Temperature dependent polarization of the thermal radiation emitted by thin, hot tungsten wires

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Abstract. We report measurements of the temperature $T$ dependence of the linear polarization $\langle P \rangle$ of the thermal radiation emitted by thin, incandescent tungsten wires. We investigate an interval ranging from a little above room temperature up to melting, $T_m = 3695$ K. These are the first measurements in such wide a range. We found that $\langle P \rangle$ decreases with increasing temperature. We obtained a satisfactory agreement with the theoretical predictions based on the Kirchhoff’s law by using a Drude-type formula for the optical properties of tungsten. This formula was tested and its parameters were assessed as valid for $T \leq 2400$ K and for wavelengths in the range from visible up to $\lambda \approx 2.6 \mu$m. We have extended the range of validity of this formula for $T$ up to $T_m$ and for $\lambda$ up to $\approx 12 \mu$m.

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1. Introduction

The study of thermal emission by hot bodies is a very important topic because the celebrated Planck’s result about the spectrum of a blackbody radiator paved the way for the development of Quantum Mechanics [1]. Planck’s law is independent of the characteristics of the blackbody material and it only depends on temperature $T$, thus making pyrometry a universal thermometric technique [2].

According to Planck’s derivation, the blackbody emission consists of unpolarized, incoherent radiation for bodies whose size is larger than the typical thermal wavelength, $\lambda_T = \frac{hc}{k_B T}$, where $h$, $c$ and $k_B$ are the Planck’s constant, speed of light, and Boltzmann’s constant, respectively.

The modern availability of radiators of dimensions comparable to, if not even smaller than $\lambda_T$ has led to the discovery that thermal radiation shows a high degree of spatial and temporal coherence in the near-field region [3, 4, 5]. Suitable subwavelength patterning of the properties of metallo-dielectric surfaces at nanoscale leads to coherence properties of the thermal emission of such nanoheaters, including carbon nanotubes [6, 7, 8, 9], that have great relevance in applied physics and engineering [10, 11, 12, 13, 14].

Early, though quite inaccurate, measurements with hot, a few $\mu m$ thick, W- [15] and Ag [16] wires have shown that thermal radiation has a high degree of linear polarization, up to $\approx 30\%$ and more, orthogonal to the wire axis. The observed polarization was explained in terms of plasma oscillations of the electron gas in the metal that can scatter, absorb, and emit light. More recently, the degree of linear polarization of incandescent W wires of diameter 5 $\mu m$ to 100 $\mu m$ has been measured in the visible region [17]. This study has confirmed the early observations that the light has a degree of polarization in excess of 20%, directed perpendicularly to the wire’s axis. Unfortunately, no attempt was done to measure the wire temperature, although it was estimated to be around 2400 K.

In those studies, the wire thickness was $r \gtrsim \lambda_T$. More recent studies on wires with $r \lesssim \lambda_T$ have shown that the emitted radiation is polarized along the wire axis, becoming fully polarized as $r \to 0$ [13, 18]. The observation that standing waves of thermally generated charge oscillations in the near field occur across metallic stripes a few $\mu m$ wide has led to the explanation of the increased polarization as a manifestation of charge confinement and correlated charge fluctuations along the long axis of the nanoheater [19]. Surface plasmon polaritons propagate only in the direction of charge oscillations. So, charge oscillations driven by the thermal environment are affected in different ways whether they are parallel or perpendicular to the heater axis when $r$ is shrunken [18]. When $r < \lambda_T$, longitudinal charge fluctuations are strongly correlated by the coupling with surface plasmons and light is polarized along the heater long axis. For heaters widths $r \geq \lambda_T$ transversal charge oscillations get correlated via the interaction with surface plasmons and the emitted light turns out to be polarized perpendicular to the heater axis. Actually, a rotation of the linear polarization of light emitted by Pt...
nanoheaters has been observed when their width changes from submicron- to micron size or when $T$ is changed, the crossover occurring at a width $r$ satisfying the criterion $2\pi r/\lambda_T \sim 1.5$ [14].

In these latter studies, the nature of the nanoheaters material is not really important as only the ratio $r/\lambda_T$ determines the direction of light polarization. However, the coupling with surface plasmons is ruled by the properties of the dielectric constant of the material [20], which depends on $T$ and on the nature of the metal [21]. More generally, the optical properties of materials, including optical constants and emissivity, do depend on $T$.

