A New Orientifold of $\mathbb{C}^2/\mathbb{Z}_N$
and Six-dimensional RG Fixed Points

Angel M. Uranga

Theory Division, CERN
CH-1211 Geneva 23, Switzerland

Abstract

We discuss the consistency conditions of a novel orientifold projection of type IIB string theory on $\mathbb{C}^2/\mathbb{Z}_N$ singularities, in which one mods out by the combined action of world-sheet parity and a geometric operation which exchanges the two complex planes. The field theory on the world-volume of D5-brane probes defines a family of six-dimensional RG fixed points, which had been previously constructed using type IIA configurations of NS-branes and D6-branes in the presence of O6-planes. Both constructions are related by a T-duality transforming the set of NS-branes into the $\mathbb{C}^2/\mathbb{Z}_N$ singularity. We also construct additional models, where both the standard and the novel orientifold projections are imposed. They have an interesting relation with orientifolds of $\mathbf{D}_K$ singularities, and provide the T-duals of certain type IIA configurations containing both O6- and O8-planes.

\footnote{angel.uranga@cern.ch}
1 Introduction

One of the most interesting quantum field theory lessons that we have learned from string theory is the existence of six-dimensional supersymmetric field theories with non-trivial infrared dynamics [1, 2]. We are now familiar with the existence of large families of interacting superconformal field theories with (0, 2) and (0, 1) supersymmetry. There are two approaches which have been extensively used to construct these theories. The first is the study of type IIB D5-branes at A-D-E singularities [3, 4] and orientifolds thereof [5, 6, 7]. The second is the construction of type IIA brane configurations (in the spirit of [15]) of NS-branes, D6-branes and possibly D8-branes and orientifold planes [16, 17, 18].

There is a close relation between both constructions. In fact, Type IIA brane configurations where one of the directions (along which the D6-branes have finite extent) is compact can be T-dualized to a system of D5-branes probing orbifold and orientifold singularities (see e.g. [20]). This observation has led to a rich interplay between both approaches. For example, the construction of superconformal field theories in the Type IIB picture in [3, 5, 6, 7] was a source of information in the study of the T-dual Type IIA configurations in [17, 18]. On the other hand, some configurations in [17, 18] (those containing oppositely charged O8-planes) produced new field theories which required the existence of new orientifolds of \( C^2/Z_N \). These were constructed in [21] making use of the information available from the IIA construction.

In this paper we continue this program by constructing the Type IIB orientifolds associated to yet another set of field theories constructed in [17, 18]. As we discuss below, the new orientifold of \( C^2/Z_N \) has some unusual and amusing properties.

Before continuing with our introduction, it will be useful to know more about the relevant IIA brane configurations [17, 18]. We consider a set of \( N \) NS-branes with worldvolume along 012345, several stacks of D6-branes (along 0123456) stretched between them, and an O6-plane parallel to the D6-branes. The direction \( x^6 \), along which the D6-branes have finite extent, is taken compactified on a circle. The field theory on the non-compact part of the D6-brane worldvolume has \( D = 6, \mathcal{N} = 1 \) supersymmetry. This is the six-dimensional version of the four-dimensional models studied in [22].

As determined in [23] (see also [24] for a worldsheet derivation of this fact) the

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1Strings in orientifold backgrounds have been studied for instance in [8, 9, 10, 11, 12, 13, 14].

2A third approach, the study of F-theory on elliptically fibered singular Calabi-Yau threefolds [19] will not be so relevant for our purposes in the present paper.

3Several group theoretical features of the corresponding projection were pointed out in [3].
RR charge of the O6-plane changes sign whenever it crosses a NS-brane, and the projection it imposes on the D6-branes changes accordingly. Therefore, a consistent configuration is obtained only for an even number of NS-branes. If we place \( n_i \) D6-branes in the \( i^{th} \) interval between NS-branes, the gauge group is of the form

\[
SO(n_0) \times USp(n_1) \times \ldots \times SO(n_{N-2}) \times USp(n_{N-1})
\] (1.1)

The matter content arises from strings stretching between neighbouring D6-branes, and is of the form

\[
\sum_{i=0}^{N-1} \frac{1}{2}(\square, \square_{i+1})
\] (1.2)

where the index \( i \) is defined mod \( N \), and the \( \frac{1}{2} \) means the matter arises in half-hypermultiplets, due to the O6-plane projection. In brane configurations realising six-dimensional field theories, the NS-branes have the same number of non-compact dimensions as the D6-branes, so their worldvolume fields are dynamical. In our model, they give rise to \( N - 1 \) tensor multiplets; an additional tensor multiplet is decoupled and hence irrelevant.

As discussed in [16, 17, 18], cancellation of charges in the NS-brane worldvolume imposes a consistency condition on the configuration, which turns out to be equivalent to the cancellation of irreducible gauge anomalies [25] in the six-dimensional field theory. In our case, these conditions are satisfied for

\[
n_{2i} = N + 8 \quad ; \quad n_{2i+1} = N
\] (1.3)

The residual gauge anomaly, as discussed in [7], is cancelled by a Green-Schwarz mechanism mediated by the tensor multiplets [26].

As mentioned above, upon T-duality along \( x^6 \) the NS-branes transform, roughly speaking, into a \( C^2/Z_N \) singularity [27], while the D6-branes become a set of D5-branes sitting at the singularity. The projection imposed by the O6-plane must transform into an orientifold projection of \( C^2/Z_N \), such that the field theory (1.1), (1.2) arises on the worldvolume of the D5-brane probes. Also, the \( N - 1 \) tensor multiplets are expected to arise from closed string twisted sectors.

