New solution to the motion acceleration disturbance in MEMS-based attitude and heading reference system

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Abstract. The attitude and heading reference system based on microinertial measurement unit is becoming increasingly important in the fields of vehicle navigation, autonomous underwater vehicles, and unmanned aircraft systems. However, motion acceleration disturbance always affects attitude fusion, thereby making the study of compensation method for attitude fusion in motion important. In this work, two independent extended Kalman filters with clear structure and easy control are used to calculate attitude fusion. The linear process of the measurement equations in this structure is avoided and is more intuitive than the Euler-angle or Quaternion method. Subsequently, this work describes the motion convergence model for suppressing motion acceleration disturbance. This model is designed on the basis of gyro compensation and motion statistical analysis. First, the concrete structure of the model is introduced, and the feasibility of the scheme is analyzed. Second, the influence of small gradient acceleration on the model is elucidated. Finally, vehicle experimental results show that the model can effectively reduce motion acceleration disturbance and reflect the real attitude changes inside the vehicle body.

1. Introduction

Many previous works on microinertial measurement unit (MIMU) attitude fusion have been conducted. These works used a complementary filter [1-5] or Kalman filter [6-10] as the basic model and then suppressed the horizontal angle error by using the piecewise control method. Several sensors, such as airspeed sensor [11], vision image [12], or GPS [13-14], can also be used as aid. The key part in this research is the identification of motion acceleration. However, existing methods are not ideal on some common plane motions, such as ground and water vehicles.

The fusion algorithm in this study consists of two sets of independent extended Kalman filters for full-attitude resolution. This method directly uses the measurement values of the accelerometer and magnetic sensor as the state variables, which do not need to linearize the observation equations. The use of this algorithm is more intuitive than the use of Euler angles or Quaternion as state variables. This study first analyzes the relationship of accelerometer measurement accuracy and motion identification to solve the motion acceleration disturbance problem. Subsequently, the validity of the motion identification condition is discussed. Finally, the motion state convergence model is designed on the basis of gyro compensation and motion state statistical analysis. In the vehicle experiment, the solution achieves satisfactory results.
2. Attitude fusion method based on the extended Kalman filter
In terms of the amount of calculation, the complementary filter method is more concise, but the extended Kalman filter method can be properly improved to achieve certain robustness and ensure estimation accuracy for complicated situations. The complementary filter method needs to adaptively control K value \[15\] by designing higher-order low-pass and high-pass filters to respond to the same complicated situation \[16\]. Thus, the computational complexities of the two methods are approximately the same in this case.

The state and measurement equations of the accelerometer and magnetic sensor are

\[
\begin{align*}
g^b &= -\omega_{\alpha b}^g b^b \\
n^b &= a - g^b + n_1
\end{align*}
\]

\[ (1) \]

\[
\begin{align*}
M^b &= -\omega_{\alpha b}^M b^K b^b \\
M^b &= M^b + n_2
\end{align*}
\]

\[ (2) \]

where \(g^b\) and \(M^b\) are the gravity and magnetic field components in the body-frame coordinate system, respectively; \(\omega_{\alpha b}^M = \begin{bmatrix} 0 & -\omega_{\beta c}^b & \omega_{\gamma c}^b \\ -\omega_{\delta c}^b & 0 & -\omega_{\epsilon c}^b \\ -\omega_{\epsilon c}^b & \omega_{\delta c}^b & 0 \end{bmatrix}\) is the antisymmetric matrix constructed by gyro angular rates \(\omega_{\alpha b}^b = [\omega_{\alpha c}^b \omega_{\beta c}^b \omega_{\gamma c}^b] \); \(f^b\) and \(M^b\) are the specific force vector and the magnetic field strength vector, respectively; \(a\) is the carrier motion acceleration vector; \(g^b\) is the gravity component in the body-frame; \(M^b\) is the true magnetic field component in the body-frame; and \(n_1\) and \(n_2\) are the noise vectors of the accelerometer and magnetic sensor, respectively.

The state equation of the accelerometer is taken as an example, and the equation is discretized. The angular increment is assumed to correspond to the same sampling interval time \(T = t_k - t_{k-1}\), and the solution of the homogeneous linear equation \((g^b = -\omega_{\alpha b}^g g^b)\) is

\[
g^b(t_k) = e^{-\omega_{\alpha b}^g dt} \cdot g^b(t_{k-1})
\]

\[ (3) \]

