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Effective action for the Abelian Higgs model in FLRW

Damien P. George\textsuperscript{1}, Sander Mooij\textsuperscript{2} and Marieke Postma\textsuperscript{3}

\textit{Nikhef,}
\textit{Science Park 105,}
\textit{1098 XG Amsterdam,}
\textit{The Netherlands}

ABSTRACT

We compute the divergent contributions to the one-loop action of the U(1) Abelian Higgs model. The calculation allows for a Friedmann-Lemaître-Robertson-Walker space-time and a time-dependent expectation value for the scalar field. Treating the time-dependent masses as two-point interactions, we use the in-in formalism to compute the first, second and third order graphs that contribute quadratic and logarithmic divergences to the effective scalar action. Working in $R_{\xi}$ gauge we show that the result is gauge invariant upon using the equations of motion.

\textsuperscript{1}dpgeorge@nikhef.nl
\textsuperscript{2}smooij@nikhef.nl
\textsuperscript{3}mpostma@nikhef.nl
1 Introduction

There is only one scalar field in the standard model, but it plays a crucial role. Scalars are more abundant in most extensions of the standard model, such as the multiple Higgs fields in grand unified theories, the superpartners in supersymmetric models, and the moduli fields in extra-dimensional set-ups. To study the physics of the early universe and to test physics beyond the standard model using cosmological data, it is important to have a precise understanding of the scalar field’s dynamics. This requires to go beyond a classical treatment and include the dominant quantum effects.

The one-loop effective action for a scalar field in an expanding universe has been known for a long time \[1, 2, 3, 4\]. It describes the backreaction of the quantum fluctuations of the scalar field on the (time-dependent) background field, which can be calculated systematically in a loop expansion. If the scalar is coupled to other scalars or to fermions via e.g. a Yukawa interaction, additional scalar and fermion loops contribute \[2, 5, 6\]. In this paper we extend these results by including a coupling to a gauge field. That is, we calculate the effective action for a Higgs-like field, which is charged under a gauge symmetry, in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe. For simplicity we focus on an Abelian symmetry, but the results are easily generalized to more general gauge groups. This is the generalization of the Coleman-Weinberg potential \[7\] to time-dependent background fields in a curved space-time.

Our results are of direct importance for inflation models in which the SM Higgs or a GUT Higgs is the inflaton (or a waterfall) field (e.g. \[8, 9\]). The Coleman-Weinberg potential gives the dominant quantum correction during inflation. The time-dependent corrections may become important at the end of inflation, and during the subsequent period of reheating. Another application is the description of flat directions of the MSSM and its extensions \[10\], which are lifted by the one-loop quantum corrections. This may affect inflation models or Affleck-Dine baryogenesis models \[11, 12\] using flat directions.

The effective action for an Abelian gauge theory in de Sitter space-time has been calculated by \[13, 14, 15\] using the Landau gauge. More recently the calculation was done in the \(R_\xi\) gauge, showing gauge invariance of the effective action \[16\]. To obtain this result an adiabatic approximation was made which fails in the \(\xi \to 0\) limit. We extend these results to a generic FLRW space-time and allow for the possibility of time-dependence of the background field, which in a cosmological set-up can be displaced from its potential minimum. The calculation is done in the \(R_\xi\) gauge using a perturbative approach. We calculate the quadratic and logarithmic divergent terms in the ultraviolet (UV) limit, which come from a finite number of diagrams (and which do not depend on the perturbation being small throughout). The resulting effective action is gauge invariant only on-shell, that is after using the classical equations of motion, in agreement with the Nielsen identities \[17, 18\].

Our results agree with the expressions in the literature in the appropriate limit. In the limit of a static background field and a constant Hubble parameter our results agree with \[16\]. In the Minkowski limit we retrieve the effective action calculated in our previous work \[19\], and also the effective equations of motion found earlier in \[20, 21, 22, 23\]. Finally, taking both a static background field and a static background we get the familiar Coleman-Weinberg potential \[7\].

To properly take into account the real-time evolution of the system, and to assure the results are manifestly real, we use the in-in formalism (also known as closed-time-path (CTP) or Schwinger-Keldysh formalism) to calculate the effective action \[24, 25, 26, 27, 28, 29, 30\]. We first derive the one-loop corrected equation of motion for the Higgs field, using the tadpole
All one-loop Feynman-diagrams with one external Higgs leg contribute. For technical reasons it is easier to work in conformal time, as the resulting action has a form similar to the Minkowski action, and all the machinery developed for this \cite{19, 32} can be used. We split all two-point interactions into time-independent and time-dependent parts, treating the former as masses and the latter as interaction terms; in the loop-expansion the result will not depend on this split \cite{7}. The equations of motion can be formally integrated to obtain the effective action up to field-independent terms \cite{33}. Finally, we can rewrite the results in coordinate time.

The effective action is independent of the specific initial conditions chosen. We will argue that this is always the case, for arbitrary initial conditions, provided the initial vacuum is chosen to be that of the free theory. The different vacua can be related via a Bogoliubov transformation \cite{34}.

As already mentioned, we only calculate the UV divergent terms, as these will generically give the dominant contribution. Using a renormalization prescription, these terms (together with the wavefunction renormalization of the gauge field) suffice to derive the renormalization group equations (RGE) and find the RG improved action. We neglect the backreaction on space-time, and treat the FLRW scale factor as classical background field. Finally, we note that to apply the results to models of Higgs inflation, a non-minimal coupling to gravity has to be considered. All this is left however for future work.

In the next subsection we give a self-contained summary of the results. Following this we go through the calculation, starting in Sec. 2 describing the model, the in-in formalism, and giving the vertices and propagators needed to compute Feynman diagrams. In Sec. 3 we calculate the relevant graphs at first, second and third order which contribute to the one-loop equation of motion. These graphs are used in Sec. 4 to compute the effective action for the charged scalar. Here we also present results when additional scalar and fermions run in the loop. Our choice of initial conditions, and their generalization, is discussed in Sec. 5. We conclude in Sec. 6 and provide some further details of the calculation in a pair of appendices.

1.1 Summary of the results

Here we shall outline the model and give the main result of our calculation. It is self-contained so that one need not get caught up in the details of the derivation to make use of the final answer.

The ansatz for the space-time metric is
\[ ds^2 = dt^2 - a^2(t) d\vec{x}^2, \]
with \( a(t) \) the time-dependent scale factor of the FLRW metric. The action is that of an Abelian Higgs model, with a Higgs field charged under a U(1) gauge symmetry
\[ S_{\text{tot}} = \int d^4 x \sqrt{-g} \left[ -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + g^{\mu\nu} D_\mu \Phi (D_\nu \Phi)^\dagger - V(\Phi) \right]. \]

We expand the Higgs into a time-dependent background field (the zero-mode) \( \phi \) plus quantum fluctuations \( h \) and \( \theta \)
\[ \Phi(x^\mu) = \frac{1}{\sqrt{2}} (\phi(t) + h(t, \vec{x}) + i \theta(t, \vec{x})). \]
and in the end go on-shell, which ensures the gauge invariance of the final result. We find, up to background field-independent terms, the UV divergent contributions at one loop to be

\[ \Gamma_{1\text{-loop}} = -\frac{1}{16\pi^2} \int d^3x dt \sqrt{-g} \left[ (\tilde{V}_{hh} + \tilde{V}_{\theta\theta} + 3m_A^2)\Lambda^2 - \left( \tilde{V}_{hh}^2 + \tilde{V}_{\theta\theta}^2 + 3m_A^4 - 6\tilde{V}_{\theta\theta}m_A^2 \right) \ln(\Lambda/\bar{m})^2 \right] \]

where the “shifted scalar mass” is

\[ \tilde{V}_{\alpha\alpha} \equiv V_{\alpha\alpha} - \dot{H} - 2H^2, \]  

which is time-dependent. A subscript on \( V \) denotes a derivative with respect to that field. Further, \( \Lambda \) is the cutoff used in regulating the divergent momentum integrals (it cuts off 3-momentum \( |\vec{k}| < \Lambda \)), and the arbitrary mass \( \bar{m} \) is put in to ensure the argument of the logarithm is dimensionless (recall that we are only interested in divergent contributions to the effective action). \( H = \dot{a}/a \) is the Hubble constant, with a dot denoting a derivative with respect to time \( t \). The (time-dependent) mass of the gauge field is \( m_A^2 = g^2\phi^2 \).

Our result agrees with those found in the literature. For the Minkowski case (\( H = \dot{H} = 0 \), and thus \( \tilde{V}_{\alpha\alpha} = V_{\alpha\alpha} \)) it matches our previous result \[19\]. In the de Sitter limit \( \dot{H} = 0 \), and for a time-independent Higgs field (\( V_{\theta\theta} = 0 \) by Goldstone’s theorem), it agrees with Garbrecht \[16\]. Finally, taking both the Minkowski limit and a static background field, we retrieve the classic Coleman-Weinberg potential \[7\].

If the Higgs field couples to additional scalars \( \chi_\alpha \) and/or fermion fields \( \psi_\beta \), we get an additional contribution

\[ \delta\Gamma_{1\text{-loop}} = -\frac{1}{16\pi^2} \int d^3x dt \sqrt{-g} \left[ \sum_{\chi_\alpha} \left( \tilde{V}_{\alpha\alpha}\Lambda^2 - \tilde{V}_{\alpha\alpha}^2 \ln(\Lambda/\bar{m})^2 \right) \right] \]

\[ - \sum_{\psi_\beta} \left( m_\beta^2\Lambda^2 - (m_\beta^4 - \tilde{V}_{\theta\theta}\bar{m}_\beta^2) \ln(\Lambda/\bar{m})^2 \right) \]

where the shifted scalar mass is given by \[5\]. Here the sum is over all bosonic and fermion real degrees of freedom, where a Weyl (Dirac) fermion counts as 2 (4) degrees of freedom.

## 2 Action and formalism

The starting point for our calculation is the action of a U(1) Abelian-Higgs model in an FLRW background. The background space-time is fixed, in the sense that the backreaction of the charged scalar is assumed negligible. We work in \( R_\xi \) gauge, and therefore include in the action a gauge fixing and a Faddeev-Popov term. We work with a conformal metric, where most expressions take a form reminiscent of the Minkowski calculation. Since the background space-time, as well as the scalar vacuum expectation value (its classical value), are taken to be time-dependent, the “masses” (really two-point interactions) of the particles are also time-dependent. We deal with this by splitting these two-point interactions into a time-independent part, which we call the mass and which determines the propagator, and a time-dependent part, which is treated as a proper two-point interaction in Feynman diagrams.

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4We assume here a Yukawa interaction \( m_\psi \propto \phi \). The expression for a more general mass term is given by \[87\]. We work in a basis where both the mass and kinetic terms are diagonal; this can be easily generalized.
propagators and interaction vertices are defined in the following subsections, along with the in-in formalism for computing expectation values. We use this machinery in the next section to calculate the one-loop equation of motion.

2.1 Notation

We use a metric with signature \((+, -, -, -)\), indexed by lower Greek letters \(\mu, \nu \ldots\), and lower Latin letters for just the 3-space. The Greek letters \(\alpha, \beta, \ldots\), and \(I, J\), index the set of quantum fields. For propagators, covariant derivatives, mode function normalization etc. we use the same conventions as Peskin and Schroeder [35]. Masses \(m\) and frequencies \(\omega\) are split into time-independent and time-dependent parts, with the notation \(m^2(t) = \hat{m}^2 + \delta m^2(t)\), with \(\delta m^2(0) = 0\). A hat above a mass scale denotes the corresponding quantity rescaled by the scale factor: \(\hat{m} = am\). Derivatives with respect to conformal time \(\tau\) are denoted by a prime, and derivatives with respect to coordinate or physical time \(t\) by a dot. In momentum integrals \(\hat{dk} = dk/(2\pi)\).

2.2 The action in an FLRW background

The FLRW metric in physical and conformal coordinates is, respectively,

\[
ds^2 = dt^2 - a^2(t) d\vec{x}^2 = a^2(\tau) \left(d\tau^2 - d\vec{x}^2\right).
\]

The non-zero connections are

\[
\Gamma^i_{0i} = \Gamma^i_{i0} = \Gamma^0_{00} = \Gamma^0_{ii} = \mathcal{H}.
\]

Here we defined \(\mathcal{H} = a'/a\), analogous to the usual definition in coordinate time \(H = \dot{a}/a\). We can decompose the charged scalar field into a real and imaginary part,

\[
\Phi(x^\mu) = \frac{1}{\sqrt{2}} \left(\phi(\tau) + h(\tau, \vec{x}) + i\theta(\tau, \vec{x})\right),
\]

with \(\phi(\tau)\) the time-dependent classical background field. The action is a sum of the kinetic and potential terms, the gauge fixing term and the Faddeev-Popov term:

\[
S_{\text{tot}} = \int d^4x \sqrt{-g}(\mathcal{L} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}),
\]

with

\[
\mathcal{L} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + g^{\mu\nu} D_\mu \Phi (D_\nu \Phi)^\dagger - V(\Phi),
\]

\[
\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi} G^2, \quad G = g^{\mu\nu} \nabla_\mu A_\nu - \xi g(\phi + h) \theta,
\]

\[
\mathcal{L}_{\text{FP}} = \bar{\eta} g \frac{\delta G}{\delta \alpha} \eta.
\]

Note that \(\nabla_\mu g^{\mu\nu} = 0\) (because of metric compatibility), and thus \(g^{\mu\nu} \nabla_\mu A_\nu = \nabla_\mu g^{\mu\nu} A_\nu\) and there is no ambiguity. \(\delta G/\delta \alpha\) is the operator obtained by computing the variation of \(G\) under a \(U(1)\) gauge transformation with infinitesimal parameter \(\alpha\). A bar over \(\eta\) denotes the conjugate, not to be confused with a time-independent quantity.
Our aim is to compute the quantum corrected equation of motion for the background field $\phi$. It can be derived by demanding that all tadpole diagrams with one external $h$-leg vanish. To calculate these diagrams we need to derive both the propagators, which follow from the free and time-independent part of the action, and the interaction vertices, which include time-dependent two-point interactions, as well as three-point interactions. The key to our approach, following [32], is the choice to separate out from the two-point terms the constant, diagonal pieces. This allows us to easily solve for the scalar and gauge propagators, since the masses are then really masses: they are field-diagonal and time independent. The left-over pieces are treated as two-point interaction vertices, and put into graphs following the usual Feynman rules.

