Theory of spin accumulation and spin transfer torque in a magnetic domain wall

Tomohiro Taniguchi\textsuperscript{1,2}, Jun Sato\textsuperscript{1}, and Hiroshi Imamura\textsuperscript{1}
\textsuperscript{1} Nanotechnology Research Institute, AIST, Tsukuba, Ibaraki 305-8568, Japan, 
\textsuperscript{2} Institute of Applied Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8573, Japan  
(Dated: June 24, 2009)

We studied the spin accumulation and spin transfer torque in a magnetic domain wall by solving the Boltzmann equation for spin accumulation with the diffusion approximation. We obtained analytical expressions of spin accumulation and spin transfer torque. Both the adiabatic and the non-adiabatic components of the spin transfer torque oscillate with the thickness of the domain wall. We showed that the oscillating component plays a dominant role in the non-adiabatic torque when the domain wall is thinner than the spin-flip length. We also showed that the magnitude of the non-adiabatic torque is inversely proportional to the thickness of the domain wall.

PACS numbers: 72.25.Ba, 75.45.+j, 75.60.Ch

Spin-dependent electron transport in magnetic nanostructures results in many interesting phenomena such as the giant magnetoresistance effect \cite{1} and current-induced magnetization dynamics \cite{2,3}. Recently, such spin-dependent phenomena in magnetic domain walls have been investigated, due to great interest in their potential application to spintronics devices such as spin-motive-force memory \cite{4,5} and racetrack memory \cite{6}. In these devices, data are stored by moving the domain wall using spin transfer torque.

In 2004, Zhang and Li showed that spin transfer torque in a domain wall can be decomposed into two parts, the adiabatic and non-adiabatic torque \cite{7}. The adiabatic torque lies along the spatial gradient of the local magnetization while the non-adiabatic torque is perpendicular to this direction. Assuming that the spin accumulation obeys the phenomenological diffusion equation and is spatially independent of the domain wall, Zhang and Li showed that the ratio of the magnitudes of the adiabatic and non-adiabatic torques is determined by the precession frequency of the spin accumulation due to the exchange coupling and the spin-flip scattering time, and that the non-adiabatic torque is about two orders of magnitude smaller than the adiabatic torque.

The thickness of a domain wall is determined by the competition of the exchange coupling between the localized magnetizations and the magnetic anisotropy, and is usually on the order of 100 nm for conventional ferromagnetic metals such as Fe, Co, Ni, and their alloys. However, recent developments in the processing technology for nanostructures have allowed the production of a domain wall whose thickness is on the order of 1-10 nm, by reinforcing the shape anisotropy of a magnetic nanowire \cite{8} or trapping a domain wall in a current-confined-path geometry \cite{9}. These developments motivated us to study the transport phenomena in a thin domain wall \cite{10,11,12}. For such a thin domain wall, we cannot assume the spatial independence of the spin accumulation. Thus, it is important to estimate spin transfer torque by taking into account the spatial variation in the spin accumulation, which would be different from the estimation by Zhang and Li \cite{7}. Recently, Vanhaeverbeke and Viret calculated spin transfer torque in a thin domain wall by numerically solving the time-dependent Larmor equation of the spin accumulation in a moving frame and showed that the non-adiabatic torque is one order of magnitude larger than that estimated by Zhang and Li when the thickness of the domain wall is comparable to the Lamor precession length \cite{13}.

In this paper, we study spin accumulation and spin transfer torque in a domain wall by solving the Boltzmann equation with a diffusion approximation. We obtained the analytical expressions of spin accumulation and spin transfer torque. Both the adiabatic and the non-adiabatic components of the spin transfer torque oscillate with the thickness of the domain wall. We show that the oscillation plays a dominant role in the non-adiabatic torque when the domain wall thickness is less than the spin-flip length, which is defined by the product of the Fermi velocity and the spin-flip scattering time. For a domain wall that is much thinner than the spin-flip length, the non-adiabatic torque is about one order of magnitude larger than the adiabatic torque, which is one order of magnitude smaller than that estimated by Zhang and Li \cite{7} and qualitatively consistent with the results of Vanhaeverbeke and Viret \cite{13}. We also showed that the magnitude of the non-adiabatic torque is inversely proportional to the thickness of the domain wall.

