A Fourier series method for solving the two-stage vibration isolation system under dual-wave shock input

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Abstract. Shock isolation for on-board devices is one of the most important issues for underwater vehicles to resist underwater non-contact explosions (UNDEX). According to German military specification BV043/85, the shock input spectrum can be transformed into a time domain signal represented as a dual-wave, which is composed of a positive and a negative half-sine. Previous researches have pointed out that two-stage vibration isolation system (TSVIS) shows better isolation performance than the single-stage vibration isolation system (SSVIS). However, there is relatively rare research on the shock isolation performance of a TSVIS subjected to a dual-wave shock input. This issue is addressed in this investigation. The Fourier series method is employed to calculate the response in the time domain. This study will shed some light on the design of TSVIS in a dual-wave environment.

1. Introduction

The concept of shock is widely encountered in everyday life, and its harm is also well known. Being as the combat platform of warring sides on sea, naval vessels have to face with threat of the shock derived from attack explosion. Therefore, continuously improving the protective performance of the naval vessels to resist underwater non-contact explosions comes to be a common issue.

Many researchers have done a lot of works on the two-stage vibration isolation system, and plenty of significant conclusions are presented[1]. Harris obtained the analytical solution of the two-stage vibration isolation system excited by step velocity[2]. Yan studied the isolation properties of the two-stage vibration isolation system in the condition of two types of acceleration induced by the nuclear explosion[3]. In N. Chandra Shekhar’s paper, three different types of shock inputs were considered as the base motion to be isolated, and three different indices were used to judge the overall performance characteristics of the isolator[4].

When the shock input is a dual-wave, there usually are two kinds of method which one of them is spectrum analysis method in frequency domain, another is numerical simulating method in time domain. This paper presents an approach to obtain the key response of the system under double-wave in time domain which can improve efficiency.
2. The mathematical formulation and solution

The dynamic model of a classical two-stage vibration isolation system is set up in Fig 1, and its equation of motion can be easily written as Equation (1).

\[
\begin{align*}
    m_1 \ddot{x}_1 + c_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) &= 0 \\
    m_2 \ddot{x}_2 + c_1 (\dot{x}_2 - \dot{x}_1) + k_1 (x_2 - x_1) + c_2 (\dot{x}_2 - \dot{u}) + k_2 (x_2 - u) &= 0
\end{align*}
\]

(1)

Where \( m_1, m_2 \) are the loaded mass and the internal mass respectively. \( x_1, x_2 \) and \( u \) are the displacement of the loaded mass, the internal mass and the base respectively. \( k_1, c_1 \) are the stiffness and damping between the loaded mass and internal mass respectively. \( k_2, c_2 \) are the stiffness and damping between the loaded mass and internal mass respectively.

Equation (1) can also be written in matrix form as following formula

\[
M \ddot{X} + C \dot{X} + KX = F
\]

(2)

During the impact resistance design of on-board devices, the shock input is a typical triple-fold line spectrum which is presented in Fig 2. According to German military specification BV043/85, the shock input can be equivalent to a time domain signal represented as a double sine wave which is shown in Fig 3. The double sine wave is composed of a positive and a negative half sine. The amplitude of the positive half sine which is obtained by shock response spectrum (SRS) is about half the maximum acceleration \( a_0 \) (spectrum acceleration). The area of each half sine is also determined by SRS, and the value is about 2/3 of the maximum velocity \( v_0 \) (spectrum velocity). Make twice integral on the acceleration history, the maximum displacement \( d_0 \) (spectrum displacement) can be obtained.

The mathematical relationship of the main parameters are shown as

\[
A_1 = 0.5a_0, \quad V_1 = V_2 = \frac{2}{3}v_0, \quad \tau_1 = \frac{\pi V_1}{2a_2}, \quad \tau_2 = \frac{2d_0}{V_1} - \tau_1, \quad A_2 = \frac{\pi V_1}{2\tau_2}
\]

(3)

Where \( A_1 \) and \( A_2 \) are, respectively, the amplitude of the positive and negative half sine wave. \( \tau_1 \) and \( \tau_2 \) are, respectively, the time span of the positive and negative half sine wave.

Figure 2. Typical triple-fold line shock input spectrum
As known that \( A_1 \tau_1 + A_2 \tau_2 = 0 \), the acceleration, velocity, displacement of the base.

The Fourier expansions of \( u(t) \) and \( \dot{u}(t) \) can be written as follows, whose cycle is \( 2l \):

\[
\begin{align*}
  u(t) &= \frac{2A_1 \tau_1}{\pi l} \sum_{n=1}^{\infty} \left( P_1 \sin \left( \frac{n\pi \tau_1}{l} \right) + P_2 \sin \left( \frac{n\pi (\tau_1 + \tau_2)}{l} \right) - P_3 \cos (n\pi) \sin \frac{n\pi t}{l} \right) \\
  \dot{u}(t) &= \frac{2A_1 \tau_1}{\pi l} \sum_{n=1}^{\infty} \left( Q_1 \cos \left( \frac{n\pi \tau_1}{l} \right) + Q_2 \cos \left( \frac{n\pi (\tau_1 + \tau_2)}{l} \right) + Q_3 \sin \frac{n\pi t}{l} \right)
\end{align*}
\]  

