The Mass Function of Dark Matter Haloes in a Cosmological Model with a Running Primordial Power Spectrum

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ABSTRACT

We present the first study on the mass functions of Jenkins et al (J01) and an estimate of their corresponding largest virialized dark halos in the Universe for a variety of dark-energy cosmological models with a running spectral index. Compared with the PL-CDM model, the RSI-CDM model can raise the mass abundance of dark halos for small mass halos at lower redshifts, but it is not apparent on scales of massive mass halos. Particularly, this discrepancy increases largely with the decrease of redshift, and the RSI-CDM model can suppress the mass abundance on any scale of halo masses at higher redshift. As for the largest mass of virialized halos, the spatially flat \(\Lambda\)CDM models give more massive mass of virialized objects than other models for both of PL-CDM and RSI-CDM power spectral indexes, and the RSI-CDM model can enhance the mass of largest virialized halos for all of models considered in this paper. So we probably distinguish the PL-CDM and RSI-CDM models by the largest virialized halos in the future survey of cluster of galaxies.

Subject headings: cosmology:theory—dark matter—galaxies:halos —large-scale structure

1. Introduction

The central problem in modern cosmology is the formation of large scale structures in the universe. In the standard picture of hierarchical structure formation, dark matter dominates the universe, and a wide variety of observed structures, such as galaxies, groups and clusters of galaxies, have formed by the gravitational growth of Gaussian primordial
density fluctuations. Due to self-gravitational instability, the fluctuations of dark matter have collapsed and virialized into objects which are so-called ‘dark matter halos’ or ‘dark halos’. The larger halos are generally considered to have formed via the merger of smaller ones collapsed first. The distribution of mass in the gravitationally collapsed structures, such as galaxies and groups (or clusters) of galaxies, which is usually called the mass or multiplicity function, has been determined by observation.

As the observational data relevant to these issues improve, the need for accurate theoretical predictions increases. By far the most widely used analytic formulae for halo mass functions are based on extensions of the theoretical framework first sketched by Press & Schechter (1974). The Press-Schechter (PS) model theory did not draw much attention until 1988, when the first relative large N-Body simulation revealed a good agreement with it. The mystery of the ‘fudge factor’ of 2 in PS theory was solved by approaching the ‘cloud-in-cloud’ problem with a rigorous way (Peacock & Heavens 1990; Bond et al. 1991). The reliability of the PS formula has been tested using N-Body simulation by several authors, which turns out the PS formula indeed provides an overall satisfactory description of mass function for virialized objects. Unfortunately, none of these derivations is sufficiently rigorous such that the resulting formulae can be considered accurate beyond the regime where they have been tested against N-body simulations. Although the analytical framework of the PS model has been greatly refined and extended in recent years, in particular to allow predictions for the merger histories of dark matter halos (Bond et al. 1991), it is well known that the PS mass function, while qualitatively correct, disagrees in detail with the results of N-body simulations. Specifically, the PS formula overestimates the abundance of halos near the characteristic mass and underestimates the abundance in the high mass tail. In order to overcome this discrepancy, Jenkins et al. (2001) proposed an analytic mass function which gives a fit to their numerical multiplicity function.

In particular, a power spectrum of primordial fluctuation, \( P_p(k) \), should be assumed in advance in the calculation of mass function. Inflationary models predict a approximately scale-invariant power spectra for primordial density (scalar metric) fluctuation, \( P_p(k) \propto k^n \) with index \( n = 1 \) (Guth & Pi 1982; Bardeen et al. 1983). The combination of the first-year Wilkinson Microwave Anisotropy Probe (WMAP) data with other finer scale cosmic background (CMB) experiments (Cosmic Background Imager [CBI], Arcminute Cosmology Bolometer Array Receiver [ACBAR]) and two observations of large-scale structure (the Anglo-Australian Telescope Two-Degree Field Galaxy Redshift Survey [2dFGRS] and Lyman \( \alpha \) forest) favour a \( \Lambda \)CDM cosmological model with a running index of the primordial power spectrum (RSI-\( \Lambda \)CDM), while the WMAP data alone still suggest a best-fit standard power-law \( \Lambda \)CDM model with the spectral index of \( n \approx 1 \) (PL-\( \Lambda \)CDM) (Spergel et al. 2003; Peiris et al. 2003). However, there still exist the intriguing discrepancies between theoretical
predictions and observations on both the largest and smallest scales. While the emergence of a running spectral index may improve problems on small scales, there remain a possible discrepancy on the largest angular scales. It is particularly noted that the running spectral index model suppress significantly the power amplitude of fluctuations on small scales (Spergel et al. 2003; Yoshida et al. 2003). This imply a reduction of the amount of substructure within galactic halos (Zentner & Bullock 2002). Yoshida et al. (2003) studied early structure formation in a RSI-ΛCDM universe using high-resolution cosmological N-body/hydrodynamic simulations. They showed that the reduced small-scale power in the RSI-ΛCDM model causes a considerable delay in the formation epoch of low-mass minihalos (\(\sim 10^6 M_\odot\)) compared with the PL-ΛCDM model, although early structure still forms hierarchically in the RSI-ΛCDM model. Thus the running index probably affect the abundance of dark halos formed in the evolution of the universe.

