Phenomenological model of the Kaonic Nuclear Cluster $K^-pnn$ in the ground state

A. N. Ivanov$^{1,3}$‡, P. Kienle$^{2,3}$‡, J. Marton$^3$§, E. Widmann$^3$¶

October 9, 2018

1Atominstitut der Österreichischen Universitäten, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien, Österreich,
2Physik Department, Technische Universität München, D–85748 Garching, Germany,
3Stefan Meyer Institut für subatomare Physik, Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090, Wien, Österreich

Abstract

A phenomenological model, proposed for the Kaonic Nuclear Cluster (KNC) $K^-pp$ (or $^2_K$H), is applied to the theoretical analysis of the KNC $K^-pnn$ (or $S^0$). The theoretical values of the binding energy and the width are equal to $\epsilon_{S^0} = -197$ MeV and $\Gamma_{S^0} = 16$ MeV. They agree well with the experimental data $\epsilon_{S^0}^{\text{exp}} = -194.0^{+1.5}_{-4.4}$ MeV and $\Gamma_{S^0}^{\text{exp}} < 21$ MeV (PLB 597, 263 (2004)) and the theoretical values $\epsilon_{S^0} = -191$ MeV and $\Gamma_{S^0} = 13$ MeV, obtained by Akaishi and Yamazaki within the potential model approach.

PACS: 11.10.Ef, 13.75.Jz, 14.20.Jn, 25.80.Nv
1 Introduction

Recently [1] Suzuki et al. have announced the experimental discovery of the bound state \( K^-pnn \), named \( S^0(3115) \), with quantum numbers \( I(J^P) = 1(\frac{3}{2}^+ \) ), the binding energy \( \epsilon_{S^0}^{\text{exp}} = -194.0^{+1.5}_{-4.4} \text{ MeV} \) and the width constrained by \( \Gamma_{S^0}^{\text{exp}} < 21 \text{ MeV} \). The possible existence of such a Kaonic Nuclear Cluster (KNC) as well as of the KNC \( K^-pp \) (or \( \frac{2}{3} \) H), which has been observed by the FINUDA Collaboration [2], has been pointed out by Akaishi and Yamazaki within the potential model approach [3–5]. According to [1], the main decay channels of the KNC \( S^0(3115) \) are \( S^0 \to \Sigma^-pn, S^0 \to \Sigma^-d \) and \( S^0 \to \Sigma^0nn \), where \( d \) is the deuteron. The decay channel \( S^0 \to \Lambda^0nn \) is suppressed.

In this paper we extend the phenomenological quantum field theoretic model for the KNC \( \frac{2}{3} \) H, proposed in [6], to the description of the KNC \( S^0(3115) \). The binding energy and the width of the KNC \( S^0(3115) \) we define as [6]

\[
- \epsilon_{S^0} + i \frac{\Gamma_{S^0}}{2} = \int d\tau \Phi_{S^0} M(K^-pnn \to K^-pnn) \Phi_{S^0}, \tag{1.1}
\]

where \( d\tau \) is an element of the phase space of the \( K^-pnn \) system, \( \Phi_{S^0} \) is the wave function of the KNC \( S^0(3115) \) and \( M(K^-pnn \to K^-pnn) \) is the amplitude of \( K^-pnn \) scattering.

In our model for the calculation of the scattering amplitude of the constituents of the KNC we use the chiral Lagrangian with a non–linear realization of chiral \( SU(3) \times SU(3) \) symmetry and derivative meson–baryon couplings [7], accepted for the analysis of strong low–energy interactions in ChPT [8] (see also [9–10]). We calculate these amplitudes in the tree–approximation to leading order in the large \( N_C \) and chiral expansion, where \( N_C \) is the number of quark colour degrees of freedom [11]. According to Witten [11], the large \( N_C \) expansion is equivalent to the heavy–baryon approximation, applied in ChPT [8–10] to the analysis of low–energy meson–baryon interactions (see also [12–13]).

In our model the main part of correlations between constituents of the KNC are described by the wave function of the KNC.

