Variation of Fundamental Couplings and Nuclear Forces

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Abstract

The dependence of the nuclear force on standard model parameters plays an important role in bounding time and space variations of fundamental couplings over cosmological time scales. We discuss the quark-mass dependence of deuteron and di-neutron binding in a systematic chiral expansion. The leading quark-mass dependence of the nuclear force arises from one-pion exchange and from local quark-mass dependent four-nucleon operators with coefficients that are presently unknown. By varying these coefficients while leaving nuclear observables at the physical values of the quark masses invariant, we find scenarios where two-nucleon physics depends both weakly and strongly on the quark masses. While the determination of these coefficients is an exciting future opportunity for lattice QCD, we conclude that, at present, bounds on time and space variations of fundamental parameters from the two-nucleon sector are much weaker than previously claimed. This brings into question the reliability of coupling-constant bounds derived from more complex nuclei and nuclear processes.
I. INTRODUCTION

The recent observation suggesting that the fine-structure constant was smaller in the past \[1\] than it is today has led to renewed interest in the idea of using time (and space) variation of fundamental parameters as a probe of high-energy physics. Based on the principle that all that is not forbidden is mandatory, time variation in the electromagnetic force is perhaps not surprising. Moreover, one would expect that the other forces of nature vary with time as well. This idea is quantifiable by assuming a suitable grand unified theory (GUT) at a scale \(M_{GUT}\) or a particular brane-world scenario \[2–4\]. The assumption that the short-distance couplings of the theory are related at all times leads to relations between the variation of the strength of the electromagnetic interaction, \(\alpha_{em}\), and the variation of other standard model parameters, such as the light quark masses, \(m_q\). As a change in \(m_q\) and \(\alpha_{em}\) will naively lead to a change in the positions of nuclear energy levels, special interest has been paid to the light-element abundances predicted by big bang nucleosynthesis (BBN) and also to the abundance of isotopes produced by the Oklo “natural reactor” in the hope that these abundances can be used to constrain high-energy physics \[2,3,5–23\].

In this work we will critically analyze the two-nucleon sector using an effective field theory (EFT) that respects the approximate \(SU(2)_L \otimes SU(2)_R\) chiral symmetry of QCD and has consistent power-counting \[24–27\]. This theory allows for a systematic study, consistent with QCD, of the bound-state in the \(^3S_1–^3D_1\) coupled-channels, the deuteron, and the scattering amplitude in the \(^1S_0\) channel. This is the first study of the \(m_q\)-dependence of these quantities that is complete at next-to-leading order (NLO) in a consistent EFT. Unfortunately, due to the current lack of understanding of the low-energy behavior of QCD, only weak bounds, if any, can be placed on the time-dependence of fundamental couplings from the two-nucleon sector. We will explicitly demonstrate this by finding plausible values for couplings in the EFT for which the deuteron binding energy varies relatively little, and for which the dineutron system in the \(^1S_0\) channel is unbound, over a relatively wide range of quark masses.

The deuteron is an interesting object to study for a variety of reasons. From a fundamental point of view it is quite intriguing as it is bound by only \(B_d = 2.224644 \pm 0.000034\) MeV, which is much smaller than the typical scale of strong interactions, \(\Lambda_{QCD}\). Clearly, in order to arrive at such a small binding energy, fine-tunings are involved and consequently naive dimensional analysis (NDA) as applied to the single-nucleon sector or the mesonic sector, which we will call naive naive dimensional analysis (\(N^2DA\)), on this system is doomed to fail. Unfortunately, all previous attempts to extract bounds on the variations of fundamental couplings from the deuteron, and in general the two-nucleon sector, have implemented \(N^2DA\). From a more phenomenological standpoint, the smallness of \(B_d\) plays a key role in the synthesis of light elements in BBN. The impressive agreement between the predictions of BBN and observation suggests that new physics that would have significantly modified the deuteron at the time of BBN is absent. The \(^1S_0\) channel is quite similar to the \(^3S_1–^3D_1\) coupled-channels in one important way, its scattering length is unnaturally large.

\[\text{In the pionless EFT one finds that both the } ^1S_0 \text{ and the } ^3S_1–^3D_1 \text{ coupled-channels are very close to an unstable IR fixed point of the renormalization group. At this fixed point, the scattering lengths are infinite } \[26,28\].\]
$a^{(1S_0)} = -23.714 \pm 0.013$ fm. While there is no bound state in this channel for the physical values of the light-quark masses, a small increase in the strength of the nuclear force would bind two nucleons in this channel. The existence of a bound state in this channel, e.g., a di-neutron, $nn$, in the nucleosynthesis epoch would be quite profound and could substantially modify the predictions of BBN.

