Light scattering from self-affine fractal silver surfaces with nanoscale cutoff: Far-field and near-field calculations

José A. Sánchez-Gil and José V. García-Ramos
Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 121, E-28006 Madrid, Spain

Eugenio R. Méndez
División de Física Aplicada,
Centro de Investigación Científica y de Educación Superior de Ensenada
Ensenada, Baja California 22800, México

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Abstract—We study the light scattered from randomly rough, one-dimensional self-affine fractal silver surfaces with nanoscale lower cutoff, illuminated by s- or p-polarized Gaussian beams a few microns wide. By means of rigorous numerical calculations based on the Green theorem integral equation formulation, we obtain both the far- and near-field scattered intensities. The influence of diminishing the fractal lower scale cutoff (from below a hundred, down to a few nanometers) is analyzed in the case of both single realizations and ensemble average magnitudes. For s polarization, variations are small in the far field, being only significant in the higher spatial frequency components of evanescent character in the near field. In the case of p polarization, however, the nanoscale cutoff has remarkable effects stemming from the roughness-induced excitation of surface-plasmon polaritons. In the far field, the effect is noticed both in the speckle pattern variation and in the decrease of the total reflected energy upon ensemble averaging, due to increased absorption. In the near field, more efficient excitation of localized optical modes is achieved with smaller cutoff, which in turn leads to huge surface electric field enhancements.

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I. INTRODUCTION

Since the early days of fractality,[1] the scattering of electromagnetic (EM) waves from fractal surfaces has been a field of intense activity. Physical fractals appear ubiquitously in nature, possessing fractal properties within a broad, however finite, range of scales. Therefore the study of classical wave scattering from fractals is a problem of interest not only from a fundamental point of view, but also from the practical knowledge that probing technologies such as surface optical characterization, remote sensing, radar, sonar, etc. can yield about a wide variety of systems. In fact, it is now well understood that many naturally occurring surfaces exhibit scale invariance, particularly in the form of self-affinity.[1, 2, 3]

There exists a large amount of theoretical works devoted to wave scattering from fractal surfaces.[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] Most of them make use of approximations, such as Kirchhoff approximation (KA) or perturbation methods, in order to obtain analytical expressions that are useful in certain regimes, but thereby imposing a constraint on the length scales over which fractality might be present. Others include calculations within the Rayleigh hypothesis and/or restricted to perfectly conducting surfaces. And only very recent works are capable of dealing with arbitrarily rough metal or dielectric surfaces,[17, 18, 21, 22] on the basis on the Green’s theorem integral equation formulation for rough surface scattering.[23, 24, 25]

On the other hand, in addition to far field scattering, the near EM field on self-affine fractals has attracted a great deal of attention.[26, 27, 28, 29, 30, 31, 32] To a large extent, the interest lies in the observation of large near field intensity distributions, through photon scanning tunneling microscopy (PSTM), of large concentrations of EM field intensity on bright spots).[33, 34, 35, 36, 37, 38] The appearance of such localized optical modes is mediated by the roughness-induced excitation of surface-plasmon polaritons (SPP) on very rough surfaces with subwavelength features. The occurrence of large surface EM fields is indeed crucial to the EM mechanism in Surface-Enhanced Raman Scattering (SERS) and to other non-linear surface optical processes.[20, 29, 30, 31, 32, 38, 39] Furthermore, the predicted, enormously high values of the surface electric field intensity have been related to the recent observation of SERS single molecule probing.[40, 41, 42]

It is our aim to employ the above mentioned rigorous Green’s theorem formulation to study the far-field and near-field scattered from one-dimensionally rough, self-affine fractal Ag surfaces, restricted to one dimension for the sake of computational limitations. The scattering model is described in Sec. II. We focus on fractals preserving self-affinity from tens of microns to subwavelength dimensions. Actually, we analyze the influence of the lower scale cutoff as is reduced from ~ 100 nm (as used in Refs. [17, 18, 28, 29, 32] to a few nanometers. It
should be mentioned that the effect on the far field of the lower scale cutoff in deterministic Koch fractals has been previously studied. The results for the far-field and near-field calculations are presented in Secs. III and IV, respectively. Finally, the main conclusions derived from this work are summarized in Sec. V.

