Towards Resolution of the Enigmas of
\(P\)-Wave Meson Spectroscopy

L. Burakovsky\(^*\) and T. Goldman\(†\)

Theoretical Division, MS B285
Los Alamos National Laboratory
Los Alamos, NM 87545, USA

Abstract

The mass spectrum of \(P\)-wave mesons is considered in a nonrelativistic constituent quark model. The results show the common mass degeneracy of the isovector and isodoublet states of the scalar and tensor meson nonets, and do not exclude the possibility of a similar degeneracy of the same states of the axial-vector and pseudovector nonets. Current experimental hadronic and \(\tau\)-decay data suggest, however, a different scenario leading to the \(a_1\) meson mass \(\simeq 1190\) MeV and the \(K_{1A}-K_{1B}\) mixing angle \(\simeq (37 \pm 3)°\). Possible \(s\bar{s}\) states of the four nonets are also discussed.

Key words: quark model, potential model, \(P\)-wave mesons

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1 Introduction

The existence of a gluon self-coupling in QCD suggests that, in addition to the conventional \(q\bar{q}\) states, there may be non-\(q\bar{q}\) mesons: bound states including gluons (gluonia and glueballs, and \(q\bar{q}g\) hybrids) and multiquark states \([4]\). Since the theoretical guidance on the properties of unusual states is often contradictory, models that agree in

\(^*\)E-mail: BURAKOV@PION.LANL.GOV

\(†\)E-mail: GOLDMAN@T5.LANL.GOV
the $q\bar{q}$ sector differ in their predictions about new states. Among the naively expected signatures for gluonium are

i) no place in $q\bar{q}$ nonet,
ii) flavor-singlet coupling,
iii) enhanced production in gluon-rich channels such as $J/\Psi(1S)$ decay,
iv) reduced $\gamma\gamma$ coupling,
v) exotic quantum numbers not allowed for $q\bar{q}$ (in some cases).

Points iii) and iv) can be summarized by the Chanowitz $S$ parameter \[ S = \frac{\Gamma(J/\Psi(1S) \rightarrow \gamma X)}{\text{PS}(J/\Psi(1S) \rightarrow \gamma X)} \times \frac{\text{PS}(X \rightarrow \gamma\gamma)}{\Gamma(X \rightarrow \gamma\gamma)}, \]

where PS stands for phase space. $S$ is expected to be larger for gluonium than for $q\bar{q}$ states. Of course, mixing effects and other dynamical effects such as form-factors can obscure these simple signatures. Even if the mixing is large, however, simply counting the number of observed states remains a clear signal for non-exotic non-$q\bar{q}$ states. Exotic quantum number states ($0^{--}$, $0^{+-}$, $1^{-+}$, $2^{+-}$, ...) would be the best signatures for non-$q\bar{q}$ states. It should be also emphasized that no state has yet unambiguously been identified as gluonium, or as a multiquark state, or as a hybrid.

In this paper we shall discuss $P$-wave meson states, the interpretation of which as members of conventional quark model $q\bar{q}$ nonets encounters difficulties. We shall be concerned with the scalar, axial-vector, pseudovector and tensor meson nonets which have the following $q\bar{q}$ quark model assignments, according to the most recent Review of Particle Physics:

1) $1^1P_1$ pseudovector meson nonet, $J^{PC} = 1^{-+}$, $b_1(1235)$, $h_1(1170)$, $h'_1(1380)$, $K_{1B}$
2) $1^3P_0$ scalar meson nonet, $J^{PC} = 0^{++}$, $a_0(?)$, $f_0(?)$, $f'_0(?)$, $K^*_0(1430)$
3) $1^3P_1$ axial-vector meson nonet, $J^{PC} = 1^{++}$, $a_1(1260)$, $f_1(1285)$, $f'_1(1510)$, $K_{1A}$
4) $1^3P_2$ tensor meson nonet, $J^{PC} = 2^{++}$, $a_2(1320)$, $f_2(1270)$, $f'_2(1525)$, $K^*_2(1430)$, and start with a review of the states and their possible masses and assignments, as viewed by several groups.

1. Scalar meson nonet.
The spectrum of the scalar meson nonet is a long-standing problem of light meson spectroscopy. The number of resonances found in the region of 1–2 GeV exceeds the number of states that conventional quark models can accommodate. Extra states are interpreted alternatively as $K\bar{K}$ molecules, glueballs, multi-quark states or hybrids. In particular, except for a well established scalar isodoublet state, the $K^*_0(1430)$, the Particle Data Group (PDG) lists two isovector states, the $a_0(980)$ and $a_0(1450)$. The latter, having mass and width $1450 \pm 40$ MeV, $270 \pm 40$ MeV, respectively, was discovered recently by the Crystal Barrel collaboration at LEAR. The third isovector state (not included in the PDG), $a_0(1320)$, having mass and width $1322 \pm 30$ MeV and $130 \pm 30$ MeV, was seen by GAMS and LASS in the partial wave analyses of the $\eta\pi$ and $K_s\bar{K}_s$ data, respectively. There are four isoscalar

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1The $K_{1A}$ and $K_{1B}$ are nearly 45° mixed states of the $K_1(1270)$ and $K_1(1400)$, their masses is therefore $\approx 1340$ MeV.
states in \[1\] the \(f_0(400 - 1200)\) (or \(\sigma\)), the interpretation of which as a particle is controversial due to a huge width of 600–1000 MeV, \(f_0(980)\), \(f_0(1370)\) (which stands for two separate states, \(f_0(1300)\) and \(f_0(1370)\), of a previous edition of PDG \[3\]), and \(f_0(1500)\) (which also stands for two separate states, \(f_0(1525)\) and \(f_0(1590)\), of a previous edition of PDG), and two more possibly scalar states, the \(f_3(1710)\), \(J = 0\) or \(2\), seen in radiative \(J/\Psi\) decays, and an \(\eta\)-\(\eta\) resonance \(X(1740)\) with uncertain spin, produced in \(p\bar{p}\) annihilation in flight and in charge-exchange. Recently several groups claimed different scalar isoscalar structures close to 1500 MeV, including a narrow state with mass 1450 \(\pm\) 5 MeV and width 54 \(\pm\) 7 MeV \[2\]. The lightest of the three states at 1505 MeV, 1750 MeV and 2104 MeV revealed upon reanalyzing of data on \(J/\Psi\rightarrow\gamma 2\pi^+2\pi^-\) \[3\], and the \(f_0(1450)\), \(f_0(1500)\), \(f_0(1520)\). The masses, widths and decay branching ratios of these states are incompatible within the errors quoted by the groups. We do not consider it as plausible that so many scalar isoscalar states exist in such a narrow mass interval. Instead, we take the various states as manifestation of one object which we identify with the \(f_0(1525)\) of a previous edition of PDG.

It has been convincingly argued that the narrow \(a_0(980)\), which has also been seen as a narrow structure in \(\eta\pi\) scattering, can be generated by meson-meson dynamics alone \[11, 12\]. This interpretation of the \(a_0(980)\) leaves the \(a_0(1320)\) or \(a_0(1450)\) (which may be manifestations of one state having a mass in the interval 1350-1400 MeV) as the \(3^3P_0\) \(q\bar{q}\) state. Similarly, it is mostly assumed that the \(f_0(980)\) is a \(K\bar{K}\) molecule, as suggested originally by Weinstein and Isgur \[11\]. The mass degeneracy and their proximity to the \(K\bar{K}\) threshold seem to require that the nature of both, \(a_0(980)\) and \(f_0(980)\), states should be the same. On the other hand, the \(K\bar{K}\) interaction in the \(I = 1\) and \(I = 0\) channels is very different: the extremely attractive \(I = 0\) interaction may not support a loosely bound state. Instead, it may just define the pole position of the \(f_0(980)\) \(q\bar{q}\) resonance. Indeed, a recent analysis of all high statistics data in the neighborhood of the \(K\bar{K}\) threshold done by Pennington \[3\] indicates in an almost model-independent way that the \(f_0(980)\) is not a \(K\bar{K}\) molecule. Moreover, Morgan and Pennington \[13\] find the \(f_0(980)\) pole structure characteristic for a genuine resonance of the constituents and not of a weakly bound system. The \(I = 1\) \(K\bar{K}\) interaction is weak and may generate a \(K\bar{K}\) molecule. Alternatively, Törnqvist \[14\] interprets both the \(f_0(980)\) and \(a_0(980)\) as the members of the \(q\bar{q}\) nonet with strong coupling to the decay channels, which, however, does not account for the recently discovered \(a_0(1320)\) and \(a_0(1450)\).

