EFFECTIVENESS OF MEASURES OF PERFORMANCE DURING SPECULATIVE Bubbles

Filippo Petroni

GRAPES, B5, Sart-Tilman, B-4000 Liege, Belgium

Giulia Rotundo

Faculty of Economics, University of Tuscia, Viterbo
Faculty of Economics, University of Rome “La Sapienza”

Abstract

Statistical analysis of financial data most focused on testing the validity of Brownian motion (Bm). Analysis performed on several time series have shown deviation from the Bm hypothesis, that is at the base of the evaluation of many financial derivatives. We inquiry in the behavior of measures of performance based on maximum drawdown movements (MDD), testing their stability when the underlying process deviates from the Bm hypothesis. In particular we consider the fractional Brownian motion (fBm), and fluctuations estimated empirically on raw market data. The case study of the rising part of speculative bubbles is reported.

Key words:

1 Introduction

Hypothesis testing in time series modeling is a delicate step that relies on the precision of statistical analysis on raw data. This is most true in the field of empirical finance, where information spreading and investors’ behavior are rendered into statistical properties that market data should verify. In this framework, the analysis of correlation in raw...
market data constitutes a base step for model identification. The slow decay of autocorrelation has been evidenced through many empirical studies, and new methods were developed to measure it through the Hurst \((H)\) exponent \([1, 2, 3]\). The outcome of statistical analysis on data often shows oscillations of \(H\) outside the confidence intervals \([4]\), that seem to exclude the Bm hypothesis, and to assess the compatibility of data with fBm. It is worth remarking that the values of \(H\) are highly sensitive to the time window selected, internal time windows showing the widest variety of fluctuations. Therefore, also the fBm hypothesis constitutes a first raw approximation for modeling the entire data sets available. Moreover, even without numerical problems on the estimation of \(H\), the slow correlation decay is compatible with the most classical fractional Brownian motion as well as with other processes having an autocorrelation function depending on the time lag \([5, 6]\). Recently, it has been shown theoretically that the same kind of correlation decay can rise also from Markov models \([7, 8]\) as well as from models of relaxation with a slowly vanishing correlation \([9]\). To distinguish between Markovian vs non-Markovian behavior is relevant for its implication in term of market risk, and subsequent approach to portfolio management and performance measures.

The occurrence of either Bm or geometric Brownian motion (gBm) opens the way to many theoretical results, often expressed through fast to compute closed form formulas. Therefore, it is worth using Bm and/or gBm as a benchmarks, and to understand the behaviour of derivatives based on them.

We aim at exploring the limits of validity of the theoretical estimate of performance measures derived in the Bm case. We focus on measures of performance based on maximum drawdown. More in details, the first task is to inquiry through simulations the distance of measures of performance on fBm data from the theoretical estimate of measures of performance that are available for Bm. The second task is to examine the case study of the rising part of speculative bubbles. The first step here is to refine the analysis on the Gaussianity of increments varying the time window, so to examine more in details a necessary condition for the fBm modeling. The second step is to examine the risk measures and the related performance indices during the rising part of the bubbles, and to compare them with the theoretical estimates that hold for (f)Bm.

The paper is organized as follows. The next section resumes the risk measures, and performance indices based on maximum drawdown. Section 3. shows the results of the simulations performed on the fBm. Section 4. sums up the speculative bubble approach, it shows the behavior of risk and performance measures on the rising part of speculative bubbles, and a comparison of them with the theoretical Bm results is also performed.
2 Performance indices based on the maximum drawdown

The analysis of long sequences of drawdown market movements is relevant in itself, because long lasting decreasing trends can force small investors to abandon the market, and they can cause fund managers to loose clients. One of the most widely quoted measures of risk used by hedge funds and commodity traders advisors is peak to trough drawdown \[10\], i.e. the size of the largest loss.

Using a stochastic processes approach, the maximum drawdown at time \(T\) of a random process on \([0, T]\) can be defined formally as follows.

**Definition 1** Let \(\{X(t)\}\) be a stochastic process. The maximum drawdown \(MDD\) is given by

\[
MDD(T) = \sup_{t \in [0, T]}(\sup_{s \in [0, t]}X(s) - X(t)).
\]  

Ismail et al. (2005) comprehensively analyzed maximum drawdown as a measure of risk with emphasis on updated ratios and Brownian motion \([11, 12]\).

