Neutrino effective charge in a dense Fermi plasma

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Abstract

By using a plasma physics approach, the neutrino effective charge in a dense Fermi plasma is determined here. Its value is determined by the collective quantum plasma processes. It is found that the contribution of the ion motion can be important even in Fermi plasmas, despite the negligible weak interaction between neutrinos and ions. This results from the electron–ion couplings, which are dominant in the low-frequency limit of the equivalent charge spectrum.

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Like the electrons and photons, the neutrinos can also generate collective effects in plasmas [1]. The electromagnetic neutrino-plasma coupling can be described in terms of effective dipole moments and an electric charge induced by the medium. The concept of the neutrino induced (effective) charge independently emerged from phenomenological plasma physics calculations [2–4], as well as from a more evolved finite temperature field theory [5,6].

It is well known that in a vacuum the neutrino does not have electric charge, but possesses only the weak charge $e / \sin^2 \theta_w$ associated with the weak nuclear force, where $e$ is the magnitude of the electric charge and $\theta_w$ is the Weinberg angle [7]. However, it was shown by Oraevsky and Semikoz [4] that in a dispersive medium the neutrino acquires an actual induced electric charge due to medium polarization, irrespective of the neutrino rest-mass. The neutrino is always subject to a weak force generated by permanent perturbations of neutral currents associated with Z-bosons whose coupling with the neutrino is described by the constant $e / \sin 2\theta_w$. Since neutral weak current interactions apply to all species, the neutrino may influence a large number of charged particles. When interacting with an electron–ion plasma, neutrinos can displace electrons relative to heavy ions, giving rise to medium polarization. This kind of polarization of a dispersive medium occurs not only for a plasma, but for any medium containing free charge carriers, like metals and semiconductors. The inhomogeneous electron density arising around the neutrino is just the induced electric charge, which is determined by the plasma Debye radius $\lambda_D$ and by the coupling constant $G_F (1 + 4 \sin^2 \theta_w) / \sqrt{2}$, where $G_F$ is the Fermi weak-interaction constant. When moving together with the neutrino, this charge may induce radiative processes like Cherenkov radiation [4,8]. The effect of an external magnetic field on the neutrino induced charge and neutrino electromagnetic processes in medium has also been investigated [5,9–13].

The study of the electromagnetic properties of neutrinos in vacuum as well as in a medium has been a subject of great interest mainly because of its potentially important consequences in a variety of astrophysical and cosmological contexts. Processes such as the plasmon decay ($\gamma \rightarrow \nu \bar{\nu}$) and the neutrino pair decay ($\nu \bar{\nu} \rightarrow e \bar{e}$), which cannot be realized or are highly suppressed in vacuum, can occur in the interior of stars, for example, due to the induced electromagnetic couplings of a neutrino propagating in a background of electrons and nucleons. Hence, the neutrino electromagnetic vertex, which is related to the pho-
ton self-energy in the medium [6], plays a fundamental role, for instance, in the cooling of super dense white dwarfs [14] and neutron stars [15].

Due to the nature of the weak nuclear force, only intense fluxes of neutrinos have a significant effect on a dense plasma. Therefore, the collective neutrino-plasma interactions are important only in scenarios where huge fluxes of neutrinos are released; for example, in superdense white dwarfs and neutron stars as well as in supernovae. In the interior of white dwarfs and in the crust of neutron stars, the plasma is extremely dense and highly degenerate, and their properties differ significantly from those of a classical Maxwellian plasma. Here, the strong correlations among electrons/positrons must be taken into account, and quantum-mechanical effects are expected to be important. For such a plasma, the electron Fermi temperature $T_F$ is much higher than the temperature $T$, which is a condition for quantum effects to become important [16] (this condition is equivalent to $n_0 \lambda_B^3 \gtrsim 1$, where $\lambda_B$ is the electron de Broglie wavelength). The equilibrium particle distribution function obeys the Fermi–Dirac statistics, and an effective potential incorporating quantum effects can describe the correlations among electrons/positrons [17].

