Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Linear and non-linear dynamics of the epidemics: System identification based parametric prediction models for the pandemic outbreaks

Onder Tutsoy a,*, Adem Polat b

a Adana AlparslanTürkes Science and Technology University, Adana, Turkey
b Çanakkale Onsekiz Mart University, Çanakkale, Turkey

A R T I C L E   I N F O

Article history:
Received 3 February 2021
Received in revised form 23 June 2021
Accepted 5 August 2021
Available online 9 August 2021

Keywords:
COVID-19
Casualties
Pandemic
Prediction
Model
Non-linear dynamics
Linear dynamics

A B S T R A C T

Coronavirus disease 2019 (COVID-19) has endured constituting formidable economic, social, educational, and psychological challenges for the societies. Moreover, during pandemic outbreaks, the hospitals are overwhelmed with patients requiring more intensive care units and intubation equipment. Therein, to cope with these urgent healthcare demands, the state authorities seek ways to develop policies based on the estimated future casualties. These policies are mainly non-pharmacological policies including the restrictions, curfews, closures, and lockdowns. In this paper, we construct three model structures of the $S_{p}I_{n}I_{t}I_{b}D-N$ (suspicious $S_{p}$, infected $I_{n}$, intensive care $I_{t}$, intubated $I_{b}$, and dead $D$ together with the non-pharmacological policies $N$) holding two key targets. The first one is to predict the future COVID-19 casualties including the intensive care and intubated ones, which directly determine the need for urgent healthcare facilities, and the second one is to analyse the linear and non-linear dynamics of the COVID-19 pandemic under the non-pharmacological policies. In this respect, we have modified the non-pharmacological policies and incorporated them within the models whose parameters are learned from the available data. The trained models with the data released by the Turkish Health Ministry confirmed that the linear $S_{p}I_{n}I_{t}I_{b}D-N$ model yields more accurate results under the imposed non-pharmacological policies. It is important to note that the non-pharmacological policies have a damping effect on the pandemic casualties and this can dominate the non-linear dynamics. Herein, a model without pharmacological or non-pharmacological policies might have more dominant non-linear dynamics. In addition, the paper considers two machine learning approaches to optimize the unknown parameters of the constructed models. The results show that the recursive neural network has superior performance for learning nonlinear dynamics. However, the batch least squares outperforms in the presence of linear dynamics and stochastic data. The estimated future pandemic casualties with the linear $S_{p}I_{n}I_{t}I_{b}D-N$ model confirm that the suspicious, infected, and dead casualties converge to zero from 200000, 1400, 200 casualties, respectively. The convergences occur in 120 days under the current conditions.

© 2021 ISA. Published by Elsevier Ltd. All rights reserved.

1. Introduction

Even though pharmacological developments like vaccines have expected to yield some successful results, the COVID-19 has continued to be a dreadful threat for societies due to challenges in producing a sufficient number of vaccines [1]. The COVID-19 emerged in December 2019 in Wuhan city of China has spread to over 113 countries with 91,605,941 infected and 1,962,345 dead as of 13 January 2021 [2]. The second peak in the COVID-19 casualties, which is larger than the first peak, has caused considerable challenges for the healthcare providers since the hospitals have been overwhelmed with suspicious, infected, intensive care, intubated, and dead people [3]. To halt the spread of the virus, non-pharmacological policies such as closures, restrictions, and curfews have been re-imposed [4]. International organizations such as the World Health Organization (WHO) and also the state authorities require accurate models to understand the character of the pandemic diseases and also to estimate the future casualties [5]. In this paper, we develop a parametric model called as $S_{p}I_{n}I_{t}I_{b}D-N$ (suspicious $S_{p}$, infected $I_{n}$, intensive care $I_{t}$, intubated $I_{b}$, and death $D$ together with the non-pharmacological policies $N$) to predict the future pandemic casualties in the presence of the non-pharmacological policies.

https://doi.org/10.1016/j.isatra.2021.08.008
0019-0578/© 2021 ISA. Published by Elsevier Ltd. All rights reserved.
The recent history has witnessed severe acute respiratory syndrome (SARS), Middle East respiratory syndrome (MERS), and the COVID-19 outbreaks and more than 2000, 8000, and 91,000,000 people were infected, respectively [6]. It is reported that the COVID-19 outbreak has brought about a heavier burden than the recent 2009 influenza pandemic and seasonal influenza in terms of the hospital requirements and mortality rates [7]. Although the COVID-19 is not a new member of the pandemic diseases family as it has emerged from the SARS coronavirus, it has unseen characters including the rapid spread in the human body, high infectious speed, extreme resilience against the environmental conditions, efficient adaptation to the human body, and considerably virulent genetic variant [8]. Therefore, developing models for the COVID-19 outbreak is challenging due to these complex and time-varying dynamics [9]. Modelling of the pandemic diseases can be categorized as parametric and non-parametric where the parametric approaches are mainly based on the system identification and the non-parametric approaches are based on the statistical approaches.

