IMPACT OF PRICE CAP REGULATION ON SUPPLY CHAIN CONTRACTING BETWEEN TWO MONOPOLISTS

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Abstract. This paper considers a supply chain with an unregulated upstream monopolist (she) supplying a kind of products to a regulated downstream monopolist (he). The upstream monopolist’s production efficiency, which represents her type, is only privately known to herself. When the downstream monopolist trades with the upstream monopolist, his pricing discretion is constrained by price cap regulation (PCR). We model this problem as a game of adverse selection with the price cap constraint. In this model, the downstream monopolist offers a menu of contracts, each of which consists of two parameters: the transfer payment and the retail price. We show that private information can weaken PCR’s impact on the optimal contract, and PCR can dampen the effects of private information. We also shed light on the influences of private information and PCR on the optimal contract, the downstream monopolist’s profit, the upstream monopolist’s profit, the consumers’ surplus and the social total welfare, respectively. Finally, a numerical example is given to illustrate the proposed results.

1. Introduction. Some markets are subject to an environment where the downstream monopolist has been regulated while the upstream monopolist has been unregulated. For example, in a district of the United States, the integrated electric utility has been separated into a regulated distribution company that provides regulated retail electric service to residences and businesses, while electric generation asset has been placed in an unregulated affiliate (31). And in all regulation strategies, PCR is one of the most efficient strategies and frequently used in practice. The purpose of PCR is to eliminate excessive pricing, and induce the monopolist to achieve the desired goals (4, 25, 26, 32). Meanwhile, note that it is much less likely that the downstream monopolist will have the same access to the upstream monopolist’s production efficiency information as the upstream monopolist, that is
to say, the upstream monopolist’s production efficiency is her private information, which plays an important role in the downstream monopolist’s contracting (10) and (30). Therefore, considering the supply chain contracting under the interaction between PCR and asymmetric information is necessary and challenged.

However, despite this kind of situation is widespread in reality, the literature has devoted much less attention. To the best of our knowledge, our paper is the first to investigate the supply chain contracting under PCR and private information, and explore PCR’s impact on the supply chain contracting and the participants’ profits and investigate the interaction between PCR and private information.

To achieve the above purpose, we study a supply chain with an unregulated upstream monopolist supplying a kind of products to a regulated downstream monopolist. The downstream monopolist may not know exactly the upstream monopolist’s production efficiency, that is, the upstream monopolist’s production efficiency is her private information. To maximize his expected profit, the downstream monopolist designs a menu of contracts, each of which specifies the transfer payment and the retail price which is constrained by price cap. We consider mainly three types of PCR: relaxed PCR (i.e., RPCR), moderate PCR (i.e., MPCR), tight PCR (i.e., TPCR).

Our contributions are twofold. First, on the one hand, under no PCR, private information makes both the retail price and the transfer payment distorted. Meanwhile, the upstream monopolist benefits from private information, that is because she obtains the information rents, which results in the decrease of the downstream monopolist’s profit. On the other hand, under PCR, except for the distortion effect incurred by private information, it is worth noting that TPCR has no impact on the optimal contract in a certain interval, that is to say, private information can weaken PCR’s impact on the optimal contract.

Second, when the upstream monopolist’s type is full information, if TPCR or MPCR takes effect, it will make the retail price lower and the transfer payment higher than those of without PCR, and meanwhile, sacrifice the downstream monopolist’s profit, but increase both the consumers’ surplus and the social total welfare. However, when the upstream monopolist’s type is private information, if the regulator implements MPCR, it will realize semi-separating equilibria contract; if the regulator implements TPCR, it will raise pooling equilibria contract. And MPCR and TPCR always result in lower profits of the downstream monopolist and the upstream monopolist than that of without PCR. However, the consumers’ surplus always benefit from PCR. As for the social total welfare, the influences of PCR are complicated. Only in the case of TPCR, the social total welfare is higher than that of without PCR. In addition, by comparing the optimal contract without PCR with that under PCR, the optimal retail price keeps invariable in a certain interval with respect to the upstream monopolist’s type, meaning that PCR can dampen the effect of private information on the optimal contract.

This research is mainly in the nexus of two streams of literature: The literature on PCR and the literature on supply chain contracting. We now discuss how this paper borrows from and contributes to these streams.

The first stream of literature on PCR is fairly abundant, which can be classified into four streams of papers: A first line of the research deals with the case that PCR dampens the effects of cost information asymmetry between monopolists and regulators. Sibley (34) presents a regulatory incentive mechanism problem that the information on cost or demand is only known to the firm but not to the regulator.
Armstrong and Vickers [1] consider an optimal design problem of price cap. They show that, on the one hand, when there is private information only about cost, the firm should always have a degree of discretion over its pricing policy. On the other hand, when uncertainty concerns demand, whether discretion is desirable depended on how demand elasticities varies with the scale of demand. A second line of the literature focuses on the social total welfare effects of tighter price cap. Armstrong and Vickers [2] show that in a single product case, tighter price cap always improves the social total welfare. However, in a multi-product setting, lowering the price cap can reduce the social total welfare since it can result in prices set below marginal cost. Law [22] and Kanga et al. [21] examine whether the consumers necessarily benefit from tighter price cap with independent or interdependent demand, respectively. A third line deals with the influence of providing monopolists with incentives to acquire cost information. Inssa and Stroffolini [19] demonstrate that PCR provides weak incentives to acquire cost information since the price can not be fully adjusted upward. Inssa and Stroffolini [18] also study that, on the basis of [19], introducing revenue sharing with PCR can improve incentives of regulated firms to acquire cost information. A fourth line considers that PCR has very desirable properties in term of allocated efficiency. Reitzes [31] finds that the imposition of a downstream price cap with an appropriate profit-sharing rate could eliminate the upstream affiliate’s exercise of market power. However, it is less desirable to fully mitigate affiliate market power when upstream rivals also behave strategically. In addition, Iozzi et al. [20] propose a general form of price cap formula and shows that it ensures the convergence to optimal prices in the long run equilibrium for virtually any form of the social total welfare function.

As comprehensive as it is, these papers limit themselves to how to design the price cap so as to realize their expected aims, e.g., the expected welfare, allocated efficiency, eliminate asymmetric information’s influence and so on. However, they do not address supply chain contracting under PCR, which has been identified as a limitation. Thereby, we extend the current PCR literature by considering the downstream monopolist’s contracting under the double effects of private information and PCR, assuming that the price cap is exogenous. As a result, we highlight the impact of PCR on the supply chain contracting which leads to interesting results.

The remainder of the paper is structured as follows. Section 2 gives the problem formulation. Section 3 analyzes the problem when the upstream monopolist’s production efficiency is full information, and compares the optimal contracts and the participants’ profits without PCR and with PCR, respectively. Section 4 concerns
the case that the upstream monopolist’s production efficiency is her private information, and analyzes the impacts of information structure and PCR on the optimal contract and the participants’ profits, respectively. Section 5 gives a numerical example. Finally, Section 6 summarizes the findings of the paper. All the proofs are in Appendix.

2. Basic structures. Consider a supply chain consisting of an unregulated upstream monopolist and a regulated downstream monopolist. The downstream monopolist (he) purchases a kind of products at a transfer payment \( t \) from the upstream monopolist (she), and then sells the products at a retail price \( p \) to the consumers. The upstream monopolist’s type \( x \), representing her production efficiency, is her private information. For the downstream monopolist, he designs a contract \((p,t)\) for the upstream monopolist in order to maximize his expected profit. Meanwhile, the retail price of the downstream monopolist is constrained by the price cap \( p \). Accordingly, we will focus the discussion here on the impacts of information structure and PCR on the optimal contract.

Even though the downstream monopolist does not know exactly the upstream monopolist’s type, he can make an assessment about it, which is represented by a stochastic variable \( X \) defined on the set \([x, x]\). The probability distribution and the density function of \( X \) is \( F(x) \) and \( f(x) \), respectively. Without loss of generality, presume that \( F(x) \) satisfies the non-decreasing hazard rate condition, that is, \( \frac{d}{dx} \left( \frac{f(x)}{1 - F(x)} \right) \geq 0 \). This is equivalent to the assumption that \( F(x) \) is log-concave, which is commonly used in mechanism design problems. Most parametric single-peak distributions have a nondecreasing hazard rate, including uniform, normal, exponential, and power function distributions (see [5], [7], [8], [23], [27] for details).

