Missing monopole strength of the Hoyle state in the alpha inelastic scattering

T Kawabata¹, T Kadoya¹, N Yokota¹, S Adachi¹, T Baba¹, T Furuno¹, H Fujimura², M Fujiwara³, K Hatanaka³, Y Ishii¹, M Itoh⁴, T Ito³, Y Maeda⁵, Y Matsuda³, M Murata¹, T Sato³, A Tamii³, M Tsumura¹, H Watanabe¹, and J Zenihiro⁶

¹Department of Physics, Kyoto University, Sakyo, Kyoto 606-8502, Japan.
²Department of Physics, Wakayama Medical University, Wakayama 641-8509, Japan.
³Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan.
⁴Cyclotron and Radioisotope Center, Tohoku University, Sendai, Miyagi 980-8578, Japan.
⁵Faculty of Engineering, University of Miyazaki, Miyazaki 889-2155, Japan.
⁶RIKEN (The Institute for Physical and Chemical Research), Wako, Saitama 351-0198, Japan.

E-mail: kawabata@scphys.kyoto-u.ac.jp

Abstract. Cross sections for the alpha inelastic scattering exciting the low-lying monopole states in ¹²C, ¹⁶O, ²⁴Mg, ²⁸Si, and ⁴⁰Ca were measured and compared with the distorted-wave Born-approximation (DWBA) calculation to examine the puzzle of the missing monopole strength of the Hoyle state. It was found the DWBA calculation using the density-dependent αN interaction systematically overestimates the cross sections for the ⁰⁺ transitions, and the puzzle is a universal problem in light nuclei but not special in the Hoyle state. Since the DWBA calculation using the density-independent interaction reasonably well reproduces the experiment, this puzzle might be related to the density dependence of the effective interaction. The coupled-channel effect for the alpha inelastic scattering is also examined. The coupled-channel effect reduces the calculated cross section, and solve the puzzle of the missing monopole strengths in part, but the improvement of the density dependence of the effective interaction is still necessary to solve the puzzle.

1. Introduction
The second ⁰⁺ state in ¹²C, known as the Hoyle state, is the most famous 3α cluster state, and locates at an excitation energy higher than the 3α-decay threshold by 0.4 MeV. The Hoyle state plays a very important role in the nucleosynthesis in the universe. This state is of great interest in the last decade because this state is theoretically described by introducing a novel concept of the nuclear structure, i.e., this state has a dilute-gas-like structure where three α clusters are weakly interacting and are condensed into the lowest s-orbit [1]. Those condensed α clusters have a very sharp delta-function-like momentum distribution, thus, their density distribution expands into large volume and becomes dilute [2].

The structure of the Hoyle state was extensively studied from both the theoretical and experimental sides. The electron scattering data were precisely compared with the model calculations, and these calculations reasonably well reproduce the experiment. On the other hand, the cross sections for the alpha inelastic scattering exciting the Hoyle state were...
significantly overestimated by the theoretical calculation, thus, the monopole strength for the Hoyle state had to be scaled down by a factor of 2–4 to explain the experimental data. D. T. Khoa et al. claimed this puzzle of the missing monopole strength should be attributed to the exotic nature of the Hoyle state. Since the loosely bound dilute structure of this state enhances absorption in the exit $\alpha^+{^{12}}C^*(0^+_2)$ channel, the strong imaginary part of the transition potential must be introduced to solve the puzzle [3].

Recently, it was demonstrated that the large monopole strength is a signature of the spatially well-developed cluster state in light nuclei [4]. The alpha inelastic scattering was extensively utilized to measure monopole strengths and to examine the cluster structures in light nuclei. It is, therefore, an important question whether the puzzle of the missing monopole strength is a special phenomena to the Hoyle state only as suggested in Ref. [3] or is a universal phenomena observed in various nuclei.

