Modeling the Morning Commute Problem With Stochastic Travel Time in a Bottleneck Model

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ABSTRACT

This paper studies the traffic behavior of travelers who drive from living areas to workplaces through bottleneck roads during the early rush hours when the travel time on the road is uncertain. Based on the travel time which is assumed stochastic and follows a uniform distribution, the equilibrium properties of the proposed model are derived and individual travel cost is obtained. According to the assumptions, three possible departure-time intervals have been considered when users always arrive early, arrive early or late, always arrive late. Travelers’ travel choice behaviors are analyzed in detail and equilibrium is achieved with these three situations. The results show that even if they start at the same time, they may arrive early or late, but not necessarily early or late. Numerical examples verify the theoretical analysis, and it is found that the time interval of arriving at the workplace also changes when the stochastic degree of travel time increases.

INDEX TERMS Bottleneck model, departure time choice, morning commute problem, stochastic travel time.

I. INTRODUCTION

The well-known bottleneck model was originally developed by Vickrey [1]. This model is a common situation during the morning rush hour, where a fixed and very large number of travelers travel from home to workplace along the same stretch of road. This road has a single bottleneck with a fixed capacity. If the arrival rate at the bottleneck exceeds its capacity, a queue forms. Although all the travelers wish to arrive at the destination at the same time, this is not physically possible because the bottleneck capacity is finite. Consequently, some travelers may decide to depart earlier or later to avoid the cost of waiting in the queue and pay the penalty cost for doing so. Therefore, each traveler faces a trade-off between travel time cost and the schedule delay cost while choosing an optimal departure time to minimize the total travel cost. At equilibrium, the total travel expenses of all travelers are equal, and no one can reduce his/her commuting cost by changing his/her departure time.

The bottleneck model depicts the commuting behavior of travelers during morning rush hours on a simple and direct way, and clearly describes the formation and dissipation of queuing congestion and the departure time choice behavior of travelers. Subsequently, the morning commuting problem has been extended by many others. Arnott, de Palma, and Lindsey [2] and Braid [3] developed the basic bottleneck model to consider elastic demand, and Lindsey [4] and Ramadurai et al. [5] extended a single bottleneck model with heterogeneous travelers.

Most of the existing literature, however, is based on deterministic settings, with either fixed capacity or demand [6], [7]. In reality, the variations in, including the behavior of individual drivers, the performance of vehicles, on the driving conditions like weather or lighting, etc., lead to the unpredictability or the unreliability of travel time. The travel time may also vary for physical or operational reasons, such as road work, accidents, or vehicle breakdown. Therefore, it is necessary to study the travelers’ choice behavior when travel time is stochastic.

There are two kinds of bottleneck models based on uncertainty, one is the stochastic bottleneck capacity, and the other is the stochastic travel time. In the study of capacity randomness, different scholars give different results depending on different situations. Arnott, de Palma, and Lindsey [8] examined the case where the ratio of demand to capacity is stochastic and examined the effect of information on the total...
social cost. Xiao et al. [9] investigated a bottleneck model in which the capacity of the bottleneck is assumed stochastic and follows a uniform distribution. Li et al. [10] presented an analytical investigation of departure time choice under stochastic capacity. Fosgerau [11] investigated the distribution of delays during a repeatedly occurring demand peak in a congested facility with random capacity and demand. Zhang et al. [12] presented time-varying capacity in the bottleneck model and derived the corresponding UE states. Zhu et al. [13] studied the dynamical evolution of travelers’ departure time choice in a bottleneck-constrained roadway system with stochastic capacity. Liu et al. [14] presented a bottleneck model in which the capacity of the bottleneck is constant within a day but changes stochastically from day-to-day.

