Impact Factors for Reggeon-Gluon Transitions

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Abstract

General expressions for the impact factors up to terms vanishing at the space-time dimension $D \to 4$ are presented. Their infrared behaviour is analysed and calculation of exact in $D \to 4$ asymptotics at small momenta of Reggeized gluons is discussed.

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1 Introduction

Reggeon-gluon impact factors are natural generalization of particle-particle ones. In the BFKL approach \[1\]-\[4\], discontinuities of elastic amplitudes are given by the convolutions of the Green functions of two interacting Reggeized gluons with the impact factors of colliding particles describing scattering of these particles due to interaction with the Reggeized gluons. Similarly, discontinuities of many-gluon amplitudes in the multi-Regge kinematics (MRK) contain the Reggeon-gluon impact factors, which describe transitions of Reggeons (Reggeized gluons) into particles (ordinary gluons) due to interaction with the Reggeized gluons. These impact factors appeared firstly \[5\] in the derivation of the bootstrap conditions for the gluon Reggeization (more precisely, for the multi-Regge form of the many-gluon amplitudes). The idea of this form is the basis of the BFKL approach. It can be proved using the s-channel unitarity. Compatibility of the unitarity with the multi-Regge form leads to the bootstrap relations connecting discontinuities of the amplitudes with products of their real parts and gluon trajectories \[6\]. It turns out that fulfillment of an infinite set of these relations guarantees the multi-Regge form of scattering amplitudes. On the other hand, all bootstrap relations are fulfilled if several conditions imposed on the Reggeon vertices and the trajectory (bootstrap conditions) hold true \[6\]. Now fulfillment of all bootstrap conditions is proved. The most complicated condition, which includes the impact factors for Reggeon-gluon transition, was proved recently, both in QCD \[7\]-\[9\] and in supersymmetric Yang-Mills theories \[10\].

Discontinuities of \(n\)-gluon amplitudes in the MRK at \(n \geq 6\) can be used \[11\] for a simple demonstration of violation of the ABDK-BDS (Anastasiou-Bern-Dixon-Kosower — Bern-Dixon-Smirnov) ansatz \[12, 13\] for amplitudes with maximal helicity violation (MHV) in Yang-Mills theories with maximal supersymmetry (N=4 SYM) in the planar limit and for the calculations of the remainder functions to this ansatz. There are two hypothesis about the remainder functions: the hypothesis of the dual conformal invariance \[14\]-\[20\], which asserts that the MHV amplitudes are given by the products of the BDS amplitudes and the remainder functions depending only on the anharmonic ratios of kinematic invariants, and the hypothesis of scattering amplitude/Wilson loop correspondence \[18, 19\], \[21\]-\[24\], according to which the remainder functions are given by the expectation values of the Wilson loops. Both these hypotheses are not proved. They can be tested by comparison of the BFKL discontinuities with the discontinuities calculated with their use \[25\]-\[28\].

The discontinuities of many-particle amplitudes are interesting also because they are necessary for further development of the BFKL approach. They do not need for derivation of the BFKL equation in the next-to-leading logarithmic approximation (NLLA), because they are suppressed by one power of some of large logarithms in comparison with the real parts of the amplitudes and therefore in the NLLA they don’t contribute in the unitarity relations. But their account in the next-to-next-to-leading logarithmic approximation (NNLLA) is indispensable.

All this makes calculation of discontinuities of the MRK amplitudes to be very important. Since the discontinuities contain the Reggeon-gluon impact factors, the calculation requires knowledge of these impact factors and investigation of their properties very important. Here I discuss the current situation with the Reggeon-gluon impact factors.
2 Reggeon-gluon impact factors in the bootstrap scheme

As it is known, in the next-to-leading order (NLO) impact factors are scheme dependent. In the Yang-Mills theories of general form they contain contributions of gauge bosons (gluons), fermions and scalars. In the scheme adapted for verification of the bootstrap conditions (bootstrap scheme) these contributions were calculated in [8], [7] and [10] respectively. Using these results, one can obtain in these scheme the NLO Reggeon-gluon impact factors in the Yang-Mills theories with fermions and scalars in any representations of the colour group.

