ON TRIVIALITY OF DIRAC-HARMONIC MAPS

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ABSTRACT. Dirac-harmonic maps \((f, \varphi)\) consist of a map \(f : M \to N\) and a twisted spinor \(\varphi \in \Gamma(\Sigma M \otimes f^*TN)\) and they are defined as critical points of the super-symmetric energy functional. The Dirac-harmonic map is called \(\mathcal{R}\)-trivial or uncoupled, if \(f\) is a harmonic map. We show that under some minimality assumption Dirac-harmonic maps defined on a compact domain are \(\mathcal{R}\)-trivial. This raises the question whether all Dirac-harmonic maps with compact domain are \(\mathcal{R}\)-trivial.

We can apply similar arguments to the heat flow for Dirac-harmonic maps, for which short-time existence and uniqueness was proven by Chen, Jost, Sun and Zhu [8] for \(\partial M \neq \emptyset\) and by Wittmann [18] for \(\partial M = \emptyset\). We show that this heat flow is just an extension of the classical heat flow for harmonic maps.

1. Introduction

Let \(M\) and \(N\) be Riemannian manifolds, and \(f : M \to N\) a \(C^1\). If \(M\) is compact, we can define the energy of of as

\[
E_1(f) = \frac{1}{2} \int_M |df|^2 \, d\text{vol}_g
\]

considered as a functional on \(C^1(M, N)\). Critical points of this functional are called harmonic maps, and there is an extensive literature about harmonic maps, with many interesting applications, also weak solutions were studied.

In the recent years, there is a growing number of publications about a super-symmetric analogue of harmonic maps, called Dirac-harmonic maps. They are defined as critical points of the supersymmetric energy functional \(\mathcal{E}\) defined in (1), see Section 2 for more details. In particular, a Dirac-harmonic map is a pair \((f, \varphi)\) of a map \(f : M \to N\) and a twisted spinor \(\varphi \in \Gamma(\Sigma M \otimes f^*TN)\). An early article [6] about such maps was written by Chen, Jost, Li and Wang, studying regularity issues for Dirac-harmonic maps, followed by [7, 9] and stimulating many associated questions and results. Between 2015 and September 2022, MathSciNet lists about 45 publications with “Dirac-harmonic” in the title. Many questions answered for harmonic maps can be discussed in the Dirac-harmonic context, often it is hard to include the spinorial part into the estimates, and good progress was achieved.

Obviously, Dirac-harmonic maps with \(\varphi \equiv 0\) are uninteresting, as then \((f, \varphi)\) is Dirac-harmonic if and only if \(f\) is harmonic; such solution are called spinor-trivial. Similarly, solutions with \(f\) constant, called map-trivial solutions, are uninteresting as well; in this case the problem is equivalent to finding harmonic spinors in the classical sense, see e.g. [11, 4, 1]. We say that a Dirac-harmonic map is uncoupled.
or $\mathcal{R}$-trivial if $f$ is harmonic. This is equivalent to the vanishing of the $\mathcal{R}$-term, see Section 2. A major question discussed in this article is whether non-$\mathcal{R}$-trivial Dirac-harmonic maps with compact domain exist. Let us discuss examples of Dirac-harmonic maps in the literature first.

Progress in constructing Dirac-harmonic maps was achieved in [13] by using (untwisted) harmonic spinors, twistor spinors and similar solutions of other spinorial equations, although it remained unclear, how one could get solutions of these spinorial equations. This was analyzed later by Ginoux and the author in [3]: we showed – see [3, Theorems 1.1 and 1.3] – that the conditions in [13, Theorems 1 and 3] can be satisfied only in exceptional cases.\footnote{Note that the proof of [13, Theorems 1] uses that the immersion is isometric, although this is not stated in this theorem [13, Theorems 1] explicitly.} In particular, for $\dim M \geq 3$ and $M$ complete, one concludes that $M$ has to be simply-connected of constant negative curvature. Other strong obstructions exist for $\dim M = 2$, in particular $f$ has to be necessarily harmonic, thus any Dirac-harmonic map obtained this way is $\mathcal{R}$-trivial.

The solutions in [13, Theorems 2], reproven as [3, Corollary 2.3] are $\mathcal{R}$-trivial as well.

