Noise in pulsar timing arrays

Yan Wang

School of Physics, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan, Hubei Province 430074, China
Center for Advanced Radio Astronomy, University of Texas at Brownsville, 1 West University Boulevard, Brownsville, Texas 78520, USA
E-mail: ywang12@hust.edu.cn

Abstract. To successfully detect gravitational waves with pulsar timing arrays, we need to have a comprehensive understanding of the physical origins and statistical characteristics of the noise in pulse arrival times and identify mitigation methods to reduce the noise. In this paper we will review radiometer noise, phase jitter noise and timing noise in the noise budget of pulsar timing and show various efforts used to reduce them. We will briefly discuss the results of an overall assessment of the components and physical causes of the timing residuals for millisecond pulsars in the North American Nanohertz Observatory for Gravitational Waves (NANOGrav).

1. Introduction

Pulsar timing arrays (PTAs) are striving to detect very low frequency ($10^{-9} - 10^{-6}$ Hz) gravitational waves (GWs) by observing a set of extremely stable millisecond pulsars (MSPs). Detection can be achieved by observing 20–40 pulsars over 5–10 years, assuming monthly observation cadence and 100 ns for root mean square (RMS) of timing residuals which is presumably dictated by white Gaussian noise [1]. However, detection can be delayed by up to about 10 years depending on the level of red noise and the possible errors (e.g., ephemeris, polarization calibration, time transfer) in the highest timing precision [2]. So far, upper limits have been reported for the amplitude of the stochastic GW background [3–5] and for continuous GW sources [6; 7] by three major PTAs (NANOGrav [4], PPTA [8] and EPTA [9]). Results from the International Pulsar Timing Array (IPTA) [10–12] which combine the data sets from all PTAs will also become available in the near future.

As in any gravitational wave detection experiment (e.g., LIGO [13], Virgo [14], eLISA [15]), noise characterization and mitigation are the central issues that need to be addressed in order to confidently detect and characterize the relatively weak GW signals, and carry out detailed astrophysical interpretations of them. For PTAs, a comprehensive understanding of the noise error budget of pulse arrival time is crucial to guide us to improve instruments, design experiments, and develop algorithms.

To achieve this goal, we need to build a complete measurement model that accounts for the end to end errors in the highest timing precision. Here, one end is a rotating pulsar, which produces regular radio pulses. The spacially coherent radiation carrying these pulses will be distorted by turbulent plasma in interstellar medium (ISM), causing dispersion, scattering, refraction, diffraction, etc. of the radio waves [16], along the line of propagation to the other end, a telescope. The faint EM signal is subsequently collected and focused by the large reflector,
received by the radiometer and recorded by the backend system at the observatory. From emission to reception, there are various errors that can be introduced into the final estimation of pulse time of arrivals (TOAs).

Here, we shall not discuss the propagation effects pertaining to the interstellar medium [17; 18], since it is covered in the account by L. Levin in this volume. Instead, we focus on the other errors such as radiometer noise, phase jitter noise and timing noise, which are currently under intensive scrutiny of the pulsar timing community [19–21]. The rest of this paper is organized as follows. In Sec. 2 we provide an overview of these noise sources and methods of mitigation, and in Sec. 3 we discuss briefly the efforts and results in the NANOGrav collaboration on noise analysis. The paper is concluded in Sec. 4.

2. Noise

In timing analysis, it is a common practice to transform TOAs measured in the topocentric reference frame centered at a telescope to the inertial reference frame centered at the Solar System Barycenter [16; 22]. This transformation includes terms representing deterministic effects such as clock correction, time delay due to interstellar medium dispersion, geometric time delay (Römer delay), relativistic time delay (Shapiro delay, Einstein delay), etc. In addition, various error terms should be also added into the time transformation. A comprehensive list of timing errors can be found in Table 1 of [23].

In general, we can classify the errors into the ones pertinent to the time tagging of pulses (i.e. TOA measurement by template fitting) and the ones pertinent to the physical properties of pulsar or ISM. An example of the latter is the timing of a pulse with infinite SNR and known profile which can be measured to an arbitrary precision (no time tagging error), but the irregularity of pulsar rotation or the stochastic fluctuation of dispersion measure may introduce additional random components. The error can be achromatic which means its influence is independent of the observation frequency (timing noise), weakly chromatic (radiometer noise, jitter noise), or strongly chromatic (effects rooted from interstellar medium).

