Microwave-assisted switching: optimal microwave field for a nanomagnet with surface anisotropy

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Abstract. We compute the optimal microwave field that induces the magnetization reversal of a nanomagnet modeled as a macrospin, while minimizing the total injected energy. Two complementary approaches, numerical and analytical, are presented. We then optimize the microwave field for a nanomagnet with a mixed uniaxial and cubic anisotropy. The optimal microwave fields are found to be modulated both in amplitude and frequency. We find that the cubic anisotropy, which is mainly due here to the surface contribution, has a major effect on the frequency profile, related to the variation of the effective field.

1. Introduction
Magnetic recording is a key technology in the field of high density information storage. In order to increase thermal stability small nanoparticles with high anisotropy may be used. However, high fields are then needed to reverse the magnetization but these are difficult to achieve in current devices. To overcome this so called magnetic recording trilemma, switching can be assisted by a microwave (MW) field. In 2003 Thirion et al.[1] showed that the combination of a DC applied field, well below the Stoner-Wohlfarth switching field, with a small MW field pulse can reverse the magnetization of a nanoparticle. Further experimental and theoretical studies proved that this process is more efficient with a chirped MW field [2]. Indeed, the MW field frequency can then synchronize with the magnetization precession, so that the energy pumping between the MW field and the magnetization can occur through a resonant process.

The aim of this work is to determine the optimal MW field that allows for the magnetization reversal of a nanomagnet while minimizing the total injected energy. We consider a single nanomagnet with homogeneous or nearly collinear magnetization that can be modeled as a macrospin with a mixed uniaxial and cubic anisotropy. The cubic contribution may stem from the cubic magneto-crystalline anisotropy or from surface effects as was shown in Ref. [3]. The study is carried out at zero temperature.

After introducing the model used to describe the dynamics of the magnetization, we present the two complementary approaches that we developed to optimize the MW field. Finally, we numerically compute the optimal MW field for a nanomagnet with a mixed anisotropy and investigate the effect of the cubic contribution.

2. Model and dynamics equation
The magnetic state of nanoparticle of volume $V$ is modeled by a macroscopic magnetic moment (macrospin) with constant magnitude $M = M_S m$, where $M_S$ is the saturation magnetization
and $\|\mathbf{m}\| = 1$. The nanomagnet is characterized by a mixed-anisotropy energy comprising a uniaxial term along the $z$ axis and a cubic term. In the presence of a DC magnetic field $\mathbf{H}_{DC}$, the energy density of the nanomagnet can then be written as

$$E_0/V = -K_2 m_z^2 + \frac{K_4}{2} \left( m_x^4 + m_y^4 + m_z^4 \right) - \mathbf{m} \cdot \mathbf{H}_{DC}.$$  \hspace{1cm} (1)

Introducing the anisotropy field $H_{an} = 2K_2/\mu_0 M_S$, the reduced field $\mathbf{h}_{DC} = \mathbf{H}_{DC}/H_{an}$ and the cubic anisotropy parameter $\zeta = K_4/K_2$, we define the reduced energy density of the nanomagnet by

$$\mathcal{E}_0 \equiv \frac{E_0}{2K_2V} = -\frac{m_z^2}{2} + \frac{\zeta}{4} \left( m_x^4 + m_y^4 + m_z^4 \right) - \mathbf{m} \cdot \mathbf{h}_{DC}.$$  \hspace{1cm} (2)

The trajectory of the magnetization is described by the Landau-Lifshitz-Gilbert (LLG) equation

$$(1 + \alpha^2) \frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{eff} - \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}_{eff})$$  \hspace{1cm} (3)

where $\alpha$ is the damping parameter of the nanomagnet, $\tau \equiv (\gamma H_{an}) t$ is the reduced (dimensionless) characteristic time, $\gamma \simeq 1.76 \times 10^{11}$ rad/(T.s) the gyromagnetic ratio, and $\mathbf{h}_{eff} = -\partial \mathcal{E}_0/\partial \mathbf{m}$ is the static effective field.

In the presence of a reduced microwave field $\mathbf{h}_{MW}(\tau)$, in Eq. (3) $\mathbf{h}_{eff}$ is replaced by $\mathbf{h}_{eff} + \mathbf{h}_{MW}(\tau)$. Then, the energy variation of the system is given by

$$\frac{\partial \mathcal{E}}{\partial \tau} = -\frac{\alpha}{1 + \alpha^2} \|\mathbf{m} \times (\mathbf{h}_{eff} + \mathbf{h}_{MW}(\tau))\|^2 - \mathbf{m} \cdot \frac{d\mathbf{h}_{MW}(\tau)}{d\tau}.$$  \hspace{1cm} (4)

Note that, contrary to a DC field which always tends to decrease the energy of the system, a MW field can be a source of energy. If properly designed, it can induce the switching of the magnetization from one minimum to another.

### 3. Optimization of the microwave field

Setting the magnetization initially in an energy minimum we seek the optimal MW field that triggers its reversal to another minimum in a given time $\tau_f$ while minimizing the total injected energy, defined by $E_{inj} = \int_0^{\tau_f} \left\| \mathbf{h}_{MW}^2(\tau) \right\| d\tau$. This amounts to finding the best compromise between the MW field pulse intensity and duration, so that the subsequent heating of the system will be minimized. For that, we proposed two complementary approaches [4, 5].

