Topological superfluid in a trapped two-dimensional polarized Fermi gas with spin-orbit coupling

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We study the stability region of the topological superfluid phase in a trapped two-dimensional polarized Fermi gas with spin-orbit coupling and across a BCS-BEC crossover. Due to the competition between polarization, pairing interaction and spin-orbit coupling, the Fermi gas typically phase separates in the trap. Employing a mean field approach that guarantees the ground state solution, we systematically study the structure of the phase separation and investigate in detail the optimal parameter region for the preparation of the topologically non-trivial superfluid phase. We then calculate the momentum space density distribution of the topological superfluid state and demonstrate that the existence of the phase leaves a unique signature in the trap integrated momentum space density distribution which can survive the time-of-flight imaging process.

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I. INTRODUCTION

The study of non-Abelian topological order has attracted a great amount of interest recently, due to the potential applications in fault-tolerant quantum computation [1,2]. In addition to systems with intrinsic chiral p-wave pairing order, e.g. fractional quantum hall systems [3,5], chiral p-wave superconductors [6,7], p-wave superfluidity in ultracold fermions [8,11] etc., it has been shown that a topologically non-trivial superfluid phase that supports non-Abelian excitations can be induced from an underlying s-wave superfluidity. An example is the semiconductor/superconductor heterostructures with spin-orbit coupling (SOC), s-wave pairing superfluidity and an external Zeeman field [12,14]. With the rapidly developing toolbox available for the quantum control of ultracold atomic systems, the elements above can now be implemented experimentally in ultracold Fermi gases. Importantly, the spin-orbit coupling in ultracold Bose gases has been made possible by the recent experimental achievement of synthetic gauge field in ultracold atoms and has generated considerable amount of theoretical interests [17,20]. On the other hand, the pairing superfluidity in an ultracold Fermi gas and the quantum phase transition in a polarized Fermi gas have been investigated extensively during the past decade [21,25]. With clean environment and highly tunable parameters, ultracold Fermi gases may serve as an ideal platform for the observation of topological superfluidity and for the study of the interesting physics therein. In particular, it has been suggested that the Majorana zero modes, which have eluded experimental observation for decades, may be detected in an ultracold atomic system with s-wave interactions [29,32].

Spin-orbit coupled Fermi gas has been under intensive theoretical study recently [31,33,37]. For an unpolarized Fermi gas near a wide Feshbach resonance, the SOC has been found to result in a BCS-BEC type crossover even on the BCS side of the resonance [35,37]. Furthermore, it has been suggested that the topological superfluid (TSF) phase can be stabilized in a spin polarized Fermi gas in the presence of SOC [31,33,36,39,41]. For a polarized Fermi gas without SOC, phase separation takes place near a wide Feshbach resonance in a uniform gas, due to the competition between population imbalance and pairing interactions [25,45,50]. With the introduction of SOC, the phase separation develops a rich structure involving the topologically non-trivial superfluid state [39,41]. For a polarized Fermi gas in an external trap, which is always the case in experiments, the various phases naturally separate in real space, with different phases occurring at different places in the trapping potential [21,22,45]. The important questions here are whether the topological superfluid is stable in a trapping potential and what the detailed structure of the phases in a trap is. The phase structure involving the topologically non-trivial superfluid phase in a trapped three-dimensional (3D) polarized Fermi gas with SOC has been examined previously, where it has been found that two distinct types of topological superfluid may be stabilized [39,41]. In this work, we focus on the phase structure of a spin-orbit coupled two-dimensional (2D) polarized Fermi gas near a wide Feshbach resonance in a trapping potential.

We study the system using a BCS-type mean field theory at zero temperature. Due to the existence of metastable or unstable solutions of the gap equation that are typical in the presence of population imbalance, we directly minimize the thermodynamic potential [28,41]. In contrast to the 3D case where there are two distinct topologically non-trivial phases with either two or four
gapless points in the quasi-particle excitation spectrum, in 2D we find that there is only one topologically non-trivial superfluid phase which is always protected by an excitation gap away from its phase boundary against the conventional superfluid state (SF). This agrees with the previous calculations [12, 16]. As has been shown in Ref. [12], when a vortex is created in this phase, a Majorana zero center of mass momentum in our mean field theory is not quantitatively reliable near a wide Feshbach resonance. We have also not considered the possibility of pairing states with non-resonance in 2D due to large fluctuations. We have also demonstrated that the pairing mean field interaction Hamiltonian $H_{\text{int}}$ [32, 33, 36]:

$$H = \sum_{\sigma} \mu_{\sigma} N_{\sigma} = H_{0} + H_{\text{soc}} + H_{\text{int}}$$

$$= \sum_{k,\sigma} (\epsilon_{k} - \mu_{\sigma})a_{k,\sigma}^{\dagger}a_{k,\sigma} + \sum_{k} \alpha k \left( e^{-i\varphi_{k}} a_{k,\uparrow}^{\dagger}a_{k,\downarrow} + \text{H.C.} \right)$$

$$+ \frac{U}{\sqrt{N}} \sum_{k,k'} a_{k,\uparrow}^{\dagger}a_{-k,\downarrow}^{\dagger}a_{-k',\uparrow}a_{k',\downarrow},$$

(1)

where the kinetic energy $\epsilon_{k} = \hbar^{2}k^{2}/(2m)$, $\mu_{\sigma}$ is the chemical potential for atoms with spin $\sigma = \{\uparrow, \downarrow\}$, $N_{\sigma}$ denotes the total number of particles with spin $\sigma$, $a_{k,\sigma}^{\dagger}$ ($a_{k,\sigma}$) annihilates (creates) a fermion with momentum $k$ and spin $\sigma$, $V$ is the quantization area in 2D, and H.C. stands for Hermitian conjugate. The Rashba spin-orbit coupling strength $\alpha$ can be tuned via parameters of the gauge-field generating lasers [18], while $\varphi_{k} = \arg (k_{x} + ik_{y})$.

In writing the interaction Hamiltonian $H_{\text{int}}$, we assume an $s$-wave contact interaction between the two fermion species, with the bare interaction rate $U$ renormalized following the standard relation in two dimensions $[51]$:

$$\frac{1}{U} = -\frac{1}{V} \sum_{k} \frac{1}{2\epsilon_{k} + E_{b}}.$$  

(2)

Here, $E_{b} > 0$ is the binding energy of the two-body bound state in two dimensions without SOC. By tuning through a Feshbach resonance from a high-field BCS side, $E_{b}$ increases from zero and becomes large in the low-field BEC limit. Therefore, we use the variation of $E_{b}$ to represent the BCS-BEC crossover in the following discussions. One should notice that the $E_{b}$ we use in this manuscript is not the binding energy of the two-body bound state in the presence of SOC.

The non-interacting Hamiltonian $H_{0} + H_{\text{soc}}$ can be diagonalized in the helicity basis:

$$a_{k,\uparrow} = \frac{1}{\sqrt{2}} e^{-i\varphi_{k}} (a_{k,+} + a_{k,-}),$$  

(3)

$$a_{k,\downarrow} = \frac{1}{\sqrt{2}} (a_{k,+} - a_{k,-}),$$  

(4)

where $a_{k,\pm}$ ($a_{k,\pm}^{\dagger}$) are the annihilation (creation) operators for the dressed spin states with different helicities ($\pm$). Under this basis, the interaction Hamiltonian can be written as

$$H_{\text{int}} = \frac{U}{4} \sum_{k,k'} e^{i\varphi_{k}} \left( a_{k,\uparrow}^{\dagger}a_{-k',\uparrow}^{\dagger} - a_{k,\downarrow}^{\dagger}a_{-k',\downarrow}^{\dagger} \right) \times e^{-i\varphi_{k'}} \left( a_{-k',\uparrow}^{\dagger}a_{k',\downarrow}^{\dagger} - a_{-k',\downarrow}^{\dagger}a_{k',\uparrow}^{\dagger} \right).$$

(5)

Taking the pairing mean field

$$\Delta = \frac{U}{2} \sum_{k} \left\langle e^{-i\varphi_{k}} (a_{-k,+}a_{k,+} - a_{-k,-}a_{k,-}) \right\rangle$$

$$= U \sum_{k} \langle a_{-k,\downarrow}a_{k,\uparrow} \rangle,$$

(6)

