Abstract

Pure leptonic and semileptonic rare B decays, $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow X_s \mu^+\mu^-$ in the minimal supersymmetric Standard Model (MSSM), in particular, gluino and neutralino contributions to the decays, are discussed under $\text{Br}(B \rightarrow X_s \gamma)$ and other experimental constraints. The general scalar quark mass matrices as the new sources of flavor violation are considered. We present Wilson coefficients of $bs$ transitions from $\gamma$, $Z$, and neutral Higgs boson penguin diagrams by using vertex mixing method to deal with scalar down-type quark flavor changing and also give their expressions in MIA to show different sources of enhancements. We find that under the experimental constraints, with large mixing of left-handed and right-handed sbottom, $C_{10}^{D(\prime)}$ can be enhanced by 10% compared with SM, in two cases, heavy gluino and fine-tuning between $\delta_{23}^{LL}(\delta_{23}^{RR})$ and $\delta_{23}^{LR}(\delta_{23}^{RL})$ terms in $C_7^{D(\prime)}$. Particularly, $C_{10}$ and $C_{10}^{\prime}$ can reach a 20% enhancement in some regions of parameters under experimental constraints. When CP-odd Higgs $A^0$ is not too heavy ($\sim 250\text{ GeV}$), and $\tan\beta$ is large ($\sim 40$), neutral Higgs boson penguins with gluino and down-type squark in the loop can significantly contribute to the $bs$ transition and the contributions can compete with those due to the chargino and up-type squark loop.
1 Introduction

Pure leptonic and semileptonic flavor changing neutral current (FCNC) rare B decays,

\[ B_s \to \mu^+\mu^-, \quad B \to X_s\mu^+\mu^- \]  

have received much attention recent years due to clear backgrounds and ongoing experiments at BaBar [1] and BELLE and forthcoming projects at Tevatron [2] and LHC [3] as well as sensitivity to models beyond the Standard Model (SM). The current experimental result of \( \text{Br}(B \to X_s\mu^+\mu^-) \) by the BELLE collaboration [4] is

\[ \text{Br}(B \to X_s\mu^+\mu^-)_{\text{exp}} = 7.9 \pm 2.1^{+2.1}_{-1.5} \times 10^{-6} \]  

and CDF [5] upper limit on \( \text{Br}(B_s \to \mu^+\mu^-) \) is

\[ \text{Br}(B_s \to \mu^+\mu^-)_{\text{exp}} < 2.6 \times 10^{-6} \text{ at 90\%C.L.} \]  

In the SM, these processes vanish at tree level, while they occur at one-loop level with the charged gauge boson \( W^\pm \) and up-type quarks in the loop. In the minimal supersymmetric Standard Model (MSSM), there are five kinds of contributions to partonic level process \( b \to s\mu^+\mu^- \) at one-loop level, depending on specific particles propagated in the loop, (1) Standard Model gauge boson \( W^\pm \) and up-type quarks (SM contribution); (2) charged Higgs \( H^\pm \) and up-type quarks (charged Higgs contribution); (3) chargino and scalar up-type quarks (chargino contribution); (4) neutralino and scalar down-type quarks (neutralino contribution); (5) gluino and scalar down-type quarks (gluino contribution). The flavor structure of the sfermion sector in MSSM depends on the soft terms which are determined by the supersymmetry breaking mechanism, in addition to the superpotential. In the minimal flavor violation (MFV) scenarios of MSSM, squarks are assumed to rotate in flavor bases aligned with the corresponding quark sector and the only source of flavor violation is the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix in SM. In some MFV scenarios such as the constrained MSSM (mSUGRA, string-inspired flipped SU(5), etc.) and gauge mediation supersymmetry breaking (GMSB) where the soft terms at some high scale (the grand unification scale or Plank scale or messenger scale) are characterized by the universality of sfermion masses and the proportionality of the trilinear terms, the flavor violation in sfermion sector at the electroweak (EW) scale is generated radiatively and consequently, in general, small \(^1\). Therefore, comparing with the first three kinds of contributions, the last two kinds of contributions, i.e., neutralino and gluino contributions, are negligible [13].

There are new sources of flavor violation in MSSM. Besides the CKM matrix, the the \( 6 \times 6 \) squark mass matrices are generally not diagonal in flavor (generation) indices in the super-CKM basis in which superfields are rotated in such a way that the mass matrices of the quark field components of the superfields are diagonal. This rotation non-alignment in the quark and squark sectors can induce large flavor off-diagonal couplings such as the coupling of gluino to the quark and squark which belong to different generations. There exist two different kinds of methods to deal with flavor changing vertices induced by flavor mixing in the squark mass matrices in the literature [9]. One works in quark and squark mass eigenstates with induced flavor changing couplings, so called "vertex mixing" (VM). The other method, "mass insertion approximation" (MIA) [10], works in flavor diagonal gaugino couplings \( \tilde{g}q \bar{q} \) and diagonal quark mass matrices with all the flavor changes rested on the off-diagonal sfermion propagators. The MIA can be obtained in VM through Taylor expansion of nearly degenerate squark masses \( m_{\tilde{q}} \) around the common squark mass \( m_{\tilde{q}} \), \( m_{\tilde{q}}^2 \approx m_{\tilde{q}}^2(1 + \Delta_q) \). Thus MIA can work well for nearly degenerate squark masses and, in general, its reliability can be checked only a posteriori. However, for its simplicity, it has been widely used as a model independent analysis to find the constraints on different off-diagonal parts of squark mass matrices from experiments [11]. It is clear that VM remains valid even when flavor off-diagonal squark mass matrix elements are large and there is no approximation which has been assumed.

Because the experiment of \( \text{Br}(B \to X_s\gamma) \) only constrains \( |C_7(m_b)|^2 + |C_9^f(m_b)|^2 \), with the overall sign flip of \( C_7(m_b) \) from MSSM vs SM, the branch ratio of \( B \to X_s l^+l^- \) can be enhanced from the term \( \text{Re}(C_7(m_b)C_9^f ) \) in the constrained MSSM with gluino and neutralino contributions neglected [12, 13, 15], while in these analysis \(^1\)In some large \( \tan \beta \) regions of the parameter space it becomes non-negligible [14].
supersymmetry contributions to $C_9$ and $C_{10}$ are at most changed by $\pm 5\%$ compared with the SM values. The chargino contribution in the extended MFV model is analyzed in ref [16]. $B \to X_s l^+ l^-$ has been analyzed in the left-right supersymmetric model recently [17]. In MSSM, gluino induced FCNC process $b \to s l^+ l^-$ is studied in ref [18], and chargino, gluino induced effects are studied in ref [19] in MIA. In these works the contributions from exchanging neutral Higgs bosons (NHBs) are not included. However, for $l = \mu, \tau$, when $\tan \beta$ is large and $M_A^0$ is not too large (say, 250 GeV), the NHBs contributions can become significant due to the $\tan^2 \beta$ enhancement of the corresponding Wilson coefficients in some regions of the parameter space [20,21] and the NHBs contributions to $B \to X_s l^+ l^-$ and $B_q \to l^+ l^-$ ($q = d, s, l = \mu, \tau$) in the constrained MSSM have been investigated in refs. [20,21,22,23]. Using the VM method, the gluino and neutralino induced FCNC processes $b \to s \mu^+ \mu^-$ [24] and $B_q \to \mu^+ \mu^-$ [25] have recently been analyzed, including the NHBs contributions, in the constrained MSSM and in MFV models respectively. In SUSY models with non-minimal sources of flavor mixing, the constraints on different flavor violation parameters from $\text{Br}(B \to X_s \gamma)$ have been considered in ref. [26,27,28], and $B_{s,d} \to l^+ l^-$ at large $\tan \beta$ has been investigated in ref. [29].

It is well-known that the effects of the primed counterparts of usual operators are suppressed by $\frac{m_{\text{primed}}}{m_b}$ and consequently negligible in SM because they have the opposite chiralities. In MFV models their effects are also negligible, as shown in ref. [25]. However, in MSSM their effects can be significant, since the flavor non-diagonal squark mass matrix elements are free parameters. Part of the primed counterparts of usual operators relevant to $B$ rare leptonic and semileptonic decays have been considered in ref. [19].

In this paper, we extend our previous analyses to include gluino and neutralino contributions and all operators responsible for $B$ rare leptonic and semileptonic decays in MSSM. We calculate the Wilson coefficients using the VM method and also give their expressions in MIA to show different sources of enhancements. In numerical analyses we take into account constraints from $\text{Br}(B \to X_s \gamma)$, $\Delta M_{B_s}$ and the lower bounds of superpartner masses and Higgs masses as well as $B \to X_s g$ and hadronic charmless $B$ decays. We have carefully analyzed different sources of enhancements of $C_{10}^{(i)}$ (arising from the $Z$ penguin) and $C_{Q_{1,2}}^{(i)}$ (arising from the NHB penguins), related to the general scalar-down quark mass matrix. We find that under the experimental constraints, with large mixing of left-handed and right-handed bottom, $C_{10}^{(i)}$ can be enhanced by about 10% compared to the Standard Model in two cases, heavy gluino and fine-tuning between $\delta_{23}^{LL}(\delta_{23}^{RR})$ and $\delta_{23}^{LR}(\delta_{23}^{RL})$ terms in $C_7^{(i)}$. In particular, $C_{10}$ and $C_{Q_{1,2}}^{(i)}$ can reach a 20% enhancement in some regions of parameters under experimental constraints. When CP-odd Higgs $A^0$ is not too heavy ($\sim 250 \text{GeV}$), and $\tan \beta$ is large ($\sim 40$), neutral Higgs boson penguins with gluino and down-type squark in the loop can significantly contribute to the $b s$ transition and the contributions can compete with those due to the chargino and scalar up-type quark loop.

