Hybrid algorithm for rotor angle security assessment in power systems

D. Prasad Wadduwage1, Udaya D. Annakkage1, Christine Qiong Wu2

1Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, R3T 5V6, Canada
2Department of Mechanical and Manufacturing Engineering, University of Manitoba, Winnipeg, MB, R3T 5V6, Canada
E-mail: wdprasadmr@gmail.com

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Abstract: Transient rotor angle stability assessment and oscillatory rotor angle stability assessment subsequent to a contingency are integral components of dynamic security assessment (DSA) in power systems. This study proposes a hybrid algorithm to determine whether the post-fault power system is secure due to both transient rotor angle stability and oscillatory rotor angle stability subsequent to a set of known contingencies. The hybrid algorithm first uses a new security measure developed based on the concept of Lyapunov exponents (LEs) to determine the transient security of the post-fault power system. Later, the transient secure power swing curves are analysed using an improved Prony algorithm which extracts the dominant oscillatory modes and estimates their damping ratios. The damping ratio is a security measure about the oscillatory security of the post-fault power system subsequent to the contingency. The suitability of the proposed hybrid algorithm for DSA in power systems is illustrated using different contingencies of a 16-generator 68-bus test system and a 50-generator 470-bus test system. The accuracy of the stability conclusions and the acceptable computational burden indicate that the proposed hybrid algorithm is suitable for real-time security assessment with respect to both transient rotor angle stability and oscillatory rotor angle stability under multiple contingencies of the power system.

1 Introduction

The reliability is a major concern of an electrical power utility to ensure adequate electricity supply to consumers. Every power utility has standards to express its reliability. For example, North American Electric Reliability Corporation (NERC) establishes the standards for the United States, Canada and the northern portion of Baja California in Mexico. NERC defines the following fundamental concepts for the reliability of a bulk-power system [1]:

Adequacy: The ability of the electric system to supply the aggregate electric power and energy requirements of the electricity consumers at all times, taking into account scheduled and reasonably expected unscheduled outages of system components.

Operating reliability: The ability of the electric system to withstand sudden disturbances such as electric short circuits or unanticipated loss of system components.

The above requirements are directly related with the ability of the power system to survive from a list of credible contingencies. The security of the power system is an instantaneous time-varying condition reflecting its robustness to survive from imminent disturbances [2] and is directly related to the operating states shown in Fig. 1 [3]. These five operating states are the extensions of the normal, emergency and restorative states originally proposed in [4].

Under normal operating conditions, the total load is adequately supplied by the power system and all the variables are within their desired limits. The system enters into an alert state when its security level falls below an acceptable threshold subsequent to minor disturbances such as changes in loads of the system. Preventive control actions can be initiated to bring it back to the normal state. A sufficiently severe disturbance can shift the power system into an emergency state where the inequality constraints are now violated, but still the total load is supplied by the generation. The emergency control actions are required to bring the system at least into an alert state. Depending on the severity of the disturbance, the in-extremis state may be directly reached from the alert state or it may be due to the delay or failure in emergency control actions. Now, a large portion of the consumers are without power. The emergency control actions are required to avoid a system-wide blackout. If the blackout is prevented, the next objective is to supply electricity to the consumers gradually and reconnect the system. It is said that the power system is now operating in a restorative state. Eventually, the result could be that the system state is transferred to an alert or the normal state.

The ability of the power system to withstand a list of credible contingencies is studied when it is operating in the normal or the alert states. This is known as the security assessment of the power system which has two steps [2]: (i) determining whether the equality and inequality constraints are satisfied at the new operating point reached after the contingency (static security assessment), and (ii) determining whether the new operating point can be reached (dynamic security assessment).

Subsequent to a disturbance, the power system can be insecure due to rotor angle, voltage and/or frequency instability. The focus of this paper is the security of the power system due to rotor angle stability. The power system is rotor angle stable if it maintains the synchronous operation subsequent to a contingency [2]. If not, the system can become unstable, showing a periodic increase in rotor oscillations in the first swing due to insufficient synchronising torque, which is referred to as the transient rotor angle stability problem. If the power system is transient stable, then the generator rotor angles exponentially decay with time. The damping ratios of the dominant oscillatory modes present in these trajectories provide the indication about the oscillatory stability of the post-fault power system. The damping ratio of a poorly-damped oscillatory mode can become negative subsequent to a cascading contingency, thereby causing the oscillations to grow slowly leading to the loss of synchronism in the power system as happened in the Western Interconnection in 1996 [5]. This scenario is referred to as an oscillatory stability problem. On the other hand, the transient instability in a large interconnected power system may occur beyond the first swing due to the presence of inter-area oscillations or non-linear effects [2]. Such a scenario is referred to as a multi-swing transient stability problem.

