Particle production in \( pA \) collisions beyond leading order

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Abstract

We describe the origin of, and the solution to, the negativity problem which occurs in the perturbative calculation of the cross-section for single-inclusive particle production in proton-nucleus collisions at next-to-leading order.

Keywords: Proton-nucleus collisions, Color glass condensate, Higher-order calculations

1. Introduction

Particle production at forward rapidities and semi-hard transverse momenta in proton (or deuteron)–nucleus collisions at RHIC and the LHC is an important source of information about the small-\( x \) part of the nuclear wavefunction, where gluon occupation numbers are high and non-linear effects like gluon saturation and multiple scattering are expected to be important. On the theory side, the cross-section for single-inclusive particle production has been computed \cite{1} up to next-to-leading order (NLO) in the framework of the so-called ‘hybrid factorization’ \cite{2}, but the result is problematic: the cross-section suddenly turns negative when increasing the transverse momentum of the produced hadron, while still in the semi-hard regime \cite{3}. Various proposals to fix this difficulty, by modifying the scale for the rapidity subtraction, have only managed to push the problem to somewhat larger values of the transverse momentum (see \cite{4} for a recent discussion and more references). In a recent paper \cite{5}, we have argued that this negativity problem is an artifact of some of the approximations inherent in hybrid factorization, as needed to obtain a result which looks local in rapidity. On that occasion, we have also proposed a more general factorization scheme (within the CGC effective theory \cite{6}), which is non-local in rapidity but yields a manifestly positive cross-section to NLO accuracy.

2. Leading order formalism

To leading order (LO) in the CGC effective theory, quark production at forward rapidities in \( pA \) collisions proceeds as follows: a quark from the wavefunction of the incoming proton, which is initially collinear with
the proton and carries a relatively large longitudinal fraction \( x_p \), scatters off the dense gluon distribution in the nuclear target and thus acquires a transverse momentum \( k \). The quark distribution is computed as

\[
\frac{dN^{pA\to qX}}{d^2k\, d\eta} \bigg|_{\text{LO}} = \frac{1}{(2\pi)^2} x_p q(x_p) S(k, X_y), \quad S(k, X_y) = \int d^2r \, e^{-ik\cdot r} S(r, X_y),
\]

where \( \eta \) is the rapidity of the produced quark in the center-of-mass frame and \( X_y \) is longitudinal momentum fraction carried by the gluons from the target that are involved in the collision. Energy-momentum conservation implies \( x_p = (k_x/\sqrt{s}) e^{\eta} \) and \( X_y = (k_z/\sqrt{s}) e^{-\eta} \). The forward kinematics corresponds to the situation where \( \eta \) is positive and large, which implies \( X_y \ll x_p < 1 \). Thus, forward particle production explores the small-\( X_y \) part of the nuclear wavefunction, as anticipated. Furthermore, \( x_p q(x_p) \) is the quark distribution of the proton and \( S(r, X_y) \) is the elastic \( S \)-matrix for the scattering between a ‘color dipole’ (a quark-antiquark pair in a color-singlet state) and the nucleus. Its Fourier transform \( S(k, X_y) \) plays the role of an unintegrated gluon distribution in the nuclear target, as probed by particle production in dilute-dense collisions.

To the same accuracy, the dipole \( S \)-matrix is obtained by solving the LO version of the BK equation

\[ \tag{1} \]

which resums an arbitrary number of soft gluon emissions in the scattering between the dipole and the nuclear target, in the eikonal approximation. For what follows, it is convenient to chose a Lorentz frame in which the ‘primary’ gluon (the one which is closest in rapidity to the dipole) is emitted by the dipole, whereas all the other, even softer, gluons belong to the nuclear wavefunction. Then, the LO dipole \( S \)-matrix \( S(r, X_y) \equiv S(x, y; X_y) \) (with \( r = x - y \)) admits the following integral representation (see also Fig. 1)

\[ \tag{2} \]

where \( \hat{a} = \alpha S \), \( S \) is the quark distribution of the nuclear target and \( X_y \) is the corresponding initial condition (say, as given by the McLerran-Venugopalan model), \( x \ll 1 \) is the fraction of the dipole longitudinal momentum taken by the primary gluon, and \( X(x) \equiv X_y / x \) is the longitudinal momentum fraction of the gluons from the target which scatter off the projectile made with the dipole and the primary gluon.

