Composite Immersion and Invariance Based Adaptive Attitude Control of Asteroid-Orbiting Spacecraft in Elliptic Orbit

Keum W. Lee\(^1\) and Sahjendra N. Singh\(^2\)

\(^1\) Department of Electronic Engineering, Catholic Kwandong University, Gangwon 25601, Republic of Korea
kwlee@cku.ac.kr

\(^2\) Department of Electrical and Computer Engineering, University of Nevada Las Vegas, Las Vegas, NV 89154, USA
sahaj@ee.unlv.edu

Abstract. This paper proposes a new composite noncertainty-equivalence adaptive (CNCEA) control system for the attitude (roll, pitch, and yaw angle) control of a spacecraft in an orbit around a uniformly rotating asteroid based on the immersion and invariance (I\&I) theory. For the design, it is assumed that the asteroid’s gravitational parameters and the spacecraft’s inertia matrix are not known. In contrast to certainty-equivalence adaptive (CEA) or noncertainty-equivalence adaptive (NCEA) systems, the CNCEA attitude control system’s composite identifier uses the attitude angle tracking error, a nonlinear state-dependent vector function, and model prediction error for parameter estimation. The Lyapunov analysis shows that in the closed-loop system, the Euler angles asymptotically track the reference attitude trajectories. Interestingly, there exist two parameter error-dependent attractive manifolds, to which the closed-loop system’s trajectories converge. Moreover, the composite identifier using two types of error signals provides stronger stability properties in the closed-loop system. Simulation results are presented for the attitude control of a spacecraft orbiting in the vicinity of the asteroid 433 Eros. These results show precise nadir pointing attitude regulation, despite uncertainties in the system.

Keywords: Composite adaptive attitude control; asteroid-orbiting satellite; immersion and invariance-based attitude control; composite noncertainty-equivalence adaptive control; composite parameter identifier; attitude dynamics around asteroid; nonuniform gravitational potential

1 Introduction

The study of asteroids and comets is important for discovering more about the solar system. Additionally, asteroids may provide precious materials not found on Earth. Several missions, such as Galileo (NASA Mission to Jupiter via asteroids Gaspra and Ida (1989)), Near (NASA Mission to asteroid 433 Eros (1996)), and Dawn (NASA Mission to asteroids Ceres and Vesta (2007)), have been already completed, and future missions (DART, Lucy, and Psyche) have been planned. Unlike Earth, asteroids have irregular shapes and nonuniform gravitational potential. Thus, a spacecraft orbiting in vicinity of an asteroid experiences rapidly varying force. Several researchers have studied the effects of asteroids’ gravity fields on spacecraft [1-5]. Control systems have also been designed for orbit and hovering control, as well as landing on an asteroid [6-13].

Additionally, researchers have analyzed the attitude dynamics of spacecraft in asteroids’ gravitational fields [14-19]. The authors showed that irregularly-shaped asteroids can cause attitude resonance [14-16] and chaotic phenomenon [17]. The authors of [18] found natural Sun-synchronous orbits with Sun-tracking attitude motions for an orbit-attitude coupled system around a small rotating body [18].

A variety of control systems for attitude control, and coupled position and attitude motion control of a spacecraft in the vicinity of asteroids have been developed in the past. A linear attitude control law for roll, pitch, and yaw control was designed, and attitude stabilization for a satellite orbiting around Vesta, Ida, Gasra, Eros, and Castalia was demonstrated [16]. For the attitude control of spacecraft with a large captured body (asteroid), the authors of [20] designed (i) a robust nonlinear control law, (ii) an adaptive controller, and (iii) a derivative plus proportional-derivative control system. Simulated results, obtained by the use of shape models of Eros and Itokawa, showed that performances of the control laws can vary widely with the level of model uncertainties, measurement errors, and on-board computational power [20]. Researchers have also
developed control systems for body-fixed hovering over asteroids using finite-time control, variable structure control, and Lyapunov stability-based control techniques [21-25].

Recently, the immersion and invariance (I&I)-based methodology [26, 27] was applied for position trajectory (orbit) control [28, 29] and attitude control [30, 31] of asteroid-orbiting spacecraft. The control laws of [26-31] belong to the class of noncertainty-equivalence adaptive (NCEA) control systems. The NCEA attitude control laws of [30] and [31] are for the control of Euler angles and quaternions, respectively. For simplicity, the controllers of Refs. [29-31] were synthesized using filtered signals, as proposed in Ref. [32]. Research shows that the NCEA systems have stronger stability properties, compared with certainty-equivalence adaptive (CEA) systems [33]. The main reason is that the parameter estimates of NCEA systems include state-dependent algebraic functions as well as integral components.

For CEA systems [33], composite adaptive laws were developed in the past [34-37]. Composite CEA system’s identifiers use information on the tracking error and model prediction error to enhance closed-loop system performance. For traditional CEA systems, authors showed improvement in tracking performance and reduction in control input oscillations by simulation [35, 37], with the use of composite adaptation laws.

A composite noncertainty-equivalence adaptive law has been designed for a single-input system consisting of a chain of integrators with a known input coefficient [38]. In recent papers, generalized composite noncertainty-equivalence adaptive control laws for the orbit (position) and hovering control of a spacecraft in the vicinity of an asteroid, as well as a prototypical wing section equipped with a single trailing-edge flap, were designed, and performance improvement was shown [39, 40]. The adaptation laws of these generalized composite NCEA systems include not only the update laws of NCEA systems, but also use integral update rules of CEA systems, as well as gradient-based functions [39, 40].

To date, it appears from the literature that a composite noncertainty-equivalence adaptive (CNCEA) attitude control system has not been developed. In view of the composite CEA [34-37] and generalized composite NCEA systems’ stronger stability properties [39, 40], it is of interest to design a CNCEA attitude control law for asteroid-related missions.

The purpose of this paper is to develop a new three-axis attitude control system for a spacecraft on an elliptic orbit in the vicinity of a uniformly rotating asteroid with a constant angular velocity. It is assumed here that the spacecraft’s inertia matrix is not known. For this study, Euler angles (roll, pitch, and yaw) are considered for describing the spacecraft’s orientation. It should be noted that unlike the Earth or the Moon, which are nearly spherical, the gravitational torque acting on an asteroid-orbiting spacecraft is a complicated nonlinear time-varying function of the attitude angles. Moreover, the gravitational coefficients of irregularly-shaped asteroids are imprecisely known. Therefore, for precise attitude control, it is essential to nullify the adverse effect of the gravitational torque on the spacecraft, despite uncertainties in the system parameters. Here, it is assumed that the attitude angles and angular velocity components of spacecraft are available for feedback. Certainly, an adaptive control law will be appropriate for the attitude control of asteroid-orbiting spacecraft.

