Study of $B \rightarrow K_0^*(1430)\ell\bar{\ell}$ decays

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We examine the exclusive rare decays of $B \rightarrow K_0^*(1430)\ell\bar{\ell}$ with $K_0^*(1430)$ being the $p$-wave scalar meson and $\ell = e, \mu, \tau$ in the standard model. The form factors for the $B \rightarrow K_0^*$ transition matrix elements are evaluated in the light-front quark model. For the decays of $B \rightarrow K_0^*\ell\bar{\ell}$, the branching ratios are found to be $(11.6, 1.63, 1.62, 0.029) \times 10^{-6}$ with $\ell = (e, \mu, \tau)$ and the integrated longitudinal lepton polarization asymmetries $(-0.97, -0.95, -0.03)$ with $\ell = (e, \mu, \tau)$, respectively.

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I. INTRODUCTION

The suppressed inclusive flavor-changing neutral current (FCNC) process of $B \rightarrow X_\ell^\ell^-$, induced by electroweak penguin and box diagrams in the standard model (SM), has been observed by both BABAR [1] and Belle [2] with the branching ratio (BR) of $(4.5 \pm 1.0) \times 10^{-6}$ [3] for dilepton masses greater than 0.2 GeV, where $\ell$ is either an electron or a muon and $X_\ell$ is a hadronic recoil system that contains a kaon. The exclusive decays of $B \rightarrow K^{*+}\ell^-$ and $B \rightarrow K^{*}(892)^{+}\ell^-$ have also been measured with the BRs [3] of $(0.54 \pm 0.08) \times 10^{-6}$ and $(1.05 \pm 0.20) \times 10^{-6}$ [4,5], which agree with the theoretically estimated values [6–9], respectively.

There have been many investigations of rare $B$ semi-leptonic decays of induced by the FCNC transition of $b \rightarrow s$ [10] since the CLEO observation [11] of $b \rightarrow s\nu\bar{\nu}$. The studies are even more complete if similar studies for the $p$-wave mesons of $B$ decays such as $B \rightarrow K^*_{1,2,3}(1430)\ell\bar{\ell}$ and $B \rightarrow K_{1,2,3}(1430)\ell\bar{\ell}$ are also included. In fact, the study of $B \rightarrow K_{1,2}^*(1430)\ell^+\ell^-$ has been done in Ref. [12]. It is clear that these FCNC rare decays are important for not only testing the SM but probing new physics. In this report, we concentrate on the exclusive rare decays of $B \rightarrow K_0^*\ell\bar{\ell}$, where $K_0^*$ represents the $p$-wave scalar meson of $K_0^*(1430)$ and $\ell$ stands for a charged lepton or neutrino. To obtain the decay rates and branching ratios, we need to calculate the transition form factors of $B \rightarrow K_0^*$ due to the axial-vector and axial-tensor currents, respectively, in the standard model. We will use the framework of the light-front quark model (LFQM) [13–15] to evaluate these form factors.

This report is organized as follows: We present the relevant formulas in Sec. II. First, we give the effective Hamiltonians for $B \rightarrow K_0^*\ell\bar{\ell}$ induced by $b \rightarrow s\ell\bar{\ell}$. Then, we calculate the hadronic form factors for the $B \rightarrow K_0^*$ transition in the LFQM. Finally, we study the branching ratios and polarization asymmetries of the decays. In Sec. III, we show our numerical results on form factors and the physical quantities of the decays. We give our conclusions in Sec. IV.
for the initial $s$-wave pseudoscalar ($P_i$) and final $p$-wave scalar ($S_f$) mesons of $B$ and $K'_0$, respectively, with

$$
\begin{align}
\hat{M}_0 &= \sqrt{M_0^2 - (m_1 - m_2)^2}, \\
M_0^2 &= \frac{m_1^2 + k_1^2}{(1 - x)} + \frac{m_2^2 + k_2^2}{x},
\end{align}
$$

In our calculations, we use the Gaussian type wave functions

$$
\phi(x, k_\perp) = 4 \left( \frac{\pi}{\omega_M^3} \right)^{3/4} \frac{dk_z}{dx} \exp\left(-\frac{k_z^2}{2\omega_M^2}\right),
$$

for the $s$-wave and $p$-wave mesons, respectively, where $\omega_M$ is the meson scale parameter and $k_z$ is defined through

$$
1 - x = \frac{e_1 - k_z}{e_1 + e_2}, \quad x = \frac{e_2 + k_z}{e_1 + e_2},
$$

