Compression as a tool to detect Bose glass in cold atoms experiments

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Condensed matter models have found recently a wonderful testbed in cold atoms in optical lattices physics.\textsuperscript{[1],[2]} Cold atoms allow for an optimal control of parameters of the system: by changing intensities or detunings of laser beams, one may modify the depth of the optical lattices; interactions between atoms can be controlled via external magnetic field and Feshbach resonances. An example is the Bose-Hubbard tight binding model,\textsuperscript{[3]} which can be realized using ultracold atoms and where the quantum phase transition between a superfluid (SF) phase and a Mott insulator (MI) phase can be observed by varying the depth of the optical lattice as predicted in a seminal work,\textsuperscript{[4]} and realized few years later.\textsuperscript{[5]}

Controllability of the system makes studies of disorder induced effects particularly interesting. Disorder (or pseudo-disorder) may be created in cold atoms systems in a repeatable way using optical potentials created by laser speckles or multichromatic lattices.\textsuperscript{[6],[7]} This exciting possibility attracted soon a lot of research - Anderson localization being one of the main targets for weakly or non-interacting atoms.\textsuperscript{[6],[10]} Even more exciting is another regime of strongly interacting bosons in a disordered potential. Seminal studies of the disordered Bose-Hubbard model revealed the existence of a novel insulating phase called Bose glass (BG) phase. Contrary to the MI, the BG is characterized by gapless excitation spectrum and is compressible. As far as we know it has not been observed in a "traditional" condensed matter settings. Its unambiguous observation is certainly an important milestone yet to be seen.

In the first attempt to produce a BG with ultracold atoms, a bichromatic quasi-disordered optical lattice was used.\textsuperscript{[12]} The authors measured both the absorption spectrum of the system and the long-range spatial coherence. They observed a smearing of the absorption peaks in the presence of "disorder" as well as a decreased long-range coherence, which they interpreted as a manifestation of the existence of a BG phase. The interpretation of absorption spectra is however difficult: as discussed by us elsewhere,\textsuperscript{[13]} the initial state was not the ground state of the system (this would be the case if the lattice were ramped up adiabatically as supposed in Ref.\textsuperscript{[12]}). Also the amplitude of the lattice modulation was strong, so that the absorption was not in the linear regime.

Another very recent attempt\textsuperscript{[14]} has shown – in the three-dimensional case – that the presence of the disorder leads to a significant decrease of the condensate fraction both for SF or coexisting SF and MI phases. This measurement follows in fact the original proposition of to address the disappearance of the condensate fraction as a possible signature of the BG presence. Here, again the preparation of the initial state is a key question.

While both these experiments are important studies of strongly interacting bosonic systems in the presence of disorder, it is desirable to have a clear signature of a BG phase. Of special interest is a direct measurement of the compressible or incompressible nature of the system. An interesting possibility is to access the central density of the atomic cloud and measure the changes of that density when the external trapping potential is varied (trap squeezing spectroscopy).\textsuperscript{[15]} That measurement supplemented with coherent fraction measurement provides an interesting and clear proposition for an "ideal" experimental procedure. There are two requirements. One is to make the trap geometry independent from laser beams forming the lattice, so that one can vary the trap frequency independently of the lattice height. The second one is to use an additional focused laser to monitor the density in the center of the trap. The latter while feasible seems quite difficult. The aim of this letter is to propose a much simpler experimental scenario, which has already been used in demonstrating the Mott phase for a cloud of fermions.\textsuperscript{[14]} We propose to measure the radius of the atomic cloud while changing the trap frequency. This provides a simple and direct measurement of the compressibility of the system! More precisely, it makes it possible to monitor the appearance and disappearance of incompressible phases. Complemented with measurements of the long-range phase coherence, it would allow for an unambiguous characterization of the MI-BG-SF phase diagram.

