Reliability Analysis of the Air Transportation Network when Blocking Nodes and/or Connections Based on the Methods of Percolation Theory

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Received xxxxxx
Accepted for publication xxxxxx
Published xxxxxx

Abstract

The paper shows that to study the reliability and fault tolerance of air transportation networks, methods of percolation theory can be used, in which any aviation transport structure can be represented as a random non-planar, incompletely connected graph (nodes are airports, arcs are airlines). In the theory of percolation, one can consider the solution of the problems of finding the shares of blocked nodes and blocked connections for networks with various random and regular structures, in which they decompose into unconnected areas. The share of blocked nodes (in the node problem) or connections (in the connection problem), at which the conductivity between two arbitrarily selected network nodes disappears, is called the percolation (flow) threshold. For the same structure, the values of percolation thresholds for the bond problem and the node problem have different meanings. The percolation threshold value depends on the average number of connections per network node (density), and is a criterion for its reliability, i.e. determines the percentage of blocked nodes and/or communications that the network will lose the necessary level of performance ability. The dependence of the blocking threshold (percolation) on the network connection density can be expressed mathematically. Using a map of a real aviation transport network, it is possible to determine the average number of connections per one node and then calculate the threshold value of its predetermined reliability value. If the reliability threshold needs to be increased, then the necessary number of additional links can be calculated.

Keywords: aviation transport network, network connection density, network percolation threshold, increasing reliability, increasing throughput

1. Introduction

Recently, to the study of flow balancing and ensuring the smooth operation of transport systems much attention has been paid. It should be noted that a significant number of works in this area are devoted to land transport [1–14] and there are few studies in the field of air transportation. First, the problem is that the number of vehicles is constantly growing, and the development of road infrastructure is late. As a result, the applied control models become obsolete and fail. Thus, the
need arises either to search for new management tools, or to modernize the physical basis of the existing transport network.

For aviation, there is also the problem of increasing traffic with limited bandwidth available airports. All this also entails the need to develop models and methods for controlling and balancing air traffic flows, considering the fact that the infrastructure of air systems is significantly different from ground-based ones. In a generalized sense, it is necessary to solve the dynamic problem of the redistribution of flows (traffic) in transport networks, considering their topology. Moreover, if ground networks can be reconstructed due to relatively small investments (construction of flyovers and interchanges), then modernization of aviation infrastructure requires significant costs (expansion of existing and construction of new airports), compared to ground systems. Therefore, the main instrument for solving the problem of ensuring the smooth operation of airlines can only be changing command models.

Control and balancing of flows in transport networks is one of the most important problems of ensuring uninterrupted aviation traffic. Violations in the work lead to disruptions in the transportation of passengers and goods, which can be considered as a decrease in reliability, leading to significant financial losses.

It should be noted that local control, which is based on the statistically estimated characteristics of traffic flows, is not applicable in this case, because the result is provided by obtaining an estimate of the efficiency of the functioning of traffic flows at one transport hub, excluding neighboring ones. Guidance system is needed that will allow for optimizing the functioning of traffic flows in an area that includes many transport nodes. A change in control actions on one transport node inevitably causes a change in the characteristics of traffic flows on neighboring ones.

The bulk of all ongoing studies of transport traffic, its analysis and development of control models is aimed at solving emerging problems at the local level, without considering the entire transport network.

In our opinion, it is necessary to use a broader approach that considers the transport properties of the entire network. Ensuring the smooth operation of aviation transport networks is possible based on their analysis using the methods of percolation theory, which allows us to evaluate the conductivity of the entire network using the probabilities of blocking individual nodes or links.

2. Problem Statement

The transport network of modern aviation companies can have a very large, complex and branched structure (see, for example, Figure 1, which shows the United Airlines airline communications diagram taken from the resource: https://happynewyear2018cards.com/united-airlines-flight-map/united-airlines-flight-map-stunning-us-airways-flight-map-europe-reference-united-airlines-route-map/, which can be represented as a random non-planar, incompletely connected graph (nodes — airports, and ribs — transport lines, see Figure 2).
Not the planarity and incompleteness of connectivity of the air transportation networks is obvious, but the randomness of their graph is not in the randomness and randomness of the movement of airplanes between airports, but in the fact that an arbitrarily chosen node can have a different number of connections (central nodes (for example, Chicago, San Francisco or Denver there are significantly more than the rest).

**Figure 2.** Representation of the air traffic network in the form of a random network structure.

When simulating air traffic, it is necessary to consider the dynamics of changes in air traffic congestion (daily change in the flow rate) and the fact that all elements of the transport network graph (nodes and edges) have different characteristics (throughput).

