THE CNOC2 FIELD GALAXY LUMINOSITY FUNCTION. I. A DESCRIPTION OF LUMINOSITY FUNCTION EVOLUTION

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ABSTRACT

We examine the evolution of the galaxy luminosity function (LF) using a sample of over 2000 galaxies, with $0.12 < z < 0.55$ and $17.0 < R_C < 21.5$, drawn from the Canadian Network for Observational Cosmology Field Galaxy Redshift Survey (CNOC2), at present the largest such sample at intermediate redshifts. We use $UBVRI_C$ photometry and the spectral energy distributions (SEDs) of Coleman, Wu, and Weedman to classify our galaxies into early, intermediate, and late types, for which we compute LFs in the rest-frame $B$, $R_C$, and $U$ bandpasses. In particular, we adopt a convenient parameterization of LF evolution including luminosity and number density evolution and take care to quantify correlations among our LF evolution parameters. We also carefully measure and account for sample selection effects as functions of galaxy magnitude and color.

Our principal result is a clear quantitative separation of luminosity and density evolution for different galaxy populations and the finding that the character of the LF evolution is strongly dependent on galaxy type. Specifically, we find that the early- and intermediate-type LFs show primarily brightening at higher redshifts and only modest density evolution, whereas the late-type LF is best fit by strong number density increases at higher $z$ with little luminosity evolution. We also confirm the trend seen in previous smaller $z \lesssim 1$ samples of the contrast between the strongly increasing luminosity density of late-type galaxies and the relatively constant luminosity density of early-type objects. Specific comparisons against the Canada-France and Autofib redshift surveys show general agreement among our LF evolution results, although there remain some detailed discrepancies. In addition, we use our number count and color distribution data to further confirm the validity of our LF evolution models to $z \sim 0.75$, and we also show that our results are not significantly affected by potential systematic effects such as surface brightness selection, photometric errors, or redshift incompleteness.

Subject headings: cosmology: observations — galaxies: evolution — galaxies: fundamental parameters — galaxies: luminosity function, mass function — surveys

1. INTRODUCTION

The luminosity function (LF) is a basic and fundamentally important statistic used to study galaxy populations and their evolution. In particular, measurement of the field galaxy LF at different redshifts provides a simple means of describing the global changes seen in the galaxy population with look-back time. These LF data, together with complementary information from galaxy number counts and color distributions, among others, supply some of the key observations that help shape our picture of how galaxies evolve (e.g., see reviews by Koo & Kron 1992; Ellis 1997). It remains an important problem and a difficult theoretical challenge to properly interpret the variety of galaxy evolution data, including the LF, in terms of models of galaxy formation and evolution that self-consistently incorporate relevant physical processes such as star formation, feedback, and gravity (e.g., Baugh et al. 1998; Cole et al. 1994; Kauffmann, Nusser, & Steinmetz 1997).

Recent observational progress in measuring the field galaxy LF has spanned a very wide range of redshifts, including improved estimates using larger local ($z \sim 0$) and intermediate-redshift ($z \lesssim 1$) samples, as well as the first observations for high-redshift ($z \sim 3$) galaxies. In the local $z \lesssim 0.2$ regime, the galaxy LF is now determined routinely using redshift survey samples containing thousands of galaxies, and the LF is most commonly measured for the rest-frame optical $B$ ( Loveday et al. 1992; Marzke, Huchra, & Geller 1994b; da Costa et al. 1994; Zucca et al. 1997; Ratcliffe et al. 1998; Colless 1998) and $R$ bands (Lin et al. 1996a; Geller et al. 1997). However, despite the very large sample sizes, there remains controversy in the determination of the local LF with regard to both its normalization and shape. The controversy results from potential systematic effects that may adversely affect LF measurements in local surveys at bright magnitudes; these include surface brightness selection effects, systematic photometric errors, possible local galaxy underdensities, and the small volumes over which intrinsically faint galaxies are visible (see discussion and review in Ellis 1997). The uncertainties in the local LF normalization and shape make it more difficult to...
use the local results as low-redshift anchors for models of galaxy evolution. Nonetheless, a robust result from local LF measurements appears to be the dependence of the LF on galaxy type, in the sense that the faint end of the LF is consistently dominated by galaxies of later morphology (Marzke et al. 1994a; Marzke et al. 1998), later spectral type (Bromley et al. 1998; Colless 1998), stronger line emission (Lin et al. 1996a; Zucca et al. 1997), or bluer color (Marzke & da Costa 1997; Metcalfe et al. 1998).

The LF situation is somewhat less controversial at intermediate redshifts \((0.2 \leq z \lesssim 1)\). Recent deep redshift surveys, with samples of typically hundreds of galaxies, have consistently found similar trends in the evolution of different types of galaxies (Lilly et al. 1995b; Ellis et al. 1996; Cowie et al. 1996; Lin et al. 1997; Heyl et al. 1997; Small, Sargent, & Hamilton 1997; Liu et al. 1998; Hogg et al. 1998; de Lapparent et al. 1997). Namely, there exists a distinct contrast between the rapid evolution seen in the LF of late-type, blue, star-forming galaxies and the relatively mild changes observed in the LF of early-type, red, quiescent objects. Also, type-dependent LF differences similar to those seen at low redshifts are also observed at intermediate \(z\). The depth of the photometry required for intermediate-redshift surveys renders them less susceptible than their local counterparts to surface brightness selection effects, and multicolor data are also typically available for moderate-redshift samples, permitting more accurate galaxy classification. On the other hand, the sample sizes are smaller, and the redshift completeness is not as high as for low-\(z\) surveys, so that the random errors on the LF are in general larger than those for local LFs.

A much larger intermediate-redshift sample will, however, be provided by the Canadian Network for Observational Cosmology (CNOC) Field Galaxy Redshift Survey, hereafter denoted CNOC2. The CNOC2 survey has as its primary goal the study of the evolution of galaxy clustering and galaxy populations at intermediate redshifts \(0.1 \lesssim z \lesssim 0.7\). In order to accomplish its objectives, CNOC2 will acquire some 5000 galaxy redshifts at \(R_c < 21.5\), thus making a dramatic improvement over other intermediate-\(z\) surveys in terms of sample size. In addition, nearly all CNOC2 galaxies have multicolor \(UBVR_cI_C\) photometry, which is more extensive color coverage than is available for the vast majority of other redshift surveys. This multicolor information permits galaxy classifications using fits to broadband colors computed from model galaxy spectral energy distributions (SEDs). Consequently, LFs may be calculated and studied for different galaxy populations, as well as for a number of different rest-frame bandpasses such as \(B, R_C,\) and \(U\). In addition, the multicolor photometry permits more accurate computation of \(k\)-corrections and related quantities and also allows detailed checks of the survey's redshift completeness as functions of galaxy color and type.

In this paper we will examine the evolution of the LF for different galaxy populations, using an interim but statistically complete sample of over 2000 CNOC2 galaxies with \(R_c < 21.5\). Although this is only half of the ultimate CNOC2 survey sample, our interim data set nevertheless comprises the largest intermediate-\(z\) redshift survey at present. The combination of large sample size, multicolor data, and careful control of redshift selection effects should allow CNOC2 to give the best quantitative LF constraints thus far at intermediate redshifts.

The outline of the paper is as follows: In § 2 we describe details of the CNOC2 data sample, and we will also discuss redshift success rates, use of statistical weights to correct for incompleteness, and surface brightness selection effects. Then in § 3 we detail our methods for galaxy classifications and for fitting the LF and its associated evolution parameters. Our LF evolution results are described in § 4, where we focus in particular on quantifying luminosity and density evolution in the LFs of different galaxy populations. In addition, we will also compute galaxy SED type distributions, number counts, and color distributions, as well as examine the impact that various potential systematic effects may have on our LF evolution results. In § 5 we compare our LFs to those derived from a number of previous intermediate-redshift galaxy surveys, in particular the two next largest samples, the Canada-France Redshift Survey (CFRS; Lilly et al. 1995a, 1995b) and the composite Autofib Redshift Survey (Ellis et al. 1996; Heyl et al. 1997; plus references therein). Finally, we summarize our conclusions in § 6.

CNOC2 is now complete with respect to data acquisition, and LF studies using the full CNOC2 sample will be forthcoming once all the data have been reduced. All the presently available, fully reduced, and statistically complete CNOC2 data are contained in the interim sample defined below in § 2, and this is the first in a series of papers studying LF evolution in the CNOC2 data set. The second paper (Lin et al. 1999; hereafter Paper II) will address LF evolution in the context of physically motivated galaxy evolution models. In contrast, in the present paper we focus on a description of the redshift-dependent changes in the LF and will not attempt to explain those changes in terms of physical processes; that is the province of Paper II. We emphasize that our use of “luminosity evolution” and “density evolution” should strictly be construed to describe the apparent changes in the LF and may or may not correspond to true physical changes in individual galaxies. Not too surprisingly, it turns out that the apparent LF evolution will be sensitive to the details of the galaxy classification scheme (and choice of SEDs as a function of \(z\)) that one adopts; this will be elaborated further in § 4.1.2 and in Paper II. Nonetheless, the descriptive approach we take in this paper is a simple and effective way of characterizing and quantifying the changes in the LFs of different intermediate-redshift galaxy populations, and it is also in essence the typical approach taken in previous LF studies.

For the Hubble constant \(H_0 = 100\ h\ \text{km s}^{-1}\ \text{Mpc}^{-1}\), \(h = 1\) should be assumed in this paper if the \(h\) dependence is not explicitly shown. We also adopt a deceleration parameter \(q_0 = 0.5\) throughout except where otherwise specified (in particular \(q_0 = 0.1\) will be used on occasion).

2. THE CNOC2 SURVEY DATA

A detailed description of the CNOC2 Field Galaxy Redshift Survey will be given in Yee et al. (1999; see also Yee, Ellingson, & Carlberg 1996; Yee et al. 1998). We summarize the relevant points here.

The survey covers four widely separated areas, hereafter denoted as "patches," on the sky. In this paper we use data from the two CNOC2 patches 0223+00 and 0920+37 (named by R.A. and decl.). Observations for these two patches were obtained during six observing runs at the Canada-France-Hawaii Telescope (CFHT) over the period 1995 February to 1997 August. Both photometry and spec-
Bp for five bands (approximate 5° north-south (see Yee et al. 1998). Mos Fields (each graph (mos). Each of these two patches is a mosaic of 19 MOS fields (of the 38 total in the 0223 and 0920 patches), which have full UBVRcIc coverage. The sky coverage of our sample is then 2490 arcmin², and the comoving sample volume is 1.13(1.50) × 10⁵ h⁻³ Mpc³ for q₀ = 0.5(0.1). Photometric reductions (object detection, star-galaxy classification, and photometry) were done using an improved version of the Picture Processing Package (PPP; Yee 1991; Yee et al. 1996). Objects were selected in the Rc band for the survey’s spectroscopic sample and we adopt Rc = 21.5 as the nominal spectroscopic completeness limit (see § 2.1 below).

Multislit spectroscopy was carried out on CFHT MOS. We used a band-limiting filter to restrict the wavelength coverage to 4400–6300 Å in order to increase the multiplexing efficiency in such a way that typically 90–100 objects may be observed per slit mask. The band-limiting filter does, however, restrict the redshift range over which spectroscopic features important for redshift measurement may be seen. We adopt the nominal redshift completeness range 0.12 < z < 0.55, based on the observability of the Ca II H and K (3968, 3933 Å) absorption feature important for early-type galaxies. Over this same redshift range, the [O ii] λ3727 emission feature important for late-type galaxies is also observable, except for z ≤ 0.2, where we expect [O iii] λλ5007, 4959 and Hβ (4861 Å) emission to substitute for the unobservable [O ii] λ3727. Spectroscopic reductions and redshift measurements were carried out using customized programs and standard IRAF routines. The rms error in the velocity measurements is about 100 km s⁻¹, as determined empirically from redundant spectroscopic observations.

