Attitude motion of space debris during its removal by ion beam taking into account atmospheric disturbance

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Abstract. The paper is devoted to the issue of active large space debris removal. The contactless method of influencing space debris through the ion flow is considered. The main advantage of this method is absence of a dangerous and complex stage of space debris capturing or docking. In this study, the project "Ion Beam Shepherd" was taken as a basis. The aim of this paper is to study the effect of the atmosphere in LEO on the space debris behaviour during its removal. The equations of plane motion of space debris under the action of the ion beam, gravitational and aerodynamic forces and torques were obtained using Lagrange formalism. For calculation ion beam force and torque the self-similar model of ion engine plume exploration and fully diffused reflection model of ions interaction with the surface were used. Simulation of deorbiting the spent upper stage of the Cosmos 3M rocket was carried out. Phase portraits of the rocket stage attitude motion for various heights were constructed and analysed. It was shown that the motion of the stage around its centre of mass has a noticeable effect on the average ion beam force and descent time.

1. Introduction

The space debris issues are widely discussed in the scientific literature in recent decades. Much attention is paid to removal of large space debris, which includes upper rockets stages and broken satellites [1]. Collisions of large space objects are very dangerous, since they lead to the formation of small debris clouds [2] and can cause the Kessler syndrome [3]. The scientific community discusses various ways of large space debris removal. Detailed reviews of diverse technical solutions and projects are given in the works [4, 5]. This study focuses on the contactless method of deorbiting space debris. The main advantage of contactless methods is the absence of a complex and dangerous stage of space debris capture by an active spacecraft-cleaner. To date, a few methods of contactless impact on a space object for the purpose of changing the parameters of its motion are considered: using Coulomb interaction [6, 7], by means of the ion beam [8, 9], due to laser action [10, 11]. In this study, the project "Ion Beam Shepherd" was taken as a basis [8, 12, 13]. A space debris object is removed from orbit by an active spacecraft (shepherd), which is equipped with two ion engines. The spacecraft is approaching the object at a distance of about ten meters. Thereafter, one of the engines is directed toward the object. The ion beam created by this engine blows the space object and thus momentum transfer occurs. The second engine is needed to compensate for the thrust of the first one and to hold the spacecraft near the space debris. Modern ion thrusters allow exerting a force of the order of a dozen mN on a space object [14]. Calculations show that Ion Beam Shepherd can deorbit space debris with a mass of several tons from an orbit with a radius of one thousand kilometers per
few months [8]. The attitude motion of space debris has a significant effect on its descent time. This factor was considered in [15]. Another important factor that influences the motion of a body at a given altitude is the effect of the atmosphere. A rough estimation shows that for a body on a circular orbit of 250 km altitude the magnitude of the drag force is comparable to the force created by the ion flow. The purpose of this work is to study the effect of the atmosphere on the space debris behavior when it is removed from the orbit by an Ion Beam Shepherd.

2. Mathematical model

The plane motion of space debris under the influence of gravitational, aerodynamic and ion beam forces and torques is considered. The spacecraft-shepherd is considered as a material point (point A on figure 1). It is held at a constant distance from the space debris by its control system. In the orbital reference frame associated with the space debris $B_{x_o}y_o$, the radius vector of the shepherd has coordinates $\rho_A = [0, \rho_A]^T$. The task of the spacecraft control is beyond the scope of this study. It is supposed that the space debris is a rocket stage which is modeled by a rigid body of cylindrical shape with the center of mass at point B. The inertial frame is denoted as $O_{x_p}y_p$. The axis $x_p$ passes through the pericenter of initial space debris orbit. Axis $x_o$ of the orbital frame lies along the radius vector $r$ of the space debris center of mass, and the axis $y_o$ is directed towards the orbital flight. The body frame $B_{x_b}y_b$ is fixed relative to the space debris. Its axes coincide with the principal axes. The ion beam reference frame $A_{x_a}y_a$ is connected with shepherd. The axis $x_a$ is directed along ion beam axis to flight direction, and the axis $y_a$ lies in the plane of the orbit and completes the right-handed set.

![Space debris motion](image)

Figure 1. Space debris motion under the influence of aerodynamic, and ion beam forces and torques.