The orientifold of \( C^2/Z_N \) whose existence is predicted by this argument has a number of unusual features. For instance, it must exist only for even \( N \). Also, the closed string sector must give rise to one tensor multiplet per twisted sector. This is

\[4\]When \( k \) D6-branes sit on top of an O6\(^-\)-plane (which carries \(-4\) units of D6-brane charge, as counted in the double cover) their gauge group is projected down to \( SO(k) \). When they sit on top of an O6\(^+\) (with charge \(+4\)) their gauge group is \( USp(k) \).
in sharp contrast with the usual orientifold projection considered in \cite{14,5,21}, where invariant fields are combinations of modes appearing in oppositely twisted sectors of the type IIB theory. These orientifolds provide one tensor multiplet and one hypermultiplet per pair of oppositely twisted sectors. The $N - 1$ tensors in our model suggests that the required orientifold projection is rather different from the familiar one, in that it must map each twisted sector to itself.

A second interesting fact is that matter arises in half-hypermultiplets. Before the orientifolding, the matter in the worldvolume of D5-branes at a $\mathbb{Z}_N$ orbifold singularity appears in full hypermultiplets, with the two ‘halves’ being associated to the two complex planes in $\mathbb{C}^2/\mathbb{Z}_N$. In order to obtain half-hypermultiplets after orientifolding, $\Omega$ must be accompanied by a geometric action $\Pi$ which maps the two complex planes to each other.

Finally, the gauge group on the D5-branes suggests the Chan-Paton matrix associated to the orientifolds projection contains symmetric and antisymmetric pieces. This contradicts the usual rule that D-branes of the same dimension suffer orientifold projections of the same kind.

In Section 2 we define the new orientifold projection $\Omega \Pi$ of Type IIB on $\mathbb{C}^2/\mathbb{Z}_N$, and show it has all the features just described. In Section 3 we discuss the construction of models where both the $\Omega \Pi$ and the usual $\Omega$ projections are imposed.

2 Orientifold construction

Here we propose a Type IIB T-dual realization of the field theory in the previous Section, in terms of D5-branes probing a new kind of orientifold of $\mathbb{C}^2/\mathbb{Z}_N$. As discussed below, we propose that the orientation reversing element has a geometric action on the $\mathbb{C}^2/\mathbb{Z}_N$, $xy = v^N$, given by $x \leftrightarrow y, v \to -v$. This action is a symmetry only when $N$ is even, and involves the exchange of the two complex planes. This choice can be heuristically motivated by directly T-dualizing the IIA model described above. Following \cite{27}, upon T-duality the set of $N$ NS-branes becomes a $N$-centered Taub-NUT space, which can be described as $xy = v^N$ in suitable complex coordinates, while the D6-branes transform into D5-brane probes. The directions 89 can be identified with $v$. The directions 7 and 6\textsuperscript{'} (the T-dual of 6, with asymptotic radius $R_6$) are related to $x$ and $y$ in a complicated manner. Roughly speaking, for large $x$ and fixed $y$ one has $x \sim e^{i(x^7 + ix^6')/R_6'}$ and for large $y$ at fixed $x$ one has $y \sim e^{-(x^7 + ix^6')/R_6'}$. The

\footnote{Orientifold projections involving exchange of complex planes have also appeared in \cite{28} \cite{29} in a different kind of models.}
T-dual of the O6-plane action is expected to flip the signs of the coordinates 6’789, which is certainly the case for the action proposed above.

This argument should be considered a heuristic motivation. Stronger checks will arise from the detailed discussion of the model. We therefore turn to constructing the new orientifold of $C^2/Z_N$ and to showing that the world-volume field theory on D5-brane probes is the one described in Section 1. Let us consider type IIB theory on $C^2/Z_N$, modded out by $\Omega\Pi$, where $\Omega$ is world-sheet parity and $\Pi$ acts as

$$\Pi : z_1 \rightarrow z_2 , \ z_2 \rightarrow -z_1$$

(2.1)

with $z_1, z_2$ parametrizing the two complex planes in $C^2/Z_N$. Consistency requires $(\Omega\Pi)^2$ to belong to the orbifold group $Z_N$. Since it acts as $z_i \rightarrow -z_i$, this is the case when $N$ is even, $N = 2^k$. This condition, which agrees with a constraint found in the T-dual type IIA brane configuration, will be reobtained below from a different point of view. The orientifold defined above preserves $N = 1$ supersymmetry in six dimensions.

### 2.1 Closed string spectrum

As already mentioned, a first peculiar feature of this orientifold appears in the computation of the closed string twisted sectors. In order to appreciate this, it will be convenient to compare the situation with the usual $\Omega$ projection.

In type IIB theory on $C^2/Z_N$, the left and right moving states are labeled, in the bosonic description, by a vector $r_k = (r_1, r_2, r_3, r_4)$ belonging to the $SO(8)$ weight lattice shifted by the twist vector $v_k = (k/N, -k/N, 0, 0)$, and constrained by $\sum_{i=1}^{4} r_i = 1 \mod 2$. At the massless level, these states are

**NS**:

$$|\Phi_k^{(1)}\rangle_L \otimes |\Phi_k^{(2)}\rangle_R = \left| -1 + \frac{k}{N}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle \left| 1 - \frac{k}{N}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle$$

(2.2)

Their $\theta$ eigenvalue is given by $r_k \cdot v_k = -\frac{k}{N}(1 - 2k/N)$. The NS-NS and R-R $Z_N$ invariant states are

**NS - NS**:

$$|\Phi_k^{(1)}\rangle_L \otimes |\Phi_k^{(2)}\rangle_R = 4(1,1) i, j = 1, 2$$

**R - R**:

$$|\Psi_k^{(1)}\rangle_L \otimes |\Psi_k^{(2)}\rangle_R = (1, 1) + (1, 3) k = 1, \ldots, N - 1$$

The NS-R and R-NS states can be determined from these by supersymmetry.

