Black hole analogues in braneworld scenario

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Abstract

We construct analogue black hole solutions in the braneworld scenario. The quantum fluctuations of condensate gravitons propagating around a 4+\(n\) -dimensional gravitational potential are found yielding a metric similar to higher dimensional Schwarzschild black hole line-element. Black hole analogue solutions in Randall-Sundrum and Dvali-Gabadadze-Porrati brane world models are also constructed. The properties of such black hole analogues are discussed.

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1 Introduction

A lot of interest in recent years has been raised for field theories where the standard model of high-energy physics is assumed to live on a 3-brane embedded in a larger space-time, while only the gravitational fields are in contrast usually considered to live in the whole spacetime \cite{1,2,3,4}. There mainly three kind of pictures in the brane world scenario. The first picture is proposed by Arkani-Hamed et al (ADD model) \cite{2}, who suggested that the traditional Planck scale, \(M_p\), is only an effective energy scale derived from the fundamental one in (4 + \(n\))-dimensional space, \(M_*\), through the following relation \(M_p^2 \sim M_*^{2+n}R^n\), where \(R\) is the size of each extra dimensions. The fact that we do not see experimental signs of the extra dimensions despite that the compactification scale of the extra dimensions would have to be much smaller than the weak scale, implies that only gravity can propagate in the extra-dimensional spacetime and all ordinary matter: electromagnetic, weak and strong forces, is restricted to live on a (3+1) dimensional hypersurfaces,

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a 3-brane. In the second picture which we refer as the Randall-Sundrum (RS) model, a new higher dimensional mechanism for solving the hierarchy problem was proposed. The weak scale is considered to be generated from the Planck scale through an exponential hierarchy. A positive tension 3-brane embedded in an 5-dimensional AdS bulk and the cross over between 4-dimensional and 5-dimensional gravity is set by the AdS radius. The third picture is based on the work of Dvali, Gabadadze, and Porrati (DGP), where the 3-brane is embedded in 5-dimensional Minkowski space with an infinite size extra dimension.

The presence of extra dimensions in brane world gravity models will inevitably change the properties and physics of black holes. In ref. [5], the authors found that Schwarzschild black holes in ADD model with horizon radius smaller than the size of the new spatial dimensions, $R$, are bigger, colder and longer-lived than a usual $(3 + 1)$-dimensional black hole of the same mass. Physicists expect to observe Tev scale black holes at Large Hadron Collider (LHC) in the near future [6]. However, exact solutions describing a black hole bound to a 3-brane in RS and DGP models is not yet known.

On the other hand, the remarkable work of Unruh, in 1981, developed a way of mapping certain aspects of black holes in supersonic flows and pointed out that propagation of sound in a fluid or gas turning supersonic [7], is similar to the propagation of a scalar field close to a black hole, and thus experimental investigation of the Hawking radiation is possible. From then on, several candidates have been considered for the experimental test of the analogue of black holes [8,9].

In the present study, we construct black hole analogue metrics in the brane world models and investigate their properties and physics. In ref. [3], the authors studied the linearized gravitational fluctuations in the 5-dimensional space time while the wave equation is analogous to non-relativistic quantum mechanics equation. Then they computed the effective non-relativistic gravitational potential between two particles. In this paper, we find that the non-relativistic quantum mechanics equation to describe the propagation of gravitational waves and discuss quantum fluctuations of the propagation of gravitational waves. In this way, we obtain the metric of black hole analogues and then study their behaviors in ADD, RS and DGP models.

