Exponentiating Higgs

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Abstract

The scalar models with exponential interaction, introduced in arXiv:1506.00987, include theories with $\langle \phi \rangle \neq 0$. Here, we investigate the partition function of scalar theories with normal ordered exponential interactions. The vev of the normal ordering of the exponential of the integrated potential

$$e^{-Z_R[J]} = \langle 0 | :e^{-\int d^4xV(\phi)} : |0\rangle_J$$

generates the lowest order contributions to the $N$-point function. We first investigate the case $V(\phi) = \mu^D \exp(-\alpha \phi)$, and then focus on $V(\phi) = -2\nu m^4 \sinh(\phi/\nu m)$. The latter can be seen as an effective theory with dimensionless scale parameter $\nu$. The standard normalization choice for finite theories suggests setting $\nu = \sqrt{\Delta_{F,\Lambda}(0)/m}$, with $\Delta_{F,\Lambda}(x)$ the Feynman propagator with cut-off on the momenta. For large $\Lambda$ the model is well-described by $Z_R[J]$

$$\langle \phi \rangle = 2m$$

in agreement with the experimental data. In such a limit the propagator corresponds to the free one. The investigation suggests a natural way to get the lagrangian of the Standard Model, with a different Higgs lagrangian, that may be tested in future experiments at LHC.
1 Introduction

The Higgs mechanism \([1]-[6]\) is a basic step in the formulation of the Standard Model \([7, 8]\). This has been confirmed by the spectacular experimental results at LHC \([9, 10]\). Despite this, there are still some open questions. The most important one is that the vev of the Higgs field is evaluated at the classical level. On the other other hand, there are models with non-trivial minima for the classical potential, with no order parameter. The point is that spontaneously symmetry breaking is a strictly nonperturbative phenomenon, concerning infinitely many degrees of freedom. As such, even radiative corrections to \(\langle \phi \rangle\) should be considered with particular attention. In this respect, one should also recall that, against the evidence coming from the perturbative expansion, \(\lambda \phi^4\) is believed to be a free theory. A related aspect is that the mass term of the initial lagrangian has the opposite sign.

Other questions concern the hierarchy problem, with the LHC data that do not seem to confirm that supersymmetry may be the answer, and understanding whether there is a mechanism giving mass to the Higgs.

In trying to answer the above questions one should keep in mind that the main reason for the use of Higgs model is the one of providing a nontrivial vev for the scalar field. The natural question is then if there is a simpler way to get such a vev that may avoid the above mentioned questions. This is of considerable experimental interest, since a possible alternative to the standard Higgs mechanism can be verified in future experiments at LHC. In particular, it should be possible to make the crucial check of the \(\eta^3\) term, and, even if much more difficult, of the \(\eta^4\) self-interaction.

We will start the investigation by considering the parametrization of the Higgs field \(\Phi\). In particular, we will consider a simple model, which is naturally suggested once one chooses the parametrization

\[
\Phi(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi(x) \end{pmatrix}.
\]  

(1.1)

The point is that, as a matter of fact, \(\phi\) is tacitly considered to take real values rather than non-negative ones. Such a choice is usually justified by the fact that one is considering perturbation theory around a minimum. In the next section we will consider a more direct justification for considering \(\phi\) a real field. This suggests a simple form for the scalar lagrangian of the Standard Model. Next, we will consider exponential interactions. In this respect, in \([11]\), it has been shown that these lead to a nontrivial \(\langle \phi \rangle\). An outcome of \([11]\), is that there appear terms coming from the normal ordering of the exponentiated scalar. In this respect, we note that the interaction term \(V\) in the path-integral formulation can be seen as the vev, in the free theory in an external source, of \(\text{exp}(\int d^D x : V(\phi) : )\), that is

\[
W[J] = \langle 0 | T \text{exp}(\int d^D x : V(\phi) : ) | 0 \rangle_J.
\]  