We correct our photometry for extinction from the Milky Way using the dust maps of Schlegel, Finkbeiner, & Davis (1998). We convert Schlegel et al.’s E(B − V) values to magnitudes of extinction in the UBVRcIc bands using the procedure described in their Appendix B, adopting the Milky Way extinction curves of O’Donnell (1994) and Cardelli, Clayton, & Mathis (1989). The extinction variation within each of the 0223 and 0920 patches is small, so we simply apply a single correction for each patch as a whole:

\[ \Delta U = -0.171, -0.059 \]
\[ \Delta B = -0.140, -0.048 \]
\[ \Delta V = -0.108, -0.037 \]
\[ \Delta R_c = -0.083, -0.029 \]
\[ \Delta I_c = -0.064, -0.022 \]  

where the first value corresponds to the 0223 patch and the second to the 0920 patch.

### 2.1 Redshift Success Rates and Statistical Weights

For reasons of observational efficiency, we cannot target for spectroscopy all galaxies in our fields, and, like the majority of other redshift surveys, we do not successfully measure a redshift from every spectrum. We thus need to derive a set of statistical weights so that we can account for incompleteness in the CNOC2 redshift sample for our LF and other analyses. Figure 1 (top left-hand panel) shows our redshift sampling rate as a function of apparent magnitude R_c, where the redshift sampling rate is defined as the fraction of galaxies with redshifts among all galaxies in our photometric catalog. The differential redshift sampling rate is about 20% at the nominal spectroscopic completeness limit R_c = 21.5, and the cumulative sampling rate is about 50% for 17.0 < R_c < 21.5. Since we do not put a spectroscopic slit on every object, the redshift sampling rate is different from the redshift success rate, defined as the fraction of spectroscopically observed galaxies with redshifts. Our redshift success rate is also plotted in Figure 1 (middle left-hand panel). As expected, the redshift success rate declines with fainter apparent magnitude and hence decreasing signal-to-noise ratio in our spectra. The raw success rate ranges from over 90% for R_c < 19 to about 50% at R_c = 21.5, with an overall cumulative success rate of 70% for 17.0 < R_c < 21.5. However, this raw success rate is biased low by galaxies with redshifts outside the nominal CNOC2 0.12 < z < 0.55 completeness range. As we will show at the end of this section, we can correct for this bias and estimate a corrected redshift success rate solely for 0.12 < z < 0.55 galaxies: this improves the success rate to 70% at R_c = 21.5 and to about 85% cumulatively for 17.0 < R_c < 21.5.

The simplest way to derive a statistical weight is just to use the inverse of the redshift sampling rate. This will be correct if, at each value of R_c, the spectroscopically failed objects constitute the same population as the spectroscopically successful ones in terms of the distribution of both their spectral types and redshifts. However, this will not be true in general, because our ability to measure a redshift will be a function of the spectral type and redshift of a particular galaxy. For example, given an early- and a late-type galaxy with the same apparent magnitude R_c, the early-type galaxy will yield a lower signal-to-noise ratio CNOC2 spectrum (4400–6300 Å) because of its redder SED. Moreover, our finite spectral window means that our redshift failures will be biased toward objects outside of our nominal 0.12 < z < 0.55 completeness range. We illustrate these points by plotting in Figure 1 the redshift success rate as a function of B − R_c (top right-hand panel) and R_c − I_c (middle right) colors, showing that there are indeed some obvious color and hence galaxy type dependences in our success rate (but note that the same observed color can result from galaxies of somewhat different types spread out across a range of redshifts). Note in particular that the steep decline seen in the success rate for B − R_c ≥ 3 or R_c − I_c ≥ 1 is caused by higher redshift (z ≥ 0.55) early-type galaxies whose Ca II H and K features have shifted out of our spectral window (see § 3.1). Also, the success rate drops for B − R_c ≤ 1 because of lower redshift (z ≤ 0.2) late-type galaxies for which the [O ii] λ3727 emission line is likewise outside the spectral window.

One way to proceed is to apply photometric redshift methods (e.g., Sawicki, Lin, & Yee 1997; Connolly et al. 1995; Koo 1985) on our UBVRcIc data to obtain approximate redshifts and spectral types for all galaxies in our photometric catalog (with or without spectroscopic redshifts) and subsequently derive an estimate of our spec-
This particular magnitude- and color-dependent weighting scheme does in fact account for our somewhat complicated redshift selection effects; that we obtain sensible results will be shown below in § 4.3, where we compare our LF-computed number counts and color distributions with the observations. Note also that we are using the sample as a whole to calculate our weights. We are thus ignoring some real field-to-field variations in our redshift success rate, primarily because of observational factors, particularly seeing. These variations need to be accounted for in galaxy clustering analyses but should not be important for the LF analysis of this paper. A more detailed discussion of our selection effects will be found in Yee et al. (1999).

Next we show in Figure 1 (bottom right-hand panel) the fraction of galaxies as a function of within our redshift completeness range, computed using the best-fit evolving B-band LF that we will obtain below in § 4.1. This fraction peaks at about 90% at but declines to about 60% at and overall, the cumulative fraction is about 75% for . We can then compute, as a function of , a corrected redshift success rate for corrected
galaxies using

\[ f_{\text{corrected}}(R_c) = \frac{N_f(R_c)}{N_{\text{obs}}(R_c) F_f(R_c)}, \]

where \( N_f \) is the number of galaxies with redshifts in the range \( 0.12 < z < 0.55 \), \( N_{\text{obs}} \) is the total number of spectroscopically observed galaxies, and \( F_f \) is the LF-derived fraction of galaxies with \( 0.12 < z < 0.55 \). This more relevant corrected success rate \( f_{\text{corrected}} \) is also plotted in Figure 1 (bottom left-hand panel), where we see that the differential rate improves to about 70% at \( R_c = 21.5 \), and the cumulative rate increases to about 85% for \( 17.0 < R_c < 21.5 \). These success rates are comparable to those obtained in other large intermediate-\( z \) surveys (e.g., Crampton et al. 1995; Ellis et al. 1996). Also, in §4.4.3 we discuss the impact of any potential residual \( z \)-dependent incompleteness on our results. Finally, note that \( f_{\text{corrected}} \) is computed for illustrative purposes only; it is not used to weight the data in any of our analyses.

2.2. Surface Brightness Selection Effects

Unaccounted surface brightness selection effects may seriously bias calculation of the LF, especially for low-redshift samples with relatively shallow photometry and bright limiting isophotes (e.g., Ferguson & McGaugh 1995; Dalcanton 1998). Although surface brightness selection effects are less problematic for intermediate-\( z \) surveys like CNOC2 with deeper imaging, it is nonetheless important to quantify the survey’s effective surface brightness limits. We do so in Figure 2, where we plot apparent magnitude \( R_c \) versus a central aperture magnitude \( R_c \) (aperture) for CNOC2 objects classified as galaxies or probable galaxies by PPP. \( R_c \) (aperture) will serve as our measure of the central surface brightness, and the aperture used is a circle with diameter 1.32 (corresponding to 3 pixels for our largest pixel-size STIS2 CCD). The vertical line indicates the nominal \( R_c = 21.5 \) spectroscopic limit, and the horizontal line is our estimate of the central surface brightness completeness limit, \( R_c(\text{aperture}) = 24.0 \) (or 24.3\( R_c \) mag arcsec\(^{-2}\)). The latter limit is conservatively estimated as 0.5 mag brighter than the turnover in the number count histogram for \( R_c \) (aperture). Also plotted is the track as a function of redshift for a fiducial face-on exponential disk galaxy with the Freeman (1970) central surface brightness value \( \mu_{B,0}(0) = 21.5 \) mag arcsec\(^{-2}\) and an absolute magnitude \( M_{B,0} = -19.5 + 5 \log h \approx M_B^* \). The assumed seeing is a Moffat profile with 1” FWHM, nearly the CNOC2 average (0.9”). We calculate \( k \)-corrections using an Sbc galaxy SED (Coleman, Wu, & Weedman 1980, hereafter CWW); we have checked that using an E or Im SED instead makes little difference for our conclusions.

The vast majority of our galaxies lie brightward in central surface brightness relative to the \( M^* \) Freeman disk track, even though we should be sensitive to lower surface brightness objects; this is very similar to what Lilly et al. (1995a) found in a completely analogous plot for the CFRS. Note that our Freeman disk model is a pure exponential disk only, so that the addition of a bulge component needed for a more realistic galaxy would result immediately in a higher central surface brightness. Likewise, a sub-\( M^* \) Freeman disk or an inclined \( M^* \) Freeman disk will also have tracks that are everywhere brighter in \( R_c \) (aperture) (vertically below in the plot) compared to the face-on \( M^* \) Freeman disk track shown. These other tracks pass more centrally through the observed galaxy distribution, but the \( M^* \) Freeman disk track serves as a useful central surface brightness lower bound for the vast majority of galaxies.

Given the redshift track of a particular type of galaxy, our survey will be flux-limited with respect to that galaxy type if the track first crosses the \( R_c = 21.5 \) vertical boundary instead of the \( R_c(\text{aperture}) = 24.0 \) horizontal boundary; otherwise we will need to consider the surface brightness limit explicitly in our analyses. Figure 2 shows that we are indeed flux-limited with respect to the \( M^* \) Freeman track across the entire CNOC2 nominal redshift range 0.12 < \( z < 0.55 \), and since that Freeman track is basically a lower surface brightness bound for the bulk of our galaxies, we may conclude that the \( R_c < 21.5 \) CNOC2 photometric sample is essentially free of central surface brightness selection effects.

Also shown in Figure 2 is the track for an \( M^* \) low surface brightness (LSB) disk galaxy with \( \mu_{B,0}(0) = 24.0 \) mag arcsec\(^{-2}\), 10 times fainter than that of a Freeman disk. This LSB disk crosses our central surface brightness boundary and exits our sample by \( z \approx 0.25 \). We are thus not complete in surface brightness to this type of LSB galaxy over the full CNOC2 redshift range. Nevertheless, as Figure 2 shows, we should still be sensitive to galaxies with somewhat higher surface brightnesses (but still fainter than the \( M^* \) Freeman track) over a fairly broad range of redshift and apparent
magnitude. However, very few of these LSB galaxies faintward of the $M_*$ Freeman track are detected within our sample. Hence the number of these LSB galaxies is apparently quite small compared to that of the more “normal” objects to which the survey is complete. We have not attempted to check whether the number density of our LSB galaxies is quantitatively consistent with recent results at low redshifts (e.g., Sprayberry et al. 1997; see review by Impey & Bothun 1997), since that would take us too far afield, requiring us to examine detailed issues of surface brightness measurements, LSB galaxy definitions, surface brightness evolution from intermediate to low redshifts, and so on. We will, however, explore these issues in future analyses of the galaxy surface brightness distributions in the CNOC2 sample.

3. METHODS

3.1. Galaxy Classification

We classify CNOC2 galaxies using least-squares fits of our $UBVR_cI_c$ colors to those computed from the galaxy SEDs of CWW. The CWW colors are computed using filter transmission curves taken from Buser & Kurucz (1978) for $UBV$ and from Bessel (1990) for $R_c$ and $I_c$. As shown in Figure 3, we assign numerical values to the four CWW SEDs as follows: $0 = \text{E}$ (average of the M31 bulge and M81 bulge SEDs), $1 = \text{Sbc}$, $2 = \text{Scd}$, $3 = \text{Im}$. We linearly interpolate between neighboring SEDs using 50 equal steps in the computed broadband magnitudes and also allow linear extrapolations to SED types $-0.5$ and $+3.5$. The best-fitting SED type is plotted against redshift in Figure 3 for CNOC2 galaxies with $R_c < 21.5$. The galaxies are assigned to three categories, “Early,” “Intermediate,” and “Late” according to the following:

- Early: $-0.50 \leq \text{SED type} < 0.50$
- Intermediate: $0.50 \leq \text{SED type} < 1.50$
- Late: $1.50 \leq \text{SED type} \leq 3.50$

Before fitting, we also add $-0.05$ mag to the $I_c$ magnitudes computed from the E, Sbc, and Scd SEDs (but not the Im SED) in order to empirically match the observed $R_c-I_c$ colors of CNOC2 galaxies, but otherwise we make no further adjustments to the CWW SEDs. This is an ad hoc procedure and may be symptomatic of a general limitation of the CWW SED set, specifically, that they are based on a very small number of observed local galaxies, which should not be expected to represent the full galaxy population in every exacting detail. However, lacking a better SED set, we nonetheless choose the CWW set for simplicity and note

![SED type vs. redshift for CNOC2 galaxies with $R_c < 21.5$. The SED types are determined by least-squares fits of $UBVR_cI_c$ magnitudes to those computed from the SEDs of CWW, as described in the text. Numerical SED types for the four original CWW SEDs are assigned as indicated by the dotted horizontal lines, and the boundaries defining the “Early” (squares), “Intermediate” (circles), and “Late” (triangles) CNOC2 galaxy categories are indicated by the solid horizontal lines. The dotted vertical lines indicate the CNOC2 nominal redshift completeness range $0.12 < z < 0.55.$]
that aside from the above exception the CWW galaxy colors do give a reasonable match to the observed CNOC2 galaxy colors. In § 4.1.2, we will discuss the general implications of the particular choice of SED set and classification scheme on our galaxy evolution results. We also do not attempt to fit for dust extinction in the CNOC2 galaxies themselves (though we do correct for Milky Way extinction as described in § 2); this will be addressed instead in Paper II.

