Application of decision–making parametric structures in the analysis of a compound planetary gear

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Abstract. The design process of planetary gears can be divided into two main stages, i.e. conceptual design and construction, that is the preparation of construction documentation. In the first stage, the constructor has a very limited number of project data. In the case of a reduction or multiplier transmission these are usually output parameters (torque and speed), characteristics of the driving and the driven machine. Therefore, the stage of gearbox design must be supported by preliminary calculations, which supports the selection of the correct technical solution, leading to the stage of the structural design preparation. In the method of systematic search, we used an algorithm for generating induction decision trees, based on entropy growth as a method related to machine learning. In the next step, the decision-making parametric structures were employed. In the future, the presented solution will allow further analyses and syntheses such as checking the isomorphism of the proposed solutions, determining the validity of constructions and/or operating parameters of the analysed gears.

1. Introduction
Graph-theoretical ideas are highly utilised by computer science applications. Especially in research areas of computer science data mining, clustering, images capturing, network etc; however, the scope of their applications includes studying and modelling in various areas such as, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. In the last dozen or so years, we can observe an extremely rapid trend for graph theory applications, decision trees and networks in various fields of science and technology: the theory of operational research, genetics, linguistics, econometrics, electronics to name a few. Thus, the need for further development of this theory and its methods has been growing. From a mathematical point of view, the theory of graphs and decision trees does not bring revolutionary innovation. The advantage of these methods, however, is the adaptation of mathematical methods and apparatus to the convenient modelling of phenomena of the contemporary world. Graphs, networks and trees are convenient as a type of a formal apparatus for machine system modelling, in hydraulic and transmission systems [1, 2, 3].

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Other attempts have been made to model planetary gears by means of diagrams e.g. Wolf’s pictograms [4], but the evolution of these methods has been halted by the lack of generalisation and lack of connections with other branches of mathematics [5, 6]. Also, the method based on signal flow graph theory for modelling of gears [7, 8] has not been quite popular. On the contrary, the graph-based methods have been independently and extensively developed for several recent years all over the world [7-9]. Unlike graphs, dendrite-tree structures do not have cycles, but there may be a different number of initial vertices. Therefore, a different approach has been developed as a translation of a directed graph of dependence, among others, for parametrically acting structures. This approach is different from the established solutions described in the literature on parametric automation machines and their applications, related to control systems, operating systems, importance analysis of construction and/or operating parameters, and analysis of gears previously modelled using other types of graphs.

2. Analysis of planetary gears

The planetary gear has derived its name from the similarity to the planetary system, with which it shares the common feature: the "sun," the central component around which the other elements are orbiting like planets. It is made of two concentric gear wheels: a central (solar) wheel with external toothing, fixed permanently on the shaft, and a crown wheel (also called the central one) with internal toothing. Other elements are 3 to 5 small gears – satellites connected with a yoke (a jumper). The design and model diagram of the selected variant of the transmission [10-11] is shown in figure 1.

One of the most characteristic features of a planetary gear is the ability to transfer power based on three gear ratios. The system of internal and external wheels transfers the drive with high torque even from engines with high powers, while maintaining a small size. Thanks to this, the gear unit can be placed in a small drive medium, which does not burden the machine structure. The advantages of planetary gears include the following:
- one of the highest efficiencies (98-99%),
- high reliability and low consumption,
- high power transfer,
- constant gear ratio (this is the only advantage of the chain transmission),
- other transmissions have slip gears that change the transmission ratio.
A simple gearing has a limited ratio. Typically, the absolute value of the base ratio does not exceed 10 [12]. When a larger gear ratio is needed, multi-stage systems or coupled systems are used. figure 2 shows a diagram of a two-stage ring gear [13].
Figure 3 presents an exemplary constructional solution of the transmission from figure 2.
In circulating gears, as well as in other types of gear transmissions, not all parameters can be determined simultaneously at the very beginning of the design, particularly when the contour graph method is used.

