The thermal dilepton rate at NLO at small and large invariant mass

Jacopo Ghiglieri

Institute for Theoretical Physics, Albert Einstein Center, University of Bern, Sidlerstrasse 5, CH-3012, Bern, Switzerland

Abstract

We report on a recent next-to-leading order perturbative determination of the dilepton rate from a hot QCD plasma for frequency and momentum of the order of the temperature and for much smaller invariant mass $M \sim gT$. We briefly review the calculation, which generalizes the previous one for the photon case ($M = 0$). We then analyze the consequences of the new calculation for the extraction of the photon rate from the small mass dilepton measurements. We then review a recent NLO determination at large $M$ and we show how to match and merge its results with the low-mass ones, resulting in a single rate which is NLO-accurate over the phenomenologically relevant region.

Keywords: Dileptons, Hard Probes, Quark-Gluon Plasma, High order calculations, Lattice QCD

1. Introduction

Electromagnetic (EM) probes have long been considered a key hard probe of the medium produced in ultrarelativistic heavy-ion collisions. Their chief advantage is that they are weakly coupled to the plasma, so that their reinteractions with it can be considered negligible. EM probes hence carry direct information about their formation process to the detectors, unmodified by hadronization or other late time physics.

In this contribution we will concentrate on dileptons. Compared to photons, the kinematics of dileptons is described by two parameters, the frequency $k_0^0$ and the momentum $k$, with the related invariant mass $M \equiv \sqrt{k_0^0 - k^2}$. From an experimental point of view, dileptons, compared to photons, have the advantage of a smaller background from meson decays, which needs to be subtracted. For this reason, experimentalists have also focused on small-mass dileptons, which can be thought of as massive off-shell photons. Provided the mass of the pair is above the pion mass, the pion decay background is absent and the foreground rates are under much better control. For this reason, $e^+e^-$ pairs with $M$ somewhat above $m_{\pi}^2$ have been measured, to serve as an ersatz photon rate measurement [1, 2, 3].

In this contribution we will then first illustrate a recent perturbative calculation of the thermal dilepton rate at small $M$ (and for $k \sim T$) to NLO [4], extending the previous work on real photons [5], aiming also at understanding whether the rate, as a function of $M$, is smooth enough in going from $M = 0$ to finite $M$, so that the ersatz photon rate measurements are meaningful. We will afterwards show the results of an NLO calculation at larger $M$ [6] and then show how the small- and large-$M$ computations can be merged [7, 4], resulting in a rate that is reliably NLO for most invariant masses. Another motivation for these NLO calculation is to assess the reliability of the pQCD rates, widely employed in phenomenological analyses, when extrapolated to $\alpha_s \sim 0.3$ where the coupling $g$ is not small. We will then conclude by remarking on the implications of the results on this matter, with an outlook to comparisons with non-perturbative lattice data. In all cases the starting point is the formula giving the dilepton production rate per unit phase space at leading order in QED (in $\alpha$) and to all orders in QCD. It reads (see for instance Ref. [8])

$$\frac{d\Gamma_{d\ell}}{d^2K} = -\frac{2\alpha}{3(2\pi)^4K^2}W^c(K)\theta((k_0^0)^2 - K^2),$$

where $K^2 = (k_0^0)^2 - k^2 = M^2$ is the virtuality of the dilep-
ton pair, assumed much greater than $4m_T^2$. The rate is given in terms of the photon polarization $W^\omega(K)$, which reads

$$W^\omega(K) = \int d^3 X e^{ikX} Tr J^\mu(0) J_\mu(X). \quad (2)$$

Here $J^\mu = \sum_{\text{quads}} e_\nu \gamma^\nu q\bar{q}$ is the EM current and we work in thermal equilibrium, so that the $\rho = e^{-\beta T}$ and the Hilbert space trace becomes a thermal average.\(^1\) We will work perturbatively in the strong coupling $g$, meaning that we treat the scale $gT$ (the soft scale) as parametrically smaller than the scale $T$ (the hard scale).

