THE COMBINED EFFECTS OF TWO-BODY RELAXATION PROCESSES AND THE ECCENTRIC KOZAI-LIDOV MECHANISM ON THE EMRI RATE

Smadar Naoz\textsuperscript{1,2}, Sannea C. Rose\textsuperscript{1,2}, Erez Michaeley\textsuperscript{1,2}, Denyz Melchor\textsuperscript{1,2}, Enrico Ramirez-Ruiz\textsuperscript{3}, Brenna Mockler\textsuperscript{3}, Jeremy D. Schnitman\textsuperscript{4,5}

Draft version March 23, 2022

ABSTRACT

Gravitational wave (GW) emissions from extreme-mass-ratio inspirals (EMRIs) are promising sources for low-frequency GW-detectors. They result from a compact object, such as a stellar-mass black-hole (BH), captured by a supermassive black hole (SMBH). Several physical processes have been proposed to form EMRIs. In particular, weak two-body interactions over a long time scale (i.e., relaxation processes) have been proposed as a likely mechanism to drive the BH orbit to high eccentricity. Consequently, it is captured by the SMBH and becomes an EMRI. Here we demonstrate that EMRIs are naturally formed in SMBH binaries. Gravitational perturbations from an SMBH companion, known as the eccentric Kozai-Lidov (EKL) mechanism, combined with relaxation processes, yield a significantly more enhanced rate than any of these processes operating alone. Since EKL is sensitive to the orbital configuration, two-body relaxation can alter the orbital parameters, rendering the system in a more EKL-favorable regime. As SMBH binaries are expected to be prevalent in the Universe, this process predicts a substantially high EMRI rate.

1. INTRODUCTION

Extreme-mass-ratio inspirals (EMRIs) arise from the capture of a stellar mass compact object by a supermassive black hole (SMBH). The Gravitational Wave (GW) emission from such a system is expected to be at the mHz band, thus a promising signal for the Laser Interferometer Space Antenna (LISA), as well as other mHz detectors, such as TianQin (e.g., Pan & Yang 2021).

Observations of AGN pairs, which are typically few kpc (and more) apart, suggest that these configurations may lead to the formation of SMBH binaries with sub-parsec separations (e.g., Komossa et al. 2003, Bianchi et al. 2008). Consequently, it is captured by the SMBH and becomes an EMRI. Here we demonstrate that EMRIs are naturally formed in SMBH binaries. Gravitational perturbations from an SMBH companion, known as the eccentric Kozai-Lidov (EKL) mechanism, combined with relaxation processes, yield a significantly more enhanced rate than any of these processes operating alone. Since EKL is sensitive to the orbital configuration, two-body relaxation can alter the orbital parameters, rendering the system in a more EKL-favorable regime. As SMBH binaries are expected to be prevalent in the Universe, this process predicts a substantially high EMRI rate.

Of particular interest here is the formation of EMRIs in SMBH binaries. Thanks to the hierarchical nature of galaxy formation, and since almost every galaxy hosts a SMBH at its center, SMBH binaries are expected to be a common phenomenon (e.g., Di Matteo et al. 2005, Hopkins et al. 2006, Robertson et al. 2006, Callegari et al. 2009, Li et al. 2020). Observations of AGN pairs, which are typically few kpc (and more) apart, suggest that these configurations may lead to the formation of SMBH binaries with sub-parsec separations (e.g., Komossa et al. 2003, Bianchi et al. 2008). A similar process is often considered for the formation of EMRIs (e.g., Bode & Wegg 2014, Haster et al. 2014), which can lead to the production of tidal disruption events (e.g., Chen & Liu 2013, Generozov & Madigan 2020). A SMBH companion gravitationally perturbs the orbit of a stellar-mass BH via the Eccentric Kozai-Lidov mechanism (EKL, e.g., Kozai 1962, Lidov 1962, Naoz 2016, see latter for review). These perturbations can result in extreme eccentricities (e.g., Li et al. 2014b, Naoz & Silk 2014), which can lead to the formation of EMRIs (e.g., Bode & Wegg 2014, Haster et al. 2016). A similar process is often considered for the production of tidal disruption events (e.g., Chen et al. 2008, 2009, 2011, Chen & Liu 2013, Li et al. 2015). A SMBH companion gravitationally perturbs the orbit of a stellar-mass BH via the Eccentric Kozai-Lidov mechanism (EKL, e.g., Kozai 1962, Lidov 1962, Naoz 2016, see latter for review). These perturbations can result in extreme eccentricities (e.g., Li et al. 2014b, Naoz & Silk 2014), which can lead to the formation of EMRIs (e.g., Bode & Wegg 2014, Haster et al. 2016). A similar process is often considered for the production of tidal disruption events (e.g., Chen et al. 2008, 2009, 2011, Chen & Liu 2013, Li et al. 2015).

The EKL approach often neglects collective dynamical interaction because these interactions operate on much longer timescales. In particular, relaxation by gravitational encounters typically takes place on such long time scales, compared to other physical processes (see below), and thus often is neglected when considering EKL processes. Here we show that the combined effect of EKL and relaxation processes enhances the EMRI formation efficiency more than any of these processes operating alone. Furthermore, the two-body relax-
two-body relaxation and EKL, where we consider the relative importance of the two-body relaxation processes compared to EKL. We emphasize that the physical processes described below are scalable beyond the fiducial system adopted here. In particular, we expect that two-body relaxation will play a critical role in the EKL process of a population of stars and a wide range of compact object masses surrounding SMBHs. The stellar mass BHs ($m_*$) density profile $\rho(r_*)$ is calibrated by the $M-\sigma$ relation (Tremaine et al. 2002):

$$\rho(r_*) = \frac{3 - \alpha}{2\pi} \frac{m_1}{r_*^3} \left( \frac{Gm_1 M_0}{\sigma_0 r_*^3} \right)^{-3+\alpha},$$

where $M_0 = 10^8 M_\odot$, $r_0 = 200$ km sec$^{-1}$, are scaling factors. Below we adopt a Bahcall & Wolf (1976) profile, i.e., $\alpha = 1.75$. Note that these values have been slightly modified recently (e.g., van den Bosch 2016; McConnell & Ma 2013). However, it does not affect the underlying physical processes described below, and only may slightly change the relaxation timescale (see Section 2.4).

Each BH ($m_*$) undergoes eccentricity and inclination excitations due to the far away SMBH companion ($m_2$) according to the EKL mechanism. Additionally, general relativity effects induce precession and can also circularize and shrink the orbit through gravitational wave (GW) emission. Finally, collective relaxation interactions with the sea of objects in the sphere of influence tend to change the angular momentum and energy of the orbit by an order of themselves over long timescales. Below we specify these different physical processes and outline the methodology of including them in our analysis.