Assume that the value of a portfolio follows a gBm:

\[
ds = \hat{\mu}sdt + \hat{\sigma}sW,\quad 0 \leq t \leq T
\]  

Taking a logarithmic transformation \(x = \log s\), then \(x\) follows a Bm. Defining \(\mu = \hat{\mu} - \frac{1}{2}\hat{\sigma}^2\), and \(\sigma = \hat{\sigma}\):

\[
dx = \mu dt + \sigma dW,\quad 0 \leq t \leq T
\]

where time is measured in years, \(\mu\) is the average return per unit time, \(\sigma\) is the standard deviation of the returns per unit time and \(dW\) is the usual Wiener increment.

Theoretical estimates of the expected value of \(MDD(T)\), \(E(MDD)\), have been shown in the Bm case \([11, 12]\):

\[
E(MDD(T)) = \begin{cases} 
\frac{2\sigma^2}{\mu}Q_p(\frac{2\mu^2T}{2\sigma^2}) \rightarrow_{T \to \infty} \frac{2\sigma^2}{\mu} \left(0.63 + 0.5 \log T + \log \frac{\mu}{\sigma} \right) & \text{if } \mu > 0 \\
1.25\sigma\sqrt{T} & \text{if } \mu = 0 \\
\frac{2\sigma^2}{\mu}Q_n(\frac{2\mu^2T}{2\sigma^2}) \rightarrow_{T \to \infty} -\mu - \frac{\sigma^2}{\mu} & \text{if } \mu < 0
\end{cases}
\]  

The need to consider the order of data alongside their most classical statistical properties has already led to the introduction of MDD into risk measures \((\text{Drawdown at Risk (Dar) and Conditional Drawdown at Risk (CDaR) alongside the most classic VaR, ES and CVaR, A. Chekhlov et al. (2000))}\)\([13, 14]\). The same need to accomplish the investors risk perception in measures of
performance has led to the modification of risk adjustment in measures of performance of portfolios. The classic Sharpe ratio is defined as follows:

**Definition 2** The Sharpe ratio is defined as

\[
Shrp(T) = \frac{\mu_{[0,T]}}{\sigma_{[0,T]}}
\]  

(4)

The risk is measured through the standard deviation, and the returns are adjusted w.r.t. it. Therefore, the Sharpe ratio is not suitable for describing the adversion of investors to the occurrence of downward movements. The MDD has been introduced giving rise to measures of performance like the Calmar ratio, that is defined as follows:

**Definition 3** The Calmar ratio is a risk-adjusted measure of performance that is given by the formula:

\[
Calmar(T) = \frac{\mu T}{E(MDD)}
\]  

(5)

The Calmar ratio has been shown to be closely related to the Sharpe ratio

\[
Calmar(T) = \frac{T^{2}Shrp^{2}}{Q_{p}(T^{2}Shrp^{2})}
\]

The result \[3\] shows how the expected MDD is related to the mean return and the standard deviation of the returns in the Bm case, and it can be used at first to state that

\[
E(MDD) = \frac{2Q_{p}(T^{2}Shrp^{2})}{Shrp}
\]

and then to conclude that \[11, 12\]

\[
Calmar(T) \to_{T \to \infty} \frac{TShrp^{2}}{0.63 + 0.5logT + log(Shrp)}.
\]

Theoretical estimates are not available for fBm or geometrical fBm (gfBm), and there is a lack of sensitivity analysis to deviations from Bm and gBm hypotheses.
3 Measuring performance in the case of fBm

We aim at extending the results of [11] to the case in which the underlying asset obeys a fBm. Some remarks are worth to be added about the suitability of the measures listed in the previous section as a good tool for the estimate of the performance when the underlying portfolio obeys a fBm. The common characteristic of performance measures is to consider return/risk ratios. Such ratios are different from each other because of the way in which the risk is defined. A relevant remark is that the Sharpe ratio captures only the risk that can be expressed through the variance.

We are going to consider the fBm, that is a Gaussian process. The main difference between fBm and regular Brownian motion is that while the increments in Brownian motion are independent, in fBm case they are dependent. However, in both cases increments are Gaussian. It is well known that such distribution is completely defined if the first and second order moments are fixed. Therefore the Sharpe ratio totally captures the relevant moments of the probability distribution of increments also in the fBm case, and the differences from the Bm case are due only to the correlation structure. This statement does not hold for other distributions of the increments and for other processes that have been considered in literature for modeling the dynamics of the returns. Weak stationarity is a necessary condition for the validity of the above performance measures. As an example, the definition of Sharpe ratio does not represent entirely the risk associated to the Martingale stochastic process examined in [8] due to the nonstationarity of the increments.

