HYPERON BULK VISCOSITY IN THE PRESENCE OF ANTIKAON CONDENSATE

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ABSTRACT

We investigate the hyperon bulk viscosity due to the nonleptonic process \( n + p = p + \Lambda \) in \( K^- \) condensed matter and its effect on the \( r \)-mode instability in neutron stars. We find that the hyperon bulk viscosity coefficient in the presence of antikaon condensate is suppressed compared with the case without the condensate. The suppressed hyperon bulk viscosity in the superconducting phase is still an efficient mechanism to damp the \( r \)-mode instability in neutron stars.

Subject headings: dense matter — instabilities — stars: neutron — stars: oscillations

1. INTRODUCTION

The study of \( r \)-modes in rotating neutron stars has generated great interest in recent times from two points of view. On one hand, \( r \)-modes of rotating neutron stars could be possible sources of detectable gravitational waves. On the other hand, the gravitational radiation–driven instability of \( r \)-modes (Chandrasekhar 1970; Friedman 1978; Friedman & Schutz 1978a, 1978b; Friedman & Morsink 1998; Lindblom et al. 1998; Andersson 1998, 2003; Andersson et al. 1999; Andersson & Kokkotas 2001; Stergioulas 2003) might play an important role in regulating spins of young as well as old accreting neutron stars in low-mass X-ray binaries (LMXBs). The latter situation has been strengthened by the fact that the spin distribution of pulsars has an upper limit at 730 Hz (Chakrabarty 2005; Chakrabarty et al. 2003). It is worth mentioning here that the fastest rotating pulsar has a spin frequency of 716 Hz (Hessels et al. 2006).

The bulk viscosity of neutron star matter is an important issue in connection with the damping of the \( r \)-mode instability in rotating neutron stars. The \( r \)-mode instability may be suppressed through different instabilities by the large hyperon bulk viscosity coefficient could be an efficient mechanism (Jones 2001a, 2001b; Lindblom & Owen 2002; Haensel et al. 2002; van Dalen & Dieperink 2004; Drago et al. 2005; Nayyar & Owen 2006; Chatterjee & Bandyopadhyay 2006, 2007a). However, it may be suppressed by several orders of magnitude when neutrons, protons, or hyperons are superfluid (Haensel et al. 2000, 2001, 2002; Nayyar & Owen 2006; Andersson 2007). This study was later extended to the calculation of bulk viscosity due to other exotic forms of matter such as unpaired quark matter (Madsen 1992; 2000; Dong et al. 2007a, 2007b), quark-hadron mixed phase (Drago et al. 2005; Pan et al. 2006), and color superconducting quark matter (Alford & Schmitt 2007; Alford et al. 2007, 2008; Sa’d et al. 2007a, 2007b). Another possibility of damping the \( r \)-mode instability is the mutual friction between interpenetrating neutron and proton superfluids (Lindblom & Mendell 2000; Andersson 2007).

The superfluidity of neutron star matter plays a significant role on the damping of the \( r \)-modes in neutron stars. It was shown that superfluid particles taking part in nonleptonic weak processes involving hyperons might reduce the hyperon bulk viscosity coefficient by several orders of magnitude (Haensel et al. 2002; Andersson 2007). Recently, we have investigated the bulk viscosity due to the nonleptonic weak process \( n + p = p + \Lambda \) in \( K^- \) condensed matter (Chatterjee & Bandyopadhyay 2007b). The kaon bulk viscosity was suppressed in the condensed phase by several orders of magnitude and could not damp the \( r \)-mode instability. This motivates us to further investigate how the bulk viscosity due to the nonleptonic process \( n + p = p + \Lambda \) behaves in antikaon condensed matter and how it influences \( r \)-modes of neutron stars.

We organize the paper in the following way. In § 2 the field theoretical models of strong interactions, different phases of dense matter, the bulk viscosity coefficient, and the corresponding timescale are described. Results of our calculation are discussed in § 3. Section 4 gives the summary and conclusions.

2. FORMALISM

Our motivation is to calculate the hyperon bulk viscosity due to the nonleptonic process \( n + p = p + \Lambda \) in a superconducting phase, i.e., antikaon condensed phase. This process was extensively investigated in hadronic matter by several groups (Jones 2001a, 2001b; Lindblom & Owen 2002; van Dalen & Dieperink 2004; Nayyar & Owen 2006; Chatterjee & Bandyopadhyay 2006, 2007a). In this case, we consider a first-order phase transition from hadronic to \( K^- \) condensed matter. Both the pure hadronic and \( K^- \) condensed matter are described within the framework of relativistic field theoretical models. The constituents of matter in both phases are neutrons (n), protons (p), \( \Lambda \) hyperons, electrons, and muons. The baryon-baryon interaction is mediated by the exchange of \( \sigma, \omega, \rho \) mesons and two strange mesons, the scalar meson \( f_0(975) \) (denoted hereafter as \( \sigma^0 \)) and the vector meson \( \phi(1020) \) (Schafran & Mishustin 1996). Both phases maintain local charge neutrality and \( \beta \)-equilibrium conditions, whereas these conditions are satisfied globally in the mixed phase. Further, baryons are embedded in the \( K^- \) condensate phase. We describe the equation of state (EOS) in the antikaon condensed phase using the following Lagrangian density for baryons (Schafran & Mishustin 1996),

