Estimations of Radiative Heat Transfers in Enclosures

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Abstract. In large closed spaces of buildings, the heat radiation dominates over the other two transfer mechanisms, i.e. over conduction and convection. This fact may be easily illustrated on the basis of the Stefan-Boltzmann law, Fourier's law and convective correlations of Nusselt’s number. For this reason, when the computations of heat losses of buildings are performed, the radiative heat transfers running between inner sources of heat and inner surfaces of walls should not be overlooked. This conference contribution is devoted to such problems.

In the preceding two conference contributions called “Radiative Heat Transfer in Buildings” and “Radiosity Model and Compensation Theorem”, a general model for radiative heat transfer in inner spaces of buildings has been developed and completed by the compensation theorem. This theorem mirrors the basic property of closed building envelopes stating that the total radiative heat flow in a closed envelope is zero. The present conference contribution thematically continues the preceding two contributions and applies their theoretical results to the case of radiative heat flows established within a living room consisting of 6 different grey interior surfaces. The surfaces mutually differ in temperatures and emissivities. Such an arrangement represents a real room without too much simplification. The corresponding matrix of view factors has 36 elements determined on the basis of formulae and graphs published in the technical literature. The system of 6 equations determines the 6 values of radiosities belonging to 6 interior surfaces. The radiosities represent auxiliary values that facilitate finding the heat fluxes and heat flows associated with each inner surface of the room. The sum of all the obtained positive and negative values of the heat flows has resulted in zero value which documents validity of the compensation theorem also in this complicated enclosure consisting of 6 thermodynamically different surfaces.

1. Introduction

In the preceding two conference contributions called “Radiative Heat Transfer in Buildings” [1] and “Radiosity Model and Compensation Theorem” [2], a general model for radiative heat transfer in inner spaces of buildings has been developed and completed by the compensation theorem. This theorem mirrors the basic property of closed building envelopes stating that the total radiative heat flow in a closed envelope is zero. The present conference contribution thematically continues the preceding two
contributions and applies their theoretical results to the case of radiative heat flows established within a living room consisting of 6 different grey interior surfaces. The surfaces mutually differ in temperatures and emissivities. Such an arrangement represents a real room without too much simplification.

In the preceding conference contribution [1] the following general equations for radiative heat fluxes $q_i$ and flows $\Phi_i$ have been derived by means of radiosities $W_i$ and view factors $F_{ij}$

$$W_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^{N} F_{ij} W_j \quad \text{(Watt/m}^2\text{)}$$

$$q_i = W_i - \sum_{j=1}^{N} F_{ij} W_j \quad \text{(Watt/m}^2\text{)}$$

$$\Phi_i = S_i q_i \quad \text{(Watt)}$$

where $\varepsilon_i$ and $\rho_i$ are emissivity and reflectivity of surfaces with areas $S_i$. The symbol $E_{bi}$ stands for the Stefan-Boltzmann term $E_{bi} = 5.67 \cdot 10^{-8} \cdot T^4$ (Watt/m$^2$). These general Equations are valid for both the black and grey surfaces as well as for their combinations. It has been shown [1] that the system of Equations (1) – (2) may be reduced to a simpler version (4) – (6) valid only for grey surfaces (compare complicated Equation (2) with simplified Equation (5) specifying $q_i$):

$$W_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^{N} F_{ij} W_j \quad \text{(Watt/m}^2\text{)}$$

$$q_i = \frac{\varepsilon_i}{\rho_i} (E_{bi} - W_i) \quad \text{(Watt/m}^2\text{)}$$

$$\Phi_i = S_i q_i \quad \text{(Watt)}$$

In [1], we have also formulated the compensation theorem that ascribes zero value to the total radiative heat flow $\sum_{i=1}^{n} \Phi_i$ in enclosures (e.g. in closed envelopes of rooms or buildings)

$$\sum_{i=1}^{n} \Phi_i = \sum_{j=1}^{n} \left( S_j W_j \cdot \left[ 1 - \sum_{i=1}^{n} F_{ij} \right] \right) = \begin{cases} = 0 & \text{(for closed envelopes only)} \\ \neq 0 & \text{(for open envelopes only)} \end{cases}$$

There is no doubt that heat and moisture transports within building structures [3-5] are the topics that are often discussed and analyzed by building technologists. Among heat transport processes, it is the radiative heat transport [6 – 8] that represents a dominant thermal transport especially in larger enclosures like room or building interiors.

