Distribution Network Low Voltage Prediction Method Based on Least Squares Support Vector Machine

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Abstract. Due to the lack of a direct functional relationship between voltage and voltage related parameters in the distribution network, it is difficult to scientifically carry out low voltage prediction. Therefore, a low voltage prediction method based on least squares support vector machine is proposed. The method uses the parameters related to low voltage and voltage in the distribution network as the basic data, constructs the optimal problem equation, forms the decision function, uses the decision function to predict the predicted samples, and outputs the lowest voltage value at the end of the low voltage side of the distribution transformer. Finally, the voltage value is analyzed and the low voltage of the distribution network is predicted. The actual distribution network data is used to simulate and verify the proposed method. The results show that the predicted value and the actual value error meet the requirements of estimation accuracy. It is demonstrated that the low-voltage prediction method based on least squares support vector machine is practical.

1. Introduction

In many areas of China, the distribution network is too long, the power line end loss is too high, the power load is dispersed, the distribution transformer power supply capacity is insufficient, and the power line grid structure is weak, which causes the voltage of the distribution network to be low. In recent years, with the improvement of the living standards of residents, the gradual use of large-capacity electrical appliances, the power load is rapidly increased, and the voltage value at the end of the power line is further reduced. If this situation is not controlled, the voltage qualification rate is far below the national demand value, and it is difficult for the power supply enterprise to meet the needs of the people for high-quality electricity use.

In paper [1], the BP neural network method is used to estimate the voltage of all nodes using the voltage data of a few nodes, which solves the problem of difficult collection of operating parameters in the distribution area. The paper [2] proposed the concept of load moment margin, which can provide theoretical basis for the increase of load in the distribution area and the determination of distribution transformer capacity. However, the problem of load access caused by factors such as power line diameter and power line length is not considered. In paper [3], a method for assessing the health status of low-voltage distribution network based on order-entropy-weight method is proposed. In paper [4], a low-voltage prediction model based on self-organizing competitive neural network is
trained to realize automatic clustering of low-voltage risk. The papers [5-8] uses the artificial intelligence algorithm to estimate the line voltage to achieve the purpose of early warning of low voltage. In summary, the domestic work on the low voltage prediction of the distribution network area has done a lot of work, but most of them focus on theoretical research and model derivation. The case analysis is still relatively small, and most of the research does not fully consider the main factors affecting the voltage. The accuracy of voltage estimation is difficult to guarantee.

On the basis of summarizing and analyzing the papers in recent years, this paper analyzes the main reasons leading to the low voltage in the distribution area, and carries out research on low voltage prediction of distribution network, and proposed A low voltage prediction method for distribution network based on least squares support vector machine (LS-SVM). The voltage-related parameters of the distribution network area, such as the distribution capacity, the lowest voltage at the end of the power supply line, the longest power supply radius on the low-voltage side, the longest trunk line diameter on the low-voltage side, and the total number of users of the distribution transformer are used as the basic data. The vector machine prediction method is to use these parameters to construct the optimal problem equation and form the decision function, use the decision function to predict the prediction sample, calculate the lowest voltage value at the end of the power line, and complete the target of predicting and analyzing the low voltage of the distribution network. The case analysis of the distribution area data of a distribution area in Guangxi is carried out to verify the effectiveness of the method.

2. Causes of low voltage
There is resistance and reactance in the transmission line of the distribution network. When the power is transmitted on the transmission line, the voltage of the line is divided, so that there is a voltage difference at the first end, resulting in a decrease in the voltage at the end of the line. The equivalent circuit of the low-voltage line of the distribution network is shown in Fig. 1:

![Figure 1. Transmission line equivalent circuit and vector diagram.](image)

