Flexural Strength of Functionally Graded Nanotube Reinforced Sandwich Spherical Panel

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Abstract: The flexural behaviour of the functionally graded sandwich spherical panel under uniform thermal environment has been investigated in the present work. The face sheets of the sandwich structure are made by the functionally graded carbon nanotube reinforced material and the core face is made by the isotropic and homogeneous material. The material properties of both the fiber and matrix are assumed to be temperature dependent. The sandwich panel model is developed in the framework of the first order shear deformation theory and the governing equation of motion is derived using the variational principle. For the discretization purpose a suitable shell element has been employed from the ANSYS library and the responses are computed using a parametric design language (APDL) coding. The performance and accuracy of the developed model have been established through the convergence and validation by comparing the obtained results with previously published results. Finally, the influence of different geometrical parameters and material properties on the flexural behaviour of the sandwich spherical panel in thermal environment has been investigated through various numerical illustrations and discussed in details.

1. Introduction

Conventional laminated composites consist of various layers of lamina that are homogenous and are held together to achieve enhanced mechanical properties. Thus, they are light in weight and at the same time possess higher strength/stiffness to weight ratio. However, since there is abrupt changes in material properties in between different layers of lamina, high inter-laminar stresses are induced those cause failure of structure due to delamination. This failure is more common when these structures serve under severe thermal load. To overcome this problem, Functionally Graded Materials (FGMs) are used in which the properties of materials are varied smoothly and continuously along thickness direction. In general, the FGMs are constructed by ceramic and metal, in such a way that properties are varied by changing volume fraction of constituent materials along thickness of plate. Owing to excellent thermal resistance property FGMs are widely used in various high performance engineering structures/structural components as in aerospace, automotive, marine and nuclear industries.

Recently, in the family of advanced materials, carbon nanotubes (CNTs) are very much appreciated because of its distinct mechanical, thermal and electrical properties in comparison to other existing advanced materials. The CNT Reinforced Functionally Graded materials (FGCNTs) are preferably used as plates, beams or shells, in advanced structural components. The sandwich type construction has also become much more attractive due to their outstanding properties. Therefore, it is highly essential to understand the mechanical behavior of the sandwich structures made up of FGCNT for their better design in real life application. Many researchers have attempted to investigate the linear and nonlinear behaviour of CNT reinforced FG sandwich plates and shells numerically/analytically by...
using various existing and refined theories. In this regard, the first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) are more popular in comparison to refined/layer wise theories due to simplicity in formulation [1]. Zenkour [2] presented the bending analysis of simply supported FG sandwich plates under sinusoidal loading using sinusoidal, third-order, first-order and classical shear deformation theories. Shen [3] studied the nonlinear bending behavior of FGCNT reinforced composite (FGCNTRC) plates in thermal environments using the HSDT and von-Karman nonlinear kinematics. Ke et al. investigated the nonlinear free vibration [4] and dynamic stability [5] behaviour of FGCNTRC beams in the framework of the Timoshenko beam theory and von-Karman type strains. Analytical solutions for the nonlinear bending and free vibration responses of simply supported CNTRC [6] and sandwich plates [7] in thermal environment are presented by Wang and Shen using the HSDT and general von-Karman type equations and considering temperature dependent material properties. Zhu et al. [8] presented the static and free vibration analysis of CNTRC plates using the FSDT and finite element method (FEM). Lei et al. [9] studied the nonlinear deflection behaviour of FGCNTRC plates using element-free kp-Ritz method. The formulation is based on the FSDT mid-plane kinematics and von-Karman nonlinearity. Zhang et al. [10] studied the flexural strength of FGCNTRC cylindrical panels in the frame work of the FSDT. The static and dynamic responses of sandwich plates with CNT reinforced face sheets have been investigated by Natarajan et al. [11] using the HSDT mid-plane kinematics. Mehar et al. [12] examined the nonlinear free vibration of CNTRC flat panel under uniform thermal environment using the HSDT and Green-Lagrange geometrical nonlinearity.

