Relativistic Coulomb excitation within Time Dependent Superfluid Local Density Approximation

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Within the framework of the unrestricted time-dependent density functional theory, we present for the first time an analysis of the relativistic Coulomb excitation of the heavy deformed open shell nucleus \(^{238}\text{U}\). The approach is based on Superfluid Local Density Approximation (SLDA) formulated on a spatial lattice that can take into account coupling to the continuum, enabling self-consistent studies of superfluid dynamics of any nuclear shape. We have computed the energy deposited in the target nucleus as a function of the impact parameter, finding it to be significantly larger than the estimate using the Goldhaber-Teller model. The isovector giant dipole resonance, the dipole pygmy resonance and giant quadrupole modes were excited during the process. The one body dissipation of collective dipole modes is shown to lead a damping width \(\Gamma_1 \approx 0.4\) MeV and the number of pre-equilibrium dipole modes emitted has been quantified.

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Coulomb excitation represents an ideal method to probe the properties of large amplitude nuclear motion, because the excitation process is not obscured by uncertainties related to nuclear forces. The excitation probabilities are governed by the strength of the Coulomb field only and they can be fully expressed in terms of the electromagnetic multipole matrix elements \([3–5]\). From the theoretical point of view, Coulomb excitation can be treated as a textbook example of a nuclear system being subjected to an external, time-dependent perturbation. However, in order to be able to probe nuclear collective modes involving multi-phonon states for example \([6, 8]\), a large amount of energy has to be transferred to the nuclear system. Thus the interaction time should be relatively short and the velocity of the projectile has to be sufficiently large for an efficient excitation of nuclear modes of frequency \(\omega\), the collision time \(\tau_{coll} = b/\gamma v\) has to fulfill the condition that \(\omega \tau_{coll} \sim 1\). Here \(b\) is the impact parameter, \(v\) is the projectile velocity, and \(\gamma = (1 - v^2/c^2)^{-1/2}\) is the Lorentz factor. One of the best known examples of collective nuclear motion is the isovector giant dipole resonance (IVGDR). A reasonably good estimate of the IVGDR vibrational frequency is \(\hbar \omega \approx 80\text{MeV}/A^{1/3}\) for spherical nuclei. It implies that the excitation of a heavy nucleus to such energies requires a relativistic projectile.

We report on a new and powerful method to study relativistic Coulomb excitation and nuclear large amplitude collective motion in the framework of Time Dependent Superfluid Local Density Approximation (TD-SLDA). This is a fully microscopic approach to the problem based on an extension of the Density Functional Theory (DFT) to superfluid nuclei and time-dependent external probes, where all the nuclear degrees of freedom are taken into account on the same footing, without any restrictions and where all symmetries (translation, rotation, parity, local Galilean covariance, local gauge symmetry, isospin symmetry, minimal gauge coupling to electromagnetic (EM) fields) are correctly implemented \([3–10]\). The interaction between the impinging \(^{238}\text{U}\) projectile and the \(^{238}\text{U}\) target is very strong (\(\propto Z_p Z_t \alpha \approx 62\), where \(\alpha\) is the fine structure constant), which thus require a non-perturbative treatment, and the excitation process is highly non-adiabatic. We assume a completely classical projectile straight-line motion since its de Broglie wavelength is of the order of 0.01 fm for \(\gamma \sim 1.5 – 2\). In evaluating the EM-field created by the uranium projectile with a constant velocity \(v = 0.7c\) along the \(z\)-axis, we neglect its deformation. The projectile produces an EM-field described by scalar \(A_0\) and vector \(A\) Lienard-Wiechert potentials (note an additional \(e\)),

\[
A(r, t) = \frac{\mathbf{v}}{c} A_0(r, t) = \frac{\mathbf{v}}{c} \frac{e \gamma Ze^2}{\sqrt{(x-b)^2 + y^2 + \gamma^2(z - vt)^2}}
\]

with the reaction plane in the \(xz\)-plane. These fields couple to a deformed \(^{238}\text{U}\) target nucleus residing on a spatial lattice, see \([11]\). The interaction leads to a CM motion of the target as well as to its internal excitation and full 3D dynamical deformation of the target. In order to follow the internal motion for a long enough trajectory that allows the extraction of useful information, we perform a transformation to a system in which the lattice moves with the CM. The required transformation for each single particle wave function reads\(\phi_n(r, t) = \exp(i\mathbf{R}(t) \cdot \hat{p}) \psi_n(r, t)\), with \(\mathbf{R}(t)\) describing the CM motion and \(\hat{p}\) the momentum operator. The equa-
tion of motion (simplified form here) for $\phi_n$ becomes

$$ih\dot{\phi}_n(r, t) = \left[ \hat{H}(r + \mathbf{R}(t), t) + \mathbf{R}(t) \cdot \mathbf{p} \right] \phi_n(r, t), \quad (1)$$

where $\mathbf{R}(t) = \int d^3r \, j(r, t)/M$ is the CM velocity and $j(r, t)$ the total current density.

The target nucleus is described within the SLDA and its time evolution is governed by the TD meanfield-like equations (spin degrees of freedom are not shown):

$$ih \frac{\partial}{\partial t} \begin{pmatrix} U(r, t) \\ V(r, t) \end{pmatrix} = \begin{pmatrix} h(r, t) & \Delta(r, t) \\ \Delta^*(r, t) & -h^*(r, t) \end{pmatrix} \begin{pmatrix} U(r, t) \\ V(r, t) \end{pmatrix}, \quad (2)$$

The single-particle Hamiltonian $h(r, t)$ and the pairing field $\Delta(r, t)$ are obtained self-consistently from an energy functional that is in general a function of various normal, anomalous, and current densities, and which also contains the time-dependent external fields. The external electromagnetic (EM) field has the minimal gauge coupling $\mathbf{A} = \nabla - ia/c (\text{and similarly for the time-component})$ in all terms with currents, as well as in the definition of the momentum operator $\mathbf{p}$ in Eq. (1), details in [11]. In the current calculation, the Skyrme SLy4 energy functional [12] was adopted, with nuclear pairing as introduced in Ref. [13], which provides a very decent description of the IVGDR in $^{238}$U [10]. The coupling between the spin and the magnetic field was neglected. The Coulomb self-interaction between protons of the target nucleus is taken into account using the modification of the method described in Ref. [14], so as not to include contributions from images in neighboring cells. For the description of the numerical methods see Refs. [13, 16] and many other technical details can be found in [11].

The DFT approach to quantum dynamics has some peculiar characteristics, which might appear strange at the beginning. Unlike a regular quantum mechanics (QM) treatment one does not have access to wave functions, but instead to various one-body densities and currents. Within a DFT approach quantities, which are trivial to evaluate in QM, become basically impossible to calculate. For example, by solving the Schrödinger equation one can evaluate at any time the probability that a system remained in its initial state from $P(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2$, where $\Psi(t)$ is the solution of the Schrödinger equation (or some of its approximations). Within a DFT framework one has access to the one-body (spin-)density $\rho(r, t)$ and one-body (spin-)current $j(r, t)$ and there is no formula yet for the probability $P(t)$ that the system remained in its initial state. One can calculate for example a quantity such as $\int d^3r \rho(r, 0) \rho(r, t)$, but there is no obvious way to relate it to the probability $P(t)$. One might try to define $P(t)$ instead through the overlap of the initial and current “Slatter determinants” constructed through the fictitious single-particle wave functions entering the DFT formalism, which is a rather arbitrary postulate. One can find quite often in literature various formulas used within DFT treatment of nuclei, which are simply “copied” from various mean field approaches, without any solid justification provided. As we will show here, restrictions inherent to a DFT approach, prevent us from being able to calculate various quantities, which within a QM approach are easy to evaluate. Even though we evaluate accurate densities and currents well beyond the linear regime, within a DFT approach we cannot separate for example the emission of one and two photons from an excited nucleus, which however could be estimated relative easily within a perturbative linear response approach such a (Q)RPA. On the other hand a DFT approach has unquestionable advantages, allowing us to go far into the non-linear regime and describe large amplitude collective motion.