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6The NS-R and R-NS states can be determined from these by supersymmetry.
The second column gives the representation under the spacetime little group $SU(2) \times SU(2)$. We obtain one hyper and one tensor multiplet of $D = 6 \, \mathcal{N} = 1$ supersymmetry per twisted sector.

In the usual $\Omega$ orientifolds studied in the literature \cite{14}, and for generic twisted sectors ($k \neq N/2$), the states surviving the projection are

$$\begin{align*}
\text{NS} - \text{NS} : & \quad |\Phi^{(i)}_k\rangle_L \otimes |\Phi^{(j)}_{N-k}\rangle_R + |\Phi^{(j)}_{N-k}\rangle_L \otimes |\Phi^{(i)}_k\rangle_R & \sim (4, 1) \\
\text{R} - \text{R} : & \quad |\Psi^{(i)}_k\rangle_L \otimes |\Psi^{(j)}_{N-k}\rangle_R - |\Psi^{(j)}_{N-k}\rangle_L \otimes |\Psi^{(i)}_k\rangle_R & \sim (1, 1) + (1, 3)
\end{align*}$$

Notice that invariant combinations are a mixture of states appearing in oppositely twisted sectors of the type IIB theory. The orientifolded model produces one hyper and one tensor multiplet per such pair of oppositely twisted sectors.

For the $k = N/2$ twisted sector, which does not mix with any other, the action of $\Omega$ may be defined with an additional $(-1)$ sign \cite{30}. This $\mathbb{Z}_2$ choice determines the type of multiplet surviving the projection. One gets a tensor multiplet or a hypermultiplet if the additional sign is present or not. As discussed in \cite{30}, consistent coupling between closed and open strings implies the following constraint on the D-brane Chan-Paton matrices

$$\gamma_{\theta^{N/2}} = \mp \gamma_{\Omega}^{T} \gamma_{\theta^{N/2}} \gamma_{\Omega}^{-1}$$

where the positive sign is taken when $\Omega$ includes the additional $(-1)$ sign, and the negative sign is taken otherwise. The two possibilities correspond to D-brane gauge bundles with or without vector structure \cite{31}.

Let us now turn to the $\Omega\Pi$ orientifold. It is easy to realize that $\Omega\Pi$ invariant states do not require mixing different twisted sectors of the type IIB theory. Because of that, for every sector there is a priori a $\mathbb{Z}_2$ ambiguity in defining the orientifold action. Actually, below we will show that the only consistent possibility is to include the additional $(-1)$ sign in the action of $\Omega\Pi$. Assuming momentarily this claim, the resulting invariant states are

$$\begin{align*}
\text{NS} - \text{NS} : & \quad |\Phi^{(1)}_k\rangle_L \otimes |\Phi^{(2)}_{N-k}\rangle_R - |\Phi^{(2)}_{N-k}\rangle_L \otimes |\Phi^{(1)}_k\rangle_R & \sim (1, 1) \\
\text{R} - \text{R} : & \quad |\Psi^{(1)}_k\rangle_L \otimes |\Psi^{(1)}_{N-k}\rangle_R \\
& \quad |\Psi^{(1)}_k\rangle_L \otimes |\Psi^{(2)}_{N-k}\rangle_R + |\Psi^{(2)}_{N-k}\rangle_L \otimes |\Psi^{(1)}_k\rangle_R & \sim (1, 3)
\end{align*}$$

We obtain one tensor multiplet per twisted sector. We now show that this choice is the only consistent one, by using the constraints coming from consistent coupling of open and closed strings. Consider a set of D5-branes at the $\mathbb{C}^2/\mathbb{Z}_N$ singularity, before
The orientifold projection is imposed \([\Omega \Pi]\). Their Chan-Paton matrix \(\gamma_{\theta,5}\) has \(N\) different eigenvalues \(s_i\) with multiplicities \(n_i\). It can be shown that \(\gamma_{\Omega \Pi,5}\) is block-diagonal (with \(n_i \times n_i\) blocks) in this basis, and therefore \(\gamma_{\theta^k,5}\) and \(\gamma_{\Omega \Pi,5}\) commute. The fact that \(\gamma_{\Omega \Pi}\) acts diagonally on the index \(i\) can be derived e.g. by looking at the Chern-Simons couplings of open string states and closed string RR modes in the \(\mathbb{Z}_N\) orbifold \([3]\)

\[
\int \text{tr} (\gamma_{\theta^k,5} \lambda_i) C_k \wedge e^{F_i} \tag{2.7}
\]

where \(C_k\) is a formal sum of the twisted RR forms, and \(\lambda_i\) selects the entries of the Chan-Paton matrix corresponding to the \(i^{th}\) set of D-branes. Since \(\Omega \Pi\) is a symmetry of the orbifold theory, the coupling must be invariant. Recalling that \(\Omega \Pi\) acts diagonally in \(k\), it follows that its action on the open string sector is diagonal in \(i\) \([4]\).

