Stochastic Quantization for the Edwards Measure of Fractional Brownian Motion with $Hd = 1$.

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**Abstract**

In this paper we construct a Markov process which has as invariant measure the fractional Edwards measure based on a $d$-dimensional fractional Brownian motion, with Hurst index $H$ in the case of $Hd = 1$. We use the theory of classical Dirichlet forms. However since the corresponding self-intersection local time of fractional Brownian motion is not Meyer-Watanabe differentiable in this case, we show the closability of the form via quasi translation invariance of the fractional Edwards measure along shifts in the corresponding fractional Cameron-Martin space.
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## 1 Introduction

In its original form the Edwards model was a proposal to modify the Wiener measure $\mu_0$ for d-dimensional Brownian motion by a factor which would exponentially suppress self-intersections of sample paths. Informally

$$d\mu_g = Z^{-1} e^{-gL} d\mu_0,$$

where $L$ is the self-intersection local time of Brownian motion, see e.g. [2], [3], [9], [10], [11], [19], [27], [29], [31], [40]-[45], and $Z$ is a normalization constant. Motivation for this construction came from polymer physics ("excluded volume" effect), while Symanzik [40] introduced the self-intersection local times as a tool in constructive quantum field theory, see also [13].

A mathematically well-defined version of this ansatz was first given by Varadhan [41] for $d = 2$, and then by Westwater [43] for $d = 3$.

"Stochastic quantization" addresses the - largely unresolved - challenge of constructing random fields $\varphi$ whose probability measure obeys certain physical postulates from quantum field theory. As introduced by Parisi and Wu [38], this construction is attempted by introducing an extra parameter $\tau$ and a stochastic differential equation with regard to this parameter in such a way that for large $\tau$ the asymptotic distribution of the Markov process $\varphi_\tau$ will satisfy those postulates.

Conversely, for admissible measures $\mu$, local Dirichlet forms give rise to such Markov processes with $\mu$ as their invariant measure. For the 2-dimensional Brownian motion Albeverio et. al. in [2] have proven the admissibility of the Edwards measure, properly renormalized as elaborated by Varadhan [41].

In this article we show in the framework of Dirichlet forms, that there exists a Markov process which has the fractional Edwards measure as invariant
measure for the case that the Hurst parameter $H$ and the dimension $d$ fulfill $Hd = 1$. An analogous construction for $Hd \leq 1$ can be found in [15] using integration by parts techniques which are not available in this more singular case. Instead the closability of the local pre-Dirichlet form will be shown by quasi-translation-invariance w.r.t. shifts along the Cameron-Martin space of fractional Brownian motion.

In Section 2 we shall introduce the required concepts and properties, so as to then present our results and their proof in Section 3.

2 Preliminaries

2.1 Fractional Brownian Motion

For $d \in \mathbb{N}$ and Hurst parameter $H \in (0, 1)$ a fractional Brownian motion (fBm) in dimension $d$ is a $\mathbb{R}^d$-valued centered Gaussian process $(B^H_t)_{t \geq 0}$ with covariance, in case $d = 1$:

$$\text{cov}_H(t, s) := \mathbb{E}[B^H_t B^H_s] = \frac{1}{2} \left( t^{2H} + s^{2H} - |t - s|^{2H} \right), \quad s, t \in [0, \infty). \quad (1)$$

In $d$ dimensions we consider $d$ identical independent copies of one-dimensional fractional Brownian motion.

In order to study the quasi translation invariance of the fractional Edwards measure (introduced below), we need to define the Cameron-Martin space associated to it. The main role of the Cameron-Martin space is played by the fact that it characterizes precisely those directions in which translations leave the fractional Edwards measure "quasi-invariant" in the sense that the translated measure and the original measure have the same null sets. Here we give an abstract definition of the Cameron-Martin space for a Gaussian measure $\mu$ on a separable reflexive Banach space $(B, \| \cdot \|)$ and later will realize it for the case at hand. The topological dual of the Banach space $B$ is denoted by $B'$.

