Phase oscillations between two superconducting condensates in cuprate superconductors

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Abstract

Implications of the small Fermi surface are discussed. We demonstrate that superconductivity in this system can be described in terms of two coupled condensates. The two condensates result in a collective excitation corresponding to the relative phase oscillation - a phason. We discuss the possibility of searching for this collective excitation in the dynamic resistance of the SQUID.

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I. INTRODUCTION

There is a lot of controversy about the shape of the Fermi surface in cuprate superconductors. In the early days it was believed that it is a small Fermi surface of the doped Mott insulator [1]. Later many of the results from photoelectron spectroscopy (PES) have been interpreted as favoring a large Fermi surface in agreement with Lattinger’s theorem [2]. On the other hand most recent PES data [3–6] once more give indications of a small Fermi surface for underdoped samples. In the present paper we consider the scenario with a small Fermi surface consisting of hole pockets around \((\pm \pi/2, \pm \pi/2)\), see Fig.1a. It is widely believed that the \(t – J\) model describes the main details of the doped Mott insulator. To fit the experimental hole dispersion one needs to extend the model introducing additional hopping matrix elements \(t', t''\) (see, e.g. Refs. [7–9]), but basically it is the \(t – J\) model. Superconducting pairing induced by spin-wave exchange in the \(t – J\) model has been considered in the papers [10,11]. It was demonstrated [10] that there is an infinite set of solutions for the superconducting gap. All the solutions have nodes along the lines \((1, \pm 1)\), see Fig.1a. (It is very convenient to use the magnetic Brillouin zone, but it can certainly be mapped to the full zone.) Using translation by the vector of the inverse magnetic lattice the picture can be reduced to two hole pockets centered around the points \((\pm \pi/2, \pi/2)\), see Fig.1a. The superconducting pairing is the strongest between particles from the same pocket, and the lowest energy solution for the superconducting gap has only one node line in each pocket. Having this solution in a single pocket one can generate two solutions in the whole Brillouin zone taking symmetric or antisymmetric combinations between pockets. The symmetric combination corresponds to the d-wave (Fig.1b), and the antisymmetric combination corresponds to the g-wave (Fig.1c) pairing. We would like to note that the possibility of generating new solutions by taking different combinations between pockets was first demonstrated by Scalettar, Singh, and Zhang in the paper [12]. The energy splitting between the d- and g-wave solutions has been investigated numerically in the Ref. [11]. The g-wave solution disappears and only the d-wave one survives as soon as the hole dispersion is degenerate.
along the face of the magnetic Brillouin zone. Actually in this situation there are no pockets and one has a large open Fermi surface at an arbitrary small hole concentration. However for small well separated pockets the d- and g-wave solutions are almost degenerate. In pure \( t - J \) or \( t - t' - t'' - J \) model the d-wave solution always has the lower energy. However if one extends the model including the nearest sites hole-hole Coulomb repulsion the situation can be inverted. The nearest sites repulsion does not influence the g-wave pairing and substantially suppresses the d-wave pairing. So it is quite possible that the real ground state has the g-wave superconducting gap. We would like to note that the g-wave ground state does not contradict the existing experimental data on Josephson tunneling \[13\]. The matter is that in this case the tunneling current pumps g-wave into d-wave in a thin layer near the contact, and this gives the interference picture very close to that for a pure d-wave ground state \[14\]. Anyway whatever the symmetry of the cuprate superconducting ground state (d-wave or g-wave), in the present paper we consider the scenario of the Fermi surface with separated hole pockets. According to the microscopic picture described above in this case we should consider simultaneously two coupled superconducting condensates which is equivalent to coexistence of the d- and g-wave pairings. A very strong evidence in favor of multicomponent condensate has been the recent observation of coherent Josephson response in microwave impedance of YBa\(_2\)Cu\(_3\)O\(_{6.95}\) single crystals \[15\].