The paper is arranged as following. In section 2, we define our notations and consider the effective Hamiltonian and branching ratios of pure leptonic and semileptonic rare $B$ decays. In section 3 we briefly recall the squark mass matrices and discuss the choice of parameters. In section 4, we present our numerical analysis on the possible enhancement of $C_{10}^{(i)}$ and $C_{Q_{1,2}}^{(i)}$ in the case of switching on only the gluino (or neutralino) and SM contributions. We search for maximums of $C_{10}$ and $C_{Q_{1,2}}^{(i)}$ under experimental constraints, switching on all the contributions. Section 5 is devoted to give the numerical results for $B \to X_s l^+ l^-$ and $B_s \to l^+ l^-$ in MIA. In the Appendix, Wilson coefficients at $m_{\text{ew}}$ scale are given.

### 2 Effective Hamiltonian

The effective Hamiltonian for $B \to X_s l^+ l^-$ and $B_s \to l^+ l^-$ can be written as

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_t \left[ \sum_{i=1}^{6} C_i(\mu) O_i(\mu) + \sum_{i=7}^{10} (C_i(\mu) O_i(\mu) + C_i'(\mu) O_i'(\mu)) + \sum_{i=11}^{10} (C_{Q_1}(\mu) Q_i(\mu) + C_{Q_1}'(\mu) Q_i'(\mu)) \right]$$

where $\lambda_t = V_{tb} V_{ts}^*$. $O_i$ and $Q_i$ ($i=1,...,10$) can be found in ref. [30] and [31] respectively, and the primed operators, the counterpart of the unprimed operators, are obtained by replacing the chiralities in the corresponding unprimed operators with opposite ones. The explicit expressions of the operators governing $B \to X_s l^+ l^-$ and $B_s \to l^+ l^-$ are given by

$$O_7 = \frac{e}{16\pi^2} m_b (s \sigma_{\mu \nu} P_R b) F^{\mu \nu}, \quad O_7' = \frac{e}{16\pi^2} m_b (s \sigma_{\mu \nu} P_L b) F^{\mu \nu}$$

3
Wilson coefficients of the prime operators are suppressed by left-handed and right-handed sbottoms is large. In eqs. (7) and (8) the first two terms are suppressed by the statement is also true in MFV scenarios of MSSM [25]. However, in MSSM, the statement is, in general, not been used, $\tan \beta$ and $\gamma$. We also consider the operators
\[ O_8 = \frac{g_a}{16\pi^2} m_b(\tilde{s}_a T_{\alpha\beta}^0 \sigma_{\mu\nu} P_R b_b) G_{\alpha\mu
u}, \quad O_8' = \frac{g_a}{16\pi^2} m_b(\tilde{s}_a T_{\alpha\beta}^0 \sigma_{\mu\nu} P_L b_b) G_{\alpha\mu
u} \]
in order to include the constraints from $B \rightarrow X_s \gamma$ and hadronic charmless B decays into the analysis. In SM the Wilson coefficients of the prime operators are suppressed by $\frac{m_h}{m_b}$ with respect to those of unprimed operators and the statement is also true in MFV scenarios of MSSM [25]. However, in MSSM, the statement is, in general, not valid due to the presence of new sources of flavor violation. The running of Wilson coefficients $C_1$ and $C_{Q_1}$ from $m_w$ to $m_b$ in the leading order approximation (LO) is given in refs [30] and [31] respectively. The evolution of part of the primed operators has been given in ref. [26]. Although the mixing between $O_1$ in the next-to-leading order (NLO) has been studied, the mixing of $O_i$ with $Q_1$ in NLO has not been given. So we shall use only the LO results for consistency. We present $m_w$ scale Wilson coefficients $C_1^{(\ell)}$ and $C_{Q_1}$ in Appendix. In order to see the dependences of Wilson coefficients on new flavor violation parameters, as an illustration, we present all the relevant Wilson coefficients at the $m_b$ scale induced by gluino in MIA as follows
\[ C_7^{(\ell)}(m_b) = 18.7 \frac{400 \text{GeV}}{m_{\tilde{g}}} \delta^{dLR}_{23} \delta^{dLL}_{23} - 34.6 \frac{400 \text{GeV}}{m_{\tilde{g}}} \delta^{dLR}_{23} + 0.07 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLL}_{23} - 0.04 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLR}_{23} \delta_{33} \]
(7)
\[ C_8^{(\ell)}(m_b) = 38.5 \frac{400 \text{GeV}}{m_{\tilde{g}}} \delta^{dLR}_{33} - 92.9 \frac{400 \text{GeV}}{m_{\tilde{g}}} \delta^{dLR}_{33} + 0.15 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLR}_{33} - 0.08 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLR}_{33} \]
(8)
\[ C_9^{(\ell)}(m_b) = -0.21 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLR}_{23} + 0.17 \left( \frac{400 \text{GeV}}{m_{\tilde{g}}} \right)^2 \delta^{dLR}_{33} + 0.67 \delta^{dLL}_{23} (\delta^{dLR}_{23})^2 - 1.68 \delta^{dLR}_{23} \delta^{dLR}_{33} \]
(9)
\[ C_{10}^{(\ell)}(m_b) = -8.37 \delta^{dLR}_{23} (\delta^{dLR}_{23})^2 + 21.0 \delta^{dLR}_{23} \delta^{dLR}_{33} \]
(10)
\[ C_{Q_1}(m_b) = -4.61 \left( \frac{200 \text{GeV}}{m_{\tilde{A}^0}} \right)^2 \left( \frac{\tan \beta}{40} \right)^2 \frac{m_{\tilde{g}}}{400 \text{GeV}} \delta^{dLL}_{23} \delta^{dLR}_{33} \]
(11)
\[ C_{Q_2}(m_b) = -\frac{1}{m_{\tilde{g}}} \delta^{d\ell}_{12} \]
(12)
Where $\delta^{qAB}_{ij} = \frac{(M^2_{qAB})_{ij} - \tilde{m}^2 \delta_{ij} \delta^{AB}_{\tilde{m}}}{m^2_{\tilde{m}}} \quad (M^2_{qAB} \text{ is the scalar quark mass matrix}, q = u, \ell, A, B = L, R, \ell, i, j = 1, 2, 3, \tilde{m} \text{ is the common scalar quark mass})$, the one-loop functions (which are given in appendix) at $x = \frac{m^2_{\tilde{g}}}{m^2_{\tilde{m}}} = 1$ have been used, $\tan \beta$ is the ratio of two Higgs vacuum expectation value $v_U$ and $v_D$, $\tan \beta = \frac{v_U}{v_D}$, and $m_{\tilde{H}^0} = m_{\tilde{A}^0}$, which is a good approximation in the case of $m^2_{\tilde{H}^0}/m^2_{\tilde{A}^0} \gg 1$, has been assumed. Here we have expanded the Wilson coefficients to the double MI, as investigated in ref [27], which is non-neglectable if the mixing between left-handed and right-handed sbottoms is large. In eqs. (7) and (8) the first two terms are suppressed by $\frac{1}{m_{\tilde{g}}}$ as
\[ Q_{1,2} = \frac{1}{m_{\tilde{g}}} \frac{e^2}{16\pi^2} O_{S,P} \text{ and } O_{S,P} \text{ are used in some papers (see, e.g., ref.[32]).} \]
can be seen from Appendix, although they are enhanced by a factor \( \frac{m_3^{\pm}}{m_3} \), they suffer from the \( \frac{m_3^2}{m_3^2} \) suppression.

In eq.\((9)\) for \( C_9^{(f)}(m_b) \), the first two terms come from the \( \gamma \) penguin, which is suppressed by \( \frac{m_3^2}{m_3^2} \). But eq.\((10)\) for \( C_{10}^{(f)}(m_b) \) and the last two terms in eq.\((9)\) from the \( Z \) penguin is not suppressed, which is noticed as non-decoupling of the \( bsZ \) coupling in ref. \([15]\). Therefore, there is a possibility that \( C_{10} \) can be enhanced even if the gluino is heavy. In eq.\((11)\) and eq.\((12)\) for \( C_{Q_{1,2}}^{(f)} \), we present \( \delta_2^{\text{LL}(RR)} \) contribution in the MIA, which is enhanced by a factor \( \frac{m_6^{\pm}}{m_6^{\pm}} \).

Compared to the charged penguin contribution, the gluino contribution is the same important provided that \( m_b \tan \beta \sim m_{\tilde{g}} \). We can read that \( C_{Q_{1,2}}^{(f)} \) can be large if CP-odd Higgs \( A^0 \) is not too heavy (say, \( \leq 400 \) GeV) and tan \( \beta \) is large.

The leading order \( B_s \to X_s \gamma \) branching ratio normalized to \( \text{Br}(B \to X_s e\bar{\nu}) \) is given as

\[
\text{Br}(B \to X_s \gamma) = \frac{\alpha_{\text{em}}}{\pi} \frac{|V_{tb} V_{ts}^\ast|}{V_{cb}} 2 \text{Br}(B \to X_s e\bar{\nu})(|C_7(m_b)|^2 + |C_7'(m_b)|^2),
\]

where \( \sqrt{s} = m_{\text{pole}}^{\text{pole}}/m_b^{\text{pole}} \), \( f(z) \) is the phase space function.