For the construction of the wave function \( \Phi_{S^0} \) one has to specify the structure of the KNC \( S^0(3115) \). The simplest structure of the bound \( K^-pnn \) system with quantum numbers \( I(J^P) = 1(\frac{3}{2}^+ \) ) and the decay mode \( S^0 \to \Lambda^0nn \) suppressed is \( (K^-p)_{I=0} (nn)_{I=1} \), where the \( K^-p \) pair is in the state with isospin \( I = 0 \), i.e. \( (K^-p)_{I=0} \), and the \( nn \) pair is in the \( P \)–wave and spin–triplet state with isospin \( I = 1 \), i.e. \( (nn)_{I=1} \).

The paper is organised as follows. In Section 2 we calculate the binding energy of the KNC \( S^0(3115) \). We get \( \epsilon_{S^0} = -197 \text{ MeV} \). According to [6], the dominant contribution comes from the Weinberg–Tomozawa term for \( K^-pnn \) scattering. In Section 3 we calculate the partial widths of the decay modes \( S^0 \to \Sigma^-pn, S^0 \to \Sigma^-d, S^0 \to \Sigma^0nn \) and \( S^0 \to \Lambda^0nn \) and the total width of the KNC \( S^0(3115) \). We get \( \Gamma_{S^0} = 16 \text{ MeV} \). We show that the dominant decay mode is \( S^0 \to \Sigma^-pn \) with the partial width \( \Gamma(S^0 \to \Sigma^-pn) = 15 \text{ MeV} \). We calculate the invariant–mass spectrum of the \( S^0 \to \Sigma^-p \) decay. In the Conclusion we discuss the obtained results. In the Appendix we define the wave function \( \Phi_{S^0} \) of the KNC \( S^0(3115) \).
Figure 1: Feynman diagrams giving in the tree–approximation contributions to the amplitude of the reaction $K^-pnn \to K^-pnn$ to next-to-leading order in the large $N_C$ and chiral expansion with respect to the Weinberg–Tomozawa term.

### 2 Binding energy of the KNC $S^0(3115)$

According to [6], the binding energy of the KNC $S^0(3115)$ is given by

$$-\epsilon_{S^0} = \frac{5}{4} \frac{1}{F_\pi^2} \left( \frac{\mu_1^3 \omega_1^3 \Omega_1}{\pi^3} \right)^{1/2} = 197 \text{ MeV},$$

where $F_\pi = 92.4 \text{ MeV}$ is the PCAC constant of pseudoscalar mesons. Other parameters $\mu_1$, $\omega_1$ and $\Omega_1$ are related to the wave function $\Phi_{S^0}$ of the KNC $S^0(3115)$ and defined in the Appendix. The theoretical value agrees well with the experimental one $\epsilon_{S^0}^{\text{exp}} = -194.0_{-1.4}^{+1.2} \text{ MeV}$. The binding energy is defined by the contribution of the Weinberg–Tomozawa term. This term gives the dominant contribution to the real part of the amplitude of elastic $K^-pnn$ scattering to leading order in the large $N_C$ and chiral expansion.

The correction of next-to-leading order in the large $N_C$ and chiral expansion to the Weinberg–Tomozawa term is defined by the diagrams in Fig. 1. The calculation of diagrams in Fig. 1 is carried out using the chiral Lagrangian with a non–linear realization of chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings [6]. For $N_C = 3$ such a correction makes up a few percent of the Weinberg–Tomozawa term.

### 3 Width of the KNC $S^0(3115)$

In this section we calculate the partial widths of the decay modes $S^0 \to \Sigma^-pn$, $S^0 \to \Sigma^-d$, $S^0 \to \Sigma^0nn$ and $S^0 \to \Lambda^0nn$ and the total width $\Gamma_{S^0}$ of the KNC $S^0(3115)$. For the total width we get the value $\Gamma_{S^0} = 16 \text{ MeV}$, agreeing well with the experimental data $\Gamma_{S^0} < 21 \text{ MeV}$ and the potential model prediction $\Gamma_{S^0} = 13 \text{ MeV}$ [4] (see also [5]).

