Centrifugally driven electrostatic instability in extragalactic jets

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The stability problem of the rotation induced electrostatic wave in extragalactic jets is presented. Solving a set of equations describing dynamics of a relativistic plasma flow of AGN jets, an expression of the instability rate has been derived and analyzed for typical values of AGNs. The growth rate was studied versus the wavelength and the inclination angle and it has been found that the instability process is much efficient with respect to the accretion disk evolution, indicating high efficiency of the instability.

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I. INTRODUCTION

Considering the problem of the escaping radiation from active galactic nuclei (AGN), one has to note that we have a strong observational evidence of a complex picture of emission spectra, which starts from the radio, the optical up to X-ray and γ-ray, with the bolometric luminosity power of AGNs lying in the range: \( \sim 10^{40} - 10^{47} \text{erg/s} \) (see Ref. 1). Origin of emission is supposed to be energy of an accreted matter and a spinning black hole, however, the details of the conversion process of this energy into electromagnetic radiation is still unknown. The discovery of blazars (see Ref. 2) as sources of ultra-high energy radiation, has revealed that the formation and acceleration of relativistic jets are key processes in understanding the energy conversion mechanism.

An innermost region of AGNs is characterized by rotational motion, and it is obvious that the rotation affects acceleration of plasmas in jets, which consequently may influence a process of radiation. The problem of a role of centrifugal acceleration (CA) as the rotational effect, on a theoretical level has been studied in Ref. 3 for the Schwarzschild black hole and it has been found that under certain conditions the centrifugal force may change its direction from centrifugal to centripetal. The same effect was shown in Ref. 4 where authors studied dynamics of particles, sliding along rotating straight channels. It has been argued that due to the centrifugal force, the process of acceleration might be very efficient for rotation energy extraction. On the other hand it is obvious that no physical system can preserve rigid rotation nearby the light cylinder surface (LCS) (an area where the linear velocity of rotation is equal to the speed of light) where magnetic field lines must deviate from the straight configuration. Generalization of Ref. 4 for curved field lines has been presented in Ref. 5 and it was found that for the Archimedes spiral a centrifugal outflow may reach infinity, avoiding the light cylinder problem. According to the spin paradigm (stating that, to first order, it is the normalized black hole angular momentum, that determines whether or not a strong radio jet is produced) (see Ref. 6) rotation is supposed to be fundamental in formation of outflows. Blandford & Payne (see Ref. 7) considering the problem of creation of the outflows, pointed out that in this process the centrifugal acceleration may play an important role depending on the inclination angle of magnetic field lines. A series of works dedicated to effects of rotation, examine its role in the nonthermal emission. One of the important examples from this point of view is the work of Blandford & Znajek (see Ref. 8), where they show that the rotational energy of the black hole can be extracted electromagnetically as a Poynting flux. In the context of AGN, Gangadhara & Lesch (see Ref. 9) considered the problem for understanding how efficient the rotation is for producing X-ray and γ-ray photons. It was shown that nearby the light cylinder surface, centrifugally accelerated particles, due to scattering against low energetic photons, may provide the mentioned radiation. Reconsidering the same problem for explaining the TeV energy emission from a certain class of blazars, in Ref. 10 it has been argued that the CA might be so strong, that could provide ultra-relativistic electrons.

All mentioned cases exhibit the possibility of energy pumping from rotation and in cases of the centrifugal acceleration they show its high efficiency in the energetics of relativistic electrons in rotating plasmas accelerating up to energies corresponding to the Lorentz factors of the order of \( 10^{5-8} \) (see Ref. 10). Thus, the amount of energy contained within a centrifugally accelerated plasma flow is very big and if there are mechanisms for the conversion of at least a small fraction of this energy into the the variety of instabilities - one might find a number of interesting consequences related to the problem of plasma energy conversion into radiation.

Generally speaking, for the energy pumping process, one has three fundamental stages: (a) energy conversion from rotation into kinetic energy of the plasma background flow, (b) kinetic energy conversion into energy of electrostatic waves and (c) conversion of the energy of waves into radiation by means of the process of the inverse Compton scattering (ICS) of electrostatic waves to photons \( \gamma = e + l + t \), which is thought to be responsible for creation of nonthermal radiation (see Ref. 11). By Tautz & Lerche (see Ref. 12) the electrostatic/electromagnetic mixed mode has been considered for relativistic jets in the context of Gamma-Ray Bursts.
to determine low-frequency instabilities. But our aim is to consider only electrostatic waves and investigate the second stage of the process [see (b)] and show how efficient it is. Therefore the electrostatic wave, playing an important role in the process of radiation might be very effective if one makes it unstable and as we will see the centrifugal force may be very sufficient in this context.

