ZERO-NORM STATES AND SUPERSTRINGY SYMMETRIES

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(Dated: September 9, 2018)

Abstract

We identify spacetime symmetry charges of the bosonic sector of 10D superstring theory from an infinite number of zero-norm states (ZNS) in the old covariant first quantized string spectrum. We give evidences to support this identification. These include supersymmetric sigma-model calculation, 2D super-Liouville theory calculation and, most importantly, three methods of high-energy scattering amplitude calculations. These calculations generalize the previous bosonic string calculations, which explicitly prove Gross’s conjectures in 1988 on high energy symmetry of string theory. Moreover, we discover new high energy scattering amplitudes which are, presumably, related to the high energy massive spacetime fermionic scattering amplitudes in the R-sector of the theory.
I. INTRODUCTION AND OVERVIEW

One of the fundamental issues of string theory is its spacetime symmetry. It has long been believed that quantum string theory, which is UV finite and contains an infinite number of states without any free parameter, consists of a huge hidden spacetime symmetry. Historically, the first key progress to understand symmetry of string theory is to study the high energy, fixed angle behavior of string scattering amplitude [1–3] as suggested by Gross in 1988. The second key idea to uncover the fundamental symmetry of string theory was the identification of symmetry charges from an infinite number of stringy zero-norm states (ZNS) with arbitrarily high spins in the old covariant first quantized (OCFQ) string spectrum [4].

In the context of $\sigma$-model approach of string theory, massive inter-particle symmetries were calculated by using two types of ZNS. The corresponding on-shell Ward identities on the scattering amplitudes were constructed in [5, 6]. On the other hand, ZNS were also shown [7] to carry the spacetime $\omega_\infty$ symmetry [8] charges of 2D string theory [9]. Incidentally, it was also shown [10, 11] that off-shell gauge transformations of Witten string field theory [12], after imposing the no-ghost condition, are identical to the on-shell stringy gauge symmetries generated by two types of ZNS in the massive $\sigma$-model approach of string theory [4].

Recently high-energy Ward identities derived from the decoupling of ZNS of the 26D open bosonic string, which combines the previous two key ideas of probing stringy symmetry, were used to explicitly prove Gross’s conjectures [13–20]. One utilizes the decoupling of ZNS to obtain nonlinear relations or on-shell Ward identities among string scattering amplitudes. In the high energy limit, many simplications occur and one can derive linear relations among high-energy scattering amplitudes of different string states at each fixed mass levels. Moreover, these linear relations can be used to fix the ratios among high energy scattering amplitudes of different string states at each fixed mass level algebraically. This explicitly shows that there is only one independent component of high-energy scattering amplitude at each mass level. On the other hand, a saddle-point method was developed to calculate the general formula of tree-level high-energy scattering amplitudes of four arbitrary string states to verify the ratios calculated above. This general formula expresses all high-energy string scattering amplitudes in terms of that of four tachyon as conjectured by Gross in 1988 [2].

In this report, we consider the high-energy scattering amplitudes for the NS-sector of 10D open superstring theory [19]. Based on the calculations of 26D bosonic open string
(13–17), all the three independent calculations of bosonic string, namely the decoupling of high-energy zero-norm states (HZNS), the Virasoro constraints and the saddle-point calculation, can be generalized to scattering amplitudes of string states with polarizations on the scattering plane of superstring. All three methods give the consistent results. In addition, we discover new leading order high-energy scattering amplitudes, which are still proportional to the previous ones, with polarizations orthogonal to the scattering plane. These scattering amplitudes are of subleading order in energy for the case of 26D open bosonic string theory.

II. ZERO-NORM STATE CALCULATIONS

In this section, we review the calculations of superstring symmetries from ZNS without taking the high-energy limit. In the OCFQ spectrum of the NS-sector of 10D open superstring theory, the solutions of physical states conditions include positive-norm propagating states and two types of ZNS. The latter are

Type I : \( G_{-\frac{1}{2}} |x\rangle \), where
\[
G_{\frac{1}{2}} |x\rangle = G_{\frac{3}{2}} |x\rangle = 0, \quad L_0 |x\rangle = 0;
\]

