In this paper, the design of a controller for the altitude and rotational dynamics is presented. In particular, the control problem is to maintain a desired altitude in a fixed position. The unmanned aerial vehicle dynamics are described by nonlinear equations, derived using the Newton–Euler approach. The control problem is solved imposing the stability of the error dynamics with respect to desired position and angular references. The performance and effectiveness of the proposed control are tested, first, via numerical simulations, using the Pixhawk Pilot Support Packages simulator provided by Mathworks. Then, the controller is tested via a real-time implementation, using a quadrotor Aircraft F-450.

1. Introduction

Quadrotors have recently attracted the attention of many researchers due to their interesting applications. As a matter of fact, the potential applications of such devices are countless. Examples of such applications include searching and surveillance, monitoring, and rescuing tasks. From a methodological point of view, the interest relies on the fact that a quadrotor is a complex underactuated system with high nonlinearities and strong dynamical couplings. Furthermore, it is affected by aerodynamic disturbances, unmodeled dynamics, and parametric uncertainties. Therefore, the quadrotors represent an interesting testbed for testing new control techniques.

There are a large number of works dealing with quadrotors. As far as the mathematical model is concerned, in Bouabdallah et al. [1], Zeng and Zhao [2], and Nagaty et al. [3], a Newton–Euler model was presented. Furthermore, Magnussen et al. [4] and Valenti et al. [5] considered quaternions to describe the angular kinematics, whilst Antonio-Toledo et al. [6] applied the Euler–Lagrange equations to obtain the whole quadrotor mathematical model. Regarding the control of quadrotors, many control techniques have been proposed. In Panomruttanarung et al. [7] and Pounds et al. [8], the linear quadratic regulator control and the proportional integral derivative control, respectively, were exploited to design a control law. However, these controllers ensure only local stability. In order to enlarge the basin of attraction, nonlinear control techniques have also been considered. Examples are sliding model control Luque–Vega et al. [9], backstepping Bouabdallah and Siegwart [10], and adaptive control Matouk et al. [11]. Moreover, a global fast dynamic terminal sliding mode control method was proposed for position and attitude tracking control in Xiong and Zhang [12]. An adaptive command filtered backstepping control law was designed for trajectory tracking in Choi and Ahn [13]. In Liu et al. [14], a robust adaptive attitude tracking control for a quadrotor with an unknown inertia matrix and bounded external disturbances was proposed. A command filtered
Implementation of an adaptive backstepping was proposed in Dong et al. [15], and the stability of the closed loop system was proved via the Lyapunov direct method. Finally, in Islam et al. [16], an observer-based adaptive fuzzy backstepping controller was designed for trajectory tracking, in the case of a quadrotor undergoing wind gusts and with parametric uncertainties. All these aforementioned methods provide good dynamic performance and robust stability and are tested mainly considering numerical simulations.

The main contribution of this paper is the design of a controller for the attitude and altitude of a quadrotor helicopter. This controller has been designed using the backstepping technique and has been tested using numerical simulations and real-time experimentation. In particular, the focus of this paper is to obtain a controller ensuring hovering so that

1. The reference attitude is zero for the Euler angles describing the quadrotor angular position
2. The reference altitude is a constant value

The performance and effectiveness of the proposed controller has been first tested with numerical simulations using the Pixhawk Pilot Support Package (PSP). Then, the real-time implementation has been performed implementing the proposed controller on a real F-450 quadrotor equipped of a Pixhawk and tested under environmental perturbations.

The paper is organized as follows. Section 2 introduces the description and the mathematical model of the quadrotor. In Section 3, the control problem is solved. In Section 4, numerical simulations and real-time tests are provided to show the effectiveness of the proposed controller. Finally, some concluding remarks are commented in Section 5.

2. Mathematical Model

The quadrotor considered in this work consists of a rigid frame equipped with four rotors. The rotors generate the propeller force $F_i = b_i \omega_i^2$ proportional to the propeller angular velocity $\omega_i$, $i = 1, 2, 3, 4$. The propellers 1 and 3 rotate counterclockwise, and the propellers 2 and 4 rotate clockwise.