Where \(\Delta\Theta = \int_{t_{k-1}}^{t_k} \omega_{\alpha b}^g dt = \int_{t_{k-1}}^{t_k} \begin{bmatrix} 0 & -\omega_{\beta c} & \omega_{\gamma c} \\ \omega_{\delta c} & 0 & -\omega_{\epsilon c} \\ -\omega_{\epsilon c} & \omega_{\delta c} & 0 \end{bmatrix} dt = \begin{bmatrix} 0 & -\Delta\theta_x & \Delta\theta_y \\ \Delta\theta_y & 0 & -\Delta\theta_z \\ -\Delta\theta_z & \Delta\theta_y & 0 \end{bmatrix}\), \(\Delta\theta_x, \Delta\theta_y, \Delta\theta_z\) are the angular increments of the gyro in the sampling interval \([t_{k-1}, t_k]\). Equation(3) is expanded by Taylor series as follows:

\[
g^b(t_k) = e^{-\omega_{\alpha b}^g \cdot \Delta\Theta} \cdot g^b(t_{k-1}) = I + \frac{-\Delta\Theta}{1!} + \frac{(-\Delta\Theta)^2}{2!} + \ldots \cdot g^b(t_{k-1})
\]

\[ (4) \]

The second-order approximation is taken as

\[
g^b(t_k) = I + \frac{-\Delta\Theta}{1!} + \frac{(-\Delta\Theta)^2}{2!} \cdot g^b(t_{k-1})
\]

\[ (5) \]
Where \(-\Delta \theta\)^2 = \begin{bmatrix}
-\Delta \theta_1^2 + \Delta \theta_2^2 & \Delta \theta_1 \Delta \theta_2 & \Delta \theta_1 \Delta \theta_3 \\
\Delta \theta_1 \Delta \theta_2 & -\Delta \theta_1^2 + \Delta \theta_3^2 & \Delta \theta_2 \Delta \theta_3 \\
\Delta \theta_1 \Delta \theta_3 & \Delta \theta_2 \Delta \theta_3 & -\Delta \theta_1^2 + \Delta \theta_2^2 
\end{bmatrix}.

After calibrating the gyroscope is in the laboratory, its angular rate output can be expressed as

\[
\omega_{ab}^b = \bar{\omega}_{ab}^b + \varepsilon^b + w^b
\]

Where \(\bar{\omega}_{ab}^b\) is the true angular rate; \(\varepsilon^b\) is the zero drift after power; \(w^b\) is the gyroscope measurement noise. Equation (6) is substituted for Equation (5), the effective value of the angular rate is separated from the measurement noise term, and the influence of the two-order term on \(w^b\) is neglected, thereby yielding:

\[
\begin{bmatrix}
g_x^b(k) \\
g_y^b(k) \\
g_z^b(k)
\end{bmatrix} = \Phi_{k,k-1} \begin{bmatrix}
g_x^b(k-1) \\
g_y^b(k-1) \\
g_z^b(k-1)
\end{bmatrix} + G(k-1)w^b
\]

\[
\Phi_{k,k-1} = \begin{bmatrix}
-\Delta \theta_1^2 + \Delta \theta_2^2 & \Delta \theta_1 \Delta \theta_2 & \Delta \theta_1 \Delta \theta_3 \\
\Delta \theta_1 \Delta \theta_2 & -\Delta \theta_1^2 + \Delta \theta_3^2 & \Delta \theta_2 \Delta \theta_3 \\
\Delta \theta_1 \Delta \theta_3 & \Delta \theta_2 \Delta \theta_3 & -\Delta \theta_1^2 + \Delta \theta_2^2 
\end{bmatrix}, \quad G(k-1) = \begin{bmatrix}
0 & -g_x^b(k-1) & g_y^b(k-1) \\
g_x^b(k-1) & 0 & -g_z^b(k-1) \\
g_y^b(k-1) & g_z^b(k-1) & 0 
\end{bmatrix}
\]

When the motion acceleration is near zero, the measurement data of the accelerometer are valid, and the discrete measurement equation is

\[
Z^k = \begin{bmatrix}
g_x^a(k) \\
g_y^a(k) \\
g_z^a(k)
\end{bmatrix} + R_a
\]

Where \(R_a\) is the noise variance matrix of the accelerometer. This step do not need to linearize the observation equation. After the aforementioned analysis, the discrete Kalman filter model of the accelerometer and gyroscope is expressed as

\[
\begin{bmatrix}
g_x^a \\
g_y^a \\
g_z^a
\end{bmatrix} = \Phi_{k,k-1} \cdot \begin{bmatrix}
g_x^a \\
g_y^a \\
g_z^a
\end{bmatrix} + Q_{g,k-1}
\]

\[
Z^k = g^a + R_g
\]

Where \(Q_{g,k-1} = L \cdot W \cdot L^T = G(k-1)W = \text{diag}[\sigma_x^2, \sigma_y^2, \sigma_z^2]\), and \(\sigma_x^2, \sigma_y^2, \sigma_z^2\) are the noise variances of a three-axis gyroscope.

