Janus Field Theories from Non-Linear BF Theories for Multiple M2-Branes

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Abstract

We integrate the nonpropagating $B_\mu$ gauge field for the non-linear BF Lagrangian describing $N$ M2-branes which includes terms with even number of the totally antisymmetric tensor $M^{IJK}$ in arXiv:0808.2473 and for the two-types of non-linear BF Lagrangians which include terms with odd number of $M^{IJK}$ as well in arXiv:0809:0985. For the former Lagrangian we derive directly the DBI-type Lagrangian expressed by the $SU(N)$ dynamical $A_\mu$ gauge field with a spacetime dependent coupling constant, while for the low-energy expansions of the latter Lagrangians the $B_\mu$ integration is iteratively performed. The derived Janus field theory Lagrangians are compared.
1 Introduction

Inspired by Bagger and Lambert [11] and Gustavsson [2] (BLG) who constructed the worldvolume theory of multiple coincident M2-branes following earlier works [3, 4], the multiple M2-branes have been extensively studied. The BLG theory is described by a three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons gauge theory with manifest SO(8) R-symmetry based on 3-algebra with a positive definite metric, that is, the unique nontrivial $A_4$ algebra [5]. However, this Chern-Simons gauge theory expresses two M2-branes on a $R^8/Z_2$ orbifold [6].

A class of models based on 3-algebra with a Lorentzian metric have been constructed by three groups [7, 8, 9] where the low-energy worldvolume Lagrangian of $N$ M2-branes in flat spacetime is described by a three-dimensional superconformal BF theory for the $su(N)$ Lie algebra. Using a novel Higgs mechanism of ref. [10] the BF membrane theory has been shown to reduce to the three-dimensional maximally supersymmetric Yang-Mills theory whose gauge coupling is the vev of one of the scalar fields [7, 9, 11]. For the prescription of the ghost-like scalar fields a ghost-free formulation has been proposed by introducing a new gauge field for gauging a shift symmetry and then making the gauge choice for decoupling the ghost state [12, 13, 14]. In ref. [15] starting from the maximally supersymmetric three-dimensional Yang-Mills theory and using a non-Abelian duality transformation due to de Wit, Nicolai and Samtleben (dNS) [16], the Lorentzian BLG theory has been reproduced.

The relation between the $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory [17] and the $\mathcal{N} = 8$ Lorentzian BLG theory has been studied [18, 19, 20, 21]. The various investigations related with the BLG theory have been performed [22, 23, 24, 25].

There has been a construction of a manifestly SO(8) invariant non-linear BF Lagrangian for describing the non-Abelian dynamics of the bosonic degrees of freedom of $N$ coincident M2-branes in flat spacetime, which reduces to the bosonic part of the BF membrane theory for $SU(N)$ group at low energies [26]. This non-linear Lagrangian is an extension of the non-Abelian DBI Lagrangian [27, 28] of $N$ coincident D2-branes and includes only terms with even number of the totally antisymmetric tensor $M^{IJK}$. Further, two types of non-linear BF Lagrangians have been presented such that they include terms with even and odd number of $M^{IJK}$ [29]. A different kind of non-linear gauged M2-brane Lagrangian has been proposed for the Abelian case [30].

As a related work, it has been shown that starting with the $\mathcal{N} = 8$ supersymmetric Yang-Mills theory on D2-branes and incorporating higher-derivative corrections to lowest nontrivial order, the Lorentzian BF membrane theory including a set of derivative corrections is constructed through a dNS duality [31] (see [32]). The higher-derivative corrections to the Euclidean $A_4$ BLG theory have been determined [33] by means of the novel Higgs mechanism and also shown to match the result of [31]. The couplings of the worldvolume of multiple M2-branes to the antisymmetric background fluxes have been investigated by using the low-energy Lagrangian for multiple M2-branes [34, 35] as well as the non-linear BF Lagrangian [36]. There have been proposals for the non-linear Lagrangians for describing the M2-brane-anti-M2-brane system [37] and the unstable M3-brane [38].

We will perform the integration over the redundant $B_\mu$ gauge field for the non-linear BF Lagrangians of ref. [26] and ref. [29], to see how the Lagrangians are described by the dynamical $A_\mu$ gauge field. We will carry out the $B_\mu$ integration directly for the non-linear
Lagrangian of ref. [26], while the $B \mu$ integration will be iteratively performed for the two types of non-linear BF Lagrangians of ref. [29]. These three $B \mu$ integrated Lagrangians will be compared.

2 Non-linear BF Lagrangian with even number of $M^{IJK}$

We consider the non-linear BF Lagrangian for $SU(N)$ group which describes the non-Abelian dynamics of the bosonic degrees of freedom of $N$ M2-branes in flat spacetime [26]

\[ L = -T_2 \text{Str} \left( \sqrt{\det \left( \eta_{\mu\nu} + \frac{1}{T_2} \tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} \tilde{D}_\nu X^J \right)} \right) \left( \det \tilde{Q} \right)^{1/4} \]

\[ + \text{Tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} \right) + (\partial_\mu X^I - \text{Tr}(X^I B_\mu)) \partial^\mu X^I_+ \]

\[ - \text{Tr} \left( \frac{X^+_+ X^+_+}{X^+_+} \tilde{D}_\mu X^I \partial^\mu X^I_+ - \frac{1}{2} \left( \frac{X^+_+ X^+_+}{X^+_+} \right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+ \right), \]

(1)

where $X^+_+ = X^+_+ X^+_+$ and the M2-brane tension $T_2$ is related to the eleven-dimensional Planck length scale $l_p$ as $T_2 = 1/(2\pi)^2 l_p^3$. The two non-dynamical gauge fields $A_\mu$, $B_\mu$ and the scalar fields $X^I$ ($I = 1, \cdots, 8$) are in the adjoint representation of $SU(N)$ and $X^I_\pm$ are $SU(N)$ singlets. The covariant derivative $\tilde{D}_\mu$ is defined by

\[ \tilde{D}_\mu X^I = \dot{D}_\mu X^I - \frac{X^+_+}{X^+_+} \partial_\mu X^I_+, \quad \dot{D}_\mu X^I = D_\mu X^I - X^+_+ B_\mu, \quad D_\mu X^I = \partial_\mu X^I + i[A_\mu, X^I] \]