The centrifugally driven parametric instability has been introduced in Ref. 13 where it was shown that the centrifugal force acting on particles inside the pulsar magnetospheres, causes separation of charges, leading to generation of an electrostatic field, which excites the corresponding instability. The increment of the linear stage has been estimated and analyzed for the Crab pulsar and it turned out that the linear regime was extremely efficient and short in time, indicating a need of saturation of a growth rate. This mechanism is called parametric, being efficient and short in time, indicating a need of saturation of a growth rate. This mechanism is called parametric, because an "external" force - the centrifugal force, playing a role of the parameter, acts on plasma particles, changes in time and gives rise to the instability. By applying the same approach, in Ref. 14 (to be published in Mon. Not. R. Astron. Soc.) the problem of the reconstruction of the pulsar magnetospheres on large scales nearby the LCS has been considered. We have studied a new parametric mechanism and it was shown that for curvature drift waves, corresponding instability might be extremely efficient.

In this paper a hydrodynamic approach will be used to study the parametric mechanism of rotation induced electrostatic wave generation in extragalactic jets, applying the approach developed in Ref. 13 and Ref. 14 (to be published in Mon. Not. R. Astron. Soc.).

The paper is arranged as follows. In [II] we derive the dispersion relation, in [III] the corresponding results are present and in [IV] we summarize our results.

II. THEORY

Throughout the paper it is supposed that magnetic field lines are straight and inclined by the angle $\alpha$ with respect to the rotation axis. The plasma is in the frozen-in condition, co-rotating together with the field lines with the angular velocity $\Omega$.

It is easy to start our consideration by the local non inertial co-rotating frame of reference. The corresponding interval in the rigidly rotating frame will have the form (see Ref. 10):

$$ds^2 = -(1 - \Omega^2_{e,f} r^2) dt^2 - dr^2,$$

(1)

where $\Omega_{e,f} = \Omega \sin \alpha$ is the effective angular velocity of rotation. Here we use units $c = 1$. In this paper we apply a method developed in Ref. 13 where relativistic plasma motion was described by the single particle kinematics. According to this approach, by using 1+1 formalism (see Ref. 15), in the co-rotating frame of reference the equation of motion of a particle is given by:

$$\frac{dp}{d\tau} = \gamma g + \frac{e}{m}(E + v \times B),$$

(2)

where $\gamma = (1-v^2)^{-1/2}$ is the Lorentz-factor, $v = dr/d\tau$ is the velocity of the particle determined in the 1+1 formalism, $p \rightarrow p/m$ - the dimensionless momentum, $g \equiv -\frac{\Omega}{r}$, $\gamma g$ - the effective centrifugal force (see Ref. 13) and $\tau \equiv \xi t$ ($\xi \equiv \sqrt{1 - \Omega^2_{e,f} r^2}$) - the universal time. Taking into account an identity $d/dr \equiv \partial / (\xi \partial t) + (v \cdot \nabla)$ and a fact that Lorentz factors in the co-rotating frame of reference and in the inertial-laboratory frame (LF) relate to each other: $\gamma = \xi \gamma'$ (prime denotes a physical quantity in the LF), one can rewrite the equation of motion in the LF by following:

$$\frac{\partial p_i}{\partial t} + (v_1 \cdot \nabla)p_i = -\gamma \xi \nabla \xi + \frac{e}{m}(E + v_1 \times B),$$

(3)

complemented by the continuity equation:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v_1) = 0,$$

(4)

and the Poisson equation:

$$\nabla \cdot E = 4\pi \sum_i e_i n_i,$$

(5)

$i = e, p, b$

here $e, p, b$ denote electrons, positrons and the bulk components respectively. Eq. (3) in the zeroth approximation (leading state), by applying the frozen-in condition $E_0 + v_{0i} \times B_0 = 0$, can be reduced to (see Ref. 4):

$$\frac{d^2 r}{dt^2} = \frac{\Omega^2_{e,f} r}{1 - \Omega^2_{e,f} r^2} \left[1 - \Omega^2_{e,f} r^2 - 2 \left(\frac{dr}{dt}\right)^2\right].$$

(6)

In Ref. 4 it has been shown that for ultra-relativistic particles ($\gamma \gg 1$) Eq. (6) has following solutions:

$$r(t) = \frac{V_0}{\Omega_{e,f}} \sin \Omega_{e,f} t,$$

(7)

$$v_0(t) = V_0 \cos \Omega_{e,f} t,$$

(8)

for initial conditions: $r_0 = 0$ and $V_0 \sim 1$. As we see from Eq. (3), the effective centrifugal force $-\gamma \xi \nabla \xi$ changes in time due to Eqs. (7,8) and therefore the parametric mechanism switches on, giving rise to the electrostatic instability (as we will see).