The security of the power system due to the rotor angle stability problem is a major concern of many power utilities. The major blackouts experienced by large power systems were the impetus behind this motivation. Thus, new algorithms are developed for...
determining the security of the current operating point of the power system subsequent to an anticipated set of contingencies which are selected based on their probabilities of occurrences [2]. The main contribution of this paper is to propose a hybrid algorithm to be used in a real-time dynamic security assessment (DSA) program to assess the security of the power system with respect to both transient rotor angle stability and oscillatory rotor angle stability problems. The objectives of this paper are (i) summarise the literature on rotor angle stability assessment giving due consideration to the applicability into real-time DSA, (ii) develop a new security measure based on the concept of Lyapunov exponents (LEs) to conclude the transient stability, (iii) improve the conventional Prony algorithm to extract dominant modes in transient stable power swings and conclude the oscillatory stability, (iv) propose a hybrid algorithm to perform the security assessment and (v) evaluate the performance of the hybrid algorithm using test power systems. The rest of this paper is organised in the same order as the objectives were presented.

2 Background

The rotor angle stability analysis involves studying the stability of electromechanical oscillations in power systems. Different methods available in the literature for studying the said stability can be classified as shown in Fig. 2. These methods are briefly summarised in following sections.

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**Fig. 1** Operating states of a power system [3]. E – equality constraints, I – inequality constraints

**Fig. 2** Classification of rotor angle stability assessment methods
2.1 Transient stability assessment

The power system is modelled using a set of first-order differential and algebraic equations given in (1) to study the rotor angle stability:

\[
\begin{align*}
\frac{d}{dt} x &= f(x, V) \\
I(x, V) &= YV 
\end{align*}
\]

where \( x, V, I \) and \( Y \) are the state vector, bus voltage vector, current injection vector and the node admittance matrix, respectively.

These equations are numerically solved using an explicit or implicit integration method in the time domain simulation (TDS) approach [6]. This method provides plenty of information provided that the system has been accurately and satisfactorily modelled. The stability conclusion in the TDS is derived by using a security measure. For example, the post-fault power system can be concluded as rotor angle stable (\( \eta > 0 \)) or unstable (\( \eta \leq 0 \)) using the criterion \( \eta = (360 - \delta_{\text{max}}/360 + \delta_{\text{max}}) \), where \( \delta_{\text{max}} \) is the maximum rotor angle separation between the generators in the post-fault system [7]. This paper proposes a new criterion based on a theoretically sound non-linear stability analysis technique to derive the stability conclusion using the time domain simulated trajectories of the post-fault power system.

The synchronous generators in power systems gain/loss energy during the faulted period due to acceleration/deceleration of their rotors. After clearing the fault, the post-fault system remains stable if, \( E(T_f) < E_{\text{cr}} \), where \( E(T_f) \) is the total energy at the fault clearing time and \( E_{\text{cr}} \) is the maximum energy that the post-fault system can absorb. Thus, the scalar function \( E \) representing the total energy is a Lyapunov function and the surface \( E(T_f) = E_{\text{cr}} \) is a Lyapunov surface [8]. The equal area criterion (EAC) and the transient energy function method are the applications of this approach into transient stability assessment in power systems.

The EAC method studies the transient stability of a single generator connected to an infinite bus and is used to understand the fundamental concepts behind this stability in power systems [6]. The generalised extended EAC (GEEAC) method can be used to study the transient stability in a multi-machine power system. This method first uses the TDS to distinguish the generators as critical and non-critical machines subsequent to a contingency. These two groups of machines are then represented by their single machine equivalents and the stability is assessed using the EAC method [9]. This method accommodates the detailed generator models in the TDS and provides the stability margin of the power system, i.e. the closeness of the system to instability. Therefore, the GEEAC method satisfies the requirements of a real-time DSA program.

