Integrated Field Equations of Heterotic Supergravities

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Abstract
The first-order bosonic field equations of the $D$-dimensional effective low energy theory which describes the massless background coupling of the $D$-dimensional fully Higgsed heterotic string are derived.

1 Introduction

The ten-dimensional $\mathcal{N} = 1$ supergravity [1, 2] which is coupled to 16 gauge multiplets with the gauge group either $O(32)$ or $E_8 \times E_8$ is the low energy effective limit theory which describes the massless background coupling of the ten-dimensional heterotic string [3]. If one chooses Abelian gauge multiplets then one obtains the maximal torus sub-theory of the ten-dimensional $O(32)$ or $E_8 \times E_8$ Yang-Mills supergravity theory. In this case the full gauge group is broken down to its maximal torus subgroup $U(1)^{16}$, whose Lie algebra is the Cartan subalgebra of the non-abelian gauge groups mentioned above. This mechanism is due to the general Higgs vacuum structure of the heterotic string which causes a spontaneous symmetry breakdown. Thus in
this respect upon compactification one obtains the $D$-dimensional fully Higgsed massless heterotic string coming from the maximal torus sub-theory of the ten-dimensional $O(32)$ or $E_8 \times E_8$ Yang-Mills supergravity. In either of the $O(32)$ or the $E_8 \times E_8$ heterotic string theories in order to obtain the $D$-dimensional massless heterotic string in the fully Higgsed compactification only the ten-dimensional 16 Cartan gauge fields are kept in the reduction since the non-cartan gauge fields lead to massive fields following the compactification. In other words only the Cartan gauge fields are kept since they are the only fields which will remain massless for generic values of the Wilson lines.

In [4] one can refer to the torodial compactification of the bosonic sector of the ten-dimensional $\mathcal{N}=1$ supergravity which is coupled to 16 Abelian gauge multiplets. As we have discussed above such a reduction gives us the $D$-dimensional bosonic low energy theory of the massless sector of the $D$-dimensional fully Higgsed heterotic string. In a formalism which treats the scalar manifolds as generic $G/K$-cosets and which uses the solvable Lie algebra parametrization [5] [6] [7] the field equations of these bosonic theories are studied in [8].

Starting from the bosonic field equations of the $D$-dimensional effective massless fully Higgsed heterotic string which is the $D$-dimensional heterotic supergravity in this note we derive the first-order field equations by locally integrating the second-order field equations obtained in [8]. By integration we mean cancelling an exterior derivative on both sides of the equations. Therefore we obtain a first-order formulation of the theory. We will effectively make use of the results derived in [9] which states that there exists a one-sided on-shell decoupling between the coset scalars and the gauge fields of the heterotic supergravities. For this reason to obtain the first-order field equations of the coset scalars we will adopt the general formulation of [7] which works out the first-order field equations of the pure symmetric space sigma model. We will also give a brief discussion how one can make use of the first-order field equations to perform the on-shell bosonic coset construction of the $D$-dimensional heterotic supergravity.

2 The First-Order Field Equations

The bosonic field content which constitutes the low energy effective Lagrangian that describes the bosonic sector of the $D$-dimensional massless
background coupling of the fully Higgsed heterotic string can be given as [8]

\[ \{C^I_1, A^{(2)}, \phi, \phi^i, \chi^\alpha \}. \] (2.1)

The fields \( C^I_1 \) are \((20 - 2D + 16)\) one-forms, and \( A^{(2)} \) is a two-form, the rest of the fields are scalars. The scalar field \( \phi \) is decoupled from the rest of the scalars which are the coset ones. The coset scalars \( \{\phi^i, \chi^\alpha\} \) parametrize the coset manifold

\[ O'(10 - D + 16, 10 - D)/O(10 - D + 16) \times O(10 - D), \] (2.2)

whose elements which are \((20 - 2D + 16)\)-dimensional real matrices in the fundamental representation satisfy

\[ \nu^T \Omega \nu = \Omega. \] (2.3)