The branching ratio (Br) \( B \to X_s \mu^+ \mu^- \) normalized to \( \text{Br}(B \to X_s e\bar{\nu}) \) is given as

\[
\frac{d\text{Br}(B \to X_s \mu^+ \mu^-)}{ds} = \text{Br}(B \to X_s e\bar{\nu}) \frac{\alpha_{\text{em}}^2}{4\pi^2 f(z)} (1 - s)^2 \sqrt{1 - \frac{4t^2}{s}} \frac{|V_{tb} V_{ts}^\ast|^2}{|V_{cb}|^2} D(s),
\]

where \( t = \frac{m_0^{\pm}}{m_0} \) and \( s = \frac{M_B^2 - \mu^+ \mu^-}{M_B^2} \). In SM, \( C_7(m_b) \) and \( C_8^{\text{eff}}(m_b) \) have opposite sign, so the term \( \text{Re}(C_7(m_b)C_8^{\text{eff}}(m_b)) \) decreases the overall results above. In MSSM, there exist some parameter regions, where supersymmetry contributions to \( C_7 \) can have opposite sign compared with \( C_7^{\text{SM}}(m_b) \), and even make the sign of \( C_7(m_b) \) opposite to \( C_7^{\text{SM}} \) but its size approximately equal to \( C_7^{\text{SM}}(m_b) \). In this case the term \( \text{Re}(C_7(m_b)C_8^{\text{eff}}(m_b)) \) adds constructively to \( \frac{d\text{Br}(B \to X_s \mu^+ \mu^-)}{ds} \), which can make the Br of \( B \to X_s \mu^+ \mu^- \) enhanced by approximately 50% compared to that in SM \([15]\). From the formula above, we can see that with large \( C_{Q_{1,2}}^{(f)} \), \( B \to X_s \mu^+ \mu^- \) can be enhanced greatly.

The branching ratio \( B_s \to \mu^+ \mu^- \) is given as

\[
\text{Br}(B_s \to \mu^+ \mu^-) = \frac{G_F^2 \alpha_{\text{em}}}{64 \pi^3} m_{B_s}^3 \tau_{B_s} f_{B_s}^2 |\lambda_3| 2 \sqrt{1 - 4m_b^2[(1 - 4m_b^2)]} |C_{Q_1}(m_b) - C_{Q_1}'(m_b)|^2 + |C_{Q_1}(m_b) - C_{Q_2}(m_b) + 2\hat{m}(C_{10}(m_b) - C_{10}'(m_b))|^2.
\]

where \( \hat{m} = m_\mu/m_{B_s} \). With large \( C_{Q_{1,2}}^{(f)} \), \( \text{Br}(B_s \to \mu^+ \mu^-) \) can be enhanced by several order of magnitude.

### 3 Squark mass matrices

The \( 6 \times 6 \) squark mass-squared matrices in the super-CKM basis have the structure

\[
\mathcal{M}_S^2 = \begin{pmatrix}
\mathcal{M}_{qL}^2 & \mathcal{M}_{qL}^{2LL} \\
\mathcal{M}_{qL}^{2QR} & \mathcal{M}_{qR}^2 & \mathcal{M}_{qR}^{2RL} & \mathcal{M}_{qR}^{2RR}
\end{pmatrix}
\]

\( \delta_2^{\text{LL}(RL)} \) contributions to \( C_{Q_{1,2}}^{(f)} \) from self-energy type and penguin diagrams are cancelled provided that \( m_{A^0} = m_{\mu^0} \) and MIA is valid.
where $\tilde{q} = \tilde{u}(\tilde{d})$ represent the up (down)-type squark. Note that differently from $M_{qL,R}^2$, $M_{qL,R}^2$ is not hermitian. In general, $M_{qL,R}^2$ are off-diagonal. The 3 × 3 submatrices are given by

$$
\begin{align*}
(M_{qL}^2)_{ij} &= (m_{qL}^2)_{ij} + \delta_{ij}(m_{qL}^2 + \cos 2\beta m_2^Z (I_q^3 - Q_q \sin^2 \theta_W)) \\
(M_{qR}^2)_{ij} &= (m_{qR}^2)_{ij} + \delta_{ij}(m_{qR}^2 + Q_q \cos 2\beta m_2^Z \sin^2 \theta_W) \\
(M_{uL}^2)_{ij} &= v_U A_{uij} - \delta_{ij} m_{ui} \mu \cot \beta \\
(M_{dL}^2)_{ij} &= v_D A_{dij} - \delta_{ij} m_{di} \mu \tan \beta
\end{align*}
$$

(17)

where $m_q$ with $q = d, u$ is the quark mass of generation $i$, $I_q^3$ and $Q_q$ are the third component of weak isospin and electric charge of quark $q$ respectively, $\mu$ is the Higgs superfield mixing parameter, $A_{uij}$, $A_{dij}$ are trilinear higgs-squark-squark coupling.

Because of SU(2) gauge invariance, the squark mass matrix $m_{qL}^2$ is intimately connected to $m_{dL}^2$ via

$$
m_{qL}^2 = V_{CKM}^\dagger m_{dL}^2 V_{CKM}
$$

(18)

Then, from $m_{dL}^2$, we can get $m_{qL}^2$, and vice versa.

Furthermore, we assume for the sake of simplicity that there are no new CP-violating phases, besides the single CKM phase. Thus, we in general have twenty seven new flavor violation parameters totally from squark mass matrices. However, only five of them are involved in the transition $b \to s$ in our analysis (see below).

Because we concentrate on the $b \to s$ transition, only left-left, right-right and left-right $2 \to 3$ mixing terms are directly dependent. In order to simplify the analysis we shall keep only these $2 \to 3$ mixing terms non-zero and set all the $1 \to 2$ and $1 \to 3$ mixing terms to 0. We also keep the third generation $3 \to 3$ left-right mixing term non-zero. The first generation $1 \to 1$ and second generation $2 \to 2$ left-right mixing terms are set to 0 for simplicity. We parametrize the non-vanishing $2 \to 3$ off-diagonal term as

$$
\begin{align*}
(m_{dL}^2)_{23} &= \delta_{23}^{dL} \sqrt{(M_{dL}^2)_{22}(M_{dL}^2)_{33}}, \\
(M_{dL}^2)_{23} &= \delta_{23}^{dR} \sqrt{(M_{dL}^2)_{22}(M_{dL}^2)_{33}}, \\
(m_{qL}^2)_{23} &= \delta_{23}^{UL} \sqrt{(M_{qL}^2)_{22}(M_{qL}^2)_{33}}
\end{align*}
$$

(19)

In the super-CKM basis the fields $\tilde{q}_{Li}, \tilde{q}_{Ri}$ ($i = 1, 2, 3$) are related to the mass eigenstates $\tilde{q}_a$ ($a = 1, \ldots, 6$) by

$$
\tilde{q}_{L,R} = \Gamma_{qL,R}^\dagger \tilde{q}_a,
$$

(20)

Where the matrix $\Gamma_{qL,R}^\dagger$ is a $3 \times 6$ mixing matrix.

In order to simplify the discussion further, we assume all the diagonal elements of squark mass matrices are equal to a common SUSY scale at electroweak scale, $m_q = m_{\tilde{u}_{Li}} = m_{\tilde{d}_{Li}} = m_{\tilde{d}_{Li}} = m_{\tilde{d}_{Li}}$ ($i = 1, 2, 3$). Then we have three flavor conserved parameter $m_{\tilde{g}}, \delta_{23}^{UL}, \delta_{23}^{UL}$ and five flavor violation parameters involved in the $b$ to $s$ transition, $\delta_{23}^{UL}, \delta_{23}^{UL}, \delta_{23}^{UL}$, $\delta_{23}^{UL}, \delta_{23}^{UL}$, from the squark mass matrices. In addition to these parameters, we have also the following free parameters: gaugino masses $M_i$ ($i=1,2,3$), CP-odd Higgs boson mass $M_{A_0}$, $\mu$, and tan $\beta$.

4 Numerical analysis of Wilson coefficients $C_{10}^{(f)}$ and $C_{Q_{1,2}}^{(f)}$

As analyzed in section 2, $C_{10}^{(f)}$ from the Z penguin and $C_{Q_{1,2}}^{(f)}$ from neutral Higgs boson penguins can be significantly enhanced. In this section, we present our numerical analysis of Wilson coefficients $C_{10}^{(f)}$ and $C_{Q_{1,2}}^{(f)}$ versus flavor violating parameters under experimental bounds, particularly $\text{Br}(B \to X_s \gamma)$, in the case of switching on only the gluino (or neutralino) and SM contributions in the first three subsections (fourth section), and then
switching on all the contributions from the W boson, charged Higgs, chargino, neutralino, and gluino in the last subsection. We show there are two cases, the heavy gluino, and fine-tuning (i. e., to finely tune $\delta^{dLL(RR)}$ and $\delta^{dLR(RL)}$ terms in $C_i^{(t)}$ makes the constraint from $b \rightarrow s\gamma$ satisfied) to enhance $C_9^{(t)}$. In general $C_{9}^{(t)}(m_b)$ in MSSM is enhanced by at most five percent compared to SM. We can see the reason why it can not have a large enhancement from the expressions of $C_9^{(t)}$ in the Appendix. The gamma penguin and box contributions to $C_9^{(t)}$ are suppressed by a factor of $m_b^2/m_\tilde{g}^2, \tilde{\epsilon}^\delta$. And the Z- penguin contributions to $C_9^{(t)}$ are suppressed by ($-1 + 4s^2_w$). So we shall not discuss its dependence on new flavor violation parameters in most of part of the numerical analysis hereafter. In our numerical analysis we use the expressions of Wilson coefficients obtained by the VM method, i. e., those given in Appendix. In order to simplify our notation and express new physics effects we introduce the following quantities,

$$R_i = \frac{C_i^{\text{mass}}(m_w) - C_i^{\text{sm}}(m_w)}{C_i^{\text{sm}}(m_w)}, \quad R'_i = \frac{C_i^{\text{mass}}(m_w) - C_i^{\text{sm}}(m_w)}{C_i^{\text{sm}}(m_w)}$$

where $C_i^{\text{mass}}$ is the Wilson coefficient of the operator $O_i^{(t)}$ in MSSM and $C_i^{\text{sm}}$ in SM, and $i = 7, 8, 9, 10$.