In this contribution, we analyze the behavior of radiative heat flows in the envelope of a real room (see Figure 1) consisting of 6 main interior surfaces (heated floor, cool side walls and cool ceiling).
These planar surfaces possess not only different temperatures but also different emissivities, i.e. we will analyze radiative heat transfers caused by surfaces whose thermodynamical parameters are mutually independent (see Table 1). This will lead to a view matrix consisting of 36 elements and the system of 6 algebraic equations for radiosities. As a result, 6 radiative heat flows will be determined and discussed from the viewpoint of the compensation theorem.

![Figure 1. Six-surface room.](image)

| Parameter | Floor no.1 (heated) | Wall no.2 (cool) | Wall no.3 (cool) | Wall no.4 (cool) | Wall no.5 (cool) | Ceiling no.6 (cool) |
|-----------|---------------------|-----------------|-----------------|-----------------|-----------------|-------------------|
| $\varepsilon$ | 0.95               | 0.75            | 0.89            | 0.81            | 0.72            | 0.80              |
| $\rho$    | 0.05               | 0.25            | 0.11            | 0.19            | 0.28            | 0.20              |
| $S$ (m$^2$) | 48                 | 24              | 18              | 24              | 18              | 48                |
| $T$ (K)   | (35 °C)            | (17 °C)         | (16 °C)         | (18 °C)         | (19 °C)         | (15 °C)           |

$$\varepsilon E_{\bar{\sigma}}^{\frac{T}{100}} = 484.740750 \quad 300.771245 \quad 352.017650 \quad 329.336627 \quad 296.788426 \quad 312.063516$$

2. Matrix of view factors
Figure 1 shows the model of the six-surface room. The numbering of surfaces is as follows: the heated floor no. 1, the side walls nos. 2,3,4, 5, and the ceiling no. 6. Input data can be found in Table 1. The whole computational procedure has been explained in the previous conference contributions [1] and [2].
Some of the view factors $F_{ij}$ have been calculated from published formulae [6-8], e.g. $F_{16}$, $F_{24}$, $F_{35}$, $F_{13}$, $F_{12}$, and $F_{23}$ whereas others have been deduced on the basis of the symmetry relation $S_i F_{ij} = S_j F_{ji}$ or regular rectangular symmetry of the room geometry:

$F_{ij} = 0$ (perfect planes), $F_{16} = F_{61}$, $F_{24} = F_{42}$, $F_{35} = F_{53}$, $F_{41} = F_{14}$, $F_{51} = F_{15}$, $F_{56} = F_{65}$, $F_{23} = F_{54}$.

As the consequence of this symmetry, many values of the view factors emerge repeatedly in the view matrix $\tilde{F}$:

$$
\tilde{F} = \begin{pmatrix}
0 & 0.155992 & 0.114183 & 0.155992 & 0.114183 & 0.155992 \\
0.311984 & 0 & 0.119003 & 0.138026 & 0.119003 & 0.311984 \\
0.304488 & 0.158671 & 0 & 0.158671 & 0.073685 & 0.304488 \\
0.311984 & 0.138026 & 0.119003 & 0 & 0.119003 & 0.311984 \\
0.304488 & 0.158671 & 0.073685 & 0.158671 & 0 & 0.304488 \\
0.459651 & 0.155992 & 0.114183 & 0.155992 & 0.114183 & 0
\end{pmatrix}
$$

## 3. Radiosities

Radiosities $W_i$ of gray surfaces are computed by using the system of equations specified by Relation (4):

$$
W_i = \varepsilon_i E_0 + \rho_i \sum_{j=1}^{6} F_{ij} W_j, \quad i = 1, 2, 3, ..., 6
$$

