Tunable subwavelength strong absorption by graphene wrapped dielectric particles

Bing Yang\textsuperscript{1,2}, Tong Wu\textsuperscript{1}, Yue Yang\textsuperscript{1} and Xiangdong Zhang\textsuperscript{1}

\textsuperscript{1} School of Physics, Beijing Institute of Technology, Beijing, 100081, People’s Republic of China
\textsuperscript{2} School of Physical Science and Information Engineering, Liaocheng University, 252059, Shandong, People’s Republic of China, and Shandong Provincial Key Laboratory of Optical Communication Science and Technology, 252059, Shandong, People’s Republic of China

E-mail: zhangxd@bit.edu.cn

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Abstract

The optical absorption properties of graphene-wrapped dielectric particles have been investigated within the framework of the Mie theory. It is shown that subwavelength strong absorption in infrared spectra can take place in such systems due to the excitation of plasmon resonance in graphene. The absorption characteristics and efficiency are tunable by either varying the Fermi level and the damping constant of graphene, or by changing the size and the dielectric constant of small particles. These characteristics also depend on the separation distance between particles in the cluster. These extreme light resonances and absorptions in graphene-wrapped nanostructures have great potential for optoelectronic devices.

Keywords: graphene, plasmon, absorption, nanoparticle

1. Introduction

In recent years, there has been a great deal of interest in studying the optical properties of graphene due to its abundant potential applications within a wide spectral range, from terahertz (THz) to visible frequencies \cite{1, 2}. As an ultrathin two-dimensional carbon material, graphene is widely used in transparent electrodes and optical display materials \cite{3–5}, and it has also been applied to optoelectronic components such as photodetectors \cite{6–8}, optical modulators \cite{9, 10}, and more \cite{11–13}. In these applications, the strength of the interaction between graphene and electromagnetic (EM) waves plays a central role. However, a single sheet of homogeneous graphene absorbs only about 2.3% of normal incidence light in the visible and near-infrared range \cite{14}.

The weak optical absorptions block the potential applications of graphene in optoelectronic devices. Thus, various methods to improve the interaction between graphene and EM waves have been proposed. Three main strategies have been developed in recent years for this purpose. The first strategy is to combine a graphene layer with conventional plasmonic nanostructures \cite{15–18}. The plasmonic resonance of these conventional nanostructures confines and enhances the EM radiation, which leads to enhanced interaction between graphene and EM waves. The second method is to place graphene in different kinds of optical microcavities \cite{7, 8, 19, 20}. The EM waves are localized by the microcavities, which can strengthen the interaction between graphene and EM waves. The third strategy is to fabricate periodically patterned graphene islands \cite{9, 21–26}. In this case, graphene islands can support plasmon oscillations, and graphene island arrays can improve the interaction between graphene and EM waves. Because the charge carriers of patterned graphene can be easily tuned by electrical gating or chemical doping, tunable light absorption can be easily realized in this case. To date, many kinds of graphene islands have been proposed experimentally and theoretically, including graphene ribbons, nanodisks, nanorings, nanotubes, and so on \cite{9, 24–26}.

In this work, we propose a method that consists of wrapping small dielectric particles with graphene to improve the interaction between graphene and EM waves. Within the framework of the Mie theory, we study the absorption characteristics of these particles. Our calculated results show that subwavelength strong optical absorptions in the infrared spectra can take place in these systems, and the absorption
properties are tunable by changing the parameters of the graphene-wrapped nanostructures.

2. Theory and method

We consider a cluster with \( N \) monolayer graphene-wrapped dielectric spheres embedded in a background medium with relative electric permittivity, \( \varepsilon_b \), and relative magnetic permeability, \( \mu_b \). The spheres have radii \( a \), relative permittivity \( \varepsilon \), and relative permeability \( \mu \). The geometry of the problem is shown in figure 1(a). Note that recently, similar systems have been successfully fabricated where the Raman imaging of graphene-wrapped gold nanoparticles has been investigated [27]. When the system we consider is irradiated by an external electromagnetic field with wavevector \( \mathbf{k} \), the scattering field can be obtained by means of Mie scattering theory and the multiscattering method, as briefly described below.