**Definition 1 ([16]).** The Cameron-Martin space $K_\mu$ of a Gaussian measure $\mu$ on a separable reflexive Banach space $(B, \| \cdot \|)$ is the completion of the linear subspace $\tilde{K}_\mu \subset B$ defined by

$$\tilde{K}_\mu := \left\{ h \in B \mid \exists h^* \in B' \text{ with } \int_B h^*(x) \ell(x) d\mu(x) = \ell(h), \forall \ell \in B' \right\}$$

with respect to the norm $\|h\|_\mu^2 := \int_B |h^*(x)|^2 d\mu(x)$. It becomes a Hilbert space when provided with the inner product $(h_1, h_2)_\mu := \int_B h_1^*(x) h_2^*(x) d\mu(x)$.
Remark 2. The norm $\|h\|_\mu$, hence the inner product $(h_1, h_2)_\mu$ in $\tilde{K}_\mu$, is well defined, that is they do not depend on the corresponding elements $h^*, h_1^*, h_2^*$ in $B'$, see Remark 3.26 in [16].

To realize the fBm process let $\Omega = X := C_0([0, T], \mathbb{R}^d)$ be the Banach space of all continuous paths in $\mathbb{R}^d$, null at time 0, equipped with the supremum norm. Let $\mathcal{B}_H$ denote the $\sigma$-algebra on $X$ generated by all maps $X \ni \omega \mapsto B^H_t(\omega) \in \mathbb{R}^d$, $t \geq 0$. The fractional Wiener measure on $X_H := (X, \mathcal{B}_H)$ we denote by $\nu_H$ and the expectation w.r.t. $\nu_H$ is abbreviated by $E_H(\cdot)$. Let $X'_H$ be the topological dual space of $X_H$ and $L^2 := L^2([0, T], \mathbb{R}^d)$ the space of square integrable $\mathbb{R}^d$-valued functions on $[0, T]$. Moreover let $\mathcal{H}_H$ be the Hilbert space defined by

$$\mathcal{H}_H = \{f \in L^2([0, T], \mathbb{R}^d) \text{ such that } \|M_H f\|_{L^2} < \infty\}.$$ 

Here the operator $M_H$ is given by $(M_H f)(t) = \int_0^T \Lambda_H(t, s) f(s) \, ds$, where $\Lambda_H$ is the fractional integral kernel, see [8, eq. (2.2)] and [37]. We denote by $\langle \cdot, \cdot \rangle_H = \langle M_H \cdot, M_H \cdot \rangle_{L^2}$ the inner product on $\mathcal{H}_H$. By identifying the Hilbert space $\mathcal{H}_H$ with its dual we obtain the rigging $X_H \subset \mathcal{H}_H \subset X'_H$. The dual pairing in a natural way generalizes the inner product on $\mathcal{H}_H$.

Following [34] a fractional version of the Cameron–Martin space $K_H$ of $\nu_H$ is hence given by

$$K_H := \left\{k : [0, T] \longrightarrow \mathbb{R}^d \mid \exists h \in L^2([0, T], \mathbb{R}^d), k_t = \int_0^t R_H(t, s) h(s) \, ds \right\},$$ 

where $R_H$ is the square integrable kernel defined by

$$R_H(t, s) := C_H s^{\frac{1}{2} - H} \int_s^t (u - s)^{H - \frac{3}{2}} u^{H - \frac{3}{2}} du, \quad t > s,$$

with $C_H = \sqrt{\frac{H(2H-1)}{\beta(2-2H,H-\frac{1}{2})}}$ and $\beta$ denotes the beta function. For $t \leq s$ we put $R_H(t, s) = 0$. The kernel $R_H$ is related to the covariance function of fBm in (1) through the identity

$$\text{cov}_H(t, s) = \int_0^{t \wedge s} R_H(t, r) R_H(s, r) \, dr.$$

For a fBm $B^H = \{B^H_t, t \geq 0\}$ in $\mathbb{R}^d$ a shift along the Cameron-Martin space $X^{H,u,k}$ is defined by

$$X^{H,u,k} := \{X^{u,k}_t := B^H_t + uk_t, t \geq 0\}, \quad u \in \mathbb{R}, \ k \in K_H.$$
We use the notation

\[ \int \dot{k} \, dB^H := \int_0^T \dot{k}(s) \, dB^H_s, \]

which is defined as in e.g. [8]. Note that \( \dot{k} \) is a well defined function in \( L^2([0,T],\mathbb{R}) \) due to [34].

