Analysis and Optimization of Long Baseline Acoustic Positioning Error of Synchronous Responsive Type

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Abstract. In order to solve the problem of large time delay and low accuracy of position determination when the long-baseline positioning system is operating at large depths, the underwater acoustic positioning technology of the long-baseline array is studied and the long-baseline synchronization positioning error analysis is carried out, and the simulation verification is carried out by MATLAB. A method for optimizing the underwater acoustic positioning algorithm is proposed. After optimizing the positioning algorithm, in the case of a water depth of 6000 M and a positioning area of 1000*1000m², the total positioning error of the sitting-bottom long baseline is reduced from the meter level to the decimeter level, and the positioning accuracy is greatly improved. An improved synchronous response type is determined. The underwater acoustic positioning can realize the underwater large-scale, high-precision and high-real-time positioning.

Keywords: Acoustic positioning; Long baseline positioning; Positioning signal system; Underwater moving target; Precision analysis.

1. Introduction

The movement direction, movement speed, movement trajectory and other parameters of the subsea operation mining robot are the main control basis for the operation mother ship operator. At present, the accurate position of the underwater target is mainly obtained by the acoustic positioning method, using the propagation characteristics of sound waves in the water and the acoustic ranging principle, combined with highly developed modern semiconductor technology and artificial intelligence technology, makes it possible to achieve successful target positioning under complex underwater conditions.

At present, the underwater target positioning system has been manufactured and serialized abroad, such as Norway Kongsberg, Australia Nautronix, the United States LinkQuest and other products[1,2], and has been widely used in Marine engineering. China is relatively late in development. After decades of development, excellent engineering products have been developed. However, the positioning system of domestic underwater targets is characterized by low positioning accuracy, weak anti-background noise performance, small positioning area and insufficient generalization.

In order to complete the real-time high-precision position positioning of underwater moving targets, this article provides a feasible scheme for the high-precision position positioning of mining carts for
seabed mineral resources, and the positioning accuracy of this scheme is analyzed and optimized in detail.

2. Selection of acoustic positioning mode
In order to realize high accuracy underwater acoustic positioning of harvester, a reasonable positioning mode is needed. The design requirements for the positioning mode shall be: the positioning area shall be no less than 1000*1000m²; Positioning accuracy: <1m; Refresh rate of positioning data is no more than 0.5hz; It is difficult to put and recover the array. Commonly used acoustic positioning modes include short baseline positioning (SBL), ultra-short baseline positioning (USBL) and long baseline positioning (LBL) [3-5].

The bottom-mounted base array is the best for the positioning of the seabed mining truck. The ultra-short base line array cannot meet the full coverage requirements of the actual test because its ideal positioning range is within ±60° of the plane normal direction, so it can be excluded. ; The aperture of the short baseline array is several 10 meters to tens of meters, and the measurement accuracy is related to the overall size of the mining truck. The seabed mining robot is not easy to be enlarged, so the distance between the installation array elements is small, so the short baseline positioning error is large, which does not meet the requirements; Considering the large area of the positioning area and sub-meter requirements for positioning accuracy, the underwater acoustic positioning of the seabed mining truck adopts a bottom-mounted long baseline synchronous positioning mode. This mode can effectively avoid the sound velocity measurement error caused by sound ray correction. The positioning accuracy is much higher than that of the water surface based positioning matrix, and this matrix is very stable. It does not need to be operated again during work, and it is dependent on the outside world. And the interference is very small.

3. Principle of responsive long baseline underwater acoustic synchronization localization
When the harvester is working under water, the distance between the four transponders \( l_1, l_2, l_3, l_4 \) is determined through synchronous method \( (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) \) and \( (x_4, y_4, z_4) \) has been obtained on time in acoustic self-calibration, where \((x_1, y_1, z_1)=(0,0,0)\) and \(y_2=0\). \((x, y, z)\) is the position coordinates of the underwater operating equipment to be solved.

![Figure 1. Long baseline positioning diagram](image)

According to the three-dimensional space coordinate distance formula (1):

\[
\begin{align*}
    l_1^2 &= x^2 + y^2 + z^2 \\
    l_2^2 &= (x - x_2)^2 + y^2 + (z - d_{21})^2 \\
    l_3^2 &= (x - x_3)^2 + (y - y_3)^2 + (z - d_{31})^2 \\
    l_4^2 &= (x - x_4)^2 + (y - y_4)^2 + (z - d_{41})^2
\end{align*}
\]

(1)

After simplified to obtain the following ternary equation (2):
By solving the matrix, the upper ternary equations can be solved to obtain the position coordinates of the underwater working equipment.

