Forecasting models of agricultural process based on fuzzy time series

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Abstract. Today, the problem of increasing the validity and accuracy of forecasts based on the analysis of time series under conditions of uncertainty is very important. The models and methods used to predict the dynamics of agricultural processes are built on quantitative information and are implemented as part of a statistical approach. In this approach, time-based forecasting models are constructed on several requirements for the initial data, the main of which are the requirements of comparability, sufficient representativeness to reveal regularity, uniformity, and stability. Only keeping these requirements, uncertainty can be interpreted in terms of randomness and appropriate statistical forecasting methods can be applied. However, the real dynamic processes taking place in agriculture are represented by time series, for which these requirements are rarely feasible, due to the great uncertainty of the factors determining their dynamics. The problem of forecasting such series is particularly relevant for agricultural science and practice. The article touches upon the possibilities of using fuzzy modeling tools to predict the dynamics of processes in the agricultural sector.

1. Introduction

Agricultural production is mainly carried out under conditions of a high degree of uncertainty under market economy conditions. Uncertainty arises because of the influence of many factors, not only industrial, economic and social, characteristic of any production, but as a result of the influence of natural and biological factors to a large extent. The combined influence of all these factors leads to agricultural production indicators fluctuate both in space and in time. Therefore, the problem of making managerial decisions under uncertainty in the field of agricultural production is extremely important and relevant. One of these problems is increasing the validity and accuracy of forecasts. The models and methods that are currently used in predicting agricultural processes taking place under uncertainty are built on quantitative information and are implemented as part of a statistical approach. At the same time, the statistics that serve as initial information for forecasting are either spatial (regression models) or temporal (time series models). Regression modeling of agrarian processes was discussed in previous work [1]. Here we will focus on time series and forecasting the dynamics of processes based on them.

In the statistical approach, the forecasting models are based on the number of source data requirements. The main requirements are comparability, sufficient representativeness to identify patterns, uniformity, and stability. Only observing these requirements, uncertainty can be interpreted in terms of randomness and the appropriate statistical forecasting methods can be applied. However, the real dynamic processes
taking place in agriculture are represented by time series, for which these requirements are rarely feasible, which is associated with a large uncertainty of the factors determining their dynamics [2]. Non-statistical uncertainty of the time series of real dynamic processes is expressed in a combination of such properties as:

1) short row length (less than 30 - 50);
2) unsteady time-series behavior;
3) incompleteness and inaccuracy of the values of the series;
4) unknown probabilistic characteristics of the stochastic process describing the time series;
5) nonlinear nature of the desired relationship between the levels of the series.

Given these properties, time series is said to have a high degree of uncertainty, reflecting the dynamics of poorly structured processes. [3]. The problem of forecasting precisely such series is particularly relevant for agricultural science and practice, since traditional statistical approaches to solving this problem are not suitable.

2. Problem definition and decision methods

A new direction has been actively developing the last two decades on the basis of methods and models of artificial intelligence, in particular, using fuzzy models. The development of this trend in relation to time series with a high degree of uncertainty has led to the creation of fuzzy time series models, which are based on fuzzy time series [4].

In 1993, the Song and Chissom first defined fuzzy time series models in [5, 6] and discussed their practical use. From that moment, the formation of the theory and practice of modeling fuzzy time series (FTS) began, which was developed in the works of such foreign scientists as S. Chen, Q. Tanaka, M. Sah, V. Pedrich, V. Novak, A. Jilani, Q. Hwang et al. Among domestic scientists in the field of fuzzy time series models are the studies of N. Yarushkina, I. Batyrshin, S. Kovalev, Yu. Kudinov, K. Degtyarev and others.

Fuzzy time series (FTS) is defined as a sequence of observations ordered at equally spaced points in time if the values that a certain value takes at the observed point in time are expressed using a fuzzy label. A fuzzy label is a fuzzy set defined on a universal set.

Two possibilities occur in practice: historical data (available statistics) is set either in the form of linguistic quantities or as real numbers. In both cases, the available data can be represented by fuzzy time series based on the fuzzy approximation theorem [7].