3. Subjects of investigations

3.1 2DOF planetary gear
The first considered gear is graphically presented as a functional scheme (Figure 4). It consists of three pseudo-stages, despite having 3 satellite wheels and 3 arms, referred to as carriers. In fact, there are not three classical stages because the system has a serial-parallel layout. In the classical meaning, the stages are serially arranged. The gear consists of eight movable parts, among them: three pinions, – i.e. sun gears denoted in Figure 4 as 1, 4 and 7, three planet gears 2, 5 and 8, three-ring gears 3, 6 and 9 as well as three arms h3, h5 and h8. However, it should be highlighted that gears 3 and 4 as well as wheels 6 and 7 are connected, creating the element that is simultaneously the sun and ring wheel. Similarly, all arms are united creating one arm b, which drives the output shaft II. The input shafts are these two – marked as I and III, respectively. Moreover, shaft I drives the sun gear 1, and independently shaft III drives the ring gear 9. Therefore, this is 2 DOF planetary gear, which is also calculated below, based upon an adequate formula. The kinematical scheme of the gear – grasping precisely all the above-listed relationships and descriptions – is shown in figure 1. It allows for execution of necessary structural and kinematic analyses. Moreover, all rotational pairs (full joints) are equipped in ball bearings (one for every kinematical pair) in accordance with recommendations of the classical mechanism and machine science.

\[ M = 3 \cdot n - 2 \cdot c_5 - c_4 = 3 \cdot 8 - 2 \cdot 8 - 6 = 2, \]

where: \( n = 8 \) – the number of moving links, \( c_5 = 8 \) – the number of joints of class 5 (full joints), \( c_4 = 6 \) – the number of joints of class 4 (half joints).

In general, the considered system has two DOF gears, as in [14, 15]:
The geometrical and structural data are as follows:
a) tooth numbers for consecutive geared wheels:
\[ z_1 = z_4 = z_7 = 18, \quad z_2 = z_5 = z_8 = 45, \quad z_3 = z_6 = z_9 = -108 \]
b) moduli of geared wheels (the dimensions are given in appropriate standards):

\[ \begin{align*}
I: m_1 &= m_2 = m_3 = 1 \text{ mm}, & m_4 &= m_5 = m_6 = 1.5 \text{ mm}, & m_7 &= m_8 = m_9 = 2 \text{ mm}, \\
II: m_4 &= m_5 = m_6 = 1.5 \text{ mm}, & m_7 &= m_8 = m_9 = 2 \text{ mm}, \\
III: m_4 &= m_5 = m_6 = 1.5 \text{ mm}, & m_7 &= m_8 = m_9 = 2 \text{ mm},
\end{align*} \]

c) the number of satellite wheels in consecutive parallel planes: \( s_2 = s_3 = s_8 = s = 3 \). For example, the pitch radiiuses of adequate geared wheels have the following values:

\[ \begin{align*}
r_1 &= \frac{m_1 \cdot z_1}{2} = 9 \text{ mm}, & r_2 &= \frac{m_1 \cdot z_2}{2} = 22.5 \text{ mm}, & r_3 &= \frac{m_1 \cdot z_3}{2} = 54 \text{ mm}, & r_4 &= \frac{m_1 \cdot z_4}{2} = 13.5 \text{ mm}
\end{align*} \]

3.1 1 DOF planetary gear

The second object of kinematic analysis is identical to the previous planetary gear except for the ring gear \( g \) that here is braked (Figure 5). What follows is that the gear has now one input shaft \( I \) and one output shaft \( II \), because the second input shaft \( III \) is immobilised [15].

![Figure 5. Kinematic scheme of compound 1 DOF planetary gear.](image)

The system possesses one DOF, because:

\[ M = 3 \cdot n - 2 \cdot c_5 = 6.7 - 2 \cdot 7 - 6 = 1, \]

where: \( n = 7 \) – the number of moving links, \( c_5 = 7 \) – the number of class 5 joints (full joints), \( c_4 = 6 \) – the number of class 4 joints (half joints).

Moreover, it should be noted that in the case of ring gear braked \( g \), the rotational kinematic pair (full joint) between this wheel and the base is inactive, and as such is not taken into account in structural considerations.

