Deterministic and Efficient Quantum Cryptography Based on Bell’s Theorem

Zeng-Bing Chen,1, 2 Qiang Zhang,1 Xiao-Hui Bao,1 Jörg Schmiedmayer,2 and Jian-Wei Pan1, 2

1 Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
2 Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany

(Dated:)

We propose a novel double-entanglement-based quantum cryptography protocol that is both efficient and deterministic. The proposal uses photon pairs with entanglement both in polarization and in time degrees of freedom; each measurement in which both of the two communicating parties register a photon can establish one and only one perfect correlation and thus deterministically create a key bit. Eavesdropping can be detected by violation of local realism. A variation of the protocol shows a higher security, similarly to the six-state protocol, under individual attacks. Our scheme allows a robust implementation under current technology.

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Entanglement and nonlocality lie at the heart of modern understanding of quantum foundations. One of the most striking aspects of entanglement is that certain statistical correlations derived for entangled states can be in conflict with local realism, as quantitatively shown by Bell’s inequalities (BI) [1]. These fundamental issues were originally considered at the very boundary of physics and philosophy. Yet, they have found practical applications in quantum information science. In a remarkable paper by Ekert [3], BI have a profound utility in quantum cryptography (QC) (or, quantum key distribution, QKD) [4, 5, 6, 7]. Actually, there is a fascinating link [3, 7, 8, 9] between security of certain quantum communication protocols and BI. However, quantum violations of local realism also occur in an “all-versus-nothing” (AVN) form [10, 11, 12, 13], which is more striking in the sense that the contradiction between quantum mechanics and local realism arises even for definite correlations. It thus remains to be seen if such an AVN nonlocality can have any application in quantum information, particularly, in QC. Most of the QC protocols [4, 5, 6, 7] proposed so far are non-deterministic as only less than 50% qubits detected can be further used as key bits. This may be a practical problem, e.g., in the one-time-pad secret-key cryptosystem [3]. Such a problem may be eliminated by deterministic QC protocol and secret direct communication, which attracted some recent interest [14, 15].

For QC experiments realized with faint laser pulses, they may be insecure under the so-called beamsplitter (BS) attack [5]. This is because the currently available photon sources have a finite probability of emitting more than one photon (or more than one entangled photon pair for entangled photon sources). An eavesdropper (usually called Eve) could in principle use a channel with lower photon loss or without loss and only allow those attenuated pulses containing \( n (n \geq 2) \) photons to reach the receiver. For these pulses she can use a BS to steal at least one photon, thus getting full information without being detected. However, the entanglement-based QC protocols (such as Ekert’s [3] and ours to be described below) exploit entanglement as certain “security resource” and do not suffer from this kind of problem as the security therein is guaranteed by the violations of BI.

In this paper, based on the previously proved two-party AVN nonlocality (or inseparability) for two doubly-entangled photon pairs [13], we propose a novel QC protocol which is efficient and deterministic: Each detected photon pair can establish a key bit with the help of classical communications. This deterministic feature of our protocol stems from the very nature of the two-party AVN nonlocality: The two communicating parties always have perfect quantum correlations for whatever measurement bases they choose. An eavesdropper can be detected by observing the violation of local realism for the quantum channel. A variation of the present protocol is similar to the six-state protocol [14] and shows a higher security under simple individual attacks. A remarkable advantage of the present scheme is that all required measurements can be done with linear optical elements and as such, the experimental realization of the protocol is within the reach of current technology.
In our protocol (see Fig. 1) “doubly-entangled” photon pairs (photon-1 and photon-2) in the state

\[ |\Psi\rangle_{12} = \frac{1}{2}( |\uparrow\rangle_{1} |H\rangle_{2} + |V\rangle_{1} |V\rangle_{2} (|\uparrow\rangle_{1} |\downarrow\rangle_{2} + |\downarrow\rangle_{1} |\uparrow\rangle_{2}) \]  

