Q-ball collisions in the MSSM: gravity-mediated supersymmetry breaking

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Abstract

Collisions of non-topological solitons, Q-balls, are studied in a typical potential in the Minimal Supersymmetric Standard Model where supersymmetry has been broken by a gravitationally coupled hidden sector. Q-ball collisions are studied numerically on a two dimensional lattice for a range of Q-ball charges. Total cross-sections, as well as cross-sections for fusion and charge-exchange are calculated. The average percentage increase in charge carried by the largest Q-ball after a collision is found to be weakly dependent on the initial charge.

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1 Introduction

Stable non-topological solitons [1], Q-balls [2], can be present in several field theory models. In particular the supersymmetric extensions of the Standard Model may contain them. A Q-ball is a coherent state of a complex scalar field that carries a conserved charge, typically a $U(1)$-charge. In the sector of fixed charge a Q-ball is a ground state so that the conservation of charge assures that the Q-ball is stable. In the Minimal Supersymmetric Standard Model (MSSM) Q-balls carrying lepton or baryon number are present due to the existence of flat directions in the scalar sector of the theory [3, 4].

The cosmological significance of Q-balls can present itself in many forms. Stable (or long living) Q-balls are natural candidates for dark matter [5] and the decay of Q-balls can explain the baryon to dark matter ratio of the universe [6]. Q-balls can also protect the baryon asymmetry from sphalerons at the electroweak phase transition and decaying Q-balls may be responsible for the baryon asymmetry of the universe [4]. Furthermore, Q-balls can play an important role in considering the stability of neutron stars [7].

The mechanism by which supersymmetry (SUSY) is broken in the theory is significant for the charges and stability of Q-balls in the theory. If SUSY is broken by a gauge-mediated mechanism, the baryon number carrying B-balls can have very large charges due to to the flatness of the potential. Assuming that the charge is large enough, they can then be stable against decay into nucleons [3]. If, however, SUSY is broken by a gravitationally coupled hidden sector, the potential is not completely flat but Q-balls may still exist due to radiative corrections [4, 6]. In this case the Q-balls can decay (evaporate [8, 9]) into baryons or supersymmetric particles.

The formation of Q-balls from an Affleck-Dine (AD) condensate in the early universe has been studied recently with numerical simulations [10, 11]. In these simulations both the gauge- and gravity-mediated SUSY breaking scenarios have been considered. In both cases it was found that Q-balls do form from the AD condensate. In the gravity-mediated case it was especially noted that the formed Q-balls have non-zero velocities and can hence collide with each other [11]. Q-ball collisions were also simulated on a one dimensional lattice and it was found that Q-balls typically merge, exchange charge or pass through each other [11]. Since the charge can change due to collisions, they may play an important role in the determination of the Q-ball charge distribution after their formation. On the other hand the charge distribution is important in evaluating the significance of Q-balls for the evolution of the universe. It is hence worthwhile to study Q-ball collisions in more detail. Q-ball collisions have also been studied previously in various potentials in [12]-[16] but not to our knowledge in either the gauge- or gravity-mediated scenarios.
In this paper we have studied numerically collisions of Q-balls in the gravity-mediated scenario on a two dimensional lattice. The gauge mediated scenario will be analyzed in a forthcoming paper.

2 Q-ball solutions

Consider a field theory with a U(1) symmetric scalar potential, \( U(\phi) \), with a global minimum at \( \phi = 0 \). The complex scalar field \( \phi \) carries a unit quantum number with respect to the \( U(1) \)-symmetry. The charge and energy of a given field configuration \( \phi \) are given by

\[
Q = \frac{1}{i} \int (\phi^* \partial_t \phi - \phi \partial_t \phi^*) d^Dx
\]

and

\[
E = \int \left[ |\dot{\phi}|^2 + |\nabla \phi|^2 + U(\phi^* \phi) \right] d^Dx.
\]

The single Q-ball solution corresponds to the minimum energy configuration at a fixed charge. If it is energetically more favourable to store charge in a Q-ball compared to free particles, the Q-ball will be stable against radiative decays into \( \phi \)-scalars. Hence, for a stable Q-ball, condition

\[
E < mQ,
\]

where \( m \) is the mass of the \( \phi \)-scalar, must hold.