With respect to the non-parametric approaches, Zhu and Chen introduced a statistical disease transmission model to predict the early-stage transmissibility of the COVID-19 outbreak in China which yielded 4.2 for the infectious period with a 95% confidence interval [10]. Gupta et al. analysed the relationship between the COVID-19 stemmed mortality and air pollution with the variance and regression models which revealed a positive correlation for the nine Asian cities [11]. Similarly, Redon and Aroca reviewed the role of the climate change and the COVID-19 spread with the generalized linear models which showed that the hot weather impacts on the transmission of the virus are insignificant [12]. Rozenfeld et al. performed a multivariable statistical approach to analyse the infection of the COVID-19 based on the age, obesity, gender, race, ethnicity, and income. The results highlighted that the communities having poor housing, insecure transportation, and chronic disease make people more vulnerable to the COVID-19 [13]. The key disadvantage associated with the statistical approaches is that they do not account for the possible presence of the temporal trends in the data; however, they all have a transient period before exerting their full impacts.

Neural network (NN) is a non-parametric machine learning (ML) approach mostly considered for prediction. Wieczorek et al. trained an NN with available spread rates and used the resulted model to predict the COVID-19 spread for various regions [14]. Melin et al. constructed an NN to perform predictions under various conditions and utilized a fuzzy logic algorithm to make a decision depending on the predictions [15]. Ahmad and Asad provided an NN-based prediction for the infected, recovered, and death casualties [16]. NN approaches require a suitable training model selected based on the characteristic of the historical data, which necessitate intuitive insights about the data and re-training since the pandemic outbreaks usually have time-varying dynamics.

With regard to the parametric approaches, Chen et al. used the susceptible–infected–recovered (SIR) model with available transmission and recovery rates to predict the casualties in China [17]. Calafiore et al. considered the susceptible–infected–recovered–dead (SIRD) model with the time-varying parameters to estimate the casualties in Italy [18]. Mahajan et al. implemented the susceptible–exposed–symptomatic–purely asymptomatic-hospitalized–recovered–deceased (SIPHERD) model to predict the casualties in India [5]. All these models are first-order non-linear and do not take into account the non-pharmacological policies. Recently, we developed the suspicious–infected–death (SpID) model having second-order linear coupled dynamics learned from the available data [19]. We showed that the SpID model can efficiently represent the second-order dynamics such as the distinct peak and performed eigenvalue-based character analysis. In addition, we proposed the SpID-N model with the parametrized non-pharmacological policies N and extensively analyse the role of each non-pharmacological policy on the reported casualties [20]. This research proved that the second-order dynamics of the COVID-19 occur due to non-pharmacological policies and they are not intrinsic.

Recent researches have addressed the external impacts such as the weather on the COVID-19. Coskun et al. investigated the role of the population density and the climatic properties including the temperature, humidity, wind, and the number of sunny days on the COVID-19 spread [21]. Similarly, Sahin examined the interactions between the COVID-19 and temperature, dew point, humidity, and wind [22]. Ozer analysed the distance education efforts during the COVID-19 outbreak [23]. Morgul et al. examined the relationship between the COVID-19 outbreak and psychological fatigue as a mental health issue [24]. Satici et al. assessed the COVID-19 stemmed fear and psychological distress and life satisfaction [25].

Based on these expressed gaps in the corresponding literature, we can summarize the contributions of this paper as:

(1) We construct the three S\(I_p\)I\(D-N\) model structures; namely, linear S\(I_p\)I\(D-N\) model, non-linear S\(I_p\)I\(D-N\) model, and strongly non-linear S\(I_p\)I\(D-N\) model to reveal the linear and non-linear characters of the COVID-19 casualties under the comprehensive non-pharmacological policies. All the parametric models expressed above have non-linear structures, but they do not consider the non-pharmacological policies. It is a fact that non-pharmacological policies are the essential tools to control pandemic casualties. To the best of our knowledge, this is the first paper examining the linear and non-linear properties of a pandemic disease under such extensive non-pharmacological policies.

(2) We modify the non-pharmacological policies since their characters have changed with the occurrence of the second peak in the COVID-19 casualties. Re-opening of the schools partial-by-partial and imposed self-curfews, for instance, are modelled and incorporated into the S\(I_p\)I\(D-N\) models.

(3) We enrich the SpID-N model with the intensive care I\(b\) and intubated I\(t\) to estimate the healthcare requirements.

