Low-frequency resonant vibration energy harvester using piezoelectric stacks with force magnification frames

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Abstract. The piezoelectric stack energy harvester (PSEH) is often used to transform ambient large amplitude vibrational energy, such as road vibrations and pressure fluctuations, because of its ability to withstand large external excitation force. However, due to the high natural frequency of the piezoelectric stack, it is difficult to generate resonance phenomenon, which leads to low energy efficiency for the energy harvester. In this paper, a piezoelectric stack energy harvester with force magnification frame (FMF) is proposed and studied for energy harvesting from low-frequency pressure ripples. First, a force magnification frame is designed to enhance the pressure transfer ability. The FMF is able to, not only amplify the exciting force acting on the piezoelectric stack but also decrease the natural frequency of the PSEH. Based on finite element method (FEM) analysis, the natural frequency of the PSEH with FMF is obtained. The natural frequency of the PSEH is successfully reduced, and the open-circuit voltage output of the PSEH under low-frequency resonance conditions is much higher than for off-resonance conditions. The results validate that the PSEH with FMF is efficient for lower frequency vibration environments.

1. Introduction

The requirement of power sources for low-power electronic products has attracted widespread attention due to the development of microelectronic technology. The energy obtained from a vibration system is only at the microwatt level, but it is sufficient for low-energy electronic products. In recent years, numerous researchers have studied vibration energy harvesting technology, and various transduction mechanisms have been reported for vibration energy harvesting, such as electrostatic[1, 2], electromagnetic[3, 4], and piezoelectric[5-7]. The piezoelectric vibration energy harvester is the most studied mechanism due to its feasibility and efficiency advantages for small electronic components and wireless applications.

In our daily life, pressure ripple is a common phenomenon occurring in pipes carrying fluid because pumps or other actuators operate imperfectly. Pressure ripple causes vibration and noise in the pipeline. Using piezoelectric materials, it is possible to convert the mechanical energy generated by vibrations into electrical power for low-power electronic products, such as pressure sensors. This concept may lead to the development of self-powered, remote monitoring systems. Cunefare et al[8-12] proposed an energy harvester made from a piezoelectric stack (HPEH) and used this to convert mechanical energy,
generated by pressure ripple from a hydraulic system, into electrical energy. An HPEH can power a wireless pressure sensor, but it works in an off-resonance condition. A piezoelectric stack energy harvesters (PSEH) are often used to transform ambient, large-amplitude vibrational energy, such as road vibrations and pressure fluctuations, because of their ability to withstand large external excitation forces. However, due to the high natural frequency of the piezoelectric stack, it is difficult to generate resonance phenomenon, resulting in relatively low energy efficiency for these energy harvesters.

To improve the harvesting efficiency of the PSEH, the force magnification frame (FMF) was proposed. Feenstra et al[13] proposed an energy harvesting backpack, employing a mechanically amplified piezoelectric stack, with a mean power output from the energy harvester of approximately 0.4 mW. Optimal design of the force magnification frame for a piezoelectric stack energy harvester was studied by Chen et al[14]. They found that the tilt angle and thickness of the beam used for force magnification are the main factors affecting energy conversion efficiency. Chen’s team found that adding the force magnification frames to piezoelectric stacks can reduce the natural frequency of the system.

In this work, a low-frequency (under 100 Hz) resonant vibration energy harvester using piezoelectric stacks with a force magnification frame is proposed and studied. The force magnification principle and the frequency reduction effect of the force magnification frame are analyzed. The finite element method (FEM) is used to obtain the modes of the PSEH with FMF, and the effects of the structure parameters are discussed based on the FEM results. To reduce the natural frequency of the system to resonant condition, the method of connecting FMFs in series and adding a second-stage FMF is proposed. Because pressure ripple cannot directly affect an FMF, a pressure interface film is used to transport force from the pressure ripple into the FMF. Finally, the conclusions are briefly given.