The argument in \([30]\) now allows us to use this information on the open string sector to constrain the closed string sector. Since \(\gamma_{\Omega \Pi}\) commutes with \(\gamma_{\theta^k}\), we have

\[
\gamma_{\theta^k,5} = + \gamma_{\Omega \Pi,5} \gamma_{\theta^k,5} \gamma^{-1}_{\Omega \Pi,5} \tag{2.8}
\]

Comparing with \((2.3)\), we learn that the projection \(\Omega \Pi\) in the closed string sector must include the additional \((-1)\) sign, as claimed above. This fixes the \(\mathbb{Z}_2\) ambiguity in all twisted sectors.

The bottomline of this argument is that the closed string spectrum of the \(\Omega \Pi\) orientifold of \(\mathbb{C}^2/\mathbb{Z}_N\) produces \(N - 1\) tensor multiplets.

### 2.2 Open string spectrum

We now turn to the open string spectrum and further consistency conditions on \(\gamma_{\Omega \Pi,5}\). It is convenient to introduce a concrete expression for \(\gamma_{\theta,5}\), which we take \([5]\)

\[
\gamma_{\theta,5} = \text{diag} \left( 1_{n_0}, e^{2\pi i \frac{1}{N} 1_{n_1}}, \ldots, e^{2\pi i \frac{N-1}{N} 1_{n_{N-1}}} \right) \tag{2.9}
\]

An interesting constraint on \(\gamma_{\Omega \Pi,5}\) can be derived by requiring \((\Omega \Pi)^2 = \theta^{N/2}\) in the open string sector. The action of \(\Omega \Pi\) on a state \(|\psi, ij\rangle\), corresponding to an open string stretching between the \(i^{th}\) and \(j^{th}\) D-brane, is

\[
\Omega \Pi : |\psi, ij\rangle \rightarrow (\Omega \Pi |i\rangle_{ii'}) (\Omega \Pi \psi, j'\rangle (\gamma_{\Omega \Pi}^{-1})_{j'j} \tag{2.10}
\]

\(^7\)The \(\Omega \Pi\) orientifold does not allow for the introduction of D9-branes. In Section 3 D9-branes may appear in some models where both the \(\Omega \Pi\) and the usual \(\Omega\) projections are imposed. The consistency conditions for D9-branes in those cases can be derived by simple modifications in our arguments below.

\(^8\)In other words, \(i\) labels the different kinds of fractional branes \([32]\), which can be understood as higher-dimensional branes wrapped on the collapsed two-cycles of the orbifold singularity. Since the orientifold acts diagonally on the cycles, so it does on the wrapped branes.

\(^9\)In the \(\Omega\) orientifolds, a different model would be obtained for \(\gamma_{\theta N,5} = -1\). It is easy to show that they are identical in the \(\Omega \Pi\) orientifold.
We are interested in comparing the actions of $(\Omega \Pi)^2$ and $\theta^{N/2}$, given by

$$(\Omega \Pi)^2 : |\psi, i j \rangle \rightarrow (\gamma_{\Omega \Pi, 5} (\gamma_{\Omega \Pi, 5}^T)^{-1})_{i i'} |\Omega \Pi \cdot \psi, i' j' \rangle (\gamma_{\Omega \Pi, 5}^T \gamma_{\Omega \Pi, 5}^{-1})_{j' j}$$

$$\theta^{N/2} : |\psi, i j \rangle \rightarrow (\gamma_{\theta^{N/2}, 5})_{i i'} |\theta^{N/2} \cdot \psi, i' j' \rangle (\gamma_{\theta^{N/2}, 5}^T \gamma_{\theta^{N/2}, 5}^{-1})_{j' j}$$

Let us split the set of D-branes into two types, denoted ‘even’ and ‘odd’, according to whether their $\gamma_{\theta^{N/2}}$ eigenvalue is +1 or −1. In the even-even and odd-odd sectors, $\theta^{N/2}$ acts as +1, and (2.11) imposes $\gamma_{\Omega \Pi} = \pm \gamma_{\Omega \Pi}$, with independent choices of sign for ‘even’ and ‘odd’ D-branes. In the even-odd and odd-even sectors, $\theta^{N/2}$ acts as −1 and (2.11) implies the symmetry of $\gamma_{\Omega \Pi}$ must be opposite for ‘even’ and ‘odd’ branes. Without loss of generality we get the conditions

$$\gamma_{\Omega \Pi, 5} = +\gamma_{\Omega \Pi, 5}^T \quad \text{for even branes}$$

$$\gamma_{\Omega \Pi, 5} = -\gamma_{\Omega \Pi, 5}^T \quad \text{for odd branes}$$

(2.12)

An appropriate choice of $\gamma_{\Omega \Pi, 5}$ is

$$\gamma_{\Omega \Pi, 5} = \text{diag} (1_{n_0}, \epsilon_{n_1}, \ldots, 1_{n_{N-2}}, \epsilon_{n_{N-1}})$$

(2.13)

Notice that this type of constraint is satisfied only for $N$ even. We also would like to point out that the different symmetry of $\gamma_{\Omega \Pi, 5}$ for ‘even’ and ‘odd’ branes is related by T-duality to the change of sign of the O6-plane whenever it crosses a NS-brane. The $\Omega \Pi$ orientifold hence provides a geometrical description of such process in a IIB T-dual realisation. It would be interesting to compare it with the geometries proposed in [33].

