ABSTRACT

We address the deviations of the scaling relations of elliptical galaxies from the expectations based on the virial theorem and homology, including the “tilt” of the “fundamental plane” and the steep decline of density with mass. We show that such tilts result from dissipative major mergers once the gas fraction available for dissipation declines with progenitor mass, and derive the scaling properties of the progenitors. We use hydrodynamical simulations to quantify the effects of major mergers with different gas fractions on the structural properties of galaxies. The tilts are driven by the differential shrinkage of the effective stellar radius as a function of dissipation in the merger, while the correlated smaller enhancements in internal velocity and stellar mass keep the slope of the velocity-stellar mass relation near $V \propto M^{1/4}$. The progenitors match a straightforward model of disc formation in ΛCDM haloes. Their total-to-stellar mass ratio within the effective radius varies as $M/M \propto M^{-0.2-0.3}$, consistent with the effect of supernova feedback. They are nearly homologous in the sense that the dark-matter fractions within the effective and virial radii scale with mass in a similar way, with only a weak decline of density with mass. The progenitor radius is roughly $R \propto M^{0.3}$, compatible with today’s intermediate late-type galaxies. This may indicate that the latest dissipative mergers in the history of ellipticals involved relatively big discs. The dissipative gas-to-stellar mass ratio is predicted to decline $\propto M^{-(0.4-0.6)}$, similar to the observed trend in today’s blue-sequence galaxies. Such a trend should be observed in relatively massive gaseous galaxies at $z \sim 1-4$. The corresponding “baryonic” relations are consistent with homology and spherical virial equilibrium, $V \propto R \propto M^{1/3}_{\text{bar}}$.

Key words: dark matter — galaxies: ellipticals — galaxies: evolution — galaxies: formation — galaxies: haloes — galaxies: mergers

1 INTRODUCTION

The global properties of elliptical galaxies show non-trivial correlations whose origin is one of the most interesting issues in the study of galaxy formation. These correlations may tell us whether the ellipticals could have indeed formed by mergers, and if so what were the properties of their initial progenitors. We address these issues in this paper.

We appeal here to three observable global properties of spheroidal stellar systems: the total luminosity $L$, the “effective” radius $R$ encompassing half the total luminosity in projection, and the effective velocity dispersion $\sigma$.\footnote{In the literature, the radius is sometimes replaced by surface brightness. The use of “effective” surface brightness $I = L/R^2$ is equivalent to using $R$, but not exactly so when $I$ and $\sigma$ refer instead to the “central” quantities. Nevertheless, as long as the density profile is close to universal, the central quantities should roughly scale with the effective quantities.} The galaxies lie on a “Fundamental Plane” (FP) in the three-dimensional parameter space defined by the logarithms of these quantities$^{1}$, with very little scatter about it. The projections of this plane onto the planes defined by the logarithms of the axes $\sigma - L$ and $R - L$ are well fit by straight lines, namely “scaling relations”, following$^{2}$ Faber & Jackson (1976) and Kormendy (1977).

As specified in the following section, the fundamental plane is “tilted” compared to the plane implied by the virial theorem. While part of this tilt can be attributed to systematic variations in stellar populations$^{3}$, the other part indicates a breakdown of the homol-

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$^{1}$ Dressler et al. (1987), Diomoge\'ski & Davis (1985), Bernardi & et al. (2003).

$^{2}$ Faber & Jackson (1976) and Kormendy (1977).

$^{3}$ Faber et al. (1987).
ology, or self-similarity, of the elliptical family, which can be expressed as a systematic increase of dark-matter fraction within the effective radius as a function of galaxy mass (Padmanabhan et al. 2004, Onorbe et al. 2003). Another indication of a breakdown in homology is provided by the scaling relations, which imply a steep decline of mean effective density with mass, steeper than the moderate decline expected based on the properties of dark-matter haloes in the standard ΛCDM cosmology (Bullock et al. 2001). Supporting evidence for non-homology comes from the fact that the surface-brightness profiles of larger ellipticals appear to be steeper, with a larger Sersic index (Caon et al. 1993; Graham & Guzmán 2003). We seek understanding of these deviations from homology.

In this paper, we address the structural changes induced by dissipative mergers. Our simple analytic analysis is based on power-law recipes for how the changes in the structure parameters due to mergers depend on the effective gas fraction in the progenitors. We deduce these recipes from a suite of disc merger simulations with a varying degree of gas fraction. This analysis allows us to connect the observed FP and scaling relations of ellipticals with the properties of their progenitors via simple equations. We use them to put constraints on the properties of these progenitors, their scaling relations and especially the systematic trend of gas fraction with mass in them.

In §2 we spell out the observed slopes of the scaling relations and fundamental plane, and the implied deviations from homology. In §3 we discuss the possible origin of these structural deviations, and motivate the study of the role in homology. In §4 we formulate the merger process in terms of simple power-law relations and derive equations that connect the observed relations for elliptical galaxies with the properties of their original progenitors, including the variation of gas fraction with mass. In §5 we use merger simulations to calibrate the necessary power-law approximations for the dependence of the structural changes in mergers on the gas fraction. In §6 we use the recipes derived from the simulations to solve for the properties of the progenitors given the observed structural relations. In §7 we compare the required progenitor properties to what might be expected based on other considerations. In §8 we summarize our conclusions.