It can be seen from Fig. 1 that the starting voltage $\hat{U}_1$ and the terminal voltage $\hat{U}_2$ of the line have the following relationship:

$$\triangle \hat{U}_1 = \hat{U}_1 - \hat{U}_2 = (R + jX) \hat{I} \quad (1)$$

The voltage drop $\triangle U_i$ of the equation (1) is split into the projection $\mathbb{V} U_i$ in the vertical direction of $\hat{U}_2$ and the projection $\mathbb{V} U_2$ in the same direction as $\hat{U}_2$, that is, the transverse component and the longitudinal component of $F$, and the formula is:

$$\triangle U_2 = R I \cos \phi + X I \sin \phi \quad (2)$$

$$\triangle U_h = X I \cos \phi - R I \sin \phi \quad (3)$$

Assume that the end power is:
The power representations $V_{U_2}$ and $V_{U_2}$ are:

\[
\begin{align*}
\triangle U_2 &= \frac{PR + QX}{U_2} \\
\triangle U_{U_2} &= \frac{PR - QR}{U_2}
\end{align*}
\]  

(5)

It can be known from equation (5) that the voltage loss is related to the grid component parameters. The smaller the power line diameter, the longer the power supply distance, and the greater the power transmitted, the greater the voltage loss generated. In this paper, the distribution transformer capacity, the longest power supply radius, the longest trunk line diameter, and the total number of users of the distribution transformer are taken as the main factors affecting the voltage level at the end of the distribution line.

3. Principle of least squares support vector machine

3.1. Overview of least squares support vector machine

The least square support vector machine is a machine learning method developed on the basis of statistical learning theory, and has a good effect in solving the problem of complex parameter relationships in power systems. It is used to integrate the relationship between the parameters related to the voltage of the distribution network in this paper, and has a very good effect on predicting the voltage at the end of the distribution line. LS-SVM has many unique advantages in solving small samples, high dimensionality, nonlinearity, local minimum values, etc, especially reducing computational complexity and speeding up the solution, so it has good application effect.

3.2. Principle of least squares support vector machine

3.2.1. Definition of optimization function. Since the distribution network voltage prediction method in this paper is a nonlinear system, consider the linear regression function:

\[ f(x) = (\omega, \phi(x)) + b \]  

(6)

Set a set of data points $(x_i, y_i), i=1,\ldots, l, x_i \in \mathbb{R}^d$ is related to the minimum voltage at the end of the low-voltage side of the forecasting station. Such as the distribution capacity, the lowest voltage at the end of the line on the low-voltage side, the longest power supply radius on the low-voltage side, the longest main line diameter on the low-voltage side, and the total number of users in the distribution transformer. $d$ is the dimension of the selected input variable, $y_i \in \mathbb{R}$ is the expected value of the forecast, $l$ is the total number of known data points. $\Phi(x)$ is a nonlinear mapping from the input space to the high dimensional feature space. According to the structure minimization principle, the LS-SVM optimization target can be expressed as:

\[
\min \frac{1}{2} \|\omega\|^2 + \frac{1}{2} \gamma \sum_{i=1}^l e_i^2 
\]

(7)

s.t. $\omega^T \phi(x_i) + b + e_i = y_i, i = 1, \ldots, l$
Among them, $e_i$ is the error, $e \in \mathbb{R}^l$ is the error vector, $\gamma$ is the regularization parameter, and the degree of punishment for the error is controlled.

### 3.2.2. Definition of the Lagrange function.

Introducing Lagrange multipliers, $\lambda \in \mathbb{R}^l$, Equation (7) can be converted into:

$$
\min J = \frac{1}{2} \|e\|^2 + \frac{1}{2} \gamma \sum_{i=1}^l e_i^2 - \sum_{i=1}^l \lambda_i (\phi(x_i)^T \phi(x_i) + b + e_i - y_i)
$$

By KKT conditions, you get:

\[
\begin{align*}
\frac{\partial J}{\partial \omega} &= 0 \rightarrow \sum_{i=1}^l \lambda_i \varphi(x_i) \\
\frac{\partial J}{\partial b} &= 0 \rightarrow \sum_{i=1}^l \lambda_i = 0 \\
\frac{\partial J}{\partial e_i} &= 0 \rightarrow \lambda_i = \gamma e_i, i = 1, 2, \ldots, l \\
\frac{\partial J}{\partial \lambda_i} &= 0 \rightarrow \omega^T \varphi(x_i) + b + e_i - y_i = 0, i = 1, 2, \ldots, l
\end{align*}
\]

