Parton recombination effect in polarized parton distributions

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Abstract

Parton recombination corrections to the standard spin-dependent Altarelli-Parisi evolution equation are considered in a nonlinear evolution equation. The properties of this recombination equation and its relation with the spin-averaged form are discussed.

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1 Introduction

The spin problem of the nucleon has been investigated extensively since the discovery of the EMC spin effect, which indicates that a small fraction of the nucleon spin is from the quark helicity [1]. This surprising result leads to extensive study on the gluon- and sea quark-polarization. The current parameterizations suggest the non-vanished polarization in gluon. It implies that the spin-correlation exists among polarized partons. Determination of polarized parton distribution functions is crucial for understanding the spin structure of the nucleon. The knowledge about polarized parton distributions in the nucleon is mainly extracted from global analysis of polarized experimental data at different values of $Q^2$ based on the linear (spin dependent) Altarelli-Parisi (AP) equation [2]. However, the contributions of spin-correlation among initial polarized partons are excluded in the linear evolution equation. Therefore, it is necessary to modify the AP equation to include such correlation effect.

In the unpolarized case, the parton recombination as a leading correlation effect of partons in QCD evolution equation has been studied at leading logarithmic ($Q^2$) approximation ($LLA(Q^2)$) and in a whole $x$ region by [3-4]. We call it the recombination evolution equation. The parton recombination effect is generally considered as a phenomenon at the small $x$ region, where the rapid increase of parton densities can be obviously suppressed, i.e., the saturation is predicted [5]. The behavior of polarized parton distributions at small $x$ is an important issue because the experimental determination of the first moment of polarized gluon distribution depending on it is rather strongly [1,6]. Moreover, the parton recombination can also influence the polarized parton distributions at a larger $x$ region. For example, the helicity of a fast gluon may be changed after combining with a slower gluon, although its momentum is almost unchanged. Therefore, in this paper we generalize the recombination equation to the spin-dependent case. This equation can
work in a whole $x$ region. Thus, the recombination equation can be used to study the spin effect. In particular, we show that the polarized recombination effect disappears at the small $x$ limit. We also find that different from the linear evolution equation, the correlation form of polarized partons in the spin-dependent recombination is relevant to the structure of the spin-independent evolution kernel. Therefore, this research is also useful for understanding the shadowing dynamics in the unpolarized parton distributions.

The paper is organized as follows. In Section 2 we construct the spin-dependent recombination equation using the same way as refs. [3,4]. In Section 3 the polarized recombination functions are evolute. Then the discussion and concluding remarks are given in Section 4.
2 Recombination evolution equation

The elementary process is one-parton splitting to two-partons in the AP equation. At very large number densities of partons, for example in the small $x$ region, the wave functions of partons can overlap. In this case the contributions of two-partons-to-two-partons subprocess (i.e., the parton recombination) should be considered in the QCD evolution equations. A complete derivation for such equations need to sum the contributions from real and interference Feynman diagrams and corresponding virtual diagrams. Gribov, Levin and Ryskin in [7] use the AGK cutting rule [9] to count the contributions of interference diagrams. On the other hand, the real diagrams are calculated at the double leading logarithmic approximation ($DLLA$) in a covariant way (the cut vertex technique) by Mueller and Qiu in [8]. This nonlinear evolution is called the GLR-MQ equation.

Unfortunately, the application of the AGK cutting rule in the GLR-MQ equation was questioned since it breaks the evolution kernels [3]. To avoid this disadvantage, the simple relations among the relevant high twist amplitudes are derived by using the time ordered perturbation theory (TOPT) at the leading logarithmic approximation ($LLA(Q^2)$) in [3]. Then the recombination functions are calculated in a whole $x$ region based on the same TOPT-framework in [4]. This is the recombination evolution equation.

The helicity amplitudes in the recombination equation satisfy the same TOPT-relations as in the unpolarized case. Thus, using the arguments in [3] we can directly write down the spin-dependent evolution equations which are

\[
\frac{dxG_\pm(x,Q^2)}{d\ln Q^2} = \frac{\alpha^2 K}{Q^2} \int_{x_1+x_2 \geq x} dx_1 dx_2 [Fig.1a + Fig.1b + Fig.1c] \\
- 2 \int_{(x_1+x_2)/2 \geq x} dx_1 dx_2 [Fig.1a + Fig.1b + Fig.1c], \tag{1a}
\]
Fig. 1a = \( f_{G_+G_+}(x_1, x_2, Q^2)R_{G_+G_+ \rightarrow G_+}(x_1, x_2, x) \)

\[ + f_{G_+G_-}(x_1, x_2, Q^2)R_{G_+G_- \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{G_-G_+}(x_1, x_2, Q^2)R_{G_-G_+ \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{G_-G_-}(x_1, x_2, Q^2)R_{G_-G_- \rightarrow G_+}(x_1, x_2, x), \]  

(1b)

Fig. 1b = \( f_{q_q^{-}}(x_1, x_2, Q^2)R_{q_q^{-} \rightarrow G_+}(x_1, x_2, x) \)

\[ + f_{q_q^{+}}(x_1, x_2, Q^2)R_{q_q^{+} \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{q_q^{+}}(x_1, x_2, Q^2)R_{q_q^{+} \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{q_q^{-}}(x_1, x_2, Q^2)R_{q_q^{-} \rightarrow G_+}(x_1, x_2, x), \]  

(1c)

Fig. 1c = \( f_{G_+q_q^{+}}(x_1, x_2, Q^2)R_{G_+q_q^{+} \rightarrow G_+}(x_1, x_2, x) \)

\[ + f_{G_+q_q^{-}}(x_1, x_2, Q^2)R_{G_+q_q^{-} \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{G_-q_q^{+}}(x_1, x_2, Q^2)R_{G_-q_q^{+} \rightarrow G_+}(x_1, x_2, x) \]

\[ + f_{G_-q_q^{-}}(x_1, x_2, Q^2)R_{G_-q_q^{-} \rightarrow G_+}(x_1, x_2, x), \]  