The possibility of “two-gap” superconductivity has been suggested for conventional superconductors a long time ago \[16\]. Collective excitation corresponding to relative phase oscillations between two condensates in such superconductors has been considered by Legget \[17\]. He also pointed out the difficulty of experimental observation of this excitation. This basic difficulty is that the excitation can be revealed only in oscillations of the relative density of the electrons in two bands, and there is no external probe which is coupled directly to this quantity.

In the present work we demonstrate existence of the phase collective excitation in high \( T_c \) cuprate superconductors and calculate its dispersion in the long wavelength limit. We estimate the energy of this excitation and compare it with the superconducting gap. We
discuss also the ways to search for the phase collective excitation. Due to peculiar symmetry properties of the condensates (d-wave, g-wave) the Josephson current through the tunnel contact between conventional and cuprate superconductor can pump one condensate to another in some layer near the contact \[14\]. This gives the possibility of observing phase collective excitation in the dynamic resistance of the contact or in the dynamic resistance of SQUID.

II. Ginzburg-Landau Lagrangian and Free Energy

In the present paper we consider the scenario of the Fermi surface with well separated hole pockets in cuprate superconductors. According to the microscopic picture described above we should consider simultaneously the d- and g-wave pairings. Let us formulate an effective Ginzburg-Landau theory describing this situation. In the first approximation we can neglect the interaction between pockets in k-space. Then a half of the holes belong to one pocket and the rest belongs to the other pocket, and we should introduce two macroscopic condensates corresponding to the pockets, \( \Psi_1 = |\Psi_1|e^{i\phi_1}, \Psi_2 = |\Psi_2|e^{i\phi_2} \), where \(|\Psi_1| = |\Psi_2| = |\Psi| = \sqrt{N_h}/2\), \( N_h \) is the number density of condensate holes. The effective Lagrangian of the system in an external electric field \( E \) is of the form (hereafter we set \( \hbar = 1 \))

\[
L = \sum_{n=1,2} \frac{i}{2} \left( \Psi_n^{*}\dot{\Psi}_n - \Psi_n \dot{\Psi}_n^{*} \right) - F
\]

(1)

where \( F \) is Ginzburg-Landau free energy

\[
F = \int \left\{ \sum_{n=1,2} \left( \frac{1}{{2m^*}}|\nabla \Psi_n|^2 - a |\Psi_n|^2 + b |\Psi_n|^4 + 2e\varphi \left[ |\Psi_n|^2 - N_h/4 \right] \right) + \frac{E^2}{8\pi} \right\} dV + F_{\text{int}},
\]

(2)

with \( \varphi \) a scalar potential, and \( F_{\text{int}} \) a small interaction between pockets. Following Legget \[17\] we use the simplest form of this interaction

\[
F_{\text{int}} = \gamma \int (\Psi_1^{*}\Psi_2 + \Psi_1 \Psi_2^{*}) dV,
\]

(3)
where \( \gamma \ll a \) is a small parameter of the interaction. For the homogeneous case
\[ F_{\text{int}} \to 2\gamma V |\Psi_1| |\Psi_2| \cos(\phi_1 - \phi_2), \]
where \( V \) is the total volume. The Hamiltonian corresponding to the Lagrangian (1) is just Ginzburg-Landau free energy (2). Note, that in the free energy we have neglected mass anisotropy within the Fermi surface pocket. The anisotropy definitely exists, and it is probably very important for nonlinear microwave response observed in the Ref. [15]. However the anisotropy does not influence qualitatively the effects considered in the present work, and therefore we neglect it for the sake of simplicity.

The equilibrium values of the order parameters are
\[ |\Psi_1|^2 = |\Psi_2|^2 = |\Psi|^2 = a + |\gamma| \approx a/2b, \] (4)
The ground state phase difference \( \Delta\phi = \phi_2 - \phi_1 \) is determined by the sign of \( \gamma \): if \( \gamma > 0 \), then \( \Delta\phi = \pi \) (g-wave); if \( \gamma < 0 \), then \( \Delta\phi = 0 \) (d-wave). It is convenient also to introduce the d- and g-wave condensates \( \Psi_d = \Psi_1 + \Psi_2 = 2|\Psi|\cos(\Delta\phi/2)e^{i\phi} \), and \( \Psi_g = \Psi_1 - \Psi_2 = 2|\Psi|\sin(\Delta\phi/2)e^{i(\phi-\pi/2)} \), where \( \phi = (\phi_1 + \phi_2)/2 \). So, the ground state has either d or g-wave symmetry: if \( \gamma > 0 \) then \( \Psi_d = 0 \), \( \Psi_g \neq 0 \) and if \( \gamma < 0 \) then \( \Psi_d \neq 0 \), \( \Psi_g = 0 \).

