Constraints on Neutrino Mixing Parameters By Observation of Neutrinoless Double Beta Decay

Hisakazu Minakata ∗ and Hiroaki Sugiyama †
Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan

Abstract

Assuming positive observation of neutrinoless double beta decay together with the CHOOZ reactor bound, we derive constraints imposed on neutrino mixing parameters, the solar mixing angle $\theta_{12}$ and the observable mass parameter $\langle m \rangle_\beta$ in single beta decay experiments. We show that $0.05 \text{ eV} \leq \langle m \rangle_\beta \leq 2 \text{ eV}$ at the best fit parameters of the LMA MSW solar neutrino solution by requiring the range of the parameter $\langle m \rangle_{\beta\beta}$ deduced from recently announced double beta decay events at 95 % CL with ±50 % uncertainty of nuclear matrix elements.

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∗E-mail: minakata@phys.metro-u.ac.jp
†E-mail: hiroaki@phys.metro-u.ac.jp
I. INTRODUCTION

While the good amount of evidences for the neutrino mass and the lepton flavor mixing have been accumulated \cite{1,2,3}, we still lack observational indications of how large is the absolute mass of the neutrinos. To our understanding to date it may show up in only a few places, the single \cite{4} or the double beta decay \cite{5} experiments as well as future cosmological observations \cite{6}. Other potential possibilities for hints of absolute mass of neutrinos include Z-burst interpretation of highest energy cosmic rays \cite{7}.

Among these various experimental possibilities the neutrinoless double beta decay experiments seems to have relatively higher sensitivities. The most stringent bound on effective mass parameter $\langle m \rangle _{\beta \beta}$ (see eq. (2) for definition) is now $\langle m \rangle _{\beta \beta} < 0.35$ eV, which comes from Heidelberg-Moscow group \cite{8}. Furthermore, a wide variety of proposals for future facilities as well as ongoing attempt with greater sensitivities are actively discussed. They include NEMO \cite{9}, GENIUS \cite{10}, CUORE \cite{11}, MOON \cite{12}, XMASS \cite{13}, and EXO \cite{14} projects. These high-sensitivity experiments open the enlighting possibility of discovering neutrinoless double beta decay events, not just placing an upper bound on $\langle m \rangle _{\beta \beta}$ by its nonobservation. Therefore, it is of great importance to completely understand what kind of informations can be extracted if such discovery is made.

We discuss in this paper in a generic three flavor mixing framework the constraints on neutrino masses and mixing by positive observation (as well as nonobservation) of neutrinoless double beta decay. The constraints imposed on neutrino mixing parameters by neutrinoless double beta decay have been discussed by many authors. They include the ones in early epoch \cite{15}, those in ”modern era” in which real constraints on solar mixing parameters are started to be extracted \cite{16,17}, and the ones in ”post-modern era” where the analyses are performed in a comprehensive manner in the framework of generic three flavor neutrino mixing \cite{18}.

In a previous paper, we have made a final step in the series of analyses by proposing a way of expressing the constraints solely in terms of observables in single and double beta decay \cite{19}. By using the framework, we discussed the possibility of placing lower bound on $|U_{e3}|^2$ assuming positive observation in direct mass measurement in single beta decay and an upper limit on $\langle m \rangle _{\beta \beta}$ in double beta decay experiments. It is a natural and logical step
for us to examine next the alternative case of positive observation of neutrinoless double beta decay events.

Timely enough, an evidence of the neutrinoless double beta decay has just been reported by Klapdor-Kleingrothaus and collaborators [20]. Since the confidence levels of the claimed evidence are about 2 and 3 $\sigma$ in Bayesian and Particle Data Group methods, respectively, we must wait for confirmation by further data taking, or by other groups to conclude that neutrinos are Majorana particles. Nevertheless, we feel that the peak in the relevant kinematic region in their experiments is too prominent to be simply ignored.

As will become clear as we proceed it is essential to combine the constraint on $|U_{e3}|^2 = s_{13}^2$ imposed by the reactor experiments [21]. One of the key points in our subsequent discussion is that the double beta and the reactor bounds cooperate to produce a stringent constraint on absolute mass scale of neutrinos and the mixing angle $\theta_{12}$ which is responsible for the solar neutrino problem.