The incoming projectile excites various modes in the target nucleus and the axial symmetry of the initial ground state is lost. Because $^{238}$U is highly deformed the energy of the first $2^+$ is 45 keV, corresponding to a very long rotational period, and thus during simulation time considered here ($\approx 10^{-20}$ sec.) it can be considered fixed. The identification of these modes requires certain care, since during the collision the system is beyond the linear regime and the analysis using the response function is not applicable in general. However, the information about the excited nuclear modes is carried in the subsequent EM radiation leading to nuclear de-excitation. De-excitation to the ground state via photon emission requires times of about $10^{-16}$ sec., which is four orders of
magnitude longer than in the current calculations. However, it is possible to compute the spectrum of the pre-equilibrium neutrons and gamma radiation, which allows the identification of the excited nuclear modes. We can accurately treat the system as a classical source of electromagnetic radiation and the time dependence of proton current governs the rate of emission, see Refs. [11,17,18]:

\[ P(t + r/c) = \frac{e^2}{\pi c} \sum_{l,m} \int_{-\infty}^{\infty} b_{lm}(k, \omega) e^{-i\omega t} d\omega \],

with

\[ b_{lm}(k, \omega) = \int dt d^3v e^{-i\omega t} \nabla \times j(r, t) j_l(kr) Y_{lm}^*(\hat{r}), \]

where \( \omega = kc, j_l(kr) \) is the spherical Bessel function of order \( l \), and \( j(r, t) \) is the proton current. The emission rate \( P \) is plotted in Fig. 1. The magnitude of this quantity indicates that the total amount of radiated energy during the evolution time (about 2500 fm/c) is rather small compared to the total absorbed energy and does not exceed 1 MeV, which is about 2 – 3% of the deposited energy reported in Table I below. This implies that the effect of damping of nuclear motion due to the emitted radiation can be neglected for such short time intervals. Consequently, the decreasing intensity of the radiation, see Fig. 1 is merely related to the rearrangements of the intrinsic structure of the excited nucleus caused by damping of collective modes due to the one-body dissipation mechanism.

TDSLDA provides the EM power spectrum [11,17,18],

\[ \frac{dE}{d\omega} = \frac{4e^2}{c} \sum_{l,m} |b_{lm}(k, \omega)|^2, \]

arising from various multipoles in Eqs. (3) and (4). This quantity is different from what one would compute within a linear response approach or first order perturbation theory, see e.g. Refs. [1,2], which provides the excitation probability only \( \propto \int d^3r \rho_{\gamma}(r)V_{ext}(r)^2 \), where \( \rho_{\gamma}(r) \) is the transition density and \( V_{ext}(r) \) - the external field. \( dE/d\omega \) is proportional to the excitation probability, here in the non-linear regime, and the subsequent photon emission probability as well. A typical example of the emitted EM radiation for a given impact parameter is shown in Fig. 2(a) due to the internal excitation of the system alone. The EM radiation due the CM motion has been calculated separately (see Table II and [11]).

In Fig. 2(a) the emitted radiation shows a well defined maximum at energy 10 – 12 MeV which corresponds to the excitation of IVGDR. We have applied a smoothing of the original calculations using the half-width of 1 MeV. Therefore, the original separate peaks split due to the deformation of \( ^{238}U \) merge into a single wider peak. However, at larger frequencies another local maximum exists which we associate with the isovector giant quadrupole resonance (IVGQR). In order to rule out other possibilities we have repeated the calculation by retaining only the dipole component of the electromagnetic field produced by the projectile [11]. The results are shown in Fig. 2(b). In this case, the high-energy structure above 20 MeV disappears, evidence that the high energy peak is related to the IVGQR. Moreover, noticeable contribution to the total radiation is coming from the quadrupole component of radiated field.

At low energies a change of slope occurs at about \( \hbar \omega \approx 7 \text{ MeV} \) and it is also present at the same energy for other impact parameters and orientations, see [11]. It clearly indicates that there is a considerable amount of strength at low energies, giving rise to an additional contribution to the EM radiation. We attribute this additional structure to the excitation of the pygmy dipole resonance (PDR). The inset of the figure 2 shows the spectrum of emitted radiation due to this mode. The contribution to the total radiated energy coming from the PDR is rather small and reads: 1.7, 1.5 and 0.8 keV for impact parameters 12.2, 16.2 and 20.2 fm, respectively. It corresponds to about 0.07%, 0.14%, 0.15% of the emitted radiation (due to internal motion) respectively, which is increasing with the impact parameter. The relatively small amount of E1 strength obtained in our calculations, in the region where the PDR is expected, agrees with the experimental findings in the recent measurements performed on \( ^{238}U \) [19].

The comparison between the average energy transferred to the internal motion of the target nucleus for

![Fig. 2](image-url)
TABLE I. Internal excitation energy in TD-SLDA ($E_{\text{int}}$) and in the Goldhaber-Teller model ($E_{\text{GT}}$) for three values of impact parameters $b$ and two orientations of the nucleus with respect to the beam. In the third column we list the ratios of the internal excitation energies to the total transferred energy, while the energy radiated from the excited nucleus during time interval $\delta t = 2500 \text{ fm}/c$ after collision is shown in the fourth column. The ratio of the energy emitted due to the internal motion to the total energy radiated in the same interval is shown in column five.

| $b$ (fm) | $E_{\text{int}}$ (MeV) | $E_{\text{int}}/E_\gamma$ | $E_{\text{int}}^{(\perp)}$ (MeV) | $E_{\text{int}}^{(\perp)}/E_\gamma$ | $E_{\text{GT}}$ |
|----------|-------------------------|--------------------------|-------------------------------|---------------------------------|--------------|
| 12.2 || 25.11 | 0.568 | 0.5 | 0.941 | 17.05 |
| 16.2 || 8.966 | 0.470 | 0.217 | 0.939 | 7.33 |
| 20.2 || 3.798 | 0.367 | 0.106 | 0.934 | 3.47 |
| 12.2 ⊥ | 39.29 | 0.668 | 0.911 | 0.960 | 19.33 |
| 16.2 ⊥ | 12.87 | 0.547 | 0.411 | 0.963 | 8.6 |
| 20.2 ⊥ | 5.413 | 0.444 | 0.199 | 0.961 | 4.21 |

...three values of the impact parameter obtained within TDSLDA and also within a simplified Goldhaber-Teller model [20], see Table I shows that significantly more energy is deposited by the projectile within the TDSLDA. The Goldhaber-Teller model is equivalent to a linear response result, assuming that all isovector transition strength is concentrated in two sharp lines, corresponding to an axially deformed target. An exact QRPA approach would therefore severely underestimate the amount of internal energy deposited, one reason being the non-linearity of the response, naturally incorporated in TDSLDA. The other reason being the fact that the present microscopic framework describing the target allows for many degrees of freedom, apart from pure dipole oscillations, to be excited. At the same time, the CM target energy alone is approximately the same as obtained in a simplified point particles Coulomb recoil model of both the target and projectile.

The average energy radiated due to the internal excitation represents only part of the total radiated energy. (One should remember that a straightforward DFT approach provides no measure for the variance.) Also, because of the spreading of the strength due to one-body dissipation only a fraction of the energy $\Gamma_\gamma/\Gamma$ (where $\Gamma_\gamma$ is the EM-width alone and $\Gamma$ the total width of the IVGDR) is emitted as a pulse, as shown in Fig. I. A subsequent pulse of reduced amplitude is to be expected after a delay $\tau = \Gamma/(\Gamma_\gamma\omega_{\text{IVGDR}}) \approx 10^5 \ldots 10^6 \text{ fm}/c$. Since our simulations times are much shorter we are not able to see emission of the second photon, as reported in experiment [10], where two photons were measured in coincidence. In our calculations we have followed the nuclear evolution during approximately 2500 fm/c after collision, see [11]. The other component of the EM radiation arises from the CM acceleration as a result of collision (Bremsstrahlung). This process takes place only during the relatively short time interval $\tau_{\text{coll}} = b/v_\gamma$, see [11]. The radiation emitted from the internal motion has a much longer time scale.

We can estimate the cross section for the emission of radiation by means of

$$\sigma_\gamma \propto 2\pi \int \left( \frac{2}{3} E_{\perp}(b) + \frac{1}{3} E_{\parallel}(b) \right) bdb \approx 127 \text{ mb, (6)}$$

where $E_{\perp}(b)$ and $E_{\parallel}(b)$ are the total energies transferred to the target nucleus during the collision at the impact parameter $b$ and for two orientations of the target nucleus: perpendicular and parallel to the beam, respectively. A detailed comparison of intensities of radiation for various impact parameters and orientations is shown in Table I. It is evident that although the intensity of radiation decreases with increasing impact parameter, the ratio between the intensities due to the internal modes with that of the CM motion remains fairly constant. It is also slightly different for two orientations. In the case of the perpendicular orientation the target nucleus is more efficiently excited which results in a larger ratio. Interestingly, the ratio of the energy radiated (fourth column) to the energy deposited (second column) increases with increasing impact parameter. Eq. (6) can be decomposed into contributions from various frequencies of emitted radiation and results are shown in [11].