Type II : \( (G_{-\frac{1}{2}} + 2G_{-\frac{1}{2}} L_{-1}) |\bar{x}\rangle \), where
\[
G_{\frac{1}{2}} |\bar{x}\rangle = G_{\frac{3}{2}} |\bar{x}\rangle = 0, \quad (L_0 + 1) |\bar{x}\rangle = 0.
\]

While type I states have zero-norm at any space-time dimension, type II states have zero-norm only at D=10. In the supersymmetric \( \sigma \)-model approach of string theory, a spacetime symmetry transformation \( \delta \Phi \) for a NS-NS bosonic background field \( \Phi \) can be generated by [22]

\[
T_\Phi + \delta T = T_{\Phi + \delta \Phi},
\]

where \( T_\Phi \) is the worldsheet superstress tensor with background fields \( \Phi \) and \( T_{\Phi + \delta \Phi} \) is the new superstress tensor with new background fields \( \Phi + \delta \Phi \). It can be shown that for each ZNS in the NS-NS sector, one can construct a superconformal deformation \( \delta T \) such that Eq.(3) is satisfied to the first order of weak field approximation in the \( \beta \) function calculation. As an example, the massless ZNS solution of Eq.(1) can be used to derive the on-shell gauge symmetries of massless graviton \( h_{\mu \nu} \) and antisymmetric tensor \( b_{\mu \nu} \) in the weak field approximation.
Another evidence to support ZNS as the origin of symmetry charge was demonstrated for the 2D super-Liouville theory. The spacetime symmetry of 2D super-Liouville theory was known to be the $w_\infty$ algebra \[21\]

\[
\int \frac{dz}{2\pi i} \psi^+_{j_1 M_1}(z) \psi^+_{j_2 M_2}(0) = (J_2 M_1 - J_1 M_2) \psi^+_{(j_1+j_2-1)(M_1+M_2)}(0) \tag{4}
\]

generated by the discrete supersymmetric Polyakov states $\psi^+_{JM}$. Alternatively, one can explicitly construct a set of discrete ZNS $\Omega^+_{JM}$ in the worldsheet supersymmetric form and show that they form a $w_\infty$ algebra \[7\]

\[
\int \frac{dz}{2\pi i} \Omega^+_{j_1 M_1}(z) \Omega^+_{j_2 M_2}(0) = (J_2 M_1 - J_1 M_2) \Omega^+_{(j_1+j_2-1)(M_1+M_2)}(0). \tag{5}
\]

This seems to strongly suggest that ZNS are closely related to the spacetime symmetry of string theory.

III. HIGH ENERGY ZNS CALCULATIONS

Recently a further evidence to support ZNS as the spacetime symmetry charge of string theory was obtained by taking the high-energy, fixed angle limit of stringy Ward identities derived from the decoupling of ZNS on the scattering amplitudes. The conjectures of Gross were then explicitly proved. We first review the bosonic string case. At a fixed mass level $M^2 = 2(n - 1)$ of 26D open bosonic string theory, it was shown that \[16, 17\] a four-point function is at the leading order at high-energy limit only for states of the following form

\[
|n, 2m, q\rangle \equiv (\alpha_{-1}^T)^{n-2m-2q}(\alpha_{-1}^L)^{2m}(\alpha_{-2}^L)^q |0, k\rangle.
\]

where $n \geq 2m + 2q, m, q \geq 0$. Note that, in the high energy limit, the scattering process becomes a plane scattering, and we have defined the normalized polarization vectors of the second string state to be \[13, 14\] to be $e_P = \frac{1}{m_2}(E_2, k_2, 0) = \frac{k_2}{m_2}$, $e_L = \frac{1}{m_2}(k_2, E_2, 0)$ and $e_T = (0, 0, 1)$ in the CM frame contained in the plane of scattering. By using the decoupling of two types of bosonic string ZNS in the high energy limit, an infinite linear relations among
string scattering amplitudes at mass level $M^2 = 2(n - 1)$ can be derived [16, 17]

$$
\mathcal{T}^{(n,2m,q)} = \left(-\frac{1}{M}\right)^{2m+q} \left(\frac{1}{2}\right)^{m+q} (2m-1)!! \mathcal{T}^{(n,0,0)},
$$

where $\mathcal{T}^{(n,2m,q)}$ stands for the 4-point function with one vertex in Eq.(6), and the other three any string vertex which we have omitted their tensor index. Moreover, these linear relations can be used to fix the ratios among high energy scattering amplitudes of different string states at each mass level algebraically.

