ABSTRACT

We point out that the moduli sector of the (2,2) string compactification with its nonperturbatively preserved non-compact symmetries is a framework to study global topological defects. Based on the target space modular invariance of the nonperturbative superpotential of the four-dimensional $N = 1$ supersymmetric string vacua, topologically stable stringy domain walls are found. Explicit supersymmetric solutions for the modulus field and the metric, which saturate the Bogomol'nyi bound, are presented. They interpolate between non-degenerate vacua. As a corollary, this defines a new notion of vacuum degeneracy of supersymmetric vacua. Nonsupersymmetric stringy domain walls are discussed as well. The moduli sectors with more than one modulus and the non-compact continuous symmetry preserved allow for global monopole-type and texture-type configurations.

1. Introduction

* Lectures Presented at Summer School on Particle Physics, Trieste, Italy, June 15 – July 30, 1991
Topological defects occur during the spontaneous break-down of gauge symmetries, as a consequence of the nontrivial homotopy group \( \Pi_n \) of the vacuum manifolds. Their existence has important cosmological implications. In particular global topological defects, like textures\(^{1,2} \) and more recently, global monopoles\(^{3,4} \) as well as global \( \Pi_2 \) textures\(^{5,6} \) were proposed as a source of large scale structure formation. On the other hand, in the framework of theories with extended gauge structures it is often not aesthetically appealing to impose the existence of additional global (instead of local) gauge symmetries, which would in turn allow for formation of global topological defects. Here, we shall study the moduli sector of \((2,2)\) string compactifications which provides a natural framework for such global defects, with its potentially important physical implications.

A new distinctive feature of superstring theories is that gravity and other moduli and matter fields are on an equal footing, so the effects of gravity can yield distinctly new features. With the advent of deeper understanding of semi-classical superstring theories in a topologically nontrivial sector, various stringy topological defects were discovered: stringy cosmic strings\(^{7,8} \), axionic instantons\(^{9-12} \) as well as related heterotic five-branes and other solitons\(^{13,11,14} \) among others.

In this paper we will confine our attention to the moduli sector of superstring vacua in four dimensions. In \((2,2)\) string compactifications, where \((2,2)\) stands for \( N = 2 \) left-moving as well as \( N = 2 \) right-moving world-sheet supersymmetry, there are massless fields – moduli \( T_i \) – which have no potential, i.e. \( V(T_i) \equiv 0 \), to all orders in string loops\(^{15} \). Thus, perturbatively there is a large degeneracy of string vacua, since any vacuum expectation value of moduli corresponds to the vacuum solution. On the other hand it is known that nonperturbative stringy effects, like gaugino condensation\(^{16} \) and axionic string instantons\(^9 \), give rise to the nonperturbative superpotential.

In the case of the modulus \( T \) associated with the internal size of the compactified space for the so-called flat background compactifications (\( e.g. \), orbifolds, self-dual lattice constructions, fermionic constructions) the generalized target space duality is characterized by noncompact discrete group \( PSL(2, \mathbb{Z}) = SL(2, \mathbb{Z})/\mathbb{Z}_2 \) specified by

\[
T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad \{a, b, c, d\} \in \mathbb{Z}.
\]

If one assumes that the generalized target space duality is preserved even nonperturbatively\(^{17,18} \), the form of the nonperturbative superpotential is very restrictive\(^{18} \). The fact that this is an exact symmetry of string theory even at the level of nonperturbative effects is supported by genus-one threshold calculations\(^{19,20} \), which in turn specify the form of the gaugino condensate\(^{21} \).

This phenomenon has intriguing physical implications leading to the stable
supersymmetric domain walls\textsuperscript{[22,23]} . This physics of modulus $T$ is actually a generalization of the well known axion physics\textsuperscript{[24]} introduced to solve the strong $CP$ problem in QCD. Spontaneously broken global $U(1)$ Peccei-Quinn symmetry is non-linearly realized through a pseudo-Goldstone boson, the invisible axion $\theta$. Non-perturbative QCD effects through the axial anomaly break explicitly $U(1)$ symmetry down to $Z_{N_f}$, by generating an effective potential proportional to $1 - \cos N_f \theta$. This potential leads to domain wall solutions\textsuperscript{[25]} with $N_f$ walls meeting at the axionic strings\textsuperscript{[24]}.