2 General formalisms

Without loss of generality, we consider a system of Bose-Einstein condensate particles, which may not be the standard model particles and are permitted to propagate in an arbitrary dimensional space-time (gravitons, scalar particles etc.). A Bose-Einstein condensate is the ground state of a second quantized many body Hamiltonian for $N$ bosons trapped by an external potential $V(\vec{x})$. 
When the temperature is low enough, most of all the particles can be described by the same single-particle quantum state \( \psi(\vec{x}, t) \) and the interactions between particles become sufficiently small. In this sense, the evolution of \( \psi(\vec{x}, t) \) is described by the Schrödinger equation in \((4+n)\)-dimensions,

\[
[-\frac{\hbar^2}{2m}\nabla^2 + V(\rho, \vec{x})] \psi(\vec{x}, t) = i\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t),
\]

where \( m \) is the mass of the particles, \( \psi(\vec{x}, t) \) can be considered as a macrostate of condensate scalar particles (gravitons etc.), \( V(\rho, \vec{x}) \) is a potential in \((4+n)\)-dimensions and we normalize the total number of particles \( \int d^3x |\psi(\vec{x}, t)|^2 = N \).

The mass of particles and the total number of particles has the following relation

\[
m = N m',
\]

where \( m' \) is mass per particle. In general, \( V(\rho, \vec{x}) \) in quantum mechanics can be electric Coulomb potential or some potentials else. We will specify \( V(\rho, \vec{x}) \) to be the effective non-relativistic gravitational potentials in brane world models lately.

The Lagrangian corresponding to the Schrödinger equation can be defined as \( \mathcal{L} = i\hbar \psi^* \frac{\partial \psi}{\partial t} - \frac{\hbar^2}{2m} \nabla \psi^* \cdot \nabla \psi - V(\vec{x}) \psi^* \psi \), which can be further rewritten as

\[
\mathcal{L} = -\psi^* \left\{ \frac{\hbar}{i} \frac{\partial}{\partial t} \ln \psi + \frac{\hbar^2}{2m} \nabla (\ln \psi^*) \cdot \nabla (\ln \psi) + V(\rho, \vec{x}) \right\}.
\]

Comparing Eq.\((2)\) with the Jacobi-Hamilton equation, i.e.

\[
\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\rho, \vec{x}) = 0,
\]

where \( S \) is the action of the whole system, one obtains \( S = \frac{\hbar}{i} \ln \psi \) and \( S^* = -\frac{\hbar}{i} \ln \psi^* \). Assumed \( S = S_r + iS_i, \psi \) can be rewritten as

\[
\psi(\vec{x}, t) = e^{\frac{i(S_r + iS_i)}{\hbar}} = \rho^{1/2}(\vec{x}, t)e^{i \frac{S_r(\vec{x}, t)}{\hbar}},
\]

where \( \rho = \psi^* \psi \) is the probability density. The total particle number is then given by \( N = \int \rho \, d^3x \). Substituting Eq.\((4)\) into the Schrödinger equation, we have

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j}(\vec{x}, t) = 0,
\]

\[
\frac{\partial S_r}{\partial t} + \frac{(\nabla S_r)^2}{2m} + V(\rho, \vec{x}) - \frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} = 0,
\]

