Charged lepton spectra from hot-spot evaporation

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Abstract

Spectra for the emission of charged leptons from evaporating hot-spots of preconfining phase in SU(2) Yang-Mills thermodynamics are computed. Specifically, we consider charged single and dileptons with their spectra being functions of energy and invariant mass, respectively. In the former case, our results relate to narrow and correlated electron and positron peaks measured in supercritical heavy-ion collisions performed at GSI in the 1980ies. In the latter case, we point out how strongly the spectra depend on typical kinematic cuts (CDF analysis of Tevatron Run II data). We also propose a scenario on how muon events of anomalously high multiplicity and large impact-parameter modulus arise in the Tevatron data.
1 Introduction

Pure SU(2) Yang-Mills thermodynamics possesses a narrow preconfining phase characterized by condensed monopole-antimonopole pairs. The latter originate as isolated defects through (anti)caloron dissociation in the deconfining phase at higher temperatures \[T_H\]. Shortly above the critical temperature \(T_H\) for the Hagedorn transition towards the confining phase the preconfining thermodynamics of the spatially infinitely extended gauge-theory system is dominated by this thermal ground state: The only propagating massive, dual gauge mode decouples, that is, acquires a very large Meissner mass. This domination leads to negative pressure of order \(-T_H^4\). Once temperature falls below \(T_H\) the infinitely extended monopole-antimonopole condensate decays nonthermally into single and selfintersecting center-vortex loops (CVLs) which are interpreted as spin-1/2 fermions \[1, 3, 4, 5\]. Only one-fold selfintersecting CVLs are absolutely stable in an SU(2) Yang-Mills theory, see \[5, 6, 7\]. Thus it is these fermions that are associated with charged leptons and their antiparticles. As a consequence, there is one SU(2) Yang-Mills theory for each lepton family with the respective Yang-Mills scale \(\Lambda\) matching the mass of the charged lepton: \(\Lambda_e \sim m_e, \Lambda_\mu \sim m_\mu, \Lambda_\tau \sim m_\tau\). This scenario, although at first sight ruled out by the apparent experimentally inferred pointlikeness of charged leptons, has predictive power under conditions where the creation of large-sized hot-spots of preconfining ground state is feasible. These conditions include very high energies in particle collisions \[4\], high temperatures in extended systems \[3\] and/or local energy densities exceeding \(m_e^4 \sim 10^{11}\) keV\(^4\). Recall that, according to SU(2) Yang-Mills thermodynamics, the apparent pointlikeness of leptons when probed by high-momentum transfer Standard-Model (SM) processes is a consequence of the presence of a large number of instable excitations in the interaction region. Their combined effect together with the free shiftability of vortex intersection points is efficiently and successfully described perturbatively by the SM. In this context, a succession of vertices and off-shell propagators (Feynman diagrams) mediates the transformation of structureless initial into the according final states.

Recently, anomalous, high-multiplicity muon events of symmetrically (positive-negative) distributed and power-like decaying (as opposed to exponentially cut-off) rate as a function of impact factor \(R\) were reported to occur in high-energy particle collisions \[8\]. These data are generated by protons and antiprotons colliding head-on at \(\sqrt{s} = 1.96\) TeV (Tevatron Run II). The invariant mass spectrum of dimuons created outside the vacuum of the beam pipe exhibits anomalous structure in the GeV region. Also, taking an additional muon into account, the according experimental rate is not explicable by any known SM processes. We are aware of the fact that the experimental situation in the case of anomalous multimuon events produced at the Tevatron as of yet is not clear cut and contested \[8, 9, 10\]. Motivated by existing experiments \[8, 9\], our theoretical dimuon spectra, obtained by investigating SU(2)\(_\mu\) hot-spot evaporation under certain kinematic cuts, are intended to offer guidelines for future analysis of collider date in the TeV realm.
We also consider single-particle spectra for $e^+, e^-$ emission from evaporating SU(2)$_e$ hot spots. Here the motivation arises due to the observation of universal narrow-peak structures in supercritical heavy-ion collisions at Gesellschaft für Schwerionenforschung Darmstadt (GSI) more than two decades ago.

