Reduced order models for wind turbine blades with large deflections

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Abstract. Non-intrusive nonlinear reduced order modeling (ROM) techniques have been applied by researchers to obtain computationally cheap and yet accurate structural responses of aircraft panels. However, its application to wind turbine blades is new and challenging due to much larger deflections of wind turbine blades. This study improves a non-intrusive nonlinear ROM method for wind turbine blades going through large deflections. In the nonlinear ROM, the nonlinear stiffness is described by the quadratic and cubic functions, and the secondary motions induced by the primary large deflections are described by the modal derivative vectors in the reduction basis. The non-intrusive nature of the method requires a geometrically nonlinear solver, and HAWC2 is chosen in this study for the computation of nonlinear stiffness terms. Two examples, including a cantilever beam example and the NREL 5MW wind turbine blade model, are used to evaluate the accuracy and computational effectiveness of the nonlinear ROMs. The cantilever beam example shows that the nonlinear ROM can accurately capture the axial displacements due to large deflections reaching 20% of span length as well as the torsion coupled with flapwise and edgewise motions. The NREL blade example shows that the nonlinear ROM is accurate for the tip displacements more than 5.9 m. Because the size of the nonlinear ROM is much smaller than that of HAWC2 model, a speedup factor of 8.5 for computational time is observed for the NREL blade example.

1. Introduction
This study aims to develop an effective non-intrusive method for nonlinear reduced order modelling of long and flexible wind turbine blades with large deflections and nontrivial bend-twist coupling. In the non-intrusive approach, the nonlinear Reduced Order Model (ROM) is generated like an identification procedure based on the results from a number of high-fidelity simulations. In this study, these high-fidelity simulations are carried out based on the load analysis tool for wind turbines HAWC2 [1] developed at DTU Wind Energy. Modern wind turbine blades can go through large deflections, which are more than 20% of the blade span length [2]. Large deflections alter the blade effective length as well as the coupling between the blade flapwise/edgewise and torsional motion. Hence, the large deflection effects need to be included in an accurate load analysis for wind turbines with long and flexible blades. The load analysis tool HAWC2 can capture the large blade deflections by dividing the blades into several sub-structures [3]. The blade models generally constitute more than 80% of the degrees-of-freedom (DoFs) of the turbine structural model in HAWC2 [4]. As the model size expands in the structural analysis, the computation time and the required memory also increase. The motivation of this study is to develop an alternative approach to capture the...
large deflection effects with much fewer DoFs compared to the current HAWC2 models so as to reduce the computational cost. One promising way to achieve this is to incorporate a pre-computed nonlinear ROM of the blade into the multi-body formulation [5]. However, very few studies have examined the nonlinear blade ROMs with realistic blade deflections where the tip displacements reaches up to 10% or more of the blade span length. The method introduced in this study is based on a non-intrusive approach rather than the intrusive method used in [5] where a geometrically nonlinear finite element code was modified to generate the ROM.

In a non-intrusive approach, any simulation solver capable of capturing large deflections can be used to compute the nonlinear ROM stiffness parameters [6] without modifying the source code. Although the non-intrusive ROM techniques have been studied extensively for aircraft panel structure in aero-thermo-elastic and aero-acoustic analysis [7, 8], there has been very little research investigating the non-intrusive approach for slender cantilever structures with complex geometries like wind turbine blades. This may be due to the difficulties in identifying the nonlinear stiffness terms for cantilever structures and numerical instabilities, see e.g., [9, 10].

In order to fill this gap, this study aims to develop a non-intrusive way to build the nonlinear reduced order model (ROM) for wind turbine blades. In contrast to the linear ROM, the nonlinear ROM includes an extra set of base vectors and an extra set of coefficients that capture the aforementioned nonlinear coupling effects such as the torsional motion induced by the coupled edgewise and flapwise motions, and the longitudinal motion due to the large deflections. The extra set of base vectors and the computation of nonlinear stiffness terms are explained in Section 2. The proposed method is applied to a straight cantilever beam and the NREL 5 MW wind turbine blade [11] with dynamic loads in the flapwise, edgewise and torsional directions. The results are presented in Section 3. Finally, the discussions and conclusions are drawn in Section 4 and 5, respectively.

2. Methodology
In this section, we first introduce the basic methodology for generating the nonlinear reduced order models. Then we present a modified non-intrusive approach to identify the quadratic and cubic coefficients that are essential to describe the nonlinear stiffness in the reduced order model. Finally, we present the reduction basis enriched with modal derivatives.

