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An Advanced Extremum Seeking Scheme for the Target Trajectory in Electromagnetic Micromirror System

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Abstract—Now two problems result in bad control in the development of the electromagnetic micromirror system. One is that theoretical model in electromagnetic micromirror system is difficult to be determined; Another is that parameters in common control need to be tuned according to the experience. In this paper, cost function concept is proposed to determine the model order in slow-scan axis control of the electromagnetic micromirror. Then recursive least square scheme is built to off-line identify this model. Furthermore, an advanced extremum seeking scheme along with backtracking line search is exploited, which can automatically identify the best parameter value before each extremum search to improve the controllability based on this model for the target trajectory in slow-scan axis control. And the convergence of it is proved. Finally, the experiments and the simulations verify this method proposed valid.

Index Terms—MEMS, the model order, recursive least square, backtracking line search, advanced extremum seeking scheme

I. INTRODUCTION

Electromagnetic micromirror as a MEMS (Micro-Electro-Mechanical System) device, which has a prominent advantage over other micromirrors in aspects of lower power consumption, larger deflection angle and so on [1]–[3]. The attempt has been made by some researchers now. Design and fabrication of electromagnetic micromirror were focused on in Ji’s research [4], but the control was located in open-loop control. The sensitivity of four terminal piezo-resistive sensor on electromagnetic micromirror was emphasized in Chen’s study [5], but control was still in open-loop control. Newton’s method to determine the harmonic coefficients of electromagnetic micromirror was focused on in Steve’s paper [6], but controlling the mirror presented a significant engineering challenge. PID control and a low pass filter (LPF) were proposed in Han’s research [7], but choosing appropriate gains for PID controller is very difficult to manage and the rings from driving coils were difficult to be eliminated. In our previous work [8], it showed that the slow axis in electromagnetic micromirror system participated in several resonant motions under the signals of resonant frequencies, especially in the resonant movement on slow-scan axis shown in fig. 1, which is the main reason of the instability in the control of electromagnetic micromirror. Therefore, there is a requirement of reliability in slow-scan axis control compared to the one of reliability in quick-scan axis control and a more advanced control method need to be exploited for the slow-scan axis control compared to common control at present, in which the parameters are tuned by experience. Extremum seeking control has an advantage to autonomously finding an optimal system behavior (e.g., set point or trajectory to be tracked) for the system in the control field, while at the same time maintaining stability and boundedness of signals [9], [10]. So it is first considered for the slow-scan axis control in this paper. Now two problems are faced in exploiting extremum seeking control for slow-scan axis control. One is that theoretical model is difficult to be determined due to the structure and the fabrication [5] of it; Another is that parameters in extremum control [9] on it need to be tuned according to the experience.

Fig. 1 Main Resonant Movement in Slow-scan Axis Control
In this work, schematic in electromagnetic micromirror system is proposed and cost function concept is first used to determine model-order \([11]\) in slow-scan axis control. Then from the mathematical point of view, recursive least square algorithm is applied in identifying this model. So the first problem can be solved mathematically. Furthermore, a new extremum seeking scheme with backtracking line search is exploited to solve the problem of the instability caused by resonant movement in slow-scan axis control and to improve the ability of extremum seeking control based on this model for the target trajectory in this slow-scan axis control. This method can automatically identify the best extremum step value in candidate step lengths at the beginning of each extremum search. The second problem can also be solved. Finally, the simulations off-line show that this algorithm is effective by using the experiment data.

II. SCHEMATIC IN ELECTROMAGNETIC MICROMIRROR SYSTEM AND IDENTIFICATION

A. Schematic in Electromagnetic Micromirror System

Schematic in electromagnetic micromirror system shown in a of Fig. 2 is proposed in this paper. In slow-scan axis control, \(V_{in}(t)\) and \(V_{out}(t)\) are the input voltage \(u(t)\) and the output voltage \(y(t)\) of slow axis in this electromagnetic micromirror system, respectively. Similarly, in quick-scan axis control, \(V_{in}(t)\) and \(V_{out}(t)\) are the input voltage \(u(t)\) and the output voltage \(y(t)\) of quick axis in this electromagnetic micromirror system, respectively. The work in this paper revolves around this schematic (see Fig. 2.a) in slow-scan axis control. Input voltage \(u(t)\) as tuning voltage from DA is linearly converted to drive electric current \(I(t)\) to drive the slow-scan axis of the electromagnetic micromirror. Four terminal piezo-resistive sensor is used to monitor the angle of electromagnetic micromirror. Meanwhile, it provides the output voltage \(y(t)\) as feedback voltage for the slow-scan axis control after being scaled up linearly. Its simplified schematic of electromagnetic micromirror system is seen in b of Fig. 2 in order to mathematically identify its model.

