CORRELATIONS WEAK AND STRONG: DIVERS GUISES OF THE TWO-DIMENSIONAL ELECTRON GAS

A.H. MACDONALD

Physics Department, Indiana University, Bloomington, IN 40475, USA
E-mail macdonal@indiana.edu

The three-dimensional electron-gas model has been a major focus for many-body theory applied to the electronic properties of metals and semiconductors. Because the model neglects band effects, whereas electronic systems are generally more strongly correlated in narrow band systems, it is most widely used to describe the qualitative physics of weakly correlated metals with unambiguous Fermi liquid properties. The model is more interesting in two space dimensions because it provides a quantitative description of electrons in quantum wells and because these can form strongly correlated many-particle states. We illustrate the range of possible many-particle behaviors by discussing the way correlations are manifested in 2D tunneling spectroscopy experiments.

1 Introduction

In metals and semiconductors electrons interact among themselves and with nuclei which are localized, in ordered solids, near lattice sites. Even after ignoring the translational degrees of freedom of the nuclei, we are left with a host of different many-body problems for interacting electron systems. In elemental solids, we have \( \sim 100 \) different interacting electron problems specified, for example, by the atomic numbers and lattice constants of the periodic table on the inside front cover of the text by Ashcroft and Mermin. Problems of current interest in many-electron physics are more likely to be found in complicated compounds such as the quaternary cuprate superconductors, or in artificially fabricated systems such as metallic magnetic multilayers or semiconductor quantum dots. The set of possible many-electron problems is endless and, fortunately for this scientific subfield, the history of condensed matter physics teaches that discoveries of unanticipated properties in newly synthesized materials will also be endless.

Given the large number of many-electron problems, the hopeless difficulty of each, and the difference between the largest energy scales of these problems and the energy scales of the excitations that determine physical properties at room temperature and below, a common strategy of many-body theories is to work with models that are intended to faithfully describe essential aspects of the observed behavior of classes of materials without introducing incidental complications. Perhaps the most commonly studied models are the Hubbard model, which emphasizes the physics of strong repulsion between electrons near the same lattice site, and the electron gas model which mistreats atomic-like correlations at short length scales and captures instead the physics of screening and long-length scale correlations in metals. All electron \textit{ab-initio} calculations are generally limited to the consideration of physical properties and systems for which self-consistent-field approximations, still present in practice even in modern density-functional based electronic-structure calculations, are adequate. This situation frequently creates a difficulty in judging whether discrepancies between many-body theories and experiment are due to inadequacies of
a model, or due to inadequacies of the approximations used to estimate the properties of a model. The case of electrons in semiconductor quantum wells is unusual in this respect. At energies below the subband splitting of the quantum well, these systems are extremely accurately modeled by a two-dimensional (2D) version of the electron-gas model. In this paper we discuss some recent experimental and theoretical work on the many-body physics of the 2D electron gas. We limit our attention to the one-particle Greens function, which can be probed experimentally in these systems by 2D-2D tunneling spectroscopy.

The paper is organized as follows. In section II we discuss the case of a 2D electron system in a strong magnetic field with a partially filled Landau level. In this limit, the electronic ground state is not a Fermi liquid and the one-particle Greens function exhibits strongly non-perturbative behavior in which a large gap surrounding the Fermi energy is present in the tunneling density-of-states. This behavior is characteristic of strongly correlated electronic states. In section III we discuss the case of zero magnetic field, where experimental results confirm the Fermi-liquid nature of the electronic state, and tunneling experiments imply quasiparticle lifetimes in semi-quantitative agreement with simple random-phase-approximation estimates. Section IV offers some speculations on changes that should be expected in the zero-field tunneling density of states as the electron density is lowered and the electronic system becomes more strongly correlated. We conclude in Section V with a brief summary.

2 One-particle Greens Function at Strong Magnetic Fields

For a non-interacting electron system, the one-particle Greens function $G_i(\epsilon)$ has a single pole at $\epsilon = \epsilon_i$, where $\epsilon_i$ is an eigenvalue of the one-body Hamiltonian. Since many observables can be expressed entirely in terms of the one-particle Greens function, the way in which it is altered by interactions is a central topic of many-body theory. In 2D systems tunneling experiments have proved particularly valuable for experimental studies of the one-particle Greens function. In a tunneling-Hamiltonian formalism, the tunneling current between weakly linked electronic systems is given at zero temperature by

