Kondo effect in mesoscopic systems

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The Kondo effect may develop in those cases where there are non-commuting operators describing the interaction between the conduction electrons and impurities or defects with internal degrees of freedom. This interaction may involve spin or orbital variables. There are cases, where the conduction electrons have conserved quantum numbers which do not appear in the coupling. An example is where the Kondo effect involves orbital degrees of freedom, and the interaction is independent of the real spins of the electrons, which is conserved and leads to the two-channel Kondo (2CK) problem. The low temperature behavior is very different for the one and two-channel cases, as in the first case a Fermi liquid is formed while in the second one strong deviations appear and a non-Fermi liquid state is realized. Mesoscopic samples provide a unique possibility to study a few or even a single Kondo impurities. In this paper we first review how the original spin Kondo problem is affected by surface anisotropy and the fluctuations of the density of states in point contacts. We discuss the physics of a single magnetic impurity on the surface of a sample. The orbital 2CK effect due to dynamical defects is considered. The nature of these defects is not known but they are excellent candidates to describe the zero-bias anomalies with non-Fermi liquid character in point contacts, and the dephasing time and transport in short wires. There are two main concerns with this interpretation: Firstly, the tunneling centers formed by heavy impurities may produce too small Kondo temperature, and secondly, the splittings seen in the experiments are much smaller than expected from this model. It would be therefore extremely important to identify the microstructure of these two level systems and find new realizations for them.

1. Introduction

Since the discovery of the anomalous low temperature resistivity increase exhibited by some metallic samples [1] these anomalies attracted considerable interest. The first theoretical work to explain them was due to Kondo, who demonstrated that the scattering rate of electrons in metals by magnetic impurities has an anomalous third order contribution, which increases logarithmically as the temperature is reduced and leads to the break-down of perturbation theory [2]. Since then this phenomenon is known as the Kondo effect. Following Kondo's original work a lot of theoretical effort has been devoted to understand this phenomenon in detail. Wilson's numerical renormalization group to treat the strong coupling limit [3] and Nozières' Fermi liquid theory [4] turned out to be the most important milestones in this development.

Recently, the number of papers related to the Kondo effect showed a significant increase with broader and broader applications of the model. Various dilute and dense U and Ce based metallic alloys have been suggested as Kondo systems with both magnetic and orbital features [5]. In these systems at low temperature very strong correlations build up, hence they became known as strongly correlated systems. Other new developments were in the direction of the observation of Kondo effect in mesoscopic systems such as thin layers and point contacts, and also artificial mesoscopic atoms (quantum dots). In these latter nanofabricated devices the d-level of the magnetic impurity in the metal is mimicked by degenerate states of a quantum dot, which is coupled to metallic or semiconducting leads.

Nanotechnology itself is a very fast developing field, which opens up new perspectives and of-
fers new possibilities to study magnetic impurities and strongly correlated systems. Of course, its extensive overview or a discussion of the physics of nanofabricated artificial atoms is out of the scope of our review. Here we only focus on the study of magnetic and dynamical impurities in mesoscopic systems.

The Kondo effect, in general, originates from the scattering of conduction electrons by a localized object (magnetic or substitutional impurity) or some topological defect with some internal degrees of freedom (e.g., spin, two close atomic positions, dislocation kink). The typical Hamiltonian of Kondo-like problems is

\[ H = \sum_{k} \epsilon_{k} c_{k\mu}^{\dagger} c_{k\mu} + \sum_{\alpha} \epsilon_{\alpha} b_{\alpha}^{\dagger} b_{\alpha} + \sum_{k,k',\mu\rho\sigma\beta} V_{\mu\rho}^{\alpha\beta} c_{k\mu}^{\dagger} c_{k\rho} b_{\sigma}^{\dagger} b_{\beta} \]  

(1)

where \( \epsilon_{k} \) is the electron kinetic energy with momentum \( k \), \( c_{k\mu}^{\dagger} \) creates an electron spherical wave with radial momentum \( k \) and internal quantum numbers \( \mu \) and \( b_{\alpha}^{\dagger} \) creates a heavy object with quantum number \( \alpha \) (\( \alpha \) being the spin, the position, or a crystal field label of the impurity). Note that the internal indices \( \mu, \nu \) of the conduction electrons may also represent magnetic spin or orbital indices or a combination of them as well. \( V_{\mu\rho}^{\alpha\beta} \) denotes the interaction potential and a band cutoff \( D \) (usually of the order of Fermi energy) is applied for the conduction electrons.

The first corrections to the electron-impurity scattering matrix are given by the two time-ordered diagrams shown in Fig. 1. The direction of time corresponds to the direction of the lines on the heavy objects. Assuming an interaction independent of \( k, k' \), the scattering amplitude for an incoming electron with energy \( \omega \) is

\[ V^{(2)}_{\mu\rho}^{\alpha\beta}(\omega) = \sum_{\rho\gamma} [V_{\mu\rho}^{\alpha\gamma} V_{\rho\gamma}^{\gamma\beta} - V_{\rho\gamma}^{\alpha\gamma} V_{\gamma\beta}^{\gamma\rho}] \ln(D/\omega), \]  

(2)

where the quantum numbers of the internal lines are summed over and the negative sign arises from the fermion anticommutation relations (note the crossed lines in the second diagram). The logarithm above was first identified by Kondo. The divergence of this term as \( \omega \to 0 \) reflects the break down of perturbation theory.

Figure 1. The two leading diagrams for the scattering of a light electron on a heavy object. The light electron is represented by the light line and the heavy object by the double line. The wavy lines indicate the interaction matrix element \( V \). The quantum numbers characterizing the particles are also shown.

Different impurity models can be classified by the value of the “commutator” of Eq. (2). (i) For the commutative model \( [V_{\mu\rho}^{\alpha\gamma} V_{\rho\gamma}^{\gamma\beta} - V_{\rho\gamma}^{\alpha\gamma} V_{\gamma\beta}^{\gamma\rho}] = 0 \), and no Kondo logarithms appear. An example for this is the dissipative tunneling system where the interaction between the heavy object and conduction electrons is diagonal in the internal indices. (ii) For a non-commutative model \( [V_{\mu\rho}^{\alpha\gamma} V_{\rho\gamma}^{\gamma\beta} - V_{\rho\gamma}^{\alpha\gamma} V_{\gamma\beta}^{\gamma\rho}] \neq 0 \), and logarithmic terms appear in the scattering matrix. In the following we only concentrate to the second case.