II. FORMALISM

We first consider the model Hamiltonian for a uniform 2D polarized Fermi gas with Rashba spin-orbit coupling near a wide Feshbach resonance. The Hamiltonian can be expressed as a sum of three parts: the unperturbed Hamiltonian $H_{0}$, the SOC Hamiltonian $H_{\text{soc}}$ and the interaction Hamiltonian $H_{\text{int}}$ [32, 33, 36]:

$$H = \sum_{\sigma} \mu_{\sigma} N_{\sigma} = H_{0} + H_{\text{soc}} + H_{\text{int}}$$

$$= \sum_{k,\sigma} (\epsilon_{k} - \mu_{\sigma})a_{k,\sigma}^{\dagger}a_{k,\sigma} + \sum_{k} \alpha k \left( e^{-i\varphi_{k}} a_{k,\uparrow}^{\dagger}a_{k,\downarrow} + \text{H.C.} \right)$$

$$+ \frac{U}{\sqrt{N}} \sum_{k,k'} a_{k,\uparrow}^{\dagger}a_{-k,\downarrow}^{\dagger}a_{-k',\uparrow}a_{k',\downarrow},$$

(1)

where the kinetic energy $\epsilon_{k} = \hbar^{2}k^{2}/(2m)$, $\mu_{\sigma}$ is the chemical potential for atoms with spin $\sigma = \{\uparrow, \downarrow\}$, $N_{\sigma}$ denotes the total number of particles with spin $\sigma$, $a_{k,\sigma}^{\dagger}$ ($a_{k,\sigma}$) annihilates (creates) a fermion with momentum $k$ and spin $\sigma$, $V$ is the quantization area in 2D, and H.C. stands for Hermitian conjugate. The Rashba spin-orbit coupling strength $\alpha$ can be tuned via parameters of the gauge-field generating lasers [18], while $\varphi_{k} = \arg (k_{x} + ik_{y})$.

In writing the interaction Hamiltonian $H_{\text{int}}$, we assume an $s$-wave contact interaction between the two fermion species, with the bare interaction rate $U$ renormalized following the standard relation in two dimensions $[51]$:
the mean field Hamiltonian becomes
\[ H_m = \sum_{\sigma} \mu_\sigma N_\sigma = \sum_{k,\lambda=\pm} \xi_k a_{k,\lambda}^\dagger a_{k,\lambda} \]
\[ + \frac{\Delta}{2} \sum_k \left[ e^{-i\xi_k} (a_{k,+} a_{k,+} - a_{k,-} a_{k,-}) + h.c. \right] \]
\[ - \frac{h}{2} \sum_k \left( a_{k,+}^\dagger a_{k,-} + h.c. \right) - V |\Delta|^2 U, \]
where we have defined the chemical potentials \( \mu = (\mu_\uparrow + \mu_\downarrow)/2 \) and \( h = \mu_\uparrow - \mu_\downarrow \); and \( \xi_k = \xi_k \pm \alpha k \) with \( \epsilon_k = \epsilon_k - \mu \). The mean field Hamiltonian is quadratic and can be diagonalized in the helicity basis:
\[ \{ a_{k,+}, a_{k,-}, a_{k,+}^\dagger, a_{k,-}^\dagger \}^T \]
\[ H_m = \sum_{\sigma} \mu_\sigma N_\sigma = \sum_{k,\lambda=\pm} E_{k,\lambda} \alpha_{k,\lambda}^\dagger \alpha_{k,\lambda} \]
\[ + \frac{1}{2} \sum_{k,\lambda=\pm} (\xi_{k,\lambda} - E_{k,\lambda}) - |\Delta|^2 U. \]
Here, \( \alpha_{k,\sigma} \) (\( a_{k,\sigma}^\dagger \)) is the annihilation (creation) operator for the quasi-particles. The quasi-particle excitation spectra take the form
\[ E_{k,\pm} = \sqrt{\xi_k^2 + \alpha^2 k^2 + |\Delta|^2 + \frac{h^2}{4} \pm 2E_0}, \]
where \( \delta_\pm = -\delta_\pm = -1 \), and we have taken \( \Delta \) to be real for simplicity. The ground state of the system at zero temperature is given by the global minimum of the thermodynamic potential in Eq. (10) under the number constraints Eq. (12). For a uniform gas, one has to explicitly consider the possibility of phase separation and introduce a mixing coefficient in order to get the correct ground state. In an external trapping potential, the various phases naturally separate in real space. [23] [41] [50].