The contribution of this paper is three-fold. First, based on the immersion and invariance theory, a new state variable feedback composite noncertainty-equivalence adaptive (CNCEA) control system for the trajectory control of the Euler angles of an asteroid-orbiting spacecraft, is designed. In contrast to CEA or NCEA systems, the CNCEA system’s composite identifier utilizes not only the state-dependent nonlinear algebraic vector function and attitude angle tracking error, but also the model prediction error, derived based on the rotational dynamics of spacecraft. Second, based on the Lyapunov stability analysis, asymptotic convergence of the Euler angles to the origin is established. Interestingly, unlike the NCEA systems, the CNCEA attitude control system provides two parameter error-dependent attractive manifolds in an extended state-space, to which the closed-loop system’s trajectories can converge asymptotically. For NCEA systems, there exists a single attractive manifold. Moreover, the composite identifier using two types of error signals achieves stronger stability properties in the closed-loop system. Third, simulation is done for the control of a spacecraft in an orbit around the asteroid 433 Eros. These simulated responses show that the CNCEA control law achieves precise roll, pitch and yaw angle trajectory control on elliptic orbits, despite parameter uncertainties and large angle rotational maneuvers.
2 Attitude Dynamics of Asteroid-orbiting spacecraft

Consider a spacecraft on a planar elliptic orbit in the asteroid’s equatorial plane (see Fig. 1). The asteroid is rotating about $Z_1$ with a constant angular velocity $\Omega$. The inertial ($X_I, Y_I, Z_I$), orbital ($X_O, Y_O, Z_O$), and body-fixed ($X_B, Y_B, Z_B$) coordinates systems are shown in Fig. 1. The orbital frame is obtained by a rotation of the inertial frame through $\eta$ (the true anomaly) about the $Z_I$ axis. The satellite’s body-fixed frame is obtained from the orbital frame by three independent successive rotations $\theta_3$ (yaw), $\theta_2$ (pitch), and $\theta_1$ (roll).

For this study, satellite attitude dynamics derived in Refs. [14, 15] are considered. (Readers may refer to [14, 15] for the details.) Let $\omega = (\omega_1, \omega_2, \omega_3)^T$ be the angular velocity of the spacecraft relative to the inertial frame. Define $\theta = (\theta_1, \theta_2, \theta_3)^T$. Then, the derivative of $\theta$ can be shown to satisfy:

$$\dot{\theta} = A_1(\theta)\omega + A_1(\theta)A_2(\theta)\dot{\eta} = A_1(\theta)\omega + A_3(\theta)\dot{\eta}$$

where $\dot{\eta}$ is the orbital rate (derivative of the true anomaly), $A_3(\theta) = A_1(\theta)A_2(\theta)$, and $A_1(\theta)$ and $A_2(\theta)$ are:

$$A_1 = \frac{1}{\cos \theta_2} \begin{bmatrix} \cos \theta_2 \sin \theta_1 \sin \theta_2 & \cos \theta_1 \sin \theta_2 \\ 0 & \cos \theta_1 \cos \theta_2 - \sin \theta_1 \cos \theta_2 \\ 0 & \sin \theta_1 \end{bmatrix}$$

$$A_2(\theta) = \begin{bmatrix} \cos \theta_2 \sin \theta_3 \\ \sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \\ \cos \theta_1 \sin \theta_2 \sin \theta_3 - \sin \theta_1 \cos \theta_3 \end{bmatrix}$$

The matrix $A_1(\theta)$ has singularity at $\theta_2 = \pm \pi/2$. We are interested in the region of Euler angles away from the singular points.

The Euler’s equations for the rotational motion are:

$$J\dot{\omega} = -S(\omega)J\omega + M_g(\theta) + u$$

where $J = \text{diag}\{J_1, J_2, J_3\}$ is the diagonal principal inertia matrix of the satellite, $M_g = [M_{g1}, M_{g2}, M_{g3}]^T$ is the gravity gradient torque vector, $u = [u_1, u_2, u_3]^T$ is the vector of control torques, and $S(\omega)$ denotes the skew symmetric matrix:

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

The gravitational potential $U$ of any arbitrary primary can be expressed as:

$$U = \frac{\mu}{R} \left[ 1 + 0.5(r_0/R)^2C_{20}(3\sin^2 \delta - 1) + 3(r_0/R)^2C_{22}\cos^2 \delta \cos(2\lambda) 
+ 0.5(r_0/R)^3C_{30}\sin \delta(5\sin^2 \delta - 3) + \ldots \right]$$

(3)

where $\mu$ is the asteroid’s gravitational parameter, $R$ is the distance of the orbiting particle from the center of primary, $r_0$ is the characteristic length, and $\delta$ and $\lambda$ are the latitude and longitude of the orbiting particle expressed in an asteroid-fixed frame (not shown in Fig. 1). Define:

$$\phi = [-1.5C_{20} + 9C_{22}\cos(2\lambda_c)](r_0/R_c)^2$$

(4)

$$\chi = 6C_{22}\sin(2\lambda_c)(r_0/R_c)^2$$

(5)

where the longitude of the center of mass of the satellite is given by:

$$\lambda_c = \eta \pm \Omega t$$

(6)

where $R_c$ is the distance between the centers of mass of the asteroid and satellite. In Eq. (6), the plus and minus signs are used for retrograde and prograde (direct) orbits, respectively.
Based on the potential function (3), the gravity gradient torque components \( M_{gi} \) are given by [14, 15]:

\[
M_{g1}(\theta) = \frac{\mu}{R_c^3} [(3 + 5\phi)(J_3 - J_2) \cos \theta_1 \cos^2 \theta_2 \cos \theta_1 + 5\chi (-0.4J_1 \cos \theta_1 \sin \theta_3 + (J_1 - J_2 + J_3) \sin \theta_1 \cos^2 \theta_3)]
\]

\[
\equiv \psi_{a1}(\theta, t)p
\]

\[
M_{g2}(\theta) = \frac{\mu}{R_c^3} [(3 + 5\phi)(J_3 - J_1) \cos \theta_1 \cos \theta_2 \sin \theta_2 + 2\chi J_2 (\sin \theta_1 \sin \theta_3 - \cos \theta_1 \cos \theta_3) + 5\chi (J_2 - J_1 + J_3) (\sin \theta_1 \sin \theta_2 \sin \theta_3 + \sin^2 \theta_2 \cos \theta_1 \cos \theta_3) + 5\chi (J_2 - J_3 + J_1) \cos \theta_1 \cos^2 \theta_2 \cos \theta_3]
\]

