HYBRID MESONS FROM THE LATTICE

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I review lattice QCD results for hybrid mesons, including a discussion of their hadronic decays.

1. Introduction

Quantum Chromodynamics has emerged as the unique theory to describe hadronic physics. It is formulated in terms of gluonic and quark fields. The only free parameters are the scale of the coupling (usually called $\Lambda_{QCD}$) and the quark masses defined at some conventional energy scale.

Where large momentum transfers occur, the effective coupling becomes weak and a perturbative treatment is valid: in this domain the theory has been tested directly by experiment. However, because the effective coupling is weak for these processes that can be described by perturbation theory, they are necessarily not the dominant hadronic processes. A typical hadronic process will involve small momentum transfers and so has to be treated non-perturbatively.

In this non-perturbative régime, the description of hadrons is quite far removed from the description of the gluonic and quark fields in the QCD Lagrangian. Because only colour-singlet states survive, the hadrons are all composites of quarks and gluons. One example emphasises this: the nucleon has a mass which is very much greater than the sum of the quark masses of the three valence quarks comprising it. This extra mass comes from the gluonic interactions of QCD. Another way to view this is that the naïve quark model is a useful phenomenological tool but has constituent quarks with masses much greater than the QCD masses (ie masses as defined in the Lagrangian). It is important to understand why this is approximately what QCD requires and to find where QCD departs from the naïve quark
model.

One way to characterise the manner in which QCD goes beyond the naïve quark model is through the concept of exotic states. Here exotic is taken to mean ‘not included in the naïve quark model’. In order to discuss exotic states, we need to summarise what the naïve quark model contains. Basically the degrees of freedom are the valence quarks (i.e. quark-antiquark for a meson and 3 quarks for a baryon) with masses and interactions given by some effective interaction. The consequences of this are that only certain $J^{PC}$ values will exist and that the number of states with different quark flavours is specified. So, concentrating on mesons made of the three flavours of light quarks ($u, d, s$), one expects a nonet of mesons with the flavours ($\bar{u}d, \bar{d}u, \bar{u}u \pm \bar{d}d, \bar{s}s, \bar{u}s, \bar{d}s, \bar{s}u, \bar{s}d$). This is indeed what is found for vector mesons ($\rho, \omega, \phi, K^*$). It is also possible within the quark model for the flavour-singlet states ($\bar{u}u + \bar{d}d, \bar{s}s$) to mix, as found for the pseudoscalar mesons. What would be exotic is for a tenth state to exist. For mesons with orbital angular momentum $L$ between the quark and antiquark the allowed $J^{PC}$ values are shown below. Thus spin-parity combinations such as $0^{--}, 0^{+-}, 1^{+-}, 2^{+-}$ are termed spin-exotic since they cannot be made from a quark plus antiquark alone.

$$
\begin{array}{c|c|c|c|c}
 L & J^{PC} & J^{PC} & J^{PC} & J^{PC} \\
 0 & 0^{--} & 1^{--} & & \\
 1 & 1^{+-} & 0^{++} & 1^{++} & 2^{++} \\
 2 & 2^{+-} & 1^{--} & 2^{--} & 3^{--} \\
\end{array}
$$

It has been a considerable challenge to build a machinery that allows non-perturbative calculations in QCD with all systematic errors determined. The most controlled approach to non-perturbative QCD is via lattice techniques in which space-time is discretized and time is taken as Euclidean. The functional integral is then evaluated numerically using Monte Carlo techniques.

Lattice QCD needs as input the quark masses and an overall scale (conventionally given by $\Lambda_{QCD}$). Then any Green function can be evaluated by taking an average of suitable combinations of the lattice fields in the vacuum samples. This allows masses to be studied easily and matrix elements (particularly those of weak or electromagnetic currents) can be extracted straightforwardly. Unlike experiment, lattice QCD can vary the quark masses and can also explore different boundary conditions and
sources. This allows a wide range of studies which can be used to diagnose the health of phenomenological models as well as casting light on experimental data.

Here we will concentrate on lattice results for the spectrum of spin-exotic mesons: which are commonly called “hybrid mesons” since they must have additional degrees of freedom (such as gluonic) to achieve those spin-parity values.

