On Neutrino Masses and Family Replication

P. Q. Hung

Dept. of Physics, University of Virginia, Charlottesville, Virginia 22901

Abstract

The old issue of why there are more than one family of quarks and leptons is reinvestigated with an eye towards the use of anomaly as a tool for constraining the number of families. It is found that, by assuming the existence of right-handed neutrinos (which would imply that neutrinos will have a mass) and a new chiral $SU(2)$ gauge theory, strong constraints on the number of families can be obtained. In addition, a model, based on that extra $SU(2)$, is constructed where it is natural to have one “very heavy” fourth neutrino and three almost degenerate light neutrinos whose masses are all of the Dirac type.
The mystery of family replication can simply be paraphrased by the celebrated question of I. I. Rabi [1] concerning the discovery of the muon: “Who ordered that?” It is fair to say that, after 61 years, this question remains unanswered and is further complicated by the discovery of a third family of quarks and leptons. It is reasonable to expect that any solution to this problem will necessarily lie outside of the realm of the Standard Model (SM).

The second mystery has to do with the question of why neutrinos are massless or almost so. Any mass, in particular very tiny ones, will definitely point to physics beyond the SM. Most recently, new results from the Super Kamiokande collaboration [2] appear to give credence to this possibility. Is there a symmetry which can explain the smallness of neutrino masses, if present?

Is it possible somehow to envision a scenario in which these two mysteries are intimately intertwined? The answer might be yes. If symmetries beyond the SM are gauge symmetries, one might be able to exploit powerful constraints such as the freedom from both local (perturbative) triangle and global (nonperturbative) anomalies. To set the tone, we first briefly review some known facts about anomalies in the SM.

Let us suppose that the SM were described by $SU(N)_c \otimes SU(2)_L \otimes U(1)_Y$ instead of the usual $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Quarks now have $N$ colors. If the hypercharge assignment for the left-handed lepton doublet is $-1/2$, then the cancellation of the triangle anomaly implies $Y_{qL} = 1/2N$ for the left-handed quarks. This alone says nothing about what $N$ should be. Some other argument is needed.

Witten [3] has made the observation that an $SU(2)$ gauge theory with an odd number of doublets of Weyl fermions is anomalous in the sense that the fermionic determinant $\sqrt{\det i \nabla(A_\mu)}$ changes sign under a “large” gauge transformation $A'_\mu = U^{-1}A_\mu U - iU^{-1}\partial_\mu U$. This would make the partition function $Z$ vanish and the theory would be ill-defined. This nonperturbative anomaly would then require an even number of Weyl doublets in order for chiral $SU(2)$ to be consistent. (This ambiguity in sign stems from the fact that the fourth homotopy group $\Pi_4(SU(2)) = Z_2$.) Other groups that also have similar non-trivial constraints are $Sp(N)$ for any $N$ and $O(N)$ for $N \leq 5$. 

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There are $1 + N$ chiral doublets in the SM (1 for the leptons and $N$ for the quarks). Witten’s anomaly requires $1 + N$ to be even. In consequence, the number of colors must be odd, i.e. $N = 3, 5, ...$ (excluding 1). Experimentally, we think that $N = 3$, but more deeply it could come from some Grand Unified scheme such as $SU(5)$. It is amusing to envision a world in which $N = 5$ for example (quarks with charges $3/5$ and $-2/5$).

Let us apply this simple lesson to the family replication problem. To do this, we shall make a number of assumptions. The first two are:

1) There is a family gauge group.

2) There are right-handed neutrinos.

Once the $SU(2)$ anomaly constraint of the SM has been satisfied, there is nowhere else one can think of in using it: one needs another chiral $SU(2)$. $SU(2)_R$ of the well-known Left-Right model \cite{4} comes to mind. However, it is easy to see that both local and global anomalies are satisfied in the Left-Right model for each family, just as it is in the SM.

Let us now suppose that an extra $SU(2)$ does exist and that it is not the one associated with Left-Right models. We now propose that it is the right-handed neutrinos, and only them, that interact with this extra $SU(2)$, to be denoted by $SU(2)_{\nu R}$ from hereon, under which they transform as doublets to be denoted by $\eta_R = (\nu^R_\alpha, \tilde{\nu}^R_\alpha)$, where $\alpha$ refers to possible family indices. At this point, it is irrelevant as to which member of the doublet pairs with the left-handed neutrino to form a mass.

