Accretion and propeller torque in the spin-down phase of neutron stars: The case of transitional millisecond pulsar PSR J1023+0038

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ABSTRACT

The spin-down rate of PSR J1023+0038, one of the three confirmed transitional millisecond pulsars, was measured in both radio pulsar (radio millisecond pulsar, RMSP) and X-ray pulsar (low-mass X-ray binary, LMXB) states. The spin-down rate in the LMXB state is only about 27 per cent greater than the spin-down rate in the RMSP state. The inner disc radius, \( r_{\text{in}} \), obtained recently by Ertan for the propeller phase, which is close to the co-rotation radius, \( r_{\text{co}} \), and insensitive to the mass-flow rate, can explain the observed torques together with the X-ray luminosities, \( L_x \). The X-ray pulsar and radio pulsar states correspond to accretion with spin-down (weak-propeller) and strong-propeller situations, respectively. A several times increase in the disc mass-flow rate takes the source from the strong propeller with a low \( L_x \) to the weak propeller with a higher \( L_x \) powered by accretion on to the star. The resultant decrease in \( r_{\text{in}} \) increases the magnetic torque slightly, explaining the observed small increase in the spin-down rate. We have found that the spin-up torque exerted by accreting material is much smaller than the magnetic spin-down torque exerted by the disc in the LMXB state.

Key words: accretion, accretion discs – pulsars: individual (PSR J1023+0038, XSS J12270–4859, IGR J18245–2452).

1 INTRODUCTION

Observations of transitional millisecond pulsars (tMSPs) in their radio pulsar (RMSP) and X-ray pulsar (LMXB) states provide an excellent opportunity to test the torque and accretion luminosity models. At present, there are three confirmed tMSPs (Archibald et al. 2009; Papitto et al. 2013; Bassa et al. 2014). These sources undergo occasional transitions between LMXB and RMSP states within time-scales of about days to weeks (Papitto et al. 2013; Stappers et al. 2014), and remain in one or the other state for months to years, much longer than dynamical or viscous time-scales (see e.g. Jaodand et al. 2016).

The three tMSPs, namely PSR J1023+0038, XSS J12270–4859, and IGR J18245–2452, show remarkably similar behaviour. In the RMSP state, the sources show radio pulses and orbitally modulated X-ray emission. In the LMXB state, radio pulses disappear, and the systems exhibit three well-defined characteristic modes. Coherent X-ray pulsations are observed only in the high mode with an X-ray luminosity \( L_x \sim 5 \times 10^{33} \text{ erg s}^{-1} \), which is ~5–7 times greater than the \( L_x \) in the low mode. In the LMXB state, the sources remain in the high mode for about 70 to 80 per cent of the time, and in the low mode in the remaining time. The stars also exhibit occasional short X-ray flares with luminosities of \( L_x \sim 5 \times 10^{33} \text{ erg s}^{-1} \) (Linares 2014; Papitto et al. 2015; Jaodand et al. 2016). The most likely origin of the pulsed X-ray luminosities of XSS J12270–4859 and PSR J1023+0038 (hereafter J1023) seems to be the mass flow on to the neutron star channelled by the field lines (Archibald et al. 2015; Papitto et al. 2015). All three sources are spinning down with \( \dot{P} \lesssim 6.83 \times 10^{-20} \text{ s s}^{-1} \) for J1023 (Archibald et al. 2013 ), \( \dot{P} \lesssim 1.11 \times 10^{-20} \text{ s s}^{-1} \) for XSS J12270–4859 (Ray et al. 2015), and \( \dot{P} < 1.3 \times 10^{-17} \text{ s s}^{-1} \) for IGR J18245–2452 (Papitto et al. 2013) in the RMSP state. A measurement of \( \dot{P} \sim 8.7 \times 10^{-21} \text{ s s}^{-1} \) has been reported for the high mode of the LMXB state of J1023 (Jaodand 2016).

