The radiation reaction effects in the BMT model of spinning charge and the radiation polarization phenomenon

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Abstract

The effect of radiation polarization (RP) attended with the motion of spinning charge in the magnetic field could be viewed through the classical theory of self-interaction. The quantum expression for the polarization time follows from semiclassical relation $T_{QED} \sim \hbar c^3/\mu_B^2 \omega^3$, and needs quantum explanation neither for the orbit nor for the spin motion. In our approach the polarization emerges as a result of natural selection in the ensemble of elastically scattered electrons, among which the group of particles that bear their spins in the 'right' directions has the smaller probability of radiation.

1. Introduction

The rise in popularity of the classical spin models was stimulated by the difficulties with high spin wave equations accounting for the interaction of particle with external EM field [1]. The close relation of the (pseudo)classical models of spinning particles to the string theory raises a new phase of interest in this topic [2, 3]. The criterion used by different authors to check the spin degrees of freedom are described correctly, is the possibility for one to obtain, at least with some approximations involved, the Bargmann-Michel-Telegdi (BMT) or the Frenkel-Nyborg equations determining the spin evolution (see e.g. [2, 4]). The reason for this is the universal character of the BMT equation and its well established experimental applicability. Here we consider the problem of self-interaction of the BMT particle and its relation to the RP phenomenon.

The effect of preferable polarization emerges when the relativistic ($\sim 1 \text{GeV}$) electrons execute the motion in magnetic field during the polarization time

$$T_{QED} = \frac{8\sqrt{3}}{15} a_B c^{-2} \left(\frac{H_c}{H}\right)^3$$

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2) It is worth noting that the account for radiation through the local ALD-type equation for spinning charge seems hardly to have a practical meaning (see the extensive study of that topic in [5]).

3) With some obvious exceptions we use the system of units with $c = 1$, $\hbar = 1$, $\alpha = e^2/4\pi\hbar c$, $a_B = 4\pi\hbar^2/me^2$ and $H_c = m^2c^3/\epsilon \hbar$ ($\alpha$, $a_B$ and $H_c$ being the fine structure constant, the Bohr radius and critical QED field strength).
on the laboratory clocks \([6]\). RP manifests itself through the asymmetry of the probability of the spontaneous spin-flip transitions

\[
w^{\uparrow \downarrow} = \frac{1}{T_{QED}} \left( 1 + \zeta_3 \frac{8\sqrt{3}}{15} \right)
\]

w.r.t. the value of the initial polarization \(\zeta_3 = \pm 1\ \[7\]. The physical ground for that asymmetry is, of course, radiation process so that the phenomenon itself could be considered as a back-reaction effect. The latter one can describe with the help of the semiclassical elastic scattering probability \([8]\)

\[
\exp \left( -\frac{2}{\hbar} \Im \Delta W \right) < 1
\]

where the classical self-action of the charge \([4]\)

\[
\Delta W = \frac{1}{2} \int \int J_\mu(x) \Delta_{c}(x, x') J_\mu(x') \, dx \, dx'
\]

should have a positive imaginary part \((\Im \Delta W > 0)\) pointing to the presence of radiation. The photon Green function \(\Delta_{c}(x, x') = i(2\pi)^{-2}/[(x - x')^2 + i0]\) and the source

\[J_\mu = j_\mu + \partial_\nu M_{\mu\nu}\]

includes the orbit \((j_\mu)\) and the spin \((\partial_\nu M_{\mu\nu})\) contributions.

Below we clarify in short technical points of calculations performed and discuss the results and some differences from the original considerations in \([6, 11]\).

2. The mass shift and the internal geometry of the world lines

With the help of eqn.(5) the self-action \(\Delta W\) could be decomposed as follows:

\[
\Delta W = \Delta W_{or} + \Delta W_{so} + \Delta W_{ss}.
\]

The orbit part \(\Delta W_{or}\) does not contain the spin degrees of freedom and for the case of constant homogeneous EM field (which is the matter of interest to us here) was studied in \([8]\). The spin-orbit part

\[
\Delta W_{so} = -\frac{\mu e^2}{2\pi^2} \int d\tau \int d\tau' \zeta_{\alpha\beta\gamma\delta}(x - x') \alpha \dot{x}_\alpha(\tau) \beta \dot{x}_\beta(\tau') S_\gamma(\tau') \left( x - x' \right)^2
\]

was discussed in \([9, 10]\), and

\[
\Delta W_{ss} = \frac{\mu^2}{2} \int d\tau \int d\tau' u_{[\beta'} S_{\beta]} u_{[\beta} S'_{\beta]} \partial_\gamma' \partial_\gamma \Delta_{c}(x, x')
\]

\[4\) The subtraction of UV divergences corresponding to the definition of the observable mass is implied in (4) \([8, 9]\).
(unlike $\Delta W_{ss}$ self-action $\Delta W_{so}$ contains no UV divergences). We use the following notations in (5), (7) and (8):

