Squark Mixing Contributions to CP violating phase $\gamma$

Darwin Chang$^{a,b}$, We-Fu Chang$^{a,c}$, Wai-Yee Keung$^{d}$
Nita Sinha$^{e}$ and Rahul Sinha$^{e}$

$^a$NCTS and Physics Department, National Tsing-Hua University,
Hsinchu 30043, Taiwan, R.O.C.

$^b$Lawrence Berkeley Laboratory, University of California, Berkeley, CA, USA

$^c$TRIUMF, Vancouver, BC V6T2A3, Canada

$^d$Physics Department, University of Illinois at Chicago, Illinois 60607-7059, USA

$^e$The Institute of Mathematical Sciences, Taramani, Chennai 600 113, India

Abstract

We investigate the possibility that the CP violation due to the soft supersymmetry breaking terms in squark mixing can give significant contributions to the various $\gamma$ related parameters in $B$ decays, different from those of the Standard Model. We derive the new limits on $(\delta_{12}^u)_{LL,LR,RR}$ and on $(\delta_{23}^d)_{LL,LR,RR}$ from the recent data on $D^0-\bar{D}^0$ oscillation as well as those on $B_s^0-\bar{B}_s^0$ oscillation. We show that, together with all the other constraints, the currents limits on these parameters still allow large contributions to the CP violating phases in $B_s^0-\bar{B}_s^0$ as well as $D^0-\bar{D}^0$ oscillations which will modify some of the proposed measurements of $\gamma$ parameters in CP violating $B$ decays. However, the current constraints already dictate that the one-loop squark mixing contributions to various $B$ decay amplitudes cannot be competitive with that of the Standard Model (SM), at least for those $B$ decay modes which are dominated the tree level amplitudes within the SM, and therefore they are not significant in contributing to CP asymmetries in the corresponding $B$ decays.

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1 Introduction

With the $B$ factories producing physics at full steam, it is important to investigate critically the possibility of distinguishing standard Kobayashi-Maskawa (KM) [1] model from the other alternatives of CP violation. In particular, one needs to investigate if there is any non-KM mechanism that contributes to the measurement of CP violating phases which within the KM context, are labeled as $\alpha$, $\beta$ and $\gamma$ (or $\phi_{1,2,3}$ in another popular convention in the literature). In fact, it is interesting to note that some of the CP violating asymmetries in $B$ decays, which within the KM mechanism are considered identical, may correspond to different numerical asymmetries in an alternative theory of CP violation.

One of the leading extensions of the Standard Model is the supersymmetric version of the theory. The addition of supersymmetric partners, as well as the necessity of supersymmetry breaking creates large variations of such models. In this paper we shall assume that the spectrum of the supersymmetric extension is minimal in the sense that no additional supermultiplet is introduced beyond the usual two copies of the Higgs doublet needed for fermion masses. In this theory, the new parameters are the coupling constants related to soft supersymmetry breakings. It is well known that these soft breaking parameters can give rise to new sources of CP violation.

Among these soft breaking parameters, the ones most relevant to CP violation in $K$ or $B$ decays are the dim-2 soft squark mixing parameters (the matrices $M_Q^2$ for left-handed squarks, and $M_U^2$, $M_D^2$ for right-handed squarks), as well as the dim-3 trilinear scalar Yukawa couplings $(Y_u^A, Y_d^A)$ which, after the spontaneous symmetry breaking, creates the mixings between left- and right-squarks (the matrices $Y_u^A \langle H_u \rangle$ and $Y_d^A \langle H_d \rangle$, here we follow the notation of Ref.[2]). There are many discussions in the literature about whether the new supersymmetric contributions can give large enough $\epsilon$ and $\epsilon'$ [2, 3, 4].

In Ref.[2], it was pointed out that even though the natural value of $(\delta_{12}^d)_{LR}$ in generic models may be of order $10^{-5}$, its contribution is big enough to saturate the experimental value for $\epsilon'_K$. One may wonder whether it is possible to use $(\delta_{12}^d)_{LR}$ to saturate both $\epsilon_K$ and $\epsilon'_K$ in kaon system [2, 3]. However it was pointed out [6] that in order to saturate both, the absolute value of $(\delta_{12}^d)_{LR}$ has to be about $3 \times 10^{-3}$ which is larger than its generic value.

