Target Tracking Algorithm of Automotive Radar Based on Iterated Square-root CKF

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Abstract. An iterated square-root cubature Kalman filter (ISRCKF) algorithm for target tracking of automotive radar is proposed in this paper, which inherits the fast and robust advantages of SRCKF, combines with Gauss-Newton iterative theory and design a algorithm to iterate update the measurement process. The filtering accuracy of target tracking algorithm of automotive radar can be further improved as the newest measurement is fully utilized. Monte-Carlo simulation experiments were carried out aim at the automotive radar target tracking problem, in which used the algorithm to compare with classical algorithms such as square-root unscented Kalman filter (SRUKF) and SRCKF. The experimental results shows that the overall filtering accuracy of this algorithm is much improved compare with other classical filtering algorithms, and the filtering accuracy can be improved with the increase of iteration number.

1. Introduction

Advanced driver assistance system (ADAS) is getting more and more attention in recent years[1], which is based on radar sensor technology. Accurate environmental perception is a key issue in the ADAS system, its core is to accurately detect, track and identify barriers. Measurements obtained by automotive radar need to be non-linear transformed in ADAS, therefore, the filter in target tracking should be non-linear.

The most classical method to solve the non-linear filtering problem is the extend Kalman filter (EKF) [2-3], the EKF uses a first-order Taylor series expansion of the nonlinear function and ignores higher-order terms, then is processed by standard Kalman filtering. The EKF can lead to large errors and even become unstable when the system is highly non-linear. Unscented Kalman Filtering (UKF) is also a popular non-linear filter [4-5], the UKF obtain the statistical characteristics of random variables by sampling points with different weights (Sigma points) through non-linear functions and performing unscented transformation (UT), the filtering accuracy can reach the second order with the same performance in the EKF, and the computational efficiency is higher since it don’t need to calculate the jacobian matrix. However, the filtering performance of the UKF may be poor or even divergent when the state dimension is high. Arasaratnam and Haykin proposed the cubature Kalman Filtering (CKF) [6], the CKF uses the cubature rules for those multi-dimensional integrals to approximate the recursive Bayesian estimation integrals under the Gaussian assumption. Compared with UKF, CKF is simpler to implement, has higher computational efficiency, higher filtering accuracy and numerical stability [7].
The square-root cubature Kalman filter (SRCKF) algorithm [8] is based on the CKF framework, propagates the square-root factors of the predictive and posterior error covariance to avoid matrix square-rooting operations and to preserve the positive semi-definiteness of the error covariance, which effectively improves the accuracy and stability of the filtering.

In the target tracking of automotive radar, the process is affected by all kinds of noise and non-linear transformation, which requires fastness, robustness and accuracy of the filter. Based on the above research, an iterative square-root cubature Kalman filter (ISRCKF) algorithm for target tracking of automotive radar is proposed, which draws the advantages of SRCKF algorithm and iteratively optimizes the process of SRCKF measurement and updating by Gauss-Newton iterative method.

2. Description of system model

In ADAS system, the state of target can be defined as a 6-dimensional vector in the \( x - y \) Cartesian coordinate system: \( X_k = [x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}]^T \), the components of the state vector respectively indicate the target’s distance, velocity and acceleration on the horizontal and vertical axes at time instant \( k \). The measurement by a single period of radar is \( z_k = (r, \phi, v)^T \), meaning distance \( r \), azimuth \( \phi \) and radial velocity \( v \) of the target, respectively. The kinematics model and measurement model in target tracking of automotive radar are shown in equation (1):

\[
\begin{align*}
X_k &= F \cdot X_{k-1} + w_{k-1} \\
z_k &= h(X_k) + v_k
\end{align*}
\] (1)

