Mode transition in Bubbly Taylor-Couette flow measured by PTV

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Abstract. The drag acting to the inner cylinder in Taylor-Couette flow system can be reduced by bubble injection. In this research, relationship between drag reduction and change of vortical structure in a Taylor-Couette flow is investigated by Particle Tracking Velocimetry (PTV). The velocity vector field in the $r$-$z$ cross section and the bubble concentration in the front view ($z$-$\theta$ plane) are measured. This paper describes the change of vortical structures with bubbles, and the mode transition that is sensitively affected by the bubbles is discussed. The bubbles accumulate in the three parts relative to vortex position by the interaction between bubbles and vortices. The status of bubble’s distribution is different depending on position. This difference affects mode transition as its trigger significantly. The presence of bubbles affects the transition from toroidal mode to spiral mode but does not induce the transition from spiral mode to toroidal mode. Further we found that Taylor vortex bifurcates and a pair of vortices coalesces when the flow switches between spiral mode and toroidal mode.

1. INTRODUCTION

The use of bubbles as a way for reducing skin frictional drag reduction in shear flows has become a focus for engineering. This method is expected that large energy-saving effect is obtained by applying to large ships and pipelines whose drags are occupied almost with the frictional drag. So far, several mechanisms such as a variation of effective viscosity, decrease of mean density and modulation of turbulent vortices caused by bubbles motion have been reported to date as major factors reducing frictional drag. The actual physical mechanism, however, has been less understood owing to a complicated dynamic process existing in bubbly two-phase turbulent shear flows. To clarify this mechanism, several studies have been performed via experiment. Moriguchi et al. (2002) reported that the drag reduction effect becomes large in high void fraction regions and is generally independent of bubble’s diameter. Gabillet et al. (2002) showed that the Reynolds stress rather increases under some conditions with injection bubbles. These studies dealt with channel flow, i.e. spatially developed boundary layer.

In this study, we chose Taylor-Couette flow driven by rotating concentric cylinders. The adopting of this configuration of flow takes following advantages in investigating mechanism of bubble-originated drag reduction. i) It is possible to evaluate the drag reduction mechanism in quasi-steady state because the flow is periodic and closed so that the global energy balance is also closed inside the system. ii) The flow near the bubble generation part doesn’t affect the main section of interest so significantly, thus a high reproducibility of experiment is guaranteed.
iii) We can discuss carefully the flow field altered by bubble injection because a number of researches have been published for the structure of single-phase Taylor-Couette flow to be compared with. Upon these advantages, several studies about two-phase Taylor-Couette flow have been performed. Djeridi et al. (2004) observed that the period of circumferential wave is shortened by bubbles in wavy vortex flow region and the axial wave length becomes around 1.5 times as that without bubbles. Further, they also reported that the bubbles accumulate near the center of Taylor vortex in low Re number and out-flow region in the vicinity of the inner cylinder wall in high Re number. Berg et al. (2003) measured the frictional drag in bubbly Taylor-Couette turbulent flow, and found that bubbles reduce the friction while solid particles did not do so effectively. Murai et al. (2008) presented that the power gain of the drag reduction gets largest when the toroidal mode and the spiral mode coexist (we call this region the composite region). This implies that we may get more understanding of the drag reduction mechanism by investigating the composite region, and this can be best target to physically elucidate the drag reduction mechanism. In addition, the interaction between two phases significantly modifies the original flow structure in this region because the effect of bubbles, which is mainly buoyancy, and inertia force of liquid become comparable, or Froude number is in order of unity. This corresponds with the fact that the flow switches between toroidal mode and spiral mode in time. To understand this interaction in detail, some studies by numerical simulation also have been reported recently. Climent et al. (2007) numerically analyzed the periodic bubble accumulation under one-way coupling approach in wavy vortex mode. Sugiyama et al. (2007) has also carried out nearly direct numerical simulation in this mode and has shown more accurate dynamics in two-phase interaction by means of two-way coupling approach.