Let \( C(Q(p), x) \) be the upstream monopolist’s cost function, where \( Q(p) \) denotes the market demand of the products, satisfying \( Q'(p) < 0 \) and \( Q''(p) < 0 \), which represents that the market demand is decreasing in \( p \) and the marginal demand is decreasing in \( p \). Here, the market demand equals to production quantity. For each \( x \in [x, x] \), assume that \( \frac{\partial^2 C(Q, x)}{\partial x \partial Q} > 0 \), \( \frac{\partial C(Q, x)}{\partial Q} \geq 0 \), \( \frac{\partial C(Q, x)}{\partial x} \leq 0 \), \( \frac{\partial^2 C(Q, x)}{\partial x^2} \leq 0 \), \( \frac{\partial^2 C(Q, x)}{\partial x \partial Q} \geq 0 \). Note that the first two assumptions mean that the upstream monopolist’s cost increases with production quantity at a nondecreasing rate, similarly, \( \frac{\partial C(Q, x)}{\partial x} \leq 0 \) and \( \frac{\partial^2 C(Q, x)}{\partial x^2} \leq 0 \) mean that the upstream monopolist’s cost is non-decreasing in the production efficiency at a nondecreasing rate, \( \frac{\partial^2 C(Q, x)}{\partial x \partial Q} \geq 0 \) represents that the high production efficiency increases the magnitude of the marginal cost-increase production quantity, that is, it increases \( \frac{\partial C(Q, x)}{\partial Q} \).

The upstream monopolist’s profit

\[
\Pi(p, t, x) = t - C(Q(p), x). \tag{1}
\]

The downstream monopolist’s profit

\[
U(p, t, x) = (p - c)Q(p) - t, \tag{2}
\]

where \( c \) is the downstream monopolist’s unit cost, and \( p > c \).

The consumers’ surplus

\[
S(p, t, x) = V(Q(p)) - pQ(p), \tag{3}
\]

where \( V(Q) = \int_0^Q P(y)dy \) is the consumers’ utility function (32 and 36).
Therefore, the social total welfare
\[ W(p, t, x) = S(p, t, x) + U(p, t, x) + \Pi(p, t, x). \] (4)

3. Full information: A benchmark. In this section, we consider the full information case as a benchmark in which the downstream monopolist knows the upstream monopolist’s type, denoted by \( x \). The downstream monopolist’s problem can be modeled as
\[
\begin{align*}
\max_{(p(x), t(x))} & \quad U(p, t, x) \\
\text{subject to:} & \quad p(x) \leq \bar{p}, \\
& \quad \Pi(p, t, x) \geq 0, \forall x \in [\bar{x}, \bar{x}].
\end{align*}
\] (5)

In Model (5), the first constraint is the price cap constraint, and the second constraint is the upstream monopolist’s participation constraint, implying that when the downstream monopolist designs the contract, he must take into account the upstream monopolist’s participation, that is to say, he must ensure the upstream monopolist to obtain at least as great as her reservation profit. In this paper, we normalize her reservation profit as zero without loss of generality ([24], [30]). Of course, this assumption is analytically convenient but not crucial to the analysis, we can establish similar results when the reservation profit is (strictly) positive.

Definition 3.1. Let \( p^*(x) \) be the optimal retail price without PCR. There exists three different definitions of PCR’s type.
1) PCR is relaxed (RPCR) when \( \bar{p} \geq p^*(x), \forall x \in [\bar{x}, \bar{x}] \).
2) PCR is moderate (MPCR) when there exists \( x' \in [\bar{x}, \bar{x}] \) such that \( p^*(x') = \bar{p} \).
3) PCR is tight (TPCR) when \( \bar{p} < p^*(x), \forall x \in [\bar{x}, \bar{x}] \).

To analyze the price cap’s influence on the optimal contract clearly, we will solve Model (5) under two cases: one is without PCR, i.e., omitting the price cap constraint, and the other is with PCR.

3.1. Without PCR.

Theorem 3.2. 1) The optimal contract \((p^*(x), t^*(x))\) satisfies
\[
\begin{align*}
Q(p^*(x)) + \left( p^*(x) - c - \frac{\partial C(Q(p^*(x)), x)}{\partial Q} \right) Q'(p^*(x)) &= 0, \\
t^*(x) &= C(Q(p^*(x)), x).
\end{align*}
\] (6) (7)

2) The downstream monopolist’s profit \( U^*(p^*(x), t^*(x), x) \), the upstream monopolist’s profit \( \Pi^*(p^*(x), t^*(x), x) \), the consumers’ surplus \( S^*(p^*(x), t^*(x), x) \) and the social total welfare \( W^*(p^*(x), t^*(x), x) \) are as follows.
\[
\begin{align*}
U^*(p^*(x), t^*(x), x) &= (p^*(x) - c)Q(p^*(x)) - C(Q(p^*(x)), x), \\
\Pi^*(p^*(x), t^*(x), x) &= 0, \\
S^*(p^*(x), t^*(x), x) &= V(Q(p^*(x))) - p^*(x)Q(p^*(x)), \\
W^*(p^*(x), t^*(x), x) &= V(Q(p^*(x))) - cQ(p^*(x)) - C(Q(p^*(x)), x).
\end{align*}
\] (8) (9) (10) (11)

Note that \( \frac{dS(p^*(x), t^*(x), x)}{dp} = (V' - p)Q'(p) - Q(p) = -Q(p) < 0 \), meaning that \( S(p, t, x) \) is decreasing in the retail price \( p \). At meanwhile, \( \frac{dW(p^*(x), t^*(x), x)}{dp} = \left( V' - c - \frac{\partial C(Q(p^*(x)), x)}{\partial Q} \right) Q'(p) \leq 0 \), that is, the social total welfare \( W(p, t, x) \) is
nonincreasing in the retail price \( p \). That is to say, the lower the retail price is, the more the consumers’ surplus and the social total welfare are.

**Proposition 1.** If \( \frac{\partial C(Q(x))}{\partial x} \geq 0 \), the retail price \( p^*(x) \) is nondecreasing in \( x \), and the transfer payment \( t^*(x) \) is nonincreasing in \( x \).

Since the high production efficiency increases the marginal production cost, the retail price \( p^*(x) \) is nondecreasing in \( x \). The increase of the retail price results in lower market demand, which yields less production cost. Thereby, the downstream monopolist pays the upstream monopolist less transfer payment \( t^*(x) \), hence, the transfer payment \( t^*(x) \) is nonincreasing in \( x \).

3.2. With PCR. In this subsection, we consider the case with price cap constraint.

**Theorem 3.3.** Under full information and PCR, the optimal contract \((p^{**}(x); t^{**}(x))\) takes the following three cases.

1) When \( \overline{p} \geq p^*(x) \), \( \forall x \in [\underline{x}, \overline{x}] \), the optimal contract

\[
p^{**}(x) = p^*(x), \quad t^{**}(x) = t^*(x).
\]

2) There exists \( x_0 \in [\underline{x}, \overline{x}] \) such that \( p^*(x_0) = \overline{p} \). The optimal contract

\[
p^{**}(x) = \begin{cases} p^*(x), & \underline{x} \leq x < x_0 \\ \overline{p}, & x_0 \leq x \leq \overline{x}. \end{cases}
\]

\[
t^{**}(x) = \begin{cases} t^*(x), & \underline{x} \leq x < x_0 \\ \overline{C}(Q(\overline{p}), x), & x_0 \leq x \leq \overline{x}. \end{cases}
\]

3) When \( \overline{p} < p^*(x) \), \( \forall x \in [\underline{x}, \overline{x}] \), the optimal contract

\[
p^{**}(x) = \overline{p}, \quad t^{**}(x) = \overline{C}(Q(\overline{p}), x).
\]

The explanation of Theorem 3.3 is as follows. In the case of RPCR, PCR has no impact on the optimal contract, that is to say, the regulator grants the downstream monopolist full pricing discretion. Under MPCR, the regulator affords the downstream monopolist partial pricing discretion. To be more detailed, the regulator sets a price cap \( \overline{p} \) resulting in type \( x_0 \) as a threshold, such that \( p^*(x_0) = \overline{p} \). Furthermore, on one hand, when the upstream monopolist’s type \( x \geq x_0 \), the retail price is set as \( \overline{p} \). As for the transfer payment \( t^{**} \), it will vary with \( x \) only, which is similar to the case without PCR. On the other hand, when \( x < x_0 \), PCR has no impact on the optimal contract, it is worth noticing that, different from the above case but the same as the case without PCR, both \( x \) and \( p^* \) determine \( t^{**} \). To conclude, to prevent the downstream monopolist from abusing his monopoly pricing power, the regulator implements MPCR so as to induce the downstream monopolist to employ the price cap when the upstream monopolist’s type lies in the interval, which is determined by the upstream monopolist’s cost type. In the case of TPCR, because the price cap constraint is binding, the regulator instructs the downstream monopolist to offer the fixed price, which has nothing to do with the upstream monopolist’s type.