#### 3.1 The $S^0 \to \Sigma^-pn$ decay mode

The Feynman diagrams of the reaction $K^-pnn \to \Sigma^-pn$ are depicted in Fig. 2. In the tree–approximation these diagrams give the main contribution to the amplitude of the $S^0 \to \Sigma^-pn$ decay to leading order in the large $N_C$ and chiral expansion. The calculation of these diagrams is carried out using the chiral Lagrangian with chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings (see Appendix B of Ref.[6]). The partial
width of the $S^0 \rightarrow \Sigma^-pn$ decay is equal to
\[
\Gamma(S^0 \rightarrow \Sigma^-pn) = \left(\frac{\mu^2 \omega_2 \Omega_1}{\pi^3}\right)^{1/2} \left(\frac{\mu_2 \omega_2}{\pi}\right)^{3/2} \left(\frac{\mu_3 \omega_3}{\pi}\right)^{5/2} \frac{9}{32\pi^2} \frac{g_{\pi NN}^2}{m_K m^4_F \rho_\pi} \sqrt{\frac{2\mu^3}{m_N}} \times \varepsilon^2 f(\varepsilon) = 15 \text{ MeV},
\]
where $g_{\pi NN} = g_{AMN}/F_{\rho} = 13.3$ is the $\pi NN$ coupling constant \cite{14, 15}, $\varepsilon = m_{S^0} - 2m_N - m_\Sigma = 44$ MeV is the energy excess calculated for $m_N = 940$ and $m_\Sigma = 1193$ MeV, $\mu = 2m_N m_\Sigma/(2m_N + m_\Sigma) = 730$ MeV and $f(\varepsilon) = 0.78$ is the contribution of the phase volume of the $\Sigma^-pn$ state. The parameters $\mu_i, \omega_i$ for ($i=1,2,3$) and $\Omega_1$ of the wave function $\Phi_{S^0}$ of the KNC $S^0(3115)$ are defined in the Appendix.

### 3.2 Invariant–mass spectrum of the $S^0 \rightarrow \Sigma^-pn$ decay

Following \cite{16} we calculate the invariant–mass spectrum of the $S^0 \rightarrow \Sigma^-pn$ decay. For this aim we use the following variables \cite{16}
\[
X = m^2_{np} = (E_n + E_p)^2 - (\vec{k}_n + \vec{k}_p)^2, \quad Y = m^2_{2\Sigma p} = (E_{\Sigma^+} + E_p)^2 - (\vec{k}_\Sigma + \vec{k}_p)^2,
\]
which have the meaning of the invariant–squared masses of the $np$ and $\Sigma^-p$ pairs, respectively. At the rest frame of the KNC $S^0(3115)$ in terms of the variables $(X, Y)$ the kinetic energies of the neutron and the proton are given by
\[
T_n = \frac{(m_{S^0} - m_N)^2 - Y}{2m_{S^0}}, \quad T_p = \varepsilon - \frac{(m_{S^0} - m_\Sigma)^2 + (m_{S^0} - m_N)^2 - X - Y}{2m_{S^0}}.
\]

The invariant–mass spectrum or the Dalitz density distribution of the $S^0 \rightarrow \Sigma^-pn$ decay is defined by
\[
m^3_{S^0} \frac{d^2 \Gamma(S^0 \rightarrow \Sigma^-pn)}{dXdY} = \left(\frac{\mu^2 \omega_2 \Omega_1}{\pi^3}\right)^{1/2} \left(\frac{\mu_2 \omega_2}{\pi}\right)^{3/2} \left(\frac{\mu_3 \omega_3}{\pi}\right)^{5/2} \frac{9}{128\pi^2} \frac{g_{\pi NN}^2 m_{S^0} m_{\Sigma}}{m_K m^4_F \rho_\pi} \times |A(X, Y) + B(X, Y) + C(X, Y) + D(X, Y)|^2 = 10 |A(X, Y) + B(X, Y) + C(X, Y) + D(X, Y)|^2.
\]