Different from the network with random capacity, some researchers have studied the bottleneck model when the travel time is stochastic. Noland and Small [15] proposed the value of travel time variability (VTTV) in a case relatively general, thereby providing a sound theoretical foundation for this notion. Fosgerau and Karlström [16] studied the marginal social cost of travel time variability, which takes the feedback of travel time unreliability on the congestion profile into account. Coulombel and André de Palma [17] proposed the cost of travel time variability for car users when congestion is modeled as an endogenous phenomenon. Xiao et al. [19] presented the valuation of travel time reliability in the presence of endogenous congestion and the role of scheduling preferences. Jenelius [20] extended the analysis of the value of mean travel time (VMTT) and travel time variability (VTTV) from bottleneck model to a more general multi-trip setting, incorporating the effects of flexibility in activity scheduling and dependence of travel times across trips.

In those stochastic capacity bottleneck models, they transform all the random events into the changes of capacity and construct their bottleneck models. Then, when there is an accident on the road, the biggest impact is the change in travel time. Therefore, only considering the change of bottleneck capacity has certain limitations. On the other hand, in those stochastic travel time bottleneck models, most of them assumed everyone may arrive early or late, regarded the congestion as endogenous existence, and consider the reliability of travel time. But in real life, the earliest travelers are bound to arrive early, and the latest travelers are bound to arrive late. Based on this phenomenon, this paper studies the travelers’ travel behavior in the stochastic bottleneck model. Different from previous studies, this paper focuses on the morning commuting problem along a single way with stochastic free-flow travel time. We limit our analysis to day-to-day fluctuations in travel time and assumed assume that the travel time within the day is constant and that the bottleneck is severely congested over the peak period. According to the early arrival or late arrival of travelers in this paper, we respectively consider all three possible departure-time intervals in a congested bottleneck under the following three cases: users always arrive early, users may arrive early or late, and users always arrive late. According to different situations of travelers, the choice of their travel behavior is given.

The rest of this paper is organized as follows. In the next section, we provide an overview of the deterministic bottleneck model. In Section III, the travelers’ travel costs and departure-time choice in a single bottleneck with stochastic travel time are formulated. Some properties of the bottleneck model are examined in Section IV. Numerical examples are presented in section V. Finally, section VI concludes the paper. The notations in this paper are listed in table 1.

**TABLE 1. Model parameters and decision variable.**

| Parameters and general variables |  |
|---|---|
| $t$ | Departure time |
| $r(t)$ | The departure rate |
| $R(t)$ | The cumulative departure |
| $Q(t)$ | Queue length |
| $w(t)$ | Queuing time |
| $\alpha$ | The value of travel time |
| $\beta$ | The unit cost of schedule delay early |
| $\gamma$ | The unit cost of schedule delay late |
| $t'$ | The preferred arrival time |
| $t_e$ | The departure time of the traveler who departs and arrives at the workplace on time $t'$ |
| $t_{10}$ | The departure time for the first traveler in classical bottleneck model |
| $t_{1s}$ | The departure time for the first traveler in stochastic bottleneck model |
| $t_{1c}$ | The departure time for the last traveler in classical bottleneck model |
| $t_{1s}$ | The departure time for the last traveler in stochastic bottleneck model |
| $T_0$ | The free flow time in classical bottleneck model |
| $T_{10}$ | The free flow time in stochastic bottleneck model |
| $T_{1s}$ | The free flow time in stochastic bottleneck model |
| $X$ | The uniform random variable |
| $f(x)$ | The probability density function |
| $F(x)$ | The cumulative probability density function |
| $t_1$ | The dividing point between necessarily early and possibly early or late |
| $t_2$ | The dividing point between necessarily late and possibly early or late |
| $C(t)$ | The generalized travel cost at time $t$ |
| $E[C(t)]$ | The mean trip cost |

**II. OVERVIEW OF THE CLASSICAL BOTTLENECK MODEL**

Let us consider a highway between a residential area and a CBD where travelers commute. If the arrival rate exceeds the capacity of the bottleneck on the highway, a queue develops. Let $s$ be capacity of the bottleneck. $\lambda$ is the travel demand from the residential district to the CBD.

By definition, the cumulative departure $R(t)$ can be formulated as follows:

$$R(t) = \int_{t_b}^{t} r(x)dx$$  (1)
where \( r(t) \) is the departure rate at time \( t \), and \( t_b \) is the beginning time of rush hours.

