Determination of $Z'$ Gauge Couplings to
Quarks and Leptons at Future Hadron Colliders

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ABSTRACT
We point out that at future hadron colliders the ratio of cross sections for $pp \to Z' \to \ell^+\ell^-$ in two rapidity bins is a useful probe of the relative couplings of the $Z'$ to $u$ and $d$ quarks. Combined with the forward-backward asymmetry, the rare decay modes $Z' \to W\ell\nu_\ell$, and three associated productions $pp \to Z'V$ ($V = Z, W, \gamma$), and assuming inter-family universality, small $Z - Z'$ mixing, and the $Z'$ charge commuting with the $SU_{2L}$ generators, three out of four normalized couplings could be extracted. An analysis of the statistical uncertainties expected for the above probes at the LHC for typical models with $M_{Z'} \simeq 1$ TeV shows that one lepton coupling and two combinations of quark couplings could be determined to around 5%, 20%, and 30%, respectively. This allows for a clear distinction between models.

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A heavy gauge boson $Z'$ could be produced and clearly detected via leptonic decays $pp \rightarrow Z' \rightarrow \ell^+\ell^- \ (\ell = e, \mu)$ at the LHC and SSC if its mass does not exceed around 5 TeV. The immediate goal after the discovery of a new gauge boson would be to understand its origin and properties, including its couplings to ordinary fermions, the nature of the symmetry breaking, and its couplings to exotic fermions and supersymmetric partners.

Recently there has been a renewed interest in diagnostic probes of the couplings of possible heavy $Z'$ gauge bosons to ordinary fermions at future hadron colliders. The forward-backward asymmetry in the main production channel $pp \rightarrow Z' \rightarrow \ell^+\ell^- \ (\ell = e \ or \ \mu)$ has long been known to be useful. It is now understood, however, that several complementary probes would be useful for $M_{Z'} < (1 - 2) \text{TeV}$. In particular, rare decays were recognized and studied in detail. Such decays involve $Z' \rightarrow f_1\overline{f}_2V$, with ordinary gauge bosons $V = (Z, W)$ bremsstrahled from one of the fermionic ($f_{1,2}$) legs. A background study of such decays revealed that the only useful mode without large standard model and QCD backgrounds is $Z' \rightarrow W\ell\nu_\ell$ and $W \rightarrow \text{hadrons}$, with the imposed cut $m_{T\ell\nu_\ell} > 90 \text{ GeV}$ on the transverse mass of the $\ell\nu_\ell$. (This assumes that there is a sufficiently high efficiency for the reconstruction of $W \rightarrow \text{hadrons}$ in events tagged by an energetic lepton.) The same mode with $W \rightarrow \ell\nu_\ell$ may also be detectable if appropriate cuts are applied. These modes probe a particular combination of $Z'$ gauge couplings to leptons.

Associated productions $pp \rightarrow Z'V$ with $V = (Z, W)$ and $Z' \rightarrow \ell^+\ell^-$ were recently proposed to probe the gauge couplings to quarks, and are thus complementary to rare decays. The associated $Z'$ production with $V = \gamma$ was also
proposed.\(^\text{10}\)

Rare decays and associated production involve processes with four-fermion final states, and thus have suppressed rates compared to the main production channels \(pp \rightarrow Z' \rightarrow \ell^+ \ell^-\).

In this paper we point out that due to the harder valence \(u\)-quark distribution in the proton relative to the \(d\)-quark, the ratio \(r_{y_1}\) of production cross-sections in the two rapidity bins (\(|y| = \{0, y_1\}\) and \(|y| = \{y_1, y_{\text{max}}\}\)) is a useful complementary probe for separating the \(Z'\) couplings to the \(u\) and \(d\) quarks. We choose \(y_1\) in such a way that each bin has a comparable number of events in order to minimize the statistical error.

Another purpose of this paper is to examine how well the various \(Z'\) couplings could be extracted from the above six signals at future colliders. For definiteness, we consider the statistical uncertainties for a 1 TeV \(M_{Z'}\) at the LHC with a projected luminosity of \(10^{34}\text{cm}^{-2}\text{s}^{-1}\). At the SSC with \(10^{33}\text{cm}^{-2}\text{s}^{-1}\) one expects about half as many events. Eventually, the uncertainties associated with the detector acceptances and systematic errors will have to be taken into account.