We considered electron transport in a one-dimensional magnetic nanowire with a 180° domain wall which lies over \(-d/2 \leq x \leq d/2\), where \(d\) is the thickness of the domain wall. We assume that the interaction between the conducting (s-like) electrons and the localized (d-like) electrons is described by an sd exchange interaction, \(H_{sd} = -(J/2)\hat{\sigma} \cdot \hat{S}\), where \(\hat{\sigma}\) is the vector of the Pauli matrices, \(J\) is the sd exchange coupling constant and \(\hat{S}(x) = (0, -\sin\theta, \cos\theta)\) is the unit vector pointing in the direction of the localized spin angular momentum. The angle \(\theta(x)\) is given by \(0\) for \(x < -d/2\), \(\pi/d)(x + d/2)\) for \(-d/2 < x < d/2\), and \(\pi\) for \(x > d/2\), respectively.

Following Šimánek and Rebei \cite{14,15}, we employ the rotating frame where basic unit vectors are defined as \(\hat{e}_x = \alpha^{-1} \hat{S} \times (\partial\hat{S}/\partial x)\), \(\hat{e}_y = -\alpha^{-1} \partial\hat{S}/\partial x\) and \(\hat{e}_z = \hat{S}\), respectively. We assume that the direction of the lo-
calized spin varies slowly compared to the Fermi wavelength $\lambda_F$, i.e., $\alpha = d\theta/dx \ll 2\pi/\lambda_F$; thus, we could neglect the higher-order terms of $\alpha$ in the following calculation. The spin accumulation and spin transfer torque in a domain wall are obtained by solving the Boltzmann equation for the Wigner function defined as $\tilde{f}(x, p_x) = \frac{[f(x, p_x) + g(x, p_x) - \sigma]}{2}$, where $f(x, p_x)$ and $g(x, p_x)$ represent the charge and spin distribution functions, respectively. The spin accumulation $s$ and the spin current density $j$ are defined as

$$s = \int g \frac{d^3p}{(2\pi\hbar)^3}, \quad (1)$$

$$j = \int v_x g \frac{d^3p}{(2\pi\hbar)^3}, \quad (2)$$

respectively. It should be noted that the dimensions of $s$ and $j$ are density and density times velocity, respectively. The diffusion approximation, $\int v_x^2 g d^3p/(2\pi\hbar)^3 \approx (v_x^2/3)s$, is applied to the Boltzmann equation [14]. Up to the first order of $\alpha$, the transverse components of the spin accumulation, $s_x$ and $s_y$, and spin current, $j_x$ and $j_y$, obey the following equations;

$$\frac{\partial s_x}{\partial x} = -\frac{1}{2D} j_x + \frac{\omega_j \tilde{T}}{D} j_y, \quad (3)$$

$$\frac{\partial s_y}{\partial x} = -\frac{\omega_j \tilde{T}}{D} j_x - \frac{1}{2D} j_y, \quad (4)$$

$$\frac{\partial j_x}{\partial x} - \omega_j s_y + \frac{2}{\tau_{sf}} s_x = 0, \quad (5)$$

$$\frac{\partial j_y}{\partial x} + \omega_j s_x + \frac{2}{\tau_{sf}} s_y = \alpha j_z, \quad (6)$$

where $\omega_j = J/h$ is the Larmor precession frequency, $\tilde{T}$ is the momentum relaxation time, $\tau_{sf}$ is the spin-flip scattering time and $D = v_x^2 \tilde{T}/3$ is the diffusion constant [14]. The longitudinal spin current $j_z$ in Eq. (6) is given by $j_z = \beta j_x/(-e)$, where $\beta$ and $j_e$ are the spin polarization factor and the electric current density, respectively. In our definition, the positive electric current corresponds to the electron flow along the $-z$ direction.