Currently, various models of fuzzy forecasting are proposed. Fuzzy first-order models are considered in this article. The first-order FTS model is defined by the equation:

\[ F(t) = F(t-1)^\cdot R(t, t-1), \]  

where \( F(t) \) – the forecast value of the indicator during a time period \( t \), \( F(t-1) \) – the real value during a time period \( t-1 \), \( R \) – fuzzy relationship on successive instants.

There are two basic models among the set of fuzzy models proposed by various authors for forecasting, which differ by computational methods for determining the fuzzy ratio \( R \) in the model (1). The first model involves calculating \( R \) using the operation of fuzzy sets defined as the Mamdani implication. The second model is based on using arithmetic operations [8,9].

The common characteristic of most fuzzy approaches is the sequential performance of the main three procedures:
1) definition of the universe and fuzzification of historical data;
2) identification of internal fuzzy relationships and the definition of their groups;
3) finding fuzzy forecasts, and their defuzzification.

In this article, the basic basis of fuzzy forecasting is the method proposed in [7], using the second model. Let us consider the procedure for fuzzy time series forecasting on a specific example of a time series of potato yields. Potatoes are one of the leading crops in Russian agricultural production. Potato is not only a food product, but also an industrial raw material and feed for agricultural animals. Russia currently lags behind developed countries in terms of potato yields. However, the experience of these countries shows that Russia has good prospects for the development of this industry [10]. The correct forecast of
the most important indicators of the potato industry, in particular, productivity, is important. So let us turn to the example.

**The example.** There are historical data on potato productivity in agriculture of the Russian Federation (Table 1) [11].

**Table 1.** Potato yields.

| Year | c/ha | Year | c/ha |
|------|------|------|------|
| 2005 | 124  | 2012 | 136  |
| 2006 | 133  | 2013 | 147  |
| 2007 | 132  | 2014 | 153  |
| 2008 | 138  | 2015 | 164  |
| 2009 | 144  | 2016 | 158  |
| 2010 | 100  | 2017 | 163  |
| 2011 | 150  | 2018 | 170  |

The traditional model for forecasting time series is first applied to conduct a comparative analysis. Let us consider the simplest model of 1st-order autoregression:

\[ y_t = a y_{t-1} + \epsilon, \]  

where \( t \) – the period number, \( y_t, y_{t-1} \) – yields during a time period \( t \) (\( t-1 \)), \( a \) – the model parameter, \( \epsilon \) – the random component.

Parameter \( a \) was calculated using the least-squares method, which gave the following results:

\[ a = 1.0158 \rightarrow \hat{y}_t = 1.0158 \cdot t. \]  

The parameter turned out to be greater than unity (\( a > 1 \)) in the model (3), which indicates the unsteady behavior of the investigated time series. The average forecasting error rate (AFER) was used to assess the quality of the constructed equation (3):

\[ AFER = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n} \]  

where \( y_i \) – the real value of the indicator at the \( i \)-th time, \( \hat{y}_i \) – the calculated value at the same time, \( n \) – the number of observations.

The average forecasting error rate for the model (3) was 9.8%.

Now let us turn to the construction of a fuzzy time series model. In the fuzzy version of the model (3), there corresponds a fuzzy model of the first order (1):

\[ F(t) = F(t-1) \circ R(t, t-1). \]  

To build the forecast, a method based on the analysis of the fuzzy time series, proposed in [8], was chosen. Here is a step-by-step description of the algorithm for constructing a fuzzy time series and obtaining predictive estimates based on it.

**Step 1.** Definition of a universal set \( U \), on which all fuzzy sets will be determined. For this, the maximum (\( D_{\text{max}} \)) and minimum (\( D_{\text{min}} \)) values are determined on the set of historical data. Based on them, a universal set \( U = [D_{\text{min}} - D_1, D_{\text{max}} + D_2] \) is constructed, where \( D_1, D_2 \) are two positive real numbers. In our example, \( D_{\text{min}} = 100, D_{\text{max}} = 170 \). For the convenience of calculations, we take \( D_1 = 4, D_2 = 6 \), then \( U = [100-4, 170+6] = [96, 176] \).

