Galaxy clustering in the Legacy Survey and its imprint on the CMB

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ABSTRACT

We use data from the DESI Legacy Survey imaging to probe the galaxy density field in tomographic slices covering the redshift range 0 < z < 0.8. After careful consideration of completeness corrections and galactic cuts, we obtain a sample of 4.9 × 10^7 galaxies covering 17739 deg^2. We derive photometric redshifts with precision σ_z/(1 + z) = 0.012 – 0.015, and compare with alternative estimates. Cross-correlation of the tomographic galaxy maps with Planck maps of temperature and lensing convergence probe the growth of structure since z = 0.8. The signals are compared with a fiducial Planck ΛCDM model, and require an overall scaling in amplitude of A_L = 0.901 ± 0.026 for the lensing cross-correlation and A_{SW} = 0.984 ± 0.349 for the temperature cross-correlation, interpreted as the Integrated Sachs-Wolfe effect. The ISW amplitude is consistent with the fiducial ΛCDM prediction, but lies significantly below the prediction of the AvERA model, which has been proposed as an alternative explanation for cosmic acceleration. Within ΛCDM, our low amplitude for the lensing cross-correlation requires a reduction either in fluctuation normalization or in matter density compared to the Planck results, so that \( \Omega_m^{0.75} \sigma_8 = 0.297 \pm 0.009 \). In combination with the total amplitude of CMB lensing, this favours a shift mainly in density: \( \Omega_m = 0.274 \pm 0.024 \). We discuss the consistency of this figure with alternative evidence. A conservative compromise between lensing and primary CMB constraints would require \( \Omega_m = 0.296 \pm 0.006 \), where the 95% confidence regions of both probes overlap.

Key words: Cosmology; Cosmic Microwave Background – Cosmology: Gravitational Lensing – Cosmology: Large-Scale Structure of Universe

1 INTRODUCTION

The temperature fluctuations in the Cosmic Microwave Background (CMB) offer rich information about conditions in the early Universe at \( z \approx 1080 \) (e.g. Planck Collaboration et al. 2018a). Photons from the CMB also interact through gravity with the large-scale structures (LSS) that they traverse, inducing two major secondary effects: gravitational lensing and the Integrated Sachs-Wolfe effect (ISW). CMB lensing consists of the deflection of CMB photons by foreground LSS; the strength of the effect is quantified via the lensing convergence \( \kappa \), which provides a measure of the projected matter density fluctuations between last scattering and the present. When general relativity holds, \( \kappa \) is related to the 3D gravitational potential \( \Phi \) projected along the line of sight:

\[
\kappa(\hat{n}) = \frac{1}{c^2} \int_0^{r_{LS}} \frac{r_{LS} - r}{r_{LS}} \psi^2 \Phi(\hat{n}, r) \, dr,
\]

where \( \hat{n} \) is the position vector on the sky, \( r = \int c/H(z) \, dz \) is the line-of-sight comoving distance, \( \psi^2 \) is the comoving Laplacian, and \( r_{LS} \) is the distance to the last scattering surface; a flat geometry is assumed. Lensing distorts the background Gaussian CMB sky and creates non-Gaussian signatures, whose detection allows the reconstruction of a map of the convergence (see e.g. Lewis & Challinor 2006). The Integrated Sachs-Wolfe effect (Sachs & Wolfe 1967; Martínez-González et al. 1990) arises from the time-dependent gravitational potential \( \Phi \) causing the CMB temperature \( T_{\text{CMB}} \) to change. The induced temperature fluctuation \( \Delta T(\hat{n}) \) is proportional to the line-of-sight integral of \( \Phi \):

\[
\frac{\Delta T(\hat{n})}{T_{\text{CMB}}} = \frac{2}{c^2} \int \Phi(\hat{n}, r) \, dt = \frac{2}{c^3} \int \Phi(\hat{n}, r) \, a \, dr,
\]

where \( T_{\text{CMB}} \) is the mean temperature of the CMB at \( z = 0 \) and \( a = 1/(1 + z) \) is the cosmic scale factor. In the linear regime, the ISW signal is non-zero when the matter density of the Universe, \( \Omega_m \), deviates from unity. It is therefore sensitive to the linear growth of structure and dark energy. These two effects offer spatial and temporal information about gravitational potential fluctuations. They couple the CMB with foreground LSS, and can be detected via cross-correlation measurements. This will be the main focus of our present study.

Observations of CMB lensing have progressed hugely in recent years, with a full sky map of lensing convergence delivered by Planck (Planck Collaboration 2014; Planck Collaboration et al. 2016a, 2018c), and over 2100 deg^2 by ACTpol (Darwish et al. 2016a, 2018b), and South Pole Telescope (SPT) (Dunkley et al. 2013). The tomographic approach delivers galactic and cosmic variance in the large-scale, non-foreground components of the CMB lensing signal, and is sensitive to additional cosmological information beyond conventional CMB probes. The DESI Legacy Survey (LS) imaging is the first high-redshift, large-scale 3D tomographic galaxy survey, enabling the construction of high-surface brightness galaxy maps over a 2000 deg^2 area. Here we use DESI LS data to probe the galaxy density field for \( 0 < z < 0.8 \). We first describe the DESI Legacy Survey imaging and data analysis in Section 2.
Here, we correlate the Planck lensing and temperature maps with LSS traced by galaxies. A particular aim is to measure the ISW effect, which has the attraction of providing an independent probe of dark energy. However, ISW detections have been challenging because the signal is largest at low multipoles where substantial cosmic variance is unavoidable; the effect has therefore been detected with only modest significance (e.g. Ho et al. 2008; Giaannantonio et al. 2008; Planck Collaboration et al. 2016b). The uncertainty for measurements for the redshift range beyond $z > 0.5$ is particularly large, with some having null, or anti-correlations between LSS and the CMB (Sawangwit et al. 2010). This regime is of particular interest as it may provide key evidence for distinguishing $\Lambda$CDM from early dark energy or modified gravity models (e.g. Renk et al. 2017).

In this study, we will exploit galaxy samples from the newly released DESI Legacy imaging survey (Dey et al. 2019). This covers over 1/3 of the sky, with depth substantially greater than alternative large-area imaging such as SDSS or Pan-STARRS, and it is therefore invaluable for CMB cross-correlation studies. The photometric precision and wide wavelength coverage permits the construction of robust photometric redshifts, allowing us to perform cross-correlations between galaxy samples in multiple tomographic redshift bins and both the CMB lensing convergence map and the CMB temperature map. This provides more information by constraining the evolution of both the ISW and lensing signals, and in principle allows an empirical determination of the growth history of density fluctuations. Recent examples of this sort of work include Stöllner et al. (2018) for the ISW effect and Giaannantonio et al. (2016); Singh et al. (2017); Doux et al. (2018); Peacock & Bilicki (2018) for the ISW effect and Giannantonio et al. (2016) for the ISW effect. This can be alleviated by using the full nonlinear matter power spectrum in Eq. 7, while still assuming a linear coupling between density fluctuations. Recent examples of this sort of work include Cai et al. 2009; Smith et al. 2009; Cai et al. 2010; Carbone et al. 2018).

An initial motivation for the present work was suggestions that foreground perturbations of the temperature field have an amplitude that is substantially in excess of the standard $\Lambda$CDM ISW prediction (Granett et al. 2008; Kovács et al. 2019). These claims were based on stacking the signal from specially chosen `superstructures', but we shall not consider this detailed approach in the present paper. Rather, our aim here is to establish the Legacy Survey as a tool for two-point CMB tomography and to present the basic cross-correlation results. Much of our analysis is therefore devoted to the issue of photometric redshifts and their calibration. Having established to our satisfaction that this can be done, we use the galaxy autocorrelation results to eliminate galaxy bias, allowing the cross-correlation measurements to yield a direct probe of foreground mass fluctuations. In the simplest $\Lambda$CDM cosmology, this yields constraints on the $\Omega_m - \sigma_8$ plane, which turn out to be in some tension with the parameter values inferred from the primary CMB fluctuations.

This paper is structured as follows: Section 2 collects the necessary theory for predicting cross-correlation signals; Section 3 presents the Legacy Survey data, including the derivation of independent photometric redshift estimates; Section 4 presents the observed harmonic-space correlation results; Section 5 discusses the implications of our results; Section 6 sums up.

## 2 CROSS-CORRELATION THEORY

To measure the lensing and ISW signals associated with our galaxy sample, we will employ galaxy auto-clustering ($gg$) and the cross-correlations with CMB lensing ($gk$) and with CMB temperature ($gT$). The theoretical predictions for these quantities in the $\Lambda$CDM model are as follows; we work in spherical harmonic space and follow the notation in Peacock & Bilicki (2018). The galaxy harmonic auto-correlation in the Limber–Kaiser approximation (Limber 1953; Kaiser 1992) is given by

$$
\ell(\ell+1) \over 2\pi} C_{gg}^\ell = \frac{\pi}{\ell} \int \left\{ \frac{\partial^2 \Delta^2(k, \ell/z)}{\partial \ell^2} \right\} P_{\ell}^2(k, z) \frac{H(z)}{c} r \, dz,
$$

where $b$ is galaxy bias, $\Delta^2(k, z)$ is the dimensionless matter power spectrum at redshift $z$ ($\Delta^2(k, z) = k^3 P_{\delta\delta}(k, z)/2\pi^2$), and $P(z)$ is the redshift probability distribution function: $\int p(z) dz = 1$. We use Halofit (Smith et al. 2003; Takahashi et al. 2012) as implemented in CAMB (Lewis et al. 2000) to model the non-linear matter power spectrum. Note that the corresponding expression, $(7)$, in Peacock & Bilicki (2018) is misprinted and lacks the factor $(\ell(\ell+1)/2\pi)$. For the case of galaxy cross-correlations between different tomographic slices, $P^2(z) \rightarrow p_1(z)p_2(z)$ in Eq. 3, where $p_1(z)$ and $p_2(z)$ are the redshift probability distributions of the two slices. We would also in principle have different biases for the two slices, $b^2 \rightarrow b_1b_2$, although for tomographic slices with a single sample selection, the bias is purely a function of redshift.

