EFFECTIVE MAJORANA NEUTRINO MASSES

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Abstract

A generalisation of the neutrinoless double beta decay process is presented. Neutrinoless double beta decay measures only one out of nine possible effective Majorana neutrino masses in case of three flavours. Limits obtained for all the matrix elements - some of them for the first time - are presented using data from $\mu - e$ conversion, neutrino-nucleon scattering, HERA and rare kaon decays. An outlook towards future possibilities to improve on the bounds is given.

1 Introduction

Investigation of lepton–number violating processes is one of the most promising ways of probing physics beyond the standard model. A particular aspect of this topic is lepton–number violation in the neutrino sector, which in the case of massive neutrinos would allow a variety of new phenomena. For recent reviews see [1].

Such processes emerge immediately in case of Majorana masses of the neutrinos, which are predicted in most GUT–theories. Reactions associated with Majorana neutrinos are typically characterised by violating lepton number by two units ($\Delta L = 2$).

The gold plated channel to search for massive Majorana neutrinos is neutrinoless double beta decay of nuclei ($\Delta L_e = 2$)

$$ (A, Z) \rightarrow (A, Z + 2) + 2e^- $$

A lot of activity was spent over the past decades to push half-life limits for this decay beyond $10^{25}$ yrs and therefore the measuring quantity, called effective Majorana mass $\langle m_{ee} \rangle$ down below 1 eV. The best limit currently available is obtained for $^{76}Ge$ and results in an upper limit of $\langle m_{ee} \rangle$ of about 0.2 eV [2]. The quantity $\langle m_{ee} \rangle$ is given by

$$ \langle m_{ee} \rangle = \left| \sum U_{em}^2 m_{\nu m} h_{m}\right| $$

\footnote{most of the work done in collaboration with M. Flanz and W. Rodejohann}

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where $m_{\alpha}$ are the mass eigenvalues, $\eta_{m}^{\text{CP}} = \pm 1$ the relative CP–phases and $U_{em}$ the mixing matrix elements.

In general, there is a $3 \times 3$ matrix of effective Majorana masses, the elements being

$$\langle m_{\alpha\beta} \rangle = |(U \, \text{diag}(m_1\eta_1^{\text{CP}}, m_2\eta_2^{\text{CP}}, m_3\eta_3^{\text{CP}})U^T)_{\alpha\beta}|$$

$$= \left| \sum_{m} m_{\eta m}^{\text{CP}} U_{\alpha m} U_{\beta m} \right| \text{ with } \alpha, \beta = e, \mu, \tau. \quad (3)$$

In contrast to neutrinoless double beta decay little is known about the other matrix elements. In this paper the current status of the full matrix is reviewed as well as suggestions for future improvements are given.

The basic Feynman-graph under investigation is shown in Fig. 1.

2 \( \mu - e \) conversion and \( \langle m_{e\mu} \rangle \)

Muon - positron conversion on nuclei

$$\mu^- + (A, Z) \rightarrow e^+ + (A, Z - 2) \quad (4)$$

is a process closely related to double beta decay and, within the context discussed here, is measuring $\langle m_{e\mu} \rangle$. The current best bound is coming from SINDRUMII and is given by $\Gamma(Ti + \mu^- \rightarrow Ca^{GS} + e^+) < 1.7 \cdot 10^{-12} \quad (90\% \text{CL}) \quad (5)$

which can be converted in a new limit of $\langle m_{e\mu} \rangle < 17 \text{ (82) MeV}$ depending on whether the proton pairs in the final state are in a spin singlet or triplet state. Correction factors of the order one for the difference in Ti and S as given in [4] might be applied. As can be seen this limit is already about 8 orders of magnitude worse than neutrinoless double beta decay. Improvements on $\langle m_{e\mu} \rangle$ by an order of magnitude can be expected by a new run of SINDRUM II in 1999. A further step forward might be the proposed AGS- E940 experiment (MECO) at BNL and PRISM at the Japanese Hadron Facility.

Notice that a process like $\mu \rightarrow e\gamma$ does not give direct bounds on the quantities discussed here, because it measures $m_{e\mu} = \sqrt{\sum U_{ei}U_{\mu i}m_i^2}$. Therefore without specifying a neutrino mixing and mass scheme, the quantities are rather difficult to compare. On the other hand, if this is done, such an indirect bound is more stringent.

