The Ciliary Propulsion of An Electrically Conducting Johnson-Segalman Physiological Fluid Through A Porous Medium in An Inclined Symmetric A Channel with Slip

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Abstract. The bionic systems are distinguished by electromagnetic pumping, which offers important advantages over the well-known traditional designs, whose clever ideas are inspired by the vital characteristics of many channels in living organisms. The complex features and characteristics that we observe in the mechanism of fluid transport in channels for the bodies of living organisms and the presence of cilia in the moving of these fluids and simulate this in the design of channels for non-Newtonian fluids. In our paper, we discussed this simulation in a mathematical model of the movement of a viscous, flexible, electrically conductive fluid in an inclined channel contains cilia in which a porous medium under the influence of a magnetic field. The fluidity conduct of the fluid is emulation through the Johnson-Segalman constituent model which allows interior wall slip. The coordinated and uniform motion of the ciliated brinks (which line an interior wall of the channel) is exemplified by a converging wave motion which generates a 2-dimensional velocity of the fluid. The system of partial differential equations obtained from this mathematical model with exists boundary value conditions through long wavelength and Reynolds number concepts is solved using a perturbation method.

Keywords: Johnson-Segalman fluid, Ciliated channel, Magnetohydrodynamics, Perturbation method, porous medium

1. Introduction

Considered as the cilia is a essential components of the organism bio-system, and the cilia have the vital multiple essential functionalities for transporting liquids, foodstuffs, and others. The type of this cilia is a form of axial axonemes, which they consist of a microtubule combination of motility ingredient and cytoskeletal structure, it vestige conserved in the organism biosystem through various operation [1,2,3,4]. These processes contain the oviducts, backbone and brain ventricular apparatus, the airways, and the embryonic node. Furthermore, specific cilia occurrence is found near each cell type; they are recognized as the key hub for the crossing, propagation, and separation of signal flows and for finding both right and left asymmetries in the embryo node [5,6] and linked to moving, non-moving cilia in and around the embryo node. Cilia can however be present in the body anywhere. In a various biological system, cilia thrust is also growing, and these form thin, convoluted structures, which are projected out from the wall of a specific size approx (10-12 mm) cilia...
can easily flexion and carry a number from complicated transport mechanics. Fluid transport resulting from ciliary movement has therefore gathered important attendance in bio-fluid dynamics in the body of living beings for decades [7]. Many studies have been interested in numerous physiological and bioprocesses distinguished by a ciliary transport inclusive the ovum motion in fallopian canal and spermatozoa transport dynamics and transfer of mucus in the respiratory channels [8,9,10,11,12,39]. Each cilium moves or beats continuously with a two-beating movement [see Figure 1] in the locomotion system. Interesting are the analogies that have been studied recently by [13,14], between ciliary beating and flapping in birds and small insects (hummingbirds, species of butterflies, bumblebees, etc). The cilia are closed concurrently in rows and the adjacent cilia control in a harmonic pattern with a different little phase. In this method, the ends of the ciliary are used to shape a continual wave symmetrical movement termed the metachronal wave [15]. Many scholars researchers also studying, the influence of cilia beating for various presumptions of hydrodynamic characteristics and properties of fluids dynamics and cilia effectiveness. In the actual, are two kinds of models for the payment and to transmit fluids which are: ‘the cilia and sublayer models’ [16,17,18]. In the scientific literature, as we mentioned, there are two kinds of models for transport, and propulsion transport fluids include: "cilia sublayer models" or "discrete cilia models" and "volume force distribution" or "volume force models". In the detached cilia model, each cilium is transacted independently [19]. Magnetohydrodynamics (MHD) is considered as a significant field in modernistic intelligent bionic systems, as well as characteristics and advantages of the flow of fluids that conduct electricity in fields of medical technologies, the most important of them: MRI, EMG, IMF, GMR, etc. Where it provides "the precise and non-invasive therapy" of numerous physiology and biology conditions. By the biological thrust, MHD fluxs have been processed for types of bionical systems, inclusive ciliary magnetohydraulics in softness robotic [20], respiratory magnetic therapy [21], peristaltic magnetofluid pumping [22,23,24], magneto-robotic microswimmers [25], magnetic blood pumps [14,26], biomagnetic curved arterial fluid mechanics [27] and as well as regarding our topic this cilia-assisted magnetic hemodynamic processes [28,29,30]. The motive of the paper is to discuss the theoretical effact of the exterior magnetic field on the ciliary transport of Johnson Segalman fluid, which is an electrically-conducting in the symmetric cannal. In extension to that, will be we consider slip impacts with in the fluids, which is a characteristic that exists in the Johnson-Segalman paradigm. Also, In most of the physiological, biological, industrial, and other systems, with be a channels inclined at a certain angle, where this verity motivate researchers to Study in the fluxs confined of an inclined channels [30,31,32,33,34]. Moreover, the influence of inclina in peristaltic flow be worth too attention specialling in the existence of non-Newtonian fluids. Also the flow with exist a porous medium in the channals possess been analyzed by many researchers using Darcy law, and analyzed the flow of the viscous fluids via a porous medias, and studied free convections and mass transportation flows of a viscous fluid through a porous medium by the vertical and horizontal surfaces [35,36,37,38]. In this study, we will analyze the influence of the magnetohydrodynamics (MHD) of peristaltic flows of Johnson-Segalman nanofluid passing through a ciliated channel and in an inclined, and we consider the influence viscosity on the fluid flow through porous miduom, in addition to the effect of slip this fluid on its flow in the channel. The governing problems is modeled and formulate as a non-linear system of the arising partial differential equations are solve by using the stream function and perturbation method through applying low Reynold number approximation and long wavelength, to get to a converted into a simplified form, and then we design to the results by graphically of the different parameters of physically large important, include a length of cilia.
2. Mathematical model

