Research Article

On \(\inf\)-Hesitant Fuzzy \(\Gamma\)-Ideals of \(\Gamma\)-Semigroups

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The notions of an \(\inf\)-hesitant fuzzy \(\Gamma\)-ideal and a \((\sup, \inf)\)-hesitant fuzzy \(\Gamma\)-ideal, which are a generalization of an interval-valued fuzzy \(\Gamma\)-ideal, of a \(\Gamma\)-semigroup are introduced and some properties are investigated. Characterizations of the notions are provided in terms of sets, fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and hesitant fuzzy sets. Furthermore, characterizations of a \(\Gamma\)-ideal of a \(\Gamma\)-semigroup are given in terms of \(\inf\)-hesitant and \((\sup, \inf)\)-hesitant fuzzy \(\Gamma\)-ideals.

1. Introduction

The notion of a fuzzy set, proposed by Zadeh [1], has provided a useful mathematical tool and method for describing the behavior of complex and ill-defined systems. The notion has huge applications in decision making, artificial intelligence, automata theory, control engineering, finite state machine, expert, graph theory, robotics, and many branches of pure and applied mathematics (cf. [2]). Nevertheless, there are limitations for using the notion to deal with vague and imprecise information when different sources of vagueness appear simultaneously. In order to overcome such limitations, Torra and Narukawa [3, 4] proposed an extension of the notion so-called a hesitant fuzzy set which is a function from a reference set to a power set of the unit interval. Hesitant fuzzy set theory has been applied to several practical problems, primarily in the area of decision making (see [5–9]) and different algebraic structures, for example, Jun and Ahn [10] introduced hesitant fuzzy subalgebras and hesitant fuzzy ideals of BCK/BCI-algebras and investigated related properties. Mosrijai et al. [11–13] studied hesitant fuzzy sets on UP-algebras. Kim et al. [14] studied the concepts and properties of a hesitant fuzzy subgroupoid (left ideal, right ideal, and ideal) of a groupoid, a hesitant fuzzy subgroup (normal subgroup and quotient subgroup) of a group, and a hesitant fuzzy subring (left ideal, right ideal, and ideal) of a ring. Jittburus and Julatha [15] proposed the concepts of a sup-hesitant fuzzy ideal of a semigroup and its sup-hesitant fuzzy translations and sup-hesitant fuzzy extensions. They showed that the sup-hesitant fuzzy ideal is a general concept of a hesitant fuzzy ideal and an interval-valued fuzzy ideal and gave its characterizations in terms of sets, fuzzy sets, hesitant fuzzy sets, and interval-valued fuzzy sets. Julatha and Iampan [16] introduced sup-types of hesitant fuzzy sets based on ideal theory of ternary semigroups and examined their properties via a fuzzy set, an interval-valued fuzzy set, and a hesitant fuzzy set.

In 1981, Sen [17] introduced the concept and notion of the \(\Gamma\)-semigroup as a generalization of the plain semigroup and ternary semigroup. Many classical notions and results of (ternary) semigroups have been extended and generalized to \(\Gamma\)-semigroups, by many mathematicians, for instance, Siripitukdet and Iampan [18, 19], Siripitukdet and Julatha [20], Dutta and Adhikari [21, 22], Saha and Sen [23–25], Hila [26, 27], and Chinram [28, 29]. Simuen, Iampan, Chinram, Sardar, Majumder, Dutta, and Davvaz [30–35] studied theory of \(\Gamma\)-semigroups via fuzzy subsets. Uckun et al. [36] studied theory of \(\Gamma\)-semigroup via intuitionistic fuzzy subsets. Abbasi et al. [37] introduced hesitant fuzzy left (resp.,
right, bi-, interior, and two-sided) $\Gamma$-ideals of $\Gamma$-semigroups and characterized simple $\Gamma$-semigroups by hesitant fuzzy sets. Julatha and Lampam [38] introduced a sup-hesitant fuzzy $\Gamma$-ideal, which is a general concept of an interval-valued fuzzy $\Gamma$-ideal and a hesitant fuzzy $\Gamma$-ideal, of a $\Gamma$-semigroup and studied its properties via level sets, fuzzy sets, interval-valued fuzzy sets, and hesitant fuzzy sets.

In this paper, the notions of an inf-hesitant fuzzy $\Gamma$-ideal and a (sup, inf)-hesitant fuzzy $\Gamma$-ideal, which are a general notion of an interval-valued fuzzy $\Gamma$-ideal, of a $\Gamma$-semigroup are introduced and their properties are investigated. Equivalent conditions for a hesitant fuzzy set to be an inf-hesitant fuzzy $\Gamma$-ideal and a (sup, inf)-hesitant fuzzy $\Gamma$-ideal are provided in terms of sets, fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and hesitant fuzzy sets. We show that every interval-valued fuzzy set on a $\Gamma$-semigroup is an interval-valued fuzzy $\Gamma$-ideal if and only if it is a (sup, inf)-hesitant fuzzy $\Gamma$-ideal. Furthermore, characterizations of a $\Gamma$-ideal of a $\Gamma$-semigroup are given in terms of inf-hesitant and (sup, inf)-hesitant fuzzy $\Gamma$-ideals.

2. Preliminaries

We will introduce some definitions and results that are important for study in this paper.