If we analyse the percolation model of the transport aviation network, we can ensure the necessary flow balancing and reliability of the entire structure, even though its individual elements can be blocked due to overload or denial of service. In this case, by ensuring operability and reliability, we mean that between any two arbitrary network nodes (not necessarily central ones) there is at least one free path from the unblocked elements of the graph.

In the theory of percolation (probability theory on graphs), the solution of the node problem and the connection problem [15–18] is studied for networks with different, both regular and random structures. When solving the problem of bonds, they determine the proportion of bonds that must be broken so that the network breaks up into at least two unconnected parts (or vice versa, the fraction of conductive bonds when conductivity arises). In the problem of nodes, the fraction of blocked nodes is determined, in which the network breaks up into unconnected clusters within which connections are preserved (or vice versa, the fraction of conductive nodes when conduction occurs). The fraction of non-blocked nodes (in the node problem) or unbroken bonds (in the connection problem), at which conductivity arises between two arbitrarily selected network nodes, is called the percolation (leakage) threshold. For the same structure, the values of percolation thresholds for the bond problem and the node problem have different meanings. Note that when a node is blocked, all its connections are blocked, while when a connection is blocked, only one connection between neighbouring nodes.

Using the concept of shares of blocked nodes or links is equivalent to the concept of the probability of a randomly selected node (or link) being in a blocked state. Therefore, it can be accepted that the percolation threshold value determines the probability of passing through the entire network as a whole if some part of its nodes (or links) is blocked (excluded), i.e. the average probability of blocking a single node (or link) is given.

Percolation loss can be defined as the proportion of blocked network nodes, in which the entire network loses its conductivity.

Reaching the percolation threshold in the network corresponds to a cluster in which there are connections between any of its arbitrary nodes. A so-called endless or contracting conducting cluster is formed.

For structures of finite size, conductivity can occur at different proportions of conductive nodes (or bonds, see Figure 3). However, if the network size (the number of nodes L) is directed to infinity, then the region of transition to the conducting state becomes compact (see Figure 3, curve I for a small-sized structure, II is an infinite network).

It should be noted that terrestrial transport structures can be represented as random, almost planar, incompletely connected graphs, while air ones do not have planarity. It is important that the percolation thresholds of planar and non-planar networks at the same connection density differ significantly [19–22].

**Figure 3.** The probability of occurrence of percolation, depending on the magnitude of the fraction of conductive nodes (or bonds).

For structures of finite size, the percolation threshold $\xi_c(L)$ can be determined from a given value of the probability of the entire network transitioning to the conducting state. In Figure 3, this probability is chosen equal to 0.5.

However, the required value of the reliability value of the entire network as a whole may be set, for example, the value...
of 0.95 or 0.99 (then the percolation threshold will correspond to the specified criterion for the reliability of the network), i.e. it is possible to determine at what proportion of blocked nodes and/or communications the network as a whole will lose the necessary level of reliability.

It should be noted that in the analysis of reliability, central nodes or links can be assigned increasing weighting factors, thereby increasing their contribution to ensuring the overall reliability of the network.

3. Percolation properties of random non-planar, incompletely connected networks

The main problem in the study of the percolation properties of network structures having a random structure is that at present there are no general analytical methods for this, and their study is possible mainly only with the use of computer modelling methods.

3.1 Network percolation threshold calculation algorithm

To determine the percolation threshold of networks, the following algorithm can be used [19]:

1. First, build a random non-planar, incompletely connected network. Next, randomly select two network nodes A and B, considering the limitation that there must be at least one intermediate node between them.

2. Set the probability of blocking a single node (in the task of nodes) or communication (for the problem of communications) and randomly block the proportion of network nodes (or communications) equal to this probability.

3. Check the presence of at least one “free” path (path from not excluded nodes or links) from node A to node B. If there is no free path (the number of “free” paths is 0), write 0. Otherwise, write 1.

4. Increase the value of the probability of blocking a single node (in the task of nodes) or communication (for the problem of communication) by a certain value. Then randomly block the share of network nodes (or links) equal to a given probability value. Next, determine which specific network nodes were excluded.

5. Return to step No. 3 until all nodes of the network are enumerated.

6. Return to point No. 2 and perform points No. 3 – No. 5 — Q times (for example, several hundred times). From the first to the last steps (in cases where the entire network is blocked), for all experiments. Find the number of realizations for which at least one “free” path was found (we call this number n). For example, at h = 18 step in 8, 12, 19, 56, 58, 76, 80 and 89 experiments from Q there was at least one “free” path, then the number n (5) = 18 (8 is the total number of “free” ways). For each step, we find \( \mathcal{P}(h) = n(h)/Q \), where \( h \) is the step number. Calculate the average cluster size of excluded nodes, the number of such clusters, etc. (for all \( N \) experiments at each step). The average cluster size can be defined as the ratio of the sum of all the average values obtained at this clusterization step (for all \( Q \) experiments) to the total number of \( Q \) experiments. For an explanation, consider an example. Suppose that at \( h = 6 \) step in the 1st experiment, 4 clusters having a size of 15 nodes each were obtained, in the 2nd — 3 clusters, in the 3rd — 2, etc. in the 100th — 20. Then the average number of clusters having a size of 10 blocked nodes will be: \((4 + 3 + 2 + 5)/100\).

