Applications of effective field theory to nucleon-nucleon scattering, quarkonia decay and production and B meson decay are discussed. Some unresolved issues are considered.

1 Introduction

The use of effective field theories is a standard tool for dealing with strong interaction phenomena in the nonperturbative regime. The basic idea is to construct the most general Lagrangian consistent with the symmetries of Quantum Chromodynamics (QCD) out of fields that destroy the relevant degrees of freedom in the problem. For example, in the chiral Lagrangian for pion self interactions, those degrees of freedom are the pion fields, while in the heavy quark effective theory (HQET) and nonrelativistic quantum chromodynamics (NRQCD) they are quark and gluon fields. If this were all that was done one would have no more predictive power than that based on the symmetries and unitarity of the S-matrix. To endow the method with more predictability, there must be a power counting scheme where the effective Lagrangian is expanded in a small parameter to reduce the number of operators that occur in it. In the case of chiral perturbation theory for pion interactions the small parameter is the pion momentum divided by a typical hadronic scale. For HQET the expansion parameter is a typical hadronic scale divided by the heavy quark mass, and for NRQCD the expansion is in the typical velocity of the heavy quarks divided by the speed of light. Often the leading term in the expansion has new (approximate) symmetries that were not manifest in the QCD Lagrangian. In chiral perturbation theory (CPT) the approximate symmetry is the $SU(3)_L \times SU(3)_R$ chiral symmetry that occurs when the light quark masses are neglected. For HQET they are the heavy quark spin-flavor symmetries that occur when the heavy quark masses are taken to infinity with their four velocities fixed. In NRQCD there are also heavy quark spin symmetries.

At a practical level, the power counting takes somewhat different forms depending on what type of regulator is used for the effective field theory. Consider the case of the effective field theory for Cabibbo allowed nonleptonic charm decays that results from integrating out the W-boson. At tree level the effects of the Feynman diagram in Fig. 1 are reproduced by the effective Lagrangian

$$\mathcal{L} = C_1 O_1 + C_2 O_2 + ..., \quad (1)$$

where

$$C_1 = g_2^2 / 8 M_W^2, \quad C_2 = g_2^2 / 8 M_W^4. \quad (2)$$

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The ellipses in Eq. 1 refer to operators with more derivatives. In Eq. 2, $M_W$ is the W-boson mass and $g_2$ is the weak $SU(2)$ coupling. The operators $O_1$ and $O_2$ are

$$O_1 = [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma_\mu(1-\gamma_5)d],$$

$$O_2 = [\bar{s}\gamma_\mu(1-\gamma_5)c] [\bar{u}\gamma_\mu(1-\gamma_5)d].$$

(3)

At tree level the effects of $O_2$ on the $c \rightarrow \bar{d}us$ decay rate are suppressed by $p^2/M_W^2$, compared with those of $O_1$, where $p$ is a typical momentum in the decay.

When perturbative order $\alpha_s$ corrections are considered it is necessary to regulate the theory because of ultraviolet divergences coming from the large momentum part of the loop integration in Feynman diagrams. Imagine using a momentum cutoff, $\Lambda$, for the regulator. The size of the contribution of $O_2$ depends on the value of the cutoff. Its contribution, for $\Lambda^2 \sim M_W^2$, is suppressed by $\alpha_s$ compared with that of $O_1$. It no longer appears to be suppressed by $p^2$. This large part of its contribution can be absorbed into a redefinition of the coefficient of $C_1$. If a momentum cutoff is used as the regulator, the value of $C_1$ extracted from experiment in the theory with $O_2$ included may differ from its value in the theory without $O_2$ by an amount of order $\alpha_s$. Nonetheless the net new effect on the physics due to $O_2$ is suppressed by $p^2/M_W^2$. Even with a dimensionful cutoff there is a subtraction procedure that has all the effects of $O_2$ suppressed by $p^2/M_W^2$. This occurs if renormalized operators are defined so that, for the renormalized version of $O_2$ (i.e. $O_2^R$), the part of its matrix elements that grow as $\Lambda^2$ and are proportional to matrix elements of $O_1$ are subtracted away. For the Lagrangian, $\mathcal{L} = C_1^R O_1^R + C_2^R O_2^R + \ldots$, the extracted value of $C_1^R$ at order $\alpha_s$ will not depend on whether $O_2^R$ is included or not since all its effects are of suppressed by $p^2/M_W^2$. The value of the cutoff can be arbitrarily large. However, if such renormalized operators are not introduced for $\Lambda^2 \gg M_W^2$, the “physical” $c \rightarrow dus$ amplitude will arise from a delicate cancellation between the contribution of $C_1$ and the one loop contribution of $O_2$ proportional to $\alpha_s \Lambda^2/M_W^2$.

