On the Dynamics of the Error Floor Behavior in (Regular) LDPC Codes

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Abstract—It is shown that dominant trapping sets of regular LDPC codes, so called absorption sets, undergo a two-phased dynamic behavior in the iterative message-passing decoding algorithm. Using a linear dynamic model for the iteration behavior of these sets, it is shown that they undergo an initial geometric growth phase which stabilizes in a final bit-flipping behavior where the algorithm reaches a fixed point. This analysis is shown to lead to very accurate numerical calculations of the error floor bit error rates down to error rates that are inaccessible by simulation. The topology of the dominant absorption sets of an example code, the IEEE 802.3an (2048,1723) regular LDPC code, are identified and tabulated using topological relationships in combination with search algorithms.

Index Terms—absorption sets, error floor, Low-Density Parity-Check codes.

I. SUMMARY

The error floor in modern graph-based error control codes such as low-density parity-check codes is caused by inherent structural weaknesses in the code’s interconnect network. The iterative message passing algorithm cannot overcome these weaknesses and gets trapped in error patterns which are easily identifiable as erroneous (in LDPC codes), and are thus not valid codewords, but difficult to overcome or correct [1], [2]. These weaknesses were termed trapping sets by Richardson in [3], a summary definition for the patterns on which the message passing algorithm fails for Gaussian channels. These trapping sets are dependent on the code, the channel used, and to a lesser degree also on the details of the decoding algorithm. Prior work in identifying the weaknesses of LDPC codes on erasure channels led to the definition of stopping sets in [4]. Stopping sets, being the weaknesses of LDPC codes on erasure channels, also play a role on Gaussian channels, but are not typically the dominant error mechanisms. In [5] the authors define absorption sets, which are the subgraphs of the code graph on which the Gallager bit-flipping decoding algorithms fail for binary symmetric channels. The authors observed that these absorption sets also show up as the dominant trapping sets in certain structured LDPC codes. In [6] they devise post-processing methods to reduce the effects of these absorption sets and lower the error floor of the codes in question.

In this paper we present a linear algebraic approach to the dynamic behavior of absorption sets. We show that these sets follow a geometric growth phase during early iterations where messages inside the absorption set grow towards a largest eigenvector which characterizes the absorption set. The seemingly erratic behavior of the messages at early iterations is due to the decreasing influence of lesser eigenvectors. We define the gain of an absorption set and show how it affects the influence of the extrinsic messages that flow into the absorption set at each iteration from the remainder of the code network. The importance of set extrinsic information was already informally observed in [7], who reported a lowering of the error floor with increased extrinsic connectivity. We use our analysis to produce accurate error formulas for the error floor BER/FER and support these results with importance sampling simulations targeting the absorption sets.

As illustration we carefully identify and classify absorption sets of the regular (2048,1723) LDPC code recently designed in [8], which is used in the IEEE 802.3an standard.

Topological features of dominant absorption sets are identified and a search algorithm is presented which finds the leading dominant sets.

II. BACKGROUND

Stopping sets completely determine the performance of graph-based decoding of LDPC codes on erasure channels, i.e., on channels where the transmitted binary symbols are either received correctly, or are erased. A complete statistical treatment of stopping sets was given in [4]. Aptly named, a stopping set is a subset of uncorrected variable nodes where the decoder stops, i.e., makes no further correction progress. It is simply defined as:

Definition 1. A stopping set \( S \) is a set of variable nodes, all of whose neighboring check nodes are connected to the set \( S \) at least twice.

Fig. 1 shows an example of a stopping set. It is quite straightforward to see that if erasure decoding is performed following Gallager’s decoding algorithm [9] the variable values in the stopping set cannot be reconstructed. Valid codewords are trivially stopping sets, but the set of stopping sets is larger than the set of valid codewords.

An absorption set is an extension of the notion of a stopping set to the binary-symmetric channels [5], [6], and is defined as:

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Definition 2. An absorption set $A$ is a set of variable nodes, such that the majority of each variable node’s neighbors are connected to the set $A$ an even number of times.

Fig. 2 shows an example absorption set. It can be verified that Gallager-type bit flipping decoding will not be able to correct an absorption set, since a majority of messages impinging on each variable node will retain the erroneous sign for each iteration. Consequently, the algorithm locks up.

III. LDPC Codes on the Gaussian Channel

The Gaussian channel is different from the binary symmetric and binary erasure channels and causes a more complicated error behavior on LDPCs. Richardson [3] first seriously explored the error floor of LDPCs on Gaussian channels and defined trapping sets as the failure mechanism. Noting that typically very few trapping sets dominate the error floor region he proposed a semi-analytical method which amounts to a variant of importance sampling to numerically predict the error floor from the knowledge of a code’s trapping sets.

While finding trapping sets remained a largely open problem, [5] observed that in certain structured LDPCs the dominant trapping sets are absorption sets, i.e., the failure mechanism of the code on binary symmetric channels. In [6], algorithmic modifications were proposed to “eliminate” the error floor caused by these absorption sets.

Due to its popularity and extensive exposure we will concentrate on the (2048, 1723) regular LDPC code [8] used in the IEEE 802.3an standard. This code has been extensively analyzed. It has a low error floor that appears at $E_b/N_0 \approx 5$dB at a BER of $10^{-12}$, that is too low to be efficiently explored using conventional simulations.\footnote{Even an FPGA-based simulation running at 100Mb/s requires about a week for a single data point.}

Fig. 3 shows the structure of the dominant absorption set of this code (see also [6, Fig.2]). There are 14,272 such sets in the (2048, 1723) code of [8]. They dominate the error floor since they are the minimal absorption sets in this code (for definition of minimal and dominant, see Definition 4).

A. Finding Dominant Absorption Sets

The (2048, 1723) rate 0.8413 regular LDPC code [8] considered here has a structured parity-check matrix:

$$H = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1,32} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2,32} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3,32} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\sigma_{61} & \sigma_{62} & \sigma_{63} & \cdots & \sigma_{6,32}
\end{bmatrix}_{384 \times 2048}$$

where each $\sigma_{ij}$ is a 64 x 64 permutation matrix.

Definition 3. Let $(a, b)$ denote an absorption set, where $a$ is the size of the set (number of variable nodes) and $b$ is the extrinsic message degree (EMD), i.e., the cardinality of the set of the neighboring check nodes that are connected to the set an odd number of times (the unsatisfied checks).

For example, Fig. 2 and Fig. 3 show (4, 4) and (8, 8) absorption sets, respectively.

