Mode-dependent event-triggered tracking control for uncertain semi-Markov systems with application to vertical take-off and landing helicopter

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Abstract
The aim of this manuscript is to tackle the tracking problem for uncertain semi-Markov systems. More precisely, the mode-dependent event-triggering communication approach is introduced for networked controller design procedure, which can considerably increase the signal transmission efficiency. Based on the Lyapunov–Krasovski method, mode-dependent sufficient conditions are derived and the tracking errors could achieve the asymptotic mean-square stability, and mode-dependent controllers could be further calculated by convex optimization. A numerical simulation with application to vertical take-off and landing helicopter would be performed for verifying the availability and usefulness.

Keywords
Semi-Markov system, mode-dependent event-triggered control, mode-dependent uncertainties

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Introduction
In the past years, Markov jump systems (MJSs) have drawn a lot of research attention owing to their ability for modeling practical systems with jumping parameters governed by a Markov chain.\textsuperscript{1–5} To name a few, power systems,\textsuperscript{6} neural systems,\textsuperscript{7} robotic systems\textsuperscript{8} can be effectively described by MJSs with corresponding analysis and synthesis results. It is worth mentioning that often transition probability may be not kept constant in certain scenario, then we formulated the semi-Markovian jump systems (SMJSs).\textsuperscript{9} It can be found that by introducing the SMJS, more applicable results with less conservatism can be obtained when modeling the practical applications. As a results, some initial efforts have been paid to SMJS with remarkable methods including stability problem,\textsuperscript{10} state estimation problem,\textsuperscript{11} and synchronization problem.\textsuperscript{12} Meanwhile, it is noticed that the parameter uncertainties can always affect the practical systems. Furthermore, sometimes, these uncertainties can exhibit mode-dependent characteristics for MJS or SMJS. Therefore, some effective control approaches for dealing with these mode-dependent uncertainties have been addressed.\textsuperscript{13,14}

In the meantime, advanced networked control systems (NCSs) have been extensively investigated and used in all kinds of real-world applications.\textsuperscript{15–17} It should be pointed out that certain network constraints can be found during networked information transmission, such as transmission delays,\textsuperscript{18} bandwidth limitations,\textsuperscript{19} and signal fading effects.\textsuperscript{20} Especially, in order to cope with network burden, the event-triggered communication methods have been significantly developed instead of traditional time-triggered schemes in recent years. The innovation of event-triggered strategy is formulated by certain triggering conditions, which implies that the signal transmission frequency over the network can be decreased and the network utilization efficiency can be increased. By considering these advantages, many event-based control strategies have been studied and can be found.\textsuperscript{21–23} Moreover, for the MJS or

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SMJS, it is reasonable and urgent to design the mode-dependent event-triggered strategy for more applicable results. Recently, remarkable results can be found,24,25 where effective event-triggered control strategies have been developed. In addition, since the vertical take-off and landing (VTOL) helicopters are capable of taking-off, landing within limited field, and stable hovering over target region, many related researches have been investigated.26,27 This has distinguishing advantages compared with traditional unmanned air vehicles. To the best of the authors’ knowledge, unfortunately, the challenging and open problems for the tracking control problem of SMJS with effective event-triggered mechanism still exists, especially with the mode-dependent strategies.

Inspired by what have been discussed above, the tracking problem of SMJS accompanied by mode-dependent parameter uncertainties is studied by proposing a mode-dependent event-triggered transmission scheme. In comparison with most reporting literature, the novelties in this paper are presented by the following aspects: (1) for the sake of reducing the transmission load, an effective mode-dependent event-triggering communication approach together with corresponding sampling interval is proposed. (2) Sufficient conditions are established by selecting a reasonable mode-dependent Lyapunov–Krasovski functional, and then the tracking error would reach mean-square asymptotic stability with prescribed property.

