Mass of the Lightest Higgs Boson in the Minimal Supersymmetric Standard Model with an Additional Singlet

W. T. A. ter Veldhuis

Department of Physics
Purdue University
West Lafayette, IN 47907, U.S.A.

Abstract

An upperbound on the mass of the lightest neutral scalar Higgs boson is calculated in an extended version of the minimal supersymmetric standard model that contains an additional Higgs singlet. We integrate the renormalization group equations of the model, and impose low energy boundary conditions consistent with present experimental results, and ultra-violet conditions following from triviality. Radiative corrections induced by a large top quark Yukawa coupling are included in our analysis, and we find the allowed values for the mass of the Higgs boson as a function of the mass of the top quark. Typically, for a top quark mass $m_t = 150 \text{ GeV}$, the upper bound on the Higgs boson mass is about 25 GeV higher than in the minimal model.
The predictions of the standard model are in excellent agreement with current experimental results. However, many questions remain unanswered, and in particular the situation in the Higgs sector is theoretically unsatisfactory. The standard model is not natural, in the sense that it does not explain why the electroweak scale is so tiny compared to the Planck scale. Moreover, the presence of an elementary scalar leads to quadratic divergencies, and as a consequence the scalar mass parameter has to be fine tuned in all orders of perturbation theory. Supersymmetry provides an attractive solution to this technical aspect of the naturalness problem. It is well known that the minimal supersymmetric standard model predicts a neutral Higgs boson with a mass below $M_Z$ at tree level. Recently, it was recognized that radiative corrections due to a large top Yukawa coupling can give a significant contribution to the mass, raising its limit, typically by 25 $GeV$ \cite{1, 2, 3}. This upper bound can be regarded as a very attractive feature, making the model subject to experimental verification. However as the experimental lower bound on the mass of a neutral Higgs boson increases, the question whether it is possible to extend the minimal model in a simple way to raise the upper bound on the Higgs boson mass becomes more relevant. In the present letter we therefore calculate an upper bound on the mass of the lightest neutral Higgs boson in the minimal supersymmetric standard model with an additional Higgs singlet. This extended model has been studied in some detail \cite{4, 5, 6}, however, as the top quark was thought to be much lighter at the time, radiative corrections due to a large top Yukawa coupling were not included. The implications of unitarity constraints at the unification scale on the mass of the lightest neutral Higgs boson were studied using renormalization group techniques by Durand and Lopez \cite{7}. We follow a similar tack, but the bound we set on the mass of the lightest
neutral scalar Higgs boson follows from the requirement that perturbation theory remains valid up to a cut-off scale $\Lambda$. Moreover, we include the above mentioned radiative corrections in our analysis. Typically, we take a supersymmetry breaking scale $M_{SUSY} = 10^3 \text{ GeV}$ and the cut-off scale $\Lambda = 10^{16} \text{ GeV}$, although we study the mass of the Higgs boson as a function of both $M_{SUSY}$ and $\Lambda$. Espinosa and Quiros [8], in contrast, calculated bounds on the Higgs boson mass by requiring perturbation theory to be valid below $M_{SUSY}$. Our approach is analogous to the calculation of the familiar triviality bound in the regular standard model [9, 10]. An important role is played by a so called infrared quasi-fixed point, which gives rise to strong limits on the parameter space at low energy, insensitive to the precise value of the cut-off $\Lambda$. Recently, Ellwanger et al. [11] used a crude method to obtain an estimate for the range of all Higgs masses in this extended model, taking into account the heavy top quark radiative corrections.