Denote $RC(O, e_1, e_2, e_3)$ and $\Gamma(R, e_1, e_2, e_3)$ as the frames fixed with the Earth and the quadrotor, respectively, with $\Omega$ coincident with the center of mass of the quadrotor (see Figure 1). The quadrotor absolute position in RC is described by $p = (x, y, z)^T$, whereas its attitude is described by the Euler angles $\alpha = (\phi, \theta, \psi)^T$, where $\phi, \theta, \psi \in (-\pi/2, \pi/2)$ are the pitch, roll, and yaw angles, respectively. The sequence 3–2–1 has been considered by Hughes [17]. Moreover, $\nu = (\nu_1, \nu_2, \nu_3)^T$ and $\omega = (\omega_1, \omega_2, \omega_3)^T$ are the linear and angular velocities of the center of mass of the quadrotor, expressed in RC and in $\Gamma$, respectively.

The translation dynamics (in RC) and rotation dynamics (in the $\Gamma$) of the quadrotor are:

$$\dot{p} = v,$$

$$\dot{v} = \frac{1}{m} \mathbf{R}(\alpha) F_{\text{prop}} + \frac{1}{m} F_{\text{grav}} + \frac{1}{m} F_d,$$

$$\dot{\omega} = M(\alpha) \omega,$$

$$\dot{\omega} = J^{-1} ( -\omega J \omega + \tau_{\text{prop}} - \tau_{\text{gyro}} + M_d ),$$

where $m$ is the mass of the quadrotor, $J$ is the inertia matrix of the quadrotor, and $J = \text{diag}[J_x, J_y, J_z]$ (expressed in $\Gamma$), and

$$F_{\text{prop}} = \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{pmatrix},$$

$$\tau_{\text{prop}} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} I(F_2 - F_4) \\ I(F_3 - F_1) \\ c(F_1 - F_2 + F_3 - F_4) \end{pmatrix},$$

are the forces and moments produced by the propellers (inputs), with $I$ as the distance between the center of mass to the rotor shaft. Moreover, $F_{\text{grav}} = (0 \ 0 \ -mg)^T$ is the force due to the gravity expressed in RC.

The vectors expressed in $\Gamma$ are transformed into vectors in RC by the rotation matrix:
\[ R(\alpha) = \begin{pmatrix} c_\phi s_\psi & s_\phi s_\psi - c_\psi s_\theta & c_\phi c_\psi + s_\phi s_\theta \\ c_\phi s_\psi & s_\phi s_\psi + c_\psi s_\theta & -c_\phi c_\psi + s_\phi s_\theta \\ s_\phi & s_\theta s_\psi & c_\phi s_\theta \end{pmatrix}, \]

(4)

with \( c_\phi = \cos(\gamma), s_\phi = \sin(\gamma), \) and \( \gamma = \phi, \theta, \psi. \) The angular velocity dynamics are expressed using the matrix:

\[ M(\alpha) = \begin{pmatrix} 1 & s_\phi t g_\theta & c_\phi t g_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi s_\theta & c_\phi s_\theta \end{pmatrix}, \]

(5)

with \( t g_\gamma = \tan(\gamma) \) and \( s c_\gamma = \sec(\gamma). \)

The rolling torque \( \tau_1 \) is produced by the forces \( F_2 \) and \( F_4. \) Similarly, the pitching torque \( \tau_2 \) is produced by the forces \( F_1 \) and \( F_3. \) Due to Newton’s third law, the propellers produce a yawing torque \( \tau_3 \) on the body of the quadrotor, in the opposite direction of the propeller rotation. Moreover,

\[ \tau_{\text{gyro}} = \sum_{i=1}^{4} (-1)^{i+1} J_{\rho i} \omega_i \omega_i e_3, \]

(6)

is the gyroscopic torque due to the propeller rotations, with \( J_{\rho i} \) the propeller moment of inertia with respect to its rotation axis. Finally, \( F_i \) and \( M_i \) are the forces and torques due to the external disturbances, which are assumed negligible here.