Here, the notation of Refs.[7]-[10] are used, in particular, the momentum expansion $p = p^+ n_1 + p^- n_2 + p_\perp$, where $n_{1,2}$ are the light-cone vectors, $(n_1, n_2) = 1$, and $\perp$ means transverse to the $n_1, n_2$ plane components. For amplitudes with the negative signature, the impact factor of the transition of the Reggeon $R$ into the gluon $G$ in the interaction with the Reggeized gluons $G_1$ and $G_2$ is antisymmetric with respect to the $G_1 \leftrightarrow G_2$ exchange. It can be written as the difference of the $s$ and $u$ parts

$$\langle GR_1 \rangle = \langle GR_1 \rangle_s - \langle GR_1 \rangle_u, \quad \langle GR|G_1G_2\rangle_u = \langle GR|G_2G_1\rangle_s. \quad (2.1)$$

In the NLO each of the parts contains two colour structures. In the light-cone gauge $(e(k), n_2) = 0$,

$$e = e_\perp - \frac{(e_\perp k_\perp)}{k^+} n_2 \quad (2.2)$$

for the gluon $G$ with the momentum $k$ and the polarization vector $(e(k))$, the $s$ -part has the form

$$\langle GR_1 | G_1 G_2 \rangle_s = g^2 \delta(q_1^\perp - \vec{k} - \vec{r}_1 - \vec{r}_2) e^\perp \left[ (T^a T^b)_{c_1c_2} \left( 2\tilde{C}_1 + \tilde{g}^2 \tilde{F}_1(q_1, k; \vec{r}_1, \vec{r}_2) \right) + \frac{1}{N_c} Tr (T^{c_2} T^a T^{c_1} T^b) \tilde{g}^2 \tilde{F}_2(q_1, k; \vec{r}_1, \vec{r}_2) \right]. \quad (2.3)$$

Here $g$ is the bare coupling constant, $\tilde{g}^2 = g^2 \Gamma(1 - \epsilon)/(4\pi)^{2+\epsilon}$, $\Gamma(x)$ is the Euler gamma-function, $\epsilon = (D - 4)/2$, $D$ is the space-time dimension, $T^a$ are the colour group generators in the adjoint representation, $q_1$, $k$, $r_1$, $r_2$ and $a$, $b$, $c_1$, $c_2$ are the momenta and colour indices of the Reggeon $R_1$, the gluon $G$ and the Reggeized gluons $G_1$ and $G_2$ respectively, the vector sign is used for transverse components of vectors,

$$\tilde{C}_1 = q_1 \frac{q_1^2}{(q_1 - r_1)^2}. \quad (2.4)$$

In the bootstrap scheme with the dimensional regularization

$$\tilde{F}_1(q_1, k; \vec{r}_1, \vec{r}_2)_s = \tilde{C}_1 \left( \ln \left( \frac{(q_1 - \vec{r}_1)^2}{k^2} \right) + \ln \left( \frac{r_1^2}{k^4} \right) + \ln \left( \frac{(q_1 - \vec{r}_1)^2}{q_1^2} \right) + \ln \left( \frac{r_1^2}{q_1^2} \right) \right) - 4 \frac{(\vec{k}^2)^{\epsilon}}{\epsilon^2} + 6\zeta(2) \right) + \tilde{C}_2 \left( \ln \left( \frac{k^2}{r_2^2} \right) + \ln \left( \frac{(q_1 - \vec{r}_1)^2}{r_2^2} \right) + \ln \left( \frac{q_2^2}{q_1^2} \right) + \ln \left( \frac{k^2}{q_2^2} \right) \right). \quad (2.5)$$
\[ +2 \left[ \vec{C}_1 \times [\vec{q}_1 \times \vec{r}_1] \right] I_{\vec{q}_1, -\vec{r}_1} + 2 \left[ \vec{C}_2 \times [\vec{q}_1 \times \vec{k}] \right] I_{\vec{q}_1, -\vec{k}} - 2 \left( \vec{C}_1 - \vec{C}_2 \right) \times [\vec{k} \times \vec{r}_2] \right] I_{\vec{k}, \vec{r}_2}. \]

