Excitation of oscillation modes by tides in close binaries: constraints on stellar and orbital parameters

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Accepted 2003 August 27. Received 2003 July 29; in original form 2003 March 17

ABSTRACT

The parameter space favourable for the resonant excitation of free oscillation modes by dynamic tides in close binary components is explored using qualitative considerations to estimate the order of magnitude of the tidal force and the frequency range covered by the tidally induced oscillations. The investigation is valid for slowly rotating stars with masses in the interval between 2 and 20 \( M_\odot \), and an evolutionary stage ranging from the beginning to the end of the main sequence. Oscillation modes with eigenfrequencies of the order of five times the inverse of the dynamical time-scale \( \tau_{\text{dyn}} \) of the star, i.e. the lowest-order \( p \)-modes, the \( f \)-mode and the lowest-order \( g^+ \)-modes, are found to be outside the favourable parameter space since their resonant excitation requires orbital eccentricities that are too high for the binary to stay detached when the components pass through the periastron of their relative orbit. Resonances between dynamic tides and \( g^+ \)-modes with frequencies of the order of half of the inverse of the dynamical time-scale of the star on the other hand are found to be favourable for orbital periods up to \( \sim 200 \tau_{\text{dyn}} \), provided that the binary mass ratio \( q \) is larger than \( 1/3 \), and the orbital eccentricity \( e \) is larger than \( \sim 0.25 \). This favourable range comes down to orbital periods of up to 5–12 d in the case of 2–20 \( M_\odot \) zero-age main-sequence binary components, and orbital periods of up to 21–70 d in the case of terminal main-sequence binary components.

Key words: binaries: close – stars: oscillations.

1 INTRODUCTION

In close binaries with non-synchronously rotating components, each star experiences the time-dependent tidal force exerted by its companion. The tidal force gives rise to forced non-radial oscillations with frequencies that are determined by the orbital period and the rotation rates of the component stars. The forcing frequencies may, at any time, come close to the eigenfrequencies of the free oscillation modes of the stars because of secular changes in the orbital parameters due to orbital and rotational angular momentum losses or because of changes in the eigenfrequencies due to stellar evolution. Such resonances lead to an enhanced tidal action and result in the excitation of the oscillation mode involved in the resonance.

The resonant excitation of free oscillation modes by the tidal action of a companion was first suggested in a seminal paper by Cowling (1941). Since then, various authors have approached the problem from both an analytical and a numerical point of view with the aim of explaining some of the evolutionary aspects and the observational characteristics of binary stars (e.g. Zahn 1970, 1975; Savonije & Papaloizou 1983, 1984; Willems, Van Hoolst & Smeyers 2003 and references therein). The subject recently received a new impetus, first from the discovery of giant planets in close orbits around solar-type stars (see Willems et al. 1997 and references therein), and subsequently from the increasingly accurate large-scale surveys of pulsating stars in close binaries (e.g. Harmanec et al. 1997; Aerts et al. 1998, 2000). The potential of tides as an excitation mechanism was furthermore found to be particularly promising for the excitation of gravity modes in hot B subdwarfs by Fontaine et al. (2003). Existing and upcoming space missions such as WIRE, MOST, MONS, COROT and Eddington are likely to contribute even further to the rising interest in tidally excited oscillation modes.

The occurrence of resonances and their effects on the evolution and the observational properties of a binary depend on the period and the eccentricity of the orbit, on the masses and the radii of the component stars, and on the properties of the oscillation mode involved in the resonance. The latter in turn depend sensitively on the internal stellar structure of the tidally distorted star. The multitude of these dependences and the complexity of their interaction with each other make it difficult to form a concise intuitive picture on the possibility of exciting non-radial oscillations by tides in close binaries. A systematic study unravelling the role of the different stellar and orbital parameters would therefore provide a valuable addition to the theory of tidally induced oscillations as well as to observational campaigns dedicated to the search of free or forced oscillations in binary stars.

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In this paper, we make a first step in an attempt to understand the parameter space of stellar and orbital parameters for which the circumstances for the resonant excitation of free oscillation modes are favourable. The primary aim is to clarify the qualitative behaviour of the acting parameters and the interaction between them without detailed calculations involving the numerical integration of the systems of differential equations governing free and forced oscillations in close binary components. The results of this study are then to serve as a starting point for detailed numerical calculations which will be presented in a follow-up investigation.

The plan of the paper is as follows. In Section 1, we present the basic assumptions and the basic ingredients of the theory of forced oscillations in close binaries. Some particular attention is paid to the Fourier decomposition of the tide-generating potential, which plays a crucial part in the investigation. In Section 2, we present a detailed analysis of the Fourier coefficients as functions of the orbital eccentricity. In Section 3, we examine the ratio of the tidal force to the gravity at the equator of the star. In Section 4, the results of the preceding sections are used to constrain the range of forcing frequencies that is favourable for the resonant excitation of free oscillation modes by dynamic tides in close binaries. In the final section, we summarize our results and present some concluding remarks.

