The moduli problem at the perturbative level

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Abstract – This paper demonstrates that existing upper bounds on the magnitude of cosmological dark matter and baryon isocurvature fluctuations can be translated into stringent constraints on the parameter space of moduli fields of supersymmetric theories, defined in terms of $m_\sigma$ (modulus mass) and $\sigma_\text{inf}$ (modulus vacuum expectation value at the end of inflation). For the sake of concreteness, we assume here a quadratic modulus potential and we focus on high-scale inflationary models. These constraints are complementary to previously existing bounds from big-bang nucleosynthesis, therefore the moduli problem becomes worse at the level of cosmological perturbations. In particular, if the inflationary scale $H_\text{inf} \sim 10^{13}$ GeV, particle physics scenarios which predict high moduli masses $m_\sigma \gtrsim 10^{-100}$ TeV are plagued by the perturbative moduli problem, even though they evade big-bang nucleosynthesis constraints. This perturbative moduli problem is somewhat relaxed if the modulus is made heavy during inflation, with effective mass $H_\text{inf}$. 

Introduction. – The phenomenology of moduli fields of supersymmetric and superstring theories, or more generally, of late decaying flat directions of the scalar potential is strongly constrained by cosmological arguments [1,2]. Most notably, successful big-bang nucleosynthesis (BBN) sets very stringent upper bounds on their energy density at temperatures $T \lesssim 1$ MeV [3]. Such a small modulus abundance can be achieved through the dilution of the pre-existing moduli by late time entropy production or late inflation [4,5]; or, if the supergravity-induced potential is such that the moduli has always stayed close to its low-energy minimum [6,7]. Alternatively, the BBN constraints can be evaded if the moduli are massive enough to decay at high temperatures $T_d \simeq 2.8$ MeV ($m_\sigma/100$ TeV)$^{3/2}$, a property which is considered as a strong motivation for particle physics models which realize this hierarchy $m_\sigma \gtrsim 10$–$100$ TeV $\gg M_\text{W}$ (weak scale), such as no-scale supergravity, anomaly mediation or string inspired models [8–11].

These constraints directly relate to the background evolution of the modulus field. However, observational cosmology now offers powerful constraints on the power spectrum of density perturbations and the nature of these perturbations. At the perturbative level, a modulus behaves similarly to curvaton fields [12,13], insofar as it transfers its perturbations to the fields it decays into, which may lead to the generation of strong isocurvature fluctuations between dark matter and radiation or between baryons and radiation. In the present letter, we show for the first time that the existing observational upper limits on the magnitude of isocurvature modes can be translated into stringent constraints on moduli cosmology, which directly impact on high-energy physics model building in the modulus sector.

The constraints obtained depend on the shape of the effective potential of the moduli during and after inflation, and can be formulated most clearly in the parameter space $(\sigma_\text{inf}, m_\sigma)$. In the following we discuss three prototypical cases in which the modulus mass receives
supergravity corrections of order $H$ or not, and whether the inflaton produces moduli during its decay or not. We focus explicitly on quadratic moduli potentials, as it provides the most natural potential shape when the modulus lies not far from its minimum. Including self-interactions and couplings to other fields provides an interesting generalization of the present work, but it involves greater complexity and greater dimensionality of parameter space. On general grounds, and up to the particular case of a quartic potential $\lambda \phi^4$, such couplings are furthermore suppressed by Planck scale contributions, and therefore unlikely to affect the modulus dynamics as soon as $\sigma \ll M_P$. In order to give concrete estimates, we also assume $m_\phi^2 H^2$ inflation whose mass has been normalized by COBE/WMAP to $m_\phi = 10^{13}$ GeV so that the Hubble parameter at the end of inflation is $H_{\text{inf}} = 10^{13}$ GeV. Such models indeed provide the prototypical realization of inflation with a scalar index in accord with CMB measurements [14] and they will be tested experimentally in the near future through their non-ambiguous prediction of a detectable gravitational wave signal. Our results hardly depend on the way inflation is realized, but they depend on the scale of inflation. We also assume that the modulus remains light during inflation; we briefly discuss the dependences of our results with respect to these assumptions at the end of this paper. They also depend slightly on other cosmological and particle physics parameters, such as the post-inflationary reheating temperature and the branching ratio of moduli to dark matter. These dependences are made manifest in our analytical expressions in the following. The cosmological scenario depicted above is natural and general enough to encapsulate the salient features of the moduli problem discussed here.