We consider that the highway is congested during the rush hours, and that the capacity of the bottleneck will have been fully utilized. The length of the queue before the bottleneck is therefore

\[
Q(t) = \max \{ R(t) - s(t - t_b), 0 \}
\]

(2)

The queuing time can be formulated as follows

\[
w(t) = \frac{Q(t)}{s}
\]

(3)

The generalized travel cost \( C(t) \) of travelers commuting who left home at time \( t \) would be expressed:

\[
C(t) = \alpha(w(t) + T_0) + \max \{ \beta(t^* - T_0 - w(t) - t), 0 \}
+ \max \{ \gamma(t + w(t) + T_0 - t^*), 0 \}
\]

(4)

At equilibrium, the driver incurs the same travel cost no matter when he leaves home. During the entire operating, the bottleneck has full capacity load operation. This condition implies all the travelers’ generalized travel costs are the same. This equilibrium condition implies

\[
\frac{\partial C(t)}{\partial t} = 0
\]

(5)

The waiting time can be explicitly expressed as:

\[
w(t) = \begin{cases} \frac{\beta}{\alpha - \beta}(t - t_b), & t \in [t_b, t_n] \\ \frac{\alpha - \beta}{\alpha + \gamma}(t_e - t), & t \in (t_n, t_e) \end{cases}
\]

(6)

where \( t_b \) and \( t_e \) represent the departure time for the first traveler and the last traveler, \( t_b \) is the departure time of the traveler who departs and arrives at the workplace on time \( t^* \). Meanwhile, they can be obtained:

\[
t_b = t^* - T_0 - \frac{N}{\beta + \gamma} \cdot \frac{N}{s}
\]

(7)

\[
t_n = t^* - T_0 - \frac{\beta \gamma}{\alpha(\beta + \gamma)} \cdot \frac{N}{s}
\]

(8)

\[
t_e = t^* + \frac{\beta}{\beta + \gamma} \cdot \frac{N}{s} - T_0
\]

(9)

III. BOTTLENECK MODEL WITH STOCHASTIC TRAVEL TIME

A. ASSUMPTIONS

Due to the impact of natural disasters, traffic accidents, road construction, and other random events, it’s always possible that the travelers are affected during the process of free driving. Therefore, the free flow time of travelers is no longer a fixed value, but in a random distribution. Throughout this paper, the following three assumptions are adopted:

I Travelers are homogeneous with the same value of time and the same values of schedule delays.

II The free flow time of travelers is in a fixed time interval, obeying a uniform distribution [16]. Let \( T_0(x) \) follows the same law, \( \tilde{T}_0(x) = T_0 + x \), where \( T_0 \) is the mean, and \( x \) obey the uniform distribution in \( [\epsilon_0, \epsilon_1] \). \( T_0 + \epsilon_0 \) is the minimum free flow time, \( T_0 - \epsilon_1 \) is the maximum free flow time. The probability density function is \( f(x) \), and the cumulative probability density function is \( F(x) \).

III Travelers are aware of the stochastic travel time probability, and their departure time choice follows the UE principle in terms of mean trip cost.

IV The bottleneck is set at the starting point of the road.

Unlike the Vickrey [1] model, we assume the capacity of the single bottleneck is stochastic, although the commuters’ departure-time choice is made deterministically based on mean trip cost. The assumption (II) implies that the current model accounts for random events that happened before the peak started, but not for random events occurring during the peak period. We assume further that the commuters learn the random events probability from their day-to-day travel (III), and adjust their departure time to minimize their expected travel costs. The assumption (IV) implies that the travelers obey FIFO (first input first Output) principle: If the bottleneck is at other points on the road, travelers may not obey the FIFO principle when arriving at the bottleneck (travelers may have the situation of early departure and late arrival at the bottleneck). In order to better describe the impact of travel time uncertainty on travelers, this paper constructs a bottleneck model based on stochastic travel time.