2 BASIC CONCEPTS AND ASSUMPTIONS

Consider a close binary system of stars orbiting each other under the influence of their mutual gravitational potential. Let $P_{orb}$ be the orbital period, $a$ the semimajor axis and $e$ the orbital eccentricity. The first star, with mass $M_1$ and radius $R_1$, is assumed to be a uniformly rotating main-sequence star with a rotation axis perpendicular to the orbital plane. The magnitude of the rotational angular velocity $\Omega$ is assumed to be small in comparison to the inverse of the dynamical timescale of the star:

$$\Omega \ll \frac{1}{\tau_{dyn}} \equiv \left( \frac{GM_1}{R_1^3} \right)^{1/2}. \quad (1)$$

where $G$ is the Newtonian constant of gravitation. The second star, with mass $M_2$, is treated as a point mass.

Furthermore, let $r = (r, \theta, \phi)$ be a system of spherical coordinates with respect to an orthogonal frame of reference that is corotating with the star, and let $e_T W(r, t)$ be the potential giving rise to the tidal force. Here, $e_T$ is a small dimensionless parameter which is proportional to the ratio of the tidal force to the gravity at the equator of the star and is defined as

$$e_T = \left( \frac{R_1}{a} \right)^3 \frac{M_2}{M_1}. \quad (2)$$

(e.g. Tassoul 1987).

The tide-generating potential can be expanded in terms of unnormalized spherical harmonics $Y_{\ell}^m(\theta, \phi)$ and in Fourier series in terms of multiples of the mean motion of the companion $n = 2\pi/P_{orb}$ as

$$e_T W(r, t) = -e_T \frac{GM_1}{R_1} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{k=-\infty}^{\infty} \left( \frac{r}{R_1} \right)^{\ell} Y_{\ell}^m(\theta, \phi) \exp[i(\sigma_T t - k\tau)], \quad (3)$$

where $\sigma_T = kn + m\Omega$ is a forcing angular frequency with respect to the corotating frame of reference and $\tau$ is a time of periastron passage. The Fourier coefficients $c_{\ell,m,k}$ are determined by

$$c_{\ell,m,k} = \frac{(\ell - |m|)!}{(\ell + |m|)!} \frac{P_{\ell}^m(0)}{(\ell + |m|)!} \frac{R_1^3}{a^{\ell+2}} \frac{1}{(1 - e^2)^{\ell+1/2}} \times \frac{1}{\pi} \int_0^\pi \left( 1 + e \cos v \right)^{\ell-1} \cos(kM + mv) \, dv.$$

where $P_{\ell}^m(x)$ is an associated Legendre polynomial of the first kind, and $M$ and $v$ are, respectively, the mean and the true anomaly of the companion in its relative orbit (e.g. Pollfiet & Smeyers 1990; Smeyers, Willems & Van Hoolst 1998). The Fourier coefficients obey the property of symmetry $c_{\ell,-m-k} = c_{\ell,m,k}$ and are different from zero only for even values of $\ell + |m|$. In the particular case of a binary with a circular orbit they are different from zero only for $k = -m$.

It follows that, for each degree $\ell$ of the spherical harmonics $Y_{\ell}^m(\theta, \phi)$, the tide-generating potential generates an infinite number of partial dynamic tides with forcing angular frequencies $\sigma_T \neq 0$. When one of these forcing frequencies is close to the eigenfrequency of a free oscillation mode, the tidal action exerted by the companion is enhanced and the oscillation mode is resonantly excited. The occurrence of resonances between partial dynamic tides and free oscillation modes is particularly relevant for the excitation of free oscillation modes $g^+$ since, for binaries with short orbital periods, their eigenfrequencies may be in the range of the forcing frequencies induced by the companion.

Since the terms associated with the third- and fourth-degree spherical harmonics in the expansion of the tide-generating potential contain an additional small factor $R_1/a$ or $(R_1/a)^2$ with respect to the second-degree terms, the latter ones are usually dominant. For the remainder of the paper, we therefore restrict ourselves to the terms associated with $\ell = 2$. The only non-zero Fourier coefficients in expansion (3) of the tide-generating potential are then the coefficients $c_{2,-2,0}, c_{2,0,0},$ and $c_{2,2,2}$. These coefficients are independent of the ratio $R_1/a$ so that they are solely determined by the orbital eccentricity $e$. Since they also obey the symmetry property $c_{2,-m-k} = c_{2,m,k}$, it is furthermore sufficient to consider positive values of $k$ only.