Isocurvature fluctuations from moduli fields. –

Moduli acquire their own spectrum of density fluctuations through inflation, with $\delta \sigma_{\text{inf}} / \sigma_{\text{inf}} \sim H_{\text{inf}} / (2 \pi \sigma_{\text{inf}})$, assuming that $m_\sigma \ll H_{\text{inf}}$ during inflation. One crucial observation is that these fluctuations are independent of those of the radiation, dark matter and baryon fluids, which inherit those of the inflaton at reheating, with an energy density contrast of order $10^{-5}$. As the modulus decays, it transfers most of its energy to radiation, a fraction $B_\chi \ll 1$ to dark matter, and another part into baryons and anti-baryons. As the final fractional density perturbations of radiation ($\Delta_\gamma$), dark matter ($\Delta_\chi$) and baryons ($\Delta_b$) comprise different mixtures of the initial inflaton and modulus perturbations, isocurvature modes between those fluids exist. To go one step further, a significant dark-matter–radiation isocurvature perturbation is generated when a significant part of dark matter has been produced by the modulus while the radiation fluid has remained unaffected, i.e. $B_\chi \Omega_{\gamma}^d \gg \Omega_\chi^d$ and $\Omega_b^d \ll 1$ (the superscripts $<d$ (respectively, $>_d$) mean immediately before (respectively, after) modulus decay, the subscripts $\chi$ (respectively, $\sigma$) refer to dark matter (respectively, modulus)). However, if the modulus decay preserves the baryon number (as we assume here), its decay cannot affect the perturbations of “net baryon number”, hence the baryon isocurvature mode is generated when the modulus significantly reheats the Universe through its decay, i.e. $\Omega_{\gamma}^d / \Omega_{\sigma} < 1$. The fact that dark matter and baryon isocurvature modes are produced at very different values of $\Omega_{\gamma}^d$ explains the power of the constraints derived in this work.

The final isocurvature fluctuations, to be tested against CMB data are explicitly written as

$$\delta_{\chi \gamma} \simeq \frac{1}{1 + \Upsilon_{\chi}} \left( \frac{B_\chi \Omega_{\chi}^d}{\Omega_\chi^d} - \Omega_\chi^d \right) S_{\sigma \gamma}^{(1)}, \quad (1)$$

$$\delta_{b \gamma} \simeq -\Omega_{\gamma}^d S_{\sigma \gamma}^{(1)}, \quad (2)$$

with $S_{\alpha \beta} \equiv \delta_{\alpha \beta} - \zeta_{\alpha \beta}$ (the subscript $\gamma$ refers to radiation) and $S_{\sigma \gamma}^{(1)} = 2 S_{\log} / \sigma_{\text{inf}} [15]$; $\Upsilon_{\chi}$ denotes the ratio of the dark-matter annihilation rate to the expansion rate immediately after the modulus decay, see refs. [16,17]. We quantify the amount of matter-radiation isocurvature fluctuation through $\sigma_{\gamma}^m$:

$$\sigma_{\gamma}^m \equiv \frac{\zeta_{\chi \gamma} - \zeta_{d \gamma}}{\zeta_{\chi \gamma} + \zeta_{d \gamma}}. \quad (3)$$

where $\zeta_{\chi} \equiv \Omega_b / \Omega_{\chi} \zeta_b + \Omega_\chi / \Omega_{\chi} \zeta_{\chi}$ with $\Omega_m \equiv \Omega_\chi + \Omega_b$. Using the results of ref. [18], one finds that various CMB data imply $-0.12 < \sigma_{\chi \gamma} < 0.089$ at 95% C.L. (with $\Omega_\chi h^2 \simeq 0.12$ and $\Omega_b h^2 \simeq 0.0225$).

Consider now a concrete case with a time-independent potential $V(\sigma) = m_\sigma^2 \sigma^2 / 2$. Then one finds

$$\frac{\Omega_{\gamma}^d}{\Omega_{\sigma}^d} \simeq 6 \times 10^{10} \left( \frac{\sigma_{\text{inf}}}{M_P} \right)^2 \left( \frac{m_\sigma}{100 \text{ TeV}} \right)^{-3/2} \left( \frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right), \quad (4)$$

assuming that the modulus starts to oscillate in its potential (at $H = m_\sigma$) before reheating, an assumption which is valid throughout the present parameter space provided $T_{\text{rh}} \lesssim 10^{40}$ GeV.

