A macroscopic origin of the Hall anomaly in the mixed state of type-II superconductors

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The Hall voltage in the mixed state close to the superconducting critical temperature is determined in the framework of the macroscopic approach. Only flux flow and macroscopic excitations motion are taken into account. The Hall anomaly can appear for rather low magnetic field values.

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The Hall effect in the mixed state of many type-II superconductors has a puzzling feature. Namely, in most of cuprates and some conventional superconductors the Hall voltage changes its sign as the materials enter the superconducting state. The classical theories of vortex motion as Bardeen- Stephen and Nozieres- Vinen models seem to be inadequate to explain this effect. There exist many different attempts to explain this curious phenomenon. The theory based on the time dependent Ginzburg- Landau equation shows that the Hall conductivity in the flux flow state can be expressed as the sum of the contribution due to the quasiparticles and the Hall term due to the vortex flow. If these contributions have opposite signs then the anomaly can take place. It depend on the electronic structure of the material. In other models the anomaly is ascribed to the pinning force or due to the difference of the electron density between the center and outside region of vortices.

Our aim is to show that the macroscopic excitation (ME) motion gives an important contribution to the sign reversal of the Hall voltage. As it will be seen in the following the sign of the Hall voltage produced by the ME motion is always opposite to that one due to the flux flow (or quasiparticles). When the ME motion gives the sufficiently large rise to the measured voltage then the anomaly occurs.

The macroscopic approach describes the mixed state in the intermediate field region \((H_{c1} \ll H < H_{c2})\), where \(H_{c1}\) and \(H_{c2}\) is the lower and upper critical field, respectively. The magnetic induction vector \(B\) and macroscopic current density \(J\) are averaged over the volume with dimension larger than the London penetration depth \(\lambda\). The characteristic length of the macroscopic current penetration \(\delta = \lambda(B/4\pi M)^{1/2}\), where \(4\pi M \approx \Phi_0(8\pi\lambda^2)^{-1}\ln(H_{c2}/B)\) and \(\Phi_0\) is the flux quantum. For the considered magnetic field range \((\lambda^2/\delta^2) \equiv p \ll 1\). In the absence of an external current the homogeneous mixed state with induction \(B_0 = const\) is treated as the ground state of the system. Spontaneous inhomogeneities which minimize the energy of the system are the macroscopic excitations (MEs). The excitation can have the form of “vortex” ring or “vortex”-“antivortex” pair (in thin slab). The energy of the unit length of ME has the following form

\[ E_M \approx \left( \frac{p}{2} \right) \left( \frac{\Phi_0}{4\pi \delta} \right)^2 \ln \left( \frac{1}{p} \right) \]

The excitations carry the magnetic flux quanta, thus take part in the creep processes, both thermal and quantum. Moreover, MEs motion gives rise to the electrical resistivity, which is independent on \(J\) (Ohm low) for small current density \(J\) in thin samples. Of course, the contributions of the MEs to the different processes depend on the density of the excitations. In the low temperature this amount is exponentially small. At elevated temperature the resistivity of the mixed state strongly increases. This effect is usually ascribed to the ”melting” of the flux line lattice (FLL). In sufficiently thin samples placed in a perpendicular magnetic field this process is treated as the example of the Berezinskii- Kosterlitz- Thouless (BKT) type transition. Since the energy of the usual (Abrikosov) vortex is large even for thin films the transition is often considered as dislocation mediated one. The FLL dislocation energy is considerably less than the usual vortex one, but comparison with the experiment on NbGe shows that it is still a few orders of magnitude too large to explain the experimental data. Let us notice that \(E_M\) is less than the energy of the unit length of the usual vortex by a factor of \((4\pi M/B)^2\) \(\equiv p^2 \ll 1\). The temperature dependence of \(E_M\) in the vicinity of the superconducting critical temperature \(T_{c0}\) is \((1-t)^3\), where \(t \equiv T/T_{c0}\). For usual vortex or FLL dislocation the corresponding dependence is \((1-t)\). It means that for sufficiently high temperature the macroscopic ”vortex” pairs can mediate BKT-type melting transition even in the slab of a quite large thickness. In this case BKT transition (crossover) temperature \(T_M\) is determined as usual by the following relationship

\[ kT_M \simeq gp \left( \frac{\Phi_0}{4\pi \delta} \right)^2, \]

where \(g\) is the sample thickness and \(k\) is the Boltzmann constant. For \((4\pi M/B)\) \(\ll 1\), \((1-t_M)=T_M/T_{c0}\) we then obtain the well known relation for the melting (irreversibility) line:

\[ (1-t_M)^3 \sim B^2. \]
Let us consider a thin slab of type II superconductor placed in the perpendicular magnetic field \( H_c \). An electric current with density \( J \) is flowing through the sample parallèl to its surface. In the system there are MEs, both ’vortex’ rings and ’vortex’-’antivortex’ pairs. If \( J \) is sufficiently small then the current is able to drive only the ’vortex’ pairs\(^{14,15} \). The ’vortex’ rings are too small and collapse. If the macroscopic ’vortex’ is at the point \( r_i \) on the slab then the equation describing the magnetic field distribution connected with the local mixed state inhomogeneity has the following form\(^{14,15} \):

\[
\delta^2 \nabla \times \nabla \times \mathbf{b} + \mathbf{b} = z \Phi_0 \delta (r - r_i)
\]  
(1)

where \( \mathbf{b} = \mathbf{B} (r) - \mathbf{B}_0, \mathbf{B}_0 = \text{const} \) and \( \delta^2 \rho = \lambda^2 \). \( z \) is the unit vector in \( \mathbf{b} \) direction and \( \delta (r) \) is the two dimensional Dirac delta. Eq.(1) can be rewritten as

\[
\mathbf{j} = \frac{c}{4\pi \delta^2} \left[ \frac{\Phi_0}{2\pi} \nabla \varphi - \mathbf{a} \right]
\]  
(2)

where \( \nabla \times \mathbf{a} = \mathbf{b} \) and \( 4\pi \mathbf{j} = c\nabla \times \mathbf{b} \). \( \varphi \) is the macroscopic phase which gives the flux quantization. In Eqs (1) and (2) it appears the length \( \delta \) instead of \( \lambda \) of the usual London equation. It means that in the macroscopic current participates only the small portion \( \delta \rho \) of the total charge density \( \rho_s \) of the superconducting condensate. We assume that under action of the external current the mixed state in our slab is in a flux flow state. The temperature is larger than the melting one. So, MEs density is large and their motion also gives rise to the measured voltage. The forces acting on the usual vortex and the macroscopic one are of the same kind (see Fig. 1). \( \mathbf{F}_L = c^{-1} \Phi_0 (J \times z) \) is the Lorentz force and

\[
\mathbf{F}_H = -\alpha (\mathbf{v} \times \mathbf{z})
\]

is the Hall force (or part of the Magnus force). \( \mathbf{v} \) is the vortex (or ME) velocity with respect to the laboratory frame of reference. \( \mathbf{F}_v = -\eta \mathbf{v} \) is the viscous force. Of course, the velocity and viscosity are different for usual vortex and ME. In a steady viscosity the total force acting on vortex (or ME) is equal to zero. It can be written as follows

\[
\mathbf{F}_{tot} = \mathbf{F}_L + \mathbf{F}_H + \mathbf{F}_v = 0
\]  
(3)

In the almost ideal case the coefficient \( \alpha \) for the single vortex in the Meissner state is equal to \( \alpha_1 = \Phi_0 \rho_s / c \). According to the above mentioned in the mixed state the charge density that takes part in the macroscopic current is \( \rho_s << \rho_s \). Thus, in the considered case the coefficient \( \alpha = \rho \alpha_1 \), both for vortex and ME. We assume that the viscous drag coefficient \( \eta \) for usual vortex is determined by the Bardeen-Stephen expression

\[
\eta_f = \frac{\Phi_0 H_c 2 \sigma_n}{c^2}
\]

where \( H_c \) is the low temperature value of the upper critical field. \( \sigma_n \) is the normal state conductivity. For ME the corresponding coefficient has the form\(^{14,15} \)

\[
\eta_M \approx \frac{\Phi_0}{4\pi \delta B} \eta_f.
\]  
(4)

Let us notice that \( \eta_M / \eta_f \) is very small for \( B >> H_c \). Thus, the ME motion is much faster than the usual vortex one. The forces acting on macroscopic ”vortex” are sketched in Fig.1. Reflecting this figure with respect to \( \mathbf{J} \)-axis we obtain the situation sketch for ”antivortex”. The motion of the macroscopic ”vortices” and ”antivortices” produces an electric field. The velocity components parallel to the current direction seem to be the same for ”vortex” and ”antivortex”. It means that in a rough approximation the MEs motion give no contribution to the Hall voltage. However, the electric current is connected with the magnetic induction gradient. The magnetic field free energy associated with the macroscopic vortex (or antivortex) has the following form\(^{14,15} \)

\[
f_1 = \frac{1}{2} \left( \frac{\Phi_0}{4\pi \delta} \right)^2
\]  
(5)

\( f_1 \) is the energy per unit length and it depends on \( B \) through \( \delta \). It is the origin of an additional force acting on macroscopic vortex and antivortex. This force has the only one direction (towards larger \( B \)). The result is equivalent to the following replacement

\[
J \Rightarrow J \left( 1 \pm \frac{\Phi_0}{8\pi \delta B^2} \right),
\]  
(6)

where minus is for ”vortex” case and plus for ”antivortex” one. So, the net Hall voltage due to MEs motion is proportional to the small parameter \( (\Phi_0 / 8\pi \delta B^2) \) and it has always the opposite sign to the corresponding voltage due to flux flow and quasiparticles. We take into account only two main contributions to the Hall voltage: flux flow and MEs motion. Denoting by \( \theta_f \) the Hall angle
between the Lorentz force direction and the usual vortex velocity (flux flow) we have

$$x \equiv \tan \theta_f = \frac{p \xi}{\eta_1}$$  \hfill (7)