3 Trimuon production in $\nu N$ - scattering and $\langle m_{\mu\mu} \rangle$

The process under study is muon lepton–number violating ($\Delta L_\mu = 2$) trimuon production in neutrino–nucleon scattering via charged current reactions (CC)

$$\nu_\mu N \rightarrow \mu^- \mu^+ \mu^+ X \quad (6)$$

where $X$ denotes the hadronic final state. The measured mixing matrix element is $\langle m_{\mu\mu} \rangle$. Detailed calculations can be found in [5]. Taking the fact that in
past experiments no excess events of this type were observed on the level of $10^{-5}$ with respect to $\nu_\mu$ CC, a limit of $\langle m_{\mu\mu}\rangle \lesssim 10^4$ GeV can be deduced. This has to be compared to $\langle m_{\mu\mu}\rangle < 1.1 \cdot 10^5$ GeV as obtained from earlier K–decay data [6]. Even being an order of magnitude improvement the obtained bound is – like the kaon bound – still in an unphysical region because it is a simple extrapolation from small neutrino masses upwards neglecting effects from the propagator term. This means that the real cross section scales with

$$\sigma \propto \left| \sum_m \frac{m m_{\mu\mu} \eta_{m} \epsilon_{\mu m}^2}{(q^2 - m^2_m)} \right|^2.$$  \hspace{1cm} (7)$$

which leads to a $m_m^{-2}$ behaviour in case that $m_m^2 \gg q^2$. The total cross section for various neutrino energies is shown in Fig. 2.

Current experiments like CHORUS and NOMAD might improve on this quantity by an order of magnitude.

4 Rare kaon decays and $\langle m_{\mu\mu}\rangle$

As already mentioned, a further possibility to probe $\langle m_{\mu\mu}\rangle$ is the rare kaon decay

$$K^+ \to \pi^- \mu^+ \mu^+.$$  \hspace{1cm} (8)$$

Detailed calculations can be found in [7, 8]. Using old data a branching ratio of

$$\frac{\Gamma(K^+ \to \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \to \text{all})} < 1.5 \cdot 10^{-4} \text{ (90\% CL)}$$  \hspace{1cm} (9)$$

was obtained [8]. Combined with the theoretical calculations of [4] a limit of $\langle m_{\mu\mu}\rangle < 1.1 \cdot 10^5$ GeV could be deduced [6]. In the meantime new sensitive kaon experiments are online and using the E865 experiment at BNL a new upper limit on the branching ratio of

$$\frac{\Gamma(K^+ \to \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \to \text{all})} < 3 \cdot 10^{-9} \text{ (90\% CL)}$$  \hspace{1cm} (10)$$

could be deduced [8], an improvement by a factor 50000. Assuming the branching ratio scales with $\langle m_{\mu\mu}\rangle^2$ (valid as long as $m_m \ll q^2$) this can be converted in the limit $\langle m_{\mu\mu}\rangle \lesssim 500 \text{ GeV}$ [10], a factor of eight better than the existing limits coming from HERA (see sec. 5) and three orders of magnitude better for this particular decay channel. Also here the propagator term is neglected and therefore even this improvement still does not imply any serious limit on heavy neutrinos [11]. To improve on $\langle m_{\mu\mu}\rangle$ the decay of charmed mesons could be considered as well. Among the Cabibbo favoured modes are $D^+ \to \pi^- \mu^+ \mu^+$, $D_s^+ \to K^- \mu^+ \mu^+$ or $D_s^+ \to \pi^- \mu^+ \mu^+$. The existing limits on the branching ratio for these processes are $1.7 \cdot 10^{-5}$, $1.9 \cdot 10^{-4}$ and $8.2 \cdot 10^{-5}$ respectively [12]. While being competitive with the old bound for the kaon decay discussed, the new kaon branching ratio limit is now four orders of magnitude better. Therefore, to obtain new information on $\langle m_{\mu\mu}\rangle$ from $D^+$-decays, analyses of new data sets have to be done.

To improve significantly towards lighter neutrino masses ($\langle m_{\mu\mu}\rangle \lesssim 1$ GeV) one
might consider other processes. The close analogon to double beta decay and also a measurement of $\langle m_{\mu\mu}\rangle$ using nuclear scales would be $\mu^- - \mu^+$ capture by nuclei with $\mu^+$ in the final state as discussed in [13]. No such experiment was performed yet, probably because of the requirement to use radioactive targets due to energy conservation arguments. The ratio with respect to standard muon capture can be written in case of the favoured $^{44}\text{Ti}$ and light neutrino exchange ($m_\nu \ll q^2$) as

$$R = \frac{\Gamma(\mu^- + \text{Ti} \rightarrow \mu^+ + \text{Ca})}{\Gamma(\mu^- + \text{Ti} \rightarrow \nu_\mu + \text{Sc})} \approx 5 \cdot 10^{-24}\left(\frac{\langle m_{\mu\mu}\rangle}{250\text{keV}}\right)^2.$$  \hspace{1cm} (11)

Assuming that a branching ratio of the order of muon-positron conversion (eq.5) can be obtained, a bound $\langle m_{\mu\mu}\rangle \gtrsim 150\text{ GeV}$ results. Unfortunately this is only a slight improvement on the bound obtained from kaon decay and to make real progress a branching ratio of the order $R \approx 10^{-17}$ has to be measured. Assuming heavy neutrino exchange ($m_\nu \gg q^2$) for the muon capture, would result in a rate another four orders of magnitude lower than for light neutrino exchange.