In the $(X,Y)$–plane the Dalitz domain is bounded by the curve with $X_{\min} = 4m^2_N$, $X_{\max} = (m_{S^0} - m_\Sigma)^2$, $Y_{\min} = (m_\Sigma + m_N)^2$ and $Y_{\max} = (m_{S^0} - m_N)^2$ \cite{16}. The invariant–mass spectrum of the $S^0 \rightarrow \Sigma^-pn$ is represented in Fig. 3.
Figure 3: The invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay in the variables $X = m_{np}^2$ and $Y = m_{\Sigma p}^2$. [16]

### 3.3 The $S^0 \to \Sigma^- d$ decay mode

The partial width of the $S^0 \to \Sigma^- d$ decay is equal to [17]

$$
\Gamma(S^0 \to \Sigma^- d) = \left( \frac{\mu_1^2 \omega_1^2 \Omega_1}{\pi^3} \right)^{1/2} \left( \frac{\mu_2 \omega_2}{\pi} \right)^{3/2} \left( \frac{\mu_3 \omega_3}{\pi} \right)^{5/2} \frac{9 g_{\pi NN}^2 m_{\Sigma d}^2 Q_{\Sigma d}}{4 m_K m_N m_{S0}^2 m_{\pi}^2 F_{\pi}^8} \times \left| \int \frac{d^3k}{(2\pi)^3} \frac{\Phi_d(k)}{1 - \frac{1}{2} r_{np}^t a_{np}^t k^2 - i a_{np}^t k} (A + B + C + D) \right|^2 = 1 \text{ MeV}, \tag{3.5}
$$

where the amplitudes $A$, $B$, $C$ and $D$ are defined by the corresponding Feynman diagrams in Fig. 2, $Q_{\Sigma d} = 266$ MeV is a relative momentum of the $\Sigma^- d$ pair, $\Phi_d(k) = \sqrt{8\pi\gamma}/(\gamma^2 + k^2)$ is the wave function of the deuteron in the ground state in the momentum representation and $\gamma = 1/R_d = 46$ MeV [18]. The amplitude of the $np$ scattering is defined according to [19] (see also [17]) and taken in the effective range approximation

$$
k \cot \delta_{np}(k) = -\frac{1}{a_{np}^t} + \frac{1}{2} r_{np}^t k, \tag{3.6}
$$

where $\delta_{np}(k)$ is the phase shift of the $np$ scattering in the $^3S_1$ state, $a_{np}^t = (5.424 \pm 0.004)$ fm and $r_{np}^t = (1.759 \pm 0.005)$ fm are the S–wave scattering length and effective range of $np$ scattering in the $^3S_1$ state [18].

The main contribution to the integral comes from the relative momenta $k \sim 100$ MeV, and 94% of the value of the integral are concentrated in the region $0 \leq k \leq 200$ MeV. For the relative momenta $k = 100$ MeV and $k = 200$ MeV the phase shift of the $np$ scattering, defined by the effective range approximation (3.6), is equal to $\delta_{np} = 85.3^0$ and $\delta_{np} = 54.6^0$, respectively. These values agree well with the SAID analysis of the experimental data on the $np$ scattering in the $^3S_1$ state [20]: $\delta_{np} = 85.84^0$ and $\delta_{np} = 48.84^0$, respectively.
3.4 The $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decay modes

The amplitudes of the $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decays are defined by the integrals of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$ with the wave function $\Phi_{S^0}$ of the KNC $S^0(3115)$. Feynman diagrams of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$ are depicted in Fig.4. In the tree–approximation these diagrams give the main contribution to leading order in the large $N_C$ and chiral expansion. The contributions of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$, calculated to leading order in the large $N_C$ and chiral expansion, to the amplitudes of the $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decays vanish. This means that the decay modes $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ are suppressed to leading order in the large $N_C$ and chiral expansion.

3.5 Width of the KNC $S^0(3115)$

The width of the KNC $S^0(3115)$ is defined by the sum of the partial widths of all modes. Since the dominant modes are $S^0 \to \Sigma^- pn$ and $S^0 \to \Sigma^- d$, the width of the $S^0(3115)$ is equal to

$$\Gamma_{S^0} = 16 \text{ MeV}. \quad (3.7)$$

This agrees well with the experimental constraint $\Gamma_{S^0}^{\text{exp}} < 21 \text{ MeV}$ and the theoretical value $\Gamma_{S^0} = 13 \text{ MeV}$, predicted by Akaishi and Yamazaki within the potential model approach [4] (see also [5]).

