Stress-strain state of the elastic strip with nearly rectangular cross section

N V Minaeva
Professor, Department of Mechanics and Computer Modeling, Voronezh State University, Universitetskaya pl.1, Voronezh, 364018, Russia
E-mail: nminaeva@yandex.ru

Abstract. The article considers behavior of elastic strip in the framework of plane strain under compression. Conditions are formulated in integral form on the boundaries where the forces are imposed. All the boundary conditions are imposed on the boundary of the body in the strained state, which is necessary for investigating the continuous dependence of the solution to the corresponding task on the functions describing the difference between the shape of cross-section of the strip and a rectangle. The study of the analyticity of the problem solution with respect to small near zero parameters is carried out. The solution is found by perturbation method up to the first order of terms.

1. Introduction
When studying the behavior of elastic bodies, it is often assumed that the displacements and strains are small. Within the framework of made assumptions, this approach yields useful results in different problems [1-3]. Deformation of strips and plates by various external forces under diverse types of support is considered in [4-6].

In [7] it is shown that taking into account the strain of the boundary surface allows one to solve the problems of stability of the elastic bodies by the methods of the mathematical theory of elasticity. Using as an example the compression of a strip in the direction of an infinite extension, it is shown that the results obtained on the basis of the methods of material resistance are limiting for critical loads determined by the mathematical theory of elasticity. This approach was implemented in [8], where the components of rotation were taken into account not only in the boundary conditions, but also in the equilibrium equations.

Since it is necessary to take into account the peculiarities of a real construction in the most complete and accurate way, further development of the methods for calculating and analyzing the solutions obtained is still an urgent task.

In this paper, we consider the problem of analyzing the continuous dependence of the solution describing the stress-strain state of an elastic strip under compression on the functions that determine the contours of the cross section. It is assumed that these characteristics are known up to small parameters.

In such cases, to find the solution the perturbation asymptotic methods are usually used [9-11]. In some problems it is possible to estimate the expansion error [12]. The accuracy of the found solution is demonstrated in [13] by the comparison with the accurate known solution. In some special cases, this question has been analyzed. In the works of H Wendt, and even earlier of M V Keldysh and F I Frankl [14], the convergence of the series obtained for purely subsonic flow was shown.
Poincare [15] demonstrated that for the second-order equation describing periodic oscillations, when a small parameter enters the right-hand side under the condition of analyticity of the right-hand side, the small parameter method is a convergent one.

For quasilinear systems with non-analytic characteristics of nonlinearity by S N Shimanov and I G Malkin [16] the method for constructing periodic solutions by means of successive approximations in the case of multiple degrees of freedom has been developed, the convergence of these approximations to the periodic desired solution has been proved, and the conditions for the existence of such a solution in the case of resonance has been shown.

Solving the problems the authors have restricted themselves, as a rule, with only one parameter. The idea of finding the solution by expansion with respect to two or more small parameters has been used in the study of oscillations of quasilinear systems [17].

In [18] (for some solution spaces and initial data [19-21]) the conditions are stated when the solution is analytic one with respect to small parameters in the vicinity of zero.

2. Continuous dependence of the solution characterizing the stress-strain state of the strip on the initial data

On the basis of the results obtained in [7, 8], various mathematical models of boundary conditions in stresses are presented in [22]. As a result of the performed analysis, it has been found that in the study of the continuous dependence of the solution on the initial data (functions, parameters entering the mathematical model) the static boundary conditions are required to be stated on the boundary of the real body in the strain state. In [22], special cases of differential operators used in solving problems of quasi-static deformation of solids are considered. For some Banach spaces with the corresponding norms, usually used in solving the problems of continuum mechanics (Helder spaces, Hilbert space, \( C^2([a,b], \mathbb{R}^n) \), \( C^4([a,b], \mathbb{R}^n) \), etc.), in which Frechet derivative is an isomorphism [20, 21], the conditions are obtained for the continuous dependence of the solution on the initial data.

Let us consider the deformation of a compressed elastic strip under the conditions of a plane strain. The strip is subject to the forces under which accurate boundary conditions for normal stresses cannot be set. Similar situations arise, for example, if the load is a concentrated or distributed force applied along the line running along the strip. Let them be reduced to the principal vector equal modulo 2 \( \pi \).