2.3. General relativity and Gravitational Waves

The 1st post Newtonian effects induced by $m_1$ cause $m_*$ to precess on a characteristic timescale

$$t_{\text{1PN}} \sim \frac{P_\bullet c^2 a_0 (1-e^2)}{6\pi G (m_1+m_*)},$$

where $e$ is the speed of light. When this timescale is shorter than the quadrupole timescale from Equation 2 eccentricity excitations are typically suppressed (e.g., Ford et al. 2000; Naoz et al. 2013b; Will & Maitra 2017; Lim & Rodriguez 2020). However, when these two timescales are similar, the precession may excite eccentricities and even re-trigger the EKL behaviour of extreme eccentricity and inclination flips.

Different population may result in slightly different density profiles, see Aharon & Perets (2016).
by destabilizing the quadrupole level resonance (see, Hansen & Naoz 2020). The timescales of \( m_\bullet \) for a fiducial system are shown in Figure 1.

We include in our calculations both 1st post Newtonian effects from the primary \( m_1 \) and the secondary \( m_2 \). As mentioned in Naoz & Silk (2014) and Li et al. (2015), we choose to focus on the BHs around the less massive SMBH to minimize the part of the parameter space in which 1st pN precession suppresses the EKL’s eccentricity excitations. As we highlight below, in the presence of two-body relaxation this suppression is minimized.

In addition to 1pN precession we also include the shrinking and circularization of the stellar BH orbit due to gravitational wave emission following Peters & Mathews (1963). The characteristic timescale to merge an EMRI is:

\[
\tau_{GW} \approx \frac{5.8 \times 10^9 \text{yr}}{\left( \frac{m_1}{10^9 \text{M}_\odot} \right)^2 \left( \frac{m_\bullet}{10 \text{ M}_\odot} \right)^{-1} \left( \frac{a_\bullet}{10^{-4} \text{ pc}} \right)^4} \times f(e_\bullet)(1-e_\bullet^2)^{3/2},
\]

(4)

where \( f(e_\bullet) \) is a function of \( e_\bullet \) and for all values of \( e_\bullet \) is between 0.979 and 1.81 (Blaes et al. 2002). We show this timescale for our fiducial system in Figure 1 for \( e_\bullet = 0.9 \).

2.4. Two-body relaxation

Scattering relaxation interactions of a target black hole with the sea of objects are modeled by considering the two-body relaxation timescale (e.g., Binney & Tremaine 2008):

\[
\tau_{\text{relx}} = 0.34 \frac{\sigma^2}{GM_1^2 \rho(m_{\text{scat}}) \ln \Lambda},
\]

(5)

where \( \langle m_{\text{scat}} \rangle \) is the mass of the average scatterer, \( \sigma \) is the velocity dispersion of BHs around the SMBH

\[
\sigma^2 = \frac{GM_1}{r_\bullet(1+\alpha)},
\]

(6)

where \( \alpha \) is the slope of the density profile. The coulomb logarithm is:

\[
\Lambda = \frac{r_\bullet \sigma^2}{2Gm_\bullet},
\]

(7)

For simplicity we adopt \( \langle m_{\text{scat}} \rangle \approx m_\bullet \). However, if \( \langle m_{\text{scat}} \rangle < < m_\bullet \), mass segregation may migrate the BHs inwards.

The relaxation time from Equation (5), is the timescale for a change of energy of the stellar mass BH around the SMBH \( m_1 \) by an order of its orbital energy, or a change in angular momentum by an order of its circular angular momentum. We show the relaxation timescale in Figure 1 (solid black line on top), which for large part of the parameter space is much larger than the EKL timescale. As mentioned, this motivated many studies to ignore the contribution of two-body relaxation when considering EKL effects.

The typical change in the BH’s velocity \( v_\bullet = \sqrt{Gm_1(2/r_\bullet-1/a_\bullet)} \) due to one encounter is:

\[
\Delta v = v_\bullet \sqrt{\frac{P_\bullet}{\tau_{\text{relx}}}}.
\]

We model this change as a random walk, applying a single isotropically oriented kick to the BH velocity once per orbit around the SMBH. Each directional component of this 3D kick is drawn from a Gaussian distribution with a zero average and a standard deviation of \( \Delta v/\sqrt{3} \) (see Bradnick et al. 2017).

![Figure 2. Time Evolution of an example system in the presence of different physical processes.](Image)

We show, from top to bottom, a stellar mass black hole separation around an SMBH, inclination with respect to the outer perturber, argument of perihelion, and longitude of ascending nodes. Left side: We consider a stellar mass black hole \( (m_\bullet = 10 \text{ M}_\odot) \) orbiting an SMBH \( (m_2 = 10^9 \text{ M}_\odot) \), at \( a_\bullet = 8000 \text{ au} \), initially with \( e_\bullet = 0.02 \), \( \omega_\bullet = 45^\circ \), \( \Delta = 10^7 \). We also consider a population of stellar mass black holes around \( m_1 \), following a Bahcall & Wolf (1976) profile (i.e., \( \alpha = 1.75 \)). We normalize the density profile according to the \( m-\sigma \) relation, (see Equation (1)), which result in two-body relaxation timescale of \( \tau_{\text{relx}} \sim 3.5 \times 10^{7} \) yrs. We show the resulting orbital evolution of the stellar black hole in the thick red line. We also introduce a binary SMBH with mass \( m_2 = 10^9 \text{ M}_\odot \) set on 1 pc separation, with eccentricity of \( e_\bullet = 0.7 \). The evolution that includes both the two-body relaxation and the EKL from the outer orbit (as well as GR precession on the inner orbit) is shown in thin blue line. As depicted this system reached extreme eccentricities induced by a combination of two-body relaxation and EKL and pushed toward the SMBH, producing a GW source.

We also consider the case of which we ignore the contribution of two-body relaxation processes and consider only the EKL (+GR) in light grey. This system never reached high eccentricity to become an EMRI. At the right side we consider the same system, only this time we arbitrary increased the relaxation timescale to \( 4.3 \times 10^{11} \) yrs (by assuming scatter masses of \( 5 \times 10^{-3} \text{ M}_\odot \)). As depicted this system qualitatively follows the EKL (+GR) behaviour.

for a similar approach for binaries around a single SMBH). We assume that the kick is instantaneous at some random phase of the BH’s orbit

\[
r_\bullet = \frac{a_\bullet(1-e_\bullet^2)}{1+e_\bullet \cos f_\bullet},
\]

where \( f_\bullet \) is the true anomaly. Thus, the vector \( \vec{r}_\bullet \) in the invariable plane \( \vec{r} \) can be considered constant during the encounter. See appendix A for full set of the two-body relaxation equations.

3. Dynamical evolution

3.1. Example system and revisiting the time-scale argument

Note that the system evolves due to EKL and thus we need to project the separation vector on the invariable plane. For similar analysis see (e.g., Lu & Naoz 2019).
The EKL mechanism tends to excite high eccentricities and inclination. However, only about 30% of the parameter space in the aforementioned configuration is available to reach the extreme eccentricities needed to drive an object into the black hole, and cross its Schwarzschild radius (e.g., [Li et al. 2015]).

