Multiscale analysis of sub-wavelength imaging with metal-dielectric multilayers

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Imaging with a layered superlens is a spatial filtering operation characterized by the point spread function (PSF). We show that in the same optical system the image of a narrow sub-wavelength Gaussian incident field may be surprisingly dissimilar to the PSF, and the width of PSF is not a straightforward measure of resolution. FWHM or std. dev. of PSF give ambiguous information about the actual resolution, and imaging of objects smaller than the FWHM of PSF is possible. A multiscale analysis of imaging gives good insight into the peculiar scale-dependent properties of sub-wavelength imaging. © 2010 Optical Society of America

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Fig. 1. Intensity transmission and reflection coefficients \((T, R)\) of the multilayer as a function of the filling factor, plotted over the transfer function (in vertical cross-sections of the color map). Phase isolines are distanced by \(\pi/4\).

Since the seminal paper by Pendry [1] and the first experimental demonstration of sub-wavelength imaging through a 40nm silver slab [2, 3], sub-wavelength imaging at visible wavelengths has been investigated in much thicker low-loss layered silver-dielectric structures [4–10]. A variety of physical models may be applied to explain the mechanism of transmission: 1. the effective medium anisotropic approximation of the sub-wavelength multilayer [4] combined with the Fabry-Perot resonant condition tuned to be independent of the angle of incidence [8, 9]; 2. negative refraction at the boundary of silver slab [1] multiple interfaces of the layered structure resulting in diffraction-free propagation [6]; 3. resonant tunneling through the bandgap material formed by the periodic metal-dielectric multilayer [5]. Enhancement of evanescent fields needed for sub-wavelength imaging may be also explained in various ways: 1. as the result of collective coupling between plasmon modes at subsequent metallic layers [11, 12] - if we look to the internal field distributions; 2. as self-collimation [13] - if we look to the band diagrams of the multilayer; 3. as the result of large effective permittivity \(\epsilon_\perp \approx \infty\) - when the homogenized anisotropic model of the system is valid.

Here, we calculate numerically the transfer function (TF) of the multilayer system. Let us emphasize that the structures analyzed are inaccurately characterized with the homogenization model which underestimates the value of transmission.

Imaging through a layered superlens consisting of uniform and isotropic materials is a linear and shift invari...
The TF is the ratio of the spatial spectra of the output and incident fields and corresponds to the amplitude transmission coefficient of the multilayer

\[ \hat{H}_y(k_x, z = L) = TF(k_x) \cdot \hat{H}_y^{inc}(k_x, z = 0). \]  

(2)

Due to reflections, the incident field \( \hat{H}_y^{inc}(k_x, z = 0) \) differs from the total field \( \hat{H}_y(k_x, z = 0) \).

PSF of an imaging system can be often straightforwardly interpreted, and provides clear information about the resolution, loss or enhancement of contrast, as well as the characteristics of image distortions. For instance, the resolution may be usually linked to the width of PSF. When the input signal and PSF are positive functions with a limited region of support (region with non-zero values), the regions of support of convolved functions simply add together, contributing to the broadening of the filtered signal. This can be expressed more conveniently using the following relation on the \( L_0 \) norms,

\[ \|H_y(x) * PSF(x)\|_0 = \|H_y(x)\|_0 + \|PSF(x)\|_0, \]  

(3)

where \( \|f(x)\|_0 = lim_{y \to 0^+} \int |f(x)|^2 dx \).

On the other hand, for simple Gaussian PSF and input, the output has the width (variance) equal to the sum of variances of PSF and input,

\[ \exp(-x^2/\sigma_1^2) \ast \exp(-x^2/\sigma_2^2) \propto \exp(-x^2/(\sigma_1^2 + \sigma_2^2)). \]  

(4)

Therefore once again the width of PSF has a clear link to the resolution of the imaging system. However, formulas (4) and (3) are often invalid for diffractive systems with complex PSF.

From now on, we focus on a \( SrTiO_3 - Ag \) multilayer with \( N = 20 \) periods, and the total thickness \( L = 1.15\mu m \). The elementary cell consists of an \( Ag \) layer symmetrically coated with \( SrTiO_3 \). Strontium Titanate is an isotropic material with a high refractive index \( n = 2.674 + 0.027i \) at the wavelength \( \lambda = 430nm \) [14]. The refractive index of silver at the same wavelength is equal to \( n_{Ag} = 0.04 + 2.46i \) [15]. Fig. 1 shows how the TF of the multilayer depends on the silver-filling fraction, and the corresponding PSF is shown in the upper part of Fig 2. The evanescent part of the TF has a large magnitude, which is the necessary condition for sub-wavelength imaging. The shape of TF is generally regular with the exception of the phase discontinuity in the vicinity of \( k_x/k_0 \approx 1 \), as well as the strong phase modulation below \( d_{Ag}/\Lambda \lesssim 0.35 \) which suppresses the super-resolving properties of the PSF in that range. The phase step at \( k_x/k_0 = 1 \) in the TF influences the shape of the corresponding PSF which, with the increase of filling-factor, evolves from a narrow sub-wavelength maximum to a shape dominated by the side-lobes. The response to a narrow sub-wavelength Gaussian signal is entirely different from the PSF (Fig2, bottom). PSF does not resemble a Gaussian function and its width measured with \( FWHM \) is different from the doubled standard deviation. The off-axis background of PSF results in the...
In Fig. 5 we show how a narrow Gaussian beam, as well as a beam originating from a point source propagate through the two discussed multilayers. The simulations only confirm the behavior described in Fig. 4 but are a good illustration to the peculiarities encountered in sub-wavelength imaging. While one of the multilayers allows for approximately diffraction-free propagation independently of the size of the source, the other behaves in the same way for broader sources only, and shows strong diffraction when the shape of the source approaches a δ-function.

In conclusion, we have studied the transmission of sub-wavelength incident Gaussian field through a thick ($L \sim n\lambda$) silver-dielectric superlens. We have demonstrated that the response to narrow sub-wavelength Gaussian signal may be surprisingly different from the PSF of the system. Multiscale analysis provides the means to distinguish between diffraction-free propagation for various ranges of object sizes.

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