Mass generation for abelian spin-1 particles via a symmetric tensor

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Abstract

In the topologically massive BF model (TMBF) the photon becomes massive via coupling to an antisymmetric tensor, without breaking the $U(1)$ gauge symmetry. There is no need of a Higgs field. The TMBF model is dual to a first-order (in derivatives) formulation of the Maxwell-Proca theory where the antisymmetric field plays the role of an auxiliary field. Since the Maxwell-Proca theory also admits a first-order version which makes use of an auxiliary symmetric tensor, we investigate here a possible generalization of the TMBF model where the photon acquires mass via coupling to a symmetric tensor. We show that it is indeed possible to build up dual models to the Maxwell-Proca theory where the $U(1)$ gauge symmetry is manifest without Higgs field, but after a local field redefinition the vector field eats up the trace of the symmetric tensor and becomes massive. So the explicit $U(1)$ symmetry can be removed unlike the TMBF model.
1 Introduction

In the usual description of massive spin-1 particles via a Maxwell-Proca (MP) action the
gauge symmetry is explicitly broken. It is of interest to search for alternatives to the Higgs
mechanism to preserve the gauge symmetry while generating a mass for a spin-1 particle
specially for the nonabelian case. Here we address this question in the simpler case of the
abelian $U(1)$ gauge symmetry. Dualization methods can help in investigating this problem. It
is convenient for those methods to rewrite the Maxwell action in a first-order form by using
auxiliary fields. In $D = 1 + 1$ we can achieve that with help of a scalar field which interacts
with the vector field via a topological term $\phi \epsilon^{\mu\nu} \partial_\mu A_\nu$. By using the master action approach of
[1] as a dualization procedure it can be shown [2] that the first-order MP theory in $D = 1 + 1$
is dual to a local action with manifest $U(1)$ gauge symmetry. It corresponds to the bosonized
form of the Schwinger model whose effective action, after elimination of the auxiliary scalar
field, is written down in our formula (27). Although non local, the effective action is manifest
$U(1)$ invariant.

In $D = 2 + 1$ we replace the scalar field by a vector field $B_\mu$ and the topological coupling
term becomes $\epsilon^{\mu\nu\alpha} B_\mu \partial_\nu A_\alpha$. After some trivial field redefinition we end up, see master action
in [3] with equal masses, with a dual theory to MP which consists of a couple of noninteracting
Maxwell-Chern-Simons actions with the same mass but with opposite helicities. This theory
is manifest $U(1)$ symmetric and represents one massive spin-1 particle with helicities $\pm 1$ just
like the MP theory in $D = 2 + 1$.

In $D = 3 + 1$ we can use an antisymmetric tensor with the so called topological BF coupling
$\epsilon^{\mu\nu\alpha\beta} B_{\mu\nu} \partial_\alpha A_\beta$. The theory dual to MP is the topologically massive BF model (TMBF), also
named Cremmer-Scherk model [4]. It can be obtained from the first-order MP theory via
both master action [5] and Noether gauge embedment [6]. The TMBF model is unitary [7]
and explicitly $U(1)$ invariant. Unfortunately, as shown in [8], a nonabelian generalization of the
TMBF model without extra fields will necessarily lead to power-counting nonrenormalizable
couplings as in [9], see however, [10] [11] where the extra field is nonpropagating and [12] for
a recent suggestion which makes use of tensor gauge fields. In [13] the geometrical origin of
tensor gauge connections is investigated. Thus, it is welcome to try alternatives to the TMBF
model. Here we follow this route for the abelian $U(1)$ case as a laboratory for a possible
non-abelian generalization.

In fact, in [14] there appears a new first-order form of the Maxwell action which makes
use of a symmetric auxiliary field $W_{\mu\nu} = W_{\nu\mu}$. By adding the Proca mass term we build up
a first-order version of the MP theory, see [15]. Now we have the coupling term $W_{\mu\nu} \partial^\mu A^\nu$,
though nontopological, this term by itself has no particle content. In the next section we use
this first-order formulation of the MP theory as a starting point to obtain via master action
and Noether gauge embedment alternative dual theories to the MP theory. In section III we
start with an Ansatz quadratic in the fields $A_\mu$ and $W_{\mu\nu}$ and second-order in derivatives. We
analyze its particle content and the presence of $U(1)$ gauge symmetry. In section IV we draw
our conclusions.
2 Master action and Noether gauge embedment

We begin with an alternative derivation of the TMBF model as given in [5]. We first define in $D = 3 + 1$ a master action depending on four different fields:

$$S_M[A, \tilde{A}, B, \tilde{B}] = - \int d^4x \left[ \frac{m^2}{2} A_\mu A^\mu + \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} + \frac{m}{2} \epsilon^{\mu\nu\alpha\beta} B_{\mu\nu} \partial_\alpha A_\beta \right. \right.
\left. \left. - \frac{m}{2} \epsilon^{\mu\nu\alpha\beta} \left( B_{\mu\nu} - \tilde{B}_{\mu\nu} \right) \partial_\alpha \left( A_\beta - \tilde{A}_\beta \right) \right] \quad (1) \right.$$

The first three terms of (1) correspond to a first-order version of the Proca theory. So their particle content is one massive spin-1 particle. The last term of (1) mixes the fields $(A, B)$ with the dual ones $(\tilde{A}, \tilde{B})$. After the shifts $\tilde{B}_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} + B_{\mu\nu}$ and $\tilde{A}_\mu \rightarrow \tilde{A}_\mu + A_\mu$ it decouples from the first three terms. Its spectrum is empty (topological BF-term). So we conclude that the spectrum of (1) consists of one massive spin-1 particle. The mixing term does not contribute to the particle content as usually in the master action approach.