Many more Dirac-harmonic maps $(f, \phi)$ were constructed in [2]. Here, the domain $M$ of $f$ is closed. The method starts with a harmonic map $f : M \to N$ and then one uses index theory to get a non-vanishing harmonic spinor $\phi \in \Gamma(\Sigma M \otimes f^* T^* N)$. In particular, these solutions are $\mathcal{R}$-trivial.

Summarizing this, the author is not aware of any publication in which the existence of any non-$\mathcal{R}$-trivial Dirac-harmonic map with compact domain was proven.

In the current article, we will show, that “generically” every Dirac-harmonic map with closed domain is $\mathcal{R}$-trivial, see Corollary 3 for details. The argument may be adapted to compact manifolds $M$ with boundary for many suitable boundary conditions, an extension of the results that we will not work out.

It thus remains questionable whether non-$\mathcal{R}$-trivial Dirac-harmonic maps with compact domain actually exist. Suppose such solutions did not exist, then the main results in several recent publications would not provide new result, compared to what is known already for harmonic maps.

Our method also applies to the Dirac-harmonic map heat flow as discussed in [8] and [18]. In this situation the genericity condition is trivially satisfied. As a consequence it turns out that several results in [8] and [18] may be obtained as corollaries from well-known statements about the heat flow for harmonic maps.

\textbf{What we do not discuss here.} In order to avoid misunderstandings, we want to add here some issues that we do not prove in this article. First, there is a variant of the super-symmetric energy functional that includes a quartic spinor term, depending on the curvature of the target manifold $N$, see e.g. [12, (2.4)]. Orally, I was told, that this variant is even more important from the perspective of applications to physics. I am unable to adapt the arguments of the current article to this modified functional. On the other hand I expect that the articles [8], [18] and many publications about Dirac-harmonic maps are sufficiently robust in order to be applicable also to critical points of this modified functional. Similar arguments apply to other perturbations. As a consequence, I consider the articles listed above as valuable contributions, in spite of the fact, that some of the main results directly follow from previously known facts.
Second, we did not yet discuss the case of non-compact domains. Our argument relies essentially on the fact that the Euler-Lagrange equations (2) describe the stationary points of a well-defined functional, that $\mathcal{D}^\flat$ is self-adjoint and that we do not get boundary terms by partial integration. In [13] and [3] an example of a Dirac-harmonic map with non-vanishing $\mathcal{R}$-term is (implicitly) given: here $M^m$ is a simply connected complete Riemannian manifold of constant negative sectional curvature $-4/(m+2)$ (i.e. a space form), and $f$ is an isometric embedding into $(m+1)$-dimensional hyperbolic space $N$, for which $f(M)$ is totally umbilic with parallel shape tensor in $N$. Due to the conformal covariance of the Dirac operator and of the Penrose operator, $M$ carries many twistor spinors and harmonic spinors, thus the Jost-Mo-Zhu method may be applied, and one easily checks that $\tau(f)$ is a non-zero constant. However, in this case $|df|^2$ is not integrable, and thus the super-symmetric energy functional $\mathcal{E}$ does not converge. The Dirac-harmonic map $(f, \varphi)$ solves the system of partial differential equations (2) without being the stationarity equation of a functional defined on all pairs of maps with spinors.

Third, we only considered solutions to the Dirac-harmonic map equation in the strong sense. We do not know to which extent our methods also generalize to weak settings. For the harmonic maps, weak solutions gave rise to involved research, see e.g. Bethuel’s work [5].

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2. Main idea and $\mathcal{R}$-triviality of Dirac-harmonic maps

Let $(M,g)$ be a compact Riemannian spin manifold, and $(N,h)$ a Riemannian manifold. For a map $f : M \to N$ and a twisted spinor $\varphi \in \Gamma(\Sigma M \otimes f^*TN)$ we define

$$
\begin{align*}
\mathcal{E}_1(f) & := \frac{1}{2} \int_M |df|^2 \operatorname{dvol}^g \\
\mathcal{E}_2(f, \varphi) & := \frac{1}{2} \int_M \langle \varphi, \mathcal{D}^\flat \varphi \rangle \operatorname{dvol}^g \\
\mathcal{E}(f, \varphi) & := \mathcal{E}_1(f) + \mathcal{E}_2(f, \varphi).
\end{align*}
$$

(1)

