EMC effect with different oscillator-model parameters $h\omega$ for different shells by considering difference between proton and neutron structure functions.

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Abstract

The magnitude of binding energy used in the conventional nuclear theory to explain the EMC experimental data, seems to be larger than the one expected. In this paper to get sufficient depletion in the binding energy, different oscillator-model parameters $h\omega$ for different shells and the proton (neutron) structure function that have good agreement with experimental data are used. The extracted results for $^{4}\text{He}$, $^{12}\text{C}$, $^{40}\text{Ca}$ and $^{56}\text{Fe}$ nuclei show that one can get improved results in medium x ranges by less binding energy.

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1 Introduction

In 1983 the European Muon Collaboration (EMC) reported [1] their measurement of muon scattering cross sections per nucleon of iron to deuterium nuclei. The ratios were clearly different from unitary and totally unexpected because of high momentum transfer involved ($Q^2$ up to 200 GeV). Several phenomena were proposed to explain theoretically the EMC effect and each of them had some successes in limited $x$ (the Bjorken variable) range. The conventional nuclear theory proposed by Akulinichev et al., [2] and Dunne et al., [3] seems to be able to explain the experimental data in medium and large $x$ ranges. Akulinichev et al., used a harmonic-oscillator model with the energy levels taken from the compilation of experimental data for the $(e, ep)$ reaction [4]. They showed that the nuclear interaction of nucleons resulting in their binding and Fermi motion, plays an important role in deep-inelastic lepton-nuclear scattering. But the average value of the potential for these oscillator levels was about -60 MeV. Dunne and Thomas also used a harmonic oscillator model with the levels fitted to measured separation energies [3]. So they failed to get sufficient depletion for corresponding single particle energy. If in that time someone had used measured separation energies within the naive single-particle model, the discrepancy would have gotten worse. The authors of references [2, 3], used only one oscillator-model parameter $\hbar \omega$ for different shells inside the investigated nuclei. So they tried to understand the EMC effect in term of the change in the mass scale of a bounded nucleon [5]. Also the extracted results by the light front formalism that developed by Miler and Smith [6, 7], is not consistent with the binding effect not only in the magnitude of the effect, but also in the dependence on the number of nucleons. It should be mentioned that they used not only one momentum distribution for any nuclear level but also one nucleon structure function parametrization. The used parametrization for nucleon structure function in reference [7] considers no difference between protons and neutrons structure function inside the investigated nuclei. To compare it with other parametrization of nucleon structure functions, it is plotted in figure 1 by dash curve. Also the convolution approach represented in reference [8], no difference between protons or neutrons momentum distribution on different state has been considered. As different shells have different root mean square radius one can choose different oscillator-model parameters $\hbar \omega$ for different shells inside a nucleus. Also it should be mentioned that the nucleon structure function used by the authors of references [2, 3, 6, 7], is far from the experimental data (see figure 1). So in this paper it is tried to extract the EMC results in the conventional nuclear theory for the nuclei $^{56}\text{Fe}$, $^{40}\text{Ca}$, $^{12}\text{C}$, $^4\text{He}$, first by considering ‘the different oscillator-model parameters $\hbar \omega$ for different shells related to their root mean square radius’, and second: ‘the free GRV’s [9] neutron and proton structure functions that have good agreement with experimental data’.

2 Theoretical formalism

One can obtain the nuclear structure function of nucleus by main equation [2]

$$F_2^A(x) = \sum_{N=n,p} \sum_{nl} \int_x^\infty dz g_{nl}^N f^N(z)_{nl} F_2^N(x/z),$$