\[
\equiv \psi_{b1}(\theta, t)p
\]

\[
M_{g3}(\theta) = \frac{\mu}{R_c^3} [(3 + 5\phi)(J_1 - J_2) \cos \theta_2 \sin \theta_1 \sin \theta_2 + 2\chi J_3 (\sin \theta_1 \cos \theta_3 - \cos \theta_1 \sin \theta_2 \sin \theta_3) + 5\chi (J_2 - J_1 + J_3) (\sin \theta_1 \cos \theta_1 \sin \theta_3 - \sin^2 \theta_2 \sin \theta_1 \cos \theta_3) - 5\chi (J_1 - J_2 + J_3) \sin \theta_1 \cos^2 \theta_2 \cos \theta_3]
\]

\[
\equiv \psi_{c1}(\theta, t)p
\]

where nonlinear vectors \( \psi_{s}\) \( s \in \{a, b, c\}, j = 1 \), are defined in equations (7) to (9); and \( p \) is a constant parameter vector given by:

\[
p = [J_1, J_2, J_3, C_{20}J_1, C_{20}J_2, C_{20}J_3, C_{22}J_1, C_{22}J_2, C_{22}J_3]^T \in R^9
\]

Define:

\[
\psi_1(\theta, t) = [\psi_{a1}^T, \psi_{b1}^T, \psi_{c1}^T]^T \in R^{3 \times 9}
\]

Then, the gravity gradient torque vector can be written in a compact form as:

\[
M_g(\theta) = \psi_1(\theta, t)p
\]

Now, using Eq. (11) in (2) gives:

\[
J\dot{\omega} = -S(\omega)J\omega + \psi_1(\theta, t)p + u
\]

It is assumed that \( \psi_1(\theta, t) \) is known, but the parameter vector \( p \) is not known.

Suppose that \( \theta_r(t) \in R^3 \) is a given smooth bounded reference trajectory. We are interested in deriving an adaptive control law for the system (1) and (12), such that in the closed-loop system, the attitude angle error \( \bar{\theta} = \theta - \theta_r \) asymptotically converges to zero, despite uncertainties in the parameter vector \( p \). For nadir pointing attitude control, the selected reference trajectory \( \theta_r(t) \) converges to zero.
3 A Filtered NCEA Attitude Control Law

First, the derivation of a filtered control input is described briefly. The derivation process is similar to that of Ref [30]. (For the details of derivation, readers may refer to [30].) However, it must be noted that even though the derived filtered input is similar, the actual control input \( u \) for the CNCEA system (computed later) will differ from the control signal derived in Ref. [30] for the NCEA system. The design of the filtered control torque is completed in two steps of a backstepping design procedure.

**Step 1:**
The tracking error \( \hat{\theta} = \theta - \theta_r \) evolves according to:
\[
\dot{\hat{\theta}} = A_1(\theta)\omega + A_3(\theta)\hat{\omega} - \dot{\theta}_r
\]
For regulation of \( \hat{\theta} \), a virtual (stabilizing) signal \( \omega_s \) is chosen as:
\[
\omega_s = A_1^{-1}[-k_1\dot{\hat{\theta}} + \dot{\theta}_r] - A_2\hat{\eta}; k_1 > 0
\]
Substituting Eq. (14) in (13) gives:
\[
\dot{\hat{\theta}} = -k_1\dot{\hat{\theta}} + A_1\omega_e
\]
where \( \omega_e = \omega - \omega_s \).

**Step 2:**
Consider the angular velocity error dynamics given by:
\[
J\dot{\omega}_e = -S(\omega)J\omega + \psi_1(\theta, t)p + u - J\dot{\omega}_s
\]
Now, the objective is to choose \( u \) to force \( \omega_e \) and \( \hat{\theta} \) to zero. Adding and subtracting \( J[k_2\omega_e + k_3(s + \alpha)(A_1^T\hat{\theta})] \) in equation (16) gives:
\[
\dot{\omega}_e = J^{-1}[-S(\omega)J\omega + \psi_1(\theta)p - J\dot{\omega}_s + J[k_2\omega_e + k_3(s + \alpha)(A_1^T\hat{\theta})] + u] - [k_2\omega_e + k_3(s + \alpha)(A_1^T\hat{\theta})]
\]
where \( k_2, k_3, \) and \( \alpha \) are positive design parameters, and \( s \) denotes the Laplace variable. Defining:
\[
-S(\omega)J\omega - J\dot{\omega}_s + J[k_2\omega_e + k_3(s + \alpha)(A_1^T\hat{\theta})] = \psi_2(\omega, \omega_s, \theta, t)p
\]
and \( \Psi(\theta, \omega, t) = \psi_1 + \psi_2 \), Eq. (17) can be written as:
\[
\dot{\omega}_e = J^{-1}[\Psi p + u] - k_2\omega_e - k_3(s + \alpha)(A_1^T\dot{\theta})
\]
Consider the filtered signals generated by the following equations:
\[
\dot{\Psi}_f = -\alpha\Psi_f + \Psi(\theta, \omega, t), \Psi_f(0) = 0
\]
\[
\dot{\omega}_{ef} = -\alpha\omega_{ef} + \omega_e, \omega_{ef}(0) = 0
\]
\[
\dot{u}_f = -\alpha u_f + u, u_f(0) = 0
\]
The signal \( (\Psi_f, \omega_{ef}, u_f) \) can be obtained by filtering \( (\Psi, \omega_e, u) \) through a stable transfer function \( H(s) \) where:
\[
H(s) = \frac{1}{s + \alpha}, \alpha > 0
\]
Of course, Eq. (22) will not be implemented. Filtering signals of Eq. (19), and ignoring decaying signal \( \omega_e(0)exp(-\alpha t) \) because it will not affect asymptotic stability properties, one obtains:
\[
\dot{\omega}_{ef} = J^{-1}[\Psi_f p + u_f] - k_2\omega_{ef} - k_3A_1^T\dot{\theta}
\]
Let an estimate of \( p \) be \( \hat{\rho} = \hat{p}_Tg + \beta(\omega_{ef}, \Psi_f) \), and the parameter error be \( z = \hat{\rho} - p \). Later, \( \hat{p}_Tg \) and the algebraic vector function \( \beta \) will be obtained. Now, in view of Eq. (24), the filtered control signal \( u_f \) is selected as:
\[
u_f = -\Psi_f\hat{p}
\]
Using Eq. (25) in (24) gives:

$$\dot{\omega}_{ef} = -J^{-1}\Psi_f z - k_2\omega_{ef} - k_3A^T_1\hat{\theta}$$

(26)

