Selective transport of atomic excitations in a driven chiral-coupled atomic chain

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1. Introduction

Manipulating light–matter interactions by precisely positioning quantum emitters has made progress in versatile platforms, including photonic crystal waveguide [1], optical microtraps [2, 3], and diamond nanophotonics systems [4]. This promises to tailor the properties of quantum interface from the bottom up, which is in stark contrast to the atomic ensemble of many randomly distributed emitters [5]. Recently, one-dimensional (1D) atom-nanophotonic waveguide system [6–10] presents another potential paradigm to engineer light–matter interactions. This 1D coupled system features strong and infinite-range couplings in the resonant dipole–dipole interactions (RDDI) [11, 12], which is difficult to reach in a free-space atomic system. Only recently, superradiance [13, 14] is observed in two atomic clouds above the nanofibers [15], demonstrating the infinite-range RDDI in the 1D atom-fiber coupled system. This system is also proposed to realize mesoscopic entangled states [8], show photon–photon correlations [16, 17], and allow universal atomic dynamics [9].

In addition to the advantage of strong coupling in the 1D atom-fiber or atom-waveguide systems, it can further construct a chiral quantum network [18–21], which enables non-reciprocal decay channels and directional emissions. This chiral coupling breaks the time-reversal symmetry that should be preserved in conventional light–matter interacting systems in free space, and emerges due to the locking of transverse spin angular momentum and light propagation direction [22, 23]. The chirality can be engineered via either controlling the atomic internal states [24] or applying external magnetic fields [24–26]. This 1D chiral-coupled system can be used as complementary single-photon devices which are fundamental in quantum internet [27], and can potentially operate CNOT gates [26] which are essential to quantum computation.

In such dynamical system of chiral-coupled atomic chain, the steady-state preparations should highly depend on the positions of the atoms, the driven field detunings, and its directionality of the decay channels. However, the interplay or competition between these parameters is less explored. Here we investigate the selective and controllable transport of atomic excitations to locate the parameter regimes resilient to position fluctuations, which are advantageous to precise and genuine state preparations. The effect of position fluctuations is also discussed, and it shows that the system functions more...
stable with less than 1% fluctuations. Our results demonstrate potentially deterministic state preparations in the driven and dissipative system and hold promises to manipulate many-body spin dynamics [28].

2. Effective chiral-coupled interactions

For atoms in free space, the spontaneous emissions from excited atoms initiate from system-reservoir interactions, which decay as an exponential function with a characteristic time constant. This decay rate represents an intrinsic property of the independent atoms, which can be hugely modified when atoms are close to each other within a transition wavelength. In this regime, strongly-coupled RDDI show up since light can rescatter multiple times before leaving the whole medium. RDDI in a three-dimensional (3D) reservoir are responsible for the collective phenomena of superradiance [12–14, 29–38] and subradiance [39–51] in a dense atomic system.

RDDI in a 3D reservoir in general are reciprocal couplings which preserve the time-reversal symmetry, and in the long range behave asymptotically as the inverse of mutual atomic separation (see appendix A). By contrast for RDDI in a 1D reservoir, it shows infinite-range couplings in sinusoidal forms [8, 9], and can be further structured [19, 52, 53] to show non-reciprocal decay channels in the atom-fiber or atom-waveguide coupled systems. Here we consider a driven chiral-coupled atomic chain as shown in figure 1, where the system dynamics can be described by the effective chiral master equation [20]

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\hat{H}_S + \hat{H}_L + \hat{H}_{R,\rho}] + \hat{L}_L[\rho] + \hat{L}_R[\rho].$$

The external light–matter interaction is

$$\hat{H}_S \equiv \hbar \sum_{\mu} (\sigma_\mu^+ + \sigma_\mu^-) - \hbar \sum_{\mu} \delta_\mu \sigma_\mu^+ \sigma_\mu^-,$$

which drives the system of N two-level quantum emitters \(\langle g(\epsilon) \rangle\) for the ground and excited states respectively and \(\sigma_\mu^+ \equiv |g_\mu\rangle \langle h_\mu|\) with a uniform Rabi frequency \(\Omega\) and spatially dependent detunings \(\delta_\mu\). The coherent parts are

