Short preliminary analysis of student teachers’ mathematical knowledge of limit of a function

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Abstract. The purpose of this short preliminary study is to investigate mathematics student teachers’ mathematical and didactical knowledge of the limit of a function, particularly in evaluating the value of the limit. This study modeled these investigations using the Anthropological Theory of Didactic, which describes mathematical activity in terms of the mathematical organizations or praxeologies and considers the teacher the director of the didactic process. In this study, the case study design was used, and four mathematics student teachers were examined within the context of evaluating the limit's value. Mathematics student teachers' result work, video recordings of the participants' work in which they explained how to assess the value of limit and audio recordings of task-based interviews were used to gather data. Participants showed different ways of evaluating the value of the limit of a function and explained how they decided to answer based on their understanding.

1. Introduction

Study on students’ knowledge on limit of function has been examined by numerous researchers in the past decade. Several works point to testing their competencies regarding their content knowledge and misconceptions, particularly student teachers’ limit instruction and their reflection [1]. Other studies give more attention to students’ perception of instructional practices [2]. Other researches have examined the use of instructional media in teaching limit [3,4]. There are also studies about didactic restriction on the teachers’ practice that explained an outline of epistemological and didactic model [5].

Student teacher as a prospective teacher who will teach the limit in high school should have knowledge of limit, how to teach and evaluate it. Related to the knowledge, Shulman [6] emphasized Pedagogical Content Knowledge (PCK) relies on teachers’ capacity to transform the subject matter knowledge into other forms which can enable their students to understand the subject. One of the PCK framework is the Knowledge Quartet which allows student teacher to be evaluated and developed their teaching [7].

In this study, we investigate student teachers’ mathematical and didactical knowledge of limit of function, especially on evaluating the value of limit. The study based on The Anthropological Theory of Didactic (ATD) introduced by Chevallard [8]. This framework pointed that knowledge is considered as institutionally situated and it is studied through praxeological reference models.

The Anthropological Theory of Didactic (ATD) is known as a general epistemological model of mathematical knowledge that can be used to observe human mathematical activities [8]. The object of knowledge that will be learnt by a human related to mathematics can be identified into two aspects, a practical block and a knowledge block, which are main components of praxeological reference.
models. The practical block is formed by a type of task (T) and a technique (τ). A type of task (T) is a specific class of problems such as evaluating the value of limit of function. The students need a technique (τ) to solve this problem such as substitute a number to the function, algebraic simplification or describe using a graph. Then, the knowledge block consists of a technology (θ) used to explain the practical block and a theory (Θ) to justify and reason about the technology (θ). The technology (θ) for the case is that if two different functions f and g agree in all but one value c, someone can find \[ \lim_{x \to c} f(x) = \lim_{x \to c} g(x). \] While the theory of using a graph to support the fact that two functions agreeing in all but one point have the same limit behavior is uses to justify the technology. Those four elements (T, τ, θ, Θ) are interdependent.

2. Method

The task was chosen based on pupils’ difficulties to evaluate the value of limit of function. The given to the students as presented in Table 1.

Table 1. Task Design

| Evaluate the value of limit of these functions: |
|-----------------------------------------------|
| a. \( \lim_{x \to 1} 5x = 5 \)                |
| b. \( \lim_{x \to 2} \frac{2x^2-3x-2}{x-2} = 5 \) |

Explain your answer!

In this study, 30 mathematics student teachers from Universitas Serang Raya participated in solving those tasks. All students wrote their answer but only few students responded the last order. We collected four different students’ work with a variety ways completion and explanation. I recorded their explanation about the answer if they were a teacher. We also interviewed them to find out more about their understanding of the limit and how to teach the concept.

The assignment specified can be represented into praxeological reference models. There are two possible tasks can be expressed as shown:

\[ T_1 = \text{evaluate the following limit: } \lim_{x \to c} \frac{P(x)}{G(x)} = L. \]

\[ T^* = \text{given tasks and student feedbacks to the type of task } T \text{ and figure out what perception as a teacher to explain students.} \]

The first type of task \( T_1 \) is employed to estimate mathematics student teachers’ mathematical knowledge about evaluate the value of limit of function. Whereas the type of task \( T^* \) is purposed to classify their didactical knowledge related to teach of evaluating the value of limit of function.

The mathematicical techniques to the type of task \( T_1 \) can be described as for:

- \( \tau_{11} \) = substitution a number to the function to check whether the result 0/0.
- \( \tau_{12} \) = algebraic simplification, factorization.
- \( \tau_{13} \) = drawing the graph.
- \( \tau_{14} \) = using the approximation table of value of function.

Additionally, there are probabilities that one exposes an appropriate mathematical technique that uncommon used.

