Lone Higgs at the LHC

Ken Hsieh
in collaboration with C.-P. Yuan
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Argonne National Laboratory
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Motivation

The Lone Higgs Scenarios

- MSSM
- LHT
- MUED

Synthesis

- Synthesis
- Bonus Materials

Back-up slides
MOTIVATION
The Standard Model works wonders, but unitarity in $WW \rightarrow WW$ says we should see something below 1.2 TeV.

If this something is a Higgs boson, the hierarchy problem motivates seeing something else.

But what if we only see a Higgs boson? What then?
The Standard Model works wonders, but unitarity in $WW \rightarrow WW$ says we should see something below 1.2 TeV.

If this something is a Higgs boson, the hierarchy problem motivates seeing something else.

But what if we only see a Higgs boson? What then?

That is a hard question...
An Easier Question

We will instead attempt to answer an easier question:

Suppose we discover only a light Higgs boson (∼120 GeV) after 10 fb⁻¹...

From the absence of new resonances and the measurements of \( \sigma(p p \rightarrow (g g \rightarrow h X)) \) and \( \text{Br}(h \rightarrow \gamma \gamma) \), can we identify this Higgs boson as that of the SM, the MSSM, LHT, or MUED?

**SM** → Standard Model
**MSSM** → Minimal Supersymmetric Standard Model
**LHT** → Littlest Higgs with \( T \)-parity
**MUED** → Minimal Universal Extra Dimensions

\[ B\sigma(gg \rightarrow h \rightarrow \gamma \gamma) \equiv \sigma(p p \rightarrow (g g \rightarrow h X)) \text{Br}(h \rightarrow \gamma \gamma) \]
A Related Question

We can frame some related questions from a more theoretical-oriented perspective...

- Higgs boson with observable modifications from new physics → low-scale new physics.
- Seeing only the Higgs (SM or non-SM) → high-scale new physics. (Disclaimer: we assume no ’hiding’ of new, light physics.)
- For a given model, as I raise the scale of new physics, which happens first? Do I lose hints of new physics in the Higgs boson (but see new resonances)? Or do I lose the resonances (but with a non-SM Higgs)?
- How does the decoupling behavior vary across the different models?
This is a variant of the Inverse LHC Problem.
Arkani-Hamed, Kane, Thaler, and Wang. JHEP 0608, 070 (2006).

Instead of mapping data to the parameter space of a model, we try to map to regions in the 'model space.'

Similar questions (distill traces of new physics from collider signatures of the Higgs boson) have been asked in the literature.
Mantry, Ramsey-Musolf, and Trott. Phys. Lett. B 660, 54 (2008).
Mantry, Trott, and Wise. Phys. Rev. D 77, 013006 (2008).
Randall. JHEP 0802, 084 (2008).
The Basic Strategy

We follow the steps

- Use the SM as the yard stick: express results as deviations from the SM.
- Examine regions where $B \sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ differs significantly from the SM.
- Examine regions consistent with not-seeing-anything-else (lone Higgs scenario).
- Find out if there is an overlap between these two regions.
Motivation

The Lone Higgs Scenarios

Synthesis

Measurement of Higgs Production and Decay

- Sources of uncertainty: luminosity, PDF, scale-dependence, etc.
- Ratios of measurements reduce dependence on luminosity
- PDF-induced uncertainties may be reduced with correlations.

P. M. Nadolsky et al., arXiv:0802.0007 [hep-ph].

- With 100 fb$^{-1}$ of data per experiment, $B\sigma$ can be determined to 10% to 15% for $100 < m_h (\text{GeV}) < 150$. (Statistical errors dominate: $\Delta \sigma / \sigma = \sqrt{N_S + N_B / N_S}$.)

D. Zeppenfeld et al., Phys. Rev. D 62, 013009 (2000)

- With only 10 fb$^{-1}$, we scale this up by $1/\sqrt{0.1} \sim 3$ and treat deviations greater than 30% as ‘significant’.*
THE LONE HIGGS SCENARIOS
Each SM particle is extended with a 'superpartner'.

