A Driver-in-the Loop Fuel Economic Control Strategy for Connected Vehicles in Urban Roads

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May 23, 2017

Abstract
In this paper, we focus on developing driver-in-the loop fuel economic control strategy for multiple connected vehicles. The control strategy is considered to work in a driver assistance framework where the controller gives command to a driver to follow while considering the ability of the driver in following control commands. Our proposed method uses vehicle-to-vehicle (V2V) communication, exploits traffic lights’ Signal Phase and Timing (SPAT) information, models driver error injection with Markov chain, and employs scenario tree based stochastic model predictive control to improve vehicle fuel economy and traffic mobility. The proposed strategy is decentralized in nature as every vehicle evaluates its own strategy using only local information. Simulation results show the effect of consideration of driver error injection when synthesizing fuel economic controllers in a driver assistance fashion.

1 INTRODUCTION
Connected vehicle (CV) technology is considered to be a multi-faceted solution to the current issues of the transportation system as pointed out by the Intelligent Transportation Systems Joint Program Office (ITS-JPO). The CV technology proposed by the ITS-JPO allows vehicles to communicate with other vehicles (V2V communication) and with transportation infrastructure (vehicle to infrastructure communication (V2I)) via wireless communications. For ITS-JPO, the major areas of concern are safety, mobility, and the environmental impact of the vehicles. According to the ITS-JPO website, traffic fatalities are more than 30 thousand every year and traffic congestion costs around 87.2 billion to the U.S. economy, with 4.2 billion hours and 2.8 billion gallons of fuel
spent sitting in traffic [1]. Moreover, deterioration of traffic mobility affects the environment since vehicles that are stationary, idling, and traveling in a stop-and-go pattern emit more greenhouse gases than those traveling in free-flow conditions [1]. Although the primary aim of CV systems is to improve safety (V2V and V2I communication addresses 79% of all vehicle crashes [2]), the V2V and V2I communication can be utilized to improve many different aspects of the transportation system such as improvement of fuel economy and traffic mobility, which in turn would improve vehicle emissions [1].

In recent years, a lot of research in intelligent transportation system and automotive engineering has focused on developing more fuel efficient vehicles. The fuel efficiency of a vehicle depends on a number of factors such as vehicle aerodynamic drag, vehicle engine characteristics and powertrain system, and weather and road conditions. Apart from them, driving behavior of vehicles have been seen to influence fuel efficiency by a good margin [3]. It has been observed, that fuel efficiency of a vehicle improves when they move at a constant cruising velocity and that is why most fuel economic control strategies in the literature tries to minimize vehicle acceleration and braking [3–10].

Developing fuel efficient control strategies for urban roads is difficult because of traffic lights at regular intervals. However, reduction of red light idling have shown to improve vehicle’s fuel efficiency [11, 12]. That is why, in recent years, authors [11, 16] have developed control strategies that utilize traffic light timing information to avoid red light idling. Authors [11, 13, 17] have developed methods to optimally control vehicle acceleration and velocities while moving through multiple traffic signals. A probabilistic decision making algorithm using model predictive control is shown in [12] that addresses noise in SPAT information. Fuel optimal control strategy using data driven fuel consumption model is shown in [15], while a multi-stage dynamic programming based control strategy is provided by the authors in [16]. Field testing of fuel optimal control strategies using SPAT information with real vehicles is shown by the authors in [18]. When traffic light timings are not known, researchers in [19] have shown a stochastic online estimation method to estimate the parameters of traffic lights. Most of the above mentioned works are designed for single vehicle systems which for urban roads is a limiting condition. Although some literature [10] have considered two vehicle scenarios, the velocity and acceleration prediction strategy for the preceding vehicle might not be feasible for congested roads. Also, all the above works consider full autonomous vehicles where the evaluated control commands are exactly executed, which is a hard assumption when human drivers are present. Full autonomous driving in urban roads is a complex task [20] and a percentage of the population can always be expected to drive by choice. This demands for development of control methods that work together with the human driver and addresses their imperfection while following given commands.