2. Principle of FMF
FMFs are used to magnify the force on piezoelectric stacks. The initial structure of an FMF with a piezoelectric stack is shown in Figure 1. In Figure 1, $F_{in}, F_{out}, \theta$ and $d$ represent the force applied vertically to the FMF, the horizontal force output in the piezoelectric stack, the angle of each of the four beams with respect to the horizontal direction and the thickness of the beams, respectively.

![Figure 1. Initial structure of the FMF with a piezoelectric stack](image)

According to the static analysis of a PSEH with FMF, $F_{out}$ can be expressed by $F_{in}$ and $\theta$ as:

$$F_{out} = F_{in} \cot(\theta)$$  \hspace{1cm} (1)

When $F_{in}$ acts on the FMF, elastic deformation will occur in the four beams and lead to a change of $\theta$. The change of $\theta$ is represented by $\Delta \theta$. The expression for $F_{out}$ is:

$$F_{out} = F_{in} \cot(\theta + \Delta \theta)$$  \hspace{1cm} (2)
When the FMF with piezoelectric stack undergoes vibration, $\Delta \theta$ will change with $F_{in}$. $F_{in}$ is assumed to take the form of a simple harmonic vibration and can be expressed as:

$$F_{in} = F_0 \sin(\omega t)$$

(3)

$F_0$ is the amplitude of vibration, $\omega$ is the frequency of vibration, and $t$ is the vibration time. $\Delta \theta$ is related to $F_0$, $\omega$ and $t$. Under the off-resonance condition, the magnitude of $\Delta \theta$ has a strong correlation with $F_0$. If $F_0$ is very small, then $\Delta \theta \ll \theta$ and the expression for $F_{out}$ can be written as:

$$F_{out} = F_0 \sin(\omega t) \cot(\theta)$$

(4)

Under the resonance condition, the magnitude of $\Delta \theta$ will increase substantially, even when $F_0$ is small. The expression of $F_{out}$ can be written as:

$$F_{out} = F_0 \sin(\omega t) \cot(\theta + \Delta \theta)$$

(5)

$\Delta \theta$ has a strong correlation with $\omega$ when the PSEH with FMF is operating at the resonance condition. The amplification factor is defined as $M(\theta, F_0, \omega)$, which is a function about $F_0$, $\omega$ and $t$. The amplification factor is equal to the amplitude of $\cot(\theta + \Delta \theta)$. The amplitude of $F$ is $F_{out}$. The relationship between $F$, $F_0$, $\omega$ and $t$ can be expressed as:

$$F = F_0 M(\theta, F_0, \omega)$$

(6)

When $\omega$ is close to the natural frequency of the PSEH with FMF, $F$ will increase substantially. As a result, the system working at the resonance condition will significantly improve the force applied to the FMF, and lead to an increase of the piezoelectric stack voltage output. Lowering the natural frequency of the system will be the next step.

3. Validation of the FMF effectiveness

With the help of finite element software, FEM is used to simulate the PSEH with FMF. The emphasis of this chapter is on reducing the natural frequency of the system by improving its structure.

3.1. Simplified model of the PSEH

The PSEH consists of piezoelectric stacks, an energy conversion circuit and an energy storage circuit. The finite element model of a PSEH only considers the piezoelectric stacks. The energy harvesting efficiency of the device is measured by the open-circuit voltage output of the piezoelectric stack. The natural frequency of the piezoelectric stack is approximately 50 kHz. Because the piezoelectric stack consists of more than several hundred piezoelectric ceramic sheets and copper guided sheets, even the use of finite element software will require long calculation times. For this reason, a simplified model can be used for analyzing the frequency reduction produced by the frame while saving calculation time. Figure 2 shows the simplified model of PSEH.
Table 1. Materials and dimensions of the simplified model of the PSEH

| Material                        | Length(mm) | Wide(mm) | Height(mm) |
|---------------------------------|------------|----------|------------|
| piezoelectric ceramic layer     | PZT-5H     | 5        | 5          | 1          |
| copper guided layer             | Copper alloy | 5        | 5          | 0.5        |
| contacting part                 | Structure steel | 5        | 5          | 2          |