**Step 2.** The choice of the number of intervals and the partition of the universal set into intervals of equal length \( u_i \).

The following method was proposed to select the optimal value for the length of the interval. The first differences are calculated based on table 1. The average modulus of the difference between most of the
obtained difference values is approximately 8 units. We accepted this value as the length of the intervals. When choosing such an interval length, the number of intervals is \((176 – 96)/8=10\).

The following partition \(U\) was obtained: \(u_1=[96, 104]\), \(u_2=[104, 112]\), \(u_3=[112, 120]\), \(u_4=[120, 128]\), \(u_5=[128, 136]\), \(u_6=[136, 144]\), \(u_7=[144, 152]\), \(u_8=[152, 160]\), \(u_9=[160, 168]\), \(u_{10}=[168, 176]\).

The average values of the intervals were calculated to perform the following steps: \(u_{av}^1=100\), \(u_{av}^2=108\), \(u_{av}^3=116\), \(u_{av}^4=124\), \(u_{av}^5=132\), \(u_{av}^6=140\), \(u_{av}^7=148\), \(u_{av}^8=156\), \(u_{av}^9=164\), \(u_{av}^{10}=172\).

**Step 3.** The definition of fuzzy sets \(A_i\) \((i=1\div10)\) on the universal set \(U\). \(A_i\) is determined using the formula: \(A_i=(d_i-1, d_i, d_{i+1}, d_{i+2})\), \(i=1\div10\), where \(d_i–1\) is the lower bound \(u_i\) reduced by the length of the interval, \(d_i\) is the lower bound \(u_i\), \(d_{i+1}\) is the lower bound \(u_{i+1}\), \(d_{i+2}\) is the upper bound \(u_{i+1}\). Moreover, \(A_i\) \((i=1\div10)\) has a linguistic interpretation (from a very small value to a very large value), i.e. are the linguistic terms of the set \(U\). The following fuzzy sets are obtained:

\[
A_1 = [88, 96, 104, 112], \\
A_2 = [96, 104, 112, 120], \\
A_3 = [104, 112, 120, 128], \\
A_4 = [112, 120, 128, 136], \\
A_5 = [120, 128, 136, 144], \\
A_6 = [128, 136, 144, 152], \\
A_7 = [136, 144, 152, 160], \\
A_8 = [144, 152, 160, 168], \\
A_9 = [152, 160, 168, 176], \\
A_{10} = [160, 168, 176, 184].
\]

**Step 4.** Fuzzification of historical data. It is carried out according to the rule: if the historical value falls into the interval \(u_i\), \(i=1\div10\), then it belongs to the fuzzy set \(A_i\). The results of historical data belonging to fuzzy sets are presented in Table 2.

| Year | c/ha | \(A_i\) | Year | c/ha | \(A_i\) |
|------|------|--------|------|------|--------|
| 2005 | 124  | \(A_4\) | 2012 | 136  | \(A_6\) |
| 2006 | 133  | \(A_5\) | 2013 | 147  | \(A_8\) |
| 2007 | 132  | \(A_5\) | 2014 | 153  | \(A_8\) |
| 2008 | 138  | \(A_6\) | 2015 | 164  | \(A_{10}\) |
| 2009 | 144  | \(A_7\) | 2016 | 158  | \(A_9\) |
| 2010 | 100  | \(A_1\) | 2017 | 163  | \(A_9\) |
| 2011 | 150  | \(A_8\) | 2018 | 170  | \(A_{10}\) |

**Step 5.** The formation of fuzzy logical dependencies. To build dependencies (relations), sequential fuzzified data of the table are considered in Table 2.

If the value of the time series \(F(t – 1)\) is fuzzy as \(A_i\) and \(F(t)\) as \(A_k\), then \(A_i\) is associated with \(A_k\). This relationship is denoted as \(A_i\rightarrow A_k\), where \(A_i\) is the current state and \(A_k\) is the following one.

| \(A_4\rightarrow A_5\) | \(A_5\rightarrow A_6\) | \(A_6\rightarrow A_7\) | \(A_7\rightarrow A_1\) | \(A_1\rightarrow A_8\) | \(A_8\rightarrow A_6\) | \(A_9\rightarrow A_9\) | \(A_9\rightarrow A_{10}\) | \(A_{10}\rightarrow A_9\) |
Step 6. Formation of groups of fuzzy logical dependencies. Dependencies with the same left parts are grouped.