On the other hand, in Ref.[4], it was pointed out that if one takes into account the
isospin breaking effect of the supersymmetric $\Delta S = 1$ box diagrams, it is possible to account for $\epsilon'$ even using the $\text{Im}(\delta^d_{12})_{LL}$ with a mild fine-tuning. Note that, as emphasized in Ref.[3], while $\delta_{LR}$'s should be generically small due to its $SU(2)$ breaking character, $\delta_{LL}$ and $\delta_{RR}$ do not have to be small. This opens up the possibility to use $(\delta^d_{12})_{LL}$ alone to saturate both $\epsilon$ and $\epsilon'$.

In Ref.[4], it was pointed out that while it is possible for $(\delta^d_{13})_{LR}$ to contribute to $\beta$ parameter in $B$ decays due to the $B^0_d - \bar{B}^0_d$ oscillation, however its generic value is typically too small to account for the recent data from $B$ factories, $\sin 2\beta = 0.59 \pm 0.14 \pm 0.05$ (for Babar[8]) and $\sin 2\beta = 0.99 \pm 0.14 \pm 0.06$ (for Belle[9]).

The purpose of this paper is to address the issue whether these new supersymmetric sources of CP violation can give rise to asymmetries which are usually associated with $\gamma$. We will consider only those $B$ or $B_s$ decay modes whose amplitudes are dominated by tree amplitudes within Standard Model (that is, the modes without “Penguin pollution”). We call these decays, non-Penguin type. We shall briefly comment on modes with Penguin contributions later on.

We first derive constraints on squark mixing parameters based on the most recent data on $B_s - \bar{B}_s$ and $D^0 - \bar{D}^0$ oscillations. Within SUSY, $\Delta B = 1$ box diagrams also arise. These can contribute even to modes that have only tree level contributions within the SM. Using the constraints obtained on the squark mixing parameters, we show that the contributions of $\Delta B = 1$ SUSY box diagrams to the these (non-Penguin) $B$ decays are negligible. However, we find that SUSY contribution can give rise to CP asymmetries through either the initial state $B^0_s - \bar{B}^0_s$ oscillation or the final state $D^0 - \bar{D}^0$ oscillation. We compare these asymmetries with the predictions of KM model.

## 2 New Limits on Squark Mixings

In a comprehensive paper[10], Gabbiani et. al. work out various limits on the flavor changing couplings $(\delta^q_{ij})_{LL,LR,RR}$, where $q$ can be $u$(up-type), $d$(down-type) or $\ell$(lepton), and $ij$ are generation indices. The $\delta$'s are dimensionless parameters defined as

$$ (\delta_{ij})_{AB} = (m^2_{ij})_{AB}/m^2_q, \quad (1) $$
where $AB$ and $ij$ stand for the chirality and flavor respectively. For our purpose we will need only $(\delta^u_{12})_{LL,LR,RR}$ and $(\delta^d_{23})_{LL,LR,RR}$. The earlier limits on these parameters, given in Ref.[10], are summarized in Tables 1 and 2.

Recently there are new measurements on the $D^0-\bar{D}^0$ oscillation as well as on the $B_s^0-\bar{B}_s^0$ oscillation. They can be translated into new limits on these $\delta$ parameters. In particular, for $D^0-\bar{D}^0$ oscillation, the new measurements give $\Delta m_D < 0.461 \times 10^{-10}$MeV (in Ref.[11]). For $B_s^0-\bar{B}_s^0$, the data have so far not been able to set a solid upper limit on the oscillation frequency, $\Delta M_s$, due to the error in the measurement of the higher frequency region [12]. The combined data from LEP and SLD give $\Delta M_{B_s} > 15$ ps$^{-1}$ at 95% C.L. However, a hint of oscillation is observed (with large error) around $\Delta m_{B_s}$ of 17 ps$^{-1}$ [12]. The new measurements from LEP and CDF can also be combined to give $\Delta \Gamma_s/\Gamma_s = 0.16^{+0.08}_{-0.09}$, or $\Delta \Gamma_s/\Gamma_s < 0.31$, at 95% C.L. This can be combined with the lattice calculation of $\Delta \Gamma_s/\Delta M_{B_s} = 3.5^{+0.94}_{-1.55} \times 10^{-3}$ in SM, to give $\Delta m_{B_s} = (29^{+16}_{-21})$ ps$^{-1}$. In presence of SUSY contributions, we expect, this lattice estimate to change. To obtain reasonable limits on the $\delta_{23}$ parameter, we use the suggestive values of $\Delta m_{B_s} = 8, 17, 45$ ps$^{-1}$ as typical value in our study. Note that these are not yet serious experimental limits, however as commented later, our physical conclusions on $B$ decays in the next section, are not significantly altered even if $\Delta M_{B_s}$ turns out to be one order of magnitude larger.