In this study modal derivatives are used to describe the secondary motions induced by the primary deflections. For example, there is a secondary axial motion associated with the primary bending motion in a cantilever beam. Another example is that there can be a secondary torsional motion associated with the primary bending motions occurring in two directions at the deflected state of a symmetric beam. These are further illustrated later in the illustration of the modal derivatives and the numerical results.

2.1. Nonlinear reduced order model
Assume that the nonlinear internal forces in a structure such as a blade are described by the quadratic and cubic functions of displacements. Then the equation of motion of the structure can be written in a compact form by using the entries of the related matrices as

\[
M_{pr} \ddot{u}_r + C_{pr} \dot{u}_r + K_{pr}^{(1)} u_r + \sum_{r,s} K_{pr,s}^{(2)} u_r u_s + \sum_{r,s,v} K_{pr,s,v}^{(3)} u_r u_s u_v = f_p(t) \tag{1}
\]

where \( p, r, s, v = 1, \ldots, N \) with \( N \) denoting the number of DoFs. The associated matrices \( M \in \mathbb{R}^{N \times N} \) and \( C \in \mathbb{R}^{N \times N} \) are the symmetric inertia and damping matrices, and \( f(t) \in \mathbb{R}^N \) is the external force vector for the \( N \)-DoFs system. The quantities of \( K^{(1)} \in \mathbb{R}^{N \times N} \), \( K^{(2)} \in \mathbb{R}^{N \times N \times N} \) and \( K^{(3)} \in \mathbb{R}^{N \times N \times N \times N} \) stand for the linear, quadratic and cubic stiffness coefficients. The displacements, velocities and accelerations are represented by \( u \in \mathbb{R}^N \), \( \dot{u} \in \mathbb{R}^N \) and \( \ddot{u} \in \mathbb{R}^N \), and \( t \) is the time.
To obtain a reduced order model, the spatial and temporal components of the displacements can be separated as

\[ u(x, t) \approx \sum_{i=1}^{M} \phi_i(x)q_i(t) = \Phi(x)q(t) = \begin{bmatrix} \Phi_B(x) & \Phi_E(x) \end{bmatrix} \begin{bmatrix} q_B(t) \\ q_E(t) \end{bmatrix} \]

(2)

where \( \phi_i(x) \in \mathbb{R}^N \) is the \( i^{th} \) base vector and \( q_i(t) \) is its amplitude. All base vectors are collected in the columns of the matrix \( \Phi \in \mathbb{R}^{N \times M} \) and the corresponding amplitudes in the vector \( q \in \mathbb{R}^M \). The chosen base vectors in \( \Phi \) include linear bending modes (\( \Phi_B \)) and the vectors of an extra set of base vectors (\( \Phi_E \)) which are elaborated in Section 2.3. The nonlinear ROM is obtained by substituting equation (2) into equation (1) and multiplying by \( \Phi^T \), which yields

\[
\tilde{M}_{ij}\dot{q}_j + \tilde{C}_{ij}\dot{q}_j + \tilde{K}_{ij}^{(1)}q_j + \tilde{K}_{ijk}^{(2)}q_jq_k + \tilde{K}_{ijk}^{(3)}q_jq_kq_l = \tilde{f}_i(t)
\]

(3)

where \( i, j, k, l = 1, \ldots, M \) with \( M \) denoting the number of the generalized DoFs in ROM, and \( p, r, s, v = 1, \ldots, N \). The reduced quantities are expressed as

\[
\tilde{M} = \Phi^T M \Phi, \quad \tilde{C} = \Phi^T C \Phi, \quad \tilde{f}(t) = \Phi^T f(t), \quad \tilde{K}^{(1)} = \Phi^T K^{(1)} \Phi, \quad \tilde{K}^{(2)}_{ijk} = \sum_{p,r,s} K_{prs}^{(2)} \Phi_i(p) \Phi_j(r) \Phi_k(s), \quad \tilde{K}^{(3)}_{ijk} = \sum_{p,r,s,v} K_{prs}^{(3)} \Phi_i(p) \Phi_j(r) \Phi_k(s) \Phi_l(v)
\]

(4)