Our manuscript would be arranged with this content. The tracking control problem for SMJS is first formulated and the mode-dependent controllers are designed with novel mode-dependent event-triggered mechanism. Then, main theoretical results are given by matrix convex optimization method. In addition, the effectiveness of our control strategy can be illustrated and supported with the VTOL helicopter simulation example. Finally, the manuscript is concluded with some further prospect.

Notations:

\( \mathbb{R}^n \) denotes the \( n \) dimensional Euclidean space. 
\( A - B > 0 \) means \( A - B \) is a positive definite. 
\( \text{diag}\{ \cdots \} \) denotes the block-diagonal matrix. 
\( \mathbb{E}() \) represents the expectation operator. 
\( (\Omega, \mathcal{F}, \mathcal{P}) \) is a probability space.

**Preliminaries and problem formulation**

Taking into account the below SMJS with mode-dependent uncertainties:

\[
\begin{align*}
\dot{x}(t) &= (A(\sigma(t)) + \Delta A(\sigma(t)))x(t) + (B(\sigma(t)) + \Delta B(\sigma(t)))u(t) \\
&\quad + (B_s(\sigma(t)) + \Delta B_s(\sigma(t)))w(t) \\
y(t) &= C(\sigma(t))x(t) \\
x(0) &= x_0
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) denotes the system state, \( u(t) \in \mathbb{R}^m \) represents the control input, and \( w(t) \in L[0, \infty) \) is the external disturbance. \( \{\sigma(t), \ t \geq 0\} \) indicates a continuous-time discrete-state semi-Markov process on \( (\Omega, \mathcal{F}, \mathcal{P}) \) taking values in a finite set \( \mathcal{I} = \{1, \ldots, N\} \). Furthermore, the transition probability matrix \( \Theta := (\pi_{ij}(k)), \ k > 0, \forall i, j \in \mathcal{I} \) is defined as follows

\[
\Pr(\sigma(t + k) = j | \sigma(t) = i) = \begin{cases} 
\pi_{ij}(k) + \alpha(k), i \neq j \\
1 + \pi_{ii}(k) + \alpha(k), i = j
\end{cases}
\]

where \( \lim(\alpha(k)/k) = 0, \pi_{ij}(k)\geq 0, i \neq j, \) is the transition rate from mode \( i \) at time \( t \) to mode \( j \) at time \( t + k, \) satisfying \( \pi_{ij}(k) = -\sum_{t \neq j}^{N} \pi_{ij}(k), \forall i \in \mathcal{I} \).

For a fixed-system mode \( \sigma(t), A(\sigma(t)), B(\sigma(t)), B_s(\sigma(t)), \) and \( C(\sigma(t)) \) are known constant matrices, and \( \Delta A(\sigma(t)), \Delta B(\sigma(t)), \) and \( \Delta B_s(\sigma(t)) \) denote the parameter uncertainties, respectively, which can be given as follows

\[
\begin{align*}
\Delta A(\sigma(t)) &= MA(\sigma(t))\Phi(\sigma(t), t)N_A(\sigma(t)) \\
\Delta B(\sigma(t)) &= MB(\sigma(t))\Phi(\sigma(t), t)N_B(\sigma(t)) \\
\Delta B_s(\sigma(t)) &= MB_s(\sigma(t))\Phi(\sigma(t), t)N_{B_s}(\sigma(t))
\end{align*}
\]

Note that \( M_A(\sigma(t)), M_B(\sigma(t)), M_{B_s}(\sigma(t)), N_A(\sigma(t)), N_B(\sigma(t)), \) and \( N_{B_s}(\sigma(t)) \) are known constant matrices and \( \Phi(\sigma(t), t) \) is a unknown time-varying matrix satisfying

\[
\Phi^T(\sigma(t), t)\Phi(\sigma(t), t) \leq I
\]

In addition, the output-tracking reference model is given by

\[
\begin{align*}
\dot{x}_r(t) &= A_r(\sigma(t))x_r(t) + B_{r_1}(\sigma(t))u(t) \\
y_r(t) &= C_r(\sigma(t))x_r(t) \\
x_r(0) &= x_{r0}
\end{align*}
\]

where \( x_r(t) \in \mathbb{R}^q \) denotes the desired tracking reference and \( r(t) \in L[0, \infty) \) denotes the external disturbance. Without loss of generality, it is assumed that the tracking system and the system are with the same system mode with synchronous control during the event-triggered control procedure.