II. EFT FOR TWO NUCLEONS

During the last decade there has been a significant effort to construct an EFT to describe nuclear physics. While it is straightforward to write down all possible terms in the effective Lagrange density for two or more nucleons, arriving at the correct power-counting proved to be a difficult task. Weinberg’s (W) original proposal [24] for an EFT describing multi-nucleon systems was to determine the nucleon-nucleon (NN) potentials using the organizational principles of the well-established EFT’s describing the meson-sector and single-nucleon sector (chiral perturbation theory, $\chi$PT), and then to insert these potentials into the Schrödinger equation to solve for NN wavefunctions. Observables are computed as matrix elements of operators between these wavefunctions. W power-counting has been extensively and successfully developed during the past decade to study processes in the few-nucleon systems. This method is intrinsically numerical and is similar in spirit to traditional nuclear-physics potential theory. Unfortunately, there are formal inconsistencies in W power-counting [25], in particular, divergences that arise at leading order (LO) in the chiral expansion cannot be absorbed by the LO operators. Problems persist at all orders in the chiral expansion, and the correspondence between divergences and counterterms appears to be lost, leading to uncontrolled errors in the predictions for observables. This formal issue was partially resolved by Kaplan, Savage and Wise (KSW) who introduced a power-counting in which pions are treated perturbatively [26]. The NN phase-shifts and mixing angle in the $1S_0$ and $3S_1 - 3D_1$ coupled-channels have been computed to next-to-next-to-leading order (N$^2$LO) in the KSW expansion by Fleming, Mehen and Stewart (FMS) [29] from which it can be concluded that the KSW expansion converges slowly in the $1S_0$ channel and does not converge in the $3S_1 - 3D_1$ coupled-channels. Therefore, neither W or KSW power-counting provide a complete description of nuclear interactions (for recent reviews see Ref [30–34]).

The problems with W and KSW power-counting appear to have been resolved in the work by Bedaque, van Kolck and the authors [27], which from this point on we will refer to as BBSvK. It was realized in FMS that the contributions to the amplitude that lead to non-convergence in the $3S_1 - 3D_1$ coupled-channels persist in the chiral limit (it is the chiral limit of iterated one-pion-exchange (OPE) that is troublesome). Therefore, in BBSvK power-counting the scattering amplitude is an expansion about the chiral limit. This recovers KSW power-counting in the $1S_0$ channel, where FMS found it to be slowly converging. However, in the $3S_1 - 3D_1$ coupled-channels, the chiral limit has contributions from both local four-

† In KSW power-counting the momentum-independent four-nucleon operators are LO and are resummed to yield the LO scattering amplitude, while pions are NLO and treated in perturbation theory.
nucleon operators and from the chiral limit of OPE. It is these two contributions that must be resummed using the Schrödinger equation to provide the LO scattering amplitude in the \( ^3S_1 - ^3D_1 \) coupled-channels.

The LO Lagrange density describing the single-nucleon sector and the pseudo-Goldstone bosons in two-flavor QCD is

\[
\mathcal{L} = \frac{1}{8} (f_\pi^{(0)})^2 \text{Tr} \left[ \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right] + \lambda \text{Tr} \left[ m_q \Sigma^\dagger + m_q \Sigma \right] + N^\dagger \left( i\partial_0 + \nabla^2 / 2M_N^{(0)} \right) N + g_A^{(0)} N^\dagger \sigma \cdot A N ,
\]

where \( f_\pi^{(0)} \) is the pion decay constant in the chiral limit, \( M_N^{(0)} \) is the nucleon mass in the chiral limit, \( g_A^{(0)} \) is the axial coupling constant in the chiral limit, \( \Sigma \) is the exponential of the pion field, and \( A \) is the axial-vector field. The properties of the nucleons and mesons have been studied in the first few orders of the chiral expansion (a good review can be found in Ref. [36]). If one is interested in the \( m_q \)-dependence of the nuclear force, as we are, one needs to have the chiral expansion for the nucleon mass, for the axial coupling and for the pion decay constant up to NLO. Each of these observables has been studied extensively, the results of which can be found in Refs. [36–39],

\[
f_\pi = f_\pi^{(0)} \left[ 1 + \frac{m_\pi^2}{8\pi^2 (f_\pi^{(0)})^2} T_4 + \mathcal{O} \left( m_\pi^4 \right) \right],
\]

