Comment on 'Photoproduction of \( \eta \)-Mesic \(^3\)He'

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In a recent paper by the TAPS collaboration \[1\] a first measurement of a bound system of an \( \eta \) meson and a \(^3\)He nucleus was reported. In this comment we critically reexamine the interpretation of the data and show that the data prefers a solution where there is no bound state present. Given the low statistics of the measurement, however, it does not exclude the existence of a bound state.

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The interaction of \( \eta \) mesons with nucleons is strong and attractive due mainly to the presence of the \( S_{11}(1535) \) resonance that strongly couples to this system. Consequently it is expected that the \( \eta \) meson should be bound in sufficiently heavy nuclei. So far, however, it is unclear what mass number is sufficient. Some authors predicted a bound state to occur on nuclei as light as \(^3\)He \[2, 3, 4, 5, 6, 7\], whereas others expect binding only for heavier nuclei \[8, 9, 10, 11\]. Until recently no direct experimental evidence for the existence of \( \eta \)-mesic nuclei was available. Only the presence of a strong \( \eta \)-nucleus interaction was seen experimentally in strong final state interaction effects in reactions like \( pn \rightarrow \eta d \[12\], \( pd \rightarrow \eta ^3\text{He} \[13, 14\], and \( dd \rightarrow \eta ^4\text{He} \[15\).

Thus it was a big step forward from the experimental side when this year the TAPS collaboration reported positive evidence for \( \eta \)-mesic \(^3\)He. Besides a strong deviation in the angular shape of \( \gamma ^3\text{He} \rightarrow \eta ^3\text{He} \) from the expectation for quasi–free production (the cross section is flat instead of forward peaked), a structure was observed in the cross section \( \gamma ^3\text{He} \rightarrow \pi^0 pX \) just below the \( \eta \) production threshold. These signatures were taken as strong evidence for the existence of \( \eta \)-mesic \(^3\)He.

It should be clear that the former evidence—a flat \( \eta ^3\text{He} \) angular distribution in the close–to–threshold regime is a hint solely for a strong \( s \)-wave \( \eta ^3\text{He} \) interaction that leads to a relative suppression of the impulse term with respect to the \( s \)-wave multiple scattering terms. Thus, given what we already know about the strong \( \eta ^3\text{He} \) interaction, a flat angular distribution in the close–to–threshold regime should be expected. A closer look at the structure in the cross section \( \gamma ^3\text{He} \rightarrow \pi^0 pX \) is the focus of this comment.

The structure reported by the TAPS collaboration was fitted with a Breit–Wigner function. In its non–relativistic form the scattering amplitude then is

\[
f_{BW} \propto \left( E - E_R + \frac{i}{2} \Gamma \right)^{-1}, \tag{1}\]

where \( \Gamma \) is assumed to be constant. The parameters deduced were \((4.4 \pm 4.1) \text{ MeV} \) and \((25.6 \pm 6.1) \text{ MeV} \) for the binding energy and the width, respectively. However, since the position of the signal coincides with the \( \eta \) production threshold, one might wonder whether this is more a cusp than the signal of a bound state.

Already in 1976, when studying the light scalar mesons \( a_0 \) and \( f_0 \), Flatté observed that in the presence of thresholds the Breit–Wigner form of Eq. \(\text{(1)}\) is to be modified to include the momentum–dependence of the elastic width \[16\] (for a more recent discussion of threshold effects in various systems we refer to Ref. \[17\]). Thus Eq. \(\text{(1)}\) should be changed according to

\[
\Gamma \rightarrow \Gamma_{\text{inel}} + \Gamma_{\text{el}},
\]

where \( \Gamma_{\text{inel}} \) and \( \Gamma_{\text{el}} \) denote the inelastic and the elastic width (in our case with respect to the \( \eta ^3\text{He} \) channel) of the resonance respectively. Here \( \Gamma_{\text{inel}} \) can be assumed constant; however, \( \Gamma_{\text{el}} \) has to vanish at the elastic thresh-
old! Thus, for an $s$–wave structure, one gets

$$\Gamma_{el} = g_{eff}/k,$$

where $k$ denotes the momentum of the $\eta$ relative to the $^3$He nucleus and, above the production threshold, may be written as $k = \sqrt{2\mu E}$. Here $\mu$ denotes the reduced mass of the $\eta$He system and $E$ is its kinetic energy. In the region below threshold, however, $k = i\sqrt{2\mu E}$. Thus, we find that if a structure is predominantly inelastic, a Breit–Wigner might still be a good approximation, even in the proximity of a threshold; however, if a structure is predominantly elastic, using a Breit–Wigner is not justified.