To be concrete, let us consider the Higgs sector of the softly broken supersymmetric standard model with an additional singlet field [4, 5]:

$$\Gamma = \int dV \left[ \left( 1 + \hat{m}_T^2 \theta^2 \bar{\theta}^2 \right) H_T \bar{H}_T + \left( 1 + \hat{m}_B^2 \theta^2 \bar{\theta}^2 \right) \bar{H}_B H_B + \left( 1 + \hat{\mu}^2 \theta^2 \bar{\theta}^2 \right) S \bar{S} \right] +$$

$$\int dS \left[ \xi S + \frac{\mu}{2} (1 + b_s \theta^2) S^2 + \frac{\lambda_s}{3} (1 + a_s \theta^2) S^3 \right] + h.c. +$$

$$\int dS \left[ m(1 + b \theta^2) H_T \bar{H}_B + g(1 + a \theta^2) S H_T \bar{H}_B \right] + h.c.. \quad (1)$$

Here $H_T$ and $H_B$ are $SU(2)$ doublets, $S$ is the additional singlet, and we have included general soft breaking terms [12]. It is interesting to note that, in contrast with the minimal supersymmetric model, it is possible to induce the correct electroweak symmetry breaking with only dimensionless supersymmetric coupling constants (i.e. $\xi = 0$ and $\mu = m = 0$) as discussed by Nilles [13]. However, here we will analyze the more general case.

The $SU(2) \otimes U(1)_Y$ symmetry is spontaneously broken into $U(1)_{EM}$, because the scalar...
components of the Higgs superfields develop vacuum expectation values:

\[
< A_B > = \frac{1}{\sqrt{2}} \begin{pmatrix} v_B \\ 0 \end{pmatrix}, \quad < A_T > = \frac{1}{\sqrt{2}} (v_T, 0), \quad < A_S > = \frac{1}{\sqrt{2}} v.
\]  

(2)

We will always assume that the coupling constants are real. As was shown by Romao [14], it then necessarily follows that the vacuum expectation values of all the Higgs fields are real and the model does not exhibit explicit CP violation. In fact, the weakness of the observed CP violation gives a physical motivation for this assumption.

The particle spectrum of this model contains, apart from the three Goldstone bosons which give rise to the weak gauge boson masses, three neutral scalar, two pseudoscalar, and two charged Higgs bosons. Their mass structure has been studied by Drees [5] and the symmetric mass matrix for the neutral scalars in the basis \( \frac{1}{\sqrt{2}} \text{Re}(A_T)_1, \frac{1}{\sqrt{2}} \text{Re}(A_B)_1, \frac{1}{\sqrt{2}} \text{Re}(A_S) \) takes the form:

\[
(M_S^2) = \begin{pmatrix}
-C_1 \tan \beta + M_Z^2 \cos^2 \beta & C_1 + 2 \left( g^2 - \frac{1}{4} g_1^2 - \frac{1}{4} g_2^2 \right) \frac{v_T v_B}{2} M_W (C_5 \cos \beta + C_4 \sin \beta) \\
& -C_1 \cot \beta + M_Z^2 \sin^2 \beta & M_W (C_5 \sin \beta + C_4 \cos \beta) \\
& & -C_6 
\end{pmatrix}.
\]  

(3)

Here

\[
C_1 = \left( \lambda_s g \frac{v^2}{2} + g a \frac{v}{\sqrt{2}} + \xi g + \mu \frac{v}{\sqrt{2}} + mb \right), \\
C_4 = \left( 2 \lambda_s g v + g a \sqrt{2} + \mu g \sqrt{2} \right), \\
C_5 = \left( 2 g^2 v + 2 m g \sqrt{2} \right), \\
C_6 = -g \frac{M_Z^2}{g_2^2} (\sin(2\beta)(a + \mu) + 2m) \frac{v}{\sqrt{2}} + 2 \lambda_s^2 v^2 + \lambda_s \frac{v}{\sqrt{2}} (a_s + 3\mu) - \frac{v}{\sqrt{2}} \mu \xi.
\]

We will consider the case in which there is only one light Higgs boson. This situation is realized if \( |C_1| >> M_Z^2 \), and the lightest scalar mass eigenstate is approximately, to order
\[ \left( \frac{M_Z^2}{C_1^2} \right) , \frac{1}{\sqrt{2}} Re \left( \cos \beta A_T + \sin \beta A_B \right) \] with corresponding mass:

\[ M_H^2 = M_Z^2 \left( \cos^2(2\beta) + 2g_1^2 g_2^2 \sin^2(2\beta) \right). \]  

(4)

Even in the case that the condition \(|C_1| >> M_Z^2\) is not fulfilled, equation (4) still provides a useful upper bound on the mass of the lightest neutral Higgs boson. This can easily be seen by diagonalizing the two by two submatrix in the left hand top corner of eq. (3), and calculating the minimum of the lowest eigenvalue as a function of \(C_1\).

