TESTING MODELS FOR STRUCTURE FORMATION

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Abstract

I review a number of tests of theories for structure formation. Large-scale flows and IRAS galaxies indicate a high density parameter $\Omega \simeq 1$, in accord with inflationary predictions, but it is not clear how this meshes with the uniformly low values obtained from virial analysis on scales $\sim 1$ Mpc. Gravitational distortion of faint galaxies behind clusters allows one to construct maps of the mass surface density, and this should shed some light on the large vs small-scale $\Omega$ discrepancy. Power spectrum analysis reveals too red a spectrum (compared to standard CDM) on scales $\lambda \sim 10 - 100 \, h^{-1}$ Mpc, but the gaussian fluctuation hypothesis appears to be in good shape. These results suggest that the problem for CDM lies not in the very early universe — the inflationary predictions of $\Omega = 1$ and gaussianity both seem to be OK; furthermore, the COBE result severely restricts modifications such as tilting the primordial spectrum — but in the assumed matter content. The power spectrum problem can be solved by invoking a cocktail of mixed dark matter. However, if gravitational lensing fails to reveal extended dark mass around clusters then we may be forced to explore more radical possibilities for the dark matter.

1 Introduction

Current theories for structure formation, as exemplified by the standard cold dark matter model, are based on two quite distinct pieces of physics: First there is the inflationary phase in the very early universe when $H$ is on the order of the Planck or GUT mass, and when it is assumed i) that the universe is dominated by a nearly spatially constant scalar field $\phi_0$ which is rolling slowly down the potential $V(\phi)$ and which drives inflation, thus preparing the universe with $\Omega$ very close to unity and ii) that there are zero-point fluctuations of the field $\delta \phi$ which, at horizon crossing, leave their imprint as a very nearly scale-invariant spectrum of gaussian density fluctuations $\delta \rho/\rho$. The second piece of physics describes the modification of the initially featureless spectrum as the universe passes from radiation to matter domination: $P(k) \rightarrow T^2(k)P(k)$, and where the transfer function $T(k)$ depends critically on the dark matter content.
2 Tests for $\Omega$

2.1 $\Omega$ from large-scale flows

There is a growing consensus that i) there is a positive correlation between large-scale peculiar motions and the gravity inferred from the galaxy distribution (as would be expected under gravitational instability) and ii) that the relative amplitude of the peculiar motions and the (mainly IRAS) galaxy density contrast implies a high density parameter $\Omega \simeq 1$ if these galaxies fairly trace the mass.

The POTENT comparison with IRAS galaxies has been discussed elsewhere in these proceedings. These results are in broad agreement with the $\Omega$ inferred from the QDOT redshift survey \[1\], which in turn reinforced the high $\Omega$ inferred from the angular dipole moments (\[2\] and references therein). While there are relative advantages and disadvantages of all these approaches, there is generally good agreement (though similar efforts using optical galaxies tend to give lower $\Omega$, perhaps implying that the optical galaxies are somewhat positively biased).

The QDOT approach was to estimate the smoothed gravity from the 1-in-6 IRAS 0.6Jy redshift survey, correct this for redshift space distortion effect, and then compare this with the peculiar velocity smoothed on the same scale. This avoids the inhomogeneous Malmquist bias effect which enhances the density contrast in the POTENT maps, but at the price of sacrificing the locality of the POTENT velocity-divergence comparison. We tried to minimise the effect of structures beyond the survey volume by using predicted velocities relative to the local group. A comparison with the motions of a set of clusters with fairly precise distant estimates \[3\] is shown in figure 1.

Figure 1: Comparison of the smoothed velocity predicted from the IRAS QDOT survey (assuming $b = \Omega = 1$) with the observed motions of a set of clusters from Frenk et al., 1993.

This shows (somewhat more convincingly than in our previously published scatter plot) that if IRAS predicts that a cluster should be moving towards or away from us then it will with great probability be found to be moving in the expected sense. For $\Omega = 1$ the points should scatter about a line of slope 45°, which looks about right. The formal error on $\Omega$ in this and other studies is typically 30%. This is of course questionable, but it is clear
at least that a density parameter equal to the baryonic density predicted by big bang nucleosynthesis $\Omega_{\text{BBN}} \simeq 0.015h^{-2}$ would be very hard to reconcile with this result — we would need a strong antibias — so we have evidence here that the dark matter is non-baryonic.