All the model parameters are assigned as unknown and learned from the available data by using the batch type least-squares (BLS) approach. In this respect, the S\(I_p\)I\(D-N\) model is adaptive since it updates its parameters as the new data are available. In addition, further linear and non-linear model structures can be constructed, but as this paper aims to analyse the character of the pandemic dynamics, we have considered only three S\(I_p\)I\(D-N\) model structures.

In the rest of the paper, Section 2 introduces the proposed model structures, Section 3 provides the modified non-pharmacological policies, Section 4 derives the BLS to learn the unknown parameters, Section 5 analysis the models and predicts the future pandemic casualties and Section 6 summarizes the key contributions of the paper.

2. Linear and non-linear parametric model structures

In this section, we introduce three parametric model structures; namely, a linear S\(I_p\)I\(D-N\) model, a nonlinear S\(I_p\)I\(D-N\) model, and a strongly non-linear S\(I_p\)I\(D-N\) model which are all extensively analysed in terms of representing the characteristics of the pandemic diseases, specifically the COVID-19. It is important to note that various alternative structures for the linear and non-linear S\(I_p\)I\(D-N\) model can be formed. However, since this paper focuses on revealing the existence of linear or non-linear dynamics, we have constructed and analysed only three model structures.
2.1. Linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model

The linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model architecture is shown in Fig. 1. As can be seen from Fig. 1, the linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model consists of five sub-models. The suspicious S\textsuperscript{P} sub-model represents the people tested daily. Some of the suspicious people become infected \( I^p \) with an unknown parameter \( a_{12} \), and a number of the infected people can spread the virus with an unknown parameter \( a_{23} \), which excites the number of the suspicious S\textsuperscript{P} casualties. The infected \( I^p \) individuals can move to the intensive care \( I^a \) unit with the parameter \( a_{23} \), and a number of them leave the intensive care unit \( I^a \) but remain infected \( I^p \) with the unknown parameter \( a_{32} \). Some of the intensive care \( I^a \) patients become intubated \( I^b \) with the unknown parameter \( a_{32} \). The intubated \( I^b \) patients can be dead \( D_n \) with the parameter \( a_{33} \), and the changes in the death is reflected on the intubated \( I^b \) casualties with the \( a_{52} \) unknown parameter. Since the S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model does not cover the recovered and asymmetrical casualties, the model represent them as the unmodelled dynamics with the \( a_{12}, a_{23}, a_{31}, a_{41}, a_{51} \) coupling parameters. To explicitly represent such epidemiological properties, the S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model should be enhanced. The non-pharmaceutical policies \( u_k \) is the external input and it manipulates the sub-models with the \( b_1, b_2, b_3, b_4 \), and \( b_5 \) unknown parameters. These unknown parameters are learned in Section 0.

The linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model in Fig. 1 can be represented by considering only the arrows leaving out each sub-model as

\[
S^p_{k+1} = a_{11}S^p_k + a_{12}I^a_k + b_1u_k \tag{1}
\]

\[
I^a_{k+1} = a_{21}I^a_k + a_{22}S^p_k + a_{23}I^b_k + b_2u_k \tag{2}
\]

\[
I^b_{k+1} = a_{31}I^b_k + a_{32}I^a_k + a_{33}I^p_k + b_3u_k \tag{3}
\]

\[
b^k_{k+1} = a_{41}I^k_k + a_{42}I^b_k + a_{43}D_k + b_4u_k \tag{4}
\]

\[
D_{k+1} = a_{51}D_k + a_{52}I^b_k + b_5u_k \tag{5}
\]

Table 1 introduces the components of the linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N.

All the sub-models of the linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model have corresponding internal dynamics and also the coupling dynamics associated with the neighbouring sub-models. These dynamics are represented with the parameters learned from the data (reported pandemic casualties); therefore, the parameters of the pandemic diseases such as the infectious rate and recovery rate are learned implicitly. The next sub-section introduces the non-linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model.

2.2. Non-linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model

As can be seen from Figs. 1 and 2, the only difference between the linear and non-linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N models is the non-linear coupling of the suspicious S\textsuperscript{P} and infected \( I^p \) sub-models through the \( a_{11} \) and \( a_{21} \) unknown parameters. The suspicious casualties vary based on the number of the people tested daily, the daily testing capacity of the healthcare centres and whether the tests are made in the presence of strong symptoms. If the capacities of the healthcare centres are constant or less than the daily requirements, then it is expected to have a non-linear relationship between the suspicious S\textsuperscript{P} and infected \( I^p \) casualties. The non-linearity occurs because the proportional input of the model does not yield a proportional output. However, if the healthcare centres are flexible to meet the daily test requirements and the tests are performed in the presence of certain symptoms, then it is anticipated to have a linear relationship between the suspicious S\textsuperscript{P} and infected \( I^p \) casualties. In this case, the proportional inputs of the model produce proportional outputs. Thus, the results presented in the analyses section can vary from country to country based on the healthcare infrastructure of the countries and policies for handling the outbreaks. However, since the proposed S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N models learn the parameters from the available data, they are adaptive and can be used to estimate the corresponding pandemic casualties.

