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To cite this article: L Wang et al 2006 J. Phys.: Conf. Ser. 48 1312

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Abstract. Theoretical improvement and practical application of table-rotating method to optimize fuzzy control rules are proposed in this paper. The theorem is developed that one m×n rule table need to rotate 2(m+n-2) times at most to get all the rule tables, which are the different combinations of all the m×n×q rules. Then the fuzzy controller based on improved fuzzy table-rotating optimization is designed and applied to the systems with time delay. The simulation results show the feasibility and validity of the fuzzy controller based on the improved table-rotating rules. Compared with the conventional PID controllers and Smith predictor controllers, the advantages of this optimizing rules approach are less overshoot, shorter setting time and better robust.

1. Introduction
Since L.A.Zadeh founded fuzzy sets theories in 1965, the fuzzy controller has acquired much research and valuable achievement in theories. [1] Furthermore, it has been widely applied to industrial processes. Rules table is a key problem in fuzzy controller designs. [2] Then the optimization of fuzzy sets and fuzzy rules is the core of research in fuzzy controller theory [3-5].

In order to improve the performance of fuzzy controller, a theoretical improvement of table-rotating optimization method is proposed in this paper. Its practical application in open-loop time-delay systems is studied.

2. Another section of your paper
Consider the fuzzy controller with error \( E \) and error change \( EC \) as inputs and control \( u \) as output, shown in figure 1. The rules table of fuzzy controller is shown in table 1.

![Figure 1. Fuzzy control system.](image)

The fuzzy table-rotating optimization method is composed of two parts. [6,7] One is rotating in outer circle just as \( E=NB \cup PB \) and \( EC=NB \cup PB \) and the other is rotating in inner circle just as
$E=NS \cup PS$ and $EC=NS \cup PS$. In order to detect the level of the system errors, the objective function is provided.

**Table 1.** fuzzy rules table.

| E | N | Z | P |
|---|---|---|---|
| N | N | N | Z |
| Z | N | Z | P |
| P | Z | P | P |

$$J(k, y(k)) = \| g(k) - y(k) \| \rightarrow \text{min}$$

where $y(k)$ is the actual output of system and $g(k)$ is the expected output of system.

Firstly, the theorem is developed that one $m \times n$ rule table needs to rotate $2(m+n-2)$ times at most to get all the rule tables. The table-rotating method is composed of inner circle and the outer circle rotating rules. The both rules assemble a whole rules which is close to the optimization rules.

Secondly, through the farther research of the rotation algorithm is shown in figure 3. We get the following conclusions.

**Lemma 1:** One $3 \times 3$ rules table is only composed of eight rules, the possible rule number of one $3 \times 3$ rules table is 27, which the optimum rules combinations consist of 8 rules given by lemma 1.

**Lemma 2:** The position clockwise and converse clockwise is equal completely.

**Lemma 1** and **lemma 2** can be described as lemma 3:

**Lemma 3:** According to one $3 \times 3$ rules table, only eight rotations is needed.

Mathematics induction method shows that lemma 1 to lemma 3 can be generalized as theorem 1:

**Theorem 1:** The all possible rules combinations of one $m \times n$ rules table are obtained only by rotating $2(m+n-2)$ times clockwise or counter clockwise.

**Table 2.** Rule-rotating of fuzzy controller.

| E  | NB | NS | ZO | PS | PB |
|----|----|----|----|----|----|
| NB |    |    |    |    |    |
| NS |    |    |    |    |    |
| ZO |    |    |    |    |    |
| PS |    |    |    |    |    |
| PB |    |    |    |    |    |

3. The applications in time delay system

Based on improved fuzzy table-rotating optimization, the fuzzy controller is designed and applied to the open-loop systems with pure time delay $\tau$ in figure 2.

$$A \xrightarrow{u} R \xrightarrow{y} S \xrightarrow{e^{-\tau}} D \xrightarrow{x}$$

Figure 2. Open-loop time-delay systems.