\[
M_N = M_N^{(0)} - 4m_\pi^2 c_1 + \mathcal{O} \left( m_\pi^3 \right),
\]

\[
g_A = g_A^{(0)} \left[ 1 - \frac{2(g_A^{(0)})^2 + 1}{8\pi^2 (f_\pi^{(0)})^2} m_\pi^2 \log \left( \frac{m_\pi^2}{\lambda^2} \right) + \mathcal{O} \left( m_\pi^2 \right) \right],
\]

where \( T_4 = 4.4 \pm 0.2 \) [37,38], \( c_1 \sim -1 \text{ GeV}^{-1} \) [34] are \( m_q \)-independent constants, and \( m_\pi = 139 \text{ MeV} \) has been used to determine the constants in the chiral limit. We use \( g_A = 1.25 \), \( M_N = (M_n + M_p) / 2 \) and \( f_\pi = 135 \text{ MeV} \). We have retained only the leading chiral-logarithmic contribution to \( g_A \), and have chosen a renormalization scale of \( \lambda = 500 \text{ MeV} \). This point requires discussion. Extraction of the counterterm relevant to the \( m_q \)-dependence of \( g_A \) at one-loop order presently yields \( \lambda \sim 100 \text{ MeV} \) (see Ref. [38] and references therein). As this anomalously small value seemingly indicates a breakdown of the chiral expansion, we assume that this is a problem with the extraction and use a natural value of \( \lambda = 500 \text{ MeV} \) for this analysis. This, of course, introduces an uncertainty at NLO in the EFT calculation.

\[\text{‡} \]

A technical issue that remains to be solved is that while dimensional regularization can be used to regulate the \( ^1S_0 \) channel, so far no one has managed to dimensionally regulate the \( ^3S_1 - ^3D_1 \) coupled-channels due to the presence of the \( 1/r^3 \) component of the nucleon-nucleon potential. Therefore, in this work we will dimensionally regulate the \( ^1S_0 \) channel and use position-space square-well regularization in the \( ^3S_1 - ^3D_1 \) coupled-channels. It has been shown that a singular \( 1/r^n \) potential (\( n \geq 2 \)), can be regulated and renormalized by a single momentum-independent square-well [33].
The interactions between two nucleons arise from pion exchange, resulting from the Lagrange density in eq. (1), and also from local four-nucleon interactions, which for s-wave interactions result from a Lagrange density of the form,

\[
\mathcal{L} = -\frac{1}{2} C_S \left( N^\dagger N \right)^2 - \frac{1}{2} C_T \left( N^\dagger \sigma^i N \right)^2 \\
-\frac{1}{2} D_{S1} \left( N^\dagger \mathcal{M}_{q+N} N \right) \left( N^\dagger N \right) - \frac{1}{2} D_{T1} \left( N^\dagger \mathcal{M}_{q+N} \sigma^i N \right) \left( N^\dagger \sigma^i N \right) \\
-\frac{1}{2} D_{S2} \left( N^\dagger N \right) \left( N^\dagger N \right) \text{Tr} [\mathcal{M}_{q+N}] - \frac{1}{2} D_{T2} \left( N^\dagger \sigma^i N \right) \left( N^\dagger \sigma^i N \right) \text{Tr} [\mathcal{M}_{q+N}] + \ldots ,
\]

where \( \mathcal{M}_{q+N} = \frac{1}{2} \left( \xi^\dagger m_q \xi + \xi m_q \xi \right) \), and \( m_q = \text{diag}(m_u, m_d) \). The ellipses denote operators involving two derivatives \([10]\), which are the same order in the power-counting as a single insertion of \( m_q \), and also higher-dimension operators. The \( m_q \)-independent coefficients \( C_i, D_i \) and so forth, are to be determined from experimental data. An important point to notice is that one cannot separate the contributions from the \( C_i \) and the \( D_i \) using NN scattering data alone, as both operators are momentum independent. However, the \( D_i \) operators contribute to interactions between two nucleons and two or more pions, while the \( C_i \) operators only contribute to interactions between two nucleons.

### III. THE \( ^1S_0 \) CHANNEL AND THE DI-NEUTRON

One might imagine that grave violence would be done to the predictions of BBN if a di-neutron or di-proton were to be stable during the nucleosynthesis epoch. Thus it is interesting to know just how much the couplings in the Lagrange density describing the interactions between two nucleons in the \( ^1S_0 \) channel can change before a di-neutron state becomes stable \([8]\). However, from the point of view of setting bounds on the variation of fundamental couplings, what is important to know is if there is a set of “reasonable” couplings for which the scattering length remains unnaturally large and the di-neutron is unbound over a large range of pion masses. In this situation the pion mass can vary substantially, yet still not significantly modify the physics of two nucleons in the \( ^1S_0 \) channel. Indeed, such a parameter set exists.