Note that our SED types are "stellar population" types derived from broadband galaxy colors and would be more closely related to classifications derived from galaxy spectra than from galaxy morphologies. We stress that our galaxy SED types are not and should not be interpreted as morphological types. Of course, there are correlations between galaxy types derived separately from colors, spectra, and morphologies; we postpone an examination of the similarities and differences among these various classification schemes to future CNOC2 papers.

The visual impression from Figure 3 is that there are no obvious type- or redshift-dependent incompletenesses, except for the lack of galaxies earlier than Sbc at redshifts $z \gtrsim 0.6$ (Ca II H and K redshifts out of our spectral window 4400–6300 Å) and the dearth of intermediate-type galaxies at $z \lesssim 0.05$; both cases are outside our redshift completeness range $0.12 < z < 0.55$. Also, there are no redshifts at $z > 0.7$ because [O II] $\lambda\lambda$3727 redshifts beyond the red end of our spectra. (The exceptions are a handful of higher-$z$ active galactic nuclei/quasi-stellar objects that are excluded from our analysis and are not plotted.) Figure 4 compares the $B-R_C$, $R_C-I_C$, $V-R_C$, and $U-R_C$ colors observed for CNOC2 galaxies with those computed from the CWW SEDs, showing that the SEDs do indeed span the range of actual galaxy colors.

We will compute absolute magnitudes, $k$-corrections, and other needed quantities using the best-fitting (interpolated or extrapolated) CWW SED for each individual galaxy. Note in particular that the absolute magnitudes we use (in $B$, $R_C$, or $U$) are calculated directly from the best-fitting SED (thus making full use of the available $UBVR_CI_C$ data), rather than from any single apparent magnitude. For example, the $U$ absolute magnitude would not be derived by adding $-25 - 5 \log d_L$ and a $k$-correction to the $U$ apparent magnitude; instead, we would calculate the $U$ absolute magnitude by direct integration of the best-fit SED convolved with the $U$-band filter response function.

![Figure 4](image-url)

**Fig. 4.—** Plot of various observed colors vs. redshift for CNOC2 galaxies with $R_C < 21.5$: $B - R_C$ (top left), $R_C - I_C$ (top right), $V - R_C$ (bottom left), and $U - R_C$ (bottom right). The upper and lower solid curves show the colors for the original CWW E and Im SEDs, respectively. The dashed curves (corresponding to Fig. 3, solid lines) show the colors for those interpolated and extrapolated CWW SED types which define the boundaries of early (squares), intermediate (circles) and late (triangles) CNOC2 galaxies (the same classifications as in Fig. 3). The dotted vertical lines indicate the CNOC2 nominal redshift completeness range $0.12 < z < 0.55$. 


3.2. Computing the LF

We compute the LF using standard maximum-likelihood methods (Sandage, TAMMANN, & YAHIL 1979; EFSTATHIOU, ELLIS, & PETERSON 1988, hereafter EEP), which are unbiased by density inhomogeneities in the galaxy distribution. Our procedure essentially follows that given in LIN et al. (1996a, 1997) and is only summarized briefly here, except that we will describe in more detail our present methods for parameterizing and fitting the evolution of the LF.

Given a survey of $N$ galaxies at redshifts $z_i$, we form the likelihood $\mathcal{L}$ for those galaxies to possess their observed absolute magnitudes $M_i$:

$$
\ln \mathcal{L} \equiv \ln p(M_1, \ldots, M_N \mid z_1, \ldots, z_N) = \sum_{i=1}^{N} W_i \ln p_i + \text{constant} .
$$

Here $W_i$ is the weight described previously in §2.1, and $p_i$ is the individual conditional probability,

$$
p_i \equiv p(M_i \mid z_i) \propto \phi(M_i) \int_{M_{\min}(z_i)}^{M_{\max}(z_i)} \phi(M) dM ,
$$

where $M_1$ and $M_2$ are the global absolute magnitude limits we impose on the sample ($M_1 < M < M_2$), $M_{\min}$ and $M_{\max}$ are the absolute magnitude limits at $z_i$ that correspond to the survey’s apparent magnitude limits, and $\phi(M)$ is the differential LF, whose parameters we determine by maximizing $\ln \mathcal{L}$.

For the form of $\phi(M)$, we adopt the usual Schechter (1976) parameterization,

$$
\phi(M) = (0.4 \ln 10) \phi^*(10^{0.4 M - M^*})^1 + \alpha \exp(-10^{0.4 M - M^*}) ,
$$

with characteristic magnitude $M^*$, faint-end slope $\alpha$, and normalization $\phi^*$. We also use the nonparametric “stepfunction” of EEP,

$$
\phi(M) = \phi_k , \quad M_k - \Delta M/2 < M < M_k + \Delta M/2 ,
$$

$k = 1, \ldots, N_p ,

which we refer to hereafter as the SWML (stepwise maximum-likelihood) LF. The details for computing $\phi(M)$ via maximum likelihood and for estimating errors are as given in EEP and LIN et al. (1996a, 1997).

To parameterize evolution in the LF, we adopt the following simple model for the redshift dependence of the Schechter parameters:

$$
M^*(z) = M^*(0) - Qz = M^*(z = 0.3) - Q(z - 0.3) ,
$$

$$
\alpha(z) = \alpha(0) ,
$$

$$
\rho(z) = \rho(0) 10^{-0.4 Pz} .
$$

We thus take $M^*$ to vary linearly with redshift at a rate quantified by $Q$, which we call the $M^*$ or luminosity evolution parameter. Note that we will fit for $M^*(z = 0.3)$, since this is a better constrained quantity than $M^*(0)$, given the mean redshift $z \approx 0.3$ for CNOC2 galaxies. We also make the null assumption that $\alpha$ does not change with redshift, so that the shape of the LF stays the same. Since $\alpha$ is fixed, the normalization parameter $\phi^*$ and the total galaxy number density $\rho = \int \phi(M) dM$ are essentially equivalent. We then take $\rho$ to vary with $z$ as determined by the density evolution parameter $P$ defined above. The expression for $\rho$ in equation (10) is chosen for convenience, so that the luminosity density $\rho_L = \int L \phi(M) dM$ (where $L \propto 10^{-0.4 M}$) may be written as

$$
\rho_L(z) = \rho(z) 10^{-0.4 P + Qz} ,
$$

where $P + Q$ then measures the linear rate of evolution of $\rho_L$ with redshift. Also, note that $\rho$ as defined above may be approximated by $\rho(z) \approx \rho(0)(1 + Pz)$, so that $P$ is merely the coefficient of the linear term in the expansion of $\rho$ in powers of $z$.

We first estimate $M^*(z = 0.3)$, $\alpha$, and $Q$ together using the usual maximum likelihood method, and by design this is independent of density fluctuations or density evolution, so that both $\phi^*(0)$ and $P$ have to be determined separately, beginning with $P$. In the case of a non-evolving LF, it is possible to derive maximum-likelihood estimates of $\rho(z)$ without prior knowledge of the LF (SAUNDERS et al. 1990; LOVEDAY et al. 1992; FISHER et al. 1992). This is completely analogous to the case above, where we may estimate $M^*$ and $\alpha$ independently of galaxy density variations. For the more general evolving LF defined above, we may still determine $P$ without knowing $M^*(z = 0.3)$ or $\alpha$, but not without first knowing $Q$. Given a value of $Q$, we may convert an observed absolute magnitude $M_i$ at $z = z_i$ to an evolution-corrected absolute magnitude at some fiducial redshift, say, $z = 0$ (the actual redshift does not matter): $M_i(0) = M_i(z_i) + Q z_i$. Then, with any given $Q$ and the resulting set of $M_i(0)$, we compute the likelihood that those galaxies will have their observed redshifts $z_i$:

$$
\ln \mathcal{L}' \equiv \ln p[z_1, \ldots, z_N \mid M_i(0), \ldots, M_N(0), Q] = \sum_{i=1}^{N} W_i \ln p_i + \text{constant} .
$$

Here the individual conditional probabilities are

$$
p_i \equiv p(z_i \mid M_i(0), Q) \propto \phi(M_i(0), z = 0) \rho(z_i) \int_{\min(z_{\max}(M_i(0), z_i))}^{\max(z_{\min}(M_i(0), z_i))} \phi(M, z = 0) dV dz dz
$$

$$
= \rho(z_i) \int_{\min(z_{\max}(M_i(0), z_i))}^{\max(z_{\min}(M_i(0), z_i))} 10^{0.4 Pz_i} dV dz dz
$$

(see SAUNDERS et al. 1990; FISHER et al. 1992), where $\phi^*$ is $\phi$ with $\phi^*$ set to unity ($\phi^*$ has units of mag$^{-1}$), $z_1$ and $z_2$ are the global redshift limits we impose on the sample, and $z_{\min}$ and $z_{\max}$ are the redshift limits over which galaxy $i$ may be observed, given the survey’s apparent magnitude limits and the assumed rate of evolution specified by $Q$. The value $P$ may then be readily determined using maximum likelihood once given the previously found best-fit value for $Q$. The fifth and final parameter $\phi^*(0)$ is then computed via straightforward summation:

$$
\phi^*(0) = \frac{1}{V} \sum_i \frac{W_i}{S(z_i) 10^{-0.4 P z_i}} \int_{M_1}^{M_2} \phi(M, z = 0) dM
$$

(see LIN et al. 1996a, 1997), where $V$ is the survey volume and $S(z)$ is the selection function, defined by

$$
S(z) = \int_{\min(z_{\max}(z, M_2))}^{\max(z_{\min}(z, M_1))} \phi(M, z) dM .
$$
Once we have fit for all the LF parameters, we will calculate luminosity densities $\rho_L$ as a function of redshift using

$$\rho_L(z_a < z < z_b) = \frac{1}{V(z_a < z < z_b)} \sum_{z_a < z < z_b} \frac{W_i 10^{-0.4 M_i}}{S_L(z_i)},$$

where

$$S_L(z) = \int_{\min [M_{\text{max}}(z), M_z]}^{\max [M_{\text{min}}(z), M_z]} 10^{-0.4 M} \phi(M, z) dM \int 10^{-0.4 M} \phi(M, z) dM.$$

That is, we sum over the luminosities of our observed galaxies but weight by the factor $S_L(z)$, which uses the LF $\phi$ to extrapolate for the luminosity of unobserved galaxies lying outside the accessible survey flux limits. Also, we will express $\rho_L$ in units of $h insdes dm^{-3}$ using the conversion given in Lilly et al. (1996b); specifically, one $M_{B_{RA}} = -19.5 + 5 \log h$ galaxy $\text{Mpc}^{-3}$ produces a luminosity density of $2.85 \times 10^{21} h^{-2} W \text{ Hz}^{-1} \text{ Mpc}^{-3}$.