(2)

and the SO(8) tensor $\tilde{Q}^{IJ}$ is given by

\[ \tilde{Q}^{IJ} = S^{IJ} + \frac{X^+_+ X^+_+}{X^+_+} (\det S - 1), \quad S^{IJ} = \delta^{IJ} + \frac{i}{\sqrt{T_2}} \frac{m^{IJ}}{\sqrt{X^+_+}}, \]

(3)

where $m^{IJ}$ is expressed as

\[ m^{IJ} = X^K M^{IKJK}, \quad M^{IKJK} = X^K [X^J, X^K] + X^J [X^K, X^I] + X^+ [X^I, X^J]. \]

(4)

In (1) $\tilde{Q}^{-1}_{IJ}$ denotes the matrix inverse of $\tilde{Q}^{IJ}$ and Str is the symmetrized trace [27]. The non-linear Lagrangian $L$ is invariant under the obvious global SO(8) transformation and the SU(N) gauge transformation associated with the $A_\mu$ gauge field, and further the non-compact gauge transformation associated with the $B_\mu$ gauge field

\[ \delta X^I = X^I_+ \Lambda, \quad \delta B_\mu = D_\mu \Lambda, \quad \delta X^I_+ = 0, \quad \delta X^I_- = \text{Tr}(X^I \Lambda). \]

(5)

The terms except for the first non-linear term and the second BF-coupling term in (1) are added to have consistency with the low-energy Lagrangian. In the non-linear Lagrangian $L$ only the symmetric part of $\tilde{Q}^{-1}_{IJ}$ is taken into consideration.
We introduce a Lagrange multiplier $p$ to rewrite the square root term in (1) as

$$- T_2 \sqrt{- \det \left( \eta_{\mu \nu} + \frac{1}{T_2} \tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} \tilde{D}_\nu X^J \right) \det \tilde{Q}}^{1/4}$$

$$\to \left( \frac{T_2^2}{2p} \det(\eta_{\mu \nu} + \frac{1}{T_2} \tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} \tilde{D}_\nu X^J) - \frac{p}{2} \right) \det \tilde{Q}^{1/4},$$  \hspace{1cm} (6)$$

where a matrix can be treated as a c-number within the symmetrized trace. Owing to $\tilde{Q}_{IJ}^{-1} = \tilde{Q}_{JI}^{-1}$ the relevant tensor is rearranged as

$$\eta_{\mu \nu} + \frac{1}{T_2} \tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} \tilde{D}_\nu X^J = g_{\mu \nu} + \tilde{B}_\mu \tilde{B}_\nu,$$  \hspace{1cm} (7)$$

where

$$g_{\mu \nu} = \eta_{\mu \nu} + \frac{1}{T_2} \tilde{D}_\mu X^I \tilde{P}_{IJ} \tilde{D}_\nu X^J, \quad \tilde{D}_\mu X^I = D_\mu X^I - X^I - \tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} X^J$$

and

$$\tilde{B}_\mu = \sqrt{\frac{X^I \tilde{Q}_{IJ}^{-1} X^J}{T_2}} \left( B_\mu - \frac{\tilde{D}_\mu X^I \tilde{Q}_{IKL} X^L}{X^I \tilde{Q}_{IKL} X^L} \right)$$  \hspace{1cm} (8)$$

with

$$\tilde{P}_{IJ} = \tilde{Q}_{IJ}^{-1} - \frac{\tilde{Q}_{IK} X^I \tilde{Q}_{JL} X^L}{X^I \tilde{Q}_{IKL} X^L},$$  \hspace{1cm} (9)$$

which is orthogonal to $X^I$ as $X^I \tilde{P}_{IJ} = 0$.

The expression (6) together with (7) is quadratic in $B_\mu$ so that the equation of motion for the auxiliary field $B_\mu$ is given by

$$g^{\mu \nu} \left( B_\nu - \frac{\tilde{D}_\nu X^I \tilde{Q}_{IJ}^{-1} X^J}{X^I \tilde{Q}_{IKL} X^L} \right) = \frac{p}{T_2 \det g(X^I \tilde{Q}_{IKL} X^L)(\det \tilde{Q})^{1/4}} \left( x^\mu - \frac{1}{2} \epsilon^{\mu \nu \lambda} F_{\nu \lambda} \right),$$  \hspace{1cm} (10)$$

where

$$x^\mu = \partial^\mu X_+^I P_{IJ} X^J$$  \hspace{1cm} (11)$$

with a projection operator

$$P_{IJ} = \delta_{IJ} - \frac{X^I X^J}{X_+^2}.$$  \hspace{1cm} (12)$$

Substituting the expression (10) back into the starting Lagrangian accompanied with the replacement (6) and solving the equation of motion for $p$ we get

$$L = \text{STr} \left[ -T_2 (\det \tilde{Q})^{1/4} \sqrt{- \det g} \left\{ 1 + \frac{1}{2T_2 (X^I \tilde{Q}_{IKL} X^L)(\det \tilde{Q})^{1/4}} \frac{1}{\sqrt{2T_2 (X^I \tilde{Q}_{IKL} X^L)(\det \tilde{Q})^{1/4}}} F_{\mu \rho} F_{\nu \sigma} g^{\mu \rho} g^{\nu \sigma} \right\} \right. \left. + \frac{\tilde{D}_\mu X^I \tilde{Q}_{IJ}^{-1} X^J}{X^I \tilde{Q}_{IKL} X^L} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} F_{\nu \lambda} - x^\mu \right) \right] + L_0,$$  \hspace{1cm} (13)$$
where

\[ F_{\mu\nu} = F_{\mu\nu} - \frac{1}{\det g} \epsilon_{\mu\nu\lambda} x^\lambda, \]  

\[ L_0 = \partial_\mu X^I_+ \partial^\mu X^I_+ - \text{Tr} \left( \frac{X_+ \cdot X}{X_+^2} D_\mu X^I \partial^\mu X^I_+ - \frac{1}{2} \left( \frac{X_+ \cdot X}{X_+^2} \right)^2 \partial_\mu X^I \partial^\mu X^I_+ \right). \]  