Table I shows the first few absorption sets of the code in [8].

Definition 4. (i) Let the ratio $b/a$ denote the average EMD for an $(a, b)$ absorption set. (ii) An $(a, b)$ absorption set is called minimal if no $(a', b')$ absorption set exists with $a' < a$ and $b'/a' \leq b/a$, i.e., less variable nodes and smaller average EMD. (iii) A minimal $(a, b)$ absorption set is called dominant if no $(a, b')$ absorption set exists with $b' < b$, i.e., smaller EMD.

The smaller the absorption set, the more severe the effect on the error floor. Thus our target is to find the dominant absorption sets in terms of $a$, $b$ and $b/a$. Since the variable node degree $d_v = 6$, $a \geq 5$ by the definition of absorption sets and the code is 4-cycle free. In addition, $b \in [0, 2a]$ and must be even. So let us start with $a = 5$ to develop the numbers in Table I. The coefficient $5 - b/a$ is the gain of the absorption set, which determines how fast the extrinsic information enters the set — see later.
TABLE 1
ABSORPTION SETS OF IEEE 802.3AN LDPC CODE.

| a | b | Existence | Multiplicity | Gain   |
|---|---|-----------|--------------|--------|
| < 5 | 10 | No        |              |        |
| 5  | 6  | No        | 1            |        |
| 6  | 8  | No        |              |        |
| 6  | 12 | No        |              |        |
| 7  | 8  | Yes       | 65,472       | 3.29   |
| 7  | 10 | No        |              |        |
| 8  | 2  | No        |              |        |
| 8  | 4  | No        |              |        |
| 8  | 6  | No        |              |        |
| 8  | 8  | Yes       | 14,272       | 4      |
| 8  | 12 | No        |              |        |
| 9  | 4  | No        |              |        |
| 9  | 6  | No        |              |        |
| 9  | 8  | Yes       | ?            | 3.66   |
| 9  | 10 | Yes       | ?            | 3.44   |
| 9  | 12 | Yes       | ?            | 3.22   |
| 9  | 14 | Yes       | ?            | 3      |
| 9  | 16 | Yes       | ?            | 3      |
| 9  | 18 | Yes       | ?            | 3.2    |
| 9  | 20 | Yes       | ?            | 3      |
| 10 | 2  | No        |              |        |
| 10 | 4  | No        |              |        |
| 10 | 6  | No        |              |        |
| 10 | 8  | Yes       | ?            | 3.6    |
| 10 | 12 | Yes       | ?            | 3.4    |
| 10 | 14 | Yes       | ?            | 3.2    |
| 10 | 16 | Yes       | ?            | 3      |
| 10 | 18 | Yes       | ?            | 3      |
| 10 | 20 | Yes       | ?            | 3      |

Fig. 4. The only possible topology of (5, 10) absorption sets.

1) a = 5: Clearly b can only equal 10 and there is only one possible connecting topology shown in Fig.4(b).

Lemma 1. There are no size-5 absorption sets.

Proof: See Appendix A.

2) a = 6: Let us introduce additional notations needed to prove the existence of absorption sets.

Definition 5. (i) For any variable node v in an absorption set, let \(\deg(v)\) denote the number of neighboring check nodes of v that are connected to the set an even number of times.

\(\deg(v)\) is the degree of vertex v in the topology graph with check nodes hidden. (ii) Let an unordered array \([\deg(v_i) : i = 1, 2, \ldots, a]\) denote a class of \((a, b)\) absorption sets, where \(\deg(v_i) \in \{4, 5, 6\}\) and \(\sum_{i=1}^{a} \deg(v_i) = 6a - b\).

It is difficult to find absorption sets, even by making use of Algorithm [1] type methods (see Appendix A), due to their extremely low appearance. Hence we need Definition 5 to classify the absorption sets first. For each pair \((a, b)\), there may be several classes of absorption sets, and each class may exhibit several topologies. What we are trying to do is to reduce one unknown absorption set to a smaller absorption set whose non-existence is known by eliminating nodes from the original set. We can then argue that there is only a limited number of topologies that have to be searched algorithmically.

Theorem 2. There are no size-6 absorption sets.

Proof: See Appendix B.

3) a = 7:

Theorem 3. There are no \((7, b)\) absorption sets with \(b < 12\). (7, 12) and \((7, 14)\) absorption sets do exist.

Proof: See Appendix C.

4) a = 8: First we show

Lemma 4. For \(b < 8\), there exist no \((8, b)\) absorption sets.

For \(b = 8\), there exists no \((8, 8)\) absorption set that contains a degree-6 variable node.

Proof: See Appendix D.

Then the only possible class of \((8, 8)\) absorption set would have connectivity \([5, 5, 5, 5, 5, 5, 5, 5, 5]\). We claim that

Claim 1. Graphically, there exist five possible topologies for \([5, 5, 5, 5, 5, 5, 5, 5, 5]\) absorption sets, shown in Fig.7(b) [18] and 22.

Proof: See Appendix E.

Theorem 5. The number of \((8, 8)\) absorption sets is 14,272 and they all have the topology of Fig. 22(b).

Proof: By searching all topologies in Claim 1 on the H matrix of [8].

Since these are the dominant absorption sets, let us sketch their connections in Fig. E one more time.

The average multiplicity of each variable node appeared in such sets is \(14272 \times 8/2048 = 55.75\). Because of the block structure of the H matrix, certain groups of variable nodes do share the same multiplicity, as listed in Table II.

The ratio \(b/a = 1\). The next possible absorption set with these parameters would be the \((10, 10)\) set.

B. Less Dominant Absorption Sets

There exist larger and less dominant absorption sets. See Appendix F for details.

3As a corollary, since there is no \((8, 0)\) absorption set, the minimum distance bound of this LDPC code [8] is strengthened to \(d_{\min} \geq 10\).

4It took approximately ninety minutes on an AMD Opteron Processor (64bits/2.4GHz) to search the topology Fig. F.
IV. Dynamic Analysis of Absorption Sets

We now present a linearized analysis to gain insight into the behavior of dominant absorption sets starting with the leading \((8,8)\) absorption set. First we note that the variable nodes perform simple addition. Furthermore, the check nodes basically choose the minimum of the incoming signals. If we make the reasonable assumption that the absorption set converges slower than the remaining nodes in the code, and due to the fact that each (satisfied) check node is connected exactly to two absorption set variables, the minimum absolute-value signal into the participating check nodes will come from one of the absorption set variables. If this is true, the check nodes simple exchange the signals on the connections to the absorption set variable nodes. We will refine this approximation below.