Similarly, the theoretical galaxy-lensing convergence cross power spectrum is computed by

$$
\ell(\ell+1) \over 2\pi} C_{gT}^\ell = \frac{\pi}{\ell} \int b \Delta^2(k, \ell/z) p(z) K(r) d\ell \, dz,
$$

where the lensing kernel is given by

$$
K(r) = \frac{3H_0^2\Omega_m r (r_{LS} - r)}{2c^2 a},
$$

Finally, the galaxy ISW cross-correlation is given by

$$
\ell(\ell+1) \over 2\pi} C_{gT}^\ell = T_{CMB} \frac{2\pi}{c^2} \int b \Delta^2 \delta \phi(k, \ell/z) k p(z) d\ell \, dz.
$$

$\Delta^2 \delta \phi(k, z)$ is the dimensionless matter-$\Phi$ cross-power spectrum. In linear theory, it is given by

$$
\Delta^2 \delta \phi(k, z) = \frac{3H_0^2\Omega_m H(z) (1 - f_s(z))}{4k^2 a} \Delta^2(k, z),
$$

where $\Delta^2(k, z)$ is again the dimensionless matter power spectrum, $D(z)$ is the linear growth factor, $a$ is the scale factor, and $f_s = d\ln D/d\ln a = \Omega_m^{1/2}(z)$ is the growth rate (e.g. Crittenden & Turok 1996; Afshordi 2004; Ho et al. 2008; Giaannantonio et al. 2008; Planck Collaboration et al. 2016b). N-body simulations have suggested that small deviations from linear theory for $C_{gT}^\ell$ occur at $\ell \geq 50$, and Eq. 6 becomes inaccurate (Seljak 1996; Cooray 2002; Cai et al. 2009; Smith et al. 2009; Cai et al. 2010; Carbone et al. 2016). This can be alleviated by using the full nonlinear matter power spectrum in Eq. 7, while still assuming a linear coupling between the density and velocity fields (Cai et al. 2010). Thus Halofit is used to model $\Delta^2(k, z)$ in Eq. 7.

The above expressions for angular power spectra assume spatial flatness. The Limber-Kaiser approximation is inaccurate for $\ell < 10$; but in practice we exclude those large-scale modes from our fitting, to allow for possible complications from combining several surveys in the sky (see section 3). Because of cosmic variance, those very large-scale perturbations contain little statistical power. Note also that in principle the bias parameter may depend on scale, although it should tend to a constant in the linear limit as $k \rightarrow 0$; in practice we do allow for this scale dependence (see Section 4.1).

In summary, Eqs. 3, 4 & 6 are the theoretical predictions to be compared with our measurements from observations. The combinations of them should in principle allow us to determine both...
cosmological parameters and nuisance parameters such as galaxy bias and uncertainties in the true redshift distribution of the galaxy samples. Most directly, we can determine the amplitudes of the CMB lensing and ISW signals associated with the late-time LSS galaxies, relative to the prediction of a fiducial cosmological model. We take this to be the Planck 2018 cosmology, with \( \sigma_8 = 0.965 \), \( \Omega_m = 0.315 \), \( \Omega_b = 0.0493 \), and \( H_0 = 67.4 \) (Planck Collaboration et al. 2018b). The cross-correlation measurements are made using the CMB temperature and lensing \( \kappa \) maps and masks from the 2018 Planck data (Planck Collaboration et al. 2018a,c), together with maps of galaxy number densities from the DESI Legacy Survey. We detail our galaxy sample in the next section.

3 LEGACY SURVEY DATA

3.1 Selection

The Legacy Imaging Survey (Dey et al. 2019) is a combination of four different projects observed using three different instruments on three different telescopes: the Dark Energy Camera Legacy Survey (DECaLS) observed using the Dark Energy Camera (Flaugher et al. 2015) including data from DES (The Dark Energy Survey Collaboration 2005), the Mayall \( z \)-band legacy Survey (MzLS) observed by the Mosaic3 camera (Dey et al. 2010) and the Beijing-Arizona Sky Survey (BASS) observed by the 90Prime camera (Williams et al. 2004). Altogether covering an area of 17,739 deg\(^2\), the survey is divided around Dec = 33\(^\circ\) in J2000 coordinates, with the southern part included in DECaLS, and the northern part covered by BASS and MzLS. We use the publicly available Data Release 8\(^1\). The sources are processed and extracted using Tractor\(^2\) (Lang et al. 2016), with three optical bands, (\( g, r, z \)), and three WISE (Wright et al. 2010) fluxes (\( W_1, W_2, W_3 \)) available. Because of the shallower effective depth of the \( W_2 \) and \( W_3 \) bands, we only make use of \( W_1 \) (3.4 \( \mu \)m). We apply the following selections to the data:

(i) PSF type objects are excluded. This step eliminates most stars and quasars.

(ii) \( \text{FLUX}_G/R/Z/W1 > 0 \), i.e. fluxes for all four bands are detected. This is to ensure successful determination of photometric redshifts.

(iii) \( \text{MW_TRANSMISSION}_G/R/Z/W1 \) are applied to the fluxes for galactic extinction correction.

(iv) Magnitude cuts are applied with \( g < 24, r < 22, \) and \( W_1 < 19.5 \), where all magnitudes are computed by \( m = 22.5 - 2.5 \log_{10} (\text{flux}) \). The cuts in \( g \) and \( r \) are chosen as reasonable completeness limits from inspection of the number counts. The cut in \( W_1 \) further removes faint objects that are not well covered by the calibration sample. We experimented with imposing a brighter cut, and found that our main results were essentially unchanged if all limits were made 0.5 mag. brighter.

In addition, Bitmasks\(^3\) are used to generate a survey completeness map, with bits = (0, 1, 5, 6, 7, 11, 12, 13) masked. These masks cover foreground contamination at pixel level, including bright stars, globular clusters, and incompleteness in the optical bands. To convert the mask to appropriate resolution for this work, we generate large number of randoms and bin them into a healpix map (Górski et al. 2005) with \( N_{\text{side}} = 128 \), corresponding to a pixel area of \( 0.2 \) deg\(^2\). The completeness map is obtained by the ratio of the number of randoms in each healpix pixel with and without masking. The map is then upgraded to \( N_{\text{side}} = 1024 \) which is the resolution used for most of our analyses.

3.2 Photometric redshifts

One of the key pieces of information needed for interpreting observations of CMB-galaxy cross-correlations is the redshift distribution of the galaxy sample. A variety of methods have been developed over many years to estimate either the redshifts of individual galaxies or the redshift distribution of a galaxy sample using broad band photometry (see Schmidt et al. 2020 for a review). Generally photometric estimates are either template based (e.g. LePHARE Arnouts et al. 1999; BPZ Benítez 2000; EAZY Brammer et al. 2008) or data-driven methods (e.g. TFZ Carrasco Kind & Brunner 2013; SkyNet Graff et al. 2014; GPZ Almosallam et al. 2016; ANNz2 Sadeh et al. 2016; META-PHOT Cavuoti et al. 2017; Duarré Leistedt & Hogg 2017; CMN Graham et al. 2018; CHIPPR Malz & Hogg 2020). There have been several attempts to compare the accuracy and precision of various photometric redshift methods (Hildebrandt et al. 2010; Rau et al. 2015; Sánchez et al. 2014; Bonnett et al. 2016) with no strong winner. Our approach is direct and empirical, based on using observed spectroscopy to assign a redshift to a given location in multi-colour space. In parallel with this work, a public catalogue of photometric redshifts for the Legacy Survey was made available by Zhou et al. (2020); 220 hereafter. Their approach differs somewhat from ours, being based on machine learning. The advantage of this is that we are able to look in detail at the sensitivity of our results to the properties of the photometric redshifts.

The following spectroscopic surveys are used for redshift calibration: GAMA (DR2: Liske et al. 2015), BOSS (DR12: Alam et al. 2015), eBOSS (DR16: Ahumada et al. 2020), VIPERS (DR2: Scodellio et al. 2018), and DEEP2 (Newman et al. 2013). In addition, we also include COSMOS (Ilbert et al. 2009) and DESY1A1 redMaGiC (Cawthon et al. 2018) for their highly accurate photometric redshifts. The redshifts from the original calibration samples will be referred to as ‘spectroscopic’ or ‘true’ from this point onward, in order to make a distinction with the inferred photometric redshifts. The majority of these datasets overlap with DECaLS, and galaxies in the calibration data sets are matched with DECaLS objects based on their nearest neighbours using cKDTree within a distance of 0.5\(^\circ\). The GAMA sample has been rejection sampled to remove the dip in density around \( z = 0.23 \); this is known to represent a rare LSS fluctuation, which we do not wish to imprint on our photometric redshifts. For the COSMOS sample, only objects with \( r - z < 2.3 \) are used to match with the DECaLS sample. The whole calibration sample roughly covers the redshift range \( 0 < z < 1 \).