Here we use the identity for 3 \times 3 matrices \( g_{\mu\nu} + a F_{\mu\nu} \) with \( F_{\mu\nu} = -F_{\nu\mu} \)

\[ \det (g_{\mu\nu}) \left( 1 + \frac{1}{2} a^2 F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \right) = \det (g_{\mu\nu} + a F_{\mu\nu}) \]

(16)

to obtain a DBI-type Lagrangian

\[ L = \text{STr} \left[ -T_2 (\det Q)^{1/4} \sqrt{\frac{1}{T_2 (X^K Q_{KL}^{-1} X^L) (\det Q)^{1/4}}} F_{\mu\nu} \right] + \bar{D}_\mu X^I \tilde{Q}^{-1}_{IJ} X^J_+ + L_0. \]  

(17)

The inverse matrix of \( \tilde{Q}^{-1}_{IJ} \) in (3) is given by

\[ \tilde{Q}^{-1}_{IJ} = P_{IJ} + \frac{X^I_+ X^J_+}{X_+^2} \frac{1}{\det S} + \left( \frac{m_0}{1 - m_0} \right)^{IJ}, \]

(18)

where an orthogonal relation \( m^{IJ} X^J_+ = 0 \) is used and

\[ \left( \frac{m_0}{1 - m_0} \right)^{IJ} = m_0^{IJ} + (m_0^2)^{IJ} + (m_0^3)^{IJ} + \cdots, \]

\[ m_0^{IJ} = -\frac{i}{\sqrt{T_2 X_+^2}} m^{IJ} \]

(19)

with \( (m_0^2)^{IJ} = m_0^{IK} m_0^{KJ} \). Since only the symmetric part of matrix \( \tilde{Q}^{-1}_{IJ} \) is taken into account in the Lagrangian, the expression (18) is modified to be

\[ \tilde{Q}^{-1}_{IJ} = P_{IJ} + \frac{X^I_+ X^J_+}{X_+^2} \frac{1}{\det S} + \left( \frac{m_0^2}{1 - m_0^2} \right)^{IJ}, \]

(20)

which obeys \( \tilde{Q}^{-1}_{IJ} \) = \( \tilde{Q}^{-1}_{JI} \) and includes only terms with even number of \( M^{IJK} \) as expressed by

\[ \left( \frac{m_0^2}{1 - m_0^2} \right)^{IJ} = (m_0^2)^{IJ} + (m_0^4)^{IJ} + (m_0^6)^{IJ} + \cdots. \]

(21)

From this expression the following SO(8) invariant factors are simplified as

\[ X^I_+ \tilde{Q}^{-1}_{IJ} X^J_+ = X_+^2 \frac{1}{\det S}, \]

\[ \bar{D}_\mu X^I \tilde{Q}^{-1}_{IJ} X^J_+ = \frac{1}{\det S}. \]  

(22)
and the tensor $\tilde{P}_{IJ}$ in (9) is also given by

$$\tilde{P}_{IJ} = \tilde{Q}_{IJ}^\dagger \frac{X_I X_J}{X_+^2} \frac{1}{\det S}. $$

(23)

The relations in (22) together with $\det \tilde{Q} = (\det S)^2$ make the DBI-type Lagrangian (17) a simple form

$$L = -T_2 S \text{Tr} \left( -\det \left( g_{\mu\nu} + \frac{1}{\sqrt{T_2 X_+^2}} F_{\mu\nu} \right) (\det S)^{1/2} \right)$$

$$+ \text{Tr} \left( \frac{D_\mu X^I X^J}{X_+^2} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - x^\mu \right) \right) + L_0, \quad (24)$$

where $g_{\mu\nu}$ defined in (8) is rewritten by

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{T_2} \bar{D}_\mu X^I \left( P_{IJ} - \frac{1}{T_2 X_+^2} \left( \frac{m^2}{1 + \frac{m^2}{T_2 X_+^2}} \right)^{IJ} \bar{D}_\nu X^J \right) \bar{D}_\nu X^J \quad (25)$$

and there is a relation derived from (14)

$$\frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} = \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - x^\mu. \quad (26)$$

Thus from the non-linear BF Lagrangian with two nonpropagating gauge fields $A_\mu, B_\mu$ we have integrated the auxiliary $B_\mu$ gauge field to extract the DBI-type Lagrangian expressed in terms of the $SU(N)$ dynamical $A_\mu$ gauge field.

Now to perform the low-energy expansion for the non-linear Lagrangian (24), we calculate $\det S_{IJ}$ for $8 \times 8$ matrices by making the $1/T_2$ expansion as

$$\det S = 1 + \frac{1}{2T_2 X_+^2} (m^2)^{IJ} - \frac{1}{4T_2^2 (X_+^2)^2} (m^4)^{IJ} - \frac{1}{2} ((m^2)^{IJ})^2 + \cdots, \quad (27)$$

where $m^{IJ} = -m^{JI}$ is taken into account and $(m^2)^{IJ} = -X_+^2 M^{IJ} M^{JK}/3$. There is the following identity with finite terms for any $3 \times 3$ matrices $A_{\mu\nu}$

$$\det(\eta_{\mu\nu} + A_{\mu\nu}) = \det \eta \left( 1 + \text{tr}(\eta^{-1} A) - \frac{1}{2} \text{tr}(\eta^{-1} A)^2 + \frac{1}{2} \left( \text{tr}(\eta^{-1} A) \right)^2 \right)$$

$$+ \frac{1}{3} \text{tr}(\eta^{-1} A)^3 - \frac{1}{2} \text{tr}(\eta^{-1} A) \text{tr}(\eta^{-1} A)^2, \quad (28)$$