The effects of resonant dynamic tides on the evolution and the observational characteristics of a binary depend on the strength of the tide and on the properties of the oscillation mode involved in the resonance (e.g. Smeyers et al. 1998; Willems et al. 2003). The strength of the tide is determined by the mass ratio $q = M_2/M_1$ and the orbital separation $a$ via the dimensionless parameter $e_T$, and by the orbital eccentricity $e$ via the Fourier coefficients $c_{\ell,m,k}$. The properties of the oscillation mode contribute to the effects of the resonance via the so-called overlap integral, which is proportional to the ratio of the work done by the tidal force to the kinetic energy of the mode (for a precise definition see, for example, Press & Teukolsky 1977; Kumar, Ao & Quataert 1995; Smeyers et al. 1998). For main-sequence stars, the behaviour of the overlap integral is such that the coupling between the oscillation mode and the tidal force generally weakens with increasing radial order of the mode.

In the following sections, we focus on the role of the stellar and orbital parameters affecting the strength and the frequency of the tides. The role of the internal structure of the star and the properties of the oscillation modes will be the subject of a forthcoming investigation.

3 THE ORBITAL ECCENTRICITY

The orbital eccentricity contributes to the determination of the strength of the partial dynamic tides through the Fourier coefficients
Figure 1. Left: the logarithm of the absolute value of the coefficients $c_{2,-2,k}(e)$ as a two-dimensional function of the Fourier index $k$ and the orbital eccentricity $e$. The solid lines indicate the values of $k$ and $e$ for which $|c_{2,-2,k}(e)| \approx 10^{-2}, 10^{-3}, 10^{-4}$. Right: cross-section of the variation of the coefficients $c_{2,-2,k}(e)$ for the orbital eccentricities $e = 0.4, 0.6, 0.8$.

Figure 2. As Fig. 1, but for the coefficients $c_{2,0,k}(e)$.

c_{2,m,k}$. For a given eccentricity, the coefficients generally decrease with increasing values of $k$, but the decrease is slower for higher orbital eccentricities. In addition, for a given value of $k$, the coefficients $c_{2,m,k}$ generally increase with increasing values of $e$. Hence, there are a finite number of non-trivially contributing terms in the expansion of the tide-generating potential which increases with increasing orbital eccentricities.

In order to estimate the value of $k$ beyond which the contributions to the expansion of the tide-generating potential become negligible as a function of the orbital eccentricity, the logarithms of the absolute values of the non-zero coefficients $c_{2,m,k}$ are displayed in the left-hand panels of Figs 1–3 as a two-dimensional function of $k$ and $e$. For clarity, lines for which $|c_{2,m,k}(e)| \approx 10^{-2}, 10^{-3}, 10^{-4}$ are added to the left-hand panels of the figures and one-dimensional variations of the coefficients $c_{2,m,k}$ as a function of $k$ are shown in the right-hand panels in the case of the orbital eccentricities $e = 0.4, 0.6$ and 0.8.

The smallest Fourier coefficients considered have absolute values of the order of $10^{-11}$.

As mentioned in the beginning of this section, the general tendency of the coefficients $c_{2,m,k}$ is to decrease rapidly with increasing values of $k$. The behaviour is slightly more complex for the coefficients $c_{2,-2,k}$ for which, depending on the value of the orbital eccentricity, a local minimum and a local maximum may occur prior to the rapid decrease with increasing values of $k$. This is particularly clear in the cross-sections shown in the right-hand panel of Fig. 1. A similar behaviour is observed for the coefficients $c_{2,2,k}$, albeit somewhat less pronounced. For a given value of the orbital eccentricity $e$

\[ \ln \approx \Omega_p, \text{ with } \Omega_p = n(1 + e)/(1 - e)^{3/2} \text{ the orbital angular velocity at the periastron of the relative orbit.} \]
and sufficiently large values of the Fourier index $k$, the largest contributions to the expansion of the tide-generating potential usually stem from the terms associated with the coefficients $c_{2,0,k}$, while the contributions of the terms associated with the coefficients $c_{2,2,k}$ are usually negligible. For large orbital eccentricities and small values of $k$, the dominant contributions generally stem from the terms associated with the coefficients $c_{2,0,k}$.

Although the value of $k$ beyond which the contributions to the expansion of the tide-generating potential are considered to become negligible depends on the desired accuracy of the Fourier decomposition, a reasonable estimate for a cut-off value can be obtained by observing the rapid decrease of the absolute value of the Fourier coefficients from values of the order of $10^{-2}$ to values of the order of $10^{-4}$. If we focus on the coefficients associated with $m = -2$, the maximum $k$-value that needs to be considered increases from $k \approx 5$–$10$ in the case of the orbital eccentricity $e = 0.2$ to $k \approx 25$–$45$ in the case of the orbital eccentricity $e = 0.6$. In the latter case there are thus up to five times more forcing frequencies induced in the star that may be resonant with the free oscillation modes of the star.