As an example, we consider in Figure 2 a system whose EKL eccentricity excitations do not result in values sufficient to cross the SMBH’s Schwarzschild radius (gray lines in both columns). For this system, the EKL timescale ($t_{EKL} \approx 1.4 \times 10^4$ yr) is shorter than the GR precession timescale ($t_{GR} \approx 6 \times 10^4$ yr).

However, as can be seen in Figure 2 left column, a two-body relaxation process combined with EKL results in aggravated EKL eccentricity and inclination excitations. We note that we include GR precession for the inner and outer orbit. The former suppresses the EKL eccentricity excitations when two-body relaxation is not included (gray lines). However, in this example we do not include GW emission. To avoid clutter, GW is included in the Monte Carlo analysis below. In this example (Figure 2 left column) we consider a black hole population with a Bahcall & Wolf (1976) distribution (i.e., $\alpha = 7/4$). The two-body relaxation timescale from Equation (5) is $\tau_{relx} \approx 3.5 \times 10^6$ yrs, well above the the EKL timescale (see also Figure 1 for this case it’s about four orders of magnitude larger). By definition, over the $\approx 1.5$ Myrs run, the relaxation timescale is insufficient to change the angular momentum by an order of itself (because the timescale is shorter than $t_{relx}$ in this case). However, the combined effect of two-body relaxation and EKL results in higher eccentricity and inclination amplitude modulations.

In fact, the eccentricity excitations were large enough to drive this stellar mass BH onto the SMBH, thus forming an EMRI. The higher eccentricity values reached are correlated with the BH semi-major-axis slightly drifting to higher values, due to two-body relaxation, thus getting closer to the secondary SMBH ($m_2$). This process yields a shorter EKL timescale (recall Eq. (2) dependency on the inner orbital period). Furthermore, as the inner orbit gets closer to the secondary SMBH, the octupole-level of approximation dominates more. This behaviour is expressed by the pre-factor of the octupole-level Hamiltonian $\epsilon$ (e.g., Lithwick & Naoz 2011a):

$$\epsilon = \frac{a_\bullet}{a_{bin}} \frac{\epsilon_{bin}}{1 - c_{bin}^2}$$

Thus, as $a_\bullet$ increases, so does $\epsilon$, which excites the eccentricity of the BH toward larger values (e.g., Li et al. 2014b).

The obvious questions from this result are why these diffusion processes create such a large effect, and will it always happen regardless the value of $t_{relx}$. The answers to both of these questions can be understood by examining Equation (8), which suggests that $h/\Delta h_{relx} \approx \sqrt{relx}$, where $h$ is the angular momentum and $\Delta h$ is the change of the angular momentum due to a small kick over the particle orbit around $m_1$. However, the angular momentum changes due to the EKL are $h/\Delta h_{EKL} \approx t_{EKL}$ (e.g., Naoz et al. 2013a). Thus, effectively, we should compare $\sqrt{relx}/P_{EKL}$ to $P_{EKL}/P_\bullet$. We show this comparison in Figure 1 bottom panel, where we compare $h/\Delta h$, due to the different processes. Using this picture, it is clearer that two-body relaxation is relevant to a large part of the parameter space.

In the example depicted in the left column of Figure 2, even though $h/\Delta h_{relx} > h/\Delta h_{EKL}$, it is only by a factor of 20, which yields this cumulative effect (examining the bottom panel in Figure 1 helps clarify the comparison between the two effects). About an order of magnitude difference can still lead to a significant cumulative effect. This behavior is similar to the way that GR precession destabilizes the quadrupole resonance, even when it’s timescale is much longer than the quadrupole level (e.g., Naoz et al. 2017). We note of course that for this system, two-body relaxation would have eventually change the energy and angular momentum of the orbit by an order of themselves, regardless of EKL. However, this does not guarantee an orbit that will plunge onto $m_1$. In our case we have adopted a Bahcall & Wolf (1976), i.e., $\alpha = 7/4$, which results in zero net flux, thus, the BHs are expected to undergo diffusion, but not preferentially migrate.

We emphasize that the two-body relaxation effect on the orbital configuration is indeed small compared to the long-term EKL eccentricity excitation. This is highlighted in Figure 2 for the two-body relaxation-only case (red lines), which does not excite the eccentricity to any meaningfully high values during the simulation run-time. Instead, the BH simply undergoes diffusion in its energy and angular momentum. However, since the EKL is sensitive to the orbital configuration, the diffusion in energy and angular momentum due to two-body relaxation can still contribute to large effects on the BH orbit. If the small changes in the orbit’s energy and angular momentum can cause a change of the angular momentum of about 10–15%, the effects on EKL are substantial.

For comparison, we consider the same system in Figure 2 (right column), only this time we artificially increased the relaxation timescale, for illustration purposes. In this example $\tau_{relx} \approx 4.3 \times 10^4$ yrs, which is also longer than the lifetime of the system, and the BH simply undergoes diffusion. As clearly depicted in the Figure, the diffusion in this system is insignificant and does not trigger larger EKL effects. Furthermore, in this example, we find that $h/\Delta h_{relx} \approx 700 \times h/\Delta h_{EKL}$. Thus, the relaxation effects, according to this comparison, results in a negligible change. In this panel, we again over-plot the two-body relaxation-only effect (+1pN), as shown by the thick red lines. Note that the apparent drift in $\omega$ in this case is due to the 1pN precession, a similar drift, is depicted in the left column, only modulated by the diffusion processes.

As depicted in the bottom two panels in Figure 3, the nominal suppression of eccentricity excitations due to 1pN precession does not take place. To guide the eye we have outline the $f_{EKL} \approx f_{1pN}$ line for a $\epsilon_\bullet = 2/3$. Indeed, without two-body relaxation processes, eccentricity excitations are suppressed in the presence of GR precession (e.g., Ford et al. 2000). However, the small kicks result in a diffusion, thus allowing the eccentricity excitation to take place over a wide range of the parameter space.

Lastly, a striking feature of Figure 2 is that in the presence of two-body relaxation the system moves in and out libration regime, not in-sync with EKL. The resonant angle, $\omega$, is known to change from libration to circulation in EKL (e.g., Li et al. 2014b). However, as depicted, the diffusion process changes these processes, even when the two-body relaxation effects are insignificant. These small kicks allow the (already chaotic) system to transfer zones.

### 3.2. Monte Carlo Proof-of-concept

As mentioned, two-body relaxation processes are often neglected when analyzing the EKL like systems. On the other
The stellar mass BH pericenter crossed a critical distance, which we adopt as $R_{\text{crit}} = 8g\sigma_{\text{BH}}/c^2$, (following Naoz & Silk 2014, Naoz et al. 2019), which is inside the inside the Kerr black hole’s inner-most retrograde stable orbit. These are represented by red points below the solid line. We label them as “GW sources.” In the EKL + GR, about 31% of all systems crossed the critical radius, while 50% (53%) of all systems in the EKL + GR + two-body relaxation (+GW) run have ended up as GW sources.