Actually, a theoretical study addresses also the issue of how the optical properties of the material, not only its size, influence the features of the radiation emitted by long cylinders [22]. In particular, it is shown that the polarization curves for W may shift by a factor of 10 when $T$ is changed from 300 K to 2400 K.

In this paper, we report measurements of the degree of linear polarization of the light emitted by W wires heated by Joule effect in a temperature range from room-up to melting temperature in a wavelength band across the infrared and visible region. Two types of wires, of radius $r = 9 \mu m$ and $25 \mu m$, respectively, are investigated. Their size is such that the light emitted is always polarized perpendicularly to their axis, so that the variation of the degree of polarization can solely be ascribed to the temperature dependence of the optical properties of tungsten.

The paper is organized as follows: in Sect. 2 we describe the experimental apparatus. In Sect. 3 we present the experimental data and compare them with the theoretical predictions. Finally, the conclusions are drawn in Sect. 4.

2. Experimental Details

In this section we describe the experimental apparatus: at first, the mechanical and optical setup and, then, the electronics needed to power the wires and to reveal and analyze the detector signal.

2.1. Mechanical and Optical Setup

The experimental apparatus is schematically shown in Fig. 1. The tungsten wires (HW), supplied by LUMA (9 \mu m) and SIT (25 \mu m), are mounted in parallel inside a 50 cm long metal cylinder with a diameter of 2.5 cm evacuated down to a working pressure $p \leq 10^{-3}$ Pa. The wires are stretched and clamped on supports connected to the electrical power supply by means of suitable vacuum feedthroughs.

The wires used in this experiment are $\approx 7$ mm long and their radius is either 25 \mu m or 9 \mu m. We used four wires at once in order to increase the amount of light impinging on the detector while keeping their electrical resistance at a manageable low value. The wires are mounted with their cylindrical axes perpendicular to the axis of the vacuum cylinder whose internal surface is mat and coated with aquadag in order to
minimize polarized reflections from the inner wall. Three equally spaced optical baffles (ob) consisting of drilled washers with a central hole of $\approx 6$ mm in diameter are located along the optical axis in order to further prevent internally reflected light from reaching the detector and to reduce the contribution of non paraxial rays. The light eventually exits through a ZnSe optical window ($W$) of $\approx 1$ cm in diameter, located at a distance of $\approx 30$ cm from the wires.

Two ZnSe lenses, $L_1$ and $L_2$, image the wires on the liquid $N_2$ cooled, photovoltaic HgCdTe detector (Fermionics, mod. PV-12-0.5) working in the spectral range $2 \mu m \leq \lambda \leq 12 \mu m$, whose active area is a circle of $1 \text{mm}^2$.

The degree of the polarization of the light emitted by the wires is analyzed by means of a ZnSe wire grid, infrared (IR) polarizer (WP25H-Z, Thorlabs) mounted on a rotary frame coupled to a d.c. motor by means of a scaler gear so that it can be continuously rotated about the optical $z$-axis of the system. The rotational period is varied by changing the driving voltage of the d.c. motor. Typically, the polarizer is rotated at an angular speed of $\approx 0.7^\circ/s$. The rotation angle $\theta$ is measured by a digital encoder ($E$) interfaced to a PC.

A second, identical polarizer can be inserted, if necessary, in the optical path in order to verify that no residual light outside the spectral range of the polarizers still reaches the detector. It is also used to determine the direction of the polarization of the emitted light relative to the wire axis. It turns out that the polarization is always directed perpendicularly to that axis.
2.2. Electronics

The electronics required to power the wires and to measure the emitted light is shown in Fig. 2. The light emission is modulated by superimposing a small, low-frequency ($\approx 2$ Hz) a.c. current to a steady d.c. current that sets the average wire temperature. Owing to their negligible thermal inertia, the wire temperature (and emission) instantaneously follows the current changes, whereas the surrounding environment remains at constant temperature because of its huge thermal inertia and its emission does not change as long as the d.c. current in the wires is kept constant. In this way, the wire signal is completely decoupled from the environment contribution and standard lock-in amplification techniques are able to detect it.

In the previous experiment in the visible range [17] light was modulated by means of a mechanical chopper. This technique cannot be exploited in the infrared range, in which all the surfaces emit detectable radiation. In this case, the tiny emitting area of the wires is negligible with respect to the enormously larger area of the chopper blades that would then obscure the wire signal.

