Long-Horizon Multi-Robot Rearrangement Planning for Construction Assembly
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Abstract—Robotic construction assembly planning aims to find feasible assembly sequences as well as the corresponding robot-paths and can be seen as a special case of task and motion planning (TAMP). As construction assembly can well be parallelized, it is desirable to plan for multiple robots acting concurrently. Solving TAMP instances with many robots and over a long time-horizon is challenging due to coordination constraints, and the difficulty of choosing the right task assignment. We present a planning system which enables parallelization of complex task and motion planning problems by iteratively solving smaller subproblems. Combining optimization methods to jointly solve for manipulation constraints with a sampling-based bi-directional space-time path planner enables us to plan cooperative multi-robot manipulation with unknown arrival-times. Thus, our solver allows for completing subproblems and tasks with differing timescales and synchronizes them effectively. We demonstrate the approach on multiple construction case-studies to show the robustness over long planning horizons and scalability to many objects and agents. Finally, we also demonstrate the execution of the computed plans on two robot arms to showcase the feasibility in the real world.

Index Terms—Manipulation planning, multi-robot systems, robotics and automation in construction, task planning.

I. INTRODUCTION

As robots become ubiquitous in manufacturing and production processes, more robust algorithms to coordinate their work are needed. When multiple robots are employed to achieve a desired goal, two main problems have to be solved: 1) assigning tasks to individual robots, and 2) coordinating movements of robots to allow effective execution of those tasks. Combined task and motion planning (TAMP) approaches provide a suitable framework to jointly solve such problems. Scaling these methods to robotic teams consisting of multiple agents and to long-horizon problems remains a major challenge.

We focus on multi-robot planning problems in the context of building construction (Fig. 1): as one of the largest industries worldwide, building construction can benefit from autonomous robots and planning processes [1], and the increased efficiency they bring. While robotic construction processes have gained more use in this industry [2], [3], particularly in off- and on-site prefabrication, an integrated autonomous decision-making and robot motion planning approach is missing.

Previous work on autonomous assembly planning showed promising results in problems, such as furniture assembly [4], [5], [6] or building construction [7], [8], [9]. Although TAMP formulations are theoretically suitable for such problems, existing approaches do not scale well with an increasing number of robots [7], [8] and/or are only demonstrated on problems spanning short time-horizons [10], [11].

(Construction) Assembly planning can be thought of as rearrangement planning [12], [13] with additional difficulties: 1) The ordering of objects is crucial to finish a task, since the feasibility to place a part is highly dependent on previously placed objects [7], [14] (e.g., for stability-reasons, or object availability).
2) Objects and object orderings might impose constraints on the robots (e.g., if an object cannot be transported by a single robot or temporary support of the intermediate structure is necessary). These dependencies are amplified when using multiple robots to parallelize the process, due to the nonsequential assembly process.

This article extends prior work [8], [15] in robotic assembly planning to provide effective coordination of potentially heterogeneous robotic teams in long-horizon settings. We frame the problem as a TAMP problem and use the framework of logic-geometric programming (LGP) [10] to formalize it.

Our algorithm decomposes the overall planning problem by sequentially considering smaller, limited horizon problems with only a subset of robots to enable scalability. These subproblems aim to solve the task sequencing and motion planning while keeping previously planned robot trajectories fixed. A heuristic prioritizes the order in which subproblems are solved. The path planning for one subproblem needs to account for the previously computed, concurrent motion of other robots. We present a novel bi-directional space-time embedded rapidly exploring random trees (RRT) method that allows planning to an unknown arrival time and enables accounting for previously planned robot-trajectories. The goals for path planning are generated using an optimization-based approach that jointly solves for all manipulation constraints within the subproblem.

Summarizing, our core contribution is a method to solve time-embedded TAMP problems, which is enabled by two main novelties as follows:

1) A time-embedded keyframes optimization that samples time embeddings of manipulation constraints and employs optimization methods to jointly solve for them in the limited horizon subproblems.

2) A novel bi-directional space-time motion planner that finds paths between keyframes in combined space-time with unknown arrival times, thereby integrating sampling-based path planning with optimization-based methods.

Using these methods we can decompose an assembly problem into a series of limited horizon subproblems which only contain a subset of agents while accounting for the constraints implied by previously solved subproblems. This enables us to compute solutions to TAMP problems with multiple robots and long planning horizons.

We demonstrate our algorithm on various construction assembly problems with up to 12 heterogeneous agents, including two long-horizon case-studies using real architectural models where up to 113 parts have to be placed. Our evaluations show that this decomposition-based approach scales well with the number of robots and can efficiently coordinate robotic teams. We also show the feasibility of the plans computed by our algorithm on real robots.

II. RELATED WORK

Construction robotics [16] is a growing industry which aims to automate building construction. Specialized robots, such as bricklaying robots [17] or automated hydraulic excavators [18] recently emerged. Such robots have been used in various constructions settings [19], [20], [21], [22]. However, the algorithmic aspects of construction planning when dealing with multiple robots have only begun to be studied.

A. Multi-Robot Motion Planning

On the lowest level, construction planning needs to solve a multi-robot motion planning problem for heterogeneous robot teams [23]. This problem is often divided into two categories: first, one can plan roadmaps in parallel for individual robots, combine those roadmaps into an (implicit) joint state space roadmap, and eventually search this roadmap using algorithms, such as M* [24], or discrete RRT [25], [26]. Those algorithms can often be significantly improved using heuristics learned from prior experience [27].