$$I = \frac{2\pi e}{h} \sum_{i_L, i_R} |t(i_L, i_R)|^2 \int_{\mu}^{\mu+eV} \epsilon d\epsilon A^+_{i_L}(\epsilon) A^-_{i_R}(\epsilon - eV).$$

where $\mu$ is the Fermi energy, $L$ and $R$ labels are used to distinguish 2D systems on opposite sides of the tunneling barrier, $A^+_i(\epsilon)$ and $A^-_i(\epsilon)$ are the particle and hole contributions to the spectral weight function for single-particle label $i$ and $V$ is the bias voltage. The spectral weights are related to the exact one-particle Greens function as described below. When interactions are neglected $A^+_i(\epsilon) = \delta(\epsilon - \epsilon_i)$ if state $i$ is empty, $A^-_i(\epsilon) = \delta(\epsilon - \epsilon_i)$ if state $i$ is occupied and the spectral weights are otherwise zero. In strongly correlated systems, the spectral function can be qualitatively altered.

In the presence of a perpendicular magnetic field, the single-particle kinetic-energy spectrum consists of Landau levels with energy $\epsilon_n = \hbar\omega_c(n + 1/2)$ and
macroscopic degeneracy $N_\phi = AB/\Phi_0$. Here $\omega_c = eB/mc$ is the cyclotron frequency, $A$ is the system area, $B$ is the magnetic field strength, and $\Phi_0 = \hbar c/e$ is the magnetic flux quantum. The one-particle Greens function is defined by

$$G_n(\epsilon) = \int_\infty^{-\infty} \frac{dt}{\hbar} \exp(i\epsilon t/\hbar)G_n(t) = -i \int_\infty^{-\infty} \frac{dt}{\hbar} \exp(i\epsilon t/\hbar)\langle\Psi_0|T[c_{n,m}(t)c_{n,m}^\dagger(0)]|\Psi_0\rangle$$

(2)

where $c_{n,m}$ and $c_{n,m}^\dagger$ are respectively fermion annihilation and creation operators for one of the degenerate states within the $n$'th Landau level, and $|\Psi_0\rangle$ is the ground state of the interacting electron system. We restrict our attention here to $n = 0$ and drop the Landau level index on $G_n(\epsilon)$. The Greens function can then be written in the form

$$G(\epsilon) = \int d\epsilon' \left[ \frac{A^+(\epsilon')}{\epsilon - \epsilon' + i\eta} + \frac{A^-(\epsilon')}{\epsilon - \epsilon' - i\eta} \right]$$

(3)

where the particle and hole spectral functions $A^+$ and $A^-$ have the following formal expression in terms of exact eigenstates of the Hamiltonian:

$$A^+(\epsilon) = \sum_\alpha |\langle\Psi_\alpha(N + 1)|c_m^\dagger|\Psi_0(N)\rangle|^2 \times \delta(\epsilon - [E_\alpha(N + 1) - E_0(N)])$$

$$A^-(\epsilon) = \sum_\alpha |\langle|\Psi_\alpha(N - 1)|c_m|\Psi_0(N)\rangle|^2 \times \delta(\epsilon - [E_0(N) - E_\alpha(N - 1)])$$

(4)

When interactions are neglected, states in the lowest Landau level are degenerate and are occupied with probability $\nu$ where $\nu = N/N_\phi$ and $N$ is the number of electrons in the many-particle system. It follows that $A_+ (\epsilon) = (1 - \nu)\delta(\epsilon)$ and $A_- (\epsilon) = \nu\delta(\epsilon)$. When these spectral functions are inserted in Eq.(1) we find that $I \propto \nu(1 - \nu)\delta(eV)$. If interactions were unimportant, a sharp peak in the tunneling current would occur near zero bias. The experimental result of Eisenstein et al. shown in Fig.[1] evidence quite the opposite behavior; the tunneling current is immeasurably small at zero bias and is peaked instead near a bias voltage of $\sim 5\text{mV}$. Since $I(V)$ depends on the product of electron and hole spectral functions at energies within $eV$ of the Fermi energy, it follows that both are extremely small close to the Fermi energy. Similar conclusions can be drawn on the basis of 3D-2D tunneling experiments by Ashoori and collaborators. This experimental result can be understood in qualitative terms as a consequence of strong correlations in the ground state of the electron system. When an additional electron is added to the ground state of an $N$ electron system, it is not strongly correlated with the electrons already present. The resulting $N + 1$ particle state will have a small overlap with the ground state or any low energy states of the strongly correlated $N + 1$ electron system. $A_+ (\epsilon)$ should therefore be peaked at energies well above the energy difference between the ground states of the $N + 1$ and $N$ particle systems, the chemical potential $\mu$. 