Similarly, the discrete Kalman filter model of the magnetic sensor and gyroscope is expressed as

\[
\begin{bmatrix}
M_x^k \\
M_y^k \\
M_z^k
\end{bmatrix} = \Phi_{k,k-1} \cdot \begin{bmatrix}
M_x^k \\
M_y^k \\
M_z^k
\end{bmatrix} + Q_{m,k-1}
\]

\[
Z_{m,k} = M^k + R_m
\]
Where \( Q_{n,k} = L_n \cdot \mathbf{W} \cdot (L_n) \), \( L_n = \begin{bmatrix} 0 & -M_y^{n,k}(k-1) & M_x^{n,k}(k-1) \\ M_y^{n,k}(k-1) & 0 & -M_z^{n,k}(k-1) \\ -M_x^{n,k}(k-1) & M_z^{n,k}(k-1) & 0 \end{bmatrix} \), and \( R_{n,k} \) is the noise variance matrix of the magnetic sensor.

According to the transformation relations between the navigation-frame coordinate system and the body-frame \( g^b = C^b_n g^n \), where \( C^b_n \) is the direction cosine matrix of the navigation-frame to the body-frame, and \( g^b \) and \( g^n \) are the gravity components in the body-frame and navigation-frame, respectively. thereby yielding

where \( g = g_z^n \) is the scalar value of the gravity acceleration. The formula is simplified as

\[
\begin{align*}
g^b_x &= -g \sin \theta \\
g^b_y &= g \cos \theta \sin \phi \\
g^b_z &= g \cos \theta \cos \phi
\end{align*}
\]

(11)

The pitch (\( \theta \)) and roll (\( \phi \)) angle of the carrier is given by Equation (11), and the results are presented as follows:

\[
\begin{align*}
\theta &= -\arcsin(g^b_y / g) \\
\phi &= \begin{cases} 
\pi + \arctan(g^b_y / g^b_z), & g^b_y > 0 \text{ and } g^b_z < 0 \\
\pi / 2, & g^b_y > 0 \text{ and } g^b_z = 0 \\
\arctan(g^b_y / g^b_z), & g^b_y > 0 \\
-\pi / 2, & g^b_y < 0 \text{ and } g^b_z = 0 \\
-\pi + \arctan(g^b_y / g^b_z), & g^b_y < 0 \text{ and } g^b_z < 0 \end{cases}
\end{align*}
\]

(12)

The projection output of the magnetic sensor in the magnetic navigation-frame is \( M^b = \begin{bmatrix} M_x & 0 & M_y \end{bmatrix} \), and the projection output in the body-frame is \( M^b = \begin{bmatrix} M_x & M_y & M_z \end{bmatrix} \). The pitch and roll angle obtained by Equation (12) are substituted into \( M^b = C^b_n |_{\psi = \psi_m} \cdot M^n \), where \( C^b_n |_{\psi = \psi_m} \) is the direction cosine matrix of the magnetic navigation-frame to the body-frame, and \( \psi_m \) is the magnetic azimuth angle, that is:

\[
\begin{bmatrix} M_x^b \\ M_y^b \\ M_z^b \end{bmatrix} = \begin{bmatrix} M_x \cos \theta \cos \psi_m - \sin \theta M_y \\ (\sin \phi \sin \theta \cos \psi_m - \cos \phi \sin \psi_m)M_x + \sin \phi \cos \theta M_y \\ (\cos \phi \sin \theta \cos \psi_m + \sin \phi \sin \psi_m)M_x + \cos \phi \cos \theta M_y \end{bmatrix}
\]

(13)

Equation (13) is simplified as

\[
\begin{align*}
M_x \cos \psi_m &= M_x^b \cos \theta + M_y^b \sin \theta \sin \phi + M_z^b \sin \cos \phi \\
-M_x \sin \psi_m &= M_x^b \cos \phi - M_y^b \sin \phi
\end{align*}
\]

(14)

According to Equation (14), the magnetic azimuth angle can be given by
\[
\begin{align*}
\psi_m &= \begin{cases} 
\arctan(-Y_H / X_H), & X_H > 0 \text{ and } Y_H \leq 0 \\
\pi/2, & X_H = 0 \text{ and } Y_H < 0 \\
\pi + \arctan(-Y_H / X_H), & X_H < 0 \\
3\pi/2, & X_H = 0 \text{ and } Y_H > 0 \\
2\pi + \arctan(-Y_H / X_H), & X_H > 0 \text{ and } Y_H > 0
\end{cases} 
\end{align*}
\]

(15)

Where \( X_H = M_H \cos \theta + M_z \sin \theta \sin \phi + M_y \sin \theta \cos \phi \), and \( Y_H = M_y \cos \phi - M_x \sin \phi \). The magnetic North Pole is not completely consistent with the geographical North Pole; hence, correcting the measured azimuth is necessary. If the magnetic declination angle is \( \Delta \psi \), then the true azimuth angle is \( \psi = \psi_m + \Delta \psi \).

