to satisfy the bound coming from g-2.

The choice of \( \tan\beta \sim 30 \) is a standard one in grand unified theories with unification of the Yukawa coupling constants of the 3d generation of fermions.
Table 1: mSUGRA solutions

| tanβ | 10  | 20  | 30  |
|------|-----|-----|-----|
| m_{1/2} | 400 | 350 | 350 |
| m_0   | 100 | 120 | 170 |
| μ     | 475 | 415 | 415 |
| δμ_μ × 10^{10} | 14  | 33  | 43  |

| M_{chargino1} | 305  | 265  | 265  |
| M_{neutralino1} | 160  | 140  | 140  |
| M_{slepton_R}   | 190  | 186  | 221  |
| M_{neutralino2} | 395  | 270  | 295  |
| M_{chargino2}   | 500  | 440  | 440  |
| M_{neutralino2} | 305  | 265  | 265  |
| M_{squark}      | 900  | 800  | 800  |
| M_{gluino}      | 900  | 790  | 790  |

| Rate/100kg/day Ge | 0.05 | 0.25 | 0.7 |

Higher values of tanβ cannot be accommodated with the low values of m_0 and m_{1/2} needed for g-2 according to [4]. The conclusion is that with the g-2 result, an e^+e^- collider operating at 500 GeV can very likely observe sleptons and neutralinos (first + second lightest neutralino).

At 800 GeV there is also access to charginos. Very similar numbers can be found in [7].

Figure 1 and 2 give a more general picture of the coverage offered by a LC operating at 800 GeV. The LSP relic abundance satisfies the cosmological bounds in a small band at the frontier of the excluded band (co-annihilation with staus with a mass close to the LSP). Combining this cosmological requirement with the g-2 measurement one can conclude that the preferred region falls well inside the area covered by the LC for right handed sleptons and of the 2 lightest neutralinos. The coverage for the lightest charginos and left handed sleptons is also favoured but with smaller confidence.

These conclusions are based on the mSUGRA approach which is still consistent with the main requirements dictated by the Higgs limits/signal from LEP2, g-2 and DM. There are however several motivations to relax the tight unification of the mSUGRA scheme and this can be done without contradicting experimental constraints. Recall for instance that EW baryogenesis [8] requires a light stop, of order 100 GeV, which is acceptable if one relaxes the unification of scalar masses at GUT. One could then have a light right handed stop and a very heavy left-handed stop (providing the necessary input for the Higgs mass at ∼115 GeV).

Relaxing the mSUGRA connection between the slepton and the squark masses, one could ignore the Higgs limit from LEP2 which mainly comes from
Table 2: mSUGRA without the Higgs constraint

| tanβ   | 10 |
|--------|----|
| $m_{1/2}$ | 200 |
| $m_0$   | 50  |
| $\delta a_\mu \times 10^{10}$ | 54  |
| $M_{\text{chargino1}}$ | 136 |
| $M_{\text{neutralino1}}$ | 71  |
| $M_{\text{sleptonR}}$ | 102 |
| $M_{\text{sneut}}$ | 137 |
| $M_{\text{chargino2}}$ | 284 |
| $M_{\text{neutralino2}}$ | 138 |
| Rate/100kg/day Ge | 0.8 |

the stop sector, and allow for lighter chargino/slepton to have a g-2 value consistent with the tan$\beta$=10 solution (table 2). Nothing therefore forbids at present the possibility to observe with $e^+e^-$ a more complete SUSY spectrum than suggested by the mSUGRA scheme.

Although still speculative, given the present error on g-2 (which however should be reduced by two with the existing data), this discussion illustrates the impact of the BNL result. If confirmed, it could herald a brilliant future of major discoveries for a LC operating up to $\sim$1 TeV and for dark matter searches.

APPENDIX

In this appendix, I will briefly describe the various assumptions made to derive above results.

Concerning the derivation of $\mu^2$ from the ESWB relation, I have used the approximation valid for large tan$\beta$:

$$
\mu^2 \sim 1.5m_{1/2}^2 - 0.5M_Z^2 - 0.04m_0^2
$$

As noted in [9] one can almost neglect the dependence with $m_0$ for the large values of tan$\beta$ considered here.

One can in principle also derive $m_A$ using (standard notations):

$$
m_A^2 = m_1^2 + m_2^2 \sim m_{H_1}^2 + \mu^2
$$

At moderate values of tan$\beta$, one should have:

$$
m_{H_1}^2 = m_0^2 + 0.52m_{1/2}^2 + \Delta m^2
$$
Table 3: Approximate values for $m_A$

| $\tan\beta$ | 10  | 30  | 50  |
|-------------|-----|-----|-----|
| $m_{1/2}$   | 250 | 350 | 400 |
| $m_0$       | 100 | 170 | 350 |
| $m_A$ approx| 369 | 525 | 667 |
| $m_A$ from [7] | 380 | 475 | 460 |

where $\Delta m^2$ is a correction expected to be negative and important at large $\tan\beta$. Checking with the results given in [7] there seems to be good agreement even up to $\tan\beta=30$ as shown in table 3. This implies that the CP-odd Higgs boson will be light enough that one will be able to measure sizeable deviations on the light Higgs branching ratios [10]. This important conclusion has to be checked more precisely.

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Figure 1: $g$-2 constraints at $±1\sigma$ (full red curve) and $±1.5\sigma$ (dashed red curve) for $\tan\beta=20$. The external green contour represents the $e^+e^-$ reach with 800 GeV for right-handed sleptons+neutralino1+neutralino2. The internal green contour also includes the coverage for left-handed sleptons and the lightest chargino. The yellow shaded region is excluded by the requirement that the LSP be neutral. The small blue band is the region with satisfies both the $±1.5\sigma$ constraint on $g$-2 and gives the correct LSP relic abundance for cosmology (co-annihilation with staus with a mass close to the neutralino LSP).
Figure 2: g-2 constraints at ±1σ and ±1.5σ for tanβ=30. Same conventions as for figure 1.