2 OBSERVED SCALING RELATIONS

We write the observed scaling relations for elliptical galaxies as

\[ \sigma \propto L^{\tilde{\alpha}} , \quad R \propto L^{\tilde{\beta}} , \]

where \( L \) is the absolute luminosity in a given band. The slopes \( \tilde{\alpha} \) and \( \tilde{\beta} \) were determined by Bernardi et al. (2003) for a large sample from the Sloan Digital Sky Survey (SDSS), applying a maximum-likelihood analysis for the conditional average of the log of the relevant quantity at a given luminosity, namely

\[ \langle \log \sigma | L \rangle = \tilde{\alpha} \log L + \text{const.} , \quad \text{etc.} \]

Unless specified otherwise, we assume hereafter analogous definitions of slopes for all the power-law relations. The most recent values based on revised photometry in the \( r \) band (M. Bernardi, private communication) are \( \tilde{\alpha} = 0.230 \pm 0.012 \) and \( \tilde{\beta} = 0.704 \pm 0.025 \), with conditional standard deviations of the logs 0.063 and 0.115 respectively. The slopes in the other SDSS bands are not very different.

The Fundamental Plane as determined from SDSS by Bernardi & et al. (2003) is similar to the original FP by Dressler et al. (1987) and Djorgovski & Davis (1987) and can be approximated by

\[ \sigma^2 R \propto L^{1 + \tau} , \]

with \( \tau \simeq 0.17 \) and a very small scatter about it. The way the slopes are determined, the value of \( \tau \) is expected to be related to the slopes of the scaling relations via \( 1 + \tilde{\tau} = 2 \tilde{\alpha} + \tilde{\beta} \). The revised slopes of the scaling relations indeed imply \( \tilde{\tau} \simeq 0.164 \).

For the purpose of studying structural and dynamical issues we rather replace the luminosity \( L \) with the corresponding stellar mass \( M \) (which for ellipticals roughly constitutes the whole baryonic mass). We parameterize the systematic variation in stellar mass-to-light ratio due to stellar populations by

\[ \frac{M}{L} \propto L^{\tau} , \]

and for the sake of simplicity ignore the scatter about this relation. The structural relations for ellipticals within the effective radius become

\[ \sigma \propto M^{\alpha} , \quad R \propto M^{\beta} , \quad \sigma^2 R \propto M^{1 + \tau} , \]

with

\[ \alpha = \frac{\tilde{\alpha}}{1 + \tau_*} , \quad \beta = \frac{\tilde{\beta}}{1 + \tau_*} , \quad 1 + \tau = \frac{1 + \tilde{\tau}}{1 + \tau_*} . \]

The value of \( \tau_* \) is believed to be on the order of 0.1 (e.g. Shen et al. 2003). It can therefore make only a \( \sim 10\% \) difference to the scaling slopes \( \alpha \) and \( \beta \), but a major difference to the FP tilt \( \tau \).

The scaling relations are non-trivial in two ways. First, the FP is tilted relative to the virial plane,

\[ \sigma^2 R \propto M , \]

which is expected to be roughly valid for the total mass within the effective radius if the ellipticals are a homologous family. The tilt reflects a systematic variation of the total mass-to-light ratio within the effective radius, \( M/L \propto L^{\tau} \). This could be partly due to stellar populations and partly a structural tilt,

\[ \frac{M}{M} \propto M^{\tau} , \quad \tau = 2\alpha + \beta - 1 . \]

If indeed \( \tau > 0 \), it represents a breakdown of homology for these systems, indicating that the mass fraction of dark matter within the effective radius is increasing with mass. Furthermore, any non-negative tilt \( (\tau \geq 0) \) for the merger remnants is non-trivial because the progenitors are actually expected to show a negative tilt relative to the virial plane. This is if the variation of \( M/M \) with progenitor mass is attributed to the decreasing effectiveness of supernova feedback in suppressing star formation as a function of mass (Dekel & Silk 1986; Dekel & Woo 2003).

\[ \text{revised from the earlier published values of 0.255 and 0.632.} \]
Second, the virial relation also implies that the mean total mass density within the effective radius varies with $M$ as
\[ \rho \propto \frac{\sigma^2}{R^2} \times M^\alpha, \quad \delta = 2\alpha - 2\beta. \] (9)

The observed values of $\delta$ and $\beta$ correspond to $\delta \approx -0.95/(1 + \tau_e)$, namely a rather steep decline of density with mass, which provides an even stronger evidence for a homology breakdown. The simplest self-similar model of simultaneous spherical collapse predicts virial equilibrium at a constant mean density, with $\alpha = \beta = 1/3$. Indeed, numerical studies of dark-matter haloes in ΛCDM cosmological simulations reveal only a weak systematic density variation, weaker than $\delta \approx -0.25$ (based on Bullock et al. 2001). The steep decline observed for ellipticals thus implies a breakdown of homology. Unlike the tilt of the FP, the density decline cannot be explained by a variation in the stellar $M/L$ for any sensible value of $\tau_e$.

In our analysis below we adopt $\tau_e = 0.1$, but this particular choice within the range of uncertainty does not have a significant effect on our conclusions. The targets for our modeling thus become $\alpha \approx 0.21$ and $\beta \approx 0.64$, with the associated $\tau_e \approx 0.06$ and $\delta = -0.86$.