Using dimensional regularization (with mass independent $\overline{\text{MS}}$ subtraction) is a little like taking the momentum cutoff to infinity. But with this regulator power divergences are automatically subtracted and the effects of $O_2$ are manifestly suppressed by $p^2/M_W^2$. Using almost any type of ultraviolet regulator is fine. However, one might encounter technical issues with one type of regulator that are not there with another, for example adding counter terms to restore gauge invariance in the case of a momentum cutoff. For the remainder of this lecture, I assume that dimensional regularization with minimal subtraction is used.

My lecture will be divided into five Sections. In Section 2 I discuss an issue in the application of chiral perturbation to nucleon-nucleon scattering that needs to be resolved.
before this can be claimed to be a useful systematic method. Section 3 discusses some recent applications of NRQCD that show that terms that are naively suppressed by powers of \( \nu/c \) can become important in certain kinematic regions. Section 4 deals with recent developments in NRQCD formalism and finally Section 5 briefly describes a recent application of HQET to semileptonic B decay to excited charmed mesons.

2 Chiral Perturbation Theory for Nucleon-Nucleon Scattering

Weinberg first suggested using CPT for nuclear physics. The basic idea was that a power counting can be established for the nucleon-nucleon potential which is then used to calculate properties of systems of nuclei, e.g. \( \text{NN} \) phase shifts. (For a review of some applications see Ref. [2].) In the \( ^1S_0 \) channel the leading nucleon-nucleon potential is supposed to be

\[
V_0(p, p') = \tilde{C} - 4\pi \alpha_\pi/(q^2 + m^2_\pi), \tag{4}
\]

where \( p \) and \( p' \) are the relative three momenta of the initial and final \( \text{NN} \) pair, \( q = p - p' \), and

\[
\alpha_\pi = g_A^2 m^2_\pi / 8\pi^2 f^2_\pi. \tag{5}
\]

In Eq. 5 \( g_A \approx 1.25 \) is the axial current coupling and \( f_\pi \approx 132 \text{MeV} \) is the pion decay constant. The second term in Eq. 4 comes from one pion exchange. You might not recognize its form because pions are derivatively coupled. In Eq. 4 the numerator was arrived at by writing \( q^2 \) as \( (q^2 + m^2_\pi) - m^2_\pi \). The first of these two terms gives a constant in the potential that was absorbed into \( \tilde{C} \) by the definition, \( \tilde{C} = C + g_A^2 / 2 f^2_\pi \). The constant \( C \) is the coefficient of a four nucleon operator of the form \( N^\dagger N N^\dagger N \). Note that four nucleon operators containing derivatives or insertions of the quark mass matrix are supposed to give contributions to the potential that are suppressed.

![Two loop diagram contributing to \(^1S_0\) nucleon-nucleon scattering. The dashed line represents virtual pion exchange.](image)

Imagine using the potential in Eq. 4 to calculate the \(^1S_0\) \( \text{NN} \rightarrow \text{NN} \) phase shift. This is done by solving the Schrodinger equation or equivalently summing the ladder diagrams. The potential is singular (in coordinate space the constant term in \( V_0(p, p') \) corresponds to a delta function potential) and ultraviolet divergences are encountered. For example Fig. 2 gives rise to a divergent amplitude proportional to

\[
\frac{1}{d - 4} \tilde{C}^2 \alpha_\pi M^2_N / 4\pi, \tag{6}
\]
where $d$ is the space-time dimension. (If a cutoff were used instead of dimensional regularization then the factor of $1/(d-4)$ would be replaced by $\ln(m_\pi^2/\Lambda^2)$.) The divergence in Eq. 4 is removed by adding a counter term. The subtraction point dependence associated with the finite part of Fig. 2 is canceled by the subtraction point dependence in the coefficient of the counter term operator. In this case the counter term is of the form

$$L_{ct} \sim C_{ct} Tr(\Sigma m_q + \Sigma^\dagger m_q) N \dagger N N \dagger N.$$  

(7)

Eq. 7 is necessary as a counter term for the leading order calculation. The only possible power counting prescription that could make its contribution to the potential subdominant to that in Eq. 4 would be one based on considering logarithms of $(m_q/\Lambda_{QCD})$ as large. This situation is similar to what happens in pion self interactions. There as one includes higher and higher loops, terms with more insertions of the quark mass matrix and/or higher derivatives must also be included in the chiral Lagrangian. (Actually in the case of $NN^1 S_0$ scattering, no divergences corresponding to operators with derivatives occur. However the work of Ref. [4] makes it seem unlikely that a reasonable power counting can be developed for which such operators are subdominant.)

The problem of elastic nucleon-nucleon scattering itself is not that interesting. However, if a systematic approach using CPT is possible for this problem, then one can contemplate using it also for inelastic pion production and for the study of nuclear matter. It seems worth exploring further whether a systematic approach to nucleon-nucleon scattering based on CPT is possible.

