Effective fiber-coupling of entangled photons for quantum communication

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We report on theoretical and experimental demonstration of high-efficiency coupling of two-photon entangled states produced in the nonlinear process of spontaneous parametric down conversion into a single-mode fiber. We determine constraints for the optimal coupling parameters. This result is crucial for practical implementation of quantum key distribution protocols with entangled states.

Entangled-photon pairs generated in the nonlinear process of spontaneous parametric down conversion (SPDC) are proven to be a highly desirable means for practical quantum cryptography. The main difficulty of practical utilization of such system usually stems from a relatively low photon collection efficiency because of the complex spatial distribution of SPDC radiation and due to the broad spectral width of entangled-photon wave packets.

The problem of coupling entangled photons into a fiber has been considered before by Kurnsieber et al. Assuming the pump to be a plane wave the emission angle of the SPDC has been calculated as a function of the wavelength. The waist of the focused pump beam has been chosen to maximally overlap the “impression” of the Gaussian mode of a single-mode fiber on the crystal. It has been pointed out that the coupling efficiency may be significantly affected by transverse walk-off.

In this letter we present a significantly modified approach allowing us to achieve a high-efficiency coupling of the SPDC pairs into single mode fibers. In particular, we demonstrate how the pump beam waist, crystal length, optical system magnification and the fiber mode field diameter (MFD) must obey a precise joint relation in order to ensure high coupling efficiency. We describe a specific model that allows us to determine a scaling law accounting for all the real experimental parameters. We demonstrate that obtained experimental data are in good agreement with the proposed model.

The two-photons state can be written as

$$|\Psi\rangle_{\text{SPDC}} = \int d\omega_o \ d\eta_o \int d\omega_e \ d\eta_e \Phi(\eta_o, \omega_o; \eta_e, \omega_e) \cdot \hat{a}^\dagger(\omega_o, \eta_o) \hat{a}_e(\omega_o, \eta_e) |0\rangle.$$ (1)

The function $\Phi(\eta_o, \omega_o; \eta_e, \omega_e) = \tilde{E}_p(\eta_o + \eta_e, \omega_o + \omega_e) \tilde{\chi}(\eta_o, \omega_o; \eta_e, \omega_e)$ accounts for the phase matching conditions. $\tilde{E}_p(\cdot)$ represents the amplitude of the plane-wave expansion of the pump field and $\tilde{\chi}(\eta_o, \omega_o; \eta_e, \omega_e)$ with $\Delta k = k_p - k_o - k_e$. Inside the crystal the $z$-component of the wave-vector is defined as $k_z(\eta, \omega) = \sqrt{|\omega^2(n(\eta, \omega)/c)^2 - |q|^2}$. All the information on the state is given by the amplitude $A_{1,2}(x_1, t_1; x_2, t_2)$ of detecting the SPDC two-photons in space-time events at $(x_1, t_1)$ and $(x_2, t_2)$. The Fourier transform with respect to $t_1$ and $t_2$ of the two-photon amplitude $A_{1,2}(x_1, t_1; x_2, t_2)$ is given by

$$\tilde{A}_{1,2}(x_1, \omega_o; x_2, \omega_e) = \int d\eta_o \ d\eta_e \tilde{\Phi}(\eta_o, \omega_o; \eta_e, \omega_e) \cdot \mathcal{H}_1(x_1; \eta_o, \omega_o) \mathcal{H}_2(x_2; \eta_e, \omega_e).$$ (2)

$\mathcal{H}_j(x; q, \omega) (j = 1, 2)$ being the Fourier transform of impulse response functions $h_j(x; j; \omega)$ of the optical systems through which the two photons propagate from the output face of the crystal to the detection plane.