Table 4. Fuzzy link groups.

| №  | Fuzzy link | №  | Fuzzy link | №  | Fuzzy link | №  | Fuzzy link |
|----|------------|----|------------|----|------------|----|------------|
| 1. | $A_4 \rightarrow A_5$ | 3. | $A_6 \rightarrow A_7$, $A_8$ | 5. | $A_1 \rightarrow A_8$ | 7. | $A_9 \rightarrow A_9$, $A_{10}$ |
| 2. | $A_5 \rightarrow A_5$, $A_6$ | 4. | $A_7 \rightarrow A_1$ | 6. | $A_8 \rightarrow A_6$, $A_8$, $A_{10}$ |

Step 7. Definition of fuzzy forecasts and their defuzzification. If $F(t-1)=A_j$, then forecasting $F(t)$ is performed using the following heuristic rules.

Rule 1. If the group of heuristic fuzzy logical dependencies (FLD) is empty, i.e. $A_j \rightarrow 0$, then the forecast estimate is defined as $A_j$, i.e. midpoint of the interval $[u_{j-1}, u_j, u_{j+1}, u_{j+2}]$.

Rule 2. If the group of heuristic fuzzy logical dependencies is displayed “one to one”, i.e. $A_j \rightarrow A_p$, then the forecast estimate is found as $A_p$, i.e. defined by the midpoint of the interval $[u_{p-1}, u_p, u_{p+1}, u_{p+2}]$.

Rule 3. If a group of heuristic FLD displays “one to many”, then the predictive estimate is found as

$$\frac{A_{p_1} + A_{p_2} + \ldots + A_{p_k}}{k},$$

i.e. calculated as the midpoint of the interval

$$\left[\frac{u_{p_1-1} + u_{p_2-1} + \ldots + u_{p_k-1}}{k}, \frac{u_{p_1} + \ldots + u_{p_k}}{k}, \frac{u_{p_1+1} + \ldots + u_{p_k+1}}{k}, \frac{u_{p_1+2} + \ldots + u_{p_k+2}}{k}\right]$$

Observed and predicted values are shown in Table 5.

Table 5. Historical and projected potato yields.

| Year | The historical value (c/ha) | Predictive estimate (c/ha) | Year | The historical value (c/ha) | Predictive estimate (c/ha) |
|------|-----------------------------|---------------------------|------|-----------------------------|---------------------------|
| 2005 | 124                         |                           | 2013 | 147                         | 148                       |
| 2006 | 133                         | 132                       | 2014 | 153                         | 148                       |
| 2007 | 132                         | 132                       | 2015 | 164                         | 156                       |
| 2008 | 138                         | 136                       | 2016 | 158                         | 164                       |
| 2009 | 144                         | 148                       | 2017 | 163                         | 164                       |
| 2010 | 100                         | 100                       | 2018 | 170                         | 168                       |
| 2011 | 150                         | 156                       |      |                             |                           |
| 2012 | 136                         | 156                       |      |                             | Forecast for 2019         | 172                       |

To assess the quality of the forecast, the average forecasting error rate (AFER) was used (similar to formula (4)):

$$AFER = \frac{\sum_{i=1}^{n} |(F_i - T_i)|/T_i}{n} \cdot 100\%,$$

(6)

where $F_i$ and $T_i$ are the predicted and real values for the i-th period, n is the number of values of the time series. The lower this estimation, the better the model.
Table 6. Calculation of the average forecasting error.

| Year | Historical data | Forecast value | Average error |
|------|-----------------|----------------|---------------|
| 2006 | 133             | 132            | 0.00752       |
| 2007 | 132             | 132            | 0             |
| 2008 | 138             | 136            | 0.01449       |
| 2009 | 144             | 148            | 0.02778       |
| 2010 | 100             | 100            | 0             |
| 2011 | 150             | 156            | 0.04          |
| 2012 | 136             | 156            | 0.14706       |
| 2013 | 147             | 148            | 0.00680       |
| 2014 | 153             | 148            | 0.03268       |
| 2015 | 164             | 156            | 0.04878       |
| 2016 | 158             | 164            | 0.03797       |
| 2017 | 163             | 164            | 0.00613       |
| 2018 | 170             | 168            | 0.01176       |
| Total|                 |                | 0.38098       |

The average forecasting error rate is 2.9%.