### 3 POSSIBLE ORIGIN OF THE TILTS

When addressing the possible physical origin of the structural tilts, we adopt the standard paradigm of galaxy formation. It assumes that the accretion of cold gas in dark-matter haloes leads to central gaseous discy systems in which stars form (White & Rees 1978; Fall & Efstathiou 1980; Mo et al. 1998). Major mergers between such systems of comparable masses produce more stars, consume the available cold gas, and transform the stellar systems into elliptical configurations (Fall 1979; Jones & Efstathiou 1977; Aarseth & Fall 1980; Barnes 1999). In haloes more massive than $\sim 10^{12}M_\odot$, there is a shutdown of gas cooling and star formation, allowing the stellar population to age passively and turn “red and dead” (e.g. Birnboim & Dekel 2003; Binnen 2004; Keres et al. 2005; Dekel & Birnboim 2006; Croton et al. 2006; Cattaneo et al. 2004). These ellipticals may grow further by non-dissipative mergers with other ellipticals. One of the open questions is whether the dissipative mergers in the history of big ellipticals involved big systems in a relatively late epoch or small systems earlier on, followed by major non-dissipative mergers.

As a simple and general model for the mergers that transform the initial systems into ellipticals, we assume that progenitor galaxies of type “d” are transformed by mergers to remnant galaxies of type “e”. Each galaxy is characterized by the three global quantities: a stellar mass $M$, an effective stellar radius $R_e$, and a characteristic velocity $V_c$ (circular velocity for discs or velocity dispersion for spheroids, up to a multiplicative factor of order unity). The dimensionless growth factors describing the structural changes due to mergers, $R_e/R_d$, $V_c/V_d$ and $M_*/M_d$, are determined by the merger dynamics. If the progenitors were completely homologous, and all the processes involved in a merger were scale free, then these factors were independent of mass, and the mergers would have led from power-law scaling relations to similar power laws, though in general with different zero-points. This would have meant that the structural plane in $M$-$R$-$V$ and its projections are preserved under mergers, but perhaps with an increased scatter.

In particular, if the elliptical galaxies are homologous, and the merger dynamics involves gravitational dynamics only, then the FP should be preserved under a few major mergers. Such a behavior has indeed been noticed in N-body simulations of dissipationless mergers (Boylan-Kolchin et al. 2005; Robertson et al. 2006). The question is thus how did the elliptical galaxies make it to the FP in the first place. This requires a breakdown of the self-similarity of at least one property of the progenitors or the processes acting during the mergers. The dimensionless growth factors should depend on mass due to some important merger characteristic that varies strongly with mass.

The gravitational process associated with spherical collapse and virial equilibrium are not expected to break the self-similarity of the progenitors in a major way. The systematic variation of the characteristic collapse time with mass introduces a relatively weak variation in halo concentration which is not enough for explaining the deviations from homology. The origin of angular momentum via tidal torques is also largely self-similar (Fall & Efstathiou 1980; Bullock et al. 2001), implying disc sizes which are close to self-similar. The distribution of merger orbit parameters is not expected to vary significantly with mass either (Vitvitska et al. 2002).

On the other hand, gas dissipation may lead to a more substantial breakdown of homology. Once the progenitors contain gas, the dissipation involved in the merger leads to a more centrally concentrated stellar system than in the non-dissipative case. The tidal interactions during the close passages induce collisions and shocks which condense and heat the gas. This is followed by enhanced radiative cooling and star formation, ending up with a more compact stellar system deeper in the potential well.

Dissipation is indicated, for example, by the high phase-space density in the centers of ellipticals (Hernquist et al. 1993; Robertson et al. 2006), and by the tendency for oblateness and kinematic misalignment of ellipticals compared to simulated merger remnants (Cox et al. 2006).

A property of the gas which depends on galaxy mass and could therefore produce structural tilts in the scaling relations is the cooling efficiency on a dynamical timescale, which declines with halo mass (e.g., Rees & Ostriker 1977; Blumenthal et al. 1984). A tilt of this sort has been seen in remnants of hydrodynamical merger simulations by Robertson et al. 2006). Their Fig. 11 shows a tilt in the $R$-$M$ relation when the gas fraction is 0.4 (or higher), but the change from $\beta \approx 0.44$ to 0.51 is small compared to the required tilt. A careful inspection of this figure reveals that for a fixed gas fraction the tilt is noticeable only for giant ellipticals with stellar masses above $\sim 5 \times 10^{11}M_\odot$, while the observed tilts extend far below this mass.