3. The BH semi-major axis changed due to two-body relaxation such that $\epsilon > 0.1$ (pink points, to the right of the dashed line). This is only possible when the two-body relaxation is turned on.

While it is clear that the systems whose pericenter crossed $R_{\text{crit}}$ are GW sources (i.e., EMRI candidates), it may be less obvious to understand what is the outcome of those with $\epsilon > 0.1$. We emphasize that this condition for hierarchy is based on the octupole pre-factor and therefore is somewhat arbitrary (e.g., Lithwick & Naoz 2011). Furthermore, it was suggested in Bhaskar et al. (2021) that violating this role often results in even higher eccentricities. Thus, we refer to those systems as possible EMRIs candidate as well.

We find that between $\approx 50 - 100\%$ of the BHs (corresponding to pericenter smaller than $R_{\text{crit}}$, to $\epsilon > 0.1$) become a GW source.

For peri-centers smaller than $R_{\text{crit}}$, Kerr geometry may cause the BHs to spend a lot of time on the SMBH’s ergosphere (Schmitman 2015) where GW emission may shrink their separations. Furthermore, special relativity effects should also be taken into account (Yunes et al. 2008, Berry & Gair 2013).

As can be seen from Figure 3, the combination of EKL with two-body relaxation allows the system to access a larger part of the parameter space, thus triggering the EKL mechanism. In general, the number of objects that undergo high eccentricity excitation depend on the density distribution (e.g., Li et al. 2011). Furthermore, it was suggested that the efficiency of the combined system will depend on the underlying density distribution (see Melchor et al. in prep.).

4. EMRI RATE ESTIMATION

The rate estimation is very sensitive to the steady state number of BHs around the SMBH. It varies over three orders of magnitude between the various assumptions for EMRIs formation processes (e.g., Freitag 2001, Hopman & Alexander 2005, Hopman 2009, Amaro-Soae et al. 2011, Aharon & Perets 2016, Bar-Or & Alexander 2016, Babak et al. 2017). Thus, here we aim to highlight the efficiency of the proposed mechanism by utilizing the $M - \sigma$ relation for the number of BHs. We then compare to similar approaches in the literature for the two-body relaxation process.

The EKL-only runs compared to the ones with two-body relaxation processes yield a significantly different flux of GW source formation. This is shown in the top panel of Figure 4 where a striking feature is the EKL (+GR)-only result. This feature is consistent with a “burst”-like behavior that depletes the stellar mass BHs, which could otherwise become

---

*Note that systems that crossed the Roche limit (or the Hill Sphere) of the secondary may also be considered as systems that descend toward the SMBH (either the primary or secondary) following Chen et al. (2008, 2009, 2011). Similar arguments were done for systems for which $\epsilon > 0.1$ (e.g., Bhaskar 2021). Furthermore, Zhang et al. in prep. showed that even in the case of this Roche limit crossing of a tertiary the system may not change its energy or angular momentum at the order of itself for long timescales. In other words, the system may still considered “stable” and the eccentricity may continue to increase via EKL. Thus, the combined effect of EKL + two-body relaxation processes may continue to occur for BHs for which $\epsilon > 0.1$, until resulting in possible EMRIs (see Appendix B).*
GW sources (a similar behavior was found for TDEs and dark matter particle depletion Li et al. 2015; Naoz & Silk 2014). Thus, for a relatively short time (6 × 10^6 yr, corresponding to the width of the distribution), the rate is high, but on the timescale it takes to replenish the stellar mass BH population, the rate is low. Replenishment of BHs can take place via mass segregation, which brings BHs in from the sphere of influence (e.g., Hopman & Alexander 2006). The corresponding timescale at the order of the two-body relaxation timescale up to a factor of the mass ratio between the BHs and the stars. Another source of replenishment is star formation, which for our galactic center is estimated to occur every few × 10^6 yr (Lu et al. 2013). Unlike the EK (+GR)-only result, the inclusion of two-body relaxation expands the timescales at which GW sources can form, thus allowing for the replenishment of stellar-mass BHs to take place. To estimate the number of black holes, n_{BH} (≤ r_k), within a distance r_{max}, we use the M − σ relation:

$$n_{BH}(≤ r_k) = f_{BH} M(≤ r_{max}) \langle m_* \rangle$$

where $M(≤ r_{max}) = f^{10} \rho(r') 4\pi r'^2 dr'$ and $\rho$ is the density profile form Equation (1). Furthermore, $\langle m_* \rangle$ is the average mass

Note that in these cases, during the long timescales the SMBH binary’s separation is expected to shrink, yielding an enhancement to the EMRI rate (e.g., Iwasa & Seto 2016). The inclusion of this effect is beyond the scope of this paper.

of the stars and $f_{BH}$ is the fraction of BHs from the overall stellar population, where we adopt $f_{BH} = 3.2 \times 10^{-3}$ (e.g., Aharon & Perets 2016). In our fiducial system $r_{max} = 0.07$ pc, which corresponds to the $\epsilon = 0.1$, and the number of BHs within this radius is about 330.

As highlighted in previous studies, it is straightforward to scale the system to a wide range of primary masses, for a constant mass ratio, while holding the quadrupole moment (Eq. 2) constant [10] and considering the number of BHs up to $r_{max}$, for $\epsilon = 0.1$ (e.g., Naoz & Silk 2014; Naoz et al. 2019). Thus, in Figure 4 bottom panel, we show the number of stellar mass BHs that are sunk onto the SMBH, for the run that includes all of the aforementioned physical processes. In this scaling, proof of concept, $r_{max}$ is then mass dependent and it takes the following form:

$$r_{max} = \left( \frac{15}{16} \right) \left( \frac{t_{EKL}}{q} \right) \left( \frac{q}{0.01} \right)^{2/3} \left( \frac{1 - e_{BH}^2}{0.7} \right) \left( \frac{m_1}{10^7 M_\odot} \right)$$

where $q = m_1/m_2$ is the mass ratio. We note that both in Figure 3 and below we refer to these objects as GW sources, and EMRIs.

The EMRI rate is then estimated by:

$$\Gamma \approx \Gamma_{EKL} \times f_{EKL} \times f_{EMRI} \times f_{BH}(≤ r_{max})$$

where $f_{EMRI}$ is the fraction of systems that may become an EMRI rather than a plunged orbit, $f_{EKL}$ is the fraction of systems that have their eccentricity excited to cross $R_{sch}$ and $\Gamma_{EKL}$ is the rate estimated in the simulation. We estimate the latter by calculating the average accretion rate and estimating ±68% of it from our fiducial simulations (i.e., taking 1σ of the accretion rate, estimated from Figure 2) and normalized to the range of primary masses as described above (see Figure 3 bottom panel). As highlighted in Figure 3, a large fraction of systems sink onto the SMBH when both EKL and two-body relaxation operate, i.e., $f_{EKL} \sim 0.5$ – 1.

