Photoproduction of $\eta'$ mesons off nuclei and their properties in the nuclear medium

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Abstract

We study the photoproduction of $\eta'$ mesons from nuclei near the threshold within the collision model based on the nuclear spectral function. The model takes properly into account both primary photon–nucleon $\gamma N \rightarrow \eta'N$ and secondary pion–nucleon $\pi N \rightarrow \eta'N$ production processes as well as the effect of the nuclear $\eta'$ mean-field potential on these processes. We find that the secondary channel $\pi N \rightarrow \eta'N$ plays an insignificant role in the $\eta'$ photoproduction off nuclei. We calculate the transparency ratio for $\eta'$ mesons and compare it to the available experimental data. The comparison indicates an inelastic $\eta'N$ cross section of the order of 6–10 mb. We show that the existing transparency ratio measurements do not allow one to distinguish between two adopted $\eta'$ in-medium modification scenarios. Our studies also demonstrate that the momentum distribution and excitation function for $\eta'$ production in $\gamma A$ reactions reveal some sensitivity to these scenarios, which means that such observables may be an important tool to get a valuable information on the $\eta'$ in-medium properties.
1. Introduction

The study of the modifications of the hadronic properties (masses and decay widths) in a strongly interacting environment is one of the most important subjects in contemporary hadron and nuclear physics owing to the expectation to observe a partial restoration of chiral symmetry in nuclear medium. The production of mostly light vector mesons $\rho$, $\omega$, $\phi$ in nuclear reactions with elementary probes (photons, protons) as well as in heavy–ion collisions has been investigated to search for possible in-medium modifications of their properties (see, for example, [1–13]).

Another interesting case of hadron medium renormalization is that of the pseudoscalar $\eta'(958)$ meson, whose mass, as is believed, is generated by the interplay of the QCD $U_A(1)$ anomaly and chiral symmetry breaking. The $\eta'$ mass reduction induced by the combined effect of the density-independent $U_A(1)$ anomaly and partial restoration of chiral symmetry in nuclear matter is predicted to be of the order of 150 MeV at the saturation density $\rho_0$ [14, 15]. The possible suppression of the $U_A(1)$ anomaly effect in nuclear medium leads to further mass reduction of the $\eta'$ here, which is expected to be in the 100 MeV area at the normal nuclear matter density [14, 15]. A reduction of the $\eta'$ mass of 200 MeV in medium formed in heavy–ion collisions was recently deduced from experiments at RHIC [16]. On the other hand, it is expected to be only a few ten MeV at saturation density [15] if it is estimated in the linear density approximation by using the $\eta'N$ scattering length extracted from the study of the $pp \rightarrow pp\eta'$ reaction near the threshold at COSY [17]. The $\eta'$ mass reduction in matter due to the possible suppression of the $U_A(1)$ anomaly effect in it is not accompanied by the appearance of an additional inelastic $\eta'N$ processes in the nuclear medium [14, 15]. Hence, the $\eta'$ absorption can be small, which is in line with the recent theoretical [18] and experimental [19] findings. Thus, the $\eta'$ in-medium properties should provide us valuable information both on the partial restoration of chiral symmetry at finite density and on the behavior of the $U_A(1)$ anomaly in the nuclear medium.

The production of $\eta'$ mesons in photon collisions with C, Ca, Nb, and Pb targets has been studied via the $\eta' \rightarrow \pi^0\pi^0\eta \rightarrow 6\gamma$ decay channel by the CBELSA/TAPS Collaboration in [19]. The variation of the $\eta'$ nuclear transparency ratio normalized to carbon with atomic number $A$ and with $\eta'$ momentum $p_{\eta'}$ as well as the momentum distribution of $\eta'$ mesons produced off a C target for the incident photon energy range of $E_\gamma$ = 1.5–2.2 GeV have been measured. A comparison of the data for the transparency ratio as a function of the nuclear mass number $A$ at photon energies of 1.7, 1.9 and 2.1 GeV with the theoretical calculations assuming a single step process for $\eta'$ creation and a one-body $\eta'$ absorption process as well as describing the propagation of $\eta'$ mesons in nuclei in the eikonal approximation, yielded a width of the $\eta'$ meson of the order of 15–25 MeV in the nuclear rest frame at normal nuclear matter density $\rho_0$ and for an average momentum $p_{\eta'} = 1.05$ GeV/c [19]. In the low-density approximation, this corresponds to an in-medium $\eta'N$ inelastic cross section of $\sigma_{\eta'N} \approx 6–10$ mb [19]. In an extreme case, providing the lower boundary for the determination of $\sigma_{\eta'N}$, when the one- and two-body $\eta'$ absorption mechanisms contribute equally, the extracted inelastic cross section is reduced to $\sigma_{\eta'N} \approx 3–5$ mb [19]. This allowed one to estimate in [19] the inelastic $\eta'N$ cross section as 3–10 mb to take into account the uncertainties arising from the unknown strength of the two-nucleon $\eta'$ absorption mechanism. The small $\eta'$ in-medium width, deduced in [19], implies the feasibility of an experimental observation of $\eta'$ bound states in nuclei [14, 15, 20]. In order to get a deeper insight into the interaction of the $\eta'$ meson with nuclear matter, it is important to analyze not only the $A$, but also the momentum dependences of the transparency ratio and the $\eta'$ production cross section measured in [19]–the main aim of the present work.

