Prompt Photon Plus Jet Photoproduction at HERA at
Next-to-Leading Order in QCD

L. E. Gordon
High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439

Abstract

The cross section for photoproduction of an isolated prompt photon in association with a jet is studied in Next-to-Leading Order. The kinematics are those appropriate for the DESY $ep$ collider HERA. The effects on the cross section of various experimental cuts including isolation cuts on the photon is examined. Comparisons with the ZEUS preliminary data using two parametrizations of the photon structure function is made, and good agreement is found. The data is not yet precise enough to make a distinction between various models for the photon structure function.
I. INTRODUCTION

It has long been anticipated that the DESY ep collider HERA would provide a good opportunity to study prompt photon production in photoproduction processes \[1\]. Over the past few years various calculations of this process have been performed leading to continuous improvements in their theoretical precision \[2–4,6\]. In the most recent studies \[6,7\] the inclusive cross section for producing a single photon was calculated fully in NLO with photon isolation effects incorporated. Gordon and Vogelsang \[6,7\] use an approximate but nevertheless accurate analytic technique \[8,9\] for including isolation effects in the NLO calculation, including the fragmentation contributions. This analytic technique is only applicable to single inclusive prompt photon production and cannot be applied when a jet is also observed.

The ZEUS Collaboration have reported prompt photon data \[10\] and have first chosen to analyse events with a jet balancing the transverse momentum \(p_T^{\gamma}\) of the photon. In order to compare with this data a new calculation is necessary as described in outline in the next section.

In all previous studies of prompt photon production at HERA, one of the common themes was the possibility of using it for measuring the photon distribution functions, particularly the gluon distribution, \(g^\gamma(x, Q^2)\) which is presently poorly constrained by the available data. This latter fact is still true even with the availability of jet photoproduction data from both HERA and TRISTAN. Prompt photon production is particularly attractive since it is dominated in Leading Order (LO) by the hard scattering subprocess \(qq \to \gamma q\), resulting in a cross section which is very sensitive to the gluon distribution.

At HERA the situation is more complicated than at hadron colliders for two reasons. Firstly there are two particles involved in the reaction, namely the quasi-real photon (emitted by the electron which scatters at a small angle) and the proton. Both particles have distinct gluon distribution functions \(g^\gamma\) and \(g^p\), hence two different \(qg\) initiated subprocesses are present, \(q^p g^\gamma \to \gamma q\) and \(q^\gamma g^p \to \gamma q\). Since they contribute to the cross section in different regions of pseudo-rapidity, \(\eta\), it has been proposed that this may provide a means of sepa-
rating them, but this has proven to be difficult to implement in the experiments. Secondly, there are two contributions to the cross section in photoproduction processes, usually labelled the direct and resolved. In the former case the quasi-real photon participates directly in the hard scattering subprocess and gives up all its energy, while in the latter, resolved, case it interacts via its partonic substructure. Thus the resolved subprocesses are sensitive to the photon structure functions whereas the direct are not. Again it was proposed that they may be separated experimentally with suitable rapidity cuts, but these studies assumed a fixed initial photon energy. Since the initial photon energy is not fixed but forms a continuous spectrum, then even this separation is not straightforward [4]. This is because the spectrum of initial photon energies causes the sharply separated peaks in the rapidity spectrum of the resolved and direct components, present when the initial photon energy is fixed, to become smeared out and so less sharply defined. Separation of the resolved and direct processes is better achieved by tagging of the spectator jet from the resolved photon.

In section II a brief outline of the theoretical background to the cross section as well as the technique of calculation is given. In section III numerical results are presented and in section IV the summary and conclusions are presented.

II. THE INCLUSIVE PHOTON PLUS JET CROSS SECTION

A. Contributing Subprocesses

In addition to the direct and resolved photon contributions to the cross section there are the non-fragmentation and fragmentation contributions. In the former case the observed final state photon is produced directly in the hard scattering whereas in the latter it is produced by long distance fragmentation off a final state parton. The fragmentation processes involve the functions which cannot be calculated and must be taken from experiment. So far they have not been satisfactorily measured. There are various parametrizations of these functions available using different models for the input distributions. As the numerical re-
sults will show in the next section these contributions are small at HERA energies and so do not provide a significant source of uncertainty in the present calculation. This point has already been noted in previous studies and will be returned to below.