It is evident from the brief review that, many efforts have already been made in past on the theoretical developments for the numerical or analytical solutions of linear/nonlinear flexural behaviour of CNT reinforced FGM sandwich structures. To the best of the authors’ knowledge, no work has been reported in the open literature on flexural behaviour of FGCNT reinforced sandwich spherical plate. The aim of the present investigation is to study the bending behavior of sandwich spherical panels with FGCNT face sheets. In order to do so, a simulation model for the FG sandwich panel has been developed using APDL code and discretized using Shell 281 element from ANSYS elemental library. Numerical illustrations are presented to show the effect of various parameters on the flexural response of FGCNT reinforced sandwich spherical panel and discussed in detail.

2. Methodology
In the present study, two types of CNT reinforced sandwich panels have been considered with length “a”, width “b”, thickness “h” and principal radius of curvature “R”, as shown in Figure 1 (a) and (b). The notations UD and FG stands for uniform distribution and functionally graded distribution of the carbon nanotubes along the thickness direction of the sandwich panel. Due to convenience, the Mori-Tanaka scheme [8] is employed to estimate the material properties of two-phase nano-composites (mixture of CNTs and isotropic polymer) by using CNT efficiency parameter. Thus, the material properties can be written as

\[ E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \]  
\[ \frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^m} \]  
\[ \frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^m} \]

where, \( E_{11}^{CNT}, E_{22}^{CNT} \) and \( G_{12}^{CNT} \) represents the Young’s moduli and shear modulus of single walled CNTs, respectively and \( E^m \) and \( G^m \) indicate the corresponding properties of the isotropic matrix.

The UD and FG carbon nanotubes reinforcement composite are related as follows:

\[ V_{CNT}(z) = 2 \left( \frac{2|z|}{h} \right) V_{CNT}^{*} \]  

(UD) \[ (FG) \]
where, 

\[ V_{\text{CNT}}^* = \frac{W_{\text{CNT}}}{W_{\text{CNT}} + (\rho_{\text{CNT}}^n - \rho_{\text{CNT}}^m)W_{\text{CNT}}} \]

\[ W_{\text{CNT}} = \text{mass fraction of the carbon nanotube in the composite panel} \]
\[ \rho_{\text{CNT}}^n = \text{density of matrix} \]
\[ \rho_{\text{CNT}}^m = \text{density of carbon nanotube} \]

\( (6) \)

**Figure 1.** Configuration and gradation of FGCNT sandwich spherical panel (a) UD (b) FG

In order to compute the desired responses a simulation model for the sandwich spherical panel has been developed using APDL code in ANSYS 15.0 environment. In ANSYS mechanical APDL, various element types are available for modeling of layered structures. For the present analysis, Shell 281 element has been taken. Shell 281 is known for its robustness and suitability for the analysis of thin to slightly thick shell structures. The considered element has total eight nodes having six degrees of freedom per each node say, translation along \( x, y, \) and \( z \) axes, and rotation about \( x, y, \) and \( z \) axes. This element gives satisfactory results for linear, large rotation, and/or large strain nonlinear solutions. It also accounts for load stiffness effects of distributed pressure. It frameworks and carry the solutions based on the FSDT and logarithmic strain and true stress measures. Thus, the displacements at any point in the panel \( u, v \) \( \text{and} \) \( w \) along \( x, y \) \( \text{and} \) \( z \) directions, respectively can be represented as:

\[ u(x, y, z) = u^0(x, y) + z\theta_x(x, y) \]
\[ v(x, y, z) = v^0(x, y) + z\theta_y(x, y) \]
\[ w(x, y, z) = w^0(x, y) + z\theta_z(x, y) \]

where, \( u^0, v^0 \) \( \text{and} \) \( w^0 \) are the displacements of any point in the mid-plane along \( x, y \) \( \text{and} \) \( z \) directions, respectively and \( \theta_x, \theta_y \) \( \text{and} \) \( \theta_z \) are the shear rotations. Figure 2 represents the geometrical parameters such as node location and element coordinate system for this element. The element is defined by shell section information and by eight nodes (I, J, K, L, M, N, O and P).