In this paper, we study the inclusive $\eta'$ creation in photon–nucleus collisions on the basis of the nuclear spectral function approach [21–24]. We present detailed predictions for the absolute and relative $\eta'$ meson production cross sections from these collisions in the threshold energy region obtained within this approach in two scenarios for its in-medium modification by considering cor-
responding primary photon–nucleon and secondary pion–nucleon production processes as well as compare part of them with the available data [19].

2. Framework

2.1. One-step \( \eta' \) production mechanism

Since we are interested in the incident photon energy range up to approximately 2 GeV, we accounted for the following direct elementary processes, which have the lowest free production threshold (\( \approx 1.446 \) GeV):

\[
\begin{align*}
\gamma + p & \rightarrow \eta' + p, \\
\gamma + n & \rightarrow \eta' + n.
\end{align*}
\]

We will not consider the \( \eta' \) creation processes with an additional pion in the final state because their total cross section is less than those of the reactions (1) and (2) by a factor of about 3 [25] in the energy domain of our interest. In the following calculations we will include the medium modification of the \( \eta' \) mesons, participating in the production processes (1), (2) by using, for reasons of simplicity, their average in-medium mass defined as:

\[
<m_{\eta'}^*>= \int d^3r \rho_N(r)m_{\eta'}^*(r)/A,
\]

where \( \rho_N(r) \) and \( m_{\eta'}^*(r) \) are the local nucleon density and \( \eta' \) effective mass inside the nucleus, respectively. Assuming in line with [14, 15] that

\[
m_{\eta'}^*(r) = m_{\eta'} + V_0 \frac{\rho_N(r)}{\rho_0},
\]

we can readily rewrite Eq. (3) in the form

\[
<m_{\eta'}^*>= m_{\eta'} + V_0 \frac{\rho_N}{\rho_0}.
\]

Here, \( m_{\eta'} \) is the \( \eta' \) free space mass and \( \langle \rho_N \rangle \) is the average nucleon density. For the potential depth at saturation density \( V_0 \approx -100 \) MeV [14, 15, 26] as well as for \( \langle \rho_N \rangle = \rho_0/2 \) [27], Eq. (5) leads to

\[
<m_{\eta'}^*>= m_{\eta'} + 1/2 V_0 \approx m_{\eta'} - 50 \text{ MeV}.
\]

In order to see the sensitivity of the \( \eta' \) production cross sections from the one-step processes (1), (2) to the \( \eta' \) mass shift (6), we will also ignore it in our calculations. The total energy \( E_{\eta'}' \) of the \( \eta' \) meson inside the nuclear medium can be expressed through its average effective mass \( <m_{\eta'}^*> \) defined above and in-medium momentum \( p_{\eta'}' \) as in the free particle case, namely:

\[
E_{\eta'}' = \sqrt{(p_{\eta'}')^2 + (m_{\eta'}^*)^2}.
\]

The momentum \( p_{\eta'}' \) is related to the vacuum one \( p_{\eta'} \) by the following expression:

\[
\sqrt{(p_{\eta'}')^2 + (m_{\eta'}^*)^2} = \sqrt{p_{\eta'}^2 + m_{\eta'}^2}.
\]

---

1) Not considered quantitatively in [19].
2) We will quote namely this mass shift in figures 7–12, presented below. However, it should be mentioned that in the literature to quantify the hadron in-medium mass shift it is usually quoted at normal nuclear matter density.
Finally, neglecting the distortion of the incident photon and describing the \( \eta' \) meson final-state absorption by in-medium cross section \( \sigma_{\eta'N} \), as well as using the results given in [21–24], we can represent the inclusive differential and total cross sections for the production of \( \eta' \) mesons with the momentum \( p_{\eta'} \) off nuclei in the primary photon–induced reaction channels (1), (2) as follows:

\[
\frac{d\sigma^{(\text{prim})}_{\gamma N \to \eta' X}(E_\gamma)}{dp_{\eta'}} = I_V[A] \\
\times \left[ Z \left\langle \frac{d\sigma_{\gamma p \to \eta' p}(p_\gamma, p'_{\eta'})}{dp_{\eta'}} \right\rangle_A + N \left\langle \frac{d\sigma_{\gamma n \to \eta' n}(p_\gamma, p'_{\eta'})}{dp_{\eta'}} \right\rangle_A \right] \frac{dp_{\eta'}}{dp_{\eta'}},
\]

\[
\sigma^{(\text{prim})}_{\gamma A \to \eta' X}(E_\gamma) = I_V[A] \left[ Z \left\langle \sigma_{\gamma p \to \eta' p}(p_\gamma) \right\rangle_A + N \left\langle \sigma_{\gamma n \to \eta' n}(p_\gamma) \right\rangle_A \right];
\]

where

\[
I_V[A] = 2\pi A \int_0^R \frac{r d^3r}{\sqrt{R^2-r^2}} \int d\rho(\sqrt{r^2+z^2}) \exp \left[ -\sigma_{\eta'N} A \int \frac{\sqrt{R^2-r^2}}{z} \rho(\sqrt{r^2+x^2}) dx \right],
\]

\[
\left\langle \frac{d\sigma_{\gamma N \to \eta' N}(p_\gamma, p'_{\eta'})}{dp_{\eta'}} \right\rangle_A = \int \int P_{A}(p_t, E)dp_t dE \left[ \frac{d\sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >, p'_{\eta'})}{dp_{\eta'}} \right],
\]

\[
\left\langle \sigma_{\gamma N \to \eta' N}(p_\gamma) \right\rangle_A = \int \int P_{A}(p_t, E)dp_t dE \sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >)
\]

and

\[
s = (E_\gamma + E_t)^2 - (p_\gamma + p_t)^2,
\]

\[
E_t = M_A - \sqrt{(-p_t)^2 + (M_A - m_N + E)^2}.
\]