In Fig. 2, resistors $R_1 \approx 12$ k$\Omega$ and $R_2 \approx 27$ k$\Omega$, and capacitor $C \approx 6.9 \mu$F accomplish the net for summing together the d.c. voltage $V_{dc}$ supplied by a power supply (Agilent, mod. 3620A) and the a.c. voltage $V_{ac}$ supplied by a signal generator (HP, mod. 3312A). The resistance and capacitance values have been chosen by taking into account the input impedance of the power amplifier $PA$ (Techron, mod. 5515), $R_i \approx 23$ k$\Omega$, thus yielding an effective time constant of the input circuit $\tau \approx 0.16$ s.

The output of $PA$ directly feeds the wires whose resistance ($0.2 \Omega \leq R \leq 1 \Omega$ at room temperature) is measured with the standard Kelvin technique by using the ammeter $A$ (Tektronix, mod. DMM914) and the voltmeter $V$ (Keithley, mod. 195A).

The light emitted by the wires crosses the optical setup and is focused on the photovoltaic detector $MCT$. The alignment of the optical system is achieved by using a

![Figure 2. Electronics of the hot wire experiment. $V_{ac}$ = sinusoidal a.c. voltage generator, $V_{dc}$ = d.c. voltage supply, $PA$ = power amplifier, $A$ = ammeter, $V$ = voltmeter, $MCT$ = HgCdTe photovoltaic detector, $TIA$ = transimpedance amplifier, $LA$ = linear amplifier, $LIA$ = lock-in amplifier.](image)
laser pointer and finely positioning the detector with an $x - y$ translation stage.

The photodiode current is converted to voltage by the transimpedance amplifier TIA (Fermionics, PVA-500-10) whose output is linearly amplified by the amplifier LA (EG&G PARC, mod. 113). The output of LA is fed to the lock-in amplifier LIA (Stanford Research Systems, mod. SR830), whose reference signal is supplied by the signal generator Vac. In order to maximize the LIA output, we manually adjusted the lock-in phase reference as the relative phase of light signal and reference voltage is not known because the working frequency is low and the summing net and the power amplifier introduce a frequency-dependent phase shift, and because the wires act as a natural low-pass filter whose time constant is unknown. We take advantage of the fact that the working frequency is kept constant throughout the experiment and is constantly monitored by a frequency meter (Agilent, mod. 34401A).

3. Experimental Results and Discussion

3.1. Experimental Procedure

When a good vacuum is reached in the vacuum container, the d.c. voltage is set to the desired working point and wires and environment are allowed to reach the working temperature. As the tungsten resistivity depends on $T$ [23], the approach to equilibrium is monitored by recording the wires resistance $R$. Once steady-state is reached, $R$ remains constant.

It has to be noted that it is more appropriate to speak about steady-state conditions rather than equilibrium. Actually, the wires are clamped to supports that are strongly thermally coupled with the environment and the balance between Joule heating and the heat dissipation by thermal conduction over the wire boundaries and by emission of radiation leads to a strongly non uniform temperature profile along the wires. Under steady-state conditions, the temperature profile is still non uniform but does no longer change in time.

When steady state is reached, the wires resistance is measured and the a.c. modulation is turned on so that the modulation signal produced by the detector can be observed and monitored.

3.2. Signal Formation and Analysis

The detector signal is proportional to the intensity of the light emitted by the wires, which, in turn, depends on their temperature that depends on the current flowing in them. Thus, a relationship between the detector signal and the current intensity is needed.

Let us assume for a while that the wires temperature is uniform. This is not a necessary condition for the following arguments and, later, this assumption can be relaxed. Moreover, let $i$ be the current in the wires and $I$ the emitted light intensity.
Let $V_{\text{d.c.}} = V_0$ be the d.c. component of the voltage across the wires and $V_{\text{a.c.}} = V_1 \cos \omega t$ the a.c. component. The total current in the wires is thus

$$i = i_0 + i_1 \cos \omega t$$

in which $i_0 = V_0/R$ and $i_1 = V_1/R$. The modulation amplitude is always smaller than the strength of the d.c. component. In the worst case, at low temperature, when the light emission is very weak (in this case, the glow of the wires cannot even be seen by the eyes), $i_1/i_0 = V_1/V_0 \leq 0.1$. Usually, $V_1/V_0 \approx 5 \times 10^{-2}$ or less.

Owing to the smallness of the modulation amplitude, we can assume that the wire temperature is determined, at least to first order, by the d.c. current, $i_0$. Hence, also the wire resistance $R$ can be assumed to be constant for a given $i_0$ at steady state.