These discoveries are rather surprising, because, according to conventional models, neutron stars are expected to be in the propeller phase without any accretion for low X-ray luminosities (Illarionov & Sunyaev 1975). In many theoretical models, the inner radius of the disc, \( r_{\text{in}} \), is estimated to be close to the conventional Alfvén radius, \( r_{\text{A}} \), while mass accretion on to the star is expected when the innermost disc extends inward of the co-rotation radius, \( r_{\text{co}} \), at which the speed of field lines co-rotating with the star equals the Kepler speed of the disc matter. This is in sharp contrast with the properties of tMSPs observed in quiescence at low X-ray luminosities. For J1023, the high mode with pulsed X-rays suggests accretion with spin-down, in a weak-propeller state with \( r_{\text{in}} > r_{\text{co}} \), while the \( r_{\text{in}} \) is estimated to be about six times greater than the \( r_{\text{co}} \). Such an \( r_{\text{A}} \) has no physical meaning since it remains even outside the light cylinder. This clearly indicates that the actual \( r_{\text{in}} \) in the spin-
down phase could be much smaller than the $r_A$. As shown recently (Ertan 2017), the maximum $r_a$ at which the propeller mechanism can work is much smaller than the $r_A$, but not much larger than the $r_{\text{co}}$. The critical accretion rate, $\dot{M}_{\text{crit}}$, for the transition to accretion with spin-down (weak propeller) is orders of magnitude smaller than the rate corresponding to $r_A \approx r_{\text{co}}$. This is consistent with the luminosities of tMSPs during the transitions between the RMSP and LMXB states. In the propeller phase, the disc mass-flow rate, $\dot{M}_{\text{in}}$, dependence of $r_a$ is much weaker than that of $r_A$, and variations in magnetic torque in response to changes in $\dot{M}_{\text{in}}$ are much smaller than in conventional torque models with $\dot{M}_{\text{in}} \propto r_A$. These predictions of the model can be tested with precisely determined properties of J1023 in different states of the spin-down phase. We pursue these results obtained by Ertan (2017) to model the spin-down torques and the luminosities of the tMSPs. Our model is described in Section 2. In Section 3, we test the model with the rotational properties and X-ray luminosities of J1023 in the LMXB and RMSP states, and estimate the dipole field strengths and $\dot{M}_{\text{crit}}$ for the tMSPs in compact LMXBs (also known as ‘redbacks’; see Linares 2014 for a review) that have been observed only in the RMSP state. We discuss our results in Section 4.

2 THE SPIN-DOWN PHASE

The magnetosphere of the star is defined as the region of closed field lines in which matter and the field lines rotate together. The inner disc radius, $r_{\text{in}}$, is expected to be close to the radius of the magnetosphere. The inner disc and the field lines interact in a boundary between $r_a$ and $r_{\text{in}} + \Delta r$. In the interaction region, the field lines cannot slip through the disc, because the diffusion time-scale of the magnetic field lines (which is comparable to the viscous time-scale, $t_{\text{visc}}$) is much longer than the interaction time-scale $t_{\text{in}} = |\Omega_\ast - \Omega_k|^{-1}$ (Fromang & Stone 2009), where $\Omega_\ast$ is the rotational angular velocity of the star and $\Omega_k$ is the Keplerian angular velocity of the disc matter. The field lines interact with the disc inflate and open up on the interaction time-scale (Aly 1985; Lovelace, Romanova, & Bisnovatyi-Kogan 1995; Hayashi, Shibata, & Mashimoto 1996; Miller & Stone 1997; Uzdensky, Königl, & Litwin 2000; Uzdensky 2004). In the propeller phase, the matter leaves the disc along open field lines, and the field lines reconnect on dynamical time-scale completing the cycle (Lovelace, Romanova, & Bisnovatyi-Kogan 1999; Ustyugova et al. 2006). This could be imagined as a continuous process since $t_{\text{dyn}} < t_{\text{visc}}$. Numerical simulations indicate that the field lines outside a radially narrow boundary remain disconnected from the disc (Lovelace et al. 1995).