The paper is organized as follows: In Chapter 2 we describe global supersymmetric domain walls as a warming up for the stringy domain walls with gravity included. In Chapter 3 local stringy domain walls are described, including non-supersymmetric domain walls. In addition, implications of nonperturbative stability of supersymmetric vacua are discussed. Possibility of other topological defects in the moduli sector of string theory is discussed in Chapter 4.

2. Global Stringy Domain Walls

As an instructive example let’s first consider a global supersymmetric theory with

$$L = K_{TT} |\nabla T|^2 + K^{TT} |\partial_T W(T)|^2$$

Here, $K_{TT} \equiv \partial_T \partial_T K(T, \bar{T})$ is the positive definite metric on the complex modulus space and the superpotential, $W$, is a rational polynomial $P(j(T))$ of the modular-invariant function $j(T)$\textsuperscript{[26]}, i.e. a modular invariant form of $PSL(2, \mathbb{Z})$. The potential

$$V \equiv K_{TT} |\partial_T W(T)|^2 = G^{TT} |\partial_j P(j) \partial_T j(T)|^2$$

has at least two isolated zeros at $T = 1$ and $T = \rho \equiv e^{i\pi/6}$ in the fundamental domain $\mathcal{D}$ for $T$ (see. Fig. 1),

i.e. when $|\partial_T j(T)|^2 = 0$\textsuperscript{[26]} . Other isolated degenerate minima might as well arise when $|\partial_j P(j)|^2 = 0$. Then, the mass per unit area of the domain wall can be written as: \textsuperscript{[27,28]}

$$\mu = \int_{-\infty}^{\infty} dz G_{TT} |\partial_z T - e^{i\theta} G^{TT} \partial_T W(T)|^2 + 2Re(e^{-i\theta} \Delta W)$$

where $\Delta W \equiv W(T(z = \infty)) - W(T(z = -\infty)).$ The arbitrary phase $\theta$ has to be chosen such that $e^{i\theta} = \Delta W/|\Delta W|$, thus maximizing the cross term in Eq. (2). Then, we find $\mu \geq K \equiv 2|\Delta W|$, where $K$ denotes the kink number. Since $\partial_T W$ is analytic in $T$, the line integral over $T$ is path independent as for a conservative force. The minimum is obtained only if the Bogomol’nyi bound $\partial_z T(z = \infty) = \partial_T j(T(z = \infty))$ and $e^{i\theta} = \Delta W/|\Delta W|$ is satisfied.
Figure 1. Fundamental domain for $PSL(2, \mathbb{Z})$.

$G \bar{T} e^{i\theta} \bar{T} \partial_{\bar{T}} W(T(z))$ is saturated. In this case $\partial_z W(T(z)) = G \bar{T} e^{i\theta} |\partial_T W(T(z))|^2$, which implies that the phase of $\partial_z W$ does not change with $z$. Thus, the supersymmetric domain wall is a mapping from the $z$-axis $[-\infty, \infty]$ to a straight line connecting between two degenerate vacua in the $W$-plane. We would like to emphasize that this result is general; it applies to any globally supersymmetric theory with disconnected degenerate minima that preserve supersymmetry.

For the superpotential, e.g. $W(T) = j(T)$ the potential has two isolated degenerate minima at $T = 1$ and $T = \rho \equiv e^{i\pi/6}$ (see fig. 2 for the potential along the geodesic $T = e^{i\phi}, \phi = \{-\pi/6, \pi/6\}$). At these fixed points, $j(T = \rho) = 0$ and $j(T = 1) = 1728$. Therefore, the mass per unit area is $\mu = 2 \times 1728$. The explicit solution for $T = e^{i\phi(z)}$ is displayed on Fig. 3.

