Gluon production in current-nucleus and nucleon-nucleus collisions in a quasi-classical approximation

Yuri V. Kovchegov*  
and  
A.H. Mueller*

Physics Department, Columbia University  
New York, New York 10027

Abstract

We calculate gluon production in deep inelastic scattering of the current \( j = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} \) off a large nucleus and in nucleon-nucleus collisions. In a covariant gauge calculation the transverse momentum spectrum of the gluon is determined by the final state interactions of the gluon with the nucleons in the nucleus. In a light-cone gauge calculation final state interactions are absent and these effects come from the light-cone wavefunction of the nucleus. We work in an approximation which neglects QCD evolution of gluons in the nucleon, that is in a quasi-classical approximation.

1 Introduction

A few years ago L. McLerran and R. Venugopalan [1] introduced an interesting program of studying the small \( x \) gluon distribution, and gluon production, involving a very large nucleus. The essential idea is to use the valence quarks of the nucleus as a source for a light-cone gauge calculation, involving only tree graphs, of the Weizsäcker-Williams (WW) field of the nucleus. The higher momentum components of this WW field, interpreted in terms of gluon quanta, would then be included with the valence quarks to give a new source which could be used to calculate another momentum layer of WW quanta, etc. This program is being actively pursued [2, 3]. It was observed in this (quasi-classical) WW approximation that the gluon distribution for a large nucleus is strongly modified from what one would expect from adding the distributions of the individual nucleons incoherently. An apparent saturation of the gluon distribution sets in at moderate to low gluon transverse momenta while the high transverse momentum part of the WW single particle gluon distribution remains additive. However, some time ago [6] it was observed that there are no shadowing corrections, in this equivalent gluon approximation. It is one of the purposes of this paper to show the compatibility of the no shadowing result with the result of Ref.5 and to interpret that latter result. To do this we introduce a current \( j(x) = -\frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) \) and calculate deep inelastic scattering of this current off a large nucleus in an approximation where the nucleons in the nucleus have no QCD evolution included in their gluon distributions. We suppose \( \alpha \ll 1 \) but the parameter \( \alpha^2 A^{1/3} \), with \( A \) the atomic number, will generally be taken to be large. We do our calculation first in covariant gauge and then in an appropriate light-cone gauge in order to interpret the result of Ref.5.

It is convenient to consider two distinct regimes of gluon production in the deep inelastic scattering of the current \( j \) off a large nucleus. If \( Q^2 = -q_\mu q^\mu \) is the virtuality of the current and if \( \sqrt{<\ell_1^2>} \) is the typical transverse momentum that a high energy gluon obtains by multiple scattering with the nucleons as it passes through the nucleus [5], then these two regimes are characterized by \( Q^2 \gg <\ell_1^2> \) and by \( Q^2 < <\ell_1^2> \). We begin by describing our results when \( Q^2 \gg <\ell_1^2> \).

*This work is supported in part by the Department of Energy under GRANT DE-FG02-94ER-40819.
In small $x$ deep inelastic scattering, and in covariant gauge, the current $j$ produces a gluon locally in the nucleus. This gluon then multiply scatters, both elastically and inelastically, as it passes through the nucleus. The equation for the transverse momentum distribution of the produced gluon is given in (6) below. The solution, given in (18) and (21), is exactly the one found in [5] for the gluon correlation function. This distribution does not reflect shadowing, but rather the probability conserving final state interactions which modify the transverse momentum distribution of the produced gluon but not the cross section for its production. In an $A_+ = 0$ light-cone gauge calculation, in a frame where the nucleus is right-moving and the current left-moving, we find a complete absence of final state interactions and we give a physical picture of why this happens. Here the effect of the final state interaction which we found in covariant gauge is encoded in the light-cone wavefunction of the nucleus. It is quite remarkable that the distribution of gluons in the WW wavefunction gives the correct transverse momentum distribution of produced gluons without any final state interactions whatever. The traditional gluon distribution $xG_A(x, Q^2)$ can be obtained by integrating the produced gluon distribution over all transverse momenta obeying $\ell_+^2 \leq Q^2$. Because $Q^2 \gg \ell_+^2$, in the logarithmic approximation, one can view the current $j$ as measuring the gluon distribution in the nuclear wavefunction.

When $Q^2 < < \ell_+^2$ the covariant gauge calculation of the produced gluon spectrum does not change in form from that when $Q^2 \gg \ell_+^2$ since the large $\ell_+^2$ comes from final state interactions which appear after the production of the gluon occurs. The light-cone gauge calculation now appears more complicated. The three and four gluon terms in $j$ become important and final state interactions are not negligible. The current is no longer sufficiently pointlike to be viewed as measuring the partonic gluon distribution of the nucleus. If we construct $xG_A(x, Q^2)$ from its moments, defined in terms of nuclear matrix elements of the normal local gluon operators, with the counter terms taken to be the same as for nucleon matrix elements, then $xG_A(x, Q^2)$ is additive $xG_A(x, Q^2) = A x G(x, Q^2)$. It appears that this object has no direct connection with partonic gluons in the light-cone wavefunction of the nucleus when $Q^2 < < \ell_+^2$. It is perhaps surprising that the gluon distribution in the light-cone wavefunction is still the same as the produced gluon spectrum. Finally, while the gluon distribution in the light-cone wavefunction is very different from the incoherent nucleon light-cone distributions when $\ell_+^2 < < \ell_+^2$ the nuclear modifications are simply probability conserving redistributions in phase space and so are unrelated to nuclear shadowing.

In Sec.3 we calculate gluon production in a nucleon-nucleus collision. In a calculation equivalent to a covariant gauge calculation we view the gluon as being produced in one of two ways. (i) The gluon may be present in the wavefunction of the hadron as the hadron reaches the front face of the nucleus. In that case interactions with nucleons in the nucleus free the gluon and multiple scattering in the nucleus broadens its transverse momentum spectrum. (ii) The gluon may be radiated in the final state from fragments of the incident hadron. In that case the transverse momentum spectrum of the gluon is not affected by the nucleus. Interference terms between initial and final state emission are small. In an $A_+ = 0$ light-cone gauge calculation of gluon production from the collision of a right-moving nucleus with a left-moving nucleon no final state interactions appear. If $A_\mu^+ \perp$ is the field of the nucleus while $A_\mu^\perp$ is the field of the nucleon gauge rotated by those nucleons in the nucleus having the same impact parameter then the gluon production cross section is obtained by coupling $A_\mu^+ \perp$ and $A_\mu^\perp$ to the QCD radiation field in a way determined by the QCD Lagrangian. A very simple expression emerges for the cross section as given by (73)-(75).

2 The gluon distribution and gluon production in deep inelastic scattering off a large nucleus

In this section, we consider deep inelastic scattering off a large nucleus where the (gauge invariant) current initiating the scattering is taken to be $j(x) = -\frac{1}{4} F^\mu\nu F_{\mu\nu}^i$ with $F_{\mu\nu}$ the usual QCD field strength tensor. We choose this current because it furnishes the most direct way to measure the gluon distribution and to produce a gluon and follow its interactions with the nucleons of the nucleus as it passes into the final state. We neglect QCD evolution in the interaction of $j$ with the individual nucleons in the nucleus. Our calculation of the gluon distribution is closely related to that of Ref.5 with which we are in agreement. Our motivation for neglecting QCD evolution, and we shall explain below in what sense we neglect it, is the same as in Ref.1. We are interested in a model to illustrate particular small $x$ and nuclear effects. We shall later comment on...
our expectations in a realistic nucleus.

There is a surprisingly different picture of gluon production in covariant gauge as compared to light-cone gauge. In covariant gauge, and in the rest systems of the nucleus, the current \( j \) produces a gluon off a nucleon in the nucleus in a reasonably local way. This gluon then has final state interactions with nucleons along its path as it passes through the nucleus\[10, 12\]. The final state interactions change the transverse momentum distribution of the produced gluon, but they do not modify the production rate. Thus the gluon distribution, and the structure function, of the nucleus is additive in the atomic number and there is no shadowing. In light-cone gauge, at least in a particular light-cone gauge, there are no final state interactions whatever. The transverse momentum distribution of the produced gluon is already encoded in the wavefunction of the nucleus. After deriving this result we shall give a simple physical picture why final state interactions are not present in our light-cone gauge.

### 2.1 Covariant gauge calculation of gluon production of the nuclear gluon distribution

We begin with the scattering of \( j = -\frac{1}{4}F_{\mu\nu} \) off a single nucleon as illustrated in Fig.1. Define a structure function

\[
W = (2\pi)^3 2E_p \int d^4x \, e^{i q \cdot x} \langle p|j(x)j(0)|p \rangle \tag{1}
\]

in analogy with deep inelastic lepton nucleon scattering. Then, from the graph in Fig.1 it is straightforward to show

\[
W(s, Q^2) = \frac{s}{4} W_0 \ln s/\mu^2 \tag{2a}
\]

which can be factorized as

\[
W(s, Q^2) = \frac{s}{4} xG(x, Q^2) + \frac{s}{4} W_0 \ln 1/x \tag{2b}
\]

with \( x \) the usual Bjorken variable, \( s = (p + q)^2 \) and \( W_0 \) a constant. (The Feynman rules for coupling \( j \) to two gluons are given in Ref.6.) The first term on the right-hand side of (2b) corresponds to gluon operators in the operator product expansion while the second term corresponds to quark operators. As discussed in some detail in Ref.6 the nucleons are additive in our approximation so that

\[
W_A = AW \tag{3}
\]

and

\[
xG_A = AxG \tag{4}
\]

Where \( W_A \) and \( G_A \) are the structure function and the gluon distribution for the nucleus. Indeed, additivity is closely related to our treatment of the gluon-hadron amplitude, the lower blob in Fig.1. We suppose that there are no \( \ln k^2/\lambda^2 \) factors in that amplitude and we also assume that the gluon-nucleon amplitude vanishes as \( k \cdot p \) becomes large. That is, we suppose that high mass intermediate states are not present in the gluon-nucleon scattering amplitude. These assumptions limit the longitudinal coherence length of the forward current-nucleon scattering amplitude to be not significantly larger than the nucleon size, which means that the interaction is local in the nucleus.