where \( \vec{j}(\vec{x}, t) = \frac{\rho}{m} \nabla S_r \) and the last term in Eq.\((6)\) corresponds to the quantum effect without classical correspondence. We may drop out this term by assuming that this term is small compare to other terms and then obtain Euler equation for classical liquid. If we set \( \vec{v} = \nabla S_r/m \), then Eq.\((6)\) can be
rewritten as
\[ \frac{\partial \vec{v}}{\partial t} + \nabla (\vec{v}^2/2) = -\nabla V(\rho, \vec{x})/m, \] (7)
which is the exact equation of irrotational fluid. Defining $\Phi = S_r/m$, we have
\[ \frac{\partial \Phi}{\partial t} + \vec{v}^2/2 = -V(\rho, \vec{x})/m. \] (8)
By further defining $\xi = ln\rho$, we have the new forms of Eq.(5) and Eq.(8)
\[ \frac{\partial \xi}{\partial t} + \vec{v} \cdot \nabla \xi + \nabla \cdot \vec{v} = 0, \] (9)
\[ \frac{\partial \Phi}{\partial t} + \vec{v}^2/2 + V(\xi, \vec{x})/m = 0. \] (10)
Linearizing Eq.(9) and Eq.(10) around the assumed background $(\xi_0, \Phi_0)$, with $\xi = \xi_0 + \xi$, $\Phi = \Phi_0 + \Phi$ and $V(\xi, \vec{x}) = V(\xi_0, \vec{x}) + V'(\xi_0, \vec{x})\xi + \ldots$, we obtain
\[ \rho_0^{-1} \left[ \frac{\partial \rho_0 \xi}{\partial t} + \nabla \cdot (\rho_0 \vec{v} + \rho_0 \nabla \Phi) \right] = 0, \frac{\partial \Phi}{\partial t} + \vec{v} \cdot \nabla \Phi + \xi V'(\xi_0)/m_0 = 0, \] (11)
which result in an equation for $\tilde{\Phi}$,
\[ \rho_0^{-1} \left\{ \frac{\partial}{\partial t} \left( \frac{m_0 \rho_0}{V'(\xi_0)} \frac{\partial \tilde{\Phi}}{\partial t} + \frac{m_0 \rho_0 \vec{v}_0}{V'(\xi_0)} \cdot \nabla \tilde{\Phi} \right) \right. \\
+ \nabla \cdot \left( \frac{m_0 \rho_0 \vec{v}_0}{V'(\xi_0)} \frac{\partial \tilde{\Phi}}{\partial t} - \rho_0 \nabla \tilde{\Phi} + \vec{v}_0 \frac{m_0 \rho_0}{V'(\xi_0)} \vec{v}_0 \cdot \nabla \tilde{\Phi} \right) \right\} = 0, \] (12)
where $m_0 = m' \int \rho_0 d^3x$. The above equation is identified with a massless scalar field equation describing the sound wave in the curved spacetime background,
\[ \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu \nu} \partial_{\nu} \tilde{\Phi}) = 0 \] (13)
with the background metric, $g_{\mu \nu} = \left( \frac{\rho_0}{c} \right) \left( \begin{array}{cc} -c^2 + v_0^2 & -v_i v_0 \\ -v_i & \delta_{ij} \end{array} \right)$, which is a $(n + 4) \times (n + 4)$ matrix. The local speed of sound is defined by $c^2 \equiv |V'(\xi_0)/m_0|$. The metric can be written as $ds^2 = \frac{\rho_0}{c} \left[ (c^2 - v_0^2) dt^2 + 2 \vec{v}_0 \cdot d\vec{x} dt - d\vec{x} \cdot d\vec{x} \right]$. Assuming that the background flow is a spherically symmetric, stationary, and convergent flow, we can define a new time coordinate by $d\tau = dt + \frac{\vec{v}_0 \cdot d\vec{x}}{c^2 - v_0^2}$. Substituting this back into the line element gives
\[ ds^2 = \frac{\rho_0}{c} \left[ -(c^2 - v_0^2) d\tau^2 + \frac{c^2}{c^2 - v_0^2} dr^2 + r^2 d\Omega_{n+2}^2 \right], \] (14)
This metric denotes the physics of analogue black holes—solid or liquid systems that trap scalar particle waves in a way similar to real black holes. Here, $c$
plays the role of escaping velocity just as what the speed of light means to real black holes. The horizon of the analogue black hole is defined by \( c^2 = v_0^2 \). Comparing Eq. (14) with the 4-dimensional Schwarzschild metric,

\[
 ds^2 = -(c^2 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{c^2r})^{-1}dr^2 + r^2d\Omega_2^2, \tag{15}
\]

If we assume \( \frac{\rho_0}{c^2}ds^2 = ds^2 \) and \( v_0^2 = \frac{2GM}{r} \), they identify with each other. In the following, we will see that Eq. (14) describing such a gravitational wave trap are similar to those for a black hole’s funnel-shaped space time. Once such black hole analogues are formed, they should emit Hawking-like radiation from the edge of the horizon in the form of phonons. Analogues of black holes thus can be used to detect extra dimensions and even study the nature of quantum gravity. In the following, we will specify our discussions about the potential on gravitational potentials, since gravitons are the only known particles that can propagate in extra dimensions [2].