The experimental measurements of mass differences in $B^0_s - B^0$ system give the following bound [34]

$$\Delta M_s \geq 14.4 \text{ps}^{-1}.$$  

(22)

We consider the constraints of $\Delta M_s$ on $\delta$'s as in ref. [36]. Current experimental measurements of inclusive decay $B \rightarrow X_s\gamma$ at ALEPH [6], CLEO [7] and BELLE [8] produce the world average value

$$\text{Br}(B \rightarrow X_s\gamma)^{\text{exp}} = (3.23 \pm 0.41) \times 10^{-4}.$$  

(23)

We use $2\sigma \text{Br}(B \rightarrow X_s\gamma)$ bound in our numerical analysis.

### 4.1 Heavy gluino contributions to $C_9^{(t)}$

As we have noticed in section 2, though $C_7^{(t)}$ has $\frac{m_a}{m_b}$ enhancement, it still suffers from $\frac{m_b^2}{m_\tilde{g}^2}$ suppression. Then $C_7^{(t)}$ is suppressed by $\frac{1}{m_\tilde{g}}$, as presented in Eq.(7). Nevertheless the Z penguin contribution to $C_9^{(t)}$ is non-decoupled when $m_\tilde{g}$ is large. Therefore, we expect that when the gluino is heavy, the $b \rightarrow s\gamma$ constraint can be easily satisfied and a large enhancement of $C_9^{(t)}$ can occur. Including, in addition to the SM contribution, only the gluino contribution, we show $\delta^{dLL}_{23}, \delta^{dLR}_{23}, \delta^{dRR}_{23}$, and $\delta^{dRL}_{23}$ dependences of $C_9^{(t)}$ respectively, i. e., in each case only one non-zero off-diagonal parameter enters, in Fig. 1, where $M_\tilde{g} = M_\tilde{f} = 800 \text{GeV}, M_1 = 100 \text{GeV}, M_2 = 1200 \text{GeV}, M_3 = 3000 \text{GeV}, \mu = 3200 \text{GeV}$ and $\tan \beta = 50$. The choice of large $\mu$ and $\tan \beta$ is to ensure large left-handed and right-handed sbottom mixing, $\delta^{dLL}_{23} = -0.75$.

In Fig. 1.a, we give $R_10$ as a function of $\delta^{dLL}_{23}$. When $\delta^{dLL}_{23}$ is near 0.6, $R_10$ is the order of 10%, where the sign of $C_7(m_b)$ is flipped, and the lightest sbottom mass is near the low bound obtained in experiments. We show $R_10$ as a function of $\delta^{dLL}_{23}$ in Fig. 1.b. $R_10$ can also reach the order of 10%.

When we switch on only $\delta^{dRR}_{23}$ or $\delta^{dRL}_{23}$, we assume some mechanism to render $C_7(m_b)$ to be 0, e. g., some non-zero $\delta^{dLL}_{23}$ or $\delta^{dLR}_{23}$ contribution to $C_7(m_w)$ nearly cancels the SM contribution. We present $R_10$ as a function of $\delta^{dRR}_{23}$ in Fig. 1.c. When $\delta^{dRL}_{23}$ is near $-0.6$, $R_10$ can be as large as 7%. $R_10$ as a function of $\delta^{dRL}_{23}$ are shown in Fig. 1.d. When $\delta^{dRL}_{23}$ is near $-0.15$, $R_10$ can be as large as 7%.

### 4.2 Fine-tuning of $\delta^{dLL(RR)}_{23}$ and $\delta^{dLR(RL)}_{23}$ terms in $C_7^{(t)}$ and gluino contributions to $C_9^{(t)}$

It is obvious from Eq.(7) that $C_7^{(t)}$ can be finely tuned to zero with the large cancellation between $\delta^{dLL(RR)}_{23}$ and $\delta^{dLR(RL)}_{23}$ terms. However, in this case the $\delta^{dLL(RR)}_{23}$ and $\delta^{dLR(RL)}_{23}$ contributions to $C_9^{(t)}$ can be large.

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The analysis in ref. [36] is based on the old experimental bound, $\Delta M_s \geq 15.0 \text{ps}^{-1}$ [35]. An update analysis in MSSM is in progress [37].
We show the correlations of $\delta_{23}^{dLL}$ vs $\delta_{23}^{dLR}$ and $R_9$ vs $R_{10}$ in Fig. 2.a, where fine-tuning of $\delta_{23}^{dLL}$ and $\delta_{23}^{dLR}$ terms in $C_7$ have been carried out. The other parameters are $M_\tilde{q} = 500$GeV, $M_3 = 500$GeV, $\mu = 1200$GeV, and tan $\beta = 50$. Large $\mu$ and tan $\beta$ is to ensure large sbottom mixing $\delta_{23}^{dLR} = -0.72$. We find that the largest $R_{10}$ is 40%. However, when we impose the constraints from $B \to X_s g$ and hadronic charmless B decays which require $|R_8|$ cannot be larger than 10 [38, 16], $R_{10}$ can only be as large as 8%.

In order to analyze $\delta_{23}^{dRR(RL)}$, we set $C_7(m_b)$ to 0 and keep $C_9(m_b)$ and $C_{10}(m_b)$ to be their corresponding SM values. We make the fine-tuning of $\delta_{23}^{dRR}$ and $\delta_{23}^{dRL}$ in $C_7'$ so that the constraint from $b \to s\gamma$ is satisfied. With $M_\tilde{q} = 500$GeV, $M_3 = 500$GeV, $\mu = 1200$GeV, tan $\beta = 50$, and sbottom mixing $\delta_{23}^{dLR} = -0.72$, the gluino contributions to $R_9$ and $R_{10}'$ are shown in Fig. 2.b. $R_{10}'$ can reach 40% and 8% without and with the constraints from $B \to X_s g$ and hadronic charmless B decays respectively.

### 4.3 Gluino contributions to $C_{Q1,2}^{(t)}$

As stressed in section 2, when the CP-odd Higgs is not too heavy and tan $\beta$ is large, neutral Higgs contribute significantly via $C_{Q1,2}^{(t)}$ to the process (1). As we have stated before, only $\delta_{23}^{dLL}$ have sizable contribution to $C_{Q1,2}^{(t)}$. The contribution from $\delta_{23}^{dRR(RL)}$ can be neglected due to the cancellation between contributions of the soft energy type and penguin diagrams. Because the loop functions decrease when the mass of gluino increases, gluino contributions to $C_{Q1,2}^{(t)}$ in the heavy gluino case are less significant than those in the not too heavy gluino case. So in this subsection we consider only the case where gluino is not too heavy.

We show in Fig. 4.a the gluino contribution to $C_{Q1}$ vs $\delta_{23}^{dLL}$ with $M_3 = 500$GeV, $M_{A0} = 250$GeV, $\mu = 800$GeV, tan $\beta = 40$, and sbottom left-right mixing $\delta_{23}^{dLR} = -0.38$. The other parameters are $M_2 = 500$GeV and the SU(5) gaugino mass relation at electroweak scale $M_2 : M_1 : M_3 = 1 : 2 : 7$ for simplicity. $C_{Q1}(m_b)$ can be as large as 2.5, when $\delta_{23}^{dLL}$ is near 0.04. We can see that due to the constraint of Br($B \to X_s \gamma$), $\delta_{23}^{dLL}$ is restricted to the order of 0.04.

In order to analyze $C_{Q1,2}'$, we set $C_7(m_b)$ and $C_8(m_b)$ to 0, and keep $C_9(m_b)$ and $C_{10}(m_b)$ unchanged from their SM values. We show in Fig. 4.b the gluino contribution to $C_{Q1}'$ vs $\delta_{23}^{dRR}$ with $M_\tilde{q} = 500$GeV, $M_{A0} = 250$GeV, $M_3 = 500$GeV, $\mu = 800$GeV, tan $\beta = 40$, and sbottom mixing $\delta_{23}^{dLR} = -0.38$. In the figure the solid and dot curves (which denote the gluino contribution and all contributions respectively) are almost overlapping. $C_{Q1}'(m_b)$ can be as large as 3.6, when $\delta_{23}^{dRR}$ is near 0.06.