Here the indices of \( \Phi_i(p) \) mean an entry at column \( i \) and row \( p \) of the \( \Phi \) matrix. The system in equation (3) has \( M \) DoFs, while the system in equation (1) has \( N \) DoFs. In equation (3), the quantities \( \tilde{M} \in \mathbb{R}^{M \times M}, \tilde{C} \in \mathbb{R}^{M \times M}, \tilde{K}^{(1)} \in \mathbb{R}^{M \times M} \) and \( \tilde{f} \in \mathbb{R}^M \) are computed from their counterparts \( M, C, f \) and \( K^{(1)} \). In contrast, the quadratic and cubic coefficients \( \tilde{K}^{(2)}_{ijk} \) and \( \tilde{K}^{(3)}_{ijk} \) are not computed from their counterparts since these are generally unknown or difficult to obtain. Instead, these quadratic and cubic coefficients are computed by the non-intrusive method described in the following subsection.

### 2.2. Non-intrusive method for identification of nonlinear stiffness terms

In the literature, there are two types of non-intrusive methods to identify the aforementioned quadratic and cubic coefficients, including force-based non-intrusive method [12, 8, 6] and displacement based method [6, 9, 10]. The force-based method is found more suitable to cantilever structure than the displacement-based method, because secondary motions occur naturally in force-based method without applying explicit force in secondary motion directions done in displacement-based method [8]. A force-based method is modified in this study to suit its applications in wind turbine blades.

In the force-based method, a high-fidelity solver is first called to generate a set of data capturing the relation between applied force and output deflections. Then, the data are used to approximate the quadratic and cubic coefficients \( \tilde{K}^{(2)}_{ijk} \) and \( \tilde{K}^{(3)}_{ijk} \). It is known that the static load cases can be generated by \( K^{(1)} \) and the combination of chosen bending modal amplitudes by pairs. Equation (5) shows the computation of the \( r^{th} \) load case vector \( f^r \), where \( \Phi_B^i \) is \( i^{th} \) column of \( \Phi_B \), and \( q_B^i \) is the \( i^{th} \) modal amplitude of the \( r^{th} \) load case.

\[
f^r = K^{(1)} [q_B^i \Phi_B^i + q_B^j \Phi_B^j]
\]

(5)

A zero-force condition in torsion/axial components of applied loading is proposed in this study to suit the applications of the force-based method in wind turbine blades. It means the applied loading only includes force components in main bending directions and no force components in
torsion or axial directions. Since the bending mode shapes can have nonzero torsion and axial motions for pre-bend/twist structures, the above zero-force condition is satisfied by putting zeros in the applied force at the entries corresponding to torsion and axial directions.

After the static analyses, the corresponding modal amplitudes of load case deflections, i.e., \( q_j \), \( q_k \), \( q_l \), are computed by the least-squares method, and the applied force results are projected onto the reduction basis to obtain \( \hat{f}_l \). Equation (6) shows the \( r \)th static load case with linear and nonlinear stiffness terms, while the inertia and damping terms in equation (3) are dropped in the static analyses.

\[
\hat{K}_{ijk}^{(2)} q_j q_k + \hat{K}_{ijkl}^{(3)} q_j q_k q_l = \hat{f}_l - \hat{K}_{ij}^{(1)} q_j
\]

Here \( \hat{f}_l, \hat{K}_{ij}^{(1)}, q_j, q_k \) and \( q_l \) are known. The unknown \( N_k \) number of stiffness terms and right hand side of the equation (nonlinear force vector) can be written as vectors and modal amplitude multiplications on the left hand side can be written in a matrix form. By collecting all \( N_k \) number of the load cases results in one modal amplitude matrix \( D(q) \in \mathbb{R}^{N_k \times N_s} \), one unknown vector \( k \in \mathbb{R}^{N_k} \) and one force vector \( f_{nl} \in \mathbb{R}^{N_s} \), the nonlinear stiffness terms in \( k \) can be determined by the least-squares method as

\[
D(q)k = f_{nl} \rightarrow \text{minimize } \| f_{nl} - D(q)k \|^2
\]

where the superscript 2 denotes the power of 2, and \( || \) denotes the 2-norm of the vector.