(1)
where the first sum is over the proton and neutron cases. The second sum is over the quantum number of each energy levels. The $g^N_{nl}$ is the occupation number of energy level $\epsilon_{nl}$ for proton ($N=p$) and neutron ($N=n$). The nucleon distribution inside the nucleus define as

$$f^N(z)_{nl} = \int_{|m_N(z-\epsilon_{nl})|}^{\infty} dp \, pm_N |\phi_{nl}(p)|^2 / (2\pi)^2,$$

with $z = p_{nl}q/m_Nq_0$ and $x = Q^2/2m_Nq_0$ the bjorken variable for free nucleon. The effects of the momentum and energy distribution of the nucleon in the nucleus are included in Eq. (2) through $\phi_{nl}(p)$ and $\epsilon_{nl}$, respectively. The magnitude of nuclear binding energy ($\epsilon_{nl}$) mainly effects the structure functions in the intermediate $x$ region. The function $f^A(z)_{nl}$ describes the momentum and energy distribution of nucleons inside nuclei and satisfies the normalization rule

$$\sum_{N=p,n} \sum_{nl} \int_0^\infty dz g^N_{nl} f^N(z)_{nl} = A. \quad (3)$$

Akulinichev et al. [2] used harmonic-oscillator nuclear wave function to calculated $f^A(z)_{nl}$. For the oscillator-model parameter $\overline{\hbar}\omega$ they used only one parameter for any quantum number $n, l$. We knew in the heavier nuclei the deeper closed shells have different root mean square radius and for this purpose, in the harmonic-oscillator model one could use [10] (see appendix A)

$$<r^2>_{nl} = \frac{1}{\alpha^2} (2n + l + \frac{3}{2}), \quad (4)$$

where

$$\alpha^2 = \frac{m_N\omega}{\hbar} \quad (5)$$

and by considering $m_N = 938.905 \text{ MeV}$, one can find in the natural unit

$$\hbar\omega = \frac{41.33}{<r^2>_{nl}} (2n + l + \frac{3}{2}), \quad (6)$$

where $<r^2>_{nl}^{1/2}$ and $\hbar\omega$ expressed respectively in Fermi and MeV unit. Table 1 shows the calculated $\hbar\omega$ for the nuclei that are investigated here and the table 2 contains the brackets that shows the occupation number for different levels.

The resulting expression for the nucleon distribution $f^N(z)_{nl}$ inside the nucleus is [2]

$$f^N(z)_{nl} = \frac{1}{2} \left( \frac{m_N}{\hbar\omega} \right)^{1/2} \frac{n!}{\Gamma(n + l + \frac{3}{2})} \sum_{t_1=0}^{n} \sum_{t_2=0}^{n} \frac{(-1)^{t_1+t_2}}{t_1!t_2!} \left( \frac{n+l+\frac{3}{2}}{n-t} \right)$$

$$\times \left[ \frac{n+l+\frac{3}{2}}{n-t} \right] \Gamma(l + t_1 + t_2 + 1, \frac{m_N(z - 1 - \epsilon_{nl})}{\hbar\omega}) \quad (7)$$

For the $F_2^N(x/z)$ the GRV’s LO free proton and neutron structure functions parameterizations used [9]

$$\frac{1}{x} F_2^{ep}(x, Q^2) = \sum_q e_q^2 \left\{ q(x, Q^2) + \overline{q}(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \right\}$$

$$\times \left[ C_{q,2}^s(q + \overline{q}) + 2C_{g,2}^s q \right]$$

$$+ \frac{1}{x} F_2^{nc}(x, Q^2, m_c^2) \quad (8)$$
with

\[ C_{g,2}(z) = \frac{1}{2} \left( (z^2 + (1-z)^2) \ln \frac{1-z}{z} - 1 + 8z(1-z) \right) \]

(10)

and the \( \Sigma_q \) extended over all light quarks \( q = u,d,s \). The convolution product and the convolution product with \( [\ ]_+ \) are defined as usual

\[ C^*q = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right) q(y,Q^2), \]

(11)

\[ \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) \left[ g(y) - \frac{x}{y} g(x) \right] - g(x) \int_0^x dy f(y) \]

(12)

and

\[ \frac{\alpha_s(Q^2)}{2\pi} = \frac{2}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{2\beta_1 \ln \ln(Q^2/\Lambda^2)}{[\ln(Q^2/\Lambda^2)]^2} \]