To make explicit all factors of the scale factor we now write $g^{\mu\nu} = a^{-2} \eta^{\mu\nu}$ with $\eta^{\mu\nu}$ the Minkowski metric, which is the metric in the comoving frame with coordinates $(\tau, \vec{x})$. In the expressions below, all indices are raised and lowered using the Minkowski metric. We denote all mass scales in comoving coordinates with a hat. In particular we define:

$$\hat{\phi}_i = a \phi_i, \quad \hat{V} = a^4 V(\hat{\phi}),$$

with $\phi_i = \{ \phi, h, \theta, \eta \}$ the scalars in the theory. The hatted fields are canonically normalized in the comoving frame. Since the gauge field kinetic terms are conformally invariant, there is no rescaling of the gauge field. These comoving fields feel a potential $\hat{V}$. All the comoving quantities map directly to the equivalent set-up in Minkowski, and we can use the usual Minkowski machinery to calculate Feynman diagrams.

The action (10) is expanded in quantum fluctuations around the background. Here we state the results at each order; for details see Appendix A. The one-point vertex is (104)

$$S^{(1)} = \int d^4x \left( -\hat{\lambda}_h \hat{h} \right),$$

where

$$\hat{\lambda}_h = (\partial^2 - (H' + H^2)) \hat{\phi} + \hat{V}_\phi = a^3 \left[ \ddot{\phi} + 3H \dot{\phi} + V_\phi \right].$$

The quadratic action is (105)

$$S^{(2)} = \int d^4x \left\{ -\frac{1}{2} A_\mu \left[ -(\partial^2 + \hat{m}_i^2(\mu)) \eta^{\mu\nu} + \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] A_\nu - A_0 (\hat{m}^2)^{0\mu}_\delta A_\mu \\
- \hat{m}_\lambda^2 A_\lambda A_0 - \frac{1}{2} \sum_{\phi_i = \{h, \bar{h}, \theta, \eta\}} \hat{\phi}_i (\partial^2 + \hat{m}_\phi^2) \hat{\phi}_i - \hat{\eta}(\partial^2 + \hat{m}_\eta^2) \hat{\eta} \right\}.$$ (17)

The explicit form of the two-point interactions, with $\hat{m}^2 = a^2 m^2$, are:

$$\hat{m}_i^{(\mu)} = g^2 \hat{\phi}^2 + \frac{2}{\xi} (H' - 2H^2) \delta_{\mu 0} = a^2 \left[ g^2 \phi^2 + \frac{2}{\xi} \left( \dot{H} - H^2 \right) \delta_{\mu 0} \right],$$

$$\hat{m}_i^h = \hat{V}_{hh} - (H' + H^2) = a^2 \left[ V_{hh} - (\dot{H} + 2H^2) \right],$$

$$\hat{m}_i^\theta = \hat{V}_{\theta\theta} + \xi g^2 \phi^2 - (H' + H^2) = a^2 \left[ V_{\theta\theta} + \xi g^2 \phi^2 - (\dot{H} + 2H^2) \right],$$

$$\hat{m}_i^\eta = \xi g^2 \phi^2 - (H' + H^2) = a^2 \left[ \xi g^2 \phi^2 - (\dot{H} + 2H^2) \right],$$ (18)

There are also four-point interactions but these do not contribute to the one-loop tadpoles.
where we used $\mathcal{H}^2 = a^2H^2$ and $\mathcal{H}' = a^2(\dot{H} + H^2)$. The off-diagonal two-point terms are:

\[
\hat{m}_{A\theta}^2 = 2g(\partial_\tau - \mathcal{H})\dot{\phi} = a^2 \left[2g\dot{\phi}\right], \quad (\hat{m}^2)^{0i} = (\hat{m}^2)^{i0} = \frac{2}{\xi} \mathcal{H} \partial^i.
\]

(19)

The mixing between the spatial $A_i$ and temporal $A_0$ gauge field contains a first derivative, which survives in all gauges except for unitary gauge $\xi \to \infty$. However, we will not work in unitary gauge as this is known to give false results if not done carefully (see for example [36]), and we are thus forced to deal with this extra complication.

As stated above, we choose to split the two-point interactions into a time-independent and time-dependent part:

\[
\hat{m}^2(\tau) = \hat{m}^2 + \delta\hat{m}^2(\tau).
\]

(20)

The first term contributes to the free Lagrangian from which the propagator is constructed, whereas the time-dependent term is treated as a two-point interaction. The loop expansion is independent of the split of the two-point terms into a free and interacting part [7]. The split is defined by requiring the interaction to vanish at the initial time, which we choose without loss of generality to be at $t_0 = 0$:

\[
\delta\hat{m}^2(0) = 0.
\]

(21)

For the diagonal two-point interactions, (21) serves only to define $\hat{m}^2$ unambiguously, and does not constrain anything physically meaningful, like, for example, the initial conditions of the background scalar and scale factor. However, due to the presence of off-diagonal two-point interactions, and the fact that we want to retain a Minkowski-like propagator structure to simplify the calculation, we will need to make some choice. In particular, we choose initial conditions such that the initial off-diagonal two-point interactions and Lorentz violating contribution to the gauge boson mass (the term proportional to $\delta \mu_0$) vanish at the initial time: $m_{\text{off-diag}}^2(0) = 0$. This implies the initial conditions

\[
\delta\phi(0) = \delta\phi'(0) = \mathcal{H}(0) = \mathcal{H}'(0) = 0,
\]

(22)

where we wrote $\phi(t) = \bar{\phi} + \delta\phi(t)$. As we will argue in Sec. 5 the results we obtain do not depend on the specific initial conditions chosen, provided we define the initial vacuum as that of the free theory. It is therefore no real limitation that the boundary conditions above are not the most physically motivated ones in actual cosmological settings.

The two-point interactions for the scalars, $\hat{m}_h^2$, $\hat{m}_\theta^2$ and $\hat{m}_\eta^2$, are split as per (20). The diagonal gauge boson two-point interaction is split as a constant degenerate piece, a time-dependent degenerate piece, and an extra piece for $A_0$ denoted by $\hat{m}_0^2$:

\[
\hat{m}_{(\mu)}^2 = \hat{m}_A^2 + \delta\hat{m}_A^2 + \delta\hat{m}_0^2 \delta_{0\mu},
\]

(23)

with

\[
\delta\hat{m}_0^2 = \frac{2}{\xi}(\mathcal{H}' - 2\mathcal{H}^2).
\]

(24)

The off-diagonal two-point interactions do not have a time-independent part, so $\delta\hat{m}_{A\theta}^2 = \hat{m}_{A\theta}^2$ and $(\delta\hat{m})^{0i} = (\hat{m}^2)^{0i}$. For these terms, we shall use the notation with and without the $\delta$ interchangeably.

The three-point interaction vertices are (106)

\[
S^{(3)} = \int d^4x \left[-\frac{1}{2} \tilde{\lambda}_{h\alpha\dot{h}} \tilde{\hat{h}}(\dot{\psi}_\alpha)^2 - \tilde{\lambda}_{\dot{h}h\eta} \tilde{\hat{h}} \tilde{\eta} \dot{\eta} - \tilde{h} \tilde{\lambda}_{hA\theta} A_\theta \right],
\]

(25)
with the sum over all fields $\phi$ and $D$ fields is defined with an overall minus sign we have
\begin{equation}
(17). The corresponding quadratic Lagrangian can be written in the form

\end{equation}

$D_{\pm}$ with the usual in-out formalism, this approach gives results that are manifestly real.

\begin{equation}
\text{as closed-time-path or Schwinger-Keldysh formalism) [24, 25, 26, 27, 28, 29, 30]. In contrast with time, rather than scattering amplitudes, we will use the in-in formalism (also known}
\end{equation}

\begin{equation}
2.3 \text{ In-in formalism and propagators}
\end{equation}

Since we are interested in expectation values of the background field, and their evolution with time, rather than scattering amplitudes, we will use the in-in formalism (also known as closed-time-path or Schwinger-Keldysh formalism) [24, 25, 26, 27, 28, 29, 30]. In contrast with the usual in-out formalism, this approach gives results that are manifestly real.

Expectation values are computed using an action $S = S[\phi^+] - S[\phi^-]$, with boundary condition $\phi^+_i(t) = \phi^-_i(t)$. That is, we double the fields, and take the action for the plus-fields as for the minus-fields, given by the equations in the previous subsection. All fields, propagators and vertices are labeled by $\pm$ superscripts. In a Feynman diagram the propagator $D^{\pm \pm}(x - x')$ connects between a $\lambda^{\pm}(x)$ and a $\lambda^{\pm}(x')$ vertex. Since the action of the minus-fields is defined with an overall minus sign we have
\begin{equation}
[m^2_{\alpha\beta}]^- = -[m^2_{\alpha\beta}]^+, \quad \lambda^-_{h\alpha\beta} = -\lambda^+_{h\alpha\beta}.
\end{equation}

We construct the propagators from the free, time-independent part of the quadratic action [17]. The corresponding quadratic Lagrangian can be written in the form
\begin{equation}
\mathcal{L}_{\text{free}}^{\phi^+\alpha} = -(1/2) \sum_{\alpha, \beta} \phi^+_\alpha(x^\mu) \tilde{K}^{\alpha\beta}(x^\mu) \phi^+_\beta(x^\mu),
\end{equation}
with the sum over all fields $\phi_\alpha = \{h, \theta, \eta, A^\mu\}$. For example, the scalar fields have $\tilde{K}^{\alpha\beta} = (\partial^2 + m^2)\delta^{\alpha\beta}$. The propagators are then defined as the solutions of
\begin{equation}
\begin{pmatrix}
\tilde{K}^{\alpha\beta}(x^\mu) \\
0
\end{pmatrix}
\begin{pmatrix}
D_{\beta\gamma}^{++}(x^\mu - y^\mu) \\
D_{\beta\gamma}^{+ -}(x^\mu - y^\mu)
\end{pmatrix}
= -i\delta^\alpha_\gamma \delta(x^\mu - y^\mu) I_2.
\end{equation}

This defines $D^{++}$ as the usual Feynman propagator, $D^{-+}$ as the anti-Feynman propagator, and $D^{++}$ and $D^{+-}$ as Wightman functions. Our initial conditions (21, 22) are such that the
\begin{equation}
\text{(106)} also contains a term $-2gA^i\partial^i h$. Since the final expression of each tadpole graph is independent of the spatial coordinates, this three-point interaction does not contribute to the overall result. We have checked this by explicit computation.\footnote{Equation (106) also contains a term $-2gA^i\partial^i h$. Since the final expression of each tadpole graph is independent of the spatial coordinates, this three-point interaction does not contribute to the overall result. We have checked this by explicit computation.}
FLRW propagators in conformal coordinates are analogous to the usual Minkowski expressions.

It turns out convenient to rewrite all the propagators in terms of Wightman functions. We introduce the shorthand for the Wightman function

\[ D_I,ab \equiv D^{-+}_I(x_a - x_b), \quad D_{\mu\nu,ab} \equiv D^{-+}_{\mu\nu}(x_a - x_b), \]

for the propagator of a type-I scalar, and the gauge boson propagator, respectively. Using this the propagators are (suppressing the \( I \) or the Lorentz indices)

\[
D^{++}(x_a - x_b) = D_{ab}\Theta_{ab} + D_{ba}\Theta_{ba}, \\
D^{--}(x_a - x_b) = D_{ab}\Theta_{ba} + D_{ba}\Theta_{ab}, \\
D^{+-}(x_a - x_b) = D^{+-}(x_b - x_a) = D_{ab},
\]

with \( \Theta_{ab} = \Theta(\tau_a - \tau_b) \) the usual step function. We Fourier transform the propagator with respect to comoving three-momentum

\[
D(x_a - x_b) = \int d^3k D(\tau_a - \tau_b, \vec{k}) e^{i\vec{k} \cdot (\vec{x}_a - \vec{x}_b)},
\]

with \( dk = dk/(2\pi) \). The time dependence \((\tau_a - \tau_b)\) in the Fourier propagator will from now on be suppressed. For a scalar field the solution for the Fourier transformed Wightman function is

\[
D_I(\vec{k}) = \frac{1}{2\omega_I(k)} e^{-i\hat{\omega}_I(k)(\tau_a - \tau_b)},
\]

where \( \hat{\omega}_I^2(k) = \hat{m}_I^2 + \vec{k}^2 \) and \( \hat{m}_I^2 \) is the (conformal) mass-squared of the appropriate scalar. The Fourier transformed Wightman function for the gauge boson propagator is

\[
D_{\mu\nu}(\vec{k}) = -\left( \eta_{\mu\nu} - \frac{\vec{k}_\mu\vec{k}_\nu}{\hat{m}_A^2} \right) D_A(\vec{k}) - \frac{\xi}{\hat{m}_2^2} D_\xi(\vec{k}),
\]

with \( D_A \) and \( D_\xi \) scalar propagators with mass-squared \( \hat{m}_A^2 \) and \( \hat{m}_2^2 = \xi\hat{m}_A^2 \) respectively. In the \( \xi = 1 \) gauge the gauge boson propagator is diagonal, \( D_{\mu\nu} = -\eta_{\mu\nu} D_A \).