Driver assistance systems have gained a lot of popularity in the recent years and they primarily focus on vehicle safety (such as giving warning signals). These assistance systems work with the driver by either giving them commands to follow, such as maintaining a velocity, or provide active control for a short-while when required. The common driver assistance systems include brake as-
istance system, driver status monitoring, blind spot warning, lane departure warning and lane keeping assistance system, speed control, and night vision system \cite{21}. These systems have evolved over time and some exploit the recent advances in CV technology \cite{20,22,23}. Some recent works on driver assistant systems that focus on vehicle fuel economy are available in \cite{24,27}. Authors \cite{24} have provided optimal velocity and acceleration information to the driver by minimizing an approximate fuel consumption model. A driver training and analysis system is developed in \cite{25} where the drivers are taught to reduce sharp acceleration and braking. A context aware driver assistance system is proposed in \cite{26} that considers driver, environmental, and in-vehicle conditions for its decision making. Authors in \cite{27} have presented a driver assistance system that focuses on fuel economy using data driven and manufacturer independent model. Most of the above mentioned works on driver assistance systems do not consider the driver behavior or his/her capabilities in following commands from the assistance system. Apart from that, most of the work in the literature focuses on developing fuel economic control strategies for a single vehicle without considering the impact of a vehicle’s driving behavior on another.

Some previous research on semi-autonomous control of vehicles are available in \cite{28,31}. Authors \cite{28} have used stochastic model predictive control (SMPC) for energy management in hybrid electric vehicles (HEVs) where Markov chain model is used to model driver future power request. Authors in \cite{29,31} have developed a controller that only takes over the vehicle when it predicts some danger, e.g., vehicle entering wrong lane, and have modeled the human driver as a feedback controller. In CV systems and urban road conditions, longitudinal vehicle control in a driver assistance framework that focus on fuel economy and system mobility while considering driver behavior has not been addressed before. Thus, in this paper, we develop a driver-in-the loop fuel economic control strategy for multiple CVs in urban road conditions that also focus on the improvement of system mobility by reducing red light idling using SPAT information. Since the controller design is driver specific, the assistance system can be considered to be a *customized/personalized driver assistance system*. The developed strategy is decentralized in nature as every vehicle can develop its own control using only local V2V and V2I information. First we model the driver error with a Markov chain model and then employ a scenario based stochastic model predictive control strategy \cite{28,32,36} for evaluating the optimal control policy in presence of a human driver.

In some of our previous works \cite{37,40}, we have developed model predictive control based fuel economic control strategies for a group of conventional and hybrid electric vehicles \cite{38} in urban roads. In this paper, we extend those works by developing the control strategy as a driver assistance system while considering the driver behavior. The paper contributions can be listed as: (i) a longitudinal vehicle control strategy is developed in a driver assistance framework considering the driver behavior, especially the imperfection associated with the driver when following the instructions from the assistance system; (ii) consideration of computational tractability by employing sampling based scenario tree generation and use of discounted cost for the SMPC; and (iii) consideration
of multiple CVs and an urban environment (presence of traffic lights) for fuel economic controller development. While improving fuel efficiency of vehicles, we also improve system mobility which in turn improves vehicle environmental impact \cite{1,38}. We have compared our proposed method with a passive driver assistance system (does not consider human error injection) and with two popular optimal control methods for stochastic systems: (i) certainty equivalence control \cite{41}, and (ii) frozen time model predictive control \cite{28,36}.

The paper is organized as follows: in Section 2 we mathematically introduce the problem and in Section 3 we describe the stochastic optimal control strategy that uses V2I SPAT information and V2V neighboring vehicle information. Finally, in Sections 4 and 5 we provide the simulation results and the conclusion and the future potentials of this work.

