Tests for the extraction of Boer-Mulders functions

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Abstract. At present, the Boer-Mulders (BM) functions are extracted from asymmetry data using the simplifying assumption of their proportionality to the Sivers functions for each quark flavour. Here we present two independent tests for this assumption. We subject COMPASS data on semi-inclusive deep inelastic scattering on the $\langle \cos \phi_h \rangle$, $\langle \cos 2\phi_h \rangle$ and Sivers asymmetries to these tests. Our analysis shows that the tests are satisfied with the available data if the proportionality constant is the same for all quark flavours, which does not correspond to the flavour dependence used in existing analyses. This suggests that the published information on the BM functions may be unreliable.

The $\langle \cos \phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$ asymmetries receive contributions also from the, in principle, calculable Cahn effect. We succeed in extracting the Cahn contributions from experiment (we believe for the first time) and compare with their calculated values, with interesting implications.

1. Introduction

At present it is already recognized that the collinear picture of the parton model, according to which quark momenta are parallel to proton momentum, is a rather rough approximation for the nucleon structure – quarks have also transverse momentum. This leads especially to a completely new type – T-odd, parton densities (pdf’s). Here we shall focus on two of them – the Boer-Mulders (BM) and Sivers transverse momentum dependent (TMD) parton densities.

In present analyses [1] it is assumed that the BM functions are proportional to the Sivers functions. This much simplifies the analysis, but it is clearly model dependent – a different assumption would lead to different BM functions.

Here we suggest two independent tests for the above assumption using only measurable quantities – relations between the $\langle \cos \phi_h \rangle$, $\langle \cos 2\phi_h \rangle$ and Sivers asymmetries. We work with the so-called difference asymmetries i.e. the difference between the production of particles and
their anti-particles. We then utilise COMPASS data on semi-inclusive deep inelastic scattering (SIDIS) on a deuteron target in the formulated tests. Further, as the \( \langle \cos \phi_h \rangle \) and \( \langle \cos 2\phi_h \rangle \) azimuthal asymmetries receive contributions from both the BM and Cahn effects, we are able to extract information on the Cahn effect from experiment - as far as we know for the first time. The details are presented in [2].

2. BM and Sivers functions
The BM parton densities \( \Delta^N f_{q/p}(x_B, k_{\perp}) \) describe the distribution of transversely polarized quarks \( q^+ \) in an unpolarized proton [3]. The Sivers parton densities \( \Delta^N f_{q/p}^{Siv}(x_B, k_{\perp}) \) describe the distribution of unpolarized quarks in a transversely polarized proton \( p^\uparrow \) [4].

For the BM and Sivers functions, as well as for all TMD functions that enter our cross sections, we use the standard parametrization. For SIDIS on deuterium target only the sum of the valence quarks, \( Q_V \equiv u_V + d_V \), enters and we have [5]:

\[
\Delta f_{qV}^J(x_B, k_{\perp}, Q^2) = \Delta f_{qV}^{BM}(x_B, Q^2) \frac{e^{-k_{\perp}^2 / (2\alpha)^2}}{\pi \langle k_{\perp}^2 \rangle_J}, \quad J = BM, Siv
\]

with

\[
\Delta f_{qV}^{BM}(x_B, Q^2) = 2 \mathcal{N}_{qV}(x_B) Q_V(x_B, Q^2), \quad \langle k_{\perp}^2 \rangle_J = \frac{\langle k_{\perp}^2 \rangle_J}{\langle k_{\perp}^2 \rangle_J + M_J^2}
\]

Here the \( \mathcal{N}_{qV}(x_B) \) are unknown functions, and \( M_J \), or equivalently \( \langle k_{\perp}^2 \rangle_J \), are unknown parameters.

