Thermo field dynamics approach for entropy of spacetime

Yu-Zhu Chen, Wen-Du Li, and Wu-Sheng Dai

Department of Physics, Tianjin University, Tianjin 300072, P.R. China
LiuHui Center for Applied Mathematics, Nankai University & Tianjin University, Tianjin 300072, P.R. China

Abstract: A thermo field dynamics approach for calculating the entropy of a spacetime is suggested. It is exemplified through the Rindler spacetime, the Milne spacetime, the Boulware spacetime, and the Minkowski spacetime with a moving mirror that the entropy of a spacetime is equal to the entropy of a thermo quantum field with the same temperature of the spacetime we study. This in fact suggests a thermo field dynamics approach of calculating the entropy of a spacetime. In this approach, the entropy of a spacetime is an expectation value of the entropy operator on a thermo vacuum state. The thermo vacuum state is the vacuum state on the maximal manifold which is the maximal analytic extension of the spacetime we study.

Keywords: thermo field dynamics, entropy of spacetime, thermo vacuum, maximal manifold

1daiwusheng@tju.edu.cn.
1 Introduction

Thermo field dynamics is a quantum field theory at finite temperature [1, 2]. In thermo field dynamics, the statistical average of a physical quantity is represented by an expectation value of an operator expressed by field operators on a temperature-dependent thermo vacuum state $|0(\beta)\rangle$. For example, the entropy is the expectation value of an entropy operator on a thermo vacuum state $|0(\beta)\rangle$. In this paper, we apply thermo field dynamics to calculate the entropy of spacetime.

In our scheme, for the calculation of the entropy of a spacetime, the entropy operator is constructed by thermo field dynamics and the thermo vacuum state $|0(\beta)\rangle$ is defined as the vacuum state on the maximal manifold which is the maximal analytic extension of the spacetime we study.

The validity of the approach is exemplified by applying the approach to calculate the entropy of four kinds of spacetimes: the Rindler spacetime, the Milne spacetime, the Boulware spacetime, and the Minkowski spacetime with a moving mirror. We show that the result agrees well with literature.

Many approaches are developed for the calculation of the entropy of a spacetime [3]. The first calculation is due to the similarity between the property of a black hole and the law of thermodynamics [4, 5]. The entropy of a black hole is then calculated by Euclidean quantum field theory [6]. By regarding the entropy of a black hole as an entanglement entropy between the exterior and interior of the horizon, one can also calculate the entropy of a spacetime [7, 8]. The black hole entropy can also be calculated by quantum geometry [9]. In string theory, the entropy of the $AdS$ spacetime is obtained through the $AdS/CFT$...
correspondence [10] due to the equivalence between the \( n + 1 \)-dimensional AdS and the conformal field theory on the boundary [11]. A relation between the black hole entropy and the conformal field on the horizon is discussed in ref. [12]. In ref. [13], the correspondence between AdS\(_2\) and CFT\(_1\) is provided. The entropy of the BTZ black hole is discussed in ref. [14]. The entropy of a spherically symmetric system is given by the Liouville theory in ref. [15]. The central charge of the conformal field which is related to the entropy is obtained according to the symmetry of the system in ref. [16]. The entropy can also be obtained by counting D-brane states [17]. A derivation of the entropy based on the microstates of CFT is given by [18]. In loop quantum gravity [9], the entropy is obtained by counting the microscopic states of a black hole and some efforts are devoted to match the loop-quantum-gravity entropy with the Bekenstein-Hawking entropy [19–24]. The entropy of a black hole can also be calculated as a Noether charge associated with the horizon Killing field [25–27].

In section 2, we provide a brief review of thermo field dynamics. In section 3, we construct the thermo field dynamics approach for the calculation of the entropy of a spacetime. In sections 4, 5, 6, and 7, as exemplification, we calculate the entropy of the Rindler spacetime, the Milne spacetime, the Boulware spacetime, and the Minkowski spacetime with a moving mirror. The conclusion is given in section 8.

2 Calculating entropy: A brief review of thermo field dynamics

In this section, we provide a brief review of thermo field dynamics [1, 28, 29].

In thermo field dynamics, the statistical average of a physical operator \( \hat{F} \) at a temperature \( T = 1/\beta \) is replaced by an expectation value of \( \hat{F} \) on a thermo vacuum state:

\[
\langle \hat{F} \rangle = \frac{1}{Z(\beta)} \text{Tr} \left( \hat{F} e^{-\beta \hat{H}} \right) = \langle 0(\beta) | \hat{F} | 0(\beta) \rangle ,
\]

where the thermo vacuum state \( |0(\beta)\rangle \) is a temperature-dependent vacuum state defined by

\[
|0(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n} |n\rangle |\tilde{n}\rangle .
\]

Here, for zero chemical potential cases, \( Z(\beta) = \sum_n e^{-\beta E_n} \) is the partition function, the eigenvalue \( E_n \) and eigenvector \( |n\rangle \) are determined by \( \hat{H} |n\rangle = E_n |n\rangle \), and \( |\tilde{n}\rangle \) is the eigenvector of the corresponding fictitious system [1]. The fictitious system represented by the tilde state \( |\tilde{n}\rangle \) is interpreted as the hole of the physical particle in \( |n\rangle \) [1].

