Stellar Mass Function From SIM Astrometry/Photometry

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Abstract.

By combining SIM observations with ground-based photometry, one can completely solve microlensing events seen toward the Galactic bulge. One could measure the mass, distance, and transverse velocity of \( \sim 100 \) lenses to \( \sim 5\% \) precision in only \( \sim 500 \) hours of SIM time. Among the numerous applications are 1) measurement of the mass functions (MFs) of the bulge and disk 2) measurement of the relative normalizations of the bulge and disk MFs (and so their relative contribution to the Galactic potential), 3) measurement of the number of bulge white dwarfs and neutron stars (and so the initial MF well above the present turnoff). SIM astrometric measurements are simultaneously photometric measurements. SIM astrometry determines the angular size of the Einstein ring on the sky, and comparison of SIM and ground-based photometry determines the size of the Einstein ring projected onto the observer plane. Only by combining both of these measurements is it possible to completely solve the microlensing events.

1. Introduction

Microlensing observations toward the Galactic bulge are yielding important clues about the structure of the Milky Way (Udalski et al. 1994; Alcock et al. 1997). However, the only useful parameter that is usually extracted from a microlensing event is its timescale, \( t_e \), which is a complicated combination of the three parameters one would like to know about the lens, its mass \( M \), its distance \( d_L \), and its proper motion relative to the observer-source line of sight \( \mu \). Specifically,

\[
t_e = \frac{\theta_e}{\mu}, \quad \theta_e = \sqrt{\frac{4GM}{c^2D}},
\]

where \( \theta_e \) is the angular Einstein radius (the characteristic angular size over which the lens has a significant effect),

\[
D \equiv \frac{d_L d_s}{d_s - d_L},
\]

and \( d_s \) is the distance to the source. Thus, if one wants to use microlensing observations, for example, to measure the bulge mass function (MF), one can analyze the distribution of timescales (Zhao, Spergel, & Rich 1995; Han & Gould...
1996), but to do so one must make a whole series of model-dependent assumptions, such as the distributions of the source-lens relative velocities, the source distances, and the lens distances, and the proportion of events that are due to foreground lenses in the disk rather than in the bulge itself.

The scientific return from bulge microlensing observations would be increased many fold if it were possible to measure $M$, $D$, and $\mu$, separately for each event, especially if this were combined with measurements of $d_s$ and $\mu_s$, the distance and proper motion of the source. With these additional pieces of information, one could determine both $d_l$ and the absolute transverse velocity of the lens.

First, bulge and disk lenses could be separately identified (from their distances and kinematics) so that the bulge and disk MFs could be measured separately and unambiguously. Second, the relative normalizations of the bulge and disk MFs could be determined so that one would know how much of the Galactic potential was attributable to each structure.

Third, it would be possible to measure the number of white dwarfs and neutron stars in the bulge (Gould 1999). These stars are substantially too faint to be detected optically in the crowded bulge fields, but they would be easily revealed in a census of masses of bulge microlensing events. White dwarfs would show up as a spike in the MF at $M \sim 0.6 M_\odot$, and neutron stars have masses that are higher than those of turnoff stars. Note that the sharp white dwarf feature in the MF is spread out to a fractional width of $O(1)$ in the $t_e$ distribution, so the white dwarfs cannot be picked out from the timescales. The same is basically true of neutron stars since they are only $\sim 1.5$ times heavier than turnoff stars. Since white dwarfs and neutron stars are remnants of main-sequence stars with masses respectively $M_\odot < M_{\text{ms}} < 8 M_\odot$ and $M_{\text{ms}} > 8 M_\odot$, the specific frequency of these remnants would in turn yield information about the initial MF to very high masses.

Fourth, one could determine whether the bulge contains massive objects other than those associated with the observed stars. A very puzzling (but often overlooked) fact is that kinematic studies of ellipticals and spiral bulges typically yield mass-to-light ratios $M/L_V \sim 10 h \sim 7$, much higher than the only two populations for which we have unambiguous measurements: dynamical studies of globular clusters yield $M/L_V \sim 2–3$ (Pryor & Meylan 1993), and a complete census of stars in the disk (Gould, Bahcall, & Flynn 1997) combined with a surface brightness of the disk (Binney & Tremaine 1987) yield $M/L_V \sim 2$. It is usually assumed that the bulge MF differs dramatically from these other populations. However, it is quite possible that the bulge contains substantial quantities of dark matter, either in compact objects or in diffuse material (WIMPs). The MF of the luminous stars in the bulge has now been measured in both the optical (Holtzman et al. 1998) and the infrared (Zoccali et al. 1999), so that if the total MF were measured from microlensing it would be possible to distinguish among these various competing scenarios.

