Corrigendum

Corrigendum: Correction for stress-induced optical path length changes in a refractometer cell at variable external pressure (2019 Metrologia 56 015001)

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Corrigendum: 3. Stress-induced optical path length changes

In the derivation of equation (10) a term is missing, which takes into account the replacement of glass material by the gas medium when the refractometer cell is compressed by the external pressure. As a consequence, instead of the change of the optical path length, i.e. the distance between two fixed points independent of the material boundaries, unintentionally only the change of the optical thickness is represented by equation (10). This can be resolved by setting up equation (4) as follows:

\[ n_{\text{air}} - 1 = \left[ l_{\text{c}, \text{out}} \cdot \left( 1 - \frac{k}{3} p_{\text{out}} \right) + \delta l_{\text{c}, \text{bend}} (\delta p_{\text{out}}) \right] \]

\[ - \left( \delta l_{\text{w}, \text{out}} - \delta l_{\text{w}, \text{in}} \right) \left[ A \otimes \mathcal{V} \right] \]

\[ + \left( \mu_{\text{fs}} - 1 - L_{\text{fs}} \right) \cdot l_{\text{w}, \text{out}} \cdot \frac{k}{3} \delta p_{\text{out}} \]

\[ + \left( \mu_{\text{fs}} - 1 - L_{\text{fs}} \right) \cdot \delta l_{\text{w}, \text{in}} (\delta p_{\text{out}}) \]

\[ + \left( l_{\text{c}, \text{out}} - l_{\text{c}, \text{in}} \right) \cdot \frac{k}{3} \delta p_{\text{out}} \]

\[ - 2 \cdot L_{\text{fs}} \cdot \left( l_{\text{w}, \text{out}} - l_{\text{w}, \text{in}} \right) \cdot \frac{k}{3} \delta p_{\text{out}} \]

(2)

which is approximated by

\[ n_{\text{air}} - 1 \approx \frac{[A \otimes \mathcal{V}]}{l_{\text{c}, \text{out}} \cdot \left( 1 - \frac{2}{3} \delta p_{\text{out}} \right)} \]

\[ - \left[ \mathcal{V} \otimes \mathcal{V} \right] \]

\[ + \frac{\left( \mu_{\text{fs}} - 1 - L_{\text{fs}} \right) \cdot l_{\text{w}, \text{out}} \cdot \frac{2}{3} \delta p_{\text{out}}}{l_{\text{c}, \text{out}} \cdot \left( 1 - \frac{2}{3} \delta p_{\text{out}} \right)} \]

\[ + \frac{\left( \mu_{\text{fs}} - 1 - L_{\text{fs}} \right) \cdot \delta l_{\text{w}, \text{in}} (\delta p_{\text{out}})}{l_{\text{c}, \text{out}} \cdot \left( 1 - \frac{2}{3} \delta p_{\text{out}} \right)} \]

(3)

in analogy to the previous equation (13). Note that the factor \( \left( \mu_{\text{fs}} - 1 - L_{\text{fs}} \right) \) now involves a ‘−1’.

However, a comparison with an all-FEM-based approach performed at NIST by Egan et al (see appendix A in [1]) has...
Corrigendum: Appendix. Refractive index changes

Following [2] and [3] we applied the Lorentz–Lorenz relation to calculate the change of the refractive index of fused silica induced by external pressure. Unfortunately, this approach is in conflict with experimental data of Vedam et al [4], Ritland [5], Spinner et al [6] and Waxler et al [7]. These publications provide experimental evidence that the relation between the refractive index and the density of solid materials, in particular of fused silica, is not compatible with the Lorentz–Lorenz relation (i.e. with the assumption of a constant value of the polarizability). Therefore, instead of the previous equation (A.2) the relation between the relative change of the refractive index and the relative density change should be expressed by

\[ \frac{dn}{n} = a \cdot \frac{d\rho}{\rho} \]  

(similar to [8]) with \( a = 0.226 \pm 0.010 \) being a fit parameter which is determined by linear regression of the experimental results from [4] for fused silica. Consequently, the experimentally-based factor \( \mathcal{L} \) must read

\[ \mathcal{L} = a \cdot n. \]  

Conclusion

Considering the corrections described above, the partially FEM-based approach from [9] yields the updated figure 6 which shows the dependence between the resulting corrections and the external gas pressure affecting the cell windows of our particular cell geometry. However, as mentioned above, the all-FEM-based approach from Egan et al [1] provides more reliable results and yields the pressure-dependent correction shown in figure 7.

Compared to the previously published correction in [9] its magnitude is decreased so that, for instance, at 1000 hPa

The resulting implications are pointed out below.
the contribution to the air refractivity is of the order of $2.4\text{nm}/420\text{mm} \approx 6 \times 10^{-9}$ which corresponds to a relative effect of approximately $2 \times 10^{-5}$ at standard conditions.

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We appreciate a thorough and critical discussion of our previously published results from [9] with J Stone and P Egan from NIST, USA. This interaction has lead to the revealing of the corrections shown in the present corrigendum and the FEM data from P Egan enabled us to improve the accuracy of the correction.

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