The power spectrum of the noise can be white or red. The TOA fluctuations caused by the stochastic GW background have red spectra for individual pulsars, which are angularly correlated between pairs of pulsars [24]. Thus, for detecting this background, it is imperative to assess and reduce, if possible, the confusing red noise components rooted from other sources, for example, pulsar and ISM.

2.1. Radiometer noise

Radiometer noise is instrumental in origin (thermal electron fluctuation) combining the contribution from the sky background (dominated by the synchrotron-radiating electrons in the plane of the Galaxy). Radiometer noise with a Gaussian probability density function is additive to the pulse profile, of any of the Stokes parameters. Usually, the Stokes I (intensity) is used to measure the TOAs. Radiometer noise is weakly chromatic if the declination of pulse flux density with increasing of observation frequency cancels out much of the variation of the sky background temperature. The resulting TOA is estimated from template fitting of integrated pulse profile with theoretical or integrated template. The minimum RMS error for TOA estimation due to finite SNR and sampling rate is [23]:

$$\sigma_{\text{SNR}} = 1 \mu s \left( \frac{W}{1 \text{ms}} \right) \left( \frac{N}{10^6} \right)^{-1/2} \left( \frac{1}{\text{SNR}_1} \right) \left( \frac{\Delta f}{W} \right)^{1/2} \left( \frac{P}{1 \text{ms}} \right)^{-1} \left( \frac{f}{1.4 \text{GHz}} \right)^{-\alpha} \left( \frac{\Delta f}{1 \text{GHz}} \right)^{-1/2} \left( \frac{N}{10^6} \right)^{-1/2} \left( \frac{S_{\text{sys}}}{S_{1400}} \right).$$

(1)

$$= 0.71 \mu s \left( \frac{W}{1 \text{ms}} \right)^{3/2} \left( \frac{P}{1 \text{ms}} \right)^{-1} \left( \frac{f}{1.4 \text{GHz}} \right)^{-\alpha} \left( \frac{\Delta f}{1 \text{GHz}} \right)^{-1/2} \left( \frac{N}{10^6} \right)^{-1/2} \left( \frac{S_{\text{sys}}}{S_{1400}} \right).$$

(2)

It could be strongly chromatic if the pulsar spectrum is flat in the observation band.
Here $W$ is the effective pulse width which equals to $W_{\text{FWHM}}/\sqrt{2\pi \ln 2}$ for a Gaussian pulse profile, $W_{\text{FWHM}}$ is the full width at half maximum of the pulse, $N$ is the number of pulse averaged synchronously to yield an integrated profile, SNR$_1$ is the single pulse SNR, $\Delta$ is the sampling interval, and $P$ is the spin period. Radio frequency $f$ and bandwidth $\Delta f$ are in GHz, $S_{\text{sys}}$ and $S_{1400}$ are the system equivalent flux density in Jy and mean flux density of the pulsar at 1.4 GHz in mJy. Note that optimal result can be archived by increasing the bandwidth and integration time, and choosing bright pulsars with short duty cycle. The second equality is obtained by using the radiometer equation [25]:

$$\Delta T_{\text{sys}} = \frac{T_{\text{sys}}}{\sqrt{n_p t \Delta f}},$$

(3)

and assuming that the flux density of pulsar follows a power law with spectral index $\alpha$. $T_{\text{sys}}$ and $\Delta T_{\text{sys}}$ is the system temperature of the receiver and its RMS fluctuation, $n_p$ is the number of polarization, and $t$ is the integration time.

Eq. 1-2 are obtained under the assumption that there is no variance for the integrated pulse template over the frequency band, i.e. the profile has the same shape at high frequency as at low frequency. However, the profile evolution can arise from phenomena intrinsic to pulsar emission beam or ISM (DM change, scattering and scintillation). Ignoring it will degrade the timing precision promised by the modern broad band receivers and backends (see Table 1). This so called “the large-bandwidth problem” [26] is demonstrated in a recent 24-hour global observation for the millisecond PSR 1713+0747 (c.f. Fig. 4 in [20]). Current treatment of this problem is to generate templates for each frequency channel, add additional fitting parameters (JUMPs) to handle the possible offsets between them, and allow the standard TOA analysis packages such as tempo and tempo2 to find out the best fit value for them [4]. A more consistent and efficient method may be to create a two-dimensional pulse portrait (rather than one-dimensional profiles at different frequencies) which takes into account of differential profile evolution and time offset as a smooth function of frequency. This strategy has been explored for broad band data in [27; 28].