With respect to the \(f_0(1370)\) (or two separate states, \(f_0(1300)\) and \(f_0(1370)\), according to a previous edition of PDG), we follow the arguments of Morgan and Pennington \[13\] and assume that the \(\pi\pi\) interaction produces both very broad, \(f_0(1000)\), and narrow, \(f_0(980)\), states, giving rise to a dip at 980 MeV in the squared \(\pi\pi\) scattering amplitude \(T_{11}\). In this picture, the \(f_0(1370)\) is interpreted as the high-mass part of the \(f_0(1000)\) (the low-mass part may be associated with the \(\sigma\) of recent PDG). In experiments, the \(f_0(1000)\) shows up at \(\sim 1300\) MeV because of the pronounced dip in \(|T_{11}|^2\) at \(\sim 1\) GeV. The \(f_0(1000)\) has an extremely large width; thus the resonance interpretation is questionable. It could be generated by \(t\)-channel exchanges instead.
of inter-quark forces [15].

The \( f_0(1500) \) state has a peculiar decay pattern [16]

\[
\pi\pi : \eta\eta : \eta\eta' : K\bar{K} = 1.45 : 0.39 \pm 0.15 : 0.28 \pm 0.12 : < 0.15.
\]  

This pattern can be reproduced by assuming the existence of a further scalar state which is mainly \( s\bar{s} \) and should have a mass of about 1700 MeV, possibly the \( \tilde{f}_J(1710) \), and tuning the mixing of the \( f_0(1500) \) with the \( f_0(1370) \) \( n\bar{n} \) and the (predicted) \( f_0(1700) \) \( s\bar{s} \) states [16]. In this picture, the \( f_0(1500) \) is interpreted as a glueball state with strong mixing with the close-by conventional scalar mesons. An interpretation of the \( f_0(1500) \) as a conventional \( q\bar{q} \) state, as well as a qualitative explanation of its reduced \( K\bar{K} \) partial width, were given by Klempt et al. [17] in a relativistic quark model with linear confinement and an instanton-induced interaction. A quantitative explanation of the reduced \( K\bar{K} \) partial width of the \( f_0(1500) \) was given in a very recent publication by the same authors [18].

The above arguments lead one to the following spectrum of the scalar meson nonet (in the order: isovector, isodoublet, isoscalar mostly singlet, isoscalar mostly octet),

\[
a_0(1320) \text{ or } a_0(1450), \quad K_0^*(1430), \quad f_0(980) \text{ or } f_0(1000), \quad f_0(1500).
\]  

This spectrum agrees essentially with the \( q\bar{q} \) assignments found by Klempt et al. [17], and Dmitrasinovic [19] who considered the Nambu–Jona-Lasinio model with a \( U_A(1) \) breaking instanton-induced ’t Hooft interaction. The spectrum of the meson nonet given in [17] is

\[
a_0(1320), K_0^*(1430), f_0(1470), f_0(980),
\]  

while that suggested by Dmitrasinovic, on the basis of the sum rule

\[
m_{f_0}^2 + m_{f_0'}^2 + m_\eta^2 + m_{\eta'}^2 = 2(m_{K^*}^2 + m_{K}^2)
\]  

derived in his paper, is [13]

\[
a_0(1320), K_0^*(1430), f_0(1590), f_0(1000).
\]  

The \( q\bar{q} \) assignment obtained by one of the authors by the application of the linear mass spectrum discussed in ref. [20] to a composite system of the two, pseudoscalar and scalar nonets, is [21]

\[
a_0(1320), K_0^*(1430), f_0(1525), f_0(980),
\]  

in essential agreement with (3) and (5).

2. Axial-vector meson nonet.

1) One of the uncertainties related to the axial-vector nonet is the still undefined properties of its \( I = 1 \) member, the \( a_1(1260) \) meson. This meson has a huge width of \( \sim 400 \) MeV, due to strong coupling to a dominant decay channel \( a_1 \rightarrow \rho\pi \), which makes the determination of its mass rather difficult. A decade ago Bowler [22] argued that, according to his parametrization of the couplings as functions of mass,
the $\tau$-decay and hadronic data as of 1986 were entirely consistent as far as the $a_1$
mass was concerned, and concluded that the $a_1$ mass and width are safely within
the ranges $\simeq 1235 \pm 40$ MeV and $400 \pm 100$ MeV, respectively. These values are in
excellent agreement with those currently adopted by PDG [4]: $1230 \pm 40$ MeV and
$\sim 400$ MeV. Historically, there exists a respectable old prediction by Weinberg [23],

$m(a_1) \simeq \sqrt{2} m(\rho)$, which places the mass of the $a_1$ around 1090 MeV, in apparent
disagreement with experiment. It is reasonable to think that the discrepancy be-
tween Weinberg’s prediction and the currently adopted value can be explained if one
includes possible contributions from the radial and orbital excitations of $\rho$ and $a_1$ to
the spectral sum rules used for the derivation of the $a_1$ mass. These excited states
can, in fact, possess nonnegligible matrix elements to the vacuum through the vector
and axial-vector currents, as indicated by the appreciable rate of the $\rho \rightarrow e^+e^-$ decay
[4]. An alternative derivation of the $\rho-a_1$ mass relation using the both Weinberg’s and
KSFR [24] sum rules done by Li [25] leads to $m(a_1) = 1.36 \pm 0.23$ GeV, in better
agreement with experiment. The value of the $a_1$ mass provided by QCD sum rules is
$1150 \pm 40$ MeV. Recently Oneda et al. put forward theoretical arguments which
suggest a simultaneous mass degeneracy of the axial-vector and pseudovector nonets
in the isovector and isodoublet channels [27]. The mass relations obtained in [27] in
the algebraic approach to QCD developed in ref. [28],

\[
\begin{align*}
  m^2(K^+) - m^2(\rho) &= m^2(K_{1A}) - m^2(a_1) = m^2(K_{1B}) - m^2(b_1), \\
  m(K_{1A}) &= m(K_{1B}),
\end{align*}
\]

lead to $m(a_1) = m(b_1)$. Since, according to the recent PDG analysis, $m(b_1) = 1231 \pm 10$
MeV, degeneracy of the $a_1$ and $b_1$ meson masses seems to be a real possibility.

2) The $q\bar{q}$ model predicts a nonet that includes two isoscalar $1^3P_1$ states with masses
below $\sim 1.6$ GeV. Three “good” $1^{++}$ objects are known, the $f_1(1285)$, $f_1(1420)$ and
$f_1(1510)$, one more than expected. Thus, one of the three is a non-$q\bar{q}$ meson, and
the $f_1(1420)$ is the best non-$q\bar{q}$ candidate, according to ref. [24]. In this case, it
may be a multiquark state in the form of a $KK\pi$ bound state (“molecule”) [30],
or a $KK^*$ deuteron-like state (“deuson”) [31]. On the other hand, Aihara et al.
[32] have argued that, assuming that both the $f_1(1285)$ and $f_1(1420)$ belong to the
same nonet and using several additional hypotheses, the octet-singlet mixing an-
gle obtained is compatible with the $f_1(1420)$ being mostly $s\bar{s}$ and the $f_1(1285)$ being
mostly $(u\bar{u} + d\bar{d})/\sqrt{2}$, although both require large admixtures of other $q\bar{q}$ components.