4 THE PARAMETER $\varepsilon_T$

The small dimensionless factor $\varepsilon_T$ in the definition of the tide-generating potential can be used as a first estimate for the order of magnitude of the tidal force exerted on a binary component by its companion. In particular, the largest term in expansion (3) of the tide-generating potential, evaluated at the surface of the star, is of the order of

$$\varepsilon_T R_1 \frac{GM_1}{M_2} \max_{m,k} |c_{2,m,k}(e)|.$$

(5)

where the maximum is taken over all admissible values of $m$ and $k$. For orbital eccentricities less than or equal to 0.8 the maximum takes values between 0.5 and 2.5 (see equation 4 and Figs 1–3). The eccentricity therefore affects the order of magnitude by less than a factor of 2.5.

Definition (2) of the small parameter $\varepsilon_T$ can be rewritten by means of Kepler’s third law as

$$\varepsilon_T = \frac{4\pi^2 R_1^3}{P_{orb}^2 GM_1} \frac{q}{1 + q}.$$  

(6)

The parameter is thus proportional to the square of the dynamic time-scale of the star and inversely proportional to the square of the orbital period. In addition, it depends on the mass ratio $q$ by the factor $q/(1 + q)$, which is always smaller than 1. In the case of a binary consisting of two $10M_\odot$ zero-age main-sequence stars (ZAMS) with an orbital period of 6 d, the parameter $\varepsilon_T$ is of the order of $10^{-3}$.

For detached binaries, an upper limit for $\varepsilon_T$ can be derived from the requirement that the radius of the star must be smaller than the volume equivalent radius of its Roche lobe. The latter may be approximated by means of Eggleton’s (1983) fitting formula

$$R_{eq} = \frac{0.49q^{-2/3}}{a^{2/3} + \ln \left(1 + q^{-1/3}\right)}.$$  

(7)

where $q = M_2/M_1$. The relative error of the approximation is smaller than 2 per cent for $0 < q < \infty$. By means of Definition (2) and equation (7), one then derives the inequality

$$\varepsilon_T < \frac{0.12}{q \left[0.6q^{-2/3} + \ln \left(1 + q^{-1/3}\right)\right]}.$$  

(8)

Taking into account the 2 per cent uncertainty in the fitting formula given by equation (7), it follows that $0 < \varepsilon_T < 0.13$.

Equation (7) for the Roche lobe radius of a binary component is strictly speaking only valid for binaries with circular orbits. In the case of a binary with an eccentric orbit, the radius of the Roche lobe varies periodically in time, with the smallest value being reached when the companion passes through the periastron of its relative orbit. An instantaneous estimate for the Roche lobe radius at a time of periastron passage is obtained by replacing the semimajor axis $a$ in equation (7) with the periastron distance $r_p = a(1 - e)$. The requirement that the star fits within its Roche lobe when the companion is located in the periastron of its relative orbit then yields

$$\varepsilon_T < \frac{0.12(1 - e)^3}{q \left[0.6q^{-2/3} + \ln \left(1 + q^{-1/3}\right)\right]}.$$  

(9)

The variation of the right-hand member of this inequality as a function of the mass ratio $q$ is illustrated in Fig. 4 for orbital eccentricities
The maximum values of the small parameter $\varepsilon_T$ for which a binary with an orbital eccentricity $e$ and a mass ratio $q$ is still detached.

The critical orbital periods separating detached from semidetached systems for $2-20\, M_\odot$ ZAMS stars and for binary mass ratios $q = 0.1$ (dashed lines) and $q = 1$ (solid lines). The grey-shaded region in between the lines associated with the two different mass ratios connects the lines associated with the same orbital eccentricity.

Figure 5. As Fig. 4, but for TMS stars.

Figure 6. The critical orbital periods separating detached from semidetached systems for $2-20\, M_\odot$ ZAMS stars and for binary mass ratios $q = 0.1$ (dashed lines) and $q = 1$ (solid lines). The grey-shaded region in between the lines associated with the two different mass ratios connects the lines associated with the same orbital eccentricity.

The eigenfrequencies of the lower-order second-degree $g^+$-modes in 2- and 20-$M_\odot$ main-sequence stars at the beginning and at the end of the main sequence are shown in Fig. 7, in units of the inverse of the dynamical time-scale of the star. The stellar models ranging from 0.0 to 0.6. It follows that, for a given value of $e$, the parameter $\varepsilon_T$ takes values in the range $0 < \varepsilon_T < 0.13(1 - e)^3$. The maximum value of $\varepsilon_T$ that can be reached by detached binaries decreases by approximately one order of magnitude when $e$ increases from 0 to 0.5, and by approximately two orders of magnitude when $e$ increases from 0 to 0.8.