The direct methods of transient stability analysis use the well-known numerical energy function to calculate \( E(T_f) \). Different direct methods are available depending on the method used to determine \( E_{\text{cr}} \). The potential energy boundary surface, the closest unstable equilibrium point (UEP), the controlling UEP and the boundary of stability region-based controlling unstable equilibrium point are examples of such methods. Details of these methods can be found in [10].

The computational burden and the difficulty of accommodating complex generator models are considered as limitations of the method for transient security assessment in power systems. A significant attention has been made in the recent literature to apply artificial intelligence-based techniques to real-time transient security assessment in power systems. In this approach, a database is created for a given network using TDS and a classifier is trained using a machine learning technique. This learning may be done using the fuzzy-logic rules [11], neural networks [12], decision trees [13], extreme learning machines [14], support vector machines [15] and so on. It is clear that the transient stability can be determined in a less computing time once the classifiers have been trained extensively in an offline environment making these techniques more suitable for the real-time DSA. However, the effort required to train the network may be significant. The recent publication [16] shows that the size of the training data set can be reduced using the input features as the energy terms of the transient energy function.

This paper proposes a new security measure based on the concept of LEs for transient security assessment in power systems.

2.2 Oscillatory stability assessment

The presence of a negatively-damped oscillation leads to the oscillatory instability of the power system. Thus, the oscillatory stability subsequent to a contingency can be determined by calculating the eigenvalues of the state matrix obtained by linearising the DAEs around the equilibrium point of the post-fault power system. However, the size of the state matrix increases with the number of dynamic devices and modelling details. The excessive computing time is a drawback of the use of eigenvalue analysis. For example, the number of states in the linearised model cannot exceed 1000 in small-signal analysis tool [17]. Thus, a significant attention has been given in the literature to determine the dynamic performance using the outputs of the transient simulation.

The Prony [18], matrix pencil [19], eigensystem realisation algorithm [20] are linear parametric methods which first select a possible model for the observed data. In contrast, the algorithms based on Hilbert–Huang and wavelet transformations determine the instantaneous frequency and the damping of the modes. These two methods have been proposed for real-time oscillation monitoring purposes [21]. However, they can still be used to determine the oscillation modes, their frequencies and damping from simulated power swings. The non-parametric methods work on the data itself to estimate the characteristics without making any assumptions. The Fourier transform (FT) [22] and the discrete short-time FT [23] are two such methods. These algorithms accurately determine the frequencies of the modes. However, their damping estimations are more sensitive to the length of the data window.

In proposing most of these algorithms, the Prony algorithm has been treated as the reference for comparing the results. This paper uses an improved Prony algorithm to extract only the dominant modes present in the transient stable power swings and determines their parameters to conclude the oscillatory security state subsequent to different contingencies in a power system.

3 Transient stability assessment using a LEs-based algorithm

This section presents a new security measure which can be used for transient security assessment in power systems.

The LEs assess the stability of non-linear dynamic systems using the exponential rates of divergence or convergence of trajectories in the state space. The theory of the LEs and their properties are explained in detail in [24]. The procedure of calculating a spectrum of LEs is summarised below:

1. Numerically integrate the dynamic equations of the post-fault power system starting from the state values at the fault clearing time \( T_f \) to determine the reference trajectory.
2. Numerically solve the linearised equations to determine the evolution of the nearby trajectories. A volume of nearby trajectories is evolved by starting from an \( n \)-hypersphere with unity magnitude principal axes.
3. Steps (1) and (2) give rise to the simultaneous solution of the following set of equations:

\[
\begin{align*}
\frac{d}{dt} x &= f(x) \\
\frac{d}{dt} Ax &= -4Ax 
\end{align*}
\]
where \( x \) and \( \Delta x \) are the state vector and the perturbed state vector, \( f \) is an autonomous function and \( A \) is the Jacobean matrix

\[
A_{ij} = \frac{\partial f_j}{\partial x_i} |_{x=x_0, t=0}
\]

The initial conditions of the above integration are \( x_0 = x_f \) and \( \Delta x_0 = I \), where \( I \) is an \( n \times n \) identity matrix with \( n \) equals the order of the dynamic model.