Note that for the nonlinear ROM in this study, the nonlinear stiffness terms only includes one- and two-mode couplings. The stiffness terms of triple-mode couplings involving three different modes are ignored because their contributions are generally very weak [6]. It is important to choose the load cases according to the real-life deflections, which means large flapwise displacements (up to 20\% of blade span for very flexible blades) and edgewise displacements (up to 5\% of blade span). The hardening effects are defined by the cubic stiffness terms, and the softening effects are defined by the quadratic stiffness terms. The quadratic terms are zeros for straight symmetric structures, whereas they are generally non-zero for asymmetric pre-bend structures. In this study, nonlinear static analyses needed to compute the nonlinear stiffness terms (see equation (6)) are carried out in HAWC2. HAWC2’s structural part uses a multi-body formulation with linear Timoshenko beam elements in bodies [13]. When a continuous structure is modelled by the same number of sub-bodies as beam elements, it is similar to a co-rotational formulation [3]. The modal amplitudes, linear and nonlinear ROM stiffness (see equation (7)) terms are computed outside HAWC2 by using in-house code programmed in Python 3.6.

2.3. Reduction basis and computation of modal derivatives

The nonlinear stiffness terms include the nonlinear force effects such as hardening/softening, however additional displacement/rotation vectors apart from bending directions are needed to capture the physical displacements accurately. As a structure goes through large deflections, its motion also occurs in axial and torsional directions due to large deflections. An extra set of base vectors is used to capture these secondary motions. Since the secondary motions are induced by primary deflections mainly in the bending directions, the extra set of base vectors and amplitudes are functions of the primary deflections. There are different schemes in literature to select the extra set of base vectors [6] such as Expansion Modes and Dual Modes [9]. In this study, Modal Derivatives [5] are used as extra set of base vectors.

For an interpretation of the modal derivatives [5], they are linked to the second term in the Taylor series expansion of displacements with respect to modal amplitudes, as shown in equation (8). They represent the secondary motions of large deflections, such as the axial and torsional motion of a cantilever beam.

\[
\frac{\partial u}{\partial q} \cdot q + \frac{1}{2} \left( \frac{\partial^2 u}{\partial q \partial q} \cdot q \right) \cdot q + \mathcal{O}(\|q\|^3) = \Phi \cdot q + \frac{1}{2} \left( \frac{\partial \Phi}{\partial q} \cdot q \right) \cdot q + \mathcal{O}(\|q\|^3)
\]
Calculation of the modal derivatives starts by taking the derivative of the eigenvalue problem as shown in Equation (9), where $K$, $M$ are the stiffness and mass matrices and $\omega_i$ is $i^{th}$ eigenvalue. It is a common practice to neglect inertia effects in many applications since their effects are much smaller compared to the stiffness. The modal derivatives without inertia effects which are called Static Modal Derivatives [14] are used in this work. For convenience, they are called as Modal Derivatives thereafter. When the inertia effects are neglected, Equation (10) is obtained and static modal derivatives can be computed by numerical implementation of Equation (10).

$$\frac{\partial}{\partial q_j} \left( (K - \omega_i^2 M) \phi_i \right) = \left( \frac{\partial K}{\partial q_j} - \frac{\partial (\omega_i^2 M)}{\partial q_j} \right) \phi_i + (K - \omega_i^2 M) \frac{\partial \phi_i}{\partial q_j} = 0 \quad (9)$$

$$\frac{\partial \phi_i}{\partial q_j} = -K^{-1} \frac{\partial K}{\partial q_j} \phi_i = -K^{-1} \frac{\partial K}{\partial \mathbf{u}} \phi_i \phi_j = -K^{-1} \frac{\partial^2 f_{int}}{\partial \mathbf{u} \partial \mathbf{u}} \phi_j \phi_i = -K^{-1} \frac{\partial^2 f_{int}}{\partial q_j \partial q_i} \quad (10)$$

Since the linear stiffness and inertia matrices are known, the only unknown in equation (10) is the derivative of the stiffness matrix with respect to the modal amplitudes. In this study, the second-order derivative of internal forces ($f_{int}$) with respect to modal amplitudes are computed by the central finite difference given in equation (11). The displacement constraints in the mode shape vector directions are applied in HAWC2 by amplitudes of $\delta_i$ and $\delta_j$ and the internal force vector $f_{int}$ are obtained from HAWC2.

$$\frac{\partial^2 f_{int}}{\partial q_j \partial q_i} = \frac{f_{int}(\phi_i \delta_i + \phi_j \delta_j) + f_{int}(-\phi_i \delta_i - \phi_j \delta_j) - f_{int}(-\phi_i \delta_i + \phi_j \delta_j) - f_{int}(\phi_i \delta_i - \phi_j \delta_j)}{4\delta_i \delta_j} \quad (11)$$