| x  | $\sqrt{\Re(\delta^u_{12})_{LL}}$ | $\sqrt{\Re(\delta^u_{12})_{LR}}$ | $\sqrt{\Re(\delta^u_{12})_{LL} (\delta^u_{12})_{RR}}$ |
|----|---------------------------------|---------------------------------|--------------------------------------------------|
| 0.3| $4.7 \times 10^{-2}$            | $6.3 \times 10^{-2}$            | $1.6 \times 10^{-2}$                             |
| 1.0| $1.0 \times 10^{-1}$           | $3.1 \times 10^{-2}$            | $1.7 \times 10^{-2}$                             |
| 4.0| $2.4 \times 10^{-1}$           | $3.5 \times 10^{-2}$            | $2.5 \times 10^{-2}$                             |

Table 1: Limits on Re $(\delta^u_{12})_{AB} (\delta^u_{12})_{CD}$ from $\Delta m_D$, with $A,B,C,D = (L,R)$, for an average
squark mass $m_{\tilde{q}} = 500\text{GeV}$ and for different values of $x = \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}$.

| $x$ | $|\left(\delta_{23}^d\right)_{LL}|$ | $|\left(\delta_{23}^d\right)_{LR}|$ |
|-----|-----------------|-----------------|
| 0.3 | 4.4             | $1.3 \times 10^{-2}$ |
| 1.0 | 8.2             | $1.6 \times 10^{-2}$ |
| 4.0 | 26              | $3.0 \times 10^{-2}$ |

Table 2: Limits on the $|\delta_{23}^d|$ from $b \to s\gamma$ decay for an average squark mass $m_{\tilde{q}} = 500\text{ GeV}$ and for different values of $x = \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}$. For different values of $m_{\tilde{q}}$, the limits can be obtained multiplying the ones in the table by $(m_{\tilde{q}}/500\text{ GeV})^2$.

For $B_s^0 - \bar{B}_s^0$ oscillation, the $(\delta_{23}^d)_{ab}$ mixing contributes to the operator

$$Q_1 = \bar{s}_L^\alpha \gamma_\mu L^\alpha s_L^\beta \gamma_\mu b_R^\beta,$$

$$Q_2 = \bar{s}_R^\alpha b_L^\beta s_R^\beta b_L^\alpha,$$

$$Q_3 = \bar{s}_R^\alpha b_L^\beta s_R^\beta b_L^\alpha,$$

$$Q_4 = \bar{s}_R^\alpha b_L^\beta s_R^\beta b_L^{\prime\alpha},$$

$$Q_5 = \bar{s}_R^\alpha b_L^{\prime\beta} s_R^{\prime\beta} b_L^\alpha,$$

plus the operators $\tilde{Q}_{1,2,3}$ obtained from the $Q_{1,2,3}$ by the exchange $L \leftrightarrow R$. Here $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$, and $\alpha$ and $\beta$ are color indices. The color matrices normalization is $\text{Tr}(t^A t^B) = \frac{1}{2}\delta^{AB}$. The $\Delta B = 2$ effective Hamiltonian reads:

$$\mathcal{H}_{\text{eff}} = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} \left\{ \left(\delta_{23}^d\right)_{LL}^2 \left(24 Q_1 x f_6(x) + 66 \bar{Q}_1 \bar{f}_6(x)\right) + \left(\delta_{23}^d\right)_{RR}^2 \left(24 Q_2 x f_6(x) + 66 \bar{Q}_2 \bar{f}_6(x)\right) + \left(\delta_{23}^d\right)_{LL} \left(\delta_{23}^d\right)_{RR} \left(504 Q_4 x f_6(x) - 72 Q_4 \bar{f}_6(x) + 24 Q_5 x f_6(x) + 120 Q_5 \bar{f}_6(x)\right) + \left(\delta_{23}^d\right)_{RL}^2 \left(204 Q_2 x f_6(x) - 36 Q_3 x f_6(x)\right) + \left(\delta_{23}^d\right)_{LR}^2 \left(204 \bar{Q}_2 x f_6(x) - 36 \bar{Q}_3 x f_6(x)\right) + \left(\delta_{23}^d\right)_{LR} \left(\delta_{23}^d\right)_{RL} \left(-132 Q_4 \bar{f}_6(x) - 180 Q_5 \bar{f}_6(x)\right) \right\},$$

