Development of Three-Dimensional Force Measurement Instrument Using Zero-Compliance Mechanism

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A three-dimensional force measurement instrument is designed and developed considering the basic principle of zero compliance mechanism. In the proposed system, the point of force and the detection point are connected in series and suspended with leaf springs. One of the suspensions operates to cancel the displacement of the other. If any force is applied at point of force that causes displacement, the detection point displaces proportionally to cancel out the displacement of the point of force. As a result, zero compliance is achieved at the point of force and applied force can be measured from the displacement of the detection point. This paper presents the design and construction of a three-dimensional force measurement instrument and the effectiveness of the proposed system is demonstrated by the experimental result of the vertical-direction force measurement.

Keywords: Zero-compliance, force measurement, infinite stiffness, electromagnetic force.

1. Introduction

In scientific research and industrial application, force measurement has significant impact since it is one of the prime physical quantities. Nowadays, measurement of small forces gets higher priority in the development of new materials such as paints, inks, cosmetics, drugs, etc. because interaction forces between molecules and surfaces are essential to know in these works. There are several techniques established for force measurement explained by Stefanescu [1]. In case of measuring small forces like micro force, atomic force microscope cantilever is the widely used tool so far. It measures the attraction and repulsion forces from the deflection of the cantilever. Similar to atomic force microscope cantilever, most of the present methods computed force from the displacement of the point of force. There are some works estimating force from the control signal of the suspension by maintaining the original position of the object like Boyden et al. [2]. However, the force measurement from control signal has limitation in measuring small force. In contrast, zero compliance mechanism can estimate the force without displacing the point of force and maintain good accuracy even in case of small force. Mizuno et al. [3] proposed to apply double series magnetic suspension to measure force.

There are several systems built based on the principle of zero compliance force measurement. Mizuno et al. [4] developed a single-dimensional force measurement device using double series magnetic suspension system. In this device, an electromagnet was installed at the top and the first floator which is also denoted as detection point was suspended with leaf springs. In contrast, the second floator that is also known as point of force was suspended completely without contact. At first, both floators were suspended in series and then force was applied to the second floator. After achieving zero-compliance, the force was estimated both from the displacement of the first floator and from the control current. The results demonstrated that the force measured from the displacement of the first floator had higher resolution than that measured from the control current. Mizuno et al. [5] built an apparatus for the demonstration of one dimensional force measurement. In this apparatus, the second floator was suspended with leaf springs similar to the first floator. Hayashi et al. [6] presents a force measurement instrument with zero-compliance mechanism using a cantilever. Besides these single-dimensional force measurement systems, the first work on multidimensional force measurement has been conducted by Ishii et al. [7]. This apparatus focuses on measuring tri-axial force by using double series magnetic suspension. Like the structure of the first apparatus [4], the first floator was attached with leaf springs and the second floator was suspended completely without contact. Due to the suspension of the point of force, the second floator rotated, which disturbed the measurement even if in the zero-compliance condition. Additionally, there was some nonlinearity effect of permanent magnet in the measurement result. Iida et al. [8] developed another three-dimensional force measurement apparatus which focused on measuring vertical direction axial force and rotation force along x and y axis. Voice coil motor was used in this system as linear actuator to reduce the nonlinearity effect of permanent magnet.

To achieve stable zero compliance condition, this work focuses on designing and construction of a three-dimensional force (x-y-z axial force) measurement.

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system by using the principle of zero-compliance mechanism. In this system, the second floator is suspended with leaf springs as well as the first floator to achieve stable zero compliance and to constrain its motions into translations. Voice coil motors are used to reduce the nonlinearity effect of the permanent magnet in force measurement but complexity of the structure increases much more with the voice coil motor in all three directions. To avoid an excessive complexity, Electromagnet and permanent magnet are used only in the z-direction.

2. Principle of Measurement

2.1 Zero-compliance Mechanism

The force measurement using zero-compliance mechanism is proposed by Mizuno et al. [5]. The detection point and the point of force are suspended by series connection as shown by Fig. 1. The constant \( k_c \) is the stiffness of the whole suspension whereas the constant \( k_1 \) and \( k_2 \) are the individual stiffness of Suspensions I and II. The total stiffness \( k_c \) can be denoted as

\[
k_c = \frac{k_1k_2}{k_1 + k_2}
\]

From this equation, it is seen that if normal springs are connected, the combined stiffness is smaller than that of individual suspension. However, if the suspensions have stiffness such as

\[
k_2 = -k_1,
\]

the combined stiffness will be infinite. It means that even if any force is applied to the point of force, it maintains its position. In contrast, the point of detection is displaced in proportion to the applied force. From this displacement, the force can be estimated.