The coupling of the photon pairs into fibers can be considered a problem of maximizing the overlap between...
the two-photon amplitude $A_{1,2}$ of entangled-photon state in the detector plane with the field profiles of single-mode fibers. Assuming a quasi-monochromatic and quasi-plane wave travelling in the $z$-direction and defined on the two-dimensional continuous space of the detector planes, we can express the electromagnetic field operator $\hat{E}^{(\pm)}(\mathbf{x}, \omega)$ in terms of a linear superposition of electromagnetic field operators, $\hat{c}_l(\omega)$ and $\hat{c}_k(\omega)$ associated with a complete orthonormal set of functions $\phi_l(\mathbf{x})$, $\phi_k(\mathbf{x})$.

Choosing conveniently the noncontinuous set of guided modes $\psi_{lm}(\mathbf{x}, \omega)$ and the continuous set of the radiation modes $\psi_k(\mathbf{x}, \omega)$, respectively, the field operator can be decomposed as $\hat{E}^{(\pm)}(\mathbf{x}, \omega) = \sum_{lm} \psi_{lm}(\mathbf{x}, \omega) \hat{c}_l(\omega) + \int d\mathbf{k} \psi_k(\mathbf{x}, \omega) \hat{c}_k(\omega)$ where the new operators $\hat{c}_l(\omega)$ are defined as $\hat{c}_l(\omega) = \int d\mathbf{x} \psi_l^*(\mathbf{x}, \omega) \hat{E}^{(\pm)}(\mathbf{x}, \omega)$ with $l = (l, m)$ or $k$, and obey the usual bosonic commutation relations $[\hat{c}_l(\omega), \hat{c}_k^\dagger(\omega)] = \delta_{l\alpha, \beta}$. The amplitude in Eq. (2) can be expanded in terms of guided and radiation modes. The coefficients of the expansion for two guided modes, $(l, m)$ and $(l', m')$, are given by

$$A_{lm, l'm'}^{(1,2)}(\omega_o, \omega_e) = \int dx_1 dx_2 \psi_{l,m}(x_1, \omega_o, x_2, \omega_e) \cdot \psi_{l',m'}^{(1,2)*}(x_1, \omega_o) \psi_{l,m}^{(2)*}(x_2, \omega_e).$$

Coupling into a single-mode fiber can be quantified by the coupling efficiency parameter $\eta_c$ defined as the ratio of the probability to find two photons in the guided modes over the square root of the product of the probability to find one photon in a guided mode independently of the detection of the other photon. In the case of a single-mode fiber, when only the linearly-polarized fundamental mode $LP_{01}$ is allowed, the coupling efficiency takes the simple form

$$\eta_c = \frac{\mathcal{P}(1,2)}{\sqrt{\mathcal{P}(1)} \cdot \sqrt{\mathcal{P}(2)}}.$$

The numerator of this expression is given by

$$\mathcal{P}(1,2) = \int d\omega_o d\omega_e |A_{lm, l'm'}^{(1,2)}(\omega_o, \omega_e)|^2$$

where $A_{lm, l'm'}^{(1,2)}(\omega_o, \omega_e)$ is given by Eq. (3). The contributions at the denominator of Eq. (4) are given by

$$\mathcal{P}(1) = \int d\omega_o d\omega_e \int dx_2 \int dx_1 |A_{lm, l'm'}(\omega_o, \omega_e) \cdot \psi_{lm}^{(1)*}(x_1, \omega_o)|^2$$

and analogous expression for $\mathcal{P}(2)$. The maximum coupling, i.e. $\eta_c = 1$, is reached when the two-photon amplitude in Eq. (2) is the product of single-mode field profiles of two fibers.