\[
\mathcal{L}_B = \sum_B \bar{\Psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\omega B} \sigma - g_{\rho B} \gamma_5 \omega^\mu + g_{\rho B} \gamma_5 \rho^\mu \cdot \rho^\mu) \Psi_B + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^2_\sigma \sigma^2 \right)
- U(\sigma) - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} - \frac{1}{2} m^2_\omega \omega_{\mu \nu} \omega^{\mu \nu} - \frac{1}{4} \rho_{\mu \nu} \cdot \rho^{\mu \nu} + \frac{1}{2} m^2_\rho \rho_{\mu \nu} \cdot \rho^{\mu \nu} + \mathcal{L}_{YY},
\]

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where \( \psi_B \) denotes the Dirac bispinor for baryons \( B \) with vacuum mass \( m_B \) and the isospin operator is \( t_B \). The scalar self-interaction term (Boguta & Bodmer 1977) is

\[
U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4.
\]  

(2)

The Lagrangian density for hyperon-hyperon interaction (\( L_{YY} \)) is given by

\[
L_{YY} = \sum_B \bar{\Psi}_B \left( g_{\sigma^+ \sigma^+} \gamma_\mu \phi^\mu + \frac{1}{2} \left( \partial_\mu \sigma^+ \partial^\mu \sigma^+ - m_\sigma^2 \sigma^2 \right) \right) - \frac{1}{4} \phi_\mu \phi^\mu - \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu.
\]  

(3)

As nucleons do not couple with strange mesons, \( g_{\sigma^+ N} = g_{\sigma^+ Y} = 0 \).

Similarly, we treat the (anti-) kaon-baryon interaction in the same footing as the baryon-baryon interaction. The Lagrangian density for (anti-) kaons in the minimal coupling scheme is (Glendenning & Schaffner-Bielich 1998, 1999; Banik & Bandyopadhyay 2001a, 2001b)

\[
L_K = D_\mu \bar{K} D^\mu K - m_K^2 \bar{K} K,
\]  

(4)

where the covariant derivative is \( D_\mu = \partial_\mu + ig_{\omega_\mu} \gamma_\mu + ig_{\phi_\mu} \phi_\mu + ig_{\kappa_\mu} \kappa_\mu \) and the effective mass of (anti-) kaons is \( m_K^2 = m_K - g_{\sigma^+ K} \sigma - g_{\sigma^+ \kappa} \kappa \).

We perform this calculation in the mean field approximation (Serot & Walecka 1986). The mean meson fields in the condensed phase are denoted by \( \sigma, \sigma^+, \omega_0, \phi_0, \) and \( \rho_3 \). The expressions for mean fields can be found in Banik & Bandyopadhyay (2001b) and Chatterjee & Bandyopadhyay (2007b). The in-medium energy of \( K^- \) mesons for s-wave \( (k = 0) \) condensation is given by

\[
\omega_{K^-} = \mu_{K^-} = m_{K^-}^2 - g_{\omega_0 K} \omega_0 - g_{\phi_0 K} \phi_0 + I_{3K^-} g_{\rho_3 K} \rho_3,
\]  

(5)

where \( \mu_{K^-} \) is the chemical potential of \( K^- \) mesons and the isospin projection is \( I_{3K^-} = -1/2 \). The chemical potential for baryons \( B \) is given by

\[
\mu_B^K = \left( k_B^2 + m_B^2 k_B \right)^{1/2} + g_{\omega_0 B} \omega_0 + g_{\phi_0 B} \phi_0 + I_{3B} g_{\rho_3 B} \rho_3,
\]  

(6)

where the effective baryon mass is \( m_B^e = m_B - g_{\sigma^+ B} \sigma - g_{\sigma^+ \rho_3} \sigma \) and isospin projection for baryons \( B \) is \( I_{3B} \). We obtain the mean fields in the hadronic phase putting source terms for \( K^- \) mesons equal to zero in corresponding equations of motion (Banik & Bandyopadhyay 2001b; Chatterjee & Bandyopadhyay 2007b).