(4) The principal axes of the hypersphere tend to fall along the direction of the most rapid growth. Thus, Gram–Schmidt reorthonormalisation (GSR) is done at each time step to avoid the misalignment of vectors. This is illustrated in Fig. 3 for a two-dimensional system.

In the case of an \( n \)-dimensional system, the generalised equations of GSR at the \( j \)th step are as follows: (see (3))

where \( \langle \rangle \) stands for the inner product of vectors.

(5) Determine the growth rates of the principal axes at each time step

\[
\Lambda_{ij}^j = \frac{1}{\Delta t} \ln(U_{ij})
\]

(6) Average individual \( \Lambda_{ij}^j \) for a sufficiently long time to determine the \( i \)th LE, \( \Lambda_i \) as follows:

\[
\Lambda_i = \lim_{k \to \infty} \left\{ \frac{1}{k\Delta t} \sum_{j=1}^{k} \ln(U_{ij}) \right\} \quad i = 1, 2, \ldots, n
\]

where \( \Delta t \) is the integration time step.

The above algorithm was implemented in MATLAB and used to study the transient security of the 3-generator 9-bus test system [25] shown in Fig. 4. For dynamic simulations, the generators were modelled using the classical model with the constant admittance load model. Further, bus 1 was considered as an infinite bus. The following observations were done based on this study:

(a) The LEs converged to negative values under stable scenarios. Otherwise, the largest LE (LLE) was a positive number.
(b) Subsequent to a given contingency, the LEs converged to a same negative value irrespective of the fault clearing time up to the critical clearing time beyond which the system becomes unstable.
(c) The LEs converged to different values under different \( (N-1) \) contingencies in the system. A \( (N-1) \) contingency represents an outage of a single element in the power system after the disturbance [2].
(d) Even though the convergence of the LEs is a mandatory requirement to derive the valid stability conclusions, the convergence time was significant. The post-fault system had to be simulated more than 1000 s to get the convergence up to the third decimal place. Such a long simulation time is not acceptable for transient security...
assessment studies. The reason for this long computational time required for the convergence is due to the oscillatory nature of the trajectories in the post-fault power system.

Table 1 shows the transient security assessment results of the test system under three fault scenarios with two fault clearing times (100 and 250 ms). Visual inspection of the trajectories showed that the post-fault system was stable for the fault clearing time of 100 ms and unstable for 250 ms. The LLEs converged to negative and positive values, respectively, in the stable and unstable cases. Table 1 shows the average values of the LLEs calculated over finite time intervals. These finite time LLEs are referred to as the largest average exponential rates (LAERs) in this paper.

The LAERs have accurately derived the stability conclusion of the equilibrium points under different contingencies. The post-fault power system has exponentially decaying oscillations under stable contingencies and diverging trajectories under unstable contingencies. The LEs are related to the exponential decaying rate. Thus, the finite-time LEs remain negative/positive subsequent to stable/unstable scenarios, respectively, thereby accurately predicting the system stability of the post-fault power system same as determining the true LEs. The modified algorithm proposed in this paper collects the data only up to several seconds from $T_c$ to determine the local average convergence rate. This approach significantly reduces the computing time while preserving the invariance property of the conventional LEs associated with the stable equilibrium points of the post-fault power system. The suitability of the modified algorithm for transient security assessment is further illustrated using larger power systems in Section 6.

4 Oscillatory stability assessments using an improved Prony algorithm

This section presents an improved Prony algorithm to assess the security of the power system with respect to oscillatory stability problem. The Prony algorithm approximates an input signal $y(t)$ as

$$y(t) = \sum_{i=1}^{p} a_i e^{-\sigma_i t} \cos(2\pi f_i t + \varphi_i)$$

where $\sigma_i, f_i, a_i$, and $\varphi_i$ are the damping, frequency, strength and the phase angle of the $i$th mode, respectively, and $p$ is the order of the Prony model [18]. The procedure of determining the mode parameters by this algorithm is mentioned below:

1. Construct a discrete linear prediction model that best fits the observed signal.
2. Determine the coefficients of the prediction model via a least-squares approach. The roots of the characteristic polynomial associated with the prediction model determine $\sigma_i$ and $f_i$.
3. Determine $a_i$ and $\varphi_i$ using the least-squares solution of the original set of equations.

