Bound on a diffuse flux of ultra-high energy neutrinos in the ADD model

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Abstract

The search for ultra-high energy downward-going and Earth-skimming cosmic neutrinos by the Surface Detector array of the Pierre Auger Observatory (PAO) is analyzed in the ADD model with \(n\) extra flat spatial dimensions. We assumed that the diffuse neutrino flux \(dN_\nu/dE_\nu\) is equal to \(kE_\nu^{-2}\) in the energy range \(10^{17}\) eV – \(2.5 \times 10^{19}\) eV. Taking into account that no neutrino events were found by the PAO, we have estimated an upper bound on a value of \(k\). It is shown that this bound can be stronger than the upper bound on \(k\) recently obtained by the Pierre Auger Collaboration, depending on \(n\) and (\(n+4\))-dimensional gravity scale \(M_D\).

1 Introduction

Ultra-high energy (UHE) cosmic neutrinos play an important role in particle physics and astrophysics. They help us to determine the composition of UHE

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cosmic rays, as well as their origin. In particular, the detection of UHE neutrino candidates by the Pierre Auger Observatory (PAO) in coincidence with gravitational wave (GW) events could constrain the position of the source of GW [1]. Measuring the scattering of UHE cosmic neutrinos off atmospheric nucleons can probe a new physics that could modify the neutrino-nucleon cross section at energies above $10^{17}$ eV. The first observation of high-energy astrophysical neutrinos was done by the IceCube Collaboration in 2014 [2]. It was found that the neutrino-nucleon cross section agrees with predictions in the range 6.3 TeV – 980 TeV [3].

To detect neutrino events with energies above $10^{17}$ eV, more powerful cosmic rays facilities such as the PAO [4] and Telescope Array [5] are needed. Recently, the Pierre Auger Collaboration reported on searches for downward-going (DG) UHE neutrinos [6]. The DG incline air showers [7]–[9] are initiated by cosmic neutrinos moving with large zenith angle which interact in the atmosphere near the Surface Detector (SD) array of the PAO. Note that the background from hadronic showers is very small at $E_\nu > 10^{17}$ eV and negligible above $10^{19}$ eV [10]. The data were collected by the SD in the zenith angle bins $60^\circ$ – $75^\circ$ and $75^\circ$ – $90^\circ$ for a period which is equivalent of 6.4 years of a complete PAO SD working continuously.

The PAO also searched for Earth-skimming (ES) air showers [11]–[12] induced by upward tau neutrinos at zenith angles $90^\circ$ – $95^\circ$ which interact in the Earth producing tau leptons. In their turn, the tau leptons escape the Earth and initiate showers close to the SD.

No neutrino candidates were found. Assuming the diffuse flux of UHE neutrinos to be

$$\frac{dN}{dE_\nu} = k E_\nu^{-2}$$

in the energy range $1.0 \times 10^{17}$ eV – $2.5 \times 10^{19}$ eV, the upper single-flavor limit to the diffuse flux of UHE neutrinos was obtained by the Pierre Auger Collaboration

$$k < 6.4 \times 10^{-9} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.$$  \hspace{1cm} (2)

This bound is approximately four times less than the Waxman-Bachall bound on cosmic neutrino production in optically thin sources [13]. Some cosmogenic neutrino models with a pure proton composition injected at the sources were rejected by the Auger limit (2). The maximum sensitivity of the SD of the PAO lies at the neutrino energies around 1 EeV [6]. The IceCube fit of the diffuse single-flavor astrophysical neutrino flux [14], if extrapolated
to 1 EeV, would give $E_\nu^2 dN/dE_\nu = 0.3 \times 10^{-9}$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$.

The calculations of the exposure of the SD array of the PAO were done under assumption that neutrino-nucleon collisions in the atmosphere are described by the SM interactions (in CC and NC channels).

The goal of the present paper is to estimate the single-flavor bound on the diffuse flux of UHE cosmic neutrinos in the model with extra dimensions. Namely, the ADD model [15] with $n$ extra flat spatial dimensions will be considered. We will assume that neutrino energy spectrum is of the form $E_\nu^{-2}$ in the range $10^{17}$ eV – $2.5 \times 10^{19}$ eV.

