Possibility of experimental study on nonleptonic $B_c^*$ weak decays

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Abstract

The ground vector $B_c^*$ meson has not yet been experimentally discovered until now. Besides the dominant electromagnetic decays, nonleptonic weak decays provide another choice to search for the mysterious $B_c^*$ mesons. Inspired by the potential prospects of $B_c^*$ mesons in future high-luminosity colliders, nonleptonic $B_c^*$ weak decays induced by bottom and charm quark decays are studied within the SM by using a naive factorization approach. It is found that for $B_c^* \to B_{s,d} \pi, B_{s,d}^* \pi, B_{s,d} \rho, B_s K, B_s^* K, \eta_c(1S, 2S) \pi, \eta_c(1S, 2S) \rho$ and $\psi(1S, 2S) \pi$ decays, a few hundred and even thousand of events might be observable in CEPC, FCC-ee and LHCb@HL-LHC experiments.

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I. INTRODUCTION

According to the $q\bar{q}$ quark model assignments for mesons, the bottom-charmed mesons are unique particles consisting of two heavy quarks with different flavors, because it is generally assumed that the top quark being the heaviest element fermion of the standard model (SM) has a very short lifetime and decays before hadronization. The bottom-charmed mesons are isospin singlets, and have nonzero additive quantum numbers, $Q = B = C = \pm 1$, where $Q$, $B$ and $C$ are respectively electric charge, bottom and charm. The bottom-charmed mesons, analogous with hidden-flavored charmonium $c\bar{c}$ and bottomonium $b\bar{b}$, are often considered as nonrelativistic bound states. The hyperfine splitting interactions divide the ground bottom-charmed states into the spin singlet $1^1S_0$ and triplet $1^3S_1$, corresponding to the pseudoscalar $B_c$ meson with the spin-parity $J^P = 0^-$ and vector $B_c^*$ meson with $J^P = 1^-$, respectively.

The pseudoscalar $B_c$ meson was first discovered via $B_c \to J/\psi \pi$ and $J/\psi \pi\pi^+\pi^-$ decay modes with hadronic $Z^0$ decay sample data collected by the DELPHI detector at the $e^+e^-$ collider LEP in 1997 [1]. The natural properties of the $B_c$ meson including its mass, lifetime, and spin-parity have now been well determined [2]. In the meantime, the vector $B_c^*$ meson has not been definitively identified by experimental physicists yet. All the information on the $B_c^*$ meson come only from theoretical calculation for the moment. Undoubtedly, it is a matter of great urgency to find and identify the $B_c^*$ meson in experiments, which is the basis and prerequisite to obtain a deeper insight into the desired $B_c^*$ meson, and distinguish different theoretical models.

The production probability of bottom-charmed mesons is far less than that of heavy-light $B_{u,d}$ mesons, and heavy-heavy charmonium and bottomonium. First, two heavy quark pairs of both $b\bar{b}$ and $c\bar{c}$ should be almost simultaneously produced in a high energy process, and then the $b$ (or $\bar{b}$) quark from a $b\bar{b}$ pair and $\bar{c}$ (or $c$) quark from a $c\bar{c}$ pair should be lucky enough to combine and finally form bottom-charmed mesons [3]. In positron-electron collisions, the bottom-charmed mesons can be produced via the $Z^0$ boson decay,

$$e^+ + e^- \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c,$$

(1)

for example, the observation of $B_c$ mesons by DELPHI [1], ALEPH [4] and OPAL [5] detectors at LEP experiments. In hadron-hadron collisions, the bottom-charmed mesons can be
produced via gluon-gluon fusion and quark-antiquark annihilation,

\[ g + g \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c, \]  

(2)

\[ q + \bar{q} \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c, \]  

(3)

for example, the observation of \( B_c \) meson by CDF [6] and D0 [7] at Fermilab Tevatron, by LHCb [8] at LHC experiments. More and more \( B_c^* \) mesons are expected to be accessible in the future. Given the branching ratio of \( Z^0 \) boson decay \( \mathcal{B}_r(Z^0 \rightarrow b\bar{b}) = 15.12(5)\% \) [2] and the bottom quark fragmentation fraction \( f(b \rightarrow B_c^*) \sim 6 \times 10^{-4} \) [9–11], there will be more than \( 10^8 \) \( B_c^* \) mesons from \( 10^{12} Z^0 \) boson decays at the Circular Electron Positron Collider (CEPC) [12], and more than \( 10^9 \) \( B_c^* \) mesons from \( 10^{13} Z^0 \) boson decays at the Future Circular Collider (FCC-ee) [13]. Assuming the \( B_c^* \) production cross section is about 100 nb for pp collisions at \( \sqrt{s} = 13 \) TeV [14], more than \( 3 \times 10^{10} \) \( B_c^* \) mesons will be available with a total integrated luminosity of 300 fb\(^{-1}\) at LHCb of HL-LHC [15]. The huge amount of experimental data provides an excellent opportunity and solid foundation to carefully study the \( B_c^* \) mesons.

Because of the nonrelativistic nature, the mass of the \( B_c^* \) meson is approximately equal to the sum of the mass of bottom and charm quarks, \( i.e., m_{B_c^*} \approx m_b + m_c \). A recent lattice calculation gives \( m_{B_c} = 6331(7) \) MeV [16], which agrees basically with many other theoretical estimations with various theoretical models (\( e.g., \) references [150-207] of Ref. [17]). Owing to the hierarchical relationships among mesonic mass, \( m_{B_c^*} < m_B + m_D \) and \( m_{B_c^*} < m_{B_c} + m_\pi \), the \( B_c^* \) meson can not decay through the strong interactions. The dominant decay of the \( B_c^* \) meson is the radiative transition process, \( B_c^* \rightarrow B_c + \gamma \). This partial decay width can be written as [18],

\[ \Gamma(B_c^* \rightarrow B_c \gamma) = \frac{4}{3} \alpha_{em} k_\gamma^3 |\mu_{B_c^* B_c}|^2, \]  

