Muons $g-2$ and CP violation in MSSM

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Abstract

We study the constraints of the CP violation in the muon $g-2$ preferred region of the minimal supersymmetric standard model assuming a universal slepton masses within first two generations. We present two particular scenarios where the $g-2$ anomaly is predicted within 2 $\sigma$ level mainly through the chargino loop or the bino loop. We found that for both cases the electron EDM experiment already highly constrained the CP phase of the parameters: either the Arg[$\mu M_1$] or Arg[$\mu M_2$] should be smaller than $O(2-3) \times 10^{-5}$. If the muon $g-2$ anomaly is explained by the MSSM, a particular SUSY breaking mechanism is needed to guarantee the small CP phase of SUSY parameters. Otherwise, a tuning of $O(10^{-5})$ is needed to cancel the phase in a general CP violated SUSY model.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) as the minimal extension of the standard model in the framework of supersymmetry, remains one of the well-motivated models beyond the standard model. It not only alleviates the Higgs mass hierarchy problem but also provides a dark matter candidate as well as predicts the unification of the gauge couplings at a high energy scale. Therefore it is also one of the most important targets to search for at high-energy physics experiments. However, the large hadron collider (LHC) already set a very strong limit on the masses of SUSY particles. For example, the gluino and first two generation squarks should be heavier than 2 TeV, and the electroweak particles should beyond a few hundred GeV \[1-4\].

Since there are still no signatures of SUSY particles in direct search experiments. It is intriguing to consider that whether the precision measurement could provide any indirect signature of the SUSY particles. For example, experiments to measure the lepton flavor-related process such as \(\mu \rightarrow e\gamma\), muon \(g-2\), electron EDM, etc, can reach very high precision and may provide a good signature of high scale physics. Interestingly, the Fermilab just announced the new measurement of the muon anomalous magnetic momentum \([5]\) and confirm previous search results \([6]\), indicating a large deviation of SM prediction at 4.2\(\sigma\) \([7]\) after combining:

\[
\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}
\] (1)

It would be very interesting if such deviation can be explained in the framework of supersymmetry. Indeed there are already numerous studies on these \([8-12]\) and it shows that if smuon masses are less than TeV such anomaly can be easily explained.

On the other hand, to avoid the large FCNC process in the quark sector, a flavor-blinded mediation of SUSY breaking models are usually preferred. For example, in the gauge mediation models, the gaugino mediation models, or the anomaly mediation models, a universal squark masses is usually predicted at a high energy scale. However, such kinds of models also predict a universal masses for the slepton sector, resulting in a close mass parameters for the first two generation sleptons at low energy scale\(^1\). If the SUSY parameter is generally CP violated, it would unavoidably induce large CP violation in the electron sector as well. The measurement of electron EDM already highly constrained the CP violation effect in the electron sector \([13]\)

\[
|d_e| < 1.1 \times 10^{-29}
\] (2)

\(^1\) The third generation sfermions could be much different at low energy scale due to the large RGE effect from the Yukawa sector \([10]\).
This fact intriguing us that the muon $g - 2$ and electron EDM might be correlated and the CP phase of the SUSY parameters should be constrained.

In this paper, we study the constraints of electron EDM on the CP violation phase of the SUSY parameter space where the muon $g - 2$ anomaly can be explained. This article is organized as follows, in Sec. II we briefly discuss the physics related to the muon $g - 2$ and electron EDM. In Sec. III we present our strategy of numerical calculation and the result. We draw our conclusion in Sec. IV.

II. MUON $G - 2$ AND ELECTRON EDM IN MSSM

In the MSSM, if the selectron and smuon share same mass parameters, the similar diagram contributing the muon $g - 2$ would also contribute to the electron EDM if the parameters are generally CP violated. In the following, we discuss the physics related to muon $g - 2$ and electron EDM.