Here the \((20 - 2D + 16) \times (20 - 2D + 16)\) matrix \( \Omega \) is

\[ \Omega = \begin{pmatrix} 0 & 0 & -1_{(10-D)} \\ 0 & 1_{(16)} & 0 \\ -1_{(10-D)} & 0 & 0 \end{pmatrix}, \] (2.4)

where \( 1_{(n)} \) is the \( n \times n \) unit matrix. Following the notation of [8] we use a prime in (2.2) which stands for the particular representation of \( O(10 - D + 16, 10 - D) \) defined through (2.3) that is generated by the indefinite signature metric (2.4). We should also state that we separate \( N = 16 \) in our expressions to emphasize the number of Abelian matter multiplets coupling to the ten-dimensional \( N = 1 \) type I supergravity which forms when coupled to these 16 \( U(1) \) vector multiplets the low energy effective limit of the ten-dimensional fully Higgsed heterotic string. The coset parametrization can be constructed in the solvable Lie algebra gauge [5, 6, 7, 8] as

\[ \nu = e^{\frac{1}{2} \phi^i H_i} e^{\chi^\alpha E_\alpha}, \] (2.5)

where \( i = 1, \cdots, r \) and \( \alpha = 1, \cdots, n \). Here \( H_i \) are the Cartan generators and \( E_\alpha \) are the positive root generators of the solvable Lie algebra which takes part in the Iwasawa decomposition of \( O'(10 - D + 16, 10 - D) \) [10].}

\footnote{On the other hand the unprimed notation \( O(10 - D + 16, 10 - D) \) is used in [8] for the usual representation of the generalized orthogonal group generated by the diagonalized indefinite signature metric \( \eta = \text{diag}(-, -, \ldots, -, +, +, \ldots, +) \).}
can find a more detailed study of the solvable Lie algebra parametrization in [6] [7] [8]. The field equations of the bosonic fields (2.1) are already derived in [8]. They read

\[
\begin{align*}
(-1)^D d(*d\phi) &= \frac{1}{2} \sqrt{\frac{8}{(D-2)}} e^{-\sqrt{\frac{8}{D-2}} \phi} * F(3) \wedge F(3) \\
&\quad + \frac{1}{2} \sqrt{\frac{2}{(D-2)}} e^{-\sqrt{\frac{2}{D-2}} \phi} M_{IJ} * H^I_{(2)} \wedge H^J_{(2)}, \\
&\quad d(e^{-\sqrt{\frac{2}{(D-2)}} \phi} F(3)) = 0, \\
&\quad d(e^{-\sqrt{\frac{2}{(D-2)}} \phi} M_{IJ} * H^I_{(2)}) = (-1)^D e^{-\sqrt{\frac{2}{(D-2)}} \phi} \Omega^I_{IJ} H^J_{(2)} \wedge *F(3), \\
&\quad d(\gamma^i \phi^i \wedge U^\gamma) = \sum_{\alpha-\beta=-\gamma} N_{\alpha-\beta} U^\alpha \wedge e^{\beta,\phi^i} * U^\beta, \\
&\quad d(*d\phi) = \frac{1}{2} \sum_{\beta \in \Delta^+_{nc}} \beta^i e^{\frac{1}{2} \beta_i \phi^i} U^\beta \wedge e^{\frac{1}{2} \beta_i \phi^i} * U^\beta \\
&\quad - \frac{1}{2} (-1)^D e^{-\sqrt{\frac{2}{(D-2)}} \phi} * H(2) \wedge \nu^T H_i \nu H(2),
\end{align*}
\]

