Noise and decoherence induced by gravitons

Sugumi Kanno*, Jiro Soda♭ and Junsei Tokuda♭

* Department of Physics, Osaka University, Toyonaka 560-0043, Japan
♭ Department of Physics, Kobe University, Kobe 657-8501, Japan

Abstract

We study quantum noise and decoherence induced by gravitons. We derive a Langevin type equation of geodesic deviation in the presence of gravitons. We calculate the noise correlation in squeezed coherent states and find that the squeezed state enhance it compared with the vacuum state. We also consider the decoherence of spatial superpositions of massive objects caused by gravitons in the vacuum state and find that gravitons might give the leading contribution to the decoherence. The decoherence induced by gravitons would offer new vistas to test quantum gravity in tabletop experiments.
1 Introduction

An understanding of the nature of gravity has been a central issue in physics since the discovery of general relativity and quantum mechanics. Nevertheless, no one has succeeded in constructing quantum theory of gravity. In particular, the existence of gravitons is still obscure [1]. In these situations, it is legitimate to doubt the necessity of canonical quantization of gravity [2]. Hence, it is worth seeking an experimental evidence of quantum gravity.

Usually, theorists explore the field of quantum gravity at energy scales near the Planck scale. However, it is far beyond the capacity of the current or future particle accelerators. Recently, tabletop experiments of quantum gravity are drawing attention [3, 4, 5]. Remarkably, based on the development of quantum information, several ideas to test the quantum nature of gravity through a laboratory experiment are proposed [6, 7]. It is also argued that noise in the lengths of the arms of gravitational wave detectors can be a probe of gravitons [8] and may be a hint of quantum gravity.

The noise is usually associated with the decoherence which is induced by quantum entanglement between a system and gravitons. Thus, as an approach to testing quantum gravity, it
is important to understand the noise induced by gravitons and then the decoherence caused by the noise profoundly.

The decoherence due to gravity in the context of quantum superposition of massive objects has been investigated. For instance, the decoherence rate was derived based on effective field theory approach [9]. The decoherence due to quantum fluctuations of geometry caused by gravitons is discussed in [10]. Following the paper in the context of electromagnetic dynamics [11], the effect of gravitons on the destruction process of quantum superposition has been studied [12,13,14,15,16,17,18,19]. In these literature, an open quantum system in the graviton reservoir has been studied. The interaction between a quantum mechanical system and an environment of gravitons produces the noise and leads to the decoherence of the system [20].

In this paper, we focus on the quantum noise and decoherence to probe gravitons and ultimately quantum gravity. Firstly, we target on gravitational wave detectors. When gravitational waves arrive at the laser interferometer, the suspended mirrors interact with the gravitational waves. The mirror interacts with an environment of gravitons quantum mechanically. We will evaluate the effect of quantum noise induced by gravitons on the suspended mirrors. We show that the noise in the squeezed state can be sizable. Secondly, we consider a tabletop experiment by using two massive particles, one of which is superposed spatially, so called, the quantum state of Schrödinger’s cat. It would be interesting to study how the entanglement is created and how a spatial superposition of a massive object is destroyed by gravitons. We give a simple estimate of the decoherence rate induced by gravitons. We expect that these studies could give a hint to perform a tabletop experiment for finding the evidence of quantum nature of gravity or gravitons in the future.

The organization of the paper is as follows: In section 2, we describe geodesics in the graviton background and derive a Langevin type equation of the system by eliminating the environment of gravitons. In section 3, we evaluate the noise correlation functions and show that the noise can be observable if the gravitons are in the squeezed state. In section 4, we discuss the decoherence induced by gravitons and detectability of gravitons. The final section is devoted to the conclusion. A detailed calculation of a momentum integral is presented in the Appendix.

2 Quantum mechanics in the graviton background

In this section, we present a model to study quantum mechanics in the graviton background. It gives rise to the basis for studying the noise and the decoherence due to low energy gravitons. In particular, we derive the quantum Langevin equation.
2.1 Gravitational waves

We consider gravitational waves in the Minkowski space. The metric describing gravitational waves in the transverse traceless gauge is expressed as

\[ ds^2 = -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j, \]

where \( t \) is the time, \( x^i \) are spatial coordinates, \( \delta_{ij} \) and \( h_{ij} \) are the Kronecker delta and the metric perturbations which satisfy the transverse traceless conditions \( h_{ij,j} = h_{ii} = 0 \). The indices \((i, j)\) run from 1 to 3. Substituting the metric Eq. (2.1) into the Einstein-Hilbert action, we obtain the quadratic action

\[ \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R \simeq \frac{1}{8\kappa^2} \int d^4x \left[ \dot{h}_{ij} \dot{h}_{ij} - h_{ij,k} h_{ij,k} \right], \]

(2.2)

where \( \kappa^2 = 8\pi G \) and a dot denotes the derivative with respect to the time. We can expand the metric field \( h_{ij}(x^i, t) \) in terms of the Fourier modes

\[ h_{ij}(x^i, t) = \frac{2\kappa}{\sqrt{V}} \sum_{k,A} h_{ij}^A(t) e^{i\mathbf{k} \cdot \mathbf{x}} e_A^{ij}(\mathbf{k}), \]

(2.3)

where we introduced the polarization tensor \( e_A^{ij}(\mathbf{k}) \) normalized as \( e_{ij}^{AB}(\mathbf{k}) e_{ij}^{BC}(\mathbf{k}) = \delta^{AB} \). Here, the index \( A \) denotes the linear polarization modes \( A = +, \times \). Note that we consider finite volume \( V = L_x L_y L_z \) and discretize the \( k \)-mode with a width \( k = (2\pi n_x/L_x, 2\pi n_y/L_y, 2\pi n_z/L_z) \) where \( n = (n_x, n_y, n_z) \) are integers. Substituting the formula (2.3) into the quadratic action (2.2), we get

\[ S_g \simeq \int dt \sum_{k,A} \left[ \frac{1}{2} h^A(\mathbf{k}, t) \dot{h}^A(\mathbf{k}, t) - \frac{1}{2} k^2 h^A(\mathbf{k}, t) h^A(\mathbf{k}, t) \right]. \]

(2.4)

Note that we used \( k = |\mathbf{k}| \). We see that a gravitational wave consists of an infinite number of harmonic oscillators.

