Effect of the single-scattering phase function on light transmission through disordered media with large inhomogeneities

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Abstract. We calculate the total transmission coefficient (transmittance) of a disordered medium with large (compared to the light wavelength) inhomogeneities. To model highly forward scattering in the medium we take advantage of the Gegenbauer kernel phase function. In a subdiffusion thickness range, the transmittance is shown to be sensitive to the specific form of the single-scattering phase function. The effect reveals itself at grazing angles of incidence and originates from small-angle multiple scattering of light. Our results are in a good agreement with numerical solutions to the radiative transfer equation.

1. Introduction

Multiple light scattering in disordered media is a fundamental problem with many applications \([1, 2]\). An interrelation between characteristics of multiply scattered light and parameters of scattering inhomogeneities is of considerable interest both for understanding of the physical process and for practical use. This interrelation is a basis for numerous techniques for characterization of turbid media (see, e.g., \([3, 4, 5, 6]\)).

In media with large-scale inhomogeneities, the transport mean free path \(l_{tr}\) appears to be much greater than the mean free path \(l\), \(l_{tr} \gg l\), and two regimes of multiple light scattering can be distinguished. The diffusive regime develops over spatial scales exceeding \(l_{tr}\). Diffusely scattered light retains no information about the single-scattering phase function (\(l_{tr}\) is the only retrievable parameter). The small-angle regime of multiple scattering is more informative. The angular and spatial distributions of multiply scattered light \([7, 8, 9]\) relate directly to the single-scattering law. As is shown in \([10]\), if the medium thickness is less than the transport mean free path, the total transmittance also appears to be sensitive to the single-scattering phase function. The effect reveals itself at grazing angles of incidence \((\zeta_0 \ll 1, \zeta_0\) is the angle between the direction of light incidence and the medium boundary) and originates from small-angle multiple scattering of light.

In this paper, we provide a detailed description of the asymptotic method of the total transmittance calculations outlined in \([10]\). An analytic formula for the total transmittance is derived for an arbitrary relation between the angle \(\zeta_0\) and the characteristic angle of single scattering.
2. General relations

Consider transmission of a broad collimated beam of light through a nonabsorbing plane-parallel medium with large-scale inhomogeneities. The intensity of light inside the medium $I(z, \Omega)$ obeys the radiative transfer equation

$$\left(\cos \theta \frac{\partial}{\partial z} + \sigma\right) I(z, \Omega) = \sigma \int d\Omega' p(\Omega, \Omega') I(z, \Omega')$$

(1)

where $\sigma = l^{-1}$ is the scattering coefficient, $\Omega$ is the unit vector in the direction of light propagation, $p(\cos \vartheta)$ is the single-scattering phase function normalized by the condition $\int_0^\pi 2\pi \sin \vartheta p(\cos \vartheta) d\vartheta = 1$. The $z$-axis is perpendicular to the medium boundaries, $\cos \theta = \Ω$. At grazing angles of incidence, $\zeta_0 \ll 1$, the radiative transfer equation (1) can be simplified using the small-angle approximation (see, e.g., [8])

$$\left(\zeta \frac{\partial}{\partial z} + \sigma\right) I(z, \zeta, \varphi) = \sigma \int_{-\infty}^{\infty} d\zeta' d\varphi' p(\vartheta) I(z, \zeta', \varphi')$$

(2)

In deriving Eq. (2), we put $\cos \theta = \sin \zeta \approx \zeta$ and $2(1 - \Omega_0) \approx \vartheta^2 = (\zeta - \zeta')^2 + (\varphi - \varphi')^2$ where $\varphi$ is the azimuthal angle of the direction $\Omega$, $\zeta = \pi/2 - \theta$ is the angle between the direction $\Omega$ and the medium boundary. Angles $\zeta > 0$ and $\zeta < 0$ correspond to light propagating in the forward and backward directions, respectively. As is usual in the small-angle approximation, the angles $\zeta$ and $\varphi$ are formally considered to vary within infinite limits provided that light propagates predominantly at small angles to the direction of incidence $\Omega_0$ ($\cos \theta_0 \approx \zeta_0$, $\varphi_0 = 0$), and the intensity decreases rapidly for $|\zeta|, |\varphi| > 1$.

