Experimental Realization of a PhotonicBell-State Analyzer

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Abstract

Efficient teleportation is a crucial step for quantum computation and quantum networking. In the case of qubits, four different entangled Bell states have to be distinguished. We have realized a probabilistic, but in principle deterministic, Bell-state analyzer for two photonic quantum bits by the use of a non-destructive controlled-NOT (CNOT) gate based on entirely linear optical elements. This gate was capable of distinguishing between all of the Bell states with higher than 75% fidelity without any noise substraction due to utilizing quantum interference effects.

Many quantum communication and quantum computation schemes, including quantum teleportation [1, 2], dense coding [3, 4], and quantum cryptography [5, 6, 7] are based on the maximally-entangled two-particle quantum states called Bell states. For both the creation and discrimination of these states, two-qubit quantum logic gates are necessary. For quantum communication the use of single photons to encode quantum information is most promising due to the their robustness against decoherence and the possibility of photon broadcasting. However, it has been very difficult to achieve the requisite logic operations between two individual photonic qubits due to their weak physical interaction [8].

Remarkably, Knill, Laflamme, and Milburn [9] found a way to circumvent this problem and to implement efficient quantum computation based on linear optics. They showed that the measurement process itself was enough to induce strong nonlinearities at the single-photon level.

One of the most important two-qubit gates for quantum computation is the controlled-NOT (CNOT) gate, which flips the second (target) bit if and only if the first (control) bit has the logical value 1, while the control bit remains unaffected: $|0\rangle_c |0\rangle_t \rightarrow |0\rangle_c |0\rangle_t$, $|0\rangle_c |1\rangle_t \rightarrow |0\rangle_c |1\rangle_t$, $|1\rangle_c |0\rangle_t \rightarrow |1\rangle_c |1\rangle_t$, $|1\rangle_c |1\rangle_t \rightarrow |1\rangle_c |0\rangle_t$. Very recently, the first demonstrations of these linear optics based CNOT gates have been demonstrated [10, 11, 12, 13, 14] to generate entanglement. The same CNOT operation can also be used to distinguish the four Bell states [15]:

$$|\phi^\pm\rangle_{1,2} = \frac{1}{\sqrt{2}} (|0\rangle_1 |0\rangle_2 \pm |1\rangle_1 |1\rangle_2)$$

$$|\psi^\pm\rangle_{1,2} = \frac{1}{\sqrt{2}} (|0\rangle_1 |1\rangle_2 \pm |1\rangle_1 |0\rangle_2),$$

where the subscripts 1 and 2 label different qubits.

In the present work we explicitly realized the controlled disentangling operation of the CNOT gate for input Bell states from pulsed laser sources. Our implementation of this gate utilizes the CNOT gate proposed by Pittman et al. [16] and experimentally demonstrated by Gasparoni et al. [12]. In particular this gate requires an entangled ancillary photon-pair which in principle gives access to classical feedforward of the output states as demanded for quantum information tasks [9]. This is in contrast to other destructive linear optics gates [10, 11] which necessarily have to destroy their output states. Through the usage of polarization entangled input pairs our experiment entirely differs from other previous teleportation experiments [14] where the CNOT was applied to independent qubits, as well as for quantum process tomography experiments [13] which were constraint to differently polarized input states of continuous laser sources. Our CNOT gate is probabilistic with an ideal success rate of $\frac{1}{4}$ when monitoring all four combinations of outputs for the ancilla pair. In the present version only fixed linear polarizers were used and thus decreasing the actual success rate to $\frac{1}{16}$. When a Bell state enters a CNOT gate (Fig. 1) in modes 1 and 2, the state transforms as:

$$|\phi^\pm\rangle_{1,2} \rightarrow |\pm\rangle_a |0\rangle_b$$

$$|\psi^\pm\rangle_{1,2} \rightarrow |\pm\rangle_a |1\rangle_b,$$

where $|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$ represents the complementary basis, 1 and 2 label the two input modes, while a and b the two output modes. In the experiment we used the polarization of photons to represent the qubits with $|0\rangle$ denoting the horizontal polarization state $|H\rangle$ and $|1\rangle$ the vertical polarization state $|V\rangle$. In Figure 1, the two input photons and the two photons from an ancilla pair in the entangled state $|\phi^+\rangle_{3,4}$ are superimposed at two polarizing beamsplitters (PBSs). The PBS is an optical device that transmits horizontally-polarized photons and reflects vertically-polarized photons. In this experiment, the PBS implements a two-qubit parity check: if two photons enter the PBS from the two different input ports, then they must have the same logical value, $|0\rangle |0\rangle$ or $|1\rangle |1\rangle$, in order to pass to the two different output ports. The lower PBS, acting in the $|\pm\rangle$ basis (indicated by the circle within the square) performs the logical function of a destructive CNOT gate, where one of the input qubits is destroyed in the lower detector (mode d) [12]. This destruction of one of the input qubits is compensated by first encoding the value of that qubit onto two output qubits. This happens by utilizing the parity check.
at the upper polarizing beamsplitter and exploiting the entanglement between the modes 3 and 4. Thus conditioned on the successful detection of one and only one photon in the output mode c in the state \(|+\rangle_c\), an arbitrary input state in mode 1 undergoes the following encoding transformation: \(\alpha|0\rangle_1 + \beta|1\rangle_1 \rightarrow \alpha a|0\rangle_a|0\rangle_4 + \beta|1\rangle_a|1\rangle_4\). One of the encoded qubits is issued as the input in mode 4 for the destructive CNOT gate, while the remaining copy serves as one of the required logical outputs in mode a. The scheme works in those cases where one and only one photon is found in each of the modes a, b, c, and d with a theoretical probability of \(\frac{1}{4}\) when the two possible outcomes for each photon can be distinguished. Therefore, all of the failures in this CNOT gate corresponds to the emission of more than one photon into the same spatial output mode, which can be arbitrarily reduced by using the quantum Zeno effect \([17]\). In the case of photons, where the emission of two photons into the same output mode must be suppressed, this quantum Zeno effect can be implemented by utilizing photon-atom interactions, i.e. by an optical fibre, whose core is containing a single atom. For our proof-of-principle demonstration, where we used polarizers in the output beams, the success rate is reduced to \(\frac{1}{16}\).

An ultraviolet laser pulse with a wavelength of 395nm passes through a nonlinear \(\beta\)-barium borate (BBO) crystal \([18]\), reflects at a moveable delay mirror, and passes through the crystal for a second time (Fig. 2). Such an alignment allows the emission of polarization-entangled photon pairs into the forward pair of modes 3 and 4, as well as into the backward pair of modes 1 and 2. To counter the effect of birefringence from the BBO crystal, compensators are used in each mode. These compensators, composed of a half-wave plate (HWP) performing a 90° rotation and an additional BBO crystal, erase the longitudinal and transversal walk-off of the down-converted photons. Final HWPs, one for each photon pair, and the tilt of the compensation crystals allow for the production of any of the four Bell states. The modes of the forward emitted pairs 1 and 2 and the modes of the backward emitted pairs 3 and 4 are coherently combined at polarizing beamsplitters (PBSs) by adjusting the position of the delay mirror. The indistinguishability between the overlapping photons is improved by introducing narrow bandwidth (3nm) spectral filters at the outputs of the PBSs and by monitoring the outgoing photons by single-mode fiber-coupled detectors (Fig. 3). After each PBS a quarter-wave plate (QWP) compensates birefringence effects in the PBSs.

The destructive CNOT gate is realized by rotating the input states along modes 2 and 4 by 45° through HWPs. Additional HWPs can be used to identify the outcomes according to the original proposal \([16]\). These HWPs in the output modes are not necessary if the measurements are made directly in the complementary \(|\pm\rangle\) basis.

For the Bell state analysis, two Bell states have to be prepared in different modes and at the same time. These pairs, the input Bell state and the ancilla pair, have to overlap at the two PBSs. The HWP in mode 4 transforms the ancilla Bell state to \(|\phi^+\rangle_{3,4} = \frac{1}{\sqrt{2}}(|0\rangle_3|+\rangle_4 - |1\rangle_3|−\rangle_4\), while the HWP in mode 2 transforms each of the individually prepared input Bell states to

\[
|\phi^\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}(|0\rangle_1|+\rangle_2 + |1\rangle_1|−\rangle_2) \\
|\psi^\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}(|0\rangle_1|−\rangle_2 ± |1\rangle_1|+\rangle_2). \quad (3)
\]

Recalling the action of the each PBS as parity check the four different input states result in:

\[
|\phi^+\rangle_{1,2}, |\phi^+\rangle_{3,4} \rightarrow |\phi^+\rangle_{a,c}|\phi^+\rangle_{b,d} \\
|\psi^+\rangle_{1,2}, |\phi^+\rangle_{3,4} \rightarrow |\phi^+\rangle_{a,c}|\phi^−\rangle_{b,d}, \quad (4)
\]

if one photon is detected in each of the four outputs. Projective measurements in modes c and d onto the state \(|+\rangle\) complete the action of the CNOT and transforms each input Bell state to a different output as follows: \(|\phi^\pm\rangle_{1,2} \rightarrow |\pm\rangle_a|+\rangle_b\) and \(|\psi^\pm\rangle_{1,2} \rightarrow |\pm\rangle_a|−\rangle_b\). Note the similarity between this transformation and Eq. 2.

