Materials Research Express

PAPER

Application of non-contact strain measurement based on CCD camera in PMMA material constitutive model

Xiaohui Qian1,3, Xiaoyan Ma1,3,∗, Yuekun Heng1,2,3, Wei He1, Hongqiang Tang4, Shaojing Hou1,2 and Yatian Pei1,2
1 Institute of high energy physics, Chinese Academy of Sciences, Beijing, People’s Republic of China
2 University of Chinese Academy of Science, Beijing, People’s Republic of China
3 State Key Laboratory of Particle Detection and Electronics, Beijing, People’s Republic of China
4 Jiangsu Donchamp New Materials Technology Co., Ltd., Taixing, People’s Republic of China
∗ Author to whom any correspondence should be addressed.
E-mail: maxy@ihep.ac.cn

Keywords: JUNO, non-contact strain measurement, PMMA, bonding area, constitutive model

Abstract

The Jiangmen Underground Neutrino Observation (JUNO) will build a polymethyl methacrylate (PMMA) spherical vessel of a diameter of 35.4 meters. The constitutive model of PMMA is a key parameter for the design of PMMA structure. The bulk polymerization bonding area is often the weak point of the PMMA structure, so it is useful to understand the constitutive model of bonding areas for FEA. However, the traditional contact strain measurement methods, such as the strain extensometer and resistance strain gauge, will have an impact on the strain of PMMA, as the lossy measurement. Traditional measuring methods also can’t measure the small-sized bonding areas as 3 mm in most structures. The non-contact strain measurement method based on the CCD camera is studied. The tensile test result shows that the influence of environment on the strain value is less than 0.01%. The two strain measurement methods, the CCD camera and strain extensometer, are compared. The strain curves obtained by the two methods are highly consistent, and the maximum strain difference is 8.53 e-4. The fracture strain of the PMMA tensile specimen is 4.32% and slight plastic deformation has occurred. The Zhu-Wang-Tang (ZWT) nonlinear viscoelastic constitutive model of PMMA is obtained by fitting the stress-strain data. The tensile test result of PMMA specimen with bulk polymerization bonding area shows that the constitutive equations are different when the length ratio of the bonding area is different. By analyzing the relationship between the length ratio and the coefficients of constitutive equation, the constitutive equation of the bonding area is finally obtained. The results show that the coefficient $E_0$ of the constitutive equation of the bonding area is smaller than that of the mother material. The fracture strain of PMMA specimen with bonding area is 2.60%, lower than that of mother material, which makes the coefficient $\beta$ of the bonding area constitutive equation opposite to the sign of the mother material. The tensile strength of specimen with bonding area is about 85.68% of that of the mother material. The lower tensile strength makes the bonding area become one of the weak points of the structure.

1. Introduction

For the mechanical testing of materials, strain measurement is very important to determine some parameters such as the elastic modulus, bending modulus, and Poisson’s ratio. When carrying out strain tests, it is necessary to ensure that the measurement method or testing tool will not affect the properties of the material itself, or the effect should be controllable. The most common methods used for strain measurement are mainly contact methods such as strain extensometer and resistance strain gauge, both of them can meet the mechanical test requirements of most materials. The strain extensometer [1] is a standard measuring tool for measuring the...
relative deformation (strain) between two points of the specimen, which is convenient to use, but it also has some limitations. For a certain type of extensometer, the gauge distance is fixed, and when the length of gauge distance of specimen is less than the gauge distance of extensometer, strain measurement cannot be carried out. When the specimen yields, the extensometer must be removed, otherwise it is easy to cause damage to the extensometer. Therefore, the appropriate extensometers should be selected for strain measurement according to the characteristics of different materials. Since the strain extensometer fastens two knife edges on the specimen during strain measurement and affects the strain or strength of materials, this kind of method is not suitable for some materials such as rubber, film, and brittle plastic. There are also some limitations in using resistance strain gauges for strain measurement. Strain gauges pasted on the sample surface may bring damage to the material or change the properties of materials, which will also affect the measurement result. For example, when measuring the brittle PMMA, the fracture point often occurs at the area of strain gauges, as shown in figure 1. Another issue of the strain gauges is they can’t measure the ultra-large deformation materials such as nylon films. Finally, when measuring, the strain gauge needs to be welded with signal wires and pasted on the surface of the test piece, the operation is complicated and the gauge consumption is large.