II. CONSTRAINTS FROM NEUTRINOLESS DOUBLE BETA DECAY

Let us start by defining our notations. We use throughout this paper the standard notation of the MNS matrix [22]:

$$
U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.
$$

Using the notation, the observable in neutrinoless double beta decay experiments can be expressed as

$$
\langle m \rangle_{\beta\beta} = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| = \left| m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta} + m_3 s_{13}^2 e^{i(3\gamma - 2\delta)} \right|,
$$

where $m_i$ (i=1, 2, 3) denote neutrino mass eigenvalues, $U_{ei}$ are the elements in the first low of the MNS matrix, and $\beta$ and $\gamma$ are the extra CP-violating phases characteristic to Majorana neutrinos [23], for which we use the convention of Ref. [16].

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We define the neutrino mass-squared difference as $\Delta m^2_{ij} \equiv m_j^2 - m_i^2$ in this paper. In the following analysis, we must distinguish the two different neutrino mass patterns, the normal ($\Delta m^2_{23} > 0$) vs. inverted ($\Delta m^2_{23} < 0$) mass hierarchies. We use the convention that $m_3$ is the largest (smallest) mass in the normal (inverted) mass hierarchy so that the angles $\theta_{12}$ and $\theta_{23}$ are always responsible for the solar and the atmospheric neutrino oscillations, respectively. We therefore often use the notations $|\Delta m^2_{23}| \equiv \Delta m^2_{\text{atm}}$ and $\Delta m^2_{12} \equiv \Delta m^2_{\odot}$ to emphasize that they are experimentally measurable quantities. Because of the hierarchy of mass scales, $\Delta m^2_{\odot}/\Delta m^2_{\text{atm}} \ll 1$, $\Delta m^2_{12}$ can be made always positive as far as $\theta_{12}$ is taken in its full range $[0, \pi/2]$.[24]

In order to derive constraint on mixing parameters we need the classification.

Case I: \[ |m_1 c^2_{12} c^2_{13} e^{-i\beta} + m_2 s^2_{12} c^2_{13} e^{+i\beta}| \geq m_3 s^2_{13} \] \hspace{1cm} (3)

Case II: \[ |m_1 c^2_{12} c^2_{13} e^{-i\beta} + m_2 s^2_{12} c^2_{13} e^{+i\beta}| \leq m_3 s^2_{13} \] \hspace{1cm} (4)

However, examination of the case II reveals that it does not lead to useful bounds. Therefore, we only discuss the case I in the rest of this paper.

A. Joint constraint by upper bounds on $\langle m \rangle_{\beta\beta}$ and reactor experiments

Since we try to utilize the experimental upper bound on $\langle m \rangle_{\beta\beta}$, $\langle m \rangle_{\beta\beta} \leq \langle m \rangle^{\text{max}}_{\beta\beta}$, we derive the lower bound on $\langle m \rangle_{\beta\beta}$. It can be obtained in the following way;

\[ \langle m \rangle_{\beta\beta} \geq c^2_{13} \left| (m_1 c^2_{12} + m_2 s^2_{12}) \cos \beta - i(m_1 c^2_{12} - m_2 s^2_{12}) \sin \beta \right| - m_3 s^2_{13} \]
\[ = c^2_{13} \sqrt{m^2_1 c^4_{12} + m^2_2 s^4_{12} + 2m_1 m_2 c^2_{12} s^2_{12} \cos 2\beta - m^2_3 s^4_{13}}. \] \hspace{1cm} (5)

Noticing that the right-hand-side (RHS) of (5) has a minimum at $\cos 2\beta = -1$, we obtain the inequality

\[ \langle m \rangle_{\beta\beta} \geq c^2_{13} \left| m_1 c^2_{12} - m_2 s^2_{12} \right| - m_3 s^2_{13}. \] \hspace{1cm} (6)