II. SCATTERING MODEL

The scattering geometry is depicted in Fig. 1. A monochromatic, linearly polarized Gaussian beam of frequency ω and 1/e amplitude half width W impinges at an angle θ0 on a randomly rough surface \( z = \zeta(x) \). The rough interface separates vacuum from a semi-infinite silver volume occupying the lower half-space \( z < \zeta(x) \) and characterized by an isotropic, frequency-dependent homogeneous dielectric function \( \varepsilon(\omega) \).

With the aim of solving this scattering problem for arbitrarily large roughness parameters, we make use of the scattering integral equations based on the the application of Green’s second integral theorem. It is well known that this full vectorial formulation is considerably simplified when restricted to 1D surfaces and linear polarization, thereby being reduced to four scalar integral equations for the only nonzero component of either the electric field amplitude

\[
\mathbf{E}^{(s)} \equiv \mathbf{E}^{(s)}(\mathbf{r}, \omega) \hat{y},
\]

with

\[
\mathbf{H}^{(s)} \equiv H_x^{(s)}(\mathbf{r}, \omega) \hat{x} + H_z^{(s)}(\mathbf{r}, \omega) \hat{z},
\]

for \( s \) polarization, or the magnetic field amplitude

\[
\mathbf{H}^{(p)} \equiv H_x^{(p)}(\mathbf{r}, \omega) \hat{x} + H_z^{(p)}(\mathbf{r}, \omega) \hat{z},
\]

with

\[
\mathbf{E}^{(p)} \equiv E_x^{(p)}(\mathbf{r}, \omega) \hat{x} + E_z^{(p)}(\mathbf{r}, \omega) \hat{z},
\]

for \( p \) polarization. Two of these surface integral equations can be used as extended boundary conditions, which (recalling the continuity relations across the interface) lead to a system of two coupled integral equations for the surface field and its normal derivative. This can be solved numerically in the form of linear equations by means of a quadrature scheme. Once these source functions are obtained, it is straightforward to calculate the far field scattered intensity, and in general the electric (magnetic) field amplitude for \( s \) \( (p) \) polarization at any point in vacuum or inside the metal from two of the integral equations. In addition, from the latter integral equations, the corresponding expressions for the magnetic (electric) field amplitudes for \( s \) \( (p) \) polarization can be simply obtained by making use of Maxwell equations (cf. Ref. in the case of \( p \) polarization). These expressions can be useful in near electric or magnetic field calculations. However, they exhibit non-integrable singularities when approaching the surface profile.

In this regard, the calculation of the surface magnetic field intensity for \( s \) polarization has been done in a similar manner by exploiting the connection between its normal and tangential components on the vacuum side

\[
H_n^{(s)}(x) = \frac{ic}{\omega} \gamma_m^{-1} \frac{dE^{(s)}(x)}{dx},
\]

\[
H_t^{(s)}(x) = \frac{ic}{\omega} \gamma_m^{-1} F^{(s)}(x),
\]

with the surface electric field and its normal derivative from the vacuum side,

\[
E^{(s)}(x) = E^{(s,>)}(x, \zeta(x)),
\]

\[
F^{(s)}(x) = \gamma \frac{\partial E^{(s,>)}(x)}{\partial n} \Big|_{z=\zeta^{(+)}}(x),
\]

with \( \gamma = [1 + (\zeta'(x))^2]^{1/2} \). The corresponding expressions for the surface electric field components for \( p \) polarization are given by:

\[
E_n^{(p)}(x) = \frac{ic}{\omega} \gamma_m^{-1} \frac{dH^{(p)}(x)}{dx},
\]

\[
E_t^{(p)}(x) = -\frac{ic}{\omega} \gamma_m^{-1} L^{(p)}(x),
\]

where

\[
H^{(p)}(x) = H^{(p,>)}(x, \zeta(x))
\]

\[
L^{(p)}(x) = \gamma \frac{\partial H^{(p,>)}(x)}{\partial n} \Big|_{z=\zeta^{(+)}}(x),
\]

As mentioned above, \( E^{(s)}(x), F^{(s)}(x), H^{(p)}(x), L^{(p)}(x) \) constitute the source functions of the integral equations in this scattering configuration.

