Nonmachine energy separation in channel with permeable walls

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Abstract. New type of nonmachine energy separation method was considered. This method is based on the suction of “cold” portion of compressible boundary layer in channel with permeable walls. Numerical model of energy separation device (converged nozzle and permeable walls channel) was developed. The model was based on the Reynolds averaged Navier-Stokes (RANS) equations with additional equations of the turbulence model. Axisymmetrical approach was used for the analysis.

The quantitative measure of the energy separation (temperature separation) can be defined as a difference between the total mass-average temperatures of the flow at the “hot” and “cold” outputs and at the input of the device.

Validation of the developed model was conducted against of available measurement data. Parametric study was provided by using the developed model. A wide range of flow regimes was analyzed: from an impermeable wall to an asymptotic suction. The influence of the initial Mach number as well as the molecular Prandtl number on the energy separation is shown.

1. Introduction
Energy separation is a rearrangement of the total energy (total temperature) of the flow under adiabatic conditions and without the mechanical work exchange (nonmachine). As a result, cold and hot regions appear in the flow. According to Eckert [1] classification, there are two types of energy separation. The first one is primarily caused by pressure forces acting on the fluctuating curved streamlines with a minor contribution from viscous forces. Main examples of such energy separation type are Ranque-Hilsch vortex tube [2] and Sprenger resonance tube [3]. The second type of energy separation is caused by the action of heat conduction and viscous stress. The present paper is devoted to investigation of this energy separation type.

As it is well known [4–6] the total temperature distribution in compressible boundary layer looks like the following (see figure 1). As it can be seen from the figure, the temperature at the wall is lower than temperature of free flow. This temperature is called ”recovery temperature” or ”adiabatic wall temperature” and can be obtained by following equation:

$$T^{\ast}_{aw} = T^{\ast}_{\infty} \frac{1 + r \frac{k-1}{2} M^2}{1 + \frac{k-1}{2} M^2} \quad ,$$  
(1)

where $T^{\ast}_{\infty}$ is total temperature of free flow, $r$ is temperature recovery factor, $k$ is heat capacity ratio, $M$ is Mach number.
The recovery factor is function of molecular Prandtl number mainly. For gases $Pr \leq 1$ (for air $Pr = 0.71$) and therefore $r \leq 1$.

Moreover, there is a maximum of total temperature at certain distance from the wall. The main idea of the investigated energy separation method consists in a removing of "cold" layers from the near wall flow. One of the way to realize such method is to use permeable walls. In this case "cold" layers will be removed from the flow but the rest part of the flow will have higher total temperature. In other words, the energy separation of the flow will take place.

![Figure 1. Boundary layer total temperature distribution](image)

2. Problem formulation
Let us consider the device which was experimentally studied as reported in [7]. The device consists of converged nozzle (see figure 2) and cylindrical porous tube with permeable walls.

![Figure 2. Scheme of the investigated device](image)

Inner diameter of porous tube is $d_{in} = 3.5 \text{ mm}$, outer diameter is $d_{out} = 10.5 \text{ mm}$, length of the tube is $L = 150 \text{ mm}$. It should be noted that critical diameter of the converged nozzle is $d_{cr} = 3.2 \text{ mm}$, so that there was a step $\Delta = 0.15 \text{ mm}$ between nozzle and tube.

3. Numerical model
Axisymmetrical approach was used for the modelling. The discretization of the Reynolds averaged Navier-Stokes equations (RANS), the energy equation and equations of the
corresponding turbulence model was performed on the basis of the control volume method. Two-equations $k - \omega$ differential turbulence model was used to close the main system of equations. The problem was solved by using of ANSYS Fluent.

Air and low Prandtl mixtures were considered as a fluid. Moreover, all fluids were treated as a compressible ideal gas, obeying the Mendelev-Clapeyron equation. The thermodynamic and transport properties were determined according to [8].

Total pressure and total temperature were applied at inlet boundary. Inlet total pressure was varied in a wide range. Static pressure equal the atmospheric pressure was applied at outlet.

The permeable wall was not modeled explicitly. The mass flux and heat flux were applied at the internal cylindrical surface of the porous tube. The value of mass flux was obtained on the basis of the Darcy-Forchheimer equation (see figure 3):

$$\frac{p_{amb}^2 - p_{in}^2}{2\Delta rRT} = \alpha \mu \frac{r_{in}}{\Delta r} \ln \left( \frac{r_{out}}{r_{in}} \right) j_w + \beta \frac{r_{in}}{r_{out}} j_w^2,$$

(2)

where $p_{amb}$ is ambient pressure, $p_{in}$ is pressure at inner surface of porous tube (permeable wall), $r_{in}$ is inner radius of porous tube, $r_{out}$ is outer radius of porous tube, $\Delta r = r_{out} - r_{in}$, $R$ is gas constant, $\mu$ is viscosity, $\alpha$ is viscous coefficient, $\beta$ is inertial coefficient, $j_w = \rho_w u_w$ is mass flux.

The values of the viscous $\alpha$ and inertial $\beta$ coefficients were determined on the basis of the experimental data [7].

Figure 3. Massflow characteristic of porous tube

4. Model Validation
Static pressure distribution along the length of the porous tube was measured in [7], as well as the local temperature of the outer surface of the porous tube.

