Screw-pitch effect and velocity oscillation of a domain wall in a ferromagnetic nanowire driven by spin-polarized current

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Abstract

We investigate the dynamics of a domain wall in a ferromagnetic nanowire with spin-transfer torque. The critical current condition is obtained analytically. Below the critical current, we get the static domain wall solution, which shows that the spin-polarized current cannot drive a domain wall moving continuously. In this case, the spin-transfer torque plays both the anti-precession and anti-damping roles, which counteracts not only the spin precession driven by the effective field but also Gilbert damping of the moment. Above the critical value, the dynamics of the domain wall exhibits the novel screw-pitch effect characterized by the temporal oscillation of domain wall velocity and width, respectively. Both the theoretical analysis and numerical simulation demonstrate that this novel phenomenon arises from the conjunctive action of Gilbert damping and spin-transfer torque. We also find that the roles of spin-transfer torque are completely opposite for the cases below and above the critical current.

(Some figures in this article are in colour only in the electronic version)
that spin-polarized current can cause DW motion, the current-driven DW dynamics is not well understood. The dynamics of magnetization described by the LLG equation admits the static solutions for DW motion. In the presence of spin torque and the external magnetic field, it is difficult to derive the dynamic solutions. A circumvented approach is Walker solution analysis [22] for the moving DW in response to a steady magnetic field smaller than some critical value. However, this approximation applying to DW motion driven by electric current is unclear, and its reliability has to be verified theoretically and numerically.

In this paper, we report analytically the critical current condition for anisotropic ferromagnetic nanowire driven only by spin-transfer torque. Below the critical current, the ferromagnetic nanowire admits only the final static DW solution, which implies that the spin-polarized current cannot drive a DW moving continuously. When the spin-polarized current exceeds the critical value, the dynamics of the DW exhibits a novel screw-pitch effect with periodic temporal oscillation of DW velocity and width. A detailed theoretical analysis and numerical simulation demonstrate that this novel phenomenon arises from the natural conjunction action of Gilbert damping and spin-transfer torque. We also observe that the spin-transfer torque plays entirely opposite roles in the above two cases. Finally, our theoretical prediction can be confirmed by numerical simulation in terms of the RKMK method [23].

We consider an infinitely long uniaxial anisotropic ferromagnetic nanowire, where the electronic current flows along the long length of the wire, defined as the \( x \) direction, which is also the easy axis of the anisotropic ferromagnet. For convenience, the magnetization is assumed to be nonuniform only in the direction of the current. Since the magnetization varies slowly in space, it is reasonable to take the adiabatic limit. Then the dynamics of the localized magnetization can be described by the modified LLG equation with spin-transfer torque

\[
\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + b_j \frac{\partial \mathbf{M}}{\partial x},
\]

where \( \mathbf{M} = \mathbf{M}(x, t) \) is the localized magnetization, \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is the damping parameter, and \( \mathbf{H}_{\text{eff}} \) represents the effective magnetic field. The last term of equation (1) denotes the spin-transfer torque, where \( b_j = P_j \mu_0/(eM_s) \), \( P_j \) is the spin polarization of the current, \( j_k \) is the electric current density and flows along the \( x \) direction, \( \mu_0 \) is the Bohr magneton, \( e \) is the magnitude of electron charge, and \( M_s \) is the saturation magnetization. For the uniaxial ferromagnetic nanowire the effective field can be written as

\[
\mathbf{H}_{\text{eff}} = \left(2A/M_s^2\right)\beta^2 \mathbf{M}/\partial x^2 + \hat{H}_i M_s/M_e \mathbf{e}_i - 4\pi M_s \mathbf{e}_z,
\]

where \( A \) is the exchange constant, \( \hat{H}_i \) is the anisotropy field, and \( \mathbf{e}_i \), \( i = x, y, z \), is the unit vector, respectively. Introducing the normalized magnetization, i.e., \( \mathbf{m} = \mathbf{M}/M_s \), equation (1) can be simplified as the dimensionless form

\[
\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}_{\text{eff}})
\]

\[
+ ab_1 \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x} + b_1 \frac{\partial \mathbf{m}}{\partial x},
\]

where \( \alpha_1 = (1 + \alpha^2) \) and \( b_1 = b_j M_s/\mu_0 \). The time \( t \) and space coordinate \( x \) have been rescaled by the characteristic time \( t_0 = 1/(16\pi \gamma M_s) \) and length \( l_0 = \sqrt{A/(8\pi M_s^2) \} \). Respectively, the dimensionless effective field becomes

\[
\mathbf{h}_{\text{eff}} = \beta^2 \mathbf{m}/\partial x^2 + C_1 \mathbf{m} \mathbf{e}_z - C_2 \mathbf{m} \mathbf{e}_x,
\]

with \( C_1 = H_i/M_s/(16\pi M_s) \) and \( C_2 = 0.25 \).

