**B → K^*\ell^+\ell^-** as a probe of Universal Extra Dimensions

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1 Introduction

The idea of the existence of extra dimensions has recently obtained a lot of attention [1]. In part, this interest is because the scale at which the extra-dimensional effects can be relevant could be around a few TeV, even hundreds of GeV in some cases, clearly a challenging possibility for the next generation of accelerators. Moreover, this new point of view has permitted to study many long-standing problems (as the hierarchy problem) from a new perspective.

An interesting model is that proposed by Appelquist, Cheng and Dobrescu with so-called universal extra dimensions (UED) [2], which means that all the Standard Model (SM) fields may propagate in one or more compact extra dimensions. The compactification of the extra dimensions introduces in the four-dimensional description of the theory an infinite tower of states for every field. Such states are called Kaluza-Klein (KK) particles and their masses are related to compactification radius according to the relation

\[ m_n^2 = m_0^2 + \frac{n^2}{R^2}, \]

with \( n = 1, 2, ... \). We consider the simplest Appelquist, Cheng and Dobrescu (ACD) scenario, characterized by a single extra dimension. It presents the remarkable feature of having only one new parameter with respect to SM, the radius \( R \) of the compactified extra dimension.

Rare \( B \) transitions can be used to constrain this scenario [3]. Buras and collaborators have investigated the impact of universal extra dimensions on the \( B_{d,s}^0 - \bar{B}_{d,s}^0 \) mixing mass differences, on the CKM unitarity triangle and on inclusive \( b \rightarrow s \) decays for which they have computed the effective Hamiltonian \[4, 5\]. In particular, it was found that \( B(B \rightarrow X_c\gamma) \) allowed to constrain \( 1/R \geq 250 \) GeV, a bound updated by a more recent analysis to \( 1/R \geq 600 \) GeV at 95% CL, or to \( 1/R \geq 330 \) GeV at 99% CL [6].

In [7] several \( B_{d,s} \) and \( A_b \) decays induced by \( b \rightarrow s \) transitions were analyzed, finding that in many cases the hadronic uncertainties do not hide the dependence of the observables on \( R \). In the following Sections we shall discuss some of these results, such as the dependence on \( R \) of the branching ratio, the
forward-backward asymmetry and the $K^*$ helicity distributions for the decay modes $B \rightarrow K^*\ell^+\ell^-$, with $\ell^- = e^-, \mu^-$, and the tau polarization asymmetries for the mode $B \rightarrow K^*\tau^+\tau^-$. 

2 The decays $B \rightarrow K^*\ell^+\ell^-$

In the Standard Model the effective $\Delta B = -1$, $\Delta S = 1$ Hamiltonian governing the rare transition $b \rightarrow s\ell^+\ell^-$ can be written in terms of a set of local operators:

$$H_W = 4\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{10} c_i(\mu)O_i(\mu)$$

where $G_F$ is the Fermi constant and $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa mixing matrix; we neglect terms proportional to $V_{ub}V_{us}^*$ since the ratio $|V_{ub}|/|V_{us}|$ is of the order $10^{-2}$. We show only the operators $O_i$ which are relevant for the decays we consider here:

$$O_7 = \frac{e}{16\pi^2}m_b(\bar{s}_L\alpha\sigma^{\mu\nu}b_R)F_{\mu\nu} \quad O_9 = \frac{e^2}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\bar{\ell}\gamma_\mu\ell$$
$$O_{10} = \frac{e^2}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\bar{\ell}\gamma_\mu\gamma_5\ell,$$

where $\alpha, \beta$ are colour indices, $b_{R,L} = \frac{1 \pm \gamma_5}{2} b$, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$; $e$ is the electromagnetic coupling constant, while $F_{\mu\nu}$ denotes the electromagnetic field strength tensor.

