We propose that femto scale QCD analogs of Fullerene type excitations may exist with “magic numbers” 8, 24, 48, and 120. These “J-balls” are the most symmetric closed networks of gluonic baryon junctions and anti-junctions with zero net baryon number. CP odd J-Moebii and other femto-tubes and tori gluonic structures may also exist. We estimate the relative binding energies and discuss possible experimental signatures of the formation of these new topological gluonic states.

1 Introduction

In QCD, baryons are made of quarks. However, to construct a gauge-invariant wave function of the baryon, one has to introduce also an interesting configuration of gauge fields – the “baryon junction”. The junction is built out of Wilson lines or strings, $U(x, \bar{x}) = P \exp \left( ig \int_x^{\bar{x}} dx^\mu A_\mu \right)$, with gauge field $A$. It ensures that the nonlocal baryon wave function, with quarks at $x_i$ and a junction at $x$ is invariant under gauge transformations. It has a color tensor structure $J_{j_1j_2j_3}(x_1, x_2, x_3) = \epsilon_{k_1k_2k_3} U_{j_1k_1}(x, x_1) U_{j_2k_2}(x, x_2) U_{j_3k_3}(x, x_3)$.

A color neutral baryon with quark flavors $(1,2,3)$ is then constructed as the contraction $B_{123}(x) = q_1(x_1)q_2(x_2)q_3(x_3)J(x_1, x_2, x_3)$. It is interesting that junctions in $SU(N)$ gauge theories call for the existence of a new family of glueball–like bound states in the confined phase, with masses scaling with the number of colors, $M \sim N$. (Note that “ordinary” glueballs, described by the closed strings, have masses $M \sim O(1)$. ) In QCD, the simplest bound state of this kind is a quarkless “baryonium” J-ball, formed by connecting a $J(x; \ldots)$ with an anti-$J(\bar{x}; \ldots)$.

$$M_0^J(x, \bar{x}) = Tr(J(x)J^\dagger(\bar{x})) = \epsilon^{j_1j_2j_3} U_{j_1k_1}(x, \bar{x}) U_{j_2k_2}(x, \bar{x}) U_{j_3k_3}(x, \bar{x}) \epsilon_{k_1k_2k_3}$$
While this state is constructed only from the gluon field, and thus has a zero net baryon number, it has a large coupling to the baryon wave functions, and couples to the $B - \bar{B}$ state.

In high-energy collisions, the valence quarks carry a large fraction of the incident baryon’s momentum, and thus always hadronize in the fragmentation regions. This, undoubtedly correct, statement is at the origin of the widely accepted belief that the central rapidity region of high energy proton and nuclear collisions should be “baryon–free”, i.e. should have a zero net baryon number. The existence of baryon junctions invalidates this naive picture, since gluonic junctions on the average carry only small fraction of the baryon’s momentum, and thus can be easily transferred to the central rapidity region, where the string break-up dresses them by quarks. In other words, this picture implies that the baryon number effectively resides in purely non-perturbative (topological) configuration of gluon fields, rather than in the valence quarks. In heavy ion physics, the interest in these configurations stems from the possibility that they lead to substantial baryon asymmetry in the mid-rapidity region of ultra-relativistic heavy ion collisions, leading also to new diquark breaking mechanisms which could transfer the baryon number over many units of rapidity. These configurations provide a novel mechanism for strangeness enhancement and for production of hyperon pairs, too. The resulting junction model of baryons can also be formulated in a Poincare invariant dynamical model for a baryon, where three valence quarks and a junction that represents sea quarks and all gluonic degrees of freedom interact as classical particles via a quasi potential. In spite of its simplicity, this junction model is able to describe some features of the baryon (binding energy, structure functions) surprisingly well. In addition, preliminary RHIC data were consistent with the high baryon stopping power predicted by $M^*_J$ exchange.

In this study we explore the combinatorial consequences of the existence of networks or closed cages involving junction and anti-junction type of configurations in QCD (and non-Abelian gauge theories in general). In particular, we argue that higher excitations of the QCD vacuum can be constructed, that are characterized by remarkable geometrical and topological structures. Certain “magic numbers” exist with high symmetry similar to Fullerene structures made of carbon, discovered in ref. However, in the QCD case the three valence connections are replaced by Wilson lines as in Eq. (1), representing the flow of color flux between adjacent junctions and anti-junctions.

Although the building blocks and the QCD Lagrangian are CP even, states can be constructed that are CP odd. We characterize the topological structure of these novel structures in QCD.
2 Fullerene

New forms of the element carbon - called fullerenes- in which the atoms are arranged in closed shells were discovered in 1985 by R. F. Curl, H. W. Kroto and R. E. Smalley. The number of carbon atoms in the shell can vary and for this reason numerous new carbon structures have become known.