When studying the problem for the leading terms, plasma oscillations have not been considered and only
where the effects of the centrifugal acceleration have been taken into account (see Ref. 4). Generally speaking, different species of plasmas [electrons, positrons and protons (bulk)] experience the centrifugal force, as a result they separate, which will lead to the generation of the electrostatic field, creating the Langmuir waves. This process can be considered as a next step of the approximation. The aim is to investigate the linear perturbation theory of the electrostatic instability and estimate its role in relativistic AGN jets.

We start the analysis by introducing small perturbations around the leading state:

\[ \Psi \approx \Psi^0 + \Psi^1, \]  

where \( \Psi = (n, v, p, E, B) \).

Perturbing all physical quantities by following:

\[ \Psi^1(t, r) \propto \Psi(t) \exp \left[i (k \mathbf{r})\right], \]

Eqs. (11) will get the form:

\[ \frac{\partial p^1_i}{\partial t} + ikv_0 p^1_i = n_0 \Omega_{ef} r p^1_i + \frac{e}{m_i} E^1, \]

\[ \frac{\partial n^1_i}{\partial t} + ikv_0 n^1_i + ikn_0 v^1_i = 0, \]

\[ i k E^1 = 4\pi \sum_i e_i n^1_i, \]

Here \( E^1 \) is the electric field in the LF, induced by separation of charges.

In order to reduce the above system into a single equation let us use an ansatz:

\[ n^1_i \equiv n_i e^{-i \frac{\Omega_{ef}}{\gamma_{0c}} \sin(\Omega_{ef} t)}, \]

then from Eqs. (11,12,13) one gets:

\[ \frac{\partial^2 N_i}{\partial t^2} = - \frac{e_i n^0}{m_i \gamma_i} e^{i R_i} k E^1, \]

\[ i k E^1 = 4\pi \sum_i e_i N_i e^{-i R_i}, \]

where \( R_i = \frac{\Omega_{ef}}{\gamma_{0c}} \sin(\Omega_{ef} t) \).

Restoring the speed of light and introducing a new variable \( N \equiv N_p - N_e \), after making the Fourier transformations, one can easily reduce Eqs. (15,16):

\[ \omega^2 N_0(\omega) = - \frac{m_0^0}{2M n^0_{\gamma_{0c}}} \sum_s (\omega - s \Omega)^2 J_s(a) N(\omega - s \Omega_{ef}), \]

\[ \omega^2 - \frac{\omega_{\gamma_{0c}}^2}{\gamma_{0c}} \] \( N(\omega) = - \frac{\omega_e^2}{2\gamma_{0c}} \sum_s J_s(a) N_0(\omega + s \Omega_{ef}), \]

where \( M \) and \( m \) are masses of protons and electrons/positrons respectively, \( a \equiv c k / \Omega_{ef} \) and \( \omega_e = \sqrt{8\pi n^0_e e^2 / m} \) - electron/positron plasma frequency. For deriving Eqs. (17,18), the identity

\[ e^{ \pm i \Omega_{ef} t} = \sum_s J_s(x) e^{ \pm i \Omega_{ef} t}, \]

has been used. Direct substitution of Eq. (17) into Eq. (18), yields:

\[ \left( \omega^2 - \frac{\omega_{\gamma_{0c}}^2}{\gamma_{0c}} \right) N(\omega) = \frac{\omega_b^2}{2\gamma_{0c}} \sum_{s,l} J_s(a) J_l(a) \left( \omega + (s - l) \Omega_{ef} \right)^2 \times \]

\[ \times N(\omega + (s - l) \Omega_{ef}). \]

where \( \omega_b = \sqrt{8\pi n^0_e e^2 / M} \) is the plasma frequency corresponding to the bulk component.

Generally speaking in order to solve Eq. (20) one has to consider similar equations, rewriting Eq. (20) for \( N(\omega \pm \Omega_{ef}), \ N(\omega \pm 2 \Omega_{ef}), \) etc., thus for solving the problem exactly, one needs to solve a system with the infinite number of equations, which makes the problem impossible to handle. Therefore the only way to overcome this difficulty and to gain extended view of the general behavior of the instability, is to consider physics close to the resonance condition, which provides the cutoff of the infinite row in Eq. (20) and makes the problem solvable (see Ref. 16).

