A possible minimum toy model with negative differential capacitance for self-sustained current oscillation

Gang Xiong,1 Z. Z. Sun,2 and X. R. Wang2

1Physics Department, Beijing Normal University, Beijing 100875, P. R. China
2Physics Department, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong SAR, China

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We generalize a simple model for superlattices to include the effect of differential capacitance. It is shown that the model always has a stable steady-state solution (SSS) if all differential capacitances are positive. On the other hand, when negative differential capacitance is included, the model can have no stable SSS and be in a self-sustained current oscillation behavior. Therefore, we find a possible minimum toy model with both negative differential resistance and negative differential capacitance which can include the phenomena of both self-sustained current oscillation and \( I - V \) oscillation of stable SSSs.

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I. INTRODUCTION

Following the early pioneering study1,2 on vertical electron transport in superlattices (SLs), one of the recent surprising discoveries is self-sustained current oscillations (SSCOs) under a dc bias3.4. A large number of experimental and theoretical studies have focused on different aspects of these oscillations. Experimentally, it is known that SSCOs can be induced by varying the doping density5, temperature and magnetic field6,7. Theoretically, it is understood that SSCOs are accompanied by the motion of boundaries of electric field domains (EFDs)8,9. A discrete drift (DD) model capable of describing both the formation of stationary EFDs and SSCOs emerged after many tedious analyses and numerical calculations10,11,12. Our understanding of SSCOs was greatly advanced through numerical investigations of this model13. However, this nonlinear dynamical model is too difficult to be solved analytically in a way that one can understand its different types of attractor solutions and their bifurcations. Thus, a minimum model with certain solvability may give some help for understanding these phenomena.

Recently Wang and Niu(WN)14 proposed a simple model with many nice features. This model is capable of revealing analytically the source of instabilities in the electron transport. However, we can show that this model always has at least one stable steady-state solution (SSS) under a dc bias. Thus it does not support a SSCO phenomenon. The question that we want to address is whether it is possible to find a minimum model which is simpler than the DD model. We shall show that a generalized WN model, by including the negative differential capacitance, may have no stable SSS, thus it can be a possible minimum toy model for SSCO in superlattices. This paper is organized in the following way. In Sec.III we shall show that there is always a stable SSS in the original WN’s model no matter what kind of \( I - V \) curve is used for each barrier. Then we will derive a generalized WN model by including negative differential capacitance in Sec.IV. In Sec.IV we shall show that this toy model with both negative differential capacitance and NDR can include both \( I - V \) oscillations of stable SSSs and SSCO behaviors if the model parameters are properly selected.

II. EXISTENCE OF A STABLE STEADY-STATE SOLUTION IN THE ORIGINAL WN MODEL

Let us first briefly review WN’s model. WN employed the following three assumptions for their model of a superlattice of \( N \) quantum wells. (1) Inside each quantum well, the electronic states are described fully by time-independent quantum mechanics. (2) Charge carriers are in local equilibrium with each well so that a chemical potential can be defined locally. The chemical potential difference between two adjacent wells is call the bias \( V \) between them. (3) For a given bias \( V \) between two adjacent wells, a current \( I(V) \) passes through the barrier between them. In the following, \( I_i(t) \) and \( V_i(t) \) denote the electric current and bias through the \( i \)-th barrier, \( n_i(t) \) the free charge in the \( i \)-th well. The current \( I_i(t) \) is supposed to depend on the bias \( V_i(t) \) only. WN thus obtain the following dynamical equations

\[
\frac{dn_i(t)}{dt} = I_{i-1}(t) - I_i(t),
\]

\[
V_i(t) - V_{i-1}(t) = kn_i(t),
\]

\[
\sum_i V_i(t) = U,
\]

where

\[
k = \frac{4\pi}{eS}
\]

Eq.(1) and Eq.(2) are the charge conservation law and
For Eq. (1)–(3), there is at least one stable SSS when the total bias $U = U(I)$ is always a stable SSS when the total bias $U$. If the above SSS itself is stable, then it can be the SSS we look for. On the contrary, if the above SSS is unstable, then by Eq. (10), we have $U(I) = \sum_{i=1}^{N} V_i(I) \leq NV_{max}$ which means that $U(I)$ is in the region $[0, NV_{max}]$. Thus there is also a stable SSS in this case since we have shown that there is always one stable SSS when $U = [0, NV_{max}]$. Therefore, there is always a stable SSS when the total bias $U = U(I)$. Since $I$ is regarded as infinitely small, by Eq. (11) the value $U(I) - NV_{max}$ is also infinitely small. Thus we can say that we have proven the existence of a stable SSS when $U \in [0, U_{max}]$. Now let us further decrease $I$ by $dI$ and consider the corresponding SSS of the total bias $U = U(I_{max} - 2dI)$ with only the first barrier in the NDR region. Similar to Eqs. (9)–(11), we can obtain that this SSS in unstable when and only when $U(I_{max} - 2dI) \leq U(I_{max} - dI)$. Therefore, we can prove the existence of a stable SSS when $U \in [0, U(I_{max} - 2dI)]$. By decreasing $I$ step by step, we can go on with this procedure before $I = I_{min}$. When $I = I_{min}$, the SSS we consider is as follows: the bias of the first barrier is $V_1 = V_{A}$ and the biases of all other barriers are $V_i = V_{B}$. Thus we have proven the
existence of a stable SSS for $U \in [0, V_A + (N - 1)V_{\min}]$.

