Splitting the split supersymmetry

Kingman Cheung

Department of Physics and NCTS, National Tsing Hua University, Hsinchu, Taiwan, R.O.C.

Cheng-Wei Chiang

Department of Physics, National Central University, Chungli, Taiwan 320, R.O.C. and Institute of Physics, Academia Sinica, Taipei, Taiwan 115, R.O.C.

(Dated: November 26, 2018)

Abstract

In split supersymmetry, the supersymmetric scalar particles are all very heavy, at least at the order of $10^9$ GeV, but the gauginos, Higgsinos, and one of the neutral Higgs bosons remain below a TeV. Here we further split the split supersymmetry by taking the Higgsino mass parameter $\mu$ to be very large. In this case, the $\mu$ problem is avoided and we keep the wino as a dark matter candidate. A crude gauge coupling unification is still preserved. Dark matter signals and collider phenomenology are discussed in this $\mu$-split SUSY scenario. The most interesting dark matter signal is the annihilation into monochromatic photons. In colliders, chargino-pair and the associated chargino-neutralino production cross sections have a certain ratio due to gauge couplings, and the chargino has long decays.

PACS numbers:
Supersymmetry (SUSY) is one of the most elegant solutions, if not the best, to the gauge hierarchy problem. The fine-tuning argument in the gauge hierarchy problem requires SUSY particles at work at the TeV scale to stabilize the gap between the electroweak scale and the grand unified theory (GUT) scale or the Planck scale. The most recent lower bound on the Higgs boson mass has been raised to 114.4 GeV \cite{1}. This in fact puts some stress on the soft SUSY parameters, known as the little hierarchy problem. Since the Higgs boson receives radiative corrections dominated by the top squark loop, the mass bound requires the top squark mass to be heavier than 500 GeV. From the renormalization-group (RG) equation of $M^2_{H_u}$, the magnitude of $M^2_{H_u} \sim M_i^2 \gtrsim (500 \text{ GeV})^2$. Thus, the parameters in the Higgs potential are fine-tuned at a level of a few percent in order to obtain a Higgs boson mass of $O(100) \text{ GeV}$.

Recently, Arkani-Hamed and Dimopoulos adopted a rather radical approach to SUSY \cite{2}. They essentially discarded the hierarchy problem by accepting the fine-tuning solution to the Higgs boson mass. They argued that since the cosmological constant problem needs much more serious fine-tuning that one has to live with, one may as well let go of the much less serious fine-tuning in the gauge hierarchy problem. The only criteria in setting up the scenario are (i) the dark matter constraint imposed by the WMAP data \cite{3}: $\Omega_{\text{DM}} h^2 = 0.094 - 0.129$ (2$\sigma$ range), and (ii) the gauge-coupling unification. The scenario is coined as “split SUSY” \cite{4} with the spectrum specified by the feature of the following distinct scales:

1. All the scalars, except for a CP-even Higgs boson, are very heavy. One usually assumes a common mass scale for them at $\tilde{m} \sim 10^9 \text{ GeV}$ to $M_{\text{GUT}}$.

2. The gaugino masses $M_i$ and the Higgsino mass parameter $\mu$ are comparatively much lighter and of the order of TeV in order to provide an acceptable dark matter candidate.

In this work, we propose a further split in the split SUSY by raising the $\mu$ parameter to a large value which could be about the same as the sfermion mass or the SUSY breaking scale. We call it the $\mu$-split SUSY scenario. In this scenario, we do not encounter the notorious $\mu$ problem \cite{5}. At the same time, our scenario can still achieve the gauge coupling unification and provide a viable dark matter candidate. The gauge coupling unification is satisfied because the RG running of the gauge couplings is mainly determined by the standard model parameters.
(SM) particle and gaugino contributions. Whether the Higgsinos are very heavy has a milder effect. The dark matter constraint requires $M_2$ to be smaller than $M_1$; i.e., the dark matter is wino-like.

We summarize the differences between the split SUSY and our $\mu$-split SUSY scenarios as follows.

1. The Higgsino mass parameter $\mu$ is raised to a very high scale in our scenario while in split SUSY it is at the electroweak scale.

2. The lightest supersymmetric particle (LSP) has to be the wino or gluino instead of the bino in our scenario because the bino would give a too large relic density, whereas the LSP can be the bino with $M_1 \sim \mu$ in split SUSY.

3. The wino dark matter can reach an interesting level of indirect detection, particularly the monochromatic photon signal, due to strong annihilation cross sections, and similarly for the anti-proton and positron detection. On the other hand, the signal for direct detection is vanishingly small because of the absence of light squarks or Higgsino couplings. In split SUSY both the direct and indirect detection signals are present, depending on the nature of the LSP.

4. In our $\mu$-split SUSY scenario, only chargino-pair production ($\tilde{\chi}_1^+ \tilde{\chi}_1^-$) and chargino-neutralino associated production ($\tilde{\chi}_1^\pm \tilde{\chi}_1^0$) are possible at hadron colliders. The cross sections are in a certain ratio in terms of gauge couplings. Moreover, at $e^+e^-$ colliders only the chargino-pair production is possible. In split SUSY all pair production channels are possible.