2 PROBLEM DESCRIPTION

We focus on a group of CVs in urban road conditions that consist of traffic lights at regular intervals. In this CV scenario, wireless V2V communication allow vehicles to share their position and velocity information with other CVs. In every vehicle, the driver assistance system is considered to provide velocity command to the driver which the driver needs to follow to improve its fuel economy. For human drivers, following these commands exactly at every time instant is difficult and their actions would introduce random errors which would be driver specific. Fig. 1 shows a schematic of the mentioned scenario and Fig. 2 shows the driver assistant system in every vehicle.

![Figure 1: Schematic of the problem](image-url)
2.1 System Description

The discrete time longitudinal dynamics of any vehicle $i$ with time step $\Delta t$ is given by [10]:

$$
    x_i(k + 1) = x_i(k) + \Delta t f_i(x_i(k), u^f_i(k))
$$

with

$$
    f_i(x_i(k), u^f_i(k)) = \begin{bmatrix}
    v_i(k) \\
    -\frac{1}{2M_i} C_D \rho_a A_i^v v_i^2(k) - \mu g - g\theta + u^f_i(k)
    \end{bmatrix}
$$

(1)

Here $x_i(k) \in \mathbb{R}^{n_x}$ and $u^f_i(k) \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$, where $n_x = 2$ and $n_u = 1$. The vehicle state is $x_i(k) = [s_i(k) \ v_i(k)]^T$, where $s_i(k)$ is the position of a vehicle while $v_i(k)$ is its velocity at time instant $k$. In the above equation, $M_i$ is the mass of the vehicle $i$, and $A_i^v$ is its frontal area. The terms $C_D$, $\rho_a$, $\theta$ and $\mu$ are the drag coefficient, air density, the road gradient, and the rolling friction coefficient respectively. It is assumed that road gradients are mild, so $\sin(\theta) = \theta$ and $\cos(\theta) = 1$. The effective control strategy, $u^f_i(k)$, of a vehicle $i$ is its braking or traction force per its unit mass which is considered to be a sum of the control, $u_i(k)$, suggested by the assistance system and the random error, $\omega_i(k) \in \mathcal{W}$, injected by the human driver. The constraint sets $\mathcal{U}$ and $\mathcal{W}$ are considered to be compact. We can consider that the assistance system provides velocity information to the driver to follow so that at $k$, it would ask the driver to reach velocity $v_i(k + 1) = v_i(k) + \Delta t u_i(k)$ when the vehicle velocity is $v_i(k)$.

As mentioned before, the fuel consumption in a vehicle depends on a number of factors such as torque, engine speed, and gear ratio [9]. Many papers approximate the cost of fuel consumption as a function of velocity and acceleration of the vehicle. Following [10], we represent the rate of fuel consumption (ml/s) by:
\[ F_{\text{fuel}} = (1 - \zeta)(f_{\text{cruise}}^i + f_{\text{accel}}^i) + \zeta F_d^i \quad (2a) \]
\[ f_{\text{cruise}}^i = b_0 + b_1 \dot{v}_i + b_2 v_i^2 + b_3 v_i^3 \quad (2b) \]
\[ f_{\text{accel}} = \dot{a}_i(c_0 + c_1 v_i + c_2 v_i^2) \quad (2c) \]
\[ \dot{a}_i = -\frac{1}{2M_h}C_D \rho_a A_i v_i^2 - \mu g + u_f^i \quad (2d) \]
\[ \zeta = \begin{cases} 1 & \text{if } \dot{v}_i = 0 \text{ or } u_f^i < 0 \\ 0 & \text{otherwise} \end{cases} \]

In Eq. (2), \( \dot{f}_{\text{cruise}}^i \) is the rate of fuel consumed while cruising and \( \dot{f}_{\text{accel}}^i \) is the rate of fuel consumed when accelerating. Apart from that, a constant fuel consumption \( F_d^i \) is considered when red light idling and braking [9]. The binary term \( \zeta \) is 1 during braking or red light idling and 0 otherwise. The constant terms \( b_0, b_1, b_2, b_3, c_0, c_1 \) and \( c_2 \) are specific to a vehicle and could be found to approximate the fuel map of a vehicle.