With the development of computer image recognition and processing technology, non-contact strain measurement based on digital image processing has been widely used [2, 3], and commercial products are available. Through the non-contact strain measurement method, the gauge distance and measuring range of the specimen are not limited, and it is especially suitable for strain measurement of super-conventional scale (such as 3 mm-length PMMA bonding area) and soft materials (such as rubber, plastic, fiber, etc). Compared with traditional measurement methods, non-contact measurement can effectively measure the strain of materials without damage to the tested products [2]. These advantages of non-contact measurement are more obvious for brittle materials. According to the test of PMMA in figure 1, the PMMA shows the property of brittle fracture that it has little plastic deformation after fracture. Contact measurement may cause the damage to the specimen, and the brittle materials are more sensitive to notches, thus affecting the test results.

The Jiangmen Underground Neutrino Observation (JUNO) will build a PMMA spherical vessel of a diameter of 35.4 m [4–6]. The constitutive model and other mechanical properties of PMMA are very important for the design of this huge vessel, the quality control of PMMA panels, and the long-term running state of detectors. A lot of research has been carried on PMMA materials, such as its process ability [7], creep and aging properties in special liquid [8, 9] and other mechanical properties at different temperatures [10–13]. However, there are few literatures about the properties of PMMA with bulk polymerization bonding area. The PMMA vessel of JUNO needs to be assembled and bonded on-site for the limitation of shipping. To improve the strength of the bonding area, the bulk polymerization bonding method is usually used. For this bonding process, a cavity with a length of about 3 mm between two PMMA panels needs to be made to fill MMA cement. When the MMA is polymerized, the material at the bonding area has the same chemical formula as the panels [14]. However, because the polymerization condition (Polymerization temperature and annealing temperature, etc.) are different, characteristics are different between the bonding area and the mother panels [15]. Bonding area always becomes one of the weak points of structures for the lower strength than mother material [16]. In structural design, it is necessary and useful to understand the constitutive model of the material, so that the stress and strain of the structure can be simulated more accurately.

In this paper, the strain of PMMA is measured in a non-contact method based on a CCD camera, and the test results are compared with the strain extensometer. According to the measurement results of tensile stress and strain of PMMA, the ZWT model is applied to fit the function of the PMMA constitutive model. The tensile test of PMMA material with bulk polymerization bonding area is carried out too, and the constitutive equation of the material at 3 mm bonding area is obtained by fitting and deducing the constitutive parameters.
2. Principle of strain measurement method based on CCD camera

The working principle of non-contact strain measurement based on the CCD camera is shown in figure 2. Firstly, the measurement points need to be marked on the surface of the specimen, and then the digital images of the mark points are acquired in real time by the CCD camera. The pixel coordinates of the mark points in the images are determined by image processing, and then the strain values can be calculated according to the coordinates of the mark points at different times.

In figure 3, the distance between mark points A and B is defined as $L$. The initial distance is $L_0$. After stretching the sample, the distance becomes $L_n$, and $n$ represents the serial number of the shot image. The strain could be calculated by the following equation.

$$
\varepsilon_n = \frac{(L_n - L_0)}{L_0} \quad n = 1, 2, 3...
$$

(1)

Since the strain is dimensionless, the $L$, $L_0$ and $L_n$ in equation (1) can be the pixel distance on the picture. The pixel coordinate position of the mark point can be determined by the gray gravity center method [17]. Figure 4 shows a binary image of the mark point. The gray value calculation formula of the binary image is as follows:

$$
f_{ij} = \begin{cases} 
0, & \text{Greyscale } e < T_i, j \in S \\
1, & \text{Greyscale } e \geq T_i, j \in S 
\end{cases}
$$

(2)

