Theory and Applications of Active Constellation Extension

WEI-LUN LIN¹ AND FAN-SHUO TSENG², (Member, IEEE)
¹Department of Communications Engineering, Feng Chia University, Taichung 40724, Taiwan
²Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung 510275, Taiwan
Corresponding author: Wei-Lun Lin (weilunlin@mail.fcu.edu.tw)

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ABSTRACT Active constellation extension (ACE) was originally developed to reduce the peak-to-average-power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems with quadrature amplitude modulation (QAM). Alternatively, ACE can be a promising approach for optimizing various possible aspects of the 2-D constellation signals. However, the literature lacks a rigorous theoretical framework for ACE, and hence its applications are currently limited. This study proposes a formal mathematical framework and theory for ACE for the general case of the 2-D constellation signals. The proposed framework is used to demonstrate the roles of ACE in reducing the PAPR in the OFDM systems with QAM and improving the error performance and throughput of phase shift keying under fading channels.

INDEX TERMS Active constellation extension (ACE), orthogonal frequency division multiplexing (OFDM), quadrature amplitude modulation (QAM), phase shift keying (PSK).

I. INTRODUCTION

Active constellation extension (ACE) was first proposed in 2003 as a means of reducing the peak-to-average power ratio (PAPR) in quadrature amplitude modulated orthogonal frequency division multiplexing (OFDM) systems [1]. The principle of ACE is to move the quadrature amplitude modulated frequency-domain symbols within so-called permissive regions so as to emulate the corresponding clipped-version signal. In [1] and [2], permissive regions are described (without formal definition) as the regions within which the outer constellation points of 4-ary quadrature amplitude modulation (4-QAM) and 16-QAM signals can be moved without decreasing the minimum distance of the constellation points. It was shown in [1] and [2] that ACE achieves a significant reduction in the PAPR at the cost of a limited signal power growth in OFDM systems to less than 1 dB.

In tackling the PAPR problem, many ACE-based approaches are proposed following the concept of moving signals within permissive regions to reduce PAPR [1]–[17]. Much effort is spent in reducing the computation complexity, such as smart gradient project ACE [1], [3], least square approximation ACE [4], ACE with frame interleaving [5], clipping-based ACE [6], ACE with the bounded distortion [7], ACE with parabolic peak cancellation [8], [9], a constellation extension-based ACE [10], ACE with signals of high-order constellation [11], and neural network aided ACE [14]. Apart from OFDM, filter-bank multi-carrier (FBMC) is also considered as the signal waveform for ACE to reduce PAPR [12], [13].¹ In addition to reducing PAPR in OFDM with QAM, ACE has also been used in DVB-T2 systems [15], in space-time block coded (STBC) systems to improve the decoding performance [16], and in encrypted transmission to ensure security [17].

Although the above-mentioned research of ACE has done much progress in various system performance [1]–[16], the approach of ACE was again given descriptively without formal definition. Due to the enhanced degree of freedom which it provides, ACE enlightens a promising opportunity for optimizing various aspects of QAM systems, including power/resource allocation, precoding, channel pre-distortion, and preamble insertion for synchronization purposes.

¹ Since OFDM with ACE possesses the same PAPR reduction as that of FBMC, without loss of generality, we only address our design in an OFDM system [12].
However, to realize this potential, a rigorous theoretical framework for ACE is required for arbitrary constellation signals. Accordingly, this study proposes a mathematical framework and the corresponding theories to perform ACE for any generalized 2-D constellation. The proposed framework is demonstrated by illustrating the roles of ACE in reducing the PAPR in the OFDM systems with QAM, especially when specific constellation size is adopted, and in improving the error performance and throughput of phase shift keying (PSK) under fading channels.

In this paper, we propose the mathematical framework of ACE for any 2D-constellation in Section II. Specific constellation is adopted following the proposed ACE mathematical framework as an example for reducing PAPR of the OFDM signal in Section III. Sections IV proposes a new pre-equalization model based on the ACE mathematical framework to fight the flat fading channel, and further extension in Section V is also proposed to take advantage of the proposed mathematical framework to seek for extra throughput. Section VI concludes the paper.