2.2 Action for two test particles

When gravitational waves arrive at the laser interferometers, the suspended mirrors interact with the gravitational waves. Let us regard the mirror as a point particle for simplicity. A single particle, however, does not feel the gravitational waves because of the Einstein’s equivalence principle at least classically. To see the effect of the gravitational waves, we need to consider two massive particles and measure the geodesic deviation between them.

In this subsection, we evaluate the effect of gravitational waves on the two particles by introducing an appropriate coordinate system called the Fermi normal coordinates along one
Figure 1: Two neighboring timelike geodesics (γτ, γτ') separated by ξi are depicted in the blue lines and the green lines show two neighboring spacelike geodesics (γs, γs') orthogonal to the geodesic γτ in the spacelike hypersurface Σs. We introduce the Fermi normal coordinate system using the orthogonal geodesics at the point P(0, t).

of their geodesics γτ (See Figure 1). The Fermi normal coordinate system represents a local inertial frame. The dynamics of the other geodesic of particle γτ' is described by the position x\(^i\)(t) = ξ\(^i\)(t) in the vicinity of the point P(0, t) \(^{23}\) and ξ\(^i\) represents the deviation.

The action for the two test particles along the geodesics γτ, γτ' is given by

\[
S_p = -m \int_{γτ'} dτ = -m \int_{γτ'} dt \sqrt{-g_{μν}(ξ^i, t) \dot{ξ}^μ \dot{ξ}^ν}. \tag{2.5}
\]

where ξ\(^μ\) = (t, ξ\(^i\)(t)). Note that we omit the action for the particle along γτ because it’s in the inertial frame and then the action has no dynamical variables. The metric \(g_{μν}\) up to the second order of \(x^i\) in the Fermi coordinates is computed as

\[
ds^2 ≃ (-1 - R_{00j} x^i x^j) dt^2 - \frac{4}{3} R_{0ijk} x^j x^k dt dx^i + \left( δ_{ij} - \frac{1}{3} R_{ikjℓ} x^k x^ℓ \right) dx^i dx^j. \tag{2.6}
\]

Here the Riemann tensor is evaluated at the origin \(x^i = 0\) in the Fermi normal coordinate system. Substituting the metric (2.6) into the action (2.5), the action for the two particles up to the second order of ξ\(^i\) is expressed as

\[
S_p ≃ \int_{γτ'} dt \left[ \frac{m}{2} \dot{ξ}^2 - \frac{m}{2} R_{00ij}(0, t) ξ^i ξ^j \right]. \tag{2.7}
\]

Because the Riemann tensor \(R_{00ij}\) is gauge invariant at the leading order in the metric fluctuation \(h_{ij}\), we can evaluate it in the transverse traceless gauge to get \(R_{00ij}(0, t) = -\ddot{h}_{ij}(0, t)/2\). We then finally obtain the action for the geodesic deviation

\[
S_p ≃ \int_{γτ'} dt \left[ \frac{m}{2} \ddot{ξ}^2 + \frac{m}{4} \dddot{h}_{ij}(0, t) ξ^i ξ^j \right]. \tag{2.8}
\]
Notice that, when considering gravitation waves with wavelength smaller than the characteristic separation length $\xi$, an approximation (2.6) cannot be used. However, we do not need to take into account the effect from such gravitational waves in this study because of the equivalence principle. Thus, we consider the action of the form

$$S_p \approx \int dt \left[ \frac{m}{2} \dot{\xi}^2 + \frac{m}{2} \sqrt{V} \sum_A \sum_{k \leq \Omega_m} \ddot{h}^A(k, t) e_{ij}^A(k) \xi^i \xi^j \right],$$

(2.9)

where the metric $h_{ij}(0, t)$ is replaced by the Fourier modes in Eq. (2.3) and $\sum_{k \leq \Omega_m}$ represents the mode sum with the UV cutoff $\Omega_m \lesssim \xi^{-1}$. We see a cubic derivative interaction appeared in the above action.

### 2.3 Particles in an environment of gravitons

From Eqs. (2.4) and (2.9), the total action $S = S_g + S_p$ we consider is given by

$$S \approx \int dt \sum_{k, A} \left[ \frac{1}{2} \dot{h}^A(k, t) \dot{h}^{*A}(k, t) - \frac{1}{2} k^2 h^A(k, t) h^{*A}(k, t) \right]$$

$$+ \int dt \left[ \frac{m}{2} \dot{\xi}^2 + \frac{m}{2} \sqrt{V} \sum_A \sum_{k \leq \Omega_m} \ddot{h}^A(k, t) e_{ij}^A(k) \xi^i \xi^j \right].$$

(2.10)

Now we canonically quantize this system. We can expand the interaction picture field $\dot{h}^A_1(k, t)$, whose time evolution is governed by the quadratic action, in terms of the creation and annihilation operators as

$$\dot{h}^A_1(k, t) = \hat{a}_A(k) u_k(t) + \hat{a}^+_A(-k) u_k^*(t),$$

(2.11)

where the creation and annihilation operators satisfy the standard commutation relations:

$$[\hat{a}_A(k), \hat{a}^+_A(k')] = \delta_{k, k'} \delta_{AA'}, \quad [\hat{a}_A(k), \hat{a}^+_A(k')] = [\hat{a}^+_A(k), \hat{a}^+_A(k')] = 0$$

(2.12)

and $u_k(t)$ denotes a mode function properly normalized as

$$\dot{u}_k(t) u_k^*(t) - u_k(t) \dot{u}_k^*(t) = -i.$$ 

(2.13)

The Minkowski vacuum $|0\rangle$ is defined by $\hat{a}_A(k) |0\rangle = 0$, with choosing the mode function as

$$u_k(t) = \frac{1}{\sqrt{2k}} e^{-ikt} \equiv u^M_k(t).$$

(2.14)

$^1\delta_{k, k'}$ is replaced by $\delta^{(3)}(k - k')$ when taking the infinite volume limit $L_x, L_y, L_z \to \infty$. 