All calibration samples except DESY1A1 redMaGiC (Cawthon et al. 2018) are binned in 3-dimensional grids of \( g-r, r-z, \) and \( z - W_1 \) with a pixel width of about 0.03. The range of the colours are: \(-0.5 < g - r < 2.5, -2 < r - z < 3, \) and \(-2 < z - W_1 < 4 \). Pixels containing more than 5 objects from the calibration samples are assigned the mean redshift of these objects. The DES sample is processed in the same way to fill out pixels that are not calibrated in this initial pass. We then apply this calibration to the full Legacy Survey: objects that fall in pixels that lack a redshift calibration are excluded, thus selecting objects that occupy the same colour space as our calibration sample. The assigned photometric redshift is the mean redshift for the colour pixel, plus a random top-hat dither of \( \pm 0.005 \) so that digitization artefacts are not apparent in the \( N(z) \) dist-

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1. http://legacysurvey.org/dr8/
2. https://github.com/dstndstn/tractor
3. http://legacysurvey.org/dr8/bitmasks

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MNRAS 0000, 1–17 (2020)
Photometric redshifts are assigned to 78.6% of the selected Legacy Survey objects, yielding a primary sample of approximately 49 million galaxies (see Table 1 for details). The lost 21.4% objects will lead to higher shot noise, but this is a small price to pay for excluding objects where the photometric redshift cannot be reliably calibrated. The redshift distribution of our final sample is shown in Fig. 2. We can compare this distribution with the corresponding $N(z)$ for the public Legacy Survey photometric redshifts made available by Zhou et al. (2020); this is shown in Fig. 2. The two distributions are generally in good consistency with each other; both distributions show some weak features, indicating that LSS in the calibrating samples has still propagated into the final photo-zs to some extent. With broad tomographic bins, we expect that such structure will be unimportant, but it will be helpful to compare the results from two rather different photo-z catalogues. We divide our samples into four tomographic slices, illustrated by the grey dotted lines in Fig. 2. The redshift ranges are: bin 0: 0 < $z$ < 0.3; bin 1: 0.3 < $z$ < 0.45; bin 2: 0.45 < $z$ < 0.6; bin 3: 0.6 < $z$ < 0.8. Our photo-z data and accompanying software will be made public after this paper has been accepted.

3.2.1 Photometric redshift error distribution

For the calibration sample, the distribution of $\delta z = z_{\text{spec}} - z_{\text{phot}}$ as a function of $z_{\text{phot}}$, can be well modelled by the modified Lorentzian function,

$$L(x) = \frac{N}{(1 + ((x-x_0)/\sigma)^2/2a)^\alpha},$$

where $x_0$, $\sigma$, and $a$ are parameters that control the mean, width, and fall-off of the distribution, and $N$ is the normalization such that $\int_{-\infty}^{+\infty} L(x) \, dx = 1$. For each of the tomographic bins, we fit $\sigma$ and $a$, while $x_0$ is fixed to zero. These parameters are summarized in Table 1. The inferred true redshift distribution $p(z)$ is then estimated by convolution of the raw distribution with the Lorentzian function. Schaan et al. (2020) have recently proposed a similar approach to marginalizing over photo-z errors while restricting themselves to the case of Gaussian fields with an ad-hoc mixing matrix.

However, galaxies fainter than the calibration sample may not

![Figure 1. Photometric redshifts inferred from $g - r$, $r - z$, and $z - W_1$ colours, versus the spectroscopic redshifts for the calibration samples. The contour shows the 95% interval. The colour bar indicates the number of galaxies in each pixel.](image1)

![Figure 2. Photometric redshift distribution of galaxies after selection, in the DECALS (yellow) and BASS-MzLS (green) regions respectively. We compare our photometric redshifts with the corresponding redshifts from Zhou et al. (2020). Grey dotted lines show our four tomographic redshift bins in 0 < $z$ ≤ 0.8.](image2)

![Figure 3. Redshift distribution function, normalized such that for each redshift bin $\int p(z) \, dz = 1$. The dotted lines show the raw photometric redshift distribution with |$\Delta z$| < 0.05, the solid lines show the mean distribution (see text for details) and their 1-$\sigma$ deviation using the 2-bias model, and the dashed lines show the distribution using parameters from spectroscopic calibration sample.](image3)
Table 1. Summary of the four tomographic redshift slices. The first row shows the number of galaxies in each redshift slice. The second row shows the effective volume of the redshift slice. The third and forth rows are parameters for the Lorentzian function (Eq. 8) fitted to redshift errors in each redshift bin derived from the calibration data sets; and the last two rows show the best-fit parameters derived empirically from the cross-correlations between the different tomographic bins (noting that \( \sigma \) is not varied in this exercise). The best-fit parameters refer to our photo-z data clipped with \(|\Delta z| < 0.05\).

| Redshift bin | 0: \(0 < z \leq 0.3\) | 1: \(0.3 < z \leq 0.45\) | 2: \(0.45 < z \leq 0.6\) | 3: \(0.6 < z \leq 0.8\) |
|--------------|------------------|------------------|------------------|------------------|
| Number of galaxies | 14 363 105 | 11 554 242 | 13 468 310 | 7 232 579 |
| Volume \([\text{Mpc}^3]\) | 1.047 | 2.084 | 3.431 | 20.37 |
| \(\sigma^\text{spec}\) | 0.0122 | 0.0151 | 0.0155 | 0.0265 |
| \(\Delta^\text{spec}\) | 1.257 | 1.319 | 1.476 | 2.028 |
| \(\delta^\text{spec}\) | 1.257 | 1.104 | 1.476 | 2.019 |
| \(X^2_\text{hi}\) | -0.0010 | 0.0076 | -0.0024 | -0.0042 |

Follow this \(\delta z\) distribution exactly. There is an irreducible scatter that arises because galaxy spectra are not universal in shape; but photometric measuring errors will increase the scatter for fainter objects. As shown below in Section 4.1, we are able to diagnose this using the galaxy cross-correlations between the different tomographic redshift slices. The width of the error distribution controls the degree of cross-correlation between the different tomographic slices, which is observed to be larger than predicted when using the directly calibrated \(p(z)\) parameters from Table 1. The largest discrepancy occurs in the cross-correlation between redshift bin 1 and bin 2, which is almost double the predicted value. We therefore model the true error distribution in the photometric redshifts by allowing the tail \(\sigma\) to that determined by the spectroscopic sample. We also allow a change in the mean \(\Delta z\) of each bin, while requiring the sum of the mean shifts in the four bins to be zero. This results in 7 systematic nuisance parameters to marginalize over. We take 10 samples in each dimension of the 7-parameter space with appropriate upper and lower bounds, and for each point in the grid, we compute the \(\chi^2\) of the 10 galaxy auto- and cross-correlation between different redshift slices. The galaxy bias parameters in each case are fixed at the lowest-\(\chi^2\) values from the auto-correlation (which we fit using the 2-bias model up to \(\ell = 500\)). This is sufficient given the size of the error bar in the auto-correlations; the galaxy bias is very tightly constrained. Constraints on the cross-correlations amplitudes can then be marginalized over the photo-z parameters, i.e., weighted by the likelihoods of each set of parameters. The mean and 1-\(\sigma\) deviation of \(p(z)\) weighted by the likelihoods of the \(p(z)\) parameters are shown in Fig. 3. The procedure is detailed in Section 4.1.

### 3.3 Comparison with Zhou et al. (Z20)

It is interesting to compare our redshift estimates with those of Z20: Zhou et al. (2020). This is studied in some detail in Appendix A, but we summarise the main features here. Firstly note that this comparison is only possible for the 78.6% of galaxies that lie in regions of multicolour space for which calibration data exist. Z20 give photometric redshifts for additional galaxies, and these are probably to be considered less reliable. Nevertheless, we can perform clustering analyses that use all the Z20 data, or just their redshifts for the same set of objects that we use, and this can give useful insight into the robustness of our conclusions. For the objects in common, the median redshift difference is \(|\Delta z| = 0.023\), and 68% of objects agree in photometric redshift to within 0.038. The difference distribution has non-Gaussian tails, and we also therefore consider a ‘clipped’ selection where we retain only objects where the two estimates agree to within \(|\Delta z| < 0.05\); this is about twice as large as our photo-z 1-\(\sigma\) uncertainty, so the effect is to remove outlying objects in the tails of the error distribution. This removes a further 23.4% of the sample, but should provide a cleaner selection in the sense that object are more likely to lie in their nominal tomographic bin. The cross-correlations between the different bins confirm that this strategy is successful.

### 3.4 Galaxy maps and systematic corrections

Galaxies in each tomographic slice are binned in healpix maps with \(N_{\text{side}} = 1024\). The density fluctuation, \(\delta\), in each pixel is then computed by

\[
\delta = \frac{n}{\bar{n}} - 1,
\]

where \(n\) is number of galaxies in the pixel, and \(\bar{n}\) is the mean number of galaxies per pixel. Due to the slight differences in the photometric passbands for DECam, BASS, and MzLS, the surface density of the tomographic slices varies slightly, between \(2\%\) and \(5\%\), in the north and south regions. For our purpose here, we compute \(\delta\) for the north and south regions separately, and join the two regions at Dec = 33\(^\circ\).

The density maps are correlated with various systematics, including observational conditions, survey depth, stellar density, and galactic extinction. Most foreground contamination is captured by the completeness map. In addition, we use the ALLWISE total density map as a proxy for stellar density. We find little correlation with the \(E(B-V)\) extinction map. The following corrections are applied to the density map to remove possible systematics.