1.1 Experimental situation: Multilepton events

*Electron and positron emission in supercritical heavy-ion collisions (GSI).* In supercritical heavy-ion collision systems such as U+Th, Th+Th, Th+Cm, U+U, U+Cm of combined nuclear charge $Z_u$ between 180 and 188 and thus well above criticality at $Z_{u,c} = 173$ the emission of narrow positron peaks of several ten keV width and center at 250 keV, 330 keV, and 400 keV was detected [11, 12, 13]. These narrow peaks sit on a broad background which is explained by conventional, parameter-free theory. Later the correlated emission of narrow peaks of electrons with similar characteristics was observed [13]. Both signatures decisively were ruled out to be due to any nuclear reactions when bombarding energies near the Coulomb barrier are applied. The emergence of these narrow peaks was theoretically approached by postulating large delay times of the order $3 \times 10^{-21}$ s which, however, are hard to reconcile with the smaller average delay time in deep inelastic reactions, see [11] and references therein.

The width of the narrow $e^+, e^-$ peaks in sum energy is narrower than that of the single positron lines. Also, from an analysis of Doppler broadening one concludes that the emitting source moves very slowly ($v/c \leq 0.05$). At the same time, compared to the situation of a weak constraint on opening angle (between 40 and 170 degrees) no sizable detector activity was seen in the search for correlated back-to-back emission of electrons and positrons. The combination of the latter two signatures excludes the *pair* (two-particle) decay of a neutral particle state in the mass range between 1.5 MeV to 2 MeV which from an analysis of the invariant mass spectrum alone would not be excluded!

Furthermore, the prediction of a strong dependence of peak energies and intensities on the combined nuclear charge $Z_u$, derived from the hypothesis of a long-lived, giant dinuclear complex needed for spontaneous pair creation$^1$, does not match with the experimental data. Rather, the data point to an independence of the peak features on $Z_u$. This universality is a puzzle in conventional strong-field QED which predicts strong power laws in $Z_u$.

Considering all signatures mentioned above (large delay times $\rightarrow$ formation of spatial regions with negative pressure pointing to the existence of droplets of pre-confining ground state of an SU(2) Yang-Mills theory of scale $T_H \sim m_e = 511$ keV; no pair decay but still a heavy neutral primary $\rightarrow$ droplets of energy content several times $m_e$ evaporating isotropically; universality of peak energies and widths in single

$^1$Strong-field QED, local decay of the QED vacuum: occupation of the 1s state of binding energy (use of natural units $c = \hbar = k_B = 1$) $E_b \sim -2m_e$ by a state of equal energy transferred from the Dirac sea.
positron and pair spectra → again, new phase of matter, preconfining-phase droplets whose decay is insensitive to the physics of their creation) we tend to attribute them to the formation and subsequent evaporation of SU(2)\(_e\) preconfining-phase hot-spots whose one-particle thermal emission spectrum is simply computed, see Sec. 2.

Let us now estimate the typical energy density associated with the Coulomb fields at closest approach of the two nuclei. Notice that the spontaneous creation of positrons in strong-field QED considered in explaining the anomalously high yield requires electronic binding energies \(E_b \sim -2 m_e\). Since \(Z_u \alpha \sim 1\) the associated Bohr radius \(a_0\) is \(a_0 \sim |E_b|^{-1}\), and the maximal energy density \(\rho_p\) in supercritical heavy-ion collision is \(\rho_p \sim a_0^{-4} \sim 10 m_e^4\) which is larger (but not hierarchically larger!) than what is minimally needed to traverse the Hagedorn transition in SU(2)\(_e\). Thus preconfining-phase hot-spots likely are generated in the form of small droplets (spatial extent comparable with \(a_0\)).