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For later convenience, we define a “classical” piece \( h_{cl}(k, t) \equiv \langle \hat{h}^A_t(k, t) \rangle \) and a “quantum” piece as

\[
\delta \hat{h}^A_t(k, t) = \hat{h}^A_t(k, t) - h_{cl}(k, t).
\]

(2.15)

Here, \( \langle \hat{X} \rangle \) denotes an expectation value of an operator \( \hat{X} \) for a given quantum state. In this way, one can describe the gravitational quantum fluctuation \( \delta \hat{h} \) on top of a given classical gravitational wave background \( h_{cl} \). We may identify \( \delta \hat{h} \) as gravitons. Similarly, we promote the position \( \xi^i(t) \) to the operator \( \hat{\xi}^i(k, t) \) below.

### 2.4 Langevin type equation of geodesic deviation

The variation of the action Eq. (2.10) with respect to \( h^A_t \) and \( \xi^i_t \) gives the following equations of motion for the operators in the Heisenberg picture:

\[
\ddot{h}^A(k, t) + k^2 h^A(k, t) = \frac{\kappa m}{2\sqrt{V}} e^A_{ij}(k) \frac{d^2}{dt^2} \left\{ \dot{\xi}^i(t) \dot{\xi}^j(t) \right\},
\]

(2.16)

\[
\ddot{\xi}^i(t) = \frac{\kappa}{\sqrt{V}} \sum_A \sum_{k \leq \Omega_m} e^A_{ij}(k) \dot{h}^A(k, t) \dot{\xi}^j(t).
\]

(2.17)

Eq. (2.16) is solved by standard Green’s function techniques. Specifically we consider the setup where the interaction between \( \hat{h} \) and \( \xi^i \) is turned on at \( t = 0 \). Under this setup, the formal solution of Eq. (2.16) is given by

\[
\hat{h}^A(k, t) = \hat{h}^A_{cl}(k, t) + \delta \hat{h}^A_t(k, t) + \frac{\kappa m}{2\sqrt{V}} e^A_{ij}(k) \int_0^t dt' \sin k(t - t') \frac{d^2}{dt'^2} \left\{ \dot{\xi}^i(t') \dot{\xi}^j(t') \right\}.
\]

(2.18)

The last nonhomogeneous solution describes the gravitational waves emitted from the particles. Substituting the formal solution Eq. (2.18) into Eq. (2.17), we have

\[
\ddot{\xi}^i(t) = \frac{1}{2} \hat{h}^A_{cl}(0, t) \dot{\xi}^j(t) - \frac{\kappa}{\sqrt{V}} \sum_A \sum_{k \leq \Omega_m} k^2 e^A_{ij} \delta \hat{h}^A_t(k, t)
\]

\[
- \frac{\kappa^2 m}{4V} \sum_{k \leq \Omega_m} \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] \int_0^t dt' \sin k(t - t') \frac{d^2}{dt'^2} \left\{ \dot{\xi}^k(t') \dot{\xi}^\ell(t') \right\}
\]

\[
+ \frac{\kappa^2 m}{4V} \sum_{k \leq \Omega_m} \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] \dot{\xi}^j(t) \frac{d^2}{dt^2} \left\{ \dot{\xi}^k(t) \dot{\xi}^\ell(t) \right\},
\]

(2.19)

where we have defined

\[
h^A_{ij}(x^i, t) \equiv 2 \frac{\kappa}{\sqrt{V}} \sum_{k, A} e^A_{ij}(k) h_{cl}(k, t) e^{ikx},
\]

(2.20)
and introduced the projection tensor \( P_{ij} = \delta_{ij} - \bar{k}_i \bar{k}_j \) orthogonal to the unit wave number \( \bar{k}_i = k_i / k \) and used

\[
\sum_A \epsilon^{A}_{ij}(k) \epsilon^{A}_{kl}(k) = \frac{1}{2} \left[ P_{ik} P_{jl} + P_{il} P_{jk} - P_{ij} P_{kl} \right].
\]

Here, the UV-regulated mode sum \( \sum_{k \leq \Omega_m} \) in the second and the third line of (2.19) can be performed by taking the continuum limit of the \( k \)-mode by removing the width introduced in Eq. (2.3):

\[
\frac{1}{V} \sum_{k \leq \Omega_m} \rightarrow 1 \int_{\Omega_m} d^3 k.
\]

The momentum integral is computed in Appendix A, and the result is

\[
\ddot{\xi}_i(t) - \frac{1}{2} \ddot{h}_{ij}(0, t) \dot{\xi}_j(t) + \frac{\kappa^2 m}{40 \pi} \left[ \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right] \ddot{\xi}_j(t) \frac{d^5}{dt^5} \left\{ \dot{\xi}_k(t) \dot{\xi}_\ell(t) \right\} = -\delta \dot{N}_{ij}(t) \dot{\xi}_j(t) + \frac{\kappa^2 m}{20 \pi^2 \Omega_m} \left[ \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right] \dot{\xi}_j(t) \frac{d^4}{dt^4} \left\{ \dot{\xi}_k(t) \dot{\xi}_\ell(t) \right\}.
\]

where we have defined

\[
\delta \dot{N}_{ij}(t) \equiv \frac{\kappa}{\sqrt{V}} \sum_A \sum_{k \leq \Omega_m} k^2 \epsilon^{A}_{ij}(k) \delta h^A_I(k, t).
\]

The third term on the left hand side represents the force of radiation reaction. On the right hand side, the first term is the random force induced by gravitons, which is nothing but quantum noise. The second term is not relevant in the discussions of the noise and the decoherence.