(13)

with \( \beta_0 = 11 - 2f/3, \beta_1 = 102 - 38f/3 \) and \( \Lambda^{f=4}_{LO} = 0.2 \text{ GeV} \). The LO charm quark contribution in Eq. (8) is defined as

\[ \frac{1}{x} F_2^c(x,Q^2,m_c^2) = 2e^2 \frac{\alpha_s(\mu'^2)}{2\pi} \int_{ax}^1 \frac{dy}{y} C_{g,2} \left(\frac{x}{y},\frac{m_c^2}{Q^2}\right) g(y,\mu'^2) \]

(14)

with \( a = 1 + 4m_c^2/Q^2 \) and in LO

\[ C^c_{g,2}(z,\frac{m_c^2}{Q^2}) = \frac{1}{2} \left\{ \left[ z^2 + (1-z)^2 \right] + z(1-3z) \frac{4m_c^2}{Q^2} - z^2 \frac{8m_c^2}{Q^4} \right\} \ln \frac{1+\beta}{1-\beta} + \beta \left[ -1 + 8z(1-z) - z(1-z) \frac{4m_c^2}{Q^2} \right] \}

(15)

where \( \beta^2 = 1 - (4m_c^2/Q^2)z(1-z)^{-1} \), \( \mu' = 4m_c^2 \) and \( m_c = 1.5 \text{ GeV} \). Figure 1 shows the extracted GRV's free proton and neutron structure function that used in Eq. (1). The related LO GRV's parton distribution Fortran code can be found in [11]. The calculated ratio

\[ R_{EMC}^A(x) = F_2^A(x)_{\text{per nucleon}} / F_2^H(x)_{\text{per nucleon}} \]

(16)
presented in figure 2. To better express deformation of bonded nucleon structure function in comparison with free nucleon structure function, the ratio

\[ R_A(x) = \frac{F_A^2(x)}{Z F_p^2 + N F_n^2} \]  

is calculated. Where here \( N \) is the number of neutron and the \( Z \) is the atomic number so \( A = Z + N \). The extracted results for \( R_A(x) \) are plotted in figure 4.

3 result, discussion and conclusion

Figure 1 shows the free nucleon structure function (read curve) that is used by S. V. Akulinichev et al, [2]. This is far from the GRV’s free nucleon structure function ((\( F_2^{GRV} + F_2^{GRV} \))/2) that is shown by brown curve [5,7]. It seems, if the corrected nucleon structure functions were used in Eq. (1), the calculated \( R_{EMC}^A(x) \) could be improved. It is necessary to notice that we can not use the nucleon structure function like read curve even brown in figure 1, instead of proton and neutron structure functions for the nuclei like nucleus \(^{56}\text{Fe} \) because of \( Z \neq N \). In this paper for extracting all the results, the GRV’s proton and neutron structure functions are used. The difference between the root mean square of shells encourages me to consider different \( \hbar \omega \) parameters for different shells inside a nucleus (see table 1). The plotted results in figure 2, 3, 4 extracted by using: a) Proton and neutron structure functions that have good agreement with experimental data, b) The \( \hbar \omega \) parameter that is related to shell’s root mean square for investigated nuclei. By considering the cases mentioned above, the binding energies \( \varepsilon_{nl} \) (see table 2) that are used to obtain the results in figure 2, seems to be less than those used in previous published paper. For example, compare \( \varepsilon_{nl} = -40 \text{ MeV} \) for \(^{56}\text{Fe} \) [2, 3, 16] with \( \varepsilon_{nl} = -26 \sim -32 \text{ MeV} \) which are used here to extract the result for the \(^{56}\text{Fe} \) that is shown in figure 2 (see table 2). The figure 3 shows that with this new used \( \hbar \omega \) parameters (see table 1), the contribution of only Fermi motion effects specially for \(^{56}\text{Fe} \) (full curve) is similar to the previous work [15] that is plotted by dash curves in this figure but we should notice the full curves rise up about 0.05 \sim 0.02 more than related Bjorken parameter scale. The reason of this effect is that the \( \hbar \omega \) parameters related to the root mean square radius are a bit larger than the \( \hbar \omega \) parameters for example used in 2. We see that the used harmonic oscillator parameters by Akulinichev, et al., are related to larger radius than those that the experimental data shows. In the medium and large \( x \) ranges the neutron structure function is smaller than the proton structure function so the extra 4 neutrons in the \(^{56}\text{Fe} \) nucleus cause the EMC ratio of iron comes down about 3 percent more than the other nuclei even by putting \( \varepsilon_{nl} = 0 \) in medium \( x \) range. In the figure 4 according to the Eq. (17) the ratio of investigated nuclei’s structure function to simple sum of equal free protons and neutrons GRV’s structure function plotted. One can find a bit difference in the extracted results for each nucleus because of different radius in the case of neglecting binding energy and considering only Fermi motion effect (dash curve). We can trust the results from \( x \sim 0.2 \) to \( x \sim 0.7 \) [17] But when the effect of binding energy added to the Fermi motion effect the difference between the ratio for different nuclei near the \( x \sim 0.7 \) is decreased (full curves). The magnitude of difference between full curve near the \( x \sim 0.7 \) is small enough that one interpret this effect as a result of saturation for large \( x \) ranges. The figure 4 shows the binding effect plays important role in medium \( x \) ranges.
Appendix A