\[ + \frac{\beta_0}{N_c} \left[ \vec{C}_2 \ln \frac{q_2^2 (\vec{q}_1 - \vec{r}_1)^2}{q_1^2 q_2^2 r_2^2} - \vec{C}_1 \left( \frac{1}{\epsilon} + \ln \left( \frac{(\vec{q}_1 - \vec{r}_1)^2}{q_1^2} \right) \right) \right] + \vec{C}_1 \left( \frac{67}{9} - \frac{10 a_f}{9} - \frac{4 a_s}{9} \right) \]

\[ + \left[ \frac{\beta_0}{N_c} \left( \vec{C}_2 \vec{q}_1^2 + \vec{q}_2^2 \vec{k}^2 + \frac{2 \vec{q}_2^2 \vec{q}_1^2}{k^2 q_1^2 q_2^2} \ln \frac{q_1^2}{q_2^2} \right) + \vec{C}_0 \left( \vec{C}_2 \frac{2k^2}{(q_1^2 - q_2^2)^2} - \vec{k} (2k^2 - q_1^2 - q_2^2) \right) \times \left( \frac{q_1^2 + q_2^2}{q_1^2 - q_2^2} - \frac{2q_2^2 q_1^2}{q_1^2 - q_2^2} \ln \frac{q_1^2}{q_2^2} \right) + \frac{k}{q_1^2} \right) \right] \cdot (\vec{q}_1 \to \vec{q}_1 - \vec{r}_1, \vec{q}_2 \to \vec{r}_2, \vec{k} \to \vec{k}). \] (2.4)

Here the subscript * denotes the bootstrap scheme, \( \zeta(n) \) is the Riemann zeta-function \( (\zeta(2) = \pi^2/6) \),

\[ \vec{C}_2 = \vec{q}_1 - \vec{k} \frac{q_1^2}{k^2}, \] (2.5)

\([\vec{a} \times c [\vec{b} \times \vec{c}]]\) is a double vector product,

\[ I_{\vec{p}, \vec{q}} = \int_0^1 \frac{dx}{(\vec{p} + x\vec{q})^2} \ln \left( \frac{\vec{p}^2}{x^2 \vec{q}^2} \right), \quad I_{\vec{p}, \vec{q}} = I_{-\vec{p}, -\vec{q}} = I_{\vec{q}, \vec{p}} = I_{-\vec{q}, -\vec{p}} \] \( \beta_0 = \frac{11}{3} N_c - \frac{2}{3} a_f - \frac{1}{6} a_s, \quad \beta_0 = \frac{1}{3} N_c - \frac{1}{3} a_f + \frac{1}{6} a_s, \]

\( a_f = 2\kappa_f n_f T_f, \quad a_s = 2\kappa_s n_s T_s, \quad T_f \) and \( T_s \) are defined by the relations

\[ \text{Tr} \left( T^a_f T^b_f \right) = T_f \delta^{ab}, \quad \text{Tr} \left( T^a_s T^b_s \right) = T_s \delta^{ab}, \] (2.6)

where \( T^a_f \) and \( T^a_s \) are the colour group generators for fermions and scalars, respectively, and \( \kappa_f \) (\( \kappa_s \)) is equal to 1/2 for Majorana fermions (neutral scalars) in self-conjugated representations and 1 otherwise. In the case of \( n_M \) Majorana fermions and \( n_s \) scalars in the adjoint representation \( a_f = n_M N_c, \quad a_s = n_s N_c. \) For \( N \)-extended SYM \( n_M = N, \quad n_s = 2(N - 1). \) Remind that the result (2.4) is obtained in the dimensional regularization, which differs from the dimensional reduction used in supersymmetric theories. For \( N = 4 \) SYM in the dimensional reduction one has to take \( n_s = 6 - 2\epsilon. \) In this case the terms with \( \beta_0, \vec{\beta}_0 \) and \( \left( \frac{67}{9} - \frac{10 a_f}{9} - \frac{4 a_s}{9} \right) \) in (2.4) disappear. Note that the expression (2.4) is obtained with the accuracy up to terms vanishing at \( \epsilon \to 0. \) With the same accuracy

\[ \Phi_2(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) = \vec{q}_1^2 \int_0^1 dx_1 \left\{ \frac{(\vec{q}_1 - \vec{r}_1)}{(\vec{q}_1 - \vec{r}_1)^2} \left[ \frac{(\vec{k}^2)}{x_1^{1-2\epsilon}} - \zeta_2 + \frac{(\vec{r}_2^2 - x_1 \vec{k}^2)}{(\vec{r}_2 + x_1 \vec{k})^2} \right] \right\} \]