In this formula, $S$ is a pixel of the $M \times N$ matrix, and $T$ is a threshold constant between 0 and 1. When the gray value is less than the threshold constant, the picture gray value is forcibly set to 0. Otherwise, when the gray value is greater than the threshold constant, the gray value is forcibly set to 1. Therefore, the coordinates of the
marked points in figure 4 can be calculated by the following formula:

\[
\begin{align*}
    x_n &= \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} f_{ij}}{\sum_{i=1}^{M} \sum_{j=1}^{N} f_{ij}}, \\
    y_n &= \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} y_{ij} f_{ij}}{\sum_{i=1}^{M} \sum_{j=1}^{N} f_{ij}}
\end{align*}
\]

(3)

It can be seen from equation (3) that the coordinates are calculated according to the center mass of the pixel of the mark point, so square, triangle or even irregular shape can be used as the mark point.

Figure 5 is a flow chart of non-contact strain measurement based on the CCD camera. Before testing, mark points should be made on the test piece, which should have a large contrast with the color of the specimen as far as possible, and there is no special requirement on the shape of the mark points. Then the CCD camera is connected with the computer, and focused to the mark point. Image acquisition and storage are carried out in real time during the test. In order to reduce the influence of environment on measurement, such as illumination change and wind, the test is conducted in the laboratory. The images are stored for post-processing during the test. The data processing will calculate the coordinates of the mark points and convert them into strain according to equation (1). The coordinates and strains are also stored in real time. During the whole test process, it will also judge whether the test is finished or not in real time. When the specimen is broken, the strain or coordinates will change suddenly, so they can be used as the indicator of test finished or not.

3. Tensile strain test of PMMA

Figure 6 is a tensile stress-strain testing device. The PMMA specimen is clamped by the upper and lower clamp heads of the stretching machine. The PMMA specimen is made according to ASTM D638–02a plastic tensile test standard [18], and its dimensions are shown in figure 7. The full length of the specimen is 200 mm, the thickness is 14 mm, the gauge width is 19 mm, and the gauge length is 57 mm. Within the range of standard distance, some
points are marked in the form of an equidistant array, as shown in figure 8. The strain is measured by a strain extensometer located on the right side of the specimen. The strain measuring distance of the extensometer is 50 mm. The CCD camera is hung on the fixed beam at the top of the stretcher, and the distance from the specimen is about 150 mm. The resolution of camera in the width × length is 3000 × 4000 pixels and the SNR (signal noise ratio) is more than 38 dB. During testing PMMA material, the lower fixture head moves downward at a speed of 5 mm min⁻¹.

Figure 8(a) shows the mark points of the specimen. The distance between two adjacent mark points is 6 mm in the vertical direction and 7 mm in the horizontal direction. The array of mark points can be drawn by the standard module shown in figure 9. The large black area in figure 8(a) is the shadow of the extensometer. The
image binarization results are shown in figure 8(b). Four mark points in the gauge section of the specimen are selected for strain calculation. During the tensile test, the specimen moves down, and the selected mark points cannot exceed the camera’s visual field. The connecting line between points P1 and P2 is in the y axis direction, which is the same as the stretching direction of the material. The line between points P1 and P3 is in the direction of the x-axis. Two groups of strain in x and y directions can be obtained simultaneously through four mark points. The shooting field width shown in figure 8 is about 20 mm. According to the pixels and field size of the CCD camera, it is estimated that the measurement accuracy of the camera can be higher than 20 mm/3000 = 6.7 μm, and the strain value is better than 6.7 um/14 mm = 0.05%.

Figure 10 shows the strain measured by the CCD camera in 2D. Before the tensile test, the camera begins to collect images and the strain values in x and y directions are obtained, which can reflect the measurement data errors caused by the measurement system and environmental influences. From the strain curve from 0 to 100 s, it can be found that the strain value changes within 0.01%, which indicates that the measuring system is less affected by environmental factors. After the tensile test, the strain curves between Y1 and Y2 are highly coincident, and the same happens between strain X1 and strain X2, which shows that the selection of mark points has little influence on strain results.

Figure 11 is a comparison of the y-direction strain measured by the CCD camera and strain extensometer. From the curve, the strain values obtained by the two measurement methods are highly coincident. Figure 12 shows the strain difference measured by two measurement methods. From the curve, it can be seen that the difference between them increases with the strain. This is mainly due to the change of environmental conditions during the movement of the mark points, and the non-perpendicularity between the axis of the CCD camera and the surface of the marking points. However, the maximum strain difference caused by the superposition of these two factors is 8.53 e-4, the relative error is 1.91%. Therefore, it can be considered that the difference between the two measurement methods is very small.