The Wilson coefficients $c_i$ appearing in (1) are modified in the ACD model because the KK states can contribute as intermediate states in penguin and box diagrams. As a consequence, the Wilson coefficients can be expressed in terms of functions $F(x_t, 1/R)$, $x_t = \frac{m_b^2}{M_W^2}$, which generalize the corresponding SM functions $F_0(x_t)$ according to $F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n)$, where $x_n = \frac{m_n^2}{M_W^2}$ and $m_n = \frac{n}{R}$.

The description of the decay modes $B \rightarrow K^*\ell^+\ell^-$ involves the hadronic matrix elements of the operators appearing in the effective Hamiltonians (1). We use for them two sets of results: the first one, denoted as set A, obtained by three-point QCD sum rules based on the short-distance expansion [8]; the second one, denoted as set B, obtained by QCD sum rules based on the light-cone expansion [9].

With these ingredients we can calculate the branching fraction as a function of $1/R$, as depicted in Fig. [1]. The hadronic uncertainty is evaluated considering the two set of form factors and taking into account their errors. Comparing the theoretical prediction with the horizontal band representing experimental...
data, we obtain that set A of form factors does not allow to establish a lower bound on $1/R$, while, as we can see in Fig. 1 for set B one gets $1/R > 200$ GeV. The present discrepancy between BaBar and Belle measurements does not permit stronger statements.

Fig. 1. Left: $BR(B \to K^* \ell^+ \ell^-)$ versus $1/R$ using set B of form factors. The two horizontal regions correspond to BaBar [10] (lower band) and Belle (upper band) [11] results. Right: forward-backward lepton asymmetry in $B \to K^* \ell^+ \ell^-$ versus $1/R$ using set A. The dark band correspond to the SM results, the intermediate band to $1/R = 250$ GeV, the light one to $1/R = 200$ GeV.

Important information could be gained from the forward-backward asymmetry, defined as

$$A^{FB}(q^2) = \frac{\int_0^1 \frac{d^2 \Gamma}{dq^2 d\cos \theta_\ell} d\cos \theta_\ell - \int_{-1}^0 \frac{d^2 \Gamma}{dq^2 d\cos \theta_\ell} d\cos \theta_\ell}{\int_0^1 \frac{d^2 \Gamma}{dq^2 d\cos \theta_\ell} d\cos \theta_\ell + \int_{-1}^0 \frac{d^2 \Gamma}{dq^2 d\cos \theta_\ell} d\cos \theta_\ell}, \quad (3)$$

where $\theta_\ell$ is the angle between the $\ell^+$ direction and the $B$ direction in the rest frame of the lepton pair (we consider massless leptons). This asymmetry is a powerful tool to distinguish between SM and several extensions of it. Belle Collaboration has recently provided the first measurement of such an observable [12]. We show in the right part of Fig. 1 our predictions for the SM, $1/R = 250$ GeV and $1/R = 200$ GeV. A relevant aspect is that the zero of $A_{fb}$ is sensitive to the compactification parameter, so that its experimental determination would constrain $1/R$.

We investigate another observable, the fraction of longitudinal $K^*$ polarization in $B \to K^* \ell^+ \ell^-$, for which a new measurement in two bins of momentum transfer to the lepton pair is available in case of $\ell = \mu, e$ [13]:

$$f_L = 0.77^{+0.63}_{-0.30} \pm 0.07 \quad 0.1 \leq q^2 \leq 8.41 \text{ GeV}^2$$

$$f_L = 0.51^{+0.22}_{-0.25} \pm 0.08 \quad q^2 \geq 10.24 \text{ GeV}^2. \quad (4)$$
The dependence of this quantity on the compactification parameter provides us with another possibility to constrain the universal extra dimension scenario. In fact, we obtain that the value $q^2$ where this distribution has a maximum is sensitive to $R$, as we can see in the left part of Fig. 2.