Equations of the space debris can be obtained with by the means of Lagrange equations of the second kind.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j$$  (1)
where $L$ is the Lagrangian, $q_j = \{v, r, \varphi\}$ is the generalized coordinates, $v$ is the true anomaly angle, $r$ is the distance between the center of Earth and the space debris center of mass, $\varphi$ is the deflection angle of the space debris longitudinal axis from its local vertical $OB$, $Q^\prime_j$ is the non-potential generalized forces. The Lagrangian of the space debris can be written in the form

$$L = \frac{m(r^2 \dot{v}^2 + \dot{\varphi}^2)}{2} + \sum_j \left( \frac{\mu m}{r} \right) - \frac{3 \mu (J_x \cos^2 \varphi + J_y \sin^2 \varphi + J_z)}{2r^3}. \tag{2}$$

Here $\mu$ is the gravitational constant of the Earth, $m$ is the mass of the space debris, $J_x$ is the longitudinal moment of inertia of the space debris, $J_y$ and $J_z$ are the transversal moments of inertia.

Generalized forces can be calculated through the virtual work

$$\delta W = (F^i + F^o) \cdot \delta r + (L^i + L^o) \cdot \delta \varphi = \sum_j Q^\prime_j \delta q_j \tag{3}$$

where

$$F^i = [F^x, F^y]^T$$

is the ion beam force, which is specified in the orbital reference frame, $F^o = [-A, N]^T$ is the aerodynamic force in body frame, $A$ is the axial aerodynamic force, $N$ is the normal aerodynamic force, $L^i$ is the ion beam torque, $L^o$ is the pitch moment, $\psi = v - \varphi$.

$$Q^x = -A \cos \varphi + N \sin \varphi + F^i, \quad Q^y = Ar \sin \varphi + Nr \cos \varphi + F^o r + \frac{L^i + L^o}{2}, \quad Q^\varphi = -\frac{L^o}{u} - L^i. \tag{4}$$

Aerodynamic force and torque can be calculated as

$$A = \frac{\rho V^2}{2} S C_A(\alpha), \quad N = \frac{\rho V^2}{2} S C_N(\alpha), \quad L^o = \frac{\rho V^2}{2} S C_{\psi o}(\alpha) \tag{5}$$

where $\rho$ is the density of the atmosphere, $V$ is the velocity of the space debris, $S$ is the space debris cross-section area, $l$ is the characteristic length of the space debris, $C_A(\alpha), C_N(\alpha), \text{ and } C_{\psi o}(\alpha)$ are dimensionless aerodynamic coefficients of the axial force, normal force and pitch moment, $\alpha$ is the angle of attack.

$$\alpha = \text{sign}(v \sin \psi - v_y \cos \psi) \arccos \left( \frac{v_y \cos \psi + v \sin \psi}{V} \right) \tag{6}$$

Here $V_x$ and $V_y$ are projections of the velocity vector on the axis of the inertial reference frame $Ox_py_p$.

For calculation ion beam force and torque the self-similar model of ion engine plume exploration and fully diffused reflection model of ions interaction with the surface can be used. The surface of the space debris can be divided into triangles and force acting on each $j$-th triangle can be calculated as [14, 15]

$$F^j = -m_0 n_0 R_s^2 S_j \frac{3(y^2_{ij} + z^2_{ij})}{x^2_{ij} \tan^2 \alpha_0} \left( V_j \cdot \hat{N}_j \right) V_j \tag{7}$$

where $m_0$ is the ion mass, $n_0$ is the plasma density at the beginning of the far region, $R_s$ is the radius of the beam at the beginning of the far region, $\alpha_0$ is the divergence angle of the ion beam, $V_j = \left[ -u_{0x}, -u_{0y}, -u_{0y}, -u_{0z} \right]$ is the velocity of the ion flux at the barycenter $P^j$ of $j$-th triangle, given by its coordinates in $Ax_yz_o$ frame (figure 1); $u_0$ is the axial component of the ion flux.
velocity, the radius vector of the barycenter point $P_j$ in $Ax_uy_3z_3$ frame has coordinates $d_j = \left[-x_{aj}, y_{aj}, z_{aj}\right]^T$, $S_j$ is the area of $j$-th triangle. The total ion beam force and torque vector, relative to the space debris center of mass, can be calculated as

$$F_j = \sum_{j \in J} F_j, \quad L_i = \sum_{j \in J} d_j \times F_j$$

(8)

where $J$ is an index set including the subset of triangles inside the ion beam.