The total energy density and pressure in the antikaon condensed phase are given by

\[
e^K = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_{\kappa^2} n_K^- - \sum_{\mathbf{k}_B n_p \Lambda} \frac{2 J_B}{2 \pi^2} \int \frac{d^3 k}{k^3} \left( k^2 + m_B^2 k^2 \right)^{1/2} k^2 dk + \sum_{l=e, \mu} \frac{1}{\pi^2} \int \frac{d k^l_{K^-}}{k^l_{K^-} (k^2 + m_{K^-}^2 k^2)^{1/2} k^2 dk + m_{K^-} n_{K^-}},
\]  

(7)

\[
p^K = -\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} m_{\kappa^2} n_K^- + \frac{1}{3} \sum_{\mathbf{k}_B n_p \Lambda} \frac{2 J_B}{2 \pi^2} \int \frac{d^3 k}{k^3} \left( k^2 + m_B^2 k^2 \right)^{1/2} k^2 dk + \frac{1}{3} \sum_{l=e, \mu} \frac{1}{\pi^2} \int \frac{d k^l_{K^-}}{k^l_{K^-} (k^2 + m_{K^-}^2 k^2)^{1/2} k^2 dk + m_{K^-} n_{K^-}}.
\]  

(8)

We describe the mixed phase of hadronic and \( K^- \) condensed matter using the Gibbs conditions for thermodynamic equilibrium along with global charge and baryon number conservation laws (Glendenning 1992; Glendenning & Schaffner-Bielich 1999).

Now we focus on the calculation of bulk viscosity due to the nonleptonic process. It was shown that the part of the bulk viscosity coefficient was related to the relaxation times of microscopic processes (Landau & Lifschitz 1999; Lindblom & Owen 2002) by the relation

\[
\zeta = \frac{P(\gamma_\infty - \gamma_0) \tau}{1 + (\omega \tau)^2},
\]  

(9)

where \( P \) is the pressure, \( \tau \) is the net microscopic relaxation time, and \( \gamma_\infty \) and \( \gamma_0 \) are “infinite” and “zero” frequency adiabatic indices, respectively, and their difference

\[
\gamma_\infty - \gamma_0 = -\frac{m_b^2 \partial P}{P} \frac{d \tilde{\epsilon}_n}{\partial n_b}.
\]  

(10)
is determined using the EOS as input. Here, \( \bar{n}_n = n_n/m_b \) gives the neutron fraction in the equilibrium state and \( n_b = \Sigma_b n_B \) is the total baryon density. We are interested in (l, m) \( r \)-modes. In this case, the angular velocity (\( \omega \)) of the mode is related to the angular velocity (\( \Omega \)) of a rotating neutron star as \( \omega = (2ml/[(l+1)])\Omega \) (Andersson 2003).

The partial derivatives of pressure with respect to neutron fraction in both phases are calculated using the Gibbs-Duhem relation (Chatterjee & Bandyopadhyay 2007b). In the pure hadronic phase, this relation gives

\[
\frac{\partial P_h}{\partial n_n} = n_n^h \alpha_{nn}^h + n_p^h \alpha_{pn}^h + n_{\Lambda}^h \alpha_{n\Lambda}^h,
\]

(11)

where \( \alpha_{ij} \) is defined by \( \alpha_{ij} = \left( \partial \mu_i/\partial n_j \right)_{n_k=\beta} \). Similarly, in the pure condensed phase it is given by

\[
\frac{\partial P^K}{\partial n_n} = n_n^K \alpha_{nn}^K + n_p^K \alpha_{pn}^K + n_{\Lambda}^K \alpha_{n\Lambda}^K + n_K \alpha_{K-n}^K.
\]

(12)

In the mixed phase, this relation has the form

\[
\frac{\partial P}{\partial n_n} = \frac{\partial P_h}{\partial n_n} + \frac{\partial P^K}{\partial n_n}.
\]

(13)

where \( n_n = (1 - \chi) n_n^h + \chi n_n^K \).

Next, we calculate the relaxation times of the nonleptonic process in different phases. In this case, we express all perturbed quantities in terms of the variation in neutron number density \( (n_n) \) in the respective phase. The relaxation time (\( \tau \)) for the nonleptonic process is given by (Lindblom & Owen 2002)

\[
\frac{1}{\tau} = \frac{\Gamma_{\Lambda}}{\delta \mu} \frac{\delta \mu}{\delta n_n^h}.
\]

(14)

where \( \delta n_n^h = n_n^h - \bar{n}_n^h \) is the departure of neutron fraction from its thermodynamic equilibrium value \( \bar{n}_n^h \) in the \( j \)-th (= hadron, \( K^- \) condensed) phase. The reaction rate per unit volume for the nonleptonic process in question was already calculated by others (Lindblom & Owen 2002; Nayyar & Owen 2006). The relaxation time (\( \tau \)) for the process in the hadronic phase is given by (Lindblom & Owen 2002)

\[
\frac{1}{\tau} = \frac{(kT)^2}{192\pi} p_\Lambda \left( |M| \right)^2 \frac{\delta \mu}{\delta n_n^h},
\]

(15)

along with

\[
\frac{\delta \mu}{\delta n_n^h} = \left( \alpha_{nn}^h - \alpha_{n\Lambda}^h \right) + \left( \alpha_{np}^h - \alpha_{n\Lambda}^h \right) \frac{\delta n_p^h}{\delta n_n^h} + \left( \alpha_{n\Lambda}^h - \alpha_{\Lambda\Lambda}^h \right) \frac{\delta n_{\Lambda}^h}{\delta n_n^h}.
\]

(16)

where \( p_\Lambda \) is the Fermi momentum for \( \Lambda \) hyperons and \( |M| \) is the angle-averaged matrix element squared in the hadronic phase given by Lindblom & Owen (2002) and Nayyar & Owen (2006). In the pure hadronic phase, the second term in equation (16) vanishes because \( \delta n_{\Lambda}^h = 0 \). However, the calculation of the hadronic part of the mixed phase is a little bit involved and is described below.