There has been a great deal of interest in investigating collective interactions in dense Fermi plasmas also due to their potential applications in ultrasmall electronic devices [18]. Recently, Haas et al. [19] showed that strongly coupled plasmas can be described by a quantum hydrodynamical model derived from the nonlinear Wigner–Poisson system. For a strongly coupled plasma, collisions are, in principle, as important as collective effects. Fortunately, the Pauli blocking effect dramatically reduces the collision rates in most cases of interest [16]. In Ref. [19] the authors studied the two-stream instabilities in an ultracold Fermi plasma and showed that a new purely quantum branch appears. The quantum statistical temperature effect was included at a later stage [20]. Anderson et al. [21] also analysed two-stream instabilities in quantum plasmas and included statistical effects associated with the finite width of the probability distribution function; Shukla and Stenflo [22] considered the nonlinear interaction between large-amplitude electromagnetic waves and low-frequency plasma oscillations in a quantum plasma. Marklund and Brodin [23] derived the governing equations for spin-1/2 quantum plasmas.

In the present work, we present a calculation of the induced electric charge of the electron-type neutrino propagating in a dense Fermi electron–ion plasma. Neutrinos are treated as quasi-classical particles [24], and this is valid only if the neutrino De Broglie wavelength is much shorter than the typical length scale of the plasma density perturbation $\lambda_{pe} = 2\pi c/\omega_{pe}$, where $c$ is the speed of light in vacuum, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency, $n_0$ is the equilibrium electron number density and $m_e$ is the electron rest mass. For the description of the electron–ion Fermi plasma, we use a quantum hydrodynamical model which accounts for the quantum statistical pressure law and the quantum force arising from the electron correlation [25]. We obtain relations for the electron number density perturbations coupled with the neutrino density fluctuations, and derive the effective charge for the electron-type neutrino. Our discussion includes the influence of the ions on the low-frequency response of the Fermi plasma in the presence of neutrino fluxes. At a first sight, this contribution could be considered negligible because the electron neutrinos do not directly interact with the ions (protons), but it is shown here that it can be relevant in the low-frequency limit, since the electron motion is strongly coupled with the ions due to the space charge electric field reinforced by the neutrino driving force.

Let us consider a cold streaming gas of electron-type neutrinos propagating through an unmagnetized electron–ion Fermi plasma. The equations describing the Fermi plasma dynamics are [19,24]

$$\frac{\partial n_\sigma}{\partial t} + \nabla \cdot \mathbf{J}_\sigma = 0,$$

and

$$\frac{\partial p_\sigma}{\partial t} + (\mathbf{v}_\sigma \cdot \nabla) p_\sigma = -\frac{\nabla P_\sigma}{n_\sigma} + \mathbf{F}_{Q\sigma} + q_\sigma \mathbf{E},$$

where

$$\mathbf{J}_\sigma = \bar{e} \mathbf{q}_\sigma \times \mathbf{B}_\sigma.$$

The coupling between the neutrinos and the plasma is given by the term $G_{\sigma \nu} = \sqrt{2} G_F [\delta_{\sigma \nu} \delta_{\nu v} + (I_\nu - 2 Q_\nu \sin^2 \theta_W)]$ [26], where $G_{\sigma \nu} = -G_{\bar{\sigma} \nu} = G_{\sigma \bar{\nu}} = G_{\bar{\sigma} \bar{\nu}}$. Here, $\sigma$ (\$\bar{\sigma}\$) denotes the plasma particles (antiparticles), $\sin^2 \theta_W \approx 0.23$, $I_\nu$ is the weak isotopic spin of the particle of specie $\sigma$ (equals $-1/2$ and $1/2$ for the electrons and protons, respectively), and $Q_\nu = q_\nu/e$ is the particle normalized electric charge. The first term in $G_{\sigma \nu}$ is due to the charged weak currents and applies only to the electrons and electron-type neutrinos, while the remaining terms are due to the neutral weak currents and apply to all species. Here we consider only the electron-type neutrinos, but the inclusion of additional particle species or neutrino (antineutrino) flavors is straightforward [26].

