Electromagnetic wave propagation in spatially homogeneous yet smoothly time-varying dielectric media

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Abstract

We explore the propagation and transformation of electromagnetic waves through spatially homogeneous yet smoothly time-dependent media within the framework of classical electrodynamics. By modelling the smooth transition, occurring during a finite period $\tau$, as a phenomenologically realistic and sigmoidal change of the dielectric permittivity, an analytically exact solution to Maxwell’s equations is derived for the electric displacement in terms of hypergeometric functions. Using this solution, we show the possibility of amplification and attenuation of waves and associate this with the decrease and increase of the time-dependent permittivity. We demonstrate, moreover, that such an energy exchange between waves and non-stationary media leads to the transformation (or conversion) of frequencies. Our results may pave the way towards controllable light-matter interaction in time-varying structures.

Keywords: wave propagation, time-dependent media, amplification and attenuation of waves, energy exchange, frequency conversion, exactly solvable systems

1. Introduction

Controlling the optical properties of photonic structures has been a topic of significant interest for last few decades both in fundamental \cite{1,2} and applied research \cite{3,4,5}. Recent advances in technology and instrumentation have made it possible to realize such control systems via ultrafast switching of the time-dependent dielectric permittivity (or the refractive index) \cite{6,7,8,9,10,11,12}.

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The two most relevant mechanisms of modifying the permittivity in the time domain are the excitation of the so-called free charge-carriers and the electronic Kerr effect. While the free-carriers are typically induced by strong pump pulses creating electron-hole pairs in semiconductors [13, 14, 15, 16], the Kerr effect arises from the non-linear (instantaneous) response of bound electrons to the applied field [17, 18]. These mechanisms enable one to employ the optical switching of the refractive index for various purposes, such as quantum interference [19], information processing [20, 21, 22, 23], material science [24, 25, 26], control of spontaneous emission [27], and several others [28, 29, 30, 31, 32, 33].

The investigation of dynamics of both optical and matter waves in instantaneously time-varying structures have also been in the focus of intense research throughout the last decades. Until now, several effects have already been proposed, for instance, to account for the stimulated electron-light interaction [34] and photon generation [35], to investigate irregular alternation of phases of electromagnetic waves [36] and the energy exchange between waves and non-stationary media [37]. In other scenarios, modulation and conversion of frequencies as well as the transformation of waves have been explored in non-stationary waveguides, resonators and plasmas [cf. books [38, 39, 40] and references therein]. The idea of looking at such phenomena stems from the pioneering papers by Morgenthaler [41] and Ginzburg and Tsytovich [42, 43], who considered the velocity modulation of waves and the transition radiation of charged particles in time-dependent environments. Recently, the dynamics of sound waves have also been examined in non-stationary fluids with either a sudden [37] or smooth [44, 45] change of the medium, leading to the frequency conversion of waves that have already become accessible in an experiment [46].

In most of the studies related to the transformation of electromagnetic waves, the parameters describing the medium are assumed to vary sharply (or step-like) with time, i.e., when the time duration during which the medium experiences a change is much shorter than the propagation period of the wave, \( \tau \ll 2\pi/\omega \). Although such an assumption simplifies the theory and describes the relevant effects, it rarely describes the reality of switching processes and should be replaced with a smooth transition. This is especially important for experiments in which the switching duration is comparable to the period of light, so that \( \tau \approx 2\pi/\omega \).

In this paper, we re-visit the problem of transformation of electromagnetic waves in time-dependent media and provide a step forward towards understanding of the impact of non-stationary environments, continuously changing in time, on the dynamical properties of waves. To this end, we examine the propagation and transformations of waves in time-varying and, at the same time, spatially homogeneous (i.e., uniform) dielectric media. A particular emphasis is placed on studying how a smoothly time-dependent dielectric permittivity affects the energy (flux) and the frequency of transformed waves. In view of this, we derive a generalized wave equation for the electric displacement and obtain an analytically exact solution expressed via hypergeometric functions for a judiciously chosen sigmoidal change of the permittivity, explicitly accounting for the finite transition period \( \tau \). Using this solution, we show that an energy exchange oc-
curs between electromagnetic waves and non-stationary media, which is further demonstrated to lead to either amplification or attenuation of waves depending on whether the refractive index decreases or increases as a function of time. For a monochromatic incident wave, moreover, such an energy non-conservation gives rise to the transformation (or conversion) of frequency due to the (more or less) abrupt change of the refractive index, quite similar to that of the sound wave frequency [44, 45].

The paper is organized as follows. In the next section, we derive a generalized wave equation for the electric displacement when the time-varying dielectric permittivity remains uniform in space. While continuity conditions for the electric displacement and magnetic induction are used to account for the sudden transition (Subsection 2.1), rigorous exact solutions to the time-dependent wave equation are obtained in the case of the smooth transition (Subsection 2.2). These solutions are then discussed in Section 3 and a few effects are predicted, such as the energy exchange, amplification and attenuation as well as frequency conversion of waves. In Section 4, we conclude with future research directions.

2. Theory of time-dependent propagation and transformation of electromagnetic waves

In order to describe the temporal dynamics of light in spatially homogeneous and isotropic media, we start from the source-free Maxwell equations in Gaussian units

\[ \nabla \times H = \frac{1}{c} \frac{\partial D}{\partial t}, \]  

\[ \nabla \times E = -\frac{1}{c} \frac{\partial H}{\partial t}, \]  

\[ \nabla \cdot D = 0, \]  

\[ \nabla \cdot H = 0, \]

where \( \nabla \) is the vector differential operator, “cross” and “dot” mean vector and inner products, respectively. Maxwell’s equations self-consistently characterize the electromagnetic field state only in a vacuum. In a general medium, however, constitutive relations must be added to Eqs. (1-4) to provide a complete description of waves [47]. For an isotropic medium, the electric field \( E \) and displacement \( D \) are related via the standard constitutive relation \( D = \varepsilon E \), with \( \varepsilon \) being the scalar dielectric permittivity. For non-magnetic media, moreover, the magnetic permeability \( \mu = 1 \) as in most of the experiments on optical switching, so that the magnetic field \( H \) equals to the induction \( B \). In this particular case, therefore, the dielectric permittivity \( \varepsilon \) and the refractive index \( n \) are related by a simple formula \( \varepsilon = n^2 \).