### 3. Beyond leading order

The quark multiplicity in Eq. 1 receives two types of next-to-leading order (NLO) corrections (see Figs. 2 and 3): those related to the dipole evolution — i.e. corrections of \( O(\alpha_s) \) to the kernel of the BK equation — and those related to the impact factor — corrections of \( O(\alpha_s) \) which arise when the emission of the primary gluon is computed “with exact kinematics”, i.e. beyond the eikonal approximation.

The NLO corrections to the BK kernel are generated by the ensemble of the genuine one-loop corrections to the emission of a soft (\( x \ll 1 \)) primary gluon. For instance, Fig. 2 illustrates the effect of a gluon loop where the secondary gluons are both soft, but their longitudinal momentum fractions are comparable to each other: \( x_1 \sim x_2 \ll 1 \). (The situation where \( x_2 \ll x \approx x_1 \) contributes to the second step of the LO evolution and must be properly subtracted when computed the NLO correction to the evolution kernel.) Accordingly,
the 3-gluon vertices visible in Fig. 2 must be computed with exact kinematics, and similarly for the other one-loop diagrams. The ensemble of such corrections has been computed in [7]. The strict NLO version of the BK equation turns out to be unstable [8], due to the presence of large NLO corrections enhanced by collinear logarithms. All-order resummations which overcome this problem have been devised in [9, 10, 11].

In what follows we shall focus on the NLO correction to the impact factor, which as we shall see is responsible for the negativity problem mentioned in the introduction. To isolate this correction, one must compute the primary emission with the exact kinematics (as valid for any \( x \leq 1 \)) and subtract away the respective contribution of an eikonal emission (strictly correct for \( x \ll 1 \) alone), that was already included in the LO BK evolution. This is illustrated in Fig. 3. To simplify the discussion, we shall keep the dipole evolution at LO (possibly amended by running coupling corrections; see below). Also, we shall not present the NLO corrections in detail (these can be found in the literature; see e.g. [5, 12]), rather we shall use symbolic notations, which are both compact and suggestive.

To start with, we shall rewrite the LO BK equation (2) as [we recall that \( r = x - y \) and \( X(x) = X_g/x \)]

\[
S(r, X_g) = S_0(r) + \bar{\alpha}_s \int_{X_g/X_0}^1 \frac{dx}{x} K(r; 0) S(r, X(x)),
\]

(3)

where the kernel \( K(r; 0) \) succinctly denotes the convolution in transverse space and the terms quadratic in \( S \) are kept implicit. Clearly, a similar representation can be written in momentum space, with \( K \rightarrow K(k; 0) \). In these symbolic notations, the LO cross-section (1) reads

\[
\frac{dN_{pA \rightarrow qX}}{d^2 k \, d\eta} \bigg|_{LO} = S_0(k) = S_0(k) + \bar{\alpha}_s \int_{X_g/X_0}^1 \frac{dx}{x} \mathcal{K}(k; 0) S(k, X(x)),
\]

(4)

where we also have omitted the quark distribution, to simplify notations. Notice that the last expression in (4) is the integral representation for \( S(k, X_g) \) obtained by taking a Fourier transform in (3).

This last expression can be generalized to include the NLO correction to the impact factor [5]:

\[
\frac{dN_{pA \rightarrow qX}}{d^2 k \, d\eta} \bigg|_{NLO} = S_0(k) + \bar{\alpha}_s \int_{X_g/X_0}^1 \frac{dx}{x} \mathcal{K}(k; x) S(k, X(x)),
\]

(5)

where \( \mathcal{K}(k; x) \) is the kernel describing a ‘non-eikonal’ emission, that is, the emission of a gluon with a generic value of \( x \), as computed without any kinematical approximation. [As the notation suggests, the ‘eikonal’ kernel \( \mathcal{K}(k; 0) \) which enters the LO BK evolution is obtained by letting \( x \to 0 \) inside the more general kernel \( \mathcal{K}(k; x) \).] Eq. (5) can be viewed as the sum of Figs. 1 and 3; the contribution of the eikonal
primary emission cancels out in this sum, so we are left with 2 diagrams (the bare dipole and a dipole dressed by a non-eikonal gluon emission) corresponding to the 2 terms visible in the r.h.s. of Eq. (5).