In the loaded state, the upper and lower edges are characterized by the functions \( y = g_i(x) \) (i = 1,2). In the unloaded state, by \( y = f_j \) (j = 1,...,4), respectively (see figure 1).

![Figure 1. The cross section of the strip in unloaded and loaded conditions](image-url)
The behavior of the plate will be described by the solution to the system of equations [7]:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0, \quad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} = 0,
\]

\[
\sigma_x = \lambda \theta + 2 \mu \frac{\partial u}{\partial x}, \quad \sigma_y = \lambda \theta + 2 \mu \frac{\partial v}{\partial y},
\]

\[
\tau = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \quad \theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}.
\]

(1)

We state the boundary conditions in the following form

\[
P_x \bigg|_{y=g} = P_x \bigg|_{y=g_2} = 0,
\]

\[
P_y \bigg|_{x=b} = P_y \bigg|_{y=g_2} = 0.
\]

(2)

\[
\int_{A_i} P_i \bigg|_{x=q_1} \, ds = \int_{C_i} P_i \bigg|_{x=q_2} \, ds = -2ph,
\]

\[
\int_{A_i} P_i \bigg|_{x=q_1} \, y \, ds = \int_{C_i} P_i \bigg|_{x=q_2} \, y \, ds = 0,
\]

\[
\int_{A_i} v \bigg|_{x=f_1} \, ds = \int_{C_i} v \bigg|_{x=f_2} \, ds = 0.
\]

(3)

Since in (3) \( ds = \sqrt{1 + (q_i')^2} \, dy \) \( (i=1,2) \), (2), (3) will take the form [7, 22]

\[
\left( \sigma_x g_1' - \tau \right)_{y=g} = \left( \sigma_x g_2' - \tau \right)_{y=g_2} = 0,
\]

\[
\left( \sigma_y - \tau g_1' \right)_{y=g} = \left( \sigma_y - \tau g_2' \right)_{y=g_2} = 0.
\]

(4)

\[
\int_{y_1}^{y_2} \left( \sigma_x - \tau q_1' \right)_{x=q_1} \, dy = \int_{y_1}^{y_2} \left( \sigma_x - \tau q_2' \right)_{x=q_2} \, dy = -2ph,
\]

\[
\int_{y_1}^{y_2} \left( \sigma_x - \tau q_1' \right)_{x=q_1} \, y \, dy = \int_{y_1}^{y_2} \left( \sigma_x - \tau q_2' \right)_{x=q_2} \, y \, dy = 0,
\]

\[
\int_{y_1}^{y_2} v \bigg|_{x=f_1} \, dy = \int_{y_1}^{y_2} v \bigg|_{x=f_2} \, dy = 0.
\]

(5)

\[
\eta_1 = f_1(\xi_1), \quad \xi_1 = f_2(\eta_1), \quad \eta_2 = f_2(\xi_2), \quad \xi_2 = f_1(\eta_2),
\]

\[
\eta_3 = f_1(\xi_3), \quad \xi_3 = f_4(\eta_3), \quad \eta_4 = f_2(\xi_4), \quad \xi_4 = f_4(\eta_4),
\]

\[
y_1 = \eta_1 + v(\xi_1, \eta_1), \quad y_2 = -\eta_1 + v(\xi_2, -\eta_1),
\]

\[
y_3 = \eta_3 + v(-\xi_3, \eta_3), \quad y_4 = -\eta_1 + v(-\xi_2, -\eta_3).
\]

(6)

Let us consider a special case when the cross section in an unloaded state differs only slightly from a rectangular one.
\[ f_1(x) = h + h^{(1)}(x) \quad f_2(x) = -f_1(x), \]
\[ f_3(y) = \ell + \ell^{(1)}(y) \quad f_4(y) = -f_3(y), \]
\[ h = \text{const}, \quad \ell = \text{const}. \]  