In connection with the smallness of the droplets, an important question concerns the narrowness of the observed peaks. Notice that the observed width of the electron and/or positron peaks corresponds to a lifetime of about \(10^{-20}\) s. This, however, is the predicted lifetime for SU(2)\(_e\) preconfining-phase hot-spot evaporation (with a slow dependence on the deposited energy), see Eq. (10) below. If the energy deposited during the formation of a hot-spot is larger but comparable to the peak energy of the single-particle spectrum for thermal electron or positron emission\(^2\) then the hot-spot and its decay signatures must be interpreted quantum mechanically. The droplet’s thus computed lifetime\(^4\), however, still should coincide with the inverse width seen in the experimental decay spectra. From the fact that the observed peaks are narrow in contrast to what is seen in the computed spectra we would again conclude that the experimental hot-spots are small droplets with a position uncertainty characterized by typical recoils. That is, their mass is only several times larger than \(m_e\). This is compatible with the argument on available energy density for hot-spot creation given above.

For the case, where the deposited energy in hot-spot creation would exceed \(m_e\) by many orders of magnitude, the observed peaks should broaden, and asymptotically the experimental one-particle spectra should match those computed in Sec. 2.

**Multimuon events at large impact parameter (Tevatron Run II).** For anomalous multimuon events ignited by proton-antiproton primary collisions at \(\sqrt{s} = 1.96\) TeV (Tevatron Run II, total integrated luminosity \(2100\) pb\(^{-1}\)) and analyzed by the CDF collaboration (however, not verified by DO\(^9\) who seem to apply a much tighter constraint on the range of investigated impact parameter\(^10\)), we are interested in the large number of dimuon events originating outside the beam pipe. Namely, an anomalous rate of dimuons, nearly independent of the sign of the impact parameter \(R\) was detected for \(|R| > 1.5\) cm = \(|R_{BP}|\). Notice that known QCD processes account

\(^2\)By thermal we mean that the hot-spot is considered sufficiently large to not wobble about indeterministically. That is, one can assume a smoothly shrinking spherical surface due to recoil-free evaporation.
for the dimuons created within the vacuum of the beam pipe while the counts for
dimuons originating from vertices with \(|R| > |R_{BP}|\) are in excess to SM predictions.

The motivation and precise implementation philosophy of Ref. \[8\] in imposing
certain kinematic cuts during data analysis remains largely obscure to the present
authors. In part this is explicable by a profound lack of experience in addressing
the complex data situation posed by TeV range primary collisions. Nevertheless,
we would like to point out what our evaporating hot-spot model predicts about the
shape of the dimuon invariant mass spectrum at large impact-parameter modulus,
see below, when applying typical kinematic constraints chosen by the experimental-
ists.

In \[8\] a subset of the Run-II data, subject to certain kinematic constraints in
transverse momentum and opening angle (integrated luminosity now only 742 pb\(^{-1}\))
was analyzed, and so-called ghost events were isolated. These events share the
following features: neither the approximate symmetry under \(R \rightarrow -R\) and the
decay of the rate distribution of multimuon events with increasing \(|R|\), nor the large
magnitude of the rate of events containing two or more muons for \(|R| > |R_{BP}|\), nor
the distribution of rate for a given charge composition of dimuon events in invariant
mass is explained by known SM processes. Probably depending on the imposed
kinematic cuts \((p_T \geq 2 \text{ GeV})\) the measured invariant mass spectra of dimuons at
large \(|R|\) typically have large weight in the GeV region. To link this to our SU(2)\(_\mu\)
approach, we will in Sec.\[3\] present a number of theoretical results on dimuon spectra
based on a single evaporating SU(2)\(_\mu\) hot-spot when varying kinematic cuts.