Next, we focus on the phase separation in the presence of an external trapping potential \( V(r) \), due to its experimental relevance. Assuming the potential to be slowly varying and taking the local density approximation (LDA), we can write the chemical potentials at each spatial location \( r \) as: \( \mu_\uparrow(r) = \mu_\uparrow + h/2, \mu_\downarrow(r) = \mu_\uparrow - h/2, \) and \( \mu_r = \mu - V(r) \), where the chemical potential at trap center \( \mu \) and the chemical potential imbalance \( h \) are related to the total particle number \( N = N_\uparrow + N_\downarrow \) and the polarization \( P = (N_\uparrow - N_\downarrow)/N \). The total particle number for each spin species can be determined from a trap integration: \( n_\sigma = \int d^3r n_\sigma(r) \), where the local density \( n_\sigma(r) \) can be calculated from Eq. (12) with \( \mu_r \) replaced by \( \mu_\sigma \), and with the local pairing order parameter \( \Delta(r) \) determined from the global minimum of the thermodynamic potential at each spatial location \( r \). Without loss of generality, we assume \( N_\uparrow > N_\downarrow \) throughout this work such that \( h > 0 \) and \( P > 0 \).
state is reached. To avoid this complication, we directly minimize metastable or unstable states, in addition to the ground state of the system changes from one local minimum to another a topological superfluid with the same symmetry, there exists in the integrand of the gap equation (11) over considerably large parameter regions. So long as this singularity exists, the gap equation always has at least one finite solution regardless of the SOC strength $\alpha$ and chemical potential combinations ($\mu$, $h$). In order to understand this picture, we show in Fig. 2(a) the behavior of the function $G \equiv (-1/2\Delta)\partial\Omega/\partial\Delta$, which is proportional to the left-hand side of the gap equation (11). In the presence of the singularity, for arbitrary SOC strength, the function $G$ is always diverging as $\Delta \to 0$, and tends to large negative values as $\Delta \to \infty$. Therefore, there is at least one solution to the gap equation under these conditions, indicating the presence of gapped phases, as has been pointed out previously in Ref [16]. Further analysis shows that one of the gapped phases is the global minimum of the thermodynamic potential. This observation shows that superfluidity can survive arbitrary polarization, provided that an SOC is introduced. In Fig. 2(b), we present the pairing gap $\Delta$ of the ground state as a function of SOC strength $\alpha$. The numerical result suggests that the pairing gap decreases super-exponentially as $\alpha$ approaches zero.

The singularity responsible for the divergence of $G$ comes from the terms proportional to $E_{k,-}^{-1}$, which diverges at $\Delta = 0$. As $E_{k,-} = ||\xi_k| - \sqrt{\alpha^2 k^2 + h^2/4}||$ at $\Delta = 0$, the solutions of the equation $|\xi_k| = \sqrt{\alpha^2 k^2 + h^2/4}$ give the singularity points in momen-
IV. PHASE DIAGRAM IN THE $\alpha$-$\mu$ PLANE

From the discussions in the previous section, we see that the presence of SOC can lead to a rich structure of phases in a trapping potential. As a first step to understand the spatial distribution of the various phases in the trap, we consider in this section a homogeneous system and investigate the phase diagram as a function of $(\alpha, \mu)$ for given $E_b$ and $\hbar$. Under LDA while assuming both spin species experience the same harmonic potential, a downward vertical line in such a phase diagram represents a trajectory from a trap center to its edge, with the chemical potential at the trap center fixed by that at the starting point of the line. To this end, we only need to minimize the thermodynamic potential in Eq. (10) for given SOC strength $\alpha$ and chemical potential difference $\mu$ while sweeping the chemical potential $\mu$.

In Fig. 3, we show a typical phase diagram in the $\alpha$-$\mu$ plane for $E_b/\hbar = 0.5$. Notice that there is only one topologically non-trivial superfluid phase, which is clearly different from the 3D case as discussed before. The TSF phase is separated from the conventional superfluid phase by two kinds of phase boundaries. The solid curve in Fig. 3 represents a first order phase boundary, along which the states corresponding to the two local minima of the double well structure in the thermodynamic potential are degenerate in energy. Compared to the 3D case, the first order phase boundary is dramatically extended. The other kind of TSF-SF phase boundary is of second order, given by $h/2 = \sqrt{\mu^2 + \Delta^2}$, along which the pairing gap remains finite and the excitation gap vanishes. To determine the phase boundary for $\Delta = 0$, we need to examine the existence of divergence in the gap equation as $\Delta$ approaches zero. As discussed in the previous section, the singularities go away when $\mu < -(\alpha^4 + h^2)/(4\alpha^2)$ or $\mu < \min(-h/2, -\alpha^2/2)$. The phase boundary of the superfluid phases with $\Delta = 0$ can be calculated by solving the gap equation in these parameter regions. We note that the maximum value of the chemical potential satisfying these relations is $-h/2$, below which the chemical potential of both spin species $\mu_{\sigma}$ become negative. Hence there will not be a phase boundary between a superfluid phase (SF or TSF) and a normal phase. Instead, only phase boundaries between a superfluid phase (SF or TSF) and vacuum (VAC) exist. For the calculations above, we always check the thermodynamic potential to ensure that states along the phase boundary with $\Delta = 0$ represent ground state solutions.

