An Argumentation-Based Approach for Computing Inconsistency Degree in Possibilistic Lightweight Ontologies

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Abstract. Reasoning with inconsistent ontologies plays an important role in Semantic Web applications. An important feature of argument theory is that it is able to naturally handle inconsistencies in ontologies and allows a user to represent information in the form of an argument. In argumentation, given an inconsistent knowledge base, arguments compete to decide which are the accepted consequences. In this paper, we are interested in using the argumentation for the inconsistency degree of uncertain knowledge bases expressed in possibilistic DL-Lite (the key notion in reasoning from a possibilistic DL-Lite knowledge base) without going through the negative closure. In the present work, the terminological base is assumed to be fully certain and the uncertainty is only considered on the assertion based. We proved that it is coherent and feasible to use Dung's abstract argumentation theory to compute the inconsistency degree and how argumentation semantics relate to the state of the art of handling inconsistency.

1. Introduction
In real world applications, the information arriving from different sources led to the continuous evolution of the ontology. We are often confronted with uncertainty in the information used in the continual need to integrate or to match them. Description Logics (DLs for short), mostly based on first order logic, are recognized as powerful formal frameworks for representing and reasoning on ontologies.

In this work, we focus on main fragments of DL-Lite. Namely DL-Lite\textsubscript{core} [1,2]. Most of the existing works have focused on the study of different inference strategies based on productivity criteria and computational complexity. In these studies, closely related to works on consistent query answering from inconsistent databases, there is a lack of studies on how existing inference strategies can be placed within the space of possible inference strategies. Another way to deal with inconsistency is the argumentative based-approach, which has become one of the solutions implemented to solve the problem of the inconsistency. The argumentation approach allows representing the data in a format that is simplest, and most adequate for the user to understand [3,4]. In the literature, several works have been developed to use the argumentation approach with ontology. The work [5] propose a framework for handling inconsistent DL ontologies. The authors involved expressing a DL ontology in a Defeasible Logic Programs program, they provided a semantic interpretation of ontologies as DeLP programs. The work presented in [6] is the first proposition that uses argumentation with various ontologies. It permits agents to review the answers to queries without one agent copying the entirety of his ontology to the other and without an exhausting time and efforts to integrate this ontology to theirs. The work in [7], uses the argumentation-based approach with description logic ALC. More recently, in [8] an approach for reasoning with inconsistent possibilistic DL is proposed. It relies on the translation of the ontology into the language of possibilistic logic programming and aggregate of arguments for the same conclusion.
through argument accrual. This approach deals with knowledge bases expressed in a general DLs while in this paper we handle knowledge bases expressed in lightweight DLs.

In a real situation, we use the argumentation to argue an idea, to defend a thesis, to persuade an audience of the merits of a certain decision [3]. In this paper, we use the structured argumentation for handling inconsistency in possibilistic lightweight ontologies because this work permits the user to better understand information since it is simply represented in form of an argument. It allows to track the provenance of different pieces of information used to conclude a given formula and to see which pieces of information are not compatible together.

The rest of this paper is organized as follows: Section 2 provides some basic concepts of DL-Lite. Section 3 presents our argumentation-based procedure. First, we show how to transform the knowledge base to a possibilistic arguments then how to define the relation between them and lastly we get the inconsistency degree from the result of the argumentation framework. We conclude in Section 4.

2. Possibilistic DL-Lite
DL-Lite is a family of DLs that aims to capture some of the most popular conceptual modelling formalisms. A DL KB $K = \langle T, A \rangle$ consists of a set $T$ of concept and role axioms and a set $A$ of assertional facts. In this paper, we only consider the DL-Lite$_{core}$. The syntax of the main fragment DL-Lite$_{core}$ language is given as [9]:

$$
\begin{align*}
B & \rightarrow A \exists R & C & \rightarrow B \neg B \\
R & \rightarrow P P & E & \rightarrow R \neg R
\end{align*}
$$

Where $A$ is an atomic concept, $P$ an atomic role, $P^\ast$ the inverse of the atomic role $P$. $B$ (resp. $C$) are called basic (resp. complex) concepts and $R$ (resp. $E$) are called basic (resp. complex) roles.