Two independent extended Kalman filters are used to calculate attitude. When the state parameters are updated, the pitch \( \theta_k \) and roll \( \phi_k \) angles are calculated by using Equation (12). These parameters \((\theta_k, \phi_k, M_k)\) are substituted into Equation (15) to calculate the magnetic azimuth angle \( \psi_m \). Finally, the magnetic azimuth angle is corrected into the true north azimuth angle \( \psi = \psi_m + \Delta \psi \).

This method directly uses the measurement values of the accelerometer and magnetic sensor as the state variables, which do not need to linearize the observation equations. The use of this structure is more intuitive than the use of Euler angles or quaternion as the state variables in the Kalman filter framework. Some papers are relatively simple to derive R (Observation noise) and Q (Process noise) in this respect. Because when using Euler angle or Quaternion as the state variables, it is very difficult to linearly separate the sensor noise from the state variables. At the same time, the linearization process should be at least second order for dynamic measurements. In fact, these decompositions are critical for dynamic measurements.

3. Solution to the motion acceleration disturbance

In the design of the extended Kalman filter, the accelerometer measurement equation is only valid when the motion acceleration is near zero. Therefore, the problem on the accurate identification of the motion acceleration must be solved. This problem is essentially the same as the K setting process of the complementary filter, because the size of K value is set according to the motion acceleration. In many papers, K value is dynamically corrected by piecewise control according to the size of the motion acceleration that can be identified.

3.1. Relationship between motion identification and accelerometer measurement accuracy

Whether it is the dynamic correction of K value in the complementary filter or the effectiveness of the measurement equation in the Kalman filter, a basic judgment formula, \( \| \mathbf{f} \|_2 < \sigma \), which is the comparison of the resultant force of the three-axis accelerometer and the gravity, is needed. Only when \( \| \mathbf{f} \|_2 \) is close to the gravity value will the attitude measurement error of the accelerometer be guaranteed within a certain range. Low-cost MIMU gyroscope and accelerometer are used to analyze the effective range of this judgment formula and the size of the attitude angle error produced. The inertial measurement sensors used in this study are L3G4200D (three-axis gyroscope) and LSM303DLH (three-axis accelerometer and three-axis reluctance) from the ST company. Figure.1 shows the static outputs of the three-axis accelerometer, and 1 s smooth filtering is applied to eliminate high-frequency noise.
Fig. 1 Static output of accelerometer

In Figure 1, the measurement accuracy of the accelerometer became less than 2 mg after eliminating high-frequency noise. According to the basic judgment formula, $\| f \| - g < \sigma$, the identification degree should be within 2 mg, that is, $\sigma = 2\text{mg}$. The attitude and heading reference system (AHRS) is assumed to be horizontally placed in the carrier, and the carrier accelerates along the X-axis. The maximum disturbance acceleration at the X-axis based on the basic judgment formula is $\sqrt{(63.277\text{mg})^2 + 0^2 + (1000\text{mg})^2} - 1000\text{mg} 
\approx 2\text{mg}$, that is, $f_x = 63.277\text{mg}$. According to Equation (12), the pitch angle error that corresponds to this acceleration is $\theta = -\arcsin\left(\frac{63.277\text{mg}}{1000\text{mg}}\right) \approx -3.628^\circ$. This error is mainly due to the excessively large Z-axis weight of the gravity acceleration relative to the X- and Y-axes. When the carrier moves in a horizontal direction, the acceleration of the Z-axis is almost unchanged in motion, and the Z-axis weight of the gravity acceleration is considerably large. Consequently, the acceleration changes in the X-axis (or Y-axis) are not evident in the resultant force, and the final output pitch angle (or roll angle) error is excessively large. The general attitude fusion algorithm is prone to having a large horizontal attitude error angle for this motion type. Fig.2 shows the X-axis acceleration data of the car at random acceleration/deceleration on the horizontal road and the changes in the pitch angle after using the complementary filter.

Fig. 2 Outputs of the X-axis accelerometer and the pitch angle
When the gravity acceleration is basically projected on the Z-axis, the sensitivity of the basic formula \( mgg f' b \ll 2mg \) is rather low for the X-axis motion acceleration. Therefore, the output value of the pitch angle varies with the changes in the X-axis acceleration, and the road surface information cannot be accurately reflected when the vehicle is moving.