In the following, we seek the exact DW solutions of equation (2), and then study the dynamics of magnetization driven by spin-transfer torque. With this purpose we make the ansatz

\[
m_x = \tanh \Theta_1, \quad m_y = \frac{\sin \phi}{\cosh \Theta_1}, \quad m_z = \frac{\cos \phi}{\cosh \Theta_1},
\]

where \( \Theta_1 = k_1 x - \omega_1 t \), with the temporal and spatial independent parameters \( \phi, k_1, \) and \( \omega_1 \) to be determined, respectively. Substituting equations (3) into equation (2) we have

\[
k_1^2 = C_1 + C_2 \sin^2 \phi,
\]

\[
-\omega_1 (1 + \alpha^2) = b_1 k_1 + C_2 \sin \phi \cos \phi,
\]

\[
\alpha b_1 k_1 \cos \phi = \alpha (C_1 - k_1^2) \sin \phi,
\]

\[
\alpha b_1 k_1 \sin \phi = -\alpha C_2 \sin^2 \phi \cos \phi.
\]

From the above equations we can get three cases of DW solutions for equation (2). Firstly, in the absence of damping equations (4)–(7) admit the solution

\[
k_1 = \pm \sqrt{C_1 + C_2 \sin^2 \phi}, \quad \omega_1 = -b_1 k_1 - \frac{C_2}{2} \sin 2\phi,
\]

with the arbitrary angle \( \phi \). This solution shows that the spin-transfer torque contributes a dimensionless velocity \( -b_1 \) only without damping. The velocity of DW is formed by two parts, i.e., \( v = -(C_2 \sin 2\phi)/(2k_1) - b_1 \), which can be affected by adjusting the angle \( \phi \) and the spin-transfer torque. Secondly, in the absence of spin torque, we have the solution of equations (4)–(7) as \( \omega_1 = 0, \phi = \pm \pi/2, k_1 = \pm \sqrt{C_1}, \) i.e., the static DW solution. In terms of the RKMK method [23] we perform direct numerical simulation for equation (2) with various initial conditions, and all numerical results show that the damping drives the change of \( \phi \) which in turn affects the DW velocity and width defined by \( 1/|k_1| \). Finally, \( \phi = \pm \pi/2, \omega_1 = 0, \) i.e., the DW loses movement, and the DW width attains its maximum value \( \sqrt{C_1} \), which confirms the Walker’s analysis [22] that the damping prevents the DW from moving without the external magnetic field or spin-transfer torque. However, as shown later, the presence of damping is prerequisite for the novel screw-pitch property of the DW driven by spin-transfer torque. Finally, we consider the case of the presence of damping and spin-transfer torque. Solving equations (4)–(7) we have

\[
k_1 = \pm \frac{1}{2} (B_1 - \sqrt{B_2}), \quad \omega_1 = 0, \quad \sin 2\phi = -\frac{2b_1 k_1}{C_2},
\]

where \( B_1 = 2C_1 + C_2 - b_1^2; \quad B_2 = (C_2 - b_1^2)^2 - 4C_1 b_1^2 \).

It is clear that equations (9) imply the critical spin-polarized current condition, namely

\[
b_j \leq \frac{1}{2} (\sqrt{C_1 + C_2 - \sqrt{C_1} l_0/\mu_0},
\]
which is determined by the characteristic velocity $l_0/t_0$, the anisotropic parameter $C_1$, and the demagnetization parameter $C_2$. Below the critical current, i.e., $b_1^2 \leq (\sqrt{C_1^2 + C_2^2} - \sqrt{C_1})^2$, the DW width falls into the range where $1/\sqrt{C_1^2 + C_1C_2} \leq 1/|k_1| \leq 1/C_1$. From equations (9) we get four solutions of $\phi$, i.e., $\phi = \pm \pi/2 + 1/2 \arcsin(2b_1k_1/C_2)$ for $k_1 > 0$ and $\phi = \pm \pi/2 - 1/2 \arcsin(2b_1|k_1|/C_2)$ for $k_1 < 0$. In fact, the signs '+' and '-' in equations (9) denote kink and antikink solutions, respectively, and the corresponding solution in equations (3) represents the static tail-to-tail or head-to-head Néel DW, respectively. This result shows that, below the critical current, the final equilibrium DW solution must be realized by the condition that $m \times h_{\text{eff}} = b_1 \partial m/\partial x$. It clearly demonstrates that the spin-transfer torque has two interesting effects. One is that the term $b_1 \partial m/\partial x$ in equation (2) plays the anti-precession role counteracting the precession driven by the effective field $h_{\text{eff}}$. However, the third term on the right-hand side of equation (2), namely $a b_1 m \times \partial m/\partial x$, has the anti-damping effect counteracting the damping term $-a m \times (m \times h_{\text{eff}})$. That is to say that, below the critical value, the spin-polarized current cannot drive DW moving continuously without the applied external magnetic field.