![Fig. 2](image)

Fig. 2. Left: longitudinal $K^*$ helicity fraction in $B \to K^* \ell^+ \ell^-$ obtained using set A of form factors. Right: Transverse $\tau^-$ polarization asymmetry in $B \to K^* \tau^+ \tau^-$ for set A of form factors. The dark region is obtained in SM; the intermediate one for $1/R = 500$ GeV, the light one for $1/R = 200$ GeV.

3 Lepton polarization asymmetries in $B \to K^* \tau^+ \tau^-$

As first noticed in [1], the process $B \to K^* \tau^+ \tau^-$ is of great interest due to the possibility of measuring lepton polarization asymmetries which are sensitive to the structure of the interactions, so that they can be used to test the Standard Model and its extensions.

To compute lepton polarization asymmetries for $B$ decays in $\tau$ leptons we consider the spin vector $s$ of $\tau^-$, with $s^2 = -1$ and $k_1 \cdot s = 0$, $k_1$ being the $\tau^-$ momentum. In the rest frame of the $\tau^-$ lepton three orthogonal unit vectors: $e_L$, $e_N$ and $e_T$ can be defined, corresponding to the longitudinal $s_L$, normal $s_N$ and transverse $s_T$ polarization vectors:

$$s_L = (0, e_L) = \left(0, \frac{k_1}{|k_1|}\right) , \quad s_N = (0, e_N) = \left(0, \frac{p' \times k_1}{|p' \times k_1|}\right) ,$$

$$s_T = (0, e_T) = (0, e_N \times e_L) . \quad (5)$$

In eq. (5) $p'$ and $k_1$ are respectively the $K^*$ meson and the $\tau^-$ three-momenta in the rest frame of the lepton pair. Choosing the $z$-axis directed as the $\tau^-$ momentum in the rest frame of the lepton pair: $k_1 = (E_1, 0, 0, |k_1|)$ and boosting the spin vectors $s$ in eq. (5) in the same frame, the normal and transverse polarization vectors $s_N, s_T$ remain unchanged: $s_N = (0, 1, 0, 0)$ and $s_T = (0, 0, -1, 0)$,
while the longitudinal polarization vector becomes: 
\[ s_L = \frac{1}{m}\left(|k_1|, 0, 0, E_1\right) \].

For each value of the squared momentum transferred to the lepton pair, \( q^2 \), the polarization asymmetry for the negatively charged \( \tau^- \) lepton is defined as:

\[ A_A(q^2) = \frac{d\Gamma}{dq^2}(s_A) - \frac{d\Gamma}{dq^2}(-s_A) \]
\[ \frac{d\Gamma}{dq^2}(s_A) + \frac{d\Gamma}{dq^2}(-s_A) \]

(6)

with \( A = L, T \) and \( N \). In the right part of Fig. 2 the transverse polarization asymmetry \( A_T \) is shown for different values of \( R \). It decreases (in absolute value) by nearly 15% with the decrease of \( 1/R \) down to \( 1/R = 200 \) GeV.

In deriving the expressions of polarization asymmetries it is possible to exploit some relations among form factors that can be obtained in the large energy limit of the final meson for \( B \) meson decays to a light hadron [15]. We obtain that, as a consequence of such relations, the polarization asymmetries become independent of form factors; this is a remarkable observation, which renders the polarization asymmetries important quantities to measure.

4 Conclusions

We have analyzed the branching fraction as well as the forward-backward lepton asymmetry in \( B \rightarrow K^* \ell^+ \ell^- \), finding that these observables are promising in order to constrain \( 1/R \). We have also considered the longitudinal \( K^* \) helicity fractions, for which some measurements are already available when the leptons in the final state are \( \ell = e, \mu \). For the mode \( B \rightarrow K^+ \tau^+ \tau^- \), we have found that the dependence of the \( \tau^- \) polarization asymmetries on \( 1/R \) is mild but still observable, the most sensitive ones being the transverse asymmetry. Finally, during our investigation we have shown that in the exclusive modes the polarization asymmetries are free of hadronic uncertainties if one considers the Large Energy limit for the light hadron in the final state.

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