![Schematic in Electromagnetic Micromirror System](image)

**Fig. 2. Schematic in Electromagnetic Micromirror System**

B. Model-order Identification

Suppose this model is linear discrete time invariant single input single output system based on this structure (Fig. 2), which can be described as

\[
y(k) = \hat{y}(k) + e(k),
\]

with

\[
\hat{y}(k) = \psi^T(k) \hat{\theta}(k),
\]

transposed parameter vector

\[
\hat{\theta}^T(k) = \begin{bmatrix} \hat{a}_1^k & \hat{a}_2^k & \cdots & \hat{a}_m^k & \hat{b}_1^k & \hat{b}_2^k & \cdots & \hat{b}_m^k \end{bmatrix}_{1 \times 2m}
\]

and data vector

\[
\psi^T(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & \cdots & -y(k-m) & u(k-1) & u(k-2) & \cdots & u(k-m) \end{bmatrix}_{1 \times 2m}
\]

Where \(u(k)\) and \(y(k)\) are tuning voltage and output voltage sampled at time step \(k\) in electromagnetic micromirror system,
respectively. \( \hat{\theta}(k) \) is parameter vector at time step \( k \) and \( m \) is the model order. \( e(k) \) shown as (5), is an error between real output voltage \( y(k) \) in electromagnetic micromirror system and output voltage \( \hat{y}(k) \) of the model at time step \( k \).

\[
e(k) = y(k) - \hat{y}(k). \tag{5}
\]

Cost function \( V(m, N) \) on it is given as

\[
V(m, N) = E^T(k)E(k) = \sum_{k=1}^{N} e^2(k), \tag{6}
\]

with the transposed error vector

\[
E^T(k) = [e(1) \ e(2) \ \cdots \ e(k)]_{k \times 1} \tag{7}
\]

of error vector \( E(k) \). Here \( N \) is sample number. The change \( \Delta V(\hat{m}, N) \) as follows (8) can be used to estimate the model order \( m \). We set the estimated model order as \( \hat{m} \).

\[
\Delta V(\hat{m}, N) = V(\hat{m}, N) - V(\hat{m} + 1, N). \tag{8}
\]

And \( \hat{m} \) can be to look for

\[
\Delta V(\hat{m} + 1, N) \ll \Delta V(\hat{m}, N). \tag{9}
\]

Equation (9) means that no significant improvement of the cost function can be obtained, or that basically \( V(\hat{m}, N) \) does not diminish much more, such that

\[
V(\hat{m} + 1, N) \ll V(\hat{m}, N). \tag{10}
\]

Then we adopt

\[
m = \hat{m}. \tag{11}
\]

C. Recursive Least Square Identification

Recursive least square algorithm as the method on the identification of dynamic process is widely used in the engineering application of system. [12, 13] Based on this schematic (Fig. 2) and Sec. II-B, we considers the application of the recursive least square method to the problem of identifying the model in slow-scan axis control of electromagnetic micromirror. It can update the coefficients recursively that minimize the cost function corresponding to output voltage \( y(k) \) of electromagnetic micromirror system at time step \( k \) in the process of identification. Recursive least square algorithm can be described as

\[
\hat{\theta}(k) = \hat{\theta}(k - 1) + K(k)(y(k) - \psi^T \hat{\theta}(k - 1)), \tag{12}
\]

\[
K(k) = P(k - 1)\psi(k)\psi^T(k)P(k - 1)\psi(k) + 1, \tag{13}
\]

\[
P(k) = (I - K(k)\psi^T(k))P(k - 1) \tag{14}
\]

Where \( K(k) \) is a \( 2m \times 1 \) gain matrix at time step \( k \), \( P(k) \) is a \( 2m \times 2m \) covariance matrix at time step \( k \) and \( \hat{\theta}(k) \) is a parameter vector at time step \( k \). We set the transposed consistent estimation as

\[
\theta^T(k) = [a_1 \ a_2 \ \cdots \ a_m \ b_1 \ b_2 \ \cdots \ b_m]_{k \times 2m}. \tag{15}
\]

