Structure of the low-lying states in some $N=80$ isotones

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Abstract. The quasiparticle-phonon model using a separable interaction deduced from a Skyrme force is adopted to compute low-lying spectra and the giant dipole resonance in the $N=80$ isotones. It is shown that the Skyrme interaction not only reproduces the fragmentation of the giant resonance but also yields results comparable to the ones obtained when a Woods-Saxon potential is used in the description of the mixed symmetry states recently discovered in these nuclei. This test is encouraging in view of future extensions of the method to neutron-rich nuclei far from stability.

1. Introduction

Skyrme effective interactions are widely used for the study of nuclear properties. Attention is focused mostly on the bulk properties, like binding energies, mean square radii, energy centroids, and on the strength distributions of Giant Resonances [1, 2]. Recently, a tensor term was incorporated in the Skyrme force [3, 4]. Such a term came out to improve significantly the single particle level spectra.

Recently, methods for obtaining separable approximations to Skyrme forces have been developed. One is anchored to the mean field self-consistency [5], the other is based on a finite rank approximation [6]. Both methods have been applied to deformed [5] as well as spherical nuclei [6, 7, 8]. The finite rank approximation procedure was used to calculate the properties of low-lying quadrupole excitations in the chain of tin isotopes [9].

The Quasiparticle-Phonon Model ($QPM$) of Soloviev and collaborators [10] has been applied successfully to the study of the low-lying excitations and giant resonances in spherical and deformed nuclei. The main ingredients of the model are phonons obtained in the quasi-particle random-phase approximation ($QRPA$) interacting by means of a residual interaction. The $QPM$ unifies the description of the structure of low-lying states in terms of single and multiple phonon states. Within the model, the consistency between the mean field and residual interaction is ensured by the relation between the radial part of the interaction and the derivative of the mean...
field potential, 
\[ f(r) = \frac{dU(r)}{dr}. \]  

(1)

In the QPM one adopts generally a Woods-Saxon one-body potential. This is a good choice for stable nuclei, where the Woods-Saxon parameters can be inferred from a fit of the experimental quantities, like single-particle energies. This is no longer possible in nuclei far from stability. In neutron rich nuclei, in fact, the proton mean field potential becomes much deeper than the neutron well because of the enhancement of the proton-neutron interaction. For those nuclei, it is safer to generate the mean field self-consistently. This can be done by Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) methods using Skyrme forces.

A preliminary condition for the reliability of the one-body potentials so obtained is that they reproduce the low-energy properties and spectra of stable nuclei. A meaningful test is provided by the work presented here. We have, in fact, performed a QPM calculation using a two-body interaction of separable form derived from a Skyrme force. Here, a detailed comparison with an analogous calculation using a Woods-Saxon potential shows that the separable version of the Skyrme force is successful in the study of low-lying collective states in spherical nuclei as well as in reproducing the distribution of the strength in the domain of the Giant Dipole Resonance (GDR).

2. Main ingredients of the model

The intrinsic QPM Hamiltonian has the following structure

\[ H = H_{sp} + V_{pair} + V_{ph}^{M} + V_{SM}^{ph} + V_{pp}^{M}. \]  

(2)

\( H_{sp} \) is the one-body Hamiltonian, \( V_{pair} \) is the monopole pairing force, \( V_{ph}^{M} \) and \( V_{SM}^{ph} \) are respectively the particle-hole separable multipole and spin-multipole interactions, and \( V_{pp}^{M} \) is the particle-particle interaction.

As pointed out already, most of the QPM calculations have been performed using Woods-Saxon one-body potentials. In the case of spherical symmetry, the parameters of these potentials have been fitted for several domains of mass number [10]. The comparison of a typical Woods-Saxon potential with the corresponding SHF potential, obtained through a SIII parametrization, is shown in Fig. 1 for \( ^{136}\text{Ba} \). We have to keep in mind that, in the case of SHF potential, the effective mass \( m^{*}(r)/m \) is smaller than 1. Thus, the term of comparison is not the central part of the potential but \( m^{*}_{cm}(r) V_{central}(r) \) [4].

The close similarity between the two potentials has been proved quantitatively in Ref. [11] and exploited for the study of the pygmy resonance. In the mentioned study, the mean field part was calculated by means of the Hartree-Fock-Bogoliubov method and the parameters of Woods-Saxon potentials were fitted to reproduce the single-particle separation energies, the charge radii and the difference between proton and neutron mean square radii.

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**Figure 1.** A mean field potential calculated within the Skyrme-HF approximation with SIII parameterization compared to the typical Woods-Saxon potential for \( ^{136}\text{Ba} \).
The residual interaction in the particle-hole form has the following expression,

\[ V \approx \sum_l \kappa_l Q^\dagger_{lm} Q_{lm}, \tag{3} \]

where

\[ Q^\dagger_{lm} = \sum_{j_1j_2} \langle j_1| f(r) Y_{lm}(\Theta, \Phi) | j_2 \rangle a^\dagger_{j_1} a_{j_2}, \tag{4} \]

\( a^\dagger(a) \) being creation(annihilation) particle operators. If \( f(\vec{r}) = r^l Y_{lm}(\Theta, \Phi) \) Eq.(4) yields the well known standard multipole operator. The parameters \( \kappa_l \) are chosen according to prescriptions of the QPM [10].