First, we recall the definition of $\Gamma$-semigroups which is defined by Sen and Saha [25]. By a $\Gamma$-semigroup, we mean a nonempty set $G$ with a nonempty set $\Gamma$ and a mapping $G \times \Gamma \times G \rightarrow G$, written as $(u, \gamma, v) \rightarrow uv\gamma$ satisfying the identity $(uvw)\delta w = u\gamma v\delta w$ for all $u, v, w \in G$ and $\gamma, \delta \in \Gamma$. From now on throughout this paper, $G$ is represented as a $\Gamma$-semigroup and $X$ a nonempty set unless otherwise specified. For nonempty subsets $U$ and $V$ of $G$, let $UTV = \{uv\mid u \in U, v \in V, \gamma \in \Gamma\}$. By a $\Gamma$-ideal (IId) of $G$, we mean a nonempty subset $V$ of $G$ such that $GTV \subseteq V$ and $VTG \subseteq V$. Then, a nonempty subset $V$ of $G$ is an IId of $G$ if and only if $uv\gamma \subseteq V$ for all $u \in G$, $v \in V$, and $\gamma \in \Gamma$.

A fuzzy subset (FS) $\{0, 1\}$ of $X$ is a function from $X$ into the unit segment of the real line $[0, 1]$. A FS $\phi$ of $G$ is called a fuzzy $\Gamma$-ideal (IFTId) of $G$ if

$$\max\{\phi(u), \phi(v)\} \leq \phi(uv\gamma), \text{ for all } u, v \in G \text{ and } \gamma \in \Gamma. \tag{1}$$

An intuitionistic fuzzy set (IFS) $A$ [39] in $X$ is an object having the form $A = \{(x, \phi(x), \psi(x))\mid x \in X\}$, where the functions $\phi$ and $\psi$ are defined on $X$ into $[0, 1]$ and $\phi(x) + \psi(x) \leq 1$ for all $x \in X$. An IFS $A = \{(x, \phi(x), \psi(x))\mid x \in X\}$ in $X$ can be identified to an ordered pair $(\phi, \psi)$ in $[0, 1]^2 \times [0, 1]^2$. For a FS $\phi$ of $X$, we define a FS $\phi(2)$ by $\phi(2)(x) = (\phi(x)/2)$ for all $x \in X$. Then, $((\phi/2), (2))$ is an IFS in $X$ for all FSs $\phi$ and $\psi$ of $X$. An IFS $(\phi, \psi)$ in $G$ is called an intuitionistic fuzzy $\Gamma$-ideal (IFTId) [36] of $G$ if the following two conditions hold:

(i) (IFTId1) $\phi(uv\gamma) \leq \min\{\phi(u), \phi(v)\}$ for all $u, v \in G$ and $\gamma \in \Gamma$

(ii) (IFTId2) $\phi(uv\gamma) \leq \min\{\phi(u), \phi(v)\}$ for all $u, v \in G$ and $\gamma \in \Gamma$

By an interval number $\overline{a}$, we mean an interval $[a^-, a^+]$, where $0 \leq a^- \leq a^+ \leq 1$. The set of all interval numbers is denoted by $\mathcal{D}[0, 1]$. Especially, we denote $T := [1, 1]$ and $\overline{b} := [0, 0]$. For two elements $\overline{a} = [a^-, a^+]$ and $\overline{b} = [b^-, b^+]$ in $\mathcal{D}[0, 1]$, define the operations $\cdot, =, \leq$ and rmax in case of two elements in $\mathcal{D}[0, 1]$ as follows:

(i) $\overline{a} \cdot \overline{b} \rightarrow a^+ \leq b^+$

(ii) $\overline{a} = \overline{b} \rightarrow a^- = b^- \text{ and } a^+ = b^+$

(iii) $\overline{a} \cdot \overline{b} \rightarrow a^+ \leq b^+$ and $\overline{a} \cdot \overline{b} \rightarrow a^- \geq b^-$

(iv) $r \max\{\overline{a}, \overline{b}\} = [\max\{a^+, b^+\}, \max\{a^-, b^-\}]$

Denote the case that $a^+ > b^-$ or $a^+ > b^+$ by $\overline{a} \not\leq \overline{b}$. A function $\omega: X \rightarrow \mathcal{D}[0, 1]$ is called an interval-valued fuzzy set (IVFS) [40] on $X$, where $\omega(x) = [a^-(x), a^+(x)]$ for all $x \in X$ and $\omega^-$ and $\omega^+$ are FSs of $X$ such that $\omega^-(x) \leq \omega^+(x)$ for all $x \in X$. Let IVFS($X$) be the set of all IVFSs on $X$. An IVFS $\omega$ on $G$ is called an interval-valued fuzzy $\Gamma$-ideal (IFTId) of $G$ if

$$r \max\{\omega(u), \omega(v)\} \leq \omega(uv\gamma), \text{ for all } u, v \in G \text{ and } \gamma \in \Gamma. \tag{2}$$

Then, $\omega$ is an IVFID of $G$ if and only if $\omega(u) \leq \omega(uv\gamma)$ and $\omega(v) \leq \omega(uv\gamma)$ for all $u, v \in G$ and $\gamma \in \Gamma$.

