Quantum coherence of double-well BEC: a SU(2)-coherent-state path-integral approach

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Macroscopic quantum coherence of Bose gas in a double-well potential is studied based on SU(2)-coherent-state path-integral. The ground state and fluctuations around it can be obtained by this method. In this paper, one can obtain macroscopic quantum superposition states for attractive Bose gas. The coherent gap of degenerate ground states is obtained with the instanton technique. The phenomenon of macroscopic quantum self-trapping is also discussed.

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I. INTRODUCTION

Quantum tunneling at mesoscopic scale is one of the most fascinating phenomena in condensed matter physics. The double-well potential provides a simple and yet physically relevant example for studies of quantum tunneling in mesoscopic systems. Recently, there have been great experimental and theoretical interests in studying the coherent quantum tunneling between two Bose-Einstein condensates (BEC) in a double-well potential. It was found that the ground state changes from a coherent state to a Fock state as the interaction between particles is increased in a repulsive Bose gas[1, 2]. It was numerically shown that the attractive Bose gas in a double-well potential has Schrödinger Cat-like ground states[3]. A similar model can be found in Refs.[4, 7] and [8].

In this paper we study quantum coherence of BEC in a double-well potential by mapping the two site boson model onto an anisotropic spin model in an external magnetic field[9]. Then the coherence properties of Bose system can be studied in the SU(2)-coherent-state path-integral representation. From the effective classical energy, it is easy to show that the phenomenon of macroscopic quantum self-trapping (MQST) exists in both the repulsive and attractive interaction cases. The point separating the coherent ground state and macroscopic quantum superposition state can be obtained analytically. The number fluctuation and relative phase fluctuation between two-well condensates are given through path-integral technique. Within the instanton technique, we obtain the tunnel splitting of degenerate ground states in the attractive interaction case. It is noted that the model presented here is general (see Eq.(3)), including BEC in symmetric or non-symmetric double-well potential, and in different regimes of parameters. We emphasize that the quantum coherence properties of BEC depend on the parameters of system distinctly.

II. PHYSICAL MODEL AND SU(2) COHERENT STATE

For the Bose gas in an external potential, \(U(\vec{r})\), the Hamiltonian can be written in the second quantized form as

\[
H = \int d\vec{r} \phi^\dagger \left( -\frac{\hbar^2}{2m} \nabla^2 + U \right) \phi + \frac{g}{2} \int d\vec{r} \phi^\dagger \psi \psi^\dagger \phi. \tag{1}
\]

In Eq. (1) we have used a shape-independent form for the atom-atom interaction with \(\phi = 4\pi \hbar^2 a_{sc}/m\), where \(a_{sc}\) is the s-wave scattering length for repulsive \((g > 0)\) or attractive \((g < 0)\) interactions. Under the two-mode approximation[6], \(\psi\) can be expanded as

\[
\psi = \phi_L b_L + \phi_R b_R,
\]

where \(\phi_L\) and \(\phi_R\) are real, and describe the mainly left- and mainly right-well populated states respectively. With the help of Schwinger boson representation for angular momentum[4],

\[
J_+ = J_x + i J_y = b_L^\dagger b_R, \quad J_- = J_+^\dagger, \quad J_z = \frac{1}{2} (b_L^\dagger b_L - b_R^\dagger b_R),
\]

\[
N = b_L^\dagger b_L + b_R^\dagger b_R, \quad J^2 = \frac{N}{2} \left( \frac{N}{2} + 1 \right), \tag{2}
\]

we can map the Bose model (1) to a new Hamiltonian:

\[
H = \varepsilon N - 2t' J_x + \frac{\beta_L}{2} \left( \frac{N}{2} + J_z \right)^2 + \frac{\beta_R}{2} \left( \frac{N}{2} - J_z \right)^2 + u \left( 2J^2_x - J^2_z + \frac{N^2}{4} \right) + v_L (N J_x + J_z J_x + J_x J_z) \\
+ v_R (N J_x - J_z J_x - J_x J_z), \tag{3}
\]

where \(\varepsilon = \int d\vec{r} \phi_L(L) \left( -\frac{\hbar^2}{2m} \nabla^2 + U \right) \phi_L(R)\), \(\beta_L(L) = g \int d\vec{r} \phi_L(L) \left( -\frac{\hbar^2}{2m} \nabla^2 + U \right) \phi_L(R)\), \(u = g \int d\vec{r} \phi_L^2 \phi_R^2\), \(v_L(L) = 2g \int d\vec{r} \phi_L^2 \phi_L(R)\). One can easily estimate that \(|\beta_L(L)| > |u|, |v_L(L)|\). And the condition \(t' > 0\) can always be satisfied by choosing proper signs.