Figure 1 presents the (blue colored) contours of the $\delta_{m \gamma}$ quantity calculated numerically through the integration of the evolution equations for background and perturbations as in refs. [16,19], assuming $T_{\text{rh}} = 10^9$ GeV, $H_{\text{inf}} = 10^{13}$ GeV; we set $B_\chi = m_\chi / m_\sigma$ (i.e., one modulus particle produces one dark-matter particle at decay assuming instantaneous thermalization of the dark matter, which is a good approximation [17]) in regions of parameter space where the present-day dark-matter density has the correct magnitude, and decrease $B_\chi$ accordingly in order to match this constraint otherwise. The dashed yellow area is excluded by BBN; in order to draw this region, we have used the results of refs. [3,20] for a hadronic branching ratio of $10^{-3}$ and initial jet energy 1 TeV. One sees clearly that a large portion of the parameter space is excluded by the generation of isocurvature perturbations, well beyond the bounds of BBN. For $\sigma_{\text{inf}} > 4 \times 10^{-6} (m_\sigma / 100 \text{ TeV})^{3/4} M_P$ (with a
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Fig. 1: (Colour on-line) Constraints in the \((\sigma_{\text{inf}}, m_{\sigma})\)-plane. The yellow shaded region shows the constraints derived from the analysis of the effect of moduli decay on BBN. The blue contours give the value of \(\delta_{\text{m}}\), for the matter-radiation isocurvature mode, constrained to be less than \(\sim 10\%\) by CMB data. The white dashed line indicates the place where \(\Omega_{\phi}^{\text{eq}} = 0.5\). These calculations assume \(H_{\text{inf}} = 10^{13} \text{ GeV}\) and \(T_{\text{bh}} = 10^6 \text{ GeV}\). The dotted orange line indicates the standard deviation of \(\sigma\) on large scales due to stochastic motion during (chaotic \(m^2 \phi^2\)) inflation.

Numerical prefactor scaling as \((T_{\text{bh}}/10^9 \text{ GeV})^{-1/2}\), one finds \(\Omega_{\phi}^{\text{eq}} \sim 1\) which leads to a strong baryon isocurvature mode but no dark-matter–radiation isocurvature perturbation. On the contrary, if \(\Omega_{\phi}^{\text{eq}}\) is small and \(\sigma_{\text{inf}} \geq 10^{-10}(m_{\sigma}/100 \text{ TeV})^{1/2}M_{\text{Pl}}\) (with a numerical prefactor scaling as \((T_{\text{bh}}/10^9 \text{ GeV})^{-1}\)), there is no baryon isocurvature mode but a large dark-matter isocurvature mode. In both regions, such large fractions should have been seen in current CMB data.

Note that the typical displacement \(\sigma_{\text{inf}}\) of the inflaton is bounded below by the standard deviation associated with the stochastic motion of the modulus during inflation, whose magnitude is quite large: \(\langle \delta \sigma^2 \rangle^{1/2} \sim H_{\text{inf}}\sqrt{2}N_{\tau}/(2\pi) \geq 4 \times 10^{-5}M_{\text{Pl}}\) for chaotic \(m^2 \phi^2\) inflation, with \(N_{\tau}\) denotes the total number of \(e\)-folds; the last inequality uses \(H_{\text{inf}} = 10^{13} \text{ GeV}\) and \(N_{\tau} \geq 60\). This value is indicated in dotted lines in fig. 1. The above estimate of \(\langle \delta \sigma^2 \rangle^{1/2}\) furthermore represents a lower limit to \(\sigma_{\text{inf}}\), as the modulus may have been initially displaced from its minimum. Actually, most of the region above this bound is also excluded by the normalization of the total curvature perturbation to CMB data.

Let us note at this stage the implications for moduli cosmology and model building. First of all, even if moduli are massive enough to evade BBN constraints (\(i.e., m_{\sigma} \gtrsim 100 \text{ TeV}\)), they are bound to produce strong isocurvature fluctuations between baryons and radiation (at high values of \(\sigma_{\text{inf}}\)) or between dark matter and radiation (at small values of \(\sigma_{\text{inf}}\)). More generally, the present results (under the present assumptions) forbid any large amount of entropy production by late time decaying scalars, since this would precisely be accompanied by strong baryon isocurvature fluctuations. This result bears strong implications for the dilution of unwanted relics through moduli decay. The present perturbative moduli problem requires to reduce quite significantly the Hubble scale of inflation, as discussed at the end of this paper. This would mean, of course, that tensor modes would become inaccessible to the upcoming generation of CMB instruments.

We now discuss a second prototypical case, in which the potential receives supergravity corrections \(V'' \sim H^2\) associated with the breaking of supersymmetry by the finite energy density in the early Universe [6]:

\[
V(\sigma) \simeq \frac{1}{2}m_{\sigma}^2 \sigma^2 + \frac{1}{2}c^2 H^2 \sigma^2. \tag{5}
\]

Such a potential is realized after inflation, for instance, in D-term inflation if \(\sigma\) corresponds to the supersymmetry breaking Polonyi field, the field \(\sigma\) remaining light during inflation [17]. The modulus abundance at the onset of oscillations \((H = m_{\sigma})\) is smaller than in the previous case by a factor \((m_{\sigma}/H_{\text{inf}})^{2(\mu+1-c)}\), with \(\mu = -1/2\), \(\nu^2 = 1/4 - 4c^2/9\) (we assume \(c < 3/4\) here). Consequently, the constraints in the modulus parameter space shift to increasing values of \(\sigma_{\text{inf}}\), as shown in fig. 2.