### 2.2 $\Omega$ from lensing

The large $\Omega$ implied from large-scale ($\gtrsim 30h^{-1}\text{Mpc}$) studies stands in contrast to the low mass-to-light ratios $M/L \sim 100 - 300h$ (implying $\Omega \lesssim 0.2$) on scales $\sim 1\text{Mpc}$ inferred from virial analysis of clusters and from the cosmic virial theorem (Davis and Peebles, [5]). This may be partly reflecting a relative bias of IRAS and optical galaxies, but this most probably also reflects a real dependence on scale. An attractive possibility is that the small scale estimates are biased downwards because the galaxies are more concentrated than the dark matter in clusters: i.e. clusters, like galaxies, have dark halos.

We can test this hypothesis with gravitational lensing: Dark mass in the outskirts of clusters will produce small, but coherent, distortions of the background galaxies. The effect is weak at large radii (the distortion falls roughly like the projected surface mass density), and this is where we are most interested in probing the mass. However, the potential sensitivity improves at large angle due to the increase in the number of background galaxies. For a space density profile $\rho \propto 1/r^2$, the signal to noise should be independent of radius. The effect is clearly seen in the inner parts of clusters so a $\rho \propto 1/r^2$ continuation to large scales should also be clearly seen (and steeper profiles could be measured statistically by stacking the results from a number a clusters).

Here’s how it works. One measures quadrupole moments of the faint background galaxies and constructs ellipticity or polarization parameters $e_1, e_2$ which describe stretching of the galaxy images along the axes and diagonals respectively. The galaxy ellipticities provide noisy estimates of a gravitationally induced ellipticity pattern which is just the convolution of the projected mass density field with the pattern from a point mass: $e_i \propto \chi_i(\bar{\theta}) = \{\cos 2\phi, \sin 2\phi\}/|\theta|^2$:

$$e_i = \chi_i \otimes \Sigma$$  \hspace{1cm} (1)

from which we can reconstruct the projected surface density $\Sigma(\bar{\theta})$ (measured in units of the critical density). In fact [6], the optimal inversion is just the
convolution and contraction of the observed ellipticity with the same kernel \( \chi_i \):
\[
\Sigma = -\chi_i \otimes e_i
\]  
(2)

The assumed random, intrinsic ellipticities of the background galaxies provide a source of noise, and this means that one must smooth the dark matter map in some way. A further complication is that the method cannot determine the average surface density so in practice this means that about half of the data are used to set the baseline surface density.

This probe of DM was pioneered by Tyson, Valdes and Wenk \cite{Tyson92} with their study of A1689, though using a different convolving kernel. More recently Ellis and Smail \cite{Ellis94} have obtained interesting results for Cl1455-22 using \cite{Ellis94}; their dark matter map shows, in addition to the main cluster, a second blob, which is also seen as an enhancement in the ROSAT X-ray image. Fahlman, Squires, Woods and myself \cite{Fahlman92} have applied this technique to CFH images of A2218 and MS1224.

No prominent giant arcs are seen in this cluster, yet the mass reconstruction clearly shows the dark mass in the centre of this not exceptionally massive cluster (the central velocity dispersion is 800 km/s). This is the largest field studied so far, with about 1000 background galaxies measurable, and is the first to allow any view of the dark mass beyond about 1/2 Mpc. Unfortunately (for inflation), our preliminary analysis indicates that the DM is, if anything, more concentrated than the galaxies in this cluster.

This is a very promising probe of the DM which will surely take off with the advent of large format CCD mosaics. A failure to detect an increase of M/L with radius would be very interesting, and would point to one of two radical conclusions:
1) \( \Omega \) is really low and nearly all of the large-scale structure studies reported at this meeting are incorrect — heaven forbid!
2) \( \Omega \) is really unity but for some reason the dark mass cannot cluster on scales less than tens of Mpc.
3 Power Spectrum and Gaussianity

At a qualitative level it is a pleasant prediction of models like CDM that the primordial spectrum should break from the primordial $n = 1$ spectrum at the scale of the horizon at $z_{eq}$ to something flatter. Several studies though have shown that at a more quantitative statistical level the break occurs at too large a scale; to make standard CDM match what is seen would require $\Omega h \approx 0.2 - 0.25$ which, with $\Omega = 1$, is an unacceptable Hubble parameter. The first really strong evidence for extra large-scale power came from the APM survey\[10\]. Several groups have now performed power-spectrum analysis on redshift surveys. The spectrum extracted from the QDOT redshift survey by Feldman et al. \[11\] is shown in figure 3.

![Figure 3: Power spectrum of QDOT redshift survey.](image)

This survey, being deep and of $\sim 3\pi$ steradians is particularly well suited to studying $P(k)$ at large scales. A new feature of this analysis is that the statistical uncertainty is calculated analytically under the assumption that the large-scale fluctuations are gaussian, and this allowed us to optimise the weighting scheme. The survey provides a fairly accurate $P(k)$ at wavelengths $\lambda \approx 30 - 150h^{-1}\text{Mpc}$ where linear theory should be accurate, and yet the spectrum clearly has the wrong shape for standard CDM. Figure 3 also shows that as a phenomenological fix, the 70/30 cold/hot mixed dark matter model \[12\] would fit the data quite well.