4 Conclusion

The phenomenological quantum field theoretic model of the KNC $K^- pp$ (or $\tilde{2}_K H$), proposed in [6], is extended to the description of the KNC $S^0(3115)$ with quantum numbers $I(J^P) = 1(\frac{3}{2}^+)$ [4]. The wave function of the KNC $S^0(3115)$ is taken in the oscillator form. The frequencies of oscillations are related to the frequency $\omega$, describing longitudinal oscillations of the $K^-$ meson with respect to the $pp$ pair in the KNC $\tilde{2}_K H$ [6]. The value of the frequency $\omega = 64 \text{ MeV}$ has been calculated in [6] by fitting the width of the $\Lambda^0(1405)$ hyperon, treated as a bound $K^- p$ state $\frac{1}{2}_K H$.

We would like to make clear the title of our model namely “The phenomenological quantum field theoretic model”. On the one hand our model is “phenomenological”, since
we construct the wave function of the KNC, the parameters of which are fixed in terms of the experimental data on the width and mass of the $\Lambda(1405)$ hyperon. On the other hand our model is “quantum field theoretic”, since to the calculation of scattering amplitudes of the constituents of the KNC we apply the chiral Lagrangian with chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings, ChPT and the large $N_C$ expansion.

The binding energy $\epsilon_{S^0} = -197.0$ MeV and the width $\Gamma_{S^0} = 16$ MeV, calculated in our model, agree well with the experimental data $\epsilon_{S^0}^{\text{exp}} = -194.0^{+1.5}_{-4.4}$ MeV and $\Gamma_{S^0}^{\text{exp}} < 21$ MeV. We have found that the decay mode $S^0 \rightarrow \Sigma^- pn$ is dominant, whereas the decay modes $S^0 \rightarrow \Sigma^0 nn$ and $S^0 \rightarrow \Lambda^0 pn$ are suppressed. The calculation of the amplitudes of elastic and inelastic $K^- pnn$ scattering we have carried out using the chiral Lagrangian with the non–linear realization of chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings (see Appendix B of Ref.[6]). We have kept the contributions of leading order in the large $N_C$ and chiral expansion [6]. In more detail the experimental analysis of the theoretical amplitude of the $S^0 \rightarrow \Sigma^- pn$ decay can be obtained through the measurements of the invariant–mass spectrum [16]. The theoretical invariant–mass spectrum of the $S^0 \rightarrow \Sigma^- pn$ decay is given by Eq.(3.4) and represented in Fig. 3.

We would like to accentuate that our theoretical predictions for the binding energy $\epsilon_{S^0} = -197.0$ MeV and the width $\Gamma_{S^0} = 16$ MeV of the KNC $S^0(3115)$ agree well with results, obtained by Akaishi and Yamazaki within the potential model approach [4] (see also [5]): $\epsilon_{S^0} = -191.0$ MeV and $\Gamma(S^0 \rightarrow \Sigma^- pn) = 13$ MeV.

However one can show that in our model the nuclear matter density of the KNC $S^0(3115)$ is smaller by factor 3 than $n_{S^0}(0) = 1.56$ fm$^{-3}$, predicted in the potential model approach [5]. Indeed, using the wave function of the KNC $S^0(3115)$ one can calculate the nuclear matter density at the center of mass of the $K^- pnn$ system. This gives

$$n_{S^0}(0) = 3 \left( \frac{\mu_1^2 \omega_1^2 \Omega_1}{\pi^3} \right)^{1/2} = 0.53 \text{ fm}^{-3}. \quad (4.8)$$

The obtained result is larger by a factor 4 than the normal nuclear matter density $n_0 = 0.14$ fm$^{-3}$, but it is smaller by factor 3 than $n_{S^0}(0) = 1.56$ fm$^{-3}$, predicted by Akaishi and Yamazaki.