A hesitant fuzzy set (HFS) [3, 4] on $X$ in terms of a function $\overline{\psi}$ is that when applied to $X$ returns a subset of $[0, 1]$, that is, $\overline{\psi}: X \rightarrow \mathcal{P}[0, 1]$, where $\mathcal{P}[0, 1]$ denotes the set of all subsets of $[0, 1]$. Let HFS($X$) be the set of all HFSs on $X$, that is, HFS($X$) = $\{\overline{\psi} \mid \overline{\psi}: X \rightarrow \mathcal{P}[0, 1]\}$ and let HFS*($X$) = $\{\overline{\psi} \in \text{HFS}($$X$$)\mid \overline{\psi}(x) \neq \emptyset \text{ for all } x \in X\}$. Then, IVFS($X$) $\subseteq$ HFS*($X$) $\subseteq$ HFS($X$). A HFS $\overline{\psi}$ on $G$ is called a hesitant fuzzy $\Gamma$-ideal (HFTId) [37] of $G$ if

$$\overline{\psi}(u) \cup \overline{\psi}(v) \subseteq \overline{\psi}(uv\gamma), \text{ for all } u, v \in G \text{ and } \gamma \in \Gamma. \tag{3}$$

Then, $\overline{\psi}$ is a HFTId of $G$ if and only if $\overline{\psi}(u) \subseteq \overline{\psi}(uv\gamma) \cup \overline{\psi}(uv\gamma)$ for all $u, v \in G$ and $\gamma \in \Gamma$.

For $\overline{\psi}$ in HFS($X$) and $\overline{\psi} \supseteq \mathcal{P}[0, 1]$, we define the element $\text{SUP}(\overline{\psi})$ of $[0, 1]$, the subset $\text{SUP}(\overline{\psi}) \cap \mathcal{P}([0, 1])$ for all $x \in X$. We denote $HFS(\mathcal{P}[0, 1])$ by $HFS(\mathcal{P}[0, 1])$, and then, $HFS(\mathcal{P}[0, 1]) \subseteq HFS(\mathcal{P}[0, 1])$.

Julatha and Lampam [38] introduced a sup-hesitant fuzzy $\Gamma$-ideal, which is a generalization of the concepts of an IVFID and a HFTId, of a $\Gamma$-semigroup and studied its properties in terms of FSs, IFSs, HFSs, and IVFSs in the following.

Definition 1 (see [38]). Given $V \in \mathcal{P}([0, 1])$, a HFS $\overline{\psi}$ on $G$ is called a sup-hesitant fuzzy $\Gamma$-ideal of $G$ related to $V$ (briefly,
Theorem 3

Every \( \text{IvFIIId} \) of \( G \) is a sup-\( \Gamma \)Id of \( G \).

Lemma 1 (see [38]). For any \( \psi \in \text{HFS}(G) \), the following are equivalent:

1. \( \psi \) is a sup-\( \Gamma \)Id of \( G \)
2. \( \mathcal{F} \psi \) is a \( \Gamma \)Id of \( G \)
3. \( \mathcal{H} \psi \supset \) is a HFIIId of \( G \)
4. \( \mathcal{H} \psi \supset \) is an invHFIIId of \( G \)
5. \( \mathcal{H} \psi \supset \) is a sup-HFIIId of \( G \)
6. \( \mathcal{H} \psi \supset \mathcal{V} \supset \) is a HFIIId of \( G \) for all \( \forall \in \mathcal{P}([0,1]) \)

3. inf-Hesitant Fuzzy \( \Gamma \)-Ideals

For a HFS \( \psi \) on \( X \) and an element \( \forall \in \mathcal{P}([0,1]) \), define INF and \( [\psi;\forall] \) by

\[
\text{INF} = \begin{cases} \inf \forall, & \text{if } \forall \neq \emptyset, \\ \emptyset, & \text{otherwise,} \end{cases}
\]

Given \( \psi \in \text{HFS}(X) \), the HFS \( \mathcal{H} \psi \), defined by \( \mathcal{H} \psi(x) = [1 - \text{SUP}(x)] \) for all \( x \in X \), is called the suprema complement [13, 38] of \( \psi \) on \( X \). Then, \( \text{SUP}(x) = 1 - \text{SUP}(x) \) for all \( x \in X \) and \( (\mathcal{F} \psi, \mathcal{V} \psi) \) is an IFS in \( X \).

Theorem 4 (see [38]). For a nonempty subset \( Y \) of \( G \), the following are equivalent:

1. \( Y \) is a \( \Gamma \)Id of \( G \)
2. The ClvFS CI\(_Y\) is a sup-HFIIId of \( G \)
3. The CHFS CH\(_Y\) is a sup-HFIIId of \( G \)
4. \( \chi_{Y}^{(\Delta,\forall)} \) is a sup-HFIIId of \( G \) for all \( \forall \in \mathcal{P}([0,1]) \) with \( \text{SUP} < \text{SUP} \).

\[ \text{INF} = \{ x \in X | \text{INF}(x) \geq \text{INF} \}, \]

Note that for all \( x \in X \) and for all \( \omega \in \text{IVFS}(X) \), we have \( \text{INF}(x) = \text{INF}(x) = \min \omega(x) = \omega(x) \). Now, we introduce the notion of an inf-hesitant fuzzy \( \Gamma \)-ideal of a \( \Gamma \)-semigroup in the following definition.