$$M_{\alpha\beta}(x) = \int d\tau \mu_{\alpha\beta}\left(\delta^{(4)}(x - x(\tau))\right)$$

is the polarization density; Frenkel polarization tensor

$$\mu_{\alpha\beta} = i\mu\varepsilon_{\alpha\beta\gamma\delta}\dot{x}_\gamma(\tau) S_\delta(\tau)$$

with $\mu = \frac{1}{2} g \mu_B$, and $g$, $\mu_B = e/2m$ being the $g$ factor and Bohr magneton correspondingly. 4-velocities $u \equiv \dot{x}(\tau)$, $u' \equiv \dot{x}(\tau')$ and spin 4-vectors $S \equiv S(\tau)$, $S' \equiv S(\tau')$ are determined from Lorentz and BMT equations:

$$\dot{u} = \kappa \hat{F} \cdot u,$$

$$\dot{S} = \frac{1}{2} g \kappa \hat{F} \cdot S + (\frac{g}{2} - 1) \kappa u (u \cdot \hat{F} \cdot S),$$

($\kappa \equiv e/m$) where the dot from above means derivative w.r.t. proper time.

For the constant homogeneous background the translational symmetry entails in

$$\Delta W = -\Delta m T,$$

with $\Delta m$ denoting the mass shift (MS) and T corresponding to the proper time interval of the charge’s stay in external field. In application of eqn.(13) it is, generally, supposed that the formation (proper) time of the non-local $\Delta m$ is much less than $T$.

The important property of the motion in the constant field is the ‘isometry’ property of the world lines [8]:

$$(x(\tau) - x(\tau')^2 = f(\tau - \tau').$$

Here the function $f$ is an even function of the proper time difference $\Delta \tau = \tau - \tau'$. Given this difference, the integrands in expressions (7) and (8) preserve their value along the world line so that these non-local geometrical characteristics exhibit some kind of ‘rigidity’ which eventually gives rise to eqn.(13).

To compute the invariants present as integrands in the self-actions $\Delta W_{so}$ and $\Delta W_{ss}$ one can exploit Frenet-Serret (FS) formalism adapted to the case of constant homogeneous EM field in [12]. Let $e^A (A = 0, ..., 3)$ be a FS tetrad with $e^{(0)} \equiv u(\tau)$. For every element of tetrad the Lorentz equation is valid:

$$\dot{e}^A = \kappa \hat{F} \cdot e^A(\tau).$$

Combining the eqn.(15) with the basic equation of FS formalism,

$$\dot{e}^A = \Phi^A_B e^B(\tau),$$

one can turn the action of the Lorentzian matrix $\hat{F}$ into the tetrad basis. Of first importance now is the constancy of Frenet matrix $\Phi^A_B$. The non-zero elements of $\Phi^A_B$ are the curvature ($k$), the first ($t_1$) and the second ($t_2$) torsions, which have their representations directly in terms of electric and magnetic fields [12].
3. Results

Below we concentrate on the plane motion in the purely magnetic field and \( g = 2 \) — those conditions are simplest one to make possible the comparison with standard QED [6] and semiclassical QED [11] approaches. For that case \( k = \kappa H v_\perp \gamma_\perp = v_\perp t_1 \) and \( t_2 = 0 \), so that the MS \( \Delta m_{so} \) and \( \Delta m_{ss} \) corresponding to the self-actions (7) and (8) can be transformed into forms:

\[
\Delta m_{so} = -i \frac{\mu c}{2\pi^2} S_3 \omega_c^2 f_{so}(v_\perp),
\]

\[
\Delta m_{ss} = \begin{cases}
  k_0 + k_{13} - k_{13} \zeta_3^2 + (k_{12} - k_{13}) \zeta_\perp^2, \\
  k_0 - k_{12} - k_{12} \zeta_3^2 - (k_{12} - k_{13}) \zeta_\perp^2.
\end{cases}
\]

The upper and lower representations in eqn.(18) are equivalent since the spin vector \( \vec{\zeta} \) in the rest frame of the particle satisfies the relation

\[
\vec{\zeta}^2 = \zeta_3^2 + \zeta_\parallel^2 + \zeta_\perp^2 = 1.
\]