Now let us follow a gluon, produced by an interaction of \( j \) with a nucleon in the nucleus, as it scattering with other nucleons in the nucleus as illustrated Fig.2 where we have shown examples of both inelastic and elastic scattering in the nucleus. It is convenient to define a probability distribution for the gluon to have a transverse momentum \( \ell \) as it passes through the nucleus\[13\]. If the gluon is produced with a transverse momentum \( \ell_0 \) at a longitudinal coordinate \( z_0 \) and at an impact parameter \( b \) with respect to the center of the nucleus, let \( f(b, z_0, \ell_0, z, \ell) \) be the probability distribution for the gluon to have transverse momentum \( \ell \) at a longitudinal position \( z \). Of course
\[ \int d^2 \ell f(b, z_0, \ell_0, z, \ell) = 1 \] (5)

while the number density of produced gluons in the nucleus is

\[ \frac{dN(\ell)}{d^2 \ell} = \int d^2 \ell_0 \frac{dN_0}{d^2 \ell_0} \rho(b, z_0) f(b, z_0, \ell_0, z, \ell) \bigg|_{z=\sqrt{R^2-b^2}} \] (6)

where the integration \( d^2 b dz_0 \) goes over the volume of the nucleus, \( \rho \) is the nuclear density normalized to the atomic number \( A \) by

\[ \int d^2 b dz_0 \rho(b, z_0) = A, \] (7)

and we take the nuclear density to be uniform throughout the nucleus for simplicity. The initial distribution resulting from an interaction of the current \( j \) with a nucleon in the nucleus is normalized to be given by the unintegrated gluon distribution of the nucleon

\[ \frac{dN_0(\ell)}{d^2 \ell} = \frac{1}{\pi} \frac{dN_0}{d\ell^2} = \frac{1}{\pi} \frac{\partial}{\partial \ell^2} [xG(x, \ell^2)] \] (8)

where, in our approximation, there is no \( x \)-dependence in \( xG(x, \ell^2) \) and there is no \( \ell^2 \)-dependence in \( \ell^2 \frac{\partial}{\partial \ell^2} xG(x, \ell^2) \) for large \( \ell^2 \).

Now \( f \) obeys the equation (See Eqs. 2.7 and 2.8 of Ref. 12. Appendix A of that reference gives a derivation.)

\[ \frac{\partial}{\partial z} f(z, \ell) = -\frac{1}{\lambda} f(z, \ell) + \rho \sigma \int d^2 \ell' V(\ell') f(z, \ell - \ell') \] (9)

with

\[ f(z_0, \ell_0) = \delta(\ell - \ell_0) \] (10)

and where

\[ V(\ell) = \frac{1}{\sigma} \frac{d\sigma}{d^2 \ell} \] (11)

is the normalized gluon-nucleon scattering amplitude with \( \ell \) the momentum transfer and where

\[ \lambda = [\rho \sigma]^{-1}, \] (12)

is the mean free path of gluons in the nuclear matter. For notational simplicity we suppress the \( b, z_0, \ell_0 \) dependence of \( f \). The first (inelastic) interaction of the produced gluon shown in Fig. 2 is contained in the second term in (9) while the second (elastic) interaction comes from the first term in (9). We work in an approximation where the gluon-nucleon cross section is energy independent, at high energy, and where the gluon’s helicity is conserved during the scatterings in the nucleus.

Eq. 9 is solved by going to a transverse coordinate representation

\[ \tilde{f}(z, x) = \int d^2 \ell e^{-i(\ell-\ell_0) \cdot x} f(z, \ell). \] (13)

As usual in the high energy limit the transverse coordinate of the gluon will not change as the gluon passes through the nucleus, and thus, using (13) in (9) gives

\[ \frac{\partial}{\partial z} \tilde{f}(z, x) = -\frac{1}{4\lambda} x^2 \delta(x) \tilde{f}(z, x) \] (14)

with

\[ \tilde{f}(z_0, x) = 1 \] (15)
and where
\[ \tilde{v}(x) = \frac{4}{x^2}(1 - \tilde{V}(x)) \] (16)
with
\[ \tilde{V}(x) = \int d^2\ell e^{-i\hat{\ell}\cdot x} V(\ell). \] (17)

Now defining
\[ \tilde{N}(x) = \int d^2\ell e^{-i\hat{\ell}\cdot x} \frac{dN(\ell)}{d^2\ell} \] (18)
one easily finds from (6) that
\[ N(x) = \int d^2b d\rho_0 \rho_0 \tilde{N}(x) \tilde{f}(z, x)|_{z_0=\sqrt{R^2-b^2}} \] (19)
The solution to (14) and (15) is clearly
\[ \tilde{f}(z, x) = \exp\left\{ -\frac{(z-z_0)^2}{4\lambda x^2} \tilde{v}(x) \right\} \] (20)
leading to
\[ \tilde{N}(x) = \int d^2b \frac{N_c^2 - 1}{\pi^2 \alpha N_c x^2} (1 - \exp\left\{ -\frac{\sqrt{R^2-b^2} x^2}{2\lambda} \tilde{v}(x) \right\}) \] (21a)
or
\[ \tilde{N}(x) = \int d^2b \frac{N_c^2 - 1}{\pi^2 \alpha N_c x^2} (1 - \exp\left\{ -\frac{2\pi^2 \sqrt{R^2-b^2} x^2}{N_c^2 - 1} \rho x G(x, x^2) \right\}) \] (21b)
where we have used (See Eq.4.7 and Appendix B of Ref.12.)
\[ \frac{\tilde{v}(x)}{\lambda} = \frac{4\pi^2 \alpha N_c}{N_c^2 - 1} \rho x G(x, x^2), \] (22)
valid in a logarithmic approximation for small \( x^2 \). (Our notation is such that \( xG(x, x^2) \) is not the Fourier transform of \( xG \), but rather stands for \( xG(x, 1/x^2) \).)

Eq.(21) is the same result found in Ref.5. In the limit of very small \( x^2 \) one can keep only the first two terms in the expansion of the exponential in (21) to arrive at
\[ N(x) = Ax G(x, x^2) \] (23)
which, with the identification of the nuclear gluon distribution with \( N(x^2) \), gives
\[ xG_A(x, x^2) = Ax G(x, x^2). \] (24)
At first sight it would seem that (24) no longer holds for extremely large nuclei at fixed \( x^2 \) where
\[ N(x) \approx \frac{N_c^2 - 1}{\pi \alpha N_c} \frac{R^2}{x^2}. \] (25)
However (24) is still correct. What fails is the identification of \( N(x) \) with \( xG_A(x, x^2) \) when \( A \) is very large. The physics here is quite clear. The current \( j \) makes a (local) hard interaction with one of the nucleons in the nucleus and produces a gluon. The coherence length for this production is small and so the production cross section, and hence the nuclear gluon distribution \( xG_A(x, Q^2) \), is incoherent among the nucleons in the nucleus\[\text{ref.13}]. The quantity which is calculated in (6) and (21) is the momentum distribution of produced gluons, and the \( \hat{\ell} \) in (21) is conjugate to the \( \ell \) of the produced gluon but has no natural connection with
the \( Q^2 \) of the current when \( A \) is very large. When \( A \) is not too large, and the final state interactions do not change the momentum of the produced gluon too much, and if one takes a cutoff on \( \ell^2 \) equal to the \( Q^2 \) of the current, then \( N(x^2 = 4/Q^2) \) can be identified with \( xG_A(x, x^2) \) and (23) holds. However, when the final state interactions give a broadening of the transverse momentum distribution \( \Delta \ell^2 \) which is large compared to \( Q^2 \) then \( N(x^2 = 4/Q^2) \) has no relation to the nuclear gluon distribution.

### 2.2 Light-cone gauge calculation of gluon production and of the nuclear gluon distribution

In a light-cone gauge calculation of gluon production one must also arrive at (21) and (24). Indeed (21) has already been obtained in Ref.5 using a light-cone gauge although the interpretation of what exactly was calculated is, perhaps, not so straightforward. In this section, we calculate gluon production in \( \eta \cdot A = A_+ = 0 \) light-cone gauge. We restrict our discussion to \( N \) light-cone gauge. We now carry out an integration over \( k \) and if \( k_\perp \) is the way we choose the \( a' \)s in the light-cone gauge propagator. With the momentum as indicated in Fig.3 our light-cone gauge propagator is

\[
D_{\alpha\beta}(k) = \frac{-i}{k^2 + i\epsilon} [g_{\alpha\beta} - \frac{\eta_\alpha k_\beta}{\eta \cdot k - i\epsilon} - \frac{k_\alpha \eta_\beta}{\eta \cdot k + i\epsilon}].
\] (26)

This choice was used in Ref.11 and implicitly in Refs.4 and 5. If \( \eta \cdot k = k_+ \) then the choice of \( i\epsilon \)'s in (26) dictates that a charge at an \( x_- = \frac{x_0 - x_3}{\sqrt{2}} \) coordinate \( x_\perp(0) \) gauge rotates all charges having \( x_- < x_\perp(0) \) but does not gauge rotate charges with \( x_- > x_\perp(0) \).