The numerical approach [4] is based on the optimal control theory. We define the cost functional

$$\mathcal{F} \left[ \mathbf{m}(\tau), \mathbf{h}_{MW}(\tau) \right] = \frac{1}{2} \| \mathbf{m}(\tau_f) - \mathbf{m}_f \|^2 - \frac{\eta}{2} \int_0^{\tau_f} \left\| \mathbf{h}_{MW}^2(\tau) \right\| d\tau$$  \hspace{1cm} (5)

where $\mathbf{m}(\tau_f)$ is the magnetization reached at the final time $\tau_f$, $\mathbf{m}_f$ is the magnetization at the second minimum (the target), and $\eta$ is a numerical control parameter. Introducing a Lagrange parameter, we then minimize this cost functional along the trajectory given by the driven LLG equation. Because of the non linearity of the problem this is done numerically using the Conjugate Gradient technique augmented by a Metropolis algorithm.

In the analytical approach [5], we first determine the critical MW field that exactly compensates for the effects of damping, thus maintaining the magnetization precession at any time. Assuming that the latter and the optimal MW field we seek have a similar shape, we find that the optimal MW field is given by

$$\mathbf{h}_{MW}^{opt}(\tau) = \frac{2\alpha}{1 + \alpha^2} \left[ -\mathbf{m} \times \mathbf{h}_{eff} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}_{eff}) \right].$$  \hspace{1cm} (6)
In the presence of this MW field, the magnetization trajectory can be described by the LLG equation with a negative damping parameter, which corresponds to an “amplified” precession with a frequency \( \omega = \frac{\| h_{\text{eff}} \|}{(1 + \alpha^2)} \).

Both approaches have been favorably compared in the case of a nanomagnet with uniaxial anisotropy \( (\zeta = 0) \). The optimal MW field is found to be modulated both in amplitude and frequency, and synchronized at any time with the magnetization precession. Its role is to drive the magnetization from the first minimum to a saddle point, then the damping induces the relaxation down to the second minimum. Moreover, we showed that the total injected energy \( E_{\text{inj}} \) is proportional to the damping parameter and to the static energy barrier between the saddle point and the first minimum.

4. Effect of the cubic anisotropy

Here we drop the DC field and assume that the nanomagnet is characterized by a mixed anisotropy energy. The cubic contribution can result from the magneto-crystalline anisotropy or stem from surface effects. We will restrict our study to moderate values of \( \zeta \) \((-1 < \zeta < 1)\) that have been observed in real nanoparticles [6]. The potential energy surface presents two minima at \( \mathbf{m} = (0, 0, 1) \) (up state) and \( \mathbf{m} = (0, 0, -1) \) (down state) separated by four saddle points with the same energy located in the \( xy \) plane, along the cube diagonals if \( \zeta > 0 \) or along the \( x, y \) axes if \( \zeta < 0 \). The magnetization is initially set in the up state. For \( \zeta \neq 0 \) the time evolution of the optimal MW field can not be expressed analytically. Hence, only the numerical approach has been used to optimize the MW field.

The optimal MW field was firstly determined for \( \zeta = 0.3 \) (Fig. 1). It is found to be modulated both in amplitude and frequency, and to lie mainly in the \( xy \) plane. Moreover, it is synchronized with the magnetization. It drives the magnetization away from the up state and to climb up the potential well, until it reaches the saddle point defined by \( \mathbf{m} = \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right) \) at \( \tau \approx 680 \). Next, the MW field vanishes and the magnetization, driven by damping, relaxes towards the down state. More runs of the program with the same parameters rendered nearly superposable MW fields, the magnetization trajectory crossing randomly any of the four saddle points.

![Figure 1. Optimal MW field (upper panel) and corresponding magnetization trajectory (lower panel) obtained numerically for \( \zeta = 0.3 \). The blue dotted line indicate the crossing of the saddle point. Parameters: \( \alpha = 0.05, t_f = 800, \eta = 0.01, \) number of points \( N = 15000 \).](image)

We optimized the MW fields for different values of \( \zeta \) (Fig. 2). As before, the optimal fields are found to be chirped and synchronized with the magnetization precession. As can be seen, the main effect of \( \zeta \) on the optimal MW field is to modify its frequency profile. This can be related to the variation of the effective field with \( \zeta \). When the magnetization goes from the up
state to one of the saddle points, the effective field magnitude $|\mathbf{h}_{\text{eff}}|$ varies from $1 - \zeta$ to $|\zeta|$. For low or negative values of $\zeta$, we thus expect the MW field frequency to decrease continuously during the pulse. On the other hand, for high values of $\zeta$ the frequency should be low at the beginning and increase during the pulse. This is what is observed for example for $\zeta = 0.8$.

![Figure 2](image)

**Figure 2.** Optimized MW fields obtained for different values of $\zeta$ (left) and corresponding frequencies (right). Same parameters as Fig. 1.

5. Conclusion
In this work, we computed the optimal MW that induces the switching of a nanomagnet modeled as a macrospin in an effective potential comprising a mixed anisotropy and a Zeeman term. After presenting the two complementary approaches that we developed, we used the numerical approach to optimize the MW field for different values of the cubic anisotropy parameter $\zeta$. We find that the optimal field is modulated both in amplitude and frequency, and that the frequency profile strongly depends $\zeta$. This result can be used to design MW fields to induce switching in real nanoparticles that often exhibit a complex energy anisotropy due to *inter alia* surface effects. From a fundamental point of view, it is also interesting to consider higher values of $\zeta$, since secondary minima can appear, creating many possible trajectories for the magnetization. Finally, it will be necessary to introduce the effects of temperature and determine if they can favor or prevent the microwave-assisted switching.

References
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