Additionally, each absorption set variable node is singly connected to a “lone” floating extrinsic parity check node, all of whose other connections go to other, set-external variable nodes. The messages through these eight extrinsic check nodes are the extrinsic messages into the absorption set, and are of crucial importance. Algorithmically, they play exactly the same role as the intrinsic channel values which are fed into the variable nodes by virtue of the summation function executed at the variable nodes.

Fig. 6 shows an example of the dynamic behavior of the absorption set variables close to its decision threshold boundary. The seemingly erratic behavior resolves after a number of iterations when all variables follow highly correlated trajectories. This observation is the basis for the following analysis.

Denote the outgoing \textit{solid edge values} from the variable nodes (Fig. 3) by \(x_i, i.e., x_1, \cdots, x_8\) leave variable node \(v = 0, x_6, \cdots, x_{10}\) variable node \(v = 1, \) etc. Collect the \(x_i\) in the length-40 column vector \(x, \) which is the vector of outgoing variable edge values in the absorption set. Likewise, and analogously, let \(y_j\) be the \textit{incoming edge values} to the variable nodes, such that \(y_j\) corresponds to the reverse-direction message. Now, at iteration \(i = 0\)

\[
x_0 = \lambda
\]

where the initial input is the vector of channel intrinsics \(\lambda = [\lambda_1, \cdots, \lambda_1, \lambda_2, \cdots, \lambda_2, \cdots, \lambda_8, \cdots, \lambda_8]^T\) duplicated onto the outgoing messages. It undergoes the following operation at the check node:

\[
y_0 = Cx
\]

where \(C\) is a permutation matrix that exchanges the absorption set signals as discussed above. At iteration \(i = 1\) we obtain

\[
x_1 = V C \lambda + \lambda_1^{(ex)}
\]

where \(V\) is the variable node function matrix, i.e., each output is the sum of the other four inputs from the check nodes plus the intrinsic input. The extrinsic inputs from the remainder of the code graph are contained in \(\lambda_1^{(ex)}\). Following the linear model, at iteration \(I = j\) extrinsic signals are injected into the absorption set as \(\lambda_j^{(ex)} = [\lambda_j^{(ex)}(1), \lambda_j^{(ex)}(2), \cdots, \lambda_j^{(ex)}(8)]^T\) via the extrinsic check nodes. By induction we obtain at iteration \(i = I\)

\[
x_I = \sum_{i=0}^{I} (VC)^i \lambda + \sum_{j=1}^{I} \sum_{i=j}^{I} (VC)^i \lambda_j^{(ex)}
\]
Applying the spectral theorem we obtain
\[(VC)^T \lambda \rightarrow \mu_{\text{max}} \left( \lambda^T v_{\text{max}} \right) v_{\text{max}} \]
where \(v_{\text{max}}\) is the unit-length eigenvector of the maximal
eigenvalue \(\mu_{\text{max}}\) of the matrix \(VC\).

The following lemma holds:

**Lemma 6.** The largest eigenvalue of \(VC\) for the \((8, 8)\) set
is \(\mu_{\text{max}} = d_v - 2 = 4\), and its associated eigenvector is
\(\lambda_{\text{max}} = [1, \cdots, 1]^T\).

**Proof:** First write \(VC = 4M\). By inspection \(M\) is a
probability matrix, i.e., the sum of all rows equals unity.
As a special case of the Perron-Frobenius theorem it is
known that the largest eigenvalue of a probability matrix is 1,
therefore the largest eigenvalue of \(VC\) equals 4. By inspection
\(VC[1, \cdots, 1]^T = 4[1, \cdots, 1]^T\).

The absorption set in question falls in error if
\[
\beta = \lambda^T v_{\text{max}} + \sum_{j=1}^{I} \left( \lambda_{ji}^{(ex)} + \lambda_i \right) \frac{v_{\text{max}}}{\mu_{\text{max}}} \leq 0
\]
or, in the case of the \((8, 8)\) absorption set
\[
\beta = \sum_{i=1}^{8} \left( \lambda_i + \sum_{j=1}^{I} \lambda_{ji}^{(ex)} \right) \frac{v_{\text{max}}}{\mu_{\text{max}}} \leq 0
\]
The eigenvalue \(\mu_{\text{max}} = d_v - 2\) is the gain of the absorption
set and it is determined by the variable node degree.

Exact knowledge of \(\lambda_{ji}^{(ex)}\) is not available to the analysis,
since these values depend on the received signals. However,
assuming that the code structure extrinsic to the absorption set
operates "regularly", we may substitute average values for the
\(\lambda_{ji}^{(ex)}\). Note that \(\lambda_i\) is Gaussian distributed from the channel,
and that we may assume that \(\lambda_{ji}^{(ex)}\) is also Gaussian distributed
as is customary in density evolution analysis [9], [10]. Further-
more, like \(\lambda_i\), we assume that \(\lambda_{ji}^{(ex)}\) has a consistent Gaussian
distribution with \(m = 2\sigma^2\), where \(m\) is the mean. We therefore
only need the mean of \(\lambda_{ji}^{(ex)}\), which we can calculate from a
Gaussian density evolution calculation\(^5\) i.e.,
\[
m^{(i)}_{\lambda^{(ex)}} = \phi^{-1} \left( 1 - \left\{ 1 - \phi \left( m_\lambda + (d_v - 1)m_{\lambda^{(ex)}} \right) \right\}^{d_v - 1} \right)
\]
where \(m_\lambda = 2E_\mu/\sigma^2\) is the mean of \(\lambda_i\), \(m^{(i)}_{\lambda^{(ex)}}\) is the mean
of the extrinsic signal \(\lambda_{ji}^{(ex)}\), and \(\phi\) is the check node mean
transfer function [9].