Table 2. Materials properties of simplified model of PSEH

| Material       | Density (kg m\textsuperscript{3}) | Young’s Modulus (Pa) | Piezoelectric constant (Pc N\textsuperscript{-1}) |
|----------------|-----------------------------------|----------------------|-----------------------------------------------|
| PZT-5H         | 7600                              | 1.15 10\textsuperscript{11} | 660                                           |
| Copper alloy   | 8300                              | 1.1 10\textsuperscript{11}   | ___                                            |
| Structural steel | 7850                            | 2 10\textsuperscript{11}     | ___                                            |

3.2. Initial design of the PSEH with FMF

The emphasis of this paper is on reducing the natural frequency of the system by installing an FMF on the piezoelectric stacks. The initial structure of the PSEH with FMF is shown in Figure 3.

![Figure 3. Initial structure of the PSEH with FMF](image)

Figure 3. Initial structure of the PSEH with FMF

![Figure 4. Open-circuit voltage frequency-response curves of the PSEH and the PSEH with FMF](image)

Figure 4. Open-circuit voltage frequency-response curves of the PSEH and the PSEH with FMF

The important dimensions of the FMF are given in Table 4, where $d$ is the thickness of the beams, $l$ is the length of the beams, $\theta$ is the angle of each of the four beams with respect to the horizontal direction, $L$ is the distance between contacting surfaces, $H$ is the height of the FMF, and $W$ is the width of the FMF. To verify the effectiveness of the FMF, the natural frequencies and open-circuit voltage of the PSEH and PSEH with FMF are simulated by FEM. The open-circuit voltage frequency-response curves are shown in Figure 4 under 10 kPa pressure at frequencies of 0-300 Hz. The natural frequency of the system ranges from approximately 55.1 kHz to 1.89 kHz due to the addition of the FMF on the PSEH. Figure 4 also shows that the open-circuit voltage of the PSEH with FMF is almost
five times higher than the PSEH in the 0-300 Hz frequency range. Static analysis of the FMF with PSEH is carried out by FEM, and the elastic strain of the structure under a pressure of 100 kPa is obtained. In Figure 5, most of the elastic strain is concentrated on the four beams. Therefore, the stiffness of the beams will significantly influence the deformation of the FMF, and the natural frequency of the PSEH with FMF is closely related to the dimensions of the beams.

4. Study on reducing the natural frequency of the PSEH with FMF

4.1. Effect of the length and thickness of the FMF

According to the static analysis result, the dimensions of the four beams will influence the natural frequency of the PSEH with FMF. Figure 6 shows the open-circuit voltage frequency-response curves, produced by FEM, of the PSEH with FMF under 10 kPa pressure at frequencies of 0-300 Hz for (a) different beam lengths and (b) different beam thicknesses. Table 3 shows the natural frequencies.

| Length of beams (mm) | Thickness of beams (mm) | Natural frequency (Hz) |
|----------------------|-------------------------|------------------------|
| 6.5                  | 0.5                     | 1888.9                 |
| 9                    | 0.5                     | 1266.1                 |
| 11.5                 | 0.5                     | 966.59                 |
| 14                   | 0.5                     | 797.26                 |
| 14                   | 0.4                     | 714.75                 |
| 14                   | 0.3                     | 641.26                 |
| 14                   | 0.2                     | 568.63                 |

In Figure 6, the open-circuit voltage increases as the beam length increases and as the beam thickness decreases. The natural frequency of the PSEH with FMF decreases as the beam length increases and as the beam thickness decreases. These results show that the natural frequency of the system can be effectively reduced by a reasonable design of the beam structure. The improved design of the PSEH with FMF is called the first-stage FMF. Considering the structural strength of the beam, the first-stage FMF dimensions are shown in Table 4.

| Dimension | Initial design | First-stage | Second-stage |
|-----------|----------------|-------------|--------------|
| d (mm)    | 0.5            | 0.3         | 0.3          |
| l (mm)    | 6.5            | 14          | 20           |
| θ (°)     | 6              | 6           | 6            |
| L (mm)    | 18             | 18          | 42           |
| H (mm)    | 16             | 14          | 14           |
| W (mm)    | 5              | 5           | 5            |
4.2. Connection of the PSEH with FMF in series

The natural frequency of first-stage FMF is three times lower than the initial design of the PSEH with FMF, but it cannot satisfy the condition of low-frequency resonance (applied force at a frequency under 100 Hz). Connecting first-stage FMFs in series can lower the natural frequency of the system because the equivalent stiffness of the structure decreases. The principle is similar to connecting spring in series to lower the natural frequency of the total structure. The structure of the three first-stage FMF in series is shown in Figure 7.