Because the graphene layer is an ultrathin, two-dimensional material with only one atom thickness (≈0.3 nm) in the radial direction on the sphere surface, the effect of graphene on the scattering properties of the graphene-wrapped dielectric sphere can be attributed to the interfacial effect—shown in figure 1(b)—which is reflected by a boundary condition in Mie scattering theory. For example, the magnetic field condition for a monolayer graphene-wrapped dielectric sphere can be expressed as

\[
\hat{n} \times \left( \mathbf{H}_0 + \mathbf{H}_r - \mathbf{H}_i \right) = \mathbf{J},
\]

where \( \hat{n} \) is the outwardly directed unit normal to the particle surface and \( \mathbf{H}_0 \), \( \mathbf{H}_r \), and \( \mathbf{H}_i \) represent the incident, scattered, and internal magnetic fields of the dielectric sphere, respectively. \( \mathbf{J} = \sigma_i \mathbf{E}_i \) is induced by the tangential component, \( \mathbf{E}_i \), of the electric field and is proportional to the optical conductivity, \( \sigma_i \), of the graphene layer, which can be derived from Kubo formula [28–32]. For illustration and simplicity, we only consider the situation where the absolute value of the Fermi level, \( E_f \), of the graphene is much larger than \( k_BT \), where \( k_B \) is the Boltzmann constant and \( T \) is the absolute temperature (or for low-temperature conditions) [29, 32]. So we chose \( \sigma_i = \sigma_i^1 + i\sigma_i^2 \) describing intraband processes and \( \sigma_i = \sigma_i^1 + i\sigma_i^2 \) describing interband contributions with

\[
\sigma_d = \frac{4E_f}{\pi (\Gamma_c - i\hbar \omega)},
\]

\[
\sigma_i' = \frac{1}{\pi} \left[ 1 + \frac{1}{\pi} \arctan \left( \frac{\hbar \omega - 2E_f}{\Gamma_c} \right) - \frac{1}{\pi} \arctan \left( \frac{\hbar \omega + 2E_f}{\Gamma_c} \right) \right],
\]

\[
\sigma_i'' = -\frac{1}{2\pi} \ln \left( \frac{(2E_f + \hbar \omega)^2 + \Gamma_c^2}{(2E_f - \hbar \omega)^2 + \Gamma_c^2} \right).
\]

Here, \( \sigma_0 = e^2/4\hbar \) is called the universal conductivity of graphene with the electronic charge, \( e \), and the reduced Plank constant, \( \hbar \). \( \Gamma_c \) is the damping constant and \( \omega \) is angular frequency of the incident field. The optical conductivity is the function of the incident wave frequency and the Fermi level, \( E_f \), of graphene, as \( \Gamma_c \) is a constant. This scattering problem is similar to the charged sphere, which has been discussed in some references [33–35]. By using the electric field boundary condition in Mie scattering theory, \( \hat{n} \times (\mathbf{E}_0 + \mathbf{E}_r - \mathbf{E}_i) = 0 \). \( \mathbf{E}_0, \mathbf{E}_i \), and \( \mathbf{E}_r \) represent the incident, scattered, and internal electric fields of the dielectric sphere, respectively. The coefficients of the scattering field of the single graphene-wrapped sphere can be expressed as

\[
a_{mn}^s = \left( \psi_n^r (x) \psi_n^0 (mx) - m \psi_n^i (x) \psi_n^0 (mx) \right) + i m \psi_n^0 (mx) \psi_n^i (x),
\]

\[
b_{mn}^s = \left( \psi_n^r (x) \psi_n^0 (mx) - m \psi_n^i (x) \psi_n^0 (mx) \right) + i m \psi_n^0 (mx) \psi_n^i (x),
\]