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3 Phase mixing and other gravitational relaxation processes may eventually push such systems toward the virial plane obeyed by the total mass, but this is apparently a long term process that requires many successive mergers, and, anyway, it does not produce a tilt toward the FP.
A more pronounced violation of the self-similarity of the dissipative processes may arise from a systematic variation of the gas fraction with progenitor mass. The relative amount of gas in the merging systems has a strong effect on the resulting stellar mass and structure of the remnant. More gas naturally results in more new stars, and the higher degree of dissipation involved in the merger leads to an enhanced shrinkage in the radius $R$ compared to the non-dissipative case. Based on a combination of physical considerations and numerical simulations we have learned that the fraction of new stars and the radiative energy loss during the merger are both roughly proportional to the effective gas fraction $V$ (Covington et al. 2006). These two effects make the decline in $R/M$ due to mergers depend strongly on the gas fraction. Together, the increase in $V$ is not enough to compensate for the strong decrease in $R/M$, yielding a strong gas-dependent decline in the combination relevant to the dissipation processes. Based on a combination of physical considerations and recent simulations in Cox (2004), two identical spiral galaxies are put in collision and supernova feedback (Springel 2000) is treated using the entropy-conserving, gravitating, Smoothed Particle Hydrodynamics (SPH) code GADGET (Springel et al. 2001). Gas cooling, star formation and supernova feedback are treated using simplified recipes that were calibrated to match observed star-formation rates. The fiducial case is G3-G3 from the suite of merger simulations in Cox (2004). Two identical spiral galaxies are put on a parabolic orbit, each consisting of stellar and gaseous discs and a stellar bulge, all embedded in a ΛCDM halo. The gravitational and hydrodynamical evolution is followed using the entropy-conserving, gravitating, Smoothed Particle Hydrodynamics (SPH) code GADGET (Springel et al. 2001). Gas cooling, star formation and supernova feedback are treated using simplified recipes that were calibrated to match observed star-formation rates. The structure of the fiducial progenitor disc galaxies mimics Sb-Sb spirals today, similar to the Milky Way. The dark-matter halo virial mass is $116 \times 10^{10} M_\odot$, represented by 120,000 particles of $(0.97 \times 10^5 M_\odot$ each). The virial radius is 272 kpc, corresponding to a virial velocity of $135 \text{ km s}^{-1}$. The halo density profile is NFW (Navarro et al. 1997), with a concentration parameter $C = 6$, and a spin parameter $\lambda = 0.05$. In the fiducial case, the baryonic fraction of the total

$$\frac{(V_e/V_d)}{(M_e/M_d)^\gamma} \propto M_e^{\gamma_e - \alpha_d}, \quad \frac{(R_e/R_d)}{(M_e/M_d)^\beta} \propto M_e^{\beta_e - \beta_d}. \quad (11)$$

We then parameterize the structural growth factors in the mergers as functions of $g$ by the power laws

$$\frac{R_e}{R_d} \propto g^r \frac{V_e}{V_d} \propto g^r \frac{M_e}{M_d} \propto g^m. \quad (12)$$

Finally, we assume that the mean dependence of $g$ on progenitor mass can be parameterized by,

$$g \propto M_d^{-\gamma}. \quad (13)$$

When substituting these power laws in eq. (10) and eqs. (11), we obtain useful relations between the power indices involved,

$$2\nu + r = (1 + \tau_e) m - \gamma^{-1}(\tau_e - \tau_d), \quad (14)$$

$$\gamma(-\nu + m\alpha_d) = (\alpha_e - \alpha_d), \quad (15)$$

$$\gamma(-\nu + m\beta_d) = (\beta_e - \beta_d). \quad (16)$$

These three equations are degenerate; only two are independent, with $\tau = 2\alpha + \beta - 1$ for “$d$” and for “$e$”. Once we know the growth-factor indices $r$, $\nu$ and $m$ from theory, these equations connect the final structural tilts to the properties of the progenitors, which are described by three independent variables: the gas-fraction gradient index $\gamma$, and the slopes $\alpha_d$ and $\beta_d$ (one of which can be replaced by $\tau_d$ or $\delta_d$). For given values of the observed parameters $\alpha_e$ and $\beta_e$ (or $\tau_e$), the solutions for the progenitors are a one-parameter family. Namely, for any assumed value of $\gamma$ there is a unique solution for $\alpha_d$, $\beta_d$, etc.

5 MERGER SIMULATIONS: GAS DEPENDENCE

In order to determine the dependence of the structural growth factors on the gas fraction [eq. (12)], we ran two sets of simulations of typical mergers of disc galaxies, in which the initial conditions were identical except for the gas fraction in the disc; the initial gas-to-stars mass ratio $g$ ranges from ~0.1 to 3. The two sets differ only in their bulge-to-disc ratio. The fiducial case is G3-G3 from the suite of merger simulations in Cox (2004). Two identical spiral galaxies are put on a parabolic orbit, each consisting of stellar and gaseous discs and a stellar bulge, all embedded in a ΛCDM halo. The gravitational and hydrodynamical evolution is followed using the entropy-conserving, gravitating, Smoothed Particle Hydrodynamics (SPH) code GADGET (Springel et al. 2001). Gas cooling, star formation and supernova feedback are treated using simplified recipes that were calibrated to match observed star-formation rates. The structure of the fiducial progenitor disc galaxies mimics Sb-Sb spirals today, similar to the Milky Way. The dark-matter halo virial mass is $116 \times 10^{10} M_\odot$, represented by 120,000 particles of $(0.97 \times 10^5 M_\odot$ each). The virial radius is 272 kpc, corresponding to a virial velocity of $135 \text{ km s}^{-1}$. The halo density profile is NFW (Navarro et al. 1997), with a concentration parameter $C = 6$, and a spin parameter $\lambda = 0.05$. In the fiducial case, the baryonic fraction of the total
mass is 0.054. The total baryons are divided into a stellar
disc fraction of 0.66, a gaseous disc fraction of 0.20, and
a stellar bulge fraction of 0.14. The stellar disc mass of \(4.11 \times 10^{10} M_\odot\) is represented by 50,000 particles (of \(0.822 \times 10^5 M_\odot\) each). The surface density profile is exponential, with a scale
radius \(R_d = 2.85\) kpc, and the stability is ensured by a
Toomre \(Q\) parameter of 2. The fiducial gas disc of \(1.22 \times 10^{10} M_\odot\) is represented by 50,000 particles (of \(2.44 \times 10^5 M_\odot\) each). The gas is distributed in an exponential disc of a scale
radius \(3R_d\) extending out to 5 gas scale radii. The fiducial bulge is a sphere containing \(0.89 \times 10^{10} M_\odot\) of stars in 20,000 particles. Its density profile is exponential with scale radius 0.62 kpc. The smoothing length is \(h = 100\) pc for the gas and star particles and 400 pc for the dark-matter particles, with the force becoming Newtonian at \(\simeq 2.3h\).