Define the tracking error as

\[
e(t) = y(t) - y_r(t)
\]

such that the reference signal \( y_r(t) \) can be tracked when the \( e(t) \) is asymptotically stable in the mean-square sense.

For simplification, denote \( \sigma(t) \in \mathcal{I} \) as \( i \) index and the augmented system can be obtained as follows

\[
\begin{align*}
\dot{\xi}(t) &= A_i\xi(t) + B_{i_1}u(t) + B_{i_2}v(t) \\
e(t) &= C_i\xi(t)
\end{align*}
\]

where

\[
\begin{align*}
\xi(t) &= \begin{bmatrix} x^T(t), x_r^T(t) \end{bmatrix}^T \\
v(t) &= \begin{bmatrix} w^T(t), r^T(t) \end{bmatrix}^T \\
A_i &= \begin{bmatrix} A_i + \Delta A_i & 0 \\ 0 & A_{i_1} \end{bmatrix}
\end{align*}
\]
\[
B_i = \begin{bmatrix} (B_i + \Delta B_i) \\ 0 \end{bmatrix}, \\
B_{ni} = \begin{bmatrix} (B_{ni} + \Delta B_{ni}) & 0 \\ 0 & B_{ni} \end{bmatrix}, \\
C_i = \begin{bmatrix} C_i & -C_{ri} \end{bmatrix}
\]

Under the networked environment, it is supposed that the sensor is time-driven with mode-dependent sampling period \( t_k h(\sigma(t)) \) according to mode \( \sigma(t) \). Then the event generator updates the released signals with \( t_k h(\sigma(t)) \), \( k = 0, 1, 2, \ldots \). Correspondingly, the event-triggering function is proposed as follows

\[
e(t(t)) = \xi(t_k h(\sigma(t))) - \xi(t_k h(\sigma(t)) + j h(\sigma(t)))
\]

where \( \xi(\cdot) = \xi_1(\cdot) + j \xi_2(\cdot) \)

\[
\Omega(\sigma(t)) = \text{diag} \left( \Omega_1(\sigma(t)), \Omega_2(\sigma(t)) \right) > 0 \text{ denotes the mode-dependent triggering scale matrix and } 0 \leq \varepsilon(\sigma(t)) < 1.
\]

**Remark 1.** It is marked that the mode-dependent event-triggering strategy is adopted in this paper, which can effectively deal with the system jumping behaviors accordingly by mode information.

**Remark 2.** Compared with the mode-independent sampling schemes, the mode-dependent sampling strategy can be more applicable with different modes and would lead to less conservatism with different triggering conditions by utilizing the mode information.

The mode-dependent tracking controller can be designed by

\[
u(t) = K(\sigma(t))\xi(t_k h(\sigma(t)))
\]

where \( K(\sigma(t)) \) denotes the mode-dependent controller gains.

By applying the virtual delay approach and letting

\[
\tau_i(t) = t - t_k h_i - j h_i
\]

the event-triggering function can be rewritten by

\[
e(t(t)) = e(t(t) - (\tau_i(t)) - \tau_i(t))
\]

where \( 0 \leq \tau_i(t) \leq \tau \). Moreover, it can be obtained that

\[
\dot{\xi}(t) = \hat{A}_i \xi(t) + \hat{B}_i \hat{K}_i e(t(t))
\]

\[
+ \hat{B}_i \hat{K}_i \xi(t(t) - \tau_i(t)) + \hat{B}_i \nu(t)
\]

\[
e(t(t)) = \hat{C}_i \xi(t)
\]

Furthermore, the following \( H_{\infty} \) tracking performance is introduced

\[
\int_v^T(t(t))e(t(t))dt \leq \gamma^2 \int_v^{\infty} v(t(t))v(t(t))dt
\]

where \( \gamma > 0 \) denotes the performance scalar.