3.2. Effective identification condition of the model in motion

When the horizontal attitude of the vehicle is relatively stable, the condition for the effective application of the basic judgment formula \( mgg f' b \ll \sigma^2 (\sigma = 2mg) \) is analyzed. The condition states that the maximum angular error must be less than 1° of that detected in the motion direction. The following conclusion can be drawn through the introduction of the previous section: When the gravity acceleration on the three axes of the accelerometer is evenly distributed, the judgment formula will be highly sensitive to the identification of motion acceleration. Only when the gravity acceleration projection is centered on a certain axis (e.g., Z-axis) will the judgment formula be insensitive to the motion acceleration on other axes (e.g., X- and Y-axes). Thus, the distribution of gravity acceleration on the three axes determines the sensitivity of the judgment formula to motion acceleration. In vehicles, such as two-wheel balanced vehicles and indoor unmanned vehicles, the AHRS is generally placed horizontally in the carrier, and the coordinate X-axis points to the direction of the vehicle movement. The X-axis \( f_x = \min(f_x, f_y, f_z) \) is assumed to be the minimum distribution coordinate axis of gravity acceleration. The minimum acceleration that the basic judgment formula can detect on this coordinate axis is the largest, and the corresponding error angle (pitch) is the largest. In the following analysis, when the error angle is 1° (pitch), the minimum acceleration can be detected in the direction of this coordinate axis.

The motion acceleration generated on the X-axis \( (f_x) \) is assumed to be \( a^*g \); when \( \sigma = 2mg \), it is substituted into the judgment formula as:

\[
\sqrt{(f_x + a^*g)^2 + (f_y)^2 + (f_z)^2 - g^2} < 0.002g
\]  

(16)

The coordinate axis \( (f_x) \) corresponds to the pitch angle. According to Equation (12), when the acceleration is \( a^*g = 17mg \), the corresponding pitch angle error is approximately \( \Delta \theta = -\arcsin(\frac{17mg}{1000mg}) = -1° \). \( a^*g = 17mg \) is thus taken into Equation (16), that is,

\[
f_x < \frac{0.004004 - a^2}{2} \quad g = 0.10926g
\]  

(17)

When \( f_x \geq 0.10926g \), if the motion acceleration on the X-axis is \( a^*g = 17mg \), then \( \sqrt{(f_x + a^*g)^2 + (f_y)^2 + (f_z)^2 - g^2} > 2mg \), that is, the judgment formula can detect the motion acceleration of 17 mg on the coordinate axis \( (f_x) \). The corresponding identification angle is approximately 1°. Actually, if \( f_x = 0.10926g \), according to Equation (12), then the pitch angle of the AHRS is \( \theta = -\arcsin(\frac{109.26mg}{1000mg}) \approx -6.273° \). In other words, if the fluctuation in the road surface is within 6.273°, then the minimum acceleration that can be detected by using the judgment formula will be more than 17 mg, and it cannot guarantee that the motion disturbance angle error (pitch) is less than 1° on the X-axis. Therefore, when the AHRS is used in the ground test, the output attitude angles are inconsistent with the fluctuation in the road surface. At this point, solving the problem on motion acceleration
disturbance by only relying on the judgment formula to modify the K value gain is insufficient. This study will solve this problem through gyro compensation and statistical model.

3.3. Gyro compensation and statistical model
When the AHRS is placed horizontally in the vehicle, the Y-axis and X-axis angular velocities of the gyroscope are insensitive to motion acceleration. In other words, when the accelerometer continuously measures the stationary, acceleration, uniform speed, deceleration, and other motion states of the vehicle, the pitch and roll angles calculated by the gyroscope are almost free from motion acceleration disturbance. Although the gyro attitude calculation contains accumulated errors, it can guarantee certain measurement accuracy in a short time. During this period, if the state that the vehicle does not include the motion acceleration is detected, then the gyro state equation can be updated by using the accelerometer measurement equation. In other words, when the judgment formula is satisfied but still cannot effectively identify the motion acceleration, the gyroscope extends the time for detecting the motion state of the vehicle. The first step is to determine the time length.

