A Note on CP Invariance and Rephasing Invariance

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Abstract

CP invariance condition and rephasing invariance in KM scheme of CP violation is discussed. The CP violation measure in B physics, corresponding to the three angles of unitarity triangle are given in rephasing invariant form. With the obtained results it is shown that the sum of the three CP angles to be measured at B factories becomes $\pi$ even if the Kobayashi-Maskwa matrix is not a $3 \times 3$ unitary matrix under reasonable assumptions.

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1 CP invariance

It is often said that CP invariance holds if all the coupling constants in the theory are real. This is true, but simultaneously we can make some of coupling constants complex by rephasing complex fields without changing physics. Therefore more accurate definition of CP invariance is necessary in particular when we deal with the rephasing invariance of physical quantities. In this note we discuss the CP violation scheme of Kobayashi-Maskawa (KM) by taking the freedom of rephasing quark fields into account, and give rephasing invariant expressions of the CP violation measures at B meson system, so called the CP angles of the unitarity triangle, in the standard model and its extensions.

CP transformation of a quark field $q$ is defined as

$$(CP)q(CP)^{-1} = e^{i\tilde{q}\gamma^0}qC = e^{i\tilde{q}\gamma^0}C(\bar{q})^T,$$

where $C$ is the charge conjugation matrix and $e^{i\tilde{q}}$ is a phase factor which in general can be taken arbitrarily and depend on the field. The interaction among quark charged currents and $W$ boson is given in quark mass eigenstates as

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \left[ u_{Li} \gamma^\mu (V)_{ij} d_{Lj} W^+_{\mu} + d_{Lj} \gamma^\mu (V^*)_{ij} u_{Li} W^-_{\mu} + (h.c.) \right],$$

where $i, j$ are the flavor indices and $V$ is KM matrix. By the CP transformation given in eq. (1) and $(CP)W^\pm(CP)^{-1} = -W^\pm$ we have

$$(CP)\mathcal{L}_W(CP)^{-1} = \frac{g}{\sqrt{2}} \left[ d_{Lj} \gamma^\mu (V)_{ij} e^{i(d_{Lj} - \tilde{u}_{Li})} u_{Li} W^{-\mu} + (h.c.) \right].$$

We take $\tilde{q}_L = \tilde{q}_R \equiv \tilde{q}$ so that the mass term might be CP invariant, then CP invariance requires that the following conditions should hold

$$(V)_{ij} = (V^*)_{ij} e^{-i(\tilde{d}_j - \tilde{u}_i)},$$

by suitably choosing the CP phases $\tilde{q}$. If we take $\tilde{u}_i = \tilde{d}_j$ for any quark flavor, the condition of CP invariance becomes that $V_{KM}$ should be real, the usual one. On the contrary CP invariance means $V$ is pure imaginary if we take $\tilde{u}_i = \tilde{d}_j + \pi$. CP violation occurs when any choice of $\tilde{q}$ cannot satisfy the condition (4). Say in other words, if we can find a set of $\tilde{q}$ which satisfy the condition (4), we can make KM matrix real by the redefinition of the phases of quarks and the theory is CP invariant in the usual manner, which shall be explained in the next section.
2 Rephasing

Let us consider redefinition of the phases of quarks;

\[ q' = e^{-i\hat{q}}q, \]  

(5)

where the rephasing angle \( \hat{q} \) is arbitrary and depends on the filed. Note that \( \hat{q} \) is independent of the CP phase \( \tilde{q} \) discussed before. Physics does not change under this rephasing. The Lagrangian (3) becomes by rephasing as follows;

\[ L_W = \frac{g}{\sqrt{2}} [u'_L i \gamma^\mu (V'_i)_{ij} d'_L W^\mu_j + \text{(h.c.)}], \]  

(6)

where the transformed KM matrix element is given as \( (V'_i)_{ij} = e^{i(d_j - \tilde{u}_i)} (V^*)_{ij} e^{-i(d_i - \tilde{u}_j)} = (V'^*)_{ij} e^{-i((d_j - 2d_i) - (\tilde{u}_i - 2\tilde{u}_j))}. \)  

(7)

So we should change the CP phase \( \tilde{q} \) to \( \tilde{q} - 2\hat{q} \) under the rephasing of quarks. If the CP invariant condition (4) is satisfied, KM matrix can be made real by the rephasing of the quark fields with \( \hat{q} = \frac{\tilde{q}}{2} \).

