Multi-Higgs Doublet Model:
Can It Account for $l^+l^-\gamma\gamma$ Events?

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ABSTRACT
Motivated by recent experiments at LEP ($Z \rightarrow l^+l^-\gamma\gamma$, with $M_{\gamma\gamma} \simeq 59 GeV$), we examine the possibility that such events are explained by a Bjorken process with a resonant Higgs ($M_{h^0} \simeq 59 GeV$) decaying to $\gamma\gamma$. Our model is an $N$-doublet-Higgs model without CP-violation. Under the simplifying assumption that $(N-k)$ doublets $(k \geq 1)$ decouple from the fermion sector, we could explain the observed events if: i) $(N-1) \geq 2$ and ii) the masses of the corresponding $(N-1)$ neutral scalars are either completely or almost degenerate, i.e. the decay $Z \rightarrow l^+l^-\gamma\gamma$ proceeds now via several overlapping resonances.
Recently, the L3 Collaboration has found four events $Z \rightarrow l^+l^-\gamma\gamma$ with the invariant mass of the photon pairs $M_{\gamma\gamma} \simeq 59\text{GeV}$ \cite{1}. The total number of the corresponding analyzed $Z$-decays is $N_{\text{tot}} = 1.6 \times 10^6$ \cite{2} (up until mid-November 1992).

In this letter, we investigate the possibility that this process is realized via the Bjorken process $Z \rightarrow Z^* h^0$, where $Z^* \rightarrow l^+l^-$ and $h^0 \rightarrow \gamma\gamma$ ($M_{h^0} \simeq 59\text{GeV}$), within the simplest possible extensions of the minimal standard model (SM) - those with $N$ scalar isodoublets ($N \geq 2$). The observed decays can’t be explained within the minimal SM ($N = 1$), among other things because the branching ratios $\Gamma(h^0 \rightarrow \gamma\gamma)/\Gamma(h^0 \rightarrow \text{all}) \ll 1$ (e. g., the decays $h^0 \rightarrow \bar{b}b$ would be much more frequent than $h^0 \rightarrow \gamma\gamma$).

There are, in general, three problems when one tries to explain these high photon mass events by a Bjorken process. Each of these problems is connected to one of the vertices in the decay diagram.

1. The first problem is the deficit of the $Z \rightarrow \nu\bar{\nu}\gamma\gamma$ and $Z \rightarrow q\bar{q}\gamma\gamma$ events, relative to $Z \rightarrow l^+l^-\gamma\gamma$ events. Since the experimental results might be too tentative to draw definite conclusions on this point, we will not discuss it here.

2. The branching ratio $B(h^0 \rightarrow \gamma\gamma)$ should be of the order one. This is excluded in the minimal SM, but could be arranged in a multi-doublet-Higgs model (see below).

3. The third problem concerns the on-shell production rate for $Z \rightarrow Z^* h^0 \rightarrow l^+l^- h^0$. In SM, one predicts 1.3 such events if $M_{h^0} \simeq 59\text{GeV}$ and the total number of $Z$-decays is $1.6 \times 10^6$. This would make it difficult to reconcile the observed four events with the theory, even when $B(h^0 \rightarrow \gamma\gamma) \simeq 1$.

Below, we will concentrate primarily on the issue 3., taking the optimal assumption that $B(h_j^0 \rightarrow \gamma\gamma)$ is roughly one for all neutral ($CP = +1$) Higgses contributing to the process.

If we allow for $N \geq 2$, we can in principle find a scenario in which only one neutral ($CP = +1$) scalar (for example, the Nth one) couples to fermions, while all the other neutral scalars ($\{h_j^0\}, j = 1, \ldots, N - 1$) do not. The decoupling of ($N - 1$) neutral scalar Higgses from the fermion sector could ensure that $\Gamma(h_j^0 \rightarrow \gamma\gamma)$ is one of the dominant
decay channels. In order to have \( B(h_j^0 \rightarrow \gamma\gamma) \simeq 1 \), one has to tune the parameters of the theory. We will find that even under these optimal conditions, the experimental result can be explained by the Bjorken-type process only if the \( N \)-doublet model displays some “exotic” features like degeneracy of masses.