(4)

where the center-of-mass momentum of a photon in the rest frame of the \( B_c^* \) meson and the magnetic dipole (M1) momentum are respectively defined as,

\[ k_\gamma = \frac{m_{B_c^*}^2 - m_{B_c}^2}{2 m_{B_c^*}} \approx 56 \text{ MeV}, \]  

(5)

\[ \mu_{B_c^* B_c} = \langle B_c | \sum_{i=b,c} \frac{Q_i}{2 m_i} \sigma_{iz} | B_c^* \rangle = \frac{1}{6} \left( \frac{2}{m_c} - \frac{1}{m_b} \right). \]  

(6)

It is clear that kinematically, the parity and angular momentum conservation in electromagnetic interactions require the orbital angular momentum between the \( B_c \) meson and photon.
to be \( L = 1 \). In addition, the photon is very soft, resulting in a very small phase space. Dynamically, the decay width is proportional to the fine-structure constant \( \alpha_{\text{em}} \) of the electromagnetic interactions and the module square of the magnetic dipole momentum, while the magnetic dipole momentum is inversely proportional to the mass of heavy quarks. The combined effects of both kinematical and dynamical factors produce a very narrow decay width, \( \Gamma(B_c^* \to B_c \gamma) \approx 60 \text{ eV} \) [17]. A good approximation is the full decay width of the \( B_c^* \) meson \( \Gamma_{B_c^*} \approx \Gamma(B_c^* \to B_c \gamma) \). Experimentally, the electromagnetic process \( B_c^* \to B_c \gamma \) with an occurrence probability of almost 100% should be easily detected at the \( e^+e^- \) CEPC and FCC-ee colliders, thanks to the excellent photon resolution of the electromagnetic calorimeter, and thanks to the fine performance in reconstruction technology and method of the charged particle tracks. Because the masses of \( B_c^* \) and \( B_c \) mesons are very close, the detection of the photon from the \( B_c^* \to B_c \gamma \) process plays a critical role in distinguishing between the \( B_c^* \) and \( B_c \) mesons. The identification of the soft photon will be very challenging. What’s more, the photon from the M1 transition \( B_c^* \to B_c + \gamma \) decay is bound to be seriously affected by those from bremsstrahlung radiation and chaotic electromagnetic backgrounds, which results in identification inefficiencies. Besides the electromagnetic interactions, the \( B_c^* \) meson can also decay through the weak interactions in the standard model (SM) of elementary particles. The narrowness of the full width of the \( B_c^* \) meson affords a great potential for experimental investigations on the \( B_c^* \) weak decays. The \( B_c^* \) weak decays will provide a good opportunity and an important and useful alternative to find and explore the foreseeable \( B_c^* \) mesons with some novel ways at the future high-luminosity and high-precision experiments.

Compared with the electromagnetic \( B_c^* \) decays, there are plenty of \( B_c^* \) meson weak decay processes. Based on the weak interaction couplings among particles, both component quarks of the \( B_c^* \) meson, the heavy bottom and charm quarks can transmit into first and second family quarks lighter than themselves. These \( B_c^* \) weak processes can be classified into three types, similar to those for the pseudoscalar \( B_c \) meson, (1) the bottom quark decays while the charm quark remain quiescent as a spectator; (2) the charm quark decays while the bottom quark remain inactive as a spectator; (3) the bottom and charm quarks annihilate into a virtual charged \( W \) boson. The purely leptonic \( B_c^* \) decays belonging to type (3) will suffer additional complications from the final neutrinos. The charged hadrons from \( B_c^* \) weak decays, such as pions and kaons, are relatively easy to identify at sensitive particle detectors. In this paper, we will estimate the branching ratio for \( B_c^{*+} \to BP, BV, B^*P, \psi P, \eta_c P, \)
η_cV decays arising from external W emission with the factorization approach, where P (V) denotes the positively charged pion and kaon (ρ+ and K∗+). Some studies [19–22] have shown that these processes in question have relatively large branching ratios among B_c^* meson weak decays, and should have the priority to be investigated experimentally. Here, we hope to provide a feasibility analysis of searching for the B_c^* meson via some particular nonleptonic weak decays in the future high-energy and high-luminosity experiments.

The remainder of this paper is organized as follows. The effective Hamiltonian for the nonleptonic B_c^* weak decays is given in Section II. Branching ratios and our comments are presented in Section III. Section IV is devoted to a brief summary. The decay amplitudes are listed in Appendix.

II. THEORETICAL FRAMEWORK

The effective Hamiltonian for the concerned nonleptonic B_c^* decays is written as [23],

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{uq} \{ C_1 O_{1b}^b + C_2 O_{2b}^b \} \]

\[ + \frac{G_F}{\sqrt{2}} V_{c\bar{q}2}^* V_{uq3} \{ C_1 O_{1c}^c + C_2 O_{2c}^c \}, \]

where Fermi constant \( G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2} \) [2] and Wilson coefficients \( C_{1,2} \) are process-independent couplings. Eq.(7) and Eq.(8) correspond to type (1) and (2) B_c^* decays, respectively. \( V_{cq} \) and \( V_{uq} \) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [24, 25], and their magnitudes have been determined experimentally [2].

\[ |V_{ud}| = 0.97370(14), \]

\[ |V_{us}| = 0.2245(8), \]

\[ |V_{cd}| = 0.221(4), \]

\[ |V_{cs}| = 0.987(11), \]

\[ |V_{cb}| = 0.0410(14). \]

The expressions of effective tetra-quark operators are written as,

\[ O_{1b}^b = [\bar{b}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{1\beta}], \]

\[ O_{2b}^b = [\bar{b}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{1\alpha}], \]
\[ O_1^c = \left[ \bar{q}_2 \gamma^\mu (1 - \gamma_5) c_\alpha \right] \left[ \bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_3 \beta \right], \quad (16) \]
\[ O_2^c = \left[ \bar{q}_2 \gamma^\mu (1 - \gamma_5) c_\beta \right] \left[ \bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_3 \alpha \right], \quad (17) \]

where \( \alpha \) and \( \beta \) are the color indices, \( q_{1,2,3} = d \) and \( s \).