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(positive or negative) for real $\phi_L$ and $\phi_R$. The Hamiltonian describes an anisotropic spin system in an external magnetic field, which has been studied extensively in quantum tunneling in mesoscopic magnets. The terms involving $N$ but independent of components of $J$ are constants and can be dropped in $H$. The Hamiltonian is general, valid for the symmetric and non-symmetric wells as well. By non-symmetric well we mean the difference in the depth and shape between left and right wells, which can be tuned by the external potential $U(r)$.

For simplicity, we consider the symmetrical-well case, $\beta_L = \beta_R = \beta'$, $v_L = v_R = v$. Hence the Hamiltonian can be simplified as $H = -2tJ_z + \beta J_z^2 + 2uJ_z^2$, where $\beta = \beta' + u$ and $t = t' - vN$. In this paper, we consider only the case $|\beta| \gg |u|$ and $t > 0$. In fact, the two-mode approximation is invalid for large $vN$ for the occupation of other modes. The Hamiltonian now reads:

$$H = -2tJ_z + \beta J_z^2. \quad (4)$$

In the SU(2)-coherent-state path-integral representation, the Euclidean transition amplitude from an initial state to a final state can be written as

$$\langle \Omega_f | e^{-H(\tau_f - \tau_i)/\hbar} | \Omega_i \rangle = \int [d\Omega(\tau)] \exp \left\{ -\frac{1}{\hbar} S_E(\theta, \phi) \right\}, \quad (5)$$

where

$$S_E(\theta, \phi) = \int_{\tau_i}^{\tau_f} d\tau \left[ i\hbar \frac{N}{2} (1 - \cos \theta) \left( \frac{d\phi}{dt} \right) + E(\theta, \phi) \right]. \quad (6)$$

The first term in Eq. (6) is the Wess-Zumino term, and the effective classical energy $E(\theta, \phi)$ is

$$E(\theta, \phi) = -tN \sin \theta \cos \phi + \frac{\beta N^2}{4} \cos^2 \theta. \quad (7)$$

It is noted that the action describes the $(1 \oplus 1)$-dimensional dynamics in the Hamiltonian formulation, which consists of the canonical coordinates $\phi$ and the canonical momentum $p_\phi = i\hbar N (1 - \cos \theta)/2$. In this picture, $\langle J_z \rangle = \frac{N}{2} \sin \theta \cos \phi$, $\langle J_y \rangle = \frac{N}{2} \sin \theta \sin \phi$, $\langle J_x \rangle = \frac{N}{2} \cos \theta = (N_L - N_R)/2$, where $N_L$ and $N_R$ are the left and right-well particle number respectively. Therefore, the effective classical energy $E(\theta, \phi)$ can be described by the energy contour. In Figs. 1(a)-(d), we plot the classical orbits for different parameters $\beta N/2t$. The classical orbits show the interesting phenomenon of self-maintained population imbalance, i.e., macroscopic quantum self-trapping. This phenomenon was first found by Smerzi et al. in both repulsive and attractive interaction cases by using the canonical conjugate variables approximation and numerical calculation, and was explained as a nonlinear phenomenon induced by the interaction between atoms. Here with the help of effective classical energy, we study this MQST phenomenon in both repulsive and attractive interaction cases, and present the condition for MQST analytically. From the effective classical energy $E(\theta, \phi)$ one can easily obtain the two dividing points $\beta N/2t = \pm 1$, which correspond to the existence of two degenerate energy maxima at $\phi = \pi$ (i.e. the phase difference between double-well BEC is $\pi$) or minima at $\phi = 0$ (i.e. the phase difference is 0). When $-1 < \beta N/2t < 1$, Fig. 1 show that MQST is forbidden and the particle number of each well oscillates around $N/2$. However the phenomena of MQST exist in the cases of $\beta N/2t > 1$ and $\beta N/2t < -1$. As a result, we conclude that the phenomenon of MQST is permitted when the interaction between atoms is strong enough to obtain degenerate energy maxima or minima $(|\beta N|/2t > 1)$ in both repulsive and attractive interaction cases. Our results of repulsive case agree well with the results in Refs. and .