Fullerenes are formed when vaporized carbon condenses in an atmosphere of inert gas. The gaseous carbon is obtained e.g. by directing an intense pulse of laser light at a carbon surface. The released carbon atoms are mixed with a stream of helium gas and combine to form clusters of some few up to hundreds of atoms. The gas is then led into a vacuum chamber where it expands and is cools. The carbon clusters can then be analyzed with mass spectroscopy.

Curl, Kroto and Smalley performed such an experiment and were able in particular to produce clusters with 60 and 70 carbon atoms. They found high stability in $C_{60}$ which suggested a molecular structure of great symmetry. It was suggested that $C_{60}$ could be a “truncated icosahedron cage”, a polyhedron with 20 hexagonal surfaces and 12 pentagonal (5 angled) surfaces. The pattern of European football has exactly this structure, as does the geodetic dome designed by the American architect R. Buckminster Fuller for the 1967 Montreal World Exhibition. The researchers named the newly discovered structure “Buckminsterfullerene” after him, subsequently the nick-named as “Buckyball”.

This discovery opened the way to a completely new branch of chemistry creating new forms of matter with unusual properties and to the 1996 Nobel prize in chemistry to Curl, Kroto and Smalley, see ref. for further details.

3 QCD Buckyballs

The QCD junctions and anti-junctions can be combined in higher excited states which can be called as QCD fullerenes (or Buckyballs), that are similar to the nano-structures formed in carbon chemistry, as summarized in the previous section. The QCD fullerenes (or junction balls) can be characterized by the topology of their structure. Due to the existence of junctions ($J$) and anti-junctions ($\bar{J}$), the QCD fullerenes may have only even number of vertices along any faces, hence their geometrical structure will be different from the carbon fullerenes, that contain 12 pentagonal and a varying number of hexagonal faces.

Let us recall Euler’s theorem that relates the number of faces ($F$), the number of edges ($E$) and the number of vertices ($V$) in a polyhedron as

$$V + F = 2 + E \quad (1)$$
where the number of faces is the sum of the number of squares ($N_4$), the number of hexagons ($N_6$), the number of octagons ($N_8$), etc,

$$F = N_4 + N_6 + N_8 + \ldots$$  \hspace{1cm} (2)

Each edge belongs to two faces. As junctions are color sinks, while anti-junctions are color sources, each junction has to be surrounded by three anti-junctions, and vice-versa, thus each vertex belongs to three faces:

$$E = \frac{1}{2} \left(4N_4 + 6N_6 + 8N_8 + \ldots\right),$$ \hspace{1cm} (3)

$$V = \frac{1}{3} \left(4N_4 + 6N_6 + 8N_8 + \ldots\right).$$ \hspace{1cm} (4)

The resulting Diophantian equations are solvable. The solution is independent from the number of hexagons, $N_6 = \text{any}$, the remaining constraint is

$$N_4 - \sum_{i=4}^{\infty} (i - 3)N_{2i} = 6$$  \hspace{1cm} (5)

This implies that if octagons, decagons and other structures with higher even number of vertices are neglected, 6 squares and any number of hexagons can be utilized to build up Euler polyhedra. These correspond to QCD fullerenes
where the junctions and anti-junctions alternate on the neighboring vertices. Such a variety of structures is similar to the case of carbon nano-structures, where any number of hexagons and exactly 12 pentagons can be combined to create a rich family of carbon nano-structures.

4 Magic numbers of QCD fullerenes

The formation of QCD fullerenes is a complicated dynamical process. A necessary condition for such process is the (symmetric) presence of the building blocks, the baryon and anti-baryon junctions. Thus QCD fullerenes may be excited in processes with vanishing (or small) net baryon number and high energy density sufficient to create the anti-baryons in big enough number.

A similar process is illustrated on Fig. 2 for the case of carbonic fullerenes. Here we will not speculate further on the dynamics that may lead to the formation of QCD J-balls. Our aim is simply to try to identify the most stable configurations and their “magic” (vertex) numbers.
As in the case of carbon fullerenes, we assume that the most symmetric (Archimedean polyhedra) configurations are the most stable ones. We estimate the energy of a particular configuration and the relative binding energies in the next paragraph. It is straightforward to solve the Euler equations for the case of the regular polyhedra. There exists only four Archimedean polyhedra consisting of even number of junction and anti-junction pairs. These numbers together with relative size and shape are shown in Figure 3. Many other, less regular shapes, of course, may exists, but these are the ones with highest symmetry. In SU($N$) gauge theories these structures are replaced by the most symmetric cages formed with each vertices connecting to exactly $N$ edges. It follows that in SU(2) only ring-like structures can be created.