5. In our $\mu$-split SUSY scenario, charginos can have long decays. Since the mass difference between the chargino and the neutralino can be less than the pion mass, the chargino may travel more than a meter or so before it decays, and therefore producing ionized tracks in central silicon detectors. In split SUSY, the chargino decays promptly in general.

The paper is organized as follows. In the next section, we discuss a few issues in raising the $\mu$ parameter. We discuss the effects on gauge coupling unification in Sec. III, dark matter requirements in Sec. IV, and collider phenomenology in Sec. V. We summarize our findings in Sec. VI.
II. RAISING $\mu$

In the minimal supersymmetric standard model (MSSM) a dimensionful superpotential parameter $\mu$ is associated with the Higgs superfields. A natural choice for the value of this parameter should be either zero or the scale of the ultra-violet (UV) theory, say, the Planck scale, GUT scale, or SUSY breaking scale. Nevertheless, phenomenological analyses give us a different result. Once the electroweak symmetry is broken, the weak scale, characterized by the $Z$ boson mass, is given by the $\mu$ parameter and other soft SUSY breaking parameters:

$$\frac{M_Z^2}{2} = \frac{M_{H_d}^2 - M_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \tag{1}$$

Based upon the naturalness argument, $\mu$ along with other soft SUSY breaking parameters are required to fall in a range near the weak scale. This discrepancy between scales of $\mu$ based on the two different naturalness arguments is the so-called $\mu$ problem \[5\]. It should be emphasized that although the value of $\mu$ has a lower bound set by the chargino mass, its upper bound purely comes from the naturalness requirement as outlined above.

Since in the split SUSY model the fine-tuning of the light Higgs boson mass is accepted, as is the case of the cosmological constant, we then do not need to worry about the possibility of an unnatural cancellation between the two terms on the right-hand side of Eq. (1). Moreover, $\mu$, $M_{H_d}^2$, and $M_{H_u}^2$ are now raised to more natural values, such as the SUSY breaking scale, thus alleviating the $\mu$ problem.

Another issue is in the Higgs potential given by

$$V_{\text{scalar}} = (M_{H_u}^2 + \mu^2)|H_u|^2 + (M_{H_d}^2 + \mu^2)|H_d|^2 + \mu B (\epsilon_{ij} H_u^i H_d^j + \text{h.c.}) + V_D, \tag{2}$$

where $\epsilon_{12} = 1$, $V_D$ is the $D$-term contribution and much smaller than the other terms in both the split SUSY and the $\mu$-split SUSY scenarios. In order to have a light Higgs boson near the electroweak scale, a finely-tuned relation among all three terms is required \[2\]. In the split SUSY, since $\mu$ remains small, $B$ has to be extremely large ($\gg \tilde{m}$) such that $\mu B$ is comparable to $M_{H_u}^2$ and $M_{H_d}^2$ (modulo an extra small factor $\sim 1/\tan \beta$). In the $\mu$-split SUSY, however, $\mu$ is of the order $\tilde{m}$, the value of $B$ can just be of the same order such that $\mu B \sim \tilde{m}^2$, the same order as $M_{H_u}^2$ and $M_{H_d}^2$. In this sense, our $\mu$-split SUSY is better than split SUSY.
Moreover, one minimization condition of the Higgs potential, Eq. (2), gives

$$\sin 2\beta = \frac{2\mu B}{M_{H_u}^2 + M_{H_d}^2 + 2\mu^2}.$$  \hspace{1cm} (3)

As pointed out in Ref. [6], from the above expression split SUSY predicts that $\tan \beta \sim \mathcal{O}(\tilde{m}^2/M_{\text{weak}}^2)$, thereby relating the two scales given the phenomenological constraint $0.5 \lesssim \tan \beta \lesssim 100$. This is because in split SUSY the light gaugino masses are achieved by a softly broken $R$ symmetry. However, such an $R$ symmetry that allows a supersymmetric $\mu$ term forbids a nonvanishing $B$. Therefore, $|B| \sim \mathcal{O}(M_{\text{weak}})$ and gives the above prediction of $\tan \beta$. In our scenario, we have already assumed $\mu \sim \tilde{m}$, which is natural in the context of the $\mu$ problem, and the natural scale for $B$ would be $M_{\text{SUSY}} \sim \tilde{m}$, e.g., in gravity-mediated models.\footnote{In our $\mu$-split SUSY scenario, we only need an $R$ symmetry to forbid the gaugino masses. Then, the $\mu$ parameter naturally takes on $\tilde{m}$ and so does the $B$ parameter.} Thus, $\tan \beta \sim \mathcal{O}(1)$, which fits easily within the phenomenological constraint $0.5 \lesssim \tan \beta \lesssim 100$.