Equations (6) and (9) can furthermore be combined to derive the shortest orbital period for which a binary is still detached when the components are located at the periastron of their relative orbit. The resulting critical orbital periods separating detached from semidetached systems are displayed in Figs 5 and 6 for $2-20\, M_\odot$ main-sequence stars at the beginning (ZAMS) and at the end (TMS) of the main sequence, respectively. The dashed lines represent the critical orbital periods in the case of the binary mass ratio $q = 0.1$, while the solid lines represent the critical orbital periods in the case of the binary mass ratio $q = 1$. The grey-shaded region in between the lines associated with the two mass ratios only serves as a visual aid to identify the lines associated with the same orbital eccentricity. Furthermore, we note that (for brevity) we here neglect the short phase of core contraction that takes place between the exhaustion of hydrogen in the core and the start of the Hertzsprung gap, and we refer to the stage where hydrogen is exhausted in the core as the end of the main sequence. The ZAMS and TMS stellar radii appearing in equation (6) are determined by means of the analytic approximation formulae derived by Hurley, Pols & Tout (2000).

Since the critical orbital periods separating detached from semidetached systems are proportional to the dynamical time-scale of the star (see equations 6 and 9), they increase with increasing mass of the star. For a given orbital eccentricity and a given mass ratio, the same dependence yields longer orbital periods for stars on the TMS than for stars on the ZAMS. In addition, the critical orbital periods increase with increasing values of the mass ratio $q$ because the inner Lagrangian point $L_1$ moves closer to the star with mass $M_1$. In the next section, we will use the behaviour of the critical periods to constrain the range of forcing frequencies induced by the tidal force exerted by the companion.

5 THE FORCING FREQUENCIES

In order for a free oscillation mode $g^+$ to be resonantly excited by a dynamic tide, the forcing frequency of the tide must be close to the eigenfrequency of the mode. Since the forcing frequencies depend on the orbital period and the rotational angular velocity of the tidally distorted star, the requirement that the forcing frequencies be in the range of the eigenfrequencies of the lower-order $g^+$-modes imposes constraints on the parameter space available for the occurrence resonances.

The eigenfrequencies of the lower-order second-degree $g^+$-modes in 2- and 20-$M_\odot$ main-sequence stars at the beginning and at the end of the main sequence are shown in Fig. 7, in units of the inverse of the dynamical time-scale of the star. The stellar models...
consist of a convective core and a radiative envelope and have a ZAMS chemical composition \((X, Z) = (0.7, 0.02)\). Since, for main-sequence stars, the influence of non-adiabatic effects on the oscillation frequencies is known to be small (e.g. Unno et al. 1989), the eigenfrequencies are determined in the isentropic approximation.

The frequencies of the lower-order \(g^+\)-modes typically range from 0.1 to \(10^{-1}\) days, so that the condition that the forcing frequencies be in the range of the eigenfrequencies can be expressed as

\[
kn + m\Omega \approx f_{\text{dyn}}^{-1}
\]

(10)

with \(0.1 \lesssim f \lesssim 10\). For rotational angular velocities \(\Omega \ll \tau_{\text{dyn}}^{-1}\), the \(k\)th harmonic in the expansion of the tide-generating potential may thus give rise to resonances with lower-order \(g^+\)-modes for orbital periods

\[
P_{\text{orb}} \approx \frac{2\pi k}{f_{\text{dyn}}}
\]

(11)

In order to constrain the forcing frequencies leading to significant resonances between dynamic tides and free oscillation modes, we consider equation (11) in the cases where \(f = 0.5\) and 5.0. The former choice represents the bulk of the lower-order \(g^+\)-modes displayed in Fig. 7, while the latter choice represents an upper limit on the frequency range of the \(g^+\)-modes. For presentation purposes, the two values of \(f\) are conveniently chosen to differ by a factor of 10. The choice is convenient because, for a given stellar model, the orbital periods associated with \(k\) and \(f\) are equal to those associated with 10\(k\) and 10\(f\). The variations of the resulting orbital periods as a function of the stellar mass are displayed by the solid lines in Figs 8 and 9, for stars at the beginning and at the end of the main sequence, respectively. The value of \(k\) corresponding to each line is indicated in the tables on the right-hand side of the figures. It follows that, for ZAMS stars in binaries with orbital periods of the order of a few days, the lower-order harmonics in expansion (3) of the tide-generating potential give rise to forcing frequencies of the order of \(\sim 0.5\tau_{\text{dyn}}^{-1}\), while higher-order harmonics give rise to forcing frequencies of the order of \(\sim 5.0\tau_{\text{dyn}}^{-1}\). For stars on the TMS, the corresponding orbital period range increases to a few tens of days due to the longer dynamical time-scales associated with these stars.