3
Figure 1: Low temperature tunneling I-V characteristics for bulk 2D to 2D tunneling in the fractional Hall regime. The traces are at magnetic fields separated by 0.25 Tesla and for this sample cover a range of filling factors from $\nu = 0.48$ to $\nu = 0.83$. The higher bias potential peak is due to Landau level mixing while the lower bias potential peak is due to tunneling within the lowest Landau level. The tunneling current is the convolution of particle and hole spectral weight functions, at energies separated by $eV$. This experiment shows that the tunneling current is strongly suppressed near zero bias and hence that the spectral weight functions are suppressed at energies near the Fermi energy over a broad range of filling factors. Features in the tunneling data associated with particular incompressible states, for example the one that occurs at $\nu = 2/3$, are weak. The inset shows the onset and peak values of the intra Landau level tunneling currents. (After Eisenstein et al. in Ref.11.)

Despite the achievement of a number of valuable insights, we still do not have a completely satisfactory theory for the spectral weight function in the strong magnetic field limit. Nevertheless, the connection between the energy at which the tunneling current is peaked and correlations in the ground state of the electron system suggested by the preceding words can be made more precise. The following sum rules for the zeroth and first moments of $A(\epsilon) \equiv A_-(\epsilon) + A_+ (\epsilon)$ follow from Eqs. (4):

$$\int d\epsilon A(\epsilon) = \langle \Psi_0 | c^+_m c_m + c_m c^+_m | \Psi_0 \rangle = 1$$ (5)

and

$$\int d\epsilon \epsilon A(\epsilon) = \langle \Psi_0 | [H, c^+_m] c_m + c_m [H, c^+_m] | \Psi_0 \rangle = \epsilon_{HF}.$$ (6)
In Eq. (6),

$$\epsilon_{HF} = \nu \sum_{m'} \left[ \langle m, m' | V | m, m' \rangle - \langle m', m | V | m, m' \rangle \right]$$  (7)

is the single-particle Hartree-Fock energy for this system. The last identity in Eq. (6) follows from translational invariance which requires that $\langle \Psi_0 | c_m^\dagger c_m | \Psi_0 \rangle = \nu$ for all $m$. These sum rules should be compared with those for $A$:

$$\int d\epsilon A_-(\epsilon) = \langle \Psi_0 | c_m^\dagger c_m | \Psi_0 \rangle = \nu$$  (8)

and

$$\int d\epsilon \epsilon A_-(\epsilon) = \langle \Psi_0 | [H, c_m^\dagger] c_m | \Psi_0 \rangle = 2\epsilon_0(\nu).$$  (9)

where $\epsilon_0(\nu) = E_0/N_0$ is the many-particle ground state energy per lowest-Landau-level orbital. The last identity is an adaptation, to the case of Hamiltonians that include only an interaction term, of the expression for the ground state energy in terms of the one-particle Greens function, derived using an equation of motion approach in many-body theory texts. Its application here rests on the observation that $\langle \Psi_0 | [H, c_m^\dagger] c_m | \Psi_0 \rangle$ is independent of $m$. The integrated weight of the hole part of the spectral weight function is $\nu$ and its mean energy is therefore $\langle \epsilon_- \rangle = 2\epsilon_0(\nu)/\nu$. For $A(\epsilon)$, the integrated weight is 1 and the mean energy is $\epsilon_{HF}$. It follows that the mean energy in the particle part of the spectral weight function is

$$\langle \epsilon_+ \rangle = \langle \epsilon_- \rangle + \frac{\nu\epsilon_{HF} - 2\epsilon_0(\nu)}{\nu(1 - \nu)}$$  (10)

The second term on the right hand side of this equation is a sum rule estimate of the gap separating peaks in the hole and particle portions of the spectral weight, $\Delta_{sr}$. $\Delta_{sr}$ will give an accurate value for the peak position whenever the peak is sharp and its position well defined; a property established for the present case by experiment. The dependence of $\Delta_{sr}$ on $\nu$ is illustrated in Fig.[2]; when corrected for finite quantum well width effects these results agree well with experiment.