3. Pseudovector meson nonet.

Experimental information on the $h_1$ and $h_1'$ mesons is rather restricted. A very wide
$\rho\pi$ resonance with $I = 0$, $J^{PC} = 1^{+-}$ has been seen in three experiments as having
mass and width $1170 \pm 20$ MeV and $360 \pm 40$ MeV, respectively, which is identified
with the dominantly $(u\bar{u} + d\bar{d})/\sqrt{2}$ meson $h_1$. Its $s\bar{s}$ partner $h_1'$ is expected to decay
dominantly into $KK\pi$. So far, in a single experiment by LASS, a candidate has been
observed in $K^-p \rightarrow K_S K^\pm \pi^\mp \Lambda$ with mass and width $1380 \pm 20$ MeV and $80 \pm 30$ MeV,
respectively, decaying into $(K^*K + K^*K)$ [33]. More recently, the Crystal Barrel collaboration has studied the reaction $\bar{p}p \rightarrow K_L K_S \pi^0 \pi^0 (\rightarrow 8\gamma + \text{missing energy}$ and
4. Tensor meson nonet.

The two $^3P_2$ $q\bar{q}$ states are likely the well-known $f_2(1270)$ and $f_2'(1525)$ currently adopted by PDG, although the observation by Breakstone [35] of the $f_2(1270)$ production by gluon fusion could indicate that it has a glueball component. At least five more $J^{PC} = 2^{++}$ states have to be considered: the $f_2(1520)$, $f_2(1810)$, $f_2(2010)$, $f_2(2100)$ and $f_2(2340)$. Of these, the $f_2(1810)$ is likely to be the $2^3P_2$, and the three $f_2$’s above 2 GeV could possibly be the $2^3P_2$ $s\bar{s}$, $1^3F_2$ $s\bar{s}$, and $3^3P_2$ $s\bar{s}$, but a gluonium interpretation of one of the three is not excluded. The remaining $f_2(1520)$ was seen in 1989 by the ASTERIX collaboration [36] as a $2^{++}$ resonance in $p\bar{p}$ $P$-wave annihilation at 1565 MeV in the $\pi^+\pi^-\pi^0$ final state. Its mass is better determined in the $3\pi^0$ mode by the Crystal Barrel collaboration [37] to be 1515 MeV, in agreement with that seen previously [38].

It has no place in a $q\bar{q}$ scheme mainly because all nearby $q\bar{q}$ states are already occupied. Dover [39] has suggested that it is a “quasinuclear” $N\bar{N}$ bound state, and Törnqvist [31] that it is a deuteron-like $(\omega\omega+\rho\rho)/\sqrt{2}$ “deuson” state.

5. Let us also discuss the $I = 1/2$ $^1P_1$ and $^3P_1$ mesons, $K_1(1270)$ and $K_1(1400)$, with masses $1273 \pm 7$ MeV and $1402 \pm 7$ MeV, respectively [4]. It has been known that their decay satisfies a dynamical selection rule

$$\Gamma (K_1(1270) \to K\rho) \gg \Gamma (K_1(1270) \to K^*\pi),$$

$$\Gamma (K_1(1400) \to K^*\pi) \gg \Gamma (K_1(1400) \to K\rho),$$

which prompted experimentalists to suspect large mixing (with a mixing angle close to 45°) between the $I = 1/2$ members of the axial-vector and pseudovector nonets, $K_{1A}$ and $K_{1B}$, respectively, leading to the physical $K_1$ and $K_1'$ states [41]. Carnegie et al. [41] obtained the mixing angle $\theta_K = (41 \pm 4)^\circ$ as the optimum fit to the data as of 1977. In a recent paper by Blundell et al. [42], who have calculated strong OZI-allowed decays in the pseudoscalar emission model and the flux-tube breaking model, the $K_{1A}$-$K_{1B}$ mixing angle obtained is $\simeq 45^\circ$. Theoretically, in the exact $SU(3)$ limit the $K_{1A}$ and $K_{1B}$ states do not mix, similarly to their $I = 1$ counterparts $a_1$ and $b_1$. For the $s$-quark mass greater than the $u$- and $d$-quark masses, $SU(3)$ is broken and these states mix to give the physical $K_1$ and $K_1'$. If the $K_{1A}$ and $K_{1B}$ are degenerate before mixing, the mixing angle will always be $\theta_K = 45^\circ$ [43].

However, as pointed out by Suzuki [43], the recent data of the TPC/Two-Gamma collaboration on $K\pi\pi$ production in $\tau$-decay appear to contradict this simple picture: if $\theta_K = 45^\circ$, production of the $K_1(1270)$ and $K_1(1400)$ would be one-to-one up to the kinematic corrections, since in the $SU(3)$ limit only the linear combination $(K_1(1270) + K_1(1400))/\sqrt{2}$ would have the right quantum number to be produced.

momentum) [14]. Its final state is dominated by the strange resonances $K_1(1400)$, $K_1(1270)$, $K_0'(1430)$ and $K^*(892)$. However, albeit with a small intensity, the best fit requires also a $1^+$ state at 1385 MeV with width 200 MeV, decaying into $K^*K$. The $K_SK_L\pi^0$ Dalitz plot for a mass window of $\pm 50$ MeV around 1380 MeV shows a clearly destructive interference between the two $K^*$ bands, which is consistent with an $I = 0$, $J^{PC} = 1^{+-}$ state.
there. After the phase-space correction, the $K_1(1270)$ production would be favored over the $K_1(1400)$ one by nearly a factor of 2. Actually, $K\pi\pi$ production is dominated by the $K_1(1400)$, with little evidence for the $K_1(1270)$. As found by Suzuki \cite{45}, the $K_1(1270)/K_1(1400)$ production ratio observed favors $\theta_K \approx 33^\circ$, although some $SU(3)$ breaking effects are needed to obtain good qualitative agreement between the theory and experiment.

Since the experimentally established isodoublet states of the scalar and tensor meson nonets, $K_0^*$ and $K_1^*$, are mass degenerate, 1429 ± 6 MeV and 1429 MeV, respectively, and different models (like those considered in refs. \cite{17, 19, 21}) lead to the $q\bar{q}$ assignment for the scalar nonet which includes both the $a_0(1320)$ and $f_0(1525)$ mesons which are mass degenerate with the corresponding tensor mesons $a_2(1320)$ and $f_2(1525)$, the question naturally suggests itself as to whether the scalar and tensor nonets are intrinsically mass degenerate\cite{43}. Similar questions may be asked regarding the mass degeneracy of the axial-vector and pseudovector nonets in the $I = 1$ and $I = 1/2$ channels. If this mass degeneracy of two pairs of nonets, $^{3}P_0-^{3}P_2$ and $^{3}P_1-^{1}P_1$, is actually the case, it should be reproduced in a simple phenomenological model of QCD, e.g., in a nonrelativistic constituent quark model. The purpose of this work is to apply the latter model for $P$-wave meson spectroscopy in order to establish whether mass degeneracy of the two pairs of nonets discussed above actually occurs.

2 Nonrelativistic constituent quark model

In the constituent quark model, conventional mesons are bound states of a spin 1/2 quark and spin 1/2 antiquark bound by a phenomenological potential which has some basis in QCD \cite{46}. The quark and antiquark spins combine to give a total spin 0 or 1 which is coupled to the orbital angular momentum $L$. This leads to meson parity and charge conjugation given by $P = (-1)^{L+1}$ and $C = (-1)^L$, respectively. One typically assumes that the $q\bar{q}$ wave function is a solution of a nonrelativistic Schrödinger equation with the generalized Breit-Fermi Hamiltonian\cite{46},

$$H_{BF}\psi_n(r) \equiv (H_{kin} + V(p, r))\psi_n(r) = E_n\psi_n(r),$$

where $H_{kin} = m_1 + m_2 + p^2/2\mu - (1/m_1^2 + 1/m_2^2)p^4/8, \mu = m_1m_2/(m_1 + m_2)$, are the constituent quark masses, and to first order in $(v/c)^2 = p^2c^2/E^2 \simeq p^2/m^2c^2,$ $V(p, r)$ reduces to the standard nonrelativistic result,

$$V(p, r) \simeq V(r) + V_{SS} + V_{LS} + V_T,$$

\footnote{In the scenario suggested in refs. \cite{17, 19}, due to instanton effects, the mass of the $f_0$ meson is shifted down to $\sim 1$ GeV, as compared to the mass $\approx 1275$ MeV of its tensor “partner” $f_2$.}