As a model describing many naturally occurring surface growth phenomena exhibiting self-affine fractality, we have chosen that given by the trace of a fractional Brownian motion through Voss’ algorithm. In addition, this model yields self-affine fractal structures that resemble fairly well the properties of some SERS metal substrates. The ensembles of realizations thus generated are characterized by their fractal dimension \( D = 2 - H \) (\( H \) being the Hurst exponent) and rms height \( \delta \). (In order to avoid the inherent ambiguity in the definition of the rms height for self-affine fractals, \( \delta \) refers in our calculations to the rms height defined over the entire fractal profile with length \( L_f = 51.4 \mu m \). Recall that \( \delta \) depends on the length \( \Delta x \) over which it is measured through \( \delta = l^{1-H} \Delta x^H \), \( l \) being the topothesy. Thus the topothesy \( l \) will be also given. From each generated fractal profile with \( N_f \) points and length \( L_f \), sequences of \( N \) points (with constant \( N/N_f \)) are extracted to obtain similar profiles with identical properties except for the lower scale cutoff as determined by \( \xi_L = L_f/N_f \). (Strictly speaking, \( \xi_L \) should be given by the minimum
length scale above which the ensemble of such realizations exhibit self-affinity, resulting in a value typically larger than the mere discretization cutoff (29).

From now on, we will focus below on the fractal dimension $D = 1.9$. This choice is justified for we expect the influence of decreasing $\xi_L$ to be more significant for larger fractal dimensions. And recall that the effect of varying $D$ has been already studied on the far field (18) and the near field (29), albeit for a relatively large lower cutoff $\xi_L$. The values of the lower scale cutoffs hitherto considered are $\xi_L = 51.4, 25.7, 12.85,$ and $6.425$ nm, resulting from sequences of $N = 102, 205, 410,$ and $819$ points extracted from profiles with $N_f = 1024, 2048, 4096,$ and $8192$ points. The length of all realizations is thus $L = L_f(N/N_f) = 5.14$ $\mu$m. Actually, the final number of sampling points per realization used in the numerical calculations is significantly higher for the sake of accuracy: $N_p = n_i N$ as obtained by introducing $n_i = 4-10$ cubic-splined interpolating points, the latter being chosen on the basis of numerical convergence tests.

### III. FAR FIELD

First, we present in Fig. 2 the angular distributions of the speckle pattern intensities scattered from self-affine Ag surface profiles with identical properties ($D = 1.9$, $L = 5.14$ $\mu$m, and $\delta$) except for $\xi_L = 51.4, 25.7, 12.85,$ and $6.425$ nm, as mentioned in the preceding section. Both s- and p-polarized incident ($\theta_0 = -10^\circ$) Gaussian beams are considered with wavelength $\lambda = 2\pi c/\omega = 629.9$ nm and $W = (L/4) \cos \theta_0$, the dielectric constant of Ag at this frequency being $\epsilon = -15 + i$. The upper plots correspond to moderately rough surfaces with $\delta = 51.4$ nm and topohesy $l = 24$ nm (the corresponding surface profiles will be shown in Fig. 4), beyond the reach of standard approximate treatments such as the KA or perturbation theories. No significant differences are found (note the semi-log scale in order to enhance them) for s polarization, nor even away from the bright specular peak. In the case of p polarization, a weak decrease in the specular peak with decreasing $\xi_L$ is barely observed, whereas small differences appear in the speckle pattern away from the specular peak.

We now consider rougher surfaces with $\delta = 257$ nm and topohesy $l = 143$ nm (see Figs. 4c and 4d). Appreciable changes are observed in the s-polarized speckle with decreasing $\xi_L$, although the patterns are qualitatively similar. However, in the case of p-polarized speckle patterns, differences are both quantitative and qualitatively notorious. Note the absence of specular peaks in both polarizations. Therefore, in light of Figs. 3a and 3b, it is seen that the details at nanometer scales produce no change in the s-polarized speckle patterns for moderately rough self-affine profiles, appearing only small quantitative variations for very large rms height. For p polarization, nonetheless, small changes are already observed for moderately rough profiles, becoming dramatic for large roughness.