For time-varying and space-independent media, i.e., when the dielectric permittivity is only a function of time, \( \varepsilon(t) \), an exact wave equation can be derived for the electric displacement from Eqs. (1-3)

\[ \nabla^2 D(r, t) - \frac{\varepsilon(t)}{c^2} \frac{\partial^2 D(r, t)}{\partial t^2} = 0. \]  

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Given that the medium is spatially uniform, we seek for the solution of this equation in the form

\[ D(r, t) \equiv \hat{d} D(r, t) = \hat{d} e^{ik \cdot r} D(t), \]

(6)

where \( \hat{d} \) is a unit vector along the displacement \( D \), while \( k \) and \( r \) are the wave and position vectors, respectively. The ansatz (6), which is necessary to assure time evolution of the system, characterizes the transformation of waves and accounts for a modification of frequencies of transformed waves. This is reminiscent of the counterpart scenario, when the space dependence of the medium implies \( D(r, t) = \hat{d} e^{-i\omega t} D(r) \) for a monochromatic wave traveling with frequency \( \omega \).

Next, by combining Eqs. (5) and (6), we obtain a one-dimensional equation for \( D(t) \)

\[ \frac{d^2 D(t)}{dt^2} + \frac{c^2 k^2}{\varepsilon(t)} D(t) = 0, \]

(7)

where the permittivity appears in the denominator of the prefactor of \( D \), in contrast to the position dependent wave equation with the permittivity in the nominator [cf., e.g., Eqs. (88.3-4) of Ref. [47]]. Built upon the explicit form of \( \varepsilon(t) \), Eq. (7) allows solutions which characterize dynamics of waves in non-stationary media independent of the nature of switching or tuning of the permittivity. Moreover, an analogous generalized equation for sound waves can also be derived from Euler’s and continuity equations in non-stationary fluids [cf. Eq. (34) of Ref. [45]].

In the following, we shall investigate solutions of Eq. (7) for two distinct cases: when the time-dependent dielectric permittivity experiences either sudden or smooth change. On each of these scenarios, we shall (i) apply continuity conditions for electric displacement and magnetic induction or (ii) rigorously solve the differential equation with a phenomenologically realistic sigmoidal change of the permittivity, judiciously chosen to qualitatively coincide with experimentally determined behaviour. Our approach is based on the direct and exact integration of the time-dependent wave equation, in contrast to other theoretical methods, such as Green’s functions representation [48], the Wentzel-Kramers-Brillouin-Jeffreys approximation [49, 50] or the Volterra integral equation approach [51, 52] (see also Refs. [40, 53]).

2.1. Sudden change of the dielectric permittivity

Before we examine the dynamics of waves in spatially uniform media with a smoothly changing time-dependent permittivity, we re-visit the case of the sudden change, as reported in Ref. [37]. In this scenario, it is assumed that the permittivity undergoes a discrete change at some time \( t_s \) from \( \varepsilon_1 \) (for \( t < t_s \)) to \( \varepsilon_2 \) (for \( t > t_s \)) [cf. Fig. 1]. For these constant values of the permittivity, the solution of the wave equation (7) can be expressed in terms of plane monochromatic
waves. For a wave with the initial frequency \( \omega_I = ck/\sqrt{\varepsilon_1} \) and the constant amplitude \( D_{I0} \)

\[
D_I (r, t) \equiv D_{t<t_s} (r, t) = D_{I0} e^{i(k \cdot r - \omega_I t)},
\]

the sudden change of the permittivity gives rise to the superposition of two – reflected (“R”) and transmitted (“T”) – waves

\[
D_{t>t_s} (r, t) = D_{R0} e^{i(k \cdot r - \omega_R t)} + D_{T0} e^{i(k \cdot r - \omega_T t)}
\]

propagating with opposite frequencies \( \omega_R = -ck/\sqrt{\varepsilon_2} \) and \( \omega_T = ck/\sqrt{\varepsilon_2} = -\omega_R \) and distinct amplitudes \( D_{R0} \) and \( D_{T0} \). In accordance with our adopted assumption about the spatial homogeneity, the frequencies of the initial and transmitted waves are related via the conversion relation

\[
\omega_In_1 = \omega_Tn_2
\]

where the constant numbers \( n_1 = \sqrt{\varepsilon_1} \) and \( n_2 = \sqrt{\varepsilon_2} \) represent the refractive indices of the medium before \( (t < t_s) \) and after \( (t > t_s) \) the change [cf. Ref. [45] for a detailed discussion].

The reflected wave in Eq. 9 can be interpreted as a propagation backward-in-time with positive frequency \( \omega_R \) as \( -i\omega_R t = -i\omega_T (-t) \), or else, propagation forward-in-time with negative frequency \( \omega_R \) as \( -i\omega_R t = -i(-\omega_T) t \). We can nevertheless demonstrate that the reflected wave describes an actual reflection.
in space by calculating the Poynting vector of the three waves \([8]-[9]\)

\[
S_T = \frac{c^2 |D_{T0}|^2}{4\pi \omega \varepsilon_1^2} \mathbf{k} \uparrow \uparrow \mathbf{k},
\]

\[
S_R = -\frac{c^2 |D_{R0}|^2}{4\pi \omega \varepsilon_2^2} \mathbf{k} \uparrow \downarrow \mathbf{k},
\]

\[
S_T = \frac{c^2 |D_{T0}|^2}{4\pi \omega \varepsilon_2^2} \mathbf{k} \uparrow \uparrow \mathbf{k}.
\]

As it can be readily seen, \(S_R\) is anti-parallel to the corresponding Poynting vectors in the initial and transmitted waves. Moreover, the phase velocities

\[
v_I^{(ph)} = \frac{\omega_I}{k}, \quad v_R^{(ph)} = -\frac{\omega_T}{k} = -v_T^{(ph)}
\]

also indicate that the reflected wave propagates opposite to the propagation direction of the initial and transmitted waves. This theoretical construct for waves is still possible to interpret in terms of dynamical observables by taking into account the fundamental relation between the homogeneity of time and the conservation of energy, known as Noether’s theorem [54], as we show below.

Being a solution to the wave equation (5), Eqs. (8)-(9) allow us to reveal the behaviour of waves in non-stationary media. To discuss this, we shall construct the reflectivity and transmittivity which are correspondingly defined as ratios of (space) averaged energy fluxes of reflected and transmitted waves to the averaged flux of the initial wave [37]

\[
\mathcal{R} = \frac{n_2 |D_{R0}/\varepsilon_2|^2}{n_1 |D_{I0}/\varepsilon_1|^2}, \quad \mathcal{T} = \frac{n_2 |D_{T0}/\varepsilon_2|^2}{n_1 |D_{I0}/\varepsilon_1|^2}.
\]

A similar definition, leading to the famous Fresnel’s formulae, is provided in Ref. [47] to describe the transformation of waves in spatially inhomogeneous and time-independent media. Moreover, Refs. [55] and [56] are dedicated to some modified Fresnel’s formulae constituting the transformation of resonant light and polarized matter waves, respectively. Finally, another type of Fresnel’s formulae accounting for a time-dependent spatial reflection of waves from non-stationary interfaces are derived in Refs. [49, 57].