In the mathematical impersonation of the models, we considering an incompressible of the magnetohydrodynamics controlled flows of Johnson-Segalman nanofluid through a ciliary channels, with moving cilia extension for a both walls, and the cilia are constantly beating by dynamically and a tip of cilia is a graph elliptical path centered at \((\bar{X}, a_0)\): The situation of the cilia is given by the resulting parametrically representation [3].

\[ \bar{X} = F(\bar{X}, t) = \bar{X}_0 + \epsilon a_0 \alpha \sin \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right] \]

\[ \bar{Y} = H(\bar{X}, t) = + \left[ a_0 + \epsilon a_0 \cos \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right] \right] = +L \]

(1) \quad (2)

Here \( \epsilon a_0 \alpha \) is the fundamental axis in \( \bar{X} \) direction and \( \epsilon a_0 \) is its secondary axis in \( \bar{Y} \) direction and \( 2L \) is a width of channel and after determin the situation of a cilia inclination, and we calculating the vertical and horizontal velocity components, where the both a horizontal \( \bar{U} \) and vertical \( \bar{V} \) velocity are obtained by the time \( t \) derivative at \( \bar{X}, \bar{V} \) coordinates respectively i.e.

\[ \bar{U} = \frac{-2\pi}{\lambda} \left( \frac{\epsilon \bar{x}_0 \alpha \cos \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right]}{1 - 2\pi \left( \epsilon \bar{x}_0 \alpha \cos \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right] \right)} \right) \]

\[ \bar{V} = \frac{-2\pi}{\lambda} \left( \frac{\epsilon \bar{x}_0 \alpha \sin \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right]}{1 - 2\pi \left( \epsilon \bar{x}_0 \alpha \cos \left[ \frac{2\pi}{\lambda} \left( \bar{X} - ct \right) \right] \right)} \right) \]

(3) \quad (4)

Using the Johnson-Segalman fluid paradigm with MHD [40], we will situation:

\[ \text{div } U = 0, \; \rho \frac{dU}{dt} = \text{div } \sigma + \rho f \quad \text{where} \quad \rho f = (\sigma B_0^2 \bar{U}, 0, 0) \]

(5)

and \( \sigma = -\bar{P} \bar{I} + T \) and \( T = 2 \mu D + S, S \) m \( \left( \frac{dS}{dt} + S(W - aD) + (W - aD)^T S \right) = 2 \mu D \),

(6)
\[ D = \frac{1}{2} (L + L^T); \quad W = \frac{1}{2} (L - L^T); \quad L = \text{grad} U; \quad U = (\bar{U}(x, y, z), \bar{V}(x, y, z), 0) \]

(7)

The above equations contain "the scalar pressure \( \bar{P} \), the identity tensor \( I \), the relaxation time \( m \), the slip parameter \( a \) and the respective symmetric and skew symmetric part of velocity gradient \( D \) and \( W \)."

