The cosmological constant filter without big bang singularity

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Abstract

In the recently proposed cosmological constant (CC) filter mechanism based on modified gravity in the Palatini formalism, gravity in the radiation, matter and late-time de Sitter eras is insensitive to energy sources with the equation of state \( p = -\rho \). This implies that finite vacuum energy shifts from phase transitions are filtered out too. In this work, we investigate the CC filter model at very early times. We find that the initial big bang singularity is replaced by a cosmic bounce, where the matter energy density and the curvature are finite. In a certain case, this finiteness can be already observed on the algebraic level.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The core of the old cosmological constant (CC) problem [1–3] is the huge hierarchy between the observed value \( \Lambda_0 \sim 10^{-47} \text{GeV}^4 \) of the CC in the \( \Lambda \text{CDM} \) model and the value \( \Lambda \) inferred from theory. While the latter is difficult to calculate, it is usually expected to be dominated by the ultraviolet sector of the theory implying an enormous magnitude \(|\Lambda| \gg \Lambda_0\). For example, the most reliable contribution to \( \Lambda \) comes from the electro-weak phase transition giving rise to the relation \(|\Lambda| \sim (10^2 \text{GeV})^4 \sim 10^{45} \Lambda_0\) [4]. A common way to deal with this problem is by adding a CC counterterm, which subtracts off the huge vacuum energy density and permits a reasonable low-energy universe. The price to pay is the enormous finetuning of the counterterm, i.e. the precise choice of tens of digits, which is an undesired property for any theory. It is important to note that replacing the CC by a dynamical dark energy source generally does not avoid the old CC problem [5]. However, it may help modelling the late-time accelerated expansion and other cosmological epochs [6–13].
Instead of removing the large CC by a static counterterm, we consider a more dynamical approach to neutralize the effects of $\Lambda_1$. It is easy to see that in standard general relativity, the large CC would dominate the evolution of the universe at early times. Hence, one has to introduce nontrivial changes in gravity [14, 4, 15, 16] or in the matter sector [17–19] in order to permit a low curvature cosmos. Further ways to address the old CC problem can be found in [20–26], where, for instance, quantum gravity effects or self-adjustment mechanisms are important. Finally, neutralizing the effect of vacuum energy was investigated in [27–32] from different perspectives.

In this work, we continue exploring the CC filter model proposed in [33]. This approach to solve the old CC problem is based on $f(R, Q)$ modified gravity in the Palatini formalism [34–38], where the gravity action is formulated in terms of the Ricci scalar $R$ and the squared Ricci tensor $Q$ [39–42]. Palatini theories have a more involved algebraic structure than the corresponding theories in the metric formalism [43–51], which permits the construction of a filter for vacuum energy. Moreover, in our setup, we find only second order field equations just as in general relativity; thus, extra degrees of freedom are avoided.

So far, only the region where the matter energy density $\rho$ is smaller in magnitude than the CC $\Lambda_1$ has been studied, which includes the radiation, matter and late-time de Sitter epochs. We found on the algebraic level that energy sources with the equation of state (EOS) $-1$ do not contribute to the curvature at leading order, which is just a manifestation of the filter effect. In the following, we will extend the analysis by discussing the very early cosmic epoch where $\rho$ may be of the order of $\Lambda_1$ or larger. As the main result, we find that the filter model avoids the occurrence of an initial curvature singularity, which is instead replaced by a cosmic bounce with finite matter energy density and finite curvature. For a certain case, this finiteness can also be seen on the algebraic level. Finally, it is worth mentioning that the absence of an initial singularity has been observed also for other Palatini models in [52, 53], and a connection to loop quantum cosmology was suggested [54].

The paper is organized as follows: in section 2, we briefly review how to solve the gravity field equations in the Palatini formalism, and the procedure will be applied to the CC filter model in section 3. We solve the main equations and discuss the cosmological solutions in section 4 before we conclude in section 5.

In this work, the speed of light $c$ and the Planck constant $\hbar$ are set to unity, and the signature of the metric is $(−1, +1, +1, +1)$. The tensor indices $a, \ldots, f, m, n$ run from 0 to 3, whereas $i, j = 1, 2, 3$ are related to spatial coordinates.