Using relations (13), we can establish the energy balance between the transformed waves \([9]\) and the medium if we, without loss of generality, assume that the dielectric permittivity (or the refractive index) suffer an abrupt change at the time \(t_s = 0\). For such an assumption, the continuity conditions for the electric displacement and magnetic induction

\[
D_{t<0}(r,0) = D_{t>0}(r,0),
\]

\[
B_{t<0}(r,0) = B_{t>0}(r,0),
\]

6
lead to the Fresnel-type formulae in the time domain \[37\]

\[
\mathcal{R} = \frac{n_1(n_1 - n_2)^2}{4n_2^2} = \frac{\omega_T(\omega_T - \omega_I)^2}{4\omega_T^3}, \tag{15}
\]

\[
\mathcal{T} = \frac{n_1(n_1 + n_2)^2}{4n_2^2} = \frac{\omega_T(\omega_T + \omega_I)^2}{4\omega_T^3}. \tag{16}
\]

To get a deeper insight, we add the expressions (15) and (16) and find

\[
\mathcal{R} + \mathcal{T} = \frac{n_1(n_1^2 + n_2^2)}{2n_2^2} = \frac{\omega_T(\omega_T^2 + \omega_I^2)}{2\omega_T^3} \neq 1, \tag{17}
\]

that shows non-conservation of energy for the wave. As the energy is conserved for the whole “wave + medium” system, the expression (17) can be nevertheless interpreted as an energy (flux) exchange between the wave and the time-dependent medium. Depending on whether the wave propagates to optically denser \((n_2 > n_1)\) or rarer \((n_2 < n_1)\) medium, it is either attenuated \((\mathcal{R} + \mathcal{T} < 1)\) or amplified \((\mathcal{R} + \mathcal{T} > 1)\), as also illustrated on Fig 2. This behaviour is quite in contrast to the propagation of sound waves in non-stationary fluids, when the wave is only amplified despite the increase or decrease of the appropriate quantities, such as distributions of the mass density and the sound velocity \[37, 44, 45\].

It is important to realize that in the absence of the time-dependent change of the permittivity, i.e., when the temporal inhomogeneity is “switched off” \((\varepsilon_1 = \varepsilon_2 \text{ or } n_1 = n_2)\), the reflectivity \(\mathcal{R}\) vanishes, while \(\mathcal{T} = 1\), the energy of the wave is conserved \((\mathcal{R} + \mathcal{T} = 1)\), the black line on Fig 2 and no frequency conversion occurs \((\omega_T = \omega_T)\), as one would expect.
Table 1: Comparison of dynamics of electromagnetic waves in time and space domains. Refractive indices and the reflectivity/transmittivity (for normal incidence) in the case of the spatial transformation are marked with upper cases \( N_1, N_2 \) and \( R, T \) to distinguish from the temporal transformation.

| Reflection | Spatial homogeneity and temporal inhomogeneity | Spatial inhomogeneity and temporal homogeneity |
|------------|-----------------------------------------------|-----------------------------------------------|
| \( R = \frac{n_1(n_1 - n_2)^2}{4n_2^2} \) | \( R = \left( \frac{N_1 - N_2}{N_1 + N_2} \right)^2 \) | \( R = \left( \frac{N_1 + N_2}{N_1 - N_2} \right)^2 \) |
| Transmission | \( T = \frac{n_1(n_1 + n_2)^2}{4n_2^2} \) | \( T = \frac{4N_1N_2}{(N_1 + N_2)^2} \) |
| Energy balance | \( R + T \neq 1 \) | \( R + T = 1 \) |

For a better understanding, a comparative analysis of the transformation of waves in the time domain and of its spatial counterpart is summarized in Table 1. For the sake of simplicity, we compare the Fresnel-type formulae (15) and (16) with the conventional Fresnel’s formulae for the case of the normal incidence of a wave on the interface between two different spatially homogeneous media with refractive indices \( N_1 \) and \( N_2 \). While the wave does not conserve energy throughout the propagation in the non-stationary medium, the energy of the wave is conserved in a stationary medium, even if it is spatially inhomogeneous. These energy-related effects are a direct manifestation of either violation or observance of Noether’s conservation theorems.

After this overview, in the next subsection, we extend our studies of transformation of waves in abruptly-varying media to the case of the adiabatic change of the dielectric permittivity. In view of this, we shall assume that the permittivity varies smoothly during some finite transition period \( \tau > 0 \), which also plays the role of the switching duration. Such a smoothness is then modelled by a sigmoidal function, and an analytically exact treatment to the transformation of waves is developed based on the time-dependent wave equation (7).

2.2. Smooth change of the dielectric permittivity

In a more realistic case, the dielectric permittivity, instead of an abrupt variation, often experiences a change during a finite transition period \( \tau \). In order to account for such a finiteness, it is no longer sufficient to consider continuity conditions (14): we need to solve the time-dependent wave equation (7) where the sudden change of the permittivity is replaced by a judiciously chosen and smoothly time-varying function, as depicted in Fig. 3. We may therefore model the switching of the permittivity by a phenomenological sigmoidal function

\[
\varepsilon(t) = \frac{\varepsilon_1\varepsilon_2 (1 + e^{t/\tau})}{\varepsilon_2 + \varepsilon_1 e^{t/\tau}},
\]

(18)
to assure that the asymptotic values \( \varepsilon_1 \) (for \( t < 0 \)) and \( \varepsilon_2 \) (for \( t > 0 \)) are recovered when \( \tau \to 0 \). Choosing this shape of the permittivity, we derive an
Figure 3: Transformation of electromagnetic waves in spatially homogeneous yet time-dependent dielectric media when the permittivity either increases (a) or decreases (b) smoothly during some finite transition period $\tau$. Smooth changes are modelled via Eq. (18).

The exact second order linear differential equation from Eq. (7)

$$
\zeta (1 - \zeta) \frac{d^2 D}{d\zeta^2} + (1 - \zeta) \frac{d D}{d\zeta} + \left( \frac{\alpha + \beta}{\zeta} - \alpha \right) D = 0,
$$

(19)

where a new variable is introduced, $\zeta \equiv -e^{t/\tau}$, which converges with $t \to -\infty$.