II. FORMULATION OF ACTIVE CONSTELLATION EXTENSION

Consider a set of \( M \) 2-D constellation points \( \{X_0, X_1, \ldots, X_{M-1}\} \), which can be regarded as the modulated symbols mapped from the input information. Specifically, the decision region of \( X_i \) is denoted as \( \mathcal{R}(X_i) \) and represented by a Voronoi cell \([18]-[19]\) as

\[
\mathcal{R}(X_i) = \{x \in \mathbb{C} : |x - X_i| \leq |x - X_j|, \forall j \in \{0, 1, \ldots, M - 1\} \neq i\},
\]

where \( \mathbb{C} \) represents the set of all complex numbers and \( |x| \) denotes the Euclidean norm of the complex number \( x \). Notably that each Voronoi cell is simply connected, i.e., every closed path in \( \mathcal{R}(X_i) \) encloses only points belonging to \( \mathcal{R}(X_i) \), and the boundary of the cell comprises piecewise straight line segments [20]. For convenience, in classifying the constellation points in the Voronoi cells, let a bounded set first be defined as follows:

**Definition 1:** A set \( Q \) is said to be bounded if all the elements in \( Q \) satisfy

\[
\{x \in Q : |x| \leq P\}
\]

where \( P \) is a finite real number.

**Definition 2:** When all the boundary segments of a Voronoi cell are bounded, the cell is said to be closed and the corresponding constellation point is referred to as an inner point. By contrast, when certain boundary segments of the cell are unbounded, the cell is said to be open and the corresponding constellation point is referred to as an outer point.

Fig. 1(a) shows a specific 2-D constellation with \( M = 5 \). Based on Definitions 1 and 2, \( \mathcal{R}(X_0) \) in Fig. 1 (a) is a closed cell, and thus \( X_0 \) is an inner point; By contrast, \( \{\mathcal{R}(X_i^o) | i = 1, 2, 3, 4\} \) are open cells, and thus \( \{X_i^o | i = 1, 2, 3, 4\} \) are all outer points. Note that each open cell has at most two unbounded boundary segments, as given in the following lemma.

**Proposition 1:** Two possible cases exist for the unbounded boundary of an open cell for 2-D constellation, namely a single unbounded segment and two unbounded segments.

**Proof:** If there exists a straight line which connects all the constellation points in the provided constellation, every Voronoi cell within the constellation is a strip-shaped unbounded region [20] (also shown in Fig. 2(a)). In such a case, the two constellation points located at the edges have
Consider a certain outer point \( X^o \). Let the permissive region of \( X^o \) be denoted as \( \mathcal{R}^p(X^o) \) and be defined as follows:

**Definition 3:** \( \mathcal{R}^p(X^o) \) is an unbounded region and has two permissive boundary segments which start from point \( X^o \) and run parallel to the one or two unbounded boundary segment(s) of \( \mathcal{R}(X^o) \).

Fig. 1(b) presents an illustrative example of a specific 2-D constellation with \( M = 5 \), in which the permissive regions \( \{\mathcal{R}^p(X^o_i)\}_{i=1,2,3,4} \) are indicated by the shaded regions.

Using **Definition 3**, the permissive regions for any 2-D constellation can be easily obtained. One of the most fundamental properties of a permissive region is given in the following.