The first term in the right-hand side of Eq. (2) is the force due to the pressure of a zero-temperature Fermi–Dirac plasma. For a degenerate plasma, we can assume $P_\sigma/P_0 = (n_\sigma/n_0)^\gamma$, where $P_0$ is the plasma equilibrium pressure, $n_\sigma$ is the number density of the species $\sigma$, and $\gamma = (D + 2)/D$ is the exponent related to the dimensions of the system. In 3D, the equilibrium pressure of a quantum gas at zero temperature is $P_0 = 2n_0 T_F/5$, where $T_F = h^2(3\pi^2 n_0)^{2/3}/2m_e$ is the Fermi temperature (in energy units) of the plasma of species $\sigma$ and $h$ is the Planck constant divided by $2\pi$. However, the correct exponent to use in the equation of state is $\gamma = 3$, like in the 1D case, since linear wave propagation is essentially a 1D phenomenon [16]. The equation of state then becomes $P_\sigma = m_\sigma v_F^2 n_\sigma^3/5n_0^2$, where $v_F = (2T_F/m_\sigma)^{1/2}$ is the Fermi speed. The quantum force is given by $F_{Q\sigma} = \frac{\hbar^2}{2m_\sigma} \nabla \left[ \sqrt{2\pi/(\bar{e}^2 \lambda_{pe})} \right]$. The last term in Eq. (2) is the force acting on the Fermi plasma due to all types of neutrinos, and $\mathbf{E}_\nu = -\nabla n_\nu - (1/c^2)\partial \mathbf{J}_\nu /\partial t$ and $\mathbf{B}_\nu = (1/c) \nabla \times \mathbf{J}_\nu$ are the "weak-electromagnetic" fields, respectively. $\mathbf{J}_\nu = n_\nu \mathbf{q}_\nu$ and $\mathbf{J}_\nu = n_\nu \mathbf{v}_\nu$ are the $\sigma$ species and neutrino currents, and $\mathbf{J}_\nu$ and the neutrino density $n_\nu$ are related by the continuity equation

$$\frac{\partial n_\nu}{\partial t} + \nabla \cdot \mathbf{J}_\nu = 0.$$

[19,24]
The space charge electric field $\mathbf{E} = -\nabla \phi$, where $\phi$ is the electrostatic potential, is determined from the Poisson equation, $\nabla \cdot \mathbf{E} = 4\pi \sum \rho_n \rho_e$. We can assume a collisionless Fermi plasma if we consider only electrostatic modes on a timescale $\tau \leq \omega_{pe}^{-1}$, with $\omega_{ei} \ll \omega_{pe}$, where $\omega_{ei}$ is the collision frequency.

First, we analyse the neutrino density perturbations coupled with the electron density fluctuations in a background of fixed ions. By using the standard perturbation analysis and writing all the dynamical quantities in the form $\delta = f_0 + \delta f$, where $\delta f \ll f_0$, we obtain the neutrino continuity equation

$$\frac{\partial \delta n_{\nu}}{\partial t} + n_0 \nabla \cdot \nabla v_{\nu} = 0,$$

and the equations governing the electron dynamics

$$\frac{\partial \delta n_{e}}{\partial t} + n_0 \nabla \cdot \nabla v_{e} = 0$$

and

$$\frac{\partial \delta v_{e}}{\partial t} = \frac{e}{m_e} \nabla \phi - \frac{1}{n_0 m_e} \left( \frac{6T_{Fe}}{5} - \frac{\hbar^2 \nabla^2}{4m_e} \right) \nabla \delta n_e - \sqrt{2G_F \sum} (\nabla \delta n_e + \frac{n_0 \delta \delta v_{e}}{c^2} + \frac{v_{e0} \partial \delta n_{\nu}}{c^2} - \frac{\delta \delta v_{e}}{\partial t}),$$

where $\nabla^2 \phi = 4\pi e \delta n_e$. Combining Eqs. (4)–(6), we obtain the driven electron plasma wave equation

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - C_{se}^2 \nabla^2 + \frac{\hbar^2 \nabla^4}{4m_e^2} \right) \delta n_e$$

$$= \frac{\sqrt{2G_F n_0}}{m_e c^2} \left( c^2 \nabla^2 - \frac{\delta^2}{\partial t^2} \right) \delta n_e,$$

where $C_{se} = (6T_{Fe}/5m_e)^{1/2} = \sqrt{3/5} v_{Fe}$ is the electron thermal speed. Eq. (7) describes the parametric coupling between intense neutrino bursts and the electron plasma waves in the Fermi plasma.

In general, we can consider $\delta n_{e(v)}$ such that

$$\delta n_{e(v)} = \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \delta n_{e(v)}(\omega, \mathbf{k}) e^{i(k \cdot \mathbf{r} - \omega t)},$$

where $\delta n_{e(v)}$ are the spectral fluctuations of the electron (neutrino) population. For each component of the neutrino density perturbation $\delta n_{\nu}$, there is an associated spectral component $\delta n_{e}$ of the electron density fluctuation, which can be derived by Fourier transforming Eq. (7). We have

$$\left( \omega^2 - \omega_{pe}^2 - k^2 C_{se}^2 - \frac{\hbar^2 k^4}{4m_e^2} \right) \delta n_e$$

$$= \frac{\sqrt{2G_F n_0}}{m_e c^2} (k^2 c^2 - \omega^2) \delta n_{\nu}.$$  

(9)

