Computational modeling of the conjugated thermomechanical and aerogasdynamics processes for composite structures of high speed vehicles

Yu Dimitrienko¹, A Zakharov¹ and M Koryakov¹

¹Bauman Moscow State Technical University, Computational Mathematics and Mathematical Physics, Russian Federation

E-mail: dimit@bmstu.ru, azaharov@bmstu.ru, mkoryakov@bmstu.ru

Abstract. A conjugated problem statement for aerodynamics and thermomechanics of heat-shielding structures from thermally decomposing polymer composite materials is proposed. It is based on the iterative solution of the three types of detached problems: aerogasdynamics problem for viscous heat-conducting flows, internal heat and mass transfer and thermoelasticity of shell constructions. An example of the numerical solution of the conjugated problem is given. It is shown that due to the high temperatures of the aerodynamic heating of the structure made of a polymer composite material there can appear a polymer phase thermodecomposition and intensive internal gas generation into the structure of the material.

1. Introduction

The learning of hypersonic speeds is one of the most promising complex problems of the high-tech development. This problem can be identified such components as: the study of hypersonic aerodynamics of flights, study of the heat transfer on surfaces of aircraft constructions, thermophysics research of constructional materials, thermal strength study, analysis and development of constructional materials for hypersonic aircrafts, and the problems of hypersonic aeroelasticity, control and others. Significant amount of work [1, 5, 9, 10] described researches of hypersonic aerodynamics conditions, less studied problems of heat transfer [2, 6, 8] under hypersonic speeds. The more difficult problem is the high-temperature behavior of composite materials based on heat-resistant filaments and masters [3]. The complex conjugated problem of aerothermodynamics, heat transfer, thermal physics and thermal strength of hypersonic constructions is still practically uninvestigated, and there are relatively recent work that studied aeroelasticity of constructions under hypersonic speeds. However under the actual operating conditions of hypersonic vehicles the problems of aerothermodynamics, heat transfer and thermophysics of the constructions are conjugated through the boundary conditions on the surface of the constructions, so the parameters of the heat flux acting on the materials depend on the properties of these materials. By turn, thermal properties of the materials at high temperatures may depend on the mode of deformation of the constructions. So the significant level of thermal stresses in composite materials leads to the filament microcracking long before the full macrodestruction of constructions, thereby the gas permeability and thermal conductivity materials and temperature fields in the constructions are changed. Thus the development of the methods for solving the conjugated problem for the study of the real processes taking place in the constructions of hypersonic vehicles is necessary.

This work is licensed under the Creative Commons Attribution 3.0 Unported License. To view a copy of this license, visit http://creativecommons.org/licenses/by/3.0/ or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.
The general formulation of the conjugated problem of aerothermodynamics and thermomechanics consists of the three systems of equations

- the Navier-Stokes equations of an external gas flow;
- the internal heat and mass transfer equations;
- the equations of thermoelasticity of a shell.

2. Mathematical Formulation

2.1. System of Gasdynamics Equations

Consider the system of equations of a viscous heat-conducting gas (the Navier-Stokes equations) [1, 5, 7]:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + p \mathbf{E} - T \mathbf{I}) = 0, \\
\frac{\partial \rho \varepsilon}{\partial t} + \nabla \cdot ((\rho \varepsilon + p) \mathbf{v} - T \mathbf{I} \cdot \mathbf{v} + \mathbf{q}) = 0,
\]

where \(\rho\) is the gas density, \(t\) is the time, \(\mathbf{v}\) is the velocity vector, \(p\) is the pressure, \(E\) is the identity tensor, \(\varepsilon\) is the total energy per unit volume.

This system adds relations for the perfect gas, viscous stress tensor and heat flux vector

\[
p = \rho \frac{R}{M} \theta, \quad e = c_v \theta, \quad E = e + \frac{1}{2} |\mathbf{v}|^2, \\
T = \mu_1 (\nabla \cdot \mathbf{v}) \mathbf{E} + \mu_2 (\nabla \otimes \mathbf{v} + \nabla \otimes \mathbf{v}^T), \\
\mathbf{q} = -\lambda \nabla \theta,
\]

where \(R\) is the universal gas constant, \(M\) is the molecular weight of gas, \(\theta\) is the gas temperature, \(c_v\) is the specific heat at constant volume, \(\mu_1\) and \(\mu_2\) are the coefficients of viscosity, \(\lambda\) is the thermal conductivity of gas. The coefficients of viscosity and thermal conductivity are functions of temperature.