The noise of gravitons \( \delta \dot{N}_{ij} \) always exists if the dynamical component of the gravity is quantized, and this piece encodes the quantum effects on the separation \( \dot{\xi}_i \). If one detected the noise when observing the gravitational waves from classical sources, it could be the evidence for the presence of gravitons. In usual situation, however, the amplitude of the noise will be negligibly small, unless the quantum fluctuations of gravitons are very enhanced compared with those of zero-point fluctuations for some reasons, as we will see in the next section.

### 3 Quantum noise induced by gravitons

#### 3.1 Quantum noise correlations

In this section, we compute the amplitude of the noise for various quantum states.

The anticommutator correlation function of \( \delta \dot{N}_{ij}(t) \) can be computed by using Eqs. (2.21) and (2.12) in the infinite volume limit \( L_x, L_y, L_z \rightarrow \infty \) as

\[
\left\langle \left\{ \delta \dot{N}_{ij}(t), \delta \dot{N}_{kl}(t') \right\} \right\rangle = \frac{\kappa^2}{10 \pi^2} \left( \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right) \int_0^{\Omega_m} dk k^6 P_{\delta h}(k, t, t'),
\]

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where we defined the anticommutator symbol \{ \cdot, \cdot \} as \{ \hat{X}, \hat{Y} \} \equiv (\hat{X}\hat{Y} + \hat{Y}\hat{X})/2, and \(P_{\delta h}\) is given by

\[
\left\{ \delta h_A^\dagger(k, t), \delta h_A'(k', t') \right\} = \delta_{A,A'} \delta_{k+k',0} P_{\delta h}(k, t, t').
\] (3.2)

Below we compute the noise correlation functions when the graviton is in a squeezed-coherent state and discuss the Minkowski vacuum as a special case. The coherent state or squeezed state can be realized when the squeezing parameter or the coherent parameter goes to zero, respectively.

### 3.1.1 Squeezed coherent states

The definition of the squeezed coherent state \(|\zeta, B\rangle\) is

\[
|\zeta, B\rangle \equiv \hat{S}(\zeta) \hat{D}(B) |0\rangle,
\] (3.3)

where \(\hat{S}(\zeta)\) and \(\hat{D}(B)\) are the squeezing and the displacement operators, respectively. They are expressed by

\[
\hat{S}(\zeta) \equiv \exp \left[ \frac{1}{V} \sum_{k,A} \left( \zeta_k^* \hat{a}_A(k) \hat{a}_A(-k) + \zeta_k \hat{a}_A^\dagger(k) \hat{a}_A^\dagger(-k) \right) \right],
\] (3.4)

\[
\hat{D}(B) \equiv \exp \left[ \frac{1}{V} \sum_{k,A} \left( B_k \hat{a}_A^\dagger(k) - B_k^* \hat{a}_A(k) \right) \right],
\] (3.5)

where \(\zeta_k \equiv r_k \exp[i\varphi_k]\) and \(r_k\) is the squeezing parameter. The coherent parameter \(B_k\) is written as \(B_k \equiv |B_k| \exp[i\theta_k]\). Here, we assume that the parameter \(\zeta_k\) and \(B_k\) only depend on \(k\) and are independent of the direction of \(k\). These operators are unitary, and satisfy the following relations:

\[
\hat{D}(B)\hat{S}(\zeta) \hat{a}_A(k) \hat{S}(\zeta) \hat{D}(B) = (\hat{a}_A(k) + B_k) \cosh r_k - \left( \hat{a}_A^\dagger(-k) + B_k^* \right) e^{i\varphi_k} \sinh r_k,
\] (3.6)

\[
\hat{D}(B)\hat{S}(\zeta) \hat{a}_A^\dagger(-k) \hat{S}(\zeta) \hat{D}(B) = \left( \hat{a}_A^\dagger(-k) + B_k \right) \cosh r_k - (\hat{a}_A(k) + B_k) e^{-i\varphi_k} \sinh r_k.
\] (3.7)

The vacuum expectation value of the above operators become

\[
\langle 0 | \hat{D}(B)\hat{S}(\zeta) \hat{a}_A(k) \hat{S}(\zeta) \hat{D}(B) |0\rangle = B_k \cosh r_k - B_k^* e^{i\varphi_k} \sinh r_k,
\] (3.8)

\[
\langle 0 | \hat{D}(B)\hat{S}(\zeta) \hat{a}_A^\dagger(-k) \hat{S}(\zeta) \hat{D}(B) |0\rangle = B_k^* \cosh r_k - B_k e^{-i\varphi_k} \sinh r_k.
\] (3.9)

