Modeling flows of underexpanded jets in a slot space

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Abstract. In the present paper, we report on the results of a study of supersonic gas flows in a slot space. It is shown that the structure of the ejected jet exhausting into a slot ambient space substantially depends on the friction force. The latter force leads to a substantial change of the supersonic jet, which acquires, instead of a barrel-shaped, a fan-shaped form. The results of numerical simulation are consistent with experimental data.

1. Introduction
Until recently, supersonic flows in the slot space did not attract very much attention of researchers. To our knowledge, only one work [1] dealing with a supersonic flow in a radial nozzle formed by a gap between two parallel disks, has so far been published. Currently, the situation has changed after publication [2] in which it was shown that radial nozzles could be used in the cold gas spraying technology for depositing coatings onto the inner surface of pipes. In [3, 4], a pseudo-shock structure formed in a radial nozzle was studied and in [5], supersonic jets discharged into a slot space from a rectangular channel were investigated. The present paper is a continuation of the study of [5]. Here, new results concerning the structure of supersonic jets discharged from a rectangular channel into a slot space filled with quiescent air are reported.

2. Statement of the problem
In the present study, experimental and numerical modeling of the submerged gas flow both in the rectangular channel and in the slot space was carried out. A diagram of the rectangular channel and the slot space is shown in figure 1, \(a\) and \(b\). Two disks, \(I\) and \(2\), both of radius \(r = 36\) mm, spaced apart by a distance \(h\), are fitted onto a central rod \(3\) of radius \(r_0 = 5\) mm. The distance between the disks was varied in various experiments and numerical calculations. A gasket with a cut was inserted in between the disks (this gasket is shown in figure 1, \(a\) and \(b\) as a shaded area). The cut made in the gasket formed a rectangular channel of width \(b\), length \(a\), and height \(h\). The gas having temperature \(T_0\) was supplied from chamber 4 into the channel through the slot \(x_i < x < r_i\), \(-b/2 < y < b/2\) under pressure \(p_0\). In the experiments and calculations, the parameters \(a, b\) and \(h\) were varied parameters, whereas the parameters \(x_i = 6\) mm and \(r_i = 9\) mm were fixed quantities. In the channel, the gas was accelerated to the speed of sound; as a result, an underexpanded supersonic jet was ejected into the slot.
space \( x > r_2 \). The latter space was filled with air under normal conditions, \( p_\infty = 0.1 \text{ MPa} \) and \( T_\infty = 300 \text{ K} \).

**Figure 1.** Diagram of the gas flow in the channel and in the slot space drawn in the planes \((x, z)\) and \((x, y)\) (respectively, a and b). Part c of the figure shows the computational domain in the plane \((x, y)\).

The working gas was air with temperature \( T_0 = 300 \text{ K} \) and pressure \( 0.9 \text{ MPa} \leq p_0 \leq 1.0 \text{ MPa} \). In the experiment, a flow with converging boundary layers formed between the disks; that is why in the numerical study the gas flow in the plane \((x, y)\) was calculated using equations averaged over the channel width [3, 4]:

\[
\frac{\partial A}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + f = 0, \quad p = \rho RT, \quad e = c_v T, \quad q^2 = u^2 + v^2, \\
A = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho (e + q^2 / 2) \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho u (e + p / \rho + v^2 / 2) \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho v (e + p / \rho + v^2 / 2) \end{pmatrix}, \quad f = \begin{pmatrix} 0 \\ \tau_x \\ \tau_y \\ 0 \end{pmatrix}, \quad (1)
\]

\[\tau_x = C_f \rho u q / \mu, \quad \tau_y = C_f \rho v q / h, \quad C_f = 12 / \text{Re} + 0.06 / (2 \text{Re}^{0.25}), \quad \text{Re} = \rho |v| h / \mu.\]

In equations (1), \( \rho, \ p, \ e \) and \( T \) are the gas density, the gas pressure, the specific internal energy, and the gas temperature; \( u \) and \( v \) are the velocity components respectively along the axes \( x \) and \( y \); \( q^2 \) is the squared velocity modulus; \( \tau_x \) and \( \tau_y \) are the components of the friction force acting on the gas from the side of the disks; \( C_f, \ \mu \) and \( C_v \) are the drag coefficient, the dynamic viscosity, and the heat capacity; \( \gamma \) is the Poisson adiabatic index; and \( \text{Re} \) is the Reynolds number. The drag coefficient \( C_f \) involved two terms. The first term described the dependence of \( C_f \) on the Reynolds number \( \text{Re} \) for laminar flow, and the second term, the same dependence for turbulent flow. The numerical modeling was carried out in the domain shown in figure 1 c (for convenience in presenting the calculated and experimental data, the coordinate axes \((x, y)\) in figure 1 c are turned through angle 90° compared to figure 1 b). The system of equations (1) was solved numerically using the modified Lax – Wendroff scheme [3]. At the lateral boundaries \( DABC \) (hatched ones in figure 1 c), the conditions of impermeability were set, and at the axis \( x_i < x < r_e, \ y = 0 \) and line \( CG \), the conditions of symmetry were posed. The gas entered the computational domain through the inlet section \( x = x_i, \)
at which the conditions \( p_0, T_0, v = 0 \) were set; the fourth condition was derived from the condition of conservation of the Riemann invariant \( J_\gamma = u - 2c / (\gamma - 1) \). The gas left the slot space through the boundary \( FG \), at which the symmetry condition was specified in the case of supersonic flow with \( M = q / c > 1 \), and the pressure \( p_\infty \), in the case of subsonic flow with \( M < 1 \); the rest conditions were derived from the conditions of conservation of the Riemann invariants reaching the boundary \( FG \) [3 - 5]. Like in [3 - 5], the steady flow in the channel and in the slot space was calculated using the relaxation method.