Another interesting observation concerns macroscopic quantum superposition state in BEC with attractive interaction. One can easily show that the system has different energy minima for different parameters $\beta N/2t$. The energy minima appear at $\phi = 0$ and different $\theta$ (i.e. different population imbalance). The $\theta$-dependence of effective classical energy $E(\theta, \phi = 0)$ is plotted in Fig. 2 for

- **III. EFFECTIVE CLASSICAL ENERGY**

In this section, we will show that the effective classical energy and the classical equations of motion give many interesting results about quantum coherence properties of BEC in a double-well potential. It is natural to investigate the classical orbits on the Bloch sphere, which can be described by the energy contour. In Figs. 1(a)-(d), we plot the classical orbits for different parameters $\beta N/2t$. The classical orbits show the interesting phenomenon of self-maintained population imbalance, i.e., macroscopic quantum self-trapping. This phenomenon was first found by Smerzi et al. in both repulsive and attractive interaction cases by using the canonical conjugate variables approximation and numerical calculation, and was explained as a nonlinear phenomenon induced by the interaction between atoms. Here with the help of effective classical energy, we study this MQST phenomenon in both repulsive and attractive interaction cases, and present the condition for MQST analytically. From the effective classical energy $E(\theta, \phi)$ one can easily obtain the two dividing points $\beta N/2t = \pm 1$, which correspond to the existence of two degenerate energy maxima at $\phi = \pi$ (i.e. the phase difference between double-well BEC is $\pi$) or minima at $\phi = 0$ (i.e. the phase difference is 0). When $-1 < \beta N/2t < 1$, Fig. 1 show that MQST is forbidden and the particle number of each well oscillates around $N/2$. However the phenomena of MQST exist in the cases of $\beta N/2t > 1$ and $\beta N/2t < -1$. As a result, we conclude that the phenomenon of MQST is permitted when the interaction between atoms is strong enough to obtain degenerate energy maxima or minima $(|\beta N|/2t > 1)$ in both repulsive and attractive interaction cases. Our results of repulsive case agree well with the results in Refs. and .

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different parameter $\beta N/2t$. When $\beta N/2t > -1$, there is only one energy minimum at $\theta = \pi/2$; while when $\beta N/2t < -1$, there are two degenerate energy minima at $\theta = \theta_0$ and $\theta = \pi - \theta_0$, where $\sin \theta_0 = 2t/|\beta|N$. The former case favors a coherent ground state or a Fock ground state, the latter case favors a Schrödinger cat-like ground state which is the superposition of two SU(2) coherent states $|\theta_0, 0\rangle$ and $|\pi - \theta_0, 0\rangle$. This kind of definition describes the same Schrödinger cat-like state as that in Ref.[3], where the result was obtained by numerical calculation. Here the dividing point separating the coherent ground state and the Schrödinger cat-like state is analytically obtained in a simple and clear approach. Moreover, the tunnel splitting can be obtained by applying the instanton technique in the SU(2)-coherent-state path-integral representation, as shown in the next section.

IV. COHERENT QUANTUM TUNNELING

As shown in Fig. 2, there will be two degenerate energy minima when $\beta N/2t < -1$ for the attractive BEC. Now the system in question performs coherent quantum tunneling (i.e., quantum coherence or coherent superposition) between two degenerate energy minima. The tunneling removes the degeneracy of the original ground states, and the true ground state (i.e., Schrödinger cat-like state) is a superposition of the previous ground states. Tunneling between neighboring degenerate vacua can be described by the instanton configuration and leads to a level splitting of the ground states. Here we evaluate the tunnel splitting of two degenerate ground states by applying the instanton technique.

After adding some constants, we can rewrite the effective energy as

$$E(\theta, \phi) = \frac{|\beta|^2 N^2}{4} (\sin \theta - \sin \theta_0)^2 + tN \sin \theta (1 - \cos \phi),$$

where $\sin \theta_0 = 2t/|\beta|N$. From $\delta S_E(\theta, \phi) = 0$, we obtain the instanton solution as

$$\cos \theta = -\cos \theta_0 \tanh(\omega_b \tau),$$
$$\sin \phi = -\frac{i}{2} \frac{\cot^2 \theta_0 \operatorname{sech}^2(\omega_b \tau)}{[1 + \cot^2 \theta_0 \operatorname{sech}^2(\omega_b \tau)]^{1/2}},$$

(9)

corresponding the transition from $\theta = \theta_0$ to $\theta = \pi - \theta_0$, where $\omega_b = |\beta| \cos \theta_0/2\hbar$. The associated classical action is found to be

$$S_{cl} = N \left[ -\cos \theta_0 + \frac{1}{2} \ln \left( \frac{1 + \cos \theta_0}{1 - \cos \theta_0} \right) \right],$$