On the other hand, the transformation of the operator \(\delta h_A^\dagger(k, t)\) is given by definition as

\[
\hat{D}(B)\hat{S}(\zeta) \delta h_A^\dagger(k, t) \hat{S}(\zeta) \hat{D}(B) = \hat{D}(B)\hat{S}(\zeta) \hat{h}_A^\dagger(k, t) \hat{S}(\zeta) \hat{D}(B)
- \langle 0 | \hat{D}(B)\hat{S}(\zeta) \hat{h}_A^\dagger(k, t) \hat{S}(\zeta) \hat{D}(B) |0\rangle.
\] (3.10)
Because $\hat{h}^A_i(k,t)$ consists of the $\hat{a}_A(k)$ and $\hat{a}_A^+(k)$, we see that the right hand side of the above relation is independent of the coherent parameter $B_k$ and we have

$$
\hat{D}^i(B)\hat{S}^i(\zeta) \hat{h}^A_i(k,t) \hat{S}(\zeta) \hat{D}(B) = \left( \hat{a}_A(k) \cosh r_k - \hat{a}_A^+(-k)e^{i\varphi_k} \sinh r_k \right) u_k^c(t) \\
+ \left( \hat{a}_A^+(-k) \cosh r_k - \hat{a}_A(k)e^{-i\varphi_k} \sinh r_k \right) u_k^s(t) \\
= u_k^s(t)\hat{a}_A(k) + u_k^s(t)\hat{a}_A^+(-k) \\
= \hat{S}^i(\zeta) \hat{h}^A_i(k,t) \hat{S}(\zeta),
$$

where the mode function in the squeezed state is given in terms of that in the Minkowski space in Eq. (2.14) such as

$$
u_k^{sq}(t) \equiv u_k^M(t) \cosh r_k - e^{-i\varphi_k} u_k^{M*}(t) \sinh r_k.
$$

Hence, the anticommutator correlation function of $\delta \hat{N}_{ij}(t)$ in the squeezed coherent state in Eq. (3.1) becomes independent of the coherent state such as

$$
\langle \zeta, B \left| \left[ \delta \hat{N}_{ij}(t), \delta \hat{N}_{kl}(t') \right] \right| \zeta, B \rangle
= \frac{\kappa^2}{10\pi^2} \left( \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - 2\frac{3}{3}\delta_{ij}\delta_{kl} \right) \int_0^{\Omega_m} dk \kappa^6 \text{Re} \left[ u_k^{sq}(t)u_k^{sq*}(t') \right],
$$

where

$$
P_{\delta h}(k, t, t') = \text{Re} \left[ u_k^{sq}(t)u_k^{sq*}(t') \right] \\
= \frac{1}{2k} \left[ \cos \{k(t - t')\} \cosh 2r_k - \cos \{k(t - t') - \varphi_k\} \sinh 2r_k \right] \sinh 2r_k.
$$

In general, the squeezing parameter $r_k$ and the phase $\varphi_k$ depend on $k$. However, for simplicity, we regard these variables as constants. Then, plugging this into Eq. (3.13), we obtain

$$
\langle \zeta, B \left| \left[ \delta \hat{N}_{ij}(t), \delta \hat{N}_{kl}(t') \right] \right| \zeta, B \rangle
= \frac{\kappa^2\Omega_m^6}{20\pi^2} \left( \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - 2\frac{3}{3}\delta_{ij}\delta_{kl} \right) F(\Omega_m(t - t'), r, \varphi),
$$

where

$$
F(x, r, \varphi) = \frac{1}{x^6} \left[ \left\{ (5x^4 - 60x^2 + 120) \cos x + x (x^4 - 20x^2 + 120) \sin x - 120 \right\} \cosh 2r \\
- \left\{ (5x^4 - 60x^2 + 120) \cos (\varphi - x) - x (x^4 - 20x^2 + 120) \sin (\varphi - x) \\
- 120 \cos \varphi \right\} \sinh 2r \right].
$$

Note that $F(x, r, \varphi)$ converges to zero for large $x$ and the function $F(x, r, \varphi)$ for small $x$ can be expanded as

$$
F(x, r, \varphi) = \frac{1}{6} \left( \cosh 2r - \cos \varphi \sinh 2r \right) - \frac{10}{7} \sin \varphi \sinh 2r x \\
- \frac{25}{4} \left( \cosh 2r - \cos \varphi \sinh 2r \right) x^2 + \mathcal{O}(x^3).
$$

(3.17)
We find that quantum noise correlations increases as $\Omega_m$ increases.

### 3.1.2 The Minkowski vacuum state

For comparison, let us see the correlation functions of the quantum noise in the Minkowski vacuum state which is obtained by taking $r_k \to 0$ and then we have

$$P_{sh}(k, t, t') = \text{Re} [u_k^M(t)u_k^M(t')] = \frac{1}{2k} \cos \{k(t - t')\}. \tag{3.18}$$

Substituting this into Eq. (3.1), we get

$$\langle 0 \mid \{ \delta \hat{N}_{ij}(t), \delta \hat{N}_{k\ell}(t') \} \mid 0 \rangle = \frac{k^2 \Omega_m^6}{20\pi^2} \left( \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{k\ell} \right) F(\Omega_m(t - t')) \text{,} \tag{3.19}$$

where

$$F(x) \equiv \frac{1}{x^6} \left[ (5x^4 - 60x^2 + 120) \cos x + x (x^4 - 20x^2 + 120) \sin x - 120 \right]. \tag{3.20}$$

Note that, for small $x$, the function $F(x)$ can be expanded as

$$F(x) = \frac{1}{6} - \frac{25}{4} x^2 + O(x^4). \tag{3.21}$$

Comparing this with the result of squeezed coherent state, we see the quantum noise correlations are enhanced exponentially by the squeezing parameter.

### 3.2 Detectability of the quantum noise

In this subsection 3.2, we roughly estimate the effective strain $h_{\text{eff}}$ corresponding to the quantum noise $\delta \hat{N}_{ij}$ and discuss the detectability of the quantum noise. For a given quantum state, the amplitude of the quantum noise in frequency domain can be characterized as

$$\delta N(f) \equiv \left( \int_{-\infty}^{\infty} dt \left\langle \{ \delta N^{ij}(t), \delta N^{ij}(0) \} e^{2\pi if t} \right\rangle \right)^{1/2}. \tag{3.22}$$

From Eq. (2.22), it is found that the response of $\xi^i$ to presence of the classical gravitational wave and the quantum noise is proportional to $\ddot{h}_{\text{cl}}(t, 0)$ and $\delta N(t)$, respectively. Here we omitted spatial indices. This suggests that we can discuss the detectability of the noise by using the effective strain $h_{\text{eff}}(f) \equiv (2\pi f)^{-2}\delta N(f)$ in frequency domain.