**Proposition 3:** The permissive region of a certain outer point \( X^o_i \), \( \mathcal{R}^p(X^o_i) \), forms an open region containing points providing larger Euclidean distances to \( X^o_i \) than \( X^o_j \) to \( X_j \), i.e.,

\[
\mathcal{R}^p(X^o_i) = \{ x \in \mathcal{R}(X^o_i) : |x - X_j| \geq |X^o_i - X_j|, \forall j \in \{0,1,\ldots,M-1\} \neq i \},
\]

**Proof:** Since the shared boundary of two neighboring Voronoi cells coincides with the whole or partial perpendicular bisector of the associated constellation points (see **Proposition 2**), the two unbounded boundary segments of \( \mathcal{R}^p(X^o_i) \), i.e., when the equality holds in (3), are parallel to the unbounded boundary segments of \( \mathcal{R}(X^o_i) \) and are thus perpendicular to the line connecting the two corresponding neighboring constellation points \( X_i \) and \( X_j \), respectively. Consequently, for \( x \in \mathcal{R}^p(X^o_i) \), angles \( \angle xX^o_i X_j \) and \( \angle xX^o_j X_k \) are obtuse angles and thus are opposite to the longest sides of the triangles \( \Delta xX^o_i X_j \) and \( \Delta xX^o_j X_k \), respectively. As a result,

\[
|X^o_i - X_j| \leq |x - X_j| \text{ and } |X^o_i - X_k| \leq |x - X_k| \quad (4)
\]

for \( x \in \mathcal{R}^p(X^o_i) \).

Note that for an inner point, the corresponding permissive region is an empty set, i.e., \( \mathcal{R}^p(X_0) \in \phi \) (empty set), as given in the following proposition.

**Proposition 4:** A closed cell has no permissive region.

**Proof:** For any point \( x \) in the Voronoi cell of an inner point \( X_i \), a neighboring constellation point \( X_j \) can always be found such that

\[
|x - X_j| \leq |X_j - X_i|, \forall x \in \mathcal{R}(X_i).
\]

In other words, no point in the Voronoi cell of \( X_i \) satisfies (3).

Fig. 3 shows the ACE treatment of a QAM \((M = 32)\) signal. Note that the shaded regions again indicate the permissive regions, \( \mathcal{R}^p(X^o_i) \), of the constellation points \( X^o_i \).
It is seen that the magnitude of the points in the permissive regions increases with an increasing constellation size of QAM by comparing Figs. 1 and 3, which results in fewer choices of points with smaller magnitude.

**III. USE OF ACE TO REDUCE PAPR IN OFDM SYSTEMS**

Consider an OFDM system with \( N \) subcarriers. The frequency-domain modulated signal can be expressed as

\[
S = [S[0], S[1], \ldots, S[N - 1]]^T,
\]

where \( S[k] \) is the modulated symbol on the \( k \)th subcarrier.

Multiplexing \( S \) through an inverse discrete-time Fourier transform operation, the OFDM time-domain signal

\[
s = [s[0], s[1], \ldots, s[N - 1]]^T
\]

can be obtained as

\[
s[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S[k] \exp \left\{ \frac{j2\pi nk}{N} \right\}.
\]

Within the respective permissive regions, ACE [1] adjusts the outer points of the considered signal constellation in such a way as to achieve the PAPR value of a clipped signal. Denoting the clipped signal as \( \bar{s} \), the \( n \)th element of \( \bar{s} \) is represented as

\[
\bar{s}[n] = \begin{cases} 
    s[n], & |s[n]| \leq A \\
    A \cdot \exp\{j\theta[n]\}, & |s[n]| > A 
\end{cases}
\]

for \( n = 0, 1, \ldots, N - 1 \), where \( \theta[n] \) denotes the phase of \( s[n] \), i.e.,

\[
s[n] = |s[n]| \exp\{j\theta[n]\},
\]

and \( A \) is the clipping level. The difference between the clipped signal \( \bar{s} \) and the original signal \( s \) is defined by the clipped-off portion signal, i.e.,

\[
c_{\text{clip}} \triangleq \bar{s} - s.
\]