The critical condition for a steady propeller effect is defined by the first principles: (i) at a radius greater than $r_1 = 1.26 r_{\text{co}}$, the speeds of the field lines co-rotating with the star exceed the escape speed, $v_{\text{esc}}$, and (ii) the angular momentum transferred to the gas at $r_{\text{in}} > r_1$ should accelerate the matter to the speed of the field lines within $t_{\text{out}}$. The maximum inner disc radius, $r_{\text{in,max}}$, at which the propeller condition is satisfied is estimated as

$$R_{\text{in,max}}^{25/8} (1 - R_{\text{in,max}}^{-3/2}) \simeq 8.4 \alpha^{25/12} M_1^{-7/6} M_{12}^{-7/20} \mu_{26} P_{-3}^{-13/12}$$

(Ertan 2017), where $R_{\text{in,max}} = r_{\text{in,max}}/r_{\text{co}}$, $\alpha = (\sigma/0.1)$, $M_1 = (M/M_\odot)$, $M_{12} = M_\odot/(10^{12} \text{M}_\odot)$, $\mu_{26} = \mu/10^{26} \text{G cm}^3$, and $P_{-3} = P/10^{-3}$ is the rotational period of the star in milliseconds. We have obtained equation (1) rearranging equation (9) in Ertan (2017), which gives $r_{\text{in,max}}$ in terms of $r_A$. We write $r_{\text{in,max}} \approx \eta r_{\text{in,max}}$, with $\eta \lesssim 1$ likely to be close to unity as magnetic stresses decrease sharply with $r$. The $M_{\text{in}}$ dependence of $r_{\text{in}}$ is much weaker than that of $r_A \propto M^{-2/7}$.

The radial width of the outflow region, $\delta r$, is likely to be very narrow. Estimated as the radial diffusion length of the inner disc within several $t_{\text{in}}$, it is found to be even smaller than the pressure scale height of the disc $h$. The interaction does not have to take place only in such a tiny boundary where the propeller condition is satisfied. While matter can leave the system from a radially very thin region of width $\delta r$ at $r_a$, matter expelled by field lines in a wider region, between $r_{\text{in}}$ and $r_{\text{in}} + \Delta r$, could return back to the disc at larger radii. We will use the term ‘backflow’ for matter that is expelled from the boundary and returns back to the disc, and reserve ‘outflow’ for the matter leaving the system from $r_{\text{in}} < r < r_{\text{in}} + \Delta r$ with speeds greater than $v_{\text{esc}}$. A steady propeller state is reached, as shown in Fig. 1, when the net mass-flow rate at each point along the disc becomes equal to $\dot{M}_{\text{in}}$. When there is a continuous mass backflow from the boundary to outer radii, the total flow along the disc is inwards. Despite the pile-up resulting from the backflow, $M_{\text{out}} = M_{\text{back}} + (\dot{M}_{\text{in}} - \dot{M}_{\text{acc}})$, the gas co-rotates with the field lines, but not reaching $v_{\text{esc}}$, which is possible when $r_{\text{in}} > r_1$ with $M_{\text{out}} = M_{\text{in}}$. For a constant $M_{\text{in}}$, what happens if $r_{\text{in}}$ is instantaneously set up between $r_{\text{co}}$ and $r_1$? There could be an efficient backflow from the boundary. Without any accretion or outflow, the pile-up outside $r_{\text{in}}$ grows in time and pushes $r_{\text{in}}$ toward $r_{\text{co}}$. Backflowing matter from the boundary to larger radii of the disc moves inwards and piles up outside $r_{\text{in}}$ on the viscous time-scale across this region. This could take a long time due to the low $M_{\text{in}}$. Furthermore, both $r_{\text{in}}$ and the field strength increase as $r_{\text{in}}$ decreases toward $r_{\text{co}}$, which requires more pile-up to push $r_{\text{in}}$ further inward. It is likely that $r_{\text{in}}$ decreases very slowly for a steady and low $M_{\text{in}}$. This problem could be studied through numerical simulations. In this work, we assume that a long-lasting propeller state could be maintained when $r_{\text{co}} < r_{\text{in}} < r_1$, as well as when $r_1 < r_{\text{in}} < r_{\text{in,max}}$.