In this study, we experimentally investigate the interaction between two phases in the composite region. As far as we surveyed, few studies have succeeded in measuring the velocity and vorticity in liquid phase in this composite region under two-phase situation. Ordinary PIV, which is based on cross correlation interrogation, cannot acquire the liquid behavior in unsteady bubbly Taylor-Couette system because of difficulties in optical separation of visualized image into two phases. Our approach is succeeded by utilizing PTV, i.e. particle-tracking type of PIV, which is combined with spatio-temporal post-processing of discrete velocity vector information. From the parametric study enabled by the present measurement, we are going to discuss how the original vortex structure in Taylor-Couette flow is altered by presence of bubbles, and to conclude which factor has the most important responsibility to promote the drag reduction by bubbles in this paper.

2. EXPERIMENTAL METHOD

2.1 Experimental setup

Figure 1 shows the schematic diagram of experimental setup. The rotating inner cylinder has 120 mm outer diameter and is made of resin painted black to aid flow visualization. The immobilized outer cylinder has 144 mm inner diameter and a thickness 6 mm, and is made of transparent acrylic resin. The gap distance between cylinders is 12 mm. The effective height filled with working fluid is 390 mm; therefore aspect ratio is 32.5, thus the number of Taylor vortex pairs is estimated more than 10. This helps to relax the dependency of initial condition to measuring the two-phase flow in quasi-steady state, or fully developed state. The upper surface is open to atmosphere to release bubbles on the surface. Silicone oil of kinematic viscosity 10 cSt and density 935 kg/m$^3$ was used as working fluid. Use of silicone oil removes uncertainties to guarantee experimental reproducibility. It restricts bubble coalescence and has few effect of contamination. Bubbles were generated by porous body set up under the cylinder, and flowed up continuously in the gap because of its buoyancy. The gas flow rate was varied from 0 to 1.17x10$^{-6}$ m$^3$/s, which corresponds to 5% in the bulk mean void fraction at the maximum. The bubble shape is almost spherical because of low capillary number less than 0.05, and of low
Weber number less than unity. The spherical bubble’s mean diameter was 0.6 mm in Sauter mean value. A stepper motor was mounted on the shaft connected to the inner cylinder. The rotational speed was varied from 8.3 to 62.5 rad/s, which provides centrifugal acceleration in bubbly mixture stronger than the acceleration of gravity in the fastest case. The apparatus was submerged in rectangular oil jacket to reduce the influence of refraction and temperature change. As tracer particles for PTV, a glass-made fine particle named “FLO-BEADS (CL-2507)” was mixed in the working fluid. The particle properties are 180 µm in average diameter and 0.918 kg/cm³ in density. The test section of r-z cross section was illuminated by metal halide light sheet. The motion of bubbles and tracer particles are recorded by high-speed digital video camera at 1000 frames per second. The detailed photographing condition and experimental condition are shown in Table 1 and 2, respectively.

![Fig. 1 Schematic of experimental setup](image)

### Table 1 Photographing conditions

|                         |                  |
|-------------------------|------------------|
| Frame rate              | 1000 fps         |
| Shutter speed           | 1/2000 s         |
| F number                | 5.6              |
| Frame size              | 200x1024 pix     |

### Table 2 Experimental conditions

|                              |                  |
|------------------------------|------------------|
| Inner cylinder radius        | Rᵢ = 60 mm       |
| Outer cylinder radius        | Rₒ = 72 mm       |
| Radius ratio                 | Rᵢ/Rₒ = 0.833   |
| Aspect ratio                 | L/d = 32.5       |
| Density of liquid            | ρᵢ = 935 kg/m³ (at 298 K) |
| Kinematic viscosity          | ν = 10x10⁻⁶ m²/s (at 298 K) |
| Surface tension              | σ = 19.7x10⁻³ N/m |
| Mean bubble diameter         | dᵢ = 0.6 mm      |
| Gas flow rate                | Qᵢ = 0 to 1.17x10⁻⁶ m³/s |
| Re number                    | Re = 600 to 4500 |
| Ta number                    | Ta = 7.92x10⁴ to 3.88x10⁶ |
| Fr number                    | Fr = 0.65 to 4.89 |
2.2 Experimental range targeted