**Formalism and Typical Models.** The neutral current gauge interaction term in the presence of an additional \(U_1\) is

\[
- L_{NC} = eJ_{em}^\mu A_\mu + g_1J_1^\mu Z_{1\mu} + g_2J_2^\mu Z_{2\mu},
\]

with \(Z_1\) being the \(SU_2 \times U_1\) boson and \(Z_2\) the additional boson in the weak eigenstate basis. Here \(g_1 \equiv \sqrt{g_L^2 + g_Y^2} = g/\cos \theta_W\), where \(g_L, g_Y\) are the gauge couplings of \(SU_{2L}\) and \(U_{1Y}\), and \(g_2\) is the gauge coupling of \(Z_2\). The currents are:

\[
J_j^\mu = \frac{1}{2} \sum_i \bar{\psi}_i \gamma^\mu \left[ \hat{g}_V^{ij} - \hat{g}_A^{ij} \gamma_5 \right] \psi_i, \quad j = 1, 2,
\]

where the sum runs over fermions, and the \(\hat{g}^{ij}_{V,A}\) are the vector and axial vector couplings of \(Z_j\) to the \(i^{\text{th}}\) flavor. Analogously,

\[
\hat{g}^{ij}_{L,R} = \frac{1}{2} (\hat{g}_V^{ij} \pm \hat{g}_A^{ij} ).
\]
We consider the following typical GUT, left-right symmetric, and superstring-motivated models: (i) $Z_\chi$ occurs in $SO_{10} \to SU_5 \times U_{1\chi}$, (ii) $Z_\psi$ occurs in $E_6 \to SO_{10} \times U_1$, (iii) $Z_\eta = \sqrt{3/8}Z_\chi - \sqrt{5/8}Z_\psi$ occurs in superstring inspired models in which $E_6$ breaks directly to a rank 5 group, (iv) $Z_{LR}$ occurs in left-right (LR) symmetric models. Here we consider the special value $\kappa = g_R/g_L = 1$ of the gauge couplings $g_{L,R}$ for $SU_{2L,2R}$, respectively.

In the rest of the paper we assume family universality and neglect $Z - Z'$ mixing (as suggested from experiments). We also assume $[Q', T_1] = 0$, where $Q'$ is the $Z'$ charge and $T_1$ are the $SU_{2L}$ generators, which holds for $SU_2 \times U_1 \times U'_1$ and LR models. The relevant quantities\(^9\) to distinguish different theories are the charges, $\hat{g}_L^u = \hat{g}_L^d \equiv \hat{g}_L^e$, $\hat{g}_R^u$, $\hat{g}_R^d = \hat{g}_L^f$, $\hat{g}_R^f$, and the gauge coupling strength $g_2$. The overall scale of the charges (and $g_2$) depends on the normalization convention for $\text{Tr}(Q'^2)$, but the ratios characterize particular theories. The signs of the charges will be hard to determine at hadron colliders. Some information is possible in principle from $\gamma$ and $Z$ interference effects, but this is expected to be small. Other possibilities include precision experiments and possible future $e^+e^-$ colliders. We therefore concentrate only on the four “normalized” observables:\(^9\)

\[
\gamma_L^\ell = \frac{(\hat{g}_L^\ell)^2}{(\hat{g}_L^f)^2 + (\hat{g}_L^R)^2}, \quad \gamma_L^q = \frac{(\hat{g}_L^q)^2}{(\hat{g}_L^f)^2 + (\hat{g}_L^R)^2}, \quad \hat{U} = \left(\frac{\hat{g}_R^u}{\hat{g}_L^f}\right)^2, \quad \hat{D} = \left(\frac{\hat{g}_R^d}{\hat{g}_L^f}\right)^2.
\]

The values of $\gamma_L^\ell$, $\gamma_L^q$, $\hat{U}$, and $\hat{D}$ for models (i)-(iv) are listed in Table I.