In many cases, the general form of the interaction Hamiltonian can be simplified by introducing appropriate variables and truncating the Hilbert space to

\[ H_{int} = \sum_{\alpha,\beta=x,y,z} \sum_{i=s,1,...,n} V^{i}_{\alpha\beta} \sigma^{i}_{\alpha\beta} b_{\alpha}^{\dagger} b_{\beta} c_{\mu s}^{\dagger} c_{\mu s} c_{k\rho s} c_{k\nu s} , \]  

(3)

where \( \sigma^{i} \)'s are the Pauli operators, \( V^{i} \)'s are anisotropic couplings and the conduction electron may have an additional channel index \( s \) which is conserved and does not occur in the coupling itself. As it has been pointed out by Nozières and Blandin its mere existence can change the nature of the low temperature behavior drastically. According to the possible different values of that channel indices we speak about \( n \)-channel Kondo problem \( (n = 1, 2, \ldots) \).

In the original spin Kondo problem \( V^{i} = J/2 \) where \( J \) is the exchange coupling between the localized spin described by the spin indices \( \alpha, \beta \) and
the conduction electron spins labeled by $\mu, \nu, \beta$ creates the localized electron state with spin $S$, for simplicity $S = 1/2$. In that model there is no additional channel, thus $n = 1$. In this case the localized $S = 1/2$ spin is screened by the “compensation cloud” of the electrons and finally a singlet is formed. The binding energy is proportional to the Kondo temperature $T_K$ which for the isotropical case can be written as

$$T_K = D (2\varrho_0 J)^{1/2} \exp(-1/2\varrho_0 J),$$

with $\varrho_0$ the density of states at the Fermi level for one spin direction. At $T \ll T_K$ thermally excited conduction electrons cannot break the singlet, which acts as a rigid potential scatterer and Nozières' Fermi liquid theory holds [4].

The actual size of the compensation cloud is given by the Kondo coherence length

$$\xi_K \sim \frac{v_F}{T_K},$$

with $v_F$ the Fermi velocity. For $T_K \sim 1K$ it can easily exceed $1\mu m$, which is in the range of the size of a mesoscopic sample or even can be much larger. This observation triggered the experimental study of the dilute Kondo alloys in thin films, wires and point contacts (see Sec. 2.1 and 2.2).

Contrary to the $n = 1$ case, for the two channel Kondo model ($n = 2$) the ground state is not a singlet because two electrons with different channel indices compete to screen the impurity spin. The complexity of the ground state is reflected in a residual entropy of $S = k_B \ln \sqrt{2}$. It shows that the impurity is only partially screened even at $T = 0$ and that small energy excitations are not frozen out even there. As a result, the linear specific heat coefficient $C(T)/T$ and the impurity susceptibility were found to diverge logarithmically. A further surprising result arose from the conformal field theory approach: Affleck and Ludwig [11] showed that (i) the impurity contribution to the resistivity shows a $\sqrt{\max[\omega, T]}$ singularity (as opposed to the $\sim \omega^2, T^2$ Fermi liquid behavior) (ii) that the amplitude of scattering to a one-electron state vanishes at $T = \omega = 0$, and an incoming electron “evaporates” to infinitely many electron-hole excitations once it hits the impurity (3) (see Table I).

It must be emphasized that for $n = 2$ an arbitrarily small splitting of the impurity states (produced by magnetic field or a strain field for dynamical defects) provides a cutoff for the non-Fermi liquid behavior and ultimately leads to a crossover to a Fermi liquid state.

2. Spin Kondo effect in mesoscopic devices

2.1. Size dependence in films and wires

In the last decade, many experiments [12–15] were performed on thin films and narrow wires of dilute magnetic alloys in search of the Kondo compensation cloud. In these experiments (see Fig. 2) no essential change in the Kondo temperature was observed, however, in most of them [12–13] a suppression of the Kondo resistivity amplitude was observed for small sample sizes. Covering a thin layer of magnetic alloys by another pure metal layer, a partial recovery of the Kondo signal was found [14,15] which was smaller for more disordered overlayers [18]. The first natural explanation concerning the compensation cloud [18] was ruled out both theoretically [20,21] and experimentally [17] as the Kondo singlet is formed whenever the level spacing is small compared to the Kondo temperature: $\delta \epsilon < T_K$. The effect of local density of states (LDOS) fluctuations close to the surface (discussed in the next subsection) is also probably relatively small for the investigated alloys [22].

Two theories have been developed, that seem to explain the two limiting cases in the experiments: The first, based on weak localization [23], may be valid in disordered samples [12,16], where the smallest system size is large compared to the elastic mean free path and the Kondo anomaly depends on the level of disorder. The other explanation, the theory of spin-orbit-induced surface anisotropy [24] explains all the experiments performed in ballistic samples (i.e., when the size of the sample is in the ballistic region). This surface anisotropy is developed in samples with strong spin-orbit interaction on the non-magnetic host atoms [24]. In this case electrons can mediate information about the geometry of the sample resulting in an anisotropy for the impurity spin nearby the surfaces, but only in those cases.
where the angular momenta of the localized orbital $l \neq 0$ (e.g. $l = 2$). According to Ref. [24] this anisotropy is non-oscillating in the leading order and inversely proportional to the distance $d$ measured from the surface. For flat surfaces it is described by the Hamiltonian

$$H_a = K_d (\mathbf{n} \mathbf{S})^2$$

where $\mathbf{n}$ is the normal vector of the surface and $\mathbf{S}$ is the spin operator of the impurity. The anisotropy factor $K_d$ is positive and is in the range of \( \frac{0.01}{(\text{d}/\text{Å})}\) eV < $K_d$ < \( \frac{1}{(\text{d}/\text{Å})}\) eV [24]. An elegant extension of these calculations to general geometries was performed by Fomin and coworkers [25] who investigated the dependence of the anisotropy on the roughness of the surface as well.

The Kondo resistivity of thin films was calculated [26,27] assuming that the two surfaces of the thin films contribute to the anisotropy in an additive way, e.g. $K_{d,t} = \alpha_d + \alpha_t - d$, which was justified later in Ref. [25]. The Kondo temperature was found only slightly affected [26] in a sample of finite size, in agreement with the experiments [12,13].

