The Design of Power System Stability Controller Based on the PCH Theory and Improved Genetic Algorithm

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Abstract. Low frequency oscillation is still frequently happened in the power system and it affects the safety and stability of power system directly. With the continuously expending of the interconnection scale of power grid, the risk of low frequency oscillation becomes more and more noticeable. Firstly, the basic theory of port-controlled Hamilton (PCH) and its application is analyzed. Secondly, based on the PCH theory and the dynamic model of system, from the viewpoint of energy, the nonlinear stability controller of power system is designed. By the improved genetic algorithm, the parameters of the PCH model are optimized. Finally, a simulation model with PCH is built to vary the effectiveness of the method proposed in this paper.

1. Introduction

In recent years, with the development of the society and economy, the acceleration of the construction of power grid and the expansion of the scale of interconnected power system become the trend of the electric power industry\cite{1}. Although it has brought huge benefits, it also causes serious stability problems at the same time. The most prominent stability problem is the low frequency oscillation\cite{2}.

At first, this paper studies the basic port-controlled Hamilton (PCH) theory and its application. And then, based on the theory of PCH and the dynamic model of system, a port-controlled Hamilton model of the single-machine infinite system is established. The parameter $k$ in the PCH model is usually selected on the basis of experience, therefore, this paper optimizes the value of $k$ in the PCH model by the improved genetic algorithm to make the PCH device’s effect on suppressing low frequency oscillation achieve the best. Finally, through simulation, the feasibility of the optimized PCH device is verified.

2. The theory of PCH

The PCH theory uses the energy functions to design nonlinear controllers\cite{3}. As an open and dynamic system, Hamilton control system has energy exchange with the external environment, and it has dissipation and generation of energy. The PCH theory plays an important role in the development of nonlinear control theory \cite{4}.

2.1. The basic theory of PCH
By introducing the concept of dissipation, the affine nonlinear systems as equation (1) can be used to describe the Port-controlled Hamilton system[5].

\[
\begin{aligned}
\dot{x} &= \left[ J(x) - R(x) \right] \frac{\partial H}{\partial x} (x) + g(x)u \\
y &= g^T (x) \frac{\partial H}{\partial x} (x)
\end{aligned}
\]  

(1)

Where \( x \) means the state vector; \( x \in \mathbb{R}^n \); \( u, y \in \mathbb{R}^m \) means the input and output vectors respectively, which belongs to the conjugate variables, and its product is power; \( R(x) \) is the semi-positive definite symmetric matrix depended on \( x \), and \( R(x) = R^T (x) \geq 0 \) means dissipation. All of these reflect the interconnected structure of the system. \( J(x) \) is the anti-symmetric matrix, which satisfy \( J(x) = -J^T (x) \), and it reflects the interconnected structure of the system. \( H(x) \) means the total stored energy function of the system.

2.2. Using PCH design the dynamic system stabilizer[6-7]

**Definition 1** For the affine nonlinear dynamic system showed in equation (1), if \( u(t) = 0 \), any solution of \( x(t) \) which satisfy \( y(t) = 0 \) can tend to the balance point \( x_0 \), and the system (1) is zero-state detectable.

**Definition 2** For the affine nonlinear dynamic system showed in equation (1), there exists control strategy, as follows.

\[
u_f = -KG(x)g^T \frac{\partial H}{\partial x} (x)
\]  

(2)

Where \( K \) is the positive definite matrix, it makes the nonlinear dynamic system (1) asymptotically stable at balance point \( x_0 \).

Under the action of the control law (2), the closed loop feedback system as follows.

\[
\dot{x} = \left[ J(x) - R(x) - KG(x) \right] g^T \frac{\partial H}{\partial x} (x)
\]  

(3)

If \( \tilde{H}(x) = H_{\text{eq}}(x) - H(x) > 0 \), then

\[
\dot{\tilde{H}}(x) = G^T (x) \left[ \frac{\partial H}{\partial x} (x) \right]^T x = \left[ \frac{\partial H}{\partial x} (x) \right]^T (-R) \left[ \frac{\partial H}{\partial x} (x) \right] + [G^T (x) \frac{\partial H}{\partial x} (x)]^T (-K) [G^T (x) \frac{\partial H}{\partial x} (x)]
\]  

(4)

\( K \) is the positive definite matrix and \( R \) is the positive semi-definite matrix, so \( \tilde{H}(x) \leq 0 \). \( \tilde{H}(x) \) is a Lyapunov function of the closed-loop feedback system (3). \( y = 0, u = 0 \) are derived easily from \( \tilde{H}(x) = 0 \). Because the system (1) is zero-state detectable, the controlled nonlinear dynamic system (4) is stable at the balance point \( x_0 \) under the action of control law \( u_f \).