(3)
where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, $m_{\tilde{q}}$ is the average squark mass involved in the box diagram, $m_{\tilde{g}}$ is the gluino mass and the functions $f_{\tilde{g}}(x)$ and $\tilde{f}_{\tilde{g}}(x)$ are given by:

\[
\begin{align*}
    f_{\tilde{g}}(x) &= \frac{6(1 + 3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}, \\
    \tilde{f}_{\tilde{g}}(x) &= \frac{6x(1 + x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}.
\end{align*}
\]

(4)

Note that $f_{\tilde{g}}(x = 1) = 1/20$ while $\tilde{f}_{\tilde{g}}(x = 1) = -1/30$, therefore they cancel a lot in the combination $24x f_{\tilde{g}}(x) + 66 \tilde{f}_{\tilde{g}}(x) = -1$ for $x = 1$.

The matrix elements of the operators $Q_1$, $Q_2$ are defined as

\[
\begin{align*}
    \langle \bar{B}_s|Q_1|B_s \rangle &= \frac{2}{3}f_{\bar{B}_s}^2 M_{\bar{B}_s}^2 B, \\
    \langle \bar{B}_s|Q_2|B_s \rangle &= -\frac{5}{12}f_{\bar{B}_s}^2 M_{\bar{B}_s}^2 \left(\frac{M_{\bar{B}_s}}{\bar{m}_b + \bar{m}_s}\right)^2 B_S, \\
    &\equiv -\frac{5}{12}f_{\bar{B}_s}^2 M_{\bar{B}_s}^2 \bar{B}_S.
\end{align*}
\]

(5, 6)

The matrix elements of the other operators in Eq.(2) can be obtained in terms of those given in Eqs.(5,6). The bag factors $B$ and $B_S$ parameterize the non perturbative contributions to the matrix elements and have been evaluated \[14\] on the lattice to be $B = 0.9 \pm 0.1$ and $\bar{B}_S = 1.25 \pm 0.1$.

Similar equations apply to the calculation of the mass difference of $D^0-\bar{D}^0$ system. Using the same naive estimate of hadronic matrix element used in Ref.[10], the results may be summarized in Table 3. Note that, while for the $B_s$, the hadronic matrix elements discussed above, have been obtained with some rigor, those for the $D$ meson assume an universal bag parameter of unity.

As one can see by comparing Table 3 with Table 1 and Table 2, the recent (and coming) data do provide significant improvement on the limits on $(\delta_{12}^{B_s})_{AB}$. 

6
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$x$ & $\sqrt{\text{Re} (\delta_{12}^{u})_{LL}^{2}}$ & $\sqrt{\text{Re} (\delta_{12}^{u})_{LR}^{2}}$ & $\sqrt{\text{Re} (\delta_{12}^{u})_{LL} (\delta_{12}^{u})_{RR}}$ \\
\hline
0.3 & $2.58 \times 10^{-2}$ & $3.43 \times 10^{-2}$ & $8.52 \times 10^{-3}$ \\
1.0 & $5.46 \times 10^{-2}$ & $1.72 \times 10^{-2}$ & $9.49 \times 10^{-3}$ \\
4.0 & $1.28 \times 10^{\alpha|B-1}$ & $1.90 \times 10^{-2}$ & $1.34 \times 10^{-2}$ \\
\hline
\end{tabular}
\caption{Values of $\text{Re} (\delta_{ij})_{AB} (\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$. The upper part of the table are derived from saturating $\Delta m_{D} < 0.479 \times 10^{-10}$ MeV by squark mixing contribution. The lower part of the table are based on suggestive values $\Delta m_{B_{s}} = 8, 17, 45$ ps$^{-1}$. We use an average squark mass $m_{\tilde{q}} = 500$ GeV and choose different values of $x = m_{\tilde{q}}^{2}/m_{\tilde{q}}^{2}$. The constraints on $(\delta_{ij})_{RR}$ are the same as those on $(\delta_{ij})_{LL}$ in the Table.}
\end{table}