Here, \( d\sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >, p'_{\eta'})/dp_{\eta'} \) and \( \sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >) \) are, respectively, the off-shell differential and total cross sections for the production of \( \eta' \) with reduced mass \( < m_{\eta'}^* > \) in reactions (1) and (2) at the \( \gamma N \) center-of-mass energy \( \sqrt{s} \); \( p_\gamma \) is the momentum of the initial photon; \( \rho(r) \) and \( P_{A}(p_t, E) \) are the local nucleon density and the spectral function \( A \) normalized to unity; \( Z \) and \( N \) are the numbers of protons and neutrons in the target nucleus \( A = N + Z \), \( M_A \) and \( R \) are its mass and radius, respectively; \( m_N \) is the bare nucleon mass.

Following [21–24], we assume that the off-shell cross sections \( d\sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >, p'_{\eta'})/dp_{\eta'} \) and \( \sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >) \) for \( \eta' \) production in reactions (1) and (2) are equivalent to the respective on-shell cross sections calculated for the off-shell kinematics of the elementary processes (1), (2) as well as for the \( \eta' \) in-medium mass \( < m_{\eta'}^* > \). Accounting for the two-body kinematics of these processes, we can get the following expression for the former differential cross section:

\[
\frac{d\sigma_{\gamma N \to \eta' N}(\sqrt{s}, < m_{\eta'}^* >, p'_{\eta'})}{dp_{\eta'}} \times \frac{1}{(\omega + E_t)} \delta \left[ \omega + E_t - \sqrt{m_N^2 + (Q + p_t)^2} \right],
\]

where

\[
I_2(s, m_N, < m_{\eta'}^* >) = \frac{\pi}{2} \lambda(s, m_N^2, < m_{\eta'}^* >^2) / s,
\]

\[3\]In the full phase space without any cuts on angle and momentum of the observed \( \eta' \) meson.

\[4\]Which represents the probability to find a nucleon with momentum \( p_t \) and removal energy \( E \) in the nucleus.

\[5\]It is determined from the relation \( \rho_N(R) = 0.0390 \) [28], and is equal to 4.0, 5.259, 5.763, 7.128, 8.871, 9.214 fm, respectively, for \(^{12}\text{C}, ^{27}\text{Al}, ^{40}\text{Ca}, ^{93}\text{Nb}, ^{208}\text{Pb}, ^{238}\text{U}\) target nuclei considered in the present work.
Here, \( d\sigma_{\gamma N \rightarrow \eta' N}(\sqrt{s}, <m_{\eta'}^*>, \theta_{\eta'}^*)/d\Omega_{\eta'} \) is the off-shell differential cross section for the production of an \( \eta' \) meson with mass \(<m_{\eta'}^*>\) under the polar angle \( \theta_{\eta'}^* \) in the \( \gamma N \) c.m.s.

The currently available recent experimental information on the angular distributions of the outgoing \( \eta' \) mesons in the \( \gamma p \rightarrow \eta' p \) [25, 29–31] and \( \gamma n \rightarrow \eta' n \) [25] reactions in the photon energy range \( E_\gamma \leq 2.5 \text{ GeV} \) can be fitted with Legendre polynomials as [25]:

\[
\frac{d\sigma_{\gamma p \rightarrow \eta' p}(\sqrt{s}, m_{\eta'}, \theta_{\eta'}^*)}{d\Omega_{\eta'}} = \begin{cases} 
\frac{\sigma_{\gamma N \rightarrow \eta' N}(\sqrt{s}, m_{\eta'})}{4\pi} & \text{for } E_\gamma^{\text{thr}}(m_{\eta'}) \leq E_\gamma < 1.525 \text{ GeV}, \\
\frac{p_{\eta'}^*(m_{\eta'})}{p_{\gamma N}^*(m_{\gamma N})} \sum_{i=0}^{3} A_i^{\gamma p \rightarrow \eta' p} P_i(\cos \theta_{\eta'}^*) & \text{for } 1.525 \text{ GeV} \leq E_\gamma \leq 2.5 \text{ GeV},
\end{cases}
\]

where

\[
\sigma_{\gamma p \rightarrow \eta' p}(\sqrt{s}, m_{\eta'}) = \begin{cases} 
3.856 \left[ (E_\gamma - E_\gamma^{\text{thr}}(m_{\eta'}))/\text{GeV} \right]^{0.5} \text{ [mb]} & \text{for } E_\gamma^{\text{thr}}(m_{\eta'}) \leq E_\gamma < 1.51 \text{ GeV}, \\
26.602 - 16.973(E_\gamma/\text{GeV}) & \text{for } 1.51 \text{ GeV} \leq E_\gamma < 1.525 \text{ GeV},
\end{cases}
\]

\[
A_0^{\gamma p \rightarrow \eta' p} = 0.751(E_\gamma/\text{GeV})^{-2.908} \text{ [mb/sr]},
\]