3.4. Time factor analysis of gyro compensation
The angular velocity of the gyroscope after calibration is expressed as

\[ \omega_{\text{raw}}^b = \omega_{\text{cor}}^b + \dot{\psi} + w^b \]  \hspace{1cm} (18)

Where \( \omega_{\text{cor}}^b \) is the true angular velocity, \( \dot{\psi} \) is the gyro zero-drift after power, and \( w^b \) is the gyro measurement noise. The gyro measurement noise \( w^b \) belongs to the high-frequency white noise part in the fast drift and can be basically eliminated after integral. The gyro zero-drift \( \dot{\psi} \) belongs to the low-frequency part related to time in the slow drift. Under normal circumstances, the angular velocity error caused by the slow drift is less than \( \pm 0.1^\circ/s \) (the performance of the L3G4200D gyro used in this study). After the analysis, the calibrated gyroscope can guarantee at least 1° cumulative error within 10 s. This time period (i.e., 10 s) will provide the sample support for the accelerometer motion statistical model, in which the output frequency of the accelerometer and gyroscope is 100 Hz. The gyro zero-drift \( \dot{\psi} \) walks in the vicinity of zero at the most time, and this period of time is usually greater than 10 s. In Figure 3, the cumulative attitude increment distribution of every 10 s calculated by the three-axis gyroscope is given, where \( \Delta \text{pitch}, \Delta \text{roll}, \Delta \text{yaw} \) are the accumulated values of the pitch, roll, and azimuth angles every 10 s.

![Figure 3 Incremental value (\( \Delta \text{pitch}, \Delta \text{roll}, \Delta \text{yaw} \)) of 10s](image-url)
In Figure 3, the fluctuation in the accumulated error is not more than 0.5°. If the gyroscope calibration error in the laboratory is less than 0.05°/s, the gyro attitude drift angle guaranteed in 10 s is less than 1°.

3.5. **Probability factor analysis of the statistical model**

The previous analysis indicated that in the range of 1° (the attitude angle error), the gyroscope provides at least 10 s (the time is used for the motion acceleration detection). The acceleration data (10 s) will provide the required sample size for the statistical model of the motion state. Only the samples that pass the statistical model are considered the steady-state samples, in which the samples have high expected probability at the zero-motion acceleration position; otherwise, the samples are the interference samples that have high expected probability in the motion acceleration. Therefore, the statistical model is mainly embodied in the general law for the vehicle movement, that is, the probability distribution of two states (e.g., steady and acceleration states) in the entire movement process. The probability distribution of two states in the general movement is discussed in this work.

The running state of the vehicle is divided into stationary, low acceleration, acceleration, uniform speed, low deceleration, and deceleration. From the perspective of energy consumption, the vehicles with acceleration and deceleration will increase energy consumption, whereas those with stationary, low acceleration, uniform speed, and low deceleration will tend to keep the energy consumption low. Based on experience, drivers could not always keep on accelerating or decelerating, thereby causing the car speed to have very fast growth or the car to stop. The drivers also do not frequently switch acceleration and deceleration in a short time because they could not stand this state. From the angle of energy consumption and drivers’ experience, the entire driving process of the vehicles tends to the steady states, namely, stationary, low acceleration, uniform speed, and low deceleration. The probability of the steady states is greater than that of the non-steady states in the entire driving process. The probability distribution of the two states after collecting a large number of driving samples is given below (Table 1). In the entire process, the AHRS is placed horizontally, the Z-axis is placed vertical downward, and the road surface is basically horizontal. The samples are smoothed for 0.5 s (eliminating vibration noise), and the X-axis and Y-axis of the accelerometer measure the head and lateral accelerations of the vehicle, respectively.

| Table 1. Probability Distribution of the Vehicle-Running State |
|-----------------------------|--------|--------|--------|
| X-axis (mg) | <17 | <25 | <34 | <51 | <68 |
| Probability | 61% | 73% | 81% | 89% | 94% |
| Y-axis (mg) | <17 | <25 | <34 | <51 | <68 |
| Probability | 76% | 85% | 90% | 95% | 98% |

Table 1 shows that the car is running in a relatively stable state most of the time. This statistical rule shows that after the sample passes the statistical model of the motion state, the model will eventually converge to the steady state with the increase in time and sample. In other words, the system exhibits a rough initial state and a certain threshold range. The initial state of the system is constantly updated by the continuous input of the samples. As time goes on, the state value of the final model will fall on the large value of the probability distribution, that is, the steady state. The acceleration/deceleration cycle of these carriers (such as ground-surface vehicles and water-surface vehicles) is usually brief. Consequently, the state convergence process is fast.

3.6. **Design of the state convergence model**

The following condition is based on the analysis in the previous section. The X-axis accelerometer that corresponds to the pitch angle is taken as an example, and the specific scheme is introduced in detail. The output frequency of the entire system is 100 Hz.
Step 1. The initial value. The pitch and roll angles are solved by the gyroscope. If the angular increments are less than 2° in 5 s, the acceleration increments of the X-axis and Y-axis are less than 10 mg in 5 s, and the resultant force of the three-axis acceleration conforms to the basic judgment formula \( \sum|a| < \sigma (\sigma = 2mg) \), then the motion state is considered approximately stationary. The 5 s data of the X-axis accelerometer are mathematically averaged to obtain the rough initial value \( f_{x0} \) of the steady state at this time.