Under the assumption of small angles, matrix \( M(\alpha) \) reduces to the identity matrix. This assumption is justified by the fact that the control objective is to maintain the quadrotor in an hover position Nagaty et al. [3]. This leads to a simplified mathematical model of the quadrotor given by

\[ \begin{align*}
\dot{x} &= \frac{1}{m}(c_\phi s_\psi + s_\phi s_\psi)u_1, \\
\dot{y} &= \frac{1}{m}(c_\phi s_\psi - s_\phi c_\psi)u_1, \\
\dot{z} &= \frac{1}{m}c_\phi c_\psi u_1 - g, \\
\dot{\phi} &= \frac{J_y - J_z}{J_x} \hat{\phi} \psi - \frac{J_y}{J_x} \omega_\phi \hat{\theta} + \frac{1}{J_x} \tau_1, \\
\dot{\theta} &= \frac{J_z - J_x}{J_y} \hat{\theta} \phi + \frac{J_z}{J_y} \omega_\theta \hat{\psi} + \frac{1}{J_y} \tau_2, \\
\dot{\psi} &= \frac{J_x - J_y}{J_z} \hat{\psi} \theta + \frac{1}{J_z} \tau_3,
\end{align*} \]

(7)

where \( X = (\phi, \theta, \psi, \phi_1, \theta_1, \psi_1, \phi_2, \theta_2, \psi_2) \) are the measured signals. The control objective is to match the UAV attitude \( X \) to a desired reference:

\[ X_{\text{ref}} = (\phi_{1,\text{ref}}, \phi_{2,\text{ref}}, \theta_{1,\text{ref}}, \theta_{2,\text{ref}}, \psi_{1,\text{ref}}, \psi_{2,\text{ref}}). \]

(11)

To this aim, let us define the tracking errors as

\[ \begin{align*}
e_{\phi,1} &= \phi_1 - \phi_{1,\text{ref}}, \\
e_{\phi,2} &= \phi_2 - \phi_{2,\text{ref}} + k_1 e_{\phi,1}, \\
e_{\theta,1} &= \theta_1 - \theta_{1,\text{ref}}, \\
e_{\theta,2} &= \theta_2 - \theta_{2,\text{ref}} + k_2 e_{\theta,1}, \\
e_{\psi,1} &= \psi_1 - \psi_{1,\text{ref}}, \\
e_{\psi,2} &= \psi_2 - \psi_{2,\text{ref}} + k_3 e_{\psi,1},
\end{align*} \]

(12)

with \( k_1, k_2, k_3 > 0. \) Note that \( e_{\phi,1} = 0 \) implies \( e_{\phi,2} = 0, \)

\( j = \phi, \theta, \psi. \) Deriving the error system (12) with respect to (10) is possible to calculate the error dynamics:
Using the following control law [18],
\[
\tau_1 = I_x \left( \frac{J_y - J_z}{J_x} \dot{\phi}_2 \dot{\psi}_2 + \frac{J_y}{J_x} \omega_p \dot{\theta}_2 + \frac{1}{J_x} \dot{\tau}_1 \right)\]
\[
+ \frac{1}{J_x} \dot{\phi}_2 + k_1 (\dot{e}_\phi - k_1 e_\phi - \dot{\phi}_{\text{ref}})
\]
\[
\tau_2 = I_y \left( \frac{J_z - J_x}{J_y} \dot{\phi}_1 \dot{\psi}_2 + \frac{J_z}{J_y} \omega_p \dot{\theta}_1 + \frac{1}{J_y} \dot{\tau}_2 \right)\]
\[
+ \frac{1}{J_y} \dot{\phi}_1 + k_2 (\dot{e}_\theta - k_2 e_\theta - \dot{\theta}_{\text{ref}})
\]
\[
\tau_3 = I_z \left( \frac{1}{J_z} \omega_p \dot{\psi}_1 \dot{\theta}_1 - k_3 (\dot{e}_\psi - k_3 e_\psi - \dot{\psi}_{\text{ref}})\right)
\]
\[
- \frac{1}{J_z} \omega_p + k_4 (\dot{e}_\psi - k_4 e_\psi - \dot{\psi}_{\text{ref}})
\]