The relaxation time in the antikaon condensed phase has the same form as in equation (15). In this case, the angle-averaged matrix element \( \langle |M|^2 \rangle \) and \( p_\Lambda \) are to be calculated in the condensed phase. In addition, we calculate \( \delta \mu/\delta n_n^K \) from the chemical potential imbalance due to the nonleptonic hyperon process \( n + p = p + \Lambda \), and it is given by

\[
\delta \mu = \delta \mu^K - \delta \mu^K_A = \left( \alpha_{nn}^K - \alpha_{n\Lambda}^K \right) \frac{\delta n_n^K}{\delta n_n^h} + \left( \alpha_{np}^K - \alpha_{n\Lambda}^K \right) \frac{\delta n_p^h}{\delta n_n^h} + \left( \alpha_{n\Lambda}^K - \alpha_{\Lambda\Lambda}^K \right) \frac{\delta n_{\Lambda}^h}{\delta n_n^h}.
\]

(17)

We express \( \delta \mu \) in terms of \( \delta n_n^K \) and obtain \( \delta \mu/\delta n_n^K \) using the following constraints,

\[
\left( \frac{\delta n_n^K}{\delta n_p^h} + \frac{\delta n_p^K}{\delta n_A} \right) = 0, \quad \left( \frac{\delta n_p^h}{\delta n_A} + \frac{\delta n_A}{\delta n_n^K} \right) = 0,
\]

(18)

and the chemical equilibrium in the strangeness-changing process \( n = p + K^- \),

\[
\delta \mu_n^K - \delta \mu_n^K - \delta \mu_n^K = \left( \alpha_{nn}^K \delta n_n^K + \alpha_{np}^K \delta n_p^K + \alpha_{n\Lambda}^K \delta n_{\Lambda}^K + \alpha_{nK}^K \delta n_K^K \right) - \left( \alpha_{nn}^K \delta n_n^K + \alpha_{np}^K \delta n_p^K + \alpha_{p\Lambda}^K \delta n_{\Lambda}^K + \alpha_{pK}^K \delta n_K^K \right)
\]

(19)

\[
- \left( \alpha_{n\Lambda}^K \delta n_{\Lambda}^K + \alpha_{nK}^K \delta n_K^K + \alpha_{n\Lambda}^K \delta n_{\Lambda}^K + \alpha_{nK}^K \delta n_K^K \right) = 0.
\]
Next, we calculate $\alpha_{ij}$ in the hadronic as well as the $K^-$ condensed phases using the EOS. We can write down these quantities for both phases in generalized forms. For $B = B'$, we get

$$
\alpha^p_{BB'} = \frac{\partial \mu^p_{B'} }{ \partial n^p_{B'} } = \frac{1}{4} \left( \frac{g_{SB}}{m_{\sigma}} \right) ^2 + \left( \frac{g_{SB}}{m_{\rho}} \right) ^2 + \left( \frac{g_{SB}}{m_{\rho'}} \right) ^2 - \frac{\pi^2}{k_F s} \sqrt{ \frac{k_F^2 + m_B^{p^2} }{k_F^2 + m_B^{p^2} } } \left( g_{SB} \frac{\partial \sigma}{\partial n^p_{B'} } + g_{SB'} \frac{\partial \sigma^*}{\partial n^p_{B'} } \right),
$$

(20)

and for $B \neq B'$,

$$
\alpha^p_{BB'} = \frac{\partial \mu^p_{B'} }{ \partial n^p_{B'} } = \left( g_{SB} g_{SB'} \right) \left( g_{SB} g_{SB'} \right) - \frac{m_B^{p^2}}{k_F s} \sqrt{ \frac{k_F^2 + m_B^{p^2} }{k_F^2 + m_B^{p^2} } } \left( g_{SB} \frac{\partial \sigma}{\partial n^p_{B'} } + g_{SB'} \frac{\partial \sigma^*}{\partial n^p_{B'} } \right),
$$

(21)

along with the following relations applicable for both cases

$$
\frac{\partial \sigma}{\partial n^p_{B'} } = \left( g_{SB}/m_{\sigma} \right) \left( \frac{m_B^{p^2}}{k_F^2 + m_B^{p^2} } \right),
$$

(22)

$$
\frac{\partial \sigma^*}{\partial n^p_{B'} } = - D'' \frac{\partial \sigma}{\partial n^p_{B'} },
$$

(23)

where

$$
D = 1 + \frac{1}{m^2_{\sigma}} \frac{d^2 U}{d \sigma^2} + \sum_{B = \pi, \rho} \frac{2 J_B + 1}{2 \pi^2} \left( \frac{g_{SB}}{m_{\sigma}} \right)^2 \int_0^{k_F s} \frac{k^4 dk}{(k^2 + m_B^{p^2} )^{3/2}},
$$