The Kondo signal becomes reduced because close to the surface the motion of the Kondo spins is hindered by the spin-anisotropy [26,27], but the size dependence in the Kondo resistivity amplitude $B(t)$ defined by $\Delta \rho_{\text{Kondo}} = -B(t) \ln T$ is different for integer and half-integer spins. For integer spins (e.g. $S = 2$ for Fe) an impurity close enough to the surface is frozen to the $nS = 0$ state and the Kondo effect is impossible. Therefore the amplitude $B(t)$ is reduced with respect to its bulk value and goes to zero as the film thickness is decreased (see Fig. 3) [28]. For half integer spin (e.g. $S = 5/2$ for Mn), on the other hand, the lowest energy state is the $nS = \pm 1/2$ doublet, thus the impurity still produces Kondo resistivity. As a consequence, the size dependence is much weaker, and $B(t)$ remains finite even for $t \to 0$ (see Fig. 3) [27]. These results are in agreement with the experiments [12,13]. For $S = 1/2$ spin alloys (e.g. La$_{1-x}$Ce$_x$ films) no size dependence is expected due to the surface anisotropy in agreement with the experiment of Ref. [15]. The proximity effects [16,18] can be well explained by the surface anisotropy as the number of available
Figure 3. Calculated size dependence in the Kondo resistivity amplitude due to surface anisotropy. Solid line for \( S = 2 \) and \( T_K = 0.3 \) K (i.e., \( Au(Fe) \)), dashed line for \( S = 5/2 \) and \( T_K = 10^{-3} \) K (i.e., \( Cu(Mn) \)). The minima in the half-integer case may be only a sign of the breakdown of the weak coupling calculation \[27\].

spin-orbit scatterers is increased by the overlayer and the magnetic impurities are in further distances from the surface of the samples, but only if the overlayer is in the ballistic region as well. In a new experiment of Giordano \[28\] different multilayers composed of \( Au \) and \( Au(Fe) \) films were examined (where the overlayer was positioned only on one side, or on both sides of the film), giving good agreement also quantitatively with the predictions of the theory of surface anisotropy.

There are experiments where quantities different from the Kondo resistivity were measured in order to test the theory of anisotropy as well. First Giordano measured the magnetoresistance \[29\] of thin films and found also a size dependence as the splittings due to the magnetic field and the surface anisotropy compete. Magnetoresistance calculations \[30\] gave excellent agreement with these measurements. Thermopower \[31\] and impurity spin magnetization measurements \[32\] on samples with reduced dimensions can also be explained by the theory of surface anisotropy.

2.2. Size dependence in point contacts

Parallel to the thin film experiments, a thorough study of the Kondo effect in ultra small \( CuMn \) point contacts (PCs) has been carried out \[33\]. Rather surprisingly, in this case not a suppression but an orders of magnitude increase of both the Kondo signal and the Kondo temperature has been reported.

As shown in Ref. \[22\], these anomalies can be well explained by the presence of LDOS fluctuations: For a small PC, even a weak channel quantization induces huge LDOS fluctuations \[22\] which become larger and larger with decreasing contact sizes (see Fig. 4). As \( T_K \) depends on the LDOS exponentially (see Eq. \( 4 \)), this may produce an extremely wide distribution of the Kondo temperatures for impurities in the contact region. The zero bias anomaly of the PC, however, turns out to be dominated by magnetic impurities with the largest \( T_K \), since these are the ones that show a well-developed Kondo resonance. Indeed, in Ref. \[22\] the effect of these fluctuations was taken into account through a modified renormalization procedure, and a perfect agreement was found between the calculated and experimentally determined anomalous amplitude of the Kondo signal (see Fig. \[3\]).

It was also predicted by the theory \[22\] that
Figure 5. Size dependence of the amplitude of the dimensionless Kondo conductance $\Delta g = \Delta G \ h/e^2$ for the model PC of Ref. [22] as a function of the contact radius. Diamonds denote the experimental data taken from Ref. [33] while theoretical results (with no adjustable parameter) are indicated by crosses. The dashed line indicates the results without LDOS fluctuations ($g \sim R^3$) while the continuous line corresponds to the best fit to the data of Ref. [33]: $g \sim R^{2.17}$.

this effect should be much less pronounced for alloys with large $T_K$ as $T_K$ is less sensitive to the change in $\rho_0$ in that case (see Eq. (4)), which has indeed been later confirmed by the experiments studying Cu(Fe) alloys.

2.3. Kondo resonance in the density of states measured by STM

It has been known for a long time [35] that the local electron density of states nearby a magnetic Kondo impurity has a specific structure due to the Kondo resonance. In the early experimental attempts a change in the electron density of states due to a layer of dilute magnetic alloys fabricated inside a metal has been measured [36] by an oxide tunnel junction placed in a few atomic distances from that layer, and the Kondo structure was indeed observed.

Recently, several groups have demonstrated using scanning tunneling microscopy (STM) [37] that a magnetic Kondo impurity adsorbed on the surface of a normal metal produces a narrow, resonance-like structure in the electronic surface density of states (DOS), whose asymmetric line shape resembles that of a Fano resonance [10]. The experiments were performed with single Ce atoms on Ag [37] as well as with single Co atoms on Au [38] and Cu [39] surfaces by measuring the I-V characteristics of the tunneling current through the tip of a STM placed close to the surface and at a small distance $R$ from the magnetic atom (see Fig. 3 (a)). A systematic study of the
local electronic structure of individual transition-metal impurities on Au surfaces was performed by Jamneala et al. [11] who showed that for elements near the end of the 3d row (Ti, Co, and Ni) the above mentioned narrow resonance structure appears, whereas for the elements around the center of the row (V, Cr, Mn, Fe) the electronic structure is found to be featureless. In these experiments the electron tunnels from the tip into the metal, travels to the impurity and, after scattering off it, goes back to the tip, resulting in an interference between the unperturbed and scattered electrons. There is also a possibility that the electrons tunnel from the tip directly to the magnetic impurity, more precisely into the d- or f-level of the atom. However, the tunneling rate for the latter process is probably very small, especially for f-levels, which are deeply inside the atom.