4. Method of contour equations (graphs)

Graphs play an important role in the kinematic analysis. Kinematic analysis is the study of relative motions associated with the links of a mechanism or machine and is a critical step towards the proper design of mechanisms. The problem can be formulated by the graphical, vector algebra, matrix, or other mathematical methods. The general idea of the graph-based modelling of mechanical systems consists of the following steps [5]:

- discretisation of a mechanical system. This means that appropriate simplifications have to be made. Some aspects are omitted. Some structural elements are considered essential and they are interpreted as graph vertices.
- assignment of the graph to the mechanism (especially the planetary gear) from special rules.
- derivation of special subgraphs, e.g. f-cycles or contours. These subgraphs can be singled out based upon the graph-theoretical algorithms, which causes that the approach is simple and algorithmic,
- listing the codes of these graph elements. The encoding rules are clearly defined, thus preventing mistakes,
- generation of a system of equations in an algorithmic way using codes.
- solving the obtained system in a chosen algebraic way to obtain needed angular velocities, ratios, forces, accelerations, etc.
4.1. Method of contour equations for the analysis of 2 DOF gear

Kinematic data for the first considered planetary gear have been established from or related to the structural layout of the considered planetary gear:

rotational velocity of shaft I: \( \omega_1 = \omega_3 = 150.08 \text{rad/s} \),

rotational velocity of shaft III: \( \omega_{m} = \omega_9 = -0.314 \text{rad/s} \),

additional data resulting from the gear structure:

\( \omega_4 = \omega_3 \), \( \omega_{h_2} = \omega_{h_5} = \omega_{h_6} = \omega_8 = \omega_{9} = \omega_2 \).

The following problems can be solved:

- find the absolute output rotational velocity \( \omega_2 \) of the output shaft II (that is also arms \( h_2, h_5, h_6 \) forming the main arm \( h \) connected to the shaft II - Figure 1), if the input shafts I (sun 1) and II (ring gear 9) rotate with input rotational velocities \( \omega_1 = \omega_3 \) and \( \omega_9 \), respectively,

- find the transmission ratio \( i_{1,h} \),

- find the absolute rotational velocities \( \omega_2, \omega_3 = \omega_4, \omega_5, \omega_6 = \omega_7 \), and \( \omega_8 \) of the other movable elements, i.e. gears 2, 3, 4, 5, 6, 7 and 8.

The formulated tasks will be first solved by means of the contour graph method. As mentioned, the basics of the method may be found in the book [16]. Therefore here, the equations are used without detailed explanations or derivations. The further considerations are related to the contour graph assigned to the analysed gear (Figure 6).

![Figure 6. Contour graph model of 2 DOF planetary gear.](image)

Aiming for increasing the usefulness of graph modelling as well as to improve the understanding of the derived equations, the descriptions of relative rotational velocities were placed near the adequate arcs. For example, \( \omega_3^i \) denotes the vector of the relative rotational velocity of wheel 1 in relation to wheel 2. We use here notation rule that vector variables are written in bold instead of a traditional arrow above them.

The number of independent contours is determined based on the following formula [16]:

\[ N = c - n = 14 - 8 = 6 \],

where \( c \) – the number of joints (\( c = c_3 + c_4 = 14 \)).

The algorithm for the method consists of contour equations for consecutive contours [16]:

\[ \text{(contour equation)} \]
for contour I:

\[ \omega_0^{2} + \omega_2^{b2} + \omega_1^{2} + \omega_0^{1} = 0, \]

\[ (r_1 + r_2) \times \omega_2^{b2} + r_1 \times \omega_1^{2} = 0, \] (1)

for contour II:

\[ \omega_0^{2} + \omega_2^{b2} + \omega_1^{3} + \omega_0^{3} = 0, \]

\[ (r_1 + r_2) \times \omega_2^{b2} + (r_1 + 2r_2) \times \omega_1^{3} = 0, \] (2)

for contour III:

\[ \omega_0^{4} + \omega_4^{b4} + \omega_0^{b4} = 0, \]

\[ r_4 \times \omega_4^{b4} + (r_4 + r_5) \times \omega_0^{b4} = 0, \] (3)