3 Rephasing invariant expressions of CP angles

In the case of 3 generations KM matrix is a \( 3 \times 3 \) unitary matrix so that the following condition holds:

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \]  

(8)

We have so-called unitarity triangle by expressing the above condition in complex plane. The parameters in KM matrix can be determined through the measurements of the sides and the angles of this triangle. We can test the accuracy of the 3 generation standard model by over-checking the consistency of the triangle. If the test fails, we can explore new physics beyond the standard model from the inconsistency[3]. This is one of the main aims of the B factory projects at KEK[4] and SLAC[5].

The angles of the unitarity triangle are geometrically defined as

\[ \phi_1 (\beta) \equiv \arg[V_{cb}^* V_{cd}] - \arg[V_{tb}^* V_{td}] - \pi, \]  

(9)

\[ \phi_2 (\alpha) \equiv \arg[V_{tb}^* V_{td}] - \arg[V_{ub}^* V_{ud}] + \pi, \]  

(10)

\[ \phi_3 (\gamma) \equiv \arg[V_{ub}^* V_{ud}] - \arg[V_{cb}^* V_{cd}] + \pi. \]  

(11)

These angles do not necessarily agree with the CP angles to be measured in experiments, which we call \( \tilde{\phi}_i (i = 1, 2, 3) \) below, if a new physics contributes significantly to CP
violation. New physics can affect the CP angles through (i) $B^0 - \bar{B}^0$ mixing as in the case of SUSY standard models\[6\], (ii) $b$ decay as in $SU(2)_L \times SU(2)_R \times U(1)$ models\[7\] or (iii) deformation of KM matrix as in 4 generation models\[8\] or extra vector-like quark models\[9\]. Let us see the differences between $\phi_i$ and $\tilde{\phi}_i$ ($i = 1, 2, 3$) under the following assumptions;

* Quark decay amplitude has the same phase as those given by the corresponding tree level $W$ boson exchange diagram up to minor corrections except for the $\Delta I = 1/2$ penguin type contribution.

* $D$ meson decay is dominated by the tree level $W$ boson interaction with negligible CP violation.

Without the first assumption the CP angles will become irrelevant to KM matrix elements and the details of new physics should be necessary for discussion. The second one is necessary because we use D meson decay to obtain $\tilde{\phi}_3$ at B factories.

### 3.1 $\phi_1$ ($\beta$)

The CP angle $\tilde{\phi}_1$ corresponding to $\phi_1$ ($\beta$) can be measured through time dependent CP asymmetry of the neutral $B$ meson decay into a CP eigenstate, $J/\Psi K_S$ \[10\]:

$$\text{Asy}[J/\Psi K_S] \equiv \frac{\Gamma[B^0(t) \to J/\Psi K_S] - \Gamma[\bar{B}^0(t) \to J/\Psi K_S]}{\Gamma[B^0(t) \to J/\Psi K_S] + \Gamma[\bar{B}^0(t) \to J/\Psi K_S]}$$

$$= \frac{2}{(2 + c_d)} \left[ \text{Im}(\frac{q}{\rho}) \sin(\Delta M_B t) - \frac{c_d}{2} \cos(\Delta M_B t) \right], \quad (13)$$

Figure 1: Unitarity triangle
where
\[ \frac{q}{p} \equiv \frac{|M_{B_12}|}{M_{B_12}}, \quad M_{B_12} \equiv \langle B^0| H^{\Delta B=2}|B^0 \rangle, \]
\[ \rho \equiv \frac{A(B^0 \to J/\Psi K_S)}{A(B^0 \to J/\Psi K_S)}, \quad |\rho|^2 \equiv 1 + c_d, \]
and we have neglected the absorptive part of \( \langle B^0| H^{\Delta B=2}|B^0 \rangle \), which is a good approximation in B meson system. The assumption given before allows us to express \( \rho \) by KM matrix elements.