**Rapidity Ratio.** In the main production channels $pp \to Z' \to \ell^+\ell^- \ (\ell = e, \mu)$ we define the ratio:

\[
r_{y_1} = \frac{\int_{-y_1}^{y_1} \frac{d\sigma}{dy} dy}{\left(\int_{-y_{\text{max}}}^{y_{\text{max}}} - \int_{y_{\text{min}}}^{y_{\text{max}}} \frac{d\sigma}{dy} dy\right)}, \quad \frac{\int_{-y_1}^{y_1} [F(y) + B(y)] dy}{\left(\int_{-y_{\text{max}}}^{y_{\text{max}}} [F(y) + B(y)] dy\right)},
\]

\(^{1}\)
where $F(y) \pm B(y) = [\int_{0}^{1} \pm \int_{-1}^{0}] d \cos \theta (d^{2} \sigma / dy \ d \cos \theta)$ and $\theta$ is the $\ell^{-}$ angle in the $Z'$ rest frame. The rapidity range is from $\{-y_{\text{max}}, y_{\text{max}}\}$. $y_{1}$ is chosen in a range $0 < y_{1} < y_{\text{max}}$ so that the number of events in the two bins are comparable. At the LHC $y_{\text{max}} \simeq 2.8$ for $M_{Z'} \simeq 1$ TeV, and $y_{1} = 1$ turns out to be an appropriate choice.

$r_{y_{1}}$ can be expressed in terms of $\tilde{U}$ and $\tilde{D}$. The expression for $M_{Z'} = 1$ TeV at the LHC is given in the first line of the Table II, using the quark distributions of Ref.$^{15}$. This expression and those for the other probes are adequate for illustration. The use of other structure functions leads to somewhat different expressions. If a $Z'$ is actually observed it would be necessary to recalculate the expressions using updated distribution functions (which should by then be known to a few%), and QCD corrections to the $Z'$ production would have to be included.

The numerator and denominator involve different combinations of $\tilde{U}$ and $\tilde{D}$, reflecting the harder distribution of valence $u$ quarks. In particular, the dependence on $\tilde{U}$ and $\tilde{D}$ is sufficiently different from that of the forward-backward asymmetry $A_{FB}$ (see the second line$^{16}$ of Table II),$^{9}$ that $r_{y_{1}}$ provides a complementary probe. For the typical models described here the values for $r_{y_{1}}$ and their statistical errors$^{17}$ (for $Z' \rightarrow [e^{+}e^{-} + \mu^{+}\mu^{-}]$) are given for $M_{Z'} = 1$ TeV at the LHC in the first line of Table II. The statistical errors are sufficiently small for $r_{y_{1}}$ to be useful for distinguishing between the models.

Another potential possibility is the ratio of forward-backward asymmetries in the two rapidity bins. We define:

$$A_{FB_{y_{1}}} = \frac{(\int_{0}^{y_{1}} - \int_{0}^{y_{1}})[F(y) - B(y)]dy}{(\int_{y_{1}}^{y_{\text{max}}} - \int_{y_{1}}^{y_{\text{max}}})[F(y) - B(y)]dy}, \quad (3)$$

where $F(y), B(y)$ are defined after Eq.(2). $A_{FB_{y_{1}}}$ can be viewed as “a refinement
of a refinement” in the main production channel, since it involves the angular distribution of $\ell^\pm$ as well as the rapidity distribution.\(^{18}\) The expression for $A_{FB_{\nu_1}}$ is given in the third line of Table II. From the expression and the explicit numerical values for typical models (see Table II) it is apparent that $A_{FB_{\nu_1}}$ is not a sensitive enough function of the gauge couplings to provide useful information for the projected luminosities.

For completeness in Table II we also quote results for the rare decay mode and the associated productions. For the “gold-plated” events\(^{19}\) $Z' \to W\ell\nu_\ell$ and $W \to \text{hadrons}$ the ratio\(^ {20}\) $r_{\ell\nu W} \equiv \frac{B(Z'\to W\ell\nu_\ell)}{B(Z'\to \ell^+\ell^-)}$ is defined, in which one sums over $\ell = e, \mu$ and over $W^+, W^-$. $r_{\ell\nu W}$ is rewritten in terms of the gauge couplings in the the fourth line of Table II,\(^ {21}\) along with the values and statistical error bars for the typical models. It is apparent that this decay is an excellent probe of $\gamma_L^\ell$.