According to the phase diagram in Fig. 3, the stability region for the TSF phase appears to be significant. Yet this can be misleading for experimental detection. In fact, the size of the pairing gap in the TSF phase with small SOC strength is typically vanishingly small. This can be seen from the dashed curve traversing the TSF phase in Fig. 3, which is solved from the gap equation by setting $\Delta/\hbar = 10^{-3}$. To the left of the curve, the pairing gap $\Delta/\hbar < 10^{-3}$ and decreases exponentially fast as

![Phase diagram](image-url)
The distributions of number densities $n_\sigma(\tilde{r})$ (a-e) and the order parameter $\Delta(\tilde{r})$ (f-j) are shown versus dimensionless distance from the trap center $\tilde{r} = r/R$. The parameters for each column are: (a) $E_b/E_F = 0.32$, $\alpha k_F/E_F = 0.3$, $h/E_F = 1$, $P = 0.624$; (b) $E_b/E_F = 0.5$, $\alpha k_F/E_F = 0.4$, $h/E_F = 1$, $P = 0.287$; (c) $E_b/E_F = 0.5$, $\alpha k_F/E_F = 0.7$, $h/E_F = 1$, $P = 0.188$; (d) $E_b/E_F = 0.5$, $\alpha k_F/E_F = 0.8$, $h/E_F = 1$, $P = 0.176$; (e) $E_b/E_F = 0.5$, $\alpha k_F/E_F = 0.6$, $h/E_F = 1.45$, $P = 0.662$.

The bottom row (k-o) illustrates the shell structure of phase separation. The solid black (dashed red) curves in the density subplots represent spin up (down) species. The thin dotted lines in the first two rows illustrate the TSF-SF or the SF-SF boundary. The units of energy $E_F$ and of length $R$ are defined in the text, and the unit of density is $n_0 = m E_F/(\pi \hbar^2)$.

$\alpha$ approaches zero. The order parameter $\Delta$ only becomes significant when $\alpha$ is further increased toward the phase boundary between TSF and SF. Given the fluctuations in 2D systems at finite temperatures, experimental observation of the TSF phase is only possible to the right of the dashed curve and with reasonably large pairing gap $\Delta$.

Figure 5 also provides information regarding the structure of the phase separation in a trapping potential. When the SOC is small, the Fermi gas will phase separate into two regions, a conventional superfluid core surrounded by a TSF phase with large spin polarization and vanishingly small pairing order. The phase boundary between them is of first order. As the SOC increases, the local minima in the thermodynamic potential corresponding to the TSF and the SF states move closer as the pairing gap of TSF state increases. The two local minima merge at a critical end point beyond which the double-well structure in the thermodynamic potential disappears and the phase boundary between TSF and SF becomes second order. Further increasing the SOC, there may be a parameter window where the TSF phase appears as a ring structure in the trap. Finally, when the SOC is large enough, phase separation no longer occurs and the trap is filled with a superfluid of rashbons.

We have also calculated the $\alpha-\mu$ phase diagram for a homogeneous system with different bound state energies $E_b$ (see Fig. 4). Toward the BCS side [Fig. 4(a)], the stability region of the TSF phase increases while the first order phase boundary between TSF and SF no longer exists. For small SOC and large chemical potential, there may exist two different SF phases at the trap center, separated by a first-order-like boundary. In this case as the symmetries of the two SF phases are the same, the boundary is merely a crossover. On the phase diagram, this first-order-like crossover boundary ends at a critical end point where the two potential wells in the double well structure of the thermodynamic potential merge. Immediately below this first order crossover boundary and with small SOC, an SF phase with vanishingly small order parameter and small polarization appears where the chemical potentials of both spin species are positive. This corresponds to an SOC induced SF phase out of a normal phase with two spin species without SOC. Toward BEC side [Fig. 4(b-d)], the stability region of the TSF phase becomes smaller and eventually disappears from the phase diagram. The trap is then occupied by superfluid of rashbons.
on the dimensionless parameters \( \{ \alpha, \kappa \} \). It is obvious that the properties of the system only depend on the dimensionless distance from the trap center, \( \tilde{r} = r/R \), and \( \beta = \sigma R \), where \( \sigma \) is the number density given by the number equation \( \sigma = \frac{n}{\beta} \) and \( \beta \) is defined as \( \beta = E_{\mu}/E_{\text{F}} \).