In general, a frequency-domain process over \( c_{\text{clip}} \) provides the means to approach

\[
\bar{s} = s + c_{\text{clip}}
\]

without adversely affecting the error performance of the original signal. Thus, in ACE, a discrete-time Fourier transform operation over \( c_{\text{clip}} \) is performed to obtain

\[
c_{\text{clip}} = [C_{\text{clip}}[0], C_{\text{clip}}[1], \ldots, C_{\text{clip}}[N - 1]]^T.
\]

\( C_{\text{clip}} \) is then moved in the respective permissive regions to form the signal

\[
C = [C[0], C[1], \ldots, C[N - 1]]^T
\]

in accordance with the following constraints:

\[
C[n] = \begin{cases} 
    C_{\text{clip}}[n], & \text{if } S[n] \text{ is an outer point} \\
    S[n] + C_{\text{clip}}[n] \in \mathbb{R}^p(S[n]) & \text{otherwise} 
\end{cases}
\]

for \( n = 0, 1, \ldots, N - 1 \). With \( c \) being inverse discrete-time Fourier transform of \( C \),

\[
s + c
\]

can be adopted as an approximation of

\[
\bar{s} = s + c_{\text{clip}}.
\]

Applying the convex optimization algorithm [21]–[23], the following optimization problem with step size \( \zeta \) can be introduced to find the minimum PAPR:

\[
||\bar{s}||^2_\infty = \min_{\zeta \in \mathbb{R}^+} \left\{ ||s + \zeta c||^2_\infty \right\},
\]

where \( \mathbb{R}^+ \) is a set of non-negative real numbers and

\[
\bar{s} = [\bar{s}[0], \bar{s}[1], \ldots, \bar{s}[N - 1]]^T.
\]

Notice that in (15) the region \( \mathbb{R}^p(S[n]) \) provides a larger Euclidean distance than \( S[n] \) to any other constellation point. Thus, when the clipped points \( C_{\text{clip}}[n] \) for \( n = 0, 1, \ldots, N - 1 \) in (13) falling within \( \mathbb{R}^p(S[n]) \), \( C_{\text{clip}}[n] + S[n] \) for \( n = 0, 1, \ldots, N - 1 \) can be used to emulate a clipped OFDM signal without destroying the orthogonality between the OFDM subcarriers, and the minimum PAPR can be obtained through (16).

Consider the OFDM signals with 256 subcarriers optimized based on the ACE approach as in (16). The PAPR performance for the ACE aided OFDM with \( M \)-QAM signals is shown in Fig. 4, where the OFDM signals are oversampled 8 times to approximate the analog PAPR and the clip level \( A = 6.85 \text{dB} \) above the average power was used for 16-QAM, and \( A = 7.23 \text{dB} \) was used for 64-QAM to achieve...
FIGURE 4. Comparison of PAPR for ACE aided OFDM systems with different QAM constellations.

FIGURE 5. Comparison of PAPR for ACE-OFDM systems with different number of subcarriers.

similar PAPR without ACE. Obviously, the PAPR performance decreases with the increase of the constellation size $M$, which is reasonable that the permissive regions $R^p(S[n])$'s to the regions of interior points shrink in a smaller ratio for the OFDM signal with higher constellation size, resulting in less ACE flexibility and therefore less PAPR reduction [1]. Further, the nontypical 5-QAM is examined in Figs. 4 and 5, where the above-mentioned PAPR performance trend is also verified.

IV. USE OF ACE TO PRE-EQUALIZE THE FADING CHANNEL FOR $M$-PSK

Without loss of generality, we consider a slow and flat Rayleigh fading channel with its channel gain denoted by $h$. We assume both the transmitter and receiver are perfectly synchronized and can perfectly estimate the channel status. The received signal $r$ can be represented by

$$r = hs + w$$

where $w$ is the additive Gaussian white noise (AWGN) distributed with zero mean and unit variance.