The above equation is rewritten in matrix form:

\[
\begin{bmatrix}
    x_2 & y_2 & z_2 \\
    x_3 & y_3 & z_3 \\
    x_4 & y_4 & z_4
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
= \begin{bmatrix}
    l_1^2 - l_2^2 + d_{21}^2 \\
    l_1^2 - l_3^2 + d_{31}^2 \\
    l_1^2 - l_4^2 + d_{41}^2
\end{bmatrix}
\]

Hypothesis:

\[
A = \begin{bmatrix}
    x_2 & y_2 & z_2 \\
    x_3 & y_3 & z_3 \\
    x_4 & y_4 & z_4
\end{bmatrix}
\]

Both sides are simultaneously multiplied by the inverse matrix of \( A \), and the result is:

\[
A^{-1} A \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
    l_1^2 - l_2^2 + d_{21}^2 \\
    l_1^2 - l_3^2 + d_{31}^2 \\
    l_1^2 - l_4^2 + d_{41}^2
\end{bmatrix}
\]

Therefore, solving \((x, y, z)\) becomes a solution to the inverse matrix of \( A \). According to:

\[
A^{-1} = \frac{A^*}{|A|}
\]

\[
A^* = \begin{bmatrix}
    y_3 z_4 - z_3 y_4 & y_2 z_4 - z_2 y_4 & y_2 z_3 - z_2 y_3 \\
    x_3 z_4 - z_3 x_4 & x_2 z_4 - z_2 x_4 & x_2 z_3 - z_2 x_3 \\
    x_3 y_4 - y_3 x_4 & x_2 y_4 - y_2 x_4 & x_2 y_3 - y_2 x_3
\end{bmatrix}
\]

\[
|A| = x_2(y_3 z_4 - z_3 y_4) - x_3(y_2 z_4 - z_2 y_4) + x_4(y_2 z_3 - z_2 y_3)
\]

Substituting \( A^{-1} \) gives the coordinates \((x, y, z)\) of the anchor point.

4. Long baseline synchronization positioning error analysis

Generally, error analysis has two methods: formula method and simulation method. Among them, the formula method is to derive an expression formula of error, and then determine the range of error by calculating the extremum; the simulation method is to traverse the error of each point to form an error distribution map, and judge the error range through the error distribution map. This article uses the latter method.

The principle of simulation error analysis is as follows:

Suppose you have the following expression \((3)\):

\[
y = f(x_1, x_2, ..., x_n)
\]

\( f \) is a function related to \( x_1, x_2, ..., x_n \). When it is necessary to calculate the error caused by \( x_k \), the partial derivative of \( x_k \) is first obtained for the function \( f \). According to the following formula:
\[
\frac{\Delta f(x_1,x_2,...,x_n)}{\Delta x_k} = \frac{\partial f(x_1,x_2,...,x_n)}{\partial x_k} 
\]

Error formula: \[
\Delta f(x_1,x_2,...,x_n) = \frac{\partial f(x_1,x_2,...,x_n)}{\partial x_k} \cdot \Delta x_k
\]

\(\Delta f\) is the error caused by \(x_k\). \(\Delta x_k\) is the measurement error of \(x_k\). This is the principle formula on which the error simulation calculation is based.

4.1. Error analysis model

Theoretically, the array aperture is a square of \(l \times w = 1000m \times 1000m\). The actual layout of the array shape is most likely an irregular quadrilateral. If the error is calculated using this irregular quadrilateral as the matrix, it will inevitably increase the amount of unnecessary calculation. Then, considering the simplified calculation, the irregular quadrilateral is converted into a rectangle as shown in FIG.2 by plane coordinate transformation, that is, the irregular matrix shape is transformed into a rectangular matrix. To further simplify the calculation, the different depths of the four subsea transponders are transformed to the same depth by spatial coordinate transformation. This reduces the spatial three-dimensional coordinate system into a two-dimensional coordinate system.

As shown in Fig. 2, let \(l\) and \(w\) be the length and width of the rectangle. Where \((x_1, y_1) = (0,0)\) is the origin of the coordinates, \(y_2 = 0, x_4 = 0\), ie the x-axis is directed to the submarine transponder No.2 by the submarine transponder No.1 and the submarine transponder is the y-axis Point to the No.4 submarine transponder. \(z_1 = z_2 = z_3 = z_4 = z\), \(x_3 = l, y_3 = w\). According to the distance formula between plane coordinates:

\[
\begin{align*}
(cT_1)^2 &= x^2 + y^2 \\
(cT_2)^2 &= (x - l)^2 + y^2 \\
(cT_3)^2 &= (x - l)^2 + (y - w)^2 \\
(cT_4)^2 &= x^2 + (y - w)^2 
\end{align*}
\]

\(c\) is the sound speed in the water, measured by the sound speed measurement module; \(T_1\) to \(T_4\) are the water sound propagation time of the sync pulse signal to the four transducers.