The first theory proposed in Ref. [38,42] takes into account both processes. Recently, it has been shown in Ref. [13] however, that the Fano resonance can develop even if one neglects the direct tunneling to the impurity. In this theory the physics is governed by the unperturbed one-electron Green’s function at the surface of the metal \( G_R^{(0)}(\omega - i\delta) \) and the scattering amplitude \( t_\sigma(\omega - i\delta) \) due to the impurity. The latter, given by \( \frac{\Delta}{\pi\rho_0} G_{d,\sigma}(\omega - i\delta) \) in the Anderson model [41,43], can be approximated as

\[
t_\sigma(\omega - i\delta) = \frac{\Delta}{\pi\rho_0} \left( \frac{Z_d}{\omega - \tau_d - i\Delta} + \frac{Z_U}{\omega - \tau_d - U - i\Delta} + \frac{Z_K}{\omega - \epsilon_K - iT_K} \right),
\]

where \( G_{d,\sigma}(\omega) \) is the d-electron Green’s function, \( Z_d, Z_U \) and \( Z_K \) are the appropriate strength of the poles, \( \tau_d, U, \Delta \), and \( \epsilon_K \) are the energies of the singly and doubly occupied orbitals of the effective model [2], the broadening of the d-level, and the position of the Kondo resonance, respectively [13]. The final expression for the tunneling density of states reads

\[
\delta\rho_R(\omega) = \frac{|\text{Im} G_R^{(0)}(\omega^-)|^2}{\pi\rho_0} \left\{ \frac{(q_R + \epsilon^2)^2}{\epsilon^2 + 1} - 1 + C_R \right\},
\]

where the spin index \( \sigma \) was dropped, \( \omega^- = \omega - i\delta \), \( \epsilon = (\omega - \epsilon_K)/T_K \) and \( q_R = \text{Re} G_R^{(0)}(\omega^-)/\text{Im} G_R^{(0)}(\omega^-) \). \( C_R \), which depends on \( Z_d, \tau_d, \Delta, \) and \( q_R \) [43], arises from potential scattering on the d-level and corresponds to a weakly energy dependent Friedel oscillation. The first part of Eq. (8) coming from the scattering by the Kondo resonance gives a Fano line shape in the tunneling LDOS, controlled by the parameter \( q_R \). The fit on the experimental data for a Co atom on a Au (111) surface [18] gave excellent agreement with fitting parameters being consistent with the predictions of an NCA calculation combined with band structure results [13].

In the experiment of Jamneala et al. [11] the Kondo resonances were not observed in case of V, Mn, Cr, and Fe atoms. In case of Mn, the Kondo temperature is small for bulk samples and it is further reduced by the weaker exchange coupling at the surface, thus the resonance cannot be expected on meV scale. In case of Fe and Cr the surface anisotropy described by Eq. (9) [23] may be also reduced, but even in that case, that may make impossible to see spin \( S = 2 \). In case of Co according to the electronic structure calculations [13], the spin on the surface is close to \( S = 1/2 \), where the anisotropy does not play a role.

To calculate the distance dependence of \( q_R \) and \( C_R \), i.e., of the line shape, the tunneling of electrons from the tip (1) into the 3-dimensional Au bulk states as well as (2) into the 2-dimensional Au(111) surface band [45] was considered. In both cases a free electron-like band structure was assumed [13]. Whereas the periodic changes of the line shape between Fano and Lorentzian ones and the decrease in the overall amplitude with increasing distance were demonstrated (see Fig. 1 (b)), the precise dependence of the line shape on \( R \) is not reproduced by our simplifying assumption of a free electron band structure [13]. That will require taking into account the detailed band structure as well as the additional scattering phase shift induced by the charge of the Co ion and the charge distribution around the Co ion.

Finally, it is worth to comment on the Fano parameter. In the original paper of Fano [40] the parameter \( q \) is defined in a way where it is proportional to \( q^2 \sim |\langle \Phi|T|i\rangle|^2/|\langle \Psi_E|T|i\rangle|^2 \). The discrete level and the continuum state of energy \( E \) have the wave function \( \varphi \) and \( \Psi_E \),
3. Possibility of two-channel Kondo effect due to structural defects

It is well established by now that scattering on fast dynamical defects can produce Kondo-like anomalies \cite{47,48}. In the simplest model the defect atom tunnels between two positions and thus forms a two-level system (TLS). These two levels are typically split due to the spontaneous tunneling between the positions and the asymmetry of them, resulting in a typical splitting of $\Delta \sim 1 - 100K$ (see Fig. 6). In the TLS Kondo model the coordinate of the dynamical impurity is coupled to the angular momentum of the conduction electrons through an effective exchange interaction, and the real spins of the conduction electrons act as silent channel indices. Consequently – in the absence of splitting – the physics of the TLS is described by the two-channel Kondo model predicting a NFL behavior below $T_K$. In this model the spin-flip scattering of the original Kondo model is replaced by electron assisted tunneling.

3.1. Point contacts

Several experiments have been reported where the observed low temperature anomalies were attributed to TLS Kondo defects \cite{17,18,22} (see Fig. 8). In all these experiments the observed anomalies were unambiguously due to dynamical structural defects: they disappear under annealing and not or only slightly depended on magnetic field.

A logarithmic increase of the resistivity attributed to the presence of dislocations or substitutional tunneling impurities has been observed in various systems \cite{16,17,18}. However, the
most spectacular experiments were carried out in $Cu$ and $Ti$ point contacts where a two-channel Kondo-like $\sim \sqrt{T}$ and $\sqrt{V}$ non-Fermi liquid scaling behavior due to non-magnetic scatterers has been observed in the contact resistance \[44,45\]. The widths of the zero bias anomalies were associated with the Kondo temperature, $T_K \sim 5K$. In another beautiful experiment a fluctuation of the zero bias anomaly between two curves due to some slow TLS’s has been observed in amorphous point contacts \[41\], which could be consistently explained assuming that a slow fluctuator influences the splitting of one or two fast Kondo two-level systems close to it \[52\]. There is a further experiment \[3\] where an alternating voltage was superimposed on a constant bias $V_0$, $V(t) = V_0 + V_1 \cos(\omega t)$. As far as the characteristic frequency (e.g. Kondo temperature) of the mechanism responsible for the zero bias anomaly is large compared to $\hbar \omega$ the measured $I-V$ characteristic is just the time average of the current: $\langle I(t) \rangle = I(V_0) + \frac{1}{4} \left( \frac{\partial^2 I}{\partial V^2} \right)_{V=V_0} V_1^2$. For frequencies higher than this scale an $1/\omega$ dependence is expected. No deviations have been observed experimentally even for $\nu = 60$ GHz (2.4 K) implying that $T_K > 5K$. This lower bound is in agreement with the value of $T_K$ estimated from the width of the zero-bias anomaly.