The mixed two-point interaction \((\delta\hat{m}^2)^{0i}\) contains a spatial derivative; it acts on the gauge boson propagator by

\[
(\delta\hat{m}^2)^{0i}(\tau_a) D_{ij,ab}(\vec{k}) = -i\frac{\hat{k}^i}{\xi} H(\tau_a) D_{ij,ab}(\vec{k}), \\
(\delta\hat{m}^2)^{0i}(\tau_b) D_{ij,ab}(\vec{k}) = -i\frac{\hat{k}^i}{\xi} H(\tau_b) D_{ij,ab}(\vec{k}),
\]

where we note that \((\delta\hat{m}^2)^{00}\) and \(D_{\mu\nu,ab}\) are diagonal in the Lorentz indices. The boundary conditions \((21, 22)\) imply that \((\hat{m}^2)^{00} = \hat{m}_A^2 = \hat{m}_0^2 = 0\); the off-diagonal two-point interactions and the Lorentz violating \(A_0\) mass contribution only enter the interaction Lagrangian.

### 3 One-loop equation of motion

The equation of motion for the background Higgs field \( \phi(t) \) follows from the vanishing of the tadpole. In terms of diagrams, these are all one-particle irreducible tadpole graphs with
one external $h^+$ leg. In this section we compute these diagrams, and thus the quantum corrected equation of motion, at the one-loop level. We are here concerned only with the UV divergent contributions to the graphs, and thus need to consider only the diagrams with up to three vertices. Throughout this work we use a cutoff regularization scheme for the momentum integrals. For the goal of this work, this seems the most intuitive approach, since the momentum integrals are over three-momentum $\vec{k}$, and we cut off $|\vec{k}| < \Lambda$. Other regularization methods, such as for instance dimensional regularization, would give equivalent answers. The calculation is done in the conformal frame, in terms of hatted fields and mass scales, conformal time and momenta. For notational convenience, in this section we drop the hat on all quantities; it shall be reinstated at the end when we give the results.

The calculation is analogous to the one for Minkowski [19], but with two-point interactions (18) that now depend on the FLRW scale factor. This is straightforward to incorporate for the diagrams with a scalar running in the loop. There are however some new technical difficulties that come in with the gauge boson loops:

1. The mass of the temporal gauge boson $m^2_0$ gets FLRW corrections but the mass of the spatial components $m^2_i$ does not. This is possible because Lorentz symmetry is broken by the time-dependent background. Consequently the diagrams with $A_0$ and $A_i$ contribute differently.

2. The off-diagonal gauge boson two-point interaction $(\delta m^2)^{0i}$ is non-zero. This results in new diagrams with both two and three two-point insertions.

3. The formalism is set up in such a way that the two-point interactions vanish at the initial time (21). This avoids divergences that depend on the initial conditions. We will argue in Sec. 5 that this is always an allowed choice, for arbitrary initial conditions, provided the initial vacuum is chosen accordingly.

The one-loop equation of motion can be extracted from the series of tadpole diagrams with one external $h^+$ leg, see Fig. 1. This gives

\[
0 = A_4 + A_1 + A_2 + A_3 + \text{finite}
\]

\[
= \lambda_h^+(x) + \sum \lambda_{h\alpha\beta}^+(x) \left[ S_{\alpha\beta} D^{++}_{\alpha\beta}(0) - i S_{\alpha\beta\gamma\delta} \int d^4x' D^{++}_{\alpha\gamma}(x - x') \left( \delta m^2_{\gamma\delta}(x') \right)^\pm D^{++}_{\delta\beta}(x' - x) - S_{\alpha\beta\gamma\delta\rho\sigma} \int d^4x' \int d^4x'' D^{++}_{\alpha\gamma}(x' - x') \left( \delta m^2_{\gamma\delta}(x') \right)^\pm D^{++}_{\delta\beta}(x' - x) \left( \delta m^2_{\rho\sigma}(x'') \right)^\pm D^{++}_{\sigma\beta}(x'' - x) \right] + \text{finite}
\]

where $A_i$ labels the $i$th order contribution to all tadpole diagrams with $i$ vertices. We labeled the classical contribution $A_4$, which comes from a tree-level diagram. Indices $\{\alpha, \beta, \ldots\}$ are compound indices denoting field-type as well as possible Lorentz indices for the gauge field. The sum is over these compound indices (all fields and their Lorentz indices) as well as all possibilities for $\pm$. The $S_{\alpha\beta\ldots}$ are appropriate symmetry factors, derived in Appendix B $\delta m^2_{\alpha\beta}$ and $\lambda_{h\alpha\beta}$ are the two- and three-point interaction vertices, respectively, as defined in Sec. 2.2.

The tree-level tadpole diagram contributes (10), and we recover the classical equations of motion (remember we dropped the hat for conformal coordinates and scales):

\[
0 = \lambda_h^+ = \phi'' - (\mathcal{H}' + \mathcal{H}^2)\phi + V_\phi.
\]
\[ \sum A_i = h^+ \lambda_h + h^+ \lambda_{h\alpha\beta} D_{\alpha\beta} + h^+ \lambda_{h\alpha\beta} \delta m_{\rho\sigma}^2 D_{\sigma\beta} + h^+ \lambda_{\alpha\beta} \delta m_{\rho\sigma}^2 D_{\alpha\rho} D_{\sigma\beta} \]

\[ \sum A_i = h^+ \lambda_h + h^+ \lambda_{h\alpha\beta} D_{\alpha\beta} + h^+ \lambda_{h\alpha\beta} \delta m_{\rho\sigma}^2 D_{\sigma\beta} + h^+ \lambda_{\alpha\beta} \delta m_{\rho\sigma}^2 D_{\alpha\rho} D_{\sigma\beta} \]

Figure 1: Tree-level tadpole giving the classical equation of motion and the first, second and third order diagrams respectively. The summation is over all fields, for the gauge bosons also over Lorentz indices, and over \( \pm \) at the two-point vertices.

Given the equation of motion, we want to find the corresponding action. We do this by simply writing down an action which, upon using the Euler-Lagrange equations, yields the equation of motion \([37]\). This action is then transformed from the comoving to the physical frame, thereby obtaining the effective action. This will be done in Sec. 4. The idea is to apply this procedure to the quantum corrected equations of motion in order to determine the quantum corrected effective action.

We divide the calculation based on the order of the contributing graphs, which is the number of vertices in the loop of the tadpole. As discussed above, we must work to third order. Independent of this, we can distinguish three classes of diagrams depending on how they contribute to the answer. First there is the contribution that is fully analogous to the Minkowski calculation \( A_{\text{Mink}}^1 = A_{\text{Mink}}^1 + A_{\text{Mink}}^2 \), the only difference is that the mass term of the scalars now depends on the scale factor. Second is \( A_{\text{mass}} = A_{\text{mass}}^2 \), which arises from the extra Feynman diagrams due to the FLRW mass correction of the temporal gauge field \( \delta m_0^2 \); see \([23]\). And finally \( A_{\text{mix}} = A_{\text{mix}}^1 + A_{\text{mix}}^2 \) gives the diagrams with one and two off-diagonal vertices \((\delta m_0^2)^{ij}\) connecting the temporal and spatial gauge fields, also absent in Minkowski.

### 3.1 First order contribution \( A_1 \)

The calculation of the first order diagrams proceeds analogously to the equivalent calculation in Minkowski, which can be found in \([19]\). At first order, four diagrams contribute, with \( \psi_\alpha = \{h, \theta, \eta, A^\mu\} \) running in the loop. The result only depends on the time-independent part of the two-point interaction, as there is no vertex insertion. For each diagram the result has the same structure, given by \([113]\)

\[ (A_{1}^{\text{Mink}})_\alpha = \frac{1}{2} \partial_\phi m_\alpha^2 D^{++}(0). \]  

\[ (A_{1}^{\text{Mink}})_\alpha = \frac{1}{2} \partial_\phi m_\alpha^2 D^{++}(0). \]  

Just as in Minkowski the gauge loop can be expressed in terms of scalar propagators \([34]\) via

\[ -\eta^{\mu\nu} D^{++}_{\mu\nu}(0) = 3 D^{++}_{\alpha}(0) + \xi D^{++}_{\alpha}(0). \]  

\[ -\eta^{\mu\nu} D^{++}_{\mu\nu}(0) = 3 D^{++}_{\alpha}(0) + \xi D^{++}_{\alpha}(0). \]
The sum of all first-order diagrams is

\[
A_1^{\text{Mink}} = \frac{1}{2} \sum_{\alpha} \partial_\phi m_\alpha^2 D_\alpha^+(0) = \frac{1}{2} \sum_{\alpha} S_\alpha \partial_\phi m_\alpha^2 \frac{1}{4\pi^2} \int_0^\Lambda k^2 \frac{dk}{k} \left[ \frac{1}{k} - \frac{1}{2} \frac{\bar{m}_\alpha^2}{k^2} + \ldots \right]
\]

\[
= \frac{1}{16\pi^2} \sum_{\alpha} S_\alpha \partial_\phi m_\alpha^2 \left[ \Lambda^2 - \frac{1}{2} \bar{m}_\alpha^2 \ln(\Lambda/\bar{m})^2 + \text{finite} \right].
\]  

(40)

In the momentum integrals here and below, the variable \( k \) is the comoving momentum, \( \Lambda \) is a comoving cutoff, and we have \( k < \Lambda \). (Recall that graphs in this section in the comoving frame, and all quantities are actually hatted quantities. The cutoff regularisation we apply here is equivalent to a physical cutoff on physical momentum.) The sum is over \( \alpha = \{ h, \theta, \eta, A, \xi \} \), and \( S_\alpha = \{ 1, 1, -2, 3, 1 \} \) counting the real degrees of freedom (with a minus sign for the anti-commuting ghost). Further \( m_A^2 = g^2\phi^2 \) and \( m_\xi^2 = \xi m_A^2 \). Note that the factor \( \partial_\phi m_\alpha^2 \) is time-dependent, and evaluated at \( \tau \); hence \( A_1^{\text{Mink}} \) is a function of \( \tau \). The finite terms that we have neglected remain finite as \( \Lambda \to \infty \).

### 3.2 Second order contribution \( A_2 \)

At second order the loop diagrams with one two-point insertion contribute. We split them into three parts. The first part, \( A_2^{\text{Mink}} \), contains all scalar loops, and the gauge boson loop where only the diagonal part of (23) is inserted. In addition there is a mixed \( \theta A^0 \)-loop (115). This part is analogous to the equivalent Minkowski calculation. The second part \( A_2^{\text{mass}} \) contains the gauge boson loop with a \( \delta m_0^2 \), and the third part \( A_2^{\text{mix}} \), with a \( (\delta m^2)^{ab} \). These last two diagrams are both absent in Minkowski.