The physics behind Eqs. (3)-(6) are as follows. Traveling through the domain wall, the conducting electrons vary the direction of their spin along the localized spin angular momentum $\mathbf{S}$. Then, spin accumulation and spin current polarized along the $y$-direction ($\propto \partial \mathbf{S}/\partial x$) are induced; see Eq. (6). The accumulated spins precess around $\mathbf{S}$ due to the sd exchange coupling with the precession frequency $\omega_j$. Then, the $x$-components of the spin accumulation and spin current are induced [see Eq. (5)]. Equations (3) and (4) relate the spin accumulation and spin current by the diffusion constant.

Before estimating the spin accumulation and spin transfer torque in a domain wall, we should emphasize the validity of our calculations. Since Eqs. (3)-(6) are obtained by applying the diffusion approximation to the Boltzmann equation, they are valid for $d \geq l_{mfp}$, where $l_{mfp} = e\tilde{T}/D$ is the mean-free-path of the conducting electrons. For a domain wall the thickness of which is much smaller than the mean free path, i.e., $d \ll l_{mfp}$, the Boltzmann equation should be solved without the diffusion approximation. Moreover, in such a very thin domain wall, we cannot neglect the higher-order terms of $\alpha$.

We assume that the transverse spin accumulation and spin current are such that they vanish at the limit of $|x| \to \infty$ and are continuous at $x = \pm d/2$. Then, solving Eqs. (3)-(6), the transverse spin accumulations in the domain wall are obtained as $s_x = \text{Re}[s_+]$ and $s_y = \text{Im}[s_+]$, where

$$s_+ = \frac{\pi(1 + i\zeta)j_z}{\omega_j d(1 - \zeta^2)} \left[1 - \exp\left(-\frac{d}{2\ell}\cosh\left(\frac{x}{\ell}\right)\right)\right]. \quad (7)$$

Here $\zeta = 2i/(\omega_j \tau_{sf})$ and $\ell$ is given by

$$\frac{1}{\ell} = \sqrt{\frac{1}{2D} \left[1 + 2\omega_j \tilde{T} \left(\omega_j + \frac{2}{\tau_{sf}}\right)\right]}, \quad (8)$$

where $k_s = \text{Re}[1/\ell]$ and $k_i = \text{Im}[1/\ell]$ characterize the oscillation and damping of $s_x$ and $s_y$ due to the sd exchange coupling and the spin-dependent scattering, respectively. As shown in Eq. (7), the transverse spin accumulations can be decomposed into spatially independent (first) and dependent (second) parts.

Figure 1 shows the spatial variation of the transverse spin accumulations, $s_x$ and $s_y$, for thick ($d = 100$ nm) and thin ($d = 10$ nm) domain walls; (a) $s_x$ for $d = 100$ nm, (b) $s_y$ for $d = 100$ nm, (c) $s_x$ for $d = 10$ nm and (d) $s_y$ for $d = 10$ nm, respectively. The magnitudes of $s_x$ and $s_y$ are divided by $j_x/\omega_J$, see Eq. (7).
and \( l \) (d), for a thin domain wall, the spin accumulations vary in the domain wall, and we cannot assume the spatial independence of the spin accumulations.