The strength of large-scale power is $P \approx 10^4(h^{-1}\text{Mpc})^3$ at $k \approx 0.05h/\text{Mpc}$ ($\lambda \approx 120h^{-1}\text{Mpc}$). It it perhaps more meaningful to express this as a contribution to the variance:

$$\langle \sigma^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P(k) = (0.25)^2 \int d\ln k (k/0.05)^3 (P(k)/10^4)$$

So this level of power corresponds to a rms density fluctuation of about 25%.

It is also interesting to relate this power to other probes of large-scale structure: peculiar velocity; microwave anisotropy and coherent gravitational distortions. A quantitative prediction for the rms signal in a given experiment requires one to calculate the ‘window function’ which determines the sensitivity as a function of spatial wavelength. This is usually straightforward but is experiment dependent. One can get a good idea of how much signal might be detected by estimating the contribution per log interval of
wave number as in (3). Assuming \( b_{\text{IRAS}} \simeq 1 \) and \( \Omega = 1 \) the observed power would drive streaming motions of rms amplitude \( \simeq 500 \text{ km/s} \):

\[
\langle v^2 \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{k^2} P(k) = (500 \text{ km s}^{-1})^2 \int d\ln k (k/0.05)(P(k)/10^4)
\]

so there should be substantial motions even on these rather large scales.

Relating \( P(k) \) to the microwave anisotropy is more model dependent. Naively applying the Sachs-Wolfe result gives an rms anisotropy of \( \delta T/T \simeq 5 \times 10^{-6} \):

\[
\langle (\Delta T/T)^2 \rangle = \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \frac{H^4}{k^4} P(k) = (5 \times 10^{-6})^2 \int d\ln k (k/0.05)^{-1}(P(k)/10^4)
\]

The Sachs Wolfe result is not, however, directly applicable on these scales, but what this means is that extrapolating to the \( \sim 1000h^{-1}\text{Mpc} \) scales probed by COBE with a scale-invariant \( P(k) \propto k \) would give roughly what is observed.

Finally, these density fluctuations would cause distortions of distant galaxies coherent over degree scales at about the 1% level:

\[
\langle e^2 \rangle = \frac{12\pi w_s^3}{5} \int \frac{d^3k}{(2\pi)^3} \frac{H}{k} P(k) = (9 \times 10^{-3})^2(w_s/0.3)^3 \int d\ln k (k/0.05)^2(P(k)/10^4)
\]

with \( w_s \equiv 1 - (1 + z_s)^{-1/2} \). Hudon et al. [13] have obtained limits at about this level, and the prospects for improving sensitivity here are good.

It is apparent from the foregoing equations that the power spectrum provides a very convenient vehicle for comparison with other probes of large-scale structure. The comparison can be made precise simply by inserting the appropriate window function (or rather it’s fourier transform squared) in the above equations. A further important advantage of power spectrum over correlation function is that the former effectively diagonalises the error matrix and this greatly facilitates the calculations of likelihood ratios for competing theories, which is after all surely the ultimate goal of these studies.

We have also used the power spectrum to test the gaussian hypothesis. While the mean value of the power is void of information regarding gaussianity, the fluctuations about the mean can reveal some traces of primordial non-gaussianity. We have estimated the 1-point distribution of the power. According to the gaussian hypothesis this should be exponential, and the observed distribution is in very good agreement with this.
Figure 4: 1-point distribution of the power from the IRAS-QDOT redshift survey

We have also looked at the two point function of the power spectrum: \( \chi(\delta k; k) = \langle P(\vec{k})P(\vec{k} + \delta k) \rangle \). Under the gaussian hypothesis this is a bell-shaped function localised around \( \delta k = 0 \) with width \( \sim 1/D \), the inverse effective depth of the survey. An excess width to the two-point function can arise in models where one has one gaussian random field with strong low-spatial frequency power amplitude modulating a second independent gaussian field with strong high frequency fluctuations; crudely speaking \( \chi(\delta k; k) \) measures the extent to which modes of wavenumber \( k \) are being modulated by modes of wavenumber \( \delta k \). Such a model has been suggested by Peebles [14] as phenomenologically attractive, and this behaviour can arise in certain inflationary models (Bardeen, personal communication). Again, in the data, we see no evidence for non-gaussian behaviour.