3. The transverse quark momenta
Further in our considerations the parameters \( \langle k_{\perp}^2 \rangle \) and \( \langle p_{\perp}^2 \rangle \), which appear in the unpolarized cross sections that normalize all TMD asymmetries will enter. They are interpreted as the average transverse quark and hadron momenta and are determined from multiplicities. At present the obtained values are rather controversial:

- From old measurements we have:
  1) \( \langle k_{\perp}^2 \rangle \approx 0.25 \text{GeV}^2 \) and \( \langle p_{\perp}^2 \rangle \approx 0.20 \text{GeV}^2 \) [6], from the EMC and FNAL data;
  2) \( \langle k_{\perp}^2 \rangle = 0.18 \text{GeV}^2 \) and \( \langle p_{\perp}^2 \rangle = 0.20 \text{GeV}^2 \) [7].

- The more recent data from HERMES and COMPASS separately, gives quite different values [8]:
  3) \( \langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{GeV}^2 \) and \( \langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{GeV}^2 \), extracted from HERMES data;
  4) \( \langle k_{\perp}^2 \rangle = 0.61 \pm 0.20 \text{GeV}^2 \) and \( \langle p_{\perp}^2 \rangle = 0.19 \pm 0.02 \text{GeV}^2 \), extracted from COMPASS data.

Further we shall comment on this controversial situation, since the Cahn effect, which we extract from data, is calculable, and depends sensitively on \( \langle k_{\perp}^2 \rangle \) and \( \langle p_{\perp}^2 \rangle \).

4. The difference asymmetries
We consider the production of charged hadrons \( h^\pm \) in SIDIS of charged leptons on an unpolarized and a transversely polarized deuteron target:

\[
l + d \rightarrow l' + h^\pm + X, \quad l + d^\uparrow \rightarrow l' + h^\pm + X
\]

We work with the so called difference asymmetries:

\[
A^{h^+ - h^-} = \frac{\Delta \sigma^{h^+} - \Delta \sigma^{h^-}}{\sigma^{h^+} - \sigma^{h^-}} \tag{4}
\]
where $\sigma^{h^+, h^-}$ and $\Delta \sigma^{h^+, h^-}$ are the unpolarized and polarized SIDIS cross sections, respectively.

The difference asymmetries do not present a new measurement – they are expressed in terms of the usual asymmetries $A^{h^+, h^-}$ and the ratio of the corresponding multiplicities $r$ [9]:

$$A^{h^+, h^-} = \frac{1}{1 - r} \left( A^{h^+} - r A^{h^-} \right), \quad A^{h^+} = \frac{\Delta \sigma^{h^+}}{\sigma^{h^+}}, \quad A^{h^-} = \frac{\Delta \sigma^{h^-}}{\sigma^{h^-}}, \quad r = \frac{\sigma^{h^-}}{\sigma^{h^+}}. \quad (5)$$

As shown in ref.[5], the advantage of using the difference asymmetries is that, based only on charge conjugation (C) and isospin (SU(2)) invariance of the strong interactions, they are expressed purely in terms of the best known valence-quark distributions and fragmentation functions; sea-quark and gluon distributions do not enter. For a deuteron target there is the additional simplification – independently of the final hadron, only the sum of the valence-quark distributions enters.

5. The azimuthal asymmetries

$\Delta f_{BM}$ and $\Delta f_{Siv}$ are measured via the dependence on the azimuthal angle $\phi_h$ of the final hadron:

$$d\sigma^h(x, z, Q^2, P_T, \phi) = d\sigma_0^h \left\{ 1 + A_{BM}^h \cos \phi_h + \Delta f_{BM}^h \cos 2\phi_h + ... \right. \left. + ST \left[ A_{Siv}^h \sin(\phi_h - \phi_t) + ... \right] \right\} \quad (6)$$

where $d\sigma_0^h$ is the unpolarized, $\phi_h$-independent cross section, $d\sigma_0^h \propto f_q(x) \otimes D_q^h(z)$.