Where

\[
\begin{align*}
P_1 &= \frac{l^4 (\tau_2^2 - \tau_1^2)}{\pi^2 (l^2 - n^2 \tau_1^2)(l^2 - n^2 \tau_2^2)},
P_2 &= \frac{l^4}{n^2 \pi^2 (l^2 - n^2 \tau_2^2)},
P_3 &= \frac{(\tau_1 + \tau_2) l}{n\pi},
Q_1 &= \frac{nl^3 (\tau_1^2 - \tau_2^2)}{\pi (l^2 - n^2 \tau_1^2)(l^2 - n^2 \tau_2^2)},
Q_2 &= \frac{-l^3}{n\pi (l^2 - n^2 \tau_2^2)},
Q_3 &= \frac{l^3}{n\pi (l^2 - n^2 \tau_1^2)}
\end{align*}
\]

Substituting Eq. (4) and Eq. (5) into Eq. (1), the solution of the above dynamic equations can be change into a new issue which superposes a series of two-stage system under harmonic excitation. The equation is presented as

\[
M \ddot{x} + C \dot{x} + K x = \frac{2A_1 \tau_1}{\pi l} (k_2 O_1 + c_2 O_2)
\]  

Where

\[
O_1 = \left[ \sum_{n=1}^{\infty} \left( P_1 \sin \left( \frac{n\pi \tau_1}{l} \right) + P_2 \sin \left( \frac{n\pi (\tau_1 + \tau_2)}{l} \right) - P_3 \cos (n\pi) \sin \frac{n\pi t}{l} \right) \right]
\]

\[
O_2 = \left[ \sum_{n=1}^{\infty} \left( Q_1 \cos \left( \frac{n\pi \tau_1}{l} \right) + Q_2 \cos \left( \frac{n\pi (\tau_1 + \tau_2)}{l} \right) + Q_3 \sin \frac{n\pi t}{l} \right) \right]
\]

When a two-stage vibration system excited by a sine wave

\[
M \ddot{x} + C \dot{x} + K x = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \sin \omega t
\]  

Its solution can be expressed as a form like

\[
y = U a - V b + W \gamma + g \cos \omega t + h \sin \omega t
\]

\(y\) is the solution when the equations have been made canonical transformation. And every vector can be obtained through literature [5]. We can see that the solution can be divided into three parts. The
first part is the free vibration created by initial condition, the second part is the free vibration created by forced excitation, the third part is the steady-state vibration created by forced excitation.

In this paper, \( f_1 = 0 \) thus, we can finally get the solution of Eq. (6), like the following

\[
X = \sum_{n=1}^{N} (a_n \mathbf{u} - V_n \mathbf{b} + W_n \gamma_n + g_n \cos \omega_n t + h_n \sin \omega_n t)
\]  

(9)

Where \( \omega_n = \frac{n\pi}{l} \)

3. Numerical verification

The solution of Eq. (1) has been obtained by using Fourier expansion method. Since this problem can also be solved by other methods such as Runge-Kutta method, in this section, an example is presented here to illustrate the application and the correction of the method. Parameters involved in the illustration are listed in Table 1. And \( a_0 = 1250 \text{m/s}^2 \), \( v_0 = 1.2 \text{m/s} \), \( d_0 = 0.02 \text{m} \), \( l = 10(\tau_1 + \tau_2) \), \( \xi_1 = c_1 / 2 \sqrt{k_1 m_1} \), \( \xi_2 = c_2 / 2 \sqrt{k_2 m_2} \).

| \( m_1 (t) \) | \( m_2 (t) \) | \( k_1 (N/m) \) | \( k_2 (N/m) \) | \( \xi_1 \) | \( \xi_2 \) |
|-----------|-----------|-------------|-------------|--------|--------|
| 50        | 25        | 1e9         | 1e8         | 0.1    | 0.1    |

Figure 4 shows that very good agreement has been obtained compared with the results by Runge-Kutta method in a Fourier cycle \( l \). When the Fourier cycle is chosen appropriately, the peaks of the response such as the displacements can be always obtained.

4. Conclusion

In this paper, an approach has been developed to solve the system dynamic equation and get the peak responses of the system. These works can provide some idea when engineers design this kind of shock isolation system.

Acknowledgements

This work was supported by the State Key Laboratory of Mechanical System and Vibration (Grant No. MSV202002), Open Fund of Defense Key Disciplines Laboratory of Ship Equipment Noise and Vibration Control Technology (VSN201801), Shandong Provincial Natural Science Foundation (ZR2018MC017), Funds of national key research and development for 13th year plan
(2018YFD0700604), Innovation team fund for fruit industry of modern agricultural technology system in Shandong Province (SDAIT-06-12, SDAIT-06-1), Research project-2017 on intelligent agricultural mechanization equipment of Shandong Province (2017YF003), and Funds of Shandong “Double Tops” programs (SYL2017XTTD07).

References
[1] Jacobsen L S 1966 Study of shock isolation for hardened structures Department of the army 6 639303
[2] Harris C M 2002 Harris shock and vibration handbook New York: McGraw-Hill 5 1025-1083
[3] Yan D, Tang D and Qian Q 2001 Isolation properties of double mass-spring system under blast shock and vibration Journal of Vibration Engineering 14 340-344
[4] Shekhar N C, Hatwal H and Mallik A K 1999 Performance of non-linear isolators and absorbers to shock excitations Journal of Sound and Vibration 227 293-307
[5] Qin J and Tang J 2006 Exact solution of non-classical damping system Journal of Dynamics and Control 4 136-144