To model highly forward scattering in the medium we take advantage of the two-parameter phase function proposed in [11]. The small-angle form of this phase function can be written as

$$p(\vartheta) = \frac{\alpha - 2}{2\pi} \frac{\vartheta_0^{\alpha-2}}{\left(\vartheta_0^2 + \vartheta^2\right)^{\alpha/2}}, \quad (\alpha > 2)$$

(3)

where $\vartheta_0$ is the characteristic angle of single scattering, $\vartheta_0 \ll 1$. The phase function (3) decreases with angle $\vartheta$ as $p(\vartheta) \sim 1/\vartheta^\alpha$, and unifies a number of scattering models (see, e.g. [7, 12]). For $\alpha = 3$, Eq. (3) corresponds to the Henyey-Greenstein phase function which is used very widely to model scattering of light in natural media [7].

Equation (2) integrated over the azimuthal angle can be presented in the following form

$$\zeta \frac{\partial}{\partial z} I(z, \zeta) = \frac{(\alpha - 2)\sigma\vartheta_0^{\alpha-2}}{2\pi} \int_{-\infty}^{\infty} d\varphi' d\zeta'[I(z, \zeta') - I(z, \zeta)]$$

(4)

where $I(z, \zeta)$ is the azimuth-integrated intensity of light, $I(z, \zeta) = \int_{-\infty}^{\infty} I(z, \zeta, \varphi)d\varphi$. Boundary conditions for Eq. (4),

$$I(z = 0, \zeta > 0) = \delta(\zeta - \zeta_0); \quad I(z = L, \zeta < 0) = 0,$$

(5)

determine the intensity within the half-ranges of the angles that correspond to light incident upon the medium (we do not consider the effects caused by Fresnel reflections at the boundaries). Quantities $I(z = 0, \zeta < 0)$ and $I(z = L, \zeta > 0)$ (sometimes referred to as the reflection and the transmission functions, respectively [1]) describe the angular distribution of reflected and transmitted light and should be determined in the course of solving Eqs.(4)-(5). A closed system of integral equations for these functions can be formulated using the eigenfunction method [13]. However, an analytical solution of this system is possible only in the diffusion approximation where the integral term in Eq.(4) is replaced by the differential operator.
The total transmittance (total transmission coefficient) is a ratio of the transmitted flux in the semi-infinite medium, \( R \), to the incident flux at \( z = 0 \). Light obviously increases with increasing the medium thickness \( L \). The characteristic single-scattering angle \( \vartheta_0 \), for which the value \( \vartheta \) is found in [8] just under assumption that the characteristic multiple-scattering angle exceeds the single-scattering angle, \( \vartheta_0 \), is the angle of incidence \( \vartheta_0 \) less than the characteristic single-scattering angle, \( \vartheta_0 \), and \( \vartheta_0 > \vartheta \), light is mainly reflected from the medium if the angle of incidence \( \vartheta_0 \) is less than \( \vartheta_0 \); otherwise, \( \vartheta_0 > \vartheta_0 \), light is transmitted through the medium.

3. Asymptotic expressions for the total transmittance

The total transmittance (total transmission coefficient) is a ratio of the transmitted flux \( \int_0^\infty \zeta I(z = L, \zeta) d\zeta \) to the incident flux (the latter equals \( \zeta_0 \)). In a nonabsorbing medium, the condition of the total flux conservation, \( \int_0^\infty \zeta I(z = L, \zeta) d\zeta + \int_0^\infty |\zeta| I(z = 0, -|\zeta|) d|\zeta| = \zeta_0 \), enables us to express the total transmittance in terms of the intensity of reflected light

\[
T = \frac{1}{\zeta_0} \int_0^\infty d|\zeta| R(|\zeta|, \zeta_0) \tag{8}
\]

where we define the reflection function as \( R(|\zeta|, \zeta_0) = |\zeta| I(z = 0, -|\zeta|) \). According to the reciprocity principle [1], \( R(|\zeta|, \zeta_0) = R(\zeta_0, |\zeta|) \).