In the present work, a systematic measurement of the alpha inelastic scattering exciting the low-lying monopole states in $^{12}C$, $^{16}O$, $^{24}Mg$, $^{28}Si$, and $^{40}Ca$ was carried out and the measured cross sections were compared with the model calculations.

2. Experiment and results
The experiment was performed at the Research Center for Nuclear Physics, Osaka University, using a 130-MeV $\alpha$ beam. The $\alpha$ beam extracted from the AVF cyclotron was achromatically transported to the target. Scattered $\alpha$ particles were momentum analyzed by the high-resolution spectrometer Grand Raiden [5]. The $^{nat}C$, $^{24}Mg$, $^{nat}SiO_2$, $^{nat}Si$, and $^{nat}Ca$ targets with a thickness of 1–5 mg/cm$^2$ were used.

The cross sections for the alpha elastic scattering from $^{12}C$, $^{16}O$, $^{24}Mg$, $^{28}Si$, and $^{40}Ca$ were measured at $\theta_{lab} = 3.7^\circ$–$55^\circ$, whereas the cross sections for the alpha inelastic scattering were measured at $\theta_{lab} = 0^\circ$–$13.4^\circ$. The measured cross sections for the elastic scattering are shown in Fig. 1, and the cross sections for the inelastic scattering exciting some of the low-lying $0^+$ and $2^+$ states are shown in Fig. 2.

3. Discussion
3.1. Elastic scattering
The measured cross sections for the alpha elastic scattering from $^{12}C$, $^{16}O$, $^{24}Mg$, $^{28}Si$, and $^{40}Ca$ are compared with the optical-model calculation in Fig. 1. The optical potentials $U(r)$ were obtained by the single folding prescription where the ground-state density distribution $\rho_0(r)$ is folded by the phenomenological $\alpha N$ interaction $V(|r - r'|, \rho_0(r')).$

$$U(r) = \int dr' \rho_0(r') V(|r - r'|, \rho_0(r')).$$

The ground-state density distributions are taken from the electron scattering data [6]. The $\alpha N$ interaction is parametrized by

$$V(|r - r'|, \rho_0(r')) = -V \left(1 + \beta \rho_0^{2/3}(r')\right) \exp\left(-\frac{|r - r'|^2}{\alpha_V}\right)$$

$$-iW \left(1 + \beta \rho_0^{2/3}(r')\right) \exp\left(-\frac{|r - r'|^2}{\alpha_W}\right),$$

where $\beta$ determines the density dependence of the interaction. In the present work, the two different values of $\beta$ were used. $\beta = -1.9$ taken from Ref. [7] gives the density-dependent (DD)
interaction, and $\beta = 0$ gives the density-independent (DI) interaction. The four parameters $\alpha_V$, $\alpha_W$, $V$, and $W$ are determined to fit the elastic scattering data. The optical-model calculation with the DI and DD interactions are shown by the thin solid and thick dashed lines, respectively. Both the DI and DD interactions give the almost same results and reasonably well describe the experimental data.

3.2. Inelastic scattering

The measured cross sections for the alpha inelastic scattering exciting some of the low-lying $0^+$ and $2^+$ states in $^{12}$C, $^{16}$O, $^{24}$Mg, $^{28}$Si, and $^{40}$Ca are compared with the distorted-wave Born-approximation (DWBA) calculation in Fig. 2. The transition potentials $\delta U_\lambda(r, E_x)$ used in the DWBA calculation were obtained by folding the macroscopic transition densities $\delta \rho^{(\lambda)}(r, E_x)$ with the phenomenological aN interaction which is determined to fit the elastic scattering.

$$\delta U_\lambda(r, E_x) = \int dr' \delta \rho^{(\lambda)}(r, E_x) \left[ V(|r-r'|, \rho_0(r')) + \rho_0(r') \frac{\partial V(|r-r'|, \rho_0(r'))}{\partial \rho_0(r')} \right].$$

The macroscopic transition densities for the $0^+$ and $2^+$ states are given as following.

