The characteristics of failure among students who experienced pseudo thinking

D Anggraini, T A Kusmayadi and I Pramudya

Mathematics Education, Sebelas Maret University
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, Indonesia

Email: dellaramandha@gmail.com

Abstract. The purpose of this research is to describe the thinking process of students who experienced pseudo thinking when solving the straight line equation. The result of this study shows the characteristics of error that caused students to experience pseudo thinking when solving the problem and their relation with students’ metacognition skill. This qualitative research was conducted in State 16 Junior High School in Surakarta, Indonesia during the odd semester of 2017/2018 academic year. The subjects of the study were students Junior High School students of 8th grade chosen using purposive sampling technique. Data were collected through the administration of think aloud method. The result showed that the characteristics of errors among the subjects are: 1) the answers resulted from pseudo thinking when solving the problem were obtained from the spontaneous, fast, unconscious and uncontrolled thinking process; 2) students had misconception; 3) students had tendency to memorize the formula and imitate the completion procedure; 4) students experienced fuzzy memory when solving the problem. From the mistakes among students who experienced pseudo thinking, their metacognition ability could be inferred.

1. Introduction

When solving a math problems, there are two possible results students that can be given by the students, correct and incorrect answers. However, the correct answers may not be merely resulted from the right thinking process as the incorrect ones are not merely resulted from the wrong thinking process. In other words, when solving math problems, students can experience pseudo thinking. The pseudo thinking term in Indonesian educational field was firstly introduced by Subanji in 2007 on his dissertation entitled “The Thinking Process of Pseudo's Reasonable Reasoning in Constructing the Dynamic Genesis Functions Chart”.

The pseudo thinking has been studied by many experts using different terms and contexts: Pseudo analytic versus Pseudo conceptual in the context of routine mathematical problem solving [1]; Established Experience (EE) versus Plausible Reasoning (PR) in the context of problem solving non routine [2]; Direct Translation Approach (DTA) versus Meaning Based Approach (MBA) in the context of solving word problems [3]; Dual Process Theory of Kahneman (Process System 1 versus Process System 2) in the context of solving algebra problems [4]; The true pseudo construction versus the false pseudo construction in constructing mathematical concepts and solving mathematical problems [5].

The given answers of the pseudo thinking are divided into two; the true pseudo thinking and the false pseudo thinking [6]. The true pseudo thinking happens when the students get the correct answers
but in fact students’ reasoning is wrong while the false pseudo thinking happens when the students’ answers are incorrect, but after reflection, the students are able to give reasons correctly [7].

The pseudo thinking is one of students’ errors in constructing mathematical concepts [8]. The results of Subanji and Nusantra’s research [9] show that students often seem to understand the concept well based on the final results they obtained when solving the problem, but after deeper tracing, it turns out that the process of thinking is wrong. These kinds of thinking process are known as the pseudo thinking process.

Based on the preliminary study conducted in State 16 Junior High School in Surakarta, Indonesia among 8th grade students when the students were asked to determine the gradient line $6x - 3y + 10 = 0$, most of the students determined the line gradient by using the formula $m = -\frac{a}{b}$ so obtained $m = 2$.

However, when students were asked why they used the formula and how they got it, and what $a$ and $b$ meant, students were unable to explain. Furthermore, when students were given similar problem as the previous to determine the gradient line $8y - 4x + 16 = 0$, most of the students replied that the line gradient was the same as the line gradient in the previous problem that is $m = 2$, despite their knowledge that in determining the gradient of the line students being able to use the formula $m = -\frac{a}{b}$.

However, students’ thinking was dominated by thinking fast without the reflection process so that they chose the incorrect answer. These conditions show that students experienced pseudo thinking when determining the line gradient.

The pseudo thinking is defined as the visible result of the problem solving process, and is not the result of a genuine thinking process, but rather comes from a pseudo thinking process that occurs when students learn or when solving a problem [10]. The mistakes among students experiencing pseudo thinking in constructing concepts and solving mathematical problems need to get attention as if they are not resolved soon, those errors will have a consequential impact on subsequent math problems [11].

