3D Finite Element Stress Analysis of Reinforced Double-Tapered Glulam Beams

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Abstract. The paper is dedicated to the stress analysis in reinforced double-tapered glulam beams within 3D computational model of orthotropic linear elasticity theory. Standard Eurocode design recommendations are the reference data to the finite element results. The results are presented for beams with a single and double line of reinforcement and compared with a non-reinforced beam. 3D elasticity model is an underestimation in comparison to the equivalent 2D model. Qualitative as well as quantitative analysis of the influence of reinforcements for the stress state of the beam is possible within a 3D finite element model. The proposed technique is especially recommended to estimate reduction of perpendicular to grain stress for a double line reinforcement in glulam beams.

1. Introduction
Analysis of double-tapered beams is a standard task while designing glulam structures [1]. Such beams are often used as roof girders [2]. According to the design rules, presented in paragraph 2 of the present paper, it is recommended to check normal stress perpendicular to the grain in the apex zone and to design appropriate reinforcement if such stress is too big. However, the distribution of this stress is approximated in the design standard and therefore, a proper verification and confirmation using a more detailed analysis is required.

A numerical analysis using the finite element method for two specific types of loading is presented in [3]. Wider parametric analyses taking into account geometrical (slope angle – apex height) and physical parameters (orthotropic parameters of timber) are presented in [4,5]. In the paper [6] a beam with a non-standard geometry and real loading was analysed. In [7] a model that takes into account behaviour of the reinforcement within 2D elasticity theory was proposed. The papers [3-7] justify the use of the finite element method for calculation of stresses in beams with orthotropic material properties.

The present paper is dedicated to the finite element stress analysis of real double-tapered glulam beam within 3D elasticity theory with the influence of orthotropic material properties. The main objective is to verify the validity and expedience of 3D model in the context of the 2D model as well as design codes. The qualitative and quantitative analysis is performed using the finite element method [8] in the computer system Abaqus [9]. A simple model of reinforcement is proposed. The reinforcement is applied in one and two lines.
2. Design of double tapered glulam beams

Simply supported beams are one of the most commonly used structural members in timber engineering. They can be tapered or have constant cross-section. Tapered beams are often used as roof elements, as their geometry naturally provides a desired roof inclination. Furthermore, the material of such beams is used with better efficiency, because the geometry tends to reflect the bending stress distribution under most load cases. In the present paper double-tapered glulam beams are considered, with a standard geometry described in Eurocode 5 [1] (tailor 1). In the design and analysis of such beams, apart from shear and bending stresses, tensile stress perpendicular to the wood fibres should be taken into account and reinforcement against this stress should be considered if needed.

Figure 1. Double-tapered beam considered in Eurocode 5 with apex zone.

According to the design standard [1] normal stress perpendicular to grain should be evaluated in the apex zone, which is marked in figure 1. The width of this zone is equal to the apex height. Eurocode 5 provides two alternative formulae for calculation of the stress perpendicular to grain, depending on the load type:

\[ \sigma_{t,90,d} = k_p \frac{6M_{ap,d}}{bh_{ap}^2} \]

\[ \sigma_{t,90,d} = k_p \frac{6M_{ap,d}}{bh_{ap}^2} - 0.6 \frac{P_d}{b} \]

where: \( M_{ap,d} \) is the design moment at the apex, \( b \) is the width of the beam, \( h_{ap} \) is the height of the beam at the apex, \( p_d \) is the uniformly distributed load acting on the top of the beam over the apex area and \( k_p = 0.2 \tan(\alpha_{ap}) \). Whereas the first expression (1) is accepted and recommended (unless the National annex states otherwise) for all load cases, the second one (2) may be used only when the beam is loaded with a uniform load applied to the top of the apex zone.

According to Eurocode 5, the biggest tensile stress perpendicular to the grain should satisfy the expression

\[ \sigma_{t,90,d} \leq k_{ds} f_{t,90,d} \]

where: \( f_{t,90,d} \) is the design tensile strength perpendicular to grain, \( k_{ds} \) is a factor which takes into account the effect of the stress distribution in the apex zone and for double-tapered beams equals 1.4, \( k_{vol} \) is a volume factor given by the formula

\[ k_{vol} = \left( \frac{V_0}{V} \right)^{0.2} \]

where: \( V_0 \) is the reference volume of 0.01m³, \( V \) is the volume of the apex zone (see figure 1) and should not be taken greater than \( 2V_0 / 3 \), where \( V_0 \) is the total volume of the beam.

3. Results and discussions of 3D computational model

A 3D numerical model of the beam was prepared in the Abaqus/Standard [9] environment using the finite element method. The following geometry of the beam was modelled: width 30 cm, span 36 m, height over the supports 1 m and in the apex 3.4 m. The beam was loaded with self-weight, density 425 kg/m³, and concentrated forces applied in lines to the nodes of the upper surface with values 32.5 kN (16.25 kN at sides). The orthotropic properties of wood were adopted: \( E_1=12600 \) MPa, \( E_2=E_3=300 \) MPa, \( G_{12}=G_{13}=G_{23}=650 \) MPa, \( \mu_{12}=\mu_{13}=\mu_{23}=0.35 \).
The structure was divided into 210269 20-node quadratic brick elements. The boundary conditions were applied at support nodes, displacements in all directions were blocked. The beam model with load and boundary conditions is presented in figure 2a.