For the associated productions one defines\(^ {9}\) the ratios: $R_{Z'V} = \frac{\sigma(pp\to Z'V)B(Z'\to \ell^+\ell^-)}{\sigma(pp\to Z')B(Z'\to \ell^+\ell^-)}$ with $V = (Z,W)^9$ and $V = \gamma^{10}$ decaying into leptons and quarks, and $\ell$ includes both $e$ and $\mu$. The expressions\(^ {22}\) and values for these ratios are given in Table II for $M_{Z'} = 1$ TeV at the LHC. (For $R_{Z'\gamma}$ a transverse momentum cut $p_{T\gamma} > 50$ GeV is imposed.) The ratios $R$ yield direct information on the couplings of quarks to $Z'$. $R_{Z'(Z,W)}$ primarily single out a combination $\sim 2\bar{U} + \bar{D}$, which is the same quantity that is probed by $A_{FB}$ (for a known $\gamma_L^\ell$), but the extra information provides a welcome consistency check. The numerator in the expression for $R_{Z'Z}$ has a weak dependence on $\bar{U}$ and $\bar{D}$, due to the fact that they are weighted by the squares of the (small) gauge couplings of the right-handed quarks to the $Z$. $R_{Z'\gamma}$ has a strong dependence on $\bar{U}$ in the numerator. For the typical models (except LR) $\bar{U} = 1$, and thus $R_{Z'\gamma}$ by itself is not a useful discriminant between these models.

Within the assumptions of interfAMILY universality, negligible $Z - Z'$ mixing,
and \([Q', T_i] = 0\) one sees from Table II that the six quantities \(r_{y_1}, A_{FB}, r_{\ell \nu W},\) and \(R_{Z'V}\) with \((V = Z, W, \gamma)\) yield significant information on three \((\gamma^\ell_L, \tilde{U} \text{ and } \tilde{D})\) out of four normalized gauge couplings\(^{23}\) of ordinary fermions to the \(Z'\). The relative size of the \(Z'\) couplings to quarks and leptons could be determined by a measurement of the branching ratio \(B(Z' \to q\bar{q})\). In particular, the ratio \(\frac{1}{3} \frac{B(Z' \to q\bar{q})}{B(Z' \to \ell^+\ell^-)} = \gamma^q_L (2 + \tilde{U} + \tilde{D})\) (counting all 3 families) would yield the left-handed quark coupling \(\gamma^q_L\).

However, this appears difficult.\(^{24}\)

\textit{Determination of the Couplings.} To study with what precision these couplings could be determined, we have performed a combined \(\chi^2\) analysis of the observables \(r_{y_1}, A_{FB}, r_{\ell \nu W},\) and \(R_{Z'V}\) with \(V = (Z, W, \gamma)\) for each of the models. We have included only the statistical uncertainties (from Table II), and have ignored correlations between the observations.\(^{25}\) The resulting uncertainties for the couplings are given in Table I.

In particular, \(\gamma^\ell_L\) can be determined very well (between 2\% and 8\% for the \(\chi, \psi,\) and \(\eta\) models), primarily due to the small statistical error for the rare decay mode \(Z' \to W\ell\nu_\ell\). On the other hand the quark couplings have larger uncertainties, typically 20\% for \(\tilde{U}\), and an absolute error of \(\sim 0.3 - 0.6\) for \(\tilde{D}\) (except \(Z_{LR}\)).