The average energy deposited in the target nucleus is of the order of the neutron separation energy. In Fig. I we plot the total number of neutrons inside the sphere of radius 15 fm around the target nucleus as a function of time. Both the perpendicular (solid lines) and parallel (dashed lines) are shown, for three values of the impact parameters: 12.2, 16.2, and 20.2 fm.

FIG. 3. (color online) The number of neutrons inside the sphere of radius 15 fm around the target nucleus as a function of time. Both the perpendicular (solid lines) and parallel (dashed lines) are shown, for three values of the impact parameters: 12.2, 16.2, and 20.2 fm.
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Supplemental material to:

Relativistic Coulomb excitation within Time Dependent Superfluid Local Density Approximation

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Density functional in TDSLDA and coupling to electromagnetic field

Here we present various definitions and conventions which we have used in the manuscript. The density functional is constructed from the following local densities and currents:

- density: \( \rho(r) = \rho(r, r') |_{r=r'} \)
- spin density: \( \tilde{s}(r) = \tilde{s}(r, r') |_{r=r'} \)
- current: \( \tilde{j}(r) = \frac{1}{2}(\nabla - \nabla')\rho(r, r') |_{r=r'} \)
- spin current (2nd rank tensor): \( J(r) = \frac{1}{2i}(\nabla - \nabla') \otimes \tilde{s}(r, r') |_{r=r'} \)
- kinetic energy density: \( \tau(r) = \nabla \cdot \tilde{\nabla}' \rho(r, r') |_{r=r'} \)
- spin kinetic energy density: \( \tilde{T}(r) = \nabla \cdot \tilde{\nabla}' \tilde{s}(r, r') |_{r=r'} \)
- anomalous density: \( \chi(r) = \chi(r, r' \sigma') |_{r=r', \sigma=1, \sigma'=-\sigma} \)

where

\[
\begin{align*}
\rho(r, r') &= \sum_{\mu} \left( V_{\mu}^*(r, +) V_{\mu}(r', +) + V_{\mu}^*(r, -) V_{\mu}(r', -) \right) \\
\tilde{s}_x(r, r') &= \sum_{\mu} \left( V_{\mu}^*(r, +) V_{\mu}(r', -) + V_{\mu}^*(r, -) V_{\mu}(r', +) \right) \\
\tilde{s}_y(r, r') &= i \sum_{\mu} \left( V_{\mu}^*(r, +) V_{\mu}(r', -) - V_{\mu}^*(r, -) V_{\mu}(r', +) \right) \\
\tilde{s}_z(r, r') &= \sum_{\mu} \left( V_{\mu}^*(r, +) V_{\mu}(r', +) - V_{\mu}^*(r, -) V_{\mu}(r', -) \right) \\
\tau(r, r') &= \sum_{\mu} \left( \nabla V_{\mu}^*(r, +) \cdot \nabla V_{\mu}(r', +) + \nabla V_{\mu}^*(r, -) \cdot \nabla V_{\mu}(r', -) \right) \\
\tilde{j}(r) &= -Im \left( \sum_{\mu} V_{\mu}^*(r, +) \cdot \nabla V_{\mu}(r, +) + V_{\mu}^*(r, -) \cdot \nabla V_{\mu}(r, -) \right) = Im \left( \sum_{\mu} V_{\mu}(r, +) \cdot \nabla V_{\mu}^*(r, +) + V_{\mu}(r, -) \cdot \nabla V_{\mu}^*(r, -) \right) \\
J_x(r) &= Im \left( \frac{\partial}{\partial y} \tilde{s}_x(r, r') - \frac{\partial}{\partial z} \tilde{s}_y(r, r') \right) |_{r=r'} \\
J_y(r) &= Im \left( \frac{\partial}{\partial z} \tilde{s}_x(r, r') - \frac{\partial}{\partial x} \tilde{s}_z(r, r') \right) |_{r=r'} \\
J_z(r) &= Im \left( \frac{\partial}{\partial x} \tilde{s}_y(r, r') - \frac{\partial}{\partial y} \tilde{s}_z(r, r') \right) |_{r=r'} \chi(r, \sigma, r' \sigma') = \sum_{\mu} V_{\mu}^*(r, \sigma) U_{\mu}(r', \sigma')
\end{align*}
\]
The coupling of the nuclear system to the electromagnetic field:

\[
\vec{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}
\]

\[
\vec{B} = \nabla \times \vec{A}
\]

requires the following transformation of proton densities and currents (subscript \(A\) denotes the quantities in the presence of electromagnetic field): The coupling of the nuclear system to the electromagnetic field:

\[
e\vec{E} = -\nabla A_0 - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}
\]

\[
e\vec{B} = \nabla \times \vec{A}
\]

(we included the electron charge \(e\) into the definition of EM potentials) requires the following transformation of proton densities and currents:

- density: \(\rho_A(r) = \rho_A(r)\)
- spin density: \(\vec{s}_A(r) = \vec{s}(r)\)
- current: \(\vec{j}_A(r) = \vec{j}(r) - \frac{1}{\hbar c} \vec{A} \rho(r)\)
- spin current (2nd rank tensor): \(\vec{J}_A(r) = \vec{J}(r) - \frac{1}{\hbar c} \vec{A} \otimes \vec{s}(r)\)
- spin current (vector): \(\vec{j}_A(r) = \vec{j}(r) - \frac{1}{\hbar c} \vec{A} \times \vec{s}(r)\)
- kinetic energy density: \(\tau_A(r) = \left( \nabla - i \frac{\hbar c}{\mu_0} \vec{A} \right) \cdot \left( \nabla' + i \frac{\hbar c}{\mu_0} \vec{A} \right) \rho(r, r') \big|_{r=r'}\)
  \[= \tau(r) - 2 \frac{\hbar c}{\mu_0} \vec{A} \cdot \vec{j}(r) + \frac{\hbar^2}{\mu_0^2 |\vec{A}|^2} \rho(r)\]
- spin kinetic energy density: \(\vec{T}_A(r) = \left( \nabla - i \frac{\hbar c}{\mu_0} \vec{A} \right) \cdot \left( \nabla' + i \frac{\hbar c}{\mu_0} \vec{A} \right) \vec{s}(r, r') \big|_{r=r'}\)
  \[= \vec{T}(r) - 2 \frac{\hbar c}{\mu_0} \vec{A}^T \cdot \vec{J}(r) + \frac{\hbar^2}{\mu_0^2 |\vec{A}|^2} \vec{s}(r)\]

As a result the proton single particle hamiltonian has the form:

\[
h_A(r) = -\nabla_A \cdot \left( \vec{B} + \vec{A} \cdot \vec{C}(r) \right) - \vec{A}(r) + U_A(r) + \frac{1}{2i} \left( \vec{W}(r) \cdot (\nabla_A \times \vec{A}) + \vec{A} \cdot (\vec{A} \times \vec{W}(r)) \right)
\]

\[
+ \vec{U}_0^A(r) \cdot \vec{s} + \frac{1}{i} \left( \vec{\nabla}_A \cdot \vec{U}_0^A(r) + \vec{U}^A_A(r) \cdot \vec{\nabla}_A \right)
\]

and

\[
U_A(r) = U(r) - C T \frac{\hbar c}{\mu_0} \nabla \cdot [\vec{A} \times \nabla(r)] - C T \left( 2 \frac{1}{\hbar c} \vec{A} \cdot \vec{j}(r) + \frac{\hbar^2}{\mu_0^2 |\vec{A}|^2} \rho(r) \right)
\]

\[
\vec{U}_0^A(r) = \vec{U}_0^A(r) - C T \frac{\hbar c}{\mu_0} \nabla \cdot [\vec{A} \rho(r)] - C T \left( 2 \frac{1}{\hbar c} \vec{A} \cdot \vec{J}(r) + \frac{\hbar^2}{\mu_0^2 |\vec{A}|^2} \vec{s}(r) \right)
\]

\[
\vec{U}_0^A(r) = \vec{U}_0^A(r) - C_T \frac{\hbar c}{\mu_0} \vec{A} \rho(r)
\]

\[
\vec{\nabla}_A \cdot \left( \vec{B} + \vec{A} \cdot \vec{C}(r) \right) \vec{\nabla}_A = \left[ \vec{\nabla}_A \left( \vec{B} + \vec{A} \cdot \vec{C}(r) \right) \right] \cdot \vec{\nabla}_A +
\]

\[
\left( \vec{B} + \vec{A} \cdot \vec{C}(r) \right) \left[ \Delta - i \frac{\hbar c}{\mu_0} (\vec{A} \cdot \vec{\nabla}_A + \vec{\nabla}_A \cdot \vec{A}) + \frac{\hbar^2}{\mu_0^2 |\vec{A}|^2} \right]
\]