The Lagrangian is constructed in such a way that, because of the virtual contributions of the superpartners, there is no quadratic sensitivity in the Higgs mass.

\[
\frac{y_t^2}{16\pi^2} \Lambda^2 \quad \text{and} \quad \frac{y_t^2}{16\pi^2} \Lambda^2
\]

The superpartners have SUSY-breaking masses, leading to many free parameters.
In MSSM, $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ depends sensitivity on the details of the s-top sector and is in general quite complicated. (There are also dependencies from all other superpartners through the $D$-terms.)
$B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ in the MSSM - an example

Scanned parameters:
- $300 \text{ GeV} \leq M_\tilde{t} \leq 1.5 \text{ TeV},$
- $-4 M_\tilde{t} \leq A_\tilde{t} \leq 4 M_\tilde{t}$

Fixed parameters:
- $M_\tilde{\chi} = 100 \text{ GeV},$
- $M_\tilde{\chi} = \mu = 200 \text{ GeV},$
- $M_\tilde{g} = M_\tilde{Q} = 500 \text{ GeV}.$
We need to make some simplifying assumptions.

Lone Higgs scenario implies 'heavy' superpartners, so let us constrain some superpartner masses.
(We assume no hiding of light particles.)

We classify the new resonances of the MSSM into four classes:
colored superpartners (squarks and gluino), sleptons, neutralinos, and Higgs bosons.

We turn to our attention to decouple some of these resonances from the LHC.
Motivation
The Lone Higgs Scenarios
Synthesis

Lone Higgs scenarios in the MSSM

Evading discovery at the LHC requires:
2.5 TeV for $\tilde{g}$ and $\tilde{q}$,
300 GeV for $\tilde{\ell}$ and $\tilde{\chi}$.

This leaves us with the Higgs bosons ($H^0, \pm$ and $A$), with masses of order $M_A$. 
Let us decouple s-fermions and neutralinos/charginos:

\[\mu = M_\tilde{w} = M_\tilde{\ell} = 1 \text{ TeV}\]

\[M_\tilde{g} = M_\tilde{Q} = 0.5 A_t = 2.5 \text{ TeV (both s-tops are heavy)}\]

\[\delta B \sigma(gg \rightarrow h \rightarrow \gamma\gamma)\] in the MSSM

- \[\text{Br}(h \rightarrow \gamma\gamma)\] is always suppressed due to \([\tan \beta]\)-enhanced \(h\bar{b}b\) and \(h\tau\tau\) couplings when s-tops decouple.

- For \(M_A < 350 \text{ GeV}\), there is much suppression!
What does a lone Higgs scenario require of $M_A$ and $\tan \beta$?

The region $M_A < 350$ GeV is allowed if $\tan \beta$ is not too large!

- This parameter space of the MSSM contains lone Higgs scenarios with a Higgs boson having a suppressed $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$. 

Ken Hsieh  Lone Higgs at the LHC
LHT Overview

- Littlest Higgs with $T$-parity protects Higgs mass by two independent global symmetries.
  Arkani-Hamed, Cohen, Katz, and Nelson. JHEP 0207, 034 (2002)
  Cheng and Low, JHEP 0309, 051 (2003), 0309, 051 (2003) Low, 0410, 067 (2004)
- Extended gauge and top sectors cancel quadratic corrections to the Higgs mass from SM at one-loop.
- $T$-parity gives loop-suppressed corrections to EW observables.
  $gf$: scale of extended gauge sector;
  $\kappa f$: scale of fermion partners;
  $\lambda f$: scale of extended top sector.
- We need $f > 500$ GeV to be consistent with EWPT.
  Hubisz, Meade, Noble, and Perelstein, JHEP 0601, 135 (2006)
In LHT, $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ only depends on $f$, and dependencies to the parameters in the extended top sector cancel.

C. R. Chen, K. Tobe and C. P. Yuan, Phys. Lett. B 640, 263 (2006).

- $m_h = 110$ (top line), 120, 130 GeV.
- Note the suppression as in the MSSM.
- To have a large deviation, we need small $f$ with $f < 560$ GeV.
With 500 GeV < f < 560 GeV, can we have a viable lone Higgs scenario?