This recursive least square algorithm can give strongly consistent estimation [12], that is

\[
\hat{\theta}^T(k) \rightarrow \theta^T(k). \tag{16}
\]

III. A NEW EXTREMUM SEEKING SCHEME

Based on Sec. II, we propose a new extremum seeking scheme along with backtracking line search. The convergence of it is proved referring to Appendix A. And its schematic diagram is shown in Fig. 3. In this diagram (see Fig. 3), \( y(k) \) as the output voltage of slow-scan axis in electromagnetic micromirror system at time step \( k \), is assigned to \( y_s(k) \) as the starting value of the iteration in this new extremum seeking algorithm. It is calculated for predictive output voltage \( y_s(k + 1) \) in the expression of the model on slow-scan axis control of electromagnetic micromirror. Then \( y_s(k + 1) \) participates in termination condition (seen in equations (17) and (30)). \( J_1(k + 1) \) is the cost function (defined in equation (30)). If \( J_1(k + 1) \) satisfied

\[
J_1(k + 1) \leq 10^{-8}, \tag{17}
\]

\( u_s(k + 1) \) is the tuning voltage for target trajectory \( r_s(k + 1) \) at time step \( k \), otherwise extremum seeking control based this model continues iterative calculation. Backtracking line search provide the best extremum step length \( x_h \) in its candidate step lengths for each extremum search at time step \( k \).
Fig. 3. Schematic Diagram on a New Extremum Seeking Scheme Along With Backtracking Line Search

A. **Extremum Seeking Control Algorithm**

Relative differential equation of slow-scan axis in electromagnetic micromirror system based on the model (see Sec. II) is depicted by

\[
A_i(z^{-1}) y_i'(k) = B_i(z^{-1}) x_i'(k),
\]

with

\[
A_i(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_m z^{-m}
\]

and

\[
B_i(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}, \quad (b_0 = 0).
\]

By left multiplying (18) by \(z\), we obtain \(zA_i(z^{-1}) y_i'(k) = zB_i(z^{-1}) x_i'(k)\).

The corresponding discrete-time state space representation in slow-scan axis control is given as

\[
X_i'(k+1) = F_i X_i'(k) + G_i u_i'(k),
\]

\[
y_i'(k+1) = C_i X_i'(k+1),
\]

with the state vector

\[
X_i'(k+1) = \begin{bmatrix}
      x_i'(k+1) \\
      x_i'(k+1) \\
      \vdots \\
      x_i'(k+1) \\
      x_i'(k+1)
\end{bmatrix}
\]

in \(i\)th iteration at step time \(k+1\), the state vector

\[
X_i'(k) = \begin{bmatrix}
      x_i'(k) \\
      x_i'(k) \\
      \vdots \\
      x_i'(k) \\
      x_i'(k)
\end{bmatrix}
\]

in \(i\)th iteration at step time \(k\), the tuning voltage \(u_i'(k)\) in \(i\)th iteration at step time \(k\), the output voltage \(y_i'(k+1)\) in \(i\)th iteration at step time \(k+1\), the state matrix

\[
F_i = \begin{bmatrix}
      0 & 1 & 0 & \cdots & 0 \\
      0 & 0 & 1 & \cdots & 0 \\
      \vdots & \vdots & \vdots & \cdots & \vdots \\
      0 & 0 & 0 & \cdots & 1 \\
      -s a_m & -s a_{m-1} & -s a_{m-2} & \cdots & -s a_1
\end{bmatrix}_{m \times m}
\]

the input matrix
\[ G_s = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{m-1} \\ h_m \end{bmatrix} \] (27)

and the output matrix
\[ C_s = [1 \ 0 \ \cdots \ 0]_{v \times m}. \] (28)

The parameters \( s h_1, \ s h_2, \ s h_3, \ \cdots, \ s h_m \) in the input matrix can be expressed as
\[ s h_2 = b_2 - a_1 s h_1 \]
\[ s h_3 = b_3 - a_2 s h_1 - a_1 s h_2 \]
\[ \vdots \]
\[ s h_m = b_m - a_{m-1} s h_1 - \cdots - a_1 s h_{m-1} \] (29)