### 2.2 Optimal Control Problem

To maximize ‘miles per gallon’ (mpg), we can minimize fuel consumption per unit distance. The fuel consumed per unit distance for a group of \( n \) vehicles is given by \( \sum_{i=1}^{n} \sum_{k} (F_{\text{fuel}}(k)\Delta t)/(v_i(k)\Delta t) \). Generally, this problem is solved as a receding horizon problem, with a time horizon \( T \) secs, where the cost includes fuel consumption per unit distance, collision avoidance penalty, vehicle’s desired velocity tracking, and the cost of applying control. The optimal control problem for every vehicle \( i \) is given by [9]:

\[
\min_{u_i} J_i(k) \quad (3a)
\]

\[
J_i(k) = E\left[ \sum_{t=k}^{k+M-1} c_1 \frac{F_{\text{fuel}}(t)\Delta t}{v_i(t)\Delta t} + c_2(t)R_{ij}(t)^2 + c_3(v_i(t) - v_i^{\text{target}}(k))^2 + c_4 u_f^i(t)^2 \right] \quad (3b)
\]

\[
R_{ij}(t) = S_0 + t_{hd}(v_i(t) - v_j(t)) + (s_i(t) - s_j(t)) \quad (3c)
\]

\[
v_{\min} \leq v_i(t) \leq v_{\max} \quad (3d)
\]

\[
u_{f,\min}^i \leq u_f^i(t) \leq u_{f,\max}^i \quad (3e)
\]

In the above equation, \( M = (T/\Delta t) \) is the discrete time horizon and \( E[\cdot] \) is the expectation. Since the human driver introduces the random error while trying to follow the command of the driver assistance system, the overall goal here is to minimize the expected cost. In Eq. (3b), the first term inside the expectation minimizes fuel consumed per unit distance, the second term minimizes the deviation from a desirable distance between vehicle \( i \) and its preceding vehicle \( j \), the third term tries to minimize the velocity deviation from its target
velocity \( v^i_{\text{target}}(k) \), and the last term minimizes the control effort. The target or desired velocity of vehicle \( i \) at time \( k \) is given by \( v^i_{\text{target}}(k) \) and is generally chosen as the road speed limit. The terms \( S_0 \) and \( t_{hd} \) in Eq. (3c) are pre-defined critical distance and headway time respectively. In Eq. (3b), \( c_1 \), \( c_3 \) and \( c_4 \) are constant weights while \( c_2(t) \), similar to [10], is chosen as a function of the relative distance, \( (s_j(t) - s_i(t)) \), so that it increases as the relative distance decreases and vice versa.

3 METHODOLOGY

Our proposed methodology works in two phases: (A) : we evaluate the target velocity of a vehicle so that it avoids red light idling at the upcoming traffic signal, and (B) : a scenario tree based stochastic model predictive control strategy is proposed for each vehicle that aims at improving their performance.

3.1 Target Velocity

We consider the SPAT information of only the upcoming traffic signal is known by each vehicle, via V2I communication. Rather than using speed limit (maximum allowable velocity) as the target velocity as in [10], each vehicle evaluates its target velocity so that it can avoid red light idling at the upcoming traffic signal. The target velocity \( v^i_{\text{target}}(k) \) is computed by each vehicle \( i \) as [11,37,40]:

\[
v^i_{\text{target}}(k) = \begin{cases} 
\frac{d_{iq}(k)}{K_w t_{cycle} - t_g - k} & \text{if light = red} \\
v_{max} & \text{if light = green and} \\
\frac{d_{iq}(k)}{K_w t_{cycle} + t_r - k} & \text{if light = green and Otherwise} \\
\end{cases} \tag{4a}
\]

\[
\text{light} = \begin{cases} 
\text{red} & \text{if } 0 \leq \text{mod}\left(\frac{k}{t_{cycle}}\right) \leq t_r \\
\text{green} & \text{if } t_r < \text{mod}\left(\frac{k}{t_{cycle}}\right) < t_{cycle} \\
\end{cases} \tag{4b}
\]

\[
v_{\text{min}} \leq v^i_{\text{target}}(k) \leq v_{max} \tag{4c}
\]

\[
t_{cycle} = t_r + t_g \tag{4d}
\]

\[
K_w > \frac{k}{t_{cycle}} \tag{4e}
\]