3. Discussion of calculated and experimental data

Figure 2a shows the results of numerical calculation of a supersonic underexpanded jet ejected from a rectangular channel \((a = 12 \text{ mm}, b = 4 \text{ mm}, h = 0.2 \text{ mm})\) into a slot space width \( h = 0.2 \text{ mm} \). The gas parameters in the prechamber were \( p_0 = 1.0 \text{ MPa} \) and \( T_0 = 300 \text{ K} \). Figure 2b shows soot-oil visualization data in the form of a photograph taken in an experiment with two jets simultaneously discharged from channels of width \( b = 4 \text{ mm} \) and \( b = 17 \text{ mm} \). The remaining parameters in the experiment were the same as those in the numerical calculation with \( h = 0.2 \text{ mm} \). Both in the calculation and in the experiment, the nozzle pressure ratio was \( p(r_2) / p_0 = 5 \), where \( r_2 = 20 \text{ mm} \). Figure 2c shows the dependence of non-dimensional pressure \( p(x) / p_0 \) on the coordinate \( x \) (along the jet centerline) obtained in two independent experiments and in numerical calculations. The scatter of experimental data permits an estimate of the experimental error, which was mainly related with the non-parallelism of the two disks.

![Figure 2](image-url)

Figure 2. The calculated and experimental data obtained for the steady flow in the channel and in the slot space: the calculated distribution of Mach numbers \( M(x, y) \) (a); soot-oil visualization data for the flow on the surface of the external disk (photo) (b); the distribution of non-dimensional pressure \( p(x, y) / p_0 \) along the jet axis \( y = 0 \) (c). In part c of the figure, the circles and rectangles show the experimental data, and the solid and dashed line, the data calculated respectively with and without allowance for the friction force.

As it is seen in figure 2a, a diverging supersonic jet is discharged from the channel; the boundary of this jet is inclined at an angle \( \varphi = 35^\circ \) to the \( x \)-axis. A similar picture was observed in the experiment (see figure 2b). If the friction force were absent \( (C_f = 0) \), then the jet would have the barrel-shaped form shown in figure 3b of publication [5]. The formation of the diverging jet is a result of the action of the friction force due to the disk walls on the ejected supersonic jet. If the jet leaves the channel at point \( B \) (see figure 1c), there forms a centered rarefaction wave, which undergoes reflection from the centerline of the jet and extends to the jet boundary. Under the action of the friction force, the
rarefaction wave gets attenuated; therefore, when this wave reaches the jet boundary, it becomes rather weak and does not disturb this boundary. From figure 2c, it follows that, due to the deceleration of the supersonic jet, a shock wave is formed in the vicinity of the exit from the slot space. The calculated distribution of pressure along the jet centerline is in satisfactory agreement with the pressure values that were measured in the experiments (see figure 2c).

The friction force that acts on the jet flow from the side of the disk walls is inversely proportional to the spacing $h$ between the disks (see formulas (1) for the friction-force components $\tau_x$ and $\tau_y$). A decrease in the distance between the disks leads to an increased friction force and to a change of the flow pattern in the jet. Figure 3 shows the calculated and experimental data obtained for a supersonic underexpanded jet with $p(r_s)/p_0 = 5$ ejected from a rectangular channel into a slot space with a smaller spacing between the disks, $h = 0.15$ mm.

![Figure 3](image)

**Figure 3.** The calculated and experimental data for the steady flow obtained at $a = 12$ mm, $b = 7$ mm, $h = 0.15$ mm, $p_0 = 0.95$ MPa, and $T_0 = 300$ K: the calculated distribution of Mach numbers $M(x, y)$ (a); the calculated distribution of density gradient $|\nabla \rho(x, y)|$, kg/m$^3$ (b); and the soot-oil visualization data for the flow on the surface of the external disk (photo) (c).

Evidently, an increase in friction force leads to an amplification of the shock wave and to its displacement towards the channel outlet. Upstream of the shock wave, the discharged jet has a rectilinear boundary inclined to the centerline of the jet. Behind the shock, we have a subsonic flow in which the pressure increases and the jet velocity decreases. This causes an intense lateral expansion of the gas. As a result, the boundary of the jet becomes fan-shaped. The results of calculating the jet boundary (see figures 3a and 3b) are in satisfactory agreement with the soot-oil visualization obtained data (see figure 3c).

4. Conclusion
The paper presents results of a numerical and experimental study of supersonic underexpanded jets discharged from a rectangular channel into a slot space formed by two parallel disks. As the gas flow leaves the channel and enters the slot space, at the corner there arises a centered rarefaction wave in which the gas flow undergoes acceleration to a supersonic speed. Under the action of the friction force, the rarefaction wave rapidly decays. As a result, the rarefaction wave reflected from the jet centerline does not perturb the jet boundaries. In this case, the initial length of the jet boundary is a rectilinear streamline inclined at some angle to the jet axis. The angle of inclination is defined by the turn of the velocity vector that occurs as the streamline intersects the centered rarefaction wave. Depending on the separation between the disks, a curvilinear section of the jet, over which a most profound expansion of the jet occurs, may be adjoined to its rectilinear section. The emergence of this former section is related with the formation of a shock wave closing the zone of supersonic flow.
Behind this shock wave, the jet flow is subsonic. The subsonic stream expands intensively towards the lateral regions in which the pressure is equal to the external pressure. If the subsonic region is large enough, then a second fan-shaped section of the jet boundary can arise with the occurrence of an intensive expansion of the gas to the sides. The results of numerical calculations proved to be in satisfactory agreement with experimental data in terms of the distribution of pressure along the jet centerline and in terms of the shape of jet boundaries.

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