Define the cost function \( J_i'(k+1) \) in extremum seeking control of slow-scan axis in \( ith \) iteration at step time \( k+1 \) as
\[ J_i'(k+1) = \frac{1}{2} (r_i'(k+1) - y_i'(k+1))^2. \] (30)

Supposed that \( J_i'(k+1) \) and \( u_i'^{i+1}(k) \) are both \(nth\) order continuously differentiable in interval \([u_i'(k), u_i'(k+1)]\). Where \( r_i'(k+1) \) is the target trajectory in slow-scan axis control at step time \( k+1 \). \( u_i'^{i+1}(k) \) based on the steepest descent method at step time \( k \) for \((i+1)th\) iterations is designed as
\[ u_i'^{i+1}(k) = u_i'(k) + \Delta u_i'(k), \] (31)
with
\[ \Delta u_i'(k) = -\lambda_i^+ \nabla J_i'(k+1) \] (32)
and
\[ \nabla J_i'(k+1) = \frac{\partial J_i'(k+1)}{\partial u_i'(k)} = -2 \times \frac{1}{2} (r_i'(k+1) - y_i'(k+1)) \frac{\partial y_i'(k+1)}{\partial x_i(k+1)} \times \frac{\partial x_i(k+1)}{\partial u_i'(k)} = -(r_i'(k+1) - y_i'(k+1)) \frac{\partial y_i'(k+1)}{\partial x_i(k+1)} \times \frac{\partial x_i(k+1)}{\partial u_i'(k)} = -(r_i'(k+1) - y_i'(k+1)) \frac{\partial y_i'(k+1)}{\partial x_i(k+1)} \times \frac{\partial x_i(k+1)}{\partial u_i'(k)} = -(r_i'(k+1) - y_i'(k+1)) \frac{\partial x_i(k+1)}{\partial u_i'(k)} \] (33)

Where \( \nabla J_i'(k+1) \) is the gradient in \( ith \) iteration at step time \( k+1 \) and \( r_i'(k+1) \) is the target trajectory at step time \( k+1 \). The appropriate step length \( \lambda_i^+ \) at step time \( k \) is chosen by using a so-called backtracking line search approach seen as Sec. III-B in detail. Repeat (33), (32), (31), (22), (23) and (30) at step time \( k+1 \) until satisfying with (17). Then
\[ u_i'(k+1) = u_i'^{i+1}(k). \] (34)
Note that \( x_i'(k+1) \) and \( x_i'^{i+1}(k+1) \) are separately assigned to \( x_i'^{i+1}(k) \) and \( x_i'^{i+1}(k+1) \) after each calculation in (22) and (23).

**B. Automatically Choosing the Best Extremum Step Length**

Before each extremum seeking algorithm in slow-scan axis control at time step \( k \), backtracking line search technique [14] is applied in choosing the best extremum step length \( \lambda_i^+ \) during the candidate step lengths, and its essential backtracking proceed is shown in Fig. 4. In the flow diagram (see Fig. 4), after starting,
\[ \lambda > 0, \]
\[ \rho, c \in (0,1). \] (35) (36)
Then
\[ \lambda = \lambda. \]  
Repeat
\[ \lambda = \rho \lambda, \]  
until
\[ J_{11}^i (k+1)(u^i(k) + \lambda p^{k+1}) \leq J_{11}^i (k+1)(u^i(k)) + c \lambda \nabla J_{11}^i (k+1)p^{k+1}. \]  
(39)
In the end,
\[ \lambda^* = \lambda. \]  
(40)

IV. EXPERIMENT AND SIMULATION RESULTS

In this section, experiments are all based on dsPACE platform including DA (DS2012) and AD (DS2004). Its connection between dsPACE and electromagnetic micromirror system (Fig. 2) is shown in Fig. 5. Sampling frequencies in this section are all 5 kHz.

A. Second Order Model in Slow-scan axis control

The electromagnetic micromirror system model in slow scan-axis was modelled with the parameters \( \theta^f(k) \) (see eq. (16)) captured from results of identification on model order and recursive least square.