We neglect in our approach the coupling of the magnetic field to nucleon magnetic moments as it represents the higher order correction and does not influence the results presented in the manuscript. Consequently the neutron single particle hamiltonian is not affected by the electromagnetic field.
Numerical Implementation

We build a spatial three-dimensional Cartesian grid in coordinate space with periodic boundary conditions, and derivatives evaluated in momentum (Fourier-transformed) space. This method represents a flexible tool to describe large amplitude nuclear motion as it contains the coupling to the continuum and between single-particle and collective degrees of freedom. For the present problem, we have considered a box size of 40\(^3\) with the lattice constant 1 fm. The time step has been set to 0.076772 fm/c with a total time interval of about 4000 fm/c. The projectile is initially placed at such a distance from the target nucleus that the collision occurs after 1600–1700 fm/c. Even though initially the projectile is far enough from the target and hence the EM fields are weak, spurious excitations produced by a sudden switch of the EM interaction at \(t = 0\) are possible. They were avoided by multiplying the EM potentials in Eqs. (29, 30) by the smoothing function \(f(t) = 1/[1 + \exp((r(t) - R_0)/a_0)]\), where \(R_0 = 800\) fm, \(a_0 = 120\) fm. This ensures that the EM field varies smoothly within the distance \(2R_0\).

Coulomb potential on the lattice

The Coulomb self-interaction of protons on the lattice has to be treated with care in order to avoid spurious interaction with the proton density images in the neighboring cells. Such effect arises due to the long tail of the Coulomb term. Here we present the method used to describe the Coulomb self-interaction of the target nucleus.

Consider the charge distribution \(e\rho(r)\):

\[
\nabla^2 A_0(r) = 4\pi e^2 \rho(r) \tag{19}
\]

\[
A_0(r) = \int d^3r' \frac{e^2 \rho(r)}{|r - r'|} \tag{20}
\]

Note that above we have defined \(A_0\) as \(eA_0\) (note \(e^2\) in the formula). After the Fourier transform one gets:

\[
A_0(r) = \int \frac{d^3k}{(2\pi)^3} \frac{e^2 \rho(\vec{k})}{k^2} \exp(i\vec{k} \cdot \vec{r}) \tag{21}
\]

The above prescription generates however the spurious interaction between neighboring cells.

Therefore we define the modified potential \((N_x, N_y, N_z\) denote number of equidistant lattice points in each direction, \(L_i = N_i\Delta x, i = x, y, z, \Delta x\) is the lattice constant):

\[
f(r) = 1/r \text{ for } r < \sqrt{L_x^2 + L_y^2 + L_z^2} \]

\[
f(r) = 0 \text{ otherwise} \tag{22}
\]

Clearly the Fourier transform is:

\[
f(k) = 4\pi \frac{1 - \cos(k\sqrt{L_x^2 + L_y^2 + L_z^2})}{k^2} \tag{23}
\]

and moreover

\[
A_0(r) = \int \frac{d^3k}{(2\pi)^3} \frac{e^2 \rho(\vec{k})}{k^2} \exp(i\vec{k} \cdot \vec{r}) = \frac{1}{27N_xN_yN_z} \sum_{\vec{k} \in L_xL_yL_z} e^2 \rho(\vec{k})f(k) \exp(i\vec{k} \cdot \vec{r}) \tag{24}
\]

where in the last term \(\rho(\vec{k})\) is the Fourier transformed density on the lattice \(27L_xL_yL_z\). In practice it means that one has to perform forward and backward Fourier transforms on the lattice three times bigger in each direction.

This may however be avoided if one realizes that the Fourier transform of the density in the larger lattice can be expressed through the Fourier transforms in the smaller lattices:

\[
\rho_{lm}(\vec{k}) = \sum_{r \in L^3} \rho(x, y, z) \exp \left(-i \left( k \frac{2\pi}{3L_x} x + l \frac{2\pi}{3L_y} y + m \frac{2\pi}{3L_z} z \right) \right) \exp(-i\vec{k} \cdot \vec{r}) \tag{25}
\]

and we need to perform 27 FFTs on the smaller lattice \(L\) for \(k, l, m = 0, 1, 2\) of the following quantities:

\[
\rho(x, y, z) \exp \left(-i \left( k \frac{2\pi}{L_x} x + l \frac{2\pi}{L_y} y + m \frac{2\pi}{L_z} z \right) \right) \]
FIG. 1. Coordinate system used to describe the reaction.

Subsequently we obtain the potential through the relation:

\[ A_0(r) = \frac{1}{27N_xN_yN_z} \sum_{k,l,m=0}^2 \left[ \sum_{\vec{k} \in L^3} e^2 \rho_{klm}(\vec{k}) f \left( \vec{k} + \left( k \frac{2\pi}{3L_x}, l \frac{2\pi}{3L_y}, m \frac{2\pi}{3L_z} \right) \right) \exp(i\vec{k} \cdot r) \right] \times \exp \left( i \left( k \frac{2\pi}{3L_x}x + l \frac{2\pi}{3L_y}y + m \frac{2\pi}{3L_z}z \right) \right). \]  

(26)

Dipole component of the electromagnetic field produced by the projectile

Coordinates (see fig. 1):

\[ R = (b, 0, vt), \quad r = (x, y, z). \]  

(27)

The interaction energy:

\[ V_{E}(t) = \int d^3r A_0(r,t) \rho_c(r,t), \]  

(28)

where

\[ A_0(r,t) = \frac{\gamma Ze^2}{\sqrt{(x-b)^2 + y^2 + \gamma^2(z-vt)^2}}, \]  

(29)

and \( \gamma = (1 - v^2/c^2)^{-1/2} \). \( \rho_c(r) = \Psi^*(r)\Psi(r) \) is the charge density of the nucleus at location \( r \) and \( \Psi(r) \) are proton wavefunctions. The vector potential is given by

\[ A(r,t) = \frac{v}{c} A_0(r,t). \]  

(30)

In order to extract the dipole component we used the interaction Hamiltonian:

\[ \mathcal{H}_{int}(r,t) = \frac{\gamma Ze^2}{\sqrt{(x-b)^2 + y^2 + \gamma^2(z-vt)^2}} - \frac{\gamma Ze^2}{\sqrt{b^2 + \gamma^2 v^2 t^2}}, \]  

(31)

where one subtracts the second term which is responsible for the c.m. scattering (i.e. monopole field).
Consequently the dipole term reads:

$$H_{E1m}(\mathbf{r}, t) = \sqrt{\frac{2\pi}{3}} r Y_{1m}(\hat{r}) \frac{\gamma Ze^2}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \left\{ \begin{array}{l} \mp b, \quad (\text{if } m = \pm 1) \\ \sqrt{2}vt, \quad (\text{if } m = 0) \end{array} \right. ,$$  

(32)

where $\mathbf{r}$ is the coordinate of one of the protons in the target. A sum over $m$ has to be performed, i.e.

$$H_{E1}(\mathbf{r}, t) = \sum_{i=\text{protons}} \sum_{m} H_{E1m}(\mathbf{r}_i, t) = \sum_{i=\text{protons}} \sum_{m} r_i^1 Y_{1m}(\hat{r}_i) f_{1m}(t),$$  

(33)

where $f_{1m}(t)$ is the part of the interaction which does not involve the intrinsic structure of the nucleus:

$$f_{1m}(t) = \sqrt{\frac{2\pi}{3}} \frac{\gamma Ze^2}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \left\{ \begin{array}{l} \mp b, \quad (\text{if } m = \pm 1) \\ \sqrt{2vt}, \quad (\text{if } m = 0) \end{array} \right. .$$  