\footnote{The most widely used potential models are the relativized model of Godfrey and Isgur \cite{47} for the $q\bar{q}$ mesons, and Capstick and Isgur \cite{48} for the $qqq$ baryons. These models differ from the nonrelativistic quark potential model only in relatively minor ways, such as the use of $H_{kin} = \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}$ in place of that given in (8), the retention of the $m/E$ factors in the matrix elements, and the introduction of coordinate smearing in the singular terms such as $\delta(r)$.}
with \( V(r) = V_T(r) + V_S(r) \) being the confining potential which consists of a vector and a scalar contribution, and \( V_{SS}, V_{LS} \) and \( V_T \) the spin-spin, spin-orbit and tensor terms, respectively, given by \([10]\)

\[
V_{SS} = \frac{2}{3m_1m_2} \mathbf{s}_1 \cdot \mathbf{s}_2 \Delta V_V(r),
\]

\[
V_{LS} = \frac{1}{4m_1^2m_2^2} \frac{1}{r} \left\{ \left[ (m_1 + m_2)^2 + 2m_1m_2 \right] \mathbf{L} \cdot \mathbf{S}_+ + (m_2^2 - m_1^2) \mathbf{L} \cdot \mathbf{S}_- \right\} \frac{dV_V(r)}{dr} - \left[ (m_1^2 + m_2^2) \mathbf{L} \cdot \mathbf{S}_+ + (m_2^2 - m_1^2) \mathbf{L} \cdot \mathbf{S}_- \right] \frac{dV_S(r)}{dr},
\]

\[
V_T = \frac{1}{12m_1m_2} \left( \frac{dV_V(r)}{dr} - \frac{d^2V_V(r)}{dr^2} \right) S_{12}.
\]

Here \( \mathbf{S}_+ \equiv \mathbf{s}_1 + \mathbf{s}_2, \mathbf{S}_- \equiv \mathbf{s}_1 - \mathbf{s}_2, \) and

\[
S_{12} \equiv 3 \left( \frac{(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})}{r^2} - \frac{1}{3} \mathbf{s}_1 \cdot \mathbf{s}_2 \right).
\]

For constituents with spin \( s_1 = s_2 = 1/2, S_{12} \) may be rewritten in the form

\[
S_{12} = 2 \left( 3 \frac{(\mathbf{s} \cdot \mathbf{r})^2}{r^2} - r^2 \right), \quad \mathbf{S} = \mathbf{S}_+ \equiv \mathbf{s}_1 + \mathbf{s}_2.
\]

Since \((m_1 + m_2)^2 + 2m_1m_2 = 6m_1m_2 + (m_2 - m_1)^2, m_1^2 + m_2^2 = 2m_1m_2 + (m_2 - m_1)^2,\) the expression for \( V_{LS}, \) Eq. (11), may be rewritten as follows,

\[
V_{LS} = \frac{1}{2m_1m_2} \frac{1}{r} \left\{ \left[ 3 \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right] + \frac{(m_2 - m_1)^2}{2m_1m_2} \left( \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \right\} \mathbf{L} \cdot \mathbf{S}_+ \quad + \quad \frac{m_1^2 - m_1^1}{4m_1^2m_2^2} \frac{1}{r} \left( \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \mathbf{L} \cdot \mathbf{S}_- \equiv V_{LS}^+ + V_{LS}^-.
\]

Since two terms corresponding to the derivatives of the potentials with respect to \( r \) are of the same order of magnitude, the above expression for \( V_{LS}^+ \) may be rewritten as

\[
V_{LS}^+ = \frac{1}{2m_1m_2} \frac{1}{r} \left( 3 \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \mathbf{L} \cdot \mathbf{S} \left[ 1 + \frac{(m_2 - m_1)^2}{2m_1m_2} \cdot O(1) \right].
\]

### 2.1 \( S \)-wave spectroscopy

Let us first apply the Breit-Fermi Hamiltonian to the \( S \)-wave which consists of the two, \( ^1S_0 \) \( J^{PC} = 0^- \) pseudoscalar and \( ^3S_1 \) \( 1^- \) vector, meson nonets. We shall consider only the \( I = 1 \) and \( I = 1/2 \) mesons which are pure \( n\bar{n} \) and \( (n\bar{s}, s\bar{n}) \) states, respectively. Since the expectation values of the spin-orbit and tensor terms vanish for \( L = 0 \) or \( S = 0 \) states \([10]\), the mass of a \( q\bar{q} \) state with \( L = 0 \) is given by

\[
M = m_1 + m_2 + E + K \frac{s_1 \cdot s_2}{m_1m_2},
\]

(17)
where $E$ is the nonrelativistic binding energy. As shown below, the sum of just the constituent quark masses and the quark-quark hyperfine interaction term describes the masses of the $S$-wave mesons extremely well. Moreover, Eq. (17) with no $E$ is consistent with the empirical mass squared splitting $\Delta M^2 \equiv M^2(3S_1) - M^2(1S_0) \approx \text{const}$ for all the corresponding mesons composed by the $n, s$ and $c$ quarks, aside from charmonia. Physically, these observations mean that $E$ is small compared to $m_1$ and $m_2$ or approximately constant over all of these states, and so may be absorbed in the definition of the latter. For the higher $L$ nonets, $E$ decreases with the increasing quark masses, according to the Feynman-Hellmann theorem; it therefore may be absorbed into the constituent quark mass defined for every $L$.

Since $s_1 \cdot s_2 = -3/4$ for spin-0 mesons and $+1/4$ for spin-1 mesons, one has the four relations (in the following, $\pi$ stands for the mass of the $\pi$ meson, etc., $n$ and $s$ for the masses of the non-strange and strange quarks, respectively, and we assume $SU(2)$ flavor symmetry, $m_u = m_d = n$),

$$\pi = 2n - \frac{3}{4} \frac{\Lambda}{n^2}, \quad \text{(18)}$$
$$\rho = 2n + \frac{1}{4} \frac{\Lambda}{n^2}, \quad \text{(19)}$$
$$K = n + s - \frac{3}{4} \frac{\Lambda}{ns}, \quad \text{(20)}$$
$$K^* = n + s + \frac{1}{4} \frac{\Lambda}{ns}. \quad \text{(21)}$$

It then follows from these relations that

$$n = \frac{\pi + 3\rho}{8}, \quad \text{(22)}$$
$$s = \frac{2K + 6K^* - \pi - 3\rho}{8}, \quad \text{(23)}$$
$$\frac{\Lambda}{n^2} = \frac{\rho - \pi}{2}, \quad \text{(24)}$$
$$\frac{\Lambda}{ns} = \frac{K^* - K}{2}. \quad \text{(25)}$$

By expressing the ratio $n/s$ in two different ways, directly from (22),(23) and dividing (25) by (24), one obtains the relation

$$\frac{n}{s} = \frac{\pi + 3\rho}{2K + 6K^* - \pi - 3\rho} = \frac{K^* - K}{\rho - \pi}. \quad \text{(26)}$$

For the physical values of $\pi, \rho, K$ and $K^*$ (in MeV), 138, 769, 495 and 892, respectively, the above relation reads $0.629 = 0.627$, i.e., the result is consistent within the accuracy provided by the assumption of exact $SU(2)$ flavor symmetry. The values of $n, s$ and $K$ provided by (22)-(25) are $n = 306$ MeV, $s = 487$ MeV, $K = 0.0592$ GeV$^3 = (390$ MeV)$^3$. The values of the meson masses, as calculated from (18)-(21), are (in MeV) $\pi = 137.8, \rho = 770.0, K = 495.0, K^* = 892.3$. The relation (17) may also be applied successfully to the $^3S_1 I = 0$ mesons too, assuming that they are pure...
n\bar{n} and s\bar{s} states. In this case, as follows from (19), \( \omega = \rho = 770 \text{ MeV} \), and \( \phi = 2s + K/(4s^2) = 1036 \text{ MeV} \). Both numbers are within 1.5% of the physical values 782 and 1019 MeV, respectively.