3 SUSY contributions to $\gamma$

The clean measurement of $\gamma$ has been a challenge, leading to several attempts at providing feasible techniques to measure it. SUSY contributions to $\gamma$ therefore vary, depending on the method used. We therefore first discuss the various methods proposed to measure $\gamma$. The original suggestion \[15\] for cleanly measuring $\gamma$ involved the decays $B^{\pm} \rightarrow D^{0} K^{\pm}$ and $D^{0}_{CP} K^{\pm}$, where $D^{0}_{CP}$ stands for the CP eigenstate of $D$. However, since it is virtually impossible to tag the flavor of the $D$ meson, the method was improved. Ref. \[16\] considered the $D^{0}$ produced, to subsequently decay to at least two final states. The mode $B^{\pm} \rightarrow D^{0*} K^{*\pm}$ was proposed in Ref. \[17\]. For the purpose of this paper the arguments made to the generic mode $B^{\pm} \rightarrow D^{0} K^{\pm}$ applies to all the methods in Ref.\[15, 16, 17\]. Alternative modes involving $B_{s}$ mesons have also been suggested to measure $\gamma$. They
include $B_s^0/\bar{B}^0_s \rightarrow D_s^\pm K^\pm$ [18] (see Fig. 1) and its final state vector meson analogue $B_s^0/\bar{B}^0_s \rightarrow D_{s}^{*\pm} K^{*\pm}$ [19]. One may note that all the modes discussed above only have tree level contributions. Other methods involving $B \rightarrow K\pi$ together with $B \rightarrow KK$, etc., which include penguin contribution, have also been considered [20]. However, they involve theoretical assumptions like the inherent use of SU(3) or factorization assumption.

All CP asymmetries arise from a relative phase between two decay channels to the same final state. This may arise either due to two or more contributions to the direct decay or, due to the oscillation of the initial or final state neutral meson.

Let us consider the CP asymmetry appearing in charged decay modes $B^\pm \rightarrow D^0 K^\mp$ or $\bar{D}^0 K^\mp$. In SM, the first stage of the decay subprocess at quark level is either due to $b \rightarrow u(\bar{c}s)$, which is severely Cabbibo suppressed but complex in the Wolfenstein (or Chau-Keung) convention [21], or, due to $b \rightarrow c(\bar{u}s)$ which is doubly Cabbibo suppressed and real in the same convention. By observing decays of $D^0$ and $\bar{D}^0$ to a common final state $f$ (which may be a CP eigenstate), one achieves the required interference. The relevant effective Hamiltonian for the quark level process $b \rightarrow u(\bar{c}s)$, in SM is

$$\frac{1}{2} \left( \frac{g_2^2}{M_W^2} \right) V_{cs}^* V_{ub} (\bar{u}_L \gamma_\mu b_L)(\bar{s}_L \gamma_\mu c_L) , \quad (7)$$

where $V_{cs}^* V_{ub} \sim A \lambda^3 e^{-i\gamma}$; while for the quark level process $b \rightarrow c(\bar{u}s)$, it is,

$$\frac{1}{2} \left( \frac{g_2^2}{M_W^2} \right) V_{us}^* V_{cb} (\bar{c}_L \gamma_\mu b_L)(\bar{s}_L \gamma_\mu u_L) , \quad (8)$$

where $V_{us}^* V_{cb} \sim A \lambda^3$. The relative phase gives rise to CP violating parameter $\gamma$ directly.

In the scenario of squark mixing, the effective Hamiltonian for $b \rightarrow u(\bar{c}s)$ is

$$\mathcal{H}_{\text{eff}}^{b\rightarrow u(\bar{c}s)} = - \frac{\alpha_s^2}{108m_\tilde{q}^2} (\delta_{12}^u)_{LL}(\delta_{23}^d)_{LL} \left( 24x f_6(x) + 66 \tilde{f}_6(x) \right)(\bar{u}_L \gamma_\mu b_L)(\bar{s}_L \gamma_\mu c_L) + \cdots , \quad (9)$$

with $x = m_{\tilde{u}}^2/m_{\tilde{d}}^2$. We only list the contribution from the channel LL in chirality. Other amplitudes due to the insertion of other $\delta$ parameters are easily obtained from Eq. (3).

For SUSY contribution to the $\Delta B = 1$ amplitude to be relevant to CP asymmetry, it has to be a sizable contribution to the decay amplitude. To estimate the SUSY contribution we take $x = 1$, $24x f_6(x) + 66 \tilde{f}_6(x) = -1$ and optimistically use $\alpha_s^2 \sim 0.02$, $|V_{ub}| \sim 0.003$, $(\delta_{12}^u)_{LL} \sim 0.06$, $(\delta_{23}^d)_{LL} \sim 0.7$. The amplitude ratio of the SUSY to the SM is about

$$\frac{(\alpha_s^2/108)10^{-4} \text{ GeV}^{-2}(100 \text{ GeV}/M_\tilde{q})^2 \times 0.06 \times 0.7}{(4G_F/\sqrt{2}) \times 0.003} \sim \left( \frac{100 \text{ GeV}}{M_\tilde{q}} \right)^2 \times 10^{-2} . \quad (10)$$
Assuming that the SUSY $\Delta B = 2$ box diagrams dominate in the $B_s - \overline{B}_s^0$ and $D^0 - \overline{D}^0$ oscillations, the same ratio can be expressed more directly in terms of measured quantities as,

$$\frac{3\sqrt{\Delta M_D \Delta M_{B_s}}}{f_D f_{B_s} \sqrt{M_D M_{B_s}}} \sqrt{2} \left| \frac{4G_F |V_{ub}|}{\sqrt{2}} \right| \sim 10^{-4}. \quad (11)$$