2 Space-time with large extra dimensions (the ADD model)

Let us briefly remind readers the main features of the ADD model. The large extra dimensions scenario was postulated in refs. [15]. Its metric looks like

$$ds^2 = g_{\mu\nu}(x) \, dx^\mu \, dx^\nu + \eta_{ab} \, dy^a \, dy^b,$$

where $\mu, \nu = 0, 1, 2, 3$, $a, b = 1, \ldots, n$, and $\eta_{ab} = (-1, \ldots, -1)$. All $n$ extra dimensions are compactified with a size $R_c$.

There is a hierarchy relation between the fundamental mass scale in $D = 4 + n$ dimensions, $M_D$, and 4-dimensional Planck mass, $M_{Pl}$,

$$M_{Pl} = V_n \, M_D^{2+n},$$

where $V_n$ is a volume of the compactified dimensions. $V_n = (2\pi R_c)^n$ if the extra dimensions are of a toroidal form. In order $M_D$ to be of order one or few TeV, the radius of the extra dimensions should be large. The compactification scale $R_c^{-1}$ ranges from $10^{-3}$ eV to 10 MeV as $n$ runs from 2 to 6.

All SM gauge and matter fields are assumed to be confined to a 3-dimensional brane embedded into a $(3 + n)$-dimensional space, while the gravity lives in all $D$-dimensional space-time called bulk.

In linearized gravity we present $D$-dimensional metric $G_{AB}$ in the form ($A, B = 0, 1, \ldots, 3 + n$)

$$G_{AB}(x, y) = \eta_{AB} + \frac{2}{M_D^{1+n/2}} \, h_{AB}(x, y).$$
Performing the KK mode expansion of the gravitational field $h_{AB}(x, y)$, we obtain the graviton interaction Lagrangian density

$$L_{\text{int}}(x) = -\frac{1}{M_{\text{Pl}}} T^{\mu\nu}(x) \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x) ,$$

(6)

where $n$ labels the KK excitation level and $\bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi}$ is a reduced Planck mass. $T^{\mu\nu}(x)$ is the energy-momentum tensor of the matter on the brane. The masses of the KK graviton modes $h^{(n)}_{\mu\nu}$ are

$$m_n = \frac{\sqrt{n_a n^a}}{R_c} , \quad n_a = (n_1, n_2 \ldots n_n) .$$

(7)

So, a mass splitting is $\Delta m \sim R_c^{-1}$ and we have an almost continuous spectrum of the gravitons.

One can see from (6) that the coupling of both massless and massive graviton is universal and very small ($\sim 1/\bar{M}_{\text{Pl}}$). Nevertheless, all cross sections with real and virtual production of the massive KK gravitons are defined by the gravity scale $M_D$, but not by $\bar{M}_{\text{Pl}}$.

### 3 Neutrino-nucleon cross sections

We intend to consider ultra-high energies of cosmic neutrino, $E_\nu > 10^{17}$ eV. It corresponds to a large center-of-mass energy of the neutrino-proton collision, $\sqrt{s} \gtrsim 14$ TeV. Thus, we are in a transplanckian region $\sqrt{s} \gg M_D$. At the transplanckian energies a scattering is described by classical physics [17] - [18], provide an impact parameter $b$ is larger than the $D$-dimensional Schwarzschild radius $R_S$

$$R_S(s) = \frac{1}{\sqrt{\pi}} \frac{1}{M_D} \left[ \frac{8 \Gamma \left( \frac{n+3}{2} \right) \sqrt{s}}{n + 2} \frac{M_D}{M_D} \right]^{\frac{1}{n+1}} .$$

(8)

$R_S$ as a function of the neutrino energy $E_\nu$ is presented in tabs. 1-3 in Appendix A ($s = 2m_N E_\nu$). The transplanckian regime corresponds to the conditions

$$\sqrt{s} \gg M_D , \quad \theta \sim (R_S/b)^{n+1} ,$$

(9)

where $\theta$ is the scattering angle [17].