This method works beautifully for fermions,\textsuperscript{[16]} where...
in the Mott state one has at most one fermion per spin state. In such a case the radius of the cloud becomes practically independent of the trap frequency, clearly demonstrating the incompressibility of the MI state. The situation is quite different for bosons where in typical experiments \[12\] one may have up to three bosons per site of the optical lattice and the density profile in the trap resembles that of a wedding cake - see e.g. \[17\]. Between Mott regions with single, double, or triple occupancies, there are superfluid “shells”. Those lead to a finite compressibility of the total sample and could make radius measurements useless. As shown below, this is not a major problem.

Let us start with a one-dimensional Bose-Hubbard model in the presence of a trapping potential and a diagonal disorder. The Hamiltonian is \[12\]:

\[
\hat{H} = -J \sum_{(j,j')} \hat{b}_j^\dagger \hat{b}_{j'} + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) + \sum_j \epsilon_j \hat{n}_j,
\]

(1)

where \( \hat{b}_j \) (\( \hat{b}_j^\dagger \)) is the destruction (creation) operator of one particle in the \( j \)-th site, \( \hat{n}_j = \hat{b}_j^\dagger \hat{b}_j \) is the number operator, and \((j,j')\) indicates the sum on nearest neighbors. \( U \) is the interaction energy and \( J \) the hopping energy. The energies at sites, \( \epsilon_j \) are given by the sum of the energy shift due to the the harmonic potential and the additional disorder:

\[
\epsilon_j = \frac{1}{2} m \omega^2 a^2 (j - j_0)^2 + x_j U \Delta
\]

(2)

where \( m \) is the particle mass, \( a \) the lattice spacing, \( \omega \) the trap frequency and \( j_0 \) the position of the trap center. \( \Delta \) is a dimensionless parameter measuring the strength of the disorder (in units of the interaction energy), while \( x_j \) is a (pseudo-)random variable. We consider two different types of disorder. For a truly random disorder, the \( x_j \)'s are chosen as independent variables with uniform distribution in the interval [-1,1]. For a secondary optical lattice as used in \[12\], \( x_j \) is a sine function with incommensurate frequency: \( x_j = \sin(\lambda j) \) resulting in pseudo-random correlated variables with distribution \[18\]:

\[
P(x) = \frac{1}{\pi \sqrt{1 - x^2}}.
\]

(3)

The system properties in such a pseudo-random disorder may differ from the truly random situation \[12, 20, 21\].

We employ the parameters originating from the Florence \[12\] experiment with the exception that we assume the possibility of independent change of trap frequencies. In particular we concentrate on the deep Mott regime \( J/U \approx 0.027 \).

As shown below, in this regime – and as long as we look at the compressibility of the system and not at long range phase coherence – we can use a local density approximation (LDA) to describe the system \[16, 18\]. This amounts at neglecting tunneling between neighboring sites and assuming a Fock state at each site with the occupation determined by the local chemical potential \( \mu - \epsilon_j \). Determining the shape of the atomic cloud is a simple minimization procedure for the total energy, i.e., the sum of independent contributions for various sites, constrained by a fixed total number of particles.

In the absence of disorder and at low trapping frequency, the ground state is a pure MI with an unit occupation number at all sites around the trap center. If we define the r.m.s radius of the trap (in units of the lattice spacing \( a \)) as:

\[
R = \sqrt{(\langle r^2 \rangle - \langle r \rangle^2)} = \sqrt{\frac{\sum_j j^2 n_j}{N} - \left( \frac{\sum_j j n_j}{N} \right)^2}
\]

(4)

where \( N \) is the total number of particles, it is clear that, for large \( N \) and low trap frequency, the radius will be \( R_c = N/2\sqrt{3} \) independently of the trap frequency, a clear-cut manifestation of the incompressibility of the MI phase. When the trap frequency is increased, the energy of the outest particles \( m \omega^2 a^2 N^2/8 \) increases until the point where it is cheaper to pay the interaction energy \( U \) and transfer the particle at the trap center, creating doubly occupied sites. Beyond this point, \( R \) decreases with frequency, implying global compressibility. In the large \( N \) limit, a straightforward calculation shows that the parameters obey scaling laws. The critical frequency is (from the two estimates above) \( \omega_c = \sqrt{\frac{3 \mu}{m \omega}} \). We thus define the scaled frequency: \( \omega_0 = \omega/\omega_c \) and the scaled r.m.s. radius, \( R_0 = R/R_c \).