Finally, we estimate uncertainties in $\rho_L$ and $\phi(0)$ using both bootstrap resampling and an estimate of the uncertainty contributed by galaxy density fluctuations. We apply bootstrap resampling (e.g., Barrow, Bhavsar, & Sonoda 1984) to the full photometric sample (not just to those galaxies with redshifts), recalculate statistical weights $W_i$ (as in § 2.1) anew for galaxies with redshifts in each bootstrap resample, and then refit our LF evolution model and compute luminosity densities. This process should account for the uncertainties in $\phi(0)$ and $\rho_L(z)$ contributed by sampling and weighting fluctuations and by our fitting procedure. This does not account for the additional uncertainty arising from galaxy density fluctuations, which we estimate instead using an integral over the galaxy clustering power spectrum $P(k)$; see § 3 of Lin et al. (1997) for details. For $P(k)$ we adopt the local result from the Las Campanas Redshift Survey (Lin et al. 1996b, their eqs. [23], [24]) but adjusted (only) for the linear clustering evolution at the higher redshifts sampled in CNOC2; this is done as appropriate for both the $q_0 = 0.5$ and $0.1$ cosmologies we consider. We then take the overall error on $\phi(0)$ and $\rho_L$ to be the quadrature sum of the bootstrap resampling and density fluctuation error contributions.

### TABLE 1

| Sample | $N^a$ | $M^*(z = 0)^b$ | $q_0 = 0.5$ | $q_0 = 0.1$ |
|--------|-------|----------------|-------------|-------------|
| Early  | 611   | $-19.06 \pm 0.12$ | $-19.38 \pm 0.08$ | $-19.38 \pm 0.06$ |
| Intermediate | 518 | $-19.38 \pm 0.16$ | $-19.51 \pm 0.17$ | $-19.38 \pm 0.16$ |
| Early plus intermediate | 1129 | $-19.19 \pm 0.10$ | $-19.32 \pm 0.10$ | $-19.32 \pm 0.10$ |
| Late | 1016 | $-19.26 \pm 0.16$ | $-19.26 \pm 0.16$ | $-19.26 \pm 0.16$ |

$^a$ All tabulated errors are $1 \sigma$ one-parameter errors. See Figs. 7 and 9 for the joint two-parameter $M^*-z$ and $P-Q$ error contours, respectively.

$^b$ We apply apparent magnitude limits $17.0 < R_a < 21.5$, absolute magnitude limits $-22.0 < M_{B_{RA}} < -16.0$, and redshift limits $0.12 < z < 0.55$ defining our samples.

$^c$ We take Hubble constant $h = 1$.

$^d$ Units are $h^2 \text{ Mpc}^{-2} \text{ mag}^{-1}$.

### 4. RESULTS

#### 4.1. Evolution of the $B_{AB}$-band LF

We apply the LF fitting methods and evolution model of § 3.2 to our nominally complete $17.0 < R_a < 21.5$ sample, subdivided into early, intermediate, and late galaxy types as described in § 3.1. The sample details and fit parameters are given in Table 1. We first concentrate on our LF results in the $B$ band, shown in Figure 5 for the three galaxy types and for each of three redshift bins in the range $0.12 < z < 0.55$. For ease of comparison against previous surveys, we will report our $B$-band LF results in the $AB$ system (Oke 1972) using the transformation $B_{AB} = B - 0.14$ (Fukugita, Shimizu, & Ichikawa 1995). The points in the figure show the nonparametric SWML LF estimates in each individual type-redshift bin, and the solid lines indicate the results of our five-parameter LF evolution model $(M^*(z = 0), z, \phi(0), P, Q)$, fitted to the full redshift completeness range $0.12 < z < 0.55$ for each of the three galaxy types. Figure 5 should allow us to judge how well our parametric LF model matches the nonparametric LF estimates, which we obtained without making any assumptions about the form that the LF evolution takes.

#### 4.1.1. Some Technical Considerations

There are, however, a number of subtleties involved in comparing the SWML LF estimates $\phi_L$ to the parametric evolving LF estimate $\phi(M, z)$. The nonparametric $\phi_L$ are binned in both $M$ and $z$, but the parametric $\phi(M, z)$ is not; it may be unclear at what redshift we should evaluate $\phi(M, z)$ in order to compare against the $\phi_L$. For example, a simple procedure such as plotting the parametric LF models evaluated at the average redshift of each redshift bin in Figure 5 will actually result in noticeable discrepancies (at the bright and faint ends of the LF) when compared against the $\phi_L$, even when there should be none. The proper thing to do is actually to calculate a weighted average of $\phi(M, z)$ over the appropriate intervals in $M$ and $z$. Specifically, we follow a procedure given by EEP but modified for our sample. We note first that, in general, $\phi_L \neq \phi(M = M_k)$ even in the absence of LF evolution. As shown by EEP in their equation (2.15), the $\phi_L$ are actually related to the parametric $\phi(M)$ by a weighted integral over $\phi(M)$, where the

\[ \int \phi(M) dM = \phi(M = M_k). \]
weights are just the expected number $N(M)$ of galaxies of absolute magnitude $M$ observable by the survey. In the limit that the bin size $\Delta M \to 0$ and with no evolution, $\phi_k$ would indeed converge to $\phi(M = M_k)$. For CNOC2 we need to modify EEP’s original equation (2.15) to account for our use of an evolving LF as well as for cosmological and $k$-correction effects important for our intermediate-$z$ sample. Specifically, for an absolute magnitude bin $M_k - \Delta M/2 < M < M_k + \Delta M/2$ and a redshift bin $z_1 < z < z_2$, we may define the quantity

$$
\phi_{\text{parametric}, k}(z_1 < z < z_2) = \frac{\min_{[z_{\max}(M), z_2]} \phi(M, z) \frac{dN}{dz} dM(M, z) dz dM}{\int_{M_k - \Delta M/2}^{M_k + \Delta M/2} \left[ \frac{dN}{dz} dM \right](M, z) dz dM}
$$

where $(dN/dM)(M, z) = \phi(M, z) dV$ is the expected number of galaxies per unit magnitude at redshift $z$, and $z_{\min}(M)$ and $z_{\max}(M)$ are the minimum and maximum redshift, respectively, at which a galaxy of absolute magnitude $M$ may be seen, given our survey’s apparent magnitude limits and cosmological and $k$-correction effects. The parametric LF estimates we plot in Figure 5 and elsewhere are those given by equation (19); this is the right way to compare our parametric LF fits against the directly computed nonparametric SWML estimates.

In addition, in Figure 5 we also show low-redshift fiducial LFs (dotted curves) to facilitate comparison from one redshift bin to another. For this purpose we use $\phi(M, z = 0.175)$ (i.e., $\phi$ evaluated at nearly the average redshift of the lowest $z$ bin) but appropriately averaged using equation (19) over the higher redshift intervals. Also, to facilitate bin-to-bin comparisons, we show extrapolations (dashed curves) of the parametric LFs faintward of the faintest absolute magnitude accessible in each redshift bin. For this purpose we simply choose $\phi[M, z = (z_1 + z_2)/2]$ (i.e., $\phi$ evaluated at the average redshift of the bin), since $\phi_{\text{parametric}, k}$ defined above is zero at these magnitudes (no galaxies observable there!). Note the slight disconnections between the solid and dashed curves for the late-type parametric LFs in the two highest $z$ bins in Figure 5; these are
For clarity in seeing the evolution trends, we have purposefully matched the normalizations of the SWML and parametric LF fits in each redshift bin of Figure 5. Specifically, we set

\[ N_\text{parametric} \frac{V(M_k)}{k} = N_\text{SWML} \frac{V(M_k)}{k}, \]

where \( V(M_k) \) is the volume (within the redshift limits of each bin) over which a galaxy of absolute magnitude \( M_k \) may be seen in our survey. We do this to take out the effects of strong density fluctuations present in the survey, clearly seen in the (weighted) redshift histograms shown in Figure 6 (left-hand panels), particularly for early- and intermediate-type galaxies in the \( 0.25 < z < 0.4 \) bin. Note from Figure 6 (right-hand panels) that the ratio of actual to LF-computed redshift distributions are reasonably centered on unity and do not show conspicuous systematic trends with redshift, indicating that the normalization \( \phi^*(0) \) and the number density evolution parameter \( P \) in our fits are indeed good matches to the data. (To construct the galaxy redshift histograms, we have first weighted each galaxy by \( W_i \) to correct for redshift incompleteness; this makes construction of the corresponding LF-computed redshift histogram much simpler, since we are then freed from modeling the somewhat complicated magnitude- and color-dependent selection effects of our survey.)

4.1.2. Description of the LF Evolution

Setting the above technical considerations aside and returning to Figure 5, we may note that our LF model does indeed appear to be a reasonable description of the data, as seen in the good agreement between the parametric and nonparametric SWML fits (but recall we have matched their normalizations, so that we are really only assessing the validity of the \( M^*, a, \) and \( Q \) parameters). The comparison also shows that our simple assumption of a fixed \( a \) is quite reasonable, although of course at higher redshifts it becomes increasingly difficult to constrain the faint-end slope of the LF. Note also that our fixed-\( a \) result differs...
from that found in the Autofib Redshift Survey (Ellis et al. 1996; Heyl et al. 1997); see § 5.3 below.

Figure 5 also shows that the three galaxy types have conspicuously different LFs, with faint-end slopes $\alpha$ steepening from $\alpha = +0.1$ for early types to $\alpha = -1.2$ for late types (Table 1). These clear LF differences are indeed significant, as shown in Figure 7 (top panel), where we see nonoverlapping or barely-touching 2 $\sigma$ error contours in $M^*(z = 0.3)$ and $\alpha$ for the three galaxy types.

Figure 5 demonstrates that the LFs for all three galaxy types do indeed evolve. The impression is that the early- and intermediate-type LFs are not changing much in number density but rather are brightening in $M^*$ at higher redshifts. For the late-type LF, it is harder to discern visually (because of the steepness of the LF) whether the definite changes seen result from increasing number density, brightening $M^*$, or a combination of the two. We can isolate the luminosity evolution component of the LFs by rescaling the dotted fiducial $z = 0.175$ LF in each panel by the factor $10^{[0.4P(z_1 + z_2)/2 - 0.175]}$ to explicitly take out the effect of the number density evolution parameter $P$. We do this in Figure 8, where we confirm our earlier impression that the early- and intermediate-type LFs are evolving primarily in $M^*$. In contrast, the rescaled low-$z$ fiducial late-type LF is a good match to the results in the two higher redshift bins, indicating that the observed late-type LF evolution seen before in Figure 5 is driven primarily by number density changes. (Note that an apparent change in number density does not necessarily imply mergers; changes in the star formation duty cycle for late-type galaxies may also mimic the effect of true mergers.)

Now an important consideration mentioned in § 1 needs to be kept in mind, namely, the sensitivity of the LF evolution results to the precise choice of SEDs used to classify galaxies. In particular, the present choice of nonevolving CWW SEDs obviously does not account for the evolution of the colors of galaxies with time, so that during the course of its evolution, a particular galaxy may actually cross the type boundaries we have defined. It is thus better to use more physically motivated evolving galaxy SEDs (e.g., produced by models such as those of Bruzual & Charlot 1996) to properly track the paths different galaxies may take in the space of redshift versus color. Not surprisingly, the resulting galaxy classifications will in general differ from the ones we make based on the CWW SEDs, and, importantly, the conclusions we draw on the rates of luminosity and

![Figure 7](image_url)
density evolution for different galaxy populations will also be different in general. However, there are a myriad of possible evolving SEDs that one may choose by varying parameters such as star formation history, stellar initial mass function, epoch of galaxy formation, metallicity, dust content, and others, so that there is no unique set of SEDs that one should obviously pick a priori. In our next LF paper (Paper II), we will examine evolution in the CNOC2 sample using these physically motivated evolving galaxy SED models. In the present paper, though, we will use only the nonevolving CWW SEDs for galaxy classification. Thus the LF evolution constraints we derive here should strictly be considered as descriptions of galaxy evolution within the framework of nonevolving SEDs rather than as explanations of galaxy evolution in terms of more physically motivated processes. In other words, we are using the terms “luminosity evolution” and “density evolution” purely to describe the changes in the LFs of the galaxy populations we have defined, and those terms may not correspond to the true evolutionary processes those galaxies are actually undergoing. (The latter is not precluded, though. As we will find in Paper II, the luminosity evolution we see in early and intermediate galaxies still holds true when we use physically motivated evolving SEDs.)