As the reduction basis includes both the bending modes and the modal derivatives, the coupling stiffness terms between them appear and must be computed for the nonlinear ROMs in this work. An alternative method is to have a reduction basis without the modal derivatives or the extra base vectors and use them in post-processing through, e.g. Implicit Condensation Method [6]. This approach can reduce the number of unknown stiffness terms. However, a model without the modal derivatives is not able to give reaction to the loads in the directions of modal derivatives i.e. axial and torsional loads. Torsional loads and deflections are particularly important for wind turbine blades. Therefore, modal derivatives are included in the reduction basis in this study.

3. Results
A straight cantilever beam and the NREL 5MW wind turbine blade are used to evaluate the accuracy and computational cost of the proposed nonlinear ROM method. HAWC2 results are used as a reference solution, and load cases for nonlinear ROM stiffness terms are run in HAWC2 (ver. 12.8). The Newmark time integration scheme is used for the ROM analyses with values of integration parameters $\gamma = 0.51$ and $\beta = 0.27$ same as the default values in HAWC2 [1]. The ROM time integration is carried out in a Python 3.6 script, and analysis time is recorded. The modal based blade models used in FAST Elastodyn [15] and FLEX [16] have 2 flapwise and 1 or 2 edgewise modes. To mimic the choice of the modes in FAST and FLEX in this study, the base vectors for the ROMs also consist of the first 2 flapwise modes and the first 2 edgewise modes together with selected modal derivative vectors. The linear ROM also has the same number of modes (including modal derivatives). However, it does not include the nonlinear stiffness terms in the nonlinear ROM.
3.1. Straight beam

To demonstrate the effectiveness of nonlinear ROM (NL-ROM) to capture the effects of large deflections, the proposed NL-ROM is first applied to a straight cantilever beam, see Figure 1. The considered problem is a straight aluminium cantilever beam with applied loads in the two lateral directions, as shown in the figure. While $F_x=100$ N is constant, $F_y$ is a time-periodic load whose magnitude and frequency is $132.435$ N and $0.1667$ Hz, respectively. The value of the frequency corresponds to a gravity load caused by a rotation speed of 10 rpm. Such a loading condition is to mimic the loading of a wind turbine blade. For a wind turbine blade, it bears aerodynamic and gravity load in the flapwise and edgewise directions.

The HAWC2 model of the beam consists of 10 sub-bodies, and each body consists of one beam element. The resulting HAWC2 model has 60 finite element DoFs and 60 constraint equations. The NL-ROM and linear ROM (L-ROM) have 8 DoFs, including 4 bending modes and 4 modal derivative vectors. The difference between the L-ROM and NL-ROM lies in whether the nonlinear stiffness terms are included.

Note that in total there are 10 modal derivative vectors for 4 linear bending modes. However, the NL-ROM and the L-ROM include only the modal derivatives of the first bending mode in the $x-$direction, since the large deflection occurs in the $x$-direction in this example.

Figure 1: The straight aluminium cantilever beam with applied loads, cross-section dimensions and material properties.

For an intuitive interpretation of the concept of modal derivatives, Figure 2 shows the first two modal derivatives of the first bending mode of the beam. It can be seen that the modal derivative of the first bending mode in $x$-direction with respect to its own modal amplitude $q_1$, denoted as $\frac{\partial \phi}{\partial q_1}$, shows an axial motion. In contrast, the modal derivative of the first bending mode $\phi$ with respect to the amplitude of the first bending mode in $y$-direction, denoted as $\frac{\partial \phi}{\partial q_2}$, shows a torsional motion. These axial and torsional motions demonstrate the capacity of the modal derivatives to capture the secondary motions when used in a nonlinear ROM.

Figure 2: First two modal derivatives of the first bending mode of a straight beam
Figure 3 shows a comparison of the simulated steady time-periodic responses obtained from HAWC2, L-ROM and NL-ROM. The dynamic response was simulated for 200 s with a time step size of 0.05 s. The beam damping ratios were increased by 10 times to damp out the initial beam vibration fast in beam structural analysis with HAWC2 and reduced order models. The results are shown in Figure 3 for the responses at the last 20 s.