To calculate the entropy of a spacetime by thermo field dynamics is just to calculate the thermo vacuum expectation value of the entropy operator of the spacetime, i.e.,

\[
S = \langle 0(\beta) | \hat{S} | 0(\beta) \rangle .
\]

The entropy operator in thermo field dynamics reads [1]

\[
\hat{S} = -\sum_p \left[ \hat{b}_p^\dagger \hat{b}_p \ln \sinh^2 \theta_p (\beta) - \hat{b}_p \hat{b}_p^\dagger \ln \cosh^2 \theta_p (\beta) \right] , \quad \text{bosonic,}
\]

\[
\hat{S} = -\sum_p \left[ \hat{b}_p^\dagger \hat{b}_p \ln \sin^2 \theta_p (\beta) - \hat{b}_p \hat{b}_p^\dagger \ln \cos^2 \theta_p (\beta) \right] , \quad \text{fermionic,}
\]
where \( b^\dagger_p \) and \( b_p \) are creation and annihilation operators and [1]

\[
\sinh^2 \theta_p(\beta) = \frac{1}{e^{\beta \Omega_p} - 1}, \quad \text{bosonic},
\]

\[
\sin^2 \theta_p(\beta) = \frac{1}{e^{\beta \Omega_p} + 1}, \quad \text{fermionic},
\]

with \( \Omega_p = |p| \), are Bose-Einstein and Fermi-Dirac distributions, respectively.

### 3 Thermo field dynamics approach for entropy of spacetime

In this section, we construct the approach for calculating the entropy in the frame of thermo field dynamics. In next sections, as exemplification, we will calculate the entropy of the Rindler spacetime, the Milne spacetime, the Boulware spacetime, and the Minkowski spacetime with a moving mirror and compare them with literature.

The key step in thermo field dynamics is to construct the thermo vacuum state \( |0(\beta)\rangle \).

To calculate the entropy of a spacetime, we choose the thermo vacuum state as the vacuum state on the maximal manifold, or, the maximal spacetime.

The maximal manifold is the maximal analytic extension of the spacetime we study. In the maximal manifold every geodesic is either of infinite length in both directions or else ends or begins on a singularity [30]. The maximal manifold is described by the complete atlas [31]. The spacetime we study is a submanifold of the maximal manifold, and the maximal manifold can be achieved by the maximal analytic extension of the spacetime we study. Take the Rindler spacetime as an example. The Rindler spacetime is the region that an accelerated observer with constant acceleration sees. The boundary of the Rindler spacetime has no singularity at all. The Rindler spacetime can be analytically continued to the Minkowski spacetime by a coordinates transformation [32]. The Minkowski spacetime covers not only the Rindler spacetime, but also the region besides the Rindler spacetime, or, the Rindler spacetime is a subspace or submanifold of the Minkowski spacetime. No other spacetime covers larger region than the Minkowski spacetime does. Therefore, the Minkowski spacetime is a maximal manifold. Moreover, the Minkowski spacetime is the unique maximal spacetime corresponding to the Rindler spacetime according to the uniqueness of analytic continuation.

After analytically continuing the spacetime we study to the corresponding maximal manifold, we have two kinds of spacetimes. On each of them, we can define a set of creation and annihilation operators, respectively: on the maximal manifold, \( a^\dagger_\omega \) and \( a_\omega \); on the spacetime we study, \( b^\dagger_\Omega \) and \( b_\Omega \). Furthermore, we can define two vacuum states by the corresponding annihilation operators:

\[
 a_\omega |0_a\rangle = 0
\]

defines the vacuum state \( |0_a\rangle \) on the maximal manifold;

\[
 b_\Omega |0_b\rangle = 0
\]

defines the vacuum state \( |0_b\rangle \) on the spacetime we study.
In the approach, the thermo vacuum state is chosen as the vacuum state on the maximal manifold, $|0_a\rangle$.