$$\delta \rho^{(0)}(r') = -\delta_0 \left( 3 + r' \frac{d}{dr'} \right) \rho_0(r'), \quad \delta \rho^{(2)}(r') = -\delta_2 \frac{d}{dr'} \rho_0(r').$$

The cross sections calculated by the DI and DD interactions are shown by the thin solid and thick dashed lines, respectively. The normalization factors for the macroscopic transition densities were determined to reproduce the known electromagnetic transition strengths. The shaded bands in Fig. 2 show uncertainties of the calculated cross sections due to the errors in the electromagnetic transition strengths. It should be noted that all the parameters in the present DWBA calculation are determined from the previous experimental data and no parameters are tuned to fit the present results in Fig. 2.
For the $2^+$ states, the DWBA calculations using the DI and DD interactions predict the almost same cross sections and both the calculations reasonably reproduces the experimental results. On the other hand, the DWBA calculations using the DI and DD interactions give quite different results for the $0^+$ states. The DD interaction gives much larger cross sections than the DI interaction, especially for the lighter nuclei such as $^{12}$C and $^{16}$O. The DWBA calculation using the DD interaction systematically overestimates the cross sections for the $0^+$ states, and this result is quite similar to the previous result in Ref. [3]. It is worth to mention that the previous calculation in Ref. [3] was carried out using the density-dependent NN interaction. The preset results suggest the puzzle of the missing monopole strengths is not a special problem in the Hoyle state, but a universal problem in light nuclei.

For further clarification of the problem, the transition potentials for the $0_2^+$ and $2_1^+$ states are shown in Figs. 3(a) and (b). The transition potentials calculated by the DI and DD interactions are shown by the thin and thick lines, and the real and imaginary parts of the transition potentials are given by the solid and dashed lines, respectively. It is remarkable that the transition potentials with the DD and DI interactions are completely different for the $0_2^+$ state although they are quite similar to each other for the $2_1^+$ state. This difference comes from the radial dependence of the transition densities as seen in Fig. 3(c). The transition density for the $0^+$ transition has a finite value at the origin where the matter density is high, whereas the transition density for the $2^+$ transition is zero at the origin. Because the Pauli blocking effect...
Figure 3. (a) Transition potentials for the $0^+_2$ state in $^{12}$C. The thin and thick lines show the transition potentials calculated by using the DI and DD interactions, and the solid and dashed lines show the real and imaginary parts of the transition potentials, respectively. (b) Same with (a) but for the $2^+_1$ state. (c) Macroscopic transition densities for the $0^+$ (solid) and $2^+$ (dashed) transitions.

attenuates the effective interaction around the origin, the DD transition potentials for the $0^+$ transitions become shallower.

Since the imaginary part of the DD transition potential for the $0^+$ transition is much shallower than that of the DI transition potential, the DD interaction suppresses the absorption to enhance the cross section. This is the reason that the DD interaction overestimates the cross sections for the $0^+$ transitions and causes the puzzle of the missing monopole strengths. It should be noted the DI interaction gives relatively better results for the $0^+$ transitions than the DD interaction in Fig. 2. It should be considered that this puzzle is related to the density dependence of the effective interaction, and the density dependence of the effective interaction must be improved to solve the puzzle.

3.3. Uncertainties in the DWBA calculation

Uncertainties in the DWBA calculation mainly come from (a) the effective $\alpha$N interaction, (b) transition densities, and (c) coupled-channel effects. Since the effective interaction has been discussed in the previous subsection, the uncertainties in the transition densities and coupled-channel effects are discussed in this subsection.

Since the transition densities for the DWBA calculation are obtained by the macroscopic model in the present work, the radial distribution of the transition densities is determined by the spin and parity only. Therefore, the state-by-state variation of the nuclear structure is ignored. To examine the uncertainties of the DWBA calculation come from the transition densities, Fig. 4 shows the calculated cross sections for the $0^+_2$ and $2^+_1$ states in $^{12}$C using the macroscopic transition densities compared with those using the microscopic transition densities, which are obtained from the THSR model [8]. The solid and dashed lines show the calculated cross sections using the macroscopic and microscopic transition densities, respectively, whereas the thin and thick lines present the calculated cross section using the DI and DD interactions. The macroscopic and microscopic transition densities give similar cross sections, therefore, the uncertainties due to the transition densities are quite small in the present DWBA calculation.