(1.2)
It is worth noticing that, as emphasized in [12], the difficulties in quantizing some non-renormalizable field theories, concern the non-uniqueness of the solution, rather than its existence. In such a context, let us remind that in [13], using the ultraviolet cutoff $\gamma^{-N}$, $\gamma > 1$, $N > 0$, have been investigated scalar theories with interaction $\lambda \exp(\alpha \phi)$ :. It turns out that for $d > 2$, for all $\alpha$, and for $d = 2$, with $|\alpha| > \alpha_0$, the Schwinger functions converge to the free Schwinger functions. The essential point in the investigation of [13] is that $\Delta F$, $\Lambda(0)$, with $\Delta F$, $\Lambda(x)$ the Feynman propagator with cut-off on the momenta, grows sufficiently fast to kill the fluctuations of $\phi$, so that $\exp(\alpha \phi) := \exp\left(-\frac{\alpha^2}{2} \Delta F(0)\right) \exp(\alpha \phi)$ vanishes in the limit $\Lambda \to \infty$. We are not aware if such findings have been reproduced in the standard lattice regularization. However, as we will see, we will get nontrivial expectation values of $\phi$. It would be of considerable interest if this may correspond to a gaussian theory.

We will consider the $D$-dimensional euclidean partition function of the scalar theory with potential $V(\phi) = e^{-\mu D \int d^D x \exp(-\alpha \phi(x))}$, and then consider

$$W_R[J] = e^{-Z_R[J]} = \langle 0 | e^{\int d^D x V(\phi)} | 0 \rangle_J . \quad (1.3)$$

This can be seen as filling-in the free vacuum in an external source by scalar modes. By using an iterative algorithm, reproducing the Wick theorem, we derive a simple way to implement such a normal ordering in the path-integral approach. The results in [11] straightforwardly imply that

$$Z_R[J] = Z_0[J] + \mu^D \int d^D x e^{-\alpha \int d^D y J(y) \Delta F(y-x)} , \quad (1.4)$$

where $\Delta F(x-y)$ is the Feynman propagator. It turns out that $Z_R[J]$ generates the lowest order contributions in $\alpha$ to the $N$-point point function. In particular

$$-\frac{\delta Z_R[J]}{\delta J(x)} = \frac{\alpha \mu^D}{m^2} . \quad (1.5)$$

Next, we consider the lagrangian density

$$L_\phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - m^2 \Phi^\dagger \Phi + 2 \nu m^4 \sinh \left( \frac{\sqrt{2 \nu}}{vm} (\Phi^\dagger \Phi)^{\frac{1}{2}} \right) , \quad (1.6)$$

and then, as usual, parameterize the scalar doublet in polar coordinates. In the unitary gauge, the scalar part of $L_\phi$ reduces to

$$L_\phi = \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + 2 \nu m^4 \sinh \left( \frac{\phi}{\sqrt{2 \nu}} \right) . \quad (1.7)$$

We will see that this leads to

$$\langle \phi(x) \rangle = 2m + O(\nu^{-1}) . \quad (1.8)$$
When $N$ is even, $Z_R[J]$ generates only the lowest order contribution to the $N = 2$ point function

$$\frac{\delta^2 Z_R[J]}{\delta J(x_1)\delta J(x_2)} = \Delta_F(x_1 - x_2) .$$

(1.9)

We then consider the model as an effective theory. An example of possible choice is to identify $\nu$ with $\sqrt{\Delta F}/m$, so that

$$\nu = \frac{1}{4\pi} \sqrt{\frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2}{m^2}} + O[(\Lambda^{-1})^0] .$$

(1.10)

2 A simple model

In the unitary gauge, the Higgs field $\Phi$ has only one non-vanishing component, the field $\phi(x)/\sqrt{2}$. Note that although $\eta = \phi - \langle \phi \rangle \geq -\langle \phi \rangle$, the field $\eta$ is usually considered as taking real values. This seems a subtle point since the lagrangian of the Standard Model contains the term $\eta^3$ and the linear one in $\eta$ in the Yukawa couplings. The usual argument is that this is justified when considering perturbation around a minimum. This would suggest considering the lagrangian density $L_\Phi = (D_\mu \Phi)^\dagger(D^{\mu} \Phi) - m^2 \Phi^\dagger \Phi + \sqrt{8} m^3 \sqrt{\Phi^\dagger \Phi}$.