Table 1. Wide band receivers and backends currently used or planned for pulsar observation.

| Telescope | D(m) | Receiver | $f$(GHz) | $T_{\text{sys}}$ | Backend | $\Delta f$(MHz) | Ref. |
|-----------|------|----------|---------|----------------|---------|----------------|-----|
| GBT       | 100  | L-band   | 1.15-1.73 | 20           | GUPPI   | 800            | [29]|
| Arecibo   | 305  | L-band   | 1.15-1.73 | 25           | PUPPI   | 800            | [30; 31]|
| Parkes    | 64   | Multibeam| 1.23-1.53 | 28           | APSR    | 1000           | [8] |
| Effelsberg| 100  | UBB      | 0.6-3    | 24           | ASTERIX | 512            | [32; 33]|
| FAST      | 500  | L-band   | 1.15-1.72 | 25           | –       | –              | [34]|

2.2. Pulse phase jitter and amplitude modulation noise

Increasing the SNR of the integrated profile will reduce the radiometer noise, however it is not the only justification for using integrated pulse profile in TOA measurement. Individual pulse usually jitters in phase at the level of single pulse width, and its amplitude can change more than 100% from pulse to pulse (c.f. Fig. 4 of [35] for PSR J1740+1000). Thus measuring individual pulses even with large SNR will result in a TOA uncertainty in the order of the pulse width. Phase jitter and amplitude modulation are related to the stability of the integrated pulse shape which is determined by the shape of individual pulse and the probability density function of the
phase jitter. A stable pulse profile can be obtained by summing over at least several hundreds of individual pulses, and the associated TOA uncertainty due to jitter roughly scales inversely as square root of the number of the individual pulses [36].

Pulse phase jitter and amplitude modulation appear in all well studied pulsars. It is weakly chromatic. Jitter noise is not additive in nature as the radiometer noise. In fact, it changes the integrated pulse profile in a statistical manner [21]. For a simple case in which we assume the pulse profile is Gaussian shape and the phase jitter follows a Gaussian distribution, the RMS error caused by jitter noise can be written as [23]

\[ \sigma_J = 0.28 \mu s \left( \frac{W_1}{1 \text{ ms}} \right) \left( \frac{N}{10^6} \right)^{-1/2} \left( \frac{f_I}{1/3} \right) \left( \frac{1 + m_1^4}{2} \right)^{1/2}. \]  

Here, \( m_1 \) is the amplitude modulation index defined as the ratio of the standard deviation of the amplitude to the mean at different pulse phases and \( m_1 \approx 1 \). \( f_I \) is the dimensionless jitter parameter defined as the ratio of the standard deviation of the phase of single pulse to the intrinsic pulse width \( W_1 \) of the template and \( f_I \approx 1/3 \). The RMS of total error \( \sigma_I \) for estimated TOA is the quadratic summation of radiometer noise and jitter noise, i.e. \( \sigma_I^2 = \sigma_{I\text{NR}}^2 + \sigma_J^2 \).

By equating Eq. 1 with Eq. 4, we find that jitter noise will become more important than radiometer noise when SNR \( \approx 25 \) exceeds only a few tenths. This sensitivity is accessible by the future radio telescopes such as FAST [37] and SKA [38] that have larger collecting areas and lower system temperatures. In this scenario, the noise will not be reduced by increasing the observation bandwidth. Increasing the observation time will become an inevitable choice. As a result, PTAs will need to request more observation time of radio telescopes.