Step 2. The sample data. The 10 s data of the X-axis accelerometer and the attitude incremental data of gyro solution (pitch angle increment) are divided into 10 equal parts. Each part of the data, such as the X-axis accelerometer, is mathematically averaged as \( f_{x1}, f_{x2}, \ldots, f_{x10} \), and the pitch angle increment of the gyro solution is \( \Delta \theta_1, \Delta \theta_2, \ldots, \Delta \theta_{10} \).

Step 3. If the gyroscope pitch angle increment is \( \Delta \theta_1 + \Delta \theta_2 + \ldots + \Delta \theta_{10} \leq 1 \) and \( f_{x1}, f_{x2}, \ldots, f_{x10} \) are verified by the inequality \( f_{x0} - \Delta f < f_x < f_{x0} + \Delta f \), \( f_x = f_{x1}, f_{x2}, \ldots, f_{x10} \), where \( \Delta f = 25mg \) is the width of the steady state value \( f_{x0} \) and the size is determined according to Table 1, then the verified data and \( f_{x0} \) value are mathematically averaged as the next initial value \( f_{x0} = \frac{f_{x0} + f_{x1} + \ldots + f_{x10}}{n+1} \), and the system returns to the second step. As time goes on, according to the large probability event of the steady state, it \( f_{x0} \) will gradually converge to the value with non-motion acceleration disturbance. If the pitch angle increment of gyro calculation is greater than 1°, that is, the change in the pavement’s ups and downs in 10 s can be fully reflected by gyroscope attitude calculation. At this point, the acceleration of the non-motion disturbance should be based on the pavement–slope state value (10th s) as the reference that is, the initial value \( f_{x0} \) and the values of \( f_{x1}, f_{x2}, \ldots, f_{x10} \) are mapped to the 10th s state value. The method is presented as follows: First, the corresponding pitch angles, namely, \( \theta_{x0}, \theta_{x1}, \theta_{x2}, \ldots, \theta_{x10} \) are calculated according to Equation (12). Then, the pitch angle increments \( \Delta \theta_1, \Delta \theta_2, \ldots, \Delta \theta_{10} \) of gyro calculation are used to compensate for the pitch angles, and the new acceleration sample values are obtained as

\[
\begin{align*}
  f'_{x0} &= -g^* \sin(\theta_{x0} + \Delta \theta_1 + \ldots + \Delta \theta_{10}) \\
  f'_{x1} &= -g^* \sin(\theta_{x1} + \Delta \theta_2 + \ldots + \Delta \theta_{10}) \\
  & \vdots \\
  f'_{x10} &= -g^* \sin(\theta_{x10} + \Delta \theta_{10}) 
\end{align*}
\]

Finally, the updated samples are verified by the inequality \( f_{x0} - 25mg < f_x < f_{x0} + 25mg, f_x = f_{x1}, f_{x2}, \ldots, f_{x10} \), the mathematical average of the verified samples and the initial value \( f_{x0} \) is taken as the next initial value \( f_{x0} = \frac{f_{x0} + f_{x1} + \ldots + f_{x10}}{n+1} \), and the system returns to the second step.

Step 4. The pitch angle \( \theta = -\arcsin(f_{x0}/g^*) + \Delta \theta \) of non-motion acceleration disturbance is obtained, where \( \Delta \theta \) is the cumulative increment in the pitch angle (gyro solution) at an output time point. According to the aforementioned design method, we can effectively discriminate the motion acceleration and reduce the influence on the attitude fusion process.

3.7. Influence of small gradient acceleration on the model

In common plane motion carriers, such as vehicles, two-wheel balanced vehicles, and indoor unmanned vehicles, the AHRS is horizontally placed in the vehicles, and the motion acceleration usually acts on the X- and Y-axes. The acceleration/deceleration process is usually a slight gradient
process; consequently, outputting high-precision attitude angle by using the piecewise control method based on the judgment formula \( |r^p - g| < \sigma \) (\( \sigma = 2mg \)) is difficult for AHRS.

In this scheme, the maximum error angle of the horizontal motion direction is first less than 3.628° based on the judgment formula \( |r^p - g| < \sigma \) (\( \sigma = 2mg \)), and the horizontal error angle is then converged to 1° by the gyro compensation and the motion-state convergence model. Here, in the design of the motion-state model, a threshold setting exists at \( \Delta f = 25mg \), which represents the width of the steady-state value \( f_{o,0} \). If the motion acceleration runs in a certain gradual manner within 25 mg, then the steady-state value \( f_{o,0} \) may go along with the gradual change of the motion acceleration.