Next, the influence of nanoscales on the mean scattered intensities is analyzed. This is done in Figs. 3 after averaging speckle pattern distributions over an ensemble of $N_{\text{real}} = 200$ realizations. Experimentally, this can be achieved either by also ensemble averaging, or by illuminating a very large surface and collecting a large amount of speckles at every scattering angle (or a combination of both). In the case of the self-affine fractals with $\delta = 51.4$ nm (see Figs. 3a and 3b), the diffuse component of the mean scattered intensity in s polarization is not altered at all with decreasing lower scale cutoff. As for the specular, which amounts to $\sim 32\%$, and not shown in Fig. 3b, the same is true. The p-polarized diffuse component in Fig. 3b shows no significant variation at angles away from the specular direction, whereas a very small, but observable, decrease in the specular direction is observed when the lower scale cutoff is reduced. On the other hand, the p-polarized specular component, not shown in Fig. 3b, exhibits a larger decrement (from $\sim 32\%$ down to $\sim 24\%$) with diminishing $\xi_L$. Interestingly, despite the specular components are not included in Figs. 3a and 3b, the resulting angular distributions of diffusely scattered intensities present pronounced peaks at the specular directions. These diffuse specular peaks have been already discussed in Ref. (18) for self-affine fractals with various fractal dimensions but a higher lower scale cutoff; the results in Figs. 3a and 3b demonstrate that such peaks are not substantially affected by the fractal scales below a hundred nanometers.

Ensemble averaging of the speckle pattern scattered from very rough ($\delta = 257$ nm) self-affine fractal surfaces also tends to wash out differences arising from nanoscale fractal details (see Figs. 4c and 4d). In fact, no differences are found in the angular distribution of s-polarized, mean scattered intensities (leaving aside some spurious speckle noise due to the limited number of realizations employed), even though quantitative changes have been found above in the speckle patterns for single realizations (see Fig. 3b). Furthermore, the remarkable qualitative and quantitative changes in the p-polarized speckle spots seen in Fig. 4b reduce to significant, though only quantitative, variations upon ensemble averaging as shown in Fig. 3b. And if the effect of absorption, defined as $A = 1 - S$ (S being the normalized reflectance) is taken into account, the resulting renormalized angular distributions of p-polarized mean scattered intensities, $(1/\theta)/S$, exhibit little differences with decreasing fractal lower scale cutoff (see Fig. 4). Incidentally, note that the remaining speckle noise in Figs. 3b and 4hinders the possible observation of a backscattering peak due to the multiple scattering of SPP.

### IV. NEAR FIELD

We now turn to the investigation of the influence on the near EM field of the lower scale cutoffs of the self-affine
fractal surfaces whose far-field scattering properties have been studied in the preceding section. Near field intensity distributions are relevant for the information they provide on the scattering process, and can be measured through near-field optical microscopy.

First, we calculate the intensities of all the EM field components on the surface. The results for the moderately rough fractals with $\delta = 51.4$ nm employed in the speckle pattern calculations in Fig. 2 are shown in Figs. 5 and 6 for $s$ and $p$ polarization, respectively. The corresponding surface profiles are shown in the bottom, Figs. 3d and Figs. 4d. The nonzero components of the EM field being plotted are (cf. Sec. II): the tangential, perpendicular to the incident plane, electric (respectively, magnetic) field and the normal and tangential (in the plane of incidence) magnetic (respectively, electric) field, in the case of $s$ (respectively, $p$) polarization. For the sake of clarity, only the central part of the illuminated surface is shown.

Before analyzing the spatial distributions, let us recall what is expected within the KA, namely, the sum of the incident and the specularly (locally) reflected fields. Since the angular variation of the Fresnel coefficients for metals at this frequency is small, the local variations of the specular field can be neglected. Therefore, the KA field intensities are approximately given by those for a planar surface, namely:

$$E^{(s,K)}_n(\xi,\psi) \approx |1 + R_n|^2 E^{(s,i)}(\xi,\psi)$$

$$H_n^{(s,K)}(\xi,\psi) \approx \sin^2 \theta_0 |1 + R_n|^2 H^{(s,i)}(\xi,\psi)$$

$$H_t^{(s,K)}(\xi,\psi) \approx \cos^2 \theta_0 |1 - R_n|^2 H^{(s,i)}(\xi,\psi),$$

for $s$ polarization; and

$$H_n^{(p,K)}(\xi,\psi) \approx |1 + R_p|^2 H^{(p,i)}(\xi,\psi)$$

$$E_n^{(p,K)}(\xi,\psi) \approx \sin^2 \theta_0 |1 + R_p|^2 E^{(p,i)}(\xi,\psi)$$

