Dynamic modelling and controlling Unmanned Surface Vehicle

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Abstract. Unmanned Surface Vehicle (USV) is vessel that can move without human to control. USV is capable of moving using the thrust force and can maneuver using the deflection angle of rudder. USV has a 6-degree of freedom which are non-linear system that can be approached with the 3-degrees of freedom. The purpose of dynamic modeling of USV is to achieve the models and parameters of real USV. Non-linear least square Levenberg-Marquardt method can be applied to estimate USV model as a non-linear system. The purpose of controlling USV is to set the surge velocity and the yaw-rate so that the USV movement can be controlled. Proportional integral controller can be used to manipulate the velocity and to accelerate the response speed of the USV’s velocity. Parameters validation results show that the dynamic modeling methods could generate the parameters nearly equal to the true parameters. This is evidenced by the relatively small RMSE value in x-axis acceleration, y-axis acceleration, and yaw-axis acceleration. The result of controller testing illustrate that the designed controller can adjust the surge velocity and yaw-rate and fit the time constant controller design specification. Further, the testing show that the responses have zero steady state errors.

1. Introduction

Unmanned surface vehicle or USV is a craft that can move without human to drive it or in general is designated as autonomous [1]. USV can be classified according to its body form, which could be monohull, catamaran, or trimaran. USV movements can be controlled by controlling the thruster machine and deflection of the rudder angle. Controlling a USV need the model of the USV itself as reference to design the corresponding controller. USV essentially is a non-linear dynamic system that need parameter estimation methods to model its behavior.

Nowadays, many methods to solve non-linear least square problem have been discovered. Those are, for examples, vector fitting method, Gauss-Newton method, gradient descent method, and Levenberg-Marquardt method. From the research by Knockaert [2] it is shown that Levenberg-Marquardt method has better accuracy than vector fitting method, although the former need more time to be completed. According to Ranganathan [3], Levenberg-Marquardt method need less time than gradient descent method to accomplish the solution of non-linear problem, and, furthermore, Levenberg-Marquardt method is well functioned to practical problem. According to Gavin [4], Gauss-Newton method needs initial estimation because this method using the assumption that optimized function is locally quadratic and finding the local minimum from that quadratic function. While Levenberg-Marquardt method does not need initial estimation because this method can move between
local minimum on global minimum. Referring to the advantages of the Levenberg-Marquardt method the current research adopts Levenberg-Marquardt method to model the non-linear behaviour of USV.

Specifically in this research, Levenberg-Marquardt Non-Linear Least Square method is employed to achieve the model of USV using the input-output data from some experiments. After the model is acquired, linearization is made to that non-linear model and design the PI controller. After the controller is done, simulation is performed and some analysis is conducted on the results of USV simulation.

USV model can be approached with a 3-degree of freedom non-linear system. To control the USV, the model of USV, machine model, and rudder model is are needed, which can be achieved from experiments. Moreover, the controller is needed to control the thruster machine and rudder so the USV can move according to its mission.

The purposes of this research are to design and estimate the USV LSS-01, as shown in Figure 1, model’s parameters and also the model of rudder and thruster machine using compatible parameter identification method. A further purpose is to design the controller for controlling the surge velocity and yaw-rate of the USV.

![Figure 1. USV LSS-01](image)

2. Research Method

2.1 Unmanned Surface Vehicle (USV)

USV is ship that can move without human to drive it or called autonomous [1]. The general dynamic model of USV is expressed in 6-degree of freedom. The following are the general USV dynamic model equations [5]:

\[ M \ddot{v} = -C(v)v - D(v)v - g(\eta) + \tau + \tau_E \]  \(\text{(1)}\)

with,

\[ M = M_{RB} + M_A \]  \(\text{(2)}\)

\[ C(v) = C_{RB}(v) + C_A(v) \]  \(\text{(3)}\)

\[ D(v) = D + D_R(v) \]  \(\text{(4)}\)

\[ \tau_E = \tau_E^C + \tau_E^{wa} + \tau_E^{wi} \]  \(\text{(5)}\)

In this research, several assumptions were used so that the USV dynamics model could be simplified into a model with 3-degree of freedom. The following are the assumptions taken in this research:

a. Roll, pitch, and heave movements are ignored.

b. The vessel has a homogeneous mass distribution and symmetry in the xz-plane so \(I_{xy} = I_{yz} = 0\).

c. The center of gravitational force and the center of buoyancy are located in one vertical line, that is the z-axis.