We can represent the non-linear S\textsuperscript{P}t\textsuperscript{P}t\textsuperscript{P}D-N model with five sub-models where the suspicious S\textsuperscript{P} and infected \( I^p \) sub-models are non-linearly coupled as

\[
S^p_{k+1} = a_{11}S^p_k + a_{12}I^a_k + b_1u_k \tag{6}
\]

\[
I^a_{k+1} = a_{21}I^a_k + a_{22}S^p_k + a_{23}I^b_k + b_2u_k \tag{7}
\]

\[
I^b_{k+1} = a_{31}I^b_k + a_{32}I^a_k + a_{33}I^p_k + b_3u_k \tag{8}
\]

\[
b^k_{k+1} = a_{41}I^k_k + a_{42}I^b_k + a_{43}D_k + b_4u_k \tag{9}
\]

\[
D_{k+1} = a_{51}D_k + a_{52}I^b_k + b_5u_k \tag{10}
\]
The next section reviews the parametrized non-pharmacological policies imposed to battle against the pandemic diseases. These non-pharmacological policies are (A) Curfews on the people with chronic diseases, age over 65, and age under 20, (B) Curfews on the weekends and holidays, (C) Closure and re-opening of the schools and universities. Since the data for these non-pharmacological policies are not directly available, it is necessary to develop mathematical models which imitate the response of them.

3.1. Curfews on people with chronic diseases, age over 65, and age under 20

As people with chronic diseases and age over 65 are highly defenceless against the outbreaks, curfews are implemented on them primarily. Even though people age under 20 are resilient against the outbreaks, as they are super spreaders of the virus, they are put under curfews as well.

To model such curfews, consider these facts about the pandemics:

- It is reported that symptoms of a pandemic diseases can appear in 14 days where the peak point is around day 7 as reported by the WHO [26]. Therefore, a non-pharmacological policy should have a transient ascent part that reaches the peak point around day 7 and a transient descent part that converges to zero at day 14 as shown in Fig. 4.
- If the curfew continues for a period of time, then the response of the non-pharmacological policy has a steady-state behaviour just after the transient ascent part as can be seen in Fig. 4.
- Since it is likely that the curfew can be violated by an uncertain number of people, the response of the curfew covers random non-parametric uncertainties as shown in Fig. 5.

We can model the uncertain transient ascent and steady-state parts of the response as

\[ u_k = n' \left(1 - \alpha^{k-k_i} + \sigma_k^* \right) \]  

where

- \( u_k \) is the response of the curfew (in closed form solution),
- \( n' \) is the number of the people under the curfew,
- \( k \) is the number of the days and \( k_i \) is the start day of the curfew,
- \( \alpha \) is the discount factor of the response, where \( \alpha^k \approx 0 \) for \( \alpha = 0.71 \) and \( k = 14 \),
- \( \sigma_k^* \) is the random non-parametric uncertainty in the response.
Table 2
Components of the non-linear SplIttBd-N.

| Component   | Description                                                                                                                                 |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| $a_{11}, a_{22}$ | Unknown internal parameters non-linearly associated with the $S^0$ and $I^0$.                                                               |
| $a_{11}, a_{33}, a_{41}, a_{51}$ | Unknown internal parameters of the $I^0$, $I^i$, and $D$, respectively.                                                                     |
| $a_{22}, a_{12}, a_{32}, a_{42}, a_{52}$ | Unknown coupling parameters of the $S^0$, $I^0$, $I^i$, and $D$.                                                                            |
| $b_1, b_2, b_3, b_4, b_5$ | Unknown parameters of the non-pharmacological policies for the $S^0$, $I^0$, $I^i$, $I^i$, and $D$ sub-models, respectively. |

Table 3
Components of the strongly non-linear SplIttBd-N.

| Component   | Description                                                                                                                                 |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| $a_{11}, a_{22}, a_{32}, a_{41}, a_{51}$ | Unknown internal parameters non-linearly associated with the $S^0$, $I^0$, $I^i$, $I^i$, and $D$, respectively. |
| $a_{22}, a_{32}, a_{42}$ | Unknown coupling parameters of the $I^i$, $I^i$, and $I^i$, respectively.                                                                   |
| $b_1, b_2, b_3, b_4, b_5$ | Unknown parameters of the non-pharmacological policies contributing to the $S^0$, $I^0$, $I^i$, $I^i$, and $D$ sub-models, respectively. |