On the other hand, if we Gaussian integrate the $(A, B)$ fields we obtain the TMBF model in terms of the dual fields $(\tilde{A}, \tilde{B})$:

$$S_{TMBF} = \int d^4x \left[ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{12} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} + \frac{m}{4} \epsilon^{\mu\nu\alpha\beta} B_{\mu\nu} \tilde{F}_{\alpha\beta} \right] \quad (2) \right.$$

Where $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ and $\tilde{H}_{\mu\nu\lambda} = \partial_\mu \tilde{B}_{\nu\lambda} + \partial_\nu \tilde{B}_{\lambda\mu} + \partial_\lambda \tilde{B}_{\mu\nu}$. The action $S_{TMBF}$ is invariant under the independent gauge transformations:

$$\delta_\phi \tilde{A}_\mu = \partial_\mu \phi \quad ; \quad \delta_\phi \tilde{B}_{\mu\nu} = 0 \quad (3)$$
$$\delta_\Lambda \tilde{A}_\mu = 0 \quad ; \quad \delta_\Lambda \tilde{B}_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \quad (4)$$

It is convenient for future comparison to recall the equations of motion of the TMBF model. Skipping the tildes for convenience we have from (2):

$$\partial_\alpha H^{\mu\nu\alpha} + m \epsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta = 0 \quad (5)$$
$$\partial_\nu F^{\mu\nu} - \frac{m}{6} \epsilon^{\mu\nu\alpha\beta} H_{\nu\alpha\beta} = 0 \quad (6)$$

Following [7], we first solve (5). Recalling that in four dimensions and for vanishing mass a two-form is dual to a scalar field $(\eta)$ we have the general solution $H_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\delta} (\partial^\delta \eta - m A^\delta)$. Back in (6) we have

$$\partial_\mu F^{\mu\nu} - m^2 A^\mu + m \partial^\mu \eta = 0 \quad (7)$$

Since $\delta_\phi H_{\alpha\beta\gamma} = 0$ we must have $\delta_\phi \eta = m \phi$. In terms of the gauge invariant vector field $\bar{A}^\mu = A^\mu - \partial^\mu \eta / m$ the equation (7) becomes the well known Maxwell-Proca equation without manifest $U(1)$ gauge invariance:

\[1\text{In this work we use mostly plus } D\text{-dimensional signature } \eta_{\mu\nu} = (-,+,\cdots,+).\]
\[ \partial_\mu F^{\mu\nu} - m^2 A^\nu = 0 , \]

In summary, the gauge invariant equations of motion (5) and (6) show that \( B_{\mu\nu} \) can be eliminated in terms of a gauge invariant vector field which satisfies the Proca equation and consequently we have only one massive spin-1 particle in the spectrum of the TMBF model as expected from the master action approach.

In the first three terms of (11) we have written the Maxwell-Proca theory in a first-order form with the help of a totally antisymmetric rank two tensor \( (B_{\mu\nu}) \). It is possible \(^{14, 15}\) to replace the two form field by a totally symmetric tensor \( W_{\mu\nu} = W_{\nu\mu} \) and a master action similar to (1) can be written down in arbitrary \( D \)-dimensions as

\[
S_M [A, \tilde{A}, W, \tilde{W}] = \int d^D x \left[ W^{\mu\nu} W_{\mu\nu} - \frac{W^2}{D - 1} + 2 W^{\mu\nu} \partial_\mu A_\nu - \frac{m^2}{2} A^\mu A_\mu \right. \\
\left. - 2 \left( W^{\mu\nu} - \tilde{W}^{\mu\nu} \right) \left( \partial_\mu A_\nu - \partial_\mu \tilde{A}_\nu \right) \right]
\]

The first four terms of (9) correspond to a first-order form of the Maxwell-Proca theory while the last term mixes the \( (A, W) \) fields with the duals \( (\tilde{A}, \tilde{W}) \). After the shift \( \tilde{A}_\mu \to \tilde{A}_\mu + A_\mu \) and \( \tilde{W}_{\mu\nu} \to \tilde{W}_{\mu\nu} + W_{\mu\nu} \) the last term of (9) decouples and becomes \( \mathcal{L}_{\tilde{A}, \tilde{W}} = -2 \tilde{W}^{\mu\nu} \partial_\mu \tilde{A}_\nu \).

Thus, the particle content of the master action (9) corresponds to one massive spin-1 particle plus the content of \( \mathcal{L}_{\tilde{A}, \tilde{W}} \). Minimizing the action \( S_{A,W} = \int d^D x \mathcal{L}_{A,W} \) we have the equations of motion:

\[ \partial_\mu A_\nu + \partial_\nu A_\mu = 0 \quad (10) \]
\[ \partial^\mu W_{\mu\nu} = 0 \quad (11) \]

It is easy to convince oneself, assuming vanishing fields at infinity, that the solution of (10) is trivial \( A_\mu = 0 \) while (11) is solved \(^{16}\) by \( W_{\mu\nu} = \partial^\alpha \partial^\beta R_{\alpha\beta\mu\nu} \) where \( R_{\alpha\beta\mu\nu} \) is a tensor with the index symmetries of the Riemann curvature tensor but otherwise arbitrary. However, since the action \( S_{A,W} \) is itself invariant under \( \delta \Lambda W_{\mu\nu} = \partial^\alpha \partial^\beta \Lambda_{\alpha\beta\mu\nu} \) where \( \Lambda_{\alpha\beta\mu\nu} \) has the same properties of \( R_{\alpha\beta\mu\nu} \) we can say that the general solution of (11) is pure gauge. Therefore, the last term of (9) has no particle content and the whole master action (9) contains only one massive spin-1 particle in the spectrum. Following the master action approach if we Gaussian integrate over the fields \( (A, W) \) we have the dual action to the first-order Maxwell-Proca model:

\[ S^* = \int d^D x \left[ -\frac{1}{4} F_{\mu\nu}^2 (\tilde{A}) + \frac{2}{m^2} \left( \partial^\alpha \tilde{W}^{\alpha\nu} \right)^2 - 2 \tilde{W}^{\mu\nu} \partial_\mu \tilde{A}_\nu \right] \]

The action \( S^* \) is invariant under the gauge transformations \( \delta \Lambda \tilde{W}_{\mu\nu} = \partial^\alpha \partial^\beta \Lambda_{\alpha\beta\mu\nu} \) with \( \delta \Lambda \tilde{A}_\mu = 0 \). The equations of motion of (12) are:

\[ \partial_\mu \tilde{q}_\nu + \partial_\nu \tilde{q}_\mu = 0 \quad (13) \]
\[ \Box \partial_\mu \tilde{A}_\nu - m^2 \tilde{A}_\mu + m^2 \tilde{q}_\mu = 0 \quad (14) \]