Other cases can be worked out analogously $^{[23]}$. 
3. Local Stringy Domain Walls

The case with gravity restored* has a Kähler potential $K = -3 \log(T + \bar{T})$ and the superpotential should transform as a weight $-3$ modular function under modular transformations\cite{17,18}. The simplest choice, with supersymmetric vacua is with the superpotential

$$W(T) = \frac{\Omega(S)j(T)}{\eta(T)^6}. \quad (3)$$

Here, $\eta(T)$ is the Dedekind eta function, a modular form of weight $1/2$ and $j(T)$ is a modularly invariant function\cite{26}. The potential is of the following form:†

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* We use the conventions: $\gamma^\mu = e^\mu_a \gamma^a$ where $\gamma^a$ are the flat spacetime Dirac matrices satisfying $\{\gamma^a, \gamma^b\} = 2\eta^{ab}; \quad e^\mu_a e^\nu_b = \delta^\nu_b; \quad a = 0, ... 3; \quad \mu = t, x, y, z; \quad \overline{\psi} = \psi^\dagger \gamma^t; \quad (+, -, -, -)$ space-time signature; and dimensions such that $8\pi G_N \equiv 1$.

† Note, that the potential depends on a prefactor $\Omega(S)$ which depends exponentially on the dilaton field $S$. Here we assume that supersymmetry is not broken in the $S$ sector, i.e. $D_S W = 0$. On figures $\Omega = 1$ was used.
\[
V(T, \bar{T}) = \Omega(S) \frac{3|j|^2}{(T + \bar{T})^3|\eta|^2}(\frac{(T + \bar{T})}{3} (\frac{\partial_T j}{j} + \frac{3}{2\pi} \hat{G}_2)^2 - 1)
\]

where \( \hat{G}_2 = -4\pi \partial_T \eta / \eta - 2\pi / (T + \bar{T}) \) is the Eisenstein function of weight 2 \([26]\). The scalar potential (4) has two isolated supersymmetric minima one at \( T = 1 \) and one at \( T = \rho \) \([18]\). At these two supersymmetric minima, the superpotential takes values \( j(\rho) = 0 \) and \( j(1) = 1728 \). This in turn implies that the supersymmetric minima of the potential are non-degenerate. At \( T = 1 \) one has an anti-deSitter space with cosmological constant \(-3|W(T = 1)|^2 e^{K(T=1)} \) and at \( T = \rho \) the cosmological constant is zero. Even though the two supersymmetric minima of the matter potential are not degenerate (see Fig. 4 for the scalar potential along the geodesic \( T = e^{i\phi} \), \( \phi = \{-\pi/6, \pi/6\} \)). there does exist a stable domain wall solution interpolating between them.

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Figure 4. Local modular invariant potential along the geodesic \( T(z) = e^{i\phi(z)} \). Potential \( V(\phi) \) plotted in units of \( 10^8 \).

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### 3.1 Minimal Energy Solution for Supersymmetric Stringy Domain Walls

We now minimize the domain wall mass density. Details of the derivations are given in Ref. 23. The analysis is general and can be applied to any supergravity
theory with isolated supersymmetric vacua. The bound is a generalization of
the global result. We employ the results of Ref.29 and Ref.30 which addressed the
positivity of the ADM mass in general relativity, as well as certain generalizations
to anti-de Sitter backgrounds.