We now consider a model that includes propagation of both fields through two equal infinite ideal lenses without an aperture limit. We also assume that the output plane of the crystal, at a distance $d_{sl}$ from the lenses, is imaged on the fiber plane, at a distance $d_{sl}$ from the lenses, i.e. $1/d_{sl} + 1/d_{sl} = 1/f$. The amplitude $A_{lm, l'm'}^{(1,2)}(x_1, \omega_o, x_2, \omega_e)$ becomes $A_{1,2}(x_1, \omega_o, x_2, \omega_e) \propto \Phi(\mu x_1, \mu x_2, \omega_o)$ where $\mu = d_{sl}/d_{sl} = d_{sl}/f - 1$ is the inverse of the magnification and $\Phi(\mathbf{x}', \omega_o; \mathbf{x}'', \omega_e)$ is the 2-D inverse Fourier transform of the matching function $\tilde{\Phi}(\mathbf{q}_o, \omega_o; \mathbf{q}_e, \omega_e)$, which we calculate in the paraxial and quasi-monochromatic approximation. Inside the crystal, such approximations allow us to consider only first terms in the exponential expansion of expression $\tilde{\Phi}(\mathbf{q}_o, \omega_o; \mathbf{q}_e, \omega_e)$. For a type-II non-collinear configuration and assuming a pump field to be factorable in terms of frequency and wave vectors $\tilde{\mathbf{E}}_p(\mathbf{q}_o + \mathbf{q}_e, \omega_p) = \tilde{\mathbf{E}}_p(\omega_p) \tilde{\mathbf{E}}_p(\mathbf{q}_o + \mathbf{q}_e)$, we can calculate the inverse Fourier transform with respect to the frequencies and obtain $\Phi(x', y' + t/2; x'', y'' - t/2) \sim \Pi_{DL} (t) \tilde{\mathbf{E}}_p(T - \Delta t/D) \tilde{\mathbf{E}}_p(\mathbf{x}' + \mathbf{x}'' - \mathbf{A}/D)/2 \delta(\mathbf{x}' - \mathbf{x}'' - \mathbf{B}/D)$ where we have introduced $t = t_1 - t_2$ and $T = (t_1 + t_2)/2$. The function $\Pi_{DL}(t)$ has value 1 for $0 < t < DL$ and zero elsewhere. The vectors $\mathbf{A} = 2M_p - \mathbf{M}$, $\mathbf{B} = \mathbf{M} + 2Q/K$ depend on $\mathbf{M}$ and $\mathbf{M}_o$, which are the spatial walk-off vectors for the extraordinary field at the generation and pump frequency, respectively. $\mathbf{Q}$ is the transverse wave-vector associated with perfect phase matching along the intersection of cones, and $K = 2K_oK_e/(K_o + K_e)$ represents a mean value of wave-vector for generated photons inside the crystal. We have also introduced $D = 1/u_o - 1/u_e$ and $\Lambda = 1/u_p - (1/2u_o + 1/2u_e)$ where $u_j$ is the group velocity for the $j$-polarization. The expression has a simple physical meaning. Due to the locality of the interaction, as dictated by the delta function, photons are created in pairs at each point of the crystal illuminated by the pump field, $\mathbf{E}_p(x)$. After their birth, photons propagate in the dispersive nonlinear crystal environment experiencing longitudinal $D$ and spatial walk-off, $\mathbf{M}$. They spread with respect to each other in time and in transverse direction, according to the travel distance, $z$. The transverse spread contributes through two distinct processes when multiplied by the crystal length $L$. The product $|\mathbf{A}|L$ represents the shift between the generated pairs and the pump field, which is also extraordinary and hence has a walk-off. $|\mathbf{B}|L$ is the spread between the two entangled photons generated from the same pump photon. This vector contains a contribution due to a spatial-walk-off, $|\mathbf{M}|L$, and one more term, $2|\mathbf{Q}|L/K$, which represents the transverse distance between pairs generated at the input face of the crystal with respect to the ones generated at the output face, due to the geometry of optical propagation inside the crystal. If we assume a pump beam with Gaussian profile, $\mathbf{E}_p(x) = \exp(-|x|^2/2r_p^2)$, as well as a mode field profile, $\psi_j(x) = \exp(-|x|^2/2w^2)\sqrt{\pi}w$, we can derive a closed expression for the coupling efficiency, namely

