Localization of GRBs by Bayesian Analysis of Data from the HETE WXM

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Abstract. We describe a new method of transient point source localization for coded-aperture X-ray detectors that we have applied to data from the HETE Wide-Field X-Ray Monitor (WXM). The method is based upon the calculation of the likelihood function and its interpretation as a probability density for the transient source location by an application of Bayes’ Theorem. The method gives a point estimate of the source location by finding the maximum of this probability density, and credible regions for the source location by choosing suitable contours of constant probability density. We describe the application of this method to data from the WXM, and give examples of GRB localizations which illustrate the results that can be obtained using this method.

INTRODUCTION

The HETE Wide-Field X-Ray Monitor (WXM) is composed of two crossed, one-dimensional coded-aperture cameras. GRB locations are inferred from position histograms in the X and Y cameras, using the known mask pattern and detector response.

Several processing methods for coded-aperture data have been proposed, including cross correlation [1], least-squares fitting [2], and Maximum Entropy [3, 4]. Skinner and Nottingham [5] have described a maximum-likelihood fitting technique.

In [6] we presented a Bayesian scheme for analyzing coded-aperture data from such transient events. The method is based on the calculation of the joint likelihood function for two stretches of data: the stretch covering the transient event itself, and a stretch before and/or afterwards, which provides information about the background. We interpret the likelihood thus obtained as a posterior probability density for the transient event location by an application of Bayes’ Theorem.

We have implemented the method in the HETE data-processing pipeline, where it is routinely used to obtain GRB locations. Here we present the implementation details, and some of the results that have been obtained to date.

IMPLEMENTATION

When high-resolution WXM data from a HETE trigger are received on the ground, background and burst time intervals are automatically determined by SNR maximization. The software then essentially compares the background-to-burst change in the WXM position histogram data with a Monte Carlo point source model, using the method described in [6] to calculate the posterior probability density at each interrogated source location.

Computation of the Monte Carlo point source model (or “template”) is somewhat time-consuming, so that it is not possible to pre-compute templates in a dense grid spanning all possible locations in the field-of-view. We have therefore adopted a two-stage approach:

- **Coarse Location** — coarsely spaced pre-computed templates are compared to the data to produce a location accurate to about $1/3^\circ$.
- **Fine Location** — a freshly-generated template at the coarse location is shifted around the immediate neighborhood of that location, to map the posterior probability distribution.

Coarse Location

The WXM is composed of two 1-dimensional coded-aperture cameras that resolve locations in perpendicular directions (X and Y). We can use 1-dimensional arrays of sky locations with pre-computed templates in order to get
coarse X and Y locations. Given such an array, the code simply computes the posterior density at each location and plots it as a function of template number, reporting the highest value.

One such array that we occasionally use has 22 lines of templates — 11 each in X and in Y. The lines are chosen so as to expose select combinations of adjacent WXM detector wires (the detector in each WXM camera has six wires, some of which might not be illuminated by a point source at a certain location). This allows the X-detector to supply some Y information, and vice-versa. Figure 1 shows a skymap of this template configuration.

Another template array is a simple cross - two lines of sky locations, one in the X direction and one in the Y direction, intersecting at the center of the FOV. This array uses less information, but can be run much faster, and is usually adequate for the purpose of obtaining a coarse localization.

In both cases the spacing between adjacent templates on a line is about 18', which is the fundamental resolution element of WXM. This is thus also the accuracy of a coarse location obtained in this fashion.

Figure 2 shows examples of the output from the coarse location procedure, for GRB010326B and GRB010629. Each set of two panels shows the log-likelihood profile as a function of template number, for the X and Y template lines. The two panels on the left show the result of the analysis of GRB010326B performed using the full 22-line template array. The two panels on the right show the result of the analysis of GRB010629 performed using the cross template array. In both cases the best-fit X and Y angles are marked by a vertical line.

### Fine Location

The posterior density function allows us to go beyond merely obtaining a point estimate of the location — we can also use it to infer contours of prescribed probability content around the best-fit location. That is, we can use it to get error boxes.

We proceed by calculating one very bright Monte-Carlo template at the best-fit coarse location. We fit that template with an empirical model constructed by smearing and transforming the coded mask pattern. This transformation is designed to give a simplified account of physical processes such as scattering and detector penetration. An example of such a fit is shown in Figure 3, which was produced in the analysis of GRB010629.

We then calculate the posterior density on a 20 × 20 grid of points in the neighborhood of the coarse location, using at each point a template calculated by smearing the mask pattern with the best-fit empirical parameters determined by the fit of the model to the Monte-Carlo template.

We issue not only a refined best-fit location, but also constant-density contours containing 68.3%, 95.5%, and 99.7% (statistical) probability of including the correct location. Figure 4 shows the location contours produced in this way for GRB010629.

For the purposes of quick communication, and to simplify follow-up observations, we also produce “circularized” error-boxes by calculating the radius of a circles about the best-fit location that just contain 68.3%, 95.5% and 99.7% (statistical) probability of including the correct location. Since these are not constant probability contours, they necessarily subtend slightly larger solid angles than the iso-density contours. The difference is usually not very large.
FIGURE 2. Coarse location log-likelihood profiles. Each set of two panels shows X and Y localizations. Left: GRB010326B, 22-line array of templates. Right: GRB010629, “Cross” configuration of templates.

FIGURE 3. Fit of empirical model to Monte Carlo simulation. The assumed direction of the source is the coarse location of GRB010629.

The statistical probabilities inferred here must be supplemented by the systematic location error deduced from our calibration studies.

FIGURE 4. Skymap showing location probability contours obtained by the fine location of GRB010629.

REFERENCES

1. Fenimore E. E., Appl. Opt. 17, 3562 (1978).
2. Doty J. P. “The All Sky Monitor for the X-ray Timing Explorer,” in X-ray Instrumentation in Astronomy II, edited by L. Golub, SPIE Conference Proceedings 982, Bellingham, Washington, 1988, p. 164.
3. Sims M., Turner M.J.L., and Willingale R., Space Sci. Instrum. 5, 109 (1980).
4. Willingale R., Sims M., and Turner M.J.L., Nucl. Instrum. Methods Phys. Res. 221, 60 (1984).
5. Skinner G. K., and Nottingham M. R., Nucl. Instrum. Methods Phys. Res. A333, 540 (1993).
6. Graziani, C., Lamb, D. Q., and Slawinski, R. “Determination of X-Ray Transient Source Positions by Bayesian Analysis of Coded Aperture Data,” in All-Sky X-Ray Observations in the Next Decade, edited by M. Matsuoka and N. Kawai, RIKEN, Wako, Japan, 1997, pp. 303-308.