Astrophysical constraints on primordial black hole formation from collapsing cosmic strings

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Abstract. Primordial Black Holes (PBH) may have formed from the collapse of cosmic string loops. The spectral shape of the PBH mass spectrum can be determined by the scaling argument for string networks. Limits on the spectral amplitude derived from extragalactic $\gamma$-ray and galactic $\gamma$-ray and cosmic ray flux observations as well as constraints from the possible formation of stable black holes remnants are reanalyzed. The new constraints are remarkably close to those derived from the normalization of the cosmic string model to the cosmic microwave background anisotropies.

1. Introduction

Cosmic strings (CS) are linear topological defects that are believed to originate during phase transitions in the very early Universe \cite{1,2,3}. Here, we consider the “standard” CS model \cite{4,5}, according to which the network of linear defects quickly reaches a “scaling” solution characterized by having the statistical properties of the string distribution independent of time if all lengths are scaled to the Hubble radius ($R_H = ct$, where $c$ is the speed of light). Cosmic string loops (CSL) are continually formed by the intersection and self-intersection of long CS (infinite CS or CSL with radius of curvature larger than $R_H$). After formation, a loop oscillates due its own tension and slowly decays by emitting gravitational radiation. The initial length of a CSL is $l(t) = \alpha R_H$, where $\alpha$ is expected to be $\sim G\mu/c^2$. The mass of a CSL is $m(t) = l(t)\mu$, where $\mu$ is the mass per unit of length of the string.

Since CS also lead to cosmic microwave background (CMB) anisotropies, the string model can be normalized by the recent COBE observations giving the constraint \cite{6,7}

\[ G\mu/c^2 \leq 1.7(\pm 0.7) \times 10^{-6} \quad (1) \]

Our assumption is that a distribution of PBH was formed by the collapse of a fraction $f$ of CSL \cite{8,9}. Hence, from the observational consequences of a present surviving distribution of PBH we can derive updated constraints on the CS scenario \cite{10}. These constraints are important because: \textit{i-)} They may indicate new ways to search for direct signatures from CS; \textit{ii-)} They may provide constraints on CS models with symmetry breaking scale $\mu^{1/2}$ smaller than $10^{16}$ GeV which are not constrained by CMB and large-scale structure data; and \textit{iii-)} They may provide tighter limits than the CMB on CS models with $G\mu/c^2 \sim 10^{-6}$.

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Because CS do not dominate the energy density of the Universe, the CS network must lose energy. We derive the rate of CSL production $\frac{dn_l}{dt}$ from the conservation of string energy in the “scaling” scenario,

$$\dot{\rho}_\infty - 2H\rho_\infty = -\frac{dn_l}{dt} \alpha \mu t,$$

where $\rho_\infty = \nu \mu c^{-3} t^{-2}$ is the energy density in long strings and $\nu$ is proportional to the average number of long strings crossing each Hubble volume.

Hawking [8] and Polnarev and Zembowicz [9] first postulated that a fraction $f$ of the CSL could collapse within its Schwarzschild radius and then form a BH. More recently, Caldwell and Casper [11] have performed numerical simulations to determine $f$ and found

$$f = 10^{4.9\pm0.2} \left( \frac{G\mu}{c^2} \right)^{4.1\pm0.1}.$$  \hspace{1cm} (2)

The BH are sufficiently small that they lose mass due to the Hawking evaporation process. The fraction of the critical density of the Universe in PBH today due to collapsing CSL is (see [10] and references quoted therein)

$$\Omega_{PBH}(t_0) = \frac{1}{\rho_{\text{crit}}(t_0)} \int_{t_0}^{t_*} dt' \frac{dn_{BH}}{dt'} m(t', t_0),$$ \hspace{1cm} (3)

where $t_0$ is the present age of the Universe; $t_*$ is formation time for a PBH with mass $M_\ast = 4.4 \times 10^{14} h^{-0.3}$ g, which is expiring today; $m(t', t_0)$ is the mass of a PBH formed at a time $t'$ at a later time $t$; and $h$ is the Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$.