It is clear that the calculation of hadronic matrix elements (HMEs) is the remaining central missions of decay amplitudes. HMEs trap the tetra-quark operators between initial and final mesons, and involve the long- and short-distance contributions. Recently, several attractive phenomenological methods based on power counting rules and QCD perturbative calculations have been developed to deal with HMEs, such as the QCD factorization (QCDF) approach built upon the collinear approximation [26–36], and the perturbative QCD (pQCD) approach where contributions from the transverse momentum of valence quarks and Sudakov factors for hadronic wave functions are taken into consideration [37–43]. The theme of both the QCDF and pQCD approaches is to properly factorize the perturbative and nonperturbative contributions contained in HMEs, and appropriately evaluate their shares. The main objective of this paper is to determine whether the \( B_c^* \) meson can be explored or not in the future experiments, so an approximate estimation on branching ratio is completely sufficient. In this sense, the naive factorization (NF) approach [44] will be used and applied to nonleptonic \( B_c^* \) meson weak decay in our calculation. The physics picture of the NF approach is very simple and clear. The results from the NF approach can be regarded as the leading order approximation of those from the QCDF approach. What matters is that the NF approach often has good performance for nonleptonic \( B \) and \( D \) weak decays induced by external \( W \) emission interactions. Using the NF approximation as a working hypothesis, it is usually assumed that the final state interactions and annihilation contributions might be disregarded. The product of quark currents in effective tetra-quark operators could be replaced by the product of the corresponding hadronic currents formed by the physical hadrons involved. HMEs of tetra-quark operators are separated into two HMEs of hadronic currents, which are further parameterized by hadronic decay constants and transition form factors.

Due to the relatively large mass, the vector mesons decay dominantly through the strong and/or electromagnetic interactions within the SM. The weak decays of the vector mesons are rare processes and usually draw less attention. Following the conventions in Refs. [45–49], the hadronic decay constants and transition form factors are defined by HMEs of color-singlet...
where \( \bar{q} \gamma_\mu q \), respectively;
diquark currents.

\[
j^V_\mu = \bar{q}_1 \gamma_\mu q_2, \\
\]
\[
j^A_\mu = \bar{q}_1 \gamma_\mu \gamma_5 q_2, \\
\]
\[
\langle P(k) | j^V_\mu | 0 \rangle = 0, \\
\]
\[
\langle P(k) | j^A_\mu | 0 \rangle = -i f_P k_\mu, \\
\]
\[
\langle V(k, \epsilon) | j^V_\mu | 0 \rangle = f_V m_V \epsilon_\mu, \\
\]
\[
\langle V(k, \epsilon) | j^A_\mu | 0 \rangle = 0, \\
\]
\[
\langle P(p_2) | j^V_\mu | B^*_c(\epsilon, p_1) \rangle = \varepsilon_{\mu \alpha \beta} \epsilon^\nu P^\alpha q^\beta \frac{V^{B^*_c \rightarrow P}(q^2)}{m_{B^*_c} + m_P}, \\
\]
\[
\langle P(p_2) | j^A_\mu | B^*_c(\epsilon, p_1) \rangle \\
= +i 2 m_{B^*_c} \frac{c^\nu q}{q^2} q_\mu A^\beta_{B^*_c \rightarrow P}(q^2) + i \epsilon_\mu (m_{B^*_c} + m_P) A^\beta_{B^*_c \rightarrow P}(q^2) \\
+ i \frac{c^\nu q}{m_{B^*_c} + m_P} P_\mu A^\gamma_{B^*_c \rightarrow P}(q^2) - i 2 m_{B^*_c} \frac{c^\nu q}{q^2} q_\mu A^\beta_{B^*_c \rightarrow P}(q^2), \\
\]
\[
\langle V(\epsilon_2, p_2) | j^V_\mu | B^*_c(\epsilon_1, p_1) \rangle \\
= - (\epsilon_1^* \cdot q) \left\{ P_\mu V^{B^*_c \rightarrow V}(q^2) + q_\mu V^{B^*_c \rightarrow V}(q^2) \right\} \\
+ \frac{\epsilon_1^* q}{m_{B^*_c}^2 - m_V^2} \left\{ [P^\nu - \frac{m_{B^*_c}^2 - m_V^2}{q^2} q^\nu] V^{B^*_c \rightarrow V}(q^2) + \frac{m_{B^*_c}^2 - m_V^2}{q^2} q_\mu V^{B^*_c \rightarrow V}(q^2) \right\} \\
- (\epsilon_1^* q) \epsilon_2^* \cdot q V^{B^*_c \rightarrow V}(q^2) + (\epsilon_2^* q) \epsilon_1^* \cdot q V^{B^*_c \rightarrow V}(q^2), \\
\]
\[
\langle V(\epsilon_2, p_2) | j^A_\mu | B^*_c(\epsilon_1, p_1) \rangle \\
= - i \varepsilon_{\mu \alpha \beta} \epsilon_1^\alpha \epsilon_2^\beta \left\{ [P^\nu - \frac{m_{B^*_c}^2 - m_V^2}{q^2} q^\nu] A^{\beta \gamma}_{B^*_c \rightarrow V}(q^2) + \frac{m_{B^*_c}^2 - m_V^2}{q^2} q_\mu A^{\beta \gamma}_{B^*_c \rightarrow V}(q^2) \right\} \\
- i \frac{\varepsilon_{\mu \alpha \beta} c^\alpha}{m_{B^*_c}^2 - m_V^2} \left\{ (\epsilon_1^* q) \epsilon_1^\nu A^{\beta \gamma}_{B^*_c \rightarrow V}(q^2) - (\epsilon_1^* q) \epsilon_2^* \cdot q A^{\beta \gamma}_{B^*_c \rightarrow V}(q^2) \right\}, \\
\]
where \( m_P (m_V) \) and \( f_P (f_V) \) are the mass and decay constant of final pseudoscalar (vector) mesons, respectively; \( \epsilon_i \) is the polarization vector; \( A_i \) and \( V_i \) are mesonic transition form factors; the momentum \( P = p_1 + p_2 \) and \( q = p_1 - p_2 \). There are some relationships among form factors,