A dynamically generated singularity, like a bound state, also dominates the final state interaction in the $\eta$He system. In addition, if a production reaction is short–ranged (typical momentum transfer significantly larger than any other scale of the problem) the final state interaction is universal (independent of the reaction) and can be related to the elastic scattering of the outgoing particles [18] which reads in the effective range approximation

$$f_{sc} \propto \left(1/a + r k^2 / 2 - ik\right)^{-1}. \quad (2)$$

Recently the world data set on the reaction $pd \rightarrow \eta^3$He was analyzed [19]. This study led to quite constrained values for the real and imaginary part of the $\eta^3$He scattering length, namely

$$a = (\pm 4.3 \pm 0.3, 0.5 \pm 0.5) \text{ fm}, \quad (3)$$

where the first number refers to the real part and the second number to the imaginary part—in the analysis the effective range term of Eq. (2) was neglected ($r = 0$). Note, these numbers where found from a fit to the world data set. However, this dataset is inconsistent and if we use only the newest data in the fit the scattering length is less constrained; see Ref. [19] for details. We come back to this point below. The two signs given in front of the real part indicate that $\eta$ production data can not fix the sign of the real part. A positive sign would point at a virtual state (a singularity on the unphysical sheet), whereas a negative sign would point at the existence of a bound state. In Ref. [20] it was stressed that isospin–violating ratios of pion production cross sections taken in the vicinity of the $\eta$ production threshold should be a good tool to fix the sign of the real part. It is important to understand whether or not the TAPS measurement is sufficient to decide on the sign of the real part of the scattering length.

The Flatté form discussed above can be easily matched to the effective range approximation of Eq. (2) [21]. One thus finds that neglecting the effective range term in Eq. (2) is equivalent to assume that $g_{eff}$ is sufficiently large that in the region of interest $E$ can be neglected in Eq. (1). In Ref. [21] it was argued that this should be a good approximation if the structure of interest is dynamically generated and the singularity is close to the threshold, as is the case here. In addition, the role played by the effective range term in the $\eta^3$He final state interaction is completely unclear and the data for $pd \rightarrow \eta^3$He could be very well fitted using $r = 0$.

There is one additional comment necessary before we can apply Eq. (2) to the TAPS data: there is in principle some interference with the background. Thus, what was identified as the resonance signal might well have some contribution from an interference term, and the full signal may be written as

$$N \left(2 \text{Re}(B f^{res}) + |f^{res}|^2\right), \quad (4)$$

where $B$ is some complex number parameterizing that part of the background that is allowed to interfere with the resonance signal and $N$ is a measure of the total strength of the signal. Therefore, we performed three different fits: fit 1 included only the pure resonance signal ($B = 0$; only $N$ as a free parameter); fit 2 included only the interference term ($B \rightarrow \infty$; $N$ and the phase of $B$ as a free parameter); and fit 3 considered the full structure (thus here we have 3 free parameters: $N$, $|B|$ and the phase of $B$). As it turned out, the $\chi^2$ per degree of freedom for the two scenarios (positive and negative real part of the scattering length) was almost the same in all three cases and thus for illustration in Figs. 1 and 2 we only show the results of the first two fits, where the left panel corresponds to the results after binning in accordance with that of the experiment and the right panel corresponds to the unbinned results. To keep the numbers of free parameters low we choose $a = (\pm 4, 1)$
fm. In both figures the dashed line corresponds to a negative real part (indicating the existence of a bound state) and the solid line corresponds to a positive real part (indicating a virtual state). The fit gave a $\chi^2$ per degree of freedom of 1 for the latter case, whereas it was worse than 3 in the former. Thus the data prefers the solution that corresponds to a virtual state, although the existence of a bound state can not be excluded, given the quality of the data. Note, already in Ref. [22] the interpretation of the TAPS data as a bound state was questioned.

There is one important comment to be added: the data set for $pd \to \eta\mathbf{3}\text{He}$ shows some inconsistencies. As discussed in detail in Ref. [19], a fit to just the most recent data allows for a significantly broader band of scattering lengths: then even the case of a vanishing real part is not excluded (together with $\text{Im}(a)=3.5$ fm). To illustrate the impact of this scattering length in Figs. 1 and 2 we also show the corresponding results as the dotted curve. As can be seen, this fit is almost equally good as that with the positive scattering length ($\chi^2$ per degree of freedom of about 2).

Thus we conclude that the data on $\gamma\mathbf{3}\text{He} \to \pi p X$ recently measured by the TAPS collaboration does not allow for a conclusion on the existence of a bound system of $\eta$ and $\mathbf{3}\text{He}$. To improve the situation the measurement should be redone with improved statistics to allow for smaller energy bins. In addition, to permit an unambiguous interpretation of $\gamma\mathbf{3}\text{He} \to \pi p X$, more refined information on the $\eta\mathbf{3}\text{He}$ scattering length is needed. Fortunately, this will be available soon from measurements performed at COSY [23, 24, 25]. The present paper clearly shows the usefulness of a combined analysis of data from both electromagnetic and hadronic probes.

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