for contour IV:

\[ \omega_0^{5} + \omega_5^{b5} + \omega_0^{b5} = 0, \]

\[ (r_4 + r_5) \times \omega_5^{b5} + (r_4 + 2r_5) \times \omega_0^{b5} = 0, \] (4)

for contour V:

\[ \omega_7^{7} + \omega_7^{b7} + \omega_7^{b} = 0, \]

\[ r_7 \times \omega_7^{b7} + (r_7 + r_8) \times \omega_8^{b} = 0, \] (5)

for contour VI:

\[ \omega_8^{8} + \omega_8^{b8} + \omega_8^{b} = 0, \]

\[ (r_7 + r_8) \times \omega_8^{b8} + (r_7 + 2r_8) \times \omega_0^{b} = 0. \] (6)

Since the edges of the graph (digraph) have directions, they can be called arcs. The oriented cycles are in this case referred to as contours to distinguish them from others. The contours are marked by Roman numerals.

5. Application of decision–making parametric structures

In the graphs of dependencies parametrically analysed are the so-called 'bound' decisions. The results obtained in the subsequent decisions depend on the initial decisions, which enables the creation of dynamic models. The decision process and the space of the possible states of the analysed system are described parametrically from each of the graph tops.

A graph is an ordered pair \( G= (X, R) \), in which \( X \) is a finite set of elements called vertices of a graph, and \( R \) is a set of pairs \( (x_i, x_j) \) \((x_i, x_j \in X)\) called the edges of the graph. In the case of parametric graphs, the notation introduced by Deptuła [17, 18] defines the signs: \( G= (Q, Z) \), where \( Z \) is a set of pairs \( (z_i, z_j) \) \((z_i, z_j \in Z)\). Figure 7 shows an example of a parametric graph \( G \).
The oriented game graph is shown in Figure 7 is composed of a set of vertices Q:

\[ Q = \{ q_1, q_2, q_3, q_4, q_5 \} \]  \hspace{1cm} (13)

and of a set of edges Z, that is an ordered pair of vertices:

\[ Z = \{ z_1, z_2, z_3, z_4, z_5, z_6 \} \]  \hspace{1cm} (14)

The path in the \( G=(Q, Z) \) is the edge sequence \( (z_1, z_2), (z_2, z_4), ..., (z_{k-1}, z_k) \) in which for each \( j \in \{2,3,...,k\} \), \( z_j \in R \) and vertices \( q_i, q_j, ..., q_k \) are different pairs. Vertex \( q_i \) is called the beginning of the path, and the top \( q_k \) – the end of the path. As a result of the graph distribution from the chosen vertex, a tree structure with cycles is obtained in the first step and, next, a general game tree structure is obtained. Each of them has an appropriate analytical formulation \( G^i \) and \( G^{++} \). A game tree structure is a part of the systematic searching method. A start vertex \( q_i \) is chosen in the first step. The algorithm of graph distribution of dependence on parametric structures can be found in the literature [19].

The graph in figure 7 is distributed into 5 structures forming the D set:

\[ D = \{ G^{++}_{q_1}, G^{++}_{q_2}, G^{++}_{q_3}, G^{++}_{q_4}, G^{++}_{q_5} \} \]  \hspace{1cm} (15)

Figures 8-9 show the structures from each of the vertices: \( q_1, q_2, q_3, q_4, q_5 \) of the graph from figure 7.
Each structure has its own analytical formula. For example, the structures in figure 4 are described by the following formulas:

\[ G_1 = (q_1 q_2 q_3 q_4 q_5), \]
\[ G_2 = (q_1 q_2 q_3 q_4 q_5), \]
\[ G_3 = (q_1 q_2 q_3 q_4 q_5), \]
\[ G_4 = (q_1 q_2 q_3 q_4 q_5), \]

5.1. Structural representation including game-tree structures

Kinematic synthesis is the reverse problem of kinematic analysis. In this case, the designer is challenged to devise a new mechanism that satisfies certain desired motion characteristics of an output link. The kinematic synthesis problem can be further divided into three interrelated phases:

1. Type synthesis refers to the selection of a specific type of mechanism for product development. During the conceptual design phase, the designer considers many types of mechanism as possible and decides what type shows the highest potential of meeting the design objectives.