\[
A^{B^*_c \rightarrow P}_{0}(0) = A^{B^*_c \rightarrow P}_{3}(0), \\
A^{B^*_c \rightarrow V}_{1}(0) = A^{B^*_c \rightarrow V}_{2}(0), \\
\]
The decay constants and form factors are nonperturbative hadronic parameters. These parameters are universal, and can be obtained from data and some nonperturbative methods. The experimental data on the $B_c^*$ meson are still unavailable. Phenomenologically, form factors at the pole $q^2 = 0$ are expressed as the overlap integrals of mesonic wave functions [45], where the mesonic wave functions are nonperturbative but process-independent physical quantities. For example, the form factors for the $B_c^* \to B_{d,s}$ transitions have been calculated with the Wirbel-Stech-Bauer model in Ref. [20]. Recently, the form factors for the $B_c^* \to J/\psi, B_{d,s}^*$ transitions have also been investigated with the light front quark model (LFQM) in Ref. [48]. Additionally, as noted, the latest decay constants obtained with LFQM [50, 51] are generally consistent with the data as well as those from lattice QCD simulations and QCD sum rule approaches. Following the calculation in Ref. [48], we obtain the form factors for the $B_c^* \to \psi(2S), \eta_c(1S), \eta_c(2S), B_{d,s}$ transitions with LFQM. The numerical values of the form factors are listed in Table I. The theoretical uncertainties are not particularly important or worthy of our attention for the moment. After all, the magnitude order estimation of branching ratios for the nonleptonic $B_c^*$ weak decays is quite sufficient for the present purposes.

III. BRANCHING RATIO

The branching ratio of nonleptonic $B_c^*$ decays is defined as

$$B_r = \frac{\rho_{cm}}{24 \pi m_{B_c^*}^2 \Gamma_{B_c^*}} | \mathcal{A} |^2.$$  \hspace{1cm} (31)

where $p_{cm}$ is the center-of-mass momentum of final states in the rest frame of the $B_c^*$ meson. $\mathcal{A}$ denotes the decay amplitudes, which are collected in the Appendix. With the input parameters in Table I and Table II, we obtain the branching ratios, which are listed in Table III.

Our comments on branching ratios are as follows.

(1) For the nonleptonic $B_c^*$ weak decays induced by the external $W$ emission interactions, branching ratios using the NF approach are generally of the same order of magnitude as previous estimations with the QCD-inspired QCDF and pQCD approach [19–21]. The consistency of the results with different approaches indicates that our results in this paper might
TABLE I: Mesonic decay constants [2, 17] and form factors at the pole $q^2 = 0$ [48].

| Decay          | $f_\pi =$ 130.2±1.2 MeV | $f_K =$ 155.7±0.3 MeV | $f_\rho =$ 207.7±1.6 MeV | $f_{K^*} =$ 202.5$^{+6.5}_{-6.7}$ MeV |
|----------------|--------------------------|-----------------------|--------------------------|-------------------------------------|
| $V_1^{B_c^*\to\psi(1S)}$ | 0.56                     | $V_2^{B_c^*\to\psi(1S)}$ | 0.33                     | $V_3^{B_c^*\to\psi(1S)}$ | 0.20                     | $V_5^{B_c^*\to\psi(1S)}$ | 1.17                     |
| $V_6^{B_c^*\to\psi(1S)}$ | 0.65                     | $A_{1,2}^{B_c^*\to\psi(1S)}$ | 0.54                     | $A_3^{B_c^*\to\psi(1S)}$ | 0.13                     | $A_4^{B_c^*\to\psi(1S)}$ | 0.14                     |
| $V_1^{B_c^*\to\psi(2S)}$ | 0.39                     | $V_2^{B_c^*\to\psi(2S)}$ | 0.32                     | $V_3^{B_c^*\to\psi(2S)}$ | 0.12                     | $V_5^{B_c^*\to\psi(2S)}$ | 0.79                     |
| $V_6^{B_c^*\to\psi(2S)}$ | 0.48                     | $A_{1,2}^{B_c^*\to\psi(2S)}$ | 0.37                     | $A_3^{B_c^*\to\psi(2S)}$ | 0.13                     | $A_4^{B_c^*\to\psi(2S)}$ | 0.07                     |
| $V_1^{B_c^*\to B_s^*}$ | 0.63                     | $V_2^{B_c^*\to B_s^*}$ | 1.06                     | $V_3^{B_c^*\to B_s^*}$ | 0.40                     | $V_5^{B_c^*\to B_s^*}$ | 3.52                     |
| $V_6^{B_c^*\to B_s^*}$ | 3.02                     | $A_{1,2}^{B_c^*\to B_s^*}$ | 0.53                     | $A_3^{B_c^*\to B_s^*}$ | 0.73                     | $A_4^{B_c^*\to B_s^*}$ | 0.85                     |
| $V_1^{B_c^*\to B^*}$ | 0.52                     | $V_2^{B_c^*\to B^*}$ | 1.18                     | $V_3^{B_c^*\to B^*}$ | 0.40                     | $V_5^{B_c^*\to B^*}$ | 3.15                     |
| $V_6^{B_c^*\to B^*}$ | 2.66                     | $A_{1,2}^{B_c^*\to B^*}$ | 0.43                     | $A_3^{B_c^*\to B^*}$ | 0.81                     | $A_4^{B_c^*\to B^*}$ | 0.89                     |
| $V^{B_c^*\to \eta_c(1S)}$ | 0.91                     | $A_0^{B_c^*\to \eta_c(1S)}$ | 0.66                     | $A_1^{B_c^*\to \eta_c(1S)}$ | 0.69                     | $A_2^{B_c^*\to \eta_c(1S)}$ | 0.59                     |
| $V^{B_c^*\to \eta_c(2S)}$ | 0.59                     | $A_0^{B_c^*\to \eta_c(2S)}$ | 0.43                     | $A_1^{B_c^*\to \eta_c(2S)}$ | 0.41                     | $A_2^{B_c^*\to \eta_c(2S)}$ | 0.51                     |
| $V^{B_c^*\to B_s}$ | 3.40                     | $A_0^{B_c^*\to B_s}$ | 0.69                     | $A_1^{B_c^*\to B_s}$ | 0.75                     | $A_2^{B_c^*\to B_s}$ | 0.96                     |
| $V^{B_c^*\to B}$ | 3.08                     | $A_0^{B_c^*\to B}$ | 0.60                     | $A_1^{B_c^*\to B}$ | 0.65                     | $A_2^{B_c^*\to B}$ | 0.91                     |