For a weak-propeller state, we estimate that the inner disc cannot penetrate inside $r_{\text{co}}$, and $r_{\text{in}}$ remains equal to $r_{\text{co}}$ even when $M_{\text{in}}$ is much greater than $M_{\text{crit}}$. The critical accretion rate corresponding to $r_{\text{co}} \approx r_{\text{co}}$ for the start for the spin-up phase, as long as $r_A > r_{\text{co}} \approx r_{\text{co}}$. This is because $t_{\text{in}}$ increases as $r_{\text{in}}$ approaches $r_{\text{co}}$, and the gas co-rotating with the field lines, but not reaching $v_{\text{esc}}$, can flow onto the star coupling to the field lines at $r_{\text{co}}$. The only way for $r_{\text{in}}$ to penetrate inside $r_{\text{co}}$ is that the viscous stresses should dominate the magnetic stresses at $r_{\text{co}}$, which is possible when $r_A$ comes close to $r_{\text{co}}$. This will happen at accretion rates much larger than the rates estimated for J1023 in its LMXB state (Ertan 2017). Such high accretion rates $\dot{M}_{\text{in}} \sim 10^{17} \text{g s}^{-1}$ are typical throughout the evolutionary epoch when accretion is spinning the neutron star.

Figure 1. A simplified picture for the strong-propeller phase. When $r_{\text{co}} > 1.26 r_{\text{co}}$, a steady propeller phase could be built up. A long-lasting propeller phase could prevail when $r_{\text{co}} < r_{\text{in}} < 1.26 r_{\text{co}}$, since $r_{\text{in}}$ is very insensitive to $M_{\text{in}}$ and mass accumulation. In this case (with only backflow), $r_{\text{in}}$ moves slowly inward due to increasing pile-up at the inner disk. Accretion (weak propeller) starts when $r_{\text{in}} \approx r_{\text{co}}$, and the accretion torque is negligible compared to the spin-down torque (see the text).

Spin-down phase of neutron stars
up towards millisecond periods (Alpar et al. 1982; Radhakrishnan and Srinivasan 1982). From an evolutionary point of view, tMSPs are at the end of the LMXB epoch, with \( M_{\text{in}} \) from their companions much reduced as they proceed through the transition to the RMSP epoch, though they could still show X-ray outbursts due to viscous disc instabilities like those observed in IGR J18245–2452 (Linares 2014).

In the weak-propeller phase of a tMSP, we can safely take \( r_{\text{in}} = r_{\text{co}} \) when \( \eta r_{\text{in,max}} \) becomes smaller than \( r_{\text{co}} \), and write

\[
\frac{r_{\text{in}}}{r_{\text{co}}} = \max \{ \eta r_{\text{in,max}}, r_{\text{co}} \}. \tag{2}
\]

For a narrow boundary width \( \Delta r \) and a mass accretion rate \( M_\bullet \), the total torque acting on the star becomes

\[
\Gamma = -\frac{\mu^2}{r_{\text{in}}^3} \left( \frac{\Delta r}{r_{\text{in}}} \right) + \left( GM_{\text{co}} \right)^{1/2} M_\bullet \tag{3}
\]

where \( G \) is the gravitational constant, and we take \( M_\bullet = M_{\text{in}} \) in the accretion phase (\( r_{\text{in}} = r_{\text{in}} \)) and \( M_\bullet = 0 \) in the strong-propeller phase (\( r_{\text{in}} > r_{\text{co}} \)). The first term on the right-hand side of equation (3) is the spin-down torque resulting from the disc–field interaction, while the second term is the spin-up torque due to angular momentum transfer by matter flowing onto the star. In the weak-propeller phase, the spin-down torque dominates the spin-up torque. Note that the spin-down torque in equation (3) also includes and is much greater than the angular momentum loss through mass outflow (see Ertan 2017).

