SNAP microwave optical filters

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If the originally flat bottom of a wide quantum well with multiple eigenstates is periodically modulated, its eigenvalues rearrange into denser groups separated by wider gaps. We show that this effect, if implemented in an elongated bottle microresonator (also called a SNAP microresonator) allows to design microwave photonic tunable filters with an outstanding performance.

Design and fabrication of high-quality microwave filters is a longstanding problem which attracted scientists and engineers for several decades [1-3]. The interest to this problem is motivated by critical applications of microwave filters in modern communication technologies where their accurate transmission spectrum characteristics is highly desirable to combine with small dimensions and broadband tunability. Photonics suggests several solutions to this problem based on miniature photonic circuits [4-6]. In particular, much research work was done to design and fabricate microwave filters based on coupled ring resonators [7-9], photonic crystals [10], distributed feedback resonators [11], Mach-Zehnder interferometers [12], fiber Bragg gratings [13], frequency comb generators [14], Brillouin scattering [15] and other approaches [4-6].

In many cases, it is important to create filters which transmission amplitude has maximum flatness within the predetermined bandwidth and steeply vanishes outside it. Theoretically, filters with predetermined flatness and high rejection rate can be designed by apodization of coupled microresonator circuit with sufficiently large number of elements [16, 17]. Experimentally, the intrinsic losses and insufficient fabrication precision lead to severe noise in the transmission amplitude of such circuits growing with the number of resonant elements [18] and impracticality of devices fabricated of sufficiently large number of coupled microresonators.

Even for the negligible propagation losses, at light frequency $f$ and for the pass bandwidth $\Delta f$, microresonators with characteristic dimension $d$ should be fabricated with the precision of better than $\Delta d \approx f / \Delta f / f$. For characteristic $f = 200$ THz, $\Delta f = 100$ MHz, and $d = 100 \mu$m, we have $\Delta d \approx 0.5 \AA$, not possible to achieve by conventional modern microphotonic fabrication technologies. For this reason, coupled ring resonator and other photonic infinite impulse response filters were fabricated with the aid of microheaters enable to tune the circuit elements individually (see, e.g., [9, 11, 19] and references therein).

Increasing the microresonator Q-factor allows to create filters with better passband flatness and larger rejection rate. Indeed, at optical frequency $f$, the pass bandwidth cannot be smaller than $\Delta f_{\text{pass}} \approx f / Q$, while to arrive at sufficient flatness we have to have $Q \gg f / \Delta f_{\text{pass}}$. Thus, at $f = 200$ THz, the characteristic for microwave applications passband with $\Delta f_{\text{pass}} = 100$ MHz requires $Q \gg 2 \cdot 10^6$. That high and much higher Q-factors are possible to achieve in standing along microresonators [20, 21]. However, the problem of effective combining them into a circuit of multiple elements with the predetermined dimensions and coupling remains open.

![Fig. 1. (a) Illustration of a SMR with nanoscale periodically modulated ERV coupled to microfibers MF1 and MF2 and (b) the corresponding CFV. Red lines are the axial frequency eigenvalues of SMR.](image)

Favorably, the ultraprecise fabrication precision combined with ultralow material and scattering losses required for realization of microwave photonic filters can be achieved in the surface nanoscale axial photonics (SNAP) technology [22-26] which has not yet been considered for microwave applications. In SNAP, the required microresonator circuits are fabricated in the form of coupled bottle microresonators having nanoscale effective radius variation (ERV). In Ref. [23], 30 coupled bottle microresonators were fabricated at the surface of a 19 µm radius optical fiber with better than 1 Å precision. In Ref. [25], it was shown that coupled SNAP bottle microresonators can be postprocessed with the frequency precision of better than 0.2 GHz. This fabrication precision can be
show below that the periodic SMR structure illustrated in Fig. 1 can be presented in the dimensionless form:

\[ \Psi(z) = \left( e^{z} - v(z) \right) + \lambda \delta(z) + \lambda \delta(Mz) + \lambda \delta(Mz + \pi) \Psi = 0, \quad (1) \]

