Multiple Internal Reflections in Acrylic Triangle Waveguide

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Abstract. Theoretical and experimental study of light propagation in acrylic triangle waveguides has been studied theoretically and experimentally aimed to obtain a light cone which can tunnel and bring the light to a certain place. Numerical aperture of light cones which have a length of 7 cm but different cross-section area have been measured using He-Ne laser (λ=632 nm) and the results were compared to those obtained theoretically and no significant different was observed.

1. Introduction

Over decades, multiple internal reflection effect has shown its important role in modern technology. Light propagation in an optical fiber used for optical communication is one of the well-known examples. Using ultra-low loss optical fiber, an optical signal can propagate and maintain its intensity over very long distance. Parallel to the advancement of fiber optic for long distance communication, fiber sensors have become an interesting research topic exploring the unique light propagation in fiber optic under different physical or chemical environment conditions [1-5]. In addition of that, multiple internal reflections have also been used to design light trapping [6-7]. Light is trapped inside the fiber or two reflected surface such that as the light goes deeper inside the trapper more energy will be absorbed. Tapered optical fiber is one of the fiber types mostly used for fiber sensor [8-10]. The diameter of this fiber can be changed either permanently or temporarily. Gradual change of the fiber diameter can decrease the reflection angle at core-cladding interface results in decreasing light intensity coming out of the fiber.

Similar to the tapered optical fiber, acrylic triangle waveguides presented in this paper is designed to focus the light with no light radiating out from the waveguide sides. Theoretical calculation was made such that for a given maximum incident angle and the base triangle angle, we can find the maximum length of the triangle at which no light coming out from the triangle sides.

2. Experiment

The aim of the research reported in this paper is to find an acrylic triangle configuration that for a certain value of incident angle, the light will be guided through it by multiple internal reflection mechanism. Light is incident at the triangle entrance window and coming out at the other end of it. Between these two windows, it is expected that no light is radiated out. In order to theoretically calculate the triangle
numerical aperture of a given triangle configuration that meets the above condition, data of acrylic index of refraction is required. Figure 1 is experimental setup used to measure the index of refraction designed based Brewster angle method. Transverse magnetic (TM) polarized light was incident onto the PMMA sample positioned at the center of a rotational table of a spectrometer. The reflected light intensity was detected by a photo-detector. A curve showing the intensity of reflection angle as function of incident angle was then constructed. From this curve, we can obtain the value of Brewster angle ($\theta$) defined as the incident angle results in minimum reflected light intensity. The PMMA refractive index ($n_s$) can then be calculated using equation

$$\tan \theta = \frac{n_s}{n_a}$$

where $n_a$ is the air refractive index.

Figure 1. Experimental setup used to measure refractive index.

Figure 2. Basic setup for measuring triangle numerical aperture

Figure 2 is basic experimental setup used to measure numerical apperture all acrylic triangle. Samples with the same thickness (0.5 cm) but different length (6.0 cm, 6.5 cm, and 7.0 cm) and different incline angle ($83^0$, $85^0$ and $87^0$) were put at the centre of the rotating table. Monochromatic light was incident on the wider window at two fixed points: at the edge of the window and at the axis of the triangle. Laser light was then rotated such that the incident angle observed at the side of triangle wall will get smaller and smaller. Maximum value of the incident angle giving no light radiated out from the triangle edge is defined as the numerical aperture.

3. Results and Discussion

3.1. Refractive index

Reflection coefficient of polarized light in both TE and TM modes are shown in Figure 3. Reflectivity of TM mode polarized light shows a minimum value (Brewter’s angle) at $56.33^0$. Inserting this value into equation (1) and taking the air index of refraction equals 1, we can find value of the acrylic index of refraction which is 1.500.