The electrical power $W$ dissipated into the wires by Joule effect is then given by

$$W = Ri^2 = R i_0^2 \left[ 1 + \frac{1}{2} \left( \frac{i_1}{i_0} \right)^2 (1 + \cos 2\omega t) + 2 \left( \frac{i_1}{i_0} \right) \cos \omega t \right]$$

whose average value is

$$\bar{W} = W_0 \left[ 1 + \frac{1}{2} \left( \frac{i_1}{i_0} \right)^2 \right] = W_0 + \mathcal{O} \left[ \left( \frac{i_1}{i_0} \right)^2 \right]$$

with $W_0 = R i_0^2$. $\mathcal{O} \left[ (\ldots)^k \right]$ means small terms of order $k$ or higher. In the worst case, the modulation amplitude contributes a few parts per thousands to the Joule effect. This confirms the assumption that the wire temperature is mainly determined by the d.c. current.

The intensity $I$ of the emitted radiation is given by the Stefan’s law. At steady-state, the power input $W$ is balanced by heat losses by thermal conduction over the wires’ supports and by the radiation emitted by the glowing wires. In this condition, it is intuitive to assume (and the validity of this assumption will be shown in a forthcoming paper) that the temperature $T$ of the wires is proportional to $W$, thus yielding $I \propto W^4$. By expanding $W^4$ and keeping only leading order terms in $(i_1/i_0)$, we get

$$I(t) \propto W_0^4 \left[ 1 + 14 \left( \frac{i_1}{i_0} \right)^2 (1 + \cos 2\omega t) + 8 \left( \frac{i_1}{i_0} \right) \cos \omega t + \mathcal{O} \left( \frac{i_1}{i_0} \right)^3 \right]$$

As the detector output is $\propto I$, Eqn. (4) states that the detector output contains modulation at the same frequency of the current modulation as well as at twice that frequency. The synchronous detection with the lock-in amplifier picks up only the amplitude of the $\cos \omega t$ term and the lock-in output $v$ is thus proportional to the current modulation amplitude

$$v = AW_0^4 \left( \frac{i_1}{i_0} \right)$$

Here $A$ is a suitable constant that includes the overall gain of the amplification chain, the detector sensitivity, the solid angle subtended by the wires at the detector, and so on.
As a check of the consistency of the previous approximations, the detector signal is monitored on a digital oscilloscope (Agilent, mod. DSO3102A) that also displays the signal spectrum. In all experimental conditions only the first harmonic is present, whereas second and higher order harmonics are absent. This means that all terms of order \((i_1/i_0)^2\) and higher are negligible and that the wires temperature is only set by \(i_0\), as stated by Eqn. 3.

The signal amplitude \(v\) is modulated at very low frequency by the rotation of the polarizer. A complete rotation takes place in \(\approx 520\,\text{s}\), yielding a modulation frequency \(f_p \approx 2 \times 10^{-3}\,\text{Hz}\), whereas the current modulation frequency is \(\omega/2\pi \approx 2\,\text{Hz}\). These two frequencies are quite different so as to allow an easy choice of the integration time constant of the lock-in amplifier that is large enough to give a good noise rejection but short enough so as to yield a negligible uncertainty, \(\delta \theta \leq 0.7^\circ\), on the rotation angle \(\theta\) of the polarizer.

A typical record of \(v\) is shown in Fig. 3. According to Malus' law [24], the polarizer produces a \(\cos^2(\theta)\) modulation of the light intensity at the detector, hence of the detector signal amplitude

\[
v = v_u + v_p \cos^2(\theta - \theta_0)
\]

The fitting parameters are \(v_u = 56.6\,\text{mV}, v_p = 43.1\,\text{mV}\), yielding \(\langle P \rangle \approx 27.6\%\).
where \( \theta_0 \) is the initial angle between the polarizer and wires axis when the polarizer rotation is started and has no physical relevance. \( v_u \) and \( v_p \) are the amplitudes of the detector signal due to the unpolarized and polarized components of the emitted light, respectively, and are determined by fitting Eqn. (6) to the experimental data. The (average) polarization is then computed as the polarization contrast

\[
\langle P \rangle = \frac{v_p}{2v_u + v_p}
\]

in which the factor of 2 extinction of the unpolarized light component due to the polarizer has been taken into account. A typical example of the fit of Eqn. (6) to the data is also shown in Fig. 3.

