Break-up channels in muon capture on $^3$He

J. Golak$^{1,a}$, R. Skibiński$^{1,b}$, H. Witała$^{1,c}$, K. Topolnicki$^{1,d}$, A. E. Elmeshneb$^{1,e}$, H. Kamada$^{2,f}$, A. Nogga$^{3,g}$, and L. E. Marcucci$^{4,h}$

$^1$M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30059 Kraków, Poland
$^2$Department of Physics, Faculty of Engineering, Kyushu Institute of Technology, Kitakyushu 804-8550, Japan
$^3$IKP-3, IAS-4, JCHP and JARA-HPC, Forschungszentrum Jülich, D-52425 Jülich, Germany
$^4$Department of Physics, University of Pisa, IT-56127 Pisa, Italy and INFN-Pisa, IT-56127 Pisa, Italy

Abstract. The $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ capture reactions are studied under full inclusion of final state interactions with the AV18 nucleon-nucleon potential, augmented by the Urbana IX three-nucleon force, and employing the single nucleon weak current operator. We give first realistic estimates of the total capture rates: $544\, s^{-1}$ and $154\, s^{-1}$ for the $n + d$ and $n + n + p$ channels, respectively. Our results are compared with the most recent experimental data, finding a rough agreement for the total capture rates, but failing to reproduce the differential capture rates.

1 Introduction

Muon capture reactions on light nuclei have been studied intensively both experimentally and theoretically for many years and information on earlier achievements can be found in Refs. [1–4]. Recent theoretical efforts, presented for example in Refs. [5–8], focused on the $\mu^- + ^2$H $\rightarrow \nu_\mu + n + n$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + ^3$H reactions. These calculations were based on various dynamical inputs, representing the so-called phenomenological approach, the “hybrid” chiral effective field theory ($\chi$EFT) approach and also the “non-hybrid” $\chi$EFT approach. A good agreement between the results obtained within different approaches was found, as well as between theoretical predictions and available experimental data.

In Ref. [9] we joined our expertise: in momentum space treatment of electromagnetic processes [10, 11] and in the potential model approach developed in Ref. [5] to perform a systematic study of all the $A = 2$ and $A = 3$ muon capture reactions. We compared results of new calculations carried out in the momentum space for the $\mu^- + ^2$H $\rightarrow \nu_\mu + n + n$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + ^3$H reactions with

\begin{footnotesize}
\begin{itemize}
  \item[a] a-e-mail: jacek.golak@uj.edu.pl
  \item[b] b-e-mail: roman.skibinski@uj.edu.pl
  \item[c] c-e-mail: henryk.witala@uj.edu.pl
  \item[d] d-e-mail: kacper.topolnicki@uj.edu.pl
  \item[e] e-e-mail: alaa.elmeshneb@uj.edu.pl
  \item[f] f-e-mail: kamada@mns.kyutech.ac.jp
  \item[g] g-e-mail: a.nogga@fz-juelich.de
  \item[h] g-e-mail: marcucci@df.unipi.it
\end{itemize}
\end{footnotesize}
those of Ref. [5], finding a very good agreement not only for the predictions obtained with the single nucleon current operator but also for the case, when the meson-exchange currents from Ref. [12] (Eqs. (4.16)–(4.39), without Δ-isobar contributions) were included. These calculations employed the AV18 nucleon-nucleon (NN) potential [13] supplemented with the Urbana IX three-nucleon (3N) force [14]. Results for the break-up channels in muon capture on $^3$He obtained with the same forces will be demonstrated in the next section.

Recently, improved chiral NN forces were published in Refs. [15, 16] and thus we would like to present here preliminary results for the $\mu^- + ^2$H $\rightarrow \nu_\mu + n + n$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + ^3$H reactions. The calculations used the single nucleon current operator [9], since no consistent current operators are available for these new potentials at the moment. Even if these reactions are not in the focus of the present paper and the calculations are not yet consistent, we believe they might be interesting for the reader. The spread of the results due to the different regulator parameters gets very narrow for the N4LO predictions and we obtain $\Gamma_{d}^{1/2} \in [384.6, 386.3]$ s$^{-1}$ for the total doublet capture rate in the case of the $\mu^- + ^2$H $\rightarrow \nu_\mu + n + n$ reaction and $\Gamma \in [1285, 1308]$ s$^{-1}$ for the total capture rate in the case of the $\mu^- + ^3$He $\rightarrow \nu_\mu + ^3$H process. Future calculations will combine the presently included dynamical ingredients with consistent two-body currents and three-nucleon forces, to produce complete chiral EFT calculations at high orders. The convergence pattern of such calculations can then be used to assess the theoretical uncertainty in predictions for the muon capture rates.

2 Results for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ reactions

In Ref. [9] we provided, for the first time, predictions for the total and differential capture rates of the reactions $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ and $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$. To this aim we first considered kinematically allowed regions for these two reactions (see Fig. 1) and made sure that the non-relativistic kinematics could be used consistently with the corresponding non-relativistic dynamics. Our essential results for the differential capture rates are shown in Fig. 2. We see that inclusion of the 3N force reduces the peak heights by about 20%. It is also clear that the main contributions to the total decay rates come from the regions with highest neutrino energies.

The corresponding total decay rates for the two break-up channels are displayed in Table 1. The 3N force effects for the two reactions are comparable and amount to about 10%. In the same table we collect also the experimental results from Refs. [17–20] and find a fair agreement with most of them.