**Forecasting results.** A time series of forecast values were obtained as a result of the study of the fuzzy model. The maximum forecast error was approximately 3%, which is three times lower than in the forecast according to the classical autoregression model. In addition to the above calculations, fuzzy time series constructed with a different number of intervals $u_i$ were analyzed. The calculation results are given in Table 7. The best predictive model was the fuzzy model with the number of intervals equal to 10.

Table 7. Quality rating of different models.

| Rating | Autoregression | $u=7$ | $u=8$ | $u=9$ | $u=10$ |
|--------|----------------|-------|-------|-------|--------|
| AFER   | 9.8%           | 4.8%  | 4.4%  | 3.7%  | 2.9%   |

Additional calculations of the same example using the first model were performed in order to make sure that the fuzzy model (the second model [8]) gives better results compared to the model based on the Mamdani implication.

Let us show the results obtained when dividing the universal set into seven intervals. The algorithm also consists of several steps, but the computational operations are different.

**Step 1.** Universal set $U=[96, 180]$

**Step 2.** Partitioning a universal set into intervals: $u_1=[96, 108], u_2=[108, 120], u_3=[120, 132], u_4=[132, 144], u_5=[144, 156], u_6=[156, 168], u_7=[168, 180]$ with average values:

- $u_1^*=102$, $u_2^*=114$, $u_3^*=126$, $u_4^*=138$, $u_5^*=150$, $u_6^*=162$, $u_7^*=174$.

**Step 3.** Fuzzy sets $A_i$ ($i=1\ldots7$) on the universal set $U$ are determined by the rule: $A_i=0.5/u_{r-1}+1/u_{r}+0.5/u_{r+1}$, $r=1\ldots m$ is the number of intervals (other terms have zero membership function values). For $m=7$:

- $A_1=1/u_1+0.5/u_2+0.5/u_3+\ldots+0/u_6+0/u_7$,
- $A_2=0.5/u_1+1/u_2+0.5/u_3+0/u_4+0/u_5+0/u_6+0/u_7$,
- $A_7=0/u_1+0.5/u_2+\ldots+0/u_3+0.5/u_4+1/u_5$.
Step 4. Fuzzification of historical data. It is carried out in two stages. First, the belonging of historical data to the fuzzy sets \( A_i \) is determined (Table 8). Then we define real data as fuzzy sets with membership degrees determined from the formula (7). The results are shown in Table 9.

\[
\mu_i(x_t) = \frac{1}{1 + k(x_t - u_{av}^i)^2},
\]

where \( x_t \) — the actual value of the time series during a time period \( t \);
\( u_{av}^i \) — the midpoint of interval \( i \);
\( k \) (in the example \( k=0.01 \)) is a positive constant reflecting the membership of \( x_t \) in the interval \( u_i \).

### Table 8. Identification of historical data with fuzzy sets.

| Year | c/ha | \( A_i \) | Year | c/ha | \( A_i \) |
|------|------|----------|------|------|----------|
| 2005 | 124  | \( A_3 \) | 2012 | 136  | \( A_4 \) |
| 2006 | 133  | \( A_4 \) | 2013 | 147  | \( A_5 \) |
| 2007 | 132  | \( A_4 \) | 2014 | 153  | \( A_5 \) |
| 2008 | 138  | \( A_4 \) | 2015 | 164  | \( A_6 \) |
| 2009 | 144  | \( A_5 \) | 2016 | 158  | \( A_6 \) |
| 2010 | 100  | \( A_1 \) | 2017 | 163  | \( A_6 \) |
| 2011 | 150  | \( A_5 \) | 2018 | 170  | \( A_7 \) |