With the aforementioned assumptions, the equation of the USV dynamics model becomes as follows:
\[ M\dot{v} = -C(v)v - (D + D_h(v))v + \tau + \tau_E \]  
\[ \nu = [u \nu_\tau]^T \]  
\[ M = \begin{bmatrix} m - X_u & 0 & 0 \\ 0 & m - Y_u & 0 \\ 0 & 0 & I_z - N_r \end{bmatrix} \]  
\[ C(v) = \begin{bmatrix} 0 & 0 & -m(x_g r + v) + Y_\nu v + Y_r r \\ m(x_g r + v) - Y_\nu v - Y_r r & -m u + X_u u \\ 0 & 0 & 0 \end{bmatrix} \]  
\[ D = \begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_\nu & Y_r \\ 0 & N_\nu & N_r \end{bmatrix} \]  
\[ D_h(v) = -\begin{bmatrix} X_u[u[u]]u \\ 0 \\ 0 \end{bmatrix} \]  
\[ \tau = \begin{bmatrix} \tau_u \\ 0 \end{bmatrix} \]  
\[ \tau_E = \frac{\tau_{U_E}^{wu} + \tau_{U_E}^{wa} + \frac{1}{2} \rho a V_w^2 c_x (y_u) a_F w}{\tau_{U_E}^{wu} + \tau_{U_E}^{wa} + \frac{1}{2} \rho a V_w^2 c_y (y_u) a_L w} \]  
\[ \frac{\tau_{U_E}^{wu} + \tau_{U_E}^{wa} + \frac{1}{2} \rho a V_w^2 c_a (y_u) a_L L_{OA}}{\tau_{U_E}^{wu} + \tau_{U_E}^{wa} + \frac{1}{2} \rho a V_w^2 c_a (y_u) a_L L_{OA}} \]  
From the above equation, by ignoring external disturbances, the mathematical equations for each coordinate axis are obtained as follows:

\[ \dot{u} = \frac{(m - Y_\nu)}{(m - X_u)} u - \frac{X_u u}{(m - X_u)} u + \frac{1}{(m - X_u)} \tau_u \]  
\[ \dot{v} = \frac{(m - Y_\nu)}{(m - X_u)} u - \frac{Y_\nu v}{(m - Y_\nu)} v - \frac{Y_\nu v}{(m - Y_\nu)} v - \frac{Y_\nu v}{(m - Y_\nu)} v + \frac{1}{(m - Y_\nu)} \nu_v \]  
\[ \dot{r} = \frac{(y_u - X_u)}{(I_z - N_r)} u - \frac{N_\nu v}{(I_z - N_r)} v - \frac{N_\nu v}{(I_z - N_r)} v + \frac{1}{(I_z - N_r)} \tau_r \]  

The above equation can be written as follows:

\[ f_1 = A_1 v \]  
\[ f_2 = A_2 u - A_3 \|u\|u + A_4 \|u\|u \]  
\[ f_3 = -A_1 \|v\|v - A_4 \|r\|r - A_5 \|v\|v + A_6 \|v\|v \]  

with \( f_1 \) is \( \dot{u} \), \( f_2 \) is \( \dot{v} \), and \( f_3 \) is \( \dot{r} \) [6]. In this respect the nomenclature related to the USV modeling is given in Table 1.