In Appendix B we estimate the fraction of systems that are likely to appear within the LISA band ($f_{BH}$). Roughly speaking one divides between plunging orbits which may be characterized with a short GW burst and EMRIs that have many to a few cycles before merging with the SMBH (e.g., Rubbo et al. 2006; Yunes et al. 2008; Berry & Gair 2013; Rubbo et al. 2006; Yunes et al. 2008; Berry & Gair 2013 for further discussion). In the former case, special relativity correction may need to be included (e.g., Yunes et al. 2008). Additionally, we note that the pN treatment utilized here may break down around a rotating SMBH, because the stellar mass BHs are expected to spend a lot of time close to the SMBH’s ergosphere, before continuing on their original trajectory (Schnittman 2015). Thus, GW emission may alter their orbit. Therefore, the distinction between plunging and cycling orbit represents a larger problem in this field.

Based on the above distinction (see Appendix B for more details), we find that about 40% of the systems may be defined as EMRIs. In Appendix B we also present possible SNR of an example system. Note that the fraction of systems that may

Note that we limit our analysis to systems for which $f_{EKL} < t_{rec}$, to allow for a the behavior outlined in Figure 3 to take place.
Finally, we depict the EMRI rate for the number of BH limited up to \( r_{\text{max}} \), the critical replenishment time (dark dashed line). The latter is loosely estimated as the relaxation estimated rate from Equation (14). We compare to the EKL (+GR) of BHs. We consider the case which includes EKL (+GR) + two-body relaxation. For consistent comparison, we only include the MEHR as being naturally formed in SMBH binaries with higher eccentricity, as highlighted here two-body relaxation processes cannot be neglected (see for example Figure 2). In Figure 1 we show this rate for \( r_{\text{max}} \) (the sphere of influence), dashed (solid) line.

5. DISCUSSION

EMRIs are the result of an SMBH that captures a stellar-mass compact object, such as BH. Thus, these are some of the promising GW signals for low-frequency GW detectors such as LISA. Different channels have been suggested to form EMRIs. In particular, two-body relaxation has been proposed as one of the likely physical processes to form EMRIs efficiently. In this process, weak two-body kicks from the population of stars and compact object that surrounds the SMBH can change the BH’s orbit over time, driving it into the SMBH. On the other hand, perturbations from SMBH companions, via the EKL mechanism, can excite the SMBH to high eccentricities, thereby forming EMRIs. Here we demonstrated that EMRIs are naturally formed in SMBH binaries with higher efficiency than either of these processes considered alone.

In the presence of an SMBH companion, the EKL mechanism can excite the BHs eccentricity to high values. However, the EKL mechanism’s efficiency depends to some extent on the initial conditions (e.g., [14]). Therefore, the small kicks due to two-body relaxation do not need to accumulate to change the angular momentum by order of itself. Instead, they can change the orbital parameters of the stellar mass BH, such as eccentricity, semi-major axis, and argument of periastron, rendering it in a favorable EKL regime. We show an example of such as system in Figure 2. Even if the two-body relaxation timescale is orders of magnitude longer than the EKL timescale (see Figure 1), the small-kicks are effective as long as they result in a change of angular momentum comparable to that due to EKL. In particular, we suggest that \( h/\delta h_{\text{relx}} \) needs to be within a couple of orders of magnitude (or close to) \( h/\delta h_{\text{EKL}} \). If \( h/\delta h_{\text{relx}} >> h/\delta h_{\text{EKL}} \), the angular momentum change \( \delta h \) due to the two-body relaxation can be neglected (see for example Figure 2). In Figure 1 we highlight the proposed comparison between the two-body relaxation process and EKL, using \( h/\delta h \) rather than timescales.

In general, other collective processes may also be considered. For example, resonant relaxations (Rauch & Tremaine 1996), which arise from orbit-averaged mass distribution of the objects around the primary, can be added as well (e.g., Eilon et al. 2009, Kocsis & Tremaine 2011, Snidhar & Touma 2016, Touma et al. 2019). However, scalar and vector resonant relaxation processes modify the angular momentum.
\( \Delta h / h_{RR} \sim t_{\text{res}} / P_* \), thus using their timescales to estimate their contribution may not be as misleading as the aforementioned timescale analysis of the two-body relaxation (instead of using \( \Delta h / h \)). Vector resonant relaxation processes have been added recently to the EKL context and were shown to drive low-inclination configurations to a more EKL favorable regime (e.g., [Hamers et al. 2018]). However, the latter study concluded that overall, the combined effect is not very efficient in the context of BH-BH mergers. In contrast, as high-aspirations very efficiently.

\[ \text{Stellar black holes around } \mathcal{M}_{\odot}, \mathcal{M}_{\odot} \text{ on an eccentric orbit } e_{\text{bin}} = 0.7, \text{ at 1 pc separation. We begin by considering the effect of the EKL mechanism on stellar black holes around } m_1 \text{ (Figure 3 top panel). Note that all runs include the 1Pn contribution to the inner and outer orbit.} \]

Stellar-mass BHs whose pericenter distance passed a critical value are considered as EMRs. As noted in previous studies, the efficacy of this mechanism is about 30% (e.g., [Naoz & Silk 2014]). We then systematically add two-body relaxation (middle panel in Figure 5) and gravitational wave emission (bottom panel). As a result, the efficacy increased to \( 50-100\% \), meaning nearly all of the stellar mass BHs ended up descending into the SMBH, thereby possibly forming EMRs, within a few \( 10^{13} \) yr, after a single star formation burst, i.e., not including replenishment.

To highlight the efficiency of this scenario, we extrapolate the EMRI formation rate to different SMBHs. Since EMRIs rate is highly uncertain and is sensitive to the number of BHs as a function of time, we used the \( M-\sigma \) relation. Moreover, we rescale our fiducial example by keeping the quadrupole-level of the EKL approximation constant. This means a constant power law and a constant mass ratio and \( \Delta \mathcal{M} = 10^{-7} \mathcal{M}_{\odot} \) on an eccentric orbit \( e_{\text{bin}} = 1 \), \( a_{\text{bin}} = 2.2 \) pc. Furthermore, the number of BHs inside a sphere at which \( \epsilon \leq 0.1 \) varies accordingly, for \( m_1 = \mathcal{M}_{\odot} \), \( m_1 = 10^2 \mathcal{M}_{\odot} \). \( N_{\text{BH}} \approx 331 (N_{\text{BH}} \sim 1979) \). We depict the rescaling in Figure 2.

Even for this simple scaling, it is clear that having the entire population of BHs, (or even just 50\%) becoming EMRIs has large implications on the EMRI rate. We compare the predicted EMRI rate from this scenario to the prediction from two-body relaxation only in Figure 5. As depicted in this Figure, the EMRI rate in SMBH binaries is orders of magnitude larger than in isolated SMBHs. Additionally, the dependency on the SMBH mass is different, offering a potential way to disentangle between the different scenarios. Furthermore, because SMBH binaries are expected to be ubiquitous in the Universe, our results suggest that the EMRI rate may be much higher than nominal estimations. In particular, post star burst galaxies may be interesting candidates for enhanced EMRIs formation as they possibly host a SMBH binary. Moreover, this result suggests that the observed EMRI rate may be used to constrain the prevalence of SMBH binaries in the Universe.