#### 2.2.1 Final state interactions are absent

In order to understand why final state interactions do not appear in deep inelastic scattering in light-cone gauge it is useful to begin with the example of gluon production off a quark in one nucleon and the subsequent rescattering of the gluon off a quark in another nucleon as illustrated in Fig.4. The quarks are assumed to be right-movers and the produced gluon, \( \ell \), is assumed to obey \( \ell_+ \ll p_+ \). The quarks are on-shell before and after the scattering while the quark having \( p_+ \) is taken to have its \( x_- \)-coordinate to be greater than the \( x_- \)-coordinate of \( p_1 \) so that it is natural to expect a production of the gluon off quark 1 and a later rescattering off quark 2. The + symbols along the quark lines in Fig.4 indicate that the \( \gamma_+ \) matrices will be dominant, as usual when considering soft gluon production.

The relevant factors for the graph shown in Fig.4a are

\[
G_\alpha = \left( \frac{-i}{k^2 - i\epsilon} \right)_\alpha \frac{\eta_\beta}{\eta_\beta - i\epsilon} \frac{\eta_\gamma}{(k - \ell)_\gamma + i\epsilon} + \frac{i}{2\ell_- [(k - \ell)_+ + (k - \ell)_+ - i\epsilon]} \left[ \frac{\eta_\beta}{(k - \ell)_\gamma + i\epsilon} \frac{\eta_\gamma}{(k - \ell)_+} \frac{-\eta_\gamma}{(k - \ell)_+ - i\epsilon} \right] \Gamma_{\gamma\sigma\nu} \frac{-i k_+}{k^2 (k - \ell)_+} \epsilon_\nu(\ell).
\] (27)

We now carry out an integration over \( k_+ \), recalling that \( \ell_+ \ll -q_+ \ll p_+ \), \( \ell_+ \ll q_+ \) follows from \( \ell_- = q_- \) and \( Q^2 = -q_+ q_- \gg \ell^2 \) while \( -q_+ \ll p_+ \) follows from the fact that we are considering small-x deep inelastic scattering with \( x = -q_+ / p_+ \). We take transverse components of \( q_\mu \) to be zero.) Before doing the \( k_+ \) integration we use the Slavnov-Taylor-Ward (STW) identities to drop the \( \eta_\beta(k - \ell)_\gamma \) part of the gluon propagator \( D_{\beta\gamma} \) as this term cancels between the three graphs, (a-c), of Fig.4. After this term is dropped only the \( \frac{-i k_+}{k^2 (k - \ell)_+ + i\epsilon} \) has a singularity in the lower half \( k_+ \)-plane. We can do the \( dk_+ \) integration by distorting the contour in the lower half \( k_+ \)-plane so long as the rest of the integrand, the other terms in \( G_\alpha \), vanish.
for large values of $k_+$. And if that is the case the net result will be small because the final evaluation is done at $k_+ = (\ell - q)_+ \approx -q_+$ so that inverse powers of $k_+$ correspond to higher inverse powers of $Q^2$ as compared to the case of no rescattering. In order to count the $k_+-$ factors in $G_\alpha$ it is convenient to write

$$g_{\beta\gamma} = \frac{\eta_\gamma(k - \ell)_\beta}{(k - \ell)_+ - i\epsilon} = \eta_\beta \bar{\eta}_\gamma + g_{\beta\gamma}^+ + \frac{\eta_\gamma(\eta_\beta\ell_+ - (k - \ell)_\beta)}{(k - \ell)_+ - i\epsilon}.$$  

(28)

Then in order to compensate for the $k_+ - i\epsilon$ and $(k - \ell)_+$ one would need a $k_+^\beta$ coming from the numerator terms in order not to have a small result. This means that one must get a single factor of $k_+$ (or $q_+$) each from the $\Gamma_{\gamma\sigma\nu}$ term and from the $\Gamma_{\beta\alpha}$ term in (27). In order to get a $k_+-$ factor from $\Gamma_{\gamma\sigma\nu}$ an accompanying $\bar{\eta}_\gamma$, $\bar{\eta}_\sigma$ or $\bar{\eta}_\nu$ must appear. But $\bar{\eta}_\sigma$ clearly gives zero from (27) while $\bar{\eta}_\gamma$ gives zero, or an extra $k_+$ in the denominator, from (28) so we are left with $\bar{\eta}_\nu$ as the only possibility. Here we can simplify our discussion by choosing $\bar{\eta} \cdot \epsilon = 0$. This is perhaps an unusual choice to go with $\eta \cdot A = 0$ light-cone gauge but, after all, one is free to choose the external polarizations in any convenient basis. With the choice $\bar{\eta} \cdot \epsilon = 0$ we find

$$\int dk_+ G_\alpha = 0$$  

(29)

(We could arrive at this result somewhat more directly. After observing that there is sufficient convergence to do the $k_+-$ integration by distorting in the lower half plane we could use current conservation in the $\alpha$-index of the $q-$vertex along with $(k + q - \ell)_+ \approx (k + q - \ell)_\alpha$ to get (29). However, it does not seem possible to generalize this procedure to all higher order graphs.)

The choice $\bar{\eta} \cdot \epsilon = 0$ also makes graphs b and c of Fig.4 equal to zero because one cannot have a $\gamma_+$ matrix with this polarization vector. Thus we are left only with the graph shown in Fig.4d and we must also evaluate it with $\bar{\eta} \cdot \epsilon = 0$. Now

$$G_d = \frac{-i(k - \ell + q)_\alpha}{(k - \ell + q)^2[(k + q - \ell)_+ + i\epsilon]} \frac{-ik_\sigma^+}{k^2(k_+ - i\epsilon)} \epsilon_\nu \Gamma^\alpha_{\alpha\sigma\nu}$$  

(30)

where

$$\Gamma^\alpha_{\alpha\sigma\nu} = g_{\alpha\sigma}(\ell - q - 2k)_\nu - g_{\alpha\nu}(2\ell - q - k)_\sigma + g_{\nu\sigma}(k + \ell)_\alpha.$$  

(31)

Again, in doing the $k_+$ integration we will end up with a $q_+$ in the denominator, giving a small result, unless one can take a $k_+$, or a $q_+$, from $\Gamma$ as given in (31). But, taking a $k_+$, or a $q_+$, from $\Gamma$ will result in a factor of $\bar{\eta}_\nu$, $\bar{\eta}_\sigma$ or $\bar{\eta}_\nu$ each of which give zero in (30).

We have seen that the graphs of Fig.4 give a small contribution. (We have reached the conclusion for $\ell^2 \ll Q^2$. Of course we would have found a small result for $\ell^2$ large independently of the size of $Q^2$, but that is not different than in covariant gauge. The fact that final state interactions are suppressed for large $Q^2$, and with $\ell^2 \ll Q^2$, is different than what we found in covariant gauge.) In a moment we shall give a physical argument explaining why this is not unexpected. However, before describing our physical argument consider the high energy gluon quark scattering amplitude at order $\alpha$ in $A_+ = 0$ light-cone gauge and in a frame where the quark is right-moving and the gluon left-moving. The graphs are shown in Fig.5. If we choose $\bar{\eta} \cdot \epsilon = 0$ for the initial and final gluons, then only the first graph in Fig.5 contributes. The essential factors in this graph are

$$G = \bar{u}(p - k)_{\gamma_+} u(p) \frac{i}{k^2} k_+^4 \Gamma_{\alpha\beta\gamma}^{(\lambda_+)} \epsilon_\lambda \epsilon_\beta$$  

(32)

The on-shell conditions $(\ell - k)^2 = 0$ and $(p - k)^2 = 0$ give $k_+ = \frac{(\ell - k)^2}{2\ell - k} = -k^2/2p_+$. In order to see how the high energy limit comes about it is useful to write

$$k_\alpha^+ = k_\alpha - \bar{\eta}_\alpha k_+ - \eta_\alpha k_-.$$  

(33)

$k_\alpha$ gives zero while acting in (32) while $\eta_\alpha$ gives a small factor. Using
\[ \bar{\eta}_\mu \Gamma_{\alpha\beta\gamma} \epsilon^\lambda \bar{\epsilon}^{(\lambda_f)} = 2 \ell_- \epsilon^{(\lambda_1)} \cdot \bar{\epsilon}^{(\lambda_f)} \]  

(34)
gives
\[ G = \frac{2i s}{k^2} \epsilon^{(\lambda_1)} \cdot \bar{\epsilon}^{(\lambda_f)}. \]  

(35)

Of course, the result emerges directly from an evaluation of (32) without explicitly invoking current conservation, (33). In a frame where \( \ell_+ \) is large the high energy growth, the factor of \( s \) in (35), comes partly from the smallness of \( k_\perp \) in the denominator in (32). In order to get a small denominator it is crucial to have a long region of integration over \( x_- \), the variable conjugate to \( k_\perp \). Of course the quark and gluon are far apart when \( x_- \) is large, but in axial gauge important interactions can happen in a non-causal manner. In our choice of light-cone gauge, (26), the factor \( \frac{1}{k_\perp^2} \) in (32) comes from times long before the quark and gluon have reached each other.