Combining with experiment data on output voltage \( y(k) \) and input voltage \( u(k) = 2e^{\frac{-k}{2000}} \sin(2 \pi \times 300 \times e^{\frac{-k}{2000}}) \) (attenuation amplitude 2V, decay rate 3500 and attenuation frequency from 300 Hz to 6 Hz) at time step \( k \), when
\[ \Delta V(3, N) = 0.308 \quad (N = 13927) \]  
and
\[ \Delta V(2, N) = 4.1104 \quad (N = 13927) \]  
(41) (42) (43)

![Flow Diagram on Backtracking Line Search](image-url)
in model-order identification (Sec. II-B). Satisfying with
\[ \Delta V(3, N) \ll \Delta V(2, N) \quad (N = 13927), \]  
we choose
\[ m = 2. \]  
(45)
The corresponding performance in off-line calculations of recursive least square algorithm is shown in Fig. 6. In the recursive least square algorithm of slow-scan axis control, the parameters
\[ \hat{\theta}^r(k) = [\hat{a}_1^s, \hat{a}_2^s, \hat{b}_1^s, \hat{b}_2^s]_{1 \times 4}, \]
converges to
\[ \theta^r(k) = [a_1, a_2, b_1, b_2]_{1 \times 4} \]
(46)
(see eq. (3), eq. (15) , eq. (16) and Fig. 6).

B. Performance of Advanced Extremum Seeking Scheme

Based on the parameters convergence results from Fig. 6, and along with (22) and (23), corresponding vectors in discrete-time state space representation (see Sec. III-A) in slow-scan axis control are derived as follows. State vector
\[ X'_i(k+1) = \begin{bmatrix} x'_i(k+1) \\ x'_i(k+1) \end{bmatrix}_{2 \times 1} \]  
in \( i \)th iteration at step time \( k + 1 \), state vector
\[ X'_i(k) = \begin{bmatrix} x'_i(k) \\ x'_i(k) \end{bmatrix}_{2 \times 1} \]  
in \( i \)th iteration at step time \( k \), state matrix
\[ F_s = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}_{2 \times 2} \]
\[ = \begin{bmatrix} 0 & 1 \\ -0.1371 & 0.6434 \end{bmatrix}_{2 \times 2} \]  
input matrix
and output matrix
\[ C_s = \begin{bmatrix} 1 & 0 \end{bmatrix}. \] (52)

\( s_i x_1(k+1) \) is from the output voltage at time step \( k + 1 \) for \( ith \) iterations in the model determined by (22)-(23), (45) and (48)-(52)) mathematically. We define \( s_i x_2(k+1) \) as the pseudo velocity of slow-scan axis at time step \( k + 1 \) for \( ith \) iterations in the model determined by (22)-(23), (45) and (48)-(52)) mathematically. Now two aspects are focused on to show the performance of the advanced extremum seeking scheme. Compared with the secondary filtering [8] (seen in Fig. 9 and Fig. 10), this advanced extremum seeking algorithm plays a better role in filtering (seen from Fig. 7 to Fig. 10). Based on this model, the proposed method (see Sec. III) of slow-scan axis succeed in keeping the original frequency 60 Hz for target trajectory with an amplitude 0.1 V under external disturbance, especially under electromagnetic interference similar to cell phone calls (Operating Performance in Fig. 7 and Spectrogram Performance in Fig. 8). This is the first advantage.

Fig. 7. Operating Performance of Advanced Extremum Seeking Scheme for Slow scan Axis under Environment Noise

Fig. 8. Operating Results of Advanced Extremum Seeking Scheme for Slow-scan Axis in Frequency under Environment Noise
The second advantage is that the proposed method can automatically identify the best extremum step value in candidate step lengths at the beginning of each extremum search to improve the tracking for the target trajectory in slow-scan axis control. Furthermore, tracking performance such as deflection angle, tuning voltage, state variables and so on in off-line iteration and recursion of this advanced extremum seeking scheme in slow-scan axis control is shown in Fig. 11 and in Fig. 12, in the case that input sine voltage is with an amplitude of $2\text{ V}$ and is of the frequency 60 $\text{Hz}$. The resolution of four terminal piezo-resistive sensor for slow-scan axis for this electromagnetic system is $1\text{deg}$. Maximum absolute deflection angle error $0.3113^\circ$ appears in 0.0002 second of this new extremum seeking scheme (see the second row in Fig. 11). And then it is limited in $0.0191^\circ$. Compared with the maximum absolute error of $0.67^\circ$ in open-loop control (see Fig. 13), this advanced extremum seeking scheme in slow-scan axis control succeeds in regulating the tuning voltage more quickly and more accurately. (see Fig. 12). Although the maximum absolute error is also $0.3113^\circ$ in the extremum seeking with fixed step length (see Fig. 14 and Fig. 15) through adjusting the fixed step length by experience. After 0.0002 second, it is only limited in $0.18^\circ$. So the advanced extremum seeking scheme show stronger tracking the target angle than the extremum seeking with fixed step length (see Fig. 11). Maximum deflection angle in this new extremum seeking scheme of slow-scan axis control is $\pm 6^\circ$. 
Fig. 11. Tracking Performance in Off-line Iteration and Recursion of The Advanced Extremum Seeking Scheme