which gives the $1/T_2$ expansion for $\det g_{\mu\nu}$ in (13)

$$\det g_{\mu\nu} = - \left( 1 + \frac{1}{T_2} D_\mu X^I P_{IJ} D_\nu X^J + \frac{1}{T_2^2} \left( \frac{1}{2} D_\mu X^I P_{IJ} D_\nu X^J D^K X^K P_{KL} D^K Y^L \right. \right.$$

$$\left. + \frac{1}{2} (D_\mu X^I P_{IJ} D^K X^K) + \frac{1}{X_+^2} D_\mu X^I m^{IK} m^{KJ} P_{KL} D^K Y^L \right) + O \left( \frac{1}{T_2^3} \right). \quad (29)$$
We see that the $SO(8)$ vectors $\hat{D}_\mu X^I$ are contracted with $(m^2)^IJ$ and the projection operator $P^{IJ}$. It is convenient to express the square root factor including $F_{\mu\nu}$ in (13) in terms of $F_\mu \equiv \epsilon^{\mu\nu\lambda}F_{\nu\lambda}/2 - x^\mu$ which appears as an interaction $\hat{D}_\mu X^I X^I F_\mu / X_+^2$ in (24), and expand it through (25) and (29) as

$$\sqrt{1 + \frac{1}{2T_2 X_+^2} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma}} = \sqrt{1 + \frac{1}{T_2 X_+^2} \det g} F_\mu F_\nu g_{\mu\nu}$$

$$= 1 - \frac{1}{2T_2 X_+^2} F_{\mu\nu} F_{\eta\mu} + \frac{1}{2T_2 X_+^2} \left( F_{\mu\nu} \hat{D}_\mu X^I P_{IJ} \hat{D}_\nu X^J - \frac{1}{4X_+^2} (F_\mu F_\mu)^2 \right) + O \left( \frac{1}{T_2^3} \right). \quad (30)$$

Gathering the expansions (27), (29) and (30) in (24) or (13) we obtain the low-energy effective Lagrangian whose leading part is given by

$$L = - NT_2 + \text{Tr} \left( \frac{1}{12} M^{IJK} M^{IJK} - \frac{1}{2} D_\mu X^I P_{IJ} \hat{D}_\nu X^J + \frac{1}{2X_+^2} X^I \partial^\mu X^I \hat{D}_\nu X^J \right) + L_0,$$

where $F_\mu F_\mu / 2X_+^2$ is alternatively expressed as $-f_{\mu\nu} f^{\mu\nu}/4X_+^2$ in terms of $f_{\mu\nu} \equiv F_{\mu\nu} + \epsilon_{\mu\nu\lambda} x^\lambda$. This leading Lagrangian shows the Janus field theory with a spacetime dependent coupling constant in ref. [11] (see [39]). This Lagrangian is rewritten by the following form

$$L = - NT_2 + \text{Tr} \left( \frac{1}{12} M^{IJK} M^{IJK} - \frac{1}{2} D_\mu X^I P_{IJ} \hat{D}_\nu X^J + \frac{1}{2X_+^2} X^I \partial^\mu X^I \hat{D}_\nu X^J \right)$$

$$- 2D_\mu X^J X^I - \frac{1}{4X_+^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2X_+^2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda}(D_\mu X^I X^I - X^I \partial_\mu X^I) + \partial_\mu X^I \partial^\mu X^I, \quad (32)$$

which is further compactly represented by

$$L = - NT_2 + \text{Tr} \left( \frac{1}{12} M^{IJK} M^{IJK} - \frac{1}{2} D_\mu X^I D^\mu X^I \right)$$

$$+ \frac{1}{2X_+^2} \left( \frac{1}{2} \epsilon^{\mu\lambda} F_{\nu\lambda} + D^\mu X^I X^I - X^I \partial^\mu X^I \right)^2 + \partial_\mu X^I \partial^\mu X^I. \quad (33)$$

The subleading terms of order $1/T_2$ are derived as

$$\frac{1}{8T_2} \text{STr} \left[ \frac{1}{(X_+^2)^2} m^{IJK} m^{KLM} m^{LMI} - \frac{1}{36} (M^{IJK})^2 + 2\hat{D}_\mu X^I P_{IJ} \hat{D}_\nu X^J \hat{D}^\nu X^K P_{KL} \hat{D}^\mu X^L \right.$$

$$- (\hat{D}_\mu X^I P_{IJ} \hat{D}^\mu X^J)^2 + \frac{1}{3} (M^{IJK})^2 \hat{D}_\mu X^I P_{IJ} \hat{D}_\nu X^J + \frac{1}{X_+^2} \hat{D}_\mu X^I m^{IJK} m^{KL} \hat{D}^\mu X^J$$

$$+ \frac{(F_\mu F^\mu)^2}{(X_+^2)^2} - \frac{F_\mu F^\mu}{3X_+^2} (M^{IJK})^2 + \frac{4F_\mu F^\nu}{X_+^2} \hat{D}_\nu X^I P_{IJ} \hat{D}^\nu X^J - \frac{2F_\mu F^\mu}{X_+^2} \hat{D}_\nu X^I P_{IJ} \hat{D}^\nu X^J \bigg]. \quad (34)$$

The last four terms including $F_\mu = \epsilon^{\mu\nu\lambda} f_{\nu\lambda}/2$ in (34) are expressed in terms of $f_{\mu\nu}$ as

$$\frac{1}{8T_2 X_+^2} \text{STr} \left[ \frac{1}{4X_+^2} (f_{\mu\nu} f^{\mu\nu})^2 + 4f_{\mu\nu} f_{\rho\sigma} \hat{D}^\nu X^I P_{IJ} \hat{D}^\sigma X^J \right.$$
\[ -f_{\mu \nu} f^{\mu \nu} \bar{D}_\lambda X^I P_{IJ} \bar{D}^\lambda X^J + \frac{1}{6} (M^{IJK})^2 f_{\mu \nu} f^{\mu \nu}, \]  