Our approach for the analysis of the Kaonic Nuclear Clusters $K^- p$ (or $\Lambda^0(1405)$), $K^- pp$ and $K^- pnn$ can be applied to the description of the Kaonic Nuclear Clusters $K^- pnn$ with isospin $I = 0$, $K^- pnn$ and $K^- ppp$ with isospin $I = 1$, $K^-NNNN$ and so on [3].

**Appendix A: Wave function $\Phi_{S^0}$ of the KNC $S^0(3115)$**

In the momentum representation the wave function $\Phi_{S^0}$ of the KNC $S^0(3115)$ is defined by [6]

$$\Phi_{S^0}(\vec{q}, \vec{k}, \vec{Q})_{JM} = \Phi_K(\vec{q}) \Phi_{p(nn)}(\vec{k}) \Phi_{nn}(Q) \Psi_{JM}(\vec{Q}). \tag{A-1}$$

This wave function describes the $K^- pnn$ system with total momentum $J = \frac{3}{2}$ and positive parity, where $\Phi_K(\vec{q})$, $\Phi_{p(nn)}(\vec{k})$ and $\Phi_{nn}(\vec{Q})$ are the wave functions of harmonic oscillators [6]

$$\Phi_K(\vec{q}) = \left( \frac{64\pi^3}{\mu_1^2 \omega_1^2 \Omega_1} \right)^{1/4} \exp \left( - \frac{\vec{q}_\perp^2}{2\mu_1 \omega_1} - \frac{\vec{q}_\parallel^2}{2\mu_1 \Omega_1} \right).$$
\[
\Phi_{p(nn)}(\vec{k}) = \left(\frac{4\pi}{\mu_2\omega_2}\right)^{3/4} \exp\left(-\frac{\vec{k}^2}{2\mu_2\omega_2}\right),
\]
\[
\Phi_{nn}(\vec{Q}) = \Phi_{nn}(Q) Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}}) = \frac{8}{\sqrt{3}} \left(\frac{\pi}{\mu_3\omega_3}\right)^{5/4} |\vec{Q}| \exp\left(-\frac{\vec{Q}^2}{2\mu_3\omega_3}\right) Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}}),
\]
where the \(nn\) pair is in the \(P\)-wave state, \(Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}})\) is a spherical harmonic, \(m = 0, \pm 1\) is a magnetic quantum number. The frequencies and the reduced masses are
\[
\begin{align*}
\omega_1 &= \sqrt\frac{3\mu}{2\mu_1} \omega = 76 \text{ MeV}, \quad \mu_1 = \frac{3m_Km_N}{m_K + 3m_N} = 420 \text{ MeV} \\
\Omega_1 &= \sqrt\frac{\mu}{3\omega_1} = 132 \text{ MeV}, \\
\omega_2 &= \sqrt\frac{\mu}{\mu_2} \omega = 51 \text{ MeV}, \quad \mu_2 = \frac{2}{3} m_N = 630 \text{ MeV}, \\
\omega_3 &= \frac{1}{2} \sqrt\frac{\mu}{\mu_3} \omega = 30 \text{ MeV}, \quad \mu_3 = \frac{1}{2} m_N = 470 \text{ MeV},
\end{align*}
\]
where \(\omega = 64 \text{ MeV}\) is the frequency of the longitudinal oscillation of the \(K^-\) meson relative to the \(pp\) pair in the KNC \(\pi K^-\) [6] and \(\mu = 2m_Km_N/(m_K + 2m_N) = 391 \text{ MeV}\) is the reduced mass of the \(K^-\) (\(pp\)) system, calculated for \(m_K = 494 \text{ MeV}\) and \(m_N = 940 \text{ MeV}\). The value \(\omega = 64 \text{ MeV}\) is obtained from the fit of the width of the \(\Lambda^0(1405)\) hyperon, treated as a bound \(K^-p\) state [6].