The procedure used here differs from the one adopted in Ref.[11] for studying the pygmy resonance. Here, in fact, we derive the mean field potential directly from the Skyrme-force by a Skyrme-HF method and keep the consistency between the mean field and residual interaction by enforcing the consistency condition expressed by Eq. (1).

3. Results
Recently, the QPM is the only method that allows to perform microscopic calculations in a sufficiently large configuration space and provides a unified description of low-lying and high-energy single- and multi-phonon excitation levels, including the excited states in the domain of Giant Resonances.

In recent years, the QPM has been adopted with success in disclosing the properties of the so-called mixed symmetry states discovered for the first time in Mo [12] and, since then, found in most spherical nuclei in the vicinity of the \( N = 50 \) and \( N = 82 \) neutron shell closures [13]. The QPM calculations, using a WS one-body potential, nicely reproduce the low-lying spectra in these nuclei and provide specific signatures for the symmetric and mixed symmetry states [14]. Moreover, the same QPM studies were able to point out the importance of shell structure in determining the splitting of the mixed symmetry mode in selected nuclei around \( N = 82 \) [15, 16].

In view of the extension of the method to nuclei far from stability, it is of special importance to perform a QPM study of these states using a separable interaction deduced from a Skyrme force. We have done this for the even-even N=80 isotones.

3.1. Single particle schemes
We have examined several Skyrme interactions in order to find the potential that yields the best single-particle spectrum in the N=80 chain. Our main criterion was to choose the force that best reproduces the level ordering and level density around the Fermi level for both proton and neutron subsystems. We have compared the results to those obtained by a Woods-Saxon potential, parameterized for the given mass region, and known to give a good description of the experimental spectra. The comparison for the neutron level scheme is shown in Fig.2. As seen from the figure, the best results are obtained when the SIII parametrization is used. The same SIII interaction yields also for protons a level sequence in good agreement with the one obtained using the WS potential (Fig.3). Thus, we have adopted the SIII parameterization for the QRPA and QPM calculations.

3.2. Mixed symmetry states
As it was shown in [15], the single-particle spectra have a crucial role in the formation of the structure of mixed symmetry states along the N=80 isotones as well as the changing of the structure within the isotonic chain. The nucleus \(^{138}Ce\) reveals an interesting feature. The M1 strength is split into two pieces while in \(^{136}Ba\) the corresponding M1 strength is concentrated
Figure 2. Neutron single-particle levels obtained with different Skyrme parameterizations, compared to Woods-Saxon single particle spectrum.

Figure 3. Proton single-particle levels obtained with different Skyrme parameterizations, compared to Woods-Saxon single particle spectrum.

Table 1. Energy of the lowest quasi-particle proton and neutron states of $^{136}$Ba

| State          | Woods-Saxon | SIII |
|----------------|-------------|------|
| $(1g_{7/2})_p$ | 1.31        | 1.06 |
| $(2d_{5/2})_p$ | 1.57        | 2.32 |
| $(2d_{3/2})_n$ | 1.095       | 1.176|
| $(1h_{11/2})_n$ | 1.245      | 1.164|
| $(3s_{1/2})_n$ | 1.55        | 1.62 |
| $(2d_{5/2})_n$ | 3.30        | 3.55 |

Table 2. Energy and $E2$ decay strengths of the lowest $[2^+]_{RPA}$ states. The numbers in brackets are the RPA values obtained using a WS potential

| Nucleus | $\lambda_i^+$ | $\omega \lambda_i^+$ | $B(E2)$ ↓ |
|---------|---------------|-----------------------|-----------|
| $^{136}$Ba | $2^+_1$ | 1.06 (1.03) | 15 (24.8) |
|         | $2^+_2$ | 2.5 (2.12) | 1.7 (1.7) |
|         | $2^+_3$ | 2.89 (2.25) | 0.5 (0.07) |
| $^{138}$Ce | $2^+_1$ | 1.06 (1.02) | 15 (22.2) |
|         | $2^+_2$ | 2.6 (2.21) | 5 (1.6) |
|         | $2^+_3$ | 2.89 (2.31) | 0.1 (3.96) |

on a single excitation level. This property [15] is connected with shell structure and pairing. The important single-particle states around the Fermi level are $2d_{3/2}$, $1h_{11/2}$ and $3s_{1/2}$ for neutrons and $1g_{7/2}$ and $2d_{5/2}$ for protons. The lowest proton and neutron quasi-particle energies in $^{136}$Ba are shown in Table 1. One can seen that both potentials give similar values for the quasi-particle
energies of the states around the Fermi level. Because of this close correspondence, the low-lying states of $^{136}$Ba and $^{138}$Ce, calculated by means of Woods–Saxon and SIII potentials must have a similar structure.