There remain, however, a number of puzzles. In all these experiments the estimated Kondo temperature is in the range $T_K \sim 10K$. $T_K$ has been first estimated in Ref. \[40\] assuming that TLS’s are formed by a heavy atom that tunnels within a distance of about $\sim 0.4\AA$, and was found to be in the range $\sim 0.01-1K$. It has been suggested that virtual hopping to the lowest excited states could increase $T_K$ substantially \[7\], however, in the above model this turned out to be wrong \[7,52\]. It has been shown that, in reality, $T_K$ is reduced even further if one includes the effect of all the excited states. Thus, within the original simplistic TLS model, where the TLS is formed by some heavy atom tunneling between two close positions and electron-hole symmetry is assumed, it seems to be impossible to have $T_K$ in the experimentally observed range. It has been pointed out recently, that the criticism of Ref. \[55\] is essentially based on the assumption of electron-hole symmetry in the conduction electron density of states \[37\]. Electron-hole symmetry, however, is strongly violated in any realistic band structure. As shown in Ref. \[57\], $T_K$ can be enhanced by orders of magnitude with a relatively small electron-hole symmetry breaking even when all excited states are included, and $T_K$ can be in a much broader range than expected previously.

On the other hand, one has very little knowledge about the microstructure of the TLS’s, and in order to make any quantitative prediction it would be extremely important to identify it. We expect that TLS’s with a relatively small effective mass (such as Hydrogen stuck at the surface of the sample or dislocation jogs \[58\]) could be able to tunnel over a distance of $\sim 1\AA$ and probably produce a $T_K$ in the experimentally observed range.

Another interesting question is related to the splitting of the two levels, which provides a lower cutoff for the NFL scaling. The presence of splitting and the cutoff of NFL behavior has been observed in several experiments. In particular, measurements on $Ti$ point contacts are in perfect agreement with all predictions of the TLS Kondo model. In Ref. \[49\], however, the number of TLS’s has been estimated to be about 50, for which concentration already a significant deviation from the NFL scaling should have appeared due to the presence of disorder generated splitting \[63\]. However, no such deviation has been reported in Ref. \[15\]. The resolution for this puzzle may also be related to the precise microstructure of the tunneling impurities.

3.2. Electron dephasing time $\tau_\phi$

Recent developments in mesoscopic physics raised an interesting question about the electronic dephasing time $\tau_\phi$. This is the time scale for an electron to stay in a given exact one-electron state in the presence of static impurities. The transitions between these states are due to electron-phonon, electron-electron, electron-dynamical defects (e.g. two-level system), or electron - magnetic impurity interactions. At low temperature the electron-phonon interaction freezes out and the electron-electron interaction becomes domi-
According to that theory the dephasing time tends to infinity as the temperature is lowered, as the available phase space for the electron-electron scattering gradually vanishes. However, as recently emphasized by Mohanty and Webb [61], experimentally this is not always the case. In some materials and samples, like Ag produced by the Saclay group [62], \(\tau_{\phi}\) follows very closely the predictions of the electron-electron interaction theory but in other cases there are strong deviations from the predicted behavior and the data indicate some saturations (see Fig. 9). It has been known since a long time that the saturation-like behavior depends very much on the preparation of the samples [53] and even on the substrates on which the films are deposited [54]. The early suggestion by Lin and Giordano [53] was that the dephasing is either due to the magnetic impurities or some defects which are very sensitive to the metallurgical properties of the films including thickness, annealing etc. The effect of magnetic Kondo impurities on the dephasing rate was carefully studied in those cases where the Kondo temperatures were in the relevant temperature range [55]. There the dephasing rate shows a maximum at the Kondo temperature of the magnetic impurities due to the enhancement of spin-flip scattering, but at lower temperature singlet Kondo ground state is formed and \(\tau_{\phi}\) decreases as the spin-flip rate gradually freezes out (see Table 1). However, the saturation observed has no resemblance to this behavior.

The only possibility for magnetic impurities to produce a saturation of \(\tau_{\phi}\) would be if their Kondo temperatures were much smaller than the temperature range of interest. Then the spin-flip rate is approximately temperature independent. However, in order to have \(T_K \leq 10mK\) the exchange coupling must be very weak \(J_{\rho_0} \leq 3 \times 10^{-2}\) and to produce the dephasing rate observed an enormous number of unidentified magnetic impurities should be present, which is very unlikely.

Accepting that the low temperature dephasing anomalies are intrinsic properties of the samples and far from a universal behavior it looks reasonable that some local dynamical defects as TLS’s are responsible for them.

Depending on the electron-TLS interaction two different limits must be considered: (i) For weak couplings the electron induced transition in the TLS is treated in second order perturbation theory. In that case to get an almost temperature independent dephasing rate the splittings \(\Delta\) (excitation energies) must be smaller than the measured temperature and their distribution must be peaked at very low energies [66]. However, there is no evidence for such anomalous distribution, and linear specific heat measurements on metallic glasses are consistent rather with a uniform distribution [67]. (ii) The other theoretical possibility is given by TLS’s with 2CK behavior [68]. In that case, in contrary to the magnetic Kondo problem, the scattering rate at low temperature is due to processes where the final states contain many electron-hole pairs (see Table 1), and being a dynamical scattering process, this produces dephasing. In order to get a reasonable dephasing rate less than 1 ppm 2CK defect is required. This explanation has two drawbacks: The questionable existence of such 2CK defects and the required small splitting \(\Delta\), even if this latter is renormalized downwards due to the strong interaction by a factor \(\Delta/T_K\) if \(\Delta < T_K\) [69]. On the other hand
Table 1
Comparison of the $n = 1$ and $n = 2$ channel Kondo problems.

|                | $n=1$                     | $n=2$                     |
|----------------|---------------------------|---------------------------|
| ground state   | singlet                   | non-singlet               |
| ground state entropy | $S=0$                  | $S=\frac{1}{2} \ln 2$    |
| electrical resistivity (scattering amplitude) as a function of $T$, $(\omega)$ | | |
| $T, (\omega) \geq T_K$ | logarithmic              | logarithmic              |
| l.e.c. $< T$, $(\omega) \leq T_K$ | $1- (...) T^2, (\omega^2)$ | $1- (...) T^{1/2}, (\omega^{1/2})$ |
| $T, (\omega) < l.e.c.$ | $1- (...) T^2, (\omega^2)$ | $1- (...) T^2, (\omega^2)$ |
| amplitudes of the scattering for a single electron | | |
| $T_K > \omega > l.e.c.$ | in the final state: | |
| single electron | | |
| $el + m (el + hole)$ | $\omega \rightarrow 0$ | $\omega \rightarrow 0$ |
| ampl. $\sim 1- (...) \omega^2$ | ampl. $\sim \omega^2$ | ampl. $\rightarrow \text{const.}$ |
| m. 1. | m. 1. | m. 1. |
| (m $\geq 1$) | (m $\geq 1$) | (m $\geq 1$) |

l.e.c. (low energy cutoff)
in case of 2CK defects the non-universality and metallurgical dependence are quite natural.