(24)

$$
D' = \sum_{B = \pi, \rho} \frac{2 J_B + 1}{2 \pi^2} \left( \frac{g_{SB} g_{SB'}}{m_{\sigma}^2} \right) \int_0^{k_F s} \frac{k^4 dk}{(k^2 + m_B^{p^2} )^{3/2}},
$$

(25)

$$
D'' = \sum_{B = \pi, \rho} \frac{2 J_B + 1}{2 \pi^2} \left( \frac{g_{SB} g_{SB'}}{m_{\sigma}^2} \right) \int_0^{k_F s} \frac{k^4 dk}{(k^2 + m_B^{p^2} )^{3/2}},
$$

(26)

Here, $B$ and $B'$ denote baryons and $P$ stands for the hadron ($h$) or antikaon ($K$) phase. Further, nucleons do not couple with strange mesons, i.e., $g_{\pi N} = g_{\rho N} = 0$. Similarly, hyperons do not couple with $\rho$ mesons, i.e., $g_{\rho \Lambda} = 0$. The results for other $\alpha$ quantities in the antikaon condensed phase are given below,

$$
\alpha_{BK^-} = \frac{\partial \mu^K_{BK^-} }{ \partial n^K_{BK^-} } = \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right) - \left( g_{SB} g_{SBK} \right) - \frac{m_B^{K^2}}{k_F s} \sqrt{ \frac{k_F^2 + m_B^{K^2} }{k_F^2 + m_B^{K^2} } } \left( g_{SB} \frac{\partial \sigma}{\partial n^K_{BK^-} } + g_{SB'} \frac{\partial \sigma^*}{\partial n^K_{BK^-} } \right),
$$

(27)

$$
\alpha_{K^- B} = \frac{\partial \mu^K_{K^- B} }{ \partial n^K_{K^- B} } = \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right) - \left( g_{SB} g_{SBK} \right) - \left( g_{SB} g_{SBK} \right) \left( g_{SB} \frac{\partial \sigma}{\partial n^K_{K^- B} } + g_{SB'} \frac{\partial \sigma^*}{\partial n^K_{K^- B} } \right),
$$

(28)

$$
\alpha_{K^- K^-} = \frac{\partial \mu^K_{K^- K^-} }{ \partial n^K_{K^- K^-} } = \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right) + \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right) - \left( g_{SB} g_{SBK} \right) \left( g_{SB} \frac{\partial \sigma}{\partial n^K_{K^- K^-} } + g_{SB'} \frac{\partial \sigma^*}{\partial n^K_{K^- K^-} } \right),
$$

(29)

where

$$
\frac{\partial \sigma}{\partial n^K_{K^- K^-} } = \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right), \quad \frac{\partial \sigma^*}{\partial n^K_{K^- K^-} } = \left( g_{SB} g_{SBK} \right) \left( g_{SB} g_{SBK} \right),
$$

(30)

$$
F = 1 + \sum_{B = \pi, \rho} \frac{2 J_B + 1}{2 \pi^2} \left( \frac{g_{SB} g_{SBK} }{m_{\sigma}^2} \right) \int_0^{k_F s} \frac{k^4 dk}{(k^2 + m_B^{p^2} )^{3/2}},
$$

(31)

In the second term in equations (27) and (28), the plus sign corresponds to neutrons and the minus sign is for protons. With the given $\alpha_{ij}$, we can now calculate the relaxation time for the nonleptonic process in the hadron as well as antikaon condensed phases. As soon as we know the relaxation time, we can calculate the bulk viscosity coefficient in each phase.
Now we focus on the calculation of the relaxation time and bulk viscosity in the mixed phase. For this, we have to express the chemical imbalance \( \delta n^h_{\mu} \) in the nonleptonic hyperon process as given by equation (17) in terms of \( \delta n^h_{\mu} \) from the following constraints,

\[
(1 - \chi) \left( \delta n^h_{n} + \delta n^h_{p} + \delta n^h_{\Lambda} \right) + \chi \left( \delta n^h_{K} + \delta n^K_{p} + \delta n^K_{\Lambda} \right) = 0, \quad (1 - \chi) \delta n^h_{p} + \chi (\delta n^h_{p} - \delta n^K_{p}) = 0, \\
\delta \mu^h_{p} = \delta \mu^h_{n}, \quad \delta \mu^h_{p} = \delta \mu^h_{n}, \quad \delta \mu^h_{\Lambda} = \delta \mu^K_{\Lambda}, \quad \delta \mu^h_{n} - \delta \mu^h_{p} - \delta \mu^K_{p} = 0. \tag{32}
\]