Also, we caution that the LF constraints will be weaker and the errors larger when our LF models are extrapolated outside the nominal CNOC2 redshift limits. For example, the errors are approximately doubled for $M^*(z = 0)$ compared to $M^*(z = 0.3)$; specifically, $M^*(z = 0) = −18.58 \pm 0.23$, $−19.11 \pm 0.34$, and $−19.20 \pm 0.35$ for early-, intermediate-, and late-type galaxies, respectively. There are only about 200 galaxies in our LF sample with $0.12 < z < 0.2$ to constrain the lowest $z$ behavior of our LF evolution model. These galaxies alone do in fact give an overall best fit $M_{B_{AB}}^* = −19.5$ and $α = −0.9$, in reasonable agreement with results from much larger local redshift samples (e.g., Loveday et al. 1992), and our $M^*(z = 0)$ and $α$ values for intermediate- and late-type galaxies are also in good agreement with local results (e.g., Fig. 8 of Colless 1998). However, our early-type LF may have a fainter $M^*(z = 0)$ and shallower $α$ compared to local values (e.g., Colless 1998, but see also Bromley et al. 1998). In future work we will compare in more careful detail our results with those of large local surveys in order to further check the validity of extrapolations of our LF models (see also §§ 4.3 and 5.2).

Keeping the above caveats in mind, our impression so far is that evolution in early- and intermediate-type galaxies is dominated by brightening in $M^*$ at higher $z$, whereas the evolution in late-type galaxies is caused by increasing number densities at higher redshifts. This impression is borne out in the $P$ versus $Q$ error contours shown in Figure

Fig. 8.—Same as Fig. 5 but with the fiducial low-redshift LF (dotted curves) rescaled to take out the effects of density evolution; see text for details.
late types evolve at a significantly more rapid rate
\((P + Q = 3.3)\) than that of early and intermediate types
\((P + Q = 0.5 \text{ and } 1.6)\). This is shown in more
detail in Figure 10, where we plot \(\rho_L(z)\) for the three galaxy
types individually, as well as summed together. We also
plate our luminosity density results in Table 3. Clearly,
the late-type population shows the most strongly increasing
\(\rho_L(z)\), whereas the early and intermediate types show much
milder increases at higher redshift. Note that the luminosity
densities for the three types are roughly equal at \(z \approx 0.1\), but
by \(z \approx 0.55\) late-type galaxies predominate and account for
over half of the total luminosity density. Also shown in
Figure 10 are the separate contributions to \(\rho_L(z)\) from the
luminosity and number density evolution components for
each of the three galaxy types. Compared to the \(P-Q\) plot or
even the LF plots, the curves for these individual com-
ponents most clearly illustrate the different LF evolution
trends we discussed earlier (but keeping the caveats in
mind). The late-type galaxy LF is dominated by strong
density evolution with nearly no luminosity evolution. The
intermediate-type LF shows positive luminosity evolution
plus weak positive density evolution, resulting in mild posi-
tive evolution in \(\rho_L\). The early-type LF shows positive lumi-
nosity evolution, which is nearly compensated by negative
density evolution, yielding a very weak positive evolution in
the luminosity density.

These general conclusions are not altered much by
adopting a \(q_0 = 0.1\) instead of a \(q_0 = 0.5\) cosmology. Our
\(q_0 = 0.1\) results are also tabulated in Tables 1 and 3, and the
corresponding \(P-Q\) contours and \(\rho_L(z)\) plots are shown in
Figures 11 and 12, respectively. To first order, absolute
magnitudes change with \(q_0\) as \(M(z, q_0) \approx M(z, q_0 = 0.5)
+ (q_0 - 0.5)z\), and the differential volume element varies as
\((dV/dz)(z, q_0) \approx (dV/dz)(z, q_0 = 0.5)[1 - 2(q_0 - 0.5)z]\). We
thus expect \(\Delta Q \approx +0.4\), \(\Delta P \approx -0.8\), and \(\Delta(P + Q) \approx -0.4\n\) in going from \(q_0 = 0.5\) to \(q_0 = 0.1\), and indeed that is what
we approximately find quantitatively. Qualitatively, this
means more positive luminosity evolution but more nega-
tive number density and luminosity density evolution. In
particular, no-evolution \((P = Q = 0)\) for the early and inter-
mediate types combined may be ruled out at higher signif-
cance than was possible for the \(q_0 = 0.5\) case. Otherwise,
though, the general LF evolution trends follow those for the
\(q_0 = 0.5\) cosmology.

Finally, in this subsection we make the most minimal
assumptions and fit the LF for the three galaxy types with
nonevolving Schechter functions and compare the resulting
trends of luminosity density versus redshift against those
obtained from the evolving models above; this is done in
Figure 13 (for \(q_0 = 0.5\) only). Reassuringly, we find that the
actual trend of \(\rho_L\) with redshift (the \(\text{points}\) in the figure, not
the curves) is insensitive to whether we use nonevolving or
evolving LFs (in eqs. [16] and [17]). The late-type \(\rho_L\)
always rises sharply compared to the weak increases
observed for the earlier types. However, as seen in the top
two panels of Figure 13, a constant \(\rho_L\), as required by a
nonevolving LF, is clearly a bad description of the late-type
\(\rho_L(z)\) and consequently for the total \(\rho_L(z)\) as well, since late
types make up the greatest contribution. On the other hand,
a nonevolving constant \(\rho_L(z)\) does seem to be reasonable for
the two earlier types. This is not surprising, since Figure 9
shows that although an evolving LF is preferred by the
data, a nonevolving LF is ruled out at only about 2 \(\sigma\) for early
and intermediate galaxies. A larger data set will be

\[\begin{align*}
  P \approx -0.3, \quad \text{whereas late types show strong positive density evolution,}
  P = 3.1, \quad \text{but little } M^* \text{ evolution, with}
  Q = 0.2 \text{ (Table 1). However, Figure 9 also shows that we}
  \text{need to be somewhat cautious and keep the correlated nature}
  \text{and fairly large size of the } P-Q \text{ error contours in}
  \text{mind. Not surprisingly, our ability to decouple density and}
  \text{luminosity evolution depends on the shape of the } P-Q \text{ for}
  \text{early and intermediate types, the LF has a shallow } z \text{ and}
  \text{a conspicuous } \text{“ knee” near } M^*; \quad \text{however, for late types}
  \text{the LF is steep, and it becomes correspondingly harder to}
  \text{measure subtle changes in } M^* \text{ with redshift. Thus, although}
  \text{the late-type sample is the largest among the three types, the}
  \text{late-type } P-Q \text{ contour is the most elongated and the most}
  \text{difficult one for which to separately constrain density and}
  \text{luminosity evolution. Moreover, even for the two earlier}
  \text{type samples, no-evolution } (P = 0, \quad Q = 0) \text{ is ruled out at}
  \text{only somewhat better than the } 2 \sigma \text{ level. Nonetheless, it}
  \text{does appear to be a fairly robust conclusion from Figure 9}
  \text{that late types occupy a different region of } P-Q \text{ parameter}
  \text{space than early and intermediate types, so that the form of}
  \text{the LF evolution of late-types is distinct from that of early-
and intermediate-type galaxies.}
\end{align*}\]
needed in order to make a stronger statement regarding no-evolution versus evolution, and the doubled size of the final CNOC2 sample should allow significantly improved constraints on these early- and intermediate-type galaxies. In addition, we are also calibrating photometric redshifts using the multicolor data for our spectroscopic-redshift sample. Application of photometric redshifts to those CNOC2 galaxies without spectroscopic redshifts should provide another factor of 2 increase in the number of galaxies that may be used in our LF studies, thereby allowing further improvements in our evolution constraints.

4.2. The $R_C$- and $U$-band LFs

The availability of $UBVR_CIC$ colors in conjunction with the CWW SED types makes it a straightforward matter to calculate the appropriate $k$-corrections and derive the LF in bands other than $B$, the most typical choice. We do so for the $R_C$ and $U$ bands. No extrapolations are required of our color data to derive rest-frame $U$ magnitudes. Although extrapolations are required to obtain rest-frame $R_C$ magnitudes, the needed $k$-corrections are not large ($\lesssim 1$ mag) and are well-constrained by the SED classifications. Note that here we are always using a $17.0 < R_C < 21.5$ sample; we are not varying the band used for galaxy selection (not until §4.5 and 5.3 below).

The best-fit $R_C$ and $U$ LF and evolution parameters are given in Table 2, and the luminosity densities are given in Table 3. The $R_C$- and $U$-band LFs themselves will not be plotted, since those figures would look very similar to Figure 5 for the $B_{AB}$ LFs. Essentially, the same trends observed earlier for the $B_{AB}$ LFs are seen for $R_C$ and $U$ as well, and our earlier discussion applies. We also find that the best-fit faint-end slopes $\alpha$ are independent of band. For all three galaxy types, the full range in best-fit $\alpha$ values is only about 0.2 among the three bands $B_{AB}$, $R_C$, and $U$. Thus, to convert the LF results from one band to another, it is a good approximation to keep $\alpha$ fixed and just apply an appropriate offset in $M^*$ based on the mean rest-frame color for that galaxy type. Moreover, it also turns out that the evolution parameters $Q$ and $P$ agree well from band to band. In retrospect this is not surprising. We expect that $Q$ should stay the same, because our galaxy classification scheme is based on selecting galaxies of similar rest-frame colors at different redshifts. For example, however much the average $M_B$ changes with redshift for our early-type galaxies, $M_R$ for those same galaxies should change by about

---

**Fig. 10.—**Redshift evolution of the CNOC2 rest-frame $B_{AB}$ luminosity density $\rho_L(z)$ shown for the early, intermediate, late, and total galaxy samples. We plot both the directly observed but LF-weighted (points) as well as the LF-computed (solid lines) luminosity densities. We also show the separate luminosity-evolution (dotted curves) and density-evolution (dashed curves) components of the overall LF-computed luminosity density evolution curves. Results shown are for $q_0 = 0.5$. 
the same amount, since by definition the rest-frame color $M_B - M_R$ of our early-type galaxies needs to stay about constant with redshift. Thus, for populations of similar rest-frame color, $Q$ and subsequently $P$ will be approximately independent of which band is chosen for the LF.

However, although the rate $P + Q$ of luminosity density evolution for a particular galaxy type is similar in different bands, the normalization $\rho_L(z = 0)$ is in general different. This makes the overall evolution of $\rho_L$ somewhat different for the different bands, and Figure 14 compares $\rho_L(z)$ for the three bands including subdivision by galaxy type. For $R_C$ and $U$, we have first applied $AB$ corrections $R_{CAB} = R_C + 0.169$ and $U_{AB} = U + 0.69$ (Fukugita et al. 1995) before applying the same conversion we used for $B_{AB}$ to convert to $h^2 \text{Mpc}^{-3} \text{mag}^{-1}$.

\[ F(t_1 < t < t_2) \equiv \sum_{t_1 < t < t_2} W_i S_j(z_i) / \sum_{\text{all galaxies}} W_i S_j(z_i). \]

Here we are weighting by the usual statistical weights $W_i$, as well as by the appropriate selection function $S_j(z_i)$ from equation (15), where $j = \text{early, intermediate, or late indi-
icates the category to which galaxy \( i \) belongs. The inverse selection function weighting corrects \( F(t) \) to what one would obtain for a volume-limited sample with \( -22 < M_{\text{AB}} < 5 \log h < -16 \), and there is also an implicit assumption that the LF is independent of SED type \( t \) within each of the three galaxy categories. Also, because the LF evolves, \( F(t) \) will change with redshift, but for illustrative purposes we will neglect this complication and simply plot in Figure 15 the \( F(t) \) computed over the full redshift completeness range \( 0.12 < z < 0.55 \). In our number count and color distribution calculations we actually only need the fractional distribution within each of the three galaxy categories \( j \):

\[
G_j(t_1 < t < t_2) \equiv \sum_{t_1 < t < t_2; \text{galaxy in category } j} \frac{W_i S_j(z_i)}{\sum_{\text{all galaxies in category } j} W_i S_j(z_i)}.
\]

We have explicitly checked that \( G_j \) changes only weakly with redshift, so that it is a good approximation to adopt the \( G_j \) computed for the full redshift range \( 0.12 < z < 0.55 \) in our subsequent calculations. Note that the appearance of the histogram in Figure 15 (the presence of peaks and valleys, how smooth or not it appears) may depend to some extent upon the choice of the set of original SEDs used to define the classification scheme.