For the stability analysis, consider a Lyapunov function:

$$V_s(\omega_{ef}, \hat{\theta}) = \frac{1}{2}||\hat{\theta}||^2 + ||\omega_{ef}||^2$$

(27)

Then, the derivative of $V_s$ along the solution of Eqs. (15) and (26) can be written as:

$$\dot{V}_s = -k_1||\hat{\theta}||^2 + \dot{\omega}_{ef}^T A_1 \omega + \omega_{ef}^T [-J^{-1}\Psi_f z - k_2\omega_{ef} - k_3A^T_1\hat{\theta}]$$

$$= -k_1||\hat{\theta}||^2 - k_2||\omega_{ef}||^2 + \omega_{ef}^T [-J^{-1}\Psi_f z - k_3A^T_1\hat{\theta}] + \hat{\theta}^T A_1 \omega$$

(28)

Using $\omega_c = \dot{\omega}_{ef} + \alpha\omega_{ef}$ from equation (21), selecting the design parameters to satisfy $k_3 + k_2 - \alpha = 0$, and after performing some manipulations, one shows that (see [30]):

$$\dot{V}_s \leq - \frac{1}{2}k_1||\hat{\theta}||^2 + k_2||\omega_{ef}||^2 + \left(\frac{||A_1||^2}{2k_1} + \frac{1}{2k_2}\right)||J^{-1}||^2 + ||\Psi_f z||^2 - k_3||\hat{\theta}||^2$$

(29)

In view of equation (29), it easily follows that if $\Psi_f z = 0$, then $\hat{\theta}$ and $\dot{\omega}_{ef}$ will be bounded. The expression for the control input $u_f$ given in Eq. (25) will be used later. However, the parameter estimate $\hat{p}$ is not known yet.

4 Composite Identifier and CNCEA System’s Stability

In this section, first an estimator, based on a prediction model obtained from Eq. (12), is designed. This will be followed by the design of a composite parameter identifier.

4.1 Gradient-based Update Rule

In this subsection, a gradient-based parameter estimator is designed. Solving for $u$, Eq. (12) gives:

$$u = J\dot{\omega} + S(\omega)J\omega - \psi_1(\theta)p$$

$$= J\dot{\omega} + \psi_4 p$$

(30)

where $\psi_3 p = S(\omega)J\omega$ and $\psi_4 = -\psi_1 + \psi_3$. Define filtered signals $\omega_f$ and $\psi_4 f$ satisfying the following equations:

$$\dot{\omega}_f = -\alpha\omega_f + \omega_f(0) = 0$$

(31)

$$\dot{\psi}_4 f = -\alpha\psi_4 f + \psi_4(0) = 0$$

(32)

Next, filtering both sides of Eq. (30) using the transfer function $H(s)$ gives:

$$u_f = J\dot{\omega}_f + \psi_4 fp = J[\dot{\omega} - \alpha\omega_f] + \psi_4 fp = W(\omega, \omega_f, \psi_4 f)p$$

(33)

where the regressor $W \in \mathbb{R}^{3 \times 9}$ can be computed using Eq. (33).

For the derivation of a gradient-based update rule, consider a prediction model based on Eq. (33) as:

$$\hat{u}_f = W\hat{p}$$

(34)

Define the prediction error as:

$$\tilde{u}_f = \hat{u}_f - u_f$$

(35)

Then, subtracting Eq. (33) from (34) gives the prediction error:

$$\tilde{u}_f = W\hat{p} - Wp = Wz$$

(36)
where \( z = \hat{p} - p \). An adaptation law can be derived by minimizing a performance index \( P = [\hat{u}_f^T \hat{u}_f]/2 \). Its gradient with respect to \( \hat{p} \) is:

\[
\frac{\partial P}{\partial \hat{p}} = \frac{1}{2} \frac{\partial z^T W^T W z}{\partial \hat{p}} = W^T W z = W^T \hat{u}_f
\]

An adaptation law is chosen as negative of the gradient to minimize \( P \) [33]. Thus, the update rule is:

\[
\dot{\hat{p}} = -\gamma_g W^T \hat{u}_f, \gamma_g > 0
\]  

(37)

For examining the stability properties, consider a Lyapunov function \( V = z^T z/2 \). Then, its derivative is:

\[
\dot{V}_e = z^T \dot{z} = -z^T \gamma_g W^T \hat{u}_f = -\gamma_g \hat{u}_f^T \hat{u}_f \leq 0
\]

This implies that \( z \) is bounded and \( \hat{u}_f \) is square integrable. The model prediction error-based integral adaptation law (37) will be used in the next subsection to obtain a composite identifier for the attitude control of the spacecraft.

### 4.2 Composite Adaptation Law

In this subsection, a composite adaptation law is derived. Let the full parameter vector estimate \( \hat{p} \) of the actual parameter vector \( p \) be:

\[
\hat{p} = \hat{p}_{Ig} + \beta(\omega_{ef}, \Psi_f)
\]

\[
\beta(\omega_{ef}, \Psi_f) = \gamma \Psi_f^T \omega_{ef}
\]  

(38)

Here, \( \hat{p}_{Ig} \) is the integral component and \( \beta \) is the nonlinear algebraic part of the full estimate. The vector function \( \beta(\omega_{ef}, \Psi_f) \) has been judiciously selected based on the I&I principle. Its role will be seen when the derivative of the parameter vector \( z = \hat{p} - p \) will be computed.

For the derivation of the integral part \( \hat{p}_{Ig} \), it is necessary to obtain the dynamics of the parameter error \( z \). Differentiating \( z \) and using Eq. (38) gives:

\[
\dot{z} = \dot{\hat{p}}_{Ig} + \dot{\beta} = \dot{\hat{p}}_{Ig} + \dot{\beta} = \dot{\hat{p}}_{Ig} + \gamma (\dot{\Psi}_f^T \omega_{ef} + \Psi_f^T \dot{\omega}_{ef})
\]  

(39)

Substituting for the derivatives of \( \Psi_f \) and \( \omega_{ef} \) from Eqs. (20) and (26) gives:

\[
\dot{z} = \dot{\hat{p}}_{Ig} + \gamma (-\alpha \Psi_f^T + \Psi^T) \omega_{ef} + \Psi_f^T (-J^{-1} \Psi_f z - k_2 \omega_{ef} - k_3 A_f^T \hat{\theta})
\]  

(40)