We are now in a position to see why final state interactions cannot be important in single gluon production in light-cone gauge in deep inelastic scattering. In Fig.4a and its higher order counterparts a final state interaction occurs much like the one we have just considered. In order for this final state interaction to be important one would need a trapping of the \( k_\perp \) integration contour at values of \( k_\perp \) on the order of \( \ell_+^2/2\ell_- \) but from (27) we see that we can distort the \( k_\perp \)-contour so that \( k_\perp \) is always of size \( q_+ = -\frac{Q^2}{x_-} \). Because \( Q^2 \) is a large parameter \( k_\perp \) is not forced to be small, and thus final state interactions are weak. We can say this slightly differently. The highly off-shell current has a lifetime \( \Delta x_+ \approx \frac{Q^2}{x_-} \) and this is the limiting \( x_- \) over which integrations can be done. This \( \Delta x_- \) is not large enough to generate the small light-cone denominators necessary to have final state interactions, at least so long as the target is not too long compared to its longitudinal momentum, that is so long as \( \xi^2 \ll Q^2 \).

### 2.2.2 Gluon production in deep inelastic scattering

We begin by considering deep inelastic scattering in a nucleus where two separate nucleons are involved in the scattering. The two classes of graphs are illustrated in Fig.6. The graphs of Fig.6a correspond to inelastic scattering off both nucleon 1 and off nucleon 2 while graphs in the class of Fig.6b correspond to an inelastic reaction with nucleon 1 and an elastic reaction with nucleon 2. We emphasize that in our frame where the nucleons are right-movers and the current \( q \) has \( q_+ = 0 \), the \( x_- \) coordinate of nucleon 1 is less than the \( x_- \) coordinate of nucleon 2. The interactions shown in Fig.6 are nonzero only when the propagator of the \( k_\perp \)-line is taken to be \( \frac{\bar{\eta}_\mu k_\perp}{|k_\perp + i\epsilon | |k_\perp - \sigma |} \) and similarly for the \((k_\perp - \Delta)\)-line. (We suppose \( \Delta_+ = \Delta_- = 0 \) while \( \Delta_+ \) is integrated freely over values large compared to \( \ell_+ \) as it is appropriate for a low \( x \) collision where the produced gluon has a coherence length large compared to the length of the nucleus.) Our gauge choice, as indicated in (26), allows the gluon field of a nucleon to have a pure gauge part which extends in the negative \( x_- \) direction starting from that nucleon. As shown in detail in Ref.11 this field gauge rotates the field of nucleons having smaller values of \( x_- \). Thus, the field of nucleon 2 rotates the field coming from nucleon 1.

Now let us evaluate explicitly the graphs in Fig.6 starting with those in Fig.6a. The \( k_\perp \)-line of that figure goes from nucleon 2 to nucleon 1 or to the field coming from nucleon 1. Because of the \( k_\alpha \) at the end of the \( k_\perp \)-line we can use the STW identities to evaluate these contributions. This has already been done in Ref.11 and here we need only outline the procedure and put in the normalizations appropriate for our problem. In doing the \( k_\perp \)-integration we distort over the \( \frac{1}{k_\perp^2 - \sigma^2} \) pole if the light-cone propagator which sets \( k_\perp = 0 \). Since \( k_\perp = 0 \) compared to \( \ell_\perp \) the \( k_\perp \)-line, which by the STW identities eliminates the \((\ell_\perp - q_\perp)\)-propagator, effectively inserts a transverse momentum \( k_\perp \) just before the attachment of the current \( q \). In terms of formulas

\[
\int dk_\perp \frac{k_\alpha}{k_\perp + i\epsilon} \Gamma_{\sigma\alpha\rho} D_{\rho\mu}(\ell - q) v_{\mu\lambda} = -2\pi q v_{\sigma\lambda} + \cdots \]  

(36)

where \( \Gamma_{\sigma\alpha\rho} \) is the triple gluon vertex, \( D_{\rho\mu} \) the gluon propagator and \( v_{\mu\nu} \) the vertex of the current \( q \) with the gluon lines \( \ell - q \) and \( \ell \). The omitted terms in (36) cancel with gluon, \( k_\alpha \), attachments into the lower \( \text{blob} \) of nucleon 1. In exactly the same way, and integrating over \( \Delta_+ \), the gluon line \( k - \Delta \) also eliminates the \( \ell - q \) line in the complex conjugate amplitude.
Thus, the STW identities allow one to calculate the effect of nucleon 2 on the field coming from nucleon 1. We may write the gluon production amplitude in terms of the scattering of the current off the gluon \( \ell - q \), as illustrated in Fig.7, times the unintegrated gluon distribution coming from nucleon 1 as modified by nucleon 2. Rather than immediately writing the answer for the two nucleon case we instead proceed to the general case using the “classical” field calculated in Ref.4 to generate the tree graphs contributing to the unintegrated gluon distribution coming from an arbitrary number of nucleons in the nucleus. We change notation slightly from Ref.4 to better match the problem at hand. Write \( A^\perp_\mu (q,x_-) = \sum_{\alpha=1}^{N^2-1} T^\alpha A^\perp_\mu (q,x_-) \) in light-cone gauge as

\[
A^\perp_\mu (q,x_-) = \int S(x,b_-) T^\alpha S^{-1}(x,b_-) \nabla^\perp_\mu \ln(x-b/|\mu| \hat{\rho}^\alpha(b,b_-) \theta(b_- - x_-) \nu_{b_-})
\]

with

\[
S(x,b_-) = P \exp \left\{ i g T^\alpha \int \ln(x-b/|\mu| \hat{\rho}^\alpha(b,b_-) \theta(b_- - x_-) \nu_{b_-}) \right\}
\]

where \( P \) path orders the \( b_- \)-integration with terms having smaller values of \( b_- \) coming more to the right. \( \hat{\rho}^\alpha \) is a color charge operator normalized according to

\[
< \hat{\rho}^\alpha(b,b_-) \hat{\rho}^\alpha(b',b'_-) > = \frac{\rho(b,b_-)}{N^2-1} \delta(b - b') \delta(b_- - b'_-) \delta_{\alpha\alpha'} Q^2 \frac{\partial}{\partial Q^2} \hat{G}(x,Q^2)
\]

with \( \rho(b,b_-) \) the normal nuclear density, in our boosted frame, obeying

\[
\int d^2bdb_- \rho(b,b_-) = A.
\]

\( \mu \) in (37) and (38) is an infrared cutoff which will disappear from physical quantities. We note that \( Q^2 \frac{\partial}{\partial Q^2} \hat{G}(x,Q^2) \) is independent of \( x \) and \( Q^2 \) in our approximation. The expectation indicated in (39) is an expectation in the nuclear wavefunction and we suppose that the pairwise correlation indicated there is the only nontrivial correlation. This is a natural assumption for a nucleus having weak nucleon-nucleon correlations. Define a Fourier transformed field by

\[
\Lambda(k,k_+) = \int \frac{d^2x}{2\pi} \int d\nu_{b_-} e^{i(k_+-i\nu)x_- - ik*b_-} A^\perp_\mu (q,x_-).
\]

Then the main formula which we shall need in order to evaluate \( \frac{dN}{d\ell} \) is

\[
\frac{dN}{d\ell} = -2Tr < A^\perp_\mu (\ell, \ell_+ + q_+) A^\perp_\mu (\ell, \ell_+ - q_+) > [g_{\mu\nu} \frac{(Q^2 - \ell^2)}{4} - \ell_{\mu} \ell_{\nu} Q^2 \frac{1}{\ell^2}]
\]

where the term in the brackets on the right-hand side of (42) is given by the graph in Fig.7. The \( A_\mu \) in (42) is the same as given in (37) and represents the field of the nucleus which the source \( j \) interacts with. Eq.(42) does not have any final state interactions, interactions of the produced gluon with the nucleus after the action of \( j \), which we have previously argued are small. As \( Q^2 \gg \ell^2 \) and \( \ell_+ \ll -q_+ \) then taking the leading \( Q^2 \) term in Eq.(42) and using Eq.(18) yields

\[
\tilde{N}(q^2) = -\frac{2}{\pi} \int d^2b Tr < A^\perp_\mu (b) A^\perp_\mu (b + q) >.
\]

Plugging in \( A^\perp_\mu \) from Eq.(37) one finds

\[
\tilde{N}(q^2) = -\frac{2}{\pi} \int d^2b \int d^2b' d^2b'' d^2b_-' d^2b_''
\]

\[
< \frac{b - b'}{|b - b'|^2} \cdot \frac{b + q - b''}{|b + q - b''|^2} \hat{\rho}^\alpha(b',b'_-) \hat{\rho}^\alpha(b'',b''_-)>
\]
Here we can assume that the path ordered $b_-$-integration in the $S(x, x_-)$-matrices [see Eq.(38)] is performed up to some point close to $x_-$, but excludes this point and its immediate vicinity. This statement is equivalent to what was referred to as dropping the last nucleus in Ref.4. Using these assumptions we can independently calculate the correlation of the two color densities in (44). Employing (39) and integrating over $d^2b'$ we arrive at

$$\langle Tr[S(b, b')T^aS^{-1}(b, b_-)S(b + x, b')T^aS^{-1}(b + x, b_-)] \rangle >.$$  