To obtain an unbiased mean density, we compute \(\bar{n}\) using pixels with completeness \(> 0.95\) and stellar number \(\log_{10}(N_{\text{star}}) < 2.6\), about 50% of the total unmasked pixels. The largest correlation with density comes from the completeness map. The galaxy count in each pixel is corrected by \(n/w\), where \(w\) is the completeness in each pixel. Regions with \(w < 0.86\) are masked, based on the binned one-dimensional relation between the completeness and mean density fluctuation in the bin, \(\delta\), such that the deviation of \(\delta\) from zero is smaller than 0.1. We also introduce a similar cut in stellar number at \(\log_{10}(N_{\text{star}}) < 2.83\). The residual binned one-dimensional correlation between \(\log_{10}(N_{\text{star}})\) and mean \(\delta\) in the bins is below 5% for all bins except for the highest redshift bin at the large stellar density end. We use 5th-order polynomials to fit for the residual correlation for each bin as a function of \(\log_{10}(N_{\text{star}})\) and subtract the residual mean density from the raw \(\delta\). The final corrected density maps are cross-correlated with the completeness map and stellar density map in each bin. The resultant correlation is consistent with zero for the \(\ell\) range used in the analysis. The corrected density fluctuations in the four redshift slices are shown in Fig. 4. For illustrative purpose, they are smoothed by a Gaussian symmetric beam with \(\sigma = 20\ h^{-1}\) Mpc.
Figure 4. The density fluctuation maps for the four tomographic slices. For illustrative purpose only, they are smoothed by a Gaussian symmetric beam with comoving scale of $20 h^{-1} \text{Mpc}$. These maps are made from galaxy maps via Eqn. 9, and corrected by completeness and stellar density.

in comoving distance. We note that the photometric variations and correlations with various foreground maps for our sample are relatively small. This is driven by the magnitude cuts used in our selection (see section 3.1). Kitanidis et al. (2020) provides a more detailed analysis of photometric systematics for a variety of galaxy samples.

4 RESULTS

We now compute the angular correlations between our various galaxy number density maps, and with the CMB $\kappa$ and temperature maps. We will be especially interested in comparing the amplitudes of the CMB lensing and ISW signals with the predictions of the fiducial $\Lambda$CDM model, from Eqs 4 & 6. The procedure can be summarised as follows:

- Constrain linear galaxy biases with the galaxy auto- and cross-correlations from the four redshift bins:

$$C_{\ell}^{gg} = b_i b_j C_{\ell}^{\delta \delta}.$$  

(10)

Here, we allow the pdf of photo-zs to vary with nuisance parameters that will be marginalised over.

- Measure the amplitude of the lensing and ISW signals $A_k$ and $A_{ISW}$ defined as

$$C_{\ell}^{\kappa \kappa} = A_k b C_{\ell}^{\delta \delta}; \quad C_{\ell}^{\kappa T} = A_{ISW} b C_{\ell}^{\delta T},$$  

(11)

incorporating the constrained galaxy biases from the previous step.

The angular power $C_{\ell}$ is computed by converting a pixel map into its spherical harmonics $a_{\ell m}$ in healpy. For a masked map, we use the simplest pseudo-power estimate $\hat{C}_{\ell} = C_{\ell}^{\text{masked}}/f_{\text{sky}}$. We have verified that inaccuracies in this estimate are unimportant for this large sky coverage, especially given that we exclude $\ell < 10$ as further insurance against any residual large-scale systematics. We also impose an upper cutoff: throughout the analysis, we use modes in the range $10 \leq \ell < 500$. The $\ell > 500$ modes give very noisy measurements for cross-correlations between LSS and CMB, and the $S/N$ for the amplitude of the cross-correlation signal has converged by this point. Linear bias is no longer a valid assumption beyond about $\ell = 250$, and we make allowance for scale-dependent bias as described below. We use a healpix resolution of $N_{\text{side}} = 1024$ for our analysis, and have tested that using finer maps would not alter the results. We correct for the pixel window function, although this is not a significant effect.

In the following analysis, we group every $M = 10 \ell$-modes together such that

$$\langle C_{\ell}\rangle_{\text{group}} = \frac{1}{M} \sum_{\ell'}^{\ell+M-1} C_{\ell'}, \quad \ell' = M, 2M, \ldots,$$  

(12)

and $\ell$ is the median value in each case. A simple error bar on each grouped data point can then be computed by

$$\sigma_{\ell} = \frac{1}{f_{\text{sky}}} \sqrt{\frac{(C_{\ell}^2) - (\langle C_{\ell}\rangle^2)}{M-1}}.$$  

(13)

The $f_{\text{sky}}$ factor takes care of correlations between $\ell$-modes due to the masked sky. We have verified using simulated lognormal density maps that this simple approximation indeed gives unbiased error estimates, leading to a diagonal covariance, $C = \text{diag}(\sigma_{\ell}^2)$. The $\chi^2$ of a theoretical model is defined as

$$\chi^2 = \mathbf{d}^T C^{-1} \mathbf{d},$$  

(14)

where the vector $\mathbf{d}$ has components $d_{\ell} = C_{\ell}^{\text{data}} - C_{\ell}^{\text{th}}$. The likelihood
Figure 5. The galaxy auto-correlation $C_{gg}^{S\delta}$ for each redshift slice (diagonal) and cross-correlation coefficients between different slices (off-diagonal). The last column shows the auto- and cross-correlations with the unbinned case, with shot noise subtracted. Data is presented in groups of 10 modes. The black solid line shows the theory with the best-fit $n(z)$ and redshift-dependent bias. The last column is not used in fitting $n(z)$. The $\chi^2$ is for modes in $10 < \ell < 500$.

of a model parameter set $\mathbf{x}$ is given by

$$L(\mathbf{x}) = \frac{e^{-\chi^2(\mathbf{x})/2}}{\int e^{-\chi^2(\mathbf{z})/2} d\mathbf{z}},$$  \hspace{1cm} (15)$$

where as usual we will take the likelihood to give the posterior on the parameters, assuming uninformative uniform priors.

The theory vector $C^{th}_\ell$ contains the predictions from Eqs 3, 4 & 6 and we convert them to equivalent band power before comparing with data. It has the following free parameters: $\theta = \{A_k, A_{ISW}, d', x_i'\}$. $d'$ and $x_i'$ are nuisance parameters to account for uncertainties in our photo-$z$ calibration. We impose $\sum_i x_i' = 0$, where the indices of the redshift bins are $i = 0, 1, 2, 3$, and so there are 7 degrees of freedom for the nuisance parameters. $A_k$ and $A_{ISW}$ are the key parameters of interest, which characterise the amplitudes of the lensing and ISW signals relative to the fiducial model, as discussed above. All other cosmological parameters are fixed to the Planck cosmology. Galaxy bias is a further nuisance parameter, but this will be constrained from data.

4.1 Galaxy auto- and cross-correlations

We now present the auto- and cross-correlations from the different tomographic bins. We will use the results to constrain galaxy bias and also to determine the empirical form of the photo-$z$ error distribution. The galaxy auto-power requires shot noise to be subtracted. Given $N_g$ galaxies in a redshift slice, the shot noise spectrum is given by $C_{\ell}^{\text{shot}} = 4\pi f_{sky} / N_g$. There is no correction to be made to the cross-power between the different bins. However, we also consider the cross-correlation between our data and that of Z20 and the computation of shot noise is more complicated in that case, since it
depends on the numbers of galaxies that are in common to the two catalogues (which is non-zero even for cross-correlation).

Data points with error bars in Fig. 5 show the 10 measured galaxy auto- and cross-correlations for our data. The off-diagonals show the cross-correlation coefficients, as defined as

$$r_{ab} = \frac{C_{\ell}^{ab}}{\sqrt{C_{\ell}^{aa} C_{\ell}^{bb}}}$$

where \( a, b \) refers to different redshift slices. These are independent of galaxy bias.

In this procedure of finding photometric redshift errors, we use only the large-scale modes with \( f_{\text{max}} = 500 \) as discussed above. The cross-correlation coefficients are flat over a large range of \( \ell \), and is only dependent on the redshift distribution. Specifically, using constraints from the 10 auto- and cross-correlations of galaxy redshift bins, we compute \( \chi^2 \)'s in the 7D nuisance parameter space \([a^i x^i]\) for \( p(z) \). The fitting also excludes \( \ell < 10 \) modes. We use the 2-bias model, detailed in the next section, to find the best-fit \( p(z) \). In the marginalized case, to speed up the computation, we approximate the best-fit biases by taking the ratio of the data with the linear and non-linear theory at different scales using

$$b^2 = \sum_{\ell} w(\ell) \frac{C_{\ell}^{\text{data}}}{C_{\ell}^{\text{th}}}, \quad w(\ell) = \frac{1/\sigma^2_{\ell}}{\sum_{\ell} (1/\sigma^2_{\ell})}.$$  

The transition scales are different for each redshift slice. For bias fitting, a good approximation is the scale at which the fraction of the nonlinear power becomes comparable to the measurement error. This ranges between \( \ell \sim 100 \) – 200 from low to high redshift slices. The drawback of this approximation is that the intermediate scales are hard to control, but it gives biases close to the lowest \( \chi^2 \) value. In this case, the best-fit \( n(z) \) gives \( \chi^2 = 471 \) with DOF = 483. The model parameters are shown in Table 1. The best-fit spectra are shown as black solid lines in Fig. 5, with the galaxy biases and break-down of \( \chi^2 \) printed for each case. The measured galaxy biases and their errors for each redshift slice are shown in Table 3. We have checked that with \( f_{\text{max}} = 500 \), the best-fit \( p(z) \) model and the marginalized case give almost identical amplitude constraints on the cross-correlation of CMB lensing and ISW effects. Therefore, in the following analysis, we will carry out the modelling using the best-fit \( p(z) \).