Let us analyze expression (4) as a function of \(\beta\). One has to distinguish two cases. If \(2g_{1+2} < 1\), then \(M_H\) has a maximum for \(\beta = 0\). This maximum value is independent of \(g\) and equal to the maximum value in the minimal supersymmetric standard model. On the other hand, if \(2g_{1+2} > 1\), then \(M_H\) reaches its maximum value for \(\beta = \pi/4\). In the latter case \(g\) determines the maximum value of \(M_H\), and the situation is considerably different from the minimal supersymmetric standard model.

Now that the spectrum of the Higgs sector at tree level has been discussed, we will proceed by including one loop quantum corrections in a renormalization group analysis of the mass of the lightest Higgs boson. We assume there is no new physics between the cut-off scale \(\Lambda\) and the soft supersymmetry breaking scale \(M_{SUSY}\). The existence of this so called desert enables us to use the renormalization group equations of the supersymmetric model to relate the couplings at the cut-off scale \(\Lambda\) to their values at the supersymmetry breaking scale \(M_{SUSY}\). The requirement of consistency of perturbation theory puts a constraint on the value of \(g(M_{SUSY})\), because the structure of the renormalization group equations causes the Higgs self couplings to become singular at an energy scale \(\mu < \Lambda\) if their low energy values are too big. We therefore introduce a new variable \(\tan(\alpha)\), and impose high energy boundary conditions on \(\lambda_s\) and \(g\) by requiring that at least one of them becomes non-perturbative at
the cut-off scale $\mu = \Lambda$:
\[
\tan \alpha = \frac{\lambda_s(\Lambda)}{g(\Lambda)}, \quad \text{with} \quad g^2(\Lambda) + \lambda_s^2(\Lambda) = 100.
\]

The choice of the right hand side of equation(5) is quite arbitrary, as long as the number is large, since the nature of the renormalization group equation is such that solutions which are rather far apart at high energy approach each other at low energy. Defining the scaling variable $t = \ln\left(\frac{\mu}{M_{Z}}\right)$, the evolution of the coupling constants between $\Lambda$ and $M_{SUSY}$ is given by their one loop renormalization group equations [15]:
\[
(4\pi)^2 \frac{dg^2}{dt} = (4\lambda_s^2 + 8g^2 + 6h_t^2 - 2g_1^2 - 6g_2^2) g^2,
\]
\[
(4\pi)^2 \frac{d\lambda_s^2}{dt} = (12\lambda_s^2 + 12g^2) \lambda_s^2,
\]
\[
(4\pi)^2 \frac{dh_t^2}{dt} = \left(2g^2 + 12h_t^2 - \frac{26}{9} g_1^2 - 6g_2^2 - \frac{32}{3} g_3^2\right) h_t^2,
\]
\[
(4\pi)^2 \frac{dg_i^4}{dt} = \beta_i g_i^4,
\]
with $\beta_1 = 22$, $\beta_2 = 2$ and $\beta_3 = -6$. In the case that only one Higgs boson has mass below $M_{SUSY}$, the lightest neutral scalar state and the Goldstone bosons will form the doublet that is almost a mass eigenstate in the following way:
\[
\begin{align*}
H_H &= -\sin(\beta) A_T + \cos(\beta) \overline{A}_B, \\
H_L &= \cos(\beta) A_T + \sin(\beta) \overline{A}_B.
\end{align*}
\]