For all parameters and tensors and stresses which are applied in equations (1)-(7) are defined in the terminologies. We get classical Navier-Stokes equations, where take the basic equations governing the incompressible (MHD) Johnson Segalmann fluid in the 2-dimensional flow which, are the continuity equation and the momentum equations are following form:

\[
\frac{\partial \bar{U}}{\partial \bar{x}} + \frac{\partial \bar{V}}{\partial \bar{y}} = 0
\]

(8)

\[
\rho \left[ \frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{y}} \right] = -\frac{\partial \bar{P}}{\partial \bar{x}} + \mu \left( \frac{\partial^2 \bar{U}}{\partial \bar{x}^2} - \frac{\partial^2 \bar{V}}{\partial \bar{y}^2} \right) + \frac{\partial}{\partial \bar{x}} \left( \bar{S}_{\bar{X}\bar{X}} \right) + \frac{\partial}{\partial \bar{y}} \left( \bar{S}_{\bar{X}\bar{Y}} \right) - \frac{\mu}{K_0} \bar{U} - \rho g \sin \theta
\]

(9)

\[
\rho \left[ \frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{V}}{\partial \bar{y}} \right] = -\frac{\partial \bar{P}}{\partial \bar{y}} + \mu \left( \frac{\partial^2 \bar{V}}{\partial \bar{x}^2} - \frac{\partial^2 \bar{V}}{\partial \bar{y}^2} \right) + \frac{\partial}{\partial \bar{x}} \left( \bar{S}_{\bar{Y}\bar{X}} \right) + \frac{\partial}{\partial \bar{y}} \left( \bar{S}_{\bar{Y}\bar{Y}} \right) - \frac{\mu}{K_0} \bar{V} - \rho g \cos \theta
\]

(10)

where \( \bar{S}_{\bar{X}\bar{X}}, \bar{S}_{\bar{X}\bar{Y}} \) and \( \bar{S}_{\bar{Y}\bar{Y}} \) are satisfy the following equations:

\[
2\eta \frac{\partial \bar{U}}{\partial \bar{x}} = \bar{S}_{\bar{X}\bar{X}} + m \left( \bar{U} \frac{\partial \bar{U}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{y}} \right) \bar{S}_{\bar{X}\bar{X}} - 2amS_{\bar{X}\bar{X}} \frac{\partial \bar{U}}{\partial \bar{x}} + m \left[ (1 - a) \frac{\partial \bar{V}}{\partial \bar{x}} - (1 + a) \frac{\partial \bar{U}}{\partial \bar{y}} \right] \bar{S}_{\bar{X}\bar{Y}}
\]

(11)

\[
\eta \left( \frac{\partial \bar{U}}{\partial \bar{y}} + \frac{\partial \bar{V}}{\partial \bar{x}} \right) = \bar{S}_{\bar{X}\bar{Y}} + m \left( \bar{U} \frac{\partial \bar{U}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{y}} \right) \bar{S}_{\bar{X}\bar{Y}} + \frac{m}{2} \left[ (1 - a) \frac{\partial \bar{V}}{\partial \bar{x}} - (1 + a) \frac{\partial \bar{U}}{\partial \bar{y}} \right] \bar{S}_{\bar{X}\bar{X}}
\]

(12)

\[
2\eta \frac{\partial \bar{V}}{\partial \bar{y}} = \bar{S}_{\bar{Y}\bar{Y}} + m \left( \bar{U} \frac{\partial \bar{V}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{V}}{\partial \bar{y}} \right) \bar{S}_{\bar{Y}\bar{Y}} - 2amS_{\bar{Y}\bar{Y}} \frac{\partial \bar{V}}{\partial \bar{y}} + m \left[ (1 - a) \frac{\partial \bar{U}}{\partial \bar{y}} - (1 + a) \frac{\partial \bar{V}}{\partial \bar{x}} \right] \bar{S}_{\bar{X}\bar{Y}}
\]

(13)

Where "\( \rho \) is the fluid density, \( \sigma \) is the electrical conductivity of the fluid, \( g \) is the acceleration due to the gravity, \( \mu \) and \( \eta \) are the viscosity coefficients of the Johnson-Segalman fluid, \( a \) is slip parameter, \( K_0 \) is the permeability parameter, \( B_0 \) is the magnetic field, \( \theta \) is the inclination angle of the channel".