Since in the present case there is no one-body term in the Hamiltonian, the ground state energy per orbital in the Hartree-Fock approximation is $\nu\epsilon_{HF}/2$. The gap is proportional to the amount by which the ground-state energy lies below the Hartree-Fock approximation ground-state energy, i.e., the gap is proportional to the correlation energy. We see that in the strong-magnetic-field case where the kinetic energy can be completely removed from the problem, the intuitive notion that there should be a gap in the electronic spectral function related to ground-state correlations has surprising exactness. Now we turn to the zero magnetic field limit, where interactions are in competition with the kinetic energy.

3 2D Tunneling Spectroscopy of the Fermi Liquid State

At zero magnetic field, we can label states in the 2D electron system by two-dimensional wavevectors. A unique feature of 2D-2D tunneling between quantum
Figure 2: Sum rule estimate for the gap between particle and hole contributions to the spectral weight function of a two-dimensional electron gas at strong magnetic fields as a function of Landau level filling factor $\nu$. The tunneling conductance is peaked when $eV = \Delta_{sr}$. These estimates are for the case of an ideal two-dimensional electron gas and need to be reduced to account for finite thickness effects when comparing with experimental systems. The gap is expressed in units of $e^2/\ell$ where $\ell = (hc/eB)^{1/2}$ is the magnetic length. After Mori et al. in Ref. 14.

wells in epitaxial layered semiconductors, is the near perfect translational invariance perpendicular to the tunneling direction which selects momentum-conserving tunneling processes. Accounting for spin-degeneracy at zero field, the tunneling current is

$$I = \frac{4\pi e |t|^2}{\hbar} \sum_k \int_{\mu} e^{\mu + eV} d\epsilon A^+_k(\epsilon) A^-_k(\epsilon - eV).$$  \hspace{1cm} (11)$$

For non-interacting 2D electrons, the spectral-weight function had a delta-function peak in the hole contribution for $|\vec{k}| < k_F$ and a delta-function peak in the particle contribution for $|\vec{k}| > k_F$ where $k_F$ is the Fermi wavevector. The tunneling current consequently has a delta-function peak at zero bias. At zero magnetic field, it is generally expected that the interacting 2D electron system should be in a Fermi-liquid state in which a large fraction of the spectral weight lies in a sharp Lorentzian quasiparticle peak centered at the quasiparticle energy:

$$A^+_k(\epsilon) = \frac{z_k}{\pi} \frac{\Gamma^+_k}{(\epsilon - E^+_k)^2 + \Gamma^2_k} + A^{inc}_k(\epsilon).$$ \hspace{1cm} (12)$$

Here $\Gamma^+_k = \hbar/2\tau_k$ where $\tau_k$ is the quasiparticle lifetime, $E^+_k$ is the quasiparticle energy, $z_k < 1$ is the quasiparticle normalization factor, and $A^{inc}_k$ is the part of the spectral function non included in the quasiparticle peak. In a Fermi liquid, the quasiparticle lifetime diverges in the limit of zero temperature as $k$ approaches $k_F$. 

If we ignore the renormalization of the quasiparticle energy due to interactions and the decrease of the quasiparticle lifetime on moving away from the Fermi surface, the contribution to the integrals in Eq. (11) from the quasiparticle portion of the spectral function can be evaluated analytically with the result

\[
G_{qp}(V) = \frac{I(V)}{V} = \frac{e^2}{\hbar} \frac{z_{k_F}^2 |t|^2 A \nu_0}{(eV)^2 + (\hbar/\tau)^2} \]  

(13)

where \( \nu_0 \) is the 2D Fermi-gas density of states. A finite quasiparticle lifetime turns the \( \delta \) function peak in the tunneling conductance into a Lorentzian. This is exactly what is seen in experiment, providing a very direct confirmation of the Fermi-liquid nature of the 2D electron-gas state at zero field. The zero-bias peak in the tunneling conductance at zero field stands in stark contrast with the extremely small conductance at small bias found at strong fields and discussed in the previous section. The 2D tunneling-spectroscopy experiment is a sensitive indicator of the nature of the many electron state. This simple result for the tunneling-conductance-spectroscopy line shape applies when all quasiparticle lifetimes are approximated by their value at \( |\vec{k}| = k_F \). Detailed calculations\(^{17,18} \) show that the true line shape is not Lorentzian, but that the voltage at which \( G_{qp} \) is reduced to half of its \( V = 0 \) value is increased by only 20% when the \( \vec{k} \) dependence of the quasiparticle lifetime is taken into account. Thus it is possible to read off the quasiparticle lifetime at the Fermi energy by looking at the width of the zero-bias tunneling conductance peak. As illustrated in Fig. [3], the measured lifetimes are in reasonable accord with random-phase-approximation theoretical estimates. The improved agreement supports the efficacy of exchange-correlation local-field corrections to the random phase approximation.\(^{19} \)

The sharp peak in the tunneling conductance near zero bias is due entirely to quasiparticle peaks in the spectral weight functions. As long as the 2D electron system is a Fermi liquid, this peak will be present. However, as the electron system becomes more strongly correlated, the quasiparticle normalization factor \( z_{\vec{k}} \) will become smaller and the incoherent part of the spectral function will become more dominant. We now turn our attention to some speculations on what will happen in this regime.