Here the dimensionless frequency \( \epsilon \), attenuation \( \gamma \), MF-SMR coupling parameter \( \lambda \), and distance along the SMR \( z \) are defined as:

\[ \epsilon = \frac{\Delta f}{f_0}, \quad \gamma = \frac{\sigma}{\Delta f}, \quad \delta = \frac{2}{z_0}, \quad \lambda = \frac{2\pi}{22}, \quad n_0 = \frac{n}{(f_0, \Delta f)} \],

where \( c \) is the speed of light, \( n \) is the SMR refractive index (below we consider silica SMR with \( n = 1.44 \)), \( g \) is the attenuation expressed through its Q-factor as \( g = f_0/Q \), and \( D \) is the microfiber-SMR complex-valued coupling parameter [22] which is assumed the same for MF1 and MF2. We also assume that the SMR is symmetric with respect to its center at \( z = 0 \) and MF1 and MF2 are positioned symmetrically at axial coordinates \( z = z_M \) and \( -z_M \). Then, provided that the microfiber-SMR coupling is lossless [29, 30], the transmission amplitude \( S_{12}(f, z_M) \) from MF1 to MF2 is determined as [22]

\[ S_{12}(f, z_M) = \frac{1}{1 + \epsilon S_{12}(f, z_M)} \].

Here \( \xi_M = z_M/f_0, \epsilon = \Delta f/f_0, \) and \( G(\epsilon, \xi_M) \) is the Green's function of Eq. (1). To take into account the coupling loss (which can be very small for a four-port microresonator [29]), the numerator in this equation should be reduced accordingly [22, 28]. The dimensionless form of Eqs. (1)-(3), allows us to design filters with different passbands \( \Delta f_{\text{pass}} \) by rescaling. It follows from Eqs. (2) and (3) that to design a filter with passband \( \Delta f_{\text{pass}} \), we have to rescale the CFV of the last filter by \( \sigma \), its Q-factor by \( 1/\sigma \), and the coupling parameter \( D \) by \( \sigma^{-1/2} \).

We start with the design of a 100 MHz passband filter constructed of four coupled microresonators modeled by harmonic oscillations of CFV plotted in Fig. 2(a). Fabrication of similar SMR with subangstrom precision was demonstrated in [22, 23]. The CFV precision achieved in [25] was 0.16 GHz sufficient to introduce the CFV with characteristic 2 GHz amplitude shown in Fig. 2(a). In our modelling, we assume that the intrinsic SMR Q-factor, which determines its internal losses, is \( Q = 10^8 \) [27, 31] and set the frequency \( f_0 = 193.4 \) THz corresponding to light wavelength 1550 nm. We optimize the symmetric positions of MF1 and MF2 to arrive at the best flat transmission power \( |S_{12}|^2 \) within bandwidth \( \Delta f_{\text{pass}} = 100 \) MHz and vanishing outside it. The result of optimization is shown in Fig. 2(b) along the whole SMR spectrum and in Fig. 2(c) for the spectrum in the vicinity of the passband considered. It is seen that the transmission power is quite flat within the passband and vanishes down to \(-100 \) dB in its vicinity (we show below that this value is prevailed by a greater value of nonresonant transmission). The optimized positions of MF1 and MF2 are \( \pm z_{\text{opt}} = \pm 381 \) \( \mu \text{m}, \) and coupling parameter is \( D = 0.0015 + 0.0017i \mu \text{m}^{-1}. \) This value of \( D \) is an order of magnitude less than those typically observed for the coupling of microfiber and SMR positioned in direct contact [22, 23, 32]. We suggest that this small coupling can be achieved by placing the microfiber (or a...
planar waveguide) several hundred nanometers away from the
SMR [30]. Remarkably, we found that the displacement of MF1 and
MF2 by several microns followed by optimization of \( D \) does not
significantly change the behavior of \( |S_{12}|^2 \).