(10)

and the final result of tunnel splitting is

$$\hbar \Delta = 8 \left( \frac{N}{2\pi} \right)^{1/2} |\beta| N \frac{\cos \theta_0^{5/2}}{\sin \theta_0} \left( \frac{1 - \cos \theta_0}{1 + \cos \theta_0} \right)^{1/2} \cos \theta_0 e^{-S_{cl}}.$$

(11)

It is noted that the tunnel splitting is obtained with the help of instanton technique in the SU(2)-coherent-state path-integral representation, which is semiclassical in nature, i.e., valid for large $N$. Therefore, one should analyze the validity of the semiclassical approximation. The semiclassical approximation is valid only when the energy splitting is far less than the energy barrier $N^2|\beta| (1 - \sin \theta_0)^2 / 4$ and the energy of zero point oscillation $N^2|\beta| \cos \theta_0 / 4$, which indicates that the classical action $S_{cl} \gg 1$. From Eq. (10) one can easily see $S_{cl} \gg 1$ when $\theta'_0 = \pi/2 - 0.3$ for typical particle number of attractive BEC $N = 1000$. Then the semiclassical approximation should be already rather good for $0 \leq \theta_0 \leq \theta'_0$.

Because of coherent quantum tunneling, the two degenerate energy minima ground state $|\theta_0, 0\rangle$ and $|\pi - \theta_0, 0\rangle$, which correspond to SU(2) coherent states with population imbalance $\pm \cos \theta_0$ and phase difference $0$ between condensates in the two wells, split to the two parity-different Schrödinger cat states $|\pm \rangle = (|\theta_0, 0\rangle \pm |\pi - \theta_0, 0\rangle)/\sqrt{2}$ with energy splitting $\hbar \Delta$. One can show that the energy splitting is extremely small when $\theta_0$ is away from $\pi/2$.

V. NUMBER FLUCTUATIONS AND RELATIVE PHASE FLUCTUATIONS

In this section we consider the fluctuations around the ground state. We shall evaluate the fluctuations for the parameters $\theta$ and $\phi$, which correspond to the relative number fluctuations and the phase fluctuations. Rewriting the parameters as $\theta(\tau) = \theta(\tau) + \phi(\tau)$ and $\phi(\tau) = \phi(\tau) + \phi_1(\tau)$, one obtains the Euclidean action as

$$S_E(\theta, \phi) = S_{cl} + \frac{1}{2} \delta^2 S,$$

(12)

where $S_{cl}$ is the classical action which satisfy $\delta S_{cl} = 0$ and
\[
\frac{1}{2} \beta^2 S = - \int_{\tau_i}^{\tau_f} i \hbar N \frac{d}{d\tau} \left( \sin \theta \phi \right) \phi \ d\tau
+ \frac{1}{2} \int_{\tau_i}^{\tau_f} \left( \frac{\hbar^2 N}{2} \cos^2 \theta \phi^2 \right) d\tau
+ \frac{1}{2} \int_{\tau_i}^{\tau_f} \left( E_{\theta\phi} \phi^2 + 2E_{\theta\phi\theta} \phi^2 + E_{\phi\phi} \phi^2 \right) d\tau.
\]

In the above equation, \( E_{\theta\theta} = \partial^2 E/\partial \theta^2 \), \( E_{\theta\phi} = \partial^2 E/\partial \theta \partial \phi \) and \( E_{\phi\phi} = \partial^2 E/\partial \phi^2 \), which are evaluated at the classical path. Under the condition that \( E_{\theta\phi} > 0 \), the Gaussian integration can be performed over \( \phi \), then the effective action for \( \theta \) is found to be

\[
I \left( \theta \right) = \int_{\tau_i}^{\tau_f} \left( A \phi^2 + B \phi + C \phi \right) d\tau.
\]

Using Eq. \( E_{\phi\phi} \), we have

\[
E_{\phi\phi} = tN \sin \theta \cos \phi > 0,
A = \frac{\hbar^2 N \sin \theta}{8t \cos \phi},
B = 0,
C = \frac{tN \cos \phi}{2 \sin \theta} + \frac{\beta N^2}{4} \sin^2 \theta.
\]