After eliminating and, the solution of equation (9) is:

\[
\begin{bmatrix}
0 \\
I^T \\
\Omega + \gamma I
\end{bmatrix}
\begin{bmatrix}
b \\
Y
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
Y
\end{bmatrix}
\]

among them, $\mathcal{T} = [\lambda_1, \lambda_2, \ldots, \lambda_l]^T, \mathcal{T} = [1, 1, \ldots, 1]^T$ is Dimension vector, $Y = [y_1, y_2, \ldots, y_l]^T, \Omega \in \mathbb{R}^{l^2},$ and $\Omega_y = \phi(x_i)^T \phi(x_i), \Omega_y = K(x_i, x_j), K$ is a kernel function that satisfies the Mercer condition, $K(x_i, x_j) = \phi(x_i)^T \phi(x_j)$ replace the dot product operation in the high dimensional feature space with the kernel function of the original space, simplifying the calculation.

### 3.2.3. Nonlinear prediction model.

Therefore, the expression of the nonlinear prediction model is:

$$
y = \sum_{i=1}^l \lambda_i K(x_i, x) + b
$$

Among them, $\lambda, b$ can be obtained by solving the linear equation of equation (10), and $K(\cdot, \cdot)$ is a nonlinear mapping from input space to high-dimensional feature space.

### 4. Implementation steps

#### 4.1. Principle of least squares support vector machine

#### 4.1.1. The basic data of the distribution network area in a certain area is taken from the database, including the distribution capacity, the lowest voltage at the end of the line on the low-voltage side of the transformer, the longest power supply radius on the low-voltage side, the longest main line
diameter on the low-voltage side of the distribution transformer, and the total number of users of the
distribution transformer are used as training data, the training data set is as shown in equation (12).

\[ S = \{(x_1, y_1), \ldots, (x_t, y_t)\} \subset \mathbb{R}^n \times \mathbb{R} \]  
(12)

4.1.2. Choosing the appropriate kernel function, because the radial basis function representation is
simple and well interpreted, the radial basis function is used as the kernel function in the regression
model, as shown in equation (13).

\[ K(x, x_i) = \exp\left(-\frac{\|x - x_i\|^2}{\sigma^2}\right) \]  
(13)

where: \( x \) is the \( m \)-dimensional input vector, \( x_i \) is the center of the \( i \)th radial basis function, and has
the same dimension as \( x \), \( \sigma \) is the normalized parameter, which determines the width of the function
around the center point, according to experience=2, \( \|x - x_i\| \) is the norm of the vector \( x - x_i \),
indicating the distance between the \( x \) and the \( x_i \).

4.1.3. Construct and solve the optimization problem formula, get the optimal solution
\( \alpha = (\alpha_1, \alpha_1^*, \ldots, \alpha_t, \alpha_t^*)^T \), construct the decision function, as shown in equation (14):

\[ f(x) = \sum_{i=1}^{t} (\alpha_i^* - \alpha_i) K(x_i, x) + b \]  
(14)

4.1.4. The prediction function is used to predict the predicted sample, and the lowest voltage at the
end of the low-voltage side of the output area is output, and the minimum voltage prediction at the
end of the station is completed.

4.2. Verification case

4.2.1. Forecast data. Taking the data of the distribution network in a certain area of Guangxi to
predict the minimum voltage at the end of the station, the simulation environment is MALAB2008a.
After the training sample data is formed into a training set and the construction of the decision
function is completed, the prediction function data of Table 1 can be calculated using the decision
function.