III. GAUGE COUPLING UNIFICATION

The general form of the one-loop RG equations for the gauge couplings are given by

$$\frac{1}{\alpha_i(M_X^2)} = \frac{1}{\alpha_i(M_Z^2)} - \frac{\beta_i}{4\pi} \ln \left(\frac{M_X^2}{M_Z^2}\right),$$  \hspace{1cm} (4)

where $i = 1, 2, 3$ for the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge couplings, respectively. The differences among the SM, MSSM, split SUSY, and $\mu$-split SUSY scenarios lie in the values
of $\beta_i$’s:

$$ SM: \quad (\beta)_{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ 0 \end{pmatrix} F + \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} N_H , $$

$$ MSSM: \quad (\beta)_{MSSM} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} F + \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} N_H , $$

$$ \text{Split-SUSY:} \quad (\beta)_{\text{split}|<\tilde{m}} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ 0 \end{pmatrix} F + \begin{pmatrix} \frac{5}{10} \\ \frac{5}{6} \\ 0 \end{pmatrix} , $$

$$ \mu\text{-split SUSY scenario:} \quad (\beta)_{\mu\text{-split}|<\tilde{m}} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ 0 \end{pmatrix} F + \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} N_H , $$

where $F = 3$ is the number of generations of fermions or sfermions, and $N_H$ is the number of Higgs doublets ($N_H = 1$ in the SM, $N_H = 2$ in the SUSY.) In the evolution of the gauge couplings in our scenario, we use (i) the SM $\beta_i$’s from the weak scale ($M_Z$) to the scale of gaugino masses, which we take a common value of 1 TeV; (ii) the $\beta_i$’s in our $\mu$-split SUSY scenario from the gaugino mass scale to the scalar mass scale ($\tilde{m}$), which we fix it at 10$^9$ GeV; and (iii) the $\beta_i$’s for the MSSM from the scalar mass scale ($\tilde{m}$) to the GUT scale.

In Fig. 1, we compare the scale dependence of the SM gauge couplings in the MSSM, the split SUSY, and our $\mu$-split SUSY scenarios. For simplicity we take a universal value of 1 TeV for all the gaugino masses and a value of 10$^9$ GeV for all the SUSY scalars and Higgsino mass parameter $\mu$ in Fig. 1(a). It is seen that our scenario shares a common feature with the split SUSY; that is, the gauge couplings unify at $\mathcal{O}(10^{16})$ GeV and their unified value, $\alpha_{\text{GUT}}$, is smaller than that in the MSSM. The imperfect unification can be used to account for the discrepancy between the MSSM predicted strong coupling $\alpha_3^{\text{MSSM}}(M_Z) = 0.130 \pm 0.004$, given the input of the experimental values of $\alpha_{1,2}(M_Z)$, and the measured one $\alpha_3^{\text{exp}}(M_Z) = 0.119 \pm 0.002$. Although the triangular area enclosed by the three gauge coupling curves in our scenario is larger than split SUSY, possible threshold effects from sfermions may improve the situation. This is illustrated in Fig. 1(b), where we have separated the masses of the three generations of sfermions into three different scales: 10$^7$ GeV, 10$^8$ GeV, and 10$^9$ GeV.
FIG. 1: Gauge coupling unification in the MSSM (dotted green lines), the split SUSY (dashed red lines), and our $\mu$-split SUSY (dash-dotted blue lines) scenarios. In plot (a), $\mu$ and the Higgsino masses are set the same as the sfermions at $10^9$ GeV. Note that the curves for the strong coupling in split SUSY and our scenario coincide since the beta functions are identical. In plot (b), $\mu$ and the Higgsino masses are fixed at $10^9$ GeV, while the masses of the three generations of sfermions are separated into $10^7$ GeV, $10^8$ GeV, and $10^9$ GeV in our scenario as an example to illustrate possible threshold effects on the unification. Following Ref. [2], we take $\alpha^{-1}_1(M_Z) = 58.98$, $\alpha^{-1}_2(M_Z) = 29.57$, and $\alpha^{-1}_3(M_Z) = 8.40$ in making these plots.

respectively. The two-loop contributions have minor effects on these general behaviors.

IV. DARK MATTER

A. $M_1 < M_{2,3}$

We are left with three gauginos in the TeV regime or less. First of all, the bino cannot be the lightest. It is well-known that the bino annihilation cross section is very small, so its relic density will be too large and overclose the Universe if it is the LSP. Therefore, this
possibility is ruled out.  

**B. \( M_2 < M_{1,3} \)**

In this case, the neutral wino is the LSP and has a large annihilation cross section into \( W \) pairs, \( \tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow W^+W^- \). It constitutes a substantial fraction of the dark matter in the Universe only if the wino mass is of order 2 TeV. The relic density for the wino dark matter in the case of anomaly mediated SUSY breaking \[8, 9\] is given by \[9, 10\]

\[
\Omega_{\tilde{\chi}_0} h^2 \approx 0.05 \left( \frac{M_2 \text{ TeV}}{2} \right)^2.
\]

If the effect of coannihilation from the charged wino is included, the coefficient in the above equation will be further reduced. Therefore, if the wino is the dominant dark matter component, its mass has to be of order 2 TeV. However, there is also the possibility that the wino comes from other non-thermal sources \[11\].