(1d)

\[ \frac{dxq_x^i(x, Q^2)}{d \ln Q^2} = \frac{\alpha^2 K}{Q^2} \int_{x_1+x_2 \geq x} dx_1 dx_2 [Fig. 1d + Fig. 1(e - 1) + Fig. 1(e - 2) + Fig. 1(f)] \]

\[ -2 \int_{(x_1+x_2)/2 \geq x} dx_1 dx_2 [Fig. 1d + Fig. 1(e - 1) + Fig. 1(e - 2) + Fig. 1(f)], \]  

(1e)

Fig. 1d = \( f_{G_+G_+}(x_1, x_2, Q^2)R_{G_+G_+ \rightarrow q_q^{+}}(x_1, x_2, x) \)

\[ + f_{G_+G_-}(x_1, x_2, Q^2)R_{G_+G_- \rightarrow q_q^{+}}(x_1, x_2, x) \]
\[ + f_{G^-G^+}(x_1, x_2, Q^2)R_{G^-G^+ ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{G^-G^-}(x_1, x_2, Q^2)R_{G^-G^- ightarrow q^i_{\pm}}(x_1, x_2, x), \quad (1f) \]

\[ Fig.1(e-1) = f_{q^i_{+} q^i_{+}}(x_1, x_2, Q^2)R_{q^i_{+} q^i_{+} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{+} q^i_{-}}(x_1, x_2, Q^2)R_{q^i_{+} q^i_{-} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} q^i_{+}}(x_1, x_2, Q^2)R_{q^i_{-} q^i_{+} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} q^i_{-}}(x_1, x_2, Q^2)R_{q^i_{-} q^i_{-} ightarrow q^i_{\pm}}(x_1, x_2, x), \quad (1g) \]

\[ Fig.1(e-2) = f_{q^i_{+} q^i_{+}}(x_1, x_2, Q^2)R_{q^i_{+} q^i_{+} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{+} q^i_{-}}(x_1, x_2, Q^2)R_{q^i_{+} q^i_{-} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} q^i_{+}}(x_1, x_2, Q^2)R_{q^i_{-} q^i_{+} ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} q^i_{-}}(x_1, x_2, Q^2)R_{q^i_{-} q^i_{-} ightarrow q^i_{\pm}}(x_1, x_2, x), \quad (1h) \]

\[ Fig.1f = f_{q^i_{+} G^+}(x_1, x_2, Q^2)R_{q^i_{+} G^+ ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{+} G^-}(x_1, x_2, Q^2)R_{q^i_{+} G^- ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} G^+}(x_1, x_2, Q^2)R_{q^i_{-} G^+ ightarrow q^i_{\pm}}(x_1, x_2, x) \]
\[ + f_{q^i_{-} G^-}(x_1, x_2, Q^2)R_{q^i_{-} G^- ightarrow q^i_{\pm}}(x_1, x_2, x). \quad (1i) \]

The explanations about this equation are summarized as follows. (i) We neglect the linear terms of the AP equation in Eq. 1. (ii) The first integral in Eqs. 1a and 1e is from two-parton-to-two-parton amplitudes and it leads to the positive (antishadowing) effect, while the second integral is the contributions of the interference amplitudes between the one-parton-to-two-parton and the three-parton-to-two-parton processes, and it is the negative (shadowing) effect. The contributions from the virtual diagrams are cancelled each
other in the nucleon. (iii) $R_{A_mB_n \rightarrow C_l}(x_1, x_2, x)$ are the recombination functions defined as Fig.1, where $m$ and $n$ are the helicities of parton $A$ and $B$, and we shall detail them in next Section. (iii) The nonlinear coefficient $K$ depends on the definition of the two parton correlation $f_{A_mB_n}(x_1, x_2, Q^2)$ and the geometric distributions of partons inside the target. For simplicity, we take $K$ as a free parameter.

The parton density is a concept defined at twist-2. The parton correlation function is the generalization of the parton density beyond the leading twist and it has not yet been determined either theoretically or experimentally. Similar to [4], we use a toy model (model I) to assume that

$$\chi(x_1, x_2) = \chi x_1 x_2 \theta(1 - x_1 - x_2),$$

where the correlator $\chi$ is regarded as a constant and it can be incorporated into the free parameter $K$.

The equation (1) can immediately be simplified using parity conservation in QCD, which leads to

$$R_{A_+ B_+ \rightarrow C_+} = R_{A_- B_- \rightarrow C_-}, R_{A_+ B_- \rightarrow C_+} = R_{A_- B_+ \rightarrow C_-},$$

$$R_{A_+ B_- \rightarrow C_+} = R_{A_- B_+ \rightarrow C_-}, R_{A_+ B_+ \rightarrow C_-} = R_{A_- B_- \rightarrow C_+}. \quad (3)$$

We define

$$A = A_+ + A_-, \quad \Delta A = A_+ - A_-,$$

and $A$ is the unpolarized parton (quark or gluon) density. In the meantime, the following definitions are convenient in the summation of Eq. (1).
\[ R_{AB\rightarrow C} = \frac{1}{2} [R_{A_+B_+\rightarrow C_+} + R_{A_+B_+\rightarrow C_-} + R_{A_+B_-\rightarrow C_+} + R_{A_+B_-\rightarrow C_-}] \quad (5a) \]

\[ \Delta R_{AB\rightarrow C}^I = \frac{1}{2} [R_{A_+B_+\rightarrow C_+} - R_{A_+B_+\rightarrow C_-} - R_{A_+B_-\rightarrow C_+} + R_{A_+B_-\rightarrow C_-}] \quad (5b) \]

\[ \Delta R_{AB\rightarrow C}^{II} = \frac{1}{2} [R_{A_+B_+\rightarrow C_+} - R_{A_+B_+\rightarrow C_-} + R_{A_+B_-\rightarrow C_+} - R_{A_+B_-\rightarrow C_-}] \quad (5c) \]