When the spin-polarized current exceeds the critical value, the dynamics of the DW possesses two novel properties as shown in the following section. Above the critical current, the precession term $-m \times h_{\text{eff}}$ cannot be counteracted by spin-transfer torque, and the static DW solution of equation (2) does not exist. Because the magnetization magnitude is constant, i.e., $m^2 = 1$, we have $m \cdot \partial m/\partial x = 0$, which shows that the direction of $\partial m/\partial x$ is always perpendicular to the direction of $m$, or $\partial m/\partial x = 0$. It is well known that a magnetic DW separates two opposite domains by minimizing the energy. In the magnetic DW the direction of magnetic moments gradually changes, i.e., $\partial m/\partial x \neq 0$, so the direction of $\partial m/\partial x$ should adopt the former case. Out of the region of the DW the normalized magnetization will be sited at the easy axis, i.e., $m_x = 1$ (or $-1$), on which $\partial m/\partial x = 0$.

With the above consideration we make a detailed analysis for equation (2). As a characteristic view we mainly consider the DW center, defined by $m_x = 0$. The magnetic moment must be in the $y$-$z$ plane, while the direction of $\partial m/\partial x$ should lie on the $x$-axis ($+x$-axis for $k_1 > 0$, and $-x$-axis for $k_1 < 0$). In order to satisfy equation (2) the magnetic moment in the DW center should include both the precession around the effective spin torque field $a b_1 \partial m/\partial x$ and the tendency along the direction of $\partial m/\partial x$ continuously from the last two terms on the right-hand side of equation (2). The former precession motion implies that the parameter $\phi$ will rotate around the $x$-axis continuously, while the latter tendency forces the DW center to move in the opposite direction to the current, i.e., the $-x$-axis direction, confirming the experiment [16–20] in magnetic thin films and magnetic wires. Combining the above two effects we find that this rotating and moving phenomenon is very similar to the screw-pitch effect. The continuous rotation of magnetic moment in the DW center, i.e., the periodic change of $\phi$, can result in the periodic oscillation of DW velocity and width from equations (8) under the action of the first two terms on the right-hand side of equation (2).

It is interesting to emphasize that, when the current exceeds the critical value, the term $a b_1 m \times \partial m/\partial x$ plays the role of inducing the precession, while the term $b_1 \partial m/\partial x$ has the effect of damping, which is actually entirely contrary to the case below the critical current as mentioned before. Combining the above discussions, we conclude that the motion of magnetic

**Figure 1.** The dynamics of the DW above the critical current. (a)–(c) Evolution of the normalized magnetization $m$. (d) The displacement of the DW driven only by spin-transfer torque. The parameters are $\alpha = 0.2$, $C_1 = 0.05$, $C_2 = 0.25$, $b_J = 0.6$, and the initial angle $\phi = 0.01\pi$. 


and the Gilbert-damping term, different from the common counteracts both the precession driven by the effective field and the spin-transfer torque. An external magnetic field should be applied in order to drive DW motion. We also find that the spin-transfer torque leads naturally to the novel screw-pitch effect characterized by the temporal oscillation of DW velocity and width.

The situation is now clear for the dynamics of the DW driven only by spin-transfer torque. Coming back to equation (2) we can see that this novel screw-pitch effect with the periodic oscillation of DW velocity and width occurs even at the conjunct action of the damping and spin-transfer torque. To confirm our theoretical prediction we perform direct numerical simulation for equation (2) with an arbitrary initial condition by means of the RKMK method [23] with the current exceeding the critical value. In figures 1(a)–(c) we plot the time-evolution of the normalized magnetization $m$, while the displacement of the DW center is shown in figure 1(d). The result in figure 1 entirely confirms our theoretical analysis above. The evolution of $\cos \phi$ and the periodic oscillation of DW velocity and width is shown in figure 2. From figure 2 we can see that the periodic change of $\cos \phi$ leads to the periodic temporal oscillation of DW velocity and width. From equations (8) and the third term of equation (2) we can infer that $\cos \phi$ possesses uneven change as shown in figure 2(a); i.e., the time corresponding to $0 < \phi + n \pi \leq \pi/2$ is shorter than that corresponding to $\pi/2 < \phi + n \pi < \pi$, $n = 1, 2, \ldots$, in each period, and the DW velocity has the same character. This leads to the DW displacement first increasing rapidly, and then slowly as shown in figure 1(d). This phenomenon clearly demonstrates the presence of the screw-pitch effect. The DW velocity oscillation driven by the external magnetic field has been observed experimentally [24]. Our theoretical prediction for the range of DW velocity oscillation driven by the above critical current could be observed experimentally.

In summary, the dynamics of a DW in a ferromagnetic nanowire driven only by spin-transfer torque is theoretically investigated. We obtain an analytical critical current condition, below which the spin-polarized current cannot drive a DW moving continuously and the final DW solution is static. An external magnetic field should be applied in order to drive DW motion. We also find that the spin-transfer torque counteracts both the precession driven by the effective field and the Gilbert-damping term, different from the common understanding. When the spin current exceeds the critical value, the conjunctive action of Gilbert damping and spin-transfer torque leads naturally to the novel screw-pitch effect characterized by the temporal oscillation of DW velocity and width.

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