2. Decoding Microlensing Events with SIM

SIM observations can completely solve for the physical parameters of the lens and source, $M$, $D$, $\mu$, $d_s$, and $\mu_s$, by combining two seemingly unrelated ideas:
Boden, Shao, & Van Buren (1998) showed that it was possible to measure $\theta_e$ from astrometric measurements of the apparent source position. Gould (1995) showed that it was possible to measured the projected Einstein radius $\tilde{r}_e$ from photometric measurements of the event simultaneously from the Earth and a satellite in solar orbit. It is clear that if both $\theta_e$ and $\tilde{r}_e$ are measured, then one can measure $M$, $D$, and $\mu$.

$$M = \frac{c^2}{4G} \tilde{r}_e \theta_e, \quad D = \frac{\tilde{r}_e}{\theta_e}, \quad \mu = \frac{\theta_e}{t_e}. \quad (4)$$

Moreover, as we will show below, in the course of measuring $\theta_e$ astrometrically, one automatically measures $\mu_s$ and $d_s$. Thus, if SIM can really carry out these two measurements simultaneously, bulge microlensing events can be completely solved. How does this work?

### 2.1. Astrometry

Suppose that a lens and source are separated on the sky by $u \theta_e$, where $u = (\tau, \beta)$ is the separation in units of the Einstein radius, $\beta$ is the impact parameter of the event, $\tau = (t - t_0)/t_e$, and $t_0$ is the time of closest approach. Then the source will be split into two images with positions $u_\pm \theta_e$ and magnifications $A_\pm$,

$$u_\pm = \left[ u \pm \frac{\sqrt{u^2 + 4} - 2}{2} \right] \frac{u}{u}, \quad A_\pm = \frac{A \pm 1}{2}, \quad A = \frac{u^2 + 2}{u \sqrt{u^2 + 4}}. \quad (5)$$

The separation between the images ($\sim 2\theta_e$) is of order 100s of $\mu$as and so is far too small to be resolved by SIM with its 10 mas central fringe. However, as Boden et al. (1998) showed, the displacement of the image centroid from the “true” position of the source is

$$(A_+ u_+ + A_- u_- - u) \theta_e = \frac{u}{u^2 + 2} \theta_e, \quad (6)$$

and therefore has a maximum of $\theta_e/\sqrt{8}$ (at $u = \sqrt{2}$) and so is well within SIM’s capabilities.

Of course, just measuring the apparent position of the source does not by itself yield the displacement due to lensing. One must also know where the source would have appeared in the absence of lensing. To determine this, one must measure the distance $d_s$ (i.e. the parallax $\pi_s = AU/d_s$) and proper motion $\mu_s$ of the source at late times (when its apparent position is not influenced by the lens), then project its “true” position backwards to the time of the event.

In principle, it is also possible to measure $\tilde{r}_e$ from astrometry, but since the deviations caused by Earth’s motion are a higher order effect, this is not the most practical method (Gould & Salim 1999).
2.2. Photometry

The Einstein radius projected onto the plane of the observer is typically a few AU, and so the satellite and the Earth see significantly different events, with different impact parameters $\beta$ and different times of maximum $t_0$. (The timescales $t_e$ are also slightly different, but this is a higher order effect which we will ignore for the moment.) Hence, the position in the Einstein ring will differ by

$$\Delta u = (\Delta \tau, \Delta \beta)$$

where $\Delta \tau = (-t_{0,\text{sat}} + t_{0,\oplus})/t_e$ and $\Delta \beta = \beta_{\text{sat}} - \beta_{\oplus}$. By measuring $\beta$ (from the peak magnification) and $t_0$ (from the time of peak magnification) from the Earth and satellite, one can therefore measure $\Delta u$. It is then possible to determine $\tilde{r}_e$ by using

$$\tilde{r}_e = \frac{d_{\text{sat}}}{\Delta u},$$

where $d_{\text{sat}}$ is the distance to the satellite projected onto the plane of the sky.