We thank the referee for useful comments. SN acknowledges the partial support from NASA ATP 80NSSC20K0505 and thanks Howard and Astrid Preston for their generous support. SR thanks the Nina Byers Fellowship, the Charles E Young Fellowship, and the Michael A. Jura Memorial Graduate Award for support, as well as partial support from NASA ATP 80NSSC20K0505. EM acknowledges the support of s Howard and Astrid Preston, the Mani L. Bhaimik Institute for Theoretical Physics, and as well as partial support from NASA ATP 80NSSC20K0505. DM acknowledges the partial support from NSF graduate fellowship, the Eugene Cota-Robles Fellowship, and the NASA ATP 80NSSC20K0505. B.M. is grateful for the AAUW American Fellowship, and the UCSC Presidents Dissertation Fellowship. E-R.-R. and B.M. are grateful for support from the Packard Foundation, Heising-Simons Foundation, NSF (AST-1615881, AST-1911206 and AST-1852393), Swift (80NSSC21K1409, 80NSSC19K1391) and Chandra (GO9-20122X).

**Appendix**

**A: The Post Kick Orbital Parameters**

Consider a BH orbiting SMBH. In the plane of the ellipse we can define the separation vector as \( r_{\text{inv}} = r_\bullet (\cos f_\bullet, \sin f_\bullet, 0) \), where \( f_\bullet \) is the true anomaly and

\[ r_\bullet = \frac{a_\bullet (1 - e_\bullet^2)}{1 + e_\bullet \cos f_\bullet}. \tag{A1} \]

The associated velocity vector at the plane of the ellipse is: \( v_\bullet = h / a_\bullet (-\sin f_\bullet, e_\bullet \cos f_\bullet, 0) / (1 - e_\bullet^2) \). These vectors are projected onto the invariable plane, where in the case of test-particle EKL is simply the plane of the outer orbit (e.g., [Lithwick & Naoz 2011b]). Thus, we rotate the the separation and velocity vectors at each time-step given their argument of perihelion, \( \omega \), longitude of ascending nodes, \( \Omega \) and inclination \( i \). For example, and similarly for the velocity vector, we have:

\[ r_{\text{inv}, \ell} = R_{\ell}(\Omega)R_{\ell}(\Omega)\gamma R_{\ell}(\omega) r_{\text{inv}} \, , \tag{A2} \]

where the subscript “inv” and “ell” refer to the invariable and ellipse coordinate systems, respectively. Given a rotation angle \( \theta \), the rotation matrices \( R_x \) and \( R_y \) are

\[ R_{\ell}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A3} \]

---

\( \Delta \mathcal{M} \) Note that we do not include crossing terms (e.g., Naoz et al. 2013b, Lim 
& Rodriguez 2020), because their overall effect should be minimal in this configuration.
to find the orbital parameters. Specifically, the semi-major axis of the BH post-kick is:

\[ a_{p} = \left( \frac{2}{r_{*}} - \frac{v_{*p}^2}{Gm_{1}} \right)^{-1} \]

(A6)

The post-kick eccentricity is:

\[ e_{p} = \sqrt{1 - \frac{h_{p}^2}{Gm_{1}a_{p}}} \]

(A7)

Because the z axis is defined by the outer orbit, the new inclination is \( \cos i_{p} = h_{p,z}/h_{p} \), where \( h_{p,z} \) is the z component of the post-kick angular momentum.

The post-kick longitude of ascending nodes is:

\[ \Omega_{p} = \arctan_{2} \left( \frac{\pm h_{p,1} \mp h_{p,2}}{h_{p} \sin i_{p}} \right) \]

(A8)

The post-kick true anomaly is:

\[ f_{p} = \arctan_{2} \left( \frac{a_{p}(1-e_{p}^2)}{h_{p}e_{p}}, \frac{1}{e_{p}} \left[ \frac{a_{p}(1-e_{p}^2)}{r_{*}} - 1 \right] \right) \]

(A9)

where \( \hat{R} = \pm \sqrt{v_{*p}^2 - h_{p}^2/r_{*}} \), where the sign is defined by the sign of \( r_{*} \cdot v_{*p} \) (e.g. Murray & Dermott 2000). The post-kick argument of pericenter is then:

\[ \omega_{p} = \arctan_{2} \left( \frac{r_{*}^3}{r_{*} \sin i_{p}}, \frac{r_{*}^3 \sin \omega_{p} \cos i_{p}}{r_{*} \sin i_{p}} \right) \sec \Omega_{p} - f_{p} \]

(A10)

B: PLUNGING ORBITS AND AN EXAMPLE OF SIGNAL TO NOISE IN THE LISA BAND

We first differentiate between plunging orbits and EMRIs, where the former is described as a burst associated with their pericenter passage. Our adopted stopping condition of \( \hat{R}_{sh} = 8Gm_{1}/c^2 \) means that beyond this threshold the BH trajectory will be modified by Kerr geometry and special relativity (e.g. Schnittman 2015, Schnittman et al. 2018, Yunes et al. 2008, Berry & Gair 2013). The specific trajectories are beyond the scope of this study. Nonetheless, in the presence of GW emission, we can roughly estimate the fraction of systems that are more likely to appear as EMRIs rather than GW bursts. For that, we first confirmed that all of the systems in the EKL (+GR) + two-body relaxation indeed reach the Schwarzschild radius by integrating all the systems below the solid line in the bottom panel of Figure 5.

Second, examining the integration prior to the threshold we found that \( \sim 40\% \) of the system reach a configuration for which \( \hat{R}_{sh} \leq 10 \) yr. and \( a_{p}(1-e_{p}) < 1 \) au. This specific configuration is chosen such that the characteristic strain will appear in the LISA band, resulting in mHz signals (see below). Assuming LISA lifetime to be about 10 yr. We emphasize that the 40\% estimation is rather conservative because, as mentioned, even the plunged BHs trajectories may spend a long time zooming in the SMBH’s ergosphere, where GW emission may alter their separation can result in an EMRI-like signal. Note that even when the BH period is smaller than 10 years (roughly equivalent to S0-2’s orbital period), two-body relaxation may still result in small kicks, about 0.0003 of the BH velocity, according to Eq. (8). Thus, overall the orbit will not substantially change over the BH period.

To estimate the signal to noise we follow Robson et al. (2018, 2019). The strain and thus the SNR depend on the orbital period, the eccentricity, and the luminosity distance. As a proof of concept we depict in Figure 6 the characteristic strain for all of the systems that crossed \( \hat{R}_{sh} \) in our nominal system (i.e., all the point below the line in Figure 5). For this example, we adopt a

Note that we are not taking into account star-BH collisions and tidal interactions that may result in electromagnetic signatures or larger BHs (e.g., Metzger et al. 2021, Rose et al. 2021, Kremer et al. 2022).
luminosity distance of 0.7 Mpc, and LISA observation time of 10 years. We find that 52% of the systems have a SNR > 5. Out of these systems 3% have GW dissipation timescale which is shorter than 10 years, which implies that a more careful analysis of the characteristic strain should be conducted for them (e.g., Barack & Cutler 2004). Eccentricity oscillations due to the EKL signature on the characteristic strain (e.g., Hoang et al. 2019; Deme et al. 2020) are unlikely to be detected in this configuration.