The asymmetries $A_{BM}^h$, $A_{Siv}^h$ receive contributions from both $\Delta f_{BM}$ and the Cahn effect

- $A_{BM}^h$ receives a leading BM-contribution and twist-4 $1/Q^2$-contribution from BM and Cahn effects:

$$1) \quad A_{BM}^h \approx \frac{1}{Q} \sum_q \left[ \Delta f_{BM}^q \otimes D_q^h + f_q \otimes D_q^h \right] \quad (7)$$

while $A_{Siv}^h$ receives a leading BM-contribution and twist-1 $1/Q^2$-contribution from Cahn effect:

$$2) \quad A_{Siv}^h \approx \sum_q \left[ \Delta f_{BM}^q \otimes D_q^h + \frac{1}{Q^2} f_q \otimes D_q^h \right] \quad (8)$$

The Sivers asymmetry $A_{Siv}^h$ is induced by $\Delta f_{Siv}^h$:

$$3) \quad A_{Siv}^h \approx \sum_q \Delta f_{Siv}^q \otimes D_q^h \quad (9)$$

6. Tests for the relation between the BM and Sivers functions on a deuteron target

As on a deuteron target only the sum of the valence-quarks $Q_V$ enters, in contrast to the currently used assumption of proportionality between BM and Sivers functions for each quark flavour, we assume the relation:

$$\Delta f_{BM}^{Q_V}(x, k_\perp, Q^2) = \lambda_{Q_V} \Delta f_{Siv}^{Q_V}(x, k_\perp, Q^2), \quad Q_V = u_V + d_V \quad (10)$$

where $\lambda_{Q_V}$ is a constant.

This assumption leads to relations between the BM induced contributions in $A_{BM}^h$ or $A_{Siv}^{2h}$ and the Sivers asymmetries. They are particularly simple and present predictive tests for (10).
Our analysis showed that for both tests, Eqs (11,12), we obtain a good fit in almost the same interpolation of the data. There are two ways to utilize (11) and (12), we shall follow both of them:

1) two independent direct tests of the assumed relation (10) between the BM and Sivers asymmetries on deuterium target [2]

2) two independent ways for extracting the Cahn contribution from data.

Here we present these tests.

First, we consider both

\[ A_{\cos \phi, d}^{h^{-} h^{+}}(x_B) - \Phi(x_B) \bar{C}_{BM}^{h_{\cos \phi}} A_{\cos \phi, d}^{h^{-} h^{+}}(x_B) = \Phi(x_B) \bar{C}_{BM}^{h_{\cos \phi}} \tag{11} \]

\[ A_{\cos \phi, d}^{h^{-} h^{+}}(x_B) - \Phi(x_B) \bar{C}_{BM}^{h_{\cos \phi}} A_{\cos \phi, d}^{h^{-} h^{+}}(x_B) = \frac{M_d}{\langle Q \rangle^2} \Phi(x_B) \bar{C}_{BM}^{h_{\cos \phi}}, \quad h = \pi^+, K^+, h^+ \tag{12} \]

Here the functions \( \Phi(x_B) \) and \( \Phi(x_B) \) are completely fixed by kinematics:

\[ \Phi(x_B) = \frac{\sqrt{\pi} (2 - \bar{y}) \sqrt{1 - \bar{y}}}{\langle Q \rangle [1 + (1 - \bar{y})^2]}, \quad \bar{y} = \frac{\langle Q \rangle^2}{2M_d E x_B}. \tag{13} \]

\( \langle Q \rangle^2 \) is some mean value of \( Q^2 \) for each \( x_B \)-bin, \( M_d \) is the mass of the deuterium target. \( \bar{C}_{BM}^{h_{\cos \phi}} \) and \( \bar{C}_{BM}^{h_{\cos \phi}} \) are constants, determined entirely by the collinear and Collins FFs, the explicit expressions are given in ref.[2].

Relations (11) and (12), in which \( \bar{C}_{BM}^{h_{\cos \phi}} \) are parameters, present:

1) two independent direct tests of the assumed relation (10) between the BM and Sivers functions, in which only measurable quantities enter, and no knowledge about the TMD functions is required and,

2) two independent ways for extracting the Cahn contribution from data.