\(^{2}\)We quit the tildes for while.
where \( \tilde{q}_\mu = \tilde{A}_\mu + 2\partial^\nu \tilde{W}_{\alpha\mu}/m^2 \) is gauge invariant. As in (10), due to the boundary conditions at infinity, we have the solution \( \tilde{q}_\mu = 0 \) of (13) which allows us to eliminate \( \tilde{W}_{\mu\nu} \) in terms of \( \tilde{A}_\mu \) up to gauge transformations, i.e., \( \partial^\nu \tilde{W}_{\alpha\beta} = -m^2 \tilde{A}_\beta/2 \). Back in (13) we recover the Maxwell-Proca equation confirming that \( S^* \) contains only one massive particle of spin-1 in the spectrum. Although \( S^* \) is equivalent to the Maxwell-Proca theory, the dual model (12) has no \( U(1) \) gauge symmetry (differently from \( S_{TMBF} \)). The key point is that the mixing term (BF term) in the master action (1) carries \( U(1) \) gauge symmetry contrary to the mixing term of the master action (9). We conclude that a natural application of the master action approach to the first-order Maxwell-Proca theory obtained with help of a symmetric tensor \( W_{\mu\nu} \) does not lead us to a theory with explicit \( U(1) \) gauge invariance.

Another way of obtaining \( S_{TMBF} \) from the Maxwell-Proca theory is by means of a Lagrangian Noether gauge embedding (NGE) procedure as in [6, 17], see also [18] for a Hamiltonian embedding. Let us repeat the same Lagrangian procedure here. The first four terms of (9) define the first-order Maxwell-Proca theory:

\[
S^{(0)} = \int d^D x \left[ W^{\mu\nu} W_{\mu\nu} - \frac{W^2}{D - 1} + 2 W^{\mu\nu} \partial_{(\mu} A_{\nu)} - \frac{m^2}{2} A^\mu A_\mu \right]
\]  

(15)

The first three terms of (15) are invariant under the \( U(1) \) gauge transformations:

\[
\delta_\phi A_\mu = \partial_\mu \phi \quad ; \quad \delta_\phi W_{\mu\nu} = \Box \theta_{\mu\nu} \phi
\]  

(16)

Where we define the projection operators:

\[
\theta_{\alpha\beta} = (\eta_{\alpha\beta} - \omega_{\alpha\beta}) \quad , \quad \omega_{\alpha\beta} = \frac{\partial_\alpha \partial_\beta}{\Box}
\]  

(17)

In order to preserve the \( U(1) \) symmetry we can modify the action \( S^{(0)} \) by adding linear terms in auxiliary fields:

\[
S^{(1)} = S^{(0)} + \int d^D x \left( K_\mu B^\mu + M_{\mu\nu} C^{\mu\nu} \right)
\]  

(18)

where the Euler tensors are given by

\[
K_\mu = \frac{\delta S^{(0)}}{\delta A^\mu} = -m^2 A_\mu - 2\partial^\nu W_{\mu\nu}
\]  

(19)

\[
M_{\mu\nu} = \frac{\delta S^{(0)}}{\delta W^{\mu\nu}} = 2 \left[ \partial_{(\mu} A_{\nu)} + W_{\mu\nu} - \eta_{\mu\nu} \frac{W}{D - 1} \right]
\]  

(20)

The auxiliary fields must transform with the opposite sign of the original fields such that their variations in (18) compensate the variation of \( S^{(0)} \),

\[
\delta_\phi B^\mu = -\partial^\mu \phi \quad ; \quad \delta_\phi C^{\mu\nu} = -\Box \theta^{\mu\nu} \phi
\]  

(21)

Since \( \delta_\phi M_{\mu\nu} = 0 \) and \( \delta_\phi K_\mu = -m^2 \partial_\mu \phi = m^2 \delta_\phi B_\mu \) we have

\[
\delta_\phi S^{(1)} = \int d^D x \delta_\phi \left( m^2 \frac{B^\mu B_\mu}{2} \right)
\]  

(22)
So we deduce the gauge invariant action:

\[ S^{(2)} = S^{(0)} + \int d^D x \left( K_\mu B^\mu - m^2 B^\mu B_\mu + C^{\mu\nu} M_{\mu\nu} \right) \]  

(23)

After a functional integral over the auxiliary fields we get a functional delta function \( \delta(M_{\mu\nu}) \) which allows us to further integrate over \( W_{\mu\nu} \). We end up with the following gauge invariant effective action for the vector field

\[ S_P = \frac{1}{4} \int d^D x F_{\mu\nu} \left( 1 - \frac{\Box}{m^2} \right) F^{\mu\nu} \]  

(24)

The action (24) is the Podolsky [19] action up to an overall sign. Its equations of motion can be written as

\[ \Box \left( \Box - m^2 \right) A_\mu^T = 0 \]  

(25)

where \( A_\mu^T = \theta_{\mu\nu} A_\nu \) satisfies \( \partial^\mu A_\mu^T = 0 \) which altogether with \( (\Box - m^2) A_\mu^T = 0 \) are equivalent to the Maxwell-Proca equations as expected from the embedding procedure. However, (25) also contain massless solutions to the usual free Maxwell equations \( \Box \theta_{\mu\nu} A^\nu = 0 \). An analysis of the sign of the imaginary part of the residues of both massless and massive poles of the propagator coming from (24) reveals that we have one massive physical particle and one massless ghost which violates unitarity. This is not completely unexpected from the point of view of the NGE procedure as explained in [20].

In summary the master action and the NGE methods have led us to different results and none of them is satisfactory. In the next section we use another approach for a broader investigation of this question.