Since such a calculation unrealistically assumes an impact-parameter indepen-
dence of the process creating the hot-spot and since it ignores the physics of prereq-
usite processes neither the impact-parameter dependence nor the overall normaliza-
tion of the dimuon spectra are predicted here. For a prediction of these two features
additional deliberations and assumptions must be made. It is clear, however, that a
preconfining SU(2)\(_\mu\) hot-spot of total mass \(\sim 1 \text{ TeV}\) is almost at rest and thus emits
muons \textit{isotropically} explaining the observed approximate symmetry under \(R \rightarrow -R\)
in the impact-parameter distribution of the experimental yields. (Emission towards
the beam pipe is as likely as away from it at given distance.)

We tend to believe that the reason why QCD predictions and experimental data
agree well for events originating at \(|R| < |R_{BP}|\) is the absence of heavy nuclei whose
strong electric fields trigger hot-spot creation assisted by the rare TeV photons of
electromagnetic \(p\bar{p}\) annihilation. Outside the beam pipe the detector material pro-
vides for these nuclei. Assuming the probability for the conversion of a TeV photon
into an SU(2)\(_\mu\) hot-spot inside the Coulomb potential of a heavy nucleus to be suffi-
ciently small\[3\], the almost unadulterated total photon flux decreases (geometrically)
in a power-like way as a function of \(R\) while the evaporation physics is independent

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\(3\)In principle, this is calculable under specific assumptions. For example, one could presume that
inside the Coulomb potential of the nucleus the incoming photon perturbatively creates a single pair
of TeV dimuons which in turn originate a perturbative cascade producing an ever increasing number
of low-energy muons until their number density reaches criticality for the Hagedorn transition.

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of \( R \). This would explain the slow, power-like decay of multimuon activity at large \( |R| \).

This article is organized as follows: In the next section we compute the spectrum for emission of single charged leptons, arising from evaporation of static SU(2) hot-spots, as a function of lepton energy. Specifically for SU(2), we also determine hot-spot life times in dependence of the deposited energy and observe agreement with the inverse width of the narrow peaks detected at GSI. In Sec. 3 we compute the spectra for the emission of charged dileptons from the evaporation of static SU(2) hot-spots as a function of the pair’s invariant mass and under various kinematic cuts. As a result, these spectra sensitively depend on the kinematic cuts applied. Sec. 4 discusses our results in view of present and future analysis of TeV-range collider data.

2 Single lepton spectrum from hot-spot evaporation

For temperatures just above \( T_H \) the SU(2) Yang-Mills system is ground-state dominated. Once the creation of a bubble of radius \( r_0 \) containing this new phase of matter has taken place its evaporation is simply determined by the content of stable, final particle species (number of selfintersections in center-vortex loop, spin, and charge), energy conservation, and the value of the Hagedorn temperature \( T_H \).

Since we are interested in leptons \( l \) far away from the emitting hot-spot surface we can address the problem of calculating their spectra thermodynamically setting \( T = T_H \equiv \frac{1.24}{2\pi} y m_{l=1} \) (\( l = 1 \) is the charged lepton state of the SM) where \( y \) is of order unity expressing a theoretical uncertainty. In the following we strongly appeal to the results of [4].

Leaving the angular integration in Eq. (7) of [4] explicit and setting \( l = 1 \), the number \( \eta_{l=1} \) of charged leptons emitted per unit time and surface reads

\[
\eta_{l=1} = \int_0^\infty dp \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \frac{M_{l=1}}{8\pi^3 \sqrt{p^2 + m_{l=1}^2} e^{\sqrt{p^2 + m_{l=1}^2}/T_H + 1}},
\]

where \( M_{l=1} = 2 \times 2 = 4 \) (2-fold spin degeneracy, two-fold charge degeneracy), \( p = |\mathbf{p}| \) is the modulus of the spatial on-shell momentum of a single charged lepton far away

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4 Microscopically, Lambert’s law states that at a given point of the emitting surface only the momentum component perpendicular to this surface gets Fermi-distributed at \( T_H \) far away from the hot-spot system and thus is measurable. Thus the factor 1/4 in the first half of Eq. (7) of [4] actually should be replaced by unity.