3. The PCH model of single-machine infinite system

Considering the generator’s excitation control, make an assumption that the generator adopts the three-order suitable model, so the equations as follows can be used to show the single-machine infinite system.
\[
\begin{align*}
\delta &= \omega \\
\omega &= -\frac{D}{T_f} \omega + \frac{1}{T_f} (P_e - E_q' U_s \sin \delta) \\
E_q' &= \frac{1}{T_d} (\frac{X_d}{X_{ds}} E_q' + \frac{(X_d - X_q') U}{X_{qs}} \cos \delta + E_{\mu q} + u_j)
\end{align*}
\]

(5)

In this equation, \(\delta\) means generator’s power Angle; \(\omega\) means the generator’s rotational speed; \(P_m\) means the input mechanical power generator; \(P_e\) means the generator’s output electromagnetic power; \(D\) means the damping coefficient; \(T_f\) means the generator’s inertia constant; \(E_q\) means the electric potential of generator quadrature-axis; \(E_{\mu q}\) means the excitation voltage; \(T_{d0}\) means the direct-axis transient short-circuit time constant; \(X_d\) means the generator’s direct-axis reactance; \(X_q'\) means the generator’s direct-axis transient reactance; \(U_s\) means the infinite bus voltage.

Where

\[x_{d\Sigma} = x_d' + x_L + x_T\]  
(6)

\[x_{d\Sigma} = x_d + x_L + x_T\]  
(7)

Through the the equation (5), assume that the energy function of the single-machine infinite system as follows:

\[H(\omega, \delta, E_q') = \frac{1}{2} T_f \omega^2 - [P_m (\delta - \delta_0)] + \frac{X_{d\Sigma}}{X_{d\Sigma}} E_q' + \frac{E_q' U_s}{X_{d\Sigma}} \cos \delta - \frac{E_q' U_s}{X_{d\Sigma}} \cos \delta_0 + \frac{1}{2} \frac{X_{d\Sigma}}{X_{d\Sigma}} (E_q - \frac{X_{d\Sigma} E_{\mu q}}{X_{d\Sigma}})^2\]  
(8)

The total energy function of the system is called Hamilton function. The partial derivatives of total energy function for variable \(\omega, \delta, E_q\) are derived as follows:

\[\frac{\partial H(\omega, \delta, E_q')}{\partial \delta} = -P_m + \frac{E_q' U_s}{X_{d\Sigma}} \sin \delta\]  
(9)

\[\frac{\partial H(\omega, \delta, E_q')}{\partial \omega} = T_f \omega\]  
(10)

\[\frac{\partial H(\omega, \delta, E_q')}{\partial E_q'} = -\frac{U_s}{X_{d\Sigma}} \cos \delta + \frac{X_{d\Sigma}}{X_{d\Sigma}} (E_q - \frac{X_{d\Sigma} E_{\mu q}}{X_{d\Sigma}})^2\]  
(11)

According to the equations from (1) to (11), make the mathematical model of single-machine infinite system be described as PCH model, as follows.
\[
\begin{align*}
\delta & = \left( -p_n + \frac{E_i U_i \sin \delta}{X_{ZZ}} \right) + \left( \omega_o \right) + \left( \frac{T_o \omega}{T_j} \right) \omega + \left( \frac{1}{T_j} \right) \left( \frac{p_n - E_i U_i \sin \delta}{X_{ZZ}} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) + g(x) \\
E_i & = \left( \frac{\omega_o}{T_j} \right) \omega + \left( \frac{1}{T_j} \right) \left( \frac{p_n - E_i U_i \sin \delta}{X_{ZZ}} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) + E_{\omega i} + u_j
\end{align*}
\]

\[(12)\]

From the equation (12), the equation (13) is derived.

\[
[J(x) - R(x)] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\]

\[(13)\]