**Table 1.** Prediction sample data.

| Distribution number | Distribution capacity (kVA) | Distribution longest power supply radius | Distribution longest trunk line diameter (²) | Total number of customers |
|---------------------|-----------------------------|------------------------------------------|---------------------------------------------|--------------------------|
| 1                   | 30                          | 350                                      | 25                                         | 113                      |
| 2                   | 80                          | 400                                      | 50                                         | 160                      |
| 3                   | 125                         | 780                                      | 70                                         | 110                      |
| 4                   | 80                          | 420                                      | 35                                         | 147                      |
| 5                   | 160                         | 968                                      | 35                                         | 87                       |
| 6                   | 100                         | 487                                      | 25                                         | 158                      |
| 7                   | 100                         | 670                                      | 35                                         | 110                      |
| 8                   | 80                          | 690                                      | 50                                         | 138                      |
| 9                   | 80                          | 500                                      | 50                                         | 138                      |
| 10                  | 160                         | 360                                      | 25                                         | 211                      |
4.2.2. Prediction result. The prediction function predicts the lowest voltage at the end of the distribution, and the predicted result is compared with the actual value of the actual distribution end data. The prediction results of the lowest voltage at the end of the low-voltage side of the distribution transformer are shown in Table 2:

| Distribution number | Actual value (V) | Predictive value (V) | Predictive value (V) | Relative error % |
|---------------------|------------------|----------------------|----------------------|------------------|
| 1                   | 183              | 180.59               | 2.41                 | 1.315            |
| 2                   | 189              | 190.04               | -1.04                | 0.552            |
| 3                   | 188              | 185.51               | 2.49                 | 1.324            |
| 4                   | 165              | 168.59               | -3.59                | 2.175            |
| 5                   | 190              | 187.25               | 2.75                 | 1.446            |
| 6                   | 182              | 179.95               | 2.05                 | 1.126            |
| 7                   | 187              | 183.63               | 3.37                 | 1.802            |
| 8                   | 192              | 190.07               | 1.93                 | 1.003            |
| 9                   | 190              | 192.69               | -2.68                | 1.413            |
| 10                  | 174              | 172.03               | 1.97                 | 1.130            |
| average value       | 184              | 183.04               | 0.96                 | 1.328            |

From the comparison and comparison of prediction results, we can see that: The relative prediction error of the low voltage prediction method for distribution network based on least squares support vector machine is 1.328%, the result shows that the error is small. The lowest voltage at the end of the line on the low-voltage side of the transformer and the lowest voltage at the end of the line on the low-voltage side of the transformer are shown in Figure 2:

![Figure 2. Minimum voltage comparison at the end of the line.](image)

Figure 2 shows the node data with "△" and "∗", and connects the corresponding nodes into a curve. The curve of the node symbol "△" represents the actual value of the lowest voltage at the end of the line on the low-voltage side of the distribution transformer, and the curve of the node symbol "∗" represents the lowest voltage predicted value at the end of the line on the low voltage side of the distribution transformer. It can be seen from the deviation of the 10 distribution voltage values of the two curves in the figure that the actual value and the predicted value of the lowest voltage at the end of the distribution voltage are approximated. In summary, the low voltage prediction method of the distribution network based on the least squares support vector machine has a small deviation.
between the predicted value and the actual value, and can complete the prediction of the minimum voltage at the end of the line on the low voltage side of the distribution transformer.

5. Conclusion
Low-voltage prediction method for distribution network based on least squares support vector machine, considering the parameters related to the voltage at the end of the distribution line, such as power line radius, power line length, power distribution capacity, minimum voltage at the end of the power line, and number of users of the distribution transformer, these parameters are analyzed and data pre-processed, the optimal problem equation is constructed, and the decision function is formed. The decision function is used to predict the predicted samples, and the lowest voltage value at the end of the distribution is formed to achieve the purpose of low voltage prediction. This method is used to predict the voltage of the distribution area of a distribution network in a certain area of Guangxi, and the analysis results verify the effectiveness of the prediction method. In summary, the low voltage prediction method based on least squares support vector machine based on this paper is simple and easy to implement. Can provide theoretical basis for distribution network planning, low voltage station prediction, and low voltage control.

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