(2.6)

where \(\alpha, \beta, \gamma\) whose corresponding generators enter in the solvable lie algebra parametrization of (2.5) are the elements of \(\Delta^+_{nc}\) which is the set of non-compact positive roots of \(\mathfrak{o}'(10 - D + 16, 10 - D)\) [7] [8]. The field strengths of the fields \(\{C^I_{(1)}, A^i_{(2)}, \chi^\beta\}\) are respectively defined as

\[
\begin{align*}
H^I_{(2)} &= dC^I_{(1)}, \\
F(3) &= dA^i_{(2)} + \frac{1}{2} \Omega^I_{IJ} C^I_{(1)} \wedge dC^I_{(1)}, \\
U^\alpha &= \Omega^\alpha_{\beta} d\chi^\beta.
\end{align*}
\]

(2.7)

The matrix \(\mathcal{M}\) is

\[
\mathcal{M} = \nu^T \nu.
\]

(2.8)

In the above relations \(\beta_i, \gamma_i\) are the root vector components and \(N_{\alpha,\beta}\) are the structure constants of the corresponding positive root generators of the solvable Lie algebra generated by \(\{H_I, E_\alpha\}\) [6] [7] [8]. More specifically

\[
[H_j, E_\gamma] = \gamma_j E_\gamma,
\]

(2.9)

\[\text{We adopt the notation of [7] and randomly enumerate the roots in } \Delta^+_{nc} \text{ from 1 to } n.\]
and

\[ [E_\alpha, E_\beta] = N_{\alpha,\beta} E_{\alpha+\beta}. \]  \hspace{1cm} (2.10)

From \[6, 7\] the definition of the \( n \times n \) matrix \( \Omega(\chi^\beta) \) reads\(^3\)

\[ \Omega = (e^\omega - I) \omega^{-1}, \] \hspace{1cm} (2.11)

where \( \omega \) is an \( n \times n \) matrix with components

\[ \omega^\gamma_\beta = \chi^\alpha K^\gamma_{\alpha\beta}. \] \hspace{1cm} (2.12)

Here \( K^\gamma_{\alpha\beta} \) are defined through the commutators of \( \{ E_\alpha \} \)

\[ [E_\alpha, E_\beta] = K^\gamma_{\alpha\beta} E_\gamma. \] \hspace{1cm} (2.13)

We should state that we freely lower and raise indices by using various dimensional Euclidean metrics when necessary for convenience of notation. In \[8\] it is shown that the second term on the right hand side of the last equation in (2.6) which is compactly written in matrix form can be given as

\[ -\frac{1}{2} (-1)^D e^{\sqrt{-1} D \phi^\beta} \ast H(2) \wedge \nu^T H\nu H(2) = (-1)^D \frac{\partial L_m}{\partial \phi^\beta}. \] \hspace{1cm} (2.14)

Here \( H_i \) are the \((20 - 2D + 16)\)-dimensional matrix representatives of the Cartan generators and \( H(2) \) is the vector of the field strengths defined in \[(2.7)\]. Also \( L_m \) is the matter-scalar coupling Lagrangian \[8\]. However on the other hand in \[9\] it is proven that the expression in (2.14) vanishes on-shell for the elements of the solution space indicating that the coset scalar field equations coincide with the pure sigma model ones. Therefore we can legitimately drop the second term on the right hand side of the last equation in (2.6).

As discussed in the Introduction our aim in this note is to integrate the field equations in (2.6) locally. In this respect we will use the fact that locally a closed differential form is an exact one. For this reason we first introduce the dual fields

\[ \{ \tilde{C}^I, \tilde{B}, \tilde{\phi} \}, \] \hspace{1cm} (2.15)

\(^3\)The reader should be aware that the plain \( \Omega \) with indices \( I, J, K, ... \) and the bold one \( \Omega \) with the indices \( \alpha, \beta, \gamma, ... \) are two different objects. We prefer using them together for the sake of conformity with the references.