In the unitary gauge, the purely scalar part reduces to

$$L_\phi = \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + 2m^3 |\phi| .$$

(2.1)

The corresponding vev for $\phi$ is $v = 2m$, so that, if perturbation theory justifies taking $\phi$ real, one would get the free lagrangian for $\eta = \phi - v$. It is then clear that it would be desirable finding a more natural way to justify the parametrization with $\phi \in \mathbb{R}$, so that one may consider (2.4) with $|\phi|$ replaced by $\phi$. In this respect, note that

$$\Phi(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi(x) \end{pmatrix} , \quad \phi \in \mathbb{R} ,$$

(2.2)

resembles the decomposition of complex numbers $z = \chi e^{i\theta}$, with $\chi \in \mathbb{R}$ and $\theta \in [\pi/2, \pi/2)$, a choice that avoids the multivaluedness of the arctangent parametrization of $\theta$. As in the standard polar decomposition, one may then extend $\theta$ to $\mathbb{R}$. Now the transformation $z \rightarrow -z$ can be obtained either by $\chi \rightarrow -\chi$ or by $\theta \rightarrow \theta + (2k+1)\pi, k \in \mathbb{Z}$. This represents an alternative to the periodicity $\theta \rightarrow \theta + 2k\pi, k \in \mathbb{Z}$ of the polar decomposition. Since $\phi$ in (2.2) is gauge invariant, any $V(\phi)$ is a gauge invariant potential. With the choice (2.2) one has to remind that $\phi = e^{k\pi i} |\phi| = e^{k\pi i} \sqrt{\phi^2}, k \in \mathbb{Z}$, so that $\phi = (\phi^2)^{1/2} = (\Phi^\dagger \Phi)^{1/2}$. We stress that a gauge transformation of $\Phi(x)$ by a sign corresponds to $U(x) \rightarrow e^{2(k+1)\pi i} U(x)$, so that $U(x)^\dagger U(x) = e^0$, and, as obvious, $(\Phi^\dagger \Phi)^{1/2}$ is invariant.

In the following we use the parametrization (2.2). Let us consider the lagrangian density for the scalar field $\Phi$

$$L_\Phi = (D_\mu \Phi)^\dagger(D^{\mu} \Phi) - m^2 \Phi^\dagger \Phi + \sqrt{8} m^3 (\Phi^\dagger \Phi)^{1/2} ,$$

(2.3)
and note that $\sqrt{8m^2}$ is the value of $\Phi_1^i \Phi$ that minimizes $\Phi_1^i \Phi - \sqrt{8m} (\Phi_1^i \Phi)^{\frac{1}{2}}$. By (2.2) it follows that, in the unitary gauge, the lagrangian density describing the purely scalar sector is
\[
L_\phi = \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + 2m^3 \phi ,
\]

$\phi \in \mathbb{R}$. Note that, since the theory is free, it follows that $\langle \phi \rangle$ coincides with the value of $\phi$ that minimizes $\phi^2 - 4m \phi$, that is
\[
v := \langle \phi(x) \rangle = 2m .
\]

Let us recall that it is a general fact that higher order derivatives of the logarithm of the partition function, in an external source $J$, evaluated at $J = 0$, coincide with the connected correlators of $\eta = \phi - \langle \phi \rangle$. For example,
\[
\frac{\delta^2 \ln W}{\delta J(x) \delta J(y)}|_{J=0} = -\langle \phi(x) \rangle \langle \phi(y) \rangle + \langle (\eta(x) + \langle \phi(x) \rangle)(\eta(y) + \langle \phi(y) \rangle) \rangle = \langle \eta(x) \eta(y) \rangle .
\]

This identifies $\eta$ as the true scalar field, and the lagrangian density becomes the free one
\[
L_\eta = \partial_\mu \eta \partial^\mu \eta - \frac{1}{2} m^2 \eta^2 , \quad \eta \in \mathbb{R} .
\]