If we neglect this difference, i.e. \( h^{(1)}(x) \equiv 0, \ell^{(1)}(y) \equiv 0 \), then problem (1)-(7) admits solution [4,8]:

\[ v = v^{(0)} = e_y^0 y, \quad u = u^{(0)} = -\frac{2\mu + \lambda}{\lambda} e_y^0 x, \]

\[ \sigma_x = \sigma_x^{(0)} = -\frac{4\mu(\mu + \lambda)}{\lambda} e_y^0 y, \quad \sigma_y = \sigma_y^{(0)} = 0, \quad \tau = \tau^{(0)} = 0, \]  

where the constant \( e_y^0 \) is determined from (5) by the following relation

\[ e_y^0 = -\frac{1}{2} + \frac{1}{\sqrt{4 + 4\mu(\lambda + \mu)}}. \]  

In order to find out for what values of the external load parameter \( p \) the solution (8) to the problem (1)-(7) for \( h^{(1)}(x) << h, \ell^{(1)}(y) << \ell \) could be taken as an approximate one, we must consider the question of the continuous dependence of the solution to the problem (1)-(7) on the functions \( h^{(1)}(x), \ell^{(1)}(y) \) for \( h^{(1)}(x) \equiv 0, \ell^{(1)}(y) \equiv 0 \). For this purpose, as it follows from [22], we consider the auxiliary problem with respect to the functions \( \zeta \), obtained on the basis of (1), (4), (5) \( (\sigma_x = \sigma_x^{(0)} + \zeta_1; \sigma_y = \sigma_y^{(0)} + \zeta_2; \ldots v = v^{(0)} + \zeta_5) \):

\[ \frac{\partial \zeta_1}{\partial x} \frac{\partial \zeta_3}{\partial y} = 0, \quad \frac{\partial \zeta_2}{\partial y} + \frac{\partial \zeta_3}{\partial x} = 0, \]

\[ \zeta_1 = \lambda \zeta_6 + 2\mu \frac{\partial \zeta_4}{\partial x}, \quad \zeta_2 = \lambda \zeta_6 + 2\mu \frac{\partial \zeta_5}{\partial y}, \]

\[ \zeta_3 = \mu \left( \frac{\partial \zeta_4}{\partial x} + \frac{\partial \zeta_5}{\partial y} \right), \quad \zeta_6 = \frac{\partial \zeta_4}{\partial x} + \frac{\partial \zeta_5}{\partial y}. \]  

\[ \left[ (\sigma_x^{(0)} + \zeta_1) y_1' - \zeta_3 \psi_2' \right]_{y_1 y_2} = \left[ (\sigma_x^{(0)} + \zeta_1) y_1' - \zeta_3 \psi_2' \right]_{y_1 y_2} = 0, \]

\[ \left[ \zeta_2 - \zeta_3 \psi_1' \right]_{y_1 y_2} = \left[ \zeta_2 - \zeta_3 \psi_1' \right]_{y_1 y_2} = 0. \]  

\[ \int_{y_1}^{y_2} \left( \sigma_x^{(0)} + \zeta_1 - \zeta_3 \psi_1' \right) dy = \int_{y_1}^{y_2} \left( \sigma_x^{(0)} + \zeta_1 - \zeta_3 \psi_1' \right) dy = -2ph, \]

\[ \int_{y_2}^{y_1} \left( \sigma_x^{(0)} + \zeta_1 - \zeta_3 \psi_1' \right) y dy = \int_{y_4}^{y_3} \left( \sigma_x^{(0)} + \zeta_1 - \zeta_3 \psi_1' \right) y dy = 0, \]

\[ \int_{-h}^{h} \left( v^{(0)} + \zeta_5 \right) dy = \int_{-h}^{h} \left( v^{(0)} + \zeta_5 \right) dy = 0, \]  

where

\[ y_1 = h + v^{(0)}(\ell, h) + \zeta_5(\ell, h), \quad y_2 = -h + v^{(0)}(\ell, -h) + \zeta_5(\ell, -h), \]