The coordinate transformations from a fixed coordinate system to a moving coordinate system are related as:

\[
\bar{x} = \hat{x} - ct, \quad \bar{y} = \hat{y}, \quad \bar{u}(\hat{x}, \hat{y}) = \hat{U}(\hat{x}, \hat{y}, t) - c, \quad \bar{v}(\hat{x}, \hat{y}) = \hat{V}(\hat{x}, \hat{y}, t),
\]

\[
\bar{p}(\hat{x}, \hat{y}) = \bar{P}(\hat{x}, \hat{y}, t), \quad H(\hat{x}, t) = H(\hat{x})
\]

(14)

where \( \bar{u}, \bar{v} \) are the velocity components with the wave frame \( (\hat{x}, \hat{y}) \), and assume that a flow velocity is constant \( c \).
3. Dimensionless parameter

In the dimensional analysis process, the non-dimensional amounts are following:

\[
\begin{align*}
    x &= \frac{x}{\lambda}, \\
    y &= \frac{y}{\tilde{a}}, \\
    u &= \frac{u}{\tilde{a}_0 c}, \\
    v &= \frac{v}{\tilde{a}_0 c}, \\
    t &= \frac{ct}{\lambda}, \\
    \beta &= \frac{\beta_0}{\lambda}, \\
    p &= \frac{a_0^2 p}{\lambda (\mu + \eta)}, \\
    S_{ij} &= \frac{a_0 \tilde{S}_{ij}}{c \mu}, \\
    D_a &= \frac{K_0 a_0^2}{a_0^2}, \\
    M &= \sqrt{\frac{\sigma}{\mu} a_0 B_0}, \\
    Re &= \frac{\rho c a_0}{\mu}, \\
    We &= \frac{mc}{a_0^2}.
\end{align*}
\]

(15)

Here \( \beta \) the wave number, \( M \) the Hartmann number, \( Re \) the "Reynolds’ number", \( D_a \) the Darcy number and \( We \) the "Weissenberg number".

By the equations (14)-(15), this leads to the equations (8)–(10) take the following form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

(16)

\[
\beta Re \left[(u + 1) \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right] = -\left(\frac{\mu + \eta}{\mu}\right) \frac{\partial p}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial x} (S_{xx}) \]

\[
+ \frac{\partial}{\partial y} (S_{xy}) - \left(M^2 + \frac{1}{\beta^2}\right) (u + 1) + \frac{\rho a}{c} \sin \theta.
\]

(17)

\[
\beta^3 Re \left[(u + 1) \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}\right] = -\left(\frac{\mu + \eta}{\mu}\right) \frac{\partial p}{\partial y} + \beta^4 \frac{\partial^2 v}{\partial x^2} + \beta^2 \frac{\partial^2 v}{\partial y^2} + \beta^2 \frac{\partial}{\partial x} (S_{xy}) \]

\[
+ \beta \frac{\partial}{\partial y} (S_{yy}) - \beta^2 v + \beta \cos \theta.
\]

(18)

through applying low Reynold number approximation \( Re \) and long wavelength \( \beta \), and since the flow is 2-dimensional, then by using "stream function \( \psi \)"., which is define by "Cauchy-Riemann equations":

\[
u = \frac{\partial \psi}{\partial y}, \quad u = -\frac{\partial \psi}{\partial x}.
\]

(19)

\[
\left(\frac{\mu + \eta}{\mu}\right) \frac{\partial p}{\partial x} = \frac{\partial}{\partial x} (S_{xy}) + \frac{\partial^3 \psi}{\partial y^3} - \left(M^2 + \frac{1}{\beta^2}\right) \left(\frac{\partial \psi}{\partial y} + 1\right) + \frac{\rho a}{c} \sin \theta.
\]

(20)

\[
\frac{\partial p}{\partial y} = 0
\]

(21)

\[
S_{xx} = We(1 + a) \frac{\partial^2 \psi}{\partial y^2} (S_{xy})
\]

(22)

\[
\frac{\mu + \eta}{\partial y^2} = S_{xy} + \frac{We}{2} (1 - a) \frac{\partial^2 \psi}{\partial y^2} (S_{xx}) - \frac{We}{2} (1 - a) \frac{\partial^2 \psi}{\partial y^2} (S_{yy})
\]

(23)

\[
S_{yy} = -We(1 - a) \frac{\partial^2 \psi}{\partial y^2} (S_{xy})
\]

(24)

From the equations (22)-(24) following expression can be obtained:

\[
S_{xy} = \frac{\frac{\partial \psi}{\partial y} (S_{xy})}{1 + We^2 (1 - a) \left(\frac{\partial^2 \psi}{\partial y^2}\right)^2}
\]

(25)

By substituting equation (25) in the equation (20) yields:
\[ \frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} + 3We^2 \frac{(a^2-1)\eta}{\mu + \eta} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{\mu}{\mu + \eta} \left( M^2 + \frac{1}{D_a} \right) \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{\mu}{\mu + \eta} \frac{\rho \theta}{c} \sin \theta \]  