By the planar symmetry, the most general static Ansatz for the metric in which
the domain wall is oriented parallel to \((x,y)\) plane is

\[
ds^2 = A(z)(-dt^2 + dz^2) + B(z)(dx^2 + dy^2).
\]

Consider the supersymmetry charge density

\[
Q[\epsilon] = \int_{\partial \Sigma} \bar{\epsilon} \gamma^{\mu \nu \rho} \psi_{\rho} d \Sigma_{\mu \nu}
\]

where \(\epsilon\) is a commuting Majorana spinor, \(\psi_{\rho}\) is the spin 3/2 gravitino field, and
\(\Sigma\) is a spacelike hypersurface. We take a supersymmetry variation of \(Q[\epsilon]\) with
respect to another commuting Majorana spinor \(\delta_{\epsilon} Q[\epsilon] \equiv \{Q[\epsilon], Q[\epsilon]\}\)

\[
= \int_{\Sigma} N^{\mu \nu} d \Sigma_{\mu \nu} = 2 \int_{\Sigma} \nabla_{\nu} N^{\mu \nu} d \Sigma_{\mu}
\]

where \(N^{\mu \nu} = \bar{\epsilon} \gamma^{\mu \nu \rho} \bar{\nabla}_{\rho} \epsilon\) is a generalized Nester’s form. Here \(\bar{\nabla}_{\rho} \epsilon \equiv \delta_{\epsilon} \psi_{\rho} = [2 \nabla_{\rho} + ie^K (W P_R + \bar{W} P_L) \gamma_{\rho} - i m (K_T \partial_{\rho} T) \gamma^5] \epsilon\) and \(\nabla_{\mu} \epsilon = (\partial_{\mu} + \frac{1}{2} \omega_{ab} \sigma_{ab} \epsilon) \). In (7)
the last equality follows from Stoke’s law.

We are concerned with supercharge density and thus insist upon only \(SO(1,1)\)
covariance in the \(z\) and \(t\) directions. This in turn implies that the space-like
hypersurface \(\Sigma\) in eq. \((7)\) is the \(z\)–axis with measure \(d \Sigma_{\mu} = (d \Sigma_t, 0, 0, 0) = |g_{tt} g_{zz}|^{\frac{1}{2}} dz\). The boundary \(\partial \Sigma\) are then the two asymptotic points \(z \to \pm \infty\).
Technical details in obtaining the explicit form of eq. \((7)\) are given in Ref. 23. Here
we only quote the final results.

The volume integral yields:

\[
2 \int_{\Sigma} \nabla_{\nu} N^{\mu \nu} d \Sigma_{\mu} = \int_{-\infty}^{\infty} \left[ -\delta_{\epsilon} \psi_i g^{ij} \delta_{\epsilon} \psi_j + K_T T \delta_{\epsilon} \chi \delta_{\epsilon} \chi \right] dz \geq 0
\]

where \(\delta_{\epsilon} \psi_i\) and \(\delta_{\epsilon} \chi\) are the supersymmetry variations of the fermionic fields in the
bosonic backgrounds.
Analysis of the surface integral in (7) yields two terms: (1) The ADM mass density of configuration, denoted $\sigma$ and (2) The topological charge density, denoted $C$ (see Ref. 23).

Positivity of the volume integral translates into the bound

$$\sigma \geq |C|$$

which is saturated iff $\delta_{e}Q[\epsilon] = 0$. In this case the bosonic backgrounds are supersymmetric; i.e. they satisfy $\delta_{\psi_{\mu}} = 0$ and $\delta_{\chi} = 0$ (see eq.(8)).

The solution of self-dual equations yields:

$$\partial_{z}T(z) = -\zeta\sqrt{A}|W|e^{\frac{R}{2}}K^{TT}D_{T}\frac{W}{W},$$

$$\partial_{z}\ln A = \partial_{z}\ln B = 2\zeta\sqrt{A}|W|e^{\frac{R}{2}},$$

$$(9)$$

$$Im(\partial_{z}T\frac{D_{T}W}{W}) = 0$$

where $\zeta = \pm 1$ can change only at the point where $W$ vanishes.

We now comment on these three equations.

(i) The first equation in (9) is a local generalization of the global result. It is evident that $\partial_{z}T(z) \rightarrow 0$ as one approaches the supersymmetric minima, i.e. $D_{T}W = 0$, thus indicating a domain wall configuration, however no constraint is put on the degeneracy of vacua.