\[ y_3 = h + v^{(0)}(- \ell, h) + \zeta_5(- \ell, h), \quad y_4 = -h + v^{(0)}(- \ell, -h) + \zeta_5(- \ell, -h). \]
The function \( \phi_1(x) \) is determined by the following parametric form
\[
\begin{align*}
  x &= t + u^{(0)}(t, h) + \zeta_4(t, h), \\
  y &= t + v^{(0)}(t, h) + \zeta_5(t, h),
\end{align*}
\]
(14)

the function \( \phi_2(x) \):
\[
\begin{align*}
  x &= t + u^{(0)}(t, -h) + \zeta_4(t, -h), \\
  y &= t + v^{(0)}(t, -h) + \zeta_5(t, -h),
\end{align*}
\]
(15)

the function \( \psi_1(x) \):
\[
\begin{align*}
  x &= \ell + u^{(0)}(\ell, t) + \zeta_4(\ell, t), \\
  y &= t + v^{(0)}(\ell, t) + \zeta_5(\ell, t),
\end{align*}
\]
(16)

and the function \( \psi_2(x) \):
\[
\begin{align*}
  x &= \ell + u^{(0)}(-\ell, t) + \zeta_4(-\ell, t), \\
  y &= t + v^{(0)}(-\ell, t) + \zeta_5(-\ell, t).
\end{align*}
\]
(17)

Following [22], it is necessary to perform linearization of the problem (10)–(17) with respect to \( \zeta_1 \).

The system of equations (10) is already linear. Functions \( \phi_i(x) \) and \( \psi_i(x) \) linearized with respect to \( \zeta_1 \) take the form
\[
\begin{align*}
  \phi_1 &= (1 + e_x^0) \ell + \zeta_5 \left( \frac{x}{1 + e_x^0}, h \right), \\
  \phi_2 &= -(1 + e_x^0) \ell + \zeta_5 \left( \frac{x}{1 + e_x^0}, -h \right),
\end{align*}
\]
\[
\psi_1 = (1 + e_x^0) \ell + \zeta_4 \left( \ell, \frac{y}{1 + e_y^0} \right), \\
\psi_2 = -(1 + e_x^0) \ell + \zeta_4 \left( -\ell, \frac{y}{1 + e_y^0} \right),
\]
\[
e_x^0 = -\frac{2\mu + \lambda}{\lambda} e_x^0.
\]

The boundary conditions (11) will take the form
\[
\begin{align*}
  \left[ \frac{d\zeta_5 \left( \frac{x}{1 + e_x^0}, h \right)}{dx} \right]_{x = (1 + e_x^0) \ell} &= 0; \\
  \zeta_2(x, y) \big|_{y = (1 + e_y^0) h} &= 0, \\
  \left[ \frac{d\zeta_5 \left( \frac{x}{1 + e_x^0}, -h \right)}{dx} \right]_{x = -(1 + e_x^0) \ell} &= 0; \\
  \zeta_2(x, y) \big|_{y = -(1 + e_y^0) h} &= 0.
\end{align*}
\]
(18)
And conditions (12) are as follows

\[
\begin{align*}
\int_{-\gamma}^{\gamma} \zeta_1(x) \frac{\partial \phi_1}{\partial x} dy + \sigma_0 \left[ \zeta_5(\pm \ell, h) - \zeta_5(\pm \ell, -h) \right] = 0, \\
\int_{-\gamma}^{\gamma} \zeta_1(x) \frac{\partial \phi_1}{\partial x} dy + \sigma_0 \left[ h \zeta_5(\pm \ell, h) + h \zeta_5(\pm \ell, -h) \right] = 0, \\
\int_{-h}^{h} \zeta_5(x) dx = 0,
\end{align*}
\]

where \( \gamma_1^0 = (1 + \nu_0^0) \eta \).