### 2. NLO at small $M$

By small $M$ we mean $M \sim gT$ (and $k \sim T$), so that, as we will show, the calculation shares many similarities with the one for real photons \([5]\). In a naive perturbative expansion the leading order term would be the Born term, corresponding to the amplitude of the simple diagram shown in Fig. 1. However, its contribution to $W^\omega$ scales approximately like $M^2$, so that in our case its contribution is suppressed and other processes, apparently of higher loop order, contribute at the same (leading) order in $g$. These are the 2 ↔ 2 processes shown in Fig. 2 and the collinear processes shown in Fig. 3. The former require some care when the $t$ or $u$ channel exchanged quark become soft: the resulting logarithmic divergence is cured by Hard Thermal Loop resummation \([10, 11]\). At these small virtualities the calculation is unmodified w.r.t. the real photon case.

Collinear processes are apparently suppressed w.r.t. the 2 ↔ 2 ones. However, they receive an enhancement when the quark and antiquark (in the annihilation case) or the outgoing quark and the photon (in the bremsstrahlung case) are collinear. Furthermore, the soft scatterings that induce the splitting/annihilation are so frequent that, within the photon’s formation time, many of them can occur and interfere, in what is called the Landau-Pomeranchuk-Migdal (LPM) effect. Its treatment require the resummation of an infinite number of ladder exchanges of spacelike HTL gluons. This has been done for photons first \([12]\) and later extended to small-$M$ dileptons \([13]\). In the latter case it is important to note that the Born term in Fig. 1 is the zeroth-order term in the ladder resummation series, and is thus included in the treatment of the LPM effect.

In summary, the leading-order result can be written as\(^2\)

$$W^\omega(K)_{\text{LO}} = \frac{8 \alpha_{\text{EM}} m_T(k) g T^2}{3} \left[ \ln \left( \frac{T}{m_\omega} \right) + C_{2,2} \left( \frac{k}{T} \right) \right] + C_{\text{coll}} \left( \frac{k^2}{T^3 g T} \right), \quad (3)$$

where the logarithm comes from the screening of the aforementioned divergence and $m_\omega^2 = g^2 T^2/3$ is the thermal mass of quarks. $C_{2,2}$ is the coupling-independent part of the 2 ↔ 2 processes.

At NLO both processes receive $O(g)$ corrections: the soft end of the 2 ↔ 2 region is sensitive to the addition of one extra soft gluon and similarly the collinear sector requires the resummation of soft one-loop corrections to the ladder resummation. Furthermore a new process, the semi-collinear one, contributes. It can be seen as the next order in a collinear expansion, where the angle is allowed to be a bit larger. It thus interpolates between the 2 ↔ 2 and collinear limits. The

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\(^1\)A calculation in an off-equilibrium setting relevant for heavy-ion collisions has been presented at this conference in \([9]\).

\(^2\)for QCD with $uds$ light quarks
calculation of all of these corrections requires dealing with loops of soft HTL excitations, which are known to be a computational challenge, resulting in multidimensional numerical integrals over the intricate HTL propagators and vertices. However it has recently been found [14, 5, 15] that such calculations simplify tremendously when the related operators are at light-like separations, as is the case for photons and small-mass dileptons. Furthermore, the evaluation of soft and semi-collinear corrections for low-mass dileptons is unaffected by $M$ and thus identical to the corresponding photon case. Only the NLO LPM resummation needs to be modified in order to obtain the small-$M$ NLO correction. Numerical results will be shown in Sec. 4.

3. Merging small and large $M$

An NLO perturbative calculation for $M \sim T, k \sim T$ has been presented in [6]. In this region the Born term in Fig. 1 is the leading order and NLO is given by the $2 \leftrightarrow 2$ processes in Fig. 2, together with their $1 \leftrightarrow 3$ crossings, now kinematically allowed, and with virtual corrections to the Born term, as shown in Fig. 4. The NLO evaluation is rather intricate, as one has to deal with the complicated kinematics of these processes and with the fact that the virtual corrections to the Born term and the real corrections ($2 \leftrightarrow 2$ and $1 \leftrightarrow 3$ processes) are separately IR divergent, requiring intermediate regularizations. After the kinematics and divergences have been taken care of, one is left with a set of two-dimensional numerical integrals.