We classify the new resonances in the LHT into three classes:

- Heavy gauge bosons $W_H$ and $Z_H$ ($M_{W_H} \sim gf \sim 0.65f$),
- $T$-odd fermions $Q_H$ and $L_H$ ($M_{F_H} \sim \kappa f$),
- $T$-even and $T$-odd top partners $T_{\pm}$,

and we assume a universal $\kappa = \kappa_q = \kappa_\ell$ that is large enough to forbid the decay $W_H \rightarrow q Q_H$. 

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Lone Higgs at the LHC
Evading discovery of $W_H$

- Two modes of $W_H^\pm$ pair-production (due to $T$-parity):
  - s-channel $Z^*$-exchange, and t-channel $Q^*$ exchange.

Q. H. Cao and C. R. Chen, Phys. Rev. D 76, 075007 (2007)

- The two diagrams destructively interfere, and $Z$-exchange dominates.
- With smaller $\kappa$, there is more destructive interference, and production rate drops.
- We can thus evade discovery of $W_H$ with smaller $\kappa$, leading to a smaller production cross section.
Q. H. Cao and C. R. Chen, Phys. Rev. D 76, 075007 (2007)

So we would need $\kappa \lesssim 2.2$ to evade the discovery of $W_H$ with $f < 560$ GeV.
(For $M_{W_H} < 360$ GeV, we need $M_{Q_H} < 1.75$ TeV to evade discovery of $W_H$.)
Evading discovery of $Q_H$

D. Choudhury and D. K. Ghosh, JHEP 0708, 084 (2007)

The discovery mode of $Q_H$ arises from, for example,

$$pp \rightarrow Q_H Q_H$$

$$\rightarrow \bar{q}q W^+_H W^-_H$$

$$\rightarrow qq W^+ W^- A_H A_H$$

For $f < 560$ GeV, we can evade discovery of $Q_H$ by making it heavier with larger $\kappa$ ($\kappa \gtrsim 1.4$, $M_{Q_H} \gtrsim 1.1$ TeV).
Evading discovery of top partners

- The top quark and the top partners have masses of
  \[ m_t \approx \lambda_1 \lambda_2 \left( \sqrt{\lambda_1^2 + \lambda_2^2} \right)^{-1} v_{\text{ew}}, \]
  \[ m_{T^+} \approx \left( \sqrt{\lambda_1^2 + \lambda_2^2} \right) f, \quad \text{and} \quad m_{T^-} \approx \lambda_2 f. \]

- However, \( B\sigma(gg \rightarrow h \rightarrow \gamma\gamma) \) is independent of \( \lambda \), so we can simply evade discovery of \( T^\pm \) by raising \( \lambda_2 \), and tune \( \lambda_1 \) to have viable \( m_t \).

- At the LHC, a discovery reach on top partners with masses around 900 GeV gives \( \lambda \gtrsim 2 \) to evade the discovery.

Matsumoto, Nojiri, and Nomura. Phys. Rev. D 75, 055006 (2007).
In summary, if we demand both the lone Higgs scenario and a large deviation in $B\sigma(gg \to h \to \gamma\gamma)$, we need

$$500 < f \text{ (in GeV)} < 560,$$

$$1.4 < \kappa < 2.2,$$

$$2.0 < \lambda_2.$$

In this parameter space, we have a Lone Higgs scenario with a large suppression in $B\sigma(gg \to h \to \gamma\gamma)$.

Problem (for later): how could we distinguish LHT from MSSM?
Minimal Universal Extra Dimensions extends all SM particles to propagate in one additional, flat extra dimension, with $R^{-1}$ and $\Lambda$ as the only additional parameters. (We choose $\Lambda = 10R^{-1}$.)

Appelquist, Cheng and Dobrescu, Phys. Rev. D 64, 035002 (2001)

Interesting dark matter candidate in the KK photon $B_{\mu}^{(1)}$.

G. Servant and T. M. P. Tait, Nucl. Phys. B 650, 391 (2003)

K. Kong and K. T. Matchev, JHEP 0601, 038 (2006)

The processes $gg \rightarrow h$ and $h \rightarrow \gamma\gamma$ now proceed through a tower of KK modes.
\[ \delta B\sigma(gg \rightarrow h \rightarrow \gamma\gamma) \text{ in MUED} \]

Unlike MSSM and LHT, there is an enhancement in \( \delta B\sigma(gg \rightarrow h \rightarrow \gamma\gamma) \)!