| Faulted bus | Tripped line | Fault duration, ms | $T_c-(T_c+5)$ s | $T_c-(T_c+10)$ s | $T_c-(T_c+20)$ s | $T_c-(T_c+30)$ s | $T_c-(T_c+40)$ s |
|-------------|--------------|--------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 4           | 4-5          | 100                | -0.0253         | -0.1452         | -0.0806         | -0.0876         | -0.0963         |
| 7           | 7-5          | 100                | -0.0858         | -0.0578         | -0.0384         | -0.0499         | -0.0603         |
| 9           | 9-8          | 100                | -0.1155         | -0.2447         | -0.1166         | -0.1776         | -0.1094         |
| 4           | 4-5          | 250                | 1.3049          | 0.6227          | 0.243           | 0.1632          | 0.1188          |
| 7           | 7-5          | 250                | 0.9335          | 0.4754          | 0.2588          | 0.1717          | 0.1290          |
| 9           | 9-8          | 250                | 0.2277          | 0.1396          | 0.0794          | 0.0523          | 0.0343          |

$T_c$ – fault clearing time.
A higher order Prony model is usually selected for this analysis which over fits the data [18]. Thus, the Prony algorithm produces fictitious modes in addition to the true modes. The fictitious modes do not reflect the true dynamic behaviour of the power system. This paper uses a modified Prony algorithm to extract the true dominant oscillatory modes present in the power swings.

Fig. 5 shows the rotor angle of generator 2 subsequent to a three-phase solid fault at bus 4 cleared after 100 ms without any topology change in the system shown in Fig. 4. The following approach was applied to extract the true modes present in the input signal.

The power system response soon after the disturbance is non-linear. Thus, a 15 s long data window of the input signal was selected after a 5 s delay of clearing the fault. The Prony algorithm was individually applied on the main data window and on two sub-windows inside the main data window. The sampling rate used was 10 samples/s and the order was individually applied on the main data window and on two sub-windows inside the main data window. The least-square solution was determined using the truncated singular value decomposition. The Prony algorithm on the 13, 14 and 15 s long data windows determined 23, 26 and 26 modes, respectively. Subsequently, the modes satisfying the condition given in (6) were extracted among these modes.

The modes extracted using the above approach and their parameters are shown in Table 2. Further, the results were compared against the eigenvalue analysis. It is seen that the aforementioned approach extracts only the true dominant modes present in the input signal. The rationale behind this modified approach is to fit an input signal using different order Prony models. The dominant modes of the input signal consistently appear irrespective of the order of the Prony model. These modes can then be extracted using the sorting method given in (6).

The performance of the improved Prony algorithm depends on the data window length and the number of sub-windows inside the main data window. Based on the studies performed using synthetic and simulated signals of different test systems, it was found that a four cycles long data window of the dominant mode with two sub-windows generated by reducing the length of the main data window in steps of 1 s extracts the true modes present in the input signal at an acceptable computing time for real-time DSA applications. If there is no prior knowledge about the low-frequency mode in the system, it is recommended to set the length of the data window to be 10–20 s. This is acceptable since the electromechanical oscillations are typically in the range of 0.1–2 Hz [2], thus the 10 s long data window covers at least one cycle of a 0.1 Hz mode.

Table 2 True modes of the input signal

| Mode no. | Frequency, Hz | Dam. ratio, % | Mode no. | Frequency, Hz | Dam. ratio, % |
|----------|---------------|---------------|----------|---------------|---------------|
| 1        | 1.1737        | 1.46          | 1        | 1.1736        | 1.47          |
| 2        | 2.1206        | 1.06          | 2        | 1.1223        | 1.06          |
| 3        | 0             | 100           | 3        | 0             | 100           |

5 Proposed hybrid algorithm for rotor angle security assessment in power systems

The hybrid algorithm proposed in this paper to determine the security of the power system with respect to rotor angle stability is shown in Fig. 6.

Traditionally, the current snapshot for the real-time DSA is obtained using supervisory control and data acquisition systems [26]. These data include analogue measurements such as bus voltage magnitudes, active and reactive power flows as well as logic measurements such as the status of switches and breakers. These measured data are in general corrupted with errors and may contain bad data as well. Therefore, a state estimation algorithm is used to determine the best estimate of the state variables of the power system based on the measured data [26]. A short-term load forecast using the current snapshot is useful to balance the future generation and the forecasted load. Subsequently, the security of the operating points for a list of credible contingencies can be studied.