(44)

where $\mu$ is some infrared cutoff, and we have assumed that the normal nuclear density is uniform throughout the nucleus and in the boosted frame is given by $\rho_{rel}$. Rewriting (38) according to the definition of a path-ordered product we obtain

$$S(b, b_-) = \prod_i [1 + igT^a \hat{\rho}(y, y_-)\ell n(|b - y|/\mu)d^2y\Delta y_i - (1/2)g^2T^aT^b\hat{\rho}(y, y_-)\hat{\rho}(y', y'_-)\ell n(|b - y'|/\mu)\ell n(|b - y'|/\mu)
d^2y\Delta y_i - d^2y'\Delta y'_i],$$  

(46)

where the product goes along the $y_-\text{-axis}$, such that smaller $y_-$ corresponds to greater label $i$. For a combination of matrices like $S^{-1}(b, b_-)S(b + x, b')$ we start by considering the last (rightmost) term of the product given by (46) for $S^{-1}(b, b_-)$ and the first (leftmost) term in a similar product for $S(b + x, b')$. Both terms include the same interval of the $y_-$ integration, the closest to $b'_-$. Since all $S-$matrices in (45) are taken at the same longitudinal coordinate $b'_-$, then taking these terms for both pairs of $S-$matrices in (45) we observe that nothing else in (45) depends on the longitudinal coordinates in this interval. Therefore we can average these terms independently of the rest of the expression in (45), as well as do the transverse integrations. Taking only the two-density correlation terms (We throw away the higher order correlations since they have more powers of $g$, which goes beyond the classical approximation[1]) and making use of (39) we end up with

$$\langle Tr[S(b, b')T^aS^{-1}(b, b_-)S(b + x, b')T^aS^{-1}(b + x, b_-)] \rangle =$$

$$= \left[1 - g^2\pi\rho_{rel}\frac{N_c}{4(N_c^2 - 1)}xG(x, 1/x^2)\Delta y_- \right]$$

$$+ \langle Tr[S(b, b'_- - \Delta y_-)T^aS^{-1}(b, b'_- - \Delta y_-)S(b + x, b'_- - \Delta y_-)T^aS^{-1}(b + x, b'_- - \Delta y_-)] \rangle >,$$

(47)

where $\Delta y_-$ is the absolute value of the change in longitudinal coordinate. The transverse integration $d^2y$ was done assuming that the nucleus is infinite in the transverse direction, which is a reasonable approximation for a large nucleus. In arriving at (47) we have neglected QCD evolution in $xG$. Continuing this procedure of picking small intervals along the $y_-\text{-axis}$ and taking the limit of $\Delta y_- \rightarrow 0$ shows that the trace in (47) is equal to an exponential function

$$\langle Tr[S(b, b')T^aS^{-1}(b, b_-)S(b + x, b')T^aS^{-1}(b + x, b_-)] \rangle =$$

$$= C_F N_c \exp \left(-g^2\pi\rho_{rel}\frac{N_c}{4(N_c^2 - 1)}xG(x, 1/x^2)(b'_- + b'_0) \right),$$  

(48)

with $\pm b'_0$ the upper (lower) limit of the $y_-\text{-integration}$ in the $S-$matrices in (47). Plugging this back in (45), performing the $db'_-$ integration and defining the nuclear density in the center of mass frame $\rho = \rho_{rel}/\gamma\sqrt{2}$, with $\gamma$ the Lorentz contraction factor provides us with
\[
\hat{N}(x^2) = \frac{N_c^2 - 1}{\pi^2 \alpha N_c x^2} \int d^2 b \left[ 1 - \exp \left( -\frac{2\alpha \pi x N_c x^2 \sqrt{R^2 - b^2}}{N_c^2 - 1} \rho x G(x, 1/x^2) \right) \right] ,
\]
which is exactly formula (21b). Thus one indeed is able to calculate the produced gluon spectrum from the “classical” field given in (37) and (38) and this accounts for the final state interactions which are present in covariant gauge.

3 The produced gluon distribution in nucleon-nucleus collisions

In this section we shall calculate the spectrum of gluons produced in the scattering of a nucleon off a large nucleus. If the nucleus is right-moving and the nucleon left-moving then we suppose the centrally produced gluon momentum, \( \ell \), is such that \( \ell_- \) is much less than the minus component of the nucleon momentum and that \( \ell_+ \) is sufficiently less than the momentum of a nucleon in the nucleus so that the gluon is coherent over the longitudinal extent of the nucleus. We begin our discussion with gluon production in nucleon-nucleon collisions.

3.1 Soft gluon production in nucleon-nucleon collisions

We choose \( A_- = 0 \) gauge and a center of mass system to describe gluon production. If we choose the outgoing gluon polarization such that \( \eta = \epsilon = c_+ = 0 \) then we need only consider the graph shown in Fig.8 for the gluon production amplitude. The terms occurring at the 3-gluon vertex in that graph are

\[
V^{(\lambda)} = \frac{1}{k_- (\ell - k)_-} (\ell - k)^\perp \eta_\alpha \epsilon_\beta^{(\lambda)} [-g_\alpha \gamma (2k - \ell)_\beta + g_\beta \alpha (k + \ell)_\gamma - g_\gamma \beta (2\ell - k)_\alpha] \tag{50}
\]

where \( p_{1+} \) and \( p_{2-} \) are the large components of \( p_1 \) and \( p_2 \) respectively. It is straightforward to check that

\[
V^{(\lambda)} = \frac{2}{k_- k^2} [(\ell^2) (\ell - k) - (\ell^2 - k^2) \epsilon^{(\lambda)} \cdot \ell] \tag{51}
\]

and that

\[
\sum_{\lambda=1}^{2} V^{(\lambda)} V^{(\lambda)\dagger} = \frac{4k^2 (\ell - k)^2}{k^2 \ell^2} . \tag{52}
\]

There is an \( \bar{\eta}_\rho \) at the \( \rho \)-vertex in Fig.8 while there is a \( k^\perp_\sigma \) at the \( \sigma \)-vertex. Writing

\[
k^\perp_\sigma = k_\sigma - \eta_\rho k_- - \bar{\eta}_\rho k_+
\]

and using the fact that \( k_\sigma \) gives zero by current conservation while \( \bar{\eta}_\sigma \) gives a very small result one can replace \( k^\perp_\sigma \) by \(-\eta_\sigma k_- \). The \( k_- \) will cancel the \( k_- \) denominator in (51) (and in (52)) so that in each case the vertices \( \rho \) and \( \sigma \) are multiplied by the appropriate factors for the cross section to be given by the product of the unintegrated gluon distributions of nucleons \( p_1 \) and \( p_2 \). The result is\[14] - \[19].

\[
\frac{d\sigma_0}{d^2 k_\perp dy} = \frac{4\alpha N_c}{(N_c^2 - 1)k^2} \int d^2 k \frac{\partial x_1 G(x_1, k^2)}{\partial k^2} \frac{\partial x_2 G(x_2, (\ell - k)^2)}{\partial ((\ell - k)^2)} \tag{54}
\]

where \( x_1 x_2 s = \ell^2 \) and \( y \) is the rapidity of the produced gluon. Of course (54) is difficult to take literally. If \( \ell^2 \) is not large there is no hard scale and the unintegrated gluon distributions do not have much meaning. On the other hand if \( \ell^2 \) is large then the cross section is better expressed as the product of two gluon distributions times a hard scattering cross section. Nevertheless, (54) is useful as a normalization and for comparison with the equations we are now going to derive for gluon production in nucleon-nucleus collisions. It also follows in a BFKL approximation\[20, 21].
3.2 Central region gluon production in nucleon-nucleus collisions

It is not too difficult to calculate the cross section for producing a gluon of transverse momentum $\ell$ in the collision of a nucleon with a large nucleus. It is convenient to view the calculation in the rest system of the nucleon. We suppose the longitudinal momentum of the gluon is large enough so that the gluon is coherent over the length of the nucleus. We begin by considering gluon radiation when a quark scatters on a nucleus. It will then be a simple matter to extend the result to an incoming nucleon. It is convenient to view the calculation in the rest system of the nucleon. (Technically, elastic quark-nucleon scatterings and in elastic quark-nucleon scatterings, where the nucleon breaks up, cancel.) What is unusual in (56) is that we are attributing a contribution to the cross section to the disconnected graph where the quark freely passes through the nucleus. The Feynman diagram contribution to this process would be zero by energy conservation. However, (56) corresponds to $\tau_1$ and $\tau_2$ greater than zero so we do not yet have energy conservation. There are other terms in (57) and (58) which can be viewed as cancelling this “gluon radiation without interaction” term. Indeed these are the

\begin{align}
(a) & \quad \tau_1 > 0, \tau_2 > 0 \\
(b) & \quad \tau_1 < 0, \tau_2 > 0 \\
(c) & \quad \tau_1 > 0, \tau_2 < 0 \\
(d) & \quad \tau_1 < 0, \tau_2 < 0
\end{align}