Finally, we note that the use of cross-correlations in calibrating \( p(z) \) is potentially problematic because of lensing. Even with perfect redshift selection, some cross-correlation is expected between different tomographic slices because of magnification bias; lensing by the nearer slice will imprint an image of its density fluctuations on the more distant slice. Indeed, Krolikowski et al. (2020) argue strongly that magnification bias should be allowed for in CMB lensing tomography. However, we can see that such effects are unimportant here, as they should be largest for widely separated bins, and where the bin has the largest count slope. This should affect above all bin 3, with the highest mean redshift and the highest count slope (the slopes in slices 0–3 are respectively \( z = d \log (N / dm) = 0.19, 0.29, 0.41, 0.57 \). But we see from Fig. 5 that bin 3 has no significant correlation with bins 0 and 1. The reason for our different conclusion regarding magnification bias is that our photo-zs are calibrated using the colours of spectroscopic objects, whereas Krolikowski et al. (2020) calibrated their photo-zs using the cross-clustering with a spectroscopic sample. Magnification bias can affect that cross-correlation and hence the inferred \( p(z) \), but it has no effect on the numbers of objects at a given colour.

![Figure 6. Linear and non-linear bias parameters, \( b_1 \) and \( b_2 \) (Eq. 18), as a function of mean redshift. The circles show minimum-\( \chi^2 \) bias measured in 8 sub-bins, the stars show that measured in 4 bins, and the solid lines show quadratic fits to the circles.](image-url)

| Bin    | 0     | 1     | 2     | 3     | Unbinned |
|--------|-------|-------|-------|-------|----------|
| \( z^{\text{eff}} \) | 0.21  | 0.39  | 0.52  | 0.66  | 0.42     |
| \( \delta b_1 \) | -0.010 | 0.098 | -0.033 | 0.029 | -0.005   |
| \( \delta b_2 \) | -0.022 | 0.159 | -0.056 | -0.056 | 0.027    |

4.1.1 Non-linear bias and bias evolution

The galaxy auto-power data beyond \( z = 250 \) cannot be fit well by a constant bias. Specifically, the ratio between \( C_{\ell}^{\text{data}} \) and \( C_{\ell}^{\text{DM}} \) are roughly constant at small and large \( \ell \), with a transition at intermediate scales corresponding to roughly the transition between linear and non-linear scales. We allow for this by introducing two bias parameters for the linear and non-linear regimes separately:

$$C_{\ell}^{\text{g}} = b_1^2 C_{\ell}^{\text{lin}} + b_2^2 C_{\ell}^{\text{nl}},$$

where \( C_{\ell}^{\text{lin}} \) and \( C_{\ell}^{\text{nl}} \) are computed using the linear and additional non-linear components of the CAMB power spectrum. This simple model gives an excellent fit up to \( \ell = 1000 \). The best-fit linear and non-linear biases using the best-fit \( n(z) \) are shown in Table 2. We note that \( b_2 \) is systematically larger than \( b_1 \), obeying the approximate relation \( b_2 - 1 \approx 1.9(b_1 - 1) \).

The linear and non-linear biases evolve with redshift, with \( b_1 \) increasing from 1.2 to 2.0 over redshift 0.2 to 0.7. In general, such evolution can be locally treated as a constant if the redshift bin is thin, or if the distribution is symmetric. However, for bin 3, which has a tail towards higher redshifts, and for an analysis of the unbinned sample, such an approximation breaks down, and the full bias evolution needs to be included in the kernel. To determine the bias evolution more precisely, we sub-divided each bin into two bins. We approximate the redshift distribution of each sub-bin by convolution of the raw \( n(z) \) with the best-fit photo-z error of that bin. Then for each sub-bin we fit linear and non-linear biases as above. These measurements are consistent with the 4-bin case. The biases as a function of the mean redshift in that bin can be fitted by a quadratic function (see Fig. 6). We only use the increasing part of the quadratic, and extrapolate the decreasing part beyond the function’s minimal point by a constant.

To match the auto-correlation amplitude, for each bin, we in-
produce a small correction \( b_i(z) = (1 + \delta b_i) b_i^0(z) \), where \( i = 1, 2 \), \( b_i^0(z) \) is the fitted quadratic curve, and \( \delta b_i \ll 1 \). We find \( \delta b_i \) by iteration. This is also shown in Table 2.

This model agrees with our measurements very well in general, as seen in Fig. 5, with reasonable \( \chi^2/\text{DOF} \) overall and for most individual spectra. The auto-power for bin 1 has \( \chi^2 \) on the high side, but we were unable to identify any systematics that could account for this (e.g. looking for discrepant sky sub-areas in the data). In any case, the look-elsewhere effect is clearly relevant here, with 10 spectra to consider. It is worth noting that \( \chi^2 \) is nominal for bin 3, even though this has the largest volume and the lowest errors. Indeed, the precision of this bin and bin 2 is sufficient to show a clear signal from Baryon Acoustic Oscillations (BAO).

The measured linear galaxy bias tends to increase with redshift from 1.3 to 1.9, although the trend is not quite monotonic (see Fig. 6). This is consistent with the expectation for luminosity-limited galaxy samples in which high-z galaxies are intrinsically brighter, thus those galaxies tend to occupy more massive dark matter haloes. Overall, then, these cross-correlation results reassure us that the clustering of the galaxy samples and the calibration of the underlying \( N(z) \) distributions is robust, and that the samples are ready for the cross-correlation analysis with the CMB.

### 4.2 Galaxy-lensing cross-correlations

In computing the galaxy-lensing cross-power signal, we encountered unexpected practical issues. The Planck CMB lensing data are made available as spherical harmonic coefficients, from which the required \( \kappa \) map can be obtained by using the \texttt{alm2map} routine within the \texttt{healpy} package. The maximum wavenumber is 2048 in the 2015 release and 4096 in the 2018 release. The 2015 map displays a nearly divergent spike at high \( \ell \). \( C_\ell^{\kappa} \) increases from about \( 10^{-1} \) at \( \ell = 3650 \) to zero unity at \( \ell = 4096 \). This creates numerical problems in reconstructing the map, so that e.g. making a map at \( N_{\text{side}} = 512 \) directly yields a different answer to creating a map at 2048 and downgrading to 512. The spike at \( \ell = 2048 \) can be tamed by filtering the map, but a sufficiently large FWHM is required that modes at \( \ell < 100 \) would be affected. In practice, therefore, we chose to truncate the data at \( \ell = 2048 \), consistent with the 2015 data. With the adoption of a standard resolution of \( N_{\text{side}} = 1024 \) for our analysis, the results were robust (and only slightly different from \( N_{\text{side}} = 512 \)).

A further issue concerned coordinate systems: the CMB maps are supplied in galactic coordinates, whereas we constructed our galaxy maps in equatorial coordinates. Facilities exist within \texttt{healpy} for performing the rotation in \( \ell \)-space, but we found that the rotation generated artefacts in the lensing auto-power \( C_\ell^{\kappa} \), which we attribute to the extremely noisy nature of the lensing map, dominated by fluctuations on the inter-pixel scale. After tests at a range of resolutions, we are confident that this issue does not affect the regime of our measurements, out to \( \ell = 500 \).

Fig. 7 shows the measured galaxy-\( \kappa \) cross-power, with the solid black lines showing the theory using the best-fit \( p(z) \) and biases obtained from the galaxy auto- and cross-correlations. The black lines are not fits to the data points. To quantify the consistency between data and theory, we include a scaling factor for the lensing amplitude, \( A_\kappa \), such that \( C_\ell^{\text{th}} = A_\kappa b C_\ell^{DM} \). The constraints on \( A_\kappa \) as a function of maximum \( \ell \)-mode is shown in Fig. 8. The coloured points show measurements from individual tomographic slices, the black open circle shows the product of the four likelihoods, and the black solid points show that from the unbinned case. The mean and 1-\( \sigma \) deviation for each of these likelihoods are presented in Table 3.

A tendency for the CMB lensing signal to lie below the fiducial model is seen consistently in all tomographic bins. It is also a robust feature, which does not alter with different treatments of the photometric redshifts. We summarise the results of a number of options that we considered in Fig. 11. We can consider our
photometric redshifts or those of Z20; we can further restrict the Z20 sample to objects in the calibratable region of multicolour space; we can clip the photo-z catalogues to remove objects where the estimates are discrepant (we choose a threshold of $|\Delta z| = 0.05$); we can adjust one of the photo-z catalogues to remove any offset in $\langle \Delta z \rangle$ as a function of redshift; we can remove objects that are placed in different tomographic bins by the two catalogues. All of these options potentially alter the error distribution and hence the true $n(z)$ of the selection. The nuisance parameters governing these distributions were therefore re-optimised using the galaxy cross-correlations in each case. The impact of some of these different choices is shown in Fig. 11.

Our conclusion is that all of these options consistently yield $A_k$ close to 0.9, and that the deviation from the fiducial Planck prediction is real. In order to report an overall amplitude for $A_k$, we need to combine the different redshift slices, which we do in the simple approximation that the slices are independent. Because this is not exact, we also consider an unbinned analysis in which all objects at $z < 0.8$ are combined; this gives closely consistent results to the outcome of averaging the four slices. We adopt the mean of the unbinned measurements using the two sets of photo-zs as our final result:

$$A_k = 0.901 \pm 0.026.$$  

This significant discrepancy with the fiducial model is one of the principal results of this paper, and we consider its implications below in Section 5.