The calculated very large transmission sideband rejection (down
to -100 dB in both cases shown in Fig. 2(b) and 3(b)) is violated by
non-resonant transmission of light from MF1 to MF2 not taken into
account by Eq. (1) which describes the contribution of WGMs
having a single azimuthal and radial quantum number, \( m = m_0 \)
and \( p = p_0 \), only. To estimate the contribution of WGMs with
nonresonant quantum numbers, we assume that all WGMs with
radial quantum numbers greater than \( p = 0 \) vanish (like, eg., in a
capillary SMR with sufficiently narrow walls [33]). We introduce
the separation of the cutoff frequencies \( \Delta f_{az} \) along the azimuthal
quantum number \( m \) in the vicinity \( |\Delta m| \ll m_0 \) of \( m_0 \), where
\( \Delta m = m_0 - m \). Then \( \Delta f_{az} \) is expressed through the SMR radius \( r_0 \)
as \( \Delta f_{az} = c (2 \pi m_0 r_0)^{-1} \). For fiber radii and CFVs of our concern,
\( r_0 \approx 1 \) mm and \( \Delta f_0 \approx 10 \) GHz, we have \( \Delta f_0 \ll \Delta f_{az} \). Under these
assumptions, the contribution to the nonresonant transmission \( S_{12}^{(nr)} \) of azimuthal modes with \( \Delta m < 0 \) is negligible, while the
characteristic value of non-resonant transmission, the sum over
\( \Delta f_{az} \) and \( \Delta f_{ph} \) along the azimuthal and radial quantum numbers,
is expressed through the SMR radius \( r_0 \) as

\[
\Delta f_{az} = \frac{c}{2 \pi m_0 r_0}.
\]

Increasing the number of coupled microresonators and
apodization of their initially periodic ERV allows to add more
flexibility in designing a filter and achieve the better passband
flatness and rejection rate. In the next example, we design a 1 GHz
passband filter by optimization of an SMR composed of 8 coupled
microresonators. Now, in addition to the optimization of positions
of MF1 and MF2 made above, we apodize the SMR by narrowing the
barriers between first, second, and third and, symmetrically,
between sixth, seventh, and eighth microresonators. The optimized
CVF of a SMR with the Q-factor \( Q = 10^8 \) is plotted in Fig. 3(a). While
narrowing of the barriers is achieved here by straightforward
cutting, we suggest that in a more advanced CVF optimization, the
barrier widths can be adjusted in a way more suitable for
experimental realization. It is seen from Fig. 3(a) that the length of
the designed 1 GHz filter is much smaller than that of the 100 MHz
filter shown in Fig. 2 and its CVF is much greater. Fig. 3(b) shows the
spectrum of the transmission power \( |S_{12}|^2 \) along the full SMR
bandwidth, which was obtained by optimization of the transmission
power along the passband magnified in Fig. 3(c). The determined optimized positions of MF1 and MF2 are \( z_{opt} = \pm 90 \)
\( \mu m \) and the coupling parameter is \( D = 0.0145 + 0.013 i \) \( \mu m^{-1} \).

The considered models and rescaling relations indicated above allow
to design SMRs having CVFs and transmission spectra with
other passbands. For example, to design a SMR with 500 MHz
passband from the 100 MHz passband SMR with \( Q = 10^8 \)
described above (Fig. 2), we have to magnify the frequency values
along the horizontal axis in Figs. 2(b) and (c) by five. The CVW of this
SMR is obtained by dividing the values of distance along the
horizontal axis in Fig. 2(a) by \( 5^{1/2} \) (i.e., this SMR is \( 5^{1/2} \) times
shorter) and multiplying the CFV values along the vertical axis by 5.
The Q-factor of this SMR is five times smaller; \( Q = 2 \cdot 10^7 \).

