QCD SUM RULE APPROACH FOR THE SCALAR MESONS AS FOUR-QUARK STATES

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We study the two point-function for the scalar mesons $\sigma$, $\kappa$, $f_0(980)$ and $a_0(980)$ as diquark-antidiquark states. We also study the decays of these mesons into $\pi\pi$, $K\pi$ and $KK$. We found that the couplings are consistent with existing experimental data, pointing in favor of the four-quark structure for the light scalar mesons.

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It is known that the identification of scalar mesons is difficult experimentally, and that the underlying structure of them is not well established theoretically, due to the complications of the nonperturbative strong interactions. Actually, the observed light scalar states below 1.5 GeV are too numerous \cite{1} to be accommodated in a single $q\bar{q}$ multiplet.

The experimental proliferation of light scalar mesons is consistent with two nonets, one below 1 GeV region and another one near 1.5 GeV. If the light scalars (the isoscalars $\sigma(500)$, $f_0(980)$, the isodoublet $\kappa$ and the isovector $a_0(980)$) form an SU(3) flavor nonet, in the naive quark model the flavor structure of these scalars would be:

\[
\begin{align*}
\sigma &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), & f_0 &= s\bar{s}, \\
a_0 &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), & a_0^+ &= u\bar{d}, & a_0^- &= d\bar{u}, \\
\kappa^+ &= u\bar{s}, & \kappa^0 &= d\bar{s}, & \kappa^- &= s\bar{u}.
\end{align*}
\]

Although with this model it is difficult to understand the mass degeneracy of $f_0(980)$ and $a_0(980)$, and it is hard to explain why $\sigma$ and $\kappa$ are broader than $f_0(980)$ and $a_0(980)$, its use is not yet discarded \cite{2,3,4,5,6}. Some alternative models allow a mixing between the isoscalars. However, different experimental data lead to different mixing angles \cite{5,6,7,8}.

By the other hand, the scalar mesons in the $1.3 - 1.7$ GeV mass region (the isoscalars $f_0(1370)$, $f_0(1500)$, the isodoublet $K_0^*(1430)$ and the isovector $a_0(1450)$) may be easily accommodated in an SU(3) flavor nonet. Therefore, theory and data are now converging that QCD forces are at work but with different dynamics dominating below and above 1 GeV mass. Below 1 GeV the phenomena point clearly towards an $S$-wave attraction among two quarks and two anti-quarks, while above 1 GeV it is the $P$-wave $q\bar{q}$ that is manifested. \cite{9}

Below 1 GeV the inverted structure of the four-quark dynamics in $S$-wave is revealed with $f_0(980)$, $a_0(980)$, $\kappa$ and $\sigma$ symbolically given by \cite{11}

\[
\begin{align*}
\sigma &= ud\bar{u}\bar{d}, & f_0 &= \frac{1}{\sqrt{2}}(us\bar{u}s + ds\bar{d}s), \\
a_0^- &= ds\bar{u}s, & a_0^+ &= \frac{1}{\sqrt{2}}(us\bar{u}s - ds\bar{d}s), & a_0^- &= us\bar{d}s, \\
\kappa^+ &= u\bar{d}\bar{u}s, & \kappa^0 &= u\bar{u}\bar{d}s, & \kappa^- &= ds\bar{u}\bar{d}.
\end{align*}
\]

This is supported by a recent lattice calculation \cite{12}. In this four-quark scenario for the light scalars, the mass degeneracy of $f_0(980)$ and $a_0(980)$ is natural, and the mass hierarchy pattern of the nonet is understandable. Besides, it is easy to explain why $\sigma$ and $\kappa$ are broader than $f_0$ and $a_0$. The decays $\sigma \to \pi\pi$, $\kappa \to K\pi$ and $f_0$, $a_0 \to KK$ are OZI superallowed without the need of any gluon exchange, while $f_0 \to \pi\pi$ and $a_0 \to \eta\pi$ are OZI allowed as it is mediated by one gluon exchange. Since $f_0(980)$ and $a_0(980)$ are very close to the $KK$ threshold, the $f_0(980)$ is dominated by the $\pi\pi$ state and $a_0(980)$ is governed by the $\eta\pi$ state. Consequently, their widths are narrower than $\sigma$ and $\kappa$.