We note that the RHS of (5) is a decreasing function of $s^2_{13}$, and hence takes a minimum value for the maximum value of $s^2_{13}$ which is allowed by the limit placed by the reactor experiments[21]. We denote the maximum value as $s^2_{13}(\text{CH})$ throughout this paper. Numerically,
\( s_{13}^{2} \text{(CH)} \simeq 0.03 \). (While the precise value of the CHOOZ constraint actually depends upon the value of \( \Delta m_{\text{atm}}^{2} \text{[21]} \), we do not elaborate this point in this paper.) Using the constraint we obtain

\[
\langle m \rangle_{\beta \beta}^{\text{max}} \geq \langle m \rangle_{\beta \beta} \geq |m_{1}c_{12}^{2} - m_{2}s_{12}^{2}| - \left( m_{3} + |m_{1}c_{12}^{2} - m_{2}s_{12}^{2}| \right) s_{13}^{2} \text{(CH)}. \tag{7}
\]

It can be rewritten as the bound on \( \cos 2\theta_{12} = \cos 2\theta_{\odot} \) as

\[
\frac{m_{2} - m_{1}}{m_{2} + m_{1}} - \frac{\langle m \rangle_{\beta \beta}^{\text{max}} + m_{3}s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_{2} + m_{1})c_{13}^{2} \text{(CH)}} \leq \cos 2\theta_{12} \leq \frac{m_{2} - m_{1}}{m_{2} + m_{1}} + \frac{\langle m \rangle_{\beta \beta}^{\text{max}} + m_{3}s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_{2} + m_{1})c_{13}^{2} \text{(CH)}}, \tag{8}
\]

where \( c_{13}^{2} \text{(CH)} \equiv 1 - s_{13}^{2} \text{(CH)} \).

### B. Joint constraint by lower bounds on \( \langle m \rangle_{\beta \beta} \) and reactor experiments

A positive observation of neutrinoless double beta decay will lead to the experimental lower bound on \( \langle m \rangle_{\beta \beta}, \langle m \rangle_{\beta \beta} \geq \langle m \rangle_{\beta \beta}^{\text{min}} \), which we use to place new bound on neutrino mixing parameters. Toward the goal we note, similarly as (5), that

\[
\langle m \rangle_{\beta \beta} \leq c_{13}^{2} \sqrt{m_{1}^{2}c_{12}^{4} + m_{2}^{2}s_{12}^{4} + 2m_{1}m_{2}c_{12}^{2}s_{12}^{2} \cos 2\beta + m_{3}s_{13}^{2}}, \tag{9}
\]

whose RHS is maximized by taking \( \cos 2\beta = +1 \) and \( s_{13}^{2} = s_{13}^{2} \text{(CH)} \) in the last term and \( c_{13}^{2} = 1 \) in front of the square root. (A more refined treatment entails the same excluded region.) One can then derive an inequality similar to (7);

\[
\langle m \rangle_{\beta \beta}^{\text{min}} \leq \langle m \rangle_{\beta \beta} \leq (m_{1}c_{12}^{2} + m_{2}s_{12}^{2}) + m_{3}s_{13}^{2} \text{(CH)}. \tag{10}
\]

By rewriting (10) we obtain the other upper bound on \( \cos 2\theta_{12} \);

\[
\cos 2\theta_{12} \leq \frac{m_{2} + m_{1}}{m_{2} - m_{1}} - \frac{\langle m \rangle_{\beta \beta}^{\text{min}} - m_{3}s_{13}^{2} \text{(CH)}}{\frac{1}{2}(m_{2} - m_{1})}. \tag{11}
\]

To summarize, we have derived in this section the two kinds of upper bound on \( \cos 2\theta_{12} \) (lower bound for \( \cos 2\theta_{12} < 0 \)) by using the assumed experimental constraint \( \langle m \rangle_{\beta \beta}^{\text{min}} \leq \langle m \rangle_{\beta \beta} \leq \langle m \rangle_{\beta \beta}^{\text{max}} \) imposed by neutrinoless double beta decay experiments.
III. CONSTRAINTS EXPRESSED BY EXPERIMENTAL OBSERVABLES

We rewrite the bounds on solar mixing angle in terms of measurable quantities. Toward the goal we note that three neutrino masses $m_i$ (i=1,2,3) can be expressed by the two $\Delta m^2$ and a remaining over-all scale $m_H$. We assign $m_H$ to the mass of the highest-mass state, $m_3$ in the normal mass hierarchy ($\Delta m^2_{23} > 0$), and $m_2$ in the inverted mass hierarchy ($\Delta m^2_{23} < 0$), respectively.