Let us start with Minkowski vacuum state. In this case, the amplitude of the quantum noise in frequency domain can be computed as

$$\delta N(f) = \frac{\pi^2 (2f)^{5/2}}{M_{\text{pl}}}, \quad (3.23)$$
where the reduced Planck mass is $M_{\text{pl}} \sim 10^{18}$ GeV. Corresponding effective strain is then

$$h_{\text{eff}}(f) = \frac{(2f)^{\frac{1}{2}}}{M_{\text{pl}}} \approx 2 \times 10^{-42} \left( \frac{f}{\text{1 Hz}} \right)^{\frac{1}{2}} \text{ Hz}^{-\frac{1}{2}}. \quad (3.24)$$

For instance, the characteristic frequency of LIGO is around 100 Hz. Then the amplitude of quantum noise becomes $h_{\text{eff}}(f)|_{f \sim 100 \text{ Hz}} \sim 10^{-41} \text{ Hz}^{-1/2}$. The strain sensitivity of LIGO is about $10^{-23} \text{ Hz}^{-1/2}$ for $f \sim 100 \text{ Hz}$, so the amplitude of quantum noise is too small to be detected.

However, if gravitational waves are in the squeezed state (or in the squeezed coherent state) when arriving at the detectors, the amplitude of the quantum noise is enhanced by the exponential factor of squeezing parameter as is seen in Eq. (3.16). That is,

$$h_{\text{eff}}(f) \approx 2 \times 10^{-42} \left( \frac{f}{\text{1 Hz}} \right)^{\frac{1}{2}} e^{r_k|_{k=2\pi f}} \text{ Hz}^{-\frac{1}{2}}. \quad (3.25)$$

For instance, if the squeezing parameter is large as much as $e^{r_k} \sim 10^{22}$, the amplitude of the quantum noise at the characteristic frequency of LIGO becomes $h_{\text{eff}}(f) \sim 10^{-20} \text{ Hz}^{-1/2}$, which is detectably large.

**Primordial gravitons**

One possible and well-known mechanism to produce gravitons with large quantum fluctuations is inflation. Gravitons produced during inflation experience large squeezing which leads to the detectably large noise amplitudes. In the case of primordial gravitational waves, the relation between the squeezing parameter and the current frequency $f$ is given by

$$e^{r_k|_{k=2\pi f}} \approx \left( \frac{f_c}{f} \right)^2, \quad (3.26)$$

where $f_c$ is the cutoff frequency. In the case of GUT inflation, we have $f_c \sim 10^8$ Hz. In this case, the effective strain at $f \sim 0.1$ Hz, which is the characteristic frequency of DECIGO [21][22], reads $h_{\text{eff}}(f) \sim 10^{-24} \text{ Hz}^{-1/2}$. If we could observe the noise $\delta N_{ij}$, it would mean the detection of gravitons.

## 4 Decoherence induced by gravitons

In this section, we consider a tabletop experiment by using two massive particles, one of which is in a superposition state of two spatially-separated locations and then consider the [The gravitons may undergo the quantum decoherence during inflation or during the propagation. However, when discussing the amplitude of the noise, we do not need to take care of the decoherence.]
Figure 2: The massive object with the mass $m$ is in a superposition state of two spatially-separated locations $\xi_1$ and $\xi_2$.

decoherence of the superposition state caused by the quantum noise $\hat{N}_{ij}$. We assume that
the timelike geodesic $\gamma_{\tau'}$ of a mass $m$ is spatially superposed across the distance $\xi_2 - \xi_1$ as
in Figure 2. Such a superposition state is described by

$$|\Psi\rangle = |\xi_1\rangle + |\xi_2\rangle,$$  \hspace{1cm} (4.1)

where $|\xi_1\rangle$ and $|\xi_2\rangle$ are eigenstates of the operator $\hat{\xi}^i$ satisfying $\hat{\xi}^i|\xi_1\rangle = \xi_1^i|\xi_1\rangle$ and $\hat{\xi}^i|\xi_2\rangle = \xi_2^i|\xi_2\rangle$. To discuss the rate of decoherence, we apply the influence functional method [24]. The influence functional $\exp[i\Phi]$ represents the time evolution of density operator of the
system, which is expressed by

$$\rho(t) = \exp[i\Phi]\rho(0).$$  \hspace{1cm} (4.2)

The decoherence rate $\Gamma$ is described by the time evolution of the influence phase functional $i\Phi$ such as

$$|\exp[i\Phi]| = \exp[-\Gamma].$$  \hspace{1cm} (4.3)

Then, the decoherence rate $\Gamma$ is given by

$$\Gamma(\tau) = m^2 \int_0^\tau dt \Delta(\xi^i\xi^j)(t) \int_0^{t'} dt' \Delta(\xi^k\xi^l)(t') \left\{ \delta \hat{N}_{ij}(t), \delta \hat{N}_{kl}(t') \right\}.$$  \hspace{1cm} (4.4)

We may then define the decoherence time $\tau_{\text{dec}}$ by $\Gamma(\tau_{\text{dec}}) = 1$. Here $\Delta(\xi^i\xi^j)(t) = \xi_1^i\xi_1^j - \xi_2^i\xi_2^j$ denotes a difference of $\xi^i(t)\xi^j(t)$ in the superposition. The value of $\Delta(\xi^i\xi^j)(t)$ is determined
by an experimental setup. Schematically, we can estimate the decoherence time by taking $\sqrt{\Gamma}$ of Eq. (4.4), that is,

$$m \delta N \tau_{\text{dec}} \xi^2 \simeq 10^{-1} m \frac{\Omega_m}{M_{\text{pl}}} \Omega_m^2 \tau_{\text{dec}} \xi^2,$$  \hspace{1cm} (4.5)
and if this quantity becomes order one, then the decoherence becomes effective. Here, we considered the Minkowski vacuum and estimated the noise amplitude in the time domain as

\[ \delta N \equiv \left( \langle \delta N_{ij}(t)\delta N^{ij}(t) \rangle \right)^{\frac{1}{2}} \approx \frac{\Omega_m^{3}}{10M_{pl}}, \]  