**C. \( M_3 < M_{1,2} \)**

One should also entertain the option that the gluino is the LSP, which has been discussed in the literatures \[13, 14\]. The gluino can hadronize into an \( R \)-hadron. If the lightest \( R \)-hadron is electrically neutral and its mass is of order 2 – 3 TeV, it can also form a major component of the dark matter in the Universe \[13\]. This is based on the assumption that the effective annihilation of the \( R \)-hadron is due to the annihilation of its internal gluino with the gluino from another \( R \)-hadron. Since the annihilation of gluinos is via strong interactions, the annihilation rate is typically large and therefore requires a gluino mass of order 2 – 3 TeV in order to be a dark matter. Also, the gluino LSP with a mass of 2 TeV or more is safe from the search for strongly interacting massive particles in anomalously heavy nuclei \[15\].

\[2\] There is a slight complication \[4\] when \( \mu \) is not too large, e.g., 10 TeV, and \( M_1 \) is close to \( M_2 \) such that the LSP has a non-negligible fraction of wino and the LSP mass is close to the next-to-lightest neutralino and the light chargino. In such a situation, the LSP can still annihilate efficiently to give the correct relic density. However, in our \( \mu \)-split-SUSY scenario, \( \mu \sim 10^9 \text{ GeV} \) and thus \( M_1 \) has to be extremely close to \( M_2 \) (they differ by \( \lesssim 10^{-7} \)) in order to have an effect.
D. Other non-thermal sources

Since the wino needs to be very heavy (∼2 TeV) to be the dominant dark matter, the whole SUSY spectrum can only be marginally produced at the LHC. The collider phenomenology will only be limited to a very small corner provided by a ∼2 − 3 TeV gluino. The production rate is not high enough for a good study.

On the other hand, there can be other non-thermal sources of dark matter. For example, in the context of anomaly-mediated SUSY breaking models [8, 9] the LSP is usually the neutral wino. For a relatively light neutral wino it cannot be the dominant dark matter because of its large annihilation cross sections. However, an intriguing source of non-thermal wino for compensation is the decay of moduli fields [11], which can produce a sufficient amount of neutral winos. In this case, even a light neutral wino can constitute a major fraction of the dark matter. There are also other non-thermal sources of wino dark matter discussed in the literatures [12].

E. Dark matter detection

Pure wino dark matter, as explained above, can come from both thermal and non-thermal sources, with the latter source possibly being dominant. The wino dark matter has a very interesting signal for indirect detection in view of its large annihilation cross sections, e.g., \( \tilde{\chi}^0 \tilde{\chi}^0 \rightarrow W^+ W^-, \gamma \gamma, \gamma Z \). In particular, the last two channels, though involving loop-suppressed cross sections, can give a very clean signal of monochromatic photon lines. If the resolution of the photon detectors (either ground-based or satellite-based) is high enough, a clean, sharp, unambiguous photon peak at hundreds of GeV should be observed above the background.

Here we give an estimate on the photon flux in our \( \mu \)-split SUSY scenario. We use the results given in Ref. [16]. In the limit of pure wino and very heavy sfermions, only the \( W^- \tilde{\chi}_1^+ \) loop is important. We obtain

\[
\nu \sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \gamma \gamma) \approx 14 \times 10^{-28} \text{ cm}^3\text{s}^{-1}, \quad \text{for } M_{\tilde{\chi}_1^0} = 0.5 - 2 \text{ TeV}. \tag{6}
\]

Note that for comparison purposes, the value of \( \nu \sigma \) for a pure Higgsino case is about 1 ×
The photon flux as a result of this annihilation is given by

\[ \Phi_\gamma \simeq 1.87 \times 10^{-11} \left( \frac{N_\gamma v\sigma}{10^{-29} \text{cm}^3\text{s}^{-1}} \right) \left( \frac{10 \text{GeV}}{M_{\tilde{\chi}_1^0}} \right)^2 J(\psi) \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1} \]

\[ \simeq 2 \times 10^{-10} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}, \]  

(7)

where we have used \( v\sigma = 14 \times 10^{-28} \text{ cm}^3\text{s}^{-1} \), \( M_{\tilde{\chi}_1^0} = 500 \text{ GeV} \), \( N_\gamma = 2 \), and \( J(\psi = 0) = 100 \) for the photon flux coming from the Galactic Center. The value of \( J(\psi) \) depends on the selected Galactic halo model. It ranges from \( O(10) \) to \( O(1000) \) [17]. For a typical Atmospheric Cerenkov Telescope (ACT) such as VERITAS [18] and HESS [19], the angular coverage is about \( \Delta \Omega = 10^{-3} \) and may reach a sensitivity at the level of \( 10^{-14} - 10^{-13} \text{ cm}^{-2}\text{s}^{-1} \) at the TeV scale. Therefore, the signal of pure wino dark matter annihilating into monochromatic photons is easily covered by the next generation ACT experiments.