\begin{align}
S_{12}^{(nr)} &= S_0^{(nr)} \Xi, \quad \Xi = \sum_{\Delta m} \frac{1}{\Delta m} \exp \left( 2 \beta \omega_0 z_{opt} \left( \Delta m + \frac{\Delta f_0}{\Delta f_{az}} \right)^2 \right), \\
S_0^{(nr)} &= \frac{\text{Im}(D)}{\beta_0}, \quad \beta_0 = \frac{2^{2/3} \pi n}{c} \left( f_0 \Delta f_{az} \right)^{1/2}, \quad \Delta f_{az} = \frac{c}{2 \pi m_0 r_0}.
\end{align}

Factor \( S_0^{(nr)} \) in this equation determines the characteristic value of
nonresonant transmission, while the sum over \( \Delta m \) rapidly
oscillates as a function of SMR radius \( r_0 \). For the 100 MHz passband
SMR considered above (Fig. 2) with radius \( r_0 = 20 \) \( \mu m \) we have
\( 20 \log(S_0^{(nr)}) \equiv -53 \) dB. For the 1 GHz filter with the same radius
(Fig. 3), we have \( 20 \log(S_0^{(nr)}) \equiv -35 \) dB. These values are much
greater than the rejection rates calculated above in the resonance
approximation. Therefore, they determine the sideband rejection
rate of the designed filters. The increase of the rejection rate with
the reduction of microfiber-SMR coupling determined by \text{Im}(D)
correlates with experimental observations for a ring
microresonator [35]. Numerical modelling based on Eq. (4) shows
that these sideband transmission values can be reduced by \( \sim 10 \) dB
by optimization of the fiber radius \( r_0 \).

Remarkably, connection of SMR filters in series allows to
significantly reduce the non-resonant transmission in the rejection
region and achieve the passband tunability. As an example, we
consider a device consisting of two SMR 1 GHz passband filters
designed above connected in series as illustrated in Fig. 4(a). The
transmission amplitude of this filter is found as

\[
S_{12}^{(2)} = S_{12}^{(1)}(\Delta f - \Delta f_{ph1}, z_{MF1})S_{12}^{(1)}(\Delta f - \Delta f_{ph1}, z_{MF2}).
\]

Here \( \Delta f_{ph1,2} \) are the shifts of the cutoff frequencies of SMRs which can be tuned by adjacent thermal
heaters shown in Fig. 4(a). This allows us to tune both the central
frequency and the passband width of the filter. Figs. 4(b) and (c)
show the spectra of transmission power of this filter for \( \Delta f_{ph1} = 0 \)
and \( \Delta f_{ph2} = 0 \) (1 GHz passband, red curves), \( \Delta f_{ph1} = 0.5 \) GHz (0.5
GHz passband, blue curves), and \( \Delta f_{ph2} = 0.8 \) GHz (0.2 GHz
passband, green curves). The dashed horizontal line in Fig. 4(b)
shows the estimated rejection rate \( \sim 70 \) dB limited by the non-
resonant transmission with the amplitude \( S_0^{(nr)} \) defined by Eq. 4.
the microfiber and SMR directions and by tuning the realization of SMR devices for more general microwave photonics microfiber-SMR gap. Finally, the design and experimental (EP/P006183/1). Funding. Experimentally, optimization of the MF1 and MF2 positions and their coupling to SMR can be performed by their translation along the microfiber-SMR gap. Finally, the design and experimental realization of SMR devices for more general microwave photonics spectral shaping [36] may be of special interest.

We suggest that optimization of a SMR profile with more flexible CFV parameters, as well as SMRs connected in series, will lead to filter designs with superior transmission characteristics. Experimentally, optimization of the MF1 and MF2 positions and their coupling to SMR can be performed by their translation along the microfiber and SMR directions [24] and by tuning the microfiber-SMR gap [30]. Finally, the design and experimental realization of SMR devices for more general microwave photonics spectral shaping [36] may be of special interest.

Funding. Engineering and Physical Sciences Research Council, (EP/P006183/1).

Disclosures. The author declares no conflicts of interest.

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