### 3 Kinematically Enhanced Nonperturbative Corrections in NRQCD

NRQCD is an effective field theory used to predict properties of quarkonium in an expansion in $v$, where $v$ is the magnitude of the relative velocity of the heavy $\bar{Q}Q$ quark pair. (In this section we adopt the usual particle physics convention that the speed of light is $c = 1$.) It has made surprising predictions for quarkonia production and decay because effects suppressed by powers of $v$ can be enhanced by factors of $1/\alpha_s(m_Q)$ compared with the leading term in the $v$ expansion. As an example of this phenomena consider the inclusive decay $\Upsilon \to \gamma + X$, where $X$ denotes light hadrons. The differential decay rate can be written as the imaginary part of a time ordered product of electromagnetic currents (there are some complications due to overlapping unwanted cuts, see Ref. [5] for a discussion of this.),

$$\frac{d\Gamma}{dE_\gamma} = \frac{2e_b^2}{\pi} E_\gamma \text{Im} T',$$  

(8)

where $e_b = -1/3$ is the b-quark charge and $E_\gamma$ is the photon energy. The operator product expansion (OPE) can be used to calculate the imaginary part of the time ordered product $T'$. For a given photon energy the final hadronic invariant mass squared is $m_X^2 = m_Y^2(1 - 2E_\gamma/m_Y)$ so near the endpoint $E_\gamma = m_Y/2$ only low mass hadronic final states can occur. In this region predictions based on the OPE must be smeared over a region of photon energies, $\Delta E_\gamma$, before they can be compared with experiment. If the smearing region is chosen too small, higher dimension operators in the OPE become successively more important and
the OPE breaks down. In this section we examine the type of operators that control this endpoint region and also show that analogous operators play an important role in predictions for quarkonium production based on NRQCD.

At tree level the OPE is performed by calculating the tree level Feynman diagrams in Figs. 3. Fig. 3a gives a contribution of order \( \alpha_s(m_b) \), while Fig. 3b gives a contribution of order \( \alpha_s(m_b)^2 \). The order \( \alpha_s(m_b) \) contribution to \( T' \) from Fig. 3a, denoted by \( T_8' \), is called the octet contribution. It is usually neglected because it is higher order in \( v \) than the color singlet contribution from Fig. 3b (which we denote by \( T_1' \)). However \( T_8' \) is enhanced by \( 1/\alpha_s(m_b) \) compared with \( T_1' \). The Feynman diagram in Fig. 3a gives

\[
T_8' = 4g^2 \langle \Upsilon | \left[ \bar{b} \gamma^\nu (p + iD - q + m_b) T^A \gamma_\mu b \right] \frac{i\epsilon}{(2p - q + iD)^2 + \epsilon} | \Upsilon \rangle.
\]

In Eq. \([9]\) \( g \) is the strong coupling, \( D \) denotes a covariant derivative, \( p = (m_b, 0) \) and the Upsilon states are at rest. The imaginary part comes from the gluon propagator which produces the factor \( \delta((2p - q + iD)^2) \). Expanding this in \( D \) gives a sequence of operators that are more and more singular in the endpoint region.

Making the transition to NRQCD there are two contributions depending on the spin structure of the \( b\bar{b} \) pair,

\[
T_8' = T_8'(3P_J) + T_8'(1S_0),
\]

and the expansion in \( D \) of Eq. \([9]\) gives

\[
\text{Im} T_8'(1S_0) = \frac{7\pi g^2}{18m_b} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \langle \Upsilon | \left[ \bar{\psi}^\dagger T^A \chi \right] (i\sigma \cdot D)^m \left[ \chi^\dagger T^A \psi \right] | \Upsilon \rangle \delta^{(m)}(2E_\gamma - 2m_b).\]

In Eq. \([10]\) \( n = (2p - q)/m_b \), which can be taken to be equal to \( (1, 0, 0, 1) \), is a light-like four-vector. The superscript \( m \) on the delta function denotes its \( m \)th derivative with respect to \( 2E_\gamma \) and the \( \psi \) and \( \chi \) are the two component Pauli spinor fields that destroy the bottom quark and antiquark in NRQCD. A similar expression holds for \( \text{Im} T_8'(3P_J) \).