### 3 Exponential interactions

Let us introduce some notation, which follows the one in Ramond’s book [14], and shortly reviewing the investigation in [11]. Consider the partition function in the $D$-dimensional euclidean space
\[
W[J] = e^{-Z[J]} = N \int D\phi \exp \left[ -\int d^Dx \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + V(\phi) - J\phi \right) \right] .
\]

Define
\[
\langle f(x_1, \ldots, x_n) \rangle_{x_1 \ldots x_k} = \int d^Dx_1 \ldots d^Dx_k f(x_1, \ldots, x_n) ,
\]
and denote by $\langle f(x_1, \ldots, x_n) \rangle$ integration of $f$ over $x_1, \ldots, x_n$. Let
\[
\Delta_F(x - y) = \int \frac{d^Dp}{(2\pi)^D} \frac{e^{ip(x-y)}}{p^2 + m^2} ,
\]
be the Feynman propagator and set
\[
Z_0[J] = -\frac{1}{2} \langle J(x) \Delta_F(x - y) J(y) \rangle .
\]

To compute $W[J]$ we use the Schwinger trick
\[
W[J] = Ne^{-\langle V(\phi) \rangle} e^{-Z_0[J]} .
\]
The starting point in [11] has been the observation that Schwinger’s trick can be extended to get exact results. In particular, it has been observed that exponential interactions can be obtained by acting on $\exp(-Z_0[J])$ with the translation operator. Consider the potential investigated in [11] with the opposite sign of $\alpha$

$$V(\phi) = \mu^D e^{-\alpha\phi}.$$  \hfill (3.6)

The corresponding partition function (we drop the constant $N$) is

$$W[J] = \exp \left[ -\mu^D \langle \exp(-\alpha \omega) \rangle \right] \exp(-Z_0[J])$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \mu^D \langle \exp(-\alpha \omega) \rangle^k \exp(-Z_0[J]) .$$  \hfill (3.7)

Then, we use [11]

$$\exp(-\alpha \omega J(x)) \exp(-Z_0[J]) = \exp(-Z_0[J - \alpha x]) \exp(-\alpha \omega J(x)) = \exp(-Z_0[J - \alpha x]) ,$$  \hfill (3.8)

where

$$Z_0[J - \alpha x] := -\frac{1}{2} \int d^D y d^D z (J(y) - \alpha \omega(x-y)) \Delta_F(y - z)(J(z) - \alpha \omega(x - z))$$

$$= Z_0[J] - \frac{\alpha^2}{2} \Delta_F(0) + \alpha \int d^D y J(y) \Delta_F(y - x) .$$  \hfill (3.9)

Therefore,

$$W[J] = \exp(-Z_0[J]) \sum_{k=0}^{\infty} \left[ \frac{(-\mu^D)^k}{k!} \exp \left( \frac{k\alpha^2}{2} \Delta_F(0) \right) \right]$$

$$\int d^D z_1 \ldots \int d^D z_k \exp \left( -\alpha \int d^D z J(z) \sum_{j=1}^{k} \Delta_F(z - z_j) + \alpha^2 \sum_{j>l}^{k} \Delta_F(z_j - z_l) \right) .$$  \hfill (3.10)

The terms $\exp \left( \frac{k\alpha^2}{2} \Delta_F(0) \right)$ and $\exp(\alpha^2 \sum_{j>l}^{k} \Delta_F(z_j - z_l))$ are related to normal ordering. In this respect, note that (3.7) corresponds to the expansion

$$\exp \left( \int d^D x V(\phi) \right) = \sum_{k=0}^{\infty} \frac{1}{k!} \left( \int d^D x V(\phi) \right)^k ,$$  \hfill (3.11)