3.3. Comparisons of the case of with PCR with that of without PCR.

**Proposition 2.** Under full information, let \((p^*, t^*)\) and \((p^{**}, t^{**})\), \(U^*(p^*, t^*, x)\) and \(U^{**}(p^{**}, t^{**}, x)\), \(S^*(p^*, t^*, x)\) and \(S^{**}(p^{**}, t^{**}, x)\), \(W^*(p^*, t^*, x)\) and \(W^{**}(p^{**}, t^{**}, x)\) be the optimal contracts, the downstream monopolist’s profits, the consumers’ surpluses, and the social total welfare without PCR and with PCR, respectively. Their relationships are
1) \[ p^* \geq p^{**}, \quad t^* \leq t^{**}. \quad (16) \]

2) \begin{align*}
U^{**}(p^{**}, t^{**}, x) &\leq U^*(p^*, t^*, x), 
\Pi^{**}(p^{**}, t^{**}, x) &= \Pi^*(p^*, t^*, x), 
S^{**}(p^{**}, t^{**}, x) &\geq S^*(p^*, t^*, x), 
W^{**}(p^{**}, t^{**}, x) &\geq W^*(p^*, t^*, x). 
\end{align*} \quad (17), (18), (19), (20)

The result 1) in Proposition 2 is intuitive. Indeed, PCR designed by the regulator is to reduce the retail price, thereby raising the demand quantity, which increases the upstream monopolist’s more production cost. Consequently, the downstream monopolist must pay more transfer payment for the sake of compensating the upstream monopolist’s production cost.

The result 2) in Proposition 2 implies that, PCR results in more consumers’ surplus and the social total welfare. To illustrate this, note that the social total welfare is a sum of the consumers’ surplus, the upstream monopolist’s profit and the downstream monopolist’s profit. The decrease of the retail price results in the reduction of downstream monopolist’s profit and the increase of consumers’ surplus, moreover, the increase of the consumers’ surplus outweighs the reduction of the downstream monopolist’s profit. In addition, the upstream monopolist’s profit is zero. As a result, the social total welfare is better off.

To sum up, under full information, if there is no PCR, the downstream monopolist deprives all profit of the supply chain; if there exists PCR, the downstream monopolist’s profit is reduced, whereas both the consumers’ surplus and the social total welfare are raised, and in those two cases, the upstream monopolist’s profits are always zero.

4. Private information. In this section, we consider the setting that the type of the upstream monopolist is her private information. According to Revelation Principle [28], to obtain the maximal expected profit, the downstream monopolist can restrict his attention to the direct mechanism, where the upstream monopolist’s only action is to submit a claim about her type. This direct mechanism can induce truth-telling such that the upstream monopolist accepts the contract and announces her true type. Let \((p(\cdot), t(\cdot))\) denote a contract for the upstream monopolist of type \(x\).

Given a contract \((p(\cdot), t(\cdot))\), if the upstream monopolist’s real type is \(x\) and she reports \(x\) truthfully, her profit
\[ \Pi(p(x), t(x), x) = t(x) - C(Q(p(x)), x). \quad (21) \]
If the upstream monopolist’s real type is \(x\), but she misreports it as \(\hat{x}\), her profit
\[ \Pi(p(\hat{x}), t(\hat{x}), x) = t(\hat{x}) - C(Q(p(\hat{x})), x). \quad (22) \]
To prevent the upstream monopolist to misreport, the incentive constraint must be satisfied
\[ \Pi(p(x), t(x), x) \geq \Pi(p(\hat{x}), t(\hat{x}), x), \forall x, \hat{x} \in [\underline{x}, \overline{x}]. \quad (23) \]
The upstream monopolist’s participation constraint must be satisfied
\[ \Pi(p(x), t(x), x) \geq 0, \forall x \in [\underline{x}, \overline{x}]. \quad (24) \]
The downstream monopolist’s expected profit

\[ E[U(p(X), t(X), X)] = \int_x \left[ (p(x) - c)Q(p(x)) - t(x) \right] f(x) dx. \]  

(25)

The downstream monopolist’s problem under private information can be formulated as follows

\[
\max_{(p(x), t(x))} E[U(p(X), t(X), X)] \quad \text{subject to:}
\begin{align*}
& p(x) \leq \bar{p}, \forall x \in [\underline{x}, \bar{x}] \\
& \Pi(p(x), t(x), x) \geq \Pi(\hat{p}(x), \hat{t}(x), x), \forall x, \hat{x} \in [\underline{x}, \bar{x}] \\
& \Pi(p(x), t(x), x) \geq 0, \forall x \in [\underline{x}, \bar{x}].
\end{align*}
\]  

(26)

4.1. Equivalent model. In the following, for the sake of solving, we will give the equivalent form of Model (26). To achieve it, we require technical assumptions: \( \frac{\partial^4 C(Q, x)}{\partial x^2 \partial Q^2} \leq 0 \) and \( \frac{\partial^4 C(Q, x)}{\partial x \partial Q^2} \leq 0 \) which ensure the optimal solutions satisfy the monotonicity conditions.

**Theorem 4.1.** Model (26) is equivalent to

\[
\max_{p(x)} \int_x \left\{ \left( p(x) - c \right)Q(p(x)) - C(p(x)), x \right\} f(x) dx 
\]  

subject to:

\[
\frac{dp(x)}{dx} \geq 0, \forall x \in [\underline{x}, \bar{x}]. \\
\frac{dp(x)}{dx} \leq \bar{p}, \forall x \in [\underline{x}, \bar{x}].
\]  

(27)

4.2. Without PCR.

**Proposition 3.** Under private information, in the case of without PCR,

1) The optimal retail price \( p^*_1(x) \) is determined by

\[
Q(p^*_1(x)) + \left( p^*_1(x) - c - \frac{\partial C(Q(p^*_1(x)), x)}{dp_1} \right) \frac{dQ(p^*_1(x))}{dp_1}
+ \left( 1 - \frac{F(x)}{f(x)} \right) \frac{\partial^2 C(Q(p^*_1(x)), x)}{dx \partial Q} ds \left( Q(p^*_1(x)), x \right) = 0,
\]  

(28)

and the optimal transfer payment \( t^*_1(x) \) is given by

\[
t^*_1(x) = C(Q(p^*_1(x)), x) - \int_x^\infty \frac{\partial C(Q(p^*_1(s)), s)}{ds} ds.
\]  

(29)

2) The downstream monopolist’s expected profit

\[
E[U^*_1(p^*_1(X), t^*_1(X), X)] = \int_x \left\{ (p^*_1(x) - c)Q(p^*_1(x)) - C(p^*_1(x), x) \right\} f(x) dx + \left( 1 - \frac{F(x)}{f(x)} \right) \frac{\partial C(Q(p^*_1(x)), x)}{dx} f(x) dx.
\]  

The upstream monopolist’s profit

\[
\Pi^*_1(p^*_1(x), t^*_1(x), x) = -\int_x^\infty \frac{\partial C(Q(p^*_1(s)), s)}{ds} ds.
\]  

(30)
The consumers' surplus

\[ S^*_1(p^*_1(x), t^*_1(x), x) = V(Q(p^*_1(x))) - p^*_1(x)Q(p^*_1(x)). \]  

(31)

In Eq. (29), the term \( C(Q(p^*_1(x)), x) \) identifies the upstream monopolist's cost subsidy to ensure the upstream monopolist's participation, while the term \( - \int_{x}^{x} \frac{\partial C(Q(p^*_1(x)), x)}{\partial x} ds \) is the information rents paid to the upstream monopolist in order to induce the upstream monopolist to tell the truth. Additionally, note that the information rents are nondecreasing in the upstream monopolist's type \( x \), which implies the upstream monopolist does not have an incentive to understated her type. In particular, the upstream monopolist will not obtain the information rents when \( x = \bar{x} \).

As Eq. (28) reveals, the crucial difference between the optimal retail price under private information and that under full information is the expression

\[ \frac{(1 - F(x))}{f(x)} \frac{\partial^2 C(Q(p^*_1(x)), x)}{\partial x \partial Q} \frac{dQ(p^*_1(x))}{dp_1}. \]

To understand how does the retail price affected by private information, it is useful to analyze the above expression in some detail. In the above expression, the term \( \frac{\partial^2 C(Q(p^*_1(x)), x)}{\partial x \partial Q} \frac{dQ(p^*_1(x))}{dp_1} \) identifies the change of information rents reflecting the change of retail price. In other words, if the optimal retail price under private information is determined by that under full information, then the upstream monopolist will obtain more information rents. Thereby, in order to limit the information rents, the downstream monopolist sets the optimal retail price distorted downward.

4.3. With PCR.

**Theorem 4.2.** Under private information, in the case of PCR, the optimal contract contains the following three cases.

1) When \( \bar{p} \geq p^*_1(x) \), \( \forall x \in [x, \bar{x}] \), the optimal contract

\[ p^*_1(x) = p^*_1(x), \quad t^*_1(x) = t^*_1(x). \]  

(32)

2) There exists \( x_1 \in [x, \bar{x}] \) such that \( p^*_1(x_1) = \bar{p} \), the optimal contract

\[ p^*_1(x) = \begin{cases} p^*_1(x), & x \leq x_1, \\ \bar{p}, & x_1 \leq x \leq \bar{x}. \end{cases} \]  

(33)

\[ t^*_1(x) = \begin{cases} t^*_1(x), & \bar{x} \leq x < x_1, \\ C(Q(\bar{p}), x) - \int_{x_1}^{x} \frac{\partial C(Q(\bar{p}), s)}{\partial s} ds, & x_1 \leq x \leq \bar{x}. \end{cases} \]  

(34)

3) When \( \bar{p} < p^*_1(x) \), \( \forall x \in [x, \bar{x}] \), the optimal contract

\[ p^*_1(x) = \bar{p}, \quad t^*_1(x) = C(Q(\bar{p}), x) - \int_{x}^{\bar{x}} \frac{\partial C(Q(\bar{p}), s)}{\partial s} ds. \]  

(35)

The interpretations for Theorem 4.2 are as follows:

1) In the case of RPCR, the downstream monopolist offers different contracts depending on the upstream monopolist’s type. As for the upstream monopolist, in order to maximize her profit, she will report truly her type. Therefore, the optimal contract brings about separating equilibria.