Actually, there is a bit of a complication in that the impact parameter can be on either side of the lens so that $\beta_{\text{sat}}$ and $\beta_{\oplus}$ can each be of either sign, while the measurement of $\beta$ from the light curve is sensitive only to its square (i.e., its amplitude but not its sign) (Refsdal 1966; Gould 1994). Hence $\Delta \beta = \pm(\beta_{\text{sat}} \pm \beta_{\oplus})$ and so cannot be unambiguously determined simply by measuring $\beta_{\text{sat}}$ and $\beta_{\oplus}$. However, Gould (1995) showed that this ambiguity could be resolved using the small difference in $t_e$ as measured by the Earth and satellite.

3. SIM: Simultaneous Astrometry and Photometry

In fact, although SIM is designed to do astrometry, the astrometric measurements are done by counting photons over the central fringe. The sum of these photon counts is a photometric measurement. Thus SIM simultaneously does astrometry and photometry. Of course, for most purposes, photometry using 25 cm mirrors is not very interesting. However, in the present case, the fact that SIM is making the photometric measurement at several tenths of an AU from the Earth is what is crucial. For photon-limited measurements, the ratio of the fractional photometric error $\sigma_{\text{ph}}$ to the astrometric error $\sigma_\theta$ is given by

$$\sigma_{\text{ph}} = \frac{\sigma_\theta}{\theta_f}, \quad \theta_f \equiv \frac{\lambda}{2\pi d} \sim 2.5 \text{ mas},$$

where $d \sim 10$ m is the distance between the mirrors, $\lambda$ is the wavelength of the light, in this case taken to be $\lambda \sim 0.8 \mu$m, appropriate for bulge clump giants.

Gould & Salim (1999) showed that by combining such photometric measurements that can be generated simultaneously with SIM astrometric measurements, it should be possible to measure $M$, $D$, $\mu$, $d_s$, and $\mu_s$ all to better than 5% precision with about 5 hours of SIM time for bulge microlensing events with $I = 15$ sources. Over the SIM lifetime, there should be of order 100 such events, so about 100 mass measurements are possible.
Acknowledgments. This work was supported in part by grant AST 97-27520 from the NSF and in part by grant NAG5-3111 from NASA.

References

Alcock, C., et al. 1997, ApJ, 479, 119
Binney, J. & Tremaine, S. 1987, Galactic Dynamics, Princeton: Princeton University Press
Boden, A.F., Shao, M., & Van Buren, D. 1998, ApJ, 502, 538
Gould, A. 1994, ApJ, 421, L75
Gould, A. 1995, ApJ, 441, L21
Gould, A. 1999, ApJ, submitted (astro-ph/9906472)
Gould, A., Bahcall, J.N., & Flynn, C. 1997, ApJ, 482, 913
Gould, A. & Salim, S. 1999, ApJ, 524, 000
Han, C., & Gould, A. 1995, ApJ, 447, 53
Holtzman, J.A., Watson, A.M., Baum, W.A., Grillmair, C.J., Groth, E.J., Light, R.M., Lynds, R., & O’Neil Jr., E.J. 1998, AJ, 113, 1948
Pryor, C. & Meylan, G. 1993, in Structure and Dynamics of Globular Clusters, S.G. Djorgovski & G. Meylan, San Francisco: ASP, 357
Refsdal, S. 1966, MNRAS, 134, 315
Udalski, A., Szymanski, M., Stanek, K.Z., Kaluzny, J., Kubiak, M., Mateo, M., Krzeminski, W., Paczynski, B., & Venkat, R. 1994, Acta Astronomica, 44, 165
Zhao, H.S., Spergel, D.N., & Rich, R.M. 1995, ApJ, 440, L13
Zoccali, M., Cassisi, S., Frogel, J.A., Gould, A., Ortolani, S., Renzini, A., Rich, R.M. 1999, & Stephens, A. 1999, ApJ, submitted (astro-ph/9906452)