To calculate root mean square radius we start from calculated total wave function for harmonic oscillator potential

\[ \psi(r) = R(r)Y(\theta, \varphi) = \frac{u(r)}{r}Y(\theta, \varphi). \]  

(18)

The equation governing the radial motion is

\[ -\frac{\hbar^2}{2m} \frac{d^2u(r)}{dr^2} + [l(l + 1) + V(r)]u(r) = Eu(r). \]  

(19)

in studying bound states, conditions have to be imposed on radial solution \( u(r) \):

\[ \lim_{r \to \infty} u(r) \to 0 \]

\[ u(0) = 0. \]  

(20)

the normalization of radial wave function leads to the integrals

\[ \int_0^\infty R^2(r)dr = \int_0^\infty u^2(r)dr = 1. \]  

(21)

for harmonic oscillator potential i.e., \( V(r) = \frac{1}{2}mw^2r^2 \) one can obtain the radial laguerre equation with the solution [18]

\[ u_{nl}(r) = N_{n,l}r^{l+1}e^{-\nu r^2}L_n^{l+1/2}(2\nu r^2), \]  

(22)

where \( \nu = m\omega/2\hbar \), \( N_{n,l} \) normalization factor and \( E = E_\omega(2n + l + \frac{3}{2}) \). For root mean square radius we have

\[ <r^2>_{nl} = \int \psi_n^*(r)r^2\psi_n(r)dv \]

\[ = \int \frac{u_n(r)}{r}r^2\frac{u_n(r)}{r}r^2dr \int Y^*(\theta, \varphi)Y(\theta, \varphi)d\Omega \]

\[ = \int_0^\infty u_n^2(r)r^2dr \]

\[ = \int_0^\infty \left\{ N_{n,l}r^{l+1}e^{-\nu r^2}L_n^{l+1/2}(2\nu r^2) \right\}^2 r^2dr \]

\[ = \int_0^\infty N_{n,l}^2(r^2)^{l+1}e^{-2\nu r^2}L_n^{l+1/2}(2\nu r^2)L_n^{l+1/2}(2\nu r^2)r^2dr. \]  

(23)

If we put \( x = 2\nu r^2 \) and \( dr = \frac{1}{4\nu}\sqrt{\frac{2\nu}{x}}dx \) then we have

\[ <r^2>_{nl} = \int_0^\infty N_{n,l}^2 \left( \frac{x}{2\nu} \right)^{l+1}e^{-x}L_n^{l+1/2}(x)L_n^{l+1/2}(x) \frac{x}{2\nu} \left[ \frac{\sqrt{2\nu}}{x} \right] dx \]

\[ = N_{n,l}^2 \left( \frac{1}{2\nu} \right)^{l+1} \frac{\sqrt{2\nu}}{8\nu^2} \int_0^\infty x^{l+\frac{3}{2}}e^{-x}L_n^{l+1/2}(x)L_n^{l+1/2}(x)dx. \]  