where \( a_{mn}^0, b_{mn}^0 \) are expanding coefficients of vector spherical wave functions for the incident fields of the sphere located at position \( \mathbf{r}_0(x_0, y_0, z_0) \). \( \psi_n^r (x) \) and \( \psi_n^i (x) \) are Riccati-Bessel functions. \( j_m (x) \) and \( h_{m}^{(1)} (x) \) are the Bessel and Hankel functions of the first kind. \( x = ka \) is the dimensionless size parameter with the sphere radius, \( a \), and wave number \( k = (\varepsilon_b \mu_b)^{1/2} \omega/c \), with \( c \) as the light speed in a vacuum. The prime indicates differentiation with respect to the argument in parentheses; \( m = (\varepsilon \mu)^{1/2} (\varepsilon_b \mu_b)^{1/2} \) is the complex relative refractive index of the sphere to the outside medium \( (\varepsilon_r = 1, \mu_r = \mu = 1 \text{ in our calculation}) \). The dimensionless value, \( \tau = \sigma_i (\varepsilon_0 / \varepsilon_b) \), is defined as the surface conductivity of the graphene-wrapped sphere, with \( \varepsilon_0 \) being the permeability and permittivity in a vacuum, respectively.
After the scattering problem from a scatter has been solved, the total scattering coefficients of a cluster of sphere particles shown in figure 1(a), \( a_{mn}^{\text{total}} \) and \( b_{mn}^{\text{total}} \), can be obtained by the multiscattering method. This method has been introduced in detail in [36]. Thus, the extinction and scattering efficiencies, \( Q_{\text{ex}} \) and \( Q_{\text{sc}} \), can be expressed as

\[
Q_{\text{ex}} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} n(n+1)(2n+1) \times \frac{(n-m)!}{(n+m)!} \cdot \text{Re}\left( a_{mn}^{0*}a_{mn}^{\text{total}} + b_{mn}^{0*}b_{mn}^{\text{total}} \right)
\]

\[
Q_{\text{sc}} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} n(n+1)(2n+1) \times \frac{(n-m)!}{(n+m)!} \cdot \left( |a_{mn}^{\text{total}}|^2 + |b_{mn}^{\text{total}}|^2 \right)
\]

where \( a_{mn}^{0*} \) and \( b_{mn}^{0*} \) represent the complex conjugate of \( a_{mn}^{0} \) and \( b_{mn}^{0} \), respectively. Then, the absorption efficiency, \( Q_{ab} = Q_{\text{ex}} - Q_{\text{sc}} \), can be obtained.

3. Numerical results and discussion

We first consider the absorption efficiency of the single graphene-wrapped dielectric sphere illuminated by the linear polarization incident wave. Figure 2 describes the calculated results of \( Q_{ab} \) as a function of wavelength for different tunable variables of the system. We found the strong resonance absorption in the infrared region, which depends on the system size, external electromagnetic fields, and dielectric constants of the spheres. This is because such strong absorption comes from the plasmon resonance in graphene with the external EM waves. Because of the peculiar electronic band structure, the optical conductivity is dominated by the intra-band term expressed by equation (2) in high-doped graphene \( \omega > \hbar \omega \), leading to graphene that acts like a metal in the frequencies we examined. Therefore, this resonance absorption strongly depends on the size of the particle. With a decrease in size, the resonance wavelength becomes small, the blue shift of the resonance peak occurs, the full width at half maximum of the peak becomes small, and the amplitude is improved, as shown in figure 2(a). For example, the resonance absorption peak appears at wavelength \( \mu = 0.67 \mu m \) for the dielectric sphere with \( a = 5 \mu m \) and \( \epsilon = 3.9 \). It appears at wavelength \( \mu = 8.91 \mu m \) for the dielectric sphere with \( a = 100 \mu m \), the resonance wavelength is about one hundred times as large as the radius of the sphere in such a case. Thus, it is typical subwavelength behavior, although the redshift of the resonance absorption with the increase in the size of sphere qualitatively coincides with the results in graphene nanoribbons and nanodisks [9, 24–26]. Note that the number of resonance peaks increases in our case due to the multipole
resonance of the plasmons in graphene when the sphere is big enough.

Similarly, the surface plasmon resonance also changes if we change the Fermi level of the graphene, which can be accomplished by tuning the density of the charge carriers through the external electrical gating field and/or chemical doping. The blue shift of the resonance peak also occurs with the increase of $E_F$ from 0.3 eV to 1.0 eV, as shown in figure 2(b). This phenomenon qualitatively coincides with those in the graphene ribbons and nanodisks described by other groups [9, 24–26]. It is interesting that a similar phenomenon can be found if we simply change the dielectric constant inside the sphere, as shown in figure 2(c). Blue shift and enhancement of absorption efficiencies also appear as the dielectric constant of the sphere decreases from 12.96 to 3.9. This is because the decrease of the dielectric constant inside the sphere is equivalent to the decrease in the size of the system. Of course, the damping constant of graphene, $\Gamma_c$, is also a tunable parameter. With the increase of $\Gamma_c$ from 0.2 meV to 1.5 meV, the resonance absorption becomes small and broadens in the absorption wavelength range due to the greater momentum randomization of electrons in graphene, as shown in figure 2(d).