Second, prioritization frameworks [14], [28], [29] can be used, where robot-paths are planned sequentially, imposing the previously planned movements as constraints for the next robot. Planning sequentially can require backtracking, which can be time consuming. Several approaches exist to avoid backtracking, e.g., analyzing start or goal conflicts [30].

In our approach, we use a prioritization approach by employing a heuristic to prioritize exploration of LGP subproblems. Contrary to prioritization in multi-robot motion planning we embed planning in space-time [31], [32] to enable planning in a dynamic environment and with unknown arrival times. Space-time planning has previously been investigated in the context of rendezvous planning [33]. In assembly tasks, it is often necessary to sample several keyframes at different time instances, due to blockages of the goal region from other agents. We combine space-time planning explicitly with discrete constraint switches. Our novel bi-directional space-time RRT allows us to efficiently find time-dependent plans while avoiding collisions with previously planned robots.

B. Assembly Planning

The algorithmic aspects of construction planning are studied in the field of assembly or rearrangement planning [34], [35]. Assembly planning traditionally focuses on finding valid sequences to assemble objects [36], [37]. More recently, this was also applied to masonry constructions [38], [39]. Assembly sequence planning approaches have been scaled to large number of parts. In [40], an assembly planning algorithm for a large number of bricks that are assembled in a plane is introduced, but the approach, in contrast to our work, does not take the path-planning problem into account. In [41], a method that computes assembly plans for a large numbers of objects is presented, but the approach is not general purpose, i.e., it is only applicable to grid-like structures, and the specific robot-building material pairing of TERMES [42]. A review on collective robotic construction can be found in [43].
A review of solution approaches to TAMP problems can be found in [15]. In this review, we focus on integrated task and motion planning applied to robotic assembly planning. Constraints and their intersections are explicitly enumerated in the state space [44]. Such a constraint graph can be exploited by biased sampling at constraint intersections [45], [46], and these samples can be connected along the constraint manifolds using projection methods [47], [48], [49]. Complex applications of such an approach are demonstrated in [25], such as concurrent handovers between multiple robots with multiple objects and capacity constraints.

On the other hand, symbolic assembly approaches explicitly introduce symbolic states to find task-level decisions and to factorize the problem [10], [50], [51]. Once symbolic decision sequences (skeletons) are found, lower level planners are used to execute a skeleton, using sampling-based [52], [53] or optimization-based [10] methods. This approach can be tailored toward many different applications, for example by including force constraints [51], dealing with replanning [54], [55] or handling partial observability [56].

Previous work dealt with long-horizon construction planning for a single agent [57], or two agents [8], but solving long-horizon TAMP problems using multiple robots is an open challenge. Initial steps toward solving multi-robot, multi-object rearrangement tasks in a simple configuration space for homogeneous robot teams with few objects were made in [11] and [58]. Such previous work assumes that the actions of robots are synchronized [8], [10], [11], [25], plan in the combined space of all robots, and do therefore not scale [8], are only demonstrated for simple robots and few objects and state that they do not expect to scale [58], or are not demonstrated on long-time-horizons [11], [25].

With our approach, we are able to plan for heterogeneous robot teams with complex interactions, and to scale to more objects and robots. We achieve this by combining sampling-based methods for path-planning, and optimization-based methods for finding the mode-switches. Contrary to previous work, our approach does also not assume synchronicity of actions, thereby allowing the parallelization of assembly tasks efficiently.

III. MULTI-ROBOT REARRANGEMENT PLANNING NOTATION AND PROBLEM FORMULATION

Given $n$ unique objects, indexed by $o \in O, |O| = n$, with initial poses $p_{0}^{o} \in SE(3)$ at time $t = 0$, and $m$ robots, indexed by $r \in R, |R| = m$, the aim is to rearrange all objects to their (given) goal locations $p_{T}^{o} \in SE(3)$. Each robot may have its own configuration space $Q_{r} \subset \mathbb{R}^{d_{r}}$.

We formulate the problem as a nonlinear mathematical program over the path $x : [0,T] \rightarrow \mathcal{X}$. The configuration space $\mathcal{X} = Q \times SE(3)^{m}$ consists of all robot configuration spaces $Q = Q_{r_{1}} \times \cdots \times Q_{r_{m}}$ and object configuration spaces.

Over time, different constraints on the path are active, e.g., at the end of a pick-task, the end-effector of an agent needs to fulfill gripping-constraints. Which constraints are active is determined by the task assignment $s \in S = S_{r_{1}} \times \cdots \times S_{r_{m}}$, where $S_{r}$ indicates the feasible tasks for robot $r$. Thus, the state $s$ determines the current task assignment of each robot.\footnote{This is slightly different to most TAMP literature, where $s$ is a set of grounded literals that determine, e.g., which robot is assigned to which task, and STRIPS-like rules determine feasible transitions between logical states (task assignments).}

We use $s_{r,1:K_{r}} \in S(R, O)$ to denote the discrete sequence of tasks of robot $r$, with $s_{r,j} \in S_{r}, K_{r}$ is the number of discrete states for robot $r$ in the sequence $s_{r,1:K_{r}}$. The set $S(R, O)$ denotes all valid state sequences induced by a first-order logic-language for the robots $R$ and objects $O$. For example, a handover-task necessitates a pick-task as precondition.