The constant parameters

$$
\alpha \equiv \frac{c^2 k^2 \tau^2}{\varepsilon_2} = \omega_T^2 \tau^2, \quad \beta \equiv \frac{c^2 k^2 \tau^2}{\varepsilon_1} - \alpha = (\omega_T^2 - \omega_I^2) \tau^2,
$$

(20)

expressed also by means of the initial $\omega_I$ and transmitted (transformed) $\omega_T$ frequencies, carry information about the wave and the impact of the medium upon it.

Equation (19) has a singularity at $\xi = 0$, which can be removed by making the replacement

$$
D (\xi) = \xi^\nu F (\xi),
$$

(21)

where $\nu$ is a constant and, in general, complex number, while $F$ represents an analytical function of $\xi$ and describes the time-dependent dynamics of the wave. The ansatz (21) we advocate here amounts to reducing Eq. (19) to the conventional form

$$
\xi (1 - \xi) \frac{d^2 F}{d\xi^2} + (c - (a + b + 1) \xi) \frac{d F}{d\xi} - ab F = 0,
$$

(22)

where the constant parameters

$$
a \equiv \nu - i\sqrt{\alpha}, \quad b \equiv \nu + i\sqrt{\alpha}, \quad c \equiv 2\nu + 1
$$

(23)
are introduced for the sake of brevity. Equation (22) has an exact solution expressed in terms of the hypergeometric function, \( F(\xi) = CF(a, b, c, \xi) \) with \( C \) being a constant, provided that \( \nu^2 \equiv -\alpha - \beta = -\omega^2 \tau^2 \) to ensure the convergence of the solution when \( \xi = -e^{t/\tau} \rightarrow 0 \) [58]. This means that the solution is valid for negative values of \( t \) since \( \tau > 0 \). Furthermore, we exploit Eqs. (6) and (21) in order to construct the explicit form of the electric displacement in the interval \( t < 0 \)

\[
D_{t<0}(r, t) = e^{i k \cdot r} e^{i t/\tau} F(a, b, c, -e^{t/\tau}).
\]

The asymptotic behaviour of this function defines the constant number \( C \) and the sign of \( \nu \). Given that \( F(a, b, c, \xi \rightarrow 0) \rightarrow 1 \), the anticipated initial wave (8) can be gained if

\[
C = D_{\infty} (-1)^{-\nu}, \quad \nu = -i \omega I \tau, \quad (24)
\]

such that the electric displacement itself takes the final form

\[
D_{t<0}(r, t) = D_{\infty} e^{i(k \cdot r - \omega I t)} e^{i t/\tau} F(a, b, c, -e^{t/\tau}). \quad (25)
\]

This holds for all times \( t < 0 \) for the given ‘rate of change’ of the dielectric permittivity [18].

The solution (25) does also contain the transformed waves (9) that in turn incorporates the two – reflected and transmitted – waves. For \( t > 0 \), however, the hypergeometric function diverges because of the argument \(-e^{t/\tau}\). We therefore need to employ its symmetry properties to circumvent this divergence. By building a new convergent variable \( 1/\xi = -e^{-t/\tau} \) at \( t > 0 \), in fact, one can re-write the solution (25) as

\[
D_{t>0}(r, t) = D_{\infty} e^{i(k \cdot r - \omega I t)}
\times \left[ \Gamma(c) \Gamma(b-a) \Gamma(b) \Gamma(c-a) \right] \left[ \Gamma(c) \Gamma(a-b) \Gamma(a) \Gamma(c-b) \right] \left[ e^{-a t / \tau} F(a, a+1-c, a+1-b, -e^{-t/\tau}) + e^{-b t / \tau} F(b, b+1-c, b+1-a, -e^{-t/\tau}) \right], \quad (26)
\]

that represents a superposition of two waves with time-varying complex ‘amplitudes’ expressed in terms of the hypergeometric functions. In Eq. (26), the presence of the two exponentials indicates, generally, the frequency conversion owing to the terms \(-i \omega I t - at / \tau = -i \omega_R t \) and \(-i \omega I t - bt / \tau = -i \omega_T t \) [cf. Eq. (23)], which mean that the non-stationary medium serves as a frequency transformer for waves [1]. In order to disentangle these frequency- and time-dependent terms from the hypergeometric function (that is, to ensure that they

\[1\] Similar effects related to frequency conversion, modulation and shift attracted continuously growing interest since the past decades [40]. See also Refs. [59, 60] for the frequency modulation in parity-time symmetric non-stationary structures and for the frequency conversion in the quantum regime, respectively.
occur only in the exponentials), we shall consider the dynamics of the transformed waves long after the permittivity experiences the change. In this limiting case, since $F(a, b, c, 1/\xi \to 0) \to 1$, Eq. (26) can be re-written as

$$D_{t>0}(r, t) \cong D_{i0} e^{ik \cdot r} \times \left[ \frac{\Gamma(c) \Gamma(b-a)}{\Gamma(b) \Gamma(c-a)} e^{i\omega_\tau t} + \frac{\Gamma(c) \Gamma(a-b)}{\Gamma(a) \Gamma(c-b)} e^{-i\omega_\tau t} \right],$$

(27)

which shows explicitly the occurrence of two counter-propagating waves with modified amplitudes determined by the $\Gamma$-functions. As we expect the solution (27) to take the form (9) for $t \to \infty$, further comparison of Eqs. (9) and (27) gives

$$\frac{D_{R0}}{D_{i0}} = \frac{\Gamma(c) \Gamma(b-a)}{\Gamma(b) \Gamma(c-a)}, \quad \frac{D_{T0}}{D_{i0}} = \frac{\Gamma(c) \Gamma(a-b)}{\Gamma(a) \Gamma(c-b)}.$$  

(28)

These expressions define the $\tau$-dependent amplitudes of the reflected and transmitted waves normalized to the amplitude of the initial wave (see also Eqs. (20) and (23)). In the limiting case $\tau \to 0$, moreover, our general treatment confirms the results of Refs. [34, 37, 41, 61] as $\Gamma(z) \to 1/z$ for $|z| \to 0$, but disagrees with Ref. [62] where the authors derive incorrect coefficients despite using the same continuity conditions as Eq. (14).²