Among the virialized structures, galaxy clusters are extremely useful to cosmology because they may be in detail studied as individual objects, and especially are the largest virialized structure in the universe at present. The mass of a typical rich clusters is approximately \(10^{15} h^{-1} M_\odot\), which is quite similar to the average mass within a sphere of \(8 h^{-1} \text{Mpc}\) radius in the unperturbed universe. However, the theoretical estimate of the mass of the largest collapsed object in the RSI-ΛCDM cosmological framework has still not been presented. Therefore, we will calculate the mass function of collapsed objects by J01 mass functions respectively and present the first calculation of the largest virialized object in the Universe in a RSI-ΛCDM model to explore the effect of running spectral index of primordial fluctuation on structure formation.

The reminder of this paper is organized as follows. We describe mass function of dark halos in Section 2. The largest virialized dark halos in the universe are presented in Section 3. The conclusion and discussion are given in Section 4.

2. Mass Function of Dark Halos

In the standard hierarchical theory of structure formation, the comoving number density of virialized dark halos per unit mass \(M\) at redshift \(z\) can be expressed as: 
\[ n(M, z) = \frac{dN}{dM} = \rho_0 f(M, z)/M \]
where \(\rho_0\) is the mean mass density of the universe today and, instead of PS formula in this letter, the mass function \(f(M, z)\) takes the form of an empirical fit from high-resolution simulation (Jenkins et al. 2001)

\[ f(M, z) = \frac{0.301 d \ln \sigma^{-1}(M, z)}{M} \frac{d \ln M}{d \ln M} \exp(-|\ln \sigma^{-1}(M, z) + 0.64|^{3.88}). \] (1)
Here $\sigma(M, z) = \sigma(M)D(z)$ and $D(z) = e(\Omega(z))/e(\Omega_m)(1 + z)$ is the linear growth function of density perturbation (Carroll et al. 1992), in which $e(x) = 2.5x/(1/70 + 209x/140 - x^2/140 + x^4/7)$ and $\Omega(z) = \Omega_m(1+z)^3/E^2(z)$. The present variance of the fluctuations within a sphere containing a mass $M$ can be expressed as $\sigma^2(M) = 1/2\pi \int_0^\infty P(k)W(kr_M)k^2dk$, where $W(kr_M) = 3[\sin(kr_M)/(kr_M)^3 - \cos(kr_M)/(kr_M)^2]$ is the Top-hat window function in Fourier space and $r_M = (3M/4\pi \rho_0)^{1/3}$. The power spectrum of CDM density fluctuations is $P(k) = P_p(k)T^2(k)$ where the matter transfer function $T(k)$ is given by Eisenstein & Hu (1999), and $P_p(k)$ is the primordial power spectrum of density fluctuation. The scale-invariant primordial power spectrum in the PL-ΛCDM model is given by $P_p(k) = A k^{ns}$ with index $ns=1$ and that in the RSI-ΛCDM model is assumed to be $P_p(k) = P(k_0)(k/k_0)^{ns(k)}$, where the index $ns(k)$ is a function of length scale

$$ns(k) = n_s(k_0) + \frac{1}{2} \frac{dn_s(k_0)}{d \ln k} \ln \left( \frac{k}{k_0} \right).$$

