Charmonium production in relativistic proton-nucleus collisions: What will we learn from the negative $x_F$ region?

D. Koudela$^{a}$, C. Volpe$^{a,b}$

$^a$ Institut für Theoretische Physik der Universität, Philosophenweg 19, D-69120 Heidelberg, Germany
$^b$ Groupe de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

We study the nuclear medium effects on the $c\bar{c}$ time evolution and charmonium production, in a relativistic proton-nucleus collision. In particular, we focus on the fragmentation region of the nucleus where the formation length of the charmonium mesons is shorter than the size of the nucleus. Little is known on the nuclear effects in this region. We use a quantum-mechanical model which includes a realistic potential for the $c\bar{c}$ system and an imaginary potential to describe the collisions of the $c\bar{c}$ with the nucleons. The imaginary potential introduces a transition amplitude among the charmonium states and produces an interference pattern on the charmonium survival probability, which is particularly important for $\psi'$. Our results on the suppression factors are compared with data from the NA50 and E866/NuSea Collaborations. Predictions are given for the suppression of $J/\psi$, $\psi'$, $\chi_c$ as a function of the nuclear mass and in the negative $x_F$ region, where data will be available soon.

A very exciting era in the study of strong interactions has begun with the advent of high energy heavy ion collision experiments. In particular, this has opened the way to the exploration of new states of matter, such as the colour glass condensate, corresponding to the nonlinear regime of quantum chromodynamics, or the high temperature phase of nuclear matter, the so-called quark-gluon plasma, which is also important for our understanding of the early universe. In relativistic heavy ion collisions, evidence for the formation of the quark-gluon plasma may come from the combined information of different signals, such as strangeness, dilepton, photon or charmonium production.

Matsui and Satz [1] first suggested that the suppression of charmonia could signal that a phase transition has occurred. This pioneering idea has triggered a series of experiments at the CERN SPS [2,3], whose interpretation has been very controversial. It has become clear that, to use charmonium as a signal, all possible mechanisms which can affect charmonium production need to be well understood [4,5].

In this context, proton-proton ($pp$) and proton-nucleus ($pA$) collisions have been intensively studied [2–8]. In fact, these processes are used as a reference for the case of nucleus-nucleus ($AB$) collisions where a critical energy density may be eventually attained, producing the plasma phase. This should affect various observables and in particular, it may lead to an extra (anomalous) charmonium suppression. There have been indications recently that the plasma has been produced, but it is still too early to draw definite conclusions [9]. These studies will be pursued by the running experiments at RHIC, and future measurements at LHC.

It takes a certain time for a $c\bar{c}$ produced in a collision to become a colour singlet state, then it expands to the size of a $\psi$ meson (the symbol $\psi$ stands from now on for any charmonium meson, i.e. $J/\psi$, $\psi'$ and $\chi_c$). Different models exist at present for the production mechanisms [4,10,11].

Sufficiently fast $c\bar{c}$ can traverse the nucleus before a $\psi$ meson is fully formed; whereas slow $c\bar{c}$ can traverse the nucleus as fully formed $\psi$ meson [12,13]. Experimentally one can pinpoint to these different kinematical regions by measuring charmonium production at positive and negative small Feynman $x_F$ and/or making inverse kinematics measurements [4,13].

One of the aspects which need to be well understood is the effect of the nuclear medium on the $c\bar{c}$ pair evolution. In particular, very little is known on the nuclear medium effects when the formation time of a fully developed charmonium is smaller than the time to traverse the nucleus [13]. Soon, measurements in $pA$ and $AB$ collisions will be performed by the HERA-B Collaboration [14] at DESY and the PHENIX Collaboration at RHIC [15]. Moreover the E160 Collaboration at SLAC will perform similar experiments where the charmonia are photoproduced [16].

In this letter, we focus on this region, i.e. the fragmentation region of the nucleus (negative $x_F$ region or slow $c\bar{c}$) and study the effects of the nuclear medium on the $c\bar{c}$ time evolution and on charmonium suppression in relativistic $pA$ collisions [17]. Because of the kinematical region we are interested in, we assume that the produced $c\bar{c}$ becomes quickly a colour singlet state (p meson) and on its way through the nucleus, this p meson expands to a $\psi$ meson, while experiencing collisions with the nucleons. To describe this, we use here a quantum-mechanical model where, contrary to previous works [4,12,13,18], the $c\bar{c}$ pair is bound by a realistic potential [19] and the p meson (and then meson) wave function is expanded on the basis given by the charmonium eigenstates. An imaginary potential depending on the dipole charmonium-nucleon cross section [12,20] is included to describe the collisions with the nucleons. This introduces a transition amplitude among the char-
monium states. (We assume the premeson wavefunction has a fixed angular momentum quantum number, while it has different radial components.)