In order to mitigate the impact of the fading in the channel, equalization at the receiver is a common approach to counteract the fading channel gain, however, the equalized signal suffers from noise enhancement which also seriously degrades the error performance of the system. To avoid such trade-off, ACE can be utilized at the transmitter to perform pre-equalization within permissive region in a way to invert the magnitude of the channel gain, and the equal gain equalization is adopted at the receiver to correct the distorted phase. Through the equalization at both transmitter and receiver, the fading gain can be removed completely without affecting the statistics of AWGN. Thus, it seems like that the proposed approach alters a fading channel to an AWGN one.

To obtain maximum benefits from the adoption of ACE, we consider the $M$-ary phase shift keying ($M$-PSK) signal, where every modulated $M$-PSK signal $x$ is free to be moved within $R^p(x)$ as shown in Fig. 6 with $M = 8$. Note that every constellation points in $M$-PSK are outer points. Direct inversion of the channel gain at the transmitter is not necessary, we only amplify the transmitting signal when $|h|x$ leaves $R^p(x)$, i.e., the permissive region of $x$, where $|h|$ represents the magnitude of $h$. Thus, the transmitted signal $s$ can be designed to be $Qx$ with

$$Q = \begin{cases} 
1, & \text{if } |h|x \in R^p(x) \\
|h|^{-1}, & \text{if } |h|x \notin R^p(x).
\end{cases}$$

Meanwhile, the receiver counteracts the phase of the fading channel, the received signal $r$ is thus equal-gain equalized to be $v$ as

$$v = \angle h^*r = \angle h^*|h|\angle hs + \angle h^*w = |h|s + \angle h^*w = |h|s + y$$

where $\angle h = h/|h|$ represents the phasor of $h$, and $y$ is the circularly symmetric AWGN. We assume the fading channel gain and AWGN are independent. The mean and variance of $y$, denoted by $E\{y\}$ and $Var\{y\}$, respectively, are identical to those of $w$ due to $\angle h^*$ being uniform distributed. We can

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2 Though the considered pre-equalizer is only studied for flat fading, a direct extension of the proposed approach to the multiple-carrier system under a frequency selective fading channel is also applicable.
further simplify (18) as

\[ v = \begin{cases} |h|x + y, & \text{if } |h|x \in R^p(x) \\ x + y, & \text{if } |h|x \notin R^p(x). \end{cases} \]

Through the maximal likelihood detection, ACE-aided M-PSK under the flat Rayleigh fading channel exhibits identical error performance to M-PSK under AWGN at the receiver. However, considering practical implementation, a proper outage threshold should be set to prevent infinite power compensation. Apperantly, ACE compensates the fading channel, but at the sacrifice of the extra power \( \Upsilon \) derived by

\[ \Upsilon = \int_{\mu}^{1} h^{-2} f_{|h|} (h) dh - \int_{1}^{\infty} h^2 f_{|h|} (h) dh \]

\[ = \int_{\mu}^{1} h^{-1} \exp\left(-\frac{h^2}{2\sigma^2}\right) dh - \int_{1}^{\infty} h^3 \exp\left(-\frac{h^2}{2\sigma^2}\right) dh \]

\[ = \frac{1}{2} \left[ 2Ei\left(-\frac{1}{2\sigma^2}\right) - Ei\left(\frac{\mu^2}{2\sigma^2}\right) \right] - 3 \exp\left(\frac{-1}{2\sigma^2}\right) \]

where \( \mu \) is the outage threshold, \( f_{|h|} (h) \) denotes the Rayleigh distribution, and \( Ei(x) \) is the exponential integral. With an outage threshold \( \mu \), the transmitter gives up compensating the signal when in the deep fade, i.e., \( |h| < \mu \). The error performance of the ACE-aided 8-PSK signal is provided in Fig. 7 when \( \sigma^2 = 1 \).\(^3\)