In most approaches to solve TAMP problems, the transitions between task assignments occur at fixed intervals [8], [10], [11], [25]. Our problem formulation allows for task assignments to switch (finitely often) at any time. This is achieved by the scheduling function, $k : [0,T] \rightarrow \mathcal{N} = \{1,\ldots,K_{1}\} \times \cdots \times \{1,\ldots,K_{m}\}$, that maps continuous time $t$ into a vector of indices that select the currently active task assignments for all robots, such that $s(t) = s_{k(t)}$. We use $k_{r}(t)$ to denote the scheduling function for robot $r$. The scheduling function is constrained to respect the order of the indices, i.e., $k_{r}(t_{1}) \leq k_{r}(t_{2}) \forall t_{1} \leq t_{2}$.

Therefore, we try to find the path $x$, the terminal time $T > 0$, the scheduling function $k$, and the sequences of discrete states $\{s_{r,1:K_{r}}\}_{r=1}^{m}$ to optimize

$$\min_{x,T,k,\{s_{r,1:K_{r}}\}_{r=1}^{m}} \int_{0}^{T} c(x(t), \dot{x}(t), \ddot{x}(t)) \, dt$$ (1a)

s.t.

$$x(0) = x_{0}$$ (1b)

$$\forall t \in [0,T] : g(x(t), \dot{x}(t), s_{k(t)}) \leq 0$$ (1c)

$$\{s_{r,1:K_{r}}\}_{r=1}^{m} \in S(R, O)$$ (1d)

$$g_{goal}(x(T), p_{r}^{O}, O) \leq 0.$$ (1e)

The task assignment state $s_{r,k_{r}(t)} \in S_{r}$ determines currently active constraints for each robot $r$ on the path $x$ at time $t$ via the constraint function $g$ in (1c). For example, $s_{r,k_{r}(t_{1})} = s_{r,j}$ could specify the necessary constraints for robot $r$ such that it grasps an object at time $t_{1}$ as the $j$th discrete state $s_{r,j}$ of the sequence $s_{r,1:K_{r}}$. Additionally, (1c) could describe collision constrains, or joint-limits. In the following, we refer to the discontinuities in $s$ as mode-switches, or keyframes. Fig. 2 illustrates the components of $s$, and how the sequence is mapped to continuous time.

The goal constraint (1e) specifies that at the end, all objects have to be at their target poses. The initial condition $x_{0}$ contains
the initial configuration of all robots as well as the initial poses of the objects. Finally, \( c \) is a cost function, such as path length, control cost, or minimal time. In case we are only interested in finding a feasible solution, \( c = 0 \).

A. Assumptions

We summarize the assumptions we made in the problem formulation as follows:

1) Known initial and final position of all objects, and availability of a method to sample configurations for manipulating them.
2) Monotonic rearrangement: While it is possible to handle nonmonotonicity in the logic search, we assume in this work that each object is handled a single time. However, we consider regrasping of objects, such as handovers.
3) No force and torque constraints for the robots: In this work, we assume that the parts are light compared to the allowable robot-payload. Consequently, every object can be manipulated by a single robot under this assumption.

IV. METHOD

Solving the problem described in (1) in a fully joint and global manner is intractable, and even finding a feasible solution that utilizes all robots is hard. We present our approach to decompose the problem into simpler subproblems, and to solve the subproblems such that the solutions together are a feasible solution to the original problem. Algorithms 1 to 3 and the following sections describe

1) A decomposition of the overall problem into subproblems that each contain only a subset of robots and assigned tasks. These subproblems account for the time-embedding in a scene where other robots are already moving, and represent coordination constraints for their respective subset of robots to ensure feasible cooperative manipulations, e.g., handovers;
2) An approach to generate solutions to manipulation constraints defined by the subproblems, such as pick, place, or handover constraints. We use this method to sample goals for the path planning algorithm;
3) A bi-directional space-time RRT path planner to find feasible motions between keyframes taking moving robots into account;
4) A heuristic to prioritize the order of subproblems that the overall system tries to solve, and how everything is integrated with each other.

An illustration of this system can be seen in Fig. 3.

A. Decomposition Into Time-Embedded, Limited Horizon Subproblems With A Subset of Agents

A natural decomposition of (1) into smaller subproblems emerges from the problem specification of rearranging objects, i.e., we consider subgoals of rearranging one object with potentially multiple robots. The following description focuses on clarifying the degrees of freedom (DoF) for each subproblem.

Assume that we are in step \( l \) of the planning process. The set \( \mathcal{O}^{l-1} \subset \mathcal{O} \) denotes all objects that have been successfully moved to their respective goal locations at previous planning
Algorithm 3: ST-RRT($x_0$, goal_sampler).
1 $T_0 \leftarrow x_0$, $T_\emptyset \leftarrow \emptyset$
2 while not stopped do
3 $t_b, t_o \leftarrow$ update_bounds()
4 if rnd(0, 1) $< p_{pool}$
5 $t \leftarrow$ sample($t_b, t_o$)
6 $q_s \leftarrow$ goal_sampler($t$)
7 $\{t, q_s\} \leftarrow$ add_goal($t, q_s$)
8 $q \leftarrow$ sample_valid_state()
9 $t \leftarrow$ sample_valid_time($q$)
10 if not $x_{sw} \leftarrow$ extend($[t, q], T_\emptyset$) was trapped
11 if connect($x_{sw}, T_\emptyset$) was reached
12 return extract_path()
13 swap($T_a, T_b$)
14 return $\emptyset$

Fig. 4. Problem with three agents, in which we plan for $r_1$ and $r_2$ (and object $o$) with a handover sequence. The previously fixed plans are dark grey, the red lines indicate constraints ($g_{sw}, g_{goal}$) that have to be fulfilled at the mode-switches, and the tasks and corresponding paths that have to be planned are light grey. The blue box indicates the DoF $x_{I_1}$ for the current planning problem. The active indices $I$ are $\{r_1, r_2, o\}$.

steps, i.e., a task sequence and corresponding trajectory has been planned. The set $O^{t-1} = O \setminus O^{t-1}$ denotes the objects that have no plan associated yet. A heuristic (explained in Section IV-D) selects a single new object $o_1 \in O^{t-1}$ and a set of robots $R_t \subseteq R$ that should be involved in rearranging the object $o_1$ to its target pose $p_{goal}^{o_1}$.