On the other hand, the Calmar ratio is not based explicitly on the Gaussian assumption, that is needed only when the relationship with the Sharpe ratio is examined. The estimate of the relationship between the Calmar and the Sharpe ratio is mostly based on the behavior of $E(MDD)$, that in [3] is obtained through a theoretical approach, and through the estimate of the function $Q(x)$. The main result is the detection of a phase transition when $\mu$ changes its sign, and so $\mu = 0$ is a critical value. We start examining the behavior of $E(MDD)$ of fBm through simulations.

3.1 $E(MDD)$ in the fBm case

Simulations have been performed in order to estimate $E(MDD)$ on fBm $X(t)$ by varying the Hurst exponent $H \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. Figure II shows the behavior of $E(MDD)$ for the whole set of values of $H$ considered here. For each value of $H$ the figure reports the mean of $E(MDD)$ calculated over 1000 fBm synthetic time series.

The Monte Carlo time for the synthetic time series is chosen such to be equivalent to days in real time series, we considered one year as given by 256 days.
(or Monte Carlo time steps). The plot is shown for a return of 5%/year considering a standard deviation of 5%/year (Shrp = 1). Different values for μ and σ in the range of real market portfolios, i.e. the annual return μ ∈ [1, 10]% and the annual volatility σ ∈ [1, 20]% also give the same behavior. Such range for μ gives rise to Shrp > 0.

As expected the H = 0.5 case is linear in the log-linear plot of Figure 1.a. Some departure from linearity starts already for H = 0.6 and the discrepancy increases with increasing H, for H < 0.5 the curves are very close to each other and to the linear one.

Therefore, when μ > 0, the curve of E(MDD) for H = 0.5 separates two different regimes; although we do not test any hypotheses on the functional form of the curves in the fBm case, there is a clear passage from curves slower than the logarithm to curves faster than the logarithm.

This behavior is in good agreement with the persistency of fBm for H > 0.5 and for the mean reverting behavior for H < 0.5. In the former case long range drawdowns as well as drawups have a higher probability to happen than in the latter case.

Figure 1b shows the case for Shrp = 0. Also in this case H = 0.5 separates E(MDD) growing faster (H > 0.5) and slower (H < 0.5). Figure 1c completes the analysis in the case of Shrp < 0, showing the same monotonic behavior of the curves w.r.t. H. The complete scheme is summed up in Figure 1d.

### 3.2 Performance measures in the fBm case

Equation (3) relating E(MDD) to μ plays a key role in the assessment of the relationship between the Calmar and the Sharpe ratios. Equation (3) was obtained through a theoretical approach, supported by the numerical estimate of the function Q(x). In the case of fBm the behavior of E(MDD) w.r.t. time changes as shown in the previous section. Therefore, the relationships between the Calmar and the Sharpe ratios must be analyzed considering the new behavior of E(MDD). From now on, we focus on the more interesting case of profitable portfolios, i.e. μ > 0.

Figure 2 reports the behavior of the Sharpe ratio as a function of E(MDD) for each H ∈ {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9}. Figure 3 considers the Calmar ratio, and it shows its behavior by varying H. In both cases H = 0.5 separates the two regimes H > 0.5 from H < 0.5 and the behavior is monotonic w.r.t. H. The Sharpe and the Calmar ratio are relevant also for portfolio optimization, where they play the role of objective function. The analysis of their relationship can be a basis for the study of portfolio optimization and it will be studied elsewhere.
4 The rising part of speculative bubbles

Stock market indices are a particular portfolio, based on a weighted mean of selected stocks. Buying/selling stock market indices has the meaning of buying/selling a previous selected financial product replica of the index (Exchange Traded Funds, certificates). The results of the previous sections can be used to examine the behavior of stock market indices. The analysis of the performance of the stock market indices can be more relevant than any other portfolio because they can be selected in order to represent the market risk and can be used as a benchmark for “beating the market”.

Our aim is to understand the behavior of MDD on raw data and to compare it with the results obtained on synthetic fBm. In particular we focus on stock market indices that have shown speculative bubbles due to endogenous causes, and we estimate the risk measures on them.

Following Ausloos et al. [15, 16], and Sornette et al. [17, 18, 19], we selected the indices that were well modeled through the log-periodic correction to power law. In all that cases the magnitude of the crash is proportional to the price, so it is commonly suggested to use the logarithm of market index data

$$Y(t) = \log(p_t)$$  [18, 19, 20, 21]. We start performing some statistical analysis and after we examine the behavior of $E(MDD)$.

