PARABOLIC NILRADICALS OF HEISENBERG TYPE, II

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This is a sequel to [11], where we proved that every real simple non-compact Lie algebra different from \(\mathfrak{so}(1, n)\) has an essentially unique parabolic subalgebra whose nilradical is a Heisenberg algebra of a division algebra, and deduced some consequences regarding their Tanaka prolongations.

Here we discuss their symmetries, the associated parabolic geometries, and the riemannian geometry of the harmonic spaces \(\mathbb{R}_+ N\), having the former as conformal infinities.

It is somewhat remarkable that such basic result had not been noticed before. Since [11] was written we found that it can also be deduced from [10][13][5] by identifying the building blocks of Howe’s H-tower groups\(^1\).

Still, our construction is independent of type H and representation theory, and explains part of the “high degree of symmetry” observed in [10].

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1. Algebras of type \(\text{divH}\) and associated parabolics

Recall that a normed real division algebra \(A\) determines two series of graded nilpotent algebras

\[
\mathfrak{h}_n(A) = (A^n \oplus A^n) \oplus A, \quad \mathfrak{h}'_{p,q}(A) = (A^p \oplus A^q) \oplus \Im(A)
\]

with respective brackets

\[
[x + y + t, \hat{x} + \hat{y} + \hat{t}] = \sum x_i\hat{y}_i - \hat{x}_i y_i, \\
[x + y + t, \hat{x} + \hat{y} + \hat{t}] = -\Im(\sum x_j\hat{x}_j + \sum \hat{y}_ky_k).
\]

Excluding the \(\mathfrak{h}'_{p,q}(\mathbb{R})\)'s, which are abelian, and the \(\mathfrak{h}_n(\mathbb{O})\), \(\mathfrak{h}'_{p,q}(\mathbb{O})\) for \(n > 1\) or \(p + q > 1\), which are non-prolongable (see 3.2 below), and taking the isomorphism \(\mathfrak{h}'_{p,q}(\mathbb{C}) \cong \mathfrak{h}_{p+q}(\mathbb{R})\) into account, the remaining ones are

\[
\mathfrak{h}_n(\mathbb{R}) \quad \mathfrak{h}_n(\mathbb{C}) \quad \mathfrak{h}_n(\mathbb{H}) \quad \mathfrak{h}_{p,q}(\mathbb{H}) \quad \mathfrak{h}_1(\mathbb{O}) \quad \mathfrak{h}_{1,0}(\mathbb{O}).
\]

We call these algebras and associated objects of type \(\text{divH}\), or when convenient, \(\mathfrak{h}(A)\).

Let now \(g\) be a simple Lie algebra, \(p \subset g\) a parabolic subalgebra, and \(n \subset p\) the nilradical of \(p\). If \(g\) is compact, then \(p\) is either 0 or \(g\). If \(p\) is proper and \(g\) is isomorphic to \(\mathfrak{so}(1, n)\), then \(p\) is unique up to conjugacy and \(n\) is abelian. Moreover, the \(\mathfrak{so}(1, n)\) are the only simple algebras with these
properties. Here we will be interested in the remaining ones, those which contain parabolic subalgebras with non-abelian nilradical, the set of which we which often denote by $\mathcal{S}$.

For a graded nilpotent Lie algebra $n = \mathfrak{g}^{-1} \oplus \mathfrak{g}^{-2} \oplus \ldots$ to be the nilradical of a parabolic subalgebra of a semisimple algebra is equivalent to asking that it can be “prolonged” to a finite dimensional graded semisimple algebra

$$g(n, g_0) = g^k \oplus \ldots \oplus g^1 \oplus g^0 \oplus g^{-1} \oplus \ldots \oplus g^{-k}$$

where $g^i = \theta g^{-i}$ for some Cartan involution. This already implies that $\text{Aut}(n)$ must be large enough, so as to contain such $g^0$. The associated parabolic subalgebra is $g^0 \oplus n$.

The main results of [11] can be resumed as follows.

**Theorem 1.** [11]

(a) Every simple non-compact Lie algebra not isomorphic to $\mathfrak{so}(1, n)$ has a parabolic subalgebra with non-singular nilradical.

(b) Any two are conjugate by the adjoint group.

(c) The nilradicals that appear are exactly the algebras of type $\text{div}H$.

(d) An algebra of type $H$ is of type $\text{div}H$ if and only its Tanaka prolongation is not trivial.

