Thermal radiation from lorentzian traversable wormholes

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Abstract. In this contribution we show that lorentzian dynamic wormholes emit thermal phantom-like radiation. Analogously to as it occurs for black holes, the consideration of such radiation process allows the formulation of a wormhole thermodynamics which might help in the understanding of those objects.

1. Introduction
It is well known that the universe is currently undergoing a period of accelerating expansion and, in the framework of general relativity, this phenomenon is only understandable if dark energy dominates the universal dynamics. It is even possible that the dark fluid violates the null energy condition [1], being the equation of state parameter \( w < -1 \) (with \( w = p/\rho \)). As recent studies have shown, this strange fluid, dubbed phantom energy [2], has negative temperature [3].

On the other hand, lorentzian traversable wormholes are solutions of general relativity, describing short-cuts between two regions of the same universe or connections between two universes [4]. The problem with these solutions was that, in order to be traversable, they must be supported by some stuff violating the null energy condition, which had been called exotic matter. Nevertheless, this property is precisely the same which fulfils phantom energy. In fact, it has been shown that an inhomogeneous version of phantom energy can be the exotic stuff which is required to support wormholes [5].

Due to the possible existence of wormholes in our Universe, a deeper understanding of these objects is necessary. Therefore, in the present contribution, we consider the potential thermodynamical properties of lorentzian traversable wormholes [6]. The key point to derive such properties is the study of the radiation process.

2. Preliminaries.
In order to study the thermal radiation process, in this section we summarize some previous results necessary to characterize wormholes univocally.

2.1. Dark energy accretion onto black holes and wormholes.
Babichev, Dokuchaev and Eroshenko were the first to study the accretion of dark energy onto black holes phenomenon [7]. They used a test-fluid approach considering a Schwarzschild black
hole and the dark energy described by an energy-momentum tensor of a perfect fluid, obtaining that the variation of the black hole mass with time takes the form

$$\dot{M} = 4\pi DM^2 (p + \rho),$$

(1)

where $D$ is a constant of order unity. This expression shows how the black hole mass, and with it its size, increases if the fluid fulfils the null energy condition, it decreases if $p + \rho < 0$, and remains constant in the cosmological constant case.

Soon after, Gonzalez-Diaz studied the dark energy accretion onto wormholes [8]. It can be seen that the application of the test-fluid approach to the Morris-Thorne wormhole [9] leads to

$$\dot{m} = -4\pi Qm^2 (p + \rho),$$

(2)

where $Q$ is a positive constant. Therefore, the wormhole increases by the accretion of phantom energy, it decreases if the fluid fulfils the null energy condition, and it also remains constant in the cosmological constant case.

### 2.2. 2+2-formalism and its applications.

In order to study the possible thermodynamics of wormholes, we must consider a method based on local quantities since, by definition, there is no event horizon in these spacetimes. Therefore, we consider the 2+2 formalism developed by Hayward for spherically symmetric spacetimes [10]. The metric of these spacetimes can always be expressed, at least locally, in terms of the double null coordinates, $\xi^\pm$, as

$$ds^2 = 2g_{\pm} \, d\xi^\pm \xi^\mp + r^2 d\Omega^2_{(2)},$$

(3)

where $r(\xi^+, \xi^-)$ is the radius of the spheres of symmetry. The expansions along the null directions are $\Theta_{\pm} = 2 (\partial_{\pm} r) / r$ and, since the sign of their product is an invariant, it can allow us to determine the kind of the spheres of symmetry. The sphere is trapped, marginal or untrapped, if this product is bigger, equal or less than zero. A marginal sphere (fixing $\Theta_+ = 0$) can be future ($\Theta_- < 0$), bifurcating ($\Theta_- = 0$) or past ($\Theta_- > 0$) and outer ($\partial_- \Theta_+ < 0$), degenerate ($\partial_- \Theta_+ = 0$) or inner ($\partial_- \Theta_+ > 0$). A trapping horizon is defined as the hypersurface foliated by marginal spheres and it has the same classification as the marginal spheres.

In spherically symmetric spacetimes one can consider the Kodama vector

$$k = \text{curl} \, 2r = -g^{+ -} (\partial_+ r \partial_- - \partial_- r \partial_+),$$

(4)

where the subscript 2 means the space normal to the spheres of symmetry. On the trapping horizon, this vector fulfills an equation similar to that of the temporal Killing vector in static spacetimes [10], therefore, a generalized surface gravity can be defined as [10]

$$\kappa = \frac{1}{2} \text{div} \, 2r = g^{+ -} \partial_+ \partial_- r.$$

(5)

It can be seen that $\kappa$ is positive, zero or negative if the trapping horizon is outer, degenerate or inner.

Hayward also found a generalized first law of thermodynamics for these spacetimes, which is [10]

$$L_z E = \frac{\kappa L_z A}{8\pi} + \omega L_z V,$$

(6)

where $z = z^+ \partial_+ + z^- \partial_-$ is the vector which generates the horizon, $E = r (1 - \partial^a r \partial_a r) / 2$ is the Misner-Sharp energy and $\omega$ is an invariant. It can be noted that Eq. (6) suggests a relation between the entropy and the area, $S \propto A$.