- \( \tau_{15} \) = using the formal definition of limit, \( \lim_{x \to c} f(x) = L \) means that for every \( c > 0 \), there exists a \( \delta > 0 \), such that for every \( x \), the expression \( 0 < |x - c| < \delta \) implies \( |f(x) - L| < \epsilon \).
- \( \tau_{16} \) = applying the formal definition of continuity of functions of one variable, function \( f(x) \) is continuous at point \( x = a \) if the following three conditions are satisfied: \( f(a) \) is defined, \( \lim_{x \to a} f(x) \) exists, and \( \lim_{x \to a} f(x) = f(a) \).

Those mathematical techniques expressed above are just sectional techniques that commonly used in evaluating the value of limit and can be more varied. In the meanwhile, the possible technology (θ) to justify a value of limit and the theory (Θ) are as follows:
θ = If two functions f and g agree in all but one value c then \( \lim_{x \to c} f(x) = \lim_{x \to c} g(x) \). If \( r(x) \) is a rational function and c is a real number such that \( r(c) \) exists, then \( \lim_{x \to c} r(x) = r(c) \).

Θ = In the textbook, a graph supports the fact that two functions agreeing in all but one point have the same limit behavior. An \( \varepsilon \rightarrow \delta \) proof is presented by the teacher in the course.

Those mathematical techniques conduct the authors to report some of didactical techniques that possible to be applied to solve the type of task \( T^* \). This didactical technique indicates mathematics student teacher explain to pupils using the mathematical technique of \( \tau_{11} \).

\( \tau_{17*} \) = show pupils to plug in, if plugging in \( c \) for \( x \) does not result in division by zero, then you have found the limit.

\( \tau_{18*} \) = explain to pupils if ‘plug in’ results in division by zero, algebraically simplify the expression (if possible), and then ‘plug in’ will give the students the limit if the students do not get division by zero.

\( \tau_{19*} \) = use visual representation by asking pupils to draw a graph.

\( \tau_{20*} \) = show pupils to make a table of value of function.

\( \tau_{21*} \) = describe to pupils the definition of limit using epsilon delta and illustration of graph.

\( \tau_{22*} \) = illustrate to pupils through a simple example related to everyday activities.

3. Result and Discussion

The mathematical techniques explained by mathematics student teacher were not only investigated from their explanation on the worksheets yet additionally expounded from their performing in teaching and task-based interview.

Starting from student A, a male student, he answered the task on his worksheet using the substitution process (\( \tau_{11} \)). He explained that it is the easiest way to know whether the function has a limit or not. Then he suggested to the pupils to use this method first when evaluating the limit. Then he proceeded to point, “if you found your answer 0/0 or something/0, then the next way you have to do is using algebraically method, it depends on your function you have, you can do factorization.” We categorized his didactical techniques into \( \tau_{17*} \) and \( \tau_{18*} \).

Student B, a female student, she wrote explicitly two mathematical techniques to solve the type of task \( T_1 \) on her worksheet. The first mathematical technique is clearly about finding the \( f(x) \) for every \( x \) that she chose her own domain, she made a table of value of function and created the approximation (\( \tau_{14} \)). The second technique that she proposed is drawing a graph that the data obtained from the table that she previously made (\( \tau_{13} \)). She explained to the pupils that by making a visual representation can help you to see the abstract thing more clearly. Then the didactical technique to solve the task was \( \tau_{20*} \) and \( \tau_{19*} \).

Student C, also a female student, she expressed an uncommon answer. She evaluated the value of limit of function using the formal definition of limit with epsilon and delta (\( \tau_{15} \)). We initially thought that the pupils have difficulty to understand her explanation, because this process involving two abstract symbols, epsilon and delta. We were impressed when she taught it using a graph, she drew illustration epsilon and delta on Cartesian coordinate (\( \tau_{22*} \)).

Student D, a male student, he responded the task by applying the formal definition of continuity of functions of one variable (\( \tau_{16} \)). He tried to explain the pupils what he wrote, unfortunately the pupils did not understand what he explained. Then he took an illustration of limit related to everyday activities (\( \tau_{22*} \)), and represented it into a graph (\( \tau_{19*} \)).

Table 2 shows a recapitulation of student teachers’ mathematical and didactical technique on how they explain the concept of evaluating limit.

| Students | Mathematical techniques | Didactical techniques |
|----------|-------------------------|----------------------|
| A        | \( \tau_{11} \)         | \( \tau_{17*}, \tau_{18*} \) |
| B        | \( \tau_{13}, \tau_{14} \) | \( \tau_{19*}, \tau_{20*} \) |
| C        | \( \tau_{15} \)         | \( \tau_{21*} \)     |
| D        | \( \tau_{16} \)         | \( \tau_{19*}, \tau_{22*} \) |
4. Conclusion
This short study for mathematics student teachers’ knowledge on evaluating the value of limit has not finished yet. The didactical techniques demonstrated by students are similar, consequently, it makes difficult to distinguish one and another. Hence it requires more development methods to exploring students’ thinking in describing mathematical and didactical techniques based on praxeological reference models specifically in limit concept.

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6. References
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