We will show that the interaction of the premeson and later on the meson with the nuclear medium produces an interference pattern on the charmonium survival probability. We will show how this affects the charmonium suppression both when the path in the nucleus is varied, by changing the mass number $A$, and as a function of $x_F$. These effects are particularly important for $\psi'$. We compare our results with experimental data obtained at CERN SPS by the Na50 [7] and at Fermilab by the E866/NuSea [8] Collaborations. We present predictions at CERN SPS by the Na50 [7] and at Fermilab by the

We compare our results with experimental data obtained our model to

$J/\psi, \psi'$ quantum number we do not write explicitly the dependence on magnetic energy. (We consider that our $H$ Eq.(1), one gets the eigenenergies $E_{n,\ell}$ and the corresponding wave functions $|n, \ell\rangle$ (to simplify the notations, we do not write explicitly the dependence on magnetic quantum number $m$). The spin-dependent terms are neglected. Only transitions between charmonium states with different $n$ quantum numbers are considered.

By solving the static Schrödinger equation with $H_0$ Eq.(1), one gets the eigenenergies $E_{n,\ell}$ and the corresponding wave functions $|n, \ell\rangle$ (to simplify the notations, we do not write explicitly the dependence on magnetic quantum number $m$). The spin-dependent terms are neglected. Only transitions between charmonium states with different $n$ quantum numbers are considered.

The time dependent wave function for the $c\bar{c}$ in its rest frame is expanded on the basis of eigenstates of $H_0$ Eq.(1):

$$|c\bar{c},\ell\rangle(\tau) = \sum_{n=0}^{\infty} c_{n,\ell}(\tau)e^{-iE_{n,\ell}\tau}|n, \ell\rangle.$$  

(2) $(\hbar, c = 1)$. In practice, one truncates the sum to a number $\tilde{n}$ of eigenstates, for a given $\ell$ value. As we will discuss, we have chosen $\tilde{n}$ large enough so that the results we present are not sensitive to this truncation.

To model the interaction of the $c\bar{c}$ with the nuclear medium we add an imaginary part to Eq.(1):

$$iW = i\frac{\gamma v}{2} \sigma(\vec{r}_T, \sqrt{s_{\psi N}})\rho(\vec{b}, z)$$  

(3) where $v$ is the speed of the nucleus with respect to the $c\bar{c}$ frame and $\sigma$ is the dipole cross section associated to the interaction of the $c\bar{c}$ with a nucleon $N$, $r_T$ is the transverse distance between $c$ and $\bar{c}$, $\sqrt{s_{\psi N}}$ is the energy in the center of mass of the $\psi N$ system and $\rho$ is the nuclear density evaluated at the position $(\vec{b}, z)$ of the $c\bar{c}$ center of mass, $z$ being the beam direction. For the dipole cross section we use the parametrization $\sigma(\vec{r}_T, \sqrt{s}) = \sigma_0(s)(1 - e^{-r_T^2/\alpha^2(s)})$, determined by fitting deep inelastic scattering data and which well reproduces the charmonium photoproduction data [20]. Concerning the nuclear density $\rho$, we present results obtained with a Woods-Saxon profile, with parameters chosen such as to well reproduce the nuclear radii.

The time evolution of $c\bar{c}$ wavefunction (2) in the nucleus is determined by solving the time dependent Schrödinger equation for $H = H_0 - iW$ Eqs.(1) and (3). This leads to a system of first-order coupled-channel differential equations for the amplitudes $c_{n,\ell}(t)$ :

$$\dot{c}_{n,\ell}(t) = -a \sum_{k=1}^{\tilde{n}} c_{k,\ell}(t)e^{i(E_{n,\ell} - E_{k,\ell})\tau/2} |n, \ell, \sigma| |k, \ell\rangle$$  

(4) with $a = \psi v(b, z)/2$. The time $t = \gamma \tau$ is now in the laboratory frame. We have neglected in this first calculation the higher Fock states that should emerge from the Lorentz boost [20]. We see from (4) that the imaginary potential (3) introduces a transition amplitude among the charmonium eigenstates.