Let us estimate spin transfer torque in the domain wall, which is defined as

\[
\tau = \int_{-d/2}^{d/2} \omega_J s \times \mathbf{S} \, dx.
\]  

(9)

The Landau-Lifshitz-Gilbert equation for the localized magnetization \( \mathbf{M} = -\mathbf{S} \) with the torque \( \tau \) is given by

\[
\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{B} + \alpha_0 \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \frac{\gamma h}{2\pi M} \frac{\partial \mathbf{M}}{\partial x} + \frac{\gamma h}{2\pi M} r_x \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x},
\]

(10)

where \( \gamma \) is the gyromagnetic ratio, \( \mathbf{B} \) is the effective magnetic field, \( M \) is the magnitude of the magnetization and \( \alpha_0 \) is the Gilbert damping constant. \( \tau_y = e_y \cdot \tau \) and \( \tau_x = e_x \cdot \tau \) correspond to the adiabatic and non-adiabatic torque, respectively. By using Eq. (7), we find that

\[
\tau_y = \frac{\pi \beta_\perp}{e(1 + \zeta^2)} \left[ k_x e^{-k_x d} \left( k_x \cos k_x d - k_x \sin k_x d \right) \right]
\]

(11)

\[
- \frac{\pi \beta_\perp}{e(1 + \zeta^2)} \left( k_x \cos k_x d + k_x \sin k_x d \right) \right],
\]

\[
\tau_x = - \frac{\pi \beta_\perp}{e(1 + \zeta^2)} \left[ k_x e^{-k_x d} \left( k_x \cos k_x d - k_x \sin k_x d \right) \right]
\]

(12)

\[
+ \frac{\pi \beta_\perp}{e(1 + \zeta^2)} \left( k_x \cos k_x d + k_x \sin k_x d \right) \right].
\]

The first terms of Eqs. (11) and (12) are identical to the adiabatic and non-adiabatic torque estimated by Zhang and Li \( l \), respectively. These first terms arise from the spatially independent part of the spin accumulation, i.e., the first term of Eq. (7). It should be noted that these terms are independent of the thickness of the domain wall \( d \). For a thick domain wall, these first terms are dominant for spin transfer torque, and the ratio of the magnitude of the adiabatic and non-adiabatic torque, \( \tau_x / \tau_y \), is given by \( \zeta \approx 10^{-2} \) \( l \). On the other hand, the second and third terms of Eqs. (11) and (12) arise from the spatial variation in the spin accumulation, i.e., the second term of Eq. (7). As shown in Figs. (1) (c) and (1) (d), for a thin domain wall, we cannot neglect the spatial variation in the transverse spin accumulation, and these second and third terms dominate the spin transfer torque. It should be noted that these terms are inversely proportional to the thickness \( d \). Thus, for a thin domain wall, the strength of the spin transfer torque is considerably different from that estimated by Zhang and Li \( l \).

Figure 2 shows the strength of the adiabatic torque \( \tau_y \) renormalized by \( \pi \beta_\perp e \) against the thickness of the domain wall \( d \). We denote the torque for \( d \leq l_{\text{mfp}} \) by the dotted line because our calculations are restricted for \( d \geq l_{\text{mfp}} \); thus, the torques for \( d \leq l_{\text{mfp}} \) are not valid. As shown in Fig. 2 for \( d \geq 30 \) \( \text{nm} \), spin transfer torque is nearly independent of the thickness \( d \). On the other hand, for \( d \ll 30 \) \( \text{nm} \), the strength of the non-adiabatic torque increases as the thickness \( d \) decreases. For \( d \leq 10 \) \( \text{nm} \), \( \tau_x / \tau_y \approx 10^{-1} \), which is one order of magnitude larger than that estimated by Zhang and Li \( l \). Moreover, for \( d \leq 10 \) \( \text{nm} \), the spin transfer torque oscillates against the thickness \( d \) with the period of the oscillation given by \( 2\pi / k_t \).