(26)

Now by the equations (20),(21),(25) we get:

\[ \frac{\partial^4 \psi}{\partial y^4} + 3We^2 \frac{(a^2-1)\eta}{\mu + \eta} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{\mu}{\mu + \eta} \left( M^2 + \frac{1}{D_a} \right) \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{\mu}{\mu + \eta} \frac{\rho \theta}{c} \sin \theta = 0 \]

(27)

Let \( K_1 = \frac{\mu}{\mu + \eta} \), \( K_2 = \frac{(a^2-1)\eta}{\mu + \eta} \), \( \zeta = \frac{\rho \theta}{c} \), \( N^2 = (M^2 + \frac{1}{D_a})^{0.5} \)

(28)

We substitute the values in equation (28) into equations (26)-(27), we get:

\[ \frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} + 3K_2We^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \left( \frac{\partial^3 \psi}{\partial y^3} \right) - K_1N^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) + K_1 \zeta \sin \theta \]

(29)

\[ \frac{\partial^4 \psi}{\partial y^4} + 3K_2We^2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \left( \frac{\partial^3 \psi}{\partial y^3} \right) + 2 \left( \frac{\partial^2 \psi}{\partial y^2} \right) \left( \frac{\partial^3 \psi}{\partial y^3} \right) - K_1N^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) \frac{\partial^2 \psi}{\partial y^2} = 0 \]

(30)

4. "Volumetric flow rate and boundary conditions"

The volume flux rate is the key design quantity in bionic pumping systems where is in the fixed frame by:

\[ F = \int_0^H \bar{U}(\bar{x}, \bar{y}, \bar{t}) d\bar{y} \]

(31) by the formula of transference given in equation (14), we get

\[ f = \int_0^h \bar{u}(\bar{x}, \bar{y}) d\bar{y} \]

(32)

the fixed and wave frame of volume flow rate can be define by the aid of equations (31),(32):

\[ F = f + c \bar{h} \]

(33)

The time-mean flow over a period \( \bar{T} \) at a fixed position \( \bar{X} \) is defined as:

\[ Q = \frac{1}{\bar{T}} \int_0^{\bar{T}} F d\bar{t} \]

(34)

By use the equation (33) into the equation (34), we obtain:

\[ Q = \int_0^h \left( f + c \bar{h} \right) d\bar{x} = f + c \int_0^h \bar{h} d\bar{x} \]

(35)

Now with the help of equation (2), equation (14), and equation (35), we obtain:

\[ Q = f + c a_0 \]

(36)

We define the dimensionless form of the time mean flow \( \Theta, F \):

\[ \Theta = \frac{Q}{ca_0} \]

(37)

\[ F = \frac{f}{ca_0} \]

we get:

\[ \Theta = F + 1 \]

(38)

where

\[ F = \int_0^h \frac{\partial \psi}{\partial y} dy = \psi(h) - \psi(0) \]

(39)

In this model, if taking \( \psi(0) = 0 \) at \( y = 0 \) then \( \psi(h) = F \) at \( y = h \). The boundary conditions as the following:
\[
\psi = 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \\
\psi = F, \quad \frac{\partial \psi}{\partial y} = u_0 \quad \text{at} \quad y = h
\]
(40)
where
\[
F = \int_{0}^{h} \frac{\partial \psi}{\partial y} \, dy, \quad u_0 = -1 - 2\pi \alpha \beta \cos(2\pi x), \quad h = 1 + \epsilon \cos(2\pi x)
\]
(41)

5. "Perturbation solutions"

To solve the nonlinear PDE's (29), (30) with the boundary conditions in equation (40), a perturbation technique is used, where expanding \(\psi, p\) and \(F\) around the \(\mathcal{W}_2\) leads to:

\[
\psi = \psi_0 + \mathcal{W}_2 \psi_1 + \cdots \\
p = p_0 + \mathcal{W}_2 p_1 + \cdots \\
F = F_0 + \mathcal{W}_2 F_1 + \cdots
\]
(42)

As we use a small Weissenberg number (\(\mathcal{W}_2\)), the equations (29), (30). We get the following systems:

5.1. Zero order system

\[
\frac{\partial^4 \psi_0}{\partial y^4} - K_1 N^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0
\]
(43)
\[
\frac{\partial p_0}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} - K_1 N^2 \left( \frac{\partial \psi_0}{\partial y} + 1 \right) + K_1 \zeta \sin \theta
\]
(44)
with corresponding boundary conditions
\[
\psi_0 = 0, \quad \frac{\partial \psi_0}{\partial y} = 0 \quad \text{at} \quad y = 0 \\
\psi_0 = F_0, \quad \frac{\partial \psi_0}{\partial y} = u_0 \quad \text{at} \quad y = h
\]
(45)