The X-ray luminosity powered by accretion onto the star is given by

\[
L_\bullet = \frac{GM_{\text{in}}}{R} \tag{4}
\]

where \( R \) is the radius of the star. In the strong-propeller phase, the X-rays produced by viscous heating in the disc are emitted mostly from the inner disc with luminosity

\[
L_\text{d} = \frac{GMM_{\text{in}}}{2r_{\text{in}}} \tag{5}
\]

(see e.g. Frank, King, & Raine 2002) where \( r_{\text{in}} = \eta r_{\text{in,max}} > r_{\text{co}} \) in this phase. In our model for J1023, \( L_\bullet \) with \( M_\bullet = M_{\text{in}} \), the weak propeller, represents the observed \( L_\text{d} \) in the high X-ray mode, while \( L_\bullet = L_\text{d} \) in the low X-ray mode (strong propeller) with negligible accretion onto the star. We estimate that the \( M_{\text{in}} \) values for these two modes in the LMXB state are similar. In the RMSP state, \( L_\bullet = L_\text{d} \) like in the low mode, but with a several times lower \( M_{\text{in}} \).

Recently, simultaneous X-ray and radio continuum observations of J1023 in the LMXB state clearly showed an anti-correlation between the radio brightness and X-ray luminosity (Bogdanov et al. 2018). The radio flux, which is relatively steady in the high mode, starts to increase with transition to the low mode, reaches a maximum, and decreases to the pre-transition level within about 30 s of the transition back to the high mode. This picture is consistent with transition from the weak propeller to the strong propeller on a short viscous time-scale of the innermost disc. These transitions could take place with small occasional enhancements in the mass-outflow rate which could hinder the accretion on to the star briefly. The resultant density gradients at the inner disc lead to a rapid increase in \( M_{\text{in}} \) back to the pre-enhancement level. Eventually, the accretion resumes switching on the high mode. The observed increase (decrease) in the radio brightness during the low mode is likely to be associated with the increasing (decreasing) rate of mass outflow, which is a likely source of the unpulsed radio continuum emission.

\section*{3 Application to PSR J1023+0038}

The X-ray flux of J1023 has been measured several times with XM-M–Newton in the LMXB state in both low and high X-ray modes (Archibald et al. 2015; Bogdanov et al. 2015; Jaodand et al. 2016). For the estimated distance \( d \approx 1365 \text{ pc} \) (Deller et al. 2012), \( L_\bullet \sim 3 \times 10^{31} \text{ erg s}^{-1} \) in the high mode and \( \sim (3 - 5) \times 10^{32} \text{ erg s}^{-1} \) in the low mode. In the RMSP state, \( L_\bullet \sim 1 \times 10^{31} \text{ erg s}^{-1} \) (Archibald et al. 2010). The X-ray emission in the RMSP state is modulated with the orbital period (see the discussion below). Characteristic properties of J1023 and other tMSPs in different states (Linares 2014; Archibald et al. 2015; Papitto et al. 2015; Jaodand et al. 2016) can be studied in the propeller model at hand.

The period, \( P \), and the period derivative, \( \dot{P} \), of J1023 were determined in the RMSP phase (Archibald et al. 2013). Recently measured \( P \) in the high X-ray mode shows that \( P \) increased by \( \sim 27 \) per cent in the LMXB state compared to the torque measured earlier in the RMSP state (Jaodand et al. 2016).