Definition 2. A HFS \( \psi \) on \( G \) is an inf-hesitant fuzzy \( \Gamma \)-ideal (inf-HFIIId) of \( G \) if \( \psi \) \( \in \text{IvFIIId}(G) \) for all \( \forall \in \mathcal{P}([0,1]) \) when \( [\psi;\forall] \notin \emptyset \).

Example 1. Let \( Z^- \) be the set of all negative integers, \( G = Z^- \cup \{0\} \), and \( \Gamma = 2G \). Then, \( G \) is a \( \Gamma \)-semigroup with respect to usual multiplication.

1. Define a HFS \( \psi \) on \( G \) by

\[
\psi(u) = \begin{cases} [0.5, 0.6, 0.7], & \text{if } u = 0, \\ [0.3, 0.8], & \text{if } u \in 2Z^-, \\ \emptyset, & \text{otherwise}, \end{cases}
\]

for all \( u \in G \). Then, \( \psi \) is an inf-HFIIId of \( G \) but not a sup-HFIIId of \( G \) because

\[
\text{SUP}(5)(0)(-6) = \text{SUP}(0)(0) = 0.7 < 0.8 = \max\{\text{SUP}(5), \text{SUP}(6)\}.
\]

2. Define a HFS \( \psi \) on \( G \) by

\[
\psi(u) = \begin{cases} [0.1], & \text{if } u = 0, \\ [0.4, 0.8], & \text{if } u \in 2Z^-, \\ [0.5, 0.6, 0.7], & \text{otherwise}, \end{cases}
\]

for all \( u \in G \). Then, \( \psi \) is an HFIId of \( G \) but not an inf-HFIIId of \( G \) because the nonempty subset \( [\psi; [0.5, 0.6]] \) of \( G \) is not a \( \Gamma \)-Id of \( G \), that is,

\[
-1 \in [\psi; [0.5, 0.6]], (-1)(-2)(-3) = -6 \notin [\psi; [0.5, 0.6]].
\]
By Example 1 and Lemma 2, we obtain that an inf-HFId of $G$ is not a sup-HFId and a HFIId of $G$ and a sup-HFIId of $G$ is not an inf-HFId of $G$.

**Lemma 3.** Every IvvFIId of $G$ is an inf-HFId of $G$.

**Proof.** Suppose that $\bar{\omega}$ is an IvvFIId of $G$ and $v \in \mathcal{P}([0, 1])$ such that $[\bar{\omega}; \mathcal{V}]_{\text{INF}}$ is a nonempty set. Let $u, v \in G$, $v \in [\bar{\omega}; \mathcal{V}]_{\text{INF}}$, and $\gamma \in \Gamma$. Since $\bar{\omega}$ is an IvvFIId of $G$, we get $\bar{\omega}(v) < \bar{\omega}(uv)$ and $\bar{\omega}(v) < \bar{\omega}(v\gamma u)$. Thus,

$$\text{INFV} \leq \text{INF} \bar{\omega}(v) = \omega^-(v) \leq \min\{\omega^-(uv), \omega^- (v\gamma u)\} = \min\{\text{INF}(uv), \text{INF}(v\gamma u)\},$$

which implies that $\nu v, \nu v \in [\bar{\omega}; \mathcal{V}]_{\text{INF}}$. Hence, $[\bar{\omega}; \mathcal{V}]_{\text{INF}}$ is a IId of $G$. Therefore, $\bar{\omega}$ is an inf-HFId of $G$. \[\square\]

In the following example, it is shown that the converse of Lemma 3 is not generally true.

**Example 2.** Let $G$ be a $\Gamma$-semigroup defined in Example 1. Define an IVS $\bar{\omega}$ on $G$ by for all $u \in G$,

$$\bar{\omega}(u) = \begin{cases} 1, & \text{if } u = 0, \\ [0.5, 0.7], & \text{if } u \in 2\mathbb{Z}^-, \\ [0.3, 0.8], & \text{otherwise}. \end{cases}$$

Then, $\bar{\omega}$ is an inf-HFId of $G$ but not an IvvFIId of $G$ because

$$\mathcal{R} \max(\bar{\omega}(-1), \bar{\omega}(-3)) = [0.3, 0.8] \notin [0.5, 0.7] = \bar{\omega}(-12) = \bar{\omega}((-1) - (-4))(-3)).$$

By Lemma 3 and Example 2, we obtain that an inf-HFId of a $\Gamma$-semigroup $G$ is a general concept of an IvvFIId of $G$.

For every HFS $\hat{\psi}$ on $X$, define the FS $\mathcal{F}_{\hat{\psi}}$ of $X$ by $\mathcal{F}_{\hat{\psi}}(x) = \text{INF} \hat{\psi}(x)$ for all $x \in X$. A HFS $\hat{\vartheta}$ on $X$ is called an infimum complement of $\hat{\psi}$ on $X$ if $\text{INF} \hat{\vartheta}(x) = 1 - \text{INF} \hat{\psi}(x)$ for all $x \in X$. Let $\mathcal{I}(\hat{\psi})$ be the set of all infimum complements of $\hat{\psi}$. Define the HFS $\hat{\psi}^*$ by $\hat{\psi}^*(x) = \{1 - \text{INF} \hat{\psi}(x)\}$ for all $x \in X$, and then we have $\hat{\psi}^* \in \mathcal{I}(\hat{\psi})$, $\mathcal{F}_{\hat{\psi}^*}(x) = 1 - \text{INF} \hat{\psi}(x)$, and $\text{INF}(\hat{\psi}^*) = \text{INF}(\hat{\psi})(x)$ for all $x \in X$. Note that min$\{1 - t_1, 1 - t_2\} = 1 - \max(t_1, t_2)$ for all $t_1, t_2 \in [0, 1]$.