Let us reveal the parameters which characterize the above behavior of the spin transfer torque. Assuming that \( k_t \approx \sqrt{3} / (4l_{\text{mfp}}) \ll k_i \approx \sqrt{3} \omega_J / v_F \) (Ref. [13]) and \( d \gg l_{\text{mfp}} \), we find that \( \tau_y \approx \pi \beta_\perp e \) and \( \tau_x \approx -\pi \beta_\perp e - \pi \beta_\perp e \) \( (edk_i) \), respectively. Thus, for \( d \gg l_{\text{mfp}} \), the adiabatic torque is nearly independent of the thickness \( d \). On the other hand, for \( d \ll 1 / (\zeta k_i) \approx l_{sf} / (2\sqrt{3}) \approx 40 \) \( \text{nm} \), where \( l_{sf} = v_F \tau_{sf} \) is the spin-flip length, the torque due to the spatial variation in the spin accumulation is dominant for the non-adiabatic torque. For a thin domain wall, the ratio of the adiabatic and non-adiabatic torque is characterized by \( v_F / (\sqrt{3} \omega_J d) \), which is the ratio of the precession frequency of the electrons’ spin around the localized spin angular momentum due to the sd exchange coupling and the angular velocity of the rotation of the exchange field in the domain wall. For \( d = 10 \) \( \text{nm} \), \( v_F / (\sqrt{3} \omega_J d) \approx 10^{-1} \). The oscillation period is given by

FIG. 2: (a) The strength of the adiabatic torque \( \tau_y \) renormalized by \( \pi \beta_\perp e \) against the thickness of the domain wall \( d \). (b) The strength of the non-adiabatic torque \( \tau_x \) renormalized by \( \pi \beta_\perp e \) against the thickness of the domain wall is shown. For \( d \leq l_{\text{mfp}} \), the diffusion approximation cannot be applied to the Boltzmann equation, and thus, torque below \( d \leq l_{\text{mfp}} \) denoted by the dotted line is not valid.
$2\pi/k_0 \approx 2\pi v_F/(\sqrt{3} \omega_f) \approx 2.5 \text{ nm}$. These estimations can be confirmed by the plots shown in Fig. 2.

When the precession frequency of the electrons’ spin around the exchange field, $\omega_f$, is comparable to the angular velocity of the rotation of the exchange field in space, $\pi v_F/d$, the direction of the electron’s spin cannot vary their direction adiabatically, and the non-adiabaticity, which is sometimes called the mistracking effect, plays an important role in the spin-dependent transport phenomena. For example, the terminal velocity of the domain wall motion is proportional to the ratio of the adiabatic and non-adiabatic torque [17]. The non-adiabaticity is characterized by a dimensionless parameter $\xi = \pi v_F/(2d\omega_f)$ [18]. As shown above, the ratio of the adiabatic and non-adiabatic torques for a thin domain wall, $v_F/(\sqrt{d}\omega_f d)$, is the first order of $\xi$, while the magnetoresistance due to the mistracking effect or spin accumulation is on the second order of $\xi$ [10, 14, 15]. For conventional ferromagnetic metals with $d \geq l_{\text{mfp}}$, $\xi$ is less than unity. Thus, the non-adiabaticity plays an important role in the dynamics of the localized magnetization compared to the magnetoresistance. It should be noted that for a thick domain wall the non-adiabatic torque is characterized by $\xi = 2/(\omega_f \tau_{sd})$, not $\xi$, as shown by Zhang and Li [7].

We compare our results with those of Vanhaeverbeke and Viret [13]. In Ref. [13], a time-dependent phenomenological Larmor equation for the magnetic moment in a moving frame is solved numerically, and showed that the non-adiabatic torque is shown to be one order of magnitude larger than that estimated by Zhang and Li [7] when the thickness of the domain wall is comparable to the Larmor precession length $\lambda_L = v_F/(2\pi \omega_f)$, which is on the order of a few nanometers. On the other hand, we consider the spin diffusion in the domain wall in a steady state by solving the Boltzmann equation in the rotated frame, and analytical expressions of the spin accumulation and spin transfer torque are obtained. We show that the strength of the non-adiabatic torque increases as the thickness of the domain wall decreases for $d \leq l_{\text{mfp}}/(2\sqrt{3})$. Note that the condition is determined by the spin-flip length $l_{\text{mfp}}$ instead of the Larmor precession length $\lambda_L$. We also find that the strength of the non-adiabatic torque is characterized by the first order of the non-adiabatic parameter $\xi \propto 1/d$. The non-adiabatic torque does not change its sign, as shown in Fig. 5 in Ref. [12].