Let’s denote the \( N \) doublets \( \Phi_j \) \( (j = 1, \ldots, N) \) as:

\[
\Phi_j = e^{i\delta_j} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_j^{(1)} + i\phi_j^{(2)} \\ v_j + H_j^0 + iA_j^0 \end{pmatrix}
\]

\[
\langle \Phi_j \rangle_o = e^{i\delta_j} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_j \end{pmatrix}.
\]  

(1)

From the gauge boson-scalar interaction part

\[
\mathcal{L}_{gb-sc} = \sum_{j=1}^{N} (D_{\mu}\Phi_j)^\dagger (D^\mu\Phi_j)
\]

we then obtain

\[
\mathcal{L}^{Z\mu-Z\nu} = \frac{1}{4}(g^2 + g'^2) \left[ 4 \sum_{j=1}^{N} v_j H_j^0 \right] Z^\mu Z_\nu
\]

(3)

where the \( Z \)-mass term yields the condition

\[
\sum_{j=1}^{N} v_j^2 = v^2 \quad (= (246GeV)^2).
\]  

(4)

Assuming that there is no CP-violation (or: that it is reasonably small), i.e. \( \delta_j \)'s are zero, then the physical \( CP = +1 \) scalars \( h_j^0 \) are obtained by an orthonormal transformation \( O \) of \( H_j^0 \)'s, determined by the scalar potential \( V(\{\Phi_j\}) \) (there are no substantial admixtures of \( A_j^0 \)'s which have \( CP = -1 \))

\[
h_j^0 = O_{ji} H_i^0, \quad H_j^0 = O_{ij} h_i^0.
\]  

(5)

We assume that \( O \) is a diagonal block matrix made up of one \( (N-1) \times (N-1) \) and one \( 1 \times 1 \) diagonal block, i.e. that the \( (CP = +1) \) neutral scalar component of the doublet \( \Phi_N \) (the latter couples to fermions) doesn’t have quadratic mixing terms with the corresponding components of the doublets \( \{\Phi_j\}_{j=1,\ldots,N-1} \) in the Higgs potential \( V(\{\Phi_j\}_{j=1,\ldots,N}) \).
If we assume that there is just one $h_j^o$ that has $M_{h_j^o} \simeq (59 \pm 1)\text{GeV}$ ($j$ is fixed, $1 \leq j \leq N - 1$), then we have the following relations for the process $Z \to Z^* h_j^o \to l^+ l^- \gamma \gamma$

$$\frac{N_{\text{events}}}{N_{\text{decays}}} = \frac{\sum_{l=e,\mu} \Gamma(Z \to Z^* h_j^o \to l^+ l^- \gamma \gamma)}{\Gamma(Z \to \text{all})}$$

$$= \left(\frac{g_{h_j^o ZZ}}{g_{h_j^o ZZ}^{SM}}\right)^2 \left[\sum_{l=e,\mu} \frac{\Gamma^{SM}(Z \to l^+ l^- h^o)}{\Gamma(Z \to \text{all})}\right] \left[\frac{\Gamma(h_j^o \to \gamma \gamma)}{\Gamma(h_j^o \to \text{all})}\right]$$

$$= \kappa_j (0.84 \pm 0.13) \times 10^{-6} B(h_j^o \to \gamma \gamma) \leq \kappa_j (0.84 \pm 0.13) \times 10^{-6} \cdot (6)$$

The superscript $SM$ denotes the corresponding quantities in the minimal SM, with $M_{h^o} = (59 \pm 1)\text{GeV}$. Furthermore, we denoted

$$\kappa_j = \left(\frac{g_{h_j^o ZZ}}{g_{h_j^o ZZ}^{SM}}\right)^2 \cdot$$

The values of the ratio

$$\frac{\sum_{l=e,\mu} \Gamma^{SM}(Z \to l^+ l^- h^o)}{\Gamma(Z \to \text{all})} = (0.84 \pm 0.13) \times 10^{-6}$$

were obtained by using known formulas of the minimal SM (e. g. [3]), where the upper bound corresponds to $M_{h^o} = 58\text{GeV}$ and the lower bound to $M_{h^o} = 60\text{GeV}$.