We assume that the heavy Higgs doublet $H_H$ decouples at $M_{SUSY}$, and the theory below $M_{SUSY}$ is equivalent to the regular standard model, with Higgs potential:
\[
V = m_0^2 H_L^2 + \lambda (H_L H_L)^2.
\]
This approximation makes it possible to relate the coupling constants of the standard model to those of the supersymmetric model at $M_{SUSY}$. To be concrete, we find the following boundary conditions at $M_{SUSY}$:

$$
\lambda = \frac{1}{8} \left( g_1^2 + g_2^2 \right) \left( \cos^2(2\beta) + 2 \frac{g_2^2}{g_1^2 + g_2^2} \sin^2(2\beta) \right), \quad \text{(12)}
$$

$$
h_t' = h_t \cos(\beta). \quad \text{(13)}
$$

Here $h_t'$ is the top Yukawa coupling in the standard model. In addition, below the supersymmetry breaking scale $M_{SUSY}$, the superpartners decouple and do not contribute to the renormalization group equations any more. In order to take this into account, we utilize the renormalization group equations of the standard model to run the coupling constants down from $M_{SUSY}$ to the electroweak scale. To one loop order these renormalization group equations are [17]:

$$
(4\pi)^2 \frac{d\lambda}{dt} = 24\lambda^2 + \left( 12h_t'^2 - 3g_1^2 - 9g_2^2 \right) \lambda + \frac{3}{8}g_1^4 + \frac{3}{4}g_1^2g_2^2 + \frac{9}{8}g_2^4 - 6h_t'^4, \quad \text{(14)}
$$

$$
(4\pi)^2 \frac{dh_t'^2}{dt} = 9h_t'^4 - \left( \frac{17}{6}g_1^2 + \frac{9}{2}g_2^2 + 16g_3^2 \right) h_t'^2, \quad \text{(15)}
$$

$$
(4\pi)^2 \frac{dg_i^4}{dt} = \beta_i g_i^4, \quad \text{(16)}
$$

with $\beta_1 = \frac{41}{3}$, $\beta_2 = -\frac{19}{3}$ and $\beta_3 = -14$. We thus include leading log radiative corrections proportional to $\ln(\frac{M_{SUSY}}{M_Z})$, in particular those caused by a large top Yukawa coupling $h_t$, but neglect finite corrections. Current experimental results lead us to impose the following low energy boundary conditions on the gauge coupling constants: $g_1^2(M_Z) = 0.1282$, $g_2^2(M_Z) = 0.4222$ and $g_3^2(M_Z) = 1.445$. To complete the set of input parameters, we take $M_Z = 91.0$ GeV. The vacuum expectation value $v_0$ of the light Higgs boson is then fixed to be:

$$
v_0 = 2 \frac{M_Z}{\sqrt{g_1^2 + g_2^2}}, \quad \text{(17)}
$$
and \( m_t \) and \( m_H \) are defined as follows:

\[
m^2_H = 2\lambda(m_H)v_0^2, \tag{18}
\]

\[
m^2_t = \frac{v_0^2}{2}h'_t(m_t). \tag{19}
\]

The mass of the lightest neutral Higgs boson is enhanced by radiative corrections as well as by the second term in eq.(4). An estimate of the magnitude of the radiative corrections can be obtained in the following way \cite{14}. If the top quark is heavy, the \( h'_t^4 \) term in eq.(14) will dominate. Keeping only this term and assuming \( h'_t \) to be constant, the renormalization group equation can be solved, and one obtains:

\[
(4\pi)^2\lambda(t) = (4\pi)^2\lambda(t_{\text{SUSY}}) + 6h'_t(t_{\text{SUSY}} - t). \tag{20}
\]

This leads to a positive contribution to \( m^2_H \):

\[
\delta m^2_H \approx \frac{48}{(4\pi)^2} \frac{m_t^4}{v_0^2} \ln \left( \frac{M_{\text{SUSY}}}{m_H} \right). \tag{21}
\]