Since the coherence time of the gluon is assumed to be much bigger than the radius of the nucleus we can assume that the quark, or the quark-gluon system, passes over the nucleus instantaneously compared to the magnitudes of the times of emission $\tau_1$ and $\tau_2$. Also once we separate contributions into definite times we are in effect dealing with light-cone perturbation theory rather than with Feynman graphs though in many graphs the orderings (55) simply tell whether the gluon is emitted before or after the quark, or quark-gluon system, interacts with the nucleus. The cross sections coming from the regions (55) are given by the expressions

\begin{align}
\frac{d\sigma^{(a)}}{d^2ldy} &= \frac{1}{\pi} \int d^2b d^2x_1 d^2x_2 \frac{1}{4\pi^2} \frac{\alpha C_F}{\pi} \frac{x_1 \cdot x_2}{x_1^2 x_2^2} e^{i(x_1 - x_2)} \\
\frac{d\sigma^{(b+c)}}{d^2ldy} &= \frac{-1}{\pi} \int d^2b d^2x_1 d^2x_2 \frac{1}{4\pi^2} \frac{\alpha C_F}{\pi} \frac{x_1 \cdot x_2}{x_1^2 x_2^2} \\
&\times \left( \exp \left[ \frac{x_1^2 \hat{v} \sqrt{R^2 - b^2}}{2\lambda} \right] + \exp \left[ -\frac{x_2^2 \hat{v} \sqrt{R^2 - b^2}}{2\lambda} \right] \right) e^{i(x_1 - x_2)} \\
\frac{d\sigma^{(d)}}{d^2ldy} &= \frac{1}{\pi} \int d^2b d^2x_1 d^2x_2 \frac{1}{4\pi^2} \frac{\alpha C_F}{\pi} \frac{x_1 \cdot x_2}{x_1^2 x_2^2} \exp \left[ -(x_1 - x_2)^2 \frac{\hat{v} \sqrt{R^2 - b^2}}{2\lambda} \right] e^{i(x_1 - x_2)}
\end{align}

In the above we take the transverse coordinate of the quark to be $0$ while $x_1$ and $x_2$ are the transverse coordinates of the gluon in the amplitude and complex conjugate amplitude respectively. We use a shorthand notation where $x^2 \hat{v}$ means $x^2 \hat{v}(\ell^2)$ with $\hat{v}, \lambda, R$ and $\ell$ as previously used in Sec.2. Let us now see how (56) to (58) come about.

In (56) the factor $\frac{1}{4\pi^2} \frac{\alpha C_F}{\pi} \frac{x_1 \cdot x_2}{x_1^2 x_2^2}$ is just the product of the coordinate space gluon emission amplitude times the complex conjugate amplitude. What may seem surprising in (56) is that there is no trace of the nucleus! But this is easy to understand. What passes through the nucleus is a high energy quark. The quark does not change its transverse coordinate nor does it lose a significant amount of energy as it passes through the nucleus. (Technically, elastic quark-nucleon scatterings and inelastic quark-nucleon scatterings, where the nucleon breaks up, cancel.) What is unusual in (56) is that we are attributing a contribution to the cross section to the disconnected graph where the quark freely passes through the nucleus. The Feynman diagram contribution to this process would be zero by energy conservation. However, (56) corresponds to $\tau_1$ and $\tau_2$ greater than zero so we do not yet have energy conservation. There are other terms in (57) and (58) which can be viewed as cancelling this “gluon radiation without interaction” term. Indeed these are the
$v$-independent terms of (57) and (58). However, the grouping of terms as given in (56)-(58) is convenient for our purposes.

Now turn to (58). The exponential term in (61) reflects multiple scattering of the gluon as it passes through a length $\Delta z = 2\sqrt{R^2 - b^2}$ of nuclear matter. This factor is identical to that given by (20) with $z - z_0$ taken to be $2\sqrt{R^2 - b^2}$. In this case it is the quark-gluon system which passes through the nucleus both in the amplitude and in the complex conjugate amplitude. Interactions of the quark with the nucleons in the nucleus cancel between real and virtual (production and elastic scattering) terms.

The expression (57) corresponds to a quark-gluon system passing through the nucleus in the amplitude and a quark passing through the nucleus in the complex conjugate amplitude along with a term where the amplitude and complex conjugate amplitude terms are exchanged. In Appendix A we outline how the exponential factors, involving $x_1^2$ and $x_2^2$, come about.

It is straightforward to evaluate (56)-(58) in the approximation of neglecting the $x$-dependence in $\tilde{v}$. One finds

$$\frac{d\sigma^{(a)}}{d^2\ell dy} = \int \frac{\alpha C_F}{\pi^2} \frac{1}{\ell^2} d^2b$$

$$\frac{d\sigma^{(b+c)}}{d^2\ell dy} = -2 \int \frac{\alpha C_F}{\pi^2} \left[1 - e^{-\frac{\ell^2}{<\ell^2>}}\right] d^2b$$

$$\frac{d\sigma^{(d)}}{d^2\ell dy} = \int d^2b \frac{\alpha C_F}{\pi^2} \frac{e^{-\frac{\ell^2}{<\ell^2>}}}{\ell^2} <\ell^2>$$

$$\{\ln\left[\frac{\ell^2}{4}ight] - \Gamma(0, -\frac{\ell^2}{<\ell^2>}) - \ell n\frac{-\ell^2}{<\ell^2>}\}$$

where

$$<\ell^2> = <\ell^2 (b)> = 2\sqrt{R^2 - b^2} \frac{\tilde{v}(<\ell^2>)}{\lambda}$$

$L$ is an infrared cutoff and $\Gamma(n, z)$ is the incomplete $\Gamma$-function. In arriving at (61) we have used

$$\int_0^{2\pi} d\phi(y) \frac{x + y}{(x + y)^2} = 2\pi \Theta(x - y) \frac{x}{x^2}$$

and

$$\int_0^{2\pi} d\phi(y) \frac{1}{|x + y|^2} = \frac{2\pi}{|x^2 - y^2|}$$

with $x$ and $y$ in (63) and (64) related to $x_1$ and $x_2$ in (58) by $x = x_1 - x_2, y = x_2$. The second plus third terms in the brackets on the right-hand side of (61) are non-logarithmic when $\ell^2 << <\ell^2 >$. When $\ell^2 >> <\ell^2>$ this term exactly cancels similar terms in (59) and (60). Thus in the logarithmic approximation we keep only the first term on the right-hand side of (61) and we understand that (59) holds only for $\ell^2 << <\ell^2>$.

We can now go to the case of an incident nucleon by making the replacements

$$\frac{\alpha C_F}{\pi} \frac{1}{\ell^2} \rightarrow \frac{\partial}{\partial \ell^2} xG(x, \ell^2).$$

$$\frac{\alpha C_F}{\pi} \ell n[<\ell^2 > L^2] \rightarrow xG(x, \ell^2).$$

(65a)

(65b)

In the logarithmic approximation the (b) + (c) contribution can be neglected and one finds

$$\frac{d\sigma}{d^2\ell dy} = \frac{1}{\pi} \int d^2b \left[ \frac{\partial}{\partial \ell^2} xG(x, \ell^2) + xG(x, <\ell^2>) \exp\left(-\frac{\ell^2}{<\ell^2>}\right) \right]$$

(66)
with the understanding that the first term on the right-hand side of (66) can only be used when \( \ell^2 \ll \ell^2 \).

We note that

\[
\frac{d\sigma}{dy} = \int \frac{d\sigma}{d^2 \ell dy} d^2 \ell = 2 \int d^2 b \ xG(\langle \ell^2 \rangle)
\]

(67)

There is a simple physical interpretation of (66) and (67). When an incident nucleon passes through a nucleus all the gluons in the nucleon’s wavefunction having \( \ell^2 < \langle \ell^2 \rangle \) are freed during the collision and the final state interactions broaden their distribution. This is the second term on the right-hand side of (66) and one half of (67). The nucleon remnants emerging from the nucleus have lost their gluon cloud. In rebuilding that cloud there is further gluon emission, but this gluon emission has no transverse momentum broadening. This is the first term on the right-hand side of (66) and the other half of (67).

### 3.3 Gluon production in nucleon-nucleus collisions without final state interactions

In this section we describe exactly the same process as in the previous section. However, now we choose \( A_+ = 0 \) gauge with \( \epsilon_- = 0 \). Our object is to show that, as for our deep inelastic scattering process of Sec.2, the gluon production cross section can be described completely in terms of the interaction of the “classical” fields of the incoming nucleon and of the incoming nucleus without any final state interactions.

It is relatively straightforward to derive a general formula for \( \frac{d\sigma}{d^2 \eta} \) in terms of classical fields associated with the colliding nucleon and nucleus. To see how this comes about we discuss in detail the scattering of a left-moving nucleon on two right-moving nucleons. We begin with the graphs shown in Fig.9 where we suppose that nucleons 1 and 2 are right-moving and that the \( x_- \) coordinate of nucleon 1 is less than that of nucleon 2, that is, nucleon 1 is ahead of nucleon 2. We assume these nucleons are separated by a longitudinal distance large compared to \( 1/\lambda \) in their common rest system. Then the interaction in Fig.9a must be a gauge term with the effective part of the \( k \)-propagator being \( \frac{i\eta}{\ell-\epsilon} \). The phase factor \( e^{ik_+(x_2-x_1)} \) dictates that one distort the contour in the upper-half plane with the propagator becoming \( \frac{2\pi i}{\ell} \delta(k_+)k_\perp \). Similarly for the graph in Fig.9b one can again distort the propagator in the upper half \( k_+ \)-plane getting an identical result for the \( k_- \) propagator. One now can use the STW identities to arrive at a contribution which is illustrated in Fig.10 where the dotted line brings color to the \( (\ell-k-r) \) line as well as transverse momentum, but it brings no longitudinal momentum and no momentum dependence at its vertex. This can be interpreted as a color rotation and transverse momentum convolution of the field of nucleon 1 by the gauge field from nucleon 2 followed by a scattering off the left-moving nucleon.