In the eikonal approximation [16], which is valid at small momentum transfer ($-t \ll s$) the leading part of the scattering amplitude is obtained
by sumation of all ladder diagrams with graviton exchange in the $t$-channel \cite{17} - \cite{18}. The tree-level exchange of the $D$-dimensional graviton gives the following Born amplitude

$$A_{\text{Born}}(q^2) = \frac{s^2}{M_D^{n+2}} \int \frac{d^n q_n}{t - q_n^2} = \pi^{n/2} \Gamma(1 - n/2) \left( \frac{-t}{M_D^2} \right)^{n/2 - 1} \left( \frac{s}{M_D^2} \right)^2,$$  \hspace{1cm} (10)

where $q_n$ is the momentum transfer in the extra dimensions. Summing all loop diagrams leads to the eikonal formula

$$A_{\text{eik}}(s, t) = -2is \int d^2 b e^{iqb} \left[ e^{\chi(b)} - 1 \right],$$  \hspace{1cm} (11)

with the eikonal phase

$$\chi(b) = \frac{1}{2s} \int \frac{d^2 q}{(2\pi)^2} e^{-iqb} A_{\text{Born}}(q^2).$$  \hspace{1cm} (12)

It has been calculated in \cite{17}-\cite{18} (see also \cite{20}) to be

$$\chi(b) = \left( \frac{b}{b_c} \right)^n,$$  \hspace{1cm} (13)

where

$$b_c = \left[ \frac{(4\pi)^{n/2-1} s \Gamma(n/2)}{2 M_D^{n+2}} \right]^{1/n}.$$  \hspace{1cm} (14)

An energy dependence of $b_c$ for different values of $n$ and $M_D$ is presented in tabs. 1-3 in Appendix A.

As a result, the final expression of the eikonal amplitude (11) is given by

$$A_{\text{eik}}(s, t) = 4\pi s b_c^2 F_n(b_c q),$$  \hspace{1cm} (15)

$$F_n(y) = -i \int_0^{\infty} dxx J_0(xy) \left[ e^{ix - n} - 1 \right],$$  \hspace{1cm} (16)

where $x = b/b_c$. The eikonal representation of the scattering amplitude is a good approximation, provided $b > R_S$ \cite{17}-\cite{18}.

At UHEs the neutrino interacts essentially with the quarks (antiquarks) and gluons inside the nucleon. Let us define a fraction of the neutrino energy transferred to the nucleon

$$y = \frac{E_\nu - E'_\nu}{E_\nu} = \frac{Q^2}{xs},$$  \hspace{1cm} (17)
where $E_\nu(E'_\nu)$ is the initial (final) energy of the neutrino, and $x$ is the fraction of nucleon momentum carried by parton $i$ ($i = q, \bar{q}, g$). Taking into account above mentioned formulas, we get the differential neutrino-nucleon cross section

$$
\frac{d^2\sigma}{dxdy} = \pi s \sum_i x f_i(x, \mu^2) b_i^4(s) |F_n(bcQ)|^2,
$$

(18)

where $s = xs$, and $Q = \sqrt{ys}$. The quantities $f_i(x, \mu^2)$ are the parton distribution functions (PDFs). Following ref. [20], we put $\mu^2 = Q^2$. We use the CT14 set for the PDFs [21]. For a chosen value of $n$ we take $M_D$ to be equal to a 95% CL lower limit on $M_D$ obtained recently by the CMS Collaboration (see fig. 11 in [22]). For instance, $M_D^{\text{min}} = 2.3$ TeV (2.5 TeV) for $n = 2$ (6). In order to calculate total cross sections, we integrate (18) in the region $Q_0^2 < Q^2 < R_S^{-2}$ [20]. As in [20], we put $Q_0^2 = 0.01m_W^2$, where $m_W$ is the W-boson mass.

As it was mentioned above, the eikonal approximation can be used if $Q^2 < R_S^{-2}$ ($b > R_S$). In the rest of integration region $s \geq Q^2 > R_S^{-2}$, that corresponds to the region $b < R_S$ in the impact parameter space, one expects that the neutrino and a parton inside the nucleon will form a black hole. In such a case, the cross section can be estimated as [23]-[24]

$$
\sigma_{\nu N \rightarrow BH}(s) = \pi \sum_i \int_{(M_{\text{min}}^{\text{bh}})^2/s}^1 dx f_i(x, \tilde{\mu}^2) R_S^2(s).
$$

(19)

We put $\tilde{\mu}^2 = xs$. The dependence of $\sigma_{\nu N \rightarrow BH}$ on the choice of $\tilde{\mu}^2$ and $M_{\text{min}}^{\text{bh}}$ is discussed in [25]-[26]. For chosen $n$, $M_D$, we take $M_{\text{min}}^{\text{bh}}$ to be equal to the 95% CL lower limit on $M_{\text{bh}}$ for the same $n$ and $M_D$ obtained by the CMS Collaboration [28]. As one can see from fig. 6 in [28] and tabs. 1-3 in Appendix A, $M_{\text{min}}^{\text{bh}} \gg R_S^{-1}$ for all $E_\nu$, if $2 \leq n \leq 6$, and 2 TeV $< M_D < 6$ TeV.

