Some new general integral inequalities for $P$-functions

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Abstract

In this paper, we derive new estimates for the remainder term of the midpoint, trapezoid, and Simpson formulae for functions whose derivatives in absolute value at certain power are $P$-functions. Some applications to special means of real numbers are also given.

Keywords: Convex function, $P$-function, Simpson’s inequality, Hermite-Hadamard’s inequality.

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1 Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function defined on the interval $I$ of real numbers and $a, b \in I$ with $a < b$. The following inequality holds:

$$f \left( \frac{a + b}{2} \right) \leq \frac{1}{b - a} \int_a^b f(x)dx \leq \frac{f(a) + f(b)}{2}. \quad (1.1)$$

This double inequality is well known as Hermite-Hadamard integral inequality for convex functions in the literature.

In [2] Dragomir et al. defined the concept of $P$-function as the following:

Definition 1.1. We say that $f : I \to \mathbb{R}$ is a $P$-function, or that $f$ belongs to the class $P(I)$, if $f$ is a non-negative function and for all $x, y \in I, \alpha \in [0, 1]$, we have

$$f(\alpha x + (1 - \alpha)y) \leq f(x) + f(y).$$

$P(I)$ contain all nonnegative monotone convex and quasi convex functions.

In [2], Dragomir et al., proved following inequalities of Hadamard’s type for $P$-function

Theorem 1.1. Let $f \in P(I), a, b \in I$ with $a < b$ and $f \in L[a, b]$. Then the following inequality holds

$$f \left( \frac{a + b}{2} \right) \leq \frac{2}{b - a} \int_a^b f(x)dx \leq 2 \left[ f(a) + f(b) \right]. \quad (1.2)$$

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The following inequality is well known in the literature as Simpson’s inequality.

Let \( f : [a, b] \to \mathbb{R} \) be a four times continuously differentiable mapping on \((a, b)\) and \( \left\| f^{(4)} \right\|_\infty = \sup_{x \in (a, b)} |f^{(4)}(x)| < \infty \). Then the following inequality holds:

\[
\frac{1}{3} \left[ \frac{f(a) + f(b)}{2} + 2f \left( \frac{a + b}{2} \right) \right] + \frac{1}{b - a} \int_a^b f(x)dx \leq \frac{1}{2880} \left\| f^{(4)} \right\|_\infty (b - a)^4.
\]

In recent years many authors have studied error estimations for Simpson’s inequality and Hermite-Hadamard inequality; for refinements, counterparts, generalizations, see (11-10).

In [3], Iscan obtained a new generalization of some integral inequalities for differentiable convex mapping which are connected Simpson and Hadamard type inequalities by using the following lemma.

Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) such that \( f' \in L[a, b] \), where \( a, b \in I \) with \( a < b \) and \( a, \lambda \in [0, 1] \). Then the following equality holds:

\[
\lambda (af(a) + (1 - \lambda) f(b)) + (1 - \lambda) f(aa + (1 - \lambda)b) - \frac{1}{b - a} \int_a^b f(x)dx = (b - a) \left[ \int_0^{1-a} (t - a\lambda) f'(tb + (1 - t)a) \, dt + \int_{1-a}^1 (t - 1 + \lambda (1 - a)) f'(tb + (1 - t)a) \, dt \right].
\]

The aim of this paper is to establish some new general integral inequalities for functions whose derivatives in absolute value at certain power are \( P \)-functions. Some applications of these results to special means is to give as well.

Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a differentiable function on \( I^0 \), the interior of \( I \). Throughout this section we will take \( I_f (\lambda, a, a, b) \)

\[
= \lambda (af(a) + (1 - \lambda) f(b)) + (1 - \lambda) f(aa + (1 - \lambda)b) - \frac{1}{b - a} \int_a^b f(x)dx
\]

where \( a, b \in I^0 \) with \( a < b \) and \( a, \lambda \in [0, 1] \).