We have argued in our previous paper [19] that in a reasonable approximation one can regard $m_H$ as the observable $\langle m \rangle_\beta$ in direct mass measurements in single beta decay experiments. We have noticed that the identification is exact in two extreme cases of degenerate and hierarchical mass spectra. Then, the three mass eigenvalues of neutrinos can be represented solely by observables; $\Delta m^2_{\text{atm}}$, $\Delta m^2_{\odot}$, and $\langle m \rangle_\beta$ in a good approximation.

In each neutrino mass pattern, we have the expressions of three mass eigenvalues:

Normal mass hierarchy ($\Delta m^2_{23} > 0$);

$$m_1 = \sqrt{m_H^2 \pm \Delta m^2_{\text{atm}} - \Delta m^2_{\odot}}, \quad m_2 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}}}, \quad m_3 = m_H \simeq \langle m \rangle_\beta. \quad (13)$$

Inverted mass hierarchy ($\Delta m^2_{23} < 0$);

$$m_1 = \sqrt{m_H^2 - \Delta m^2_{\odot}}, \quad m_2 = m_H \simeq \langle m \rangle_\beta, \quad m_3 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}}}. \quad (14)$$

It is instructive to work out the form of constraint in the degenerate mass approximation, $m_i^2 \simeq m^2 \gg \Delta m^2_{\text{atm}}, \Delta m^2_{\odot}$. It is easy to show that in the degenerate mass limit the bound (8) becomes

$$|\cos 2\theta_{12}| \leq \sec^2 \theta_{13}(\text{CH}) \left[ \frac{\langle m \rangle_{\beta \beta}^{\text{max}}}{\langle m \rangle_\beta} + s_{13}^2(\text{CH}) \right]. \quad (15)$$

On the other hand, the bound (9) gives the inequality $\langle m \rangle_\beta \geq \langle m \rangle_{\beta \beta}^{\text{min}}$ in the degenerate mass limit. (To show this one may go back to (9), rather than using (11).)

*While we used the linear formula derived by Farzan, Peres and Smirnov [25]

$$\langle m \rangle_\beta = \frac{\sum_{j=1}^{n} m_j |U_{ej}|^2}{\sum_{j=1}^{n} |U_{ej}|^2} \quad (12)$$

with $n$ being the dimension of the subspace of (approximately) degenerate mass neutrinos, this point remains valid even if we use an alternative quadratic expression [26].

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IV. ANALYSIS OF THE DOUBLE BETA-REACTOR JOINT CONSTRAINTS

We analyze in this section the joint constraints derived in the foregoing sections and try to extract the implications. Let us start by examining the case of recent observation announced in [20] which gives rise to $0.11 \text{ eV} \leq \langle m \rangle_{\beta \beta} \leq 0.56 \text{ eV}$ and $0.05 \text{ eV} \leq \langle m \rangle_{\beta \beta} \leq 0.84 \text{ eV}$ if $\pm 50\%$ uncertainty of the nuclear matrix elements are considered, each at $95\%$ CL. In Fig. 1 we present on $\langle m \rangle_{\beta} - \cos 2\theta_{12}$ plane the constraint (8) by the thick solid lines (solid line) and (11) by the thick dashed line (dashed line) for cases with (without) uncertainty of the nuclear matrix elements, respectively. The regions surrounded by these lines are allowed. The slope of $\langle m \rangle_{\beta}$-dependence of (11) is so large that the dashed line looks like a vertical line, which implies the inequality $\langle m \rangle_{\beta} \geq \langle m \rangle_{\beta \beta}^{\text{min}}$. We have derived it earlier in the degenerate mass limit, but it is generically true if $\Delta m_{23}^2$ is smaller than other relevant mass squared scales. Only the case of normal mass hierarchy ($\Delta m_{23}^2 > 0$) is shown in Fig. 1; the case of inverted hierarchy ($\Delta m_{23}^2 < 0$) gives an almost identical result except for a slight shift of the dashed line toward smaller $\langle m \rangle_{\beta}$ by $\simeq 10\%$.