(35)

where a \( f_{\mu \nu} \) is accompanied with a factor \( 1/\sqrt{X_+^2} \). The trace is taken symmetrically between all the matrix ingredients \( f_{\mu \nu}, \bar{D}_\mu X^I, M^{IJK} \) so that the expression (35) is described by

\[
\frac{1}{12 T_2 X_+} \text{Tr} \left[ -\frac{1}{2} (2 f_{\mu \nu} f^{\mu \nu} \bar{D}_\lambda X^I \bar{D}^\lambda X^J + f_{\mu \nu} \bar{D}_\lambda X^I f^{\mu \nu} \bar{D}^\lambda X^J) P_{IJ} \right. 
\]

\[ + \left. \left( 2 f^{\rho \mu} f_{\mu \nu} f_{\rho \sigma} f^{\mu \nu} f^{\rho \sigma} + f_{\mu \nu} f_{\rho \sigma} f^{\mu \nu} f^{\rho \sigma} \right) P_{IJ} \right]+ \frac{1}{8 X_+} \left( 2 f_{\mu \nu} f_{\rho \sigma} f^{\mu \nu} f^{\rho \sigma} + \frac{1}{12} (2 f_{\mu \nu} f^{\mu \nu} (M^{IJK})^2 + f_{\mu \nu} M^{IJK} f^{\mu \nu} M^{IJK}) \right]. \]

The potential part in (34) is also expanded as

\[
\frac{1}{24 T_2} \text{Tr} \left[ \frac{(m^2)^I}{(X^2_+)^2} (m^2)^I + 2(m^2)^I (m^2)^I \right] - \frac{1}{36} \left( M^{IJK} M^{LMN} M^{IJK} M^{LMN} + 2((M^{IJK})^2)^2 \right). \]

(37)

Here we write down the remaining terms

\[
\frac{1}{12 T_2} \text{Tr} \left[ \bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}^\nu X^K \bar{D}^\mu X^L + \bar{D}_\mu X^I \bar{D}_\nu X^K \bar{D}^\nu X^J \bar{D}^\mu X^L + \bar{D}_\mu X^I \bar{D}_\nu X^K \bar{D}^\mu X^L \bar{D}^\nu X^J 
\]

\[ - \bar{D}_\mu X^I \bar{D}^\mu X^J \bar{D}_\nu X^K \bar{D}^\nu X^L - \frac{1}{2} \bar{D}_\mu X^I \bar{D}_\nu X^K \bar{D}^\mu X^J \bar{D}^\nu X^L \right] P_{IJ} P_{KL} \]

\[ + \frac{1}{12 T_2} \text{Tr} \left[ \frac{2(m^2)^I (\bar{D}_\mu X^I \bar{D}^\mu X^J + \bar{D}_\mu X^J \bar{D}^\mu X^I)}{X_+^2} + \frac{1}{X_+^2} (\bar{D}_\mu X^I m^{IK} \bar{D}^\mu X^J m^{JK} + m^{KI} \bar{D}^\nu X^I m^{KJ} \bar{D}^\nu X^J) \right]+ \frac{1}{6} (2(M_{MN})^2 \bar{D}_\mu X^I P_{IJ} \bar{D}^\mu X^J + M_{MN} \bar{D}_\mu X^I M_{MN} \bar{D}^\mu X^J P_{IJ}). \]

3 Two non-linear BF Lagrangians with even and odd number of \( M^{IJK} \)

There are propositions of two types of non-linear BF Lagrangians for multiple M2-branes, which include terms with even number as well as odd number of \( M^{IJK} \) [29]. One type is presented by

\[
L_1 = -T_2 S \text{Tr} \left[ \sqrt{\text{det} \left( \eta_{\mu \nu} + \frac{1}{T_2} \bar{D}_\mu X^I \bar{D}^{IJ} \bar{D}_\nu X^J \right)} (\text{det} S_1) \right] \]

\[ + \frac{1}{2} \epsilon_{\mu \nu \lambda} \left( \text{Tr}(B_\mu F_\nu) - i \frac{1}{T_2} S \text{Tr}(\bar{D}_\mu X^K \bar{D}_\nu X^I M^{IKN} (S_1^{-1})^{NJ} \bar{D}^\lambda X^J) \right) \]

\[ + (\partial_\mu X^I - \text{Tr}(X^I B_\mu)) \partial^\mu X^I - \text{Tr} \left( \frac{X \cdot X}{X_+^2} \bar{D}_\mu X^I \partial^\mu X^I X^I - \frac{1}{2} \left( \frac{X \cdot X}{X_+^2} \right)^2 \partial_\mu X^I \partial^\mu X^I \right), \]

(39)
where the symmetric tensor $\tilde{R}^{IJ}$ is defined by

$$
\tilde{R}^{IJ} = (S_1^{-1})^{IJ} + \left[ \frac{1}{\sqrt{\det S_1}} - 1 \right],
$$

$$
S_1^{IJ} = \delta^{IJ} - \frac{1}{T_2} M^{IKM} M^{JKN} \left( \frac{X^M X^N}{X_+^2} \right).
$$

(40)

Because of $\det S_1 = (\det S)^2$ the symmetric tensor $\tilde{R}^{IJ}$ is identical to $\tilde{Q}^{-1}_{IJ}$ in (20), and $(\det S_1)^{1/4} = (\det Q)^{1/4}$, so that the Lagrangian $L_1$ except for terms with odd number of $M^{IKJ}$ reduces to $L$ in (1). For this topological BF Lagrangian we consider the integration over the $B_\mu$ gauge field to obtain a dynamical gauge theory Lagrangian. Since the type one Lagrangian $L_1$ contains not only the mass term of $B_\mu$ but also the cubic term, we cannot perform the $B_\mu$ integration directly. Instead, we begin to make the low-energy expansion for the non-linear term in (39) up to $1/T_2$ order

$$
- T_2 N + \text{Str} \left[ -\frac{1}{2} \tilde{D}_\mu X^I \tilde{D}^\mu X^I + \frac{1}{4} A^{II} \right]
+ \frac{1}{T_2} \left( -Z(B_\mu) + \frac{1}{8} (A^{II} \tilde{D}_\mu X^I \tilde{D}^\mu X^I + A^{IJ} A^{JI} - \frac{1}{4} (A^{II})^2) \right),
$$