Solutions (25)-(27) show how the smooth change of the dielectric permittivity (18) manifests itself in the electric displacement, and therefore, affects the dynamical properties of electromagnetic waves in a non-stationary medium. Since these solutions – established already for the entire time axis $t \gtrless 0$ – depend explicitly on the switching duration (but not the mechanism!) of the refractive index, the transformation of waves and their energy exchange with the non-stationary medium will also depend on the switching duration $\tau$ and the conversed frequency $\omega_\tau$. To demonstrate this, we employ the recurrence relation and Euler’s reflection formula for the $\Gamma$-function

$$\Gamma(\vartheta + 1) = \vartheta \Gamma(\vartheta),$$

$$|\Gamma(i\vartheta)|^2 = \Gamma(i\vartheta) \Gamma(-i\vartheta) = \frac{\pi}{\vartheta \sinh(\pi\vartheta)},$$

use Eqs. (13) and (28), we finally obtain the $\tau$-dependent reflectivity and transmittivity

$$\mathcal{R} = \frac{\omega_T^2}{\omega_T^2} \frac{\sinh^2(\pi(\omega_I - \omega_\tau)\tau)}{\sinh(2\pi\omega_I\tau) \sinh(2\pi\omega_\tau\tau)},$$

(29)

$$\mathcal{T} = \frac{\omega_T^2}{\omega_T^2} \frac{\sinh^2(\pi(\omega_I + \omega_\tau)\tau)}{\sinh(2\pi\omega_I\tau) \sinh(2\pi\omega_\tau\tau)}.$$  

(30)

²This mistake is later corrected in the subsequent paper [61].
Table 2: Comparison of transformation of electromagnetic and sound waves waves in smoothly time-dependent media described by either the refractive index \((n_1, n_2)\) or the mass density \((\rho_1, \rho_2)\) and the sound velocity distributions \((V_1, V_2)\). \(\tau^{(s)}\) and \(R^{(s)}\) correspondingly represent the energy flux transmission and reflection coefficients for sound waves, i.e., the sound reflectivity and transmittivity.

|                        | Electromagnetic waves | Sound waves \([45]\) |
|------------------------|-----------------------|---------------------|
| **General, \(\tau\)-dependent** | \(\checkmark\) Eqs. (29)-(30) | \(\checkmark\) Eqs. (53)-(54) of \([45]\) |
| **Fresnel-type formulae** | | |
| Energy balance         | \(\checkmark \ n_1 \neq n_2\) | \(\checkmark \ \rho_1 \neq \rho_2, \ V_1 \neq V_2\) |
|                        | [Eq. (31)] | [Eq. (56) of \([45]\)] |
| Amplification of waves | \(\checkmark \ n_1 > n_2\) | \(\checkmark \ \rho_1 \gtrsim \rho_2, \ V_1 \gtrsim V_2\) |
| Attenuation of waves   | \(\checkmark \ n_1 < n_2\) | \(\times\) |
| Energy difference      | \(\checkmark \ T - R = n_1^2/n_2^2\) | \(\checkmark \ T^{(s)} - R^{(s)} = 1\) |
|                        | [Eq. (32)] | [Eq. (57) of \([45]\)] |

This is one of the main results of this work. By generalizing the Fresnel-type formulae \([15]\) and \([16]\) from sudden \((\tau \to 0)\) to smooth \((\tau > 0)\) transition of the permittivity, the expressions \((29)-(30)\) describe the energy transport of an electromagnetic wave when propagating through a spatially homogeneous yet smoothly time-varying medium for the specific time dependence \((18)\). We will utilize formulae \((29)-(30)\) in the next section in order to discuss the energy balance between waves and non-stationary media, which depends on the switching duration and reveals the amplification and attenuation of waves.

3. Results and discussion

For both the sudden and smooth changes of the permittivity, the wave exchanges energy with the medium \([\text{cf. Eq. (17)}]\) and, as a result, is either amplified or attenuated. For a smooth change, that occurs during the finite period \(\tau\), the sum of the reflectivity \((29)\) and the transmittivity \((30)\) takes the form

\[
R + T = \frac{\omega_T^2}{\omega_I^2} \left( \frac{\tanh(\pi \omega_T \tau)}{2 \tanh(\pi \omega_I \tau)} + \frac{\tanh(\pi \omega_I \tau)}{2 \tanh(\pi \omega_I \tau)} \right) \neq 1. \tag{31}
\]

As the expression in the parentheses is always larger than unity, we immediately see that the wave is either amplified or attenuated depending on whether the conversed frequency is increased or decreased (as compared to the initial one), or else, the refractive index is decreased or increased \([\text{cf. Eq. (10)}]\). A similar situation also holds for the sudden change of the medium, as described by Eq. \((17)\). Being one of the main results of this paper, the expression \((31)\) generalizes Eq. \((17)\) to explicitly include the switching duration \(\tau\) and shows the universal nature of the amplification and attenuation of electromagnetic waves also in the case of smooth change of the medium. This is in contrast with sound
Figure 4: Amplification (a) and attenuation (b) of electromagnetic waves in suddenly (dashed lines) and smoothly (solid curves) changing dielectric media. Comparison is made between different ratios $\delta$ of refractive indices before and after the change; $\delta > 1$ and $\delta < 1$ correspond to the decrease (a) and increase (b) of the refractive index. The case $\delta = 1$, when no energy exchange occurs, is not shown on the figure.

waves which due to their inherent structure are only amplified, irrespective, whether the mass density and sound velocity distributions increase or decrease as functions of time. In Table 2, a comparison is made in terms of generalized Fresnel-type formulae for the electromagnetic and sound waves.

Another interesting feature can be obtained from Eqs. (29)-(30) if we calculate the difference of the transmittivity and reflectivity

$$\mathcal{T} - \mathcal{R} = \frac{\omega^2}{\omega^2} = \frac{n_2^2}{n_1^2},$$

which is independent of $\tau$ and maintains the same form as that for the sudden change [37]. While the wave travels from optically rarer to denser medium ($n_2 > n_1$), the transmitted wave carries an energy flux smaller than the sum of the energy fluxes in the reflected and initial waves, and vice versa for $n_2 < n_1$: the energy flux of the transmitted wave surpasses that of the two other waves. This is again in contrast to sound waves where the transmitted wave carries an energy flux exactly equal to the sum of the fluxes of the reflected and initial waves [37, 45] [cf. Table 2].