It was reported that various flow patterns take place in single phase Taylor-Couette flow depending on inner- and outer-cylinder Reynolds numbers (Andereck et al., 1986). Figure 2 shows the mode transitions along Reynolds number of rotating inner cylinder at fixed outer cylinder, calculated for the present radius ratio of the coaxial cylinders. In this case, Reynolds number is defined by

\[
Re = \frac{R_o \omega (R_o - R_i)}{\nu} \tag{1}
\]

where \(\nu\) is the kinematic viscosity of pure liquid without bubbles. The uncertainty of \(Re\) is 5%, which is estimated by the temperature-dependent kinematic viscosity of the silicone oil. The effective viscosity changes slightly when bubbles are mixed. Such effect is quantitatively known in simple shear flows as that of Stokes flow regime (Rust et al., 2002). The two-phase flow inside the present bubbly Couette-Taylor system is considerably inhomogeneous, and thus the original kinematic viscosity of liquid is used for the definition of \(Re\) number. In this experiment, \(Re\) number was varied from 600 to 4500. The single phase Taylor-Couette flow in this range has five flow modes; i.e., Circular Couette Flow (\(Re < 92\), CCF), Taylor Vortex Flow (92 < \(Re < 138\), TVF), Wavy Vortex Flow (138 < \(Re < 1020\), WVF), Modulated Wavy Vortex Flow (1020 < \(Re < 1380\), MWVF), Turbulent Taylor Vortex Flow (1380 < \(Re\), TTV). These critical \(Re\) numbers are adopted from the result of Djeridi et al. (2004) due to the radius ratio is almost same.

When the interaction between bubbles and vortices is investigated, the ratio of centrifugal acceleration to gravity become important parameter and it is defined by the Froud number

\[
Fr = \frac{\omega (R_i)}{g} \tag{2}
\]

\(Fr\) number increases with \(Re\) number because both of those are functions of \(\omega\).

![Fig. 2 Flow regime in single phase Taylor-Couette flow](image)

2.3 Measurement of liquid flow field by PTV

The method to obtain velocity vectors of liquid phase is explained in this section. Figure 3 (a) is the raw image photographed, and (b) is the path line of the tracer particle and bubbles. The bubbles exist in the parts of red circle, which are recorded as higher brightness and larger than that of tracer particles. We obtained the velocity vector from photographed images by PTV. The binary cross correlation method was used as PTV algorithm. In this method, the determination of the same particle between two images is enabled by using geometrically resemblance of particle pattern. The binary image is used for detecting the particle information. We distinguished between bubbles and tracer particles by the size. In this experiment, the threshold value of the area between bubbles and tracer particles was determined 15 pixels. This value was determined by using Otsu’s method on diameter distribution of bubbles and tracer particle
obtained from images. The accuracy of particle detection was also enhanced by doing similar process on brightness. Then we got the vectors from the motion of tracer particles. Further, the obtained vector was rearranged by spatio-temporal Laplace equation (Ido and Murai, 2007) to get regularly arranged vector data as shown in Figure 3(c). Figure 3 (d) shows the contour map of the stream function calculated from the velocity vector obtained. It is confirmed with these figure that the present combination of binarization, PTV, and its post-processing derive fine structure of vortical cells. In the section 3, we discuss about some topics using these quantities measured by the present method.

Fig. 3 Instantaneous two-phase flow structure measured in r-z section of bubbly Taylor-Couette flow: (a)Original image, (b)Path line of the tracer particle and bubbles, (c)Velocity vector measured, (d)Color contour map of stream function.

3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1 Vortical structure changed by mixing of bubbles

Figure 4 shows the spatio-temporal map of stream function along the center line in the gap (Re=1800, Fr=1.96, Qg=0 and 0.17 m³/s). The horizontal axis and vertical axis respectively represent time and vertical position z. Since vortex centers are located near the center line in the gap, the magnitude and the sign of the stream function indicates the vortex intensity and the direction of the rotation. With this representation, spatio-temporal flow structure of vortical cells is easier identified. From these maps, we can see the difference of vortical structure between with and without bubbles. Red and blue bands represent the clockwise vortex and counter clockwise vortex, respectively. We can find that the presence of bubbles provides elongation of vortical wavelength to be around 1.4 times as that without bubbles. This elongation implies the occurrence of the drag reduction, because momentum exchange is restricted by the elongation of vortical wavelength or the decrease in radial momentum transfer of the azimuthal flow. Another change by the mixing of bubbles is that the time fluctuation of vortical structure enlarges. Especially, it occurs at the out flow region, namely around the border between the downside of the blue band and the upside of the red band. This region of the change corresponds with the region in which the bubbles frequently exist. Hence it is simply concluded that the local change by the mixing of bubbles takes place at the region of bubble presence. In one-way numerical
simulation, it was found that bubbles are accumulated periodically on the wavy vortex in the azimuthal direction. In experiment, the bubbles accumulated there drive the change of the stripe pattern of the array of cells as shown here. Namely, this alternation of the cell structure is caused by two-way interaction between bubbles and vortex motion, and it would significantly affect the mode transition of Taylor-Couette flow as following discussion.