This completes the discussion on consistency conditions. Let us turn to computing the massless open string spectrum. The projection on the Chan-Paton factors for gauge bosons is

$$\lambda = \gamma_{\theta, 5} \lambda \gamma_{\theta, 5}^{-1}$$

$$\lambda = -\gamma_{\Omega \Pi, 5} \lambda^T \gamma_{\Omega \Pi, 5}^{-1}$$

(2.14)

and leads to a gauge group

$$SO(n_0) \times USp(n_1) \times \ldots \times SO(n_{N-2}) \times USp(n_{N-1})$$

(2.15)

The projection on the matter Chan-Paton factors is

$$Z_1 = e^{2\pi i/N} \gamma_{\theta, 5} \gamma_{\theta, 5}^{-1} Z_1 \gamma_{\theta, 5}^{-1}$$

$$Z_2 = \gamma_{\Omega \Pi, 5} \gamma_{\Omega \Pi, 5}^T \gamma_{\Omega \Pi, 5}^{-1} \gamma_{\Omega \Pi, 5}^{-1}$$

$$Z_2 = e^{-2\pi i/N} \gamma_{\theta, 5} \gamma_{\theta, 5}^{-1} \gamma_{\theta, 5}^{-1}$$

(2.16)
We obtain the following hypermultiplet matter content

$$\sum_{i=0}^{N-1} \frac{1}{2}(\mathcal{Q}_i,\mathcal{Q}_{i+1})$$

(2.17)

Notice that the action of $\Pi$ as exchange of the two complex planes is essential in obtaining half (rather than full) hypermultiplets.

Thus, the spectrum in the closed and open string sectors agrees with the field theory constructed in Section 1 from type IIA brane configurations of NS- and D6-branes in the presence of an O6-plane.

This field theory is potentially anomalous. We now show that, in analogy with the models in [6, 7, 21], the irreducible gauge anomaly vanishes once tadpole cancellation conditions are imposed. The reader not interested in these details is encouraged to skip the computation. The tadpoles can be obtained using the general techniques in [13, 14], and we only stress the differences between the $\Omega$ and $\Omega\Pi$ projections. Since the cylinder diagrams do not involve crosscaps, their contribution to the tadpoles is the familiar one

$$\mathcal{C} = \sum_{k=1}^{N-1} 4 \sin^2\left(\frac{\pi k}{N}\right)(\text{Tr} \, \gamma_{\theta^k,5})^2$$

(2.18)

where the untwisted tadpole $k = 0$ vanishes in the non-compact limit and is therefore ignored. The tadpoles from the Möbius strip diagrams are quite similar to those in [14]. The only difference is that the eigenvalues of the twists $\theta^k\Pi$ that act along with $\Omega$ are $e^{\pm 2\pi i/4}$ (and independent of $k$) rather than $e^{\pm 2\pi ik/N}$, and this modifies the trigonometric coefficient of the tadpole. Concretely we have

$$\mathcal{M} = -16 \sum_{k=0}^{N-1} 4 \sin^2\left(\frac{\pi k}{N}\right)\frac{\pi}{4} \text{Tr} (\gamma_{\theta^k,5}^T \Omega \gamma_{\theta^k,5}^{-1}) = -32N\text{Tr} \, \gamma_{\theta^N/2}$$

(2.19)

where we have used the property $\text{Tr} (\gamma_{\theta^k,5}^T \Omega \gamma_{\theta^k,5}^{-1}) = \text{Tr} \, \gamma_{\theta^N/2}$. The Klein bottle amplitude $\mathcal{K}(\theta^n, \theta^k)$ is evaluated by tracing over the $\theta^n$-twisted closed string spectrum with an insertion of $\theta^k\Omega\Pi$. Since $\Omega\Pi$ acts diagonally in the twisted sector index, all values of $n$ contribute. We must take into account that, as in the Möbius strip computation, the eigenvalues for the twist $\theta^k\Pi$ are $e^{\pm 2\pi i/4}$, and that for $n = 0$ zero mode factors from momentum states should be included. The different contributions $\mathcal{T}(\theta^n, \theta^k)$ to the tadpoles add up to

$$\mathcal{K} = \sum_{k=0}^{N-1} \mathcal{T}(1, \theta^k) + \sum_{n=1}^{N-1} \sum_{k=0}^{N-1} \mathcal{T}(\theta^n, \theta^k)$$

\footnote{Recall that in the $\Omega$ orientifolds only the pieces with $n = 0, N/2$ give a net contribution. For other twists, the contributions from symmetric and antisymmetric combinations of $\theta^n$ and $\theta^{-n}$ twisted sectors cancel in the trace.}
\[
\sum_{k=0}^{N-1} \frac{4 \sin^2 \frac{2\pi k}{N}}{4 \sin^2 \frac{\pi k}{N}} + 64 \sum_{n=1}^{N-1} \sum_{k=0}^{N-1} 1 = 64N^2
\] (2.20)

The complete expression \( C + M + K \) can be factorized as

\[
\sum_{k=1}^{N-1} \frac{1}{4 \sin^2 \frac{\pi k}{N}} \left( 4 \sin^2 \frac{\pi k}{N} \text{Tr} g_{\theta^{k,5}} - 16N\delta_{k,N/2} \right)^2 = 0
\] (2.21)

and leads to the constraint

\[
4 \sin^2 \frac{\pi k}{N} \text{Tr} g_{\theta^{k,5}} - 16N\delta_{k,N/2} = 0
\] (2.22)

The tadpole cancellation conditions can be expressed in terms of the integers \( n_r \), yielding the condition

\[-2n_r + 16(-1)^r + n_{r-1} + n_{r+1} = 0 \] (2.23)

This is the condition of cancellation of the irreducible anomaly in the field theory (2.15), (2.17). Their solution is \( n_{2i} = N + 8, n_{2i+1} = N \). This solution reproduces the result based on RR charge conservation in the type IIA brane configuration discussed in the introduction. The factorization of the remaining anomalies and their cancellation by a Green-Schwarz mechanism involving the \( N-1 \) tensor multiplets [26] has been discussed in [3], and we will not repeat it here.