4 Strong Correlations at Zero Magnetic Field

A 2D electron system becomes increasingly correlated at low electron densities. The density is usually parameterized in terms of the electron gas parameter \( r_s \) defined by the following equation:

\[
n = \frac{N}{A} = \frac{\pi r_s^2 a_0^2}{\epsilon} 
\]  

(14)

where \( a_0 = \hbar^2 c / m^* e^2 \) is the host semiconductor Bohr radius, \( m^* \) is the band-structure effective mass of the electrons, and \( \epsilon \) is the host semiconductor dielectric constant. When \( r_s \) is large, the typical interaction-energy scale is much larger than the typical kinetic-energy scale, and the electron system is strongly correlated. For
Figure 3: Half-width at half-maximum for the zero-bias 2D-2D tunneling spectroscopy peak in the Fermi liquid state of a 2D electron gas. This width is inversely proportional to the quasiparticle lifetime at the Fermi energy. Theoretical results including local-field corrections to the RPA (full line), theoretical results in the RPA (dashed line), and experimental results (dotted line) reported by Murphy et al. in Ref. 17. After Jungwirth et al. in Ref. 17.

$r_s$ larger than about 30 a phase transition between the Fermi-liquid electronic state and a Wigner-crystal state with broken translational symmetry is expected to occur. Existing tunneling experiments in 2D electron systems have been performed on samples with $r_s < 2$, which have moderate correlations adequately described by the random-phase approximation. We expect that when experiments are performed in samples with much lower density, the zero-bias Fermi-liquid peak in the tunneling conductance will be embedded in a broader structure with a minimum near zero bias similar to what has already been observed for strongly correlated states in partially filled Landau levels. This expectation follows from a sum-rule analysis similar to that in Section II.

At zero magnetic field, the one-particle Green's function is diagonal in a representation of wavevectors and spins, $\vec{k}$ and $\sigma$. Although, unlike the strong field case, the Green's function is dependent on its single-particle state labels, it still satisfies similar identities for the zeroth and first moments of the full spectral function:

$$\int_{-\infty}^{\infty} A_{\vec{k},\sigma}(\epsilon) = \langle \Psi_0 | c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} + c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} | \Psi_0 \rangle = 1$$  \hspace{1cm} (15)$$

and

$$\int d\epsilon \epsilon A_{\vec{k},\sigma}(\epsilon) = \langle \Psi_0 | [H, c_{\vec{k},\sigma}^\dagger] c_{\vec{k},\sigma} + c_{\vec{k}\sigma}^\dagger [H, c_{\vec{k}\sigma}] | \Psi_0 \rangle$$

$$= \epsilon_k + \frac{N}{A} V(\vec{q} = 0) - \frac{1}{A} \sum_{\vec{k}'} V(\vec{k} - \vec{k}') n_{\vec{k}',\sigma}$$
Note that in Eq. (16), \( n_{\vec{k},\sigma} = \langle \Psi_0 | c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma} | \Psi_0 \rangle \) rather than the non-interacting Fermi gas occupation number. If not for this distinction, \( E_{\vec{k}} \) would be the Hartree-Fock approximation to the quasiparticle energy. Below we compare the total energy of the electron system with the quantity

\[
\tilde{E} = \frac{1}{2} \sum_{\vec{k},\sigma} n_{\vec{k},\sigma} (\epsilon_{\vec{k}} + E_{\vec{k}}).
\]

(17)

If \( n_{\vec{k}} \) were equal to its non-interacting value, \( \tilde{E} \) would be the Hartree-Fock approximation to the ground-state energy. Actually, \( \tilde{E} \) will be larger than the Hartree-Fock ground-state energy, since the kinetic energy is increased and the exchange energy reduced in magnitude by the partial occupation of states outside the Fermi sea. The difference between the exact ground-state energy \( E \) and the approximate value obtained in a Hartree-Fock approximation is generally referred to as the correlation energy of an interacting electron system. In the following, we will appropriate this language in referring to the difference between \( E \) and \( \tilde{E} \).