2) MPCR imposes an important impact on the transfer payments, as shown in Eqs. (34). Concretely, when \( x \geq x_1 \), the transfer payments are constant with respect to \( x \). That is to say, the optimal transfer payments keep constant regardless of the
upstream monopolist’s type, which realizes pooling equilibria contract. The reason for this is that the reduction of the cost in $x$ offsets the increase of information rents in $x$. While when $x < x_1$, the transfer payments vary with $x$, the optimal contract realizes the separating equilibria. From the above, we can conclude the case 2) realizes semi-separating equilibria.

3) TPCR makes the optimal contract turn to be pooling equilibria, i.e., the optimal contract is invariable with the upstream monopolist’s type.

4.4. The effects of information structure. Now we will analyze the information structure’s impacts on the optimal contract and the participants’ profits without PCR and with PCR, respectively.

4.4.1. Comparing the optimal contracts and the participants’ profits in the case of without PCR.

Proposition 4. In the case of without PCR, let $(p^*, t^*)$ and $(p^*_1, t^*_1)$, $S^*(p^*, t^*, x)$ and $S^*_1(p^*, t^*_1, x)$, $\Pi^*(p^*, t^*, x)$ and $\Pi^*_1(p^*, t^*_1, x)$ be the optimal contracts, the consumers’ surpluses and the upstream monopolist’s profits under full information and private information, respectively. We can obtain

$$p^*_1 \leq p^*,$$
$$t^*_1 > t^*.$$  

$$S^*_1(p^*_1, t^*_1; x) \geq S^*(p^*, t^*; x),$$

$$\Pi^*_1(p^*_1, t^*_1; x) \geq \Pi^*(p^*, t^*; x) = 0.$$

Proposition 4 indicates that, under private information, the optimal retail price $p^*_1$ is distorted from that under full information. The reason is that the presence of private information may give rise to the information rents, in order to limit the information rents, the downstream monopolist lowers the retail price from that under full information except at $x = x_1$. To illustrate this, consider the information rents $-\int_{x}^{x_1} \frac{\partial C(Q(p^*_1), s)}{\partial s} \, ds$, note that $-\frac{d}{dp} \int_{x}^{x_1} \frac{\partial C(Q(p^*_1), s)}{\partial s} \, ds \geq 0$ represents that the information rents are nondecreasing in $p$, therefore, the downstream monopolist has to lower the retail price to reduce the information rents. Especially, when the upstream monopolist’s type is $x_1$, $p^*_1 = p^*$, which implies the optimal retail price is not distorted when $x = x_1$.

The transfer payment $t^*_1$ under private information is more than that $t^*$ under full information, to see why, note that $t^*$ is to ensure the upstream monopolist’s participation, called by cost subsidy. However, $t^*_1$ has two roles: One is to ensure the upstream monopolist’s participation, the other is to induce the upstream monopolist to report truly private information. Moreover, the retail price $p^*_1$ becomes less, thereby, the market demand increases, which results in the more production cost, paid by the downstream monopolist. Hence, the transfer payment $t^*_1$ increases. In addition, the information rents make $t^*_1$ even more, consequently, $t^*_1 \geq t^*$.

As for the consumers’ surplus under private information, it is no less than that under full information, the reason is that private information makes the optimal retail price distorted downward, which brings about the increase of demand resulting in more consumers’ surplus. In other words, under private information, the downstream monopolist lowers the retail price to avoid giving more information rents, whereas the consumers obtain more surplus from it.
With the upstream monopolist’s profit, a key difference between private information and full information is information rents, the information rents should be employed to motivate the upstream monopolist’s truth telling, whereas under full information, the downstream monopolist deprives all profit of the supply chain, therefore, $\Pi_1^*(p_1^*, t_1^*, x) > \Pi_1^*(p^*, t^*, x)$. In particular, when $x = \bar{x}$, the upstream monopolist can not obtain the information rents, thus, $\Pi_1^*(p_1^*, t_1^*, x) = \Pi_1^*(p^*, t^*, x) = 0$.

4.4.2. Comparing the optimal contracts and the participants’ profits in the case of with PCR.

**Proposition 5.** In case of PCR, let $(p^{**}(x), t^{**}(x))$ and $(p^{*\ast}(x), t^{*\ast}(x))$, $S^{**}(p^{**}, t^{**}, x)$ and $S^{*\ast}(p^{*\ast}, t^{*\ast}, x)$, $\Pi^{**}(p^{**}, t^{**}, x)$ and $\Pi^{*\ast}(p^{*\ast}, t^{*\ast}, x)$ denote the optimal contracts, the consumers’ surpluses, the upstream monopolist’s profits under full information and private information, respectively.

- When $x \in [\underline{x}, \underline{x}_0]$, $p_1^{*\ast}(x) < p^{**}(x)$, $t_1^{*\ast}(x) > t^{**}(x)$.
- When $x \in [\underline{x}_0, \underline{x}_1]$, $p_1^{*\ast}(x) < p^{**}(x)$, $t_1^{*\ast}(x) > t^{**}(x)$.
- When $x \in [\underline{x}_1, \bar{x}]$, $p_1^{*\ast}(x) = p^{**}(x)$, $t_1^{*\ast}(x) > t^{**}(x)$.
- When $p^*(x), p_1^*(x) \leq \overline{p}$, $p_1^{*\ast}(x) \leq p^{**}(x)$, $t_1^{*\ast}(x) > t^{**}(x)$.

While $p^*(x), p_1^*(x) > \overline{p}$, $p_1^{*\ast}(x) = p^{**}(x)$, $t_1^{*\ast}(x) > t^{**}(x)$.

$S_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, x) \geq S^{**}(p^{**}, t^{**}, x)$,

$\Pi_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, x) \geq \Pi^{**}(p^{**}, t^{**}, x)$.

Proposition 5 reveals that the impact of private information on the optimal contract under PCR. To concrete, PCR makes the optimal retail price $p^{**}(x)$ no less than $p_1^{*\ast}(x)$. Further, note that the optimal retail prices under full information and private information are equal in the interval $[\underline{x}_1, \bar{x}]$, different from the result in Proposition 4 where if and only if $x = \bar{x}$, $p_1^*(x) = p^*(x)$. This fact demonstrates that PCR weakens private information’s impact on the optimal contract. Another feature which is worth noticing is that when $x \in [\underline{x}_0, \underline{x}_1]$, PCR has not restrained the retail price under private information, but interestingly, the retail price under full information is controlled, i.e., equal to price cap. That means that private information can dampen the effects of PCR on the optimal contract.

The result 2) in Proposition 5 implies that, although the optimal contracts under full information and private information are both influenced by PCR, private information eventually raises the consumers’ surplus. As for the upstream monopolist, even though PCR limits her information rents, she still obtains more profit than that under full information.

4.5. The effect of PCR. We are now in a position to analyze the PCR’s influences on the optimal contract and the participants’ profits under private information, respectively.

**Proposition 6.** Under private information, let $p_1^*$ and $p_1^{*\ast}$, $E[U_1^*(p_1^*, t_1^*, X)]$ and $E[U_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, X)]$, $\Pi_1^*(p_1^*, t_1^*, x)$ and $\Pi_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, x)$, $S_1^*(p_1^*, t_1^*, x)$ and $S_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, x)$ be the optimal retail prices, the upstream monopolist’s profits, the downstream monopolist’s expected profits, the consumers’ surpluses without PCR and with PCR, respectively. We can deduce the following results.

$p_1^* \geq p_1^{*\ast}, \quad (38)$

$\Pi_1^*(p_1^*, t_1^*, x) \geq \Pi_1^{*\ast}(p_1^{*\ast}, t_1^{*\ast}, x), \quad (39)$
consumers’ surplus and the social total welfare under full information as shown Tables 2 and 3, PCR’s impacts on the downstream monopolist’s profit, the impact on the optimal contracts under full information and private information as shown Table 1, PCR’s profit $\Pi^*$, obeys uniform distribution on the support $[9,10]$. By solving Model (26), we can deduce that the optimal retail price $p^*_1$, the optimal transfer payment $t^*_1$, the downstream monopolist’s expected profit $E[U^*_1(p^*_1,t^*_1,X)]$, the upstream monopolist’s profit $\Pi^*$, the consumers’ surplus $S^*(p^*_1,t^*_1,x)$, the social total welfare $W^*(p^*,t^*,x)$ without PCR.