B. STOCHASTIC BOTTLENECK MODEL

Under the stochastic condition, the generalized travel cost \( C(t) \) of travelers commuting who left home at time \( t \) would be expressed:

\[
C(t) = \alpha(w(t) + \tilde{T}_0(x)) + \beta \max\{t^* - t - w(t) - \tilde{T}_0(x), 0\}
+ \gamma \max\{t + w(t) + \tilde{T}_0(x) - t^*, 0\}
\]

(10)

The mean trip cost for the departure time \( t \) under stochastic condition can be formulated as:

\[
E[C(t)] = \alpha[w(t) + T_0]
+ \beta E[\max\{t^* - t - w(t) - T_0, 0\}]
+ \gamma E[\max\{t + w(t) + T_0 - t^*, 0\}]
\]

(11)

Due to the uncertainty of travel time on the road, even if the travelers leave home at the same time, they may arrive at the workplace early or late. According to the early arrival or late arrival of travelers to the workplace, the travel time interval can be divided into three types: (I) travelers always arrive early; (II) travelers may arrive early or late; (III) travelers always arrive late. The three situations occur consecutively, and we use \( t_1 \) and \( t_2 \) to denote the watershed lines that separate the three situations. In the following, this paper analyzes the travel choice behavior of travelers in these three time intervals.
(I) Travelers always arrive early in the time interval \([t_0, t_1]\). In this situation, no travelers experience schedule delay late subject to all possible values of the travel time. For these early travelers, the generalized travel cost at time \(t\) can be expressed as:

\[
E[C(t)] = E[\alpha(w(t) + T_0 + x) + \beta(t^* - t - w(t) - T_0 - x)]
\]

where \(\alpha\) and \(\beta\) are the travel cost coefficients. For the left side of formula (14), calculate:

\[
E[C'(t)] = 0
\]

Take formula (12) to (13), and we can get:

\[
[\alpha(w(t) + T_0) + \beta(t^* - t - w(t) - T_0)]\int_{-\infty}^{t^*-t-w(t)-T_0} \varphi(x)dx
\]

\[
-\beta \int_{-\infty}^{t^*-t-w(t)-T_0} x\varphi(x)dx]' = 0
\]

(II) Travelers always arrive late in the time interval \([t_0, \bar{t}_e]\). In this situation, no travelers experience schedule delay late subject to all possible values of the travel time. For the late travelers, the generalized travel cost at time \(t\) can be expressed as:

\[
E[C(t)] = E[\alpha(w(t) + T_0 + x) + \gamma(t + w(t) + T_0 + x - t^*)]
\]

\[
= E[\alpha(w(t) + T_0 + x) + \gamma(t + w(t) + T_0 + x - t^*)]
\]

\[
+ \gamma \int_{t^*-t-w(t)-T_0}^{+\infty} \varphi(x)dx
\]

We can get the derivative of the waiting time:

\[
w'(t) = \frac{\beta}{\alpha - \beta}
\]

According to (17), the waiting time function of travelers at time \(t\) is:

\[
w(t) = \frac{\beta}{\alpha - \beta}(t - t_0), \quad t \in [t_0, t_1]
\]

Because \(t_1\) is the critical value of early arrival departure time, that is to say, when \(x = \epsilon_1\), travelers will not be late. Therefore, we get:

\[
t^* - t_1 - w(t_1) - T_0 - \epsilon_1 = 0
\]

Take formula (18) to (19), we can get the critical value that all travelers always arrive early:

\[
t_1 = \frac{\alpha - \beta}{\alpha}(t^* - T_0 - \epsilon_1) + \frac{\beta}{\alpha}t_0
\]

(III) Travelers may arrive early or late in the time interval \([t_0, \bar{t}_e]\). If the travel time is \(T(x)\) short enough, travelers will be early. If the travel time is \(\bar{T_0}(x)\) long enough, travelers will
be late. The generalized travel cost at time $t$ can be expressed as:

$$E[C(t)] = E[\alpha(w(t) + T_0) + \beta \int_{-\infty}^{t - t - w(t) - T_0} (t^* - t - w(t) - T_0 - x)\phi(x)dx + \gamma \int_{t - t - w(t) - T_0}^{+\infty} (t + w(t) + T_0 + x - t^*)\phi(x)dx]$$

(26)