To better perceive the energy balance between the electromagnetic wave and the non-stationary medium, let us re-write the expressions (31) and (17) in the
Figure 5: Energy balance versus ratio between refractive indices before and after the change of the medium for different values of the dimensionless time $\eta$. The pink point illustrates the area where the energy of the wave is conserved, $\delta = 1$.

dimensionless form

\[
R + T = \delta^2 \left( \frac{\tanh(\pi \eta)}{2 \tanh(\pi \delta \eta)} + \frac{\tanh(\pi \delta \eta)}{2 \tanh(\pi \eta)} \right) \neq 1, \quad (33)
\]

\[
(R + T)_{\eta \to 0} = \frac{\delta}{2} (1 + \delta^2) \neq 1. \quad (34)
\]

Here, $\eta = \omega_T \tau$ represents the dimensionless transition period, while the parameter $\delta \equiv \omega_T / \omega_I = n_1 / n_2$ shows the ratio between the refractive indices before and after the change, so that $\delta$ equals to unity when the tuning of the refractive index is “switched off”. Figure 4 demonstrates the energy balance between the wave and the medium as a function of $\eta$ for different values of $\delta$ as well as for both the sudden (dashed lines) and smooth (solid curves) changes of the dielectric permittivity. The fact that the sum of the reflectivity and transmittivity is either greater (Fig. 4(a)) or less (Fig. 4(b)) than the unity is a signature of the wave amplification and attenuation, respectively. This change in the sum of the energy fluxes of transformed waves is quantified by means of the ratio $\delta$ between the refractive indices. As seen, the larger the ratio $\delta$ or $1/\delta$, the stronger is the amplification or attenuation of the wave. The inclination of curves for the smooth change is more pronounced as $\delta$ increases (decreases) from unity due to the variation $d(R + T)/d\eta \approx -\pi^2 \delta (\delta^2 - 1)^2 \eta/3$ for small $\eta$. Particularly, the increase of 7% in $\delta$ corresponding to the decrease of the refractive index ($\delta = 1.07$) gives rise to the change of $\sim 15\%$ in the amplification ($R + T \approx 1.15$) [cf. black curves in Fig. 4(a)]. Whereas the decrease of 7% in $\delta$ results in the decrease of $\sim 13\%$ in the sum of the reflectivity and transmittivity and leads to an attenuation of waves ($R + T \approx 0.87$) [cf. grey curves in Fig. 4(b)]. Such an asymmetry between the increase and decrease of energy fluxes is due to the fact that Eqs. (33) and (34) do not remain symmetric under the interchange $\delta \leftrightarrow 1/\delta$ (or $n_1 \leftrightarrow n_2$). As expected, moreover, when the transition period is much less than the period of the initial wave ($\tau \ll 2\pi / \omega_I$) both curves for sudden and smooth changes at given $\delta$ merge to each other as $\tanh(\pi \delta \eta) / \tanh(\pi \eta) \to \delta$ for $\eta \to 0$. Figure 5 in turn, illustrates the dependence of the energy balance on the ratio $\delta$.
between the refractive indices for selected values of the dimensionless time \( \eta \). As the dimensionless time approaches its asymptotic value \( (\eta \to 0) \), representing the sudden change of the permittivity, the difference between curves decreases. The dependence on \( \eta \) is markedly pronounced in the domain of amplification, which is again due to the \( \delta \leftrightarrow 1/\delta \) asymmetry of the energy balance (33).

Thus, apart from being a frequency transformer, the non-stationary medium acts as a ‘source or sink of energy’ for electromagnetic waves. The prediction of an increase and decrease of normalized energy flux for the wave, which can correspondingly be interpreted as an amplification and attenuation due to an energy exchange with a medium, can be tested experimentally, if it is possible to disentangle the energy transport through the dielectric from its internal energy. A microscopic theory should be developed in order to explicitly reveal the source of energy, as in our phenomenological approach the time-dependent permittivity characterizes only a ‘net’ structure, which can be generated, for instance, by laser pulses in photonic systems [63, 64].

4. Conclusion

We have derived an analytically exact theory to describe the propagation and transformation of electromagnetic waves in spatially homogeneous yet smoothly time-varying dielectric structures. The emphasis has been put on exploring how the finite transition period \( \tau \) for the dielectric permittivity influences the dynamical properties of waves, such as the energy (flux) exchange between waves and non-stationary media and the conversion of frequencies of transformed waves. The exchange is shown to lead to the \( \tau \)-dependent amplification or attenuation of waves correspondingly linked to the wave propagation from optically denser to rarer medium or vice versa. We have provided a detailed comparison between predictions of our generalized theory and those of the sudden change approximation. The peculiar differences in transformations of electromagnetic and sound waves in smoothly-varying media are also pointed out. Being manifestations of the temporal inhomogeneity, both the energy exchange and the transformation of frequencies can be tested experimentally should the switched dielectric permittivity follow the sigmoidal shape, as shown in Fig. 3. However, a rigorous study of ubiquitous processes of relaxation following the switching of the refractive index would constitute modification of the sigmoidal change, and therefore correction to the reflectivity (29) and transmittivity (30).

Although our results are valid for a wide range of frequencies (from radio frequency up to ultraviolet) and for various types of dielectric media, a generalized approach is needed to simultaneously account for smoothly time-dependent dielectric permittivity and magnetic permeability, especially relevant for studying transformation of waves in non-stationary plasmas [65, 66, 67, 68] and magnetoelectric systems [69]. Such a general study would enable one to resolve the debate about various ways of deriving the reflection and transmission coefficients in suddenly changing media [70, 71]. In recent years, moreover, controlling waves in both space and time domains has raised considerable interest [72, 73]. Simultaneous investigation of space- and time-dependent transformation of waves
will lead to an intriguing ‘interplay’ of energy and momentum exchange between waves and spatially inhomogeneous and non-stationary media.

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References

[1] Boyd RW. Nonlinear Optics. Academic Press; 2008.

[2] Joannopoulos JD, Johnson SG, Winn JN, Meade RD. Photonic crystals: Molding the flow of light. Princeton University Press; 2008.

[3] Hulin D, Mysyrowicz A, Antonetti A, Migus A, Masselink WT, Morkoç H, Gibbs HM, Peyghambarian N. Ultrafast all-optical gate with subpicosecond on and off response time. Appl Phys Lett 1986;49:749-51.

[4] Jewell JL, McCall SL, Scherer A, Houh HH, Whitaker NA, Gossard AC, English JH. Transverse modes, waveguide dispersion, and 30 ps recovery in GaAs/AlAs microresonators. Appl Phys Lett 1989;55:22-4.

[5] Rivera T, Ladan FR, Izraël A, Azoulay R, Kuszelewicz R, Oudar JL. Reduced threshold all-optical bistability in etched quantum well microresonators. Appl Phys Lett 1994;64:869-71.

[6] Yamamoto T, Yoshida E, Nakazawa M. Ultrafast nonlinear optical loop mirror for demultiplexing 640 Gbit/s TDM signals. Electron Lett 1998;34:1013-4.