This is due to the additive effects of the KK modes.

F. J. Petriello, JHEP 0205, 003 (2002)[hep-ph/0204067]

\[ \frac{\delta(B\sigma)}{(B\sigma)_{SM}} = \frac{130 \text{ GeV}}{M_h} \]

To have a significant deviation, we need \( R^{-1} < 650 \text{ GeV} \).

For a 100 fb\(^{-1}\) lone Higgs scenario, need \( R^{-1} < 1.2 \text{ TeV} \) distinguish from the SM.
We need small $R^{-1}$ to see a modified Higgs boson, and $R^{-1}$ is the mass scale of the 1st KK modes.

Using MSSM reaches, $g^{(1)}$ and $q^{(1)}$ have to be roughly 2 TeV or heavier to evade discovery.

This sets $R^{-1}$ to be roughly 1.8 TeV, and the deviation in $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ is too small to be significant, even at the 10% level.

Unlike the LHT and the MSSM, there is no lone Higgs scenario with a modified Higgs boson in the MUED!
SYNTHESES
Putting it all together

- If we see a small deviation... then we may take this as a hint and cross our fingers.
- If we see a large enhancement, then is it MUED? Why haven’t we seen the KK modes? Need to perform consistency checks.
- If we see a large suppression, then we favor LHT and MSSM over MUED.
Disentangling LHT and MSSM

We can use the amount of suppression as a discriminant, even before seeing any new resonances. The MSSM lone Higgs scenarios can give rise to larger suppression than LHT.

\[ \delta \left( \frac{B(\gamma \gamma \to \gamma \gamma)}{B(\gamma \gamma \to \gamma \gamma)_{SM}} \right) \]

\[ M_A (\text{GeV}) \]

\[ \tan(\beta) = 10 \]
\[ \tan(\beta) = 20 \]
\[ \tan(\beta) = 30 \]
In a MSSM lone Higgs scenario large suppression $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$, even with more luminosity we are not guaranteed to see new resonances, but we know where to look.
With LHT, the gap in $\kappa_q$ shrinks with further running, and we may discover $W_H$ and/or $Q_-$ for low enough value of $f$ with more luminosity.
Find only one Higgs boson with mass between 120 GeV and 130 GeV. Compare the measurement of $\sigma(pp(gg \rightarrow h) X) \times Br(h \rightarrow \gamma \gamma)$ with SM prediction. Does the measurement deviate from the SM prediction by 30% or more? No. The deviation in the measurement is not great enough to claim evidence of new physics.

Yes

Is the deviation an enhancement or a suppression? Enhancement

Strong disfavor MUED, LHT, and the MSSM. Seek alternative models of new physics for this observed enhancement.

Suppression

Favor the MSSM and the LHT over the MUED.

Is the suppression greater than 40%? Yes

Favor MSSM over LHT as the model of new physics.

No

Both MSSM and LHT are possible models of new physics. In the case of the LHT, we should discover either $W_H$ or $Q'_H$. 

Yes
BONUS MATERIAL
Other ways of distinguishing the models

- Suppose we discover new resonances in addition to a light Higgs boson, how do we tell the models apart?
- Much exists in literature already on this topic!
- For examples, we use spin correlations to tell apart the MSSM gluino and the MUED KK gluon.
- We contribute our 2-cents 1.4 cents here.
Of the three models of new physics we study. Only the LHT has top partners that can be singly-produced.

Due to mixing with the SM top quark, this can also lead to a change in the SM single-top production rate.