7. Tests using the COMPASS data for \( h^\pm \) production on a deuterium target

Here we use relations (11) and (12) between the difference asymmetries to test assumption (10) between the TMD functions, using the COMPASS data on deuterons for production of charged hadrons \( h^\pm \) for the angular distributions \( A_{\cos \phi, d}^{h^\pm}(x_B) \) and \( A_{\cos \phi, d}^{h^\pm}(x_B) \) [10], and the single-spin Sivers asymmetry data \( A_{\cos \phi, d}^{h^\pm}(x_B) \) [11]. We proceed in 3 steps:

1) Via Eq.(5), we form the difference asymmetries \( A_{j^+ j^-}^{h^\pm}, J = \cos \phi, \cos 2\phi, Siv \) from the measured asymmetries \( A_{j^+}^{h^\pm} \) and \( A_{j^+}^{h^\pm} \) for positive and negative charged hadron production, \( r \) is given in [9]. The obtained asymmetries are presented on Fig.1. As seen from the Figure, the Sivers asymmetry \( A_{\cos \phi, d}^{h^\pm}(x_B) \) is determined with large relative errors and is close to 0. This suggests that \( \bar{C}_{BM}^{h_{\cos \phi}} \) and \( \bar{C}_{BM}^{h_{\cos \phi}} \) may be poorly determined. The same arguments hold for \( A_{\cos \phi, d}^{h^\pm}(x_B) \) and we expect tests with \( A_{\cos \phi, d}^{h^\pm}(x_B) \) to give more precise information.

2) We choose the \( Q^2 \) interval where the \( Q^2 \)-dependence of the collinear pdf’s and FFs can be neglected. In the COMPASS kinematics to each value of \( \langle Q^2 \rangle \) corresponds one definite value of \( (x_B) \), thus fixing the \( Q^2 \) interval we fix also the \( x_B \)-interval. Using the available CTEQ parametrizations for the pdf’s and the parametrization in [12] for FFs we see that aside from the small values of \( Q^2 < 1.5 GeV^2 \), the \( Q^2 \)-dependence is weak. This implies that Eqs (11,12) can be applied for \( x_B \geq 0.014 \).

3) Finally, we fit the parameters in Eqs (11,12) from the data, using \( \chi^2 \)-analysis with linear interpolation of the data. There are two ways to utilize (11) and (12), we shall follow both of them:

4) We consider both \( \bar{C}_{BM}^{h_{\cos \phi}} \) and \( \bar{C}_{BM}^{h_{\cos \phi}} \) (respectively \( \bar{C}_{BM}^{h_{\cos \phi}} \) and \( \bar{C}_{BM}^{h_{\cos \phi}} \)), as fitted parameters. Our analysis showed that for both tests, Eqs (11,12), we obtain a good fit in almost the same kinematic interval \( x_B \geq 0.014 \), the results are presented on Fig.2.
Figure 1. The difference asymmetries with their statistical errors: a) $A_{\cos \phi,d}^{h^+ - h^-} (x_B)$ (solid line) and $A_{\cos \phi,d}^{h^+ - h^-} (x_B)$ (dashed line) b) $A_{\cos \phi,d}^{h^+ - h^-} (x_B)$ (solid line) and $A_{\cos \phi,d}^{h^+ - h^-} (x_B)$ (dashed line)

Figure 2. The fits for $x_B \geq 0.014$: a) of Eq.(11), the dashed white line is for $F(x_B) \equiv A_{\cos \phi,d}^{h^+ - h^-} (x_B) - C_{BM}^{h^-} \Phi(x_B) A_{\cos \phi,d}^{h^-} (x_B)$, the black line is $C_{Cahn}^{h^+} \Phi(x_B)$; b) of Eq.(12), the white line is $\hat{F}(x_B) \equiv A_{\cos \phi,d}^{h^+ - h^-} (x_B) - C_{BM}^{h^-} \Phi(x_B) A_{\cos \phi,d}^{h^+} (x_B)$, the black one is $M M_d \Phi(x_B) \hat{C}_{Cahn}^{h^+} / (Q)^2$