We shall assume the family group to be $SO(N_f)$ except for $N_f = 6$ with a spinor representation. The reason for such a choice is because, if $\eta_R = (\nu^R_\alpha, \tilde{\nu}^R_\alpha)$ were to carry family indices, the family group will not be vector-like (unlike QCD for example) and one would encounter a problem with the usual perturbative triangle anomaly unless the group is of the type $SO(n)$ ($n \neq 6$ if spinor representations are used) and $E_6$. We are not considering cases where the anomaly of different representations cancel each other. We shall assume that the usual left-handed and right-handed quarks and leptons, as well as $\eta_R$, transform as a vector representation of $SO(N_f)$, i.e. as a vector with $N_f$ components. This is free from the triangle anomaly even if $N_f = 6$. 

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Our model is described by: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SO(N_f) \otimes SU(2)_{\nu_R}$; $q_L = (3, 2, 1/6, N_f, 1); q_R = (3, 1, 2/3, N_f, 1); u_R = (3, 1, -1/3, N_f, 1); l_L = (1, 2, -1/2, N_f, 1); e_R = (1, 1, -1, N_f, 1); \eta_R = (1, 1, 0, N_f, 2)$. This model does not have any perturbative triangle anomaly. The only remaining anomaly one should be concerned with is Witten’s anomaly associated with $SU(2)_{\nu_R}$. There are two possibilities:

1) With $\eta_R = (1, 1, 0, N_f, 2)$, global anomaly freedom dictates $N_f = 2, 4, 6, 8, ...$

2) If there exists a family singlet $\eta'_R = (1, 1, 0, 1, 2)$ in addition to $\eta_R = (1, 1, 0, N_f, 2)$, global anomaly freedom dictates $N_f = 3, 5, 7, ...$ (We exclude the one family case.)

These options will be referred to as the even and odd options respectively. The arguments presented below are not meant to “fix” the number of families but are simply meant to indicate what the phenomenological implications for each choice of $N_f$ might be. The correct choice is undoubtedly fixed by a yet-unknown theory of everything. Here we simply attempt to see what physical differences between the even and odd options might be in the context of $SU(2)_{\nu_R}$, leaving the deeper question to someone else.

One might require that gauge couplings are free from Landau singularities below the Planck scale in such a way that unification of the SM gauge couplings, if it exists, occurs in the perturbative regime. This requirement rules out a large number of possible choices. With this criterion, one can see that the even option can only accommodate $N_f = 2, 4, 6$, while the odd option can only accommodate $N_f = 3, 5$. This is because for $N_f \geq 7$, one or more gauge couplings will “blow up” before the Planck scale. There are no reasons, in the absence of a deeper theory, to rule out any of the above choices. This will require other yet-unknown conditions. The only thing one can say, in the context of our model, is that electroweak precision experiments appear to rule out $N_f \geq 5$ and that existential facts tell us that $N_f$ is at least three. This leaves us with the choice $N_f = 4$ for the even option and $N_f = 3$ for the odd option.

There are real physical differences between the even and odd options. The former predicts the existence of a fourth generation whose consequences have been recently discussed in Refs. [5] and [6]. The latter predicts the existence of a neutral family-singlet $\eta'_R$ (doublet under
\( SU(2)_{\nu_R} \) which could probably have cosmological consequences. In addition, as we point out below, it appears that the even option prefers almost degenerate light neutrinos while the odd option prefers a hierarchical structure for the light neutrinos.