In order the solution to the initial problem depends continuously on the functions \( h^{(1)}(x) \), \( \ell^{(1)}(y) \) it is necessary that the problem (10) and (12) has only a trivial solution \([20, 22]\). It is rather difficult to carry out the study in the general case, therefore we will further consider the case, which is often encountered in practice, when \( \frac{\mu}{\kappa} \ll 1 \). Then, according to (9), the strain is also small

\[ \varepsilon_y^0 \ll 1, \quad \varepsilon_x^0 \ll 1. \]

In this case, the boundary conditions (18) and (19) can be replaced by the following approximate ones

\[
\begin{align*}
\zeta_3(x, \pm h) - \sigma_0 \frac{\partial \phi_1}{\partial x} (x, \pm h) = 0, \\
\zeta_2(x, \pm h) = 0, \\
\int_{-h}^{h} \zeta_1(\pm \ell, y) dy + \sigma_0 \left[ \zeta_5(\pm \ell, h) - \zeta_5(\pm \ell, -h) \right] = 0, \\
\int_{-h}^{h} \zeta_1(\pm \ell, y) dy + \sigma_0 \left[ h \zeta_5(\pm \ell, h) + h \zeta_5(\pm \ell, -h) \right] = 0, \\
\int_{-h}^{h} \zeta_5(\pm \ell, y) dy = 0.
\end{align*}
\]

The boundary condition (20) coincides with the results in \([7, 8]\).

Following \([7, 8]\) we seek the solution to the problem (10), (20), (21) in the form

\[
\begin{align*}
\zeta_4 &= \left[ A \sinh \alpha y + D \left( \frac{\lambda + 3\mu}{\lambda + \mu} \sinh \alpha y + \alpha \cosh \alpha y \right) \right] \sin \alpha x, \\
\zeta_5 &= \left[ A \cosh \alpha y + D \alpha \sinh \alpha y \right] \cos \alpha x, \quad \alpha = \frac{\pi}{2\ell},
\end{align*}
\]

Since, on the basis of (22) and (10), \( \zeta_4 \) and \( \zeta_5 \) for \( x = \pm \ell \) turn to zero, the conditions (21) are satisfied. We obtain that the problem (10), (20), (21) has a nontrivial solution for
\[ p = p^* = 2\mu \frac{\text{ch}\beta \text{sh}\beta - \beta}{\lambda + \mu} \frac{\mu}{\text{ch}\beta \text{sh}\beta + \beta} \]  

where \( \beta = \frac{\pi h}{2\ell} \). This result coincides, to within the notations, with the value of the critical force in [7].

Figure 2 shows the graph (23) \( \left( \frac{P}{\mu} \ll 1 \right) \) for \( \lambda = 117.68 \text{GPa} \) and \( \mu = 78.45 \text{GPa} \). If we compare \( p^* \) with the critical load under the loss of stability of a compressed rectangular plate [4] for \( \beta = 0.04 \), then we obtain insignificant discrepancies: \( p^* = 119.544 \text{MPa} \), but \( p_0 = 119.543 \text{MPa} \). For the values \( \beta > 0.1 \) the condition \( \frac{P}{\mu} \ll 1 \) is still fulfilled, but the stresses already exceed the elastic limit of real materials, therefore the graph is not given.

Thus, we obtain that for \( p < p^* \) the solution to the problem (1)-(7) depends continuously on \( h^{(i)}(x) \), \( \ell^{(i)}(y) \) for \( h^{(i)}(x) \equiv 0 \), \( \ell^{(i)}(y) \equiv 0 \), and the stress-strain state (under small strain) of the strip will be close to the homogeneous one, described by (8).

\[ \text{Figure 2. The graph of the function } p(\beta). \]

3. Application of the perturbation method

Let in (1) - (3) the functions characterizing the deviation of the unloaded cross-section edges from straight lines be known to within small parameters

\[ h^{(i)}(x) = e_i f^{(1)}(x) \text{, } \ell^{(i)}(y) = e_3 f^{(2)}(y) \quad (i = 1,2) \text{, } (|e_j| < 1 \text{, } j = 1,2,3). \]

Then, according to [18, 19], for \( p < p^* \) the solution to problem (1) - (3) are analytic one with respect to the parameters \( e_i, e_3 \) in the vicinity of the point \( e_i = e_3 = 0 \).