The dynamic simulation in power systems is started using a power flow solution [6] which determines the key parameters satisfying the condition that the total power generation equals the
system load and losses. The operating point is acceptable if the power flow solution converges without violating the operational constraints. However, for the operating point to be secure, it must be able to survive from a list of credible contingencies. The proposed hybrid algorithm first uses the LEs-based modified algorithm presented in Section 3 to determine the transient rotor angle security and the improved Prony algorithm presented in Section 4 to determine the oscillatory rotor angle security. The input signal to the improved Prony algorithm among the set of rotor angle waveforms were selected using the below mentioned ranking logic.

Fig. 7 shows the relative rotor angle of a generator subsequent to a contingency in a power system. Consider a data window in length $T$ s from the first peak of the waveform. $\delta_0$ and $\delta_m$ are the magnitudes of the angle at the first peak and at time $t = t_1 + T$, respectively. $\delta_0$ and $\delta_m$ can be related as: $\delta_m = e^{-\alpha_{avg} T} \delta_0$, where $\alpha_{avg}$ is the average decaying rate over the data window which is calculated as given in the following equation

$$
\alpha_{avg} = \frac{1}{T} \ln \left( \frac{\delta_m}{\delta_0} \right)
= \frac{1}{T} \ln \left( \frac{\delta_1}{\delta_0} \cdot \frac{\delta_2}{\delta_1} \cdot \ldots \cdot \frac{\delta_m}{\delta_{m-1}} \right)
= \frac{1}{n \Delta T} \sum_{t=1}^{n} \ln \left( \frac{\delta_{it}}{\delta_{i-1}t} \right)
$$

(7)

The rotor angle waveform with the least negative $\alpha_{avg}$ value was used to extract the dominant modes and to determine their parameters by the improved Prony algorithm. In this study, the length $T$ was selected to cover the first ten peaks of the waveform.

Fig. 6 Proposed rotor angle security assessment tool, $C$ – Number of contingencies, $\xi$ – percentage damping ratio, $\tau$ – threshold of damping
Results and discussion

The proposed hybrid algorithm was implemented in MATLAB in the PC environment and the performance was evaluated using a 16-generator 68-bus test system [27] and a 50-generator 470-bus test system [28]. For the dynamic simulations, the synchronous generators were modelled using the detailed generator models with their auxiliary controls and the constant admittance load model was used. However, the time-varying Jacobean matrix required in the LEs-based algorithm was derived by assuming the classical generator model. The time step used in TDS was (1/120) s.

Assuming that there is no prior knowledge about the dominant mode in the test systems, a 20 s long data window was selected for the improved Prony algorithm at a 5 s delay of clearing the fault as explained in Section 4. The sampling rate used was 10 samples/s. Two sub-windows were generated inside the main window by reducing the length in steps of 1 s and the threshold used to extract the true modes was 0.01. Furthermore, the dc component of the input signal was removed before applying the improved Prony algorithm.

The security assessments of the test systems were done using different contingencies. The transient security assessment results derived using the finite time LEs (i.e. LAERs) were compared with the standard TDS and the GEEAC described in Section 2.1. The decision of stability using TDS was made by visual inspection.

![Fig. 7](image-url) Selection of rotor angle trajectories for the improved Prony algorithm

![Fig. 8](image-url) Single line diagram of the 16-generator 68-bus test system
Section 6.1 Transient rotor angle security assessment: Table 3 shows the transient security assessment results of the test system. With reference to the TDS result, Table 3 shows that all the stable and unstable scenarios have been correctly classified by the LES-based algorithm except the last two cases. The LAERs are indicators of the transient security of the post-fault power system. If the LAER is negative, the post-fault power system is transient stable, and unstable, if it is positive. The PSSI is a positive number subsequent to stable cases and a negative number, otherwise.