#### 3.2.1 \( A_2^{\text{Mink}} \)

The scalar Higgs loop with one two-point insertion gives (114)

\[
A_{2,h}^{\text{Mink}} = -\frac{i}{2} \partial_\phi m_h^2(\tau_a) \int d^4 x_b \delta m_h^2(\tau_b) \left[ D_h^{++}(x_a-x_b)D_h^{++}(x_b-x_a) - D_h^{-+}(x_a-x_b)D_h^{-+}(x_b-x_a) \right]
\]

\[
= -\frac{i}{2} \partial_\phi m_h^2(\tau_a) \int d^4 x_b \delta m_h^2(\tau_b) \Theta_{ab} \left[ D_{h,ab}^2 - D_{h,ba}^2 \right],
\]

(41)

where we expressed the result in terms of Wightman functions, using the notation introduced in Sec. 2.3. Fourier transforming as in (32) and performing the \( d^3x \) integral gives a \( \delta^3(\vec{k} + \vec{p}) \). And thus:

\[
A_{2,h}^{\text{Mink}} = -\frac{i}{2} \partial_\phi m_h^2(\tau_a) \int_0^{\tau_a} d\tau_b \delta m_h^2(\tau_b) \int \frac{d^3k}{(2\omega_h)^2} \sin \left[ 2\omega_h(\tau_a - \tau_b) \right]
\]

\[
= -\partial_\phi m_h^2(\tau_a) \int_0^{\tau_a} d\tau_b \delta m_h^2(\tau_b) \int \frac{d^3k}{(2\omega_h)^2} \sin \left[ 2\omega_h(\tau_a - \tau_b) \right],
\]

(42)

where we used the explicit form of the propagator (33). Now use integration by parts to extract the UV divergent piece (we give here the general formula, which can also be applied
to the gauge boson loop discussed below)

\[
\int_{0}^{\tau_a} d\tau_b f(\tau_b) \frac{d^3k}{(2\omega_I)(2\omega_J)} \sin[(\omega_I + \omega_J)(\tau_a - \tau_b)] = f(\tau_a) \int d^3k \frac{1}{(2\omega_I)(2\omega_J)(\omega_I + \omega_J)} + \ldots
\]

(43)

where we assumed \(f(0) = 0\) and the ellipses denote higher order terms \(\mathcal{O}(\vec{k}^{-4})\). Putting it all back together we find for the scalar loop

\[
A_{2,h}^{\text{Mink}} = -\partial_\phi m_h^2(\tau) \delta m_h^2(\tau) \int \frac{d^3k}{(2\omega_h)^3} = -\partial_\phi m_h^2(\tau) \delta m_h^2(\tau) \int \frac{d^3k}{8\kappa^3} = -\partial_\phi m_h^2(\tau) \delta m_h^2(\tau) \frac{1}{32\pi^2} \ln(\Lambda/\bar{m})^2 + \text{finite},
\]

(44)

where we rewrote the time variable \(\tau_a = \tau\). To get to the second expression we expanded in large momentum.

The Goldstone boson and ghost loops give a similar contribution, although for the ghost with an overall factor \((-2)\) to take into account that these are two real anti-commuting degrees of freedom. The calculation of the gauge boson loop follows the same steps, except that now care has to be taken of the Lorentz structure. We find

\[
A_{2,A}^{\text{Mink}} = -\frac{i}{2} \partial_\phi m_A^2 \int d^4 x_b \delta m_A^2(\tau_b) \eta^{\mu\nu} \eta^{\rho\sigma} \left[ D^{\mu+}(x_a - x_b) D^{\sigma+}_{\nu}(x_b - x_a) - D^{\mu-}(x_a - x_b) D^{\sigma-}_{\nu}(x_b - x_a) \right]
\]

\[
\int_{0}^{\tau_a} d\tau_b m_A^2(\tau_b) \int \frac{d^3k}{(2\omega_I)(2\omega_J)} \sin [(\omega_I + \omega_J)(\tau_a - \tau_b)]
\]

\[
= -\partial_\phi m_A^2(\tau) \delta m_A^2(\tau) \frac{(3 + \xi^2)}{32\pi^2} \ln(\Lambda/\bar{m})^2 + \text{finite},
\]

(45)

Again, we rewrote \(\tau_a = \tau\). The relevant propagator combination, which defines \(C_{IJ}\), is

\[
\eta^{\mu\nu} \eta^{\rho\sigma} \left[ D^{\mu+}(\vec{k}) D^{\nu-}_{\sigma}(\vec{p}) \right] |_{\vec{k} = -\vec{p}} = C_{IJ}(k) D_I D_J
\]

(46)

\[
= \left( 3 + \frac{4\epsilon^2(\omega_A^2)}{m_A^4} \right) D_A(\vec{k})^2 + \xi^2 \left( 1 + \frac{4\epsilon^2(\omega_A^2)}{m_A^4} \right) D_\xi(\vec{k})^2 - 2\xi \frac{\epsilon^2(\omega_A^2 + \omega_\xi^2)}{m_A^2 m_\xi^2} D_A(\vec{k}) D_\xi(\vec{k}),
\]

with \(I, J = A, \xi\), and \(D_{I,ab} = (2\omega_I)^{-1} e^{-i\omega_I(\tau_a - \tau_b)}\).

Finally, the mixed \(\theta A^0\)-loop gives \(\langle 115 \rangle\)

\[
A_{2,\theta A}^{\text{Mink}} = -i \lambda_{\theta A}(\tau_a) \int d^4 x_b \delta m_A^2(\tau_b) \left[ D^{++}_{\theta ba} D^{++}_{\theta ba} - D^{+-}_{\theta ba} D^{+-}_{\theta ba} \right]
\]

\[
= -2 \lambda_{\theta A}(\tau_a) \int_{0}^{\tau_a} d\tau_b m_A^2(\tau_b) \int d^3k \sum \frac{C_I}{(2\omega_I)(2\omega_\theta)} \sin[(\omega_I + \omega_\theta)(\tau_a - \tau_b)]
\]

\[
= 2 \lambda_{\theta A}(\tau) \delta m_A^2(\tau) \frac{(3 + \xi)}{128\pi^2} \ln(\Lambda/\bar{m})^2 + \text{finite},
\]

(47)

with \(\lambda_{\theta A}(\tau) = 2g(-\partial_\tau - \mathcal{H}(\tau))\) the appropriate three-point vertex\footnote{Note that \(\int d\tau \lambda_{\theta A}(\tau) \delta m_A^2(\tau) = \int d\tau \partial_\tau m_A^2(\tau) \delta m_A^2(\tau)\) after integration by parts, which we will use in the expression for the effective action in Sec.\(\ref{sec:effective-action}\) to rewrite \((A_{2,\theta A}^{\text{Mink}})_{\theta A}\) in the same form as the other contributions.}

In the second line we identified

\[
D_{00} = \sum C_I D_I = - \left( 1 - \frac{\omega_\xi^2}{m_A^2} \right) D_A - \xi \frac{\omega_\xi^2}{m_\xi^2} D_\xi.
\]

(48)
In the final step we performed integration by parts \((13)\), and took the large \(k\) limit.

Adding everything together gives

\[
A_{2}^{\text{Mink}} = -\frac{1}{32\pi^2} \sum_{\alpha} S_{\alpha} \partial_{\phi} \delta \mu_{\alpha}^{2} \delta \mu_{\alpha}^{2} \ln \Lambda^2 + \frac{(3 + \xi)}{64\pi^2} \lambda_{h,A\phi} \delta \mu_{A\phi} \ln (\Lambda/\bar{m})^2 + \text{finite}. \tag{49}
\]

### 3.2.2 \(A_{2}^{\text{mass}}\) and \(A_{2}^{\text{mix}}\)

The last two diagrams to contribute at second order are those with an \(m_{0}^2\) and an \((m^{2})_{Aji}\) insertion, the Lorentz violating mass and off-diagonal gauge boson interaction respectively. Neither diagram is present in Minkowski. The loop with \(m^{2}_{0}\) gives \((114)\)

\[
A_{2}^{\text{mass}} = -\frac{i}{2} \partial_{\phi} m_{A}^{2}(\tau_{a}) \int d^{4}x_{b} \delta m_{0}^{2}(\tau_{b}) \eta^{\mu\nu} \left[ D_{\mu,ab}^{++} D_{0b,ba}^{++} - D_{\mu,ab}^{+-} D_{0b,ba}^{-+} \right] \\
= -\partial_{\phi} m_{A}^{2}(\tau_{a}) \int_{0}^{\tau} d\tau_{b} \delta m_{0}^{2}(\tau_{b}) \int d^{3}k \left( \sum_{I,J} \frac{C_{IJ}}{(2\omega_{I})(2\omega_{J})} \sin [(\omega_{I} + \omega_{J})(\tau - \tau_{b})] \right) \\
= -\partial_{\phi} m_{A}^{2}(\tau) \delta m_{0}^{2}(\tau) \frac{(3 + \xi^{2})}{4 \times 32\pi^2} \ln (\Lambda/\bar{m})^2 + \text{finite}. \tag{50}
\]

Here we defined the relevant propagator combination by

\[
\eta_{\mu\nu}^{00} D_{\mu,ab}^{00}(k) D_{0b,ba}(p) \mid_{k = -p} = \sum_{I,J} C_{I,J} D_{I}(k) D_{J}(k) \tag{51}
\]

\[
\frac{k^{2} (2\bar{k}^{2} + \bar{m}^{2}_{A})}{m_{A}^{4}} \frac{\omega_{I}^{2} + \bar{k}^{2}}{m_{\xi}^{4}} \omega_{J}^{2} D_{A}(k) D_{A}(\bar{k}) D_{I}(\bar{k}) D_{J}(\bar{k}),
\]

with as before \(I, J = A, \xi,\) and \(D_{I}(-k) = D_{I}(k) = 1/(2\omega_{I})e^{-i\omega_{I}(\tau - \tau_{b})}\).

The off-diagonal interaction \((\delta \mu^{2})_{Aji}\) contains a spatial derivative, and brings down a factor of the momentum. The diagram is given by \((116)\)

\[
A_{2}^{\text{mix}} = i \partial_{\phi} m_{A}^{2}(\tau_{a}) \int d^{4}x_{b} (\delta m_{0}^{2})(\tau_{b}) \eta^{\mu\nu} \left[ D_{\mu,ab}^{++} D_{\nu,ba}^{++} - D_{\mu,ab}^{+-} D_{\nu,ba}^{-+} \right] \\
= -\frac{2}{\xi} \partial_{\phi} m_{A}^{2}(\tau_{a}) \int_{0}^{\tau} d\tau_{b} \eta_{\mu\nu} H(\tau_{b}) \int d^{3}k p^{\mu} \eta^{\nu} \left[ D_{ab,\mu0}(\bar{k}) D_{ab,\nu0}(p) + D_{ba,\mu0}(\bar{k}) D_{ba,\nu0}(p) \right] \mid_{k = -p} \\
= -\frac{2}{\xi} \partial_{\phi} m_{A}^{2}(\tau_{a}) \int_{0}^{\tau} d\tau_{b} \eta_{\mu\nu} H(\tau_{b}) \int d^{3}k \left( \frac{C_{IJ}}{(2\omega_{I})(2\omega_{J})} \right) 2 \cos [(\omega_{I} + \omega_{J})(\tau_{a} - \tau_{b})] \\
= -\frac{2}{\xi} \partial_{\phi} m_{A}^{2}(\tau_{a}) \int d^{3}k \left[ \frac{2H'(\tau_{a})}{(2\omega_{I})(2\omega_{J})(\omega_{I} + \omega_{J})^{2}} C_{IJ} \right] + \text{finite} \\
= \partial_{\phi} m_{A}^{2}(\tau) \frac{3H'(\tau)(1 - \xi)^{2}}{64\pi^{2} \xi} \ln (\Lambda/\bar{m})^2 + \text{finite}. \tag{52}
\]

As we now have a cosine instead of a sine in the expression on the third line above, we integrate by parts twice to isolate the leading term in the UV limit. This is why the result is proportional to \(H'\). The relevant propagator contribution is defined by

\[
p^{\mu} \eta^{\mu\nu} D_{\mu0}(k) D_{\nu0}(p) \mid_{k = -p} \equiv \sum_{I,J} C_{I,J} D_{I} D_{J} \\
= -\frac{k^{2}\omega_{A}(2\bar{k}^{2} + \bar{m}^{2}_{A})}{m_{A}^{4}} D_{A}^{2} - \frac{k^{2}\omega_{\xi}(2\bar{k}^{2} + \bar{m}^{2}_{\xi})}{m_{\xi}^{4}} D_{\xi}^{2} + \frac{k^{2}(\omega_{A} + \omega_{\xi})(2\bar{k}^{2} + \bar{m}^{2}_{A} + \bar{m}^{2}_{\xi})}{m_{A}^{4}} D_{A} D_{\xi}.
\]
3.3 Third order contribution $A_3$

The third order diagrams with two two-point insertions are UV finite, which can be easily checked by power counting. The only exception to this is the diagram with two off-diagonal $(m^2)^0j$ insertions, because each insertion contains a spatial derivative, and thus brings down a power of momentum. We thus consider the third order diagram with two mixed-interaction insertions \([117]\):

$$A_3^{\text{mix}} = \frac{1}{2} \partial_\tau m_A^2(\tau_a) \int d^4 x_b d^4 x_c (\delta m^2)^{(i)}(\tau_b)(\delta m^2)^{(j)}(\tau_c) \sum_{\rho,a} \eta^{\mu\nu} D_{\mu\rho,ab} D_{\sigma\kappa,ba} D_{\tau\nu,ca},$$

where the sum $\sum_{\rho,a}$ is over the four possibilities for the Lorentz indices

$$\begin{aligned}
(\rho, \sigma, \kappa, \tau) = (i, 0, j, 0), (0, i, 0, j), (0, i, j, 0), (i, 0, 0, j)
\end{aligned}
$$

and also over the four possibilities for plus and minus fields. The $x_a$ vertex is a $+\text{-vertex}$. There are then 4 possibilities for the vertices:

$$\begin{aligned}
(a, b, c) = (+ + +), (+ - +), (+ + -), (+ - -).
\end{aligned}
$$

The propagator between a $(\pm)$-vertex and a $(\pm)$-vertex is $D^{\pm\pm}$, and we write them out in terms of Fourier transformed Wightman functions using the notation of section (2.3). Taking the action of the spatial derivatives in $(m^2)^{ij}$ will then bring down powers of momentum, as per (55). Finally, the $\vec{x}_b$ and $\vec{x}_c$ integrals give delta functions encoding momentum conservation.