![Diagram](image.png)

**FIG. 1.** LO contribution to the scattering amplitude in the \( ^1S_0 \) channel. The small solid circle denotes an insertion of \( C_0 \).

\[8\]In this initial study, we do not consider the di-proton system due to the coulomb potential that further destabilizes it. However, for a meaningful study of constraints from BBN in this channel, the possibility of \( nn, np \) and \( pp \) bound states would have to be considered.
FIG. 2. NLO contributions to the scattering amplitude in the $^1S_0$ channel. The small solid circles denote an insertion of $C_0$ or $g_A$. Dashed lines are pions and the large solid circle (square) corresponds to an insertion of $C_2$ ($D_2$).

The scattering amplitude for two nucleons in the $^1S_0$ channel has been computed out to N$^2$LO [28,29], but for our purposes it will be sufficient to work with the amplitude at NLO [26]. The scattering amplitude thus has an expansion of the form

$$A = A_{-1} + A_0 + A_1 + \ldots$$  \hspace{1cm} \text{(4)}$$

where $A_n$ is of order $Q^n$ in the dual $m_q$ and momentum expansions. The LO amplitude arises from the diagrams in Fig. 1 and is given by

$$A_{-1} = \frac{-C_0}{1 + \frac{C_0 M}{4\pi} (\mu + ip)}$$ \hspace{1cm} \text{(5)}$$

while the NLO amplitude arises from the diagrams in Fig. 2 and is given by the sum of

$$A_0^{(a)} = \frac{-C_2 p^2}{\left[1 + \frac{C_0 M}{4\pi} (\mu + ip)\right]^2}$$

$$A_0^{(b)} = \left(\frac{g_A^2}{2f_\pi^2}\right) \left(-1 + \frac{m_\pi^2}{4p^2} \ln \left(1 + \frac{4p^2}{m_\pi^2}\right)\right)$$

$$A_0^{(c)} = \frac{g_A^3}{f_\pi^2} \left(\frac{m_\pi M A_{-1}}{4\pi}\right) \left(-\frac{\mu + ip}{m_\pi} + \frac{m_\pi}{2p} \left[\tan^{-1} \left(\frac{2p}{m_\pi}\right) + \frac{i}{2} \ln \left(1 + \frac{4p^2}{m_\pi^2}\right)\right]\right)$$

$$A_0^{(d)} = \frac{g_A^3}{f_\pi^2} \left(\frac{m_\pi M A_{-1}}{4\pi}\right)^2 \left(1 - \left(\frac{\mu + ip}{m_\pi}\right)^2 + i \tan^{-1} \left(\frac{2p}{m_\pi}\right) - \frac{1}{2} \ln \left(\frac{m_\pi^2 + 4p^2}{\mu^2}\right)\right)$$

$$A_0^{(e)} = \frac{-D_2 m_\pi^2}{\left[1 + \frac{C_0 M}{4\pi} (\mu + ip)\right]^2},$$  \hspace{1cm} \text{(6)}$$
where we have chosen to work in the isospin limit, \( m_u = m_d \), and we have turned off the electromagnetic interaction. These amplitudes are manifestly renormalization-scale independent order-by-order in the EFT expansion. The coefficients \( C_0 \), \( D_2 \) and \( C_2 \) are the combinations of couplings from the Lagrange density in eq. (3) appropriate for the \(^1S_0\) channel. \( C_0 \) corresponds to a four-nucleon operator that is independent of \( m_q \) and momentum, \( D_2 \) corresponds to a four-nucleon operator that has a single insertion of \( m_q \) and no derivatives, and \( C_2 \) corresponds to a four-nucleon operator that is independent of \( m_q \) and has two derivatives. The quantity \( \mu \) is the renormalization scale, and we have used dimensional regularization and the power-divergence subtraction procedure (PDS) \([26]\) to renormalize the theory; \( p \) is the magnitude of the three momentum of each nucleon in the center-of-mass frame. In addition, we have used the LO relation between the quark masses and the square of the pion mass so that the amplitude at NLO is written entirely in terms of \( m_\pi \). It is important to note that the operator with coefficient \( D_2 \) is required at NLO. It is this operator that absorbs the scale dependence of \( A_{0(d)}(0) \).

