An Optimal Distributed Algorithm with Operator Extrapolation for Stochastic Aggregative Games

Tongyu Wang\textsuperscript{1, 3}, Peng Yi\textsuperscript{1, 2, 3},
\textsuperscript{1}Department of Control Science and Engineering, Tongji University, Shanghai, 201804, China;
\textsuperscript{2}The Shanghai Research Institute for Intelligent Autonomous Systems, Shanghai, 201804, China;
\textsuperscript{3}Shanghai Institute of Intelligent Science and Technology, Tongji University, Shanghai 200092, China.

Corresponding Author: Peng Yi Email: yipeng@tongji.edu.cn

Abstract—This work studies Nash equilibrium seeking for a class of stochastic aggregative games, where each player has an expectation-valued objective function depending on its local strategy and the aggregate of all players’ strategies. We propose a distributed algorithm with operator extrapolation, in which each player maintains an estimate of this aggregate by exchanging this information with its neighbors over a time-varying network, and updates its decision through the mirror descent method. An operator extrapolation at the search direction is applied such that the two step historical gradient samples are utilized to accelerate the convergence. Under the strongly monotone assumption on the pseudo-gradient mapping, we prove that the proposed algorithm can achieve the optimal convergence rate of $O(1/k)$ for Nash equilibrium seeking of stochastic games.

Index Terms—Aggregative games, Stochastic game, Distributed algorithm, Operator extrapolation, Optimal convergence rate

I. INTRODUCTION

In recent years, the non-cooperative games have been widely applied in decision-making for networked systems, where selfish networked players aim to optimize their own objective functions that have coupling with other players’ decisions. Nash equilibrium (NE) is one of the most widely used solution concept to non-cooperative games, under which no participant can improve its utility by unilaterally deviating from the equilibrium strategy. Specifically, the aggregative games is an important class of non-cooperative games, in which each player’s cost function depends on its own strategy and the aggregation of all player strategies. This aggregating feature emerges in numerous decision-making problems over networks systems, when the individual utility is affected by network-wide aggregate. Hence, aggregative games and its NE seeking algorithm get wide research, with application in charging coordination for plug-in electric vehicles, multidimensional opinion dynamics, communication channel congestion control, and energy management in power grids, etc. It also worth pointing out that stochastic game should be taken as a proper decision-making model when there are information uncertainties, while the NE seeking for stochastic games with various sampling schemes are recently studied. Hence, when the game has both information uncertainties and aggregating-type objective functions, stochastic aggregative game and its NE seeking are studied recently [1].

Existing works on distributed NE seeking largely resided in best-response (BR) schemes and gradient-based approaches. In BR schemes, players select a strategy that best responds to its rivals’ current strategies. For example, [2] proposed an inexact generalization of this scheme in which an inexact best response strategy is computed via a stochastic approximation scheme. In gradient-based approaches, the algorithm is easily implementable with a low computation complexity per iteration. For instance, [3] designed an accelerated distributed gradient play for a strongly monotone game and proved its geometric convergence, while it needs to estimate all players’ strategies. To reduce the communication costs, [4] proposed a consensus-based distributed gradient algorithm for aggregative games, in which players only need to estimate the aggregate in each iteration. Furthermore, fully asynchronous distributed algorithm and continuous-time distributed algorithm for aggregative games have also been studied. Moreover, fast convergence rate is indispensable bonus for distributed Nash equilibrium seeking algorithm, since it implies less communication rounds. Particular, for stochastic games, distributed NE seeking with a fast convergence rate is highly desirable since its also implies low sampling cost of stochastic gradients. Motivated by this, an extra-gradient method is designed to improve the speed of NE seeking, while it needs two steps of operator evaluations and two step of projections at each iteration. To further reduce the computation burden, another method designed a simpler recursion with one step of operator evaluation and projection at each iteration, given by $x_{t+1} = \arg\min_{x \in X} \gamma_t \left(F(x_t + \beta_t (x_t - x_{t-1})) + V(x_t, x)\right)$, called projected reflected gradient method. But the iterate $x_t + \beta_t (x_t - x_{t-1})$ may sit outside the feasible set $X$, so it needs to impose the strong monotone assumption on the pseudo-gradient over $\mathbb{R}^n$. Recently, [5] proposed an operator extrapolation (OE) method to solve the stochastic variational inequality problem, for which the optimal convergence rate can be achieved through one operator evaluation and a single projection per iteration.