Superimposed in Fig. 1 are the $95\%$ CL allowed regions of $\cos 2\theta_{12}$ for the large mixing angle (LMA) MSW solution (indicated by the shaded region between thin solid lines) and the low (LOW) MSW solution (indicated by the shaded region between thin dashed lines) of the solar neutrino problem [27]. There are several up to date global analyses of the solar neutrino data with similar results of allowed region of mixing parameters [28]. Therefore, we just quote the result obtained by Krastev and Smirnov in the last reference in [28].

Figure 1 illustrates that for a given value of $\cos 2\theta_{12}$ the single beta decay observable $\langle m \rangle_{\beta}$ has to fall into a region bounded by $\langle m \rangle_{\beta}^{\text{min}} \simeq \langle m \rangle_{\beta \beta}^{\text{min}}$ and $\langle m \rangle_{\beta}^{\text{max}}$, which are dictated by (11) and (8), respectively. Thus, we have a clear prediction for direct mass measurements using a single beta decay with observation of double beta decay events. With use of the numbers given in [20], for example, the observable $\langle m \rangle_{\beta}$ must fall into the region $0.05 \text{ eV} \leq \langle m \rangle_{\beta} \leq 2 \text{ eV}$ ($0.11 \text{ eV} \leq \langle m \rangle_{\beta} \leq 1.3 \text{ eV}$) with (without) uncertainty of nuclear matrix elements at the best fit parameters of the LMA MSW solution. (The best fit value is $\tan^2 \theta_{12} = 0.35$, or $\cos 2\theta_{12} = 0.48$ in the last reference in [28].) Within the allowed region the cancellation between three mass eigenstates can take place for appropriate values of Majorana phases that allow (typically) a factor of 2-3 larger values of $\langle m \rangle_{\beta}$ compared with the measured value.

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of $\langle m \rangle_{\beta\beta}$. At around maximal mixing ($\cos 2\theta_{12} \approx 0$), which is allowed by 95 % CL in the LOW solution, the cancellation is so efficient that much larger values of $\langle m \rangle_{\beta}$ is allowed. Therefore, there are still ample room for hot dark matter mass neutrinos both in the LMA and the LOW solutions.

It should be emphasized that a finite value of $\langle m \rangle_{\beta\beta}$ does imply a lower bound on $\langle m \rangle_{\beta}$, as indicated in Fig. 1; a vanishingly small $\langle m \rangle_{\beta}$ cannot be consistent with finite $\langle m \rangle_{\beta\beta}$ in double beta decay experiments. The sensitivity of the proposed KATRIN experiment is expected to extend to $\langle m \rangle_{\beta} \leq 0.3$ eV [29]. On the other hand, the present 68 % CL limit quoted in [20] without nuclear element uncertainty is $0.28$ eV $\leq \langle m \rangle_{\beta\beta} \leq 0.49$ eV. Therefore, if the limit is further tightened by additional data taking in the future, both experiments can become inconsistent, giving another opportunity of cross checking.

In Fig. 2, we demonstrate the approximate scaling relation obeyed by the constraint (8) by taking $\langle m \rangle_{\beta}/\langle m \rangle_{\beta\beta}^{\text{max}}$ as the abscissa in a wide range of the $\langle m \rangle_{\beta\beta}^{\text{max}}$ in degenerate mass region, $0.1$ eV $\lesssim \langle m \rangle_{\beta\beta}^{\text{max}} \lesssim 1$ eV. The scaling is exact in the degenerate mass limit as shown in (15). The relation is useful to estimate the allowed region of $\langle m \rangle_{\beta}$ for a given value of $\langle m \rangle_{\beta\beta}^{\text{max}}$ which is not explicitly examined in this paper.

The most stringent bound to date on $\langle m \rangle_{\beta}$ is from the Mainz collaboration [30], $\langle m \rangle_{\beta} \leq 2.2$ eV (95 % CL). (A similar bound $\langle m \rangle_{\beta} \leq 2.5$ eV (95 % CL) is derived by the Troitsk group [31].) As we can see in Fig. 2 that the double beta bound with the CUORE sensitivity region $\langle m \rangle_{\beta\beta} \lesssim 0.3$ eV [11] becomes stronger than the Mainz bound for the LMA MSW solution but not for the LOW MSW solution in their 95 % CL regions.