The pivot scale $k_0=0.05$ h Mpc$^{-1}$, $n_s(k_0)=0.93$, and $dn_s/d \ln k=-0.03$ are the best-fit values to the combination data of the recent CMB experiments and two other large-scale structure observations (Spergel et al. 2003). For both PL-ΛCDM and RSI-ΛCDM models, the amplitude of primordial power spectrum, $A$ and $P(k_0)$, are normalized to $\sigma_8 = \sigma(r_M = 8h^{-1}\text{Mpc})$, which is the rms mass fluctuations when present universe is smoothed using a window function on a scale of $8h^{-1}\text{Mpc}$. In this section, we assume spatially flat ΛCDM models characterized by the matter density parameter $\Omega_m$, vacuum energy density parameter $\Omega_\Lambda$. For both PL-ΛCDM and RSI-ΛCDM models, we take cosmological parameters to be the new result from the WMAP: Hubble constant $h = 0.71$, $\Omega_m = 0.27$, $\sigma_8 = 0.84$ (Bennett et al. 2003; Spergel et al. 2003).

The mass function of dark halos directly involve the calculation of primordial power of density fluctuation. In order to explore the difference between the two kinds of primordial power spectrum, we first calculate the mass function of dark matter halos in a wide range of redshift, which are plotted in Fig.(1). It is noted that there is a slight difference between the PL-CDM model and RSI-CDM model at lower redshifts. Compared with the PL-CDM model, the RSI-CDM model can raise the mass abundance of dark halos for small mass halos at lower redshifts, but it is not apparent on scales of massive mass halos. Particularly, this discrepancy increases largely with the decrease of redshift. Similar to the result(Yoshida et al. 2003), the RSI-ΛCDM model can suppress the mass abundance on any scale of halo masses at higher redshift. According to the hierarchical formation theory of structure, there is fewer higher mass halos at higher redshift and the higher mass halos are formed by the merger of lower mass haloes at the relative late stage. As pointed previously, the RSI model can suppress the power spectrum at small scale, so this just leads to a considerable delay in the formation of low mass haloes instead of high mass haloes. Therefore, compared with the
PL-CDM model, the mass function is uniformly lower for the RSI model at higher redshift of \( z = 6 \).

3. The Largest Virialized Dark Halos In the Universe

Based on the theoretical expression above, we can easily get the total number \( N \) of the virialized objects with the mass larger than \( M \)

\[
N = \int_0^\infty \left[ \int_M^\infty \frac{dn}{dM} dM \right] \frac{dV}{dz} dz, \tag{3}
\]

where \( dV \) is the comoving volume element for the Friedmann-Robertson-Walker metric and \( \frac{dV}{dz} \) takes the form

\[
d\frac{V}{dz} = \begin{cases}
4\pi \frac{c^3}{H_0^2} \frac{D_a^2 (1+z)^2 \cos[\sqrt{\Omega_k} f]}{\sqrt{1+\Omega_k D_a^2 (1+z)^2 E(z)}} & \text{for } \Omega_k < 0 \\
4\pi \frac{c^3}{H_0^2} \frac{D_a^2 (1+z)^2}{E(z)} & \text{for } \Omega_k = 0 \\
4\pi \frac{c^3}{H_0^2} \frac{D_a^2 (1+z)^2 \cosh[\sqrt{\Omega_k} f]}{\sqrt{1+\Omega_k D_a^2 (1+z)^2 E(z)}} & \text{for } \Omega_k > 0,
\end{cases} \tag{4}
\]

where \( D_a = d_A H_0 / c \), \( d_A \) is the angular diameter distance and \( f = \int_0^z dz / E(z) \).