Let us note that, although Eq. (18) contains no information on chiral symmetry, one may deal with the chiral limit \( \pi = 0 \) by the introduction of the so called “dynamical” quark mass \( m_{\text{dyn}} \), defined as the solution of \( 2m_{\text{dyn}}^2 - 3K/(4m_{\text{dyn}}^2) = 0 \).

Although this does not restore chiral symmetry, it does incorporate the masslessness of the pion, in accord with common understanding of the latter as the Nambu-Goldstone boson of broken chiral symmetry, as well as calculating the chiral limit values of \( \rho \) and \( K^* \), in agreement with other models [50].

3 \ P-wave spectroscopy

We now wish to apply the Breit-Fermi Hamiltonian to the \( P \)-wave mesons. By calculating the expectation values of different terms of the Hamiltonian defined in Eqs. (10),(14),(15), taking into account the corresponding matrix elements \( \langle L \cdot S \rangle \) and \( S_{12} \) [46], one obtains the relations [42]

\[
\begin{align*}
M(3P_0) &= M_0 + \frac{1}{4}\langle V_{SS} \rangle - 2\langle V_{LS}^+ \rangle + \langle V_T \rangle, \\
M(3P_2) &= M_0 + \frac{1}{4}\langle V_{SS} \rangle + \langle V_{LS}^+ \rangle + \frac{1}{10}\langle V_T \rangle, \\
M(a_1) &= M_0 + \frac{1}{4}\langle V_{SS} \rangle - \langle V_{LS}^+ \rangle - \frac{1}{2}\langle V_T \rangle, \\
M(b_1) &= M_0 - \frac{3}{4}\langle V_{SS} \rangle,
\end{align*}
\]

\[
\left( \begin{array}{c} M(K_1) \\
M(K_1') \end{array} \right) = \left( \begin{array}{cc} M_0 + \frac{1}{4}\langle V_{SS} \rangle - \langle V_{LS}^+ \rangle - \frac{1}{2}\langle V_T \rangle & \sqrt{2}\langle V_{LS}^- \rangle \\
\sqrt{2}\langle V_{LS}^- \rangle & M_0 - \frac{3}{4}\langle V_{SS} \rangle \end{array} \right) \left( \begin{array}{c} K_{1A} \\
K_{1B} \end{array} \right),
\]

where \( M_0 \) stands for the sum of the constituent quark masses in either case. The \( V_{LS}^- \) term acts only on the \( I = 1/2 \) singlet and triplet states giving rise to the spin-orbit mixing between these states\(^4\), and is responsible for the physical masses of the \( K_1 \) and \( K_1' \). Let us assume, for simplicity, that

\[
\sqrt{2}\langle V_{LS}^- \rangle(K_{1B}) \simeq -\sqrt{2}\langle V_{LS}^- \rangle(K_{1A}) \equiv \Delta.
\]

\(^4\)The spin-orbit \( 3P_1 - 1P_1 \) mixing is a property of the model we are considering; the possibility that another mechanism is responsible for this mixing, such as mixing via common decay channels [44] should not be ruled out, but is not included here.
The masses of the $K_{1A}$, $K_{1B}$ are then determined by relations similar to those for the $a_1$, $b_1$ above, and $K_1 \simeq K_{1A} + \Delta$, $K_1' \simeq K_{1B} - \Delta$, or

$$\Delta \simeq K_1 - K_{1A} \simeq K_{1B} - K_1'.$$

We consider, therefore, the following formulas for the masses of all eight $I = 1, 1/2$ $P$-wave mesons, $b_1, a_0, a_1, a_2, K_{1B}, K_{0}, K_{1A}, K_{2}':$

\begin{align*}
M^{(1P_1)} & = m_1 + m_2 - \frac{3}{4} \frac{a}{m_1 m_2}, \\
M^{(3P_0)} & = m_1 + m_2 + \frac{1}{4} \frac{a}{m_1 m_2} - \frac{2b}{m_1 m_2} + \frac{c}{m_1 m_2}, \\
M^{(3P_1)} & = m_1 + m_2 + \frac{1}{4} \frac{a}{m_1 m_2} - \frac{b}{m_1 m_2} - \frac{2}{m_1 m_2}, \\
M^{(3P_2)} & = m_1 + m_2 + \frac{1}{4} \frac{a}{m_1 m_2} - \frac{b}{m_1 m_2} + \frac{c}{10 m_1 m_2},
\end{align*}

where $a, b$ and $c$ are related to the matrix elements of $V_{SS}, V_{LS}$ and $V_T$ (see Eqs. (10),(12),(16)), and assumed to be the same for all of the $P$-wave states. In the above expressions, the nonrelativistic binding energies are absorbed in the constituent quark masses, as discussed above. The same constituent quark masses appear also in the denominators of the hyperfine interaction terms in Eqs. (12)-(15) below, similar to $S$-wave spectroscopy considered in a previous section. Since this is usually done only for the lowest $S$-wave states, we briefly review the precedent and argument for the generality of these forms.

It was shown in [51] that a pure scalar potential contributes to the effective constituent quark mass. Bag models suggest that the kinetic energy also contributes to the effective constituent quark mass in the case of no potential [52]. These results were generalized further by Cohen and Lipkin [53] who have shown that both the kinetic and potential energy are included in the effective mass parameter which appears also in the denominators of the hyperfine interaction terms in the case of a scalar confining potential. The analyses of experimental data suggest that the non-strange and strange quarks are mainly subject to scalar part of the confining potential (whereas charmed and bottom quarks are more dominantly affected by Coulomb-like vector part) [10]. Moreover, the generality of the arguments by Cohen and Lipkin [53] allows one to apply them to any partial wave. Therefore, the constituent quark masses can be defined for any partial wave, through relations of the form (28)-(31); in this case they vary with the energies of the corresponding mass levels. Such an energy dependence of the constituent quark masses was considered in refs. [54, 55].

Actually, as follows from Eq. (45) below,

$$\frac{K_1 - K_{1A}}{K_{1B} - K_1} = \frac{K_1 + K_{1B}}{K_1 + K_{1A}} \approx \frac{2K_{1B}}{2K_{1A}} \approx 1,$$

since the deviations $K_1 - K_{1A}, K_{1B} - K_1'$ are small compared to $K_{1A}, K_{1B}$, and the mixing angle is $\sim 45^\circ$. 

11
Also, a QCD-based mechanism which generates dynamical quark mass growing with $L$ in a Regge-like manner was considered by Simonov [56].

The correction to $V_{LS}^\pm$ in the formula (16), due to the difference in the masses of the $n$ and $s$ quarks, is ignored. Indeed, these masses, as calculated from (28)-(31), are

$$n = \frac{3b_1 + a_0 + 3a_1 + 5a_2}{24},$$

$$s = \frac{6K_{1B} + 2K_0^* + 6K_{1A} + 10K_0^* - 3b_1 - a_0 - 3a_1 - 5a_2}{24}.$$  \hspace{1cm} (33)

With the physical values of the meson masses (in GeV), $a_1 \simeq b_1 \simeq 1.23$, $a_0 \simeq a_2 \simeq 1.32$, $K_{1A} \simeq K_{1B} \simeq 1.34$, $K_0^* \simeq K_2^* \simeq 1.43$, the above relations give

$$n \simeq 640 \text{ MeV}, \quad s \simeq 740 \text{ MeV},$$

so that the abovementioned correction, according to (16), is $\sim 100^2/(2 \cdot 640 \cdot 740) \simeq 1\%$, i.e., comparable to isospin breaking on the scale considered here, and so completely negligible. It follows from (28)-(31) that

$$\frac{9a}{m_1m_2} = M(3P_0) + 3M(3P_1) + 5M(3P_2) - 9M(1P_1),$$

$$\frac{12b}{m_1m_2} = 5M(3P_2) - 3M(3P_1) - 2M(3P_0),$$

$$\frac{18c}{5m_1m_2} = 2M(3P_0) + M(3P_2) - 3M(3P_1).$$ \hspace{1cm} (37)