\(^4\)Of course by comparing (2.13) with (2.10) one may relate \( N_{\alpha,\beta} \) to \( K^\gamma_{\alpha\beta} \).
where $\tilde{C}^I$ are $(D - 3)$-forms, $\tilde{B}$ is a $(D - 4)$-form and $\tilde{\phi}$ is a $(D - 2)$-form. It is a straightforward operation that if one applies the exterior derivative on both sides of

$$e^{-\sqrt{(D-2)/2}} \cdot F(3) = d\tilde{B}, \quad (2.16)$$

one obtains the second equation in $(2.6)$. Thus $(2.16)$ is our first first-order field equation. Next let us consider

$$e^{-\sqrt{(D-2)/2}} M^I J * H_{(2)}^J = (-1)^D (d\tilde{C}^I + \Omega^I_K C^K_{(1)} \wedge d\tilde{B}). \quad (2.17)$$

If we take the exterior derivative of both sides we get

$$d(e^{-\sqrt{(D-2)/2}} M^I J * H_{(2)}^J) = (-1)^D \Omega^I_K dC^K_{(1)} \wedge d\tilde{B}. \quad (2.18)$$

By using $(2.16)$ in it $(2.18)$ gives the third equation in $(2.6)$. Finally let us take the first-order equation

$$d\star \phi = d\tilde{\phi} - \frac{1}{2} \sqrt{8/(D-2)} A_{(2)} \wedge d\tilde{B} + \frac{1}{2} \sqrt{2/(D-2)} \delta_{I,J} C^I_{(1)} \wedge d\tilde{C}^J. \quad (2.19)$$

If we apply the exterior derivative on both sides of $(2.19)$ we get

$$d(\star d\phi) = -\frac{1}{2} \sqrt{8/(D-2)} dA_{(2)} \wedge d\tilde{B} + \frac{1}{2} \sqrt{2/(D-2)} \delta_{I,J} dC^I_{(1)} \wedge d\tilde{C}^J. \quad (2.20)$$

By using $(2.7)$ and $(2.16)$ the above equation can be written as

$$d(\star d\phi) = \frac{1}{2} \sqrt{8/(D-2)} e^{-\sqrt{(D-2)/2}} (-1)^D \star F(3) \wedge F(3)$$

$$+ (-1)^D \frac{1}{2} \sqrt{2/(D-2)} e^{-\sqrt{(D-2)/2}} M_{IK} \star H_{(2)}^K \wedge H_{(2)}^I$$

$$+ \frac{1}{2} \sqrt{2/(D-2)} e^{-\sqrt{(D-2)/2}} \Omega_{I,J} C^I_{(1)} \wedge H_{(2)}^J \wedge \star F(3)$$

$$- \frac{1}{2} \sqrt{2/(D-2)} e^{-\sqrt{(D-2)/2}} \Omega_{K,I} C^K_{(1)} \wedge H_{(2)}^I \wedge \star F(3). \quad (2.21)$$

Since $\Omega$ is a symmetric matrix the last two terms cancel and as $M$ is also a symmetric matrix this equation gives us the first equation in $(2.6)$. The first-order formulation of the coset scalar field equations in $(2.6)$ is a straightforward task. Following our discussion above when we drop the second term on the right hand side of the last equation in $(2.6)$ we obtain the pure sigma
model field equations which are the same with the ones derived for a generic coset manifold in [7]. As we have remarked before this fact is a consequence of the on-shell conditions satisfied by the general solutions of the theory which are rigorously derived in [9]. The first-order field equations of the general non-split [10] scalar coset are already derived in [7]. In general the coset manifolds in (2.2) are also in non-split form. Therefore we can adopt the results of [7] for the scalar sectors of the heterotic supergravities. For the sake of completeness we will repeat the first-order scalar field equations of [7] here. From [7] we have

\[
* \Psi = (-1)^D e^\Gamma e^A \tilde{A},
\]

(2.22)

Here we define the \((r + n)\)-dimensional column vectors \(\tilde{\Psi}\) and \(\tilde{A}\) whose components can be given as

\[
\Psi^i = \frac{1}{2} d\phi^i, \quad \text{for} \quad i = 1, \ldots, r, \quad \Psi^{\alpha+r} = e^{\frac{i}{2} \phi^i} \Omega_{\gamma}^\alpha d\chi^\gamma, \quad \text{for} \quad \alpha = 1, \ldots, n,
\]