Where \( k \) is the discrete time; \( X_k \) is the state vector of the system at time instant \( k \); \( z_k \) is the measurement; \( F \) and \( h(\cdot) \) are the state equation and measurement function of the system; \( w_{k-1} \) and \( v_k \) denote the process and measurement Gaussian white noise, which with zero means and covariance \( Q_{k-1} \) and \( R_k \), respectively, \( \{w_{k-1}\} \) and \( \{v_k\} \) are mutually uncorrelated. The motion of target can be assumed to be a normally accelerating motion. Therefore, the state equation \( F \) of the system model is based on the normal acceleration (CA) model [9-10], and the time interval \( dt \) between two measurements of the radar as the equation of state parameters, the \( F \) can be expressed as:

\[
F = \begin{bmatrix}
1 & dt & \frac{1}{2} dt^2 & 0 & 0 & 0 \\
0 & 1 & dt & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & dt & \frac{1}{2} dt^2 \\
0 & 0 & 0 & 0 & 1 & dt \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\] (2)

The process noise covariance matrix \( Q \) is initialized to \( Q = \text{diag}[\sigma_x, \sigma_y, \sigma_x, \sigma_y, \sigma_y, \sigma_y] \)

The measurement function \( h(\cdot) \) completes the transformation of the state vector \( X_k \) to the measurement vector \( z_k \), as defined below:

\[
h(X_k) = \begin{bmatrix}
\sqrt{x_k^2 + y_k^2} \\
\frac{\arctan(y_k/x_k)}{y_k/x_k} \\
(x_k \dot{x}_k + y_k \dot{y}_k)/\sqrt{x_k^2 + y_k^2}
\end{bmatrix}
\] (3)

The measurement noise covariance matrix \( R \) is initialized to \( R = \text{diag}[\sigma_r, \sigma_r, \sigma_r] \).

3. Theory of iterative square-root CKF

In a nonlinear discrete dynamic systems described in equation(1), \( \tilde{x}_k \sim N(x_k, P_k) \), \( z_k \sim N(h(x_k), R_k) \); From the Bayesian perspective we have \( P(x_k|z_k) = \eta P(z_k|x_k)P(x_k) \), where \( \eta = P_{z_k|x_k}^{-1} \) amounts to a normalizing factor, \( P(x_k) \) is the prior, and \( P(z_k|x_k) \) is the likelihood:
Defining the likelihood surface function (LSF) as

\[ P(x_k) = \frac{1}{\sqrt{(2\pi)^n|\mathbf{P}_k|}} \exp \left( -\frac{1}{2} (x_k - \hat{x}_k)^T \mathbf{P}_k^{-1} (x_k - \hat{x}_k) \right) \]

(4)

\[ P(z_k|x_k) = \frac{1}{\sqrt{(2\pi)^n|\mathbf{R}_k|}} \exp \left( -\frac{1}{2} (z_k - h(x_k))^T \mathbf{R}_k^{-1} (z_k - h(x_k)) \right) \]

(5)

Defining the likelihood surface function (LSF) as

\[ J(x_k) = \frac{1}{2} \left[ (x_k - \hat{x}_k)^T \mathbf{P}_k^{-1} (x_k - \hat{x}_k) + (z_k - h(x_k))^T \mathbf{R}_k^{-1} (z_k - h(x_k)) \right] \]

(6)

Obtaining the maximum likelihood estimate is equivalent to solving the minimum value of \( J(x_k) \).

Gauss-Newton nonlinear iterative method is introduced to linearize the measurement equation to obtain the updated measurement iteration equation of \( J(x_k) \):\[ x_k^{(i+1)} = \hat{x}_k + \hat{P}_k (H_k^{(i)})^T (\mathbf{H}_k^{(i)} \hat{P}_k (H_k^{(i)})^T + \mathbf{R}_k)^{-1} [z_k - h(\hat{x}_k^{(i)}) - H_k^{(i)} (\hat{x}_k^{(i)} - \hat{x}_k^{(i)})] \]

(7)