### Table 9. Degrees of belonging of historical data to fuzzy sets.

| Year | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( A_4 \) | \( A_5 \) | \( A_6 \) | \( A_7 \) |
|------|----------|----------|----------|----------|----------|----------|----------|
| 2005 | 0.7      | 0.9      | 1        | 0.8      | 0.6      | 0.4      | 0.3      |
| 2006 | 0.5      | 0.7      | 0.9      | 1        | 0.8      | 0.5      | 0.4      |
| 2007 | 0.5      | 0.7      | 1        | 1        | 0.7      | 0.5      | 0.4      |
| 2008 | 0.4      | 0.6      | 0.9      | 1        | 0.9      | 0.6      | 0.4      |
| 2009 | 0.4      | 0.5      | 0.7      | 1        | 1        | 0.7      | 0.5      |
| 2010 | 1        | 0.8      | 0.6      | 0.4      | 0.3      | 0.2      | 0.1      |
| 2011 | 0.3      | 0.4      | 0.6      | 0.9      | 1        | 0.9      | 0.6      |
| 2012 | 0.5      | 0.7      | 0.9      | 1        | 0.8      | 0.6      | 0.4      |
| 2013 | 0.3      | 0.5      | 0.7      | 0.9      | 1        | 0.8      | 0.6      |
| 2014 | 0.3      | 0.4      | 0.6      | 0.8      | 1        | 0.9      | 0.7      |
| 2015 | 0.2      | 0.3      | 0.4      | 0.6      | 0.8      | 1        | 0.9      |
| 2016 | 0.2      | 0.3      | 0.5      | 0.7      | 0.9      | 1        | 0.8      |
| 2017 | 0.2      | 0.3      | 0.4      | 0.6      | 0.8      | 1        | 0.9      |
| 2018 | 0.2      | 0.2      | 0.3      | 0.5      | 0.7      | 0.9      | 1        |

Step 5. Based on the results of table 8, fuzzy logical relationships are formed: \( A_3 \rightarrow A_4 \); \( A_4 \rightarrow A_4 \); \( A_4 \rightarrow A_1 \); \( A_1 \rightarrow A_5 \); \( A_5 \rightarrow A_4 \); \( A_4 \rightarrow A_5 \); \( A_5 \rightarrow A_5 \); \( A_5 \rightarrow A_6 \); \( A_6 \rightarrow A_6 \); \( A_6 \rightarrow A_7 \).

Step 6. The formation of the resulting relationship \( R \) is the union of the logical relationships obtained in step 5.
Let us denote: \( R_i = A_i^T x A_i; \) \( R_2 = A_i^T x A_i; \) \( R_3 = A_i^T x A_i; \) \( R_4 = A_i^T x A_i; \) \( R_5 = A_i^T x A_i; \) \( R_6 = A_i^T x A_i; \) \( R_7 = A_i^T x A_i. \) When evaluating \( R_j (j=1\div10), \) the following rule is used:

the ratio \( D = B \cdot C \) for arbitrary vectors \( B \) and \( C \) is calculated using the formula: \( d_j = b_i^T x c_j = \min(b_i,c_j). \)

Having calculated all \( R_j \), \( R \) is defined as \( R = \bigcup_{j=1}^{10} R_j. \) We obtain:

\[
R = \begin{bmatrix}
0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 \\
0 & 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \\
0.5 & 0.5 & 0.5 & 1 & 0.5 & 0 & 0 \\
1 & 0.5 & 0.5 & 1 & 0.5 & 0.5 & 0.5 \\
0.5 & 0.5 & 0.5 & 1 & 1 & 0.5 & 0 \\
0 & 0 & 0 & 0.5 & 0.5 & 1 & 1 \\
0 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5
\end{bmatrix}
\]

**Step 7.** Definition of fuzzy forecasts and their defuzzification. Fuzzy forecast is carried out using the rule:

\[ A_i = A_{i-1} \cdot R, \]

where \( A_{i-1} \) – the fuzzy historical value during the period \( t-1, \)

\( A_i \) – the fuzzy forecast value during the period \( t \) in terms of degrees of belonging to these sets, \( \cdot \) operation «max-min».