![Figure 4](image_url)

Fig. 4 The change of vortical structure by the presence of bubbles \((Re=1800, Fr=1.96)\). (a) without bubble, (b) with bubble \((Q_g=0.17\times10^{-6} \text{ m}^3/\text{s})\).

### 3.2 Mode transition

On focusing on the apparent pattern of bubble distribution, there are three modes found in Taylor-Couette flow; i.e., uniform, toroidal, and spiral modes. Murai et al (2008) investigated the transition diagram of the bubble distribution as functions of gas flow rate and Reynolds number. The first transition from the uniform to the toroidal mode is caused due to liquid-to-bubble interaction while strong Taylor vortex is formed. The second transition from the toroidal to the spiral mode happens due to bubble-to-liquid interaction since local high void fraction in toroidal distribution begins to drive the cell structure resulting in destroying the toroidal mode. In order to find out more detailed process of it, we have captured the switching process between these two modes.

Figure 5 shows temporal change of bubble distribution in this switching region, i.e. coexisting region of toroidal and spiral modes. This image is obtained by rearranging the line image extracted from the front visualized image along the time direction. The Reynolds number is 1800, the Froud number is 1.96 and the gas flow rate is 0.50 \(\times 10^{-6} \text{ m}^3/\text{s}\). Horizontal axis represents time and vertical axis represents vertical position, respectively. In Figure 5(a), the toroidal mode occurs at \(t<4\). In this region, torus-shape bubble cluster moves upward gradually by the buoyant effect. Owing to this displacement, then the flow transits to the spiral mode in the lower half of the gap. The bubble cluster connects spirally from the bottom to the top. After the spiral mode is formed, bubble distribution shows a downward phase velocity while individual bubbles rise up continuously in the spiral path. The reason of the downward phase velocity is explained by the azimuthal phase velocity of the spiral structure being slower than the azimuthal speed of the inner cylinder. Namely, the spiral path of the bubbles is also rotated at a certain characteristic speed. The mechanism of the spiral accumulation of bubbles is discussed in Murai et al (2008). Their explanation is based on spatial matching between original Taylor vortex wavelength and the axial pitch of the bubble trajectory that turns around the cylinder during the rotation.
In Figure 5(a), as time elapses at $t>8$, the toroidal mode begins to be restored from the downside of the gap. That is, the mode transition from spiral to toroidal mode occurs. We found that the ordinary transition from toroidal to spiral mode (T-S transition) can occur from anywhere in the gap. To the contrary, the most opposite transition (S-T transition) occurs from the downside of the gap. Once the ordinary T-S transition occurs, flow pattern keeps the spiral mode whether bubbles exist or not (Fig. 5 (b)). The opposite S-T transition arises when the toroidal mode, which is generated from the lower part of the gap, propagates upward. It is therefore considered that the presence of bubbles significantly affects the vortical structure in the ordinary T-S transition but doesn’t affect it in the opposite S-T transition so much. This fact is consistent to the discussion mentioned above, i.e. the ordinary T-S transition is caused by bubble-to-liquid interaction. In other words, the opposite S-T transition is a process in which two-phase interaction ceases for decrease of bubbles in the flow. It is worth noting that there is another independent transition from spiral to toroidal modes at high Re number. This is called third transition and it happens because kinetic energy of turbulent Taylor vortex exceeds the buoyant energy of bubbles. Since bubbles are diffused widely in the gap due to turbulent mixing at this status, the body force is dissipated resulting in the null effect. The S-T transition in the coexisting region is different from it, and it is mainly realized by disappearance of bubbles due to high-speed exhausting of bubbles through the spiral path.