Where \( \hat{x}_k^{(i)} \) is the state estimation at the \( i \)-th iteration, \( H_k^{(i)} \) is the jacobian matrix of \( h(\hat{x}_k^{(i)}) \), we consider the iterative filter gain \( K_k^{(i)} = \hat{P}_k (H_k^{(i)})^T (\mathbf{H}_k^{(i)} \hat{P}_k (H_k^{(i)})^T + \mathbf{R}_k)^{-1} \); the terms \( (H_k^{(i)} \hat{P}_k (H_k^{(i)})^T + \mathbf{R}_k) \) and \( \hat{P}_k (H_k^{(i)})^T \) are innovation covariance and cross covariance, they are expressed as \( P_{zz}^{(i)} = H_k^{(i)} \hat{P}_k (H_k^{(i)})^T + \mathbf{R}_k \), \( P_{xz}^{(i)} = \hat{P}_k (H_k^{(i)})^T \); \( P_{zz}^{(i)} \) and \( P_{xz}^{(i)} \) are obtained by linearizing the measurement function \( h(\cdot) \) with a first-order taylor expansion the linearized error is introduced due to high-order terms being truncated. To decrease the propagated linearized error, we recalculate innovation covariance and cross covariance based on the SRCKF in the following way:

\[ Z_{j,k}^{(i)} = h(\chi_{j,k}^{(i)}) \]

(8)

\[ \hat{Z}_k^{(i)} = \sum_{j=1}^{m} w_j Z_{j,k}^{(i)} \]

(9)

\[ \mathbf{X}_k^{(i)} = [\chi_{1,k}^{(i)} - \hat{x}_k^{(i)} , \chi_{2,k}^{(i)} - \hat{x}_k^{(i)} , ..., \chi_{m,k}^{(i)} - \hat{x}_k^{(i)}] / \sqrt{m} \]

(10)

\[ \eta_k^{(i)} = [\mathbf{Z}_k^{(i)} - \mathbf{Z}_k^{(i)} , \mathbf{Z}_k^{(i)} - \hat{Z}_k^{(i)} , ..., \mathbf{Z}_k^{(i)} - \hat{Z}_k^{(i)}] / \sqrt{m} \]

(11)

\[ S_{zz,k}^{(i)} = q \rho \left( \eta_k^{(i)} , \sqrt{\mathbf{R}_k} \right) \]

(12)

\[ P_{xz,k}^{(i)} = \chi_{k}^{(i)} \eta_k^{(i)} \]

(13)

Where \( \chi_{j,k}^{(i)} \) in equation (8) is the cubature points, \( m = 2n \), \( w_j = 1/m \) is the weight of the cubature points[6]; Derived from the above, the state estimation in equation(7) can be derived as follows:

\[ x_k^{(i+1)} = \hat{x}_k + P_{xz,k}^{(i)} (S_{zz,k}^{(i)} S_{zz,k}^{(i)})^{-1} [z_k - h(\hat{x}_k^{(i)}) - (P_{xz,k}^{(i)})^T \hat{P}_k^{-1} (\hat{x}_k - \hat{x}_k^{(i)})] \]

(14)

4. The algorithm of iterative square-root CKF

Combined with the Gauss-Newton iterative method, the improved innovation covariance and cross covariance algorithm, the specific process of ISRCKF algorithm is as follows:

**Step 1:** Using the three-order cubature rule to evaluate the cubature points and weights:

\[ \xi_j = \sqrt{(m/2)[1]} , w_j = 1/m, j = 1, ..., m \]

(15)

Where \( n \) is the dimension of state vector, we denote the \( n \)-dimensional unit vector as \( e = [1,0,...,0]^T \), use the [1] to represent a complete fully symmetric set of points that can be obtained by permutating and changing the sign of the generator \( e \) in all possible ways. The [1] \( j \) is denoted the \( j \)-th point from the set [1].

**Step 2:** Time update;
Assuming the posterior density function \( p(x_{k-1}|z_{k-1}) = N(x_{k-1}; \hat{x}_{k-1}, P_{k-1}) \) at \( k - 1 \) instant time is known, where \( P_{k-1} = (S_{k-1})^T(S_{k-1}) \)

1) Calculate the cubature points:
\[ X_{j,k-1} = S_{k-1} \xi_j + \hat{x}_{k-1} \]  
(16)

2) Propagate the cubature points through the state equation:
\[ X'_{j,k} = f(X_{j,k-1}) \]
(17)

3) Evaluate the predicted state and square-root of the predicted covariance
\[ \hat{x}'_k = \sum_{j=1}^{m} w_j X'_{j,k} \]
\[ \chi'_k = [X'_{1,k} - \hat{x}'_k, X'_{2,k} - \hat{x}'_k, ..., X'_{m,k} - \hat{x}'_k] / \sqrt{m} \]
\[ \hat{S}_k = q \Gamma \{ [\chi'_k, \chi'_k] \} \] 
(18)