In contrast to the case of the single graphene-wrapped dielectric sphere, the absorption spectra of a cluster with many graphene-wrapped dielectric spheres exhibit richer phenomena. Figures 3(a) and (b) show the absorption efficiency of two-sphere systems at the incident direction, $k$, of the linear polarized wave perpendicular or parallel to the longitudinal axis of the system, respectively. (See the insets in the corresponding subplots; the polarization of the incident wave is denoted by red arrows with $\mathbf{E}_i$.) Here, the radius and the dielectric constant of the two spheres are taken as $a = 100$ nm and $\varepsilon = 12.96$; the parameters of graphene are taken as $E_F = 1.0$ eV and $\Gamma_c = 1.0$ meV. The grass-green, red, cyan, magenta, and yellow lines represent the separation distance between the two spheres: $d = 5$ nm, 20 nm, 50 nm, 100 nm, and 300 nm, respectively. For comparison, the case of the single sphere is plotted with a blue line. When the separation distance is large enough, the absorption phenomenon is identical to the case of the single sphere. With the decrease in the separation distance, the red shift of the resonance absorption peak occurs at the incident direction of the wave perpendicular to the longitudinal axis of the system (see figure 3(a)). In contrast, the blue shift appears at the incident direction of the wave parallel to the longitudinal axis of the system (see figure 3(b)). This is because the coupling effect between the two particles strengthens with the decrease of the separation distance. In comparison to the single sphere, the effective transverse size along the polarization direction of the incident wave becomes large for the case where the incident direction of the wave is perpendicular to the longitudinal axis (figure 3(a)). The resonance absorption peak shifts to the long wavelength. For the case at the incident direction of the wave parallel to the longitudinal axis of the system (figure 3(b)), the effective longitudinal size becomes large, which is equivalent to decreasing the effective transverse size. The resonance absorption peak shifts to the short wavelength.

The absorption efficiency depends not only on the parameters of scatters, but also on the incident wave. The above discussions only focus on the linear polarized incident wave. Figure 3(c) shows the absorption efficiency of two-sphere systems at the circular polarization incident wave with $k$ perpendicular to the longitudinal axis of the system (see right inset of figure 3(c)). Here, the parameters of scatters are assumed to be identical to those in figure 3(a). Because the circular polarized incident wave includes two kinds of polarized information—parallel and perpendicular to the longitudinal axis of the system—the absorption spectra in figure 3(c) exhibit double resonant absorption peaks, which are caused by the two kinds of coupling resonances, as shown in figures 3(a) and (b), respectively. Due to the subwavelength behavior, these resonant absorption peaks are mainly determined by the dipole term in the multiscattering calculation. Although the dipolar effects domain the resonance absorptions in the present cases, the multipole effects can also exhibit in the absorption spectra. In the left inset of figure 3(c), the multipole signal is enlarged. We find that it sticks out with the decrease of the separate distance between the two spheres.

For the cluster with many graphene-wrapped dielectric spheres, we choose two representative cases, as shown in the insets of figures 4(a) and (b). For the first case, seven spheres are arranged in a plane with six spheres at the hexagon vertex and another at the center of the hexagon. In the second case, 27 spheres are arranged in three-dimensional space in $3 \times 3 \times 3$ stacks. The direction of the incident EM waves is perpendicular to the sphere layer plane, with linear polarization along the longitudinal axis of the two adjacent spheres. For comparison, the absorption efficiency of the single sphere is also plotted with a blue line. The parameters of the spheres and the graphene are taken to be identical to those in figure 3(a). Similar to the phenomena for two-sphere systems in figures 3(a) and (b), in figures 4(a) and (b) we can also observe the red shifts of the resonance absorption peaks as the separation distances between the spheres decrease. The difference is that the number of resonance peaks increases due to the complex coupling interactions (the coupling effect with different directions) of plasmon resonances in the cluster. In fact, varying the Fermi level and the damping constant of the graphene layer, and varying the dielectric constant and the radii of the spheres can also tune the absorption properties of the cluster with many graphene-wrapped dielectric spheres. Results show that these tunable variables have similar effects in cluster as they do in the single graphene-wrapped dielectric sphere.