Let us concentrate, in this paper, on neutrino masses within the even option with \( N_f = 4 \). In addition to the fermions, there is the SM Higgs field which transform as \( \phi = (1, 2, 1/2, 1, 1) \). We shall require that all fermions be endowed with a global \( B-L \) symmetry. Since we are dealing only with leptons in this manuscript, a global \( L \) symmetry is sufficient. This global \( L \) symmetry would prevent a Majorana mass term of the type \( \eta_i^{\alpha R} \eta_i^{\alpha R} \), where \( i = 1, 2 \) and \( \alpha = 1, .., 4 \). In fact, in this scenario, only Dirac masses are allowed. It the follows that the only Yukawa coupling that one can have (for the lepton sector) is \( \mathcal{L}_Y = g_E \bar{\nu}_L^\alpha \phi e^{\alpha R} + \text{h.c.} \), where \( \alpha = 1, .., 4 \) is the family index. This is unsatisfactory for two reasons. First, it gives equal masses to all four charged leptons. Second, all four neutrinos are massless. We know that the charged leptons are not degenerate in mass. Furthermore, the width of the Z boson tells us that, if there were a fourth neutrino, its mass would have to be larger than half the Z mass. We therefore need to lift the degeneracy among the charged leptons and to give a mass to the neutrinos (at least to the fourth one). There are probably many ways to achieve this and we shall present one of such scenarios here.

To achieve the above aim, we introduce the following set of vector-like (heavy) fermions: \( F_{L,R} = (1, 2, -1/2, 1, 1) \); \( M_{1L,R} = (1, 1, -1, 1, 1) \) and \( M_{2L,R} = (1, 1, 0, 1, 1) \) as well as the following scalar fields: \( \Omega^\alpha = (1, 1, 0, 4, 1) \) and \( \rho_i^\alpha = (1, 1, 0, 4, 2) \), where \( \alpha \) and \( i \) are \( SO(4) \) and \( SU(2)_{\nu_R} \) indices respectively. Notice that \( F_{L,R} \) and \( M_{1L,R} \) are vector like under \( SU(2)_L \otimes U(1)_Y \), while \( M_{2L,R} \) is singlet under everything. As a result, they can have arbitrary gauge-invariant bare masses.

The Yukawa part of the Lagrangian involving leptons can be written as

\[
\mathcal{L}_{\text{Lepton}}^Y = g_E \bar{\nu}_L^\alpha \phi e^{\alpha R} + G_1 \bar{\nu}_L^\alpha \Omega e^{\alpha R} + G_{M1} \bar{\nu}_L \phi M_{1R} G_{M2} \bar{\nu}_L \phi M_{2R} + G_2 \bar{\nu}_L \Omega e^{\alpha R} + G_3 \bar{\nu}_L \rho_i^\alpha \eta_{i R} + M_F \bar{\nu}_L e_R + M_1 \bar{\nu}_L M_1 + M_2 \bar{\nu}_L M_2 + \text{h.c.}
\]

As we have stated above, the assumption of an unbroken \( L \) symmetry forbids the presence
of Majorana mass terms. For reasons to be discussed below, we shall assume that \( M_{F,1,2} \sim \) the scale of the family \( SO(4) \) breaking. After integrating out the \( F, M_1, \) and \( M_2 \) fields, the effective Lagrangian below \( M_{F,1,2} \) reads

\[
\mathcal{L}_{\text{Y,eff}}^{\text{Lepton}} = g_{E} \bar{l}_\alpha e_R \phi_\alpha + G_{E} \bar{l}_\alpha \phi \Omega^\beta e_\beta R + \]

\[
G_{N} \bar{l}_\alpha \tilde{\phi} \rho^\beta \eta_\beta R + h.c. + \text{higher dimensional operators},
\]

(2)

where

\[
G_{E} = \frac{G_{1} G_{M_{2}} G_{2}}{M_{F} M_{1}}; \quad G_{N} = \frac{G_{1} G_{M_{2}} G_{3}}{M_{F} M_{2}}.
\]

The above effective Lagrangian accomplishes two things: 1) If \( \Omega \) develops a non-vanishing vacuum expectation value (VEV), the second term in Eq. (2) might lift the degeneracy of the charged lepton masses; 2) If both \( \Omega \) and \( \rho \) develop non-vanishing VEV’s, the third term in Eq. (2) might give rise to a Dirac neutrino mass. These extra mass terms are linked to the breakdown of \( SO(4) \otimes SU(2)_{\nu R} \). We would like to now show that it is rather straightforward to obtain a “heavy” fourth neutrino and three very light ones.

Let us assume: \(< \Omega >= (0, 0, 0, V) \) and \(< \rho >= (0, 0, 0, V' \otimes s_1) \), where \( s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).