The boundary conditions on the solid surface, which is the interface of the gas and solid domains, are as follows

\[
\mathbf{v} = \mathbf{0}, \quad \theta = \theta_w, \\
-\lambda \nabla \theta \cdot \mathbf{n} = -\lambda_s \nabla \theta_w \cdot \mathbf{n} + \varepsilon_g \sigma \theta_{\text{max}}^4 - \varepsilon_s \sigma \theta^4_u,
\]

where \(\theta_w\) is the temperature of the solid surface, \(\theta_{\text{max}}\) is the maximum temperature into the boundary layer, \(\nabla \theta_{\text{w}}\) is the temperature gradient on the solid wall from the construction, \(\varepsilon_g\) and \(\varepsilon_s\) are the emissivity of the heated gas and solid surface respectively and \(\sigma\) is the Stefan-Boltzmann constant.

2.2. System of Equations of Internal Heat and Mass Transfer

We consider a typical element which is made of a heat-resistant composite material consisting of a polymer master with heat-resistant filaments. There are physical and chemical processes of thermal decomposition in such composite master under high temperatures of aerodynamic heating. In these processes the gaseous products of thermal decomposition are generated, then they are accumulated in the pores of the material and filtered into the outer gas flow, as well as a new solid phase is formed. It is the phase of pyrolytic master which has significantly lower elastic-strength properties than the original polymer phase. The four-phase model to describe the internal heat and mass transfer and deformations of such composite is proposed in [3]. This model consists of:
- the equation of change of mass of polymer master phase:
  \[
  \rho_b \frac{\partial \phi_b}{\partial t} = -J; \tag{4}
  \]

- the equation of filtration of gaseous products of thermodestruction in pores of the composite material:
  \[
  \frac{\partial \rho_f \phi_g}{\partial t} + \nabla \cdot \rho_f \phi_g v_f = J \Gamma; \tag{5}
  \]

- the heat transfer equation in the thermodestruction composite:
  \[
  \rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot \mathbf{q} - c_g \nabla \theta - \rho_b \phi_b \mathbf{v}_b - J \Delta e^0; \tag{6}
  \]

where \( \phi_b, \phi_g \) are the volume concentrations of the phase of the initial polymer master and gas phase; \( \rho_b \) is the density of the phase of the initial polymer master which is assumed to be constant; \( \rho_g \) is the average pore density of the gas phase (variable); \( c_g \) is the specific heat of the gas phase at constant volume; \( \rho \) and \( c \) are the density and specific heat of the composite as a whole; \( \mathbf{q} \) is the heat flux vector; \( \theta \) is the composite temperature for all phases in common; \( v_f \) is the velocity vector of the gas phase; \( \Delta e^0 \) is the specific heat of the master thermodecomposition; \( J \) is the mass velocity of the master thermodecomposition and \( \Gamma \) is the gas-producing factor of the master.

The equations (4)-(6) are added the relations between the heat flux vector \( \mathbf{q} \), the velocity vector of the gas in pore \( v_f \) with the temperature gradient \( \nabla \theta \) and the pressure gradient \( \nabla p \) using the Fourier and Darcy laws, as well as the Arrhenius relation for the mass velocity of the master thermodecomposition \( J \) and Mendeleev-Clapeyron equation for the pore pressure of the gas phase \( p \):

\[
\mathbf{q} = -\mathbf{A} \cdot \nabla \theta, \quad \rho_f \phi_g v_f = -K \cdot \nabla p, \quad J = J_0 \exp \left( \frac{-E_A}{R_0} \right), \quad p = \frac{\rho_b R_0}{M_g}, \tag{7}
\]

where \( J_0 \) is the pre-exponential factor, \( E_A \) is the activation energy of the thermodecomposition process, \( M_g \) is the molecular weight of the gas phase, \( \mathbf{A} \) is the thermal conductivity tensor, \( K \) is the permeability tensor of the composite. They depend on the phase concentration.