Minimizing the energy is straightforward and is easily done by using Lagrange multipliers. The Q-ball solution can be shown to be of the form

\[
\phi(x, t) = e^{i\omega t} \phi(r),
\]

where \( \phi(x) \) is now time independent and real, \( \omega \) is the Q-ball frequency, \( |\omega| \in [0, m] \) and \( \phi \) is spherically symmetric. The charge of a spherically symmetric Q-ball in \( D \)-dimensions reads

\[
Q = 2\omega \int \phi(r)^2 d^Dr.
\]

The equation of motion at a fixed \( \omega \) is

\[
\frac{d^2 \phi}{dr^2} + \frac{D - 1}{r} \frac{d\phi}{dr} = \phi \left( \frac{\partial U(\phi^2)}{\partial \phi^2} \right) - \omega^2 \phi.
\]

To obtain the Q-ball profiles we must solve (6) with boundary conditions \( \phi'(0) = 0, \ \phi(\infty) = 0 \).

In the present paper we consider a potential of the form

\[
U(\phi) = m^2 \phi^2(1 - K \log(\frac{\phi^2}{M^2})) + \lambda \phi^{10},
\]
where the parameter values are chosen to be $m = 100$ GeV, $K = 0.1$ and $\lambda = M_{Pl}^6$, where $M_{Pl}$ is the reduced Planck mass. This choice of potential corresponds to a D-flat direction in the full scalar potential in the MSSM where supersymmetry has been broken by a gravitationally coupled hidden sector \cite{4}. The large mass scale $M$ is chosen such that the minimum is degenerate

$$M = \left(\frac{1}{4} K m^2 \lambda^{-1} \exp(-1 - 4 \frac{K}{\lambda})\right)^{\frac{1}{2}} \sim 10^{11} \text{ GeV}. \quad (8)$$

We have calculated the charge and energy of Q-balls for different values of $\omega$. Energy vs. charge curves are shown in Figure \ref{fig:Q-ball_energy_charge}(a). The axis scales are chosen differently for two and three dimensions; for two dimensions, $Q_0 = 8000(M/\text{GeV})^2$, $E_0 = 8 \times 10^5 M^2 \text{ GeV}^{-1}$ and for three dimensions, $Q_0 = 600(M/\text{GeV})^2$, $E_0 = 6 \times 10^4 M^2 \text{ GeV}^{-1}$. The dashed line is the stability line, $E = mQ$, that indicates that the Q-balls considered here are stable with respect to scalar decays. From the figure it can be seen that the energy vs. charge curves are of a similar shape in two and three dimensions. Q-ball profiles are plotted in Figure \ref{fig:Q-ball_profiles}(b) for different values of $\omega$ in two and three dimensions. The profiles appear very similar in these two cases. These figures suggest that the collision processes calculated in two spatial dimensions are likely to be similar to collisions in three spatial dimensions. Here it is worth noting that, as from the equation of motion \cite{3} can be seen, a non-zero dissipation term is present for dimensions larger than one. This suggests that collisions in one dimension may differ quite significantly from collision processes in higher dimensions. This also seemed to be the case in the one dimensional simulations we have done.
3 Collisions

We have studied collisions of Q-balls with equal charges in the potential (7) [17]. The studied values of $\omega$ were $\omega/m = 0.50, 0.60, 0.75, 0.90, 0.99$. These values correspond to charges $1450, 645, 110, 12.6, 2.24$ in units of $(M/\text{GeV})^2$ in the case with two spatial dimensions so that in terms of $\phi$-scalars the considered range of charges is $\sim 10^{24} - 10^{26}$. For the same range of $\omega$, the charges in the three dimensional space are in the range $\sim 10^{23} - 10^{26}$.

The relative phase of the Q-balls is also accounted for. By the relative phase, we mean the difference in individual phases at the point where the distance between Q-balls is at a minimum assuming there is no interaction between them. Position is defined as the point of the maximum value of the amplitude of $\phi$. The relative phase is allowed to have values in the range $0 \leq \Delta \phi \leq 2\pi$. The impact parameter is also varied to study the cross-sections. The Q-balls studied here are of the thick-wall type so that there is no natural definition of the Q-ball size. Therefore we have defined the size of a Q-ball by a Gaussian fit; we fit a Gaussian $\phi = Ae^{-Br^2}$ to the profiles and define the radius of the ball as $R = B^{-\frac{1}{2}}$. The cross-sections quoted here are three dimensional cross-sections with the interaction radius taken from the two-dimensional simulations.

Collisions were simulated on a 2+1 -dimensional lattice. The lattice size typically used was $\sim 200^2$ with continuous boundary conditions. A 9-point Laplacian operator and a step size of $5 \times 10^{-3}$ was used in all calculations. Collisions were studied for different initial velocities of Q-balls, $v = 10^{-3}$ and $v = 10^{-2}$.