Since the small parameter \(\varepsilon_T\) appearing in the definition of the tide-generating potential is inversely proportional to the square of the orbital period, the tidal force decreases rapidly with increasing orbital separations. From equations (6) and (11), one derives that the values of \(\varepsilon_T\) associated with the orbital periods represented by the solid lines in Figs 8 and 9 are given by

Figure 8. The orbital period range that may give rise to resonances with lower-order \(g^+\)-modes in 2–20 \(M_\odot\) ZAMS stars. The solid lines correspond to the orbital periods obtained from equation (11) by setting \(f = 0.5\) and \(k = 1, 2, 3, \ldots, 13\), or \(f = 5.0\) and \(k = 10, 20, 30, \ldots, 130\). For each value of \(k\), the table on the right-hand side of the figure lists the minimum values of the orbital eccentricity \(e_{-2}\) and \(e_{-4}\) for which \(|\varepsilon_{-2}(e)| \gtrsim 10^{-2}\) and \(|\varepsilon_{-4}(e)| \gtrsim 10^{-4}\), respectively. The grey-shaded background indicates the orbital periods for which a binary with a mass ratio \(q = 1\) and an orbital eccentricity \(e = 0.5\) becomes semidetached when the stars pass through the periastron of their relative orbit.
For binaries with mass ratio \( q = 1 \), the parameter \( \varepsilon_T \) takes the value \( 10^{-2} \) when \( f = 0.5 \) and \( k \approx 3-4 \) (or \( f = 5.0 \) and \( k \approx 35-36 \)) and decreases to \( 10^{-3} \) when \( f = 0.5 \) and \( k \approx 11-12 \) (or \( f = 5.0 \) and \( k \approx 111-112 \)). Smaller mass ratios yield smaller values of \( k \).

As discussed in Section 3, the rapid decrease of the Fourier coefficients \( c_{2,m,k} \) in the expansion of the tide-generating potential puts an upper limit on the maximum value of \( k \) that needs to be considered for a given value of the orbital eccentricity \( e \). Vice versa this implies that for each value of \( k \) there is a minimum orbital eccentricity for which the term associated with the Fourier coefficient \( c_{2,m,k} \) provides a non-negligible contribution to the expansion of the tide-generating potential. We determined this minimum value for each value of \( k \) listed in the tables on the right-hand side of Figs 8 and 9. To this end, we restricted ourselves to the terms associated with the azimuthal number \( m = -2 \) since they generally provide the largest contributions. We furthermore adopted two different thresholds to separate contributing from non-contributing terms: \( |c_{2,-2,2}(e)| \approx 10^{-2} \) and \( |c_{2,-2,2}(e)| \approx 10^{-4} \) (cfr. the solid lines in Fig. 1). The resulting values of \( e \) are listed in the columns labelled \( e_{-2} \) and \( e_{-4} \), respectively.

The table shows, for example, that the minimum orbital eccentricity required for the absolute value of the Fourier coefficient \( c_{2,-2,2} \) to be larger than or equal to \( 10^{-2} \), and thus for the associated term in the expansion of the tide-generating potential to be non-negligible, is \( e = 0.17 \). When the absolute value of the Fourier coefficient \( c_{2,-2,2} \) is required to be larger than or equal to \( 10^{-4} \), the condition on the orbital eccentricity is relaxed to \( e \gtrsim 0.04 \).

The high-order harmonics required to reach forcing frequencies of the order of \( \sim 5.0 \tau_{\text{dyn}}^{-1} \) need very high orbital eccentricities in order for the associated terms to contribute non-trivially to the tide-generating potential. These high orbital eccentricities in combination with short orbital periods yield small periastron distances that may cause the star to overflow its Roche lobe when it passes through the periastron of its relative orbit. The critical orbital periods separating detached from semidetached systems were derived in Section 4 and presented in Figs 5 and 6 for different combinations of the binary mass ratio \( q \) and the orbital eccentricity \( e \). For illustration, the orbital periods leading to semidetached systems in the case of the binary mass ratio \( q = 1 \) and the orbital eccentricity \( e = 0.5 \) are displayed in Figs 8 and 9 by means of the grey-shaded background. In this example, the critical orbital periods separating detached from semidetached systems increase from \( \sim 1.5 \) d for a 2-M\(_{\odot}\) ZAMS star to \( \sim 3 \) d for a 20-M\(_{\odot}\) ZAMS star, and from \( \sim 5 \) d for a 2-M\(_{\odot}\) TMS star to \( \sim 20 \) d for a 20-M\(_{\odot}\) TMS star. The minimum orbital eccentricity \( e = 0.52 \) required to have a non-negligible contribution to the tide-generating potential from the terms associated with \( k \)-values up to 30 can therefore not be reached by short-period systems without becoming semidetached at the periastron. From Figs 5 and 6, it follows that the same holds true for the other high-order harmonics required to reach forcing frequencies of the order of \( \sim 5.0 \tau_{\text{dyn}}^{-1} \). The situation improves somewhat for binaries with mass ratios \( q \) smaller than unity, but even for \( q = 0.1 \) the systems can barely avoid a semidetached state when the stars are located at the periastron of their relative orbit.