One limitation of the lattice approach to QCD is in exploring hadronic decays because the lattice, using Euclidean time, has important contributions from low lying thresholds which can obstruct the study of decay widths. The finite spatial size of the lattice implies that two-body states are actually discrete. By measuring their energy very precisely as the spatial volume is varied, it is possible to extract the scattering phase shifts and hence decay properties. For on-shell transitions, it is possible to estimate hadronic transition strengths more directly and this has recently been checked for the case of $\rho$ meson decay to $\pi\pi$. This approach has been used to explore hybrid meson decay rates.

2. Hybrid Mesons

A hybrid meson is a meson in which the gluonic degrees of freedom are excited non-trivially. The most direct sign of this would be a spin-exotic meson, since that could not be created from a $q\bar{q}$ state with unexcited glue. A spin-exotic meson could, however, be a $q\bar{q}q\bar{q}$ or meson-meson state and that possibility will be discussed. I first discuss hybrid mesons with static heavy quarks where the description can be thought of as an excited colour string. The situation concerning light quark hybrid mesons is then summarised.

2.1. Heavy quark hybrid mesons

This topic has a long history: the first paper was published 20 years ago. The basic ideas have not changed, and I summarise them here.

Consider $Q\bar{Q}$ states with static quarks in which the gluonic contribution may be excited. We classify the gluonic fields according to the symmetries of the system. This discussion is very similar to the description of electron wave functions in diatomic molecules. The symmetries are (i) rotation around the separation axis $z$ with representations labelled by $J_z$ (ii) CP with representations labelled by $g(\pm)$ and $u(\mp)$ and (iii) $C\mathbb{R}$. Here $C$ interchanges $Q$ and $\bar{Q}$, $P$ is parity and $\mathbb{R}$ is a rotation of $180^\circ$ about the
Figure 1. The potential energy between static quarks at separation $R$ (in units of $r_0 \approx 0.5$ fm). The symmetric gluonic field configuration is shown by the lower points while the $\Pi_u$ excited gluonic configuration is shown above. The energy levels in these potentials for $b$ quarks are shown using the adiabatic approximation.

mid-point around the $y$ axis. The $\mathcal{CR}$ operation is only relevant to classify states with $J_z = 0$. The convention is to label states of $J_z = 0, 1, 2$ by $\Sigma, \Pi, \Delta$ respectively. The ground state ($\Sigma^+_0$) will have $J_z = 0$ and $CP = +$.

The exploration of the energy levels of other representations has a long history in lattice studies $^4,6$. The first excited state is found to be the $\Pi_u$. This can be visualised as the symmetry of a string bowed out in the $x$ direction minus the same deflection in the $-x$ direction (plus another component of the two-dimensional representation with the transverse direction $x$ replaced by $y$), corresponding to flux states from a lattice operator which is the difference of U-shaped paths from quark to antiquark of the form $\sqcup - \sqcap$.

The picture of the gluon flux between the static quarks suggests that the excited states of this string may approximate the excited potentials found...
from the lattice. In the simplest string theory, the first excited level has \( \Pi_u \) symmetry and is at energy \( \pi/R \) above the ground state. This is indeed approximately valid and a closer approximation is to use a relativistic version \( ^5 \) (namely \( E_m(R) = (\sigma^2R^2 + 2\pi\sigma(m - 1/12))^{1/2} \) for the \( m \)-th level), see also ref. \( ^7 \) for a recent comparison of this expression.

Recent lattice studies \( ^7 \) have used an asymmetric space/time spacing which enables excited states to be determined comprehensively. These results confirm the finding that the \( \Pi_u \) excitation is the lowest lying and hence of most relevance to spectroscopy.

From the potential corresponding to these excited gluonic states, one can determine the spectrum of hybrid quarkonia using the Schrödinger equation in the Born-Oppenheimer approximation. This approximation will be good if the heavy quarks move very little in the time it takes for the potential between them to become established. More quantitatively, we require that the potential energy of gluonic excitation is much larger than the typical energy of orbital or radial excitation. This is indeed the case \( ^4 \), especially for \( b \) quarks. Another nice feature of this approach is that the self energy of the static sources cancels in the energy difference between this hybrid state and the \( QQ \) states. Thus the lattice approach gives directly the excitation energy of each gluonic excitation.