7. Next, return to the implementation of paragraph No. 1, and repeat the execution of steps No. 2 – 6 another \( W \) times. For each of the \( W \) experiments, we find the quantity \( \mathcal{P}(h) = n(h)/Q \). The \( W \) index determines which of the \( W \) tests we are considering.

8. After the simulation is completed, for each of the \( h \) steps find the quantity \( \mathcal{P}(h) = n(h)/W \) — is the average value of the probability of passing through the network in general, for non-blocked nodes (or links for the task of blocking links) at each of the steps (taking into account various possible path configurations).

9. Find the magnitude of the percolation loss, which is equal to the fraction of blocked nodes (or connections) at which the network conductivity disappears (calculated by the formula: unit minus the share of conductive nodes, in the problem of nodes (or connections, in the connection problem)).

Performing calculations using this algorithm allows you to obtain an array of data for the dependence of the average value of the probability of passing through the network \( \mathcal{P}(h) \) as a whole on the share of blocked network nodes (or connections for the task of blocking connections), with a different average number of connections per node (density network).

Determination of the dependence of percolation thresholds of networks on their density (average number of links per node).

The results of numerical modelling and calculation of percolation thresholds for random non-planar, incompletely connected networks with different average number of links at each node for tasks of blocking nodes and links are presented in Table 1 [20–22]. In this case, to determine the percolation threshold, a reliability value of 0.5 (50%) was chosen. However, we note again that you can set any desired value of the reliability value of the entire air traffic network, for example, the value of 0.95 or 0.99 and determine the percolation threshold for a given value of reliability.

The data presented in Table 1 are linearized well in the coordinates: \( \ln P(x) \) is the natural logarithm of the percolation threshold depending on \( z = 1/x \) (the reciprocal of
the average number of bonds $x$ (network density) per node, see Fig. 4. The points in Figure 4 indicate the experimental data, and the solid lines correspond to linear equations.

In the task of blocking nodes, the dependence of the natural logarithm of the percolation threshold $\ln P(x)$ on the reciprocal of the network density $\frac{1}{x}$ can be described by the equation:

$$\ln P_{\text{node,unreg}}(x) = -\frac{1.70}{x} + 0.04$$  \hspace{1cm} (1)

with the value of the correlation coefficient of numerical data and the equation of linear dependence equal to 0.99 (see line 2 in Figure 4).

Fig. 4. The dependence of the logarithm of the percolation threshold in a random non-planar, incompletely connected network on the reciprocal of the average number of connections (network density) per node (line 1 for the problem of blocking nodes and line 2 for the task of blocking connections).

Discussion of the results

Equations (1) and (2) make it possible to evaluate fault tolerance and compare random non-planar, incompletely connected networks with different connection densities, with a given value of reliability. In this case, these equations make it possible to give an estimate of operability for a value of the reliability level of 0.5 (50%), which is very small. However, can be set any desired value of the level of reliability of operability, for example, 0.95; 0.99 or 0.999. Then it is...
possible to build random non-planar, incompletely connected networks with different connection densities, and using the algorithm described above, calculate the values of their percolation thresholds at a selected level of reliability. Further, using the results obtained, it is possible to linearize the data and determine the corresponding linear dependences like equations (1) and (2).

Using equations (1) and (2), compare the reliability of air transportation of any randomly selected sufficiently large airlines (for example, the American company United Airlines and the Indian Jet Airways).

The United Airlines fleet consists of 787 aircraft (for 2019: http://www.airlines.id1945.com/1usr/united-ru.htm) and it is one of the four largest US airlines (in terms of fleet size it is in third place in the world, by the number of passengers carried in the fourth (at the end of 2017)). Jet Airways (data taken from the resource: https://india.ru/jetairways/jetairways-map.html) is the second largest airline in India (now its fleet has about 120 aircraft: https://www.skyscanner.ru/airline/airline-jet-airways-9w.html).

The United Airlines airline communications network, the diagram of which is shown in Figure 1, has a link density per node of about 7.4, and the graph of the Jet Airways airline network (see Figure 5) is about 3.2 (there are many hanging peaks).