### 4.4 Neutralino contributions to $C_{10}^{(t)}$ and $C_{Q1,2}^{(t)}$

In the heavy gluino case, because we choose $M_1 = 100$GeV, the lightest neutralino should be light so that it contributes to $R_{10}'$ greatly and constructively with the gluino contribution, as can be seen from the dot curve in Fig. 1 which corresponds the case of including all contributions. We now turn to the fine-tuning case. In the case neutralinos can also have large effect as long as the lightest neutralino is light enough. We show in Fig. 3.a the correlation of $\delta_{23}^{dLL}$ vs $\delta_{23}^{dLR}$ and $R_9$ vs $R_{10}$. With $M_\tilde{q} = 500$GeV, $M_1 = 80$GeV, $M_2 = 300$GeV, $\mu = 1200$GeV, tan $\beta = 50$, sbottom mixing $\delta_{23}^{dLR} = -0.72$, $R_{10}$ can reach 10%. And $R_9$ can reach only 3%, as expected. We show the neutralino contributions to $R_9$ and $R_{10}$ and the correlation $\delta_{23}^{dRR}$ vs $\delta_{23}^{dRL}$ in Fig. 3.b with the same values of parameters as those in Fig. 3.a. $R_{10}'$ can reach the order of 6%. And $R_9$ can reach only 2%.

Neutralinos can also have large contributions to $C_{Q1,2}^{(t)}$. The neutralino contribution to $C_{Q1}$ vs $\delta_{23}^{dLL}$, with $M_\tilde{q} = 500$GeV, $M_{A0} = 250$GeV, $M_1 = 100$GeV, $M_2 = 300$GeV, $M_3 = 1000$GeV, $\mu = 800$GeV, tan $\beta = 40$, and sbottom mixing $\delta_{23}^{dLR} = -0.38$, is shown in Fig. 5. The $C_{Q1}(m_b)$ can be as large as 1.5, when $\delta_{23}^{dLL}$ is near $-0.7$. It is too hard for the non-zero $\delta_{23}^{dLL}$ to generate sizable neutralino contributions to $C_{Q1,2}'$ under the severe Br($B \to X_s \gamma$) experimental constraint.
4.5 Wilson coefficients $C^{(l)}_{10}$ and $C^{(l)}_{Q_{1,2}}$ in MSSM

In this subsection we switch on all the contributions of charged gauge boson $W^\pm$, charged Higgs $H^\pm$, chargino $\tilde{\chi}^\pm$, neutralino $\tilde{\chi}^0$ and gluino $\tilde{g}$ and all the $\delta s$, $\delta_{23}^{dLL}$, $\delta_{23}^{dLR}$, $\delta_{23}^{dRR}$, and $\delta_{23}^{dRL}$, under the experimental constraints. Specifically, we are interested in calculating the maximum enhancements of the Wilson coefficients $C^{(l)}_{10}$ and $C^{(l)}_{Q_{1,2}}$ that SUSY can provide, which is important to discriminate the small tan $\beta$ and large tan $\beta$ scenarios by the measurement of $\text{Br}(B_s \to \mu^+ \mu^-)$.

First, we limit ourself to the case of only one non-zero $\delta_{23}^{dAB}$ and including all the contributions under the experimental constraints. The results are presented in Figs. 1, 4 and 5. The dependences of $C^{(l)}_{10}$ on $\delta$s are shown by dot curves in Fig. 1. It is clear that the allowed ranges of $\delta_{23}^{dLL(RR)}$ and $R_{10}^{(l)}$ are reached 15%. The dependences of $C^{(l)}_{Q_{1,2}}$ on $\delta$s are shown by dot curves in Fig. 4 and Fig. 5 with different values of gaugino masses. In the case of $M_3 = 500$ GeV, because non-zero $\delta_{23}^{dLL}$ can induce scalar up-type quark 2–3 left-left mixing, the chargino can have large contributions and $C^{(l)}_{Q_{1,2}}$ can reach about 3.5, as shown in the Fig. 4.a. As for the dependence on $\delta_{23}^{dRR}$, not like the non-zero $\delta_{23}^{dLL}$ case, both the chargino and neutralino contributions are small, as shown in the Fig. 4.b. In the case of $M_3 = 1000$ GeV, both chargino and gluino contributions are important, $C_{Q_{1,2}}$ can reach about $-2$, and the dependence on $\delta_{23}^{dRR}$ is quite different from that in the case of switching on only the neutralino contribution, as shown in Fig. 5.

Next, we switch on all the contributions and all the $\delta s$, $\delta_{23}^{aLL}$, $\delta_{23}^{dLL}$, $\delta_{23}^{dLR}$, $\delta_{23}^{dRR}$, and $\delta_{23}^{dRL}$, under the experimental constraints. We perform a Monte Carlo scan of the parameter space with the ranges, $|\delta_{23}^{aLL}| \leq 1$, $|\delta_{23}^{dLL(RR)}| \leq 1$, $|\delta_{23}^{dLL(RR)}| \leq 0.02$. There are three gaugino masses $M_{1,2,3}$ which are free parameters in MSSM. For simplicity, we employ the SU(5) gaugino mass relation at the electroweak scale $M_Z$, $M_1 : M_2 : M_3 = 1 : 2 : 7$ and take $M_3 = 1000$ GeV in the scan. We fix $M_{\tilde{q}}, \mu, A_{(u,d),33}$ with $M_{\tilde{q}} = 500$ GeV, $\mu = 500$ GeV, $A_{u,33} = 250$ GeV, and $A_{d,33} = 0$ and consider two cases, small tan $\beta = 4$ and large tan $\beta = 50$. The results of the scan are presented in correlations, $R_{10}$ vs $R'_{10}$ and $C_{Q_1}$ vs $C'_{Q_1}$, in Fig. 6. In the small tan $\beta = 4$ case, $R_{10}$ can reach 12%, $R'_{10}$ is nearly zero, and $|C_{Q_1}(m_{\tilde{b}})|$ and $|C'_{Q_1}(m_{\tilde{b}})|$ are smaller than 0.1 in some parameter region due to the smallness of tan $\beta$. In the large tan $\beta$ case, $R_{10}$ can reach 20%, $R'_{10}$ can reach 1.5%, and $C_{Q_1}(m_{\tilde{b}})$ and $C'_{Q_1}(m_{\tilde{b}})$ can be as large as 3.6 in some parameter region.

Only six Wilson coefficients, $C^{(l)}_{10}$, $C^{(l)}_{Q_{1,2}}$, affect $\text{Br}(B_s \to \mu^+ \mu^-)$. Because $C^{(l)}_{Q_{1,2}}$ cannot be large in the small tan $\beta$ case, the main contribution to $\text{Br}(B_s \to \mu^+ \mu^-)$ comes from $C^{(l)}_{10}$ which can have at most an enhancement of order of 20% compared with the Standard Model $C_{10}$. But in the large tan $\beta$ case, if the CP-odd Higgs boson is not too heavy, $C^{(l)}_{Q_{1,2}}$ can be as large as order of one, which strongly enhances $\text{Br}(B_s \to \mu^+ \mu^-)$ by a factor of $10^3 - 10^5$ depending on the values of parameters, as shown in next section.

5 Numerical results for $B_s \to \mu^+ \mu^-$ and $B \to X_s \mu^+ \mu^-$

In the numerical calculations, the following input parameters have been used:

- $\alpha_s(m_z) = 0.118$, $\alpha_s(m_b) = 0.215$, $\alpha_{em}(m_z) = \frac{1}{127}$, $\alpha_{em}(m_b) = \frac{1}{127}$, $\lambda_t = -0.038$,
- $V_{cb} = 0.04$, $\text{Br}(B \to X_c \ell \nu) = 0.11$, $m_{\mu} = 0.1057$ GeV, $m_{\mu}(m_z) = 2.9$ GeV, $m_{\mu}(m_b) = 4.2$ GeV,
- $m_{\text{pole}}^{\mu} = 175$ GeV, $m_{\text{pole}}^{\mu}/m_{\text{pole}}^{b} = 0.29$, $m_{B_s} = 5.370$ GeV, $f_B = 0.22$ GeV, $\tau_B = 1.46$ ps.

As can be seen from eq. (14), the branching ratio of $B_s \to X_s \mu^+ \mu^-$ depends on the sign of $C_7(m_b)$ and, as mentioned before, in MSSM there exist some parameter regions, where supersymmetric contributions to $C_7$ can make its sign opposite to $C_7^{\text{SM}}(m_b)$. Therefore, under the experimental constraint from the branching ratio of $B \to X_s \gamma$, two separate regions for the correlation between the branching ratios of the $B \to X_s \gamma$ and $B \to X_s \mu^+ \mu^-$ are allowed. One corresponds to the case in which the sign of $C_7(m_b)$ is the same as that in SM, and the other corresponds to the case when the sign of $C_7(m_b)$ is opposite to that in SM, which is similar to the results given in ref. [15]5. Our numerical results verify the analysis.