**Figure 2.** SHELL281 geometry (ANSYS15.0)
3. Results and Discussions

In this section, the flexural responses of FGCNT sandwich spherical panel under thermal environment have been investigated using the present simulation model. The material properties used for the computation purpose have been provided in Table 1. For the computation purpose PMMA and the single walled carbon nanotube (SWCNT) of armchair (10, 10) configuration are considered as the matrix material and the reinforcement phase, respectively. The effective properties and the effective parameters for the PMMA and SWCNT can be seen in [12]. The ability and efficacy of the present model has been established through the convergence and validation studies. Subsequently, various parametric studies have been provided to bring out their significance. If not stated otherwise, uniformly distributed mechanical load \( q_0 = 1 \text{MPa} \) and temperature of 300K is assumed throughout for the present study. To restrict the rigid body motion and to reduce the number of unknowns for finding the solution, the various boundary conditions used in the present analysis are given as:

1. All sides simply supported (SSSS)
   \[ v^0 = w^0 = \theta_x = \theta_y = \theta_z = 0 \text{ at } x = 0 \text{ and } a; \quad u^0 = w^0 = \theta_x = \theta_y = \theta_z = 0 \text{ at } y = 0 \text{ and } b. \]

2. All sides clamped (CCCC)
   \[ u^0 = v^0 = w^0 = \theta_x = \theta_y = \theta_z = 0 \text{ at both } x = 0 \text{ and } a; \quad y = 0 \text{ and } b. \]

3. All sides free (FFFF)
   \[ u^0 \neq v^0 \neq w^0 \neq \theta_x \neq \theta_y \neq \theta_z \neq 0 \text{ at both } x = 0 \text{ and } a; \quad y = 0 \text{ and } b. \]

The transverse central deflection of the FGCNT sandwich spherical panel in nondimensional form is expressed using the formula as:
\[ w_{\text{nondimensional}} = \frac{w_{\text{central}}}{h}. \]

3.1 Convergence Study

The convergence behavior of the present simulation model has been tested in this example. In order to do so, FGCNT reinforced square \((a/b=1)\) sandwich spherical panels \((h=2\text{mm} \ a/h =50 \text{ and } R/a=2, \ V_{\text{CNT}}=0.11)\) subjected to uniformly distributed load \((q_0 = 1\text{MPa})\) with both UD and FG grading condition and different core to face thickness ratios \((0.5, 1 \text{ and } 2)\) are considered. The nondimensional central deflections values computed over various mesh refinements under SSSS and CCCC support conditions are plotted in Figure 3 (a) and (b), respectively. It is clearly observed that the responses computed using the present simulation model are converging well with mesh refinement. Based on the convergence study a 10\times10 mesh has been chosen for determining the responses further throughout the present study.

![Convergence Study Graph](image-url)
3.2 Comparison Study
In order to validate the present simulation model, FGCNT reinforced sandwich flat panels as in [8] have been considered. This is because only flat panel examples are available in the concerned domain and the flat panels are considered to be the simplest form of the shell panels. The responses are computed for uniformly distributed load \( q_0 = 0.1 \text{MPa} \), diverse volume fraction of CNT \( V_{\text{CNT}} = 0.11, 0.14 \) and 0.17, SSSS and CCCC boundary conditions and considering both UD and FG gradient. For the sake of computation, the geometry, material properties, support conditions and nondimensional formula for the maximum deflection used are same as considered reference [8]. The comparison of results presented in Figure 4 depicts that the responses computed using the present model are as good as the reference values.

![Figure 3](image3.png)

**Figure 3.** Convergence behaviour of nondimensional central deflection \((w/h)\) of FGCNT reinforced sandwich spherical panel subjected to uniformly distributed load under (a) SSSS and (b) CCCC boundary condition.