Using the following control law [18],
\[
\tau_1 = I_x \left( \frac{J_y - J_z}{J_x} \dot{\phi}_2 \dot{\psi}_2 + \frac{J_y}{J_x} \omega_p \dot{\theta}_2 + \frac{1}{J_x} \dot{\tau}_1 \right)\]
\[
- \dot{\phi}_{\text{ref}} + k_1 (e_\phi - k_1 e_\phi)
\]
\[
\tau_2 = I_y \left( \frac{J_z - J_x}{J_y} \dot{\phi}_1 \dot{\psi}_2 + \frac{J_z}{J_y} \omega_p \dot{\theta}_1 + \frac{1}{J_y} \dot{\tau}_2 \right)\]
\[
- \dot{\theta}_{\text{ref}} + k_2 (e_\theta - k_2 e_\theta)
\]
\[
\tau_3 = I_z \left( \frac{1}{J_z} \omega_p \dot{\psi}_1 \dot{\theta}_1 - k_3 (e_\psi - k_3 e_\psi)\right)
\]
\[
- \frac{1}{J_z} \omega_p + k_4 (e_\psi - k_4 e_\psi)
\]

where \( e_X = \left( e_\phi, e_\theta, e_\psi, \right) \), which converge exponentially to zero. In fact, considering the Lyapunov candidate
\[
V_X(t, e_X) = \frac{1}{2} e_X^T P_X e_X,
\]
and differentiating along the error dynamics (15), one obtains
\[
\dot{V}_X(t, e_X) = e_X^T A_X^T P_X + P_X A_X e_X = -e_X^T Q_X e_X < 0,
\]
so that the tracking error converges globally exponentially to zero [19].

### 3.2. Altitude Control
For the altitude control, let us consider the altitude dynamics:
\[
\dot{z}_1 = z_2,
\]
\[
\dot{z}_2 = g + c_\phi c_\theta \frac{1}{m} u_1.
\]

The control problem is to maintain the quadrotor at a desired constant altitude \( z_{1,\text{ref}} \), \( z_{2,\text{ref}} = 0 \). The tracking errors are defined as
In this section, we describe the robotic platforms. Its processor can run to 168 MHz/252 MIPS multi-rotors, helicopters, cars, boats, and other mobile robotics systems that are based on sensor measurements. The onboard electronics system consists of a flight controller, a hardware setup, and some ultrasonic sensors.

4. Simulation and Experimental Results

4.1. Simulation Results. For the numerical simulations, the quadrotor model provided by the Pixhawk PSP, in Simulink, has been used. This model captures the attitude and altitude flight control model called px4 demo attitude control, which shows a good performance in predicting the dynamic quadrotor behavior, very close to the real drone dynamics.

Controllers (14) and (21) use the nominal values of Table 1 and the gains of Table 2. The simulations have been performed in two steps. In the first one, the quadrotor is stabilized in altitude. In Figure 2, the altitude \( z \) is shown. In the second step, the quadrotor is stabilized in attitude. The pitch, roll, and yaw angles are shown in Figure 3. The initial conditions considered are \( \dot{z}(0) = 0 \), \( \phi(0) = -5.73^{\circ} \), \( \theta(0) = 5.73^{\circ} \), and \( \psi(0) = 5.73^{\circ} \). The reference values are \( z_{ref} = 2 \text{ m} \) and \( \phi_{ref} = \theta_{ref} = \psi_{ref} = 0 \).

4.2. Experimental Results. In this section, we describe the physical setting of the embedded control that allows stabilizing the quadrotor. An embedded control system generally consists of three elements: sensors, actuators, and a microcontroller. The microcontroller interacts with the continuous dynamics of the plant via the sensors and actuators, and its major function is to compute and generate control commands for the actuators that are based on sensor measurements. The onboard electronic system consists of a flight controller, a hardware setup, and some ultrasonic sensors.