By processing the strain of P1 ~ P4 mark points in the x direction, the strain relationship curve of material in 2D can be obtained as shown in figure 13.
4. Constitutive model of PMMA

At room temperature, PMMA shows nonlinear viscoelastic constitutive relation \([19, 20]\), which can be described by the following ZWT model \([21]\).
Where \( \sigma \) and \( \varepsilon \) are true stress and true strain respectively, and the relationship between true stress-strain and nominal stress-strain is as follows:

\[
\sigma = E_0 \varepsilon + \alpha \varepsilon^2 + \beta \varepsilon^3 + E_1 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t - \tau}{\theta_1}\right) d\tau + E_2 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t - \tau}{\theta_2}\right) d\tau.
\]

(4)

In equation (4), \( E_0, \alpha, \beta \) are nonlinear elastic constants, \( \dot{\varepsilon} \) is strain rate, \( E_1 \) and \( E_2 \) are elastic constants, \( t \) is time, and \( \tau \) is relaxation time. In this equation, \( E_0 \dot{\varepsilon} + \alpha \varepsilon^2 + \beta \varepsilon^3 \) mainly describes the nonlinear elastic characteristics of materials, \( E_1 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t - \tau}{\theta_1}\right) d\tau \) and \( E_2 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t - \tau}{\theta_2}\right) d\tau \) are the viscoelastic responses of materials under low and high strain rate respectively.

The tensile test rate of the material is 5 mm min\(^{-1}\). It is a quasi-static tensile process. Therefore, the ZWT model in equation (4) can be simplified as a nonlinear elastic model:

\[
\sigma = E_0 \varepsilon + \alpha \varepsilon^2 + \beta \varepsilon^3
\]

(7)

### 4.1. Constitutive model of PMMA without bonding area

Figure 14 shows the true stress-strain curve of PMMA material measured by the CCD camera. The tensile strength of the material is 69.84 MPa and the fracture strain is 4.32%. The true stress-strain curve is fitted by equation (7), and the red solid curve in the figure is obtained. It can be seen from this figure that the test data consistent with the fitting curve and the correlation coefficient of the fitting function reaches 0.9999, which is very close to 1, indicating that the fitting result is very good. The stress-strain constitutive relation of the reduced ZWT model of PMMA is as follows:

\[
\sigma = 3.46e3 \varepsilon - 5.05e4 \varepsilon^2 + 1.72e5 \varepsilon^3
\]

(8)

### 4.2. Constitutive model of PMMA with bonding area

Figure 15 shows PMMA specimen with bonding area in the middle of the specimen under polarized light. There is a great contrast between the color of the bonding area and the mother panel under the polarized light, which indicates that the properties between them will be different due to the different polymerization conditions, resulting in the material discontinuity. The thickness of specimen is 14 mm and the width of the gauge length of the specimen is 19 mm, as shown in figure 7. The dimension of bonding area is 14 mm \(\times\) 19 mm \(\times\) 3 mm.

Before testing PMMA specimen with bonding area, mark the specimen with standard block, the distance between the mark points along the stretching direction is 6 mm, and the length of bonding area is 3 mm. By selecting four mark points P1-P4 as strain measurement marks as shown in figure 16, the strain of P1P2, P2P3, P1P3 and P1P4 can be obtained. The length ratio of the bonding area to the measured gauge section is 0, 3/6, 3/12 and 3/18, respectively.
Figure 17 shows the fitting of the strain in each measurement section by the ZWT reduced model. Table 1 gives the coefficients of the fitting equation for each measuring section. The correlation coefficient between each fitting function and test data is 0.9999. The true stress-strain curves of each measuring section are very close, and if the curves are enlarged locally, shown in figure 17, it can be found that there are some differences among the fitting curves. In addition, the fitting curves are regularly distributed in the length ratio of the bonding area. The strain of the P1P2 section without bonding area is at the top, followed by the P1P4 section, P1P3 section and P2P3 section. The P1P2 section has no bonding area, and the difference of parameter between the P1P2 section
and PMMA bulk panel in equation (8) is 4.6%, which is mainly caused by the difference of test specimens. In equation (8), the coefficient of constitutive model of PMMA without bonding area is positive, but the coefficient of constitutive model of PMMA specimen with bonding area is negative. Comparing the curves of figures 14 and 17, it can be seen that the fracture strain of PMMA without bonding area is 4.32%, and the material has slight plastic deformation after fracture. However, the fracture strain of the specimen with bonding area is 2.60%, and there is no obvious plastic deformation.