The gradual change process is elucidated in subsequent parts of this work. If the motion acceleration is always increasing in the gyro compensation time (10s), but the increment is less than 25 mg, then all the samples pass the threshold judgment. According to the statistical average process (without considering the error of the initial steady-state value), the steady-state value \( f_{o,0} \) of the next stage will increase the acceleration error by less than 25 mg. If the vehicle runs in this way, then the steady-state value \( f_{o,0} \) will gradually be far away from the actual value, and the horizontal attitude error angle (pitch) will become increasingly large. However, this motion law is a continuous increase or decrease acceleration process within 10–20s, and the increment or decrement range is no more than 25 mg in 10 s. For several common plane motion carriers, such as vehicles, two-wheel balanced vehicles, and indoor unmanned vehicles, motion behavior is unlikely to occur. The continuous acceleration (or deceleration) process of such vehicles is usually less than 7–8s, and the continuous acceleration (or deceleration) increment is more than 25 mg after 7–8s. Hence, the motion-state convergence model designed in this study can completely adapt to their motion law.

4. Analysis of the vehicle experiment

In this section, we will combine the previous attitude fusion algorithm and the state convergence model to carry out the vehicle test. The attitude and heading reference system (AHRS) is placed horizontally in the vehicle, and it is better to keep away from the engine position for reducing the superposition of the external noise. The X-axis coordinate is consistent with the heading of the vehicle. The attitude and heading reference system (Model: 3DM-S10A) built by the inertial sensor chip (L3G4200D and LSM303DLH, from STMicroelectronics). As shown in Figure 4.

![Fig.4 Attitude and heading reference system (3DM-S10A)](image)

Experiment 1: The vehicle accelerates (or decelerates) along the straight line at the horizontal plane. The new design scheme based on the extended kalman filter and the traditional method based on the complementary filter are compared, mainly compared with the pitch angle change related to the vehicle motion acceleration. Figure 5 is the pitch angle outputs of the traditional complementary filter algorithm (the blue curve), and the pitch angle changes of the gyro solution (the red curve).
Figure 6 is the contrast curve of the pitch angle outputs, where the blue curve is the complementary filter algorithm, the red curve is the new design scheme based on the extended kalman filter algorithm.

In Figure 5 and Figure 6, the traditional complementary filter method is not ideal about suppressing the motion acceleration disturbance, and the output error of the pitch angle is about 3.3 degrees, while the influence of the motion acceleration on the attitude data of gyro calculation is not obvious. Note that in Figure 5, a slightly larger fluctuation with the gyro attitude data is mainly caused by the shock absorption effect during the acceleration or deceleration, which completely reflects the real attitude change inside the vehicle body. In Figure 6, the motion state convergence model designed by the new scheme can suppress the motion acceleration disturbance in the range of 1 degree. The blind spot of the traditional method is solved through this new scheme in low-motion acceleration. Experiment 2: The normal driving does not set the road condition and the movement way, and the pitch angle changes of the different attitude fusion methods are compared. Figure 7 is the vehicle trajectory recorded by the GNSS receiver. Figure 8, the upper part is the pitch angle outputs of two methods and the lower part is the changes of the pitch angle by gyro calculation.
Fig. 7 The trajectory of the vehicle in the experiment

Fig. 8 Pitch angle output of two methods and gyro attitude

In Figure 8, the attitude fusion method designed by the new scheme can suppress the motion acceleration disturbance and accurately reflect the driving attitude changes of the vehicle. It shows that the motion state convergence model in the new scheme is effective. In addition, there are several angular jitter in the changes of gyro attitude data, mainly due to the existence of several damping belts on the road surface.

5. Conclusions
This study describes the state convergence model for suppressing the motion acceleration disturbance. The state convergence model is designed based on gyro compensation and statistical model. The feasibility of the model is determined and verified, and the parameters of the model are analyzed in detail. In addition, the influence of small gradient acceleration on the model is discussed. It shows that the motion state convergence model can completely adapt to the motion law, such as vehicles, two wheel balanced vehicles and indoor unmanned vehicles, etc. The vehicle experimental results show that it is good to reduce the motion acceleration disturbance and completely reflect the real attitude changes inside the vehicle body. The blind spot of the traditional method is solved through this new scheme in low-motion acceleration. In this paper, two independent extended Kalman filters are used for attitude calculation. The measurement values of the accelerometer and magnetic sensor are used as the state variables of the Kalman filter model, so the linear process is avoided on the measurement
equations. This is more intuitive than Euler angles or Quaternion as the state variables. In future studies, we plan to implement the method in surface water vehicle and unmanned aerial vehicle, and compare its performance aspects in depth.

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