PBH formed at times $t < t_*$ ($M < M_\ast$) do not contribute to this integral because they will have evaporated by today. If we assume for simplicity that PBH with mass $M > M_\ast$ will have evaporated little by the present time, we can approximate $m(t', t_0)$ by $\alpha \mu ct'$.

2. Galactic and extragalactic flux constraints

It is well known [12, 13, 14] that the extragalactic $\gamma$-ray flux observed at 100 MeV provides a strong constraint on the population of black holes evaporating today. Too many black holes would lead to an excess of such radiation above the observed value. In particular, it was shown that if the present day number density distribution of black holes of mass $M$ is proportional to $M^{-2.5}$ for PBH formed in the radiation-dominated era, then comparing the Hawking emission from the black hole distribution with the $\gamma$-ray background observed by the EGRET experiment implies the limit on the present black hole density of [15]

$$\Omega_{PBH} < \left( 5.1 \pm 1.3 \right) \times 10^{-9} h^{-1.95\pm0.15}.$$ \hspace{1cm} (4)

From the rate of formation of CSL it follows

$$\frac{dn_{BH}}{dt} = f \frac{dn_l}{dt} = \frac{\nu}{\alpha} f c^{-3} t^{1/2} \dot{t}^{-5/2}$$ \hspace{1cm} (5)

which is also proportional to $M^{-2.5}$. Thus, we can apply the limit (4) to (3). Taking into account the formation rate of CSL given by (5), we can determine an upper bound on the fraction $f$ of
CSL which collapses to form PBH \[10\]

\[
f \lesssim 6.8(\pm 1.7) \times 10^{-19} \left[ \frac{\nu}{40} \right]^{-1} \left[ \frac{\gamma}{100} \right]^{-\frac{1}{2}} \left[ \frac{M_\ast}{4.4 \times 10^{14} \ h^{-0.3} \ \text{gm}} \right]^{\frac{1}{2}} \\
x \left[ \frac{G\mu/c^2}{1.7 \times 10^{-6}} \right]^{-2} h^{-0.1 \pm 0.15} \left[ \frac{t_{eq}}{3.2 \times 10^{10} \ h^{-4} \ \text{sec}} \right]^{-1/2}, \tag{6}
\]

where \(\gamma\) is a dimensionless coefficient describing the strength of gravitational radiation generated by string loops (\(\alpha = \gamma G\mu/c^2\)).

Now, if we assume the validity of the Caldwell and Casper simulations \[11\], we can deduce an upper bound on the value of \(G\mu/c^2\). Combining (2) and (6) and taking into account (4), we have

\[
G\mu/c^2 \leq 2.1(\pm 0.7) \times 10^{-6}. \tag{7}
\]

This limit is very close to those from the normalization of the CS model to the CMB.

We can also apply the limits on \(\Omega_{PBH}(t_0)\) coming from the observations of the Galactic \(\gamma\)-ray, antiproton, electron and positron fluxes. These limits, however, are less certain than the diffuse extragalactic \(\gamma\)-ray flux because of the dependence on the unknown degree to which PBH cluster in the Galaxy and on the propagation and modulation of emitted particles in the Galaxy and Solar System. Assuming a halo model in which the spatial distribution of black holes is proportional to the isothermal density distribution of dark matter within the Galactic halo and simulating the diffusive propagation of antiprotons in the Galaxy, Maki et al. \[16\] derive an upper limit on \(\Omega_{PBH}\) of

\[
\Omega_{PBH} < 1.8 \times 10^{-9} h^{-4/3}
\]

based on antiproton data from the BESS '93 balloon flight. This value would imply a limit on \(f\) in (6) that is stronger by a factor of about 3 and a corresponding limit on \(G\mu/c^2\) in (7) of

\[
G\mu/c^2 < 1.8(\pm 0.5) \times 10^{-6}.
\]

3. Limits on black hole remnants

The final stage of an expiring BH is unknown. It is possible that the evaporation may stabilize at or before the BH mass reaches the Planck mass, \(m_{pl}\). Thus, a mass \(M_{\text{relic}}\) would remain, implying that the present fraction of the critical density in BH relics is

\[
\Omega_{\text{relic}} = \frac{M_{\text{relic}}}{m_{pl}} \frac{\rho_{\text{crit}}(t_0)}{\rho_{\text{crit}}(t_0)} \int_{t_i}^{t'(t_0)} dt' \frac{dn_{BH}}{dt'}.
\]