### 4.3 Galaxy-temperature cross-correlations

Fig. 9 shows the measurements of galaxy-temperature cross-correlations. The signal is dominated by noise at $\ell > 50$. The black solid line shows the theory prediction using the best-fit $p(z)$ and bias from galaxy-auto correlations. As with the lensing case, we introduce an ISW amplitude $A_{ISW}$ in order to compare theory and data, such that $C_\ell^{GG} = A_{ISW} b_{CDM}$. The likelihood for $A_{ISW}$ is then computed for each set of $p(z)$, then marginalized over. The marginalized likelihood for $A_{ISW}$ is almost identical with that of the best-fit model, as shown in Table 3. Fig. 10 shows the likelihoods of $A_{ISW}$ for each redshift slice (coloured) and combined (black) in the marginalized (solid line), mean parameter (circles), and best-fit (dotted line) model cases. The mean and width of individual curve are presented in Table 3. The combined likelihood shows a clear detection of the ISW signal, with $A_{ISW} = 0.984 \pm 0.349$, excluding zero at 2.8$\sigma$.

### 5 IMPLICATIONS FOR THE COSMOLOGICAL MODEL

#### 5.1 Implications of low $A_k$

We first consider the simplest interpretation of our low $A_k$ amplitude for the galaxy-CMB lensing cross correlation in terms of parameters within the CDM model. The lensing signal at low $z$ has a direct linear dependence on the matter density fluctuation, which is proportional to the mean density times the relative fluctuation – i.e. to $\Omega_m r_s$. At non-zero redshifts, the dependence on $\Omega_m$ becomes nonlinear as this parameter influences distances and evolution of density fluctuations. For our range of redshifts, the em-
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Table 3. The linear and non-linear bias and constraints on $A_k$ and $A_{ISW}$ for various cases at $\ell_{\text{max}} = 500$. The first row shows the case where all $p(z)$ parameters are marginalized over. The second row shows the case for best-fit $p(z)$ parameters. The third and fourth rows show the cases using the photo-$z$ from Zhou et al. (2020) (Z20) and that with the applied offset. The last row shows the case of using the AvERA model described in Beck et al. (2018).

| Parameters | bin0 | bin1 | bin2 | bin3 | combined | Un-binned |
|------------|-----|-----|-----|-----|--------|--------|
| Redshift   | $0 < z \leq 0.3$ | $0.3 < z \leq 0.45$ | $0.45 < z \leq 0.6$ | $0.6 < z \leq 0.8$ | - | $0 < z \leq 0.8$ |
| Marginalized over $p(z)$ | | | | | | |
| $b_1$      | $1.25 \pm 0.01$ | $1.53 \pm 0.02$ | $1.54 \pm 0.01$ | $1.86 \pm 0.02$ | - | - |
| $b_2$      | $1.27 \pm 0.01$ | $1.85 \pm 0.03$ | $1.82 \pm 0.01$ | $2.23 \pm 0.02$ | - | - |
| $A_k$      | $0.91 \pm 0.05$ | $0.82 \pm 0.04$ | $0.94 \pm 0.04$ | $0.90 \pm 0.04$ | $0.89 \pm 0.02$ | - |
| $A_{ISW}$  | $0.52 \pm 0.78$ | $1.20 \pm 0.63$ | $1.48 \pm 0.61$ | $0.18 \pm 0.67$ | $0.91 \pm 0.33$ | - |
| Best-fit $p(z)$ | | | | | | |
| $b_1$      | 1.25 | 1.56 | 1.53 | 1.83 | - | 1.43 |
| $b_2$      | 1.26 | 1.88 | 1.84 | 2.19 | - | 1.59 |
| $A_k$      | $0.91 \pm 0.05$ | $0.80 \pm 0.04$ | $0.94 \pm 0.04$ | $0.91 \pm 0.04$ | $0.88 \pm 0.02$ | $0.91 \pm 0.03$ |
| $A_{ISW}$  | $0.52 \pm 0.75$ | $1.17 \pm 0.58$ | $1.44 \pm 0.52$ | $0.18 \pm 0.67$ | $0.91 \pm 0.33$ | $0.99 \pm 0.35$ |
| Zhou et al. | | | | | | |
| $b_1$      | 1.25 | 1.54 | 1.55 | 1.90 | - | 1.44 |
| $b_2$      | 1.26 | 1.87 | 1.90 | 2.21 | - | 1.62 |
| $A_k$      | $0.91 \pm 0.06$ | $0.81 \pm 0.04$ | $0.93 \pm 0.04$ | $0.87 \pm 0.04$ | $0.87 \pm 0.02$ | $0.89 \pm 0.03$ |
| $A_{ISW}$  | $0.50 \pm 0.79$ | $1.03 \pm 0.59$ | $1.37 \pm 0.55$ | $0.20 \pm 0.63$ | $0.82 \pm 0.33$ | $0.98 \pm 0.35$ |
| Offset     | | | | | | |
| $b_1$      | 1.28 | 1.52 | 1.54 | 1.89 | - | 1.45 |
| $b_2$      | 1.30 | 1.86 | 1.87 | 2.20 | - | 1.64 |
| $A_k$      | $0.89 \pm 0.05$ | $0.81 \pm 0.04$ | $0.93 \pm 0.04$ | $0.89 \pm 0.04$ | $0.87 \pm 0.02$ | $0.88 \pm 0.03$ |
| $A_{ISW}$  | $0.45 \pm 0.81$ | $1.05 \pm 0.58$ | $1.32 \pm 0.56$ | $0.25 \pm 0.46$ | $0.83 \pm 0.33$ | $0.99 \pm 0.35$ |
| AvERA model | | | | | | |
| $b_1$      | 1.16 | 1.34 | 1.25 | 1.46 | - | 1.23 |
| $b_2$      | 1.11 | 1.50 | 1.45 | 1.75 | - | 1.33 |
| $A_k$      | $0.97 \pm 0.06$ | $0.80 \pm 0.04$ | $0.91 \pm 0.04$ | $0.85 \pm 0.04$ | $0.87 \pm 0.02$ | $0.91 \pm 0.03$ |
| $A_{ISW}$  | $0.24 \pm 0.35$ | $0.48 \pm 0.25$ | $0.55 \pm 0.23$ | $0.07 \pm 0.24$ | $0.35 \pm 0.13$ | $0.39 \pm 0.14$ |

Figure 11. Measurements of $A_k$ and $A_{ISW}$ for various data selections at $\ell_{\text{max}} = 500$ using the appropriate best-fit $n(z)$ for each set. The blue dashed line and band shows our default result, which is the average of the first two data points in each column. These represent a single unbinned analysis, as opposed to the average of the results for the various tomographic shells. The ‘offset’ results refer to the impact of the mean differences between our photo-$z$s and those of Z20 (see Appendix A).

Empirical density dependence of the amplitude is as $\Omega_m^{0.78}$, so that our result for $A_k$ produces the following constraint:

$$\sigma_8 \Omega_m^{0.78} = 0.297 \pm 0.009.$$  

It is interesting to note that total CMB lensing itself produces a constraint of a similar form, but with a different density dependence:

$$\sigma_8 \Omega_m^{0.25} = 0.589 \pm 0.020.$$  

A straightforward combination of these two results yields

$$\Omega_m = 0.275 \pm 0.024; \quad \sigma_8 = 0.814 \pm 0.042;$$  

the same normalization as Planck, but a somewhat lower density.

It is interesting to compare these results with analogous constraints from weak galaxy lensing. Here the dependence on density is intermediate in strength. The constraints from the cosmic shear measurement of KIDS-1000 (Asgari et al. 2020) and DES Y1 (Troel et al. 2018) are as follows:

$$\sigma_8 \Omega_m^{0.5} = 0.416 \pm 0.013 \quad \text{KIDS – 1000}$$  

$$\sigma_8 \Omega_m^{0.5} = 0.428 \pm 0.015 \quad \text{DES – Y1}$$  

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which is in close consistency with what would be deduced from the CMB lensing results: $\sigma_8 \Omega_m^{0.5} = 0.427$, as opposed to the fiducial 0.455. In combination, these three lensing results then give a clear preference for a model with a rather lower density than the Planck fiducial model, as illustrated in Fig. 12:

$$\Omega_m = 0.274 \pm 0.024; \quad \sigma_8 = 0.804 \pm 0.040.$$  

It can be noted that the KiDS-1000 papers preferred to interpret their results in terms of a reduced $\sigma_8$, but a shift purely in normalization is disfavoured by the total CMB lensing amplitude, quite apart from our current results.

The conflict of this result with Planck is marked: $\Delta \chi^2 = 12$ on 2 degrees of freedom, which represents a $p$ value of $2 \times 10^{-3}$. In these circumstances, we should be cautious in accepting the formal combination of the above lensing result with Planck, which is

$$\Omega_m = 0.296 \pm 0.006; \quad \sigma_8 = 0.798 \pm 0.006.$$  

In fact, this unimaginative compromise model is arguably not ridiculous: it lies within the 95% confidence contours of both our combined lensing result and Planck. Nevertheless, agreement this weak is asking a lot of bad luck: we may be fairly sure that systematics are by definition impossible to rule out, and therefore we think it is plausible that the compromise solution with $\Omega_m \approx 0.296$ may be close to the truth. If we look at CMB constraints independent of Planck, ACT+WMAP yields $\Omega_m = 0.313 \pm 0.016$, which is easily consistent with 0.296; this work also has $\Lambda_{\text{dark}}$ very close to unity (Aiola et al. 2020).

A slightly reduced matter density would also have the advantage of reducing the other tension that is currently the subject of much discussion: the Hubble parameter. The most robust inference concerning $H_0$ from the CMB comes from the main acoustic scale, which can be taken empirically as measuring the combination $\Omega_m h^3$ with negligible error. If we use this as a basis for rescaling the fiducial model, the compromise $\Omega_m = 0.296$ would require $H_0 \approx 69 \text{ km s}^{-1} \text{Mpc}^{-1}$. This 2% increase from the fiducial value is still significantly below the direct determination of 74.03 ± 1.42 (Riess et al. 2019), but again would only require a modest level of systematics for consistency. Furthermore, taking seriously the $\Omega_m \approx 0.274$ from the combined lensing data would imply a completely consistent $H_0 \approx 71$.