Simultaneously, we compare the information structure’s impact on the optimal contracts under full information and private information as shown Tables 2 and 3, PCR’s impacts on the downstream monopolist’s profit, the consumers’ surplus and the social total welfare under full information as shown

$$E[U^*_1(p^*_1,t^*_1,X)] \geq E[U^{**}_1(p^{**}_1,t^{**}_1,X)], \quad (40)$$

$$S^*_1(p^*_1,t^*_1,x) \leq S^{**}_1(p^{**}_1,t^{**}_1,x). \quad (41)$$

As Inequality (38) reveals that PCR limits the retail price determination, however, there is no obvious relationship between the transfer payment $t^*_1$ and $t^{**}_1$. Implicit reason for this result is the two roles of transfer payment: cost subsidy and information rents, specially, the retail price falls, but the cost subsidy is increased and the information rents are decreased. Furthermore, there is no pronounced relationship between cost subsidy’s increase and the information rents’ decrease.

Inequality (40) illustrates that under private information, PCR results in the downstream monopolist’s profit worse off. The intuition for this result is straightforward. In order to limit the downstream monopolist’s pricing discretion, PCR is implemented at the expense of the downstream monopolist’s expected profit. Inequality (41) shows that PCR benefits the consumers, in other words, pooling equilibrium contract or semi separate equilibrium contract are optimal for the consumers.

As for the upstream monopolist’s profit, because the information rents are non-decreasing in the retail price, in other words, the information rents are limited by PCR, hence, PCR results in the reduction of the upstream monopolist’s profit.

5. Numerical Example. In this section, a numerical example is provided to illustrate PCR’s impact on the optimal contracts under full information and private information, respectively, and presents the influences on the upstream monopolist’s profit, the downstream monopolist’s profit, the consumers’ surplus and the social total welfare.

For simplicity, we assume that the upstream monopolist’s cost function $C(Q(p), x) = Q(p) + xQ(p) - 0.2x^2$, where $Q(p) = a - bp$ denotes the demand function, $a$ is a demand parameter, representing the market capacity of the products, here set $a = 20$, $b$ measures a demand sensitivity in response to the price change, without loss of generality, set $b = 1$ in this numerical example. Thus $C(Q(p), x) = (x + 1)(20 - p) - 0.2x^2$. This cost function assumption is analytically convenient. However, it is not crucial to the analysis. We can establish similar results if use other cost functions. Moreover, assume that $p \leq 20$, ensuring that the demand is nonnegative, and the downstream monopolist’s unit production cost $c = 5$.

Presume that tight price cap $\bar{p} = 16.75$, relaxing price cap $\bar{p} = 18.25$, and moderate price cap $\bar{p} = 17.75$.

Firstly by solving Model (5) under full information, we can obtain that the optimal retail price $p^*$ and the optimal transfer payment $t^*$, and the downstream monopolist’s profit $U^*(p^*,t^*,x)$, the upstream monopolist’s profit $\Pi^*(p^*,t^*,x) = 0$, the consumers’ surplus $S^*(p^*,t^*,x)$, the social total welfare $W^*(p^*,t^*,x)$ without PCR.

Under private information, for simplicity, we presume that stochastic variable $X$ obeys uniform distribution on the support $[9,10]$. By solving Model (26), we can deduce that the optimal retail price $p^*_1$, the optimal transfer payment $t^*_1$, the downstream monopolist’s expected profit $E[U^*_1(p^*_1,t^*_1,X)]$, the upstream monopolist’s profit $\Pi^*_1(p^*_1,t^*_1,x)$, the consumers’ surplus $S^*_1(p^*_1,t^*_1,x)$ without PCR.
Table 4, PCR’s impacts on the upstream monopolist’s profit, the downstream monopolist’s profit and the consumers’ surplus under private information as shown in Table 5.

Table 1 Information structure’s impact on contract

| Scenario       | p       | t       |
|----------------|---------|---------|
| full information | 0.5x + 13 | -0.7x² + 6.5x + 7 |
| private information | x + 8   | -0.5x² - x + 63.3 |

Table 2 Price cap’s impact on the optimal contract under full information

| Scenario       | p**     | t**     |
|----------------|---------|---------|
| RPCR           | 0.5x + 13 | -0.7x² + 6.5x + 7 |
| TPCR           | 16.75   | -0.2x² + 3.25x + 3.25 |
| MPCR when x ≥ x₀ | 17.75   | -0.2x² + 2.25x + 2.25 |
| MPCR when x < x₀ | 0.5x + 13 | -0.7x² + 6.5x + 7 |

Table 3 Price cap’s impact on the optimal contract under private information

| Scenario       | p₁**    | t₁**    |
|----------------|---------|---------|
| RPCR           | x + 8   | -0.5x² - x + 63.3 |
| TPCR           | 16.75   | 16.3    |
| MPCR when x ≥ x₁ | 17.75   | 5.175   |
| MPCR when x < x₁ | x + 8   | -0.5x² - x + 63.3 |

Table 4 Price cap’s impacts on the downstream monopolist’s profit, the consumers’ surplus and the social total welfare under full information

| Scenario       | U**     | S**     | W**     |
|----------------|---------|---------|---------|
| RPCR           | 0.45x² - 7x + 49 | 0.125x² - 3.5x + 24.5 | 0.575x² - 10.5x + 73.5 |
| TPCR           | 0.2x² - 3.25x + 49.94 | 5.28 | 0.2x² - 3.25x + 40.22 |
| MPCR when x ≥ x₀ | 0.2x² - 2.25x + 26.44 | 2.53 | 0.2x² - 2.25x + 28.97 |
| MPCR when x < x₀ | 0.45x² - 7x + 49 | 0.125x² - 3.5x + 24.5 | 0.575x² - 10.5x + 73.5 |

Table 5 Price cap’s impacts on the downstream monopolist’s profit, the consumers surplus and the social total welfare under private information

| Scenario       | U₁**    | π₁**    | S₁**    | W₁**    |
|----------------|---------|---------|---------|---------|
| RPCR           | 22.53   | 0.7x² - 12x + 51.3 | 0.5x² - 12x + 72 | 1.2x² - 24x + 145.83 |
| TPCR           | 21.89   | 0.2x² - 3.25x + 13.05 | 5.28 | 0.2x² - 3.25x + 40.22 |
| MPCR when x ≥ x₁ | 22.52   | 0.2x² - 2.25x + 2.93 | 2.53 | 0.2x² - 2.25x + 27.98 |
| MPCR when x < x₁ | 22.52   | 0.7x² - 12x + 51.3 | 0.5x² - 12x + 72 | 1.2x² - 24x + 145.83 |

Fig. 1 shows that the private information’s impact on the optimal contract under no PCR. On the one hand, to limit information rents, the retail price p₁ under private information is reduced, i.e., p* > p₁*, and moreover, p* and p₁* are increasing with respective to x, thereby, p* and p₁* are upward to maximum at x, respectively, and p*(x) = p₁*(x), that is to say, the retail price is not distorted at
The retail price under full information

\( p_1^{\text{TPCR}} \) \( p_1^{\text{RPCR}} \) \( p_1^{\text{MPCR}} \)

The transfer payment under full information

\( t_1^{\text{RPCR}} \) \( t_1^{\text{TPCR}} \) \( t_1^{\text{MPCR}} \)

Fig. 2A presents the change of the optimal retail price under full information under different PCR. From Fig 2B, we can observe that with TPCR, \( t^* \) is strictly more than that with RPCR, the reason is that TPCR makes the retail price lowest, which brings about the most demand quantity, therefore, the downstream monopolist must pay most payment to the upstream monopolist. In the case of MPCR, when \( x \geq x_0 \), the reduction of \( p^* \) results in the increase of \( t^* \), whereas when \( x < x_0 \), \( t^* \) is not influenced.

Fig. 3 shows that how do the participants’ profits change as the price cap changes in the case of full information. In the case of TPCR, the downstream monopolist’s profit is least, nonetheless, the consumers’ surplus and the social total welfare are up to maximum, respectively. And, it’s worth noting that even when PCR happens, both the downstream monopolist’s profit and the social total welfare do not alter monotonicity in \( x \), whereas the consumers’ surplus keeps invariable. The reason is that as long as the retail price is given, the demand quantity is determined, i.e., both the retail price and demand quantity will not alter with the type \( x \), which generates the invariable consumers’ surplus.
Fig. 3 characters that how does PCR affect the optimal contract under private information, in which, the retail price's change is the same as that under full information, it is worth noticing that the difference of the transfer payments between under private information and that under full information. To see this in more retail, as we know, the retail price is equal to the price cap under binding price cap constraint. Furthermore, under full information, the transfer payment is determined by cost subsidy, which is dual, i.e., output and production efficiency $x$. Thus, the transfer payment falls as the type $x$ becomes high. Nevertheless, under private information, the transfer payment $t^*_1$ is determined by cost subsidy and information rents. Further, the retail price is fixed by price cap. On the one hand,
the production cost is determined. On the other hand, as $x$ becomes high, the increase of information rents is equal to the cost’s reduction resulting in the invariable transfer payment.