The reinforcement was modelled as 3-node quadratic beam elements in space. Linear displacements were tied with 3D elements and the rotations of the nodes were allowed. Two cases of reinforcement distribution in the apex zone were considered (figure 2b, 2c): in one line (6ϕ9 mm, area of one bar 63.6 mm², Young’s modulus 210 GPa, bars spacing 0.75 m) and in two lines (6ϕ6.36 mm, area of one bar 31.8 mm², Young’s modulus 210 GPa, bars spacing 0.75 m).

![Figure 2. Double-tapered beam: geometry, load and boundary conditions (a), reinforcement: single (b) and double lines (c).](image)

The distributions of all stresses in the 3D model without the reinforcement do not show any variations in the horizontal direction across the fibres. The differences between 3D and the corresponding 2D model of the beam are as follows:

- normal stress along the grain – approx. 3 %,
- normal stress perpendicular to the grain – approx. 6 %,
where bigger values were obtained in the 3D model.

The stresses in the horizontal direction are significantly smaller than others and therefore, will not be discussed. For example, in figure 3 the distribution of normal stress along the grain is presented.
Figure 3. Stress parallel to grain with no reinforcement: general view (a), apex zone view (b).

Distribution of normal stress perpendicular to grain in the beam with no reinforcement is presented in figure 4. Delaminating stresses appear in the apex zone with the maximum value 0.16 MPa in the upper part of the beam close to its central axis. The zone in tension obtained numerically is consistent with the recommendation of the design standard (figure 1). Figure 4b depicts a constant distribution of the stress under consideration in the horizontal direction perpendicular to the grain.
Figure 4. Stress perpendicular to grain (positive values only) with no reinforcement: general view (a), apex zone view (b).

Distribution of normal stress perpendicular to grain in the beam with reinforcement located in a single line is presented in figure 5. Delaminating stresses appear again in the apex zone, however the maximum values are reached around the bottom ends of reinforcing bars – this is an important qualitative difference compared to the beam with no reinforcement. The extreme value 0.16 MPa occurs in the area around the inner pair of reinforcing bars. However, the reinforcement placed in a single line did not reduce the maximum value of delaminating stress – it just moved to another location. Figures 5b, 5c and 5d present distributions of stresses under consideration in the cross-sections determined by the reinforcement.
Figure 5. Stress perpendicular to grain (positive values only) with single line of reinforcement: general view (a), apex zone views (b), (c), (d).

Distribution of normal stress perpendicular to grain in the beam with reinforcement located in two lines is presented in figure 6. Delaminating stresses appear again in the apex zone, however the maximum values are reached around the bottom ends of both lines of reinforcing bars – this is an important qualitative difference compared to the beam with no reinforcement and with a single line of reinforcement. The extreme value 0.13 MPa occurs in the area around the inner pairs of reinforcing bars and is reduced by 17 %, which is enough to fulfil the design criteria if the stress exceeds the allowed
value. Figures 6b, 6c and 6d present distributions of stresses under consideration in the cross-sections determined by the reinforcement.
Figure 6. Stress perpendicular to grain (positive values only) with double line of reinforcement: general view (a), apex zone views (b), (c), (d).

Distributions of stresses presented in figures 5 and 6 prove that in the case of the reinforced beams a 3D model analysis leads to the results that differ both qualitatively and quantitatively from the ones obtained in the corresponding 2D model. These results allow to justify in terms of quantity the effectiveness of the applied reinforcement.
4. Conclusions
The paper is dedicated to the stress analysis in glulam reinforced double tapered beams within 3D computational model of orthotropic linear elasticity theory. Standard Eurocode design recommendations are the reference data to the finite element results. The results are presented for beams with a single and double line of reinforcement and compared with a non-reinforced beam. 3D elasticity model is an underestimation in comparison to the equivalent 2D model. Qualitative as well as quantitative analysis of the influence of reinforcements for the stress state of the beam is possible within a 3D finite element model.

It is very interesting that the maximum value of normal stress perpendicular to grain occurs in a different location when the reinforcement is applied. Extreme values appear around ends of reinforcing bars.

The performed analyses show that the way the reinforcing bars are placed in the beam is very important. In both considered cases the same total area of reinforcement was applied. However, the bars placed in a single line were not enough to reduce the maximum value of delaminating stress – the extreme stress appeared in a different area. In the second analyzed case, the maximum stress not only changed its location but also the value, which was enough to satisfy the design requirements.

The proposed technique is especially recommended to estimate reduction of perpendicular to grain stress for a double line reinforcement in glulam.

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