with $e_i = \sqrt{m_i^2 + k_i^2}$. Then, we have

$$
M_0 = e_1 + e_2, \quad k_z = \frac{xM_0}{2} - \frac{m_1^2 + m_2^2}{2xM_0},
$$

and

$$
\frac{dk_z}{dx} = \frac{e_1e_2}{x(1 - x)M_0}.
$$

We normalize the meson state as

$$
\langle B(P', S', S'_f)|B(P, S, S_f)\rangle = 2(2\pi)^3 P^+ \delta^3(\vec{P}' - \vec{P}) \delta_{SS_f}\delta_{S'_f},
$$

so that the normalization condition of the momentum distribution function can be obtained by

$$
\int \frac{dxdk_z}{2(2\pi)^3} |\phi(x, k_\perp)|^2 = 1.
$$

We are now ready to calculate the matrix elements of the $P_i \rightarrow S_f$ transition, which can be defined by

$$
\langle S_f(p_f)|A_\mu|P_i(p_i)\rangle = -i[u_+(q^2)P_\mu + u_-(q^2)q_\mu],
$$

$$
\langle S_f(p_f)|T^\mu_{PS_f}q^\nu|P_i(p_i)\rangle = \frac{-i}{m_{P_f} + m_{S_f}} \left[ q^2 P_\mu - (P \cdot q)q_\mu \right] F_T(q^2),
$$

where $A_\mu = \bar{q}_f \gamma_\mu \gamma_5 q_i$, $T^\mu_{PS_f} = \bar{q}_f \sigma_{\mu\nu} q_i$, $P = p_i + p_f$ and $q = p_f - p_i$ with the initial (final) meson bound state $q_i \bar{q}_i (q_f \bar{q}_f)$. We note that all form factors will be studied in the timelike physical meson decay region of $0 \leq q^2 \leq (m_{P_f} - m_{S_f})^2$. The form factors in Eq. (16) are found to be
\[ u_+(q^2) = \frac{1 - r_-}{r_+ - r_-} \text{H}(r_+) - \frac{1 - r_+}{r_+ - r_-} \text{H}(r_-), \]
\[ u_-(q^2) = \frac{1 + r_-}{r_+ - r_-} \text{H}(r_+) - \frac{1 + r_+}{r_+ - r_-} \text{H}(r_-), \]
\[ F_T(q^2) = - \int_0^1 dx \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{M_0^2}{2\sqrt{3}M_0} \phi_{S_i}(x', k_{\perp}) \phi_{P_i}(x, k_{\perp}) \frac{m_{P_i} + m_{S_i}}{1 + 2r} \frac{A}{x^2 + (m_{P_i}^2 - m_{S_i}^2)} \frac{A}{2} \sqrt{A_{P_i}^2 + k_{\perp}^2}, \]
\[ \text{where } r = p_+^2 / p_+^2, x' = x / r, \]
\[ r_\pm = \frac{m_{P_i}}{m_{S_i}} \left[ \frac{v_i \cdot v_f + \sqrt{(v_i \cdot v_f)^2 - 1}}{v_i \cdot v_f - \frac{m_{P_i}^2 + m_{S_i}^2 - q^2}{2m_{P_i}m_{S_i}}} \right], \]
\[ H(r) = - \int_0^1 dx \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{M_0^2}{2\sqrt{3}M_0} \phi_{S_i}(x', k_{\perp}) \phi_{P_i}(x, k_{\perp}) \times \left[ \frac{[m_{q_i} x + m_{q_i} (1 - x)] [m_{q_i} x' + m_{q_i} (1 - x')] + k_{\perp}^2}{\sqrt{A_{P_i}^2 + k_{\perp}^2} \sqrt{A_{S_i}^2 + k_{\perp}^2}} \right], \]
\[ A = \frac{1}{\sqrt{x'x}(1 - x)(1 - x')} \left[ [x m_{q_i} + (1 - x) m_{q_i}] [x m_{q_i} + (1 - x') m_{q_i}] [x (1 - x) m_{q_i} - x' (1 - x) m_{q_i}] \right] \]
\[ + \frac{1}{k_{\perp}^2} \left[ -x' (1 - x)(2x - 1) m_{q_i} + (1 - x') (x + x' - 2xx') m_{q_i} + x (1 - x)(1 - 2x') m_{q_i} \right], \]
\[ A_{P_i}(x_i) = m_{q_i}(x_i)^2 + m_{q_i}(1 - x_i). \]

The sign +(-) of \( r_{+(-)} \) represents the final meson recoiling in the positive (negative) \( z \) direction relative to the initial meson.