Typically, for each \( i_0 \) settings, i.e., for each \( T \), a long experimental run is carried on by recording, on average, 20 revolutions of the polarizer in order to improve the statistical accuracy of the experiment. At the end of the run, the current modulation is turned off and the wire resistance is measured again and compared with the value it had before the current modulation was turned on. In this way, we always check that the modulation is small enough not to change the wire temperature more than 1%.

The lock-in output \( v \) and the encoder output \( \theta \) are fetched by a PC over a GPIB-IEEE 488 bus and are stored for offline processing. The data set of the long run is divided in subsets corresponding each to one single polarizer turn. The data of each subset is fitted to Eqn. (6) by using well-known nonlinear least-squares algorithms [25]. For each subset \( j \), the polarization \( \langle P_j \rangle \) is computed with the aid of Eqn. (7). Finally, the polarization for the long run corresponding to \( i_0 \) is obtained as the weighted average of the individual \( \langle P_j \rangle \)'s.

### 3.3. Wire Temperature Determination

The measured polarization has to be connected to the wire temperature. Whereas the determination of \( \langle P \rangle \) is quite easy, the assessment of the wire temperature is not straightforward. Even worse, \( T \) is an ill-defined quantity.

The wires are clamped at their ends on massive supports, which are in good thermal contact with the room temperature parts of the apparatus that act as the thermal equivalent of an electrical ground. In this situation, the steady-state balance between the heat input by Joule heating and the heat loss by thermal conduction through the supports and radiation emission leads to the buildup of a non uniform temperature profile along the wires.

This intuitive expectation is confirmed by the analysis of the visible image of the wires produced by a CCD camera (Lumenera, mod. Skynix2-1). In Fig. 4 we show the camera output \( I_{CCD} \), recorded when the wires were glowing reddish, viz., at an estimated central temperature about \( T \approx 2000 \text{ K} \) [26], as a function of the position \( px \) in pixels along the wires. The emission is maximum in the center and rapidly decreases towards the ends. A rough estimate of the temperature is obtained by applying the Stefan’s law as \( T \propto I_{CCD}^{1/4} \) and the corresponding profile is also shown in Fig. 4. The temperature
profile appears to be quite flat in the center where the temperature, $T_M$, is much higher than $T_0$ at the boundaries.

This picture is confirmed by the direct numerical integration of the differential equation stemming from the energy balance (details will be given in a forthcoming paper). We can anticipate the results for the temperature and intensity profiles when $T_M$ equals the melting temperature of Tungsten: $T_m = 3695$ K [23]. In this condition, $T_M/T_0 > 7$. Hence, the intensity of the light radiated from the central part of the wires is a few thousands of times larger than that radiated from the outer parts of the wires. Only the fraction of light emitted from the central part of the wires is collected by the detector, whose size is smaller than that of the wires image on its plane, as qualitatively shown in Fig. 4. Thus, the light impinging on the detector is originated from a wire region, over which $T$ is roughly constant. In the worst case, i.e., when $T_M \approx T_m$, the value of the temperature $\langle T \rangle$ averaged over the wire region projected onto the detector plane differs by no more than a few % from $T_M$. We thus assign $T = T_M$ as the temperature, at which $\langle P \rangle$ is measured for a given setting of the d.c. current $i_0$.

A further step is, however, required to determine $T$ because we actually measure the electrical power dissipated into the wires $W_0 = V_0i_0$. According to the results of the
numerical integration of the differential equation stemming from the energy balance, the relationship between $W_0$ and $T$ is linear and can be cast in the following form

$$T = T_a + \left( \frac{T - T_a}{W_{0,m}} \right) W_0$$

(8)

$T_a$ is the room temperature and $W_{0,m}$ is the electrical power at melting. We have measured $W_{0,m} = 0.675$ W for the 9 µm wires and $W_{0,m} = 3.014$ W for the 25 µm wires.

3.4. Polarization Data and Comparison with Theory

We are now able to present the polarization data as a function of the estimated wire temperature. In Fig. 5 we show $\langle P \rangle$ for the 9 µm- and 25 µm radius wires. $\langle P \rangle$ for both wire types strongly decreases with increasing $T$. At low $T$ the two data sets approach a polarization value of $\langle P \rangle \approx 35\%$, whereas the polarization decreases towards $\langle P \rangle \approx 15\%$ for $T$ near $T_m$. At low $T$ the error bars are very large because the signal is tiny and the signal-to-noise ratio is very unfavorable, whereas it becomes $\gg 1$ at high $T$.