In the four-quark scenario it is also easier to understand why, in some three-body decays of charmed mesons, the intermediate light scalar meson accounts for the main contribution to the total decay rate. For example, about half of the total decay rate of $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+$ and $D^*_+ \to \pi^-\pi^+\pi^+$ comes from $D^+ \to \sigma\pi^+$, $D^+ \to \kappa\pi^+$ and $D^*_+ \to f_0(980)\pi^+$ respectively \cite{13}, while the light scalar mesons are hardly seen in the semileptonic decays $D^+ \to \pi^-\pi^+\ell^+\nu_\ell$, $D^+ \to K^-\pi^+\ell^+\nu_\ell$ and $D^*_+ \to \pi^-\pi^+\ell^+\nu_\ell$ \cite{14}.
FIG. 1: The $D^+ \to \kappa^0$ transition where $\kappa^0$ is described as a $s\bar{d}$ state. a) and b) nonleptonic decay; c) semileptonic decay.

Consider, for example, the decays $D^+ \to \kappa^0\pi^+ \to K^-\pi^+\pi^+$ and $D^+ \to \kappa^0\ell^+\nu_\ell \to K^-\pi^+\ell^+\nu_\ell$. If $\kappa^0$ is a quark anti-quark state ($\kappa^0 = s\bar{d}$) the above mentioned decays can proceed through the diagrams in Fig. 1, from where we see that one has two possible diagrams for the nonleptonic decay (Figs. 1a and 1b), and only one diagram for the semileptonic decay (Fig. 1c). However, if $\kappa^0$ is a four-quark state ($\kappa^0 = u\bar{s}\bar{u}\bar{d}$), from Fig. 2 we see that there are four possible diagrams for the nonleptonic decay, while there is still only one diagram for the semileptonic decay. Therefore, in the four-quark scenario the probability to form a scalar meson from the decay of charmed mesons, is much bigger for the nonleptonic decay as compared with the semileptonic decay. A trend that seems be followed by the experimental data [14].

FIG. 2: The $D^+ \to \kappa^0$ transition where $\kappa^0$ is described as a $u\bar{s}\bar{u}\bar{d}$ state. a), b), c) and d) nonleptonic decay; e) semileptonic decay.

In this work we use the method of QCD sum rules (QCDSR) [15] to study the two-point function and the three-point function, associated with the meson decay constant and hadronic coupling constants of the scalar mesons, considered as four-quark states. The first evaluation of the $f_0(980)$ as a four-quark state in the QCDSR formalism was done in [16] for the meson-vacuum decay constant, and in [17] for the hadronic coupling constants (for a review see [18]). We extend these works by considering all the scalar mesons in the nonet and by considering different currents.

We follow ref. [19] and consider that the lowest lying scalar mesons are $S$-wave bound states of a diquark-antidiquark pair. As suggested in ref. [20] the diquark is taken to be a spin zero colour anti-triplet, flavour anti-triplet. Therefore, the $(q)^2(\bar{q})^2$ states make a flavour $SU(3)$ nonet. The corresponding interpolating fields are:

\[
\begin{align*}
  j_\sigma &= \epsilon_{abc}\epsilon_{def}(u_a^T C\gamma_5d_b)(\bar{u}_d\gamma_5C\bar{d}_e^T), \\
  j_f &= \frac{\epsilon_{abc}\epsilon_{def}}{\sqrt{2}} [(u_a^T C\gamma_5s_b)(\bar{u}_d\gamma_5C\bar{s}_e^T) + u \leftrightarrow d], \\
  j_a &= \frac{\epsilon_{abc}\epsilon_{def}}{\sqrt{2}} [(u_a^T C\gamma_5s_b)(\bar{u}_d\gamma_5C\bar{s}_e^T) - u \leftrightarrow d], \\
  j_\kappa &= \epsilon_{abc}\epsilon_{def}(u_a^T C\gamma_5d_b)(\bar{q}_d\gamma_5C\bar{s}_e^T), \quad \bar{q} = \bar{u}, \bar{d},
\end{align*}
\]

where $a, b, c, \ldots$ are colour indices and $C$ is the charge conjugation matrix. The other members of the nonet are easily constructed.

The coupling of the scalar meson $S$, to the scalar current $j_S$, can be parametrized in terms of the meson decay constant $f_S$ as [10]:

\[
\langle 0|j_S|S \rangle = \sqrt{2}f_sm_S^4.
\]
In order to compute this parameter by QCDSR, we consider the two-point correlation function

\[ \Pi(q) = i \int d^4x \ e^{i q \cdot x} \langle 0 | T[j_S(x) j_S^\dagger(0)] | 0 \rangle. \]  

(5)

In the QCD side we work at leading order and consider condensates up to dimension six. We deal with the strange quark as a light one and consider the diagrams up to order \( m_s \). In the phenomenological side we consider the usual pole plus continuum contribution. Therefore, we introduce the continuum threshold parameter \( s_0 \) \cite{21}. In the \( SU(2) \) limit the \( f_0 \) and \( a_0 \) states are, of course, mass degenerate, and we get the same decay constant for them. After doing a Borel transform the two-point sum rules are given by:

\[
\begin{align*}
2f_0^2 m_{a_0}^8 e^{-m_{a_0}^2/M^2} &= \frac{M_{10}^{10} E_4}{2^{10} 3 \pi^6} + \frac{\langle g^2 G^2 \rangle M^6 E_2}{2^{6} \pi^4} + \frac{\langle \bar{q} q \rangle^2 M^4 E_1}{12 \pi^2}, \\
2f_0^2 m_{a_0}^8 e^{-m_{a_0}^2/M^2} &= \frac{M_{10}^{10} E_4}{2^{10} 5 \pi^6} - \frac{m_s (2 \langle \bar{q} q \rangle - \langle \bar{s} s \rangle)}{2^{6} \pi^4} + \frac{\langle g^2 G^2 \rangle M^6 E_2}{2^{10} 3 \pi^6} + \frac{\langle \bar{q} q \rangle^2 + \langle \bar{q} q \rangle \langle \bar{s} s \rangle}{24 \pi^2} M^4 E_1, \\
2f_0^2 m_{f_0}^8 e^{-m_{f_0}^2/M^2} &= \frac{M_{10}^{10} E_4}{2^{7} 3 \pi^4} + \frac{m_s (\bar{s} g \sigma G \bar{s})}{2^{6} \pi^4} - \frac{m_s (\bar{q} g \sigma G \bar{q})}{2^{7} \pi^4} M^4 E_1 + \frac{3}{2} \ln(M^2/\Lambda^2) M^4 E_1, \\
2f_0^2 m_{f_0}^8 e^{-m_{f_0}^2/M^2} &= \frac{M_{10}^{10} E_4}{2^{7} 3 \pi^4} + \frac{m_s (\bar{s} g \sigma G \bar{s})}{2^{6} \pi^4} + \frac{m_s (\bar{q} g \sigma G \bar{q})}{2^{7} \pi^4} \left( \frac{3}{2} \ln(M^2/\Lambda^2) \right) M^4 E_1,
\end{align*}
\]

(6)

where

\[ E_n = 1 - e^{-s_0/M^2} \sum_{k=0}^{n} \left( \frac{s_0}{M^2} \right)^k \frac{1}{k!}, \]

(7)

which accounts for the continuum contribution.