The optimization problem we solve in step $l$ therefore only optimizes over a part of the path $x$. We use $x_{I_1}^l$ to denote the degrees of freedom in the subproblem, where $I_1 = \{R_t, o_1\}$ is the set of indices of the path $x$ that correspond to the robots $R_t$ and the object $o_1$. Not all degrees of freedom in $x_{I_1}$ necessarily become active at the same time, since some of the robots might be involved in previously planned motions up to different times (Fig. 4: $r_1$ and $r_2$ become active at different times). Similarly, the subproblem in step $l$ is temporally embedded into a scene where robots and objects not part of the current planning problem may follow previously computed plans ($r_3$ in Fig. 4).

To define the optimization problem for the subproblem, we therefore also need to specify how the inactive indices of $x$ are defined. In order to do so, let $T_r^{l-1}, T_o^{l-1} \in \Re$ denote the time until which paths for robot $r$ and object $o$ have been planned in the $l - 1$ previous planning steps. If no path has yet been planned for a robot/object, its time is set to zero. This allows us to define the $i$th component of the path variable $x^l$ in the planning step $l$ at time $t$ as

$$
x_{i}^{l}(t) = \begin{cases} 
    x_{i}^{l-1}(t) & t \leq T_r^{l-1} \\
    x_{i}^{l-1}(T_r^{l-1}) & t > T_r^{l-1}, i \notin I_1 \\
    x_{i}^{l-1}(T_o^{l-1}) & t > T_o^{l-1}, i \in I_1,
\end{cases}
$$

(2)

Therefore, the degrees-of-freedom $x_{I_1}^l$ in planning step $l$ are those of $R_t$ and $o_1$ from the point in time where they have no associated planned trajectory yet (i.e., $t > T_r^{l-1}, i \notin I_1$). In the other cases, they move according to previously computed plans ($t \leq T_r^{l-1}$) or remain at the last planned configuration ($t > T_r^{l-1}, i \notin I_1$), i.e., they correspond to inactive degrees of freedom. The same holds for the scheduling function $k$ which has the effective degrees-of-freedom $k_{I_1}$. Similarly, the search over the symbolic task state for the selected robots happens over $\{s_{r, K_{r}^{l-1}+1}, ..., s_{r, K_{r}}\} \subseteq R_{T_t}$ only. The complete task state sequence of robot $r$ is then the concatenation $s_{r, K_{r}} = (s_{r, 1}, ..., s_{r, K_{r}^{l-1}}, s_{r, K_{r}^{l-1}+1}, ..., s_{r, K_{r}})$ of the sequences that have been determined in the steps up until step $l - 1$ and the new sequence. We show an illustration that serves to explain the free and fixed parts, respectively, in planning step $l$ in Fig. 4.

This leads to the following limited horizon optimization problem in step $l$ for the chosen object $o_1$ and robots $R_t$:

$$
\min_{T', x_{I_1}^l(\cdot), \dot{x}_{I_1}^l(\cdot)} \int_{T'}^{T' + 1} c(x^l(t), \dot{x}^l(t), \ddot{x}^l(t)) \, dt
$$

(3a)

s.t. $x^l$ as defined in Eq.(2)

(3b)

$$
\forall t \in [T' + 1, T'] : g(x^l(t), \dot{x}^l(t), \ddot{x}^l(t)) \leq 0
$$

(3c)

$$
\forall r \in R_t : s_{r, K_{r}^l} \in S(R_t, \{o_1\})
$$

(3d)

Here, $T' + 1 = \min_{t \in R_t} T_r^{l-1}$ is the earliest time for which no plan of a robot in $R_t$ exists yet. If $R_t$ contains more than one robot, the final time $T'$ that is being optimized for is the maximum time of all robots $R_t$ that are involved in the current planning step, as one robot could fulfill all its constraints earlier than the others. Consequently, the final times $T_r^{l-1} \leq T'$ are assigned by extracting the minimum times where each individual robot and object $j \in I_1$ fulfill their constraints.

B. Time-Embedded Keyframes Optimization to Jointly Solve for Sequential Transition Constraints

Equation (3) is nonconvex due to collision avoidance, manipulation constraints, the time-embedding, and the discrete task-assignment. To robustly find feasible solutions to (3), we combine a search for a valid sequence with nonlinear optimization and a sampling-based planner. The optimizer solves for configurations at the transition between two task assignment states, while the motion planner, described in Section IV-C, iteratively finds paths between the keyframes.
Assume we are given robots $R$ and the sequences of discrete states $\{s_{r,j}^i\}_{j \in J_r} \forall r \in R$ with $J_r^t = \{K_r^t-1, \ldots, K_r^t\}$. The problem

$$\min_{x_{l_1}(\cdot)} \sum_{r \in R_1, j \in J_r} c_d(x_{l_1}(t_{r,j}))$$

(4a)

s.t. $\forall r \in R_r \forall j \in J_r : g_{sw}(x_{l_1}(t_{r,j}), s_{r,j}, s_{r,j-1}) \leq 0$ (4b)

$$\exists r \in R_r \exists j \in J_r : g_{goal}(x_{l_1}(t_{r,j}), p_{oi}, o_l) \leq 0$$

(4c)

thus describes the configurations of the involved robots and the object $o_l$ at the mode switching times $t_{r,j}$. The constraint (4b) is the discrete version of (3b) at the transition from $s_{r,j-1}$ to $s_{r,j}$. The cost function $c_d$ is the discrete version of $c$. Solving (4) (using, e.g., [59]) generates a set of keyframes that jointly fulfill interdependent constraints, e.g., finding consistent hand positions in pick and place poses.