$$\hat{H}_L \equiv -\frac{i\hbar \gamma_L}{2} \sum_{\mu<\nu} (\sigma_\mu^+ \sigma_\nu^- - \sigma_\nu^+ \sigma_\mu^- - \text{h.c.}),$$

$$\hat{H}_R \equiv -\frac{i\hbar \gamma_R}{2} \sum_{\mu>\nu} (\sigma_\mu^+ \sigma_\nu^- - \sigma_\nu^+ \sigma_\mu^- - \text{h.c.}),$$

which denote the collective energy shifts, and the Lindblad forms of

$$\hat{L}_L[\rho] \equiv -\gamma_L \sum_{\mu} [e^{-i\hbar \epsilon_{g\mu} t} - e^{i\hbar \epsilon_{h\mu} t}] (\sigma_\mu^+ \sigma_\mu^-) \rho - \sigma_\mu^- \rho \sigma_\mu^+ + \text{h.c.},$$

$$\hat{L}_R[\rho] \equiv -\gamma_R \sum_{\mu} [e^{-i\hbar \epsilon_{g\mu} t} - e^{i\hbar \epsilon_{h\mu} t}] (\sigma_\mu^- \sigma_\mu^+) \rho - \sigma_\mu^- \rho \sigma_\mu^+ + \text{h.c.},$$

characterize the collective decay behaviors. \(k = 2\pi/\lambda\) is the wave vector for the transition wavelength \(\lambda\), and the subscripts \(L\) and \(R\) respectively label the left- and right-propagating decay channels. For a 1D atomic chain, we can order them as \(x_1 < x_2 < \ldots < x_{N-1} < x_N\) for convenience. The above Lindblad forms do not include the non-guided decay or other non-radiative losses of the atoms, which would compromise the light detection efficiency via fibers or the fidelity of state preparations.

Next we further assume a weakly driven system with \(N\) atoms, such that the Hilbert space can be confined in the ground \(|g\rangle^\otimes N\) and singly excited states \(|\psi_\mu\rangle = (\sqrt{N})^{-1} \sum_{\mu=1}^N \sigma_\mu^+ |g\rangle^\otimes N\). This is similar to the coherent dipole model with weak laser excitations [32] or Green’s function approach in low saturation regime [54], where the ground state population is much larger than the one of the excited state, that is \(\langle \sigma_\mu \sigma_\mu^- \rangle \approx 1 \gg \langle \sigma_\mu^+ \sigma_\mu \rangle\). In this limit, the state of the system can be expressed as

$$|\Psi(t)\rangle = \frac{1}{\sqrt{1 + \sum_{\mu=1}^N |A_\mu(t)|^2}} \left(|g\rangle^\otimes N + \sum_{\mu=1}^N A_\mu(t) |\psi_\mu\rangle\right),$$

where the probability amplitude \(A_\mu(t)\) can be obtained by

$$\dot{A}_\mu(t) = -i\Omega + \sum_{\nu=1}^N V_{\mu\nu} A_\nu(t).$$
and the chiral-coupled interaction \( V \) reads

\[
V = \begin{bmatrix}
\frac{\gamma_{\mu}}{2} & -\gamma_{\mu} e^{-i[k_{\mu}]} & -\gamma_{\mu} e^{-i[2k_{\mu}]} & \cdots & -\gamma_{\mu} e^{-i[Nk_{\mu}]} \\
-\gamma_{\mu} e^{-i[k_{\mu}]} & \frac{\gamma_{\mu}}{2} & -\gamma_{\mu} e^{-i[2k_{\mu}]} & \cdots & -\gamma_{\mu} e^{-i[Nk_{\mu}]} \\
-\gamma_{\mu} e^{-i[2k_{\mu}]} & -\gamma_{\mu} e^{-i[2k_{\mu}]} & \frac{\gamma_{\mu}}{2} & \cdots & -\gamma_{\mu} e^{-i[Nk_{\mu}]} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\gamma_{\mu} e^{-i[Nk_{\mu}]} & -\gamma_{\mu} e^{-i[Nk_{\mu}]} & -\gamma_{\mu} e^{-i[Nk_{\mu}]} & \cdots & \frac{\gamma_{\mu}}{2}
\end{bmatrix}
\]

(9)

where \( x_{\mu,0} \equiv x_{\mu} - x_{\mu} \).