The numerical result is based on the central values of the parameters, $\Delta M_D = 0.07$ ps$^{-1}$, $f_D = 0.2$ GeV, and $f_{B_s} = 0.23$ GeV, as well as the suggestive value $\Delta M_{B_s} = 45$ ps$^{-1}$. If one takes $\tilde{m} = 500$ GeV in Eq. (10), the number comes up to be $10^{-4}$ as in Eq. (11).

Therefore even after taking the parameters to be favorable to the SUSY contribution, such one loop contribution is still much smaller than the highly KM suppressed SM tree amplitude. (Even if the color suppression of the contribution of the KM operator to $B^+ \to D^0 K^+$ is taken into account, SUSY is still very much a minor contribution to decay amplitudes). Therefore we conclude that SUSY contribution to the $\Delta B = 1$ box diagram (Fig. 2) is irrelevant to the CP asymmetry as long as we limit our consideration to $\delta_{d_{23}}$ and $\delta_{u_{12}}$. The same conclusion can be applied to all the $B$ decay modes.

One may hence conclude that the only possible SUSY contributions to the CP asymmetries arise from initial state $\Delta B = 2$ or final state $\Delta D = 2$ transition, or both. Among the various methods to determine $\gamma$, $B_s \to D_s^\pm K^\mp$ get contributions from initial state $B_s^0 - \overline{B}_s^0$ oscillation, $B^\pm \to D^0 K^\mp$ get contributions from final state $D^0 - \overline{D}^0$ oscillation and and $B_s \to D^0 \phi$ get contributions from both $B_s^0 - \overline{B}_s^0$ and $D^0 - \overline{D}^0$ oscillations.

We first consider the contributions from initial state $B_s^0 - \overline{B}_s^0$ oscillation. Since SUSY particles are heavy, we assume that $\Gamma_{12}$ is not modified by SUSY and parameterize the SUSY contributions by $M_{12}^{SUSY} = M_{12}^{SM} y e^{i\eta}$. Hence, we have,

$$\frac{\Gamma_{12}}{M_{12}} = \frac{\Gamma_{12}^{SM}/M_{12}^{SM}}{1 + M_{12}^{SUSY}/M_{12}^{SM}} = \frac{s e^{i\phi}}{1 + y e^{i\eta}} . \quad (12)$$

$s$ and $\phi$ are SM parameters, with $s \sim O(10^{-2})$ and $\phi \approx 0$. In terms of the SUSY parameters discussed earlier, for the $LL$ chirality we have,

$$y = \frac{\alpha_s}{216 \tilde{m}_q^2} \frac{1}{3} M_{B_s} f_{B_s} f_{D_s} B_S \left( \frac{24 x f_6(x) + 66 \tilde{f}_6(x)}{M_{12}^{SM}} \right) \left| \left( \delta_{d_{12}} \right)^2_{LL} \right| .$$

The expression for $M_{12}^{SM}$ may be taken from Ref.[22]. Using,

$$\Delta M = -2 \text{Re} \left( \frac{j}{p} (M_{12} - \frac{i}{2} \Gamma_{12}) \right) , \quad (13)$$
the expressions for $\Delta M$ including SUSY may be written as,

$$\Delta M = 2|M_{12}^{SM}| \text{Re}(1 + 2y \cos \eta + y^2 - \frac{s^2}{4} - is(\cos \phi + y \cos(\phi - \eta)))^{\frac{1}{2}}$$

$$= 2|M_{12}^{SM}| \sqrt{(1 + 2y \cos \eta + y^2)(1 + O(s^2))} \quad (14)$$

The effect of SUSY on the meson mixing parameter $(q/p)$ can be expressed as,

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^{SM*}(1 + ye^{-i\eta})}{M_{12}^{SM}(1 + ye^{i\eta})}. \quad (15)$$