Consider first a situation where the angle of incidence is greater than the characteristic single-scattering angle, \( \vartheta_0 > \vartheta_0 \). In this case, the reflected flux is governed by light scattered through angles exceeding the value \( \vartheta_0 \), and \( \vartheta_0 \) can be neglected in the denominator of the phase function in Eq.(4). As follows from Eqs. (7) and (8), the \( L \)-dependence of the total transmittance has a single scale \( l_{tr}\zeta_0^{-1} \) (\( T \) depends on the ratio \( \zeta_0/\vartheta_0 \)).

To calculate the total transmittance for \( L > l_{tr}\zeta_0^{-1} \), we take advantage of the solution to Eq.(4) in the semi-infinite medium. The explicit form of the reflection function (for \( L \rightarrow \infty \)),

\[
R_\infty(|\zeta|, \zeta_0) = \frac{2(\alpha - 1)}{\pi \alpha} \left( \frac{|\zeta_0|}{\zeta} \right)^{2(\alpha - 1)/\alpha} \frac{\sin(2\pi/\alpha)}{\left( \frac{\sin(\pi/\alpha)}{\sin(\pi/\alpha)} \right)^{2(\alpha - 1)/\alpha} - 2 \cos(2\pi/\alpha)} \tag{9}
\]

is found in [8] just under assumption that the characteristic multiple-scattering angle exceeds the value \( \zeta_0 \) (see Appendix). For given values of the angles \( \zeta \) and \( \zeta_0 \), the intensity of reflected light obviously increases with increasing the medium thickness \( L \). Therefore, the solution for the semi-infinite medium, \( R_\infty(|\zeta|, \zeta_0) = R_\infty(|\zeta|/\zeta_0) \), is a majorant for the reflection function \( R(|\zeta|, \zeta_0) \), that is, with allowance for scaling (7),

\[
R(|\zeta|, \zeta_0) = R_\infty(|\zeta|/\zeta_0) F \left( |\zeta|/\zeta_0, \zeta_0/\vartheta_0 \right) \tag{10}
\]
where \(0 \leq F(x,x_0) \leq 1\) for all values \(x\) and \(x_0\), \(F(x,x_0) = F(x_0,x)\). In the limit \(x,x_0 \to 0\) (i.e., \(L \to \infty\)), \(F(0,0) = 1\). Taking into account the flux conservation in a semi-finite medium, 
\[
\int_0^\infty d|\zeta|R_\infty(|\zeta|/\zeta_0) = \zeta_0,
\]
we can present the total transmittance in the following form
\[
T = x_0^{-1} \int_0^\infty dx R_\infty(x/x_0)[1 - F(x,x_0)], \quad x_0 = \zeta_0/\zeta_L
\]
(11)

For \(x_0 \ll 1\), Eqs. (9) and (11) give a simple asymptotic formula \(T = c_\alpha x_0^{1-2/\alpha}\) or, in the initial notations,
\[
T = c_\alpha \left(\frac{\zeta_0}{\sigma_{tt} L}\right)^{\frac{\alpha-2}{\alpha-1}},
\]
(12)
where the numeric constant \(c_\alpha = 2(\alpha - 1)(\pi \alpha)^{-1}\sin(2\pi/\alpha) \int_0^\infty x^{2/\alpha-2}[1 - F(x,0)]dx\) depends only on the parameter \(\alpha\). The value \(c_\alpha\) can be determined only by solving the radiative transfer equation with the relevant boundary conditions. For the Heney-Greenstein phase function \((\alpha = 3)\), analysis of the results of our numerical calculations gives \(c_3 \approx 0.7\) [10].