Then, let us consider the following series of SSSs and increase the equilibrium current $I$ from $I_{\min}$ to $I_{\max}$: the first barrier in a part of its upper PDR region, i.e., $V_1 \in (V_{\min}, V_B)$ and all other barriers in a part of their lower PDR regions, i.e., $V_i \in (V_A, V_{\max})$. It is obvious that this series of SSSs are stable because all barriers are in PDR regions. Thus we have proven the existence of a stable SSS for $U \in [0, V_B + (N - 1)V_{\max}]$ when $I$ is increased to $I_{\max}$. At this point, the first barrier is in its upper-bias PDR region while all other barriers are at the cross point between the lower-bias PDR region and the NDR region. To go further, we can decrease $I$ and keep the first barrier in its upper-bias PDR region, let the second barrier in its NDR region and all other N-2 barriers in their lower-bias PDR regions. Following the above procedure, by decreasing $I$ step by step we can arrive at an SSS that the biases of the first and second barrier are $V_1 = V_2 = V_{\min}$ while all other N-2 barriers are at $V_i = V_A$. Thus we have proven the existence of a stable SSS for $U \in [0, 2V_A + (N - 2)V_{\min}]$. Then, follow the above procedure and consider the following series of SSSs and increase the equilibrium current $I$ from $I_{\min}$ to $I_{\max}$: $V_1 \in (V_{\min}, V_B)$ and $V_2 \in (V_{\min}, V_B)$ while $V_i \in (V_A, V_{\max})$ for all other barriers. Thus we have proven the existence of a stable SSS for $U \in [0, 2V_B + (N - 2)V_{\max}]$ when $I$ is increased to $I_{\max}$. Then, we decrease $I$, and keep the first and second barriers in their upper-bias PDR regions, let the third barrier in its NDR region and all other N-3 barriers in their lower-bias PDR regions. Following the above procedure, we can prove the existence of a stable SSS for $U \in [0, 3V_B + (N - 3)V_{\max}]$ and so on so forth. Finally, we can prove the existence of a stable SSS for $U \in [0, NV_{\max}]$. QED.

Summarize the above proof, we can find the following general method to prove the existence of a stable SSS. Suppose that for some $U_0 > 0$ we have proven the conclusion that there is always a stable SSS for $U \in (0, U_0)$. Then, in order to extend the conclusion to a larger region $U \in [0, U_1]$ with $U_1 > U_0$, all we need to do is to find a series of SSSs that fulfill the following two conditions: (1) the maximum number of barriers in NDR regions for each SSS is one; (2) the total bias of the series of SSSs covers the region between $U_0$ and $U_1$. Using this method, we can extend the conclusion of Claim 1 to a general case and obtain

Claim 2. For Eq (4 - 3), there is always one stable SSS for each value of $I$ when $V$ curves of barriers are not necessarily the same while each barrier has only one NDR region.