According to Kirchhoff's law [27], the absorptivity and emissivity of a body in thermodynamic equilibrium with the radiation field are equivalent. This conclusion has been proved true also if the body is freely radiating to the outside environment, provided

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{$\langle P \rangle$ vs $T$. Closed symbols (experiment) and solid line (theory): 25 µm-wires. Open symbols (experiment) and dashed line (theory): 9 µm-wires.}
\end{figure}
that the local temperature of the body is well defined so that the energy distribution
over the material states of the body is the equilibrium distribution [28, 29, 30].

Thus, the calculation of the wires emissivity proceeds via the calculation of the
absorption efficiency (cross section per unit area) of a wire on which a electromagnetic
wave is impinging [31]. The wire is assumed to be a homogeneous circular cylinder of
length $l$ and radius $r \ll l$. The radiation scattered by the wire is observed at a distance $d$
from the wire in the plane crossing the wire at its midpoint and perpendicular to its axis.
The experimental conditions are such that $r \sim 10^{-5} \text{m}$, $\lambda \sim 10^{-5} \text{m}$, $l \sim 10^{-2} \text{m}$, and
$d \sim 10^{-1} \text{m}$ that yield the following inequalities: $r^2/\lambda \sim 10^{-4} \text{m} \ll d \sim 10^{-1} \text{m} \ll l^2/\lambda \sim 10^{2} \text{m}$. Under this conditions the scattered wave mainly has a cylindrical character. The
scattered field is then obtained as the far-field solution for an infinitely long circular
cylinder [16]. The optical properties of the wire are described by a complex, $\lambda$ and $T$
dependent, relative permittivity $\epsilon(\lambda, T)$. The cylinder surface is considered a sharp
boundary between the wire and the vacuum with $\epsilon = 1$. As tungsten is a non magnetic
material, its relative magnetic permeability is $\mu = 1$.

Owing to the cylindrical symmetry of the problem, the electromagnetic field can be
decomposed into transverse electric (TE)- and transverse magnetic (TM) modes. TE
modes are polarized with electric field vector perpendicular to the cylinder axis, whereas
TM modes are polarized with electric field vector parallel to the cylinder axis. For light
of intensity $I_0$, incident perpendicularly upon a wire, the intensity of light scattered at
an angle $\psi$ is given by

$$I^\dagger(\psi) = I_0 \left(\frac{2}{\pi k d}\right) |T^\dagger(\psi)|^2$$

where $T^\dagger(\psi)$ is the scattering amplitude of the mode at hand and $k = 2\pi/\lambda$ is the wave
number in vacuo.

For TE modes, $T^\perp$ is given by

$$T^\perp(\psi) = a_0 + 2 \sum_{m=1}^{\infty} a_m \cos(m\psi)$$

in which the coefficients $a_m$ are obtained by enforcing the boundary condition that the
electric field parallel to the cylinder axis vanishes at the wire surface, thus yielding

$$a_m = \frac{J'_m(nkr)J(kr) - nJ_m(nkr)J'_m(kr)}{J'_m(nkr)H^{(2)}_m(kr) - nJ_m(nkr)H^{(2)}'_m(kr)}$$

Here, $n = \sqrt{\epsilon}$ is the complex index of refraction of the material. $J_m$ are Bessel functions
of the first kind and $H^{(2)}_m$ are Hankel functions of the second kind [32]. The prime
indicates differentiation with respect to the argument.

Similarly, for TM modes, for which the component of the magnetic field parallel to
the wire axis vanishes at the wire surface, $T^\parallel$ is given by

$$T^\parallel(\psi) = b_0 + 2 \sum_{m=1}^{\infty} b_m \cos(m\psi)$$
with
\[ b_m = \frac{n J_m'(nkr) J_m(kr) - J_m(nkr) J_m'(kr)}{n J_m'(nkr) H_m^{(2)}(kr) - J_m(nkr) H_m^{(2)\prime}(kr)} \]  

The absorption efficiency factor \( Q_{\text{abs}}^{\dagger} \), i.e., the absorption cross section divided by the geometrical cross section of the wire, is given in terms of the extinction efficiency factor \( Q_{\text{ext}}^{\dagger} \) and of the scattering efficiency factor \( Q_{\text{sca}}^{\dagger} \) as
\[ Q_{\text{abs}}^{\dagger} = Q_{\text{ext}}^{\dagger} - Q_{\text{sca}}^{\dagger}. \]