The adaptation law for \( \dot{\hat{p}}_{Ig} \) is selected as:

\[
\dot{\hat{p}}_{Ig} = -\gamma (-\alpha \Psi_f^T + \Psi^T) \omega_{ef} - \gamma \Psi_f^T [-k_2 \omega_{ef} - k_3 A_f^T \hat{\theta}] - \gamma_g W^T \hat{u}_f
\]  

(41)

where, for synthesis, one sets \( \hat{u}_f = \hat{u}_f - u_f = W \hat{p} + \Psi_f \hat{\theta} \) (see Eqs. (25), (34), and (35)) in Eq. (41). We note that the adaptation gain \( \gamma \)-dependent terms of Eq. (41) form the integral component of the I&I-based identifier. Thus, the integral parameter adaptation law in Eq. (41) combines the I&I-based adaptation law, as well as the \( \gamma \)-dependent gradient-based law derived in Eq. (37). Of course, the net parameter estimate is \( \dot{\hat{p}} = \dot{\hat{p}}_{Ig} + \beta \). Substituting Eq. (41) in (40) gives:

\[
\dot{z} = -\gamma \Psi_f^T J^{-1} \Psi_f z - \gamma_g W^T \hat{u}_f
\]  

(42)

Next, consider a Lyapunov function:

\[
V_e = z^T z/2
\]

Now, its derivative will be computed along the solution of Eq. (42) of the composite identifier. Its derivative is:

\[
\dot{V}_e = -\gamma z^T \Psi_f^T J^{-1} \Psi_f z - z^T \gamma_g W^T \hat{u}_f
\]

\[
\leq -\gamma \lambda_{\text{min}}(J^{-1}) ||\Psi_f z||^2 - \gamma_g ||W||^2 ||(43)\]

where \( \lambda_{\text{min}}(.) \) denotes the minimum eigenvalue of \( J^{-1} \). This implies that the equilibrium point \( z = 0 \) of Eq. (42) is uniformly stable.

### 4.3 CNCEA System’s Stability
Now, for analyzing the stability of the CNCEA attitude control system (including control input Eq. (25) and composite estimation law (41)), consider a Lyapunov function:

$$V_c = V_s + \nu V_r; \nu > 0$$  \hspace{1cm} (44)

Then, using Eqs. (29) and (43) gives:

$$\dot{V}_c \leq -\frac{1}{2} [k_1 (||\dot{\theta}||^2 + k_2 ||\omega_{ef}||^2) + \left(\frac{||A_1||^2}{2k_1} + \frac{1}{2k_2}\right) ||J^{-1}|^2||\psi_f z||^2 - k_3 ||\dot{\theta}^T A_1||^2 + \nu [-\gamma \lambda_m (J^{-1})||\psi_f z||^2 - \gamma_0 ||Wz||^2]$$

Because $\nu$ is an arbitrary positive number, certainly there exists $l^* > 0$, such that for a sufficiently large value of $\nu$, one obtains:

$$\left(\frac{||A_1||^2}{2k_1} + \frac{1}{2k_2}\right) ||J^{-1}|^2 - \nu \gamma \lambda_m (J^{-1}) \leq -l^* < 0$$

For such a choice of $\nu$, Eq (45) yields:

$$\dot{V}_c \leq -\frac{1}{2} [k_1 (||\dot{\theta}||^2 + k_2 ||\omega_{ef}||^2) - l^* ||\psi_f z||^2 - \gamma_0 ||Wz||^2]$$  \hspace{1cm} (46)

In the following analysis, it will be assumed that the closed-loop system’s trajectories remain away from $\theta_2 \neq \pi/2$, where Eq. (2) is not defined. Furthermore, it is assumed that $\theta_r(t)$ is a smooth bounded function converging to zero and remains away from $\pm \pi/2$. Thus, the stability properties established here will be valid only locally around $\theta = 0$. The function $V_c$ is a positive definite function of $\dot{\theta}$, $\omega_{ef}$, and $z$. Since $V_c$ is negative semi-definite function, it follows that $V_c$ has a finite limit. Therefore, $\dot{\theta}$, $\omega_{ef}$, and $z$ are bounded. Because $\omega_{ef}$ is bounded, Eq. (21) implies that $\omega_c$ is bounded. Thus, all the signals in the closed-loop system will be bounded. Because $\dot{\theta}$, $\omega_{ef}$, $\omega_c$, $\psi_f$, and $z$ are differentiable, it follows that $\dot{V}_c$ exists. Then, using Brabala’s lemma [33], one concludes that $\dot{\theta}$, $\omega_c$, $\psi_f z$, and $Wz$ tend to zero, as $t$ tends to $\infty$. Of course, as $\omega_{ef}$ tends to zero, $\omega_c$ tends to zero. This establishes the convergence of the attitude angles to zero for the choice of $\theta_r$ converging to zero. Thus, the nadir pointing attitude is attained. (It will be seen later that even for large attitude perturbations, desired attitude regulation can be accomplished.)

**Remark 1:** It should be pointed out that the CNCEA attitude control law provides two attractive manifolds $\Omega_a$, and $\Omega_b$, defined by:

$$\Omega_a = \{(\psi_f \in R^{3 \times 9}, z \in R^9) : \psi_f z = 0\}$$

$$\Omega_b = \{W \in R^{3 \times 9}, z \in R^9) : Wz = 0\}$$  \hspace{1cm} (47)

to which the trajectories of the closed-loop system converge asymptotically. For the closed-loop CNCEA system, obtained by setting the adaptation gain $\gamma_g$ to zero, there exists a single attractive manifold $\Omega_a$. Of course, for CEA systems, there does not exist any $z$-dependent attractive manifold. This clearly shows a special feature of the designed composite identifier.

**Remark 2:** It is interesting to note that the Lyapunov derivative in Eq. (46) includes two $z$-dependent functions $-||\psi_f z||$ and $-||Wz||$. For this reason $V_c$ decays faster. Thus, it causes a faster convergence of the system’s trajectories.