Proof. Since $I - V$ curves of barriers are not necessarily the same, let us denote $(I_{\max,i}, V_{\max,i})$ and $(I_{\min,i}, V_{\min,i})$ as the maximum and minimum currents and the corresponding biases in the I-V curve of the $i$-th barrier. Without loss of generality, let us suppose that $I_{\max,1} \leq I_{\max,2} \leq \cdots \leq I_{\max,N}$ as shown in Fig 2. Let us start by considering a series of SSSs with the first barrier lying between the origin and $C_1$ while all other barriers lying in their low-bias PDR regions. Let us increase the equilibrium current of these SSSs $I$ continuously from zero to $I_{\max,1}$. This series of SSSs are stable because all barriers are in PDR regions. Now we arrive at an SSS with the first barrier at $C_1$ while all other barriers in their low-bias PDR regions. Let us denote the total bias of this SSS as $U_0$. Then, we have proven that there is always a stable SSS for $U \in [0, U_0]$. The rest of the proof is to find a series of SSSs that fulfill: (1) the maximum number of barriers in NDR regions for each SSS is one; (2) the total bias of the series of SSSs covers the region $(U_0, +\infty)$. Following the method in Claim 1, let us decrease $I$ from $I_{\max,1}$ to $I_{\min,1}$ and consider a series of SSSs with the first barrier lying between $C_1$ and $D_1$ and the second lying between $D_2$ and $C_2$ while all other barriers lying in their low-bias PDR regions. Then we arrive at an SSS with the first barrier at $D_1$ and the second barrier at $C_2$ while all other barriers lying in their low-bias PDR regions. Let us denote the total bias of this SSS as $U_1$. Thus we have proven that there is always a stable SSS for $U \in [0, U_1]$. Then, keep the first barrier in its high-bias PDR region, let $I$ increase, and move the second barrier along the trace $C_2 \rightarrow D_2 \rightarrow E_2 \rightarrow F_2$ and other barriers correspondingly. If $I_{\min,2} \geq I_{\min,1}$, then the second barrier can cross $G_2$ when the first barrier is still in its high-bias PDR region. If $I_{\min,2} < I_{\min,1}$, then the second barrier reaches $F_2$ when the first barrier reaches $D_1$. At this point, the second barrier can not move continuously from $F_2$ to $H_2$ with the above method applicable. In order to fill the sudden jump from $F_2$ to $H_2$, we can do as follows: move the first barrier from $D_1$ to $B_1$ while all other barriers are unmoved. This decreases the total bias. According to the above proof, there is a stable SSS for this total bias. Then, we can first move the second barrier from $F_2$ to $H_2$ while keep all other barriers in their low-bias PDR regions, then move the first barrier along $B_1 \rightarrow C_1 \rightarrow D_1$ while keep the second barrier in its high-bias PDR region and other barriers in their low-bias PDR regions. By doing this, we let the second barrier cross its minimum current point while keeps the number of NDR barriers in this series of SSSs less than

FIG. 2: The curve of $I$ vs. $V$ for (a) the first barrier, (b) the second barrier, and (c) the third barrier. The minimum current and maximum currents of them are not same.
or equal to one.

Now let us consider the third barrier. If \( I_{\text{min},3} \) is larger than \( I_{\text{min},1} \) and \( I_{\text{min},2} \), then it is easy to let the third barrier cross its minimum current point. If \( I_{\text{min},3} \) is less than one of \( I_{\text{min},1} \) and \( I_{\text{min},2} \), say, \( I_{\text{min},3} < I_{\text{min},1} \) while \( I_{\text{min},3} > I_{\text{min},2} \) as shown in Fig 2, then we can use the method in the proceeding paragraph to let the third barrier cross its minimum current point. If \( I_{\text{min},3} \) is less than both \( I_{\text{min},1} \) and \( I_{\text{min},2} \), then we can do as follows.

When the third barrier reaches \( E_3 \), move the first barrier from \( D_1 \) to \( B_1 \). Then, move the second barrier from \( G_2 \) to \( B_2 \) when the third barrier reaches \( F_3 \). After that, let the third barrier cross its minimum current point while keeping all other barriers in their low-bias PDR regions. Then let the first barrier cross its minimum current point while keeping the third barrier in its high-bias PDR region and the second barrier in its low-bias PDR region. When this is finished, let the second barrier cross its minimum barrier with the use of the above method. Thus the first, second and third barriers cross their minimum current point. Keep this go on, we can cross the NDR regions of all \( N \) barriers. This finishes the proof because the conclusion is obvious when all NDR regions are crossed.

**QED.**

In the above proof, we restrict to the case that there is only one NDR region in \( I - V \) curve of each barrier. It is straightforward to generalize the above proof for general \( I - V \) curves that may have a lot of NDR regions separated by PDR regions. In fact, one can show that the above method works if the following condition is fulfilled: each barrier always lies in PDR region when the absolute value of its bias is above some upper-limit value. This condition is generally fulfilled. Therefore, we can come to the conclusion that there is always a stable SSS in WN’s original model no matter what kind of \( I - V \) curve is used for each barrier.