![Figure 3a: Beam tip axial displacement](image)

![Figure 3b: Beam tip torsion deflection](image)

![Figure 3c: Beam tip x-displacement](image)

![Figure 3d: PSD of the beam tip x-displacement](image)

Figure 3: Comparison of the three models for the straight cantilever beam example: HAWC2 model, linear ROM (L-ROM) and nonlinear ROM (NL-ROM). The figures show the time histories of the secondary motions such as axial displacement, torsion deflection and x-displacement at tip node, and the power spectrum density (PSD) of the tip x-displacement.

Figure 3a shows the axial tip displacement of the beam due to the large deflections. The L-ROM (linear ROM) estimates zero axial displacements, although L-ROM (linear ROM) has the first modal derivative vector, which is in the axial direction. The L-ROM needs an axial load to result in a non-zero axial displacement, and there is no axial load in this example. On the other hand, the NL-ROM (nonlinear ROM) and HAWC2 estimate more than 0.2 m axial tip displacement due to the large deflections in bending directions. Although NL-ROM and L-ROM include the same mode shape and modal derivative vectors, the secondary motions are captured by the nonlinear stiffness terms defined in NL-ROM. Figure 3b shows the tip torsion of the beam for three models. The beam has torsional motions due to the coupling between x- and y-directions at the deflected state. The L-ROM estimates zero torsional motion, whereas HAWC2 and NL-ROM estimate a periodic torsional motion, which has an amplitude about 1.5°. Figure 3c shows the x-displacement at the tip of the beam. The L-ROM overestimates the x-displacement up to 0.08 m without any periodic motion. The NL-ROM estimates the periodic motion in x-direction, but its maximum amplitude is about 0.01 m shorter than HAWC2. Since the beam has a periodic motion in y-direction, the secondary motions in axial, x- and torsional...
directions are also periodic. The periodic behavior of the secondary motions can be seen in the Power Spectrum Density (PSD) results of them. Figure 3d shows the PSD of tip $x-$displacement for three models. The HAWC2 and NL-ROM have the same peak at 0.33 Hz, which is twice of the excitation frequency of 0.1667 Hz in the $y$-direction. On the other hand, L-ROM has a weak peak at 0.41 Hz, which is the first bending frequency of the beam in $x-$direction, and the peak is observed due to the initial vibrations.

3.2. NREL 5MW wind turbine blade

NREL 5MW blade [11] is also modelled by using the two linear ROMs, two nonlinear ROMs and HAWC2. The length of the blade is 61.5 m. Although the NREL 5MW blade does not have a large prebend, it is not perfectly symmetric. Hence, the flapwise and edgewise vibration modes of the blade also include small vibration motion in the axial and torsional directions in contrast to the straight beam example. The blade is loaded with flapwise, edgewise and torsional loads. These loads are computed in HAWC2 for NREL 5MW turbine model with a steady wind speed of 11 m/s. To calculate the loads, the full NREL 5MW turbine model is used, where only the blades are modelled as flexible bodies, while the rest structures are treated as rigid bodies. The tower shadow effects are switched off in the aerodynamic load calculation.

The resulting time histories of the loads are displayed in the left sub-figure in Figure 4. These time histories are measured at a location in the blade at 0.77 of the length from the root. The right sub-figure in Figure 4 shows the spatial distribution of the three loads along the length of the blade. The spatial distributions of the loads correspond to a time point when time is 1.36 s when the maximum flapwise load is observed in time. In the spatial distribution of the loads, the maximum flapwise load occurs around 0.77 of the blade length, which is the location where the time histories in the left sub-figure are measured. Besides, it is also seen that the edgewise load reaches its maximum magnitude at the root of the blade since it is mainly due to the gravity load.

![Time history of loads at 0.77 of the blade span](image1)

![Distribution of loads over the span at 1.36 s](image2)

Figure 4: NREL 5MW blade steady wind loads in edgewise, flapwise and torsional directions. Figure (a) shows the time history of loads at 0.77 of the blade span, and Figure (b) shows the distribution of loads over the blade span when the maximum flapwise load occurs at 1.36 s.

The gravity load in the edgewise direction at 0.77 of the blade span is 1.8 times larger than the aerodynamic load, although the aerodynamic loads at that section are very large compared to the rest of the blade. Towards the root sections, this ratio increases and reaches up to 8 at 10 m of the blade span. The aerodynamic forces at first 10 m of the blade are not large, and the ratio is more than 25 at those regions. The periodic change of the edgewise load is caused by the rotation of the rotor. Additionally, it is seen that the torsional load is small compared to the
flapwise and edgewise load. In contrast to flapwise and edgewise load, the sign of the torsional load can change in the spatial distribution along the length of the blade.