We now consider the CNOC2 photometric sample with \( R_c < 21.5 \), the nominal spectroscopic limit. The galaxy number counts in the \( UBVRI_c \) bands for this sample are plotted in Figure 16. Note the turnover at faint magnitudes in the \( UBVRI_c \) bands is due to our explicit cut and \( UBVRI_c \) is not a result of incompleteness in the photometry in these bands. We also plot the number counts computed using the evolving LFs and the fractional distributions derived previously. In particular, galaxies with redshifts \( z_1 < z < z_2 \) will contribute to the number counts in an apparent magnitude interval of \( m_1 < m < m_2 \) of a particular band according to

\[
N(m_1 < m < m_2; z_1 < z < z_2) = \sum_j \sum_t G_j(t) \int_{z_1}^{z_2} \left( \frac{dV}{dz} \right) dz \int_{M_{\text{min}}(z, m_1, t)}^{M_{\text{max}}(z, m_2, t)} \phi_j(M, z) dM ,
\]

where \( M_{\text{min}}(z, m_1, t) \) and \( M_{\text{max}}(z, m_2, t) \) are the absolute magnitude limits observable at redshift \( z \), given the apparent magnitude limits \( m_1 \) and \( m_2 \) and the \( k \)-corrections connecting absolute \( B_{\text{AB}} \) magnitudes to the apparent magnitudes for the band in question. These \( k \)-corrections depend on the galaxy type \( t \) and are calculated using our
usual CWW SEDs. Also, in the second sum above, $G_j$ is evaluated using bins of width $\Delta t = 0.2$ (as in Fig. 15). We calculate $N$ first considering only the contribution of galaxies with $0.12 < z < 0.55$, the nominal redshift completeness range adopted for the LF analysis. We can clearly see the shortfall compared to the actual counts at both bright and faint magnitudes, resulting from neglect of low- and high-$z$ galaxies, respectively. The match between the observed and LF-computed counts is much improved by extending the redshift range to $0 < z < 0.75$, and further extension to $0 < z < 1$ makes little difference. The good agreement seen is not a circular result, since the LF is fitted only for galaxies within $0.12 < z < 0.55$ so that including the LF-extrapolated contribution from galaxies outside that redshift range serves as an independent check on the validity of our LF and evolution models.

We then repeat the same exercise but using various color distributions, as shown in Figure 17. The LF-computed color distributions are calculated using an expression analogous to equation (23) but augmented with the appropriate limits in the observed colors. The LF-computed results again converge by $z = 0.75$, and the match to the observed color distributions is good for all four colors shown: $B - R_C$, $R_C - I_C$, $V - R_C$, and $U - R_C$. (We should recall here that we did adjust the CWW SED $I_C$ magnitudes to improve the match to the $R_C - I_C$ distribution, as mentioned in § 3.1.) The overall reasonable agreement between the $0 < z < 1$ LF-computed color distributions and the observed distributions provides further validation of our evolving LF model and of our magnitude- and color-dependent weighting scheme defined back in § 2.1.

4.4. Potential Systematic Effects

Here we will consider a number of potentially important systematic sources of error that may affect our LF and evolution fits, specifically, (1) differences between the 0223 and 0920 patches, (2) random photometric errors, (3) potential redshift incompleteness, and (4) potential apparent magnitude incompleteness. We will find that typically our LF and evolution parameters are biased at less than the 1 $\sigma$ level.

4.4.1. Patch-to-Patch Variation

Since there are large-scale density fluctuations in our survey (Fig. 6), we should check how well our results for the 0223 and 0920 patches agree. We do so in some detail,
comparing the $M^* - z$ (Fig. 7) and $P - Q$ error contours, as well as the trend of $\rho_L(z)$ by galaxy type. Encouragingly, the LF parameters $M^*, \alpha, P,$ and $Q$ for the two patches are all consistent within their respective 2 $\sigma$ error contours. Examination of the $\rho_L(z)$ comparison shows excellent agreement of the LF-computed luminosity density evolution trends for all three galaxy types, despite the obvious density fluctuations seen in both patches.

4.4.2. Random Photometric Errors

Random photometric errors will, in general, cause an Eddington-type effect on the LF in such a way that $M^*$ is biased brighter and $\alpha$ is biased steeper (see EEP). This effect is appreciable for photographic-plate-based surveys with magnitude errors $\sigma_m \sim 0.3$ mag (e.g., Loveday et al. 1992; Marzke et al. 1994b), but it is essentially negligible for CCD-based surveys with magnitude errors $\sigma_m \sim 0.1$ mag (e.g., Lin et al. 1996a), although one can correct for it nonetheless by taking the LF to be a Schechter function convolved with a Gaussian magnitude error distribution with dispersion $\sigma_m$ (see EEP). For CNOC2, $\sigma_{R_C} < 0.1$ mag at the nominal $R_C = 21.5$ spectroscopic limit, and the consequent effects on $M^*$ and $\alpha$ should be small. However, we should also confirm that the impact of photometric errors are likewise negligible for the $P$ and $Q$ evolution parameters. Moreover, since our galaxy classifications (and consecutive derivation of $k$-corrections and absolute magnitudes) also make use of the $UBVIC$ magnitudes apart from just $R_C$, the photometric error distributions in these various bands will affect our derivation of the LF and evolution parameters in a complicated way. The median magnitude errors for our $R_C < 21.5$ galaxies are 0.04 mag for $R_C$ and $I_o$, 0.05 mag for $V$, 0.08 mag for $B$, and 0.16 mag for $U$. One could estimate the potential biases by fitting the LFs of Monte Carlo mock CNOC2 galaxy catalogs, generated using the best-fit type-dependent LF and evolution parameters of the real sample, combined with the appropriate photometric error distributions in each of the CNOC2 bands. We will, however, use a less complicated procedure and simply see what happens if we artificially increase the photometric errors of the real CNOC2 sample. Specifically, for each photometric band of each galaxy, we modify the observed magnitude by adding a Gaussian magnitude error distribution with zero mean and

### Table 3

| Sample          | $\rho_L(z = 0)$ | $\rho_L(0.12 < z < 0.25)$ | $\rho_L(0.25 < z < 0.40)$ | $\rho_L(0.40 < z < 0.55)$ |
|-----------------|-----------------|---------------------------|---------------------------|---------------------------|
| $B, q_o = 0.5$  |                 |                           |                           |                           |
| Early           | 0.258 ± 0.042   | 0.301 ± 0.074             | 0.387 ± 0.071             | 0.265 ± 0.045             |
| Intermediate    | 0.159 ± 0.028   | 0.217 ± 0.055             | 0.351 ± 0.063             | 0.259 ± 0.045             |
| Late            | 0.189 ± 0.030   | 0.390 ± 0.094             | 0.580 ± 0.109             | 0.720 ± 0.123             |
| Total           | 0.606 ± 0.078   | 0.907 ± 0.213             | 1.318 ± 0.230             | 1.244 ± 0.191             |
| $B, q_o = 0.1$  |                 |                           |                           |                           |
| Early           | 0.252 ± 0.042   | 0.282 ± 0.072             | 0.360 ± 0.069             | 0.223 ± 0.040             |
| Intermediate    | 0.155 ± 0.030   | 0.203 ± 0.053             | 0.317 ± 0.061             | 0.224 ± 0.040             |
| Late            | 0.183 ± 0.030   | 0.364 ± 0.092             | 0.521 ± 0.106             | 0.615 ± 0.114             |
| Total           | 0.591 ± 0.076   | 0.849 ± 0.208             | 1.197 ± 0.225             | 1.063 ± 0.177             |
| $R_r, q_o = 0.5$|                 |                           |                           |                           |
| Early           | 0.785 ± 0.130   | 0.896 ± 0.221             | 1.115 ± 0.202             | 0.750 ± 0.126             |
| Intermediate    | 0.351 ± 0.066   | 0.472 ± 0.118             | 0.764 ± 0.139             | 0.565 ± 0.096             |
| Late            | 0.320 ± 0.050   | 0.657 ± 0.158             | 1.012 ± 0.199             | 1.225 ± 0.215             |
| Total           | 1.455 ± 0.186   | 2.024 ± 0.475             | 2.891 ± 0.510             | 2.539 ± 0.386             |
| $R_r, q_o = 0.1$|                 |                           |                           |                           |
| Early           | 0.778 ± 0.134   | 0.841 ± 0.217             | 1.006 ± 0.194             | 0.620 ± 0.112             |
| Intermediate    | 0.346 ± 0.066   | 0.444 ± 0.116             | 0.692 ± 0.133             | 0.486 ± 0.088             |
| Late            | 0.302 ± 0.050   | 0.604 ± 0.152             | 0.901 ± 0.187             | 1.055 ± 0.198             |
| Total           | 1.426 ± 0.189   | 1.889 ± 0.463             | 2.599 ± 0.489             | 2.160 ± 0.356             |
| $U_r, q_o = 0.5$|                 |                           |                           |                           |
| Early           | 0.086 ± 0.014   | 0.102 ± 0.025             | 0.136 ± 0.024             | 0.092 ± 0.016             |
| Intermediate    | 0.077 ± 0.014   | 0.106 ± 0.026             | 0.171 ± 0.031             | 0.127 ± 0.022             |
| Late            | 0.118 ± 0.019   | 0.241 ± 0.057             | 0.344 ± 0.064             | 0.438 ± 0.076             |
| Total           | 0.281 ± 0.036   | 0.449 ± 0.104             | 0.651 ± 0.114             | 0.657 ± 0.104             |
| $U_r, q_o = 0.1$|                 |                           |                           |                           |
| Early           | 0.084 ± 0.014   | 0.096 ± 0.025             | 0.123 ± 0.024             | 0.079 ± 0.014             |
| Intermediate    | 0.075 ± 0.014   | 0.099 ± 0.026             | 0.154 ± 0.030             | 0.109 ± 0.020             |
| Late            | 0.113 ± 0.019   | 0.224 ± 0.057             | 0.310 ± 0.062             | 0.381 ± 0.070             |
| Total           | 0.273 ± 0.035   | 0.419 ± 0.103             | 0.587 ± 0.110             | 0.569 ± 0.096             |

* Units are $10^{20}$ h W Hz$^{-1}$ Mpc$^{-3}$. As discussed in the text, the tabulated 1 $\sigma$ errors include both bootstrap resampling errors (accounting for uncertainties in the LF fits and in galaxy sampling) and estimated uncertainties due to large-scale galaxy density fluctuations.
Fig. 14.—Redshift evolution of the CNOC2 rest-frame luminosity density $\rho_L(z)$ shown for the $B$, $R_c$, and $U$ bands. The top left-hand panel compares the total $\rho_L(z)$ for the three bands, and the other three panels break down each band into results by galaxy type. Note that unlike in previous figures, $\rho_L$ is plotted here on a logarithmic scale to facilitate comparison of the rates of luminosity density evolution among the three different bands.

4.4.3. Redshift Incompleteness

As described in § 2, our nominal redshift limits $z = 0.12$ and 0.55 are set by the observability of important absorption and emission features over the 4400–6300 Å spectroscopic range. Examination of Figure 6 shows that, as expected, the observed (weighted) redshift distribution outside the $0.12 < z < 0.55$ range tends to lie low compared to the distribution from the best-fit LF model, although the effect is primarily seen for early and intermediate types at higher redshifts (compare also Figs. 3 and 4). However, in the highest $z$ bin $0.5 < z < 0.55$ within our nominal redshift range, there is already a noticeable dip in the redshift distributions for early and intermediate galaxies. This perhaps indicates some unaccounted residual redshift incompleteness in that bin, and we should check what happens if we exclude that bin from our analysis. Also, we note that the $0.1 < z < 0.2$ bins may suffer from incompleteness in late-type galaxies, if H$\beta$ and [O III] $\lambda\lambda$5007, 4959 do not adequately pick up for the unobservable [O II] $\lambda$3727 line.