(4.6)

and assumed that the quadratic component of \( \Delta(\xi^i\xi^j) \) is of the order of \( \xi^2 \). Under this condition, the decoherence time is computed as

\[ \tau_{\text{dec}} \simeq 10 \, \text{s} \left( \frac{1\,\text{Hz}}{\Omega_m} \right) \left( \frac{1}{(\Omega_m \xi)^2} \right) \left( \frac{10^{-5}\,\text{g}}{m} \right), \]  

(4.7)

where we used the reduced Planck mass \( M_{pl} \sim 10^{-5} \, \text{g} \). We notice that

\[ \tau_{\text{dec}} \simeq \frac{1}{m(\Omega_m \xi)^2 \hbar} \simeq \frac{1}{(\Delta E) \hbar}, \]  

(4.8)

where \( \Delta E \sim m(\Omega_m \xi)^2 \) is the energy difference of the quantum superposition. This is consistent with the result in [10].

### 4.1 Decoherence due to gravitons in a tabletop experiment

In this subsection, we suppose that gravitational fields are quantized and then behaves as the noise as in Eq. (4.6). This setup enables us to perform a tabletop experiment of quantum gravity through the decoherence of the superposition caused by the quantized gravitational fields.

Let us consider some proposed tabletop experiments. In [6], the separation of quantum superposition of a massive object with the mass \( m = 10^{-14} \, \text{kg} \) is given by \( \Delta \xi = 250 \, \mu\text{m} \). Since the cutoff \( \Omega_m \) satisfies the relation \( \Omega_m \xi = 1 \), we have \( \Omega_m \sim 10^{12} \, \text{Hz} \) in this case. By using Eq. (4.7), we estimate the time of decoherence due to the noise of gravitons would be \( \tau_{\text{dec}} \simeq 10^{-5} \, \text{s} \). In this paper, the time of collisional and thermal decoherence is estimated as \( 3.5 \, \text{s} \) under the temperature \( 0.15 \, \text{K} \) and the pressure \( 10^{-15} \, \text{Pa} \). It turns out that the decoherence due to the noise of gravitons takes place faster than the collisional and the thermal decoherence in this experiment. Another paper [7] considered the mass \( m = 10^{-12} \, \text{kg} \) and the separation \( \Delta \xi \sim 1 \, \mu\text{m} \). For these parameters, we find \( \Omega_m = 10^{14} \, \text{Hz} \). Hence, the time of decoherence due to the noise of graviton is estimated as \( \tau_{\text{dec}} \simeq 10^{-9} \, \text{s} \). Since the time-scale for the decoherence due to entanglement between two quantum systems is expected to be in the range of \( \mu\text{s} \) to \( \text{ms} \) in this paper, the decoherence induced by the noise of gravitons takes place faster than their estimation.

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3 Strictly speaking, we may have to take into account the intrinsic gravitational decoherence proposed by [25] [26].
The decoherence caused by the noise of gravitons would offer new vistas to test quantum gravity in the tabletop experiment.

5 Conclusion

We considered quantum mechanics in the graviton background. We derived the Langevin type equation of geodesic deviation in Eq. (2.22). We found that the gravitons give rise to the noise to the dynamics of particles. We also found that the force of radiation reaction came in this system. We calculated the noise correlation in squeezed coherent states and find that the squeezed state enhance it compared with the vacuum state. We then discussed the detectability of the noise of gravitons. It turned out that the amplitude of the noise of gravitons in the case of the Minkowski vacuum is too small to be detected by the current detectors. However, in the squeezed state, we found that the noise of gravitons is enhanced by the exponential factor of squeezing parameter and then we may be able to detect the noise of gravitons for sufficiently large squeezing. Hence, the primordial gravitational waves that experience the large squeezing during inflation tend to make the amplitude of the noise sizable.

The noise is usually associated with the decoherence. The decoherence is a phenomena of the appearance of classical world from a quantum world. It is often argued that gravity has a universal role in the process of the decoherence. In particular, a superposition state of two spatially-separated locations has attracted interest of researchers as an approach to testing quantum gravity. We estimated the time of decoherence by gravitons and found that gravitons might be in fact the leading source of decoherence for some setup. In other words, we may be able to test quantum gravity by tabletop experiments through the process of decoherence due to the noise of gravitons in the future.

The Hanbry-Brown-Twiss interferometers measure the intensity-intensity correlation functions (fourth order correlation functions) [27, 28] and supply experimental evidence for quantum nature of gravitons [29, 30], hence we expect the higher order correlation functions of the noise might contain the further information of the nonclassicality of gravitational waves.