On physical grounds, it is a priori clear that the r.h.s. of Eq. (5) must be positive-definite: the emission of the primary gluon can only increase the unintegrated gluon distribution of the target. This is confirmed by the numerical evaluation of Eq. (5) in Ref. [12], which yields the “unsubtracted” curve in the left panel of Fig. 4. However, Eq. (5) is not exactly the same as the hybrid factorization in Ref. [1], for which the negativity problem occurs. To obtain the latter, one should first separate the LO contribution to the cross-section from the NLO correction to the impact factor. This can be done by combining Eqs. (4) and (5) to deduce

\[
\frac{dN^{pA-\eta X}}{d^2 k \, d\eta} \bigg|_{\text{LO}} = S(k, X_\eta) + \tilde{\alpha}_s \int_{X_\eta/X_0}^1 \frac{dx}{x} \left[ K(k; x) - K(k; 0) \right] S(k, X(x)) .
\]

Since obtained via exact manipulations, the result in Eq. (6) is identical to that in Eq. (5) — in particular, it is still positive. This is confirmed by the numerical results in [12] (see the “subtracted” curve in the left panel of Fig. 4), which also show that the second term in Eq. (6) — the NLO correction to the impact factor — is by itself negative, but smaller than the LO contribution. The negativity problem only occurs after additional approximations, which are specific to hybrid factorization and aim at obtaining a result which looks local in rapidity (here, in X): (i) The integral over x in Eq. (6) is controlled by x \sim 1, hence to the NLO accuracy of interest, one can replace X(x) \rightarrow X(1) = X_g inside the S-matrix. (ii) After the previous step, one can extend the integral over x down to x = 0. We are thus led to the NLO factorization proposed in [1]:

\[
\frac{dN^{pA-\eta X}}{d^2 k \, d\eta} \bigg|_{\text{CXY}} = S(k, X_g) + \tilde{\alpha}_s \int_0^1 \frac{dx}{x} \left[ K(k; x) - K(k; 0) \right] S(k, X(x)) .
\]

However, this result suddenly turns out negative when increasing k_\perp, as visible too in the left panel of Fig. 4. As discussed in [5], this is an artifact of the additional approximations (i), (ii), which artificially enhance the negative contribution of the NLO impact factor [e.g., for any x \leq 1, one has S(k, X_g) \geq S(k, X(x))].

The LO approximation to the dipole evolution, cf. Eq. (2), is very bad in practice. A considerably better approximation, which allows for realistic applications to phenomenology, is obtained by including the QCD running-coupling (RC) corrections. The respective generalization of the BK equation (2) is rather straightforward: it suffices to replace there \tilde{\alpha}_s \rightarrow \tilde{\alpha}_s(\alpha_s) \equiv \min \{ |x-y|, |x-z|, |y-z| \} and \tilde{\alpha}_s(\alpha_s) is the one-loop RC evaluated at \alpha_s^2 = 4/\pi^2. But the corresponding generalization of the cross-section for particle production is complicated by the potential mismatch between the treatment of RC corrections in transverse coordinate space, and in momentum space, respectively [5].

Specifically, the NLO cross-section in (5) is written in momentum space, hence the coupling \alpha_s, which appears there is naturally interpreted as \tilde{\alpha}_s(\alpha_s^2) [5]. It may be tempting to perform a similar substitution in the
‘subtracted’ version, Eq. (6), but this is not fully right: in the presence of RC corrections, the second equality in Eq. (4) is only approximately satisfied⁴ hence there is some mismatch between the LO quantities which are ‘added’ and ‘subtracted’ when going from Eq. (5) to Eq. (6). Since these quantities are individually large, such a mismatch may easily lead to unphysical results. And indeed, whereas Eq. (5) with \( \bar{\alpha}_s \rightarrow \bar{\alpha}_s(r_{\text{min}}) \) is still guaranteed to be positive-definite, this is not the case anymore for Eq. (6) [with \( \bar{\alpha}_s \rightarrow \bar{\alpha}_s(k_2^2) \)], which turns negative at sufficiently large \( k_2 \), as illustrated in the right panel of Fig. 4 (taken again from Ref. [12]).

To summarize, our result for the quark multiplicity, as presented (in symbolic notations) in Eq. (5), is correct to NLO accuracy and positive-definite. As such it is suitable for applications to the phenomenology.

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⁴The momentum-space function \( S(k, X_g) \) is obtained by first solving the BK equation (2) with \( \bar{\alpha}_s \rightarrow \bar{\alpha}_s(r_{\text{min}}) \) and then performing a Fourier transform; but the effect of the latter is not faithfully reproduced by replacing \( \bar{\alpha}_s \rightarrow \bar{\alpha}_s(k_2^2) \) in the r.h.s. of Eq. (3).