that is

$$W[J] = \langle 0 | T e^{\mu^D \int d^D x \exp(-\alpha \omega(x))} | 0 \rangle_J$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \mu^D \int d^D x_1 \ldots d^D x_k \langle 0 | T e^{-\alpha \omega(x_1)} \ldots e^{-\alpha \omega(x_k)} | 0 \rangle_J .$$  \hfill (3.12)
where the vacua are the ones of the free scalar theory coupled to the external source \( J \). The fact that the normal ordering problem is the cause of some of the infinities arising in perturbation theory, suggests considering

\[
W_R[J] = \langle 0 | e^{-\mu^D \int d^D x \exp(-\alpha \phi(x))} | 0 \rangle_J
\]

\[
= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \mu^D \int d^D x_1 \ldots d^D x_k \langle 0 | e^{-\alpha \phi(x_1)} \ldots e^{-\alpha \phi(x_k)} | 0 \rangle_J .
\] (3.13)

To check the consequence of taking the normal ordering, note that

\[
\langle e^{-\alpha \phi(x)} \rangle := e^{-\frac{\alpha^2}{2} \Delta F(0)} e^{-\alpha \phi(x)} ,
\] (3.14)

and

\[
T : e^{-\alpha \phi(x_1)} \ldots e^{-\alpha \phi(x_k)} := e^{\alpha^2 \sum_{j=1}^{k} \Delta_F(x_j-x_1)} e^{-\alpha \phi(x_1)} \ldots e^{-\alpha \phi(x_k)} .
\] (3.15)

Therefore,

\[
e^{-\alpha \phi(x_1)} \ldots e^{-\alpha \phi(x_k)} := e^{-\alpha^2 \left( \sum_{j=1}^{k} \Delta_F(x_j-x_1) \right) T} e^{-\alpha \phi(x_1)} \ldots e^{-\alpha \phi(x_k)} .
\] (3.16)

It follows that the expansion on right hand side in (3.13) exponentiates. Actually, (3.10), (3.12), (3.13) and (3.16) yield

\[
W_R[J] = \exp(-Z_R[J]) ,
\] (3.17)

where

\[
Z_R[J] = Z_0[J] + \mu^D \int d^D x e^{-\alpha \int d^D y J(y) \Delta_F(y-x)} .
\] (3.18)

Interestingly, removing the term \( \exp(\alpha^2 \sum_{j=1}^{k} \Delta_F(x_j-x_1)) \), coming from the normal ordering in (3.15), is equivalent to remove a term \( \langle \exp(-\alpha \phi) \rangle \) in (3.7). To show this, recall that for any suitable function \( F \), if \( A \) and \( B \) are operators, then \( A^{-1} F(B) A = F(A^{-1} B A) \). Therefore,

\[
W[J] = \exp \left[ -\mu^D \langle \exp(-\alpha \frac{\delta}{\delta J}) \rangle \right] \exp(-Z_0[J])
\]

\[
= \exp(-Z_0[J]) \left[ -\mu^D \exp(Z_0[J]) \langle \exp(-\alpha \frac{\delta}{\delta J}) \rangle \exp(-Z_0[J]) \right]
\]

\[
= \exp(-Z_0[J]) \left[ -\mu^D \langle \exp \left( -\alpha \langle J(y) \Delta_F(x-y) \rangle_y \right) \rangle \exp(-\alpha \frac{\delta}{\delta J}) \right] .
\] (3.19)

where

\[
\mu^D = \mu_0^D \exp \left( -\frac{\alpha^2}{2} \Delta_F(0) \right) .
\] (3.20)

Eq. (3.19) differs from \( W_R[J] \) by the term \( \langle \exp(-\alpha \phi) \rangle \) in the last member, and by the relabeling of \( \mu_0 \). The latter is equivalent to consider the normal ordering of \( \exp(-\alpha \phi) \). Therefore,

\[
W[J, : e^{-\alpha \phi} :] = W_R[J] + \ldots ,
\] (3.21)
where the dots denote the terms in (3.10) coming from the expansion
\[
\sum_{n=1}^{\infty} \frac{\alpha^{2n}}{n!} \left( \sum_{j>l}^{k} \Delta_F(z_j - z_l) \right)^n.
\] (3.22)