(34)

**Goldhaber-Teller model of Coulomb excitation**

We present here the simple model for the description of the Coulomb excitation based on the Goldhaber-Teller model. The results obtained within this model have been listed in Table 1 in the manuscript and served as a reference for a fully microscopic treatment of the Coulomb excitation. Within the model it is assumed that both protons and neutrons are represented by rigid density distributions which can oscillate against each other. Thus the parameters describing the nucleus are represented by masses of neutrons and protons $M_n, M_p$, proton charge $Ze$ and frequencies of IVGDR: $\omega_x, \omega_z$. The Coulomb excitation can therefore be treated in terms of classical motion of two bodies representing protons and neutrons bound by the harmonic potential and perturbed by an external field produced by the incoming projectile. In turn the classical equation of motion reads:

$$M_p \frac{d^2 x_p}{dt^2} = F_x - \frac{d}{dt} \left( \mu \omega_x^2 (x_p - x_n) \right)$$

$$M_p \frac{d^2 z_p}{dt^2} = F_z + \frac{d}{dt} \left( \mu \omega_z^2 (z_p - z_n) \right)$$

$$M_n \frac{d^2 x_n}{dt^2} = \mu \omega_x^2 (x_p - x_n)$$

$$M_n \frac{d^2 z_n}{dt^2} = \mu \omega_z^2 (z_p - z_n)$$

(35)

where $\mu = \frac{M_p M_n}{M_p + M_n}$ is the reduced mass and

$$F_x = -\frac{d}{dx} (f(t) A_0(\mathbf{r}, t))$$

$$F_z = -\frac{d}{dz} (f(t) A_0(\mathbf{r}, t))$$

$$B_y = (\nabla \times \frac{v}{c} f(t) A_0(\mathbf{r}, t))_y.$$  

(36)

Smoothing function $f(t)$ has been described in the section: Numerical implementation.

In this model the target nucleus possess only two types of degrees of freedom: those related to the CM motion and those describing the internal harmonic excitation of GDR. In fig. 2 the results of the ultrarelativistic Coulomb excitation within this model have been shown. It corresponds to the collision of $^{238}$U on $^{238}$U (the same as described in the manuscript) at the impact parameter $b = 12.2$ fm. The frequencies of the IVGDR have been assumed to be $\hbar \omega_x = 10$ MeV and $\hbar \omega_z = 12$ MeV. The figure 3 shows the energy deposited in the target nucleus and corresponding to the internal motion (i.e. vibrations of protons against neutrons) as a function of the impact parameter.
Electromagnetic radiation from a nucleus described within TDSLDA

In this section we will use the notation $(\phi, \vec{A})$ instead of $(A_0 = e\phi, \vec{A})$ to denote the EM potentials.

Let us consider the proton density and current (we use Gauss units):

$$\rho(\vec{r}, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \rho(\vec{r}, \omega) \exp(-i\omega t)$$

$$\vec{j}(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \vec{j}(\vec{r}, \omega) \exp(-i\omega t)$$

where

$$\rho(\vec{r}, \omega) = \int \frac{d^3k}{(2\pi)^3} \rho(k, \omega) \exp(i\vec{k} \cdot \vec{r})$$

$$\vec{j}(\vec{r}, \omega) = \int \frac{d^3k}{(2\pi)^3} \vec{j}(\vec{k}, \omega) \exp(i\vec{k} \cdot \vec{r})$$

Maxwell equations:

$$\nabla \cdot \vec{E}(\vec{r}, t) = 4\pi \epsilon \rho(\vec{r}, t)$$

$$\nabla \cdot \vec{B}(\vec{r}, t) = 0$$

$$\nabla \times \vec{E}(\vec{r}, t) = -\frac{1}{c} \frac{\partial}{\partial t} \vec{B}(\vec{r}, t)$$

$$\nabla \times \vec{B}(\vec{r}, t) = \frac{1}{c} \left( \frac{\partial}{\partial t} \vec{E}(\vec{r}, t) + 4\pi \epsilon \vec{j}(\vec{r}, t) \right)$$
FIG. 3. (color online) Energy deposited in the target nucleus $^{238}$U for three values of the impact parameter: 12.2, 16.2, 20.2 fm and for two nuclear orientations: nuclear symmetry axis being parallel (squares) and perpendicular (circles) to the trajectory of incoming projectile. The same quantity is shown for the Goldhaber-Teller model, assuming that the frequencies of the dipole oscillations are $\hbar \omega = 10$ MeV and $\hbar \omega = 12$ MeV parallel (blue-dashed line) and perpendicular (red-solid line) to the nuclear symmetry axis, respectively.

and spatial Fourier transforms:

\[
\begin{align*}
  i\vec{k} \cdot \vec{E}(\vec{k}, t) &= 4\pi e\rho(\vec{k}, t) \\
  i\vec{k} \cdot \vec{B}(\vec{k}, t) &= 0 \\
  i\vec{k} \times \vec{E}(\vec{k}, t) &= -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}(\vec{k}, t) \\
  i\vec{k} \times \vec{B}(\vec{k}, t) &= \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}(\vec{k}, t) + 4\pi e\vec{j}(\vec{k}, t)\right)
\end{align*}
\]

Hence clearly:

\[
\begin{align*}
  \vec{E} &= \vec{E}_\parallel + \vec{E}_\perp \\
  \vec{B} &= \vec{B}_\perp
\end{align*}
\]

where

\[
\begin{align*}
  \nabla \times \vec{E}_\parallel (r, t) &= 0 \\
  \nabla \cdot \vec{E}_\parallel (r, t) &= 0 \\
  \nabla \cdot \vec{B}_\perp (r, t) &= 0
\end{align*}
\]

and

\[
\begin{align*}
  \vec{E} &= -\nabla \phi(r, t) - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}(r, t) \\
  \vec{B} &= \nabla \times \vec{A}(r, t)
\end{align*}
\]
Clearly
\[ \vec{E}_\parallel(r, t) = -\nabla \phi(r, t) - \frac{1}{c} \frac{\partial}{\partial t} \vec{A}_\parallel(r, t) \quad (56) \]
\[ \vec{E}_\perp(r, t) = -\frac{1}{c} \frac{\partial}{\partial t} \vec{A}_\perp(r, t) \quad (57) \]
\[ \vec{B} = \nabla \times \vec{A}_\perp(r, t) \quad (58) \]

Therefore one has a freedom to choose \( \vec{A}_\parallel(r, t) \) (gauge transformation) whereas \( \vec{A}_\perp(r, t) \) is the gauge invariant part of the vector potential.

We choose the Coulomb gauge:
\[ \vec{A}_\parallel(r, t) = 0 \Leftrightarrow \vec{A}(r, t) = \vec{A}_\perp(r, t) \quad (59) \]

Hence
\[ e\vec{E}_\parallel(r, t) = -\nabla \phi(r, t) \quad (60) \]
\[ e\vec{E}_\perp(r, t) = -\frac{1}{c} \frac{\partial}{\partial t} \vec{A}_\perp(r, t) \quad (61) \]
\[ e\vec{B} = \nabla \times \vec{A}_\perp(r, t) \quad (62) \]

and only perpendicular components of electric and magnetic fields are responsible for emission of radiation. The important equation in this case is the fourth Maxwell equation:
\[ \nabla \times \vec{B}_\perp(r, t) = \frac{1}{c} \left( \frac{\partial}{\partial t} \vec{E}_\perp(r, t) + 4\pi e \vec{j}_\perp(r, t) \right) + \frac{1}{c} \left( \frac{\partial}{\partial t} \vec{E}_\parallel(r, t) + 4\pi e \vec{j}_\parallel(r, t) \right) \quad (63) \]

Since the lhs represents the vector of type \( \perp \) therefore:
\[ \frac{\partial}{\partial t} \vec{E}_\parallel(r, t) + 4\pi e \vec{j}_\parallel(r, t) = 0 \quad (64) \]

and
\[ \nabla \times \vec{B}_\perp(r, t) = \frac{1}{c} \left( \frac{\partial}{\partial t} \vec{E}_\perp(r, t) + 4\pi e \vec{j}_\perp(r, t) \right) \quad (65) \]