We seek it in the form of power series, and for zero approximation we use (8):
\[ u = \sum_{k,l,m=0}^{\infty} e_1^k e_2^l e_3^m u_{klm} (x,y), \quad v = \sum_{k,l,m=0}^{\infty} e_1^k e_2^l e_3^m v_{klm} (x,y), \]
\[ \sigma_x = \sum_{k,l,m=0}^{\infty} e_1^k e_2^l e_3^m \sigma_{x_{klm}} (x,y), \quad \sigma_y = \sum_{k,l,m=0}^{\infty} e_1^k e_2^l e_3^m \sigma_{y_{klm}} (x,y), \quad \tau = \sum_{k,l,m=0}^{\infty} e_1^k e_2^l e_3^m \tau_{klm} (x,y). \]

(24)

As a result of substitution (24) into (1) - (3), according to the small parameter method, we obtained the relations for finding the corresponding approximations.

For example, for the components “100” they have the form

\[ \sigma_{x_{100}} = \lambda \Theta_{100} + 2\mu \frac{\partial u_{100}}{\partial x}, \quad \sigma_{y_{100}} = \lambda \Theta_{100} + 2\mu \frac{\partial u_{100}}{\partial y}, \quad \tau_{100} = \mu \left( \frac{\partial v_{100}}{\partial x} + \frac{\partial u_{100}}{\partial y} \right), \quad \Theta_{100} = \frac{\partial u_{100}}{\partial x} + \frac{\partial v_{100}}{\partial y}, \]

with the boundary conditions

\[ \tau_{100} (x,h) - \sigma_{x_{100}} \frac{\partial v_{100}}{\partial x} (x,h) = \sigma_{x_{100}} \frac{df}{dx}, \quad \sigma_{x_{100}} (x,\pm h) = 0, \]

\[ \tau_{100} (x,-h) - \sigma_{x_{100}} \frac{\partial v_{100}}{\partial x} (x,-h) = \sigma_{y_{100}} (x,\pm h) = 0. \]

(26)

\[ \int_{-h}^{h} \sigma_{x_{100}} (\pm l, y) dy + \sigma_{y} \left[ v_{100} (\pm l, h) - v_{100} (\pm l, -h) \right] = 0, \]

\[ \int_{-h}^{h} \sigma_{x_{100}} (\pm l, y) dy + \sigma_{y} h \left[ v_{100} (\pm l, h) - v_{100} (\pm l, -h) \right] = 0, \]

\[ \int_{-h}^{h} v_{100} (\pm l, h) dy = 0. \]

(27)

For the remaining components of the first approximation, the system of differential equations are the same as (25).

The conditions (27) are analogous for the approximation “010”, and (26) take the form:

\[ \tau_{100} (x,h) - \sigma_{x_{100}} \frac{\partial v_{100}}{\partial x} (x,h) = \sigma_{y_{100}} (x,\pm h) = 0, \]

\[ \tau_{100} (x,-h) - \sigma_{x_{100}} \frac{\partial v_{100}}{\partial x} (x,-h) = \sigma_{y_{100}} (x,\pm h) = 0. \]

For the components “001” all boundary conditions are trivial one.

Let \( f^{(1)} (x) = \sin ax \), then we seek the solution to the problem (25)-(27), following [7], in the form

\[ u_{100} = g_{100} (ay) \sin ax, \quad v_{100} = f_{100} (ay) \sin ax. \]

(28)

As a result of the substitution of (28) into (25) we obtained the system of equations
\[
\mu \dddot{g}^{100} - (\lambda + 2\mu) g^{100} + (\lambda + \mu) \ddot{g}^{100} = 0, \\
(\lambda + \mu) \dddot{f}^{100} - \mu \ddot{f}^{100} + (\lambda + \mu) \dot{g}^{100} = 0,
\]

where the dot denotes the finding of the derivative.