$$E_t^{(p,K)}(\xi,\psi) \approx \cos^2 \theta_0 |1 - R_p|^2 E^{(p,i)}(\xi,\psi),$$

for $p$ polarization, $R_n$ and $R_p$ being the corresponding Fresnel coefficients for $\theta_0$ on a planar metal surface. The latter field intensities are plotted in Figs. 5 and 6. As expected, the KA does not hold even for the moderately rough surface profiles used therein, nor does perturbation theory (recall that the planar surface field intensities can be considered the zeroth order approximation in the small-amplitude perturbation expansion of the field), but they provide the background about which the actual surface EM field intensities strongly vary.

In the case of $s$ polarization, Fig. 6 reveals the appearance of narrow and bright spots where an enhancement of the field intensity of nearly two orders of magnitude occur, despite the relatively low value of $\delta$. This clearly manifests the crucial role played by the small scale cutoff in the excitation of localized optical modes. (6c)

What if the surface roughness parameter is increased? In Figs. 3 and 4, the surface EM field intensity distributions become more complicated, with larger variations from one realization to another with diminishing $\xi_L$, and no resemblance whatsoever with the surface profiles. These variations are considerably larger than those for the far-field speckle patterns, indicating the crucial role played by the evanescent components. Furthermore, the excitation of SPP propagating along the surface and reradiating into vacuum mediates the scattering process. In fact, the spatial frequency of the large oscillations about the background in the near-field intensities is related to the SPP wavelength; this is more easily observed in the surface magnetic field in Fig. 4 (also in the total electric field, not shown here) for the profiles with higher $\xi_L$. On the other hand, the surface normal electric field component for the smaller $\xi_L$ (see Fig. 4b) reveals the appearance of narrow and bright spots where an enhancement of the field intensity of nearly two orders of magnitude occur, despite the relatively low value of $\delta$. This clearly manifests the crucial role played by the lower scale cutoff in the excitation of localized optical modes. (6c)

In the case of $s$ polarization, Fig. 5 reveals the appearance of narrow and bright spots where an enhancement of the field intensity of nearly two orders of magnitude occur, despite the relatively low value of $\delta$. This clearly manifests the crucial role played by the lower scale cutoff in the excitation of localized optical modes. (6c)
that no localized optical excitations seem to appear, as
expected since no SPP can be excited, nor significant
EM field enhancements are found. Note that, thanks to
the semi-logarithmic scale used in Figs. 5 for the surface
field intensities, a weak qualitative resemblance of posi-
tive contrast with the surface profiles can be seen.

The $p$-polarized surface EM field intensities presented
in Fig. 8 reveal very strong changes in all field compo-
ments with lower $\xi_L$. These changes are considerably
stronger than those for the smoother surfaces (see Fig. 3).
As mentioned above, narrow, large peaks tend to con-
centrate at bright spots in the form of localized optical
modes, exhibiting very large electric field enhancements
preferentially polarized along the normal to the surface,
in agreement with the SPP polarization, although sig-
ificantly large enhancements of the tangential electric
field intensity are also found. In addition, large bright
field intensities abound, not necessarily at the same elec-
tric field bright spots. The polarization of the modes
has been discussed in Ref. 12 for rougher self-affine frac-
tals with large, but yet subwavelength, $\xi_L$. It should
be emphasized that the nanoscale lower cutoff largely con-
tributes to the building and strengthening of such optical
excitations.

Let us plot the EM field in the near vicinity of the
rough interface. This is done in Figs. 9 and 10, where
the near-field intensity maps in a logarithmic scale for
$s$ and $p$ polarization, respectively, are shown in a region
around the origin of the surface profile with $\xi_L = 12.85$
mm in Figs. 9i and 10i. All the EM field components
are included for the sake of completeness; recall that it
has been recently reported that the magnetic field inten-
sity can be also probed through PSTM for certain ex-
perimental configurations. 32 The actual surface profile,
though not explicitly depicted, can be inferred from the
fairly black, metal regions due to the evanescent behav-
ior of the EM fields inside metal with a small skin depth
($d \approx 25$ nm).