The black hole production by cosmic rays was studied in a number of papers (see, for an example, [23], [25]-[27]).

As for the SM neutrino interaction, we adopt the neutrino-nucleon cross sections in [29], since the Pierre Auger Collaboration [6] has obtained limit with the use of these SM cross sections.

The total cross sections as functions of the $D$-dimensional mass scale $M_D$ and number of the extra dimensions $n$ are shown in figs. [2]. Let us note
that at $E_{\nu} > 10^{19}$ eV the cross section $\sigma_{\nu N \rightarrow BH}$ rises with $n$, while the eikonal cross section decreases. The combined effects of these two factors is that the difference of the cross sections for $n = 4$ and $n = 6$ tends to zero as $E_{\nu}$ grows (see fig. 2).

Our calculations of the cross sections is not an end in itself but it will enable us to estimate exposures for both DG and ES neutrino events at the SD array of the PAO in the ADD model and thus to put limits on the diffuse single-flavor flux of UHE neutrinos.

4 Limits on diffuse flux of UHE neutrinos in the ADD model

In [30] the following functional dependence of the DG event rate on the new physics cross section $\sigma_{\text{NP}}$ was proposed for UHE neutrino events

$$\mathcal{E}_{\text{BSM}}^{\text{DG}}(E_{\nu}) = \mathcal{E}_{\text{SM}}^{\text{DG}}(E_{\nu}) \frac{\sigma_{\text{eff}}^{\text{SM}}(E_{\nu}) + \sigma_{\text{NP}}(E_{\nu})}{\sigma_{\text{eff}}^{\text{SM}}(E_{\nu})},$$

(20)

where $\mathcal{E}_{\text{BSM}}^{\text{DG}}$ ($\mathcal{E}_{\text{SM}}^{\text{DG}}$) is the exposure of the SD of the PAO with (without) account of the new interaction. In addition, instead of $\sigma_{\text{CC}}$, an effective SM
cross section $\sigma_{SM}^{eff}$ is introduce in (20):

$$\sigma_{SM}^{eff} = \sigma_{CC} \sum_{i=e,\mu,\tau} m_{iCC}^i + 3 \sigma_{NC} m_{NC}^i + \sigma_{CC} m_{mount}.$$  (21)

Here $m_{CC}^i$ and $m_{NC}^i$ are relative mass apertures for charged current (CC) and neutral current (NC) interactions of the DG neutrinos at the PAO. The mass aperture $m_{mount}$ corresponds to the CC interaction of a $\tau$ neutrino within the mountains around the PAO. The relative mass apertures as functions of the neutrino energy where calculated using the data in Table I of ref. [31]. Note that $\sum_{i=e,\mu,\tau} m_{CC}^i + 3 m_{NC}^i + m_{mount} = 1$.

In contrast to the DG neutrino exposure, the exposure of the ES neutrinos decreases with the rise of the neutrino total cross section [30]

$$\mathcal{E}_{BSM}^{ES}(E_{\nu}) = \mathcal{E}_{SM}^{ES}(E_{\nu}) \frac{\sigma_{CC}^2(E_{\nu})}{[\sigma_{CC}(E_{\nu}) + \sigma_{NP}(E_{\nu})]^2}.$$  (22)

The formulas (20) and (22) allowed us to calculate exposures of the SD of the PAO for the period 1 January 2004 – 20 June 2013 expected in the ADD model. The PAO data on the exposures for the SM neutrino interactions in the region from $\log(E_{\nu}/eV) = 17$ to 20.5 were used (see fig. 3 taken from ref. [6]). The results of our calculations are presented in figs. 4-5.

We assume that the astrophysical flux arrives isotropically from all directions, and neutrino flavor composition is $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$. Following
Pierre Auger Collaboration, we also assume that the flux is described by a power law of the form \( N = \frac{N_{up}}{2} \). Then the upper limit on the value of \( k \) can be estimated as:

\[
k = \frac{N_{up}}{\int E_{\nu}^{-2} \mathcal{E}_{\nu} dE_{\nu}},
\]

where \( N_{up} \) is an actual value of the upper limit on the signal events which depends on the number of the observed events and total exposure:

\[
\mathcal{E}_{\nu} = \mathcal{E}_{\nu, BSM}^{DG} + \mathcal{E}_{\nu, BSM}^{ES},
\]

see eqs. (20) and (22). Since the PAO sees no events, we put \( N_{up} = 2.39 \), assuming a number of expected background events to be zero.