With the Gaussian assumptions, the probability of (2) happening
can be calculated as
\[
P_{\text{AS}} = \Pr (\beta \leq 0) = Q \left( \frac{2m_\lambda + 2 \sum_{j=1}^{I} \left( \frac{m^{(j)}_{\lambda^{(ex)}} + m_\lambda}{\mu_{\text{max}}} \right) \prod_{l=1}^{l} \frac{1}{g_i}}{\left( 1 + \sum_{j=1}^{I} \frac{1}{g_j \mu_{\text{max}}} \right) \mu_{\text{max}} \sum_{j=1}^{I} \left( \prod_{l=1}^{l} \frac{1}{g_i} \right)^2} \right)
\]

Two refinements can be added to this analysis. The exchange
of extrinsics through the matrix \(C\) is an approximation
in two ways: (i) As long as the remaining \(d_v - 2 = 30\) inputs
to the check node are relatively small, the entries of \(C\) are
strictly less than unity, and, (ii) in case one of the extrinsic
incoming check node messages has the wrong polarity, the
returned signal to the absorption set switches polarity. Case (i)
is approached as follows. Using a Taylor series approximation
we show that the quintessential check node operation
\[
\tanh^{-1} \left( \tanh(x) \prod_{i=1}^{d_v - 2} \tanh(x_i) \right) = \prod_{i=1}^{d_v - 2} \tanh(x_i) x + O \left[ x^3 \right]
\]
where \(\prod_{i=1}^{d_v - 2} \tanh(x_i)\) can be interpreted as a "check node
gain". If we use for \(x_i\) the mean \(m^{(i)}_{\mu^{(ex)}}\) of the signals \(\mu^{(ex)}\)
from the variable to the check nodes, an average gain can be
computed as
\[
g_i = E \left[ \prod_{i=1}^{d_v - 2} \tanh \left( \frac{m^{(i)}_{\mu^{(ex)}}}{2} \right) \right]^{d_v - 2}
\]
where the last equality results from the definition of the density
evolution function \(\phi(\cdot)\). With this result the probability in (3)
is modified to (4). In the case of general sets we need to work
with (4) instead, and compute \(\mu_{\text{max}}\) and \(v_{\text{max}}\) numerically
using the set topology.

Case (ii) can be handled by the linear analysis as well in
the following way. If an external variable to the absorption set
has an incorrect sign, this reverses the polarity of the signal returned to the absorption set from that particular check node. During the first iteration, these extrinsic signals are basically the received channel LLRs from the connected variable nodes. The probability that these are in error is given by the raw bit error rate

\[ P_e = Q \left( \sqrt{\frac{2E_s}{N_0}} \right) \]  

(8)

There are \( d_e - 2 = 30 \) external inputs impinging on each check node of the absorption set, therefore the probability that a returned signal experiences a polarity reversal is given by

\[ P_p = \sum_{k=1}^{14} \left( \frac{30}{2k+1} \right) P_e^{2k+1} (1 - P_e)^{20-2k} \]  

(9)

The model in (7) can now be expanded by injecting a correction value into the absorption set node whenever an external value is in error. We assume that if a polarity reversal occurs, the minimum value of the check node is likely close to zero, therefore the injected correction value needs to cancel the absent feedback signal and is set to \(-\lambda_{ex,i}/\mu_{max}\). If \( k \) check nodes are in error, \( 2k \) correction values are injected, one for each message going back to the absorption set. The injected correction values will alter the mean value of the decision variable to

\[
\text{mean} \to 8 \left( m_\lambda \left( 1 - \frac{k}{4\mu_{max}} \right) + \sum_{j=1}^{f} \frac{m_{(j)}(\lambda_{ex})}{\mu_{max}} \prod_{l=1}^{j} \frac{1}{g_l} \right)
\]

and the variance is adjusted accordingly, where care needs to be taken how the correction values accumulate. We have used an upper bound on the variance.

Note that these modifications only include check node polarity reversal at the first iteration, but an extension to subsequent iterations is straightforward if messy. Furthermore, as seen in Fig.8 (dashed curves), the addition of this mechanism has only a minor effect on the results.

The probability \( P_{AS} \) needs to be multiplied with the multiplicity factor of 14, 272 in order to obtain a union bound. In order to compute a BER estimate, we further multiply this number by 8/1723, since there are eight errors that occur in a frame of 1,723 bit errors due to this absorption set.

Fig.7 shows \( P_{AS} \) for the first most dominant absorption sets. Also shown are general tendencies of \( P_{AS} \) as a function of \( a \) and \( b \):

\[ P_{AS} = Q(f(a,b)) \]  

(10)

where \( f(a,b) \) is defined by (11). It can be shown that \( \mu_{max} \approx 5 - b/a \) is a close approximation (exact for symmetric sets with \( a = b \)) to the gain of the set, and this was used in Fig.7 to plot the curves.

It can be seen that the \((8,8)\) absorption set is the most dominant, which is consistent with numerical observations. Multiplicities also affect a set’s impact — see Fig.8. Additionally, some sets, like the majority of \((7,12)\) sets, are “contained” in larger sets, that is, such \((7,12)\) absorption sets are not stable under bit flipping and will evolve into \((8,8)\) sets, of which they are subgraphs.

V. NUMERICAL VERIFICATION

Fig.8 shows the analytical error floor calculation using (7) and the multiplicity of 14, 272. Lesser absorption sets have an impact more than an order of magnitude lower. And they are not considered. The figure also shows hardware simulations using an FPGA platform, as well as importance sampled simulations using the same absorption sets as bias targets. Regular mean-shift importance sampling was utilized and each of the absorption sets containing a specific variable node was biased separately. As evidenced by the figure, our linearized analysis provides an accurate picture of the error floor behavior of this code and illustrates the dominance of the \((8,8)\) absorption sets.

VI. CONCLUSION

We have presented an analytical analysis of the dynamic behavior of the dominant absorption sets in LDPC message-passing decoders. These absorption sets cause the infamous error floor at high signal-to-noise ratios, and we have identified the dominant such sets for the example regular LDPC code used in the IEEE 802.3an standard via topological arguments and searches. Using importance sampling with the dominant sets accurately predicts the error floor of this code.
Appendix A

Proof of Lemma 1

Matrix $H$ is searched observing the constraints imposed by the absorption set topology. In addition, some of the properties listed in [11], [12] for array-based LDPC codes apply, as well. The following algorithm is used:

Input: Parity-check matrix $H_{384 \times 2048}$ and Fig. 4
Output: Absorption sets.

```
foreach variable node $v \in \{0, 1, 2, \ldots, 2047\}$ do
    Pick 4 out of 6 neighboring check nodes of $v$
    denoted $c_0, c_1, c_2$ and $c_3$, respectively;
    foreach one out of 31 neighboring variable nodes
    other than $v$ of $c_0$, denoted as $v_1$ do
        foreach one out of 31 neighboring variable nodes
        other than $v$ of $c_1$, denoted as $v_2$ do
            if $v_2$ and $v_1$ are not connected then
                re-pick $v_2$;
            else
                foreach one out of 31 neighboring
                variable nodes other than $v$ of $c_2$,
                denoted as $v_3$ do
                    if $v_3$ and $v_1$ or $v_3$ and $v_2$ are not
                    connected then
                        re-pick $v_3$;
                    else
                        foreach one out of 31
                        neighboring variable nodes other
                        than $v$ of $c_3$, denoted as $v_4$
                        if $v_4$ and $v_1$ or $v_4$ and $v_2$
                        or $v_4$ and $v_3$ are not connected
                        then
                            re-pick $v_4$;
                        else
                            follow Definition 2 to
determine if this candidate
set is an absorption set;
```

Algorithm 1: Algorithm for finding $(5, 10)$ absorption sets.