**Lemma 4.** If $\hat{\psi} \in \text{HFS}^*(G)$ is a HFIIId of $G$, then $\hat{\vartheta}$ is an inf-HFIIId of $G$ for all $\hat{\vartheta} \in \mathcal{I}(\hat{\psi})$.

**Proof.** Suppose that $\hat{\psi} \in \text{HFS}^*(G)$ is a HFIIId of $G$ and $\hat{\vartheta} \in \mathcal{I}(\hat{\psi})$. Let $v \in \mathcal{P}([0, 1]), u \in G, v \in [\hat{\vartheta}; \mathcal{V}]_{\text{INF}}$, and $\gamma \in \Gamma$. Then, $\hat{\psi}(v) \leq \hat{\psi}(uv)$ and $\hat{\psi}(v) \leq \hat{\psi}(v\gamma u)$, and since $\hat{\psi} \in \text{HFS}^*(G)$, we get

$$\text{INF} \hat{\psi}(v) \geq \max\{\text{INF} \hat{\psi}(uv), \text{INF} \hat{\psi}(v\gamma u)\},$$

$$\text{INFV} \leq \text{INF} \hat{\psi}(v) = 1 - \text{INF} \hat{\psi}(v) \leq 1 - \max\{\text{INF} \hat{\psi}(uv), \text{INF} \hat{\psi}(v\gamma u)\} = \min\{1 - \text{INF} \hat{\psi}(uv), 1 - \text{INF} \hat{\psi}(v\gamma u)\} = \min\{\text{INF} \hat{\vartheta}(uv), \text{INF} \hat{\vartheta}(v\gamma u)\}.$$
\[ \text{INF}_\psi(\omega \nu) = 1 - (1 - \text{INF}_\psi(\omega \nu)) = 1 - \text{INF}_\psi^*(\omega \nu) \geq 1 - \min[\text{INF}_\psi^*(u), \text{INF}_\psi^*(v)] = 1 - \min[1 - \text{INF}_\psi(u), 1 - \text{INF}_\psi(v)] = 1 - (1 - \max[\text{INF}_\psi(u), \text{INF}_\psi(v)]) = \max[\text{INF}_\psi(u), \text{INF}_\psi(v)]. \]

**Theorem 5.** If an IvFS \( \bar{\omega} \) of G is an HFTId of G, then \( \mathcal{F}_{\bar{\omega}} \) is a FTId of G.

**Proof.** It follows from Lemmas 3 and 5. \(\square\)

**Theorem 6.** If \( \bar{\psi} \in HFS^* \) (G) is a HFTId of G, then \( \mathcal{F}_{\bar{\psi}} \) is a FTId of G for all \( \bar{\psi} \in IC(\bar{\psi}). \)

**Proof.** It follows from Lemmas 4 and 5. \(\square\)

**Theorem 7.** A HFS \( \bar{\psi} \) of G is an inf-HFTId of G if and only if for all \( t \in [0, 1] \), a nonempty subset \( U_{\text{INF}}(\bar{\psi}; t) \) of G is a \( \Gamma \)Id of G.

**Proof.** Let \( t \in [0, 1] \) and \( U_{\text{INF}}(\bar{\psi}; t) \neq \emptyset \). Choose \( \forall \in \mathcal{P}([0, 1]) \) such that \( \text{INF} \psi = t \), and we get \( [\bar{\psi}; \nu]_{\text{INF}} = U_{\text{INF}}(\bar{\psi}; t) \). Since \( \bar{\psi} \) is an inf-HFTId of G, we get that \( U_{\text{INF}}(\bar{\psi}; t) = [\bar{\psi}; \nu]_{\text{INF}} \) is a \( \Gamma \)Id of G.

Conversely, let \( \forall \in \mathcal{P}([0, 1]) \) and \( [\bar{\psi}; \nu]_{\text{INF}} \neq \emptyset \). Choose \( t := \text{INF} \psi \), and by the assumption, we obtain that \( [\bar{\psi}; \nu]_{\text{INF}} = U_{\text{INF}}(\bar{\psi}; t) \) is a \( \Gamma \)Id of G. Therefore, \( \bar{\psi} \) is an inf-HFTId of G. \(\square\)

**Corollary 1.** Let \( \bar{\omega} \) be an IVFTId of G. Then, for all \( t \in [0, 1] \), a nonempty subset \( U_{\text{INF}}(\bar{\omega}; t) \) of G is a \( \Gamma \)Id of G.