5In ref. [15] the neutral Higgs boson contributions to the branching ratio of $B_s \to \mu^+ \mu^-$ in the large tan $\beta$ case was missed since they set the mass of a muon equal to zero.
As an illustration, in Tab. 1 we present numerical results of \( \text{Br}(B_s \to \mu^+\mu^-) \) and \( \text{Br}(B \to X_s\mu^+\mu^-) \) for the two set of parameter values which are in the region of the parameter space where the sign of \( C_7(m_b) \) is the same as that in SM. The other parameters which are not given in the table are \( M_3 = 1000\text{GeV} \) and the SU(5) gaugino mass relation at the electroweak scale \( M_Z, M_1 : M_2 : M_3 = 1 : 2 : 7, M_{\tilde{q}} = 500\text{GeV} \), and \( \mu = 500\text{GeV} \).

| \( \tan\beta \) | \( \delta_{23}^{uLR}/\delta_{23}^{dLR} \) | \( C_{10}/C_{10}' \) | \( C_{Q_1}/C_{Q_1}' \) | \( C_{Q_2}/C_{Q_2}' \) | \( C_7/C_7' \) | \( C_8/C_8' \) | \( C_9/C_9' \) |
|---|---|---|---|---|---|---|---|
| A | 0.79/−0.046/0.0039 | 0.52/0.002 | −0.0011/0.0013 | 0.0008/0.0014 | 4.28 \times 10^{-6} | 4.29 \times 10^{-6} |
| B | 0.68/−0.044/0.018 | 5.5/0.036 | −0.37/−0.0065 | 0.40/0.16 | 0.14/0.076 | 4.39/−0.0055 |
| 50 | 0.34/0.10 | 2.0/−2.7 | 0.15/0.06 | 4.3/−0.003 |

One can see from the table that in case B, i.e., the large \( \tan\beta \) case, \( \text{Br}(B_s \to \mu^+\mu^-) \) can be enhanced by a factor of \( 10^3 \), compared to SM. But in the case A where \( \tan\beta \) is small, \( \text{Br}(B_s \to \mu^+\mu^-) \) is the same as that in SM. These two case can be discriminated at Tevatron Run II. If the observed \( \text{Br}(B_s \to \mu^+\mu^-) \) is larger than the Standard Model expectation value by a factor of 10 or larger and one assumes that new physics is SUSY, then this will unambiguously signal the large \( \tan\beta \) case. As for the semileptonic decay \( \text{Br}(B \to X_s\mu^+\mu^-) \), there is a 50% enhancement for the values of the set of parameters in the large \( \tan\beta \) case, which is closer to the central value of the experiment result, (2), than SM.

### 6 Conclusions

We have examined pure leptonic and semileptonic rare B decays, \( B_s \to \mu^+\mu^- \) and \( B \to X_s\mu^+\mu^- \), under \( \text{Br}(B \to X_s\gamma) \) and other experimental constraints in the minimal supersymmetric Standard Model. In particular, we have in detail analyzed the dependence of the relevant Wilson coefficients on new flavor violation parameters, off-diagonal scalar quark mass matrix elements. We find that under all the relevant experimental constraints, if only the gluino and SM contributions are included and assuming large mixing of left-handed and right-handed sbottom, \( C_{10}' \) can be enhanced by a factor of 10% in two cases, the heavy gluino and fine-tuning between \( \delta_{23}^{uLR}(\delta_{23}^{dLR}) \) and \( \delta_{23}^{uRL}(\delta_{23}^{dRL}) \). In particular, it is found that \( C_{10}' \) can be enhanced by 40% compared with SM in the fine-tuning case under all experimental constraints except for that from \( B \to X_s\mu^+\mu^- \). When all the contributions are included and all experimental constraints are taken into account, \( C_{10}' \) can be enhanced at most by 20% compared with SM. When CP-odd Higgs \( A^0 \) is not too heavy (\( \sim 250\text{GeV} \)), and \( \tan\beta \) is large (\( \sim 40 \)), neutral Higgs boson penguins with gluino and down-type squark in the loop can significantly contribute to the \( h \) transition and the contributions can compete with those due to the chargino and up-type squark loop. Comparing with the constrained MSSM, the Wilson coefficient \( C_{10}' \) can reach a larger value due to the gluino and neutralino contributions, but the largest value of \( C_{Q_1,2}' \) allowed by all the experimental constraints is of the same order. From the above results, the following conclusions can be drawn:

**A.** Although a 20% enhancement of the Wilson coefficient \( C_{10}' \), compared to SM, which is twice of that in CMSSM can be reached in MSSM, it alone is far from the explanation of data if \( \text{Br}(B_s \to \mu^+\mu^-) = 2 \times 10^{-8} \) will be observed at Tevatron run II.

**B.** \( C_{Q_1,2}' \) can reach order of one and order of 0.01 in the large and small \( \tan\beta \) case respectively and consequently can lead to an enhancement of \( \text{Br}(B_s \to \mu^+\mu^-) \) by a factor of \( 10^3 \) in the large \( \tan\beta \) case. Therefore, if \( \text{Br}(B_s \to \mu^+\mu^-) \geq 10^{-8} \) is observed, there should be new physics and \( \tan\beta \) must be large if new physics is the MSSM.
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Wilson coefficients $C_t(m_w), C'_t(m_w)$ ($i = 7, 8, 9, 10, Q_{i,2}$) of gluino contribution are as follows

$$C^x_{7}(m_w) = - \frac{1}{72 \lambda_t} G^{LSS}_{d_{\alpha} \bar{u}_{\beta} \chi_{i}} \frac{m_{\tau}^2}{m_{\chi_i}} (G^{Lb}_{u_{\alpha} \bar{u}_{\beta} \chi_{i}} (3 f_1(x_{\bar{u}_{\alpha} \chi_i}) + 2 f_2(x_{\bar{u}_{\alpha} \chi_i})) + 8 G^{Rb}_{u_{\alpha} \bar{u}_{\beta} \chi_{i}} \frac{m_{\chi_i}}{m_{b}} (3 f_3(x_{\bar{u}_{\alpha} \chi_i}) + f_4(x_{\bar{u}_{\alpha} \chi_i})))$$

$$C^0_{7}(m_w) = \frac{1}{72 \lambda_t} G^{Rss}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} \frac{m_{\tau}^2}{m_{\chi_i}} (G^{Lb}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} f_2(x_{\bar{d}_{\alpha} \chi_i}) + 4 G^{Rb}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} \frac{m_{\chi_i}}{m_{b}} f_4(x_{\bar{d}_{\alpha} \chi_i}))$$

$$C^0_{7}(m_w) = \frac{1}{72 \lambda_t} G^{Rss}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} \frac{m_{\tau}^2}{m_{\chi_i}} (G^{Lb}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} f_2(x_{\bar{d}_{\alpha} \chi_i}) + 4 G^{Rb}_{d_{\alpha} \bar{d}_{\beta} \chi_{i}} \frac{m_{\chi_i}}{m_{b}} f_4(x_{\bar{d}_{\alpha} \chi_i}))$$
\[ C_7^\bar{g} (m_w) = \frac{1}{27 \lambda g d_{a \bar{g}}} G_{L^s} m_w^2 G_{d_{a \bar{g}}} f_2(x_{a \bar{g}}) + 4 G_{R_{d_{a \bar{g}}} m_b} f_4(x_{a \bar{g}}) \]

\[ C_8^\bar{g} (m_w) = \frac{1}{27 \lambda g d_{a \bar{g}}} G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} f_2(x_{a \bar{g}}) + 4 G_{L_{d_{a \bar{g}}} m_b} f_4(x_{a \bar{g}}) \]

\[ C_8^\bar{g}_{w} (m_w) = -\frac{1}{24 \lambda} G_{L^s} m_w^2 G_{d_{a \bar{g}}} f_2(x_{a \bar{g}}) + 4 G_{R_{d_{a \bar{g}}} m_b} f_4(x_{a \bar{g}}) \]

\[ C_8^\bar{g}_{w} (m_w) = -\frac{1}{24 \lambda} G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} f_2(x_{a \bar{g}}) + 4 G_{L_{d_{a \bar{g}}} m_b} f_4(x_{a \bar{g}}) \]

\[ C_8^\bar{g}_{w} (m_w) = \frac{1}{27 \lambda g d_{a \bar{g}}} G_{L^s} m_w^2 (9 f_1(x_{a \bar{g}}) + f_2(x_{a \bar{g}}) + 8 G_{R_{d_{a \bar{g}}} m_b} (9 f_3(x_{a \bar{g}}) + \frac{1}{2} f_4(x_{a \bar{g}})) \]

\[ C_8^\bar{g}_{w} (m_w) = \frac{1}{27 \lambda g d_{a \bar{g}}} G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} f_2(x_{a \bar{g}}) + 4 G_{L_{d_{a \bar{g}}} m_b} f_4(x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = -\frac{1}{36 \lambda} G_{d_{a \bar{g}}} G_{d_{a \bar{g}}} m_w^2 (9 f_5(x_{a \bar{g}}) - 2 f_6(x_{a \bar{g}})) \]

\[ C_9^\bar{\chi}_{\gamma} = -\frac{1}{36 \lambda} G_{L^s} m_w^2 f_6(x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = -\frac{1}{36 \lambda} G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} m_w^2 f_6(x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = -\frac{2}{27 \lambda g d_{a \bar{g}}} G_{L^s} G_{d_{a \bar{g}}} m_w^2 f_6(x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = -\frac{2}{27 \lambda g d_{a \bar{g}}} G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} m_w^2 f_6(x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = \frac{1}{16 \lambda s_w^2} (-1 + 4 s_w^2) G_{L^s} G_{d_{a \bar{g}}} \left[ -3 \delta_{ij} (\Gamma_{\alpha \beta}^{d_{a \bar{g}}} \Gamma_{\alpha \beta}^{d_{a \bar{g}}} + f_6(x_{a \bar{g}}, x_{a \bar{g}})) \right] \]

\[ + \delta_{ij} (2 (U_{11} U_{33}^*) f_0(x_{a \bar{g}}, x_{a \bar{g}}) + 3 (V_{12} V_{13}) f_0(x_{a \bar{g}}, x_{a \bar{g}})) \]

