Revisiting Bohr’s principle of complementarity using a quantum device

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Bohr’s principle of complementarity lies at the central place of quantum mechanics, according to which the light is chosen to behave as a wave or particles, depending on some exclusive detecting devices. Later, intermediate cases are found, but the total information of the wave-like and particle-like behaviors are limited by some inequalities. One of them is Englert-Greenberger (EG) duality relation. This relation has been demonstrated by many experiments with the classical detecting devices. Here by introducing a quantum detecting device into the experiment, we find the limit of the duality relation is exceeded due to the interference between the photon’s wave and particle properties. However, our further results show that this experiment still obey a generalized EG duality relation. The introducing of the quantum device causes the new phenomenon, provides a generalization of the complementarity principle, and opens new insights into our understanding of quantum mechanics.

Bohr’s principle of complementarity (BPC) has been the cornerstone of quantum theory since it was proposed in 1928 [1, 2]. This principle states that some physical objects have multiple properties, but these properties are exhibited depending on some types of exclusive detecting devices. One well-known example is the wave-particle duality, by considering a single particle in a two-way interferometer [3]. One can choose to observe the wave-like or particle-like behaviors of the particle by using different detection arrangements. Interference fringes have been observed for massive particles such as neutrons [4], electrons [5], atoms [6, 7] and molecules [8], all thought to be only particle-like before. These observations show the unfamiliar wave-like side of these particles. In the case of light, both the anti-bunching effect and its interference fringes—associated with its particle-like and wave-like properties respective—have been previously demonstrated [9–11].

Besides these all-or-nothing situations, there actually exists some intermediate stages [12–16], where the which-path knowledge corresponding to the particle-like property is partially detected, resulting in the reduced interference visibility. This issue was first discussed by Wooters and Zurek in 1979 [12]. Later, an inequality was experimentally shown by Greenberger and Yasin in some unbalanced neutron interferometry experiments [17, 18], and theoretically derived by Jaeger et al. [19] and Englert [20, 21] independently. This inequality is written as

\[ V^2 + D^2 \leq 1, \]

where \( V \) is the visibility of the interference fringes, and \( D \) is the path distinguishability of the particle, which stands for the available quantity of which-path knowledge from the system. This inequality is also known as the EG duality relation. Plenty of experiments have demonstrated this inequality with atoms [22], nuclear magnetic resonance [23, 24], faint laser [25], and also single photons in a delayed-choice scheme [26]. Recently, this duality relation has been extended to the more general case of an asymmetric interferometer where only a single output port is considered, and this inequality still holds [27].

One of the most efficient quantum systems for testing BPC is the single photons in a Mach-Zehnder interferometer (MZI). In Ref. [26], a series of unbalanced beam splitters (BS) were randomly chosen in the MZI, including the extreme cases with reflection coefficients of \( R = 0 \) and 0.5. However, we notice that beam splitters of this type are all classical devices. Mapping to the quantum BS (q-BS) scheme recently proposed by Ionicioiu and Terno [28, 29], the same results will come out when the q-BS is selected to collapse on a set of eigenstates. These eigenstates of the q-BS can be the same as the previously-mentioned classical devices.

In our experiment, the q-BS stays at the quantum superposition states of the extreme eigenstates—notated as |a⟩ (\( R = 0 \)) and |p⟩ (\( R = 0.5 \))—corresponding to the absence and presence of a balanced BS, respectively. We introduce this q-BS into the MZI, and not only the eigenstates but also the quantum-superposition states of the q-BS are selected as the bases to collapse on at detection. The particles are single photons emitted from an InAs/GaAs self-assembled quantum dot [30, 31]. Our result shows that the EG duality relation is exceeded when some certain detecting basis of the q-BS is chosen. This exceeding is caused by the interference between the wave and particle properties of the photons. In order to derive a generalized EG duality relation, we consider both of the two orthogonal detecting bases, then we find the generalized EG duality relation holds for our results.

The experimental setup is sketched in Fig. 1(a). The single photons are split by a 50:50 BS into two paths, followed by a \( \varphi \) phase, then combined by a q-BS. The use of the q-BS is the main difference between this setup and a regular MZI. The photons are detected by the single-photon avalanche photodiodes (APD).