Consider the field
\[
\phi_{cl}(x) := -\frac{\delta Z_R[J]}{\delta J(x)},
\] (3.23)

and note that by (3.18)
\[
\phi_{cl}(x) = \langle J(y) \Delta_F(x-y) \rangle_y + \alpha \mu^D \langle \Delta_F(y-x) \exp(-\alpha \langle J(z) \Delta_F(y-z) \rangle_z) \rangle_y,
\] (3.24)

that satisfies the equation of motion
\[
(-\partial_\mu \partial_\mu + m^2) \phi_{cl}(x) = J(x) + \alpha \mu^D \exp(-\alpha \langle J(y) \Delta_F(x-y) \rangle_y).
\] (3.25)

By (3.24) it follows that \( \Gamma_R[\phi_{cl}] = Z_R[J] - \langle J(x) \phi_{cl}(x) \rangle_x \), reads
\[
\Gamma_R[\phi_{cl}] = Z_R[J] - \langle J(x) \Delta_F(x-y) J(y) \rangle_{xy} - \alpha \mu^D \langle J(x) \Delta_F(x-y) \exp(-\alpha \langle J(z) \Delta_F(z-y) \rangle_z) \rangle_{xy}
\] (3.26)

By (3.24) it follows that at the first order in \( \alpha \)
\[
\langle \phi(x) \rangle_0 = -\frac{\delta Z_R[J]}{\delta J(x)} \bigg|_{J=0} = \frac{\alpha \mu^D}{m^2},
\] (3.27)

where we used
\[
\langle \Delta_F(x-y) \rangle_y = \frac{1}{m^2}.
\] (3.28)

Such a relation follows, for example, by
\[
\int d^D x (-\partial_\mu \partial_\mu + m^2) \Delta_F(x-y) = 1,
\] (3.29)

and then using the translation invariance of the Feynman propagator to check that there are no additional terms in (3.28), solutions of the homogeneous Klein-Gordon equation.

It follows that the higher derivatives of \( Z_R[J] \), evaluated at \( J = 0 \), correspond, to the lowest order contribution in the \( \alpha \) expansion, to the connected Green functions associated to
\[
\eta(x) = \phi(x) - \frac{\alpha \mu^D}{m^2},
\] (3.30)

that is
\[
(-1)^N \frac{\delta^N Z_R[J]}{\delta J(x_1) \ldots \delta J(x_N)} \bigg|_{J=0} = \langle 0|T \eta(x_1) \ldots \eta(x_N)|0 \rangle,
\] (3.31)

and by (3.18), for \( N > 1 \),
\[
\langle 0|T \eta(x_1) \ldots \eta(x_N)|0 \rangle = \delta_{N2} \Delta_F(x_1-x_2) + \alpha^N \mu^D \int d^D y \Delta_F(y-x_1) \ldots \Delta_F(y-x_N).
\] (3.32)

Note that higher order contributions in \( \alpha \) come from the expansion (3.22).
\( v = 2m \)

The above model can be extended to more general interactions, such as

\[ V(\phi) = \sum_{k=1}^{n} \mu_k^D \exp(\alpha_k \phi) \]  

(4.1)

In order to find the explicit expression of \( W_R[J] \) in the case of the potential (4.1), one first notes that

\[ W[J] = \left[ \prod_{k=1}^{n} \exp[-\mu_k^D \langle \exp(\alpha_k \frac{\delta}{\delta J}) \rangle] \right] \exp(-Z_0[J]) \]  

(4.2)

then, uses (3.19) iteratively. In the first step one has

\[ \exp \left[ -\mu_n^D \langle \exp(\alpha_n \frac{\delta}{\delta J}) \rangle \right] \exp(-Z_0[J]) = \exp(-Z_0[J]) \exp \left[ -\mu_{n0}^D \langle \exp \left( \alpha_n \langle J(y) \Delta F(x - y) \rangle \right) \rangle \right] \]  

(4.3)