\[
A_1^{\gamma p \rightarrow \eta' p} = \begin{cases} 
0.0021(E_\gamma/\text{GeV})^{6.305} \text{ [mb/sr]} & \text{for } 1.525 \text{ GeV} \leq E_\gamma < 1.9 \text{ GeV}, \\
1.3305(E_\gamma/\text{GeV})^{-3.748} \text{ [mb/sr]} & \text{for } 1.9 \text{ GeV} \leq E_\gamma < 2.5 \text{ GeV}, \\
-0.02 \text{ [mb/sr]} & \text{for } 1.525 \text{ GeV} \leq E_\gamma < 1.7 \text{ GeV},
\end{cases}
\]

\[
A_2^{\gamma p \rightarrow \eta' p} = \begin{cases} 
-0.53 + 0.3(E_\gamma/\text{GeV}) & \text{for } 1.7 \text{ GeV} \leq E_\gamma < 2.1 \text{ GeV}, \\
0.88(E_\gamma/\text{GeV})^{-2.93} \text{ [mb/sr]} & \text{for } 2.1 \text{ GeV} \leq E_\gamma < 2.5 \text{ GeV},
\end{cases}
\]

\[
A_3^{\gamma p \rightarrow \eta' p} = \begin{cases} 
0.1 [(E_\gamma/\text{GeV}) - 2.0] & \text{for } 1.525 \text{ GeV} \leq E_\gamma < 2.0 \text{ GeV}, \\
0 & \text{for } 2.0 \text{ GeV} \leq E_\gamma \leq 2.5 \text{ GeV},
\end{cases}
\]

and

\[
\frac{d\sigma_{\gamma n \rightarrow \eta' n}(\sqrt{s}, m_{\eta'}, \theta_{\eta'}^*)}{d\Omega_{\eta'}} = \begin{cases} 
\frac{d\sigma_{\gamma p \rightarrow \eta' p}(\sqrt{s}, m_{\eta'}, \theta_{\eta'}^*)}{d\Omega_{\eta'}} & \text{for } E_\gamma^{\text{thr}}(m_{\eta'}) \leq E_\gamma < 1.525 \text{ GeV}, \\
\frac{p_{\eta'}^*(m_{\eta'})}{p_{\gamma N}^*(m_{\gamma N})} \sum_{i=0}^{3} A_i^{\gamma n \rightarrow \eta' n} P_i(\cos \theta_{\eta'}^*) & \text{for } 1.525 \text{ GeV} \leq E_\gamma \leq 2.5 \text{ GeV},
\end{cases}
\]

\[
A_0^{\gamma n \rightarrow \eta' n} = \begin{cases} 
4.278(E_\gamma/\text{GeV})^{-7.277} \text{ [mb/sr]} & \text{for } 1.525 \text{ GeV} \leq E_\gamma < 1.7 \text{ GeV}, \\
0.204(E_\gamma/\text{GeV})^{-1.546} \text{ [mb/sr]} & \text{for } 1.7 \text{ GeV} \leq E_\gamma < 2.2 \text{ GeV}, \\
1.287(E_\gamma/\text{GeV})^{-3.888} \text{ [mb/sr]} & \text{for } 2.2 \text{ GeV} \leq E_\gamma \leq 2.5 \text{ GeV},
\end{cases}
\]

\[
A_i^{\gamma n \rightarrow \eta' n} = A_i^{\gamma p \rightarrow \eta' p}, \quad i = 1, 2, 3.
\]

Here,

\[
E_\gamma^{\text{thr}}(m_{\eta'}) = \frac{m_{\eta'}(2m_N + m_{\eta'})}{2m_N}.
\]
\[ p_{\eta'}^*(m_{\eta'}) = \frac{1}{2\sqrt{s}} \lambda(s, m_N^2, m_{\eta'}^2), \]  
\[ p_{\gamma}^*(m_N^2) = \frac{1}{2\sqrt{s}} \lambda(s, 0, m_N^2). \]  

When the reaction $\gamma N \rightarrow \eta' N$ occurs on an off-shell target nucleon, then instead of the incident photon energy $E_{\gamma}$, appearing in the Eqs. (20)–(27), and of the nucleon mass squared $m_N^2$ in the Eq. (31), we should use, respectively, the effective energy $E_{\gamma}^{\text{eff}}$ and the quantity $E_t^2 - p_t^2$. The latter one should be calculated in line with Eq. (15). The energy $E_{\gamma}^{\text{eff}}$ can be expressed in terms of the collision energy squared $s$, defined by Eq. (14), as follows:

\[ E_{\gamma}^{\text{eff}} = \frac{s - m_N^2}{2m_N}. \]  

The $\eta'$ meson production angle $\theta_{\eta'}^*$ in the $\gamma N$ c.m.s. in this case and in the $\eta'$ medium modification scenario is defined by:

\[ \cos \theta_{\eta'}^* = \frac{p_{\gamma}^*(E_t^2 - p_t^2)p_{\eta'}^*(<m_{\eta'}^*>)}{p_{\gamma}^*(E_t^2 - p_t^2)p_{\eta'}^*(<m_{\eta'}^*>)}. \]  