$$
\begin{align*}
W_1 &= 484.740750 + 0.05 \cdot (0 \cdot W_1 + 0.155992 \cdot W_2 + 0.114183 \cdot W_3 + 0.155992 \cdot W_4 + 0.114183 \cdot W_5 + 0.459651 \cdot W_6) \\
W_2 &= 300.771245 + 0.25 \cdot (0.311984 \cdot W_1 + 0 \cdot W_2 + 0.119003 \cdot W_3 + 0.138026 \cdot W_4 + 0.119003 \cdot W_5 + 0.311984 \cdot W_6) \\
W_3 &= 352.017650 + 0.11 \cdot (0.304488 \cdot W_1 + 0.158671 \cdot W_2 + 0 \cdot W_3 + 0.158671 \cdot W_4 + 0.073685 \cdot W_5 + 0.304488 \cdot W_6) \\
W_4 &= 329.336627 + 0.19 \cdot (0.311984 \cdot W_1 + 0.138026 \cdot W_2 + 0 + W_3 + 0.119003 \cdot W_5 + 0.311984 \cdot W_6) \\
W_5 &= 296.788426 + 0.28 \cdot (0.304488 \cdot W_1 + 0.158671 \cdot W_2 + 0.073685 \cdot W_3 + 0.158671 \cdot W_4 + 0 \cdot W_5 + 0.304488 \cdot W_6) \\
W_6 &= 312.063516 + 0.20 \cdot (0.459651 \cdot W_1 + 0.155992 \cdot W_2 + 0.114183 \cdot W_3 + 0.155992 \cdot W_4 + 0.114183 \cdot W_5 + 0 \cdot W_6)
\end{align*}
$$

After rearranging the above system of equations, the normal form of the system of linear algebraic equations appears:
By solving this system of equations, the following results can be determined:

\[ W_1 = 505.0927335 \text{ Watt/m}^2, \quad W_2 = 410.1917622 \text{ Watt/m}^2, \quad W_3 = 400.18279915 \text{ Watt/m}^2, \]
\[ W_4 = 412.4373302 \text{ Watt/m}^2, \quad W_5 = 419.002342788 \text{ Watt/m}^2, \quad W_6 = 402.86890699 \text{ Watt/m}^2. \]

4. Radiative heat flows and the compensation theorem

By using Equations (5) and (6), the radiative heat flows \( \Phi_i \) may be specified:

\[ q_i = S_i \frac{E_i}{\rho_i} (E_{wi} - W_i), \quad i = 1, 2, 3, ..., 6 \]  

\[ \Phi_1 = +4706.547048 \text{ Watt}, \quad \Phi_2 = -659.7673584 \text{ Watt}, \quad \Phi_3 = -678.2794689 \text{ Watt}, \]
\[ \Phi_4 = -598.4350057 \text{ Watt}, \quad \Phi_5 = -314.5667621 \text{ Watt}, \quad \Phi_6 = -2455.586285 \text{ Watt}. \]

From results (13), it can be seen that the heated floor emits power \( \Phi_1 = +4706.547048 \text{ Watt} \) into the room whereas all other surfaces take powers from the room. Surprisingly, the total sum of all the radiative heat flows is not accurately zero, i.e. \( \sum_{i=1}^{6} \Phi_i \) as the compensation theorem would require. One must ask for the reason of this discrepancy. The explanation is easy. Some of the sums of row elements in the view matrix \( \sum_{i=1}^{6} F_{ji} \) do not exactly equal one and this contradicts the concept of view factors related to enclosures [2]. According to the compensation theorem (7), the numerical imbalance of the view factors may be the reason for such a discrepancy. This may be verified by performing summation (7) prescribed by the compensation theorem for closed envelopes:

\[ \sum_{i=1}^{n} \Phi_i = \sum_{j=1}^{n} \left( S_j W_j \right) \left[ 1 - \sum_{i=1}^{n} F_{ji} \right] = 48 \cdot 505.0927335 \cdot [1-1.000001] + 24 \cdot 410.1917622 \cdot [1-1] +
+18 \cdot 400.18279915 \cdot [1-1.000003] + 24 \cdot 412.4373302 \cdot [1-1] + 18 \cdot 419.002342788 \cdot [1-1.000003] +
+48 \cdot 402.86890699 \cdot [1-1.000001] = -0.0878 \text{ Watt} \]

As can be seen, the compensation theorem has confirmed our assumption that the small discrepancy (-0.0878 Watt) is caused by tiny numerical imbalances of the elements in the view matrix (8).
5. Conclusions
In this paper, the radiative heat flows established in a living room have been computed and discussed. The room has been surrounded by 6 grey interior surfaces. The surfaces have differed in temperatures and emissivities. The corresponding matrix of view factors has contained 36 elements determined on the basis of formulae published in the technical literature. The system of 6 equations has determined the 6 values of radiosities belonging to 6 interior surfaces. The radiosities have represented auxiliary values that have facilitated finding the heat fluxes and heat flows associated with each inner surface of the room. The sum of all the obtained positive and negative values of the heat flows has not resulted in exact zero value but a small discrepancy has occurred. The discrepancy has been explained on the basis of tiny numerical imbalances of the elements in the view matrix. The verification of the origin of the discrepancy has been performed by means of the computations based on the compensation theorem.

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