See Fig. 5 for the solution $T = e^{i\phi(z)}, \phi = \{-\pi/6, 0\}$.

(ii) The second equation in (9), i.e. the equation for the metric, implies that we can always rescale the space-time coordinates to bring $A = B$. Thus, our metric Ansatz is reduced to a class of conformally flat metrics with $z$-dependent conformal factor. The asymptotic behaviour of the metric depends on whether the supersymmetric vacuum is Minkowski ($|W_{\pm \infty}| = 0$) or anti-deSitter ($|W_{\pm \infty}| \neq 0$). In the first case the metric equation gives $A \rightarrow const$, while in the second case $A \rightarrow const'/z^{2}$, which are the proper asymptotic behaviours in Minkowski and anti-deSitter space-times, respectively. In the case of $W = j(T)\eta^{-6}(T)$, at $T = e^{i\pi/6}$ the metric goes to a constant ($W = 0$) and at $T = 1$ the metric falls off at $1/|W(1)|^{2}e^{K(1)/z^{2}}$. See Fig. (6) for the explicit form of the metric $A(z)$.

(iii) The third equation in (9) describes a geodesic path between two supersymmetric vacua in the supergravity potential space $e^{K/2W} \in \mathbb{C}$ when mapped from the $z$-axis ($-\infty, +\infty$). Here, we would like to contrast the geodesic equation

\* $\delta_{e}Q[\epsilon] = 0$ seems to only require $\delta_{e}\psi_{i} = 0$ with $i \neq t$. However, in order for $\delta_{e}\psi_{i} = 0$ for an arbitrary space-like hypersurface, one in fact requires $\delta_{e}\psi_{\mu} = 0$ for $\mu = t, x, y, z$.\footnote{29}
in (9) with the geodesic in the global supersymmetric case. In the global case the geodesics are straight lines in the $W-$plane (see discussion after eq. (2)). On the other hand, the local geodesic equation in the limit $G_N \to 0$ (global limit of the local supersymmetric theory) leads to the geodesic equation $\text{Im}(\partial_z W) \equiv \partial_z \vartheta = 0$ where $W$ has been written as $W(z) = |W|e^{i\vartheta}$. This in turn implies that as $G_N \to 0$ the geodesic equation reduces to the constraint that $W(z)$ has to be a straight line passing through the origin; i.e. the phase of $W$ has to be constant mod $\pi$.

In the case of $W = j(T) \eta^{-6}(T)$ one can prove that the geodesic corresponds to $T = e^{i\phi}(z)$. First note $\partial_T j$ and $\hat{G}_2$ are both modular forms of weight 2 while $j$ is the absolute modular invariant function. The results $\partial_T j(\frac{1}{T}) = -T^2 \partial_T j(T)$, $\hat{G}_2(\frac{1}{T}) = -T^2 \hat{G}_2(T)$, and $j(e^{i\phi}) = j(e^{-i\phi})$ imply $\text{Im}(\frac{D_T W}{W} \partial_z T) = 0$ for $T = e^{i\phi}(z)$. Therefore $T = e^{i\phi}(z)$ satisfies the geodesic equation. Thus, the geodesic equation is the same as in the global case.

The energy density of the minimal energy solution can be written as

$$\sigma = |C| \equiv 2|\langle \zeta |W e^K \rangle_{z=+\infty} - \langle \zeta |W e^K \rangle_{z=-\infty}| = 2|\Delta(\zeta |W e^K \rangle)|$$

Again in the case with $W$ as defined in (3), $\sigma = 2W(T = 1)e^{K(1)/2}$. Eq. (10)
constitutes a generalization of the global case.

Comparing this local example with the corresponding global supersymmetric modular invariant theory, both cases are similar; e.g., the two isolated supersymmetric minima are at $T = \rho$ and $T = 1$ and the geodesic is the same in both cases. However, a significant difference is that in the local case the minima are not degenerate; i.e. at $T = \rho$ the cosmological constant is zero, while at $T = 1$ the cosmological constant is negative. In addition, these domain walls represent a new class of domain walls beyond those classified in Ref. 32; they are static, reflection asymmetric domain walls interpolating between non-degenerate vacua.