The result equation is

\[
\frac{dG(x, Q^2)}{d\ln Q^2} = \frac{a_s^2 K}{Q^2} \int_{x_1+x_2 \geq x} dx_1 dx_2 \frac{x_1 x_2}{x_1 + x_2} \big[ Eq. 6b \big] \\
-2 \int_{(x_1+x_2)/2 \geq x} dx_1 dx_2 \frac{x_1 x_2}{x_1 + x_2} \big[ Eq. 6b \big], (6a)
\]

\[ Eq. 6b = G(x_1)G(x_2)\theta(1 - x_1 - x_2)R_{GG\rightarrow G}(x_1, x_2, x) \]

\[ + \sum_i q^i(x_1)\bar{q}^i(x_2)\theta(1 - x_1 - x_2)R_{q\bar{q}\rightarrow G}(x_1, x_2, x) \]

\[ + \sum_i G(x_1)q^i(x_2)\theta(1 - x_1 - x_2)R_{Gq^i\rightarrow G}(x_1, x_2, x), (6b) \]

\[
\frac{dx \Delta G(x, Q^2)}{d\ln Q^2} = \frac{a_s^2 K}{Q^2} \int_{x_1+x_2 \geq x} dx_1 dx_2 \frac{x_1 x_2}{x_1 + x_2} \big[ Eq. 7b \big] \\
-2 \int_{(x_1+x_2)/2 \geq x} dx_1 dx_2 \frac{x_1 x_2}{x_1 + x_2} \big[ Eq. 7b \big], (7a)
\]

\[ Eq. 7b = G(x_1)\Delta G(x_2)\theta(1 - x_1 - x_2)\Delta R_{GG\rightarrow G}^I(x_1, x_2, x) \]

\[ + \Delta G(x_1)G(x_2)\theta(1 - x_1 - x_2)\Delta R_{GG\rightarrow G}^{II}(x_1, x_2, x) \]
\[\begin{align*}
&+ \sum_i q^i(x_1) \Delta \overline{q}^i(x_2) \theta(1 - x_1 - x_2) \Delta R^I_{q^i} G(x_1, x_2, x) \\
&+ \sum_i \Delta q^i(x_1) \overline{q}^i(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{q^i} G(x_1, x_2, x) \\
&+ G(x_1) \Delta q^i(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{Gq^i} G(x_1, x_2, x) \\
&+ \sum_i \Delta G(x_1) q^i(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{Gq^i} G(x_1, x_2, x), \quad (7b) \\
\end{align*}\]

\[
\frac{dx q^i(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s^2 K}{Q^2} \int_{x_1 + x_2 \geq x} dx_1 dx_2 \frac{x x_1 x_2}{x_1 + x_2} [Eq.8b] \\
-2 \int_{(x_1 + x_2)/2 \geq x} dx_1 dx_2 \frac{x x_1 x_2}{x_1 + x_2} [Eq.8b], \quad (8a) \\
\]

\[\begin{align*}
Eq.8b &= G(x_1) G(x_2) \theta(1 - x_1 - x_2) R_{GG \rightarrow q^i} G(x_1, x_2, x) \\
&+ q^j(x_1) q^k(x_2) \theta(1 - x_1 - x_2) R_{q^j q^k \rightarrow q^i} G(x_1, x_2, x) \\
&+ q^j(x_1) G(x_2) \theta(1 - x_1 - x_2) R_{q^j G \rightarrow q^i} G(x_1, x_2, x), \quad (8b) \\
\end{align*}\]

\[
\frac{dx \Delta q^i(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s^2 K}{Q^2} \int_{x_1 + x_2 \geq x} dx_1 dx_2 \frac{x x_1 x_2}{x_1 + x_2} [Eq.9b] \\
-2 \int_{(x_1 + x_2)/2 \geq x} dx_1 dx_2 \frac{x x_1 x_2}{x_1 + x_2} [Eq.9b], \quad (9a) \\
\]

\[\begin{align*}
Eq.9b &= G(x_1) \Delta G(x_2) \theta(1 - x_1 - x_2) \Delta R^I_{GG \rightarrow q^i} G(x_1, x_2, x) \\
&+ \Delta G(x_1) G(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{GG \rightarrow q^i} G(x_1, x_2, x) \\
&+ q^j(x_1) \Delta \overline{q}^j(x_2) \theta(1 - x_1 - x_2) \Delta R^I_{q^j \overline{q}^i} G(x_1, x_2, x) \\
&+ \Delta q^j(x_1) \overline{q}^j(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{q^j \overline{q}^i} G(x_1, x_2, x) \\
&+ q^j(x_1) q^k(x_2) \theta(1 - x_1 - x_2) \Delta R^I_{q^j q^k \rightarrow q^i} G(x_1, x_2, x) \\
&+ q^j(x_1) \Delta q^k(x_2) \theta(1 - x_1 - x_2) \Delta R^{II}_{q^j q^k \rightarrow q^i} G(x_1, x_2, x) \\
\end{align*}\]
\[ + \Delta q^j(x_1)q^k(x_2)\theta(1 - x_1 - x_2)\Delta R^{II}_{q^j q^k \to q^i}(x_1, x_2, x) \]

\[ + q^i(x_1)\Delta G(x_2)\theta(1 - x_1 - x_2)\Delta R^{I}_{q^i G \to q^i}(x_1, x_2, x) \]

\[ + \Delta q^i(x_1)G(x_2)\theta(1 - x_1 - x_2)\Delta R^{II}_{q^i G \to q^i}(x_1, x_2, x). \]