FIG. 3: The scalar meson decay constants as a function of the Borel mass. Solid: \( f_{f_0} = f_{a_0} \); dashed: \( f_s \); dots: \( f_{a_0} \). In the numerical analysis of the sum rules, the values used for the strange quark mass and condensates are: \( m_s = 0.13 \) GeV, \( \langle \bar{q} q \rangle = -(0.23)^3 \) GeV\(^3\), \( \langle \bar{s} s \rangle = 0.8 \langle \bar{q} q \rangle \), \( \langle \bar{q} g \sigma G \bar{q} \rangle = m_0^2 \langle \bar{q} q \rangle \) with \( m_0^2 = 0.8 \) GeV\(^2\) and \( \langle g^2 G^2 \rangle = 0.5 \) GeV\(^4\). We estimate the decay constants by using the experimental values for the smaler meson masses \[13\]: \( m_0 = 0.5 \) GeV, \( m_s = 0.8 \) GeV, \( m_{f_0} = 0.98 \) GeV. For the continuum thresholds we use \( s_0 = 1.0 \) GeV\(^2\), \( s_0 = 1.2 \) GeV\(^2\), \( s_0 = 1.5 \) GeV\(^2\).

From Fig. 1 we see that we get a very stable result, as a function of the Borel mass, for \( f_{f_0} \) and \( f_{a_0} \). In the case of \( f_s \) the stability is not so good, but it is still acceptable. The problem with these sum rules, as already noticed in ref. \[10\], is that the continuum contribution is only smaller than the pole contribution for small values of the Borel
This method was also applied to the nucleon-kaon-hyperon coupling \(^{(2, 3, 24)}\), to the \(J/\psi\) the quark condensate. Therefore, working up to dimension eight in this case we get proportional to \(1\)

In order to study the scalar-pseudoscalar-pseudoscalar vertex associated with the \(\sigma \to \pi^+\pi^-, \kappa \to K^+K^-, f_0(a_0) \to K^+K^-\) and \(f_0 \to \pi^+\pi^-\) decays, we consider the three-point function

\[
T_{\mu\nu}(p, p', q) = \int d^4x \ d^4y \ e^{i\cdot(p' - x)} e^{i\nu \cdot y} \langle 0|T\{j_{\gamma\mu}^F(x)j_{\gamma\nu}^F(y)J_S^0(0)\}|0\rangle,
\]

where \(p = p' + q\), \(S\) denotes the scalar meson and \(P_1, P_2\) are the two pseudoscalar mesons in the vertex, for which we use the axial currents:

\[
j_{\gamma\mu}^S = i\gamma_\mu \gamma_5 u_a, \quad j_{\gamma\mu}^\pi = \bar{u}_a \gamma_\mu \gamma_5 u_a, \quad j_{\gamma\mu}^K = \bar{s}_a \gamma_\mu \gamma_5 s_a, \quad j_{\gamma\mu}^\kappa = \bar{u}_a \gamma_\mu \gamma_5 s_a.
\]

In order to evaluate the phenomenological side we insert intermediate states for \(P_1, P_2\) and \(S\), and we use the definitions in Eqs. (4) and (11) bellow:

\[
\langle 0|j_{\gamma\mu}^F|P_1(p)\rangle = i p_\mu F^\mu_{P_1}.
\]

We obtain the following relation

\[
T_{\mu\nu}^{\text{phen}}(p, p', q) = \frac{\sqrt{2}F_{P_1} F_{P_2} m_S^2 f_S}{(m_S^2 - p^2)(m_{P_1}^2 - p'^2)(m_{P_2}^2 - q^2)} g_{SP_{1}P_{2}} p'_\mu q_\nu + \text{ contributions of higher resonances},
\]

where the coupling constant \(g_{SP_{1}P_{2}}\) is defined by the matrix element

\[
\langle P_1(p')P_2(q)|S(p)\rangle = g_{SP_{1}P_{2}}.
\]