As discussed in Ref. [27], we need to derive the photon
The q-BS state is then collapsed on an arbitrary basis by measuring the detecting basis of the q-BS. The polarizer with a $\beta$ oriented axis selects the detecting basis of the q-BS.

state after the q-BS and know the probabilities of each path taken by the photon, in order to calculate the visibility. The state of the q-BS is $|\text{qbs}\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |p\rangle)$; hence we derive the photon state (before the q-BS state is detected) as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\text{particle}\rangle|a\rangle + \frac{1}{\sqrt{2}}|\text{wave}\rangle|p\rangle$$

according to Ionicioiu and Terno [28], with $|\text{particle}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\phi}|2\rangle)$ corresponding to the particle state, and $|\text{wave}\rangle = e^{i\frac{\pi}{4}}(\cos\frac{\phi}{2}|1\rangle - i\sin\frac{\phi}{2}|2\rangle) e^{i\delta_1}$ corresponding to the wave state. $\delta_1$ and $\delta_2$ are two additional constant phases, which can be adjusted in the experiment. The q-BS state is then collapsed on an arbitrary basis $|b\rangle = \sin\beta|a\rangle + \cos\beta|p\rangle$, which means the photon state becomes $\rho = \tilde{\rho}/Tr(\tilde{\rho})$, where $\tilde{\rho} = Tr_{q-BS}(P_b|\psi\rangle\langle\psi|)$ with $P_b = |b\rangle\langle b|$ as the projection operator. Here we derive the probability that the photon takes Path 2 as $p_2(\varphi) = Tr(|2\rangle\langle 2|\rho)$. From this probability, we have the visibility of Path 2,

$$V = \frac{p_{\text{max}} - p_{\text{min}}}{p_{\text{max}} + p_{\text{min}}}$$

As shown in Fig. 1(a), each photon has four possible sub-paths to reach the APDs from the first BS (P11, P12, P21, P22). The photons that finally appear on Path 2 can come from either P12 or P22, each of which represents a totally different which-path knowledge. Assuming that the probabilities of the photons coming from P12 and P22 are respectively $w_{12}$ and $w_{22}$, the distinguishability of Path 2 can be written as

$$D = |w_{12} - w_{22}|.$$
Similarly, the q-BS state is detected on the other eigenstate of the closed MZI. Therefore, the photons behave as a wave, and the visibility (shown in Fig. 3(a)) of the interference fringe reaches $0.961 \pm 0.004$. This result coincides with the classical-BS-experiment result found in Ref. [26]. Fig. 2(c) corresponds to the $\beta = \frac{\pi}{4}$ case. Similarly, the q-BS state is detected on the other eigenstate $|b⟩ = |p⟩$, which is associated with the open MZI. Thus, the photons behave as particles. The result is also the same as in the classical BS case. However, $\beta = \frac{3\pi}{16}$ for Fig. 2(b), so the detecting basis here is a quantum-superposition state, which is related to the MZI staying in both a closed and an opened state. The visibility in this case is $0.707 \pm 0.017$. The photons behave as a quantum superposition of wave and particle, which is well illustrated by the expression describing the photons’ state $\rho$, i.e., $C_1(\sin\beta|\text{particle}\rangle + \cos\beta|\text{wave}\rangle)$ (where $C_1$ is a coefficient). This phenomenon does not have a counterpart in the classical BS experiment. The differences between the experimental and theoretical values are caused by the counting statistics, the imperfection of the optical glasses, the dark and background counts, and the tiny instability of the MZIs.

To measure the distinguishability $D$, we first block Path 1 after the BS in Fig. 1(a) and detect the number of photons coming from P22 ($N_{22}$), then block Path 2, and detect the photon number from P12 ($N_{12}$). Hence, the distinguishability of Path 2 can be calculated using $D = \frac{N_{12} - N_{22}}{N_{12} + N_{22}}$ according to Eq. (4). The result is shown in Fig. 3(b) with larger dots, and the smaller-dot line is the theoretical simulation. When $\beta = 0$ (the closed MZI), then $D = 0.045 \pm 0.024$ and no which-path knowledge is available. However, when $\beta = \frac{\pi}{4}$ (the open MZI), then $D = 0.97751 \pm 0.0038$ and full which-path knowledge is detected. This result is in accord with the wave-like and particle-like behavior of the photons previously discussed. For these all-or-nothing cases, the q-BS collapses on the eigenstates, which means these situations give the same results as the classical BS experiment; the inequality $V < 1$ holds, and the upper bound is reached (See in Fig. 3(c)). On the other hand, in the quantum intermediate case of $\beta = \frac{3\pi}{16}$, the value of $V^2 + D^2$ goes beyond the limit of the EG duality relation (1, the blue dashed line in Fig. 3(c)) by 10 deviations to reach $1.428 \pm 0.043$. This result coincides with the results from the theoretical simulation.

