Stability analysis of offshore wind power monopile foundation under wind load

Jianli Jin¹,³ and Cong Hu²

¹Tibet agriculture and Animal Husbandry College, Linzhi, 860000, Tibet, China, ²China Electrical Construction International Engineering Co., Ltd. Beijing, 100036, China
³Email:495688405@qq.com

Abstract. Based on the calculus and blade element theory, the formulas for calculating wind load on the blade of offshore wind turbine are deduced. The time history curve and wind speed time history curve, power spectrum and wind load of wind turbines are simulated by using MATLAB software with the derived formulas and wind observation data of London Array Wind Farm. Based on the above analysis results, the stability of offshore wind power monopile foundation structures under wind loads is analyzed and studied by using ABAQUS finite element software. It is concluded that the structural stability and ultimate load of offshore wind power structures under horizontal, lower bound and ultimate loads are the same. With the increase of internal friction angle, the shakedown load and ultimate load increase gradually with the increase of pile foundation length.

1. Introduction

The offshore wind power industry has developed rapidly in recent years, and China is rich in wind energy resources. But in China, the offshore wind power industry is still under development, and the associated design and construction about offshore wind farms are waiting for matured technologies. Therefore, this paper analyzes the stability of offshore wind power single pile structure under the wind load alone through theoretical and numerical simulations, and uses the London array wind farm as an engineering example, and then studies the offshore wind power single pile structure stability analysis method.

The infrastructure like monopile foundation is subjected to repeated action of various dynamic loads for a long time, which results in tower vibration. Under the action of long-term cyclic loads, the structure will form cracks and the cracks will continue to expand, which will eventually lead to the overall fatigue damage of the structure. Therefore, it is very important to analyze the structural deformation performance of wind power foundation and improve the fatigue strength of foundation to avoid the loss caused by fatigue damage in engineering.

Zhang Hao etc. [1, 2] analyzed the mechanical characteristics and dynamic response of offshore wind power pile foundation under the action of sea waves, Guan Dawei and Liu Chao[3, 4] studied the deformation characteristics of monopile foundation of wind power pile under the action of sea waves, Zhang Zihua etc.[5] used finite element model to study the wind power structure under extreme weather conditions. Ma Hongwang et al. [6] analyzed the influence of long-term repeated load on monopile foundation of offshore wind power, obtained the natural frequency of wind power foundation and the law of pile deformation. Wang Fei[7] applied the classical stability theory to the
stability analysis of offshore platform foundation, and obtained the offshore platform foundation by using this theory. Huang et al.[8] studied the displacement of pile body and pile head under the action of wave load, pointed out that increasing pile diameter or installing wing plate on monopile can reduce the permanent deformation of pile body. Chong et al. [9-11]studied the displacement of monopile body under the action of wave load. The volumetric strain, shear strain, strain cumulative rate and stress inclination of pile foundation under long-term cyclic loading are analyzed. The influence of pile stiffness on horizontal stress and displacement of pile body is pointed out.

The shakedown analysis is an important part of plastic deformation analysis in plastic mechanics. The plastic behavior of ideal elastic-plastic structures under cyclic loading is mainly studied. The general stability state is described as: the cumulative plastic deformation of an ideal elastic-plastic structure under cyclic loads within a certain range tends to be stable. As long as the loads remain within this range, the structure is no longer sensitive to cyclic loads after a certain number of cycles, that is, no new plastic deformation will occur. During the subsequent cycle of cyclic loading, the stress-strain characteristics of the structure show a pure elastic state, which means that the elastic-plastic structure reaches a stable state under the cyclic loading. Melan [12] extended the Bleich shakedown theorem to a more general three-dimensional case for ideal elastic-plastic materials, namely the famous static shakedown theorem, also known as the lower bound shakedown theorem. In the 1960s, Koiter [13] put forward the theory of dynamic shakedown. From the point of view of energy, the upper limit shakedown theorem is obtained under the condition that the total energy dissipated in the plasticity of the structure is less than the work done by the external cyclic load on the structure. Melan's static shakedown theorem and Koiter's mobile shakedown theorem are the lower bound and upper bound approximate solutions of the real value of the structural shakedown solution respectively. These two theorems together constitute the classical shakedown analysis theory. Zouain [14] introduced the application of shakedown theory in mechanical engineering, and discussed the numerical method of shakedown solution based on mathematical programming technology. Auricchio et al. [15] studied the cyclic response of SAMARAS under macro-elastic stability, and put forward the criterion of high cycle fatigue of SAMARAS. Qian et al. [16] carried out a series of experiments on Shanghai clay using a dynamic hollow cylinder testing machine, proposed a new method to determine the stability boundary by using effective cyclic stress ratio, and it was pointed out that the cumulative undrained response of clay with different stress levels can be described by shakedown method. Xiao et al. [17] in plastic shakedown state, there are less broken ballast particles, and the strain rate of the sample decreases rapidly with the increase of loading cycles; in plastic creep state, the ballast particles break at the beginning of loading cycle, and then tend to be stable. In the incremental collapse state, ballast particles are broken under initial cyclic loading, but the plastic deformation of specimens is mainly caused by rapid shear failure. The stress limit of ballast specimens under different stable conditions increases with the increase of confining pressure, but the effect of loading frequency on the stress limit is related to the stress level of confining pressure. Based on this, an empirical exponential model is established. At present, the application of stability theory mainly concentrates on machinery, road engineering and other fields. The application of shakedown analysis in offshore engineering, especially the study of shakedown analysis method of offshore wind power monopile foundation under cyclic loads such as wind load and wave load, needs qualitative and quantitative analysis.

Therefore, in this paper, considering wave load alone, the shakedown analysis of monopile foundation is carried out, and the distribution law of shakedown load under wave load with the change of the depth of pile foundation and the internal friction angle of foundation soil is obtained. On the basis of introducing the calculation method of wind loads on offshore wind turbines in detail, the London Array Wind Farm is selected as an engineering example. Taking the 120 m diameter Siemens 120 wind turbines used in the wind farm as an example, based on the blade element theory, the calculus idea is used to derive the calculation of offshore wind loads. Formulas for calculating wind loads of power generators. Based on the wind observation data of London Array Wind Farm, the wind
speed time history curve, power spectrum and wind load time history curve of wind turbine are simulated by MATLAB, and the offshore wind junction under wind load is further analyzed.

2. Research model

At present, the size of offshore wind turbines used in various countries is different. The common manufacturers and models of offshore wind turbines and related parameters are shown in Table 1. The physical drawings of SWT-4.0-120 offshore wind turbines used in London Array Wind Farm are shown in Figure 1.

**Table 1. Technical indicators of Offshore Wind Turbines.**

| Fan type       | cut-in wind speed | rated wind speed | Cut-out wind speed | Wind power rating | wind turbine diameter | sweep area | blade length | overall machine quality | hub height | design life | survival wind speed | applicable environment |
|----------------|-------------------|------------------|-------------------|-------------------|----------------------|------------|--------------|------------------------|------------|-------------|---------------------|------------------------|
| SWT-4.0-120    | 3-5m/s            | 13-14m/s         | 32m/s             | IEC IA            | 120m                 | 11300m²    | 58.5m        | 240E3Kg                | 90m        | 20years     | 70m/s               | offshore waters         |

**Figure 1.** Physical drawings of SWT-4.0-120 offshore wind turbines.

In order to simulate the monopile structure of wind power generation more truly, the whole generator set on the foundation of steel pipe is considered in this paper, and the wind load is directly applied on the generator blade, which is more close to the real situation. Figure 2 is numerical model of offshore wind power monopile structure. The length of fan blade is 58.5 m, the width of root is 3.2m, the width of tip is 0.2m and the thickness is 0.9m; the dimension of engine room is 13.25m × 3.60 m ×4.60 m; the height of fan tower is 90 m, the diameter of bottom is 7.5 m, the diameter of top is 2.5 m.
and the thickness is 0.09 m; the diameter of pile foundation is 7.5 m, the thickness is 0.09 m and the length is 60 m; the foundation soil is a cylindrical shape with a diameter of 90 m and a thickness of 45 m. Pile side Mohr-Coulomb penalty function form, in the form of pile bottom in contact with hard soil boundary conditions: a bottom fixed constraint, radially outward displacement constraints. The physical parameters of each component are shown in Table 2.

3. Classical wind load calculation theory

At present, the numerical simulation methods of wind load mainly include: linear filtering method [18-20] proposed by SAMARAS & MIGNLET, harmonic superposition method [21, 22] proposed by SHINOZUKA & TAN, inverse Fourier transform method proposed by FARGE & HAYASHI and wavelet generation method [23, 24].

3.1. Calculation method of Fundamental Load on Blades

Because the traditional empirical formula is used to calculate the wind load on the wind turbine, the accuracy is too low, and in the stability analysis, the accuracy of the cyclic load on the structure is higher. Therefore, based on the leaf element theory, this paper will use the calculus idea to derive the calculation formula of wind load on wind turbine blades with higher accuracy.

Blade-element theory (BET) [25-27] was put forward by Richard Froude in 1889 on the basis of momentum theory. Its specific expression is that the wind turbine blades are divided into many micro-segments along the extension direction, which are called leaf element. In fact, this is the idea of calculus proposed by Newton. The radius of the annular blade element is \( r \), and the spreading width is \( Dr \). The blade element theory assumes that the airflow acting on each blade element does not interfere with each other. The force acting on the blade is decomposed into lift and drag. The angle of attack between the resultant direction of each blade element and the blade plane is called the angle of attack. The characteristic coefficient of airfoil varies with the angle of attack.

For the blade, chord length and torsion angle will change along the blade extension direction. The blade rotates at a certain angular speed. The expression of synthetic air velocity acting on the blade element during the operation of wind turbine is as follows:

\[
W = \sqrt{U^2 (1-a)^2 + \omega^2 r^2 (1+b)^2} \quad (1)
\]

Among them, \( \omega \) is the angular velocity of blade rotation, \( U \) is the upwind inflow velocity of blade, \( r \) is the radius of any blade element, \( r \) is the tangent velocity of the blade element, and \( \phi \) is the angle between the synthetic velocity and the rotating plane. The relationship between them is as follows:

\[
\begin{align*}
\sin \phi &= \frac{U(1-a)}{W} \\
\cos \phi &= \frac{U(1+b)}{W}
\end{align*} \quad (2)
\]

In formula (1), there are two coefficients \( a \) and \( b \), which are the axial and tangential interference coefficients of the blade, respectively. Its physical meaning is to describe the degree to which the airflow velocity is affected by the wind wheel when the airflow passes through the wind wheel. Because the aerodynamic performance of generator blades is not considered in this paper, only the wind loads acting on the blades are calculated. Therefore, these two disturbance coefficients can be neglected.

Based on the blade element theory, the forces and moments acting on the blade element are first calculated, and then the forces and moments acting on the whole blade are obtained by integration along the extension direction. So we can get the theoretical accurate calculation value and avoid the huge error caused by the empirical formula. The angle between the lift component and the synthetic velocity of the unit annular width \( dr \) is 90 degrees. The expressions of the lift component and the drag component are as follows:
Combining Formula 3 and Formula 4, we can get more concise expressions:

\[
\begin{align*}
    dL &= \frac{1}{2} \rho W^2 c \, dl \\
    dD &= \frac{1}{2} \rho W^2 c \, dr \\
    dF_x &= dL \cos \varphi + dD \sin \varphi \\
    dF_y &= dL \sin \varphi - dD \cos \varphi
\end{align*}
\]

Combining Formula 3 and Formula 4, we can get more concise expressions:

\[
\begin{align*}
    dF_x &= \frac{1}{2} \rho W^2 c \, dl C_L \\
    dF_y &= \frac{1}{2} \rho W^2 c \, dr C_D
\end{align*}
\]

Among them:

\[
\begin{align*}
    C_x &= C_L \cos \varphi + C_D \sin \varphi \\
    C_y &= C_L \sin \varphi - C_D \cos \varphi
\end{align*}
\]

Among them: \( c \) is the chord length, \( dL \) is the lift component of leaf element, \( dD \) is the drag component, \( C_L \) is the lift coefficient, \( C_D \) is the drag coefficient.

3.2. Wind load integral method based on BET

The accuracy of traditional empirical formulas for calculating wind loads on wind turbines is too low. In stability analysis, the accuracy of cyclic loads on structures is required to be high. Therefore, based on the blade element theory, this paper uses the calculus idea to derive the wind load calculation formula of wind turbine blades with higher accuracy.

The cross-sectional force analysis of wind turbine blades is shown in Figure 3, where \( dL \) and \( dD \) are wind load lifting element and drag element respectively, and \( W \) is the combined speed of blade rotation. Formula 5-6 has deduced the expressions of lift element and drag element. By integrating each element along the span length of wind turbine blades, the formula for calculating wind load on wind turbine blades can be obtained accurately. However, because the surface of wind turbine blades is irregular, the integration path in this paper is irregular. Therefore, the wind load on the blades of wind turbines can be calculated by numerical integration with MATLAB.

![Diagram of forces acting on blades and wind turbines of offshore wind turbines.](image)

**Figure 3.** Diagram of forces acting on blades and wind turbines of offshore wind turbines.

The formula for calculating the wind load on the blades of wind turbines accurately can be obtained by integrating the elements along the span length of the blades of wind turbines as follows:
Formula 1 can be substituted for Formula 7:

\[
\begin{align*}
F_X &= C_x \int \frac{1}{2} \rho W^2 c d r \\
F_Y &= C_y \int \frac{1}{2} \rho W^2 c d r
\end{align*}
\]  

Formula 8 is the wind load calculation formula for wind turbine blades obtained by integral method. Later, numerical integration is carried out in MATLAB to calculate wind load for further stability analysis of offshore wind power monopile foundation.

3.3. Wind load analysis of wind generator based on MATLAB

Based on the wind power observation data of the generator of the London Array Wind Farm\[28]\ , the wind speed time history curves of the impeller vertex and the bottom point of the wind generator of the London Array Wind Farm are obtained by using the harmonic superposition method in MATLAB. The power spectrum of the vertex and the bottom point is shown in Figures 4 and 5, and in Figures 6 and 7. From Figures 4 and 5, it can be seen that the wind speed at the top and bottom of the impeller is random distribution, but from the wind speed time history curve, it can be seen that the wind speed value is actually a periodic random process within a certain range, which is a necessary parameter to simulate the wind load time history curve in MATLAB. Because of the randomness and cyclic characteristics of wind load, it is necessary to analyze the stability of offshore wind power monopile foundation under wind load in this paper.

\[
\begin{align*}
F_X &= C_x \int \frac{1}{2} \rho (U^2 + \omega^2 r^2) c d r \\
F_Y &= C_y \int \frac{1}{2} \rho (U^2 + \omega^2 r^2) c d r
\end{align*}
\]  

Figure 4. Wind speed time history curve of generator impeller vertex.  

Figure 5. Wind speed time history curve of bottom point of generator impeller.
4. Stability analysis of wind power monopile foundation under wind load

4.1. FEM model parameters

In order to eliminate the influence of boundary effect on calculation results as much as possible, the radial dimension of foundation soil is 12 times the diameter of pile, and the vertical boundary is 15 m below the bottom of pile. The three-dimensional eight node (C3D8) solid element is adopted for both pile and soil elements. The pile-soil contact type in the numerical model is contact, and the total number of grids is 258000. Mohr-Coulomb yield criterion is used for soil failure criterion, and correlation flow rule is used for soil flow criterion.

When using ABAQUS for the stability analysis of the foundation of the wind turbine single pile foundation, this paper uses the dichotomy method proposed by Sun Yang [29] based on the Melan lower limit stability theorem and the Koiter upper limit stability theorem to determine the stability load of the foundation, that is, the foundation is determined Elasto-plastic load, and then use the dichotomy method to gradually approximate the real stable load of the structure.

This paper studies the stability of the offshore wind power generation structure in three planes (Ph-Pv plane, Ph-M plane and M-Pv plane) composed of horizontal load Ph, vertical load Pv and bending moment M under the combined action of wind load. The influence of load distribution law and various influencing factors (friction angle of soil in foundation, buried depth of pile foundation, etc.) on the stability load of foundation.

4.2. Stability analysis numerical simulation working condition

The stability analysis working conditions under wind load are shown in Table 3.

| Table 3. Working condition table under wind load. |
|-----------------------------------------------|
| Analysis of Influence of Internal Friction Angle $\phi$ (°) on Stable Load |

| Model number | Internal friction angle $\phi$ (°) | Ultimate load | Stable load | Remarks |
|--------------|-----------------------------------|---------------|-------------|---------|
| FF1          | 5                                 | 1.1357        | 0.8498      |         |
| FF2          | 15                                | 1.8184        | 1.3605      |         |
| FF3          | 25                                | 2.9013        | 2.1707      |         |
| FF4          | 35                                | 4.3049        | 3.2210      |         |
| FF5          | 45                                | 4.6204        | 3.4570      | Standard Group |
| FF6          | 55                                | 4.8608        | 3.6369      |         |

Figure 6. Generator impeller vertex wind power spectrum.