(1)

are generated via spontaneous parametric down conversion (SPDC) and sent, respectively, to two communicators, Alice and Bob. Here \(|H\rangle (|V\rangle)\) stands for photons with horizontal (vertical) polarization; \(|\uparrow\rangle\) and \(|\downarrow\rangle\) span an orthonormal basis for either time or path states of photons. \(|\Psi\rangle_{12}\) is maximally entangled both in polarization and in time/path degrees of freedom of photons. The creation of polarization-path entanglement was discussed in [12, 17]. With a pump interferometer in Fig. 1 one can generate polarization-time double entanglement (More details are given at the end of this paper).

To create secure keys, each party needs to measure observables involving the spin-type operators \(x = |H\rangle \langle V| + |V\rangle \langle H|\) (for polarization) and \(x' = |\uparrow\rangle \langle \downarrow| + |\downarrow\rangle \langle \uparrow|\) (for time/path). Particularly, the two parties should measure nine observables:

\[
\begin{align*}
A_1 & \left\{ \begin{array}{l}
\frac{z_1}{x_1}, \quad x_1' \cdot \frac{z_1'}{x_1'} \\
\frac{z_1}{x_1} \cdot \frac{x_1'}{x_1'} \cdot \frac{z_1'}{x_1'} \\
\end{array} \right. \\
A_2 & \left\{ \begin{array}{l}
\frac{z_1'}{x_1'} \quad \frac{z_1'}{x_1'} \cdot \frac{x_1'}{x_1'} \\
\frac{z_1}{x_1} \cdot \frac{x_1'}{x_1'} \cdot \frac{z_1'}{x_1'} \\
\end{array} \right. \\
A_3 & \left\{ \begin{array}{l}
\frac{z_1'}{x_1'} \quad \frac{z_1'}{x_1'} \cdot \frac{x_1'}{x_1'} \\
\frac{z_1}{x_1} \cdot \frac{x_1'}{x_1'} \cdot \frac{z_1'}{x_1'} \\
\end{array} \right. \\
B_1 & \left\{ \begin{array}{l}
\frac{x_2'}{x_2} \quad \frac{x_2'}{x_2} \cdot \frac{x_1'}{x_1'} \\
\frac{x_2'}{x_2} \cdot \frac{x_2'}{x_2} \cdot \frac{x_1'}{x_1'} \\
\end{array} \right. \\
B_2 & \left\{ \begin{array}{l}
\frac{z_2'}{x_2'} \quad \frac{z_2'}{x_2'} \cdot \frac{x_1'}{x_1'} \\
\frac{z_2'}{x_2'} \cdot \frac{z_2'}{x_2'} \cdot \frac{x_1'}{x_1'} \\
\end{array} \right. \\
B_3 & \left\{ \begin{array}{l}
\frac{z_2'}{x_2'} \quad \frac{z_2'}{x_2'} \cdot \frac{x_1'}{x_1'} \\
\frac{z_2'}{x_2'} \cdot \frac{z_2'}{x_2'} \cdot \frac{x_1'}{x_1'} \\
\end{array} \right. \\
\end{align*}
\]

(2)

Here Alice (Bob) arranges her (his) local observables into three groups \(A_1, A_2\) and \(A_3\) \((B_1, B_2\) and \(B_3\)), each of which has three operators. As in Ref. [15], the three operators of each group can be measured by one and the same apparatus (to be described below). This is crucial in the AVN argument of nonlocality without the necessity of an additional assumption of noncontextuality [15].