### 4.4 CNCEA System’s Control Input Torque $u$

For implementation, it is necessary to determine the actual control torque vector $u$ from the filtered signal $u_f$. The control law Eq. (25) provides a filtered signal:

$$u_f = -\psi_f \dot{\hat{\theta}} = -\psi_f [\hat{\phi}_g + \beta]$$

In view of Eqs. (20) to (22), $u$ can be written as:

$$u = \dot{u}_f + \alpha u_f$$

$$= -\psi_f \dot{\hat{\theta}} - \psi_f \dot{\hat{\theta}} - \alpha \psi_f \dot{\hat{\theta}}$$
\[ -\Psi p - \Psi f \dot{p} \]  

Now, one computes the derivative of \( \dot{p} = \dot{\hat{p}}_{Ig} + \beta \). The derivative of \( \beta \) is:

\[ \dot{\beta} = \gamma \{ \Psi^T f \omega_{ef} + \Psi^T f (-\alpha \omega_{ef} + \omega_e) \} \quad (49) \]

Using Eq. (49) and the derivative of \( \dot{\hat{p}}_{Ig} \) from Eq. (41) gives:

\[ \dot{\hat{p}} = \dot{\hat{p}}_{Ig} + \dot{\beta} = \gamma \Psi^T f \left( (k_2 - \alpha) \omega_{ef} + k_3 A_1^T \dot{\theta} + \omega_e \right) - \gamma g W^T \dot{u}_f \quad (50) \]

Finally, substituting equation (50) in (48) gives:

\[ u = -\Psi (\dot{\hat{p}}_{Ig} + \beta) - \gamma \Psi \dot{\Psi}_f^T \left( (k_2 - \alpha) \omega_{ef} + k_3 A_1^T \dot{\theta} + \omega_e \right) + \gamma g \Psi_f W^T \dot{u}_f \quad (51) \]

where for implementation, one uses:

\[ \dot{\alpha}_f = \dot{u}_f - u_f = (W + \Psi_f) \dot{p} = (W + \Psi_f)(\dot{\hat{p}}_{Ig} + \beta) \quad (52) \]

in Eq. (51). It is interesting to note that the control law \( u \) in Eq. (51) utilizes both the tracking error \( \dot{\theta} \), as well as the model prediction error \( \dot{u}_f \). This control signal differs from the control input computed for the NCEA law in Ref. [30] for the control of Euler angles. This completes the control law derivation.

5 Simulation Results

This section presents numerical results. The satellite is assumed to be in an orbit around asteroid 433 Eros. The spacecraft’s principal moments of inertia are \((J_x, J_y, J_z) = (110, 115, 100) \text{ [Kg m}^2]\), and its mass is 600 [kg]. The remaining parameters of the model given in [14] are used for simulation. These are: \( r_0 = 9.933 \text{ [km]}, C_{20} = -0.0878, \text{ and } C_{22} = 0.0439 \). The asteroid’s gravitational parameter and its rotation rate used for simulation are \( \mu = 4.4650 \times 10^{-4} \text{ [km}^3/\text{s}^2] \) and \( \Omega = 3.312 \times 10^{-4} \text{ [rad/s]} \), respectively. Assuming that satellite is in an elliptical orbit, its radial distance from the asteroid’s center of mass is:

\[ R_c(\eta) = \frac{a(1 - e^2)}{1 + e \cos \eta} \quad (53) \]

where \( a \) is the semi-major axis, and \( e \) is the eccentricity. The orbital rate is:

\[ \dot{\eta} = \left( \sqrt{\frac{\mu}{p_h}} \right) (1 + e \cos \eta)^2 \quad (54) \]

where the semilatus rectum is \( p_h = a(1 - e^2) \).

For smooth attitude regulation, reference attitude angle trajectories \( \theta_{ri}, (i = 1, 2, 3) \), were generated by a fourth-order command generator of the form:

\[ (s^2 + 2 \zeta_1 \omega_{r1}s + \omega^2_{r1})(s^2 + 2 \zeta_2 \omega_{r2}s + \omega^2_{r2})\theta_{ri}(t) = \omega^2_{r1} \omega^2_{r2} \theta^*_{ri}(t) \quad (55) \]

where \( \zeta_1 = \zeta_2 = 1 \), and \( \omega_{r1} = \omega_{r2} = 0.001 \). The initial values are: \( \theta_{ri}(0) = \theta_i(0), \dot{\theta}_{ri}(0) = \dot{\theta}_i(0) = d^3 \theta_{ri}(0)/dt^3 = 0 \). Here, for nadir pointing, \( \theta^*_{ri} \) is set to zero.

The initial attitude angles and angular velocities were assumed to be \( (\theta(0) = (60, 60, 50)^T \text{ [deg]}), \text{ and } \omega(0) = (0.0004, 0.0004, 0.0004)^T \text{ [rad/sec]} \), respectively. The initial value of parameter estimate was arbitrarily set as \( \hat{p}(0) = 0.0x_1 \). The filter initial values were \( \omega_{ef}(0) = 0.0x_1 \) and \( \Psi_f(0) = 0.00x_9 \).

The controller gains were set as \( k_i = 0.1 \) \((i = 1, 2, 3)\), and the adaptation gain was assumed to be \( \gamma = 1 \). The parameter \( \alpha \) of the filter was chosen to satisfy \( k_2 + k_3 - \alpha = 0 \). Different values of \( \gamma \in \{0, 10^3, 10^4\} \) will be considered for simulation to examine the effect of the gradient-based integral update law on the responses. These design parameters were selected by the observation of simulated responses.
5.1. Composite adaptive attitude control in a prograde elliptic orbit: $e = 0.2, a = 30 \text{ [km]}$

The closed-loop system including the attitude dynamics (Eqs. (1) and (2)), the control law (Eq. (51)), and the adaptation law (Eq. (41)) was simulated. First, the adaptation gain $\gamma_g$ was chosen to be $10^3$. The responses are shown in Fig. 2. It can be seen that the attitude angles $\theta_i, (i = 1, 2, 3)$, tracking error vector $\tilde{\theta}$, and $\omega_{ef}$ converge to zero. The maximum control torque is 0.01 (Nm). The response time is of the order of three hours. It can also be seen that $\Psi_f z$ and $W_z$ converge to zero, as expected.

5.2. Composite adaptive attitude control in a prograde elliptic orbit: $\gamma_g = 10^4, e = 0.2, a = 30 \text{ [km]}$

To examine the effect of adaptation gain $\gamma_g$, it was set as $\gamma_g = 10^4$. The remaining parameters and initial conditions of Case 5.1 were retained. Selected responses are shown in Fig. 3. Again, the attitude angles converged to zero. One observes that the larger value of $\gamma_g$ causes a drastic reduction in the transient period in which the attitude angle trajectories undergo oscillations. Of course, the control magnitude is larger because the response time is shorter compared to Case 5.1. Additionally, one observes faster convergence of the trajectories to the manifolds $\Omega_a$ and $\Omega_b$.