If one considers in Eq. (20) only resonance terms, corresponding to the following frequency \( \omega_{res} \equiv \omega e / \gamma_{0c}^{1/2} \), then preserving leading terms and taking into account the resonance conditions: \( \omega \approx - s_0 \Omega_{ef}, \ s_0 = l_0 \equiv \left[ \frac{\omega}{\gamma_{0c} \Omega_{ef}} \right] \) (here \( [A] \) means the integer part of \( A \)), the dispersion relation can be reduced into a following single term specifying the growth rate \( \Delta \equiv \omega - \omega_{res} \) of the instability:

\[ \Delta^3 \approx \frac{\omega_b^2 \omega_e}{4 \gamma_{0c} \gamma_{0c}^2} J^2_{s_0}(a). \]

Since we are interested in imaginary parts of \( \Delta \), it is easy to see that the following solution (see Ref. 17):

\[ \Delta \approx \frac{1}{2} \left(1 - i \sqrt{3} \right) \left[ \frac{\omega_b^2 \omega_e}{4 \gamma_{0c} \gamma_{0c}^2} J^2_{s_0}(a) \right]^{1/2} \]

is responsible for the instability, the increment of which is given by:
The values of the wave length and the angle respectively are minimum and maximum values of the wave length and the angle for a wider range of parameters. Here $\lambda_{1,2}$ and $\alpha_{1,2}$ are minimum and maximum values of the wave length and the angle respectively for each interval.

In spite of that jets usually have small opening angles, it is better to study a dependence of the growth rate on inclination for a wider range of angles, in order to understand a general behaviour of the increment. In Fig. 2 we show the dependence of logarithm of the increment versus $\alpha$. The set of parameters is the same as for Fig. 1 except a wider range of $\alpha$ and fixed $\lambda \equiv \lambda_0 = 10^6 cm$.

As we see from the present figure, the instability is less efficient for smaller angles. Let us study now a dependence of the growth rate on the wave length for the their ambient, obviously having density of the order of $n_{am} \sim 1cm^{-3}$.

An interesting feature of the result shown in Fig. 1 is the sensibility of the growth rate on the wave length. As it is clear from the plot, a small change of $\lambda$, drastically changes the situation: for certain values of the wave length the increment reaches its maximum level and for slightly different values - it becomes equal to 0. Therefore it is better to examine an average value for each interval with a single peak, taking into account both parameters: $\lambda$ and $\alpha$, calculate the growth rate with respect to them:

$$\Gamma \equiv \frac{1}{(\lambda_2 - \lambda_1)(\alpha_2 - \alpha_1)} \int_{\lambda_1}^{\lambda_2} \int_{\alpha_1}^{\alpha_2} d\lambda d\alpha \delta(\lambda, \alpha),$$

and based on discrete data interpolate it for a wider range of parameters.

III. DISCUSSION

We consider a central black hole with mass $M_{BH} = 10^5 M_\odot$ and an angular rate of rotation $\Omega = 3 \times 10^{-5} s^{-1}$ of an AGN wind, making the light cylinder located at a distance $r_L \approx 10^{13} cm$ from the center of rotation. These values are typical for active galactic nuclei (see Ref. 18).

AGN winds and extragalactic jets are supposed to be composed of a bulk component having the Lorentz factor of the order of $\sim 20$ and relativistic electron-positron pairs with Lorentz factors: $\sim 10^5$ (see Ref. 10).

As it was already explained, the centrifugal force leads to the generation of the Langmuir waves with the wave vector $k (|| \vec{B} ||)$, the growth rate of which we are going to estimate versus the wave length $\lambda$ and the inclination angle $\alpha$ in order to investigate the corresponding instability.

Let us suppose a jet with the opening angle $\beta = 2\alpha = 2^\circ$. In Fig. 1 we show the dependence of logarithm of the instability increment on the wave length $\lambda \equiv \lambda_0 + \Delta \lambda$, where $\lambda = 10^6 cm$ and $\Delta \lambda = [0, 0.1] cm$. The set of parameters is: $\alpha = 1^\circ$, $M_{BH} = 10^5 M_\odot$, $\Omega = 3 \times 10^{-5} s^{-1}$, $\gamma_0 = 20$, $\gamma_\infty = 10^5$ and $n_0 = n_\infty = 0.001 cm^{-3}$. The values of $n_0$ and $n_\infty$ are typical for extragalactic jets, which are supposed to be under dense with respect to
inclusion angle $1^\circ$ when the instability is minimal (as it is seen in Fig. 4).