**III. A GENERALIZED MODEL WITH DIFFERENTIAL CAPACITANCE**

In this section, we shall derive a generalized WN model and introduce negative differential capacitance. In the generalized model, we consider each barrier as a narrow homogeneous dielectric material. Then, instead of using the Poisson equation, we make use of the Gauss’ law for dielectric materials

\[
[D_i(t) - D_{i-1}(t)]S = 4\pi n_i(t) \tag{14}
\]

where \( D_i(t) \) the electric displacement in the \( i \)-th barrier and \( S \) the transverse area of the superlattice. We assume that \( D_i(t) \) only depends on the average electric field \( E_i(t) = V_i(t)/L_i \), \( L_i \) the thickness of the \( i \)-th barrier. Eq. (14) still holds since it is the charge conservation law. Eliminate \( n_i(t) \) by Eq. (14) and Eq. (14), we have

\[
C_i(V_i) \frac{dV_i}{dt} - C_{i-1}(V_{i-1}) \frac{dV_{i-1}}{dt} = I_{i-1}(V_{i-1}) - I_i(V_i) \tag{15}
\]

and

\[
\sum_{i=1}^{N} \frac{dV_i(t)}{dt} = 0 \tag{16}
\]

where

\[
C_i(V_i) = \frac{S}{4\pi} \frac{dD_i}{dV_i} = \frac{S}{4\pi L_i} \frac{dD_i}{dE_i} \tag{17}
\]

One can see that \( C_i(V_i) \) serves as the differential capacitance of the \( i \)-th barrier because \( dD_i/dE_i \) can be viewed as the differential dielectric function of it. When the conventional relation \( dD_i/dE_i = \epsilon \) is substituted into Eq. (17), it returns to WN’s original model. Thus the above model is a generalized version of WN’s model with bias-dependent differential capacitance \( C_i(V_i) \).

Take a SSS of Eq. (17) and (16)

\[
V_i(t) \equiv V_i^0 \tag{18}
\]

which should satisfy

\[
I_i - I_{i-1}(V_i^{0}) = I_i(V_i^{0}) \tag{19}
\]

and

\[
\sum_{i=1}^{N} V_i^0 = U \tag{20}
\]

Follow the standard analysis of linear stability analysis as WN did, we take a time-dependent perturbation of the following form

\[
V_i(t) = V_i^{0} + A_i \exp(\lambda t) \tag{21}
\]

Substitute it into Eq. (15) and (16) and take the linear terms of \( \delta V_i(t) \), we have

\[
C_i^{0} \lambda + G_i^{0} A_i = C_{i-1}^{0} \lambda + G_{i-1}^{0} A_{i-1} - \sum_{i=1}^{N} A_i = 0 \tag{22}
\]

where

\[
C_i^{0} = C_i(V_i^{0}) \tag{23}
\]

\[
G_i^{0} = \frac{dA_i}{dV_i} \bigg|_{V_i=V_i^{0}} \tag{24}
\]

are the differential capacitance and differential conductance of the \( i \)-th barrier at \( V_i = V_i^{(0)} \). The solution of Eq. (22) has the following form

\[
A_i = \frac{A}{C_i^{0} \lambda + G_i^{0}} \tag{25}
\]

where \( A \) is a non-zero constant. Substitute it back into Eq. (22), we find that a non-zero solution should satisfy

\[
F(\lambda) = \sum_{i=1}^{N} \frac{1}{C_i^{0} \lambda + G_i^{0}} = \sum_{i=1}^{N} \frac{1/C_i^{0}}{\lambda + C_i^{0}/G_i^{0}} = 0 \tag{26}
\]

The SSS is stable if and only if none of \( \lambda_i \) satisfying Eq. (26) is positive.
FIG. 3: Curves of $I$ and $D$ vs. $V$ for two special cases which can lead to SSCO. (a) Barriers with negative differential capacitance and with no NDR region; (b) barriers with two bias regions of negative differential capacitance and with an NDR region in between.

IV. EFFECT OF DIFFERENTIAL CAPACITANCE

In this section, we shall discuss the effect of differential capacitance on stability of the SSSs. We shall first consider the case when all the differential capacitances are positive, i.e., $C_i^0 > 0$. As shown in Eq. (26) in Sec III, an SSS is stable if and only if none of $\lambda_i$ satisfying $F(\lambda_i) = 0$ is positive. In the case when $C_i^0 > 0$ for all barriers, let us denote $\lambda_i = -G_i^0/C_i^0$ and $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$. WN found a method to analyze the root of $F(\lambda) = 0$, and it is easy to show that their method works when all $C_i^0$’s have the same sign. Following WN’s method we obtain : (1) $F(\lambda) = 0$ has no negative root when $\lambda_i > 0 > 0$ for all barriers; (2) when $\lambda_1 < 0$ and $\lambda_2 > 0$, $F(\lambda) = 0$ has one negative root if and only if $F(\lambda = 0) > 0$. Since $C_i^0 > 0$ for all barriers, the above conclusions lead to the same conclusions mentioned in Sec III: (1) an SSS is stable if all barriers have PDR, i.e., $G_i^0 > 0$; (2) if one and only one barrier has NDR, then the SSS is stable when $F(\lambda = 0) = \sum_{i=1}^{N} [G_i^0]^{-1} = \sum_{i=1}^{N} [F_i'(V_i^0)]^{-1} > 0$. (27)

Therefore, the same proof as in Sec III leads us to the conclusion that in the generalized WN model of Eq. (15) and (16) there is always a stable SSS in the absence of negative differential capacitance.