Since the wino annihilation into the \( W^+W^- \) pair is very effective, one can also measure the excess in anti-protons and positrons [20], which can be measured in anti-matter search experiments, e.g., AMS-II [21]. We end this section by noting that the direct search signal for our \( \mu \)-split SUSY scenario is very difficult because of the absence of light squarks or the Higgsino components in the lightest neutralino.

V. COLLIDER PHENOMENOLOGY

The collider phenomenology is mainly concerned with the production and detection of neutralinos, charginos, and gluinos. We restrict our discussions below to cases with exact or approximate \( R \)-parity symmetry as follows.

A. Neutralinos and Charginos

Since the Higgsino mass parameter \( \mu \) is very large, only the first two neutralinos and the first chargino are light. We will concentrate on their phenomenology in this section. Let us first examine their relevant couplings to gauge bosons and the Higgs bosons.

- The \( Z-\tilde{\chi}_{1,2}^0\tilde{\chi}_{1,2}^0 \) couplings only receive contributions from the Higgsino-Higgsino-gauge couplings. In the limit of very large \( \mu \) the Higgsino component of \( \tilde{\chi}_1^0 \) and \( \tilde{\chi}_2^0 \) are essentially zero. Therefore, they are zero.
• The $H-\tilde{\chi}_{1,2}^0\tilde{\chi}_{1,2}^0$ couplings have sources from the Higgs-Higgsino-gaugino terms. Therefore, in the limit of large $\mu$, these couplings are also zero.

• The $W^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$ couplings have sources from the Higgsino-Higgsino-gauge couplings and from the gaugino-gaugino-gauge couplings. In the limit of large $\mu$, the former contribution goes to zero while the latter remains. Therefore, the $W^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$ couplings contain only the gaugino-gaugino-gauge part. Since only the wino component couples to the $W$ boson, thus only $W^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$ is nonzero if $M_2 < M_1$, and vice versa.

• The $H^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$ couplings have sources from the Higgs-Higgsino-gaugino couplings. In the limit of large $\mu$, they do not contribute to $H^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$, which thus vanishes.

• The $\gamma(Z)-\tilde{\chi}_1^+\tilde{\chi}_1^-$ coupling has sources from the Higgsino-Higgsino-gauge couplings and from the gaugino-gaugino-gauge couplings. In the limit of large $\mu$, the former contribution goes to zero while the latter remains. Therefore, the $\gamma(Z)-\tilde{\chi}_1^+\tilde{\chi}_1^-$ coupling contains only the gaugino-gaugino-gauge part.

• The $H-\tilde{\chi}_1^+\tilde{\chi}_1^-$ couplings have sources from the Higgs-Higgsino-gaugino couplings. In the limit of large $\mu$, they do not contribute to $H^-\tilde{\chi}_1^+\tilde{\chi}_1^-$, which thus vanishes.

The underlined are the only couplings that survive in the limit of large $\mu$ and large sfermion masses. The phenomenology of the two light neutralinos and the lightest chargino depends on the above nonzero couplings.

The parameter space of the SUSY spectrum relevant for phenomenology consists of the bino ($M_1$), wino ($M_2$), and gluino ($M_3$) masses. In the rest of the paper, we only discuss the case of $M_2 < M_1$ because, as explain in the last section, the bino LSP would overclose the Universe.

As have discussed above, under the assumptions of large sfermion masses, large $\mu$, and $M_2 < M_1$, the only sizable couplings to a pair of neutralinos and charginos are

$$\gamma(Z)\tilde{\chi}_1^+\tilde{\chi}_1^- \quad W^-\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+.$$ 

Note that the second lightest neutralino is almost a bino and thus has no coupling to the $W$ boson. In view of these couplings, the only noticeable pair production channels at hadronic colliders are $\tilde{\chi}_1^+\tilde{\chi}_1^-$ and $\tilde{\chi}_{1,2}^0\tilde{\chi}_1^+$. We show the production cross sections of these channels versus $M_2$ (at the weak scale) in Fig. 2. Note that in the scenario with very large $\mu$ and $M_2 < M_1$
the lightest neutralino and chargino have masses very close to $M_2$. In fact, the conventional $U$ and $V$ matrices (that diagonalize the chargino mass-squared matrix) are unit matrices. Therefore, their masses are simply equal to $M_2$ before radiative corrections are taken into account. Radiative corrections lift the mass degeneracy and make the neutral wino lighter than the charged wino \[22\]. Note also that the production cross sections shown in Fig. 2 are independent of $\tan \beta$. This very special scenario can then be checked by comparing gaugino-pair production cross sections, shown in Fig. 2 because they are simply given by the gauge couplings.

In $e^+e^-$ linear colliders, it is not possible to produce neutralino pairs because of the absence of the Higgsino couplings. Instead, only the chargino pair production is possible. This is another interesting check on our $\mu$-split SUSY scenario. In contrast, both neutralino-pair and chargino-pair production are possible in the split SUSY scenario.