**Proof.** It follows from Lemma 3 and Theorem 7. \(\square\)

**Theorem 8.** Let \( \bar{\psi} \in HFS(G) \) and \( \bar{\theta} \in IC(\bar{\psi}). \) Then, \( \bar{\theta} \) is an inf-HFTId of G if and only if for all \( t \in [0, 1] \), a nonempty subset \( L_{\text{INF}}(\bar{\psi}; t) \) of G is a \( \Gamma \)Id of G.

**Proof.** Let \( t \in [0, 1] \) and \( L_{\text{INF}}(\bar{\psi}; t) \neq \emptyset \). There exists \( \forall \in \mathcal{P}([0, 1]) \) such that \( \text{INF} \psi = 1 - t \) and then \( [\bar{\theta}; \nu]_{\text{INF}} = L_{\text{INF}}(\bar{\psi}; t) \). Since \( \bar{\theta} \) is an inf-HFTId of G, we obtain that \( L_{\text{INF}}(\bar{\psi}; t) = [\bar{\theta}; \nu]_{\text{INF}} \) is a \( \Gamma \)Id of G.

Conversely, let \( \forall \in \mathcal{P}([0, 1]) \) be such that \( [\bar{\theta}; \nu]_{\text{INF}} \neq \emptyset \). Choose \( t := 1 - \text{INF} \psi \), and by the assumption, we obtain that \( [\bar{\theta}; \nu]_{\text{INF}} = L_{\text{INF}}(\bar{\psi}; t) \) is a \( \Gamma \)Id of G. Hence, \( \bar{\theta} \) is an inf-HFTId of G. \(\square\)

**Corollary 2.** If \( \bar{\psi} \in HFS^* \) (G) is a HFTId of G, then for all \( t \in [0, 1] \), a nonempty subset \( L_{\text{INF}}(\bar{\psi}; t) \) of G is a \( \Gamma \)Id of G.

**Proof.** It follows from Lemma 3 and Theorem 8. \(\square\)

In the following theorem, we give conditions for a HFS of a \( \Gamma \)-semigroup to be an inf-HFTId via IFSs.

**Theorem 9.** For \( \bar{\psi} \in HFS(G) \), the following are equivalent:

1. \( \bar{\psi} \) is an inf-HFTId of G
2. \( (\mathcal{F}_{\bar{\psi}}, \mathcal{F}_{\bar{\psi}}) \) is an HFTId of G for all \( \bar{\psi} \in IC(\bar{\psi}) \)
3. \( (\mathcal{F}_{\bar{\psi}}, \mathcal{F}_{\bar{\psi}}) \) is an HFTId of G

**Proof.** It follows from Lemma 5. \(\square\)

**Corollary 3.** If an IvFS \( \bar{\omega} \) of G is an HFTId of G, then \( (\mathcal{F}_{\bar{\omega}}, \mathcal{F}_{\bar{\omega}}) \) is an HFTId of G for all \( \bar{\omega} \in IC(\bar{\omega}). \)

**Proof.** It follows from Lemma 3 and Theorem 9. \(\square\)

**Corollary 4.** If \( \bar{\psi} \in HFS^* \) (G) is a HFTId of G, then \( (\mathcal{F}_{\bar{\psi}}, \mathcal{F}_{\bar{\psi}}) \) is an HFTId of G for all \( \bar{\psi} \in IC(\bar{\psi}). \)

**Proof.** It follows from Lemma 4 and Theorem 9. \(\square\)

For \( \bar{\psi} \in HFS(X) \) and \( \forall \in \mathcal{P}([0, 1]) \), we define the HFS \( H_{\text{INF}}(\bar{\psi}; \forall) \) on \( X \) by

\[ H_{\text{INF}}(\bar{\psi}; \forall)(x) = \{t \in \forall \mid \text{INF}_{\bar{\psi}}(x) \geq t\} \text{ for all } x \in X, \]

and we denote \( H_{\text{INF}}(\bar{\psi}; [0, 1]) \) by \( \text{INF}_{\bar{\psi}} \). Then, the following statements hold:

1. \( H_{\text{INF}}(\bar{\psi}; \forall)(x) \subseteq \forall \text{ for all } x \in X \)
2. \( \forall \in \text{IVFS}(X) \)
3. \( 0 = \min H_{\text{INF}}(\bar{\psi}; \forall)(x) \leq \text{INF}_{\bar{\psi}}(x) = \max H_{\text{INF}}(\bar{\psi}; \forall)(x) \text{ for all } x \in X \)

In the following theorem, we give conditions for a HFS of a \( \Gamma \)-semigroup to be an inf-HFTId in terms of IvFSs and HFSs.