**Table 1. Nomenclature of USV**

| Symbols | Explanation |
|---------|-------------|
| \( Y_\nu \) | Y-axis added mass caused by \( \nu \) |
| Symbols | Explanation |
|---------|-------------|
| \(X_u\) | X-axis added mass caused by \(u\) |
| \(Y_v\) | Y-axis added mass caused by \(v\) |
| \(N_r\) | N-axis added mass caused by \(r\) |
| \(X_u\) | X-axis linear damper caused by \(u\) |
| \(Y_v\) | Y-axis linear damper caused by \(v\) |
| \(N_r\) | N-axis linear damper caused by \(r\) |
| \(I_z\) | Inertial moment with respect to \(O_bZ_b\) |
| \(X_{u|u}\) | X-axis non-linear damper caused by \(u\) |
| \(Y_{v|r}\) | Y-axis non-linear damper caused by \(v\) and \(r\) |
| \(N_{r|v}\) | N-axis non-linear damper caused by \(v\) and \(r\) |
| \(N_{r|r}\) | N-axis non-linear damper caused by \(r\) and \(r\) |
| \(\rho\) | Sea water density |
| \(L\) | Length of the vessel |
| \(B\) | Breadth of the vessel |
| \(T\) | Draft of the vessel |
| \(\rho_a\) | Air density |
| \(A_{pW}\) | Wind Frontal projected area |
| \(A_{LW}\) | Wind Lateral projected area |
| \(A_{PC}\) | Water Frontal projected area |
| \(A_{LC}\) | Water Lateral projected area |
| \(L_{OA}\) | Vessel length of overall |
| \(H_{pW}\) | Centroid of \(A_{pW}\) above waterline |
| \(H_{LW}\) | Centroid of \(A_{LW}\) above waterline |
| \(C_x(yw)\) | X-axis wind coefficient |
| \(C_y(yw)\) | Y-axis wind coefficient |
| \(C_N(yw)\) | Yaw-axis wind coefficient |

Parameters \(A_1\) to \(A_{16}\) will be estimated by Levenberg-Marquardt non-linear least square method from experimental data in the form of speed, acceleration, thrust force, and moment of inertia due to deflection angle of the rudder [7].

2.2 The Levenberg-Marquardt Method
The Levenberg-Marquardt method can updates the parameter using Gauss-Newton update and the gradient descent update adaptively according to the value of algorithmic parameter. If the value of algorithmic parameter is small, the Gauss-Newton update is used. Otherwise, the gradient descent update is used. Algorithmic parameter in this method is initialized to be large so the gradient descent update is used. If the iteration comes to the worst case, then the algorithmic parameter is increased. When the solution is getting better, the algorithmic parameter is decreased, hence the Gauss-Newton update is used, and the solution accelerates to the local minimum [4,7].

2.3 Model of Thruster Engine
The thruster engine is a machine that provides thrust force onto USV so that it can move forward. The equation used to model the engine thruster is as follows:

\[
\frac{\Omega_{motor}(s)}{\%Throttle(s)} = \frac{f(\%Throttle)}{\tau_{BLDC}s + 1} \\
\tau_u = K_{Thrust} \cdot \rho \cdot D^4 \cdot n^2 - F_{rudder}
\]
With \( f(\% \text{Throttle}) \) is a mathematical approach function obtained from experimental data measurement of motor rotational velocity, while \( T_{BLDC} \) is a time constant of rotational velocity of motor [8]. \( K_{\text{Thrust}} \) as given in equation (22) is a thrust constant that will be estimated by data from the result of straight motion experiment, \( \rho \) is the water density which has a value of 1025 kg/m\(^3\), \( D \) is a propeller diameter which has a value of 0.05 m, \( n \) is the propeller rotational speed in rps (revolution per second), and \( F_{\text{rudder}} \) is the force produced due to the deflection angle of the rudder combined with the surge velocity of the ship whose direction is perpendicular to the heading of the USV. The value of \( F_{\text{rudder}} \) is determined by equation (24) [9].

\[
K_{\text{Thrust}} = \frac{B \cdot u}{\rho \cdot D^4 \cdot n^2}
\]  
(22)

With \( B \) is a linear damper of USV whose value is equal to the mass of USV divided by the time constant of the \( x \)-axis velocity, and \( u \) is the surge velocity of USV. The value of \( n \) depends on the value of the thruster engine’s throttle, the value of \( B \) is estimated with time constant data of USV \( x \)-axis velocity from straight-motion experiment, and \( u \) is the steady-state surge velocity experienced by the ship during a straight-motion experiment.