Note that no bright spot whatsoever is observed in the
EM field intensity maps in the case of $s$ polarization,
Fig. 3, which exhibits no relevant features in the near
field. The actual interface appears slightly blurred, since
the minima have been reduced to enhance contrast. Inci-
didentally, the continuity of the (tangential) electric field
is satisfied in Fig. 3b, as well as the (dis)continuity of the
(normal) tangential magnetic field in Figs. 3f and 3i.

On the other hand, a very bright spot is found at the
local maximum in the central part of the intensity map
in Fig. 10a: The intensity field enhancement at such spot
is $|E|^2 / |E(0)|^2 \sim 10^4$ (recall that the maximum scale of
EM field intensities in Fig. 8 is about 3 orders of magni-
tude lower than those in Fig. 10). Note that there seems
to be a bright, though weaker, magnetic spot associated
with the optical mode (see Fig. 10b), which is nonetheless
slightly shifted to the right of the electric field maximum,
on the surface local minimum nearby. The electric field
intensity decays rapidly upon moving into vacuum away
from the bright spot, faster than expected for the evanes-
cent decay of SPP propagating on a plane. This may
help to explain why localized optical modes experimen-
tially observed through PSTM yield considerably smaller
enhancement factors, 15 leaving aside the fact that no
direct comparison with theoretical calculations for the
actual experimental surface profile are available. With
regard to the electric field orientation on the bright spot,
Figs. 10c and 10d indicate that the normal electric field
component is responsible for the bright spot, in agree-
ment with Figs. 3 and 5c; this component corresponds to $|E_x|^2$
in regions near surface maxima and minima (see Fig. 10h)
and to $|E_z|^2$ near vertical surface walls (see Fig. 10f). It can be also observed that, as expected, the
normal electric field is discontinuous across the interface,
the tangential component being continuous. The contin-
unity of the (tangential) magnetic field for $p$ polariza-
tion is evident in Fig. 10n.

The statistics of the surface electric field enhance-
ment has been studied elsewhere, 48 confirming a sig-
nificant increase with decreasing $\xi_L$. It should be
remarked that such large surface electric fields plays a
decisive role in SERS and other surface non-linear op-
tical processes. 26, 29, 39 In particular, local values
of $\sim 10^4$ can help to explain the existence of bright
spots at which SERS single-molecule detection has been
claimed. 10, 11, 12

V. CONCLUDING REMARKS

To summarize, we have studied the influence of the
nanoscale (in the region below a hundred nanometers)
lower cutoff $\xi_L$ on the scattering of light from one-
dimensional, self-affine fractal Ag surfaces with large
fractal dimension $D = 1.9$ and for both moderately ($\delta = 
51.45$ nm) and large ($\delta = 257$ nm) surface height devia-
tions. Since no approximate methods are applicable for
such roughness parameters, numerical calculations have
been carried out on the basis of the Green’s theorem in-
tegral equation formulations, extended to account for all
EM field components in the near field region. Both far-
field and near-field distributions have been obtained for
$s$ and $p$ polarization (no depolarization takes place in
this scattering geometry). We have presented far-field in-
tensity distributions corresponding to single realizations
(specular patterns) and averages over an ensemble of re-
alizations.

It has been shown that the details corresponding
to scales below a hundred nanometers in moderately
rough self-affine profiles produce no change in the far
field speckle pattern, appearing only small quantitative
variations for very large rms height. For $p$ polarization,
nonetheless, small changes are already observed for
moderately rough profiles, becoming dramatic for large
roughness. Ensemble averaging tends to wash out the
differences. In the case of $s$ polarization, the angular
distributions of mean scattered intensities are practically
identical, not only for $\delta = 51.45$ nm, but also for $\delta = 257$
nm. For $p$ polarization, the differences encountered in the speckle patterns are largely suppressed in the mean scattered intensity distributions; the only effect of the varying nanoscale cutoff $\xi_L$ manifests in the total reflectance (which is smaller for decreasing $\xi_L$ due to larger absorption).