Fig. 5 presents PCR’s impacts on the participants’ profits in the case of private information. With TPCR, the downstream monopolist’s expected profit, the consumers’ surplus and the social total welfare are strictly less than, more than, more than that with MPCR and RPCR, respectively. With MPCR, there exists a jump point $x_1$, when $x = x_1$, the upstream monopolist’s profit is zero, i.e., the downstream monopolist deprives the upstream monopolist’s all information rents. And, MPCR induces the social total welfare to be worse off, which implies that the merits of MPCR for the social total welfare are less clear cut.

6. Conclusions. This paper explores a supply chain contracting with PCR, and analyzes three different PCR’s and private information’s impacts on the optimal contract. We obtain the following conclusions:

First, on the one hand, under no PCR, private information makes both the retail price and the transfer payment distorted. Meanwhile, the upstream monopolist benefits from private information. On the other hand, under PCR, except for the distortion effect incurred by private information, TPCR has no impact on the optimal contract in a certain interval, that is to say, private information can weaken PCR’s impact on the optimal contract.

Second, when the upstream monopolist’s type is full information, if using TPCR or MPCR, it will make the retail price lower and the transfer payment higher than those of without PCR, and meanwhile, sacrifice the downstream monopolist’s profit, but increase both the consumers’ surplus and the social total welfare. However,
when the upstream monopolist’s type is private information, if the regulator implements MPCR, it will realize semi-separating equilibria contract; if the regulator implements TPCR, it will raise pooling equilibria contract. And MPCR and TPCR always result in lower profits of the downstream monopolist and the upstream monopolist than that without PCR. However, the consumers’ surplus always benefit from PCR. As for the social total welfare, the influences of PCR are complicated. Only in the case of TPCR, the social total welfare is higher than that without PCR. In addition, by comparing the optimal contract under no PCR with that under PCR, the optimal retail price keeps invariable in a certain interval with respect to the upstream monopolist’s type, meaning that PCR can dampen the effect of private information on the optimal contract.

In this paper, we only consider PCR as an exogenous variable. However, if regarding PCR as a decision of the regulator whose objective is to maximize the social total welfare, the downstream monopolist’s contracting will be influenced by the constraint of maximizing social total welfare. In particular, the retail price determined by the downstream monopolist must ensure that the social total welfare is maximized. In other words, the regulator allows the downstream monopolist to choose any retail price that makes the social total welfare maximized, such consideration will affect the downstream monopolist’s contracting. We expect that the regulator regards the retail price maximizing the social total welfare as a price cap.

In the future work, we can incorporate dual PCR into two monopolists’ pricing problem. For instance, if the upstream monopolist has pricing discretion, the regulator may set price cap to limit the upstream monopolist’s production pricing, and to pursue the maximization of social total welfare, the regulator also sets price cap for the downstream monopolist’s production pricing, then examine different price cap’s effects on the optimal contract.

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Appendix. Proof of Theorem 3.2

Proof. 1) At the optimum, the participation constraint of Model (5) is binding; otherwise, the downstream monopolist can decrease $t$ until it binds, i.e.,

$$\Pi^*(p, t, x) = 0.$$  \(42\)

Then we can obtain

$$t = C(Q(p), x).$$  \(43\)

Substituting $t$ into the objective function of Model (5) yields

$$U(p, t, x) = (p - c)Q(p) - C(Q(p), x).$$  \(44\)

In the following, we will verify the concavity of the downstream monopolist’s objective function \(44\). Since the second order derivative of $U(p, t; x)$ with respect to $p$ is

$$Q'(p) + \left(1 - \frac{\partial^2 C(Q(p), x)}{\partial Q^2}Q'(p)\right)Q'(p) + \left(p - c - \frac{\partial C(Q(p), x)}{\partial Q}\right)Q''(p),$$
from the above expression, we can see that it is necessary to judge whether the expression \( p - c - \frac{\partial C(Q(p), x)}{\partial Q} \) is positive or negative. However, note that

\[
p - c - \frac{\partial C(Q(p), x)}{\partial Q} \geq 0. \tag{45}
\]

Because, under full information, the downstream monopolist will undertake the upstream monopolist’s cost and his production cost. Moreover, the retail price is no less than marginal cost, then it is clearly that \( p - c - \frac{\partial C(Q(p), x)}{\partial Q} \geq 0 \). In addition, it follows from \( Q'(p) < 0, Q''(p) < 0 \) and \( \frac{\partial^2 C(Q(p), x)}{\partial q^2} \geq 0 \) that \( U(p, t, x) \) is concave with respect to \( p \). Therefore, the optimal retail price \( p^* \) satisfies the first-order condition

\[
Q(p^*(x)) + \left( p^*(x) - c - \frac{\partial C(Q(p^*(x)), x)}{\partial Q} \right) Q'(p^*(x)) = 0.
\]

By Eq. (43), the optimal transfer payment \( t^*(x) = C(Q(p^*(x)), x) \).

2) It is evidently shown by Eqs. (44), (42), (3) and (4). The proof is complete. \( \square \)

**Proof of Proposition 1**

**Proof.** From Theorem 3.2 we can see that both the optimal retail price \( p^* \) and the transfer payment \( t^* \) are implicit functions in \( x \), respectively, therefore, differentiating Eqs. (6) and (7) with respect to \( x \), respectively, we can obtain that

\[
2Q'(p) + \left( p(x) - c - \frac{\partial C(Q(p), x)}{\partial Q} \right) Q''(p(x)) p'(x) = \frac{\partial^2 C(Q(p), x)}{\partial x \partial x} Q'(p),
\]

and

\[
t'(x) = \frac{\partial C(Q(p), x)}{\partial Q} Q'(p) p'(x) + \frac{\partial C(Q(p), x)}{\partial x}. \tag{46}
\]

In the case of complements, according to \( Q'(p) < 0, Q''(p) < 0 \), \( p - c - \frac{\partial C(Q(p), x)}{\partial Q} \geq 0 \), \( \frac{\partial^2 C(Q(p), x)}{\partial x \partial Q} \geq 0 \), it is easy to know \( p'(x) \geq 0 \). Using the same method, we have \( t'(x) \leq 0 \). The proof of Proposition 1 is complete. \( \square \)

**Proof of Theorem 3.3**

**Proof.** According to the price cap constraint in Model (5), it is shown that \( p^{**} = \min\{p^*, p\} \), and combining Proposition 1 and Eq. (7) yields Theorem 3.3 immediately. \( \square \)

**Proof of Proposition 2**

**Proof.** 1) It is immediately obtained that \( p^* \geq p^{**} \) from Theorem 3.3. Since \( t = C(Q(p), x) \), differentiating with respect to \( p \) yields \( \frac{\partial C(Q(p), x)}{\partial p} = \frac{\partial C(Q(p), x)}{\partial Q} Q'(p) < 0 \), that is, the transfer payment is decreasing in \( p \), hence, \( t^* \leq t^{**} \). The proof is complete.

2) Since increasing a price cap constraint condition will shrink the feasible region, then Inequality (17) can be obtained, immediately.

The consumers’ surplus under no PCR \( S^*(p^*(x), t^*(x); x) = V(Q(p^*(x))) - p^*(x)Q(p^*(x)) \), comparing it with PCR and without PCR, it is clearly that

if \( p \geq p^*(x), \forall x \in [x, \bar{x}] \), we have \( S^{**}(p^{**}(x), t^{**}(x), x) = S^*(p^*(x), t^*(x), x) \);

if \( p < p^*(x), \forall x \in [x, \bar{x}] \), then \( S^{**}(p^{**}(x), t^{**}(x), x) > S^*(p^*(x), t^*(x), x) \);
In the case of complements, if \( \exists x_0 \in \{x, \overline{x}\} \), such that \( p^*(x_0) = \overline{p} \), \( x_0 \leq x \leq \overline{x} \), \( S^{**}(p^*(x), t^*(x), x) \geq S^*(p^*(x), t^*(x), x) \); when \( \underline{x} \leq x < x_0 \), \( S^{**}(p^*(x), t^*(x), x) = S^*(p^*(x), t^*(x), x) \). Therefore, \( S^{**}(p^*(x), t^*(x), x) \geq S^*(p^*(x), t^*(x), x) \).

Through the same method, we can show Inequality \((20)\) holds. The proof is complete.