As one can see in fig. 1, in the ADD model the cross sections rise more rapidly with the neutrino energy than the SM cross sections. As a result, the exposure for the DG events, \( \mathcal{E}_{\nu, BSM}^{DG} \), rises, while the exposure for the ES events, \( \mathcal{E}_{\nu, BSM}^{ES} \), decreases as \( E_{\nu} \) grows (see figs. 4, 5). The expected ratio of the ES neutrinos to the DG neutrinos with zenith angle \( 75^\circ < \theta < 90^\circ \) is shown in fig. 6.

As a result, for some values of \( n \) and \( M_D \), the total expected exposure in the ADD model can be larger than the Auger exposure calculated on the assumption that the neutrino-nucleon scattering is defined by the SM interactions only. Correspondingly, an upper bound on \( k \) defined by eq. (23) can be even stronger than the bound obtained by the Pierre Auger Collaboration.
Figure 4: Left panel: the expected exposures of the SD array of the PAO for the DG neutrinos with zenith angle $75^\circ < \theta < 90^\circ$ in the ADD model. Right panel: the expected exposures of the SD array of the PAO for the ES neutrinos in the ADD model.

Figure 5: The same as in fig. 4 but for $n = 6$.

Collaboration (2). It is demonstrated by figs. 7-8, in which the PAO upper bound on the value of $k$ is also shown.

5 Conclusions

Using the exposure of the PAO for the period equivalent of 6.4 years of the complete PAO SD array working continuously, we have estimated the exposures for the neutrino induced events expected in the scenario with the large flat extra dimensions of the space-time. Both downward-going and Earth-skimming UHE cosmic neutrinos are considered.

The exposures are defined by the neutrino-nucleon cross sections in the ADD model. In the transplanckian region and large impact parameters
Figure 6: The expected ratio of the ES neutrinos to the DG neutrinos with zenith angle $75^\circ < \theta < 90^\circ$ at the SD array of the PAO as a function of the gravity scale $M_D$ for two values of $n$.

Figure 7: Left panel: the upper bound on the value of $k$ as a function of D-dimensional Planck scale $M_D$ for $n = 2$ in the ADD model. Dashed line is the PAO upper limit [6]. Right panel: the same as on the left panel, but for $n = 6$.

$b > R_S$ the eikonal approximation is valid. In such a case, the scattering amplitude is given by the exchanges of the $t$-channel massive gravitons. At small $b < R_S$, the eikonal approximation breaks down, and the production of the black holes is assumed. The dependence of the exposures on the number of extra dimensions $n$ and the gravity scale $M_D$ is obtained (figs. 4-5).

Our main goal was to calculate the single-flavor upper limit on the diffuse neutrino flux in the presence of the massive graviton interactions in the ADD model. We assumed that the flux of UHE neutrinos is proportional to $E_{\nu}^{-2}$ [1]. Our results demonstrate us that in the ADD model the upper bound on the diffuse neutrino flux can be stronger that the PAO limit [2], depending
Figure 8: Left panel: the upper bound on the value of $k$ as a function of number of extra dimensions $n$ for $M_D = 2.3\text{ TeV}$. Right panel: the same as on the left panel, but for $M_D = 4.0\text{ TeV}$.

on the parameter of the ADD model, $n$ and $M_D$. As one can see in fig. 7, it takes place for $M_D < 3.01\text{ TeV}$ (2.38 TeV), if $n = 2$ (6). For $M_D = 2.3\text{ TeV}$ it is true for $n \leq 3$ and $n \geq 6$ (left panel of fig. 8). However, with the increase of $M_D$ our bound becomes weaker than the PAO bound for all $n$ (right panel of fig. 8).