### 3 A general Ansatz

Another way of figuring out the particle content of the TMBF model is to integrate in the two form field \( B_{\mu\nu} \) in the path integral and obtain an effective action for the vector field \( A_\mu \). One ends up, see [21], with a four dimensional version of the well known Schwinger model which appears in \( D = 1 + 1 \) dimensions due to the non-conservation of the axial current, namely:

\[ \exp^{iS_{eff}[A]} = \int D[B_{\mu\nu}] \exp^{iS_{TMBF}[A,B]} \]  

(26)

Where

\[ S_{eff}[A] = S_{Schw} = -\frac{1}{4} \int d^4 x F_{\mu\nu} \left( \frac{\Box - m^2}{\Box} \right) F^{\mu\nu} \]  

(27)

The Schwinger model is of course \( U(1) \) gauge invariant and a careful analysis of the analytic properties of the propagator reveals that we have only one massive (spin-1) particle in the spectrum as in the initial TMBF model. In what follows we start with a more general (second-order in derivatives) Ansatz for a local quadratic action containing the fields \( (A_\mu, W_{\mu\nu}) \) and integrate over \( W_{\mu\nu} \) in order to deduce a \( D \) dimensional effective action for the vector field.

Let us start with the Ansatz:
\[ S[A, W] = \int d^Dx \left[ a (\partial \cdot A)^2 + b \left( \partial_{(\mu} A_{\nu)} \right)^2 + c_1 (\partial^\nu W_{\mu\nu})^2 + c_2 \partial^\nu W \partial^\mu W_{\mu\nu} + c_3 \partial^\mu W \partial_{\mu} W + c_4 \partial^\mu W_{\mu\nu} \partial_{\alpha} W^{\mu\nu} + d W_{\mu\nu} W^{\mu\nu} + e W^2 \right] + f W_{\mu\nu} \partial^\mu A^\nu + g W \partial \cdot A \] (28)

where \((a, b, c, e, f, g)\) are so far unknown real constants. We can rewrite the Ansatz as:

\[ S[A, W] = \int d^Dx \left[ a (\partial \cdot A)^2 + b \left( \partial_{(\mu} A_{\nu)} \right)^2 + W_{\mu\nu} G^{\mu\nu}_{\alpha\beta} W^{\alpha\beta} + W_{\alpha\beta} T^{\alpha\beta} \right] . \] (29)

Where

\[ T^{\alpha\beta} = f \partial^\alpha A^\beta + g \eta^{\alpha\beta} \partial \cdot A \] (30)

\[ G^{\mu\nu}_{\alpha\beta} = \left\{ (d - c_4 \Box) P^{(2)}_{SS} + \left( d - \frac{c_1 \Box}{2} - c_4 \Box \right) P^{(1)}_{SS} \right. \]
\[ + \left. [d + e - (c_1 + c_2 + c_3 + c_4)] P^{(0)}_{WW} + [d - c_4 \Box + (e - c_3 \Box)(D - 1)] P^{(0)}_{SS} \right. \]
\[ + \left. \sqrt{D - 1} \left[ e - c_3 \Box - \frac{c_2 \Box}{2} \right] \left( T^{(0)}_{SW} + T^{(0)}_{WS} \right) \right\}_{\alpha\beta}^{\mu\nu} \] (31)

where the projection operators \(P^{(s)}_{ij}\) of spin-\(s\) and the transition operators \(T^{(0)}_{SW}, T^{(0)}_{WS}\) are defined as:

\[ \left( P^{(2)}_{SS} \right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{2} \left( \theta^{\lambda\alpha} \theta^{\mu\beta} + \theta^{\mu\alpha} \theta^{\lambda\beta} \right) - \frac{\theta^{\lambda\mu} \theta^{\alpha\beta}}{D - 1} \] , (32)

\[ \left( P^{(1)}_{SS} \right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{2} \left( \theta^{\lambda\alpha} \omega^{\mu\beta} + \theta^{\mu\alpha} \omega^{\lambda\beta} + \theta^{\lambda\beta} \omega^{\mu\alpha} + \theta^{\mu\beta} \omega^{\lambda\alpha} \right) \] , (33)

\[ \left( P^{(0)}_{SS} \right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{D - 1} \theta^{\lambda\mu} \theta^{\alpha\beta} \] , \( \left( P^{(0)}_{WW} \right)_{\alpha\beta}^{\lambda\mu} = \omega^{\lambda\mu} \omega^{\alpha\beta} \) , (34)

\[ \left( T^{(0)}_{SW} \right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{\sqrt{D - 1}} \theta^{\lambda\mu} \omega^{\alpha\beta} \] , \( \left( T^{(0)}_{WS} \right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{\sqrt{D - 1}} \omega^{\lambda\mu} \theta^{\alpha\beta} \) , (35)

From (29), integrating over the fields \(W_{\mu\nu}\) in the path integral we obtain the effective action

\[ S_{eff}[A] = \int d^Dx \left[ a (\partial \cdot A)^2 + b \left( \partial_{(\mu} A_{\nu)} \right)^2 - \frac{1}{4} T^{\alpha\beta}(A) \left( G^{-1} \right)^{\mu\alpha}_{\gamma\beta} T^{\gamma\beta}(A) \right] \] (36)

where, suppressing the indices for convenience, we have

\[ G^{-1} = \frac{P^{(2)}_{SS}}{(d - c_4 \Box)} + \frac{P^{(1)}_{SS}}{d - \Box \left( c_4 + \frac{c_2}{2} \right)} + \frac{[d - c_4 \Box + (e - c_3 \Box)(D - 1)] P^{(0)}_{WW}}{K} \]
\[ + \frac{[d + e - (c_1 + c_2 + c_3 + c_4) \Box] P^{(0)}_{SS}}{K} + \frac{\sqrt{D - 1}}{K} \left( e - c_3 \Box - \frac{c_2 \Box}{2} \right) (T^{(0)}_{SW} + T^{(0)}_{WS}) \] (37)

With
\[ K = [d + e - (c_1 + c_2 + c_3 + c_4) \Box] [d - c_4 \Box + (e - c_3 \Box)(D - 1)] \]
\[ - (D - 1) \left( e - c_3 \Box - \frac{c_2 \Box}{2} \right)^2. \] 
(38)