From the explicit dependence of the probes on \(\gamma^\ell_L, \tilde{U}\) and \(\tilde{D}\) (see Table II) one sees that the correlation between \(\tilde{U}\) and \(\tilde{D}\) is appreciable, while \(\gamma^\ell_L\) is weakly correlated with \(\tilde{U}\) and \(\tilde{D}\) because of the small statistical error on \(r_{\ell \nu W}\), which singles out \(\gamma^\ell_L\). Explicit calculation shows that this is the case for all the models studied except for \(Z_{\chi}\). In this case the statistical errors on \(r_{\ell \nu W}\) and \(A_{FB}\) (which depends on all three variables) are comparable, inducing sizable correlations. The fitted correlation coefficient between \(\tilde{U}\) and \(\tilde{D}\) is given for each model in the last line of Table I.
In Figs. 1a, 1b and 1c the $1\sigma (\Delta \chi^2 = 1)$ and 90\% confidence level ($\Delta \chi^2 = 4.6$) contours are plotted for $\tilde{D}$ versus $\gamma_L^U$, $\tilde{U}$ versus $\gamma_L^L$, and $\tilde{D}$ versus $\tilde{U}$, respectively. The statistical error bars are for $M_{Z'} = 1$ TeV at the LHC for the $\eta$, $\psi$ and $\chi$ models. The $LR$ model has $\tilde{U}$ and $\tilde{D}$ in different region of parameter space (see Table I). From Figures 1a and 1b it is clear that one can distinguish well between different models. In Figure 1c the correlations between $\tilde{U}$ and $\tilde{D}$ are evident, while from Figures 1a and 1b the correlation between $\gamma_L^U$ and $(\tilde{U}, \tilde{D})$ is significant only for the $Z_\chi$.

**Conclusions.** In this note we have explored possible experimental signals which probe $Z'$ gauge couplings to ordinary fermions at hadron colliders. In addition to the forward-backward asymmetry $^1 A_{FB}$, and the more recently proposed rare decay modes $Z' \to W\ell\nu_\ell$ $^5$ and associated productions $pp \to Z'V$ with $V = Z, W$ $^9$ and $V = \gamma$ $^10$, we point out that the ratio $r_{y_1}$ of the cross-sections in the two rapidity bins in the main production channels is a useful complementary probe of the relative $Z'$ couplings to $u$ and $d$ quarks.

To test the sensitivity of the six proposed signals we express them in terms of the normalized gauge couplings to quarks and leptons; as an example we chose $M_{Z'} = 1$ TeV at the LHC. The error analysis shows that under the assumptions of family universality, small $Z - Z'$ mixing, and $[Q', T_i] = 0$ the magnitude of three out of four gauge couplings could be determined with precisions of around 5\% for $\gamma_L^U$, around 20\% for $\tilde{U}$, and somewhat higher for $\tilde{D}$, allowing for the clear identification of particular models.

For higher $Z'$ masses the number of events drops rapidly. For $M_{Z'} = 2$ TeV, the statistical errors on $r_{y_1}$, $A_{FB}$, and $r_{\ell\nu W}$ increase by a factor of 4, while those on $R_{Z'V}$ increase by a factor of 3. From Tables I and II we see that reasonable
discrimination between models and determination of the normalized parameters is still possible. However, for $M_{Z'} = 3$ TeV the statistical errors on the first three quantities are by a factor of 13 larger than for 1 TeV, and there are not enough events expected for $R_{Z'V}$ to allow a meaningful measurement. For $M_{Z'} \geq 3$ TeV, there is therefore little ability to discriminate between models.

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Table I

| $\gamma^\ell_L$ | $\chi$   | $\psi$   | $\eta$   | $LR$  |
|----------------|----------|----------|----------|-------|
| $0.9 \pm 0.018$ | $0.5 \pm 0.03$ | $0.2 \pm 0.015$ | $0.36 \pm 0.007$ |       |
| $0.1$          | $0.5$    | $0.8$    | $0.04$   |       |
| $1 \pm 0.18$   | $1 \pm 0.27$ | $1 \pm 0.14$ | $37 \pm 8.3$ |       |
| $9 \pm 0.61$   | $1 \pm 0.41$ | $0.25 \pm 0.29$ | $65 \pm 14$ |       |
| $-0.19$        | $-0.24$  | $-0.66$  | $0.93$   |       |

Table I Values of $\gamma^\ell_L$, $\gamma^q_L$, $\bar{U}$, and $\bar{D}$ for the $\chi$, $\psi$, $\eta$, and $LR$ models. The error bars indicate how well the coupling could be measured at the LHC for $M_{Z'} = 1$ TeV. $\rho_{ud}$ indicates the correlation coefficient between $\bar{U}$ and $\bar{D}$. Except for the $\chi$ model the correlation between $\gamma^\ell_L$ and ($\bar{U}$, $\bar{D}$) are negligible.