Its solution is as follows

\[
g^{100}(\eta) = A^{100} \cosh + B^{100} \sinh + C^{100} \eta \cosh + D^{100} \eta \sinh, \\
f^{100}(\eta) = A_i^{100} \cosh + B_i^{100} \sinh + C_i^{100} \eta \cosh + D_i^{100} \eta \sinh,
\]

where the arbitrary constants are connected by the relations:

\[
A_i^{100} = B^{100} - \frac{\lambda + 3\mu}{\lambda + \mu} C^{100}, \\
B_i^{100} = A^{100} - \frac{\lambda + 3\mu}{\lambda + \mu} D^{100}, \\
C_i^{100} = D^{100}, \\
D_i^{100} = C^{100}.
\]

The boundary conditions (27) for \( a = n \frac{\pi}{l} \) are satisfied. Substituting (29) into (26) we will obtain the system for determining the constants

\[
\begin{align*}
\alpha_{11} A^{100} + \alpha_{12} B^{100} + \alpha_{13} C^{100} + \alpha_{14} D^{100} &= -p, \\
- \alpha_{11} A^{100} + \alpha_{12} B^{100} + \alpha_{13} C^{100} - \alpha_{14} D^{100} &= 0, \\
\alpha_{31} A^{100} + \alpha_{32} B^{100} + \alpha_{33} C^{100} + \alpha_{34} D^{100} &= 0, \\
\alpha_{31} A^{100} - \alpha_{32} B^{100} - \alpha_{33} C^{100} + \alpha_{34} D^{100} &= 0,
\end{align*}
\]

where

\[
\begin{align*}
\alpha_{11} &= (2\mu + p) \text{h} \beta, & \alpha_{12} &= (2\mu + p) \text{ch} \beta, \\
\alpha_{13} &= (2\mu + p) (\beta \text{sh} \beta - \frac{\mu}{\lambda + \mu} \text{ch} \beta) - p \frac{\lambda + 2\mu}{\lambda + \mu} \text{ch} \beta, \\
\alpha_{14} &= (2\mu + p) (\beta \text{ch} \beta - \frac{\mu}{\lambda + \mu} \text{sh} \beta) - p \frac{\lambda + 2\mu}{\lambda + \mu} \text{sh} \beta, \\
\alpha_{31} &= 2\mu \text{ch} \beta, & \alpha_{32} &= 2\mu \text{sh} \beta, & \alpha_{33} &= 2\mu (\beta \text{ch} \beta - \frac{\lambda + 2\mu}{\lambda + \mu} \text{sh} \beta), \\
\alpha_{34} &= 2\mu (\beta \text{sh} \beta - \frac{\lambda + 2\mu}{\lambda + \mu} \text{ch} \beta), & \beta &= ah.
\end{align*}
\]

As a result of the transformations, (30) is divided into two systems, from which all the unknowns are determined:

\[
\begin{align*}
\alpha_{11} A^{100} + \alpha_{14} D^{100} &= -\frac{1}{2} p, \\
\alpha_{31} A^{100} + \alpha_{34} D^{100} &= 0.
\end{align*}
\]

To find \( u^{010} \) and \( v^{010} \) in the form which is analogous to (28), the systems for arbitrary constants take the form
\[
\begin{align*}
\alpha_1 A^{010} + \alpha_4 D^{010} &= 0, \\
\alpha_3 A^{010} + \alpha_4 D^{010} &= -\frac{1}{2} p .
\end{align*}
\]

For the functions \( g^{001}(ay) \) and \( f^{001}(ay) \) the system of equations is homogeneous one for finding arbitrary constants. Since its determinant for \( p < p^* \) is not equal to zero, all arbitrary constants, and consequently, the components themselves, will be equal to zero.

Thus, the functions

\[
\sigma_x = \sigma_x^0 + \varepsilon_1 \sigma_x^{100} + \varepsilon_2 \sigma_x^{010} + \ldots , \quad v = v^0 + \varepsilon_1 v^{100} + \varepsilon_2 v^{010},
\]

according to Saint-Venant’s principle, at a sufficient distance (because of the integral form of the boundary conditions) from the edges of the strip for \( x = \pm l \), describe the stress-strain state of the strip for \( p < p^* \) to within the first-order quantities of smallness, since the remaining terms of the series (24) are the second-order quantities of smallness.

4. Conclusion
The limitation on the compressive forces parameter is obtained for a strip with a cross-sectional shape close to a rectangle. If the loads are such that they do not exceed the limit found, then the similarity of the cross-sectional shape is also retained during deformation. If the value found is exceeded, the stress-strain state will no longer be homogeneous one, corresponding to (8). For such values, one needs either consider another solution to the nonlinear problem posed, or change the mathematical model, for example, use equilibrium equations in the deformed state, nonlinear physical relationships, etc.