(41)

where

$$
A^{IJ} = M^{IKM} M^{JKN} \left( \frac{X^M X^N}{X_+^2} \right) = \frac{1}{X_+^2} (m^2)^{IJ},
$$

$$
Z(B_\mu) = \frac{1}{8} \left( (\tilde{D}_\mu X^I \tilde{D}^\mu X^I)^2 - 2 \tilde{D}_\mu X^I \tilde{D}_\nu X^I \tilde{D}^\nu X^J \tilde{D}^\mu X^J 
+ 4 \tilde{D}_\mu X^I \left( A^{IJ} + \frac{X^I X^J}{2X_+^2} A^{KK} \right) \tilde{D}^\mu X^J \right).
$$

(42)

The algebraic equation of motion for $B_\mu$ reads

$$
X_+^I (\tilde{D}^\mu X^I) - X_+^I B_\mu \right) + \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - x^\mu
$$

$$
= \frac{1}{T_2} \left( \frac{1}{4} A^{II} X_+^I (\tilde{D}^\mu X^J - X_+^J B_\mu) + \frac{\delta Z}{\delta B_\mu} + \frac{i}{2} \epsilon^{\mu\nu\lambda} \frac{\delta X_{\nu\lambda}}{\delta B_\mu} \right)
$$

(43)

with $X_{\mu\nu\lambda}(B_\mu) = \tilde{D}_\mu X^K \tilde{D}_\nu X^I M^{IKJ} \tilde{D}_\lambda X^J$. The solution can be iteratively derived by $B_\mu = B_\mu^0 + B_\mu^1/T_2$, with

$$
B_\mu^0 = \frac{1}{X_+^2} \left( X_+^I \tilde{D}^\mu X^I + \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - x^\mu \right)
$$

(44)

and

$$
B_\mu^1 = -\frac{1}{X_+^2} \left( \frac{1}{4} A^{II} X_+^I (\tilde{D}^\mu X^J - X_+^J B_\mu^0) + \frac{\delta Z}{\delta B_\mu^0} \bigg|_{B_\mu^0} + \frac{i}{2} \epsilon^{\mu\nu\lambda} \frac{\delta X_{\nu\lambda}}{\delta B_\mu^0} \bigg|_{B_\mu^0} \right),
$$

(45)
where the expression of $B_0^\mu$ \[44\] is inserted into the last two derivative terms. Substituting this solution back into the low-energy Lagrangian of $L_1 \[39\]$ we obtain the same leading Lagrangian as \[31\] through a relation
\[
\bar{D}^\mu X^I - X^I_+ B_0^\mu = P^{IJ} \bar{D}^\mu X^J - \frac{1}{X^2_+} X^I_+ F^\mu
\] (46)
and the following correction terms of order $1/T_2$
\[
\frac{1}{T_2} \text{STr} \left( (D_\mu X^I - X^I_+ B_0^\mu) X^I_+ B_1^\mu + \frac{1}{8} A^{IJ}(D_\mu X^J - X^J_+ B_0^\mu)(D^\mu X^J - X^J_+ B_0^\mu) 
\right. \\
+ \frac{1}{8} (A^{IJ} A^{II} - \frac{1}{4} (A^{II})^2) - Z(B_0^\mu) + F^\mu B_1^\mu - \frac{i}{2} \epsilon^\mu_{\nu\lambda} X_{\nu\lambda}(B_0^\mu) \right). 
\] (47)
The subleading terms except for the terms including $X_{\nu\lambda}(B_0^\mu)$ and $\delta X_{\nu\lambda}/\delta B_\mu|_{B_0^\mu}$ are described by
\[
\frac{1}{T_2} \text{STr} \left[ \frac{1}{8} A^{KK} \bar{D}_\mu X^I P_{IJ} \bar{D}_\mu X^J - A^{II} \frac{F_\mu_F^\mu}{X^2_+} + A^{IJ} A^{II} - \frac{1}{4} (A^{II})^2 \right] \\
+ \frac{1}{4} \left( \bar{D}_\mu X^I P_{IJ} \bar{D}_\nu X^J \bar{D}_\nu X^K P_{KL} \bar{D}_\mu X^L + \frac{2 F_\mu F_\nu}{X^2_+} \bar{D}_\mu X^I P_{IJ} \bar{D}_\nu X^J + \frac{(F_\mu F_\mu)^2}{(X^2_+)^2} \right) \\
- \frac{1}{8} \left( \bar{D}_\mu X^I P_{IJ} \bar{D}_\mu X^J \right)^2 + \frac{2 F_\mu F_\mu}{X^2_+} \bar{D}_\nu X^I P_{IJ} \bar{D}_\nu X^J + \frac{(F_\mu F_\mu)^2}{(X^2_+)^2} \\
+ \frac{1}{2 X^2_+} \bar{D}_\mu X^I m^{IK} m^{JK} \bar{D}_\mu X^J . \right] 
\] (48)
It is noted that the $SO(8)$ vectors $\bar{D}_\mu X^I$ are contracted with $(m^2)^{IJ}$ and the projection operator $P_{IJ}$ which is due to \[46\]. The expression \[48\] is confirmed to agree with \[31\]. Thus we have observed that these subleading terms obtained by the iterative procedure for the $B_\mu$ integration in the low-energy Lagrangian reproduces the previous expression \[31\] which is derived by the low-energy expansion of the effective DBI-type Lagrangian generated by the exact $B_\mu$ integration.