To solve (4), we first sample the times $t_{r,j}$ where the transition from the discrete state $s_{r,j-1}$ to $s_{r,j}$ occurs uniformly between $t_{th,j}$ and $t_{ab,j}$ with $t_{r,j-1} < t_{r,j}$ and $T^{l-1} < t_{r,j}$ for $j \in J_r$. The lower bounds of the intervals are estimated by using the minimum possible arrival time and the upper bound is a fixed multiple of it. The intervals are gradually enlarged during path-planning if no solution can be found. We then use an optimization based solver to find configurations fulfilling the constraints. Optimization-based solvers are strong to resolve equality constraints, but are prone to local optima. We alleviate this problem by repeatedly solving (4) to generate various consistent keyframes. There are two reasons for why the solution to (4) is randomized and generates varieties of solutions: First, whenever we solve (4), we sample the times $t_{r,j}$, which leads to a different time embedding and corresponding constraints. Second, we initialize the optimizer with randomly sampled configurations, which helps to find various local optima.

In this view, solving (4) generates feasible keyframes that can be used as goals for bi-directional path planning.

**C. Bi-Directional Space-Time Path Planning to Connect Keyframes**

We compute the path for a given task sequence in a sequential manner (Fig. 5): the path planner first aims to find a path between the first two keyframes; when one is found, it moves on to find a path to a third keyframe that is consistent with the first two. It can always query the keyframes optimizer for more keyframes consistent with given previous keyframes, i.e., (4) is solved for the remaining mode-switch configurations. Solving the full remaining problem, instead of only the constraints implied by the next mode-switch, excludes keyframes that are feasible in the next mode switch, but lead to an infeasible problem later on, e.g., a pick configuration, which does not correspond to a feasible place-configuration.

Since the arrival time at which the robot can reach a goal keyframe configuration is unknown, we uniformly sample a range of candidate time-embeddings $t_{r,j}$ as input to the keyframes optimizer. During the planning process, the path planner continuously extends the range of time-embedding samples to allow for consideration of larger time-spans, i.e., to enable “waiting.” The arrival times that are found in this fashion correspond to the scheduling function $k$ for the robots involved in the current subproblem.

**Path-Planning:** We finally present the bi-directional path planner to connect the keyframe-configurations. Following the standard notation for time-embedded path planning [32], [33], let $Q_{R_1}$ denote the configuration space for the robots $R_1$, and $T \subset \mathbb{R}_{\geq 0}$ the time dimension. Our path planning problem is the problem of finding a collision-free path through the combined space-time $\mathcal{Y} = Q_{R_1} \times T$ from an initial keyframe configuration $(x(t_{r,j-1}), t_{r,j-1})$ at time $t_{r,j-1}$, to a set of candidate goal keyframe configurations, each a pair $(x(t_{r,j}), t_{r,j})$ in space-time with varying $t_{r,j}$.

The free configuration space $Q_{free}$ for the robots $R_1$ is time-dependent, as objects and other agents might move on previously planned paths. Additionally, it can be the case that some of the agents we are currently planning for move on a fixed path for some time, i.e., they only become a degree of freedom at time $t > T^{l-1}$. We deal with this by using a constrained path-planner [60], with the constraints defined in (2).

Specific care has to be taken due to the time-dimension, and the direction-dependent distance function in the configuration space:

$$d(y_1, y_2) = \begin{cases} \lambda d_{Q_{R_1}}(q_1, q_2) + (1-\lambda)(t_2-t_1) & \text{if } t_1 < t_2, v \leq v_{max} \\ \infty & \text{else} \end{cases}$$

(5)

where $d_{Q_{R_1}}$ can be any valid metric in $Q_{R_1}$, and $v$ is an estimate of the velocity. We use $\lambda \in (0,1]$ to describe the importance of
the path-length and the needed time, respectively. This distance-
metric encodes the inability to move from a configuration \( q_1 \) at
time \( t_1 \) to a configuration \( q_2 \) at time \( t_2 \) if either the required
speed \( v \) is too high, or the robot would need to move backwards
in time.

We then extend bi-directional RRT [61] to space-time\(^2\): this
extension consists of the previously described keyframe sampler
which generates the goals, time dependent collision queries, and
configuration-sampling bounded by the sampled keyframe time.
Specific care has to be taken when connecting edges using the
goal-centered tree: we move “backwards” in this case, and, thus,
the distance function has to be adapted. This bi-directional
space-time RRT formulation allows us to efficiently find paths.

After finding a path using the outlined approach, we post-
process the path by shortcutting [63] the obtained path, and
smoothing it. Shortcutting of the path works by repeatedly
choosing two discrete states of the path and checking if they can
be connected with a straight line while fulfilling the constraints,
e.g., collision, kinematics, or velocity limits. If that is the case,
the straight line path replaces the part of the path between the
chosen discrete states.

For smoothing, we use an optimizer [64] that takes the con-
straints and the cost function of the original problem as input and
is thus able to take the dynamics and constraints into account.
Taking this approach is similar to separating path planning and
trajectory planning, and is a common approach to find feasible
paths in cluttered environments [65], [66].