The above example is representative of a situation where the study of a global supersymmetric domain wall is readily generalizable to a local supersymmetric theory. One may be tempted to conclude that all the supersymmetric domain walls in the global supersymmetric theory automatically remain as supersymmetric domain wall solutions even after gravity is turned on. However, this is not always the case.

Consider another modular invariant superpotential:

$$W = j(T)(j(T) - 1728)$$

(11)

There are three isolated global supersymmetric minima at $T = 1, \rho$ and $\partial W/\partial j = 2j(T) - 1728 = 0$. Therefore, we expect two domain walls interpolating between each of the two adjacent vacua. In the supergravity case we find the minima $T = 1$ and $T = \rho$ remain supersymmetric minima. They both have zero cosmological constant since the superpotential vanishes at these two points. Additionally, there is a local minimum with positive cosmological constant at $T = T_3$ which is in the neighborhood of the point $j^{-1}(864) \in D$. However, this point is not supersymmetric since $D_T W = [\partial_T W + \frac{3}{2\pi} \hat{G}_2 W]|_{T = T_3} \neq 0$. Thus, the domain wall interpolating between $T = 1$ and $T_3$ (or between $T_3$ and $T = e^{i\pi/6}$) is not stable since the minimum at $T_3$ is a non-supersymmetric de-Sitter minimum. Also, the wall interpolating directly between the supersymmetric vacua at $T = 1$ and $T = e^{i\pi/6}$ does not exist either as the superpotential vanishes at these vacua and thus there is no energy associated with such a wall.

### 3.2 Non-perturbative Stability of Supersymmetric String Vacua

The analysis of the above local supersymmetric domain wall solution interpolates between two non-degenerate vacua of the supergravity matter potential, e.g. one with zero and another with negative cosmological constant. The existence of such static domain walls has strong implications for the non-perturbative stability of supersymmetric vacua, and thus also for supersymmetric superstring

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* Note, again, that $j(\rho) = 0$ and $j(1) = 1728$. 

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The obtained result is intimately related to the $O(4)$ symmetric bubbles of the false vacuum decay in the presence of gravity. In Ref. 33, Coleman and DeLuccia found that a false vacuum decay from the Minkowski space-time to anti-deSitter space-time cannot take place unless the matter vacuum energy difference $\epsilon = -V(true)$ meets an inequality

$$\epsilon \geq \frac{3}{4} \sigma^2$$

(12)

in which $\sigma$ denotes the energy density stored in the bubble wall. In the case of false vacuum decay from anti-deSitter to anti-deSitter space-time one arrives at the same equation (12) with $\epsilon \equiv (\sqrt{-V(true)} - \sqrt{-V(false)})^2$

The residual energy after materializing the bubble wall accelerates the wall asymptotically to the speed of light. Also as the energy difference $\epsilon$ approaches the minimum of the Coleman-DeLuccia bound (12) the radius of the $O(4)$ invariant bubble wall becomes indefinitely large. Precisely at the saturation limit,

$$\sigma = \sigma_c \equiv 2 \sqrt{\frac{\epsilon}{3}}.$$  

(13)

No kinetic energy is available for the wall to accelerate to the speed of light, and the wall radius becomes infinite, i.e. becomes planar. The resulting configuration of the $O(4)$ bubble is a time-independent and infinite planar domain wall dividing the Minkowski space-time from the anti-deSitter space-time. In the supergravity theory, $\sigma_c = 2 \sqrt{\frac{\epsilon}{3}} = 2 e^{K/2} |W(true)|$ which coincides with the topological charge $|C| = 2 |\Delta(\zeta e^{K/2}|W|)|$ (see eq. (10)). Thus, the critical Coleman-DeLuccia bubble wall in supergravity theory saturates the Bogomol’nyi bound, and hence, this is a special class of the supersymmetric domain wall described above. This result has strong implications for the stability of supersymmetric vacua in general, and superstring vacua in particular; namely, supersymmetric vacua are non-perturbatively stable against false vacuum decay. This result completes a perturbative analysis $\kappa = 8 \pi G_N \rightarrow 0$ of Weinberg.