The remaining terms lead to
\[
- \frac{i}{2 T_2} \text{STr} \left( \epsilon^\mu_{\nu\lambda} X_{\nu\lambda}(B_0^\mu) + \epsilon^\mu_{\nu\lambda} \delta X_{\nu\lambda} \delta B_\mu |_{B_0^\mu} \left( \frac{X^I_+}{X^2_+} (\bar{D}_\mu X^I - X^I_+ B_0^\mu) + \frac{1}{X^2_+} F_\mu \right) \right) \\
= \frac{i}{2 T_2} \epsilon^\mu_{\nu\lambda} \text{STr} \left( \tilde{M}^{IK} \bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}_\lambda X^K - \frac{3 m_{IJ}}{X^2_+} F_\mu \bar{D}_\nu X^I \bar{D}_\lambda X^K \right),
\] (49)
where $\tilde{M}^{IK}$ is a totally antisymmetric tensor defined by
\[
\tilde{M}^{IK} = M^{IK} - \frac{1}{X^2_+} (m^{IJ} X^K + m^{IK} X^J + m^{JK} X^I),
\] (50)
which is orthogonal to $X^I_+$ as $\tilde{M}^{IK} X^I_+ = 0$. This expression including single $M^{IK}$ is rewritten by
\[
\frac{i}{2 T_2} \text{STr} (\epsilon^\mu_{\nu\lambda} \tilde{M}^{IK} \bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}_\lambda X^K + 3 m^{IJ} f^{\mu\nu} \bar{D}_\mu X^I \bar{D}_\nu X^J).
\] (51)
We see that there is a coupling between the gauge field strength $F^{\mu\nu}$ and $m^{IJ} \bar{D}_\mu X^I \bar{D}_\nu X^J$. The symmetrization in (51) is taken as

$$\frac{i}{2T_2} \text{Tr}(\epsilon^{\mu\nu\lambda} M^{IK} \bar{D}^I \bar{D}^\mu X^I \bar{D}^\nu X^J + m^{IJ} f^{\mu\nu} \bar{D}_\mu X^I \bar{D}_\nu X^J$$

$$+ m^{IJ} \bar{D}_\mu X^I f^{\mu\nu} \bar{D}_\nu X^J + m^{IJ} \bar{D}_\mu X^I \bar{D}_\nu X^J f^{\mu\nu}),$$

(52)

where the first term remains intact.

Now we turn to the other type of non-linear Lagrangian for non-Abelian BF membranes

$$L_2 = -T_2 \text{Str} \left( \sqrt{-\det(\eta_{\mu\nu} + \frac{1}{T_2} \bar{D}_\mu X^I (S_2^{-1})_{IJ} \bar{D}_\nu X^J)} (\det S_2)^{1/6} \right)$$

$$+ \frac{1}{2} \epsilon^{\mu\nu\lambda} \left( \text{Tr}(B_\mu F_{\nu\lambda}) - \frac{i}{T_2} \text{Str}(\bar{D}_\mu X^K \bar{D}_\nu X^L M^{IK} M^{LJ} (S_2^{-1})_{NJ} \bar{D}_\lambda X^J) \right)$$

$$+ (\partial_\mu X^I - \text{Tr}(X^I B_\mu)) \partial^\mu X^I_+ - \text{Tr} \left( \frac{X_+ \cdot X}{X^2_+} \bar{D}_\mu X^I \partial^\mu X^I_+ - \frac{1}{2} \left( \frac{X_+ \cdot X}{X^2_+} \right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+ \right) \right),$$

(53)

where the symmetric tensor $S_2^{IJ}$ is defined by

$$S_2^{IJ} = \delta^{IJ} - \frac{1}{2T_2} B^{IJ}$$

(54)

with $B^{IJ} = M^{IK} M^{JK} \equiv (M^2)^{IJ}$. We make the following low-energy expansion for the non-linear term in (53)

$$- T_2 N + \text{Str} \left[ -\frac{1}{2} \bar{D}_\mu X^I \bar{D}^\mu X^I + \frac{1}{12} B^{IJ}$$

$$+ \frac{1}{T_2} \left( -W(B_\mu) + \frac{1}{24} \left( B^{IJ} \bar{D}_\mu X^J \bar{D}^\mu X^J + \frac{1}{2} B_{IJ} B^{IJ} - \frac{1}{12}(B^{IJ})^2 \right) \right) \right],$$

(55)

where

$$W(B_\mu) = \frac{1}{8} \left( (\bar{D}_\mu X^I \bar{D}^\mu X^I)^2 - 2 \bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}^\nu X^J \bar{D}^\mu X^J + 2 \bar{D}_\mu X^I B^{IJ} \bar{D}^\mu X^J \right).$$

(56)

The algebraic equation of motion for $B_\mu$ is also given by

$$X^I_+ \left( \bar{D}_\mu X^I - X^I_+ B^\mu \right) + \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - x^\mu$$

$$= \frac{1}{T_2} \left( \frac{1}{12} B^{IJ} X^J_+ (\bar{D}_\mu X^J - X^J_+ B^\mu) + \frac{\delta W}{\delta B_\mu} \bigg|_{B^\mu_0} + \frac{i}{2} \epsilon^{\rho\nu\lambda} \frac{\delta X^{\rho\nu\lambda}}{\delta B_\mu} \bigg|_{B^\mu_0} \right),$$

(57)

whose solution is iteratively derived by $B^\mu = B^\mu_0 + \tilde{B}_{1\mu}^\mu / T_2$ where $B^\mu_0$ is the same expression as (44) and $\tilde{B}_{1\mu}^\mu$ is

$$\tilde{B}_{1\mu}^\mu = -\frac{1}{X^2_+} \left( \frac{1}{12} B^{IJ} X^J_+ (\bar{D}_\mu X^J - X^J_+ B^\mu_0) + \frac{\delta W}{\delta B_\mu} \bigg|_{B^\mu_0} + \frac{i}{2} \epsilon^{\rho\nu\lambda} \frac{\delta X^{\rho\nu\lambda}}{\delta B_\mu} \bigg|_{B^\mu_0} \right).$$