We note that to evaluate the form factors, we have to fix the meson scale parameters \( \omega_{9g} \) and \( \omega_{K_0} \) in the meson wave functions in Eq. (10) by some known parameters such as the meson decay constants, defined by
\[ f_{P_i} = \sqrt{24} \int dx d^2 k_{\perp} \phi(x, k_{\perp}) \frac{m_{q_i} x + m_{q_i} (1 - x)}{\sqrt{A_{P_i}^2 + k_{\perp}^2}} \]
\[ f_{S_i} = \sqrt{24} \int dx d^2 k_{\perp} \phi(x', k_{\perp}) \frac{m_{q_i} x' + m_{q_i} (1 - x')}{{\sqrt{A_{S_i}^2 + k_{\perp}^2}}}, \]
\[ \begin{align*}
\frac{d \Gamma(B \rightarrow K_0^* \ell \bar{\nu})}{ds} &= \frac{G_F^2 |\lambda_1|^2 \alpha_{em}^2 |D(x_s)|^2 m_B^5}{2 \pi \sin^2 \theta_W} \frac{\alpha_{K_0^*}^{1/2} |u_+|^2}{s^{1/2}} \left[ 1 - \frac{4 t}{s} - \frac{4 t}{s} \frac{\alpha_{K_0^*}^{1/2} \left[ 1 + \frac{2 t}{s} \right] \alpha_{K_0^*}^{-1/2}}{\alpha_{K_0}^{1/2}} + t \delta_{K_0}^* \right], \\
\frac{d \Gamma(B \rightarrow K_0^{*+} \ell^-)}{ds} &= \frac{G_F^2 |\lambda_1|^2 m_B^3 \alpha_{em}^2}{3 \cdot 2 \pi} \left( \frac{4 t}{s} \right)^{1/2} \frac{\alpha_{K_0^*}^{1/2} \left[ 1 + \frac{2 t}{s} \right] \alpha_{K_0}^{-1/2}}{\alpha_{K_0}^{1/2}}, \\
\end{align*} \]
\[ \text{where } \]
\[ s = q^2 / m_B^2, \quad t = m_B^2 / m_B^2, \quad r_{K_0} = m_{K_0}^2 / m_B^2, \quad \varphi_{K_0} = (1 - r_{K_0})^2 - 2x(1 + r_{K_0}) + x^2, \]
\[ \alpha_{K_0^*} = \varphi_{K_0^*} \left[ C_{eff} u_+ - \frac{2 C_{T} F_T}{1 + \sqrt{\delta_{K_0}}} |u_+|^2 \right], \]
\[ \delta_{K_0} = 6 |C_{10}|^2 \left[ 2(1 + r_{K_0}) - s \right] |u_+|^2 + 2(1 - r_{K_0}) \text{Re}(u_+ u_+^*) + s |u_-|^2. \]

When the polarization of the charged lepton is in the longitudinal direction, i.e. \( \vec{n} = \vec{e}_L = \vec{p}_{\ell}/|\vec{p}_{\ell}| = \pm 1 \), we can also define the longitudinal lepton polarization asymmetry in \( B \rightarrow K_0^{*+} \ell^- \) as follows [24,25]:
\[ P_L(s) = \frac{d \Gamma(s, \ell^-)}{ds} / \frac{d \Gamma(s, \ell^-)}{ds}, \quad (22) \]
From Eq. (22), we find that
\[ P_L = \frac{2(1 - \frac{4 t}{s})^{1/2}}{1 + \frac{4 t}{s} \alpha_{K_0^*}^{-1/2} + t \delta_{K_0}} \times \text{Re} \left[ \frac{\varphi_{K_0^*} \left( C_{eff} u_+ - \frac{2 C_{T} F_T}{1 + \sqrt{\delta_{K_0}}} \right)(C_{10} u_+^*)}{\alpha_{K_0}^{1/2}} \right], \]
in \( B \rightarrow K_0^* \ell^+ \ell^- \). We remark that there is no forward-
backyard asymmetry for $B \to K_0^0 \ell^+ \ell^-$ in the SM similar to other pseudoscalar to pseudoscalar dilepton decays such as $P_i \to P_f \ell^+ \ell^-$ with $P_i = K(B)$ and $P_f = \pi(K)$ [26].

### III. NUMERICAL RESULTS

In our numerical study of the hadronic matrix elements for the $B \to K_0^0$ transition, we fix the quark masses to be $m_b = 4.64$, $m_s = 0.37$, and $m_{u,d} = 0.26$ GeV and use the meson decay constants to determine the meson scale parameters as shown in Table I. We note that the current direct measurement of $f_B$ is $0.229^{+0.036+0.034}_{-0.031-0.037}$ GeV [27], whereas there is no experimental information on $f_{K_0^0}$. Since $f_B$ and $f_{K_0^0}$ are fixed to be 0.18 and 0.021 GeV in Ref. [15], respectively, we will also use these values in our numerical analysis and briefly discuss other values at the end. Our results for the form factors at $q^2 = 0$ are given in Table II. In the table, as a comparison, we have also shown the value of $u_+(0)$ used in Ref. [15]. To give explicit $q^2$ dependent form factors, we fit our results to the form