Another technical issue is to detect the chargino in either chargino-pair production or the associated production with the lightest neutralino. The decay time of the chargino into the neutralino and a virtual $W$ boson depends critically on the mass difference $\Delta M \equiv M_\chi^+ - M_\chi^0$. The phenomenology in this case had been studied in great details in Ref. \[23\]. We give highlights in the following paragraph.

The partial width of $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 f \bar{f}'$ is given by \[23\]

\[
\Gamma(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 f \bar{f}') = \frac{N_c(f)G_F}{(2\pi)^3} \left[ M_\chi^+ \left( (O^L_{11})^2 + (O^R_{11})^2 \right) \right. \\
\times \int_{(M_\chi^0 + m_f)^2}^{M_\chi^+_f} dq^2 \left( 1 - \frac{M_\chi^0 + m_f^2}{q^2} \right)^2 \left( 1 - \frac{q^2}{M_\chi^+_f} \right)^2 \sqrt{\lambda(q^2, M_\chi^0, m_f^2)} \\
- 2M_\chi^0 O^L_{11} O^R_{11} \int_{m_f^2}^{(M_\chi^0 - M_\chi^+_f)^2} M_\chi^+_f \left( 1 - \frac{m_f^2}{q^2} \right)^2 \sqrt{\lambda(M_\chi^+_f, M_\chi^0, q^2)} \right],
\]

where $(f, f')$ is, for example, $(u, d)$ or $(e, \nu_e)$, $N_c = 3(1)$ if $f$ is a quark (lepton), and $\lambda(a, b, c) = (a + b - c)^2 - 4ab$. The above formula is valid for (i) leptonic decays, and (ii) hadronic decays when $\Delta M \gtrsim 2$ GeV. For hadronic decays with $\Delta M \lesssim 1 - 2$ GeV, one has to explicitly sum over exclusive hadronic final states. We have to include the partial widths into one, two, and three pions. The explicit formulas can be found in Ref. \[23\]. In our $\mu$-split SUSY case, $O^L_{11} = O^R_{11} = 1$. The detection of the chargino depends on the magnitude of $\Delta M$.
FIG. 2: Production cross sections versus $M_2$ (the wino mass at the weak scale) for the $\tilde{\chi}_1^+ \tilde{\chi}_1^-$, $\tilde{\chi}_1^0 \tilde{\chi}_1^+$, and $\tilde{\chi}_1^0 \tilde{\chi}_1^-$ channels at (a) the LHC and (b) the Tevatron. Note that at the Tevatron the production cross sections of $\tilde{\chi}_1^0 \tilde{\chi}_1^+$ and $\tilde{\chi}_1^0 \tilde{\chi}_1^-$ are the same.
1. $\Delta M < m_\pi$. In this case, the only available decay modes are $e^+\nu_e$ and $\mu^+\nu_\mu$. We use Eq. (8) to estimate the decay width to be $\mathcal{O}(10^{-7})$ eV or less. Therefore, the chargino will travel a distance of the order of a meter or longer before decaying, and will leave a heavily-ionized track in the vertex detector. The leptons coming out are too soft to be detectable. This signal of ionized tracks is essentially SM background-free.

2. $m_\pi < \Delta M < 1$ GeV. This is the most difficult regime, and very much depends on the design of the central detector. The important criteria are the decay length $c\tau$ of the chargino and the momentum of the pion from the chargino decay. The decay length $c\tau$ may be long enough to travel through a few layers of the silicon vertex detector. For example, if $m_\pi < \Delta M < 190$ MeV the chargino will typically pass through at least two layers of silicon chips [23]. Since the pion is derived from the chargino decay, it is a non-pointing pion. That is, the backward extrapolation of the pion track does not lead to the interaction point. The resolution on the impact parameter $b_{\text{res}}$ depends on the momentum of the pion $p_\pi \sim \sqrt{(\Delta M)^2 - m_\pi^2}$ in the chargino rest frame. The higher the momentum is, the better the resolution $b_{\text{res}}$ will be [23]. Thus, detecting the signal involves the combination of detecting a track left in only a few layers of the silicon and identifying a nonzero impact parameter of the pion coming out of the chargino. A detailed simulation is beyond the scope of the present paper.

3. $\Delta M \gtrsim 1-2$ GeV. We can use Eq. (8) to estimate the total decay width of the chargino. The decay width is large enough that it decays promptly, producing soft leptons, pions, or jets, plus large missing energies. The problem is on the softness of the leptons, pions, or jets. Experimentally, their detection is difficult. Only if $\Delta M$ is sufficiently large to produce hard enough leptons or jets can the chargino decay be detected. Otherwise, one has to rely on some other methods, such as $e^+e^- \rightarrow \gamma\tilde{\chi}_1^+\tilde{\chi}_1^- \rightarrow \gamma + E_T$, a single photon plus large missing energy above the SM background $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ [23]. Unfortunately, the signal rate is $\mathcal{O}(\alpha_{\text{em}})$ smaller than the chargino-pair production. Such a method of detecting the single photon plus missing energy is more difficult at hadronic colliders.