Figure 4 shows a comparison of the measured and calculated static pressure distribution, as well as the outer surface temperature distribution along the porous tube. As it can be seen from the figure, the model demonstrate a good match to experimental data. A difference is observed at the beginning of porous tube for $x/d_h \leq 5$. It can be explained by the presence of shock waves generated by step between nozzle and tube (see figure 5). As it was mentioned above, in numerical model the porous tube was not modeled explicitly. Therefore, peaks of the wall temperature was observed, while in the experiment the influence of shock waves on the wall temperature was smoothed by thermal conductivity of the porous tube.
Figure 4. Distribution of static pressure (left) and outer wall temperature (right) along the porous tube. $P_0^* = 4.0$ atm. Solid curves are numerical results, dashed lines are ambient conditions, symbols are experimental data [7]

Figure 5. Numerical Schlieren

The quantitative measure of the energy separation (temperature separation) can be defined as a difference between the total mass-average temperatures of the flow at the "hot" and "cold" outputs and at the input of the device. In this regards the total temperatures for both outputs where measured in [7].

The calculations results compared against the experimental data are shown in figure 6. The developed model demonstrate a good agreement with experiment. Thus, the model adequately describes the physical processes taking place in the device and can be used for the further research.

5. Parametric study

In the study the calculations were performed for several values of the total pressure at the nozzle inlet, namely, $P_0^* = 1.01 - 100.0$ atm. The results of the simulation are shown in figure 7 as a difference in the total temperatures at the outlets and inlet of the permeable tube versus massflow ratio $m_w/m_0$, where $m_w$ is massflow though permeable wall, $m_0$ is inlet massflow. As it can be seen from the figure, there are three representative regimes. The first regime corresponds to minimum total pressure at inlet ($P_0^* \rightarrow 1$ atm and consequently $m_w/m_0 \rightarrow 0$, see figure 7), for this condition the energy separation effect is nearly zero. The second regime is extremum: maximum of $\Delta T_h^*$ and correspondingly minimum of $\Delta T_c^*$. Note that locations of extremum are
Figure 6. Comparison of the quantitative measure of the energy separation effect. Solid curves are numerical results, symbols are experimental data [7]

different for hot and cold flows. And, finally, the third regime corresponds to maximum of inlet total pressure (maximum value of the massflow though permeable wall). In this case, the inlet total pressure is so large that it leads to an asymptotic suction and energy separation effect tends to zero.

Figure 7. The quantitative measure of the energy separation effect. Molecular Prandtl number influence

Moreover, figure 7 shows the influence of the molecular Prandtl number on the value of energy separation. As it was noted above, the recovery factor is a strong function of the molecular Prandtl number. As can be seen from the figure, usage of $H_2-Xe$ mixture can lead to the increase of the effect up to three times.

The second way of the energy separation effect increase is to increase the inlet Mach number. It can be achieved by using different supersonic nozzles. Figure 8 shows results of calculation for
two different nozzles: converged $M = 1.0$ and converged-diverged $M = 1.35$. As it can be seen from figure 8, the value of energy separation effect is higher for increased inlet Mach number.

![Diagram](image.png)

**Figure 8.** The quantitative measure of the energy separation effect. Inlet Mach number influence

6. Conclusions

Based on the results of the study the following conclusions can be drawn:

- For the considered configuration of the device, the effect of the nonmachine energy separation is observed;
- in the wide range of the device operation parameters ($P_0^* = var$) three representative regimes can be highlighted;
- the use of low Prandtl mixtures leads to significant increase of the energy separation effect;
- the increase of inlet Mach number also leads to the increase of the energy separation effect.

Acknowledgments

The author would like to thank Ph.D. A. G. Zditovets for his comments and for a number of suggestions. He would also like to thank Ph.D. N. A. Kiselev for help with the preparation of the paper.

The research was supported by RSF (project No. 14-19-00699).

References

[1] Eckert E R G 1987 *Wärme- und Stoffübertragung* 21 73–81
[2] Ranque G J 1933 *J. Phys. Radium* 4 112–114
[3] Sprenger H S 1954 *Mitteilungen aus dem Institut für Aerodynamik ETH* 21 18–35
[4] Spivack H 1950 *Experiments in the turbulent boundary layer of a supersonic flow* Tech. Rep. CM-615 (AL-1052)
[5] Lobb R K, Winkler E M and Persh J 1955 *Journal of Aeronautical Sciences* 22 1–9
[6] Mabey D G, Meier H U and Sawyer W G 1974 *Experimental and theoretical studies of the boundary layer on a flat plate at Mach numbers from 2.5 to 4.5* Tech. Rep. 3784
[7] Zditovets A G, Leontiev A I, Kiselev N A, Vinogradov Y A and Strongin M M 2018 *Proceedings of the 16th International Heat Transfer Conference, IHTC-16*, Beijing, China IHTC16-21878
[8] Bell I H, Wronski J, Quoilin S and Lemort V 2014 *Industrial & Engineering Chemistry Research* 53 2498–2508 (*Preprint* http://pubs.acs.org/doi/pdf/10.1021/ie4033999)