Working out the expression (36) we have

\[ S_{\text{eff}} [A] = \int d^D x \left[ a (\partial \cdot A)^2 + b (\partial_{\mu} A_{\nu})^2 + (\partial \cdot A) H(\Box)(\partial \cdot A) - \frac{1}{16} F_{\mu \nu} \frac{f^2}{d - \Box (c_4 + \frac{c_1}{2})} F^{\mu \nu} \right] \]
(39)

where

\[ H(\Box) = \frac{-1}{4K} \{ (D - 1)g^2 [d + e - (c_1 + c_2 + c_3 + c_4) \Box] + (f + g)^2 [d - c_4 \Box + (e - c_3 \Box)(D - 1)] \]
\[ + 2(D - 1)g(f + g) [(c_3 + c_2/2) \Box - e] \]  
(40)

In order to have \( U(1) \) gauge invariance in (39) the constants in our Ansatz (28) must be such that

\[ H(\Box) = -(a + b) \]  
(41)

Consequently we end up with the gauge invariant theory

\[ S_{\text{eff}} [A] = -\frac{1}{16} \int d^D x F_{\mu \nu} \frac{4b \left[ (c_4 + \frac{c_1}{2}) \Box - d \right] + f^2}{d - (c_4 + \frac{c_1}{2}) \Box} F^{\mu \nu}. \]  
(42)

By adding a gauge fixing term we can obtain the propagator and calculate the saturated two point amplitude in momentum space \( A(k) \) from which we can read off the particle content of the theory. Explicitly,

\[ A(k) = J^*_\mu(k) \langle A^\mu(-k)A^\nu(k) \rangle J_\nu(k) = -\frac{i}{2} J^*_\mu(k) \left[ G^{-1}(k) \right]^{\mu \nu} J_\nu(k) \]
\[ = -\frac{i}{2} \frac{J^*(k) \cdot J(k) \left[ (c_4 + \frac{c_1}{2}) k^2 + d \right]}{k^2 \left[ 4b \left( c_4 + \frac{c_1}{2} \right) k^2 + 4bd - f^2 \right]} \]  
(43)

Note that the contribution of the gauge fixing term \( \lambda (\partial \cdot A)^2 \) drops out from \( A(k) \) due to the transverse nature of the sources \( (k \cdot J = 0) \) as required by gauge invariance.

We may have one or two poles in \( A(k) \). Since our aim is to obtain only one physical massive particle in the spectrum we impose henceforth:

\[ d = 0 ; \quad b \left( c_4 + \frac{c_1}{2} \right) \neq 0 ; \quad f \neq 0 \]  
(44)

In this case:

\[ A(k) = -\frac{i J^* \cdot J}{8b(k^2 + m^2)}. \]  
(45)
Where
\[ m^2 = -\frac{f^2}{4b(c_4 + c_1/2)}. \] (46)

The imaginary part of the residue of \( A(k) \) at the pole \( k^2 = -m^2 \) becomes \(-J^*(k) \cdot J(k)/(8b)\) evaluated at \( k^2 = -m^2 \). In the rest frame \( k_\mu = (m, 0, \cdots, 0) \), due to \( k \cdot J(k) = 0 \), we must have \( J_0(k) = 0 \). So we can easily check that the frame independent quantity \( J^*(k) \cdot J(k) \) is positive. Consequently, in order to have a physical particle as required by unitarity \((\text{Im Res}(A(k)) > 0)\) and be free of tachyons, see (46), we must further assume that:
\[ b < 0 \quad ; \quad c_4 + \frac{c_1}{2} > 0 \] (47)

According to the above requirements the effective action (42) becomes exactly, fixing \( b = -1 \), the Schwinger model effective action (27). Clearly, we have to inspect the restrictions imposed by the gauge invariance condition (41). Namely,
\[ (D - 1)e = 0 \] (48)
\[ (a + b) [(D - 1)(c_2f - 2c_1g) - 2c_4(f + Dg)] = 0 \] (49)
\[ (D - 1)f^2c_3 = g(D - 1)(c_2f - c_1g) - c_4(f^2 + 2fg + Dg^2). \] (50)

For future use we recall that in deducing (48), (49) and (50) we have assumed \( d = 0 \), \( c_4 \neq 0 \), \( c_4 + c_1/2 \neq 0 \) and that \( K \neq 0 \) which means, using \( e = 0 \) according to (48), that
\[ K = \Box^2 \left\{ (c_1 + c_2 + c_3 + c_4) [(D - 1)c_3 + c_4] - (D - 1) \left( c_3 + \frac{c_2}{2} \right)^2 \right\} \neq 0 \] (51)

Although there are several solutions for (49) and (50) some of them are related via trivial field redefinitions in our Ansatz (28). Here we stick to the simplest case. First of all we solve (49) by choosing \( b = -a \) which implies that the first two terms of (28) build up the Maxwell Lagrangian density. So the gauge symmetry is present before the integration over the \( W_{\mu\nu} \) fields. This is also the case of the TMBF model. For definiteness we choose \( a = 1 \) and \( b = -1 \). Moreover, if \( b = -a \) the field redefinition \( A_\mu \rightarrow A_\mu + c_2\partial_\mu W/f \) in the Ansatz (28) will bring \( c_2 \rightarrow 0 \). The coefficient \( c_3 \) also changes but we rename it \( c_3 \) again without loss of generality. Regarding \( c_4 \), we might think of choosing \( c_4 = 0 \) for simplicity. However, since \( d = 0 \), that leads to a zero in the denominator of (37). This indicates the appearance of a gauge symmetry. Indeed, if \( c_4 = 0 \), the Ansatz (28) becomes invariant under any transformation which preserves both the trace \( W \) and \( \partial^\mu W_{\mu\nu} \). They are given by the local transformations:
\[ \delta_\Lambda W_{\mu\nu} = \left[ \Box^2 P^{(2)}_{SS} \right]_{\mu\nu}^{\alpha\beta} \Lambda_{\alpha\beta} \rightarrow \delta_\Lambda W = 0 \] (52)
where \( \Lambda_{\alpha\beta} = \Lambda_{\beta\alpha} \) is an arbitrary symmetric tensor. In practice we can set \( c_4 = 0 \) but in order to integrate over \( W_{\mu\nu} \) the term \( c_4 \partial^\mu W_{\alpha\beta} \partial_\mu W^{\alpha\beta} \) must be replaced by a local gauge fixing term like
\[ \mathcal{L}_{GF}^{(2)} = \lambda_2 \left( \Box^2 \left[ P^{(2)}_{SS} \right]_{\mu\nu}^{\alpha\beta} W_{\alpha\beta} \right)^2 \] (53)
Now the denominator \((d-c_2\Box)^4\) of the first term of (37) is replaced by \(\lambda_2\Box^4\). Since \(T_{\mu\nu}\left[P^{(2)}_{SS}\right]_{\alpha\beta}\Gamma^{\alpha\beta}_{\mu\nu} = 0\) the effective action, see (36), does not depend on the arbitrary real constant \(\lambda_2\) as expected. Returning to our \(U(1)\) gauge invariance conditions, from (50) we must have \(c_3 = -(g/f)^2c_1\). Thus, we may choose