\[ \text{Im}(\frac{q}{p}) = \text{Im} \left[ \frac{|M_{B_12}|}{M_{B_12}} V_{cb}V_{cs}^* \left( \frac{q_K}{p_K} \right)^* \right], \]

where
\[ \frac{q_K}{p_K} = \frac{[(M_{K_12}^K - (i/2)\Gamma_{K_12}^K)(M_{K_12}^{K*} - (i/2)\Gamma_{K_12}^{K*})]^{1/2}}{M_{K_12}^K - (i/2)\Gamma_{K_12}^K}, \]
with \( M_{K_12}^K - (i/2)\Gamma_{K_12}^K \equiv \langle K^0| H^{\Delta S=2}|K^0 \rangle \). It is experimentally known that CP violation in K meson system is tiny, \( O(10^{-3}) \), so that we neglect it here. Then we can take \( M_{K_12}^K/\Gamma_{K_12}^K \) to be real, and
\[ \frac{q_K}{p_K} = \frac{|\Gamma_{K_12}^K|}{\Gamma_{K_12}^K} = V_{ud}V_{us}^* \frac{V_{ud}}{V_{us}}. \]

The phase of \( \Gamma_{K_12}^K \) is calculated from W boson exchange tree decay diagram since we neglected CP violation in K meson system. Let us define the phase discrepancy between the KM factors as
\[ \delta_1 \equiv \text{arg}[V_{ud}V_{us}^*] - \text{arg}[V_{cd}V_{cs}^*] + \pi, \]
where \( \delta_1 = O(10^{-3}) \) in the 3 generation standard model. We have
\[ \frac{\text{Asy}[J/\Psi K_S]}{\sin(\Delta M_B t)} = \text{Im} \left[ \frac{|M_{B_12}^B|}{M_{B_12}^B} V_{cb}V_{cs}^* V_{cd}V_{cs}^* \right] e^{-2i\delta_1} = -\sin(\phi_M + 2\phi_c + 2\delta_1), \]
where \( \phi_M \equiv \text{arg}[M_{B_12}^B], \ \phi_c \equiv \text{arg}[V_{cb}V_{cd}^*]. \) In the case of the 3 generation standard model \( \phi_M = -2\text{arg}[V_{cb}^*V_{td}] \), so that the righthand-side of eq.(20) becomes \(-\sin 2\phi_1\) up to tiny correction of \( \delta_1 \). If a new physics affects \( \phi_M \) or \( \delta_1 \), then the CP angle to measure,
\[ \tilde{\phi}_1 = \frac{1}{2} \phi_M + \phi_c + \delta_1 - \pi \ (\text{mod} \ \pi), \]
can deviate from the geometrical angle, \( \phi_1 \). This CP angle \( \tilde{\phi}_1 \) is rephasing invariant because \( M_{B_12}^B \rightarrow e^{2i(b-d)}M_{B_12}^B \) under the redefinition of quark phases.
3.2 $\phi_2 (\alpha)$

The $CP$ angle $\bar{\phi}_2$ is measured in a similar manner as $\bar{\phi}_1$ by using $b(\bar{b}) \to u\bar{d}(d)$ decay. A typical $CP$ eigenstate is $\pi\pi$. There is a $\Delta I = 1/2$ penguin contribution in this decay mode. But it can be removed by isospin analysis. The weak phase of the resulting decay amplitude is controlled by KM matrix elements following the assumption. The $CP$ asymmetry is given by

$$\text{Im}(\frac{q}{p})_{\pi\pi} = -\text{Im}\left[\frac{|M_{12}|}{|M_{12}|} V_{ub} V_{ud}^*\right] = \text{Im}(\phi_M + 2\phi_u),$$

(22)

where $\phi_u \equiv \arg[V_{ub}^* V_{ud}]$. In the case of the standard model the righthand-side of eq.(22) becomes $\sin[2(\pi - \phi_2)] = -\sin 2\phi_2$. With new physics effects on $M_{12}$, the $CP$ angle to measure,

$$\tilde{\phi}_2 = -\frac{1}{2} \phi_M - \phi_u + \pi \text{ (mod } \pi),$$

(23)

can deviate from $\phi_2$. This $\tilde{\phi}_2$ is also rephasing invariant as it should be.