**Lemma 3.** For a Gaussian measure \( \nu \), in particular for \( \nu_H \), the shifted measure \( \nu \circ \tau_{sk} \), where \( \tau_{sk}(\omega) = \omega + sk, s \in \mathbb{R} \) for \( k \) from the corresponding Cameron-Martin space \( K_H \) is indeed quasi-translation invariant, hence absolutely continuous w.r.t. \( \nu \), see e.g. [20]. The Radon-Nikodym derivative, in the case of fractional Wiener measure \( \nu_H \), is given by

\[ \frac{d\nu_H \circ \tau_{sk}}{d\nu_H} (B^H) = \frac{1}{\mathbb{E}(\exp(s\langle dB^H, \dot{k}\rangle_H))} \exp(s\langle dB^H, \dot{k}\rangle_H), \quad s \in \mathbb{R}, \]

where the first expression may be considered as an \( L^2(\nu_0) \) limit, \( dB^H \) denotes the fractional white noise process and \( \dot{k} \) the derivative of the function from the Cameron-Martin space \( K_H \). See also [37].

### 2.2 The Edwards Model

The self-intersection local time of a fractional Brownian motion \( B^H \) is given informally by

\[ L(T) := L(T, B^H) := \int_0^T dt \int_0^t ds \, \delta(B^H_t - B^H_s). \]

However it is well known that, for \( Hd = 1 \) one has \( L(T) = \infty \) \( \nu_H \)-a.e., see e.g. [24]. Therefore a renormalization procedure is needed. Let us use the heat kernel for the approximation of the \( \delta \)-function

\[ p_\varepsilon(x) := \frac{1}{(2\pi \varepsilon)^{d/2}} e^{-\frac{|x|^2}{2\varepsilon}} = \frac{1}{(2\pi)^d} \int_{\mathbb{R}^d} e^{-\frac{|y|^2}{2\varepsilon} + i(y,x)} \, dy, \quad x \in \mathbb{R}^d, \]

which leads to the approximated self-intersection local time, see also [24]

\[ L_\varepsilon(T) := \int_0^T dt \int_0^t ds \, p_\varepsilon(B^H_t - B^H_s) = \frac{1}{(2\pi)^d} \int_0^T dt \int_0^t ds \int_{\mathbb{R}^d} e^{-\frac{|y|^2}{2\varepsilon} + i(y,B^H_t - B^H_s)} \, dy. \]
Moreover, as in [41] one has to center the local time in order to perform the limit later on. Hence we define:

\[ L_{\varepsilon,c}(T) := L_{\varepsilon}(T) - \mathbb{E}(L_{\varepsilon}(T)). \]

In [24] it is shown that for \( \varepsilon \to 0 \) there exist a limit of \( L_{\varepsilon,c}(T) \) in the space of square integrable functions. We denote:

\[ L_{\varepsilon,c}(T) \to L_c = L_c(T), \quad \varepsilon \to 0. \]

In the case \( Hd = 1 \), it is shown in [15] that, under certain conditions on the coupling constant \( g \), one has that the random variable \( e^{-gL_c} \) is a well defined object as an integrable function w.r.t. \( \nu_H \). Hence we can define the fractional Edwards measure in this case by

\[ d\nu_{H,g} := \frac{1}{\mathbb{E}(e^{-gL_c})} e^{-gL_c} d\nu_H. \]

**Remark 4.**
1. Note that by this definition \( \nu_{H,g} \) is indeed a probability measure which is absolutely continuous w.r.t. the fractional Wiener measure \( \nu_H \). We will hence use several times that properties are holding \( \nu_H \)-a.e. and hence \( \nu_{H,g} \)-a.e.
2. Notice also that the existence of the density as an \( L^1(\nu_H) \) function is not trivial due to the fact that, after centering the random variable \( L_c \) can indeed take negative values and the exponential could become infinity. The ensurance of integrability, at least for mild assumptions on \( g \) is done in [15].
3. The existence of certain exponential moments of \( L_c \) was studied in [26]. Due to this property the measure \( \nu_{H,g} \) is also defined at least for some negative \( g \).