Table II

| $r_{y_1}$                   | $\chi$   | $\psi$   | $\eta$   | $LR$  |
|-----------------------------|----------|----------|----------|-------|
| $1.55^{+0.64 \bar{U} + 0.36 \bar{D}}_{-0.73 \bar{U} + 0.27 \bar{D}}$ | $1.79 \pm 0.02$ | $1.55 \pm 0.04$ | $1.49 \pm 0.03$ | $1.62 \pm 0.014$ |
| $10^{-3} \frac{7.94 + 0.96 \bar{U} + 0.11 \bar{D}}{1 + 0.68 \bar{U} + 0.32 \bar{D}}$ | $0.060 \pm 0.0014$ | $0.034 \pm 0.002$ | $0.013 \pm 0.001$ | $0.024 \pm 0.0008$ |
| $R_{Z'W} = 10^{-3} \frac{25.7}{1 + 0.68 \bar{U} + 0.32 \bar{D}}$ | $0.0056 \pm 0.0004$ | $0.013 \pm 0.001$ | $0.015 \pm 0.001$ | $0.00055 \pm 0.00010$ |

Table II. The quantities $r_{y_1}$, $A_{FB}$, $A_{FB_{\mu \gamma}}$, $r_{\ell\nu W}$, and $R_{Z'V}$ with $V = (Z, W, \gamma)$ and their numerical values (with statistical errors) for $M_{Z'} = 1$ TeV at the LHC. Error bars for $r_{y_1}$, $r_{\ell\nu W}$, $R_{Z'V}$ are for $e + \mu$ channels, while $A_{FB}$ and $A_{FB_{\mu \gamma}}$ are
for $e$ or $\mu$. 
Figure caption

Figure 1. 1 $\sigma$ ($\Delta \chi^2 = 1$) contours (solid lines) and the 90% confidence level ($\Delta \chi^2 = 4.6$) contours (dotted lines) for the $\chi$, $\psi$ and $\eta$ models are plotted for $\tilde{D}$ versus $\gamma^f_L$ (Figure 1a), $\tilde{U}$ versus $\gamma^f_L$ (Figure 1b), and $\tilde{D}$ versus $\tilde{U}$ (Figure 1c). The input data are for $M_{Z'} = 1$ TeV at the LHC and include statistical errors only.
REFERENCES

1. P. Langacker, R. Robinett, and J. Rosner, Phys. Rev. D 30, 1470 (1984).

2. V. Barger, et al., Phys. Rev. D 35, 2893 (1987).

3. L. Durkin and P. Langacker, Phys. Lett. B166, 436 (1986); F. del Aguila, M. Quiros, and F. Zwirner, Nucl. Phys. B 287, 419 (1987); 284, 530 (1987); J. Hewett and T. Rizzo in Proceedings of the 1988 Snowmass Summer Study on High Energy Physics in the 1990’s, Snowmass, CO 1988; P. Chiappetta et al., in the Proceedings of the Large Hadron Collider Workshop, Aachen, Germany, 1990; J. Hewett and T. Rizzo, Phys. Rep. 183, 193 (1989).