Associated with this electron density fluctuation there is an electric charge fluctuation $-e \delta n_{e}$, which can be seen in the Poisson equation as the source of the electrostatic potential. From Eq. (9), we observe that this electric charge is proportional to $\delta n_{\nu}$. Therefore, we can establish the identity

$$-e \delta n_{e} = q_\nu \delta n_{\nu},$$

where $q_\nu$ is the neutrino spectral effective charge. Defining $\omega_{\nu}^2 = \omega_{pe}^2 + k^2 C_{se}^2 + k^2 C_{Qe}^2$, with $C_{Qe} = \hbar k/2m_e$, we have from Eq. (9)

$$q_\nu = \frac{\sqrt{2G_F n_0}}{m_e c^2} \left( \frac{\omega_{pe}^2 - 2k^2 C_{se}^2}{\omega^2 e^2(\omega, \mathbf{k})},$$

(11)

where $\epsilon(\omega, \mathbf{k}) = 1 - \omega_{\nu}^2 / \omega^2$ is the dielectric constant of an electron Fermi plasma. In Fig. 1 we show the frequencies $\omega_{pe}$, $\omega_{se} = kC_{se}$ and $\omega_{Qe} = kC_{Qe}$ for a Fermi plasma with an electron number density typical of dense white dwarfs, $n_0 = 10^{29}$ cm$^{-3}$. We observe that the quantum term $C_{Qe}/k$ becomes important for values of $k$ of the order of the Compton wavenumber $k_C = 2.59 \times 10^{10}$ cm$^{-1}$. In Fig. 2 the parameter $H = C_{Qe}/C_{se}$ is plotted as a function of $n_0$ for $k = k_C$. For quantum plasmas with $n_0 \lesssim 10^{30}$ cm$^{-3}$ the effect of the Bohm potential is more significant or comparable to the effect associated with the Fermi pressure. For greater values of $n_0$ (for the crust of neutron stars $10^{29}$ cm$^{-3} < n_0 < 10^{30}$ cm$^{-3}$ [27]) the Fermi speed is extremely high and the term $C_{se}$ is always dominant.
Now, we analyse the effective charge in different limits. First, we investigate the high-wavenumber (or short-wavelength) limit $\omega^2 \ll k^2 \epsilon^2$, $\omega^2$. In this case we have from Eq. (11) $q_v \approx \sqrt{2G_F k^2/4\pi e} (1 + k^2 \lambda_D^2 + k^2 \lambda_{Qe}^2)$, where $\lambda_D = \lambda_{se}/\omega_{pe}$ and $\lambda_{Qe} = \lambda_{Qe}/\omega_{pe}$. The expression for $q_v$ has the form similar to that obtained for a classical plasma [3,24,28]; the term $\lambda_{Qe}$ has been kept because it can be comparable to $\lambda_D$ for a quantum plasma, since it depends on the value of $k$ (Fig. 1).

Second, in the quasistatic limit $\omega \rightarrow 0$, and for $k^2 \lambda_D^2$ and $k^2 \lambda_{Qe}^2 \gg 1$, we obtain from Eq. (11)

$$q_v = \frac{\sqrt{2G_F}}{4\pi e (\lambda_D^2 + \lambda_{Qe}^2)},$$

which has the form similar to that obtained for classical plasmas [24,6,26,28]. Third, in the high-frequency limit $\omega^2 \gg \omega_{pe}^2$, we obtain from Eq. (11) $q_v \approx -\sqrt{2G_F k^2 \omega_{pe}^2 (k^2 \epsilon^2/\omega^2 - 1)}/4\pi e$, where $k_{pe} = \omega_{pe}/c$. This result is the same obtained by Mendonça et al. [28] for the neutrinos interacting with a classical electron–ion plasma, and indicates that the plasma nature has little effect in this regime. Finally, Eq. (11) reveals that, for very high frequencies and very high wavenumbers, such that $\omega^2 \approx k^2 \epsilon^2$, the value of the neutrino charge goes to zero. This means that when the influence of the plasma on the fluctuations becomes negligible, the induced neutrino charge disappears. It has been shown by many authors [4,5,8,13,29] that different radiative processes can occur due to the electromagnetic properties of the neutrinos in a medium. Processes like the Cherenkov radiation can be important under certain conditions [8]. The influence of the quantum term $\lambda_{Qe}$ on these electromagnetic processes remains to be analyzed.