In table 1, the relationship between the length ratio of bonding area and each parameter is fitted by quadratic curve, and the fitting results are shown in figures 18–20. It can be seen that the relationship between the length ratio of bonding area and parameters and shows a monotonic decreasing trend, while the relationship between the length ratio and parameter shows a monotonic increasing trend. When fitting with the quadratic function, the function has three unknown parameters. There are four groups of data, so the least square method can be used for fitting. The fitting correlation coefficients are all 1, indicating that the fitting function shown below is completely correlated with the data.

| Section | Length ratio of bonding area | $E_0$/MPa | $\alpha$/MPa | $\beta$/MPa |
|---------|-----------------------------|-----------|--------------|-------------|
| P1P2    | 0                           | 3301      | $-3.583e4$   | $-6.862e4$  |
| P2P3    | 3/6                         | 3161      | $-2.821e4$   | $-2.342e5$  |
| P1P3    | 3/12                        | 3228      | $-3.179e4$   | $-1.592e5$  |
| P1P4    | 3/18                        | 3252      | $-3.306e4$   | $-1.311e5$  |
In the equations (9) to (11), x represents the length ratio of the bonding area, and its value ranges from 0 to 1. When \( x = 0 \), it means that there is no bonding area in the measured section; when \( x = 1 \), it indicates that the measuring section is a bonding area. Therefore, through the extrapolation of fitting equations (9) to (11), the parameters of the bonding area constitutive model can be obtained as follows.

\[
y_E = 39.60x^2 - 299.44x + 3.30e3
\]  
(9)

\[
y_o = -3.90e3x^2 + 1.7167e4x + 3.58e4
\]  
(10)

\[
y_b = 1.27e5x^2 - 3.95e5x - 6.87e4
\]  
(11)

4.3. Comparison of PMMA mother material and bonding area

The constitutive equations of PMMA mother material and bonding area are shown in equations (8) and (12). The coefficient \( E_0 \) of the constitutive equation of the bonding area is about 92.12% of that of the mother material. Because there is a slight plastic deformation when the mother material is fracture, the sign of coefficient \( \beta \) is opposite to that of the bonding area. The fracture strains of mother material specimen and the specimen with bonding area are 4.32% and 2.60%. The tensile strength of specimen with bonding area is 59.84 MPa, which is about 85.68% of that of the mother material.

Figure 21 shows the fracture sections of mother material and specimen with bonding area. The mother material cracks from the surface of the specimen and then extends to the inside. The innermost circle marked in the figure 21(a) shows a smooth cracking point. The surface away from the crack point is rough. The area between two circles is a transition region from smooth to rough. In figure 21(b), the fracture point of the specimen with bonding area is inside the specimen. The whole fracture section is relatively flat. This shows that the bonding area and the mother material are insufficiently polymerized, which makes its strength lower than that of the mother material.
5. Conclusion

The tensile strain of PMMA is measured by the CCD camera in a non-contact way. The influence of the environment on the strain value is less than 0.01%. Two strain measurement methods, CCD camera and strain extensometer, are compared. The strain curves obtained by the two methods are highly consistent that the difference is less than 0.01%. Two strain measurement methods, CCD camera and strain extensometer, are compared. The strain curves obtained by the two methods are highly consistent that the environment on the strain value is less than 0.01%.