2. $f(R, Q)$ modified gravity in the Palatini formalism

The CC filter mechanism is based on $f(R, Q)$ modified gravity1 in the Palatini formalism, where $R$ is the Ricci scalar and $Q$ the squared Ricci tensor. In terms of the spacetime metric $g_{ab}$ and its determinant $g = \det g_{ab}$, the complete action is given by

$$S = \int d^4x \left[ \sqrt{|g|} \frac{1}{2} f(R, Q) \right] + S_{\text{mat}}[g_{ab}, \phi]$$

with the matter fields $\phi$ in $S_{\text{mat}}$ coupled minimally to $g_{ab}$. Furthermore, here the connection $\Gamma_{bc}^a$ and the Ricci tensor $R_{ab}$ are restricted to be symmetric2:

$$R_{ab} = \Gamma_{ab,c}^c - \Gamma_{eb,a}^c + \Gamma_{ab}^e \Gamma_{fe}^c - \Gamma_{af}^e \Gamma_{eb}^c$$

1 The Palatini gravity model discussed in this work does not introduce extra degrees of freedom. Therefore, Ostrogradski-type [55] instabilities do not occur despite the squared Ricci tensor $Q$ in the action.
2 See [56] for generalizations.
\[ R = g^{ab} R_{ab}, \quad Q = R^{bc} R_{bc} = g^{ab} g^{cd} R_{ab} R_{cd}. \] (3)

In the metric formalism, the connection would be identified with the Lévi-Civita connection
\[ \Gamma^a_{bc} = \frac{1}{2} g^{ad} \left( g_{dc,b} + g_{bd,c} - g_{bc,d} \right), \] (4)

and the Ricci tensor and all derived quantities would be functionals of the metric only. Differently, the Palatini formalism treats the connection as an independent object in the beginning. Therefore, the variational principle \( \delta S = 0 \) yields two equations of motion for gravity, one for the metric:
\[ f_R R_m^n + 2 f_Q R_m^a R^n_a - \frac{1}{2} \delta_m^n f = T_m^n, \] (5)

and another one for the connection:
\[ \nabla_a \sqrt{|g|} (f_R g^{mn} + 2 f_Q R_{mn}) = 0, \] (6)

where \( \nabla_a \) denotes the covariant derivative in terms of the yet unknown connection and \( \Gamma^a_{bc} \) and \( f_{R,Q} \) are the partial derivatives of \( f \) with respect to \( R \) and \( Q \), respectively.

In the following, we briefly summarize how to solve these equations, and we point the reader to [33, 57] for a detailed exposition. Since we focus on the background cosmology, the matter sector can be described by a perfect fluid with the energy–stress tensor given by
\[ \rho = T_m^m, \] (7)

where \( \rho \) is the energy density and \( p \) the pressure of ordinary matter with the 4-velocity vector \( u_m \) normalized by \( u_m u^m = -1 \). \( \Lambda \) denotes the large CC vacuum energy density.

From equation (5), two algebraic equations for the two unknowns \( R \) and \( Q \) can be derived; the first one is the trace of (5)
\[ f_R R + 2 f_Q f = T, \] (8)

where \( T = T_m^m \), and the second equation follows from treating (5) as a matrix equation for the Ricci tensor \( \hat{P} := R^m_n \) in the matrix form\(^3\)
\[ (2 f_Q)^2 \left( \hat{P} + \frac{1}{4} \frac{f_R}{f_Q} \mathbb{I} \right)^2 = X^2 \mathbb{I} + 2 f_Q (\rho + p) u_m u^n, \] (9)

with
\[ X^2 := 2 f_Q (p - \Lambda) + f_Q f + \frac{1}{4} \frac{f_R^2}{f_Q}, \quad Y := X - c_a \cdot \sqrt{X^2 - 2 f_Q (\rho + p)}. \] (10)