In Fig. 3, we present the similar allowed regions for hypothetical discovery of neutrinoless double beta decay events which would produce the experimental bounds $0.01$ eV $\leq \langle m \rangle_{\beta\beta} \leq 0.03$ eV. It is to examine how the constraint changes in some other situation of discovery with different mass parameter ranges. We note that even such deep region of sensitivity will be explored by several experiments [10,12–14].

For this case, the bounds for the normal and the inverted mass hierarchies start to split as shown in Fig. 3. In the case of inverted mass hierarchy the lower bound on $\langle m \rangle_{\beta}$ is replaced by the trivial bound $\langle m \rangle_{\beta} \geq \sqrt{\Delta m_{\text{atm}}^2}$ which is more restrictive. The latter is indicated by the dash-dotted line in Fig. 3b. It is also evident that the constraint from double beta decay
is so stringent that the limit on $\langle m \rangle_\beta$ is tightened to be $\langle m \rangle_\beta \lesssim 0.2$ eV for the LMA MSW solution.

In conclusion, we have demonstrated in this and the previous papers the mutual intimate relationship between observation and/or nonobservation in single beta decay and neutrinoless double beta decay experiments. We hope that it stimulates even richer future prospects not only in double beta decay experiments but also in direct mass measurements using single beta decay.

Finally, some remarks are in order:

(1) If the LMA MSW solution is the case and if the KamLAND experiment [32] that just started data taking can measure $\cos 2\theta_{12}$ within 10 % level accuracy, the upper limit of $\langle m \rangle_\beta$ can be accurately determined with $\sim 20$ % accuracy.

(2) In this paper we have derived constraints imposed on neutrino mixing parameters by observation of neutrinoless double beta decay events and the CHOOZ reactor bound on $|U_{e3}|^2$ in the generic three flavor mixing framework of neutrinos. Suppose that neutrinoless double beta decay events are confirmed to exist and the single beta decay experiments detect neutrino mass outside the region of the bound derived in this paper. What does it mean? It means either that double beta decay would be mediated by some mechanisms different from the usual one with Majorana neutrinos, or the three flavor mixing framework used in this paper is too tight.

Nore added:

After submitting the first version of our paper to the electronic archive, we became aware of the works which address relatively model-independent implication of the results reported in [20], or critically comment on the interpretation of the events. References [33] and [34] are the incomplete lists of them.

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FIG. 1. The constraints imposed on neutrino mixing parameters $\theta_{12}$ and the observable mass parameter $\langle m \rangle_\beta$ in single beta decay experiments by recent observation of neutrinoless double beta decay events. The solid and the dashed lines represent the bounds (8) and (11), respectively; the allowed region is inside these three lines. The bold and the normal lines are for the ranges of mass parameter $0.05 \text{ eV} \leq \langle m \rangle_\beta \leq 0.84 \text{ eV}$ and $0.11 \text{ eV} \leq \langle m \rangle_\beta \leq 0.56 \text{ eV}$ corresponding, respectively, with and without $\pm 50\%$ uncertainty of nuclear matrix elements. The mixing parameters are fixed as $\Delta m^2_{\text{atm}} = 3 \times 10^{-3} \text{ eV}^2$ and $\Delta m^2_\odot = 4.8 \times 10^{-5} \text{ eV}^2$. Also shown as shaded region are the allowed regions of $\cos 2\theta_{12}$ at 95% CL for the LMA (the region between thin solid lines) and LOW (the region between thin dashed lines) MSW solutions.
FIG. 2. The approximate scaling relation obeyed by the bound (8) for a wide range of $\langle m \rangle_{\beta \beta}$, $0.1 \text{ eV}$ (dashed lines) $\leq \langle m \rangle_{\beta \beta}^{\text{max}} \leq 1 \text{ eV}$ (solid lines). The bound (11) is schematically drawn by vertical thin dashed line. The shaded regions are the same as in Fig. 1.
FIG. 3. The same as in Fig. 2 but with assumed observed mass parameter $\langle m \rangle_{\beta\beta}$ in the range $0.01 \text{ eV} \leq \langle m \rangle_{\beta\beta} \leq 0.03 \text{ eV}$. Fig. 3a and 3b for the normal and the inverted mass hierarchies, respectively. The dash-dotted line in Fig. 3b denotes the trivial bound $\langle m \rangle_{\beta} \geq \sqrt{\Delta m_{atm}^2}$. 