Note however, the situation here differs from the minimal supersymmetric standard model, because \( h_t \) affects the running of \( g \) above \( M_{\text{SUSY}} \). It can be easily seen from equation(13) that \( h_t \) becomes larger if \( \beta \) increases for a fixed value of the top quark mass \( m_t \). Moreover, according to equation(3), in that case the value of \( g(M_{\text{SUSY}}) \) decreases for a given value of \( g(\Lambda) \). However, it follows from boundary condition(12) that if \( 2\frac{g}{g_1+g_2} > 1 \), \( \lambda(M_{\text{SUSY}}) \) is maximal for a given value of \( g(M_{\text{SUSY}}) \) for \( \beta = \frac{\pi}{4} \). We therefore expect that the mass of the lightest neutral scalar Higgs boson reaches its maximum for a value of \( \beta \) somewhere in between 0 and \( \frac{\pi}{4} \) for sufficiently small \( \tan(\alpha) \).

Before introducing our results, we now outline our computational procedure. We choose values for the free parameters \( \tan\alpha, \tan\beta, \Lambda \) and \( M_{\text{SUSY}} \), and we pick a value for the top
quark mass $m_t$ in the currently expected range. Subsequently, we numerically integrate the system of differential equations and find a solution that satisfies all boundary conditions. The mass of the lightest neutral Higgs boson is then extracted using equation (18). Since we neglect the bottom quark Yukawa coupling, our results are valid if $h_t >> h_b$, which means $\tan(\beta) >> \frac{m_b}{m_t} \approx 0.03$.

In figure 1 we plot the mass of the lightest neutral Higgs boson as a function of $\tan \beta$. Motivated by grand unification, we choose a cut-off scale $\Lambda = 10^{16} \text{ GeV}$ and $M_{SUSY} = 10^3 \text{ GeV}$. For small values of $\tan(\beta)$, the first term in equation (12) dominates. This term is independent of $\lambda_s$ and $g$, and so for all values of $\tan(\alpha)$ we find that the mass of the lightest neutral Higgs boson is equal to the mass in the minimal supersymmetric standard model enhanced by the heavy top quark radiative corrections. However, if we follow the Higgs mass towards larger values of $\tan(\beta)$, we observe that for small values of $\tan(\alpha)$ the mass reaches a maximum of $130 \text{ GeV}$ for $\tan(\beta) = 0.6$. On the other hand, for large values of $\tan(\alpha)$, the Higgs mass decreases monotonously as a function of $\tan(\beta)$. In fact, in the limit $\tan(\alpha) \rightarrow \infty$ the singlet decouples, and we obtain the minimal supersymmetric standard model.

A cutoff scale of $10^{16} \text{ GeV}$ is attractive because the flow of the renormalization group in combination with the low energy boundary conditions forces the trajectories of the three gauge couplings to intersect at one point. However, we do not limit ourselves to this case, and in figure 2 we plot the triviality bound on the mass of the lightest neutral scalar Higgs boson as a function of the cut-off scale $\Lambda$. Again we chose $m_t = 150 \text{ GeV}$, $M_{SUSY} = 1 \text{ TeV}$ and we took $\tan(\beta) = 0.6$ and $\tan(\alpha) = 0.1$, since the Higgs boson mass reaches its maximum for these values. As the cut-off scale $\Lambda$ is lowered, the upper bound on the lightest neutral Higgs mass increases, and in the limit $\Lambda \rightarrow M_{SUSY}$ the upper bound on the lightest Higgs boson approaches $M_H = 800 \text{ GeV}$, the triviality bound in the regular standard model.
In figure 3 we show a triviality plot of $m_H$ versus $m_t$ for a cut-off scale $\Lambda = 10^{16} \text{ GeV}$ and a supersymmetry breaking scale $M_{\text{SUSY}} = 1 \text{ TeV}$. The boundary of the enclosed area indicates that either $\lambda_s$, $g$ or $h_t$ becomes non-perturbative for $\mu \leq \Lambda$ for all values of $\tan \beta$ and $\tan \alpha$. In other words, there exist finite values of the coupling constants $\lambda_s(\Lambda)$, $g(\Lambda)$ and $h_t(\Lambda)$ that give combinations of the top quark mass $m_t$ and the lightest neutral Higgs boson mass $M_H$ within this region. The dashed line indicates combinations of $m_t$ and $M_H$ that can only be reached if $h_t(\Lambda)$ becomes singular. The dotted lines below the solid line show $M_H$ as a function of $m_t$, for $\tan(\alpha) = 10.0$ and various values of $\tan(\beta)$. As we have discussed before, in the limit of large $\tan(\alpha)$ we obtain the minimal supersymmetric standard model. Hence, points on the dashed line represent the predictions of the supersymmetric top quark condensate model [18, 19, 20]. In particular, the fact that $h_t(\Lambda) \to \infty$ corresponds exactly to the compositeness condition in that model. The solid line shows the relation between $M_H$ and $m_t$ for $\tan(\alpha) = 10.0$ and $\tan(\beta) = 0.1$. Consequently, the dashed line and the solid line encompass the area of allowed $m_t$ and $M_H$ in the minimal supersymmetric standard model.