The \( \Pi_u \) symmetry state corresponds to excitations of the gluonic field in quarkonium called magnetic (with \( L^{PC} = 1^{++} \)) and pseudo-electric (with \( 1^{--} \)) in contrast to the usual P-wave orbital excitation which has \( L^{PC} = 1^{--} \). Thus we expect different quantum number assignments from those of the gluonic ground state. Indeed combining with the heavy quark spins, we get a degenerate set of 8 states:

| \( L^{PC} \) | \( J^{PC} \) | \( J^{PC} \) | \( J^{PC} \) | \( J^{PC} \) |
|---|---|---|---|---|
| 1^{-+} | 1^{--} | 0^{--} | 1^{-+} | 2^{++} |
| 1^{--} | 1^{++} | 0^{++} | 1^{--} | 2^{--} |

Note that of these, \( J^{PC} = 1^{-+}, 0^{++} \) and \( 2^{++} \) are spin-exotic and hence will not mix with \( QQ \) states. They thus form a very attractive goal for experimental searches for hybrid mesons.

The eightfold degeneracy of the static approach will be broken by various corrections. As an example, one of the eight degenerate hybrid states is a pseudoscalar with the heavy quarks in a spin triplet. This has the same overall quantum numbers as the S-wave \( QQ \) state (\( \eta_b \)) which, however, has
the heavy quarks in a spin singlet. So any mixing between these states must be mediated by spin dependent interactions. These spin dependent interactions will be smaller for heavier quarks. It is of interest to establish the strength of these effects for b and c quarks. Another topic of interest is the splitting between the spin exotic hybrids which will come from the different energies of the magnetic and pseudo-electric gluonic excitations.

One way to go beyond the static approach is to use the NRQCD approximation which then enables the spin dependent effects to be explored. One study finds that the $L^P C = 1^+ -$ and $1^+ -$ excitations have no statistically significant splitting although the $1^+ -$ excitation does lie a little lighter. This would imply, after adding in heavy quark spin, that the $J^P C = 1^+ -$ hybrid was the lightest spin exotic. Also a relatively large spin splitting was found among the triplet states considering, however, only magnetic gluonic excitations. Another study explores the mixing of non spin-exotic hybrids with regular quarkonium states via a spin-flip interaction using lattice NRQCD.

Confirmation of the ordering of the spin exotic states also comes from lattice studies with propagating quarks which are able to measure masses for all 8 states. I discuss that evidence in more detail below.

Because of the similarity of the lightest hybrid wavefunction with that of the 2S state (which has $L = 1$), it is convenient to quote mass differences between these states. Within the quenched approximation, the lattice evidence for $b\bar{b}$ quarks points to a lightest hybrid spin exotic with $J^{PC} = 1^- +$ at an energy given by $(m_H - m_{2S}) r_0 = 1.8$ (static potential); 1.9 (static potential, NRQCD); 2.0 (NRQCD). These results can be summarised as $(m_H - m_{2S}) r_0 = 1.9 \pm 0.1$. Using the experimental mass of the $\Upsilon(2S)$, this implies that the lightest spin exotic hybrid is at $m_H = 10.73(7)$ GeV including a 10% scale error. Above this energy there will be many more hybrid states, many of which will be spin exotic.

The results from a study with $N_f = 2$ flavours of sea-quarks show very little change in the static potential as shown by SESAM and as illustrated in fig. 2 and also relatively little change in NRQCD determinations of mass ratios such as $(m_H - m_{2S})/(m_{1P} - m_{1S})$. Expressed in terms of $r_0$ (using $r_0 = 1.18/\sqrt{\sigma}$) this gives $(m_H - m_{2S}) r_0 = 2.4(2)$, however. This is significantly larger than the quenched result and, using the $1P - 1S$ mass difference to set the scale, yields a prediction for the lightest hybrid mass of 11.02(18) GeV.
2.2. Hybrid meson decays

Within this static quark framework, one can explore the decay mechanisms. One special feature is that the symmetries of the quark and colour fields about the static quarks must be preserved exactly in decay, hence the light quark-antiquark pair produced must respect these symmetries. This has the consequence that the decay from a $\Pi_u$ hybrid state to the open-$b$ mesons ($B\bar{B}$, $B^*\bar{B}$, $BB^*$, $B^*\bar{B}^*$) will be forbidden if the light quarks in the $B$ and $B^*$ mesons are in an S-wave relative to the heavy quark (since the final state will have the light quarks in either a triplet with the wrong $CP$ or a singlet with the wrong $J_z$ where $z$ is the interquark axis for the heavy quarks). The decay to $B^{**}$-mesons with light quarks in a P-wave is allowed by symmetry but not energetically.