Writing the invariant $t$–the squared four-momentum transfer between the incident photon and the secondary $\eta'$ meson—in the l.s. and in the $\gamma N$ c.m.s. and equating the results, we obtain:

\[ \cos \theta_{\eta'}^* = \frac{p_{\gamma}p_{\eta'} \cos \theta_{\eta'} + (E_{\gamma}^*E_{\eta'}^* - E_{\gamma}E_{\eta'})}{p_{\gamma}^*(E_t^2 - p_t^2)p_{\eta'}^*(<m_{\eta'}^*>)}, \]  

where

\[ E_{\gamma}^* = p_{\gamma}^*(E_t^2 - p_t^2), \quad E_{\eta'}^* = \sqrt{|p_{\eta'}^*(<m_{\eta'}^*>)|^2 + (<m_{\eta'}^*>)^2}. \]  

In Eq. (34), $\theta_{\eta'}$ is the angle between the initial photon three-momentum $p_{\gamma}$ and the in-medium $\eta'$ three-momentum $p_{\eta'}^*$ in the l.s. frame. For the sake of numerical simplicity, we will assume that the direction of the $\eta'$ meson three-momentum remains unchanged as it propagates from its creation point inside the nucleus to the vacuum far away from the nucleus. As a consequence, this angle does not deviate from the angle $\theta_{\eta'}$ between the incident photon momentum $p_{\gamma}$ and vacuum $\eta'$ momentum $p_{\eta'}$ in the l.s. In this case, the quantity $dp_{\eta'}/dp_{\eta'}$, entering into Eq. (9), can be put in the simple form $p_{\eta'}^*/p_{\eta'}$.

For the $\eta'$ production calculations in the case of $^{12}\text{C}$ and $^{27}\text{Al}$, $^{40}\text{Ca}$, $^{93}\text{Nb}$, $^{208}\text{Pb}$, $^{238}\text{U}$ target nuclei reported here we have employed for the nuclear density $\rho(r)$, respectively, the harmonic oscillator and the Woods-Saxon distributions, given in [32]. For the latter one, we take $R_{1/2} = 3.347$ fm for $^{27}\text{Al}$, $R_{1/2} = 3.852$ fm for $^{40}\text{Ca}$, $R_{1/2} = 5.216$ fm for $^{93}\text{Nb}$, $R_{1/2} = 6.959$ fm for $^{208}\text{Pb}$, $R_{1/2} = 7.302$ fm for $^{238}\text{U}$. The nuclear spectral function $P_A(p_t, E)$ for the $^{12}\text{C}$ target nucleus was taken from [21]. The single-particle part of this function for $^{27}\text{Al}$, $^{40}\text{Ca}$, $^{93}\text{Nb}$ and $^{238}\text{U}$ target nuclei was assumed to be the same as that for $^{208}\text{Pb}$. The latter was taken from [22]. The correlated part of the nuclear spectral function for these target nuclei was borrowed from [21].

The absorption cross section $\sigma_{\gamma'N}$ can be extracted, for example, from a comparison of the calculations with the transparency ratio of the $\eta'$ meson, normalized to carbon, as measured in [19]

\[ T_A = \frac{12}{A} \frac{\sigma_{\gamma A \rightarrow \eta'X}}{\sigma_{\gamma C \rightarrow \eta'X}}. \]  

Here, $\sigma_{\gamma A \rightarrow \eta'X}$ and $\sigma_{\gamma C \rightarrow \eta'X}$ are inclusive total cross sections for $\eta'$ production in $\gamma A$ and $\gamma C$ collisions, respectively. If the primary photon–induced reaction channels (1), (2) dominate in the $\eta'$ production in $\gamma A$ interactions, then, according to (10) and (11), we have:

\[ T_A = \frac{12}{A} \frac{I_V[A]}{I_V[C]} \left[ \frac{1}{2} \langle \sigma_{\gamma p \rightarrow \eta'p}(p_{\eta'}) \rangle_A + \frac{N}{A} \langle \sigma_{\gamma n \rightarrow \eta'n}(p_{\eta'}) \rangle_A \right]. \]
Ignoring the medium and isospin effects\,\footnote{It is worth noting that these effects lead to only small corrections to the ratio $T_A$. They are within several percent, as our calculations by (37), (38) for the nucleus with a diffuse boundary showed.}, from (37) we approximately obtain:

$$T_A \approx \frac{12}{A} \frac{I_V[A]}{I_V[C]}.$$  \hspace{1cm} (38)

The integral (11) for $I_V[A]$ in the case of a nucleus of a radius $R = r_0 A^{1/3}$ with a sharp boundary has the following simple form:

$$I_V[A] = \frac{3A}{2a_1} \left\{ 1 - \frac{2}{a_1^2} \left[ 1 - (1 + a_1) e^{-a_1} \right] \right\},$$  \hspace{1cm} (39)

where $a_1 = 2R/\lambda_{\eta'}$ and $\lambda_{\eta'} = 1/(\rho_0 \sigma_{\eta'} N)$. In particular, it can be easily obtained from the more general expression (28) given in [33] in the limit $a_2 \to 0$. The transparency ratio $\tilde{T}_A$ can be defined also as the ratio of the inclusive nuclear $\eta'$ photoproduction cross section divided by $A$ times the same quantity on a free nucleon [34]. According to (10) and (39), for this quantity one gets:

$$\tilde{T}_A = \frac{1}{A} I_V[A] = \frac{\pi R^2}{A \sigma_{\eta'} N} \left\{ 1 + \left( \frac{\lambda_{\eta'}}{R} \right) e^{-\frac{2\rho_{\eta'}}{\lambda_{\eta'}}} + \frac{1}{2} \left( \frac{\lambda_{\eta'}}{R} \right)^2 \left[ e^{-\frac{2\rho_{\eta'}}{\lambda_{\eta'}}} - 1 \right] \right\}.$$  \hspace{1cm} (40)

This formula was derived also in [34] and was reported in [19] in $\eta'$ photoproduction from nuclei.

