Uncertainty Analysis of Thermostatically Controlled Loads Response Potential

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Abstract. Thermostatically Controlled Loads (TCLs) are an important type of dispatchable resources to participate in power balancing in short-time scale, which have huge response potential and fast response speed. But since the response of TCLs is uncertain, it is difficult for the control center to get them directly. To solve this problem, this paper proposed a novel method to evaluate the uncertainty of TCLs response potential based on the aggregated model, which divided the response potential into two parts: expectation value and distribution characteristics. According to probability distribution, the uncertainty of TCLs potential was analyzed. Simulation results show the effectiveness of the method.

1. Introduction

With the fast development in communication and control systems, flexible loads are emerging as low-cost alternatives to conventional generation resources. Flexible loads are got to participate in the operation of the power grid by load aggregators (LA). Thus, from the view of LA, it is a prerequisite to get the potential of the inner loads. So, it is important to quantify the response potential of flexible loads.

Thermostatically controlled loads (TCLs) such as air conditioners, refrigerators and water heaters, represent about 20% of the total electricity consumption in the United States, while the number is 10% in China. Due to their thermal energy storage characteristics, TCLs are very suitable as fast flexible loads. From recent studies, TCLs are widely considered to possess huge demand response (DR) potential and flexibility. A few pilot studies have proven that TCLs can offer more lucrative fast ancillary services such as frequency regulation or load-following. [1,2].

The modeling and potential assessment of TCLs have been studied a lot in the literatures. For a single TCL, it can be modeled by first-order ordinary differential equations using equivalent thermal parameters (ETP). But for massive TCLs, the aggregation model is quite difficult. Ref. [3] transformed the conventional dynamic model of the TCL into the continuous power model and then modeled the aggregated TCLs as a stochastic battery model with dissipation. Ref. [4] developed a Lyapunov-stable sliding mode controller to model the TCLs.
The two-state resistance-capacitance (RC) model and physical-statistical approach are also commonly used in the aggregation of TCLs [5,6]. Ref. [5] uses physically-based load models to inform the development of a new theoretical model that accurately predicts changes in load resulting from changes in the thermostat temperature set points. Ref. [6] use data collected from residential refrigeration units operating in 214 different households to propose a strategy to select parameters when simulating a TCL population. Ref. [7] presents a fitted regression model based on a combination of Energy Plus and two-state models for TCLs. first, DR potential for commercial and residential buildings are got from Energy Plus models and two-state RC models respectively. Subsequently, regression models are fit to each dataset to predict DR potential. This method is quite fast while ensuring the high accuracy, which is a novel idea for the LA or the control center to get the potential of inner loads.

However, although there are many studies have done on aggregation modeling of TCLs, further studies are still needed. Since the aggregated power of a TCL population is uncertain with the different parameters, acceptance and ambient temperature etc, which is considered insufficient in the current studies.

In this paper, a DR potential assessment method of TCLs is proposed. By this method, the distribution characteristics and response uncertainty of TCLs potential are also analyzed.

The rest of this paper is organized as follows. Individual and population are modeled established separately in Section II. In Section III, the evaluation method and process are provided. In Section IV, the response potential and distribution characteristics of air-conditionings are simulated as case study, and conclusions are drawn in Section V.

2. Modeling approach

2.1. Individual TCL Model

An Individual TCL can be modeled by first-order ordinary differential equation as follows [8]:

$$\dot{T}_i(t) = \frac{1}{C_i R_i} [T_{e,i} - T_i(t) - s_i(t) R_i P_i], \quad i=1,2,\ldots,N_L$$

(1)

Where, $T_i$ and $T_{e,i}$ are the internal and ambient temperatures, respectively (in °C). $C_i$ and $R_i$ are the thermal capacitance (kWh/°C), in series with a thermal resistance (°C/kW). $s_i(t)$ is a binary variable, $s_i(t) = 1$ denotes TCL is on while $s_i(t) = 0$ denotes TCL is off. $P_i$ is the load’s cooling (or heating) power (kW).