As a matter of fact, the solid line gives the sum of $M_Z$ and the radiative corrections to the Higgs mass as a function of $m_t$ in the minimal supersymmetric model. For small values of $m_t$, the maximum value of $M_H$ is reached for $\beta = \frac{\pi}{4}$, but as the top mass increases, $M_H$ is maximal for lower values of $\beta$. Indeed, for very large values of $m_t$ the highest value for $M_H$ is obtained for $\beta = 0$. The dotted lines above the solid line show $M_H$ as a function of $m_t$ for $\tan(\alpha) = 0.1$ and various values of $\tan(\beta)$. The envelop of these dotted lines gives the maximum of $M_H$ as a function of $m_t$ in the extended model.

Finally, we show in figure 4 how the mass of the lightest neutral Higgs boson depends on the SUSY breaking scale $M_{\text{SUSY}}$ for various values of $\tan \beta$ and $\tan \alpha$. It is clear that the upper bound on the lightest neutral Higgs boson increases as $m_{\text{SUSY}}$ becomes larger, since
the radiative corrections become stronger.

In conclusion, we have studied the mass of the lightest neutral scalar Higgs boson in an extended version of the minimal supersymmetric standard model, with an additional Higgs singlet. We have included radiative corrections in the leading log approximation. Since we have assumed hard decoupling of the super-partners, we have neglected threshold effects. Moreover, we have assumed that one Higgs boson has mass below $M_{\text{SUSY}}$, and that the heavy doublet and singlet decouple at $M_{\text{SUSY}}$. However, even if this is not the case realized in nature, our upper bounds remain valid. At tree level it is obvious that the lightest neutral scalar mass eigenstate becomes lighter if other Higgs bosons do not decouple. Quantum effects would manifest themselves by a different running of the coupling constants below $M_{\text{SUSY}}$, but the running of the top Yukawa coupling, the main source of radiative corrections, would not be affected significantly. We have checked this explicitly by running the coupling constants down with the appropriate renormalization group equations for a model with two Higgs doublets and one Higgs singlet below $M_{\text{SUSY}}$. The upper bound on the lightest neutral scalar Higgs calculated in this way is virtually identical to the mass we calculate in our model with only one light Higgs doublet. As a consequence the upper bounds we calculate are independent of the details of the soft supersymmetry breaking parameters. Furthermore we have ignored the bottom quark Yukawa coupling. In light of the large mass difference between the top and bottom quark, this assumption seems very reasonable. The results of our calculations show that, in particular for a low top quark mass, the lightest neutral scalar Higgs boson can be significantly heavier than in the heavy top enhanced minimal supersymmetric standard model. To be more specific, for $m_t = 100$ GeV, $M_H$ can be as much as 55 GeV heavier than in the minimal model, and for $m_t = 150$ GeV, a value more likely, the upper bound on $M_H$ is about 25 GeV higher.
Acknowledgement

The author would like to thank Tom E. Clark for useful suggestions and interesting comments.
References

[1] J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257** (1991) 83.