Concretely, consider a real scalar field. Define creation and annihilation operators, $b^\dagger_\Omega$ and $b_\Omega$, on the spacetime we study, such as the Rindler spacetime, etc. Furthermore, define creation and annihilation operators, $a^\dagger_\omega$ and $a_\omega$, on the corresponding maximal manifold. Note that $\omega \in (-\infty, \infty)$ and $\Omega \in (-\infty, \infty)$. For conformally flat cases, these two sets of creation and annihilation operators can be related by the Bogoliubov transformation [32, 33],

$$b_\Omega = \int_{-\infty}^{\infty} d\omega \left( \alpha_{\omega\Omega} a_\omega - \beta_{\omega\Omega} a^\dagger_\omega \right),$$
$$b^\dagger_\Omega = \int_{-\infty}^{\infty} d\omega \left( \alpha^*_{\omega\Omega} a^\dagger_\omega - \beta^*_{\omega\Omega} a_\omega \right). \tag{3.3}$$

The transformation coefficients $\alpha_{\omega\Omega}$ and $\beta_{\omega\Omega}$ satisfy

$$\int_{-\infty}^{\infty} d\omega \left( \alpha_{\omega\Omega} \alpha^*_{\omega'\Omega'} - \beta_{\omega\Omega} \beta^*_{\omega'\Omega'} \right) = \delta (\Omega - \Omega'). \tag{3.4}$$

Particularly, for $\Omega = \Omega'$, we have

$$\int_{-\infty}^{\infty} d\omega \left( |\alpha_{\omega\Omega}|^2 - |\beta_{\omega\Omega}|^2 \right) = \delta (0). \tag{3.5}$$

For a real scalar field, by eqs. (2.4) and (2.3), the entropy $S = \langle 0_a | \hat{S} | 0_a \rangle$, the expectation value of the entropy operator $\hat{S}$ on the vacuum state $|0_a\rangle$:

$$S = -\sum_p \left[ \langle 0_a | b^\dagger_\Omega b_\Omega | 0_a \rangle \ln \sinh^2 \theta_p (\beta) - \langle 0_a | b^\dagger_\Omega b^\dagger_\Omega | 0_a \rangle \ln \cosh^2 \theta_p (\beta) \right]. \tag{3.6}$$

Substituting the Bogoliubov transformation (3.3) into eq. (3.6), we arrive at

$$S = \sum_p \int_{-\infty}^{\infty} d\omega \left[ |\alpha_{\omega\Omega}|^2 \ln \cosh^2 \theta_p (\beta) - |\beta_{\omega\Omega}|^2 \ln \sinh^2 \theta_p (\beta) \right]. \tag{3.7}$$

Substituting the Bose-Einstein distribution (2.6) and $\cosh^2 \theta_p (\beta) = e^{\beta \Omega_p} / (e^{\beta \Omega_p} - 1)$ which is given by eq. (2.6) into eq. (3.7), we obtain

$$S = \sum_p \left( \ln \frac{e^{\beta \Omega_p}}{e^{\beta \Omega_p} - 1} \int_{-\infty}^{\infty} d\omega |\alpha_{\omega\Omega}|^2 - \ln \frac{1}{e^{\beta \Omega_p} - 1} \int_{-\infty}^{\infty} d\omega |\beta_{\omega\Omega}|^2 \right), \tag{3.8}$$

where $\Omega_p = |\Omega|$.

In a word, once the Bogoliubov transformation (3.3) is constructed, one can directly obtain the entropy by eq. (3.8).

It should be emphasized that though the Bogoliubov transformation (3.3) is valid for conformally flat cases, the method constructed above can be applied to more general cases. We will discuss the application of this method to non-conformally flat cases further later.
4 Entropy of Rindler Spacetime

The Rindler spacetime is the region that an accelerated observer with constant acceleration $a$ sees, characterized by the metric

$$ds^2 = e^{2a\xi} (d\tau^2 - d\xi^2) - dy^2 - dz^2. \quad (4.1)$$

The maximal manifold of the Rindler spacetime is the Minkowski spacetime:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2. \quad (4.2)$$

The coordinates in the metric (4.1), $(\tau, \xi)$, and the coordinates in the metric (4.2), $(t, x)$, are connected by [32]

$$t = \frac{1}{a} e^{a\xi} \sinh (a\tau), \quad x = \frac{1}{a} e^{a\xi} \cosh (a\tau). \quad (4.3)$$

The real scalar field. Consider real scalar fields in the Minkowski spacetime and in the Rindler spacetime. In the Minkowski spacetime, denote the creation and annihilation operators as $a_\omega^\dagger$ and $a_\omega$; in the Rindler spacetime, denote the creation and annihilation operators as $b_\Omega^\dagger$ and $b_\Omega$.

The vacuum state in the Minkowski spacetime $|0_M\rangle$ is defined by

$$a_\omega |0_M\rangle = 0. \quad (4.4)$$

For massless cases, we have $|\omega| = |k|$ in the Minkowski spacetime and $|\Omega| = |p|$ in the Rindler spacetime.