![Figure 4](image4.png)

**Figure 4.** Comparison of nondimensional central deflection of FGCNT reinforced composite flat panel under uniform distributed load.
3.3 Numerical Illustrations

The stability and accuracy of the present developed model has been confirmed in the convergence and the comparison study. In this section, some more numerical investigations have been carried out to demonstrate the applicability of the proposed model and to bring out the effect of parameters (curvature ratio, thickness ratio, boundary conditions and core thickness to face thickness ratio) on the flexural behavior of FGCNT reinforced sandwich spherical panel under thermal environment.

3.3.1 Effect of curvature ratio (R/a) and temperature

It is a fact that membrane strength of the panel structure greatly depends on its curvature. The effect of curvature ratio on the bending behavior of FGCNT reinforced sandwich spherical panel in thermal environment is investigated in this example. Responses are computed for different curvature ratios \((R/a = 5, 20, 50 \text{ and } 100)\) with both UD and FG gradient composition separately for different volume fraction of CNT \((V_{\text{CNT}} = 0.11, 0.14 \text{ and } 0.17)\) and temperature \((T=300K, 500K \text{ and } 700K)\). The geometrical parameters and support condition considered as: \(a/b= 1, a/h=10\), core thickness to face thickness ratio = 0.5, SSSS boundary condition and the results are presented in Table 1. It is observed that the central deflection values increase with curvature ratio whereas decrease with increasing volume fraction of CNT. It is also worthy to note that as the temperature increases the maximum deflection value also increases.

| Temperature | 300K | 500K | 700K |
|-------------|------|------|------|
| \(V_{\text{CNT}}\) | UD | FG | UD | FG | UD | FG |
| 0.11        | 5  | 36.6750 x 10^{-3} | 31.6410 x 10^{-3} | 36.994 x 10^{-3} | 31.882 x 10^{-3} | 37.154 x 10^{-3} | 32.003 x 10^{-3} |
|             | 20 | 37.8450 x 10^{-3} | 32.5160 x 10^{-3} | 38.184 x 10^{-3} | 32.771 x 10^{-3} | 38.353 x 10^{-3} | 32.898 x 10^{-3} |
|             | 50 | 37.9085 x 10^{-3} | 32.635 x 10^{-3} | 38.248 x 10^{-3} | 32.819 x 10^{-3} | 38.418 x 10^{-3} | 32.947 x 10^{-3} |
|             | 100| 37.9175 x 10^{-3} | 32.5705 x 10^{-3} | 38.257 x 10^{-3} | 32.826 x 10^{-3} | 38.428 x 10^{-3} | 32.954 x 10^{-3} |
| 0.14        | 5  | 32.7145 x 10^{-3} | 28.6445 x 10^{-3} | 32.972 x 10^{-3} | 28.837 x 10^{-3} | 33.101 x 10^{-3} | 28.933 x 10^{-3} |
|             | 20 | 33.6790 x 10^{-3} | 29.3825 x 10^{-3} | 33.952 x 10^{-3} | 29.586 x 10^{-3} | 34.088 x 10^{-3} | 29.687 x 10^{-3} |
|             | 50 | 33.7315 x 10^{-3} | 29.4230 x 10^{-3} | 34.005 x 10^{-3} | 29.626 x 10^{-3} | 34.142 x 10^{-3} | 29.728 x 10^{-3} |
|             | 100| 33.7390 x 10^{-3} | 29.4285 x 10^{-3} | 34.013 x 10^{-3} | 29.632 x 10^{-3} | 34.150 x 10^{-3} | 29.734 x 10^{-3} |
| 0.17        | 5  | 25.9980 x 10^{-3} | 22.3620 x 10^{-3} | 26.204 x 10^{-3} | 22.516 x 10^{-3} | 26.307 x 10^{-3} | 22.594 x 10^{-3} |
|             | 20 | 26.8470 x 10^{-3} | 22.9990 x 10^{-3} | 27.066 x 10^{-3} | 23.162 x 10^{-3} | 27.175 x 10^{-3} | 23.244 x 10^{-3} |
|             | 50 | 26.8925 x 10^{-3} | 23.0330 x 10^{-3} | 27.112 x 10^{-3} | 23.197 x 10^{-3} | 27.222 x 10^{-3} | 23.279 x 10^{-3} |
|             | 100| 26.8985 x 10^{-3} | 23.0380 x 10^{-3} | 27.118 x 10^{-3} | 23.202 x 10^{-3} | 27.228 x 10^{-3} | 23.284 x 10^{-3} |