The \( m \)'th term in the sum of Eq. \([10]\) scales as \( v^{7+2m} \) according to the NRQCD counting rules. Here we are assuming that \( \Lambda_{QCD}/m_b \) is of order \( v^2 \). The contribution to the total \( \Upsilon \to \)

![Figure 3: Feynman diagrams contributing to ImT'.](image-url)
\(\gamma + X\) rate is dominated by the \(m = 0\) term which gives a contribution of order \((\alpha_s(m_b)/\pi)v^7\). Note that the color singlet contribution to the decay rate is of order \((\alpha_s(m_b)/\pi)^2v^3\). If we take \(\alpha_s(m_b)/\pi\) to be of order \(v^2\), then the color singlet term dominates the rate. But the octet contribution is very important near the endpoint. If we smear the endpoint region of the differential decay rate over photon energies \(\Delta E_\gamma \ll m_b v^2\), then successive terms in the sum over \(m\) in Eq. (11) become more important. On the other hand if we smear the differential decay rate over a region of photon energies \(\Delta E_\gamma \gg m_b v^2\), then successive terms in the sum over \(m\) in Eq. (11) become less important and the color singlet contribution dominates. For \(\Delta E_\gamma \sim m_b v^2\) all terms in the sum are of comparable importance to the color singlet contribution. Similar remarks hold for the contribution from \(Im T'_8(\bar{\beta} P_f)\). Note that the region of photon energies, \(\Delta E_\gamma \sim m_b v^2\) corresponds to a range of final hadronic masses \(\Delta m_X \sim m_b v\) which is much greater than the QCD scale. Similar remarks hold for \(\psi\) decay.

An analogous calculation in the color singlet case gives a sum of the form

\[
Im T'_1 = g^4 G(E_\gamma) \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \langle \Upsilon | \left[ \psi^\dagger \sigma_k \chi \right] (in \cdot \partial)^m \left[ \chi \sigma_k \psi \right] | \Upsilon \rangle \theta(m)(2m_b - 2E_\gamma). \tag{12}
\]

In Eq. (12) \(G(E_\gamma)\) is a smooth function of the photon energy and the superscript \(m\) on the theta function denotes its \(m\)'th derivative with respect to \(2E_\gamma\). Inserting a complete set of states between the bilinears of NRQCD fields, one finds that the vacuum state dominates and \(in \cdot \partial\) produces a factor of the binding energy. Consequently Eq. (12) becomes

\[
Im T'_1 = g^4 G(E_\gamma) \langle \Upsilon | \left[ \psi^\dagger \sigma_k \chi \right] \left[ \chi \sigma_k \psi \right] | \Upsilon \rangle \theta(m \Upsilon - 2E_\gamma). \tag{13}
\]

In the color singlet case the sum of singular terms has converted the parton kinematics to hadron kinematics.

The endpoint region of the photon energy spectrum in \(\Upsilon \rightarrow \gamma + X\) decay is determined by a sum of leading twist operators. The situation is very analogous to the endpoint region of electron energies in semileptonic \(B\) meson decay. A similar phenomenon also occurs in quarkonium production. For example, at large transverse momentum \(p_\perp\) quarkonium production at the Tevatron is controlled by gluon fragmentation. The differential cross section for \(\psi\) production is

\[
\frac{d\sigma}{dp_\perp} = \int_{\hat{z}_{\text{min}}} \frac{d\hat{z}}{\hat{z}} K(\hat{z}, p_\perp) D_{g \rightarrow \psi}(\hat{z}, p_\perp). \tag{14}
\]

In Eq. (14), \(\hat{z} = p_+ / k_+\), where \(k\) is the four-momentum of the fragmenting gluon and \(p\) is the four-momentum of the \(c\bar{c}\) quark pair in the \(\psi\) (i.e., in the rest-frame of the \(\psi\), \(p = (2m_c, 0))\).

The hat is placed on \(z\) because it is defined in terms of parton kinematics instead of hadron kinematics. The function \(K\) depends on the parton cross section and parton distributions. In the region \(p_\perp \sim 15\text{GeV}, K \sim \hat{z}^5\) so the integral in Eq. (14) weights the fragmentation function \(D_{g \rightarrow \psi}(\hat{z}, p_\perp)\) towards \(\hat{z} = 1\).

Including the most singular terms as \(\hat{z} \rightarrow 1\) the gluon fragmentation function is
\[ D_{g \rightarrow \psi}(\hat{z}, 2m_c) = \frac{\pi \alpha_s(2m_c)}{24m_c^3} \sum_m \frac{1}{m!} \delta^{(m)}(1 - \hat{z}) \sum_X \langle 0 | \left[ \chi^\dagger \sigma_i T^A \psi \right] | \psi + X \rangle \langle \psi + X | (in \cdot D/(2m_c))^m \left[ \psi^\dagger \sigma_i T^A \chi \right] | 0 \rangle. \] (15)

In this case the light-like vector is \( n = (1, 0, 0, -1) \) and the superscript \( m \) on the delta function denotes its \( m \)’th derivative. Terms in the sum are of order \( v^{7+2m} \) and so if one focuses on a region near the endpoint \( \hat{z} = 1 \), of size \( \Delta z \sim v^2 \), then all terms in the sum are of comparable importance. One crude way to gauge the importance of the higher order terms is to imagine that their effect is to shift the delta function at \( \hat{z} = 1 \) to a delta function at \( \hat{z} = 1 - O(v^2) \). Then with \( K \sim \hat{z}^5 \) we see that there is a correction to the leading \( m = 0 \) term in the sum of order \( 4v^2 \sim 1 \). Of course this is just dimensional analysis. Without a more precise estimate of the higher order terms in the sum, we cannot be confident that they are numerically important for \( \psi \) production at the Tevatron at large \( p_\perp \). Similar sums occur in all quarkonia production processes and are important near the boundary of phase space where there is sensitivity to the difference between parton and hadron kinematics.