3.1 Numerical Results

Collisions can roughly be divided into three types; fusion, charge exchange and elastic scattering. Fusion is defined as a process where most of the initial charge is in a single Q-ball after the collision and the rest of the charge is lost either as radiation or as small Q-balls. By charge exchange we mean a process where Q-balls exchange some of their charge while the total amount of charge carried by the two balls is essentially conserved. An elastic scattering is defined to be a process where less than 1% of the total charge is exchanged. After the collision the ratio of the charge in the largest Q-ball to the total initial charge as a function of the relative phase has been plotted in Figures 2 and 3. From the figures the two different types of processes can be distinguished. In a fusion process typically $\sim 10 - 20\%$ of the initial charge is lost as radiation and small Q-balls and the rest of the charge is in a single Q-ball. On the other hand, if charge is exchanged the larger Q-ball carries usually less than 70% of the total charge. As from Figs. 2 and 3 can be seen, fusion occurs generally only when the relative phase is small and is more likely to occur with smaller $\omega$. The amount
of exchanged charge decreases substantially with increasing relative phase. Increased velocity does not seem to have a large effect for the range of $\omega$ where fusion occurs. However, more charge is exchanged between balls of equal size when the initial velocity is larger. The relative changes in size are weakly dependent on $\omega$ (in both cases the standard deviation is less than 1%) and hence on the size of the Q-balls. Averaging over the relative phase (assuming a random distribution for the $\Delta\phi$'s) and $\omega$:s, the relative change in the size of a Q-ball is 10% for $v = 10^{-3}$ and 14% for $v = 10^{-2}$.

We are now ready to calculate the fusion cross-section, $\sigma_F$, and the cross-section for the charge exchange, $\sigma_Q$, as a function of $\omega$. These are plotted in Figure 4 with the geometrical cross-section, $\sigma_G$. Clearly the fusion cross-section is strongly dependent on $\omega$; larger Q-balls fuse more easily than smaller ones. This effect is clearly not explained by the different geometrical sizes of the Q-balls as can be noted from the geometrical cross-section. The fusion cross-section decreases with increasing $\omega$ because the Q-balls with higher $\omega$ have more regions with differing relative phases. The field dynamics cannot then even out the relative phase differences quickly enough to keep the colliding balls together. From the simulations it can be seen that balls with larger $\omega$ are less likely to fuse than balls with the same phase difference but smaller $\omega$. The

Figure 2: The fraction of initial charge in the largest Q-ball after a collision for different values of $\Delta\phi$, $v = 10^{-3}$.
Figure 3: The fraction of initial charge in the largest Q-ball after a collision for different values of $\Delta \phi$, $v = 10^{-2}$.

The effect of increasing the initial velocity on the cross-sections can also be noted from Fig. 4. The fusion cross-section is slightly decreased as velocity increases while the charge exchange cross-section increases. The increase in $\sigma_Q$ is due to the fact that now the Q-balls have more kinetic energy to overcome the repulsion resulting from the relative phase difference.

The total cross-section including all the cases i.e. when the balls fuse, exchange charge or scatter elastically, is also dependent on $\omega$, but only weakly. Averaging over the $\omega$:s, $\sigma_{\text{tot}} = 0.27 \pm 0.01$ GeV$^{-2}$ ($v = 10^{-3}$) and $\sigma_{\text{tot}} = 0.19 \pm 0.01$ GeV$^{-2}$ ($v = 10^{-2}$).

We have also studied Q-ball collisions with larger initial velocities for a more limited set of parameter values. When the initial velocity is increased to $v = 10^{-1}$, the fusion cross-section is reduced significantly from its value when $v = 10^{-3}$. Furthermore, at such high velocities we also see processes where the Q-balls pass through each other essentially without exchanging any charge. This is a similar process that was reported to occur in one dimension in [11] and which we have also observed in our one dimensional simulations.

Charge exchange also affects the final velocities of the Q-balls. As charge is ex-
changed the speed of the Q-balls typically increase. The final velocity of the smaller ball can be quite large; we have often noted final velocities ten times larger than the initial velocity.