4. J. Hewett and T. Rizzo, MAD/PH/649/91.

5. M. Cvetič and P. Langacker, Phys. Rev. D 46, R14 (1992).

6. M. Cvetič, B. Kayser, and P. Langacker, Phys. Rev. Lett. 68, 2871 (1992).

7. F. del Aguila, B. Alles, Ll. Ametller and A. Grau, University of Granada preprint, UG-FT-22/92/Rev (December 1992).

8. J. Hewett and T. Rizzo, Argonne National Laboratory preprint, ANL-HEP-PR-92-33 (June 1992).

9. M. Cvetič and P. Langacker, Phys. Rev. D46, 4943 (1992).

10. T. Rizzo, Phys. Rev. D47, 965 (1993).

11. See also F. del Aguila and J. Vidal, Int. Journal of Math. Phys. A4, 4097 (1989).

12. T. Rizzo, Phys. Lett. B 192, 125 (1987).
13. The $\tau$ polarization in $pp \rightarrow Z' \rightarrow \tau^+\tau^-$ would be another useful probe if it can be measured. See J. Anderson, M. Austern, and B. Cahn, Phys. Rev. D46, 290 (1992), Phys. Rev. Lett. 69, 25 (1992). Similarly, if proton polarization were available the measurements of the corresponding asymmetries in $pp \rightarrow Z' \rightarrow \ell^+\ell^-$ would also be useful. See A. Fiandrino and P. Taxil, Phys. Lett. B292, 242 (1992) and references therein.

14. P. Langacker and M. Luo, Phys. Rev. D 44, 817 (1991).

15. E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. 56, 579 (1984).

16. In Ref. 9 the coefficient $0.38 \equiv \frac{3}{4} \times 0.51$ was erroneously quoted to be $\frac{3}{4} \times 0.58$.

17. The statistical errors are based on the same (illustrative) branching ratio assumptions as in Ref. 5.

18. The quantities $r_{y1}$, $A_{FB}$, and $A_{FB y1}$ are useful for displaying the dependences of the production distribution on the underlying couplings. In practice, however, it would be more efficient to directly fit to the observed distribution in $y$ and $\theta$, folding in the detector acceptances and other systematic uncertainties.

19. The only other signal which does not suffer from a major standard model or QCD background is $Z' \rightarrow Z\ell^+\ell^-$. This, however, turns out not to be a useful diagnostic probe. See Refs. 5 and 9 for details.

20. This ratio is independent of the branching ratio $B(Z' \rightarrow \ell^+\ell^-)$ and therefore probes the gauge couplings of the ordinary fermions only.

21. The coefficient is $0.067 \equiv 0.77 a_W$, where $a_W \approx \frac{\alpha}{6\pi \cos \theta_W \sin \theta_W} \left[ \ln^2 \mu + 3 \ln \mu + 5 - \frac{\pi^2}{3} \right]$ is a kinematic factor which only depends on $\mu = M_{W}^{2}/M_{Z'}^{2}$. For $\alpha = 1/128$ and $M_{Z'} = 1$ TeV, $a_W = 0.087$. In Ref. 9 $\alpha = 1/137$ and thus $a_W = 0.080$.
were used.

22. In Ref. \(^9\) slightly different numerical values for \(R_{Z'Z} \) and \(R_{Z'W} \) were given, mainly due to less accurate numerical integrations.

23. The overall normalization is not fixed by the ratios. It could be determined by an independent measurement of the \(Z' \) width.

24. A. Henriques and L. Poggioli, ATLAS Collaboration, Note PHYS-NO-010 (October 1992); T. Rizzo, ANL-HEP-PR-93-18 (March 1993). See also P. Mohapatra, Univ. of Colorado preprint 92-0580 (June 1992).

25. In principle there is a correlation between the errors of the input quantities \(A_{FB} \) and \(r_{y_1} \), since they both depend on four quantities \(F_1 \equiv \int_{0}^{y_1} F(y)dy + \int_{-y_1}^{0} B(y)dy, F_2 \equiv \int_{y_1}^{y_{max}} F(y)dy + \int_{-y_{max}}^{-y_1} B(y)dy, \) and similar definitions for \(B_{1,2} \). However, explicit calculation yields the correlation \(\rho = [\sqrt{r_{y_1}/(1 + r_{y_1})}] \times (A_{FB_1} - A_{FB_2})/\sqrt{1 - A_{FB}^2}. \) Namely, it is proportional to a difference of forward-backward asymmetries \(A_{FB_{1,2}} \equiv (F_{1,2} - B_{1,2})/(F_{1,2} + B_{1,2}) \) in the two rapidity bins. \(A_{FB_{1,2}} \) and their differences turn out to be numerically small, and therefore the correlation is negligible.