**Proof of Theorem 4.1**

Proof. 1) We firstly will show that the incentive constraint \((23)\) can be written as

\[
\frac{dt(x)}{dx} - \frac{\partial C(Q(p(x)), x)}{\partial Q} \frac{dQ(p(x))}{dp} \frac{dp(x)}{dx} = 0, \forall x \in \{x, \overline{x}\}, \tag{47}
\]

and

\[
\frac{dp(x)}{dx} \geq 0, \forall x \in \{x, \overline{x}\}. \tag{48}
\]

Let \( L(x, \hat{x}) = t(\hat{x}) - C(Q(p(\hat{x}), x) \), which denotes the profit of the upstream monopolist when her type is \( x \) but she misreports it as \( \hat{x} \), i.e., the upstream monopolist chooses the contract \((p(\hat{x}), t(\hat{x}))\), where \( x, \hat{x} \in \{x, \overline{x}\} \) and \( x \neq \hat{x} \). Thus, for any given \( x \), Inequality \((23)\) can be written as

\[
L(x, x) \geq L(x, \hat{x}), \forall \hat{x} \in \{x, \overline{x}\}.
\]

That means that \( L(x, \hat{x}) \) obtains its maximal value at \((x, x)\), i.e., the upstream monopolist who knows its type \( x \) has no incentive to misreport its type as \( \hat{x} \), \( \hat{x} \neq x \). Thus, \( L(x, \hat{x}) \) satisfies the first-order condition \( \frac{\partial L(x, \hat{x})}{\partial \hat{x}} \bigg|_{\hat{x} = x} = 0 \) and the second-order condition \( \frac{\partial^2 L(x, \hat{x})}{\partial \hat{x}^2} \bigg|_{\hat{x} = x} \leq 0 \). It follows from the first-order condition that

\[
\frac{dt(x)}{dx} - \frac{\partial C(Q(p(x)), x)}{\partial Q} \frac{dQ(p(x))}{dp} \frac{dp(x)}{dx} = 0. \tag{49}
\]

Differentiating \((49)\) with respect to \( x \) yields

\[
\frac{d^2t(x)}{dx^2} - \frac{\partial^2 C(Q(p), x)}{\partial Q^2} \left( \frac{dQ(p)}{dp} \frac{dp(x)}{dx} \right)^2 - \frac{\partial C(Q(p), x)}{\partial Q} \frac{d^2Q(p)}{dx^2} \left( \frac{dp(x)}{dx} \right)^2
- \frac{\partial C(Q(p), x)}{\partial Q} \frac{dQ(p)}{dp} \frac{d^2p(x)}{dx^2} \frac{\partial^2 C(Q(p), x)}{\partial Q \partial x} \frac{dQ(p)}{dp} \frac{dp(x)}{dx} = 0. \tag{50}
\]

By the second-order condition, we can obtain

\[
\frac{d^2t(x)}{dx^2} - \frac{\partial^2 C(Q(p(x)), x)}{\partial Q^2} \left( \frac{dQ(p(x))}{dx} \frac{dp(x)}{dx} \right)^2
- \frac{\partial C(Q(p(x)), x)}{\partial Q} \frac{d^2Q(p(x))}{dx^2} \left( \frac{dp(x)}{dx} \right)^2
- \frac{\partial C(Q(p(x)), x)}{\partial Q} \frac{dQ(p(x))}{dp} \frac{d^2p(x)}{dx^2} \leq 0. \tag{51}
\]

Applying Inequality \((51)\) to Eq. \((50)\), together with \( \frac{\partial^2 C(Q(p(x)), x)}{\partial x \partial Q} \geq 0 \) and \( \frac{dQ(p)}{dp} < 0 \), yields

\[
\frac{dp(x)}{dx} \geq 0, \forall x \in \{x, \overline{x}\}.
\]

That is, \((23) \Rightarrow (47) \text{ and } (48)\).
On the other hand, by \( \frac{dp(x)}{dx} \geq 0, \frac{\partial^2 C(Q(p(x)), x)}{\partial x \partial Q} \geq 0 \) and \( \frac{dQ(p)}{dp} < 0 \), integrating \( (47) \) yields
\[
\begin{align*}
t(x) - t(x^*) &= \int_x^{x^*} \frac{\partial C(Q(p(s)), s)}{\partial Q} \frac{dQ(p(s))}{dp} \frac{dp(s)}{ds} ds \\
&\geq \int_x^{x^*} \frac{\partial C(Q(p(s)), x)}{\partial Q} \frac{dQ(p(s))}{dp} \frac{dp(s)}{ds} ds \\
&= C(Q(p(x)), x) - C(Q(p(x^*)), x),
\end{align*}
\]
when \( x > x^* ); and
\[
\begin{align*}
t(x^*) - t(x) &= \int_x^{x^*} \frac{\partial C(Q(p(s)), s)}{\partial Q} \frac{dQ(p(s))}{dp} \frac{dp(s)}{ds} ds \\
&\leq \int_x^{x^*} \frac{\partial C(Q(p(s)), x)}{\partial Q} \frac{dQ(p(s))}{dp} \frac{dp(s)}{ds} ds \\
&= C(Q(p(x^*)), x) - C(Q(p(x)), x),
\end{align*}
\]
when \( x^* > x \). Therefore, the incentive constraint \( (23) \) is satisfied. That is, \( (47) \) and \( (48) \Rightarrow (23) \).

2) Secondly, we will verify that the participation constraint \( (24) \) can be written as
\[
t(x) = C(Q(p(x)), x).
\]

The profit of the upstream monopolist
\[
\Pi(p(x), t(x), x) = t(x) - C(Q(p(x)), x),
\]
then,
\[
\begin{align*}
\frac{d\Pi(p(x), t(x), x)}{dx} &= \frac{dt(x)}{dx} - \frac{\partial C(Q(p(x)), x)}{\partial Q} \frac{dQ(p(x))}{dx} \frac{dp(x)}{dx} - \frac{\partial C(Q(p(x)), x)}{\partial x}.
\end{align*}
\]

It follows from Eq. \( (47) \) and \( \frac{\partial C(Q(p(x)), x)}{\partial x} \) \( \leq 0 \) that
\[
\frac{d\Pi(p(x), t(x), x)}{dx} = - \frac{\partial C(Q(p(x)), x)}{\partial x} \geq 0,
\]
which means that \( \Pi(p(x), t(x), x) \) is increasing with respect to \( x \). Consequently, the participation constraint \( (24) \) is equivalent to
\[
\Pi(p(x), t(x), x) = t(x) - C(Q(p(x)), x) \geq 0.
\]

In fact, the constraint \( (59) \) is binding under the optimal contract. Since for any feasible contract \( (p(\cdot), t(\cdot)) \) of Model \( (26) \), a new contract \( (p(\cdot), t^*(\cdot)) \) can be established, where \( t^*(x) = C(Q(p(x)), x) \) and
\[
\frac{dt^*(x)}{dx} = \frac{dt(x)}{dx}.
\]

It is easy to testify that \( (p(\cdot), t^*(\cdot)) \) is also feasible for Model \( (26) \) and \( t^*(x) \leq t(x) \) for all \( x \in [x, \bar{x}] \). Since \( \frac{dt}{dx} = -1 \), i.e., the downstream monopolist’s profit is decreasing with respect to \( t \), hence,
\[
U(p(x), t^*(x), x) \geq U(p(x), t(x), x),
\]
which means that the downstream monopolist will choose the least transfer payment satisfying the participation constraint. Thus, an optimal contract should satisfy
\[
t(x) = C(Q(p(x)), x).
\]
3) Thirdly, the objective function of Model (26) can be written as
\[ \int_{\mathbb{E}} \left\{ (p(x) - c)Q(p(x)) - C(Q(p(x)), x) + \left( \frac{1 - F(x)}{f(x)} \right) \frac{\partial C(Q(p(x)), x)}{\partial x} \right\} f(x)dx. \]

Specifically, integrating (47) yields
\[ \int_{\mathbb{E}} \left\{ (p(x) - c)Q(p(x)) - C(Q(p(x)), x) + \left( \frac{1 - F(x)}{f(x)} \right) \frac{\partial C(Q(p(x)), x)}{\partial x} \right\} f(x)dx. \]

Therefore,
\[ t(x) = t(x) - C(Q(p(x)), x) + C(Q(p(x)), x) - \int_{\mathbb{E}} \frac{\partial C(Q(p(s)), s)}{\partial s} ds. \]

By Eq. (56), \( t(x) = C(Q(p(x)), x) \), the transfer payment
\[ t(x) = C(Q(p(x)), x) - \int_{\mathbb{E}} \frac{\partial C(Q(p(s)), s)}{\partial s} ds. \]

Substituting \( t(x) \) into the profit function of the downstream monopolist \( U(p(x), t(x)); x = (p(x) - c)Q(p(x)) - t(x) \) yields
\[ U(p(x), t(x), x) = (p(x) - c)Q(p(x)) - C(Q(p(x)), x) + \int_{\mathbb{E}} \frac{\partial C(Q(p(s)), s)}{\partial s} ds. \]

It follows from Eq. (25) that the downstream monopolist’s expected profit can be expressed as
\[ E[U(p(X), t(X), X)] = \int_{\mathbb{E}} \left\{ (p(x) - c)Q(p(x)) - C(Q(p(x)), x) \right. \]
\[ \left. + \int_{\mathbb{E}} \frac{\partial C(Q(p(s)), s)}{\partial s} ds \right\} f(x)dx. \]

Integrating (61) by parts yields
\[ \int_{\mathbb{E}} \left\{ (p(x) - c)Q(p(x)) - C(Q(p(x)), x) + \left( \frac{1 - F(x)}{f(x)} \right) \frac{\partial C(Q(p(x)), x)}{\partial x} \right\} f(x)dx. \]

It follows from 1), 2), 3) that the first result of Theorem 4.1 holds. The proof is complete.