The last two scenarios are two special cases. The post-fault power systems are concluded as stable by LAERs and PSSIs in these cases. However, visual inspection of the trajectories showed oscillations with slowly increasing amplitudes. Hence, these are two oscillatory instability scenarios. The LAERs gave positive values when the length of the data window was greater than 50 s in these two cases. This shows that the LES-based algorithm may take long time to identify an oscillatory unstable scenario.

Table 4 Oscillatory security assessment under different contingencies of the 16-generator 68-bus test system

| Faulted bus | Tripped line | Input signal | Dominant modes | Conclusion |
|-------------|--------------|--------------|----------------|------------|
| 18          | —            | Gen. 5 relative angle | 0.6911 | post-fault system has inadequate damping |
| 17          | 17–43        | Gen. 2 relative angle   | 0.6824 | post-fault system has inadequate damping |
| 22          | 22–23        | Gen. 13 relative angle  | 0.6904 | post-fault system has inadequate damping |
| 43          | 43–44        | Gen. 12 relative angle  | 0.6825 | post-fault system has inadequate damping |
| 55          | 55–56        | Gen. 12 relative angle  | 0.6908 | post-fault system has inadequate damping |
| 60          | 60–61        | Gen. 15 relative angle  | 0.6666 | post-fault system has inadequate damping |
| 23          | 23–24        | Gen. 10 relative angle  | 0.6650 | post-fault system is oscillatory unstable |
| 27          | 27–37        | Gen. 12 relative angle  | 0.6719 | post-fault system is oscillatory unstable |

$T_c$ – fault clearing time; LAER – largest average exponential rate; PSSI – power swing-based stability index; TDS – time domain simulation.

Table 3 Transient security assessment results under different contingencies of the 16-generator 68-bus test system

| Faulted bus | Tripping line | $T_c$ $(T_c + 5)$ s | $T_c$ $(T_c + 10)$ s | $T_c$ $(T_c + 15)$ s | $T_c$ $(T_c + 20)$ s |
|-------------|---------------|----------------------|----------------------|----------------------|----------------------|
| 18          | —             | -0.0903              | -0.0467              | -0.0521              | -0.0428              | 61.53 stable         |
| 17          | 17–43         | -0.0810              | -0.0310              | -0.0247              | -0.0197              | 58.81 stable         |
| 22          | 22–23         | -0.0807              | -0.0339              | -0.0240              | -0.0245              | 54.72 stable         |
| 38          | 38–46         | -0.0758              | -0.0320              | -0.0247              | -0.0240              | 63.69 stable         |
| 43          | 43–44         | -0.1005              | -0.0445              | -0.0348              | -0.0282              | 64.27 stable         |
| 55          | 55–56         | -0.0765              | -0.0319              | -0.0285              | -0.0165              | 57.36 stable         |
| 60          | 60–61         | -0.1962              | -0.0360              | -0.0743              | -0.0343              | 58.25 stable         |
| 17          | 17–36         | 0.7107               | 0.7021               | 0.5593               | 0.5728               | -99.33 unstable      |
| 19          | 19–68         | 0.2338               | 0.3403               | 0.4386               | 0.5398               | -71.77 unstable      |
| 36          | 36–17         | 0.8903               | 1.0136               | 0.8150               | 0.6549               | -99.39 unstable      |
| 42          | 42–41         | 0.4774               | 0.4044               | 0.4013               | 0.3363               | -99.53 unstable      |
| 50          | 50–51         | 0.6297               | 0.9713               | 1.1080               | 1.0948               | -98.80 unstable      |
| 50          | 50–69         | 0.7247               | 0.9367               | 1.2387               | 1.4934               | -98.58 unstable      |
| 23          | 23–24         | -0.0654              | -0.0566              | -0.0135              | -0.0154              | 39.84 unstable       |
| 27          | 27–37         | -0.0440              | -0.0442              | -0.0380              | -0.0273              | 28.50 unstable       |

$T_c$ – fault clearing time; LAER – largest average exponential rate; PSSI – power swing-based stability index; TDS – time domain simulation.

of the rotor angle trajectories. The GEEAC-based assessment was done using the power swing-based stability index (PSSI) in

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Referring to the last two rows, it is seen that the oscillatory instability of the post-fault system has been correctly identified by the improved Prony algorithm. A negatively damped low-frequency mode has been excited in these cases. Furthermore, it is seen that the damping of the critical modes changes subsequent to different contingencies. This emphasises the importance of having the improved Prony algorithm in the proposed security assessment tool.