Figure 7. The open-circuit voltage frequency-response curves of the first-stage FMFs in series

FEM analysis shows that the natural frequency of the first-stage FMF is approximately 645.69 Hz, and the two first-stage FMFs in series will lower the natural frequency of the system to 410.37 Hz. Figure 8 shows the open-circuit voltage frequency-response curves for these devices under 10 kPa pressure at frequencies of 0-300 Hz. Due to a natural frequency of the three first-stage FMFs in series of 289.3 Hz, the resonance phenomenon occurs when the frequency of the vibrational force is approximately 290 Hz. The open-circuit voltage of the PSEH working at the resonance condition is much higher than the PSEH working at the off-resonance condition.

4.3. Addition of a second-stage FMF

Connecting first-stage FMFs in series can effectively reduce the natural frequency of the system, but it will increase the overall size of the device. To make the PSEH resonate at a low-frequency vibration condition, the natural frequencies of the system can be further reduced by adding a second-stage FMF to the three first-stage FMFs in series. The design and size of the second-stage FMF structure are shown in Figure 9 and Table 4, respectively.

Figure 9. The structure of three first-stage FMFs in series with a second-stage FMF

FEM analysis shows that the natural frequency of the three first-stage FMFs in series with a second-stage FMF is 63.28 Hz. According to Figure 10, under 10 kPa pressure at 61-65 Hz, the device produces resonant phenomenon. This resonance greatly increases the open-circuit voltage output of the device and satisfies the purpose of collecting energy generated by vibrations in low-frequency resonance conditions.

Figure 10. Open-circuit voltage frequency-response curves of three first-stage FMFs in series with a second-stage FMF
4.4. Interface film effect on the PSEH

The purpose of designing the low-frequency resonant vibration energy harvester using piezoelectric stacks with an FMF is to collect energy generated by pressure ripples. However, pressure ripples cannot apply force directly onto the piezoelectric stacks or FMF. By adding an interface film, the energy generated by a pressure ripple is transferred to the PSEH with FMF in the form of a mechanical vibration. The material of the interface film is aluminum. Figure 11 shows the location of the interface film on the PSEH with FMF. The influence of the interface film on energy transfer is analyzed by FEM. Figure 12 shows that under 10 kPa pressure at frequencies of 0-300 Hz, the maximum open-circuit voltage of the PSEH with FMF and interface film is lower than the PSEH with FMF. This reduction is due to the interface film limiting the energy transfer from the pressure ripple to the PSEH.

5. Conclusion

A low-frequency resonant vibration energy harvester using piezoelectric stacks with force magnification frames was proposed to collect energy from pressure ripples. The system, working at the resonance condition, will significantly improve the force applied to the FMF and lead to an increase of the piezoelectric stack voltage output. The natural frequency of the system can be reduced by adding an FMF to the PSEH. As the length and thickness of the four beams in the FMF become longer and thinner, the natural frequency of the PSEH with FMF decreases. The improved design of the PSEH with FMF is called the first-stage FMF. Connecting first-stage FMFs in series can lower the natural frequency of the system. To enable the PSEH to collect energy generated under low-frequency resonance conditions, a second-stage FMF is added to three first-stage FMFs in series to lower the natural frequency of the system to 63.28 Hz. The open-circuit voltage output of the PSEH under low-frequency resonance conditions is much higher than for off-resonance conditions. Adding an interface film, allows the energy generated by the pressure ripple to be transferred to the PSEH with FMF in the form of a mechanical vibration. The results show that the interface film limits energy transfer from the pressure ripple to the PSEH. In future work, a dynamic model and experimental test of the PSEH with FMF should be studied.

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