One consequence is that divH algebras are the most symmetric among 2-step non-singular nilpotents, in the following sense. Let $n = v \oplus z$ be a 2-graded nilpotent Lie algebra with center $z$ and let $m = \dim z$ and $n = \dim v$. Since $\text{Der}(n) = \text{Der}_{gr}(n) \oplus \text{Hom}(v, z)$,

$$\dim \text{Der}(n) = \dim \text{Der}_{gr}(n) + mn.$$ 

Generically, $\dim \text{Der}_{gr}(n) = 1$. If $n$ is of type $H$,

$$\dim \text{Der}_{gr}(n) \geq \frac{1}{2} m(m + 1).$$

Now let $N$ be the csc Lie group with Lie algebra $n$, $\mathcal{V}$ the left-invariant distribution on $N$ determined by $v$, and $\text{Inf}(n)$ the algebra of infinitesimal automorphisms of $\mathcal{V}$ at $e$, that is, germs of vector fields $X$ on $N$ near $e$ such that $L_X(\mathcal{V}) \subset \mathcal{V}$. Clearly, $\text{Inf}(n) \supset \text{Der}_{gr}(n)$. Then, generically, even among type $H$,

$$\dim \text{Inf}(n)/\text{Der}_{gr}(n) = \dim n.$$ 

For type $\text{div}H$ instead,

$$\dim \text{Inf}(n)/\text{Der}_{gr}(n) \geq 2 \dim n.$$

### 2. Langlands decompositions

Let $n$ be of type $\text{div}H$, $p$ a standard parabolic subalgebra of some simple Lie algebra $\mathfrak{g}$ having $n$ as nilradical, and

$$p = m \oplus a \oplus n$$

its Langlands decomposition. Then
Proposition 1.

\[ m = m_o \oplus \text{spin}(n), \quad a = a_o \oplus a_\delta, \quad g_0 = m_o \oplus \text{spin}(n) \oplus a \]

where

- \( m_o \) is the centralizer of \( z \) in \( m \);
- \( \text{spin}(n) \cong \text{so}(3) \) acts on \( z \) by the standard representation and on \( v \) as a sum of spin representations;
- \( a_o = a \cap \text{Der}_o(n) \), which is 0 if \( p \) is maximal and 1-dimensional otherwise; and \( a_\delta = \mathbb{R} \delta \).

The individual factors of the resulting decomposition

\[ p = (m_o \oplus a_o) \oplus \text{spin}(n) \oplus (a_\delta \oplus n) \]

are listed in Table 1.

Proof. All the assertions follow from Table 1, which is obtained applying the construction in the proof of Theorem 1 case by case. \( \square \)

Given a simple \( g \in \mathcal{S} \), denote by \( p(g) \) the parabolic subalgebra with nilradical \( n(g) \) of type \( \text{divH} \), and \( m(g) \) its Levi factor.

Corollary 1. \( g \in \mathcal{S} \) has a complex or quaternionic structure if and only if \( n(g) \in h_o(\mathbb{C}) \) or \( h_o(\mathbb{H}) \), respectively.

Proposition 2. \( p(g) \) is maximal parabolic except for \( g \cong su(1,n), su(1,n), su^*(2n), su^*(6), FII, sl(3,\mathbb{R}), sl(3,\mathbb{C}), \) or \( EIV \). It is minimal iff \( g \cong su(1,n), sp(1,n), su^*(6), FII, sl(3,\mathbb{R}), sl(3,\mathbb{C}), \) or \( EIV \).

Even if \( p(g) \) is not minimal, it contains the following distinguished minimal one. First note that any reductive Lie algebra can be uniquely decomposed as \( r = r' \oplus r'' \) where \( r' \) is semisimple with simple factors in \( \mathcal{S} \), and \( r'' \) is reductive with simple factors not in \( \mathcal{S} \).

Proposition 3.
(a) If \( g \in \mathcal{S} \), then \( m(g)' \in \mathcal{S} \).
(b) If \( g \in \mathcal{S} \) is classical, then \( m(g)' \) is classical and of the same type as \( g \).
(c) If \( n \) is \( \text{divH} \), then \( \text{Der}_o(n)' \in \mathcal{S} \).

Proof. By inspection of Table 1 \( \square \)

One obtains a filtration

\[ g = g^0 \supset g^{-1} \supset \ldots \supset g^{-k} \]

of Lie subalgebras all of class \( \mathcal{S} \), with corresponding \( \text{divH} \)-nilradicals \( n(g^{-i}) \), such that \( g^{-i-1} = m(g^{-i})' \).