In the following we shall restrict our considerations to coupling constants \( g \) such that \( e^{-gL_c} \in L^1(\nu_H) \), see [15].

### 2.3 Dirichlet Forms

For the stochastic quantization we will use classical Dirichlet forms of gradient type in the sense of [1]. We start with a densely defined bilinear form of gradient type

\[ \mathbb{E}(f,g) = \int \langle \nabla f, \nabla g \rangle dm \]
in a suitable $L^2(m)$ space and show closability. In many particular cases, as in [4], this can be done by an integration by parts argument. Here however, due to the lack of Meyer-Watanabe differentiability of the self-intersection local time for the case $H \Delta = 1$, see e.g. [23] for the fBm case, the techniques are more involved. Instead we show quasi-invariance of the fractional Edwards measure $\nu_{H,g}$ with respect to shifts in the Cameron-Martin space $K_H$ of fBm. Details on Dirichlet forms can be found in the monographs [7, 14, 32] and for the gradient Dirichlet forms, see [1].

As mentioned above we consider classical gradient Dirichlet forms, hence we have to introduce the gradient. To this end, at first we define the space of smooth cylinder functions. For a topological vector space $(\mathcal{X}, \tau)$ we define the set of smooth bounded cylinder functions

$$\mathcal{F} \mathcal{C}^\infty_b(\mathcal{X}) := \{ f(l_1, \ldots, l_n) \mid n \in \mathbb{N}, f \in C^\infty_b(\mathbb{R}^n), l_1, \ldots, l_n \in \mathcal{X}' \},$$

where $C^\infty_b(\mathbb{R}^n)$ is the space of bounded infinitely often differentiable functions on $\mathbb{R}^n$, where all partial derivatives are also bounded.

For $u \in \mathcal{F} \mathcal{C}^\infty_b(X_H)$ and $\omega \in X_H$, following the notation [1], we define

$$\frac{\partial u}{\partial k}(\omega) := \frac{d}{ds}u(\omega + sk)|_{s=0}.$$ 

By $\nabla u(\omega)$ we denote the unique element in $\mathcal{H}_H$ such that

$$\langle \nabla u(\omega), k \rangle_H = \frac{\partial u}{\partial k}(\omega), \quad \text{for all } k \in K_H.$$ 

**Theorem 5.** The bilinear form

$$\mathcal{E}_H(u, v) := \mathbb{E}_H(e^{-gL_c}\nabla u \cdot \nabla v), \quad u, v \in \mathcal{F} \mathcal{C}^\infty_b(X_H)$$

is a symmetric pre-Dirichlet form, i.e., in particular closable, and gives rise to a local, quasi-regular symmetric Dirichlet form in $L^2(X_H, \nu_{H,g})$.

The proof of Theorem 5 is given in Section 3 which contains the proofs and main results. As indicated above we show closability of the bilinear form via quasi-translation invariance along shifts in the Cameron-Martin space $K_H$.

**Remark 6.** As in [22, Cor. 10.8] we obtain that the closures of $(\mathcal{E}_H, \mathcal{F} \mathcal{C}^\infty_b(X_H))$ and $(\mathcal{E}_H, \mathcal{P})$ coincide, where $\mathcal{P} \subset L^2(X_H, \nu_{H,g})$ denotes the dense subspace of polynomials.
3 Main Results and Proofs

Crucial for the results of this paper is the following theorem.

**Theorem 7.** Let \( k \in K_H \) be given and
\[
 a_{sk} := \frac{d\nu_H \circ \tau_{sk}}{d\nu_{H,g}}, \quad s \in \mathbb{R}.
\]

Then the process \((a_{sk})_{s \in \mathbb{R}}\) has a version which has \( \nu_H \)-a.e. (and hence \( \nu_{H,g} \)-a.e.) continuous sample paths.