The two galaxies, which were set on a parabolic orbit, merge because of dynamical friction due to their massive haloes. One galaxy starts such that its disc lies in the collision orbital plane, and the other galaxy’s disc is inclined by 30°, both with a prograde orientation of spin. The initial
separation between the galaxy centres is 250 kpc, and the expected pericentre (assuming point-like galaxies) is at 13.6 kpc, namely about 5% of the virial radius. The merger results in two successive starbursts, one after the first close approach, and the other after the second, final coalescence. The starbursts occur between 2 and 3 Gyr after the begin-
ing of the simulation, and the remnant is “observed” at 4 Gyrs. The amount of stars formed during the merger is roughly proportional to the initial gas fraction, and is not too sensitive to the orbit or orientation. In the cases with moderate gas fractions, the young stars formed during the merger typically constitute \(\simeq 20\%\) of the total stars.

The remnant galaxies resemble normal elliptical galaxies in many respects, as demonstrated in [Dekel et al. 2005]. This includes the stellar density profile, the ellipticity and triaxiality, the velocity-dispersion profile as well as higher moments of the line-of-sight velocity distribution.

The galaxies in all cases are identical in their dark mat-
ter halos, baryonic fraction and merger parameters. In one
suite of simulations the bulge-to-disc ratio is the fiducial 0.17, while in the other suite it is 0.39. Each of the two suites consists of 5 cases, where the bulge-to-disc ratio is kept fixed and only the disc gas-to-stars mass ratio \(g\) is varied. In the small-bulge suite, \(g\) varies between 0.12 and 3.0 [corresponding to gas/(gas+stars) between 0.11 and 0.75], while in the suite with a more massive bulge, \(g\) varies between 0.097 and 1.72 [corresponding to gas/(gas+stars) between 0.088 and 0.63].

Figure 1 shows the growth factors in the mergers as a function of \(g\). The trends are fit by the power laws in eq. (12) with \(r \simeq -0.4\), \(v \simeq 0.12\), \(m \simeq 0.3\). Each of the two suites consists of 5 cases, where the bulge-to-disc ratio is kept fixed and only the disc gas-to-stars mass ratio \(g\) is varied. In the small-bulge suite, \(g\) varies between 0.12 and 3.0 [corresponding to gas/(gas+stars) between 0.11 and 0.75], while in the suite with a more massive bulge, \(g\) varies between 0.097 and 1.72 [corresponding to gas/(gas+stars) between 0.088 and 0.63].

When the growth factors are plotted instead as func-
tions of \(g' = \text{gas}/(\text{gas}+\text{stars})\) rather than \(g\), the correspon-
ding best-fit power indices become \(r' \simeq -0.6\), \(v' \simeq 0.2\), \(m' \simeq 0.4\). However, the power-law fits in this case are not as good. In particular, the \(R\) factor flattens at the low-
\(g'\) end, and the \(m\) factor steepens significantly at the high-
\(g'\) end. We therefore prefer to use \(g\) as the gas-content variable.

6 RESULTS: PROGENITOR PROPERTIES

We are now set to solve eqs. (14)-(16). We use the values of the growth-factor indices \(r = -0.4\), \(v = 0.12\) and \(m = 0.3\) as deduced from the merger simulations. We then adopt the observed values for ellipticals \(\alpha_4 = 0.23\) and \(\beta_4 = 0.70\) \((\tau_4 = 0.16)\) and solve for the required progenitor properties. The one-parameter family of solutions is shown in Fig. 2. For each choice of \(\gamma\), the associated values of the power indices \(\alpha_4\) and \(\beta_4\), and their linear combinations \(\tau_4\) and \(\delta_4\), can be read from the corresponding curves. For example, the verti-
cal line at \(\gamma = 0\) refers to the null case where the progenitors are identical to the final ellipticals because the available gas fraction does not vary with mass. Any other vertical line corresponds to another possible solution for the progenitor properties. Higher initial values of \(\gamma\) naturally correspond to larger changes in the power indices as a result of mergers.
with progenitor mass, based for example on the common wisdom that supernova feedback becomes less effective in massive haloes (Dekel & Silk 1986), then the requirement \( \tau_d \leq 0 \) puts a lower bound of \( \gamma \geq 0.15 \) (for \( \tau_s = 0.1 \)). This corresponds to \( \beta_d \leq 0.55 \) and \( \delta \geq -0.7 \).