$$\eta_c = \frac{4(1 + \xi^2)}{(2 + \xi^2)^2} \frac{\sigma_1}{\sigma_c} \sqrt{\frac{\sigma_1}{\sigma_2}} \frac{1}{\sqrt{\sigma_1}} \frac{1}{\sqrt{\sigma_2}}$$

where $\xi = w/\mu/r_p$ and

$$\sigma_c = \frac{L}{r_p} \sqrt{\frac{(\alpha_1 + \alpha_2)\xi^2 + \beta}{\xi^2(2 + \xi^2)}} ; \quad \sigma_j = \frac{L}{r_p} \sqrt{\frac{\alpha_j}{1 + \xi^2}}$$

The parameters $\alpha_1 = |\mathbf{M}_o|^2 + |\mathbf{Q}|^2/K^2$, $\alpha_2 = |\mathbf{M}_p - \mathbf{M}|^2 + |\mathbf{Q}|^2/K^2$, and $\beta = |\mathbf{M}|^2 + 4|\mathbf{Q}|^2/K^2$ are determined only by the crystal parameters and phase matching geometry. The expression for $\eta_c$ depends on the size of...
fibers effectively imaged onto the crystal plane, $w_p$, and the crystal length, $L$, scaled to the pump beam waist into the crystal, $r_p$.

The experimental verification of the above model has been performed using an actively mode-locked Ti:Sapphire laser, which emitted pulses of light at 830 nm. After a second harmonic generator, a 100-fsec pulse (FWHM) was produced at 415 nm, with a repetition rate of 76 MHz and an average power of 200 mW. The UV-pump radiation was focused to a beam diameter of 150 $\mu$m inside a BBO crystal cut for a non-collinear type-II phase-matching using a $f=75$ cm quartz lens (see Figure 1).

![Fig. 1. Sketch of the fiber coupling experiment.](image)

Two points of intersection of the ordinary and extraordinary cones (3.5 deg relative to the pump direction) were imaged into a single mode fiber with a MFD of 4.2 $\mu$m using two f=15.4 mm coupling lenses placed 78 cm from the output face of the non-linear crystal. We obtain a coupling efficiency of $\sim 18\%$ with a crystal of 3 mm length. We achieved $\sim 29\%$ coupling efficiency using a 1 mm crystal in the same experimental setup. Taking into account the approximately 50% detector efficiency and 85% transmittance of filters, one can determine that effective fiber coupling coefficients can reach 42% and 68%, respectively (same results have been obtained with both 11 nm bandwidth filters centered at 830 nm and pass-band filters). This result is illustrated in Figure 2 where a solid line indicates theoretical curve obtained from our model Eq. 6. The parameters used have been calculated for crystal dispersion at 830 nm: $|M_p| = 0.07631$, $|M| = 0.07243$, and $Q/K = |M|^2/2 = 0.036215$. The fiber parameter is $w = MFD/2\sqrt{2} \sim 1.48 \mu$m, the pump beam radius $r_p \sim 53 \mu$m, and $\mu \sim 49$.

Examining the dependence of $\eta_c$ vs. crystal length (see Figure 2) for different values of parameter $\mu$ one can notice, for example, that one cannot reach a coupling efficiency greater than $\sim 50\%$ for $r_p \sim 53 \mu$m and for crystals longer than 2 mm.

In conclusion, we have evaluated the dependence of coupling efficiency of SPDC pairs into single-mode fiber on several major experimental parameters. We obtained an analytical expression for the coupling efficiency $\eta_c$ that allows us to characterize the importance of the spatial walk-off and to choose appropriate values of experimental parameters to reach high coupling efficiency. Numerical inspection of the general expression 4 with a more sophisticated model of the impulse response function of our system can allow us to further increase the coupling efficiency. Acknowledgements This work was supported by MIUR (Project 67679). G.D.G and A.V.S. also acknowledge support by DARPA and NSF.

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