At the end of the day we find

$$A_3^{\text{mix}} = \frac{2}{\xi^2} \partial_\tau m_A^2(\tau_a) \int_0^{\tau_b} d\tau_b \int_0^{\tau_c} d\tau_c \mathcal{H}(\tau_b) \mathcal{H}(\tau_c) \int d^3 k \sum_{\rho} k^i k^j s_{\rho}$$

$$\times \left[ D_{ba}(\vec{k}) D_{bc}(\vec{p}) D_{ac}(\vec{q}) + c.c \right]_{\vec{k}=\vec{q}=-\vec{p}} - 2\Theta_{bc} \left( D_{ab}(\vec{k}) D_{bc}(\vec{p}) D_{ac}(\vec{q}) + c.c \right)_{\vec{k}=\vec{p}=-\vec{q}},$$

where the sum $\sum_{\rho}$ is now only over the Lorentz indices (54), which we suppressed in the above formula. The sign $s_{\rho} = (1, 1, -1, -1)$ for the four possibilities (54). The relevant propagator combinations, putting Lorentz indices back in, are

$$\begin{aligned}
\sum s_m k^i k^j D_{ba}(\vec{k}) D_{bc}(\vec{p}) D_{ac}(\vec{q}) &\bigg|_{\vec{k}=\vec{q}=-\vec{p}} = k^i k^j \eta^{\mu\nu} \left[ D_{\mu i}(\vec{k}) D_{0 j}(\vec{p}) D_{0 \nu}(\vec{q}) + D_{\mu 0}(\vec{k}) D_{0 j}(\vec{p}) D_{\nu}(\vec{q}) \
&\quad - D_{\mu 0}(\vec{k}) D_{i j}(\vec{p}) D_{0 \nu}(\vec{q}) - D_{\mu i}(\vec{k}) D_{0 0}(\vec{p}) D_{j \nu}(\vec{q}) \right]_{\vec{k}=\vec{q}=-\vec{p}} \\
&\quad = \sum C_{IJK}(\vec{k}) D_I(\vec{k}) D_J(\vec{k}) D_K(\vec{k}),
\end{aligned}$$

and

$$\begin{aligned}
\sum s_m k^i k^j D_{ab}(\vec{k}) D_{bc}(\vec{p}) D_{ac}(\vec{q}) &\bigg|_{\vec{k}=\vec{p}=-\vec{q}} = k^i k^j \eta^{\mu\nu} \left[ D_{\mu i}(\vec{k}) D_{0 j}(\vec{p}) D_{0 \nu}(\vec{q}) + D_{\mu 0}(\vec{k}) D_{0 j}(\vec{p}) D_{\nu}(\vec{q}) \
&\quad - D_{\mu 0}(\vec{k}) D_{i j}(\vec{p}) D_{0 \nu}(\vec{q}) - D_{\mu i}(\vec{k}) D_{0 0}(\vec{p}) D_{j \nu}(\vec{q}) \right]_{\vec{k}=\vec{p}=-\vec{q}} \\
&\quad = \sum D_{IJK}(\vec{k}) D_I(\vec{k}) D_J(\vec{k}) D_K(\vec{k}),
\end{aligned}$$

53
with $I, J, K = A, \xi$, and $C_{IJK}, D_{IJK} \sim k^2$. Using $D_{Iab}(\vec{k}) = (2\omega_I)^{-1}e^{-i\omega_I(\tau_a - \tau_b)}$ we have

$$A_{3a}^{\text{mix}} = \frac{4}{\xi^2} \partial_\phi m_\Lambda^2(\tau_0) \int_0^{\tau_a} d\tau_b \int_0^{\tau_a} d\tau_c \mathcal{H}(\tau_b) \mathcal{H}(\tau_c) \int d^3k \sum \frac{1}{(2\omega_I)(2\omega_J)(2\omega_K)}$$

$$\times \left[ C_{IJK} \cos \left( (\omega_K - \omega_I)\tau_a + (\omega_J + \omega_K)\tau_b - (\omega_J + \omega_K)\tau_c \right) \\
- 2\Theta_{bc}D_{IJK} \cos \left( (\omega_I + \omega_K)\tau_a - (\omega_I - \omega_J)\tau_b - (\omega_J + \omega_K)\tau_c \right) \right].$$

Now use integration by parts with respect to $\tau_b$ and $\tau_c$ to write

$$\int_0^{\tau_a} d\tau_b \mathcal{H}(\tau_b) \int_0^{\tau_a} d\tau_c \mathcal{H}(\tau_c) \cos \left( (\omega_I + \omega_J)\tau_b - (\omega_J + \omega_K)\tau_c + (\omega_K - \omega_I)\tau_a \right)$$

$$= \frac{\mathcal{H}(\tau_a)^2}{(\omega_J + \omega_K)(\omega_I + \omega_J)} + \ldots$$

where we used the initial conditions [22]. And similarly

$$\int_0^{\tau_a} d\tau_b \mathcal{H}(\tau_b) \int_0^{\tau_a} d\tau_c \mathcal{H}(\tau_c) \Theta_{bc} \cos \left( (\omega_I + \omega_K)\tau_a - (\omega_I - \omega_J)\tau_b - (\omega_J + \omega_K)\tau_c \right)$$

$$= - \int_0^{\tau_a} d\tau_b \frac{\mathcal{H}(\tau_a)^2}{(\omega_J + \omega_K)(\omega_I + \omega_K)} \sin \left( (\omega_I + \omega_K)\tau_a - (\omega_I + \omega_K)\tau_b \right)$$

$$= - \frac{\mathcal{H}(\tau_a)^2}{(\omega_J + \omega_K)(\omega_I + \omega_K)}.$$

The end result is

$$A_{3a}^{\text{mix}} = \frac{4}{\xi^2} \partial_\phi m_\Lambda^2(\tau) \mathcal{H}^2(\tau) \int d^3k \sum \frac{1}{(2\omega_I)(2\omega_J)(2\omega_K)}$$

$$\times \left[ \frac{C_{IJK}}{(\omega_I + \omega_J)(\omega_J + \omega_K)} + \frac{2D_{IJK}}{(\omega_J + \omega_K)(\omega_I + \omega_K)} \right]$$

$$= \partial_\phi m_\Lambda^2(\tau) \mathcal{H}^2(\tau) \frac{(1 - 3\xi - 3\xi^2 + \xi^3) - (1 + \xi)^3}{64\pi^2\xi^2} \ln(\Lambda/\hat{m})^2 + \text{finite}$$

$$= \partial_\phi m_\Lambda^2(\tau) \mathcal{H}^2(\tau) \frac{-6(1 + \xi)}{64\pi^2\xi^2} \ln(\Lambda/\hat{m})^2 + \text{finite}. \quad (62)$$

### 3.4 Summary of graphs

In the previous subsections we have computed all quadratically and logarithmically divergent contributions to the one-loop equation of motion. Here we collect and summarize the results, putting the hats back on the relevant variables to indicate that we are still in the conformal frame.

The first order graphs are given by [40]. The second order contributions are [49, 50, 52], and are summarized in Fig. 2. At third order there is only one piece, given by [62]. We now collect these terms into the three groups $\hat{A}^{\text{Mink}}, A^{\text{mass}}$ and $A^{\text{mix}}$.

The first and second order combined $A^{\text{Mink}} = \hat{A}_1^{\text{Mink}} + \hat{A}_2^{\text{Mink}}$ is

$$\hat{A}^{\text{Mink}} = \frac{1}{16\pi^2} \sum_\alpha S_\alpha \partial_\phi m_\alpha^2 \left[ \hat{\Lambda}^2 - \frac{1}{2} m_\alpha^2 \ln \hat{\Lambda}^2 \right] + \frac{(3 + \xi)}{64\pi^2} \hat{\lambda}_{h,h0} m_\alpha^2 \ln(\hat{\Lambda}/\hat{m})^2. \quad (63)$$
\[ \sum A^{(2)}_i = \left[ \right. \]
\[ \frac{1}{D^\alpha h} D^\alpha h \]
\[ \partial_\phi \hat{m}_h^2 \delta \hat{m}_h^2 \]  
\[ -2 \partial_\phi \hat{m}_h^2 \delta \hat{m}_h^2 \]  
\[ + \frac{1}{D^\alpha \theta} D^\alpha \theta \]
\[ \partial_\phi \hat{m}_\theta^2 \delta \hat{m}_\theta^2 \]  
\[ - \frac{1}{2} \partial_\phi \hat{m}_\theta^2 \delta \hat{m}_\theta^2 \]  
\[ + \frac{1}{D^\alpha \mu \nu} D^\alpha \mu \nu \]
\[ \partial_\phi \hat{m}^2_A \delta \hat{m}^2_A (3 + \xi^2) \]  
\[ \frac{1}{4} \partial_\phi \hat{m}^2_\theta \delta \hat{m}^2_\theta + \xi^2 \]  
\[ + \frac{1}{D^\alpha 0 \mu} D^\alpha 0 \mu \]
\[ \partial_\phi \hat{m}^2_A \delta \hat{m}^2_A (3 + \xi^2) \]  
\[ - \frac{1}{2} \partial_\phi \hat{m}^2_A \delta \hat{m}^2_A \]  
\[ + \frac{1}{D^\alpha 0 0} D^\alpha 0 0 \]
\[ \delta \hat{m}^2_A \delta \hat{m}^2_A \]  
\[ \hat{\lambda}_{h \theta} \delta \hat{m}^2_{A \theta} \frac{3 + \xi^2}{2} \]  
\[ - \frac{1}{32 \pi^2} \log \Lambda^2 \]
\[ \left. \right] \]

Figure 2: The second order tadpole diagrams and their corresponding mathematical expression (below each graph). These Feynman diagrams are in (conformal) coordinate space, with the left and right vertices at \( x_a \) and \( x_b \) respectively. The argument of each of the propagators is \( (x_b - x_a) \), and all time-dependent quantities (\( \hat{\lambda}_{h \theta}, \hat{m}^2_A, \delta \hat{m}^2_\alpha \) and \( \mathcal{H} \)) are evaluated at \( \tau \).

As expected, this is independent of how the two-point interaction is split into a free and interacting term, since the first and second order pieces combined in the sum \( \hat{m}^2_\alpha = \hat{m}^2_\alpha + \delta \hat{m}^2_\alpha \).

For \( A_0 \) mass insertions we have the second order piece \([50]\)

\[ \hat{A}^{\text{mass}} = -\partial_\phi \hat{m}^2_A \delta \hat{m}^2_\phi \frac{3 + \xi^2}{128 \pi^2} \ln(\hat{\Lambda}/\hat{m})^2. \]  

(64)

For the mixed piece we have contributions from second order \([52]\) and third order \([62]\), giving a total

\[ \hat{A}^{\text{mix}} = \partial_\phi \hat{m}^2_A \left( \frac{3 \mathcal{H}'(1 - \xi)^2}{\xi} - \frac{6 \mathcal{H}^2(1 + \xi)}{\xi} \right) \frac{1}{64 \pi^2} \ln(\hat{\Lambda}/\hat{m})^2. \]  

(65)

All factors in \([63, 64, 65]\) that are time-dependent — being the \( \hat{m}^2 \)'s, \( \hat{\lambda}_{h \theta} \) and \( \mathcal{H} \) — are understood to be evaluated at \( \tau \).
4 Effective action

The previous section found the one-loop equation of motion. The corresponding loop-corrected effective action is the one which, upon applying the Euler-Lagrange equations for \( \phi \), yields the loop-corrected equation of motion found in Sec. 3. The effective action is defined this way up to an arbitrary field-independent constant, and an overall minus sign which is fixed to obtain the correct sign kinetic terms. Our working assumption is that the background is fixed, i.e. \( a(\tau) \) not a function of the background field \( \phi(\tau) \).

The classical action is defined by

\[
\Gamma_{\text{cl}} = \int \! d^3x \, dt \, L_{\text{cl}},
\]

where \( L_{\text{cl}} \) is the equation of motion following from the Lagrangian density \( L_{\text{cl}} \):

\[
\dot{A}_{\text{cl}} = \left( \frac{\delta L_{\text{cl}}}{\delta \dot{\phi}} \right) - \frac{\delta L_{\text{cl}}}{\delta \phi}.
\]

From (37) we have that \( \dot{A}_{\text{cl}} = \dot{\lambda} + k \), so the classical action is

\[
\Gamma_{\text{cl}} = \int \! d^3x \, \sqrt{-g} \left[ -\frac{1}{2} \frac{\dot{\phi}^2}{a} - a'' - H^2 \phi - V \right],
\]

where we used that \( H' + H^2 = a''/a \), and \( H = \dot{a}/a \). In the second line we went to unhatted quantities, and in the third we changed to physical time. The measure in conformal coordinates is \( \sqrt{-g} \) conformal = \( a^4 \), and in physical coordinates \( \sqrt{-g} \) physical = \( a^3 \). Taking the Euler-Lagrange equation from the last line, using the definition (67), we get the familiar FLRW equation of motion: \( \ddot{\phi} + 3H \dot{\phi} + V_\phi = 0 \).