4 Conclusions

In this paper we have studied Q-ball collisions in the MSSM with supersymmetry broken by a gravitational hidden sector. For the studied range of charges the total cross-section was found to be approximately constant. The cross-section for fusion, $\sigma_F$, appeared to be smaller than the geometrical cross-section, $\sigma_G$, whereas the cross-section for charge exchange, $\sigma_Q$, was larger than $\sigma_G$. In a collision it is hence more probable that a charge exchanging process occurs rather than a fusion process. This probability increases with increasing $\omega$ (or with decreasing charge). Averaging over the fusion and charge exchanging processes the average charge increase of the largest Q-ball emerging from a collision was found to be approximately constant. For the considered range of charges and velocities it was $\sim 10\%$ ($v = 10^{-3}$) and $\sim 14\%$ ($v = 10^{-2}$).

In a cosmological context, Q-ball collisions may have a significant effect on the
charge distribution of Q-balls. Clearly for collisions to be important the number density of Q-balls must be high enough and the balls must have large enough velocities for the rate of interaction to be significant. In the early universe this obviously means that the interaction rate must be larger than the Hubble rate. If collisions typically do occur the resulting charge distribution can then be altered by the fusion and charge exchange processes. Based on the results presented in this paper, the relative phase, size and the initial velocity of the balls then play important roles in studying the evolution of the Q-ball charge distribution.

If the balls that are formed from the AD condensate are in the same phase, fusion processes will dominate and the average size of a Q-ball grows substantially in a collision. Since most of the charge is left in the remaining ball and the rest is in the form of several small, quickly evaporating Q-balls and radiation, the number density of Q-balls reduces rapidly. Collisions can therefore freeze the Q-ball distribution quickly in the early phases of the universe. If, on the other hand, the phases are randomly distributed the probability for fusion is greatly reduced and the distribution will not change as significantly as in the previous case. Collision processes can then also continue for a longer period of time.

The typical size of Q-balls is obviously an important factor. The total scattering cross-section depends quite weakly on the size of the Q-balls but the fusion and charge exchange cross-sections do have a strong $\omega$-dependence.

The initial velocity of the Q-balls is also significant in the evolution of the Q-ball distribution. A larger velocity means that the interaction rate is increased but on the other hand if the initial velocity is too large the cross-sections decrease due to a decreased interaction time. Collisions can also significantly change the velocities of the Q-balls so that an initially uniform velocity distribution can be spread out by the collision processes.

The effect of collisions can be important in deciding the exact role and significance of Q-balls in the evolution of the universe. In determining their importance on cosmology, more information is needed about the Q-ball distribution after their formation and also about the effects of collisions on the initial distribution. To quantify the effects of the different collision processes described in this paper, a more detailed analysis is needed which gives motivation for future work.

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References

[1] T. Lee and Y. Pang, *Phys. Rev.* **221** (1992) 251.

[2] S. Coleman, *Nucl. Phys.* **B262** (1985) 263.

[3] A. Kusenko, *Phys. Lett.* **B405** (1997) 108.

[4] K. Enqvist and J. McDonald, *Phys. Lett.* **B425** (1998) 309.

[5] A. Kusenko and M. Shaposhnikov, *Phys. Lett.* **B418** (1998) 46.

[6] K. Enqvist and J. McDonald, *Nucl. Phys.* **B538** (1999) 321.

[7] A. Kusenko *et al.*, *Phys. Lett.* **B423** (1998) 104.

[8] A. Cohen *et al.*, *Nucl. Phys.* **B272** (1986) 301.

[9] T. Multamäki and I. Vilja, [hep-ph/9908446](http://arxiv.org/abs/hep-ph/9908446), to appear in Nucl. Phys. B.

[10] S. Kasuya and M. Kawasaki, *Phys. Rev. D* **61** (2000) 041301.

[11] S. Kasuya and M. Kawasaki, [hep-ph/0002284](http://arxiv.org/abs/hep-ph/0002284).

[12] M. Axenides *et al.*, *Phys. Rev. D* **61** (2000) 085006.

[13] T. I. Belova and A. E. Kudryavtsev, Zh. Eksp. Teor. Fiz. **95** 1989 13.

[14] J. K. Drohm *et al.*, *Phys. Lett.* **B101** (1981) 204.

[15] V. G. Makhankov, G. Kummer and A. B. Shvachka, *Phys. Lett.* **A70** (1979) 171.

[16] R. A. Battye and P. M. Sutcliffe, [hep-th/0003252](http://arxiv.org/abs/hep-th/0003252).

[17] Images of Q-ball collisions are available at [http://www.utu.fi/~tuomul/qballs/](http://www.utu.fi/~tuomul/qballs/)