In Fig. 3 the behaviour of logarithm of $\Gamma$ versus $\lambda$ is shown. The set of parameters is the same as in Fig. 1 except a wider range of the wave length. The figure shows that by increasing $\lambda$, the average growth rate increases as well. Let us consider a less efficient case, thus the smallest increment (for $\lambda = 10^6 \, \text{cm}$) shown in the figure: $\Gamma \sim 10^{-5} \, \text{s}^{-1}$ ($\lambda = 10^6 \, \text{cm}$), and see how powerful the instability is. One can easily estimate the corresponding time scale by inverting the value of $\Gamma$:

$$t_{\text{inst}} = \frac{1}{\Gamma} \sim 10^5 \, \text{s}. \quad (25)$$

On the other hand jets are formed due to an accreting matter, therefore energetics of jets is defined by the efficiency of the accretion process, and in order to understand how efficient the considered instability is, one has to estimate the evolution time scale of an accreting disk and compare it with the instability time scale.

In Ref. 19 the problem of fueling of AGNs is considered and it is shown that for a given mass and luminosity of the AGN, mass of the self gravitating disk can be expressed by:

$$M_{\text{sg}} = 2.76 \times 10^5 \left( \frac{\eta}{0.03} \right)^{-2/27} \left( \frac{\epsilon}{0.1} \right)^{-5/27} \left( \frac{L}{0.1L_E} \right)^{5/27} \left( \frac{M_{\text{BH}}}{10^8 M_\odot} \right)^{23/27} M_\odot, \quad (26)$$

where $\eta$ is the standard Shakura, Sunyaev viscosity parameter (see Ref. 20), $\epsilon$ is the accretion parameter related to the accretion mass rate $\dot{M}$ and the luminosity $L$ by the following expression:

$$\epsilon \equiv \frac{L}{\dot{M} c^2}, \quad (27)$$

$L_E$ is the Eddington luminosity

$$L_E = 1.4 \times 10^{46} \frac{M_{\text{BH}}}{10^8 M_\odot} \, \text{erg/s}. \quad (28)$$

Then estimating the disk’s evolution time scale $t_{\text{evol}} \equiv M_{\text{sg}} / \dot{M}$, by taking Eq. (26) into account, one can easily get:

$$t_{\text{evol}} = 3.53 \times 10^{13} \left( \frac{\eta}{0.03} \right)^{-2/27} \left( \frac{\epsilon}{0.1} \right)^{22/27} \left( \frac{L}{0.1L_E} \right)^{-22/27} \times \left( \frac{M_{\text{BH}}}{10^8 M_\odot} \right)^{-4/27} \, \text{s}. \quad (29)$$

For taking the sense out of the considered parametric instability, let us examine typical values of the accretion disk $\eta = 0.03$, $\epsilon = 0.1$ and $M_{\text{BH}} = 10^8 M_\odot$, then considering the dependence of the evolution time scale on the luminosity, one gets the plot shown in Fig. 4. For producing the plot we have examined the following luminosity range $L/L_E \in [0.01; 1]$. As we see in the figure, the evolution time scale is a continuously decreasing function, and for the minimum value of the considered luminosity, the time scale is of the order of $10^{15} \, \text{s}$, whereas for the

\[ \begin{align*}
\text{FIG. 3: Dependence of logarithm of the average instability rate on the wave length. The set of parameters is the same as in Fig. 1 except a wider range of the wave length.}
\end{align*} \]
Eddington limit the time scale reaches its minimum level \( \sim 10^{13} \) s.
Calculating \( t_{\text{evol}}/t_{\text{inst}} \), by using Eq. (24) one can see that the present ratio lies in the range \( \sim [10^8; 10^{10}] \) s, which means that the process of the instability is much faster than typical rates of accretion, therefore the process of conversion of rotational energy into energy of the Langmuir waves is very rapid. This means that the linear stage has to be short in time, and the growth rate must saturate soon due to non linear effects, consideration of which seems to be essential.