\[
a = 1 \quad ; \quad b = -a = -1 \quad ; \quad c_2 = 0 = c_4 \quad ; \quad c_3 = -\left(\frac{g}{f}\right)^2c_1, \tag{54}
\]

which leads to the model

\[
S_I = \int d^Dx \left\{ -\frac{1}{4}F_{\mu\nu}^2 + f W_{\mu\nu}\partial^\nu A^\mu + g W \partial \cdot A + c_1 \left[ (\partial^\nu W_{\mu\nu})^2 - \frac{g^2}{f^2}\partial^\mu W\partial_\mu W \right] \right\} \tag{55}
\]

We must have, see (44), (47) and (54), \(c_1 > 0\) and \(f \neq 0\) while \(g\) is an arbitrary real constant. Unfortunately, the action \(S_I\) is not invariant under usual \(U(1)\) gauge transformation in general but rather under a higher derivative form of it,

\[
\delta_{\phi} A_\mu = \partial_\mu \Box \phi \tag{56}
\]

\[
\delta_{\phi} W_{\mu\nu} = -\frac{f}{2c_1(D-1)g} \left[ (f + g D) \partial_\mu \partial_\nu \phi - (f + g) \eta_{\mu\nu} \Box \phi \right] \tag{57}
\]

Furthermore, if \(f \neq -g D\) we can redefine the fields according to

\[
A_\mu = \tilde{A}_\mu - \frac{2c_1 g}{f(g + g D)} \partial_\mu \tilde{W} \tag{58}
\]

\[
W_{\mu\nu} = \tilde{W}_{\mu\nu} - \frac{g}{f + g D} \eta_{\mu\nu} \tilde{W} \tag{59}
\]

which is equivalent to set \(g = 0\) in \(S_I\). The new vector field \(\tilde{A}_\mu\) is gauge invariant \((\delta_{\phi} \tilde{A}_\mu = 0)\) while \(\delta_{\phi} \tilde{W}_{\mu\nu} = f(g + g D) \Box \theta_{\mu\nu} \phi / [2c_1(1 - D)g]\). This transformation preserves \(\partial^\mu \tilde{W}_{\mu\nu}\).

The \(U(1)\) gauge symmetry is no longer manifest in \(S_I(g = 0)\). However, in the \(W\) sector we have a larger symmetry now since the trace \(W\) is absent and any transformation which preserves \(\partial^\mu \tilde{W}_{\mu\nu}\) is a symmetry. So the \(U(1)\) symmetry moves to the \(W\) sector. The action \(S_I(g = 0)\) is equivalent, with the normalization \(c_1 = 2/m_2^2\) and \(f = -2\), to the dual model (12) obtained from the MP theory via master action. It is surprising to end up without explicit \(U(1)\) gauge symmetry after imposing the gauge invariance conditions (48), (49) and (50). However, the derivation of those conditions requires \(K \neq 0\) which is not valid for \(g = 0\) since, see (54), in this case \(c_2 = 0 = c_4 = c_3\).

On the other hand, if \(f = -D g\) we can choose, recalling (46), without loss of generality \(f = m^2 = -D g\) and \(c_1 = m^2/2\). Back in (55) we have:

\[
S_{II} = \int d^Dx \left\{ -\frac{1}{4}F_{\mu\nu}^2 + m^2 \left[ W_{\mu\nu} - \frac{W}{D} \eta_{\mu\nu} \right] \partial^\nu A^\mu + \frac{m^2}{2} \left( \partial^\mu W_{\mu\nu} \partial^\alpha W_{\alpha\nu} - \frac{1}{D^2} \partial^\mu W \partial_\mu W \right) \right\} \tag{60}
\]

The action \(S_{II}\) is explicitly invariant under usual (first-order) \(U(1)\) gauge transformations:
\[ \delta_\phi A_\mu = \partial_\mu \phi \quad ; \quad \delta_\phi W_{\mu\nu} = \eta_{\mu\nu} \phi \]  

(61)

After adding an appropriate gauge fixing term like (53) and integrating over \( W_{\mu\nu} \) in the path integral we end up with the effective action of the Schwinger type, see (27). Thus, \( S_{II} \) is a new action dual to the Maxwell-Proca theory with manifest usual \( U(1) \) symmetry. It is important to notice however, that since \( \delta_\phi W = D\phi \) we can always change variables to a gauge invariant vector field \( A_\mu \rightarrow A_\mu - \partial_\mu W/D \) and loose the manifest \( U(1) \) symmetry. The action now becomes:

\[ S_{II-b} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^2 + m^2 \left( W_{\mu\nu} - \frac{W}{D} \eta_{\mu\nu} \right) \partial^\mu A^\nu + \frac{m^2}{2} \left( \partial^\mu W_{\mu\nu} - \frac{\partial_\nu W}{D} \right)^2 \right] \]  

(62)

One can say that the initial massless vector field \( A_\mu \) has eaten up the trace \( W \) and became massive as in the usual Stuckelberg mechanism. Notice also that in \( S_{II-b} \) only the traceless piece of \( W_{\mu\nu} \) effectively appears contrary to (60). The action \( S_{II-b} \) is invariant under the spin-2 local transformations (52) but since the whole \( U(1) \) symmetry is shifted to the \( W_{\mu\nu} \) sector.