Finally it should be mentioned that such saturation like behavior has also been seen in degenerate semiconductor ballistic quantum dots [70]. In degenerate semiconductor as far as we know even the theory of magnetic Kondo effect has not been worked out in detail.

3.3. Energy distribution of electrons in short wires

In addition to the dephasing problem another closely related dilemma exists concerning the energy distribution of electrons in short wires with finite bias voltage. In these experiments [71,72] one measures the energy distribution function of electrons $f(E)$ by fabricating a metal-metal oxide-superconductor (M-MO-S) tunnel junction at various positions along the wire (see Fig. 10). From the $I−V$ characteristic $f(E)$ is determined by deconvolution. The typical length of the wires was $1.5\mu m$ and $5\mu m$. The measurements were carried out at $25mK$, and the samples were in the diffusive limit. In this case the typical energy relaxation processes are slow compared to the time it takes an electron to diffuse through the sample. Therefore the electron distribution exhibits typically two steps corresponding to the Fermi energy of the left and right contacts, thus their difference is proportional to the applied bias $U$ [73]. For non-interacting electrons at distance $x$ measured from one of the contacts the distribution function is

$$f_x(E,U) = \left(1 - \frac{x}{L}\right) f^0(E + eU) + \frac{x}{L} f^0(E), \quad (9)$$

where $L$ is the length of the sample and $f^0$ is the equilibrium distribution. In the diffusive limit without energy relaxation these two steps are smeared only by the temperature. At low enough temperature this smearing becomes, however, much larger than $T$, and the measured smearing gives information on the relaxation processes. In the case of long samples the smearing can be essential as the electrons spend longer time in the sample before leaving into the contacts. Typically the applied voltage $U \sim 0.1−0.2meV$ is larger than the temperature $T \sim 30mK$. The surprising results for short $Cu$ wires were that the
shape and the amplitude of the smearing could not be explained by the electron-electron interaction. Using a Boltzmann equation approach the line shape could only be reproduced by assuming an anomalously strong electron-electron interaction kernel $K$ with an anomalous dependence on the transferred energy $K \sim 1/\varepsilon^2$ \cite{74,72,71}, clearly in disagreement with the predictions of the theory of electron-electron interaction \cite{60}. On the other hand, it has been shown many years ago \cite{74} that such dependence can be due to magnetic impurity mediated inelastic scattering for $T > T_K$. The same dependence is valid for two-channel Kondo impurities. For Ag samples, however, a good agreement has been found with the electron-electron interaction theory with the expected energy dependence $K \sim \varepsilon^{-3/2}$ and amplitude.

The close similarity of this problem to the dephasing time dilemma became obvious, when the dephasing time was directly measured on samples prepared in the same way, and it was found that $\tau_\phi$ in the Ag samples \cite{74} follows again the standard electron-electron interaction theory and does not saturate.

Au wires prepared in Saclay, on the other hand, show also a strong relaxation rate and exhibit an electron distribution that could be fitted using a $K \sim \varepsilon^{-2}$ kernel. The strong anomalies in the electron distribution and the dephasing time of Cu and Au wires and the fact that Ag wires behave “regularly” both in dephasing time and electron distribution experiments strongly indicate that the anomalous dephasing time and distribution function are related to the same non-universal and material dependent processes.

It has been realized \cite{76} that assuming a $K \sim \varepsilon^{-2}$ electron-electron interaction kernel for energies $E \gg kT$ an energy independent electron relaxation rate can be derived making a connection between the distribution function dephasing time experiments. It has been shown that the Au and Cu impurities with negligible splitting \cite{74} by treating the 2CK problem in the framework of non-crossing approximation (NCA) \cite{74} and handling the non-equilibrium situation within the quantum Boltzmann equation framework.

That method had been applied earlier to the non-equilibrium transport through nano-point contacts in the presence of 2CK defects \cite{78}. Kroha also showed that the electron-electron interaction mediated by 2CK centers shows a $K \sim \varepsilon^{-2}$ energy dependence \cite{74}. That suggests that the classical Boltzmann equation with that kernel may give similar results to those obtained by using quantum Boltzmann equation and the 2CK scattering.

The most challenging feature of the experiments is that for an intermediate range of applied bias $U$, $0.1mV < U < 0.5mV$ the measured distribution function follows a scaling

$$f_x(E, U) = f_x \left( \frac{E}{U} \right).$$

In the framework of the 2CK interpretation the space-dependent non-equilibrium quantum Boltzmann equation is solved, where the collision term is expressed by the non-equilibrium electron self-energy calculated in a self-consistent way. In these calculations $T_K > 1K$ is assumed which determines the energy scale of the 2CK effect with zero applied bias. The Kondo effect is due to the sharp step of the unperturbed energy distribution function at the Fermi edge and the size of the step determines the Kondo temperature. As far as the bias $U$ is not larger than $T_K$, from the point of view of the Kondo effect there are no two separate steps and no scaling holds as the presence of $U$ influences the shape, width and amplitude of the Kondo resonance. For $T_K < U$, however the two steps are separated, and two distinct Kondo resonances are formed at the energy of each step with an effective Kondo temperature which can be essentially smaller than the equilibrium one. That bias region can be described by scaling according to Kroha’s NCA results \cite{74}. At even higher voltage other parameters of the model start to be essential and the scaling breaks down again. This theory is in good agreement with the experimental results for the complete applied voltage range $U$ and different positions of the measuring M-MO-S diodes and for samples of various length. For Cu and Au samples the anomalous smearing can be obtained but for Au a much higher concentration of 2CK centers is needed (100ppm) and
for Cu 1 – 5ppm. These values are in complete accordance with the amplitude of the measured dephasing rate. It should be emphasized that all these data, taken from the measurements of the Cu, Au samples prepared in Saclay, very likely depend a lot on the sample preparation, metalurgy and maybe also on the substrates used.