### 2.2. Two-step $\eta'$ production mechanism

Kinematical considerations show that in the incident photon energy range of our interest the following two-step production process may contribute to the $\eta'$ creation in $\gamma A$ interactions. An incident photon can produce in the first inelastic collision with an intranuclear nucleon also a pion through the elementary reaction [24]

$$\gamma + N_1 \rightarrow 2\pi + N.$$  \hspace{1cm} (41)

Then, the intermediate pion, which is assumed to be on-shell, produces the $\eta'$ meson on another nucleon of the target nucleus via the elementary subprocess with the lowest free production threshold momentum (1.43 GeV/c):

$$\pi + N_2 \rightarrow \eta' + N.$$  \hspace{1cm} (42)

It should be noted that the elementary processes $\pi N \rightarrow \eta' N \pi$ with one pion in final states is expected to play a minor role in $\eta'$ production in $\gamma A$ reactions for kinematics of interest. This is due to the following. For instance, at beam energy of 2 GeV the maximum allowable momentum of a pion in the process (41) taking place on a free nucleon at rest is 1.85 GeV/c [24]. This momentum is close to the threshold momentum of 1.72 GeV/c of the channel $\pi N \rightarrow \eta' N \pi$. Therefore, it is energetically suppressed.

Taking into account the medium effects on the $\eta'$ mass on the same footing as that employed in calculating the $\eta'$ production cross sections (9), (10) from the primary processes (1), (2) and ignoring the distortion of the initial photon inside the target nucleus as well as using the results given in [23, 24], we get the following expression for the $\eta'$ production total cross section for $\gamma A$ reactions from the secondary channel (42):

$$\sigma^{(sec)}_{\gamma A \rightarrow \eta' X} (E_\gamma) = I_V^{(sec)}[A] \sum_{\pi' = \pi^+, \pi^0, \pi^-} \int d\Omega_\pi \int_{p_{\pi}^{\text{abs}}} p_{\pi}^2 dp_{\pi} \times$$  \hspace{1cm} (43)
the primary photon-induced reaction channel (41); \(
\sigma
\)
angle in the lab frame, we assume writing the formula (44) for it that the Eqs. (43), (45), following strictly the approach [24]. The elementary \(\pi\) via the subprocess (42) at the intermediate pion; \(\pi\) is the absolute threshold momentum for the \(\eta\) production on nuclei. Isospin considerations show that the following relations among the \(\eta\)′ production on the residual nucleus by this pion.

We will calculate hereafter the differential cross sections \(d\sigma_{\gamma N \rightarrow \pi^+ X}(\sqrt{s}, \mathbf{p}_\pi)/d\mathbf{p}_\pi\), entering into Eqs. (43), (45), following strictly the approach [24]. The elementary \(\eta\)′ production reactions \(\pi^+ n \rightarrow \eta' p\), \(\pi^0 p \rightarrow \eta' p\), \(\pi^0 n \rightarrow \eta' n\) and \(\pi^− p \rightarrow \eta' n\) have been included in our calculations of the \(\eta\)′ production on nuclei. Isospin considerations show that the following relations among the total cross sections of these reactions exist:

\[
\sigma_{\pi^+ n \rightarrow \eta' p} = \sigma_{\pi^− p \rightarrow \eta' n},
\]

\[
\sigma_{\pi^0 p \rightarrow \eta' p} = \sigma_{\pi^0 n \rightarrow \eta' n} = \frac{1}{2}\sigma_{\pi^− p \rightarrow \eta' n}.
\]

For the free total cross section \(\sigma_{\pi^− p \rightarrow \eta' n}\) we have used the following parametrization:

\[
\sigma_{\pi^− p \rightarrow \eta' n}(\sqrt{s_1}, m_{\eta'}) = \begin{cases} 
223.5 \left( \sqrt{s_1} - \sqrt{s_0} \right)^{0.352} [\mu b] & \text{for } 0 < \sqrt{s_1} - \sqrt{s_0} \leq 0.354 \text{ GeV}, \\
35.6/ \left( \sqrt{s_1} - \sqrt{s_0} \right)^{1.416} [\mu b] & \text{for } \sqrt{s_1} - \sqrt{s_0} > 0.354 \text{ GeV},
\end{cases}
\]

\(\sqrt{s_1} \) and \(\sqrt{s_0} \) are the in-medium total cross sections for the production of \(\eta\)′ mesons with the reduced mass \(m_{\eta'} \) in \(\pi^N\) collisions via the subprocess (42) at the \(\pi^N\) center-of-mass energy \(\sqrt{s_1}\); \(\sigma_{\pi^N N}^{\text{tot}}\) is the total cross section of the free \(\pi N\) interaction; \(\mathbf{p}_\pi\) and \(E_\pi\) are the three-momentum and total energy \((E_\pi = \sqrt{p_\pi^2 + m_\pi^2})\) of an intermediate pion; \(p^{\text{abs}}_\pi\) is the absolute threshold momentum for the \(\eta'\) production on the residual nucleus by this pion.