Notice that each component (under \( SO(4) \)) of \( \rho \) transforms as a doublet under \( SU(2)_{\nu R} \). If we denote the 4th element of \( \eta_R \) by \( (N_R, \tilde{N}_R) \), one can use the above two VEV’s along with \(< \phi >= (0, v/\sqrt{2}) \) in Eq.(2) to write down a mass term for the 4th generation neutrino, namely

\[
\tilde{G}_{N} \frac{v}{\sqrt{2}} \tilde{N}_L N_R + h.c.; \quad \tilde{G}_{N} = G_{1} G_{M_{2}} G_{3} \frac{V' V'}{M_{F} M_{2}}.
\]

(4)

At \textit{tree level}, all other neutrinos are massless. Two remarks are in order concerning the 4th neutrino mass. First, it is a \textit{Dirac} mass. Second, the 4th neutrino could be rather \textit{heavy}. In fact, it is not unnatural to expect \( G_{1}, G_{M_{2}} \) and \( G_{3} \) to be of the order of unity. In consequence, as long as \( VV' / M_{F} M_{2} = O(1) \) (either \( V \sim V' \sim M_{F} \sim M_{2} \) or any other combination), one might expect the fourth neutrino to be even as heavy as 175 GeV. Certainly, the LEP
bound of $M_Z/2$ can easily be satisfied. In our scenario where the mass is a Dirac mass, it is natural to have a very heavy fourth neutrino and three so-far-massless neutrinos. We show below that, although they are massless at tree level, they can acquire a mass at one loop.

In the Higgs potential, there is a term which is crucial to the computation of the light neutrino masses, namely

$$\mathcal{L}_{\Omega\rho} = \lambda_{\Omega\rho}(\Omega^\alpha \rho_\alpha)(\Omega^\beta \rho_\beta).$$

(5)

It is beyond the scope of this paper to give a detailed discussion of the pattern of symmetry breaking. Without loss of generality, we will assume that the physical scalars associated with $\Omega$ and $\rho$ have a mass $M_\Omega$ and $M_\rho$ respectively. The one-loop diagram which contributes to all four neutrino Dirac masses is shown in Fig. 1, where the coupling given in Eq. (1) has been used. To make the discussion more transparent, we simplify the problem by assuming $M_F = M_2$. A general case with $M_F \neq M_2$ can easily be dealt with. It is easy to see from Fig. 1 that this one-loop diagram gives a common mass to all four neutrinos: $m_\nu = \tilde{G}_\nu \sqrt{2}$, where

$$\tilde{G}_\nu = G_1 G_M G_3 V V' \frac{\lambda_{\Omega\rho}}{M^2 \sqrt{16 \pi^2}} I(M, M_\Omega, M_\rho),$$

(6a)

$$I(M, M_\Omega, M_\rho) = M^2 \left\{ \frac{2 M^2}{(M_\rho^2 + M^2)(M_\rho^2 - M^2)} + \frac{M_\rho^2(M_\rho^2 + M^2)}{M_{\Omega\Omega}^2(M_\rho^2 - M^2)^2} \ln \left( \frac{M_\rho^2 + M^2}{M_\rho^2} \right) + \frac{M^2(M^2 - 3 M_\rho^2)}{M_{\Omega\Omega}^2(M_\rho^2 - M^2)^2} \ln \left( \frac{M_\rho^2 + M^2}{M_\rho^2} \right) \right\},$$

(6b)

and where we have already made use of the simplification $M_F = M_2 = M$.

At this stage, the mass of the first three neutrinos are simply $m_\nu = \tilde{G}_\nu \sqrt{2}$ while that of the fourth generation neutrino is $m_N = \tilde{G}_N \sqrt{2} + m_\nu$. To see that $m_\nu$ can be much smaller than the first term in $m_N$ and hence much smaller than $m_N$ itself, let us simply assume that $M_\Omega = M_\rho = M_S$ in Eq. (6b). The function $I(M, M_\Omega, M_\rho)$ becomes $I(x = M^2/M_S^2)$. It is straightforward to see that, for finite values of $x$, the function $I(x)$ has a zero at $x \approx 0.4965$. What this intriguing fact means is that $m_\nu$ can be very small when the heavy fermion mass $M$ approaches $\sqrt{0.4965} M_S$. In fact in that limit,
\[ m_\nu/m_N = (\lambda_{\Omega\rho}/16\pi^2)I(x), \]  