![Fig. 5 Temporal change of the bubble distribution. ($Re=1800, Fr=1.96, Q_g=0.50 \times 10^{-6} \text{ m}^3/\text{s}$)](image)

3.3 Temporal change of vortical structure and bubble distribution

The relationship between bubble distribution and vortical structure during the mode transition is investigated to elucidate how the bubbles affect the transition. Figure 6 (a) shows bubble
distribution in the vicinity of the inner cylinder, obtained by cutting the images of r-z cross section. The photographed region of this image is narrower than that of Figure 4 to observe the detailed process of the two-phase interaction. Figure 6 (b) shows spatio-temporal distribution of stream function along the center line in the gap. The stream function is obtained by the measured velocity distribution in r-z cross section, combined with post-processing to interpolate the scattered velocity vector information (see section 2.3). This result corresponds to the composite region of the two modes in bubble distribution, measured at Re number of 1800. The Fr number is 1.96, and gas flow rate, $Q_g$, is $6.7 \times 10^{-6}$ m$^3$/s. Red and blue bands represent the clockwise and counter-clockwise vortices, respectively. From figure 6 (a) and (b), we can find that the position of the bubble accumulation and that of vortex have a high correlation. The bubble accumulation takes place in the out-flow region of Taylor vortices. This result agrees with Djeridi et al. (2004), and can be explained by the balance between the centrifugal force of fluid and the drag force of bubbles in the local descending flow there. The most remarkable point, which is newly confirmed here, is that Taylor vortex bifurcates and a pair of vortices coalesces when the flow switches between spiral mode and toroidal mode. Such an oscillatory mechanism does not exist in ordinary Taylor vortex without bubbles.

![Image of bubble distribution in composite region of spiral mode and toroidal mode, Re=1800, Fr=1.96, Q_g=0.67x10^{-6} m^3/s.](image)

Fig. 6 (a) Variation of bubble distribution in composite region of spiral mode and toroidal mode, Re=1800, Fr=1.96, $Q_g=0.67 \times 10^{-6}$ m$^3$/s. (b) Spatio-temporal distribution of stream function on the center line of the gap. Red band represents clockwise vortex and blue band represents counter clockwise vortex.

Subsequently we measured the average bubble distribution, or probability distribution of bubbles in r-z cross section in order to know how bubbles interact with Taylor vortices. Figure 7 (a) is a sample of velocity vector distribution in a single pair of vortices, which is also measured by the present PTV. The bubble position relative to the vortex position is shown in Fig. 7 (b) to (e) for different Reynolds numbers from 800 to 2600. Since the length of the pair of vortices depends on Re number, the vertical coordinate in each figure is normalized by the individual length of it at each instant of time. With this expression, where bubbles tend to accumulate can be identified relatively to the vortex structure. At Re=800, the bubbles hardly accumulate in the vortices but distribute uniformly inside a thin layer in the vicinity of the inner cylinder. At
$Re > 1200$, this layer of the accumulation is broken, and it separates into three principal regions. One is the vicinity of the inner cylinder of out-flow part. The bubbles around this region clearly concentrate to the root of the outflow region as $Re$ number increases. Second region is seen inside the counter clockwise vortex. These bubbles are distributed in the vortex overall, and circulate with the vortex. In this situation, the vortex itself is unmodified by the bubbles while it can be lifted upward due to the buoyancy of bubbles. Third region is inside the clockwise vortex. In this part, the bubbles are distributed disproportionately in the clockwise vortex, i.e. the center of the accumulation is located near the outer cylinder. With this type of accumulation, we can deduce that the vortex is easily deformed due to the localized buoyancy. Therefore, this third region can be concluded to be a key factor that effectively changes the original vortical structure.

![Figure 7 Bubble position relative to the vortex position](image)

**Figure 7** Bubble position relative to the vortex position: (a) Sample velocity vector of liquid phase, (b),(c),(d),(e) Accumulation position of bubbles in $r$-$z$ cross section: (b) $Re$=800, (c) $Re$=1200, (d) $Re$=1800, (e) $Re$=2600