The quadratic equation (9) has the solutions
\[ c_{a,b} \cdot 2 f_Q \left( \hat{P} + \frac{1}{4} \frac{f_R}{f_Q} \mathbb{I} \right) = X \mathbb{I} + Y u_m u^n, \] (11)

where \( c_{a,b} = \pm 1 \) and the signs of the roots in \( X \) and \( Y \) follow from requiring that the final solution of the model solves equation (5). The trace of (11) yields
\[ c_a (2 f_Q R + 2 f_R) = 4 X - Y, \] (12)

and at this point we simply eliminate all roots and \( \pm 1 \) factors by squaring equation (12) twice:
\[ ((2 f_Q R + 2 f_R)^2 + 8 X^2 + 2 f_Q (\rho + p))^2 = 36 X^2 (2 f_Q R + 2 f_R)^2. \] (13)

Using (8) and (13), the Ricci scalar \( R \) and the squared Ricci tensor \( Q \) can be determined as algebraic functions of \( \rho \), \( p \) and \( \Lambda \).

Next, we derive the connection \( \Gamma \) with the help of the auxiliary metric \( h_{mn} \) defined by
\[ \sqrt{|h|} h^{mn} = \sqrt{|g|} g^{mn} (f_R \mathbb{I} + 2 f_Q \hat{P})_a^n, \] (14)

\(^3\) Here, \( \mathbb{I} = \delta_m^n \) is the identity matrix, and \( \hat{P}^2 = R^m_n R^n_s. \)
which allows us to write equation (6) in the form
\[ \nabla_a [\sqrt{|h|} h^{am}] = 0. \] (15)

Since \( \bar{P} \) follows from equation (11), one finds after some algebra that \( h_{mn} \) is related to the ‘physical’ metric \( g_{mn} \) by the disformal transformation [57]
\[ h_{mn} = \Omega \left( g_{mn} - \frac{L_2}{L_1 - L_2} u^m u^n \right), \quad h^{mn} = \Omega^{-1} \left( g^{mn} + \frac{L_2}{L_1} u^m u^n \right), \] (16)
where
\[ \Omega := \sqrt{|L_1 (L_1 - L_2)|}, \quad L_1 := c a X + \frac{1}{2} f_R, \quad L_2 := c a Y. \] (17)

Finally, the Palatini connection \( \Gamma^a \) is just the solution of (15), i.e. the Lévi-Cività connection of \( h_{mn} \):
\[ \Gamma^a_{bc}[h] = \frac{1}{2} h^{ad} (h_{dc,b} + h_{bd,c} - h_{bc,d}) \] (18)
with \( h_{mn} \) given in (16). This result solves equation (6) and eventually defines the Ricci tensor and all quantities derived from it. Note that \( h_{mn} \) does not contain spacetime derivatives because it is a function of only \( g_{ab}, \rho, p \) and \( \Lambda \). Hence, \( R_{ab}, R \) and the other quantities calculated according to equations (2) and (3) contain derivatives of at most second order of the metric \( g_{ab} \). Consequently, in this modified gravity theory in the Palatini formalism one has to solve the second order differential equation
\[ R[\Gamma] = R(\rho, p, \Lambda), \] (19)
where the right-hand side follows from the algebraic solution of equations (8) and (13), whereas the left-hand side is given in equation (3) using \( \Gamma \) from equation (18). In the next section, we will apply this procedure to the CC filter model.