We now consider the superstring case [19]. We first consider high-energy scattering amplitudes of string states with polarizations on the scattering plane. It can be argued that there are four types of high-energy scattering amplitudes for states in the NS-sector with even GSO parity. These are

$$
|n, 2m, q\rangle \otimes \left| b^T_{-\frac{1}{2}} \right\rangle \quad (8)
\equiv (\alpha^T_{-1})^{n-2m-2q}(\alpha^L_{-1})^{2m}(\alpha^L_{-2})^q(b^T_{-\frac{1}{2}})|0, k\rangle,
$$

$$
|n, 2m+1, q\rangle \otimes \left| b^T_{-\frac{1}{2}} \right\rangle \quad (9)
\equiv (\alpha^T_{-1})^{n-2m-2q-1}(\alpha^L_{-1})^{2m+1}(\alpha^L_{-2})^q(b^T_{-\frac{1}{2}})
\left| 0, k \right\rangle,
$$

$$
|n, 2m, q\rangle \otimes \left| b^L_{-\frac{1}{2}} \right\rangle \quad (10)
\equiv (\alpha^T_{-1})^{n-2m-2q}(\alpha^L_{-1})^{2m}(\alpha^L_{-2})^q(b^L_{-\frac{1}{2}})|0, k\rangle,
$$

$$
|n, 2m, q\rangle \otimes \left| b^T_{-\frac{1}{2}} b^L_{-\frac{1}{2}} \right\rangle \quad (11)
\equiv (\alpha^T_{-1})^{n-2m-2q}(\alpha^L_{-1})^{2m}(\alpha^L_{-2})^q(b^T_{-\frac{1}{2}})(b^L_{-\frac{1}{2}})
\left| 0, k \right\rangle.
$$

Note that the number of $\alpha^L_{-1}$ operator in Eq.(9) is odd. The decoupling of ZNS in Eqs.(1)
and (2) in the high energy limit implies the ratios among scattering amplitudes

\[ |n + 1, 2m, q\rangle \otimes |b_T^{\frac{1}{2}}\rangle \]
\[ \equiv \left( -\frac{1}{2M} \right)^m \left( -\frac{1}{2M} \right)^q (2m - 1)!! \left| n, 0, 0 \right\rangle \]
\[ \otimes |b^{\frac{1}{2}}\rangle , \]
\[ |n, 2m + 1, q\rangle \otimes |b^{\frac{1}{2}}\rangle \]
\[ \equiv \left( -\frac{1}{2M} \right)^m \left( -\frac{1}{2M} \right)^q (2m + 1)!! \left| n, 0, 0 \right\rangle \]
\[ \otimes |b^{\frac{1}{2}}\rangle , \]
\[ |n, 2m, q\rangle \otimes |b^{\frac{1}{2}}\rangle \]
\[ \equiv \left( -\frac{1}{2M} \right)^m \left( -\frac{1}{2M} \right)^q (2m - 1)!! \left| n, 0, 0 \right\rangle \]
\[ \otimes |b^{\frac{1}{2}}\rangle , \]
\[ |n - 1, 2m, q - 1\rangle \otimes |b^T b^L b^{\frac{1}{2}}\rangle \]
\[ \equiv \left( -\frac{1}{2M} \right)^m \left( -\frac{1}{2M} \right)^q (2m - 1)!! \left| n, 0, 0 \right\rangle \]
\[ \otimes |b^{\frac{1}{2}}\rangle . \]

These ratios can be rederived from the method of Virasoro constraints and the saddle-point calculation. Finally, it was discovered [19] that many high-energy scattering amplitudes with polarizations orthogonal to the scattering plane are at the same order in energy as the previous ones and are proportional to them. These scattering amplitudes are of subleading order in energy for the case of 26D open bosonic string theory. The existence of these new high-energy scattering amplitudes is due to the worldsheet fermion exchange in the correlation functions and is, presumably, related to the high energy massive fermionic scattering amplitudes in the R-sector of the theory.
IV. ACKNOWLEDGMENTS

I thank the collaborations of C.T. Chan and Y. Yang on calculations of high energy superstring scatterings.

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