In summary, the detection of the chargino is easier when $\Delta M$ is very small ($< m_\pi$) or when $\Delta M$ is large ($> a \text{ few GeV}$). The former gives charged tracks in the silicon detector whereas the latter gives prompt leptons or jets plus missing energies. The intermediate
range presents a challenge to experiments. The questions are how many layers of silicon that the chargino can travel and how well the resolution the non-pointing pion can be.

In the case of pure wino LSP, the lightest neutralino and chargino are essentially degenerate in mass at the tree level. The radiative corrections can lift the mass degeneracy. In our $\mu$-split SUSY scenario, the mass difference is due to the radiative corrections of the gauge boson loops [24], given by

$$\delta m_{\text{rad}} = \frac{\alpha_{\text{em}} M_{\tilde{\chi}_1^0}}{4\pi s_W^2} \left[ f\left(\frac{m_W}{M_{\tilde{\chi}_1^0}}\right) - c_W^2 f\left(\frac{m_Z}{M_{\tilde{\chi}_1^0}}\right) - s_W^2 f(0) \right], \quad (9)$$

where $s_W$ and $c_W$ are the sine and cosine of the Weinberg angle, and $f(a) = \int_0^1 dx \frac{2}{1+x} \log(x^2 + (1-x)a^2)$. Numerically, we obtain $\delta m_{\text{rad}} \approx 170$ MeV for $M_{\tilde{\chi}_1^0} = 0.5 - 2$ TeV. Therefore, $\Delta M$ may fall into the very difficult regime. However, there may be higher order corrections or effects from other sources that can further increase or decrease $\Delta M$.

Note that when $\tilde{m}$ and $\mu$ are of $O(10^9)$ GeV or above, the gluino will not decay within the detector, and the second lightest neutralino $\tilde{\chi}_2^0$ cannot be produced in the collisions. Thus, there is no production of the second lightest neutralino in colliders in such an extreme scenario. However, if $\tilde{m}$ is of order $10^6$ GeV or less, the gluino can decay within the detector. It will produce the second lightest neutralino, which in turn decays into the lightest neutralino via an intermediate squark or slepton. Therefore, the decay time is long. We will not go further into this low $\tilde{m}$ case.

### B. Gluino

In our $\mu$-split SUSY scenario, the behavior of the gluino is the same as in the split SUSY scenario. Once produced, the gluino is stable inside the detector or longer. The signature of the gluino as the $R$-hadrons has been studied in a number of papers [13, 14, 25, 26]. Essentially, once the gluino is produced it hadronizes into an $R$-hadron by combining with light quarks or a gluon. When the $R$-hadron traverses through the detector, it will lose energy to the detector material, thus producing the signal. The detectability depends on the $R$-hadron spectrum and their electric charges. In fact, it involves large uncertainties because the $R$-hadron spectrum is not clearly known. Also, the event rate has a large range depending on whether the $R$-hadron is electrically charged [27]. Further complication arises from the fact that there can be frequent swappings between various states of the $R$-hadron.
when it collides with nucleons of the detector material. Another possibly clean signature was proposed to detect the gluino-gluino bound state called the gluinonium \[27\]. Since the gluino is stable, a pair of gluinos can exchange gluons between themselves to form a bound state. The gluinonium can then annihilate into a pair of heavy top or bottom quarks, experimentally forming a sharp peak in the invariant mass spectrum of \(M_{t\bar{t}}\) or \(M_{b\bar{b}}\), provided the background can be efficiently suppressed \[27\].

VI. CONCLUSIONS

In the present paper we have considered an even more radical scenario than split SUSY. We call it \(\mu\)-split SUSY, characterized by the assumption that the \(\mu\) parameter is raised together with the scalar sfermion to the high SUSY breaking scale. The Higgsino masses are lifted accordingly. The main motivation is to solve the \(\mu\) problem. We have investigated the effects on gauge coupling unification, dark matter constraints, and collider phenomenology.

We have found that the gauge coupling unification is slightly worse than the split SUSY scenario mainly because of the change in the running of \(\alpha_2\). Nevertheless, the effect is mild. On the other hand, there is a rather big change in the dark matter requirement. Since the LSP does not have the Higgsino component any more, the LSP cannot be the bino because otherwise the relic density would be too large. The only sensible LSP is then the wino, with the wino dark matter receiving contributions from both thermal and non-thermal sources such that its mass can be less than one TeV. The wino dark matter has a large annihilation cross section, and the annihilation into two photons is expected to give rise to a very clean and sharp monochromatic photon line. The flux is well above the sensitivity of the future ACT experiments.

The collider phenomenology is also quite different from the usual MSSM and split SUSY. The only possible gaugino production channels are the gluino pair, chargino pair, and associated chargino-neutralino pair. The behavior of the gluino will be the same as in split SUSY, i.e., a long-lived gluino. However, the chargino-pair and the associated chargino-neutralino production channels are very different. The only production channels at hadron colliders are \(\tilde{\chi}_1^+ \tilde{\chi}_1^-\) and \(\tilde{\chi}_1^+ \tilde{\chi}_1^0\), whereas at \(e^+e^-\) colliders only \(\tilde{\chi}_1^+ \tilde{\chi}_1^-\) is possible. The production cross sections are in a certain ratio depending on the mass \(M_2\). No such behavior is seen in the split SUSY scenario. Furthermore, in our \(\mu\)-split SUSY scenario the chargino has a long
decay, which again is very different from the split SUSY. The differences are already detailed in the Introduction.