3 Black hole analogues in ADD model

The gravitational potential in higher dimensional space time can derived from the equation \( \nabla^2 V_{(4+n)} = 4\pi G_{(4+n)}\rho \). We integrate both sides of equation and use the divergence theorem on the left-hand and note that the volume integral on the right-hand side gives the total mass \( M \). Thus, in ADD model, the gravitational potential for two test particles with mass \( M \) and \( m_0 \) within a distance \( r \ll R \), is given by

\[
 V(\xi_0, \vec{x}) = -\frac{4\pi G_{(4+n)}Mm_0}{(n+1)r^{n+1}\Omega_{n+2}}, \tag{16}
\]

where \( G_{(4+n)} \) denotes the \((4+n)\)-dimensional Newton constant, and \( \Omega_{n+2} = \frac{2\pi^{\frac{n+3}{2}}}{\Gamma(\frac{n+3}{2})} \) denotes the volume of a unit \((n+2)\) sphere. If we further assume the velocity \( v \) is time independent, then Eq. (8) indicates that \( v_0^2 = \frac{8\pi G_{(4+n)}M}{(n+1)r^{n+1}\Omega_{n+2}} \).

The metric can be written as,

\[
 ds^2 = \rho_0 c \left[ -c^2 f(r)dr^2 + f^{-1}(r)dr^2 + r^2d\Omega_{n+2}^2 \right], \tag{17}
\]

where \( f(r) = \left(1 - \frac{8\pi G_{(4+n)}M}{(n+1)r^{n+1}c^2\Omega_{n+2}}\right) \). The above metric is conformal to Schwarzschild black hole metric in \((4+n)\) dimensions (only the coefficient is different) [11]. We should notice that when \( v_0 \) is chosen, the continuity equation \( \nabla \cdot (\rho_0\vec{v}_0) = 0 \) then it implies that \( \rho_0 \propto r^{-\frac{4+n}{2}} \). The presence of the conformal factor does not influence the properties of black hole analogues much in that the surface gravity and Hawking temperature are conformal invariants [10]. If we mainly
consider the region near the event horizon, the conformal factor $\varphi_c$ can simply be regarded as a constant, and we can ignore the contribution of the factor $[9].$ Hereafter, we focus our discussions on the near-horizon region and set $\varphi_c ds^2 = ds'^2.$ The properties of higher dimensional Schwarzschild black holes in ADD model have been discussed by Aryres, Dimopoulos and March-Russell$[5].$ We would like to review their main results here. The event horizon radius is given by

$$r_{H(4+n)} = \left( \frac{8\pi G_{(4+n)}M}{(n+1)c^2\Omega_{n+2}} \right)^{\frac{1}{n+1}}. \quad (18)$$

Note that in ADD model, $G_{(4+n)} = \frac{1}{M_{*}^{2+n}}.$ Thus, we find that

$$r_{H(4+n)} \sim \frac{1}{M_{*}} \left( \frac{M}{M_{*}} \right)^{\frac{1}{n+1}}. \quad (19)$$

When $n = 0,$ we obtain the four dimensional horizon radius,

$$r_{H(4)} = \left( \frac{2G_{(4)}M}{c^2} \right) \sim \frac{1}{M_{p}} \left( \frac{M}{M_{p}} \right) \sim \frac{M}{M_{p}^{2+n}R^n}, \quad (20)$$

where the relation $G_4^{-1} \sim M_p^2 \sim M_{*}^{2+n}R^n$ has been used. As a consequence, we have

$$\frac{r_{H(4)}}{r_{H(4+n)}} \sim \left( \frac{r_{H(4+n)}}{R} \right)^n < 1, \quad (21)$$