The entropy of the Rindler spacetime. As analyzed in section 3, the entropy $S_R = \langle 0_M | \hat{S}_R | 0_M \rangle$ is the expectation value of $\hat{S}_R$ on the corresponding thermo vacuum — the vacuum of the Minkowski spacetime $|0_M\rangle$.

The Bogoliubov transformation coefficients in eq. (3.3), $\alpha_{\omega\Omega}$ and $\beta_{\omega\Omega}$, satisfy [32]

$$|\alpha_{\omega\Omega}|^2 = e^{\beta |\Omega|} |\beta_{\omega\Omega}|^2, \quad (4.5)$$

where $\beta = 2\pi / a$. Substituting eq. (4.5) into eq. (3.5), we arrive at

$$\int_{-\infty}^{\infty} d\omega |\beta_{\omega\Omega}|^2 = \frac{1}{e^{e^{\beta |\Omega|}} - 1} \delta (0). \quad (4.6)$$

Substituting the relation (4.5) with $\Omega_p = |\Omega|$ into eq. (3.8) and using (4.6), we obtain

$$S_R = \delta (0) \sum_p \left[ \beta |\Omega| \frac{e^{\beta |\Omega|}}{e^{\beta |\Omega|}} - 1 \ln \left( e^{\beta |\Omega|} - 1 \right) \right]. \quad (4.7)$$

For 1 + 3 dimensions, converting the summation into an integral, $\sum_p \to 4\pi \int_0^\infty \Omega^2 d\Omega$, where $|\Omega| = |p|$, gives

$$S_R = \frac{16\pi^5}{45\beta^3} \delta (0). \quad (4.8)$$
Here $\delta (0)$ represents the volume of the Rindler spacetime: $\delta (0) = \int \frac{d^3 x}{(2\pi)^3} e^{i p \cdot x} \big|_{p=0} = \frac{1}{(2\pi)^3} \int d^3 x = \frac{1}{(2\pi)^3} V$. The entropy then reads
\[ S_R = \frac{2\pi^2}{45} VT^3. \] (4.9)

This result agrees with the result given in ref. [35].

5 Entropy of Milne spacetime

The $1 + 1$-dimensional Milne spacetime. The Milne spacetime is the region of the future lightcone of a given point in the Minkowski spacetime [36, 37], which can be, for example, regarded as the empty universe of the FLRW model [38]. The Milne spacetime is characterized by the metric
\[ ds^2 = e^{2 \alpha \tau} \left( d\tau^2 - d\xi^2 \right). \] (5.1)

The maximal manifold of the Milne spacetime is the Minkowski spacetime
\[ ds^2 = dt^2 - dx^2. \] (5.2)

The coordinates in the metric (5.1), $(\tau, \xi)$, and the coordinates in the metric (5.2), $(t, x)$, are connected by
\[ t = \frac{1}{a} e^{a \tau} \cosh (a \xi), \quad x = \frac{1}{a} e^{a \tau} \sinh (a \xi). \] (5.3)

The real scalar field. The creation and annihilation operators of a real scalar field in the Minkowski spacetime are $a^{\dagger}_\omega$ and $a_\omega$, and in the Milne spacetime are $b^{\dagger}_\Omega$ and $b_\Omega$.

The vacuum state in the Minkowski spacetime $|0_M\rangle$ is defined by
\[ a_\omega |0_M\rangle = 0. \] (5.4)

The entropy of the Milne spacetime. Similarly, the entropy $S_{Mil} = \langle 0_M | \hat{S}_{Mil} | 0_M \rangle$ is the expectation value of $\hat{S}_{Mil}$ on the corresponding thermo vacuum — the vacuum of the Minkowski spacetime $|0_M\rangle$.

The Bogoliubov transformation coefficients in eq. (3.3), $\alpha_\omega \Omega$ and $\beta_\omega \Omega$, satisfy
\[ |\alpha_\omega \Omega|^2 = e^{\beta |\Omega|} |\beta_\omega \Omega|^2, \] (5.5)

where $\beta = 2\pi/a$. By the same procedure in section 4, we obtain the entropy:
\[ S_{Mil} = \sum_p \left( \ln \frac{e^{\beta |\Omega|}}{e^{\beta |\Omega|} - 1} e^{\beta |\Omega|} \right) \int_{-\infty}^{\infty} d\omega |\beta_\omega \Omega|^2 \ln \frac{1}{e^{\beta |\Omega|} - 1} \int_{-\infty}^{\infty} d\omega |\beta_\omega \Omega|^2 \right). \] (5.6)

We then have
\[ S_{Mil} = \delta (0) \sum_p \left[ \beta |\Omega| \frac{e^{\beta |\Omega|}}{e^{\beta |\Omega|} - 1} - \ln \left( e^{\beta |\Omega|} - 1 \right) \right]. \] (5.7)
Converting the summation into an integral \( \sum_p \rightarrow \int_0^\infty d\Omega \), where \( \Omega = |p| \), give

\[
S_{Mil} = \frac{\pi^2}{3\beta} \delta(0).
\] (5.8)

Here \( \delta(0) \) represents the volume of the Milne spacetime: \( \delta(0) = L/(2\pi) \).