[7] Nakamura S, Ueno Y, Tajima K. Femtosecond switching with semiconductor-optical-amplifier-based symmetric Mach-Zehnder-type all-optical switch. Appl Phys Lett 2001;78:3929-31.

[8] Nielsen ML, Mørk J, Suzuki R, Sakaguchi J, Ueno Y. Experimental and theoretical investigation of the impact of ultra-fast carrier dynamics on high speed SOA-based all-optical switches. Opt Express 2006;14:331-47.

[9] Harding PJ, Euser TG, Nowicki-Bringuier Y-R, Gérard J-M, Vos WL. Dynamical ultrafast all-optical switching of planar GaAs/AlAs photonic microcavities. Appl Phys Lett 2007;91:111103-1-3.

[10] Euser TG, Molemaar AJ, Fleming JG, Gralak B, Polman A, Vos WL. All-optical octave-broad ultrafast switching of Si woodpile photonic band gap crystals. Phys Rev B 2008;77:115214-1-6.
[11] Nozaki K, Tanabe T, Shinya A, Matsuo S, Sato T, Taniyama H, Notomi M. Sub-femtojoule all-optical switching using a photonic crystal nanocavity. Nat Photonics 2010;4:477-83.

[12] Ctistis G, Yüce E, Hartsuiker A, Claudon J, Bazin M, J-M Gérard, WL Vos. Ultimate fast optical switching of a planar microcavity in the telecom wavelength range. Appl Phys Lett 2011;98:161114-1-3.

[13] Leonard SW, van Driel HM, Schilling J, Wehrspohn RB. Ultrafast band-edge tuning of a two-dimensional silicon photonic crystal via free-carrier injection. Phys Rev B 2002;66:161102(R)-1-4.

[14] Mazurenko DA, Kerst R, Dijkhuis JI, Akimov AV, Golubev VG, Kurdyukov DA, Pevtsov AB, Sel’kin AV. Ultrafast optical switching in three-dimensional photonic crystals. Phys Rev Lett 2003;91:213903-1-4.

[15] Bristow AD, Wells J-PR, Fan WH, Fox AM, Skolnick MS, Whittaker DM, Tahraoui A, Krauss TF, Roberts JS. Ultrafast nonlinear response of AlGaAs two-dimensional photonic crystal waveguides. Appl Phys Lett 2003;83:851-3.

[16] Euser TG, Vos WL. Spatial homogeneity of optically switched semiconductor photonic crystals and of bulk semiconductors. J Appl Phys 2005;97:043102-1-7.

[17] Inouye H, Kanemitsu Y. Direct observation of nonlinear effects in a one-dimensional photonic crystal. Appl Phys Lett 2003;82:1155-7.

[18] Hu X, Zhang Q, Liu Y, Cheng B, Zhang D. Ultrafast three-dimensional tunable photonic crystal. Appl Phys Lett 2003;83:2518-20.

[19] Yüce E, Ctistis G, Claudon J, Dupuy E, Buijs RD, de Ronde B, Mosk AP, Gérard J-M, Vos WL. All-optical switching of a micro cavity repeated at terahertz rates. Opt Lett 2013;38:374-6.

[20] Harris SE, Yamamoto Y. Photon switching by quantum interference. Phys Rev Lett 1998;81:3611-4.

[21] Dawes AMC, Illing L, Clark SM, Gauthier DJ. All-optical switching in rubidium vapor. Science 2005;308:672-4.

[22] Bajcsy M, Hofferberth S, Balic V, Peyronel T, Hafezi M, Zibrov AS, Vuletic V, Lukin MD. Efficient all-optical switching using slow light within a hollow fiber. Phys Rev Lett 2009;102:203902-1-4.

[23] Venkataraman V, Londero P, Bhagwat AR, Slepkov AD, Gaeta AL. All-optical modulation of four-wave mixing in an Rb-filled photonic bandgap fiber. Opt Lett 2010;35:2287-9.

[24] Yoshimura K, Yamada Y, Okada M. Optical switching of Mg-rich Mg-Ni alloy thin films. Appl Phys Lett 2002;81:4709-11.
[25] Srinivas G, Sankaranarayanan V, Ramaprabhu S. Optical switching properties of RCo$_2$-type alloy hydride based solid state device. J Appl Phys 2008;104:064504-1-5.

[26] Tajima K, Hotta H, Yamada Y, Okada M, Yoshimura K. Electrochromic switchable mirror glass with controllable reflectance. Appl Phys Lett 2012;100:091906-1-3.

[27] Thyrestrup H, Hartsuiker A, Gérard J-M, Vos WL. Non-exponential spontaneous emission dynamics for emitters in a time-dependent optical cavity. Opt Express 2013;21:23130-44.

[28] Johnson PM, Femius Koenderink A, Vos WL. Ultrafast switching of photonic density of states in photonic crystals. Phys Rev B 2002;66:081102(R)-1-4.

[29] Fushman I, Waks E, Englund D, Stoltz N, Petroff P, Vučković J. Ultrafast nonlinear optical tuning of photonic crystal cavities. Appl Phys Lett 2007;90:091118-1-3.

[30] Sivan Y, Pendry JB. Time-reversal in dynamically-tuned zero-gap periodic systems. Phys Rev Lett 2011;106,193902-1-4.

[31] Harding PJ, Bakker HJ, Hartsuiker A, Claudon J, Mosk AP, Gérard J-M, Vos WL. Observation of a stronger-than-adiabatic change of light trapped in an ultrafast switched GaAs-AlAs microcavity. J Opt Soc Am B 2012;29;A1-5.

[32] Xia K, Twamley J. All-optical switching and router via the direct quantum control of coupling between cavity modes, Phys Rev X 2013;3:031013-1-11.

[33] Sivan Y, Ctistis G, Yüce E, Mosk AP. Femtosecond-scale switching based on excited free-carriers. Opt Express 2015;23:16416-28.

[34] Avetisyan HK, Avetisyan AK, Petrosyan RG. Stimulated interaction of charged particles with electromagnetic radiation in a medium with nonstationary properties. Sov Phys – J Exp Theor Phys 1978:48;192-6.

[35] Cirone M, Rzążewski K, Mostowski J. Photon generation by time-dependent dielectric: A soluble model. Phys Rev A 1997;55:62-6.

[36] Nerukh AG. Intermittency of electromagnetic waves in a regular time-varying medium. J Phys D: Appl Phys 1999;32:2006-13.

