Effects of Muscle Length and Physiological Cross Sectional Area on Muscle Force Production: a Comparative Study

F Romero-Sánchez, F J Alonso, J Barrios-Muriel and G Rodriguez-Jiménez

1Department of Mechanical Engineering, Energy and Materials, University of Extremadura. Avda. de Elvas s/n, 06006 Badajoz, Spain
2Department of Mechanical Engineering, University of Seville, Camino de los Descubrimientos s/n. 41092 Seville, Spain.

fromsan@unex.es, fjas@unex.es, jorgebarrios@unex.es

Abstract. The estimation of muscular forces is useful in several areas such as biomedical or rehabilitation engineering. The analysis of the forces that produce a given movement (inverse dynamics, ID) or the movement induced by a set of muscle forces or activations (forward dynamics, FD) are typical problems that need the description of muscle mechanical properties. Moreover, to solve the indeterminacy problem in the ID analysis, optimization schemes are required. Several optimization methods (static optimization, dynamic optimization, augmented static optimization) and optimization criteria (minimum metabolical cost of transport, minimum sum of muscle stresses, time-integral cost of activations, torque-tracking) are available in the literature to that end. In all the aforementioned problems muscle properties, such as muscle length or physiological cross sectional area (PCSA), play a key role in the development of consistent models to perform specific tests. In this work, a comparative study of the effects on muscle force production in muscles with different ratios between length and PCSA is presented. Its relevance in the traditional problems faced in biomechanics is also studied.

1. Introduction

The analysis of muscle force activity is an essential exercise to understand how the force is generated to perform a movement. Movement Biomechanics deals with the analysis of the forces that produce a given movement (the inverse dynamics problem, ID) and the analysis of the movement produced by a given set of muscle forces (forward dynamic problem, FD). Both problems need to be described by muscle models. The work of Hill [1] established the basis for a parametric model description. Since then, different models have been described in the literature (see [2,3] for traditional models or [4,5] more complex models based on fractional order dynamics). Such models are described mainly by the muscle length, muscle contraction velocity and activation level. By using these models, once the current state is known (muscle length and contraction velocity), it is possible to obtain the output, i.e., the muscle force for different sets of activations that combined provide the net joint torque in a joint and, therefore, the movement. On the contrary, to the aforementioned description of the FD problem, in the ID problem the inputs are the spatial coordinates of the body and the ground reaction forces. Both allow to obtain the net joint torques and joint reaction forces. The muscle forces are then obtained by distributing the net joint torque between the muscles spanning the joint. To do so, different optimization schemes can be used [6-8] where the usual cost function depends on the muscle stress, which is directly related to the...
physiological cross-sectional area (PCSA). As it is shown, traditional problems in Biomechanics
depends on the muscle macroscopic structure, and therefore, either the force production in the FD
problem or the muscle force distribution in the ID are conditioned by it. For the muscles shown in figure
1, for the same level of activation the force production will be different in a slender muscle compared
to a wider muscle as well as the muscle stress. In this work, we study the differences in the resolution
of both typical problems in Biomechanics, for simple examples regarding muscles of the lower limbs
were the biggest differences between slender muscles can be found.

![Figure 1](image)

**Figure 1.** a). Differences in muscle length and physiological cross sectional area on shank muscles.
Left: Anterior view of the shank: *tibialis anterior* muscle. Right: Posterior view of the shank:
*gastrocnemius*. b) Muscle groups used in this work: 1 – Iliopsoas, 2 – Rectus Femoris, 3 – Glutei, 4 –
Hamstrings, 5 – Vasti, 6 – Gastrocnemius, 7 – Tibialis Anterior, 8 – Soleus. $T_o$ represents the given
torque at each joint.

2. **Methods**

In order to study the influence of different values of muscle length and physiological cross-sectional in
muscle force production, we first present the problems in which these terms are involved, and then the
methodology to test the influence of variations in those parameters.

2.1. **The FD problem. Description of the muscle model.**

In a previous work, the authors tested the different Hill-type dynamics formulations for muscle force
estimation [9] that accounted for muscle parameters such as muscle length, muscle contraction velocity
or activation level among others. In this work we use the description provided by Thelen [10], as the
output is implicitly related to muscle parameters.

The contractile element in a Hill-type muscle model is responsible of the active force generation in
muscle tissue. The force developed in this element (contraction dynamics) depends on two relationships,
the force-length and the force-velocity relationships:

$$F^M = F_0^M \cdot a \cdot f_l(l^M) \cdot f_v(\dot{\theta}^M)$$  \hspace{1cm} (1)
where $F_0^M$ is the maximum isometric force of the muscle, $a$ is the activation level ($a \in [0,1]$), and $f_L(\bar{l}^M)$ and $f_V(\bar{v}^M)$ are the force-length and the force-velocity relationships, respectively. On the one hand, the force length relationship is defined as:

$$f_L(\bar{l}^M) = \begin{cases} 
0 & \text{for } \bar{l}^M \leq 1 \\
 e^{-((\bar{l}^M - 1)/\gamma}) & \text{for } \bar{l}^M > 1 
\end{cases} \quad (2)$$

where $\bar{l}^M = l^M/L_{opt}$ is the normalized muscle length by the length in which the maximum force is obtained and $\gamma$ a shape factor set to 0.45 in this work.