2.3. Timing noise

Timing noise, also known as spin noise, appears as the structures with temporal correlation in timing residuals that depart greatly from the measurement error alone. It may be caused by the irregularity of rotational spin rate which could root from the changes in internal structure and/or magnetosphere of neutron star. It has been found in a number of canonical pulsars and a few MSPs (e.g., B1937+21, B1821-24) [39]. The RMS error of timing noise can be characterized by a scaling law [19]:

\[ \sigma_{TN} = C \nu^\alpha |\dot{\nu}|^\beta T^\gamma, \]  

where \( C, \alpha, \beta \) and \( \gamma \) are the fitting parameters determined from the whole populations of pulsars with measurable timing noise or upper limits, \( \nu \) and \( \dot{\nu} \) is the pulsar spin frequency and frequency derivative, \( T \) is the total span of observations. The best-fit values and \( \pm 2\sigma \) confidence limits calculated from canonical pulsar and MSP population are in \( C = 1.6 \pm 0.4, \alpha = -1.4 \pm 0.1, \beta = 1.1 \pm 0.1, \) and \( \gamma = 2.0 \pm 0.2 \) [19]. This suggests that MSPs with high spin frequency and low spin frequency derivative would have a lower level of timing noise. Eq. 5 can serve as a criterion to select excellent timers that potentially have less timing noise from newly discovered MSPs for the PTAs. Despite the timing noise is not yet detected in most MSPs, it can be a latent phenomenon that may emerge from future data with longer observation span and higher timing precision.

Timing noise is the red noise intrinsic to pulsar. For \( \gamma \approx 2 \), it has a power spectral index between \( -4 \) and \( -6 \) based on the random walk model of pulsar rotation. As a comparison, the characteristic strain \( h_c \) of the stochastic GW background scales as \( h_c = A (f/yr^{-1})^\alpha \), where \( \alpha = -2/3 \) for a background generated by the incoherent superposition of supermassive black hole binaries. The RMS residuals induced by this background grows as \( T^{5/3} \) associated with a spectral index of \( 2\alpha - 3 = -13/3 \) [40]. One of the consequences of the red noise is that the time of GW detection predicted by white noise dominated model [1] may be delayed up to 10 years.
depending on the level of red noise [2]. Besides, more MSPs will be required than previously expected in order to distinguish between the red noise from the GW stochastic background and the other red noise sources by cross-correlating the data from different pulsars, and the number of pulsars becomes a more important factor than the observation cadence and timing quality of individual pulsar when the PTA enters a regime where the lowest frequencies of the timing residuals are dominated by GWs, a likely case for the current PTAs [2]. Currently, the NANOGrav are adding 3-4 new MSPs discovered from the ongoing major pulsar surveys at Arecibo Observatory and Green Bank Telescope (e.g., PALFA and GBNCC) in the observation campaign each year.

3. Noise assessment

As discussed above, the characterization of noise in PTA plays a central role in hunting for GWs. Due to its importance, the NANOGrav has formed a noise budget working group to use complementary methods to assess the constituents of timing residuals and their physical causes. The white noise and Gaussian statistics are two essential aspects of the assessments, especially the latter is a common assumption in forming the GW detection strategies [41–44]. Blindly applying these strategies without checking the presumption may lead to unreliable results.

Methods used include autocorrelation analyses, Bayesian inference, zero-crossing tests, Gaussianity tests, etc. A memorandum to consolidate the results from the overall assessment of the NANOGrav 5-yrs ASP/GASP data set for 17 MSPs [4] is in preparation. Further studies extended to the NANOGrav 9-yrs (including 4-yrs PUPPI/GUPPI) data set for more than 30 MSPs will be also carried out once the data are available.

Initial results show that for the 5-yrs data most of the pulsars are consistent with the white noise assumption [45; 46], although it is possible that the red timing noise can appear in the 9-yrs observations which have longer span and higher precision. Different levels of departure from Gaussian statistics are shown in most of the pulsars [46], it is suspected that the diffractive interstellar scintillation is the root cause. This suggests that the robust signal detection and characterization methods that are not sensitive to the non-Gaussianity should be implemented in GW data analysis.

4. Conclusions

Noise characterization and mitigation are the central issues in detecting GWs by pulsar timing arrays. In this article, we provide an overview of the features of radiometer noise, phase jitter noise and timing noise as well as the efforts to reduce them.

The radiometer noise is dominant in the current timing precision, it will be continuously mitigated with the developments of instrument and algorithm to a level smaller than the jitter noise. The jitter noise is intrinsic to a pulsar, it can only be mitigated by extending integration time, therefore affects the strategies on telescope time application and allocation.

Timing noise is latent for most MSPs, it can potentially postpone the detection of GWs by PTAs. Finding more MSPs with excellent timing performance in the ongoing and future surveys is imperative in the competition between the red noise from GWs and pulsars themselves.

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