The boundary conditions for the equations (4)-(6) on the heated surface of the construction are as follows

\[
p = p_e, \quad \theta = \theta_e,
\]

where \( p_e, \theta_e \) are the pressure and temperature of the flow on the surface.

The boundary conditions of tightness and thermal insulation are specified on the rest of the composite surface

\[
n \cdot \nabla p = 0, \quad -n \cdot A \nabla \theta = 0.
\]

2.3. **System of Equations of Thermoelasticity**

System of equations of thermoelasticity consists of [4]:

- the equilibrium equations of the shell;
- the kinematic relations.

3. **Results and Discussion**

A numerical solution of the conjugated problem was calculated for a hypersonic flow (\( M = 6 \)) around the nose of a vehicle model flying at the altitude of 15 km.

Fig. 1 shows the distribution of the temperature of the flow near the body. The temperature reaches 1.600\(^\circ\)K at the stagnation point on the nose of the vehicle and decreases monotonically as the distance from the stagnation point, but it remains rather high value. At the maximum distance the temperature
is about 800°K for the edge and lower generatrix and 1,000°K for the upper generatrix of the vehicle which has a large value of the cone angle.

The results of numerical calculations of the fields of internal heat and mass transfer in the shell element of the hypersonic aircraft are shown in Fig. 7. The curves show the distributions along the shell thickness (the dimensionless coordinate \( x = 0.5 + q_3/h \) is introduced) at a control point on the bottom surface at the distance of 30r from the stagnation point, where r is the blunted body radius at the stagnation point. Different colors correspond to the 7 different time points \( t_1, \ldots, t_7 \) of aircraft flight.

Fig. 3 shows maximum of the transversal stresses \( \sigma_{33} \) for the shell at time point \( t_7 \).

**Figure 1.** Temperature of the flow, \( \theta \) (°K).

**Figure 2.** Pore pressure \( p \), atm.

**Figure 3.** Maximum transversal stress, \( \sigma_{33} \) (GPa).
Acknowledgments
The work was carried out with support of state task of Ministry for Science and Education of the RF
No. 9.3602.2017/ПЧ

References
[1] Anderson J D 2006 Hypersonic and High-Temperature Gas Dynamics. 2nd ed. American
Institute of Aeronautics and Astronautics (Reston, Virginia)
[2] Shumaev V V and Kuzenov V V 2017 Development of the numerical model for evaluating the
temperature field and thermal stresses in structural elements of aircrafts J. Phys: Conf. Ser.
IOP Publishing Ltd 1 891
[3] Dimitrienko Yu I 1999 Termomechanics of composites under high temperatures ed Kluwer
Academic Publishers (Dordrecht/Boston/London)
[4] Dimitrienko Y I, Minin V V and Syzdykov E K 2011 Modeling of Thermomechanical
Processes in Composite Shells in Local Radiation Heating, Composites: Mechanics,
Computations, Applications 2 147-169
[5] Kuzenov V V 2018 et al Calculating processes of laminar and turbulent heat transfer around the
elements of the aircraft J. Phys.: Conf. Ser. 980
[6] Kandinsky R O, Prosuntsov P V 2015 Modelling combined heat exchange in the leading edge
of perspective aircraft wing, MATEC Web of Conferences 23 01019
[7] Grishin Y A, Bakulin V N 2015 New calculation schemes based on the large-particle method
for modeling gas-dynamic problems, Doklady Physics 60 555-558
[8] Leont’ev A I, Kuzma-Kichta Y A, Popov I A 2017 Heat and mass transfer and hydrodynamics
in swirling flows (review), Thermal Engineering 64 pp. 111-126
[9] Gorskii V V, V A Sysenko, Dekermendzhi K Y. Accuracy of calculating the momentum-loss
thickness in a laminar boundary layer on the surface of a hemisphere in a supersonic air flow
Journal of Engineering Physics and Thermophysics 3 90 pp. 535-540, 2017.
[10] Isaev S A, Schelchkov A V, Leontiev A I 2016 Baranov P A, Gulcova M E, Numerical
simulation of the turbulent air flow in the narrow channel with a heated wall and a spherical
dimple placed on it for vortex heat transfer enhancement depending on the dimple depth
International Journal of Heat and Mass Transfer 94 pp. 426-448