Here we follow refs. \(^{17, 18}\) and work at the pion pole, as suggested in \(^{22}\) for the nucleon-pion coupling constant. This method was also applied to the nucleon-kaon-hyperon coupling \(^{23, 24}\), to the \(D^*-D-\pi\) coupling \(^{25}\) and to the \(J/\psi - \pi\) cross section \(^{26}\). It consists in neglecting the pion mass in the denominator of Eq. (12) and working at \(q^2 = 0\). In the QCD side one singles out the leading terms in the operator product expansion of Eq. (9) that mach the 1/\(q^2\) term. Up to dimension six only the diagrams proportional to the quark condensate times \(m_s\) and the four-quark condensate contribute. Making a single Borel tranform to both \(-p^2 = -p'^2 \to M^2\) we get:

\[
g_{\pi\pi + \pi} = \frac{\sqrt{2}F_{\pi} F_{\pi} m_\pi^2}{m_\pi^2 - m_\pi^2} (e^{-m_\pi^2/M^2} - e^{-m_\pi^2/M^2}) = \frac{2}{3}(\bar{q}q)\),
\]

\[
g_{\kappa K + \kappa} = \frac{\sqrt{2}F_{K} F_{K} m_K^2}{m_K^2 - m_K^2} (e^{-m_K^2/M^2} - e^{-m_K^2/M^2}) = \frac{(\bar{q}q)}{3} (\langle \bar{q}q \rangle + \langle \bar{s}s \rangle) - \frac{m_s}{8\pi^2}(\bar{q}q)M^2 \left(1 - e^{-s_0/M^2}\right),
\]

\[
g_{f_0K + f_0} = \frac{\sqrt{2}F_{f_0} F_{f_0} m_{f_0}^2}{m_{f_0}^2 - m_{f_0}^2} (e^{-m_{f_0}^2/M^2} - e^{-m_{f_0}^2/M^2}) = \frac{1}{6\sqrt{2}} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2 + \langle \bar{q}q \rangle \langle \bar{s}s \rangle) + \frac{m_s}{16\sqrt{2}} (\langle \bar{q}q \rangle + \frac{2}{3}(\bar{s}s)) M^2 \left(1 - e^{-s_0/M^2}\right).
\]

As said in the introduction, the \(f_0 \to \pi^+\pi^-\) decay is mediated by one gluon exchange. Therefore, the first diagram proportional to 1/\(q^2\) (at leading order) is the eight dimension condensate proportional to the mixed condensate times the quark condensate. Therefore, working up to dimension eight in this case we get

\[
g_{f_0\pi + \pi} = \frac{\sqrt{2}F_{\pi} F_{f_0} m_{f_0}^2}{m_{f_0}^2 - m_{f_0}^2} (e^{-m_{f_0}^2/M^2} - e^{-m_{f_0}^2/M^2}) = \frac{m_{f_0}^2(\bar{q}q)(\bar{s}s)}{12\sqrt{2}M^2}.
\]

The problem of doing a single Borel transformation is the fact that terms associated with the pole-continuum transitions are not suppressed \(^{27}\). In ref. \(^{27}\) it was explicitly shown that the pole-continuum transitions have a
different behaviour, as a function of the Borel mass, as compared with the double pole contribution: it grows with $M^2$. Therefore, the pole-continuum contribution can be taken into account through the introduction of a parameter $A$ in the phenomenological side of the sum rules in Eqs. (14), (15), by making the substitution $g_{SP_1 P_2} \rightarrow g_{SP_1 P_2} + AM^2$ \[24, 25, 26].