Proposition 4. \( p(g^{-k}) \) is a minimal parabolic subalgebra of \( g \), and \( \bigoplus_{i=0}^{k} n(g^{-i}) \) is a maximal nilpotent one.

It follows that every classical \( g \in \mathcal{S} \) fits into a strictly increasing filtration of algebras

\[ 0 \subset g^{-k} \subset \ldots \subset g^{-1} \subset g \subset g^1 \subset \ldots \]

of simple algebras of the same simple type and satisfying \( g^{-1} = (g^1)' \).

Remark 1. The \( \bigoplus_{i=0}^{j} n(g^{-i}) \) are essentially Howe’s H-tower algebras.
3. Parabolic Geometries

Among the distributions with symbol of type $\text{H}$, those with symbol of type $\text{divH}$ have compact Klein models. More precisely, let $\mathcal{V}$ be the canonical distribution on a group $N$ of type $\text{divH}$, and choose a simple $G$ and a parabolic $P$ with $N$ as nilradical. The tangent space to $G/P$ at the origin can be identified with $\bar{\mathfrak{n}} = \bar{\mathfrak{v}} \oplus \bar{\mathfrak{z}}$, and $P$ respects this grading. Let $\mathcal{V}$ be the $G$-invariant distribution on $G/P$ determined by $\bar{\mathfrak{v}}$. Therefore

**Proposition 5.** $G/P$ carries a $G$-invariant distribution locally equivalent to $\mathcal{V}$.

Consider now a parabolic geometry of type $\text{divH}$ on a manifold $M$, i.e., a Cartan geometry of type $(G, P)$ where $\mathfrak{g} = \text{Lie}(G) \in \mathfrak{S}$ and $P \subset G$ is a parabolic subgroup with unipotent radical of type $\text{divH}$. Let $\omega$ be its Cartan connection and $\kappa$ its curvature. Together with the gradings

$$\mathfrak{g} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2$$
$$\mathfrak{p} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \quad \mathfrak{n} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1},$$

$\omega$ determines a distribution $\mathcal{D}$ on $M$ and a principal $G_0$-bundle, where $G_0$ is the subgroup of $P$ that preserves the grading of $\mathfrak{g}$. $G_0$ is isomorphic to a subgroup of $\text{Aut}_{gr}(\mathfrak{n})$ and $\text{Lie}(G_0) = \mathfrak{g}_0$.

$\omega$ is called *regular* when $\mathcal{D}$ has constant symbol isomorphic to $\mathfrak{n}$ and the principal $G_0$-bundle is a reduction of the canonical $\text{Aut}_{gr}(\mathfrak{n})$-bundle. When $G_0 = \text{Aut}_{gr}(\mathfrak{n})$, we have just a distribution of constant symbol. $\omega$ is called *normal* if it satisfies $\partial^* \kappa = 0$ where $\partial^*$ is the Kostant codifferential. This condition assures the uniqueness of the Cartan connection.

Since a distribution is fat if and only if its symbol is non-singular, Theorem 1 (a) implies

**Theorem 2.** The regular normal parabolic geometries supported on fat distributions are exactly those of $\text{divH}$ type.

In fact,

(a) Distributions with symbol $\mathfrak{h}_n(\mathbb{R})$ are associated to contact parabolic geometries: Lagrangean, partially integrable almost CR, Lie contact, contact projective, and exotic contact structures \[7\].

(b) Distributions with symbol $\mathfrak{h}_n(\mathbb{C})$ are associated to the complex contact structures of Boothby \[3\] and, more generally, to partially integrable almost CR-structures of CR-codimension 2 with additional structure. These have not received much attention except for special cases \[6\].

(c) For the cases $\mathfrak{g} = \mathfrak{sp}(n, \mathbb{R})$, $\mathfrak{sp}(n, \mathbb{C})$, $\mathfrak{g}$ is not the prolongation of $(\mathfrak{n}, \mathfrak{g}_0)$, so the underlying structure they determine on the manifold is just a real or complex contact structure with the canonical $\text{Aut}_{gr}(\mathfrak{n})$-bundle. To characterize this parabolic geometries we have to consider finer underlying structures, in this case are a contact projective structure, i.e. a contact projective equivalence class of partial contact connections \[7\].

(d) For all the other $\text{divH}$ algebras the parabolic geometry is determined by the distribution alone, with no additional structure (Proposition 4.3.1 in
Quaternionic and octonionic contact structures associated to $h'_{p,q}(\mathbb{H})$ and $h'_{1,0}(\mathfrak{O})$ have been the subject of interest \[7\].