From the NLO amplitude it is easy to construct \( p \cot \delta(1S_0) \), which has a well-behaved power-series expansion for \( p < m_\pi/2 \), and thus we can determine a linear combination of \( C_0 \) and \( D_2 \) in terms of the scattering length \( a(1S_0) \), and \( C_2 \) in terms of the effective range, \( r(1S_0) = 2.73 \pm 0.03 \) fm, at the physical value of the pion mass. Once these parameters are fixed, the scattering length, effective range and phase-shift can be determined as a function of the pion mass. The scattering length is given by

\[
\frac{1}{a(1S_0)} = \gamma + \frac{g_A^2 M_N}{8 \pi f_\pi^2} \left[ m_\pi^2 \log \left( \frac{\mu}{m_\pi} \right) + (\gamma - m_\pi)^2 - (\gamma - \mu)^2 \right] - \frac{M_N m_\pi^2}{4 \pi} (\gamma - \mu)^2 D_2 ,
\]

where \( \gamma = \mu + 4\pi/M C_0 \). Given that it is only a combination of \( C_0 \) and \( D_2 \) that can be

\[ \text{FIG. 3.} \quad \text{The scattering length in the} \ 1S_0 \ \text{channel (in fm's) as a function of the pion mass, for the couplings given in eq. (8). As the scattering length is negative over this entire region, the di-neutron is unbound.} \]

fixed from NN scattering, and at present \( D_2 \) cannot be isolated with pionic processes, unique values for \( D_2 \) or \( C_0 \) that are determined experimentally do not exist. Thus, one must explore a “reasonable” range of values for \( D_2 \) and \( C_0 \) in order to gain some understanding of how
the $^1S_0$ channel is modified away from the physical values of the quark masses. It is quite straightforward to identify a set of couplings,

$$C_0(m_{\pi}^{\text{PHYS}}) = -4.09 \text{ fm}^2, \quad D_2(m_{\pi}^{\text{PHYS}}) = 0.50 \text{ fm}^4, \quad C_2(m_{\pi}^{\text{PHYS}}) = 4.94 \text{ fm}^4,$$

for which the scattering length in the $^1S_0$ channel remains unnaturally large and negative over a wide range of pion masses, as shown in Fig. 3, and hence a di-neutron is not stable in this region. We have extracted the $C_i$ and $D_2$ using eq. (7) and an analogous equation for the effective range, with $g_A$, $M_N$ and $f_\pi$ set to their chiral limit values in eq. (2). In order to determine the impact of a varying pion mass on nuclei more complex than the deuteron, it is useful to know the behavior of the phase-shift as a function of the pion mass over a relatively wide range of momentum. In Fig. 4 we show the $^1S_0$ phase-shift for $m_\pi = 60$ MeV, $m_{\pi}^{\text{PHYS}}$ and 180 MeV, for the couplings in eq. (8). One notices that the NLO phase-shift does not reproduce the results of the Nijmegen partial-wave analysis above $\sim 50$ MeV, even at the physical value of the pion mass. This is because we have fixed the parameters, $C_0$, $D_2$ and $C_2$ to the scattering length and effective range. However, this means that the dual chiral and momentum expansion of the EFT at NLO has been matched to the momentum expansion of $p \cot \delta(^1S_0)$. Consequently, both the scattering length and the effective range have their own chiral expansions that we are truncating at NLO. Thus we do not expect to reproduce the phase-shift exactly, but should be perturbatively close, as can be seen to be the case in Fig. 4. Rather than compare the phase-shifts with the Nijmegen partial-wave analysis, it is perhaps more informative to compare the phase-shifts for $m_\pi = 60$ MeV

**For convenience, we plot observables as a function of $m_\pi$ rather than $m_\pi^2 \propto m_q$, which would be more natural from the standpoint of QCD.

††For a detailed discussion of the $^1S_0$ channel description with EFT, see Refs. [26,29,42].
and 180 MeV with those at $m_\pi = m_\pi^{\text{PHYS}}$. Thus we have identified a set of NLO couplings for which the $^1S_0$ channel is quite insensitive to moderate changes in the pion mass.