It can be understood as follows. Remember that the upper limit on the neutrino diffuse flux (1) is given by formula (23). In the presence of the extra dimensions, the neutrino-nucleon cross section grows with the neutrino energy more rapidly than the SM one (fig. 4). Correspondingly, the expected exposure for the DG neutrino events, $\mathcal{E}^{\text{DG}}_{\text{BSM}}$ (20), also rises. On the contrary, the exposure for the ES neutrino events, $\mathcal{E}^{\text{ES}}_{\text{BSM}}$ (22), decreases as the energy grows (figs. 4-5). As a result, the total expected exposure of the SD array of the PAO, $\mathcal{E}_{\text{tot}}$ (24), may be larger than the total exposure obtained by the Pierre Auger Collaboration (fig. 3), provided that the integrated increase of $E^{-2}_\nu \mathcal{E}^{\text{DG}}_{\text{BSM}}(E_\nu)$ prevails over the integrated reduction of $E^{-2}_\nu \mathcal{E}^{\text{ES}}_{\text{BSM}}(E_\nu)$. As $M_D$ grows, the upper limit on the value of $k$ tends to the PAO limit (2) from above, as one can see on the right panel of fig. 7.

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Appendix A

Here we present an energy dependence of the parameter $b_c$ \( (14) \) and $D$-dimensional Schwarzschild radius squared $R_S^2$ \( (8) \) for different values of the number of the extra dimensions $n$ and $D$-dimensional gravity scale $M_D$.

Table 1. The parameter $b_c$ and Schwarzschild radius squared $R_S^2$ for $n = 2, M_D = 2.3$ TeV as a function of the neutrino energy $E_\nu$.

| $E_\nu$, eV | $b_c$, GeV$^{-1}$ | $R_S^2$, GeV$^{-2}$ |
|-------------|------------------|-------------------|
| $1.00000000 \cdot 10^{17}$ | $1.831113 \cdot 10^{-3}$ | $5.155385 \cdot 10^{-7}$ |
| $1.50356136 \cdot 10^{17}$ | $2.245307 \cdot 10^{-3}$ | $5.906109 \cdot 10^{-7}$ |
| $1.00000000 \cdot 10^{18}$ | $5.790488 \cdot 10^{-3}$ | $1.110694 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{18}$ | $7.100285 \cdot 10^{-3}$ | $1.272433 \cdot 10^{-6}$ |
| $1.00000000 \cdot 10^{19}$ | $1.831113 \cdot 10^{-2}$ | $2.392918 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{19}$ | $2.245307 \cdot 10^{-2}$ | $2.741373 \cdot 10^{-6}$ |
| $1.00000000 \cdot 10^{20}$ | $5.790488 \cdot 10^{-2}$ | $5.155385 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{20}$ | $7.100285 \cdot 10^{-2}$ | $5.906109 \cdot 10^{-6}$ |
| $2.38298316 \cdot 10^{20}$ | $8.938727 \cdot 10^{-2}$ | $6.886017 \cdot 10^{-6}$ |

Table 2. The same as in tab. 1, but for $n = 4$.

| $E_\nu$, eV | $b_c$, GeV$^{-1}$ | $R_S^2$, GeV$^{-2}$ |
|-------------|------------------|-------------------|
| $1.00000000 \cdot 10^{17}$ | $1.679949 \cdot 10^{-3}$ | $1.161700 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{17}$ | $1.860272 \cdot 10^{-3}$ | $1.260429 \cdot 10^{-6}$ |
| $1.00000000 \cdot 10^{18}$ | $2.987419 \cdot 10^{-3}$ | $1.841171 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{18}$ | $3.308083 \cdot 10^{-3}$ | $1.997645 \cdot 10^{-6}$ |
| $1.00000000 \cdot 10^{19}$ | $5.312466 \cdot 10^{-3}$ | $2.918059 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{19}$ | $5.882969 \cdot 10^{-3}$ | $3.166054 \cdot 10^{-6}$ |
| $1.00000000 \cdot 10^{20}$ | $9.447048 \cdot 10^{-3}$ | $4.624811 \cdot 10^{-6}$ |
| $1.50356136 \cdot 10^{20}$ | $1.046108 \cdot 10^{-2}$ | $5.017857 \cdot 10^{-6}$ |
| $2.38298316 \cdot 10^{20}$ | $1.173752 \cdot 10^{-2}$ | $5.501970 \cdot 10^{-6}$ |

Table 3. The same as in tab. 1, but for $n = 6, M_D = 2.5$ TeV.
Note that the invariant energy squared of the neutrino-nucleon scattering is equal to \( s = 2m_NE_\nu \).

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