Using equation (1), for the United Airlines aviation network in the nodes problem, the percolation threshold value (with a reliability level of 0.5) is 0.827, i.e. with a share of blocked nodes of 0.827 with a reliability level of 0.5 (50%), the airline’s aviation network will lose its functionality. For the aviation network of Jet Airways, we get — 0.099. Thus, the reliability of United Airlines’ operability in the communications task is also greater than that of Jet Airways by 3.32 times (0.329/0.099 = 3.32).

With an increase in the reliability level of performance capability to 99.99%, the share of blocked nodes or links at which the percolation threshold of this reliability level is significantly reduced. However, the general tendency is that the higher the network density, the higher the percolation threshold for loss of reliability is maintained.

4. Conclusion

To study the reliability and fault tolerance of aviation transport networks, one can use the methods of percolation theory, in which any transport structure can be represented as a random non-planar, incompletely connected graph.

In the theory of percolation, one can consider the solution of the problem of blocking nodes and the problem of blocking connections for networks with different structures. For the same structure, the values of percolation thresholds in solving the connection problem and the node problem have different meanings. The percolation threshold value depends on the average number of connections per network node (density), and is a criterion for its reliability, i.e. determines the percentage of blocked nodes and/or communications that the network as a whole will lose the necessary level of performance for a given value of reliability.

The dependence of the blocking threshold (percolation) on the network connection density can be expressed mathematically. This allows using the map of a real transport network to determine the average number of connections per one node and then calculate the value of its blocking threshold at a given level of reliability. If the blocking threshold needs to be increased, then the necessary number of additional links can be calculated.

5. Appreciation

The results presented in this paper were obtained within the programme of choosing scientific projects performed by scientific laboratories teams, according to the letter of the Ministry of Education and Science of Russia, 3 July 2019 No. MH-1037/AM. Project 2019-1393 “Developing new models for analysing and forecasting the dynamics of stochastic processes in sophisticated systems considering self-organisation and the presence of memory”.

Figure 5. The scheme of aviation routes of the Jet Airways company.
References

[1] Briani M and Cristiani E 2014 Networks and Heterogeneous Media (NHM) 3 519–52
[2] Hui M, Bai L, Li Y and Wu Q 2015 Mathematical Problems in Engineering 20–7
[3] Ahn G-H, Ki Y-K and Kim E-J 2014 IET Intelligent Transport Systems 2 89–95
[4] Poole A and Kotsialos A 2015 Swarm intelligence algorithms for macroscopic traffic flow model validation with automatic assignment of fundamental diagrams (School of Engineering and Computing Sciences: Durham University)
[5] Guo J, Chen F and Xu C 2017 Mathematical Problems in Engineering 125–35
[6] Jiang R, Jin C and Zhang H 2017 Transportation Research Procedia 23 157–73
[7] Danchuk V, Bakulich O and Svatko V 2017 Procedia Engineering 187 425–34
[8] Pun L S C, Albert C and Chan W F 2016 Travel Behaviour and Society 3 71–7
[9] Baranovskaya T P and Pavlov D A 2016 Polythematic network electronic scientific journal of Kuban state agrarian University 120 1686–705
[10] Pavlenko P F 2014 Problemy avtomatiki i upravleniya 27 92–7
[11] Trubitsyn V A and Golub D I 2013 Bulletin of the North Caucasus Federal University 2 89–92
[12] Vlasov A A and Chushkina Z A 2014 Regional architecture and construction 4 152–6
[13] Zyryanov V V 2013 Energy and resource saving: industry and transport 21 71–4
[14] Filippova D M, Chernyag A B and Slobodchikova N A 2013 Bulletin of Irkutsk state technical University 9 172–6
[15] Grimmet G 1999 Percolation (Berlin: Springer-Verlag)
[16] Sahimi M 1992 Applications of Percolation Theory (London: Tailor & Francis)
[17] Stauffer D and Aharony A 1992 Introduction to Percolation Theory (London: Tailor & Francis)
[18] Feder J 1988 Fractals (Plenum Press: New York, London)
[19] Zhukov D O, Andrianova E G and Lesko S A 2019 Symmetry 11 920
[20] Zhukov D, Khvatova T, Lesko S and Zaltsman A 2018 Technological Forecasting and Social Change 129 297–307
[21] Zhukov D O, Khvatova T Yu, Lesko S A and Zaltsman A D 2018 Informatics and its applications 2 90–7
[22] Khvatova T Yu, Zaltsman A D and Zhukov D O 2017 Information processes in social networks: Percolation and stochastic dynamics. CEUR Workshop Proceedings 2nd International Scientific Conference "Convergent Cognitive Information Technologies", Convergent 2017 2064 pp 277–88