At equilibrium, no commuter can reduce his or her mean trip cost by unilaterally altering his or her departure time. This condition implies that the travelers’ mean trip cost is constant with respect to a time instant, that is, the first derivative is zero.

$$E'[C(t)] = E[\alpha(w(t) + T_0) + \beta \max\{t^* - t - w(t) - T_0, 0\} + \gamma \max\{t + w(t) + T_0 - t^*, 0\}] = 0$$

(27)

According to (27), the waiting time function is obtained:

$$w(t) = t^* - \frac{\epsilon_1 - \epsilon_0}{\beta + \gamma} (\alpha + \gamma) - T_0 - \epsilon_0 - t$$

$$+ \sqrt{\left(\frac{\epsilon_1 - \epsilon_0}{\beta + \gamma} (\alpha + \gamma) - (t^* - T_0 - \epsilon_0)\right)^2 + 2at \frac{\epsilon_1 - \epsilon_0}{\beta + \gamma}}$$

(28)

The above gives the travel choice behavior of travelers in each time interval. But for each traveler, the expectation of travel cost will not change because of the change of randomness. That is to say, the expectation of the travel cost of the first traveler and the last traveler is equal, $E[C(t_0)] = E[C(t_e)]$.

$$E[C(t_b)] = E[C(t_e)]$$

(29)

$$E[C(t_e)] = E[\alpha(w(t) + T_0 + x) + \beta(t^* - t_b - w(t) - T_0 - x)]$$

$$= \alpha T_0 + \beta(t^* - t_b - T_0) + (\alpha - \beta)E[x]$$

(30)

$$E[C(t_e)] = E[\alpha(w(t_e) + T_0 + x) + \gamma(t_e + w(t_e) + T_0 + x - t^*)]$$

$$= \alpha T_0 + \gamma(t_e + T_0 - t^*) + (\alpha + \gamma)E[x]$$

During the entire operating, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:

$$\tilde{t}_e - \tilde{t}_b = N/s$$

(31)

According to (23), (24), (25) and equilibrium conditions, $E[C(t)]$, $\tilde{t}_b$, $\tilde{t}_e$ are obtained:

$$E[C(t)] = \alpha(T_0 + \frac{\epsilon_0 + \epsilon_1}{2}) + \frac{\beta \gamma}{\beta + \gamma} \frac{N}{s}$$

(32)

$$\tilde{t}_b = t^* - T_0 - \frac{\epsilon_0 + \epsilon_1}{2} - \frac{\gamma}{\beta + \gamma} \frac{N}{s}$$

(33)

$$\tilde{t}_e = t^* + \frac{\epsilon_0 + \epsilon_1}{2} + \frac{\beta}{\beta + \gamma} \frac{N}{s} - T_0$$

(34)

$\textbf{IV. PROPERTIES OF THE STOCHASTIC BOTTLENECK MODEL}$

We present the following propositions to reveal some interesting properties of the equilibrium solution of the proposed bottleneck model. Proposition 1 describes the relationship between two boundary points in stochastic bottleneck model. Proposition 2 describes the relationship between the stochastic bottleneck model and the deterministic bottleneck model.

$\textit{Proposition 1:}$ At equilibrium, the dividing points $t_1$ and $t_2$ of the three time intervals have the following properties:

$$\tilde{t}_b < t_1, t_1 < t_2, t_2 < \tilde{t}_e$$

$\text{Proof:}$

(1) First prove $\tilde{t}_b < t_1$.

Because

$$t_1 - \tilde{t}_b = \frac{\alpha}{\beta} - \frac{\beta}{\alpha} (t^* - T_0 - \epsilon_1) + \beta \frac{\epsilon_0 - \epsilon_1}{\alpha} - \frac{\epsilon_0}{\alpha} - \tilde{t}_b$$

In the time interval $[\tilde{t}_b$, $t_1]$, travelers always arrive early, thus, $t^* - T_0 - \epsilon_1 - \tilde{t}_b > 0$.

On the other hand, $\alpha > \beta$, so $t_1 - \tilde{t}_b > 0$.

(2) Second prove $t_2 < \tilde{t}_e$.