We employ an exact identity \( \tilde{E} \) that follows from the equation of motion for the one-particle Greens function and relates the hole part of its spectral weight to the total energy:

\[
E_{\text{corr}} \equiv E - \tilde{E} = -\frac{1}{2} \sum_{\vec{k},\sigma} n_{\vec{k}} (1 - n_{\vec{k}})(\langle \epsilon^+_{\vec{k}} \rangle - \langle \epsilon^-_{\vec{k}} \rangle)
\]

(19)

where \( \langle \epsilon^+_k \rangle \) is the mean energy in the particle portion of the spectral weight function so that

\[
n_{\vec{k}} \langle \epsilon^-_{\vec{k}} \rangle + (1 - n_{\vec{k}}) \langle \epsilon^+_{\vec{k}} \rangle = E_{\vec{k}}.
\]

(20)

For a low-density 2D electron gas, correlations are strong. The correlation energy is negative and comparable in magnitude to the total energy per electron. One way in which this can be consistent with Eq. (19) is if, for \( \sim N \) values of \( \vec{k} \), the spectral-weight function \( A_k(\epsilon) \) has a substantial portion of its weight distributed between non-quasiparticle peaks located near \( \langle \epsilon^-_{\vec{k}} \rangle \) and \( \langle \epsilon^+_{\vec{k}} \rangle \) and relatively little weight attached to its dispersive quasiparticle peak. This is precisely what happens in the strong field limit where, in accord with the non-Fermi-liquid character of the ground state, the quasiparticle peak is entirely absent. We now consider two quite different examples, in which this scenario is realized. The generality of the behavior convinces us that it is ubiquitous in strongly correlated fermion systems.
The one-band Hubbard model is a lattice model in which electrons interact only if they occupy the same lattice site. The one-band version of this model has been extremely widely studied in connection with the strong correlations that exist in the planar cuprate superconductors, especially in the underdoped regime. Electrons in this model become strongly correlated when the band is near half-filling and when the on-site interaction $U$ is larger than the intersite hopping energy $t$. The one-particle Greens function is readily evaluated at half-filling in the narrow-band ($t \rightarrow 0$) limit. The ground state has eigenenergy 0 and is $2^N$ fold degenerate. The zero energy states are those with one electron of either spin on each lattice site. Averaging over these states gives $n_{\mathbf{k},\sigma} = 1/2$. Removing an electron from any lattice site in any of the $2^N$ states gives a state with zero energy so that for each wavevector in the Brillouin zone, $A^-_{\mathbf{k},\sigma}(\epsilon) = (1/2)\delta(\epsilon)$ and $\langle \epsilon_{\mathbf{k}}^- \rangle = 0$. Adding an electron on any lattice site produces a state with energy $U$ so that $A^+_{\mathbf{k},\sigma}(\epsilon) = (1/2)\delta(\epsilon - U)$ and $\langle \epsilon_{\mathbf{k}}^+ \rangle = U$. These two peaks in the spectral function are referred to as the lower and upper Hubbard bands and they broaden when $t \neq 0$. The mean-field single-particle energy, calculated from the mean occupation numbers but including exchange corrections which eliminate interactions between parallel spins on the same site, is $E_{\mathbf{k}} = U/2$ and, taking note of the double-counting factor for this interaction energy, the corresponding total energy is $\tilde{E} = NU/4$. In this case, the spectral weight is entirely comprised of non-dispersive delta-function hole and particle contributions. This non-Fermi liquid state has no dispersive quasiparticle pole which sharpens on crossing the Fermi energy. In precise agreement with Eq. (19), the two peaks are split by an amount proportional to the correlation energy per particle. For finite $t$, the Hubbard model is generally expected to have a Fermi liquid ground state away from the half-filled band condition. An important issue in the many-body theory of the Hubbard model has been the question of whether the dispersive quasiparticle peaks at the Fermi energy are created by moving one of the Hubbard bands to the Fermi energy or by drawing weight away from the Hubbard bands to quasiparticle bands at the Fermi energy. Since the correlation energy can be strongly dependent on band-filling, the sum rules discussed here appear to preclude the former possibility and argue for quasiparticle bands which are energetically separate from both quasiparticle bands.

