HADRONIC CORRELATIONS ABOVE THE CHIRAL/DECONFINEMENT TRANSITION

DAVID B. BLASCHKE\textsuperscript{1,2} and KYRILL A. BUGAEV\textsuperscript{1,3}

\textsuperscript{1}Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany
\textsuperscript{2}Bogolyubov Laboratory for Theoretical Physics, JINR, 141980 Dubna, Russia
\textsuperscript{3}Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

Abstract
The statistical bootstrap model is critically revised in order to include a medium-dependent resonance width in it. We show that a thermodynamic model with a vanishing width below the Hagedorn temperature $T_H$ and a Hagedorn spectrum-like width above $T_H$ may not only eliminate the divergence of the thermodynamic functions above $T_H$, but also gives a satisfactory description the lattice quantum chromodynamics (QCD) data on the energy density above the chiral/deconfinement transition as the main result of this contribution. This model allows to explain the absence of heavy resonance contributions in the fit of the experimentally measured particle ratios at SPS and RHIC energies.

Keywords: Statistical bootstrap model, Mott transition, Spectral function, Lattice QCD thermodynamics

1. Introduction

Ultrarelativistic heavy-ion collision experiments at SPS and RHIC are performed with the aim to create conditions of temperature and density under which hadronic matter undergoes a phase transition to the hypothetical quark-gluon plasma (QGP) and to investigate the properties of this state of matter once it is created. The strongest theoretical support for the existence of the QGP comes from lattice QCD simulations at finite temperature $T$ which show a step-like enhancement of the effective number of degrees of freedom (energy density in units of $T^4$) at a critical temperature $T_c$, see Fig. 1. This behavior is conventionally interpreted as the transition from a few mesonic degrees of freedom (mainly $\pi$ and $\rho$ mesons) to those of quasifree quarks and gluons. A microscopic description of this transition is, however, still missing. On the one hand, it has been shown\textsuperscript{1} that a resonance gas model can perfectly explain the steep rise in the number of degrees of freedom at $T\approx T_c$. On the other hand, lattice QCD has also revealed that hadronic correlations persist for $T > T_c$\textsuperscript{2}. The question arises whether it is more appropriate to describe hot QCD matter in terms of hadronic correlations rather than in terms of quarks and gluons. In the present contribution, we introduce a generalization of the Hagedorn resonance gas (statistical bootstrap) model as such a description.
The statistical bootstrap model (SBM) \[3, 4, 5, 6\] is based on the hypothesis that hadrons are made of hadrons, with constituent and compound hadrons being treated on the same footing. This implies an exponentially growing form of the hadronic mass spectrum \( \rho_H(m) \approx C_H m^{-\alpha} \exp\left[\frac{m}{T_H}\right] \) for \( m \to \infty \). The parameter \( T_H \), Hagedorn temperature, was interpreted as a limiting temperature reached at infinite energy density.

The extensive investigation of the SBM has led to a formulation of both the important physical ideas and the mathematical methods for modern statistical mechanics of strongly interacting matter. Thus it has been clarified that the original SBM requires two crucial modifications:

- hadrons should be considered as composite objects (the simplest way are hadrons as MIT bags \[7\] of quarks and gluons) and, their proper volume \( v \) has to be taken into account \[8, 9\] in the partition function;
• only $SU_c(3)$ color singlet clusters of quarks and gluons contribute [10, 11, 12, 13] to the partition function of the system with their masses $m_i$ and volumina $v_i$.

The SBM with proper volume was solved analytically by the Laplace transform to the isobaric ensemble [14] and, indeed, its solution showed various possibilities for the phase transition between the QGP and the hadron gas. Since then this technique has been successfully applied not only to solve far more sophisticated versions [10, 11, 12, 13, 15] of the SBM, but also to find an analytical solution of a simplified version of the statistical multifragmentation model [16, 17] for the nuclear liquid-gas phase transition.