We denote by \( L(T, u, k) \) the self-intersection local time of \( X^{H,u,k} \) and similarly for resp. \( L_\varepsilon(T, u, k) \) and \( L_{\varepsilon,c}(T, u, k) \):
\[
\begin{align*}
L(T, u, k) &:= \int_0^T dt \int_0^t ds \delta(X_{t}^{u,k} - X_{s}^{u,k}), \\
L_\varepsilon(T, u, k) &:= \frac{1}{(2\pi)^d} \int_0^T dt \int_0^t ds \int_{\mathbb{R}^d} dy e^{-\frac{\varepsilon}{2}|y|^2 + i(y, X_{t}^{u,k} - X_{s}^{u,k})}, \\
L_{\varepsilon,c}(T, u, k) &:= L_\varepsilon(T, u, k) - \mathbb{E}(L_\varepsilon(T)).
\end{align*}
\]

To prove Theorem 7 we need the following two lemmata.

**Lemma 8.** Let \( \gamma \in (0, 1) \) and \( k \in K_H \) be given. Then there exists a positive constant \( C \) such that
\[
\|L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k)\|_{L^2} \leq C|u - v|^{1+\gamma} \tag{3}
\]
for all \( u, v \in \mathbb{R} \) and \( \varepsilon > 0 \).

**Proof.** Explicitely (3).
\[
\begin{align*}
L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k) &:= L_\varepsilon(T, u, k) - L_\varepsilon(T, v, k) \\
&= \frac{1}{(2\pi)^d} \left( \int_0^T dt \int_0^t ds \int_{\mathbb{R}^d} dy \left( e^{-\frac{\varepsilon}{2}|y|^2} \left( e^{i(y, X_{t}^{u,k} - X_{s}^{u,k})} - e^{i(y, X_{t}^{v,k} - X_{s}^{v,k})} \right) \right) dy \right) \\
&= \frac{1}{(2\pi)^d} \left( \int_0^T dt \int_0^t ds \int_{\mathbb{R}^d} dy e^{-\frac{\varepsilon}{2}|y|^2} \left( e^{i(\varepsilon y, k_t - k_s)} - e^{i(\varepsilon y, k_v - k_s)} \right) e^{i(y, B_H - B'_H)} dy \right)
\end{align*}
\]

which implies
\[
\begin{align*}
&\left| L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k) \right|^2 \\
&= \frac{1}{(2\pi)^{2d}} \int_T dt \int_{\mathbb{R}^{2d}} dy e^{-\frac{\varepsilon}{2}(|y_1|^2 + |y_2|^2)} \left( e^{i(y_1, k_t - k_s)} - e^{i(y_1, k_v - k_s)} \right) \\
&\times \left( e^{-i(y_2, k_t' - k_s')} - e^{-i(y_2, k_v' - k_s')} \right) e^{i(y_1, B_H - B'_H) - i(y_2, B'_H - B''_H)},
\end{align*}
\]