When measuring the three operators of each group, e.g., \(A_1\) (other groups are similar), one measures \(z_1\) and \(x_1'\) simultaneously with the apparatus (also labeled as \(A_1\)), thus also giving the measurement result of \(z_1' \cdot x_1'\), which is just the product of the readouts of \(z_1\) and \(x_1'\). To denote this fact we then use \((\cdot)\) to separate operators (as in \(z_1 \cdot x_1'\) or \(x_1' \cdot z_1\)) or operator products (as in \(z_1 \cdot x_1' \cdot z_1\) or \(x_1' \cdot z_1\)). Two operators \((z_1 \cdot x_1' \cdot z_1)\) can be identified as local “elements of reality” in the nonlocality argument [15]. In this way, the three operators in each group are co-measurable and measured simultaneously by the same apparatus. Totally, one thus requires six apparatuses \((A_1, A_2, A_3\) for Alice; \(B_1, B_2\) and \(B_3\) for Bob), which can be realized without any mutual conflict only by linear optical elements [15].

Now we are ready to describe the present QC protocol. For each of the emitted pairs, photon-1 (photon-2) goes to Alice (Bob) who then measures an operator group, which is chosen randomly and independently from the three groups \(A_1, A_2\) and \(A_3\) \((B_1, B_2\) and \(B_3\)). Any local outcome of the above measurements is completely random and can of course be either \(-1\) or \(+1\), representing thus one bit of information.

Now one immediately has the following: For each pair of operator groups chosen by Alice and Bob, there is one and only one pair of outcomes of the local operators (or operator products) that possesses perfect correlation; totally Alice and Bob establish nine pairs of perfectly correlated local outcomes as each of the two parties has three operator groups. For instance, if Alice (Bob) measures the three operators in \(A_1\) \((B_2)\), then only the outcomes of \(z_1\) and \(z_2\) will show perfect correlation, i.e., their product will certainly be 1. The above result stems from the fact that for the photon pairs in \(|\Psi\rangle_{12}\) one has the following nine eigenvalues [15]:

\[
\begin{align*}
&z_1 \cdot z_2 |\Psi\rangle_{12} = 1, \quad z_1' \cdot z_2' |\Psi\rangle_{12} = 1, \\
x_1 \cdot x_2 |\Psi\rangle_{12} = 1, \quad x_1' \cdot x_2' |\Psi\rangle_{12} = 1, \\
z_1 \cdot z_2 \cdot z_2' |\Psi\rangle_{12} = 1, \quad x_1 \cdot x_2 \cdot x_2' |\Psi\rangle_{12} = 1, \\
z_1 \cdot x_1' \cdot x_2 ' \cdot x_2 z_2' |\Psi\rangle_{12} = 1, \\
z_1 z_1' \cdot x_1 x_2' z_2 z_2' |\Psi\rangle_{12} = 1, \\
z_1' \cdot x_1' \cdot x_1 x_2 z_2' |\Psi\rangle_{12} = 1. \\
\end{align*}
\]

(3)

Here we have used a simplification in notions, e.g., \(z_1 \cdot z_2 |\Psi\rangle_{12} = 1\) means \(z_1 \cdot z_2 |\Psi\rangle_{12} = |\Psi\rangle_{12}\).

After the above measurements have taken place on a photon pair, Alice and Bob can announce in public by classical communications which of the three operator group they have measured. They discard all measurements in which either or both of them fail to register a photon at all. In the case where Alice and Bob have detected a photon simultaneously from the emitted photon pairs, they can establish deterministically a secure key as they can know from the classical communications which pair of their outcomes has perfect correlation. For example, let us again assume that Alice (Bob) has chosen the apparatus \(A_1\) \((B_2)\). In this case the two parties will certainly obtain \(z_1 \cdot z_2 |\Psi\rangle_{12} = 1\), from which Alice using her own outcome of \(z_1\) can predict with certainty Bob’s outcome of \(z_2\), and vice versa. Any one of this type of perfect correlations can then be used to create deterministically a secure key bit. The deterministic feature of our QC protocol is thus demonstrated. As a comparison, Ekert’s protocol is nondeterministic in the sense that successful detection of a photon by both Alice and Bob can establish at most 2/9 raw key.