Therefore, for small $(4 + n)$-dimensional black holes that can submerge into extra dimensions, the horizon radius is bigger than that of corresponding 4-dimensional black holes of the same mass, i.e. $r_{H(4)} < r_{H(4+n)} < R.$ However, the Hawking temperature $T_{H(4+n)}$ of a small $(4 + n)$-dimensional black hole is found to be colder than its 4-dimensional correspondence. The temperature of black hole analogues can be easily estimated from the standard formula of acoustic black hole temperature,

$$T_{H(4+n)} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial}{\partial r} v_0 \right|_{r=r_{H(4+n)}} \sim \frac{1}{r_{H(4+n)}}, \quad (22)$$

which is smaller compared with the 4-dimensional temperature $T_{H(4)} \sim 1/r_{H(4)}.$ For such larger and colder higher dimensional black holes, the lifetime is correspondingly longer than that of an equal mass 4-dimensional one$[5].$ The entropy of such small black hole analogues is given by

$$S_{H(4+n)} = \frac{A_{n+2}}{4G_{(4+n)}} = \frac{r_{H(4+n)}^{n+2}\Omega_{n+2}}{4G_{(4+n)}}, \quad (23)$$

while the 4-dimensional black hole entropy goes as,

$$S_{H(4)} = \frac{r_{H(4)}^{2}\Omega_{2}}{4G_{(4)}}. \quad (24)$$
Comparing Eq. (23) with Eq. (24), we obtain,

\[ \frac{S_{H(4+n)}}{S_{H(4)}} \sim \left( \frac{R}{r_{H(4+n)}} \right)^n > 1. \]  

(25)

This confirms us that for horizon radius \( r_{H(4)} < r_{H(4+n)} < R \), the \((4+n)\)-dimensional entropy is larger than the 4-dimensional one.

In summary, we are able to derive an analogue black hole metric similar to that of \((4+n)\)-dimensional Schwarzschild black holes by considering the propagations of gravitational waves around a higher dimensional gravitational potential. The quantum fluctuations of the propagations of gravitational waves yield a massless scalar field equation in curved space-time. The properties and physics of such black hole analogues are just what described in [5]: for horizon radius smaller than the size of extra dimensions, such black holes are bigger, colder and longer lifetime than their 4-dimensional correspondences of the same mass.

4 Black hole analogues in RS model

The gravitational potential in RS models behaves as,

\[ V(\vec{x}) = -\frac{G_{(4)}Mm}{r} \left( 1 + \frac{2\ell^2}{3r^2} \right), \quad \text{for } r \gg \ell, \]  

(26)

and

\[ V(\vec{x}) = -\frac{G_{(4)}Mm\ell}{r^2}, \quad \text{for } r \ll \ell \]  

(27)

where \( \ell \) is the effective size of extra dimension probed by the 5-dimensional gravitons. In the RSI scenario, there are two branes (with equal but opposite tensions) separated by a piece of AdS space between them. The effective scale on the visible brane is expressed as,

\[ M_p^2 = M_*^3 \ell [1 - e^{-2L/\ell}]. \]  

(29)

In the RSII model, there is only one positive tension brane which can be thought of arising from the negative tension brane off to infinity, \( L \rightarrow \infty \). The energy scales are then written as

\[ M_p^2 = M_*^3 \ell. \]  

(30)

We will concentrate mainly on the RSII model and discuss the properties of RS black hole analogues only for region \( r \gg \ell \), since for \( r \ll \ell \) the black hole properties is just a special case (the case when \( n = 1 \)) of what we have discussed in the last section. Eq. (26) gives the small corrections to 4-dimensional
gravity at low energy from extra-dimensional effects. These effects serve to slightly strengthen the gravitational field.