Han, Logan, McElrath and Wang, Phys. Rev. D 67, 095004 (2003)
Cao, Li and Yuan, arXiv:hep-ph/0612243
Observing single-$H/A$ production rules out MUED

- The MSSM ($\Delta S = 1/2$, superpartners) and the MUED ($\Delta S = 0$, KK modes) are very similar with respect to the added particle content.
- The MSSM has two Higgs doublets ($H_u$ and $H_d$) and so does the MUED ($0^{th}$ and $1^{st}$ KK mode).
- After EWSB, the same degrees of freedom remain:
  - MSSM: $h, H, A, H^\pm$
  - MUED: $h^{(0)}, H^{(1)}, A^{(1)}, H^{(1)\pm}$
- However, the heavy Higgs bosons in MUED are $K$-odd while the MSSM ones are $R$-even.
- Seeing an additional, singly-produced Higgs boson would rule out MUED.
Summary and Outlook

- With large deviation in $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$, we can go some distance in distilling traces of new physics.
- For examples, we show that · · ·
  - the MUED can not offer a Lone Higgs scenario with a modified Higgs boson, and
  - in the case of LHT, we should see resonances within 100 fb$^{-1}$ in a Lone Higgs scenario with a suppressed $B\sigma$.
- We can iterate this line of reasoning with more luminosity, for different models of new physics.
- One can also examine the naturalness of such regions of parameter space.
If we see a Higgs boson, we also hope to see more than a Higgs boson, but Nature may not be so kind immediately, and we have to be prepared.
BACK-UP SLIDES
We should expect to discover the Higgs boson by $10 \text{ fb}^{-1}$.
The Lone Higgs Scenarios Synthesis

Summary of MSSM Higgs search (ATLAS TDR)
Summary of MSSM Higgs search (ATLAS TDR)

- LEP2
  \( \sqrt{s} = 200 \text{ GeV} \)
  \( \mathcal{L} = 200 \text{ pb}^{-1} \)

- \( \sqrt{s} = 189 \text{ GeV} \)
  \( \mathcal{L} = 175 \text{ pb}^{-1} \)

- H/A \( \rightarrow \tau \tau \)
- t \( \rightarrow bH^+ \) with
- H' \( \rightarrow \tau \nu \)
- tth, h \( \rightarrow bb \)
- h \( \rightarrow \gamma \gamma \)

- ATLAS
  \( \mathcal{L} = 300 \text{ fb}^{-1} \)
  Minimal mixing

- H \( \rightarrow ZZ^{(*)} \rightarrow 4 \text{ lep} \)
The reality is much more complicated. There are cases where the dominant production mode is not $gg \rightarrow h$.

Belyaev, Pumplin, Tung, and Yuan. JHEP 0601:069, 2006.

This makes it very difficult to extract $\sigma(pp(gg \rightarrow hX))$. 

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**Lone Higgs at the LHC**
In the case where $\sigma(pp(\bar{b}b \rightarrow hX))$ dominate, there can be significant (15%) scale uncertainty in the NNLO level. Belyaev, Pumplin, Tung, and Yuan. JHEP 0601:069, 2006.
MSSM BACK-UP SLIDES
Motivation

The Lone Higgs Scenarios


d_\Gamma(gg \rightarrow h)

in the MSSM

\( \sigma(gg \rightarrow h) \propto \Gamma(h \rightarrow gg) \).

In SM, only need to consider the top-quark loop.

In MSSM with small s-top mixing, s-top loops interfere constructively.

R. V. Harlander and M. Steinhauser, JHEP 0409, 066 (2004) [hep-ph/0409010]

It is possible to obtain destructive interference (in an extreme case, a gluo-phobic Higgs) with large hierarchy between the two s-tops and significant mixing.

This requires large mixing in the s-top sector, leading to a light s-top. These scenarios are then inapplicable to lone Higgs scenarios.
In SM, top-quark loop and $W^\pm$ loop interfere destructively, and $W^\pm$ loop dominates.

In MSSM with small s-top mixing, there are two sources of suppression.

The s-top loops cause additional destructive interference.

The $hbb$ and $h\tau\tau$ couplings are $[\tan \beta]$-enhanced, leading to another suppression in $\text{Br}(h \rightarrow \gamma\gamma)$
We scan over $A_t = \pm 3M_{\tilde{t}}$ for two values of s-top SUSY-breaking masses.

$2M_{\tilde{t}_c} = M_{\tilde{w}} = \mu = 200,$

$M_{\tilde{g}} = M_{\tilde{Q}} = 500, M_A = 1000, t_\beta = 10.$

The variation in $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ is smaller as the s-tops decouple.