Appendix B

Proof of Theorem 2

For $a = 6$, there are four possible values for $b$ and there is only one possible topology corresponding to each of them, shown in Fig. 9.

By removing any node in Fig. 9(a) or either degree-4 node in
Fig. 9(b) we obtain the [4, 4, 4, 4, 4] set, one shown in Fig. 9(b).
By Lemma 1, Fig. 9(a) and 9(b) do not exist. After eliminating topologies Fig. 9(c) and 9(d) algorithmically, Theorem 2 follows.

Appendix C

Proof of Theorem 3

Now $a$ is large enough for the neighboring check nodes to be connected to the absorption set four times. First, we suppose that all satisfied check nodes are connected to the set twice.

A. $b = 0$

If $b = 0$, then the absorption set is a codeword. However, since $d_{\text{min}} \geq 8$ [8], $b \neq 0$.

B. $0 < b < 12$

We apply the constraints in Definition 5 and the pigeonhole principle to prove this.

1) $b = 2$, there are two classes:
   a) $[6, 6, 6, 6, 5, 5, 5]$: removing either degree-5 node
      leaves a $(6, 6)$ absorption set.
   b) $[6, 6, 6, 6, 6, 6, 4]$: removing the degree-4 node
      generates a $(6, 4)$ absorption set.

2) $b = 4$, there are three classes:
   a) $[6, 6, 6, 5, 5, 5, 5]$: removing any degree-5 node
      generates a $(6, 8)$ absorption set.
   b) $[6, 6, 6, 6, 6, 4, 4]$: removing the degree-4 node
      generates a $(6, 6)$ absorption set.
   c) $[6, 6, 6, 6, 6, 4, 4]$: this is infeasible since the group
      of five degree-6 nodes requires ten edges emanating from the
      group of degree-4 nodes.

3) $b = 6$, there are four classes:
a) \([6, 5, 5, 5, 5, 5]\): removing any degree-5 node generates a \((6, 10)\) absorption set.
b) \([6, 6, 5, 5, 5, 4]\): removing the degree-4 node generates a \((6, 8)\) absorption set.
c) \([6, 6, 5, 5, 4, 4]\): each of the degree-5 nodes and each of the degree-6 nodes needs at least one and two edges emanating from the two degree-4 nodes, respectively. That makes eight. So there is no connection between the degree-4 nodes. Thus removing either of them generates a \((6, 8)\) absorption set.
d) \([6, 6, 6, 4, 4, 4]\): each of the degree-6 nodes needs three edges emanating from the two degree-4 nodes. That makes twelve. So there is no connection among the three degree-4 nodes. Thus removing any of them generates a \((6, 8)\) absorption set.

4) \(b = 8\), there are four classes:
   a) \([5, 5, 5, 5, 5, 4]\): removing the degree-4 node generates a \((6, 10)\) absorption set.
b) \([6, 5, 5, 5, 4, 4]\): let us study the intrinsic connections between the two degree-4 nodes:
   i) Not connected as shown in Fig.10(a) removing either degree-4 node will reduce it to a \((6, 10)\) absorption set.
   ii) Connected as shown in Fig.10(b) we consider the connections between the two degree-4 nodes and the other five nodes in the set. The other five nodes need at least six edges emanating from the two degree-4 nodes, so there is no degree-5 node connected to both degree-4 nodes. Thus removing both of them generates a \((5, 10)\) absorption set.
c) \([6, 6, 5, 4, 4, 4]\): the two degree-5 nodes and the two degree-6 nodes need at least ten edges emanating from the three degree-4 nodes. Hence there should be no more than one connection among the group of degree-4 nodes:
   i) No connection as shown in Fig.11(a) removing any degree-4 node will reduce it to a \((6, 10)\) absorption set.
   ii) One connection as shown in Fig.11(b) removing the topmost degree-4 node will reduce it to a \((6, 10)\) absorption set.
   d) \([6, 6, 4, 4, 4, 4]\): the three degree-6 nodes need twelve edges emanating from the four degree-4 nodes. Hence there should be no more than two connections among the group of degree-4 nodes:
   i) No connection as shown in Fig.12(a) removing any degree-4 node will reduce it to a \((6, 10)\)

absorption set.

ii) One connection as shown in Fig.12(b), removing either topmost degree-4 node will reduce it to a \((6, 10)\) absorption set.

iii) Two connections: there are two cases:
   A) Removing the bottom-right degree-4 node in Fig.12(c) will reduce it to a \((6, 10)\) absorption set.
   B) Removing either the top or the bottom couple of degree-4 nodes in Fig.12(d) will reduce it to a \((5, 10)\) absorption set.

5) \(b = 10\), there are three classes:
   a) \([5, 5, 5, 4, 4, 4]\): the four degree-5 nodes need at least eight edges emanating from the three degree-4 nodes. Hence there should be no more than two connections among the group of degree-4 nodes:
   i) No connection as shown in Fig.11(a) removing any degree-4 node will reduce it to a \((6, 12)\) absorption set.
   ii) One connection as shown in Fig.11(b) removing the topmost degree-4 node will reduce it to a \((6, 12)\) absorption set.
   iii) Two connections as shown in Fig.11(c) No node can be removed to get another absorption set. However, it is straightforward to see that there is only one possible topology to satisfy this:

Therefore we have to go check the \(H\) matrix algorithmically.

b) \([6, 5, 5, 4, 4, 4]\): the two degree-5 nodes and the degree-6 node need at least ten edges emanating from the four degree-4 nodes. Hence there should be no more than three connections among the group of degree-4 nodes:

i) No connection as shown in Fig.12(a) removing any degree-4 node will reduce it to a \((6, 12)\) absorption set.

ii) One connection as shown in Fig.12(b) removing either topmost degree-4 node will reduce it to a \((6, 12)\) absorption set.

iii) Two connections: there are two cases:
   A) Removing the bottom-right degree-4 node in Fig.12(c) will reduce it to a \((6, 12)\) absorption set.
   B) It is straightforward that there is only one possible topology to satisfy Fig.12(d)
We have to turn to $H$ to show its non-existence.

iv) Three connections: there are three cases:

A) Removing the bottom-right degree-4 node in Fig. 12(e) will reduce it to a $(6, 12)$ absorption set.