(c) Distributions whose symbol is $h_1(\mathfrak{O})$ or $h'_{1,0}(\mathfrak{O})$ are locally isomorphic to the flat model. This is a consequence of the fact that the second generalized Spencer cohomology groups vanish in these cases \[15\].

1. Conformal infinity of harmonic spaces

Let $N$ be a group of type $H$, $A = \exp(\mathbb{R}\delta)$ the group of dilations and $S = AN$ their semidirect product. Endowing $n$ with a compatible metric induces a left-invariant Riemannian metric $g$ on $S$, called a Damek-Ricci metric \[12\]. $S$ is harmonic - hence Einstein, and any homogeneous noncompact harmonic space is isometric to $S$ for some $N$ of type $H$ \[9\]. One is interested in the asymptotic behavior of the metric $g$.

If $S$ is hyperbolic, the metric in polar form satisfies
\[
g = dt^2 + e^{t\gamma} + e^{2t\delta} + o(t) \quad (2)
\]
for $t \to \infty$, where $(\gamma, D^\delta)$ is a generalized $G$-conformal structure in the sense of Biquard-Mazzeo \[2\] on the geodesic boundary of $S$, which for the hyperbolic space is a sphere.

For the general $S$ no such formula seems to exist (cf. for example \[4\]) - unless $n$ is of type $div H$. For the first statement, consider the Poincaré-like realization of $S$ in Euclidean unit ball $B$ of the same dimension, as well as the Siegel-like one on
\[
U = \{(X, Z, t) \in \mathfrak{v} \times \mathfrak{z} \times \mathbb{R} : \ t > \frac{1}{4}|X|^2\}.
\]
The Cayley transform $C : U \rightarrow \mathbb{B}$
\[
C(X, Z, t) = \frac{1}{(1 + t)^2 + |Z|^2} ((1 + t - JZ)X, 2Z, -1 + t^2 - |Z|^2)
\]
is a diffeomorphism. It extends to the boundary of $U$ in $\mathfrak{v} \times \mathfrak{z} \times \mathbb{R}$, $\partial U = \{(X, Z, \frac{1}{4}|X|^2) : X \in \mathfrak{v}, Z \in \mathfrak{z}\}$ giving a diffeomorphism
\[
C_0 : \partial U \rightarrow S^*
\]
onto the punctured sphere. $N$ acts simply transitively on $\partial U$, hence on $S^*$, and its canonical distribution induces invariant distributions on these boundaries. Writing
\[
T_{(X, Z, \frac{1}{4}|X|^2)}(\partial U) = \{(2W, Y, <X, Y>) : Y \in \mathfrak{v}, W \in \mathfrak{z}\},
\]
the distribution is given by
\[
D_{(X, Z, \frac{1}{4}|X|^2)}^{\partial U} = \{(Y, \frac{1}{2}[X, Y], \frac{1}{2} < X, Y >) : Y \in \mathfrak{v}\}.
\]
Let now $D^S = dC_0(D^{\partial U})$ and let $\infty$ denote the puncture of $S^*$.

**Proposition 6.** $D^S$ extends smoothly over $\infty$ if and only if $S$ is a hyperbolic space.

**Proof.** If $S$ is a hyperbolic space $G/K$, $K$ is transitive on $S$ and leaves invariant the distribution, it can have no singularities.
Otherwise, $N$ does not satisfy the $J^2$ condition of $[8]$. This implies that there is a unitary triple $X \in \mathfrak{v}, Z, W \in \mathfrak{z}$ such that $[X, J_Z J_W X] = 0$. The vector fields on $\partial U \cong \mathfrak{v} \times \mathfrak{z}$

$$(v_1)_{(X,Z)} = (X, 0, \frac{1}{2}|X|), \quad (v_2)_{(X,Z)} = (J_Z X, \frac{1}{2}|X| Z, 0)$$

correspond to the copy of $\mathfrak{h}_1(\mathbb{R})$ spanned by the triple $X, J_Z J_W X, J_W X$. On $S^*$ and along the orbit $\exp(\mathfrak{h}_1(\mathbb{R})) \cdot (-\infty)$, the plane spanned by $d\mathbb{C}(v_1)$, $d\mathbb{C}(v_2)$, is horizontal and has a limit as $|X|, |Z| \to \infty$, namely the plane $(\mathbb{R}X \oplus \mathbb{R}J_Z X, 0, 0)$. Doing the same with the copy of $\mathfrak{h}_1(\mathbb{R})$ spanned by $J_Z J_W X, J_W X, Z$, the corresponding limiting plane is $(\mathbb{R}J_W X \oplus \mathbb{R}J_Z J_W X, 0, 0)$. Therefore, if the distribution extends, its value at $\infty$ must be $(\mathfrak{v}, 0, 0)$. On the other hand, the vector