Motivated by the demands for fast NE seeking algorithm of stochastic aggregative games and inspired by [5], we propose a distributed operator extrapolation (OE) algorithm via mirror descent and dynamical averaging tracking. At each stage, each player aligns its intermediate estimate by a consensus step with its neighbors estimates of the aggregate, samples an unbiased estimate of its payoff gradients, takes a small step via the
OE method, and then mirrors it back to the strategy set. The algorithm achieves the optimal convergence rate $O(1/k)$ for the class of stochastic strongly monotone aggregative games. The numerical experiments on a Nash-cournot competition problem demonstrate it advantages over some existing NE seeking methods.

The rest of the paper is organized as follows. In Section II, we give the formulation of the stochastic aggregative games, and state the assumptions. In Section III, we propose a distributed operator extrapolation algorithm and provide convergence results for the class of strongly monotone games. Finally, we give concluding remarks in Section IV.

**Notations:** A vector $x$ is a column vector while $x^T$ denotes its transpose. $(x, y) = x^T y$ denotes the inner product of vectors $x, y$. $\|x\|$ denotes the Euclidean vector norm, i.e., $\|x\| = \sqrt{x^T x}$. A nonnegative square matrix $A$ is called doubly stochastic if $A 1 = 1$ and $1^T A = 1^T$, where $1$ denotes the vector with each entry equal 1. $I_N \in \mathbb{R}^{N \times N}$ denotes the identity matrix. Let $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$ be a directed graph with $\mathcal{N} = \{1, \ldots, N\}$ denoting the set of players and $\mathcal{E}$ denoting the set of directed edges between players, where $(j, i) \in \mathcal{E}$ if player $i$ can obtain information from player $j$. The graph $\mathcal{G}$ is called strongly connected if for any $i, j \in \mathcal{N}$ there exists a directed path from $i$ to $j$, i.e., there exists a sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{p-1}, j)$ in the digraph with distinct nodes $i_m \in \mathcal{N}$, $1 \leq m \leq p - 1$.

II. Problem Formulation

In this section, we formulate the stochastic aggregative games over networks and introduce basic assumptions.

A. Problem Statement

We consider a set of $N$ non-cooperative players indexed by $\mathcal{N} \triangleq \{1, \ldots, N\}$. Each player $i \in \mathcal{N}$ choose its strategy $x_i$ from a strategy set $X_i \subset \mathbb{R}^{m_i}$. Denote by $x \triangleq (x_1^T, \ldots, x_N^T)^T \in \mathbb{R}^{m}$ and $x_{-i} \triangleq \{x_j \mid j \neq i\}$ the strategy profile and the rival strategies, respectively. In an aggregative game, each player $i$ aims to minimize its cost function $f_i(x_i, \sigma(x))$, where $\sigma(x) \triangleq \sum_{j=1}^{N} x_j$ is an aggregate of all players’ strategies. Furthermore, given $\sigma(x_{-i}) \triangleq \sum_{j=1, j \neq i}^{N} x_j$, the objective of player $i$ is to minimize its parameterized stochastic local cost:

$$\min_{x_i \in X_i} f_i(x_i, x_i + \sigma(x_{-i})) \triangleq \mathbb{E} \left[ \hat{\psi}_i(x_i, x_i + \sigma(x_{-i}); \xi_i(\omega)) \right],$$

(1)

where $\xi_i : \Omega \rightarrow \mathbb{R}^{d_i}$ is a random variable defined on the probability space $(\Omega, \mathcal{F}, P_i)$, $\hat{\psi}_i : \mathbb{R} \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}$ is a scalar-valued function, and $\mathbb{E}[\cdot]$ denotes the expectation with respect to the probability measure $P_i$.