We assume the velocity $v$ is time independent, then Eq. (8) indicates that $v^2 = \frac{2G (4) M}{r} \left( 1 + \frac{2\ell^2}{3r^2} \right)$. The line element of black hole analogues in RS models are then given by,

$$ ds^2 = \frac{\rho_0}{c} \left[ -c^2 \left( 1 - \frac{2G (4) M}{rc^2} \left( 1 + \frac{2\ell^2}{3r^2} \right) \right) + \left( 1 - \frac{2G (4) M}{rc^2} \left( 1 + \frac{2\ell^2}{3r^2} \right) \right)^{-1} \right] $$

The horizon radius is $r_{RS} = \frac{2G (4) M}{3c^2 + \left( \frac{2G (4) M \ell^2}{3c^2} \right)^{1/3} + \frac{9c^2}{3c^2}}$, where $a = 9c^2 + 4G (4) M^2 + 3c^2 \ell^2 \sqrt{9c^2 + \frac{8G (4) M^2}{3c^2}}$. The value of this radius is smaller than the 4-dimensional Schwarzschild horizon radius of the same mass. The temperature of black hole analogues in RS models is simply defined by,

$$ T_{RS} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial}{\partial r} v_0 \right|_{r=r_{RS}} = \frac{\hbar}{4\pi k_B} \left( \frac{1}{r_{RS}} + \frac{8G (4) M^2}{3r_{RS}^2} \right) $$

Note that $r_{RS} < r_{H(4)}$, thus the temperature of RS black hole analogues for $r \gg l$ is hotter than that of a 4-dimensional Schwarzschild black hole of the same mass. We can understand intuitively that the smaller black hole horizon radius is, the hotter the temperature becomes.

The lifetime of such RS black hole analogues is correspondingly shorter than that of an equal mass 4-dimensional Schwarzschild one. The rate at which energy is radiated for RS black hole analogues is of order,

$$ \frac{dE}{d\tau} \sim A_{RS} T_{RS}^5, $$

where $A_{RS}$ denotes the area of the 5-dimensional RS black hole analogues. Remember that, $T_{RS} \sim \left( \frac{1}{r_{RS}} + \mathcal{O}(1/r_{RS}) \right)$, we find that,

$$ \frac{dE}{d\tau} \sim \frac{\Delta M}{\Delta \tau_{RS}} \sim \frac{1}{r_{RS}^3}. $$

Comparing with energy decaying rate of a 4-dimensional Schwarzschild black hole with temperature $T_H \sim \frac{1}{r_{H(4)}}$,

$$ \frac{\Delta M}{\Delta \tau_H} \sim \frac{1}{r_{H(4)}^2}, $$

we find

$$ \frac{\Delta \tau_{RS}}{\Delta \tau_{H}} \sim \frac{r_{RS}^2}{r_{H(4)}^2} < 1. $$

The entropy of an RS black hole induced on the 3-brane is given by,

$$ S_{RS} = \frac{A_{RS}}{4G (4)} = \frac{r_{RS}^2 \Omega_2}{4G (4)}. $$
Once compare with the entropy of an equal mass 4-dimensional Schwarzschild black hole (see Eq.(21)), we find that
\[
\frac{S_{RS}}{S_{H(4)}} \sim \frac{r_{RS}^2}{r_{H(4)}^2} < 1.
\] (38)

Therefore, the properties of an RS black hole analogue in the region \( r \gg \ell \), compared with a 4-dimensional Schwarzschild black hole, is smaller, hotter and shorter-lived, while in the region \( r \ll \ell \), the physics of such black hole analogues becomes indistinguishable from that of an ADD black hole: bigger, colder and long-lived than an equal mass 4-dimensional Schwarzschild black hole.

5 Black hole analogues in DGP model

Gravitational potential in DGP model is the exact 4-dimensional potential at short distances whereas at large distances the potential is that of a 5-dimensional theory. We mainly interested in the large distance potential, when \( r \gg r_0 \), we have,
\[
V(\vec{x}) = -\frac{G(4)Mmr_0}{r^2},
\] (39)

where \( r_0 = \frac{M^2}{2M_5^2} \). Therefore, the line element for large distance \( r \gg r_0 \) read as,
\[
ds^2 = \frac{\rho_0}{c} \left[ -c^2(1 - \frac{2G(4)Mr_0}{r^2c^2})d\tau^2 + (1 - \frac{2G(4)Mr_0}{r^2c^2})^{-1}dr^2 + r^2d\Omega_3^2 \right] \] (40)