**Theorem 10.** For \( \bar{\psi} \in HFS(G) \), the following are equivalent:

1. \( \bar{\psi} \) is an inf-HFTId of G
2. \( H_{\text{INF}}(\bar{\psi}; \forall) \) is a HFTId of G for all \( \forall \in \mathcal{P}([0, 1]) \)
3. \( H_{\text{INF}} \) is a HFTId of G
4. \( H_{\text{INF}} \) is an IVFTId of G

**Proof.** (1)\(\Rightarrow\) (2). Let \( u, v \in G, \forall \in \Gamma, \forall \in \mathcal{P}([0, 1]) \), and \( t \in H_{\text{INF}}(\bar{\psi}; \forall)(u) \cup H_{\text{INF}}(\bar{\psi}; \forall)(v) \). Then, \( t \in \forall \) and \( t \leq \max[\text{INF}_{\bar{\psi}}(u), \text{INF}_{\bar{\psi}}(v)] \). By assumption (1) and Lemma 5, we get

\[ t \leq \max[\text{INF}_{\bar{\psi}}(u), \text{INF}_{\bar{\psi}}(v)] \leq \text{INF}_{\bar{\psi}}(u \nu v). \]
Thus, let \( t \in \mathcal{H}_{\inf}^{\Psi}(\psi; \nabla)(\eta \nu) \). Hence, \( \mathcal{H}_{\inf}^{\Psi}(\psi; \nabla)(u) \cup \mathcal{H}_{\inf}^{\Psi}(\psi; \nabla)(v) \subseteq \mathcal{H}_{\inf}^{\Psi}(\psi; \nabla)(\eta \nu) \). Therefore, we have that \( \mathcal{H}_{\inf}^{\Psi}(\psi; \nabla) \) is a HFTId of \( G \).

(2)⇒(3) is clear.

(3)⇒(4). Let \( u, v \in G \) and \( \gamma \in \Gamma \). Then, \( \inf \psi(u) \in \mathcal{H}_{\inf}^{\Psi}(u) \) and \( \inf \psi(v) \in \mathcal{H}_{\inf}^{\Psi}(v) \). By assumption (3), we get \( \inf \psi(u), \inf \psi(v) \in \mathcal{H}_{\inf}^{\Psi}(\eta \nu) \).

Thus, \[
\max \mathcal{H}_{\inf}^{\Psi}(\eta \nu) = \inf \mathcal{H}_{\inf}^{\Psi}(\eta \nu) = \max [\inf \mathcal{H}_{\inf}^{\Psi}(u), \inf \mathcal{H}_{\inf}^{\Psi}(v)]
\]

\[
\geq \max [\inf \mathcal{H}_{\inf}^{\Psi}(u), \inf \mathcal{H}_{\inf}^{\Psi}(v)] = \max \left\{ \max \left[ \inf \mathcal{H}_{\inf}^{\Psi}(u), \inf \mathcal{H}_{\inf}^{\Psi}(v) \right], \right\}
\]

\[
= \max \left\{ \max \left[ \inf \mathcal{H}_{\inf}^{\Psi}(u), \inf \mathcal{H}_{\inf}^{\Psi}(v) \right] \right\}.
\]

\[
\geq \inf \mathcal{H}_{\inf}^{\Psi}(\eta \nu).
\]

Since \( \min \mathcal{H}_{\inf}^{\Psi}(x) = 0 \) for all \( x \in G \), we have \( \min \mathcal{H}_{\inf}^{\Psi}(\eta \nu) \geq \max \left\{ \min \mathcal{H}_{\inf}^{\Psi}(u), \min \mathcal{H}_{\inf}^{\Psi}(v) \right\} \) and so \( \min \mathcal{H}_{\inf}^{\Psi}(\eta \nu) \geq \max \mathcal{H}_{\inf}^{\Psi}(\eta \nu) \).

(4)⇒(1). Let \( u, v \in G \) and \( \gamma \in \Gamma \). By assumption (4), we get \( \min \mathcal{H}_{\inf}^{\Psi}(\eta \nu) \). Then,

\[
\inf \mathcal{H}_{\inf}^{\Psi}(\eta \nu) = \max \mathcal{H}_{\inf}^{\Psi}(\eta \nu)
\]

\[
\geq \max \left\{ \max \left[ \inf \mathcal{H}_{\inf}^{\Psi}(u), \inf \mathcal{H}_{\inf}^{\Psi}(v) \right] \right\}.
\]

Therefore, \( \psi \) is an HFTId of \( G \).

(2)⇒(1) follows from Lemma 5 that \( \psi \) is an HFTId of \( G \).

Corollary 5. Let \( \omega \) be an HFTId of \( G \). Then, the following hold:

1. \( \mathcal{H}_{\inf}^{\omega}(\omega; \nabla) \) is a HFTId of \( G \) for all \( \nabla \in \mathcal{P}([0, 1]) \).
2. \( \mathcal{H}_{\inf}^{\omega} \) is both a HFTId and an HFTId of \( G \).

Proof. It follows from Lemma 3 and Theorem 10.

Corollary 6. Let \( \psi \in HFS^*(G) \) be a HFTId of \( G \). Then, the following hold:

1. \( \mathcal{H}_{\inf}^{\psi}(\psi; \nabla) \) is a HFTId of \( G \) for all \( \nabla \in \mathcal{P}([0, 1]) \) and \( \psi \in \IC(\psi) \).
2. \( \mathcal{H}_{\inf}^{\psi} \) is both a HFTId and an HFTId of \( G \) for all \( \psi \in \IC(\psi) \).

Proof. It follows from Lemma 4 and Theorem 10.
For all $t_1, t_2 \in [0, 1]$, nonempty subsets $U_{\text{INF}}(\hat{\psi}; t_1)$ and $U_{\text{SUP}}(\hat{\psi}; t_2)$ of $G$ are $\Gamma$-Iffs of $G$.

Proof. It follows from Theorems 1, 2, 3, 7, and 10 and Lemma 5.