Equation (12) reveals the effect of the phase function on the total transmittance at grazing angles of incidence within the thickness range \(l_{tt}\zeta_0^{\alpha-1} < L \ll l_{tt}\). Surprisingly, the \(L\)-dependence (12) appears nearly ”universal” for \(3 \leq \alpha < 4\). The difference of the exponent \((\alpha - 2)/\alpha(\alpha - 1)\) from \(1/6\) is less than \(3\%\) [10]. The \(\zeta_0\)-dependence, \(T \sim \zeta_0^{1-2/\alpha}\), is more sensitive to the specific form of the medium phase function.

If the phase function decreases more rapidly than \(1/\theta^4\) \((\alpha > 4)\), the radiative transfer equation can be solved analytically within the small-angle diffusion approximation. In this case, the explicit form of the \(F\)-function is found in [13], and the total transmittance is equal to \(T = 0.84...\zeta_0^3/\sigma_{tt} L^{1/6}\) [13] for \(L \gg l_{tt}\zeta_0^3\) (where \(\sigma_{tt} = \sigma \theta_0^2/4\)). For \(\alpha = 4\), Eq.(12) matches the result derived within the diffusion approximation [13].

For an arbitrary ratio \(\zeta_0/\theta_0\), the relation (10) between the reflection functions changes. According to the scaling relation (6), we can present the reflection function \(R(|\zeta|,\zeta_0)\) in the following form
\[
R(|\zeta|,\zeta_0) = R_\infty\left(\frac{|\zeta|}{\zeta_0},\frac{\zeta_0}{\theta_0}\right) \tilde{F}\left(\frac{|\zeta|}{\zeta_0},\frac{\zeta_0}{\theta_0},\frac{\theta_0}{\zeta_L}\right),
\]
(13)
where we take into account that \(\sigma L/\theta^2 \sim (\zeta_L/\theta_0)^{\alpha-1}\) and write explicitly the arguments of the \(R_\infty\)-function, \(R_\infty(|\zeta|,\zeta_0) \equiv R_\infty\left(\frac{|\zeta|}{\zeta_0},\frac{\zeta_0}{\theta_0}\right)\). The reflection function of the semi-infinite medium \(R_\infty\) obeys Eq.(A.1) (see Appendix). Equation (8) for the total transmittance takes the form
\[
T = \zeta_L\zeta_0^{-1} \int_0^\infty dx R_\infty\left(\frac{x\zeta_L}{\zeta_0},\frac{\zeta_0}{\theta_0}\right) \left[1 - \tilde{F}\left(\frac{x\zeta_L}{\zeta_0},\frac{\zeta_0}{\theta_0},\frac{\theta_0}{\zeta_L}\right)\right]
\]
(14)

For \(|\zeta|,\zeta_0,\zeta_L \gg \theta_0\), Eqs.(13) and (14) transforms into Eqs.(10) and (11), correspondingly. As follows from Eq.(14), the \(L\)-dependence of the total transmittance at \(L \gg \max\{l_\theta, l_{tt}\zeta_0^{\alpha-1}\}\) (i.e., \(\zeta_L \gg \max\{\theta_0, \zeta_0\}\)) is governed by the asymptotic form of the reflection function \(R_\infty\) at relatively large angles. Substituting Eq.(A.8) into Eq.(14), we find an asymptotic dependence of the total transmittance on the medium thickness, \(T \sim (\theta_0/\zeta_L)^{1-2/\alpha}\), or, with allowance for the definition of \(\zeta_L\),
\[
T = C_\alpha(\zeta_0/\theta_0) \left(\frac{\theta_0}{\sigma L}\right)^{\frac{\alpha-2}{\alpha-1}}
\]
(15)
Equation (15) generalizes the result (12) to the case of an arbitrary relation between the incidence angle \(\zeta_0\) and the characteristic single-scattering angle \(\theta_0\). The explicit form of the function
Figure 1. Total transmittance as a function of the medium thickness. Symbols are the results of numerical integration of the radiative transfer equation (1) for the Henyey-Greenstein phase-function. The total number of discrete ordinates is chosen equal to 200. (a) The mean cosine of the single-scattering angle $g = 0.96$. The incidence angle $\zeta_0 = 0.03$ (triangles), 0.05 (squares) and 0.1 (circles). The solid line is Eq.(12) with $c_3 = 0.7$. (b) Upper graphs: $\zeta_0 = \vartheta_0 = 0.1$ (empty) and $\zeta_0 = \vartheta_0 = 0.05$ (filled), lower graphs: $\zeta_0 = \vartheta_0/2 = 0.05$ (empty) and $\zeta_0 = \vartheta_0/2 = 0.02$ (filled). The solid lines are Eq.(15) with $C_3(1) = 0.7$ and $C_3(0.5) = 0.6$, respectively.