REFERENCES

Aharon, D., & Perets, H. B. 2016, ApJ, 830, L1, 1609.01715
Alexander, T., & Hopman, C. 2009, ApJ, 697, 1861, 0808.3150
Amaro-Seoane, P. 2018, Living Reviews in Relativity, 21, 4, 1205.5240
Amaro-Seoane, P. et al. 2017, arXiv e-prints, arXiv:1702.00786, 1702.00786
Amaro-Seoane, P., & Preto, M. 2011, Classical and Quantum Gravity, 28, 094017, 1010.5781
Antognini, J. M. O. 2015, MNRAS, 452, 3610, 1504.05957
Babak, S. et al. 2017, Phys. Rev. D, 95, 103012, 1703.09722
Bahcall, J. N., & Wolf, R. A. 1976, ApJ, 209, 214
Baker, J. et al. 2019, arXiv e-prints, arXiv:1907.06482, 1907.06482
Bar-Or, B., & Alexander, T. 2016, ApJ, 820, 129, 1508.01390
Barack, L., & Cutler, C. 2004, Phys. Rev. D, 69, 082005, gr-qc/0310125
Batcheldor, D., Robinson, A., Axon, D. J., Perlman, E. S., & Merritt, D. 2010, ApJ, 717, L6, 1005.2173
Berry, C. P. L., & Gair, J. R. 2013, MNRAS, 433, 3572, 1306.0774
Bhaskar, H., Li, G., Hadden, S., Payne, M. J., & Holman, M. J. 2021, AJ, 161, 48, 2008.04335
Bianchi, S., Chiaberge, M., Piconcelli, E., Guainazzi, M., & Matt, G. 2008, MNRAS, 386, 105, 0802.0825
Binney, J., & Tremaine, S. 2008, Galactic Dynamics: Second Edition
Blaes, O., Lee, M. H., & Socrates, A. 2002, ApJ, 578, 775, arXiv:astro-ph/0203370
Boje, N., & Wegg, C. 2014, MNRAS, 438, 573
Bogdanović, T., Eracleous, M., & Sigurdsson, S. 2009, ApJ, 697, 288, 0809.3262
Boroson, T. A., & Lauer, T. R. 2009, Nature, 458, 53, 0901.3779
Bradnick, B., Mandel, I., & Levin, Y. 2017, MNRAS, 469, 2042, 1703.05796
Callegari, S., Mayer, L., Kazantzidis, S., Colpi, M., Governato, F., Quinn, T., & Wadsley, J. 2009, ApJ-Lett, 696, L89, 0811.0615
Chen, X., & Han, W.-B. 2018, Communications Physics, 1, 53, 1801.05780
Chen, X., & Liu, K. F. 2013, ApJ, 762, 95, 1211.4609
Chen, X., Liu, K. F., & Magorrian, J. 2008, ApJ, 676, 54, 0712.0246
Chen, X., Madau, P., Sesana, A., & Liu, K. F. 2009, ApJ, 697, L149, 0904.4481
Chen, X., Sesana, A., Madau, P., & Liu, F. K. 2011, ApJ, 729, 13, 1012.4466
Comerford, J. M., Griffith, R. L., Gerke, B. F., Cooper, M. C., Newman, J. A., Davis, M., & Stern, D. 2009, ApJ-Lett, 702, L82, 0906.3517
Comerford, J. M., Nevin, R., Stemo, A., Müller-Sánchez, F., Barrows, R. S., Cooper, M. C., & Newman, J. A. 2018, ApJ, 867, 1810.11543
Deane, R. P. et al. 2014, Nature, 511, 57, 1406.6365
Deme, B., Hoang, B.-M., Nazo, S., & Kocsis, B. 2020, ApJ, 901, 125, 2005.03677
Di Matteo, T., Springel, V., & Hernquist, L. 2005, Nature, 433, 604, astro-ph/0502199
Dotti, M., Montuori, C., Decarli, R., Volonteri, M., Colpi, M., & Haardt, F. 2009, MNRAS, 398, L73, 0809.3446
Eilon, E., Kupi, G., & Alexander, T. 2009, ApJ, 698, 641, 0807.1430
Ford, E. B., Joshi, K. J., Rasio, F. A., & Zbarsky, B. 2000, ApJ, 528, 336, arXiv:astro-ph/9905347
Fragione, G., Loeb, A., Kremer, K., & Rasio, F. A. 2020, ApJ, 897, 46, 2002.02975
Freitag, M. 2001, Classical and Quantum Gravity, 18, 4033, astro-ph/0107193
Generozov, A., & Madigan, A.-M. 2020, ApJ, 896, 137, 2002.10547
GRAVITY Collaboration et al. 2020, A&A, 636, L5, 2004.07187
Green, P. J., Myers, A. D., Barkhouse, W. A., Mulchaey, J. S., Bennett, V. N., Cox, T. J., & Aldcroft, T. L. 2010, ApJ, 710, 1578, 1001.1738
Gualandris, A., & Merritt, D. 2009, ApJ, 705, 361, astro-ph/0805.4514
Gürkan, M. A., & Rasio, F. A. 2005, ApJ, 628, 236, astro-ph/0412452
Hamers, A. S., Bar-Or, B., Petrovich, C., & Antonini, F. 2018, ApJ, 865, 2, 1805.10315
Hansen, B. M. S., & Milosavljević, M. 2003, ApJ-Lett, 593, L77, astro-ph/0306074
Hansen, B. M. S., & Nazo, S. 2020, MNRAS, 499, 1682, 2011.07103
Haster, C.-J., Antonini, F., Kalogera, V., & Mandel, I. 2016, ApJ, 832, 192, 1606.07097
Hoang, B.-M., Nazo, S., Kocsis, B., Farr, W. M., & McIver, J. 2019, ApJ, 875, L31, 1903.00134
Hopkins, P. F., Hernquist, L., Cox, T. J., Di Matteo, T., Robertson, B., & Springel, V. 2006, ApJS, 163, 1, astro-ph/0506398
Hopman, C. 2009, Classical and Quantum Gravity, 26, 094028, 0901.1667
Hopman, C., & Alexander, T. 2005, ApJ, 629, 362, astro-ph/0503672
——. 2006, ApJ, 645, L133, astro-ph/0603224
Iwasa, M., & Seto, N. 2016, Phys. Rev. D, 93, 124024, 1508.05762
Kalogera, V. 2000, ApJ, 541, 319, astro-ph/9911147
Kocsis, B., & Tremaine, S. 2011, MNRAS, 412, 187, 1006.0001
Komossa, S., Burwitz, V., Hasinger, G., Predehl, P., Kaastra, J. S., & Iikebe, Y. 2003, ApJ-Lett, 582, L15, astro-ph/0212099