The entropy then reads

\[
S_{Mil} = \frac{\pi}{6\beta} L.
\] (5.9)

6 Entropy of Boulware spacetime

*The Boulware spacetime.* The Boulware spacetime is the region outside a Schwarzschild black hole which is described by the tortoise coordinate \((t, r_*)\) [32],

\[
ds^2 = \left(1 - \frac{2M}{r}\right) \left(dt^2 - dr^2_*\right) - r^2 \left(d^2\theta + \sin^2\theta d\phi^2\right),
\] (6.1)

where \( r_* = r + 2M \ln \frac{r-2M}{2M} \).

The maximal manifold of the Boulware spacetime is the Kruskal-Szekeres spacetime described by the Kruskal-Szekeres coordinate \((\tilde{t}, \tilde{r})\),

\[
ds^2 = \frac{2M}{r} e^{-r/(2M)} \left(dt^2 - d\tilde{r}^2\right) - r^2 \left(d^2\theta + \sin^2\theta d\phi^2\right).
\] (6.2)

The Kruskal-Szekeres coordinate \((\tilde{t}, \tilde{r})\) and the tortoise coordinate \((t, r_*)\) are connected by [30, 32]

\[
\tilde{t} = 4Me^{r_*/(4M)} \sinh \frac{t}{4M}, \quad \tilde{r} = 4Me^{r_*/(4M)} \cosh \frac{t}{4M}.
\] (6.3)

*The real scalar field.* In the Kruskal-Szekeres spacetime, denote the creation and annihilation operators as \(a^\dagger_\omega\) and \(a_\omega\); in the Boulware spacetime, denote the creation and annihilation operators as \(b^\dagger_\Omega\) and \(b_\Omega\).

The vacuum state in the Kruskal-Szekeres spacetime \(|0_K\rangle\) is defined by

\[
a_\omega \mid 0_K \rangle = 0.
\] (6.4)

For massless cases, we have \(|\omega| = |k|\) in the Kruskal-Szekeres spacetime and \(|\Omega| = |p|\) in the Boulware spacetime.

*The entropy of the Boulware spacetime.* As above, the entropy \(S_B = \langle 0_K \mid \hat{S}_B \mid 0_K \rangle\) is the expectation value of \(\hat{S}_B\) on the corresponding thermo vacuum — the vacuum of the Kruskal-Szekeres spacetime \(|0_K\rangle\).

The coefficients, \(\alpha_{\omega\Omega}\) and \(\beta_{\omega\Omega}\), of the Bogoliubov transformation between \(a^\dagger_\omega\), \(a_\omega\) and \(b^\dagger_\Omega\), \(b_\Omega\) satisfy [32, 39]

\[
|\alpha_{\omega\Omega}|^2 = e^{\beta|\Omega|} |\beta_{\omega\Omega}|^2,
\] (6.5)

where \( \beta = 8\pi M \). By the same procedure in section 4, we obtain the entropy of the Boulware spacetime:

\[
S_B = \frac{16\pi^5}{45\beta^3} \delta(0) = \frac{2\pi^2}{45\beta^3} V.
\] (6.6)
Taking the volume as \( V = \frac{4}{3} \pi r^3 \) gives

\[
S_B = \frac{8 \pi^3}{135 \beta^3} r^3.
\] (6.7)

This result agrees with the brick wall model [40]. In ref. [40], for calculating the entropy of a Schwarzschild black hole, t’ Hooft constructs the brick wall model. In the brick wall model, one equates the entropy of the Bose gas outside the black hole with the entropy of the Schwarzschild black hole.

7 Entropy of Minkowski spacetime with a moving mirror

Placing a moving mirror in a spacetime is equivalent to setting an appropriate boundary condition [41, 42].

The 1+1-dimensional Minkowski spacetime with a moving mirror. A Minkowski spacetime with a moving mirror as a boundary can be characterized by the metric [41, 42]

\[
ds^2 = dt^2 - dx^2,
\]

with the boundary

\[
x = Z(t) = \begin{cases} 
-t - Ae^{-2\kappa t} + B, & t > 0 \\
0, & t < 0 
\end{cases},
\] (7.1)

where \( A, B, \) and \( \kappa \) are constants.

The corresponding maximal manifold of such a spacetime is the Minkowski spacetime without boundaries. In the following, we call the Minkowski spacetime without boundaries the in spacetime and the Minkowski spacetime with a moving mirror (7.1) the out spacetime. More detailed descriptions can be found in refs [41, 42].