By expressing the ratio $n/s$ in four different ways, viz., directly from (32),(33) and dividing the expressions (35)-(37) for the $I = 1/2$ and $I = 1$ mesons by each other, similarly to the case of the $S$-wave mesons considered above, one obtains the three relations,

$$\frac{3a_1 + 3b_1 + a_0 + 5a_2}{6K_{1A} + 6K_{1B} + 2K_0^* + 10K_2^* - 3a_1 - 3b_1 - a_0 - 5a_2} = \frac{K_0^* + 3K_{1A} + 5K_2^* - 9K_{1B}}{a_0 + 3a_1 + 5a_2 - 9b_1},$$

$$\frac{K_0^* + 3K_{1A} + 5K_2^* - 9K_{1B}}{a_0 + 3a_1 + 5a_2 - 9b_1} = \frac{5K_2^* - 3K_{1A} - 2K_0^*}{5a_2 - 3a_1 - 2a_0},$$

$$\frac{5K_2^* - 3K_{1A} - 2K_0^*}{5a_2 - 3a_1 - 2a_0} = \frac{2K_0^* + K_2^* - 3K_{1A}}{2a_0 + a_2 - 3a_1}.$$ \hspace{1cm} (40)

First consider Eq. (40) which may be rewritten, by a simple algebra, as

$$(K_2^* - K_0^*)(a_2 - a_1) = (K_2^* - K_{1A})(a_2 - a_0).$$ \hspace{1cm} (41)

Since $K_2^* \simeq K_0^* \simeq 1430$ MeV, it then follows from (41) that either $K_2^* \simeq K_0^* \simeq K_{1A}$, or $a_0 \simeq a_2$. The first possibility should be discarded as unphysical, since it leads, through the relations (36),(37) applied to the $I = 1/2$ mesons, to $b = c = 0$, which would in turn, from the same relations for the $I = 1$ mesons, imply $a_0 \simeq a_1 \simeq a_2$,
in apparent contradiction with experimental data on the masses of the $a_1$ and $a_2$ mesons. The physical case corresponds, therefore, to

$$a_0 \cong a_2,$$  \hspace{1cm} (42)

i.e., the mass degeneracy of the scalar and tensor meson nonets in the $I = 1/2$ channel, $K^*_0 \cong K^*_2$, implies a similar degeneracy also in the $I = 1$ channel. Note that this relation is a general feature of the nonrelativistic quark model for the $P$-wave mesons we are considering here. Even in the presence of an extra term in (28)-(31) corresponding to the quark binding energy which we have ignored by absorbing into the constituent masses, Eqs. (36) and (37) will remain the same and again lead, through (40), to the relation (42).

With $K^*_0 = K^*_2$ and $a_0 = a_2$, Eqs. (38) and (39) may be rewritten as

$$(a_0 - a_1 + K^*_0 - K_{1A})(a_1 + b_1 + 2a_0) = 2(K^*_0 - K_{1A})(K_{1A} + K_{1B} + 2K^*_0),$$ \hspace{1cm} (43)

$$(K_{1A} - K_{1B})(a_0 - a_1) = (K^*_0 - K_{1A})(a_1 - b_1).$$ \hspace{1cm} (44)

One now has to determine the values of $a_1$, $K_{1A}$ and $K_{1B}$. The remaining equation is obtained from the mixing of the $K_{1A}$ and $K_{1B}$ states which results in the physical $K_1$ and $K'_1$ mesons; independent of the mixing angle,

$$K^2_{1A} + K^2_{1B} = K^2_1 + K'^2_1.$$ \hspace{1cm} (45)

One sees that, as follows from (44), the mass degeneracy of the $^3P_1$ and $^1P_1$ nonets in the $I = 1/2$ channel, $K_{1A} = K_{1B}$, implies similar degeneracy in the $I = 1$ channel too, $a_1 = b_1$, and vice versa, so that the model we are considering provides the consistent possibility

$$a_1 = b_1, \hspace{0.5cm} K_{1A} = K_{1B}.$$ \hspace{1cm} (46)

We now check how this possibility agrees with experimental data on the meson masses. It follows from (45) and $K_1 = 1273 \pm 7$ MeV, $K'_1 = 1402 \pm 7$ MeV that in this case

$$K_{1A} = K_{1B} = 1339 \pm 7 \text{ MeV}.$$ \hspace{1cm} (47)

With $a_1 = b_1$, $K_{1A} = K_{1B}$, Eq. (43) reduces to

$$a_0^2 - a_1^2 + (a_0 + a_1)(K^*_0 - K_{1A}) = 2(K^*_0 - K^2_{1A}),$$ \hspace{1cm} (48)

which for $a_0 = a_2 = 1318$ MeV, $K^*_0 = 1429$ MeV and $K_{1A}$, $K_{1B}$ given in (47) has the solution

$$a_1 = b_1 = 1211 \pm 8 \text{ MeV},$$ \hspace{1cm} (49)

which is only a 2-standard deviation inconsistency with the experimentally established $b_1$ meson mass $1231 \pm 10$ MeV. We also consider another solution of (43)-(45) determined by adjusting $b_1$ to the experimental value $1231$ MeV. It then follows that in this case the solution to (43)-(45) is

$$a_1 = 1191 \text{ MeV}, \hspace{0.5cm} K_{1A} = 1322 \text{ MeV}, \hspace{0.5cm} K_{1B} = 1356 \text{ MeV},$$ \hspace{1cm} (50)
with small deviations from these values for possible deviations in the input parameters; e.g., with (in MeV) \( b_1 = 1231 \pm 10 \), the actual solution is \( a_1 = 1191 \pm 10 \), \( K_{1A} = 1322 \pm 9 \), \( K_{1B} = 1356 \pm 9 \), or with \( K_1 = 1273 \pm 7 \), \( K_1' = 1402 \pm 7 \), the solution is \( a_1 = 1191 \pm 17 \), \( K_{1A} = 1322 \pm 14 \), and \( K_{1B} \) remains the same. For the solution (50), we observe the sum rule

\[
K_{1A}^2 - a_1^2 = 0.329 \text{ GeV}^2 \simeq K_{1B}^2 - b_1^2 = 0.323 \text{ GeV}^2,
\]

which is accurate to 2\% and also holds for deviations from (50) due to uncertainties in the input parameters. Relations of the type (51) may be anticipated on the basis of the formulas

\[
K_{2}^2 - \rho^2 = K_2^2 - \pi^2, \quad K_2^2 - a_2^2 = K_2^2 - \pi^2, \quad \text{etc.,}
\]

provided by either the algebraic approach to QCD [28] or phenomenological formulas

\[
m_{1/2}^2 = 2B(n + C), \quad m_{1/2}^2 = B(n + C)
\]

(where \( B \) is related to the quark condensate, and \( C \) is a constant within a given meson nonet) motivated by the linear mass spectrum of a nonet and the collinearity of Regge trajectories of the corresponding \( I = 1 \) and \( I = 1/2 \) states, as discussed in ref. [20]. Note that (51) agrees with the second of the three relations (7).

Thus, the nonrelativistic constituent quark model we are considering provides two possibilities for the mass spectra of the axial-vector and pseudovector meson nonets:

1) \( a_1 = b_1 \simeq 1210 \text{ MeV}, \quad K_{1A} = K_{1B} \simeq 1340 \text{ MeV}, \)
2) \( a_1 \neq b_1, \quad K_{1A} \neq K_{1B}, \quad K_{1A}^2 - a_1^2 \simeq K_{1B}^2 - b_1^2. \)

The second case is obviously favored by current experimental data on \( K\pi\pi \) production in \( \tau \)-decay which do not support \( \theta_K \approx 45^\circ \) and, therefore, mass degeneracy of the \( K_{1A} \) and \( K_{1B} \), as discussed above in the text. In this case, assume that the \( K_1(1270) \) belongs to the axial-vector nonet, while the \( K(1400) \) belongs to the pseudovector nonet, in accord with the recent suggestion by Suzuki [57], on the basis of the analysis of the \( \tau \)-decay mode \( \tau \to \nu, K_1 \), for the values (in MeV, as follows from discussion below Eq. (50)) \( K_1 = 1273 \pm 7, \quad K_1' = 1402 \pm 7, \quad K_{1A} = 1322 \pm 14, \quad K_{1B} = 1356. \) One then obtains, with the help of the formula [13]

\[
\tan^2(2\theta_K) = \left( \frac{K_{1A}^2(1400) - K_{1A}^2(1270)}{K_{1B}^2 - K_{1A}^2} \right)^2 - 1, \quad \theta_K = (37.3 \pm 3.2)^\circ,
\]

in good qualitative agreement with the values \( \approx 33^\circ \) suggested by Suzuki [13], and \( \approx 34^\circ \) found by Godfrey and Isgur [17] in a relativized quark model.

