Is the Difference Between the Pion Form Factor Measured in $e^+e^-$ Annihilations and $\tau^-$ Decays Due to an $H$ Propagator?

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We discuss how a charged Higgs propagator would modify the form factor in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays.

Ref.1 compared the pion form factors available from $e^+e^- \rightarrow \pi^+\pi^-(\text{CMD2 experiment})$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (ALEPH, CLEO, and OPAL experiments) in the range $0.37 < s < 0.92 \text{ GeV}^2$ where $s = m^2_{\pi\pi}$. The evaluation used CVC with estimates of isospin violation between the charged and neutral states for the isovector part. This mass range is dominated by the $\rho(770)$ resonance. Generally agreement was found within several percent, except for the higher masses, where the discrepancy was about 10%. It was pointed out that some of the discrepancy may be due to different masses and widths of the charged and neutral $\rho$ mesons [2,3]. Recently the KLOE experiment [4] has measured the pion form factor through the radiative process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ in the range $0.35 < s < 0.94 \text{ GeV}^2$. The situation has been reviewed [5] by Andreas Hocker at ICHEP 04, Beijing, Aug 16-22, 2004. He concludes that even after correcting for possible differences in the masses and widths of the charged and neutral $\rho$ mesons, there is still a discrepancy at the higher masses of 5-10%. Fig. 1 shows the pion form factor measured by the KLOE experiment [4]. The statistical errors range from $\pm 2\%$ at low mass to $\pm 0.6\%$ at higher masses. The systematic error is $\pm 1.3\%$.

The decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ can proceed through either $W^-$ exchange or $H^-$ exchange [6]:

$$\psi^2_{\pi\pi} = (\psi_w + \psi_H)^2 = \psi_w^2 + 2\psi_w\psi_H + \psi_H^2 \approx \psi_w^2 + 2\psi_w\psi_H \quad (1)$$

The $W^-$ diagram is dominated by the $\rho(770)$ resonance in the mass range $0.35 < s < 0.94 \text{ GeV}^2$. There is no $\pi\pi$ resonance with the quantum numbers of the $H^-$ in this mass range. The hadronic vector current [7] is given by:

$$J^V_h = \sqrt{2}\cos\theta \cdot F_\pi(s)(q_{\pi^+} - q_{\pi^-}) \quad (2)$$
Fig. 1. Some representative points from the pion form factor $|F_\pi(s)|^2$ vs. $s$ (GeV$^2$) measured by the KLOE experiment [4]. The interference between the broad $\rho(770)$ and the narrow $\omega(782)$ can be seen. The statistical errors range from ±2% at low mass to ±0.6% at higher masses.

$F_\pi(s)$ is dominated by the $\rho(770)$ resonance in the mass range $0.35 < s < 0.94$ GeV$^2$. The two pions are in a $P$ wave. The hadronic scalar current is given by:

$$J^S_H = f_S(s)$$  \hspace{1cm} (3)

where the two pions are in a $S$ wave. We parameterize the $s$ dependence of $F_\pi$ and $f_S$ over this mass range by Breit-Wigner amplitudes:

$$\psi_w \propto \frac{m_\rho \Gamma_\rho}{m_\rho^2 - s - i m_\rho \Gamma_\rho} \quad \psi_H \propto \frac{A_H}{m_H^2 - s - i m_H \Gamma_H} \approx \frac{A_H}{m_H^2}$$  \hspace{1cm} (4)

where for this study, we have ignored the $\rho(1450)$ [8] etc. We show in Fig. 2
as a function of $s$. This shape is in agreement with the difference of the pion form factor measured in $e^+e^-$ annihilations and $\tau$ decays within the uncertainties. This is our main result.

For the $\tau$ semi-leptonic decay model discussed in ref. 6, the additional $H^-$ couplings are given by:

$$g^s = -\frac{m_\tau (m_\mu + m_\mu)}{m_{H^-}^2} \tan^2 \beta \quad (6)$$

With the minus sign in equ. 6, we get that the $\tau$ form factor should be higher than the $e^+e^-$ form factor above the $\rho$ mass, in agreement with ref. 1. Ref. 9 gives the limit

$$\frac{\tan \beta}{m_{H^-}} < 0.4 \text{ GeV}^{-1} \quad (7)$$

coming mainly from the $B$ leptonic and semi-leptonic decays. The direct limit [9] on the mass of the charged Higgs is 79.3 GeV. If we use the “current quark mass” of $m_u + m_d = 6 - 12 \text{ MeV}$ [9], we get $|g^s| < 0.0034$.

Fig. 2. $R$ in arbitrary units as a function of $s$ (GeV$^2$). This shape is in agreement with the difference of the pion form factor measured in $e^+e^-$ annihilations and $\tau$ decays within the uncertainties. This is our main result.
From above, we get:

\[ R \approx 2 g_s \left( \frac{m_{\rho}^2 - s}{m_{\rho} \Gamma_{\rho}} \right) \]  \hspace{1cm} (8)

which gives at \( s \approx 1 \text{ GeV}^2 \), \(|R| \leq 2.5\%\).

In conclusion, we have shown that the disagreement between the measured \( \tau^- \) and CVC predicted pion form factors is consistent with the interference of the \( W^- \) and \( H^- \) diagrams in shape. This is our main result. A quick estimate of the allowed size of the effect is given for a model discussed in ref. 6, which gives about one half the observed difference in the form factors.

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