5.2. First order system

\[
\frac{\partial^4 \psi_1}{\partial y^4} + 3K_2 \left[ \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 + 2 \frac{\partial^2 \psi_0}{\partial y^2} \left( \frac{\partial^3 \psi_0}{\partial y^3} \right)^2 \right] - K_1 N^2 \frac{\partial^2 \psi_1}{\partial y^2} = 0
\]
(46)
\[
\frac{\partial p_1}{\partial x} = \frac{\partial^3 \psi_1}{\partial y^3} + 3K_2 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) - K_1 N^2 \frac{\partial \psi_1}{\partial y} = 0
\]
(47)
with corresponding boundary conditions
\[
\psi_1 = 0, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{at} \quad y = 0 \\
\psi_1 = F_1, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{at} \quad y = h
\]
(48)

5.3. Zeroth-order Solution

By using a regular perturbation technique, it is found that the zeroth-solution system through for the equations (43)–(45) is as follows:

\[
\psi_0 = c_3 + yc_4 + \frac{e^{-y/\sqrt{k_1 N}}(c_1 + e^{2y/\sqrt{k_1 N}}c_2)}{k_1 N^2}
\]
\[
\frac{dp_0}{dx} = \zeta \sin[\theta]k_1 + 2e^{y/\sqrt{k_1 N}}c_2\sqrt{k_1 N} - e^{-y/\sqrt{k_1 N}}(c_1 + e^{2y/\sqrt{k_1 N}}c_2)\sqrt{k_1 N} - k_1 (1 + c_4)
\]
\[ + \frac{2e^{\chi/k_1}c_2}{\sqrt{k_1N}} - e^{-\chi/k_1N}(c_1 + e^{2\chi/k_1N}c_2)N^2 \]

### 5.4 First-order Solution

Solving first order system as the equations : (46)–(48), we get:

\[ \psi_1 = c_7 + yc_8 + (e^{-3\lambda/k_1N} - 8e^{2\lambda/k_1N} - 1 + e^{2h/k_1N})^3 (c_5 + e^{2\lambda/k_1N}c_6) \]

\[ + 24e^{2y/k_1N}(-1 + e^{2h/k_1N})^2 (1 + e^{2h/k_1N})h(c_5 + e^{2y/k_1N}c_6)\sqrt{k_1N} \]

\[ - 24e^{2y/k_1N}(-1 + e^{2h/k_1N})(1 + e^{2h/k_1N})^2 h^2 (c_5 + e^{2y/k_1N}c_6) k_1 N^2 \]

\[ + 8e^{2y/k_1N}(1 + e^{2h/k_1N})^3 h^3 (c_5 + e^{2y/k_1N}c_6)k_1^3/2N^3 \]

\[ + e^{3h/k_1N}(-1 - 30e^{2y/k_1N} + 30e^{4y/k_1N} + e^{6y/k_1N})k_1^2N^6(F_0 - hu_0)^3 \]

\[ = \frac{dp}{dx} = 3e^{-2y/k_1N}(c_1 + e^{2y/k_1N}c_2)^2k_2(2e^{\chi/k_1N}c_2\sqrt{k_1N} - e^{-\chi/k_1N}(c_1 + e^{2y/k_1N}c_2)\sqrt{k_1N} \]

\[ - (27e^{2y/k_1N} - 4e^{2h/k_1N}(-1 + e^{2h/k_1N})^3 (c_5 + e^{2y/k_1N}c_6)) \]

\[ + 24e^{2y/k_1N}(-1 + e^{2h/k_1N})^2 (1 + e^{2h/k_1N})h(c_5 + e^{2y/k_1N}c_6)\sqrt{k_1N} \]

\[ - 24e^{2y/k_1N}(-1 + e^{2h/k_1N})(1 + e^{2h/k_1N})^2 h^2 (c_5 + e^{2y/k_1N}c_6) k_1 N^2 \]

\[ + 8e^{2y/k_1N}(1 + e^{2h/k_1N})^3 h^3 (c_5 + e^{2y/k_1N}c_6)k_1^3/2N^3 \]

\[ + e^{3h/k_1N}(-1 - 30e^{2y/k_1N} + 30e^{4y/k_1N} + e^{6y/k_1N})k_1^2N^6(F_0 - hu_0)^3 \]