The functional $\mathcal{E}$ is called the super-symmetric energy functional. We define

$$
\begin{align*}
\tau(f) & := -\frac{\partial \mathcal{E}_1}{\partial f} = \operatorname{tr} \nabla df \in \Gamma(f^*TN) \\
\mathcal{R}(f, \varphi) & := \frac{\partial \mathcal{E}_2}{\partial f} \\
\Psi(f, \varphi) & := -\frac{\partial \mathcal{E}_2}{\partial \varphi} = \mathcal{D}^\flat \varphi.
\end{align*}
$$

In these equations, $\mathcal{D}^\flat$ denotes the twisted Dirac operator acting on $\Gamma(\Sigma M \otimes f^*TN)$. Further, we used the canonical $L^2$-scalar product on $\Gamma(f^*TN)$ and on $\Gamma(\Sigma \otimes f^*TN)$, in order to identify the partial derivatives of $\mathcal{E}_1$ and $\mathcal{E}_2$, given above, with the corresponding gradients.
Stationary points of $\mathcal{E}$ are called Dirac-harmonic maps. Thus this condition is equivalent to
\begin{equation}
\mathcal{H}^f \varphi = 0 \quad \text{and} \quad \tau(f) = \mathcal{R}(f, \varphi).
\end{equation}
We say that $(f, \varphi)$ is $\mathcal{R}$-trivial or uncoupled, if $\mathcal{R}(f, \varphi) = 0$. In this publication we only consider smooth Dirac-harmonic maps.

Remark. Note that $\Sigma M \otimes f^*TN$ will always denote the real tensor product, even when $f^*TN$ and $\Sigma M$ carry natural complex structures. Note that $\Sigma M$ may be defined as a real, a complex, a quaternionic or a Cl-linear Dirac operator, this does not matter. For simplicity we only restrict to the complex version in this article, as it is classically the most studied one.

Remark. The terminology for the $\mathcal{R}$-term is standard, see e.g. [13, Page 1514, after (9)], [8, (1.3)].

**Theorem 1.** Let $(f_0, \varphi_0)$ be given. We assume that there is an open neighborhood $U$ of $f_0$, and a $C^1$-map $U \ni f \mapsto \hat{\varphi}(f)$, such that $\mathcal{H}^f(\hat{\varphi}(f)) = 0$ and such that $\hat{\varphi}(f_0) = \varphi_0$. Then $(f_0, \varphi_0)$ is $\mathcal{R}$-trivial.

**Proof.** Obviously we have $\mathcal{E}_2(f, \hat{\varphi}(f)) = 0$. In the following we write $d/df$ for the total\(^2\) derivative with respect to $f$ and $-\partial/\partial f$ for the partial derivative with respect to $f$. We thus get
\begin{align*}
-\tau(f) &= \frac{d}{df} \mathcal{E}_1(f) \\
&= \frac{d}{df} \left( \mathcal{E}(f, \hat{\varphi}(f)) \right) \\
&= \frac{\partial \mathcal{E}}{\partial f} \big|_f + \frac{\partial \mathcal{E}}{\partial \varphi} \big|_{(f, \hat{\varphi}(f))} \frac{\partial \hat{\varphi}}{\partial f} \\
&= \frac{\partial \mathcal{E}_1}{\partial f} \big|_f + \frac{\partial \mathcal{E}_2}{\partial f} \big|_{(f, \hat{\varphi}(f))} + \frac{\partial \mathcal{E}_2}{\partial \varphi} \big|_{(f, \hat{\varphi}(f))} \frac{\partial \hat{\varphi}}{\partial f} \\
&= -\tau(f) + \mathcal{R}(f, \hat{\varphi}(f)).
\end{align*}
and this obviously implies $\mathcal{R}(f_0, \varphi_0) = 0$. \hfill \qed

**Corollary 2.** Suppose $f_0$ has a neighborhood $U$ such that for all $f \in U$ we have $\dim \mathcal{H}^f \geq \dim \mathcal{H}^{f_0}$. Then for every $\varphi_0 \in \ker \mathcal{H}^{f_0}$ the pair $(f_0, \varphi_0)$ is $\mathcal{R}$-trivial.

Or as a special case we get

**Corollary 3.** Suppose $(f_0, \varphi_0)$ is a Dirac-harmonic map with $M$ closed. Then
1. $(f_0, \varphi_0)$ is $\mathcal{R}$-trivial, or
2. any neighborhood $U$ of $f_0$ contains an $f \in U$ with $\dim \mathcal{H}^f < \dim \mathcal{H}^{f_0}$.