4 Effective Lagrangian for NRQCD

NRQCD organizes contributions to the physical properties of quarkonia as an expansion in powers of \( v/c \). Clearly the limit of QCD that is appropriate in this case is the large \( c \) limit. The original formulation of Bodwin, Braaten, and Lepage was similar in some ways to the effective field theory for Cabibbo allowed charm decay that resulted from integrating out the W-boson using a momentum cutoff, \( \Lambda \sim M_W c \), as the regulator for the ultraviolet divergences. In this formulation of NRQCD with a dimensionful ultraviolet regulator, a given operator contributes at many different orders in \( v/c \), unless a subtraction procedure is adopted to remove the subdominant pieces. There is nothing wrong with such a formulation. However, recently NRQCD has been reformulated using dimensional regularization so that a given operator automatically only contributes at a fixed order in \( v/c \), much like the usual formulation of HQET where a given operator contributes at a fixed order in \( \Lambda_{QCD}/m_Q \). In this section I review this recent work. Unlike the other sections of this talk, factors of \( c \) will be explicit here.

Consider the QCD Lagrangian for gluons interacting with a heavy quark \( Q \)

\[ \mathcal{L}_{QCD} = -\frac{1}{4} G^{B\mu\nu} G_{B\mu\nu} + cQ(\not{D} - m_Q c)Q. \] (16)

In Eq. the 0 component of a partial derivative is

\[ \partial_0 = \frac{1}{c} \left( \frac{\partial}{\partial t} \right), \] (17)

and \( D \) is the covariant derivative

\[ D_\mu = \partial_\mu + \frac{iq}{c} A_\mu T^B. \] (18)
The gluon field strength tensor $G^B_{\mu\nu}$ is defined in the usual way except that $g \to g/c$.

There are many choices for how factors of the speed of light $c$ are put into Eq. 16. This occurs because the normalization of the quark and gluon fields is arbitrary. NRQCD is the effective theory that arises in the limit $c \to \infty$. Having the NRQCD Lagrangian independent of $c$ (without performing any rescaling of fields) is the motivating factor behind the placement of factors of $c$ in the Lagrangian in Eq. 16 and for the factor of $1/c$ associated with the strong coupling in covariant derivative in Eq. 18.

Although $\hbar$ has been set to unity, $c$ is explicit and so the dimensions of all quantities are expressible in units of length $[x]$ and time $[t]$ ($[E] \sim 1/[t]$ and $[p] \sim 1/[x]$). The gluon field has dimensions $[A] \sim 1/\sqrt{x/t}$ and the strong coupling $[g] \sim \sqrt{x/t}$. The fermion field has dimensions $[\psi] \sim 1/[x]^{3/2}$ and its mass has dimensions $[m_Q] \sim t/[x]^2$.

For the fermion field, the transition from QCD to NRQCD follows the usual derivation of HQET. It is rewritten as

$$Q = e^{-im_Qc^2t} \left[ 1 + \frac{iD_\perp}{2m_Qc} + \ldots \right] \psi,$$

where $\psi$ is a Pauli spinor written as a four-component object satisfying the constraint $\gamma^0 \psi = \psi$. The covariant derivative $D_\perp = (0, D)$. Using Eq. 19 the part of the QCD Lagrangian density involving $Q$ becomes

$$\mathcal{L}_\psi = \psi^\dagger \left[ i \left( \frac{\partial}{\partial t} + igA_0^BT^B \right) + \frac{\nabla^2}{2m_Q} \right] \psi + \ldots,$$

where the ellipses denote terms suppressed by powers of $1/c$. The leading term is $c$ independent and corresponds to a heavy quark interacting with a gluon potential $A_0^B$. Among the terms suppressed by $1/c$ is the fermion interaction term in the Lagrangian density

$$\mathcal{L}_{int} = \frac{ig}{m_Qc} A_0^B [\psi^\dagger T^B \nabla \psi - (\nabla \psi)^\dagger T^B \psi].$$

Note that $\mathcal{L}_\psi$ in Eq. 20 and $\mathcal{L}_{int}$ in Eq. 21 both respect a heavy quark spin symmetry. Unlike HQET there is no heavy quark flavor symmetry because the leading term in Eq. 21 depends on the heavy quark mass. There is another fermion interaction term suppressed by $1/c$ involving the color magnetic field $B_c = \nabla \times A$ that breaks the spin symmetry.