III. EXCITATIONS

Lagrange equations corresponding to (4) are
\[ i\frac{\partial \Psi_n}{\partial t} = -\frac{\Delta}{2m^*}\Psi_n + 2b\Psi_n \left( |\Psi_n|^2 - N_h/4 \right) + 2e\varphi \Psi_n + |\gamma|\Psi_n + \gamma\Psi_n, \] (5)
\[ \Delta\varphi = -8\pi e \left( |\Psi_1|^2 + |\Psi_2|^2 - N_h/2 \right), \]
where \( \bar{n} = 2 \) if \( n=1 \), and \( \bar{n} = 1 \) if \( n=2 \). Consider the case \( \gamma < 0 \) which corresponds to the d-wave ground state. In this case \( \Psi_n = \sqrt{N_h/4 + \delta\Psi_n} \), where \( \delta\Psi_n \) is the deviation from ground state value. Making a linear approximation in the deviations, the eqs. (3) can be written as
\[ i\frac{\partial \delta\Psi_d}{\partial t} = -\frac{\Delta}{2m^*}\delta\Psi_d + \frac{bN_h}{2}(\delta\Psi_d + \delta\Psi_d^*) + 2e\sqrt{N_h}\varphi, \] (6)
\[ i\frac{\partial \delta\Psi_g}{\partial t} = -\frac{\Delta}{2m^*}\delta\Psi_g + \frac{bN_h}{2}(\delta\Psi_g + \delta\Psi_g^*) + 2|\gamma|\delta\Psi_g, \]
\[ \Delta\varphi = -4\pi e \sqrt{N_h}(\delta\Psi_d + \delta\Psi_d^*), \]
where $\delta \Psi_{d} = \delta \Psi_{1} + \delta \Psi_{2}$, and $\delta \Psi_{g} = \delta \Psi_{1} - \delta \Psi_{2}$. One finds from eqs.(6) that the d-wave oscillations $\delta \Psi_{d} = A_{d} \exp(ikr - i\omega t) + B_{d}^{*} \exp(-ikr + i\omega t)$ correspond to the usual plasmon in charged Bose liquid with spectrum

$$\omega_{k}^{2} = \frac{8\pi e^{2}N_{h}}{m^{*}} + bN_{h} \frac{k^{2}}{2m^{*}} + \frac{k^{4}}{(2m^{*})^{2}}. \quad (7)$$

The phase oscillations which we are looking for are described by

$$\delta \Psi_{g} = A_{g} e^{ikr - i\omega t} + B_{g}^{*} e^{-ikr + i\omega t}. \quad (8)$$

From eq. (6) one can easily find the dispersion of this excitation

$$\omega_{k}^{2} = 2|\gamma|bN_{h} + 4\gamma^{2} + (bN_{h} + 4|\gamma|) \frac{k^{2}}{2m^{*}} + \frac{k^{4}}{(2m^{*})^{2}} \approx 2|\gamma|bN_{h} + bN_{h} \frac{k^{2}}{2m^{*}}, \quad (9)$$

and the relation between $A_{g}$ and $B_{g}$

$$B_{g} = -Z_{k} A_{g}, \quad Z_{k} = 1 - 2\frac{\omega_{k} - k^{2}/(2m^{*}) - 2|\gamma|}{bN_{h}}. \quad (10)$$

To avoid misunderstanding we have to note that the phase oscillations are always accompanied by relative density oscillations between the pockets. Solution (8) represented in terms of phase and density variations looks like

$$\phi_{1} = \phi_{2} = \phi_{0} \sin(kr - \omega t),$$

$$\frac{\delta |\Psi_{1}|}{|\Psi_{1}|} = -\frac{\delta |\Psi_{2}|}{|\Psi_{2}|} = \frac{\phi_{0}\omega_{k}}{bN_{h} + k^{2}/(2m^{*}) + 2|\gamma|} \cos(kr - \omega t), \quad \text{(11)}$$

where $\phi_{0}$ is an amplitude of the phase oscillations.