3. CC filter model

In [33], a modified gravity model was constructed that filters out vacuum energy contributions (i.e. everything with the EOS \( p = -\rho \)) to the CC. It is defined by the following action functional:
\[ f(R, Q) = \kappa R + z, \quad z := \beta \left( \frac{R^3}{B} \right)^m, \quad B := R^2 - Q. \] (20)
with the dimensionful parameters \( \kappa \) and \( \beta \) and the positive number \( m \). Note that in this setup, the term \( z \) is never considered to be a little correction to \( \kappa R \), instead both terms in \( f \) act in collaboration. From here on, we will use \( B \) instead of \( Q \). With the partial derivatives of \( f, \) \( f_R = (\kappa R + \frac{1}{2} mz) / R - 2 f_Q R, \) \( f_Q = mz / B \) and the trace \( T = -4 \Lambda + 3 p - \rho \) of the energy–stress tensor (7), equation (8) becomes
\[ \gamma z = \kappa R - 4 \Lambda + 3 p - \rho, \quad \gamma := -2 - \frac{4}{3} m, \] (21)
while equation (13) can be written as
\[ 0 = \kappa R + r - \frac{2}{9} m^2 \left( \frac{B}{R^2} \right)^2 \left[ 1 + \frac{3}{m^2} (3 \kappa R + 2r) + \frac{9}{4(m^2)^2} (\kappa R)^2 - r^2 \right] \]
\[ + \frac{8}{27} m^2 \left( \frac{B}{R^2} \right)^2 \left[ 1 + \frac{3 \kappa R}{2(m^2)} \right]^2. \] (22)
In the last equation, we introduced the variable \( r := \rho + p \), which vanishes identically for CC contributions.
For matter energy densities $\rho$ and curvatures $\kappa R$ smaller in magnitude than the large CC, the term $yz \approx -4\Lambda$ in (21) is approximately constant and equation (22) determines the curvature as a function of $r$ alone. Therefore, contributions to the CC and shifts in the vacuum energy density are filtered out. This limit of constant $z = -4\Lambda$ covers most of the radiation epoch, the matter era and the late-time de Sitter phase, and it has been analysed analytically in [33] by neglecting strongly suppressed terms like $O(r/z)$.

In the following, we complement our previous results by investigating the very early universe, where $r$ may be of the same order as $\Lambda$ or above. Consequently, $z$ in equation (21) is not constant anymore and all terms in equation (22) must be taken into account. It is convenient to work with dimensionless variables, where energy densities and the curvature term $\kappa R$ are normalized by the cosmological constant ($4\Lambda$):

\[ y := \frac{\kappa R}{4\Lambda}, \quad x := \frac{r + \rho}{4\Lambda}. \quad (23) \]

Moreover, let us define

\[ m_z := \frac{m \cdot z}{4\Lambda} = \frac{m}{1 - \gamma} (1 - y - x \cdot \omega_m) \quad \text{with} \quad \omega_m := \frac{3\omega - 1}{\omega + 1}, \quad (24) \]

where $\omega = p/\rho$ denotes the matter EOS. Finally, we introduce the variable

\[ W := \frac{R^2}{B} = y^\gamma (1 - y - x \cdot \omega_m)^\delta \cdot \delta, \quad (25) \]

where $B$ was eliminated on the right-hand side by using the definition of $z$ in equation (20).

The constant $\delta$ is completely fixed by the model parameters and the large vacuum energy density $\Lambda$,

\[ \delta := \left( \frac{2m}{-9\gamma} \right) \left( \frac{4\Lambda}{\rho_c} \right)^{\frac{1}{2}}, \quad \text{where} \quad (\rho_c)^{\gamma} := \kappa \left( \frac{2}{9 \gamma} \right) \left( \frac{4\Lambda}{\rho_c} \right)^{\frac{1}{2}}. \quad (26) \]

According to our earlier results in [33], $\delta$ is very large in magnitude because $\rho_c$ is of the order of the late-time critical energy density in the limit $yz \approx -4\Lambda$. Expressed in terms of the dimensionless variables, the main equation (22) reads

\[ 0 = y + x - m_z \left[ 1 + \frac{3(3y + 2x)}{m_z} + \frac{9y^2 - x^2}{4m_z^2} \right] + \frac{8}{W^2} \left[ \frac{1 + 3y}{2m_z} \right]^2. \quad (27) \]

For the late-time solutions mentioned above, we find $m_z = -m/\gamma = O(1)$, $|y|, |x| \ll 1$ and $|W| \gg 1$. Therefore, equation (27) simplifies considerably:

\[ y + x - \frac{2m_z}{9W} = 0, \quad (28) \]

indicating $\kappa R \rightarrow \rho_c$ for vanishing matter $r \rightarrow 0$, which justifies introducing $\rho_c$ in equation (26).

