Exponentially varying suction and injection on MHD viscous oscillatory stratified flow between parallel porous plates

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Abstract

In this paper, we consider Magnetohydrodynamic viscous oscillatory flow of incompressible linearly density stratified fluid in a long vertical rectangular channel with injection through y-axis and suction through the plate parallel to y-axis. In the undisturbed state, the density distribution is assumed to be linearly distributed. Closed-form solutions are obtained using similarity transformation. The effects on axial and transverse velocity profiles due to various parameters involved in the problem are illustrated in the figures. The graphical representations of the investigation show that the stratification effects are more on transverse velocity profiles than that of axial velocity. For constant density and zero electromagnetic induction, the result reduces to that of viscous oscillatory flow between parallel plates.

1. Introduction

The characterisation of mingling of fluids with electrical conductivity and electromagnetic forces is known as Magnetohydrodynamics (MHD). In 1937, Hartmann experimented the effects of magnetic field applied in transverse direction on the flow of fluid that is viscous incompressible and electrically conducting between two infinite parallel insulated stationary plates. Motivated by the invention that plasma could be reproduced in the laboratory during 1950, the study on MHD started its development. Due the fact that Earth has its magnetic field, crystal growth induced space weather prediction with damping of highly varying fluctuations in semiconductors melts gave way for the inventions on MHD. The problem of MHD flows are usually explained mathematically by using Navier–Stokes equations, along with its associated electrodynamics equations and Maxwell’s equations. These equations are then combined into moving electrical conductor’s Lorentz force and Ohm’s law. Fluid flows affected by the viscosity and density differences present in the fluid is described as heterogeneous or stratified flow. These fluid flows in the field of environment and ocean research draws interest in the study of stratification. Viscous fluid flowing through the boundary layer that are unsteady and convective having variable fluid properties was considered by Vajravelu, Prasad and Chiu (2013). Krishnendu Bhattacharyya and Lavek (2014) analysed using fourth order R-K method by applying shooting technique on boundary layer flow of magnetohydrodynamic nanofluid over an exponentially stretching permeable sheet. Azeem Shahzad, Ramzan Ali and Masood Khan (2012) derived the variable separable solution for two dimensional convective flow over a nonlinear sheet that is stretched radially. Inviscous heterogeneous fluid flow due to forced oscillation was analysed in research work of many authors such as Krishna & Sharma (1969), Hendershott (1969), Rao and Rao (1971), ...
Sharma and Naidu (1971), K.B Naidu (1974), Prasanna Venkatesh, Ganesh and Naidu (2014) and Prasanna Venkatesh (2018). The main focus of our research problem is to consider heterogeneous fluid which is viscous oscillatory flowing through two vertical plates with a porous wall induced by variable suction and injection at the porous wall that accommodates Boussinesq approximation for density. Variable separable solution for the problem is derived by using a convenient choice of stream function which is a multiple of suction or injection velocity.

2. Problem Formulation and Solution

The mathematical construction of the problem considers the flow of heterogeneous fluid which is viscous and oscillatory in a long channel of width ‘h’. Both the injected velocity through the plate on y – axis as well as the suction velocity through the plate placed parallel to y – axis is varying as a function of y. The fluid flow is induced only after the start of suction and injection velocities. The distance between the plates is fixed be ‘h’. The density variation is along the fluid flow which is downwards in the direction of gravity along the negative side of y – axis. The density in the motionless state is considered to be dependent on only y whereas during its motion it is assumed to be dependent of x,y & t. The following set of equation describes the mathematical model for the above said situation after applying Boussinesq’s approximation in the inertial terms in the equation for creeping flow of heterogeneous fluid.

Equation of continuity

\[ \frac{\partial \rho}{\partial x} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = 0 \]  

Equation describing Incompressibility of the flow

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = 0 \]  

Equation of conservation of motion

\[ \rho_0 \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  

\[ \rho_0 \frac{\partial v}{\partial t} = - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \sigma_e B_0^2 v - \rho' g \]  

Here \( \mu \) denotes with usual notation the viscosity coefficient and \( \rho \) denotes the fluid density in general, \( \sigma \) represents the conduction of electricity and \( B_0 = \mu H_0 \) is considered to be electromagnetic induction, \( \mu_e \) is magnetic permeability notation and \( H_0 \) is the notation for measuring transverse magnetic field. The density expression in the motionless state is taken as

\[ \rho = \rho_0(y) + \rho'(y, x, t) \]  

\[ \rho_0(y) = \rho_0' (1 - \beta y) \]  

\( \rho_0' \) represents constant density part. The density in motionless state denoted by \( \rho_0(y) \) is a linear function of y, \( \rho'(y, x, t) \) denotes the density during motion of fluid, \( \beta \) is the stratification parameter (a constant). Brunt – Vaisala frequency N defined by the relation \( N^2 = \beta g \).

\[ \frac{\partial \rho'}{\partial t} = \rho_0' \beta \nu \]  

The conditions for the boundary velocities are

\[ u(0, y) = v(y e^{ihx}), u(h, y) = v(y e^{ihx}) \]  

\[ v(0, y) = 0, \quad v(h, y) = 0 \]  

By usual procedure of eliminating Pressure and with equation of density and then finally substituting stream function the system of equation above reduces to
\[
\rho_0 \frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi = \mu \frac{\partial \psi}{\partial t} - \left( N^2 \rho_0 \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} B_0^2 \right) \frac{\partial^2 \psi}{\partial \sigma^2} \quad (9)
\]

\[
\psi(x, y, t) = \Psi(x, y) e^{i \sigma t}, \quad u = u(x, y) e^{i \omega t}, \quad v = v(x, y) e^{i \omega t}, \quad p = p(x, y) e^{i \omega t}
\]

The function \( f(\eta) \) is introduced as follows:

\[
\Psi = u_0 e^{-\frac{\eta}{u_0}} f(\eta) \quad (10)
\]

Where \( \eta = \frac{x}{u_0} \) and \( u_0 \) is the average entrance velocity.

Using (10) in (9)

\[
\left[ D^4 - \frac{\rho_0}{\mu} \left( \omega^2 - N^2 \right) \frac{\partial}{\partial \sigma} B_0^2 \right] f(\eta) = 0 \quad (11)
\]

The above equation clearly represents an ordinary differential equation of order 4 with constant coefficients which are complex numbers for which the general solution is presented as follows

\[
f(\eta) = c_1 e^{a_1 \eta} + c_2 e^{-a_1 \eta} + c_3 e^{a_2 \eta} + c_4 e^{-a_2 \eta} \quad (12)
\]

The Conditions of the boundary are converted in terms of \( f(\eta) \) are given by

\[
\frac{f(0)}{f(1)} = -1, \quad f(0) = -(1 - a) \quad \text{and} \quad f'(0) = 0, \quad f'(1) = 0 \quad (13)
\]

Where \( a = 1 - \frac{\nu_1}{\nu_2} \), \( 0 \leq \nu_1 \leq \nu_2 \) and \( u_0 \) is the average entrance velocity.

Substituting (13) in (12)

\[
c_1 + c_2 + c_3 + c_4 = -1
\]

\[
c_1 e^{a_1 \eta} + c_2 e^{-a_1 \eta} + c_3 e^{a_2 \eta} = -(1 - a)
\]

\[
c_1 a_1 - c_2 a_2 + c_3 a_2 - c_4 a_2 = 0
\]

\[
c_1 a_1 e^{a_1 \eta} - c_2 a_2 e^{-a_1 \eta} + c_3 a_2 e^{a_2 \eta} - c_4 a_2 e^{-a_2 \eta} = 0
\]

The constant values present in the above system of equations after solving are represented by

\[
A_1 = \left( e^{a_1 \eta} - e^{a_2 \eta} \right) \left( 1 + \frac{a_2}{a_1} \right), \quad A_2 = \left( e^{-a_2 \eta} - e^{-a_1 \eta} \right) \left( 1 - \frac{a_2}{a_1} \right), \quad A_3 = e^{a_1 \eta} - 1 + a
\]

\[
A_4 = 2 \left( e^{a_2 \eta - a_1 \eta} - \frac{a_2}{a_1} \right) \left( e^{a_2 \eta} - e^{-a_1 \eta} \right)
\]

\[
A_5 = 2 \left( e^{-a_2 \eta - a_1 \eta} - \frac{a_2}{a_1} \right) \left( e^{-a_1 \eta} - e^{-(a_2 + a_1)} \right)
\]

\[
A_6 = \frac{A_2 A_4 - A_3 A_5}{A_1 A_2 A_4}
\]

\[
c_1 = A_8 = \frac{A_6 + A_7}{2}
\]

\[
c_2 = A_9 = \frac{A_6 - A_7}{2}
\]

\[
c_3 = A_{10} = \frac{A_3 A_6 - A_2 A_4}{A_1 A_2 A_4}
\]

\[
c_4 = A_{11} = \frac{A_3 A_6 - A_2 A_4}{A_1 A_2 A_4}
\]

Substituting the constants

\[
f(\eta) = A_8 e^{a_1 \eta} + A_9 e^{-a_1 \eta} + A_{10} e^{a_2 \eta} + A_1 e^{-a_2 \eta}
\]

\[
u(x, y) = -\nu(x, y) e^{a_1 \eta} + A_8 e^{a_2 \eta} + A_9 e^{-a_2 \eta} + A_{11} e^{-a_2 \eta}
\]
\[ v(x,y) = \frac{1}{h} \sigma e^{\eta x} \left( a_1 \left( A_0 e^{\alpha y} - A_0 e^{-\alpha y} \right) + a_2 \left( A_1 e^{\alpha y} - A_1 e^{-\alpha y} \right) \right) \]

\[ \rho = \rho_0 \left( 1 - \beta y \right) + \frac{\rho_0}{\alpha} \left( 1 - e^{\omega t} \right) h(x,y) \]

3. Results and Discussion

For Figure 1 to Figure 6 the numerical values of \( v \) (x-direction), the velocity along the fluid flow direction have been depicted for different values of \( x \) (y – direction), \( y \) and various other parameters that are present in the problem. Figures 1 shows the variation in the velocity along the fluid flow direction for different values of \( x = 0 \) to 1 and \( y = 0.3 \) while values of \( \rho_0=1.2, \mu=1.5, \sigma_e=1.5, N=1.5, u_0=1.2, v_1=0.6, v_2=1.8 \) and \( \omega t \) (\( \omega t=0 \) to \( \pi \)). Figure 2 represents the Transverse velocity profiles for \( \omega t=0, x = 0 \) to 10 and for \( y=0 \) to 1. Figure 3 depicts the variation in Transverse velocity due to the change in magnetic parameter \( B_0 \) from 0 to 10. Figure 4 represents velocity along the fluid flow direction for different \( \beta \) values from 2 to 10. It is noted from these graphical representations that velocity along the fluid flow decreases with respect to time, depth, magnetic parameter and stratification. Figure 5 and Figure 6 shows the variations in velocity along flow direction for different \( \mu \) (0 to 1) and \( \rho_0 \) values (5 to 25) respectively. The fluid are often realigned quickly around the centre of channel which indicated that the effects of stratification are more around the centre of the channel. Figure 7 and 8 represents Axial velocity for varying \( \omega t \) and \( y \) values respectively. Axial velocity profiles varies inversely with time and depth. Axial velocity profiles for varying values of \( N \) (2 to 20) is shown by Figures 9. Figures 10 and 11 represents the changes shown in the Density function due to varying \( \omega t \) and \( y \) values respectively. Figure 12 explains velocity in flow direction for different \( \sigma_e \) values (4 to 20).

4. Conclusion

The pictorial representations of Transverse and axial velocity profiles lead to the following observations. velocity along the flow direction decreases with time, depth, MHD parameter and parameter for stratification. It increases with viscosity whereas it decreases with constant density \( \rho_0 \). Axial velocity as well as density decreases with time and depth but increases with stratification parameter. The stratification effects are more at the centre than that of boundary of the channel. The stratification effects are more for transverse velocity than that of axial velocity. Density distribution is dominated by Transverse velocity profiles.

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Figure 3. Transverse Velocity for varying $B_0$ values

Figure 4. Transverse Velocity for varying $N$ values

Figure 5. Transverse Velocity for varying $\mu$ values

Figure 6. Transverse Velocity for different $\rho_0$ values
Figure 7. Axial Velocity for varying ωt values

Figure 8. Axial Velocity for varying y values

Figure 9. Transverse Velocity for different N values

Figure 10. Density Distribution for varying ωt values
Figure 11. Density Distribution for varying $y$ values

Figure 12. Transverse Velocity for different $\sigma_e$ values