B) It is straightforward that there is only one possible topology to satisfy Fig. 12(f).

C) It is straightforward that there is only one possible topology to satisfy Fig. 12(g).

We have to turn to $H$ to show its non-existence.

C) $[6, 6, 4, 4, 4, 4]$: it is straightforward to see that there is only one possible topology in this class:

We have to turn to $H$ to show its non-existence.

$C. \ b = 12$

The extrinsic degree is large now. We will see in the $a = 8$ section that there are smaller $b$’s and $(7, 12)$ absorption sets do exist as a reduction from them.

If we allow the satisfied check nodes to be connected to the set more than twice, it is clear that only one check node could be connected to the set four times, as shown in Fig. 13(a) which is a $(7, 14)$ absorption set. Now, there can be no other intrinsic connections among the four variable nodes at the top — this would create a 4-cycle. Thus depending on the intrinsic connections among the three variable nodes at the bottom, we could obtain $(7, 12), (7, 10)$ or $(7, 8)$ absorption sets, respectively. Both $(7, 14)$ and $(7, 12)$ absorption sets exist and the $(7, 8)$ can reduce to $(5, 10)$ by removing any two degree-4 nodes and does not exist therefore. Only $(7, 10)$ sets, Fig. 13(b) need to be searched.

After searching the topologies in 5(a)iii, 5(b)iiiB, 5(b)ivB, 5(b)ivC, 5c and Fig. 13(b) with $H$ algorithmically, Lemma 5 follows.

| Variable Nodes | Six Neighboring Check Nodes |
|---------------|-----------------------------|
| 0             | 56 120 184 248 312 376      |
| 109           | 56 79 174 199 300 349      |
| 740           | 21 120 138 236 310 325    |
| 1801          | 21 87 184 202 300 374    |
| 1765          | 14 125 169 199 266 376    |
| 43            | 10 74 138 202 266 330    |
| 862           | 10 125 174 242 285 325  |

Fig. 13. One check node connecting to size-7 sets four times.

D. $b = 14$

Table III shows the existence of $(7, 14)$ sets.

Again, we first suppose that all satisfied check nodes are connected to the set twice. We apply the constraints in Definition 5 and the pigeonhole principle to prove this.

A. $b = 0$

In other words, a class of $[6, 6, 6, 6, 6, 6, 6]$ absorption sets. We obtain the perfectly symmetric Fig. 14 again as Fig. 4(b). Removing any node will reduce it to a $(7, 6)$ absorption set.
B. $b = 2$

There are two classes:

1) $[6, 6, 6, 6, 6, 5, 5]$: removing either degree-5 node generates a $(7, 6)$ absorption set.
2) $[6, 6, 6, 6, 6, 5, 4]$: removing the degree-4 node generates a $(7, 4)$ absorption set.

C. $b = 4$

There are three classes:

1) $[6, 6, 6, 5, 5, 5, 5]$: removing any degree-5 node generates a $(7, 8)$ absorption set.
2) $[6, 6, 6, 5, 5, 5, 4]$: removing the degree-4 node generates a $(7, 6)$ absorption set.
3) $[6, 6, 6, 6, 4, 4, 4]$: removing both degree-4 nodes generates a $(6, 10)$ or a $(6, 8)$ absorption set.

D. $b = 6$

There are four classes:

1) $[6, 6, 5, 5, 5, 5, 5, 5]$: removing any degree-5 node generates a $(7, 10)$ absorption set.
2) $[6, 6, 5, 5, 5, 5, 4, 4]$: removing the degree-4 node generates a $(7, 8)$ absorption set.
3) $[6, 6, 6, 5, 5, 4, 4, 4]$: let us study the intrinsic connections between the two degree-4 nodes:
   a) Not connected as shown in Fig.10(a) Removing either degree-4 node will reduce it to a $(7, 8)$ absorption set.
   b) Connected as shown in Fig.10(b) We consider the connections between the two degree-4 nodes and the other six nodes in the set. There are six edges emanating from the two degree-4 nodes and at least four of the six edges must go to the four degree-6 nodes, respectively. So at most two edges emanating from the two degree-4 nodes can be connected to the two degree-5 nodes.
   i) If either of the two degree-5 nodes is connected to the two degree-4 nodes at most once: removing both degree-4 nodes generates a $(6, 8)$ absorption set.
   ii) If one degree-5 node is connected to both degree-4 nodes: removing the other degree-5 node generates a $(7, 10)$ absorption set.
4) $[6, 6, 6, 5, 5, 4, 4, 4]$: let us study the intrinsic connections among the three degree-4 nodes. There are five degree-6 nodes, which require ten edges from the three degree-4 nodes. Thus there should be no more than one connection among them.

E. $b = 8$

There are four classes by assuming there is a degree-6 node:

1) $[6, 5, 5, 5, 5, 5, 5, 4]$: removing the degree-4 node will reduce it to a $(7, 10)$ absorption set.
2) $[6, 6, 5, 5, 5, 5, 4, 4]$: let us study the intrinsic connections between the two degree-4 nodes:
   a) Not connected as shown in Fig.10(a) Removing either degree-4 node will reduce it to a $(7, 10)$ absorption set.
   b) Connected as shown in Fig.10(b) We consider the connections between the two degree-4 nodes and the other six nodes in the set. There are six edges emanating from the two degree-4 nodes and at least four of the six edges must go to the two degree-6 nodes, respectively. So at most four and at least two edges emanating from the two degree-4 nodes can be connected to the four degree-5 nodes.
   i) Two edges between the group of degree-4 nodes and the group of degree-6 nodes:

degree-4 nodes: $\bullet \bullet \bullet$

degree-6 nodes: $\bullet \bullet \bullet$

Note that under the conditions in the above two cases, the two degree-6 nodes have to be connected with each other and either of them has to be connected to all the degree-5 nodes. In addition, there are four edges coming from the degree-4 nodes to the four degree-5 nodes. Thus,

A) if there is no degree-5 node sharing the two degree-4 nodes: removing the two degree-4 nodes will reduce it to a $(6, 10)$ absorption sets.