![Figure 2. The potential energy $V(R)$ (in lattice units with $a=0.0972$ fm) versus quark separation $R$ in fm for 2 flavours of sea quark. The energies are given for the ground state and first excited gluonic state and for the two body state of ground state potential plus scalar meson ($f_0$) in a P-wave with the minimum non-zero momentum. The on-shell transition can be evaluated when $R \approx 0.2$ fm.](image)
In the heavy quark limit, the only allowed decays are when the hybrid state de-excites to a non-hybrid state with the emission of a light quark-antiquark pair. Since the $\Pi_u$ hybrid state has the heavy quark-antiquark in a triplet $P$-wave state, the resulting non-hybrid state must also be in a triplet $P$-wave since the heavy quarks do not change their state in the limit of very heavy quarks. Thus the decay for $b$ quarks will be to $\chi_b + M$ where $M$ is a light quark-antiquark meson in a flavour singlet. This proceeds by a disconnected light quark diagram and it would be expected that the scalar or pseudoscalar meson channels are the most important (ie they have the largest relative OZI-rule violating contributions). This transition can be estimated on a lattice when the initial and final energies are similar. This is the case for the $\Pi_u$ de-excitation to ground state gluonic field plus $f_0$ meson when the interquark separation is around 0.2 fm which allows a lattice evaluation of the hadronic transition strength - see fig. 2. Indeed the dominant mode (with a width of around 100 MeV) is found to be with $M$ as a scalar meson, namely $H \rightarrow \chi_b + f_0$, whereas when $M$ is an $\eta$ or $\eta'$ meson the transition strength is less that a few MeV. There will be modifications to this analysis coming from corrections to the heavy quark limit (of order $1/m_Q$ where $m_Q$ is the heavy quark mass) which might allow hybrid meson transitions to $B \bar{B}$, etc, but these have not been evaluated yet.

In this heavy quark (or static) limit, the spin-exotic and non spin-exotic hybrid mesons are degenerate. For the latter, however, the interpretation of any observed states is less clear cut, since they could be conventional quark antiquark states. Moreover, the non spin-exotic hybrid mesons can mix directly (ie without emission of any meson $M$) with conventional quark antiquark states once one takes into account corrections (of order $1/M_Q$) to the static approximation applicable for heavy quarks with physical masses.

It is encouraging that the decay width comes out as relatively small, so that the spin-exotic hybrid states should show up experimentally as sufficiently narrow resonances to be detectable. This decay analysis does not take into account heavy quark motion or spin-flip and these effects will be significantly more important for charm quarks than for $b$-quarks.

2.3. Light quark hybrid mesons

I now focus on lattice results for hybrid mesons made from light quarks using fully relativistic propagating quarks. There will be no mixing with $q\bar{q}$ mesons for spin-exotic hybrid mesons and these are of special interest. The first study of this area was by the UKQCD Collaboration who used
operators motivated by the heavy quark studies referred to above to study all 8 $J^{PC}$ values coming from $L^{PC} = 1^{+-}$ and $1^{-+}$ excitations. The resulting mass spectrum gives the $J^{PC} = 1^{-+}$ state as the lightest spin-exotic state. Taking account of the systematic scale errors in the lattice determination, a mass of 2000(200) MeV is quoted for this hybrid meson with $s\bar{s}$ light quarks. Although not directly measured, the corresponding light quark hybrid meson would be expected to be around 120 MeV lighter.

A second lattice group has also evaluated hybrid meson spectra with propagating quarks from quenched lattices. They obtain masses of the $1^{-+}$ state with statistical and various systematic errors of 1970(90)(300) MeV, 2170(80)(100)(100) MeV and 4390(80)(200) MeV for $n\bar{n}$, $s\bar{s}$ and $c\bar{c}$ quarks respectively. For the $0^{+-}$ spin-exotic state they have a noisier signal but evidence that it is heavier. They also explore mixing matrix elements between spin-exotic hybrid states and 4 quark operators.

The first analysis to determine the hybrid meson spectrum using full QCD used Wilson quarks. The sea quarks used had several different masses and an extrapolation was made to the limit of physical sea quark masses, yielding a mass of 1.9(2) GeV for the lightest spin-exotic hybrid meson, which again was found to be the $1^{-+}$. In principle this calculation should take account of sea quark effects such as the mixing between such a hybrid meson and $q\bar{q}qq\bar{q}$ states such as $\eta\pi$, although it is possible that the sea quark masses used are not light enough to explore these features.