\[ C_9^\bar{\chi}_{\gamma} = \frac{1}{16 \lambda s_w^2} (-1 + 4 s_w^2) G_{R_{d_{a \bar{g}}} m_b} G_{d_{a \bar{g}}} \left[ -3 \delta_{ij} (\Gamma_{\alpha \beta}^{d_{a \bar{g}}} \Gamma_{\alpha \beta}^{d_{a \bar{g}}} + f_6(x_{a \bar{g}}, x_{a \bar{g}})) \right] \]

\[ + \delta_{ij} (4 G_{d_{a \bar{g}}} m_w^2 f_0(x_{a \bar{g}}, x_{a \bar{g}})) - 6 G_{d_{a \bar{g}}} m_w^2 f_0(x_{a \bar{g}}, x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = \frac{1}{16 \lambda s_w^2} (-1 + 4 s_w^2) G_{d_{a \bar{g}}} G_{d_{a \bar{g}}} \left[ -3 \delta_{ij} (\Gamma_{\alpha \beta}^{d_{a \bar{g}}} \Gamma_{\alpha \beta}^{d_{a \bar{g}}} + f_6(x_{a \bar{g}}, x_{a \bar{g}})) \right] \]

\[ + \delta_{ij} (4 G_{d_{a \bar{g}}} m_w^2 f_0(x_{a \bar{g}}, x_{a \bar{g}})) - 6 G_{d_{a \bar{g}}} m_w^2 f_0(x_{a \bar{g}}, x_{a \bar{g}}) \]

\[ C_9^\bar{\chi}_{\gamma} = \frac{1}{6 \lambda s_w^2} \frac{g^2}{g^2} G_{d_{a \bar{g}}} G_{d_{a \bar{g}}} \left[ -3 (\Gamma_{\alpha \beta}^{d_{a \bar{g}}} \Gamma_{\alpha \beta}^{d_{a \bar{g}}} + f_6(x_{a \bar{g}}, x_{a \bar{g}})) \right] \]
\[
C_{9, \text{box}}^{\pm} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{9, \text{box}}^{0} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{9, \text{box}}^{0, \prime} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{10, z}^{\pm} = \frac{-1}{1 - 4s_{\omega_{\text{w}}}} C_{9, \text{box}}^{\pm}
\]

\[
C_{10, z}^{0} = \frac{-1}{1 - 4s_{\omega_{\text{w}}}} C_{9, \text{box}}^{0}
\]

\[
C_{10, z}^{0, \prime} = \frac{-1}{1 - 4s_{\omega_{\text{w}}}} C_{9, \text{box}}^{0, \prime}
\]

\[
C_{10, z}^{\prime, \prime} = \frac{-1}{1 - 4s_{\omega_{\text{w}}}} C_{9, \text{box}}^{\prime, \prime}
\]

\[
C_{10, \text{box}}^{0} = -C_{9, \text{box}}^{0}
\]

\[
C_{10, \text{box}}^{0} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{10, \text{box}}^{0, \prime} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{10, \text{box}}^{0, \prime, \prime} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{10, \text{box}}^{0, \prime, \prime} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

\[
C_{10, \text{box}}^{0, \prime, \prime} = \frac{1}{2 \lambda_s \omega_{\text{w}}} \mathcal{G}_{Ls}^{Ls} G_{L_i}^{L_b} \left[ \frac{1}{6} \delta_{ij} m_w^2 \mathcal{G}_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right] \delta_{d \alpha} \delta_{d \beta} \left( G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} - G_{\mu_i}^{L_t} G_{\mu_j}^{L_t} \right) f_{d0}(x_{d_a}, x_{d_b}, x_{d_c}, x_{d_d})
\]

(25)
\[ C_{Q_1,\text{box}} = \frac{1}{2\lambda s_w} G_{d'_w} G_{u_w} L_{s_w} \frac{m_{m_w}}{m_{d'_w}} \left( \frac{m_{X_\alpha}}{m_{Z_0}} \right) \left( -G_{Rf} - G_{bf} \cos \alpha \right) f_{e_0} (x_{d'_w}, x_{Z_0}) \]

\[ C^0_{Q_1} = \frac{1}{2\lambda s_w} G_{d'_w} G_{u_w} L_{s_w} \frac{m_{m_w}}{m_{d'_w}} \left( \frac{m_{X_\alpha}}{m_{Z_0}} \right) \left( -G_{bf} \cos \alpha + G_{bf} \sin \alpha \right) f_{e_0} (x_{d'_w}, x_{Z_0}) \]

\[ C^0_{Q_1} = \frac{1}{2\lambda s_w} G_{d'_w} G_{u_w} L_{s_w} \frac{m_{m_w}}{m_{d'_w}} \left( \frac{m_{X_\alpha}}{m_{Z_0}} \right) \left( -G_{bf} \cos \alpha + G_{bf} \sin \alpha \right) f_{e_0} (x_{d'_w}, x_{Z_0}) \]

\[ C^0_{Q_1} = \frac{1}{2\lambda s_w} G_{d'_w} G_{u_w} L_{s_w} \frac{m_{m_w}}{m_{d'_w}} \left( \frac{m_{X_\alpha}}{m_{Z_0}} \right) \left( -G_{bf} \cos \alpha + G_{bf} \sin \alpha \right) f_{e_0} (x_{d'_w}, x_{Z_0}) \]
\[ C_{Q_2, box} = \frac{1}{2} \left( \frac{c_{L_R}^2}{c_{L_S}^2} \right) G_{S_S} G_{D_S} \left[ \frac{1}{6} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} - \frac{1}{2} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} \right] f_{\Delta 0} (x_{Q_2}, x_{Q_2} \chi_{Q_2}, x_{Q_2} \chi_{Q_2}) \]

\[ C_{Q_2} = \frac{1}{2} \left( \frac{c_{L_R}^2}{c_{L_S}^2} \right) G_{S_S} G_{D_S} \left[ \frac{1}{6} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} - \frac{1}{2} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} \right] f_{\Delta 0} (x_{Q_2}, x_{Q_2} \chi_{Q_2}, x_{Q_2} \chi_{Q_2}) \]

\[ C_0 = \frac{4}{3} \left( \frac{c_{L_R}^2}{c_{L_S}^2} \right) G_{S_S} G_{D_S} \left[ \frac{1}{6} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} - \frac{1}{2} \frac{m_c^2}{m_{Q_2}^2} G_{S_S} G_{D_S} G_{R_S} G_{D_S} \right] f_{\Delta 0} (x_{Q_2}, x_{Q_2} \chi_{Q_2}, x_{Q_2} \chi_{Q_2}) \]

where \( r_s = \frac{m_s^2}{m_{Q_2}^2}, r_p = \frac{m_p^2}{m_{Q_2}^2} \) and \( x_{ij} = m_{ij}^2 / m_{Q_2}^2, s_w = \sin \theta_w, c_w = \cos \theta_w \). We have checked \( C_{7,8,9,10} \) with ref. [15], our results agree with them except there is a minus sign difference with their box diagram. \( C_{Q_2} \) calculated here agree with ref. [25].

The one-loop functions are normalized to be 1, if all the arguments are equal and set to 1.

\[
\begin{align*}
 f_1(x) &= 2(1 - 6x + 3x^2 - 2x^3) / (1 - x)^4 \\
 f_2(x) &= 2(1 - 6x + 3x^2 - 2x^3) / (1 - x)^4 \\
 f_3(x) &= 3(1 - 4x + 3x^2 - 2x^3) / (2 - x^3) \\
 f_4(x) &= 3(1 - 2x + 3x^2 - 2x^3) / (1 - x)^3 \\
 f_5(x) &= 2(7 - 36x + 75x^2 - 16x^3) / (1 - x)^4 \\
 f_6(x) &= 2(-11 + 18x - 9x^2 + 3x^3 - 6x^4) / (1 - x)^4 \\
 f_{\Delta 0}(x) &= -x \ln x / (1 - x) \\
 f_{\Delta 0}(x, y) &= -2(1 - (1 - x)(y - x))/(1 - y)(y - x) \\
 f_{\Delta 0}(x, y) &= -2(1 - (1 - x)(y - x))/(1 - y)(y - x) \\
 f_{\Delta 0}(x, y, z) &= 6(1 - (1 - x)(y - x)/(1 - y)(y - x))/ (1 - z)(z - x)(z - y) \\
\end{align*}
\]
\[ f_{390}(x, y, z) = -3 \left[ \frac{x^2 \ln x}{(1-x)(x-y)(x-z)} + \frac{y^2 \ln y}{(1-y)(y-x)(y-z)} + \frac{z^2 \ln z}{(1-z)(z-x)(z-y)} \right] \] (27)

We follow the conventions of Haber and Kane [39], to present our Feynman rules and mass matrices of chargino, neutralino, squark, and sleptons. The chargino mass matrix \( X \) is diagonalized by two matrix \( U \) and \( V \), with \( U^*XV^{-1} = \text{diag}(m_{\tilde{c}_1}^2, m_{\tilde{c}_2}^2) \). The neutralino mass matrix \( Y \) is diagonalized by matrix \( N \), \( N^*YN^{-1} = \text{diag}(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2, m_{\tilde{\chi}_3}^2, m_{\tilde{\chi}_4}^2) \), where \( m_{\tilde{\chi}_i}(i = 1, 2, 3, 4) \) are positive.