The scattering and extinction efficiency factors are obtained as
\[ Q_{\text{ext}}^{\perp} = \frac{2}{kr} \Re \left( a_0 + 2 \sum_{m=1}^{\infty} a_m \right) \]
\[ Q_{\text{sca}}^{\perp} = \frac{2}{kr} \left( |a_0|^2 + 2 \sum_{m=1}^{\infty} |a_m|^2 \right) \]

\[ Q_{\text{ext}}^{\parallel} = \frac{2}{kr} \Re \left( b_0 + 2 \sum_{m=1}^{\infty} b_m \right) \]
\[ Q_{\text{sca}}^{\parallel} = \frac{2}{kr} \left( |b_0|^2 + 2 \sum_{m=1}^{\infty} |b_m|^2 \right) \]

The linear polarization of absorption is then defined [16] as
\[ P_{\text{abs}} = \frac{Q_{\text{abs}}^{\perp} - Q_{\text{abs}}^{\parallel}}{Q_{\text{abs}}^{\perp} + Q_{\text{abs}}^{\parallel}} = P \equiv P(\lambda, T, r) \]

and, according to Kirchhoff’s law, it is also the polarization \( P \) of the light emitted by the wires. The efficiency factors in Eqn. (16) depend on \( \lambda \) and \( T \) through the refraction index.

Actually, the measurement, for a given \( T \), is an average over \( \lambda \) of the light transmitted through the ZnSe window and IR polarizer and weighted by the detector responsivity \( D(\lambda) \). The polarizer can be approximated by a transmission coefficient of unity at maximum transmission and zero at minimum transmission in the present wavelength range and the ZnSe window transmission coefficient is nearly constant.

If \( E(\lambda, T) \) is the Planck’s formula for the intensity of the blackbody thermal radiation
\[ E(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\lambda r/\lambda} - 1} \]

the measured polarization of the light emitted by the wires is given by
\[ \langle P(T, r) \rangle = \frac{\langle Q_{\text{abs}}^{\perp} \rangle - \langle Q_{\text{abs}}^{\parallel} \rangle}{\langle Q_{\text{abs}}^{\perp} \rangle + \langle Q_{\text{abs}}^{\parallel} \rangle} \]

where the averages \( \langle Q_{\text{abs}}^{\dagger} \rangle \) are computed as
\[ \langle Q_{\text{abs}}^{\dagger} \rangle = \frac{1}{C} \int D(\lambda) E(\lambda, T) Q_{\text{abs}}^{\dagger}(\lambda, T, r) \, d\lambda \quad (\dagger = \perp, \parallel) \]

\( C = \int D E \, d\lambda \) is a normalization constant. All integrals are carried out over the wavelength range, in which \( D \) does not vanish.

In order to compute the efficiency factors as a function of \( \lambda \) and \( T \), the refraction index \( n(\lambda, T) = \sqrt{\epsilon(\lambda, T)} \) must be known. Unfortunately, the optical constants of tungsten have been measured or computed only in restricted wavelength- \((0.3 \lesssim \lambda \lesssim 3 \mu m)\) and temperature \((300 < T < 2400 K)\) ranges [33, 34, 35, 36, 37, 38, 39,...]
the agreement between their different determinations is not very satisfactory. On the contrary, our measurements dramatically extend the investigated temperature range up to the melting point of tungsten $T_m \approx 3695$ K and also extend the wavelength range because the HgCdTe detector is responsive to $\lambda$ up to $\approx 12 \mu$m.

For computational purposes, an analytical expression for $\varepsilon(\lambda,T)$ is required. Roberts [33] suggested a modified Drude-like expression for the relative permittivity

$$
\varepsilon(\lambda,T) = 1 + \sum_{j=1}^{3} \frac{K_{0,j}(T)\lambda^2}{\lambda^2 - \lambda_{s,j}(T) + i\delta_j(T)\lambda_{s,j}(T)} - \frac{\lambda^2}{2\pi c\varepsilon_0} \sum_{k=1}^{2} \frac{\sigma_k(T)}{\lambda_{r,k}(T) - i\lambda}
$$