**Theorem 1.2.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) such that \( f' \in L[a, b] \), where \( a, b \in I^0 \) with \( a < b \) and \( a, \lambda \in [0, 1] \). If \( |f'|^q \) is \( P \)-function on \([a, b] , q \geq 1 \), then the following inequality holds:

\[
\left| I_f (\lambda, a, a, b) \right| \leq (b - a) \left[ |f'(b)|^q + |f'(a)|^q \right]^{\frac{2}{q}} \times \left\{ \begin{array}{ll}
\gamma_2(a, \lambda) + \gamma_2(1 - a, \lambda) & \lambda a \leq 1 - \lambda \leq 1 - \lambda(1 - a) \\
\gamma_2(a, \lambda) + \gamma_1(1 - a, \lambda) & \lambda a \leq 1 - \lambda(1 - a) \leq 1 - a \end{array} \right.,
\]

\[
\gamma_1(a, \lambda) = (1 - a) \left[ a\lambda - \frac{(1 - a)}{2} \right],
\]

\[
\gamma_2(a, \lambda) = (a\lambda)^2 - \gamma_1(a, \lambda).
\]

**Proof.** Suppose that \( q \geq 1 \). Since \( |f'|^q \) is \( P \)-function on \([a, b] \), from Lemma [1] and using the well known power mean inequality, we have

\[
\left| I_f (\lambda, a, a, b) \right| \leq (b - a) \left[ \int_0^{1-a} |t - a\lambda| |f'(tb + (1 - t)a)| \, dt + \int_{1-a}^1 |t - 1 + \lambda (1 - a)| |f'(tb + (1 - t)a)| \, dt \right]
\]
Corollary 1.4. Identical to the inequality in [1, Theorem 2.3].

512

Imdat IŞCAN et al. / Some new general...

Corollary 1.2. Using Lemma 1 we shall give another result for convex functions as follows.

\[
\leq (b - a) \left\{ \int_0^{1-a} |t - a\lambda| \, dt \right\}^{1-\frac{1}{q}} \left\{ \int_0^{1-a} |t - a\lambda| \, [f'(tb + (1-t)a)]^q \, dt \right\}^{\frac{1}{q}} + \left\{ \int_{1-a}^1 |t - 1 + \lambda (1-a)| \, dt \right\}^{1-\frac{1}{q}} \left\{ \int_{1-a}^1 |t - 1 + \lambda (1-a)| \, [f'(tb + (1-t)a)]^q \, dt \right\}^{\frac{1}{q}}
\]

\leq (b - a) \left\{ [f'(b)]^q + [f'(a)]^q \right\}^{\frac{1}{q}} \left\{ \int_0^{1-a} |t - a\lambda| \, dt + \int_{1-a}^1 |t - 1 + \lambda (1-a)| \, dt \right\}

(1.5)

Additionally, by simple computation

\[
\int_0^{1-a} |t - a\lambda| \, dt = \begin{cases} \gamma_2(\alpha, \lambda), & a\lambda \leq 1 - \alpha \\ \gamma_1(\alpha, \lambda), & a\lambda \geq 1 - \alpha \end{cases}
\]

(1.6)

\[
\gamma_1(\alpha, \lambda) = (1 - \alpha) \left[ a\lambda - \frac{(1 - \alpha)}{2} \right], \quad \gamma_2(\alpha, \lambda) = (a\lambda)^2 - \gamma_1(\alpha, \lambda),
\]

(1.7)

Thus, using (1.6) and (1.7) in (1.5), we obtain the inequality (1.3). This completes the proof.

\[\square\]

Corollary 1.1. Under the assumptions of Theorem 1.2 with \( q = 1 \), we have

\[
\left| If(\lambda, a, a, b) \right| \leq (b - a) \left( |f'(b)| + |f'(a)| \right)
\]

\[
\times \begin{cases} \gamma_2(\alpha, \lambda) + \gamma_2(1 - \alpha, \lambda) & a\lambda \leq 1 - \alpha \leq 1 - \lambda (1 - \alpha) \\ \gamma_2(\alpha, \lambda) + \gamma_1(1 - \alpha, \lambda) & a\lambda \leq 1 - \lambda (1 - \alpha) \leq 1 - \alpha \\ \gamma_1(\alpha, \lambda) + \gamma_2(1 - \alpha, \lambda) & 1 - \alpha \leq a\lambda \leq 1 - \lambda (1 - \alpha) \end{cases}
\]