2.2 Double series magnetic suspension

In double series magnetic suspension system, the levitation of two floators are accomplished by one electromagnet as shown by Fig. 2. Sufficient pulling force to suspend Floator 1 is generated by the electromagnet. There is a permanent magnet attached at the bottom of Floator 1. Attracting force of the permanent magnet levitates Floator 2. When the control current flows through the coil of the electromagnet, it controls the position of Floator 1. The gap between the permanent magnet and Floator 2 varies with the change of position of Floator 1. This is how the position of Floator 2 can be maintained by displacing Floator 1 if any force is applied to Floator 2 and the applied force can be estimated by detecting the displacement of Floator 1.

2.3 Three-dimensional force measurement

The principle of double series magnetic suspension system is elaborated to achieve three-dimensional force measurement. The z-direction measurement structure is similar to that shown by Fig. 2. In contrast, the point of force is suspended with leaf springs as shown in Fig. 4 to prevent the rotation of the point of force.

The horizontal forces in the \( x \) and \( y \) directions are also estimated by a similar principle of double series magnetic suspension as shown by Fig. 3. In each direction, a detection point is attached in between two VCMs. The outer VCM and inner VCM control the movements of the detection point and the point of force,
respectively. The detection point and the point of force are suspended with leaf springs to restrict their motions to single direction.

3. Experimental Setup

3.1 Design and structure

A conventional-size apparatus is designed and fabricated for three-dimensional force measurement. Schematic diagrams of the constructed structure are shown in Fig. 4 to Fig. 6. Figure 7 shows a picture of the apparatus. To control the z-direction movement, there is an electromagnet affixed at the top of the frame as shown by Fig. 4. It consists of a 378-turn coil of 0.5 mm diameter. The motions of the detection point (z) and the point of force (z) are restricted by leaf springs as shown in Fig. 4. At the bottom of the extension of the detection point (z), a permanent magnet is installed to control the movement of the point of force (z). As displayed by Fig. 5, two eddy current gap sensors are attached with the top frame to measure the displacement of the detection point (z). Another similar sensor is fixed on the bottom part and focused on the point of force (z) to sense the movement of the point of force as shown in Fig. 4. An extension frame and screw is connected with the point of force (z) for adding weights to generate downward force as shown by Fig. 5.

In the horizontal x and y directions, electromagnet is replaced by voice coil motor as shown in Fig. 4 and Fig. 5. In either direction, there are two VCMs. One VCM is fixed to the outer frame to control the movement of the detection point. The other VCM is attached to the detection point and regulates the motion of the point of force. The detection point is supported with leaf springs attached from the top and the bottom frame. In a counter way, the point of force is connected with fixed outer frame by leaf springs. The measurement principle is similar to that of the z-direction. When force applied to the point of force, the detection point displaces in the
direction of force and the point of force maintains its position. This is how the zero-compliance is achieved in x and y directions. There are three sensors to sense the motion of the detection point and the point of force in each direction. Two of them are installed at the two sides of the outer VCM to estimate the movement of the detection point. The displacement is computed by summing up the signals coming from both sensors and divide by 2. The other sensor is attached at the opposite side of the frame and targeted on the point of force to measure its displacement. In Fig. 3 to Fig. 5, the center red block is the point of force for the z-direction measurement. Meanwhile, the green and blue frames are the point of force in x and y directions, respectively.

3.2 Mathematical model

The coordinate system of the device is as shown in Fig. 4. The x-axis and y-axis are in the horizontal plan, and the z-axis is in the vertical direction. This paper focuses on the z-direction measurement only. The equations of motion for the z-axis movement can be represented as:

\[ m_2 \ddot{z}_2(t) = -(h_1 - k_s)z_1 + k_m(z_1 - z_2) - k_i \]  
\[ m_2 \ddot{z}_2(t) = -h_2z_2 - k_m(z_1 - z_2) + f(t) \]

where \( m_2 \): mass of the detection point (z), \( m_1 \): mass of the point of force (z), \( h_1 \): stiffness of leaf springs connected with detection point (z), \( h_2 \): stiffness of leaf springs attached with point of force (z), \( z_1 \): displacement of the detection point (z), \( z_2 \): displacement of the point of force (z), \( k_i \): electromagnet current force factor, \( k_m \): gap-force factor of electromagnet, \( k_o \): gap force factor of permanent magnet, \( f \): external force acting on the point of force.

