Spinning Particle, Rotating Black Hole and Twistor-String *

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March 27, 2022

Abstract

Structure of the spinning particle based on the rotating black hole solution is considered. It has gyromagnetic ratio $g = 2$ and a nontrivial twistorial and stringy systems. The mass and spin appear from excitations of the Kerr circular string, while the Dirac equation describes excitations of an axial stringy system which is responsible for scattering. Complex Kerr geometry contains an open twistor-string, target space of which is equivalent to the Witten’s ‘diagonal’ of the $\mathbb{CP}^3 \times \mathbb{CP}^*^3$.

Rotating Black Hole as Spinning Particle

In this paper we consider the model of spinning particle [1] which is based on the Kerr rotating black hole solution and has a reach spinor, twistor and stringy structures. The inspired by strings and twistors methods have led to the strong progress in computation of some scattering amplitudes [2]. In the recent paper [3] Witten suggested a ‘twistor-string’ which may be an

*Talk at the SPIN 2004 Symposium at Trieste.
element of the structure of fundamental particles. We show that a version of
the ‘twistor-string’ is presented in the Kerr spinning particle.

**Twistors** may be considered as the lightlike world-lines [4]. The lightlike
momentum \( p^\mu \) may be represented in the spinor form \( p^\mu = \psi \sigma^\mu \gamma^+ \). Twistor
is a generalization of the spinor which is necessary for description of the null
word-lines carrying an angular momentum, \( x^\mu(t) = x_0^\mu + p^\mu t \) with \( x_0 \neq 0 \). It
contains two extra spinor components \( \omega_\dot{\alpha} = \psi^\dot{\alpha} x_0^\nu \sigma_{\nu \dot{\alpha}} \). The set \( \{ x^\mu, \psi^\dot{\alpha} \} \),
or the equivalent set \( \{ x^\mu, Y \} \), where \( Y = \psi_2/\psi_1 \) is a projective spinor, may
also be considered as twistors.

**Black Hole which is neither ‘Black’ nor ‘Hole’** - this joke by P.
Townsend has direct relation to the Kerr spinning particle. The ratio angular
momentum/mass, \( a = J/m \), for spinning particles is very high and the black-
hole horizons disappear revealing the naked singular ring which is the branch
line of the Kerr space on the physical and ‘mirror’ sheets. The strings and
twistors going through the Kerr ring pass into a ‘mirror’ world and look as
semi-infinite. In [5] a model of spinning particle was suggested, where the
quantum electromagnetic excitations of the Kerr ring generate the spin and
mass. It was recognized that the Kerr ring is a closed string with traveling
waves [6], and excitations of the Kerr ring lead to the appearance of the extra
axial stringy system consisting of two semi-infinite singular strings of opposite
chiralities [1], see Fig. 1. These excitations generate the chiral traveling waves

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\begin{align*}
\text{Figure 1: Kerr’s singular ring and two semistrings (twistors) of opposite}
\text{chiralities.}
\end{align*}
\]

along the axial semistrings, and the Dirac spinor
\( \Psi = \begin{pmatrix} \phi_\dot{\alpha} \\ \chi^\dot{\alpha} \end{pmatrix} \) describes in
the Weyl basis an interplay of two axial traveling waves of opposite chiralities
This axial stringy system plays an important role in the formation of the third stringy structure, the *complex* twistor-string.

**Basic properties of the Kerr spinning particle:**

- Anomalous gyromagnetic ratio $g = 2$ as that of the Dirac electron,
- Twistorial and stringy structures,
- Mass and spin of the particle appear from electromagnetic excitations of the Kerr ring (traveling waves) - the Wheeler’s ‘geon’,
- Compton region is structured by the Kerr circular string.
- Dirac equation describes traveling waves on two axial semistrings.