The simultaneous equations (4) are solved:

\[
\begin{align*}
x &= \frac{1^2 + c^2(T_2^2 - T_3^2)}{2l} \\
y &= \frac{w^2 + c^2(T_2^2 - T_4^2)}{2w}
\end{align*}
\]

Formula (5) is a long-baseline synchronized hydroacoustic localization formula for underwater equipment that is simplified by coordinate transformation.
In the long baseline hydroacoustic localization mode, it is difficult to obtain an accurate error analytic formula, that is, the formula cannot be used to solve the error. Therefore, the error analysis is performed by simulation.

4.2. Error caused by sound velocity measurement

The positioning formula (5) is used to derive the partial velocity of the sound velocity $c$. Get:

$$
\frac{\partial x}{\partial c} = \frac{(T_2^2 - T_1^2) \cdot c}{l},
\frac{\partial y}{\partial c} = \frac{(T_2^2 - T_1^2) \cdot c}{w}
$$

(6)

According to the error analysis principle, the error formula caused by the sound velocity measurement is obtained as follows. Next, the error will be simulated based on the error formula. Multiply the numerator and denominator in the formula by the speed of sound $c$ and rewrite the formula to the following form:

$$
\Delta x = \frac{\partial x}{\partial c} \cdot \Delta c = \frac{(T_2^2 - T_1^2) \cdot c}{l} \cdot \Delta c,
\Delta y = \frac{\partial y}{\partial c} \cdot \Delta c = \frac{(T_2^2 - T_1^2) \cdot c}{w} \cdot \Delta c
$$

(7)

Now take $\Delta x$ as an example to calculate the error. Where $cT_1$ and $cT_2$ are the distances of the underwater operating equipment on the x-axis to the number 1 subsea transponder and to the number 2 subsea transponder. Setting: The current x-axis coordinate of the underwater working equipment is $a$, then:

$$(cT_1)^2 - (cT_2)^2 = 2 \times 1000 \times a - 1000^2$$

Taking a value from 0 to 1000 gives the error distribution on the x-axis. Similarly, the error distribution of the y-axis can be obtained, and then according to the formula (8):

$$
R = \sqrt{\Delta x^2 + \Delta y^2}
$$

(8)

The error distribution on the plane can be calculated, and the result is shown in Figure 3:

**Figure 3.** Error in sound velocity

**Figure 4.** Error in aperture generation

According to the error distribution diagram 3 caused by the sound velocity measurement, the maximum error caused by the sound velocity measurement does not exceed 0.15 m.
4.3. Error caused by aperture measurement

The simplified positioning formula (5) is used to derive partial derivatives for \( l \) and \( w \), After further finishing. It is concluded that the error formula caused by the aperture measurement is (9):

\[
\Delta x = \frac{\partial x}{\partial l} \Delta l = \left(\frac{1}{2} - \frac{c^2(T_1^2 - T_2^2)}{2l^2}\right) \Delta l
\]

\[
\Delta y = \frac{\partial y}{\partial w} \Delta w = \left(\frac{1}{2} - \frac{c^2(T_1^2 - T_2^2)}{2w^2}\right) \Delta w
\]

Following the calculation method of the error caused by the speed of sound, the error formula (9) is calculated point by point, and the obtained simulation result is shown in Figure 4:

It can be seen from the error distribution map that the error value caused by the aperture measurement is about 1.4 m at the maximum.

4.4. Error caused by time measurement

The simplified positioning formula (5) is used to obtain partial derivatives for \( T_1 \), \( T_2 \) and \( T_4 \), and obtain (10):

\[
\Delta x = \frac{\partial x}{\partial T_1} \Delta T_1 = \frac{c^2T_1}{l} \Delta T_1
\]

\[
\Delta y = \frac{\partial y}{\partial T_2} \Delta T_2 = \frac{c^2T_2}{w} \Delta T_2
\]

\[
\Delta x = \frac{\partial x}{\partial T_4} \Delta T_4 = \frac{c^2T_4}{w} \Delta T_4
\]