Under different outage probability, the error performance of ACE-aided PSK is improved significantly comparing to that of the PSK signals in Rayleigh fading channel as shown in Fig. 7. If we consider the scenario with aggressive compensation of the fading channel (i.e., \( \mu \) is set to be 0.01), around 6dB gain in SNR for the ACE-aided PSK system over the PSK one without ACE is observed at the bit error rate (BER) being \( 10^{-2} \), and when the BER is lower, much more gain in SNR can be obtained for the ACE-aided PSK system. Alternatively, if we compensate the channel with \( \mu = 0.2 \), the error performance of the ACE-aided PSK system approaches to that of PSK in AWGN though at a sacrifice of the outage of signal transmission. Here, the outage transmission probability can be obtained through

\[ P_{out}(\mu) = \int_{-\infty}^{\mu} f_{|h|} (h) dh. \]

When \( |h| \) is Rayleigh distributed with \( \sigma^2 = 1 \), \( \mu = 0.01, 0.05, 0.1, 0.2 \), \( P_{out}(\mu) = 4.9 \times 10^{-5}, 1.2 \times 10^{-3}, 0.005, 0.0198 \), respectively. In conclusion, a reasonable outage (e.g., \( P_{out}(0.1) = 0.005 \)) tradeoff provides comparable error performance for the ACE-aided PSK system with fading to the PSK system in AWGN.

This alternative ACE approach can also be applied to M-QAM, however, the interior points increase along with the increase of the constellation size \( M \) when \( M > 4 \), which leads to less outer points that can be moved to convert the channel gain. Since the interior points are not allowed to be moved as provided in Proposition 3, the error performance gain for \( M \)-QAM can be improved but is not so significant comparing to that of \( M \)-PSK.

V. FURTHER EXTENSION BASED ON THE PROPOSED ACE MATHEMATICAL FRAMEWORK

A new design is proposed in this section to utilize the proposed ACE mathematical framework to seek for more advantage in signal transmission. In Section IV, the PSK signal is moved primarily based on the inverse of the fading gain \( |h| \) when \( |h| \leq 1 \), whereas extra benefits can be further explored by properly devising the transmitting signal in the permissible region based on the proposed ACE mathematical framework. From Fig. 8, when we move the signal along the arc with the radius being the inverse of the fading gain within the

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\(^3\)Although the channel estimation error is assumed to be perfect throughout, the permissible region \( R^p(x) \) of the transmitting M-PSK signal \( x \) permits \( \pm \pi / M \) estimation error of the phase of the channel, which is expected to outperform the conventional pre-equalization approaches when phase estimation error existed, however, this subjects to future research.
the angle the two red points \( x \) and permissive regions are parallel to each other as proved in Proposition 2 as a mathematical framework that the boundary lines for Voronoi in Proposition 2 as information by moving the signal to either \( \gamma \) with \( \phi \). Consider transmitting extra one bit information through \( b \). The extra capacity denoted by \( cap \) can be derived as

\[
\begin{align*}
C_{cap} &= -\int_{\mu}^{1/2} \int_{-\infty}^{\infty} \frac{p|h|}{p(s = x_a, |h|)} \cdot \log \left( \frac{1}{2} \left( 1 + \frac{p(r|x = x_b, |h|)}{p(r|x = x_a, |h|)} \right) \right) drdh \\
&= -\int_{\mu}^{1/2} \int_{-\infty}^{\infty} \frac{h}{2\pi} e^{-(r-\sqrt{\gamma \sin \phi})^2 + h^2} \cdot \log \left( \frac{1}{2} \left( 1 + e^{-2r\sqrt{\gamma \sin \phi}} \right) \right) drdh. 
\end{align*}
\]

The extra capacity in (22) is evaluated in Fig. 9 for different SNR. Obviously, when the constellation size \( M \) increases, the extra capacity decreases due to the Euclidean distance being smaller for the two boundary points \( x_a \) and \( x_b \). Also, when considering higher \( \mu \) (outage threshold), the extra capacity decreases for lower probability enabling the boundary points \( x_a \) or \( x_b \). Around 0.2 bits/symbol is gained by modifying the ACE pre-equalized PSK system as shown in Fig. 9, but the application of the proposed ACE mathematical framework is not limited to obtained extra information.