Now consider the graphs shown in Fig.11. We again distort the \( k_+ \)-contour of integration into the upper half plane. In Fig.11a there is also the light-cone denominators from the \( (k+r) \)-line which must be considered. The term having a singularity in the upper-half plane is proportional to \( \frac{\delta_{\ell-r}}{k_+^2} \). It is straightforward to check, using the STW identities, that this term gives zero because of our choice \( \tilde{\eta} \cdot \epsilon = 0 \). Thus in graph (a) there is only the \( \frac{k_\perp^2}{k_+^2} \) singularity of the \( k_- \)-line which is evaluated by the STW identities.

The graph shown in Fig.11b has two contributions, one being the \( \frac{k_\perp}{k_+^2} \) term from the \( k_- \) propagator and the second being a singularity in the nucleon-gluon scattering process \( (P) + (-r-k) \to (P-r) + (-k) \). This second term corresponds to a single gluon exchange final state interaction of the left-moving nucleon remnants with nucleon 2 and it cancels with corresponding two-gluon exchange terms in the amplitude and complex conjugate amplitude. The result of the terms in Fig.11 is illustrated in Fig.12 and can be interpreted as a color rotation, and convolution, of the field from the left-moving nucleon by nucleon 2 followed by a scattering off nucleon 1.

Now we consider the graphs shown in Fig.13. We shall evaluate these graphs by distorting the \( k_+ \)-contour into the lower half plane. Since we now distort in a direction opposite to that natural for the factor \( e^{ik_+(x_2-x_1)} \) it is important to check that there is sufficient convergence in the \( k_+ \)-plane so that the exponential factor is not relevant. For example, in graph (a) one has \( k_+ \) denominators \( [k_+ - i\epsilon]^{-1}, [(k+r-\ell)_+ + i\epsilon]^{-1} \) and \( [(k-\ell)_+ + \frac{\ell-\ell_1}{2\ell} + i\epsilon]^{-1} \) multiplying \( k_\perp^2 \) and \( (k+r-\ell)_p \) as well as a possible light-cone denominator from the \( (\ell-k)_+ \)-line. (Note, however, that the \( \frac{\omega_{\ell-k}}{k_+^2} \) term in the \( (\ell-k)_+ \)-propagator gives a
small contribution because of our choice of polarization, $\bar{\eta}_p \epsilon = 0$.) Thus there is sufficiently strong convergence for the $k_+-$integral unless (possibly) there is a factor of $k_+$ coming from each of the 3-gluon vertices. At the $\Gamma_{\mu\alpha\gamma}$ vertex the only possibility leads to a factor of $\bar{\eta}_\mu$. (Factors of $\bar{\eta}_a$ or $\bar{\eta}_b$ give zero because they multiply $k_+^a$ or $\epsilon_a$ respectively.) At the $\Gamma_{\lambda\mu\nu}$ vertex a factor of $\bar{\eta}_b$ leads to an extra denominator $[(k-\ell)_+^{-1}]$ as does a factor of $\bar{\eta}_\lambda$ since $(k+r-\ell)_+^\rho \bar{\eta}_\mu \bar{\eta}_b = 0$. Thus there is sufficient convergence to distort the $k_+-$ contour in the lower half plane where one picks up only the pole $[(k+r-\ell)_+^{-1} + i\ell]^{-1}$. After picking up this pole one writes $(k+r-\ell)_+^\rho = (k+r-\ell)_-^\rho - \bar{\eta}_\rho (k+r-\ell)_+^\rho$ and using $(k+r-\ell)_+^\rho = 0$ and $(k+r-\ell)_-^\rho \approx 0$ one again can use the STW identities as was done for the graphs of Fig.11. Here one obtains a result illustrated in Fig.14 along with a rescattering of the remnant of the left-moving nucleon which will cancel as discussed above. Hence, we may view the graphs of Fig.13 as a color rotation, and transverse momentum convolution of the field from the left-moving nucleon by nucleon 1 followed by a scattering off nucleon 2.

Finally, there is the graph shown in Fig.15. Here we distort the contour in the upper half plane to pick up the pole at $k_+ = 0$ with the factor $k_+$ acting at the 4-gluon vertex after the $k_+-$integration has been done.

In order to see that the graphs of Figs.9,11,13 and 15 give the same result as our previous calculation summarized in (58) it is convenient to evaluate the graphs of Fig.13 in a different way. If we evaluate these graphs by distorting the $k_+-$ contour into the upper half plane poles at $k_+ = 0$ and at $(\ell - k)_+ = (\ell^2 - k_+^2)^{1/2}$ are encountered. (There is also the term $\frac{\eta_a (k-\ell)_+^\rho}{(k-\ell)_+^{-1}}$ coming from the $(\ell - k)$-line propagator which gives an additional singularity, however this term leads to a small result since the $(k-\ell)_\nu$ term either cancels between the $a$ and $b$ parts of Fig.13 by STW or the $(k-\ell)_\nu$ eliminates the $(\ell - k - r)_-$ propagator leaving no singularities whatever in the lower half $k_+$ plane.) When the singularity at $k_+ = 0$ is taken one has a resulting $k_+$ which when combined with the results given by the graphs in Figs.10, 12 and 15 gives zero from the STW identities. The singularity at $(\ell - k)_+ = (\ell^2 - k_+)^{1/2}$ gives on-shell propagation of the $(\ell - k)_-$ line with a rescattering off nucleon 2 after production by nucleon 1. Thus we recover the result of our calculation in Sec.3.2 when that calculation is restricted to the interaction of just two nucleons in the nucleus.

However, one can also describe the scattering in terms of the “classical” $A_+ = 0$ light-cone gauge fields of the left-moving nucleon and of the 2 right-moving nucleons. In this case the left-moving nucleon has a classical field color rotated by nucleons 1 and 2 as indicated in Figs.12 and 14. The right-moving nucleons have their Weizsäcker-Williams fields where the field of nucleon 1 is rotated by that of nucleon 2. In every case the nucleons with larger values of $x_-$ rotate the fields coming from smaller values of $x_-$ according to (37) and (38). The gluon production amplitude is then given by the three and four gluon interactions of these incoming “classical” fields as described by the graphs in Figs.10, 12, 14 and 15. In Appendix B we shall outline the technical argument for this result when an arbitrary number of nucleons in the nucleus are involved in the interaction.

We now put into formulas what we described above for gluon production in a nucleon- nucleon collision calculated in $A_+ = 0$ gauge, and where the nucleon is left-moving and the nuclear is right-moving. We denote by $A'_\mu(\underline{k}, k_+, \underline{b})$ the field of the nucleus given by (37-40) and (42). The “classical” field of the nucleon is denoted by

$$A'_\mu(\underline{k}, k_-, \underline{b}) = A'(\underline{k}, k_-, \underline{b}) \bar{\eta}_\mu$$

with

$$A'(\underline{x}, x_+, \underline{b}) = -S(\underline{x}) T^a S^{-1}(\underline{x}) \delta(x_+) \delta \bar{n}[|\underline{x} - \underline{b}|] \rho^a_N$$

where $S(\underline{x})$ is the same as in (38) but with $x_- = -\infty$, and where

$$< \bar{\rho}_N^a \rho^a_N > = \frac{\delta_{a\alpha'}}{N_c^2 - 1} Q^2 \frac{\partial}{\partial Q^2} x G(x, Q^2)$$

(70)

indicates an average over internal nucleon structure. We have used current conservation at the nucleon source to eliminate the light-cone gauge denominator in $A'_\mu$ much as was done in the discussion following (53) and which led to (54), but now with the roles of $+$ components and $-$ components exchanged. In order to write a compact formula for $\frac{d^2}{dx^2 dy}$ it is convenient to define a total gluon field $A'_\mu(x)$ as
\[ A_{\mu}^{\text{tot}}(x) = A_{\mu}^\perp(x) + A_{\mu}^\prime(x, b) + A_{\mu}^{\text{free}}(x) \]  

(71)

where \( A^\perp \) and \( A' \) are the fields given in (37) and (68), respectively, while \( A_{\mu}^{\text{free}}(x) \) is a free quantized gluon field normalized according to

\[
(0|A_{\mu}^{\text{free}}(x)|\ell\lambda a) = \frac{\epsilon_{\mu}^{(\lambda)}(\ell)e^{i\ell \cdot x}}{\sqrt{(2\pi)^3 2\omega_\ell}} T^a
\]