A recent dynamical quark study from 2+1 flavours of improved staggered quarks has also produced results. They also compare their results with quenched calculations and find no significant difference, except that the ambiguity in fixing the lattice energy scale is better controlled in the dynamical simulation since different reference observables are closer to experiment. Their summary result for the $1^{-+}$ hybrid with strange quarks is 2100 ± 120 MeV, in agreement with earlier results. They note that the energies of two-meson states (such as $\pi+b_1$ or $K+K(1^+)$) with the hybrid meson quantum numbers are close to the energies they obtain. This suggests that these two-particle states, which are allowed to mix in a dynamical quark treatment, may be influencing the masses determined. A study of hybrid meson transitions to two particle states is needed to illuminate this area, using techniques such as those used for heavy quark hybrid decay and decays of light quark vector mesons 3.

The lattice calculations of the light hybrid spectrum are in good agreement with each other. They imply that the natural energy range for spin-exotic hybrid mesons is around 1.9 GeV. The $J^{PC} = 1^{-+}$
state is found to be lightest. It is not easy to reconcile these lattice results with experimental indications\textsuperscript{17} for resonances at 1.4 GeV and 1.6 GeV, especially the lower mass value. Mixing with $q\bar{q}q\bar{q}$ states such as $\eta\pi$ is not included for realistic quark masses in the lattice calculations. Such effects of pion loops (both real and virtual) have been estimated in chiral perturbation theory based models\textsuperscript{18} and they could potentially reconcile some of the discrepancy between lattice mass estimates (with light quarks which are too heavy) and those from experiment. This can be interpreted, dependent on one’s viewpoint, as either that the lattice calculations are incomplete or as an indication that the experimental states may have an important meson-meson component in them.

The light quark technique of using relativistic propagating quarks can also be extended to charm quarks, as was note above\textsuperscript{10}. Another group has explored the charm quark hybrid states also using a fully relativistic action, albeit with an anisotropic lattice formulation\textsuperscript{13}. Their quenched study is in agreement with the isotropic lattice result quoted above, finding a mass value of 4.428(41) GeV in the continuum limit for the $1^{-+}$ hybrid where the scale is set by the $1P_1−1S$ mass splitting (458.2 MeV experimentally) in charmonium. Their result is also consistent with that from NRQCD methods\textsuperscript{8} applied to this case. These results all have the usual caveat that in quenched evaluations the overall mass scale of the energy difference from the 1S state at 3.067 GeV is uncertain to 10\% or so (for example the \((2S−1S)/(1P_1−1S)\) is found to be 15\% higher than experiment) which is a major source of systematic error (approximately \(\pm 140\) MeV). They also produce estimates for other charmonium spin-exotic states: $0^{+-}$ at 4.70(17) GeV and $2^{+-}$ at 4.89(9) GeV. The $0^{--}$ state is not resolved.

Thus masses near 4.4GeV are found for the charmonium $1^{-+}$ state using relativistic quarks. The non-relativistic approach using NRQCD is expected to have big systematic errors for quarks as light as charm, but results\textsuperscript{8} do agree with this value. The heavy quark effective theory approach has a leading term which corresponds to a static heavy quark, resulting in an estimate\textsuperscript{6} of the spin-exotic charm state mass of 4.0 GeV. Here again the systematic error is potentially large for charm quarks.

3. Conclusions and Outlook

For hybrid mesons, there will be no mixing with $q\bar{q}$ for spin-exotic states and these are the most useful predictions. The $J^{PC}=1^{-+}$ state is expected in the range 10.7 to 11.0 GeV for $b$ quarks, 2.0(2) GeV for $s$ quarks and
1.9(2) GeV for $u$, $d$ quarks. Mixing of spin-exotic hybrids with $q\bar{q}q\bar{q}$ or equivalently with meson-meson is allowed and will modify the predictions from the quenched approximation. A first lattice study has been made of hybrid meson decays. For heavy quarks, the dominant mode is string de-excitation to $\chi + f_0$ where $f_0$ is a flavour singlet scalar meson (or possibly two pions in this state). The magnitude of the decay rate is found to be of order 100 MeV, so this decay mode should still leave a detectably narrow resonance to be observed. It will be possible to explore hadronic decays for light-quark hybrid mesons in the near future, and this may help to elucidate the nature of the experimental spin-exotic signals found at 1.4 and 1.6 GeV.

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