The above consideration is relevant to the d-wave ground state ($\gamma < 0$), however in the case of the g-wave ground state ($\gamma > 0$) all the results are absolutely similar.

**IV. SECOND QUANTIZATION**

From the Lagrangian (1), canonical momenta are

$$p_{n} = \frac{\partial L}{\partial \dot{\Psi}_{n}} = i\Psi_{n}^{*}, \quad (12)$$
which gives $p_g = \frac{i}{2} \Psi_g^*$. With account of the relation (10) the equation (8) can be rewritten in terms of the Heisenberg creation operators of the phason $a_k^+$:

$$\hat{\Psi}_g = \sum_k Q_k \left( Z_k^{-1/2} a_k - Z_k^{1/2} a_k^+ \right) e^{i k r}. \quad (13)$$

Quantization condition

$$[\hat{p}_g(x), \hat{\Psi}_g(y)] = \frac{i}{2} [\hat{\Psi}_g^*(x), \hat{\Psi}_g(y)] = -i \delta(x - y), \quad (14)$$

together with standard commutation relations for creation and annihilation operators

$$[a_{k_1}, a_{k_2}] = 0, \quad [a_{k_1}, a_{k_2}^+] = \delta_{k_1, k_2}, \quad (15)$$

gives the amplitude $Q_k$ in equation (13)

$$Q_k = \sqrt{\frac{2 Z_k}{V(1 - Z_k^2)}}. \quad (16)$$

After substitution of $\delta \Psi_1 = -\delta \Psi_2 = \frac{1}{2} \hat{\Psi}_g$ into Ginzburg-Landau free energy (2) one finds the quantized Hamiltonian of the system

$$\hat{F} = -\frac{b N_0^2}{8} V + \sum_k \omega_k \left( a_k^+ a_k + \frac{1}{2} \right). \quad (17)$$

The first term here is the classical ground state energy, the second term gives the spectrum of the phase excitation. We have considered the case of $\gamma < 0$: d-wave ground state and g-wave phase excitations. If $\gamma > 0$ then the ground state has g-wave symmetry and the phase excitations correspond to the d-wave. However all the results, spectra, etc. are not changed.

**V. NUMERICAL ESTIMATIONS AND DISCUSSION OF THE POSSIBILITIES FOR SEARCH OF THE PHASE EXCITATION**

Due to eq. (1) the minimal energy of the phase excitation is $\omega_0 = \omega_{k=0} \approx \sqrt{2|\gamma| b N_0} \approx 2a \sqrt{|\gamma/a|}$. Let us demonstrate that this energy is much smaller than the maximum of the superconducting gap. According to the standard relation of Ginzburg-Landau theory,
parameter $a$ is related to the superconducting correlation length $\xi$: $a = 1/(4m^*\xi^2)$. A typical value of this correlation length in cuprates is about 2-3 lattice spacing, and therefore $a \sim 0.014eV/(m^*/m_e)$, where $m_e$ is an electron mass. Taking rather arbitrarily $|\gamma/a| \sim 1/10$ and $m^*/m_e \sim 7$ we find the frequency corresponding to the phase excitation

$$\hbar\omega_0 \sim 1mV, \quad \nu_0 = \frac{\omega_0}{2\pi} \sim 300GHz.$$  

(18)

The maximum of the superconducting gap on the Fermi surface $\Delta_{\text{max}}$ can be estimated by the standard BCS relation (see also discussion in Ref. [10]): $\Delta_{\text{max}} \approx 2T_c \sim 200K$. This is probably the lowest possible estimation. This gives the following ratio of the phase excitation energy to twice the superconducting gap

$$\frac{\omega_0}{2\Delta_{\text{max}}} \sim \frac{0.75}{m^*/m_e}\sqrt{|\gamma/a|} \sim \frac{1}{30},$$  

(19)

so it is really small. This certainly does not mean that the phason decay into particle-hole excitation is forbidden. It is still allowed because the superconducting gap has nodes at the Fermi surface. However due to the smallness of ratio (19) the decay phase space is very small and therefore one should expect smallness of the decay width.