(B) In the second approach, first we calculate the Cahn constants, $C_{Cahn}^{h^+}$ or $\hat{C}_{Cahn}^{h^+}$, using their explicit expressions. For example, for $C_{Cahn}^{h^+}$ we have:

$$C_{Cahn}^{h^+} = -\langle k_{\perp}^2 \rangle \frac{\int d z_h z_h [D_{w}^{h^+}(z_h)]/\sqrt{\langle P_T^2 \rangle}}{\int d z_h [D_{w}^{h^-}(z_h)]}, \quad \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle$$

(14)

They depend on the FFs and on the parameters $\langle k_{\perp}^2 \rangle$ and $\langle p_{\perp}^2 \rangle$ which, as discussed in Section (3), vary considerably. Then we fit the same data with just one parameter $C_{BM}^{h^+}$, respectively $\hat{C}_{BM}^{h^+}$. Consequently, the main interest in this approach is to compare the calculated Cahn constants with the parameters as in (A).

The calculated values for $C_{Cahn}^{h^+}$ and $\hat{C}_{Cahn}^{h^+}$ for the different values of $\langle k_{\perp}^2 \rangle$, $\langle p_{\perp}^2 \rangle$, and the fitted parameter $C_{BM}^{h^+}$ and $\hat{C}_{BM}^{h^+}$ are presented in Tables 1. For comparison, in the last column the corresponding fitted values from approach A are also presented. The errors in the parameters are calculated using Monte Carlo simulation.

To the best of our knowledge this is the first time that the Cahn contribution has been determined from data and it is intriguing that its value is in agreement with a calculated value based on the early values of the Gaussian parameters $\langle k_{\perp}^2 \rangle = 0.18$, GeV$^2$ and $\langle k_{\perp}^2 \rangle = 0.25$, GeV$^2$ and completely disagree with the later values.
\[ \begin{array}{|c|c|c|c|c|}
\hline
& \langle k^2 \rangle \text{ [GeV}^2 \rangle & \langle p_T^2 \rangle \text{ [GeV}^2 \rangle & C_{Cahn}^h & \hat{C}_{Cahn}^h \\
\hline
\hline
& 0.25 & 0.18 & 0.57 \pm 0.08 & 0.61 \pm 0.20 \\
\hline
& 0.20 & 0.20 & 0.12 \pm 0.01 & 0.19 \pm 0.02 \\
\hline
\end{array} \]

Table 1. $C_{Cahn}^h$ and $\hat{C}_{Cahn}^h$ are calculated using FFs from LSS [12], $C_{BM}^h$ and $\hat{C}_{BM}^h$ are fitted. Their values are compared to those of $(\mathcal{A})$.

8. Conclusions
We have performed two independent tests of the assumption that the proportionality between the BM and Sivers functions holds for the sum of the valence-quark TMD distributions, using the COMPASS data on the asymmetries $A_{\cos \phi,d}^{h^+-h^-}(x_B)$, $A_{\cos 2\phi,d}^{h^+-h^-}(x_B)$ and $A_{Siv,d}^{h^+-h^-}(x_B)$. Both tests are consistent with this assumption in the same kinematic interval $x_B = [0.014,0.13]$. However, in the published extractions of the BM functions [1] the assumption made is $\Delta f_{BM}^h = \lambda_q \Delta f_{Siv}^h$ for each quark separately. It would agree with our result only if $\lambda_u = \lambda_d = \lambda_\bar{u} = \lambda_\bar{d} = \lambda_{Q'}$, which does not correspond to the values obtained in [1]. This suggests that the published BM functions may be unreliable.

We have also determined the kinematical Cahn contribution, both directly from a fit to the data (as far as we know for the first time) and from a calculation. The calculated values are very sensitive to the average transverse momentum-squared $\langle k_T^2 \rangle$. Surprisingly, the calculated values agree with the extracted ones only for the old experimental values $\langle k_T^2 \rangle \approx 0.18 \text{ GeV}^2$ and $\langle k_T^2 \rangle \approx 0.25 \text{ GeV}^2$ and completely disagree with the much bigger present-day values.

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