The wave functions \(\Phi_K(\vec{q}), \Phi_{p(nn)}(\vec{k})\) and \(\Phi_{nn}(\vec{Q})\) are normalised by
\[
\int \frac{d^3q}{(2\pi)^3} |\Phi_K(\vec{q})|^2 = \int \frac{d^3k}{(2\pi)^3} |\Phi_{p(nn)}(\vec{k})|^2 = \int \frac{d^3Q}{(2\pi)^3} |\Phi_{nn}(\vec{Q})|^2 = 1.
\]
This gives
\[
\begin{align*}
\int \frac{d^3q}{(2\pi)^3} \Phi_K(\vec{q}) &= \left(\frac{\mu_3^2\omega_1^2\Omega_1}{\pi^3}\right)^{1/4}, \\
\int \frac{d^3k}{(2\pi)^3} \Phi_{p(nn)}(\vec{k}) &= \left(\frac{\mu_2^2\omega_2}{\pi}\right)^{3/4}, \\
\int \frac{d^3Q}{(2\pi)^3} Q \Phi_{nn}(\vec{Q}) &= \frac{12\pi}{\sqrt{6}} \left(\frac{\mu_3^2\omega_3^3}{\pi}\right)^{5/4}.
\end{align*}
\]
The wave functions \(\Phi_K(\vec{q}), \Phi_{p(nn)}(\vec{k})\) and \(\Phi_{nn}(\vec{Q})\) describe oscillations of the \(K^-\) meson relative to the \(pnn\) system, the proton relative to the \(nn\) pair and the neutrons in the \(nn\) pair, respectively. The \(K^-pnn\) system with the wave function \((\text{A-1})\) possesses positive parity.

Since the total angular momentum of the KNC \(S^0(3115)\) is \(J = \frac{3}{2}\) [11,14,5], the wave functions \(\Psi_{JM}(\vec{Q})\) of the angular momentum of the KNC \(S^0(3115)\) are defined by
\[
\Psi_{\frac{3}{2},\frac{3}{2}} = \frac{1}{\sqrt{2}} Y_{1,1} \left[ \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} + \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} \right] \chi_{\frac{1}{2},\frac{1}{2}}^{(p)} - \frac{1}{\sqrt{2}} Y_{1,0} \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} \chi_{\frac{1}{2},\frac{1}{2}}^{(p)},
\]
\[
\Psi_{\frac{3}{2},-\frac{3}{2}} = \frac{1}{\sqrt{2}} Y_{1,1} \left[ \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} + \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} \right] \chi_{\frac{1}{2},\frac{1}{2}}^{(p)} + \frac{1}{\sqrt{3}} Y_{1,1} \chi_{\frac{1}{2},\frac{1}{2}}^{(1)} \chi_{\frac{1}{2},\frac{1}{2}}^{(2)} \chi_{\frac{1}{2},\frac{1}{2}}^{(p)}
\]
\[ \Psi_{\frac{1}{2}, \pm \frac{1}{2}} = \frac{1}{\sqrt{3}} Y_{1,+1} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} + \frac{1}{\sqrt{6}} Y_{1,0} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} \]

\[ - \frac{1}{\sqrt{3}} Y_{1,-1} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} - \frac{1}{\sqrt{12}} Y_{1,-1} \left[ \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} + \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \right] \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} \]

\[ \Psi_{\frac{3}{2}, \pm \frac{1}{2}} = \frac{1}{\sqrt{2}} Y_{1,0} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} - \frac{1}{2} Y_{1,-1} + \chi_{\frac{1}{2}, -\frac{1}{2}}^{(1)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(2)} \chi_{\frac{1}{2}, -\frac{1}{2}}^{(p)} \]  

(A-6)

where \( \chi_{\frac{1}{2}, \pm \frac{1}{2}}^{(a)} \) with \( a = 1, 2, p \) are the spinorial wave functions of the neutrons and the proton, respectively.

References

[1] T. Suzuki et al. (the KEK Collaboration), Nucl. Phys. A 754, 375 (2005); Phys. Lett. B 597, 263 (2004).

[2] M. Agnello et al. (the FINUDA Collaboration), Phys. Rev. Lett. 94, 212303 (2005).

[3] Y. Akaishi and T. Yamazaki, Phys. Rev. C 65, 044005 (2002); T. Yamazaki and Y. Akaishi, Phys. Lett. B 535, 70 (2002).