Here, we have \( \delta \chi = 0 \) because number densities deviate from their equilibrium values only by internal reactions (Lindblom & Owen 2002). The first two constraints follow from the conservation of baryon number and electric charge neutrality. The last constrain is the result of the chemical equilibrium involving \( K^- \) condensate as already shown by equation (19). The other constraints are due to the equality of neutron, proton, and \( \Lambda \) chemical potentials in the hadronic and condensed phases, and we can rewrite them as

\[
\begin{align*}
&\left( \alpha_{\mu n}^h \delta n^h_{n} + \alpha_{\mu p}^h \delta n^h_{p} + \alpha_{\mu \Lambda}^h \delta n^h_{\Lambda} \right) - \left( \alpha_{\mu n}^K \delta n^K_{n} + \alpha_{\mu p}^K \delta n^K_{p} + \alpha_{\mu \Lambda}^K \delta n^K_{\Lambda} \right) = 0, \\
&\left( \alpha_{\mu n}^h \delta n^h_{n} + \alpha_{\mu p}^h \delta n^h_{p} + \alpha_{\mu \Lambda}^h \delta n^h_{\Lambda} \right) - \left( \alpha_{\mu n}^K \delta n^K_{n} + \alpha_{\mu p}^K \delta n^K_{p} + \alpha_{\mu \Lambda}^K \delta n^K_{\Lambda} \right) = 0, \\
&\left( \alpha_{\mu n}^h \delta n^h_{n} + \alpha_{\mu p}^h \delta n^h_{p} + \alpha_{\mu \Lambda}^h \delta n^h_{\Lambda} \right) - \left( \alpha_{\mu n}^K \delta n^K_{n} + \alpha_{\mu p}^K \delta n^K_{p} + \alpha_{\mu \Lambda}^K \delta n^K_{\Lambda} \right) = 0. \tag{33}
\end{align*}
\]

We express \( \delta n^h_{n}, \delta n^h_{p}, \delta n^h_{\Lambda}, \delta n^K_{p}, \delta n^K_{\Lambda}, \) and \( \delta n^h_{K} \) in terms of \( \delta n^h_{\mu} \) using the above six constraints. For this purpose, we solve a 6 \( \times \) 6 matrix constructed out of the above six relations and obtain \( \delta \mu/\delta n^h_{\mu} \). Similarly, we obtain \( \delta \mu/\delta n^h_{\mu} \) in the mixed phase from the above constraints. This completes the calculation of the relaxation time and bulk viscosity in the mixed phase.

Next, we calculate critical angular velocity as a function of the temperature and mass of a rotating neutron star. The bulk viscosity damping timescale \( (\tau_r) \) due to the nonleptonic process involving \( \Lambda \) hyperons and the bulk viscosity profile as a function of \( r \) are obtained by following previous work (Lindblom et al. 1999; Lindblom & Owen 2002; Nayyar & Owen 2006; Chatterjee & Bandyopadhyay 2006). Further, we take into account timescales associated with gravitational radiation \( (\tau_{gr}) \), bulk viscosity due to the modified Urca process \( (\tau_U) \) involving only nucleons (Sawyer 1989; Andersson & Kokkotas 2001), and the shear viscosity \( (\tau_{sv}; \tau_{nu}; \tau_{nU} \) Lindblom et al. 1998; Andersson & Kokkotas 2001; Andersson 2007) and define the overall \( r \)-mode timescale \( (\tau_r) \) as

\[
\frac{1}{\tau_r} = \frac{1}{\tau_{gr}} + \frac{1}{\tau_U} + \frac{1}{\tau_{sv}}. \tag{34}
\]

Finally, solving \( 1/\tau_r = 0 \), we calculate the critical angular velocity above which the \( r \)-mode is unstable, whereas it is stable below the critical angular velocity.

3. RESULTS AND DISCUSSION

For this calculation, nucleon-meson coupling constants are taken from Glendenning & Moszkowski (1991), and this set is known as GM. Nucleon-meson coupling constants are determined by reproducing nuclear matter saturation properties such as binding energy \( E/B = -16.3 \text{ MeV} \), baryon density \( n_0 = 0.153 \text{ fm}^{-3} \), asymmetry energy coefficient \( a_{\text{asy}} = 32.5 \text{ MeV} \), incompressibility \( K = 300 \text{ MeV} \), and effective nucleon mass \( m^*_n/m_N = 0.70 \). Further, we need to know kaon-meson coupling constants and determine them using the quark model and isospin counting rule. The vector coupling constants are given by

\[
g_{\omega K} = \frac{1}{3} g_{\omega N}, \quad g_{\rho K} = g_{\rho N}. \tag{35}
\]

The scalar coupling constant is obtained from the real part of the \( K^- \) optical potential depth at normal nuclear matter density,

\[
U^*_K (n_0) = -g_{\omega K} \sigma - g_{\omega K} \omega_0. \tag{36}
\]

Antikaons experience an attractive potential, whereas kaons have a repulsive interaction in nuclear matter (Friedman et al. 1994, 1999; Koch 1994; Batty et al. 1997; Waas & Weise 1997; Li et al. 1997a, 1997b; Pal et al. 2000). The analysis of \( K^- \) atomic data using a hybrid model (Friedman et al. 1999), which combines the relativistic mean field approach in the nuclear interior and a phenomenological potential at low density, yielded the real part of the antikaon potential as large as \( U^*_K (n_0) = -180 \pm 20 \text{ MeV} \) at normal nuclear matter density. It was predicted that \( K^- \) condensation might occur in neutron star matter for strongly attractive antikaon potential \( \sim -100 \text{ MeV} \) or more. In this calculation, we adopt the value of the antikaon optical potential depth at normal nuclear matter density as \( U^*_K (n_0) = -160 \text{ MeV} \). We obtain the kaon-scalar meson coupling constant \( g_{\omega K} = 2.9937 \) corresponding to this antikaon optical potential depth.