Next, we turn our attention to the forcing frequencies of the order of \( 0.5 \tau_{\text{dyn}}^{-1} \) associated with the lower-order harmonics in expansion (3) of the tide-generating potential. These harmonics provide non-negligible contributions to the tidal potential for low to moderate values of the orbital eccentricity, so that the occurrence of Roche lobe overflow at the periastron is less of an issue.

In Section 4, we have shown that the order of magnitude of any given term in the expansion of the tide-generating potential is not only determined by the orbital eccentricity \( e \), but also by the parameter \( \varepsilon_T \), which is proportional to the ratio of the tidal force to the gravity at the equator of the star. Since the parameter \( \varepsilon_T \) is the dominant factor in binaries with low to moderate orbital eccentricities, we neglect the role of the eccentricity in what follows and, somewhat arbitrarily, set the limit of what we consider as favourable conditions for the resonant excitation of free oscillation modes to \( \varepsilon_T \gtrsim 10^{-3} \). In the case of a binary consisting of two 5-M\(_{\odot}\) ZAMS stars, this corresponds to an upper limit on the orbital period of \( \sim 5 \) d. A similar upper limit for the occurrence of noticeable resonances between dynamic tides and free oscillation modes is inferred from the tidally induced radial velocity variations presented by Willems.

\[ \varepsilon_T = \frac{f^2 - q}{k^2 (1 + q)}. \] (12)
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& Aerts (2002) and the apsidal-motion calculations performed by Smeyers & Willems (2001).

The maximum value of \( k \) for which the orbital periods represented by the solid lines in Figs 8 and 9 give rise to forcing frequencies of the order of \( 0.5 \tau_{\text{dyn}}^{1/3} \) and for which \( \epsilon_\tau \lesssim 10^{-3} \) are obtained from equation (12) as

\[
k_{\text{max}} \approx 15.8 \left( \frac{q}{1 + q} \right)^{1/2}.
\]

The corresponding orbital periods are

\[
P_{\text{orb.max}} \approx 198.7 \tau_{\text{dyn}} \left( \frac{q}{1 + q} \right)^{1/2}.
\]

In the case of a binary with a mass ratio \( q = 1 \), the upper limits take the values \( k_{\text{max}} \approx 11 \) and \( P_{\text{orb.max}} \approx 140 \tau_{\text{dyn}} \). They change by less than a factor of \( \sqrt{2} \) when \( \frac{1}{2} < q < \infty \), but decrease rapidly for binary mass ratios smaller than \( \frac{1}{2} \). The minimum orbital eccentricity required for the terms associated with \( k \approx 11 \) to provide a non-negligible contribution to the tide-generating potential is \( e \approx 0.25 \sim 0.4 \). The longest orbital periods thus still require substantial orbital eccentricities in order to be able to excite oscillation modes with eigenfrequencies of the order of \( 0.5 \tau_{\text{dyn}}^{1/3} \).

The results shown in Figs 8 and 9 can furthermore be extrapolated to resonances between dynamic tides and free oscillation modes with eigenfrequencies smaller than \( 0.5 \tau_{\text{dyn}}^{1/3} \). From equation (11) it follows that the orbital periods giving rise to such resonances increase with decreasing order of magnitude of the eigenfrequencies, so that the condition \( \epsilon_\tau \gtrsim 10^{-3} \) becomes more and more stringent until it can no longer be fulfilled for any value of \( k \) and \( q \). The smallest multiple \( f \) of the dynamical time-scale of the star for which the orbital periods resulting from equation (11) yield values of \( \epsilon_\tau \) larger than \( 10^{-3} \) is given by

\[
f_{\text{min}} \approx 0.03 \left( \frac{1 + q}{q} \right)^{1/2}
\]

(see equation 12). For binaries with a mass ratio \( q = 1 \), this imposes a lower limit of \( \sim -0.05 \tau_{\text{dyn}}^{1/3} \) on the frequency range favourable for resonances between dynamic tides and free oscillation modes. In the case of a binary with a mass ratio \( q = 0.1 \), the lower limit increases to \( \sim -0.1 \tau_{\text{dyn}}^{1/3} \). The conditions for the resonant excitation of free oscillation modes \( g^+ \) thus rapidly become less favourable with decreasing order of magnitude of the involved frequencies.