The above chiral-coupled interaction becomes reciprocal, which is \( V_{\mu,\mu} = V_{\mu,\mu} \), only when \( \gamma_{L} = \gamma_{R} \). For reciprocal interaction, \( -\gamma_{L,R} \cos(k|x_{\mu}|) \) and \( \gamma_{L,R} \sin(k|x_{\mu}|) \) respectively represent the collective energy shifts and decay rates. In general, \( VV^\dagger \neq V^\dagger V \), so \( V \) is not a normal matrix. Therefore, the eigen-decomposition does not work here, and we solve for the system evolutions directly from

\[
\frac{d}{dt} A = -i\Omega + V A,
\]

(10)

where \( A \equiv (A_{1}(t), A_{2}(t), ..., A_{N}(t)) \) with the initial conditions of \( A(t = 0) = 0 \). Below we use equation (10) to investigate the transport properties of atomic excitations and state manipulations in a driven chiral-coupled atomic chain.

### 3. Selective transport of atomic excitations

Here we quantify the transport of atomic excitations by the difference of excited state populations between the left and right sections of the atomic chain for even and odd \( N \) respectively

\[
T_{p} = \frac{\sum_{\mu=1}^{N/2(N-1)/2} P_{\mu}(\infty) - \sum_{\mu=N/2+1}^{N} P_{\mu}(\infty)}{\sum_{\mu=1}^{N} P_{\mu}(\infty)},
\]

(11)

where the excited state population is \( P_{\mu} \equiv |A_{\mu}|^{2} \), and we have excluded the central atom (the \( (N + 1)/2 \)th one) for odd \( N \). Positive or negative \( T_{p} \) means that the atomic excitations accumulate toward the left or right parts of the chain, from which we can analyze how the distributions of the excitations are manipulated and controlled by system parameters. \( T_{p} \) can be used as indicators of how efficient the light is transferred to either directions of the chain, and can further apply to quantum links between multiple atomic chains.

#### 3.1. Two atoms

Before we study a longer atomic chain, it is helpful to study the case of \( N = 2 \), where we can get some insights of how atomic excitations are distributed. The coupled equations from equation (10) are

\[
\dot{A}_{1}(t) = -i\Omega + \left(i\delta_{1} - \frac{\gamma_{1}}{2}\right) A_{1} - \gamma_{L} e^{-i\xi} A_{2},
\]

(12)
in $\xi$, which is preferential for precise and genuine state preparations.

On the other hand for reciprocal couplings of $D = 0$ in figure 2(b), similar asymmetric profile emerges and shows mirror reflection at $\xi = 0$ with $\pm 6$. Specifically at $\xi = 0$, $T_p$ becomes one as long as the left atom is resonantly driven, which means a complete suppression of the atomic excitations on the right. This dispersion-like distribution of atomic excitations enables both positive and negative transport around $\xi = 0$, making flexible steady-state preparations simply by either manipulating excitation detunings or $\xi$. Under the resonance condition $\delta_{1(2)} = 0$, $T_p$ is not defined since $A_{1(2)}(\infty)$ becomes infinite. This is due to the breakdown of the low saturation approximation in the model, which will be addressed in the end of the next subsection. Next we study a longer atomic chain and investigate the multi-atom effect on the transport property.

### 3.2. Atomic chain

For a longer atomic chain, the left- and right-propagating emissions can go through multiple scatterings of transmissions and reflections before leaving the whole array. As more atoms are included, the many-body coherences will play important roles in determining the steady-state properties. Further with varying equidistant positions $\xi$, the chiral-coupled interactions can also be modified significantly. Therefore, here we focus on the interplay between the number of atoms and their positions in the transport of atomic excitations.