Where $R$ is the model of fuzzy controller and $S$ is the controlled process model. According to the $S$ supposing that:

If $u = u_i$ then $y = y_i \quad i = 1, 2, 3, \ldots, p$

Where $u = \{u_1, u_2, \ldots, u_p\}, y = \{y_1, y_2, \ldots, y_p\}$, $u_i$, $y_i$ are normal fuzzy sets. Then the $S$ is denoted as follow:
According to the R supposing, the fuzzy rules are: if \( A = A_i \) then \( u = u_i \), \( i = 1, 2, \ldots, p \) where \( A = \{ A_1, A_2, \ldots, A_p \} \), \( u = \{ u_1, u_2, \ldots, u_p \} \). \( A_i \) is normal fuzzy sets, then fuzzy controller \( R \) is:

\[
R_i = A_i^T \times u_i = y_i^T \times u_i = \begin{bmatrix}
    y_{i1} \wedge u_{i1} & y_{i1} \wedge u_{i2} & \cdots & y_{i1} \wedge u_{im}
    
    y_{i2} \wedge u_{i1} & y_{i2} \wedge u_{i2} & \cdots & y_{i2} \wedge u_{im}
    
    \vdots & \vdots & \ddots & \vdots
    
    y_{im} \wedge u_{i1} & y_{im} \wedge u_{i2} & \cdots & y_{im} \wedge u_{im}
\end{bmatrix}
\]

(1)

Because of:

\[
u_{ij} \wedge y_{ik} = y_{ik} \wedge u_{ij}
\]

(3)

\[
S_j = R_i^T
\]

(4)

\[
S = \bigcup_{i=1}^{p} S_j, R = \bigcup_{i=1}^{p} R_i
\]

(5)

then,

\[
S = R^T, R = S^T
\]

(6)

but the fuzzy controller model obtained by expert experience doesn’t equal to transpose of the controlled process model:

\[
R \neq S^T
\]

(7)

then,

\[
y_i = A_i \circ R \circ S \neq A_i
\]

(8)

This will result in process output error. In order to minimize the error, the fuzzy controller model needs to be adjusted.

Suppose that the \( r \) is the fuzzy ruler for the time delay system, then

\[
x_i = A_i \circ r \circ S \circ D = A_i \Rightarrow A_i \circ r \circ R^T \circ D = A_i
\]

(9)

and supposing the language variable domains of \( E, EC \). \( U \) are normalized as \([-1, 1]\). According to the classical fuzzy controller structure [8], input and output variables use symmetry, even distributing, full fold triangle subjection function. Variable \( E \) and \( EC \) are divided into \( N = 2J + 1 \) that denoted as \( E_j \) and \( EC_j \), \( j = 1, 2, \ldots, N \). Variable \( U \) is divided into \( 2N-1 = 4J + 1 \geq 5 \) degree that denoted as \( U_k \), \( k = 1, 2, \ldots, 2N - 1 \). \( e_i, ec_i \) and \( b_k \) are the peak value of \( E_i, EC_i, U_k \). Supposing

\[
S = \frac{2}{N - 1} = \frac{1}{J}
\]

then fuzzy group \( E_i \) is denoted as:

\[
E_i(e) = \begin{cases}
e - e_{i-1} & e_{i-1} \leq e \leq e_i \\
ed_i - e_{i-1} & e_{i-1} \leq e \leq e_i
\end{cases}, (i = 2, \ldots, p - 1)
\]

(10)

The fuzzy ruler is described with uniform as

\[
R_{E_i}. \text{If } e \in E_i \text{ and } ec \in EC_j \text{ then } u \text{ is } U_{ij}
\]

(11)