Pseudo thinking process is related to student’s metacognition skill. Metacognition skill is a conscious knowledge and someone’s control into thinking process and result [12]. Students with pseudo thinking showed that they experienced difficulty in using metacognition skill in solving math problem. For instance, students with true pseudo thinking, even if they produced the correct answer, their thinking process in solving the answer was still wrong and they did not understand what their fault was. Those findings showed inability among students who experienced pseudo thinking process in identify the difficult metacognition.

Metacognition in this research is someone’s consciousness towards thinking process and result. Metacognition includes three aspects, which are the knowledge of strategy, the knowledge of cognitive tasks and the self-knowledge [13]. Not all students were able to use their metacognition well as well as with students who experience pseudo thinking. The inability to identify the difficulties metacognition experienced by students.

Student who experiences difficulty in using metacognition skill are those who were unable or having difficulties to use metacognitive knowledge. Metacognitive knowledge is a knowledge about consciousness in general is the same thing as vigilance and knowledge about self-knowledge [12]. Metacognitive knowledge consists of the knowledge of strategy, the knowledge of cognitive tasks and the self-knowledge [13].

Based on those problems, it is important to examine the thinking process and errors of students who experienced pseudo thinking when solving math problem in straight line equation in order to find out students’ metacognition skills. This case was influenced by students’ metacognition skill in solving the problem, especially in math problem influencing learning process and students’ performance. For those reasons, this research examines the thinking process of students who experienced pseudo thinking and the errors of students who experienced pseudo thinking in solving math problem in the straight line equation. In addition, this research was also aimed to find out metacognition skill of students who experienced pseudo thinking.
2. Research Method
This research is a qualitative research that aims to describe the thinking process of students who experienced the pseudo thinking when solving the problems of straight line equation in order to reveal what mistakes were made by students who experienced pseudo thinking in solving the problems of straight line equation.

The instruments of data collection in this study was a sheet of straight line equation and interview guidelines. Interviews were used to find out whether students experienced pseudo thinking. Based on their work actually experience pseudo thinking or not and explore more about the thinking process delivered by the students. In this research, to increase the validity of the data, the triangulation method was used in the steps of verification in which the results of test data obtained were compared with the results of interviews.

This research was conducted in State 16 Junior High School in Surakarta, Indonesia on 8th grade. The research subjects were chosen with purposive sampling method. Students chosen as the subjects were those who met the following criteria: the subjects experienced pseudo thinking when solving the straight line equation problem and was able to communicate their thought well through speaking or writing so the exploration of student learning process could be done optimally.

The data collection in this research was done by think aloud method where students were given the sheet of straight line equation to solve. In the process of solving the problems the students were asked to express what students was thinking. After the solving the problems, the researcher then grouped the students’ answers into correct and incorrect answers. Students with correct answers were asked to justify the answer. Students who were not able to justify the answer were classified as subjects experiencing the true pseudo thinking. Students with incorrect answer were given the opportunity to do the reflection. Students who were able to correct the answers after the reflection and explain the answers correctly were classified as subjects experiencing the false pseudo thinking.

3. Result and Discussion
Based on the data collected from the sheet of straight line equation given to 26 students of 8th grade in State 16 Junior High School in Surakarta, Indonesia, the student answers can be categorized as follows.

| Category             | Students |
|----------------------|----------|
| True pseudo thinking | 4        |
| False pseudo thinking| 8        |
| Totally right        | 2        |
| Totally wrong        | 12       |

Based on Table 1, it can be inferred that almost half of the students experienced pseudo thinking when solving the straight line equation. However, in this article, the researcher describe the pseudo thinking process of the students with 2 samples: a student who experienced true pseudo thinking (subject 1) and a student who experienced false pseudo thinking (subject 2). The description of those 2 subjects were expected to represent those who could describe the actual conditions in the field. The following image describes the results of the work of subject 1 and subject 2 when solving the sheet of straight line equation.