**Fig. 4.** Transient ascent, steady-state, and transient descent parts of the response without uncertainty.

**Fig. 5.** The uncertain response of the curfew modelled for Turkey with the data presented in Table 4.

- When the curfew is lifted, its effect disappears in 14 days (Fig. 4) and this constitutes the transient descent part of the model represented as
  \[ u_k^* = n^i \left( a_k - k_c + \sigma_k \right) \] (17)
where

- \( k^i \) is the terminal day of the curfew.
- Even though the curfews are abolished, a number of elderly people with chronic diseases impose self-curfews which can be expressed as

\[
u_k^i = n^P \sigma_k^s
\]  \hspace{1cm} (18)

where

- \( n^P \) represents the number of the people under self-curfew.

The next sub-section reviews the parametric model of the curfew implemented on the weekends and holidays.

3.2. Curfews on the weekends and holidays

Since the duration of the curfews on the weekends and holidays are usually for two days, their response consists of the transient ascent and transient descent parts. Therefore, the response has impulse response properties whose transient ascent part is

\[
u_{i,k}^{wh} = n^{wh} (1 - \alpha^{k-i} + \sigma_{i,k}^{wh}) \delta_{i,k} \text{ for } \begin{cases} \delta_{i,k} = 1 & \text{Curfew at ith day} \\ \delta_{i,k} = 0 & i \leq k \leq i + 6 \\ \text{Otherwise} \end{cases}
\]  \hspace{1cm} (19)

where

- \( u_{i,k}^{wh} \) is the response of the curfew on the weekends and holidays,
- \( n^{wh} \) is the number of the people under the curfews on the weekends and holidays,
- \( \sigma_{i,k}^{wh} \) is the random uncertainty in the response,

The transient descent part of the response is

\[
u_{i,k}^{wh} = n^{wh} (\alpha^{i-k} + \sigma_{i,k}^{wh}) \delta_{i,k} \text{ for } \begin{cases} \delta_{i,k} = 1 & i + 7 \leq k \leq i + 14 \\ \delta_{i,k} = 0 & \text{Otherwise} \end{cases}
\]  \hspace{1cm} (20)

The overall response \( u_{i,k}^{wh} \) is

\[
u_{i,k}^{wh} = \sum_{i=k-14}^{k} u_{i,k}^{wh}
\]  \hspace{1cm} (21)

The next sub-section introduces the closure and re-opening of the schools and universities.

3.3. Closure and re-opening of the schools and universities

While closing the schools and universities can hinder the spread of the viruses, re-opening of them reverses the contribution. Modelling the closure part of the response is same as modelling the curfews on the people with chronic diseases and age over 65. However, re-opening is usually periodic (as in Turkey) since the students partially attend the schools on certain days of the week. This can be expressed as

\[
u_k^{su} = -n^P \sigma_k^{su} \cos(2\pi k/7) (1 - \alpha^{k-k_i} + \sigma_k^{su})
\]  \hspace{1cm} (22)

where

- \( u_{i,k}^{su} \) is the response of the closure and re-opening of the schools and universities,
- \( n^P \sigma_k^{su} \) is the number of the students attending the schools partially,
- \( k_i \) is the day that the schools re-open,
- \( \sigma_k^{su} \) is the random uncertainty of the response.

Fig. 6 shows the response of the model for Turkey where the schools were closed, partially re-opened, and re-closed.

The next section presents the parametrized S\(^P\)D-N models and the batch type least squares (BLS) estimator.

4. BLS based unknown parameters learning

In this section, initially, we parametrize the S\(^P\)D-N models in terms of the known bases and unknown parameters. Then, we use the BLS optimization approach to determine the unknown parameters.

4.1. Formulation of the estimated S\(^P\)D-N models

The estimated sub-models of the S\(^P\)D-N models can be represented as

\[
\hat{y}_{SP} = u_{SP}^T \phi_S
\]  \hspace{1cm} (23)

\[
\hat{y}_{p} = u_{p}^T \phi_p
\]  \hspace{1cm} (24)

\[
\hat{y}_{I} = u_{I}^T \phi_I
\]  \hspace{1cm} (25)

\[
\hat{y}_{D} = u_{D}^T \phi_D
\]  \hspace{1cm} (26)

where \( \hat{y}_{SP}, \hat{y}_{p}, \hat{y}_{I}, \text{ and } \hat{y}_{D} \) are the estimated outputs for the suspicious, infected, intensive care, intubated and deaths sub-models respectively, \( \phi_S \) are the corresponding known bases covering the reported casualties. For instance, the bases of the nonlinear S\(^P\)D-N model by considering Eqs. (6) to (10) can be formed as