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where \( d\tau = dsdt ds'dt' \), \( dy = dy_1 dy_2 \) and
\[
T := \{(s, t, s', t') : 0 < s < t < T, 0 < s' < t' < T\}.
\]
Computing the expectation
\[
\mathbb{E}(e^{i(y_1 B_t^H - B_s^H) - i(y_2 B_{t'}^H - B_{s'}^H)}) = \prod_{j=1}^{d} \mathbb{E}(e^{i(y_1 B_{t_j}^H - B_{s_j}^H) - i(y_2 B_{t'_j}^H - B_{s'_j}^H)}) = e^{-\frac{1}{2}(y, \Sigma y)},
\]
where \( y = (y_1, y_2) \) and \( \Sigma = \begin{pmatrix} \lambda & \mu \\ \mu & \rho \end{pmatrix} \) is a symmetric matrix with
\[
\lambda = |t - s|^{2H}, \\
\rho = |t' - s'|^{2H}, \\
\mu = \frac{1}{2} \left(|t - s|^{2H} + |t' - s'|^{2H} - |t - t'|^{2H} - |s - s'|^{2H}\right).
\]
Thus, the l.h.s of (3) is equal to
\[
\|L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k)\|_{L^2}^2
= \frac{1}{(2\pi)^{2d}} \int_T \int_{\mathbb{R}^{2d}} dy e^{-\frac{1}{2}(|y, \Sigma y| + |y|^2)}
\times \left(e^{iv(y_1 k_1 - k_s)} - e^{iv(y_1 k'_1 - k_{s'})}\right)
\times \left(e^{-iv(y_2 k_2 - k_s)} - e^{-iv(y_2 k'_2 - k_{s'})}\right).\]
Notice that for any given \( \alpha \in (0, 1] \) there exists a constant \( C \in (0, \infty) \) (from now on, the constant \( C \) might be different from line to line) such that
\[
|\cos(x) - \cos(y)| \leq C|x - y|^\alpha \wedge 1, \\
|\sin(x) - \sin(y)| \leq C|x - y|^\alpha \wedge 1.
\]
On the other hand, we have
\[
\left(e^{iv(y_1 k_1 - k_s)} - e^{iv(y_1 k'_1 - k_{s'})}\right)
\times \left(e^{-iv(y_2 k_2 - k_s)} - e^{-iv(y_2 k'_2 - k_{s'})}\right)
= \left(\cos(u(y_1 k_1 - k_s)) - \cos(v(y_1 k_1 - k_s))\right)
\times \left(\cos(u(y_2 k_2 - k_s)) - \cos(v(y_2 k_2 - k_s))\right)
\times \left(\sin(v(y_1 k_1 - k_s)) - \sin(v(y_1 k_1 - k_s))\right)
\times \left(\sin(u(y_2 k_2 - k_s)) - \sin(u(y_2 k_2 - k_s))\right)
+ \text{(cross terms)},
\]
where the “cross terms” are odd functions. Hence the \( y_1, y_2 \)-integral with these functions vanishes. Finally we obtain the following estimate for the l.h.s of (3).
\[
\|L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k)\|_{L^2}^2
\leq C|u - v|^{2\alpha} \int_T \int_{\mathbb{R}^{2d}} dy e^{-\frac{1}{2}(|y, \Sigma y| + |y|^2)}(|y_1||y_2|)^{2\alpha}.
\]
Here we used the fact that functions from the Cameron-Martin space are continuous since they are given as fractional integral operators acting on square integrable functions, which allows to bound them in supremum norm, see [37].

If we denote by $I_d$ the $d \times d$ identity matrix, then the Gaussian integral is equal to

$$
\int_{\mathbb{R}^{2d}} dy e^{-\frac{1}{4}((y,\Sigma y) + \varepsilon|y|^2)(|y_1||y_2|)} = 2^{d(2\alpha + 1)} \Gamma \left( \alpha + \frac{1}{2} \right)^\frac{2d}{\det(\Sigma + \varepsilon I_d)^{\frac{1}{2} + \alpha}}
$$

Summarizing we obtain

$$
\|L_{\varepsilon,c}(T, u, k) - L_{\varepsilon,c}(T, v, k)\|^2_{L^2} \leq C|u - v|^{2\alpha} \int_T d\tau \frac{1}{[(\lambda + \varepsilon)(\rho + \varepsilon) - 2\mu]^{\frac{1}{2} + \alpha}} < \infty,
$$

by Lemma 11 in [24] and the fact that for every $\varepsilon > 0$ the above integral has no singularities. Taking $\alpha \in \left[\frac{1}{2}, 1\right)$ yields the desired statement.

**Lemma 9.** Let $Y(u, k) := L_c(B^H + uk) - L_c(B^H)$, for $u \in \mathbb{R}$ and $k \in K_H$. Then for any $\gamma \in (0, 1)$, there exists a constant $0 < C < \infty$ such that

$$
\mathbb{E}\|Y(u, k) - Y(v, k)\|^2 \leq C|u - v|^{1+\gamma}, \quad u, v \in \mathbb{R}.
$$

**Proof.** We have from [24] that $L_{\varepsilon,c}$ is convergent in $L^2(\nu_H)$ for $\varepsilon \to 0$. Hence there is a sequence $\varepsilon_n \to 0$ such that $L_{\varepsilon_n,c} \to L_c$ in probability w.r.t. $\nu_H$. Hence $L_{\varepsilon_n,c}(\cdot + uk) \to L_c(\cdot + uk)$ in probability w.r.t. $\nu_H$. Therefore $L_{\varepsilon_n,c}(\cdot + uk) - L_{\varepsilon_n,c}(\cdot) \to Y(u, k)$ in probability w.r.t. $\nu_H$. This gives immediately the desired result by Lemma 8.