The coupled-channel effects are examined in Fig. 5. The solid lines show the cross sections for the $0^+_2$ and $2^+_1$ states in $^{12}$C calculated by the DWBA, and the dashed lines present those by the coupled-channel calculation taking the couplings between the $0^+_1$, $2^+_1$, $4^+_1$, $2^+_2$, and $3^+_1$ states into account. The transition densities concerning the $3^+_1$ states are taken from the RGM calculation [9] and the other transition densities are taken from the THSR model. The cross
sections shown by the thin and thick lines are calculated by using the DI and DD interactions.

The coupled-channel effects significantly reduce the cross section for the $0^+_2$ state, but do not change the cross section for the $2^+_1$ state. According to the cluster model calculations [8, 9], the coupling between the $0^+_2$ and $2^+_1$ states is much stronger than that between the $0^+_2$ and $2^+_1$ states because the $2^+_1$ state is considered to be a single $\alpha$ excited state of the $0^+_2$ state where one $\alpha$ cluster is excited into the $d$-orbit from the lowest $s$-orbit. The strong absorption in the $\alpha + ^{12}\mathrm{C}^* (0^+_2)$ channel pointed out in Ref. [3] should be attributed to this strong coupling between the $0^+_2$ and $2^+_1$ states in part, but the improvement of the density dependence of the effective interaction is still necessary to solve the puzzle of missing monopole strengths.

4. Summary
In the present work, a systematic measurement of the alpha inelastic scattering exciting the low-lying monopole states in $^{12}\mathrm{C}$, $^{16}\mathrm{O}$, $^{24}\mathrm{Mg}$, $^{28}\mathrm{Si}$, and $^{40}\mathrm{Ca}$ was carried out and the measured cross sections were compared with the model calculations. It was found that the DWBA calculation using the DD interaction systematically overestimates the cross sections for the $0^+$ transitions and the puzzle of the missing monopole strengths is a universal problem in light nuclei not special in the Hoyle state. Since the DWBA calculation using the DI interaction reasonably well reproduces the experiment, this puzzle might be related to the density dependence of the effective interaction.

Uncertainties in the DWBA calculation are also discussed. It was found that the uncertainties due to the transition densities are quite small, but the coupled-channel effects between the $0^+_2$ and $2^+_1$ states are sizable. The strong absorption in the $\alpha + ^{12}\mathrm{C}^* (0^+_2)$ channel pointed out in Ref. [3] should be attributed to this strong coupling between the $0^+_2$ and $2^+_1$ states in part, but the improvement of the density dependence of the effective interaction is still necessary to solve the puzzle of missing monopole strengths.

References
[1] Tohsaki A, Horiuchi H, Schuck P, and Röpke G 2001 Phys. Rev. Lett. 87 192501
[2] Yamada T and P. Schuck 2005 Eur. Phys. J. A 26 185
[3] Khoa Dao T and Cuong Do Cong 2008 Phys. Lett. B 660 331
[4] Kawabata T et al. 2007 Phys. Lett. B 646 6
[5] Fujiwara M et al. 1999 Nucl. Instrum. Meths. A 422 484
[6] De Vries H, De Jager C W, and De Vries C 1987 Atomic Data and Nuclear Data Tables 36 495
[7] Satchler G R and Khoa Dao T 1997 Phys. Rev. C 55 285
[8] Funaki Y, Tohsaki A, Horiuchi H, Schuck P, and Röpke G 2003 Phys. Rev. C 67 051306
[9] Kamimura M 1981 Nucl. Phys. A 351 456