A few properties may also have relation to foundations of quantum theory:

- Wave function has a physical carrier - the axial stringy system.
- Axial string may control the motion of particle due to a topological coupling to circular string, which reproduce the old de Broglie conjecture.
- The quantum property - absence of radiation by oscillations - is exhibited here at the classical level due to the twofoldedness of the Kerr space: the loss of energy on the ‘physical’ sheet of space is compensated by ingoing radiation from the ‘mirror’ sheet.

Besides, there are the relations to the Skirme and chiral bag models: the Kerr congruence is a twisting generalization of the ‘hedgehog’ ansatz.

**Complex Kerr geometry and twistor-string**

In the *Kerr-Schild formalism* [7] metric has the form $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$, where $\eta_{\mu\nu}$ is the metric of Minkowski space, $x^\mu = (t, x, y, z)$, and $k_\mu$ is a vortex of the null field which is tangent to the Kerr’s congruence of twistors \{x$^\mu$, Y\} which is determined by the **Kerr theorem**[4, 1]. Congruence of twistors, a geodesic family of null rays, covers the Kerr space twice (see figures in [1, 6].) Two axial semistrings $z^-$ and $z^+$ are created by two twistors corresponding to $Y = 0$ and $Y = \infty$.

The complex Kerr string appears naturally in the Newman-initiated *complex representation* of the Kerr geometry [8] which is generated by a complex
world-line $X_0^\mu(\tau) \in \mathbb{C}M^4$. The complex time $\tau = t_0 + i\sigma$ forms a stringy world-sheet [9, 10]. The real fields on the real space-time $x^\mu$ are determined via a complex retarded-time construction, where the vectors $K^\mu = x^\mu - X_0^\mu(\tau)$ have to satisfy the complex light-cone constraints $K_\mu K^\mu = 0$. This allows one to select two families of twistors: left (holomorphic) $\{X_0, Y\}$ and right (antiholomorphic) $\{\bar{X}_0, \bar{Y}\}$.

In the Kerr case $X_0^\mu(\tau) = (\tau, 0, 0, i\alpha)$, and the complex retarded-time equation $t - \tau \equiv t - t_0 - i\sigma = \tilde{r}$ (where $\tilde{r} = r + i\alpha \cos \theta$ is the Kerr complex radial distance and $\theta$ is an angular direction of twistor) shows that on the real space $\sigma = -a \cos \theta$. Therefore, the light-cone constrains select a strip on $\tau$ plane, $\sigma \in [-a, +a]$, and the complex world-sheet acquires the boundary, forming an open string $X_0^\mu(t, \sigma)$.

![Figure 2: The complex twistor-string is stuck to two semistrings by imbedding into the real Kerr geometry. The subset of semi-twistors at $\phi = \text{const.}$ is shown.](image)

This string is similar to the well known $N=2$ string [9]. In $\mathbb{C}M^4$ it has only the chiral left modes $X_0(\tau)$, while the complex conjugate string has only right ones, $\bar{X}_0(\bar{\tau})$. By imbedding this string into the real space-time, the left and right structures are identified by orientifolding the world-sheet [10]. The resulting string describes a massive particle and has a broken $N=2$ supersymmetry. Its target space is equivalent to ‘diagonal’ of $\mathbb{C}P^3 \times \mathbb{C}P^3$.  

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The twistors, joined to the ends of the complex string $X_\nu^\tau(t \pm ia)$, have the directions $\theta = 0, \pi$ and are generators of the singular $z^\pm$ semi-strings, so the complex string turns out to be a D-string which is stuck to two singular semistrings of opposite chiralities, see Fig. 2. These $z^\pm$ singular semistrings may carry the Chan-Paton factors (currents) playing the role of quarks with respect to the complex string. The Kerr circular string is responsible for the mass and spin of the particle, while the axial semistrings are responsible for the scattering processes.

Acknowledgments

Author thanks the ICTP and Organizers of the SPIN2004 Symposium for invitation and financial support, and the RFBR for travel grant. Work is partially supported by grant 03-02-27190 of the RFBR.

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