Charge- and parity-projected Hartree-Fock study with the tensor force of light nuclei

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We propose a mean field framework in which the charge and the parity symmetries of a single-particle state are broken. We break these symmetries to incorporate the correlation induced by the tensor force into a nuclear mean field model. We perform the charge- and the parity projections before variation and obtain a Hartree-Fock-like equation (charge- and parity-projected Hartree-Fock equation), which is solved self-consistently. We apply the Hartree-Fock-like equation to the alpha particle and find that, by breaking the parity and the charge symmetries, the correlation induced by the tensor force is obtained.

1. Introduction

The tensor force, which is mediated by the pion, plays significant roles in the structure of nuclei. A large part of attractive energy in light nuclei is due to the tensor force \cite{1,2,3,4}. Shell model calculations show that about a half of the single-particle spin-orbit splittings in \textit{p}- and \textit{sd}-shell nuclei can be produced by the tensor force \cite{5}. Furthermore, the suppression of the effect of the tensor force around and above the normal density is an essential ingredient of the saturation mechanism of nuclear matter \cite{6}. These facts should have a strong impact on the study of the nuclear structure even in a mean field approach. Therefore, we try to construct a mean field framework in which the tensor force can be treated on the same footing as other forces like the central and the LS forces.

To treat the tensor force in a mean field model, we mix positive- and negative-parity components in a single-particle state (parity mixing). Further, we mix proton and neutron components in the single-particle state (charge mixing). As the result, our single-particle state consists of four components, each of which has a good parity and a definite charge number. The mixings of parity and charge is inspired by the specific features of the pion, which mediates the tensor force. To exploit the pseudoscalar character of the pion, we introduce the parity mixing, and to take the isovector character of the pion into account, we introduce the charge mixing. These are the basic ideas how we treat the pion in a mean field model. Recently, we applied the parity-mixing mean field to the relativistic

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mean field model to include the pion mean field and showed that the correlation by the pion can be treated with the parity-mixing mean field [7].

Because a total wave function made of parity- and charge-mixing single-particle states does not have a good parity and a definite charge number, the parity and the charge projections need to be performed to obtain the wave function with a good parity and a definite charge number. It should be noticed that because the correlation induced by the tensor force is strong and has a large effect on a nuclear mean field, these projections should be performed before variation. Taking the variation of each single-particle state to minimize the total energy with the projected total wave function, the charge- and parity-projected Hartree-Fock (CPPHF) equation for each single-particle state is obtained. We solve the CPPHF equation for each single-particle state self-consistently.

In the following, we will explain our formulation briefly and apply the CPPHF equation to the alpha particle.

2. Formulation

We assume a single-particle state with the charge and the parity mixings in the spherical case as

\[ \psi_{njm} = \sum_{t_z = \pm 1/2} \left( \phi_{nlt_z}(r)Y_{jlm}(\Omega)\zeta(t_z) + \phi_{n\bar{l}t_z}(r)Y_{\bar{j}\bar{l}m}(\Omega)\zeta(t_z) \right). \]  

This wave function consists of four terms, each of which has a good parity and a definite charge number. Here, the radial wave functions \( \phi \)'s depend only on the radial coordinate \( r \), the isospin wave functions are denoted as \( \zeta \) (\( t_z = 1/2 \) for proton and \( t_z = -1/2 \) for neutron), and the eigenfunctions of the total spin \( j = l + s \) are denoted as \( Y \)'s. In the spherical case, the good quantum numbers are \( j \) and \( m \). \( Y_{jlm} \) and \( Y_{\bar{j}\bar{l}m} \) have the same \( j \), but different orbital angular momenta, \( |l - \bar{l}| = 1 \), to implement the parity mixing.

We assume the form of a total wave function with a good parity, positive (+) or negative (-) and a definite charge number \( Z \) for a \( A \)-body system as,

\[ \Psi(\pm;Z) = \hat{P}_p(\pm)\hat{P}_c(Z)\frac{1}{\sqrt{A!}}\hat{A}\prod_{a=1}^{A}\psi_{na_ja_m}, \]  

where \( \hat{P}_p(\pm) \) and \( \hat{P}_c(Z) \) are the parity-projection and the charge-projection operators respectively, and \( \hat{A} \) is the antisymmetrization operator.