3.3 $\phi_3 (\gamma)$

The rest of the $CP$ angles $\bar{\phi}_3$ is to be measured at B factories from the decays $B^\pm \to \{D^0, D^0, D_{CP}\} K^\pm$ or $B^0(\bar{B}^0) \to \{D^0, D^0, D_{CP}\} K_S$ [12], where $D_{CP}$ is a $CP$ eigenstate of neutral $D$ meson. The $CP$ eigenstate $D_{CP}$ is given generally as

$$D_{CP} \equiv e^{i\xi_D} (D_0^0 \pm e^{i\varphi_D} \bar{D}_0^0)/\sqrt{2},$$

(24)

by defining $CP|D^0_0 = e^{i\varphi_D}\bar{D}^0_0$. The overall phase $e^{i\xi_D}$ is irrelevant, so omitted below. The meson $CP$ phase $e^{i\varphi_D}$ can be taken arbitrary, but changes its value under the rephasing of quarks. To see this let us take

$$|D^0_0 \sim (\bar{c}\gamma_5 u)|0\rangle, \quad |\bar{D}^0_0 \sim e^{i\chi}(\bar{u}\gamma_5 c)|0\rangle,$$

(25)

where $\sim$ means that r.h.s and l.h.s have the same quantum number. The phase $e^{i\chi}$ can be taken arbitrary. Then we have

$$CP|D^0_0 \sim -e^{i(\bar{u} - \bar{c})} (\bar{u}\gamma_5 c)|0\rangle \sim e^{i(\bar{u} - \bar{c} - \chi + \pi)}|\bar{D}^0_0\rangle.$$

(26)

So the meson $CP$ phase given by $\phi_D = \bar{u} - \bar{c} - \chi + \pi$ has another arbitrary factor $\chi$ other than quark $CP$ phases. It varies under rephasing as $\phi'_D = \phi_D + 2(\bar{c} - \bar{u})$ as quark $CP$ phases transform.

The decay amplitudes of $B^+ \to D^0 K^+$ and $B^+ \to \bar{D}^0 K^+$ are written as

$$A(B^+ \to D^0 K^+) = V_{ub} V_{cs} a_D e^{i\delta_D},$$

(27)

$$A(B^+ \to \bar{D}^0 K^+) = V_{cb} V_{us} a_D e^{i\delta_D},$$

(28)
where the KM matrix elements and strong phase shifts ($\delta_D, \bar{\delta}_D$) are factored out. Note that the rest of the amplitudes ($a_D, a_{\bar{D}}$) are not necessarily real depending on the convention \(23\).

\[ e^{-i\varphi_D} \frac{a_{\bar{D}}}{a_D} = -e^{-i(\bar{u}-\bar{c})} \times \text{(real factor)}. \]  

(29)

The partial decay width of $B^+$ decay into $K^+$ and CP even eigenstate of $D$ meson is given as

\[
\Gamma(B^+ \to D_{CP}+K^+) = \frac{1}{2} [\Gamma(B^+ \to D^0 K^+) + \Gamma(B^+ \to \bar{D}^0 K^+)] \\
+ \int d\Gamma |V_{ub}V_{cs}|^2 \text{Re} \left[ \frac{V_{cb}^* V_{us} V_{cs}}{V_{cb}^* V_{us} a_D a_{\bar{D}} e^{-i(\varphi_D + \delta_s)}} \right],
\]

(30)

where $\delta_s = \delta_D - \bar{\delta}_D$. The final term depends on

\[
\arg \left[ \frac{V_{cb}^* V_{us}}{V_{cb}^* V_{us}} e^{-i(\bar{u}-\bar{c}-\delta_s)} \right],
\]

(31)

which is rephasing invariant. Only $V_{ub}$ and $e^{i\delta s}$ have non-negligible phases in the 3 generation standard model with vanishing quark CP phase convention ($\bar{q} = 0$) and the Wolfenstein parameterization of KM matrix[13], so we can get the angle $\phi_3$ by combining this with CP conjugate decays. If the condition (4) is satisfied, the weak phase vanishes in the above expression (31) and we have no CP violation. But it seems strange that the physical measure of CP violation (31) depends on the arbitrary quark CP phases, $\bar{u}$ and $\bar{c}$, in the general phase convention. To solve this paradox let us remember how we identify the $D$ meson CP eigenstate in experiment. The CP even eigenstate can be identified by its decay into a $K$ meson pair or a $\pi$ meson pair. Let us consider the amplitude of CP odd eigenstate of $D$ meson decaying into a $K$ meson pair;

\[
A(D_{CP-} \to K^+ K^-) \propto \left[ \frac{V_{cs}^* V_{us}^*}{V_{cs} V_{us}} e^{i(\bar{c}-\bar{u})} - 1 \right],
\]