The real scalar field. In the in spacetime (without boundaries), denote the creation and annihilation operators as \( a^\dagger_\omega \) and \( a_\omega \); in the out spacetime (with boundaries), denote the creation and annihilation operators as \( b^\dagger_\Omega \) and \( b_\Omega \).

The vacuum state in the in spacetime \( |0_{in}\rangle \) is defined by

\[
a_\omega |0_{in}\rangle = 0.
\] (7.2)

The entropy of the out spacetime. The entropy of the out spacetime \( S_{out} = (0_{in}| \hat{S}_{out} |0_{in}\rangle \) is the expectation value of \( \hat{S}_{out} \) on the corresponding thermo vacuum —– the vacuum of the in spacetime \( |0_{in}\rangle \).

The coefficients, \( \alpha_\omega \Omega \) and \( \beta_\omega \Omega \), in the Bogoliubov transformation between \( a^\dagger_\omega \), \( a_\omega \) and \( b^\dagger_\Omega \), \( b_\Omega \) satisfy [42]

\[
|\alpha_\omega \Omega|^2 = e^{\beta \Omega} |\beta_\omega \Omega|^2,
\] (7.3)

where \( \beta \equiv 2\pi/\kappa \). By the same procedure in the above sections, we obtain the entropy:

\[
S_{out} = \sum_p \left[ \beta_\Omega \frac{e^{\beta \Omega}}{e^{\beta \Omega} - 1} \ln \left( e^{\beta \Omega} - 1 \right) \right] \delta (0).
\] (7.4)
Converting the summation into an integral \( \sum_p \rightarrow \int_0^\infty d\Omega \) with \( \Omega = |p| \) gives

\[
S_{\text{out}} = \frac{\pi^2}{3\beta} \delta(0) = \frac{\pi}{6\beta} L, \tag{7.5}
\]

where \( L \) is the volume.

In the \( \text{out} \) spacetime, the observer sees particles created from the mirror. The physical picture is that the particle in the Minkowski spacetime (\( \text{in} \) spacetime) moves to the mirror and then is reflected into the \( \text{out} \) spacetime. In other words, the \( \text{out} \) spacetime is not an isolated system, it can exchange particles and energy with the maximal manifold. The moving mirror here plays a role similar to the horizon of a black hole.

## 8 Conclusion

In this paper, we suggest a thermo-field-dynamics approach for the calculation of the entropy of spacetimes. The approach is exemplified through calculating the entropy of the Rindler spacetime, the Milne spacetime, the Boulware spacetime, and the Minkowski spacetime with a moving mirror.

In this scheme, the entropy of a spacetime is an expectation value of the entropy operator on a thermo vacuum state \( |0(\beta)\rangle \) which is the vacuum state of a quantum field in the maximal manifold of the spacetime we study. Concretely, first analytically continue the spacetime to its maximal manifold. Then define the field, in our case a scalar field, on both the spacetime and its maximal manifold. The field at the same point on the spacetime and on its maximal manifold, clearly, must be equal to each other. This determines the coefficients of the Bogoliubov transformation which connects the fields on the spacetime and its maximal manifold.

It is known that there is no generally accepted definition of the entropy of spacetime. Historically, the entropy of spacetime was introduced by a non-rigorous analogy between the horizon area and the entropy \([43–46]\). Bekenstein showed that there is a way to understand the entropy of spacetime just as that in statistical mechanics: the logarithm of the number of all the possible states which can form a black hole. Moreover, there are many attempts on the explanation and calculation of the entropy of spacetime. For example, regard the entropy of a black hole as a Noether charge \([25–27]\), explain the entropy of a black hole as a topological contribution to the Euclidean action integrals \([6, 47]\), explain the entropy as the logarithm of the number of the states of the quantum states of the geometry of the horizon in quantum geometry theory \([9]\), equal the entropy of a black hole with the logarithm of the number of the states of the conformal field on the boundary of the black hole by the \( \text{AdS/CFT} \) correspondence \([10, 12, 13]\), or regard the entropy as the logarithm of the number of the \( D \)-brane states \([17]\), etc. An important explanation of the entropy of spacetime is the entanglement entropy. The discovery of black-hole radiation \([39]\) allows us to explain the entropy of spacetime as an entanglement entropy \([48]\). The quantum field on the exterior of the black hole is in a mixed state. The entanglement entropy comes from the quantum field correlations between the exterior and interior of the horizon \([3]\). The entanglement entropy has been adopted by many authors \([7, 8, 49]\). The entropy we have
calculated above is the entropy of a thermal equilibrium massless Bose gas outside the black hole. Such an entropy is essentially an entanglement entropy.