It is interesting to ask what one might expect if nature has chosen values for the couplings other than those in eq. (8). Choosing some arbitrary values for the couplings, but ones that still respect NDA in the two-nucleon sector [27],

$$C_0(m_\pi^{\text{PHYS}}) = -4.00 \text{ fm}^2 \quad , \quad D_2(m_\pi^{\text{PHYS}}) = 0.31 \text{ fm}^4 \quad , \quad C_2(m_\pi^{\text{PHYS}}) = 4.57 \text{ fm}^4$$

we show the scattering length as a function of the pion mass in Fig. 5. One can see that there are couplings for which the scattering length becomes positive, indicating the presence of a di-neutron that is stable with respect to the strong interaction, for relatively small variations in the pion mass. Perhaps the unjustified choice of NN interaction arising from N$^2$DA used in previous works on this subject (e.g. Ref [17,18,21]) is in some way comparable to such an ad-hoc choice of couplings in the EFT.

It is clear that sufficiently little is known about the $m_q$-dependence of the $^1S_0$ channel that placing bounds on the time-variation of $m_q$ from nuclear processes sensitive to this channel is not possible at this point in time.

IV. THE DEUTERON BINDING ENERGY

As we have already discussed, the deuteron plays a central role in BBN and the production of light elements. If the deuteron was deeply bound or unbound in the nucleosynthesis epoch then the abundances of the light elements would look radically different from the predictions of BBN and from those observed in nature. Therefore, it is possible that limits can be placed upon the time-variation of fundamental couplings if the dependence of $B_d$, and relevant nuclear cross-sections on these couplings is known. First realistic attempts to understand $B_d(m_q)$ as a function of $m_q$ were made in Refs [27,43]. One motivation for
BBSvK was to understand just what was possible for the $m_q$-dependence of $B_d(m_q)$, and in particular to determine if the deuteron was bound or unbound in the chiral limit. This issue could not be resolved due to the lack of information about the $D_2$ operators that contribute to the $^3S_1 - ^3D_1$ coupled-channels. A second motivation was to understand what was required to make any sort of contact with quenched lattice QCD calculations [44] of the scattering lengths in the $^3S_1$ channel.

\[ \text{FIG. 6. Lippmann-Schwinger equation for the LO contribution to the scattering amplitude (large solid rectangle) in the } ^3S_1 - ^3D_1 \text{ coupled-channels. The small solid circles denote an insertion of } C_0 \text{ or } g_A. \text{ The “o” appearing below the OPE diagram implies keeping only the chiral limit.} \]

In the $^3S_1 - ^3D_1$ coupled-channels, OPE generates both central and tensor potentials,

\[
\begin{align*}
V_C^{(\pi)}(r; m_\pi) &= -\alpha_\pi m_\pi^2 e^{-m_\pi r} / r, \\
V_T^{(\pi)}(r; m_\pi) &= -\alpha_\pi e^{-m_\pi r} \left( \frac{3}{r^2} + \frac{3m_\pi}{r} + m_\pi^2 \right), 
\end{align*}
\]

where $\alpha_\pi = g_A^2(1 - 2m_\pi^2d_{18}/g_A)^2/(8\pi f_\pi^2)$. The constant $d_{18}$ is uncertain [36] but as our results are quite insensitive to this quantity, we set it to zero in what follows. At LO in BBSvK counting we should only keep the chiral limit of these potentials and the $C_0$ operator that contributes to the $^3S_1 - ^3D_1$ coupled-channels, as shown in Fig. 6. Deviations from the chiral limit are inserted in perturbation theory. We choose to keep the full potentials in our NLO calculation, meaning that we also include part of the N$^3$LO calculation, which we find does not modify the divergence structure. In addition, at NLO in BBSvK counting there is a contribution from the chiral limit of two-pion exchange (TPE) and from an insertion of $C_2$ and $D_2$, as shown in Fig. 7. The TPE potential in coordinate space has been computed in Ref. [45], and in the chiral limit is given by

\[
\begin{align*}
V_C^{(\pi\pi)}(r; 0) &= \frac{3(22g_A^4 - 10g^2_A - 1)}{64\pi^3 f_\pi^4} \frac{1}{r^5}, \\
V_T^{(\pi\pi)}(r; 0) &= -\frac{15g_A^4}{64\pi^3 f_\pi^4} \frac{1}{r^5}. 
\end{align*}
\]