As it has been already mentioned the appearance of the factor $p$ follows from the existence of the characteristic length $\delta$ of the macroscopic current distribution in the mixed state. If we take into account Eq. (4) then for Hall angle of ME motion we have

$$\tan \theta_M = x D,$$  \hfill (8)

where

$$D = \frac{4\pi \delta^2 B}{\Phi_0}.$$  \hfill (9)

The apparent Hall angle determining the effective Hall voltage due to MEs motion is obtained from Eqs. (6), (8) and has the following form

$$\tan \theta_{M_{\text{eff}}} = -x/2.$$  \hfill (10)

The magnitude of the competing Hall voltages depend on densities of usual vortices (flux flow) and MEs. The usual vortex density is equal to $B/\Phi_0$. With respect to the ME density let us notice that in the Meissner state if $T_{c0} - T \ll T - T_{BKT}$, where $T_{BKT}$ is the temperature of BKT-transition, then the vortex density $n_f \approx \xi^{-2}$, where $\xi$ is the superconducting coherence length. For the macroscopic vortex in the mixed state $\lambda$ plays the same role as $\xi$ for usual vortex (cutoff length). Thus, we may assume that for sufficiently high temperature the ME density $n_M \approx \lambda^{-2}$. Now, we can determine the difference between the Hall voltages (or resistivities) due to MEs motion and flux flow. Denoting by $\rho_M$ and $\rho_f$ the Hall resistivity due to MEs motion and flux flow, respectively, and using Eqs. (7)-(9) we have

$$\Delta \equiv (\rho_M - \rho_f)/\rho_f \approx \left[\frac{\Phi_0^2}{8\pi \lambda^2 \delta^2 B^2 x^2} - 1\right],$$  \hfill (11)

where we have assumed $x^2 << 1$ and $D^2 x^2 >> 1$. In the case of YBCO (90K) with $\sigma_n \sim 10^4 (\Omega cm)^{-1}$, $\lambda \sim 10^{-5} cm$ and $\kappa \sim 100$. Taking $B \sim 1 T$ and $p \sim 10^{-2}$ from Eqs. (7)-(8) we obtain $x \sim 10^{-3}$ and $D \sim 10^4$. If we assume that only two considered processes contribute to the Hall voltage then the anomaly is observable for $\Delta > 0$. It gives $x < x_0$, where $x_0 \approx \Phi_0 /[2(2\pi)^{1/2} \lambda B]$. For the above quoted YBCO parameters $x_0$ has the same order of magnitude as $x$. Let us notice that $x_0 \sim (1 - \kappa)^{1/2} B^{-3/2}$ while from Eq. (7) we have $x \sim (1 - \kappa)^2 B^{-1}$. It means that for sufficiently large $B$ or far from $T_{c0}$ we have $x > x_0$ and the ME contribution to Hall anomaly is too small to be observable. In our case the longitudinal voltage (resistivity) is the sum of two components: of flux flow and due to ME motion. They have the same sign. Each resistivity is proportional to the following expression

$$\frac{n_i}{\eta_i (1 + \tan^2 \theta_i)},$$

where $i = f, M$ denotes flux flow and ME motion, respectively. Putting here the proper quantities and assuming $x^2 << 1$ and $D^2 x^2 >> 1$ we see that flux flow resistivity is proportional to $B$ and ME one is almost independent on $B$. Together with Eqs. (7), (9) it gives the flux flow Hall voltage independent on $B$ and the effective Hall voltage due to ME motion proportional to $B^{-1}$.

We have shown that the Hall anomaly has not to follow merely from a microscopic origin as it is often suggested. We have taken into account only two competing processes: flux flow and MEs motion. Even in such simplified situation the Hall anomaly can take place. The quantitative comparison with experiment should be treated as a rough estimation. The real materials are usually anisotropic and have often the layered structure. Moreover, most of experiments have been done on thin films with thickness of order of $\lambda$ whereas the macroscopic approach used here is derived for systems with dimensions much larger than $\lambda$.

With respect to the experimental evidence of the macroscopic excitations we are convinced that the measurements of the Hall effect are little conclusive, because there are other possible mechanisms which give rise to the Hall anomaly. Only the experiments which are able to observe the motion of a single ME can give conclusive information. As it has been mentioned the ME motion is very fast in comparison with the usual vortex one. Moreover, the direction of ME motion is almost parallel to the current direction whereas the usual vortices in the flux flow regime move almost perpendicularly to this direction.

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