On the other hand, the force-velocity relationship is written as:

$$f_V(\bar{v}^M) = \begin{cases} 
0 & \text{for } \bar{v}^M \leq -1 \\
 1 + \bar{v}^M / K_{CE1} & \text{for } -1 < \bar{v}^M \leq 0 \\
 1 + \bar{v}^M / K_{CE2} & \text{for } \bar{v}^M > 0 
\end{cases} \quad (3)$$

where $\bar{v}^M = v^M/(L_{opt}/\tau_c)$ is the normalized muscle contraction velocity by a term that represents the optimal contraction velocity ($\tau_c$ is a time constant set to 0.1 in this work), and $K_{CE1}$ and $K_{CE2}$ are force-velocity shape factors (0.25 and 0.06 respectively in this work). For further details of this description see [10].

### 2.2. The ID problem. Description of the optimization scheme.

Since several muscles serve each joint of the skeletal system, muscle forces cannot be directly computed from joint moments. Several optimization methods (static optimization, dynamic optimization, augmented static optimization, large-scale static optimization) and optimization criteria (minimum metabolical cost of transport, minimum sum of muscle stresses, minimum hyper-extension of the joints, time-integral cost of activations, torque-tracking) are available in the literature. In this work we use an optimization scheme proposed by the authors in which the physiological information of the muscle is considered. The scheme is as follows:

1. In the first step, the length and velocity of each muscle are obtained from generalized coordinates of the multibody model. The maximum muscle force histories $F_{max}^M$ compatible with contraction dynamics are calculated supposing that the muscle activation are maxima at every instant

   $$A_m = [a_1, K, a_N]^T = [I, K, 1]^T,$$

   i.e., the hypothetical maximum muscle force that correspond to the current state:

   $$F_{max}^M = F_0^M \cdot 1 \cdot f_L(\bar{l}^M) \cdot f_V(\bar{v}^M)$$

2. For each muscle, the muscle activation levels are obtained by solving the following optimization scheme:

   $$\min J = \sum_{j=1}^{n} \frac{a_j \cdot F_{max,j}}{C_j} \quad (4)$$

   $$s.t. \quad R(A \cdot F^M) = T$$

   3.
where \( C_j = F_j^M / PCSA \) is the muscle stress, \( T \) is the net joint torque, \( A \cdot F^M = [a_1 \cdot F_{\text{max},1}^M, a_2 \cdot F_{\text{max},2}^M, \ldots, a_n \cdot F_{\text{max},n}^M]^T \) are the muscle forces scaled by the design variables and \( R \) is the constant matrix of equivalent moment arms of the different muscles. Once solved the optimization scheme, the \( F_{\text{max}}^M \) term can be scaled by the activations obtained in the optimization problem.

2.3. Experimental setup.

In order to test the influence of muscle length and PCSA, we defined two different experiments (figure 2a). Regarding the first experiment, to quantify the influence of muscle length, we solved the FD problem. As this problem requires the values of muscle length (\( l^M \)), muscle contraction velocity (\( v^M \)) and activation (\( a_j \)) levels to be performed, we recorded the gait from a healthy voluntary subject and then we performed an inverse kinematics analysis to obtain the muscle length and muscle contraction velocity. This process can be found in detail in, e.g., [11,12]. As stated before, there is a major number of slender muscles in the lower limbs that can be used for the proposed study. As activations cannot be measured in-vivo non-invasively, we used the expression proposed by [13] that relates the activation level with the electromyography (EMG) signal that can be measured from surface muscles. This relationship can be written as:

\[
a_j(t) = \frac{e^A \cdot u_j(t) - 1}{e^A - 1} \tag{5}
\]

where \( a_j(t) \) is the activation level, \( u_j(t) \) is the processed EMG signal and \( A \) is a non linear shape factor constrained between \(-5 < A < 0 \) [13] (in this work \( A = -4 \)). By using the measured parameters, i.e., \( l^M \) and \( v^M \) from the ID of the registered motion and EMG signals on equations (2), (3) and (5), respectively, it is possible to obtain an estimate of the muscle forces in the FD problem.