Each of the four input Bell states were prepared by rotating the HWP and tilting the compensating BBO in mode 1. The quality of the input state was quantified by correlation measurements in the \(|0\rangle\) and \(|±\rangle\) bases, yielding an average fidelity of 96% and 94%, respectively. Roughly the same visibilities were obtained for the ancilla pair when adjusting a \(|\phi^+\rangle_{3,4}\) state by using the HWP and BBO crystal in mode 3. While the projections onto \(|+\rangle\) at the output modes c and d were kept fixed, the orientations of the polarizers in the output modes a and b, were changed \(|+\rangle_a|±\rangle_b\; \rightarrow \; |+\rangle_a|+\rangle_b\; \pm \; |−\rangle_a|−\rangle_b\). Each measurement was made for 1800 s and yielded a maximum of about 450 four-fold coincidences. In contrast to previous experiments \([12]\), cases where due to the spontaneous nature of the down-conversion process two pairs are emitted into the same pair of modes, 1 and 2 or 3 and 4, can ideally never result in a four-fold coincidence. These photons must have orthogonal polarization to be split up at each PBS to contribute to a four-photon detection after the beamsplitters. In the present scheme this contribution is suppressed by destructive quantum interference in the HWP rotation used for preparing the input Bell states. To explain this in more detail, consider the case that the source emits for example a \(|\phi^+\rangle_{1,2}\) Bell state into the modes 1 and 2. When the HWP rotates the polarization by \(\theta\) the Bell state to \(\cos(\theta)|\phi^+\rangle_{1,2} + \sin(\theta)|\psi^−\rangle_{1,2}\), the four-photon contribution after the PBS evolves like \(\cos(2\theta)|0\rangle_a|0\rangle_4|1\rangle_c|1\rangle_d\) and thus does not contribute at our rotation of 45°. At this specific angle any possible four-photon state, emitted into the two modes, is excising of two photons with
the same polarization within one mode. Thus, these two photons will never be split up at the PBS and therefore never contribute to a four-fold coincidence detection.

The count rates (Fig. 4) of all 16 possible combinations clearly confirm the successful implementation of the Bell-state analyzer. The fidelity of the gate operation \( F = N_{\text{correct}} / N_{\text{total}} \) can be obtained directly from the numbers of the correct measurement results, \( N_{\text{correct}} \), divided by the sum of all counts, \( N_{\text{total}} = N_{\text{correct}} + N_{\text{incorrect}} \). The achieved fidelities of each Bell state analysis are \( F_{\psi^+} = (0.75 \pm 0.05) \), \( F_{\phi^-} = (0.79 \pm 0.05) \), \( F_{\psi^+} = (0.79 \pm 0.05) \), \( F_{\phi^-} = (0.75 \pm 0.05) \) without subtracting background due to the curious quantum interference effect. This is a remarkable improvement over former experiments [12], where the substraction of the signal from the emission of two pairs into the same pair was required to achieve high fidelity results. However, minor incorrect outcomes in our experiment originate mainly from imperfections in the PBSs where photons are guided incorrectly and from the obtained visibilities when overlapping the photons at the PBSs (Fig. 3).

For confirmation of the generality of our Bell-state analyzer the input state superposition \( |\Psi\rangle_{1,2} = |\psi^+\rangle_{1,2} + i |\phi^-\rangle_{1,2} \) was created by tilting the compensating BBO crystal to a position, where the visibility in the \( |\pm\rangle \) basis was almost 0. The output settings of the polarizer result in the highest count rates for a \( |\psi^+\rangle \) and a \( |\phi^-\rangle \) setting, 170 and 104 counts, respectively, while the settings for \( |\psi^\pm\rangle \) input state just measured noise counts. This confirms that the analyzer functions as expected, detecting both the \( |\psi^+\rangle \) and \( |\phi^-\rangle \) components from the input state.

The experiment reported here explicitly demonstrates for the first time the action of the CNOT gate on input Bell states, where all Bell states were identified with an average fidelity of 0.77\(\pm\)0.05 without any substraction of noise. Additionally, the experiment shows how quantum interference effects allow to suppress the spurious signal responsible for low raw-data fidelities in previous work.

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