It is convenient to work in Coulomb gauge $\nabla \cdot A = 0$. There are two types of transverse gluon modes that one wants to keep in the effective field theory. They are the potential modes which are typically far off shell, $\partial^2 A / \partial t^2 \ll c^2 \nabla^2 A$ and propagating modes where $\partial^2 A / \partial t^2 \sim c^2 \nabla^2 A$. Including both modes is achieved by decomposing the transverse gluon field as

$$A^B(x, t) = \hat{A}^B(x, t) + \tilde{A}^B(x/c, t)/\sqrt{c},$$

where the hat is used to denote the nonpropagating potential transverse gluons and the tilde denote the propagating transverse gluons. The NRQCD Lagrangian is

$$L_{NRQCD} = L_A + L_{\hat{A}} + L_{\tilde{A}} + L_\psi,$$
where
\[ L_A = \int d^3x \frac{1}{2} (\partial_i A^B_i)^2, \]  
(24) 
\[ L_{\hat{A}} = \int d^3x (-) \frac{1}{2} (\partial_i \hat{A}^B_i)^2, \]  
(25) 
\[ L_\psi = \int d^3x L_\psi, \]  
(26) 
\[ L_{\tilde{A}} = \frac{1}{c} \int d^3x \left( \frac{1}{c} \partial_t \tilde{A}^B_i (x/c, t) \right)^2 - (\partial_i \tilde{A}^B_i (x/c, t))^2 \]  
\[ = \int d^3y (\partial_t \tilde{A}^B_i (y, t))^2 - (\partial_i \tilde{A}^B_i (y, t))^2. \]  
(27)

Eq. 23 is independent of \( c \) and corrections to it are suppressed by factors of \( 1/\sqrt{c} \). In the second line of Eq. 27 the variable \( y \) is equal to \( x/c \) and the partial derivative, \( \partial_i \), is with respect to \( y^i \). The leading order NRQCD Lagrangian in Eq. 23 does not contain any nonabelian gluon self couplings.

At leading order in the \( 1/c \) expansion there is no mixing between \( \tilde{A} \) and \( \hat{A} \), and the two types of gluon fields separately satisfy the Coulomb gauge condition \( \nabla \cdot \hat{A} = 0 \).

In terms suppressed by powers of \( 1/\sqrt{c} \) there is mixing between the zero momentum mode of \( \tilde{A} \) and \( \hat{A} \).

The rescaled field \( \tilde{A} \) in Eq. 22 was introduced in Ref. [9]. The field \( \hat{A} \) doesn’t propogate and in Ref. [9] it was not included in Eq. 22. From the perspective of Ref. [9] the terms suppressed by factors of \( 1/c \) that arise from \( \tilde{A} \) exchange are corrections from matching full QCD onto NRQCD. The advantage of including \( \hat{A} \) in the decomposition in Eq. 22 is that it automatically performs the tree level matching. Ref. [10] has both \( \tilde{A} \) and \( \hat{A} \) in the effective field theory.

The method I have described so far does not reproduce the power counting of Bodwin, Braaten, and Lepage. Consider the transverse gluon coupling in Eq. 21. At leading nontrivial order in \( 1/c \) the part involving the propogating gluons is
\[ \mathcal{L}_{\text{int}} = \frac{ig}{m_{QCD}^{3/2}} \tilde{A}^B(0, t) [\psi^\dagger T_B \nabla \psi - (\nabla \psi)^\dagger T^B \psi]. \]  
(28)

In quarkonia these gluons typically have momenta of order \( m_{QCD} v^2/c \) and if we take \( \Lambda_{QCD} \) to be of this order their coupling should be nonperturbative. In other words for the propogating gluons one wants \( \alpha = g^2/(4\pi c) \) of order unity. This means that one should take \( g \sim \sqrt{c} \) instead of \( g \sim 1 \) in the power counting. Then nonabelian terms involving the propogating gluons are not suppressed by factors of \( 1/\sqrt{c} \). For example, the three gluon coupling is in a term in the Lagrangian of the form
\[ L_{\tilde{A} \text{ gluon}} \sim \frac{g}{c^{3/2}} \int d^3x (\nabla \tilde{A}(x/c, t)) \tilde{A}(x/c, t) \tilde{A}(x/c, t) \]  
\[ \sim \frac{g}{\sqrt{c}} \int d^3y (\nabla_y \tilde{A}(y, t)) \tilde{A}(y, t) \tilde{A}(y, t), \]  
(29)
which is leading order if \( g \) is of order \( \sqrt{c} \). With \( g \) for propagating gluons of order \( \sqrt{c} \) the interaction in Eq. 28 is suppressed by a single factor of \( 1/c \), which agrees with the power counting of Bodwin, Braaten, and Lepage. It is worth exploring further the consistency of this modification of the usual nonrelativistic \( v/c \) power counting that occurs in Quantum Electrodynamics.