(2.23)

\[
A^i = \frac{1}{2} d\tilde{\phi}^i, \quad \text{for} \quad i = 1, \ldots, r, \quad \text{and} \quad A^{\alpha+r} = d\tilde{\chi}^\alpha, \quad \text{for} \quad \alpha = 1, \ldots, n,
\]

where we have introduced the dual \((D-2)\)-forms \(\tilde{\phi}^i\) and \(\tilde{\chi}^\alpha\). In (2.22) \(\Gamma(\phi^i)\) and \(\Lambda(\chi^\beta)\) are \((n + r) \times (n + r)\) matrix functions. Their components read

\[
\Gamma_n^k = \frac{1}{2} \phi^i g_{kn}^k, \quad \Lambda_n^k = \chi^\alpha f_{an}^k.
\]

(2.24)

The real constant coefficients \(\{g_{kn}^k\}\) and \(\{f_{an}^k\}\) are already listed in [6]. They are

\[
\tilde{f}_{an}^m = 0, \quad m \leq r, \quad \tilde{f}_{a,\alpha+r}^i = \frac{1}{4} \alpha_i, \quad i \leq r,
\]

\[
\tilde{f}_{a,\alpha+r}^i = 0, \quad i > r, \quad \tilde{f}_{a,\beta+r}^i = 0, \quad i \leq r, \quad \alpha \neq \beta,
\]

\[
\tilde{f}_{a,\alpha+r}^i = 0, \quad i > r, \quad \tilde{f}_{a,\beta+r}^i = 0, \quad i \leq r, \quad \alpha \neq \beta
\]
\[ \tilde{f}_{\alpha, \beta+r}^+ = N_{\alpha, -\beta}, \quad \alpha - \beta = -\gamma, \quad \alpha \neq \beta; \]

\[ \tilde{f}_{\alpha, \beta+r}^- = 0, \quad \alpha - \beta \neq -\gamma, \quad \alpha \neq \beta, \quad (\text{2.25}) \]

and

\[ \tilde{g}^n_{im} = 0, \quad m \leq r, \quad \tilde{g}^n_{im} = 0, \quad m > r, \quad m \neq n; \]

\[ \tilde{g}_{\alpha}^\alpha = -\alpha_i, \quad \alpha > r. \quad (\text{2.26}) \]

Since as discussed in detail in [6, 7] beside being enumerated \( \alpha, \beta, \gamma, \ldots \) correspond to the set of non-compact positive roots of \( \sigma'(10 - D + 16, 10 - D) \) the conditions on them in (2.25) and (2.26) must be understood in the root sense. We should also state that likewise in [7] we assume the signature of the spacetime as \( s = 1. \) It is proven in [7] that as a consequence of the dualisation of the general symmetric space sigma model the first-order field equations in (2.22) correspond to the local integration of the last two equations of (2.6) when the term which comes from the scalar-matter coupling Lagrangian is dropped as discussed before. Therefore we have derived the entire set of first-order field equations which are obtained by locally cancelling an exterior derivative on both sides of the equations in (2.6). Namely the equations (2.16), (2.17), (2.19), and (2.22) represent the first-order formulation of the \( D \)-dimensional low energy massless background coupling of the fully Higgsed heterotic string which is the \( D \)-dimensional heterotic supergravity.