To this end, the aim of our manuscript is going to project a desired mode-dependent controller for ensuring the \( H_{\infty} \)-tracking performance, and the following lemmas are provided for deriving the main results.

**Lemma 1.** For any matrix \( A \geq 0 \), scalars \( \tau > 0 \), \( \tau(t) \) satisfying \( 0 \leq \tau(t) \leq \tau \), vector function \( \hat{x}(t) : [-\tau, 0] \rightarrow \mathbb{R}^n \) such that the concerned integrations are well defined, then

\[
-\tau \int_{t_{-\tau}}^{t} \hat{x}(s)M\hat{x}(s)ds \leq \hat{\xi}(t)\Omega(\hat{\xi}(t))
\]

where

\[
\hat{\xi}(t) = \begin{bmatrix} x(t(t)), x(t(t) - \tau(t)), x(t(t) - \tau) \end{bmatrix}^T
\]

\[
\Omega = \begin{bmatrix} -M & M & 0 \\ * & -2M & M \\ * & * & -M \end{bmatrix}
\]

**Lemma 2.** Let \( L^T = L \), \( H \) and \( E \) be real matrices of appropriate dimensions with \( F(t) \) satisfying \( F(t)F(t)^T \leq I \). Then, \( L + HFE + E^TF(t)H^T \leq 0 \), if and only if there exists a scalar \( \varepsilon > 0 \) such that \( L + \varepsilon^{-1}HFE + \varepsilon E^TF(t)H^T \leq 0 \), or equivalently

\[
\begin{bmatrix} L & H & \varepsilon E^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0
\]

**Controller design**

In this following, our mode-dependent controller design procedure should be presented in following theorems.

**Theorem 1.** The tracking problem of the semi-MJS is solved by prescribed mode-dependent controller gains \( K_i \), if there are mode-dependent matrix \( P_i > 0 \), matrices \( Q > 0 \) and \( R > 0 \), such that \( \Pi_i > 0 \) for each \( i \in I \), where

\[
\Pi_i = \begin{bmatrix} \Pi_{i1} & \Pi_{i2} & \Pi_{i3} \\ \Pi_{i1}^T & \Pi_{i2}^T & \Pi_{i3}^T \end{bmatrix}
\]

\[
\Pi_{i1} = \begin{bmatrix} 0 & P_i B_i K_i^T + R & 0 \\ * & \varepsilon \Omega_i - 2R & R \\ * & * & -Q - R \end{bmatrix}
\]

\[
\Pi_{i2} = \begin{bmatrix} P_i B_i & P_i B_{i2} & \tau K_i^T B_i^T \\ 0 & 0 & \tau K_i^T B_{i2}^T \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
\Pi_{i3} = \begin{bmatrix} -\Omega_i & 0 & \tau K_i^T B_i^T R \\ * & -\gamma^2 I & \tau B_i^T R \\ * & * & -R \end{bmatrix}
\]
Proof. Choose the below Lyapunov–Krasovski function with respective mode $i$

$$V(i, t) = V_1(i, t) + V_2(i, t) + V_3(i, t)$$

where

$$V_1(i, t) = \xi^T(\phi) P_i \xi(\phi)$$

$$V_2(i, t) = \int_{t-\tau}^{t} \xi^T(\phi) Q_i \xi(\phi) d\phi$$

$$V_3(i, t) = \hat{\tau} \int_{t-\tau}^{t} \hat{\xi}^T(\eta) \hat{R} \hat{\xi}(\eta) d\eta d\phi.$$

Moreover, the weak infinitesimal operator $\mathcal{L}$ of $V(i, t)$ is stated as follows

$$\mathcal{L}V(i, t) = \frac{\partial}{\partial t} \{ V(\sigma(t + \Delta), t + \Delta) | \sigma(t) = i \} - V(i, t)$$