Nevertheless, nanoscale features have a strong impact on the near EM field distributions due to the relevant role played by the evanescent components. The $s$-polarized surface EM field intensity exhibits oscillations with higher frequency the smaller $\xi_L$ is. The amplitude of such oscillations, and thus the changes from one profile to another with different $\xi_L$, increases with the rms height $\delta$. No significant electric field enhancements are found in this polarization. Incidentally, the surface and near electric and magnetic field intensities vaguely resemble the surface profile with positive contrast despite the large surface roughness of the profiles. In the case of $p$ polarization, drastic changes with decreasing $\xi_L$ are found in the surface EM field stemming from the roughness-induced excitation of SPP. Large, narrow peaks (localized optical modes) tend to appear with either increasing rms height or decreasing nanoscale, predominantly enhancing the normal electric field intensity (as expected from the SPP electric field orientation), and rapidly decaying into both vacuum and metal. Our near field intensity maps around optical excitations can help to interpret PSTM experimental results. In addition, the large field enhancements associated with such localized optical modes on self-affine fractal metal surfaces can shed light onto SERS and surface nonlinear optical phenomena.

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Figures

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FIG. 1: Illustration of the scattering geometry.
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[FIG. 1: Illustration of the scattering geometry.]
FIG. 2: Angular distribution of the speckle pattern intensity scattered from Ag fractal surface profiles with $L = 5.14 \mu m$, $D = 1.9$, and $\delta = 51.4 \text{ nm}$ (top, a-b) and $\delta = 257 \text{ nm}$ (bottom, c-d), differing only in $\xi_L = 51.4, 25.7, 12.85, \text{ and } 6.425 \text{ nm}$. Illuminated by a Gaussian beam with $\theta_0 = -10^\circ$, $\lambda = 629.9 \text{ nm}$, and $W = L/4 \cos \theta_0$. Left (a and c): $s$ polarization; right (b and d): $p$ polarization.

FIG. 3: Same as in Fig. 2 but for the mean scattered intensity averaged over $N_r = 200$ realizations.

FIG. 4: Same as in Figs. 3c and 3d but renormalizing by the total reflectance $S$. 

$s$ polarization

- $|E|^2$
- $|\mathbf{H}_n|^2$
- $|H|^2$
FIG. 5: Surface electric (a) and magnetic, normal (b) and tangential (c), field intensities on the center spot of the illuminated area $L = 5.14 \mu m$ for $s$-polarized scattering with $\theta_0 = -10^\circ$, $\lambda = 629.9$ nm, and $W = L/4 \cos \theta_0$, from Ag fractal surfaces with $D = 1.9$, $\delta = 51.4$ nm, and $\xi_L = 51.4, 25.7, 12.85$, and 6.425 nm. The KA field intensity is also included (see text). (d) The corresponding surface profiles.

FIG. 6: Surface magnetic (a) and electric, normal (b) and tangential (c), field intensities on the center spot of the illuminated area $L = 5.14 \mu m$ for $p$-polarized scattering with $\theta_0 = -10^\circ$, $\lambda = 629.9$ nm, and $W = L/4 \cos \theta_0$, from Ag fractal surfaces with $D = 1.9$, $\delta = 51.4$ nm, and $\xi_L = 51.4, 25.7, 12.85$, and 6.425 nm. The KA field intensity is also included (see text). (d) The corresponding surface profiles.

FIG. 7: Surface EM field intensities (in a semi-log scale) for $s$ polarization as in Fig. 5 but for rougher surface profiles (d) with $\delta = 257$ nm.
**FIG. 8:** Surface EM field intensities (in a semi-log scale) for \( p \) polarization as in Fig. 6, but for rougher surface profiles (d) with \( \delta = 257 \) nm.

**FIG. 9:** Near field intensity images in \( s \) polarization [(a) electric, (b) magnetic, the former split into (c) \( x \), horizontal component, and (d) \( z \), vertical component, all of them in a \( \log_{10} \) scale] in an area of \( 0.5 \times 0.5 \) µm\(^2\) close to the fractal surface in Fig. 7d with \( \delta = 257 \) nm and \( \xi_L = 12.85 \) nm. Other parameters as in Fig. 7.

**FIG. 10:** Near field intensity images in \( p \) polarization [(a) electric, (b) magnetic, the latter split into (c) \( x \), horizontal component, and (d) \( z \), vertical component, all of them in a \( \log_{10} \) scale] in an area of \( 0.5 \times 0.5 \) µm\(^2\) close to the fractal surface in Fig. 8d with \( \delta = 257 \) nm and \( \xi_L = 12.85 \) nm, where a strong localized optical mode is observed. Other parameters as in Fig. 8.