The only direct non-fragmentation process contributing to the cross section in LO is the so called QCD Compton process (fig.1a)

\[ q\gamma \rightarrow \gamma q. \]

The corresponding direct fragmentation processes in LO (fig.1b) are

\[ q\gamma \rightarrow gq \quad \text{and} \quad g\gamma \rightarrow q\bar{q}. \]

As discussed in many places (see eg., \cite{6}) the photon fragmentation function is formally \( O(\alpha_{em}/\alpha_s) \), thus although the hard subprocess cross sections in the fragmentation case are \( O(\alpha_{em}\alpha_s) \), after convolution with the photon fragmentation functions the process contributes at \( O(\alpha_{em}^2) \), the same as the non-fragmentation part. Thus in a fixed order calculation the two contributions must be added together to provide the physical cross section.

At NLO for the non-fragmentation part there are the virtual corrections to the LO Compton process plus the additional three-body processes

\[ q\gamma \rightarrow \gamma gq \quad \text{and} \quad g\gamma \rightarrow q\bar{q}. \]

These processes have been calculated previously by various authors. In this study the virtual corrections are taken from \cite{11}. In addition there are \( O(\alpha_s) \) corrections to the fragmentation processes to take into account, but in this calculation these processes are included in LO only.

It has been shown previously \cite{6} that the fragmentation contributions are not as significant here as at hadron colliders which generally have higher cms energies. They are also reduced drastically when isolation cuts are implemented. Thus ignoring NLO corrections to the fragmentation contributions, while in principle theoretically inconsistent, will not lead to significant error in estimates of the cross section.

In the resolved case, for non-fragmentation there are only the two processes
\[ qg \rightarrow \gamma q \quad \text{and} \quad q\bar{q} \rightarrow \gamma g. \]

in LO (fig.2). At NLO there are virtual and three-body corrections to these as well as other three-body processes, for example, \( gg \rightarrow \gamma q\bar{q} \) etc. For a complete list of these plus the fragmentation processes see, for example, ref. [3]. As with the direct case, the fragmentation contributions are included here in LO.

**B. Some Calculational Details**

The calculation was performed using the phase space slicing method which makes it possible to perform photon isolation exactly as well as to implement the jet definition in the NLO calculation. More details of parts of the calculation can be found in ref. [12]. The two-body matrix elements for the resolved case, after the soft and collinear poles have been canceled and factorized in the \( \overline{\text{MS}} \) scheme can be found in the appendices of refs. [12,13]. Those for the direct contributions can be obtained from these by appropriately removing non-abelian couplings. These matrix elements depend on the soft and collinear cut-off parameters, \( \delta_s \) and \( \delta_c \) and must be added to the three-body matrix elements, also included in the appendix of [12], in order to cancel the dependence of the cross section on these arbitrary cut-off parameters.

Following the ZEUS experiment, the cone isolation method is used to isolate the photon signal. This method restricts the hadronic energy allowed in a cone of radius \( R_\gamma = \sqrt{\Delta \phi^2 + \Delta \eta^2} \), centred on the photon to be below the value \( \epsilon E_\gamma \), where \( E_\gamma \) is the photon energy. The fixed value \( \epsilon = 0.1 \) is used in this study, which corresponds to the value used in the ZEUS analysis. By contrast the CDF collaboration in their analysis [14] uses a value of \( \epsilon = 2 \) GeV/\( p_T^\gamma \), which varies with the photon energy (\( p_T^\gamma \) is the transverse momentum of the observed photon).