2. Number synthesis deals with the determination of the number of links, type of joint, and number of joints needed to achieve a given number of degrees of freedom of a desired mechanism.

Various methodologies have been developed for the systematic enumeration of kinematic structures. A thorough understanding of the structural characteristics of a given type of mechanism is critical for the development of an efficient algorithm. In the 2000s, certain approaches for structural synthesis of kinematic chains were presented [20-25]. Nelson et al. [26] the kinematic, power flow, and efficiency analysis of epicyclic gear trains. Wojnarowski et al. [27] presented a survey of works connected with the problem of the modelling of gears by means of versatile graph theory models. Zawiślak [4] reviewed selected graph applications for gear design from the AI perspective. A single atlas of graphs can be used to enumerate an enormous number of mechanisms. An exemplary transformation is shown in figure 10.
A special group of graphs are parametric graphs. The structural topology of a mechanism can be uniquely identified. Using graph representation, the similarity and difference between two different mechanism embodiments can be easily recognised. In particular, parametrically acting graphs can be used for systematic classification of mechanisms.

For the contour graph model of 2 DOF planetary gear presented in figure 6, a dependency graph can be used (figure 12).

For the graph with a path from the entrance to the output from Figure 12, it is possible to build a set $D$ of parametrically acting structures:

$$D = \{ G_1^{++}, G_2^{++}, G_3^{++}, G_5^{++}, G_6^{++} \}$$

(16)

Each of the structures has an analytical formula describing the distribution of the dependency graph. Exemplary structures ($G_1^{++}, G_2^{++}, G_3^{++}, G_4^{++}$) have the following formulas.
In the next stage, we select the optimal structuring that is parametric. The following relationships were taken additionally into account: \( \omega_i = \omega_i^0 = \omega_j \), \( \omega_i^b = \omega_i^b = \omega_j \), \( \omega_j = -\omega_{j+1} = \omega_j \) for \( j = 1, 2, \ldots, 9 \):

\[
\begin{align*}
\omega_1 + \omega_2 + \omega_3 &= \omega_1, \\
\omega_2 + \omega_3 + \omega_4 &= 0, \\
-\omega_2 + \omega_3 + \omega_4 &= 0, \\
\omega_5 + \omega_6 + \omega_7 &= 0, \\
-\omega_5 + \omega_6 + \omega_7 &= 0, \\
\omega_1 + \omega_2 + \omega_3 &= \omega_1, \\
(r_1 + r_2) \cdot \omega_1^2 + r_1 \cdot \omega_1^2 &= 0, \\
(r_1 + r_2) \cdot \omega_1^2 + (r_1 + 2r_2) \cdot \omega_1^2 &= 0, \\
r_4 \cdot \omega_5^2 - (r_4 + r_5) \cdot \omega_5^2 &= 0, \\
(r_4 + r_5) \cdot \omega_5^2 + (r_4 + 2r_5) \cdot \omega_5^2 &= 0, \\
r_7 \cdot \omega_7^2 - (r_7 + r_8) \cdot \omega_7^2 &= 0, \\
(r_7 + r_8) \cdot \omega_7^2 + (r_7 + 2r_8) \cdot \omega_7^2 &= 0.
\end{align*}
\]

The matrix-form equation representing the above system of equations is given below (where \( \omega_1 = \omega_i = 150.08 \) rad/s and \( \omega_0 = -0.314 \) rad/s):

\[
[A] \cdot [x] = [b]
\]

Where:

\[
[x] = [\omega_1 \ \omega_2 \ \omega_3 \ \omega_4 \ \omega_5 \ \omega_6 \ \omega_7 \ \omega_8 \ \omega_9]
\]

\[
[b] = [157.08 \ 0 \ 0 \ 0 \ -0.314 \ 0 \ 0 \ 0 \ 0 \ 0].
\]

In the next stage, the Willis method should be used and the analysis carried out for gear with 2 DOF. In the work on compound planetary gears [28] for the analysed gear, a special algorithm was developed to determine the optimal number of teeth. This method can also be successfully used to analyse failure modes and causes [29-32].