5 Because of the large reservoir of to-be-emitted charged leptons in a hot-spot and the assumed absence of external fields the evaporation physics is charge and spin-orientation blind. In reality, this can not entirely be true because of a possible fluctuating, CP violating axion field (whose homogeneous incarnation is of cosmological relevance [10]) that slightly prefers the emission of negative over positive charge. However, for the total yields this effect is negligible.
from the emitting surface, and \( m_{l=1} \) can be set equal to \( m_e = 511 \text{ keV} \) for SU(2)_e and \( m_\mu = 105.6 \text{ MeV} \) for SU(2)_\mu. From Eq. (1) we have for the differential yield \( \frac{d\eta_{l=1}}{dp} \) per unit on-shell momentum, time and surface

\[
\frac{d\eta_{l=1}}{dp} = \frac{2}{\pi^2 \sqrt{p^2 + m_{l=1}^2}} \frac{p^3}{e^{\sqrt{p^2 + m_{l=1}^2}/T_H} + 1},
\]

or alternatively, the differential yield \( \frac{d\eta_{l=1}}{dE} \) per unit on-shell energy \( E \), time and surface

\[
\frac{d\eta_{l=1}}{dE} = \frac{2}{\pi^2} \frac{(E^2 - m_{l=1}^2)}{e^{E/T_H} + 1}.
\]

To obtain the differential number \( dN_{l=1}/dE \) of emitted charged leptons per bin of energy we multiply \( \frac{d\eta_{l=1}}{dE} \) by the actual surface \( 4\pi r(t)^2 \) of the spherical hot-spot and subsequently integrate over time \( t \) from zero to \( t_{ev} \) [4]:

\[
\frac{dN_{l=1}}{dE} = \left( 4\pi \int_0^{t_{ev}} dt \, r(t)^2 \right) \frac{d\eta_{l=1}}{dE}.
\]

According to Eq. (5) of [4] one has

\[
r(t) = r_0 - \frac{J(T_H)}{\rho_H} t,
\]

and according to Eq. (6) of [4]

\[
t_{ev} = \frac{\rho_H}{J(T_H)} r_0,
\]

where

\[
\rho_H = 22.48 \, y^4 m_{l=1}^4, \quad r_0 = \left( \frac{3}{4\pi} \frac{\delta \sqrt{s}}{\rho_H} \right)^{1/3}, \quad J(T_H) = J_{l=0}(T_H) + J_{l=1}(T_H),
\]

and

\[
J_l(T_H) = M_l \int_0^{\infty} dp \frac{1}{2\pi^2} \frac{p^3}{e^{p^2 + m_{l=1}^2}/T_H + 1}.
\]

In Eq. (8) one has \( m_0 = 0 \) and \( M_0 = 2 \) (neutrinos are Majorana), in Eq. (11) \( \delta \sqrt{s} \) represents the fraction of c.m. energy going into hot-spot creation (\( \delta \) of order unity), and \( y \equiv \frac{\Lambda}{m_{l=1}} \) where \( \Lambda \) is the Yang-Mills scale of the SU(2) gauge theory. This implies \( T_H = \frac{11.29}{2\pi} y m_{l=1} \), see for example [16].