Finally I note that the color magnetic field \( \tilde{B}_c = \nabla_x \times \tilde{A}(x/c, t) \) vanishes at leading order in \( 1/c \), so the spin symmetry violating term that involves \( \psi, \psi^\dagger \), and \( \tilde{B}_c \) is suppressed by an additional factor of \( 1/c \) compared with the term in Eq. 28. This is the reason for the prediction that \( \psi \) and \( \psi' \)'s produced at large \( p_\perp \) at the Tevatron should be transversely aligned. There are corrections to this prediction suppressed by powers of \( \alpha_s(2m_c) \) and by powers of \( (2m_c)/p_\perp \).

5 Excited Charmed Mesons in B Semileptonic Decay

The limit of QCD where the heavy quark mass goes to infinity with its four-velocity, \( v \), held fixed gives the heavy quark effective theory, HQET. The QCD heavy quark field \( Q \) is related to its HQET counterpart \( Q_v \) by

\[
Q(x) = e^{-im_Qvx} \left[ 1 + \frac{iD_\perp}{2m_Q} + \ldots \right] Q_v, \tag{30}
\]

where for a four vector, \( X_\mu \), its perpendicular component, \( X_\perp \mu = X_\mu - v \cdot v X_\mu \), satisfies \( v \cdot X_\perp = 0 \). The field \( Q_v \) destroys a heavy quark of four-velocity \( v \) and satisfies the constraint \( \not{v}Q = Q_v \). Putting Eq. 30 into the QCD Lagrange density gives

\[
\mathcal{L} = \mathcal{L}_{HQET} + \delta \mathcal{L} + \ldots,
\]

where the HQET Lagrange density \( \mathcal{L}_{HQET} \)

\[
\mathcal{L}_{HQET} = \bar{Q}_v i\gamma \cdot DQ_v \tag{31}
\]

has the heavy quark spin-flavor symmetry \( \mathbb{I}^3 \) and

\[
\delta \mathcal{L} = \frac{1}{2m_Q} [O_{kin,v}^{(Q)} + O_{mag,v}^{(Q)}], \tag{32}
\]

with \( O_{kin,v}^{(Q)} = \bar{Q}_v (iD_\perp)^2 Q_v \) and \( O_{mag,v}^{(Q)} = \bar{Q}_v \gamma_5 \sigma_{\alpha\beta} G^{\alpha\beta} Q_v \). The Lagrange density \( \delta \mathcal{L} \) contains the \( 1/m_Q \) corrections to the HQET Lagrangian \( \mathbb{I}^3 \). The operator \( O_{kin,v}^{(Q)} \) breaks the flavor symmetry but not the spin symmetry while \( O_{mag,v}^{(Q)} \) breaks both the spin and flavor symmetries.

In the \( m_Q \to \infty \) limit the spin of the light degrees of freedom, \( s_\ell \), is a good quantum number \( \mathbb{I}^3 \) and hadrons containing a single heavy quark come in doublets with total spins

\[
j_\pm = s_\ell \pm 1/2. \tag{33}
\]
The ground state mesons with $Q\bar{q}$ flavor quantum numbers have $s_{l}^\pi = 1/2^-$ giving for $Q = c$ the $D$ and $D^*$ mesons and for $Q = b$ the $B$ and $B^*$ mesons. Excited mesons with $s_{l}^\pi = 3/2^+$ have also been observed (in both the $Q = c$ and $Q = b$ cases). For $Q = c$ these are the $D_1$ and $D_2^*$ mesons with masses of 2240 MeV and 2260 MeV respectively. In the nonrelativistic constituent quark model these are $L = 1$ orbital excitations of the ground state doublet. These excited charm mesons are “narrow” with widths around 20 MeV. Another doublet with $s_{l}^\pi = 1/2^+$ should occur, but the mesons in it are expected to be broad. Expanding the meson masses in powers of $1/m_Q$ gives the mass formula,

$$m_{H_{\pm}} = m_Q + \bar{\Lambda} - \frac{\lambda_1^H}{2m_Q} \pm \frac{n_\pm \lambda_2^H}{2m_Q} + \ldots,$$

where the ellipsis denote terms suppressed by more factors of $1/m_Q$. In Eq. 34, $n_\pm = 2j_\pm + 1$ and $\bar{\Lambda}$ is the energy of the light degrees of freedom in the $m_Q \to \infty$ limit. The matrix elements $\lambda_1^H$ and $\lambda_2^H$ are defined by

$$\lambda_1^H = \frac{1}{2\epsilon^0 m_{H_{\pm}}}(H_\pm(v))\langle H_{mn,v}^Q | H_\pm(v) \rangle,$$

$$\lambda_2^H = \frac{\mp 1}{2\epsilon^0 m_{H_{\pm}} n_\mp}(H_\pm(v))\langle H_{mn,v}^Q | H_\pm(v) \rangle.$$