It is obvious from the Eq.(3) that the total number \( N \) decrease with the increase of the mass \( M \). Setting \( N = 1 \), we can finally obtain the largest mass \( M_{MAX} \) of virialized object

\[
1 = \int_0^\infty \left[ \int_{M_{MAX}}^\infty \frac{dn}{dM} dM \right] \frac{dV}{dz} dz. \tag{5}
\]

In this section, we consider three cold matter (CDM) models, i.e. the standard CDM (SCDM), spatially flat ΛCDM models, and an open CDM (OCDM) for both PL and RSI

| Model  | \( \Omega_m \) | \( \Omega_\Lambda \) | \( \Gamma \) | \( \sigma_8 \) |
|--------|----------------|----------------|--------|--------|
| SCDM   | 1              | 0              | 0.5    | 0.6    |
| LCDM   | 0.3            | 0.7            | 0.21   | 1.0    |
| OCDM   | 0.3            | 0              | 0.25   | 1.0    |
Fig. 1.— Mass function at redshift $z=0$, 0.6, 1.27 and 6, respectively. The solid line is the mass function for running spectral index $\Lambda$CDM model, while the dashed one is that for power law $\Lambda$CDM model.
spectrum models. The cosmological models parameters are given in Table 1. Then we calculate the largest virial mass $M_{\text{MAX}}$ in a variety of cosmological models for both PL and RSI power spectrum model, the results of which are demonstrated in Table 2. From Table 2 we can see that the different cosmological models may yield the different result about virial mass for the largest virialized halos. The spatially flat ΛCDM models give more massive mass of virialized objects than other models for both of PL and RSI power spectral models. Therefore, it can distinguish different cosmological models by the largest mass of virialized halos. Due to the accumulative effect of the integration for volume(or redshift) over the whole space in the universe, the prediction for virial mass is slightly greater than the observed typical one. In addition, we also notice that the RSI-CDM model can enhance the mass of largest virialized halos for all of models considered here.

4. Conclusions and Discussion

Motivated by the new result on the index of primordial power spectrum from a combination of WMAP data with other finer scale CMB experiments and other large-scale structure observations, we present the first study on the mass functions of J01 and their corresponding largest virialized dark halos in the Universe for a variety of dark-energy cosmological models with a running spectral index. It is well known that structures in the universe forms hierarchically in standard CDM models. The most massive structure form rather late in the universe. It is also noted that there is a slight difference between the mass abundance of PL-CDM and RSI-CDM model at lower redshifts. Compared with the PL-CDM model, the RSI-CDM model can raise the mass abundance of dark halos for small mass halos at lower redshifts, but it is not apparent on scales of massive mass halos. Particularly, this discrepancy increases largely with the decrease of redshift, and the RSI-ΛCDM model can suppress the mass abundance on any scale of halo masses at higher redshift.

Table 2. Numerical Results for the largest virial mass $M_{\text{MAX}}(10^{15}h^{-1}M_\odot)$

| Model | PL model | RSI model |
|-------|----------|-----------|
| SCDM  | 3.0      | 3.3       |
| LCDM  | 6.2      | 6.9       |
| OCDM  | 4.5      | 5.05      |
As for the largest mass of virialized halos, the spatially flat ΛCDM models give more massive mass of virialized objects than other models for both of PL-CDM and RSI-CDM power spectral models. Therefore, it can distinguish different cosmological models by the largest mass of virialized halos for both of PL-CDM and RSI-CDM models. In addition, we also notice that the RSI-CDM model can enhance the mass of largest virialized halos for all of models considered here. So we probably distinguish the PL-CDM and RSI-CDM models by the largest virialized halos in the future survey of cluster of galaxies. Therefore, the obtained largest virialized object can be referred to as the complement to the observations of CMB, SN Ia and large scale structure in the future cosmological observation.

Yoshida et al. (2003) found that although the hierarchical formation mechanism do not work well in RSI-ΛCDM model compared with that in PL-ΛCDM model and it also is not clear that the PS theory can be used in RSI-ΛCDM model, the mass function measured by high-resolution cosmological N-body/hydrodynamic simulations overall match the PS mass function for both RSI-ΛCDM and PL-ΛCDM model. In addition, because the running spectral index model predicts a significant lower power of density fluctuation on small scales than the standard PL-ΛCDM model(Spergel et al. 2003; Yoshida et al. 2003), it should also attract considerable attention in studies on strong lensing (Zhang et al. 2004; Chen 2003a,b, 2004a,b; Zhang 2004) and weak lensing by large-scale structure(Ishak et al. 2004), especially on skewness(Pen et al. 2003; Zhang et al. 2003; Zhang & Pen 2005), which characterizes the non-Gaussian property of κ field in the nonlinear regime.

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