Because $t_2 = \frac{\alpha + \gamma}{\alpha} (t^* - T_0 - \epsilon_0) - \frac{\gamma}{\alpha} \tilde{t}_e$, we can get:

$$\tilde{t}_e - t_2 = \tilde{t}_e - \left[\frac{\alpha + \gamma}{\alpha} (t^* - T_0 - \epsilon_0) - \frac{\gamma}{\alpha} \tilde{t}_e\right]$$

$$= \frac{\alpha + \gamma}{\alpha} (\tilde{t}_e + T_0 + \epsilon_0 - t^*)$$

According to the above assumption, there must be late travelers, so there must be $\tilde{t}_e + T_0 + \epsilon_0 - t^* > 0$, thus, $\tilde{t}_e - t_2 > 0$.

(3) $t_1 < t_2$.

Because

$$t_1 = \frac{\alpha - \beta}{\alpha} (t^* - T_0 - \epsilon_1) + \beta \frac{\epsilon_0 - \epsilon_1}{\alpha}$$

$$t_2 = \frac{\alpha + \gamma}{\alpha} (t^* - T_0 - \epsilon_0) - \frac{\gamma}{\alpha} \tilde{t}_e$$

Therefore:

$$t_2 - t_1 = \frac{\gamma + \beta}{\alpha} (t^* - T_0) + \frac{\alpha - \beta}{\alpha} (t^* - T_0 - \epsilon_1) - \frac{\alpha + \gamma}{\alpha} (t^* - T_0 - \epsilon_0) - \frac{\gamma}{\alpha} \tilde{t}_e$$

$$= \frac{\alpha + \gamma}{\alpha} (t^* - T_0 - \epsilon_0) - \frac{\gamma}{\alpha} \tilde{t}_e - \frac{\beta - \gamma}{\alpha} (t^* - T_0 - \epsilon_1) - \frac{\beta}{\alpha} \tilde{t}_b$$

$$= (\epsilon_1 - \epsilon_0) + \frac{\beta}{\alpha} (t^* - T_0 - \epsilon_1 - \tilde{t}_b) + \frac{\gamma}{\alpha} (t^* - T_0 - \epsilon_0 - \tilde{t}_e)$$

At equilibrium, no commuter can reduce his or her mean trip cost by unilaterally altering his or her departure time, thus, $E[C(t_b)] = E[C(t_e)]$.

Substituting (33), (34) into $E[C(t_b)] = E[C(t_e)]$, we obtain

$$\beta(t^* - T_0 - \tilde{t}_b) + \gamma(t^* - T_0 - \tilde{t}_e) = \beta \frac{\epsilon_1 + \epsilon_0}{2} + \gamma \frac{\epsilon_1 + \epsilon_0}{2}.$$
Thus,
\[
\alpha(t_2 - t_1) = \alpha(\epsilon_1 - \epsilon_0) + \beta(t^* - T_0 - \epsilon_1 - t_0) + \gamma(t^* - T_0 - \epsilon_0 - t_e)
\]
\[
= \alpha(\epsilon_1 - \epsilon_0) - \frac{\beta}{2}(\epsilon_1 - \epsilon_0) + \frac{\gamma}{2}(\epsilon_1 - \epsilon_0)
\]
\[
= (\alpha - \frac{\beta}{2})(\epsilon_1 - \epsilon_0) + \frac{\gamma}{2}(\epsilon_1 - \epsilon_0)
\]

Because \(\alpha - \frac{\beta}{2} > 0\), so \(\epsilon_1 - \epsilon_0 > 0\), 
\(\frac{\gamma}{2} > 0\), 
So, \(t_2 - t_1 > 0\).

**Proposition 2:** When \(\epsilon_0 \to 0\), \(\epsilon_1 \to 0\), the stochastic bottleneck model follows the deterministic bottleneck model.