The simplest model for the relative energy of a J-ball consisting of $n_V/2$ junctions and $n_V/2$ anti-junctions making a polyhedron characterized by $n_E$ edges with lengths $\ell_i$ is given by

$$E(\ell_i, n_{v,i}; N_V, N_E) = \sum_{i=1}^{N_E} \left( \frac{a}{\ell_i} + \kappa \ell_i \right) + \gamma \sum_v \sum_{i<j} \hat{n}_{v,i} \cdot \hat{n}_{v,j}$$ (6)

where $\hat{n}_{v,i=1,2,3}$ are the three unit vectors along the edges joining vertex $v$. 

Figure 3. The family of QCD fullerenes with magic numbers 8, 24, 48 and 120.
that define the topology. The first term is a “kinetic” vertex localization energy. The second is the confining string tension ($\kappa \approx 1 - \frac{4}{3} T^2 + \ldots \text{GeV/fm}$) term that vanishes at $T_c \approx 150$ MeV. The last postulated “strain” term is analogous to the Biot-Savart law in circuits and plays the same role as bond angle strain in carbon nanostructures.

In this model the relative binding energies are determined by the last term, given by the geometrical structure of the J-ball. For the $N_V = 8$ J-cube $\sum \hat{n}_{a,i} \cdot \hat{n}_{a,j} = 0$, while for $N_V = 24, 48, 120$, this strain energy is $-1, -(1 + \sqrt{2})/2 = -1.207, -(3 + \sqrt{5})/4 = -1.309$ in units of $\gamma$. The absolute minimum of $(J\bar{J})^{N_V/2}$ Jballs is reached of course for $N_V = \infty$, corresponding to a hexagonal (graphite-like) flat 2D lattice, as in the case of carbon fullerenes.

5 CP Odd and Even J-Ribbons

Other multi $(J\bar{J})^N$ configurations can be constructed that have definite CP symmetry transformation properties.

In case of carbons, the elementary building block of fullerene formation is a ring or a closed chain of carbons. In case of QCD, this is replaced by ribbons that are formed from $(J\bar{J})$ pairs. Closed ribbons have different CP eigenstates depending on the number of $(J\bar{J})$ pairs in them. For even number of pairs, the ribbon can be closed on itself without any twist, hence $(J\bar{J})^{2n}$ states can form CP even J-prisms as indicated on the left panel of Fig 4. However, other intriguing possibilities also exist. In particular, for odd number of $N = 2n + 1$ $(J\bar{J})$ pairs the QCD ribbon can be closed on itself with a rotation to the left or with a rotation to the right, forming a Moebius ribbon with winding number $+1$ or $-1$. These states transform into each other, a class of CP-odd J-Moebii is obtained. Note that for large values of $N$ the ribbon can be twisted not only once but many times before closing the surface on itself. It seems to be rather interesting to explore the CP and topological properties of these excitations; the closed ribbons can be characterized with their topological winding number that can be any integer number $N$, $N = 0$ corresponding to the simplest, CP even J-prisms.

The topology of the excitations of QCD may be very interesting, because not only ribbons but also tubes can be formed. These would be analogous to the carbon nano-tubes displayed on Fig. 1. The ends can be closed with caps formed by squares, octagons and decagons, or can be open, closed with the help of valence quarks. Another interesting possibility is to close the J-tube on itself, creating a toroidal structure. Carbon tubes are on the nanometer scale. The QCD excitations exist on the femtometer scale, hence J-tubes can be considered as femto-tubes.
The carbon nano-tubes can be characterized by two integers, these can be obtained from cutting a planar hexagonal lattice and using the number of steps in the direction of one of the edges in a pre-selected vertex to find the point that is identified with the same vertex in the planar hexagonal lattice, these are called the \((n, m)\) nano-tubes.

The QCD femto-tubes have different topological property due to the existence of junctions and anti-junctions. This implies that on the planar hexagonal lattice a junction has 6 other anti-junctions in the nearest neighbor position, indicating also 2 independent directions. Hence the QCD femto-tubes can also be characterized by 2 integers, indicating the number of steps in these directions to find the equivalent junctions in the planar hexagonal lattice that are identified to close the planar structure to a tube. These can structures be classified as \((n, m)\) QCD femto-tubes. The femto-tubes can be closed also by connecting the two ends of a long tube, and these ends can be rotated before

Figure 4. A CP even J-ribbon and a CP-odd J-Moebius-ring.
the connection. This gives QCD femto-tori that can be characterized by 3 winding numbers, the \((i, j, k)\) femto-tori.

6 Estimates and observables

We estimate that the structures with the smallest strain and the smallest total baryon number are the easiest to produce experimentally. This suggests that the Archimedian polygons are favored, where each vertex carries the same amount of strain. The contribution from the extra strains as compared to a planar structure depends on a parameter \(\gamma\) that is not yet known; but the relative strain energy decreases from 0 to -1 from the cube to the \(B + \overline{B} = 24\) J-ball; it decreases further by 20\% when forming the \(B + \overline{B} = 48\) polyhedron. The next Archimedian polyhedra has \(B + \overline{B} = 120\), which is probably too large even for RHIC conditions. Hence we suggest to search for clusters that decays to equal number of baryons and anti-baryons with a total number of 8, 24 and 48.