As a corollary, non-supersymmetric vacua are unstable against false vacuum decay because in this case the Coleman-DeLuccia bound (12) can be satisfied; note, that in the non-supersymmetric vacuum satisfies the inequality $V > -3|W|^2 e^K$. Thus, a non-supersymmetric vacuum would decay into a supersymmetric one if there exists a supersymmetric one.

### 3.3 Non-Supersymmetric Stringy Domain Walls

In view of non-perturbative stability of supersymmetric vacua one is compelled to search for non-perturbatively induced potentials for the modulus fields $T$ with only non-supersymmetric vacua; in this case the non-supersymmetric vacuum cannot decay into existing supersymmetric one.
It turns out that the simplest form of such a superpotential is\(^{[17,21]}\):

\[
W = \Omega(S)\eta^{-6}(T) \tag{14}
\]

Within the fundamental domain \(D\) (see Fig.1) the corresponding potential has only one minimum at \(T = 1.2\)\(^{[21,18]}\), which also breaks supersymmetry. In addition, the superpotential (14) can be derived explicitly\(^{[21,39]}\) as an effective term due to gaugino condensation of the hidden \(E_8\) gauge group in orbifold compactifications. It is thus the best motivated non-perturbatively induced superpotential in a class of superstring vacua.

The underlying \(PSL(2,\mathbb{Z})\) symmetry of the theory implies\(^{[36,37,22]}\) that there should be domain wall solutions interpolating between such degenerate vacua of different fundamental domains.

The nature and existence of such domain walls has recently been studied in Ref.\textsuperscript{38}. There are two classes of domain walls associated with the superpotential (14). The first class are domain walls is associated with the symmetry transformation \(T \rightarrow T + i\), \(i.e.\) the discrete Peccei-Quinn symmetry. Thus the domain walls interpolates between minima with \(T = 1.2\) and \(T = 1.2 + i\).

The nature of this domain wall is closely related to the domain wall that exists for the QCD induced potential of the Peccei-Quinn axion \(\theta\) with only one quark flavor, \(i.e.\) \(V = 1 - \cos \theta\). In this case there is a domain wall interpolating between \(\theta = 0\) and \(\theta = 2\pi\). The domain wall is bounded by an axionic string which emerged at the first stage of symmetry breaking of the global \(U(1)\) Peccei-Quinn symmetry. Analogously, in our case the role of the stringy axion field is played by the imaginary part of the \(T\) field; the domain wall interpolates between \(T = 1.2\) and \(T = 1.2 + i\).

The second type of domain walls are associated with \(T \rightarrow 1/T\), \(i.e.\) the generator of the non-compact symmetry transformation of \(PSL(2,\mathbb{Z})\). This domain wall interpolates between the minimum at \(T = 1.2\) and \(T = 1/1.2\). It is analogous to the domain walls associated with \(Z_2\) symmetry. It is at first puzzling that there would be such a domain wall, after all, the points in the \(T\) plane are related by the \(T \rightarrow 1/T\) symmetry. However, points associated with \(T \rightarrow 1/T\) transformation \emph{can} be probed since they correspond to a different theory (with heavy winding modes becoming light and vice versa) which happens to be equivalent to the original theory.
There are difficulties associated with the cosmological implications of the above domain walls. First, the scale of the domain wall depends on the scale $\Omega(S)$, i.e. the scale at which gaugino condensation takes place. This scale could be as low as $O(\text{TeV})$ or as high as $O(10^{16}\text{GeV})$. The latter one is more plausible, at least in the scenario where the hidden gauge group is $E_8$. Thus, domain walls have to decay rapidly in order to be consistent with observations. One obvious mechanism would be by invoking inflation. The second possibility is the decay via chopping by the stringy cosmic strings, as long as the energy scales of the two types of topological defects (the domain walls and the strings) are not too far apart.