4.2.1. Flight Controller. The 3DR-PIXHAWK is a high-performance autopilot-on-module suitable for fixed wing, multi-rotors, helicopters, cars, boats, and other mobile robotic platforms. Its processor can run to 168 MHz/252 MIPS Cortex-M4F with 256 KB in RAM and 2 MB flash and has 14 PWM/Servo outputs and abundant connectivity options for additional peripherals, such as 5x UART (serial ports), one high-power capable, 2x with HW flow control, 2x CAN, Spektrum DSM/DSM2/DSM–X® Satellite compatible input, PPM sum signal input, RSSI (PWM or voltage) input, 12C, SPI, 3.3 and 6.6 V ADC input. Moreover, the 3DR-Pixhawk has different sensors, such as ST Micro L3GD20H 16 bit gyroscopes, ST Micro LSM303D 14 bit accelerometers/magnetometers, Invensense MPU 6000 3-axis accelerometers/gyroscopes, and MEAS MS611 barometers.

4.2.2. Hardware Setup. The structure of the quadrotor is composed by an F-450 frame with integrated PCB wiring, whereas the rotors are brushless motors manufactured by E–max, with 935 rpm/V and a 10 × 4.5″ propellers. Turnigy speed drivers (ESC) are BHC type at 18 A max. The battery used in this setup is a 3S, 2800 mAh, 25C. The radio-transmitter is a Turnigy with 9 PPM channels working at 2.4 GHz.

An LV-MaxSonar-EZ4 sensor with a resolution of 2.54 cm, 20 Hz reading rate, 42 kHz ultrasonic sensor measures, a maximum Range of 645 cm, operating in the range of 2.5–5.5 VDC was used to measure the altitude.

For the real-time running, the same parameter values and gains shown in Table 2 were used. The initial condition for the altitude was \( z(0) = 2.16 \text{ m} \). For the pitch, roll, and yaw angles, the initial condition were chosen at \( \phi_{i}(0) = 1.6^{\circ} \), \( \theta_{i}(0) = 0.9^{\circ} \), and \( \psi_{i}(0) = -1.6^{\circ} \). Figures 4 and 5 show the dynamic behavior of the altitude and the pitch, roll, and yaw angles, respectively. It is worth noticing that these
Figure 3: Numerical simulation: (a) roll angle $\phi_1$ (solid) and roll angle reference $\phi_{1,\text{ref}}$ (dash); (b) pitch angle $\theta_1$ (solid) and pitch angle reference $\theta_{1,\text{ref}}$ (dash); (c) yaw angle $\psi_1$ (solid) and yaw angle reference $\psi_{1,\text{ref}}$ (dash).

Figure 4: Experimental implementation: quadrotor’s altitude $z_1$ (solid) and altitude reference $z_{1,\text{ref}}$ (dash).
experimental tests were performed in an outdoor environment, without using the GPS sensor.

5. Conclusions

In this paper, a controller based on the stabilization technique for the altitude and attitude error has been proposed for a quadrotor. The simulation results have been performed using the Pixhawk PSP. Then, the controller has been implemented on a laboratory quadrotor. The simulation and experimental results show a good performance, even in outdoor environment, showing some degree of robustness in the presence of environmental disturbances.