The parameters of the spin-spin, spin-orbit, and tensor interaction in our model may be calculated from Eqs. (35)-(37) with the meson mass values obtained above. In the isodoublet channel, e.g., one obtains

\[
\langle V_{SS} \rangle = \frac{a}{n_s} \simeq 37 \text{ MeV},
\]
\[
\langle V_{LS}^+ \rangle = \frac{b}{n_s} \simeq 27 \text{ MeV},
\]
\[
\langle V_T \rangle = \frac{c}{n_s} \simeq 89 \text{ MeV}.
\]
The expectation value \( \langle V_{LS}^- \rangle \) may be obtained from (27),(50):

\[
\sqrt{2} \langle V_{LS}^- \rangle \simeq K_1(1270) - K_{1A} = (1273 - 1322) \text{ MeV}
\]

\[
\simeq K_{1B} - K_1(1400) = (1356 - 1402) \text{ MeV} \simeq -47.5 \text{ MeV},
\]

and therefore

\[
\langle V_{LS}^- \rangle \simeq -33.5 \text{ MeV},
\]

so that both \( \langle V_{LS}^+ \rangle \) and \( \langle V_{LS}^- \rangle \) are of the very similar magnitude (but opposite in sign).

Using the obtained values of \( \langle V_{LS}^+ \rangle \) and \( \langle V_{LS}^- \rangle \), along with the values of \( n \) and \( s \) given in (34), in Eqs. (15),(16), one may establish the following relation among the expectation values of the derivatives of the potentials:

\[
\left\langle \frac{1}{r} \frac{dV_S(r)}{dr} \right\rangle \simeq 2.8 \left\langle \frac{1}{r} \frac{dV_V(r)}{dr} \right\rangle.
\]

(57)

In the case of the QCD-motivated Cornell potential [58]

\[
V(r) = -\frac{4 \alpha_s}{3} r + ar,
\]

(58)

with a spin structure \( V = V_V + V_s, V_V = -\frac{4 \alpha_s}{3} r, V_S = ar \), the relation (57) reduces to

\[
a\langle r^{-1} \rangle \simeq 3.7 \alpha_s \langle r^{-3} \rangle.
\]

(59)

Consider now the ratio [46]

\[
\rho = \frac{M(3P_2) - M(3P_1)}{M(3P_1) - M(3P_0)}.
\]

(60)

Since the measured masses of the \( K_2^* \) and \( K_0^* \) coincide (as also do those of the \( a_2 \) and \( a_0 \), as established in Section 3), the value of this ratio is

\[
\rho = -1.
\]

(61)

By equating this value of \( \rho \) with that given in [46] for the Cornell case,

\[
\rho = \frac{1}{5} \frac{8\alpha_s \langle r^{-3} \rangle - 5a\langle r^{-1} \rangle}{2\alpha_s \langle r^{-3} \rangle - 4a\langle r^{-1} \rangle},
\]

(62)

we obtain

\[
a\langle r^{-1} \rangle = 4.8 \alpha_s \langle r^{-3} \rangle.
\]

(63)

Comparison of the relation (63) with (59) shows that the nonrelativistic constituent quark model considered in this paper is completely consistent, at the 25% level, with the Cornell potential with the spin structure of a vector-scalar mixing type. We
consider this to be completely satisfactory agreement, since the expectation values and $\alpha_s$ are all purely determined in this region for light quark systems.\footnote{For $a = 1/(2\pi\alpha') \simeq 0.18$ GeV$^2$, where $\alpha' \simeq 0.9$ GeV$^{-2}$ is the universal Regge slope, it follows from the relation\footnote{\[ \Delta M^2 \equiv M^2(3S_1) - M^2(1S_0) \simeq \frac{32}{9} \alpha_s a \approx 0.56 \text{ GeV}^2 \]} that $\alpha_s \simeq 0.9$. With these values of $a$ and $\alpha_s$, and in the approximation\footnote{\[ \langle r^{-3} \rangle \sim \frac{\langle r^{-1} \rangle}{\langle r \rangle^2} \]} it then follows from (59) that $\langle r \rangle \simeq 0.9$ fm.}

One may now estimate the masses of the isoscalar mesons of the four nonets assuming that they are pure $s\bar{s}$ states, by the application of (28)-(31) with $m_1 = m_2 = s$; it then follows that

$$h'_1 \simeq f'_1 \simeq 1435 \text{ MeV}, \quad f'_0 \simeq f'_2 \simeq 1525 \text{ MeV}.$$  \hspace{1cm} (64)

Hence, the model we are considering suggests that $1^{++}$ $s\bar{s}$ state is the $f_1(1420)$ (with mass $1427 \pm 2$ MeV) rather than $f_1(1510)$ ($1512 \pm 4$ MeV) meson, in accord with the arguments of Aihara et al.\cite{32}. The value 1435 given by (64) is within 4\% of the $h'_1$ meson mass, 1380 $\pm$ 20. Also, the value 1525 given by (66) agrees with the experimentally established mass of the $f'_2$ meson, $1525 \pm 5$ MeV.

At this point we call that the nonrelativistic quark model predictions on the masses of the isoscalar states are reliable for all $P$-wave nonets except the scalar nonet. Indeed, as shown by ‘t Hooft in his study on the $U_A(1)$ problem\cite{59}, an expansion of the (euclidian) action around the one-instanton solutions of the gauge fields assuming dominance of the zero modes of the fermion fields leads to an effective $2N_f$-fermion interaction ($N_f$ being the number of fermion flavors) not covered by perturbative gluon exchange, which gives an additional contribution to the ordinary confining quark-antiquark interaction. As shown in ref.\cite{60}, due to its point-like nature and specific spin structure, the instanton-induced interaction in the formulation of ‘t Hooft acts on the states with spin zero only. The masses of the other mesons with non-vanishing spin are therefore dominantly determined by the confining interaction, leading to the conventional splitting and an ideal mixing of the $q\bar{q}$ nonets which are well reproduced by constituent quark models. The only two nonets whose mass spectra turn out to be affected by an instanton-induced interaction are spin zero pseudoscalar and scalar nonets. Quantitatively, an instanton-induced interaction for the scalar mesons is of the same magnitude but opposite in sign to that of the pseudoscalars\cite{17}. It, therefore, lowers the mass of the scalar isosinglet state, in contrast to the case of the pseudoscalar isosinglet ($\eta_0$) state the mass of which is pushed up by the instanton-induced interaction before it mixes with the pseudoscalar iso-octet ($\eta_8$) state to form the physical $\eta$ and $\eta'$ states.

Thus, the only nonrelativistic quark model prediction of the masses of the isoscalar states of the scalar nonet which may be trustworthy is that on the mass of the
mostly isoscalar octet state (which has a dominantly $s\bar{s}$ component): $f_0 \approx 1525$ MeV, as given in (64), in good agreement with the measured mass of the $f_0(1500)$. The second isoscalar state of the scalar nonet should be mostly $n\bar{n}$ but also contain a non-negligible $s\bar{s}$ component. Its mass may be determined from Eq. (4) with $f_0 = 1503 \pm 11$ MeV [4]: $f'_0 = 1048 \pm 16$ MeV.