However, up to now the formulation of the SBM has some severe problems. The first one is the absence of a width for the heavy resonances. From the Particle Data Group [18] we know that heavy resonances with masses $m \geq 3.5$ GeV may have width comparable with their masses. Taking the widths into account shall effectively reduce the statistical weight of the resonance. As we shall show below, this change may eventually remove the divergence of the SBM thermodynamics. The second problem arises while discussing the results of the hadron gas (HG) model [19, 20]. The HG model describes remarkably well the light hadron multiplicities measured in nucleus-nucleus collisions at CERN SPS [19] and BNL RHIC [20] energies. This model is nothing else as the SBM of light hadrons which accounts for the proper volume of hadrons with masses below 2.5 GeV, but neglects the contribution of the exponentially growing mass spectrum.

In other words, in order to calculate particle ratios within the HG model it is, on the one hand, necessary to consider all strong decays of resonances according their partial width collected in [18], and, on the other hand, it is necessary to truncate the hadron spectrum for masses above 2.5 GeV. Thus, one immediately faces the following problem: “Why do the heavy resonances with masses above 2.5 GeV predicted by the SBM not appear in the particle spectra measured in heavy-ion collisions at SPS and RHIC energies?” Note that the absence of heavy resonance contributions in the particle ratios cannot be due to the statistical suppression of the Hagedorn mass spectrum because the latter should not be strong in the quark-hadron phase transition region, where those ratios are believed to be formed [19, 20].

In the present contribution we suggest that the introduction of a finite width of the resonances can solve the above problems of the SBM. In the next section we formulate a simple statistical model that incorporates besides of the Hagedorn mass spectrum also medium dependent resonance widths due to the hadronic Mott effect, and analyze its mathematical structure. In Section 3 we discuss a model fit to recent lattice data of QCD thermodynamics [11] and some possible consequences for heavy-ion physics.
2. Resonance Width Model: Mott Transition

According to QCD, hadrons are not elementary, pointlike objects but rather color singlet bound states of quarks and gluons with a finite spatial extension of their wave function. While at low densities a hadron gas description can be sufficient, at high densities and temperatures, when hadronic wave functions overlap, nonvanishing quark exchange matrix elements between hadrons occur in order to fulfill the Pauli principle. This leads to a Mott-Anderson type delocalization transition with frequent rearrangement processes of color strings (string-flip [21]) so that hadronic resonances become off-shell with a finite, medium-dependent width. Such a Mott transition has been thoroughly discussed for light hadron systems in [22] and has been named soft deconfinement. The Mott transition for heavy mesons may serve as the physical mechanism behind the anomalous \( J/\psi \) suppression phenomenon [23].

We introduce the width \( \Gamma \) of a resonance in the statistical model with the Hagedorn mass spectrum through the spectral function

\[
A(s, m) = N_s \frac{\Gamma m}{(s - m^2)^2 + \Gamma^2 m^2}, \tag{1}
\]

a Breit-Wigner distribution of virtual masses with a maximum at \( \sqrt{s} = m \) and the normalization factor

\[
N_s = \left[ \int_{m_h^2}^{\infty} ds \frac{\Gamma m}{(s - m^2)^2 + \Gamma^2 m^2} \right]^{-1} = \frac{1}{\pi} \frac{1}{\frac{\pi}{2} + \arctan \left( \frac{m^2 - m_h^2}{\Gamma m} \right)}. \tag{2}
\]

The energy density of this model with zero resonance proper volume for given temperature \( T \) and baryonic chemical potential \( \mu \) can be cast in the form

\[
\varepsilon(T, \mu) = \sum_{i=\pi, \rho, \ldots} g_i \varepsilon_M(T, \mu_i; m_i) + \sum_{A=M, B} \int_{m_A}^{\infty} dm \int_{m_h^2}^{\infty} ds \rho_H(m) A(s, m) \varepsilon_A(T, \mu_A; \sqrt{s}), \tag{3}
\]

where the energy density per degree of freedom with a mass \( m \) is

\[
\varepsilon_A(T, \mu_A; m) = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2 + m^2}}{\exp \left( \frac{\sqrt{k^2 + m^2} - \mu_A}{T} \right) + \delta_A}, \tag{4}
\]

with the degeneracy \( g_A \) and the baryonic chemical potential \( \mu_A \) of hadron \( A \). For mesons, \( \delta_M = -1, \mu_M = 0 \) and for baryons \( \delta_B = 1 \) and \( \mu_B = \mu \), respectively. According to Eq. (3) the energy density of hadrons consists of the contribution of
light hadrons for \( m_i < m_A \) and the contribution of the Hagedorn mass spectrum \( \rho_H(m) \) for \( m \geq m_A \).