Moreover, if regarding the entropy of a spacetime as an entanglement entropy, one encounters a problem — the species problem [43]. The Hawking radiation emits all species of particles and then the entropy should depend on the number of the species of particles. There are some discussions of the species problem. For example, Sorkin and ’t Hooft suggest that the entropy is a summation of all contributions of all species of particles [43] and Jacobson [50] and Susskind [49] consider the possibility of correcting the gravitational constant $G$. In the future research, to study the species problem, we will consider the fermionic contribution to the entropy by the thermo field dynamics approach.

The non-conformally flat case is also an important issue [36, 51]. The method established in the present paper can also be applied to such cases, which will be discussed further later.

Furthermore, the thermo field dynamics approach suggested in the present paper can also be applied to the calculation of other thermodynamic quantities of a spacetime. All thermodynamic quantities are expectation values of their corresponding operators which is defined in thermo field dynamics on the thermo vacuum state $|0(\beta)\rangle$. In particular, the partition function $Z(\beta) = \langle 0(\beta)| e^{-\beta\hat{H}} |0(\beta)\rangle$ is the Euclidean action of the system. This means that we can study quantum field theory in curved space using the approach. Moreover, the partition function is also the global heat kernel by which one can calculate many quantities in quantum field theory, such as one-loop effective actions, vacuum energies, and spectral counting functions [52, 53].

Acknowledgments

We are very indebted to Dr G. Zeitrauman for his encouragement. We would like to express our appreciation to Dr. Slava Emelyanov for his helpful comments and suggestions. This work is supported in part by NSF of China under Grant No. 11575125 and No. 11375128.

References

[1] Y. Takahashi and H. Umezawa, Thermo field dynamics, *International Journal of Modern Physics B* 10 (1996), no. 13n14 1755–1805.

[2] Y. Hashizume and M. Suzuki, Understanding quantum entanglement by thermo field dynamics, *Physica A: Statistical Mechanics and its Applications* 392 (2013), no. 17 3518–3530.

[3] R. M. Wald, The thermodynamics of black holes, in Advances in the Interplay Between Quantum and Gravity Physics, pp. 477–522. Springer, 2002.

[4] J. M. Bardeen, B. Carter, and S. W. Hawking, The four laws of black hole mechanics, *Communications in Mathematical Physics* 31 (1973), no. 2 161–170.

[5] J. D. Bekenstein, Black holes and entropy, *Physical Review D* 7 (1973), no. 8 2333.

[6] G. W. Gibbons and S. W. Hawking, Action integrals and partition functions in quantum gravity, *Physical Review D* 15 (1977), no. 10 2752.
[7] C. Callan and F. Wilczek, *On geometric entropy*, Physics Letters B 333 (1994), no. 1 55–61.

[8] C. Holzhey, F. Larsen, and F. Wilczek, *Geometric and renormalized entropy in conformal field theory*, Nuclear Physics B 424 (1994), no. 3 443–467.

[9] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, *Quantum geometry and black hole entropy*, Physical Review Letters 80 (1998), no. 5 904.

[10] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, *Large n field theories, string theory and gravity*, Physics Reports 323 (2000), no. 3 183–386.

[11] J. Maldacena, *The large-n limit of superconformal field theories and supergravity*, International journal of theoretical physics 38 (1999), no. 4 1113–1133.

[12] S. Carlip, *Black hole entropy from horizon conformal field theory*, Nuclear Physics B-Proceedings Supplements 88 (2000), no. 1 10–16.

[13] J. Navarro-Salas and P. Navarro, *Ads 2/cft 1 correspondence and near-extremal black hole entropy*, Nuclear Physics B 579 (2000), no. 1 250–266.

[14] S. Carlip, *What we don’t know about btz black hole entropy*, Classical and Quantum Gravity 15 (1998), no. 11 3609.

[15] A. Giacomini and N. Pinamonti, *Black hole entropy from classical liouville theory*, Journal of High Energy Physics 2003 (2003), no. 02 014.

[16] S. Silva, *Black-hole entropy and thermodynamics from symmetries*, Classical and Quantum Gravity 19 (2002), no. 15 3947.

[17] A. Strominger and C. Vafa, *Microscopic origin of the bekenstein-hawking entropy*, Physics Letters B 379 (1996), no. 1 99–104.

[18] A. Strominger, *Black hole entropy from near-horizon microstates*, Journal of High Energy Physics 1998 (1998), no. 02 009.

[19] A. Ghosh and A. Perez, *Black hole entropy and isolated horizons thermodynamics*, Physical review letters 107 (2011), no. 24 241301.

[20] T. Jacobson, *A note on renormalization and black hole entropy in loop quantum gravity*, Classical and Quantum Gravity 24 (2007), no. 18 4875.