This corollary shows that the construction of non-$\mathcal{R}$-trivial Dirac-harmonic maps with compact domain is difficult. This fits to the fact that to the author’s knowledge no non-$\mathcal{R}$-trivial Dirac-harmonic map with compact domain has been found so far.

Note that these results may be easily generalized to compact manifolds with boundary with suitable boundary conditions. However, our methods are not robust\(^2\) in the sense used in Lagrangian mechanics: we derive the expression $f \mapsto \mathcal{E}(f, \hat{\varphi}(f))$. 

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\(^2\)In the sense used in Lagrangian mechanics: we derive the expression $f \mapsto \mathcal{E}(f, \hat{\varphi}(f))$. 

enough to generalize to more general type of functionals $\mathcal{E}$, e.g. to the one with quartic spinor term discussed in [12, (2.4)].

Remark 4. For studying harmonic maps, Sacks and Uhlenbeck [17] applied a perturbation of the energy functional successfully, namely they defined for $\alpha \in (1, 2)$

$$\mathcal{E}_\alpha(f) = \frac{1}{2} \int_M (1 + |df|^{2})^\alpha \, d\nu^g.$$ 

Recently, a similar perturbation was also used for the super-symmetric energy functional, i.e. one defines $\mathcal{E}(f, \varphi) = \mathcal{E}_\alpha(f) + \mathcal{E}_2(f, \varphi)$, see e.g. [14, Eq. (1.2)], [15] and [16]. Obviously Theorem 1 remains true for this modification. The condition of Theorem 1 are satisfied as index theoretical methods and minimal kernel methods are used. As a consequence all $\alpha$-Dirac-harmonic maps in [15] are $\mathcal{R}$-trivial and most of the results for $(\alpha)$-Dirac-harmonic maps in [15] directly follow from the corresponding statements for $(\alpha)$-harmonic maps of the same article. The results in [14, Eq. (1.1)] an additional perturbation $F$ is allowed, which makes our trick non-applicable, while it applies to stationary points of (1.2) in the same article. Again, it seems unanswered whether non-$\mathcal{R}$-trivial $\alpha$-Dirac-harmonic maps with compact domain exist.

3. APPLICATION TO THE HEAT FLOW FOR DIRAC-HARMONIC MAPS

We now want to apply our argument to the heat flow for Dirac-harmonic maps, as introduced in [8]. This is the following geometric-elliptic problem. One considers now maps $f_t : M \to N$ and spinors $\varphi_t \in \Gamma(\Sigma M \otimes f_t^*T N)$ depending on a parameter $t \in [0, T]$, with some suitable regularity, whose precise definition will be omitted in this short note, see [8, (1.5)] for details. Note that $M$ is non-empty, connected, oriented and spin, and furthermore $M$ might have non-empty boundary $\partial M$. The heat flow for Dirac-harmonic maps is given by the following equation

$$\begin{align*}
\partial_t f_t &= \tau(f_t) - \mathcal{R}(f_t, \varphi_t) \\
\bar{\partial} f_t \varphi_t &= 0
\end{align*}$$

(3)

on $M$ for all $t \in [0, T]$. In the literature, two separate cases are discussed, the case $\partial M \neq \emptyset$ and the case $\partial M = \emptyset$. In both cases we need additional assumptions for obtaining a well-posed problem.

The case $\partial M \neq \emptyset$ was discussed in [8] and then one has to specify boundary conditions $\mathcal{B}$. For simplicity we restrict to the case of the chirality operator $\mathcal{B} = \mathcal{B}_* := (1 \pm \mathbf{n} \cdot G)$ where a sign in $\pm$ is chosen, where $\mathbf{n}$ is the unit outward normal field, and where $G$ is a chiral operator (e.g. the usual the grading operator of the spinor bundle, in case $M$ is even-dimensional). Another example is the MIT bag boundary conditions, see [8, Subsection 2.2] for further details.