2.2. TCLs Population Model

The aggregated power of a large TCL population can be shown as:

$$P_{agg}(t) = \sum_{i=1}^{N_L} P_i s_i(t)$$

(2)

In Ref. [9], we established a simplified aggregated model of TCLs by the mathematical transformation. The upper limit and lower limit of $N_L$ TCLs can be shown as below:
Where, $P^u_{agg}$ and $P^d_{agg}$ are the upper and lower limit of aggregated power. $N$ represents the number of TCLs. $\theta_a$ is the ambient temperature. $E$ represents the mathematical expectation of random variables. Specifically, $E[\theta_{set}]$ represents the mathematical expectation of temperature setpoint. $E[\delta]$ represents the mathematical expectation of deadband. $E[R]$ represents the mathematical expectation of energy efficiency. $E[R]$ represents the mathematical expectation of thermal resistance.

Furthermore, the aggregated power of the TCLs can be estimated by any value in the interval $[P^d_{agg}, P^u_{agg}]$, which can be expressed as:

$$
\tilde{P}_{agg} = \alpha P^d_{agg} + (1 - \alpha)P^u_{agg}, \quad \alpha \in [0,1]
$$

### 3. Evaluation of TCLs Potential

In this paper, DR potential of TCLs is represented by a normal distribution, which is divided into two parts: one is the expectation value. Across end-uses, a set of flexibility index can be used to quantify the DR resource potential. The filters have three components: sheddability (S), controllability (C), and acceptability (A). The other one is the distribution variance, reflecting the uncertainty under different internal and external factors, such as user's communication reliability, equipment reliability, user's acceptance willingness and so on.

#### 3.1. Flexibility Filters

Sheddability relates to physical constraints of the end-use equipment. Controllability relates to the presence of suitable control systems at facilities. Acceptability relates to user attributes like human comfort and work schedules, and the DR mechanism, such as price and incentives. This fraction is difficult to assess. In the evaluation calculation, we have made assumptions using DR participation data from published literatures and pilots.

#### 3.2. Assessment Method

Response potential is the intersection of sheddability, controllability and acceptability ($S \cap C \cap A$). In order to simplify the calculation, acceptability and controllability are assumed to be coincident. So the relationship of the three sets is $S \supset C \supset A$ or $S \supset A \supset C$, converting the response potential into mathematical description is:

$$
Pot_h = s_h \cdot \min[c_h, a_h]
$$

Where, $h$ represents the statistical period. $s_h$, $c_h$ and $a_h$ represents the user's shed ability, controllability, and acceptability respectively, $Pot_h$ represents the user's response potential during $h$ period.
$s_h$ is calculated according to the following formula:

$$s_h = \frac{\bar{P}_{\text{base}}^{\text{agg},h} - \bar{P}_{\text{DR}}^{\text{agg},h}}{\bar{P}_{\text{base}}^{\text{agg},h}}$$

(7)

Where, $\bar{P}_{\text{base}}^{\text{agg},h}$ is the base power consumption of TCLs in $h$ period. $\bar{P}_{\text{DR}}^{\text{agg},h}$ is the average power consumption during DR event $s_h > 0$ represents TCLs load reduction potential, and $s_h < 0$ represents load increase potential.

Considering the uncertainty of TCL's actual response process, the response potential is divided into two parts: one is the expectation value, which can be represented by $P_{\text{Pot}}^{h}$, and the other one is the probability distribution, which is affected by user's communication reliability, equipment reliability, response willingness and other factors. According to the law of large numbers, in this paper, we express the TCL's response potential as a normal distribution: $DR_{h} \sim N(dr_{h},\delta_{h}^{2})$. $dr_{h}$ and $\delta_{h}$ are the expectation value and Standard deviation separately.

3.3. Evaluation Process

The evaluation process of TCLs response potential is shown in Fig. 1.