Accordingly, one has

$$\lim_{\Delta \to 0} \frac{1}{\Delta} \left( T_i(h + \Delta) - T_i(h) \right) = 0$$

$$\lim_{\Delta \to 0} \frac{1}{\Delta} \left( 1 - T_i(h + \Delta) \right) - 1 = 0$$

$$\lim_{\Delta \to 0} \frac{1}{\Delta} \left( q_{ih}(T_i(h) - T_i(h + \Delta)) \right) = \pi_{ih}(h)$$

where $h$ stands for elapsed time and $T_i(h)$ denotes cumulative distribution function and $q_{ih}$ represents probability intensity jumping.

Then, one can derive that

$$\mathcal{L}V_1(i, t) = 2\xi^T(t) P_i A_i \xi(t) + B_i K_i \xi(t)$$

$$+ B_i K_i \xi(t - \tau(i)) + B_{ni} \psi(t))$$

$$+ \sum_{j=1}^{N} \pi_{ij}(h) \hat{\xi}^T(j) P_j \hat{\xi}(j)$$

$$\mathcal{L}V_2(i, t) = \hat{\xi}^T(t) Q_i \hat{\xi}(t) - \hat{\xi}^T(t - \hat{\tau}) \hat{Q} \hat{\xi}(t - \hat{\tau})$$

$$\mathcal{L}V_3(i, t) = \hat{\tau} \hat{\xi}^T(t) \hat{R} \hat{\xi}(t) - \hat{\tau} \int_{t-\hat{\tau}}^{t} \hat{\xi}^T(\eta) \hat{R} \hat{\xi}(\eta) d\eta d\phi.$$ 

By Lemma 1, it should be derivated that

$$-\hat{\tau} \int_{t-\hat{\tau}}^{t} \hat{\xi}^T(\eta) \hat{R} \hat{\xi}(\eta) d\eta d\phi$$

$$\leq \begin{bmatrix} \xi(t) \\ \xi(t - \tau(i)) \\ \xi(t - \hat{\tau}) \end{bmatrix}^T \begin{bmatrix} -\hat{\tau} & \hat{\tau} & 0 \\ \hat{\tau} & -2\tau & \hat{\tau} \\ 0 & \hat{\tau} & -\hat{\tau} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t - \tau(i)) \\ \xi(t - \hat{\tau}) \end{bmatrix}$$

In addition, it holds that

$$\mathcal{L}V(i, t) = \delta^T(t) \begin{bmatrix} \tau A_i^T \\ \tau K_i^T B_i^T \\ \tau K_i^T B_i^T \\ \tau B_i^T \end{bmatrix} + \delta(t)$$

where $\delta(t) = [\xi^T(t), \xi^T(t - \tau(i)), \xi^T(t - \hat{\tau}), \xi(t), v^T(t)]^T$

From the event-triggering function, it can be deduced that

$$\varepsilon_i(t) \Omega_i \xi(t) < \varepsilon_i^T(t - \tau(i)) \Omega_i \xi(t) - \tau_i(t)$$

Consequently, one has

$$\mathcal{L}V(i, t) + \xi^T(t) \xi^T(t) - \varepsilon_i^T(t) v_i(t)$$

$$\leq \delta^T(t) \Omega_i \delta(t) + \tau_i^2(t) \hat{R} \hat{\xi}(t)$$

where

$$\Delta_i = \begin{bmatrix} \Pi_i & \Pi_i \Pi_i & \Pi_i \Pi_i \Pi_i \end{bmatrix}$$

$$\Pi_i = \begin{bmatrix} \Pi_{i1} & \Pi_{i2} \end{bmatrix}$$

$$\Pi_{i1} = \begin{bmatrix} \Pi_{i11} & \Pi_{i12} \end{bmatrix}$$

$$\Pi_{i2} = \begin{bmatrix} 0 & \Pi_{i21} \Pi_{i22} \end{bmatrix}$$

$$\Pi_{i3} = \begin{bmatrix} 0 & \Pi_{i31} \Pi_{i32} \end{bmatrix}$$

$$\Pi_{i4} = \begin{bmatrix} 0 & \Pi_{i41} \Pi_{i42} \end{bmatrix}$$

Therefore, it can be obtained by Schur complement lemma that if $\Pi_i < 0$ holds, then the $H_\infty$-tracking performance under zero initial conditions in the mean-square sense is achieved, then the proof could be completed.