[21] M. Domagala and J. Lewandowski, *Black-hole entropy from quantum geometry*, Classical and Quantum Gravity 21 (2004), no. 22 5233.

[22] C. Rovelli, *Black hole entropy from loop quantum gravity*, Physical Review Letters 77 (1996), no. 16 3288.

[23] I. Agulló, E. F. Borja, J. Díaz-Polo, E. J. Villaseñor, et al., *Combinatorics of the su (2) black hole entropy in loop quantum gravity*, Physical Review D 80 (2009), no. 8 084006.

[24] S. Kloster, J. Brannlund, and A. DeBenedictis, *Phase space and black-hole entropy of higher genus horizons in loop quantum gravity*, Classical and Quantum Gravity 25 (2008), no. 6 065008.

[25] R. M. Wald, *Black hole entropy is the noether charge*, Physical Review D 48 (1993), no. 8 R3427.

[26] T. Clunan, S. F. Ross, and D. J. Smith, *On gauss–bonnet black hole entropy*, Classical and Quantum Gravity 21 (2004), no. 14 3447.
[27] D. N. Vollick, *Noether charge and black hole entropy in modified theories of gravity*, Physical Review D **76** (2007), no. 12 124001.

[28] A. Das, *Finite temperature field theory*, vol. 16. World Scientific, 1997.

[29] J. I. Kapusta, *Finite-temperature field theory*. Cambridge University Press, 1993.

[30] H. C. Ohanian and R. Ruffini, *Gravitation and spacetime*. Cambridge University Press, 2013.

[31] S. W. Hawking, *The large scale structure of space-time*, vol. 1. Cambridge university press, 1973.

[32] V. Mukhanov and S. Winitzki, *Introduction to quantum effects in gravity*. Cambridge University Press, 2007.

[33] V. Mukhanov and S. Winitzki, “Introduction to Quantum Fields in Classical Backgrounds.” This is a draft version of [32], 2004.

[34] L. C. Crispino, A. Higuchi, and G. E. Matsas, *The unruh effect and its applications*, Reviews of Modern Physics **80** (2008), no. 3 787.

[35] L. Susskind and J. Lindesay, *An introduction to black holes, information and the string theory revolution*. World Scientific, 2005.

[36] S. Emelyanov, *Local thermal observables in spatially open frw spaces*, arXiv preprint arXiv:1406.3360 (2014).

[37] S. Emelyanov, *Freely moving observer in (quasi) anti–de sitter space*, Physical Review D **90** (2014), no. 4 044039.

[38] S. M. Carroll, *Spacetime and geometry. An introduction to general relativity*, vol. 1. 2004.

[39] S. W. Hawking, *Particle creation by black holes*, Communications in mathematical physics **43** (1975), no. 3 199–220.

[40] G. t Hooft, *On the quantum structure of a black hole*, Nuclear Physics B **256** (1985) 727–745.

[41] P. Davies and S. Fulling, *Radiation from moving mirrors and from black holes*, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **356** (1977), no. 1685 237–257.

[42] S. Fulling and P. Davies, *Radiation from a moving mirror in two dimensional space-time: conformal anomaly*, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **348** (1976), no. 1654 393–414.

[43] J. D. Bekenstein, *Do we understand black hole entropy?*, arXiv preprint gr-qc/9409015 (1994).

[44] D. Christodoulou, *Reversible and irreversible transformations in black-hole physics*, Physical Review Letters **25** (1970), no. 22 1596.

[45] R. Penrose and R. Floyd, *Extraction of rotational energy from a black hole*, Nature **229** (1971), no. 6 177–179.

[46] S. W. Hawking, *Gravitational radiation from colliding black holes*, Physical Review Letters **26** (1971) 1344–1346.

[47] S. W. Hawking, R. Penrose, and M. Atiyah, *The nature of space and time*. Princeton University Press Princeton, 1996.
[48] L. Bombelli, R. K. Koul, J. Lee, and R. D. Sorkin, *Quantum source of entropy for black holes*, Physical Review D 34 (1986), no. 2 373.

[49] L. Susskind and J. Uglum, *Black hole entropy in canonical quantum gravity and superstring theory*, Physical Review D 50 (1994), no. 4 2700.

[50] T. Jacobson, *Black hole entropy and induced gravity*, arXiv preprint gr-qc/9404039 (1994).

[51] S. Emelyanov, *Non-unitarity or hidden observables?*, arXiv preprint arXiv:1410.6149 (2014).

[52] W.-S. Dai and M. Xie, *The number of eigenstates: counting function and heat kernel*, Journal of High Energy Physics 2009 (2009), no. 02 033.

[53] W.-S. Dai and M. Xie, *An approach for the calculation of one-loop effective actions, vacuum energies, and spectral counting functions*, Journal of High Energy Physics 2010 (2010), no. 6 1–29.