Here \( d_{iq}(k) \) is the distance between \( s_i(k) \) (location of the \( i^{th} \) vehicle) and the traffic signal \( q \), \( t_r \) and \( t_g \) are the red and green light durations respectively so that the total cycle duration is \( t_{cycle} \). \( K_w \) is an integer describing the traffic light cycle number. The function \( \text{mod}(\cdot) \) is a modulo function which generates the residue of division \( k \) by \( t_{cycle} \). It can be seen from Eq. (4a) that when traffic light status is green, the maximum allowable speed is chosen as the target velocity unless the constraint \( \frac{d_{iq}(k)}{K_w t_{cycle} - k} \leq v_{max} \) is not satisfied. Violation of this constraint (when the signal status is green) means that the vehicle needs...
to break the speed limit to pass through the traffic light in the current green light window. In that case, the vehicle desires to pass through the traffic signal in the next green light window as shown in third case of Eq. (4a). If no feasible velocity is obtained in the consecutive green light windows, the vehicle has to stop at the given traffic light signal. Eq. (4e) shows the constraint on the traffic signal index number so that $K_w$ is increased by 1 at $k = K_w^{\text{cycle}}$.

### 3.2 Stochastic Model Predictive Control

Stochastic model predictive control (SMPC) [28, 29, 36] is a popular method for solving constrained stochastic optimal control problems. Thus, after the target velocity of a vehicle $i$ is evaluated, the driver-in the loop fuel efficient control problem is solved using SMPC. Solving problem in Eq. (3) is challenging because it involves computation of the expected cost and solving the problem over the finite horizon $T$ secs. For computational purposes, we discretize the error set $W$ (hence $W$ is a bounded countable set) into $|W|$ parts ($|W|$ is the cardinality of the set $W$). The cardinality of the set $W$ here dictates the trade-off between accuracy and computational requirement since large $|W|$ would reduce discretization error but increase computational requirement.

In the literature, human behavior has been modeled with Markov chains by many researchers [28, 29, 42–47]. Authors in [46, 47] have used Markov model for analytical representation of human cognitive process. Authors in [42] modeled human behavior as a set of dynamic models sequenced together by Markov chain. Authors in [43, 44] used hidden Markov model while authors in [45] used partially observable Markov decision process model for human behavior modeling. Thus error introduced by humans in a system can be well modeled as a Markov chain. Markov chain model of human error can further be justified from the fact that human actions are highly dependent on the current state of the system.

Thus, following previous works, we model the drive error with a Markov chain with transition probability matrix $Q_i \in \mathbb{R}^{|W| \times |W|}$ where its elements $Q_i(a, b)$ ($a^{th}$ row and $b^{th}$ column) are given by:

$$Q_i(a, b) = P(\omega_i(k + 1) = b | \omega_i(k) = a)$$

Here $Q_i(a, b)$ gives the probability of state transition from state $a$ to $b$. The transition probability matrix would be driver specific and it can be modeled by the methods shown in [28]. It can be seen from Eq. (5) that different values of $\omega_i(\cdot)$ represents different state dynamics and over a finite horizon, many different state trajectories are possible depending on the future values of $\omega_i(\cdot)$. It may be noted here that no assumption on the probability distribution of the error is made.

For evaluating the expected cost in Eq. (3b) over a finite horizon, scenario based stochastic model predictive control [28] is used where a scenario is defined by a sequence of disturbance realizations $\{\omega_i(\cdot)\}$ over the given horizon. The number of such scenarios $n_{sc}$ would be exponential to the discrete time horizon.
$M$. The scenarios can be represented by a scenario tree (each path in the scenario tree represents a scenario) where the root node is the current $\omega_i(k)$ and the leaf nodes are the disturbance states reached at the end of discrete time horizon $M$. Fig. 3 shows the possible different scenarios when $|W|$ is 3.