(9b)
3 Polarized recombination functions

The parton recombination function for $A(p_1) + B(p_2) \rightarrow C(p_3) + D(p_4)$ in Eq. (1) is defined as

$$\alpha_s^2 R_{(p_1 p_2 \rightarrow p_3)} dx_4 \frac{d^2 l_1}{d^2 l_1} = \frac{1}{16\pi^2 (x_1 + x_2)^3} |M_{p_1 p_2 \rightarrow p_3 p_4}|^2 dx_4 \frac{d^2 l_1}{d^2 l_1}, \quad (10)$$

where $l_1^2 = p_x^2 + p_y^2$. The matrix $M_{p_1 p_2 \rightarrow p_3 p_4}$ describes the subprocess (for example, Fig. 1), which are factorized from the deep inelastic scattering amplitudes due to the application of TOPT in [3]. Note that the parton indicated by ‘×’ is on-mass-shell, although is off-energy-shell) in TOPT, therefore, it has determinate helicity. Concretely, the momenta of the initial and final partons are parameterized as

$$p_1 = [x_1 p, 0, 0, x_1 p], \quad p_2 = [x_2 p, 0, 0, x_2 p],$$

$$p_3 = [x_3 p + \frac{p_x^2 + p_y^2}{2x_3 p}, p_x, p, x_3 p], \quad p_4 = [x_4 p + \frac{p_x^2 + p_y^2}{2x_4 p}, -p_x, -p_y, x_4 p],$$

$$l_L = [(x_4 - x_2)p + \frac{p_x^2 + p_y^2}{2x_4 p}, -p_x, -p_y, (x_4 - x_2)p],$$

$$l_R = [(x_4 - x_2)p + \frac{p_x^2 + p_y^2}{2x_4 p}, -p_x, -p_y, (x_4 - x_2)p]. \quad (11)$$

We take the physical axial gauge and the light-like vector $n$ fixes the gauge as $n \cdot A = 0$, $A$ being the gluon field. Now a recombination function for $G^\pm G^\pm \rightarrow q^\pm_1$ in the $t$-channel is computed in following form,

$$R_{G^\pm G^\pm \rightarrow q^\pm_1} = \langle \frac{1}{12} \rangle_{color} \frac{x_3 x_4}{(x_1 + x_2)^3} Tr[\bar{\gamma}_\alpha I_L \gamma_\mu \gamma_5 \bar{\gamma}_\beta I_R \gamma_\nu \gamma_5 \frac{1 \pm \gamma_5}{2}] \frac{\epsilon^\alpha \epsilon^\beta \epsilon^\mu \epsilon^\nu}{l_L^2 l_R^2}, \quad (12)$$

where the polarization vector of the collinear initial gluon is

$$\epsilon_\pm = \frac{[0, 1, \pm i, 0]}{\sqrt{2}}. \quad (13)$$
\[ R^t_{G_\pm G_\pm \rightarrow G_\pm} = \left( \frac{9}{8} \right) \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \tr \left[ \hat{p}_3 \gamma_\mu \left( \frac{1 \pm \gamma_5}{2} \right) \hat{p}_1 \gamma_\nu \left( \frac{1 \pm \gamma_5}{2} \right) \right] \]

\[ \times \frac{d_{\mu\nu}(l_L) d_{\nu\beta}(l_R)}{l_L^2 l_R^2} [\delta_{rs} - \frac{p^r_4 p^s_4}{|p^4|^2}], \]

where \( r, s \) are the space indices of \( \rho, \sigma \) of \( p_4 \); \( C_{\alpha\beta\sigma} \) and \( C_{\beta\lambda\sigma} \) are triple gluon vertex and the polarization vector of final gluon, which has transverse momentum takes

\[ \varepsilon_\pm = [0, \frac{1}{\sqrt{2}}, \pm i, \frac{p_x \pm ip_y}{\sqrt{2}x_3 p}], \]

The gluon polarization tensor on the Feynman propagator is summing over helicity state since it is off-shell, i.e.,

\[ d_{\mu\nu}(l) = g_{\mu\nu} - \frac{l_\mu n_\nu + l_\nu n_\mu}{l \cdot n}. \]

Similarly, we write other recombination functions in the t-channel

\[ R^t_{q_\pm q_\pm \rightarrow q_\pm} = \left( \frac{2}{9} \right) \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \tr \left[ \hat{p}_3 \gamma_\mu \left( \frac{1 \pm \gamma_5}{2} \right) \hat{p}_1 \gamma_\nu \left( \frac{1 \pm \gamma_5}{2} \right) \right] \]

\[ \times \frac{d_{\mu\nu}(l_L) d_{\nu\beta}(l_R)}{l_L^2 l_R^2}, \]

\[ R^t_{G_\pm q_\pm \rightarrow G_\pm} = \left( \frac{1}{2} \right) \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \tr \left[ \hat{p}_3 \gamma_\mu \left( \frac{1 \pm \gamma_5}{2} \right) \hat{p}_1 \gamma_\nu \left( \frac{1 \pm \gamma_5}{2} \right) \right] \]

\[ \times C_{\alpha\beta\sigma} C_{\beta\lambda\sigma} \varepsilon_\pm^\alpha \varepsilon_\pm^\beta \frac{d_{\mu\nu}(l_L) d_{\nu\lambda}(l_R)}{l_L^2 l_R^2} [\delta_{rs} - \frac{p^r_4 p^s_4}{|p^4|^2}], \]

\[ R^t_{G_\pm q_\pm \rightarrow q_\pm} = \left( \frac{1}{2} \right) \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \tr \left[ \hat{p}_3 \gamma_\mu \left( \frac{1 \pm \gamma_5}{2} \right) \hat{p}_1 \gamma_\nu \left( \frac{1 \pm \gamma_5}{2} \right) \right] \]

\[ \times C_{\alpha\beta\sigma} C_{\beta\lambda\sigma} \varepsilon_\pm^\alpha \varepsilon_\pm^\beta \frac{d_{\mu\nu}(l_L) d_{\nu\lambda}(l_R)}{l_L^2 l_R^2}. \]
where the polarization vector for the on-shell final state gluon ($p_4$), which has transverse components is

$$\varepsilon_\pm = \left[0, \frac{1}{\sqrt{2}}, \frac{\pm i}{\sqrt{2}}, \frac{p_x \pm ip_y}{\sqrt{2} x_4 p}\right]. \quad (20)$$

$$R^s_{q_\pm q_\pm \rightarrow G_\pm} = \left(\frac{16}{27}\right)^{\text{color}} \left(\frac{x_3 x_4}{x_1 + x_2}\right)^3 \frac{n^\rho n^\sigma}{(l_L \cdot n)^2 (l_R \cdot n)^2} \delta^{rs} - \frac{P_R^4 P_4^4}{|P_4|^2}$$. \quad (21)