All phase boundaries shown in this plot are of second order. First order phase boundaries show up at smaller SOC strengths and/or smaller \( E_b \). The units of energy \( E_F \) and of length \( R \) are defined in the text.

V. PHASE SEPARATION IN A TRAP

Next, we adopt the LDA and explicitly include the trapping potential in our calculation. To make our calculation universal and applicable to systems with any total particle number, we derive a dimensionless form following Ref. [48]. We take the unit of energy to be the Fermi energy \( E_F \). We take the unit of energy to be the Fermi energy \( E_F \) and the unit of momentum \( k_F \) to be the Fermi momentum \( k_F \) is defined as \( k_F = \hbar \omega_0 / 2 \). The polarization \( P \) is defined as \( P = \frac{\alpha k_F}{\hbar} \).

To better understand the phase structures in a trap, we plot in Fig. 6 the zero temperature phase boundaries in a harmonic trapping potential in the \( P-\tilde{r} \) plane, where \( P \) is the trap-integrated total polarization calculated from Eq. \( 14 \), and \( \tilde{r} = r/R \) is the dimensionless distance from the trap center. Notice that the TSF phase occupies the entire trap only when the total polarization exceeds a critical value \( \sim 0.69 \) for the chosen parameters in Fig. 6. As we will show in the next section, this regime provides an ideal setup for the detection of the TSF state in a trapped gas. When \( P \) decreases from this value, the conventional SF phase will emerge from the trap center, gradually extend to the trap edge, and eventually occupy the entire trap for small polarization case.

\[ h = E_F, \quad \text{and solve for the chemical potential} \quad \mu = \frac{\beta g F}{\sigma}. \]

The polarization \( \beta \) can then be calculated from Eq. \( 14 \), and \( \beta = \sigma R \), where \( \sigma \) is taken to be the energy unit, the unit of momentum \( k_F \), and the unit of density is defined as \( n = \frac{n_0}{(2\pi \beta^2)} \).

\[ 1 = 4 \int d^2 \tilde{r} \left[ \tilde{n}_{\uparrow}(\tilde{r}) + \tilde{n}_{\downarrow}(\tilde{r}) \right], \]

\[ P = 4 \int d^2 \tilde{r} \left[ \tilde{n}_{\uparrow}(\tilde{r}) - \tilde{n}_{\downarrow}(\tilde{r}) \right]. \]

with dimensionless number density \( \tilde{n}_\sigma = n_\sigma / n_0 \). Here, \( n_\sigma \) is the number density given by the number equation \( \beta = \sigma R \), at position \( \tilde{r} \). The Fermi momentum \( k_F \) is defined as \( k_F = \hbar \omega_0 / 2 \). It is obvious that the properties of the system only depend on the dimensionless parameters \( \{ E_b/E_F, \alpha k_F/E_F, P \} \).

Solving the dimensionless equations above, we get the typical phase structure in a trapping potential with a various sets of parameters. Note that for simplicity, we first choose an appropriate chemical potential difference, e.g. \( h = E_F \), and solve for the chemical potential \( \mu \) at the center of the trap from Eq. \( 13 \) with fixed SOC strength \( \alpha \) and \( E_b \). The polarization \( \beta \) can then be calculated from Eq. \( 14 \), and \( \beta = \sigma R \), where \( \sigma \) is taken to be the energy unit, the unit of momentum \( k_F \), and the unit of density is defined as \( n = \frac{n_0}{(2\pi \beta^2)} \).

\[ h = E_F, \quad \text{and solve for the chemical potential} \quad \mu = \frac{\beta g F}{\sigma}. \]
VI. MOMENTUM DISTRIBUTION AND THE SIGNATURE OF THE TOPOLOGICAL SUPERFLUID PHASE