So far, we have only considered slowly rotating stars for which \( \Omega \ll \tau_{\text{dyn}}^{1/3} \). In more rapidly rotating stars the rotational angular velocity \( \Omega \) may significantly affect the determination of the forcing angular frequencies induced by the companion. Resonances with eigenfrequencies of the order of \( f \) times the inverse of the dynamical time-scale of the star then occur for orbital periods

\[
P_{\text{orb}} \approx \frac{2\pi k}{f - m2\pi \tau_{\text{dyn}}},
\]

For \( m = 0 \), the equation reduces to equation (11) so that the results presented in Figs 8 and 9 generally remain valid. However, since the coefficients \( c_{2,0,4} \) decrease more rapidly with increasing values of \( k \) than the coefficients \( c_{2,-2,4} \) the associated terms in the expansion of the tide-generating potential require a higher orbital eccentricity in order to provide a non-negligible contribution. For \( m = -2 \), on the other hand, the periods represented by the solid lines in Figs 8 and 9 decrease with increasing rotational angular velocities \( \Omega \). Consequently, for both \( m = 0 \) and \( -2 \), the upper limit of \( 5.0 \tau_{\text{dyn}}^{1/3} \) on the frequency range that is favourable for the resonant excitation of free oscillation modes becomes even more stringent for rapidly rotating stars than it was for more slowly rotating stars. The upper limit on the range of orbital periods favourable for the resonant excitation of oscillation modes with eigenfrequencies of the order of \( 0.5 \tau_{\text{dyn}}^{1/3} \) is not affected by the rotational angular velocity \( \Omega \), although the value of \( k \) for which the upper limit is reached increases with increasing values of \( \Omega \). Our qualitative results thus remain valid for rotation rates that are somewhat higher than implied by equation (1). In the case of very fast rotators, the situation becomes significantly more complicated since the forcing frequencies may take negative as well as positive values. We refer to Willems & Claret (2003) for a more detailed account on the behaviour of the forcing frequencies in such stars.

Finally, we note that for a rotating star the frequencies and properties of \( g^+ \)-modes with eigenfrequencies of the order of the rotational angular velocity \( \Omega \) may be significantly altered with respect to those of a non-rotating star. In particular, stellar rotation lifts the degeneracy of the eigenfrequencies with respect to the azimuthal number \( m \), so that more modes become available that may be resonantly excited (e.g. Unno et al. 1989). Stellar rotation furthermore induces additional low-frequency modes known as inertial modes and \( r \)-modes which have frequencies of the order of the rotational angular velocity of the star \( \Omega \). The resonant excitation of these modes may lead to comparably important effects as the resonant excitation of free oscillation modes \( g^+ \) (Rocca 1982; Savonije & Papaloizou 1997; Papaloizou & Savonije 1997).

6 CONCLUDING REMARKS

In this paper, we initiated a systematic study aimed at exploring the range of stellar and orbital parameters leading to favourable conditions for the resonant excitation of \( g^+ \)-modes by tides in close binary components. We considered slowly rotating stars with masses between 2 and 20 M\(_{\odot}\), and evolutionary stages at the beginning and at the end of the main sequence.

A crucial ingredient in the investigation is the expansion of the tide-generating potential in Fourier series in terms of the mean Keplerian motion. The expansion induces an infinite number of forcing frequencies in the star which may be close to the eigenfrequencies of the free modes of oscillation of the star. In practice, the rapid decrease of the Fourier coefficients limits the number of forcing frequencies to a finite number that increases with increasing values of the orbital eccentricity. The number can be derived from the variations of the dominant Fourier coefficients shown in Figs 1–3.

For given orbital and rotational periods, the finite number of non-trivial terms in the expansion of the tide-generating potential associated with a given value of the orbital eccentricity puts an upper limit on the forcing frequencies induced in a star, which in turn puts an upper limit on the eigenfrequencies of the oscillation modes that may be resonantly excited. The limit is of the order of five times the inverse of the dynamical time-scale of the star and is particularly stringent for the \( f \)-mode and the lowest-order \( g^+ \)-modes in evolved low-mass main-sequence stars (see Fig. 7). It also contributes to the difficulties in exciting \( p \)-modes by tides in close binaries.

Resonances between dynamic tides and \( g^+ \)-modes with frequencies of the order of half the inverse of the dynamical time-scale of the star on the other hand are constrained by the rapid decrease of the strength of the tidal force with increasing orbital separations. When the limit separating favourable from less favourable conditions is set at \( \epsilon_\tau \approx 10^{-3} \), where \( \epsilon_\tau \) is a measure for the ratio of the tidal force to the gravity at the equator of the star, the highest-order harmonics in the expansion of the tide-generating potential that may give rise...
to such resonances are given by equation (13). This corresponds to an upper limit on the orbital period given by equation (14). For oscillation modes $g^+$ with eigenfrequencies much smaller than half the inverse of the dynamical time-scale of the star, the conditions for resonant excitation quickly become less favourable. However, these very low frequencies are only reached by $g^+$-modes of very high radial order, so that this behaviour does not impose severe constraints on the parameter space available for the resonant excitation of free oscillation modes.

ACKNOWLEDGMENTS

I express my sincere thanks to Antonio Claret for providing a set of theoretical stellar models and to Jarrod Hurley, Onno Pols and Chris Tout for sharing their SSE software package. The anonymous referee is acknowledged for useful remarks which led to an improvement of the paper. This research was supported by the British Particle Physics and Astronomy Research Council (PPARC) and made use of NASA’s Astrophysics Data System Bibliographic Services.

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