All QKD protocols are consisted of two parts: the quantum part producing the raw keys and the classical part (e.g., reconciliation and privacy amplification) [7]. The later is not considered here as it is the same for all cryptographic protocols [15]. Now it is ready to see that our protocol is more effective than traditional protocols (e.g., Ekert’s protocol) in the quantum part. For comparison, in Ekert’s protocol 7/9 of detected photon pairs
is of no use for establishing raw keys and will be sacrificed
to detect eavesdropping. Thus, in the quantum part our
protocol is 1/(1 − 7/9) = 4.5 times more efficient than
the original Ekert protocol.

A complete security analysis of our QKD protocol is
very difficult and beyond the scope of this paper. Here
we consider the security issue by first following Ekert’s
security analysis [8]. In Ekert’s protocol, the presence of
an eavesdropper can be detected in conjunction with
BI. This is because a possible intervention (interception,
detection, and substitution of photons) by the eavesdropper
is equivalent to introducing local elements of physical
reality into the system. Following this line of thought,
Eve’s intervention would acquire information, e.g., by
randomly measuring observables like $A \in \{A_1, A_2, A_3\}$
and $B \in \{B_1, B_2, B_3\}$ with certain results (denoted by
$\lambda$); afterwards she sends the replacement of the detected
photons to Alice and Bob. Now what Alice and Bob do
is just to measure certain predetermined values of these
operators (i.e., elements of physical reality) as measured
already by Eve. In this case, Alice and Bob would obtain the
integration may also be summation if the number of
arbitrary measuring observables like $E_{\text{Eve}}$ intervention would acquire information, e.g., by
detection, and substitution of photons) by the eavesdrop-
ning of an eavesdropper can be detected in conjunction with
BI. However, in our protocol $\text{perfect correlations play the dual role of both establishing secure keys and detecting eavesdroppers.}$ We have
thus demonstrated the link for the security against eaves-
dropping of our protocol and a two-party version of Bell’s
theorem, and the definite quantum predictions used in such
a two-party AVN nonlocality argument may have fascinat-
ing application in the deterministic QKD protocol.

Note that $|\Psi\rangle_{12}$ is a maximally entangled state in a 4\times4
dimensional Hilbert space [12]. To achieve higher security
in QKD protocols, one may use either high-dimensional
systems [19] or more alternative settings (e.g., three-base
protocol [16]). Thus one might expect that our QKD
protocol using three measurement bases per party and
high-dimensional entanglement has a bonus of higher
security. To show that this is indeed the case, recall that
Ekert’s protocol can be regarded as a variation [1] of
the BB84 protocol [4], and as such the security of the
former can be guaranteed by the security of the latter.

Similarly, let us consider the case where Alice prepares
the doubly-entangled pair herself, measures one of her
closest operator groups in [2], and sends to Bob photon-2,
which might be subject to Eve’s intercept-resend attacks.

This modified protocol is then, in some sense, similar to
the six-state (three-base) protocol [10], which is more
secure than the original BB84 protocol. For instance,
when Alice measures $A_1$, she will collapse her state onto
the basis vectors $|H\rangle_1 \{|\uparrow\rangle_1 (z_1 = 1, x_1' = 1)\}$ or $|V\rangle_1 \{|\downarrow\rangle_1 (z_1 = -1, x_1' = -1)\}$ for which $z_1 \cdot x_1' = 1$, or
$|\psi\rangle_1 \{|\psi\rangle_1 (z_1 = 1, x_1' = 1)\}$ for which
$z_1 \cdot x_1' = -1$. Here $|\uparrow\rangle = \frac{1}{\sqrt2}(|\uparrow\rangle + |\downarrow\rangle)$ and
$|\downarrow\rangle = \frac{1}{\sqrt2}(|\uparrow\rangle - |\downarrow\rangle)$. If Alice gets $|\psi\rangle_1 \{|\downarrow\rangle_1 (with the probability of 1/4), Bob’s state will be equivalently
prepared as $|\psi\rangle_2 \{|\downarrow\rangle_2$, which is exactly the equal-amplitude
superposition of the basis vectors for any of $\{B_1, B_2, B_3\}$.