4.1 Statistics on raw data

Data selection is reported in Table I together with some statistical analysis: the mean of log-returns $\mu$ (i.e. the mean of the increments of $Y(t)$), their variance, $\sigma$, the Hurst exponent $H$ measured on the log-returns $\{Y(t+1) - Y(t)\}$ through the Detrended Fluctuation Analysis (DFA) [1], the Sharpe ratio and the Calmar ratio for the whole time window considered. W.r.t. paper [22] the time series of Chile General, and the one corresponding to the crash of Mexico Ipc in 1994 were not considered for the present analysis because their length is lower than one year (256 data), therefore they were not suitable for the same parameters to be used for the DFA analysis performed on the other series. The time series of Arg Burcap was retrieved and added to the data set. The Jarque-Bera test was performed on the data set listed in Table I in order to test the Gaussianity of the increments. The Gaussian hypothesis is rarely validated. Therefore, the test was made on subwindows. For each $k$ and $j$ the Jarque-Bera test was performed on the increments estimated on $\{Y(t+1) - Y(t)\}_{t=k,k+j}$, i.e. on time windows with starting time $k$ and width $j$. The choice of the minimal time window length is 256 data, corresponding to the mean number of trading days contained into a year. Figure 5 reports the starting time $k$ on the x-axis, the time window width $j$ on the y-axis and it displays the result of the Jarque-Bera test through a color palette. The
upper right triangle of course is empty, and it has been drawn with a value equal to zero (white). In the lower triangle the white dots correspond to the acceptance of the Gaussian hypothesis, and the black ones to the rejection of the hypothesis. Figure 5 shows that there are time windows in which the Gaussian hypothesis is validated. It is not obvious that Gaussianity is reached for long enough time windows. As an example in FTSE 100 there are time windows of length 300 days that verify the Gaussianity test, while longer time series reject this hypothesis, that is verified again only for much longer time windows. In the same way, the NASDAQ 100 (1998) shows Gaussian time windows of length 400-600. A definitive convergence to the Gaussian case is shown for the DAX, FTSE 100 (1987;1998), Hang Seng, Mexico Ipc, Nasdaq (1987;2000).

4.2 Estimating MDD on raw data

We aim at analyzing the situation of a practitioneer investing on a real market portfolio, basing his risk measure on the sequence of MDD actually happened rather than on an hypothetical $E(MDD)$. We compare the MDD for the logarithm of the rising part of speculative bubbles for the indices considered in the present work with $E(MDD)$ of synthetic Brownian motions and $E(MDD)$ of synthetic fractional Brownian motions with Hurst exponent as given in Table 1 for each index. The synthetic time series were simulated by using the same mean of returns and the same variance of the real data. For each index 1000 synthetic time series were generated for both type of process (Bm and fBm) and $E(MDD)$ was estimated averaging over the MDD of each time series. For both real data and synthetic time series MDD and $E(MDD)$ were estimated by dividing each time window into six equally spaced intervals. In Figure 6 we show the comparison of the $E(MDD)$ of the synthetic Bm and fBm with the MDD of real index data. The difference of MDD estimated on indices data from the theoretical values can be addressed to different factors: 1. $E(MDD)$ is a mean value, whilst the values reported for the indices is a measure of the MDD on a single trajectory. 2. Deviations from the hypothesis of Gaussian increments (Jarque-Bera test) can also give rise to deviation of MDD from the theoretical value of $E(MDD)$ for both Bm and fBm. 3. Another factor of deviation from the theoretical values could be a result of the error in the estimation of the Hurst exponent on short time series [4].

From the same figure it is possible to notice that the MDD evaluated after a huge crash remains till an even bigger crash happens, leading to an overestimation of the risk on shorter sequences.
5 Conclusions

The present report deals with measures of performance considering the maximum drawdown. Theoretical results on $E(MDD(T))$, Sharpe and Calmar ratios available for Bm are compared with empirical behavior of $E(MDD(T))$, Sharpe and Calmar ratios on fBm. By extensive simulations we show the behavior of these performance measure as function of the Hurst exponent and the mean return sign. We show that, in the most relevant case of $\mu > 0$, $H = 0.5$ separates two different regime for $E(MDD(T))$ giving rise to a regime faster than logarithm when $H > 0.5$ and slower than logarithm when $H < 0.5$. The results are then applied to the rising part of speculative bubbles. Statistical analysis on such time series is refined on their subsequences, and the distance of the obtained results from the (f)Bm case is discussed.