Figure 8 shows the change of void fraction in the composite region. This graph is extracted from the long sampling data and just represents the moment when the bubble distribution changes from the spiral mode to the toroidal mode. Sum of the brightness in the visualized image over the mode transition, the ordinate of the graph, represents magnitude of the void fraction relatively. While we need to convert it to the real void fraction for quantitative discussion, the brightness value can be a good estimate to evaluate the temporal fluctuation in void fraction since it varies monotonically to the void fraction. As seen in the upper figure showing the stream function, the liquid flow is identified to belong to the spiral mode at $t < 2.2s$, and to the toroidal mode at $t > 2.2s$. As we proved before, the bubble distribution corresponds to the mode of the liquid flow. Thus, the bubbles are distributed as the spiral mode in the first part, and as the toroidal mode in the second part. The result leads to the following discussion. In the spiral mode, the void fraction is kept small. It increases rapidly when the flow transits from the spiral to the toroidal mode. This occurs due to the difference of the vortex structure. In the spiral mode, a single spiral vortex tube exists in the flow, which starts at the bottom and reaches the top of the fluid layer. The bubbles in this mode can rise continuously in a single path that corresponds to the spiral tube. To the contrary, the toroidal mode has an array of torus-shaped vortex tubes that are separated to each other. Under this condition, bubbles are trapped in each tube to have a higher void fraction than the spiral mode. Namely, the rise of bubbles is restricted by the vortex tubes in the toroidal mode. As far as this restriction is stronger than the buoyancy...
Effect, bubbles accumulate in any position as time elapses. When the local void fraction takes higher value than a critical value for the accumulation, the buoyancy exceeds the restriction effect of the toroidal tubes to generate upward jumping of bubbles from a tube to another. This jumping of bubbles, or the axial traverse of bubbles, reduces the local void fraction quickly. Therefore, we can recognize a periodical fluctuation in void fraction in the graph. Furthermore, the data clearly shows that the time of high void fraction corresponds to the time when each vortex displaces to the highest part during the oscillation. Therefore, it is consistent to the previous discussion that the bubbles make the individual vortex centers lifting upward. After the bubbles jump, or the void fraction decreases quickly, the liquid flow field returns to be the toroidal mode temporally (see the oblique patterns of the stream function in the graph). The bubbles then again accumulate in the toroidal mode. Consequently, the iterative process of this periodical interaction generates the highest performance of the drag reduction.

4. CONCLUSIONS

As a fundamental study on drag reduction enabled by bubbles, the change of flow pattern in bubbly Taylor-Couette flow is investigated by using Particle Tracking Velocimetry. We focus on the flow mode subjected to the competition between the inertia of rotating liquid and the buoyancy of bubbles, which has a Froude number at the order of unity. The two-phase flow in this mode has a periodical switching between toroidal bubble distributions and spiral one, and the drag reduction gets the highest sensitivity to the void fraction. Our flow field measurement targeting this composite mode reveals following remarks. The two patterns of the bubble distribution; the toroidal and the spiral modes, correspond to those of the vortical structure in liquid phase. The transition from the toroidal to the spiral mode occurs due to accumulation of bubbles in torus bands. There are three regions of the bubble localization in r-z cross section; a) the root of the outflow, b) the area inside the counter-clockwise vortex, and c) the spot near the outer cylinder inside the clockwise vortex. Among these three, the effect of b) lifts up the array of vortices, and the effect of c) makes vortex deform to expand the length of the vortex pair. After the mode transits from the toroidal to the spiral one, the local void fraction decreases since the spiral vortex tube conveys bubbles continuously from the bottom to the top. Since the decrease in void fraction allows liquid to return to the toroidal mode, periodical switching between these two modes happens to the flow. Furthermore it is observed that Taylor vortex
bifurcates and a pair of vortices coalesces during the mode switching. With these findings, we conclude that the periodical interaction across the two modes provides the best performance of the drag reduction, and this is why the composite region has the highest gain factor to the void fraction.

**NOMENCLATURE**

| Symbol | Description   | Unit       |
|--------|---------------|------------|
| R      | radius        | [mm]       |
| D      | diameter      | [mm]       |
| d      | gap distance  | [mm]       |
| g      | gravity acc.   | [m/s²]     |

**Greek Letters**

| Symbol | Description   | Unit   |
|--------|---------------|--------|
| ν      | kinematic viscosity | [m²/s] |
| α      | void fraction  | [-]    |
| ω      | angular velocity | [rad/s] |

**Subscripts**

i/o inner/outer cylinder

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