Note that any two basis vectors $|e_a\rangle$ and $|e_\beta\rangle$ belonging to
different bases in $\{B_1, B_2, B_3\}$ satisfy $|\langle e_\beta | e_a \rangle|^2 = 1/4$, i.e., the three bases $\{B_1, B_2, B_3\}$ are mutually unbiased.

If Eve, with the probability of 2/3, uses wrong bases, she
gets wrong perfect correlations with Alice and thus no
information. Explicit calculation shows that Eve can be
detected with the probability of 1/2 in this case. Thus
Bob’s error rate under the simple individual attacks is
1/3, implying that our protocol might be more secure,
similarly to the six-state protocol, but eliminates the
latter’s disadvantage of low efficiency.

A recent experiment [22] (see also [21]) has successfully
created the path-polarization-entangled two-photon
states. Recall that the two photons experiences two differ-
ent paths from the source to the detectors. Then the co-
herence of the path entanglement will be sensitive to the
relative phase that a photons would acquire as it propa-
gates along the two paths. The unavoidable fluctuations
in the relative phase may destroy the path entanglement.

To maintain the path coherence, especially in the long-
distance case, the long-distance interferometric stability is
required, which is extremely difficult in practice.

Fortunately, one can overcome the above problem by
using pulsed entanglement source where the two photons
are entangled both in time (i.e., time-bin entanglement
[22, 23]) and in polarization. To create the required en-
tanglement, a short, ultraviolet (UV) laser pulse is sent
first through an unbalanced Mach-Zehnder interferome-
ter (the pump interferometer) and then through a BBO
crystal (see Fig. 1). The pump pulse is split by the first
(50%-50%) BS (BS1) into two pulses, one propa-
gating along the short path and another along the long path. If the pulse duration is shorter than the arm length difference, the output from (50%-50%) BS2 is two pulses well separated in time. For the case where there is one and only one polarization-entangled pair [assumed to be in $\sqrt{2}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2)$ for definiteness] production after the “early” and “late” pulses pass through the BBO crystal, the polarization-time entangled two-photon state $|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2)(|e\rangle_1|e\rangle_2 + |l\rangle_1|l\rangle_2)$ is then created by adjusting the phase $\phi$. Here $|l\rangle \equiv |e\rangle$ (early time) and $|e\rangle \equiv |l\rangle$ (late time) are two orthonormal time states of photons. In Fig. 1 the pulse time detector can determine the emission time of the pump laser, giving a time fiducial signal.

Now each photon hold by Alice or Bob propagates along the same path. In this way, the time-entanglement is much more robust than the path-entanglement. Indeed, time-bin entanglement has been experimentally distributed over 50 km in optical fibers [24]. However, time-bin measurement is non-deterministic [22, 23] and may thus reduce the efficiency of the key production.

Here we propose a measurement scheme with simple linear optical elements and fast switches. The setup in Fig. 1 can measure all local observables in $|e\rangle$ by using two “time-path transmitters” (TPT) with optical paths identical to the pump interferometer. In the TPT a fast switch will reflect an incident photon into the long path of the TPT only for photons in $|e\rangle$; otherwise, it is swithed off so that the $|l\rangle$ photons simply propagate along the short path of the TPT. The fast switch is controlled according to the timing of the pulsed photons by noting that $|e\rangle$ and $|l\rangle$ are two time states distinguishable with respect to the time fiducial signal (see Fig. 1). In this way, the TPT transforms coherently $|e\rangle$ ($|l\rangle$) to $|d\rangle$ ($|u\rangle$), with $|d\rangle$ and $|u\rangle$ being two distinguishable paths of photons. Afterwards, all measurements in $|e\rangle$ can be done by the linear optics setups in Ref. [13].