In the next section, we solve the complete equation (27) requiring a numerical treatment. In practice, we work along the following procedure. First, we consider a spatially flat cosmological background with the expansion described by the scale factor $a(t)$ as a function of cosmological time. In Cartesian coordinates $(i = 1, 2, 3)$, the non-zero components of the metric are given by $g_{00} = -1$, $g_{ii} = a^2(t)$, and in the stress tensor we use $u_m = \delta^0_m$. Hence, the auxiliary metric in (16) has only the diagonal components

\[ h_{00} = \Omega^2 \left( \frac{-L_1}{L_{12}} \right), \quad h_{ii} = \Omega a^2(t), \quad L_{12} := L_1 - L_2, \quad (29) \]
which determine via (18) the symmetric components of the Palatini connection
\[
\Gamma^0_{\alpha0} = \frac{x}{4} \left( 3 \frac{L_1'(x)}{L_1} - \frac{L_1(x)}{L_1} \right),
\]
\[
\Gamma^0_{i0} = \frac{a^2}{L_1} \left( \frac{\dot{a}}{a} L_{12} + \frac{x}{4} \left[ L_{12}'(x) + \frac{L_{12}(x)}{L_1} \right] \right),
\]
\[
\Gamma^i_{j0} = \frac{a + \frac{x}{4} \left( \frac{L_{12}(x)}{L_1} + \frac{x}{L_1} \right) \dot{a}}{a}.
\]
(30)

Here, the overdot indicates a derivative with respect to the time coordinate \( t \), whereas the prime denotes the partial derivative with respect to \( x \). For simplicity we consider matter with a constant EOS \( \omega \), which obeys the standard conservation equation \( \dot{\rho} + 3 \frac{x}{a} \rho (1 + \omega) = 0 \) implying \( \rho \propto a^{-3(1+\omega)} \) and \( \dot{x} = -3 \frac{x}{a} (1 + \omega) x \) via \( x = (1 + \omega) \rho / (4 \Lambda) \). Consequently, the Ricci scalar \( R(t) \) calculated via (3) from the connection (30) is a function of \( a, \dot{a} \) and \( \ddot{a} \), whereas the algebraic solution \( y(x) = \kappa R / (4 \Lambda) \) of equation (27) is only a function of \( a \). For obtaining \( a(t) \), we solve numerically the differential equation (19):
\[
\frac{R(t)}{H_0^2} = K \cdot y(x(t)) \quad \text{with} \quad K := \frac{4 \Lambda}{\kappa H_0^2},
\]
(31)

where \( H_0 = \frac{1}{2} (t_0) \) is the Hubble rate at the arbitrary initial time \( t_0 \). In [33], we found \((-\kappa)\) to be of the order of the inverse Newton’s constant \( G \) indicating \( k H_0^2 \sim \rho_0 \) by identifying \( H_0 \) with the present Hubble rate and using the late-time solution (28). In this case, \( K \) is roughly the square root of \( \delta \) in (26); its exact value is fixed when we specify \( H_0 \) and \( x(t_0) \) at the initial time \( t_0 \). Numerically, we first solve the equation \( R^2 / B = W(y(x)) \) for \( \ddot{a}(t_0) \), where \( R^2 / B \) has to be calculated from (30) at \( t = t_0 \) and \( x = x(a_0) \). Then, with \( \dot{a}(t_0) \) the values of \( R(t_0) \) and \( y(x(t_0)) \) in equation (31) are known, which yields the numerical value of \( K \) and equation (31) can be solved to determine the evolution of the scale factor \( a(t) \) for all times \( t \).

4. Solutions

Here, we study the cosmological evolution resulting from the differential equation (31). Thereby, we use the solutions to the main equation (27), which relate the curvature \( y \) to the matter energy density and pressure in \( x \) as given in (23). We do not restrict the magnitudes of \( x \) and \( y \); however, in order to be roughly compatible with our existent universe, we consider only solutions \( y(x) \) with both a final accelerating phase, \( x \to 0 \), and a high energy decelerating epoch, \( y \approx -x \). Moreover, for the numerical examples below we set \( m = 1/3 \) in equation (20), because this parameter has only little influence on the qualitative behaviour of the solutions. With these preliminaries, there are four qualitatively different cases to study corresponding to \( \Lambda \leq 0 \) and \( \delta \leq 0 \). Note that the sign of \( \delta \) can be adjusted by a suitable choice of the parameter \( \beta \) in the action functional (20).