The quadratic terms in \( W_{\mu\nu} \) are also invariant under Weyl transformations \( \delta_\phi W_{\mu\nu} = \eta_{\mu\nu} \phi \) which require another gauge fixing term. We can choose for instance:

\[ L_{GF}^{(0)} = \lambda_0 \left[ \Box^2 \left( P_{\mu\nu}^{(0)} \right)^{\mu\nu}_{\alpha\beta} W_{\mu\nu} \right]^2 \]  

(63)

After adding \( L_{GF}^{(2)} \) and \( L_{GF}^{(0)} \) to (62) we can integrate over \( W_{\mu\nu} \) and obtain an effective action, independent of both \( \lambda_0 \) and \( \lambda_2 \), which becomes exactly the Maxwell-Proca theory:

\[ L_{\text{eff}} [A] = L_{\text{MP}} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{m^2}{2} A_\mu A^\mu \]  

(64)

It is easy to check that \( K = 0 \) for (62) which explains the loss of manifest \( U(1) \) gauge symmetry once again. The equations of motion of (62) can be written as

\[ \Box_{\mu\nu} A^\nu - m^2 v_\mu = 0 \quad ; \quad v_\mu = \partial_\rho W_{\rho\mu} - \frac{\partial_\mu W}{D} \]  

(65)

\[ \partial_\mu q_\nu + \partial_\nu q_\mu = \frac{2}{D} \eta_{\mu\nu} \partial \cdot q \quad ; \quad q_\mu = A_\mu - v_\mu \]  

(66)

Note that the vectors \( A_\mu, v_\mu \) and consequently \( q_\mu \) are \( U(1) \) gauge invariant. General coordinate transformations in a flat space time changes the metric tensor according to \( \delta_\xi g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \). Conformal transformations require that \( \delta_\xi g_{\mu\nu} = \Lambda g_{\mu\nu} \) whose trace implies \( \Lambda = 2(\partial \cdot \xi)/D \). Therefore, the general solution to the first equation of (66) corresponds exactly to conformal transformations

\[ q_\mu = A_\mu - v_\mu = a_\mu + \Lambda_{\mu\nu} x^\nu + \lambda x_\mu + 2 x_\mu (x \cdot c) - x^2 c_\mu \]  

(67)

where the antisymmetric matrix \( \Lambda_{\mu\nu} \) and \( a_\mu, b_\mu, c_\mu, \lambda \) are constant parameters. Since the fields must vanish at infinity, all those constant parameters must vanish. So \( q_\mu = v_\mu - A_\mu = 0 \).
allows us to replace $v_\mu$ by $A_\mu$ in (65) which becomes, as expected from the effective action, the Maxwell-Proca equation $\Box \theta_{\mu\nu} A^\nu - m^2 A_\mu = 0$.

After eliminating $v_\mu$ in terms of $A_\mu$ we are still left with degrees of freedom in $W_{\mu\nu}$ which do not contribute do the combination $v_\mu$ however, those are exactly the pure gauge degrees of freedom related to the symmetries of (62). So the duality between (62) and the Maxwell-Proca theory is also established at classical level as expected.

At this point one might ask whether it is possible to define a unitary theory containing only one massive spin-1 particle starting with quadratic terms in $W_{\mu\nu}$ and the vector field $A_\mu$ such that the manifest $U(1)$ symmetry can not be removed by any local field redefinition as in the TMBF model. In the unitary dual models $S_I$ and $S_{II}$ it was possible to define a gauge invariant linear combination of the vector fields $A_\mu$ and $\partial_\mu W$ and remove the manifest $U(1)$ symmetry.

One might blame the choice (54) for the existence of a $U(1)$ gauge invariant vector field which leads to the lack of manifest gauge invariance in general. Next we give a symmetry argument to show that even for the general Ansatz this will be always possible. So let us return to the general Ansatz (28) and address this question from the point of view of gauge transformations. Namely, the $U(1)$ gauge transformation which leave the Ansatz (28) invariant must be of the general form

$$\delta_\phi A_\mu = \partial_\mu \phi \quad ; \quad \delta_\phi W_{\mu\nu} = r \phi \eta_{\mu\nu} + s \Box \phi \eta_{\mu\nu} + t \partial_\mu \partial_\nu \phi$$

(68)

where $(r, s, t)$ are real constants. The variation of the Ansatz includes the following independent terms:

$$\delta S = \int d^D x \left[ 2r(D e + d) W \phi + r(f + D g) \partial^\mu A_\mu \phi + (f - 2r c_1 - D r c_2 + 2d t) \partial^\mu \partial^\nu W_{\mu\nu} \phi + \cdot \cdot \cdot \right]$$

(69)

Therefore, among other constraints, we have the following ones

$$r(D e + d) = 0 \quad \quad \quad \quad \quad (70)$$
$$r(f + D g) = 0 \quad \quad \quad \quad \quad (71)$$
$$r(2c_1 + Dc_2) - 2d t = f \quad \quad \quad \quad \quad (72)$$

For only one massive particle in the spectrum we must have $d = 0$ and $f \neq 0$, therefore $r \neq 0$ so we can rescale $r \rightarrow 1$. It also follows that $e = 0$ and $f = -Dg$ which is in agreement with our previous results $S_I$ and $S_{II}$ since we have demanded usual (first-order) $U(1)$ transformations for the vector field.

On the other hand, the field redefinition

$$W_{\mu\nu} = \tilde{W}_{\mu\nu} + s \eta_{\mu\nu} \partial \cdot A + t \partial(\mu A_\nu)$$

(73)

will absorb the $t$ and $s$ factors such that $\delta_\phi W_{\mu\nu} = \eta_{\mu\nu} \phi$, i.e., we can set $s = 0 = t$ in (68). Therefore, we conclude that we are always able to make a field redefinition $A_\mu = \tilde{A}_\mu + \partial_\mu W/D$ to a gauge invariant vector field $\delta_\phi \tilde{A}_\mu = 0$ which jeopardizes the manifest $U(1)$ symmetry.
In fact, it is easy to show that the $U(1)$ symmetry will be indeed lost after the field redefinition since there will be no more contribution coming from $f W_{\mu\nu} \partial^\mu \delta_\phi \tilde{A}^\nu$ to the last (explicit) term of (69). Consequently, the new quadratic terms in $W_{\mu\nu}$ must satisfy $c_2 = -2c_1/D$. So the gauge variation of the quadratic terms in $W_{\mu\nu}$ cancel out with no need of any contribution coming from the vector field. Since a nonvanishing mass requires $f \neq 0$, it is clear that a possible $U(1)$ variation $f W_{\mu\nu} \partial^\mu \delta_\phi \tilde{A}^\nu = f W_{\mu\nu} \partial^\mu \partial^\nu \phi$ can not be compensated by the variation of $W_{\mu\nu}$ fields and the manifest $U(1)$ symmetry is lost.