### 3 Theories with the \( \Omega\Pi \) and \( \Omega \) projections

In this section we comment on the consistency conditions in type IIB orientifolds where both the usual \( \Omega \) and the new \( \Omega\Pi \) projections are imposed. As in the construction of the \( \Omega\Pi \) orientifold, the T-dual type IIA brane configurations will provide a useful guideline. We are by now familiar with the fact that the \( \Omega\Pi \) projection corresponds to the presence of an O6-plane in the T-dual Type IIA model. Similarly, the \( \Omega \) projection corresponds to the presence of two O8-planes (along 012345789) in the IIA configuration. Hence the IIA brane configurations for the models in this section include orientifold projections by both O6- and O8-planes. The general features of these configurations have been discussed in [18], and in [34] in more detail.

We consider type IIB on \( \mathbb{C}^2/\mathbb{Z}_N \), with \( N = 2K \), modded out by the orientifold projections \( \Omega\Pi \) and \( \Omega \). A first observation is that closure requires the introduction of the element \( \Pi \) in the orbifold group. The generators \( \theta \) and \( \Pi \) satisfy the properties

\[
\theta^N = 1 \quad ; \quad \Pi^2 = \theta^K \quad ; \quad \Pi \theta^n = \theta^{-n} \Pi
\] (3.1)

These relations define the non-abelian discrete group \( D_K \). Hence we are dealing with orientifolds of \( \mathbb{C}^2/D_K \). How does this arise in the IIA brane configuration?
3.1 The IIA brane configurations

The answer goes as follows. In a type IIA brane configuration with O6- and O8-planes, the orientifolds impose the projections \( \Omega_{O6} \equiv \Omega(-1)^F R_7 R_8 R_9 \) and \( \Omega_{O8} \equiv \Omega R_6 \), respectively, where \( R_i \) acts as \( x^i \rightarrow -x^i \). By closure, the configuration is also modded out by the orbifold element \( \mathcal{R} = (-1)^F R_6 R_7 R_8 R_9 \).

It is illustrative to consider momentarily the configuration modded out only by \( \mathcal{R} \), without any orientifold projection. Such brane configurations were studied in \[35\] and shown to reproduce field theories with gauge groups and matter contents defined by \( D_K \) quiver diagrams \[36\]. Furthermore, these configurations have been argued to be T-dual to systems of D5-branes at \( C^2/D_K \) singularities \[34\]. The type IIA brane configurations we are actually interested in contain an additional orientifold projection, say, by \( \Omega_{O6} \). Hence their type IIB T-duals should correspond to orientifolds of \( C^2/D_K \) singularities, as found above.

Let us consider the IIA brane configurations in some more detail. Since they contain an O6-plane which flips charge whenever it crosses a NS-brane, \( N \) is constrained to be even, \( N = 2K \). Another consistency condition is that no NS-brane can intersect the O8-planes, since the O6-plane crossing them would not respect the \( Z_2 \) symmetry imposed by the O8-plane. For a fixed \( K \), in order to specify the model completely one has to make the following choices. First, the charge of the two O8-planes \[37\], which can be positive for one and negative for the other – in which case their RR charge cancels and no D8-branes are required – or negative for both – in which case RR charge cancellation requires 32 D8-branes, as counted in the covering space; in this case one also has to choose the distribution of the D8-branes in the different intervals, in a way consistent with the orientifold symmetries –. Second, the charge assignment for the different pieces of O6-plane in the intervals. Finally, the number of D6-branes \( n_i \) at each interval \( i \) is determined by the conditions or RR charge conservation \[18, 17\]. The solution to these conditions is unique up to an overall addition of an equal number of D6-branes in all intervals.

To make the discussion a bit more concrete, consider the case of even \( K \). There are two possible patterns for the gauge group

\[
G_0 \times USp(n_1) \times SO(n_2) \times \ldots \times USp(n_{K-1}) \times G_K
\]

\[
G_0 \times SO(n_1) \times USp(n_2) \times \ldots \times SO(n_{K-1}) \times G_K
\]

\[11\]Our convention is that an O8\(^+\)-plane has +16 units of D8-brane charge, and projects the gauge group of \( k \) coincident D6-branes down to \( SO(k) \), and that an O8\(^-\)-plane, with −16 units of D8-brane charge, produces a gauge group \( USp(k) \) on \( k \) D6-branes.
The first (resp. second) possibility corresponds to the case when the O8-planes are intersected by O6$^-$- (resp. O6$^+$)- planes. The nature of the factors $G_0$ and $G_K$ is more model-dependent and will be discussed below. When $K$ is odd, the general structure of the gauge group is

$$G_0 \times USp(n_1) \times SO(n_2) \times \ldots \times USp(N_{K-2}) \times SO(n_{K-1}) \times G_K$$

(3.3)

In any case, the general structure of the matter content is given by

$$\frac{1}{2}(R_0, \square) + \sum_{i=1}^{K-2} \frac{1}{2}(\square_i, \square_{i+1}) + \frac{1}{2}(\square_{K-1}, R_K)$$

(3.4)

where $R_0$ and $R_K$ are discussed below. If D8-branes are present, suitable flavours in fundamental representations must be added [34].

Let us now specify the structure of the ‘end’ sectors in the above spectra, where a certain number $n$ of D6-branes suffers the projection by an O8-plane and an O6-plane. There are four different cases to be considered, depending on the charges of the orientifold planes, and they lead to different group factors and matter contents [34].

i) When the projection is imposed by an O8$^-$-plane and an O6$^-$-plane, the corresponding gauge factor is $SU(n/2)$ and the representation $R_0$ or $R_K$ in (3.4) is $\square + \square$.

ii) When there is an O8$^+$-plane and an O6$^+$-plane, we obtain the same answer: the gauge factor is $SU(n/2)$ and the representation $R_0$ or $R_K$ is $\square + \square$.

iii) For an O8$^-$-plane and an O6$^+$ plane, the gauge factor is $USp(n) \times USp(n')$ and the representation $R_0$ or $R_K$ is $(\square, 1) + (1, \square)$.

iv) For an O8$^+$-plane and an O6$^-$ plane, the gauge factor is $SO(n) \times SO(n')$ and the representation $R_0$ or $R_K$ is $(\square, 1) + (1, \square)$.

Concerning the NS-brane world-volume fields, we obtain $K$ tensor multiplets from the $K$ NS-branes in the configuration. There are also twisted fields living at the fixed points of $(-1)^{F_L} R_6 R_7 R_8 R_9$. For ‘end’ sectors of the type i) or ii) each orbifold plane gives rise to a hypermultiplet. For ‘end’ sectors of the type iii) or iv), each produces one tensor multiplet.

In the resulting field theory, the ranks of the gauge groups are determined by imposing the charge cancellation conditions in the IIA brane configuration. These include the the charges with respect to the twisted fields of the orbifold plane. In the cases without D8-branes, on which we center henceforth for the sake of clarity, this last condition implies $n = n'$ in the cases iii) and iv).

In the following subsection we show how these rules arise in the construction of the Type IIB orientifolds,
3.2 The orientifold construction

Even though as explained above we are dealing with an orientifold of a $D_K$ singularity, it will be convenient to continue discussing in terms of a $Z_N$ singularity modded out by the two orientifold projections $\Omega\Pi$ and $\Omega$. This is useful since it allows to benefit from the known results about models with either of the projections, taken from Section 2 for the $\Omega\Pi$ and from [6, 21] for the $\Omega$ projections. There are nevertheless some important instances where the additional elements $\theta^k\Pi$ in the orbifold group play a role.

One of these situations is the computation of the closed string spectrum. The model contains a twisted sector for each one of the $K+2$ non-trivial conjugacy classes of the orbifold group $D_K$. These classes are

$$C_n = \{\theta^n, \theta^{-n}\} \quad n = 1, \ldots, K-1 \, , \quad C_K = \{\theta^K\}$$

$$C_{\text{even}} = \{\Pi, \theta^2\Pi, \ldots, \theta^{2K-2}\Pi\} \, , \quad C_{\text{odd}} = \{\theta\Pi, \ldots, \theta^{2K-1}\Pi\}$$

(3.5)

We see that the new elements in the orbifold group generate new twisted sectors, but also introduce identifications in the twisted sectors of $Z_N$. In type IIB theory, each sector produces a hyper and a tensor multiplet. In the orientifolded theory, the action of $\Omega$ and $\Omega\Pi$ on the classes $C_n, n = 1, \ldots, K$ selects the tensor multiplet, while the action on the remaining classes may select the tensor or hypermultiplet. The relevant cases are discussed in the examples below. Notice that the action of $\Omega$ on the $\theta^{N/2}$-twisted sector corresponds to a projection with vector structure.

Another consequence of the new elements in the orbifold group is that the D5- and D9-branes in the model generate new disk tadpoles twisted by these elements $\theta^k\Pi$. These do not have a corresponding crosscap diagram, so cancellation of these tadpoles requires conditions of the type

$$\text{Tr} \, \gamma^{\theta^k\Pi,9} - 2\text{Tr} \, \gamma^{\theta^k\Pi,5} = 0$$

(3.6)

where the factor of two arises form momentum zero modes, and the first term is required only in cases with D9-branes. This condition is the IIB counterpart of the cancellation of charge under the twisted fields of the orbifold in the IIA brane configuration. We will make sure this condition is satisfied for the matrices in our models.

Let us turn to the explicit construction. Instead of being completely general, it will be illustrative to consider two examples, which include all the different ‘end’ sectors discussed in section 3.1. Let us consider $C^2/Z_N$ with $N = 2K$ and $K$ odd, and mod it by $\Omega\Pi$ and $\alpha\Omega$, with $\alpha^2 = \theta$. As explained in [21], the presence of $\alpha$ in the $\Omega$ projection implies the model contains no D9-branes, a conveniently simple case. Let us consider
the Chan-Paton matrices
\[
\gamma_{\theta,5} = \text{diag} \left( e^{2\pi i \frac{K_{-1}}{N}} 1_{n_1}, \ldots, e^{2\pi i \frac{K_{-1}}{N}} 1_{n_{K-1}}, e^{2\pi i \frac{K_{N-1}}{N}} 1_{n_1}, \ldots, e^{2\pi i \frac{N-1}{N}} 1_{n_1} \right)
\]
\[
\gamma_{\Omega \Pi,5} = \text{diag} \left( \epsilon_0, 1_{n_1}, \ldots, \epsilon_{n_{K-1}}, \epsilon_{n_{K-1}}, \ldots, 1_{n_1} \right)
\]
\[
\gamma_{\alpha \Omega} = \begin{pmatrix}
1_{n_0} & & & & \\
\alpha_{1_{n_1}} & & & &
\alpha^{P-1} & & & &
\alpha^{-1} & & & &
\end{pmatrix}
\]

Notice that the matrices \(\gamma_{\theta^+ \Pi a} = \gamma_{\alpha^+ \Omega} \gamma_{\Pi \Omega}\) are traceless, so the additional disk tadpoles mentioned above vanish. The open string spectrum one obtains from the projection using the matrices above is

\[
U(n_0/2) \times SO(n_1) \times \ldots \times USp(n_{K-1}) \times U(n_K/2)
\]
\[
\frac{1}{2} [([\square], \square)] + \sum_{i=1}^{K-2} \frac{1}{2} ([\square], \square) + \frac{1}{2} ([\square_{K-1}], \square_K) + ([\square_{K-1}], \square_K)
\]

The closed string spectrum contains \(K\) tensor multiplets and two hypermultiplets.