The effective action reads

\[
I \left( \theta \right) = \int_{\tau_i}^{\tau_f} \left[ \frac{\hbar^2 N \sin \theta}{8t \cos \phi} \phi^2 
+ \left( \frac{tN \cos \phi}{2 \sin \theta} + \frac{\beta N^2}{4} \sin^2 \theta \right) \phi^2 \right] d\tau.
\]

In the case of \( \beta N/2t > -1 \), the classical ground state is \( \theta = \pi/2, \phi = 0 \). Now we study the \( \theta \) fluctuation near the classical ground state, \( A = \frac{\hbar^2 N}{8t} \) and \( C = \frac{tN}{2} + \frac{\beta N^2}{4} = \frac{N}{4} \left( 2t + \beta N \right) \). The effective action is

\[
I \left( \theta \right) = \int_{\tau_i}^{\tau_f} \left[ \frac{\hbar^2 N \sin \theta}{8t} \theta^2 
+ \left( \frac{tN}{4} \right) \left( 2t + \beta N \right) \theta^2 \right] d\tau.
\]

The motion of the fluctuation \( N \theta \phi \) is approximately a harmonic oscillator with mass \( \frac{\hbar^2 N}{8t} \) and frequency \( \sqrt{2t} \sqrt{2t + \beta N} \). Its characteristic length \( \sigma_{N\theta^2} \) is determined by \( \sigma_{N\theta^2} = N \sqrt{\frac{2t}{2t + \beta N}} \). Hence the number fluctuation in one well is

\[
\Delta(NL - NR) = \left( N \sin \frac{\pi}{2} \right) \Delta \theta = \sqrt{N} \sqrt{\frac{2t}{2t + \beta N}}.
\]

One can see that there is a singular point in \( \beta N/2t = -1 \), which indicates a dividing behavior between coherent state-like and Schrödinger cat state-like ground state.

In the case of \( \beta N/2t < -1 \), the classical ground state is \( \theta = \theta_0 \) or \( \pi - \theta_0, \phi = 0 \), where \( \sin \theta_0 = 2t/|\beta|N \). However, due to quantum tunneling between the two classical ground states, the true ground state is an even-parity Schrödinger cat state. Inspecting the fluctuation near the classical ground state, we find that \( A = \frac{\hbar^2 N \sin \theta_0}{8t} = \frac{\hbar^2}{4|\beta|} \) and \( C = \frac{tN}{2 \sin \theta_0} + \frac{\beta N^2}{4} \sin^2 \theta_0 = \frac{|\beta| N^2}{4} \cos^2 \theta_0 \). The effective action is

\[
I \left( \phi \right) = \int_{\tau_i}^{\tau_f} \left[ \frac{\hbar^2 N}{8|\beta|} \phi^2 + \frac{|\beta| N^2}{4} \cos^2 \phi \phi^2 \right] d\tau.
\]

The motion of the fluctuation \( N \theta \phi \) is approximately a harmonic oscillator with mass \( \frac{\hbar^2 N}{8t} \) and frequency \( \frac{|\beta| N \cos \theta_0}{\hbar} \). Its characteristic length \( \sigma_{N\theta^2} \) is determined by \( \sigma_{N\theta^2} = \frac{\beta}{\cos \theta_0} \). Hence the number fluctuation in one well is

\[
\Delta(NL - NR) = N \sin \theta_0 \Delta \theta = \sin \theta_0 \sqrt{\frac{2N}{\cos \theta_0}},
\]

which is different from the case of \( \beta N/2t > -1 \). This result is not the true number fluctuation around the Schrödinger cat state. It indicates the fluctuation around the classical ground state. The true number fluctuation is of the order of \( N^2 \), referred to Ref. [11] as a superfragmented state.

Finally we discuss the relative phase fluctuation. After performing the Gaussian integration over \( \phi \), the problem reduces to the one-dimensional path integral with the effective action:

\[
I \left( \phi \right) = \int_{\tau_i}^{\tau_f} \left[ A' \phi^2 + C' \phi^2 \right] d\tau,
\]

with

\[
A' = \frac{\hbar^2 N \sin^2 \phi}{8 \left( E_{\theta\theta} - \cot \theta E_\phi \right)},
C' = \frac{1}{2} \left( E_{\phi\phi} - \frac{E_{\phi\phi}}{E_{\theta\theta} - \cot \theta E_\phi} \right) + i \frac{d}{d\tau} \left( \frac{\hbar N \sin \phi E_{\phi\phi}}{4 \left( E_{\theta\theta} - \cot \theta E_\phi \right)} \right).
\]