4. Discussions and perspectives

The spin Kondo problem in mesoscopic systems has been thoroughly studied both experimentally and theoretically and is quite well understood. There remain, however, a few further questions to answer: Concerning the surface anisotropy, the crossover from the ballistic to the dirty limits and the effect of disorder on the surface anisotropy should be further clarified. Similarly to the surface anisotropy, local density of states fluctuations decay as $1/d$ as a function of the distance from the surface. These fluctuations give probably the dominant effect in very thin films and films with a weak spin-orbit interaction for alloys with a relatively small $T_K$. Measurements on a host with weak spin-orbit scattering could help to clarify these issues.

The observation of the Kondo resonance by STM due to a single Co atom on the surface is a very impressive technical achievement. In the future, it would be worthwhile to study magnetic impurities inside the first few surface layers to establish stronger coupling between the spin and the host metals. In order to understand the data or to make predictions further electronic calculations are required for the host metal at the surface, the charge redistribution due to the impurity and the value of spin at the impurity atom.

The anomalous behavior of the zero bias anomalies in point contacts, and the dephasing and the transport in short wires can be well described by dynamical impurities with 2CK behavior. The fact that not all the samples studied (like Ag prepared in Saclay) show the anomaly supports that the anomalous NFL behavior is intrinsic, non-uniform and very likely depends on the preparation and treatment of the samples, and even on the substrate on which the sample is deposited. The orbital 2CK interpretation of these experiments is very challenging, however, it is far from being well established.

The possibility of the 2CK effect from a heavy tunneling atom has been suggested longtime ago. The original version of this model, however, cannot explain the large Kondo temperature $T_K \approx 1 \sim 10$K. However, as realized recently, electron-hole symmetry breaking present in all realistic band structure based density of states, can increase $T_K$ substantially and is very promising to resolve the long-standing problem of $T_K$. That is certainly the most important question to answer concerning the application of the idea of the 2CK problem.

Finally, we want to remark that a more detailed literature of this topic will be found in the Proceedings of the NATO workshop on ‘Size dependent magnetic scattering’ held between 28 May and 1 July 2000 in Pécs, Hungary [80].

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REFERENCES

1. W. J. de Haas, J. de Boer and G. J. van den Berg, Physica 1, 1115 (1933).
2. J. Kondo, Prog. Theor. Phys. Osaka 32, 37 (1964).
3. K. G. Wilson, Collective Properties of Physical Systems, Nobel Symposium 24 (New York Academic Press) p. 68.
4. P. Nozières, J. Low Temp. Phys. 17, 31 (1974).
5. For a review see D. L. Cox and A. Zawadowski, Adv. in Physics 47, 604 (1998).
6. For an extensive review see A. J. Leggett et al., Rev. Mod. Phys. 59, 1 (1987).
7. P. Nozières and A. Blandin, J. Phys. Paris 41, 193 (1980).
8. See e.g. E. Müller-Hartmann, Z. Phys. 223, 277 (1969).
9. N. Andrei and C. Destri, Phys. Rev. Lett. 52, 364 (1984); P.B. Wiegman and A.M. Tsvelik, Pis’ma Zh. Exp. Teor. Fiz. 38, 489 (1983) [JETP Lett. 38, 591 (1983)].
10. I. Affleck and A. W. W. Ludwig, Phys. Rev. B 48, 7297 (1993).
11. J. M. Maldecena and A. W. W. Ludwig, Nucl. Phys. B 506, 565 (1997).
12. for a review A. Blachly and N. Giordano, Phys. Rev. B 49, 6788 (1994).
13. J. F. DiTusa et al., Phys. Rev. Lett. 68, 678 (1992); G. Neuttiens et al., Europhys. Lett. 34, 623 (1996); T. M. Jacobs and N. Giordano, Europhys. Lett. 44, 74 (1998); N. Giordano and T. M. Jacobs, Physica B 280, 434 (2000).
14. V. Chandrasekhar et al., Phys. Rev. Lett. 72, 2053 (1994).
15. C. Roth, C. Sürgersand, and H. v. Löhneysen, Phys. Rev. B 51, 12537 (1995).
16. M. A. Blachly and N. Giordano, Physica B 194-196, 983 (1994).
17. M. A. Blachly and N. Giordano, Phys. Rev. B 49, 6788 (1994).
18. M. A. Blachly and N. Giordano, Europhys. Lett. 27, 687 (1994).
19. G. Bergmann, Phys. Rev. Lett. 67, 2545 (1991).
20. V. Barzykin and I. Affleck, Phys. Rev. Lett. 76, 4959 (1996).
21. G. Zaránd, Diploma Thesis at the Eotvos University of Budapest, 1992 (unpublished).
22. G. Zaránd and L. Udvardi, Physica B 218, 68 (1996); Phys. Rev. B 54, 7606 (1996).
23. I. Martin, Yi Wan, and Philip Phillips, Phys. Rev. Lett. 78, 114 (1997).
24. O. Újsághy, A. Zawadowski, and B. L. Gyorffy, Phys. Rev. Lett. 76, 2378 (1996); O. Újsághy and A. Zawadowski, Phys. Rev. B 57, 11598 (1998).
25. V. M. Fomin et al., Solid State Commun. 106, 293 (1998).
26. O. Újsághy and A. Zawadowski, Phys. Rev. B 57, 11609 (1998).
27. O. Újsághy and A. Zawadowski, Phys. Rev. B 60, 10602 (1999).
28. T. M. Jacobs and N. Giordano, submitted to Phys. Rev. B.
29. N. Giordano, Phys. Rev. B 53, 2487 (1996).
30. L. Borda and A. Zawadowski, Phys. Rev. B 61, 3570 (2000).
31. C. Strunk et al., Phys. Rev. Lett. 81, 2982 (1998).
32. E. Seynaeve et al., cond-mat/0002467.
33. I.K. Yanson et al., Phys. Rev. Lett. 74, 302 (1995); I.K. Yanson et al., Low Temp. Phys. 20, 836 (1994).
34. N. van der Post, F. L. Mettes, J. A. Mydosh, J. M. van Ruitenbeek, I. K. Yanson, Phys. Rev. B 53, R 476 (1996).
35. F. Mezei and A. Zawadowski, Phys. Rev. B 3, 167 (1971); F. Mezei and A. Zawadowski, Phys. Rev. B 3, 3127 (1971).
36. S. Berman and C. K. So, Phys. Rev. Lett. 40, 53 (1978).
37. Jintao Li, W.-D. Schneider, R. Berndt, B. Delley, Phys. Rev. Lett. 80, 2893 (1998); W.-D. Schneider, Pramana-J. Phys., 52, 537 (1999).
38. V. Madhavan, W. Chen, T. Jamneala, M. F. Crommie, and N. S. Wingreen, Science 280, 567 (1998).
39. H. C. Manoharan, C. P. Lutz and D. M. Eigler, Nature 403, 512 (2000).
40. U. Fano, Phys. Rev. 124, 1866 (1961).
41. T. Jamneala, V. Madhavan, W. Chen, and M. F. Crommie, Phys. Rev. B 61, 9990 (2000).
42. A. Schiller and S. Hershfield, Phys. Rev. B 61, 9036 (2000).
43. O. Újsághy, J. Kroha, L. Szunyogh, and A. Zawadowski, Phys. Rev. Lett. 85, 2557 (2000).
44. A. Schiller and S. Hershfield, Phys. Rev. B 61, 9036 (2000).
45. O. Újsághy, J. Kroha, L. Szunyogh, and A. Zawadowski, Phys. Rev. Lett. 85, 2557 (2000).
46. P. A. Anderson, Phys. Rev. 124, 41 (1961).
47. W. Chen, V. Madhavan, T. Jamneala, and M. F. Crommie, Phys. Rev. Lett. 80, 1469 (1998).
48. K. Vladár and A. Zawadowski, Phys. Rev. B 28, 1564, 1582, 1596 (1983).
49. S. Katayama, S. Maekawa, and H. Fukuyama, J. Phys. Soc. Jpn. 50, 694 (1987).
50. See e.g. T. Endo and T. Kino, J. Phys. F 18, 2203 (1988).
49. D. C. Ralph and R. A. Buhrman, Phys. Rev. Lett. 69, 2118 (1992); D.C. Ralph, A.W.W. Ludwig, Jan von Delft, and R.A. Buhrman, Phys. Rev. Lett. 72, 1064 (1994); see also J. von Delft, D. C. Ralph, R. A. Buhrman, A. W. W. Ludwig, and V. Ambegaokar, Ann. Phys. (NY) 263, 1 (1998); J. von Delft, A. W. W. Ludwig, and V. Ambegaokar, Ann. Phys. (NY) 273, 175 (1999).