\[
\phi_S = [S_{1,\ldots,N-1}^b \cdot I_{1,\ldots,N-1}^b] u_{1,\ldots,N-1}
\]  \hspace{1cm} (27)

\[
\phi_p = [S_{1,\ldots,N-1}^b \cdot I_{1,\ldots,N-1}^b] u_{1,\ldots,N-1}
\]  \hspace{1cm} (29)

\[
\phi_I = [I_{1,\ldots,N-1}^b \cdot I_{1,\ldots,N-1}^b] u_{1,\ldots,N-1}
\]  \hspace{1cm} (30)

\[
\phi_D = [D_{1,\ldots,N-1}^b \cdot I_{1,\ldots,N-1}^b] u_{1,\ldots,N-1}
\]  \hspace{1cm} (32)

where \( \bullet \) is the dot product and \( u \) is the sum of all non-pharmaceutical policies. The corresponding unknown parameter vectors are

\[
w_{SP} = [a_{11} \ b_1]^T
\]  \hspace{1cm} (33)

\[
w_{p} = [a_{21} \ a_{22} \ b_2]^T
\]  \hspace{1cm} (34)

\[
w_{I} = [a_{31} \ a_{32} \ a_{33} \ b_3]^T
\]  \hspace{1cm} (35)

\[
w_{D} = [a_{41} \ a_{42} \ a_{43} \ b_4]^T
\]  \hspace{1cm} (36)

\[
w_{D} = [a_{51} \ a_{52} \ b_5]^T
\]  \hspace{1cm} (37)

The real outputs are

\[
y_{SP} = S_{1,\ldots,N}^b
\]  \hspace{1cm} (38)

\[
y_{p} = I_{1,\ldots,N}^b
\]  \hspace{1cm} (39)

\[
y_{I} = I_{1,\ldots,N}^b
\]  \hspace{1cm} (40)

\[
y_{D} = D_{1,\ldots,N}^b
\]  \hspace{1cm} (41)

4.2. The BLS formulation

Consider the error between the real outputs and estimated outputs expressed as

\[ e = y - \hat{y} \]  \hspace{1cm} (43)
where \( \hat{y} = [\hat{y}_{D} \quad \hat{y}_{I} \quad \hat{y}_{T} \quad \hat{y}_{B} \quad \hat{y}_{D}] \).

The squared error is
\[
e^2 = (y - w^T \phi)^T (y - w^T \phi)
= y^T y - 2w^T y + w^T \phi^T \phi w
\]

The slope of the squared error with respect to the unknown parameters \( w \) is
\[
\frac{\partial e^2}{\partial w} = -2\phi^T y + 2\phi^T \phi w
\]  
\[(45)\]

Since the parameter learning process terminates with the zero slope, equating \((45)\) zero yields the optimum unknown parameters formulated as
\[
w = (\phi^T \phi)^{-1} \phi^T y
\]  
\[(46)\]

The unknown parameters can be determined with Eq. \((46)\) by using the constructed bases and the outputs. The next section provides the comparative results of the three \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) models.

5. Analysis of the proposed \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) models

In this section, we analyse the proposed models by using the COVID-19 casualties reported by the Health Ministry of Turkey [2]. Turkey is chosen because the authors are able to reach the data required for the constructed models. Even though the casualties are reported daily by the Health Ministry, the data for the non-pharmacological policies are usually announced verbally, and understanding the written statements is not straightforward. To properly represent the character of the models, especially the non-pharmacological policies, the home country of the authors has been chosen. To optimize the unknown parameters of the constructed models, a batch least-squares (BLS) and a neural network (NN) approaches are considered. Initially, we introduce the background data and then present the estimated casualties with developed models. We perform the mean and the standard deviation-based analysis of the models to validate the effectiveness.

5.1. Parameters of the models

Table 4 summarizes the parameters of the models.

5.2. Analysis of the linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model

Fig. 7 presents the real and estimated outputs of the linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model with the BLS approach.

As can be seen from Fig. 7, all the sub-models of the linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model with the BLS are able to follow the real outputs including the peaks. Even though the initial intensive care \(I^k_i\) and intubated \(I^k_t\) data are not available, the linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model manages to learn the corresponding characters of the sub-models. It is also noticeable that the developed \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model closely chases the fluctuations in the real outputs. This indicates the existence of low standard deviations together with the mean errors in the estimates. Fig. 8 presents the linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model estimates with the NN approach.