In the case of \( \beta N/2t > -1 \), the classical ground state is \( \theta = \pi/2, \phi = 0 \). Now we inspect the \( \phi \) fluctuation near the classical ground state, \( A' = \frac{\hbar^2 N}{4(2t + \beta N)} \) and \( C' = \frac{tN}{2} \). The effective action is

\[
I \left( \phi \right) = \int_{\tau_i}^{\tau_f} \left[ \frac{\hbar^2 N}{4(2t + \beta N)} \phi^2 + \frac{tN}{2} \phi^2 \right] d\tau.
\]

The motion of the fluctuation \( \phi \) is approximately a harmonic oscillator with mass \( \frac{\hbar^2 N}{4(2t + \beta N)} \) and frequency
\[ \sqrt{\frac{2(2t + \beta N)}{\hbar^2}}. \]

Its characteristic length \( \sigma_{\phi} \) is determined by \( \sigma_{\phi}^2 = \frac{1}{N} \sqrt{\frac{2t + \beta N}{2t}}. \) Hence the relative phase fluctuation is

\[ \Delta \phi = \sqrt{\frac{1}{N} \sqrt{\frac{2t + \beta N}{2t}}}. \quad (22) \]

In the case of \(\beta N/2t < -1\), the classical ground state is \(\bar{\theta} = \theta_0\) or \(\pi - \theta_0\), \(\bar{\phi} = 0\), where \(\sin \theta_0 = 2t / |\beta| N\). Inspecting the fluctuation near the classical ground state, we find that \(A' = \frac{h^2 \sin^2 \theta_0}{4|\beta| \cos^2 \theta_0}\) and \(C' = |\beta| N^2 \sin^2 \theta_0\). The effective action is

\[ I(\phi_1) = \int_{t_1}^{t_f} \left[ \frac{h^2 \sin^2 \theta_0}{4|\beta| \cos^2 \theta_0} \dot{\phi}_1^2 + |\beta| N^2 \sin^2 \theta_0 \phi_1^2 \right] dt. \quad (23) \]

The motion of the fluctuation \(\phi_1\) is approximately a harmonic oscillator with mass \(\frac{h^2 \sin^2 \theta_0}{2|\beta| \cos^2 \theta_0}\) and frequency \(\frac{2|\beta| N \cos \theta_0}{h}\). Its characteristic length \(\sigma_{\phi}\) is determined by

\[ \sigma_{\phi}^2 = \frac{1}{4N} \frac{\cos \theta_0}{\sin^2 \theta_0}. \quad \text{Hence the relative phase fluctuation is} \]

\[ \Delta \phi = \sqrt{\frac{\cos \theta_0}{2N \sin^2 \theta_0}}. \quad (24) \]

VI. CONCLUSION

In summary, we study the quantum coherence phenomena in a double-well BEC based on SU(2)-coherent-state path-integral. By this method we analytically study the MQST phenomenon, the ground states of this system, and the existence of macroscopic quantum superposition states. We find that MQST will happen in both repulsive and attractive interaction cases, and analytically obtain the dividing points \(\beta N/2t = \pm 1\). When \(\beta N/2t > -1\), both repulsive and attractive Bose gases favor a coherent or a squeezed ground state, which is the coherent state for non-interaction case, the relative-number-squeezed state for repulsive case, and the relative-phase-squeezed state for the attractive case (see Eqs.(17) and (22)) respectively. However when \(\beta N/2t < -1\), attractive Bose gases favor a macroscopic quantum superposition state. The relative number fluctuation and relative phase fluctuation between two-well condensates are obtained through path-integral technique. For the attractive interaction case, the coherent gap of degenerate ground states is obtained analytically with the help of the instanton technique.

It is noted that all the discussions in this paper does not imply this kind of macroscopic quantum superposition states can be easily created. To inspect the decoherence, one has to induce the interaction between thermal and condensate. Dalvit et al. have studied the decoherence of a similar model in Ref [12], which is beyond our two-mode approximation and semiclassical approach. However, as an rough estimate, the high barrier and small splitting gap imply high rate of decoherence. In fact, to the best of our knowledge, there is no experimental evidence of BEC cats up to now, although there are some proposals to create them [12, 13]. Ref. [12] pointed out that such macroscopic quantum superposition states are extremely fragile to decoherence, and suggested that the strategy of trap engineering and symmetrization of the environment should be able to deal with that issue. The theoretical calculations performed in this paper can be extended to the Bose gas in the non-symmetric double-well potential, and an effectively two-component spinor condensate. Work along this line is still in progress.

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