(19)
\[ \chi'_k = [X'_{1,k} - \hat{x}'_k, X'_{2,k} - \hat{x}'_k, ..., X'_{m,k} - \hat{x}'_k] / \sqrt{m} \]
\[ \hat{S}_k = q \Gamma \{ [\chi'_k, \chi'_k] \} \] 
(20)

**Step 3: Measurement update:**

Measurement update is an iterative process with \( \hat{x}_k \) and \( \hat{S}_k \) as the initial value, the initial covariance matrix \( P_k = (S_k)^T(S_k) \). The estimation of state and square-root of covariance obtained by i-th iteration are \( \hat{x}_k^{(i)} \) and \( \hat{S}_k^{(i)} \), \( \hat{x}_k^{(0)} = \hat{x}_k \), \( \hat{S}_k^{(0)} = \hat{S}_k \), for \( i = 1,2,...,N_{\text{iter}} \):

1) Calculate the updated cubature points
\[ X'_{j,k} = \hat{x}_k^{(i-1)} \xi_j + \hat{x}_k^{(i-1)} \]  
(21)

2) Use equations (8-13) to evaluate the square-root of innovation covariance \( S_{zz,k}^{(i-1)} \) and cross-covariance \( P_{xz,k}^{(i-1)} \)

3) Evaluate the iterative filter gain \( K_k^{(i-1)} = P_{xz,k}^{(i-1)} (S_{zz,k}^{(i-1)})^T S_{zz,k}^{(i-1)} \)^{-1}

4) Evaluate the state estimation and the square-root of the covariance
\[ Z_k^{(i-1)} = h(\hat{x}_k^{(i-1)}) \]
\[ \hat{x}_k^{(i)} = \hat{x}_k^{(i)} + K_k^{(i-1)} [z_k - Z_k^{(i-1)} - (P_{xz,k}^{(i-1)})^T \hat{P}_k^{-1} (\hat{x}_k^{(i-1)} - \hat{x}_k^{(i-1)})] \]
\[ \hat{S}_k^{(i)} = q \Gamma \{ [\chi_k^{(i-1)} - K_k^{(i-1)} \eta_k^{(i-1)}, K_k^{(i-1)} \sqrt{R_k}] \} \]  
(22)

(23)
\[ \eta_k^{(i-1)} = [\chi_k^{(i-1)} - K_k^{(i-1)} \eta_k^{(i-1)}, K_k^{(i-1)} \sqrt{R_k}] \]  
(24)

5) Termination condition of iteration: \( \| \hat{x}_k^{(i)} - \hat{x}_k^{(i-1)} \| \leq \varepsilon \) or \( i = N_{\text{iter}} \)
 Where \( \varepsilon \) and \( N_{\text{iter}} \) is the predetermined threshold and the maximum number of iterations. The iteration number is assumed to be \( N \) when the iteration is terminated; The ultimate estimation of state and square-root of covariance at the \( k \) time instant are \( \hat{x}_k = \hat{x}_k^{(N)} \), \( \hat{S}_k = \hat{S}_k^{(N)} \).

5. **Simulation**
Totally two experiments of 50 Monte-Carlo simulations were carried out in the simulation experiment. The first experiment compares the tracking filtering accuracy of SRUKF [12], SRCKF and ISRCKF. The second compares the iteration number \( N_{\text{iter}} \) of ISRCKF to the filtering accuracy influences. The same Gaussian white noise is added into each experiment, the mean square-root error of the distances, azimuths and velocities are 2.0m, 1.0° and 0.2 m/s. The data update rate of radar is 20 Hz and the target is tracked for 30s. The initial state of the target is: \( [10, 0.5, -0.1, 10, 2, -0.01]^T \).