By taking the variation of each single-particle state to minimize the total energy with the projected wave function,

\[ \frac{\delta}{\delta \psi_{na_ja_m}} \left\{ \frac{\langle \Psi(\pm;Z)|\hat{H}|\Psi(\pm;Z)\rangle}{\langle \Psi(\pm;Z)|\Psi(\pm;Z)\rangle} - \sum_{bc=1}^{A} \epsilon_{bc} \langle \psi_{na_jb_m}|\psi_{na_jc_m}\rangle \right\} = 0, \]  

the charge- and parity-projected Hartree-Fock (CPPHF) equation for \( \psi_{na_ja_m} \), which has a Hartree-Fock-equation-like form, is obtained. In the above equation, \( \hat{H} \) is a Hamiltonian of the \( A \)-body system. Lagrangian multipliers, \( \epsilon_{ab} \) are introduced to assure the orthonormalization of the single-particle states, \( \langle \psi_{na_ja_m}|\psi_{na_jb_m}\rangle = \delta_{ab} \). We solve the CPPHF equation self-consistently with the gradient method.
3. Application to the alpha particle

We apply the CPPHF equation to the ground \((0^+\rangle\) state of the alpha particle. As a configuration space, we assume two \(j = 1/2\) states are filled, each of which accommodates two particles. As the central force, we take the Volkov No. 1 force \([8]\) with the multiplying factor \(x_{TE}\) to the attractive part of the \(3E\) channel. The Majorana parameter is fixed to 0.6. As the noncentral force, we take the G3RS force \([9]\) with the multiplying factor \(x_T\) to the isovector channel of the tensor force, which is a dominant part in the tensor force and mainly mediated by the pion. We introduce \(x_{TE}\) because we treat the tensor force explicitly and \(x_T\) to check the dependence of the results on the strength of the tensor force. The factors, \(x_{TE}\) and \(x_T\), are determined to reproduce the binding energy of the alpha particle.

Table 1
Results for the ground \((0^+\rangle\) state of the alpha particle in various cases. HF (the second and the third columns) denotes the simple Hartree-Fock scheme. PPHF (the fourth column) denotes the parity-projected Hartree-Fock scheme in which only the parity-projection is performed. CPPHF (the fifth to the last columns) denotes the charge- and parity-projected Hartree-Fock scheme in which both the charge and the parity projections are performed. The potential energy \(\langle \hat{v} \rangle\) (in MeV), the kinetic energy \(\langle \hat{T} \rangle\) (in MeV), the total energy \(E\) (in MeV), the root-mean-square matter radius \(R_m\) (in fm) and the probability of the \(p\)-state component \(P(-)\) are given. \(\langle \hat{v}_C \rangle\), \(\langle \hat{v}_T \rangle\), \(\langle \hat{v}_{LS} \rangle\), and \(\langle \hat{v}_{Coul} \rangle\) are the expectation values for the central, the tensor, the LS, and the Coulomb potentials, respectively (in MeV).

|         | HF     | PPHF   | CPPHF  | CPPHF  | CPPHF  | CPPHF  | CPPHF  | CPPHF  |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|
| \(x_T\) | 1.0    | 1.5    | 1.5    | 1.0    | 1.25   | 1.5    | 1.75   | 2.0    |
| \(x_{TE}\) | 1.0    | 0.81   | 0.81   | 0.93   | 0.88   | 0.81   | 0.73   | 0.64   |
| \(\langle \hat{v}_C \rangle\) | -76.67 | -56.85 | -61.31 | -73.60 | -70.13 | -64.75 | -58.34 | -50.69 |
| \(\langle \hat{v}_T \rangle\) | 0.00   | 0.00   | -10.91 | -12.26 | -20.23 | -30.59 | -43.86 | -60.41 |
| \(\langle \hat{v}_{LS} \rangle\) | 0.00   | 0.00   | 0.67   | 0.75   | 1.26   | 1.91   | 2.72   | 3.72   |
| \(\langle \hat{v}_{Coul} \rangle\) | 0.83   | 0.76   | 0.78   | 0.85   | 0.85   | 0.85   | 0.86   | 0.87   |
| \(\langle \hat{v} \rangle\) | -75.84 | -56.10 | -70.76 | -84.27 | -88.26 | -92.58 | -98.62 | -106.51|
| \(\langle \hat{T} \rangle\) | 48.54  | 39.98  | 49.67  | 55.81  | 59.71  | 64.39  | 70.52  | 78.19  |
| \(E\) | -27.30 | -16.12 | -21.09 | -28.46 | -28.55 | -28.19 | -28.10 | -28.32 |
| \(R_m\) | 1.48   | 1.63   | 1.50   | 1.41   | 1.39   | 1.37   | 1.34   | 1.31   |
| \(P(-)\) | 0.00   | 0.00   | 0.08   | 0.11   | 0.13   | 0.16   | 0.19   | 0.21   |