First we consider the cascaded case ($D = 1$) with uniform detunings in figure 3. On resonance, the transport profiles should be symmetric around $\xi = 0$ or $\pi$ as shown in figure 3(a), similar to figure 2(b) for $N = 2$. As $N$ increases, the width of the minimum $T_p(\xi = \pi)$ narrows, and positive $T_p$ becomes sharper near $\xi \sim \pi$. More ripples around $T_p \approx 0$ show up as $N$ increases, which indicates of multiple interferences from these quantum emitters. This also appears in figure 3(b) with finite excitation detunings, where the minimum of $T_p$ shifts toward $\xi = 2\pi$. This suggests to allow adjustable transport of excitations by controlling external fields and locating the optimal $\xi$. The narrowing distribution of $T_p$ for larger $N$, however, restricts a genuine preparation of the states if the atomic chain undergoes significant position fluctuations. For an example of $N = 10$ in figure 3(a), the full width of the half minimum $T_p(\xi = \pi)$ is $\Delta\xi \sim 1$, which provides an approximate tolerance of position fluctuations around $\pm 0.5/\pi$, i.e. $\sim \pm 15\%$ displacement around $\xi = \pi$. Thus for an atomic chain of $N > 10$ subject to more significant fluctuations $\geq 15\%$, it is more demanding in stabilizing the system to transfer the excitations with high fidelity.

In addition to the transport profiles, in figure 4 we show the spatial distribution of normalized atomic excitations $P_p(\infty) = P_p(\infty)/\sum_{p} P_p(\infty)$ for $N = 10$ as an example, in the cascaded case under resonant driving fields. As $\xi$ increases toward the minimal $T_p(\xi = \pi)$, the excitations move from an even distribution at $\xi = 0$ and form a somewhat symmetric one at $\xi \sim 2.5$ with an inversion symmetry at the center of the chain. Around this $\xi$, the distributions show obvious positive and negative $T_p$ at $\xi = 2.3$ and 2.7 respectively, and migrate toward the right end with an exponential-like increasing distribution at $\xi = \pi$. This fruitful information of atomic excitation distributions can further help characterize the state-transfer properties in the steady-state preparations.

On the other hand for the case of non-reciprocal decay channels, we consider $D = 0.5$ as an example. We show $T_p$ in figure 5 for $N = 10$ with increasing detunings. Similar to the cascaded scheme of figure 3(a), in figure 5(a), the transport of atomic excitations is symmetric at $\xi = 0$ or $\pi$ under the conditions of resonantly driven fields. The small bump at $\xi = \pi$ emerges as long as $D < 1$, which can be further enhanced as $D$ decreases. Other than the parameter regime of $\xi \sim \pi$, $T_p$ shows relatively flattened distributions with almost equal excitation populations between the left and right parts.
of the chain. By contrast when detunings are increasing in figures 5(b) and (c), the minimum of $T_p$ shifts toward $\xi = 2\pi$. Significant positive $T_p$ at $\xi \gtrsim 0$ emerges as well in figure 5(c), which is over 0.5. Furthermore in figure 5(c), the width of this negative peak of $T_p$ widens as $\delta_\mu$ increases, while its peak amplitude decreases. This shows the possibility, in the widened regions of $T_p$, to manipulate a genuine transport of atomic excitations in the non-reciprocal scheme. In addition, the red-detuned excitation fields hold a symmetric relation in the transport property, such that $T_p(\delta_\mu = -\delta, -\xi) = T_p(\delta_\mu = \delta, \xi)$.

We note that as $D \to 0$ and $N \gg 1$, the time to reach steady states prolongs and even longer at $\xi \sim \pi$. At the special parameter regime of $D = 0$, $\delta_\mu = 0$, and $\xi = \pi$, the excited atoms under reciprocal couplings become decoherence-free, and thus are pumped by external fields $\Omega$ indefinitely. This eventually violates the assumptions of small excited state populations in our models in section 2. The validity of weakly-coupled regime can be retrieved with a finite detuning $\delta_\mu = \delta$ and satisfying $\Omega/\delta \ll 1$. Under this condition, the system evolves with a generalized Rabi frequency $\sim \delta$, and exchanges between the ground and the singly-excited states, where the steady-state is never reached.

4. Effect of position fluctuations of the atomic chain

Finally we study the effect of position fluctuations on the transfer of state excitations in the driven and chiral-coupled atomic chain. On the experimental side, a precise positioning of the atoms is not easily fulfilled. Whether the spatial variations of the atoms make a notable effect or not depends on the ratio of deviation and the transition wavelength. Superconducting qubits, for example, is more resilient to this spatial fluctuation due to microwave transmission line, in contrast to the optical transition of neutral atoms.

