Recent developments in light-front dynamics

V.A. Karmanov

Lebedev Physical Institute, Leninsky Prospekt 53, 119991 Moscow, Russia

Abstract. Recent results on relativistic few body systems, obtained in the framework of light-front dynamics, are briefly reviewed. The following subjects are discussed: two scalar bosons with ladder and cross ladder kernel; two fermions with OBE kernel; relativistic scattering (elastic and inelastic); three bosons and fermions with zero-range interaction; many-body contributions.

INTRODUCTION

In relativity, the state vector of a composite system is defined, in general, on a space-like surface. A limiting case the space-like plane – light-front (LF) plane, defined by the equation\( \omega \cdot x = 0 \), with \( \omega^2 = 0 \), is very preferable. In this case, the bare vacuum state (an eigenstate of free Hamiltonian) is also an eigenstate of full Hamiltonian. This simplifies the theory a lot. Dynamics, determining the LF wave functions, is called light-front dynamics (LFD). If \( \omega \) is a general four-vector (but always \( \omega^2 = 0 \)), we get explicitly covariant version of LFD [1]. In particular case \( \omega = (1,0,0,-1) \) we recover the standard version [2].

LFD is successfully applied to relativistic few-body systems and to the field theory [1,2]. Below we present some recent applications to relativistic few-body systems.

TWO SCALAR BOSONS

The orientation of LF plane \( \omega \cdot x = 0 \) is determined by the direction of \( \hat{\omega} \), i.e., by the unit vector \( \hat{n} = \hat{\omega}/|\hat{\omega}| \). Wave functions, defined on the LF plane, depend on \( \hat{n} \). For a two-body wave function we get:

\[
\psi = \psi(\vec{k},\hat{n}).
\]

For two spinless constituents with zero angular momentum wave function depends on the scalar products: \( \psi = \psi(\vec{k}^2,\vec{n}\cdot\vec{k}) \). For systems with non-zero spins and total angular momentum the vector \( \vec{n} \) participates in construction of the angular momentum on the equal ground with the relative momentum \( \vec{k} \).

Equation for two-body wave function \( \psi(\vec{k},\hat{n}) \) is determined by the kernel \( V(\vec{k},\vec{k}',\vec{n},M^2) \) which also depends on \( \vec{n} \) and, in addition, on the two-body mass \( M \). For a given model, the kernel is calculated by the LFD graph technique [1,2]. For the ladder exchange, its nonrelativistic (NR) counterpart is the Yukawa potential \( V(r) = -\alpha \exp(-\mu r)/r \), where \( \alpha = g^2/(16\pi m^2) \) (for scalars) and \( g \) is the coupling constant in the interaction Hamiltonian \( H^{int} = -g\psi^2\phi \).
The NR and LFD calculations [3] with the ladder kernel are compared in figure 1. Calculation by the Bethe-Salpeter (BS) equation is very close to the LFD one, which differs from NR. For heavy exchanged mass $\mu$ (comparable with the constituent mass $m$) the nonrelativistic and relativistic results strongly differ from each other even for very small binding energy. This conclusion is important since $\rho$ and $\omega$ mesons, incorporated in the $NN$ potential, are heavy. Therefore, the nonrelativistic approach may be too approximate.

We calculated also [4], both by the BS and LFD equations, the binding energy incorporating sum of ladder and cross-ladder graphs. The stretched box graphs (with two non-crossed intermediate mesons) were also taken into account, but their contribution turned out to be small. The result of calculation, for exchanged mass $\mu = 0.5$, is shown in figure 2. We see that the contribution of the crossed ladder graphs, relative to the ladder exchange, is large. The difference between the LFD and BS results is still small. The influence of the BS crossed box was also analyzed in [5].
TWO FERMIONS

LF wave function of two-fermions with the zero total angular momentum $J = 0^+$ has the following general form:

$$\psi(\vec{k},\vec{n}) = \frac{1}{\sqrt{2}} \left( f_1 + i\vec{\sigma} \cdot [\vec{k} \times \vec{n}] \sin \theta f_2 \right),$$

where $\theta$ is angle between $\vec{k}$ and $\vec{n}$. It is determined by two spin components $f_1, f_2$ depending on $\vec{k}^2, \vec{n} \cdot \vec{k}$ (or, equivalently, on $k$ and $\theta$) and satisfying a system of two equations.

The wave function for $J = 1^+$ is determined by six spin components. Corresponding system of 6 equations is split in two uncoupled subsystems of 4 and 2 equations.

Two fermion bound states with the OBE kernel incorporating scalar, pseudoscalar and vector mesons were investigated in detail, exchange by exchange, in [6]. The form factors in the $NN$-boson vertices were introduced and stability relative to cutoff was studied. For the scalar exchange [7] (Yukawa model), a stable bound state with $J = 0^+$ exists without form factor, if the coupling constant $\alpha = g^2/(4\pi)$ does not exceed a critical value $\alpha_c = 3.72$. If $\alpha > \alpha_c$, cutoff is needed to obtain a finite binding energy. For pseudoscalar exchange the state $J = 0^+$ is stable, whereas for the $J = 1^+$ state a vertex form factor is required. It is always required for the vector exchange.

Relativistic deuteron wave function (with six spin components) was calculated in [8] and applied to the deuteron e.m. form factors in [9]. The experimental data [10] on $t_{20}$, later obtained at JLab, are on the curves from [9].