Equations (1) after substitution of (2) and expression of second derivatives take the form

$$\ddot{r} = \frac{\mu}{r^2} \frac{-3 \mu (3J_x \cos^2 \varphi + 3J_z \sin^2 \varphi - J_x - J_z + 2J_\phi)}{2mr^2} + \frac{Q_y}{m},$$

(9)

$$\ddot{\varphi} = -\frac{2 \dot{r}}{r} + \frac{3 \mu (J_x - J_z) \cos \varphi \sin \varphi}{mr^2} + \frac{Q_\varphi}{m},$$

(10)

$$\ddot{\varphi} = -\frac{2 \dot{r}}{r} + \frac{3 \mu (J_x - J_z) \cos \varphi \sin \varphi (J_z + mr^2)}{J_z r^2 m} + \frac{Q_\varphi}{J_z m} + \frac{Q_\varphi}{mr^2},$$

(11)

3. Results and Discussion

As an example, the removal of Cosmos 3M rocket stage from a circular orbit of 500 km altitude by ion beam is considered. The stage has a mass $m = 1400$ kg, its length is $L = 6.5$ m, and its radius is 1.2 m. The moments of inertia are $J_x = 1300$ kg m$^2$, $J_y = 6800$ kg m$^2$. The shepherd-spacecraft is held at a constant distance $\rho_A = 15$ m from the stage. It creates ion beam with the following parameters: the mass of particle (xenon) is $m_0 = 2.18 \times 10^{-3}$ kg, the plasma density is $n_0 = 2.6 \times 10^6$ m$^{-3}$, the radius of the beam at the beginning of the far region is $R_0 = 0.1$ m, the axial component of the ion flux velocity $u_0 = 38000$ m/s, the divergence angle of the beam is $\alpha_0 = 15^\circ$. Ion beam axis passes through the geometric center of the stage ($\beta = 0$ on figure 1), by which the point lying at the intersection of the axis and the plane of symmetry is meant. The Cosmos 3M rocket was modeled by a cylinder with flat butt ends, which were divided into 54 thousand triangles. The ion beam force and torque were calculated by equations (8), and they are shown in figures 2 and 3. The geometric center of the stage was chosen as the pivot point of the ion beam torque. The dimensionless aerodynamic coefficients were calculated using the Newton impact theory [16]. They are shown in figure 4. Torque coefficient $C_{mc}(\alpha)$ was calculated with respect to point $O_1$. This point is shown in figure 5.

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**Figure 2.** Dependence of ion beam force projections on deflection angle.

**Figure 3.** Dependence of ion beam torque projection $L_\alpha$ on deflection angle.
The simulation of the Cosmos 3M stage descent from the orbit was conducted by the means of equations (7)-(8). Figure 6 shows dependence of the stage radius vector \( r \) on time for the cases that take into account the influence of the atmosphere (solid line) and do not take it into account (dashed line). In the first case descent to an altitude of 100 km takes about 55 days, in the second case the process lasts more than 85 days. Figure 6 demonstrates that the atmosphere begins to have an effect comparable to the ion beam influence at a height of 300 km.

In the real situation, the center of mass of the stage does not coincide with its geometric center. Let us study the attitude motion of the rocket stage when its center of mass is at a distance of \( x_{\text{cm}} = L/4 \) from point \( O \) (figure 5). In this case, aerodynamic and ion beam torques should be recalculated relative the center of mass. We construct a phase portraits of equation (9) for fixed values of \( r \) and \( \dot{\gamma} \). Figure 7 demonstrates the phase portrait for the rocket stage altitude of 500 km. At this altitude, the aerodynamic forces and moments are negligibly small. Five areas can be distinguished in the phase portrait: three oscillation areas \( A_1, A_2, A_3 \); and two rotation areas \( A_4, A_5 \). These areas are separated from each other by separatrices that connect the saddle points \( s_1, s_2, \) and \( s_3 \). The separatrices are shown in figure 7 in bold lines. The instability of position \( s_2 \) and the existence of two oscillation areas \( A_1, A_2 \) in its vicinity is caused by the influence of the gravity gradient torque on the rocket stage. It should be noted that the rocket stage can oscillate with a large amplitude in the area \( A_3 \) or with a rather small amplitude in the areas \( A_1, A_2 \).