The massless partons with the parallel momenta can go on-mass-shell simultaneously in the collinear case and the collinear singularity may arise in the $s$-channel in calculations of the recombination function. In this case, the collinear time ordered perturbation theory, which is developed in [10] provides a safe way to evolve such channel. For example,

$$R^s_{G_\pm G_\pm \rightarrow q_\pm} = \left(\frac{3}{16}\right)^{\text{color}} \left(\frac{x_3 x_4}{x_1 + x_2}\right)^3 \frac{n^\rho n^\sigma}{(l_L \cdot n)^2 (l_R \cdot n)^2} \delta^{rs} - \frac{P_R^4 P_4^4}{|P_4|^2}$$. \quad (24)

$$R^s_{q_\pm q_\pm \rightarrow q_\pm} = \left(\frac{2}{9}\right)^{\text{color}} \left(\frac{x_3 x_4}{x_1 + x_2}\right)^3 \frac{n^\rho n^\sigma}{(l_L \cdot n)^2 (l_R \cdot n)^2} \delta^{rs} - \frac{P_R^4 P_4^4}{|P_4|^2}$$. \quad (25)
\[ R_{G_\pm q_\pm^0 \rightarrow G_\pm}^s = \frac{2}{9} \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \text{Tr} \left[ \gamma_\alpha \frac{\gamma \cdot n}{2 l_L \cdot n} \gamma_\mu \gamma_\nu \gamma_\beta \right] \]  
\[ \times \epsilon_\pm^\mu \epsilon_\pm^\nu \delta^{rs} - \frac{p_4^r p_4^s}{|p_4^2|^2}, \]  
\[ (26) \]

\[ R_{q_\pm^i q_\pm^j \rightarrow G_\pm}^s = \frac{4}{3} \frac{x_3 x_4}{\text{color} (x_1 + x_2)^3} \text{Tr} \left[ \gamma_\sigma \gamma_\alpha \right] \frac{n^\eta n^\rho}{(l_L \cdot n)^2 (l_R \cdot n)^2} \]  
\[ \times C^{\mu \eta \gamma} C^{\nu \beta \lambda} \epsilon_\pm^\mu \epsilon_\pm^\nu \delta^{rs} - \frac{p_4^r p_4^s}{|p_4^2|^2}, \]  
\[ (27) \]

The complete polarized recombination functions, which summing the contributions of the \( t-, u-, s- \) and their interference channels present in Appendix. At the small \( x \) limit, one can keep only the \( \ln Q^2 \ln(1/x) \) factor (i.e., the DLLA ) and the gluonic initial partons, the non-vanished polarized recombination functions can be simplified as

\[ R_{G_+ G_+ \rightarrow G_+} = 9 \frac{1}{4} \frac{x_1^2 + x_2^2}{x_1^2}, \]  
\[ (29) \]

\[ R_{G_+ G_+ \rightarrow G_-} = 9 \frac{x_1^4 + x_2^2 x_1^2 + x_2^4}{x x_2^2 x_1^2}, \]  
\[ (30) \]

and

\[ R_{G_+ G_- \rightarrow G_+} = R_{G_+ G_- \rightarrow G_-} = \frac{9}{4} \frac{x_1^6 + 6 x_2 x_1^5 + 3 x_2^2 x_1^4 - 4 x_2^3 x_1^3 + 3 x_2^4 x_1^2 + 6 x_2^5 x_1 + 5 x_2^6}{x (x_1 + x_2)^2 x_2^2 x_1^2}. \]  
\[ (31) \]

Further assuming \( x_1 = x_2 \), we have

\[ R_{G_+ G_+ \rightarrow G_+} = R_{G_+ G_+ \rightarrow G_-} = \frac{27}{4} \frac{1}{x}. \]  
\[ (32) \]
\[ R_{G_+ G_- \rightarrow G_+} = R_{G_+ G_- \rightarrow G_-} = \frac{27}{2} \frac{1}{x}. \quad (33) \]

In this case, Eqs. (5b), (5c) and (7) are zero. It implies that the polarized recombination effect *disappears*.

The corresponding unpolarized gluon recombination functions at the DLLA are

\[ R_{G G \rightarrow G} = \frac{9}{4} \frac{6 x_1^6 + 8 x_2 x_1^5 + 5 x_2^2 x_1^4 - 2 x_2^3 x_1^3 + 5 x_2^4 x_1^2 + 8 x_2^5 x_1 + 6 x_2^6}{(x_1 + x_2)^2 x_2^2 x_1^2 x} \quad (34) \]

for \( x_1 \neq x_2 \), and

\[ R_{G G \rightarrow G} = \frac{81}{4} \frac{1}{x} \quad (35) \]

for \( x_1 = x_2 \).
4 Discussions and summary

Comparing with the linear spin-dependent AP equation, the nonlinear recombination equation has more complicated structure, in particular this equation contains yet unknown twist-4 matrix, which is defined as the correlation function $f_{A_mB_n}(x_1, x_2, Q^2)$ in Eq. (1). The concrete form about the correlation function is an unresolved problem. Although we used a toy model (Eq. (2)), we still have other choices. For example,

Model II.

$$f_{A_\pm, B_\mp}(x_1, x_2, Q^2) \gg f_{A_\pm, B_\pm}(x_1, x_2, Q^2),$$

i.e., the recombination is dominated by two polarized partons with opposite helicity. In this case, we have

$$\Delta R^I_{AB\to CD} = -\Delta R^{II}_{AB\to CD},$$

and

$$R_{AB\to CD} \to \frac{1}{2} [R_{A_+B_-\to C_+D} + R_{A_-B_+\to C_-D}].$$

Model III.

$$f_{A_\pm, B_\pm}(x_1, x_2, Q^2) \gg f_{A_\pm, B_\pm}(x_1, x_2, Q^2),$$

i.e., the recombination partons have same helicity. In this case, we have

$$\Delta R^I_{AB\to CD} = \Delta R^{II}_{AB\to CD},$$

and

$$R_{AB\to CD} \to \frac{1}{2} [R_{A_+B_+\to C_+D} + R_{A_+B_-\to C_-D}].$$
One can find the singularities in the recombination equation (6)-(7), which arise from soft initial partons.