For $N \geq 2$ case, there are several possible ways to increase $\Gamma(h_j^o \to \gamma \gamma)$ [4], by increasing the contribution of the decay $h_j^o \to \gamma \gamma$ proceeding via loops with charged Higgses. In such a case, we may expect

$$B \left(h_j^o \to \gamma \gamma\right) = \frac{\Gamma(h_j^o \to \gamma \gamma)}{\Gamma(h_j^o \to \text{all})} \sim 1 \cdot$$

because the decays $h_j^o \to \bar{f}f$ ($j \leq N - 1$) are not allowed at the tree level, and because the decays $h_j^o \to W^+ W^-$, $Z^* Z^*$ contribute in general at most a few percent to $\Gamma(h_j^o \to \text{all})$. Namely, the latter couplings are of similar strength as those in the minimal SM where $\Gamma(h^o \to W^+ W^-)$ and $\Gamma(h^o \to Z^* Z^*)$ have been calculated [4]. The inequality in (6) could in such a case be approximated by equality if the branching ratio $B(h_j^o \to \gamma \gamma)$ is close to one. However, the factor $\kappa_j$ in (6) is severely bounded from above:

$$\kappa_j = \left(\frac{g_{h_j^o ZZ}}{g_{h_j^o ZZ}^{SM}}\right)^2 \leq \left(\sum_{i=1}^{N} O_{ji} v_i \right)^2 = \left(\frac{\left(v^{\text{rot}}_j\right)}{v}\right)^2 \leq 1 \cdot (7)$$
because
\[ \sum_{j=1}^{N} v_j^2 = \sum_{j=1}^{N} \left( v_{\text{rot}}^j \right)^2 = v^2 = (246 \text{GeV})^2. \] (8)

Therefore, in such a case we obtain

\[ \frac{\sum_{l=e,\mu} N \left( Z \rightarrow l^+l^-\gamma\gamma \right)}{N \left( Z \rightarrow \text{all} \right)} \bigg|_{M_{\gamma\gamma} \approx 59 \text{GeV}} \leq \kappa_j(0.84) \times 10^{-6} < 0.84 \times 10^{-6}. \]

This would predict for \( N(Z \rightarrow \text{all}) = 1.6 \times 10^6 \) events

\[ \langle N \left( Z \rightarrow l^+l^-\gamma\gamma \right) \rangle \bigg|_{M_{\gamma\gamma} \approx 59 \text{GeV}} < 1.33. \] (9)

Since the L3 Collaboration has found four such events (among \( 1.6 \times 10^6 \) Z-decays), such a case appears to be unlikely, although not excluded.

However, there exists a scenario which would sufficiently increase the upper bound on the r. h. s. of relation (9), by increasing the “effective” \( \kappa \) in relation (6) beyond 1. Namely, let’s assume that all (or some) of those scalars \( h_j^o \) which don’t couple to fermions are degenerate in masses. For simplicity, we take that all \( h_j^o \) \((j = 1, \ldots, N - 1)\) are either completely or almost degenerate (\( h_N^o \) is the only neutral scalar with \( CP = +1 \) that couples to fermions at the tree level)

\[ M_{h_j^o} \simeq 59 \text{GeV} \quad (j = 1, \ldots, N - 1), \]

\[ | \Delta M_{h_j^o} | < \Gamma_{h_j^o}, \] (10)

and that their decay amplitudes to two photons are approximately equal. Hence, also the amplitudes

\[ \frac{A \left( Z \rightarrow Z^*h_j^o \rightarrow l^+l^-\gamma\gamma \right)}{g_{h_j^o ZZ}} = \frac{A_j}{g_{h_j^o ZZ}} \quad (j = 1, \ldots, N - 1) \]

would be approximately equal:

\[ \frac{A_1}{g_{h_1^o ZZ}} \simeq \frac{A_2}{g_{h_2^o ZZ}} \simeq \ldots \simeq \frac{A_{N-1}}{g_{h_{N-1}^o ZZ}}. \] (11)