\[ = \frac{0(1-e^{2h/k_1N}+1+e^{2h/k_1N})h\sqrt{k_1N}}{0(1-e^{2h/k_1N}+1+e^{2h/k_1N})h\sqrt{k_1N}} \]

\[ + (27e^{2y/k_1N} - 16e^{4y/k_1N}(-1 + e^{2h/k_1N})^3 c_6\sqrt{k_1N} \]

\[ - 16e^{2y/k_1N}(-1 + e^{2h/k_1N})^3 c_6 + e^{2y/k_1N}c_6(\sqrt{k_1N} \]

\[ = 4e^{4y/k_1N}(-1 + e^{2h/k_1N})^2 (1 + e^{2h/k_1N})h(c_5 + e^{2y/k_1N}c_2)k_1 N^2 \]

For the graphs of a solutions get in the above equations, using MATHEMATICA program. Complete discussion of developments on the material quantities is given at the following.

### 6. Discussion of the results

This section deals with the graphical description of different parameters on a ciliary propulsion of Johnson-Segalman fluid physiological flux in the symmetric channel. The figures 2, 3, and 4 are plotted to envisage the influences of the important parameters in another meaning the "Hartmann number, the Weissenberg number, the cilia length, the slip parameter, the Darcy number", and the "viscosity coefficients of the Johnson-Segalman fluid" on the velocity, the average pressure rise, and the "pressure gradient" distribution, keeping rest of the other parameters fixed.

### 6.1 Velocity profile

Figure 2(a–f) illustrates the effect of the principal parameters on velocity across along the length of the channel. Obviously, although symmetrical profiles in velocity are consistently computed with respect to variations in magnetic field (\(M\)), (\(We\), (\(a\), (c), (\(Da\), and (\(\mu\)). Figure 2(a,b) observe that increasing of (\(M\)) and (\(We\)) leads to velocity decreases in central the channel (0.5<y<0.5) and increases near the walls of the channel (y<0.5 and y>0.5), while the inversion behavior can be observe with an increases in "slip parameter, cilia length, Darcy number, and the viscosity coefficient" [see figure 2(c-f)] where the velocity increases in central the channel and retreat near the walls of the channel.
6.2 Pumping characteristics

The pressure rise is expressed by $\Delta p$ where

$$\Delta p = \int_0^1 \frac{dp}{dx} \, dx$$

(49)

To calculate the result of the volume flow rate, which include the integration of $dp/dx$. Due to the complexity of the expression given in equation (49), which is difficult to find analytical value, So we turned to the software MATHEMATICA to finding numerical value for this integration.

Figure 3(a–f) illustrates are expressed the pressure rise ($\Delta p$) with time-mean volume flow rate ($\Theta$) with difference in, respectively, $(M)$, $(a)$, $(We)$, $(\epsilon)$, $(Da)$, and the inclination angle of the channel $(\theta)$, which shows the: (i) represents the pumping region, where $\Delta p > 0$ (adverse pressure gradient) and $\Theta > 0$ (positive pumping). Quadrant (ii) which $\Delta p < 0$ ("favorable pressure gradient") and $\Theta > 0$ ("positive pumping") are marked as "the co-pumping region (augmented flow)." Quadrant (iii) which ($\Delta p < 0$, $\Theta < 0$) is the region where there is no flow. Quadrant (iv) represents the region of the retrograde region (backward
pumping) where \(\Delta p > 0, \Theta < 0\) and the free pumping \(\Delta p = 0\). Figure 3(a,b), show effects of the Magnetic parameter or Hartmann number \(M\), and slip parameter \(\alpha\), on pressure rise, which shows the "retrograde pumping" where \(\Theta < 0; \Delta p > 0\). The pumping rate increases with an increase in \((\text{We}), (\epsilon), \text{and} (Da)\) respectively, in the co-pumping region \((\Delta p < 0, \Theta > 0)\) [see figure 3(c-e)]. Figure 3f shows the consistent increase in pressure rise with an increasing inclination angle of the channel.