5.3. Attitude Control using NCEA law in a prograde elliptic orbit: $\gamma = 0, e = 0.2, a = 30 \text{ [km]}$

Finally, the closed-loop system, including the simplified adaptation law obtained by using $\gamma_g = 0$, was simulated. It is noted that by setting $\gamma_g = 0$, the control law becomes an NCEA law, designed based on the I&I principle. Therefore, the gradient algorithm based term $\gamma_g W^T \tilde{u}_f$ is not used in the integral update law Eq. (41). The remaining parameters of Case 5.1 were retained. Fig. 4 show the responses. Although the attitude angles converge to zero, the response time has increased. Of course, the NCEA system of Case 5.3 does not have the attractive manifold $\Omega_b$ to which the trajectories of the closed-loop system can converge.

The performance characteristics of these three cases are summarized in Table 1. It is seen in Table 1 that the control law with $\gamma_g > 0$ requires a larger peak value of the control magnitude, but the maximum tracking error in the steady state (beyond 4 hours) is smaller, compared with the control law using $\gamma_g = 0$. Thus, the composite NCEA law achieves a smaller settling time, compared with the simplified control law with $\gamma_g = 0$. In addition, the composite control system gives smaller peak values of the tracking error $e_{max}$ and $||\Psi_f z||$. Apparently, the composite NCEA system has additional flexibility in achieving desirable trade-offs among the response characteristics.

6 Conclusion

In this paper, based on the I&I theory, the design of a composite noncertainty-equivalence adaptive (CNCEA) attitude control law for an asteroid-orbiting spacecraft was considered. It was assumed that the spacecraft’s inertia matrix and asteroid’s gravitational parameters were not known. A backstepping design method was used to derive the CNCEA law. The composite adaptation law developed here provides parameter estimates based on the information on the tracking error and the model prediction error. Of course, the full parameter estimate includes a nonlinear algebraic vector function, as well as an integral component. The Lyapunov stability analysis established asymptotic convergence of roll, pitch, and yaw angles to the origin. Simulation results were presented for the attitude control of a spacecraft orbiting in the vicinity of the asteroid 433 Eros. These results showed precise nadir pointing attitude regulation, despite uncertainties in the parameters and large angle rotational maneuvers. Furthermore, simulation results confirmed better performance of the CNCEA law, compared with the NCEA law. Additionally, it was seen that adaptation gains of the composite identifier play important role in shaping the responses.

7 Disclosure of potential conflicts of interest - ethical and financial
Conflict of Interest: The authors declare that they have no conflict of interest.
References

1. Werner, R. A., and Scheeres, D. J., “Exterior Gravitation of a Polyhedron Derived and Compared with Harmonic and Mascon Gravitation Representations of Asteroid 4769 Castalia,” Celestial Mechanics and Dynamical Astronomy, Vol. 65, 1996, pp. 313-344.

2. Herrera-Sucarrat, E., Palmer, P. L., Roberts, R. M., "Modeling the Gravitational Potential of a Non-spherical Asteroid," J. Guidance Control Dyn., Vol. 36, 2013, pp. 790-798.

3. Chauvineau, B., Farinella, P., Mignard, F., “Planar Orbits about a Triaxial Body: Application to Asteroidal Satellites,” Icarus, Vol. 105, 1993, pp. 370-384.

4. Scheeres, D. J., Williams, B. G., Miller, J. K., “Evaluation of the Dynamic Environment of an Asteroid: Applications to 433 Eros,” J. of Guidance, Control, and Dynamics, Vol. 23, 2000, pp. 466-475.

5. Tricarico, P., Sykes, M. V., “The Dynamical Environment of Dawn at Vesta,” Planetary and Space Science, Vol. 58, 2010, pp. 1516-1525.

6. Yang, H., Bao, H., “Fuel-Optimal Control for Soft Landing on an Irregular Asteroid,” IEEE Trans. on Aerospace and Electronic Systems, Vol. 51, 2015, pp. 1688-1697.

7. Guelman, M., “Closed-Loop Control of Close Orbits around Asteroids,” J. Guidance Control Dyn., Vol. 38, 2015, pp. 894-860.

8. Guelman, M., “Closed-Loop Control for Global Coverage and Equatorial Hovering about an Asteroid,” Acta Astronautica, Vol. 137, 2017, pp. 353-361.

9. Furfaro, R., Cersosimo, D., Wibben, D. R., “Asteroid Precision Landing via Multiple Sliding Surfaces Guidance Techniques,” J. of Guidance Control Dyn., Vol. 36, 2013, pp. 1075-1092.

10. Furfaro, R., "Hovering in Asteroid Dynamical Environments Using Higher-Order Sliding Control," J. of Guidance Control Dyn., Vol. 38, 2015, pp. 263-279.

11. Yang, H., Bai, X., Bao, H., “Finite-Time Control for Asteroid Hovering and Landing via Terminal Sliding-Mode Guidance,” Acta Astronautica, Vol. 132, 2017, pp. 78-89.

12. Gui, H., Ruiter, A. H. J., “Control of Asteroid-Hovering Spacecraft with Disturbance Rejection Using Position-Only measurements,” J. Guidance Control Dyn. 40 (2017) 2401-2416.

13. Lee, K. W., Singh, S. N., “Adaptive and Supertwisting Adaptive Spacecraft Orbit Control Around Asteroids,” J. Aerospace Eng., Vol. 32, No. 4, (14 pages), July 2019.

14. Misra, A. K., Panchenko, Y., “Attitude Dynamics of Satellites Orbiting an Asteroid,” Journal of Astrodynamics, Vol. 54, 2006, pp. 369-381.

15. Riverin, J. L., Misra, A. K., “Attitude Dynamics of Satellites Orbiting Small Bodies,” in: Proceedings of AIAA/AAS Astrodynamics Specialist Conference and Exhibit, AIAA2002-4520, Monterey, CA, 2002.

16. Kumar, K. D., “Attitude Dynamics and Control of Satellites Orbiting Rotating Asteroids,” Acta Mechanica, Vol. 198, 2008, pp. 99-118.

17. Wang, Y., Xu, S., “Analysis of the Attitude Dynamics of a Spacecraft on a Stationary Orbit around an Asteroid via Poincare Section,” Aerospace Science and Technology, Vol. 39, 2014, 538-545.

18. Kikuchi, S., Howell, K. C., Tsuda, Y., Kawaguchi, J., “Orbit-Attitude Coupled Motion around Small Bodies: Sun-Synchronous Orbits with Sun-Tracking Attitude Motion,” Acta Astronautica, Vol. 140, 2017, pp. 34-48.

19. Wang, Y., Xu, S., “Equilibrium Attitude and Stability of a Spacecraft on a Stationary Orbit around an Asteroid,” Acta Astronautica, Vol. 84, 2013, pp. 99-108.