The energy and $E2$-strength of the low-lying $(2^+)_RPA$ states computed in the QRPA are shown in Table 2. As discussed in [15], because of the different position of the proton chemical potential, the second and third $(2^+)_RPA$ states of $^{138}$Ce are much more collective than those in $^{136}$Ba. The total value of $E2$ transition strengths $\sum_{i=2,3} B(E2; 2^+_i \rightarrow g.s.)$ for $^{138}$Ce is much larger than the corresponding value for $^{136}$Ba. This is the reason of the splitting of $M1$ strength into two peaks corresponding to the $M1$ transitions connecting the high-lying $2^+$ excitations to the first $2^+_1$ state. Table 2 shows that the SIII potential gives the same results.

In Table 3, the QPM energies and transition strengths, obtained using the SIII force, are compared with the corresponding QPM values using a WS potential and the experimental quantities. It is seen that large $B(M1)$ values are collected by the transitions connecting the $2^+_2$ and $2^+_4$ excited states to the $2^+_1$ state in $^{138}$Ce while only one large $B(M1)$ is obtained for $^{136}$Ba. The overall comparison between QPM results and experiments is shown in Fig.4. Though the energy of the excitations is not perfectly reproduced, the Skyrme force provides an overall picture which is consistent with the results obtained using the WS potential and in fair agreement with the experimental data.

We conclude that the Skyrme-HF mean field potential incorporated in QPM is very suitable for structure calculations concerning low-lying excitations.
Table 3. QPM versus experimental strengths of $E2$ and $M1$ transitions. The $E2$ strengths are given in W.u. for $^{138}\text{Ce}$, and in $e^2b^2$ for $^{136}\text{Ba}$. The $M1$ strengths are in $\mu^2_N$.

| Nucleus | $J_i \rightarrow J_f$ | EXP | B(E2) | QPM (SIII) | QPM(WS) | EXP | B(M1) | QPM (SIII) | QPM(WS) |
|---------|-----------------|-----|-------|------------|---------|-----|-------|------------|---------|
| $^{136}\text{Ba}$ | $0^+_{gs} \rightarrow 2^+_1$ | 0.400(5) | 0.24 | 0.33 |
| | $0^+_{gs} \rightarrow 2^+_2$ | 0.016(4) | 0.09 | 0.046 |
| | $0^+_{gs} \rightarrow 2^+_3$ | 0.045(5) | 0.03 | 0.065 |
| | $2^+_2 \rightarrow 2^+_1$ | 0.09(4) | 0.12 | 0.12 |
| | $2^+_1 \rightarrow 2^+_2$ | 0.007 |
| | $2^+_1 \rightarrow 2^+_3$ | 0.26(3) | 0.21 | 0.27 |
| $^{138}\text{Ce}$ | $2^+_1 \rightarrow 0^+_{gs}$ | 21.2(14) | 11 | 19 |
| | $2^+_2 \rightarrow 0^+_{gs}$ | 1.16(8) | 4.5 | 0.60 |
| | $2^+_3 \rightarrow 0^+_{gs}$ | 3.9 | 5.8 |
| | $2^+_4 \rightarrow 0^+_{gs}$ | 1.86(16) | 0.35 | 1.6 |
| | $2^+_2 \rightarrow 2^+_1$ | 28(2) | 26 | 24 |
| | $2^+_3 \rightarrow 2^+_1$ | 6.1 | 5.8 | 0.058(6) | 0.23 | 0.10 |
| | $2^+_4 \rightarrow 2^+_1$ | 0.65(10) | 0.28 | 0.32 | 0.122(10) | 0.13 | 0.17 |

Figure 6. The distribution of the B(E1) strength for $^{136}\text{Ba}$. A SIII Skyrme parameterization has been used.

3.3. Giant Dipole Resonance
Another test of the single-particle basis and of the value of the fitted QPM parameters, as obtained from the SIII parametrization of the Skyrme force, is provided by QRPA calculations
of the \( E1 \) strength distribution. The is done for \(^{136}\text{Ba}\). As shown in Fig. 6, a realistic distribution of the \( E1 \) strength in the Giant Dipole resonance domain is obtained. The Energy Weighted Sum Rule is fulfilled by 96% up to 20MeV.

4. Conclusions
The results obtained here show that the mean field generated by Skyrme forces can be successfully incorporated into the numerical scheme of the \( QPM \). In fact, the new set of single particle states generated by these interactions allow a satisfactory description of the structure of the mixed symmetry states in the \( N = 80 \) isotones. Even the splitting of the \( M1 \) strength observed in \(^{138}\text{Ce}\) is fairly well reproduced. This result gives a very positive message for the application of the \( QPM \) to exotic nuclei.

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