Corollary 1.2. In Theorem 1.2, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = \frac{1}{4} \), then we have the following Simpson type inequality

\[
\left| \frac{1}{6} [f(a) + 4f \left( \frac{a + b}{2} \right) + f(b)] - \frac{1}{b - a} \int_a^b f(x) \, dx \right| \leq \frac{5 (b - a)^3}{36} \left( |f'(b)|^q + |f'(a)|^q \right)^{\frac{1}{q}}
\]

Corollary 1.3. In Theorem 1.2, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = 0 \), then we have following midpoint inequality

\[
\left| f \left( \frac{a + b}{2} \right) - \frac{1}{b - a} \int_a^b f(x) \, dx \right| \leq \frac{b - a}{4} \left( |f'(b)|^q + |f'(a)|^q \right)^{\frac{1}{q}}
\]

Corollary 1.4. In Theorem 1.2, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = 1 \), then we get following trapezoid inequality which is identical to the inequality in [11, Theorem 2.3].

\[
\left| \frac{f(a) + f(b)}{2} - \frac{1}{b - a} \int_a^b f(x) \, dx \right| \leq \frac{b - a}{4} \left( |f'(b)|^q + |f'(a)|^q \right)^{\frac{1}{q}}
\]

Using Lemma 1 we shall give another result for convex functions as follows.
Theorem 1.3. Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I \) such that \( f' \in L[a, b] \), where \( a, b \in I \) with \( a < b \) and \( a, \lambda \in [0, 1] \). If \(|f'|^q\) is P-function on \([a, b], q > 1\), then the following inequality holds:

\[
\left| I_f (\lambda, a, a, b) \right| \leq (b - a) \left( \frac{1}{p+1} \right)^\frac{1}{p} \tag{1.8}
\]

\[
\times \left\{ \begin{array}{ll}
\varepsilon_1^{1/p} (\alpha, \lambda, p) \left| f' (a, q) + \varepsilon_1^{1/p} (1 - \alpha, \lambda, p) k_f^{1/q} (a, q) \right|, & a \lambda \leq 1 - \alpha \leq 1 - \lambda (1 - \alpha) \\
\varepsilon_2^{1/p} (\alpha, \lambda, p) \left| f' (a, q) + \varepsilon_2^{1/p} (1 - \alpha, \lambda, p) k_f^{1/q} (a, q) \right|, & a \lambda \leq 1 - \lambda (1 - \alpha) \leq 1 - \alpha , \\
\varepsilon_1^{1/p} (\alpha, \lambda, p) \left| f' (a, q) + \varepsilon_1^{1/p} (1 - \alpha, \lambda, p) k_f^{1/q} (a, q) \right|, & 1 - \alpha \leq a \lambda \leq 1 - \lambda (1 - \alpha)
\end{array} \right.
\]

where

\[
ce_1 (\alpha, \lambda, p) = (a \lambda)^{p+1} + (1 - \alpha - a \lambda)^{p+1}, \quad \varepsilon_2 (\alpha, \lambda, p) = (a \lambda)^{p+1} - (a \lambda - 1 + a)^{p+1},
\]

and \( \frac{1}{p} + \frac{1}{q} = 1 \).

Proof. Since \(|f'|^q\) is P-function on \([a, b]\), from Lemma 1 and by Hölder’s integral inequality, we have

\[
\left| I_f (\lambda, a, a, b) \right| \leq (b - a) \left( \int_0^{1-\alpha} |t - \alpha \lambda| |f' (t b + (1-t)a)| \, dt + \int_{1-\alpha}^1 |t - 1 + \lambda (1 - \alpha)| |f' (t b + (1-t)a)| \, dt \right)
\]

\[
\leq (b - a) \left\{ \left( \int_0^{1-\alpha} |t - \alpha \lambda|^p \, dt \right)^{\frac{1}{p}} \left( \int_0^{1-\alpha} |f' (t b + (1-t)a)|^q \, dt \right)^{\frac{1}{q}} \right. \\
+ \left. \left( \int_{1-\alpha}^1 |t - 1 + \lambda (1 - \alpha)|^p \, dt \right)^{\frac{1}{p}} \left( \int_{1-\alpha}^1 |f' (t b + (1-t)a)|^q \, dt \right)^{\frac{1}{q}} \right\}. \tag{1.10}
\]