Fig. 12. Corresponding Tuning Voltage and State Variable in Advanced Extremum Seeking Scheme
Fig. 13. Performance Results in Open-loop Control of Slow-scan Axis

Deflection Angle of Slow-scan Axis in Open-loop Control

Deflection Angle Error in Open-loop Control
Maximum Absolute Angle Error is 0.67°

Deflection Angle Error in Extremum Seeking With Fixed Step Length
Maximum Absolute Angle Error is 0.3113°

Fig. 14. Performance Results in Extremum Seeking With Fixed Step Length $\lambda_j = 0.94$ of Slow-scan Axis
Fig. 15. Corresponding Tuning Voltage and State Variable in Extremum Seeking With Fixed Step Length $\lambda_f = 0.94$ of Slow-scan Axis

V. CONCLUSION

The proposed scheme presents an advanced extremum seeking scheme with backtracking line search based on recursive least square algorithm in slow-scan axis control for electromagnetic micromirror. Furthermore, the results in experiment and simulation show that the performance of the proposed method not only keep the original frequency of target trajectory but also track the target angle through the optimal actuation voltage in best extremum step length.

APPENDIX A CONVERGENCE OF NEW EXTREMUM SEEKING SCHEME

We set the cost function (30) (see Sec. III-A) on $u^{i+1}(k)$ as

$$\Gamma(\bullet) = \Gamma(u_i^t(k)) = J_i^t(k+1).$$

(A.1)

According to Taylor’s Theorem [12], and along with (31) and (32) (see Sec. III-A), it is obtained that

$$\Gamma(u_i^t(k) + \Delta u_i^t(k)) = \Gamma(u_i^t(k)) - \lambda_i^t \nabla J_i^t(k + 1) \nabla \Gamma(u_i^t(k)) + O\left[\lambda_i^t \nabla J_i^t(k + 1)\right]$$

(A.2)

$$= \Gamma(u_i^t(k)) - \lambda_i^t \nabla J_i^t(k + 1) - \lambda_i^t \nabla J_i^t(k + 1)^2 + O\left[\lambda_i^t \nabla J_i^t(k + 1)\right].$$

$O\left[\lambda_i^t \nabla J_i^t(k + 1)\right]$ is infinitesimal of $\lambda_i^t \nabla J_i^t(k + 1)$. So the term $-\lambda_i^t \nabla J_i^t(k + 1)^2$ dominates in (A.2) for small $\lambda_i^t$ and determined $\nabla J_i^t(k + 1)$ (see eq. (33)). There is positive but sufficiently small $\lambda_i^k$ (see Sec. III-B) such that $-\lambda_i^k \nabla J_i^t(k + 1)^2 < 0$.

(A.3)

Considering (A.2), it follows that

$$\Gamma(u_i^t(k) + \Delta u_i^t(k)) - \Gamma(u_i^t(k)).$$

(A.4)

Suppose $J_i^t(k + 1)$ is continuously differentiable in an open neighborhood of the local minimum solution $u_i^t(k)$. Let $\nabla J_i^t(k + 1) = 0$ (A.5)

and along with (33) it derives

$$r_i(k + 1) - y_i(k + 1) = 0$$

(A.6)

and

$$u_i^t(k) = u_i^t(k).$$

(A.7)

When

$$J_i^t(k + 1) \to 0$$

(A.8)
(see (17)),
\[ y'_i(k + 1) \rightarrow r_i(k + 1) \]  \hspace{1cm} (A.9)
and
\[ \nu'_i(k) \rightarrow \nu'_i(k) \]  \hspace{1cm} (A.10)
in this new extremum seeking control algorithm.

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Figures

Figure 1

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