(24)
\[ \int_{0}^{\infty} x^{k+1} e^{-x} L_n^k(x) L_n^k(x) dx = \frac{(n+k)!}{n!} (2n+k+1). \]  

(25)

So we can write

\[ < r^2 >_{nl} = N_{n,l} \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{\sqrt{32\nu^3}} \int_{0}^{\infty} x^{(l+\frac{1}{2})+1} e^{-x} L_n^{l+1/2}(x)L_n^{l+1/2}(x) dx. \]  

(26)

By using Eq. (25) and putting \( k = l + \frac{1}{2} \), we have

\[ < r^2 >_{nl} = N_{n,l} \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{\sqrt{32\nu^3}} \frac{(n+l+\frac{1}{2})!}{n!} (2n+l+\frac{3}{2}). \]  

(27)

To calculate normalization factor

\[ \int_{0}^{\infty} u_{nl}^2(r) dr = 1 \]

\[ = \int_{0}^{\infty} \left\{ N_{n,l} r^{l+1} e^{-\nu r^2} L_n^{l+1/2}(2\nu r^2) \right\}^2 dr \]

\[ = N_{n,l}^2 \int_{0}^{\infty} (r^2)^{l+1} e^{-2\nu r^2} L_n^{l+1/2}(2\nu r^2)L_n^{l+1/2}(2\nu r^2) dr \]

(28)

If we put \( x = 2\nu r^2 \) then we have

\[ 1 = \int_{0}^{\infty} u_{nl}^2(r) dr = N_{n,l}^2 \int_{0}^{\infty} \left( \frac{x}{2\nu} \right)^{l+1} e^{-x} L_n^{l+1/2}(x)L_n^{l+1/2}(x) \frac{1}{4\nu} \sqrt{\frac{2\nu}{x}} dx \]

\[ = N_{n,l}^2 \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{8\nu} \int_{0}^{\infty} x^{l+1} e^{-x} L_n^{l+1/2}(x)L_n^{l+1/2}(x) x^{-\frac{1}{2}} dx \]

\[ = N_{n,l}^2 \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{8\nu} \int_{0}^{\infty} x^{l+\frac{1}{2}} e^{-x} L_n^{l+1/2}(x)L_n^{l+1/2}(x) dx \]

(29)

where [19]

\[ \int_{0}^{\infty} x^k e^{-x} L_n^k(x) L_m^k(x) dx = \frac{(n+k)!}{n!} \delta_{n,m}, \]  

(30)

so

\[ 1 = N_{n,l}^2 \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{8\nu} \int_{0}^{\infty} x^{l+\frac{1}{2}} e^{-x} L_n^{l+1/2}(x)L_n^{l+1/2}(x) dx \]

\[ = N_{n,l}^2 \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{8\nu} \frac{(n+l+\frac{1}{2})!}{n!} \]  

(31)

and

\[ N_{n,l}^2 = (2\nu)^{l+1} \sqrt{8\nu} \frac{n!}{(n+l+\frac{1}{2})!}. \]  

(32)
By substituting this normalization factor in Eq. (27) one can find

\[
<r^2>_n = (2\nu)^{l+1}\sqrt{8\nu} \frac{n!}{(n+l+\frac{1}{2})!} \left( \frac{1}{2\nu} \right)^{l+1} \frac{1}{\sqrt{32\nu^3}} \frac{(n+l+\frac{1}{2})!}{n!} (2n + l + \frac{3}{2})
\]

\[
= \frac{1}{2\nu}(2n + l + \frac{3}{2})
\]

\[
= \frac{1}{2\frac{m\omega}{2\hbar}}(2n + l + \frac{3}{2})
\]

\[
= \frac{\hbar}{m\omega}(2n + l + \frac{3}{2})
\]

(33)

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Figure captions

Figure 1: The LO GRV’s structure functions for proton (blue) and neutron (pink) at $Q^2 = 4 \text{ GeV}^2$ without charm quark contribution that is used in Eq. (1). Notice to the difference between the GRV’s nucleon structure function (brown) and the red curve that shows the used nucleon structure function in reference 2. The experimental data are taken from [12, 13].