A Nash equilibrium (NE) of the aggregative game (1) is a tuple $x^* \{x^*_i\}_{i=1}^{N}$ such that for each $i \in \mathcal{N}$,

$$f_i(x_i^*, \sigma(x^*)) \leq f_i(x_i, x_i + \sigma(x^*_i)), \quad \forall x_i \in X_i.$$

We consider the scenario that each player $i \in \mathcal{N}$ knows the structure of its private function $f_i$, but have no access to the aggregate $\sigma(x)$. Instead, each player may communicate with its neighbors over a time varying graph $\mathcal{G}_k = \{\mathcal{N}, \mathcal{E}_k\}$. Define $W_k = [\omega_{ij,k}]_{i,j=1}^{N}$ as the adjacency matrix, where $\omega_{ij,k} > 0$ if and only if $(j, i) \in \mathcal{E}_k$, and $\omega_{ij,k} = 0$, otherwise. Denote by $N_{i,k} \triangleq \{j \in \mathcal{N} : (j, i) \in \mathcal{E}_k\}$ the neighboring set of player $i$ at time $k$.

B. Assumptions

We impose the following conditions on the time-varying communication graphs $\mathcal{G}_k = \{\mathcal{N}, \mathcal{E}_k\}$.

**Assumption 1:** (a) $W_k$ is doubly stochastic for any $k \geq 0$; (b) There exists a constant $0 < \eta < 1$ such that $\omega_{ij,k} \geq \eta$, $\forall j \in N_{i,k}$, $\forall i \in \mathcal{N}$, $\forall k \geq 0$; (c) There exists a positive integer $B$ such that the union graph $\{\mathcal{N}, \bigcup_{k=1}^{B} \mathcal{E}_{k+1}\}$ is strongly connected for all $k \geq 0$.

We define a transition matrix $\Phi(k, s) = W_k W_{k-1} \cdots W_s$ for any $k \geq s \geq 0$ with $\Phi(k, k+1) = I_N$, and state a result that will be used in the sequel.

**Lemma 1:** nedic2009distributed Let Assumption 1 hold. Then there exist $\theta = (1 - \eta(4N^2))^{-2} > 0$ and $\beta = (1 - \eta(4N^2))^{1/B}$ such that for any $k \geq s \geq 0$,

$$\left| \Phi(k, s)_{ij} - 1/N \right| \leq \theta \beta^{k-s}, \quad \forall i, j \in \mathcal{N}.$$  (2)

We require the player-specific problem to be convex and continuously differentiable.

**Assumption 2:** For each player $i \in \mathcal{N}$, (a) the strategy set $X_i$ is closed, convex and compact; (b) the cost function $f_i(x_i, \sigma(x))$ is convex in $x_i \in X_i$ for every fixed $x_{-i} \in X_{-i} \triangleq \prod_{j \neq i} X_j$; (c) $f_i(x_i, \sigma)$ is continuously differentiable in $(x_i, \sigma) \in X_i \times \mathbb{R}^{m_i}$.

For any $x \in X \triangleq \prod_{i=1}^{N} X_i$ and $z \in \mathbb{R}^{m}$, define

$$F_i(x_i, z) \triangleq (\nabla_{x_i} f_i(x_i, \sigma) + \nabla_{\sigma} f_i(x_i, \cdot))|_{\sigma=z},$$

$$\phi_i(x) \triangleq \nabla_{x_i} f_i(x_i, \sigma(x)), \quad \forall \sigma(x) \in X_i,$$

and $\phi(x) \triangleq (\phi_i(x))_{i=1}^{N}$.  (3)

From (3) and Assumption 2(c), it follows that the pseudogradient $\phi(x)$ is continuous. Since each player-specific problem is convex, by [6, Proposition 1.4.2], $x^*$ is a NE of (1) if and only if $x^*$ is a solution to a variational inequality problem $VI(X, \phi)$, i.e., finding $x^* \in X$ such that

$$(x - x^*)^T \phi(x^*) \geq 0, \quad \forall x \in X \triangleq \prod_{j=1}^{N} X_j.$$  (4)

Since $\phi$ is continuous, $X$ is convex and compact, the existence of NE follows immediately by [6, Corollary 2.2.5].