To see the effect of the parity and the charge mixings, we calculate three cases. The first calculation is the simple Hartree-Fock (HF) case, in which we do not perform either the parity projection nor the charge projection. The second calculation is the parity-projected Hartree-Fock (PPHF) case, in which we only perform the parity projection. The third calculation is the charge- and parity-projected Hartree-Fock (CPPHF) case, in which we perform both the parity and the charge projections. We show the calculated results in Table 1. We fix \(x_T = 1.5\) and \(x_{TE} = 0.81\) for this comparison. With these parameters the
binding energy of the alpha particle is reproduced in the CPPHF case. We also show the results for the HF calculation with the original Volkov No. 1 force and the original G3RS force ($x_T = 1.0$ and $x_{TE} = 1.0$) as a reference. In the simple HF case the energy from the tensor force is zero. It means that the result of the self-consistent calculation becomes a simple $(0s)^4$ configuration and there is no $p$-state component. In this case the expectation value of the tensor force is zero identically, because the tensor force does not act between $s$ states. If we perform the parity projection (PPHF), the energy contribution from the tensor force becomes finite. The kinetic energy becomes larger because some component of the $s$ states is going up to the $p$ state to gain the correlation by the tensor force. If we perform the charge projection further (CPPHF), the contribution from the tensor force becomes much larger. It is about three times of the one in the PPHF case. It is reasonable because in the PPHF case only the $\tau_1^0 \tau_2^0$ part in the isovector ($\vec{\tau}_1 \cdot \vec{\tau}_2$) channel of the tensor force is active, but in the CPPHF case all the $\tau_1^+ \tau_2^-$, the $\tau_1^0 \tau_2^0$, and the $\tau_1^- \tau_2^+$ parts of the tensor force are active. These results indicate that both the parity and the charge projections are very important to treat the tensor force in the single-particle picture, i.e., a mean field model. We should note that the variation-after-projection scheme, which we take here, is necessary because even if we assume the mixings of parity and charge in the simple Hartree-Fock calculation, we cannot obtain the result with parity mixing as shown in the third column in Table 1.

In Table 1 we also show the results for various $x_T$'s with the CPPHF method to show the dependence of our results on the strength of the tensor force. If we calculate the $0^-$ state with the CPPHF scheme using $x_T$'s and $x_{TE}$'s in Table 1 which reproduce the binding energy of the ground state of the alpha particle, we obtain the total energies ranging from $-2.70 \sim -2.83$ MeV.

In Fig. 1 the squared wave function in the case with $x_T = 1.5$ and $x_{TE} = 0.81$ is plotted. We only show the wave function of the state in which the neutron component is dominant. In the wave function the probability of the proton component is 17%. As a reference, the harmonic oscillator wave functions for $0s$ and $0p$ with the oscillator length $\alpha = 1.37$ (fm), which corresponds to the result with the original Volkov No. 1 force in the simple Hartree-Fock calculation. From the figure, you can see that the $p$-state component mixing into our wave function behaves differently from the harmonic oscillator $0p$ wave function. It has a much narrower width. This fact indicates that to obtain the tensor correlation, a higher-momentum component is necessary to mix.

4. Summary

We have developed a mean field framework with parity- and charge-mixing single-particle states to treat the tensor force. We showed that by mixing parities and charges in a single-particle state we can obtain the tensor correlation in a mean field framework. We found that the projection before variation is necessary to obtain the tensor correlation. We also found that a higher-momentum component is needed to gain the energy from the tensor force.

In this paper, we only treat the alpha particle with spherical symmetry but it is an interesting subject to calculate heavier nuclei with our model. The extension to the deformed case is also interesting. It might give some indication for the relation between the
Figure 1. Wave function squared for the alpha particle with $x_T = 1.5$ and $x_{TE} = 0.81$ as the function of the radial coordinate $r$ (fm). The solid curve denotes the $s_{1/2}$ neutron component, the dotted curve denotes the $p_{1/2}$ neutron component, the short-dashed curve denotes the $s_{1/2}$ proton component, and the long-dashed curve denotes the $p_{1/2}$ proton component. The harmonic oscillator wave functions with the oscillator length $\alpha = 1.37$ (fm) for $0s$ (dashed-and-dotted) and $0p$ (dashed-and-double-dotted) are plotted as a reference.

clusterization in nuclei and the tensor force. We inevitably treat the short range correlation, which affects the tensor correlation, to make our model more realistic. Through these studies, we hope we will reveal the role of the tensor force in the nuclear structure in the near future.

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