The cone algorithm is also used to define the jet. This defines a jet as hadronic energy deposited in a cone radius \( R_J = \sqrt{\Delta \phi^2 + \Delta \eta^2} \). If two partons form the jet then the kinematic variables are combined to form that of the jet according to the formulae
\[ p_J = p_1 + p_2 \]
\[ \eta_J = \frac{\eta_1 p_1 + \eta_2 p_2}{p_1 + p_2} \]
\[ \phi_J = \frac{\phi_1 p_1 + \phi_2 p_2}{p_1 + p_2}. \] (2.1)

In the ZEUS analysis \( R_\gamma = 1.0 \) and \( R_J = 1.0 \) are chosen and these values will also be used in this study.

In order to estimate the flux of quasi-real photons from the electron beam the Weiszacker-Williams approximation is used. Thus the ‘electron structure function’ \( f_e(x_e, Q^2) \) is given by a convolution of the photon structure function \( f^\gamma(x_\gamma, Q^2) \) and the Weiszacker-Williams (WW) function

\[ f_{\gamma/e}(z) = \frac{\alpha_{em}}{2\pi} \left[ \left\{ \frac{1 + (1 - z)^2}{z} \right\} \ln \frac{Q^2_{\text{max}}(1 - z)}{m^2_e z^2} - 2m^2_e z \left( \frac{1 - z}{m^2_e z^2} - \frac{1}{Q^2_{\text{max}}} \right) \right] \] (2.2)

by

\[ f_e(x_e, Q^2) = \int_{x_e}^{1} \frac{dz}{z} f_{\gamma/e}(z) f^\gamma \left( \frac{x_e}{z}, Q^2 \right). \] (2.3)

The expression for \( f_{\gamma/e}(z) \) was taken from ref. [5]. Following the ZEUS analysis the value \( Q^2_{\text{max}} = 1 \text{ GeV}^2 \) is used throughout.

III. RESULTS

A. Effect of Experimental Selections

The numerical results presented in this section are obtained using the GS96 [13] photon distribution functions, the CTEQ4M [16] parton distributions for the proton and the GRVLO [17] fragmentation functions as standard. Furthermore the two-loop expression for \( \alpha_s \) is used, four-flavours of quarks are assumed active and the factorization/renormalization scales are taken to be equal to the photon \( p_T \) (\( Q^2 = (p_T^\gamma)^2 \)). The maximum virtuality of the initial state photon is fixed at \( Q^2_{\text{max}} = 1 \text{ GeV}^2 \). The calculation is performed in the \( ep \) laboratory
frame using $P_e = 27.5$ GeV for the electron energy and $P_p = 820$ GeV for the proton energy. The electron is moving toward negative rapidity.

In order to make contact with the results of previous calculations, it is convenient to start by examining the inclusive single prompt photon cross section, $ep \rightarrow \gamma X$. As more data are taken at HERA this cross section (with isolation cuts) will certainly be measured since it is the largest cross section involving prompt photon production. In fig.3a the non-isolated single inclusive prompt photon cross section is shown as a function of photon rapidity at $p_T^\gamma = 5$ GeV. No experimental cuts are implemented. In the positive rapidity region the resolved contributions are roughly twice as large as the direct and thus this is the region of interest if information on the gluon distribution of the photon is to be obtained. At negative rapidity, the direct and resolved contributions are comparable in size.

When the WW spectrum is cut as done by the ZEUS Collaboration ($0.16 \leq z \leq 0.8$) the cross section changes as shown in fig.3b (also at the same $p_T^\gamma = 5$ GeV). Both the resolved and direct contributions remain essentially unchanged at negative rapidities but are reduced in the positive rapidity region. The effect on the direct contribution is large, being reduced by a factor of 10 at $\eta^\gamma = 2$. Thus sensitivity to the photon structure function is enhanced in this region since the resolved contribution does not fall by as much. The reason for the asymmetric response of the two contributions to this cut is that the WW distribution is largest at small-$z$ ($x_e = z$, for the direct events). Cutting out this region removes a large fraction of the direct events with lower energy initial photons. When the convolution in eq.(2.3) is taken for the resolved processes on the other hand, for a given $x_\gamma = x_e/z$, all regions of $x_e$ contribute and thus the cut on $z$ does not have the same dramatic effect in this region. In all the following results the cut on $z$ is implemented.