Komossa, S., Zhou, H., & Lu, H. 2008, ApJ-Lett, 678, L81, 0804.4585
Kozai, Y. 1962, AJ, 67, 591
Kremer, K., Lombardi, James C., Lu, W., Piro, A. L., & Rasio, F. A. 2022, arXiv e-prints, arXiv:2201.12368, 2201.12368
Li, G., Naoz, S., Holman, M., & Loeb, A. 2014a, ApJ, 791, 86, 1405.0494
Li, G., Naoz, S., Kocsis, B., & Loeb, A. 2014b, ApJ, 785, 116, 1310.6044
——. 2015, MNRAS, 451, 187, 1006.0001
Li, K., Bogdanović, T., & Ballantyne, D. R. 2020, ApJ, 896, 113, 2006.08520
Lidov, M. L. 1962, planss, 9, 719
Lim, H., & Rodriguez, C. L. 2020, Phys. Rev. D, 102, 064033, 2001.03654
Lithwick, Y., & Naoz, S. 2011a, ApJ, 742, 94, 1106.3329
——. 2011b, ApJ, 742, 94, 1106.3329
Liu, X., Greene, J. E., Shen, Y., & Strauss, M. A. 2010, ApJ-Lett, 715, L30, 1003.3467
Lu, C. X., & Naoz, S. 2019, MNRAS, 484, 1506, 1805.06897
Lu, J. R., Do, T., Ghez, A. M., Morris, M. R., Yelda, S., & Matthews, K. 2013, ApJ, 764, 155, 1301.0540
Maillard, J. P., Paumard, T., Stolovy, S. R., & Rigaut, F. 2004, A&A, 423, 155, arXiv:astro-ph/0404450
McConnell, N. J., & Ma, C.-P. 2013, ApJ, 764, 184, 1211.2816
Mei, J. et al. 2020, Progress of Theoretical and Experimental Physics, 2021, https://academic.oup.com/ptep/article-pdf/2021/5/05A107/37953035/ptaa114.pdf, 05A107
Metzger, B. D., Stone, N. C., & Gilbaum, S. 2021, arXiv e-prints, arXiv:2107.13015, 2107.13015
Miller, M. C., Freitag, M., Hamilton, D. P., & Lauburg, V. M. 2005, ApJ, 631, L117, astro-ph/0507133
Murray, C. D., & Dermott, S. F. 2000, Solar System Dynamics, ed. Murray, C. D. & Dermott, S. F.
Naoz, S. 2016, ARA&A, 54, 441, 1601.0715
Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Teyssandier, J. 2013a, MNRAS, 431, 2155, 1107.2414
Naoz, S., Kocsis, B., Loeb, A., & Yunes, N. 2013b, ApJ, 773, 187, 1206.4316
Naoz, S., Li, G., Zanardi, M., de Elía, G. C., & Di Sisto, R. P. 2017, AJ, 154, 18, 1701.03795
Naoz, S., & Silk, J. 2014, ApJ, 795, 102, 1409.5432
Naoz, S., Silk, J., & Schnittman, J. D. 2019, ApJ, 885, L35, 1905.03790
Naoz, S., Will, C. M., Ramirez-Ruiz, E., Hees, A., Ghez, A. M., & Do, T. 2020, ApJ, 888, L8, 1912.04910
Pan, Z., & Yang, H. 2021, Phys. Rev. D, 103, 103018, 2101.09146
Pesce, D. W., Braatz, J. A., Condon, J. J., & Greene, J. E. 2018, ApJ, 863, 149, 1807.04598
Peters, P. C., & Mathews, J. 1963, Physical Review, 131, 435
Preto, M., & Amaro-Seoane, P. 2010, ApJ, 708, L42, 0910.3206
Rauch, K. P., & Tremaine, S. 1996, NA, 1, 149, astro-ph/9603018
Raveh, Y., & Perets, H. B. 2010, MNRAS, 401, 319, astro-ph/9911417
Robertson, B., Bullock, J. S., Cox, T. J., Di Matteo, T., Hernquist, L., Springel, V., & Yoshida, N. 2006, ApJ, 664, 986, astro-ph/0503369
Robson, T., Cornish, N., & Liu, C. 2018, arXiv e-prints, 1803.01944
Robson, T., Cornish, N. J., & Liu, C. 2019, Classical and Quantum Gravity, 36, 105011, 1803.01944
Rodriguez, C., Taylor, G. B., Zavala, R. T., Peck, A. B., Pollack, L. K., & Roman, R. W. 2006, ApJ, 664, 99, astro-ph/0604024
Rose, S. C., Naoz, S., Gautham, A. K., Ghez, A. M., Do, T., Chu, D., & Becklin, E. 2020, ApJ, 904, 113, 2008.06512
Rose, S. C., Naoz, S., Sari, R., & Linial, I. 2021, arXiv e-prints, arXiv:2201.00022, 2201.00022
Rubbo, L. J., Holley-Bockelmann, K., & Finn, L. S. 2006, ApJ, 649, L25
Runnuc. J. C. et al. 2017, MNRAS, 468, 1683, 1702.05465
Sari, R., & Fragione, G. 2019, ApJ, 885, 24, 1907.03312
Schnittman, J. D. 2015, ApJ, 806, 264, 1506.06728
Schnittman, J. D., Dal Canton, T., Camp, J., Tsang, D., & Kelly, B. J. 2018, ApJ, 853, 123, 1704.07886
Sillanpaa, A., Haarala, S., Valtonen, M. J., Sundelius, B., & Byrd, G. G. 1988, ApJ, 325, 628
Smith, K. L., Shields, G. A., Bonning, E. W., McMullen, C. C., Rosario, D. J., & Salvianier, S. 2010, ApJ, 716, 866, 0908.1998
Sridhar, S., & Touma, J. R. 2016, MNRAS, 458, 4143, 1509.02401
Stemo, A., Comerford, J. M., Barrows, R. S., Stern, D., Assef, R. J., Griffith, R. L., & Schecter, A. 2020, arXiv e-prints, arXiv:2011.10051, 2011.10051
Touma, J., Tremaine, S., & Kazandjian, M. 2019, Phys. Rev. Lett., 123, 021103, 1907.01555
Tremaine, S. et al. 2002, ApJ, 574, 740, astro-ph/0203468
van den Bosch, R. C. E. 2016, ApJ, 831, 134, 1606.01246
Will, C. M., & Maitra, M. 2017, Phys. Rev. D, 95, 064003, 1611.06931
Yunes, N., Sopuerta, C. F., Rubbo, L. J., & Holley-Bockelmann, K. 2008, ApJ, 675, 604, 0704.2612
Zheng, X., Lin, D. N. C., & Mao, S. 2020, arXiv e-prints, arXiv:2011.04653, 2011.04653