\[ \times \ln \left( \frac{(\vec{r}_1 + x_1 \vec{k})^2 (\vec{q}_1 - \vec{r}_1)^2}{q_1^2 \vec{k}^2 x_1^2} \right) - \frac{1}{x_1} \ln \left( \frac{(\vec{r}_1 + x_1 \vec{k})^2 (\vec{r}_2 + x_1 \vec{k})^2 (\vec{r}_2 + x_1 \vec{k})^2}{x_2 \vec{r}_1 \vec{r}_2 (\vec{k} + \vec{r}_2)^2} \right) \]

\[ + \vec{k} \left[ \frac{1}{x_1} \ln \left( \frac{(\vec{r}_1 + x_1 \vec{k})^2}{\vec{r}_1^2} \right) + \frac{x_1 \vec{k}^2}{\vec{r}_2 + x_1 \vec{k}} \ln \left( \frac{(\vec{r}_1 + x_2 \vec{k})^2 (\vec{q}_1 - \vec{r}_1)^2}{q_1^2 \vec{k}^2 x_1^2} \right) \right] \]

\[ - \frac{\vec{q}_1}{q_1^2} \frac{1}{x_1} \ln \left( \frac{(\vec{r}_1 + x_2 \vec{k})^2 (\vec{r}_1 + x_1 \vec{k})^2}{(\vec{r}_1 + \vec{k})^2 \vec{r}_1^2} \right) \] (2.7)
Eqs. (2.4), (2.7) give the impact factors in the bootstrap scheme. Transition to the standard scheme and to the scheme in which the BFKL kernel in \( N = 4 \) SYM and the energy evolution parameter are invariant under Möbius transformations in the momentum space is discussed in [29].

3 Colour decomposition

To calculate discontinuities one needs to decompose the colour structures into irreducible representations of the colour group in the channel with two Reggeized gluons. The decomposition looks as follows

\[
(T^a T^b)_{c_1 c_2} = N_c \sum_R c_R \langle ab | \hat{P}_R | c_1 c_2 \rangle,
\]

\[
Tr (T^{c_2} T^a T^{c_1} T^b) = N_c \sum_R c_R (c_R - \frac{1}{2}) \langle ab | \hat{P}_R | c_1 c_2 \rangle,
\]

where \( \hat{P}_R \) are the projections operators of the two-Reggeon colour states on the irreducible representations \( R \). Explicit form of these operators and the values of the coefficients \( c_R \) can be found in [30]. In the limit of large \( N_c \) the term in (2.3) with the colour structure \( Tr (T^{c_2} T^a T^{c_1} T^b) \) disappears and with the account of (2.1) the impact factors take the form

\[
\langle GR_1 | G_2 \rangle = g^2 \delta (\vec{q}_1 - \bar{\vec{k}} - \vec{r}_1 - \vec{r}_2) \epsilon^* \left[ f^{abc} f^{cde} \left( 2\vec{q}_1 - (\vec{q}_1 - \vec{r}_1) \frac{\vec{q}_1^2}{(\vec{q}_1 - \vec{r}_1)^2} \right. \right.
\]

\[
- (\vec{q}_1 - \vec{r}_2) \frac{\vec{q}_1^2}{(\vec{q}_1 - \vec{r}_2)^2} + \left. \frac{g^2}{2} \left( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) + \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_2, \vec{r}_1) \right) \right] + d^{abc} d^{cde}
\]

\[
\times \left[ \frac{\vec{q}_1^2 (\vec{q}_1 - \vec{r}_2)}{(\vec{q}_1 - \vec{r}_2)^2} - \frac{\vec{q}_1^2 (\vec{q}_1 - \vec{r}_1)}{(\vec{q}_1 - \vec{r}_1)^2} + \frac{g^2}{2} \left( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) - \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_2, \vec{r}_1) \right) \right].
\]

4 Infrared behaviour of the impact factors

As is clear from the foregoing, Eq. (3.2) gives the impact factor up to terms vanishing in the limit \( \epsilon \rightarrow 0 \). Unfortunately, using (3.2) for calculation of discontinuities does not provide such accuracy for them. The reason is the integration measure \( d^{2+2\epsilon} r_{1\perp} d^{2+2\epsilon} r_{2\perp} / (r_{1\perp}^2 r_{2\perp}^2) \delta (q_{2\perp} - r_{1\perp} - r_{2\perp}) \) which is singular at \( \epsilon \rightarrow 0 \). To keep in the discontinuities all terms nonvanishing in the limit \( \epsilon \rightarrow 0 \) one has to calculate \( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) \) more accurately.