Acknowledgements

GR thanks Amir Atiya and Anna Maria D’Arcangelis for fruitful discussions on financial risk and related topics. FP thanks Marcel Ausloos for pertinent comments and fruitful discussions, and the European Commission Project E2C2 FP6-2003-NEST-Path-012975 “Extreme Events: Causes and Consequences” for financial support.

References

[1] K. Hu, P.Ch. Ivanov, Z. Chen, P. Carpena, H.E. Stanley, Phys. Rev. E 64 (2001) 011114.
[2] J. Beran, Statistics for Long-Memory Processes, Chapman & Hall (1994)
[3] R. N. Mantegna, H. E. Stanley, Introduction to econophysics, Cambridge University Press, Cambridge (2000)
[4] R. Weron, Physica A 312 (2002) 285 299.
[5] P. Embrechts, M. Maejima, Self Similar Processes, Princeton Univ. Press (2002)
[6] F. Petroni, M. Ausloos, G. Rotundo, Physica A, 384, (2007) 359-367.
[7] K. E. Bassler, H. G. Gunaratne, J. L. McCauley, Physica A 369 (2006) 343353
[8] J. McCauley, G. H. Gunaratne, K. E. Bassler, Physica A 379 (2007) 19.
[9] L.C. Lapas, I.V. Costa, M. H. Vainstein, F. A. oliveira, EPL, 77 (2007) 37004.
[10] G.Burghardt, R. Duncan, L. Liu, ”Understanding drawdowns”, CARR futures research note www.carrfutures.com
[11] M. Magdon-Ismail and A. Atiya, Risk Magazine 17 (2004) 99-102.
[12] M. Magdon-Ismail, A. Atiya, A. Pratap, Y. S. Abu-Mostafa, J. Appl. Prob., 41 (2004) 1-15.
[13] B. V. M. Mendes, W. Brandi, "Modeling Drawdowns and Drawups in Financial Markets". Forthcoming, Journal of Risk.
[14] F.C. Harmantzis, L. Miao, "Empirical Study of Fat-Tails in Maximum Drawdown: The Stable Paretian Modeling Approach", Quantitative Methods in Finance Conference (QMF) 2005, Sydney, Australia, December 14-17, 2005.
[15] M. Ausloos, Ph. Boveroux, A. Minguet, N. Vanderwalle, Physica A Vol. 255 1-2 (1998) 201-210.
[16] M. Ausloos, Ph. Boveroux, A. Minguet, N. Vandervalle, Eur. Phys. J. B, Vol. 4 (1998) 139-141.
[17] D. Sornette, Critical market crashes, Physics Reports 378 (1), (2003) 1–98.
[18] A. Johansen, D. Sornette, “Endogenous versus Exogenous Crashes in Financial Markets”, Contemporary Issues in International Finance, (2004). In press.
[19] D. Sornette, Why Stock Markets Crash: Critical Events in Complex Financial Systems, Princeton University Press, 2002.
[20] A. Johansen, D. Sornette, J. of Risk, 4, 2 69–110 2001/02.
[21] Johansen, A. and D. Sornette, The Nasdaq crash of April 2000: Yet another example of log-periodicity in a speculative bubble ending in a crash, European Physical Journal B 17 (2000) 319–328.
[22] G. Rotundo, M. Navarra, Physica A 382 (2007) 235246
Table 1
List of crashes. Data sets of rising part of speculative bubbles are chosen from the rise of the bubble to the expected crash time [19]. W.r.t. paper [22] the time series of Chile General, and the one corresponding to the crash of Mexico Ipc in 1994 were not considered for the present analysis because their length is lower than one year (256 data), therefore they were not suitable for the same parameters to be used for the DFA analysis performed on the other series. The time series of Arg Burcap was retrieved and added to the data set. In the table are reported respectively: index name, year of crash, mean of daily returns, standard deviation of daily returns, Sharpe ratio, Hurst exponent, maximum drawdown and Calmar ratio. All this quantities are estimated on the logarithm of each index.