This solution in fact is a special case of the black hole metric given in Eq.(17) when \( n = 1 \). However, the physics of black hole analogues in DGP model are different from that in ADD model. In ADD model, if the black hole horizon is much larger then the size of the extra dimensions, \( r_H \gg R \), the nature of the black hole is actually a 4-dimensional one. It is only when \( r_H \ll R \), the black hole becomes virtually a higher dimensional object that is submerged into the extra-dimensional space-time. The DGP model, however, presents us an inverse picture: when the horizon of a black hole is larger than the size of the extra dimension, \( r_H \gg r_0 \), the produced black hole is in fact a five-dimensional one; when \( r_H \ll r_0 \), we recover to the usual 4-dimensional Einstein gravity and the black hole becomes a 4-dimensional object.

The horizon radius can be obtained from Eq.(10),
\[
r_{DGP(5)} = \left( \frac{2G(4)Mr_0}{c^2} \right)^{1/2}
\] (41)

Following the analysis in section 1, one can find that the horizon radius in the region \( r_H \gg r_0 \) of DGP black hole analogues is larger than that of a
4-dimensional Schwarzschild black hole with the same mass, namely,

\[ t_{\text{DGP}(5)} > t_{H(4)} \]  \hspace{1cm} (42)

The temperature and life-time of such DGP black hole analogues are respectively colder and longer-lived than its four-dimensional correspondences. The entropy is then larger than that of a 4-dimensional Schwarzschild black hole.

In general speaking, the scale of \( r_0 \) in DGP model is always believed to be of order of Hubble radius. So we do not expect to find such DGP black hole analogues in experiments. The properties of DGP black hole analogues discussed above are only of theoretical significance.

6 Conclusions

In conclusion, we have constructed black hole analogue solutions in the braneworld models by considering the propagation of gravitational waves in \((4+n)\)-dimensional space time. The black hole analogue metric in ADD model is similar to the higher dimensional Schwarzschild black hole line-element. Black hole analogues in RS model are also discussed and it is found that for horizon radius larger than the size of extra dimension \( r_H \gg \ell \), the analogues are smaller, hotter and shorter-lived than that of an equal mass 4-dimensional Schwarzschild black hole, while in the region \( r \ll \ell \), the physics of such black hole analogues becomes undistinguishable from that of an ADD black hole. The properties of DGP black hole analogues are also briefly discussed. When the horizon radius is larger than the size of the extra dimension \( r \gg r_0 \), the DGP black hole analogues are found to be larger, colder and longer-lived than its equal mass 4-dimensional correspondence.

The above discussions show that gravitational wave black holes have similar properties to that of higher dimensional Schwarzschild black holes. However, experimental realizing of such black hole analogues in laboratory could be no less difficult than finding mini black holes at LHC since gravitational waves are still elusive to the experiments. But this does not mean that one can preclude such possibilities of detecting extra dimensions by using gravitational wave black hole analogues. So far, in the discussions above, we have assumed the interactions between condensate particles vanished and the local speed sound to be a constant. In fact, \( c \) should be position-dependent. It is convenient to discuss the properties of black hole analogues and to compared with that of real black holes by assuming \( c \) to be a constant. We need further investigations on these points.

The correlations between the black hole mass and its temperature, deduced from the energy spectrum of the decay products, can test Hawking’s evaporation law. The amplitude probability in the bulk and on the brane allows one to find its dependence on the number of extra dimensions \( n \) and the
angular momentum number \( l \). Kanti and March-Russell have calculated the greybody factor of \((4+n)\)-dimensional Schwarzschild for scalar particles propagating in the bulk and localized on the brane \([12]\) (for related work see also \([13]\)). We would like to calculate the grey factors of the radiation spectrum for higher dimensional black hole analogues in our forthcoming paper.

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