B) if there is one and at most one degree-5 node sharing the two degree-4 nodes: the topologies will be fixed, respectively, as
ii) Three edges between the group of degree-4 nodes and the group of degree-6 nodes:

- degree-4 nodes: 
  ![Degree-4 Nodes](image1)
- degree-6 nodes: 
  ![Degree-6 Nodes](image2)

Note that under the conditions in the above case, the two degree-6 nodes have to be connected with each other and the bottom-right degree-6 node has to be connected to all the degree-5 nodes. In addition, there are three edges emanating from the degree-4 nodes to the four degree-5 nodes. Thus,

A) if there is no degree-5 node sharing the two degree-4 nodes: removing the two degree-4 nodes will reduce it to a (6, 10) absorption sets.

B) if there is one and at most one degree-5 node sharing the two degree-4 nodes: the topology will be fixed as

Removing the two degree-4 nodes and that degree-5 node will reduce it to a (5, 10) absorption sets.

iii) Four edges between the group of degree-4 nodes and the group of degree-6 nodes:

- degree-4 nodes: 
  ![Degree-4 Nodes](image3)
- degree-6 nodes: 
  ![Degree-6 Nodes](image4)

A) if there is no degree-5 node sharing the two degree-4 nodes: removing the two degree-4 nodes will reduce it to a (6, 10) absorption sets.

B) if there is one and at most one degree-5 node sharing the two degree-4 nodes: the topology will be fixed as

Removing the two degree-4 nodes and that degree-5 node will reduce it to a (5, 10) absorption sets.

3) [6, 6, 6, 5, 4, 4, 4]: There should be no more than two intrinsic connections among the three degree-4 nodes. Otherwise, the group of degree-4 nodes will not match the group of the other five nodes remained in the set.

a) No connection as shown in Fig.11(a) removing any degree-4 node will reduce it to a (7, 10) absorption set.

b) One connection as shown in Fig.11(b) removing the topmost degree-4 node will reduce it to a (7, 10) absorption set.

c) Two connections: Now, there are eight edges coming out the group of degree-4 nodes as shown in Fig.11(c). However, each of the two degree-5 nodes and each of the three degree-6 nodes needs one and two connections emanating from the group of degree-4 nodes, respectively. That makes eight. Thus, removing the three degree-4 nodes will reduce it to a (5, 10) absorption set.

4) [6, 6, 6, 4, 4, 4, 4]: The group of degree-6 nodes need at least twelve edges from the group of degree-4 nodes. So there should be no more than two connections among the four degree-4 nodes.

a) No connection: removing any degree-4 node in Fig.12(a) will reduce it to a (7, 10) absorption set.

b) One connection: removing either topmost degree-4 node in Fig.12(b) will reduce it to a (7, 10) absorption set.

c) Two connections: there are two cases:

i) Removing the bottom-right degree-4 node in Fig.12(c) will reduce it to a (7, 10) absorption set.

ii) Removing either the top or the bottom couple of degree-4 nodes in Fig.12(d) will reduce it to a (6, 10) absorption set.

If a neighboring check node connecting to the set four times is considered, then the smallest such size-8 absorption set would be (8, 4), Fig.15 Removing any two degree-5 nodes will reduce it to a (6, 10) absorption set. To obtain such (8, 6) sets, let us remove one edge from Fig.15

By removing either the bottom-left or the bottom-right degree-5 node from Fig.16(a) or the degree-4 node from Fig.16(b) respectively, we obtain a (7, 10) absorption set.
can only be connected to the set twice. More than twice is allowed, we obtain Fig. 17(b) and 18. Let Lemma 4 results.

\[ \mathcal{H} \]

→→→→→→→→→→
↓↓↓↓↓↓↓↓↓
↗↗↗↗↗↗↗↗↗↗↗↗
↘↘↘↘↘↘↘↘↘
→→→→→→→→→

Fig. 16. One check node connecting to \((8, 6)\) sets four times.

Then to obtain such \((8, 8)\) absorption sets, first we remove one edge from Fig. 16, which gives us Fig. 17. In addition, the possible topology with two neighboring check nodes connecting to the set four times is shown in Fig. 18. Removing the degree-4 node, the bottom-left degree-4 node or the bottom-right degree-4 node in Fig. 17(a) or 17(e) respectively, reduces it to a \((7, 10)\) absorption set, while removing the two degree-4 nodes in Fig. 17(f) reduces it to a \((6, 12)\) absorption set.

Note that Fig. 17(b) and 18 are in the class [5, 5, 5, 5, 5, 5, 5], so after searching \(\mathcal{H}\) with Fig. 17(d) Lemma 4 results.

**APPENDIX E**

**PROOF OF CLAIM**

If a check node connecting to the set an even number, but more than twice is allowed, we obtain Fig. 17(b) and 18. Let us find the other three by restricting that a satisfied check node can only be connected to the set twice.

We start with one node:

As Step 2, if the bottom two nodes are

A. *not connected*

We must have Fig. 19(a)
Fig. 18. Two check nodes connecting to $(8, 8)$ set four times.

Fig. 19. First possible topology of $(8, 8)$ absorption sets.

Fig. 20. Finding possible topology of $(8, 8)$ absorption sets – Step 3.

Fig. 21. Finding possible topology of $(8, 8)$ absorption sets – Step 4.

Fig. 22. Possible topologies of $(8, 8)$ absorption sets.

Fig. 23. Some topologies of $(8, 12)$ absorption sets.

After connecting the remaining five nodes we obtain Fig. 19(b) Reorganize Fig. 19(b) into a symmetric form as Fig. 22(a).

B. connected

There is only one choice for either one of them as shown in Fig. 20(a).

As Step 3, now there are two choices for the bottom-right node as shown in Fig. 20(b), 20(c).

1) Fig. 20(b) as Step 4, the top-right node only has one choice as shown in Fig. 21(a) Then the other nodes have no choice. We obtain another possible topology as shown in Fig. 22(b).

2) Fig. 20(c) as Step 4, this top-right node has two choices:

a) Fig. 21(b) By connecting the remaining nodes, we obtain Fig. 22(c).

b) Fig. 21(c) After connecting the remaining node, this gives us Fig. 22(a) again.

So eventually we obtain the other three possible $(8, 8)$ topologies as shown in Fig. 22.

APPENDIX F

LESS DOMINANT ABSORPTION SETS

We list some examples in this section to show the existence of less dominant absorption sets.