Thus we redo our fits for the more redshift-complete range $0.2 < z < 0.5$, over which the most important redshift-identification features, Ca II H and K and [O II]...
potential unaccounted incompleteness over the 21.0
ference to our results. 

\[ B \]

...this partly in anticipation of our later comparison with the
sample compared to our usual sample. We do

\[ p \]

will make a significant difference in the LF evolution
about 5. Here we check if using a 0.5 mag brighter limit of
shift sampling rate is 0.2, so that the typical galaxy weight is
nominal spectroscopic limit is 0.5, and the red-
(uncorrected) raw differential redshift success rate at the

\[ w \]

though there are some possible hints of incompleteness in
axies.

\[ F \]

As shown in Figure 1 and discussed in § 4.4.4.
Apparent Magnitude Incompleteness

\[ a \]

are always observable. We in fact find \( M^* \) and \( \alpha \)
values to be in good agreement with the original ones, and, as shown in Figure 18 (top right-hand panel), the \( P \) and \( Q \)
parameters agree within \( \pm 1.5 \) \( \sigma \) of the original values. It thus appears that our LF parameters are not significantly biased by any residual redshift incompleteness effects, even though there are some possible hints of incompleteness in the \( 0.5 < z < 0.55 \) bin for early- and intermediate-type gal-
axies.

\[ t \]

As shown in Figure 1 and discussed in § 4.2.1, the
(uncorrected) raw differential redshift success rate at the
nominal spectroscopic limit \( R_C = 21.5 \) is 0.5, and the red-
shift sampling rate is 0.2, so that the typical galaxy weight is
about 5. Here we check if using a 0.5 mag brighter limit of
\( R_C = 21.0 \), where there is an improved raw redshift success rate of 0.6 and a smaller typical galaxy weight of about 2,
will make a significant difference in the LF evolution
results. Figure 18 (bottom left) shows that the \( P \) and \( Q \)
values for the \( 17.0 < R_C < 21.0 \) sample are always within the original 1 \( \sigma \) contours. Likewise, the \( M^* \) and \( \alpha \) values are within the original 2 \( \sigma \) contours. We thus conclude that potential unaccounted incompleteness over the \( 21.0 < R_C < 21.5 \) magnitude range does not make a significant dif-
ference to our results.

\[ r \]

4.5. B-band Selection

Here we examine the effects of using a \( B \)-selected CNOC2 sample compared to our usual \( R_C \)-selected sample. We do
this partly in anticipation of our later comparison with the
\( B \)-selected Autofib Redshift Survey in § 5.3. We define a
18 < \( B \) < 23 CNOC2 sample \((N = 1936)\) and compute new
weights, using the bound \( |B_j - B_i| \leq 0.25 \) in place of the corresponding \( R_C \) bound in equation (3). The \( B \)-selected
\( P-Q \) results are shown in Figure 18 (bottom right-hand panel), where we find agreement within 2 \( \sigma \) with the \( R_C \)-selected results, except for the early-type galaxies, which now show weak positive density evolution \( P = 0.6 \). In general, the \( B \)-selected sample shows more positive density evolution compared to the \( R_C \)-selected sample, but the luminosity evolution parameters \( Q \) are very similar. The corresponding \( M^* \) and \( \alpha \) values agree well within 1 \( \sigma \) for the early and intermediate types and are within 2 \( \sigma \) for the late types. We thus conclude that the \( B \)- and \( R_C \)-selected samples do give LF evolution results that are generally in good agreement, with the sole exception of the \( P \) value for the early types.

5. COMPARISONS WITH PREVIOUS SURVEYS

In this section we compare our LF evolution results with those obtained from three previous intermediate-redshift surveys. We first briefly compare against the field galaxy sample from the CNOC1 Cluster Redshift Survey, the immediate predecessor of CNOC2. We then continue with the two next largest intermediate-z redshift survey samples, specifically, the CFRS and the composite Autofib Redshift Survey.

5.1. CNOC1 Cluster Redshift Survey: Field Sample

The CNOC1 Cluster Redshift Survey (Carlberg et al. 1996; Yee et al. 1996) included observations of both cluster and field galaxies in the fields of 16 high X-ray luminosity clusters. The observational techniques used in the CNOC1 survey are very similar to those used in CNOC2, but CNOC1 galaxies only have Gunn \( r \) and \( g \) photometry available. Lin et al. (1997) examine the LF for a sample of 389 CNOC1 field galaxies, with redshifts \( 0.2 < z < 0.6 \) and apparent magnitudes \( 18 < r < 22 \). Nonevolving LFs in \( B_{AB} \) and Gunn \( r \) are computed for the whole CNOC1 field sample, as well as for blue and red subsets divided by the observed \( g-r \) color of a CWW Sbc galaxy. Consistent with the CNOC2 results, the CNOC1 LFs show the same trend of a steeper faint-end slope for blue galaxies relative to red ones. The CNOC1 sample is too small for the LF evolution analysis of the present paper, but Lin et al. (1997) have computed luminosity densities and have shown that the CNOC1 blue galaxy \( \rho_{B}(z) \) increases strongly with redshift, whereas the CNOC1 red galaxy \( \rho_{g}(z) \) is essentially constant with \( z \). These results are consistent with those found for the CNOC2 sample.

We have also computed nonevolving \( B_{AB} \) and \( r \) LFs for CNOC2 galaxies, using basically the same CWW Sbc cut applied to CNOC1. For each of the all, blue, and red samples and for both rest-frame bandpasses, we confirm that the CNOC1 and CNOC2 results are indeed in good quantitative agreement in \( M^*, \alpha \), and normalization. The errors of the CNOC1 LFs are fairly large, however, primarily because of its much smaller sample size, so that, unfortunately, little improvement in the LF constraints is gained by adding the CNOC1 field data into the CNOC2 sample. The present CNOC2 LF results essentially super-
 sede those obtained earlier from CNOC1.

5.2. Canada-France Redshift Survey

CFRS (Lilly et al. 1995a) consists of 591 galaxy redshifts up to \( z \sim 1 \). The sample is selected in the \( I \)-band with \( 17.5 \leq I_{AB} \leq 22.5 \) and is distributed over five widely separated fields, totaling 125 arcmin\(^2\) on the sky. Lilly et al.
(1995b) examine the evolution of the CFRS LF for $0 \leq z \leq 1$. They divide their sample by observed $(V - I)_AB$ color, also using CWW SEDs, and find rapid evolution in the LF of galaxies bluer than a CWW Sbc galaxy, contrasted with little change in the LF of redder-than-Sbc galaxies.

Lilly et al. (1995b) also split their sample into several redshift bins, including 208 galaxies in a $0.2 < z < 0.5$ bin, which overlaps most with the CNOC2 redshift limits. In Figure 19 we compare the CFRS $B_{AB}$ LF results (the “best” estimates of Lilly et al. 1995b) against those for a nearly 10 times larger sample of 1842 CNOC2 galaxies with $0.2 < z < 0.5$. We also use the CWW Sbc cut to divide our sample into red and blue subsets; we initially do not include evolution, because the CFRS results are fitted using non-evolving Schechter functions. The bottom panels in the figure show that the $M^* - z$ values for the two surveys are in good agreement (the CFRS error contours have been calculated by us using CFRS redshift catalog data kindly supplied by Simon Lilly). This is also demonstrated in the middle panels, where we have renormalized the CNOC2 LFs to match the normalizations of the CFRS LFs, using an equation analogous to equation (20), in order to focus on comparing the LF shapes. The renormalizations affect the red-galaxy LFs very little, since the CNOC2 and CFRS results agree well in the first place. However, as shown in the top panels, there is a noticeable difference in the blue-galaxy LFs, where CNOC2 shows a higher number density than CFRS.

We next add evolution into our CNOC2 LF fits, using our usual five-parameter method, in order to extrapolate our luminosity density results into the $0.5 < z < 1$ redshift range probed by CFRS, as shown in Figure 20. We have extended the upper redshift limit to $z = 0.65$ for the CNOC2 blue sample in order have an additional data point to show. Notice from Figure 3 that there does not appear to be any obvious incompleteness for bluer-than-Sbc CNOC2 galaxies for $0.55 < z < 0.65$ (as there is for redder-than-Sbc galaxies), and also note that there is no obvious dip due to incompleteness in the last CNOC2 blue-galaxy $\rho_L$ point at $z \approx 0.6$ in Figure 20. The CFRS $\rho_L$ results are taken from Lilly et al. (1996; their “LF-estimated” 4400 Å values), and the two surveys do appear consistent within the errors. The CNOC2 blue $\rho_L(z)$ and extrapolation more or less parallel the CFRS results but are about 50% higher overall. The CNOC2 red $\rho_L(z)$ and extrapolation agree well with CFRS at $z \lesssim 0.7$ but appears to overshoot CFRS in the highest redshift bin, $z \sim 0.9$.

We note that the difference in the CNOC2 and CFRS
blue-galaxy luminosity densities may be consistent with the galaxy density fluctuations expected for these two surveys. We estimate (using the procedure described at the end of §3.2) that the density fluctuations $\delta \rho / \rho$ are approximately 12% and 13% for the $0.2 < z < 0.5$ volumes in CNOC2 and CFRS, respectively. The ratio of roughly 1.5 in the blue-galaxy $\rho_L$ for the two surveys would then have a $1 \sigma$ uncertainty (assuming Gaussian galaxy density fluctuations) of about $\pm 0.27$, so the luminosity densities differ at the $< 2 \sigma$ level and even less so if we include the remaining sampling and LF-fit contributions to the total error on $\rho_L$. On the other hand, it is unclear why we do not see any differences in the respective red galaxy populations, which should show stronger density fluctuations than the blue galaxies (e.g., Fig. 6). Thus the blue-galaxy differences may be caused instead by some systematic differences in, e.g., galaxy classification and/or photometry for blue galaxies in the two surveys, although one might then have expected to see a more significant difference in the shapes of the CNOC2 and CFRS LFs.

Recently, galaxy evolution results have also been reported for a sample of 341 galaxies drawn from the CFRS and the Autofib/Low Dispersion Survey Spectrograph (LDSS) data sets, which have morphologies classified from Hubble Space Telescope images (Brinchmann et al. 1998; Lilly et al. 1998). Although there is clearly correlation between the early, intermediate, and late SED/color classifications adopted in this paper and the “elliptical,” “spiral,” and “peculiar” morphological categories, respectively, defined by Brinchmann et al. (1998), the correlations are broad enough to preclude a detailed quantitative comparison. We will defer this for a future paper on morphological classifications of CNOC2 galaxies. Here we will simply mention two LF-related results from Brinchmann et al. (1998), which are qualitatively consistent with our results: (1) the LF of the spiral CFRS/LDSS sample indicates about 1 mag of luminosity evolution in $B_{AB}$ by $z \approx 1$, similar to the $Q = 0.9$ value we find for the CNOC2 intermediate-type galaxies, which should be dominated by spirals; and (2) the peculiar/irregular CFRS/LDSS galaxies appear to be primarily responsible for the rapid rise with redshift of the blue galaxy luminosity density, a result consistent with our observation that late-type CNOC2 galaxies cause the strong observed increase in the overall $\rho_L$ with redshift.

5.3. Autofib Redshift Survey

The Autofib Redshift Survey is a composite of various galaxy survey samples (Ellis et al. 1996 and references...
Fig. 18.—Impact of various systematic effects on the best-fit values of $P$ and $Q$ for the $B_{\text{AB}}$ LF. The original $P-Q$ values (open points) plus 1 $\sigma$ and 2 $\sigma$ contours from Fig. 9 are reproduced here. The solid points show the modified $P-Q$ values resulting from use of an “error-boosted” sample (top left; see text for details) from a reduced redshift range $0.2 < z < 0.5$ (top right), from adoption of a brighter magnitude limit $R_c < 21.0$ (bottom left), and from use of a $B$-selected $18 < B < 23$ sample (bottom right).