To characterize the properties of the various phases in the phase diagram, we calculate their respective momentum distribution (see Fig. 7), which is given by the summand in the number equations \( \langle k \rangle \). In Fig. 7 we show the momentum distribution of a homogeneous system with various parameters. In the first row of Fig. 7 the binding energy is set as \( E_b/h = 0.5 \) with increasing SOC strength. In the second row, a similar evolution with \( \alpha \) is shown but with a binding energy \( E_b/h = 1.2 \), more toward the BEC regime. It is apparent that the momentum distribution in a TSF phase [c.f. Fig. 7(a-b)] is drastically different from that in an SF phase. In particular, the momentum distribution of the minority spin in the TSF phase features a dip near zero momentum, which can be explained by the observation that \( n_{k,\downarrow} = 0 \) at \( k = 0 \), where \( n_{k,\downarrow} \) is the summand in the corresponding number equation \( \langle k \rangle \).

As this dip in the momentum distribution is unique to the TSF phase, one may think of using it as a signature for the experimental detection of the TSF phase. To measure the momentum distribution experimentally, a commonly used practice is the time-of-flight imaging technique, which involves a ballistic expansion of the gas after suddenly switching off the trapping potential. As there are typically several different phases in the trapping potential, the observed momentum distribution is usually a trap-integrated distribution which includes the contribution from all the phases in the trap. In this case, the signal of the topological superfluid is washed out and cannot be detected.

One possible way to overcome this difficulty is to prepare the system in an appropriate parameter region such that the center of the trap is filled with the TSF state. An example of this is demonstrated in Fig. 8(b). The corresponding trap-integrated momentum distribution is shown in Fig. 8(a), where the signature of the TSF state apparently survives the trap integration. In comparison, we show in Fig. 8(c)(d) similar calculations for the case with an SF core surrounded by the TSF phase. As is clear from Fig. 8(c), the signature dip for the TSF state can no longer be observed in the trap-integrated momentum distribution. This suggests that the existence of the dip can serve as a signature for the existence of the TSF phase if the momentum distribution of the minority spin species can be detected.

VII. SUMMARY

We have developed a mean field theory to characterize the phases of a trapped 2D polarized Fermi gas with SOC near a wide Feshbach resonance. Under LDA, we have calculated in detail the structure of phase separation of the pairing gap in a trapping potential with various parameters. Compared to the 3D case, we find dramatically increased first order phase boundary between the SF core and the TSF phase that surrounds it, which makes it observable in experiments from the density distributions of the spin species. We then develop a universal scheme for the characterization of a trapped gas. The resulting phase and density distributions are therefore independent of the trapping geometry and the total particle number, and are determined by a set of dimensionless parameters. We explicitly calculate the density and momentum distribution of the gas in a trapping potential. Importantly, we find a parameter region where the trap is occupied by the TSF phase only. In this regime, the characteristic signature of the TSF state in the momentum distribution can survive the trap integration, rendering the signal detectable in a time-of-flight imaging process.