It should be pointed out that due to the very moderate dependence of the quantity \(I^{(\text{sec})}_V [A]\) on the \(\eta'\) production angle in the lab frame, we assume writing the formula (44) for it that the \(\eta'\) meson moves in the nucleus substantially in forward direction.
where $\sqrt{s_0} = m_{\eta'} + m_N$ is the threshold energy. The comparison of the results of calculations by (53) (solid line) with the available experimental data [35] for the $\pi^- p \rightarrow \eta' n$ (full triangles) and for the $\pi^+ n \rightarrow \eta' p$ (full circles) reactions is shown in figure 1. It is seen that the parametrization (53) fits well the existing set of data for these reactions.

Figure 1: Total cross sections for the $\eta'$ production in the $\pi^- p \rightarrow \eta' n$ and $\pi^+ n \rightarrow \eta' p$ reactions. For notation see the text.

Now, we discuss the results of our calculations for $\eta'$ production in $\gamma A$ interactions within the model outlined above.

3. Results and discussion

First of all, we consider the A-dependences of the total $\eta'$ production cross sections from the one-step and two-step $\eta'$ creation mechanisms in $\gamma A$ ($A = ^{12}\text{C}, ^{27}\text{Al}, ^{40}\text{Ca}, ^{93}\text{Nb}, ^{208}\text{Pb}, \text{and} ^{238}\text{U}$) collisions calculated for $E_\gamma = 1.85$ GeV on the basis of Eqs. (10) and (43). These dependences are shown in figure 2. It can be seen that the role of the secondary pion–induced reaction channel $\pi N \rightarrow \eta' N$ is negligible compared to that of the primary $\gamma N \rightarrow \eta' N$ processes for all considered target nuclei, which is in line with the experimental findings of [19]. This gives confidence to us that the channel $\pi N \rightarrow \eta' N$ can be ignored in our subsequent calculations.

Now, we focus upon the relative observable—the transparency ratio $T_A$ defined by Eq. (36). This ratio as a function of the nuclear mass number $A$ is shown in figures 3, 4, 5 and 6 for three initial photon energies of 1.7, 1.9, 2.1 GeV and for the photon energy range of 1.5–2.2 GeV, respectively. The experimental data in these figures are from [19]. The curves are calculations by Eq. (37) for

8) It should be noted that in the case, when the incident photon energy $E_\gamma$ belongs to the range of 1.5–2.2 GeV, the cross sections $\langle \sigma_{\gamma N \rightarrow \eta' N}(p_\gamma) \rangle_{A,C}$, entering into Eq. (37), were averaged over this energy as follows:

$$\int_{1.5 \text{ GeV}}^{2.2 \text{ GeV}} dE_\gamma \langle \sigma_{\gamma N \rightarrow \eta' N}(p_\gamma) \rangle_{A,C} / (2.2 \text{ GeV} - 1.5 \text{ GeV})$$

This treatment is absolutely correct for the description both the data on the $\eta'$ transparency ratio, given in figures 6, 9–11, and the data on the $\eta'$ momentum distribution on the $^{12}\text{C}$ target, presented in figure 8. But for the comparison to the absolute experimental cross sections, determined for this energy range, it is needed , strictly speaking, to average the calculated cross sections over the photon energy $E_\gamma$ with $1/E_\gamma$ weighting.
Figure 2: \(A\)-dependences of the total cross sections of \(\eta'\) production by 1.85 GeV photons from primary \(\gamma N \rightarrow \eta'N\) channels (1), (2) and from secondary \(\pi N \rightarrow \eta'N\) processes (42) in the full phase space in the scenario without \(\eta'\) mass shift. The absorption of \(\eta'\) mesons in nuclear matter was determined assuming an inelastic cross section \(\sigma_{\eta'N} = 11.5\) mb. The lines are included to guide the eyes.

Figure 3: Transparency ratio \(T_A\) for \(\eta'\) mesons as a function of the nuclear mass number \(A\) for their different in-medium absorption cross sections (in mb) at an incident photon energy of 1.7 GeV. For notation see the text.

different values (in mb) of the in-medium inelastic cross section \(\sigma_{\eta'N}\), as indicated by the curves, and for no \(\eta'\) mass shift. A comparison of the data to our model calculations yields \(\sigma_{\eta'N} \approx 6–10\)

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\textsuperscript{9}It should be pointed out that the inclusion of the considered \(\eta'\) mass shift (6) leads, as our calculations showed,
Figure 4: The same as in figure 3, but for the interaction of 1.9 GeV photons with the considered target nuclei.

Figure 5: The same as in figure 3, but for the interaction of 2.1 GeV photons with the considered target nuclei.

mb. This value coincides with that deduced in [19] in a low-density approximation.

In addition, we have investigated the momentum dependence of the absolute and relative $\eta'$ meson yields for incident photon energies between 1.5 and 2.2 GeV. The momentum differential cross sections for $\eta'$ production from $^{12}\text{C}$ and $^{93}\text{Nb}$, calculated on the basis of Eq. (9) in the scenario to a change of the ratio $T_A$ only by about several percent (see also figures 9–11 below). This gives confidence to us that the inelastic $\eta'N$ cross section estimated from the comparison of the measured and calculated transparency ratios is sufficiently reliable.
Figure 6: The same as in figure 3, but for the interaction of photons of energies of 1.5–2.2 GeV with the considered target nuclei.