Find the line equation through \((-2,4)\) and

a. Parallel to line \(4x - 2y + 12 = 0\)

b. Perpendicular to line \(9y - 3x + 18 = 0\)

Figure 1. The question of straight line
3.1 Subject 1 (S1)

Based on S1’s answer for the (a) question, S1 was wrong in changing $4x - 2y + 12 = 0$ to $y = mx + c$ although S1 was correct in determining gradient line $4x - 2y + 12 = 0$. Other than that, S1 mentioned $m_t$ as $4x - 2y + 12 = 0$ gradient line. Whereas on the question, not mentioned $t$ line, S1 did not put the line as a $t$ line. Furthermore, S1 also did not write a parallel two line gradient relationship, so it was unknown whether S1 really knew the gradient relationship of two parallel lines or not.

When solving the (b) question, S1 was also wrong in changing the $9y - 3x + 18 = 0$ to $y = mx + c$ even though S1 was correct in determining the gradient of line $9y - 3x + 18 = 0$. Other than that, S1 also still wrote the initial $m_t$ as the grid of the line $9y - 3x + 18 = 0$ and did not write a two line perpendicular gradient relationship, so it was unknown whether S1 really knew the two line perpendicular gradient relationship or not.

Based on the steps given by S1 when solving the problem, it seemed S1 experienced the true pseudo thinking, to convince researcher whether S1 experienced the true pseudo thinking or not based on the results of his work when completing the problem, the researcher conducted interviews with S1. There are “R” for Researcher and “S1” for Subject 1.

**Excerpt 1**

R : Are you sure that you change line equation of $4x - 2y + 12 = 0$ and $9y - 3x + 18 = 0$ into the general form of line equation is already correct?
S1: The general form of line equation is \( y = mx + c \), isn’t it?
R: Yes, now. Can you explain to me how you change those line equation?
S: Yes, I just have to change that \( y = -\frac{4}{2}x + 12 \) and \( y = \frac{3}{9}x - 18 \).
R: So, the 12 isn’t divided with negative 2 and the negative 18 isn’t divided with 9, why is it?
S1: We just need to find the gradient, so only the number in front of \( x \) will be divided with the number in front of \( y \)
R: So, what is the number in front of \( x \) and the number in front is \( y \)? Is it variable or coefficient?
S1: It is coefficient
R: Are you sure 12 and 18 should not be divided with \( y \) coefficient?
S1: I’m sure. Because we just need to find the gradient from those line

Based on the interview results, it is known that S1 has not understood how to change the form of the equation line into form correctly. Although the result was only coincidence of S1 results in determining the correct gradient of the line, the construction process was not appropriate (having an error) in changing the form of the equation line. This is because S1’s process of thinking in changing the form of the equation line was done spontaneously, quickly, automatically, unconsciously and without any control.

Excerpt 2

R: Do you remember the requirement of two parallel gradient lines?
S1: Same
R: Here you wrote requirement of two parallel gradient lines, but directly input the gradient line known into the line that will be looked for. Why is it?
S1: If the question is like this, I will directly input it into the formula, miss.

From the excerpt of the interview, it is known that S1 had understood about the relationship of two parallel lines gradient. Although the result of S1’s completion does not write the relationship of two parallel lines, it is because the habit of S1 does not write the concepts, what matter is the result.

Excerpt 3

R: So, in the \((b)\) question. It will find the perpendicular line to \( 9y - 3x + 18 = 0 \), what is the requirement of a two line perpendicular gradient?
S1: (the student was silent)
R: Here when you find the line equation that through point \((-2,4)\) and perpendicular with \( 9y - 3x + 18 = 0 \), you used \( y - y_1 = -\frac{1}{m_t}(x - x_1) \). Why did you use those formula?
S1: Because in my notebook the formula to determine the perpendicular line equation is like that
R: So, Do you know what is the requirement of two perpendicular line?
S1: I forget, miss. I remembered, but like those the formula and based on the exercise given by you also for determine the line that through points \((x_1, y_1)\) and perpendicular with the other line that using those formula

The result of the interview shows that S1 does not understand the concept of a two line perpendicular gradient relationship. This is because, S1 only memorizes the formula and imitate the completion procedure that causes the reasoning S1 in constructing the concept of lines that are perpendicular to each other is not developing optimally.