In figure 6, we plot the $T_p$ by introducing fixed position fluctuations on each atoms of the chain. The position fluctuations we choose are $\lambda/100$ and $\lambda/50$, which respectively correspond to phase fluctuations of $\pi/50$ and $\pi/25$ on the atoms. As the fluctuation increases, $T_p$ smooths out especially close to $\xi \sim 2\pi$, compared to figures 3(a) and 5(a). For the cascaded case of $D = 1$ in figure 6(a) near $\xi \sim \pi$, the transport of the atomic excitations is less affected by the fluctuations, in contrast to the non-cascaded case of $D = 0.5$ in figure 6(b). This is due to the fact that the non-cascaded scheme permits both left- and right-propagating decay channels, and therefore the driven atomic chain experiences the position fluctuations more significantly from both directions. This shows that the cascaded chiral-coupled atomic chain can better withstand the influences of atomic spatial fluctuations.

5. Discussion and conclusion

The efficiency and fidelity of state preparations can be reduced due to non-guided modes of light in the chiral-coupled systems. This limits the operation time in state
manipulations, and thus the non-guided decay rate sets the overall timescale for genuinely controlled dynamical systems. The inefficiency can be overcome, for example in an atom-fiber system, by aligning the atoms close to the nanofiber with an optimal fiber radius \([10, 57]\) to raise the interaction probability. Some recent progress has shown the potential of flexible control over the chiral-coupled systems, including the superconducting qubits without external magnetic fields \([58]\) and a bilayer atomic array in free space \([59]\). This shows rich opportunities in structuring the 1D reservoirs to manipulate the directionality of light coupling in the system. Moreover, subradiance dynamics can also emerge in such chiral-coupled atomic chain \([60]\), which can be potentially used for quantum storage of light. With the scalability of atom-fiber or atom-waveguide platforms, the chiral-coupled systems can further stimulate applications in quantum information processing. As a final remark, we note that matrix product states, used for example in the density matrix renormalization group method \([61]\), can be applied to study the 1D chiral-coupled chain beyond the weak excitation limit, allowing for investigations of multiple excitations.

In conclusion, we have investigated the distribution of the atomic excitations in a weakly driven chiral-coupled atomic chain. In this 1D system, we quantify the distributions by the transport of the excitations between the left and right parts of the chain. With controllable parameters of the external field detunings, ordered array positions, and directionality of the decay channels, we are able to make a deterministic transfer of atomic excitations to both sides. With the advantage of tunable non-reciprocal couplings, chiral-coupled system allows a selective transport of excitations in the optimal parameter regimes resilient to position fluctuations. Our results provide a fundamental study on the steadystate preparations in the driven and dissipative chiral-coupled systems, and are potentially applicable in many-body state manipulations.

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Appendix A. RDDIs in 1D reservoir

A.1. General formalism in 3D reservoir

Here we review the results of RDDIs in 1D reservoir. This can be directly obtained and extended from the RDDI in 3D reservoir \([11, 12]\). The RDDI result from a system of many atoms interacting with quantized bosonic light modes. Due to the light rescattering events in the dissipation process of the system, the RDDI that feature long-range atom-atom interactions emerge as if the whole medium are effectively resonantly induced dipoles in pairs. The RDDI characterize the coherent frequency shifts and collective decay rates of the atomic system, which can be expressed respectively as imaginary and real parts of the coupling constant \(J_{\mu,\nu}\), such that the evolutions of any atomic observables \(Q\) can be governed by Lindblad forms

\[
\dot{Q}(t) = i \sum_{\mu,\nu} \text{Im}(J_{\mu,\nu})[\sigma^\dagger_{\mu}, \sigma_{\nu}, Q] + \mathcal{L}(Q),
\]

\[
\mathcal{L}(Q) = \sum_{\mu,\nu} \text{Re}(J_{\mu,\nu})
\times [2\sigma^\dagger_{\mu}Q\sigma_{\nu} - (\sigma^\dagger_{\mu}\sigma_{\nu}Q + Q\sigma^\dagger_{\mu}\sigma_{\nu})],
\]

where \(\sigma^\dagger_{\mu} \equiv |e\rangle_{\mu}\langle g|\) and \(\sigma_{\nu} = (\sigma^\dagger_{\nu})^\dagger\) are raising and annihilating operators respectively for ground \(|g\rangle\) and excited states \(|e\rangle\). The above form can be obtained with the Born–Markov and secular approximations, which can be sustained respectively.
when the response time of the reservoir is faster than the system and the dynamical time scale of the system is longer than the time light travels throughout the whole medium. These conditions can be satisfied when the macroscopic length scale of the medium is way below several meters for rubidium atoms (intrinsic decay time ~26 ns).