Applying the same procedure to the quantum corrected equation of motion gives us the quantum corrected effective action. The relevant terms at the level of the equations of motion are summarized in Sec. 3.4 and the one-loop correction to the effective action is defined as

\[
\Gamma^{1-\text{loop}} = \int \! d^3x \, \sqrt{-g} \left( \hat{L}_{\text{Mink}} + \hat{L}_{\text{mass}} + \hat{L}_{\text{mix}} + \text{finite} \right).
\]

In determining \( \hat{L} \), we must be careful to use the same sign convention in the Euler-Lagrange equation as we did above in (67), to ensure the corrections to the effective potential have the correct relative sign.

All but one term in \( \hat{L}_{\text{Mink}} \) are polynomial, the exception being the \( \hat{\lambda}_{h, A\theta} \) term. For this term, the \( \dot{\phi} \) dependent factors are

\[
\hat{\lambda}_{h, A\theta} \dot{m}_{A\theta}^2 = 4g^2 \left( -\dot{\phi}'' + H' \dot{\phi} + H^2 \dot{\phi} \right).
\]

This expression comes from a Lagrangian

\[
-\frac{1}{2} \dot{m}_{A\theta}^4 = -2g^2 \left( \dot{\phi}^2 - 2H \dot{\phi} \dot{\phi}' + H^2 \dot{\phi}^2 \right).
\]
For the rest of the terms in $\hat{A}^{\text{Mink}}$, which are polynomial in $\hat{\phi}$, the corresponding action is found simply by integrating with respect to $\hat{\phi}$, and then negating (since the $\delta \hat{L} / \delta \hat{\phi}$ term in (67) comes with a minus sign). All terms in $\hat{L}^{\text{mass}}$ and $\hat{L}^{\text{mix}}$ are also polynomial in $\hat{\phi}$, so can be similarly integrated. Thus, from (68, 69, 70), and using (71), we obtain

$$\hat{L}^{\text{Mink}} = -\frac{1}{16\pi^2} \sum_\alpha S_\alpha \left( \dot{m}_\alpha^2 \Lambda^2 - \frac{1}{4} m_\alpha^4 \ln(\Lambda / \hat{m})^2 \right) - \frac{(3 + \xi)}{128\pi^2} m_\alpha^4 \Lambda \hat{m}^2 \ln(\Lambda / \hat{m})^2,$$

$$\hat{L}^{\text{mass}} = -\frac{1}{64\pi^2} \dot{m}_A^2 \ln(\Lambda / \hat{m})^2 \left( \frac{3 + \xi}{\xi} (2\dot{H}^2 - \dot{H}') \right),$$

$$\hat{L}^{\text{mix}} = -\frac{1}{64\pi^2} \dot{m}_A^2 \ln(\Lambda / \hat{m})^2 \left( \frac{3(1 - \xi)^2}{\xi} \dot{H}' - \frac{6(1 + \xi)}{\xi} \dot{H}^2 \right),$$

(72)

with $\alpha = \{h, \theta, \eta, A, \xi\}$, and $S_\alpha = \{1, 1, -1, 2, 3, 1\}$.

Now take out a factor $\sqrt{-g}$, and write the hatted variables in terms of their unhatted counterparts to go back to the physical frame. Use that $\dot{H}^2 = a^2 H^2$ and $\dot{H}' = a^2 (\ddot{H} + 2H \dot{H})$, and transform to physical time. All terms are proportional to the fourth power of a mass, hence we factor out an $a^4$. The result is

$$\Gamma^{1\text{-loop}} = \int d^3x dt \sqrt{-g} \left( \hat{L}^{\text{Mink}} + \hat{L}^{\text{mass}} + \hat{L}^{\text{mix}} + \text{finite} \right),$$

(73)

with

$$\hat{L}^{\text{Mink}} = -\frac{1}{16\pi^2} \sum_\alpha S_\alpha \left( m_\alpha^2 \Lambda^2 - \frac{1}{4} m_\alpha^4 \ln(\Lambda / m)^2 \right) - \frac{(3 + \xi)}{128\pi^2} m_\alpha^4 \Lambda m^2 \ln(\Lambda / m)^2,$$

(74)

$$\hat{L}^{\text{mass}} = -\frac{1}{64\pi^2} m_A^2 \ln(\Lambda / m)^2 \left( \frac{3 + \xi}{\xi} (H^2 - \dot{H}) \right),$$

(75)

$$\hat{L}^{\text{mix}} = -\frac{1}{64\pi^2} m_A^2 \ln(\Lambda / m)^2 \left( \frac{3(1 - \xi)^2}{\xi} \dot{H} - \frac{6(1 + \xi)}{\xi} \dot{H}^2 \right).$$

(76)

Whereas $\Lambda$ was a conformal cutoff on conformal three-momentum (equivalent to comoving momentum), $\Lambda$ is now a physical cutoff on physical three-momentum.

When we now plug in the FLRW-corrected two-point interactions (18), we find for the total one-loop effective action (up to field-independent terms)

$$\Gamma^{1\text{-loop}} = -\frac{1}{16\pi^2} \int d^3x dt \sqrt{-g} \left[ (V_{hh} + V_{\theta\theta} + 3m_A^2) \Lambda^2 \right.$$

$$- \left( (V_{hh} - \dot{H} - 2H^2)^2 + (V_{\theta\theta} - \dot{H} - 2H^2)^2 + 3m_A^4 \right.$$

$$+ 2\xi V_{\theta\theta} m_A^2 - (6 + 2\xi)g^2 \dot{\phi}^2 + 6m_A^2 \left( \dot{H} + 2H^2 \right) \right] \ln(\Lambda / m)^2 \frac{4}{\xi}. \]$$

(77)

Recall that $m_A^2 = g^2 \phi^2$. This result is still gauge variant, which was to be expected. Gauge invariance is only achieved on-shell. For a time-independent situation ($\phi(t) = \text{const.}$) the Nielsen identities (18)

$$\frac{\partial V_{\text{eff}}}{\partial \xi} + \frac{\partial \phi}{\partial \xi} \frac{\partial V_{\text{eff}}}{\partial \phi} = 0,$$

(78)
show that the effective potential is only gauge invariant when the background field is in a minimum of the potential. Here (just like in [19]) we want to use the time-dependent version of this statement: the effective potential is only gauge invariant when the background field satisfies its equation of motion. Going on-shell enables us to rewrite in (77) the term proportional to $\dot{\phi}^2$. This term originated from the mixed Goldstone-gauge boson “mass”, the last term in (74), and can be transformed to:

$$\int d^4 x \sqrt{-g} A_\theta = \int d^4 x \sqrt{-g} 4 g^2 \phi V_\phi = \int d^4 x \sqrt{-g} 4 m_A^2 V_\theta. \quad (79)$$

In the third step we integrate by parts and go on-shell. The last step uses Goldstone’s theorem (it exploits the fact that the potential is a function of $\Phi^\dagger$ [37, 38, 19]).

On-shell the result (77) takes the form

$$\Gamma^{1-\text{loop}} = -\frac{1}{16\pi^2} \int d^3 x dt \sqrt{-g} \left[ (V_{hh} + V_{h\theta} + 3m_A^2) \Lambda^2 - (V_{hh} - \dot{H} - 2H^2)^2 + (V_{h\theta} - \dot{H} - 2H^2)^2 + 3m_A^4 - 6m_A^2 (V_{h\theta} - \dot{H} - 2H^2) \right] \frac{\ln (\Lambda/\bar{m})^2}{4}, \quad (80)$$

which is gauge invariant, as it should be. Introducing the notation

$$\tilde{V}_{aa} \equiv V_{aa} - \dot{H} - 2H^2, \quad (81)$$

we rewrite the final result (up to field-independent terms)

$$\Gamma^{1-\text{loop}} = -\frac{1}{16\pi^2} \int d^3 x dt \sqrt{-g} \left[ (V_{hh} + \tilde{V}_{h\theta} + 3m_A^2) \Lambda^2 - (\tilde{V}_{hh} + \tilde{V}_{h\theta} + 3m_A^4 - 6\tilde{V}_{h\theta}m_A^2) \frac{\ln (\Lambda/\bar{m})^2}{4} \right]. \quad (82)$$

### 4.1 Fermions and additional scalars

It is straightforward to add fermions and additional scalars to the calculation. If these fields are coupled to the Higgs field, and thus have a $\phi$-dependent mass term, they will contribute to the effective equation of motion for the background Higgs field $\phi(t)$ and to the effective action.

We assume the extra scalars are in a basis with canonical kinetic terms and have diagonal masses, and do not mix with $h$. Similarly, we assume the extra fermions have diagonal masses. It is easy to relax these assumptions and generalize the results.

In terms of Feynman diagrams, there are extra tadpole graphs with the additional scalars and fermions running in the loop. The calculation for additional scalars is analogous to that of the Higgs fluctuations $h$ already done, with a contribution at first and second order. The result is

$$\Gamma^{1-\text{loop}}_{(\text{scalar})} = -\frac{1}{16\pi^2} \int d^3 x dt \sqrt{-g} \left[ V_{\chi\chi} \Lambda^2 - \left( V_{\chi\chi} - \dot{H} - 2H^2 \right)^2 \frac{\ln (\Lambda/\bar{m})^2}{4} \right], \quad (83)$$

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where $\chi$ is the additional real scalar and $V(\chi, \phi)$ its potential.

Just as for the bosons, the tadpole diagrams with a fermion loop can be mapped to the calculation for Minkowski space, except that the “mass” terms now depend on the FLRW scale factor. To discuss fermions in curved space-time, one has to use the vielbein formalism to transform to a local Lorentz frame, where Lorentz transformations and spin-$\frac{1}{2}$ particles are well defined. The vielbeins are defined via

$$g_{\mu\nu} = \epsilon^a_{\mu} \epsilon^b_{\nu} \eta_{ab},$$

(84)

with $\epsilon^a_{\mu} = a \delta^a_{\mu}$ for a conformal FLRW metric (7). The gamma matrices are $\{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2 g^{\mu\nu}$, with $\gamma^a = \epsilon^a_{\mu} \bar{\gamma}^\mu$ the usual Minkowski gamma-matrices. With this notation the fermionic action is [5]

$$L_f = \int d^4x \sqrt{-g} \bar{\psi} (\bar{\gamma}^\mu \nabla_\mu - m) \psi,$$

(85)

with the covariant derivative $\nabla_\mu = \partial_\mu + \Omega_\mu$, and $\Omega_\mu = (1/4) \omega_{ab\mu} \gamma^a \gamma^b$ for the conformal FLRW metric. We rescale the fermion field $\hat{\psi} = a^{3/2} \psi$ and mass $\hat{m}_\psi = a m_\psi$. The Dirac equation then becomes

$$(i \gamma^\mu \partial_\mu - \hat{m}_\psi) \hat{\psi} = 0,$$

(86)

which is of the usual Minkowski form. Hence the end result is the Minkowski [6] result but with the replacement $m_\psi \rightarrow \hat{m}_\psi = a m_\psi$ [6]:

$$\Gamma^{1\text{-loop}}_{(\text{fermion})} = \frac{1}{16 \pi^2} \sum_f \int d^3x dt \left[ \hat{m}^2_\psi \hat{\Lambda}^2 - \frac{1}{4} \left( \hat{m}_\psi^4 + \hat{m}_\psi^6 \right) \ln(\hat{\Lambda}/\hat{m})^2 \right]$$

$$= \frac{1}{16 \pi^2} \sum_f \int d^3x dt \sqrt{-g} \left[ m^2_\psi \Lambda^2 - \frac{1}{4} \left( m^4_\psi + m^2_\psi \left( \tilde{\tilde{V}}_\theta + \frac{3 H \tilde{\tilde{V}}_\theta}{m_\psi} \right) + H^2 \right) \right] \ln(\Lambda/m)^2 .$$

(87)

The sum is over all fermionic degrees of freedom, which are two (helicity) states for a Weyl fermion and four states for a Dirac fermion. The first line is the Minkowski result with the replacement $m_\psi \rightarrow \hat{m}_\psi$. In the second line we went to physical coordinates by factoring out an overall $a^4$ factor, and rewriting the $\hat{m}_\psi^6$ in terms of derivatives with respect to physical time $t$. The first contribution to the logarithmic term incorporates the expansion of the universe. The second contribution to the logarithmic term is because the $\phi$ field is rolling, and is also present in Minkowski space-time.

Again we can simplify this result by going on-shell. For a fermion mass $m_\psi = \lambda \phi$ that is linear in the Higgs field — which is the case for Yukawa interactions and also for gaugino masses in supersymmetric theories — this gives

$$\Gamma^{1\text{-loop}}_{(\text{fermion})} = \frac{1}{16 \pi^2} \sum_f \int d^3x dt \sqrt{-g} \left[ m^2_\psi \Lambda^2 - \frac{1}{4} \left( m^4_\psi - m^2_\psi \tilde{\tilde{V}}_\theta \right) \ln(\Lambda/m)^2 \right].$$

(88)

Here we have used, again, the background field equations and Goldstone’s theorem. $\tilde{\tilde{V}}_\theta$ was defined in [81].

