Tree Height and DBH Growth Model Establishment of Main Tree Species in Wuling Mountain Small Watershed

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Abstract. Taken 4 main tree species in the Wuling mountain small watershed as research objects, 57 typical sample plots were set up according to the stand type, site conditions and community structure. 311 goal diameter-class sample trees were selected according to diameter-class groups of different tree-height grades, and the optimal fitting models of tree height and DBH growth of main tree species were obtained by stem analysis using Richard, Logistic, Korf, Mitscherlich, Schumacher, Weibull theoretical growth equations, and the correlation coefficient of all optimal fitting models reached above 0.9. Through the evaluation and test, the optimal fitting models possessed rather good fitting precision and forecast dependability.

1. Introduction

As the source of nutrients and energy base of the whole forest ecosystem, forest biomass is the total amount of plant organic matter accumulated in the ecosystem [1]. The carbon sequestration of forest vegetation can be directly reflected by biomass, and the carbon sink potential of forest ecosystem will be determined by the accumulation rate of carbon [2]. Tree height and DBH are important variables of biomass measurement, and also the most basic elements for the growth and yield model research [3], which play an important role in the study of basic growth law of forest trees and the establishment of forest management schemes [4]. At present, tree height and DBH growth equations which are researched and used more include Weibull [5], Mitscherlich [6], Gompertz [7], Logistic [8], Korf [9,10] and Richards [11]. In this paper, we used different growth equations to simulate the tree height and DBH growth of the main tree species in Wuling mountain small watershed, and obtained the optimal growth equation, in order to provide scientific basis for the growth dynamic prediction of tree height and DBH of different tree species, and provide basic data for scientific and reasonable forest management scheme.

About the tree height and DBH growth model has a lot of research at home and abroad [12, 13, 14, 15], and the tree height and DBH growth curve of common tree species can be better applied in practice, for example, S.R.Hui developed Richard growth equation for the stand growth of Larix kaempferi [16], X.H.Yuan constructed the height-DBH growth curve for Cunninghamia lanceolata artificial forest [17]. However, these studies focus on a certain tree species in the artificial forest or natural forest, and the study on the tree height and DBH growth equation of multiple tree species in a
certain region is rare. In this paper, different growth equations were used to simulate the tree height and DBH growth of the four main tree species in Wuling mountain small watershed by stem analysis, in order to obtain the optimal growth equation.

2. General situation of research area
The research area is located in the Nverzhai small watershed of wuling mountain area in the Yangtze River middle reaches (110°10' E, 29°30' N), about 7 km from the Cili County northwest, for secondary small tributaries of Lishui river, near north-south direction, mizoguchi altitude of 210 m, the highest peak of 917.4 m, the main channel length of about 1.2 km, small watershed with a total area of 3.15 km² and total population of 277, belong to typical subtropical humid region. Rainfall is abundant, and the annual average rainfall is about 1400 mm. The basin elevation is 210.0-917.4 m, and the main rocks are sandstone and slate. The soil is mainly mountain red soil, yellow soil and yellow brown soil. The terrain is complex, the vertical slope of main gully is about 28.4 %, and the ravines density is about 2.6 km/km². The existing woodland is 202.32 hm², paddy field 4.13 hm², vegetable plot 1.54 hm², dry land 5.07 hm², channel water 38.80 hm², roadway 24.52 hm², homestead 4.62 hm².

The main woodland types in this basin include Pinus massoniana forest, Eucommia ulmoides artificial forest, Citrus reticulate and some miscellaneous shrub forests.

3. Method
Taken four main tree species of Wuling mountain small watershed as research object, 115 sample plots were set according to the stand type, the setting conditions and the stand composition, and the area of each sample plot was 20 m×30 m. Selected typical representative trees with various diameter classes in each sample plot as sample trees, the sample trees were analyzed by stem analysis, and 80% of the sample trees were randomly selected for modeling from each species, and the remaining data was used for testing [18], using SPSS17.0 statistical software for data processing.

| Tree species                  | Number of sample plot | Number of sample tree |
|------------------------------|-----------------------|-----------------------|
| Pinus massoniana             | 17                    | 95                    |
| Cunninghamia lanceolata      | 17                    | 96                    |
| Pinus elliottii              | 14                    | 75                    |
| Liquidambar formosana        | 9                     | 45                    |
| Total                        | 57                    | 311                   |