Even more, as it has been shown [11], this method can be used to obtain the temperature of the thermal radiation for dynamical black holes, which is

$$T_H = \frac{\kappa}{2\pi}.$$

(7)
3. Wormholes characterization.
Using the Einstein equations for spherically symmetric spacetimes [10] and the definition of a trapping horizon, the following expression can be obtained

\[
\text{sign} \left( \frac{z^+}{z^-} \right) = -\text{sign} \left[ (p + \rho)_{\text{H}} \right].
\]  
(8)

Therefore, a trapping horizon is space-like if the surrounding material has \( p + \rho > 0 \), null if \( p + \rho = 0 \), and time-like if \( p + \rho < 0 \). On the other hand, the evolution of the area can be written in terms of the expansions (one of which vanishes at the horizon) as

\[
L_z A = \int \mu \left( z^+ \Theta_+ + z^- \Theta_- \right). 
\]  
(9)

The combination of Eqs. (8) and (9) and the definitions previously introduced imply that the black hole, characterized by a future outer trapping horizon, increases if \( p + \rho > 0 \) and decreases in the opposite case. Since this result is qualitatively the same as that obtained by Babichev et al. [7], we could consider that both methods describe just the same process, which is originated from a flow of the surrounding material into the hole.

Thus, dynamic wormholes must be characterized by past outer trapping horizons [6], in order to recover the results obtained following the accretion method [9] by using the 2+2 formalism. Of course, in the static case their trapping horizons will be bifurcating.

4. Wormholes thermal radiation and thermodynamics.
We consider in the present study a general spherically symmetric and dynamic wormhole which, therefore, is described through metric (3) with a past outer trapping horizon (fixing, without loss of generality, \( \Theta_- = 0 \)). It can be seen that this metric can be consequently written in terms of the generalized retarded Eddington-Finkelstein coordinates, at least locally, as [6]

\[
ds^2 = -e^{2\Psi} C \text{d}u^2 - 2e^{\Psi} \text{d}u \text{d}r + r^2 \text{d}\Omega^2,
\]  
(10)

where \( \text{d}u = \text{d}\xi^-, \text{d}\xi^+ = \partial_u \xi^+ \text{d}u + \partial_r \xi^+ \text{d}r \), and \( \Psi \) expressing the gauge freedom in the choice of the null coordinate \( u \). Since \( \partial_r \xi^+ > 0 \), we have considered \( e^{\Psi} = -g_{++} \partial_r \xi^+ > 0 \) and \( e^{2\Psi} C = -2g_{++} \partial_u \xi^+ \). It can be seen that \( C = 1 - 2E/r \), with \( E \) being the Misner-Sharp energy.

Although a traversable wormhole possesses, by definition, a classically allowed trajectory, we can think that the existence of a trapping horizon, characterized by a non-vanished generalized surface gravity, opens the possibility for an additional quantum traversing phenomenon through the wormhole. So, we consider the tunnelling probability along a classical forbidden trajectory, \( \Gamma \propto \exp \left(-2\text{Im} \left( I \right) \right) \), where \( I \) is the action. Now, applying the method considered in Ref. [11], but taking in this case the metric (10) into account, it can be seen that \( \Gamma \) has a thermal form, \( \Gamma \propto \exp \left(-\omega_\phi/T_H \right) \), being the temperature [6]

\[
T_H = -\frac{\kappa}{2\pi},
\]  
(11)

which is negative. Therefore, like in the black hole case, wormholes radiate matter of the same kind as their surrounding material, in this case, phantom energy.

Once calculated the temperature on the wormhole horizon, we can re-write the generalized first law, Eq. (6), as

\[
L_z E = -T_H L_z S + \omega L_z V,
\]  
(12)

which implies the recovery of the usual relation between the entropy and the area, \( S = A/4 \).

At this point, we can formulate the three laws of wormhole thermodynamics [6].
First law: the change in the gravitational energy of the wormhole is equal to the sum of the energy removed from the wormhole and the work done in the wormhole.

Second law: the entropy of a wormhole can never decrease when the wormhole is placed in its most natural exotic environment.

Third law: it is impossible to reach the absolute zero for surface gravity by means of any dynamical process.

This third law can be understood taking into account that if for some hypothetical process the surface gravity is reduced to vanish, then this process would also be changing the trapping horizon to be degenerate. Since a wormhole is characterized by an outer horizon, the resulting object would not be a wormhole and, therefore, these laws will not be applicable.

5. Conclusions and further comments.
It can be concluded that wormholes possess a well-defined thermodynamics and thermally radiate phantom energy.

Like in the black hole case, the radiation process would produce a decrease of the wormhole throat size, so decreasing the wormhole entropy, too. We expect that this violation of the second law would be only apparent, because it is the total entropy of the universe what should be meant to increase.

It is worth noticing that the initial conditions in the action integral could be chosen in such a way that the wormhole would radiate ordinary matter increasing its size in the process. But, if that case would be at all possible, the thermal radiation would be always thermodynamically forbidden in front of the accretion entropically favoured process.

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