[37] Mkrtchyan AR, Hayrapetyan AG, Khachatryan BV, Petrosyan RG. Transformation of sound and electromagnetic waves in non-stationary media. Mod Phys Lett B 2010;24:1951-61.

[38] Ginzburg VL. Theoretical Physics and Astrophysics. Pergamon Press; 1979.
[39] Ginzburg VL, Tsytovich VN. Transition Radiation and Transition Scattering. Taylor & Francis; 1990.

[40] Nerukh A, Sakhnenko N, Benson T, Sewell P. Non-Stationary Electromagnetics. Pan Stanford Publishing; 2012.

[41] Morgenthaler FR. Velocity modulation of electromagnetic waves. IRE Trans Microw Theory Tech 1958;6:167-72.

[42] Ginzburg VL. Concerning a certain type of transition radiation. Sov Phys – Radiophys Quantum Electron 1973;16:386-90.

[43] Ginzburg VL, Tsytovich VN. On the theory of transition radiation in a nonstationary medium. Sov Phys – J Exp Theor Phys 1974;38:65-70.

[44] Hayrapetyan AG, Grigoryan KK, Petrosyan RG, Khachatryan BV. In: Sound Waves: Propagation, Frequencies and Effects. New York: Nova Science Publishers; 2011.

[45] Hayrapetyan AG, Grigoryan KK, Petrosyan RG, Fritzsche S. Propagation of sound waves through a spatially homogeneous but smoothly time-dependent medium. Ann Phys 2013;333:47-65.

[46] Wang Q, Yang Y, Ni X, Xu Y-L, Sun X-C, Chen Z-G, Feng L, Liu X-P, Lu M-H, Chen Y-F. Acoustic asymmetric transmission based on time-dependent dynamical scattering. Sci Rep 2015;5:10880-1-10.

[47] Landau LD, Lifshitz EM, Pitaevskii LP. Electrodynamics of Continuous Media. Butterworth-Heinemann; 1984.

[48] Felsen LB, Whitman G.M. Wave propagation in time-varying media. IEEE Trans Ant Prop 1970;18:242-53.

[49] Fante RL. Transmission of electromagnetic waves into time-varying media. IEEE Trans Ant Prop 1971;193:417-24.

[50] Averkov SI, Boldin. Waves in nondispersive nonstationary inhomogeneous media. Sov Phys – Radiophys Quantum Electron 1980;23:705-10.

[51] Fedotov FV, Nerukh AG, Benson TM, Sewell P. Investigation of electromagnetic field in a layer with time-varying medium by Volterra integral equation method. J Lightwave Tech 2003;21:305-14.

[52] Shabanov NV, Huang D, Knjazikhin Y, Dickinson RE, Myneni RB. Stochastic radiative transfer model for mixture of discontinuous vegetation canopies. J Quant Spectr Radiat Transfer 2007;107:236-62.

[53] Mishchenko MI. Directional radiometry and radiative transfer: the convoluted path from centuries-old phenomenology to physical optics. J Quant Spectr Radiat Transfer 2014;146:4-33.
[54] Landau LD, Lifshitz EM. Classical Mechanics. Butterworth-Heinemann; 1976.

[55] Glauber RJ, Prasad S. Polarium model: reflection and transmission of coherent radiation. Phys Rev A 2000;61:063815-1-16.

[56] Dragys A. Boundary conditions and transmission reflection of electron spin in a quantum well. Semicond Sci Technol 2012;27:045009-1-9.

[57] Nerukh AG. Fresnel’s formulas in time domain. IEEE Trans Ant Prop 2004;52:2735-41.

[58] Gradshteyn IS, Ryzhik IM. Table of Integrals, Series and Products. Academic Press; 2000.

[59] Hayrapetyan AG, Klevansky SP, Götte JB. Instantaneous amplitude and angular frequency modulation of light in time-dependent $\mathcal{PT}$-symmetric optical potentials. arXiv:1503.04720.

[60] Manzoni MT, Silveiro I, García de Abajo FJ, Chang DE. Second-order quantum nonlinear optical processes in single graphene nanostructures and arrays. New J Phys 2015;17:083031-1-17.

[61] Mendonça JT, Martins AM, Guerreiro A. Temporal beam splitter and temporal interference. Phys Rev A 2003;68:043801-1-4.

[62] Mendonça JT, Shulka PK. Time refraction and time reflection: two basic concepts. Phys Scr 2002;65:160-3.

[63] Mondia JP, Tan HW, Linden S, van Driel HM. Ultrafast tuning of two-dimensional planar photonic-crystal waveguides via free-carrier injection and the optical Kerr effect. J Opt Soc Am B 2005;22:2480-6.

[64] Yüce E, Cistitis G, Claudon J, Dupuy E, Boller KJ, Gérard J-M, Vos WL. Competition between electronic Kerr and free-carrier effects in an ultimate-fast optically switched semiconductor microcavity. J Opt Soc Am B 2012;29:2630-42.

[65] Kalluri DK, Goteti VR. Frequency shifting of electromagnetic radiation by sudden creation of a plasma slab. J Appl Phys 1992;72:4575-80.

[66] Kalluri DK. Frequency shifting using magnetoplasma medium: flash ionization. IEEE Trans Plasma Sci 1993;21:77-81.

[67] Lee JH, Kalluri DK. Modification of an electromagnetic wave by a time-varying switched magnetoplasma medium: transverse propagation. IEEE Trans Plasma Sci 1998;26:1-6.

[68] Kalluri DK, Jinming C. Comparison identities for wave propagation in a time-varying plasma medium. IEEE Trans Ant Prop 2009;57:2698-05.

20
[69] Zhang RY, Zhai Y-W, Lin S-R, Zhao Q, Wen W, Ge M-L. Time circular birefringence in time-dependent magnetoelectric media. Sci Rep 2015;5:13673-1-10.

[70] Xiao Y, Maywar DN, Agrawal GP. Reflection and transmission of electromagnetic waves at a temporal boundary. Opt Lett 2014;39:574-7.

[71] Bakunov MI, Maslov AV. Reflection and transmission of electromagnetic waves at a temporal boundary: comment. Opt Lett 2014;39:6029.

[72] Mosk AP, Lagendijk A, Lerosey G, Fink M. Controlling waves in space and time for imaging and focusing in complex media. Nat Photon 2012;6:283-92.

[73] Thyrrestrup H, Yüce E, Ctistis G, Claudon J, Vos WL, Gérard J-M. Differential ultrafast all-optical switching of the resonances of a micropillar cavity. Appl Phys Lett 2014;105:111115-1-4.