To explain observed changes in the \( P \) and \( L_\bullet \) of J1023, we propose the following: (1) in the high X-ray mode, the source is in the weak-propeller phase with \( r_{\text{in}} = r_{\text{co}} \), and \( L_\bullet \) is produced by the accretion to the neutron star surface with \( M_{\text{in}} = M_\bullet \); (2) \( L_\bullet \) in the low X-ray mode is produced by the inner disc with \( r_{\text{in}} \approx r_{\text{co}} \) with the same \( M_{\text{in}} \) as in the accretion phase, while \( M_\bullet = 0 \); (3) in the RMSP state, \( M_{\text{in}} \) is several times less than in the LMXB state, and the source is in the strong-propeller phase with \( L_\bullet = L_\text{d} \) and \( r_{\text{in}} > r_{\text{co}} \). We do not address the sporadic X-ray flares observed from the source for about 2 per cent of the time (see e.g. Jaodand et al. 2016 for a review of proposed models).

For the torque calculation, we assume that \( \Delta r r_{\text{in}} \) is independent of \( M_{\text{in}} \) and \( r_{\text{in}} \), and use the same \( \Delta r r_{\text{in}} \) in the LMXB and RMSP states. For the transition from the LMXB to the RMSP phase, we only decrease \( M_{\text{in}} \) by a factor which produces the observed \( L_\text{d} \) in the RMSP state. The resultant increase in \( r_{\text{in}} \) is the main reason for the decrease in the magnitude of the spin-down torque.

In the LMXB phase, since \( M_{\text{in}} \) is very close to the transition rate, the source could make occasional transitions between the weak-propeller and strong-propeller phases while \( r_{\text{in}} \approx r_{\text{co}} \). This explains the high- and low-mode luminosities. In the low mode, a small fraction of \( M_{\text{in}} \) could still be flowing onto the star preventing the pulsed radio emission. In the RMSP state, since \( r_{\text{in}} > r_{\text{co}} \), the accretion is not allowed, and \( L_\bullet = L_\text{d} \) decreases by a factor \( 2r_{\text{in}}/R \) in comparison with \( L_\text{d} \) in the weak-propeller phase.

Model parameters are compared with observational properties of J1023 in Table 1. These results are obtained with \( \eta = 0.78 \), \( \Delta r r_{\text{in}} = 0.1 \), and magnetic dipole moment \( \mu_{30} = 0.5 \). It is seen that the model can produce the \( P \) values measured in the RMSP and LMXB states, consistently with the observed X-ray luminosities. The \( M_{\text{in}} \) in the LMXB state is about six times greater than that in the RMSP state. This changes \( r_{\text{in}} \) only by a small factor of \( \sim 1.09 \) due to very weak dependence of \( r_{\text{in}} \) on \( M_{\text{in}} \). The resultant increase in the spin-down torque is also small \((\sim 27 \text{ per cent})\) in good agreement with the observations. Since the accretion and dipole torques are only a few per cent of the torque produced by the disc–field interaction, the onset of accretion with the transition to the weak-propeller phase does not affect \( P \) significantly. With \( \mu_{30} = 4.7 \), estimated in our model, the dipole torque is found to be about five and seven times smaller than the observed torques in the RMSP and LMXB states, respectively. We note that for \( r_{\text{in}} \) scaling with \( r_{\text{in}} \) in the propeller phase, the magnetic torque would be proportional to \( M_{\text{in}}^{\beta} \), implying changes by a factor of \( \sim 5 \), not in agreement with the measured torques.
For the RMSP state, the observed $L_x \sim 10^{32} \text{ erg s}^{-1}$ given in Table 1 is the 0.5–10 keV luminosity estimated by Archibald et al. (2010). From the spectral fits, Li et al. (2014) estimated that the X-ray flux was found to be modulated with the orbital motion, which is likely to be due to an intra-binary shock produced by the interaction of pulsar wind with the matter outflowing from the companion. A shock region close to the inner Lagrangian point that is eclipsed when the companion star is between the Earth and the neutron star could explain the observed modulations (Archibald et al. 2010; Bogdanov et al. 2011; see also Li et al. 2014 for a different interpretation). From the orbital modulations, it is expected that at least half of the X-rays are produced at the shock region in the RMSP state. These observations do not exclude a continuous emission from the disc with $r_{\text{in}}$ close to $r_{\text{co}}$ characterized by the conditions around the inner disc in the RMSP state. A large fraction of the total $L_x$ is emitted by the inner-binary shock modulated by the orbital motion, while the remaining smaller fraction could be produced by the inner disc. The X-ray luminosity estimated in our model is compatible with (smaller than) the continuous portion of the total X-ray luminosity in the RMSP state. We note that the disc emission spectrum could significantly change in the RMSP state due to strong propeller mechanism which can produce not only mass outflow but also hot matter around the inner disc.