Progenitors with \( \gamma \approx 0.4 - 0.6 \) are favorable not only because these values are compatible with today’s trend of gas fraction along the blue sequence, but also because they represent solutions which are close to homology, roughly obeying the virial theorem (\( \tau_s \approx 0 \)) and a weak decline of density with mass (\( \delta_d \approx 0 \)). For example, a typical value of \( \gamma = 0.55 \), compatible with today’s gas gradient along the blue sequence plus an estimated contribution from the mass dependence of the cooling efficiency, corresponds to \( \alpha_d \approx 0.25 \), \( \beta_d \approx 0.31 \), \( \tau_d \approx -0.20 \) and \( \delta_d \approx -0.15 \).

We learn from Fig. 2 that the value of \( \tau_s \), namely the way the stellar \( M/L \) varies with mass, has a surprisingly small effect on the required progenitors. Even a choice of a large stellar tilt, \( \tau_s = \tau_c = 0.16 \), which eliminates the need for a structural tilt of the FP (namely \( \tau_c = 0 \)), still implies strong changes in \( \beta \) (and \( \tau \) and \( \delta \)) between the progenitors and the remnants once \( \gamma \) is not negligible.

7 INTERPRETING THE PROGENITORS

The solutions obtained above can be compared to what might be expected based on other considerations for the properties of the early gaseous progenitors of ellipticals.

Recall that the naive model of top-hat collapse to virial equilibrium followed by disc formation conserving angular momentum, at the same epoch for all galaxies, yields \( \alpha_d = \beta_d = 1/3 \), namely progenitors that lie on the virial plane (\( \tau_s = 0 \)) with the same effective density for galaxies of all masses (\( \delta_d = 0 \)). As noticed in Fig. 2 this is not a proper solution. A solution with \( \beta_d = \alpha_d \) does exist, for \( \gamma \approx 0.67 \), but it has the “wrong” value of \( \beta_d = \alpha_d \approx 0.25 \). A value close to \( \alpha_d \approx 1/3 \) cannot be obtained for any sensible value of \( \gamma \) as long as the observed slope is \( \alpha_s \approx 0.23 \), because \( \alpha \) hardly changes in the course of a merger.

However, a somewhat more sophisticated conventional model of disc formation in ΛCDM haloes does predict structural power indices in the range required for the progenitors of ellipticals. Studies of the profiles of dark haloes in cosmological simulations (Bullock et al. 2001; Wechsler et al. 2002) based on the NFW (Navarro et al. 1997) functional form (but without limiting the generality of the analysis), yield a mean trend of halo concentration with virial mass, \( c_v \propto M_{200}^{0.13} \), reflecting the different effective formation epochs as a function of halo mass (Bullock et al. 2001; Wechsler et al. 2002). While the flat part of the circular velocity resembles the virial velocity, \( V_c \propto M_{200}^{1/3} \), the maximum NFW circular velocity is \( V_{\text{max}} \propto c_v^{2/7} V_c \propto M_{200}^{0.47} \). If the relevant velocity characterizing the progenitors is between \( V_c \) and \( V_{\text{max}} \), the model predicts a slope for the \( V_c-M_{\text{200}} \) relation in the range \( 0.30 \leq \alpha_d/(1 + \tau_s) \leq 1/3 \), where \( \tau_s \) characterizes possible variations of virial to stellar mass ratio, \( M_c/M \propto M_{\text{200}}^{\tau_s/3} \).

The effective disc radius is evaluated by (Bullock et al. 2001) based on the dark-halo properties under the assumptions of a constant spin parameter as a function of mass (as seen in the simulations) and conservation of angular momentum during the gas contraction. For a disc baryonic fraction...
that is independent of mass, and in the limit $c_v \gg 1$, the
disc radius can be approximated by $R \propto c_v^{-0.7} R_c \propto M_c^{0.42}$.
An increasing baryonic fraction with mass would tend to
counter-balance the $c_v^{-0.7}$ dependence on concentration, so
the model predicts a slope for the $R_d-M_d$ relation in the
range $1/3 \leq \beta_d/(1+\tau_v) \leq 0.42$.

This crude model based on the $\Lambda$CDM halo properties
is compatible with solutions shown in Fig. 2 for the required
properties of the progenitors of ellipticals. If we take the
model predictions to be $\alpha_d \simeq 0.32 (1+\tau_v)$ and $\beta_d \simeq 0.37 (1+
\tau_v)$, and assume as before $\tau_v \simeq 0.1$, then the proper solution of
eqs. (16) and (15) is $\tau_v \simeq -0.24$, $\gamma \simeq 0.6$, $\alpha_d = 0.24$ and
$\beta_d = 0.28$. The corresponding tilt of $M/M_\ast$ is negative, $\tau_d \simeq
-0.23$, and the density decline is rather weak, $\delta_d \simeq -0.08$.

The value of $\gamma \simeq 0.6$ is roughly compatible with today's
gas gradient along the blue sequence of $\gamma \simeq 0.4$ [Kannappan
2004]. The difference can be attributed to the mass dependence
of the cooling efficiency, when $g$ represents the gas that actually manages to cool on a dynamical time scale after the merger. This contribution to $\gamma$ can be crudely estimated from the steepening from $\beta_d \simeq 0.52$ to $\beta_e \simeq 0.44$ seen in Fig. 11 of Robertson et al. (2004) for mergers with a fixed gas fraction $g' = 0.4$ (corresponding to $g = 0.67$). When inserted in eq. (16), with $r = -0.4$ and $m = 0.3$, we obtain a crude estimate of $\gamma \simeq 0.15$ for the cooling-efficiency contribution at a fixed $g$. A value of $\gamma \sim 0.5 - 0.6$ for the gaseous progenitors is therefore not surprising.