Consideration of variations in $h$ prompts us to ask whether the predicted $\Delta \Omega$ depends on $h$. From Eq. 4, we can see that there is no explicit $h$ dependence, since $h$ times comoving distance is a function of redshift and $\Omega_m$ only. The scale at which $\sigma_8$ is determined is accessible to the range of $\ell$ under study, so changes in power-spectrum shape arising from changes in $h$ would be expected to have a minor effect. In practice, we find $\Delta \Omega \propto h^{0.24}$, which is equivalent to a negligible $\Omega_m^{0.08}$ effect when considering variations with $\Omega_m h^3$ fixed.

It is undeniably depressing to be considering the possibility that one or more of the leading current cosmological datasets could be reporting results that contain systematic errors of close to $2\sigma$, but equally we need to beware of too hastily declaring the existence of new physics as soon as we see a minor statistical discrepancy. Because there are in principle two distinct discrepancies, affecting $\Omega_m = \sigma_8$ and $H_0$, a single new addition to the cosmological model that solved both issues would demand to be taken seriously. But both the lensing and $H_0$ discrepancies have existed in the literature for some while, and it is fair to say that no compelling solution has emerged. Nevertheless, it is worth reviewing some selected candidates.

### 5.1.1 Massive neutrinos

It is known that neutrinos make a non-zero contribution to the non-relativistic density, with a summed mass of at least 0.06 eV ($\Omega_\nu h^2 > 0.00064$). Owing to free streaming, the neutrino distribution is close to homogeneous on the scales of LSS, and therefore the lensing effect is reduced in two ways: the clumped mass is only the CDM, with a density $(1 - f_\nu) \Omega_m$; this lower effective dark matter density slows growth since last scattering, reducing $\sigma_8$ today. At first sight, these effects sound as if they have the potential to close the gap between lensing results and Planck, but this is not so. Firstly note that we do not really need to be concerned with growth suppression for the interpretation of the lensing results themselves, since the lensing signal is directly proportional to the low-redshift normalization. Furthermore, the standard definition of $\sigma_8$ (adopted...
potentials satisfy the Poisson equation. The most transparent mod-
Ψ = Φ
scalar potentials
is the predicted normalization is reduced, as expected, the best-fit
lensing signal is consistent with standard gravity, because it arises
ing signal. There is a degeneracy here: for
fected in this framework, and therefore modified gravity can be
cosmological parameters inferred from the CMB should be unaf-
assumed that the modifications affect only perturbations. Thus the
Λ
time accelerated expansion, and therefore it is normally assumed
et al. 2013). The motivation for modified gravity comes from late-
ations; alternatively, we can have normal growth with
Σ
µ
assumption, and also because
Ω
suppression of the strength of grav-
Ψ
3
by Planck and CAMB) is that it is the rms fractional fluctuation in
the total matter density. The fractional fluctuation in the CDM
density is thus σ8/(1 − fℓ), and this raised amplitude compensates
for the lower clumped density, so that the lensing signal for a given
Ωm and σ8 should be independent of the neutrino fraction. The
only subtlety is that the growth between z = 1 − 2 and z = 0 will
be slightly less than in ΛCDM for the given Ωm. But this is a tiny
effect: fℓ is about Ωm(z)0.56, so the relative fℓ is (1 − fℓ)0.56, so the
mass fluctuations at z = 1 − 2 are higher by of order 1 + 0.6σ8 than
in ΛCDM for a given z = 0 normalization, which is a negligible
correction.

Therefore, all the dependence on neutrino fraction on the Ωm −
σ8 plane comes from Planck. Inspecting their chains, the effect is
approximately σ8 ∝ (1 − fℓ)2 and Ωm ∝ (1 − fℓ)−2.2. Although the predicted normalization is reduced, as expected, the best-fit
density rises and so the tension between primary CMB and lensing
is increased if there is a non-minimal neutrino fraction.

5.1.2 Modified gravity
A more effective modification of theory concerns the strength of
gravity. To avoid excessive complication, it is common to approach
this in a form that includes two linear parameters that modify the
scalar potentials Ψ and Φ, which describe fluctuations in the time
and spatial parts of the metric. In the standard model, Ψ = Φ and
the potentials satisfy the Poisson equation. The most transparent mod-
ifiation is to scale the forces for non-relativistic particles (from Ψ)
and photons (from Ψ + Φ) that result from a given mass fluctuation,
δ, so that V2Ψ ∝ (1 + µ)δ and V2Ψ/Ψ + (1 + Σ)δ (e.g. Simpson
et al. 2013). The motivation for modified gravity comes from late-
time accelerated expansion, and therefore it is normally assumed
that the modifications evolve as
(μ(z), Σ(z)) = (μ0, Σ0)Ω4(z),
(27)
so that modifications are unimportant at last scattering. Since
ΛCDM seems to describe the expansion history well, it is also assumed
that the modifications affect only perturbations. Thus the
cosmological parameters inferred from the CMB should be unaf-
ected in this framework, and therefore modified gravity can be
used to close any gap between the predicted and observed lensing
signal. There is a degeneracy here: for Σ = 0 (normal lensing
strength), we can appeal to μ < 0 to reduce the growth in fluctua-
tions; alternatively, we can have normal growth with μ = 0 and
suppress the resulting lensing signal by appealing to Σ < 0. In either
of these solutions, it would be understandable that the total CMB
lensing signal is consistent with standard gravity, because it arises
around z = 2, where the modifications are only just switching on.
To achieve Σ = 0.9 at z = 0.5, where Ω4 = 0.4, we need either
Σ0 = −0.25, or µ0 = −1.5. The large value for µ0 seems surprising
at first sight, implying close to total suppression of LSS evolution
at the present epoch. This is partly a consequence of the μ ∝ Ω4(a)
assumption, and also because µ suppression of the strength of
gravity only alters the growth rate: to achieve significant reduction in δ
at z = 0.5 would require substantial alteration to the growth rate at
much higher redshifts, which is hard to achieve in this model unless
µ0 is large. Such a model can be ruled out by other evidence, since
it would imply a very non-standard growth rate at z = 0.5, whereas
we know from redshift-space distortions that the rate is within about
10% of fiducial at this redshift (eBOSS Collaboration et al. 2020).

In summary, then, an explanation of a low lensing amplitude
via modified gravity must involve an alteration of the strength of
light deflection by a given mass concentration, rather than reducing
the amplitude of mass fluctuations. Such an explanation appears
to be consistent and not in conflict with other evidence, but one
could hardly call it compelling – not least because it has no impact
on the H0 tension; such a radical conclusion requires more than a
single piece of evidence. In due course, we will have more accu-
rate tomographic lensing and redshift-space distortion data where
changes in the growth rate and strength of lensing with redshift can
be measured, so that a progressive decline in the strength of lensing
could be measured. Without such evidence, this hypothesis is at best
provisional.

5.2 ISW and the AvERA model
An interesting approach that has been proposed with a view to
explaining the high claimed ISW signal from superstructures is the
AvERA model (Rácz et al. 2017). This is a radical framework that
postulates a critical-density universe without a cosmological con-
stant, but with averaging of an inhomogeneous expansion rate, lead-
ing to an apparent acceleration as measured by the mean effective
Hubble parameter. The model can be adjusted so that the empirical
H(z) relation rather closely matches the standard ΛCDM case –
which has the advantage that the conversion between distance and
redshift remains as in the standard model, so that inferences from
the CMB regarding density parameters and the shape of the matter
power spectrum remain valid.

On the other hand, the amplitude of the spectrum is modified in
this model, and the density growth rate fℓ ≡ d ln δ/d ln a is
rather different from ΛCDM. There is a spike above fℓ ≈ 1.0 around
z ≈ 2 and in general the rate is higher than the standard model; thus,
the required value of σ8 at z = 0 has to be increased in order to
be consistent with the amplitude of primordial fluctuations inferred
from the CMB. A convenient fitting formula for the growth rate is
fℓ(a) = exp(−2.308a2 + 0.549[1 + 11.569(ln a + 1.222)]2)−1. (28)

Integration of this expression implies that σ8(z) for AvERA is
above ΛCDM at high redshift, by as much as a factor 1.2 at z = 1.5.
Conversely, the low-redshift evolution is slower and the amplitude
of present-day matter fluctuations is about 5% lower than ΛCDM.
The two models predict identical amplitudes at z ≈ 0.08. Thus,
for redshifts relevant for our tomographic data, the AvERA model
predicts a higher density fluctuation, so that the predicted amplitude
of the linear ISW signature is greater. There will also be a greater
degree of nonlinear evolution. We treat this by assuming that the
nonlinearities can be estimated in the Halofit framework by taking
the standard ΛCDM approach and increasing σ8(z) appropriately.
This should be sufficient to indicate how important the increased
nonlinearity might be (this will be more of a potential issue for
lensing, where even weak lensing can be dominated by nonlinear
structures on small enough angular scales).