Hence, one of the two, $f_0(980)$ or $f_0(1000)$, may be associated with the remaining isoscalar, which is difficult to decide (that is, on the basis of the constituent quark model we are discussing). However, two observations support the interpretation of the $f_0(980)$ as a $q\bar{q}$ state. First, the $t$-dependence of the $f_0(980)$ and the broad background produced in $\pi^-p \to \pi^0\pi^0n$ differ substantially [61]. The $f_0(1000)$ is produced in peripheral collisions only, while the $f_0(980)$ shows a strong $t$-dependence, as expected for a $q\bar{q}$ state. Second, as remarked above, although the isoscalar mostly $SU(3)$ singlet state should have a dominant $n\bar{n}$ component, its $s\bar{s}$ component should be appreciable. The $f_0(980)$ is seen strongly in $J/\Psi \to \phi f_0(980)$, but at most weakly in $J/\Psi \to \omega f_0(980)$. On the basis of quark diagrams, one must conclude that the $f_0(980)$ has a very large $s\bar{s}$ component; its decay into $\pi\pi$ with the corresponding branching ratio 78% [4] underlines an appreciable $n\bar{n}$ component.

Thus, the constituent quark model discussed in this paper supports the $q\bar{q}$ assignment for the scalar meson nonet (6) found by one of the authors in a previous paper [21]. For this assignment, the $f_0-f'_0$ mixing angle, as calculated with the help of the Gell-Mann–Okubo mass formula for $f'_0 = 980 \pm 10$ MeV [4] and $f_0 = 1525 \pm 5$ MeV [8],

$$ \tan^2 \theta_S = \frac{4K^*_{0}^2 - a_0^2 - 3f_0^2}{3f_0^2 + a_0^2 - 4K^*_{0}^2}, \quad (65) $$

is

$$ \theta_S = (21.4 \pm 1.0)^\circ, $$

in reasonably good agreement with the value predicted by Ritter et al. [18], $\theta_S \approx 25^\circ$, for which the partial widths of the $f_0(1500)$ calculated in their paper are

$$ \pi\pi : \eta\eta : \eta\eta' : K\bar{K} = 1.45 : 0.32 : 0.18 : 0.03, $$

in excellent agreement with the experimentally observed partial widths, Eq. (1).

### 4 Concluding remarks

As we have shown, a nonrelativistic constituent quark model confirms a simultaneous mass degeneracy of the scalar and tensor nonets in the isovector and isodoublet channels, and suggests a nearly mass-degeneracy of the corresponding isoscalar mostly octet states. The mass of the remaining $0^{++}$ isoscalar mostly singlet state is probably shifted down to $\sim 1$ GeV due to instanton effects, as discussed in refs. [17, 19], thus leaving two, the $f_0(980)$ and $f_0(1000)$, mesons as candidates for this state. Out of these two, preference should be given to the $f_0(980)$, as discussed above in the text.

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7The scalar meson nonet cannot be ideally mixed, as well as the pseudoscalar one, as seen, e.g., in Eq. (4).
Let us note that, if one ignores instanton or any other effects which may cause a shift in the mass of the $f'_0$, one would arrive at a $q\bar{q}$ assignment for the scalar nonet which would be nearly mass degenerate in all isospin channels (e.g., $f_0(1300)$ in (6) in place of $f_0(980)$, as compared to $f_2(1270)$). In this case, one would have the scalar nonet almost ideally mixed, just like the tensor one is. Then, as shown by Törnqvist \[32\], flavor symmetry (which should be good in the case of such an ideally mixed nonet) would predict the total width of the $a_0(1320)$ (using the experimental $K^*_0$ width as normalization) $\Gamma > 400$ MeV, and a similar $\sim 400$ MeV width of the $f_0(1525)$, much larger than 130 MeV found by GAMS for the $a_0(1320)$ and $\approx 90$ MeV found by LASS for the $f_0(1525)$. Therefore, this case should be considered as unphysical.

Although the possibility of a simultaneous mass degeneracy of the axial-vector and pseudovector nonets in the $I = 1$ and $I = 1/2$ channels is not excluded in the model considered here, it is disfavored by current experimental data. By adjusting the mass of the $b_1$ meson to the experimentally established value, the masses of the $a_1$, $K_{1A}$ and $K_{1B}$ mesons were calculated, leading to the predictions $m(a_1) \approx 1190$ MeV, and $\theta_K \approx (37 \pm 3)^o$. While the former number naturally interpolates between various predictions and current experimental data (e.g., it is at the upper limit of the range $(1150 \pm 40)$ MeV established in \[26\], and at the lower limit of that provided by data, $(1230 \pm 40)$ MeV \[1\]), the latter one is in quantitative agreement with the predictions $\theta_K \approx 33^o$ by Suzuki \[12\] and $\approx 34^o$ by Godfrey and Isgur \[17\]. The results of the work suggest that the mostly $s\bar{s}$ state of the axial-vector nonet should be associated with the $f_1(1420)$ rather than $f_1(1510)$ meson, which supports conclusions of Aihara et al. \[32\]. We did not calculate the decay widths and branching ratios for this case, since that was done in ref. \[32\]. We wish to give yet another argument in support of this prediction. As follows from the chiral theory of mesons initiated by Li \[63\], the mixing angles of both the vector and axial-vector nonets should coincide \[64\]. The value of the mixing angle of the axial-vector nonet, as calculated from Eq. (65) for $a_1$, $K_{1A}$ given in (50), with deviations due to the input parameters $K$, $K'$, and $f_1 = 1427$ MeV, is

$$\theta_A = (42.4 \pm 5.3)^o,$$

in good agreement with $\theta_V = 39.5^o$ of the vector meson nonet, while for $f_1 = 1512$ MeV and the same $a_1$ and $K_{1A}$, Eq. (65) gives

$$\theta_A = (54.8 \pm 3.4)^o,$$

in apparent disagreement with $\theta_V$.

The values of the $a_1$ and $K_{1A}$ masses calculated in this work fix the mass of the $K_{1B}$ to be 1356 MeV. The mass of the isoscalar octet state of the $1^1P_1$ nonet is then determined by the Gell-Mann–Okubo formula

$$h_8^2 = \frac{4K_{1B}^2 - b_1^2}{3},$$

$h_8 = 1395 \pm 3$ MeV (for $b_1 = 1231 \pm 10$ MeV). Since for the pseudovector nonet Eq. (65) may be rewritten as

$$\tan^2 \theta = \frac{h_8^2 - h_1^2}{h_1^2 - h_8^2},$$

18
it is clear that \( h_1 \) and \( h'_1 \) cannot both be less than \( h_8 \). Therefore, \( h'_1 \) should be greater than \( h_8 \), and with the PDG value \( h'_1 = 1380 \pm 20 \) MeV, one is left with \( h'_1 \approx 1400 \) MeV. In this case, since the \( h'_1 \) lies slightly above the \( h_8 \), the pseudovector nonet has a small positive mixing angle (just in opposite to the case of the pseudoscalar nonet for which the \( \eta \) lies slightly below the \( \eta_8 = 566 \) MeV leading to a small negative mixing angle). The above conclusion would change if one (or both) of the \( h_1, h'_1 \) appeared to have a mass higher than the value currently adopted by PDG.

We close with a short summary of the findings of this work.
1. The nonrelativistic constituent quark model shows a simultaneous mass degeneracy of the scalar and tensor meson nonets in the \( I = 1, 1/2 \), and nearly mass degeneracy in the \( I = 0, s\bar{s} \) channels.
2. Simultaneous mass degeneracy of the axial-vector and pseudovector nonets in the \( I = 1, 1/2 \) channels is not excluded in this model, but is disfavored by current experimental data.
3. The \( q\bar{q} \) assignments for the \( P \)-wave nonets obtained on the basis of the results of the work, are

\[
\begin{align*}
1^1P_1 & \ J^{PC} = 1^{+-}, \ b_1(1235), \ h_1(1170), \ h_1(1400), \ K_{1B} \\
1^3P_0 & \ J^{PC} = 0^{++}, \ a_0(1320), \ f_0(980), \ f_0(1500), \ K_0^*(1430) \\
1^3P_1 & \ J^{PC} = 1^{++}, \ a_1(1190), \ f_1(1285), \ f'_1(1420), \ K_{1A} \\
1^3P_2 & \ J^{PC} = 2^{++}, \ a_2(1320), \ f_2(1270), \ f'_2(1525), \ K_2^*(1430)
\end{align*}
\]

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