Because \(R(x)\) is the semi-positive definite symmetric matrix depended on \(I\) and satisfies \(R(x) = R^T(x) \geq 0\), \(J(x)\) is the anti-symmetric matrix, and satisfy \(J(x) = -J^T(x)\), \(J(x)\) and \(R(x)\) are derived as follows.

\[
J(x) = \begin{bmatrix} 0 & \frac{1}{T_j} & 0 \\ \frac{1}{T_j} & 0 & 0 \\ 0 & 0 & \frac{1}{T_{\omega o}} (X_{ZJ} - X_{ZJ}') \end{bmatrix}
\]

\[(14)\]

According to equation (1), the output equation can be obtained:

\[
z_i = \frac{g_j}{d} \frac{\partial H}{d} (x) = \begin{bmatrix} 0 & 0 & \frac{1}{T_{\omega o}} \left( -p_n + \frac{E_i U_i \sin \delta}{X_{ZZ}} \right) \left( \frac{T_o \omega}{T_j} \right) \omega + \left( \frac{1}{T_j} \right) \left( \frac{p_n - E_i U_i \sin \delta}{X_{ZZ}} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{E_{\omega i} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{E_{\omega i} - X_{ZJ}'}{X_{ZJ}'} \right) \end{bmatrix}
\]

\[(15)\]

According to the feedback control law as equation (2), feedback control can be obtained:

\[
u_j = -K G_j (x) \frac{\partial H}{d} (x) = \frac{-k_j}{T_{\omega o}} \left( -p_n + \frac{E_i U_i \sin \delta}{X_{ZZ}} \left( \frac{T_o \omega}{T_j} \right) \omega + \left( \frac{1}{T_j} \right) \left( \frac{p_n - E_i U_i \sin \delta}{X_{ZZ}} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{X_{ZZ} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{E_{\omega i} - X_{ZJ}'}{X_{ZJ}'} \right) \left( \frac{E_{\omega i} - X_{ZJ}'}{X_{ZJ}'} \right) \end{bmatrix}
\]

\[(16)\]

Where K is the positive definite matrix.

4. Improved genetic algorithm

In the references, the value of K is based on experience. But in the simulation analysis, it is found that the value of K changes the oscillation amplitude and the stability time of the system. Therefore, this paper uses the improved genetic algorithm to get the optimal value of K, to achieve the best effect of suppressing the low frequency oscillations.

Genetic algorithm is a kind of random search algorithm which uses biological natural selection and genetic mechanism[8]. It has a capability of global search for nonlinear and complicated problems, and it has strong robustness and comes into widespread use. But the traditional genetic algorithm is
easy to get the local optimal solution, and the convergence rate is too slow, so this paper improves the genetic algorithm[9].

The flow diagram which get the optimal value of $K$ is shown in Figure 1.

![Flow diagram](image)

**Figure 1.** The flow diagram of the improved genetic algorithm

(1) The selection of fitness function

The index that reflects the stabilizer’s control effect on the low frequency oscillation is generally the power amplitude of the oscillation, therefore the object function is

$$J = \min \left\{ \left| P_i \right|, i = 0,1,2,\ldots, n \right\}$$

(17)

Where $\left| P_i \right|$ is the absolute value of oscillation power at time of $i$; $i = [r]+1$; $[0,r]$ is the time of oscillation.

The genetic algorithm is only aimed at the maximum of fitness function, and cannot be negative, so the fitness function takes the reciprocal of $f$.

$$f = 1 / J_{\min}$$

(18)

(2) The improvement of the selection operator and mutation operator[10]

Selection operator: Traditional method of generating initial population randomly results that the convergence speed is slow, and the algorithm is not globally convergent. Therefore, the improved genetic algorithm saves the individuals with the biggest fitness, which do not need to participate in cross and mutation for avoiding damage. These optimal individuals are mixed with the sub-generation groups to form a new population of sub-generation. This improved method can guarantee the global convergence of the algorithm.

Mutation operator: Because the traditional genetic algorithm usually uses a constant mutation probability $p_m$. But with the increase of the number of iterations, the individual differences of the population becomes smaller, the diversity of the population samples cannot be guaranteed. So adaptive strategy is adopted for the mutation operator.