In Ref. [9] we analyzed also most recent data on more differential capture rates published in Ref. [20]. The data in Ref. [20] were evaluated by means of two methods (denoted by "md I" and "md II" in Table 1). In Fig. 3 we show the second set of data points for the averaged capture rates and our theoretical results obtained without and with inclusion of 3N force effects. For the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ reaction our predictions in the whole range of the deuteron energies clearly underestimate the data by nearly a factor of 2. In the case of the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ process we underestimate the data for smaller proton energies and overshoot them for the higher proton energies.

It is very important to realize that only these data points were used to obtain the total capture rates by means of simple extrapolations. If we combine information from Figs. 1 and 2, we see that the interval $E_d \in [13, 30]$ MeV corresponds to the interval $E_p \in [10, 75]$ MeV. Thus one misses the bulk of the contribution to the total capture rate. The situation for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ process is very similar. Our results raise doubts if the extrapolations performed in [20] lead to correct results for the total capture rates.

Undeniably, further theoretical work and new precision measurements in the whole kinematical region are needed to improve our knowledge about break-up channels in muon capture on $^3$He.
Figure 1. The kinematically allowed regions in the $E_\nu - E_d$ plane for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ process (left panel) and in the $E_\nu - E_p$ plane for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ process (right panel) calculated relativistically (solid curve) and nonrelativistically (dashed curve).

Figure 2. The differential capture rates $d\Gamma_{nd}/dE_\nu$ for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ process (left panel) and the differential capture rates $d\Gamma_{nnp}/dE_\nu$ for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ reaction (right panel) calculated with the single nucleon current operator without (dashed curve) and with the 3N force (solid curve). The calculations are based on the AV18 nucleon-nucleon potential [13] and the Urbana IX 3N force [14].

Table 1. Capture rates in s$^{-1}$ for the $\mu^- + ^3$He $\rightarrow \nu_\mu + n + d$ ($\Gamma_{nd}$) and $\mu^- + ^3$He $\rightarrow \nu_\mu + n + n + p$ ($\Gamma_{nnp}$) processes and their sum ($\Gamma_{br} \equiv \Gamma_{nd} + \Gamma_{nnp}$) calculated with the AV18 [13] nucleon-nucleon potential and the Urbana IX [14] 3N force, using the single nucleon current and including final state interaction effects.

| Process     | AV18  | AV18+Urb. IX | [17]   | [18]   | [19]   | [20]-md I | [20]-md II |
|-------------|-------|--------------|--------|--------|--------|-----------|------------|
| $\Gamma_{nd}$ | 604   | 544          |        |        |        | 491 ± 125 | 497 ± 57   |
| $\Gamma_{nnp}$ | 169  | 154          |        |        |        | 187 ± 11  | 190 ± 7    |
| $\Gamma_{br}$ | 773   | 698          | 660 ± 160 | 665 $^{+170}_{-430}$ | 720 ± 70  | 678 ± 126 | 687 ± 60   |
Figure 3. The capture rates \( \langle \frac{d\Gamma_{nd}}{dE_d} \rangle \) for the \( \mu^- + ^3\text{He} \rightarrow \nu_\mu + n + d \) process (left panel) and the capture rates \( \langle \frac{d\Gamma_{np}}{dE_p} \rangle \) for the \( \mu^- + ^3\text{He} \rightarrow \nu_\mu + n + n + p \) process (right panel) averaged over 1 MeV deuteron or proton energy bins compared with the experimental data given in Tables VI and V of Ref. [20] evaluated by means of method II. Curves are the same as in Fig. 2.

Acknowledgements
This work was supported from the resources of the National Science Center (Poland) under grant DEC-2013/10/M/ST2/00420. Numerical calculations have been performed on the supercomputer clusters of the JSC, Jülich, Germany.

References
[1] R. Skibiński, J. Golak, H. Witała, and W. Glöckle, Phys. Rev. C 59, 2384 (1999)
[2] D.F. Measday, Phys. Rep. 354, 243 (2001)
[3] T. Gorringe and H.W. Fearing, Rev. Mod. Phys. 76, 31 (2004)
[4] P. Kammel and K. Kubodera, Annu. Rev. Nucl. Part. Sci. 60, 327 (2010)
[5] L.E. Marcucci et al., Phys. Rev. C 83, 014002 (2011)
[6] L.E. Marcucci, Int. J. Mod. Phys. A 27, 1230006 (2012)
[7] L.E. Marcucci et al., Phys. Rev. Lett. 108, 052502 (2012)
[8] J. Adam, Jr. et al., Phys. Lett. B 709, 93 (2012)
[9] J. Golak et al., Phys. Rev. C 90, 024001 (2014)
[10] J. Golak et al., Phys. Rept. 415, 89 (2005)
[11] R. Skibiński, J. Golak, H. Witała, W. Glöckle, and A. Nogga, Eur. Phys. J. A 24, 11 (2005)
[12] L.E. Marcucci et al., Phys. Rev. C 63, 015801 (2000)
[13] R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995)
[14] B.S. Pudliner et al., Phys. Rev. C 56, 1720 (1997)
[15] E. Epelbaum, H. Krebs, U.-G. Meißner, Eur. Phys. J. A 5, 53 (2015)
[16] E. Epelbaum, H. Krebs, U.-G. Meißner, arXiv:1412.4623 [nucl-th] (2014)
[17] O.A. Zalimendoroga et al., Phys. Lett. 6, 100 (1963)
[18] L.B. Auerbach et al., Phys. Rev. 138, B127 (1965)
[19] E.M. Maev et al., Hyp. Interact. 101/102, 423 (1996)
[20] V.M. Bystritsky et al., Phys. Rev. A 69, 012712 (2004)