Data Availability

The figures, tables, and other data used to support this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] S. Bouabdallah, P. Murrieri, and R. Siegwart, “Design and control of an indoor Micro quadrotor,” in Proceedings of the IEEE International Conference on Robotics and Automation, pp. 1–6, New Orleans, LA, USA, April-May 2004.
[2] Y. Zeng and L. Zhao, “Parameter identification for unmanned four–rotor helicopter with nonlinear model,” in Proceedings of the 2014 IEEE Chinese Guidance, Navigation and Control Conference, pp. 922–926, Yantai, China, August 2014.
[3] A. Nagaty, S. Saeedi, C. Thibault, M. Seto, and H. Li, “Control and navigation framework for quadrotor helicopters,” Journal of Intelligent & Robotic Systems, vol. 70, no. 1–4, pp. 1–12, 2013.
[4] O. Magnusson, M. Ottestad, and G. Hovland, “Experimental validation of a quaternion–based attitude estimation with direct input to a quadcopter control system,” in Proceedings of the International Conference on Unmanned Aircraft Systems (ICUAS), pp. 48–485, Atlanta, GA, USA, May 2013.
[5] R. Valenti, I. Dryanovski, and J. Xiao, “Keeping a good attitude: a quaternion-based orientation filter for IMUs and MARGs,” Sensors, vol. 15, no. 8, pp. 19302–19330, 2015.
[6] M. Elena Antonio–Toledo, A. Y. Alanis, and E. N. Sanchez, “Robust neural decentralized control for a quadrotor UAV,” in Proceedings of the International Joint Conference on Neural Networks (IJCNN), pp. 714–719, Vancouver, Canada, July 2016.

[7] B. Panomruttanarug, K. Higuchi, and F. Mora-Camino, “Attitude control of a quadrotor aircraft using LQR state feedback controller with full order state observer,” in Proceeding of the International Conference on Instrumentation, Control and Information Technology (SICE), pp. 2041–2046, Konya, Turkey, June 2013.

[8] P. E. I. Pounds, D. R. Bersak, and A. M. Dollar, “Stability of small-scale UAV helicopters and quadrotors with added payload mass under PID control,” Autonomous Robots, vol. 33, no. 1-2, pp. 129–142, 2012.

[9] L. Luque–Vega, B. Castillo–Toledo, and A. G. Loukianov, “Robust block second order sliding mode control for a quadrotor,” Journal of the Franklin Institute, vol. 349, no. 2, pp. 719–739, 2012.

[10] S. Bouabdallah and R. Siegwart, “Backstepping and sliding-mode techniques applied to an indoor Micro quadrotor,” in Proceedings of the 2005 IEEE International Conference on Robotics and Automation, Barcelona, Spain, April 2005.

[11] D. Matouk, O. Gherouat, F. Abdessemed, and A. Hassam, “Quadrotor position and attitude control via backstepping approach,” in Proceedings of the 8th International Conference on Modelling, Identification and Control (ICMI), pp. 73–79, Algiers, Algeria, November 2016.

[12] J.-J. Xiong and G.-B. Zhang, “Global fast dynamic terminal sliding mode control for a quadrotor UAV,” ISA Transactions, vol. 66, pp. 233–240, 2017.

[13] Y. C. Choi and H. S. Ahn, “Nonlinear control of quadrotor for point tracking: actual implementation and experimental tests,” IEEE/ASME Transactions on Mechatronics, vol. 20, no. 3, pp. 1179–1192, 2015.

[14] Y.-C. Liu, J. Zhang, T. Zhang, and J.-Y. Song, “Robust adaptive spacecraft attitude tracking control based on similar skew-symmetric structure,” Computers & Electrical Engineering, vol. 56, pp. 784–794, 2016.

[15] W. Dong, J. A. Farrell, M. M. Polycarpou, V. Djapic, and M. Sharma, “Command filtered adaptive backstepping,” IEEE Transactions on Control Systems Technology, vol. 20, no. 3, pp. 566–580, 2012.

[16] S. Islam, J. Dias, and L. D. Seneviratne, “Adaptive output feedback control for miniature unmanned aerial vehicle,” in Proceedings IEEE International Conference on Advanced Intelligent Mechatronics, AIM, pp. 318–322, Banff, Canada, July 2016.

[17] P. C. Hughes, Spacecraft Attitude Dynamics, Dover Publications, Inc., Mineola, NY, USA, 1986.

[18] A. Isidori, Nonlinear Control Systems, Springer-Verlag, Berlin, Germany, 3rd edition, 1995.

[19] H. K. Khalil, Nonlinear Systems, Prentice-Hall, Upper Saddle River, NJ, USA, 2002.