In such a case, we would obtain

\[ \Gamma \left( Z \rightarrow l^+l^-\gamma\gamma \right) \bigg|_{M_{\gamma\gamma} \approx 59 \text{GeV}} \simeq \left[ \Gamma \left( Z \rightarrow Z^*h_j^o \rightarrow l^+l^-\gamma\gamma \right) \left( g_{h_j^o ZZ} \right)^2 \right] \left( \sum_{j=1}^{N-1} g_{h_j^o ZZ} \right)^2, \] (12)

\footnote{The constraint on the mass difference is required for the overlapping of the \((N - 1)\) resonances.}
where \( i \) is any (fixed) index between 1 and \( N - 1 \). Since the expression in the \([\cdots]\) -brackets is independent of the \( g_{h_i^{\nu\nu}} \) couplings, we effectively get in such a case in the relation (6)

\[
\kappa_j \mapsto \kappa_{\text{eff}} = \left[ \sum_{j=1}^{N-1} \frac{g_{h_i^{\nu\nu}}}{g_{h_i^{\nu\nu}}} \right]^2 = \left[ \sum_{j=1}^{N-1} \frac{(v_{\text{rot}})^j}{v} \right]^2.
\]

Due to the “Z-mass” constraint (8), we get for a given \( v_N = O_{N_i} v_i = (v_{\text{rot}})_N \) the maximum of \( \kappa_{\text{eff}} \) at

\[
\frac{(v_{\text{rot}})^1}{v} = \cdots = \frac{(v_{\text{rot}})^{N-1}}{v} = \frac{1}{\sqrt{N-1}} \sqrt{1 - \left( \frac{v_N}{v} \right)^2},
\]

\[
\kappa_{\text{eff}}^{\text{max}} = (N-1) \left[ 1 - \left( \frac{v_N}{v} \right)^2 \right] \leq (N-1).
\]

Therefore, in such a scenario, we have instead of relation (6) the following relation for the number of events

\[
\sum_{l=e,\mu} \frac{N (Z \to l^+l^-\gamma\gamma)}{N (Z \to \text{all})} \big|_{M_{\gamma\gamma} \approx 59\text{GeV}} = \kappa_{\text{eff}}^{\text{max}} \left[ (0.84) \times 10^{-6} \right] B (h_i^0 \to \gamma\gamma) \\
\leq (N-1) \left[ 1 - \left( \frac{v_N}{v} \right)^2 \right] \left[ (0.84) \times 10^{-6} \right] \times 1 \\
< (N-1)(0.84) \times 10^{-6}.
\]

The number of expected events is thus enhanced by a factor of \( (N-1) \), due to the degeneracy of \( (N-1) \) neutral scalar Higgses which makes the summation of their amplitudes become coherent.

This scenario can be realized by the following N-Higgs-doublet model. We introduce \( N \) complex \( SU(2)_L \)-doublet scalar fields \( \Phi_i \) \( (i = 1, \ldots, N) \) with \( Y = 1 \), where only \( \Phi_N \) is the standard Higgs doublet that couples to fermions (but with VEV \( v_N < v = 246\text{GeV} \)). We assume that the other \((N - 1)\) doublets decouple from fermions, by requiring, for example, a global \( U(1) \) symmetry \( \Phi_i \to e^{i\alpha} \Phi_i \) \( (i = 1, \ldots, N - 1) \). A Higgs potential which spontaneously breaks \( SU(2)_L \times U(1)_Y \) down to \( U(1)_{em} \) and has degenerate mass for the \( N - 1 \) extra neutral scalar Higgses can be written as