In this paper, we have simply estimated the amplitude of the noise of gravitons and the time scale of the decoherence. We leave the issue of more precise calculations in realistic experimental setups for future work.
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A Momentum integral

Here, we calculate the integral in Eq. (2.19):

\[ \ddot{\xi}_i(t) = \frac{1}{2} \hat{h}^{cl}_{ij}(0, t) \hat{\dot{\xi}}_j(t) - \frac{\kappa}{\sqrt{V}} \hat{\dot{\xi}}_j(t) \sum_A \sum_{k \leq \Omega_m} k^2 e^A_{ij} \delta \hat{h}^A_k(k, t) \]

\[ - \frac{\kappa^2 m}{4} \frac{1}{(2\pi)^3} \hat{\dot{\xi}}_j(t) \int_{\Omega_m} d^3k \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] \int_0^t dt' \sin k(t - t') \frac{d^2}{dt'^2} \left\{ \hat{\xi}^k(t') \hat{\xi}^\ell(t') \right\} \]

\[ + \frac{\kappa^2 m}{4} \frac{1}{(2\pi)^3} \int_{\Omega_m} d^3k \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] \hat{\dot{\xi}}_j(t) \frac{d^2}{dt'^2} \left\{ \hat{\xi}^k(t') \hat{\xi}^\ell(t') \right\} , \tag{A.1} \]

By using the following angular integrals,

\[ \int d\Omega = 4\pi, \quad \int d\Omega k^i k^j = \frac{4}{3} \pi \delta^{ij}, \quad \int d\Omega k^i k^j k^k k^\ell = \frac{4\pi}{15} \left( \delta^{ij} \delta^{k\ell} + \delta^{ik} \delta^{j\ell} + \delta^{i\ell} \delta^{jk} \right) \tag{A.2} \]

we find

\[ \int d\Omega \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] = \frac{8\pi}{5} \left( \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right) . \tag{A.3} \]

Then we can perform the momentum integral as

\[ \int_{\Omega_m} d^3k \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] = \int_0^{\Omega_m} dk \frac{k^2}{2} \int d\Omega \left[ P_{ik} P_{j\ell} + P_{i\ell} P_{jk} - P_{ij} P_{k\ell} \right] \]

\[ = \frac{8\pi}{15} \left( \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right) \Omega_m^3 . \tag{A.4} \]

If we define the limit representation of Dirac delta function as

\[ f(t - t') = \frac{\sin \Omega_m(t - t')}{t - t'} \xrightarrow{\Omega_m \to \infty} \pi \delta(t - t') , \tag{A.5} \]
we have
\[ f(t-t') \bigg|_{t'=t} = \frac{\Omega_m}{t}, \]
\[ f(t-t') \bigg|_{t'=0} = \frac{\sin \Omega_m t}{t} \simeq 0 \quad \text{for} \quad t \neq 0, \]
\[ \frac{d}{dt'} f(t-t') \bigg|_{t'=t} = 0, \]
\[ \frac{d}{dt'} f(t-t') \bigg|_{t'=0} = \frac{\sin \Omega_m t - \Omega_m t \cos \Omega_m t}{t^2} \simeq 0 \quad \text{as} \quad \Omega_m \to \infty , \]
\[ \frac{d^2}{dt'^2} f(t-t') \bigg|_{t'=t} = -\frac{1}{3} \Omega_m^3, \]
\[ \frac{d^2}{dt'^2} f(t-t') \bigg|_{t'=0} = \frac{2 \sin \Omega_m t - 2 \Omega_m t \cos \Omega_m t - \Omega_m^2 t^2 \sin \Omega_m t}{t^2} \simeq 0 \quad \text{as} \quad \Omega_m \to \infty . \] (A.6)

For Eqs. (A.6) and (A.7), after smearing the functions with an appropriate window function, these quantities vanish. The momentum integral in the second line of the right hand side of Eq. (A.1) is written by the function \( f(t-t') \) of this form
\[
\int_0^{\Omega_m} dk k^2 \int_0^t dt' k \sin k(t-t') \frac{d^2}{dt'^2} \left\{ \xi^i(t) \xi^j(t) \right\} = \int_0^t dt' \frac{d^3}{dt'^3} f(t-t') \frac{d^2}{dt'^2} \left\{ \xi^i(t') \xi^j(t') \right\} , \quad (A.8)
\]

Using above results, we can evaluate as follows
\[
\int_0^t dt' \frac{d^3}{dt'^3} f(t-t') \frac{d^2}{dt'^2} \left\{ \xi^i(t') \xi^j(t') \right\} = \left[ \frac{d^2}{dt'^2} f(t-t') \frac{d^2}{dt'^2} \left\{ \xi^i(t') \xi^j(t') \right\} \right]_0^t - \left[ \frac{d}{dt'} f(t-t') \frac{d^3}{dt'^3} \left\{ \xi^i(t') \xi^j(t') \right\} \right]_0^t \\
+ \left[ f(t-t') \frac{d^1}{dt'^1} \left\{ \xi^i(t') \xi^j(t') \right\} \right]_0^t - \int_0^t dt' f(t-t') \frac{d^5}{dt'^5} \left\{ \xi^i(t') \xi^j(t') \right\} \\
= -\frac{1}{3} \Omega_m^3 \frac{d^2}{dt'^2} \left\{ \xi^i(t') \xi^j(t') \right\} + \Omega_m^3 \frac{d^4}{dt'^4} \left\{ \xi^i(t') \xi^j(t') \right\} - \frac{\pi}{2} \frac{d^5}{dt'^5} \left\{ \xi^i(t') \xi^j(t') \right\} . \quad (A.9)
\]

Then Eq. (A.1) becomes
\[
\ddot{\xi}^i(t) = \frac{1}{2} \hat{h}_{ij}^A(0,t) \dot{\xi}^j(t) - \frac{\kappa}{\sqrt{V}} \xi^i(t) \sum_{k,A} k^2 e_{ij}^A \left\{ \hat{h}_0^A(k) \cos kt + \hat{h}_0^A(k) \sin kt \frac{k}{k} \right\} \\
+ \frac{\kappa^2 m}{4} \frac{1}{5 \pi^2} \frac{1}{\xi^j(t)} \left( \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{k\ell} \right) \left( \Omega_m \frac{d^4}{dt'^4} \left\{ \xi^i(t') \xi^j(t') \right\} - \frac{\pi}{2} \frac{d^5}{dt'^5} \left\{ \xi^i(t') \xi^j(t') \right\} \right) . \quad (A.10)
\]

This is the Eq. (2.19).

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