This exceeding of the EG duality relation is caused by the quantum superposition of the photons’ wave and particle states—or the interference between them—introduced by the q-BS and a quantum intermediate detecting basis. To illustrate this point and derive a generalized EG duality relation, we combine the corresponding photon counts of the two orthogonal bases related to $\beta$ and $\beta + \frac{\pi}{2}$, then calculate $V^2 + D^2$ in the same way. The forms of $V_g$ and $D_g$ are the same as $V$ and $D$, respectively. However, the photon counts and the meanings are different. The former ones correspond to the sum of the counts of two orthogonal bases, and describe the behavior of photons in these two cases as a whole; the wave-particle interference becomes an internal effect here. On the other hand, the latter ones describe the behavior of photons in a single basis case. We find that the generalized inequality ($V^2 + D^2 \leq 1$) holds for our results, shown in Fig. 4(a). The solid line is the theoretical simulation. To further analyze this combination process, we calculate the final state of the photon after the combination, found to be $C_2(\sin^2\alpha|\text{particle}\rangle|\text{particle}\rangle + \cos^2\alpha|\text{wave}\rangle|\text{wave}\rangle)$, where $C_2$ is a coefficient. This state is a classical mixture of the wave and particle properties, and is independent of the chosen orthogonal basis pair (defined by $\beta$). However, the state is related to the parameter $\alpha$, which determines the state of the q-BS and also the probabilities of the photon going through the closed or open MZIs. $V^2 + D^2$
our experiment as a whole is in accord with quantum

Even though our results exceed the EG duality relation,

ment is completely unrelated to the Afshar experiment.

debate continues. We must note here that our experi-

others disagree with this interpretation [34–37], and the

coherence processes, the imperfection of optical glasses

may caused by the dark and background counts, the de-

FIG. 4: $V_\alpha^2 + D_\beta^2$ after combination of the photon numbers of two orthogonal-basis cases with (a) varying $\beta$ and fixed $\alpha = \frac{\pi}{4}$ and (b) varying $\alpha$ and arbitrary $\beta$. The generalized EG duality relation holds for these results.

is calculated to be $\sin^4 \alpha + \cos^4 \alpha$, which is not larger than 1; when $\alpha = \frac{\pi}{4}$, then $V_\alpha^2 + D_\beta^2 = 0.5$. We have also measured $V_\alpha^2 + D_\beta^2$ using various values of $\alpha$, with the result shown in Fig. 4(b), which further proves our previous discussions. There is a systematic error in Fig. 4, which may caused by the dark and background counts, the de-

Actually, the violation of BPC—and specifically the EG
duality relation—has been declared by Afshar et al. [33],
who believe that quantum mechanics is not correct, but
others disagree with this interpretation [34–37], and the
debate continues. We must note here that our experi-

molecules.

2. Arndt, M. et al. Wave–particle duality of C60

3. Feynman, R. P., Leighton, R. B. & Sands, M. L. Lectures

4. Summhammer, J., Badurek, G., Rauch, H., Kischko, U.

5. Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T. &

6. Carnal, O. & Mlynek, J. Young’s Double-Slit Experiment

7. Keith, D. W., Ekstrom, C. R., Turchette, Q. A. &

theory and is only a small extension of BPC, i.e., the

classical detecting devices are replaced with the quantum
devices for our experiment. In the original BPC, the de-

tecting devices can only be in the classical states, which

are each related to the properties that can be shown. In

contrast, the detecting devices can exist in the quantum-

superposition states in our extension by using the quan-
tum control [28]. This small change makes the originally

exclusive properties of the object appears to be quantum-

superposed, allowing for the limit of the EG relation du-

ality to be exceeded.

Besides experiments in wave-particle duality, there

are many other well known experiments whose re-
sults form the foundation of quantum mechanics: the

Bell-inequality experiments [28–40], the Kochen-Specker-

inequality experiments [11–13], and so on. The new con-

cept of using a quantum device could also be introduced

into these experiments, potentially allowing new phenom-

ena to appear, which could further our understanding of

quantum mechanics.

In conclusion, we introduce a q-BS, proposed in Ref.

[28], into the unbalanced MZI used in Ref. [26], selecting

some quantum-superposition states of the q-BS as the

collapsing bases to detect the q-BS’s states. Following

the definitions of visibility and distinguishability used in

Ref. [27], we find the limit of the EG duality relation is

exceeded. We conclude that this result is caused by the

interference between the wave and particle properties of

the photons. After we combine the corresponding photon

numbers of two mutually orthogonal collapsing bases of

q-BS, the wave-particle interference becomes an internal
effect, then the generalized EG duality relation holds.

This work is entirely within standard quantum theory,

but opens up a new way for people to understand the

quantum world by replacing the classical devices with

quantum ones.

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