As this device follows the principle of double series magnetic suspension, if there is any force acting on point of force (z), it will maintain its position whereas the detection point (z) moves in the direction of force. This condition is achieved by applying PID control to the point of force and PD control to the detection point. The control current for the constructed system is given by:

\[ I(s) = (p_d + sp_d)Z_1(s) - (q_d + sq_d + \frac{q_i}{s})Z_2(s) \]  

where \( p_d \) and \( p_c \): proportional gain and derivative gain of PD controller of the detection point (z), \( q_d \), \( q_c \), and \( q_i \): proportional gain, derivative gain, and integral gain of the point of force (z), respectively. Each of the three axes are controlled separately even though they have similar structure of control system. A block diagram of the control system is shown in Fig. 8.

3.3 Transfer function

From Eqs. (3), (4), and (5), we get:

\[ Z_1(s) = \frac{k_1q_s^2 + (k_4q_d - k_m)s + k_q}{D(s)} F(s) \]  
\[ Z_2(s) = \frac{m_2s^2 + k_1ps + k_2p_d + k_3 - k_m}{D(s)} sF(s) \]

\[ D(s) = m_1m_2s^5 + m_2k_1ps^4 + \{m_1(h_1 - k_m) + m_2(k_s1 - k_m + k_p)\}s^3 + \{k_1p_m(h_1 - k_m) + k_mk_iq_i\}s^2 + \{h_2(k_s1 + k_p - k_m) + km(k_4q_d - k_3 - k_2p_d)\}s + k_mk_iq_1 \]

The sign of the control parameters \( p_s \), \( p_d \), \( q_s \), \( q_d \), and \( q_i \) are selected by using pole placement method of the transfer function. The values of \( p_s \), \( q_s \), and \( q_i \) are positive and the values of \( p_d \) and \( q_d \) are negative for the stable suspension control of this system. For the design of controller, the values of \( k_i \), \( k_m \), \( k_o \), \( h_1 \), \( h_2 \), \( m_1 \), \( m_2 \) are required to know.

3.4 Physical parameters of the structure

![Fig. 8 Block diagram of the control system](image)
After finishing the design and construction of the structure, and before the experiment of force measurement, the physical parameters of the system such as mass of the detection point and mass of the point of force, stiffness of the leaf springs, current force factor and gap force factor of electromagnet, and gap force factor of permanent magnet are determined. The current force factor of each voice coil motor is determined by measuring force with a load cell. The mass of the detection point \( m_1 \) and the mass of point of force \( m_2 \) are estimated from the frequency response. The stiffness of the leaf springs is calculated from the displacement after adding force. The current force factor and gap force factor of the electromagnet are estimated from the resonance frequency \( \omega_r \) of the subsystem by changing P gain as shown in Fig. 9, where the subsystem consists of the detection point \( z \) and the electromagnet. The gradient of the line graph represents the value of current force factor of the electromagnet. The combined stiffness of gap force factor of electromagnet and the leaf spring stiffness connected with the detection point \( z \) are denoted by the constant value of the graph in Fig. 9. The gap force factor of permanent magnet is calculated from the displacement data after adding force at the point of force. Table 1 presents the values of the physical parameters appearing in the equations of motion of the detection point \( z \) and the point of force \( z \).

4. Results and Discussion
The experiment is done so far by applying static force at the point of force \( z \) direction. Fig. 10 and Fig. 11 present the displacement and the control current according to the addition of force. It is shown in Fig. 10 that if force is applied at point of force \( z \), the detection point \( z \) displaces but the point of force \( z \) maintains its position. The downward trend of the graph represents the downward movement of the detection point \( z \). Weights are added to the point of force \( z \) to produce static force. Weights are added one by one to increase force. The experimental results of adding 0.85-g weights one by one are displayed in Fig. 10 and Fig. 11. Another similar experiment is done by changing the unit weight from 0.85-g to 0.64-g and the results are presented in Fig. 12 and Fig. 13. The results from both experiments confirm that the point of force \( z \) maintains its position even if there is addition or reduction of force which is