[2] Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1.

[3] M.A. Diaz and H.E. Haber, *Santa Cruz preprint SCIPP-91/14* (1991).

[4] J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Phys. Rev.* **D39** (1989) 844.

[5] M. Drees, *Int. J. Mod. Phys.* **A4** (1989) 3635.

[6] J.F. Gunion and H.E. Haber, *Nucl. Phys.* **B272** (1986) 1.

[7] L. Durand and J.L. Lopez, *Phys. Lett.* **B216** (1989) 463.

[8] J.R. Espinosa and M. Quiros, *Phys. Lett.* **B266** (1991) 389.

[9] C.T. Hill, C.L. Leung and S. Rao, *Nucl. Phys.* **B262** (1985) 517.

[10] C.T. Hill, *Phys. Rev.* **D24** (1981) 691.

[11] U. Ellwanger and M. Rausch de Traubenberg, *Z. Phys.* **C53** (1992) 521.

[12] L.Girardello and M.T. Grisaru, *Nucl. Phys.* **B194** (1982) 65.

[13] H.P. Nilles, *Phys. Reports* **110** (1984) 1.

[14] J.C. Romao, *Phys. Let.* **B173** (1986) 309.

[15] J.P. Derendinger and C.A. Savoy, *Nucl. Phys.* **B237** (1984) 307.

[16] Y. Okada, M. Yamaguchi and T. Yanagida, *Phys. Lett.* **B262** (1991) 54.
[17] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647.

[18] T.E. Clark, S.T. Love and W.A. Bardeen, Phys. Lett. B237 (1990) 235.

[19] K. Sasaki, M. Carena and C.E.M. Wagner, Max Plank Institute preprint MPI-Ph/91-109 (1991).

[20] M. Carena, T.E. Clark, C.E.M. Wagner, W.A. Bardeen and K. Sasaki, Nucl. Phys. B369 (1992) 33.
FIGURE CAPTIONS

**Fig. 1.** Mass of the lightest neutral Higgs boson $M_H$ as a function of $\tan(\beta)$, for various values of $\tan(\alpha)$: $\tan(\alpha) = 0.1$ (solid line), $\tan(\alpha) = 1.0$ (dotted line), $\tan(\alpha) = 3.0$ (dashed line), $\tan(\alpha) = 6.0$ (dot-dashed line) and $\tan(\alpha) = 10.0$ (dot-dot-dashed line).

**Fig. 2.** Upper bound on the mass of the lightest neutral Higgs boson as a function of the cut-off scale $\Lambda$, for a top quark mass $m_t = 150$ GeV and a supersymmetry breaking scale $M_{SUSY} = 10^3$ GeV.

**Fig. 3.** Triviality diagram, indicating possible values of the top quark mass $m_t$ and lightest neutral Higgs boson mass $M_H$ consistent with perturbation theory for a cut-off scale $\Lambda = 10^{16}$ GeV and a supersymmetry breaking scale $M_{SUSY} = 10^3$ GeV.

**Fig. 4.** Upper bound on the mass of the lightest neutral Higgs boson $M_H$ as a function of the SUSY breaking scale $M_{SUSY}$, for a top quark mass $m_t = 150$ GeV, a cut-off scale $\Lambda = 10^{16}$ GeV and various combinations of $\tan(\alpha)$ and $\tan(\beta)$: $\tan(\alpha) = 0.1$ and $\tan(\beta) = 0.6$ (solid line), $\tan(\alpha) = 0.1$ and $\tan(\beta) = 0.1$ (dashed line), $\tan(\alpha) = 10.0$ and $\tan(\beta) = 0.6$ (dotted line), and finally $\tan(\alpha) = 10.0$ and $\tan(\beta) = 0.1$ (dot-dashed line).