For \(\Omega_{\text{relic}} \leq 1\), we obtain the constraint

\[
f \leq 1.9 \times 10^{-15} \left[ \frac{\nu}{40} \right]^{-1} \left[ \frac{\gamma}{100} \right]^{-1/2} \left( \frac{m_{pl}}{M_{\text{relic}}} \right) \left( \frac{G\mu/c^2}{1.7 \times 10^{-6}} \right)^{-11/4} h^2 \left[ \frac{t_{eq}}{3.2 \times 10^{10} \ h^{-4} \ \text{sec}} \right]^{-1/2}.
\]
If we make use of both \( f \) and \( G\mu/c^2 \) from (6) and (1) we can derive an upper limit on \( \Omega_{\text{relic}} \):

\[
\Omega_{\text{relic}} \leq 3.6(\pm 0.9) \times 10^{-4}(\frac{M_{\text{relic}}}{m_{\text{pl}}})(\frac{M_\star}{4.4 \times 10^{14} \, h^{-0.3} \, \text{gm}})^{1/2}(\frac{G\mu/c^2}{1.7 \times 10^{-6}})^{3/4} h^{-2.1 \pm 0.15}.
\]

Thus, the bound on the BH formation efficiency factor \( f \) given by (6) implies that BH remnants from collapsing CSL would contribute significantly to the dark matter of the Universe in the CS scenario of structure formation \((G\mu/c^2 \approx 1.7 \times 10^{-6})\) if the BH remnants have a relic mass larger than about \( 10^3 m_{\text{pl}} \).

4. Conclusions

We have taken advantage of the recent numerical simulations to better understand PBH formation. The observational consequences of a PBH distribution were used to constrain the CS scenario. We have found that the limits on \( G\mu/c^2 \) are comparable to those stemming from other criteria. Unless the mass of the BH remnants is larger than \( 10^3 m_{\text{pl}} \), these remnants will contribute negligibly to the dark matter of the Universe, even if the BH formation rate has the maximal value allowed by the \( \gamma \)-ray flux constraints. A remnant mass of \( 10^3 m_{\text{pl}} \), however, can arise naturally in some models \([17]\) of BH evaporation. In this case, cosmic strings could consistently provide an explanation for the origin of cosmological structure, for the dark matter, and for the origin of the extragalactic \( \gamma \)-ray and Galactic cosmic ray backgrounds around \( 100 \, \text{MeV} \).

References

[1] A. Vilenkin and E.P.S. Shellard, ‘Strings and Other Topological Defects’ (Cambridge Univ. Press, Cambridge, 1994).
[2] M. Hindmarsh and T.W.B. Kibble, Rept. Prog. Phys. 58, 477 (1995).
[3] R. Brandenberger, Int. J. Mod. Phys. A9, 2117 (1994).
[4] Ya.B. Zel’dovich, Mon. Not. R. astron. Soc. 192, 663 (1980).
[5] A. Vilenkin, Phys. Rev. Lett. 46, 1169 (1981).
[6] L. Perivolaropoulos, Phys. Lett. B298, 305 (1993).
[7] B. Allen et al., Phys. Rev. Lett. 77, 3061 (1996).
[8] S. Hawking, Phys. Lett. B231, 237 (1989).
[9] A. Polnarev and R. Zembowicz, Phys. Rev. D43, 1106 (1991).
[10] J. H. MacGibbon, R. Brandenberger and U.F. Wichoski, Phys. Rev. D57, 2158 (1998).
[11] R. Caldwell and P. Casper, Phys. Rev. D53, 3002 (1996).
[12] B.J. Carr, Ap. J. 201, 1 (1975).
[13] D. Page and S. Hawking, Ap. J. 206, 1 (1976).
[14] B.J. Carr, Ap. J. 206, 8 (1976).
[15] J.H. MacGibbon and B.J. Carr, in preparation (1998).
[16] K. Maki et al., Phys. Rev. Lett. 76, 3474 (1996).
[17] S. Coleman, J. Preskill and F. Wilczek, Mod. Phys. Lett. A6, 1631 (1991).