Before concluding we will present a discussion of an important application of the first-order field equations of the heterotic supergravities. In [11] the locally integrated first-order bosonic field equations of the maximal and IIB supergravities are used to derive the superalgebras that lead to the complete coset constructions of the bosonic sectors of these theories. Similarly the methodology of [11] can be extended to the heterotic supergravities. We will not present the complete coset construction of the heterotic supergravities here and leave it to a future work however we will discuss the outline of deriving the superalgebra of the on-shell coset construction of the heterotic supergravities. The first task in constructing the coset formalism is to assign an algebra generator to each original and dual field in the first-order equations (2.16), (2.17), (2.19), (2.22) and then to propose a coset map. In our case
this map becomes

\[ \nu = \exp(\frac{1}{2} \phi^j H_j) \exp(\chi^m E_m) \exp(\phi K) \exp(C'(1)V_i) \exp(\frac{1}{2} A(t) Y) \]

\[ \times \exp(\frac{1}{2} \tilde{B} \tilde{Y}) \exp(\tilde{C}' \tilde{V}_i) \exp(\tilde{\phi} \tilde{K}) \exp(\tilde{\chi}^m \tilde{E}_m) \exp(\frac{1}{2} \tilde{\phi} \tilde{H}_j). \]  

(2.27)

The associated Cartan-form may be defined as

\[ G = d\nu \nu^{-1}. \]  

(2.28)

From [11] we know that in the doubled formalism coset construction the Cartan-form satisfies a twisted self-duality equation

\[ *G = SG, \]  

(2.29)

with \( S \) being a pseudo-involution of the coset algebra of the generators introduced in (2.27). The key ingredient of the coset construction is the requirement that (2.29) must give us the first-order field equations of the theory. Therefore the method of revealing the coset algebra structure is to calculate (2.28) in terms of the desired structure constants, then to insert it in (2.29) and finally to compare the result with the equations (2.16), (2.17), (2.19), (2.22) to read the structure constants of the coset algebra.

3 Conclusion

In this work, by locally integrating the second-order field equations which are derived in [8] and which govern the massless sector of the \( D \)-dimensional fully Higgsed heterotic string namely the \( D \)-dimensional heterotic supergravity we have obtained the first-order field equations of the theory which contain only a single exterior derivative acting on the potentials. In these first-order field equations we have introduced dual fields which may be considered as integration constants. The dual fields are nothing but the Lagrange multipliers associated with the Bianchi identities of the field strengths when one treats these field strengths as fundamental fields instead of their potentials [12]. The fact which is derived in [9] that as an on-shell condition the coset scalar field equations can completely be decoupled from the gauge fields provides us the usage of the first-order symmetric space sigma model field equations of [7] in our formulation.
In GR the Palatini application of the Ostrogradski method of reducing the derivative order of second-order Lagrangians by including auxiliary fields is a vast research area in recent years especially for the f(R) theories of gravity. The Ostrogradski method have also been effectively used to obtain the first-order formulations of supergravity theories. First-order formulations of supergravities are studied to understand the superpotentials as well as the supersymmetry transformation laws. The reader may find examples of the first-order formalism of supergravity theories in various dimensions in. In the general first-order formalism method of these works the field strengths of the basic fields are also considered as independent fields and a first-order Lagrangian is constructed which gives first-order field equations in terms of the basic fields and their field strengths. When the field equations of the field strengths are substituted back in the Lagrangian one recovers the second-order formalism. In comparison with this scheme our first-order field equations of the D-dimensional heterotic supergravity do contain the basic fields except the graviton but on the contrary they do not include the field strengths. Instead we have introduced dual fields which may be considered as arbitrary integration constants that algebraically came into the scene as a result of abolishing an exterior derivative on both sides of the second-order field equations. Thus our approach is purely algebraic rather than being formal. We have simply reduced the degree of the field equations without increasing the number of fields to be solved and in this process arbitrary integration constants have arouse. On the other hand we have not constructed the corresponding Lagrangian which would lead to the first-order equations we have obtained. However as we have discussed above such a Lagrangian which would kinematically be different than the one that would appear within the Ostrogradski method would rather be obtained by Lagrange multiplier method that makes use of the Bianchi identities of the field strengths.

The first-order formulation of the D-dimensional heterotic supergravity presented in this note has two important implications. The first-order field equations play an important role in the coset construction of the supergravities. Thus as we have briefly discussed in the previous section the equations derived in this note can be considered to be essential ingredients of a possible coset construction of the heterotic supergravities. Secondly since the dual fields introduced in the first-order field equations can be arbitrarily varied one can make use of this fact to generate solutions. Therefore in this respect beside being first-order the integrated field equations contain-
ing parameters which can be manipulated become powerful tools in seeking solutions of the heterotic supergravities.