A difficult choice is that of the initial conditions $c_{n,\ell}(0)$ for (4), related to the mechanism of hadroproduction of charmonia, which is still badly known [4,5,10,11]. We use:

$$\phi_{c\bar{c},\ell}(0) = c_{\ell} f_{\ell}(r_T)e^{-\frac{1}{2}\beta^2(r_T^2 + z^2)}$$  

(5) where $c_{\ell}$ is a normalization constant. We describe the conversion of a gluon into a $c\bar{c}$ through a gaussian multiplied by the function $f_{\ell}(r_T) = r_T^2$ (or $r_T^4$) to account for the two (one) supplementary gluons necessary to produce $J/\psi, \psi'$ (\(\chi_c\)). The initial wave function depends on one parameter only, $\beta$. This parameter has been determined by fixing the ratio $|c_{2,0}/c_{1,0}|^2$ to the experimental ratio of $\psi'$ over $J/\psi$, produced in $pp$ collisions at 450 GeV, i.e. $B_{\psi'\rightarrow\mu\bar{\mu}}/B_{J/\psi\rightarrow\mu\bar{\mu}} = (1.60 \pm 0.04)%$ [21], where $B_{\psi'\rightarrow\mu\bar{\mu}}$ is the branching ratio to dimuon production and $\sigma_{pp\rightarrow\psi'}$ the $pp$ reaction cross section. This leads to the ratio $|c_{2,0}/c_{1,0}|^2(0) = 0.22 \pm 0.03$ and $\beta = 1.33 \pm 0.5$ GeV (the same $\beta$ has been used for the $\chi_c$ states).

The number $\tilde{n}$ of states in (2), to be included in the coupled channel equations (4), has to be large enough that the results do not depend on the truncation. We have checked how the results depend on the inclusion of extra states. Up to 4 channels have been used for the $S$-states and up to 2 for the $P$-states. The truncation always affects a little the last state included; whereas the results for the lowest energy states are practically unchanged.
Let us now come to the results, obtained in infinite nuclear matter first, i.e. by taking in (4) a constant nuclear density \( \rho = \rho_0 \). In fig.1 we show the time evolution of the probabilities associated to \( J/\psi \) and \( \psi' \) production obtained by solving (4) with the initial conditions (5), both neglecting and including the transition amplitudes between different eigenstates (non-diagonal terms). While in the former case the probabilities decrease exponentially, in the latter they present an oscillation. The interference pattern is more pronounced in the case of \( \psi' \) whereas for \( J/\psi \) the oscillations stays very close to the exponential. This effect directly influences the suppression factors in \( pA \) collisions as we will see. The dominant oscillation frequency in Eq.(4) is \( \omega = (E_{2,\ell} - E_{1,\ell})/\gamma \). We have also seen that the deviation from the uncoupled solution becomes stronger the higher \( \gamma \) is.

Let us now consider a \( \bar{c}c \) which is produced in a relativistic \( pA \) collision and evolves according to Eqs.(4). Integrating over all possible paths the ratio \( |c_0(\infty)/c_0(0)|^2 \), which gives the survival probability of the \( \psi \) in the nucleus, one gets the \( pA \rightarrow \psi \) reaction cross section:

\[
\sigma_{pA \rightarrow \psi} = \int \frac{d\vec{b}d\vec{z}}{\rho(\vec{b},z)} |\sigma_{pN \rightarrow \psi}(t(\vec{b},z))|^2 \frac{|c_0(t(\vec{b},z))|^2}{|c_0(0)|^2}
\]

where \( t(\vec{b},z) \) is the time necessary to a \( \bar{c}c \) produced at a point \( (\vec{b},z) \) to traverse the nucleus. If \( \sigma_{pA \rightarrow \psi} = A\sigma_{pN \rightarrow \psi} \), the suppression factor, defined as

\[
S_A^\psi = \frac{\sigma_{pA \rightarrow \psi}}{A\sigma_{pN \rightarrow \psi}},
\]

becomes \( S_A^\psi = 1 \).

Before making predictions, let us compare our calculations to existing data. For \( J/\psi \), we have to take into account that the dimuons measured in the detector come from the decay of the directly produced \( J/\psi \)’s as well as those produced by the decays of \( \psi' \)’s and \( \chi_c \)’s. We will denote by “\( J/\psi \)” the total number of produced \( J/\psi \), i.e.

\[
S_{pA}^{\text{“}J/\psi\text{”}} = 0.62S_{pA}^{J/\psi} + 0.30S_{pA}^{\chi_c} + 0.08S_{pA}^{\psi'} [6].
\]