Using the standard parameters, the fragmentation contribution constitutes less than 20% of the cross section at $p_T^\gamma = 5$ GeV (before isolation) and as expected, falls rapidly with increasing $p_T^\gamma$. After isolation, the fragmentation contribution is reduced to about 3% of the cross section. Fig.3c shows the contribution from fragmentation processes to the resolved and direct contributions, as well as their sum, at $p_T^\gamma = 5$ GeV before isolation cuts are
implemented.

The higher order corrections, enhance the cross section by $O(20\%)$ before isolation. As indicated by fig.3d, the corrections are numerically more significant in the positive rapidity region, but they are still modest, indicating good perturbative stability for the predictions.

In Fig.4 the single inclusive prompt photon cross section at $p_T^\gamma = 5$ GeV, with only the cut $0.16 \leq z \leq 0.8$, is compared to the photon plus jet cross section with isolation cuts and jet definition incorporated as done by the ZEUS collaboration. The rapidity and $p_T$ cuts $-1.5 \leq \eta^J \leq 1.8$ and $p_T^J \geq 5$ GeV are placed on the jet. As expected, the photon plus jet cross section is significantly smaller than the single photon cross section, but does not show much difference in shape. It could thus still potentially be used to measure the photon distributions in the positive rapidity region.

The lower dot-dashed in fig.4 is the resolved contribution to the photon plus jet cross section after the further cut $x_\gamma \geq 0.8$ is imposed. This cut essentially removes most of the resolved contribution to the cross section and therefore most of the sensitivity to the photon distribution functions. It is still nevertheless not a pure direct sample and as seen in fig.5a, it still shows sensitivity to the photonic parton distributions. One of the main differences in the GRV and GS96 photon distributions is in the quark distributions at large-$x_\gamma$. In fig.5a the rapidity distribution is plotted at $p_T^\gamma = 5$ GeV with all the cuts used in the ZEUS analysis implemented, including the cut on $x_\gamma$. At negative rapidities the photonic quark distributions are probed at large-$x$ which is where the largest differences between the results of GS96 and GRV are seen. By contrast, as fig.5b demonstrates, there is almost no differences between the results when the proton distributions are changed. This cross section may thus potentially be used to distinguish between these two models of the photon structure function.

In fig.6 the cross section is plotted vs $p_T^\gamma$ with the ZEUS rapidity cuts on the photon imposed ($-0.7 \leq \eta_\gamma \leq 0.8$). It shows the well known fact, common to this type of photoproduction process, that the resolved contribution only competes with the direct at low values of $p_T^\gamma$, while the direct dominates as $p_T^\gamma$ is increased. One thus needs to look in the lower $p_T^\gamma$. 

8
region if sensitivity to the photon structure function is desired and look at higher $p_T^\gamma$ if the aim is to eliminate the resolved events.

Fig. 7 shows a partial breakdown of the isolated photon plus jet cross section into initial state contributions as a function of $\eta^\gamma$. The photon $p_T$ is integrated between 5 and 10 GeV as done by the ZEUS Collaboration. The solid curve is the sum, the dot-dashed curve the resolved and the dashed curve the direct. The contributions to the resolved process are the labelled dotted curves. The dotted curve with error bars is the $g^\gamma q^p$ initiated process as predicted using the GRV photon distributions. Clearly it is only distinguishable from the GS96 result in the far positive rapidity region. All other features of the curves except for the absolute sizes of the contributions are similar to the results of previous studies done on single non-isolated prompt photon production in the $ep$ laboratory frame [2,4,6].

B. Comparison with HERA Data

Table 1 lists predictions for the resolved and direct contributions to the cross section and their sum for various choices of parameters. As stated above, in order to obtain a sample of direct events the ZEUS Collaboration have imposed the cut $x_\gamma \geq 0.8$ on their data. This cut which is also imposed on the results in Table 1, favours the direct contributions since they contribute at $x_\gamma = 1$, but there is still a contribution from the resolved processes and hence some sensitivity to the photon distributions chosen. In addition the cuts $5 \text{ GeV} \leq p_T^\gamma \leq 10 \text{ GeV}$, $p_T^J \geq 5 \text{ GeV}$, $-1.5 \leq \eta^J \leq 1.8$, $-0.7 \leq \eta^\gamma \leq 0.8$ and $0.16 \leq z = E_\gamma/E_\gamma \leq 0.8$ along with the isolation cuts and jet definitions discussed in Section II are imposed.