This large-$M$ calculation does not require HTL resummation; hence, it diverges logarithmically for small $M$. Conversely, the small-$M$ calculation described before behaves like $W^S \sim k^2(M^2 + T^2)$ at large $M$, whereas an OPE analysis [16] shows that no $T^2$-proportional term can exist for $M \gg T$. These drawbacks of the two calculations can be overcome by merging them in a single one which has the right behaviour both at small and large $M$. This can be done [7, 4] by taking the collinear part of the small $M$ calculation and expanding it for large $M$: the first and second term in that expansion correspond to terms that are already included in the large $M$ calculation. Thus, if the collinear part of the small $M$ calculation is added to the large-$M$ one, minus these two terms, the result is the sought after merged calculation. In [7] it was presented with the LO collinear part and in [4] the small-$M$ NLO corrections were added, leading to a merged result that is NLO-correct for $M \sim gT, M \sim \sqrt{gT}$ and $M \sim T$.

4. Results and discussion

The results of the aforementioned procedure are available online [17] and shown in Fig. 5, where the spectral function associated to $W^S(K)$ is plotted for different values of the coupling. The solid lines are the small-$M$ results only and the dashed ones are the merged ones. At the smallest coupling the spectral function varies rapidly across the light cone, whereas the extrapolation at larger couplings shows a much smoother behaviour\(^2\). This then gives more support to the use of small-$M$ dileptons as a proxy for real photons.

One might still wonder, however, how reliable these perturbative calculations are, in particular when extrapolated to $\alpha_s = 0.3$ where $g$ is not small. Fig. 6 shows that the NLO corrections represent at most a 30% increase, suggesting that the dilepton rate, similarly to the

\[^2\]The code used to obtain the collinear part of the rates at LO and NLO is available in the arXiv submission for Ref. [4].

\[^3\]The dashed are less smooth at the lightcone. However, as remarked in [7, 4], the more reliable calculation there is the one in solid lines.
photon one [5] is not plagued by the severe convergence problems affecting other observables in thermal pQCD.

A further assessment of the reliability of these calculation could come from an interplay of perturbative and non-perturbative inputs. As reported elsewhere in this conference [18], the direct extraction of the rates from lattice calculations is extremely tricky: the IQCD can only access the Euclidean $J\mu_J(\tau, \mathbf{x})$, which is related to the spectral function as

$$W_E(\tau, \mathbf{k}) \equiv \int d^3 x \left\langle J_\mu(\tau, \mathbf{x}) J_\mu(0) \right\rangle e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$= \int_0^\infty dk^0 \frac{dk^0}{2\pi} \rho_\tau(k^0, \mathbf{k}) \frac{\cosh\left(k^0(\tau - (1/2T))\right)}{\sinh\left(\frac{\alpha_s\tau}{2T}\right)}.$$  

(4)

The inversion of this convolution is an ill-defined problem. On the other hand, the continuation of the pQCD data to Euclidean spacetime is straightforward and can be used for comparisons with lattice data, as attempted in [6]. As observed there, some care is necessary, as most of the Euclidean correlator comes from the $k^0 \gg k$ region of the spectral function, which is dominated by well-understood vacuum physics, so that it might not be easy to disentangle the contribution from the more interesting $k^0 \sim k$ region.

5. Conclusions

We have shown how pQCD calculations are now available at NLO for the dilepton rate at finite $k$ in a wide kinematical range. After briefly reviewing the intricacies involved in the determination of the small and large $M$ rates, we have shown how the two can be merged in a single set, which is plotted in Fig. 5 and collected online for phenomenological use in [17]. The rates appear to be only mildly affected by NLO corrections even for $\alpha_s = 0.3$ and for these couplings are smooth across the light cone, giving support to the use of low-mass dileptons as an ersatz real photon measurement.

Acknowledgements I thank G. Moore and M. Laine for collaboration. My work is supported by the Swiss National Science Foundation (SNF) under grant 200020_155935.

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