The other two tMSPs, XSS J12270–4859 and IGR J18245–2452, also show X-ray modes similar to those of J1023. For XSS J12270–4859, $P = 1.69$ ms and $\dot{P} = 1.11 \times 10^{-20} \text{ s}^{-1}$ (Ray et al. 2015). In the RMSP state, there is only an upper limit to the pulsed X-ray luminosity ($L_{x,\text{pulsed}} < 1.6 \times 10^{31} \text{ erg s}^{-1}$). For $\mu_{26} = 0.5$, we estimate $M_{\text{crit}} \simeq 2 \times 10^{13} \text{ g s}^{-1}$, which gives $L_{x} \simeq 8 \times 10^{32} \text{ erg s}^{-1}$, and $L_{x} \simeq 4 \times 10^{33} \text{ erg s}^{-1}$ during the transition. This seems to be consistent with the observations within the distance uncertainties of the source (Linares et al. 2014). For IGR J18245–2452, $P = 3.93$ ms, $P < 1.3 \times 10^{-17} \text{ s}^{-1}$, and $L_{x} \lesssim 10^{32} \text{ erg s}^{-1}$ in the RMSP state (Papitto et al. 2013). For this source to be in the strong-propeller phase with $L_{x} = 10^{32} \text{ erg s}^{-1}$, we estimate the lower limits $\mu_{26} > 1$ and $\dot{P} > 3 \times 10^{-20} \text{ s}^{-1}$.

The tMSPs belong to the redback population of MSPs in compact LMXBs. Among the other redbacks, $L_x$, $P$, and $\dot{P}$ are known for three systems observed only in the RMSP states. These redbacks, with properties similar to tMSPs in the RMSP state, are thought to be strong tMSP candidates (Linares et al. 2014). For these sources, we have estimated $\mu_{26}$, $M_{\text{in}}$, and $M_{\text{crit}}$ (Table 2). The estimated $M_{\text{crit}}/M_{\text{in}}$ ratios are very close to unity for PSR J1723–2834 and PSR J2215+5135, indicating that these sources are indeed good candidates for transition to the accretion phase with a few times increase in $M_{\text{in}}$. For PSR J2339–0533, we estimate that $M_{\text{in}}$ is about seven times lower than $M_{\text{crit}}$.

### Table 1

| Source       | Lx,obs (10^{31} \text{ erg s}^{-1}) | Lx,model (10^{31} \text{ erg s}^{-1}) | P_{\text{obs}} (s s^{-1}) | P_{\text{model}} (s s^{-1}) | M_{\text{in}} (10^{13} \text{ g s}^{-1}) | M_{\text{crit}}/M_{\text{in}} | \mu_{26} |
|--------------|-------------------------------------|---------------------------------------|--------------------------|--------------------------|---------------------------------|-----------------------------|---------|
| J1023        | 2.94–3.31                           | 0.31–0.55                             | 2.97                     | 2.97                     | 1.6                             | ~2                          | ~0.5    |
| J1723        | 8.665 × 10^{-21}                    | 8.72 × 10^{-21}                       | 1.6                      | 1.6                      | 5.0                             | ~3                          | ~1.2    |
| J2215        | 0.16                                | 0.16                                  | 1.8 × 10^{-2}            | 0                        | 1.1                             | ~7                          | ~0.8    |