To reveal the absorption efficiency of graphene-wrapped dielectric spheres more clearly, we calculate the reflectance, transmittance, and absorbance of light in a planar structure consisting of a square array of graphene-wrapped spheres—as shown in the inset of figure 5—using the method in [37, 38]. The calculated results for the absorbance ($A$, denoted by solid lines), transmittance ($T$, denoted by dashed lines), and reflectance ($R$, denoted by cycle symbols) are plotted in...
Figure 3. Absorption efficiency of two graphene-wrapped dielectric spheres as a function of wavelength under the excitation of linear and circular polarization incident waves. The radii and dielectric constants of the spheres are taken as $a = 100\text{ nm}$ and $\varepsilon = 12.96$. The parameters for monolayer graphene are taken as $E_f = 1.0\text{ eV}$ and $\Gamma_c = 1.0\text{ meV}$. (a) The perpendicular incidence with electric polarization, $\mathbf{E}$, along the longitudinal axis of the system; (b) the parallel incidence; (c) the perpendicular incidence with circular polarization. The left inset shows the enlarged results for multipole effects.

Figure 4. Absorption efficiency of many graphene-wrapped dielectric spheres as a function of wavelength under the excitation of the linear polarization incident wave. (a) Seven-sphere system in a plane, as shown in the inset; (b) 27-sphere system in $3 \times 3 \times 3$ stack, as shown in the inset. Here, the parameters of graphene-wrapped dielectric spheres are assumed to be identical to those in figure 3. The electric polarization, $\mathbf{E}$, of the incident wave is along the longitudinal axis of the two adjacent spheres.

Figure 5. Here, the parameters of scatters are assumed to be identical to those in figure 3(a). $T$ and $R$ are defined as the ratio of the transmitted and reflected energy flux, respectively, to the energy flux associated with the incident wave. The absorptance is defined from the requirement of energy conservation, $A = 1 - T - R$. The red, cyan, magenta, and yellow lines correspond to the following cases with the separation distance between two spheres: $d = 20\text{ nm}, 50\text{ nm}, 100\text{ nm}$, and $300\text{ nm}$, respectively. Due to the interaction across the lattice, significant red shifts take place as the
separation distances of the spheres decrease. It is clearly shown that the transmittance increases and the reflectance decreases as the separation distances increase. The absorptance can reach 50% as the suitable lattice constants are taken \((d = 50 \text{ nm} \) and \(100 \text{ nm} \) in this case), showing the strong absorptions of the incident infrared wave. Note that because of the infinite extension of the planar structure of the spheres, the red shift of the resonance absorption is more serious than that of the case of the two spheres in figure 3(a) for the same separation distance between the spheres. The above studies only focus on the optical absorption properties of monolayer graphene-wrapped dielectric particles; some investigations have shown that the stack of graphene layers may strongly affect the low-frequency plasmons [39–41]. For the particles wrapped with stacked graphene layers, the absorption properties should also be affected by the stack types of the graphene layers.

4. Summary

In summary, we studied the optical absorption properties of graphene-wrapped dielectric particles based on the Mie scattering theory and the exact multiscattering method. We have demonstrated strong subwavelength absorption of graphene-wrapped dielectric spheres due to the plasmon resonance of graphene with external EM waves. The absorption efficiency and resonance wavelength can be tuned easily by either tuning the Fermi level and the damping of graphene, or changing the dielectric constant and size of the spheres. The blue and red shifts of the resonant absorption peaks have been found in the system for the linear polarized wave with different incident directions. The multipole signal has also been observed in the absorption spectra. Our research proposes a new way to realize the tunable strong absorption of graphene in the infrared spectra using subwavelength sphere configurations. The methods and results can be helpful in light modulation, solar cells, biological imaging, and even in THz communication.