Acknowledgments

This work has been supported by the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDA10010200), the Youth Innovation Promotion Association of the Chinese Academy of Sciences (Grant No. 292020000087), Xie Jialin Fund of Institute of High Energy Physics, Chinese Academy of Sciences (Grant No. 51202000107).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Xiaohui Qian https://orcid.org/0000-0002-2781-523X

References

[1] Hu X et al 2011 Selection and application of extensometer Engineering & Test S1 21–4
[2] Van R M and Becker T H 2018 High-temperature tensile property measurements using digital image correlation over a non-uniform temperature field The Journal of Strain Analysis for Engineering Design 53 117–29
[3] Zha P F et al 2017 High-accuracy biaxial optical extensometer based on 2D digital image correlation Meas. Sci. Technol. 28 085006
[4] An F et al 2016 Neutrino physics with JUNO Journal of Physics G 43 030401
[5] Qian X H et al 2020 Structure design and compression experiment of the supporting node for JUNO PMMA detector Radiat Detect Technol Methods 4 345–55
[6] Zhou F, Du Y, Chen Z P and Hou S J 2019 Creep buckling analysis of PMMA (polymethyl methacrylate) pressure vessels for application in neutrino detectors Int. J. Press. Vessels Pip. 169 170–6
[7] Sun F and Gamstedt E K 2019 Homogeneous and localized deformation in Poly(Methyl Methacrylate) nanocomposites Nanomanufacturing and Metrology 2 65–55
[8] Zhou F, Hou S J, Qian X H, Chen Z P, Zheng C C and Xu F 2016 Creep behavior and lifetime prediction of PMMA immersed in liquid scintillator Polym. Test. 53 323–8
[9] Yin W H, Xie Z Y, Yin Y M, Yi J, Liu X D, Wu H Q, Wang S, Xie Y F and Yang Y 2019 Aging behavior and lifetime prediction of PMMA under tensile stress and liquid scintillator conditions Advanced Industrial and Engineering Polymer Research 2 82–7
[10] Adel A, Sabbah A and Vadim V 2017 Temperature-dependent mechanical behaviour of PMMA: experimental analysis and modelling Polym. Test. 58 86–95
[11] Singh B K 2011 Material Characterization, Constitutive Modeling and Finite Element Simulation of Polymethyl Methacrylate (PMMA) for Applications in Hot Embossing (Ohio, USA: Mechanical Engineering Department The Ohio State University)
[12] Hu W, Guo H, Chen Y M, Xie Z R, Jing H and He P 2016 Experimental investigation and modeling of the rate-dependent deformation behavior of PMMA at different temperatures Eur. Polym. J. 85 313–23
[13] Qin J, Liu T, Su B Y, Shu X F and Li Z Q 2018 Experimental investigation on the yield behavior of PMMA Polym. Bull. 75 5335–49
[14] Ma Z B 2005 Methacrylate Resin and Its Application (Beijing: Chemical Industry Press)
[15] Wang Z Y et al 2018 Plain-strain fracture toughness tests of thick acrylic sheets at different temperatures Journal of Southeast University (Natural Science Edition) 48 864–70
[16] Wang Z Y, Wang Y Q, Du X X, Zhang T X and Yuan H X 2018 Quasi-static tensile test of thick acrylic sheets at different temperatures Dongnan Daxue Xuebao (Ziran Kexue Ban) 48 132–7
[17] Ma S P, Nie J X and Ma Q W 2013 Practice of Mechanical Professional Program (Beijing: Beijing Institute of Technology Press)
[18] ASTM 2002 ASTM D638-02 Standard test method for tensile properties of plastics
[19] Mulliken A D and Boyce M C 2006 Mechanics of the rate-dependent elastic-plastic deformation of glassy polymers from low to high strain rates Int. J. Solids Struct. 1343 1331–56
[20] Chen X et al 2018 Study on the nonlinear viscoelastic constitutive relation of a solid propellant under constant strain rates Journal of Solid Rocket Technology 41 319–24
[21] Zhu Z X, Xu D B and Wang L L 1988 Thermo-viscoelastic constitutive equation and time-temperature equivalence of epoxy resin at high strain rates Journal of Ningbo University 1 58–68
[22] Fan R H 2013 The Fitting of the Constitutive Relationship of PMMA and FEA Research Based on Tensile Tests Harbin Institute of Technology