Numerical Simulation of the Plane Electromagnetic Wave Transient Propagation in Lossy Medium

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Abstract: In the field of geophysical exploration, it is of great value to study the transient propagation characteristics of electromagnetic waves in lossy medium. The equations of electric field and magnetic field are derived. The absorbing boundary conditions and numerical stability conditions are analyzed. The two-layer medium model is established to simulate the propagation characteristics of plane waves. The simulation results show that the transient analysis of electromagnetic wave propagation can reflect the time fluctuation characteristics of electromagnetic field in multilayer media, and provide the indication of the boundary position.

1. Introduction
With the wide application of electromagnetic wave technology in communication, microwave imaging, geological exploration and many other fields, it is very important to study the propagation characteristics of plane electromagnetic wave. In the field of geophysical exploration, most of the geology is lossy medium, so the study of the propagation characteristics of plane electromagnetic wave in lossy medium is of great significance. Due to the layered structure of the earth medium, especially in the detection of underground targets, the targets buried in the multi-layered medium are often encountered. Therefore, it is necessary to study the reflection and transmission of plane electromagnetic waves in the multi-layered medium. The analysis of the transient characteristics of electromagnetic wave propagation can directly reflect the wave process in the electric field and magnetic fields of the plane electromagnetic wave with the passage of time, and can identify the position of the interface [1-4].

In order to analyze the transient characteristic of electric field and magnetic field, two layer inhomogeneous formation models are set up and the plane electromagnetic waves propagating characteristics numerical modeling are simulated.

2. Equations for solving 3-D electromagnetic fields
For the time harmonic field, substituting into the constitutive relation \( \mathbf{D} = \varepsilon \mathbf{E} \), \( \mathbf{B} = \mu \mathbf{H} \), \( \mathbf{J} = \sigma \mathbf{E} \), the equations of electric and magnetic fields can be obtain[5-9].

\[
\nabla \times \left( \frac{1}{\mu_r} \nabla \times \mathbf{E} \right) - \omega^2 \varepsilon_r \mathbf{E} = -i\omega \mathbf{J} \\
\n
\nabla \times \left( \frac{1}{\varepsilon_r} \nabla \times \mathbf{H} \right) - \omega^2 \mu_r \mathbf{H} = \nabla \times \left( \frac{1}{\varepsilon_r} \mathbf{J} \right)
\]

(1)

(2)
Where, $J$ is the impressed current, $E$ is the electric field strength, $H$ is the magnetic field strength, $D$ is the electric displacement vector, $B$ is the magnetic induction intensity, $\varepsilon_r$ is the dielectric constant, and $\mu_r$ is the magnetic permeability. When the boundary conditions is considered, we can get

$$F(E) = \frac{1}{2} \iiint_V \left[ \frac{1}{\mu_r} (\nabla \times E) \cdot (\nabla \times E) - k_0^2 \varepsilon_r E \cdot E \right] dV$$

$$+ \iint_{S_1} \left[ \frac{V}{2} (\hat{n} \times E) \cdot (\hat{n} \times E) + E \cdot U \right] dS$$

$$+ ik_0 Z_0 \iiint_V E \cdot J dV$$

$$F(H) = \frac{1}{2} \iiint_V \left[ \frac{1}{\varepsilon_r} (\nabla \times H) \cdot (\nabla \times H) - k_0^2 \mu_r H \cdot H \right] dV$$

$$+ \iint_{S_1} \left[ \frac{V}{2} (\hat{n} \times H) \cdot (\hat{n} \times H) + H \cdot V \right] dS$$

They are the equations of solving electric field and magnetic field. A single value can be assigned to each node when the solve area contains discontinuous interface. Because the boundary condition of normal field continuity is also imposed in this process, the grid at the boundary surface must be encrypted. However, grid refinement near the interface will increase the number of unknowns, and may make grid generation difficult, especially when the interface is irregular. It is also possible to introduce twice as many nodes on both sides of the discontinuity boundary.