For each $i \in \mathcal{N}$, define

$$M_i \triangleq \max_{x_i \in X_i} \|x_i\|, \quad M_H \triangleq \sum_{j=1}^{N} M_i, \quad (5)$$

$$\bar{C} \triangleq \theta M_H + 2\theta \sum_{j=1}^{N} M_j/(1 - \beta).$$  (6)

We impose the following Lipschitz continuous conditions.
Assumption 3: (a) \( \phi(x) \) is L-Lipschitz continuous in \( x \), i.e.,
\[
\| \phi(x) - \phi(y) \| \leq L\| x - y \|, \quad \forall x, y \in X.
\]
(b) For each \( i \in N \) and any \( x_i \in X_i \), \( F_i(x_i, z) \) is \( L_f \)-Lipschitz continuous in \( z \) over some compact set, particularly, for all \( z_1, z_2 \in \mathbb{R}^d \) with \( \| z_1 \| \leq NC + M_H \) and \( \| z_2 \| \leq NC + M_H \):
\[
\| F_i(x_i, z_1) - F_i(x_i, z_2) \| \leq L_f \| z_1 - z_2 \|.
\]

In addition, we require each \( \psi_i(x_i, \sigma; \xi_i) \) to be differentiable, and assume there exists a stochastic oracle that returns unbiased gradient sample with bounded variance.

Assumption 4: For each player \( i \in N \) and any \( \xi_i \in \mathbb{R}^d \), (a) \( \psi_i(x_i, \sigma; \xi_i) \) is differentiable in \( x_i \in X_i \) and \( \sigma \in \mathbb{R}^m \); (b) for any \( x_i \in X_i \) and \( z \in \mathbb{R}^m \), \( q_i(x_i, z; \xi_i) = (\nabla_x \psi_i(x_i, \sigma; \xi_i) + \nabla_{\sigma} \psi_i(x_i, \sigma; \xi_i)) |_{x=z} \) satisfies \( \mathbb{E}[q_i(x_i, z; \xi_i)|_{x_i}] = F_i(x_i, z_i) \) and \( \mathbb{E}[\|q_i(x_i, z; \xi_i) - F_i(x_i, z_i)\|_2^2|x_i] \leq \nu_i^2 \) for some constant \( \nu_i > 0 \).

III. ALGORITHM DESIGN AND MAIN RESULTS

In this section, we design a distributed NE seeking algorithm and show its optimal convergence to the Nash equilibrium in the mean-squared sense.

A. Distributed Mirror Descent Algorithm with Operator Extrapolation

We assume throughout that the paper that the regularization function \( h \) is 1-strongly convex, i.e.,
\[
h(y) \geq h(x) + \langle \nabla h(x), y - x \rangle + \frac{1}{2}\|x - y\|^2, \quad \forall x, y \in \mathbb{R}^m.
\]
We define the Bregman divergence associated with \( h \) as follows.
\[
D(x, y) \triangleq h(y) - h(x) - \langle \nabla h(x), y - x \rangle, \quad \forall x, y \in \mathbb{R}^m.
\]

Recall that the operator extrapolation method for VI in [5] requires a simple recursion at each iteration given by
\[
x_{t+1} = \arg\min_{x \in X} \gamma_t \left( F(x_t) + \lambda_t (F(x_t) - F(x_{t-1})) \right) + D(x, x_t),
\]
which only involves one operator evaluation \( F(x_t) \) and one prox-mapping over the set \( X \).