(72)

with \(|\ell\lambda a\rangle\) being a single gluon state having momentum \( \ell \), polarization \( \lambda \) and color \( a \). \( A^{\text{tot}} \) depends on the impact parameter of the collision through \( A' \). Then the gluon production cross section is

\[
\frac{d\sigma}{d^2\ell dy} = \omega_t \int d^2 b < \sum_{\lambda=1}^{N^2-1} \sum_{a=1}^{N^2} (0|S|\ell\lambda a)(\ell\lambda a|S| 0) >
\]

(73)

with

\[
S = -\frac{1}{2} \int d^4 x \ Tr F_{\mu\nu}(x) F_{\mu\nu}(x)
\]

(74)

and where

\[
F_{\mu\nu} = \partial_\mu A_{\nu}^{\text{tot}} - \partial_\nu A_{\mu}^{\text{tot}} - ig[A_{\mu}^{\text{tot}}, A_{\nu}^{\text{tot}}].
\]

(75)

The \(<>\) in (73) indicate an average over nuclear structure as indicated in (39) as well as an average over nucleon structure as given by (70). In evaluating (73) one assumes that the field \( A_{\mu}^{\text{free}} \) is much smaller than \( A' \) and \( A^\perp \). Therefore \( A_{\mu}^{\text{free}} \) is included only once in the 3 and 4 gluon vertices in \( S \). At the same time each of the other two fields has to be included at least once to allow the production of the gluon. Eq.73 is a compact, and elegant, formula for gluon production in nucleon-nucleus collisions.
Appendix A

In this appendix we shall show how the exponential factors in (57) come about. We consider the case where $\tau_1 < 0$ and $\tau_2 > 0$. We consider the last, the latest in time, interaction with a nucleon in the nucleus. In the amplitude the interaction is with a quark-gluon system with the quark at transverse coordinate 0 and the gluon at $x_1$. In the complex conjugate amplitude the interaction is only with the quark, again at transverse coordinate 0. The possible interaction terms, both real and virtual, are shown in Fig.16. The factors associated with each graph are listed below as

\begin{align}
(a) &= -C_F/2 \tilde{V}(0)/N_c \\
(b) &= N_c/2 \tilde{V}(x_1)/N_c \\
(c) &= -1/2N_c \tilde{V}(0)/N_c \\
(d) &= -N_c/2 \tilde{V}(0)/N_c \\
(e) &= N_c/2 \tilde{V}(x_1)/N_c \\
(f) &= -C_F/2 \tilde{V}(0)/N_c
\end{align}

(A.1)

where $\tilde{V}$ is given in (17). Adding all the terms one finds

\[(a) + (b) \cdots + (f) = -[\tilde{V}(0) - \tilde{V}(x_1)] = -\frac{1}{4} x_1^2 \tilde{N}(x_1).\]

(A.2)

Multiplying the right-hand side of (A.2) by $\rho \sigma = 1/\lambda$ to take account of the density of nucleons and the cross section we obtain the same type of factor as appears on the right-hand side of (14). Our convention is to use $\sigma$ and $\lambda$ for gluon scattering on nucleons. The factor of $1/N_c$ on the right-hand side of the terms in (A.1) converts the result from gluons to the appropriate partons involved in the interaction.

Appendix B

In this appendix we outline an argument for the absence of final state interactions in gluon production in nucleon-nucleus collisions. The major barrier in generalizing the discussion given in Sec.3.3, for the absence of final state interactions when a nucleon scatters on two nucleons, is the presence of light-cone gauge denominators of the type $\frac{1}{(k-\ell)_+ + \nu}$, along the gluon line carrying the large minus-component of the momentum, which hinder the distortion of $k_-$ contours into the lower half plane. In Sec.3.3, for the graphs in Fig.13 this term was eliminated by our choice of polarization vector, $\bar{\eta} \cdot \epsilon = 0$ for the produced gluon. In the general case the troublesome graphs are of the type shown in Fig.17. The graph in Fig.17a is such that cutting the $(\ell - k)$-line separates the overall graph into two parts while the graph in Fig.17b is an example of a graph where cutting the $(\ell - k)$-line does not separate the graph into two parts. There is not any particular ordering of the $x_-$ positions of the nucleons belonging to $A$ and $B$. That is, some nucleons in $A$ may have larger $x_-$ values than some of the nucleons in $B$. We begin with the graphs of the type shown in Fig.17a. For our purposes the dangerous term is the $\frac{n_c(k-\ell)_-}{(k-\ell)_+ + \nu}$ part of the $(\ell - k)$- propagator. However, once all hookings of the $(\ell - k)$-line in $B$ are included, at a given order of the coupling and with a given set of nucleons, the net result must be zero from the STW identities.

Now turn to graphs of the type shown in Fig.17b where one or more lines, in addition to the $(\ell - k)$-line connect $A$ and $B$. A particular example of such a graph is shown in Fig.18, where nucleon 1 and nucleon 3 have inelastic reactions while nucleon 2 has an elastic reaction. The potentially dangerous problem with graphs of this type is that the application of the STW identities does not directly give zero since the $k_1$ line carries color. Indeed, applying the STW identities for the $\frac{n_c(k-\ell)_-}{(k-\ell)_+ + \nu}$ term in the $(\ell - k)$- propagator leads to a term like that indicated in Fig.19 where the dashed line carries the propagator factors $\frac{1}{(k-\ell)_+ + \nu}$ and $\frac{1}{(k-\ell)_+ + \nu}$.
and inserts a momentum $\ell - k$ on the $k_1$-line as well as rotating the color factors of the $k_1$-line. However, this term is small because the scattering with nucleon 2 is the same as the high energy scattering gluon $(\ell - k - k_1)$+ nucleon $\rightarrow$ gluon $(\ell - k_2)$+ nucleon and the high energy limit of such a scattering is zero in our approximation of no evolution within nucleon 2.

Thus, the gluon line carrying the large minus component of the momentum $\ell$, the hard gluon line, will never give singularities as one distorts $k_+$-momenta into the lower half plane. This is all that is needed to arrive at our main result, described in Sec.3.3 and summarized in (73)-(75). To see this, begin with the nucleon having the largest values of $x_-$. A gluon, $g$, from this nucleon either attaches directly to the hard gluon line or to a gluon coming from another nucleon or, perhaps, directly to another nucleon. If the gluon, $g$, does not attach directly to the hard gluon one is in the case considered in Ref.11 where it was shown that $g$ rotates the fields coming from nucleons with smaller values of $x_-$. If the gluon, $g$, attaches to the left-moving nucleon, or to the hard gluon before the hard gluon has any previous interactions, one distorts the $k_+$ of $g$ into the upper half plane and uses the STW identities. If $g$ attaches to the hard gluon after the hard gluon has interaction with other nucleons one routes the momentum of $g$ through the hard gluon and back into a nucleon at the first available interaction, as illustrated for example in Fig.18. In this case one distorts the $k_+$-contour in the lower half plane and uses the STW identities not for $g$ but for the gluons carrying the momentum $k$ returning from the hard gluon. This set of operators leads to the fields $A^\perp_\mu$ and $A'_\mu$ in (71) and a cross section given by (73).

References

[1] L.McLerran and R. Venugopalan, Phys.Rev.D49 (1994) 2233; 49(1994) 3352; 50 (1994) 2225.

[2] J.Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, hep-ph/9701284 (1997), [hep-ph/9706377](1997).

[3] J.Jalilian-Marian, A. Kovner and H. Weigert, [hep-ph/9709432](1997).

[4] Yu. V. Kovchegov, Phys. Rev.D54 (1996) 5463.

[5] J.Jalilian-Marian, A. Kovner, L. McLerran and H. Weigert, Phys.Rev.D55 (1997) 5414.

[6] A.H.Mueller, Nucl.Phys.B335 (1990)115.

[7] G.T. Bodwin, S.J. Brodsky and G.P. Lepage, Phys.Rev.Lett.47 (1981) 1799.

[8] R.J. Glauber in Lectures in Theoretical Physics, ed. W.E. Brittin and L.G. Dunham (Interscience, New York, 1959) Vol.1.

[9] V.N. Gribov, Sov.J. Nucl.Phys.9 (1969) 369.

[10] M. Luo, J. Qiu and G. Sterman, Phys. Rev.D49 (1994) 4493.

[11] Yu.V. Kovchegov, Phys.Rev.D55 (1997)5445.

[12] R. Baier, Yu.L. Dokshitzer, A.H. Mueller, S. Peigné and D. Schiff, Nucl.Phys.B484 (1997) 265.

[13] L.L. Frankfurt and M.I. Strikman, Phys.Reports 76 (1981) 215.

[14] E.M. Levin and M.G. Ryskin, Phys.Rep.189 (1990) 267.

[15] E.Laenen and E. Levin, Ann.Rev.Nuc.Part.Sci.44 (1994) 199.

[16] J.F. Gunion and G. Bertsch, Phys.Rev.D25 (1982) 746.

[17] Yu.V. Kovchegov and D. Rischke, Phys.Rev.C56 (1997) 1084.

[18] M. Gyulassy and L.McLerran, Phys.Rev.C56 (1997) 2219.
[19] X. Guo (private communication).

[20] V. Del Duca, M.E. Peskin and W.-K. Tang, Phys.Lett.B306 (1993) 151.

[21] K.J. Eskola, A.V. Leonidov, and P.V. Ruuskanen, Nucl.Phys.B481 (1996) 704.
FIG. 2

FIG. 3
FIG. 4
FIG. 9

(a)

FIG. 10

(b)
FIG. 13

FIG. 14

26
FIG. 15