Substituting the potential \( \vec{A} \):
\[ \nabla \times \nabla \times \vec{A}_\perp(r, t) = -\nabla^2 \vec{A}_\perp(r, t) = \frac{1}{c^2} \left( -\frac{\partial}{\partial t^2} \vec{A}_\perp(r, t) + 4\pi e^2 \vec{j}_\perp(r, t) \right) \quad (66) \]
\[ \nabla^2 \vec{A}_\perp(r, t) - \frac{1}{c^2} \frac{\partial}{\partial t^2} \vec{A}_\perp(r, t) = -\frac{1}{c} 4\pi e^2 \vec{j}_\perp(r, t) \quad (67) \]

where
\[ \vec{j}_\perp(r, t) = \vec{j}(r, t) - \vec{j}_\parallel(r, t) = \vec{j}(r, t) + \frac{1}{4\pi e} \frac{\partial}{\partial t} \vec{E}_\parallel(r, t) = \vec{j}(r, t) - \frac{1}{4\pi e^2} \frac{\partial}{\partial t} \nabla A_0(r, t) \quad (68) \]

Therefore in the Coulomb gauge
\[ \vec{A}(r, t) = \vec{A}_\perp(r, t) = \frac{1}{c} \int d^3 r' e^{2} \vec{j}_\perp(r', t - |r - r'|/c) \quad (69) \]

and in the far zone \( r >> r' \):
\[ \vec{A}(r, \omega) = \frac{\exp(ikr)}{r} \frac{1}{c} \int d^3 r' e^{2} \vec{j}_\perp(r', \omega) \exp(-i\vec{k} \cdot \vec{r}') = \frac{\exp(ikr)}{r} e^{2} \vec{j}_\perp(\vec{k}, ck) \quad (70) \]

where \( \vec{k} = k \hat{r} \) and \( \omega = ck \) and consequently:
\[ \vec{A}(r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \vec{A}(r, \omega) \exp(-i\omega t) = \frac{e^2}{2\pi} \int_{-\infty}^{\infty} dk \vec{j}_\perp(\vec{k}, ck) \frac{\exp(ik(\vec{r} - ct))}{r} \quad (71) \]
Consequently since $\vec{B} = \nabla \times \vec{A}$ we get:

$$ e\vec{B}(r, \omega) = \nabla \times \vec{A}(r, \omega) = \nabla \times \frac{1}{c} \int d^3\rho' e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)} = $$

$$ = \frac{1}{c} \int d^3\rho' \frac{ik(\rho - \rho') \times e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)}}{|\rho - \rho'|^2} - \frac{1}{c} \int d^3\rho' (\rho - \rho') \times e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)} $$

$$ = \frac{1}{c} \int d^3\rho' \frac{ik(\rho - \rho') \times e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)}}{|\rho - \rho'|^2} - \frac{1}{c} \int d^3\rho' (\rho - \rho') \times e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)} \frac{|\rho - \rho'|^3}{|\rho - \rho'|^3}, \quad (72) $$

where in the last line we have used the fact that rotation of the vector of type $||$ is zero. For the electric field:

$$ e\vec{E}_\perp(r, \omega) = \frac{i\omega}{c} \vec{A}(r, \omega) = \frac{i\omega}{c} \int d^3\rho' \frac{e^{\frac{i}{c} \vec{j}_\perp(\rho', \omega) \exp(ik|\rho - \rho'|)}}{|\rho - \rho'|} \quad (73) $$

$$ e\vec{E}_\parallel(r, \omega) = -\nabla \phi(r, \omega) = -\nabla \int d^3\rho' \frac{e^{\frac{i}{c} \vec{j}(\rho', \omega)}}{|\rho - \rho'|} \quad (74) $$

Hence in the far zone $r >> r'$ one gets:

$$ \vec{B}(r, \omega) = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} \int d^3\rho' k \times \vec{j}_\perp(\rho', \omega) \exp(-ik' \cdot r') = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} k \times \vec{j}(k, \omega) \quad (75) $$

$$ \vec{E}(r, \omega) = \vec{E}_\perp(r, \omega) + \vec{E}_\parallel(r, \omega) = \frac{\exp(ikr)}{r} \frac{i\omega}{c^2} j_\perp(k, ck) = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} k j_\perp(k, ck) \quad (76) $$

$$ = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} k(j(k, ck) - j_\parallel(k, ck)) = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} (k \cdot \frac{r}{r} j(k, ck) - (k \cdot j(k, ck)) \frac{r}{r}) \quad (77) $$

$$ = \frac{ie^{\frac{i}{c} \exp(ikr)}}{r} k \times \left(j(k, \omega) \times \frac{r}{r} \right) \quad (78) $$

and consequently:

$$ \vec{B}(r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \vec{B}(r, \omega) \exp(-i\omega t) = \frac{ie^{\frac{i}{c} \exp(ikr)}}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{k \times \vec{j}(k, \omega) \exp(ik(r - ct))}{r} \quad (79) $$

$$ = \frac{ie^{\frac{i}{c} \exp(ikr)}}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{k \times \vec{j}(k, \omega) \exp(-i\omega(t - r/c))}{r} \quad (80) $$

$$ \vec{E}(r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \vec{E}(r, \omega) \exp(-i\omega t) = \frac{ie^{\frac{i}{c} \exp(ikr)}}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{j(k, ck) \times \vec{k}}{r} \exp(ik(r - ct)) \quad (81) $$

$$ = \frac{ie^{\frac{i}{c} \exp(ikr)}}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{j(k, \omega) \times \vec{k}}{r} \exp(-i\omega(t - r/c)) \quad (82) $$

Note that in the above expressions $\vec{k}$ and $\omega$ are related: $\omega = c|\vec{k}|$. Poynating vector reads $\vec{S} = \frac{\vec{r}}{r^2} \vec{E} \times \vec{B}$ and thus:

$$ \vec{S}(t) = \frac{c}{4\pi(2\pi)^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \vec{E}(r, \omega) \times \vec{B}(r, \omega') \exp(-i(\omega + \omega')t) $$

$$ = \frac{c}{4\pi(2\pi)^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \vec{E}(r, \omega) \times \vec{B}(r, \omega') \exp(-i(\omega - \omega')t) $$

$$ = \frac{e^2}{4\pi(2\pi)^2 c^2} \int_{-\infty}^{\infty} d\omega (\vec{k} \times \vec{j}(\vec{k}, \omega)) \cdot (\vec{k} \times \vec{j}'(\vec{k}, \omega')) \exp(-i(\omega - \omega')t + i(k - k')r) \quad (83) $$

$$ = \frac{e^2}{4\pi(2\pi)^2 c^2} \int_{-\infty}^{\infty} d\omega \left|\int_{-\infty}^{\infty} d\omega' (\vec{k} \times \vec{j}(\vec{k}, \omega')) \exp(-i\omega t + ikr) \right|^2 \quad (84) $$

$$ = \frac{c}{4\pi r} \int_{-\infty}^{\infty} d\omega \vec{E}(r, \omega) \exp(-i\omega t) \quad (85) $$

Energy per unit time emitted to the angle $d\Omega$ reads:

$$ dP(t) = \vec{S}(t) \cdot \frac{\vec{r}}{r^2} d\Omega = \frac{e^2}{4\pi c} \left\|\int \frac{d\omega}{2\pi} (\vec{k} \times \vec{j}(\vec{k}, \omega)) \exp(-i\omega t + ikr) \right\|^2 \quad (86) $$
Hence
\[
\frac{dP}{d\Omega}(t) = \frac{e^2}{4\pi c} \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} (\vec{k} \times \vec{j}(\vec{k},\omega)) \exp(-i\omega(t - r/c)) \right|^2 = \frac{c}{4\pi} r^2 \left| \vec{B}(r, t) \right|^2
\]  
(87)

Note that the radiation at time \( t \) is given by the current at time \( t - r/c \), thus a simple time shift. Therefore the total amount of radiated energy at the angle \( d\Omega \) reads:
\[
\int_{-\infty}^{\infty} \frac{dP}{d\Omega}(t) dt = \frac{c}{4\pi} r^2 \int_{-\infty}^{\infty} \left| \vec{B}(r, t) \right|^2 dt = \frac{c}{4\pi} r^2 \int_{-\infty}^{\infty} \vec{B}(r, \omega) \cdot \vec{B}(r, \omega) \exp(-i\omega t) dt
\]
\[
= \frac{c}{4\pi} r^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \vec{B}(r, -\omega) \cdot \vec{B}(r, \omega) = \frac{c}{4\pi} r^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \vec{B}^*(r, \omega) \cdot \vec{B}(r, \omega)
\]
(88)

which gives the spectral decomposition of emitted radiation:
\[
\int_{-\infty}^{\infty} \frac{dP}{d\Omega}(t) dt = \frac{c}{8\pi^2} \int_{-\infty}^{\infty} d\omega \left| \vec{B}(r, \omega) \right|^2 = \frac{c}{4\pi^2} \int_{0}^{\infty} d\omega \left| \vec{B}(r, \omega) \right|^2
\]  
(89)