![Scenario tree with all possible scenarios](image)

From the scenario tree, the cost function in Eq. (3b) of the optimization problem in Eq. (3) can now be represented as:

$$J_i(k) = \sum_{l=1}^{n_{sc}} \pi_l \sum_{t=k}^{k+M-1} \left[ c_1 \frac{\dot{w}l_i(t) \Delta t}{v_i(t) \Delta t} + c_2(t)R_{ij}(t)^2 
+ c_3(v_i(t) - v_{target}(k))^2 + c_4 u_i(t)^2 \right]$$

(6)

Here, $n_{sc}$ is the total number of scenarios and $\pi_l$ is the probability of the occurrence of scenario (path) $l$ which is evaluated by the product of the probability values associated with all the edges in the path $l$. Since $n_{sc} = |W|^M$, computation cost for solving the cost in Eq. (6) would be huge and intractable. To make the problem more computationally tractable, we use a sampling based method to generate a scenario tree where only the paths with higher probability of occurrence will be considered. The sampling based scenario tree generation method is explained in Algorithm [1] where $N_{max} < n_{sc}$ is the upper bound on the number of scenarios considered, $SC_l\{\cdot\} = \mathbb{R}^M$ contains the set of nodes in a scenario $l$, and $S$ is the set of such $SC_l\{\cdot\}$ whose probability of occurrence is more than a predefined threshold $p_{th}$. Fig. 4 shows the pruned scenario tree along with the probability of occurrence of any path $l$. 

![Pruned scenario tree](image)
**Input**: Current state of \( \omega_i(k) \): \( a \) with probability 1, transition probability matrix \( Q_i \), threshold \( p_{th} \), and \( M = \frac{T}{\Delta t} \)

**Output**: Set of scenarios: \( S = \{ SC_i \} \) and their corresponding probability of occurrence \( \pi_i \)

Initialize \( S = \emptyset \);

for \( l = 1 \) to \( N_{max} \)

| Initialize \( SC_l\{1\} = a \), \( \pi_l = 1 \);
| for \( t_a = 1 \) to \( M \)
| Choose \( SC_l\{t_a + 1\} = b \) with probability \( Q_i(SC_l(t_a), b) \);
| \( \pi_l = \pi_l \times Q_i(SC_l(t_a), b) \);
| if \( \pi_l \geq p_{th} \) then
| \( S = \{ S, SC_l\cdot \} \)
| end
| end

**Algorithm 1**: Sampling based scenario tree generation

Now, the problem in Eq. (3) can be expressed as

\[
\min_{u_i} \sum_{l=1}^{\left| S \right|} \pi_l \sum_{t=k}^{k+M-1} \left[ c_1 \frac{F_i u_i(t) \Delta t}{v_i(t) \Delta t} + c_2(t) R_{ij}(t)^2 \right] + c_3(v_i(t) - v_{target}(k))^2 + \bar{c}_4(t) u_i(t)^2
\]

\[ (7a) \]

\[
\bar{c}_4(t) = \frac{c_4}{(1 + \alpha t)}
\]

\[ (7b) \]

\[ v_{min} \leq v_i(t) \leq v_{max} \]

\[ (7c) \]

\[ u_{min} \leq u_i(t) \leq u_{max} \]

\[ (7d) \]

In the above problem, we have replaced \( n_{sc} \) in Eq. (6) with \( |S| < n_{sc} \) and we have changed the constant weight \( c_4 \) to a time varying weight \( \bar{c}_4(t) \) given by Eq. (7b). By using such a time varying weight, we are discounting the future control efforts and penalizing the ones closer to the current time instant. This discounted cost is common in many different algorithms such as in reinforcement learning. It may be noted that by only summing over the number of scenarios \( |S| \) whose probability of occurrence is higher than \( p_{th} \), we reduce the computational cost by a good amount. Thus the driver assistance system solves the problem in Eq. (7) with the modified cost in a receding horizon fashion.