Remark 3. Note that the conditions in Theorem 1 are not standard linear matrix inequalities due to time-varying $h(i)$. As a consequence, the next theorem is further provided by Theorem 1 for obtaining the desired controllers.

Theorem 2. The tracking problem of the semi-MJS is solved, if there exist mode-dependent matrices $P_i = \text{diag} \{ P_{i1}, P_{i2} \}$ and $K_i = [K_{i1}, K_{i2}]$, matrices $Q_i = \text{diag} \{ Q_{i1}, Q_{i2} \}$, $R_i = \text{diag} \{ R_{i1}, R_{i2} \}$, such that $\Xi_{i,k} < 0$, $i \in \mathbb{I}$, $k = 1, 2, \ldots, K$, where

$$\Xi_{i,k} = \begin{bmatrix} \Xi_{i1,k} & \Xi_{i2,k} \\ \ast & \Xi_{i3,k} \end{bmatrix}$$
Remark 4. For obtaining an optimized disturbance attenuation level $\gamma$, the following optimization problem can be derived, such that

$$
\min \gamma
$$

s.t. $\Xi_{i, \kappa} < 0, \ i \in T, \ \kappa = 1, 2, \ldots, K$

**Illustrative example**

In this section, the simulation results for VTOL helicopter model are given to validate our proposed controller design.

First, the VTOL helicopter system in the vertical plane can be described by the following semi-MJS
where \( x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T \) and \( x_1(t), x_2(t), x_3(t), \) and \( x_4(t) \) stand for the horizontal velocity, the vertical velocity, the pitch rate, and the pitch angle of VTOL, respectively; \( u(t) \) represents the lift of VTOL helicopter and \( w(t) \) is the helicopter disturbance. Moreover, the system parameters are with different values according to the airspeed of VTOL helicopter.

Consider the following VTOL helicopter model with two jumping modes (135 knots and 60 knots), where the dynamics are given by Narendra and Tripathi\(^{31}\)

\[
\begin{align*}
A_1 &= \begin{bmatrix}
-0.0366 & 0.0271 & 0.0188 & -0.4555 \\
0.0482 & -1.01 & 0.0024 & -4.0208 \\
0.1002 & 0.3681 & -0.707 & 1.4200 \\
0 & 0 & 1 & 0
\end{bmatrix}, \\
A_2 &= \begin{bmatrix}
-0.0366 & 0.0271 & 0.0188 & -0.4555 \\
0.0482 & -1.01 & 0.0024 & -4.0208 \\
0.1002 & 0.0664 & -0.707 & 0.1198 \\
0 & 0 & 1 & 0
\end{bmatrix}, \\
B_1 &= \begin{bmatrix}
0.4422 & 0.1761 \\
3.5446 & -7.5922 \\
-5.520 & 0.4490 \\
0 & 0
\end{bmatrix}, \\
B_2 &= \begin{bmatrix}
0.4422 & 0.1761 \\
0.9775 & -7.5922 \\
-5.520 & 0.4490 \\
0 & 0
\end{bmatrix}, \\
B_{w1} &= 0.1I, \\
B_{w2} &= 0.2I, \\
C_1 &= 0.5I, \\
C_2 &= 0.6I
\end{align*}
\]

In addition, the reference system model is given as follows

\[
\begin{align*}
A_{1r} &= -2I, \\
A_{2r} &= -1.8I, \\
B_{wr1} &= I, \\
B_{wr2} &= 0.95I, \\
C_{1r} &= 0.5I, \\
C_{2r} &= 0.6I
\end{align*}
\]