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^4)},$$

with the fitted ranges of $F_T(q^2)$ and $u_+(q^2)$ being $0 \leq q^2 \leq 12$ and $0 \leq q^2 \leq (m_B - m_{K_0^0})^2$ GeV$^2$, respectively. In Fig. 1, the form factors as functions of $q^2$ are presented, where (a) $u_+(q^2)$ and (b) $F_T(q^2)$. In Table III, we give the values of the relevant WCs at the scale of $\mu = 4.8$ GeV [9]. With $|\Lambda_i| \approx 0.041$, we illustrate the differential decay branching ratios for $B \to K_0^0 \nu \bar{\nu}$ and $B \to K_0^0 \ell^+ \ell^-(\ell = \mu, \tau)$ as functions of $s$ in Figs. 2 and 3, respectively. By integrating the differential ratios over $s = q^2/m_B^2$ for $B \to K_0^0 \nu \bar{\nu}$ and $B \to K_0^0 \ell^+ \ell^-$, we have

$$Br(B \to K_0^0 \nu \bar{\nu}) = 1.16 \times 10^{-6}$$

and

$$Br(B \to K_0^0 e^+ e^-, K_0^0 \mu^+ \mu^-, K_0^0 \tau^+ \tau^-) = 1.63 \times 10^{-7},$$

$$1.62 \times 10^{-7}, \quad 2.86 \times 10^{-9},$$

respectively. Note that the small branching ratio of the tau mode is due to the highly suppressed phase space as shown in Fig. 3. We also present the longitudinal lepton polarization asymmetries of $B \to K_0^0 \ell^+ \ell^-$ as functions of $s$ in Fig. 4. We note that our results for the electron mode are

![Figure 1](image1.png)

**FIG. 1** (color online). Form Factors as functions of $q^2$ for (a) $u_+(q^2)$ and $u_-(q^2)$ and (b) $F_T(q^2)$.

![Figure 2](image2.png)

**FIG. 2**. Differential decay branching ratio for $B \to K_0^0 \nu \bar{\nu}$ as a function of $s = q^2/m_B^2$.

![Figure 3](image3.png)

**FIG. 3**. Differential decay branching ratios for $B \to K_0^0 \mu^+ \mu^-$ and $B \to K_0^0 \tau^+ \tau^-$ as functions of $s = q^2/m_B^2$.

| WC | $C_1$ | $C_2$ | $C_7$ | $C_9$ | $C_{10}$ |
|----|------|------|------|------|--------|
|    | -0.226 | 1.096 | -0.305 | 4.186 | -4.559 |

**TABLE III.** Wilson coefficients for $m_t = 170$ GeV and $\mu = 4.8$ GeV.
similar to those for the muon one. As shown in Fig. 4, $P_L(s)[B \rightarrow K_0^0 \mu^+ \mu^-]$ is close to $-1$ except those close to the end points of $q_{\min}^2 = 4m_{\mu}^2$ and $q_{\max}^2 = (m_B - m_{K_0^0})^2$ at which they are zero and $P_L(s)[B \rightarrow K_0^0 \tau^+ \tau^-]$ ranges from $-0.5$ to $0$, while the integrated values of $P_L$ are $-0.97$, $-0.95$, and $-0.03$ for electron, muon and tau modes, respectively. It is clear that due to the efficiency for the detectability of the tau lepton, it is impossible to measure the tau lepton polarization in the near future.

Finally, we remark that our results are insensitive (sensitive) to the value of $f_B$ ($f_{K_0^0}$). For examples, $Br(B \rightarrow K_0^0 \ell \bar{\ell}) (\ell = \nu, e, \mu)$ decrease about 6% by increasing $f_B = 0.16$ to 0.20 GeV, while they increase about 50% by increasing $f_{K_0^0} = 0.021$ to 0.025 GeV.

IV. CONCLUSIONS

We have studied the exclusive rare decays of $B \rightarrow K_0^0 \ell \bar{\ell}$. We have calculated the form factors for the $B \rightarrow K_0^0$ transition matrix elements in the LFQM. We have evaluated the decay branching ratios and the longitudinal charged-lepton polarization asymmetries in the SM. Explicitly, we have found that $Br(B \rightarrow K_0^0 \ell \bar{\ell}) (\ell = \nu, e, \mu, \tau) = (11.6, 1.63, 1.62, 0.029) \times 10^{-7}$ and the integrated longitudinal lepton polarization asymmetries of $B \rightarrow K_0^0 \ell^+ \ell^- (\ell = e, \mu, \tau)$ are $-0.97$, $-0.95$ and $-0.03$, respectively. It is clear that some of the above $p$-wave $B$ decays and asymmetries can be measured at the ongoing as well as future $B$ factories.

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