Figure 2: The extracted $F^A_2(x)_{\text{Per nucleon}}/F^H_2(x)_{\text{Per nucleon}}$ for $A=^4\text{He}, ^{12}\text{C}, ^{40}\text{Ca}$ and $^{56}\text{Fe}$ from Eq. (1). The $F^N_2(x)$ are taken from GRV proton N=p and neutron N=n structure functions according to the Eq. (8) that plotted in figure 1. The used parameters $(g_{nl}^P, g_{nl}^N, \epsilon_{nl})$ and the calculated $\bar{h}\omega$ parameter are shown in table 1 and 2. The experimental data without error bar are taken from [11, 14].

Figure 3: The extracted $F^A_2(x)_{\text{Per nucleon}}/F^H_2(x)_{\text{Per nucleon}}$ for $A=^4\text{He}, ^{12}\text{C}, ^{40}\text{Ca}$ and $^{56}\text{Fe}$ from Eq. (1) by considering only Fermi motion effect. The full curves were obtained with the parameters explained in the caption of figure 2 by putting $\epsilon_{nl} = 0$. The dash curves with the same color for the same nucleus were obtained with the parameters $\bar{h}\omega$ of Reference [3].

Figure 4: The extracted $F^A_2/(NF^P + NF^H_2)$ for $A=4, 12, 40,$ and $56$ from Eq. (1). The used parameters are the same as parameters that are used to extract the results in figure 1. The dash curves with the same color for the same nucleus shows the extracted results by considering only the Fermi motion effect (i.e. $\epsilon_{nl} = 0$).
| Shell | $^2H$   | $^4He$   | $^{12}C$  | $^{28}Si$  | $^{40}Ca$  | $^{56}Fe$ |
|-------|---------|---------|---------|---------|---------|---------|
| 0s    | (2.09, 15.35) | (1.67, 22.23) | (1.67, 22.23) | (1.67, 22.23) | (1.67, 22.23) | (1.67, 22.23) |
| 0p    | (2.44, 17.36) | (2.44, 17.36) | (2.44, 17.36) | (2.44, 17.36) | (2.44, 17.36) | (2.44, 17.36) |
| 0d    | (3.10, 15.05) | (3.10, 15.05) | (3.10, 15.05) | (3.10, 15.05) | (3.10, 15.05) | (3.10, 15.05) |
| 1s    | (3.48, 11.95) | (3.48, 11.95) | (3.48, 11.95) | (3.48, 11.95) | (3.48, 11.95) | (3.48, 11.95) |
| 0f    | (3.74, 13.3)  | (3.74, 13.3)  | (3.74, 13.3)  | (3.74, 13.3)  | (3.74, 13.3)  | (3.74, 13.3)  |

Table 1: The brackets contain ($<r^2>_{nl}^{1/2}, \hbar\omega$). The oscillator-model parameters $\hbar\omega$ calculated from Eq. (6).

| Shell | $^2H$   | $^4He$   | $^{12}C$  | $^{28}Si$  | $^{40}Ca$  | $^{56}Fe$ |
|-------|---------|---------|---------|---------|---------|---------|
| 0s    | (1, 1, -1) | (2, 2, -15) | (2, 2, -22) | (2, 2, -20) | (2, 2, -30) | (2, 2, -32) |
| 0p    | (4, 4, -20) | (6, 6, -20) | (6, 6, -28) | (6, 6, -32) | (6, 6, -32) | (6, 6, -32) |
| 0d    | (6, 6, -18) | (10, 10, -26) | (10, 10, -30) | (10, 10, -30) | (10, 10, -30) | (10, 10, -30) |
| 1s    | (2, 2, -25) | (2, 2, -28) | (2, 2, -28) | (2, 2, -28) | (2, 2, -28) | (2, 2, -28) |
| 0f    | (6, 10, -26) | (6, 10, -26) | (6, 10, -26) | (6, 10, -26) | (6, 10, -26) | (6, 10, -26) |

Table 2: The brackets contain ($g_{nl}^p, g_{nl}^n, \epsilon_{nl}(MeV)$) for related shell.