Suppose that each player \( i \) at stage \( k = 1, 2, \ldots \) selects a strategy \( x_{i,k} \in X_i \) as an estimate of its equilibrium strategy, and holds an estimate \( \hat{v}_{i,k} \) for the average aggregate. At stage \( k + 1 \), player \( i \) observes its neighbors’ past information \( v_{j,k}, j \in N_i,k \) and updates an intermediate estimate by the consensus step (8), then it computes its partial gradient based on the sample observation, and updates its strategy \( x_{i,k+1} \) by a mirror descent scheme (9) with operator extrapolation [5] by setting \( q_i(x_i, 0; N\hat{v}_{i,k}; \xi_i, 0) = q_i(x_{i,1}, 0; N\hat{v}_{i,2}; \xi_i, 1) \) without loss of generality. Finally, player \( i \) updates the average aggregate with the renewed strategy \( x_{i,k+1} \) by the dynamic average tracking scheme (10). The procedures are summarized in Algorithm 1.

---

**Algorithm 1** Distributed Mirror Descent Method with Operator Extrapolation

**Initialize:** Set \( k = 1 \), \( x_{i,1} \in X_i \) and \( v_{i,1} = x_{i,1} \) for each \( i \in N \).

**Iterate until convergence**

**Consensus.** Each player computes an intermediate estimate by
\[
\hat{v}_{i,k+1} = \sum_{j \in N_i,k} w_{i,j,k} v_{j,k}.
\]

**Strategy Update.** Each player \( i \in N \) updates its equilibrium strategy and its estimate of the average aggregate by
\[
x_{i,k+1} = \arg\min_{x_i \in X_i} \left( \alpha_k \langle (1 + \lambda_k) q_i(x_{i,k}, N\hat{v}_{i,k+1}; \xi_{i,k}) - \lambda_k q_i(x_{i,k-1}, N\hat{v}_{i,k}; \xi_{i,k-1}), x_i \rangle + D(x_{i,k}, x_i) \right),
\]
\[
v_{i,k+1} = \hat{v}_{i,k+1} + x_{i,k+1} - x_{i,k}
\]
where \( \alpha_k > 0, \lambda_k > 0, \) and \( \xi_{i,k} \) denotes a random realization of \( \xi_i \) at time \( k \).

Define the gradient noise \( \zeta_{i,k} \triangleq q_i(x_{i,k}, N\hat{v}_{i,k+1}; \xi_{i,k}) - F_i(x_{i,k}, N\hat{v}_{i,k+1}), \)

\[
x_k = (x_{1,k}^T, \ldots, x_{N,k}^T)^T, \quad \text{and} \quad F_k \triangleq \{x_1, \xi_{1,i}, i \in N, l = 1, 2, \ldots, k - 1 \}. \]

Then with Algorithm 1, \( x_{i,k} \) and \( \hat{v}_{i,k} \) are adapted to \( F_k \). Define \( \zeta_k = (\zeta_{1,k}^T, \ldots, \zeta_{N,k}^T)^T \). From Assumption 4, it follows that for each \( i \in N : \)
\[
\mathbb{E}[\|\zeta_k\|_{F_k}] = 0, \quad \mathbb{E}[\|v_{i,k}\|_{F_k}^2] \leq \nu_i^2, \text{ and}
\]
\[
\mathbb{E}[\|\zeta_k\|_{F_k}^2] \leq \sum_{i=1}^N \nu_i^2 \leq \nu^2.
\]

B. Main Results

Define
\[
\tilde{D}(x, y) \triangleq \sum_{i=1}^N D(x_i, y_i) \text{ for any } x, y \in X,
\]

With the definition of the Bregman’s distance, we can replace the strong monotonicity assumption by the following assumption. This assumption taken from [5] includes \( \langle \phi(x), x - x^* \rangle \geq \mu\|x - x^*\|, \forall x \in X \) as the special case when \( h(x) = \|x\|^2/2 \).