A possibilistic DL-Lite KB $K = \langle T, A \rangle$ is a finite set of a possibilistic terminological based TBox and a finite set of a possibilistic assertional based.

The syntax of possibilistic DL-lite is represented by the concept of DL-Lite knowledge base [1,10]. A $\Pi$-DL-Lite KB $K$ is a finite set of possibilistic axioms and assertions of the form $<\Phi, \alpha>$, where:
- $\Phi$ is an axiom/ assertion and
- $\alpha \in [0,1]$ represents the certainty degree of $\Phi$.

We have always observed that the inconsistency come from the assertional based ABox. In this article, we assumed that terminological based TBox is sure (fully certain).

**Example 1:** Let $K$ be a $\Pi$-DL-Lite KB composed of the following TBox $T$ and ABox.

$T = \{ <\text{Male} \sqsubseteq \neg \text{Female}, 1.0>, <\text{Male} \sqsubseteq \text{Human}, 1.0>, <\text{Female} \sqsubseteq \text{Human}, 1.0> \}$

$A = \{ <\text{Male}(T), 0.95>, <\text{Female}(T), 0.45> \}$.

A possibility distribution is a function that attributes to each DL-lite interpretation $I$, a real number belong to the interval $[0, 1]$.

$$\pi_K(I) = \begin{cases} 
1 & \text{if } \forall \phi \in K, I \models \phi \\
0 & \text{if } \exists \phi \in T, I \models \phi \\
1 - \max \{\alpha_i \} & \text{otherwise}
\end{cases}$$

(1)

Inconsistency in DL-Lite is due to the nonexistence of an interpretation that satisfies all formulas. If there exists at least one interpretation satisfy all the formulas ($\pi(I) = 1$), we said $K$ is normalized. If it is not the case, we say that $\pi$ is sub-normalized.

$$h(\pi) = \max(\pi_K(I))$$

(2)

Where $h(\pi)$ is the consistency degree.

**Definition 1.** Let $K$ be a $\Pi$-DL-Lite KB and let $K' \subseteq K$ be maximal consistency knowledge sub-base of $K$ if:
- $T' \cup A'$ is consistency and
- $\forall S \in T, if S \neq 0$ than $T' \cup A' \cup S$ is inconsistency
- $\exists \Omega \in \Omega, \pi(K') (I) = h(\pi)$

It may exist more than one maximal consistency sub-set with the same value on the highest possibility degree of their interpretations but that does not mean they are equivalent. Since the inconsistency degree
is not used to compare knowledge bases. It is use for evaluating to what extent a KB is inconsistent. For this case, Consistent(K) include the set of all maximal consistence of K.

**Definition 2.** The inconsistency degree of K is defined as follows:

$$\text{Inc}(K) = 1 - \max(\pi_K(I))$$  \hfill (3)

**Example 2:** From the KB presented in Example 1 and the possibilistic distribution, the inconsistency degree is: Inc(K) = 0.45

The instance checking problem decide, given an individual a (resp. a pair of individuals (a, b)) a concept B (resp. a role R) and a π-DL-Lite KB K, whether B(a) (resp. R(a, b)) follows from <T, A> [3,5]. The necessity degree \(N_K(q)\) is defined as:

$$N_K(q) = 1 - \max(\pi_K(I))$$  \hfill (4)

A query q is a consequence of K if its necessity degree is greater than the inconsistency degree of the first knowledge base. Formerly:

$$K \models q \text{ if and only if } N_K(q) > \text{Inc}(K)$$  \hfill (5)

### 3. Computing the inconsistency degree

In this section, we show how to compute the inconsistency degree of a π-DL-Litecore knowledge base using argumentation framework. In order to compute the inconsistency degree, we followed the structure of the argumentation process which is determined in three steps as follows:

1. Construct the possibilistic arguments,
2. Determine the relation in term of conflict between the arguments and
3. Select the justified conclusion. The construct of the possibilistic arguments is as follow:

**Definition 3.** Let K be a π-DL-Lite KB. Let \(T = \{T_P, T_N\}\) be TBox. A possibilistic argument Arg is a tuple Arg = (S, C, level), where C is a conclusion, S is the support and level is the certainty level of a nonempty subset S satisfying:

1. S is consistent and
2. S \(\models\) C.
3. The level(Arg) is calculated as follow: level(Arg) = \(\min\{\alpha_i | (\phi_i, \alpha_i) \in S\}\)

In this paper, we use the weight to express a priority between the formulas, for the set of arguments, we use the possibilistic argument level(Arg) to compare different arguments.

**Definition 4.** Let Arg1 and Arg2 be two arguments in K.

Arg1 is preferred to Arg2 iff level(Arg1) \(\geq\) level(Arg2)  \hfill (6)

Once we have constructed the set of argument, the relationships between these arguments need to be identified. These relationships are usually captured by the idea of attack.

**Definition 5.** Let \(T_N\) be all the negative axioms. A possibilistic argument Argi attacks Argj if and only if \(\exists s \in S(\text{Arg}_i)\) such that:

1. \(\{C(\text{Arg}_i) \cup s\}\) is inconsistent and
2. level(Argj) \(\geq\) level(Argj)

Now, we define the relation between possibilistic arguments (attack relation), then we will use Dung's approach define which arguments are justified and choose the conclusions of the justified arguments in order to calculate the inconsistency degree [11,12].

**Definition 6.** Given a π-DL-Lite KB, \(\text{Arg}_K\) the set of all arguments. An argumentation framework \((AF)\) is a pair \(AF = (A, R)\), where \(A = \text{Arg}_K\) and \(R\) is an attack relation \((R \subseteq A \times A)\) defined using the Definition 5.

The notation \(aRb\) indicates that a attacks b.

When the argumentation framework is defined, the next step consist in choosing a reasonable subset of arguments from \(\text{Arg}_K\) using argumentation semantics. Some well-known Dung's argumentation semantics are defined as follows [13,14].

\(AF\) is an argumentation framework, \(\mathcal{E} \subseteq A\).

- \(\mathcal{E}\) is conflict free if \(\forall \text{Arg}_i, \text{Arg}_j \in \mathcal{E}, (\text{Arg}_i, \text{Arg}_j) \in R\).
- $\text{Arg}_i$ is acceptable if $\forall \text{Arg}_j \in A$, $(\text{Arg}_i, \text{Arg}_j) \in R, \exists \text{Arg}_c \in A$ such that $(\text{Arg}_c, \text{Arg}_i) \in R$.
- If $\mathcal{E}$ is conflict free and defends all its arguments than $\mathcal{E}$ is admissible.
- If $\mathcal{E}$ is an admissible set which contains all the arguments it defends than $\mathcal{E}$ is a complete.
- If $\mathcal{E}$ is maximal admissible set than $\mathcal{E}$ is a preferred.
- If $\mathcal{E}$ is conflict free and $\forall \text{Arg}_i \in A \setminus \mathcal{E}, \exists \text{Arg}_j \in \mathcal{E}$ such that $(\text{Arg}_j, \text{Arg}_i) \in R$ than $\mathcal{E}$ is a stable.

Let us use an example to show how to construct an argumentation framework from a given knowledge base.