[4] Y. Akaishi and T. Yamazaki, Nucl. Phys. A 684, 409c (2001); A. Doté, Y. Akaishi, and T. Yamazaki, Mod. Phys. Lett. A 18, 120 (2003); T. Yamazaki, A. Doté, and Y. Akaishi, Phys. Lett. B 587, 167 (2004); A. Doté et al., Phys. Lett. B 590, 51 (2004); Y. Akaishi, T. Yamazaki, and A. Doté, Nucl. Phys. A 738, 168 (2004); A. Doté, Y. Akaishi, and T. Yamazaki, Nucl. Phys. A 738, 372 (2004).

[5] A. Andronic, P. Braun-Munzinger, and K. Redlich, Nucl. Phys. A 765, 211 (2006) and references therein.

[6] A. N. Ivanov, P. Kienle, J. Marton, and E. Widmann, nucl–th/0512037: Invited talk at the Workshop on “Exotic hadronic atoms, deeply bound kaonic nuclear states and anti–hydrogen: present results, future challenges” at ECT* in Trento, 19–24 June 2006, hep-ph/0610201. A plenary talk at the Workshop on “QCD: Facts and Prospects” in Oberwöльц, Austria, 10–16 October 2006, http://physik.uni-graz.at/itp/oberw/.

[7] B. W. Lee, Phys. Rev. 170, 1359 (1968).

[8] J. Gasser, Nucl. Phys. Proc. Suppl. 86, 257 (2000) and references therein. H. Leutwyler, PiN Newslett. 15, 1 (1999); B. Borasoy and B. Holstein, Eur. Phys. J. C 6, 85 (1999); Phys. Rev. D 59, 094025 (1999); Phys. Rev. D 60, 054021 (1999); Ulf-G. Meißner, PiN Newslett. 13, 7 (1997); H. Leutwyler, Ann. of Phys. 235, 165 (1994); G. Ecker, Prog. Part. Nucl. Phys. 36, 71 (1996); Prog. Part. Nucl. Phys. 35, 1 (1995); Nucl. Phys. Proc. Suppl. 16, 581 (1990); E. Jenkins and A. V. Manohar, Phys. Lett. B 225, 558 (1991); A. Krause, Helv. Phys. Acta 63, 1 (1990); J. Gasser, M. Sainio, and A. Švarc, Nucl. Phys. B 307, 779 (1988); J. Gasser, Nucl. Phys. B 279, 65 (1987); J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985); Ann. of Phys. 158, 142 (1984); Phys. Lett. B 125, 321 (1983).
[9] R. Kaiser and H. Leutwyler, Eur. Phys. J. C 17, 623 (2000).

[10] M. L. M. Lutz and E. E. Kolomeitsev, Found. Phys. 31, 1671 (2001).

[11] E. Witten, Nucl. Phys. B 160, 57 (1979).

[12] T. E. O. Ericson and A. N. Ivanov, Phys. Lett. B 634, 39 (2006); hep–ph/0503277.

[13] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. C59, 451 (1999).

[14] H.–Ch. Schröder et al., Eur. Phys. J. C 21, 473 (2001).

[15] T. E. O. Ericson, B. Loiseau, and S. Wycech, Phys. Lett. B 594, 76 (2004).

[16] P. Kienle, Y. Akaishi, and T. Yamazaki, Phys. Lett. B 632, 187 (2006).

[17] A. N. Ivanov et al., Eur. Phys. J. A 23, 79 (2005), nucl–th/0406053.

[18] M. M. Nagels et al., Nucl. Phys. B 147, 189 (1979); O. Dumbrajs et al., Nucl. Phys. B 216, 277 (1983).

[19] K. M. Watson, Phys. Rev. 88, 1163 (1952); A. B. Migdal, Sov. Phys. JETP 1, 2 (1955); D. S. Koltun and A. Reitan, Phys. Rev. 141, 1413 (1966); H. Machner and J. Haidenbauer, J. Phys. G: Nucl. Part. Phys. 25, R231 (1999); A. N. Ivanov et al., nucl–th/0509055.

[20] R. A. Arndt, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C 62, 034005 (2000), nucl–th/0004039, http://gwdac.phys.gwu.edu