Acknowledgments

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References

[1] Geim A K 2009 Science 324 1530–4
[2] Bonaccorso F, Sun Z, Hasan T and Ferrari A C 2010 Nat. Photon. 4 611–22
[3] Wang X, Zhi L and Müllen K 2008 Nano Lett. 8 323–7
[4] Kim K S, Zhao Y, Jang H, Lee S Y, Kim J M, Kim K S, Ahn J-H, Kim P, Choi J-Y and Hong B H 2009 Nature 457 706–10
[5] Bae S et al 2010 Nat. Nanotechnology 5 574–8
[6] Vicarelli L, Vitiello M S, Coquillet D, Lombardo A, Ferrari A C, Knap W, Polini M, Pellegrini V and Tredicucci A 2012 Nat. Mater. 11 865–71
[7] Furchi M et al 2012 Nano Lett. 12 2773–7
[8] Ferreira A, Peres N M R, Ribeiro R M and Stauber T 2012 Phys. Rev. B 85 115438
[9] Ju L et al 2011 Nat. Nanotechnology 6 630–4
[10] Liu M, Yin X, Ulin-Avila E, Geng B, Zentgraf T, Ju L, Wang F and Zhang X 2011 Nature 474 64–7
[11] Ren L et al 2012 Nano Lett. 12 3711–5
[12] Lee S H et al 2012 Nat. Mater. 11 936–41
[13] Alonso-González P et al 2014 Science 344 1369–73
[14] Mak K F, Steir M Y, Wu Y, Liu C H, Misewich J A and Heinz T F 2008 Phys. Rev. Lett. 101 196405
[15] Grande M, Stomeo T, Bianco G V, Vincenti M A, de Ceglia D, Petruzzielli V, Bruno G, De Vittorio M, Scalora M and D’Orazio A 2013 Appl. Phys. Lett. 102 231111
[16] Zhu X, Shi L, Schmidt M S, Boisen A, Hansen O, ZJ, Xiao S and Mortensen N A 2013 Nano Lett. 13 4690–6
[17] Hashemi M, Farzad M H, Mortensen N A and Xiao S 2013 J. Opt. 15 055003
[18] Stauber T, Gómez-Santos G and García de Abajo F J 2014 Phys. Rev. Lett. 112 077401
[19] Liu J-T, Liu N-H, Li J, Li X J and Huang J-H 2012 Appl. Phys. Lett. 101 052104
[20] Sreekanth K V and Yu T 2013 J. Opt. 15 055002
[21] Thongrattanasiri S, Koppens F H L and García de Abajo F J 2012 Phys. Rev. Lett. 108 047401
[22] Wang B, Zhang X, García-Vidal F J, Yuan X and Teng J 2012 Phys. Rev. Lett. 109 073901
[23] Nefedov I S, Valaginnopoulos C A and Melnikov L A 2013 J. Opt. 15 114003
[24] Fang Z, Thongrattanasiri S, Schlather A, Liu Z, Ma L, Wang Y, Ajayan P M, Nordlander P, Halas N J and García de Abajo F J 2013 ACS Nano 7 2388–95
[25] Fang Z, Wang Y, Schlather A E, Liu Z, Ajayan P M, García de Abajo F J, Nordlander P, Zhu X and Halas N J 2014 Nano Lett. 14 299–304
[26] García de Abajo F J 2014 ACS Photonics 1 135–152
[27] Ma X, Qu Q, Zhao Y, Luo Z, Zhao Y, Ng K W and Zhao Y 2013 J. Mater. Chem. B 1 6495–500
[28] Hanson G W 2008 J. Appl. Phys. 104 084314
[29] Jablan M, Buljan H and Soljačić M 2009 Phys. Rev. B 80 245435
[30] Emani N K, Chung T-F, Ni X, Kildishev A V, Chen Y P and Boltasseva A 2012 Nano Lett. 12 5202–6
[31] Majumdar A, Kim J, Vuckovic J and Wang F 2013 Nano Lett. 13 515–8
[32] Bludov Y V, Peres N M R and Vasilevskiy M I 2013 J. Opt. 15 114004
[33] Heinisch R L, Bronold F X and Fehske H 2012 Phys. Rev. Lett. 109 243903
[34] Kocifaj M and Klačka J 2012 Opt. Lett. 37 265–7
[35] Bohren C F and Hunt A J 1977 Can. J. Phys. 55 1930–5
[36] Xu Y-L 1995 Appl. Opt. 34 4573–88
[37] Stefanou N, Yannopapas V and Modinos A 1998 Comput. Phys. Commun. 113 49–77
[38] Stefanou N, Yannopapas V and Modinos A 2000 Comput. Phys. Commun. 132 189–96
[39] Ho J H, Lu C L, Hwang C C, Chang C P and Lin M F 2006 Phys. Rev. B 74 085406
[40] Chuang Y-C, Wu J-Y and Lin M-F 2013 Sci. Rep. 3 1368
[41] Wu J-Y, Gumbs G and Lin M-F 2014 Phys. Rev. B 89 165407