Relative phase oscillations in conventional superconductors were predicted by Legget [17]. He also pointed out that direct observation of these excitations is very complicated because there is no external probe coupled to them. Fortunately the situation in high-$T_c$ cuprate superconductors is different. The phase oscillations can be excited in the tunneling contact of a conventional superconductor with a cuprate. The matter is that the supercurrent in such a contact under some conditions can pump the g-wave into d-wave and vice versa in the layer of width $l_\gamma = \hbar/\sqrt{4|\gamma|m^*}$ near the contact [14]. Therefore time dependent supercurrent can excite phase oscillations. So the idea is very simple: applying voltage $V$ to the contact one induces oscillations of the supercurrent, and at $2eV = \omega > \omega_0$ absorption should sharply increase. We repeat that this absorption arises only if the supercurrent drives the relative phase difference in cuprate. The conditions under which it happens were investigated in our previous work [14] and here we present only conclusions of this work. There are two possible
scenarios: 1) the bulk ground state of the cuprate has pairing of d-wave symmetry, 2) the bulk ground state of the cuprate has pairing of g-wave symmetry.

1) Consider first the d-wave scenario. In this case the supercurrent in a single tunnel contact of the cuprate with a conventional superconductor does not drive the phase difference between the cuprate condensates, and one needs to consider the SQUID with 90° cuprate superconducting corner. There is no driving even in the SQUID if the sides of the corner are parallel to crystal axes $a$ and $b$. So consider the corner rotated by some angle with respect to crystal axes. In this case the supercurrent drives the relative phase at zero magnetic flux in the SQUID ($\Phi = 0$) and does not drive at $\Phi = 0.5\Phi_0$. So in this situation the SQUID dynamic resistance depends on the magnetic flux and at $\Phi = 0$ one could observe the phase excitations.

2) In the case of the g-wave scenario the supercurrent drives the relative phase even for a single tunnel contact and phase excitations can be seen in the single contact dynamic resistance. Nevertheless it is interesting to consider also the SQUID with 90° superconducting corner. If sides of the corner are parallel to crystal axes $a$ and $b$, the current drives the phase difference at an arbitrary magnetic flux, producing phase excitations contributing to dynamic resistance. If the corner is rotated by some angle the situation is more interesting: There is no driving at $\Phi = 0$ and maximum driving at $\Phi = 0.5\Phi_0$. This is exactly opposite to the d-wave case.

VI. CONCLUSIONS

We have considered the scenario with the small Fermi surface consisting of hole pockets. The picture can be relevant to underdoped cuprate superconductors. The small Fermi surface together with mechanism of the magnetic pairing results in the possibility of having both the d- and the g-wave pairing. Energy splitting between these states is small. The ground state symmetry depends on the interplay between the magnetic pairing and Coulomb repulsion. We have demonstrated that these two condensates result in a collective excita-
tion corresponding to the relative phase oscillation - phason. The energy of this collective excitation is of the order of 1mV which is much smaller than the maximum superconducting gap on the Fermi surface. The possibilities for searching for the phase excitation in the dynamic resistance of a single tunnel junction and in the dynamic resistance of the SQUID are discussed. These experiments allow also to determine the symmetry of the ground state pairing.

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FIGURE CAPTIONS

Fig. 1. a. Fermi surface in magnetic Brillouin zone which is equivalent to the two-pocket Fermi surface (dashed line). b. Symmetry of the d-wave pairing in momentum space. c. Symmetry of the g-wave pairing in momentum space.
Figure 1: