The evolution of the power law k-essence cosmology

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Abstract We investigate the evolution of the power law k-essence field in FRWL spacetime. The autonomous dynamical system and critical points are obtained. The corresponding cosmological parameters, such as $\Omega_\phi$ and $w_\phi$, are calculated at these critical points. We find it is possible to achieve an equation of state crossing through $-1$ for k-essence field. The results we obtained indicate that the power law k-essence dark energy model can be compatible with observations.

Keywords k-Essence · Dark energy · Phase-space analysis

1 Introduction

Since the cosmological constant model ($\Lambda$CDM) suffers from cosmological constant problem (Carroll 2001) as well as age problem (Yang and Zhang 2010), many dynamic dark energy models have been proposed over the past years, such as quintessence, k-essence, phantom, tachyon, etc. These scalar field models can be seen as special cases of a model with Lagrangian, $\mathcal{L}_\phi = V(\phi)F(X) - f(\phi)$, with the kinetic energy $X \equiv -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ (Carroll et al. 2003; Malquart et al. 2003). This general Lagrangian has attracted much attention. For some special cases, constraints have been considered in Yang and Gao (2009, 2011a), Yang et al. (2008), Yang and Zhang (2008) and dynamics have been analyzed in Yang and Gao (2011b), Yang (2012), Yang and Qi (2012), De-Santiago et al. (2013). Geometrical diagnostic methods have been used to discriminate a class of this theory from $\Lambda$CDM (Gao and Yang 2010). Unified model of inflation, dark matter and dark energy have been discussed in Bose and Majumdar (2009), De-Santiago et al. (2013), Saitou and Nojiri (2011). Generalized tachyon models have been investigated in Unnikrishnan (2008), Yang and Qi (2012). Here we consider a model with $F(X) = -\sqrt{X + X}$, $V(\phi) \propto 1/\phi^2$, and $f(\phi) = 0$. This type of k-essence has been shown to be a phenomenologically acceptable and theoretically interesting model which can unify inflation, dark matter, and dark energy (Bose and Majumdar 2009; De-Santiago and Cervantes-Cota 2011). We investigate the possible cosmological behavior of this model in Friedmann–Robertson–Walker–Lemaître (FRWL) spacetime by performing a phase-space and stability analysis. We calculate various observable quantities, such as the density of the dark energy and the equation of state (EoS) parameter in these solutions. The results show that the model discussed here can be consistent with observations.

This paper is organized as follows: in the following section, we review k-essence dark energy models. In the third section, we consider the dynamics of the k-essence scalar field. In the fourth section, we discuss the stabilities of critical points and the model. Finally, we close with a few concluding remarks in the fifth section.
2 k-Essence cosmology

We consider k-essence dark energy models with Lagrangian

\[ L = \rho_\phi = F(X)V(\phi), \]  

where \( F(X) \) and \( V(\phi) \) are analytic functions of \( X \) and \( \phi \) respectively. \( V(\phi) \) has dimension \( M^4 \) and hence \( F(X) \) is dimensionless. Theses Lagrangians are invariant under the shift symmetry: \( \phi \rightarrow \phi + \phi_0 \). Throughout this paper we will work with a flat, homogeneous, and isotropic FRWL spacetime having signature \((-+,+,+,+)\) and in units \( c = 8\pi G = 1 \). With Lagrangians (1), we can define the energy density, the pressure, and the EoS parameter. However, only after specifying the functional form of \( F(X) \), it is possible to relate \( F(X) \) with scale factor and then other dynamic quantities such as the energy density and the EoS parameter. We introduce auxiliary variables

\[ x = \dot{\phi}, \quad y = \frac{\sqrt{V(\phi)}}{\sqrt{3}H}, \]  

to transform the cosmological equations (5) and (6) into an autonomous dynamical system as following

\[ x' = \frac{\sqrt{3}2}{\lambda x^2}y - 3x + \frac{3\sqrt{2}}{2}, \]  
\[ y' = \frac{1}{4}y(-2\sqrt{3}\lambda xy + 6 - 3\sqrt{2}x^2y^2 + 3x^2y^2), \]

for \( x > 0 \). And for \( x < 0 \), Eqs. (5) and (6) turn into

\[ x' = \frac{\sqrt{3}}{2}\lambda x^2y - 3x - \frac{3\sqrt{2}}{2}, \]  
\[ y' = \frac{1}{4}y(-2\sqrt{3}\lambda xy + 6 + 3\sqrt{2}x^2y^2 + 3x^2y^2), \]

where the prime denotes a derivative with respect to the logarithm of the scale factor, \( \ln a \), and \( \lambda \equiv -V_\phi/V^3_\phi \). Here we are interested in the case where \( \lambda \) is a constant, meaning \( V(\phi) \propto \phi^{-2} \). The density parameters of k-essence, the EoS, the sound speed, and the total EoS are reformulated as, respectively,

\[ \Omega_\phi = \frac{1}{2}x^2y^2, \]  
\[ w_\phi = 1 - \sqrt{2}|x|^{-1}, \]  
\[ c_s^2 = 1 - \frac{\sqrt{2}}{2}|x|^{-1}, \]  
\[ w_t = \Omega_\phi w_\phi = \frac{1}{2}x^2y^2 - \frac{\sqrt{2}}{2}y^2|x|. \]

Because \( 0 \leq \Omega_\phi \leq 1 \), the auxiliary variable \( x \) and \( y \) are constrained as \( 0 \leq \frac{1}{4}x^2y^2 \leq 1 \).

Equations (9) and (10), (11) and (12), form self-autonomous dynamical systems which are valid in the whole phase-space, not only at the critical points. The critical points \((x_c, y_c)\) of the autonomous system are obtained by setting the left-hand sides of the equations to zero, namely by solving \( \mathbf{X}' = (x', y') = 0 \). Eight critical points are obtained in all, as shown in Tables 1 and 2 which we also present the necessary conditions for their existence, as well as the corresponding cosmological parameters, \( c_s^2, \Omega_\phi, w_\phi, \) and \( w_t \). With these cosmological parameters, we can investigate the possible state of the universe and discuss whether there exists an acceleration phase or not.
4 Stability

As shown in Yang and Gao (2011b), Yang and Qi (2012), the stability of the critical point does not mean the stability of the model, so, we must investigate both the stability of the critical point and the stability of the model.

4.1 Stability of critical points

To discuss the stability of the critical point, we expand $X = (x, y)$ around the critical values $X_c = (x_c, y_c)$ by setting $(x, y)^T = (x, y)^T + U$ with the perturbative variables $U$ (see, for example, Copeland et al. 1998; Yang and Gao 2011b; Capozziello et al. 2006; Leon and Saridakis 2009). Up to the first order we acquire $U' = M \cdot U$ with the matrix $M$ determined by

$$M = \begin{bmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{bmatrix}. \quad (17)$$

The matrix $M$ contains the coefficients of the perturbation equations, thus its eigenvalues determine the stability of the critical points. For hyperbolic critical points, all the eigenvalues have real parts different from zero: sink for negative real parts is stable, saddle for real parts of different sign is unstable, and source for positive real parts is unstable. The conditions for the stability of the critical points are given by $\text{Tr} M < 0$ and $\det M > 0$.

For the power law k-essence dark energy we discussed here, the $M$, $\det M > 0$, and $\text{Tr} M < 0$, are found to be

$$M = \begin{bmatrix} \sqrt{3} \lambda y x - 3 \\ \frac{1}{4} y (-2 \sqrt{3} \lambda y - 3 \sqrt{2} y^2 + 6 x^2 y) \end{bmatrix} - \frac{\sqrt{3}}{4} \lambda x y + \frac{3}{2} - \frac{3 \sqrt{3}}{4} x y^2 + \frac{3}{4} x^2 y^2 + \frac{1}{4} y (-2 \sqrt{3} \lambda x - 6 \sqrt{2} x y + 6 x^2 y). \quad (18)$$

$$\det M = -\frac{9}{4} \lambda^2 x^2 y^2 + \frac{9 \sqrt{3}}{2} \lambda x y - \frac{15 \sqrt{6}}{8} \lambda x y^3 + \frac{3 \sqrt{3}}{2} \lambda x^3 y^3 - \frac{9}{2} + \frac{27 \sqrt{3}}{4} x^2 y^2 - \frac{27}{4} x^2 y^2, \quad (19)$$

$$\text{tr} M = \sqrt{3} \lambda x y - \frac{3}{2} - \frac{3 \sqrt{2}}{4} x y^2 + \frac{3}{4} x^2 y^2 + \frac{y}{4} (-2 \sqrt{3} \lambda x - 6 \sqrt{2} x y + 6 x^2 y), \quad (20)$$

for the case: $x > 0$, and

$$M = \begin{bmatrix} \sqrt{3} \lambda y x - 3 \\ \frac{1}{4} y (-2 \sqrt{3} \lambda x + 3 \sqrt{2} y^2 + 6 x^2 y) \end{bmatrix} - \frac{\sqrt{3}}{4} \lambda x y + \frac{3}{2} - \frac{3 \sqrt{3}}{4} x y^2 + \frac{3}{4} x^2 y^2 + \frac{1}{4} y (-2 \sqrt{3} \lambda x + 6 \sqrt{2} x y + 6 x^2 y). \quad (21)$$

$$\det M = -\frac{9}{4} \lambda^2 x^2 y^2 + \frac{9 \sqrt{3}}{2} \lambda x y + \frac{15 \sqrt{6}}{8} \lambda x y^3 + \frac{3 \sqrt{3}}{2} \lambda x^3 y^3 - \frac{9}{2} - \frac{27 \sqrt{3}}{4} x^2 y^2 - \frac{27}{4} x^2 y^2, \quad (22)$$

$$\text{tr} M = \sqrt{3} \lambda x y - \frac{3}{2} + \frac{3 \sqrt{2}}{4} x y^2 + \frac{3}{4} x^2 y^2 + \frac{y}{4} (-2 \sqrt{3} \lambda x + 6 \sqrt{2} x y + 6 x^2 y), \quad (23)$$

### Table 1

| Critical points $\{x_c, y_c\}$ | Existence | Stable | $c_s^2$ | $\Omega_\phi$ | $\omega_\phi$ | $\omega_{tot}$ | Acceleration |
|------------------------------|-----------|--------|---------|---------------|--------------|--------------|--------------|
| $P_{11} = (\sqrt{2}, 0)$     | arbitrary | none   | 0       | 0             | $-\sqrt{2}$ | 0            | none         |
| $P_{12} = (\sqrt{2}, \sqrt{2})$ | $\lambda > 0$ | $\lambda > \frac{\sqrt{2}}{2}$ | 1   | $\frac{\sqrt{2}}{2}$ | 0            | 0            | none         |
| $P_{13} = (\frac{\lambda}{\sqrt{2}}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \sqrt{2} - \frac{\sqrt{2}}{2}, \lambda - \frac{\sqrt{2}}{2})$ | none     | none   | $-\sqrt{2} \lambda$ | 1             | $-\sqrt{2} \lambda$ | $-1 + \sqrt{2} \lambda$ | none         |
| $P_{14} = (\frac{\lambda}{\sqrt{2}}, \frac{\sqrt{2}}{2}, \sqrt{2} - \frac{\sqrt{2}}{2}, \lambda - \frac{\sqrt{2}}{2})$ | $\lambda < \sqrt{2}$ | $\lambda < \frac{\sqrt{2}}{2}$ | $\sqrt{2} \lambda$ | 1             | $\sqrt{2} \lambda - 1$ | $-\sqrt{2} \lambda - 1$ | $0 \leq \lambda < \sqrt{2}$ |

### Table 2

| Critical points $\{x_c, y_c\}$ | Existence | Stable | $c_s^2$ | $\Omega_\phi$ | $\omega_\phi$ | $\omega_{tot}$ | Acceleration |
|------------------------------|-----------|--------|---------|---------------|--------------|--------------|--------------|
| $P_{21} = (-\sqrt{2}, 0)$     | arbitrary | none   | 0       | 0             | $-1$         | 0            | none         |
| $P_{22} = (-\sqrt{2}, -\sqrt{2})$ | $\lambda < 0$ | $\lambda < -\frac{\sqrt{2}}{2}$ | 1   | $\frac{3}{2}$ | 0            | 0            | none         |
| $P_{23} = (\frac{\lambda}{\sqrt{2}}, \frac{\sqrt{2}}{2}, \sqrt{2} - \frac{\sqrt{2}}{2}, \lambda - \frac{\sqrt{2}}{2})$ | none     | none   | $\sqrt{2} \lambda$ | 1             | $-1 + \sqrt{2} \lambda$ | $-1 + \sqrt{2} \lambda$ | none         |
| $P_{24} = (-\sqrt{2}, \frac{\sqrt{2}}{2}, \sqrt{2} - \frac{\sqrt{2}}{2}, \lambda - \frac{\sqrt{2}}{2})$ | $\lambda > -\sqrt{2}$ | $\lambda > -\frac{\sqrt{2}}{2}$ | $-\sqrt{2} \lambda$ | 1             | $\sqrt{2} \lambda - 1$ | $-\sqrt{2} \lambda - 1$ | $-\sqrt{2} < \lambda \leq 0$ |
4.2 Stability of model

The stability of model includes classical and quantum stability. We first discuss the classical stability. In a flat universe, the equation for the canonical quantization variable $v$ describing the collective metric and scalar field perturbations takes the form (Garriga and Mukhanov 1999)

$$v'' + \left( c_s^2 k^2 - \frac{\Phi''}{\Phi} \right) v = 0,$$

(24)

where $\Phi = a(\rho + p_\phi)^{1/2}/(c_s H)$ with $H$ the Hubble parameter. The increment of instability is inversely proportional to the wave-length of the perturbations, therefore the background model is violently unstable and do not has any physical significance for $c_s^2 < 0$. Another potentially interesting requirement is $c_s^2 \leq 1$, saying that the sound speed should not exceed the speed of light, otherwise the causality will be violated. Note, however, this is still an open problem (see e.g. Bruneton 2007; Kang et al. 2007; Bonvin et al. 2006; Ellis et al. 2007; Babichev et al. 2008; Gorini et al. 2008). Here we take the conditions for classical stability as: $1 \geq c_s^2 \geq 0$, namely

$$1 \geq 1 - \frac{\sqrt{2}}{2} \frac{1}{|x|} \geq 0,$$

(25)

for the case of power law $k$-essence we discussed here. From this equation, we obtain the range of $\lambda$ in which the model is classically stable: $|x| \geq \sqrt{\frac{2}{x}}$.

Now discussions for the quantum stability of the $k$-essence field are in order. Expanding $p$ at second order in $\delta \phi$, the Hamiltonian fluctuations are found to be (Armendariz-Picon and Lin 2005; Bamba et al. 2012; Kahya and Onemli 2007; Piazza and Tsujikawa 2004):

$$\delta H = p_X \frac{(\nabla \delta \phi)^2}{2} + (p_X + 2 X p_{XX}) \left( \frac{\delta \phi^2}{2} - p_{\phi\phi} \frac{(\delta \phi)^2}{2} \right),$$

(26)

where $p_{\phi\phi} \equiv d^2 p/d\phi^2$. The positivity of the first two terms in Eq. (26) leads to the following conditions for quantum stability

$$p_X \geq 0, \quad p_X + 2 X p_{XX} \geq 0.$$

(27)

The conditions for quantum stability for the power law $k$-essence dark energy discussed here are found to be: $|x| \geq \sqrt{\frac{2}{x}}$. Here $p_X \geq 0$ is the gradient-stability condition and $p_X + 2 X p_{XX} \geq 0$ is the no-ghost condition. In general, violations of the null energy condition may lead to gradient instability. One way to avoid the gradient instability is to flip the sign of the kinetic term with a minimally coupled scalar field (Caldwell 2002), however, this turns out to be catastrophic since the considered theory would inevitably develop ghost instabilities (Cline et al. 2004), and as shown in Figs. 1 and 2 in Piazza et al. (2013), a minimally coupled...
The deceleration parameter is \( q \).

It has been shown that a quintessence model with \( w_\phi \leq -1 \) can be completely stable for some conditions (Creminelli et al. 2006, 2009). For k-essence or dark energy models crossing the phantom divide, the speed of sound should be set to zero to obtain stability (Creminelli et al. 2009). For generalized and detailed discussions on the problem of the soundness of the theory against ghost-like and gradient instabilities we refer to Piazza et al. (2013). Discussions about perturbational instability for violating the null energy condition can also be found in Xia et al. (2008), Cai et al. (2010, 2012), Dubovsky et al. (2006), Rubakov (2014), Guo et al. (2003).

So it can be concluded that the model is both classically and quantum stable for \(|x| \geq \frac{\sqrt{2}}{3}\). We say the model is (classically and quantum) stable at a critical point if its \( x_0 \) is in the range of \( x \) allowed by the conditions of stability for the model, or is not stable if \( x_0 \) is not in the range of \( x \) allowed by the conditions of stability for model (Yang and Gao 2011b; Yang and Qi 2012).

### 4.3 Cosmological implications

For \( x > 0 \), the model is stable at critical points \( P_{13} \) for \(-\sqrt{6} \leq \lambda \leq 0\), \( P_{14} \) for \( 0 \leq \lambda \leq \sqrt{6}\), and \( P_{11} \) and \( P_{12} \) for arbitrary \( \lambda \). But critical points \( P_{11} \) is not stable and \( P_{13} \) does not exist, so they are not relevant from a cosmological point of view. In other words, only critical points \( P_{12} \) and \( P_{14} \) are physical interesting.

For \( \lambda > \frac{\sqrt{6}}{2} \), the critical point \( P_{12} \) is stable. At this point, the k-essence behaves like dark matter with \( \Omega_\phi = \frac{1}{2\pi^2} \), meaning the universe is partly occupied by k-essence. If \( \lambda \rightarrow +\infty \), the universe will be dominated by dark matter, while if \( \lambda \rightarrow \frac{\sqrt{6}}{2} \), the universe will be dominated by k-essence.

For \( \lambda < \frac{\sqrt{6}}{2} \), the critical point \( P_{14} \) is stable, while the range of \( \lambda \) in which the model is stable is \( 0 \leq \lambda \leq \sqrt{6} \), that is to say, only for \( 0 \leq \lambda < \frac{\sqrt{6}}{2} \), both the model and the critical point are stable. At this point, the universe is dominated by k-essence with \( \Omega_\phi = 1 \) and \( w_\phi = \frac{\sqrt{6}}{2} \lambda - 1 \). If \( \lambda = 0 \), the k-essence will behave like cosmological constant; while if \( \lambda \rightarrow \frac{\sqrt{6}}{2} \), the k-essence will behave like dark matter. The deceleration parameter is \( q = -1 + \frac{\sqrt{6}}{2} \lambda \). The final state of the universe dependents on the potential: the universe will speed up if \( 0 \leq \lambda < \frac{\sqrt{6}}{2} \), will expand with constant-speed if \( \lambda = \frac{\sqrt{6}}{2} \), and will speed down if \( \frac{\sqrt{6}}{2} < \lambda < \frac{\sqrt{6}}{2} \).

We plot the evolution of \( \Omega_\phi, \Omega_m, w_\phi \), and the deceleration parameter \( q \) for \( \lambda = 0.1 \) (namely for the case \( x > 0 \)) in Fig. 3. The initial conditions are chosen as \( x = 0.65 \) and \( y = 0.0000375 \) when \( \ln a = -7 \). In this case, an interesting result is that the EoS is smaller than \(-1\) at early times and is larger than \(-1\) at late times. The parameter \( \Omega_\phi \) is nearly zero at early times and increase to 0.68 when \( \ln a \rightarrow 0 \), which is compatible with observations.

For \( x < 0 \), the critical point \( P_{21} \) is not stable and \( P_{23} \) does not exist, while other critical points are stable for a certain range of \( \lambda \). The model is stable at critical points \( P_{23} \) for \(-\sqrt{6} \leq \lambda \leq 2\sqrt{6} \), \( P_{24} \) for \(-\sqrt{6} \leq \lambda \leq 0 \), and \( P_{21} \) and \( P_{22} \) for arbitrary \( \lambda \).

For \( \lambda < -\frac{\sqrt{6}}{2} \), the critical point \( P_{22} \) is stable, and the k-essence behaves like dark matter with \( \Omega_\phi = \frac{1}{2\pi^2} \). If \( \lambda \rightarrow -\infty \), the universe will be dominated by dark matter, while if \( \lambda \rightarrow -\frac{\sqrt{6}}{2} \), the universe will be dominated by k-essence.

For \(-\sqrt{6} < \lambda \leq 0 \), both the model and the critical point \( P_{24} \) are stable. The universe is dominated by k-essence with \( \Omega_\phi = 1 \) and \( w_\phi = -\frac{\sqrt{6}}{2} \lambda - 1 \) at this point. If \( \lambda = 0 \), the k-essence will behave like cosmological constant; while if \( \lambda \rightarrow -\frac{\sqrt{6}}{2} \), the k-essence will behave like dark matter. The deceleration parameter is \( q = -1 - \frac{\sqrt{6}}{2} \lambda \). The final state of the universe dependents on the potential: the expansion of universe will speed up if \(-\frac{\sqrt{6}}{2} \leq \lambda < 0 \), keep constant-speed if \( \lambda = -\frac{\sqrt{6}}{2} \), and will speed down if \(-\frac{\sqrt{6}}{2} < \lambda < -\frac{\sqrt{6}}{2} \).

The evolution of \( \Omega_\phi, \Omega_m, w_\phi \), and the deceleration parameter \( q \) for \( \lambda = -0.8 \) (namely for the case \( x < 0 \)) are plotted in Fig. 4 with the initial conditions \( x = -0.65 \) and \( y = 0.000034 \) when \( \ln a = -7 \). The parameter \( \Omega_\phi \) is nearly zero at early times and increase to 0.68 when \( \ln a \rightarrow 0 \), which is also compatible with observations. The EoS can also cross through \(-1\).
5 Conclusions and discussions

We have investigated the evolution of the universe when power law k-essence acts as dark energy and have examined whether there are late-time solutions compatible with observations. Critical points and the conditions for their existence and stability are obtained. The corresponding cosmological parameters, $c_s^2$, $\Omega_{\phi}$, $w_{\phi}$, and $w_1$, are calculated at these critical points. The (classical and quantum) stability of the model are also discussed.

As discussed in Yang and Gao (2011b), Yang and Qi (2012), the stability of critical points does not mean the stability of the model, vice versa. The critical points can be divided into three classes: stable points at which the model is (classically or quantum) unstable, stable points at which the model is stable, unstable points at which the model is stable (Yang and Gao2011b; Yang and Qi 2012). From a cosmological point of view, only stable points at which the model is (classically and quantum) stable are physically interesting.

Fig. 4 The evolution of $\Omega_{\phi}$, $\Omega_m$, $w_{\phi}$, and the deceleration parameter $q$ for $\lambda = -0.8$ with the initial conditions $x = -0.65$ and $y = 0.000034$ when $\ln a = -7$.
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