In order to regulate the potentials at the origin, we use a spatial square-well of radius $R$ [46,27], where the potential outside the square well is

\[ \text{‡‡In contrast with the } ^1S_0 \text{ channel, in the } ^3S_1 - ^3D_1 \text{ coupled-channels there is a local four-nucleon interaction from OPE that we have included in our definition of } C_0. \]
the chiral limit of the OPE potential, the NLO
renormalize the theory) to keep only the chiral limit of these constants. Defining $\Psi$ to be
for the chiral limit of the TPE potential it is sufficient (and in fact necessary to be able to
retained. However, for the deviations of the OPE potential from the chiral limit, and also
where
$u$
-dependence in this potential due to $g_A$, $M_N$ and $f_\pi$. For
the chiral limit of the OPE potential, the NLO $m_q$-dependence of these constants must be
retained. However, for the deviations of the OPE potential from the chiral limit, and also
for the chiral limit of the TPE potential it is sufficient (and in fact necessary to be able to
renormalize the theory) to keep only the chiral limit of these constants. Defining $\Psi$ to be
$\psi(r) = \begin{pmatrix} u(r) \\ w(r) \end{pmatrix}$, (15)
where $u(r)$ is the S-wave wavefunction and $w(r)$ is the D-wave wavefunction, the regulated
Schrödinger equation is
$\Psi''(r) + \left[ k^2 + V_L(r; m_\pi) \theta(r - R) + V_S(r; m_\pi, k^2) \theta(R - r) \right] \Psi(r) = 0$. (16)
As discussed in BBSvK, we can make an identification between the coefficients of local
operators, $C_i$ and $D_i$, and the constant potentials of the square-wells that enter into eq. (14),
$V_C$ and $V_D$,

$C_i \delta^{(3)}(r) \rightarrow \frac{3C_i \theta(R-r)}{4\pi R^3} \equiv V_C \theta(R-r)$

$D_i \delta^{(3)}(r) \rightarrow \frac{3D_i \theta(R-r)}{4\pi R^3} \equiv V_D \theta(R-r)$ . (17)

FIG. 7. Chiral limit of the crossed TPE diagram, deviations from the chiral limit of OPE, and
the $C_2$ (large solid circle) and $D_2$ (large solid square) operators, all of which contribute at NLO
in the $^3S_1 - ^3D_1$ coupled-channels. The “o” appearing below a diagram implies keeping only the
chiral limit.
FIG. 8. The deuteron binding energy at NLO in the EFT as a function of the pion mass for the couplings given in eq. (18). For this set of parameters, the deuteron is loosely bound over a wide range of pion masses.

At the physical value of the pion mass, $C_2$ and a linear combination of $C_0$ and $D_2$ are fixed to reproduce the deuteron binding energy and the effective range in the $^3S_1$-channel. Given that it is only a combination of $C_0$ and $D_2$ that we determine, we have freedom to vary $D_2$ and compensate this by a change in $C_0$, as in the $^1S_0$ channel. We find couplings $C_0$, $D_2$ and $C_2$, renormalized at the scale set by the radius of the square-well, $R^* = 0.45$ fm,

$$C_0(R^*) = -6.17 \text{ fm}^2, \quad D_2(R^*) = 0.67 \text{ fm}^4, \quad C_2(R^*) = 0.75 \text{ fm}^4,$$

that are consistent with NDA and all the low-energy phase-shift data in the $^3S_1 - ^3D_1$ coupled-channels, and for which the deuteron is loosely bound over a large range of pion masses, as shown in Fig. 8.

The phase-shifts and mixing parameter for the $^3S_1 - ^3D_1$ coupled channels are shown in Fig. 9. As expected, the $^3S_1$ phase-shift falls more steeply as the deuteron becomes more loosely bound. Further, as the pion becomes lighter, the higher partial waves and mixing parameter are expected to increase due to the longer range of the potential and their relative insensitivity to short-distance physics.

In BBSvK, the renormalization scale dependence of the theory was investigated [27]. The dependence on the width of the square-well, $R$, was found to be small, consistent with the EFT expectations. We find the same to be true at NLO.

For purposes of contrast, we present the deuteron binding energy, phase-shifts and mixing parameter for an arbitrary set of couplings that respect NDA,

$$C_0(R^*) = -5.05 \text{ fm}^2, \quad D_2(R^*) = -1.60 \text{ fm}^4, \quad C_2(R^*) = 0.75 \text{ fm}^4.$$

§§It is interesting to note that the chiral-limit value of the deuteron binding energy in the EFT coincides with that found in an analogous calculation with the Argonne V18 potential, when all coupling constants are frozen to their physical values, and only the explicit $m_\pi$ dependence in eq. (10) is considered [17].
FIG. 9. The phase-shifts, $\delta_0$ and $\delta_2$ for the $^3S_1$ channel and the $^3D_1$ channel and the mixing parameter $\varepsilon_1$ as a function of momentum, $|p|$, for pion masses of $m_\pi = 60$ MeV (dashed), $m_\pi^{\text{PHYS}}$ (dotted) and 180 MeV (dot-dashed), for the couplings in eq. (18). The solid curve corresponds to the results of the Nijmegen partial-wave analysis [41]. Note that for $\delta_0$ at $m_\pi^{\text{PHYS}}$, the NLO EFT calculation coincides with the partial-wave analysis to relatively high momenta.