Note that \( S_3 = \zeta_3, S_\parallel = \gamma_\parallel \zeta_\parallel, \) and \( S_\perp = \zeta_\perp \) are the (conserved) spin components parallel to the field \( \mathbf{H}, \) parallel to the velocity \( \mathbf{v} (= \mathbf{v}_\perp) \) and perpendicular to \( \mathbf{v} \) correspondingly. The following notations were used in formulas (17) and (18):

\[
f_{so} = \frac{v_\perp^2}{\gamma_\perp} \int_0^\infty \frac{\sin^2 w - w \sin w \cos w}{(v_\perp^2 \sin^2 w - w^2)^2} \, dw,
\]

\[
k_{12} = -i \mu^2 \omega_c^3 \frac{\omega_c^3}{4\pi^2\gamma_\perp^4} \int_0^\infty \frac{-w^2 \sin^2 w}{(w^2 - v_\perp^2 \sin^2 w)^3} \left( \frac{\gamma_\perp^6}{w^2} \right) \, dw,
\]

\[
k_{12} - k_{13} = -i \mu^2 \omega_c^3 \frac{v_\perp^2}{4\pi^2\gamma_\perp^4} \int_0^\infty \frac{(w \cos w - \sin w)^2}{(w^2 - v_\perp^2 \sin^2 w)^3} \, dw
\]

with \( \omega_c = eH/m \) and \( \gamma_\perp \) being the Lorentz factor. The term \( k_0 + k_{13} \) \( (k_0 + k_{12}) \) in the upper (lower) part of the eqn.(18) do not depend on the spin direction and is not of importance in explaining RP. As the numerical investigation shows, the functions \( k_{12} \) and \( k_{13} \) are rather close each other. Note also, that \( i(k_{12} - k_{13}) \) is positive in no dependence on the energy of the particle as well as the functions \( ik_{12} \) and \( ik_{13} \) itself.

4. Discussion

The probability of not-emitting the photons is decreasing with the proper time according to general law (see (3) and (13)):

\[
\exp(\Im \Delta m \cdot T), \quad \Im \Delta m < 0.
\]

Accounting for the positivity of the integrals in the r.h.s. of expressions (21), (22) one can guess from the eqn.(18) that particles with \( \zeta_\parallel \neq 0 \) would have a better chance to preserve
their state whereas the particles with \( \zeta_{v} \neq 0 \) such a possibility should lose just with the same rate.

Supposing the relativistic energies for electrons we find for the spin-dependent part of the total MS \( \Delta m = \Delta m_{or} + \Delta m_{so} + \Delta m_{ss} \) the following sum

\[
-\frac{1}{4\sqrt{3}} a_{B} \chi^{2} \zeta_{3} + \frac{15}{64\sqrt{3}} a_{B} \chi^{3} \zeta_{3}^{2} + i \frac{1}{64\sqrt{3}} a_{B} \chi^{2} \zeta_{v}^{2},
\]

where the first term comes from \( \Delta m_{so} \) (it would have an opposite sign for positrons) and \( \chi = \gamma_{\perp} H / H_{c} \). With the notation

\[
\lambda = -\frac{2}{\hbar} \Delta mc^{2}
\]

we arrive at the spin contribution to the decay rate in the form:

\[
\lambda_{\text{spin}} = \frac{c}{a_{B}} \chi \left[ \frac{1}{2\sqrt{3}} \chi \zeta_{3} - \frac{15}{32\sqrt{3}} \chi^{2} \zeta_{3}^{2} - \frac{1}{32\sqrt{3}} \chi^{2} \zeta_{v}^{2} \right].
\]

\( \lambda_{\text{spin}} \) would be negative for \( \zeta_{3} < 0 \). This, of course, has no effect on the positivity of the total ‘decay rate’ \( \lambda \) since \( \chi \ll 1 \).

Being the probability of radiation per unit proper time, \( \lambda \) in (25) corresponds to either change of particle’s state of motion- not only to the spin-flip transitions. The negative \( \zeta_{3} \) slightly reduces this probability, as well as two last terms in (26) do - in no dependence on the signs of \( \zeta_{3} \) and \( \zeta_{v} \). Note, that according to eqns.(18) and (26) spin-spin interaction itself does not give the preferable polarization for elastically scattering particles (‘down’ for electrons and ‘up’ for positrons). The RP effect emerges in conjunction of the spin-orbit and spin-spin interactions. The characteristic laboratory times extracted from (26) are

\[
T_{ss}^{(1)} = \frac{32\sqrt{3} a_{B}}{15} \frac{1}{c} \chi^{-3} \gamma_{\perp} \, , \quad T_{ss}^{(2)} = 15 T_{ss}^{(1)}.
\]

The presence of the last term in (26) corresponds to the incomplete polarization degree among the elastically scattered electrons estimated as \( \sim 15/16 = 0.938 \) (compare it with the dynamic value 0.924 of the polarization degree in QED). The relation \( T_{ss}^{(1)} = 4 T_{QED} \) one finds from (1) and (27), should be addressed to the lack of the direct correspondence between \( \lambda \) and \( w^{\uparrow \downarrow} \) in (2). The classical model of spin relaxation proposed in \[13, 14, 7\] gives for the polarization time \( T_{QED} \) the wanted expression up to the factor of order unity (not four) and for the polarization degree 100%. So, in what concerns classical consideration, the ‘nice’ “4” and 0 < \( T_{ss}^{(2)} < \infty \) (see (27)) are the main variation of our results from those of \[13, 14, 7\].

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