50. Shashi K. Upadhyay, Richard N. Louie, R. A. Buhrman, Phys. Rev. B 56, 12033 (1997).

51. R. J. P. Keijsers et al., Phys. Rev. Lett. 77, 3411 (1996).

52. G. Zarandy, Jan von Delft, and A. Zawadowski, Phys. Rev. Lett. 80, 1353 (1998).

53. O. P. Balkashin, R. J. Keijsers, H. van Kempen, Yu. A. Kolesnichenko, and O. I. Shklyarevskii, Phys. Rev. B 58, 1294 (1998).

54. G. Zarandy and A. Zawadowski, Phys. Rev. Lett. 72, 542 (1994).

55. I. L. Aleiner, B. L. Altshuler, Y. M. Galperin, and T. A. Shutenko, cond-mat/0007430.

56. G. Zarandy, (to be published in the Proceedings of the NATO Conference on Kondo effect in submicron structures held in Pecs, Hungary).

57. A. Zawadowski and G. Zarandy, cond-mat/0009283.

58. Tejs Vegge et al., cond-mat/0003138.

59. Igor E. Smolyarenko, Ned S. Wingreen Phys. Rev. B 60, 9675 (1999).

60. B. L. Altshuler and A. G. Aronov, in Electron-electron Interaction in Disordered Systems, ed. A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985); B. L. Altshuler, A. G. Aronov, D. F. Khmel’nitskii, J. Phys. C 15, 7367 (1982).

61. P. Mohanty, E. M. Q. Jariwala and R. A. Webb, Phys. Rev. Lett. 78, 3366 (1997); P. Mohanty and R. A. Webb, Phys. Rev. B 78, 13452 (1997).

62. A. B. Gougam, F. Pierre, H. Pothier, D. Esteve, and N. O. Birge, J. Low Temp. Phys. 118, 447 (2000).

63. J. Lin and N. Giordano, Phys. Rev. B 35, 1071 (1987).

64. J. Vranken, C. van Haesendonck and Y. Bruynseraede, Phys. Rev. B 37, 8502 (1988).

65. R. P. Peters, G. Bergmann and R. M. Müller, Phys. Rev. Lett. 58, 1964 (1987); G. Bergmann, Phys. Rev. Lett. 58, 1236 (1987); C. van Haesendonck, J. Vranken and Y. Bruynseraede, Phys. Rev. Lett. 58, 1968 (1987).

66. Y. Imry, H. Fukuyama, P. Schwab, Europhys. Lett. 47, 608 (1999).

67. J. L. Black, Metallic Glasses, ed. H. Güntherodt and H. Beck, (New York: Springer), p. 167. (1981).

68. A. Zawadowski, J. von Delft, D. Ralph, Phys. Rev. Lett. 83, 2632 (1999).

69. P. D. Sacramento and P. Schlottman, Phys. Rev. B 43, 13294 (1991).

70. see e.g. D. P. Pivin, Jr. A. Andersen, J. P. Bird and D. K. Ferry, Phys. Rev. Lett. 82, 4687 (1999).

71. H. Pothier, S. Gueron, N. Birge, D. Esteve, M. H. Devoret, Z. Phys. B 104, 178 (1997).

72. H. Pothier, S. Gueron, N. Birge, D. Esteve, M. H. Devoret, Phys. Rev. Lett. 79, 3490 (1997).

73. K. E. Nagaev, Phys. Lett. A 169, 103 (1992); Phys. Rev. B 52, 4740 (1995).

74. J. Sólyom and A. Zawadowski, Z. Physik 226, 116 (1969).

75. F. Pierre, H. Pothier, D. Esteve, M. H. Devoret, J. Low Temp. Phys. 118,437 (2000).

76. J. Kroha and A. Zawadowski, in preparation.

77. see e.g. N. E. Bickers, Rev. Mod. Phys. 59, 849 (1987); E. Müller Hartmann, J. Phys 57, 281 (1984).

78. M. H. Hettler, J. Kroha and S. Hershfield, Phys. Rev. Lett. 73, 1967 (1994).

79. J. Kroha, Adv. in Solid State Physics, 40, in press (2000).

80. Proceedings of the NATO Advanced Workshop on ‘Size dependent magnetic scattering’, 28 May-1 June 2000, Pécs, Hungary, Kluwer Academic Publishers (2000).