Excerpt 4

R: I want to ask about the gradient that you already wrote in the question \((a)\) and \((b)\) question, you wrote \( m_t \), what is it about?
Based on the results of the interviews, S1 misunderstood the writing of gradients, as several times S1 wrote $m_t$, $m_x$ or $m_y$. In other words, S1 had misconception of gradient writing. Misconception is defined as an interpretation of concepts in an unacceptable statement. Therefore, the results of the work and interview results shows that S1 experienced true pseudo thinking. Although S1 was by chance able to find the correct answer, but the reason given by S1 is not right. This can be seen from the S1’s wrong process of thinking in changing the form of the line equation $ax + by + c = 0$ into the form $y = mx + c$, wrong in understanding the gradient writing, and not understanding the concept of the two line perpendicular gradient relationship.

The mistakes made by S1 were related to metacognition skill of S1 in understanding the concept of straight line equation. Based on the work and interview result, it was found that the metacognition of S1 in self-knowledge aspect was that S1 felt confident that problem completion steps was right, despite the wrong implementation. This case was taken from S1’s result in which the mistake in changing the line form $ax + by + c = 0$ to general form of straight line equation $y = mx + c$ to find out the known gradient was found. The case with metacognition of S1 in the knowledge of cognitive tasks was that S1 knew what formula that should be applied to solving the problem, however S1 did not understand the concept applied appropriately. This is in accordance with the interview result in which the fact that S1 did not understand the concept of gradient relationship of two lines perpendicular to each other was found. The case with metacognition of S1 in aspect of strategy was that S1 could not mention the reasons why S1 used $(y - y_1) = -\frac{1}{m_t} (x - x_1)$ formula in determining straight line equation appropriately and S1 also failed to reason why S1 failed to writing $m_t$ initial appropriately.

This was caused by S1’s thinking process in solving the problem was done in spontaneous, quick, automatic, unconscious and uncontrolled manner in changing the straight line equation form. S1 also has a habit to only memorize the formula and imitate the solving procedure that caused S1’s reasoning in constructing the concept of straight line equation to be wrong even if the final result was correct.

### 3.2 Subject 2 (S2).

| Equation | Description |
|----------|-------------|
| $4x - 2y + 12 = 0$ | $b = 2$ |
| $(-2, 4)$ | $x = -2$ |
| $y = 4$ | $y - y_1 = m(x - x_1)$ |
| $y - 4 = -2(x - (-2))$ | $y = -2x + 4$ |
| $y = -2x + 4l$ | $y = -2x + 4 + 8 = 0$ |

**Figure 4.** The S2’s answer for the (a) question
Based on Figure 4, S2’s answer for the (a) question it can be inferred that S2 knew that to determine the line gradient in the form of \( ax + by + c = 0 \) is \( m = \frac{-a}{b} \), with \( b \neq 0 \). However, S2 was incorrect in determining the \( y \) coefficient on the line \( 4x - 2y + 12 = 0 \). Whereas the coefficient of \( y \) should be negative 2. In addition, S2 did not write a parallel two line gradient relationship so it cannot be known whether S2 really knows the relationship of two parallel or parallel gradient lines.

![Figure 5](image)

**Figure 5.** The S2’s answer for the (b) question

From S2’s answer when solving the (b) question, S2 made a mistake which is the same as the mistake in (a) question. S2 made mistake in determining the \( x \) coefficient on the line \( 9y - 3x + 18 = 0 \). When solving (b) question, the error made by S2 was equal to the error made in the (a) question in which S2 wrongly determined the coefficient \( x \) on the line \( 9y - 3x + 18 = 0 \). However, in the (b) question S2 wrote the two perpendicular gradient relation. S2 gave the line known as the first line having a gradient \( m_1 \) and the line sought as the second line having a gradient \( m_2 \). However, since S2 was wrong in determining the first line gradient (\( m_1 \)) then in determining the second line gradient (\( m_2 \)) S2 still got the incorrect answer.