Figure 7: Momentum differential cross sections for the production of $\eta'$ mesons from the primary $\gamma N \rightarrow \eta' N$ channel in the interaction of photons of energies of 1.5–2.2 GeV with $^{12}$C (left panel) and $^{93}$Nb (right panel) nuclei. For notation see the text.

of collisional broadening of the $\eta'$ meson characterized by the value of $\sigma_{\eta'N} = 8$ mb extracted above with an in-medium $\eta'$ mass shift (6) (dashed curve) and without it (solid curve), are depicted in figure 7. It is seen that in the case, when an in-medium $\eta'$ mass shift is included, their production cross sections increase as compared to those obtained in the scenario without it at $\eta'$ momenta $\leq 1$ GeV/c for both target nuclei. This gives an opportunity to get some information on a possible $\eta'$ mass shift in nuclear matter from a precise measurement of the momentum distribution. Such
possibility has been also discussed recently for the $\omega$ meson in [36].

The experimental data [19] on the momentum distribution of $\eta'$ mesons produced off a $^{12}$C target for $E_\gamma = 1.5–2.2$ GeV are shown in figure 8 in comparison to the calculated momentum distribution given in figure 7 and normalized to the data point at $\eta'$ momentum of 1.1 GeV/c. One can see that the available data seem do not exclude both the scenario with and the scenario without $\eta'$ mass shift, which means that the absolute experimental cross sections have to be determined to shed light on this shift.

![Figure 8: Momentum distribution of $\eta'$ mesons produced off a $^{12}$C target for $E_\gamma = 1.5–2.2$ GeV. The experimental data are from [19]. The curves are our calculations normalized to the data point at $\eta'$ momentum of 1.1 GeV/c. The notation of the curves is identical to that in figure 7.](image)

The measured (full squares) in [19] momentum dependence of the transparency ratio $T_A$ for $\eta'$

![Figure 9: Transparency ratio $T_A$ as a function of the $\eta'$ momentum for combination Ca/C and for $E_\gamma = 1.5–2.2$ GeV. For notation see the text.](image)
mesons for combinations Ca/C, Nb/C and Pb/C and for the photon energy range of 1.5–2.2 GeV is shown, respectively, in figures 9, 10 and 11 in comparison to our calculations on the basis of Eq. (9), assuming collisional broadening of the \( \eta' \) meson characterized by the value of \( \sigma_{\eta'N} = 8 \) mb extracted above from the analysis of the \( A \)-dependence of this ratio with an in-medium \( \eta' \) mass shift (6) (dashed curve) and without it (solid curve). It is clearly seen that, on the one hand, the differences between the calculations corresponding to different assumptions about the \( \eta' \) mass shift (between dashed and solid curves) are insignificant for all considered \( A/C \) combinations and, on the other hand, both these calculations describe quite well the data [19]. We, therefore, come to the conclusion that the transparency ratio measurements [19] do not allow one to discriminate between

Figure 10: The same as in figure 9, but for the combination Nb/C.

Figure 11: The same as in figure 9, but for the combination Pb/C.
Finally, we consider the excitation functions for photoproduction of \( \eta' \) mesons off \(^{12}\)C and \(^{93}\)Nb target nuclei. They were calculated for the one-step \( \eta' \) production mechanism on the basis of Eq. (10) as well as for \( \sigma_{\eta'N} = 8 \) mb and for two adopted options for the \( \eta' \) in-medium mass shift, and are given in figure 12. Looking at this figure, one can see that there are well separated predictions for this two considered scenarios for the \( \eta' \) in-medium mass shift. Therefore, one can conclude that the excitation function measurements might help to get a valuable information about this shift. It should be mentioned that an analogous possibility has been discussed before for the \( \omega \) meson in [36].

Taking into account the above considerations, we come to the conclusion that such observables as the momentum distribution and excitation function for \( \eta' \) photoproduction from nuclei can be useful to help determine a possible \( \eta' \) mass shift in cold nuclear matter.

4. Conclusions

In this paper we have investigated the production of \( \eta' \) mesons in photon-induced nuclear reactions near the threshold on the basis of an approach, which accounts for both primary photon–nucleon and secondary pion–nucleon \( \eta' \) production processes as well as two different scenarios for the \( \eta' \) in-medium mass shift. We have found that the secondary channel \( \pi N \to \eta'N \) plays a minor role in \( \eta' \) photoproduction off nuclei. Hence, it can be ignored in extracting the inelastic \( \eta'N \) cross section \( \sigma_{\eta'N} \) from the analysis within the present approach of the available data [19] on the \( \eta' \) transparency ratio. We have deduced from this analysis that \( \sigma_{\eta'N} \approx 6–10 \) mb. This value coincides with that obtained in [19] in a low-density approximation. We have shown that the transparency ratio measurements [19] do not allow one to discriminate between two adopted scenarios for the \( \eta' \) in-medium mass shift. On the other hand, we have also shown that, contrary to the transparency ratio, both the momentum distribution and excitation function for \( \eta' \) photoproduction off nuclei
are appreciably sensitive to the shift. This gives an opportunity to determine it experimentally.

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