**Figure 3:** Evolution of \(\Delta p\) against \(\Theta\) for different parameters when \(y = 0.4, t = 0.2\)

6.3. The Pressure gradient distribution

Figure 4(a–f) are illustrates the response of axial pressure gradient \((dp/dx)\) to variation of magnetic field parameter \((M)\), slip parameter \((\alpha)\), cilia length \((\epsilon)\), the inclination angle of the channel \((\theta)\), Weissenberg number \((\text{We})\), and Darcy number \((Da)\) respectively. Generally, every plot can be divided into core (middle of the metachronal wave) region and boundary (start and end of the metachronal wave) regions. As shown in figure 4(a, b) "the pressure gradient" is decreased by increasing in \(M, \alpha\). figure 3(c) shows the fact of the cilia effect: One is that the impact of cilia is reflected in the middle region mainly, but in the boundary regions, there are no important differences, and the other is that the effect of cilia is nonlinear. This phenomenon explains the mutation of fluid caused by too long cilia. figure 3(e-f) demonstrates that increasing the \(\theta, \text{We}, \text{Da}\) increases the pressure gradient consistently. Moreover, is nearly constant to the gap between the core region and boundary regions.
Figure 4: Axial pressure distribution against $x$ when $y = 0.4$, $t = 0.2$

6.4. Trapping phenomena

Figures 5–9 have been sketched to study trapping phenomenon for various values of $(M)$, $(We)$, $(\epsilon)$, $(Da)$, and $(\theta)$ respectively. Where Figure 5 illustrates the effect of $(M)$ on the streamline pattern. It is observed that the size of trapped bolus decreases with the increasing value of $M$, and also the impact of $We$ on the streamlines is seen in figure 6. We found that with the increasing values of $We$, the trapping bolus decreases in size near the upper and lower walls of the channel. The influence of $\epsilon$, $Da$ on the streamlines are studied in figures 7,8. It is seen that for increasing the value of $\epsilon$, Da the size of the trapped bolus enhances. However, the opposite behavior is observed when $\theta$ is increased [see figure 9].
Figure 5: Streamlines for $a = 0.7, We = 0.2, \epsilon = 0.1, \alpha = 0.4, \beta = 0.4, \mu = 1, \eta = 0.5, D\alpha = 1, \Theta = 0.8$; (a) $M=1$ (b) $M=2$.

Figure 6: Streamlines for $a = 0.7, M = 1, \epsilon = 0.1, \alpha = 0.4, \beta = 0.4, \mu = 1, \eta = 0.5, D\alpha = 1, \Theta = 0.8$; (a) $We = 0.1$ (b) $We = 0.5$.

Figure 7: Streamlines for $a = 0.7, We = 0.2, M = 1, \alpha = 0.4, \beta = 0.4, \mu = 1, \eta = 0.5, D\alpha = 1, \Theta = 0.8$; (a) $\epsilon = 0.1$ (b) $\epsilon = 0.2$. 
Figure 8: Streamlines for $a = 0.7, We = 0.2, M = 1, \alpha = 0.4, \beta = 0.4, \mu = 1, \eta = 0.5, \epsilon = 0.1, \theta = 0.8$ ; (a) $Da = 0.1$ (b) $Da = 0.5$.

Figure 9: Streamlines for $a = 0.7, We = 0.2, M = 1, \alpha = 0.4, \beta = 0.4, \mu = 1, \eta = 0.5, \epsilon = 0.1, Da = 1$ ; (a) $\theta = 0.8$ (b) $\theta = 0.81$.

7. Conclusions

In this paper, we have studied the cilia transport of (MHD) Johnson-Segalman physiological fluids in a 2-dimensional an inclined symmetric canal with the slip through a porous medium. The flow is created by the continuous beating of cilia in an "elliptical" pathway which produces the 2-dimensional velocity field. The solutions of the governing equations in closed form are executed with the assumption of ($Re$), ($\beta$) and higher order of $We^2$ is small enough to neglect. By used ($\psi$) to simplify the system. The obtained solutions are visualized through "MATHEMATICA software" for important various parameters and it is through these results we conclude the following:

- The velocity profile is not the same through all parts of the channel, where the velocity reduces in the core (middle) region by increasing ($M$, $We$) while enhances in a center of the channel with an increase in ($a$, $\epsilon$, $Da$) and ($\mu$), whereas the opposite occurs near the channel walls.
- By enhancing ($M$, $a$) leads to a pressure rise increases, whereas, with increases of ($\epsilon$, $We$) and ($Da$) then the pressure rise is decreasing in the co-pumping region. Whereas, the pressure rise increases continuously with an increasing of ($\theta$).
- The pressure gradient is decreasing with an increase in ($M$, $a$). While the opposite happens with an increase in ($\epsilon$), where is it strongly affected and enhanced in a middle region, but retracted in the boundary regions, and with increases for ($\theta$, $We$),...
and \((Da)\) lead to the pressure gradient is increased in the middle region and boundary regions.

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