3. Numerical simulation

In numerical simulation, the frequency is 2MHz, the space step is $\Delta z = 0.03m$ and the stability condition is $\Delta t = \frac{\Delta z}{C} \approx 10^{-10} (s)$. The field on the boundary meet the absorbing boundary condition.

3.1. Normal incidence to the ideal conductor from vacuum

According to the theory of electromagnetic field, when the plane wave is incident from vacuum to the surface of ideal conductor, it will produce total reflection. The superposition of incident wave and reflected wave on the same side of the medium produces standing wave. For the pure standing wave, the anti-node and the node are fixed, and the electric field amplitude of the wave node is zero and constant. The maximum value of electric field amplitude is in the wave inverse node. The wave i anti-node of magnetic field and the node of electric field intensity are in the same position, and vice versa. The electromagnetic wave propagates along the $z$-direction, the anti-node and node is determined respectively Equation (5), (6)

$$z_n = \frac{n \lambda}{2} \quad (n = 0, 1, 2, \cdots) \quad (5)$$

$$z_n = \frac{(2m+1) \lambda}{4} \quad (m = 0, 1, 2, \cdots) \quad (6)$$

In numerical simulation, the conductivity of vacuum and metal is 0.0S/m and $1 \times 10^7 S/m$ respectively. The transient characteristics of electromagnetic wave propagation of $\frac{1}{8} \lambda$. 

\( \frac{1}{4} \lambda, \frac{3}{8} \lambda \) and \( \frac{1}{2} \lambda \) far from interface is shown in Figure 1.

(a) Electric field

(b) Magnetic field

**Figure 1.** Electromagnetic propagating characteristic of total reflection.

### 3.2. The wave is normal incident to different resistivity medium

The study region is 0~1m, the sampling interval is 0.001m. There is the interface at 0.55m and the source is located at 0.2m. Two observation points are located at 0.35m and 0.75m respectively.

1) Plane wave is incident to the high resistivity medium from the low resistivity medium

Uniform plane wave is incident from medium 1 (\( \sigma = 0.1S/m \)) to medium 2 (\( \sigma = 1.0S/m \)) and the plane electromagnetic wave propagation transient characteristics is shown in Figure 2.
The amplitude of electric field in homogeneous medium (the amplitude of electric field when plane wave is incident from medium 1 to medium 2) is compared with that in homogeneous medium (the amplitude of electric field when plane wave is incident from medium 2 to medium 1), the former is larger than the latter. So it is obvious that the reflection phenomenon will not happen at the interface. Part of the energy of the wave is converted into heat, and the other part is the transmission wave through the interface. Wave impedance, reflection coefficient and transmission coefficient are all complex numbers, so there is phase shift between reflection field and transmission field.

2) Plane wave is incident to the low resistivity medium from the high resistivity medium

When a uniform plane wave is incident from medium 1 (\( \sigma = 1.0S/m \)) to medium 2 (\( \sigma = 0.1S/m \)), the transient characteristics of plane electromagnetic wave propagation are shown in Figure 3. When plane wave incident from medium 1 (\( \sigma = 0.1S/m \)) to medium 2, the amplitude of electric field in homogeneous medium is a little smaller than that in homogeneous medium. So it's clear that the reflection is very small at the interface.

Figure 2. Transient characteristic when the wave is normal incident from high to low resistivity medium.
Figure 3. The transient characteristic when the wave is normal incident from low to high resistivity medium.

4. Conclusions

1) When the plane electromagnetic wave is directly incident from vacuum to the surface of the ideal conductor, the total reflection occurs, and the incident wave and the reflected wave are superposed on one side of the medium to produce the pure standing wave. When a high resistivity medium is incident on an ideal conductor, there are discontinuities in the transient curves of the electric and magnetic fields, which can identify the location of the interface.

2) When the plane electromagnetic wave is directly incident into the low resistance medium from the high resistance medium, there is no reflection phenomenon, and the reflection coefficient is negative; when it is incident into the high resistance medium from the low resistance medium, there is a reflection phenomenon, the reflection coefficient is positive, part of the incident wave energy is converted into heat energy, one part into reflected wave energy, the other part into transmitted wave energy, the reflection field and transmitted field There is a phase shift between them.
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