The EMG signal was recorded in the tibialis anterior muscle by following the standard recommendations given by the SENIAM [14] using the TrignoTM Wireless System from Delsys, at a sample frequency of 2000Hz. The EMG signal was imported to Matlab (The Mathworks, Inc.) where the signal processing was performed. The EMG signal was rectified and a moving average (MOVAG) was applied to smooth the signal. Then, we used the recorded and processed EMG signals on (5) to obtain the activation levels. Then, by using equations (1)-(3) we obtained the muscle forces based on the muscle state, retrieved from the coordinates of the recorded motion. The motion was recorded by means of 12 Optitrack V100-R2 IR cameras that register the position of reflective markers placed according figure 2b. In order to solve the dynamic problem, we also recorded the ground reaction forces from a force plate Bertec FP4060-10-2000. In order to compare signals, and considering the gait as a relatively slow movement to reduce the influence of muscle contraction velocity, we used the same activation level as input for all the muscles and we studied the differences between lengths. Certainly, the obtained forces do not correspond to the ones expected during gait, nevertheless the results can be compared in terms of similar contraction velocity and activation levels, being muscle length the variable term.

Regarding the second experiment, in order to quantify the influence of PCSA, we solved the distribution of a given set of joint torques in the lower limbs between a number of muscles (figure 1b). To do so, we used the expressions presented in section 2.2. As the torque distribution provides the muscle force for the muscles acting at each joint based on the muscle stress, the results are directly comparable. Nevertheless, we solved the same problem for different values of muscle stress in order to also compare variations for the same muscle (this could be comparable to study the differences in muscle force between marathon runners and sprinters that develop the same conditions during motion, except for the value of PCSA). The data set for net joint torques was retrieved from [15], as it has been widely used in the literature.
3. Results and discussion

Regarding the results of the FD analysis (figure 3) a clear relationship between muscle force and muscle length cannot be stated. For example, a slenderer muscle such as the tibialis anterior produces less force than a wider muscle, as, i.e., the gastrocnemius. This may be due to the tridimensional structure of muscle tissue that has not been considered in this study. Furthermore, the viscoelastic properties of the tendon, that complete the muscle structure, were not considered as they do not contribute to the force production. Nevertheless, its inclusion may modify the results in terms of analysis as they may modify the slenderness of the muscle (see figure 1, the gastrocnemius has a large portion of tendon that may modify the relationship between length and PCSA).

On the contrary, for the results of the ID problem (figure 3), it is shown a clear relationship between muscle force and PCSA. The wider the muscle is, the larger force is obtained. For a smaller PCSA the muscle stress is lower, and therefore the expected force production. A wider muscle supports larger stress. Results below 100% represent not only a thinner muscle, but also a proportion of the muscle that has been activated, that is directly related with the recruitment process of muscle fibres to exert a certain amount of force. Results over 100% represent an increment in the cross-sectional area of the muscle tissue.
Figure 3. Forward dynamic solution. Muscle forces for the main actuator groups of the lower limbs. Top: red line: hamstrings; green line: gluteus maximus; blue line: medial gastrocnemius; black line: soleus. Bottom: red line: psoas; green line: rectus femoris; blue line: tibialis anterior; black line: vasti.

Figure 4. Inverse dynamic solution. Muscle forces for the main actuator groups of the lower limbs for different values of the PCSA. Top: red dashed line: 25% PCSA; green line: 50% PCSA; blue line: 75% PCSA; black line: 100% PCSA, cyan line: 125% PCSA; violet dotted line: 150% PCSA; orange dashed line: 175% PCSA.

Table 1. Fundamental parameters of the muscle [11, 12, 15]

| Parameter | Value |
|-----------|-------|
| $L_{opt}^{CE}$ | (cm) |
| PCSA | ($cm^2$) |
| $F_0^M$ | (N) |
| $L_{opt}^{CE} / PCSA$ | ($cm^{-1}$) |
Psoas  |  10.2 |  7.9 |  821 |  1,291 
RF    |  8.1 | 13.9 |  663 |   0.582 
Glu   |  20  | 30.4 | 1705 |   0.657 
Ham   | 10.4 |  4.9 | 1770 |   2.122 
Vas   |  9.3 | 16.8 |  7403|  0.553  
Gas   |  5.5 | 21.4 |  639 |  0.257  
TA    |  8.2 | 11.0 |  528 |  0.745  
Sol   |  5.5 | 58.8 | 3883 |  0.093  

4. Conclusions

In this work the influence between muscle length and muscle section in force production for the traditional problems in Biomechanics has been presented. Although this is a preliminary work, the study must be completed by including the 3D structure of muscle tissue and the effects of the tendon and the surrounding viscoelastic tissue. Furthermore, a simple non-linear relationship between EMG signal and muscle activation has been used to overcome the problematic of the activation level (a(t)). The way in which muscle fibers are recruited it is not studied here, nevertheless it must be considered in future studies as it may influence the results in terms of PCSA.

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