On the other hand, hyperon-vector meson coupling constants are determined using SU(6) symmetry of the quark model (Dover & Gal 1984; Schaffner et al. 1994; Schaffner & Mishustin 1996), and the scalar \( \sigma \) meson coupling to \( \Lambda \) hyperons is calculated from the hyperon potential depth in normal nuclear matter \( U^*_\Lambda (n_0) = -30 \text{ MeV} \) obtained from hypernuclei data (Dover & Gal 1984; Chrien & Dover 1989). The hyperon-\( \sigma^* \) coupling constant is determined from double-\( \Lambda \) hypernuclei data (Schaffner et al. 1993; Schaffner & Mishustin
The strange meson fields also couple with (anti-) kaons. The \( \sigma' - K \) coupling constant determined from the decay of \( f_0(925) \) is \( g_{\sigma'K} = 2.65 \), and the vector \( \phi \) meson coupling with (anti-) kaons \( \sqrt{2} g_{\phi KK} = 6.04 \) follows from the SU(3) relation (Schaffner & Mishustin 1996).

The onsets of \( K^- \) condensate and \( \Lambda \) hyperons in neutron star matter are sensitive to the composition of matter and the strength of the antikaon optical potential depth (Banik & Bandyopadhyay 2001b). It was further noted that the early appearance of either \( \Lambda \) hyperons or the \( K^- \) condensate delayed the onset of the other to higher densities. In this calculation, for \( U_{\bar{K}}(n_0) = -160 \text{ MeV} \), \( \Lambda \) hyperons appear just after the onset of \( K^- \) condensation. The \( K^- \) condensation sets in at a density of \( 2.23n_0 \), and the mixed phase ends at \( 4.1n_0 \). On the other hand, \( \Lambda \) hyperons appear at a density of \( 2.51n_0 \). It is worth mentioning here that we obtained qualitatively similar results with the GM parameter set corresponding to \( K = 240 \) MeV and \( U_{\bar{K}}(n_0) = -140 \) MeV. However, this led to a soft EOS resulting in a neutron star mass below the accurately measured mass (Chatterjee & Bandyopadhyay 2007b).

Negatively charged particles such as electrons and muons are depleted from the system with the onset of \( K^- \) condensation and its rapid growth thereafter. At this stage, the proton density becomes equal to the density of \( K^- \) mesons in the condensate. It is evident from the figure that \( \Lambda \) hyperons populate the system just after the onset of \( K^- \) condensation.

The relaxation time for the nonleptonic process \( n + p \rightarrow p + \Lambda \) is plotted with normalized baryon density in Figure 2 for \( U_{\bar{K}}(n_0) = -160 \text{ MeV} \) and temperature \( T = 10^{10} \text{ K} \). Here, EOSs enter as inputs in the calculation of the relaxation time in different phases. In particular, partial derivatives of pressure and chemical potentials with respect to the neutron number density are calculated using EOSs according to the prescription as discussed in \( \S \) 2. This figure shows the relaxation time in different phases, i.e., the pure antikaon condensed phase (\textit{thin solid line}) and the hadronic (\textit{thick solid line}) and antikaon condensed parts of the mixed phase (\textit{dashed line}). We find
that the values of the relaxation time in the pure and mixed antikaon condensed phases are significantly smaller than that of the hadronic phase involving nonsuperfluid A hyperons (Nayyar & Owen 2006; Chatterjee & Bandyopadhyay 2006). The relaxation time for the nonleptonic weak process involving hyperons is inversely proportional to \( T^2 \) as given by equation (15).

Figure 3 exhibits the hyperon bulk viscosity coefficient as a function of the normalized baryon density at a temperature \( T = 10^{10} \) K. Similar to Figure 2, the bulk viscosity in the pure antikaon condensed phase (thin solid line) and the hadronic (thick solid line) and antikaon condensed parts of the mixed phase (dashed line) are shown here. One can immediately see that the hyperon bulk viscosity in the antikaon condensed matter, irrespective of whether it is in the pure or mixed phase, is suppressed compared with that of the hadronic phase. This suppression may be attributed to the superconducting phase, i.e., the \( \Lambda^- \) condensed phase. The role of superfluidity on hyperon bulk viscosity was studied at length by several groups (Haensel et al. 2002; Nayyar & Owen 2006; Andersson 2007). They also obtained significant suppression in the hyperon bulk viscosity, because one or more superfluid particles were participating in nonleptonic weak processes involving hyperons. In our calculation, \( \Lambda^- \) mesons in the Bose-Einstein condensed state are not members of the nonleptonic weak process \( n + p = p + \Lambda \). Therefore, the suppression of the hyperon bulk viscosity in our case originates from the EOS in the \( \Lambda^- \) condensed phase which enters into the calculation of the chemical imbalance as given by equation (17). We further add that the factor \( \omega T \) in the bulk viscosity coefficient given by equation (15) is negligible compared with unity over the whole range of baryon densities considered here. This leads to a \( 1/T^2 \) temperature dependence of the hyperon bulk viscosity. However, inversion of the temperature dependence was observed in some calculations (Haensel et al. 2002; Nayyar & Owen 2006) when the factor \( \omega T \) is much greater than unity. We find a jump in the bulk viscosity coefficient at the upper phase boundary of the mixed phase and pure \( \Lambda^- \) condensed phase. This is attributed to kinks in the EOS and discontinuities in \( (\gamma_\infty - \gamma_0) \).