![](image1.png)

Fig. 9 Relation of resonance frequency and P gain.

| Physical parameter | Numerical value | Unit |
|--------------------|----------------|------|
| \( m_1 \)          | 0.489          | kg   |
| \( m_2 \)          | 0.241          | kg   |
| \( k_i \)          | 7.4607         | N/A  |
| \( k_{11} = h_1 - k_i \) | 8009.9         | N/m  |
| \( k_m \)          | -5023.7        | N/m  |
| \( h_2 \)          | 9142.1         | N/m  |

Table 1 Values of the physical parameter

![](image2.png)

Fig. 10 Displacement of the detection point \( z \) and point of force \( z \).

![](image3.png)

Fig. 11 Relationship of control current and force.
detection point (z) and point of force (z). Figure 12 shows the relationship between the displacement and force. It can be seen that zero compliance is achieved at the point of force (z). The applied force can be estimated from the downward displacement of the detection point (z) and it shows good linearity to the change of force. In contrast, the control current has some nonlinearity to the force that is shown in Fig. 11 and Fig. 13.

5. Conclusion

A three-dimensional force measurement instrument was designed and constructed to achieve stable zero-compliance at the point of force. All the sensors were calibrated and the physical parameters needed for the z-direction force measurement were identified. The control system for the z-direction force measurement was designed and implemented. The performance of the device was tested by applying static forces in the z-direction. Experiment was conducted by gradually increasing the applied force. The point of force (z) was stable structurally with the suspension of leaf spring. The experiment results demonstrated that with the PD control to the detection point (z) and PID control to the point of force (z), the system was stable and the point of force (z) can maintain its original position even if there was the increase or decrease of force. Moreover, force was estimated from the displacement of the detection point (z). Therefore, zero-compliance force measurement achieved by considering the principle of double series magnetic suspension. Additionally, it can be said that by selecting the stiffness of the leaf springs, the range of force measurement can be set flexibly. Leaf spring with sufficiently low stiffness coefficient can even measure micro force.

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References

[1] D. M. STEFANESCU, Handbook of Force Transducers, Springer, (2011).
[2] R. P. BOYDEN, C. P. BRITCHER and P. TCHENG, “Status of Wind Tunnel Magnetic Suspension Research,” SAE Technical Paper Series 851898, pp. 1-9, 1985.
[3] T. MIZUNO, D. SEKINE, M. TAKASAKI and Y. ISHINO, “Force Measurement using Double Series Magnetic Suspension,” Transaction of the Japan Society of Mechanical Engineers, vol. 80, no. 814, 2014, (in Japanese).
[4] T. MIZUNO, D. SEKINE, Y. ISHINO and M. TAKASAKI, “Noncontact Microforce Measurement using Series Magnetic Suspension (2nd Report: Fabrication of an Experimental Apparatus),” in Proceedings of Dynamics and Design Conference, 2011, (in Japanese).
[5] T. MIZUNO, Y. HAYASHI, Y. ISHINO and M. TAKASAKI, “Proposal of Force Measurement Using A Zero-Compliance Mechanism,” in Proceedings of XXI IMEKO World Congress ‘Measurement in Research and Industry’, 2015.
[6] Y. HAYASHI, M. TAKASAKI, Y. ISHINO, D. YAMAGUCHI, M. HARA and T. MIZUNO, “Development of Force Measurement System using Zero-Compliance Mechanism with a Cantilever (2nd Report: Fabrication of Measurement Mechanism),” in Proceedings of 25th MAGDA Conference, Kiryu, 2016, (in Japanese).
[7] K. ISHII, “Noncontact Microforce Measurement using Series Magnetic Suspension (3rd Report: Development of Tri-axial Force Transducer),” in Proceedings of 24th Symposium on Electromagnetics and Dynamics, Toyama, 2012, (in Japanese).
[8] K. IIDA, T. MIZUNO, M. TAKASAKI, Y. ISHINO, D. YAMAGUCHI and M. HARA, “Development of 3-Component Force Measurement Apparatus Using Zero-Compliance Mechanism (1st Report: Basic Concepts and Fabrication of Measurement Device),” in Proceedings of Dynamics and Design Conference, 2015, (in Japanese).