A plot of \( R_0 \) versus \( \omega_0 \) is shown in the inset of Fig.\[11\] for \( N=151 \) in the absence of disorder. For shallow trap, the
radius shows a pronounced plateau. In this range of frequencies, all particles are in the Mott phase with an unit filling - the plateau is a direct manifestation of incompressibility. This regime resembles most the fermionic case \[ \text{10} \]. At \( \omega_0=1 \), a sharp kink indicates the moment when the sample becomes compressible - at this point a double occupancy appears at the centre of the trap (this can be visualized looking directly at the occupation of sites). The compressibility becomes smaller for larger frequency reaching a second kink at \( \omega_0 = 1 + \sqrt{2} \approx 2.414 \) - here a triple occupancy in the center of the trap is born. A careful inspection reveals even the third kink at \( \omega_0 = 1 + \sqrt{\sqrt{2} + \sqrt{5}} \approx 4.146 \). From this plot, it is clear that measuring the global quantity \( R \) vs. trap frequency makes it possible to monitor the appearance of the successive MI phases. Let us note that for the Florence experiment \[ \text{12} \] the effective harmonic binding (coming from the trap and lasers’ transverse profiles) is 75 Hz, corresponding to \( \omega_0 \approx 3.44 \).

In the presence of a random disorder, the plateau disappears, showing a non zero compressibility of the sample, as seen in the main panel of Fig. 1. At low frequency, this is due to the appearance of a compressible BG phase in the external part of the cloud (where the average occupation number is between 0 and 1). The kink around \( \omega_0=1 \) is, however, a robust feature. It is due to the formation of first a BG with occupation number between 1 and 2 at the trap center, followed by the birth of a n=2 MI phase. The kink itself is the signature of the appearance of new phases. The residual tunneling between the neighboring sites has almost no effect. To prove this statement, we use the TEBD algorithm \[ \text{22} \] (also known as t-DMRG algorithm \[ \text{23} \]) with imaginary time propagation to produce the quasi-exact ground state of the Bose-Hubbard Hamiltonian in the presence of disorder from which the corresponding r.m.s. radius easily computed. The result, averaged over several realizations of the disorder, is shown in Fig. 1. It is almost indistinguishable from the result of the LDA approximation. We have checked that, for other disorder strengths – but still in the low tunneling regime – there is a similar agreement between the exact TEBD result and the LDA approximation, which we will use in the following of this paper. Most probably, this is because tunneling creates coherence between neighboring sites and smoothes the steps of the wedding cake, but does not induce macroscopic transfer of particles over long distances and thus only marginally affects the r.m.s. radius. From the point of view of compressibility, whether a BG or a SF phase is formed makes little difference.

The MI phase is expected to disappear at \( \Delta=0.5 \) in the limit of small tunneling \[ \text{3} \]. In Fig. 2 we show \( R_0 \) vs. \( \omega_0 \) for increasing disorder strengths. The kink is very clearly visible even at values very close to \( \Delta=0.5 \), showing that the proposed simple method allows one to monitor appearance and disappearance of various phases. At the critical disorder \( \Delta=0.5 \), the MI phases completely disappears and a smooth curve is obtained. It can be shown \[ \text{24} \] that its equation is:

\[
R_0 = 2^{-1/3} 3 \sqrt{3/5} \omega_0^{-2/3}.
\]