### Table 2

| Source       | P (ms) | P_{\text{obs}} (10^{-20} \text{ s}^{-1}) | Lx (10^{31} \text{ erg s}^{-1}) | M_{\text{in}} (10^{13} \text{ g s}^{-1}) | Model | M_{\text{crit}}/M_{\text{in}} | \mu_{26} |
|--------------|--------|----------------------------------------|---------------------------------|---------------------------------|-------|-----------------------------|---------|
| J1023        | 1.855a | 0.75a                                  | 2.4b                            | 7.0                             | ~2    | ~0.5                        |         |
| J1723        | 2.609c | 3.3c                                   | 2.76b                           | 5.0                             | ~3    | ~1.2                        |         |
| J1723        | 2.884d | 1.41f                                 | 2.76b                           | 1.1                             | ~7    | ~0.8                        |         |

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**4 CONCLUSIONS**

Using the inner disc radius, $r_{\text{in}}$, estimated earlier for the propeller phase by Ertan (2017), we have modelled the torque acting on J1023 in the RMSP and LMXB states. The X-ray luminosity, $L_x$, in the high mode of the LMXB state is produced by mass accretion on to the star in the weak-propeller phase, while the $L_x$ in the RMSP state is explained by the emission from the inner disc in the strong-propeller phase.

When the source is in the strong-propeller state, $M_{\text{in}} \sim 2.8 \times 10^{17} \text{ g s}^{-1}$, the disk produces $L_{x} \sim 10^{32} \text{ erg s}^{-1}$, a small fraction of the orbitally modulated total $L_{x}$. A six-fold increase in $M_{\text{in}}$ takes the source to the weak-propeller phase with an accretion luminosity from the star’s surface explaining the high $L_{x}$. The change in $r_{\text{in}}$ between the high X-ray mode in the LMXB state (weak propeller) and the RMXB state (strong propeller) is only ~ 9 per cent because of the very weak $M_{\text{in}}$ dependence of $r_{\text{in}}$. The onset of accretion produces a negligible spin-up torque in comparison with the spin-down torque in the weak-propeller phase. The small decrease
in \( r_{\text{in}} \) increases the magnitude of the spin-down torque by \( \sim 30 \) per cent, which is in good agreement with the observations (Table 1). Occasional transitions between the strong-propeller and the weak-propeller phases when \( r_{\text{in}} \approx r_{\text{co}} \) could cause the observed transitions between the low X-ray mode and the high X-ray mode. With similar \( \dot{M}_{\text{in}} \), mass accretion can produce \( L_{\times} \) in the high mode, and when accretion is hindered, \( L_{\times} \) with \( r_{\text{in}} \approx r_{\text{co}} \) can explain \( L_{\times} \) in the low mode. In the LMXB state, \( \dot{M}_{\text{in}} \) is very close to \( \dot{M}_{\text{crit}} \) for J1023. This could be the reason for occasional transitions between the high and low modes in the LMXB state.

The scale \( r_{\text{in}} \ll r_{\text{co}} \) and the weak \( \dot{M}_{\text{in}} \) dependence of \( r_{\text{in}} \) in our propeller model thus explain the torques and luminosities \( L_{\times} \) in the different states. In the RMSP state, the spin-down torque is supplied by the disc, as in the LMXB state, and not by dipole radiation. Radio pulsar activity proceeds with the disc until mass accretion disrupts it in either of the modes in the LMXB state.

Finally, we have found that the redbacks PSR J1723–2834 and PSR J2215+5135, which are observed in the RMSP state, are strong tMSP candidates, close to their propeller-accretion transition rates (Table 2). These sources could show transition to the LMXB phase with small increases in their \( \dot{M}_{\text{in}} \).

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