![Figure 3](image1.png)

**Fig. 3.**—Hyperon bulk viscosity coefficient is exhibited as a function of normalized baryon density at a temperature \( 10^{10} \) K and antikaon optical potential depth at normal nuclear matter density \( U_{k}(n_0) = -160 \) MeV. Different lines have the same meaning as in Fig. 2.

![Figure 4](image2.png)

**Fig. 4.**—Hyperon bulk viscosity profile is shown with equatorial distance for a rotating neutron star of mass \( 1.60 \) \( M_\odot \), at a temperature \( 10^{10} \) K and antikaon optical potential depth \( U_{k}(n_0) = -160 \) MeV. The hadronic and antikaon condensed parts of the mixed phase are shown by thick solid and dashed lines, respectively.
Now we discuss the results of the damping timescale due to the hyperon bulk viscosity and critical angular velocity. This calculation needs the knowledge of the energy density profile, the hyperon bulk viscosity profile, and the structure of the rotating neutron star in question. For this purpose, we consider a neutron star of gravitational mass $1.60\,M_\odot$, having baryon rest mass $1.76\,M_\odot$, and central baryon density $3.50n_0$ and rotating at an angular velocity $\Omega_{\text{rot}} = 2652 \, \text{s}^{-1}$ from the sequence of rotating neutron stars calculated by the model of Stergioulas (Stergioulas & Friedman 1995). This neutron star contains both $\Lambda$ hyperons and $K^-$ condensate in its core, as its central baryon density is well above the threshold densities of $K^-$ condensation and $\Lambda$ hyperons. The hyperon bulk viscosity profile of this neutron star as a function of equatorial distance for $T = 10^{10} \, \text{K}$ is displayed in Figure 4. The hyperon bulk viscosity in the hadronic and antikaon condensed parts of the mixed phase are shown by thick solid and dashed lines, respectively. Further, we note that the bulk viscosity profile drops to zero value beyond 3.5 km, because the baryon density beyond this distance decreases below the threshold density of $\Lambda$ hyperons.

As soon as we know the energy density and bulk viscosity profiles, we obtain the damping timescale corresponding to the hyperon bulk viscosity and critical angular velocities as a function of temperature solving $\Omega = \Omega_{\text{cr}}$ for a rotating neutron star mass of $1.60\,M_\odot$ (Fig. 5). Besides the hyperon bulk viscosity, we also consider the bulk viscosity due to the modified Urca process involving nucleons as well as the shear viscosity to the total $r$-mode timescale as given by equation (34). The bulk viscosity due to the modified Urca process plays an important role to damp the $r$-mode at higher temperatures. However, this process cannot suppress the $r$-mode instability below $10^{10} \, \text{K}$, because the corresponding damping timescale is longer than the gravitational radiation growth timescale. On the other hand, the shear viscosity coefficient is proportional to $T^{-2}$, and the damping timescale $\tau_{\text{damp}}$ is also larger than $\tau_{\text{cr}}$ in the temperature range considered here. The bulk viscosity damping timescale ($\tau_{\text{damp}}$) due to the nonleptonic process $n + p = p + \Lambda$ and $\tau_{\text{damp}}$ are comparable at $T \approx 4 \times 10^{9}$ and below. Consequently, the $r$-mode instability is damped in this temperature regime by the hyperon bulk viscosity in $K^-$ condensed matter. Although the hyperon bulk viscosity is suppressed in the antikaon condensed phase, it is still a very efficient process for damping the $r$-mode instability.

4. SUMMARY AND CONCLUSIONS

We have investigated the hyperon bulk viscosity due to the nonleptonic weak process $n + p = p + \Lambda$ in a $K^-$ condensed phase and later applied it to study the $r$-mode instability in neutron stars. For the parameter set adopted here and antikaon optical potential depth $U_K(n_0) = -160 \, \text{MeV}$, $K^-$ condensation occurs before $\Lambda$ hyperons are populated in the system. We find that the hyperon bulk viscosity coefficient in $K^-$ condensed matter is significantly suppressed compared with the nonsuperfluid hyperon bulk viscosity coefficient in the hadronic phase. Further, we note that the hyperon bulk viscosity in the superconducting phase is still an efficient process to damp the $r$-mode instability.

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