**4 SIMULATION RESULTS**

In this section, we present the simulation results of the methodology explained in the previous Section 3. We consider a simulation scenario consisting of a single lane road with traffic lights at every 500m. Each vehicle implements a
scenario based stochastic model predictive controller by solving problem in Eq. (7) so that: i) each vehicle improves its fuel efficiency, ii) avoids rear end collision with the preceding vehicle, iii) reduces red light idling, and iv) Addresses error introduced by human driver.

We have considered $|W| = 5$, so that the transition probability matrix is $Q_i \in \mathbb{R}^{5 \times 5}$. The transition probability matrix captures the driver behavior. For example, if the error is positive, an aggressive driver might overcompensate and make the error negative rather than zero. Ideally, $W$ and $Q_i$ needs to be generated and updated analyzing a driver’s ability in tracking commands from the driver assistance system. Since experiments on real vehicles are beyond the scope of this paper, we assume the data already exists to evaluate our proposed method. We consider two simulation scenarios in this paper: (i) $W$ ranges from $-0.3$ to $0.3$, and (ii) $W$ ranges from $-0.5$ to $0.5$.

First, we compare our proposed method with the ideal case where the driver induced error is zero and then with a passive controller (baseline method) where the driver assistance system does not address the driver induced error (considers the driver follows its instructions exactly). In both ideal and the baseline cases, the controller solves the problem in Eq. (3), without the expectation, using model predictive control. For the ideal case, $\omega_i(k) = 0, \forall k$ while for the baseline case, $\omega_i(k)$ follows the Markov chain but the method assumes it to be zero.

We then compare our proposed method with two optimal control methods for stochastic systems: certainty equivalence control [41] and frozen time model predictive control [28,36]. In certainty equivalence optimal control problem, the future random error realizations are assumed to be known and often replaced by the expected error. In frozen time model predictive control, the future error realizations (over the horizon) are considered to be constant and equal to the initial error. A number of papers (including our previous works) have already shown how avoiding red light idling, using V2V and V2I information, and developing controls in a model predictive framework improves vehicle fuel efficiency. Thus we are not comparing our method with the traditional car following models.
For both the scenarios, we run the simulation for 400 seconds. For simulation, most of the parameters are taken from [9] and [10]. The vehicles are considered to be identical with $A_i = 2.5 \text{ m}^2, M_i = 1200 \text{ kg}$ and $u_{i,\text{max}} = 2 \text{ m/s}^2, \forall i$. The other parameters are given by: $C_D = 0.32, \rho_a = 1.184 \text{ kg/m}^3$, and $\theta = 0$ degree. The parameters used in fuel consumption model are: $b_0 = 0.1569, b_1 = 2.450 \times 10^{-2}, b_2 = -7.415 \times 10^{-4}, b_3 = 5.975 \times 10^{-5}, c_0 = 0.07224, c_1 = 9.681 \times 10^{-2}, c_2 = 1.075 \times 10^{-3}$ and $F_i^d = 0.1, \forall i$[9]. To emulate urban road conditions, the red and green light intervals are sampled from a uniform distribution with range 37 to 43 and 12 to 17 seconds respectively for each cycle of every traffic signal. The maximum ($v_{\text{max}}$) and minimum ($v_{\text{min}}$) allowable velocities are considered to be 20 m/s and 0 m/s respectively. For simulation purposes, time horizon is chosen as $T = 5$ secs with time step $\Delta t = 0.5$ secs (thus $M = 10$) and $n = 3$ vehicles are considered.

**Scenario 1:** Here, $\omega_i(\cdot)$ is considered to take discrete values $[-0.3 \quad 0 \quad 0.15 \quad 0.3], \forall i$. Although $\omega_i(\cdot)$ is discretized into 5 discrete values for solving the stochastic model predictive control problem, the actual driver error would take any value in $-0.3 \leq \omega_i(\cdot) \leq 0.3$.