**Assumption 5:** There exists a constat \( \mu > 0 \) such that
\[
\langle \phi(x), x - x^* \rangle \geq 2\mu \tilde{D}(x, x^*), \quad \forall x \in X.
\]

With this condition, we now state a convergence property of Algorithm 1.

**Proposition 1:** Consider Algorithm 1. Let Assumptions 1-5 hold. Assume, in addition, that there exists a positive sequence \( \{\theta_k\}_{k \geq 1} \) satisfying
\[
\theta_{k+1} \alpha_{k+1} \lambda_{k+1} = \theta_k \alpha_k,
\]
\[
\theta_{k-1} \geq 16 \alpha_k^2 \lambda_k^2 L^2 \theta_k,
\]
\[
\theta_k \leq \theta_{k-1}(2\mu \alpha_{k-1} + 1) \text{ and } 8L^2 \alpha_k^2 \leq 1.
\]

---
Then
\[
\theta_t (2\mu \alpha_t + 1/2) E[\hat{D}(x_{t+1}, x^*)] + \sum_{k=1}^{t-1} \frac{\theta_k}{4} E[\hat{D}(x_k, x_{k+1})] \\
\leq \theta_1 E[\hat{D}(x_1, x^*)] + 8 \nu^2 \sum_{k=1}^{t} \theta_k \alpha^2_k \lambda^2_k + 2 \theta_t \alpha^2_t \nu^2 \\
+ \theta_1 \alpha \lambda \epsilon + 2 \sum_{k=1}^{t} \theta_k \alpha_k \epsilon[\epsilon],
\]
(17)
where
\[
\epsilon_k = 2 \sum_{i=1}^{N} L_{f_i} M_i (\|N \hat{v}_{i,k+1} - \sigma(x_k)\| + \|N \hat{v}_{i,k} - \sigma(x_{k+1})\|). 
\]
(18)

Remark 1: Consider the special case where the digraph $G_k$ is a complete graph for each time $k$ with $W_k = \frac{I_N}{\sqrt{N}}$. Then (17) becomes
\[
\theta_t (2\mu \alpha_t + 1/2) E[\hat{D}(x_{t+1}, x^*)] + \sum_{k=1}^{t-1} \frac{\theta_k}{4} E[\hat{D}(x_k, x_{k+1})] \\
\leq \theta_1 E[\hat{D}(x_1, x^*)] + 8 \nu^2 \sum_{k=1}^{t} \theta_k \alpha^2_k \lambda^2_k + 2 \theta_t \alpha^2_t \nu^2.
\]
This recovers the bound of [5, Theorem 3.3].

In the following, we establish a bound on the consensus error $\|\sigma(x_k) - N \hat{v}_{i,k+1}\|$ of the aggregate.

Proposition 2: Consider Algorithm 1. Let Assumptions 1, 2, and 3 hold. Then
\[
\|\sigma(x_k) - N \hat{v}_{i,k+1}\| \leq \theta \sigma M H N \beta^k + \theta N \sum_{s=1}^{k} \beta^{k-s} \alpha_{s-1} \sum_{i=1}^{N} \left( (1 + 2 \lambda_{s-1}) C_i + \lambda_{s-1} \|\zeta_{i,s-2}\| \\
+ (1 + \lambda_{s-1}) \|\zeta_{i,s-1}\| \right),
\]
(19)
where the constants $\theta, \beta$ are defined in (2), and
\[
C_i \triangleq N \tilde{C} L_{f_i} + \max_{x \in X} \|\phi_i(x)\|.
\]
(20)

By combining Proposition 1 and Proposition 2, we can show that the proposed method can achieve the optimal convergence rate for solving the stochastic smooth and strongly monotone aggregative games.