TABLE II: Mesonic mass (in the unit of MeV) [2], where their central values will be regarded as the default inputs unless otherwise specified.

| Mass          | $m_\pi =$ 139.57          | $m_\rho =$ 775.26±0.25     | $m_{B_d} =$ 5279.65±0.12     | $m_{B_d} =$ 5324.70±0.21     |
|---------------|---------------------------|-----------------------------|-------------------------------|-------------------------------|
| $m_K =$ 493.677±0.016 | $m_K^* =$ 891.66±0.26      | $m_{\eta_c(1S)} =$ 2983.9±0.5    | $m_{\psi(1S)} =$ 3096.9        |
| $m_{B_s} =$ 5366.88±0.14 | $m_{B_s} =$ 5415.4$^{+1.8}_{-1.5}$ | $m_{\eta_c(2S)} =$ 3637.5±1.1   | $m_{\psi(2S)} =$ 3686.10±0.06  |

be reasonable. Besides, the branching ratios for the $B_c^* \to B_s^* \pi$, $B_s^* d K$, $\eta_c(2S) \rho$, $\eta_c(2S) K^*$ decays are estimated for the first time. It is found that the $B_c^* \to B_s^* \pi$ decay has a relatively large branching ratio among the two-meson $B_c^*$ weak decays. In addition, both the initial $B_c^*$ meson and one of the final meson of the processes in question contain one and/or two of the heavy quarks. Due to the fact that the light quarks and gluon clouds are almost blind to the spin of the heavy quark with the approximation of the heavy quark limit, the heavy quark spin symmetry should be expected to relate the initial and final mesons [52]. In the
TABLE III: Branching ratios and event numbers of nonleptonic $B^+_c$ weak decays, assuming that about $10^8$, $10^9$ and $3 \times 10^{10}$ $B^+_c$ mesons will be available at the future CEPC, FCC-ee and LHCb@HL-LHC experiments, respectively; where $\lambda \approx 0.2$ is the phenomenological Wolfenstein parameter.

| decay mode | CKM factor | branching ratio | event numbers |
|------------|------------|----------------|---------------|
|            |            | unit | previous | CEPC | FCC-ee | LHCb |
| $B_s \pi^+$ | $V_{cs}V_{ud} \sim \mathcal{O}(1)$ | $10^{-7}$ | 4.4 | 4.0 [20] | 7.3 [20] | 9.8 [20] | 44 | 442 | 13256 |
| $B_s \rho^+$ | $V_{cs}V_{ud} \sim \mathcal{O}(1)$ | $10^{-6}$ | 1.9 | 0.6 [20] | 1.2 [20] | 1.7 [20] | 193 | 1930 | 57887 |
| $B^+_s \pi^+$ | $V_{cs}V_{ud} \sim \mathcal{O}(1)$ | $10^{-6}$ | 1.4 | 143 | 1428 | 42848 |
| $B_s K^+$ | $V_{cs}V_{us} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 2.2 | 2.0 [20] | 3.6 [20] | 4.9 [20] | 2 | 22 | 658 |
| $B_s K^{*+}$ | $V_{cs}V_{us} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 7.2 | 2.2 [20] | 4.1 [20] | 6.0 [20] | 7 | 72 | 2169 |
| $B^+_s K^{*+}$ | $V_{cs}V_{us} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 8.4 | 10 | 97 | 2924 |
| $B_d \pi^+$ | $V_{cd}V_{ud} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 2.1 | 2.1 [20] | 4.5 [20] | 6.5 [20] | 2 | 21 | 640 |
| $B_d \rho^+$ | $V_{cd}V_{ud} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 9.7 | 3.5 [20] | 7.5 [20] | 11.5 [20] | 10 | 97 | 2924 |
| $B^+_d \pi^+$ | $V_{cd}V_{ud} \sim \mathcal{O}(\lambda)$ | $10^{-8}$ | 6.3 | 6 | 63 | 1884 |
| $B_d K^+$ | $V_{cd}V_{us} \sim \mathcal{O}(\lambda^2)$ | $10^{-9}$ | 1.1 | 1.2 [20] | 2.4 [20] | 3.5 [20] | 0 | 1 | 34 |
| $B_d K^{*+}$ | $V_{cd}V_{us} \sim \mathcal{O}(\lambda^2)$ | $10^{-9}$ | 4.4 | 1.5 [20] | 3.1 [20] | 4.8 [20] | 0 | 4 | 133 |
| $B^+_d K^{*+}$ | $V_{cd}V_{us} \sim \mathcal{O}(\lambda^2)$ | $10^{-9}$ | 3.9 | 0 | 4 | 117 |

In the extreme non-relativistic limit, it is expected to have [52]

$$ r = \frac{Br(1^- \rightarrow 1^- \pi)}{Br(1^- \rightarrow 0^- \pi)} \approx \frac{dBr(1^- \rightarrow 1^- \ell \nu)}{dq^2} \bigg|_{q^2 = m_{\ell}^2} \approx 3. $$