Now, let us assume dim $M \geq 2$. In order to define boundary conditions, following [8], we fix a $t$-dependent map $F_t : M \to N$ and a $t$-dependent spinor $\Phi_t \in \Gamma(\Sigma M \otimes F_t^*T N)$. For simplicity of presentation we assume that $F_t(x)$ and $\Phi_t(x)$ depend smoothly on $t$ and $x$.

$$\begin{align*}
f_t(x) &= F_t(x) \quad \text{for all } (x, t) \in M \times \{0\} \cup \partial M \times [0, T] \\
\mathcal{B} \varphi_t &= \mathcal{B} \Phi_t \quad \text{on } \partial M \times [0, T]
\end{align*}$$

(4) (5)

A short-time existence and uniqueness result was derived, see [8, Theorem 1.3].
In order to explain, what we mean by “1-dimensional” here, thus says that the kernel of the involved Dirac operators remains 1-dimensional in the sense of Dahl [10]. In order to explain, what we mean by “1-dimensional” here, one uses the fact that $\Sigma M \otimes f^T N$ carries a quaternionic structure if $m := \dim M \equiv 2, 3, 4 \mod 8$. This implies that $\ker \mathcal{D}f$ is a (free) $K$-vector space with $K = \mathbb{C}$ for $m \equiv 0, 1, 5, 6, 7 \mod 8$ and $K = \mathbb{H}$ for $m \equiv 2, 3, 4 \mod 8$. Following Dahl we consider the dimension with respect to $K$, i.e. we claim $\dim K \ker \mathcal{D}f = 1$ for all $t \in [0, T]$.

Furthermore, Wittmann showed that this solution is unique “up to gauge”, provided the 1-dimensionality condition holds for all $t \in [0, T]$. More precisely, Wittmann obtained uniqueness up to a $t$-dependent factor in $S_K$, where $S_K$ is the unit sphere in $K$. Multiplication by a function $[0, T] \to S_K$ will be considered as a gauge transformation. It preserves solutions of (3) & (6) & (7), and for $\dim K \ker \mathcal{D}f = 1$ the solution is unique up to such a transformation.

Examples satisfying the 1-dimensionality condition were derived in [20].

Because of the smoothness assumption for $F_0$ and $\Phi_0$, one obtains smooth solutions $f_t$ and $\varphi_t$ in both cases (see [8] for $\partial M \neq \emptyset$ and [18] for $\partial M = \emptyset$).

In both cases the spinor $\varphi_f$ satisfying $\mathcal{D} \varphi_f = 0$ and $\int_M \langle \varphi_f, \varphi_f \rangle = 1$ is unique – for $\partial M = \emptyset$: up to gauge. In the boundary case this is discussed in [8], see Cor. 3.7 for $m = 2$ and the considerations with the weak unique continuation property (WUCP, Thm 3.9). In the boundary free case this is an immediate consequence of the assumption $\dim K \ker \mathcal{D}f = 1$. This uniqueness together with standard implicit function theorem arguments implies that one can choose $\varphi_f$ to depend smoothly on $f$ (at least locally close to some given $f_0 : M \to N$). Theorem 1 thus says that the $\mathcal{R}$-term of the solution $\varphi_t$ vanishes.

We thus have proven:

**Theorem 5.** For $\partial M \not= \emptyset$: If $(f_t, \varphi_t)$ is a solution of (3)–(5), then $f_t$ is a solution of the heat flow for harmonic maps (in the classical sense) with boundary

$$\begin{align*}
\partial_t f_t &= \tau(f_t) \text{ on } M \\
&= F_t(x) \text{ for all } (x, t) \in M \times \{0\} \cup \partial M \times [0, T]
\end{align*}$$

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3 In fact [18] was written after [8]. It was heavily inspired and motivated by [8], although it uses different analytic arguments, namely a fixed point argument derived from a contraction.

4 In fact, any module over a skew field is free, thus we follow the standard convention to say “$K$-vector space” instead of “$K$-module”, and we will use the word “dimension” instead of “rank” in the following.
For $\partial M = \emptyset$: If $(f_t, \varphi_t)$ is a solution of $(3)\&(6)\&(7)$ with $\dim_{\mathbb R} \ker Df_t = 1 \ \forall t$, then $f_t$ is a solution of the heat flow for harmonic maps, i.e. we have

$$
\begin{align*}
\partial_t f_t &= \tau(f_t) \\
    f_0 &= F_0
\end{align*}
$$

on all of $M$.

Obviously if $f_t$ is a solution to the classical harmonic map heat flow (with boundary), the uniqueness – possibly up to gauge – implies that we obtain a solution of $(3)\&(5)$ resp. $(3)\&(6)\&(7)$. Thus the main statement in [8] and [18] may also be deduced from classical results as described above.

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