By the inequality (1.12), we get

\[
\int_0^{1-\alpha} |f' (t b + (1-t)a)|^q \, dt = (1 - \alpha) \left[ \frac{1}{(1 - \alpha) (b - a)} \int_a^b |f' (x)|^q \, dx \right]^{(1-a)b+a-a} \\
\leq (1 - \alpha) \left[ |f' ((1 - \alpha) b + aa)|^q + |f' (a)|^q \right]. \tag{1.11}
\]

The inequality (1.11) also holds for \( \alpha = 1 \). Similarly, for \( \alpha \in (0, 1] \) by the inequality (1.12), we have

\[
\int_{1-\alpha}^1 |f' (t b + (1-t)a)|^q \, dt = \alpha \left[ \frac{1}{\alpha (b - a)} \int_a^b |f' (x)|^q \, dx \right]^{b-a} \\
\leq \alpha \left[ |f' ((1 - \alpha) b + aa)|^q + |f' (b)|^q \right]. \tag{1.12}
\]

The inequality (1.12) also holds for \( \alpha = 0 \). By simple computation

\[
\int_0^{1-\alpha} |t - \alpha \lambda|^p \, dt = \left\{ \begin{array}{ll}
\frac{(a \lambda)^{p+1} + (1 - \alpha - a \lambda)^{p+1}}{p+1}, & a \lambda \leq 1 - \alpha \\
\frac{(a \lambda)^{p+1} - (a \lambda - 1 + a)^{p+1}}{p+1}, & a \lambda \geq 1 - \alpha \end{array} \right.
\]
and
\[
\int_{1-a}^{1} \left| t - 1 + \lambda (1 - a) \right|^p dt = \begin{cases} \\
\frac{\left| \lambda (1-a) \right|^{p+1} + \left| \lambda - (1-a) \right|^{p+1}}{p+1}, & 1 - a \leq 1 - \lambda (1-a) \\
\frac{\left| \lambda (1-a) \right|^{p+1} - \left| \lambda (1-a) - a \right|^{p+1}}{p+1}, & 1 - a \geq 1 - \lambda (1-a) \end{cases},
\]
(1.14)
thus, using (1.11)-(1.14) in (1.10), we obtain the inequality (1.8). This completes the proof. \(\square\)

**Corollary 1.5.** In Theorem 1.3, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = \frac{1}{3} \), then we have the following Simpson type inequality
\[
\frac{1}{6} \left[ f(a) + 4f \left( \frac{a+b}{2} \right) + f(b) \right] - \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{b-a}{12} \left( \frac{1 + 2^{p+1}}{3(p+1)} \right)^{\frac{1}{2}}
\]
\[
\times \left\{ \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (a) \right|^q \right)^{\frac{1}{q}} + \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (b) \right|^q \right)^{\frac{1}{q}} \right\}.
\]

**Corollary 1.6.** In Theorem 1.3, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = 0 \), then we have the following midpoint inequality
\[
\left| f \left( \frac{a+b}{2} \right) \right| - \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{b-a}{4} \left( \frac{1}{p+1} \right)^{\frac{1}{p}}
\]
\[
\times \left\{ \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (a) \right|^q \right)^{\frac{1}{q}} + \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (b) \right|^q \right)^{\frac{1}{q}} \right\}.
\]

We note that by inequality
\[
\left| f' \left( \frac{a+b}{2} \right) \right|^q \leq \left| f' (a) \right|^q + \left| f' (b) \right|^q
\]
we have
\[
\left| f \left( \frac{a+b}{2} \right) \right| - \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{b-a}{4} \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \left\{ \left( \left| f' (b) \right|^q + 2 \left| f' (a) \right|^q \right)^{\frac{1}{q}} + \left( \left| f' (a) \right|^q + 2 \left| f' (b) \right|^q \right)^{\frac{1}{q}} \right\}.
\]

**Corollary 1.7.** In Theorem 1.3, if we take \( \alpha = \frac{1}{2} \) and \( \lambda = 1 \), then we have the following trapezoid inequality
\[
\left| f \left( \frac{a+b}{2} \right) \right| - \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{b-a}{4} \left( \frac{1}{p+1} \right)^{\frac{1}{p}}
\]
\[
\times \left\{ \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (a) \right|^q \right)^{\frac{1}{q}} + \left( \left| f' \left( \frac{a+b}{2} \right) \right|^q + \left| f' (b) \right|^q \right)^{\frac{1}{q}} \right\}.
\]