20. Bandyopadhyay, S., Chung, S. J., Hadaegh, F. Y., “Nonlinear Attitude Control of Spacecraft with a Large Captured Object,” J. of Guidance, Control and Dynamics, Vol. 39, 2016, pp. 754-769.

21. Lee, D., Sanyal, A. K., Butcher, E. A., Scheeres, D. J., “Almost Global Asymptotic Tracking Control for Spacecraft Body-Fixed Hovering over an Asteroid,” Aerospace Science and Technology, Vol. 38, 2014, pp. 105-115.

22. Lee, D., Sanyal, A. K., Butcher, E. A., Scheeres, D. J., “Finite-Time Control for Spacecraft Body-Fixed Hovering over an Asteroid,” IEEE Trans. Aerosp. Electronic Systems, Vol. 51, 2015, pp. 506-520.

23. Lee, D., Vukovich, G., “Adaptive Sliding Mode Control for Spacecraft Body-Fixed Hovering in the Proximity of an Asteroid,” Aerospace Science and Technology, Vol. 46, 2015, pp. 471-483.

24. Vukovich, G., Gui, H., “Robust Adaptive Tracking of Rigid-Body Motion with Application to Asteroid Proximity Operations,” IEEE Trans. on Aerospace and Electronic Systems, Vol. 53, 2017, pp. 419-430.
25. Wang, Y., Xu, S., "Body-Fixed Orbit-Attitude Hovering Control over an Asteroid Using Non-Canonical Hamiltonian Structure," *Acta Astronautica*, Vol. 117, 2015, pp. 450-468.
26. Astolfi, A., Ortega, A. R., “Immersion and Invariance, A New Tool for Stabilization and Adaptive Control of Nonlinear Systems,” *IEEE Trans. Autom. Control*, Vol. 48, 2003, pp. 590-606.
27. Astolfi, A., Karagiannis, D., Ortega, A. R., *Nonlinear and adaptive control with applications*, Springer-Verlag, London, 2008.
28. Zhang, B., Cai, Y., "Immersion and Invariance Based Adaptive Backstepping Control for Body-Fixed Hovering over an Asteroid," *IEEE Access*, Vol. 7, 2019, pp. 34850-34861.
29. Lee, K. W., and Singh, S. N., “Immersion and Invariance-Based Adaptive Control of Asteroid-Orbiting and Hovering Spacecraft," *Journal of Astronautical Sciences*, Vol. 66, No. 4, 2019, pp. 537-553.
30. Lee, K. W., and Singh, S. N., "Noncertainty-Equivalence Adaptive Attitude Control of Satellite Orbiting Around an Asteroid," *Acta Astronautica*, Vol. 161, 2019, pp. 24-39.
31. Lee, K. W., Singh, S. N., “Quaternion-Based Adaptive Attitude Control of Asteroid-Orbiting Spacecraft Via Immersion and Invariance," *Acta Astronautica*, Vol. 167, 2020, pp. 164-180.
32. Seo, D., Akella, M. R. “High-Performance Spacecraft Adaptive Attitude-Tracking Control through Attracting-Manifold Design,” *J. Guid., Contr., Dynamics*, Vol. 31, 2008, pp. 884-891.
33. Ioannou, P., Sun, J., *Robust Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ, 2013.
34. Duarte, M. A., Narendra, K. S., “Combined Direct and Indirect Approach to Adaptive Control,” *IEEE Trans. Autom. Control*, Vol. 34, No. 10, (1989, pp. 1071-1075.
35. Slotine, J.-J. E., Li, W., “Composite Adaptive Control of Robot Manipulators,” *Automatica*, Vol. 25, No. 4, 1989, pp. 509-519.
36. Patre, P. M., Bhasin, S., Wilcox, Z. D., Dixon, W. E., “Composite Adaptation for Neural Network-Based Controllers,” *IEEE Trans. Autom. Control*, Vol. 55, No. 4, pp. 944-950. (2010)
37. Lavretsky, E., “Combined/Composite Model Reference Adaptive Control,” *IEEE Trans. Autom. Control*, Vol. 54, No. 11, 2009, pp. 2692-2697.
38. Liu, Z., Yuan, R., Fan, G., Yi, J., “Immersion and Invariance Based Composite Adaptive Control of Nonlinear High-Order Systems,” *Proceedings of 2018 Chinese Control and Decision Conf.*, IEEE, Vol. 96-101, 2018.
39. Lee, K. W., Singh, S. N., “Generalized Composite Noncertainty-Equivalence Adaptive Control of Orbiting Spacecraft in Vicinity of Asteroid," *Journal of Astronautical Sciences*, Vol. 67, 2020, pp. 1021-1043.
40. Lee, K. W., Singh, S. N., “Generalized Composite Noncertainty-Equivalence Adaptive Control of a Prototypical Wing Section with Torsional Nonlinearity," *Nonlinear Dynamics*, Vol. 103, 2021, pp. 2547-2561.

| Fig.2: $\gamma = 1$, $\gamma_g = 10^3$ | $\omega_{max}$ | $\omega_{max}(\phi)$ | $\omega_{max}(\theta)$ | $\omega_{max}(\gamma)$ | $\Phi_{max}$ | $|W_{max}|_{\infty}$ | $|W_{max}|_{\infty}(\phi)$ | $|W_{max}|_{\infty}(\theta)$ | $|W_{max}|_{\infty}(\gamma)$ | $|W_{max}|_{\infty}(\Phi)$ |
|---------------------------------|----------------|----------------|----------------|----------------|---------|----------------|----------------|----------------|----------------|----------------|
| 0.01070 | 16.61663 | 0.14210 | 0.77420 | 0.15501 | 0.00024 | 3.7065E-07 |
| Fig.3: $\gamma = 1$, $\gamma_g = 10^4$ | 1.3065E-1 | 1.09659 | 6.21630 | 0.6524 | 4.0075E-06 | 3.7062E-07 |
| Fig.3: $\gamma = 1$, $\gamma_g = 0$ | 0.00550 | 17.09380 | 0.12172 | 6.99100 | 0.00202 | 4.5008E-07 |
Fig. 1. Spacecraft in elliptic orbit around asteroid
Fig. 2. Attitude control using CNCEA law: $\gamma = 1, \gamma_g = 10^3, k_i = 0.1, (i = 1, 2, 3)$
Fig. 3. Attitude control using CNCEA law: $\gamma = 1, \gamma_g = 10^4, k_i = 0.1, (i = 1, 2, 3)$
Fig. 4. Attitude control using NCEA law: $\gamma = 1, \gamma_g = 0, k_i = 0.1, (i = 1, 2, 3)$