We now investigate the low-frequency spectrum of oscillations, such that $\omega \ll \omega_{pe}$. In this case, the motion of the ions becomes important. After the linearization process, the equations governing the plasma dynamics are (5), (6) and

$$\frac{\partial \delta n_i}{\partial t} + n_0 \nabla \cdot \delta \mathbf{v}_i = 0,$$

(13)

and

$$\frac{\partial \delta \mathbf{v}_i}{\partial t} = - \frac{e}{m_i} \nabla \phi,$$

(14)

since $G_i \approx 0$. The quantities $\delta \mathbf{v}_i$ and $\delta n_i$ are related by the neutrino continuity equation, Eq. (4), and the electrostatic potential is given by $\nabla^2 \phi = 4\pi e (\delta n_i - \delta n_e)$. Fourier transforming the equations above, we obtain the relation

$$\left[(\omega_{pi}^2 - \omega^2) (\omega^2 - \omega_{pe}^2 - k^2 \lambda_{se}^2 - k^2 \lambda_{Qe}^2) + \omega_{pe}^2 \omega_{pi}^2\right] \delta n_e$$

$$= - \frac{\sqrt{2G_F n_0}}{m_c \epsilon^2} \left((\omega^2 - k^2 \epsilon^2) (\omega_{mi}^2 - \omega^2)\right) \delta n_v,$$

(15)

where $\omega_{mi} = (4\pi n_0 \epsilon^2 / m_i)^{1/2}$ is the ion plasma frequency. By using the charge equivalence condition (10), we obtain the expression for the neutrino spectral charge where the contributions of the ions are included, i.e.,

$$q_v = \frac{\sqrt{2G_F n_0 e}}{m_c \epsilon^2} \left((\omega^2 - k^2 \epsilon^2) (\omega_{pi}^2 - \omega^2)\right) \times \left[\frac{\omega_{pe}^2 \omega_{pi}^2 - (\omega_{pi}^2 - \omega^2) (\omega_{pe}^2 + k^2 \lambda_{se}^2 + k^2 \lambda_{Qe}^2 - \omega_{se}^2)}{\omega_{pi}^2 - \omega_{pe}^2} - \frac{\omega_{se}^2}{\omega_{se}^2 - \omega_{pi}^2}\right].$$

(16)

The expression (16) has the form similar to that derived by Mendonça et al. [28] for the neutrinos interacting with a classical electron–ion plasma. If we now assume the low-frequency oscillations of the Fermi plasma, such that $\omega^2 \ll k^2 \epsilon^2$, and also consider $\omega^2 \ll k^2 \lambda_{se}^2 + k^2 \lambda_{Qe}^2$, we obtain

$$q_v = \sqrt{2G_F n_0 \epsilon^2 k^2 / m_i (k^2 \epsilon^2 - \omega^2)},$$

where we have denoted $\epsilon^2 = m_e (\lambda_{se}^2 + \lambda_{Qe}^2) / m_i$. As before, the term $\lambda_{Qe}$ has been kept because it can be of the order of $\lambda_D$. These results indicate that the effective neutrino charge is dominated by the ion motion in the very-low-frequency limit even in the Fermi plasma, despite the negligible direct weak nuclear interaction between the protons and neutrinos. This is a consequence of the electron–ion coupling, which is dominant in the low-frequency limit of the effective charge spectrum.

To summarize, the neutrino effective charge in the Fermi plasma has been determined here through the plasma physics approach. We have used the classical fluid description for the neutrinos and the quantum hydrodynamical model for the electron–ion Fermi plasma, which takes into account the quantum statistical pressure and the quantum force. These fluid equations, together with the Poisson equation for the electrostatic potential, are used to derive the relations between the plasma density fluctuations and the neutrino density perturbations. Considering the spectral fluctuations in the electron(neutrino) population, we then determine the neutrino spectral effective charge for different frequency limits. The results have the same form of the expressions obtained before for classical plasmas that used methods from plasma physics and quantum field theory. It is found that the contribution of the ion (proton) motion can be important in the low-frequency limit also for the Fermi plasma, even if the direct interaction between neutrinos and protons is negligible. It is important to notice that the neutrino effective charge in the Fermi plasma can be smaller or of the order of that in classical plasmas, since it is inversely proportional to the term $\lambda_{Qe}$, which depends on the value of $k$. However, as in a classical plasma, the neutrino-plasma coupling in the Fermi plasma can have importance in many physical and astrophysical situations, like in processes occurring in the interior of dense and degenerate stars.

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