In practice we have checked that even for other choices different from (54), it is always possible to redefine the fields and end up without manifest $U(1)$ symmetry.

4 Conclusion

In the topologically massive BF model (TMBF), also named Cremmer-Scherk model, the photon acquires mass without need of a Higgs field while keeping the $U(1)$ gauge symmetry manifest in the action. It is not possible in this case to remove the $U(1)$ symmetry from the action by any local field redefinition. In this model the vector field is coupled to an antisymmetric tensor. Motivated by the TMBF model we have investigated here the possibility of generating mass for the photon, in a $U(1)$ invariant way, by coupling the vector field to a symmetric rank-2 tensor instead. Since the TMBF model can be interpreted as a dual version of a first-order formulation of the Maxwell-Proca theory, we have applied standard dualization methods to a first-order form of the Maxwell-Proca theory which makes use of a symmetric tensor, see [14]. In particular, we have used the master action and Noether gauge embedment methods. The later has led us to a non-unitary theory while the former method has furnished the model (12) which is dual to the Maxwell-Proca theory in arbitrary $D$ dimensions without however, manifest $U(1)$ gauge symmetry.

In section 3 we have applied another procedure. After starting with a rather general second-order (in derivatives) action, see (28), involving quadratic terms in the vector and tensor fields, we have integrated in the path integral over the tensor field and obtained an effective action for the vector field. Requiring that the effective vector theory be $U(1)$ invariant and contain only one massive physical particle in the spectrum we have deduced a set of constraints for the couplings. In particular, we have derived the $U(1)$ invariant unitary models (55) and (60). However, after a local redefinition of the vector field $A_\mu \rightarrow A_\mu - \partial_\mu W/D$, involving the trace $W = W^\mu_\mu$, the manifest $U(1)$ symmetry is lost very much like in the usual Stueckelberg formalism although our action is rather different than the usual Stückelberg form of the Maxwell-Proca theory. In our case the trace $W$ is eaten up by the vector field which becomes massive. We have also tried other solutions of the constraint equations but it turns out that it is always possible to eat up the trace and end up without explicit $U(1)$ symmetry. We have given a symmetry argument explaining that point. Clearly, one might try to include higher derivative (above second-order) terms in the action but they are expected to jeopardize unitarity.

Regarding the TMBF model, the key difference seems to be that the $U(1)$ gauge symmetry of the vector field does not need to be compensated by any transformation of the auxiliary two-
form field unlike the case investigated here where the symmetric rank-2 tensor must transform nontrivially.

We are currently investigating a non-abelian extension of our results. Moreover, in [22] the coupling of higher spin particles to the the electromagnetic field has been studied leading to some apparently universal conclusions. In [22] the usual Stückelberg formalism has been employed. It is desirable to check the universality of their results via an alternative gauge invariant formulation for massive particles as given here. We are working on a generalization of our approach to higher spin charged particles

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References

[1] S.Deser and R. Jackiw, Phys.Lett.B 139 (1984) 371.
[2] R.L.P.G. Amaral and C.P. Natividade, Phys.Rev.D58:127701 (1998).
[3] D. Dalmazi, J. of High Energy Phys. 0608:040 (2006).
[4] E. Cremmer and J. Scherk, Nucl.Phys.B72:117-124 (1974).
[5] M. Botta Cantcheff, Phys.Lett.B533:126-130 (2002).
[6] R. Menezes, J.R.S. Nascimento, R.F. Ribeiro and C. Wotzasek, Phys. Lett. B537 (2002) 321.
[7] T.J. Allen, M.J. Bowick and A. Lahiri, Modern Phys. Lett. A6 559 (1991).
[8] M. Henneaux , V.E.R. Lemes, C.A.G. Sasaki, S.P. Sorella, O.S. Ventura and L.C.Q. Vilar, Phys.Lett.B410:195-202 (1997).
[9] D. Freedman and P.K. Townsend, Nucl. Phys. B177 (1981) 282.
[10] A. Lahiri, “Generating vector boson masses”, hep-th/9301060 (1993); A. Lahiri Phys.Rev.D55:5045-5050,1997;
[11] E. Harikumar, A. Lahiri and M. Sivakumar, Phys.Rev. D63 (2001) 105020.
[12] G.K. Savvidy, Phys.Lett.B694:65-73,2010.
[13] M. Botta Cantcheff, Int.J.Mod.Phys.A20 (2005)2673.
[14] A. Khoudeir, R. Montemayor and L. F. Urrutia, Phys.Rev.D78:065041, (2008).
[15] D. Dalmazi and R.C. Santos, Phys. Rev. D 84, 045027 (2011), arXiv:1105.4364.

[16] S. Deser and P.K. Townsend, Phys.Lett.B98, 188, 1981.

[17] A. Ilha, “Aspectos e Dualidades em Teoria Quântica de Campos”, PhD Thesis (2002) UFRJ (in Portuguese); M.A. Anacleto, A. Ilha, J.R.S. Nascimento, R.F. Ribeiro and C. Wotzasek Phys.Lett.B504:268-274,2001

[18] E. Harikumar and M. Sivakumar Nucl.Phys. B565 (2000) 385-396.

[19] B. Podolsky, Phys. Rev. 62 68 (1942).

[20] A.P. Baêta Scarpelli, M. Botta Cantcheff and J.A. Helayel-Neto, Europhysics Lett. 65 (2003) 760.

[21] R. Amorim and J. Barcelos-Neto, Mod. Phys. Lett. A10: 917-924, (1995).

[22] M. Porrati, R. Rahman, Nucl.Phys. B814 (2009) 370-404.