For simplicity, we assume in our study that s-tops are not only heavy enough to evade discovery, but also heavy enough so that their contributions to $B\sigma(gg \rightarrow h \rightarrow \gamma\gamma)$ is small.
Clarifications on the s-top Parameter Space

- Large $A_t$ (relative to $M_{\tilde{Q}_3, \tilde{U}_3}$) does not necessarily mean large s-top mixing. The mixing in the s-top matrix are of the form $m_t A_t$. So we can have large $A_t / M_{\tilde{Q}_3}$ (for large Higgs mass) and yet small $m_t A_t / M_{\tilde{Q}_3, \tilde{U}_3}^2$.

- For a large Higgs mass, $A_t / M_{\tilde{Q}_3} \sim \sqrt{6}$ is optimal at one-loop, and having too large $A_t / M_{\tilde{Q}_3}$ can actually give a negative Higgs mass.

![Diagram showing the relationship between $A_t / M_{\tilde{Q}_3}$ and the Higgs mass](image)
We use **SusyBSG** to calculate \( \text{Br}(b \to s\gamma) \) in the minimal flavor violation (MFV) framework.

Degrassi, Gambino, and Slavich. arXiv:0712.3265 [hep-ph].

Ambrosio, Giudice, Isidori, and Strumia. Nucl. Phys. B 645, 155 (2002).

**We use the experimental value** \( \text{Br}(b \to s\gamma)_{\text{exp}} = (355 \pm 26) \times 10^{-6} \)

E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG)] [hep-ex/0603003]

![Graph showing the relationship between \( \text{Br}(b \to s\gamma) \) and \( M_A \)]

\[
\begin{align*}
M_{\tilde{B}} &= 550, M_{\tilde{W}} = 1000, M_{\tilde{\ell}} = 2500, \\
\mu &= 1000, M_{\tilde{q}} = 2500, A_t = -5000.
\end{align*}
\]
LHT BACK-UP SLIDES
The masses in the LHT are

Extended gauge and fermion (first 2 gen.) sector:

\[ M_{W_H} = M_{Z_H} = g f \sim 0.64f, \]
\[ M_{A_H} = \left(\sqrt{5}\right)^{-1} g' f \sim 0.16f, \]
\[ M_{Q_H,L_H} = \sqrt{2} \kappa f, \]

Extended top sector:

\[ m_t \simeq \lambda_1 \lambda_2 \left(\sqrt{\lambda_1^2 + \lambda_2^2}\right)^{-1} v_{ew}, \]
\[ M_{T_+} \simeq \left(\sqrt{\lambda_1^2 + \lambda_2^2}\right) f, \]
\[ M_{T_-} \simeq \lambda_2 f. \]
MUED BACK-UP SLIDES
With radiative corrections, those masses in MUED that are of interest here are

Cheng, Matchev, and Schmaltz. Phys. Rev. D 66, 036005 (2002)

\[
m^2_{g(n)} = n^2 R^{-2} \left( 1 + \frac{23}{2} \frac{g_3^2}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right) \sim (1.5) n^2 R^{-2},
\]

\[
m^2_{W(n)} = n^2 R^{-2} \left( 1 + \frac{15}{2} \frac{g^2}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right) \sim (1.1) n^2 R^{-2},
\]

\[
M_{\text{top}} = \begin{pmatrix} nR^{-1} + \delta m_{Q_3(n)} & m_t \\ m_t & -nR^{-1} - \delta m_{U_3(n)} \end{pmatrix} \sim \begin{pmatrix} 1.13 nR^{-1} & m_t \\ m_t & 1.09 nR^{-1} \end{pmatrix},
\]

\[
\delta m_{Q_3(n)} = nR^{-1} \left( 3 \frac{g_3^2}{16\pi^2} + 27 \frac{g^2}{1616\pi^2} + \frac{1}{16} \frac{g'^2}{1616\pi^2} - \frac{3}{4} \frac{y_t^2}{1616\pi^2} \right) \ln \frac{\Lambda^2}{\mu^2},
\]

\[
\delta m_{U_3(n)} = nR^{-1} \left( 3 \frac{g_3^2}{16\pi^2} + \frac{g'^2}{16\pi^2} - \frac{3}{2} \frac{y_t^2}{16\pi^2} \right) \ln \frac{\Lambda^2}{\mu^2}.
\]