| Index               | Year | $\mu$  | $\sigma$ | Shrp | H     | MDD  | Calmar |
|---------------------|------|--------|----------|------|-------|------|--------|
| Arg Burcap          | 1997 | 0.0012 | 0.0157   | 0.08 | 0.51  | 0.24 | 3.44   |
| Arg Merval          | 1997 | 0.0015 | 0.0173   | 0.08 | 0.47  | 0.25 | 3.70   |
| Brazil Bovespa      | 1997 | 0.0029 | 0.0136   | 0.21 | 0.65  | 0.12 | 8.46   |
| DAX 40              | 1998 | 0.0018 | 0.0124   | 0.14 | 0.63  | 0.20 | 4.43   |
| FTSE 100            | 1987 | 0.0012 | 0.0082   | 0.15 | 0.60  | 0.12 | 5.64   |
| FTSE 100            | 1997 | 0.0008 | 0.0066   | 0.12 | 0.59  | 0.06 | 8.70   |
| FTSE 100            | 1998 | 0.0008 | 0.0076   | 0.11 | 0.61  | 0.12 | 5.71   |
| Hang Seng           | 1994 | 0.0018 | 0.0133   | 0.14 | 0.55  | 0.26 | 4.31   |
| Hang Seng           | 1997 | 0.0010 | 0.0115   | 0.09 | 0.57  | 0.16 | 4.35   |
| Kuala Lumpur SE Emas| 1994 | 0.0027 | 0.0095   | 0.28 | 0.75  | 0.10 | 10.06  |
| Mexico Ipc          | 1997 | 0.0017 | 0.0124   | 0.14 | 0.52  | 0.14 | 5.74   |
| Nasdaq 100          | 1987 | 0.0012 | 0.0096   | 0.13 | 0.58  | 0.20 | 3.17   |
| Nasdaq 100          | 1998 | 0.0014 | 0.0149   | 0.09 | 0.56  | 0.20 | 6.40   |
| Nasdaq 100          | 2000 | 0.0023 | 0.0205   | 0.11 | 0.53  | 0.26 | 6.80   |
| Venezuela SE Gen    | 1997 | 0.0041 | 0.0161   | 0.26 | 0.73  | 0.13 | 17.31  |
Fig. 1. $x$-axes report the time expressed in years in a log scale. The Monte Carlo time for the synthetic time series is measured in days and we considered one year as given by 256 days (see text). For each $H \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, the curves report the values of $E(MDD)$ for: (a) $\mu = \sigma = 5$, corresponding to a return of 5%/year considering a standard deviation of 5%/year ($Shrp = 1$); (b) $\mu = 0, \sigma = 5$ ($Shrp = 0$); (c) $\mu = -5, \sigma = 5$ ($Shrp = -1$).

(d) In all three cases the behavior of $E(MDD)$ for $H = 0.5$ separates the regime for $H > 0.5$ from the one for $H < 0.5$. 

| $H$ | $H < 0.5$ | $H = 0.5$ | $H > 0.5$ |
|-----|-----------|-----------|-----------|
| $\mu > 0$ | $< \log(T)$ | $\sim \log(T)$ | $> \log(T)$ |
| $\mu = 0$ | $< \sqrt{T}$ | $\sim \sqrt{T}$ | $> \sqrt{T}$ |
| $\mu < 0$ | $< T$ | $\sim T$ | $> T$ |
Fig. 2. The $E(MDD)$ per unit $\sigma$ versus Sharpe ratio ($\mu/\sigma$) for $\mu > 0$. The time is fixed at $T = 5\text{years}$, and the curves report the values for $H \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

Fig. 3. The Calmar ratio (a) w.r.t. time; (b) as a function of the Sharpe ratio for $\mu > 0$. The case $H = 0.5$ always separates the behavior in case $H > 0.5$ from $H < 0.5$. 
Fig. 4. Rising part of speculative bubbles for the indices considered in the present work, and listed in Table 1.
Fig. 5. Jarque-Bera test on the rising part of speculative bubbles. The x-axis report the initial day of the time window; the y-axis reports the time length (>256 days). The upper triangle does not contain information because of the finite size of the time series. The lower triangle reports the result of the Jarque-Bera test. For each \((x, y)\) white corresponds to the Gaussian case; black rejects the Gaussian hypothesis on the time window \((x, x + y)\).
Fig. 6. $MDD$ for the logarithm of the rising part of speculative bubbles for the indices considered in the present work. $MDD$ (diamonds) are compared with $E(MDD)$ of synthetic Brownian motions (squares) and $E(MDD)$ of synthetic fractional Brownian motions (triangles) with Hurst exponent as given in Table I for each index. $E(MDD)$ for Bm and fBm is averaged over 1000 simulation. x-axes report the time expressed in days in a log scale.