2.4 Model of Rudder

Rudder is driven by a servo motor so that the rudder deflection angle can be controlled. By adjusting the rudder deflection angle, the amount of moment of inertia given to the ship at the point of the rudder can then be controlled. The rudder drive modelling is divided into two parts, namely modelling the rudder position response and modelling of the rudder deflection angle relationship with the moment of inertia \( \tau_r \).

Modelling the position response of rudder is approached with a 1\(^{st}\)-order transfer function. This can be done because the servo motor utilized is such a general servo motor which is equipped with a gear box, so that the response is essentially the 1\(^{st}\)-order response. The equation of the rudder drive model is given as in the following:

\[
\frac{\theta_{\text{rudder}}(s)}{\theta_{\text{ref}}(s)} = \frac{1}{\tau_{\text{rudder}} s + 1}
\]  
(23)

With \( \tau_{\text{rudder}} \) will be searched based on experimental data [10].

\[
F_{\text{rudder}} = K_{\text{efficiency}} \cdot \sin(\theta_{\text{rudder}}) \cdot L_{\text{rudder}} \cdot u
\]  
(24)

\[
\tau_r = F_{\text{rudder}} \cdot L_G
\]  
(25)

With \( K_{\text{efficiency}} \) is the rudder efficiency constant that will be assumed to have a value 0.8 for this research. \( \theta_{\text{rudder}} \) is the rudder deflection angle and \( u \) is the surge velocity which is the input of equation (24). \( L_{\text{rudder}} \) is the rudder surface area that is immersed in the water, and has a value of 6.75x10\(^{-3}\) m\(^2\) and \( L_G \) is the force arm length between the rudder position and the mass point of USV whose value is 0.673 m.

The total model of USV’s hull, thruster engine, and rudder is illustrated in Figure 2.

Figure 2. Block diagram of USV’s total model

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\[5\]
2.5 The Experiments

There are four experiments conducted in this research, each of which has a specific purpose. Those experiments comprise thruster engine experiment, rudder drive experiment, straight-motion experiment, and maneuver experiment.

Thruster engine experiment is the experiment carried out to get the relationship equation between throttle percentage and the rotational velocity that are produced by the engine. The thruster is operated in various throttle percentage values and the rotational velocity is captured so the relationship equation between throttle percentage and rotational velocity of thruster engine can be found.

Rudder drive experiment is performed to get the settling time of rudder’s deflection angle. The rudder is moved to the rightmost maximum angle from the leftmost maximum angle, and the time to complete the movement is captured using a stopwatch. This experiment must be done several times so that the most accurate value can be obtained from the mean of overall time values that has been captured.

Straight-motion experiment is carried out to get the value of thruster constant. Firstly, the USV is placed on the water surface. Then the throttle percentage is increased so the USV will move forward, then the throttle percentage is set in one value which gives a desired forward speed. The rudder value is set at zero value so the USV will move in the straight line. Then the value of throttle percentage and surge velocity of USV are captured to get the thruster constant.

The last experiment is designated the USV maneuver. On the water surface USV is moved with various throttle percentage and rudder deflection angle so the USV will maneuver on the water. Then the values of input and output induced onto USV are captured so the parameter of USV can be estimated using the Non-linear Least Square method.

2.6 Feedback Linearization Decoupler

The feedback linearization decoupler is used to linearize the non-linear USV model into a linear system. In conjunction to this the output of the x-axis velocity is influenced by \( u_r \) input and the yaw-rate is only affected by the \( r_r \) input [11]. The linearization process of the x-axis acceleration model is shown as follows:

\[
\dot{u} = -A_2 u + u_r - u_r + A_1 vr + A_4 \tau_u \\
- u_r + A_1 vr + A_4 \tau_u = 0 \\
\tau_u = F_{\text{Thrust}} - |F_{\text{Rudder}}| = \frac{1}{A_4}(u_r - A_1 vr) \\
K_{\text{Thrust}} \rho_{\text{air}} d^4 \omega^2 = \frac{1}{A_4}(u_r - A_1 vr) + |F_{\text{Rudder}}| \\
\omega = \left( \frac{1}{A_4}(u_r - A_1 vr) + |F_{\text{Rudder}}| \right)^{\frac{1}{2}} \\
\omega = f(\% \text{throttle}) \rightarrow \% \text{throttle} = f^{-1}(\omega) \\
F_{\text{Rudder}} = K_{\text{efficiency}} \sin(\theta_{\text{Rudder}}) \cdot L_{\text{Rudder}} \cdot u
\]