4 Alternatives to Collisionless Cold Dark Matter?

One cannot help but be impressed by how close the standard CDM model comes to explaining the observed structure; the halos produced can arguably account nicely for galaxies and clusters, the texture of large scale structure with its sheets and filamentary appearance is a very good match at the visual level to observed galaxy clustering. Moreover, the detection of MBR anisotropy by COBE at about the predicted level provides very strong support to the general idea that present day structure can indeed be traced back to the very early universe rather than being the outcome of astrophysical processes. The problems with the model only appear at a rather detailed statistical level of comparison. One problem stands out very clearly; the shape of the power spectrum at large scales. To this we should add the Hubble constant problem and — though these seem less convincing as yet — the baryon content of clusters and the large-scale vs small-scale \( \Omega \) discrepancy.

The qualitative success of the CDM model suggest that what is needed is tinkering rather than a complete rethink. By this I mean that one should consider modifications to either the very early universe aspects or the medieval universe, but probably not both. As the data seem quite compatible with the inflationary expectations as regards \( \Omega \) and gaussianity, and since
the possibility to modify the model by e.g. tilting the initial spectrum is now highly constrained by COBE, it would seem more promising to explore alternatives to the simple, if aesthetically pleasing, assumption that the dark matter is cold and collisionless.

One simple fix for the $P(k)$ problem is to invoke a cocktail of hot and cold dark matter, and a 70/30 cold/hot mix \[12\] gives an acceptable power spectrum. However, having to invoke two different types of dark matter is clearly a demerit point for this theory as compared to standard CDM or HDM. A significant development in this regard is the proposal of Madsen \[15\], who argued that (non-radiative) decays of a massive neutrino while still relativistic could naturally produce bosonic decay products with a substantial fraction of the particles in a bose condensate and that this could therefore create mixed dark matter with the cold and hot particles being one and the same species. Stimulated by this, but not completely convinced by Madsen’s arguments which involved some questionable assumptions, Malaney, Starkman and myself \[16\] have formulated and solved the Boltzmann equations for such a decay process. We have found that a bimodal momentum distribution for the bosonic decay products can indeed result, though by a rather different process than the thermalisation envisaged by Madsen. What we found was that under certain conditions there is a runaway process of decays to very low momentum bosons or ‘neutrino lasing’. This is quite different from the more often considered case of non-relativistic decays where the decay products are extremely hot. For relativistic decays, there is the possibility to produce rather cold products if these are emitted in the opposite direction to the motion of the decaying particle. As the phase space at low momenta is small, the occupation number rapidly becomes large. This stimulates more low momentum decays and very soon essentially all of the decays are to very cold bosons.

The lasing happens when the decaying particle is relativistic. This is followed by a second phase of decays when the massive neutrino goes non-relativistic, and these result in hot bosons. The result, if the boson has a mass of a few tens of eV, is mixed dark matter. The details of the model predictions depend on such features as whether the boson is its own antiparticle. The specific model considered in \[16\], for instance, produced a cold-fraction of 37-50%, rather less than the preferred 70% fraction. However, the hot particles in this model came out to be somewhat cooler than in the standard ad hoc. model \[12\], so this may well still be viable. If not, then there are other more or less plausible ways to increase the cold fraction within this scheme.
Mixing collisionless dark matter is only one possibility. Another interesting avenue is to drop the assumption that the dark matter is collisionless. After all, we only know that the dark matter does not interact very much with photons and baryons; whether the dark matter is self-interacting is unknown. Carlson, Machacek and Hall have explored the possibility that the DM self-interacts and that number changing interactions are effective. This yields the interesting result that after becoming non-relativistic the dark matter cools more slowly than would be the case for collisionless matter. This might seem an attractive way to solve the “Ω-discrepancy” (and the closely related problem of the baryonic fraction in clusters) as keeping the dark matter warm would effectively stop it clustering on small scales. However, it seems to be rather difficult in this scheme to arrange matters so that the matter can cluster on tens of Mpc scales. A second problem with this scenario is that there is strong suppression of the growth factor on small scales, so it may be difficult to make galaxies. Another possibility is to assume that the dark matter is extremely light (e.g. with a mass on the order of $10^{-25}$eV. Such fields have been widely considered in the context of late-time phase transitions, and have the very interesting property that the particles are forbidden from clustering on small scales because of the wave-mechanical uncertainty principle — the particles cannot be bound within a structure with velocity dispersion $\sigma$ if the de Broglie wavelength $\sim \hbar/m\sigma$ is smaller than the size of the system. There is also the possibility that the dark matter is collisional, but that number changing reactions are ineffective; if so the difference in clustering properties from the collisionless case would be more subtle, but potentially interesting nonetheless.

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