(Such singularities also exist in the derivation of the GLR-MQ equation in [8], where an unusual $i\varepsilon$ prescription and the contour deformation of integral are used to avoid the pole. However, we can not use similar technique to exclude IR singularity in Eq. (6)-(7) in the case $x_1 \neq x_2$. The reason is that if $x_1$ or $x_2$ in the parton recombination functions is integrated by contour integral, the definition $f_{A_mB_n}(x_1, x_2, Q^2) \sim A_m(x_1)B_n(x_2)$ becomes unreasonable.)

Since soft partons between the perturbative and nonperturbative parts of a DIS amplitude should be absorbed into the nonperturbative correlation function in the collinear factorization scheme, we further modify the above mentioned correlation models to avoid the contributions from soft initial partons. A such correlator between two initial partons with different values of $x$ is used in [11]. In concrete, we assume that a parton with rapidity $y = \ln(1/x_1)$ combines with an other parton with rapidity $y + \eta = \ln(1/x_2)$, where $x_2 = \xi x_1$ and $\xi = e^{-\eta}$, one can generalized the correlation function Eq. (2) to

$$(x_1 + x_2)f_{A_mB_n}(x_1, x_2, Q^2) = \chi(|\eta|)x_1x_2A_m(x_1)B_n(x_2)\theta(1 - x_1 - x_2),$$

(42)

where $\chi(|\eta|)$ is a normalized correlator of fusing partons with the rapidity difference $\eta$. For example, we can take

$$\chi(|\eta|) = \frac{a}{\sqrt{\pi}}e^{-a\eta^2}.$$  

(43)

If $a = 5$, it implies that $|x_2 - x_1|_{max} = 0.0067$.

In summary, the corrections of parton recombination to the polarized AP evolution equation are derived. The relation between spin-dependent and spin-independent recombination functions and corresponding evolution equations are discussed.
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Appendix:

For completeness we list here the results for the polarized recombination functions as calculated using equations in Section 3.

\[ G_+G_+ \rightarrow G_+ = \frac{9}{4} \frac{(x_1 + x_2 - x)^3}{x_2^2 (x_1 + x_2)^3 x_1^2} (x_1^4 - 2 x_1^3 x + x_1^2 x^2 + x_2^4 - 2 x_2^3 x + x_2^2 x^2) \]

\[ + x_1^2 x_2^2 - x_1^2 x_2 x - x_1 x_2^2 x + x_1 x_2 x^2 \]  

(A.1)

\[ G_+G_+ \rightarrow G_- = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2 (x_1 + x_2)^3 x_1^2} (6 x_1^4 x_2 x + 6 x_1^3 x_2^2 x - 3 x_1^3 x_2 x^2 - 7 x_1^2 x_2 x^3 \]

\[ + 11 x_1^2 x_2^2 x^2 + 6 x_2^4 x_1 x + 6 x_2^3 x_1^2 x - 3 x_2^3 x_1^2 x^2 - 7 x_2^2 x_1 x^3 + 2 x_1 x_2 x^4 \]

\[ + x_1^6 + x_2^6 + 2 x_1^5 x_2 + 2 x_1^4 x_2^2 - x_1^4 x^2 - 2 x_1^3 x^3 + 2 x_1^2 x^4 + 2 x_2^5 x_1 \]

\[ + 2 x_2^4 x_1^2 - 2 x_2^3 x^2 + 2 x_2^2 x^4 + 2 x_1^3 x^3 \]  

(A.2)

\[ G_+G_- \rightarrow G_+ = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2 (x_1 + x_2)^3 x_1^2} (141 x_1^7 x^2 x_2 + 42 x_1^4 x^4 x_2^2 - 100 x_1^8 x_2 \]

\[ + 19 x_1^5 x^4 x_2 - 5 x_2^8 x_1 x + 39 x_1^2 x^4 x_2^4 + 137 x_1^3 x_2^6 x - 86 x_1^6 x^3 x_2 \]

\[ + 26 x_2^9 x_1 + 5 x_1^{10} + 5 x_2^{10} + 155 x_1^4 x^2 x_2^4 - 40 x_1^2 x_2^5 x^3 - 212 x_1^6 x_2 x^3 \]

\[ - 124 x_1^3 x^3 x_2^4 - 196 x_1^4 x^4 x_2^2 - 79 x_1^2 x_2^6 x^2 + 128 x_1^4 x_2^5 x + 44 x_2^7 x_1^2 x \]

\[ - 47 x_1^5 x_2^4 - 177 x_1^5 x^3 x_2^2 - 209 x_1^7 x_2 x^2 + 40 x_1^3 x^4 x_2^3 - 33 x_1^3 x_2^5 x^2 \]

\[ + x_2^6 x_1 x^3 + 319 x_1^5 x^2 x_2^3 + 291 x_1^6 x^2 x_2^2 - 35 x_2^7 x_1 x^2 + 13 x_2^5 x^4 x_1 \]

\[ + 64 x_2^7 x_1^3 + 57 x_2^8 x_1^2 + 26 x_1^9 x_2 + 57 x_1^8 x_2^2 + 64 x_1^7 x_2^3 + 34 x_1^6 x_2^4 \]

\[ + 12 x_1^5 x_2^5 + 34 x_1^4 x_2^6 - 4 x_2^9 x + 2 x_2^6 x^4 + 2 x_2^7 x^3 + 5 x_2^8 x^2 + 30 x_1^8 x^2 \]

\[ - 20 x_1^7 x^3 + 5 x_1^6 x^4 - 20 x_1^9 x \]  