(58)
The substitution of this solution into the low-energy Lagrangian of $L_2$ \((53)\) yields the same leading Lagrangian as \((31)\) and the following subleading terms of order $1/T_2$

$$\frac{1}{T_2} \text{STr} \left( (\bar{D}_\mu X^I - X^I_+ B_{0\mu}) X^I_+ \bar{D}^\mu + \frac{1}{24} B^{II}(\bar{D}_\mu X^J - X^J_+ B_{0\mu})(\bar{D}^\mu X^J - X^J_+ B_{0\mu}^\ast) + \frac{1}{48} (B^{IJ} B^{JI} - \frac{1}{6} (B^{II})^2 - W(B_0^\ast) + F^\mu \bar{B}_{I\mu} - \frac{i}{2} \epsilon^{\mu\nu\lambda} X_{\mu\nu\lambda}(B_0^\ast) \right). \quad (59)$$

We use an identity $B^{II} = 3A^{II}$ to express \((59)\) as sum of \((49)\) and

$$\frac{1}{T_2} \text{STr} \left[ \frac{1}{8} \left( A^{KK} \bar{D}_\mu X^I P_{IJ} \bar{D}^\mu X^J - A^{II} \frac{F^\mu F^\mu}{X_+^2} + \frac{1}{6} (M^2)^{IJ} (M^2)^{JI} - \frac{1}{4} (A^{II})^2 \right) \right. + \frac{1}{4} \left( \bar{D}_\mu X^I P_{IJ} \bar{D}^\nu X^J P_{KL} \bar{D}^\mu X^L + \frac{2F^\mu P^\nu}{X_+^2} \bar{D}_\nu X^I P_{IJ} \bar{D}^\mu X^J + \frac{(F^\mu F^\mu)^2}{(X_+^2)^2} \right) \right.
$$

$$- \frac{1}{8} \left( (\bar{D}_\mu X^I P_{IJ} \bar{D}^\mu X^J)^2 \right. + \frac{2F^\mu P^\nu}{X_+^2} \bar{D}_\nu X^I P_{IJ} \bar{D}^\mu X^J \left. + \frac{(F^\mu F^\mu)^2}{(X_+^2)^2} \right) \right. - \frac{1}{4} \bar{D}_\mu X^I \bar{M}^{IJMN} \bar{M}^{IJMN} \bar{D}^\mu X^J \right], \quad (60)$$

where $\bar{M}^{IJMN} = P^{IK} M^{KMN}$ and a relation $\bar{M}^{IJMN} m^{MN} = 0$ is used. The $1/T_2$ corrections show almost similar expressions to \((48)\) with two different terms which are a potential term $(M^2)^{IJ} (M^2)^{JI}/6$ and an interaction term $-\bar{D}_\mu X^I \bar{M}^{IJMN} \bar{M}^{IJMN} \bar{D}^\mu X^J/4$.

### 4 Conclusion

Without resort to the low-energy expansion we have performed the integration over one Chern-Simons nonpropagating $B_\mu$ gauge field exactly for the non-linear Lagrangian of the BF membrane theory in ref. \[26\], which includes terms with even number of $M^{IJK}$. We have observed that there appears a non-linear DBI-type Lagrangian for the worldvolume theory of $N$ M2-branes where the other Chern-Simons $A_\mu$ gauge field is promoted to the $SU(N)$ dynamical propagating gauge field.

In the non-linear DBI-type Lagrangian the coefficient factor of the modified field strength $F_{\mu\nu}$ takes a compact form $1/\sqrt{T_2 X_+^2}$ which yields the kinetic term of gauge field $-F_{\mu\nu} F^{\mu\nu}/4X_+^2$ with a space-time dependent coupling field $X_+^I$ in the leading low-energy expansion. In the same way the linear term of $F^\mu$ also takes a compact interaction $\bar{D}_\mu X^I X^I_+ F^\mu/X_+^2$. The subleading terms including dynamical gauge field strength $F_{\mu\nu}$ are expressed in terms of $F^\mu$ or alternatively $f_{\mu\nu}$ which is a specific combination of $F_{\mu\nu}$ and an $SO(8)$ invariant contraction of scalar fields with the projection operator $\partial^\mu X^I_+ P_{IJ} X^J$. In the subleading terms the $SO(8)$ vectors $\bar{D}_\mu X^I$ are contracted with $P^{IJ}$ and $(m^2)^{IJ}$ consisting of two $M^{IJK}$, which are orthogonal to $X^I_+$. This Lagrangian is regarded as the non-linear extension of the Janus field theory Lagrangian in ref. \[11\].

For the two types of non-linear BF Lagrangians in ref. \[29\] which include terms with even and odd number of $M^{IJK}$, we have made the low-energy expansion and then carried out the
integration by solving its equation of motion in the presence of the $1/T_2$ order corrections through an iterative procedure. In the type one Lagrangian $L_1$ we have demonstrated that there appear indeed various terms at order $1/T_2$ in the iteratively $B_\mu$ integrated effective Lagrangian, but they except for terms with odd number of $M^{IJK}$ are reshuffled to be in agreement with the $1/T_2$ order terms in the low-energy expansion of the above exactly $B_\mu$ integrated Lagrangian of the DBI form. In the type two Lagrangian $L_2$ the effective Lagrangian has been observed to have almost similar $1/T_2$ order corrections except for two terms, where $\bar{D}_\mu X^I$ are contracted with $P_{IJ}$ as well as $\hat{M}^{IMN}M^{JMN}$ which is also orthogonal to $X_+^I$. The remaining terms including single $M^{IJK}$ in both effective Lagrangians consist of two kinds of interactions specified by $\epsilon^{\mu \nu \lambda} \hat{M}^{IJK} \bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}_\lambda X^K$ and $m^{IJ} f^{\mu \nu} \bar{D}_\mu X^I \bar{D}_\nu X^J$ where the SO(8) vectors $\bar{D}_\mu X^I$ are contracted with the tensors $\hat{M}^{IJK}$ and $m^{IJ}$ which are orthogonal to $X_+^I$.

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