(32)

because

\[
\langle K^+ K^- |\mathcal{H} |D^0 \rangle = \langle K^+ K^- |(CP)^{-1}(CP)\mathcal{H}(CP)^{-1}(CP) |D^0 \rangle \\
= \langle K^+ K^- |\mathcal{H}^{CP} |D^0 \rangle e^{i\varphi_D} = \frac{V_{cs}^* V_{us}^*}{V_{cs} V_{us}} e^{i\varphi_D} e^{i(\bar{c}-\bar{u})}\langle K^+ K^- |\mathcal{H} |D^0 \rangle,
\]

(33)

for $\mathcal{H} \propto V_{cs}^* V_{us}^* \gamma^\mu \gamma^5 c_L \bar{u}_L \gamma^\mu s_L + \text{(h.c.)}$. The amplitude (32) should vanish under our assumption of negligible CP violation in $D$ decay, then we have

\[
e^{i(\bar{c}-\bar{u})} = \frac{V_{cs} V_{us}^*}{V_{cs}^* V_{us}}.
\]

(34)
Now the weak phase in the formula (31) is given as

\[-\arg \left[ \frac{V_{cb}^* V_{us} V_{cs} V_{us}^*}{V_{ub}^* V_{cs} V_{cs} V_{us}} \right] = -\arg [(V_{cb}^* V_{cd})(V_{ub} V_{ud}^*)(V_{us}^* V_{ud})(V_{cs} V_{cs}^*)] \]

\[= \phi_u - \phi_c - \delta_1 + \pi, \tag{35} \]

which becomes the CP angle $\tilde{\phi}_3$ to measure. When a $\pi$ meson pair is used to identify $D_{CP}$, the results becomes as follows

\[-\arg \left[ \frac{V_{cb}^* V_{us} V_{cd} V_{ud}^*}{V_{ub}^* V_{cs} V_{cd} V_{ud}} \right] = -\arg [(V_{cb}^* V_{cd})(V_{ub} V_{ud}^*)(V_{us} V_{ud}^*)(V_{cs}^* V_{cd})] \]

\[= \phi_c - \phi_u + \delta_1 - \pi. \tag{36} \]

These two results differ by $2\delta_1$ (mod $2\pi$). But as far as our assumption holds, it should be negligible.

4 Concluding remarks

We have discussed CP invariance condition and rephasing invariance in KM scheme of CP violation in this note without assuming the 3 generation standard model. The CP violation measure in B physics, corresponding to the three angles of unitarity triangle are given in rephasing invariant fashion as follows;

\[\tilde{\phi}_1 = \frac{1}{2} \phi_M + \phi_c + \delta_1 - \pi, \tag{37} \]

\[\tilde{\phi}_2 = -\frac{1}{2} \phi_M - \phi_u + \pi, \tag{38} \]

\[\tilde{\phi}_3 = \phi_u - \phi_c - \delta_1 + \pi. \tag{39} \]

where $\phi_x = \arg[V_{xb}^* V_{xd}]$, $\phi_M = \arg[\langle B^0 | H^{D=2} | B^0 \rangle]$, $\delta_1 = \arg[V_{ud} V_{us}^*] - \arg[V_{cd} V_{cs}^*] + \pi$, and $D^0, \bar{D}^0 \rightarrow K$ meson pair is used in obtaining $\tilde{\phi}_3$. The sum of these three angels becomes $\pi$ though we have not assumed 3 generation. Therefore, we should use the informations of sides also to explore a new physics. One more interesting point is that the measure $\tilde{\phi}_3$ can differ depending on what mode is used to identify neutral D meson. The difference might be observable in a future experiment with a new physics which can contribute significantly CP violation in D meson system. A quantitative discussion of this point will be given elsewhere.\[14\].
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