In the comparison of the five models (HAWC2, L-11, NL-11, L-8, and NL-11), the responses are calculated for 50 s with a time step size of 0.02 s. The blade damping ratios were increased by 10 times to damp out the initial blade vibration fast in blade structural analysis with HAWC2 and reduced order models. The HAWC2 structural model consists of only the cantilever blade model with 108 finite element DoFs and 108 constraint equations, and the computed loads are applied as external loads. There is no turbine structure or aerodynamic model in the HAWC2 blade model used for model comparison.

The torsion results of ROMs with four modal derivative vectors of first flapwise mode shape are not very accurate, especially in the tip region. These models have 8 DoFs and labeled as L-8 and NL-8 in the plots. Three modal derivative vectors are added to the ROMs to increase torsion accuracy. These models have 11 DoFs and are labeled as L-11 and NL-11 in the plots. The added modal derivative vectors include torsional motions. Two of them are the derivative of the first edgewise mode shape with respect to the second flapwise mode amplitude and second edgewise mode amplitude. The last added modal vector represents the derivative of the second flapwise mode shape with respect to the second edgewise mode amplitude.

The NREL 5MW wind turbine blade has a much more complex geometry than the straight beam. It goes through a large flapwise deflection near 6 m at its tip, see Figure 5b. Figure 5a shows the tip axial displacement of the blade due to large blade deflections. The axial displacement can be thought of as a secondary motion since there is no axial load, but the blade has more than 0.4 m axial displacements. Therefore, this effect is not captured by the linear ROMs (L-11 and L-8). The linear ROMs result in more than 0.45 m longer blade length compared to the HAWC2. The maximum differences between nonlinear models NL-11 and NL-8 and HAWC2 in axial displacement are less than 0.0045 m and 0.006 m, respectively. Figure 5b shows the tip flapwise displacement of the blade. The linear ROMs overestimate the flapwise displacement by around 0.1 m since they cannot capture the geometric stiffening effects due to large deflections. The differences between HAWC2 and NL-11 and NL-8 models are 0.019 m and 0.015 m, respectively.

Figure 5c shows the tip torsion results of the models. The difference between 8 DoFs models (L-8, NL-8) and 11 DoFs models (L-11, NL-11) are apparent in the torsion results. The maximum torsion errors of L-8 and L-11 models compared to the HAWC2 model are 1.14° and 0.83°, respectively. The maximum error of NL-8 and NL-11 models are 0.54° and 0.26°, respectively. Torsional motion occurs as a result of torsion moment and coupling between flapwise and edgewise motion. The first effect can be captured accurately by a torsion mode. However, the couplings between the motion of the beam in different directions vary with the deflections, and the torsion due to the couplings can be considered as a secondary motion for a beam at its deflected state. This secondary motion is much more apparent in the regions with large deflections, such as the tip part of the blade. The fluctuation of the torsional motion is mainly due to the coupling between flapwise and edgewise motions. The gravity loads vary due to the rotor rotation, and they cause edgewise motions. NREL 5MW blade has stronger coupling between flapwise and edgewise motions at the deflected blade position than the undeflected blade position. Since the linear models do not update the coupling terms, they cannot capture the change in couplings and secondary motions accurately. Figure 5c shows the torsional motion of the tip node, but the torsion distribution along the blade is also very important. Most of the aerodynamic forces are produced in the middle regions of the blade. Figure 5d shows the torsion distribution along the blade span when the maximum error occurs for ROMs at 1.7 s. The difference between linear ROMs and HAWC2 is visible after 20 m blade span, whereas the nonlinear ROMs have a noticeable difference only after 43 m. The nonlinear model NL-11 with 11 DoFs has 52 % less tip torsion error than NL-8 model with 8 DoFs.
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Figure 5: Comparison of the responses obtained by HAWC2, L-11, NL-11, L-8, NL-8 models for NREL 5MW blade under steady wind loads. Figure (a)-(c) show the time history the axial, flapwise displacements and torsion deflections at the blade tip. Figure (d) shows the torsion deflections along the span at 1.7 s when the maximum discrepancy occurs among the models.