(A.3)

\[ G_+G_- \rightarrow G_- = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2 (x_1 + x_2)^7 x_1^2} (-31 x_1^7 x^2 x_2 + 27 x_1^4 x^4 x_2^2 - 7 x_1^8 x_2 + 9 x_1^5 x^4 x_2 \]

\[ - 104 x_2^8 x_1 x + 54 x_1^2 x^4 x_2^4 - 206 x_1^3 x_2^6 x + 3 x_1^6 x^3 x_2 + 26 x_2^9 x_1 + 5 x_1^{10} \]

\[ + 5 x_2^{10} + 115 x_1^4 x^2 x_2^4 - 211 x_1^2 x_2^5 x^3 + 125 x_1^6 x_2 x^3 - 192 x_1^3 x^3 x_2^4 \]

\[ - 92 x_1^4 x^2 x_2^3 + 319 x_1^2 x_2^6 x^2 - 25 x_1^4 x_2^5 x - 217 x_2^7 x_1^2 x + 136 x_1^5 x x_2^4 \]
\[-24x_1^5x^3x_2^2 + 34x_1^7x_2^2 + 36x_1^3x^4x_2^3 + 307x_1^3x_2^5x^2 - 106x_2^6x_1x^3\]
\[-41x_1^5x^2x_2^3 - 67x_1^6x^2x_2^2 + 157x_2^7x_1x^2 + 27x_2^5x^4x_1 + 64x_2^7x_1^3\]
\[+57x_2^8x_1^2 + 26x_1^9x_2 + 57x_1^8x_2^2 + 64x_1^7x_2^3 + 34x_1^6x_2^4 + 12x_1^5x_2^5\]
\[+34x_1^4x_2^6 - 20x_1^9x + 5x_2^6x^4 - 20x_2^7x^3 + 30x_2^8x^2 - 5x_1^8x^2 + 2x_1^7x^3\]
\[+2x_1^6x^4 - 4x_1^9x\]  \hspace{1cm} (A.4)

\[q^i_+q^j_+ \rightarrow G_+ = \frac{16}{27} \frac{(x_1 + x_2 - x)}{x_2(x_1 + x_2)^3x_1} (8x_1^2x_2^2 + 4x_1^4 + 4x_2^4 + x_2^3x_1 + x_2x_1^3 - x_2x_1^2x)\]
\hspace{1cm} (A.5)

\[q^i_+q^j_- \rightarrow G_- = \frac{16}{27} \frac{(x_1 + x_2 - x)}{x_2(x_1 + x_2)^3x_1} (4x_1^4 - 8x_1^3x + 8x_1^2x^2 + 4x_1^2x^2 + 4x_2^4 - 8x_2^3x\]
\[+4x_2^2x^2 + x_2^3x_1 - x_2^2x_1x + x_2x_1^3)\]  \hspace{1cm} (A.6)

\[q^i_-q^j_- \rightarrow G_+ = 0\]  \hspace{1cm} (A.7)

\[q^i_+q^j_- \rightarrow G_- = 0\]  \hspace{1cm} (A.8)

\[G_+q^i_+ \rightarrow G_+ = \frac{2}{9} (x_1 + x_2 - x) (9x_1^2x_2^2 - 18x_2^2x + 9x_2^4 + 4x_1^4)\]  \hspace{1cm} (A.9)

\[G_+q^i_+ \rightarrow G_- = \frac{1}{18} (x_1 + x_2 - x) (9x_1^2x_2^2 + 18x_2^3x_1 + 36x_2^4x + 36x_2^4x_1\]
\[+36x_2^5 + 16x_1^5 + 48x_1^4x_2 + 48x_1^3x_2^2 - 32x_1^4x - 78x_1^3x_2x + 16x_1^2x_2^3\]
\[-68x_1^2x_2^2x + 16x_1^3x^2 + 12x_1^2x_2x)\]  \hspace{1cm} (A.10)

\[G_+q^i_- \rightarrow G_+ = \frac{1}{18} (x_1 + x_2 - x) (7x_1^2x_2^2 + 18x_2^3x_1 + 9x_2^4x + 16x_1^6 + 64x_1^5x_2\]
\[+132x_1^4x_2^2 + 136x_1^3x_2^3 + 52x_1^2x_2^4 - 18x_1^3x_2x^2 - 18x_1^2x_2^3x + 18x_1^4x_2x)\]  \hspace{1cm} (A.11)

\[G_+q^i_- \rightarrow G_- = \frac{2}{9} (x_1 + x_2 - x) \frac{x_1}{x(x_1 + x_2)^3x^2} (4x_1^2 - 8x_1x + 4x^2 + 9x_2^2 - 9x_2x)\]  \hspace{1cm} (A.12)

\[G_+G_+ \rightarrow q^i_+ = \frac{1}{12} (x_1 + x_2 - x) \frac{x_1}{x(x_1 + x_2)^3x^2x_1^2} (4x_1^4 + 7x_1^3x_2 - 8x_1^3x + 2x_1^2x_2^2 - 6x_1^2xx_2\]
\[+4x_1^2x_1^2 + 4x_1^2x_2^2 - x_1^2x_2^2 + 2x_1xx_2^2 - x_1^2x_2^2)\]  \hspace{1cm} (A.13)

\[G_+G_+ \rightarrow q^i_- = \frac{1}{12} (x_1 + x_2 - x) \frac{x_1}{x(x_1 + x_2)^3x^2x_1^2} (4x_2^2x_1^2 + 4x_1^2x_2^2 + 8x_1x^2 - 8x_1xx_2^2 + 4x_2^4\]
\[-8x_2^3x + 4x_2^3x^2 - x_1^2x_2^2)\]  \hspace{1cm} (A.14)
\[ G_+ G_- \rightarrow q_+^i = \frac{1}{12} \frac{(x_1 + x_2 - x)^2}{(x_1 + x_2)^2 x_2^2 x_1^2} (4 x_1^6 x^2 + 4 x_1^4 x_2^4 - 8 x_1^7 x + 24 x_1^6 x_2^2 + 16 x_1^5 x_2^3
\[ + 16 x_1^7 x_2 + 4 x_1^8 + 24 x^2 x_2^5 x_1 + 4 x^2 x_2^6 - 10 x_1^3 x^2 x_2^3 + 33 x_1^2 x^2 x_2^4
\[ - 8 x_1^4 x_2^3 + 24 x_1^5 x^2 x_2 + 33 x_1^4 x_2^2 + 14 x_1^3 x_2^4 - 40 x_1^2 x_2^5 x_1^2 - 9 x_1^2 x_2^6) \] (A.15)