Let us first discuss charmonium production as a function of the nuclear mass. The \( \bar{c}c \) pair travels along different lenghts, spending different time intervals in the nuclei. Therefore, experiments with different \( A \) explore the time dependence of the \( c_{\alpha,\ell} \) amplitudes (Fig.1). We present results on “\( J/\psi \)” and \( \psi' \) (Fig.2) in comparison with the experimental values by the NA50 Collaboration [7], obtained for \( pA \) collisions with an impinging proton energy of 450 GeV. The data are given as branching ratio times cross section divided by the nuclear mass, which differs from the suppression by a constant. (For the comparison, we normalize the data by this constant determined from the average of the ratio experimental/calculated values.) We can see that our results are in good agreement with the experiment both for “\( J/\psi \)” and \( \psi' \). In Fig.3 we give predictions for the suppression of \( \chi_c \) as a function of the nuclear mass, for different values of \( \gamma \). The largest suppression is observed when \( \gamma \) is the lowest, corresponding to a longer time spent in the nucleus.

Let us now look at the \( x_F \) dependence. The suppression factor is often parametrized as

\[
S_A^{(x_F, \sqrt{s_{pp}})} = A^{\alpha_{pN}(x_F, \sqrt{s_{pp}}, A)} - 1 [4].
\]

In Fig.4 the \( \alpha \) values for “\( J/\psi \)” and \( \psi' \) are compared to experimental data on \( pA \) collisions, obtained by the E866/NuSea Collaboration [8]. The impinging proton energy is 800 GeV and the label \( A \) is the ratio of the atomic numbers of tungsten over beryllium. In this region of small positive and negative \( x_F \) values, our calculations are in good agreement with the experimental values for “\( J/\psi \)” whereas for \( \psi' \) they overpredict the data. In fact, in the region of small positive \( x_F \), the \( \bar{c}c \) can traverse the nucleus before developing into a \( \psi \) meson. In this kinematical region, other effects missing in our model, like for example gluon shadowing [22], can play a role.

Finally, we present predictions for the \( \alpha \) values (Fig.5) and for the suppression factors (Fig.6) for “\( J/\psi \)” and \( \chi_c \) in the negative \( x_F \) region, where experimental data from the HERA-B Collaboration [14] at DESY and the PHENIX Collaboration at RHIC [15] will soon be available. As we can see, both “\( J/\psi \)” and \( \chi_c \) present a rather flat behavior, whereas \( \psi' \) is very sensitive to the \( x_F \) values. Besides, the results on \( \psi' \) also show a stronger variation with the mass of the nucleus. This can be understood from the fact that the effect of the coupling between the charmonium eigenstates is much stronger for \( \psi' \) than for \( J/\psi \) (and \( \chi_c \)), as one can see from Fig.1.

Fig.5 also shows that the \( \alpha \) parametrization of the suppression (often used with \( \alpha \) independent of \( A \)) is especially not good in the negative \( x_F \) region, since there is a strong dependence of the \( \alpha \) values on the nuclei.

In summary, we have studied the production of charmonium (\( J/\psi \), \( \psi' \), \( \chi_c \)) in relativistic proton-nucleus collisions. We focussed on the nuclear effects on the \( \bar{c}c \) time evolution and on the charmonium production when the time for a \( \bar{c}c \) to develop to a \( \psi \) meson is short compared to the time necessary to traverse the nucleus. This kinematical region may be explored experimentally by looking at slow enough \( \bar{c}c \) and therefore charmonium produced in the negative \( x_F \) region. The quantum-mechanical model used here includes a realistic potential binding the \( \bar{c}c \) pair and an imaginary potential, describing the collision of the \( \bar{c}c \) with the nucleons and depending on the dipole charmonium-nucleon cross section. The imaginary potential introduces transition amplitudes among charmonium eigenstates. We have shown that this produces an interference pattern on the charmonium survival probability, particularly important for \( \psi' \), and affects the charmonium suppression factors. The results are compared to experimental data from the NA50 and E866/NuSea Collaborations. We have made predictions on the sup-
pression factors of $J/\psi$, $\psi'$ and $\chi_c$, for different nuclei in the fragmentation region of the nucleus, where experiments will be performed soon by the HERA-B Collaboration at DESY and the PHENIX Collaboration at RHIC. We expect that the exploration of the negative $x_F$ region will bring new information on the charmonium-nucleon cross section and on the initial conditions which are not well known, especially for $\chi_c$. A better understanding of the mechanisms for charmonium production in proton-nucleus collisions in this yet unexplored region will help us in the interpretation of the measurements on relativistic nucleus-nucleus collisions.