Acknowledgments

We thank Mihoko Nojiri and Tatsuo Kobayashi for discussions. K. C. would like to thank YITP and KEK for their hospitality where part of the work was done. C.-W. C. is grateful to the hospitality of National Center for Theoretical Sciences, Taiwan, during his visit when part of the work was done. This research was supported in part by the National Science Council of Taiwan R. O. C. under Grant Nos. NSC 93-2112-M-007-025- and NSC 93-2119-M-008-028-.

[1] R. Barate et al. [ALEPH Collaboration], Phys. Lett. B 565, 61 (2003) [arXiv:hep-ex/0306033].
[2] N. Arkani-Hamed and S. Dimopoulos, arXiv:hep-th/0405159.
[3] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207]; D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].
[4] G. F. Giudice and A. Romanino, Nucl. Phys. B 699, 65 (2004) [Erratum-ibid. B 706, 65 (2005)] [arXiv:hep-ph/0406088].
[5] J. E. Kim and H. P. Nilles, Phys. Lett. B 138, 150 (1984); Y. Nir, Phys. Lett. B 354, 107 (1995) [arXiv:hep-ph/9504312]; M. Cvetic and P. Langacker, Phys. Rev. D 54, 3570 (1996) [arXiv:hep-ph/9511378].
[6] M. Drees, arXiv:hep-ph/0501106.
[7] A. Masiero, S. Profumo and P. Ullio, arXiv:hep-ph/0412058.
[8] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [arXiv:hep-th/9810155].
[9] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998) [arXiv:hep-ph/9810442].
[10] A. Pierce, Phys. Rev. D 70, 075006 (2004) [arXiv:hep-ph/0406144].
[11] T. Moroi and L. Randall, Nucl. Phys. B 570, 455 (2000) [arXiv:hep-ph/9906527].
[12] S. Profumo and P. Ullio, JCAP 0311, 006 (2003) [arXiv:hep-ph/0309220]; R. Catena,
N. Fornengo, A. Masiero, M. Pietroni and F. Rosati, Phys. Rev. D 70, 063519 (2004) arXiv:astro-ph/0403614.

[13] H. Baer, K. m. Cheung and J. F. Gunion, Phys. Rev. D 59, 075002 (1999) arXiv:hep-ph/9806361.

[14] S. Raby and K. Tobe, Nucl. Phys. B 539, 3 (1999) arXiv:hep-ph/9807281.

[15] R. N. Mohapatra, F. I. Olness, R. Stroynowski and V. L. Teplitz, Phys. Rev. D 60, 115013 (1999) arXiv:hep-ph/9906421.

[16] L. Bergstrom and P. Ullio, Nucl. Phys. B 504, 27 (1997) arXiv:hep-ph/9706232.

[17] L. Bergstrom, P. Ullio and J. H. Buckley, Astropart. Phys. 9, 137 (1998) arXiv:astro-ph/9712318.

[18] T. C. Weekes et al., arXiv:astro-ph/9706143.

[19] J. A. Hinton [The HESS Collaboration], New Astron. Rev. 48, 331 (2004) arXiv:astro-ph/0403052.

[20] S. Profumo and P. Ullio, JCAP 0407, 006 (2004) arXiv:hep-ph/0406018.

[21] M. Aguilar et al. [AMS Collaboration], Phys. Rept. 366, 331 (2002) [Erratum-ibid. 380, 97 (2003)].

[22] S. Mizuta, D. Ng and M. Yamaguchi, Phys. Lett. B 300, 96 (1993) arXiv:hep-ph/9210241; D. Pierce and A. Papadopoulos, Nucl. Phys. B 430, 278 (1994) arXiv:hep-ph/9403240.

[23] C. H. Chen, M. Drees and J. F. Gunion, Phys. Rev. D 55, 330 (1997), Erratum-ibid. D 60, 039901 (1999) arXiv:hep-ph/9607421; Phys. Rev. Lett. 76, 2002 (1996) arXiv:hep-ph/9512230; arXiv:hep-ph/9902309.

[24] J. Hisano, S. Matsumoto, M. M. Nojiri and O. Saito, arXiv:hep-ph/0412403 H. C. Cheng, B. A. Dobrescu and K. T. Matchev, Nucl. Phys. B 543, 47 (1999) arXiv:hep-ph/9811316.

[25] W. Kilian, T. Plehn, P. Richardson and E. Schmidt, arXiv:hep-ph/0408088.

[26] J. L. Hewett, B. Lillie, M. Masip and T. G. Rizzo, JHEP 0409, 070 (2004) arXiv:hep-ph/0408248.

[27] K. Cheung and W. Y. Keung, arXiv:hep-ph/0408335.