So that the dynamic equation of the x-axis acceleration after being given feedback is as follows:

\[
\ddot{u} = -A_2 u + u_r
\]

The linearization process of the yaw-axis acceleration model is shown as follows:
\[
\dot{r} = -A_{14} r + r_r - A_{10} |v|v - A_{11} |r|v - A_{12} |v|r - A_{13} |r|r + A_{15} \tau_r + A_{16} uv
\]  
(34)

\[
0 = -r_r - A_{10} |v|v - A_{11} |r|v - A_{12} |v|r - A_{13} |r|r + A_{15} \tau_r + A_{16} uv
\]  
(35)

\[
\tau_r = \frac{1}{A_{15}} (r_r + A_{10} |v|v + A_{11} |r|v + A_{12} |v|r + A_{13} |r|r - A_{16} uv)
\]  
(36)

\[
\tau_r = K_{\text{efficiency}} \sin(\theta_{\text{Rudder}}). L_{\text{Rudder}}. u. LG
\]  
(37)

\[
\theta_{\text{Rudder}} = \sin^{-1} \left( \frac{1}{K_{\text{denominator}}} * B \right)
\]  
(38)

With,

\[
B = (r_r + A_{10} |v|v + A_{11} |r|v + A_{12} |v|r + A_{13} |r|r - A_{16} uv)
\]  
(39)

\[
K_{\text{denominator}} = A_{15}. K_{\text{efficiency}}. L_{\text{Rudder}}. u. LG
\]  
(40)

So that the dynamic equation of the yaw-axis acceleration after being given feedback is as follows:

\[
\dot{r} = -A_{14} r + r_r
\]  
(41)

**Figure 3.** Block diagram of USV’s total model and feedback linearization decoupler

2.7 Proportional Integral Controller

The controller is attached to the USV model after decoupler linearization feedback is added. Transfer function from the PI controller is as follows:

\[
\frac{U(s)}{E(s)} = K_p \left( \frac{\tau_i s + 1}{\tau_i s} \right)
\]  
(42)

Simplifying negative feedback from the controller and the x-axis velocity plant produces the transfer function as follows:

\[
\frac{U(s)}{U_{\text{ref}}(s)} = \frac{K_p \left( \frac{\tau_i s + 1}{\tau_i s} \right) \left( \frac{K_{\text{plant}}}{K_{\text{plant}} s + 1} \right)}{K_p \left( \frac{\tau_i s + 1}{\tau_i s} \right) \left( \frac{K_{\text{plant}}}{K_{\text{plant}} s + 1} \right) + 1}
\]  
(43)

With \(K_{\text{plant}}\) in the above equation has a value of \(\frac{1}{A_2}\) for the x-axis velocity dynamic equation. Then the values of \(\tau_i\) and \(K_{\text{plant}}\) are considered the same so that equation (43) above becomes:

\[
\frac{U(s)}{U_{\text{ref}}(s)} = \frac{1}{K_p K_{\text{plant}} \tau_i s + 1}
\]  
(44)

The controller parameters are obtained as follows:

\[
\frac{\tau_i}{K_p K_{\text{plant}}} = \tau_{\text{design}}
\]  
(45)
\[
K_p = \frac{\tau_i}{\tau_{\text{design}} K_{\text{plant}}} = \frac{1}{\tau_{\text{design}}} \quad (46)
\]
\[
K_i = \frac{K_p}{\tau} = \frac{A_2}{\tau_{\text{design}}} \quad (47)
\]

With \( \tau_{\text{design}} \) is the system time constant when the plant has been controlled with a PI controller. In this research the \( \tau_{\text{design}} \) used has a value of 2 seconds for the x-axis velocity and has a value of 0.5 for yaw-rate. Another design specification is that the value of steady state error is 0 for both velocities.