Unlike the final ellipticals, the gaseous progenitors are
predicted to be self-similar in the sense that the ratio of
total-to-stellar mass scales with mass in a similar way inside
the virial radius and inside the effective radius, $\tau_v \simeq \tau_d$.
The fact that this tilt is negative, $\tau_v \simeq -0.24$ is not surprising either. Such a tilt is seen below $\simeq 3 \times 10^{10} M_\odot$ (Bell et al 2003; Yang et al. 2003). A negative tilt as steep as $\tau_v \sim -0.4$ is deduced from the "fundamental line" of dwarf galaxies and predicted by crude energy considerations of supernova feedback in the limit where a small fraction of the gas has turned into stars very effective feedback [Dekel & Weinberg 2003; Dekel & Birnboim 2001].

The slope $\beta_d \simeq 0.28$ lies well inside the range of the
slopes observed for late-type galaxies in the SDSS: $\beta_d \simeq 0.15$
and $0.40$ below and above $M \simeq 4 \times 10^{10} M_\odot$ respectively
(Shen et al. 2003; Kauffmann et al. 2003). A similar slope of $\beta_d \simeq 0.29$ is also deduced from a study of a large local sample of late-type spirals, adopting $M/L_V \propto L_V^{0.15}$ for these galaxies [Courteau et al. 2006]. The $R-M$ relation for typical progenitors thus resembles that of today’s intermediate, $\sim L_V$ spirals.

The value of $\alpha_d \simeq 0.24$ is compatible with the slope of the "baryonic" Tully-Fisher relation by McGaugh (2005),
referring to the flat part of the rotation curve. It is perhaps a bit lower than the $\alpha_d \simeq 0.28$ deduced using $M/L_I \propto L_I^{0.15}$
from Courteau et al. (2006), in which the rotation velocity is measured at $2.2$ exponential disc radii.

8 BARYONIC RELATIONS

The analysis above could be performed alternatively with the stellar mass $M$ replaced by the baryonic mass $M_{\text{bar}}$ (of stars plus gas). We use the same notation as above with a prime added to each variable. This analysis is simplified by

the fact that the change in $M_{\text{bar}}$ during the merger can be
neglected, namely $m' = 0$ in the analog of eq. (12). It also
eliminates the need to deal with the poor power-law fit of
$M_{\text{bar}}/M_d$ as a function of $g$ (Fig. 1). The analogs of eqs. (16)
and eq. (18) become the explicit expressions

\begin{equation}
\alpha'_d = \alpha_d + v' \gamma', \quad \beta'_d = \beta_d + r' \gamma', \quad \gamma' \simeq \gamma_{\text{gas-to-baryon ratio}} in the progenitors,
\end{equation}

\begin{equation}
g' \propto M_{\text{bar}}^{-\gamma'}. \quad \gamma' \simeq \gamma_{\text{gas-to-baryon ratio}} in the progenitors,
\end{equation}

Figure 3 is the analog of Fig. 1 now as a function of the
gas fraction $g'$, showing reasonable power-law fits with
slopes $v' = 0.17$ and $r' = 0.42$. 

Figure 4 is the analog of Fig. 1 but for the baryons (gas+stars) replacing the stars, showing the solutions to eqs. (17).
\[ \text{The solutions of eqs.}\] 
\[ \alpha' = 0, \text{the changes in}\] 
\[ \tau' \gtrsim 0.33, \text{namely}\] 
\[ \tau' \approx 0.7.\] 
\[ \text{This is the only solution for}\] 
\[ \text{which neither the effective total-to-baryonic mass nor the}\] 
\[ \text{effective density are increasing with mass. If the baryonic}\] 
\[ \text{mass scales with the virial mass, these progenitors are the}\] 
\[ \text{homologous family predicted by the simplest model of spheri-}\] 
\[ \text{cal collapse into virial equilibrium. This indicates no signi-}\] 
\[ \text{ficant loss of baryons from the progenitors due to feedback}\] 
\[ \text{or other effects. The feedback may have an important role}\] 
\[ \text{in determining the strong variation of gas-to-baryon ratio}\] 
\[ \text{with progenitor mass.}\] 

9 CONCLUSION

We have clarified how the deviations of the structural scaling
relations of elliptical galaxies from the expectations based on
the virial theorem and an assumed homology of the elliptical
family could arise from the dissipative processes in major
mergers. The key is a systematic variation of the gas fraction
available for dissipation as a function of progenitor mass.
The tilt of the elliptical’s fundamental plane compared to
the virial plane and the steep decline of density with mass
are driven by the differential shrinkage of the effective stellar
radius as a function of the degree of dissipation involved in
the merger.

The correlated enhancements in internal velocity and
stellar mass during a merger, and the fact that the change
in velocity is rather small, lead to only a small change (~
10%) in the log-slope of the velocity-stellar mass relation,
keeping it near \( V \propto M^{1/4} \) for both the merger progenitors
and products. This is in the ballpark of the stellar Tully-
Fisher relation observed in today’s disc galaxies (McGaugh
2002; Courteau et al. 2006). The small changes in velocity
indicate that the tilts are primarily associated with structure
rather than kinematics.