Repeating this for the remaining \( n - 1 \) terms in (4.2), makes it clear that

\[ W_R[J] = \exp(-Z_R[J]) = \langle 0 | e^{-\int d^Dx \sum_{k=1}^{n} \mu_k^D \exp(\alpha_k \phi(x))} | 0 \rangle_J \]  

(4.4)

is obtained from \( W[J] \) by removing, from the final expression, the term \( \langle \exp \left( \sum_{k=1}^{n} \alpha_k \frac{\delta}{\delta J} \right) \rangle \) on the right hand side, and by canceling the \( \exp \left( \sum_{k=1}^{n} \alpha_k^2 \Delta_F(0) \right) \) term. Such a cancelation is equivalent to relabel each \( \mu_{k0} \) by \( \mu_k \). It follows that

\[ Z_R[J] = Z_0[J] + \int d^Dx \sum_{k=1}^{n} \mu_k^D e^{\alpha_k \int d^Dy J(y) \Delta_F(y - x)} \]  

(4.5)

We note that taking the normal ordering of \( \exp(\int d^Dx V(\phi)) \) may lead to well-defined \( Z_R[J] \), even in cases when \( V(\phi) \) is unbounded below. A particularly interesting case is the four-dimensional potential

\[ V(\phi) = -2\nu m^4 \sinh \left( \frac{\phi}{\nu m} \right) \]  

(4.6)

where \( \nu \) is a dimensionless scale. By (4.5), we have

\[ Z_R[J] = Z_0[J] - 2\nu m^4 \int d^4x \sinh \left( \frac{\int d^4y J(y) \Delta_F(y - x)}{\nu m} \right) \]  

(4.7)

Repeating the analysis leading to (3.27), at the zero order in \( \nu^{-1} \), (4.7) yields

\[ \langle \phi(x) \rangle = 2m \]  

(4.8)

so that, at the same order,

\[ 2^{-1/4} G_F^{-1/2} = 2m \]  

(4.9)
in agreement with the LHC data. Remarkably, we have
\[
\lim_{\nu \to \infty} V(\phi) = -2m^3 \phi ,
\] (4.10)
so that, in this limit, (4.8) corresponds to the value of $\phi$ that minimizes $m^2 \phi^2 / 2 + V(\phi)$. The resulting lagrangian density coincides with (2.4).

Making the expansion in powers of $\nu^{-1}$, one sees that the lowest order contribution to the $(2N + 1)$-point function is generated by $Z_R[J]$, so that
\[
\langle 0 | T \eta(x_1) \cdots \eta(x_{2N+1}) | 0 \rangle = 2^{2N+1} \nu^{-2N} m^{3-2N} \int d^4y \Delta_F(y-x_1) \cdots \Delta_F(y-x_{2N+1}) ,
\] (4.11)
where $\eta(x) = \phi(x) - \langle 0 | \phi(x) | 0 \rangle$. In the case of even $N$, $Z_R[J]$ contributes only to the lowest-order two-point function, giving the free propagator.

Considering the model as an effective theory suggests relating $\nu$ with the momentum cut-off $\Lambda$. To select a natural choice, note that since $\langle \phi(x) \phi(x) \rangle$ is the amplitude of finding a particle at $\vec{x}$ at the time $t$ which is in the same point at the same time, it follows that in a finite theory $G(x, x) = \langle \chi(x) \chi(x) \rangle$ should be one. This implies that the correct normalization is
\[
\langle \frac{\chi(x)}{\sqrt{G(x, x)}} | \frac{\chi(x)}{\sqrt{G(x, x)}} \rangle = 1 .
\] (4.12)
This fixes the natural normalization in the case of theories with cutoff. In the present case this suggests setting $\nu = \sqrt{\Delta_{F,\Lambda}(0)/m}$, with $\Delta_{F,\Lambda}(x)$ the regularized Feynman propagator by the momentum cut-off $\Lambda$, that is
\[
\nu = \frac{1}{4\pi} \sqrt{\frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2}{m^2} + O[(\Lambda^{-1})^0]} .
\] (4.13)
Note that for large $\Lambda$ the model is well-described by $Z_R[J]$.

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