Using $F_s = \sqrt{2}$ 93 MeV, $F_K = 160$ MeV, $m_\pi = 137$ MeV, $m_K = 490$ MeV and the scalar decay constant given by the sum rules in Eq. (8) we show, in Fig. 4 the QCDSR results for the hadronic coupling constants. We see that, in the Borel range used for the two-point functions, the QCDSR results do have a linear behaviour as a function of the Borel mass. Fitting the QCDSR results by a linear form: $g_{SP_1 P_2} + AM^2$ (which is also shown in Fig. 4), the hadronic couplings can be obtained by extrapolating the fit to $M^2 = 0$. In the limits of the continuum thresholds discussed above we obtain:

$$g_{\sigma^{+}\pi^{-}} = (3.1 \pm 0.5) \text{ GeV}, \quad g_{\kappa^{+}\pi^{-}} = (3.6 \pm 0.3) \text{ GeV},$$
$$g_{f_{0}^{+}K^{-}} = g_{a_{0}^{+}K^{-}} = (1.6 \pm 0.1) \text{ GeV}, \quad g_{f_{0}^{+}\pi^{-}} = (0.47 \pm 0.05) \text{ GeV}.$$  \[16\]

The decay width of $S \rightarrow P_1 P_2$ is given in terms of the hadronic coupling $g_{SP_1 P_2}$ as:

$$\Gamma(S \rightarrow P_1 P_2) = \frac{1}{16\pi m_S} g_{SP_1 P_2}^2 \sqrt{\lambda(m_S^2, m_{P_1}^2, m_{P_2}^2)},$$  \[17\]

where $\lambda(m_S^2, m_{P_1}^2, m_{P_2}^2) = m_S^4 + m_{P_1}^4 + m_{P_2}^4 - 2m_S^2m_{P_1}^2 - 2m_S^2m_{P_2}^2 - 2m_{P_1}^2m_{P_2}^2$.

The experimental total decay width is related with a particular decay mode through:

$$\Gamma(S \rightarrow \pi \pi) = \frac{3}{2} \Gamma(S \rightarrow \pi^+ \pi^-), \quad \text{for } S = \sigma \text{ or } f_0,$$
$$\Gamma(\kappa \rightarrow K \pi) = \frac{3}{2} \Gamma(\kappa \rightarrow K^+ \pi^-).$$  \[18\]

Therefore, using the experimental results: $\Gamma(f_0 \rightarrow \pi \pi) = 40 \pm 100$ MeV \[1\], $\Gamma(\sigma \rightarrow \pi \pi) = (338 \pm 48)$ MeV \[13\], and $\Gamma(\kappa \rightarrow K \pi) = (410 \pm 58)$ MeV \[13\] in Eqs. (17) and (18) above we get

$$g_{\sigma^{+}\pi^{-}}^{exp} = (2.6 \pm 0.2) \text{ GeV}, \quad g_{\kappa^{+}\pi^{-}}^{exp} = (4.5 \pm 0.4) \text{ GeV}, \quad g_{f_{0}^{+}\pi^{-}}^{exp} = (1.6 \pm 0.8) \text{ GeV}.$$  \[19\]

Comparing Eqs. (16) and (19) we see that, although not exactly in between the experimental error bars, the hadronic couplings determined from the QCDSR calculation are consistent with existing experimental data. The biggest discrepancy is for $g_{f_{0}^{+}\pi^{-}}$ and this can be understood since, probably in this case, $\alpha_s$ corrections could play an important role. In the case of the decay $f_0(a_0) \rightarrow K^+ K^-$, the coupling can not be experimentally measured due to the unavailable phase space.
We have presented a QCD sum rule study of the scalar mesons considered as diquark-antidiquark states. We have evaluated the mesons decay constants and the hadronic couplings associated with the $\sigma \rightarrow \pi^+\pi^-$, $\kappa \rightarrow K^+\pi^-$, $f_0(a_0) \rightarrow K^+K^-$ and $f_0 \rightarrow \pi^+\pi^-$ decays, using two-point and three-point functions respectively. We found that the couplings are consistent with existing experimental data. Therefore, we consider this result as one more point in favor of the four-quark structure for the light scalar mesons.

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