This model has a clear type IIA T-dual configuration. It contains \(N\) NS branes, two O8-planes (not intersected by any NS-brane) and one O6-plane. The \(\alpha \Omega\) projection above corresponds to the case where the O8-planes are oppositely charged [21]. The choice of Chan-Paton matrices \(\gamma_{\Omega \Pi}, \gamma_{\alpha \Omega}\) above specifies that the O8\(^+\) is intersected by an O6\(^+\), and the O8\(^-\) by an O6\(^-\). Using the rules of section 3.1 we obtain a gauge group and matter content in agreement with (3.8).

Let us turn to the discussion of cancellation of tadpoles in the type IIB side. Instead of performing it in the usual way, by factorizing Klein bottle, Möbius strip and cylinder diagrams, one can take advantage of knowing directly the contributions of the \(\alpha \Omega\) [21] and \(\Omega \Pi\) crosscaps (Section 2[7]). The tadpole condition reads

\[
4 \sin^2 \left( \pi \frac{k}{N} \right) \text{Tr} \gamma_{\theta^+ 5} - 32 \delta_{k,1} \mod 2 + 16 N \delta_{k,N/2} = 0
\]

This expression agrees with that obtained performing the standard computation. This condition can be recast in terms of the integers \(n_r\), giving

\[
-2n_r + n_{r-1} + n_{r+1} + 16\delta_{r,0} - 16\delta_{r,K} - 16(-1)^r = 0
\]

\[\text{(3.10)}\]

\[\text{[12]}\]Actually, the slightly different form of \(\gamma_{\Omega \Pi}\) in this section introduces a \(-1\) sign in the crosscap computed in Section 1. Also notice that the crosscap in [21] should be multiplied by 4, since our models are six-dimensional.
These conditions correspond precisely to the constraints from cancellation of the irreducible anomaly, and to the conditions of RR charge conservation in the T-dual type IIA brane configuration. The residual gauge anomaly is factorized and cancelled as discussed in [18, 34] by exchange of the closed string tensor multiplets [26]. Also, the $U(1)$ anomalies are cancelled by hypermultiplet exchange.

The second example we consider is a slight modification of the previous model. Let us again consider $C^2/Z_N$, with $N = 2K$ and $K$ odd, modded out by $\alpha \Omega$ and $\Omega \Pi$. Let us choose the Chan-Paton embedding of $\gamma_{\theta,5}$ and $\gamma_{\alpha \Omega,5}$ to be exactly as in (3.7), but let us take

$$\gamma_{\Omega \Pi,5} = \text{diag} (\sigma_1 \otimes 1_{n_0/2}, \epsilon_{n_1}, \ldots, 1_{n_{K-1}}, i\sigma_2 \otimes 1_{n_K/2}, 1_{n_{K-1}}, \ldots, \epsilon_{n_1}) \quad (3.11)$$

Notice that $\gamma_{\Omega \Pi,5}$ by itself is equivalent to (2.13), despite the modification of the blocks at the positions 0 and $K$. The modification is required so that the matrices $\gamma_{\theta^k \Pi \alpha}$ are traceless, as required for consistency. The final spectrum of this model is

$$[SO(n_0/2) \times SO(n_0/2')] \times SO(n_1) \times \ldots \times USp(n_{K-1}) \times [USp(n_K/2) \times USp(n_K/2')]$$

$$\frac{1}{2}(\Box_0, \Box_1) + \frac{1}{2}(\Box_0, \Box_1) + \sum_{i=1}^{K-2} \frac{1}{2}(\Box_0, \Box_1) + \frac{1}{2}(\Box_{K-1}, \Box_K) + \frac{1}{2}(\Box_{K-1}, \Box_K) \quad (3.12)$$

The closed string spectrum gives $K + 2$ tensor multiplets.

The interpretation in terms of the T-dual type IIA brane configuration is identical to the one we had for the previous model, save for a flip in the sign of the O6-plane. Hence in this model the O8$^+$-plane is intersected by an O6$^-$-plane, and the O8$^-$-plane is intersected by an O6$^+$-plane. The spectrum (3.12) agrees with the rules given in section 3.1, in particular with the ‘end’ sectors proposed in rules iii) and iv).

The tadpole cancellation condition is identical to (3.9), save for a flip in the sign of the $\delta_{k,N/2}$ contribution. They correspond to the conditions of cancellation of the irreducible anomaly and the RR charge conservation in the IIA brane configuration.

These two examples illustrate the construction of type IIB orientifolds with the $\Omega$ and $\Omega \Pi$ projections. They also provide a non-trivial check of the rules proposed for the brane configurations. A complete classification of models is straightforward to perform, but lengthy to list and does not provide further insights, therefore we spare the reader their discussion.

Finally, we would like to point out that some of the orientifolds of $C^2/D_K$ which can be obtained using our indications have already appeared in section 7 of [7]. Our discussion, however, includes several cases not considered in this reference. We see that the type IIA brane configurations, which inspired the construction of our orientifolds, have provided a really insightful guideline in our task.
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