Another difficulty with the above scenario is that the Kibble mechanism for generating stringy cosmic strings has not been established, yet. Further study is of potential cosmological implications of non-supersymmetric domain walls is in progress\cite{38}.

4. Other Topological Defects

We would now like to point out\cite{40} the existence of other global topological defects, like global monopole-type and texture-type defects in the moduli sector with more than one modulus. We shall illustrate the idea using examples based on the so called flat backgrounds, i.e. generalization of $SL(2,\mathbb{R})$.

For that purpose we shall study the simplest example of $Z_4$ manifold with continuous symmetry $SU(2,2)$ on the four moduli

$$T \equiv \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

of compactified space. Note that the moduli $T$ live on the coset $SU(2,2)/SU(2) \times SU(2) \times U(1)$. The continuous non-compact symmetry $SU(2,2)$ is an exact symmetry\cite{41} at least at the string tree-level. Note that this continuous symmetry in the modulus could be broken down to the discrete subgroup $SU(2,2,\mathbb{Z})$ due to nonperturbative effects, e.g. gaugino condensation and/or axionic instanton effects. At this point we shall assume that this non-compact continuous symmetry is preserved in the modulus sector all the way to low energies and is not broken by non-perturbative effects. However, one should keep in mind that $SU(2,2,\mathbb{Z})$ is the vacuum symmetry and thus the $T$ fields should live in the fundamental domain of $SU(2,2,\mathbb{Z})$.

The maximal compact symmetry of $SU(2,2)$ is $SU(2)_A \times SU(2)_B \times U(1) \subset SU(2)_{A+B}$. Note also that in projective coordinates\cite{42} : $Z = (1 - T)/(1 + T)$. $Z$ transforms as $1 + 3$ under $SU(2)_{A+B}$. The Ansatz $Z = \sum_{a=1}^3 \sigma_a V_a$ with $V_a = f(r)x_a/r$ ensures the map of $Z$ on the $S^2$. 

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The \( \mathbb{Z} \) fields have no potential to all orders in string loops. Thus the kinetic energy term \(^{[41]}\) shrinks \( f \to 0 \) due to Derrick’s theorem, however it could be stabilized by higher derivative terms. Such higher derivative terms arise even at the tree level of the string theory. They should respect the noncompact \( SU(2,2) \) symmetry. Also, if one sticks to terms with at most two time derivatives, one has a unique form for the terms that involve four derivatives, which is very similar in nature to the Skyrme term \(^{[43]}\) in the Skyrmion model and can serve the same role as the stabilizing term. The energy stored in such a configuration is finite and is governed by the scale of \( \alpha' \). This is different from the standard global monopole configuration\(^{[3]}\), which has linearly divergent energy and thus long range interaction relevant for large scale formation.

Texture-type configurations, can also occur within this sector. Namely, the \( \mathbb{Z} \) fields transform as \( 4 \) under the compact symmetry \( SU(2)_A \times SU(2)_B \sim SO(4) \) and thus the Ansatz: \( \mathbf{Z} = a(r) + b(r) \sum_{a=1}^{3} \sigma_a x_a / r \) is mapped onto \( S^3 \).

The above studied configurations are much milder defects than strings and domain walls and they have finite range and thus finite energy. Further study and cosmological implications of such global defects is necessary.

I would like to thank my collaborators R. Davis, S. Griffies, F. Quevedo, and S.-J. Rey, for many fruitful discussions and enjoyable collaborations. I would also like to thank the Aspen Center for Physics, the International Centre for Theoretical Physics, Trieste, and CERN for their hospitality. The work is supported in part by the U.S. DOE Grant DE–22418–281 and by a grant from University of Pennsylvania Research Foundation and by the NATO Research Grant #900–700.

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