\( b_k \) that is the centre of the subjection function \( U_k \) is to denote \( U_{ij} \), when input is \( e_i \leq e < e_{i+1}, ec \leq ec \leq ec_{j+1} \), then four rulers are activated:
If $e$ is $E_i$ and $ec$ is $EC_j$, then $u$ is $U_{ij}$;
If $e$ is $E_{i+1}$ and $ec$ is $EC_j$, then $u$ is $U_{(i+1)j}$;
If $e$ is $E_{i+1}$ and $ec$ is $EC_j+1$, then $u$ is $U_{(i+1)(j+1)}$;
\[
\mu_1 = \mu_{p1} \land \mu_{U_{ij}}, \quad \mu_2 = \mu_{p2} \land \mu_{U_{(i+1)j}} \quad \mu_3 = \mu_{p3} \land \mu_{U_{ij}}, \quad \mu_4 = \mu_{p4} \land \mu_{U_{(i+1)(j+1)}}
\]
then,
\[
U_{\text{crisp}} = \frac{b_1 \mu_1 + b_{i(j+1)} \mu_2 + b_{(i+1)j} \mu_3 + b_{(i+1)(j+1)} \mu_4}{\mu_1 \mu_2 \mu_3 \mu_4}
\]
The fuzzy controller of time delay system is designed in figure 3.

Figure 3. Fuzzy control system with time delay.

Supposing the output of $G_{c1}$ is $U_{\text{crisp}}^{(1)}$ and output of $G_{c2}$ is $U_{\text{crisp}}^{(2)}$, then
\[
U_{\text{crisp}}^{(1)}(t + \tau) = b_1 \mu_1 + b_{i(j+1)} \mu_2 + b_{(i+1)j} \mu_3 + b_{(i+1)(j+1)} \mu_4
\]
\[
U_{\text{crisp}}^{(2)}(t + \tau) = b_1 \mu_1 + b_{i(j+1)} \mu_2 + b_{(i+1)j} \mu_3 + b_{(i+1)(j+1)} \mu_4
\]

It is shown that $b_1$ of $G_{c1}$ can be adjusted to satisfy $U_{\text{crisp}}^{(2)}(t) = U_{\text{crisp}}^{(1)}(t)$. So the table-rotating method can be used to control time delay system.

4. Simulation test

Time delay system is denoted as
\[
G(s) = \frac{K}{Ts + 1} e^{-\tau} = \frac{s}{s + 1} e^{-\tau}
\]
where $e = r - y$, $r$ is the set value and $y$ is the system output, $\tau$ is the delay time, respectively. The original ruler group is described as table 3 and isosceles triangle fuzzy subjection function is shown in figure 4. Lemma 1-3 and Theorem are used to optimize the rulers.

|     | NB | NM | NS | ZO | PS | PM | PB |
|-----|----|----|----|----|----|----|----|
| NB  | PB | PB | PB | PB | PM | ZO | ZO |
| NM  | PB | PB | PB | PB | PM | ZO | ZO |
| NS  | PM | PM | PM | PM | ZO | NS | NS |
| ZO  | PM | PM | PS | ZO | NS | NM | NM |
| PS  | PS | PS | ZO | NM | NM | NM | NM |
| PM  | ZO | ZO | NM | NB | NB | NB | NB |
| PB  | ZO | ZO | NM | NB | NB | NB | NB |
Simulation test results with MATLAB tools are shown in figure 5. Compared with the conventional PID controllers and Smith predictor controllers, the advantages of this optimizing rules approach are less overshoot, shorter setting time and better robust.

5. Conclusion
Theoretical improvement and practical application of table-rotating method to optimize fuzzy control rules are proposed in this paper. The theorem is developed that one m×n rule table need to rotate 2(m+n-2) times at most to get all the rule tables, which are the different combinations of all the m×n×q rules. Then the fuzzy controller based on improved fuzzy table-rotating optimization is designed and applied to the open-loop systems with pure time delay. The simulation results show the feasibility and validity of the fuzzy controller based on the improved table-rotating rules. Compared with the conventional PID controllers and Smith predictor controllers, the advantages of this optimizing rules approach are less overshoot, shorter setting time and better robust.

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