The function of a fast switch can be accomplished by an acousto-optic modulator (AOM). Due to bulk acousto-optic interaction, an incident laser beam can be either diffracted (“first order”) by or directly transmitted (“zero order”) through a bulk acousto-optic medium, depending on whether the acoustic wave is present or not. Thus, if $|e\rangle$ ($|l\rangle$) is subject to the first-order (zero-order) process, $|e\rangle$ and $|l\rangle$ will be separated in path, acting exactly as a TPT. The intensity change (the wavelength change can be safely neglected) between the zero-order and first-order beams may be compensated by an attenuator. The current commercial AOM [24] can reach a rising time of about several nanoseconds, which is sufficient enough for our proposal. Moreover, it was already used as a fast optic switch (On/Off), e.g., by Kuzmich et al. [24] for generating nonclassical photon pairs.

To summarize, we have proposed a double-entanglement-based QC protocol that is both efficient and deterministic. The deterministic feature and high efficiency of our protocol have obvious advantages in a practical utility. Importantly, our protocol is within the reach of current technology and even allows a robust intermediate-distance realization.

Note added in proof.—Recently, M. Genovese kindly informed us their related work (Ref. 25) using path-time double entanglement in nondeterministic QKD.

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[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] J.S. Bell, Physics (Long Island City, N.Y.) 1, 195 (1964).
[3] A. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[4] C.H. Bennett and G. Brassard, in Proceedings of the IEEE International Conference on Computers, Systems and Singal Proceeding, Bangalore, India (IEEE, New York, 1984), p.175.
[5] C.H. Bennett, G. Brassard, and N.D. Mermin, Phys. Rev. Lett. 68, 557 (1992).
[6] C.H. Bennett, Phys. Rev. Lett. 68, 3121 (1992).
[7] N. Gisin, R. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. 74, 145 (2002); and references therein.
[8] B. Huttner and N. Gisin, Phys. Lett. A 228, 13 (1997); C. Fuchs et al., Phys. Rev. A 56, 1163 (1997); J.I. Cirac and N. Gisin, Phys. Lett. A 229, 1 (1997).
[9] V. Scarani and N. Gisin, Phys. Rev. Lett. 87, 117901 (2001); Phys. Rev. A 65, 012311 (2002).
[10] D.M. Greenberger, M.A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. 58, 1131 (1990).
[11] J.-W. Pan et al., Nature (London) 403, 515 (2000).
[12] A. Cabello, Phys. Rev. Lett. 86, 1911 (2001); ibid. 87, 010403 (2001).
[13] Z.-B. Chen et al., Phys. Rev. Lett. 90, 160408 (2003).
[14] A. Beige et al., J. Phys. A: Math. Gen. 35, l407 (2002).
[15] Z. Zhao et al., quant-ph/0211098.
[16] D. Bruß, Phys. Rev. Lett. 81, 3018 (1998); H. Bechmann-Pasquinucci and N. Gisin, Phys. Rev. A 59, 4238 (1999).
[17] C. Simon and J.-W. Pan, Phys. Rev. Lett. 89, 257901 (2002).
[18] A.I. Lvovsky, Phys. Rev. Lett. 88, 098901 (2002).
[19] H. Bechmann-Pasquinucci and A. Peres, Phys. Rev. Lett. 85, 3313 (2000); N.J. Cerf et al., ibid. 88, 127902 (2002).
[20] T. Yang et al., Phys. Rev. Lett. 95, 240406 (2005).
[21] J.-W. Pan et al., Nature (London) 423, 417 (2003).
[22] J. Brendel et al., Phys. Rev. Lett. 82, 2594 (1999); I. Marcikic et al., Phys. Rev. A 66, 062308 (2002).
[23] I. Marcikic et al., Phys. Rev. Lett. 93, 180502 (2004).
[24] See, e.g., http://www.a-a.fr/Acousto_optic_products/.
[25] M. Genovese and C. Novero, Eur. Phys. J. D 21, 109 (2002).