The results of the operation are given in Table 10.

| Year | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( A_4 \) | \( A_5 \) | \( A_6 \) | \( A_7 \) |
|------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 2006 | 0.8       | 0.5       | 0.5       | 1         | 0.7       | 0.6       | 0.5       |
| 2007 | 1         | 0.5       | 0.5       | 1         | 0.8       | 0.8       | 0.5       |
| 2008 | 1         | 0.5       | 0.5       | 1         | 0.7       | 0.7       | 0.5       |
| 2009 | 1         | 0.5       | 0.5       | 1         | 0.9       | 0         | 0.6       |
| 2010 | 1         | 0         | 0.5       | 1         | 1         | 1         | 0.7       |
| 2011 | 0.5       | 0.5       | 0.5       | 0.6       | 0.5       | 0.5       | 0.3       |
| 2012 | 0.9       | 0.5       | 0.5       | 0.9       | 1         | 1         | 0.9       |
| 2013 | 1         | 0.5       | 0.5       | 0.9       | 1         | 1         | 0.8       |
| 2014 | 0.9       | 0.5       | 0.5       | 0.8       | 1         | 1         | 0.9       |
| 2015 | 0.8       | 0.5       | 0.5       | 0.8       | 1         | 1         | 1         |
| 2016 | 0.6       | 0.5       | 0.5       | 0.6       | 0.8       | 1         | 1         |
| 2017 | 0.7       | 0.5       | 0.5       | 0.7       | 0.9       | 1         | 1         |
| 2018 | 0.6       | 0.5       | 0.5       | 0.6       | 0.8       | 1         | 1         |

For defuzzification (translation of fuzzy forecast values to fuzzy), the following rules apply:

1) If there is only one maximum in the row of the table 10, then the midpoint of the corresponding interval \( a_i \) is taken as the forecast.
2) If there are several consecutive maxima in the line, then the average value of the midpoints of the corresponding intervals is taken as the forecast.
3) In all other cases, the data lines of the table 10 are normalized, and the forecast is carried out as a weighted average of the midpoints of all intervals.

The results of defuzzification are shown in table 11. The forecast (columns 4, 5) estimated using the formula (4) is also given there.
Table 11. Crisp values of forecast values and their assessment.

| Year | Real value, c/ha | Forecast, c/ha | Calculation using the formula (4) | Rating AFER |
|------|-----------------|----------------|----------------------------------|-------------|
| 2006 | 133             | 138            | 0,037594                         |             |
| 2007 | 132             | 130            | 0,015152                         |             |
| 2008 | 138             | 152            | 0,101449                         |             |
| 2009 | 144             | 147            | 0,020833                         |             |
| 2010 | 100             | 81             | 0,19                             |             |
| 2011 | 150             | 140            | 0,066667                         |             |
| 2012 | 136             | 137            | 0,007353                         |             |
| 2013 | 147             | 140            | 0,047619                         |             |
| 2014 | 153             | 156            | 0,019608                         |             |
| 2015 | 164             | 158            | 0,04878                          |             |
| 2016 | 158             | 168            | 0,063291                         |             |
| 2017 | 163             | 168            | 0,030675                         |             |
| 2018 | 170             | 168            | 0,011765                         |             |
| Total|                 |                | 0,660786                         | 0,05083=5%  |

It can be seen from the table that the forecast estimate is higher (5%) than the same estimate for the second model (4.8%) (Table 7), but lower than for the autoregressive model [12].

3. Conclusion
The fuzzy time series can serve as a new tool for modeling and describing the dynamics of agrarian processes operating in conditions of “non-stochastic” uncertainty.

The number of fuzzy sets used to describe FLD significantly influences the accuracy of the forecast. The modification of the initial method proposed in this paper can be recommended for practical use in the short-term forecasting of time series.

Forecasting methods based on fuzzy TS models allow us to give the most adequate estimate for predicting future changes under uncertainty.

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