3.3.2 Effect of thickness ratio \( (a/h)\)

It is well known that the thickness ratio of the panel structures contributes significantly their strength and stiffness. In order to study the effect of thickness ratio on the flexural behavior of FGCNT reinforced sandwich spherical panel under uniform distributed load, all sides clamped square sandwich spherical panels \((R/a = 10\), core thickness to face thickness ratio = 1) with different volume fraction \((V_{\text{CNT}} = 0.11, 0.14 \text{ and } 0.17)\) and support condition \((a/b= 1.5, a/h=10)\) are considered. The responses are computed for various thickness ratio \((a/h=10, 20, 40 \text{ and } 80)\) and presented in Figure 5. It is observed that the nondimensional central deflection increases with increase in thickness ratio. It is generally expected as the stiffness of the structure decrease as the panel tends to become thin. It is also noted that the maximum deflection decreases with increasing volume fraction of the CNT.

3.3.3 Effect of support conditions

Strength and stiffness behavior of any structure/structural component largely depends on the support conditions which in turn affects their flexural responses. In this example, the effects of various support conditions \((\text{SSSS, CCCC, CSCS and CFCF})\) on the nondimensional maximum deflection of FGCNT reinforced sandwich spherical panel have been investigated. For the sake of computation, the geometrical parameters are taken as: \(R/a = 10\), core thickness to face thickness ratio = 1.5, \(a/b = 1\) and...
$a/h = 10$. Results are computed for both UD and FG gradient with three different CNT volume fractions ($V_{CNT} = 0.11, 0.15$ and $0.17$) and shown in Figure 6. It is noticed that the transverse central deflection values increase as the number of constraints at the support decrease.

![Figure 5. Effect of thickness ratio on nondimensional central deflection of functionally graded nanotube reinforced sandwich spherical panel](image)

![Figure 6. Nondimensional maximum deflection FGCNT sandwich spherical panel subjected to uniformly distributed load for various support conditions.](image)

3.3.4 Effect of core to face thickness ratio
In this example the flexural behavior of FGCNT reinforced sandwich spherical panel ($R/a = 40$, $a/h = 10$, $a/b = 1$ and under CSCS support condition) due to variation of the core thickness to face thickness ratio has been investigated. The nondimensional central deflection values are computed for four different core to face thickness ratio ($0, 0.5, 1, 1.5$ and $2$), three different CNT volume fractions ($V_{CNT} = 0.11, 0.14$ and $0.17$) and for UD and FG gradient separately. The results plotted in Figure 7 depict that in both UD and FG gradient the transverse central deflection values increase with increase in core to face thickness ratio, which is within the expected line.
4. Conclusion

In this article, the flexural behavior of FGCNT reinforced sandwich spherical panel of two different grading (UD and FG) under uniformly distributed load and thermal environment has been investigated. The sandwich panel is made of FGCNT reinforced face sheets and isotropic and homogeneous material core. The properties of the panel structure are considered to vary in the thickness direction based on the volume fraction of the CNT and are governed by Mori-Tanaka scheme. In addition to this the properties of the CNT is also assumed to be temperature dependent. For the computation of the desired responses a simulation model for the FGCNT sandwich panel has been developed in ANSYS 15.0 using APDL code. The convergence and validation study shows the efficacy of the present model in terms of performance and accuracy to obtain the desired responses. Few new examples are solved to investigate the effect of various parameters on the flexural behavior of the FGCNT reinforced sandwich spherical panel. Based on the parametric studies, it is observed that, the central deflection values decrease with increase in volume fraction of CNT. It is worthy to note that the stiffness of the spherical sandwich panel decreases with increase in temperature. For a particular value of CNT volume fraction, the FG grade show lesser deflection values as compared to UG grade. It is also seen that the transverse central deflection values decrease as the number of constraints at the support increases.

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