As in the planning itself, care has to be taken to shortcut and
smooth in a constrained manner, i.e., parts of the path that have
previously been fixed can not be altered.

D. Prioritization of Subproblems and Search Over Skeletons

In each step \( l \), the limited horizon formulation (3) requires a
selection of object \( o_l \) and robots \( R_l \). Due to the assembly tasks
we consider, the order in which objects have to be rearranged
(in particular being placed) is not obvious. For example, objects
need sufficient support in order to be placed, while placing some
objects too early might obstruct later object placements.

In planning step \( l \) we have the set \( \mathcal{O}^{l-1} \subseteq \mathcal{O} \) of previously
rearranged objects, and the set of objects \( \mathcal{O}^{l-1} \subseteq \mathcal{O} \setminus \mathcal{O}^{l-1} \) that
have no plan associated yet. From previous task assignments
and trajectory planning, the times \( T_{r_i}^{l-1} \) until which a trajectory
is already assigned to robot \( r \) are known. Based on this planning
state, the algorithm has to

1) choose an object \( o_l \in \mathcal{O}^{l-1} \) subject to object order con-
straints (expressed by \( \phi(o_l; \mathcal{O}^{l-1}) \leq 0 \));
2) choose a subset of robots \( R_l \subseteq R \) to rearrange object \( o_l \);
3) search through the space of assignment sequences
\( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in R_l \) to find a sequence that is logically
feasible, leads to the goal (i.e., object placement), and for
which the keyframes optimizer and path planner can find
solutions. If no such sequence is found, choose another robot assignment \( R_l \) (that was not chosen yet);
4) if no possible choice of \( R_l \) leads to a feasible subproblem
(3), we backtrack to rewind previous object placements,
and attempt to place \( o_l \). Backtracking is repeated until a
valid solution to place \( o_l \) is found.

This can be seen as a depth-first tree-search over the objects,
robots, and assignment sequences. These steps are explained in
more detail as follows.

1) and 2) Selection of Object and Robots: The objects \( o_l \) in
step 1) and robots \( R_l \) in step 2) are selected by a strict priori-
}r,K_l\}

This can be seen as a depth-first tree-search over the objects,
robots, and assignment sequences. These steps are explained in
more detail as follows.

\( \max_{r \in R_l} T_{r}^{l-1} \)

i.e., the latest busy time of all involved robots. This assumes
(very conservatively) that the work on this subproblem starts
only when the last of the involved robots becomes free. The
robot-selection could also be prioritized by, e.g., minimizing
the estimated finishing time of the subproblem.

3) Path Planning and Task Sequence/Robot Rejection: Finding
a task sequence is realized as a breadth-first search to check if
there exists a sequence

\( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in S(R_l,\{o_l\}) \)

which is logically feasible and leads to the symbolic goal.

With (3) fully defined by the choice of \( R_l \), \( o_l \), and \( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in R_l \), we try to find a valid path (as detailed in
Sections IV-B and IV-C). Before attempting to solve the full
problem, it is possible to evaluate lower bounds, i.e., simpler
subproblems, which, if they are infeasible, guarantee that there is
no solution to the full problem. In our case, examples for this are
1) attempting to find a placement pose, and 2) attempting to find
the configurations at the mode switches. For a more thorough
description of the notion of lower bounds, see [67].

If a problem is infeasible for a chosen \( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in R_l \),
we first attempt to solve the problem using different
\( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in R_l \) multiple times, and if still no feasible solu-
tion can be found, we restart from 2), excluding infeasible task
sequences and robots.

In general, the methods we use for generating keyframes or
motion paths cannot prove infeasibility of a specific discrete
assignment \( R_l \), \( o_l \), \( \{s_{r,K_l}^{l-1+1:1:K_l}\} \in R_l \) due to nonconvexity
of the optimization problem, or due to finite runtime of the
path planning. Hence, we keep a list of all assignments that
we determined to be infeasible before, and revisit them in a
deprioritized manner if still no solutions in the remaining dis-
crete assignments can be found. This allows us to explore more
promising decisions, while still guaranteeing that a solution will
be found eventually, if it exists. For brevity, this was left out of
the algorithm.
Fig. 6. Final configurations of the models we use for the experiments and demonstrations.

4) Backtracking: At this stage, all robots $R_l$, and possible task assignments were checked. Thus, any infeasibility at this stage must be caused by previously placed objects, since a selected part $o_l$ fulfills all the constraints to be able to be placed, i.e., $\phi(o_l; O^{l-1}) \leq 0$, and placing additional objects can never make placement of $o_l$ feasible. Instead, we backtrack to rewind previously placed objects until a valid solution to place object $o_l$ can be found.

V. DEMONSTRATIONS & RESULTS

We analyze the scalability of the algorithm, and how some scenarios benefit more than others from better parallelizability. We demonstrate the robustness of the algorithm on long-horizon scenarios, show the ability to coordinate multiple robots in a scenario where a handover sequence is necessary, and a real robot experiment. Finally, we compare the algorithm to a modified version of a classical TAMP-solver.

A. Setup

We test the algorithm on several construction scenarios as follows\(^3\):\(^4\):

1) A tower, consisting of 15 pieces, where the placements of the parts have to be in strictly sequential order.
2) A wall, consisting of 36 bricks used to analyze how well a task can be parallelized.
3) A well, consisting of 52 pieces, used to demonstrate scalability.
4) A pavilion consisting of 113 unique wooden cassettes, used to demonstrate scalability.

The final configuration of the models is visualized in Fig. 6. We assume that the pieces form a rigid body with the neighboring pieces as soon as they are placed, and thus, neglect both the fastening process, and the structural support that would be necessary. We are first and foremost interested in finding a feasible solution to (1), i.e., we use $c = 0$. For the real robot experiment, we minimize the acceleration in the smoothing-step.

1) Robots: We demonstrate our approach using three different robots (Fig. 7): a mobile manipulator, a KUKA-arm on a mobile base, and a crane. The robots are being used for the demonstrations are holonomic. If not stated differently, we model manipulation by gripping by touch: on construction sites, vacuum grippers are commonly used, which can be approximated as gripping by touch.

2) Task Assignments: The discrete task assignments are pick, place, retract, and handover. We require a place task to always be followed by a retract task, since in general, after placing an object, it is not desirable to stay in the same configuration, and possibly block the placement configurations of other agents.

3) Ordering Heuristic: We represent the buildings as a graph, with the nodes being parts, and edges between connected parts. The heuristic $h$ in (6) is chosen to find the object that maximizes the number of previously placed neighbours. The placeable parts are the set of nodes which are connected to at least one node in the graph that is already placed. This is encoded in the constraint $\phi$ in (6).

Due to the heuristic and placement constraint, no backtracking was required in our experiments. As such, there is no specific demonstration, or mention of how many times backtracking was needed in the examples.

B. Experimental Results

We provide analysis on several quantitative metrics in this section. The experiments for the analysis were done using the mobile manipulator utilizing pick and place-sequences. Videos of the assembly processes for various models with different team-sizes and various constellations of robot types can be found in the supplementary material.

1) Execution Time: The execution time, i.e., the real time of the planned movements, is expected to decrease the more agents are used for the rearrangement task. Fig. 8 plots the factor by which the execution is sped up against the number of agents for the tower and the wall, and highlights how the separability of the

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\(^3\)Simulated using https://github.com/MarcToussaint/raim
\(^4\)The experiments were run on a single core of Intel(R) Core(TM) i7-8565 U CPU @ 1.80 GHz with 16 GB RAM using Ubuntu 18.04.
model influences how adding more robots leads to diminishing returns in the speedup factor. These diminishing returns occur more quickly for the tower, where all agents need to place the objects in close vicinity.

2) Computation Time: Fig. 9 shows approximately linear scaling with the number of agents for the computation time. This supports that our approach of planning each agent separately, and assuming the previously planned agents as fixed, scales well. Since the tower has a bottleneck where all agents have to come together within a relatively small region, it scales worse than the wall. The objects making up the wall are distributed over a larger space, and thus, the agents do not impede each other as much, i.e., the planning problem is easier, since fewer agents per volume are present. Table I breaks down the average necessary computation time for the different steps of the planning process. Specifically, we want to highlight that solving the keyframe-problem takes most of the time of the whole planning process.

C. Comparison to Fixed-Time-Sampling

We did not put an emphasis on comparison with other methods, as the other methods we are aware of ([8], [11], [12], [25], [58], [68]) do not scale to the number of robots and objects we consider. We compare our method to a fixed-time sampling scheme, which decomposes the problem as the presented approach does, and uses prioritized planning, but does not allow for variable durations of the tasks, i.e., all mode-switches take place at a multiple of $T$. We first note that the fixed-time-sampling approach is extremely sensitive to the choice of $T$. If there are

5This is similar to what most TAMP solvers would do.
on the Wall-example, a similar speedup can be achieved due to the large space that is available, this is not the case in the Tower-example. The main bottleneck in the fixed-time-sampling is the keyframe generation, which becomes much harder if the space is highly congested, and thus, needs more restarts, which leads to a much longer computation time, and much worse scaling of the computation time with the number of agents we plan for.

D. Demonstrations

1) Long Horizon Assembly: We demonstrate our algorithm on two long horizon-construction tasks using the mobile manipulators, and modeling manipulation constraints as gripping by touch as follows.

a) The well, using six agents: computation time 14.1 min, execution time 2150 steps.

b) The pavilion, using 8 agents: computation time 43.2 min, execution time 3867 steps.

We show a schedule for the assembly of the pavilion using eight agents in Fig. 13 to showcase the complexities in coordinating the robot movements.

2) Handover Scenario: We consider the scenario of the tower again, but this time with three mobile bases with KUKA-arms on top. Manipulation is modeled as gripping by touch. Since the KUKA-arms are unable to reach the top of the tower, we add a tower crane. However, the crane is unable to reach the pieces on the floor. Therefore, handover sequences are necessary to place the last three parts. This scenario demonstrates the ability of our framework to handle and coordinate robots with different capabilities and explore various possible task sequences to fulfill a task. The resulting schedule, and some frames from the process can be seen in Figs. 14 and 15, respectively.

3) Real Robot Experiments: We demonstrate an experiment with two robotic arms with Robotiq grippers as end-effectors. We model the manipulation constraints of the robot as a two finger gripper. The goal is to stack six boxes using the two arms.

The average computation times over ten runs for the different parts of the algorithm were 264.8 s for keyframe-optimization, 4.2 s for path-planning, and 58.0 s for postprocessing. The longer computation times for the keyframe optimization can be attributed to the difficulty of finding valid two-finger-grasps with the naive approach that we used here. It is possible to speed this up using more specialized methods, as shown, e.g., in [69].

We execute the trajectory open-loop; we thus rely on an accurate model in the simulation, such that the trajectory can be executed without adaptions. In the cases where the execution failed, it was mainly due to violating the assumption of having an accurate model: a typical failure mode was due to inaccurate initial placement of the boxes, which lead to inaccurate final placement, and sometimes knocking over the tower during the motion to place the block. Additionally, the cables of the robot were not taken into account initially, leading to pushing over the tower in some cases. The execution time was between 35 and 45 s in all attempts.

Fig. 16 shows a sequence of images from the two robots placing the blocks. While it is visible that the boxes are not perfectly aligned, it is clear that the algorithm succeeds in effectively coordinating the robots. This experiment showcases that it is possible to account for the constraints arising in real robot experiments and that the plans and trajectories generated by our algorithm can be executed on a set of real robots.

VI. DISCUSSION AND LIMITATIONS

Multi-robot assembly is a complex problem which consists of multiple NP-hard subproblems [32]. The method development in this article is application-driven, aiming to push toward an efficient solution strategy that scales well to challenging scenarios.
The prioritization of objects and Path-planning in our algorithm works configurations with “resting” agents. We are using a depth-first place or ONCLUSION pick plan the assembly for multiple agents. Another one would be to reserve a “safe corridor” for an agent, through which the execution, both in space, and in time. One possibility to do so is present, this might be more challenging. Hence, the planning of the keyframes, e.g., through online learning of initializations for the optimizer, is one way to decrease the time [69], [70]. Reducing the number of sampled goal states is another possibility to scale down the necessary time. For speeding up the planning, the biggest possible improvement is the usage of multiquery planning. Applying multiquery path planning in a dynamic environment is not straightforward, however, and needs to be considered as future work. A first step toward multiquery planning in the construction setting was shown in [71].

While we showed that the paths can be executed by real robots directly in a controlled environment, in case uncertainty is present, this might be more challenging. Hence, the planning done in this framework might need to account for uncertainty in the execution, both in space, and in time. One possibility to do so would be to reserve a “safe corridor” for an agent, through which no other agent travels for a given time-window. Another one would be to, e.g., consider clearance during the path planning, and not only to minimize time.

Finally, when applying the approach presented here to planning for real-world construction scenarios, further static and ordering constraints of the assembly sequencing need to be taken into account. Decoupling of assembly sequencing from the lower level TAMP problem is one way to deal with such additional constraints. Finding valid assembly sequences could be done with, e.g., multibound tree search together with assembly/disassembly planning, which could try to find a feasible sequence for a single agent, and then subsequently use the approach presented here to plan the assembly for multiple agents.

While the algorithm was demonstrated on a model of a real pavilion, scaling to larger number of parts might be necessary in other real-world scenarios. The demonstrations in the previous sections support our belief that this algorithm is able to scale to larger numbers of objects, but it is part of future work to scale this even further.

A. Discussion of Theoretical Properties

We briefly discuss the properties of the algorithm, and necessary changes to achieve completeness.

1) Selection of Subproblems: The prioritization of objects and robots in combination with the backtracking can be seen as depth first search. This means that every possible assignment will be chosen at some point. The methods we use for path-planning cannot prove an infeasibility, which is why we need to continue exploring the “infeasible” nodes.

2) Choice of Action-Sequence: We are using a depth-first search over the space defined by a first-order logic-language to compute the available action sequences. Under the assumption that each object can be rearranged to its goal position within a finite number of actions, this implies that we will find a feasible action sequence, if one exists.

3) Path-Planning: Path-planning in our algorithm works sequentially through an action-sequence, and plans each part of the sequence using an RRT. This means that each segment alone is asymptotically complete in the limit, assuming that the keyframe-sampling is uniformly covering the solution manifold and that we return to a subproblem infinite number of times. It might be possible that we find a path in one part of the action sequence which does not have a corresponding feasible path in the following action, and thus, label the sequence as “infeasible.” The occurrence of this is greatly reduced by jointly sampling all keyframes. Continuously reexpanding the “infeasible” nodes covers this case.

We finally note that we need to make sure in the algorithm that we do not make subsequent placements impossible by fixing paths that do not allow for a feasible subsequent path, i.e., do not block, pick or place configurations with “resting” agents.

VII. CONCLUSION

We presented a planning system to solve long-horizon multi-robot construction assembly problems that integrates several novel components. The approach strongly exploits the factorizations of multiagent construction assembly problems, by solving simpler subproblems involving only a subset of agents to plan...
for and a single object that has to be placed at its goal location. These solutions to the separate subproblems are then used to construct a feasible solution to the overall problem. To solve the limited-horizon subproblems, we combine sampling-based path planning with joint mode-switch optimization to solve for manipulation constraints, and proposed novel methods to find time-embeddings for planned tasks. Path planning between keyframes amidst other moving, previously planned, objects, and robots is achieved using a novel bi-directional RRT-planner in space-time.

We demonstrate that our approach scales well to many robots and many objects on a variety of construction tasks. We provide both qualitative and quantitative analysis of the results. Compared to planning task assignments at fixed times, our time-embedding lead to better utilization of the robots and hence lower execution time to achieve the task, as well as lower computation times for planning the movements. Finally, we demonstrated the approach in a real robot experiment. The robotic experiments showed that it is possible to execute the paths that our approach generates.

The approach exploits decompositionality and greedily selects the next subtasks. This is successful for our application scenarios, but compromises global optimality for efficient planning and execution times, as is crucial to make multi-robot planning work.

We want to push the approach to demonstration on real construction scenarios. In this setting, more realism in the model description is required, including exact physical constraints on static stability.

VII. ACKNOWLEDGMENT

The authors would like to thank Christoph Schlopschnat for the model of the wooden pavilion.

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