NN is an iterative approach, whereas the BLS is a batch type approach. It is well known that the batch kind approaches provide accurate results for stochastic optimization problems. As can be seen from Fig. 8, that the suspicious \(S^k_i\), infected \(I^k_i\), and intubated \(I^k_t\) casualties have larger fluctuations due to the stochastic nature of the casualties. Henceforth, even though the NN can capture the dynamics of these sub-models, estimates with the NN yield slightly more fluctuations than the BLS estimates in Fig. 7. The next sub-section provides the real and estimated results of the non-linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model with the BLS and NN approaches.

5.3. Analysis of the non-linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model

Fig. 9 shows the real and estimated outputs of the non-linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model with the BLS.

The internal dynamics of the suspicious \(S^k_i\) and infected \(I^k_i\) sub-models of the non-linear \(\text{S}^\text{p}\text{I}\text{n}\text{i}\text{I}\text{P}\text{D}-\text{N}\) model are non-linearly coupled as given in Eqs. (6) and (7) and the rest of the sub-models are linearly coupled with their neighbouring sub-models. As illustrated by Fig. 9, the non-linearly coupled suspicious \(S^k_i\) and infected \(I^k_i\) sub-models have the largest estimation errors. This implies that the internal dynamics of the suspicious \(S^k_i\) and infected \(I^k_i\) sub-models are not quite non-linearly coupled for the reported casualties of Turkey. However, since the casualties of each country and also regions of the countries might have different characteristics, for other cases, these sub-models may provide much more accurate results. In addition, recursive approaches such as the NN can manage to learn the non-linear dynamics with higher accuracy than a batch kind approach. Fig. 10 exhibits that...
the NN approach is able to learn the non-linear parameters with smaller estimation errors.

As can be seen from Fig. 10, the recursive estimates of the unknown parameters associated with the non-linear couplings yield a smaller estimation error than the batch type learning shown in Fig. 9. It is known that the NN is a non-parametric modelling approach which can learn any linear or non-linear functions from the input–output data. This result shows that the NN has superior performance for optimizing the non-linear parameters. The next sub-section provides the real and estimated outputs of the strongly non-linear $S^{IP}IP$ D-N model with the BLS and NN approaches.

5.4. Analysis of the strongly non-linear $S^{IP}IP$ D-N model

Fig. 11 shows the real and estimated outputs of the strongly non-linear $S^{IP}IP$ D-N model with the BLS. All the sub-models of the strongly non-linear $S^{IP}IP$ D-N model have coupled internal dynamics as illustrated by Equations from (11) to (15) where all yield the largest estimation errors compared to the non-linear $S^{IP}IP$ D-N and linear $S^{IP}IP$ D-N models. It is noticeable that especially the death $D_k$ sub-model is unable to produce proper estimations. This indicates that it is non-linearly correlated with the intubated $I_k^p$ sub-model. The positive correlation is possibly due to the fact that the COVID-19 stemmed deaths can occur among the intubated $I_k^p$, intensive care $I_k^i$, and suspicious $S_k^p$ as well. Fig. 12 illustrates the estimates of the strongly non-linear $S^{IP}IP$ D-N model with the NN approach.

As can be seen from Fig. 12, the NN again produces smaller estimation errors but generates larger variations when the data have random characters. This is expected since the recursive approaches consider the instant data; henceforth, the latest variations can be learned while the ones in the past are forgotten. However, the batch type approaches can normalize the data and can learn the average character of them. The next sub-section compares the proposed models in terms of the corresponding mean errors and standard deviations.

5.5. Comparison analysis of the proposed models

Fig. 13 shows the mean errors and the standard deviations of the estimated outputs by the proposed models. It is clear from Fig. 13, all the sub-models of the linear $S^{IP}IP$ D-N model yield the smallest mean errors and also the smallest standard deviations for the reported casualties of Turkey. Both the mean errors and the standard deviations increase with respect to the non-linear coupling. We can deduce from these results that the linear $S^{IP}IP$ D-N model provides more accurate results for Turkey. However, the character of the non-linear dynamics depends on the healthcare infrastructure of the countries and the properties of the imposed pharmacological or non-pharmacological policies. It is also obvious from Fig. 13 that the NN approach outperforms the BLS approach in the presence of
non-linear dynamics. Nevertheless, in the case of linear dynamics, the BLS generally performs better. The NN generates more accurate results only when the casualties have less variation as in the intensive care $I^I_k$ and dead $D_k$ casualties.

5.6. Predicted future COVID-19 casualties with linear $S^P I^I P^D N$ model

Since the linear $S^P I^I P^D N$ model results in the least estimation errors, we provide the estimated future casualties with the linear $S^P I^I P^D N$ model. Fig. 14 provides predicted future casualties for Turkey with the linear $S^P I^I P^D N$ model trained with data covering the period where the non-pharmacological policies have been imposed.