(32)
The mass difference in the phase factors will furnish the above relations in Eq.(32). Here, some relative branching ratios are,

\begin{align*}
  r_1 &= \frac{\mathcal{B}r(B^+_c \to B^*_s \pi)}{\mathcal{B}r(B^+_c \to B_s \pi)} = 3.2, \\
  r_2 &= \frac{\mathcal{B}r(B^+_c \to B^*_d \pi)}{\mathcal{B}r(B^+_c \to B_d \pi)} = 2.9, \\
  r_3 &= \frac{\mathcal{B}r(B^+_c \to B^*_s K)}{\mathcal{B}r(B^+_c \to B_s K)} = 3.8, \\
  r_4 &= \frac{\mathcal{B}r(B^+_c \to B^*_d K)}{\mathcal{B}r(B^+_c \to B_d K)} = 3.4, \\
  r_5 &= \frac{\mathcal{B}r(B^+_c \to \psi(1S) \pi)}{\mathcal{B}r(B^+_c \to \eta_c(1S) \pi)} = 3.3, \\
  r_6 &= \frac{\mathcal{B}r(B^+_c \to \psi(1S) K)}{\mathcal{B}r(B^+_c \to \eta_c(1S) K)} = 3.4, \\
  r_7 &= \frac{\mathcal{B}r(B^+_c \to \psi(2S) \pi)}{\mathcal{B}r(B^+_c \to \eta_c(2S) \pi)} = 3.8, \\
  r_8 &= \frac{\mathcal{B}r(B^+_c \to \psi(2S) K)}{\mathcal{B}r(B^+_c \to \eta_c(2S) K)} = 3.9.
\end{align*}

For the charm quark decay, the ratio \( r_1 \) of the Cabibbo-favored \( B^+_c \to B^*_s \pi, B_s \pi \) decays is generally consistent with both the expectation Eq.(32) from the heavy quark spin symmetry and the ratio of semileptonic \( J/\psi \) weak decays \( \mathcal{B}r(J/\psi \to D^*_s e^+ \nu_e)/\mathcal{B}r(J/\psi \to D_s e^+ \nu_e) \approx 3.1 \) obtained with the QCD sum rules [49].

(2) For the \( B^+_c \) decays induced either by the bottom quark decay [type (1)] or by the charm quark decay [type (2)], there are some clear hierarchical relationship among branching ratios according to the CKM factors of decay amplitudes. The Cabibbo-favored \( B^+_c \to B^*_s \pi \) and \( B_s \rho \) decays have relatively maximum branching ratios which can reach up to \( \mathcal{O}(10^{-6}) \). The CKM-suppressed \( B^+_c \to \eta_c(1S, 2S) K, \psi(1S, 2S) K, \) and \( \eta_c(1S, 2S) K^* \) decays, whose amplitudes are proportional to Wolfenstein parameter \( \lambda^3 \), have relatively minimal branching ratios. In addition, there are three partial wave amplitudes for final states containing one pseudoscalar plus vector mesons, while there is only the \( p \)-wave amplitude for two final pseudoscalar mesons. Hence, there are some relationships, \( \mathcal{B}r(B^+_c \to B^*_s \pi) > \mathcal{B}r(B^+_c \to B_s \pi) \) and \( \mathcal{B}r(B^+_c \to B_s \rho) > \mathcal{B}r(B^+_c \to B_s \pi) \), and these hierarchical relationships are also true for other similar cases where final states have the same valence quark compositions but different orbit-spin couplings among quarks.
(3) A more careful theoretical investigation of these decays in question is necessary and helpful to explore the hadronic $B_c^*$ weak decays experimentally. It should be pointed out that many influences can affect the final numerical results on nonleptonic $B_c^*$ weak decays, such as the nonfactorizable contributions. It is clear from Eq. (9-13) that $|V_{cb}|$ has the largest uncertainty, about 3%, among the CKM elements involved. Except for the decay constant of $f_{K^*}$, the uncertainties from other decay constants in Table I is less than 1%. For the nonleptonic decays induced by the external $W$ boson emission interactions, the nonfactorizable contributions to the coefficient $a_1$ is about 15% for charm quark decays [47], i.e., the $B_c^* \rightarrow B_{d,s}^{(*)}$ transitions, and about 5% for bottom quark decays [35, 36], i.e., the $B_c^* \rightarrow \psi$ and $\eta_c$ transitions. Large theoretical uncertainties come mainly from the hadronic transition form factors, and some of them can reach about 40% [48]. In addition, the undetermined decay width of the $B_c^*$ meson (for example, $\Gamma(B_c^* \rightarrow \gamma B_c) = 20 \sim 135$ eV, see Table 2 of Ref. [21]) will bring very large theoretical uncertainties to branching ratios. An accurate theoretical prediction seems to be temporarily unavailable. All numbers in Table III are rough estimates and only indicative of the expected order of magnitude. Here, what matters to us is whether it is possible to study the unacquainted $B_c^*$ mesons in future experiments. Hence, a rough estimate is sufficient.

(4) As it is well known that the vector $\rho$ and $K^*$ mesons are resonances, they will decay immediately via the strong interactions, with branching ratios $Br(\rho \rightarrow \pi\pi) \sim 100\%$ and $Br(K^* \rightarrow K\pi) \sim 100\%$ [2]. The vector $\rho$ ($K^*$) meson is reconstructed experimentally by the final pseudoscalar mesons. An educated guess is that the branching ratios for the three-body decay modes, $B_c^* \rightarrow B_{s,d}\pi\pi$, $B_{s,d}\pi K$, $\eta_c\pi\pi$, $\eta_c\pi K$, should be of a similar order of magnitude to those for the $B_c^* \rightarrow B_{s,d}\rho$, $B_{s,d}K^*$, $\eta_c\rho$, $\eta_cK^*$, decays, respectively. If the contributions from other possible resonances reconstructed from the $\pi\pi$, $\pi K$, $B_{s,d}\pi$, $B_{s,d}K$, $\eta_c\pi$ and $\eta_cK$ states are taken into consideration, the above hadronic three-body $B_c^*$ decays are likely to have even larger branching ratios. All in all, it is not utopian to expected that the Cabibbo-favored $B_c^* \rightarrow B_{s}\pi$, $B_{s}\pi$, $B_{s}\rho$ decays, the singly-Cabibbo-suppressed $B_c^* \rightarrow B_{s}K$, $B_{s}K^*$, $B_{s}\pi$, $B_{s}\pi$, $B_{d}\rho$ decays, and even the CKM-suppressed $B_c^* \rightarrow \eta_c(1S)\pi$, $\eta_c(1S,2S)\rho$, $\psi(1S,2S)\pi$ decays might be observable at the future CEPC, FCC-ee and LHCb@HL-LHC experiments.
IV. SUMMARY

It has been established that the charged ground vector $B_c^*$ meson carrying explicit flavor numbers should really exist according to the quark model, but to date this has been merely on the theoretical calculation level rather than the experimental measurement level. The identification of the $B_c^*$ meson at experiments is necessary and of important significance to the quark model and SM. The signal reconstruction of the $B_c^*$ meson from the electromagnetic decay $B_c^* \rightarrow B_c \gamma$ is severely hindered by the electromagnetic background pollution. Inspired by the prospects of huge numbers of the $B_c^*$ mesons in future high-energy and high-luminosity colliders, an attractive and competitive choice might be searching for the $B_c^*$ meson from its nonleptonic weak decays, where the charged final hadrons are comparatively easily and effectively identified in experiments. In this paper, we study two kinds of nonleptonic $B_c^*$ meson weak decays resulting from external $W$ boson emission interactions, by using the factorization approximation and form factors from light front quark model, one originating from bottom quark decays, and the other from charm quark decays. It is found that the branching ratios for the Cabibbo-favored $B_c^* \rightarrow B_s^* \pi$, $B_s \rho$ decays can reach up to $O(10^{-6})$, and have the first priority to be studied at experiments. For the singly-Cabibbo-suppressed $B_c^* \rightarrow B_s K$, $B_s^* K$, $B_s K^*$, $B_d \pi$, $B_d^* \pi$, $B_d \rho$ decays and the CKM-suppressed $B_c^* \rightarrow \eta_c(1S, 2S)\pi$, $\eta_c(1S, 2S)\rho$, $\psi(1S, 2S)\pi$ decays, several hundred or even thousands of events might be observable at CEPC, FCC-ee and LHCb@HL-LHC experiments. This study provides a ready and helpful reference for experimental discovery and investigation of $B_c^*$ mesons in the future.

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Appendix A: amplitudes for nonleptonic $B_c^*$ weak decays

With the conventions of Ref. [46], the amplitudes for nonleptonic $B_c^*$ weak decays can be written as follows.

Based on the conservation of angular momentum, there are only $p$-wave amplitudes contributing to $B_c^*$ meson decay into two pseudoscalar mesons.

\[
\mathcal{A}(B_c^{*+} \to B_s^0 \pi^+) = V_{cs}^* V_{ud} f_\pi M_p^{B_s}(\epsilon_{B_c^*} \cdot p_{B_s}), \quad (A1)
\]
\[
\mathcal{A}(B_c^{*+} \to B_s^0 K^+) = V_{cs}^* V_{us} f_K M_p^{B_s}(\epsilon_{B_c^*} \cdot p_{B_s}), \quad (A2)
\]
\[
\mathcal{A}(B_c^{*+} \to B_d^0 \pi^+) = V_{cd}^* V_{ud} f_\pi M_p^{B_d}(\epsilon_{B_c^*} \cdot p_{B_d}), \quad (A3)
\]
\[
\mathcal{A}(B_c^{*+} \to B_d^0 K^+) = V_{cd}^* V_{us} f_K M_p^{B_d}(\epsilon_{B_c^*} \cdot p_{B_d}), \quad (A4)
\]
\[
\mathcal{A}(B_c^{*+} \to \eta_c \pi^+) = V_{cb}^* V_{ud} f_\pi M_p^{\eta_c}(\epsilon_{B_c^*} \cdot p_{\eta_c}), \quad (A5)
\]
\[
\mathcal{A}(B_c^{*+} \to \eta_c K^+) = V_{cd}^* V_{us} f_K M_p^{\eta_c}(\epsilon_{B_c^*} \cdot p_{\eta_c}), \quad (A6)
\]

with the common factor of $p$-wave partial amplitudes,

\[
\mathcal{M}_p^{B_s} = \sqrt{2} G_F a_1 m_{B_c^*} A_0^{B_s \to B_s}, \quad (A7)
\]
\[
\mathcal{M}_p^{B_d} = \sqrt{2} G_F a_1 m_{B_c^*} A_0^{B_d \to B_d}, \quad (A8)
\]
\[
\mathcal{M}_p^{\eta_c} = \sqrt{2} G_F a_1 m_{B_c^*} A_0^{\eta_c \to \eta_c}, \quad (A9)
\]

where coefficient $a_1 = C_1 + C_2 / N_c$ is generally influenced by nonfactorizable contributions, and can be regarded as a phenomenological parameter, especially for charm quark decays. The value of $a_1 \approx 1.2$ will be used in our calculation.

There are three partial wave amplitudes contributing to $B_c^*$ meson decay into one pseudoscalar meson plus one vector meson. The general decay amplitude is written as,

\[
\mathcal{A}(B_c^* \to VP) = \mathcal{M}(\epsilon_{B_c^*} \cdot \epsilon_V^*) + \frac{\mathcal{M}_d}{m_{B_c^*} m_V} (\epsilon_{B_c^*} \cdot p_V) (\epsilon_V^* \cdot p_{B_c^*}) + \frac{\mathcal{M}_p}{m_{B_c^*} m_V} \epsilon_{\mu \nu \alpha \beta} \epsilon_{B_c^*}^\mu \epsilon_V^\nu p_{B_c^*}^\alpha p_V^\beta, \quad (A10)
\]

where $\mathcal{M}_{s,p,d}$ correspond to the $s$-, $p$-, and $d$-wave partial amplitudes.

For $B_c^{*+} \to B_s^{*0} \pi^+$, $B_s^{*0} K^+$ decays, one has

\[
\mathcal{M}_{s}^{B_c^* \pi} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi a_1 \left( m_{B_c^*}^2 - m_{B_s^*}^2 \right) V_1^{B_s^* \to B_c^*}, \quad (A11)
\]
\[ \mathcal{M}_d^{B_s^*\pi} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi A_1 m_{B_s^*} m_{B_s^*} \left( V_4^{B_s^*\to B_s^*} - V_5^{B_s^*\to B_s^*} + V_6^{B_s^*\to B_s^*} \right), \]  
(A12)

\[ \mathcal{M}_d^{B_s^*K} = \mathcal{M}_i^{B_s^*\pi} \left( V_{ud}\to V_{us}, f_\pi\to f_K \right), \quad \text{for } i = s, p, d. \]  
(A14)

For \( B_s^{*+} \to B_d^{*0}\pi^+, B_d^{*0}K^+ \) decays, one has

\[ \mathcal{M}_s^{B_s^{*\pi}} = -i \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\pi A_1 \left( m_{B_d^{*0}}^2 - m_{\pi^0}^2 \right) V_1^{B_s^+\to \pi^+}, \]  
(A15)

\[ \mathcal{M}_d^{B_s^{*\pi}} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi A_1 m_{B_d^{*0}} m_{B_s^*} \left( V_4^{B_s^*\to B_d^{*0}} - V_5^{B_s^*\to B_d^{*0}} + V_6^{B_s^*\to B_d^{*0}} \right), \]  
(A16)

\[ \mathcal{M}_p^{B_s^{*\pi}} = -2 \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi A_1 m_{B_d^{*0}} m_{B_s^*} A_1^{B_s^*\to B_d^{*0}}, \]  
(A17)

\[ \mathcal{M}_i^{B_s^{*\pi}} \left( V_{ud}\to V_{us}, f_\pi\to f_K \right), \quad \text{for } i = s, p, d. \]  
(A18)

For \( B_s^{*+} \to \psi^{\pi^+}, \psi^K^+ \) decays, one has

\[ \mathcal{M}_s^{\psi\pi} = -i \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\pi A_1 \left( m_{\psi}^2 - m_{\pi^0}^2 \right) V_1^{B_s^+\to \psi^+}, \]  
(A19)

\[ \mathcal{M}_d^{\psi\pi} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi A_1 m_{\psi} m_{\psi} \left( V_4^{B_s^+\to \psi^+} - V_5^{B_s^+\to \psi^+} + V_6^{B_s^+\to \psi^+} \right), \]  
(A20)

\[ \mathcal{M}_p^{\psi\pi} = -2 \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi A_1 m_{\psi} A_1^{B_s^+\to \psi^+}, \]  
(A21)

\[ \mathcal{M}_i^{\psi\pi} \left( V_{ud}\to V_{us}, f_\pi\to f_K \right), \quad \text{for } i = s, p, d. \]  
(A22)

For \( B_s^{*+} \to B_s^{0}\rho^+, B_s^{0}K^{*+} \) decays, one has

\[ \mathcal{M}_s^{B_s^{*\rho}} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\rho A_1 \left( m_{B_s^{*0}} + m_{B_s} \right) A_1^{B_s^+\to B_s}, \]  
(A23)

\[ \mathcal{M}_d^{B_s^{*\rho}} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho A_1 \frac{2 m_{B_s^{*0}} m_{\rho}}{m_{B_s^{*0}} + m_{B_s}} A_1^{B_s^+\to B_s}, \]  
(A24)

\[ \mathcal{M}_p^{B_s^{*\rho}} = - \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho A_1 \frac{2 m_{B_s^{*0}} m_{\rho}}{m_{B_s^{*0}} + m_{B_s}} V_1^{B_s^+\to B_s}, \]  
(A25)

\[ \mathcal{M}_i^{B_s^{*\rho}} \left( V_{ud}\to V_{us}, f_\rho\to f_{K^*}, m_{\rho}\to m_{K^*} \right), \quad \text{for } i = s, p, d. \]  
(A26)

For \( B_s^{*+} \to B_d^{0}\rho^+, B_d^{0}K^{*+} \) decays, one has

\[ \mathcal{M}_s^{B_d^{*\rho}} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho A_1 \left( m_{B_d^{*0}} + m_{B_d} \right) A_1^{B_s^+\to B_d}, \]  
(A27)

\[ \mathcal{M}_d^{B_d^{*\rho}} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho A_1 \frac{2 m_{B_d^{*0}} m_{\rho}}{m_{B_d^{*0}} + m_{B_d}} A_1^{B_s^+\to B_d}, \]  
(A28)

\[ \mathcal{M}_p^{B_d^{*\rho}} = - \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho A_1 \frac{2 m_{B_d^{*0}} m_{\rho}}{m_{B_d^{*0}} + m_{B_d}} V_1^{B_s^+\to B_d}, \]  
(A29)
\[ M_{i A_i K^*} = M_{i A_i \rho} (V_{ud} \rightarrow V_{us}, f_{\rho} \rightarrow f_{K^*}, m_{\rho} \rightarrow m_{K^*}), \quad \text{for } i = s, p, d. \] (A30)

For \( B_{c}^{*+} \rightarrow \eta_c \rho^+, \eta_c K^{*+} \) decays, one has

\[ M_{s,\rho}^{\eta_c K^*} = -i G_F \sqrt{2} V_{cb}^* V_{ud} f_{\rho} m_{\rho} a_1 \left( m_{B_{c}^{*+}} + m_{\eta_c} \right) A_{1}^{B_{c}^{*+} \rightarrow \eta_c}, \] (A31)

\[ M_{d,\rho}^{\eta_c K^*} = -i G_F \sqrt{2} V_{cb}^* V_{ud} f_{\rho} m_{\rho} a_1 \frac{2 m_{B_{c}^{*+}} m_{\rho}}{m_{B_{c}^{*+}} + m_{\eta_c}} A_{2}^{B_{c}^{*+} \rightarrow \eta_c}, \] (A32)

\[ M_{p,\rho}^{\eta_c K^*} = - i G_F \sqrt{2} V_{cb}^* V_{ud} f_{\rho} m_{\rho} a_1 \frac{2 m_{B_{c}^{*+}} m_{\rho}}{m_{B_{c}^{*+}} + m_{\eta_c}} V_{B_{c}^{*+} \rightarrow \eta_c}, \] (A33)

\[ M_{i \rho}^{\eta_c K^*} = M_{i A_i \rho} (V_{ud} \rightarrow V_{us}, f_{\rho} \rightarrow f_{K^*}, m_{\rho} \rightarrow m_{K^*}), \quad \text{for } i = s, p, d. \] (A34)
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