(7)

where \( x = M^2/M_\rho^2 \). Taking into account the fact that \( \lambda_{\Omega\rho}/16\pi^2 \) can be very small itself, e.g. \( \lambda_{\Omega\rho}/16\pi^2 \approx 10^{-7} \) for \( \lambda_{\Omega\rho} \approx 2 \times 10^{-5} \), it is not hard to imagine that \( M \) can be close to but not necessarily equal to \( \sqrt{0.4965} M_S \) for \( m_\nu/m_N \) to be much less than unity. Alternatively, one can have \( x \sim 1 \), in which case \( I(x) \sim -0.06 \) and this will implies that \( \lambda_{\Omega\rho}/16\pi^2 \approx 10^{-7} \) for \( \lambda_{\Omega\rho} \approx 2 \times 10^{-5} \). With this simple discussion, one can see that it is not unnatural in our scenario to have one “very heavy” neutrino and three very light ones, without resorting to the famous see-saw mechanism. One could have, for example, \( m_N = O(100 \text{ GeV}) \) and \( m_\nu = O(\text{eV}) \).

We now turn to the discussion albeit a rather brief one- of neutrino oscillation [7]. In such a discussion, one of the relevant quantities is the mass difference: \( \Delta m^2_{ij} = |m^2_{\nu_i} - m^2_{\nu_j}| \), \( i, j = 1, 2, 3 \), the other ones being the oscillation angles. The oscillation angles are related to the leptonic “CKM” matrix defined by \( V_{ij} = U^*_l U_{\nu} \), where \( U_l \) and \( U_{\nu} \) are the matrices which diagonalize the charged and neutral lepton mass matrices respectively.

To lift the degeneracy of the three light neutrinos, it seems obvious that the remaining light family symmetry, \( SO(3) \) (coming from \( SO(4) \to SO(3) \)), has to be broken. To this end, let us assume: \(< \Omega > = (\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, V) \) and \(< \rho > = (\tilde{v}'_1 \otimes s_1, \tilde{v}'_2 \otimes s_1, \tilde{v}'_3 \otimes s_1, V' \otimes s_1) \), where \( s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \). It is beyond the scope of this paper to study the full dynamics of the most general Higgs potential which should constraint the values of the various VeVs. We have seen how a tree-level mass for the 4th generation neutrino arised and how a common mass for all four neutrinos was obtained at one-loop level, when \( \tilde{v}' \)‘s and \( \tilde{v}' \)‘s were assumed to be zero. The strategy now is to “crank up” those VeVs from zero to some “small” values- i.e. small compared with \( V \) and \( V' \)- and see what happens. After substituting these two VEV’s into Eq.(2), one obtains a non-diagonal \( 4 \times 4 \) neutrino mass matrix \( \mathcal{M} \) whose elements are given by: \( \mathcal{M}_{ii} = m_\nu + \tilde{G}_{i} v/\sqrt{2} \), \( \mathcal{M}_{4i} = m_\nu + \tilde{G}_N v/\sqrt{2} \), \( \mathcal{M}_{ij} = \tilde{G}_{ij} v/\sqrt{2} \), \( \mathcal{M}_{ji} = \tilde{G}_{ji} v/\sqrt{2} \), \( \mathcal{M}_{i4} = \tilde{G}_{i4} v/\sqrt{2} \), \( \mathcal{M}_{4i} = \tilde{G}_{4i} v/\sqrt{2} \), with \( \tilde{G}_{ij} = G_1 G_{M_2} G_3 (\tilde{v}_i \tilde{v}'_j)/(M_F M_2) \),
\[ \tilde{G}_{4i} = G_1 G_{M_2} G_3 (\bar{\nu}_i V') / (M_F M_2), \quad \tilde{G}_{4i} = G_1 G_{M_2} G_3 (V v'_i) / (M_F M_2), \]

where \( m_\nu \) and \( \tilde{G}_N \) are given by Eq. (6a) and Eq. (4) respectively and \( i, j = 1, 2, 3 \). The analysis of such a mass matrix for arbitrary \( \tilde{v} \) and \( \tilde{v}' \) is beyond the scope of the paper. One can however still get a glimpse of various possibilities by looking at particular cases. The discussion which follows is not meant to be complete nor realistic. A simple example will be used to show how one can partially lift the degeneracy and how one might proceed to construct a more realistic neutrino mass matrix.