**Proof of Proposition 3**

**Proof.** 1) First, the concavity of the objective function in Model (27) is proved. Since the second variation of \( E[U(p(X), t(X), X)] \) with respect to \( p \), i.e., \( \delta^2 E[U(p \]
\( (X, t(X), X) \) is
\[
\int_{\mathbb{R}} \left\{ 2Q'(p)+\left( p(x)-c-\frac{\partial C(Q(p(x), x))}{\partial Q} \right) Q''(p)-\frac{\partial^2 C(Q(p(x), x))}{\partial Q^2} \left( Q'(p) \right)^2 \right. \\
+ \left( \frac{1-F(x)}{f(x)} \right) \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q^2} \left( Q'(p) \right)^2 \\
\left. + \left( \frac{1-F(x)}{f(x)} \right) \frac{\partial^2 C(Q(p(x), x))}{\partial x \partial Q} Q''(p) \right\} \left( \delta p \right)^2 f(x) dx,
\]
according to \( Q'(p) < 0, \ Q''(p) < 0, \ \frac{\partial^2 C(Q(x), x)}{\partial Q^2} \geq 0, \ \frac{\partial^3 C(Q(x), x)}{\partial x \partial Q^2} \geq 0, \ \frac{\partial^3 C(Q(x), x)}{\partial x \partial Q} \leq 0 \) and \( p - c - \frac{\partial C(Q(p(x), x))}{\partial Q} \geq 0, \ \frac{\partial^2 C(Q(p(x), x))}{\partial x \partial Q} \geq 0 \), we can obtain \( \left( \delta^2 E[U(p(X), t(X), X)] \right) < 0, \) i.e., \( E[U(p(X), t(X), X)] \) is concave with respect to \( p \). In order to maximize the downstream monopolist’s profit, it follows from the first-order condition, i.e., \( \delta E[U(p(X), t(X), X)] = 0 \), that
\[ Q(p(x)) + \left( p(x)-c-\frac{\partial C(Q(p(x), x))}{\partial Q} \right) \frac{dQ(p(x))}{dp} \\
+ \left( \frac{1-F(x)}{f(x)} \right) \frac{\partial^2 C(Q(p(x), x))}{\partial x \partial Q} \frac{dQ(p(x))}{dp} = 0. \]  

(62)

On the other hand, Eq. (28) also means that the constraint \( \frac{dp(x)}{dx} \geq 0 \) holds. Since the derivation of (28) with respect to \( x \) is
\[
\left[ \frac{dQ(p)}{dp} - \frac{\partial^2 C(Q(p(x), x))}{\partial Q^2} \left( \frac{dQ(p)}{dp} \right)^2 + \left( p-c-\frac{\partial C(Q(p(x), x))}{\partial Q} \right) \frac{\partial^2 Q(p)}{\partial p^2} \right] \frac{dp(x)}{dx} \\
+ \left( \frac{1-F(x)}{f(x)} \right) \left[ \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q^2} \left( \frac{dQ(p)}{dp} \right)^2 + \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q} \frac{\partial^2 Q(p)}{\partial p^2} \right] \frac{dp(x)}{dx} \\
= \frac{\partial^2 C(Q(p(x), x))}{\partial Q \partial x} \frac{dQ(p)}{dp} - \frac{1-F(x)}{f(x)} \frac{\partial^3 C(Q(p(x), x))}{\partial x^2 \partial Q} \frac{dQ(p)}{dp} \\
- \frac{d}{dx} \left( \frac{1-F(x)}{f(x)} \right) \frac{\partial^2 C(Q(p(x), x))}{\partial x \partial Q} \frac{dQ(p)}{dp},
\]
by \( \frac{d}{dx} \left( \frac{f(x)}{1-F(x)} \right) \geq 0, \) we can obtain \( \frac{d}{dx} \left( \frac{1-F(x)}{f(x)} \right) \leq 0 \), thus, it follows from \( \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q} \leq 0, \ \frac{\partial^2 C(Q(p(x), x))}{\partial x \partial Q^2} \geq 0, \ \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q} \leq 0, \) \( \frac{\partial^3 C(Q(p(x), x))}{\partial x \partial Q^2} > 0, \) \( p - c - \frac{\partial C(Q(p(x), x))}{\partial Q} \geq 0 \) that \( \frac{dp(x)}{dx} \geq 0, \forall x \in [x, \mathbb{R}] \). Therefore, the proof of \( p^*_1(x) \) satisfying Eq. (28) is complete. By Eq. (60), we immediately obtain Eq. (29) holds.

2) It’s evidently shown by Eqs. (61), (11), (60) and (3). The proof of Proposition 3 is complete.

**Proof of Theorem 4.2**

*Proof.* By the price cap constraint in Model (27), it is shown that \( p^*_1 = \min\{p^*_1, p\} \), and combining Inequality (48), Eqs. (28) and (29), we immediately obtain the results of Theorem 4.2.

**Proof of Proposition 4**
Proof. 1) Differentiating $Q(p) + \left( p - c - \frac{\partial C(Q(p), x)}{\partial q} \right) \frac{dq}{dp}$ with respect to $p$ yields

$$Q'(p) + \left( p - c - \frac{\partial C(Q(p), x)}{\partial q} \right) Q''(p) + \left( 1 - \frac{\partial^2 C(Q(p), x)}{\partial q^2} Q'(p) \right) Q'(p) < 0,$$

By comparing Eq. (6) with Eq. (28), we can obtain immediately $p^*_1 \leq p^*$.

For the first term of Eq. (29), i.e., the upstream monopolist’s production cost $C(Q(p), x)$, taking the derivative with respect to $p$, we can obtain that $\frac{\partial C(Q(p), x)}{\partial p} Q'(p) < 0$, that is to say, the production cost is decreasing in $p$, furthermore, it follows from Eqs. (10), (29) and (30) that Inequality (37) holds.

2) It follows from Eqs. (10), (31), together with the fact the consumers’ surplus is decreasing in the retail price, that $S_1^*(p_1^*, t_1^*, x) \geq S^*(p^*, t^*, x)$. From Eqs. (9) and (30), it is clearly that $\Pi_1^*(p_1^*, t_1^*, x) \geq \Pi^*(p^*, t^*, x)$. The proof of Proposition 4 is complete.

Proof of Proposition 5

Proof. It can be easily derived by the results of Theorems 2 and 4.

Proof of Proposition 6

Proof. i) It is immediately obtained that $p_1^* \geq p_1^{**}$ from Theorem 4.2.

ii) Because increasing a constraint condition will shrink feasible region, we can get

$$E[U_1^*(p_1^*, t_1^*, x)] \geq E[U_1^{**}(p_1^{**}, t_1^{**}, x)].$$

iii) By Eq. (30), the upstream monopolist’s profit $\Pi_1(p, t; x) = -\int_x^p \frac{\partial C(Q(p), s)}{\partial s} ds$, differentiating the above expression in $p$ yields $-\int_x^p \frac{\partial^2 C(Q(p), s)}{\partial s^2} Q'(p) > 0$, this implies that upstream monopolist’s profit is nondecreasing with respect to $p$. From Eqs. (30), we can get

if $p \geq p_1^*(x), \forall x \in [x, \bar{x}]$, it is clearly that $\Pi_1^*(p_1^*(x), t_1^*(x), x) = \Pi_1^*(p_1^{**}(x), t_1^{**}(x), x)$.

if $p < p_1^*(x), \forall x \in [x, \bar{x}]$, $\Pi_1^*(p_1^*(x), t_1^*(x), x) > \Pi_1^*(p_1^{**}(x), t_1^{**}(x), x)$.

if $\exists x_1 \in [x, \bar{x}]$, such that $p^*(x_0) = \bar{p}$, when $x_1 \leq x \leq \bar{x}$, $\Pi_1^*(p_1^*(x), t_1^*(x), x)$

$\Pi_1^*(p_1^{**}(x), t_1^{**}(x), x)$, when $\bar{p} < x < x_1$, $\Pi_1^*(p_1^*(x), t_1^*(x), x) = \Pi_1^*(p_1^{**}(x), t_1^{**}(x), x)$.

Therefore, we conclude that $\Pi_1^*(p_1^*, t_1^*, x) \geq \Pi_1^*(p_1^{**}, t_1^{**}, x)$. The results of the case of substitutes can be presented similarly.

iv) Similar proof of Inequality (41) is given. The proof is complete.

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