Based on the steps given by S2 when solving the problem, it seems that S2 is having false pseudo thinking, to convince the researcher whether S2 had false pseudo thinking or not based on the result of his work when solving the problem, the researcher conducted interview with S2. As for “R” for Researcher and “S2” for Subject 2.

**Excerpt 1**

| R          | S2: Okay, in the (a) question and (b) question. Do you know your mistake? |
|------------|----------------------------------------------------------------------------|
| R: True.   | What you actually should do?                                               |
| S2: b value | Should be negative 2 because the y coefficient is negative 2, so the line gradient |
|            | \( 4x - 2y + 12 = 0 \) has \( m = 2 \) gradient and the line gradient \( 9y - 3x + 18 = 0 \) is \( m = \frac{-(-3)}{9} \) |
|            | because the x coefficient is negative 3, so \( a = -3 \). Is it true, miss? |
| R: true.   | How did you get the wrong answer when you do the test?                     |
| S2: I actually forget about the sign in front of the x coefficient or y coefficient can be impacting or not in determining a and b value |

Based on the interview result above, it can be seen that after doing the reflection, S2 was able to improve the answer to be the correct answer in determining the line gradient. However, because the
thinking process of S2 in constructing the concept of determining the gradient is generated from fuzzy memory, it caused the incorrect answer.

**Excerpt 2**

| R | Do you the the requirement of two parallel line? |
|---|---------------------------------------------|
| S2 | They have the same gradient |
| R | Who has the gradient? |
| S2 | The parallel line |
| R | Why did you put the gradient directly into the formula? |
| S2 | Because, it is the same. So, I just put it into the formula. I don’t have to write down the requirement. |

From the excerpts, it is known that S2 could understand the concept of the relation of two parallel line. Based on the result of S2's answer when solving the question and the interview result, it can be seen that S2 experienced false pseudo thinking.

S2’s metacognition in aspect of cognitive task knowledge was shown by the finding that S2 knew what formula that should be applied when solving the problem, but had an error in deciding determine $y$ coefficient in the $(a)$ question and $x$ coefficient in the $(b)$ question that caused S2’s answer to be incorrect in determining the gradient. However, after the reflection activity, S2 showed metacognition in self-knowledge aspect which are S2 aware of the mistakes made resulting in the final result answer and on strategy aspect, S2 was able to manage or control his knowledge to fix the answer become the correct one by rechecking the answer and fixing the mistake when determines the line gradient.

It is because S2 was actually able to reason correctly when solving the question, but as thinking process in constructing the concept of determining the gradient was generated from fuzzy memory, the final result was impacted. However, actually S2 was able to understand the concept of two parallel lines and the concept of two lines that are perpendicular to each other.

### 4. Conclusion

The results showed that there were some error characteristics students tend to have in solving straight line equations that can cause them to experience pseudo thinking. First, the thinking process that students do when solving problems results from spontaneous, fast, unconscious and unchecked (control or reflection) thinking process. Second, students tend to experience misconceptions. Third, students tend to memorize the formula and imitate the completion procedure that has been given by the teacher. Fourth, students experienced fuzzy memory when solving the problem.

Based on the conclusion of the result of this research, the suggestions given to math teachers in Junior High School level is that they are expected to know how the process of students’ thinking when getting answer in resolving the problem whether the answer produced is in accordance with the right thinking process or not to minimize the occurrence of pseudo thinking. The results of this study need to be explored deeper that it is advisable for advanced researcher to conduct research on other focus like picking up mathematical topics other than straight line equation. Advanced researcher should be able to use certain knowledge construction theory to find the characteristics of student mistakes among students who experienced pseudo thinking. In an instance, the theory of Action, Process, Object, and Schema (APOS) can be used as a knife of analysis so that the description of students’ misconception of students’ pseudo thinking becomes more complete.

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