A. Muon $g - 2$

In MSSM, there are five dominant contributions to the muon $g - 2$ [8, 9] which can be divided into the neutralino-slepton loop and chargino-sneutrino loop. In the neutralino-slepton scenario, there are four dominant contributions: the $\tilde{B} - \tilde{H} - \tilde{l}_R$, $\tilde{B} - \tilde{H} - \tilde{l}_L$, $\tilde{B} - \tilde{l}_L - \tilde{l}_R$ and $\tilde{W} - \tilde{H} - \tilde{l}_L$ loops. The first two cases require light sleptons as well as light $\tilde{B}$ and $\tilde{H}$ which is highly constrained by both LHC searches and the dark matter direct searches. However, in the $\tilde{B} - \tilde{l}_R - \tilde{l}_L$ scenario, due to the enhancement of the $\mu$-term, the slepton mass could be much heavier, providing an interesting possibility\(^2\). The corresponding mass parameters in this scenario are $\mu$, $M_1$, and slepton masses. In the case of the $\tilde{W} - \tilde{H} - \tilde{l}_L$ loop, it also associates with the chargino-sneutrino loop. The parameters related to this scenario are $\mu$, $M_2$, and slepton masses. In this papers, we mainly focus on these two scenarios and they are denoted as bino loop and chargino loop respectively, which can be summarized as [8, 9]:

$$a_\mu (\tilde{B} - \tilde{l}_L - \tilde{l}_R) \simeq \frac{\alpha_Y}{4\pi} \frac{m^2_{\tilde{\mu} L} M_1 \mu}{m^2_{\tilde{\mu} L} m^2_{\tilde{\mu} R}} \tan \beta f_N \left( \frac{m^2_{\tilde{\mu} L}}{M_1^2}, \frac{m^2_{\tilde{\mu} R}}{M_1^2} \right)$$ (3)

\(^2\) There are also cases where different scenarios mixed, in this paper, we only consider one of the particular scenarios to be dominant.
\[ a_\mu(\tilde{W} - \tilde{H} - \tilde{\nu}) \simeq -\frac{\alpha_2}{8\pi} \frac{m_\mu^2}{M_2\mu} \tan \beta f_N \left( \frac{M_2^2}{m_{\mu_L}^2}, \frac{\mu^2}{m_{\mu_L}^2} \right) \]

\[ a_\mu(\tilde{W} - \tilde{H} - \tilde{\nu}) \simeq \frac{\alpha_2}{4\pi} \frac{m_\mu^2}{M_2\mu} \tan \beta f_C \left( \frac{M_2^2}{m_{\nu_L}^2}, \frac{\mu^2}{m_{\nu_L}^2} \right) \]  

where

\[ f_N(x, y) = xy \left[ \frac{-3 + x + y + xy}{(x-1)^2(y-1)^2} + \frac{2x \ln x}{(x-y)(x-1)^3} - \frac{2y \ln y}{(x-y)(y-1)^3} \right] \]  

\[ f_C(x, y) = xy \left[ \frac{5 - 3(x + y) + xy}{(x-1)^2(y-1)^2} - \frac{2 \ln x}{(x-y)(x-1)^3} + \frac{2 \ln y}{(x-y)(y-1)^3} \right] \]

### B. Electron EDM

The operator related to the electron EDM is:

\[ \mathcal{L} = -\frac{i}{2} d_f \bar{\psi}_\mu \gamma_5 \psi F^{\mu\nu} \]  

The EDM of a particle with spinor \( \psi_i \) and a scalar \( \phi_k \) with the interaction contains CP violation can be given \[14, 15\]

\[ \mathcal{L} = L_{ik} \bar{\psi}_f P_L \psi_i \phi_k + R_{ik} \bar{\psi}_f P_R \psi_i \phi_k + H.c. \]  

\[ d_f = \frac{m_i}{16\pi^2 m_k^2} \text{Im}(L_{ik} R_{ik}^*) \left[ Q_i A \left( \frac{m_i^2}{m_k^2} \right) + Q_k B \left( \frac{m_i^2}{m_k^2} \right) \right] \]  

where

\[ A(r) = \frac{1}{2(1-r)^2} \left( 3 - r + \frac{2 \ln r}{1-r} \right) \]  

\[ B(r) = \frac{1}{2(1-r)^2} \left( 1 + r + \frac{2 \ln r}{1-r} \right) \]

The \( \psi_i \) here could be the chargino or neutralino and the \( \phi \) here could be slepton or sneutrino. The interactions can be easily calculated after rotating the charginos and neutralinos as well as the sleptons into the mass basis.

### III. NUMERICAL CALCULATION AND RESULTS

In MSSM, the related parameter could have a phase are \( \mu, A_e, A_\mu, M_1, M_2 \). However, not all the phase are independent parameters. The physics only depends on the relative phase of them. To
simplify our analysis, we take $A_e = A_\mu = 0$, then there are only two independent phase $\text{Arg}[\mu M_1]$ and $\text{Arg}[\mu M_2]$ which could affect our result. In our scan, we assume $\bar{\mu}_L = \bar{\mu}_R = \bar{\tilde{e}}_L = \bar{\tilde{e}}_R = m_0$. Except for the parameters $M_2, M_1, \mu$ and $m_0$, all the other mass parameters are set to be 5 TeV and the corresponding phases are set to be 0. We also set $\tan\beta = 50$ to accommodate the muon $g - 2$.