The first column of numbers gives the results for the standard choice of parameters, while the 2nd and 3rd columns show the effect of changing the scales. The results show a remarkable stability to scale changes. This is in contrast to, for example, the $p_T^\gamma$ distribution which generally shows significant scale sensitivity. The 4th and 5th columns show the effect of changing the photon and proton distribution functions used respectively. In the latter case, as already indicated by the results shown in figs.5a and 5b there is hardly any changes
in the predictions, while in the former case the changes are very significant. Since with these cuts the cross section is mostly sensitive to the quark distributions in the photon at large-\(x\) then this measurement may potentially be used to discriminate between the GS96 and GRV photon parametrizations which differ most significantly in this region. The preliminary experimental value given by the ZEUS Collaboration of \(15.3 \pm 3.8 \pm 1.8\) pb agrees well with the NLO theoretical predictions but the errors are still too large to make any distinction between GS and GRV.

IV. CONCLUSIONS

A NLO calculation of isolated single photon plus jet production at HERA was presented. The effects of various experimental cuts on the cross section was studied in some detail, and comparisons are made with the preliminary data from the ZEUS Collaboration where good agreement was found. The kinematic cuts chosen favour the direct contribution but there is still a significant sensitivity to the quarks distributions in the photon at large-\(x_\gamma\). At the moment the error in the data is still too large to distinguish between the GRV and GS96 photon distributions, but it is expected that analysis of more data will soon remedy this situation.

ACKNOWLEDGMENTS

I am grateful to P. Bussey, M. Derrick and T. Vaiciulis of the ZEUS Collaboration for very helpful discussions. This work was funded in part by the US Department of Energy, Division of High Energy Physics, Contract No. W-31-109-ENG-38.
REFERENCES

[1] P. Aurenche et al., Z. Phys. C 24, 309 (1984).

[2] A. C. Bawa, M. Krawczyk, W.J. Stirling, Z. Phys. C 50, 293 (1991).

[3] P. Aurenche et al., Z. Phys. C 56, 589 (1993).

[4] L.E. Gordon, J.K. Storrow, Z. Phys. C 63, 581 (1994).

[5] G.A. Schuler, CERN-TH/96-297.

[6] L.E. Gordon, W. Vogelsang, Phys. Rev. D 52, 58 (1995).

[7] L.E. Gordon, W. Vogelsang, Proc. of ‘Int Workshop on Deep Inelastic Scattering and Related Phenomena’ Rome, Italy, April 15-19, 1996.

[8] L.E. Gordon, W. Vogelsang, Phys. Rev. D 50, 1901 (1994).

[9] M. Glück, L.E. Gordon, E. Reya, W. Vogelsang, Phys. Rev. Lett. 73, 388 (1994).

[10] Zeus Collaboration, DESY Preprint, To Appear.

[11] L.E. Gordon, W. Vogelsang, Phys. Rev. D 48, 3136 (1993).

[12] L. E. Gordon, ANL-HEP-PR-96-59, In Preparation.

[13] H. Baer, J. Ohnemus, and J. F. Owens, Phys. Rev. D 42, 61 (1990).

[14] CDF Collaboration, F. Abe et al., Phys. Rev. D 48, 2998 (1993).

[15] L.E. Gordon, J.K. Storrow, Nucl. Phys. B 489, 405 (1997).

[16] CTEQ Collab., H. Lai et al., Phys. Rev. D 55, 1280 (1997).

[17] M. Glück, E. Reya, A. Vogt, Phys. Rev. D 48, 116 (1993).