Hence the energy emitted at the angle \( \Omega \) at frequency \( \omega \) reads:
\[
\frac{dE}{dtdd\omega}(\omega) = \frac{e^2}{4\pi^2 c} \left| \vec{k} \times \vec{j}(\vec{k}, \omega) \right|^2 = \frac{e^2}{4\pi^2 c} \left[ \int d^3r \left( \nabla \times \vec{j}(r, \omega) \right) \exp(-i\vec{k} \cdot r) \right]^2
\]  
(90)

In order to calculate the quantities given by the expressions: \((87)\) \((90)\) we use the multipole expansion. Namely, let us consider eq. \((100)\):
\[
\frac{dE}{d\omega} = \int d^3r \int_{-\infty}^{\infty} dt \left( \nabla \times \vec{j}(r, \omega) \right) \exp(-i\vec{k} \cdot r, +i\omega t) \left| \vec{B}(r, t) \right|^2 d\Omega
\]
(91)

Let us denote:
\[
\nabla \times \vec{j}(r, t) = \vec{b}(r, t) \quad (93)
\]
\[
\nabla \times \vec{j}(r, \omega) = \vec{b}(r, \omega) \quad (94)
\]

We expand \(\exp(-i\vec{k} \cdot r)\):
\[
\exp(-i\vec{k} \cdot r) = 4\pi \sum_{l,m} (-i)^l j_l(kr) Y_{lm}(\hat{k}) Y_{lm}^*(\hat{r})
\]
(95)

and consequently we get
\[
\frac{dE}{d\omega} = \frac{e^2}{4\pi^2 c} \int d^3r \int_{-\infty}^{\infty} dt \left( \vec{b}(r, t) 4\pi \sum_{l,m} (-i)^l j_l(kr) Y_{lm}(\hat{k}) Y_{lm}^*(\hat{r}) \right) \exp(i\omega t) \left| \vec{B}(r, t) \right|^2 d\Omega
\]
(96)

\[
= \frac{e^2}{4\pi^2 c} \int dt 4\pi \sum_{l,m} (-i)^l \vec{b}_{lm}(\hat{k}, t) Y_{lm}(\hat{k}) \exp(i\omega t) \left| \vec{B}(r, t) \right|^2 d\Omega
\]
(97)

\[
= \frac{e^2}{4\pi^2 c} \int d\omega \sum_{l,m} (-i)^l \left| \vec{b}_{lm}(\hat{k}, \omega) Y_{lm}(\hat{k}) \right|^2 d\Omega
\]
(98)

where
\[
\vec{b}_{lm}(k, t) = \int d^3r \vec{b}(r, t) j_l(kr) Y_{lm}^*(\hat{r})
\]
(99)
\[
\vec{b}_{lm}(k, \omega) = \int d^3r \vec{b}(r, t) \exp(i\omega t) dt
\]
(100)
Note that \( \vec{b}_{lm} \) is a function of \( k \) (not \( \vec{k} \)) and

\[
\frac{dE}{d\omega} = \frac{e^2}{4\pi c}(4\pi)^2 \int \left( \sum_{l,m,l',m'} (-i)^l i' \left( \vec{b}_{lm}(k,\omega) \cdot \vec{b}_{l'm'}^*(k',\omega) \right) \right) Y_{lm}(\hat{k})Y_{l'm'}^*(\hat{k}) \, d\Omega
\]

(101)

\[
= \frac{e^2}{4\pi c}(4\pi)^2 \sum_{l,m} |\vec{b}_{lm}(k,\omega)|^2 = \frac{4e^2}{c} \sum_{l,m} |\vec{b}_{lm}(k,\omega)|^2
\]

(102)

The above equation is used to calculate the spectrum of emitted radiation. In practice one needs only few multipoles.

In order to determine the rate of emitted radiation let us consider eq. (87):

\[
P(t + r/c) = \int \frac{dP}{d\Omega}(t + r/c) \, d\Omega = \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \right( \vec{k} \times \vec{j} (r, \omega) \right) \exp(-i\omega t) \right|^2 \, d\Omega
\]

(103)

\[
= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3 \vec{r}}{2\pi} \left( \vec{r} \times \vec{j} (\vec{r}, \omega) \right) \exp(-i\vec{k} \cdot \vec{r}) \right| \exp(-i\omega t) \right|^2 \, d\Omega
\]

(104)

\[
= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{l,m} (-i)^l \vec{b}_{lm}(k,\omega) Y_{lm}(\hat{k}) \right| \exp(-i\omega t) \right|^2 \, d\Omega
\]

(105)

\[
= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{l,m} (-i)^l \vec{b}_{lm}(k,\omega) Y_{lm}(\hat{k}) \right| \exp(-i\omega t) \right|^2 \, d\Omega
\]

(106)

\[
= \frac{e^2}{4\pi c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left( (4\pi)^2 \sum_{l,m} (-i)^l \left( \vec{b}_{lm}(k,\omega) \cdot \vec{b}_{l'm'}(k',\omega') \right) \right) Y_{lm}(\hat{k})Y_{l'm'}^*(\hat{k}) \times \exp(-i(\omega - \omega')t)
\]

(107)

Note that in the last two lines of the above expression \( \hat{k} = \hat{k}' \) because the vectors \( \vec{k}, \vec{k}' \) differ only by length (\( \omega = ck, \omega' = ck' \)) but have the same direction specified by the angle \( \Omega \). Therefore:

\[
P(t + r/c) = \frac{e^2}{4\pi c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left( (4\pi)^2 \sum_{l,m} (-i)^l \left( \vec{b}_{lm}(k,\omega) \cdot \vec{b}_{l'm'}(k',\omega') \right) \right) \exp(-i(\omega - \omega')t)
\]

(108)

\[
= \frac{e^2}{\pi c} \sum_{l,m} \left| \int_{-\infty}^{\infty} \vec{b}_{lm}(k,\omega) \exp(-i\omega t) d\omega \right|^2
\]

(109)

The last equation is used in practice to calculate the rate of emitted radiation.

The above prescriptions work efficiently if one considers the radiation emitted due to internal nuclear excitation. However in order to determine the contribution coming from the CM motion of the nucleus the simpler formula can be derived. In this case the proton current reads:

\[
\vec{j}_p(r, t) = \vec{V}(t) \delta (r, r_0(t))
\]

(110)

Then

\[
\vec{A}(r,\omega) = \frac{\exp(ikr)}{r} \frac{Ze^2}{c} \int d^3 \vec{r}' \vec{j}_p(r',\omega) \exp(-i\vec{k} \cdot \vec{r}')
\]

(111)

\[
= \frac{\exp(ikr)}{r} \frac{Ze^2}{c} \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V}(t) \exp(-i\vec{k} \cdot \vec{r}_0(t))
\]

(112)

where

\[
\vec{j}_p(\vec{k},\omega) = \int_{-\infty}^{\infty} dt \exp \left( i\omega \left( t - \frac{1}{c} \vec{n} \cdot \vec{r}_0(t) \right) \right) \vec{V}(t)
\]

(113)

\[
\approx \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V} \left( t + \frac{1}{c} \vec{n} \cdot \vec{r}_0(t) \right)
\]

(114)

\[
\approx \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V}(t) = \vec{V}(\omega)
\]

(115)
where $\omega = ck$. The approximation was made above that the recoil velocity is small and the total displacement of the nucleus is negligible. Therefore the perturbation of the radiation due to the change of nucleus position can be neglected. Consequently:

$$e\vec{B}(\vec{r}, \omega) = \frac{iZe^2 \exp(ikr)}{r} \vec{k} \times \vec{V}(\omega)$$  \hspace{1cm} (116)$$

and

$$\vec{B}(\vec{r}, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \vec{B}(\vec{r}, \omega) \exp(-i\omega t)$$  \hspace{1cm} (117)$$

where

$$\vec{B}(\vec{r}, \omega) = \frac{iZe}{c^2} \int \frac{d\vec{\omega}}{2\pi} \exp \left( -i\omega \left( t - \frac{r}{c} \right) \right) \vec{n} \times \vec{V}(\omega)$$  \hspace{1cm} (118)$$

$$\vec{B}(\vec{r}, t) = -\frac{Ze}{c^2} \vec{n} \times \frac{d}{dt} \vec{V} \left( t - \frac{r}{c} \right)$$  \hspace{1cm} (119)$$