Now let us see what can happen when negative differential capacitance is introduced. It should be noted that since at present we cannot have a reasonable way to expect negative differential capacitance in the original superlattice system, the model in this case can only be considered as a toy model. However, this toy model may be a minimum model for the phenomenon of self-sustained current oscillation. We shall show by a simple example that negative differential capacitance can lead to a non-stationary state. Suppose that all barriers have the same $I - V$ curves with no NDR region, i.e., $G_i > 0$, while $D - V$ curves are different and have a region of negative differential capacitance as shown in Fig. (a). It is obvious that in this case the system has only one SSS at $V_i^0 = V_i/N$. Let us consider the case when $V_i^0 = V_i/N$ drops into the negative differential capacitance region of every barrier, i.e., $C_i^0 < 0$ for all barriers. Then, with WN’s method it is easy to check that this SSS is unstable if two of $\lambda_i = -G_i^0/C_i^0 > 0$ are not equal. Thus the system has no stable SSS and must be in an SSCO state. This means that the generalized model with negative differential capacitance and without NDR can lead to an SSCO state. However, the model with only negative differential capacitance is not enough because the $I - V$ oscillation behavior of stable SSSs observed in experiments can only be included by NDR. The SSCO behaviors can also occur when both negative differential capacitance and NDR exist in the model. For example, consider the case shown in Fig. (b) where each barrier has one NDR region between two negative differential capacitance regions. In this case, the number of SSSs is more than one. One can show that all SSSs are unstable when the average $V_i = V_i/N$ drops in the region of both PDR and negative differential capacitance. Therefore, the generalized WN model with both NDR and negative differential capacitance may be considered as a possible minimum toy model for self-sustained current oscillation.

In a recent study, Sanchez et. al. have pointed out that WN’s original model will always tend to a stable stationary state if it exists. However, their argument seems not fit for our generalized model with negative differential capacitance. Following their argument, the total current of each barrier in our model is $I_{total} = C_i(V_i) \frac{dV_i}{dt} + I_i(V_i)$ (28)

which leads to a series of equations for $V_i$

$$\frac{dV_i}{dt} = [C_i(V_i)]^{-1} [I_{total} - I_i(V_i)].$$ (29)

Put them into

$$\sum_i \frac{dV_i}{dt} = 0,$$ (30)

we have

$$I_{total} = \frac{\sum_i [C_i(V_i)]^{-1} I_i(V_i)}{\sum_i [C_i(V_i)]^{-1}}.$$ (31)

Put it back into Eq. (28), we have the equations for $V_i$

$$\frac{dV_i}{dt} = \frac{1}{C_i(V_i) \sum_j [C_j(V_j)]^{-1}} \sum_j I_i(V_i) - I_j(V_j) \frac{C_j(V_j)}{C_i(V_i)}.$$ (32)

Unlike the case in the Appendix of Sanchez et. al., in the presence of negative differential capacitance $C_i < 0$, the total current $I_{total}$ in Eq. (31) can be time-dependent and the equations of $V_i$’s in Eqs. (22) are coupled together
through the factor $[C_j(V_j)]^{-1}$. Thus the existence of limit circles and other types of SSCO behaviors is possible in this toy model. Although no parameter in this toy model represents directly for some external parameters in the original SL system such as well doping, temperature and transverse magnetic field, the presence of negative differential capacitance and NDR may be viewed as the consequence of selecting appropriate values of these external parameters in the original SL system. Possible detailed relation between this toy model and the original SL system needs further study and is not the aim of this paper.

In summary, our analysis shows that a generalized WN’s model without negative differential capacitance always has a stable SSS and thus cannot have SSCO behaviors. This means that negative differential capacitance is necessary for the existence of SSCO behavior. We show by a simple example that the same model with negative differential capacitance can have SSCO behavior. However, the model with only negative differential capacitance is not enough, since NDR is necessary for the $I - V$ oscillation behavior of stable SSSs observed in experiments. This means that the generalized WN model with both negative differential capacitance and NDR can serve as a possible minimum toy model for the phenomena of both self-sustained current oscillation and $I - V$ oscillation behavior of stable SSSs.

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