(20)
in which $\varepsilon_0$ is the vacuum permittivity. The first sum represents the contribution of interband (or bound electrons) transitions, which are most important in the visible region, whereas the second sum gives the contribution of intraband (or free electron) transitions, which is dominant in the infrared region. The values of the coefficients $K_{0,j}$, $\lambda_{s,j}$, $\delta_j$, $\sigma_k$, and $\lambda_{r,k}$ are tabulated for a few temperatures up to only $T = 2400$ K [33]. This formula agrees quite well with the results of another model for the optical properties of tungsten and other metals [39] and has been successfully used in the experiment aimed at measuring the wire polarization in the visible range [17]. It has also been used for the theoretical computations of the heat radiation from long cylinders [22]. For these reasons, we have used Eqn. (20) also in the high temperature range because the coefficients are well behaved in $T$ and can be reasonably well extrapolated beyond the range given in Ref. [33].

In Fig. 5, we compare the measured polarization with the results of the calculations of the model. The agreement between experiment and theory is satisfactory. At all temperatures, even at the lowest, the radius of the wires and the dominant wavelengths are such that the experimental observations fall in the range of geometrical optics. This fact can be ascertained in the following way: the normalized distribution $\mathcal{D}E/C$ is a function strongly peaked at a wavelength $\lambda_{\text{max}} = \lambda_{\text{max}}(T)$. Owing to the shape of the detector’s response, the relationship between $\lambda_{\text{max}}$ slightly deviates from Wien’s law. We have actually found $\lambda_{\text{max}}T^B = A$, with $B \approx 1.167$ and $A = 12716 \mu$mK$^B$. For $T \sim 500$ K, the lowest temperature at which a faint, though detectable signal can still be observed, $\lambda_{\text{max}} \approx 9 \mu$m, yielding $k_{\text{min}}r = 2\pi r/\lambda_{\text{max}} \approx 6 > 1$ for the thinner wires and $k_{\text{min}}r \approx 17 \gg 1$ for the thicker ones. Moreover, in the near IR range both the real and imaginary part of the refraction index are of order 10 or larger, thus making $|nk_{\text{min}}r| \gg 1$.

In the range of geometrical optics, it is shown that $P_{\text{abs}} \rightarrow 1/3$ for large $kr$ and $nk_{\text{r}}$ [16]. Actually, the experimental data for both types of wires at low $T$ confirm the theoretical prediction. Further, the theory predicts that, in the range of geometrical optics, $P_{\text{abs}}$ should decrease if $|n|$ decreases. Actually, the Drude-Roberts formula for the relative permittivity Eqn. (20) predicts a decrease of $|n|$ with increasing $T$, mainly due to the behavior of $\text{Im}(n)$. We can conclude that the overall decrease of the observed
polarization is the result of the change of the Planck’s distribution with $T$. As $T$ is increased, it shifts to shorter $\lambda$, for which the refraction index is smaller.

The non perfect agreement of experiment and theory may be ascribed to several reasons. On one hand, the two kinds of wires are supplied by different manufacturers so that the purity of the two metals, hence their optical properties, might be different. Moreover, at such high temperatures, ageing phenomena, which might act differently on wires of different diameter and properties, cannot be excluded. For instance, though the wires are being heated in vacuo, at very high $T$ the residual atmosphere could lead to the formation of WO$_3$ flocs that may suddenly detach from the wires.

On the other hand, as mentioned before, the optical properties of tungsten in the near infrared region for $\lambda \gtrsim 3 \mu$m and for $T > 2400$ K are either unknown at all or affected by large uncertainties. Although the temperature dependence of the coefficients in the Drude-Roberts formula is quite well behaved, nobody guarantees that the extrapolation beyond $T = 2400$ K gives correct results. Nonetheless, the quite satisfactory agreement of the computed polarization with the experimental data lends credibility to the extrapolation procedure.

4. Conclusions

Thermal radiation has long been proved to be, at least, partially polarized. Spatial and temporal coherence of the light emitted by bodies of restricted geometry gives origin to phenomena, which are of interest for both fundamental physics and engineering applications. We have measured the degree of linear polarization of thermal radiation emitted by thin, long tungsten wires in an extended temperature range up to the tungsten melting point. The measurements are carried out in a wavelength band across the infrared and visible region, in which the polarization is directed perpendicularly to the wires axis. We have observed a marked decrease of the polarization when $T$ is increased. We have been able to explain the temperature dependence of the polarization by extrapolating the validity of the Drude-type formula for the dielectric constant well beyond the temperature and wavelength ranges, for which it was originally proposed in literature.

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