The improved mutation operator is as follow.

$$P_m = A - bT_k$$

(19)
\[ T_k = 1 - \overline{f_{fit}} / f_{max} \]  

Where, \( \overline{f_{fit}} \) is the average of the individual fitness function value at the generation \( k \), \( f_{max} \) is the maximum of the individual fitness function value at the generation \( k \), \( A, b \) are set to the appropriate constant, \( T_k \) is the adaptive parameters. With the increase of the number of iterations, \( T_k \) will continue to decrease, the mutation probability will be increased, so as to avoid the malignant premature convergence.

5. Simulation and Analysis

On MATLAB 7, this paper establishes the PCH model (Figure 2), and puts the PCH model to the single-machine infinite system (Figure 3). By using fling-cut switch, the power output of generator under different circumstances that the PCH device is connected to the single-machine infinite system or not can be obtained. And through file memory, the oscillogram can be showed in one figure for comparison purposes, like figure 4, figure 5 and figure 6.

**Figure 2.** The packaged internal figure of PCH model

Where

\[
K1 = \frac{(X_d - X_d')U_d'}{X_d}, \quad K2 = \frac{1}{T_d}, \quad K3 = \frac{X_d \Sigma}{X_d}, \quad K4 = -\frac{U}{X_d}, \quad K5 = \frac{X_d \Sigma}{X_d \Sigma (X_d - X_d')},
\]

\[
K6 = \frac{X_d \Sigma}{X_d \Sigma}, \quad K7 = -\frac{k_1}{T_d}, \quad K8 = 1, \quad K1 = \frac{(X_d - X_d')U_d'}{X_d}, \quad K4 = -\frac{1}{X_d \Sigma}.
\]
Figure 3. The Single machine infinite system with PCH model

The generator parameters as follows. \( X_d \) is 1.03, \( X_d' \) is 0.289, \( X_d'' \) is 0.208, \( x_q \) is 0.68, \( X_q \) is 0.230464, \( T_d \) is 2.869, \( T_d' \) is 0.097.

During simulation, the whole simulation time is set to 8s. When the simulation time is 2s, a disturbance (three-phase short-circuit fault) is put to the far end of the system to make the system oscillate. And connecting the switch of PCH device to calm down the oscillation.

Then through running the improved genetic algorithm, we can get that the model is optimized when \( P_m \) is set to 0.01 ~ 0.1. And the results show that when \( k \) is set to 1.51, the stability controller can offer the best results for suppressing low frequency oscillation. The oscillogram is shown in figure 4. And in order to make comparisons on the effect of the improved genetic algorithm, we can choose \( k = 0.01 \) and \( k = 10.2 \) at random, then we can get the figure 5 and figure 6 in sequence.

Figure 4. The oscillogram of generator output power when \( k = 1.51 \)
In the figures, the solid line shows the oscillating curve when the system connected with the PCH device, the imaginary line shows the oscillating curve when the system not connected with the PCH device.

The result analysis: From the simulated oscillogram, it is obvious that when the system is not connected with the PCH device, the maximum oscillation amplitude is about 0.61, the oscillation calms down at about 6s. When the system is connected with the PCH device, the maximum oscillation amplitude is about 0.46 on average, the oscillation calms down and the system becomes stable at about 4.5s. The results show that the maximum oscillation amplitude is much smaller when the system is connected with PCH device than without PCH device, and the PCH device can help the system calm down the oscillation effectively. And during the period of oscillation, when \( k = 1.51 \), the maximum oscillation amplitude is about 0.42. When \( k = 0.01 \), the maximum oscillation amplitude is about 0.49. When \( k = 10.2 \), the maximum oscillation amplitude is about 0.48. The results show that the optimized PCH device by the improved genetic algorithm can make the oscillation amplitude decrease.

6. Conclusion
This paper establishes a port-controlled Hamilton model of single-machine infinite system through the dynamic model of power system and the theory of PCH. And by the improved genetic algorithm, the parameters of the PCH model of single-machine infinite system are optimized. Through the connection of PCH model and the different value of K, the effects on suppressing the low frequency oscillations are compared. And the results show that the PCH device can help the system calm down the oscillation effectively and the improved genetic algorithm can make the parameter \( k \) in the PCH model get the optimal value so that the effect on suppressing the low frequency oscillations can reach the best.

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