Figure 2. Total transmittance as a function of the incidence angle. The medium optical thickness $\sigma L = 1$. Symbols are the numerical data [1] (sec.13.2, Fig.13.6) for the Henyey-Greenstein phase function with $g = 15/16$. The solid line is Eq.(12) with $c_3 = 0.7$.

$C_\alpha(\zeta_0/\vartheta_0)$ depends on the $\tilde{F}$–function which (as well as $F$) can be determined only by solving the radiative transfer equation.

Equations (12) and (15) are in a good agreement with the results of numerical integration of the radiative transfer equation (1) for the Henyey-Greenstein phase function (for which $\alpha = 3$ and $\vartheta_0 = 1 - g$, $g$ is the mean cosine of the single-scattering angle). Figure 1 shows the results of our calculations by the discrete-ordinate method [14, 15]. The data [1] are plotted in Fig.2.

For angles $\zeta_0 > \vartheta_0$ the total transmittance is proved to be nearly a universal function of the single parameter $\sigma L/\vartheta_0^2$ (see Fig.1a) in agreement with scaling relation (7). If the incidence angle $\zeta_0$ is comparable with $\vartheta_0$, the total transmittance appears to be a function of the two parameters $\zeta_0/\vartheta_0$ and $\sigma L/\vartheta_0$ (see Fig.1b). In both cases, the self-similarity fails in the crossover to the diffusive regime, $L \geq L_{tr}$, where the small-angle approximation becomes inapplicable, and the numerical solutions falls off more rapidly.
4. Conclusions
In conclusion, we have presented a theoretical study of light transmission through highly forward scattering media. An influence of the single-scattering phase function on the total transmittance has been found within a subdiffusion thickness range. The effect reveals itself at grazing angles of incidence and is governed by small-angle multiple scattering of light. The derived analytic results are in a good agreement with numerical solutions to the radiative transfer equation.

The results obtained above present a way of characterizing inhomogeneities of turbid media from the total transmittance (or reflectance) measurements. This may be useful in studies of highly forward scattering media such as biological tissues.

5. Appendix. Reflection function of a semi-infinite scattering medium
For a semi-infinite medium with the phase function (3), a closed equation for the reflection function $R_{\infty}(|\zeta|, \zeta_0) = |\zeta| R(z = 0, -|\zeta|)$ is derived in [8]:

$$\zeta_0 \Phi_{\lambda}(-\zeta_0) = \int_0^\infty d|\zeta| R_{\infty}(|\zeta|, \zeta_0) \Phi_{\lambda}(|\zeta|), \quad 0 < \lambda < \infty \quad (A.1)$$

where $\Phi_{\lambda}(\zeta)$ is the angular eigenfunction of Eq. (A.4),

$$\Phi_{\lambda}(\zeta) = \frac{\sqrt{3}}{\pi |\lambda|} \int_0^\infty d\omega \cos \left\{ \omega \zeta - \frac{1}{\lambda^2} \int_0^\omega d\omega' (1 - \hat{p}(\omega')) \right\}, \quad (A.2)$$

$\hat{p}(\omega)$ is the Fourier transform of the single-scattering phase function,

$$\hat{p}(\omega) = \int_0^\infty 2\pi \theta J_0(\omega \theta) p(\theta) d\theta = \frac{2}{\Gamma(\alpha/2 - 1)} \left( \frac{\omega \theta_0}{2} \right)^{\alpha/2 - 1} K_{\alpha/2 - 1}(\omega \theta_0), \quad (A.3)$$

$\Gamma(x)$ is the Gamma function, and $K_p(x)$ is the Macdonald function. Equation (A.1) should be satisfied for all eigenvalues $\lambda$ in the interval $0 < \lambda < \infty$.