A new element of Eq. (3) in comparison to the SBM is the presence of the \( \sqrt{s} \)-dependent spectral function. The analysis shows that, depending on the behavior of the resonance width \( \Gamma \) in the limit \( m \to \infty \), there are the following possibilities:

- For vanishing resonance width, \( \Gamma = 0 \), Eq. (3) evidently reproduces the usual SBM.
- For final values of the resonance width, \( \Gamma = \text{const} \), Eq. (3) diverges for all temperatures \( T \) because, in contrast to the SBM, the statistical factor in Eq. (3) behaves as \( \{ \exp [(m_b - \mu_A)/T] + \delta_A \}^{-1} \) so that it cannot suppress the exponential divergence of the Hagedorn mass spectrum \( \rho_H(m) \).
- For a resonance width growing with mass like the Hagedorn spectrum \( \Gamma \sim C \Gamma \exp [m T_H] \) or faster, Eq. (3) converges again.

Indeed, in the latter case the Breit-Wigner spectral function behaves as

\[
N_s \frac{\Gamma m}{(s - m^2)^2 + \Gamma^2 m^2} \bigg|_{m \to \infty} \to \frac{2}{\pi} \frac{\Gamma}{\Gamma} \sim \exp \left( -\frac{m}{T_H} \right) \tag{5}
\]

and cancels the exponential divergence of the Hagedorn mass spectrum. Hence, the energy density remains finite. Note that both the analytical properties of model (3) and the right hand side of Eq. (5) remain the same, if a Gaussian shape of the spectral function is chosen instead of the Breit-Wigner one.

It can be shown that the behavior of the width at finite resonance masses is not essential for the convergence of the energy density (4). In other words, for a convergent energy density (4) above \( T_H \) it is sufficient to have a very small probability density (5) (or smaller) for a resonance of mass \( m \) to be found in the state with the virtual mass \( \sqrt{s} \). Since there is no principal difference between the high and low mass resonances, we can use the same functional dependence of the width \( \Gamma \) for all masses. Thus, for the following model ansatz

\[
\Gamma(T) = \begin{cases} 
0, & \text{for } T \leq T_H, \\
C \Gamma \left( \frac{m}{T_H} \right)^{N_m} \left( \frac{T}{T_H} \right)^{N_T} \exp \left( \frac{m}{T_H} \right), & \text{for } T > T_H, 
\end{cases} \tag{6}
\]

the energy density (3) is finite for all temperatures and the divergence of the SBM is removed. At \( T = T_H \), depending on choice of parameters, it may have either a discontinuity or its partial \( T \) derivative may be discontinuous. As discussed above, for \( T \leq T_H \) such a model corresponds to the usual SBM, but for high temperatures \( T > T_H \) it remains finite for a wide choice of powers \( N_m \).
3. Applications for lattice QCD and heavy-ion collisions

As one can see from Fig. 1 the Hagedorn gas model correctly reproduces the lattice QCD results below the critical temperature $T_c$ and just in a vicinity above $T_c$, but not for large temperatures. Fig. 2 shows a comparison of the same lattice QCD data [1] with the Mott-Hagedorn gas (6) where the parameters of the spectral function are $N_T = 2.325, N_m = 2.5$ and $T_H = 165$ MeV and $m_a = m_b = 1$ GeV. The successful description of the lattice energy density [1] indicates that above $T_c$ the strongly interacting matter may be well described in terms of strongly correlated hadronic degrees of freedom. This result is based on the concept of soft deconfinement and provides an alternative to the conventional explanation of the deconfinement transition as the emergence of quasifree quarks and gluons.

Another interesting feature of the model (6) is that it allows to explain naturally the absence of heavy resonance contributions to the particle yields measured at highest SPS and all RHIC energies, where QGP conditions are expected [19, 20]. In order to find out whether a given resonance has a chance to survive till the freeze-
out it is necessary to compare its lifetime with the typical timescale in the system. There are two typical timescales usually discussed in nucleus-nucleus collisions, the equilibration time $\tau_{eq}$ and the formation time $\tau_f$. The equilibration time tells when the matter created in collision process reaches a thermal equilibrium which allows to use the hydrodynamic and thermodynamic descriptions. For Au + Au collisions at RHIC energies it was estimated to be about $\tau_{eq} \approx 0.5 \text{ fm}$ [24]. On the other hand in transport calculations the formation time is used: the time for constituent quarks to form a hadron. The formation time depends on the momentum and energy of the created hadron, but is of the same order $\tau_f \approx 1 - 2 \text{ fm}$ [25] as the equilibration time.