Note that the average mass,

$$\bar{m}_H = \frac{n_- m_{H_-} + n_+ m_{H_+}}{n_+ + n_-},$$

is independent of $\lambda_2$. I use the notation $\bar{\Lambda}, \lambda_1, \lambda_2$ for the ground state doublet and $\bar{\Lambda}', \lambda_1', \lambda_2'$ for the excited $s_{l}^\pi = 3/2^+$ doublet. The measured $D^* - D$ mass splitting implies that $\lambda_2 \simeq 0.10$ GeV$^2$ and the measured $D_2^* - D_1$ mass difference implies that $\lambda_2' \simeq 0.013$ GeV$^2$.

The effects of $O_{mag}^Q$ are much smaller in the $3/2^+$ doublet than in the ground state $1/2^-$ doublet. Combining the mass formulae yields expression for $\bar{\Lambda}' - \bar{\Lambda}$ and $\lambda_1' - \lambda_1$ in terms of hadron masses.

$$\bar{\Lambda}' - \bar{\Lambda} = \frac{m_b(m_b' - m_B) - m_c(m_c' - m_D)}{m_b - m_c} \simeq 0.39\text{GeV},$$

$$\lambda_1' - \lambda_1 = \frac{2m_c m_b[(m_b' - m_B) - (m_c' - m_D)]}{m_b - m_c} \simeq -0.23\text{GeV}^2.$$
The form factors $f_i$ and $k_i$ are functions of the dot product $w = v \cdot v'$ of the $B$ four-velocity $v$ and the charmed meson four-velocity $v'$. At zero recoil (i.e., $w = 1$) only $f_{V_1}$ can contribute to the matrix elements above. All other terms automatically vanish (e.g., because $v \cdot e^* = 0$ when $v = v'$). For $B \to D_1 e \bar{\nu}_e$ and $B \to D_2^* e \bar{\nu}_e$ decay all of the phase space is near zero recoil. The entire phase space is $1 < w < 1.3$.

$f_{V_1}(1)$ (i.e., the zero recoil value of the form factor $f_{V_1}$) vanishes by heavy quark spin symmetry in the $m_Q \to \infty$ limit. In this limit, the vector and axial currents are charges of the heavy quark spin flavor symmetry, and the $D_1$ and $D_2^*$ have $s_{\ell} = 3/2$ while the $B$ and $B^*$ have $s_{\ell} = 1/2$. The form factors $f_i, k_i$ are related to a single Isgur–Wise function, $\tau$, in the infinite mass limit:

$$\sqrt{6} f_A = -(w + 1) \tau, \sqrt{6} f_{V_1} = (1 - w^2) \tau, \sqrt{6} f_{V_2} = -3 \tau, \sqrt{6} f_{V_3} = (w - 2) \tau, k_V = -\tau, k_{A_1} = -(1 + w) \tau, k_{A_2} = 0 \text{ and } k_{A_3} = \tau.$$  

Note that the value of $\tau(1)$ is not fixed by heavy quark symmetry. In the infinite mass limit $f_{V_1}(1) = 0$ independent of the value of $\tau(1)$.

At order $\Lambda_{QCD}/m_Q$ the form factor $f_{V_1}(1)$ is no longer zero. Recently it has been shown that at this order it can be written in terms of $\bar{\Lambda}' - \bar{\Lambda}$ (which is known in terms of measured masses) and the Isgur–Wise function $\tau(w)$ evaluated at zero recoil. Explicitly,

$$\sqrt{6} f_{V_1}(1) = -\frac{4}{m_c} (\bar{\Lambda}' - \bar{\Lambda}) \tau(1).$$  \hspace{1cm} (44)

Note that the factor of four in the numerator of Eq. (44) means that this is quite a large correction. Furthermore its importance is enhanced over other $\Lambda_{QCD}/m_Q$ corrections since most of the phase space is near zero recoil. Recently ALEPH\textsuperscript{22} and CLEO\textsuperscript{23} have measured (with some assumptions) the branching ratio, $Br(B \to D_1 e \bar{\nu}_e) = (6.0 \pm 1.1) \times 10^{-3}$. With more experimental information on the semileptonic decays $B \to D_1 e \bar{\nu}_e$ and $B \to D_2^* e \bar{\nu}_e$ it will be possible to study in detail the applicability of results based on the $\Lambda_{QCD}/m_Q$ expansion to these decays. This may also lead to a better understanding of how exclusive semileptonic decays add up to the inclusive semileptonic decay rate and influence our understanding of decays to the ground state, $B \to D^{(*)} e \bar{\nu}_e$, through the application of $B$ decay sum rules.\textsuperscript{24}

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