The parameter uncertainties are assumed to be with

\[
\begin{align*}
M_{A1} &= 0.02I, \\
M_{A2} &= 0.01I, \\
N_{A1} &= 0.02I, \\
N_{A2} &= 0.01I
\end{align*}
\]

\[
M_{B1} = 0.02I \\
M_{B2} = 0.01I \\
N_{B1} = \begin{bmatrix} 0.02 & 0.02 \\
0.02 & 0.02 \\
0.02 & 0.02 \end{bmatrix} \\
N_{B2} = \begin{bmatrix} 0.01 & 0.01 \\
0.01 & 0.01 \\
0.01 & 0.01 \end{bmatrix} \\
M_{Bw1} = 0.02I \\
M_{Bw2} = 0.01I \\
N_{Bw1} = 0.02I \\
N_{Bw2} = 0.01I
\]

and

\[
\Phi_1(t) = \sin t, \quad \Phi_2(t) = \cos t
\]

Furthermore, it is supposed to be that \( \pi_1(h) \in [-1.6, -1.4] \) and \( \pi_2(h) \in [-1.9, -1.1] \), which involves that \( \pi_{11,1} = -1.4, \pi_{11,2} = -1.6, \pi_{22,1} = -1.1, \) and \( \pi_{22,2} = -1.9 \) with \( k = 2 \).

In this numerical simulation, it is assumed that \( h_1 = 0.05 \) s and \( h_2 = 0.1 \) s. The \( H_{\infty} \) performance index \( \gamma \) is optimized by \( \gamma = 5.4749 \) and the \( H_{\infty} \) performance index of mode-independent event-triggered scheme is \( \gamma = 6.0061 \), which shows the advantages of our proposed scheme. The event-triggering scalar is given by \( \epsilon_1 = 0.02 \) and \( \epsilon_2 = 0.04 \). The external disturbances are supposed to be \( u(t) = 0.1 \sin t \) and \( r(t) = 0.1 \sin 10t \). With these parameters, the event-triggered matrices and the desired mode-dependent controller gains and the event-triggering function are obtained as follows

\[
\begin{align*}
K_{11} &= \begin{bmatrix} -0.1003 & 0.1766 & 0.3320 & 0.2437 \\
0.0561 & 0.2235 & 0.0124 & -0.4305 \end{bmatrix}, \\
K_{12} &= \begin{bmatrix} -0.0289 & 0.1112 & 0.0041 & -0.1854 \\
0.1271 & 0.1761 & -0.2878 & -0.6661 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} 0.1619 & 0.1093 & 0.2075 & 0.0130 \\
0.2464 & 0.1767 & 0.2676 & -0.0760 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} 0.0075 & 0.0318 & -0.2531 & -0.3644 \\
0.0089 & 0.0439 & -0.4618 & -0.6041 \end{bmatrix}, \\
\Omega_1 &= \begin{bmatrix} 0.1557 & -0.0692 & -0.2305 & -0.3693 \\
-0.0692 & 0.1006 & 0.2056 & 0.1749 \\
-0.2305 & 0.2056 & 0.6780 & 0.7481 \\
-0.3693 & 0.1749 & 0.7481 & 1.0910 \end{bmatrix}
\end{align*}
\]
Moreover, the event-triggered information transmission is more effective with broadcasting instants and release intervals. Thus, these simulation results can support our proposed scheme.

Conclusion and discussions

This manuscript is considered with the mode-dependent tracking control problem of semi-Markovian jump systems with parameter uncertainties. More precisely, the mode-dependent event-triggered transmission mechanism together with the mode-dependent controllers are designed, respectively. By employing the Lyapunov–Krasovskii method, sufficient conditions are first presented, which guarantee the prescribed $H_\infty$-tracking performance. Then, the corresponding controller gains are calculated via convex optimization method. Finally, the simulation of VTOL helicopter is performed for showing the usefulness of the developed control strategy. Our future study would focus on extending this study to the cases with asynchronous mode transitions.

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