In the introduction we have noticed that the Langmuir waves may strongly influence processes of radiation through the ICS of these waves against soft photons, when due to the channel \( l + e = l' + t \) the electrostatic waves lose energy, whereas the photons gain energy. This problem was not an objective of the present paper, which seems to be the first step for understanding the processes of rotation energy pumping into instabilities. As a next step one has to study this particular problem as well and see how efficient the radiation (produced via the ICS) could be for the extremely unstable electrostatic waves. This will be an objective of a work which we are going to make soon or later.

IV. SUMMARY

1. We studied a relativistic plasma of extragalactic jets composed of the bulk flow and relativistic electron-positron components. The role of the centrifugal acceleration in generation of the parametric instability of the Langmuir waves has been considered.

2. Perturbing the Euler, continuity and induction equations, preserving only first order terms, we have derived the dispersion relation governing the instability.

3. Examining the resonance condition of the system, an expression of the growth rate has been obtained for a nearly zone of the light cylinder surface.

4. Studying dependence of increment on the wave length for typical values of extragalactic jets, we found that the instability becomes efficient only for certain and narrow ranges of the wave length.

5. Taking into account efficiency of an accretion process in AGNs, and comparing the corresponding time scale to the time scale of the electrostatic instability it has been found that even when the latter is less effective (small angles and wave lengths) the instability process is extremely efficient.

The electrostatic instability may strongly affect processes of radiation in AGN jets by means of the ICS, therefore for extending this work, it is natural to consider a next important step and study the last stage: energy conversion into radiation.

An important restriction in the present model is that we studied the problem for quasi-straight magnetic field lines, whereas in real astrophysical situations the field lines are curved and it is interesting to see what changes in the dynamics of the electrostatic wave instability, when the curvature is taken into account.

On the other hand the curvature of magnetic field lines induces the curvature drift waves, which also may affect the process of energy pumping. In the context of pulsars this problem has been considered in Ref. 14 (to be published in Mon. Not. R. Astron. Soc.). It is reasonable to examine the same problem for relativistic AGN jets and study the role of the curvature drift modes in the energy conversion process.

When studying the problem on the linear stage, the increment shows high efficiency of the electrostatic instability, therefore the non linear regime also has to be examined in order to study the problem for longer time scales when the linear approximation does not work any more. For this purpose we plan to implement a numerical magnetohydrodynamical code to study this particular case as well.

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[1] Diego F. Torres and Sebastián E. Nuza, Astrophys. J. 583, L25 (2002)
[2] Hartman et al., Astrophys. J. 385, L1 (1992)
[3] M. A. Abramowicz and A. R. Prasanna, Mon. Not. R. Astron. Soc., 245, 729 (1990)
[4] G. Machabeli and A. Rogava, Phys. Rev. A 50, 98 (1994)
[5] A. Rogava, G. Dalakishvili and Z. Osmanov, Gen. Rel. and Grav. 35, 1133 (2003)
[6] R. D. Blandford, Relativistic accretion, proc. ASP, 160 (1999)
[7] R. D. Blandford and D. G. Payne, Mon. Not. R. Astron. Soc., 199, 883 (1982)
[8] R. D. Blandford and R. L. Znajek, Mon. Not. R. Astron. Soc., 179, 433 (1977)
[9] R. T. Gangadhara and Lesch, Astron. Astrophys., 323, L45 (1997)
[10] Z. Osmanov, A. Rogava and G. Bodo, Astron. Astrophys., 470, 395O (2007)
[11] R. Schlickeiser, Astron. Astrophys., 410, 397 (2003)
[12] G. Machabeli, Z. Osmanov and S. Mahajan, Phys. Plasmas 12, 062901 (2005)
[13] R. C. Tautz and I. Lerche, Astrophys. J. 653, 447 (2006)
[14] Z. Osmanov, G. Dalakishvili and G. Machabeli, Mon. Not. R. Astron. Soc. (2007)
[15] K. Thorne, R. Price and D. A. MacDonald, eds. Black Holes: The Membrane Paradigm (Yale University Press, New Haven 1986) (1986)
[16] V. P., Silin, V. T., Tikhonchuk, J. Appl. Mech. Tech. Phys., 11, 922 (1970)
[17] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, Natl. Bur. Stand. Appl. Math. Ser. No. 55 (U.S. GPO, D. C. Washington, 1965) (1965)
[18] G. Belvedere, L. Paternó and R. Pidatella, Mon. Not. R. Astron. Soc., 237, 827 (1989)
[19] A. R. King and J. E. Pringle, Mon. Not. R. Astron. Soc., 377, 25 (2007)
[20] N. I. Shakura and R. A. Sunyaev, Astron. Astrophys., 24, 337 (1973)