\[
V (\{\Phi_i\}) = \lambda_N \left( \Phi_i^\dagger \Phi_N - v_N^2 \right)^2 +
\]
\[ + \lambda_1 \sum_{i=1}^{N-1} \left( \Phi_i^{\dagger} \Phi_i - v'^2 \right)^2 + \]
\[ + \lambda_2 \sum_{i<j}^{N-1} \left[ \left( \Phi_i^{\dagger} \Phi_j + \Phi_j^{\dagger} \Phi_i - 2v'^2 \right)^2 - 2 \left( \Phi_i^{\dagger} \Phi_i - v'^2 \right) \left( \Phi_j^{\dagger} \Phi_j - v'^2 \right) \right] \]
\[ + \sum_{i<j}^{N-1} \lambda_{ij} \left( \Phi_i^{\dagger} \Phi_j \Phi_j^{\dagger} \Phi_i - \Phi_i^{\dagger} \Phi_j \Phi_j^{\dagger} \Phi_i \right) \]
\[ - \sum_{i<j}^{N-1} \lambda'_{ij} \left( \Phi_i^{\dagger} \Phi_j - \Phi_j^{\dagger} \Phi_i \right)^2, \]
\( (17) \)

where all the \( \lambda_i, \lambda_{ij} \) and \( \lambda'_{ij} \) are real parameters, the potential is bounded from below, and the minimum of the potential is manifestly at
\[ \langle \Phi_N \rangle_o = \left( \begin{array}{c} 0 \\ v_N \end{array} \right) \quad \langle \Phi_i \rangle_o = \left( \begin{array}{c} 0 \\ v_i' \end{array} \right) \quad (i = 1, \ldots, N - 1) . \]

In this potential, the neutral \((CP = +1)\) Higgs masses are given by
\[ M_{h_i^0}^2 = 4v'^2 [\lambda_1 + (N - 2) \lambda_2] \quad (i = 1, \ldots, N - 1) \]
\[ M_{h_N}^2 = 4v_N^2 \lambda_N . \]

(19)

The charged Higgs masses are determined by \( \lambda_{ij} \)'s, and the “pseudoscalar” Higgs \((CP = -1)\) masses by \( \lambda'_{ij} \)'s.

A soft breaking of the degeneracy could occur by replacing \( \lambda_1, v' \) and \( \lambda_2 \) in (17) by slightly doublet-dependent values, and by adding (small) terms to the potential (17):
\[ \Delta V = \sum_{i<j}^{N-1} \lambda_{ij}^{(3)} \left( \Phi_i^{\dagger} \Phi_i - (v'_i)^2 \right) \left( \Phi_j^{\dagger} \Phi_j - (v'_j)^2 \right) , \]
\( (20) \)

where
\[ | \lambda_{ij}^{(3)} | \ll | \lambda_1 |, \ | \lambda_2 | \quad \text{and} \quad | v'_i - v' | \ll v' . \]

In order to have the enhancement mechanism leading to (16) also in such a case, the resulting mass differences should satisfy \(| \Delta M_{h_j^0} | < \Gamma_{h_j^0} \), in order for the coherence conditions (11) to survive.

Note that, according to the relation (16), the described mechanism would predict for \( N_{tot} = 1.6 \times 10^6 \) \( Z \)-decays (and for \( \sqrt{1 - \left( \frac{v_N}{v} \right)^2} \simeq 1 \)) the number of \(( Z \to Z^* h^0 \to l^+ l^- \gamma \gamma) \)
events (with $M_{\gamma\gamma} = M_{h^0} \simeq 59 GeV$) as the following function of $(N - 1)$ (the number of Higgs doublets which don’t couple to fermions):

$$\langle N_{\text{events}} \rangle \simeq 1.35 \times (N - 1)$$

for $N_{\text{tot}} = 1.6 \times 10^6$ and $M_{\gamma\gamma} = 59 GeV \cdot (21)$

| $N - 1$ | $\langle N_{\text{events}} \rangle \simeq$ |
|---------|-----------------|
| $1$     | $1.35$          |
| $2$     | $2.69$          |
| $3$     | $4.04$          |
| $4$     | $5.38$          |

If we now relax our condition that the branching ratios $B(h_j^0 \rightarrow \gamma\gamma)$ for the $(N - 1)$ degenerate Higgses are the same and of order one, then the required number of Higgs doublets would increase. This would make an already exotic model even more exotic.