Taking the error of the x-axis caused by \( T_1 \) as an example, \( \Delta x \) is calculated. The numerator and the denominator are multiplied by the measurement time \( T_1 \) to obtain the expression (11):

\[
\Delta f = \frac{\partial x}{\partial T_1} \Delta T_1 = \left(\frac{cT_1}{l}\right) \Delta T_1
\]

\( cT_1 \) is the distance from the underwater working equipment to the x-axis of the No.1 subsea transponder. The value of \( cT_1 \) is 0 to 1000, and the \( \Delta x \) error distribution caused by \( T_1 \) can be obtained. Similarly, the \( \Delta y \) error distribution caused by \( T_4 \) and the \( \Delta x \) error distribution caused by \( T_2 \) and the \( \Delta y \) error distribution caused by \( T_4 \) can be calculated. Finally, it is also based on formula (8). A plane distribution of the error caused by the time measurement can be obtained. The results of the simulation calculations are shown in Figures 5 and 6:
As shown in Figures 5 and 6, the errors caused by the time measurement do not exceed 0.28 m and 0.2 m.

4.5. Total error distribution
The results obtained by the above various error simulations are only the error distributions caused by the individual parameters. In order to get the total error distribution, the errors caused by each parameter need to be accumulated.

According to formula (13):

\[ e = e_c + e_t + e_{r_1} + e_{r_2} \]

(12)

The total error distribution is obtained as shown in Figure 7: The total error obtained by the formula (13) is at most 1.7 m.

5. Positioning error optimization

5.1. Positioning algorithm optimization and error analysis
Again analyzing Figure 1, the subsea location area map is divided into the form of Figure 8. It is found that the error distribution is decremented from the No.1 area to the No.3 area, and the No.2 area and the No.4 area are symmetrically distributed.
Therefore, the error analysis is performed separately for the four regions 1, 2, 3, and 4. Since No. 2 and No. 4 are symmetrically distributed, only one error simulation map is made. The results of the individual area error simulation are shown in Figures 9, 10, and 11. The analysis found that only the area 3 error distribution is the smallest, and the maximum error is only 0.9m.

Although the error falls below the required range at this time, some improvements to the algorithm are required. This improved method is to transform the origin of the array. Because the above analysis shows that the area 1 error is the largest, and the area 3 error is the smallest. When performing the positioning calculation, first determine which area the underwater working equipment is located: if it is located in the area No.3, directly perform the positioning calculation; otherwise, by the origin conversion, the area where the underwater working equipment is located is transformed into the area No.3, and here Make more accurate positioning calculations.

5.2. Depth error optimization
There is a depth inconsistency in the submarine array. Therefore, accurate measurement of depth is required. The output of the depth sensor is typically an analog quantity. Assume that the output of the depth sensor is an analog voltage of 0 to 10 V, the corresponding pressure range is 0 MPa to 100 MPa, and 1 mV corresponds to a water depth of 1 m. Since the signal processing circuit is affected by noise, the conversion depth error can reach 10 meters according to the noise amplitude ±5mVpp, which has a
great influence on the positioning accuracy, and the depth data calibration must be performed.

Depth data calibration considers segmentation compensation techniques. The so-called segmentation compensation is to have different sensor magnifications in different depth segments, and establish the model formula (14):

\[ D = K_1 T_1 + K_2 T_2 + \ldots + K_n T_n + K_0 \]  

Where: \( D \) is the actual water depth after calibration; \( K_1 \) to \( K_n \) are segmentation calibration coefficients; \( T_1 \) to \( T_n \) are segmented original depth data; \( K_0 \) is the zeroing initialization parameter.

Depth sensor signal conditioning calibrates the sensor depth data and segments it according to integer parameters (such as 0V, 1V, ..., 10V) to reduce the impact of noise interference on accuracy. In addition to the segmentation compensation technology, the power supply and signal of the signal processing circuit can be effectively isolated, thereby effectively improving the depth measurement accuracy. Therefore, it is assumed that the depth measurement error of the seafloor transponder at this time is \( \Delta D = 1 \text{m} \).

6. Conclusion

In this paper, through the comparison of several conventional underwater acoustic positioning modes and the MATLAB simulation analysis of the long-baseline underwater acoustic positioning error, and through the optimization of the positioning algorithm, the improvement of the depth measurement accuracy, the base array self-calibration and the optimization of the positioning signal system, thereby greatly improving the accuracy of underwater acoustic positioning. Ensure that the positioning error still meets the engineering operation requirements of \(<1\text{m}\) even when the background noise interference cannot be filtered out, and the accuracy can be further improved after the subsequent filtering processing of the position data.

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