As an example, let us put \( \tilde{v}_i = \tilde{v} \) and \( \tilde{v}'_i = \tilde{v}' \), \( \forall i \). We shall assume that the primary diagonal masses of the light neutrinos come from the one-loop diagram (Eq.(7)). It is then interesting to notice that the mass splitting among the three light neutrinos is related to the disparity in the breaking scales \( V^{(l)} \) (of \( SO(4) \)) and \( \tilde{v}^{(l)} \) (of \( SO(3) \)), i.e. the difference between the breaking of the full family symmetry and that of the light family symmetry. Let us take a specific example to start our discussion. Let us assume \( m_N \sim 100 \text{ GeV} \) and \( m_\nu \sim 1.4 \text{ eV} \). (The reader is referred to our earlier discussion on the reason why it is possible to have the previous masses.) The elements (except for \( M_{44} \)) of the neutrino mass matrix are expressed in terms of two ratios: \( r = \tilde{v} / V \) and \( r' = \tilde{v}' / V' \), namely \( M_{4i} = m_N r \), \( M_{4i} = m_N r' \), \( M_{ij} = M_{ji} = m_N r r' \) while \( M_{ii} = m_\nu + M_{ij} \). A few remarks are in order. First, it is easy to check that the degeneracy of the three light neutrinos is still present if the two ratios, \( r \) and \( r' \), are equal, regardless of their magnitude. Second, if we wish the bulk of the light neutrino mass to come from \( m_\nu \), then \( r r' \lesssim 10^{-12} \). As an example, we take \( r = 10^{-6} \) and \( r' = 10^{-7} \). (A similar answer is found even for \( r = 1 \) and \( r' = 10^{-12} \).) The diagonalization of the neutrino mass matrix gives the following eigenvalues: 100 GeV, 1.400013439 eV, 1.4 eV, 1.4 eV. Notice that \((1.400013439)^2 - 1.4^2 \sim 4 \times 10^{-5}\). This simple exercise simply shows that one can lift the degeneracy of at least one of the three light neutrinos in the “right” direction. To have a more “realistic” splitting, one has to examine the general case with \( r_i \) and \( r'_i \), \( i = 1, 2, 3 \), all different from one another. This will be presented elsewhere. The lesson learned from the previous example is simple: the tiny mass splitting among the three
light neutrinos is related to the large disparity in scales between the full family symmetry and that of the light families.

A preliminary investigation of the odd option, with three families and one family singlet \( \eta' \), appears to indicate that the preferred solution for the neutrino masses is that in which there is a hierarchy \( m_1 \ll m_2 \ll m_3 \). This will be presented elsewhere.

Several issues which need to be investigated are: 1) The charged lepton sector whose diagonalization matrix will be an important component of the leptonic “CKM” matrix; 2) A detailed study of neutrino oscillation using (1) combined with the above analysis; 3) the quark sector within the framework of the present model; 4) Additional roles of the vector-like heavy fermions, \( F, M_1, M_2 \), other than simply being “the mothers of all neutrino masses”. In particular, it would be interesting to study the quark counterparts of these fermions.

In summary, we have presented in this paper arguments showing how an extra symmetry among right-handed neutrinos, \( SU(2)_{\nu_R} \), might shed light on the nature of family replication. We have also presented a model, with four generations, in which it is not unnatural to have one very massive fourth neutrinos and three very light ones. Furthermore, all neutrino masses are Dirac masses, and hence there should be no such phenomenon as neutrinoless double beta decay.

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FIGURES

FIG. 1. Feynman graph showing the computation of \( \tilde{G}_\nu \), where \( m_\nu = \tilde{G}_\nu \frac{v}{\sqrt{2}} \)
\[ \nu \alpha \Omega \rho \alpha \iota \eta \alpha i \]