In addition, we suggest to search for clusters of equal number of baryons and anti-baryons that violate CP symmetry, as these clusters also appear in the spectrum of QCD Jballs. Some experimental observables that may be used to search for the formation of CP-violating domains or clusters were summarized in the talk of J. Sandweiss\(^\text{11}\).

7 Summary and outlook

In summary, fullerene type of pure glue topological configurations exist in QCD. These are termed as junction-balls (Jballs) or QCD femto-structures. All of these configurations have an equal number of junctions and anti-junctions, but have interesting geometrical and topological properties, that can be determined in a straightforward manner. Some of the QCD Bucky-balls are CP even, others like Moebius ribbons are CP odd states. Topological winding numbers can be introduced to characterize these states. The QCD femto-ribbons are characterized by a single integer \((n)\), the femto-tubes by a pair of integers \((n, m)\), while the QCD femto-tori by a triplet of integers, \((n, m, l)\).

We determined that the most symmetric (likely most stable) J-ball configurations have the magic number of baryon + anti-baryon number of \(B + \overline{B} = 8, 24, 48\) and 120. Although these configurations are likely unstable, they are expected to be more stable than clusters of baryons and anti-baryons with different junction numbers, and they may appear as peaks in the spectrum of \((BB)^n\) clusters with a given total baryon+antibaryon number. To cre-
ate them, high initial energy densities and small net initial baryon number densities and large volumes are needed. Such conditions may exist in the mid-rapidity domain of central $Au + Au$ collisions at RHIC or in the mid-rapidity domain of $p + \bar{p}$ collisions at the Tevatron.

It would certainly be of great theoretical and experimental interest if these novel gluonic states were observed in QCD lattice simulations or in baryon-antibaryon clusters in high energy reactions.

Acknowledgments

T. Cs. thanks Columbia University for hospitality during several visits and to M. Albrow for inspiring discussions. MG thanks the Collegium Budapest and KFKI. This research has been supported by a Bolyai Fellowship of the Hungarian Academy of Sciences, by the Hungarian OTKA grants T024094, T026435, T029158, T034269, by the US-Hungarian Joint Fund MAKA 652/1998, by the NWO-OTKA grant N025186 and by the US DOE under contracts no. DE-FG-02-93ER40764, and DE-AC02-98CH10886.

References

1. G.C. Rossi and G. Veneziano, \textit{Nucl. Phys. B} \textbf{123} (1977) 507; \textit{Phys. Rept.} \textbf{63} (1980) 153.
2. D. Kharzeev, \textit{Phys. Lett. B} \textbf{378} (1996) 238.
3. B. Z. Kopeliovich and B. G. Zakharov, \textit{Z. Phys. C} \textbf{43} (1989) 241.
4. A. Capella and B.Z. Kopeliovich, \textit{Phys. Lett. B} \textbf{381} (1996) 325.
5. M. Gyulassy, V. Topor Pop, S. E. Vance, \textit{Heavy Ion Phys.} \textbf{5} (1997) 299
   S. E. Vance, M. Gyulassy, X.-N. Wang, \textit{Nucl. Phys. A} \textbf{638} (1998) 395c,
   S. E. Vance, M. Gyulassy, X.-N. Wang, \textit{Phys. Lett. B} \textbf{443} (1998) 45,
   P. Csizmadia, P. Lévai, S.E. Vance, T.S. Biró, M. Gyulassy, J. Zimányi, \textit{J. Phys. G} \textbf{25} (1999) 321,
   S. E. Vance, M. Gyulassy, \textit{Phys. Rev. Lett.} \textbf{83} (1999) 1735.
6. H. W. Fricke, C. C. Noack, \textit{Phys. Rev. Lett.} \textbf{80} (1998) 3014.
7. N. Xu, \textit{Proc. QM01}, [ftp://woodstock.physics.sunysb.edu/pub/qm2001/6-Saturday/1-WhatLearned/1-Xu.pdf]
8. H. W. Kroto, R. E. Smalley and R. F. Curl, \textit{Nature} \textbf{318} (85) 165.
9. [http://www.nobel.se/chemistry/laureates/1996/press.html]
10. S. Maruyama and Y. Yamaguchi, \textit{Therm. Sci. Engng} \textbf{3} (1995) 105;
    \textit{Eur. Phys. J. D} \textbf{9} 385.
11. J. Sandweiss, \textit{Proc. XXXth ISMD}, Tihany, Hungary, 2000, \textit{(World Scientific, 2001, ed. T. Csörgő, S. Hegyi and W. Kittel).}