The predicted log-slope of the radius-stellar mass rela-
tion in the progenitors is roughly \( R \propto M^{0.3} \), compatible
with the observed relation for intermediate late-type galaxies
near \( M \sim 10^{9–11} M_\odot \) (Shen et al. 2003; Courteau et al
2006). It is steeper than the slopes observed at the faint end
(\( \beta_1 \approx 0.15 \)) and at the bright end (\( \beta_0 \approx 0.4 \)) of late-type
galaxies.

The similarity of the progenitors to today’s relatively
big discs may imply that the latest dissipative mergers in
the history of ellipticals tended to involve grown discs rather
then smaller dwarfs. This is compatible with the detections
of big gaseous discs at \( z \approx 2–4 \) (Tacconi et al. 2006),
which could be produced by cold flows (Dekel & Birnboim
2003), and with the histories of big ellipticals in cosmolog-
ical simulations that properly match the galaxy bimodality
at low and high redshifts (Cattaneo et al. 2005, Cattaneo,
Dekel & Faber, in preparation). However, it does not rule
out later non-dissipative mergers along the red sequence,
which tend to preserve the FP and the associated scaling
relations.4 The tilts can be explained by dissipative major
mergers, or related dissipative processes, somewhere in the
histories of elliptical galaxies.

The required progenitor properties match the expectations
from a simple model of disc formation in ΛCDM haloes
(Mo et al. 1993; Bullock et al. 2001). First, the progenitors
are predicted to lie on a plane that is tilted relative to the
virial plane in the opposite sense to the tilt of the fundamen-
tal plane of ellipticals, roughly \( M/M \propto M^{-0.2–0.3} \). This
is similar to the expectations from the effect of supernova feed-
back. The tilt becomes positive as a result of the dissipative
shrinkage of the baryonic component in the mergers.

Second, The progenitors are expected to be close to ho-
logous, in the sense that the dark-matter fractions within
the effective radius and the virial radius scale similarly with
mass. They should have a very weak decline of density with
mass, roughly \( \rho \propto M^{-0.1} \), which later steepens by the dif-
ferential dissipative shrinkage to near \( \rho \propto M^{-0.9} \) in the merged
ellipticals.

Our main prediction is that the variation with pro-
genitor mass of the dissipative gas-to-star ratio should be
roughly \( g \propto M^{-\gamma} \) with \( \gamma \approx 0.5-0.6 \). This is consistent with
the observed trend (\( \gamma \approx 0.4 \)) in today’s blue-sequence galaxies
(Kannappan 2004), after subtracting a certain contribu-
tion to \( \gamma \) from the dependence of cooling efficiency on mass
at a fixed gas fraction. In an ongoing study we attempt to
disentangle in more detail between the roles of gas-fraction
and cooling efficiency. This will be achieved by performing
merger simulations similar to the ones described in this pa-
per but between galaxies of higher and lower masses.

The corresponding baryonic relations, in which the stel-
lar mass is replaced by the total baryonic mass in gas and
stars, are consistent with the simplest homology and spheri-
ical virial equilibrium, \( V \propto R \propto M_{\text{bar}}^{1/3} \), namely negli-
gible variations in \( M/M_{\text{bar}} \) and in mean density. While the
baryonic fraction is hardly changing, the gas fraction is pre-
dicted to vary strongly with baryonic mass in the progeni-
ators, \( g' \propto M_{\text{bar}}^{0.7} \).

The little relevance of the bulge fraction to the slopes of
the scaling relations indicates that the gaseous progenitors
need not necessarily be pure discs, as long as they have a
proper systematic decline of gas fraction with mass.

Our qualitative conclusions are insensitive to the actual
contribution of variations in stellar mass-to-light ratio to the
tilt. A substantial variation in the stellar mass-to-light ratio,
on the order of \( M/L \propto M^{0.1} \), could in principle contribute
to the tilt of the fundamental plane, but it cannot give rise
to the observed strong decline of density with mass. This
indicates that the structural changes due to differential dis-
sipation effects in mergers should have an important role in
shaping up the properties of ellipticals, and implies that the
trend in available gas fraction as a function of progenitor
mass is a robust prediction.

High-redshift observations should identify the predicted
early progenitors of today’s ellipticals, namely relative mas-
sive, gas-rich discy galaxies with a significant decline of gas
fraction as a function of mass. Indeed, an analysis of the mor-
phology of \( z \approx 1 \) galaxies using a non-parametric classifica-

\[ ^4 \text{This is yet to be tested using simulations of several success-} \] 
\[ \text{ive non-dissipative mergers between spheroids.} \]
tion system [Lotz et al. 2006] suggests that a large fraction of these galaxies are massive discs. Since this class of object makes up a significant fraction of the star formation, they are likely to be gas-rich. Observations at higher redshifts in the range $z \sim 2-4$, both in the UV and sub-millimeter wavelengths, have revealed gas rich galaxies of $\sim 10^{13} L_\odot$, whose kinematics resemble in many cases rotating thick discs of radii $\sim 2-3kpc$ [Tacconi & et al., 2006]. These galaxies may represent the required progenitors. A straightforward test of the differential dissipation model would be the detection of the predicted systematic decline of gas fraction with mass in such high-redshift galaxies.

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