On detecting the quantum correlations in the early universe

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Abstract. The superhorizon region has been supported by the Wilkinson Microwave Anisotropy Probe experimental data. In this paper, we point out that the conventional entanglement criterion cannot be applied in the superhorizon region.

1. Introduction

In the standard scenario of the early universe, the inflation theory was individually proposed by Sato [1] and Guth [2] to resolve the fundamental problems on the early universe in the follows.

The horizon problem While the density distribution of matter in the big bang cosmology is in principle inhomogeneous, this prediction is mismatched to the experimental data.

The flatness problem According to the experimental observation, the current curvature density \(\Omega\) is nearly flat. This is mismatched to \(\Omega \propto 1/H_0^2\) with the Hubble horizon \(H_0\) due to the small horizon in the early universe.

The relic problem The unresolved issue in standard cosmology in which various theories of particle physics would invariably produce massive particles which are not observed.

The fine tuning problem The cosmological constant in the Einstein equation is mismatched to the current experimental data.

Many versions of the inflation theory to adjust the experimental cosmological data were proposed, e.g., see the review papers [3, 4, 5]. However, nobody experimentally has yet confirmed this theory. However, recent observations of the cosmic microwave background (CMB) anisotropies [6] strongly support the superhorizon region in the early universe. The superhorizon region is called the time region that the physical scale of the quantum correlation exceeds one of the event horizon. It is remarked that the superhorizon region is dealt in many alternative approaches for the inflation theory, for example, ekpyrotic universe [7], cyclic universe theory [8, 9], and brane world theory [10]. The ultimate aim of this work is to establish how to detect the quantum correlations in the superhorizon region. This may contribute to
2. Quantum correlation in the superhorizon region

Before the main discussion, we briefly recapitulate the relationship between the quantum correlation and the entanglement, which is defined such that the quantum state in the combined system, \( |\psi\rangle_{AE} \in \mathcal{H}_A \otimes \mathcal{H}_E \), cannot be described as the separable state;

\[
|\psi\rangle_{AE} \neq |\phi_1\rangle_A \otimes |\phi_2\rangle_E, \ \forall |\phi_1\rangle_A \in \mathcal{H}_A, \ \forall |\phi_2\rangle_E \in \mathcal{H}_E.
\] (1)

Here, the Hilbert spaces, \( \mathcal{H}_A \) and \( \mathcal{H}_E \), express the considered and the other systems, respectively. If the state has the quantum correlation, even the state is in general not entangled. However, if the state is entangled, the state must have the quantum correlation.

While we would like to detect the quantum correlations between beyond and not beyond the physical scale of the event horizon to be illustrated in Fig. 1, there are crucial problems:

(i) the lack of the concept of the entanglement for the quantum field theory,
(ii) how to confirm the quantum correlations between beyond and not beyond the physical scale of the event horizon.

On the statement (i), the entanglement criterion for the quantum field seems not to be well defined from the quantum mechanical sense since the quantum field is defined as the global state. There are some approaches in this context. Fuentes et al. showed that the logarithmic negative criterion for the entanglement [11] was applied to the particles created by the Bogoliubov transformation. However, this criterion cannot distinguish which particles are entangled [12]. Reznik claimed that the two Unruh-Dewitt detectors couple to the quantum field. By the entanglement between the coupled detectors, the entanglement for the inflaton field is defined [13]. This idea is also used in Ref. [14]. On the statement (ii), the entanglement criterion is essentially used in the quantum informational method. In the conventional method of quantum information science, the detection of the quantum correlation is used in the local operation and classical communication (LOCC) [15]. However, in this situation, in principle, classical communication between beyond and not beyond the event horizon cannot be used due to the causality. To propose the judgment of the quantum correlation, we have known the alternative method using dynamics of the purity, the trace of the square of the density operator, in one-side subsystem [16]. According to Ref. [16], they showed that

\[
\frac{d}{dt} \left( \text{Tr} \rho_A^2 \right) \neq 0 \Rightarrow \rho \text{ is entangled},
\] (2)
where the density operator in the considered subsystem $\rho_A := \text{Tr}_E \rho$. Intuitively, when the considered field is coupled to the other field, obviously the state in the considered system must be changed. However, this criterion also has the serious problem how to define the local Hilbert space, that is, $\mathcal{H}_A$ and $\mathcal{H}_E$. On studying the quantum correlations in the superhorizon region, the local Hilbert space has not yet well defined. Therefore, the conventional well-known entanglement criterion cannot be applied.

3. Conclusion and Outlooks
We strikingly pointed out that the conventional quantum informational methods to judge the entanglement cannot be applied to the early universe context due to the causality.

From considering this problem, many open problems on quantum information science are shown up. As mentioned in the statement (i), first, the definition, the operational meaning, and the applications of the entanglement in quantum field theory are not known very well. Since many-body quantum mechanical systems can be characterized by the quantum field theory, this problem may be crucial to build up the large-scaled quantum computer. This is essentially different from the concept of the entanglement in a few body quantum system. Second, our proposal seems to allow the superluminal communication since we can extract information. However, the similar situation can be occurred in the context of the information loss paradox [18, 19] motivated by quantum teleportation. Therefore, our result has the potential to open the discussion on the role of the entanglement in the context of the cosmological situation. For example, related to the quantum teleportation, the first author, who is the co-author of Refs. [20, 21], discussed the quantum description of the closed timelike curve. However, Refs. [20, 21] proposals are only taken as the closed timelike curve consistently but were not and were not simulated to this since the arrow of time only has one direction, that is, does not have the reverse direction and the loop structure. To summarize these, quantum circuit representation may be not related to the spacetime. If so, the classification of the entanglement by LOCC is not physical. To resolve it, we need the quantum information theory to be expanded to the context in relativistic quantum field theory. Finally, what is meaning of the local operation in quantum field theory? However, in this context, the causality can be well defined. Since the locality is disconnected to the cluster principle, the local operation has no relationship to causality.

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