**Proof:**

\[
\lim_{\epsilon_1 \to 0} \frac{\alpha - \beta}{\alpha}(t^* - T_0 - \epsilon_1) = \lim_{\epsilon_1 \to 0} t_1 - \frac{\beta}{\alpha}(t^* - T_0 - \epsilon_1)
\]
\[
+ \beta(t^* - T_0 - \frac{\epsilon_0 + \epsilon_1}{2} - \frac{\gamma N}{\beta + \gamma s})
\]
\[
= \lim_{\epsilon_1 \to 0} t_1 - \frac{\beta}{\alpha}(t^* - T_0 - \epsilon_1) + \beta(t^* - T_0 - \frac{\epsilon_0 + \epsilon_1}{2} - \frac{\gamma N}{\beta + \gamma s})
\]
\[
= t^* - T_0 - \frac{\beta \gamma N}{\alpha(\beta + \gamma)}
\]

**V. NUMERICAL EXAMPLES**

In this section, we present numerical results for the personal perception bottleneck model. According to Vickrey[1], there must be \(\beta < \alpha < \gamma\). Unless otherwise specified, throughout this section, we adopt the following three parameter values from Benezech and Coulomb[18], and some modifications are made. The unit of travel time cost \(\alpha = 1\), the unit of schedule delay early \(\beta = 0.4\), the unit of schedule delay late \(\gamma = 2\), and consider the situation with \(s = 1800\), \(N = 3000\), 
\(t^* = 9:00\), \(T_0 = 45\), \(\epsilon_0 = -5\), \(\epsilon_1 = 5\).

![Equilibrium travel cost](image_url)

Fig.1 shows equilibrium of the traveler cost and its components. It can be seen from this picture that the travel expenses mainly consist of three parts, the sum of which are equal and equal to 78.3. Travel time cost mainly includes two parts: free-flow travel time cost and queuing time cost. For early travelers, their travel costs are mainly composed of early arrival penalty costs, while for late travelers, most of their costs are late arrival penalty costs. In the middle of the peak hours, the main component is the travel time cost, but due to the stochastic travel time, their schedule delay costs exist significantly. The schedule delay early cost always decreases and the schedule delay late cost increases with respect to the \(T_0(x)\).

Fig. 2(a) shows the time distribution of the first traveler’s arrival at the workplace under stochastic conditions. Fig 2(b) shows the time distribution of the last traveler arriving at the workplace. In Fig. 2(a), the black solid line represents the time when the first traveler arrives at the workplace when the free flow time is fixed, the long black dotted line represents the travel time of the first traveler to arrive at the workplace at random, and the black dotted line represents the earliest travel time of the first traveler to the workplace. It can be seen from the graph that the greater the randomness of free flow time, the wider the range of earliest arrival time. This means that the greater the random fluctuation of travelers on the road, the more serious the situation of travelers’ arriving at workplace. Similarly, similar results can be found in Fig. 2 (b). According to fig. 2(a) and 2(b), the maximum probability of arriving at the workplace is [7.36,9.53],

\[
0 \leq x \leq 100
\]

\[
E(t(x))
\]

\[
\text{Schedule delay early cost}
\]

\[
\text{Schedule delay late cost}
\]

\[
\text{Travel time cost}
\]

\[
\text{Maximum probability of arriving at the workplace is [7.36,9.53]}
\]
the minimum time interval is [7.86,9.02], and the time interval under fixed condition is [7.61,9.28]. It can be seen from the above results that when the travel time on the road is random, the time interval of the traveler arriving at the workplace is no longer a fixed time interval, but a time interval with boundary. The reason is that with the fluctuation of travel time on the road, the time for travelers to arrive at work place is no longer a fixed value, but fluctuates within a time interval, and the greater the randomness, the greater the time interval of arriving at work place.

VI. CONCLUSION

Based on the classical bottleneck model, this paper extended the classical bottleneck model for studying the travelers’ departure-time choice behavior with stochastic travel time. The travel time of travelers is assumed to be uniformly distributed and the travelers’ departure-time choice follows the UE principle in terms of their mean trip cost. Based on the user equilibrium criterion, the departure rate, expectation value, and the critical points of corresponding time intervals are derived. The results show that with the decrease in random degree of travel time, the time interval of arriving at workplace also changes. This paper mainly considers the departure time choice behavior of travelers, but does not take into consideration the impact of heterogeneity and other related strategies on travelers. These will be the contents of our next research.

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