Since within our model the QGP is equivalent to a resonance gas with medium dependent widths, all hadronic resonances with life time $\Gamma^{-1}(m)$ shorter than $\max\{\tau_f, \tau_{eq}\}$ will have no chance to be formed in the system. Therefore, the upper limit of the the integrals over the resonance mass $m$ and over the virtual mass $\sqrt{s}$ in Eq. (3) should be reduced to a resonance mass defined by

$$\Gamma(m)^{-1} = \max\{\tau_f, \tau_{eq}\}. \quad (7)$$

This reduction may essentially weaken the energy density gap at the transition temperature or even make it vanish. Thus, the explicit time dependence should be introduced into the resonance width model (3) while applying it to nuclear collisions, and this finite time (size) effect, as we discussed, may change essentially the thermodynamics of the hadron resonances formed in the nucleus-nucleus collisions.

4. Conclusions and Remarks

The statistical bootstrap model allows to interpret the QGP as the hadron resonance gas dominated by the state of infinite mass (and infinite volume). We argue that it is necessary to include the resonance width into the SBM in order to avoid the contradiction with the experimental data on hadron spectroscopy. We found that the simple model [3]-[5] with a vanishing width below Hagedorn temperature $T_H$ and a Hagedorn spectrum-like width above $T_H$ may not only eliminate the divergence of the thermodynamic functions above $T_H$, but it is able to successfully describe the lattice QCD data [1] for energy density with three fitting parameters only. Such a model also allows to naturally explain the absence of heavy resonance contributions in the fit of the experimentally measured particle ratios at SPS and RHIC energies.

However, such a modification of the SBM requires an essential change in our view on QGP: it is conceivable that hadrons of very large masses which should be associated with a QGP cannot be formed in nucleus-nucleus collisions because of their very short lifetime.

It is also necessary to remind that presented model should be applied to analyze the experimental data with care: it can be successfully applied to describe either the quantities associated with the chemical freeze-out, i.e. particle ratios or spectra...
of Ω hyperons, φ, J/ψ and ψ' mesons that are freezing out at hadronization [26, 27, 28, 29]. But as discussed in Refs. [30, 31, 32] the model presented here should not be used for the post freeze-out momentum spectra of other hadrons produced in the nucleus-nucleus collisions. Perhaps only such weakly interacting hadrons like Ω, φ, J/ψ and ψ' will allow us to test the model presented.

The question how to derive the non-zero width below $T_H$ has to be investigated. To solve this problem it will be necessary to include into present model a non-zero proper volume of hadrons. This, however, will require to consider a mixture of hadrons of different sizes as studied recently in [33, 34] or even a relativistic modification of the excluded volume of hadrons [34, 35].

Acknowledgments

We are grateful to M. I. Gorenstein for interesting discussions and important comments. K.A.B. acknowledges the financial support of DFG grant No. 436 UKR 17/13/03 and the Ministry for Culture and Education of Mecklenburg-Western Pomerania.