*If the fermions are charged under gauge groups, there will be an additional gauge connection. These extra terms do not affect the effective action for $\phi$, and for simplicity we leave them out.
5 Initial conditions

Our interactions are time-dependent, and thus we needed to define the split between a time-independent mass and a time-dependent two-point interaction (21)

\[ m_{\alpha\beta}^2(t) = \bar{m}_{\alpha\beta}^2 + \delta m_{\alpha\beta}^2(t), \quad \delta m_{\alpha\beta}^2(0) = 0. \]  

(89)

We furthermore chose initial conditions for \( \phi(t) \) and \( a(t) \) such that the off-diagonal and Lorentz violating two-point interactions vanished completely at the initial time (22). These choices ensured the simplicity of the propagators. They also ensured the vanishing of the \( t = 0 \) boundary terms coming from integration by parts when evaluating the loop diagrams in Sec. 3. If these boundary terms did not vanish, they would yield extra contributions to the final result, contributions that depend on the initial conditions, and that diverge as \( t \to 0 \).

Our chosen initial conditions are peculiar, and are not the ones to be used in a realistic situation. The problem in straightforwardly generalizing our calculation to arbitrary initial conditions are the two-point interactions \( m_{\alpha\beta}^2(0), m_{\alpha\beta}^2, m_{\beta\alpha}^2 \). To simplify the structure of the free action, and use the standard expressions for the propagator, we have treated them as interactions \( m_{\alpha\beta}^2 = \delta m_{\alpha\beta}^2 \). To satisfy (89) then requires the initial conditions (22).

However, in principle there is nothing to stop us from also splitting these two-point interactions into a free and interacting part, as in (89). Technically, this is complicated, as Lorentz symmetry is broken, and the gauge fields and Goldstone bosons all mix at the initial time. Nevertheless, in principle we can expand all fields in mode functions, where the mode functions satisfy the off-diagonal mode equations (diagonalizing the equations will result in a momentum-dependent diagonalization). Then (89) is satisfied, all terms depending on the initial conditions vanish, and the results are the same as for our choice of initial conditions (22).

In slightly different words, we argue that the result is independent of the initial conditions as long as we choose the initial vacuum to be that of the free theory, which is defined by the split of the quadratic term into a time-independent mass and a time-dependent interaction term. That is, solve the mode equations derived from the free action with \( \bar{m}_{\alpha\beta}^2 \), and the corresponding annihilation operators annihilate the vacuum. The different vacua, corresponding to different initial conditions, are then related by a Bogoliubov transformation. For the scalar field theory this was shown by Baacke et al. (see also [39]). For U(1) model we require a more general Bogoliubov transformation, with momentum and polarization dependent coefficients that mix the fields. In principle this should be straightforward, but we will not present any further details here.

In practice, choosing the initial conditions (22) simplifies the calculation of the free field mode functions and propagators, and eliminates boundary terms, which is why we choose it. We have argued that a full treatment of initial conditions would yield the same result, at least for the divergent corrections to the equation of motion.

6 Conclusion

In this paper we have computed the one-loop divergent corrections to the effective action of a U(1) charged scalar, whose background vacuum expectation value is changing with time, and when the background space-time is of FLRW form. We used the in-in formalism and \( R_\xi \) gauge, and our main aim was to demonstrate gauge invariance of the one-loop corrections.
The gauge invariance is only manifest upon using the equations of motion, i.e. on-shell, in accordance with the Nielsen identities. Our result is given by (82), and directly generalizes a previous work [19]. In comparison with the result in a Minkowski background, one can obtain the FLRW correction by shifting all scalar masses by $2H^2 + \dot{H}$. We showed that additional scalars running in the loop can be easily accommodated (83), a result which also mimics the Minkowski case, but with a shifted mass. For fermions in the loop, the additional correction is (87). Although our assumptions for the initial conditions of the background scalar and scale factor are unrealistic, we argued that, if handled in a more general way, the result would be the same.

The computed correction can now be used to derive the renormalization group and find the RG improved action, a task we defer to a future publication. An additional task left for the future is to take into full account the backreaction of the scalar on space-time. Essentially, one must allow for spin-0 fluctuations of the metric, determine their mixing with the scalar, diagonalize to a new basis, and use this basis as the starting point of the calculation. A further generalization is to include a non-minimal coupling to gravity, so as to describe models of Higgs inflation. Finally, one could also generalize the decomposition of $\Phi$ (9) to allow for a time-dependent classical background in the imaginary direction.

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A Action in detail

Here we work out the explicit form of the action (10) to fourth order in quantum fluctuations. Using conformal coordinates the overall volume factor is $\sqrt{-g} = a^4$, and $g^{\mu\nu} = a^{-2} \eta^{\mu\nu}$. Unless otherwise stated, all indices below are raised and lowered using the Minkowski metric.

Start with the kinetic term for the gauge field. The connection cancels in the field strength $F_{\mu\nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, and thus

\begin{equation}
-\frac{1}{4} \int d^4x \sqrt{-g} F^2 = -\frac{1}{4} \int d^4x F^\mu\nu F_{\mu\nu} = \frac{1}{2} \int d^4x A_{\mu} (\partial^\rho \eta^\mu\rho - \partial^\rho \partial^\rho) A_{\nu}. \tag{90}
\end{equation}

As expected, the result is invariant under a conformal transformation of the metric. The kinetic terms and potential for the Higgs field are expanded as

\begin{equation}
\begin{aligned}
\int d^4x \sqrt{-g} (|D\Phi|^2 - V) &= \int d^4x \left\{ \frac{a^2}{2} \left[ \sum_{\varphi=\phi_R,\theta} (\partial \varphi)^2 + g^2 A^2 \varphi^2 \right] + 2gA^\mu (\partial^\rho \varphi_R + \phi_R \partial_\mu \varphi) \right\} - a^4 V \\
&= \int d^4x \left\{ \sum_{\varphi=\phi_R,\theta} \left[ -\frac{1}{2} \left( \partial^2 - \frac{a^\mu}{a} + a^2 \varphi^2 \right) \hat{\varphi} - g^2 A^2 \hat{\varphi}^2 \right] - \frac{1}{3} a V_{\varphi\varphi\varphi} \hat{\varphi} - \frac{1}{4}! V_{\varphi\varphi\varphi\hat{\varphi}} \right\} \\
&\quad + gA^\mu \left[ -a \hat{\theta} \partial_\mu \left( \frac{\hat{\varphi}_R}{a} \right) + a \hat{\varphi}_R \partial_\mu \left( \frac{\hat{\theta}}{a} \right) \right], \tag{91}
\end{aligned}
\end{equation}

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with \( \phi_R = \phi + h \). The prime denotes derivative with respect to conformal time \( \tau \). We rescaled the scalar fields \( \varphi_\alpha = \{ \phi, h, \theta \} \) as in (14).

The gauge fixing action is

\[
S_{GF} = -\frac{1}{2\xi} \int d^4x \sqrt{-g} \left[ (g^{\mu\nu} \nabla_\mu A_\nu)^2 - (g^{\mu\nu} \nabla_\mu A_\nu) 2\xi g \phi_R \theta + \xi^2 g^2 \phi_R^2 \theta^2 \right].
\] (92)

The first term becomes

\[
-\frac{1}{2\xi} \int d^4x (\partial_\mu A^\mu - \eta^{\mu\nu} \Gamma^\rho_{\mu\nu} A_\rho)^2 = \frac{1}{2\xi} \int d^4x A_\mu \left[ \partial^\mu \partial^\nu + 2\eta^{\alpha\beta} \Gamma^\mu_{\alpha\beta} \partial^\nu - \eta^{\alpha\beta} \Gamma^\mu_{\alpha\beta} \Gamma^\nu_{\rho\sigma} \right] A_\nu
\]

\[
= \frac{1}{\xi} \int d^4x \left[ \frac{1}{2} A_\mu \partial^\mu \partial^\nu A_\nu + A_0 (\mathcal{H}' - 2\dot{\mathcal{H}}^2) A_0 - A_0 2\mathcal{H} \partial^k A_i \right].
\] (93)

To get the second line we used the explicit form of the connections (8) to write

\[
A_\mu \eta^{\rho\sigma} \Gamma^\mu_{\rho\sigma} = A_0 (\eta^{00} \Gamma^{00}_{00} + \eta^{ij} \Gamma^{00}_{ij}) = A_0 \mathcal{H}(1 - 3) = -2\mathcal{H} A_0,
\] (94)

and integration by parts

\[
4 \int d^4x A_0 \mathcal{H} \partial_0 A_0 = -2 \int d^4x A_0 \mathcal{H}' A_0.
\] (95)

The second term in (92) we can partially integrate using (with \( \check{A}^\mu \equiv g^{\mu\nu} A_\nu \) to indicate the index is raised with \( g^{\mu\nu} \))

\[
\int d^4x \sqrt{-g} (\nabla_\mu \check{A}^\mu) B = - \int d^4x \sqrt{-g} \check{A}^\mu \partial_\mu B.
\] (96)

This follows from the fact that we have a covariant volume and a covariant derivative. Note also that \( \nabla_\mu g_{\mu\nu} = 0 \), and it is therefore irrelevant whether the raised index is on \( A \) or on \( \nabla \). Thus the second term in (92) can be written as

\[
- \int d^4x a^2 g A^\mu (\partial_\mu \phi_R + \phi_R \partial_\mu \theta) = - \int d^4x a \theta \partial_\mu \left( \frac{\dot{\phi}_R}{a} \right) + a \dot{\phi}_R \partial_\mu \left( \frac{\theta}{a} \right).
\] (97)

The second term above will cancel with the last term in (91). The complete gauge-fixing term is

\[
S_{GF} = \int d^4x \left\{ \frac{1}{2} A_\mu \partial^\mu \partial^\nu A_\nu + A_0 (\mathcal{H}' - 2\dot{\mathcal{H}}^2) A_0 - A_0 2\mathcal{H} \partial^k A_i \right\}
\]

\[
- g A^\mu \left[ \dot{\theta} \left( \partial_\mu - \frac{a'}{a} \delta^\mu_0 \right) \dot{\phi}_R + \dot{\phi}_R \left( \partial_\mu - \frac{a'}{a} \delta^\mu_0 \right) \theta \right] - \frac{1}{2} \xi^2 g^2 \dot{\phi}_R^2 \dot{\theta}^2.
\] (98)

Finally the Faddeev-Popov term is

\[
S_{FP} = \int d^4x a^4 \eta \left[ - \nabla^2 + \xi g^2 (\theta^2 - \phi_R^2) \right] \eta,
\] (99)

which follows from

\[
\delta_\alpha G = - \frac{1}{g} \partial^2 \alpha - \frac{1}{g} \Gamma^\mu_{\mu\rho} \partial^\rho \alpha + \xi g (\theta^2 - \phi_R^2) \alpha = - \frac{1}{g} \nabla^2 \alpha + \xi g (\theta^2 - \phi_R^2) \alpha,
\] (100)

24
where we used $\delta_{\alpha \phi R} = -\alpha \delta, \delta_{\alpha \theta} = \alpha \delta_R$, and $\delta_{\alpha} A_{\mu} = (-1/g) \partial_{\mu} \alpha$. Use $\Gamma_{\mu \rho}^{\alpha} = \partial_{\rho} \sqrt{-g}/\sqrt{-g}$ to write the first term in $\langle 99 \rangle$ as

$$- \int d^4x \sqrt{-g} \eta \nabla^2 \eta = - \int d^4x \sqrt{-g} \eta \left( \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu \nu} \partial_{\nu} \right) \eta = - \int d^4x \eta \left[ \partial^2 - \frac{\alpha''}{a} \right] \eta, \hspace{1cm} (101)$$

where in the last step we rescaled the anti-commuting scalars $\hat{\eta} = a \eta$. Hence

$$S_{FP} = - \int d^4x \eta \left[ \partial^2 - \frac{\alpha''}{a} + \xi g^2 (\hat{\phi}_R^2 - \hat{\theta}^2) \right] \hat{\eta}. \hspace{1cm} (102)$$

Putting it all together, we write the action as $S = \sum S^{(i)}$ with $i$ denoting the number of quantum fields each term in $S^{(i)}$ contains. Then

$$S^{(0)} = \int d^4x \left\{ \frac{1}{2} (\hat{\phi}')^2 + \frac{1}{2} \frac{a'}{a} \hat{\phi}^2 - a^4 V \right\}, \hspace{1cm} (103)$$

$$S^{(1)} = \int d^4x \left\{ - \hat{h} \left( \left( \partial^2 - \frac{\alpha''}{a} \right) \hat{\phi} + a^3 V_{\theta} \right) \right\}, \hspace{1cm} (104)$$

$$S^{(2)} = \frac{1}{2} \int d^4x \left\{ A_{\mu} \left[ (\partial_{\nu} \partial^\nu + g^2 \hat{\phi}^2) \eta_{\mu \nu} - (1 - \frac{1}{\xi}) \partial^\mu \partial^\nu \right] A_{\nu} + \frac{1}{\xi} [A_0 (H' - 2H^2) A_0 - A_0 2H \partial^4 A_0] - \hat{\theta} (\partial^2 - \frac{\alpha''}{a} + a^2 V_{\theta \theta} + \xi g^2 \hat{\phi}^2) \hat{\theta} - 4g A^0 \hat{\theta} \left( \partial_\tau - \frac{a'}{a} \right) \hat{\phi} - \hat{h} (\partial^2 - \frac{\alpha''}{a} + a^2 V_{hh}) \hat{h} - 2 \eta \left[ \partial^2 - \frac{\alpha'}{a} + \xi g^2 \hat{\phi}^2 \right] \hat{\eta} \right\}. \hspace{1cm} (105)$$

$$S^{(3)} = \int d^4x \left\{ - S_{\alpha \beta \gamma} a V_{\alpha \beta \gamma} \hat{\phi}_\alpha \hat{\phi}_\beta \hat{\phi}_\gamma - 2g A^0 \hat{\theta} \left( \partial_\mu - \frac{a'}{a} \delta_\mu^0 \right) \hat{h} + g^2 (A^2 - \xi \hat{\theta}^2 - 2\xi \hat{\eta} \hat{\eta}) \hat{\phi} \right\}, \hspace{1cm} (106)$$

$$S^{(4)} = \int d^4x \left\{ - S_{\alpha \beta \gamma \delta} V_{\alpha \beta \gamma \delta} \hat{\phi}_\alpha \hat{\phi}_\beta \hat{\phi}_\gamma \hat{\phi}_\delta + \frac{1}{2} g^2 A^2 (\hat{h}^2 + \hat{\theta}^2) - g^2 \xi \hat{\eta} (\hat{\theta}^2 - \hat{h}^2) \hat{\eta} - \frac{1}{2} g^2 \xi \hat{\theta} \hat{h} \right\}, \hspace{1cm} (107)$$

with $\varphi_{\alpha} = \{ h, \theta \}$, and $S_{\alpha \beta \gamma (\delta)}$ symmetry factors. For a quartic Higgs potential

$$a^4 V = a^4 \left[ \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right] = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4. \hspace{1cm} (108)$$