$$((1 + |Z|^2 - \frac{1}{10} |X|^4) J_W X, (1 + \frac{1}{4} |X|^2)|X|^2 W, 0)$$

is horizontal along the curve $1 + |Z|^2 = \frac{1}{10}|X|^4$, where it spans line $(0, \mathbb{R}W, 0)$, which is a contradiction. □

If $N$ is of type $\text{divH}$ however, the $S$-orbit of any point gives an isometric embedding into the associated symmetric space $S \hookrightarrow G/K$.

Denoting by $\partial S$ the boundary of $S$ in an appropriate compactification of $G/K$, the natural projection

$$\pi : \partial S \to S$$

onto the geodesic spherical boundary resolves the singularities of $D^{S^*}$.

As a consequence, $[2]$ holds in this case. Indeed such formula seems to characterize the $\text{divH}$ among harmonic spaces. The non-hyperbolic ones are those obtained for $\mathfrak{n} = \mathfrak{h}_n(\mathbb{C}), \mathfrak{h}'_{p,q}(\mathbb{H}) (pq \neq 0), \mathfrak{h}_n(\mathbb{H}), \mathfrak{h}_1(\mathbb{O})$, are all anisotropic, and the last three admit non-regular deformations, suited to extend the arguments of $[2]$ to obtain new Einstein metrics. Details are left for a sequel, where the boundary structures $(\gamma, \delta)$ will be described for each $\text{divH}$ type.

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### Table 1. Langlands factors of divH parabolics

| g       | m               | dima | n          | Σ            |
|---------|-----------------|------|------------|--------------|
| sl(n, R)| sl(n - 2, R)    | 2    | h_{n-2}(R)| \{α_1, α_{n-1}\}|
| sl(n, C)| sl(n - 2, C) ⊕ \mathbb{R}^2 | 2 | h_{n-2}(\mathbb{C}) | \{α_1, α_{n-1}\}|
| su*(2n) | su(2)^2 ⊕ su*(2n - 4) | 2 | h_{n-2}(\mathbb{H}) | \{α_2, α_{n-2}\}|
| su(p, q)| su(p - 1, q - 1) ⊕ \mathbb{R} | 1 | h_{p+q-2}(\mathbb{R}) | \{α_1, α_{p+q-1}\}|
| sp(n, R)| sp(n - 1, R)    | 1    | h_{n-1}(\mathbb{R}) | \{α_1\}|
| sp(p, q)| su(2) ⊕ sp(p - 1, q - 1) | 1 | h'_{p-1,q-1}(\mathbb{H}) | \{α_2\}|
| sp(n, C)| sp(n - 1, C) ⊕ \mathbb{R} | 1 | h_{n-1}(\mathbb{C}) | \{α_1\}|
| so(p, q)| sl(2, \mathbb{R}) ⊕ so(p - 2, q - 2) | 1 | h_{p+q-4}(\mathbb{R}) | \{α_2\}|
| so*(2n) | su(2) ⊕ so*(2n - 4) | 1 | h_{2n-4}(\mathbb{R}) | \{α_2\}|
| so(n, C)| sl(2, \mathbb{C}) ⊕ so(n - 4, \mathbb{C}) ⊕ \mathbb{R} | 1 | h_{n-4}(\mathbb{C}) | \{α_2\}|

- **E1**: sl(6, \mathbb{R})
- **EII**: su(3, 3)
- **EIII**: su(1, 5)
- **EIV**: so(8)
- **E_6**: sl(6, \mathbb{C}) ⊕ \mathbb{R}
- **EV**: so(6, 6)
- **EV1**: so*(12)
- **EVII**: so(2, 10)
- **E_7**: so(12, \mathbb{C}) ⊕ \mathbb{R}
- **EVIII**: EV
- **EIX**: EVII
- **EIIX**: E_7 ⊕ \mathbb{R}
- **FI**: sp(3, \mathbb{R})
- **FI**I**: so(7)
- **F_4**: sp(3, \mathbb{C}) ⊕ \mathbb{R}
- **G**: sl(2, \mathbb{R})
- **G_2**: sl(2, \mathbb{C}) ⊕ \mathbb{R}

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