(1) The absence of an initial singularity can be seen best in the case \( \delta > 0 \) and \( \Lambda < 0 \) implying \( \rho_\text{cr} < 0 \) via equation (26) and \( y < 0 \) from (21). In this case, the matter variable \( x = r / (4 \Lambda) \) is negative because of \( r = \rho + p > 0 \). Numerically, we do not find solutions for \( y < 0 \) that satisfy our requirements given above. This can also be inferred from equation (28), where all terms would be negative. On the other hand for \( y > 0 \) (or \( \kappa R < 0 \)) we find

\[\vdots\]

4 Alternatively, one could start with given values of \( x \) and \( x(t_0) \) and infer \( H_0 \) from it. This is analogous to finding \( H_0 \) from the Friedmann equation \( 3 H_0^2 = 8 \pi G \cdot \rho(t) \) in general relativity.

5 We use this equation in terms of arbitrary \( t \) to determine the correct choice of \( c_{a,b} \) in (10) and (17) after equation (31) has been solved for \( a(t) \).
equation (28) as a good low-energy description with $y(x \to 0) \to \rho_c/(4\Lambda)$ and $y \approx -x$ for $|\rho_c| \ll |x| \ll 1$. However, before $y \to 1$, the matter variable $|x|$ becomes smaller again and eventually vanishes for finite $y < 1$ as illustrated in figure 1. As a result, $|y| \approx a^{-3(\omega + 1)}$ and $y$ are bounded from above signalling the absence of the initial singularity, where $x$ or $y$ would diverge. Analytically, this boundedness can be understood from equations (24) and (25) indicating that for sufficiently large values of $y$ or $|x|$ the variables $W$ and $m_z$ vanish. Since they appear in the denominators in our main equation (27), it is clear that neither $|x|$ nor $y$ can become larger than 1. After solving the differential equation (31) for the scale factor $a(t)$, we observe that not the whole phase-space curve $y(x)$ in figure 1 is realized, but only a smaller part of it. This behaviour can be explained by observing that the function $L_1 - L_2$ vanishes at $x \approx -0.043$ as shown in figure 2. Since $L_1 - L_2$ appears in the denominator of the connection components (30) and therefore in the Ricci scalar $R(t)$, equation (31) can be fulfilled only if $\dot{a}/a \propto \dot{x}/x$ vanishes at the zero of $L_1 - L_2$ because $y(x(t))$ on the right-hand side of (31) is always finite. Accordingly, $\dot{a} = 0$ indicates the occurrence of a cosmic bounce, which is shown in the plots for the scale factor $a(t)$, the Hubble rate $H = \dot{a}/a$ and the deceleration $q(t)$ in figure 3. Also we can confirm the analytical results from [33], where it was found that at early times the universe behaves like a radiation- or dust-dominated cosmos depending on the matter EOS $\omega$. Thus, for $\omega = 1/3$ or 0 the deceleration $q$ approaches the value 1 or $1/2$, respectively. In this era, $y \approx -x$ is a good approximation. Finally, at late times ($x \to 0$), we find a de Sitter cosmos with $y \to \rho_c/(4\Lambda)$.

(2) The second case we consider is $\delta < 0$ and $\Lambda > 0$ implying $x > 0$ and $\rho_c < 0$. Here, the only reasonable solution requires $y, \kappa R < 0$. In contrast to the first case, the variables $x$ and $|y|$ are not bounded from above because $W$ and $m_z$ in equations (24) and (25) do not vanish. Instead, we find $y \to -x$ for large $x \to \infty$, see figure 4. Also the functions $L_1$ and $L_1 - L_2$ do not vanish in this region. Nevertheless, the cosmic evolution inferred from the numerical solution of (31) shows a cosmic bounce, where the scale factor $a(t)$ and the matter and curvature variables $x, |y|$ remain finite, see figure 5. As before, the universe...
shows a radiation- or matter-dominated expansion behaviour with $y \approx -x$ and a final de Sitter phase ($y \rightarrow \rho_e / (4\Lambda)$).

(3) The third possibility is characterized by $\delta < 0$ and $\Lambda < 0$ indicating $x < 0$ and $\rho_e > 0$. We did not find a reasonable solution in this case, although for $y > 0$, a $y \approx -x$ phase exists, but it cannot end in a low-energy accelerating phase with $x \rightarrow 0$ because all terms in equation (27) are positive in this limit. Hence, we do not pursue this solution any further.