5 Limits from HERA on the full mass matrix

A first set of full matrix elements including the $\tau$ - sector was given by [14] using HERA data. The process studied is

$$e^\pm p \rightarrow (\nu_\alpha) l^\pm l'^\pm X, \text{ with } (ll') = (er), (\mu\tau), (\mu\mu) \text{ and } (\tau\tau)$$ \hspace{1cm} (12)

Such a process has a spectacular signature with large missing transverse momentum ($p_T$) and two like–sign leptons, isolated from the hadronic remnants. The mass bounds given below are obtained for analysed luminosities of $L_{e^+} = 36.5\text{ pb}^{-1}$ (H1) and $L_{e^-} = 47.7\text{ pb}^{-1}$ (ZEUS) and are typically in the range $10^3 - 10^4\text{ GeV}$ and given in eq. [4]. The cross section is shown in Fig. 3 including indirect bounds on mixing elements deduced from other experiments [15].

An extension of the analysis allowing for any two final state leptons and the possibility of observing only one isolated lepton (because of applied cuts) is given in [14]. This is studied within the context of the published unusual large number of events with single isolated leptons and large $p_T$ observed by H1 [17]. Possible upgrades of HERA and a more sophisticated general analysis of $\tau$ - decays might allow an improvement by a factor 20.

As stated before, applying limits from flavour changing neutral currents like $\tau \rightarrow \mu\gamma$ might be more stringent but require the specification of a mixing and mass scheme.

6 Future prospects

Besides the already mentioned steps for improvements two ways might be worthwhile to follow. First of all more general meson decays. Improvements on the $\tau$ - sector of matrix elements, especially $\langle m_{\tau\tau}\rangle$, could be done by a search for rare B-decays. Limits on the branching ratio for decays $B^+ \rightarrow K^-\mu^+\mu^+$, $B^+ \rightarrow \pi^-\mu^+\mu^+$ of less than $9.1 \cdot 10^{-3}$ exist [18], however nobody looked into the decays $B^+ \rightarrow K^-\tau^+\tau^+$ or $B^+ \rightarrow \pi^-\tau^+\tau^+$. With the new B-factories such a search
might be possible at a level of producing limits on $\langle m_{\tau \tau} \rangle$ competitive with the ones given.

A significant step forward is possible by using the proposed neutrino factory. Such a high luminosity neutrino machine, producing $10^{13}$ charged current interactions per year in a near detector, allow to improve significantly on the trimuon production process as discussed in [19]. Furthermore in a "parasitic" mode also improved searches for rare kaon decays are foreseen, which should include $K^+ \rightarrow \pi^- \mu^+ \mu^+$. This might allow to bring $\langle m_{\mu\mu} \rangle$ in a physical useful region.

7 Summary and conclusions

The underlying physical process of neutrinoless double beta decay by exchange of a virtual massive Majorana neutrino state was extended to three flavours. Processes studied within that context are reviewed.

Combining all obtained limits, ignoring possible phases in the elements $U_{\alpha m}$ (therefore getting a symmetrical matrix $\langle m_{\alpha\beta} \rangle$) as well as skipping the intrinsic CP parities, the following bounds for the effective Majorana mass matrix emerge:

$$\langle m_{\alpha\beta} \rangle = \begin{pmatrix} \langle m_{ee} \rangle & \langle m_{ep} \rangle & \langle m_{e\tau} \rangle \\ \langle m_{ep} \rangle & \langle m_{\mu\mu} \rangle & \langle m_{\mu\tau} \rangle \\ \langle m_{e\tau} \rangle & \langle m_{\mu\tau} \rangle & \langle m_{\tau\tau} \rangle \end{pmatrix} \lesssim \begin{pmatrix} 2 \cdot 10^{-10} & 1.7(8.2) \cdot 10^{-2} & 4.2 \cdot 10^{3} \\ 500 & 4.4 \cdot 10^{3} & \langle 2.0 \cdot 10^{4} \rangle \end{pmatrix} \text{GeV.}$$

A spread over 14 orders of magnitude can be seen. Again, these are direct limits and all elements other than $\langle m_{ee} \rangle$ and $\langle m_{e\mu} \rangle$ are still in an unphysical region because propagator effects are ignored. To bring at least $\langle m_{\mu\mu} \rangle$ in the physical region as well, the neutrino factory could help a lot because improving both on trimuon production and rare kaon decays described above. This would not only allow to put upper limits on light masses, but also lower limits on heavy neutrinos can then be obtained.

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Figure 1: Fundamental Feynman diagram for the processes here (The "Lobster" - diagram).

Figure 2: Total cross section of the process $\nu_\mu N \rightarrow \mu^- \mu^+ X$ for a left–handed Majorana neutrino as a function of its mass for different neutrino beam energies. No limit on $U_{\mu m}^2$ was applied. The obtained cross sections are tiny.
Figure 3: Total cross section for the process $e^+p \rightarrow \nu e l^+l'^+X$ at HERA as a function of one eigenvalue $m_m$. No limits on $U_{lm}$ are applied, the branching ratio for taus into muons is included. The $(e\tau)$ and $(\mu\tau)$ cases are indistinguishable in this plot.