We used the reported COVID-19 casualties corresponding to the period where the non-pharmacological policies such as the closure of the schools, curfews on the weekends, and restrictions on the people with chronic diseases to determine the unknown parameters of the linear $S^P I^I P^D N$ model. Otherwise, when we use the whole data, the linear $S^P I^I P^D N$ model is unstable and
all the sub-models produce unbounded outputs. This indicates the importance of non-pharmacological policies to confine the spread of the virus. As can be seen from Fig. 14 the suspicious $S_k^p$ and infected $I_k^n$ converge to zero around 120 days under the current conditions.

6. Conclusion

This paper constructed three $S^pI^pI^bD-N$ model structures consisting of the linear and non-linear representations of the pandemic dynamic. The research has confirmed that the linear $S^pI^pI^bD-N$ model yields more than 10 times smaller mean errors and standard deviations than the nonlinear $S^pI^pI^bD-N$ model. The outperformance of the linear $S^pI^pI^bD-N$ model can stem from the inclusion of the extensive non-parametric policies, which can dominate the non-linear dynamics. Moreover, the linear $S^pI^pI^bD-N$ model predicts that the casualties will reach their minimum of around 120 days under the current conditions. As future work, the parametric $S^pI^pI^bD-N$ model should be enriched with the susceptible and recovered casualties and

Fig. 9. The real and estimated outputs of the non-linear $S^pI^pI^bD-N$ model with the BLS.

Fig. 10. The real and estimated outputs of the non-linear $S^pI^pI^bD-N$ model with the NN.
the model structure should be determined based on the more accurate epidemiological facts. In addition, the model should be equipped with the priority and age-specific vaccination policy and other pharmacological advancements. Finally, the developed models should be used with artificial intelligence algorithms to make optimum policies on the control of pandemic diseases.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Fig. 13. Mean and standard deviations of the estimation errors where the bars represent the mean errors and the lines on the bars represent the standard deviations.

Fig. 14. Predicted future casualties for Turkey with the linear $S^p F^p D^p N$ model.

References

[1] Gostin LO, Karim S A, Mason Meier B. Facilitating access to a COVID-19 vaccine through global health law. J Law Med Eth 2020;48(3):622–6.
[2] Digital transformation office of the Turkish presidency. 2020, accessed 2020.
[3] COVID I, Murray CJ. Forecasting COVID-19 impact on hospital bed-days, ICU-days, ventilator-days and deaths by US state in the next 4 months. 2020, MedRxiv.
[4] Bhattacharya S, Singh A, Hossain MM. Health system strengthening through massive open online courses (MOOCs) during the COVID-19 pandemic: An analysis from the available evidence. J Educ Health Prom 2020;9.
[5] Mahajan A, Solanki R. An epidemic model SIPHERD and its application for prediction of the COVID-19 infection for India and USA. 2020, arXiv preprint arXiv:2005.00921.
[6] Peeri NC, Shrestha N, Rahman MS, Zaki R, Tan Z, Bibi S, et al. The SARS, MERS and novel coronavirus (COVID-19) epidemics, the newest and biggest global health threats: what lessons have we learned? Int J Epidemiol 2020;49(1):176–26.

[7] Yang J, Chen X, Deng X, Chen Z, Gong H, Yan H, et al. Disease burden and clinical severity of the first pandemic wave of COVID-19 in wuhan, China. Nature Commun 2020;11(1):1–10.

[8] Das P, Choudhuri T. Decoding the global outbreak of COVID-19: the nature is behind the scene. VirusDisease 2020;1–7.

[9] Abdel-Bast M, Mohamed R, Elhoseny M. <?covid19?> a model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans. Health Inform J 2020. 1460458220952918.

[10] Zhu Y, Chen YQ. On a statistical transmission model in analysis of the early phase of COVID-19 outbreak. Stat Biosci 2021;13(1):1–17.

[11] Gupta A, Bherwani H, Gautam S, Anjum S, Musugu K, Kumar N, et al. Air pollution aggravating COVID-19 lethality? Exploration in Asian cities using statistical models. Environ Dev Sustain 2021;23(4):6408–17.

[12] Rozenfeld Y, Beam J, Maier H, Haggerson W, Boudreau K, Carlson J, et al. A model of disparities: risk factors associated with COVID-19 infection. Int J Equit Health 2020;19(1):1–10.

[13] Wieczorek M, Siłka J, Woźniak M. Neural network powered COVID-19 spread forecasting model. Chaos Solitons Fractals 2020;140:110203.

[14] Cucinotta D, Vanelli M. WHO declares COVID-19 a pandemic. Acta Bio Medic: Atenei Parmensis 2020;91(1):157.