We use Planck Collaboration et al. (2018b) Cosmological pa-
rameters, and set the power spectrum of AvERA to be identical
to ΛCDM at z = 8.55 consistent with Beck et al. (2018). We use
the fitting formula in Eq. 28, and interpolate the AvERA model
H(z) and R(z) as given by Beck et al. (2018). Fig. 13 shows the
matter auto-correlation, matter-κ, and matter-T cross-correlations in Av-
ERA and ΛCDM with both linear and non-linear power spectra, us-
ing the best-fit p(z). As expected, the AvERA prediction has a
higher amplitude than ΛCDM. The corresponding galaxy biases
are significantly smaller in the AvERA case as shown in Table 3,
but this effect is absorbed in the lensing cross-correlation, resulting
in similar constraints on Λk. The likelihoods for Λk and ΛISW are
obtained in Figs 14-15. In this case, we find Λk = 0.87 ± 0.02 for

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Figure 13. The dark matter auto-correlation (top), the matter-κ cross-correlation (middle), and the matter-temperature cross-correlation (bottom) in ΛCDM (red) and AvERA (blue) model for the four tomographic bins using the best-fit $n(z)$. The solid lines show computation using linear power spectrum, and the dashed lines show that using non-linear power spectrum from HALOFIT.

Figure 14. The constraints for $A_κ$ from the normalized likelihoods in the AvERA model using the best-fit $p(z)$ and fitted galaxy bias.

Figure 15. The constraints for $A_{ISW}$ from the normalized likelihoods in the AvERA model using the best-fit $p(z)$ and fitted galaxy bias.

the product, and $A_κ = 0.91 \pm 0.03$ for the unbinned case. In the ISW case, the AvERA prediction is about three times as large as ΛCDM. The preferred amplitude is $A_{ISW} = 0.35 \pm 0.13$ from the product of tomographic bins, and $A_{ISW} = 0.39 \pm 0.14$ from the unbinned result. Adopting the unbinned case, this ISW result excludes unity
at 4.4$r$ and we can be confident that the AvERA model greatly over-predicts the general level of ISW fluctuations.

6 SUMMARY AND DISCUSSION

We have performed a tomographic analysis of the cross-correlations between Legacy Survey galaxies and the Planck CMB lensing convergence and temperature maps, covering $17,739\,\text{deg}^2$. We obtained our own photometric redshifts for the Legacy Survey photometric lensing convergence. The results are compared with the predictions of the fiducial Planck cosmological model, marginalizing over the photometric redshift bins between $z = 0$ and $z = 0.8$. We model errors in photometric redshift with respect to calibration data sets via a modified Lorentzian function, and constrain the tails of the error distribution by requiring consistent prediction of the galaxy cross-correlation signal between different tomographic bins. This modeling incorporates a novel scheme for dealing with scale-dependent bias (Eq. 18), in which the linear and nonlinear parts of the matter power spectrum receive independent boosts to their amplitudes. The consistency of the galaxy clustering and its cross-correlations argues that the galaxy sample from the Legacy Survey is robust, and that the properties of the photometric redshifts are understood.

We then proceeded to evaluate the cross-correlation between the tomographic galaxy maps and the CMB maps of temperature and lensing convergence. The results are compared with the predictions of the fiducial Planck cosmological model, marginalizing over the photometric redshift bins between different tomographic bins. This modeling incorporates a novel scheme for dealing with scale-dependent bias (Eq. 18), in which the linear and nonlinear parts of the matter power spectrum receive independent boosts to their amplitudes. The consistency of the galaxy clustering and its cross-correlations suggests that the galaxy sample from the Legacy Survey is robust, and that the properties of the photometric redshifts are understood.

The amplitude for the ISW signal relative to the fiducial prediction is $A_{\text{ISW}} = 0.98 \pm 0.35$, consistent with $\Lambda$CDM, as found by previous works, e.g. Stöllner et al. (2018). We also explored the AvERA model (Rácz et al. 2017), which was developed in order to explain the claimed excess signal in the stacked ISW signal in superradiative CMB. We find that in this model, $A_{\text{ISW}} = 0.91 \pm 0.03$, and $A_{\text{ISW}} = 0.39 \pm 0.14$, with significantly smaller galaxy biases compared to the $\Lambda$CDM case. Thus, the AvERA model achieves its aim of predicting an enhanced superradiative signal at the price of raising the overall level of ISW power to the point where it is inconsistent with observation, even given the relatively noisy nature of the ISW signal. If the superradiative signal is found to persist in future studies, AvERA cannot be the explanation.

The amplitude of the CMB lensing signal is found to be significantly lower than the prediction of the fiducial Planck model, with a scaling factor $A_k = 0.901 \pm 0.026$. We note that this lower amplitude is consistent with the results from an analysis of cross-correlation between CMB lensing and a DESI LRG sample based on the Legacy Survey data (Kritides et al. in preparation). Our result can be translated into constraints on the parameter combination $\sigma_8 \Omega_m^{0.78} = 0.297 \pm 0.009$. The total CMB lensing signal provides an alternative constraint on this plane, of $\sigma_8 \Omega_m^{0.25} = 0.589 \pm 0.020$ (Planck Collaboration et al. 2018c), which also represents an amplitude lower than fiducial, although only by $1\sigma$. In combination, these CMB lensing figures prefer a solution with a relatively low matter density of $\Omega_m \approx 0.274$. These CMB lensing results are also in excellent agreement with the value of $\sigma_8 \Omega_m^{0.25}$ deduced from weak galaxy lensing (Troxel et al. 2018; Asgari et al. 2020).

With the compass of $\Lambda$CDM, the model that does least violence to lensing and CMB data is

$$\Omega_m = 0.296 \pm 0.006, \quad \sigma_8 = 0.798 \pm 0.006,$$

and this is consistent with the 95% confidence ranges from both datasets. It is therefore worth taking seriously the possibility that the true cosmic density is substantially on the low side of the fiducial Planck estimate. Such a reduction would also reduce the $H_0$ tension, raising the best-fitting CMB value to around $69\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$ – although this would still imply the existence of systematics in the direct $H_0$ data (see e.g. Elstathiou 2020).

We therefore face a situation where at least two of three currently dominant cosmological probes contain unrecognised systematics at the level of a few standard deviations, or the standard model must be extended. The choice between conservatism or revolution is perhaps not so easy in the current circumstances, but the next generation of experiments should settle the question beyond all doubt.

7 DATA AVAILABILITY

All of the observational datasets used in this paper are available through the Legacy Survey website http://legacysurvey.org/dr8/. The codes used in this analysis along with several processed data products will be made publicly available upon publication.

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The Legacy Surveys consist of three individual and complementary projects: the Dark Energy Camera Legacy Survey (DECaLS; NOAO Proposal ID # 2014B-0404; PIs: David Schlegel and Arjun Dey), the Beijing-Arizona Sky Survey (BASS; NOAO Proposal ID # 2015A-0801; PIs: Zhou Xu and Xiaohui Fan), and the Mayall z-band Legacy Survey (MzLS; NOAO Proposal ID # 2016A-0453; PI: Arjun Dey). DECaLS, BASS and MzLS together include data obtained, respectively, at the Blanco telescope, Cerro Tololo Inter-American Observatory, National Optical Astronomy Observatory (NOAO); the Bok telescope, Steward Observatory, University of Arizona; and the Mayall telescope, Kitt Peak National Observatory, NOAO. The Legacy Surveys project is honored to be permitted to conduct astronomical research on Iolkam Du’ag (Kitt Peak), a mountain with particular significance to the Tohono O’odham Nation.

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APPENDIX A: COMPARISON WITH Z20
PHOTOMETRIC REDSHIFTS

The purpose of this appendix is to present a detailed comparison between our photometric redshifts and those of Z20 (Zhou et al. 2020), including the impact of the different photo-z options on our cosmological results.

Fig. A1 compares the two photo-z catalogues in detail. The black dashed lines show the interval where the two photo-z has a difference smaller than 0.05. We exclude objects outside the interval, cutting 23.4% of the sample. Furthermore, there is a slight offset in the mean of the two samples, shown explicitly in the north and south part of the Legacy Survey in Fig. A2. We fit this offset for the south and north part of the survey separately using a cubic spline. Then we further create an ‘offset’ sample which has its redshifts corrected using the spline for \( \Delta_z \) to match with that of Z20. For this sample, the clipping of \( |\Delta_z| = 0.05 \) is applied after correcting for the offset, cutting 22.5% of the objects. Fig. A3 compares the raw redshift distributions of this work and Z20 for the three samples. The left panel shows the sample using redshifts inferred from \( g - r \) colours, the middle panel shows that from Z20, and the right panel shows that from the offset sample. The two photo-z distributions are close in all cases.

We find the photo-z convolution function parameters, \( (\sigma^{\text{spec}}, a^{\text{spec}}) \), for the Z20 samples using the same spectroscopic samples. We then follow the same procedures to find the best-fit \( n(z) \). The parameters are summarized in Table A1.

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| Table A1. Photo-z parameters for Z20. |
|----------------------------------------|
| bin 0 | bin 1 | bin 2 | bin 3 |
|-------|-------|-------|-------|
| \( \sigma^{\text{spec}} \) | 0.0075 | 0.0128 | 0.0150 | 0.0248 |
| \( a^{\text{spec}} \) | 1.320  | 1.484  | 1.700  | 1.502  |
| \( a^{\text{fit}} \) | 1.320  | 1.110  | 1.697  | 1.502  |
| \( x^{\text{fit}} \) | 0.000  | 0.0003 | -0.0002| -0.0001|


**Figure A2.** Photometric redshifts inferred from $g - r$, $r - z$, and $z - W_1$ colours, versus the difference from the Z20 estimates. The dotted lines show a spline fit to $\Delta z$ as a function of our photo-$z$, used for the offset correction.

**Figure A3.** The raw redshift distribution binned using photo-$z$ obtained in this work (left), in Z20 (middle), and in this work with the correction for the offset (right), after a clipping of $|\Delta z| < 0.05$. The solid line shows the distribution of photo-$z$ in this work, while the dashed line shows that from Z20.