References

[1] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C 29 (2003) 549.
[2] I. Wetzorke, F. Karsch, E. Laermann, P. Petreczky and S. Stickan, Nucl. Phys. Proc. Suppl. 106 (2002) 510; M. Asakawa, T. Hatsuda, Y. Nakahara, Nucl. Phys. A 715 (2003) 863.
[3] R. Hagedorn, Suppl. Nuovo Cimento 3 (1965) 147; R. Hagedorn and J. Ranft, Suppl. Nuovo Cimento 6 (1968) 169.
[4] R. Hagedorn, The long Way to the Statistical Model, Proceedings of a NATO Adv. Research Workshop on “Hot Hadronic Matter: Theory and Experiment”, June 27 July 1, 1994, Divonne, France, (1994) p. 13, eds. J. Letessier, H.H. Gutbrod and J. Rafelski.
[5] S. Frautschi, Phys. Rev. D 3 (1971) 2821.
[6] A. Tounsi, Statistical Bootstrap and Thermodynamical Model of High Energy Strong Interactions, in: Phenomenology of Particles at High energy, Academic Press, 1974.
[7] A. Chodos et al., Phys. Rev. D 19 (1974) 3471.
[8] J. Baacke, Acta. Phys. Pol. B 8 (1977) 625.
[9] R. Hagedorn and J. Rafelski, Phys. Lett 97 B (1980) 136.
[10] M. I. Gorenstein et al., Teor. Mat. Fiz. 52 (1982) 346.
[11] M. I. Gorenstein, S. I. Lipskikh and G. M. Zinovjev, Z. Phys. C 22 (1984) 189.
[12] J. Letessier and A. Tounsi, Phys. Rev D 40 (1989) 2914.
[13] M. Kataja et al., Z. Phys. C 55 (1992) 153.
[14] M.I. Gorenstein, V.K. Petrov and G.M. Zinovjev, Phys. Lett. 106 B (1981) 327.
[15] J. Letessier, J. Rafelski and A. Tounsi, Phys. Lett. B 328 (1994) 499 and references therein.
[16] K. A. Bugaev et al., Phys. Rev. C 62 (2000) 044320; arXiv:nucl-th/0007062
    Phys. Lett. B 498 (2001) 144; arXiv:nucl-th/0103075
[17] P. T. Reuter and K. A. Bugaev, Phys. Lett. B 517 (2001) 233.
[18] Particle Data Group, Phys. Rev. D 66 (2002).
[19] P. Braun–Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465 (1999) 15;
    G. D. Yen and M. I. Gorenstein, Phys. Rev. C 59 (1999) 2788;
    F. Becattini et al., Phys. Rev. C 64 (2001) 024901.
[20] P. Braun–Munzinger et al., Phys. Lett. B 518 (2001) 41;
    N. Xu and M. Kaneta, Nucl. Phys. A 698, (2002) 306;
    W. Florkowski, W. Broniowski and M. Michalec, Acta Phys. Pol. B 33 (2002) 761.
[21] D. Blaschke, F. Reinholz, G. Röpke and D. Kremp, Phys. Lett. 151 B (1985) 439;
    G. Röpke, D. Blaschke and H. Schulz, Phys. Rev. D 34 (1986) 3499.
[22] J. Hüfner, S.P. Klevansky and P. Rehberg, Nucl. Phys. A 606 (1996) 260.
[23] G. Burau, D. Blaschke and Yu. Kalinovsky, Phys. Lett. B 506 (2001) 297.
[24] X.-N. Wang, M. Gyulassy and M. Plümer, Phys. Rev. D 51 (1995) 3436;
    R. Baier et al., Phys. Lett. B 345 (1995) 277.
[25] see, for instance, S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 225 (1998).
[26] more references can be found in K. A. Bugaev, M. Gazdzicki, M.I. Gorenstein,
    Phys. Lett. B 523 (2001) 255; Phys. Lett. B 544, 127 (2002); and Phys. Rev. C 68 (2003) 017901.
[27] M. I. Gorenstein, K. A. Bugaev and M. Gazdzicki, Phys. Rev. Lett. 88 (2002) 132301.
[28] S. Bass and A. Dumitru, Phys. Rev. C 61, 064909 (2000);
[29] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001); and
    nucl-th/0110037 (2001).
[30] K. A. Bugaev, Nucl. Phys. A 606, 559 (1996); J. Phys. G 28 (2002) 1981;
    Phys. Rev. Lett. 90 (2003) 252301.
[31] K. A. Bugaev and M. I. Gorenstein. nucl-th/9903072
[32] K. A. Bugaev, M. I. Gorenstein and W. Greiner, J. Phys. G 25 (1999) 2147;
    Heavy Ion Phys. 10, 333 (1999).
[33] M. I. Gorenstein, A. P. Kostyuk and Y. D. Krivenko, J. Phys. G 25 (1999) L75.
[34] G. Zeeb et al., arXiv:nucl-th/0209011
[35] K. A. Bugaev et al., Phys. Lett. B 485 (2000) 121.