References

[1] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, “Ten-dimensional Maxwell-Einstein supergravity, its currents, and the issue of its auxiliary fields”, Nucl. Phys. B195 (1982) 97.

[2] G. F. Chapline and N. S. Manton, “Unification of Yang-Mills theory and supergravity in ten-dimensions”, Phys. Lett. B120 (1983) 105.

[3] E. Kiritsis, “Introduction to superstring theory”, hep-th/9709062

[4] H. Lü, C. N. Pope and K. S. Stelle, “M-theory/heterotic duality: A Kaluza-Klein perspective”, Nucl. Phys. B548 (1999) 87, hep-th/9810159.

[5] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Fre, M. Trigiante, “R-R scalars, U-duality and solvable Lie algebras”, Nucl. Phys. B496 (1997) 617, hep-th/9611014.

[6] N. T. Yilmaz, “Dualisation of the general scalar coset in supergravity theories”, Nucl. Phys. B664 (2003) 357, hep-th/0301236.

[7] N. T. Yilmaz, “The non-split scalar coset in supergravity theories”, Nucl. Phys. B675 (2003) 122, hep-th/0407006.

[8] N. T. Yilmaz, “Heterotic string dynamics in the solvable lie algebra gauge”, Nucl. Phys. B765 (2007) 118, hep-th/0701275.

[9] N. T. Yilmaz, “An implicit decoupling for the dilatons and the axions of the heterotic string”, Phys. Lett. B646 (2007) 125, hep-th/0703113.

[10] S. Helgason, “Differential Geometry, Lie Groups and Symmetric Spaces”, (Graduate Studies in Mathematics, vol. 34, American Mathematical Society, Providence, 2001).

[11] E. Cremmer, B. Julia, H. Lü and C. N. Pope, “Dualisation of dualities II: Twisted self-duality of doubled fields and superdualities”, Nucl. Phys. B535 (1998) 242, hep-th/9806106.
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[12] C. N. Pope, “Lecture Notes on Kaluza-Klein Theory”, (unpublished).

[13] F. J. de Urries and J. Julve, “Ostrogradski formalism for higher-derivative scalar field theories”, J. Phys. A31 (1998) 6949, hep-th/9802115.

[14] B. Julia and S. Silva, “Currents and superpotentials in classical gauge invariant theories. I: Local results with applications to perfect fluids and general relativity”, Class. Quant. Grav. 15 (1998) 2173, gr-qc/9804029.

[15] M. Henneaux, B. Julia and S. Silva, “Noether superpotentials in supergravities”, Nucl. Phys. B563 (1999) 448, hep-th/9904003.

[16] S. Silva, “On superpotentials and charge algebras of gauge theories”, Nucl. Phys. B558 (1999) 391, hep-th/9809109.

[17] B. Julia and S. Silva, “On first order formulations of supergravities”, JHEP 0001 (2000) 026, hep-th/9911035.

[18] S. Deser and B. Zumino, “Consistent supergravity”, Phys. Lett. B62 (1976) 335.

[19] I. Bars and S. W. MacDowell, “Gravity with extra gauge symmetry”, Phys. Lett. B129 (1983) 182.

[20] A. Higuchi, “On the new first order formalism of d = 11 supergravity”, preprint YTP 85-02 (1985).

[21] P. Fre, “On the spinor form of first order gravity and supergravity”, preprint CALT-68-662 (1978).

[22] I. Bars and A. Higuchi, “First order formulation and geometrical interpretation of d = 11 supergravity”, Phys. Lett. B145 (1984) 329.

[23] R. E. Kallosh, “Geometry of eleven-dimensional supergravity”, Phys. Lett. B143 (1984) 373.