We are very grateful to Jörg Hufner for having interested us in this problem. We thank him for his useful suggestions and comments all along the realization of this work, and for his very careful reading of the manuscript. We thank Boris Kopeliovich and Stephane Peigné for very useful discussions and comments, as well as Yuri Yvanov for his kind help. One of us (C.V.) acknowledge all the people at the Institut für Theoretische Physik of the University of Heidelberg, for the very warm hospitality during her stay.

[1] T. Matsui and H. Satz, Phys. Lett. B178, 416 (1986).
[2] J. Badier et al, Na3 Collaboration, Z. Phys. C20, 101 (1983).
[3] C. Baglin et al, NA38 Collaboration, Phys. Lett. B220, 471 (1989); Phys. Lett. B255, 459 (1991); Phys. Lett. B345, 617 (1995).
[4] C. Gerschel and J. Hufner, Annu. Rev. Nucl. Part. Sci. 49, 255 (1999).
[5] R. Vogt, Phys. Rep. 310, 197 (1999).
[6] L. Antoniazzi et al., E705 Collaboration, Phys. Rev. Lett. 70, 383 (1993).
[7] P. Cortese et al., NA50 Collaboration, Proceedings to “Quark Matter 2002”, Nantes (France), July 18-24, 2002.
[8] M. J. Leitch et al., E866/NuSea Collaboration, Phys. Rev. Lett. 84, 3256 (2000).
[9] M. C. Abreu et al, NA50 Collaboration, Phys. Lett. B477, 28 (2000); J. P. Blaizot, P. M. Dinh, J. Y. Ollitrault, Phys. Rev. Lett. 85, 4012 (2000); A. Capella, E. G. Ferreiro, A. B. Kaidalov, Phys. Rev. Lett. 85, 2080 (2000).
[10] M. Kramer, Prog. Part. Nucl. Phys. 47, 141 (2001).
[11] P. Hoyer, N. Marchal, S. Peigné, Proceedings of the CERN 2001-2002 Workshop on “Hard Probes in Heavy Ion Collisions at the LHC”, hep-ph/0209365.
[12] B. Z. Kopeliovich and B. G. Zakharov, Phys. Rev. D44, 3466 (1991).
[13] D. Kharzeev and H. Satz, Phys. Lett. B356, 365 (1995).
[14] http://www-hera-b.desy.de/general/info.
[15] Jen-Chieh Peng, private communication.
[16] Keith Griffioen, private communication.

[17] D. Koudela, Diplomarbeit, University of Heidelberg, 2002; http://www.ub.uni-heidelberg.de/archiv/3043.
[18] F. Arleo, P.-B Gossiaux, T. Gousset, J. Aichelin, Phys. Rev. C61, 054906 (2000); Y. B. He, J. Hufner, B. Z. Kopeliovich, Phys. Lett. B477, 93 (2000); J. Hufner and B. Kopeliovich, Phys. Rev. Lett. 76, 192 (1996); J. Cugnon and P. B. Gossiaux, Z. Phys. C58, 77 (1993).
[19] W. Buchmüller and S. H. Tye, Phys. Rev. D24, 132 (1981).
[20] J. Hufner, Y. P. Ivanov, B. Z. Kopeliovich and A. V. Tarasov, Phys. Rev. D62, 094022 (2000).
[21] M. C. Abreu et al., Phys. Lett. B466, 408 (1999).
[22] B.Z. Kopeliovich and A. V. Tarasov, Nucl. Phys. A710, 180 (2002).
FIG. 3. Predictions on the suppression factor of the $\chi_c$ meson, produced in relativistic $pA$ collisions, as a function of the mass of the nucleus $A$. We present results for $\gamma = 5$ (circles), 16 (squares), 21 (triangles).

FIG. 4. Comparison between calculated and experimental $\alpha$ values for "$J/\psi$" (top) and $\psi'$ (bottom) as a function of Feynman $x_F$ obtained for $pA$ collisions with $E_p = 800$ GeV. Here the $\alpha$ values are relative to the ratio $W/Be$. The data are from the E866/NuSea Collaboration [8].

FIG. 5. Predictions for "$J/\psi$" (top), $\psi'$ (middle) and $\chi_c$ (bottom), on the dependence of the $\alpha$ values as a function of $x_F$, obtained with $E_p = 800$ GeV. We present results obtained for tungsten (triangles) and beryllium (squares).

FIG. 6. Predictions for the suppression factors for "$J/\psi$" (circles), $\psi'$ (squares) and $\chi_c$ (triangles) as a function of the negative $x_F$ region, obtained in a $pA$ collision with $E_p = 800$ GeV and $A$ being the ratio of tungsten over beryllium.