Figure 7. Generators wind at the bottom of the impeller power spectrum.
Analysis of the influence of $H(m)$ buried depth of steel pipe pile on the stable load

| Model number | Buried depth of steel pipe pile $H(m)$ | Ultimate load $P_h/c$ | Stable load $P_v/c$ | Remarks |
|--------------|----------------------------------------|-----------------------|---------------------|---------|
| FH1          | 15                                     | 0                     | 5.2158              | 0       | 1.3499          |
| FH2          | 20                                     | 0.5324                | 4.7608              | 0.1442  | 1.3693          |
| FH3          | 25                                     | 1.0489                | 4.3247              | 0.2847  | 1.3679          |
| FH4          | 30                                     | 1.3901                | 3.9276              | 0.6181  | 1.3231          |
| FH5          | 35                                     | 1.9496                | 3.1721              | 0.7233  | 1.2805          |
| FH6          | 40                                     | 1.9381                | 2.2716              | 0.9063  | 0.8010          |

4.3. Numerical analysis of stability

**Figure 8.** Envelope diagram of horizontal shakedown limit load $P_h/c$ varying with internal friction angle of foundation soil.

**Figure 9.** The envelope diagram of the change of $P_h/c$ with pile length under horizontal shakedown limit load.

**Figure 10.** $P_h-P_v$ Plane Limit and Safety Load Envelope ($\phi = 35^\circ$).

**Figure 11.** $P_h-P_v$ Plane Limit and Safety Load Envelope ($\phi = 45^\circ$).
Figure 8 depicts the distribution curves of horizontal, lower bound shakedown loads and ultimate loads with the change of friction angle in the foundation soil under wind loads. From the curves, it can be seen that both upper and lower bound shakedown loads and ultimate loads increase with the increase of internal friction angle. When the internal friction angle reaches 35 degrees, the increase range is larger. It shows that the influence of the internal friction angle on the stability and ultimate load of the structure under wind load is also limited by 35 degrees. When the internal friction angle reaches 35 degrees, the stability load of the structure is no longer sensitive to the change of the internal friction angle. The variation trend is basically the same as that of the wave load studied by Wang Fei [7].

From Figure 9, it can be seen that the shakedown load and ultimate load gradually increase with the increase of the depth of pile foundation. Before the depth of monopile foundation reaches 30 m, the change rate of the shakedown load with the length of pile is larger. After 30 m, the shakedown load and ultimate load increase slowly, which indicates that the monopile foundation of offshore wind power is improving the junction. In the aspect of structural stability, the balance point of design cost and performance can be reached when the depth of pile foundation is about 30 m.

Figure 10 and 11 show the envelope of the shakedown load in the $P_h$-$P_v$ plane when the friction angle in the soil is 35 and 45 degrees, respectively. It can be seen from the Figure that the upper and lower limit shakedown areas are included in the safety area of the structure. This result satisfies the shakedown theorem. At the same time, compared with Figure 8 and 9, it can be found that when the internal friction angle is 45 degrees, the safety area, the lower limit stability area and the upper limit stability area of the structure are all larger than the corresponding areas when the internal friction angle is 35 degrees, which also conforms to the law that the shakedown load increases with the increase of the internal friction angle described in Figure 7.

The calculation method in reference [29] is compared with the limit load envelope in Figure 10 and Figure 11, although there are some differences in numerical value, the envelope obtained by the two methods is similar in shape. Therefore, qualitatively, it is reasonable to use the method proposed in this paper to solve the limit load envelope.

5. Conclusion
Based on the blade element theory and the calculus theory, the formulas for calculating the wind loads are used. According to the wind observation data of the London Array Wind Farm, the SWT-4.0-120 offshore wind turbines used in the London Array Wind Farm are used. For example, the wind speed time history curve, power spectrum and wind load time history curve of wind turbine are simulated in MATLAB. In ABAQUS finite element software, the shakedown analysis of offshore wind power monopile foundation structure under wind load is carried out. The main conclusions are as follows:

1) Under the action of wind load, the horizontal and lower limit shakedown loads and ultimate loads of structures increase with the increase of internal friction angle, and the influence of internal friction angle on the shakedown and ultimate loads of structures under wind load is limited by 35 degrees. After the internal friction angle reaches 35 degrees, the shakedown loads of structures affect the internal friction of soils. Angle changes are no longer sensitive.

2) The shakedown load and ultimate load increase with the increase of pile foundation length. In the aspect of improving the shakedown performance of offshore wind power monopile foundation, the balance point of design cost and service performance can be reached when the pile length is about 30 m.

3) In the envelope of $P_h$-$P_v$ plane, the upper and lower limit stability regions are included in the envelope of the structure when the internal friction angle is 35 degrees and 45 degrees. At the same time, when the internal friction angle is 45 degrees, the safety regions, the lower limit stability regions and the upper limit stability regions are all larger than the corresponding regions when the internal friction angle is 35 degrees.
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