Now we have all ingredients to prove the Theorem 7.

**Proof of Theorem 7.** By Lemma 9 we know that for any $k \in K_H$ and $u \in \mathbb{R}$ there is a version $\tilde{Y}(u, k)$, i.e

$$
\nu_H \left( Y(u, k) = \tilde{Y}(u, k) \right) = 1, \quad u \in \mathbb{R},
$$

such that

$$
\nu_H \left( \tilde{Y}(u, k) \text{ is continuous with respect to } u \in \mathbb{R} \right) = 1.
$$

By definition of the fractional Edwards measure

$$
\nu_{H,g} = \frac{1}{\mathbb{E}(e^{-gL_c})} e^{-gL_c} \nu_H,
$$

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it is clear that $\nu_{H,g} \circ \tau_{uk}$ is absolutely continuous w.r.t. $\nu_{H,g}$ for all $u \in \mathbb{R}$ and $k \in K_H$. Then by Lemma 3 we know that
\[ a_{uk} = e^{-uY(u,k)} \frac{\exp(u \int \dot{k} dB^H)}{E(\exp(u \int \dot{k} dB^H))}. \]

Now let
\[ \tilde{a}_{uk} = e^{-u\tilde{Y}(u,k)} \frac{\exp(u \int \dot{k} dB^H)}{E(\exp(u \int \dot{k} dB^H))}. \]

Hence we have, with the previous consideration of $\tilde{Y}(u,k)$
\[ \nu_H(\tilde{a}_{uk} \text{ is continuous with respect to } u \in \mathbb{R}) = 1, \]

and due to the absolute continuity of $\nu_{H,g}$ w.r.t. $\nu_H$ the same holds for $\nu_{H,g}$ which shows the assertion.

Proof of Theorem 5. Since the Cameron-Martin space $K_H$ is dense in $\mathcal{H}_H$ we can find an orthonormal basis $(k_n)_n$ such that the bilinear form on $\mathcal{F}_{C^\infty}(X_H)$ can be written as
\[ \mathcal{E}_H(u,v) = \sum_{n=1}^{\infty} \int \frac{\partial u}{\partial k_n} \frac{\partial v}{\partial k_n} d\nu_{H,g}. \]

From Proposition 3.7 in [32] Chapter I it suffices to show closability for every $n$ separately. However this is a direct consequence of Theorem 7 and Corollary 2.5 in [1]. Hence as in the proof of Proposition 3.5 in [32] Chapter II, Section 3a) we obtain a Dirichlet form as the closure $\left(\mathcal{E}_H, D(\mathcal{E}_H)\right)$ of the above quadratic form. For locality, see Example 1.12(ii) in [32] Chapter V and for quasi-regularity, see [32] Chapter IV Section 4b).

As a direct consequence of Theorem 3.5 in [32] Chapter IV and Theorem 1.11 in [32] Chapter V we have:

Theorem 10. There exists a diffusion process
\[ \mathbb{M}_H = (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, (X_t)_{t \geq 0}, (P_\omega)_{\omega \in X_H}) \]
with state space $X_H$ which is properly associated with $\left(\mathcal{E}_H, D(\mathcal{E}_H)\right)$. In particular, $\mathbb{M}_H$ is $\nu_{H,g}$-symmetric and has $\nu_{H,g}$ as invariant measure.
Conclusion

In this work we showed the existence of a Markov process having the fractional Edwards measure for $Hd = 1$ as an invariant measure. The process is obtained as a Hunt process associated to the symmetric Dirichlet form $\mathcal{E}_H$. Closability of the form was shown using quasi-translation invariance of the fractional Edwards measure w.r.t. shifts along the Cameron-Martin space. This generalizes the results found in [4] for the case $Hd < 1$, where the closability was proved by integration by parts. This is not possible in the present case ($Hd = 1$) due to the lack of Meyer-Watanabe differentiability of the density. The explicit representation of the generator is known in the case $Hd < 1$ by standard integration by parts techniques, see [5]. In the case $Hd = 1$ this however is unknown. To characterize the Markov process in the present situation we plan to use Mosco convergence in the Hurst parameter $H$ for approximating Dirichlet forms and hence to obtain convergence of the associated operator semigroups.

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