The explicit forms of $J_{\mu,\nu}$ are defined as

$$J_{\mu,\nu} = \sum_q |g_q|^2 \int_0^\infty d\omega \epsilon_{\mu}^q(r_{\tau} - r_{\nu})$$

$$\times [e^{i\omega_\tau - \omega_\nu} + e^{-i\omega_\tau - \omega_\nu}],$$

where the coupling constant is $g_q \equiv d/h \sqrt{k_{\omega_0}(2\epsilon_0 V)(\hat{\epsilon}_q \cdot \hat{d})}$ with dipole moment $d$ and its unit direction $\hat{d}$, field polarizations $\hat{\epsilon}_q$, and a quantization volume $V$. $P$ is the principal value of the integral. From equation (A.3), the coupling constant depends on respective atomic positions $r_{\mu,\nu}$, and thus is a long-range interaction.

For a 3D reservoir, we allow continuous modes of reservoir, that is $\sum_q \rightarrow \sum_q \int_0^\infty \frac{V}{(2\pi)^3} d^3\mathbf{q}$ with two possible field polarizations $\hat{\epsilon}_q$. In spherical coordinates, we show $J_{\mu,\nu}$ explicitly [12]

$$\text{Re}(2J_{\mu,\nu}) = \frac{3\Gamma}{2} \left\{ 1 - (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}_{\mu,\nu})^2 \frac{\sin \xi}{\xi} \right\}$$

$$+ \left[ 1 - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}_{\mu,\nu})^2 \left( \frac{\cos \xi}{\xi^2} - \frac{\sin \xi}{\xi^3} \right) \right], (A.4)$$

$$\text{Im}(J_{\mu,\nu}) = \frac{3\Gamma}{4} \left\{ -[1 - (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}_{\mu,\nu})^2] \frac{\cos \xi}{\xi} \right\}$$

$$+ \left[ 1 - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}_{\mu,\nu})^2 \left( \frac{\sin \xi}{\xi^2} + \frac{\cos \xi}{\xi^3} \right) \right]. (A.5)$$

Next we let $x_{\mu,\nu} = x_{\mu} - x_{\nu}$ and drop $q$ in $\omega_\nu$ for brevity, and we obtain

$$J_{\mu,\nu} = \int_0^\infty d\omega \left\{ \frac{\partial_\omega q(\omega)}{2\pi} \right\} L \cos(k_{\omega_\nu}x_{\mu,\nu})$$

$$\times \left[ \pi \delta(\omega - \omega_\nu) + \pi \delta(\omega + \omega_\nu) + iP(\omega - \omega_\nu)^{-1} - iP(\omega + \omega_\nu)^{-1} \right]. \quad (A.7)$$

Let $\Gamma_{\text{ID}} \equiv 2|\partial_\omega q(\omega)|^x_{\omega = \omega_{\nu}} \frac{g_{\nu}^2}{\omega_\nu}$, where we keep the dispersion relation of the 1D coupling constant with $\partial_\omega q(\omega)$ being the group velocity of light in the medium. Finally we obtain

$$J_{\mu,\nu} = \frac{\Gamma_{\text{ID}}}{2} \cos(k_{\nu}x_{\mu,\nu})$$

$$- \frac{iP}{\pi} \int_0^\infty d\omega \frac{\text{Re}[i\partial_\omega q(\omega)]g_{\nu}^2 L e^{ik_{\nu}(x_{\mu,\nu})}}{\omega - \omega_\nu}, \quad (A.8)$$

$$\Gamma_{\text{ID}} = \frac{1}{2} \left[ \text{Re} (k_{\nu}x_{\mu,\nu}) + i \text{Im} (k_{\nu}x_{\mu,\nu}) \right]. \quad (A.9)$$

where the respective real and imaginary parts should demonstrate the Kramers–Kronig relation [62].

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