(4) Finally, the last case with $\delta > 0$ and $\Lambda > 0$ implies $x > 0$, and for $y < 0$ we find a radiation/matter phase described by $y \approx -x$ for $x \rightarrow \infty$. Similar to the third case, $\rho_e > 0$
Figure 4. The solution $y(x)$ from equation (27) and the functions $L_1(x)$ and $L_{12}(x)$ in the connection in equations (30) for the case $\delta = -5 \times 10^6$, $\Lambda > 0$ and two values of the EOS $\omega = \frac{1}{3}$, 0.

Figure 5. The numerical solutions from equation (31) for the scale factor $a(t)$, the Hubble rate $H(t)$ and the deceleration factor $q = -\frac{a''}{aH^2}$ for the case $\delta = -5 \times 10^6$, $\Lambda > 0$ described in figure 4.

does not seem to allow a late-time de Sitter solution; however, the last term $\propto W^{-2}$ in equation (27) is positive giving rise to an accelerating solution with $x \to 0$. This cosmos is, however, quite exotic, because there is a contraction phase between the decelerating and the final de Sitter phase. For completeness, the expansion behaviour of this interesting but unrealistic solution is shown in figure 6.
In the examples shown in this section, we used relatively small values of $|\delta|$ for numerical reasons. According to its definition in equation (26), $\delta \sim (\Lambda / \rho_e)^{7/3}$ is related to the ratio of the large CC and the late-time energy density implying an enormous magnitude for $\delta$. However, this does not correspond to unreasonable values for the parameter $\beta$ in action (20), instead a range of common values can be obtained depending on $m$ and $\kappa$. Moreover, in this work, we have normalized energy densities like $r$ and $\kappa R$ by the constant $\Lambda$ such that our results are independent of the numerical values of $\kappa$ and $\Lambda$. For details on $\beta$ and the relation between $\kappa$ and Newton’s constant, we refer the reader to [33].

In summary, we found cosmological solutions for both signs of the large CC $\Lambda$, where the initial big bang singularity is replaced by a cosmic bounce with finite curvature and finite matter energy density. After the bounce these universes behave like being radiation or matter dominated, where the deceleration $q$ is close to 1 or $\frac{1}{2}$, respectively. Eventually, when the matter component is sufficiently diluted the final de Sitter era begins, with an effective vacuum energy density $\sim \rho_e$ much smaller in magnitude than the large CC $\Lambda$. The latter property and the absence of an initial cosmological singularity are the main benefits of the CC filter model.

5. Conclusions

The filter model for a large CC in the context of Palatini modified gravity [33] makes gravity insensitive to vacuum energy density contributions. In other words, the curvature is dominated by ordinary matter, whereas energy sources with the EOS of a CC do not contribute at leading order despite having a much larger energy density at late times.

In this work, we have explored the CC filter model in the context of the very early universe, where in classical theories one usually expects to find an initial big bang singularity at very high energies. However, in our setup, we observed that the curvature singularity is replaced by a cosmic bounce with the matter energy density, the scale factor and the curvature remaining finite. In the case shown in figure 1, the boundedness can be seen already from the solution of the algebraic equation (27), whereas the other cases show the same feature after solving the differential equation (31). Moreover, our numerical results also confirm the analytical study of the late-time evolution in [33].
The absence of an initial spacetime singularity could mean that our classical model for gravity does not require an ultraviolet completion from a theory of quantum gravity. However, in the light of the exotic form of the gravity action functional (20), it is unlikely that this setup is a fundamental theory. On the other hand, we may follow the argumentation in [54], where Palatini gravity models are proposed as effective continuum descriptions of loop quantum cosmology. In both approaches, the regularization of the initial singularity does not require the introduction of new degrees of freedom making them minimal solutions to this problem.

Independent of its theoretical origin, the CC filter model has the capability of handling two fundamental problems of gravity, the old CC problem and the avoidance of an initial singularity. Therefore, it is an attractive scenario for further investigations. In addition to the qualitative features discussed so far, it will be interesting to study more aspects of the CC filter model in the future, e.g. black hole or star interiors or anisotropies and inhomogeneities in cosmology.

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