Figure 2 further establishes the power of constraints obtained at the perturbative level, as it shows that the isocurvature constraints still extend below the lower limit on \(\sigma_{\text{inf}}\) associated with stochastic motion of the modulus in its potential during inflation. With respect to concrete moduli model building, even if the minima of the modulus effective potentials at high and low energy match one another, the isocurvature constraints close the available parameter space for the present rather generic assumptions. It has been suggested that the modulus may be kept close to the minimum of the low-energy potential when this latter is an enhanced symmetry point in moduli space, thanks to the friction that results from its coupling to other light degrees of freedom [6,7,21]. Whether this can
be realized during inflation lies beyond the scope of the present work; it calls however for a detailed study of the stochastic motion of the moduli in this context of realistic moduli potentials.

Finally, one must consider the possibility that the inflaton decays partially into the modulus sector at reheating. This would attenuate the amount of isocurvature produced moduli with $m_{\phi} = 10^{13}$ GeV.

For a prototypical cosmological inflationary scenario with $H_{\text{inf}} = 10^{13}$ GeV, we find that the moduli problem appears significantly worse at the perturbative level. In this way, this opens a window on moduli phenomenology for future high-accuracy CMB experiments. As one of our main results, we find that particle physics scenarios with heavy moduli ($m_{\sigma} \gtrsim 100$ TeV) do not escape the above constraints even though they evade BBN constraints.

In order to solve the moduli problem, one needs to achieve $H_{\text{inf}} \ll 10^{13}$ GeV and this bears strong implications for the possibility of detecting tensor modes in the CMB data with the upcoming generation of instruments. In details, if $H_{\text{inf}} \ll 10^{13}$ GeV, then the isocurvature mode $S_{\gamma, \text{inf}} \propto H_{\text{inf}}/\sigma_{\text{inf}}$ becomes sufficiently small to evade detection at the $\%$ level for $\sigma_{\text{inf}} \gg M_{\text{Pl}}$ (see also [17]). Then a window opens at $m_{\sigma} \gtrsim 100$ TeV (to evade BBN constraints) and $\sigma_{\text{inf}} \sim M_{\text{Pl}}$. As a concrete realization of this scenario, one can consider the so-called KKLT potential for modulus stabilization [23], which yields $m_{\sigma} \approx 700$ TeV for their choice of numerical values, and an initial displacement $\sigma_{\text{inf}} \sim O(0.1) M_{\text{Pl}}$, with the KKLMMT inflationary model, which gives $H_{\text{inf}} \sim 10^9$ GeV [24]. Interestingly, one may also expect to detect baryonic isocurvature perturbations in upcoming CMB experiments, if $H_{\text{inf}}$ is slightly below $10^{13}$ GeV and $\sigma_{\text{inf}} \sim M_{\text{Pl}}$.

Alternatively, if the modulus receives a Hubble effective mass $c_i H$ during inflation, the isocurvature mode is erased during inflation [25] hence the constraints obtained at the perturbative level vanish. However, the stochastic motion of the modulus in its potential is sufficiently large to disrupt BBN, if $c_i$ is not significantly larger than unity or if $m_{\sigma}$ is not larger than 100 TeV [17]. Reducing the amplitude of this quantum noise then also requires lowering the scale of inflation well below $10^{13}$ GeV.

Of course, it is also possible that, in the vacuum of broken supersymmetry all moduli are significantly heavier than $10^7$ GeV, in which case the present constraints related to the generation of dark-matter isocurvature perturbations (as well as the BBN constraints) would vanish. For instance, ref. [26] provides an explicit calculation of the moduli spectrum; depending on the string scale, moduli scalars can be stabilized at a sufficiently high scale or not. Nevertheless, let us add that the existence of one light modulus suffices to create the cosmological problems that we have discussed. The constraints derived here apply to this lightest modulus.

A large amount of entropy production at low scales $H \lesssim m_{\sigma}$ through heavy particle decay or late time inflation could in principle help, by diluting the moduli abundance to low levels [4,5]. However, the present constraints on entropy release by a modulus can be directly recast on the decaying field or late time inflaton. Since the radiation produced in the reheating inherits the fluctuations of the entropy producing fluid, a strong baryon isocurvature fluctuation is produced unless baryogenesis takes place subsequently to reheating, see [19]. Given that the
intermediate scale $\sqrt{M_W M_{Pl}} \simeq 10^{10}$ GeV, this brings in additional non-trivial requirements.

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