If the angles $|\zeta|$ and $\zeta_0$ exceed the value $\theta_0$, the $\hat{p}$-function (A.3) can be expanded in the powers of $\omega \theta_0$,

$$1 - \hat{p}(\omega) \approx \frac{\Gamma(2 - \alpha/2)}{\Gamma(\alpha/2)} \left( \frac{\omega \theta_0}{2} \right)^{2 - \alpha}, \quad (A.4)$$

and the angular eigenfunctions take the form

$$\Phi_{\lambda}(\zeta) \approx \frac{\sqrt{3}}{\pi |\lambda|} \int_0^\infty d\omega \cos \left( \omega \zeta - \frac{a_\alpha \theta_0^{-2}}{\lambda^3} \omega^{\alpha - 1} \right) \quad (A.5)$$

where $a_\alpha = \Gamma(2 - \alpha/2)/[2^{\alpha - 2}(\alpha - 1) \Gamma(\alpha/2)]$. The reflection function (9) is a solution to Eq.(A.1) in which the angular eigenfunctions are approximated by Eq.(A.5).

For $|\zeta| \gg \theta_0$ and an arbitrary ratio $\zeta_0/\theta_0$, the approximation (A.5) can be used only in the kernel of Eq.(A.1). The eigenfunction in the inhomogeneous term should be calculated with the exact equations (A.2)-(A.3). Multiplying Eq.(A.1) by $\lambda^{-3\alpha/(\alpha - 1)}$ and integrating over all $\lambda$, we find the Mellin transform, $\hat{R}_{\infty}(s, \zeta_0) = \int_0^\infty |\zeta|^{s-1} R_{\infty}(|\zeta|, \zeta_0) d|\zeta|$, of the reflection function

$$\hat{R}_{\infty}(s, \zeta_0) = \frac{\zeta_0}{\theta_0} \left[ \frac{\Gamma(1 - s) \sin \frac{\pi \alpha s}{2(\alpha - 1)} \right]^{-1} \int_0^\infty d\zeta x^{-s/\alpha - 1} \cos \left( \frac{\zeta_0}{\theta_0} x + \frac{\pi s}{2(\alpha - 1)} \right) \quad (A.6)$$

$$\left[ a_\alpha^{-1} \int_0^\infty (1 - \hat{p}(\omega')) d(\omega' \theta_0) \right]^{-s/(\alpha - 1)} \cos \left( \frac{\zeta_0}{\theta_0} x + \frac{\pi s}{2(\alpha - 1)} \right)$$
The inverse transform gives the desired reflection function

\[ R_\infty(|\zeta|, \zeta_0) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{ds}{2\pi i} \left( \frac{|\zeta|}{\vartheta_0} \right)^{-s} \hat{R}_\infty(s, \zeta_0) \]  

(A.7)

The integration in Eq. (A.7) is performed in the band \( 0 < \text{Re} s < 1 \).

Equations (A.6) and (A.7) enables us to draw a conclusion regarding the asymptotic behavior of the reflection function at relatively large angles, \(|\zeta| \gg \max\{\vartheta_0, \zeta_0\}\). The asymptotic form of \( R_\infty(|\zeta|, \zeta_0) \) is governed by poles of \( \hat{R}_\infty(s, \zeta_0) \) in the complex \( s \)-plane. As analysis of Eq. (A.6) shows, the pole with the smallest positive real part is located at \( s = 2(\alpha - 1)/\alpha \). Therefore, the asymptotic expression for the reflection function has the following form

\[ R_\infty(|\zeta|, \zeta_0) = A_\alpha \left( \frac{\vartheta_0}{|\zeta|} \right)^{2(\alpha - 1)/\alpha}. \]  

(A.8)

The explicit form of the \( A_\alpha \)-function can be found by calculating the residue of \( \hat{R}_\infty(s, \zeta_0) \) at \( s = 2(\alpha - 1)/\alpha \).

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