RELATIVISTIC SCATTERING

Two following important features of relativistic scattering equations should be emphasized [11, 12, 13]. (i) Relativistic kernel $V(\vec{k}',\vec{k},\vec{n},M^2)$ automatically takes into account inelastic channel. When $M > 2m + \mu$, then denominator in the ladder kernel may cross zero, that results in a singularity and, in its turn, in imaginary part of the phase shift $\delta$. (ii) For the exchange kernel, the amplitude is not unitary above threshold, i.e., $\delta$ does not satisfy the condition $\text{Im}(\delta) = k^2 \sigma^{\text{inel}}/(4\pi)$ (valid for small $\text{Im}(\delta)$). In l.h.-side, $\text{Im}(\delta)$, generated by the exchange kernel, does not incorporate the self-energy graphs, where the meson is emitted and absorbed by the same particle. On the contrary, $\sigma^{\text{inel}}$ in r.h.-side is determined by the inelastic amplitude squared and therefore contains such contributions. That’s why unitarity is violated. It is restored, if self energy is taken into account.

Figure 3 shows a comparison of NR and LFD calculations of the phase shift for two scalar bosons exchanging by the mass $\mu = 1$. The conclusion is similar to the bound state case: relativistic and nonrelativistic results considerably differ from each other even at the small incident momenta.
THREE BODY SYSTEMS

Three-boson relativistic system with zero-range interaction was studied in [14, 15, 16]. The input is the two-body bound state mass $M_2$, the output is the three-body mass $M_3$. As well known, corresponding NR three-body system is unstable: if, for fixed $M_2$, the interaction radius $r_0$ tends to 0, then the three-body binding energy $B_3$ tends to $-\infty$ (Thomas collapse). In relativistic case, it was found that three-body mass $M_3$ is always finite. However, if the two-body mass $M_2$ approaches to the critical value $M_c = 1.43 m$, then three-body mass $M_3$ decreases down to zero [16]. When $M_2 < M_c$, $M_3^2$ becomes negative (see figure 4). Therefore, at enough strong interaction, providing $M_2 < M_c$, the three-body system does not exist (there is no any solution with real $M_3$). This is a relativistic analog of Thomas collapse. For three fermions the situation is similar, though the critical mass $M_c$ is smaller.

General spin structure of the LF nucleon wave function composed of three quarks was studied in [17]. Applications to the hadron form factors were considered in [18].
MANY-BODY CONTRIBUTIONS

In general, the number of constituents in a relativistic system is not fixed. We can consider this system as a few-body one, if few-body sectors dominate and contribution of higher Fock sectors is small. The contributions of two-body and higher Fock sectors to the total norm and electromagnetic form factor were analyzed \[19\] in the Wick-Cutkosky model, where two massive scalar particles interact by the ladder exchanges of massless scalar particles. Two-body sector contains two massive particles. Higher sectors contain two massive and \(1, 2, \ldots\) massless constituents. It was found that two- and three-body sectors always dominate. Even for maximal value of coupling constant \(\alpha = 2\pi\), corresponding to zero bound state mass \(M = 0\), they contribute to the norm 64\% and 26\% respectively (90\% in the total). With decrease of \(\alpha\) the two-body contribution increases up to 100\%. Hence, in this model few-body relativistic system is indeed a good approximation. This result is non-trivial, since for so strong interaction one might expect just the opposite relation of few-body and many-body contributions. However, the cross ladder may correct this result. A few higher Fock sector contributions to the kernel for massive exchanges were studied in \[20\].

CONCLUSION

Relativistic few-body physics covers huge and rich domain of physical phenomena, including light nuclei at small distances and hadrons in quark models. LFD is a very efficient approach to these phenomena. Recent developments in LFD show good progress in this field.

REFERENCES

1. Carbonell J., Desplanques B., Karmanov V.A and Mathiot J.-F., *Phys. Rept.*, 300, 215 (1998)
2. Brodsky S.J., Pauli H. and Pinsky S.S., *Phys. Rept.*, 301, 299 (1998)
3. Mangin-Brinet M. and Carbonell J., *Phys. Lett.*, B474, 237 (2000)
4. Carbonell J. and Karmanov V.A., *in preparation* (2004)
5. Theussl L. and Desplanques B., *Few Body Syst.*, 30, 5 (2001)
6. Mangin-Brinet M, Carbonell J. and Karmanov V.A., *Phys. Rev.*, C68, 055203 (2003)
7. Mangin-Brinet M, Carbonell J. and Karmanov V.A., *Phys.Rev.*, D64, 027701; 125005 (2001)
8. Carbonell J. and Karmanov V.A., *Nucl. Phys.* A581, 625 (1995)
9. Carbonell J. and Karmanov V.A., *Eur.Phys.J.*, A6, 9 (1999)
10. Abbott D. et al., *Phys. Rev. Lett.*, 84, 5053, 2000
11. Ji C.R. and Surya Y., *Phys. Rev.* D48, 3565 (1992)
12. Ji C.R., Kim G.H. and Min D.P., *Phys. Rev.*, D51, 879 (1995)
13. Oropeza D., Carbonell J. and Karmanov V.A., *to be published*
14. Lindesay J.V. and Noyes H.P., *Preprint SLAC-PUB-2932*, 1986
15. Frederico T., *Phys. Lett.*, B282, 409 (1992)
16. Carbonell J. and Karmanov V.A., *Phys. Rev.*, C67, 037001 (2003)
17. Karmanov V.A., *Nucl. Phys.*, A644, 165 (1998)
18. Brodsky S.J., Hiller J.R., Hwang D.S. and Karmanov V.A., *Phys. Rev.*, D69, 076001 (2004)
19. Hwang D.S. and Karmanov V.A., *Nucl. Phys.*, B696, 413 (2004), hep-th/0405035, hep-th/0410050
20. Schoonderwoerd N.C.J, Bakker B.L.J. and Karmanov V.A., *Phys. Rev.*, C58, 3093, (1998)