The state transition matrix considered in this paper is given by Eq. (8).

$$ Q_i = \begin{bmatrix} 0.10 & 0.30 & 0.45 & 0.10 & 0.05 \\ 0.05 & 0.25 & 0.45 & 0.20 & 0.05 \\ 0.01 & 0.10 & 0.78 & 0.10 & 0.01 \\ 0.05 & 0.15 & 0.45 & 0.25 & 0.10 \\ 0.05 & 0.15 & 0.45 & 0.30 & 0.05 \end{bmatrix} \quad (8) $$

Fig. 5 shows the trajectories of the three vehicles using our proposed method and how they all avoid red light idling. The red bars in Fig. 5 are the red light intervals and the blank spaces are the green light intervals so that the vehicles need to pass a traffic light through the blank spaces. The vehicle trajectories of all the 3 vehicles look similar since they end up following each other in a fuel economic manner and no collision takes place. Fig. 6 shows the comparison between the velocity profiles for the 3 vehicles in the ideal case, when using our proposed method, and for the passive controller. It can be seen from Fig. 6 that the velocity profiles generated following our proposed method are much closer to the ones in the ideal case. This is reflected in the fuel economy of the vehicles, as it can be seen from Tab. 1 that the fuel economy of the vehicles following our proposed method are much closer to the fuel economy in the ideal case.

| Vehicle No. | Ideal Case | Passive Controller | Proposed Method |
|-------------|------------|--------------------|-----------------|
| 1           | 41.52      | 37.51              | 40.65           |
| 2           | 40.58      | 37.56              | 39.65           |
| 3           | 40.08      | 36.78              | 39.32           |
We next compare our proposed method with two different optimal control methods for uncertain systems. Since the system is stochastic in nature, different simulation runs with the same initial conditions would give different results. That is why we ran 30 different simulation runs to compare our proposed method.
method with certainty equivalence optimal control and frozen time model predictive control. The results are shown in Fig. 7. It can be seen from the Fig. 7 that certainty equivalence optimal control performs better than frozen time model predictive control while our proposed method outperforms the other two methods.

**Scenario 2:** Here, $\omega_i$ is considered to take discrete values $[-0.5, -0.2, 0.0, 0.2, 0.5]$, $\forall i$, while $Q_i$ is given by Eq. (8). Fig. 8 shows the comparison between the velocity profiles for the 3 vehicles in the ideal case, when using our proposed method, and for the passive controller (baseline case). Similar to the previous case, it can be seen from Fig. 8 that the velocity profiles generated following our proposed method are much closer to the ones in the ideal case. This is reflected in the fuel economy of the vehicles, as it can be seen from Tab. 2 that the fuel economy of the vehicles following our proposed method are much closer to the fuel economy in the ideal case.

Table 2: Individual vehicle fuel economy (mpg) after 400 seconds for Scenario 2

| Vehicle No. | Ideal Case | Passive Controller | Proposed Method |
|-------------|------------|-------------------|-----------------|
| 1           | 41.52      | 36.90             | 40.28           |
| 2           | 40.58      | 36.35             | 39.38           |
| 3           | 40.08      | 36.17             | 39.12           |

5 CONCLUSIONS

In this paper, we present a driver-in-the-loop fuel efficient control strategy for a group of CVs in urban road conditions. The control strategy works as a driver assistance system where the driver behavior and its capability is considered in controller development. Anticipation of driver behavior improves the system performance as shown by the simulation results. The strategy works in a decentralized manner as every vehicle develops its own control strategy using neighborhood V2V and V2I information. Utilization of SPAT information via V2I communication improves system mobility and in the process reduces greenhouse emissions. Some of the future research directions include improvement of computational requirement in solving the scenario based stochastic model predictive control problem and consideration of network delay and communication loss in the development of control strategy for the CVs.
Figure 6: Comparison of velocity profiles between ideal case, our proposed method, and the baseline case (passive controller) for Scenario 1
Figure 7: Comparison of miles per gallon for 30 cases when using our proposed method, certainty equivalence control, and frozen time model predictive control for Scenario 1.
Figure 8: Comparison of velocity profiles between ideal case, our proposed method, and the baseline case (passive controller) for Scenario 2
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