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[1] A. Kitaev, Ann. Phys. (N.Y.) 303, 2 (2003).
[2] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. 80, 1083 (2008).
[3] G. Moore and N. Read, Nucl. Phys. B 360, 362 (1991).
[4] N. Read and D. Green, Phys. Rev. B 61, 10267 (2000).
[5] S. Das Sarma, M. Freedman, and C. Nayak, Phys. Rev. Lett. 94, 166802 (2005).
[6] S. Das Sarma, C. Nayak, and S. Tewari, Phys. Rev. B 73, 220502(R) (2006).
[7] V. Gurarie and L. Radzihovsky, Phys. Rev. B 75, 212509 (2007).
[8] S. S. Botelho and C. A. R. Sá de Melo, J. Low Temp. Phys. 140, 409 (2005).
[9] V. Gurarie, L. Radzihovsky, and A. V. Andreev, Phys. Rev. Lett. 94, 230403 (2005).
[10] S. Tewari, S. Das Sarma, C. Nayak, C. Zhang, and P. Zoller, Phys. Rev. Lett. 98, 010506 (2007).
[11] V. Gurarie, L. Radzihovsky, Ann. Phys. 322, 2 (2007).
[12] J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma, Phys. Rev. Lett. 104, 040502 (2010).
[13] J. Alicea, Phys. Rev. B 81, 125318 (2010).
[14] Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).
[15] L. Mao, J. Shi, Q. Niu, and C. Zhang, Phys. Rev. Lett. 106, 157003 (2011).
[16] S. Tewari, T. D. Stanescu, J. D. Sau, and S. Das Sarma, New J. Phys. 13, 065004 (2011).
[17] Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. 102, 130401 (2009).
[18] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, Nature (London) 471, 83 (2011).
[19] C.-J. Wu, I. M. Shem, and X.-F. Zhou, Chin. Phys. Lett. 28, 097102 (2011).
[20] C. Wang, C. Gao, C.-M. Jian, and H. Zhai, Phys. Rev. Lett. 105, 160403 (2010).
[21] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92, 040403 (2004).
[22] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. H. Drochslag, and R. Grimm, Phys. Rev. Lett. 92, 120401 (2004).
[23] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Phys. Rev. Lett. 92, 120403 (2004).
[24] R. A. Duine and H. T. C. Stoof, Phys. Rep. 396, 115 (2004).
[25] Q.-J. Chen, J. Stajic, S. Tan, and K. Levin, Phys. Rep. 412, 1 (2005).
[26] M. W. Zwierlein, A. Schirotzek, C. Schunck, and W. Ketterle, Science 311, 492 (2006).
[27] G. B. Patridge, W. Li, R. Kamar, Y. Liao, and R. G. Hulet, Science 311, 503 (2006).
[28] D. E. Sheehy and L. Radzihovsky, Ann. Phys. 322, 1790 (2007).
[29] E. Majorana, Nuovo Cimento 5, 171 (1927).
[30] S. Tewari, S. Das Sarma, C. Nayak, C. Zhang, and P. Zoller, Phys. Rev. Lett. 98, 010506 (2007).
[31] C. Zhang, S. Tewari, R. M. Lutchyn, and S. Das Sarma, Phys. Rev. Lett. 101, 160401 (2008).
[32] M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. Lett. 103, 020401 (2009).
[33] A. K. Sengupta, P. Massignan, and M. Lewenstein, Europhys. Lett. 89, 46004 (2010).
[34] J. P. Vyasankare and V. B. Shenoy, Phys. Rev. B 83, 094515 (2011).
[35] J. P. Vyasankare, S. Zhang, and V. B. Shenoy, Phys. Rev. B 84, 041512 (2011).
[36] M. Gong, S. Tewari, and C. Zhang, Phys. Rev. Lett. 107, 195303 (2011).
[37] Z.-Q. Yu and H. Zhai, Phys. Rev. Lett. 107, 195305 (2011).
[38] H. Hu, L. Jiang, X.-J. Liu, and H. Pu, Phys. Rev. Lett. 107, 195304 (2011).
[39] M. Iskin and A. L. Subasi, Phys. Rev. Lett. 107, 050402 (2011).
[40] L. Han and C. A. R. Sá de Melo, arXiv:1106.3613.
[41] W. Yi and G.-C. Guo, Phys. Rev. A 84, 033633(R) (2011).
[42] G. Chen, M. Gong, and C. Zhang, arXiv:1107.2627.
[43] L. Dell’Anna, G. Mazzarella, and L. Salasnich, Phys. Rev. A 84, 033633 (2011).
[44] K. Seo, L. Han, and C. A. R. Sá de Melo, arXiv:1108.4068.
[45] B. Huang and S. Wan, arXiv:1109.3970.
[46] L. He and X.-G. Huang, arXiv:1109.5577.
[47] L. Jiang, X.-J. Liu, H. Hu, and H. Pu, arXiv:1110.0805.
[48] W. Yi and L.-M. Duan, Phys. Rev. A 84, 033604(R) (2011).
[49] M. Iskin and C. A. R. Sá de Melo, Phys. Rev. Lett. 97, 100404 (2006).
[50] M. M. Purish, F. M. Marchetti, A. Lamacraft, and B. D. Simons, Nat. Phys. 3, 124 (2007).
[51] M. Randeria, J.-M. Duan, and L.-Y. Shieh, Phys. Rev. Lett. 62, 981 (1989).

For simplicity, in this work we assume the external trapping potential to be symmetric with respect to the origin. However, the universal approach for the trapped gas can be extended to anisotropic cases so long as the trapping potential remains harmonic. This can be achieved by applying different scalings along different spatial directions. See Ref. [15] for details.