\[ G_+ G_- \rightarrow q_-^j = \frac{1}{12} \frac{(x_1 + x_2 - x)^2}{(x_1 + x_2)^2 x_2^2 x_1^2} (24 x_2^6 x^2 + 16 x_2^7 x_1 - 8 x_2^7 x + 16 x_2^5 x_1^3 + 4 x_2^8
\[ + 4 x_2^6 x^2 + 4 x_1^4 x_2^4 + 6 x^2 x_2^5 x_1 + 4 x^2 x_2^6 - 8 x_1^3 x^2 x_2^3 + 15 x_1^2 x^2 x_2^4
\[ - 13 x_1^4 x_2 x_2^3 + 24 x_1^5 x^2 x_2 + 51 x_1^4 x^2 x_2^2 - 35 x_1^3 x_2^4 + x_1^5 x_2^2
\[ - 35 x_2^5 x_1^2 - 22 x_2^6 x_1^2) \] (A.16)

\[ q_+^i q_+^j \rightarrow q_+^i = \frac{8}{27} \frac{x_1}{(x_1 + x_2)^2} (48 x_1^5 x_2 + 72 x_1^4 x_2^2 + 48 x_1^3 x_2^3 + 12 x_1^2 x_2^4 - 12 x_1^5 x
\[ + 3 x^2 x_1^4 + 12 x_1^6 - 46 x_1^4 x_2 x_2 - 64 x_1^3 x_2 x_2^2 - 36 x_1^2 x_2 x_2^3 - 4 x_1 x_2^4
\[ + 8 x^2 x_1^3 x_2 + 18 x_2^2 x_1^2 x_2 + 24 x_2^3 x_1 x_2^2 - 20 x_2^2 x_1 x_2 + 11 x_2^4 x
\[ - 22 x_2^3 x_1^2 + 12 x_2^2 x_2^2 + 2 x_2 x_1^2 + 2 x_2^5 x) \] (A.17)

\[ q_+^i q_+^j \rightarrow q_+^i = \frac{32}{9} \frac{x_1 (-x_1 + x_2 + x)^2 x_2}{(x_1 + x_2)^2 (x_1 + x_2)^2} \] (A.18)

\[ q_+^i q_-^j \rightarrow q_+^j = 0 \] (A.19)

\[ q_-^j q_-^j \rightarrow q_-^j = 0 \] (A.20)

\[ q_+^i q_+^j \rightarrow q_+^i = \frac{8}{9} \frac{(-2 x_1 + x)^2 x_1}{(x_1 + x_2)^2 x_2} \] (A.21)

\[ q_+^j q_+^j \rightarrow q_+^j = 0 \] (A.22)

\[ q_-^j q_-^j \rightarrow q_-^j = 0 \] (A.23)

\[ q_+^i q_-^j \rightarrow q_+^j = \frac{8}{9} \frac{(-2 x_1 + x)^2 x_2}{x_1 (x_1 + x_2)^3} \] (A.24)

\[ q_+^i G_+ \rightarrow q_+^j = \frac{1}{18 x_2^2 (x_1 + x_2)^3 x_1} (45 x_1^4 - 90 x_1^3 x + 54 x_1^3 x_2 - 54 x_1^2 x_2 x + 45 x_1^2 x^2
\[ + 81 x_1^2 x_2^2 + 16 x^2 x_2^2) \] (A.25)

\[ q_+^i G_+ \rightarrow q_-^j = 0 \] (A.26)
\[ q_+^i G^- \to q_+^i = \frac{1}{18} \frac{(x_1 + x_2 - x)}{x_1^5 x_1} \left(-22 x_2^5 x_1 - 32 x_2^5 x + 16 x_2^4 x^2 + 18 x_2^3 x^2 x_1 + 4 x_2^4 x_1 \right. \\
-58 x_1^3 x_2^3 - 85 x_1^2 x_2^4 + 9 x_1^4 x^2 - 18 x_1^5 x + 198 x_1^3 x x_2^2 + 198 x_1^2 x_2^3 x \\
\left. -18 x_1^3 x_2^2 - 20 x_1^4 x_2^2 + 18 x_1^4 x x_2 - 9 x_1^2 x_2^2 + 9 x_1^6 + 16 x_2^6 \right) \] (A.27)

\[ q_+^i G^- \to q_+^i = 0 \] (A.28)

when \( x_1 = x_2 \),

\[ GG \to G = \frac{9}{64} \frac{(2 x_1 - x)(140 x_1^2 x^2 - 116 x_1 x^3 + 29 x^4 - 48 x_1^3 x + 72 x_1^4)}{x_1^5 x} \] (A.29)

\[ q^i \bar{q}^j \to G = \frac{4}{27} \frac{(2 x_1 - x)(18 x_1^2 - 9 x_1 x + 4 x^2)}{x_1^3 x} \] (A.30)

\[ Gq^i \to G = \frac{1}{288} \frac{(2 x_1 - x)^2 (79 x^2 - 202 x_1 x + 304 x_1^2)}{x_1^4 x} \] (A.31)

\[ GG \to q^i = \frac{1}{96} \frac{(2 x_1 - x)^2 (18 x_1^2 - 21 x_1 x + 14 x^2)}{x_1^5} \] (A.32)

\[ q^i \bar{q}^j \to q^i = \frac{1}{108} \frac{(2 x_1 - x)^2 (6 x_1^2 + x_1 x + 3 x^2)}{x_1^5} \] (A.33)

\[ q^i q^k \to q^i = \frac{2}{9} \frac{(2 x_1 - x)^2}{x_1^3} \] (A.34)

\[ q^i G \to q^i = \frac{1}{288} \frac{(2 x_1 - x)(140 x_1^2 - 52 x_1 x + 65 x^2)}{x_1^4} \] (A.35)

they consist with the results of Ref. [4].
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Figure Captions

Fig. 1 The diagrams contributing to the polarized recombination functions. Here the shaded part implies all possible QCD-channels and \( \times \) means the probing place.
\[ G_L \]
\[ G_m \sim \ldots \sim q^i \sim G_m \]

\[ G_n \sim \ldots \sim G_n \]
(e-1)
\[ (e^{-2}) \]