### 6.2 Security assessment of 50-generator 470-bus test system

The 50-generator 470-bus test system has 2 areas with 14 thermal generators in area 1 and 23 thermal generators and 13 hydro generators in area 2 [28]. Table 5 shows the transient security assessment results of the system subsequent to three-phase solid faults applied at different buses in the system. In these cases, two fault clearing times were considered to simulate the primary and backup protections. The LAERs were calculated over 5 s long data windows.

Table 5 also shows that the stable and unstable cases have been accurately identified by the negative and positive LAERs, respectively. The same observation was done in the analysis of the 16-generator test system.

To illustrate the performance of the oscillatory stability identification algorithm, consider the clearing of a three-phase fault at bus 16 after 100 ms without any topology change in the network. In this case, generator 16 rotor angle shown in Fig. 9 was identified as the input to the Prony algorithm and two modes were extracted.

The frequencies and the damping ratios of the two modes were 0.8225, 1.1413 Hz and 2.84, 5%, respectively. The parameters of the corresponding modes determined by the eigenvalue analysis were 0.8228, 1.1417 Hz and 2.55, 5.32%, respectively. The speed of generator 16 highly participated in 0.8228 Hz mode. Note that in this case, the 20 s long data window was selected at $t = (5 - 25)$s. However, when the data window was selected at $t = (20 - 40)$s, the improved Prony algorithm extracted only the dominant mode with frequency 0.8223 Hz and damping ratio 2.60%.

Similarly, the oscillatory security assessment of the post-fault power system subsequent to different contingencies was accurately done by the proposed algorithm.

### 6.3 Discussion

In this study, the transient security of the power system was determined using a security measure (LAER) developed based on the concept of LEs and compared the results with a security measure based on the GEEAC (PSSI). A comparison between the two methods is given below.

Both methods employ a criterion based on the simplified generator models in predicting the stability even though the power system is modelled in detail in the TDS. In the PSSI method, the rotor angle separations between all the generators are compared at each time step to group them into the critical cluster and the non-critical cluster of generators. The two groups are further simplified into their equivalent single generator models to use the EAC and hence to derive the stability conclusion. The LEs-based algorithm is less complex compared to this approach. The LEs determine whether the rotor angles converge or not to the equilibrium points by using a time-varying Jacobean matrix to evolve the nearby trajectories in the state space.

Unlike the PSSI, the major limitation of the LEs-based approach is that the magnitude of the LAER cannot be used as a stability index. However, the LEs converge to the same negative value irrespective of the fault clearing time up to the critical clearing time, for a given contingency. This invariance property can be used to determine the critical clearing time and hence to determine the closeness to instability.

The above analyses were done on a PC having Intel Core i7 (3.40 GHz) processor with 8 GB RAM. The average computational burden of the LEs-based algorithm and the improved Prony algorithm to process one contingency was around 0.0576 s in the case.
of 16-generator 68-bus test system and 0.0644 s in the case of 50-generator 470-bus test system.

The 5 s long data window used in the LEs-based algorithm was sufficient to derive the stability conclusions accurately in this study. A longer data window can be used in this regard if the additional computational burden is acceptable. Furthermore, the length of the data window in the Prony algorithm can be set based on the frequency of the dominant mode. Thus, it is recommended to perform an offline analysis first in order to set the data window lengths before using the proposed algorithms in a real-time DSA program.

7 Conclusions

A hybrid algorithm to perform the real-time rotor angle security assessment in power systems has been proposed in this paper. The proposed hybrid algorithm first uses the LEs-based algorithm to determine the transient security assessment. The secure cases are then processed using a shrinking window improved Prony algorithm to determine the oscillatory security of the post-fault power system.

It has been shown that the security measure based on the finite-time LEs determines the transient security of the post-fault power system accurately. The change in the sign of the finite-time LEs from negative to positive was an indication about the change in the security state of the post-fault power system from secure to insecure. Further, the improved Prony algorithm in the hybrid algorithm accurately concluded the oscillatory security by extracting only the dominant modes present in the transient secure power swings. It was shown that the computational burden of the hybrid algorithm proposed in this paper is acceptable for real-time DSA studies in power systems.

8 References

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