Figure 4. Block diagram of feedback added USV’s model and controller

3. Experiment Results and Analysis

3.1 Experimental Data and Parameter Estimation

Results from experiment of the thruster engine consist of two groups of data as listed in Tables 2 and 3 below:

| Table 2. Experimental data of thruster engine |
|---------------------------------------------|
| Throttle  | Rotational velocity   |
| %          | rad/s                  |
| 25.9615%   | 1248.6784              |
| 28.2692%   | 1312.6621              |
| 32.6923%   | 1423.2462              |
| 38.4615%   | 1469.6370              |

| Table 3. Specification of USV’s straight-motion experimental data |
|---------------------------------------------------------------|
| Specification               | Value             |
| Settling Time               | 13.5797 s         |
| Steady state value of surge velocity | 2.2778 m/s       |
| Throttle Input               | 38.4615%          |

From the two groups of data as above then the following parameters are found:

a. \( f(\%\text{Throttle}) \) is modeled by equation \( f(\%\text{Throttle}) = 242.7027 (\%\text{Throttle})^{\frac{1}{2}} \) which results in MRSE value of 28.1341.

b. \( B \) and \( K_{\text{Thrust}} \) values obtained from the data presented in Table 3 and USV mass parameter which has a value of 8.4367 kg. \( B \) has a value of 3.10636 and \( K_{\text{Thrust}} \) has a value of 0.020188.

For modeling the parameters of USV hull, namely \( A_1 \) to \( A_{16} \), experimental data of maneuver experiment is used. For this the assumption is taken that the value of \( A_3 \) which is a constant non-linear damper representation is 0. This is because the USV has a taper forms at the front end, so it could be assumed that the non-linear damper has a value of 0. By using the parameter estimation program for non-linear systems based on the non-linear least square from Levenberg-Marquardt method, the following parameters are obtained as shown in Table 4.
Table 4. Estimation result of USV’s hull parameters

| Parameters | Value |
|------------|-------|
| $A_1$      | -0.0152 |
| $A_2$      | 0.1305 |
| $A_3$      | 0.0000 |
| $A_4$      | 0.0508 |
| $A_5$      | 0.6245 |
| $A_6$      | -0.0075 |
| $A_7$      | 0.1831 |
| $A_8$      | -0.0111 |
| $A_9$      | 0.0139 |
| $A_{10}$   | 0.0194 |
| $A_{11}$   | 0.0505 |
| $A_{12}$   | 0.0268 |
| $A_{13}$   | -0.4451 |
| $A_{14}$   | 0.7005 |
| $A_{15}$   | 106.4701 |
| $A_{16}$   | 0.0385 |

After the USV hull parameter is obtained, validation is performed and generates the RMSE value as shown in Table 5. It can be seen that the RMSE values generated is relatively small so that the approximate parameters can be considered to be the true parameter values.

Table 5. RMSE value of parameters validation

| Variabel | RMSE value |
|----------|------------|
| $\dot{u}$ | 0.0789 |
| $\dot{v}$ | 0.1742 |
| $\dot{r}$ | 0.3601 |

The $\tau_{rudder}$ parameter generated from the experimental data of rudder’s response experiment is shown in Table 6 as follows.

Table 6. Experimental data of rudder

| Experiment number | Settling Time |
|-------------------|---------------|
| 1                 | 0.535 s       |
| 2                 | 0.475 s       |
| 3                 | 0.456 s       |
| 4                 | 0.488 s       |
| 5                 | 0.511 s       |
| Mean              | 0.493 s       |

From data in Table 6 the $\tau_{rudder}$ value is obtained by applying the formula $\tau_{rudder} = \frac{\text{settling time}}{5}$. This produces a value of 0.0986 seconds.
To prove that the total USV model is correct, in addition to the RMSE value, an adjustment with the PI controller is used directly on the total USV model to control $x$-axis velocity and yaw-rate. The responses are shown in Figures 5 and 6.

From the two figures above, the result with the correct total USV model is achieved because it can be controlled with PI controller and produces settling time that matches the design specifications. But there is an overshoot which causes the linearization process is needed before adjusting with the PI controller on the total USV model.