Another example that is relevant to the case of a 2D electron gas occurs when the translational symmetry is broken in the Wigner-crystal ground state. The broken symmetry requires a modification of our analysis that we will not detail here, since the Greens function in no longer diagonal in wavevector. Nevertheless, the connection between strong correlations and peaks in the spectral function that are away from the chemical potential is again present. The chemical potential in this system is the energy increase upon adding a particle by slightly decreasing the lattice constant, or the energy decrease upon removing a particle by slightly increasing the lattice constant. The particle portion of the spectral function has a peak at energies far above the chemical potential since the particle can be added at an arbitrary position, including energetically unfavorable positions near one of the crystal lattice sites. The hole portion of the spectral-weight function is peaked at energies well below the chemical potential, since the state with a particle removed from a lattice...
site will have an energy substantially larger than that of the adjusted lattice constant state. The correlation energy of the Wigner crystal state is proportional to the separation of these two non-quasiparticle peaks in the spectral weight function.

The 2D electron system is believed to have a Fermi-liquid ground state at all values of $r_s$ below those at which the transition to the Wigner-crystal state occurs. For large values of $r_s$, where a relatively small fraction of the spectral weight lies in the quasiparticle peak, the correlation energy is very close to that of the Wigner-crystal state. The electronic state is nearly as strongly correlated as the quantum-fluid states that occur at strong magnetic fields in partially filled Landau levels. Based on the preceding examples and the sum rules discussed above, this suggests that the non-quasiparticle portion of the spectral function at zero field and low electron densities will be qualitatively similar to those at strong fields, leading to 2D-2D tunneling spectroscopy results which have a small Fermi-liquid peak at the chemical potential but are otherwise similar to those illustrated in Fig.[1]. Since $n_F \ll 1$ for all $\vec{k}$ in this density regime, it follows from Eq.(19) that the non-Fermi-liquid peak in the tunneling conductance should occur when $eV$ is comparable to the correlation energy per particle. The situation should be quite close to that expected in Hubbard model systems where strongly dressed quasiparticle bands are separate from the lower and upper Hubbard bands. Tunneling spectroscopy studies of 2D electron gas systems have the potential to provide even more detailed information on the spectral weight function than what has been extracted from exhaustive angle-resolved-photoemission studies in the cuprate superconductor case.

5 Summary

Using the spectral-weight function of the one-particle Greens function as an example, we have attempted to illustrate the rich many-body physics of the two-dimensional electron gas system. Compared to other strongly correlated fermion systems, the 2DEG has the advantage that it is accurately described by a simple translationally invariant model of fermions interacting via long-range forces. The band structure of the host semiconductor enters only through an effective mass parameter which is accurately known. At strong magnetic fields, the 2DEG exhibits a rich variety of strong-correlation physics that has been widely studied in connection with the fractional quantum Hall effect. In this regime, 2D tunneling-spectroscopy experiments have demonstrated that the spectral weight has a wide and deep gap around the chemical potential. The size of this gap can be related to the ground-state correlation energy of the system by invoking a first-moment sum rule for the spectral function. To date, most experiments on 2D electron systems in the absence of a field have been performed in situations where the electrons are moderately correlated. Experiments show clear Fermi liquid behavior, which is evidenced in 2D tunneling-spectroscopy experiments by a peak in the tunneling conductance at the Fermi energy. The quasiparticle lifetime can be extracted from these measurements and is in qualitative agreement with a simple random-phase approximation for the electronic self-energy in the Fermi-liquid state. Systems with lower electron densities, which are becoming available because of continuous advances in the epitaxial growth of semiconductor quantum wells, are expected to be
more strongly correlated. We propose that, in this regime, the spectral function will develop peaks away from the Fermi energy that are analogous to both those which occur at strong magnetic fields and to the lower and upper Hubbard-band peaks that occur in strongly correlated lattice models. As new frontiers in materials preparation are reached, it will be possible to confidently compare theory and experiment for extremely strongly correlated Fermi-liquid states. For the 2D electron gas, tunneling spectroscopy provides a uniquely powerful probe of a strongly correlated Fermi liquid. The open many-body physics questions in the low-density 2D electron gas have a large overlap with open issues in the many-body physics of other systems, and especially with the physics of planar high-temperature-superconductor materials.

Acknowledgments

Instructive interactions with Jim Eisenstein, Rudolf Haussmann, Tomas Jungwirth, Hiro Mori, Sheena Murphy, and Lian Zheng are gratefully acknowledged. I am grateful to Ulrich Zuelicke for helpful comments on a draft of this paper. This work was supported by the National Science Foundation under grant DMR-9714055.