2 **Some applications for special means**

We now recall the following well-known concepts. For arbitrary real numbers \( a, b, a \neq b \), we define

1. The unweighted arithmetic mean
\[
A(a, b) := \frac{a+b}{2}, \ a, b \in \mathbb{R}.
\]
2. Then \( n \)-Logarithmic mean
\[
L_n(a, b) := \left( \frac{b^{n+1} - a^{n+1}}{(n+1)(b-a)} \right)^{\frac{1}{n}}, \ n \in \mathbb{N}, \ n \geq 1, \ a, b \in \mathbb{R}, \ a < b.
\]
Now we give some applications of the new results derived in section 2 to special means of real numbers.

**Proposition 2.1.** Let \( a, b \in \mathbb{R} \) with \( a < b \) and \( n \in \mathbb{N}, n \geq 2 \). Then
\[
\left| \frac{1}{3} A(a^n, b^n) + \frac{2}{3} A^n(a, b) - L^n_n(a, b) \right| \leq \frac{5n(b-a)}{36} \left( |b|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}}
\]

**Proof.** The assertion follows from Corollary 1.2 applied to the function \( f(x) = x^n, x \in \mathbb{R} \), because \( |f'|^q \) is a \( P \)-function. \( \square \)

**Proposition 2.2.** Let \( a, b \in \mathbb{R} \) with \( a < b \) and \( n \in \mathbb{N}, n \geq 2 \). Then
\[
\left| A^n(a, b) - L^n_n(a, b) \right| \leq \frac{n(b-a)}{4} \left( |b|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}}
\]
and
\[
\left| A(a^n, b^n) - L^n_n(a, b) \right| \leq \frac{n(b-a)}{4} \left( |b|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}}
\]

**Proof.** The assertion follows from Corollary 1.3 and Corollary 1.4 applied to the function \( f(x) = x^n, x \in \mathbb{R} \), because \( |f'|^q \) is a \( P \)-function. \( \square \)

**Proposition 2.3.** Let \( a, b \in \mathbb{R} \) with \( a < b \) and \( n \in \mathbb{N}, n \geq 2 \). Then
\[
\left| \frac{1}{3} A(a^n, b^n) + \frac{2}{3} A^n(a, b) - L^n_n(a, b) \right| \leq \frac{n(b-a)}{12} \left( \frac{1 + 2p+1}{3(p+1)} \right)^{\frac{1}{q}}
\]
\[
\times \left\{ \left( |A(a, b)|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}} + \left( |A(a, b)|^{(n-1)q} + |b|^{(n-1)q} \right)^{\frac{1}{q}} \right\}.
\]

**Proof.** The assertion follows from Corollary 1.5 applied to the function \( f(x) = x^n, x \in \mathbb{R} \), because \( |f'|^q \) is a \( P \)-function. \( \square \)

**Proposition 2.4.** Let \( a, b \in \mathbb{R} \) with \( a < b \) and \( n \in \mathbb{N}, n \geq 2 \). Then
\[
\left| A^n(a, b) - L^n_n(a, b) \right| \leq \frac{n(b-a)}{4} \left( \frac{1}{p+1} \right)^{\frac{1}{q}}
\]
\[
\times \left\{ \left( |A(a, b)|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}} + \left( |A(a, b)|^{(n-1)q} + |b|^{(n-1)q} \right)^{\frac{1}{q}} \right\}.
\]
and
\[
\left| A(a^n, b^n) - L^n_n(a, b) \right| \leq \frac{b-a}{4} \left( \frac{1}{p+1} \right)^{\frac{1}{q}}
\]
\[
\times \left\{ \left( |A(a, b)|^{(n-1)q} + |a|^{(n-1)q} \right)^{\frac{1}{q}} + \left( |A(a, b)|^{(n-1)q} + |b|^{(n-1)q} \right)^{\frac{1}{q}} \right\}.
\]

**Proof.** The assertion follows from Corollary 1.6 and Corollary 1.7 applied to the function \( f(x) = x^n, x \in \mathbb{R} \), because \( |f'|^q \) is a \( P \)-function. \( \square \)

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