Theorem 1: Consider Algorithm 1. Suppose Assumptions 1-5 hold. Set
\[
c_0 = \frac{4 L}{\mu}, \quad \alpha_k = \frac{1}{\mu (k + c_0 - 1)},
\]
\[
\theta_k = (k + c_0 + 1)(k + c_0), \quad \lambda_k = \frac{\theta_{k-1} \alpha_{k-1}}{\theta_k \alpha_k}. 
\]
Then the following hold with $c_e \triangleq 4 \tilde{N} \tilde{C} \sum_{i=1}^{N} L_{f_i} M_i$,
\[
E[\hat{D}(x_{t+1}, x^*)] \leq \frac{2 (c_0 + 2)(c_0 + 1) \tilde{D}(x_1, x^*)}{(t + c_0 + 1)(t + c_0)} \\
+ \frac{8 \nu^2 + c_1}{\mu (c_0 - 1)(t + c_0 + 1)(t + c_0)} + \frac{2c_0 (c_0 + 1) c_e}{\mu (c_0 - 1)(t + c_0 + 1)(t + c_0)} \\
+ \frac{8 \nu^2 + 4 \nu^2 t}{\mu (t + c_0 + 1)(t + c_0)},
\]
(21)
where
\[
c_1 \triangleq 2 \theta \sigma M H N (1 + \beta) \mu \beta \left( \frac{c_0 - 1}{1 - \beta} + \frac{1}{(1 - \beta)^2} \right) \sum_{i=1}^{N} L_{f_i} M_i, 
\]
(22)
\[
c_2 \triangleq \frac{8 \theta \sigma N (3 - \beta) \sum_{i=1}^{N} (C_i + \nu_i)}{(1 - \beta)^2} \sum_{i=1}^{N} L_{f_i} M_i. 
\]
(23)

Corollary 1: The number of iterations (the same as communication rounds) required by Algorithm 1 for obtaining an approximate Nash equilibrium $\bar{x} \in X$ satisfying $E[\tilde{D}(\bar{x}, x^*)] \leq \epsilon$ is bounded by
\[
\max \left( \frac{L \sqrt{\tilde{D}(x_1, x^*)}}{\mu \sqrt{\epsilon}}, \sqrt{\nu^2 + c_1} \frac{L \sqrt{\epsilon}}{\mu \sqrt{\epsilon}}, \sqrt{\nu^2 + 4 \nu^2} \frac{L \sqrt{\epsilon}}{\mu \sqrt{\epsilon}} \right).
\]

IV. CONCLUSIONS

This paper proposes a distributed operator extrapolation method for stochastic aggregative game based on mirror descent, and shows that the proposed method can achieve the optimal convergence for the class of strongly monotone games. It is of interest to explore the algorithm convergence for monotone games, and extend the operator extrapolation method to the other classes of network games in distributed and stochastic settings.

REFERENCES

[1] J. Lei, P. Yi, and L. Li, “Distributed no-regret learning for stochastic aggregative games over networks,” in 2021 40th Chinese Control Conference (CCC). IEEE, 2021, pp. 7512–7519.

[2] J. Lei, U. V. Shanbhag, J.-S. Pang, and S. Sen, “On synchronous, asynchronous, and randomized best-response schemes for stochastic Nash games,” Mathematics of Operations Research, vol. 45, no. 1, pp. 157–190, 2020.

[3] T. Tatarenko, W. Shi, and A. Nedić, “Geometric convergence of gradient play algorithms for distributed Nash equilibrium seeking,” IEEE Transactions on Automatic Control, vol. 66, no. 11, pp. 5342–5353, 2021.

[4] J. Koshal, A. Nedić, and U. V. Shanbhag, “Distributed algorithms for aggregative games on graphs,” Operations Research, vol. 64, no. 3, pp. 680–704, 2016.

[5] G. Kotsalis, G. Lan, and T. Li, “Simple and optimal methods for stochastic variational inequalities, I: operator extrapolation,” arXiv preprint arXiv:2011.02987, 2020.

[6] F. Facchinei and J.-S. Pang, Finite-dimensional variational inequalities and complementarity problems. Vol. I, ser. Springer Series in Operations Research. New York: Springer-Verlag, 2003.