\[
|\phi| = \frac{\pi}{M} - \arcsin \left( \frac{|h| \sin \left( \frac{\pi}{M} \right)}{2\sqrt{\gamma \sin \phi}} \right).
\]

In (19), when \(|h|\) is small, \(\phi\) tends to \(\frac{\pi}{2}\), and when \(|h| = 1\), \(\phi = 0\), which implies when the fading is severe, the adoption of extra signal points benefits from enlarging the Euclidean distance between each other. With (19), the Euclidean distance between \( x_a \) and \( x_b \) can be immediately calculated as

\[
2\sqrt{\gamma} \sin(\phi)/|h|
\]
VI. CONCLUSION
We propose a mathematical framework for ACE for the case of a generalized 2-D constellation signal. The proposed framework has been used to demonstrate the applications of ACE in improving error performance and system throughput. The proposed framework provides a useful basis for applying ACE to a variety of possible applications in modulation field with the 2-D constellation.

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REFERENCES
[1] B. S. Krongold and D. L. Jones, “PAR reduction in OFDM via active constellation extension,” IEEE Trans. Broadcast., vol. 49, no. 3, pp. 258–268, Sep. 2003.
[2] A. Hekkala, S. Boumand, and M. Lasanen, “Exponential companding and active constellation extension comparisons for PAPR reduction,” in Proc. 26th Int. Teletraffic Congr. (ITC), Sep. 2014, pp. 1–5.
[3] L. Wang and C. Tellambura, “An adaptive-scaling algorithm for OFDM PAPR reduction using active constellation extension,” in Proc. IEEE 64th Veh. Technol. Conf., Montreal, QC, Canada, Sep. 2006, pp. 1–5.
[4] M. Laabidi, R. Zayani, and R. Bouallegue, “A quick convergence active constellation extension projection onto convex sets algorithm for reducing the PAPR of OFDM system,” in Proc. IEEE Int. Conf. Adv. Inf. Netw. Appl. Workshops, Mar. 2016, pp. 439–443.
[5] Z. Yang, H. Fang, and C. Pan, “ACE with frame interleaving scheme to reduce peak-to-average power ratio in OFDM systems,” IEEE Trans. Broadcast., vol. 51, no. 4, pp. 571–575, Dec. 2005.
[6] K. Bae, J. Andrews, and E. Powers, “Adaptive active constellation extension algorithm for peak-to-average ratio reduction in OFDM,” IEEE Commun. Lett., vol. 14, no. 1, pp. 39–41, Jan. 2010.
[7] S.-K. Deng and M.-C. Lin, “OFDM PAPR reduction using clipping with distortion control,” in Proc. IEEE Int. Conf. Commun. (ICC), Seoul, South Korea, May 2005, pp. 2563–2567.
[8] J. Liu, M. Yu, X. Zeng, M. Wang, and J. Lu, “A method of combining PPC and ACE to reduce PAPR in CO-OFDM system,” in Proc. Asia Commun. Photon. Conf. (ACP), Beijing, China, 2013.
[9] H.-B. Jeon, J.-S. No, and D.-J. Shin, “A new PAPR reduction scheme using efficient peak cancellation for OFDM systems,” IEEE Trans. Broadcast., vol. 58, no. 4, pp. 619–628, Dec. 2012, doi: 10.1109/TBC.2012.2211432.
[10] M. C. Paredes, J. J. Escudero-Garzas, and M. J. F.-G. Garcia, “PAPR reduction via constellation extension in OFDM systems using generalized benders decomposition and branch-and-bound techniques,” IEEE Trans. Veh. Technol., vol. 65, no. 7, pp. 5133–5145, Jul. 2016.
[11] Y. Liu, Y. Wang, and B. Ai, “An efficient ACE scheme for PAPR reduction of OFDM signals with high-order constellation,” IEEE Access, vol. 7, pp. 118322–118332, 2019, doi: 10.1109/ACCESS.2019.2936917.
[12] S. Eldessoki, J. Dommel, K. Hassan, L. Thiele, and R. F. H. Fischer, “Peak-to-average-power reduction for FBMC-based systems,” in Proc. 20th Int. ITC Workshop Smart Antennas (WSA), Munich, Germany, 2016, pp. 1–6.