Step 1: acquire typical parameters of TCLs in certain area, including TCLs quantity, each TCL's capacitance, resistances, energy efficiency ratio and outside air temperature. Fit the probability density function of TCLs' typical parameters by the statistics of historical data. Generally, the uniform distribution or normal distribution is adopted.

Step 2: establish approximate aggregated model, which can be represented by (3)- (5). Then use this model to estimate the aggregated power of TCLs population.

Step 3: calculate users' controllability and acceptability to a certain DR strategy by the statistics of pilots. The controllability and acceptability is between 0 and 100% separately.

Step 4: calculate the change of sheddability before and after the DR event, as the physical potential of TCLs.

Step 5: calculate the expectation value of TCLs' response potential.

Step 6: calculate the distribution characteristics of TCLs' response potential.
4. Case Study

4.1. Data
Air-conditioning is a typical fast flexible load. In the case of power imbalance or system reserve capacity is insufficient, the air-conditioning power consumption can be effectively reduced by set-point change without affecting the user’s comfort. In this paper, 10,000 air-conditionings are considered in the following simulation. Parameters are supposed to obey uniform distribution in Table 1.

| Parameter | $R$  | $C$  | $P_c$ | $\theta_{set}$ | $\delta$ |
|-----------|------|------|-------|---------------|--------|
| Value     | [1.5~2.5] | [1.5~2.5] | [6~8] | [24~26] | [1~2] |

4.2. Aggregated Power
The initial switch states are randomly selected. Under the outside air temperature is 32 °C and 32 °C, the upper and lower power limit of estimated aggregated power can be calculated by formula (3) ~ (5).
Figure 2. Aggregated power of TCLs

From Fig. 2. It can be seen when outside air temperature is 32 °C, the upper and lower limit of aggregated power is 13.4MW and 11.6MW separately, and the estimated average power is 12.5MW, which is quite similar to the real power simulated by Monte Carlo.

4.3. Expectation Value of Response Potential

The most ideal situation is firstly imitated in this paper. User’s controllability and acceptability are both hypothesized to be 100% and user’s response reliability is high. Response uncertainty is not considered in this case. When gradually increase the set value of air conditioning in different outside air temperature, the simulation results are shown in Fig. 2.

Figure 3. TCLs’ aggregated response potential under different setpoint adjustment and temperature

As can be seen from Fig. 1, aggregated response potential of TCLs has linear positive correlation with the adjustment amount ($\Delta \Theta_{net}$) of temperature set value. If $\Delta \Theta_{net}$ is constant, TCLs’ aggregated response potential will decrease as the external temperature is increasing.

4.4. Distribution Characteristics of Response Potential

It is hypothesized that there are 40% customers have terminal communication and control equipment and 60% customers are willing to participate. So the response potential expectation is $s_{h} \cdot \min[40\%, 60\%] = 0.4s_{h}$. If the external temperature is 32°C and $k = 0.01$, the aggregated potential
The probability distribution of air conditioning is shown in Fig. 3 when \( \Delta \theta_{sc} \) is increased by +1°C, +2°C and +3°C.

\[ DR_{h} \sim N(17.16\%, 0.03^2) \]

The blue curve in Fig. 2 represents the customer response distribution when the temperature set value is increased by 1°C. The red curve stands for the customer response distribution if the temperature set value is increased by 3°C. As can be seen from Fig. 2, the distribution curve moves to the right and has larger distribution when the set value of air conditioning increases. This indicates that the aggregated response potential will be improved as the temperature set value increases, but the uncertainty of customer response is also added.

5. Conclusion

This paper evaluates the aggregated response potential and its distribution characteristics when the uncertainty of residential TCL is considered. The simulation result indicates that TCLs’ potential will be distributed in a large section because of the uncertainty of actual response process. The distribution of response potential is mainly influenced by customers’ telecommunication reliability, devices’ reliability, customers’ response will and other internal and external factors. As can be seen from the evaluation results, residential TCL load has large aggregated potential by adjusting the set value of air conditioning at real-time stage, which can be used as an effective resource to participate in real-time active power balancing and system frequency stabilizing.

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