In experiment 1, the iteration number \( N_{\text{iter}} \) of ISRCKF is set to 2. Figure 1 and 2 respectively shows the MSRE curves of radial distance and azimuth of 3 filtering algorithms obtained from 50 Monte-Carlo simulations. The filtering accuracy of each algorithm is quantitatively analyzed, table 1
shows the AMSRE of $[x, \hat{x}, \ddot{x}, y, \hat{y}, \ddot{y}]$ and $[r, a, v]$. From table 1 we can see that the error of ISRCKF in every component are much lower than that of SRUKF and SRCKF.

![Figure 1. MSRE curves of radial distance for various filters.](image1)

![Figure 2. MSRE curves of azimuth for various filters.](image2)

**Table 1.** Accumulated mean square-root error (AMSRE) of various algorithms.

| AMSRE            | $[x, \hat{x}, \ddot{x}]$ | $[y, \hat{y}, \ddot{y}]$ | $[r, a, v]$ |
|------------------|--------------------------|--------------------------|-------------|
| measurement      | [0.746, 0.148, ~]        | [1.815, 0.292, ~]        | [2.005, 0.998, 0.200] |
| SRUKF            | [0.308, 0.140, 0.004]    | [0.404, 0.107, 0.004]    | [0.427, 0.832, 0.022] |
| SRCKF            | [0.280, 0.113, 0.003]    | [0.336, 0.049, 0.004]    | [0.363, 0.548, 0.017] |
| ISRCKF           | [0.116, 0.074, 0.002]    | [0.212, 0.042, 0.003]    | [0.233, 0.380, 0.013] |

In experiment 2, the iteration number $N_{iter}$ of ISRCKF is set to 1, 2, 3 and 4, respectively. Figure 3 and 4 respectively shows the MSRE curves of radial distance and azimuth with different iteration numbers obtained from 50 Monte-Carlo simulations. The filtering accuracy of each algorithm is quantitatively analyzed, table 2 shows the AMSRE of $[x, \hat{x}, \ddot{x}, y, \hat{y}, \ddot{y}]$ and $[r, a, v]$. From table 2 we can see that the filtering accuracy of ISRCKF improved with the increase of iteration number, that owing to the newest measurement is fully utilized during the iteration.

![Figure 3. MSRE curves of radial distance for ISRCKF with various iteration numbers.](image3)

![Figure 4. MSRE curves of azimuth for ISRCKF with various iteration numbers.](image4)
Table 2. Accumulated mean square-root error (AMSRE) of ISRCKF with various iteration numbers

| AMSRE  | $[x, \hat{x}, \hat{x}]$ | $[y, \hat{y}, \hat{y}]$ | $[r, \hat{r}, \hat{r}]$ |
|--------|------------------------|------------------------|------------------------|
| $N_{\text{iter}=1}$ | $[0.281, 0.113, 0.003]$ | $[0.336, 0.048, 0.003]$ | $[0.362, 0.549, 0.017]$ |
| $N_{\text{iter}=2}$ | $[0.116, 0.074, 0.002]$ | $[0.210, 0.041, 0.004]$ | $[0.232, 0.374, 0.013]$ |
| $N_{\text{iter}=3}$ | $[0.095, 0.062, 0.001]$ | $[0.150, 0.038, 0.003]$ | $[0.170, 0.156, 0.011]$ |
| $N_{\text{iter}=4}$ | $[0.077, 0.060, 0.001]$ | $[0.123, 0.037, 0.003]$ | $[0.139, 0.149, 0.010]$ |

6. Conclusion

Based on SRCKF and Gauss-Newton iteration theory, an iterative square-root cubature Kalman filter (ISRCKF) algorithm is proposed for target tracking of automotive radar. The ISRCKF use the same method as SRCKF to predict the state and the square-root of covariance. In the measurement update, the Gauss-Newton nonlinear iteration method is used to perform iterative updating until the difference between the state estimation obtained by two consecutive iterations is less than the predetermined threshold or the iteration number reaches the predetermined maximum number, the latest measurement is fully utilized during the iterations to achieve a good filtering effect. Under the simulation of the target tracking of automotive radar, the experimental results show that the filtering accuracy of ISRCKF algorithm is much better than that of the classical SRUKF and SRCKF algorithms, and the filtering accuracy increases with the increase of iteration number within a certain range.

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