To factor out the chargino loop and bino loop separately, we divide our scan into two categories. For the bino loop scenario, the related parameters are set as follows:

\[
M_2 = 0.5 \text{ TeV}, \tan\beta = 50, M_1 = m_0 \\
0.5 \text{ TeV} < m_0 < 1.3 \text{ TeV}, 3 \text{ TeV} < \mu < 10 \text{ TeV} \\
\text{Arg}[\mu] = \text{Arg}[M_2] = 0, \quad 10^{-7} < \text{Arg}[M_1] < 10^{-1}. \tag{12}
\]

The minimum of the $M_2 = 0.5$ TeV is due to the limit from LHC search for displaced chargino searches [16]. $\mu$ is larger than 3 TeV to accommodate the muon $g - 2$ for the bino loop, at the same time the chargino contribution can be diminished. Therefore the parameter space to explain muon $g - 2$ mainly from the bino loop. The scan of phase on $M_1$ is to satisfy the constraint of electron EDM on $\text{Arg}[\mu M_1]$.

For the chargino loop scenario, we set

\[
\mu = 0.4 \text{ TeV}, \tan\beta = 50, M_1 = 3 \text{ TeV} \\
0.4 \text{ TeV} < m_0 < 1.3 \text{ TeV}, 0.4 \text{ TeV} < M_2 < 2 \text{ TeV} \\
\text{Arg}[\mu] = \text{Arg}[M_1] = 0, \quad 10^{-7} < \text{Arg}[M_2] < 10^{-1}. \tag{13}
\]

The lower limit on the $\mu$ is due to the LHC sleptons searches [17]. The updated ATLAS slepton search set a very strong limit on the slepton pair production for an LSP mass less than 400 GeV. The $M_1$ is set to be 3 TeV to reduce the bino loop contribution.

In our calculation, the mass spectrum and the muon $g - 2$ and electron EDM are calculated by \texttt{CPsuperH 2.3} [18–20]. The dark matter relic density and dark matter nucleon interaction are calculated by \texttt{MicrOMEGAs 5.2} [21].

Our final results are shown in Fig. 1. On the left panel we show the parameter space to explain muon $g - 2$ for the chargino loop. We can see that the current Xenon1T search already excluded part of 1\sigma preferred region, leaving a small parameter region to explain the $g - 2$ at 1\sigma level. Since the higgsino dark matter is under abundance, a reduction factor is already included in the dark matter direct detection. In the plot we already set a limit on the electron EDM everywhere. Nevertheless, the phase of $\text{Arg}[\mu M_2]$ should smaller than $2 \times 10^{-5}$ if the muon $g - 2$ anomaly is explained by the chargino loop.
FIG. 1: The parameter space for two scenarios: chargino loop contribution (left panel) and bino loop contribution (right panel). The real lines are the constraints from the electron EDM.

On the right panel, we show the parameter space for the $g - 2$ and the constraints on the phase of $\text{Arg}[\mu M_1]$ in the case of bino loop contribution. It tells that the phase of $\text{Arg}[\mu M_1]$ should also smaller than $3 \times 10^{-5}$ if the $g - 2$ is explained primarily by the bino loop. We note this limit is rather robust with $\tan \beta$ because both value of $g - 2$ and electron EDM proportional to $\tan \beta$.

IV. SUMMARY AND CONCLUSIONS

We study the constraints of the CP violation in the SUSY parameter space where the muon $g - 2$ anomaly can be explained. We present two particular scenarios where the $g - 2$ anomaly is predicted within $2 \sigma$ level mainly through the chargino loop or the bino loop. We found that for both cases the electron EDM experiment already highly constrained the CP phase of the parameters: either the $\text{Arg}[\mu M_1]$ or $\text{Arg}[\mu M_2]$ should be smaller than $\mathcal{O}(2-3) \times 10^{-5}$. If the muon $g - 2$ anomaly is explained by the MSSM, a particular SUSY breaking mechanism is needed to guarantee the small CP phase of SUSY parameters\(^3\). Otherwise, a tuning of $\mathcal{O}(10^{-5})$ is needed to cancel the phase in a general CP violated SUSY model.

\(^3\) One of such models is shown in [22].
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