References

1. Neil W. Ashcroft and N. David Mermin, *Solid State Physics* (Saunders, Philadelphia, 1976).
2. Reiner M. Dreizler and E.K.U. Gross, *Density functional theory: an approach to the quantum many-body problem* (Springer-Verlag, Berlin, 1990).
3. J. Smoliner, E. Gornik and G. Weimann, Appl. Phys. Lett. 52, 2136 (1988); J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Appl. Phys. Lett. 58, 1497 (1991); R.C. Ashoori et al., Phys. Rev. Lett. 64, 681 (1990).
4. R.C. Ashoori et al., Phys. Rev. B 48, 4616 (1993).
5. S.Q. Murphy, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 52, 14825 (1995); Y. Katayama, D.C. Tsui, H.C. Manoharan, and M. Shayegan, unpublished.
6. For a theoretical discussion that emphasizes some unique aspects of tunneling experiments in two-dimensions see Lian Zheng and A.H. MacDonald Phys. Rev. B 47, 10619 (1993).
7. See for example, A.L. Fetter and J.D. Walecka *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971) or J. W. Negele and H. Orland *Quantum Many-Particle Systems* (Addison-Wesley, Redwood City, 1988).
8. E.L. Wolf, in *Principles of Electron Tunneling Spectroscopy* (Oxford University Press, New York, 1984).
9. See for example A.H. MacDonald, in *Proceedings of the Les Houches Summer School on Mesoscopic Physics* (Elsevier, Amsterdam, 1995), edited by E. Akkermans, G. Montambaux, and J.-L. Pichard.
10. The fact that the Greens function is independent of the single-particle state within the Landau level follows from translational invariance. See for example, A.H. MacDonald and S.M. Girvin, Phys. Rev. B 38, 6295 (1988).
assume for the present strong magnetic field discussion that the electrons are completely spin polarized and take the quantized kinetic energy of the lowest Landau level as our zero of energy.

11. J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 69, 3804 (1992).
12. H.B. Chan, P.I. Glicofridis, R.C. Ashoori, and M.R. Melloch, Phys. Rev. Lett. 79, 2867 (1997).
13. Y. Hatsugai, P.-A. Bares, and X.G. Wen, Phys. Rev. Lett. 71, 424 (1993); S. He, P.M. Platzman, and B.I. Halperin, Phys. Rev. Lett. 71, 777 (1993); P. Johansson and J.M. Kinaret, Phys. Rev. Lett. 71, 1435 (1993); A.L. Efros and F.G. Pikus, Phys. Rev. B 48, 14694 (1993); C.M. Varma, A.I. Larkin, and E. Abrahams, Phys. Rev. B 49, 13999 (1994); I.L. Aleiner, H.U. Baranger, and L.I. Glazman, Phys. Rev. Lett. 74, 3435 (1995); Rudolf Haussmann, Phys. Rev. B 53, 7375 (1996).
14. Rudolf Haussmann, Hiroyuki Mori, and A.H. MacDonald, Phys. Rev. Lett. 76, 979 (1996).
15. S.R. Renn and B.W. Roberts, Phys. Rev. B 50, 7626 (1994).
16. S.Q. Murphy, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 52, 14825 (1995); Y. Katayama, D.C. Tsui, H.C. Manoharan, and M. Shayegan, unpublished.
17. T. Jungwirth, and A.H. MacDonald, Phys. Rev. B 53, 7403 (1996).
18. L. Zheng and S. DasSarma, Phys. Rev. B 53, 9964 (1996).
19. D.R. Penn, Phys. Rev. B 22, 2677 (1980); C.A. Kukkonen and A.W. Overhauser, Phys. Rev. B 20, 550 (1979); X. Zhu and A.W. Overhauser, Phys. Rev. B 33, 925 (1986); G. Vignale and K.S. Singwi, Phys. Rev. B 32, 2156 (1985); S. Yarlagadda and G.F. Giuliani, Solid State Commun. 69, 677 (1989); A.H. MacDonald and D.J.W. Geldart, Can. J. Phys. 60, 1016 (1982).
20. B. Tanatar and D.M. Ceperley, Phys. Rev. B 39, 5005 (1989); Y. Kwon, D.M. Ceperley, and R.M. Martin, Phys. Rev. B 48, 12037 (1993). F. Rapisarda and G. Senatore, Aust. J. Phys. 49, 161 (1996).
21. For a review in which considerable emphasis is placed on the one-particle Greens function see Elbio Dagotto, Rev. Mod. Phys. 66, 763 (1994).