**Example 3:** we continue with Example 1. Now let show how to construct possibilistic argument:

$\text{Arg}_1 = ((< \text{Male}(T), 0.95 >], \text{Male}(T), 0.95)$

$\text{Arg}_2 = ((< \text{Female}(T), 0.45 >], \text{Female}(T), 0.45)$

$\text{Arg}_3 = ((< \text{Female}(T), 0.45 >, < \text{Female} \sqsubseteq \text{Human}, 1.0 > ], \text{Human}(T), 0.45)$

$\text{Arg}_4 = ((< \text{Male}(T), 0.95 >, < \text{Male} \sqsubseteq \text{Human}, 1.0 > ] \text{ Human}(T) \cap \text{Male}(T), 0.95)$

$\text{Arg}_5 = ((< \text{Male}(T), 0.95 >, < \text{Male} \sqsubseteq \text{Human}, 1.0 > ] \text{ Human}(T), 0.95)$

$\text{Arg}_6 = ((< \text{Female}(T), 0.45 >, < \text{Male} \sqsubseteq \text{Human}, 1.0 > ] \text{ Human}(T) \cap \text{Female}(T), 0.45)$

The graphic representation is given in Figure 1.

![Figure 1](image-url)

**Figure 1.** The argumentation framework

The following theorem shows the relation between consistence $\pi$-DL-Lite KB and the argumentation framework.

**Theorem 1.** Let $K$ be $\pi$-DL-Lite KB, $AF$ its associated argumentation framework and $x \in \{s, p\}$. Then:

$$\text{Ext}_x(AF) = \{ \text{Arg}_K \{K" \} \mid K" \in \text{Consistent(K)} \}, x = \{s, p\}$$

Where: $\text{Arg}_K$ represents the set of all arguments. $\text{Ext}_x(AF)$ is the set of extensions.

**Definition 7.** $K$ is $\pi$-DL-Lite KB and let $AF$ be its associated argumentation framework. The result of $AF$ is defined as follow:

$$\text{Output}_x(AF) = \begin{cases} \emptyset & \text{if } \text{Ext}_x(AF) = \emptyset \\ \bigcap_{\mathcal{E} \in \text{Ext}_x(AF)} \text{Conclusion}(\mathcal{E}) & \text{otherwise} \end{cases}$$

(7)

**Proposition 1.** Let $K$ be $\pi$-DL-Lite KB and let $AF$ be the associated the argumentation framework. Then:

$\forall \mathcal{E} \in \text{Ext}_x(AF), x = \{p, s\}$ we have $\text{Conclusion}(\mathcal{E})$ is consistent.
Example 4: Using the argumentation framework (Figure 1), we have $\text{Ext}_s(AF)$ under the stable and preferred semantics: $\mathcal{E}_1 = \{Arg_1, Arg_4, Arg_5\}$.

The output $\text{Output}_s(AF)$ of the argumentation framework is $\{Arg_1, Arg_4, Arg_5\}$.

Proposition 2. $K$ is a possibilistic DL-Lite KB, $AF$ its associated argumentation framework. $\text{Output}_s(AF)$ is the output.

$\text{Inc}(K) = \alpha$ if and only if $\alpha = \max\{\text{level}(Arg_i) | Arg_i \notin \text{Output}_s(AF)\}$.

Example 5: From Example 3, we have $\text{Ext}(AF)$ under the stable and preferred semantics:

$\mathcal{E}_1 = \{Arg_1, Arg_4, Arg_5\}$. $\text{Inc}(K) = \max\{\text{level}(Arg_1), \text{level}(Arg_3), \text{level}(Arg_6)\} = 0.45$.

4. Conclusion
In this article, we have applied Dung's argumentation semantics process to get the value of the inconsistency degree of a possibilistic lightweight ontologies. This approach offers a simple method for treating inconsistent knowledge base.

First, it transforms the inconsistent $\pi$-DL-Lite KB to a set of possibilistic arguments. Then to construct the argumentation framework, we use this set and its attack relation. The last step consists of choosing the extensions based on Dung's argumentation semantics, to finally calculate the inconsistency degree. We plan to generalize this method with a general case of $\pi$-DL-Lite (an uncertain terminology based), studies the case of conjunctive queries using the argumentation framework and use other argumentation semantics for $t$ query answering based [15].

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