[18] M. Glück, E. Reya, A. Vogt, Phys. Rev. D 45, 3986 (1992).
TABLE I. Total $\gamma + jet$ cross section in pb with ZEUS cuts (see text).

|       | standard | $Q^2 = (p^2_\gamma)/4$ | $Q^2 = 4(p^2_\gamma)$ | GRV$^\gamma$ | MRSR1 |
|-------|----------|--------------------------|-------------------------|--------------|--------|
| res   | 3.31     | 2.60                     | 4.95                    | 6.72         | 3.44   |
| dir   | 9.86     | 11.45                    | 8.18                    | 9.86         | 9.34   |
| sum   | 13.17    | 14.05                    | 13.13                   | 16.58        | 12.78  |
FIGURE CAPTIONS

[1] (a) Lowest order Feynman diagrams for the direct non-fragmentation process $\gamma q \rightarrow \gamma q$. (b) Lowest order diagrams for the direct fragmentation process $\gamma q \rightarrow gq$ and $\gamma g \rightarrow q\bar{q}$.

[2] (a) Lowest order diagrams for the resolved non-fragmentation process $qg \rightarrow \gamma q$ and $q\bar{q} \rightarrow \gamma g$. (b) Lowest order diagrams for two examples of the resolved fragmentation processes.

[3] (a) Rapidity distribution at fixed $p_T^\gamma$ for the inclusive non-isolated prompt photon cross section at HERA energies showing resolved and direct contributions. (b) Same as (a) but with the cut $0.16 \leq z \leq 0.8$ imposed on the Weiszacker-Williams spectrum. (c) Same as (b) but showing the contributions from fragmentation processes to both components as well as to the sum. (d) Same as (b) and (c) but comparing the sum as calculated in LO and NLO.

[4] Comparison of the non-isolated single photon and the isolated photon plus jet cross sections at fixed $p_T^\gamma$ showing direct and resolved contributions. The lower dot-dashed curve is the resolved component after the cut $x_\gamma \geq 0.8$ is imposed.

[5] The isolated photon plus jet cross section at $p_T^\gamma = 5$ GeV vs $\eta^\gamma$ with various cuts imposed by the ZEUS Collaboration using (a) the GRV and GS96 photon structure functions and (b) the CTEQ4M and MRSR1 proton structure functions.

[6] $p_T^\gamma$ distribution of the resolved and direct contributions to the photon plus jet cross section as well as their sum for photon rapidity in the range $-0.7 \leq \eta^\gamma 0.8$.

[7] $\eta^\gamma$ distribution of the photon plus jet cross section for $0.5 \leq p_T^\gamma \leq 10$ GeV showing resolved (dot-dashed line) and direct (dashed line) contributions and their sum (solid line). The various dotted lines show the partial breakdown of
the resolved contribution. The dotted line with error bars is the contribution from the $q^p g^\gamma$ initiated process using the GRV photon parametrization.
Fig. 1
(a)

(b)

Fig. 2
$e \, p \rightarrow \gamma + X$

$p_T^\gamma = 5 \text{ GeV}$

$\frac{d\sigma}{dp_T^\gamma d\eta^\gamma}$ [pb/GeV]

$\eta^\gamma$

**Fig. 3a**

- **sum**
- **resolved**
- **direct**
$0.16 < z < 0.8$

\[
\frac{d\sigma}{dp_T^2 d\eta^\gamma} \text{ [pb/GeV]}
\]

- **sum**
- **resolved**
- **direct**

Fig. 3b
Fig. 3c
\[ \frac{d\sigma}{dp_T^2} \eta^\gamma \] [pb/GeV]

Fig. 3d
\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{\(\frac{d\sigma}{dp_T^\gamma d\eta^\gamma}\) [pb/GeV] vs. \(\eta^\gamma\) for different processes and conditions.}
\label{fig:4}
\end{figure}

- \(p_T^\gamma = 5\) GeV
- \(e + p \rightarrow \gamma + X\)
- \(e + p \rightarrow \gamma + \text{jet}\)
- \(-1.5 < \eta^j < 1.8\)
- \(p_T^j > 5\) GeV
Fig. 5b
Fig. 7

$5 < p_T^γ < 10 \text{ GeV}$
$-1.5 < \eta^j < 1.8$

$p_T^j > 5 \text{ GeV}$