If the contours of the cross section in an unloaded state are known up to small parameters, then for compressive forces satisfying the condition obtained above, one can find the solution by the perturbation method with the required accuracy. In this case, the parameters are independent. To estimate the error of the solution obtained, one can use any of the estimates of the Taylor series. For a particular case of deviation of the cross section contour, the solution describing the stress-strain state of the strip has been obtained, up to first order terms. The nonideality of the unloaded edges does not influence significantly (is characterized by not more than second order values) the state of the strip.

References
[1] Papkovich P F 1940 Dokl. Akad. Nauk SSSR On one form of solution of the plane problem of the theory of elasticity for a rectangular strip 27 359
[2] Gotsev D V, Kovalev A V and Sporykhin A N 2001 J. of Appl. Mech. and Tech. Phys. Local instability of plates with pressed-in annular incusions at the elastoplastic behavior of materials 42(3) 505
[3] Sheidakov D N 2007 J. of Appl. Mech. and Tech. Phys. Stability of a rectangular plate under biaxial tension 48(4) 547
[4] Voľ'mir A S 1963 Stability of elastic systems (Moscow: Fizmatgiz) p 880 [in Russian]
[5] Pfluäger A 1950 Stabilitätsprobleme der Elastostatik (Springer-Verlag)
[6] Green A E and Zerna W 1968 Theoretical Elasticity (Oxford, London: Univ. Press) p 457
[7] Ishlinskii A Y 1954 Ukr. Math. J. Consideration of problems of stability of equilibrium of elastic bodies from the viewpoint of the mathematical theory of elasticity 6(2) 140
[8] Ershov LV and Ivlev D D 1961 Dokl. Akad. Nauk SSSR On stability of a strip in compression 138(5) 1047
[9] Nayfeh A H 2000 Perturbation Methods (New York: John Wiley and Sons) p 456
[10] Ivlev D D and Ershov L V 1978 The Perturbation Method in the Theory of Elasto-Plastic Body (Moscow: Nauka) p 208 [in Russian]
[11] Guz’A N and Nemish Y N 1989 *The Method of Perturbation of the Boundary Form in Continuum Mechanics* (Kiev: Vishcha Shkola) p 352 [in Russian]

[12] Il’in A M and Danilin A P 2009 *Asymptotic Methods in Analysis* (Moscow: Fizmatlit) p 248 [in Russian]

[13] Cherepanov G P 1963 *J. of Appl. Math. and Mech.* On a method of solving the elasto-plastic problem 27(3) 644

[14] Keldysh M V and Frankl F I 1934 *Izv. Akad. Nauk SSSR* The exterior problem for nonlinear elliptic differential equations with application to the theory of a wing in a compressible gas

[15] Poincare H 1971 *Selected Works* (in Russian) (in 3 volumes) vol 1, 2 (Moscow: Nauka)

[16] Malkin I G 2004 *Lyapunov and Poincare Methods in the Theory of Nonlinear Oscillations* (Moscow: Nauka) p 234 [in Russian]

[17] Proskuryakov A P 1977 *Poincare Method in the Theory of Nonlinear Oscillations* (Moscow: Nauka) p 256 [in Russian].

[18] Minaeva N V 2008 *Mechanics of Solids* Application of the perturbation method in mechanics of deformable solids 43(1) 31

[19] Krasnosel’skii M A and Zabreiko P P 1975 *Geometric Methods of Nonlinear Analysis* (Moscow: Nauka) p 512 [in Russian]

[20] Zachepa V R and Saponov Y I 2002 *Local Analysis of Fredholm Equations* (Voronezh: Voronezh State Univ.) p 185 [in Russian]

[21] Darinskii B M, Saponov Y I and Tsarev S L 2007 *J. Math. Sci.* 145 5311

[22] Minaeva N V 2006 *The Adequacy of Mathematical Models of Deformable bodies* (Moscow: Nauchnaya kniga) p 236 [in Russian]