The kink is even more visible if one considers the compressibility of the system, that is the derivative of the radius with trap frequency. In \[ \text{10} \], a global compressibility \( \kappa_R \) is defined, where the powers of \( R \) are chosen to obtain well-defined quantities in the thermodynamic limit \( N \to \infty \). As it has the dimension of the inverse of an energy we prefer to use a dimensionless global compressibility \( K \) by multiplying \( \kappa_R \) by \( U \). It has a simple expression in terms of scaled quantities:

\[
K = - \frac{dR}{R \partial U} = - \frac{3}{2 \omega_0^2} \frac{dR_0}{d\omega_0} = \frac{3}{2 \omega_0^2} \frac{dR_0}{d\omega_0}.
\] (5)

It is shown in the inset of Fig. 2 for various disorder strengths. While it vanishes below \( \omega_0=1 \) for zero disorder, it displays a well marked minimum for \( 0 < \Delta < 0.5 \), directly related to the existence of an incompressible MI phase. At \( \Delta = 0.5 \), the MI phase and the minimum disappear. Note that the observed behaviour is quite similar to the one observed for fermions in \[ \text{14} \], the role of tunneling being here replaced by disorder.

A similar behaviour is observed for the three dimensional isotropic Bose-Hubbard model. The results are shown in Fig. 3. The singularity is slightly different (the compressibility has no discontinuity, but a very fast increase), but the essential property: the existence of a kink in the \( R_0 \) vs. \( \omega_0 \) curve, or equivalently, the marked minimum in the compressibility, is present as well. The scaling laws are of course different in three dimensions: we now have \( R_c = 3^{5/6} 5^{-1/2} 2^{-1/3} \pi^{-1/3} N^{1/3} \)
and $\omega_c = 2^{7/6} \pi^{1/3} 3^{-1/3} N^{-1/3} \sqrt{U/ma^2}$ and the 3/2 coefficient in eq. (3) must be replaced by 5/6. The appearance of the second MI phase is clearly visible as a dip in the compressibility around $\omega_0 = 1.5$.

Consider now the pseudo-disorder, eq. (5), employed in the Florence experiment [12]. We have performed similar calculations, averaging over the relative phase of the two lattices. An overall similar behaviour, shown in Fig. 4, is observed, a pronounced kink yields the frequency when the double occupancy emerges at the centre of the trap. With increasing disorder, the plateau shrinks, becomes tilted, the kink moves towards smaller frequencies and eventually disappears at $\Delta = 0.5$. This translates in the second lattice depth equal to $s_2 \approx 1.014$ (in the notations of [12]), which nicely matches the estimate for the disappearance of the Mott phase and the gap in the absorption spectrum [12].

Note that another singularity (sharp peak in the compressibility) is visible at lower frequency and large disorder. This is due to the peculiarity of the distribution, eq. (5), diverging at $x = \pm 1$. Those, however, may be easily identified and distinguished from the “main” Mott end kink.

It has been argued that the MI regions may be strongly affected by temperature effects [23]. Our calculations are limited to $T = 0$. Study of the effects of the temperature as well as of large tunneling on the phenomenon discussed here are in progress.

In summary, we have shown that a simple measurement of the radius of the atomic cloud may provide a clear identification of the disappearance of the Mott phase and, in the presence of disorder, may help to unravel the BG presence. The disappearance of the MI is correlated with vanishing of the kink in the radius-frequency dependence. DD acknowledges support by IFRAF, JZ acknowledges support by Polish Ministry of Education and Sports (2008-2011). This work is realized within Marie Curie TOK scheme COCOS (MTKD-CT-2004-517186).

![Figure 3: (Color online) Same as Fig. 2 for the three-dimensional disordered isotropic Bose-Hubbard model. The behaviour is similar, with a kink in the curve around $\omega_0 = 1$ and a marked minimum of the compressibility (inset).](image1)

![Figure 4: (Color online) Same as Fig. 2 but for the pseudo-random “disorder” created by a secondary incommensurate lattice: $\Delta = 0$ (solid line), $\Delta = 0.2$ (dotted line), $\Delta = 0.4$ (dashed line), $\Delta = 0.5$ (dash-dotted line beyond which the kink and the MI phase disappear).](image2)

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