**Figure 4.** Dimensionless aerodynamic coefficients of the cylindrical stage.  
**Figure 5.** Aerodynamic forces and torque.  
**Figure 6.** Change in the Cosmos 3M stage height over time.  
**Figure 7.** The phase portrait of equation (9) for \( r = 6871 \text{ km} \).
When the altitude of the orbit decreases, the density of the atmosphere increases, and, accordingly, the values of the aerodynamic forces and moments acting on the rocket stage increase too. Figure 8 shows the phase portraits that were constructed for an altitude of 250 km taking into account the influence of the atmosphere (solid line) and without taking this influence into account (dotted line). In both cases the phase portraits do not differ qualitatively from that shown in Figure 7. The increase of the atmosphere influence leads, on the one hand, to a decrease in the areas of the oscillation regions $A_1$, $A_2$. The centers $c_1$ and $c_2$ approach the saddle point $s_2$. On the other hand, when we move away from the saddle point $s_2$, the phase trajectories become more elongated vertically (Figure 8). In Figure 9, the areas $A_1$, $A_2$ are shown enlarged. Phase portraits were obtained for a height of 150 km. It can be seen that the areas $A_1$, $A_2$ are even more compressed, and the oscillations inside the region $A_1$ can occur with a rather small amplitude around the inner separatrix.

![Figure 8](image1.png)  
**Figure 8.** Comparison of phase portraits for 250km when the atmosphere is taken into account (solid line), and when it is not taken into account (dotted line).

![Figure 9](image2.png)  
**Figure 9.** Comparison of phase portraits for 150km when the atmosphere is taken into account (solid line), and when it is not taken into account (dotted line).

The amplitude of the oscillations has a noticeable effect on the time of descent, in order to demonstrate this, we calculate the average for the period of oscillations force $F_i$ for various initial positions. The results of the calculations are shown in Table 1. The calculation was carried out for a height of 150km, taking into account the influence of the atmosphere ($V_0 = \pi/2$). Analysis of the table shows that the most favorable situation is when the stage at the initial instant of time is oriented perpendicular to the ion beam axis ($\varphi_0 = 0$). Deorbiting in oscillation mode near $s_2$ or inside $A_1$, $A_2$ areas (when $\varphi_0 \approx -\pi/2$) is not effective.

**Table 1.** The influence of the space debris amplitude of oscillations on the ion beam average force.

| $\varphi_0$ (rad) | $\varphi_0$ (rad/s) | Area          | $F_i$ (mN) | $F_0$ (N)  |
|------------------|---------------------|---------------|------------|------------|
| -1.64            | 0                   | $A_3$ near inner separatrix | -30.4      | -2.56 x 10^{-5} |
| -1.58            | 0                   | $A_1$         | -30.4      | -2.16 x 10^{-4} |
| -1.56            | 0                   | $A_2$         | -30.5      | 1.39 x 10^{-4}  |
| -0.785           | 0                   | $A_1$ inside  | -38.8      | -4.44 x 10^{-6} |
| 0                | 0                   | $A_1$ inside  | **-41.9**  | **-4.97 x 10^{-6}** |
| 1.40             | 0                   | $A_3$ near outer separatrix | -37.9      | 2.96 x 10^{-5}  |
| 1.57             | 0.002               | $A_4$ near separatrix | -35.7      | -1.81 x 10^{-5} |
4. Conclusion
The problem of large space debris removal, by an active spacecraft with two ion engines was considered in this article. The influence of the atmosphere on the space debris attitude motion under the action of the ion beam created by the active spacecraft engine was studied. The mathematical model, which describes a plane motion of space debris under the action of the ion beam, gravitational and aerodynamic forces and torques, was constructed using Lagrange formalism. For calculation ion beam force and torque the self-similar model of ion engine plume exploration and fully diffused reflection model of ions interaction with the surface were used. As an example, the removal of Cosmos 3M rocket stage from a circular orbit of 500 km altitude by ion beam was considered. The dimensionless aerodynamic coefficients were calculated using the Newton impact theory. It was shown that the presence of an atmosphere significantly reduces the time of descent from orbit. The analysis of phase portraits describing the deviation of space debris from the local vertical was carried out. It was shown that in the case of a cylindrical space debris, which center of mass lies on the axis of symmetry, taking into account the influence of the atmosphere leads to deformation of the phase portrait, but does not lead to its qualitative changes. It was found that the amplitude of oscillations of space debris has a noticeable effect on the ion beam force, averaged over the period of oscillation, and therefore it also affects total descent time. The most favorable situation is when the rocket stage at the initial instant of time is oriented perpendicular to the ion beam axis.

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