## B Tadpole method

In this appendix we give the derivation of the one-loop quantum corrected equation of motion, starting from first principles to determine the symmetry factors of the diagrams. We use the in-in formalism (see Sec. 23 and 24 25 26 27 28 29 30) where all quantum fields are doubled, and labeled by $\pm$ superscripts. Expanding the Higgs field around the classical
background as in (3), the quantum corrected equations of motion follow from the vanishing of the tadpole [30]:

$$\langle h^+(\tau, \vec{x}) \rangle = 0,$$

(109)

where $\langle \cdot \rangle$ denotes the vacuum expectation value. The vanishing of the $h^-$ component gives the same result, and does not have to be considered separately. We can write this as a path integral expression:

$$0 = \langle h^+(y) \rangle = \int D\psi^+ \overline{\psi} h^+(y) e^{iS_{\text{int}}[\psi^+]} - iS_{\text{int}}[\psi^-]$$

$$= \int D\psi^+ \overline{\psi} h^+(y) \left[ 1 + i \int d^4x (L_{\text{int}}^+ - L_{\text{int}}^-) - \frac{1}{2!} \int d^4x (L_{\text{int}}^+ - L_{\text{int}}^-) \int d^4x'(L_{\text{int}}^+ - L_{\text{int}}^-) + ... \right]$$

$$= -i \int d^4x D_{\lambda}^{++}(y - x) A(x),$$

(110)

with $\psi_\alpha$ running over all fields. The equations of motion are then

$$A(x) = 0.$$  

(111)

The relevant interactions are given in Sec. 2.2. The one-loop equations of motion can be calculated order by order in perturbation theory, that is, ordered by the number of insertions coming from $L_{\text{int}}$. The higher order expectation values can be evaluated by taking all possible Wick contractions.

**First order.** At zeroth order there is no contribution because $\langle h^+ \rangle = 0$ in the free theory. At first order there is a classical and quantum contribution. Starting with the classical tree-level contribution:

$$0 = \langle h^+(y) \rangle = \int D\psi_\alpha D\overline{\psi}_\alpha h^+(y) e^{iS_{\text{int}}[\psi^+] - iS_{\text{int}}[\psi^-]}$$

$$= \int D\psi_\alpha D\overline{\psi}_\alpha h^+(y) \left[ 1 + i \int d^4x (L_{\text{int}}^+ - L_{\text{int}}^-) - \frac{1}{2!} \int d^4x (L_{\text{int}}^+ - L_{\text{int}}^-) \int d^4x'(L_{\text{int}}^+ - L_{\text{int}}^-) + ... \right]$$

$$= -i \int d^4x D_{\lambda}^{++}(y - x) A(x),$$

(110)

with $\psi_\alpha$ running over all fields. The equations of motion are then

$$A(x) = 0.$$  

(111)

The first order one-loop quantum contribution has only diagonal two-point insertions; the off-diagonal two-point interactions only enter at higher order. Then

$$0 = -i \int d^4x \langle h^+(y) h^+(x) \rangle \lambda^+_h(x) = -i \int d^4x D_{\lambda}^{++}(y - x) \lambda^+_h(x) \Rightarrow A_{\text{el}} = \lambda^+_h(x) = 0.$$  

(112)

The first order one-loop quantum contribution has only diagonal two-point insertions; the off-diagonal two-point interactions only enter at higher order. Then

$$0 = -i \int d^4x \langle h^+(y) h^+(x) \rangle \psi_\alpha^+(x)^2 \frac{1}{2} \partial_\phi m_\alpha^2 = -i \int d^4x D_{\lambda}^{++}(y - x) D_{\alpha}^{++} \frac{1}{2} \partial_\phi m_\alpha^2$$

$$\Rightarrow A_1 = \frac{1}{2} \partial_\phi m_\alpha^2 D_{\alpha}^{++}(0),$$

(113)

as there is only one possible Wick contraction. Here the subscript on $A_n$ denotes the $n$th order contribution.

**Second order.** For convenience, in the rest of this section we will drop the $\pm$ superscript. It should be kept in mind that at all times one should sum over all possibilities, and the incoming $h$ propagator is always $D_{\lambda}^{++}(y - x)$. This summation is done explicitly in the main text.

Consider the second order contribution, with one two-point insertion. The three-point vertex connecting to the incoming $h$ field can either be in the first or second factor of $L_{\text{int}}$, as there is only one possible Wick contraction. Here the subscript on $A_n$ denotes the $n$th order contribution.
which cancels the $1/2!$ in front (symmetry under $x \leftrightarrow x'$). Consider first two diagonal two-point insertions, and the three-point vertex diagonal as well:

$$
- \frac{1}{4} \int d^4x d^4x' \partial_\mu m_\alpha^2(x) m_\beta^2(x') \langle h(y) h(x) \psi_\alpha(x)^2 \psi_\beta(x')^2 \rangle \\
= -i \int d^4x D_h(y - x) \left[ -\frac{i}{2} \partial_\mu m_\alpha^2(x) \int d^4x' m_\beta^2(x') D_\alpha(x - x') D_\beta(x' - x) \right],
$$

(114)

since there are two possible Wick contractions. The part between the square brackets is $A^\text{diag}_{2A}$. For the Minkowski contribution with $m^2_{\theta A}$ we get

$$
- \int d^4x d^4y' g_{\theta \theta}(x') \langle h(y) [(\partial_\tau - H(\tau)) h(x)] A_0(x) \theta(x) A_0(x') \theta(x') \rangle \\
= -i \int d^4x \left[ (\partial_\tau - H(\tau)) D_h(y - x) \right] \left[ -i2g \int d^4x' m_\theta^2(x') D_{\theta\theta}(x - x') D_{\theta\theta}(x' - x) \right] \\
= -i \int d^4x D_h(y - x) \left[ -i2g(\partial_\tau - H(\tau)) \int d^4x' m_\theta^2(x') D_{\theta\theta}(x - x') D_{\theta\theta}(x' - x) \right],
$$

(115)
as there is only one possible Wick contraction. To get the last expression we integrated by parts. The $A_1, \partial_\mu h$-vertex does not contribute (even at higher order and finite terms), as going through the same steps, we get a result of the schematic form $\partial^4 A(t) = 0$.

Finally, for the diagram with one $(m^2)^{0i}$ insertion we get

$$
- \frac{1}{2} \int d^4x d^4x' \partial_\phi m_{\mu \mu}^2(x)(m^2)^{0i}(x')(x') \langle A_\mu(x)^2 A_0(x') A_i(x') \rangle \\
= -i \int d^4x D_h(y - x) \left[ -i\partial_\phi m_{\mu \mu}^2(x) \int d^4x' (m^2)^{0i}(x') D_{\mu \nu}(x - x') D_{\mu \nu}(x' - x) \right],
$$

(116)
as there are two possible Wick contractions. Both Wick contractions give the same result.

**Third order.** At third order the diagram with two mixed mass $(m^2)^{0i}$ contributes to the divergent part in $A$. There is a symmetry under the interchange of the positions of the three vertices, $(x, x', x'')$, and taking a definite order removes the factor $1/3!$ in [(110)]. The result is

$$
i \int d^4x d^4x' d^4x'' \frac{1}{2} \partial_\phi m_{\mu \mu}^2(x)(m^2)^{0i}(x')(m^2)^{0i}(x'') \langle h(y) h(x) A^\mu(x)^2 A^0(x') A^i(x') A^0(x'') A^i(x'') \rangle \\
= -i \int d^4x D_h(y - x) \times \left[ -\frac{1}{2} \partial_\phi m_{\mu \mu}^2(x) \int d^4x' d^4x'' m_\mu^2(x') m_\nu^2(x'') \sum D_{\mu \rho}(x - x') D_{\sigma \pi}(x' - x'') D_{\kappa \nu}(x'' - x) \right],
$$

(117)

where the sum is over the four possible Wick contractions [(14)]. The term between brackets is $A^\text{mass}_3$.

**References**

[1] N. D. Birrell and P. C. W. Davies, Cambridge, Uk: Univ. Pr. (1982) 340p
[2] P. Candelas and D. J. Raine, Phys. Rev. D 12 (1975) 965.
[3] A. Ringwald, Annals Phys. 177 (1987) 129.
[4] A. Ringwald, Z. Phys. C 34, 481 (1987).
[5] P. B. Greene and L. Kofman, Phys. Lett. B 448 (1999) 6 [hep-ph/9807339].
[6] J. Baacke, K. Heitmann, C. Patzold, Phys. Rev. D58 (1998) 125013. [hep-ph/9806205].
[7] S. R. Coleman, E. J. Weinberg, Phys. Rev. D7 (1973) 1888-1910.
[8] F. L. Bezrukov, M. Shaposhnikov, Phys. Lett. B659 (2008) 703-706. [arXiv:0710.3755 [hep-th]].
[9] R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi, JHEP 0010 (2000) 012 [arXiv:hep-ph/0002151].
[10] K. Enqvist and A. Mazumdar, Phys. Rept. 380 (2003) 99 [hep-ph/0209241].
[11] I. Affleck and M. Dine, Nucl. Phys. B 249 (1985) 361.
[12] M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B 458 (1996) 291 [hep-ph/9507453].
[13] G. M. Shore, Annals Phys. 128 (1980) 376.
[14] B. Allen, Nucl. Phys. B 226 (1983) 228.
[15] K. Ishikawa, Phys. Rev. D 28 (1983) 2445.
[16] B. Garbrecht, Nucl. Phys. B 784, 118 (2007) [hep-ph/0612011].
[17] R. Fukuda and T. Kugo, Phys. Rev. D 13 (1976) 3469.
[18] N. K. Nielsen, Nucl. Phys. B 101 (1975) 173.
[19] S. Mooij and M. Postma, JCAP 1109, 006 (2011) [arXiv:1104.4897 [hep-th]].
[20] J. Baacke, K. Heitmann and C. Patzold, Phys. Rev. D 55 (1997) 7815 [arXiv:hep-ph/9612264].
[21] J. Baacke, K. Heitmann, Phys. Rev. D60 (1999) 105037. [hep-th/9905201].
[22] K. Heitmann, Phys. Rev. D64 (2001) 045003. [hep-ph/0101281].
[23] D. Boyanovsky, D. Brahm, R. Holman and D. S. Lee, Phys. Rev. D 54 (1996) 1763 [arXiv:hep-ph/9603337].
[24] J. S. Schwinger, J. Math. Phys. 2 (1961) 407-432.
[25] L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515 [Sov. Phys. JETP 20 (1965) 1018].
[26] R. D. Jordan, Phys. Rev. D33 (1986) 444-454.
[27] P. M. Bakshi, K. T. Mahanthappa, J. Math. Phys. 4 (1963) 1-11.
[28] P. M. Bakshi, K. T. Mahanthappa, J. Math. Phys. 4 (1963) 12-16.
[29] E. Calzetta and B. L. Hu, Phys. Rev. D 35 (1987) 495
[30] S. Weinberg, Phys. Rev. D72 (2005) 043514. [hep-th/0506236].
[31] S. Weinberg, Phys. Rev. D 9 (1974) 3357.
[32] K. Heitmann, Master's Thesis, 1996.
[33] K. Heitmann, PhD Thesis, 2000.
[34] J. Baacke, K. Heitmann, C. Patzold, Phys. Rev. D57 (1998) 6398-6405. [hep-th/9711144].
[35] M. E. Peskin and D. V. Schroeder, Reading, USA: Addison-Wesley (1995) 842 p
[36] C. Grosse-Knetter and R. Kogerler, Phys. Rev. D 48 (1993) 2865 [hep-ph/9212268].
[37] J. Goldstone, Nuovo Cim. 19 (1961) 154-164.
[38] J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. 127 (1962) 965-970.
[39] J. Baacke, D. Boyanovsky and H. J. de Vega, Phys. Rev. D 63, 045023 (2001) [hep-ph/9907337].