Table 2. Six theoretical growth equations [19]

| Growth equation | Expression | Parameter |
|-----------------|------------|-----------|
| Richard         | \( H = a(1 - \exp(-bA)^c) \) | \( a,b,c > 0, a: H \) final value (upper asymptotic value), \( c \): Absolute curve shape and inflection point, \( b \) is related to the growth rate. |
| Logistic        | \( H = \frac{a}{1 + \exp( b - cA)} \) | \( a,b,c > 0, a: H \) final value (upper asymptotic value), \( b \) is related to the growth rate, \( c \): Shape parameter. |
| Korf            | \( H = a\exp(-b/A^c) \) | \( a,b,c > 0, a: H \) final value (upper asymptotic value), \( b \) is related to the growth rate, \( c \): Shape parameter. |
| Mitscherlich    | \( H = a(1 - \exp(-bA)) \) | \( a,b > 0, a: H \) final value (upper asymptotic value), \( b \): Shape parameter. |
| Schumacher      | \( H = a\exp(-b/A) \) | \( a,b > 0, a: H \) final value (upper asymptotic value), \( b \): Shape parameter. |
| Weibull         | \( H = a(1 - \exp(-bA^c)) \) | \( a,b,c > 0, a: H \) final value (upper asymptotic value), \( b \): Scale parameter, \( c \): Shape parameter. |
Using the theoretical growth equations in table 2 to fitting, the optimal theoretical growth equations which were suitable for simulating the individual height and DBH growth process of each species were selected. According to the predicted value by model and measured value of the height and DBH of modeling samples, the total relative error and the estimated accuracy were calculated, and whether the residual distribution was random was checked to evaluate whether the model reached the intended target.

According to the predicted value by model and measured value of the height and DBH of testing samples, the total relative error $e$ and the estimated accuracy $p$ were calculated using the formula $p = (1 - e) \times 100\%$, $e = \|\hat{y} - y\|/ y$. In the case that the prediction accuracy was lower than the requirement, the reason could be analyzed, and the estimated precision of the model could be improved by increasing the sample quantity.

4. Results and Discussion

4.1. Determination of sample number for tree height and DBH growth model construction

The tree height and DBH were investigated in the sample plots, and the minimum sample number was determined according to the error theory. The total sample number, model fitting sample number and model verifying sample number of sample trees of each tree species were shown in table 3.

Table 3. The total number, fitting number and checking number of sample tree of each species

| Tree species            | Total number | Fitting number | Checking number |
|-------------------------|--------------|----------------|-----------------|
| Pinus massoniana        | 95           | 76             | 19              |
| Cunninghamia lanceolata | 96           | 76             | 20              |
| Pinus elliottii         | 75           | 60             | 15              |
| Liquidambar formosana   | 45           | 36             | 9               |

4.2. Construction of the tree height and DBH growth model of main tree species

The model equations of tree height and DBH growth were obtained by the measured data of sample trees with different diameter grade of four main tree species. The model equations and test evaluation were shown in table 4.

Table 4. The tree height and DBH growth equation of 4 main tree species and their examination evaluation and application scope

| Tree species            | Model equation | $r^2$ | $\epsilon$(%) | $P$(%) | Application scope     |
|-------------------------|----------------|-------|----------------|--------|------------------------|
| Pinus massoniana        | $H=27.8389 \times (1-e^{-0.057t})^{1.3059}$ | 0.95  | 4.30          | 95.70  | 5 years $\leq t \leq 40$ years |
|                         | $D=42.2593 \times (1-e^{-0.0299t})^{1.0044}$ | 0.93  | 6.50          | 93.50  | years                 |
| Pinus elliottii         | $H=36.585 \times (1-e^{-0.0266t})^{1.034}$  | 0.92  | 7.94          | 92.06  | 2 years $\leq t \leq 30$ years |
|                         | $D=27.382 \times (1-e^{-0.086t})^{1.286}$  | 0.91  | 8.92          | 91.08  | years                 |
| Cunninghamia lanceolata | $H=20.3793 \times (1-e^{-0.0684t})^{1.1943}$ | 0.97  | 2.60          | 97.40  | 5 years $\leq t \leq 35$ years |
|                         | $D=21.7106 \times (1-e^{-0.0538t})^{0.7626}$ | 0.98  | 2.20          | 97.80  | years                 |
| Liquidambar formosana   | $H=17.9678 \times (1-e^{-0.0471t})^{0.6987}$ | 0.97  | 2.4           | 97.6   | 3 years $\leq t \leq 36$ years |
|                         | $D=19.3969 \times (1-e^{-0.0714t})^{1.0701}$ | 0.91  | 8.2           | 91.8   | years                 |

$r^2$ is the correlation coefficient, $\epsilon$ is the total relative error, $P$ is the estimation accuracy.

5. Conclusion

According to the characteristics of forest community and species distribution in the Wuling mountain small watershed, the 57 typical sample plots were set considering the factors such as site condition, stand type and community structure. The height and DBH of 311 sample trees with different diameter grade were measured, and 6 theoretical growth equations were used to fit the height and DBH growth,
then the optimal growth model of tree height and DBH of *Pinus massoniana*, *Cunninghamia lanceolata*, *Pinus elliottii* and *Liquidambar formosana* was obtained. The estimate value of tree height and DBH by each species optimal model was consistent with the actual observed value, and the correlation coefficient of all optimal models was above 0.9. Through testing of the optimal growth model by verifying sample data, the total relative error of each model was small, the prediction accuracy was more than 95%, and each model equation had better fitting precision and estimate level.

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