FIG. 10. The deuteron binding energy at NLO in the EFT as a function of the pion mass for the couplings given in eq. (19).

For these choices, the variation of $B_d(m_q)$ with respect to $m_q$ is quite rapid, as shown in Fig. 11, and for $m_q \lesssim 90$ MeV, the deuteron is unbound. The phase-shifts and mixing parameter resulting from the couplings in eq. (19) are shown in Fig. 11, and as expected, they have a relatively strong dependence upon $m_q$.

It is important to point out that while the deuteron binding energy curves shown in Fig. 8 and Fig. 10 yield unbound deuterons in the chiral limit, there exist parameter sets consistent with NDA for which the deuteron remains bound in the chiral limit.

V. CONCLUSION

We have explored the light-quark mass dependence of low-energy nucleon-nucleon interactions. The motivation for this work was to determine if, in fact, bounds could be set on the time-variation of fundamental couplings from nuclear processes, such as those occurring during big bang nucleosynthesis. We have demonstrated the existence of sets of strong in-
FIG. 11. The phase-shifts, $\delta_0$ and $\delta_2$ for the $^3S_1$ channel and the $^3D_1$ channel and the mixing parameter $\varepsilon_1$ as a function of momentum, $|p|$, for pion masses of $m_\pi = 60$ MeV (dashed), 139 MeV (dotted) and 180 MeV (dot-dashed), for the couplings in eq. (18). The solid curve corresponds to the results of the Nijmegen partial-wave analysis [11].

interaction couplings in the low-energy effective field theory describing the nucleon-nucleon interaction that are consistent with all available data and with naive dimensional analysis for which the di-neutron remains unbound, and the deuteron remains loosely bound over a wide range of light quark masses. We do not mean to imply that these are the sets of couplings that nature has chosen, but rather that this scenario is not excluded at present. Thus, we conclude that bounds that have previously been set on the time-variation of fundamental couplings from processes in the two-nucleon sector are not rigorous and should be discarded.

Our calculation does suffer from some limitations. First, we have not included electromagnetism, and thus could not address the $pp$-system in the $^1S_0$-channel. However, we do not believe that its inclusion will modify the results we have presented in any significant way. The same we believe to be true of isospin breaking. Second, and perhaps the most important limitation, is that we have not included the strange quark in our discussions, and have worked with $SU(2)_L \otimes SU(2)_R$ chiral symmetry. Variations in the strange (and charm, bottom and top) quark mass will manifest themselves as changes in the values of the coefficients in the Lagrange density, $C_i$ and $D_i$. At this point in time the tools do not exist to analyze this scenario. For instance, one might consider developing an EFT with three light-quark flavors. However, since nuclei are such finely-tuned systems, and $SU(3)$-breaking is not so small, it is likely that a perturbative treatment of nuclei using the approximate $SU(3)_L \otimes SU(3)_R$ chiral symmetry of QCD would converge very slowly, if at all.

From a conventional nuclear physics point of view, the $m_q$-dependence of the nuclear potentials not only requires knowledge of the $m_q$-dependence of one-pion exchange, but also of the $m_q$-dependence of the $\rho$, $\omega$, $\phi$,... masses and their couplings to nucleons. This would appear to be an intractable problem. It is possible that some of the traditional treatments of the nucleon-nucleon interaction [11,18,50], and treatments of light nuclei could mimic these dependences by variations in their ad-hoc short-distance components of the nucleon-nucleon and higher-body potentials. The recently developed effective field theory tools organize this problem in a very simple way, and thus at NLO in the EFT expansion, only the $D_i$ are needed. In experimental processes examined to date that, in principle, allow one to separate the $D_i$ from the $C_i$, it is found that there are other amplitudes contributing to the process that dominate over the $D_i$ contributions, rendering such a separation exceedingly difficult. In the absence of an experimental determination of the $D_i$, it would appear that the only viable
means for determination of this $m_q$-dependence is through lattice QCD simulations. We find this to be very strong motivation to pursue a lattice QCD program focused on the two-nucleon sector.

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***A first attempt at computing nucleon-nucleon scattering lengths in quenched lattice QCD has been made in Ref. [51]. A discussion of the two-nucleon potential in quenched and partially-quenched QCD can be found in Ref. [52]. For a recent discussion, see Ref. [53].***
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