There are two classes of $(8, 12)$ absorption sets:

1) $[6, 6, 4, 4, 4, 4, 4, 4]$: 11,008 such sets;

2) $[5, 5, 5, 5, 4, 4, 4, 4]$: 33,408 such sets.

Some topologies are shown in Fig. 23.

There exist two classes of $(10, 10)$ absorption sets:

1) $[5, 5, 5, 5, 5, 5, 5, 5, 5, 5]$: 192 such sets;

2) $[6, 6, 6, 6, 5, 5, 5, 5, 5, 5, 5]$: unknown.

The average multiplicity of each variable node appeared in class $[5, 5, 5, 5, 5, 5, 5, 5, 5, 5]$ is $192 \times 10/2048 = 0.9375$.

Once again, certain groups of variable nodes share the same multiplicity, as listed in Table IX. As we can see, some groups are not involved at all. Therefore, the average multiplicity of
Table IV

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376    |
| 109            | 56 79 174 199 300 349    |
| 1084           | 2 120 174 243 301 377    |
| 116            | 76 184 243 236 372    |
| 561            | 39 112 134 248 303 334 |
| 870            | 1 76 135 192 264 334 |
| 1091           | 46 79 135 237 303 329 |
| 1970           | 0 69 134 199 264 329 |

Table V

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376    |
| 109            | 56 79 174 199 300 349    |
| 90             | 15 120 140 253 305 338 |
| 1045           | 23 110 184 199 306 336 |
| 1440           | 23 76 174 248 263 370 |
| 39             | 15 79 143 207 271 335 |
| 1048           | 19 76 143 253 262 354 |
| 1444           | 19 110 140 207 317 326 |

Table VI

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376    |
| 109            | 56 79 174 199 300 349    |
| 1563           | 29 120 150 217 264 336 |
| 1628           | 40 75 171 248 264 373 |
| 176            | 43 78 174 251 267 376 |
| 314            | 8 125 150 251 300 368 |
| 560            | 40 78 177 199 313 368 |
| 1258           | 29 79 137 213 313 370 |
| 1988           | 30 125 171 213 267 336 |

Table VII

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376    |
| 109            | 56 79 174 199 300 349    |
| 90             | 15 120 140 253 305 338 |
| 1460           | 2 79 184 238 307 365 |
| 580            | 7 102 148 253 307 376 |
| 890            | 15 87 181 229 261 349 |
| 1194           | 38 87 148 228 319 360 |
| 1775           | 2 126 176 228 261 338 |
| 1881           | 33 84 176 229 300 360 |

Table VIII

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376    |
| 109            | 56 79 174 199 300 349    |
| 90             | 15 120 140 253 305 338 |
| 629            | 15 86 161 248 268 381 |
| 1063           | 1 109 178 236 312 349 |
| 504            | 20 85 174 197 258 338 |
| 802            | 49 86 154 197 300 363 |
| 1299           | 6 124 178 247 305 381 |
| 1789           | 20 109 150 247 265 363 |

Fig. 24. Topology of $[5, 5, 5, 5, 5, 5, 5, 5]$ absorption sets.

Fig. 25. Topology of $[7, 12]$ absorption sets.

Absorption sets: each involved variable node in such sets is $192 \approx \max \mu = 4$ and $v_{\max}$ is an all-1 vector, since $VC$ is a probability matrix as well.

Table X shows the existence of another class of $(10, 10)$ absorption sets: $[6, 6, 6, 5, 5, 5, 5, 4, 4, 4]$.

Note that $(7, 12)$ and $(9, 14)$ absorption sets (though not all of them) can be obtained by removing one node from $(8, 8)$ and $(10, 10)$ absorption sets, respectively. For example, there are $179,648$ $(7, 12)$ absorption sets. They all have the topology Fig. 25 which can be obtained from Fig. 5 by removing one node. However, the $(8, 8)$ sets generate $14272 \times 8 = 114176$ $(7, 12)$ absorption sets (no duplicates). Hence there are $179648 - 114176 = 65472$ $(7, 12)$ sets that are not contained in the $(8, 8)$ ones.
TABLE X
ANOTHER CLASS OF (10, 10) ABSORPTION SETS.

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376     |
| 591            | 56 65 159 207 302 327     |
| 1405           | 32 120 159 225 314 337     |
| 1904           | 0 119 184 249 314 359     |
| 210            | 42 65 160 248 286 335     |
| 732            | 30 118 157 223 312 335     |
| 1676           | 36 77 183 223 286 376     |
| 616            | 30 119 160 194 275 373     |
| 834            | 42 85 157 249 302 373     |
| 892            | 36 118 142 225 275 327     |

TABLE XI
AN EXAMPLE OF (10, 12) ABSORPTION SETS.

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 109            | 56 79 174 199 300 349     |
| 90             | 15 120 140 253 305 338     |
| 1460           | 32 23 184 238 307 365     |
| 1320           | 46 67 140 248 307 320     |
| 1543           | 51 72 145 253 312 320     |
| 931            | 45 88 160 252 305 376     |
| 9              | 46 110 174 238 302 366     |
| 1104           | 33 67 145 252 282 349     |
| 1316           | 51 88 156 199 302 365     |

TABLE XII
AN EXAMPLE OF (10, 14) ABSORPTION SETS.

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376     |
| 109            | 56 79 174 199 300 349     |
| 90             | 15 120 140 253 305 338     |
| 1460           | 32 23 184 238 307 365     |
| 931            | 45 88 160 252 305 376     |
| 9              | 46 110 174 238 302 366     |
| 1121           | 15 110 156 198 315 321     |
| 1316           | 51 88 156 199 302 365     |
| 1432           | 33 121 160 226 315 338     |
| 1549           | 45 121 142 198 300 366     |

TABLE XIII
AN EXAMPLE OF (10, 16) ABSORPTION SETS.

| Variable Nodes | Six Neighboring Check Nodes |
|----------------|-----------------------------|
| 0              | 56 120 184 248 312 376     |
| 109            | 56 79 174 199 300 349     |
| 90             | 15 120 140 253 305 338     |
| 1045           | 23 110 184 199 306 336     |
| 176            | 43 78 174 251 267 376     |
| 39             | 15 79 143 207 271 335     |
| 305            | 23 78 132 201 259 335     |
| 1048           | 19 76 143 253 262 354     |
| 1189           | 43 76 132 234 300 326     |
| 1444           | 19 110 140 207 317 326     |

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