Since the diffusion approximation is applied to the Boltzmann equation, the present theory is not applicable to the ballistic region $d \leq l_{\text{mfp}}$. The spin transfer torque in the ballistic region is obtained by Waintal and Viret [13]. One can easily confirm that our Eq. (7) reduces to Eq. (12) of Ref. [10] in the limit of $T, \tau_{sd} \rightarrow \infty$, where they assume that the local spin transfer torque is proportional to the spin accumulation. One might expect a simple connection formula between ballistic and diffusive spin transfer torque like Wexler’s formula for conductance [21]. However, this is beyond the scope of the present paper.

In conclusion, we studied spin transfer torque in a domain wall by solving the Boltzmann equation for spin accumulation, and found their analytical expressions. For a thin domain wall whose thickness is much thinner than the spin-flip length, the ratio of the magnitude of the adiabatic and non-adiabatic torque is about $10^{-1}$, which is one order of magnitude larger than that estimated in Ref. [7] and consistent with that in Ref. [13]. We also found that the strength of the non-adiabatic torque is inversely proportional to the thickness of the domain wall.

The authors would like to acknowledge the valuable discussions they had with P. M. Levy, J. Ieda, H. Sugishita, K. Matsushita, and N. Yokoshi. This work was supported by JSPS and NEDO.

[1] M. N. Baibich, J. M. Broto, A. Fert, F. N. V. Dau, and F. Petroff, Phys. Rev. Lett. 61, 2472 (1988).
[2] J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996).
[3] L. Berger, Phys. Rev. B 54, 9353 (1996).
[4] S. Maekawa, ed., Concepts in Spin Electronics (Oxford Science Publications, 2006).
[5] S. E. Barnes, J. Ieda, and S. Maekawa, Appl. Phys. Lett. 89, 125057 (2006).
[6] S. S. P. Parkin, M. Hayashi, and L. Thomas, Science 320, 190 (2008).
[7] S. Zhang and Z. Li, Phys. Rev. Lett. 93, 127204 (2004).
[8] E. Ehels, A. Radulescu, Y. Henry, L. Piraux, and K. Omadjele, Phys. Rev. Lett. 84, 983 (2000).
[9] H. F. Fuke, S. Hachimoto, M. Takagishi, H. Iwasaki, S. Kawasaki, K. Miyake, and M. Sahashi, IEEE. Trans. Mag. 43, 2848 (2007).
[10] P. M. Levy and Z. Zhang, Phys. Rev. Lett. 79, 5110 (1997).
[11] J. Sato, K. Matsushita, and H. Imamura, IEEE. Trans. Mag. 44, 2608 (2008).
[12] K. Matsushita, J. Sato, and H. Imamura, IEEE. Trans. Mag. 44, 2616 (2008).
[13] A. Vanhaeverbeke and M. Viret, Phys. Rev. B 75, 024411 (2007).
[14] E. Šimánek, Phys. Rev. B 63, 224412 (2001).
[15] E. Šimánek and A. Rebei, Phys. Rev. B 71, 172405 (2005).
[16] B. A. Gurney, V. S. Speriosu, J.-P. Nozieres, H. Lefakis, D. R. Wilhoit, and O. U. Need, Phys. Rev. Lett. 71, 4023 (1993).
[17] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett. 49, 990 (2005).
[18] C. H. Marrows, Adv. Phys. 54, 585 (2005).
[19] X. Waintal and M. Viret, Europhys. Lett. 65, 427 (2004).
[20] G. Wexler, Proc. Phys. Soc. 89, 927 (1966).