There are several other experiments at LEP (DELPHI, ALEPH, OPAL) which will possibly also yield the events $Z \rightarrow l^+l^-\gamma\gamma$ with $M_{\gamma\gamma} \simeq 59 GeV$ (out of $1.6 \times 10^6$ Z-decays). However, due to small number of these events, the possible statistical fluctuations are and will probably remain quite large. The four events of the L3 Collaboration would suggest, within our proposed scenario, that the number of the degenerate neutral ($CP = +1$) scalars (with the mass $\simeq 59 GeV$) is two or more (i.e. $(N - 1) \geq 2$).

Other authors have discussed a similar scenario, with $N - 1 = 1$ (no degeneracy).

According to the model, the LEP-experiments should observe also events with two photons and missing energy (due to $\bar{\nu}\nu$), and events with $\bar{q}q\gamma\gamma$, at $M_{\gamma\gamma} \simeq 59 GeV$.

The arguments of this paper remain basically unchanged when we assume that $k$ scalars $h_j^0 (k \geq 1; j = N - k + 1, \ldots, N)$ couple to fermions at the tree level, and that $(N - k - l) (l \geq 0)$ scalars $h_j^0 (j = 1, \ldots, N - k - l)$ are degenerate at $M \simeq 59 GeV$. In such a case, the factor $(N - 1)$ in the relation (16) would be replaced by $(N - k - l)$ and $\left[ 1 - \left( \frac{v_{\text{rot}}}{v} \right)^2 \right]$ by $\left[ 1 - \sum_{j=N-k-l+1}^{N} (\frac{v_{\text{rot}}}{v})^2 \right]$. In such a case, the L3 results would suggest $(N - k - l) \geq 2$. Such a more general scenario would allow for additional Bjorken processes at $l$ other resonant energies $M_{\gamma\gamma} = M_{h_j^0} (j = N - k - l + 1, \ldots, N - k)$, with the corresponding $\kappa_j = \left[ \frac{(v_{\text{rot}}/v)}{\kappa} \right]^2 (< 1)$ in the relation (6).
Conclusions and Discussions

If the four events \( Z \rightarrow l^+l^-\gamma\gamma (M_{\gamma\gamma} \simeq 59\text{GeV}) \) detected by L3 Collaboration \([1]\) are Bjorken-type processes \( Z \rightarrow Z^* h^0 \rightarrow l^+l^-\gamma\gamma \) and if the theory behind them is a (minimal) extension of SM with several \((N)\) Higgs doublets and no (or small) CP-violation in the Higgs sector, then we expect \( N \geq 3 \) and two or more neutral scalars (with \( CP = +1 \)) would have degenerate masses \( M_{h^0_j} \simeq 59\text{GeV} \) (or: almost degenerate, with \( \Delta M_{h^0_j} < \Gamma_{h^0_j} \)). Within such a model, it appears unlikely that the number of the observed events can be explained without the mass degeneracy of scalars \( h^0_j \) (which don’t couple to fermions).

The possibly ideal degeneracy of the \((N - 1)\) (or: \((N - l - k)\)) Higgs scalars \( h^0_j \) could result from an as yet unknown symmetry of the Lagrangian.

Note Added

After completing this paper, we were informed that the DELPHI Collaboration \([7]\) had recently observed (out of more than \(10^6\) \(Z\) decays) two such events with \( M_{\gamma\gamma} \simeq 59\text{GeV} \). Combining these results with those of the L3 Collaboration leads again to the suggestion (within the described scenario) that \((N - 1) \geq 2\).

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