Ellis et al. (1996) give overall LFs in several redshift intervals, including $0.15 < z < 0.35$ and $0.35 < z < 0.75$ bins that overlap with CNOC2. In Figure 21, we make the same redshift cuts (but with a $z = 0.55$ upper limit) and compute non-evolving LFs for comparison. We show results both for the standard CNOC2 $17 < R_c < 21.5$ sample ($N = 2076$; filled triangles) and for a blue-selected $18 < B < 23$ CNOC2 sample ($N = 1830$; filled squares). There is a significant systematic difference between the two CNOC2 samples, where the $B$-selected sample shows a steeper $\alpha$ and brighter $M^*$ because of the increased contribution (due to $k$-correction effects) of bluer late-type galaxies in the $B$-selected sample. However, this systematic offset between the $R_c$- and $B$-selected samples is an artifact of trying to force-fit a single LF to the full galaxy population, and it does not occur if we subdivide into three populations as we did before (§ 4.5). Using a $B$-selected CNOC2 sample significantly improves the agreement with the $0.15 < z < 0.35$ Autofib results, although the 2 $\sigma$ $M^*-$ $\alpha$ error contours still do not quite overlap, since Autofib shows a steeper $\alpha$ and a brighter $M^*$. (Note that, although the CNOC2 and Autofib samples here are similar in size, the Autofib error contours are smaller because of Autofib’s much wider apparent magnitude limits compared to CNOC2.) Nonetheless, a visual comparison of the two lower-$z$ LFs (Fig. 21, top left-hand panel) does show reasonable agreement. However, in the higher $z$ bin there is a noticeable mismatch in $M^*$ and/or normalization between the CNOC2 and Autofib results.

The causes of these discrepancies are not known but can include sampling fluctuations, as well as unaccounted systematic differences in sample selection, galaxy classifications and $k$-corrections, photometry, and the like (see Lin et al. 1997 for additional discussion). Note that sample size may be an important consideration for the comparison in the higher redshift bin. Although the overall Autofib sample
contains some 1700 redshifts, the relevant sample sizes here are smaller (Ellis 1997, that paper's Fig. 6b): $N = 665$ for $0.15 < z < 0.35$ and only $N = 152$ for $0.35 < z < 0.75$. The corresponding CNOC2 ($B$-selected) sample sizes are $N = 940$ and $N = 890$, so that the high-$z$ CNOC2 sample is nearly 6 times larger than the corresponding Autofib data set. Also, large-scale galaxy density fluctuations may play a role. The values of $\delta \rho / \rho$ are estimated to be 16% and 13% for the low- and high-$z$ CNOC2 volumes, respectively, and they are presumably somewhat larger for the corresponding Autofib volumes (although we have not done the exact calculations, since we lack certain needed Autofib sample details). As we saw earlier in our CFRS comparison, such values of $\delta \rho / \rho$ do not preclude a factor of 1.5 in the relative LF normalizations, which would significantly reduce the discrepancy in the high-$z$ bin. We have also checked whether random $k$-correction errors and photometric errors in the Autofib sample might be responsible for the $M^*$ and $\alpha$ differences. The Autofib $k$-corrections are assigned primarily on the basis of galaxy spectral classifications, rather than more directly via multicolor photometry as we do. Ellis et al. (1996) estimate that their $k$-correction errors due to spectral misclassifications have a redshift dependence $\sigma_B \sim 0.5 z$ mag (our interpretation of their Fig. 6). Also, the Autofib photometry is based mainly on photographic plate data with errors typically 0.1-0.2 mag, in contrast to CNOC2 CCD photometry with errors $<0.1$ mag. Both these effects will tend to bias the Autofib results to brighter $M^*$ and steeper $\alpha$ (see §4.4.2), so we have checked the effect of adding such $k$-correction errors and photometric errors (0.2 mag Gaussian) to our $B$ magnitudes. In agreement with Ellis et al. (1996) and Heyl et al. (1997), we find that the differences are small, with $|\Delta M^*| < 0.2$ and $|\Delta \alpha| < 0.1$, not enough to significantly improve the agreement of the $M^*-\alpha$ contours in the low-$z$ bin and of the wrong sign for the high-$z$ bin. We have also tried computing $b_j$ absolute magnitudes and LFs for CNOC2 galaxies using the $b_j$ response function (instead of our usual Johnson $B$), but it makes negligible difference to our results. Additional exploration of Autofib versus CNOC2 photometry systematics likely requires us to apply our photometry codes to the original Autofib data. Such a detailed comparison may not be warranted, given that the main CNOC2/Autofib differences lie in the high-$z$ bin, where the main culprits may very well be galaxy density fluctuations and the small Autofib sample size there.

![Graphical representation of the luminosity function comparison between CNOC2 and CFRS samples.](image-url)
Heyl et al. (1997) have classified Autofib galaxies into six types based on cross-correlation against local galaxy spectral templates and examined the evolution of the LF divided by galaxy spectral type. Note that in discussing LF evolution, Heyl et al. typically use the three broader categories, “early-type E/S0,” “early-type spirals,” and “late-type spirals” (each including two of their original six types), which have obvious but broad correlations relative to our early, intermediate, and late types, respectively. Also, their LF evolution model is similar but not identical to ours, and it involves six parameters compared to our five, with the additional parameter characterizing the rate of evolution of \( z \), which we have taken as fixed with redshift. In addition, unlike our analysis, Heyl et al. do not plot error contours, like our \( P-Q \) diagrams, to show the correlations among their LF evolution parameters. Because both their classification and analysis methods are different from ours and because we have already noted some discrepancies between the CNOC2 and Autofib results above, we will not attempt a detailed quantitative comparison here. We will note, however, that generally speaking the Heyl et al. (1997) results are qualitatively consistent with ours. Specifically, they find (1) no significant evolution of the E/S0 LF out to at least \( z \approx 0.5 \); (2) modest evolution in the LF of early-type spirals, characterized by steepening of \( z \) at higher redshifts rather than by changes in \( M^* \) or \( \phi^* \); and (3) strong evolution in the LF of late-type spirals, described by steepening \( z \), brightening \( M^* \), and increasing \( \phi^* \) at higher \( z \). The main difference compared to CNOC2 lies in the steepening \( z \) observed in Autofib, contrasted with the good match of our \( z(z) = \text{constant} \) models to the CNOC2 data (see Fig. 8 in particular). Also, the \( Q \approx 1 \) luminosity evolution we find in our early- and intermediate-type LFs is somewhat different from the trends seen in the Autofib E/S0 and early-spiral LFs. It is not clear at present what is responsible for these detailed CNOC2/Autofib evolution differences, but we note in particular that the different galaxy classification schemes involved may play an important role. We will return to this
comparison again in a future paper on application of spectral classifications to the full CNOC2 sample.

6. CONCLUSIONS

In this paper we have examined the evolution of the LF for a sample of over 2000 field galaxies, with $0.12 < z < 0.55$ and $17.0 < R_c < 21.5$, drawn from two different sky patches of the CNOC2 Field Galaxy Redshift Survey. Although this sample comprises only half the ultimate CNOC2 data set, it is nonetheless the largest intermediate-redshift galaxy survey sample at present. The availability of $UBVR_CI_C$ photometry for our sample allows galaxy classifications by SED type, as well as computation of LFs in different rest-frame bandpasses. In addition, the multicolor photometry permits us to examine sample selection effects in detail and allows us to construct galaxy weights to account for our redshift success rates as functions of galaxy magnitude and color.

In particular, we have calculated LF parameters in the $B_{AB}$, $R_c$, and $U$ bands for early-, intermediate-, and late-type galaxies, classified using $UBVR_CI_C$ colors derived from the nonevolving galaxy SEDs of CWW. We present a description of the LF evolution in terms of a five-parameter model involving the usual three Schechter function parameters plus two additional parameters, $P$ and $Q$, describing number density and luminosity evolution rates, respectively (eq. 10). The best-fit parameters of our LF evolution models are given in Tables 1 and 2. We find that the faint-end slope of the LF is steeper for later type galaxies relative to earlier type objects, consistent with previous LF studies at both intermediate and low redshifts.

The principal results of this paper are the quantitative separation of luminosity and density evolution in the LFs of different galaxy populations and the finding that the character of the LF evolution is strongly type-dependent, varying from primarily luminosity evolution for early-type galaxies to predominantly density evolution for late-type objects. We quantify the rates of LF evolution using our $P$ and $Q$ parameters. Specifically, we see (for $q_0 = 0.5$) that (1) the late-type galaxy LF is best fit by strong positive density evolution ($P = 3.1$), with nearly no luminosity evolution ($Q = 0.2$); (2) the intermediate-type LF shows positive lumi-
nosity evolution ($Q = 0.9$) plus weak positive density evolution ($P = 0.7$), resulting in mild positive evolution in the luminosity density $\rho^*_L$; and (3) the early-type LF shows positive luminosity evolution ($Q = 1.6$), which is nearly compensated by negative density evolution ($P = -1.1$), resulting in a very weak positive evolution in $\rho_L$. However, we should note that the $P$ and $Q$ parameters are strongly correlated for late-type galaxies, and “no evolution” for early- and intermediate-type objects is ruled out at only about the $2\sigma$ confidence level. Nonetheless, it is a robust result that the LFs of late and early plus intermediate galaxies are evolving differently and occupy different regions of $P-Q$ parameter space. Moreover, there is a distinct contrast between the sharply rising luminosity density of late-type galaxies and the relatively constant $\rho_L$ of early- and intermediate-type objects. (This is probably not too surprising, given that one expects the rapidly evolving population to consist of those galaxies actively forming stars in the past, which are essentially the late types.) These general conclusions are little changed by adopting a different value of $d_0 = 0.1$.

The rates of luminosity evolution ($Q \approx 1$) for our early and intermediate types are in the range expected from models of galaxy evolution (e.g., Bruzual & Charlot 1996). At face value, the strong density evolution observed for late types suggests that mergers play an important role in the evolution of these galaxies. However, other processes, particularly those affecting star formation properties, may mimic the effect of mergers and cause similar changes in number density (e.g., a starbursting subpopulation among the late types at high-$z$ may be responsible for the density evolution). In Paper II, we will test various physical galaxy evolution models in detail, including the effects of different star formation histories, ages, stellar initial mass functions, dust content, and the like. Nonetheless, whatever the responsible physical mechanisms are, they will need to explain the strong correlation between galaxy type and the character of the LF evolution, in particular the strong increase in the apparent density evolution as one proceeds to later galaxy types. The relevant underlying physical variables controlling the evolution should thus be closely correlated with the galaxy SED type. On the other hand, within each of our galaxy categories, those physical variables are probably not strongly correlated with galaxy luminosity, since the data are well fitted by our fixed-$z$ evolution models (so that the evolution does not vary much with luminosity within each galaxy category). It may be a challenge for physical models to explain this combination of strong type-dependence in the LF evolution, coupled with relatively little luminosity-dependence of the evolution within each galaxy type.

We also compute SED type distributions, $UBVR_C I_C$ galaxy number counts, and various color distributions for CNOC2 galaxies. In particular, we find that extrapolations of our LF evolution models to $z \approx 0.75$ yield good matches to the observed number counts and color distributions, thus providing an additional check on the validity of our LF evolution results. In addition, we have verified that various systematic effects, specifically, patch-to-patch variations, photometric errors, surface brightness selection, redshift incompleteness, and apparent magnitude incompleteness, do not significantly affect our results ($\leq 1\sigma$ difference typically).

Finally, we note that our LF results are generally consistent with those found in previous intermediate-$z$ redshift surveys, as verified in specific comparisons against results from the next two largest samples, CFRS and Autofib. However, there are still some unresolved detailed discrepancies, particularly with respect to the $B$-selected Autofib survey, which may be due to differences in galaxy classification or sample selection methods.

In this paper, we have simply presented a description of the evolution of the LFs of different intermediate-redshift galaxies, without delving into the possible underlying physical processes. As mentioned earlier, in our second LF paper we will actually confront the CNOC2 observations against various galaxy evolution models, in order to better understand and constrain those physical mechanisms. Subsequent papers on galaxy population evolution in CNOC2 will also make use of the morphological and spectral information that will become available for CNOC2 galaxies once the appropriate data are fully reduced. Ultimately, the doubled size of the full CNOC2 sample over the present interim sample will significantly improve upon the LF evolution constraints that we have presented here. We are also in the process of deriving properly calibrated photometric redshifts, which should provide another factor of 2 increase in useful sample size for $R_C < 21.5$ galaxies. Future papers will re-examine the question of LF evolution using these even larger CNOC2 galaxy data sets.

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