[13] N. van der Neut, B. T. Maharaj, F. H. D. Lange, G. Gonzalez, F. Gregorio, and J. Cousseau, “PAPR reduction in FBMC systems using a smart gradient-project active constellation extension method,” in Proc. Int. Conf. Telecommun. (ICT), May 2014, pp. 134–139.
[14] I. Sohn, “A low complexity PAPR reduction scheme for OFDM systems via neural networks,” IEEE Commun. Lett., vol. 18, no. 2, pp. 225–228, Feb. 2014, doi: 10.1109/LCOMM.2013.123113.131888.
[15] Z. Zheng and G. Li, “An efficient FPGA design and performance testing of the ACE algorithm for PAPR reduction in DVB-T2 systems,” IEEE Trans. Broadcast., vol. 63, no. 1, pp. 134–143, Mar. 2017.
[16] F. Hu, L. Jin, J. Li, and F. Wu, “An active constellation extension architecture for STBC MIMO decoding,” in Proc. IEEE 2nd Int. Conf. Softw. Eng. Service Sci. (ICSESS), Jul. 2011, pp. 128–132.
[17] J. Zhong, X. Yang, and W. Hu, “Performance-improved secure OFDM transmission using chaotic active constellation extension,” IEEE Photon. Technol. Lett., vol. 29, no. 12, pp. 991–994, Jun. 15, 2017, doi: 10.1109/LPT.2017.2700861.
[18] E. Viterbo and E. Biglieri, “Computing the Voronoi cell of a lattice: The diamond-cutting algorithm,” IEEE Trans. Inf. Theory, vol. 42, no. 1, pp. 161–171, Sep. 1996.
[19] F. Aurenhammer, “Voronoi diagrams—A survey of a fundamental geometric data structure,” ACM Comput. Surv., vol. 23, no. 3, pp. 345–405, Sep. 1991.
[20] J. H. Conway and N. J. A. Sloane, Sphere Packing, Lattices and Groups. Berlin, Germany: Springer-Verlag, 1992.
[21] A. Aggarwal and T. H. Meng, “Minimizing the peak-to-average power ratio of OFDM signals using convex optimization,” IEEE Trans. Signal Process., vol. 54, no. 8, pp. 3099–3110, Aug. 2006.
[22] S. Boyd and L. Vandenberghe. Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2003. [Online]. Available: http://www.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf
[23] M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, “Applications of second-order cone programming,” Linear Algebra Appl., vol. 284, nos. 1–3, pp. 193–228, Nov. 1998. [Online]. Available: http://www.stanford.edu/~boyd/SOC Pavel Kholodenko received the Ph.D. degree in electrical engineering from the Technical University of Munich, Munich, Germany, in 2007. From 2007 to 2010, he was a Postdoctoral Research Fellow with the Institute of Communications Engineering, National Chiao Tung University, Hsinchu, Taiwan. Since 2016, he has been an Associate Professor with the Institute of Communications Engineering, National Chiao Tung University. Since 2016, he has been an Associate Professor with the National Sun Yat-sen University, Kaohsiung, Taiwan. His research interests include transceiver design in wireless communication systems and artificial neural networks.

WEI-LUN LIN received the Ph.D. degree in electrical engineering from National Central University, Taiwan, in 2008. From 2008 to 2011, he was a Postdoctoral Research Fellow with the Graduate Institute of Communication Engineering, National Taiwan University. He was with the Department of Communications Engineering, Feng Chia University, Taiwan, in 2012, where he has been an Associate Professor, since 2017. His research interests include transceiver design in wireless communication systems and artificial neural networks.