4. Discussion
Table 1 shows a comparison of the models used in this study (HAWC2, NL-11, NL-8, L-11 and L-8) in terms of model sizes, computational efficiency, and the accuracy of the responses. The HAWC2 model sizes represent the total number of numerical DoFs which is the sum of the finite element DoFs and the number of constraint equations. The representative computation time is measured in a computer with four Intel Core i7-6600U CPU @ 2.60GHz and 8 GB memory. For implementation, HAWC2 is written in FORTRAN, whereas linear and nonlinear ROMs are implemented in Python 3.6. The linear model L-8 is the fastest, and L-11 is the second fastest model. Nonlinear models NL-11 and NL-8 are faster than the HAWC2 model by a factor of about 8.5 and 10.7 for the NREL 5MW blade. Nonlinear ROMs can be accelerated by improving the implementation in Python or re-implementing them in a compiled language. The maximum displacement errors are compared to evaluate the accuracy of the models. The nonlinear ROMs are up to 15 times more accurate than the linear ROMs for the NREL 5MW blade displacement results because of their capacity to capture the secondary motions, especially in the axial direction.

For the straight and symmetric beam example, there is a difference of about 0.01 m between the nonlinear ROM and HAWC2 model in \( x \)-displacement. The fluctuation in the \( x \)-displacement arises from the torsional motion which is already a secondary effect. This higher order relation between \( x \)- and \( y \)-displacements are not captured by any modal derivative vector used in this study. As a result, adding more modal derivative vectors does not improve
Table 1: A comparison of the computational efficiency and accuracy of the five models, including HAWC2, NL-11, NL-8, L-11 and L-8. The key performance indices are the number of Degrees of Freedom (DoFs), the speedup factor and the maximum displacement error.

| Model     | DoFs | speedup | error [%] | DoFs | speedup | error [%] |
|-----------|------|---------|-----------|------|---------|-----------|
| HAWC2     | 120  | 1.00    | 0.00      | 216  | 1.00    | 0.00      |
| NL-11     | -    | -       | -         | 11   | 8.53    | 0.51      |
| L-11      | -    | -       | -         | 11   | 125.93  | 7.96      |
| NL-8      | 8    | 5.35    | 0.47      | 8    | 10.74   | 0.47      |
| L-8       | 8    | 73.32   | 11.77     | 8    | 145.55  | 7.94      |

the accuracy in $x$--displacement. However, for the realistic wind turbine blades such as the NREL 5MW blade, the accuracy of the nonlinear ROM in torsional direction can actually be improved by adding more modal derivative vectors as shown in the numerical results since the modal derivative vectors can represent the secondary torsional motion.

5. Conclusions
In this study, a non-intrusive method is developed to build nonlinear ROMs for wind turbine blades. The nonlinear ROM has nonlinear stiffness terms as the quadratic and cubic functions of modal amplitudes. The nonlinear stiffness terms are computed by a modified force-based non-intrusive method. HAWC2 is used as a high-fidelity geometric nonlinear solver in this study. Although similar approaches exist in the literature, it is the first time this type of method is improved and applied to wind turbine blades with large deflections up to 10% of the blade span. Case studies with a straight beam and NREL 5MW blade are carried out to demonstrate the effectiveness of nonlinear ROMs to capture the large deflection effects (secondary motions). The reduction basis of ROMs is composed of 4 lateral bending mode vectors and 4 or 7 modal derivative vectors. The results show that the proposed nonlinear ROM is 15 times more accurate than the linear ROM for NREL 5MW blade displacement results and can capture the secondary motions induced by large deflections. In terms of computation efficiency, a speedup factor of 8.5 is achieved since ROMs have much smaller model sizes than the HAWC2 models have. The speedup factor can be further improved by optimizing the implementation.

Emerging designs for 15 – 20 MW wind turbines have flexible blades longer than 100 m. Future work is to apply the proposed approach to more flexible wind turbine blades, to investigate the effects of reduction basis size on accuracy and computation time, and to incorporate the nonlinear blade ROM into the multi-body formulation. The geometric stiffening terms due to axial loads will also be added in the future.

The generated nonlinear ROMs for wind turbine blades are lightweight (i.e., small-size) models that can be used in a range of applications. We will share these nonlinear ROMs for NREL 5MW and other reference wind turbine blades in gitlab.windenergy.dtu.dk in the near future.

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