Since, $M_{12}^{SM}$ is real for $B_s$, the argument $\theta_{B_s}$ of $(q/p)$ is given by,

$$\theta_{B_s} = \frac{1}{2} \tan^{-1}\left(\frac{-y^2 \sin(2\eta) - 2y \sin \eta}{1 + y^2 \cos(2\eta) + 2y \cos \eta}\right). \quad (16)$$

$y$ may be solved using Eq. (14) as a function of $\eta$, leading to the oscillation phase $\theta_{B_s}$ as a function of $\eta$. Using the upper limit of $\Delta M_{B_s} = 45 \text{ps}^{-1}$ and theoretical estimate of $M_{12}^{SM}$, from Ref. [22], we find that values of $\theta_{B_s}$ are allowed in the full range $(-45^\circ, +45^\circ)$. If, however, $M_{12}^{SUSSY} \gg M_{12}^{SM}$, i.e. in the limit, $y \to \infty$, we get $\theta_{B_s} = -\eta = -\text{Arg} \left((\delta_{23}^d)^2\right)$.

The mode $B_s \to D_s^\pm K^\pm$ has been proposed to measure $\sin^2 \gamma$ (see Fig. 1). In the presence of SUSY contributions to $B_s^0 - \bar{B}_s^0$ oscillation, the angle $\gamma$ gets modified, $\gamma \to \gamma' = \gamma_{KM} \pm \theta_{B_s}$, where, $\theta_{B_s}$ is given by Eq.(14) and $\gamma_{KM}$ is the contribution from Kobayashi-Maskawa phase. The contribution from SUSY to both the modes $B_s \to D_s^\pm K^\pm$ and $B_s \to (\psi/J)\phi$ are identical. This is very much different from the KM predictions in which the asymmetry in $B_s^0 \to \psi/J + \phi$ is negligible while that is $B_s^0 \to D_s^- K^+$ is large. Note that the phase $\theta_{B_s}$ can be measured directly using the mode $B_s \to (\psi/J)\phi$, which has a large branching ratio and should be easier to measure.

In decays of the type $B^\pm \to D^0 K^\pm$, as discussed earlier, $\gamma$ is measured by utilizing a possibility of interference between the two quark level processes $b \to c \bar{u}s$ and $b \to u \bar{c}s$. Along the decay chain the $c$ or $\bar{c}$ produce in the final state a $D^0$ or $\bar{D}^0$ mesons respectively. The two contributions are added and interfere if both $D^0$ or $\bar{D}^0$ decay to the same final state $f_D$ and have a relative phase $\gamma$. Here, $f_D$ is one of the states that both $D$ and $\bar{D}$ can decay into, such as $K^-\pi^+$ or CP eigenstates $K^K^-, \pi^+\pi^-, K_s\pi^0$ or $K_s\phi$. In fact the whole discussion can be applied to the modes in which final state $K^+$ is replaced by $\pi^+$ or $\rho^+$. In the presence of $D^0-\bar{D}^0$ oscillation there are additional contributions to
this process. As discussed in details in Refs.[23, 24], there are many different types of CP violation that can manifest themselves in such decays. SUSY can have potentially large contribution to the $D^0-\bar{D}^0$ oscillation, even providing a large phase to the oscillation. One can estimate the $D^0-\bar{D}^0$ oscillation phase, \( \theta_D \), by repeating the procedure used to determine \( \theta_{B_s} \), except that here \( \Gamma_{12}^{SM}/M_{12}^{SM} \) cannot be ignored. In the SM model, \( \gamma \) is large, and \( \theta_D \) is small. However, if SUSY were to dominate, \( \theta_D = -\text{Arg}((\delta_{12}^u)_{AB}(\delta_{12}^d)_{CD}) \), with A,B,C,D=(L,R), could give large contributions to CP asymmetries even if \( \gamma \) is small, especially \( \text{Arg}((\delta_{12}^u)^2_{LL}) \) or \( \text{Arg}((\delta_{12}^u)^2_{RR}) \). Ref.[24] has considered the various possible CP violating asymmetries in detail for arbitrary \( \theta_D \). First of all, there is the phase in the $B^+$ decay amplitude, which in KM model, is exactly the \( \gamma \) parameter. Then there is the phase in the $D^0-\bar{D}^0$ oscillation, \( \theta_D = \text{arg}(q_D/p_D) \) (here, \( q_D \) \( p_D \) are the composition amplitudes in the $D-\bar{D}$ system). In addition, the differences in strong final state phase shifts in $B^+$ decays (\( \Delta_B \)) and in D decays (\( \Delta_D \)) may become relevant for some CP observables. There is CP violation of the type proportional to \( \sin \gamma \sin \Delta_B \) which is due to the CP asymmetry in the decays $B^\pm \to DK^\pm$, the final state phase shift is needed to produce the CP asymmetry for charged $B$ decay as expected. There is CP violation proportional to \( \sin \theta_D \sin \Delta_D \) which is similar to the CP asymmetry in the decays $D \to f_D$. There is CP violation proportional to \( \sin \theta_D \cos \Delta_D \) which is due to $D-\bar{D}$ oscillation. Finally, there is CP violation of the type proportional to \( \sin(\gamma + \theta_D) \cos \Delta_B \) which is due to interference between $B \to D$ decays and the subsequent $D-\bar{D}$ mixing. Last category is of course most interesting. In the KM model, \( \gamma \) is large, however \( \theta_D \) is small, while in the SUSY model we consider, \( \gamma \) is small but \( \theta_D = \text{arg}((\delta_{12}^u)_{AA}) \) can be quite large (here, \( (AA) \) can be either $LL$ or $RR$). Of course, in the fourth type of CP violation listed above, \( \theta_D \) in SUSY model can duplicate the effect of \( \gamma \) in KM model, however as discussed in Ref.[24] in details, there are enough CP asymmetries that one can measure to distinguish the two contributions in principle.