3.2. Numerical aperture

Numerical aperture of acrylic triangle waveguide represents an acceptable angle within which light can pass through the waveguide with no light radiate out of from the triangle wall. If light is launched into the triangle from the wider window within this acceptable angle, light is reflected back and forth inside the triangle by multiple internal reflection. No light coming out from wall. Instead, the light will only come out from the small window (Figure 4).
To formulate the relation between the triangle configuration and its corresponding numerical aperture, let’s look at figure 5. Light enters the triangle at an angle $\theta_i$ and then propagates through the triangle by multiple reflections mechanism. As the point N is shifted to the left to point K and Figure 5 is cut along the S-axis, Figure 5 can then be redrawn to become Figure 6. All points initially at LB wall (Figure 5) are mirrored by the S axis into the AC wall. As the light propagates down to the other end of the triangle, it can be seen that the reflected angles get smaller and smaller. To calculate these reflected angles, let’s see Figure 6. From $\Delta ABO$, it is found that
\[ 180^\circ = \angle B_1 + 90^\circ + \alpha \]  
\[ \text{or} \quad \angle B_1 = 90^\circ - \alpha \]  
\[ \text{Since} \quad 90^\circ = \angle B_1 + \angle B_2 \]  
By substituting equation (3) into equation (4), we found
\[ \angle B_2 = \alpha \]  
Looking back to \( \triangle ABC \) and remembering that the summation of their angles is 180°, it is found that
\[ 180^\circ = \gamma + (2 \times \angle B_2) + \angle C_1 \]  
By substituting equation (5) into equation (6) it found that
\[ \angle C_1 = 180^\circ - (2\alpha + \gamma) \]  
and
\[ \angle C_2 = (2\alpha + \gamma) - 90^\circ \]  
By applying similar procedure, other reflected angles can be calculated, they are
\[ \angle E_2 = (4\alpha + 3\gamma) - 270^\circ \quad 2^{\text{nd}} \text{reflection} \]  
\[ \angle G_2 = (6\alpha + 5\gamma) - 450^\circ \quad 3^{\text{rd}} \text{reflection} \]  
\[ \angle I_2 = (8\alpha + 7\gamma) - 540^\circ \quad 4^{\text{th}} \text{reflection} \]  
For \( i^{\text{th}} \) reflection, its corresponding angle of reflection is
\[ \beta_i = 2i\alpha + (2i - 1)\gamma - (2i - 1)90^\circ \] (9)  
By taking
\[ \gamma = 90^\circ - \varphi + \theta_r \]  
\[ \alpha = 2\varphi - 90^\circ - \theta_r \]  
\[ \theta_r = \sin^{-1}\left(\frac{n_1}{n_2} \sin \theta_1 \right) \]  
Equation (9) can be expressed as
\[ \beta_i = (2i + 1)\varphi - \theta_r - 2i \cdot 90^\circ \] (10)  
If \( \varphi \) (incline angle) and \( n_2 \) (acrylic refractive index) or \( \theta_r \) (refraction angle) are known, the \( i^{\text{th}} \) reflection angle can be calculated. The distance between the two consecutive angle points as shown in Figure 6 are
\[ X_1 = r \tan \alpha \]  
\[ X_2 = \frac{r \sin \gamma \tan \alpha}{\sin(2\alpha + \gamma)} \]  
\[ X_3 = \frac{r \sin \gamma \tan(3\alpha + 2\gamma)}{\sin(2\alpha + \gamma)} \]
in general form

\[ X_n = \frac{r \sin \gamma \tan(n\alpha + (n-2)\gamma)}{\sin(n\alpha + (n-1)\gamma)} \]  

Taking the triangle height as

\[ L = \sum X_n \]  

**Table 1.** Numerical apertures comparison between those calculated theoretically and experiment at \( \lambda = 532 \) nm.

| Sample | Inclination Angle | Triangle height (cm) | Entrance window width (cm) | Numerical Aperture |
|--------|-------------------|----------------------|---------------------------|-------------------|
|        |                   |                      |                           | Theoretical       |
|        |                   |                      |                           | Experiment \( \lambda = 632 \) nm | Percent error |
| A1     | 83°               | 6.0                  | 2.0                       | 21.69             | 21.70          | 0.05 |
| A2     | 83°               | 6.5                  | 2.0                       | 20.79             | 20.73          | 0.03 |
| A3     | 83°               | 7.0                  | 2.0                       | 18.18             | 18.17          | 0.06 |
| B1     | 85°               | 6.0                  | 2.0                       | 40.27             | 40.27          | 0.00 |
| B2     | 85°               | 6.5                  | 2.0                       | 35.18             | 35.17          | 0.03 |
| B3     | 85°               | 7.0                  | 2.0                       | 33.81             | 33.80          | 0.03 |
| C1     | 87°               | 6.0                  | 2.0                       | 55.20             | 55.20          | 0.00 |
| C2     | 87°               | 6.5                  | 2.0                       | 54.84             | 54.83          | 0.02 |
| C3     | 87°               | 7.0                  | 2.0                       | 51.44             | 51.42          | 0.04 |

Once all parameters characterize an acrylic triangle are given, its numerical aperture can be calculated. Table 1 shows the numerical aperture values between those calculated based on equation (10) and (12) and those obtained experimentally. These two results show a good agreement. Small percent error might come from mainly uncertainties in preparing sample dimension. The incline angle, triangle height, and entrance window width as shown in Table 1 are the planned dimensions. The actual prepared dimensions are slightly different from those given in Table 1. Samples with the same incline angle show an increase in their numerical aperture with the increase of their heights. In addition, sample numerical aperture increases with increasing their incline angles.
4. Conclusions

This manuscript attempts to find a basic guidance to produce acrylic triangle waveguide theoretically and experimentally. Nine acrylic triangle configurations have been made. It has been shown that the triangle numerical aperture increase with increasing the incline angle and decreases with increasing the triangle length. Results obtained experimentally are in good agreement to those calculated theoretically (percent error less than 0.1 %).

5. References

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