Example 3. Let $\mathbb{Z}^-$ be the set of all negative integers and $G = \Gamma = \mathbb{Z}^-$. Then, $G$ forms a $\Gamma$-semigroup with the usual multiplication. Define a HFS $\hat{\psi}$ on $G$ by $\hat{\psi}(u) = [(u + 1/3u), (3u + 1/3u)]$ for all $u \in G$. Then,

\[
\begin{align*}
\sup_{\hat{\psi}}(uv) & = \frac{3(uv) + 1}{3(uv)} \geq \max\left\{\frac{3u + 1}{3u}, \frac{3v + 1}{3v}\right\} \\
& = \max\{\sup_{\hat{\psi}}(u), \sup_{\hat{\psi}}(v)\},
\end{align*}
\]

\[
\begin{align*}
\inf_{\hat{\psi}}(uv) & = \frac{uv + 1}{3(uv)} \geq \max\left\{\frac{u + 1}{3u}, \frac{v + 1}{3v}\right\} \\
& = \max\{\inf_{\hat{\psi}}(u), \inf_{\hat{\psi}}(v)\},
\end{align*}
\]

for all $u, v \in G$ and $y \in \Gamma$. Hence, it follows from Theorem 13 that $\hat{\psi}$ is a $(\sup, \inf)$-HFTId of $G$. Since $\hat{\psi}$ is not an IvFS of $G$, we get that it is not an IvFId of $G$.

Lemma 6. Every IvFId of $G$ is a $(\sup, \inf)$-HFTId of $G$.

Proof. It follows from Lemmas 1 and 3.

By Example 3 and Lemma 6, we see that a $(\sup, \inf)$-HFTId of $G$ is a general concept of an IvFId of $G$.

Lemma 7. Let $\tilde{w}$ be an IvFS of $G$. Then, $\tilde{w}$ is an IvFId of $G$ if and only if $\tilde{w}$ is a $(\sup, \inf)$-HFTId of $G$.

Proof. It follows from Lemma 6.

Conversely, assume that $\tilde{w}$ is a $(\sup, \inf)$-HFTId of $G$. By Theorem 13, we get $\sup_{\tilde{w}}(uv) \geq \max\{\sup_{\tilde{w}}(u), \sup_{\tilde{w}}(v)\}$ and $\inf_{\tilde{w}}(uv) \geq \max\{\inf_{\tilde{w}}(u), \inf_{\tilde{w}}(v)\}$ for all $u, v \in G$ and $y \in \Gamma$. Thus,

\[
\begin{align*}
r\max(\tilde{w}(u), \tilde{w}(v)) &= \{\max\{\inf_{\tilde{w}}(u), \inf_{\tilde{w}}(v)\}, \\
& \quad \max\{\sup_{\tilde{w}}(u), \sup_{\tilde{w}}(v)\}\} \\
& \leq \inf_{\tilde{w}}(uv), \sup_{\tilde{w}}(uv) \\
& = \tilde{w}(uv),
\end{align*}
\]

for all $u, v \in G$ and $y \in \Gamma$. Therefore, $\tilde{w}$ is an IvFId of $G$.

In Theorem 14, equivalent conditions for an IvFS to be an IvFId are given in terms of level sets, FSs, IFSs, IvFSs, and HFSs.

Theorem 14. For $\tilde{w} \in \text{IvFS}(G)$, the following are equivalent:

1. $\tilde{w}$ is an IvFId of $G$

Proof. It follows from Theorems 7 and Theorem 13.

Theorem 15. For a nonempty subset $Y$ of $G$, the following are equivalent:

1. $Y$ is a $\Gamma$-Id of $G$
2. The ClvFS $\text{Cl}_Y$ is a $(\sup, \inf)$-HFTId of $G$
3. The CHFS $\text{CH}_Y$ is a $(\sup, \inf)$-HFTId of $G$
4. $\chi_{\Delta}(Y)$ is a $(\sup, \inf)$-HFTId of $G$ for all $\Delta, \forall \in \mathcal{P}([0, 1])$ with $\text{INF}_{\Delta} \prec \text{INF}_Y$ and $\text{SUP}_{\Delta} \prec \text{SUP}_Y$

Proof. It follows from Theorems 4, 11, and 12 and Remark 1.

4. Conclusions

In this paper, we have introduced the notions of an inf-HFTId and a $(\sup, \inf)$-HFTId, which are a generalization of an IvFId, of a $\Gamma$-semigroup and examined their characterizations in terms of sets, FSs, IFSs, IvFSs, and HFSs. Furthermore, we have discussed the relation between a $\Gamma$-Id and a generalization of the CHFS and ClvFS. From the study results, we found that the following conditions are all equivalent in a $\Gamma$-semigroup: a nonempty subset $Y$ is a $\Gamma$-Id, $\text{Cl}_Y$ is an inf-HFTId, $\text{Cl}_Y$ is a $(\sup, \inf)$-HFTId, and $\text{CH}_Y$ is a $(\sup, \inf)$-HFTId.

In the future, we will study an inf-HFTId and a $(\sup, \inf)$-HFTId in LA-semigroups and UP-algebras and examine their characterizations in terms of sets, FSs, IFSs, IvFSs, and HFSs.

Data Availability

No data were used to support this research.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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