Therefore

$$\frac{dP}{d\Omega}(t) = \frac{c^4}{4\pi^2} \left| \vec{B}(\vec{r}, t) \right|^2 = \frac{1}{4\pi^2 c^3} (Ze)^2 \left| \frac{dV}{dt}(t - \frac{r}{c}) \right|^2 \sin^2 \theta$$  \hspace{1cm} (121)$$

and

$$P(t) = \frac{2}{3} (Ze)^2 \left| \frac{dV}{dt}(t - \frac{r}{c}) \right|^2$$  \hspace{1cm} (122)$$

Spectral decomposition:

$$\frac{dE}{d\Omega d\omega} = \frac{(Ze)^2}{4\pi^2 c} \left| \vec{k} \times \vec{V}(\omega) \right|^2$$  \hspace{1cm} (123)$$

and integrating over angles

$$\frac{dE}{d\Omega} = \frac{2}{3\pi c} (Ze)^2 \left| \vec{k} \times \vec{V}(\omega) \right|^2$$  \hspace{1cm} (124)$$

where $\frac{dV}{dt}(\omega)$ is the Fourier transform of acceleration:

$$\frac{dV}{dt}(\omega) = \int \frac{dV}{dt}(t) \exp(i\omega t)$$  \hspace{1cm} (125)$$

The above derivation assumes that the moving nucleus can be treated as a point-like particle. This is a reasonable approximation although it is not difficult to include suitable corrections. Let us consider the proton current in the form:

$$\vec{j}_p(\vec{r}, t) = \vec{V}(t) \rho(\vec{r}, -\vec{r}_0(t))$$  \hspace{1cm} (126)$$

Using the same assumption as before, ie. that the motion is nonrelativistic and movement in space is negligible one gets:

$$\vec{j}_p(\vec{k}, \omega) = \vec{V}(\omega) \rho(\vec{k})$$  \hspace{1cm} (127)$$

and (see $\Box$):

$$\int \frac{dE}{d\Omega d\omega} \ d\Omega = \frac{(Ze)^2}{4\pi^2 c} \left| \vec{k} \times \vec{V}(\omega) \rho(\vec{k}) \right|^2 d\Omega = \frac{(Ze)^2}{4\pi^2 c} \frac{1}{c^2} \int |\vec{n} \times \omega \vec{V}(\omega) \rho(\vec{k})|^2 d\Omega$$  \hspace{1cm} (128)$$

$$= \frac{(Ze)^2}{4\pi^2 c^3} \int \left| \vec{n} \times \frac{dV}{dt}(\omega) \rho(\vec{k}) \right|^2 d\Omega$$  \hspace{1cm} (129)$$
where $\vec{n} = \frac{\vec{r}}{r}$. In the case of spherical density distribution it simplifies to:

$$
\int \frac{dE}{d\Omega} d\omega \frac{d\Omega}{d\Omega} = \left(\frac{Ze}{4\pi c^2}\right)^2 \int \left| \vec{n} \times \frac{d\vec{V}}{dt}(\omega) \right|^2 \left| \rho(k) \right|^2 d\Omega = \frac{2}{3\pi} \left(\frac{Ze}{c}\right)^2 \left| \frac{d\vec{V}}{dt}(\omega) \right|^2 \left| \rho(k) \right|^2
$$

(130)

The above expressions can be used to determine the spectrum of emitted radiation. In the Figures 4 and 5 the contributions to the energy spectrum coming from dipole and quadrupole terms are plotted for 3 values of impact parameter. The difference between the figures originates from two different smoothing widths that have been applied. Namely, the original curves have been convoluted with gaussians of widths 1 MeV (Fig. 4) and 0.5 MeV (Fig. 5).

FIG. 4. (color online) The energy spectrum of emitted electromagnetic radiation due to internal excitation of the target nucleus, caused by the collision at the impact parameters $b = 12.2$ fm (upper subfigure), $b = 16.2$ and $b = 20.2$ (lowest subfigure). The contributions from two orientations of the target nucleus are shown: perpendicular (squares) and parallel (circles) with respect to the incoming projectile. Dotted dashed line represents the dipole component of the radiation. Dashed line represents the quadrupole component of the radiation. In this case the smoothing width of the original curves was set to 1 MeV.

For the radiation caused by the CM acceleration after collision the decomposition into multipoles is not useful and one can apply instead eqs. (122–124). The emission occurs within much shorter time scale governed by the collision time $\tau_{\text{coll}} = \frac{b}{c\gamma}$. The results are plotted in the Figs. 6, 7, 8, 9, 10.

**Dipole dynamics and neutron emission**

The framework of TDSLDA allows to calculate various one body observables. In this case the most important is the nuclear dipole moment. Only two components of the dipole moment, lying in the reaction plane, can oscillate as a result of collision. In the Figs. 11 and 12 these two components of the dipole moment have been plotted.
During the time evolution the nucleus can emit particles. In order to investigate this effect we have calculated the number of neutrons/protons within shells of various radii. As one can see from the Figs. 14, 15, 16, 17 the number of protons in the shells outside the nucleus is negligible. Moreover this proton number is approximately constant which indicates that we rather probe the tail of the proton distribution than the emission process. On the contrary the situation is different for neutrons. The number of neutrons in the smaller shell is much larger, although it is also approximately constant. However in the larger shell the number of neutrons is constantly increasing in time with a fairly constant average rate. It indicates that the neutron emission occurs as a result of Coulomb excitation process.
FIG. 6. (color online) The gamma emission rate (left panel) due to bremsstrahlung for the collision at the impact parameter $b = 12.2\text{fm}$. The right panel shows the energy spectrum emitted. Solid and dashed lines correspond to the perpendicular and parallel orientation of the target nucleus with respect to incoming projectile.

FIG. 7. (color online) The same as in the Fig. 6 but for the impact parameter $b = 16.2\text{fm}$. 
FIG. 8. (color online) The same as in the Fig. 6 but for the impact parameter $b = 20.2 \text{fm}$.

FIG. 9. (color online) The contributions to the cross section with respect to gamma emission during 2500 fm/c after collision. The dashed line represents the Bremsstrahlung contribution. The solid line shows the contribution from intrinsic excitation modes.
FIG. 10. (color online) The distance squared between the CM of protons and the total nuclear CM: $|\Delta R_{\text{CM}}|^2$ as a function of time. The impact parameter is $b = 12.2\text{ fm}$, and the nuclear symmetry axis of the target is perpendicular to the projectile’s trajectory. The slope does not depend on the orientation and the impact parameter. The numerical fit to the maxima (squared amplitudes of dipole oscillations) with the function $\exp(-t/\tau)$ yields $\tau \approx 500\text{ fm/c}$.

FIG. 11. (color online) Two components of the dipole moment: $D_z$ and $D_y$ as a function of time. The left and right panels correspond to the collision with projectile moving along the $y$-axis and $z$-axis, respectively. Impact parameter: $b = 12.2\text{ fm}$. 
FIG. 12. (color online) The same as in the Fig. 11 but for the impact parameter $b = 16.2\text{fm}$.

FIG. 13. (color online) The same as in the Fig. 11 but for the impact parameter $b = 20.2\text{fm}$. 
FIG. 14. (color online) The number of neutrons present within the shell with inner radius 10fm and outer radius 15fm (red line). The number of neutrons present within the shell with inner radius 10fm and outer radius 20fm (blue line). Thin line corresponds to the actual number of neutrons, whereas the thick line denotes the average value. The plot corresponds to the collision with the target nucleus symmetry axis perpendicular to the trajectory of the incoming projectile. The impact parameter $b = 12.2$ fm.
FIG. 15. (color online) The same as in the Fig. 14 but for the nuclear orientation parallel with respect to the incoming projectile.

FIG. 16. (color online) The same as in the Fig. 14 but for protons.
FIG. 17. (color online) The same as in the Fig. 15 but for protons.