In the next stage, we select the optimal structuring that is parametric. The following relationships were taken additionally into account: \( \omega_i = \omega_i^0 = \omega_j \), \( \omega_i^b = \omega_i^b = \omega_j \), \( \omega_j = -\omega_{j+1} = \omega_j \) for \( j = 1, 2, \ldots, 9 \):

\[
\begin{align*}
\omega_1 + \omega_2 + \omega_3 &= \omega_1, \\
\omega_2 + \omega_3 + \omega_4 &= 0, \\
-\omega_2 + \omega_3 + \omega_4 &= 0, \\
\omega_5 + \omega_6 + \omega_7 &= 0, \\
-\omega_5 + \omega_6 + \omega_7 &= 0, \\
\omega_1 + \omega_2 + \omega_3 &= \omega_1, \\
(r_1 + r_2) \cdot \omega_1^2 + r_1 \cdot \omega_1^2 &= 0, \\
(r_1 + r_2) \cdot \omega_1^2 + (r_1 + 2r_2) \cdot \omega_1^2 &= 0, \\
r_4 \cdot \omega_5^2 - (r_4 + r_5) \cdot \omega_5^2 &= 0, \\
(r_4 + r_5) \cdot \omega_5^2 + (r_4 + 2r_5) \cdot \omega_5^2 &= 0, \\
r_7 \cdot \omega_7^2 - (r_7 + r_8) \cdot \omega_7^2 &= 0, \\
(r_7 + r_8) \cdot \omega_7^2 + (r_7 + 2r_8) \cdot \omega_7^2 &= 0.
\end{align*}
\]

The matrix-form equation representing the above system of equations is given below (where \( \omega_1 = \omega_i = 150.08 \) rad/s and \( \omega_0 = -0.314 \) rad/s):

\[
[A] \cdot [x] = [b]
\]

Where:

\[
[x] = [\omega_1 \ \omega_2 \ \omega_3 \ \omega_4 \ \omega_5 \ \omega_6 \ \omega_7 \ \omega_8 \ \omega_9]
\]

\[
[b] = [157.08 \ 0 \ 0 \ 0 \ -0.314 \ 0 \ 0 \ 0 \ 0 \ 0].
\]
6. Conclusions
The game-tree structures method can be an excellent tool for gathering and transforming data. After encoding, the data can be repeatedly reproduced and globally distributed via the programme. In expert systems, the question of structuring the inference process (obtaining knowledge) consists of two separate parts:
- mapping of deductive activities specific to a given subject field (defining the structure of inference)
- conducting the deduction process by the requesting mechanism (determination of the inference function).

This approach to the deduction process makes it possible to represent the inference structure in the form of a parametrically directed graph with a defined span (i.e. the maximum or the minimum number of arcs derived from one node).

Graph nodes in the inference structure represent individual information states, and edges are logical transition operations that result in state transformation. Initial nodes are variants of the initial problem description, end nodes are the ending of the inference process. The graph-based methods of analysis and synthesis of planetary gears provide an alternative for the accomplishing of the tasks in question. The use of graph-based methods for the analysis of the presented gear type with a closed internal loop has been described and performed step by step giving a detailed explanation to all activities. The methods are relatively uncomplicated, algorithmic and general. This confirms the usefulness of these methods for verifying the correctness of gear analysis. The basic concept of modelling accounts for selected general properties or aspects of a mechanism, e.g. analysis of kinematics. Therefore, the main rotating elements of a mechanical system are represented by graph vertices and the relations among them are modelled via edges. The applied rules of assignment start from the idea: the singled out main gear elements such as e.g. gear wheels, planets and arms are considered as vertices. In particular, all elements rotating around the main physical axis of a planetary gear are represented by the vertices of a polygon, moreover the mutual relations among them: “the rotation around the same axis” are coded via the polygon edges and their diagonals, however, the latter are not drawn for simplicity of the picture. However, they are used for the determination of f-cycles.

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