In fact, greater accuracy is required only in the region of small \( |\vec{r}_2| \), because in the limit \( |\vec{r}_1| \rightarrow 0 \) \( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) \) turns to be zero, which is seen from (2.4). In contrast, in the limit \( |\vec{r}_2| \rightarrow 0 \) \( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) \) not only does not vanish but have logarithmic singularities. To keep in the discontinuities all terms nonvanishing in the limit \( \epsilon \rightarrow 0 \) one has to know in the region of small \( |\vec{r}_2| \) terms of order \( \epsilon \) in \( \Phi_1 (\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2) \) and must not expand \((\vec{r}_2^2)^\epsilon\) in powers of \( \epsilon \).

In the NLO, the impact factor contains contributions of two types: virtual ones, which are obtained from the one-loop corrections to the Reggeon vertices and the gluon trajectory, and real contributions arising from production of two real particles. In the bootstrap scheme, the
The real contribution to $\Phi_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ can be calculated at small $|\vec{r}_2|$ exactly in $\epsilon$ using intermediate results of Refs. [7]-[10]. It is proportional to $(\vec{r}_2^2)^\epsilon$ and has the form

$$\Phi_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)_{\text{real}} = 4(\vec{r}_2^2)^\epsilon \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \left[ C_2 \left( \frac{1}{2\epsilon^2} + \frac{(\psi(1) - \psi(1+2\epsilon))}{\epsilon} \right) \right. $$

$$\left. - \frac{\Gamma(1+2\epsilon)}{\Gamma(4+2\epsilon)} \left( a_1(1+\epsilon) - a_2 \right) + \frac{2\Gamma(1+2\epsilon)}{\Gamma(4+2\epsilon)} \frac{\vec{r}_2^2 \vec{C}_2}{\vec{r}_2^2} a_2 \right], \tag{4.1}$$

where

$$a_1 = 11 + 7\epsilon - 2(1+\epsilon)a_f - \frac{a_s}{2}, \quad a_2 = 1 + \epsilon - a_f + \frac{a_s}{2}. \tag{4.2}$$

For $N = 4$ SYM, the coefficients $a_1$ and $a_2$ vanish in the dimensional reduction.

The virtual contribution to $\Phi_1(\vec{q}_1, \vec{k}; \vec{r}_1, \vec{r}_2)$ in the limit of small $|\vec{r}_2|$ can be obtained with the required accuracy using its representation [8] in terms of the Reggeon vertices and the gluon trajectory and exact in $\epsilon$ expressions for the trajectory, the gluon-gluon-Reggeon vertex and fermion and scalar contributions to the Reggeon-Reggeon-gluon vertex which can be found in Refs. [32], [33] and [34, 10] respectively and the gluon production vertex in $N = 4$ SYM computed through to $O(\epsilon^2)$ in [35]. Full expressions for the Reggeon-gluon impact factors in the region of small $|\vec{r}_2|$ in the bootstrap and the standard schemes will be given in [31].

## 5 Summary

The impact factors for Reggeon-gluon transitions are an integral part of the BFKL approach. They enter the expressions for the discontinuities of many-particle amplitudes and the bootstrap conditions for the gluon Reggeization, and enable to demonstrate in a simple way violation of the ABDK-BDS ansatz for MHV amplitudes in N=4 SYM in the planar limit and to check the hypotheses about the remainder functions to this ansatz. Their knowledge is necessary for further development of the BFKL approach.

Here the impact factors in Yang-Mills theories with fermions and scalars in any representations of the gauge group are presented up to terms vanishing at $\epsilon \to 0$. Their colour decomposition is performed and infrared behaviour is discussed.

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