Finally, we have so far avoided discussing $B$ decays that can receive significant contributions from penguin or chromo-dipole moment types of diagrams such as, at quark level, $b \to d\bar{s}s$, $b \to s\bar{s}s$, $b \to d\bar{d}d$ or $b \to s\bar{d}d$, either in KM or in SUSY models. They contribute partially to $B \to \pi\pi$, $B \to K\pi$, $B \to K\phi$ and other processes. While the $\Delta B = 1$ box diagram SUSY contribution is still negligible for these processes as long
as one considers only $\delta^d_{23}$ and $\delta^u_{12}$ parameters, as discuss in Ref.\cite{25}, the SUSY penguin and/or chromo-dipole moment types of contributions can play important role in their CP asymmetries. This issue is discussed in Ref.\cite{26}.

4 Conclusion

We list below the results of our study of $(\delta^d_{LL,RR})_{23}$ and $(\delta^u_{LL,RR})_{12}$ on the CP asymmetries in $B_d$ or $B_s$ decays.

(I) We use the stringent $D^0-\bar{D}^0$ mixing data to obtain tighter limits on $(\delta^u_{LL,RR})_{12}$. Based on suggestive values of $B_s-\bar{B}_s$\cite{12}, we illustrate the amount of improvement can be made on the constraint of $(\delta^d_{LL,RR})_{23}$ in the near future.

(II) For CP asymmetry in $B$ decays, SUSY can give large contribution to $B_s$ decays due to $B_s-\bar{B}_s$ oscillation, but cannot give large contribution to the complex phase in the decay amplitude via $\Delta B = 1$ box diagram. Thus SUSY produces CP asymmetries different from those in the KM model. For example, while in KM the $B^\pm \rightarrow D^0 K^\pm$ process has large phase $\gamma$ and $B_s \rightarrow (\psi/J)\phi$ has negligible CP violating phase, however in SUSY, both processes receive the same CP phase due to the $B_s-\bar{B}_s$ oscillation.

(III) For $B_d$ decay modes or charged $B^\pm$ decays modes, the CP asymmetries due to the above $\delta$‘s are negligible. However an exception is $B^\pm \rightarrow D^0 K^\pm$, where there can be SUSY contribution to the CP asymmetry due to the final state $D^0-\bar{D}^0$ oscillation.

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**Standard Model Contribution**

\[ \bar{B}_s(b\bar{s}) \rightarrow D_s^-(\bar{c}s) \sim V_{ub}V_{cs}^* \sim A\lambda^3 e^{-i\gamma} \]

\[ B_s(\bar{b}s) \rightarrow K^+(\bar{s}u) \]

Fig. 1 Quark diagrams for the tree-amplitudes in SM for the process \((B_s/\bar{B}_s) \rightarrow K^+D_s^-\).

**Squark Mixing Contribution**

\[ \bar{B}_s(b\bar{s}) \rightarrow D_s^-(\bar{c}s) \sim \delta_{23}^d \delta_{12}^u \]

\[ B_s(\bar{b}s) \rightarrow K^+(\bar{s}u) \sim \delta_{23}^{d*} \delta_{12}^{u*} \]

Fig. 2 The Box-graph contribution in SUSY via squark mixing to the process \((B_s/\bar{B}_s) \rightarrow K^+D_s^-\).