### 3.2 Linearization and Velocity Controlling

Once the parameters of USV hull are obtained, the linearization is done using feedback linearization decoupler. The simulation results due to the addition of feedback are shown in Figures 7 and 8 as follows.

After linearization with feedback, then it is proceeded with controlling the $x$-axis velocity and yaw-rate of USV by applying the PI controller, whose control parameters are obtained from the USV hull parameters information. The PI controller parameters are shown in Table 7 as follows.
Then controller testing is carried out which produces the response as shown in Figures 9 and 10. From Figure 9 it can be concluded that the controller manages to control the $x$-axis velocity and produce a settling time of ± 10 seconds. This means the time constant value is ± 2 seconds, which essentially is the same as the design specification. Further, from Figure 10 it can be seen that the yaw-rate control yields the output yaw-rate to equalize the reference at 52.5 seconds, that is 2.5 seconds after the input step works. This means that the output signal has a settling time of ± 2.5 seconds and a time constant of ± 0.5 seconds, which is the same as the design specification. In addition, both figures also show that the value of steady state errors is zero for both velocities.

| Parameters | Value |
|------------|-------|
| $K_p \ u$  | 0.5000|
| $K_i \ u$  | 0.0653|
| $K_p \ r$  | 2.0000|
| $K_i \ r$  | 1.4009|

| Table 7. PI controller parameters |

Figure 9. Surge velocity response of controlled USV’s system

Figure 10. Yaw-rate response of controlled USV’s system with $u_{ref}=1.5$

4. Conclusions
From the results of the dynamic modeling and controlling the $x$-axis velocity and yaw-rate of USV, two main conclusions can be taken as follows:

1. By carrying out USV modeling using Levenberg-Marquardt’s Non-Linear Least Square method, it could be concluded that this method is appropriate to model the USV non-linear behavior as evidenced by the relatively small RMSE value in the validation of $x$-axis acceleration, $y$-axis acceleration, and yaw-axis acceleration at USV LSS-01.
2. Designing a PI controller is done to adjust the $x$-axis velocity and yaw-rate of the USV. The design of the PI controller at the $x$-axis velocity and yaw-rate successfully meets the applied design specifications and produces steady state error value of 0 for both velocities.

References
[1] Dong, Z., Lei, W., Yueming, L., Yulei, L. and Jiayuan, Z. (2005). “Heading control of USV by expert fuzzy technology”. Int. J. of Control and Automation 8(12):155-166.
[2] Knockaert, L. (2009). Vector Fitting Vs Levenberg-Marquardt : Some Experiments.
[3] Ranganathan, A. (2004). The Levenberg-Marquardt Algorithm, pp. 1–5.
[4] Gavin, H.P. (2017). The Levenberg-Marquardt Method for Nonlinear Least Squares Curve-fitting Problems, pp. 1–19.
[5] Do, K.D. and Pan, J. (2009). Control of Ships and Underwater Vehicles. Springer-Verlag.
[6] Eranugroho, M.A.F. (2018). Perbandingan PID Model Reference Adaptive Control dengan L1 Adaptive Control untuk Kemudi Kapal Tanpa Awak dalam Kontrol Waypoint. Departemen of Electrical Engineering ITS, Surabaya.

[7] Madsen, M.K. and Nielsen, N.B., et al. (2004). Methods for Nonlinear Least Squares Problems, Informatics and Mathematical Modeling Technical University of Denmark, Denmark.

[8] Li, X. (2015). Model-Based Design of Brushless DC Motor Control and Motion Control Modelling for Robocup SSL Robots. Information Technology Vaasa University of Applied Sciences, Finland.

[9] Mohan S., Ragul K.R., Sriram V. and Vengat Kumar V.A. (2015). “Design and implementation of the propeller for marine propulsion using pi controller”. IJRASET, 3(5):84-90.

[10] Kumar, P.R. and Babu, V.N. (2014). “Position control of servo system using PID controller tuning with soft computing”. IJERT, 3(11):976–980.

[11] Haris, M. (2019). Perancangan Kontroler Adaptif MRAC dan LQT untuk Kestabilan dan Trayektori Tracking pada UAV berjenis Fixed-Wing, Departemen of Electrical Engineering ITS, Surabaya.