The chargino(gluino, neutralino)-squark-quark couplings are as follows:

\[
\mathcal{L} = g \sum_{i=1, \alpha = 1}^{2, 6} \tilde{\chi}_i^\alpha \bar{u}_i^a (G^L_{\alpha, \tilde{\chi}_i} P_L + G^R_{\alpha, \tilde{\chi}_i} P_R) d + g \sum_{i=1, \alpha = 1}^{4, 6} \tilde{\chi}_i^\alpha \bar{d}_i^a (G^L_{\alpha, \tilde{\chi}_i} P_L + G^R_{\alpha, \tilde{\chi}_i} P_R) d + H.C.
\]

\[
G^L_{\tilde{u}_a, \tilde{\chi}_i} = -V^*_1 \Gamma^L_{\alpha, \tilde{u}_a} + V^*_2 \Gamma^R_{\alpha, \tilde{u}_a} \frac{m_{\tilde{u}_a}}{\sqrt{2m_w s_\beta}} \text{K}_2, \quad G^R_{\tilde{u}_a, \tilde{\chi}_i} = 0
\]

\[
G^L_{\tilde{d}_a, \tilde{\chi}_i} = \sqrt{2} \left(-\frac{1}{6} t_w N^*_1 \alpha + \frac{1}{2} N^*_2 \Gamma^L_{\cal{d}_a} \alpha_{2} \right), \quad G^R_{\tilde{d}_a, \tilde{\chi}_i} = -\sqrt{2} \frac{1}{3} t_w N_1 \Gamma^R_{\cal{d}_a} \alpha_{2}
\]

The chargino(neutralino)-lepton-slepton couplings are:

\[
\mathcal{L} = g \sum_{i=1, \alpha = 1}^{2, 3} \tilde{\chi}_i^\alpha \bar{\nu}_i^a G^L_{\tilde{\chi}_i} P_L \mu + \sum_{i=1, \alpha = 1}^{4, 6} \tilde{\chi}_i^\alpha \bar{\nu}_i^a (G^L_{\tilde{\chi}_i} P_L + G^R_{\tilde{\chi}_i} P_R) \mu
\]

\[
G^L_{\tilde{\chi}_i} = -V^*_1 \Gamma^L_{\tilde{\chi}_i}, \quad G^L_{\tilde{\chi}_i} = \sqrt{2} \left(\frac{1}{2} t_w N^*_1 \alpha + \frac{1}{2} N^*_2 \right) \Gamma^L_{\tilde{\chi}_i}
\]

The Z-squark-squark couplings are:

\[
\mathcal{L} = -\frac{g}{c_w} Z \left( \tilde{\chi}_i^\alpha \nu^\alpha \tilde{\nu}_i^a \tilde{\chi}_i^\beta \right) G^z_{\tilde{\chi}_i \tilde{\nu}_i}
\]

\[
G^z_{\tilde{\chi}_i \tilde{\nu}_i} = \frac{1}{2} \left( \Gamma^L_{\alpha, \tilde{\chi}_i} \Gamma^L_{\beta, \tilde{\nu}_i} - \frac{8}{3} s_w^2 \delta_{i, \beta} \right)
\]

The Z-chargino(neutralino)-chargino(neutralino) couplings are:

\[
\mathcal{L} = -\frac{g}{c_w} Z \left( \tilde{\chi}_i^\alpha \nu^\alpha \tilde{\nu}_i^a \tilde{\chi}_i^\beta \right) G^z_{\tilde{\chi}_i \tilde{\nu}_i}
\]

\[
G^z_{\tilde{\chi}_i \tilde{\nu}_i} = \frac{1}{2} \left( \Gamma^L_{\alpha, \tilde{\chi}_i} \Gamma^L_{\beta, \tilde{\nu}_i} - \frac{8}{3} s_w^2 \delta_{i, \beta} \right)
\]

\[
G^{Lz}_{\tilde{\chi}_i \tilde{\chi}_j} = -\frac{1}{2} U_{ij} U_{j1} + \delta_{ij} (s_w^2 - \frac{1}{2}), \quad G^{Rz}_{\tilde{\chi}_i \tilde{\chi}_j} = -\frac{1}{2} V_{ij} V_{j1} + \delta_{ij} (s_w^2 - \frac{1}{2})
\]

\[
G^{Lz}_{\tilde{\chi}_i \tilde{\chi}_j} = \frac{1}{2} N_{i3} N_{j3} - \frac{1}{2} N_{i4} N_{j4}, \quad G^{Rz}_{\tilde{\chi}_i \tilde{\chi}_j} = -G^{Lz*}_{\tilde{\chi}_i \tilde{\chi}_j}
\] (31)
The Higgs-chargino(neutralino)-chargino(neutralino) couplings are

\[ \mathcal{L} = -g\tilde{\chi}_j \left[ h^0 \left( G^{Lh}_{\chi_j \chi_j} P_L + G^{Rh}_{\chi_j \chi_j} P_R \right) + H^0 \left( G^{LH}_{\chi_j \chi_j} P_L + G^{RH}_{\chi_j \chi_j} P_R \right) \right] \tilde{\chi}_j + \text{h.c.} \]

\[ + i g \tilde{\chi}_j \left[ G^{\alpha} \left( G^{LG}_{\chi_j \chi_j} P_L + G^{RG}_{\chi_j \chi_j} P_R \right) + A^0 \left( G^{LA}_{\chi_j \chi_j} P_L + G^{RA}_{\chi_j \chi_j} P_R \right) \right] \tilde{\chi}_j \]

\[ + \frac{1}{2} g \tilde{\chi}_j \left[ h^0 \left( G^{Lh}_{\chi_j \chi_j} P_L + G^{Rh}_{\chi_j \chi_j} P_R \right) + H^0 \left( G^{LH}_{\chi_j \chi_j} P_L + G^{RH}_{\chi_j \chi_j} P_R \right) \right] \tilde{\chi}_j + \text{h.c.} \]

where \( s_{\alpha} = \sin \alpha \), \( c_{\alpha} = \cos \alpha \). The Higgs-squark-squark couplings are

\[ \mathcal{L} = -g\tilde{q}_i^{\dagger} t^0 \left( G^{t0}_{q_i q_j} \tilde{q}_j + i g\tilde{q}_i^{\dagger} \left( G^{t0}_{q_i q_j} + A^0 \right) \tilde{q}_j \right) + \text{h.c.} \]
Figure 1: Dependence of $R_{10}^{(1)}$ on $\delta_{23}^{dLL,dLR(dRR,dRL)}$. Gluino and SM contributions are denoted by solid curves, and dot curves denote all the contribution. The other parameters are $M_{\tilde{q}} = 800\text{GeV}$, $M_3 = 3000\text{GeV}$, $M_2 = 1200\text{GeV}$, $M_1 = 100\text{GeV}$, $\mu = 3200\text{GeV}$ and $\tan \beta = 50$. 
Figure 2: Correlations between $\delta_{23}^{dLL}$ and $\delta_{23}^{dLR}$ and between $R_9^{(i)}$ and $R_{10}^{(i)}$ switching on only gluino and SM contributions to $R_9^{(i)}$ and $R_{10}^{(i)}$ in the fine-turning case. The other parameters are $M_4 = 500\text{GeV}$, $M_3 = 500\text{GeV}$, $\mu = 1200\text{GeV}$, and $\tan \beta = 50$. Triangle points are ruled out by $B \to X_s g$ and hadronic charmless $B$ decays which require $|R_8| \leq 10$. 
Figure 3: Correlations between $\delta_{23}^{dLL(dRR)}$ and $\delta_{23}^{dLR(dRL)}$ and between $R_9^{(\prime)}$ and $R_{10}^{(\prime)}$ switching on only neutralino and SM contributions to $R_9^{(\prime)}$ and $R_{10}^{(\prime)}$ in the fine-turning case. The other parameters are $M_1 = 500\text{GeV}$, $M_1 = 80\text{GeV}$, $M_2 = 300\text{GeV}$, $\mu = 1200\text{GeV}$, and $\tan \beta = 50$. 
Figure 4: Dependence of $C^{(r)}_{Q_1}(m_b)$ on $\delta_{23}^{d_{LL}(d_{RR})}$. Gluino and SM contributions are denoted by solid curves, and dot curves denote all the contributions. The parameters are $M_{\tilde{q}} = 500\text{GeV}, M_{A^0} = 250\text{GeV}, \mu = 800\text{GeV}$ and $\tan \beta = 40$ as well as $M_3 = 500\text{GeV}$ and the SU(5) gaugino mass relation at the electroweak scale $M_Z$, $M_1 : M_2 : M_3 = 1 : 2 : 7$.

Figure 5: Dependence of $C_{Q_1}(m_b)$ on $\delta_{23}^{d_{LL}}$. The solid curve denotes neutralino and SM contributions, and the dot curve denotes all the contributions. The other parameters are $M_{\tilde{q}} = 500\text{GeV}, M_{A^0} = 250\text{GeV}, M_1 = 100\text{GeV}, M_2 = 300\text{GeV}, M_3 = 1000\text{GeV}, \mu = 800\text{GeV}$ and $\tan \beta = 40$. 
Figure 6: Correlations between $R_{10}$ vs $R'_{10}$ and between $C_{Q1}$ vs $C'_{Q1}$, switching on all the contributions and all $\delta_{23}^{qAB}$s. (a) and (b) for $\tan\beta=4$ and 50 respectively. The other parameters are $M_{\tilde{q}}=500\text{GeV}$, $\mu=500\text{GeV}$, $M_3=1000\text{GeV}$ and the SU(5) gaugino mass relation at the electroweak scale $M_Z$, $M_1 : M_2 : M_3 = 1 : 2 : 7$. 