The typical inverse width

\[
\Gamma^{-1} \sim \frac{1}{70 \text{ keV}} = 9.4 \times 10^{-21} \text{ s}
\]

of single electron or positron peaks seen in supercritical heavy-ion collisions [11] compares well with the life time \( t_{ev} \) of hot-spots belonging to SU(2)_e.
calculated by assuming recoil-free, that is, thermal emission. Recall that by Eqs. (6) and (7) $t_{ev}$ varies rather weakly with $\sqrt{s}$. Setting $\delta = y = 1$, we have

\[
\sqrt{s} = 10 \text{ MeV} \Rightarrow t_{ev} = 1.0 \times 10^{-21} \text{ s}; \quad \sqrt{s} = 100 \text{ MeV} \Rightarrow t_{ev} = 2.15 \times 10^{-21} \text{ s}; \\
\sqrt{s} = 1 \text{ GeV} \Rightarrow t_{ev} = 4.63 \times 10^{-21} \text{ s}; \quad \sqrt{s} = 10 \text{ GeV} \Rightarrow t_{ev} = 1.0 \times 10^{-20} \text{ s}; \\
\sqrt{s} = 100 \text{ GeV} \Rightarrow t_{ev} = 2.15 \times 10^{-20} \text{ s}; \quad \sqrt{s} = 1 \text{ TeV} \Rightarrow t_{ev} = 4.63 \times 10^{-20} \text{ s}.
\]

(10)

Notice that according to Eq. (4) the quantities $\delta$ and $\sqrt{s}$ enter only into the normalization of the spectrum and not into its shape. For reasons of better interpretability it is advantageous to factor out the dimensionful quantity $c_{l=1;y,\delta} = \frac{1}{\gamma y} m_{l=1}^{-1}$ where $c_{l=1;y,\delta}$ denotes the number obtained by rescaling $E$ and $T_H$ by $m_{l=1}^{-1}$ in the first factor of Eq. (4). Explicitly, we have

\[
c_{l=1:y,\delta} = \pi^2 \delta \sqrt{\frac{s}{m_{l=1}^2}} \left[ \int_0^\infty d\tilde{x} x^3 \left( \frac{1}{e^{\gamma \tilde{x}} + 1} + \frac{2}{e^{\gamma \sqrt{\tilde{x}^2 + 1}} + 1} \right) \right]^{-1},
\]

(11)

where $\gamma \equiv \frac{2\pi}{11.24 y}$. For $\text{SU}(2)_e$ and $\sqrt{s} = 10 \text{ MeV}$ this yields

\[
c_{l=1:y=1,\delta=1/2} = 0.569661, \quad c_{l=1:y=1,\delta=1} = 1.13932, \\
c_{l=1:y=1/2,\delta=1/2} = 9.88092, \quad c_{l=1:y=1/2,\delta=1} = 19.7618.
\]

(12)

The remaining factor in Eq. (4) then is a dimensionless function $S_{l=1}$ of its dimensionless arguments $x = E/m_{l=1}$ and $y$:

\[
\frac{dN_{l=1}}{dE} = c_{l=1:y,\delta} m_{l=1}^{-1} \times S_{l=1}(x, y).
\]

(13)

In Fig. 1 the function $S_{l=1}(x, y)$ is plotted in dependence of $x$ for $y = 1/2$ and $y = 1$. Notice the broad spectral shape of $S_{l=1}$. As mentioned in Sec. 1.1, this is a consequence of our assumption that the hot-spot can be treated as a classical, recoil-free object during the entire history of its thermal evaporation. Obviously, this assumption breaks down when the mass of the hot-spot becomes comparable to $m_{l=1}$: The emission of charged leptons then is subject to a quantum mechanical decay with the width of the latter still being described by the life time computed from recoil-free, thermal emission.

For the example of $\text{SU}(2)_e$, at $\sqrt{s} = 10 \text{ MeV}$ and at $\delta = 1$, Eq. (12) and Fig. 1 tell us that the number $N_{l=1}^{\text{max}}$ of single positrons and electrons per energy bin of $m_e = 511 \text{ keV}$ and emitted at the spectral maximum is

\[
N_{l=1}^{\text{max}}(\delta = 1, y = 1) = 1.14 \times 0.3 = 0.342, \\
N_{l=1}^{\text{max}}(\delta = 1, y = 1/2) = 19.8 \times 0.064 = 1.27.
\]

(14)

Appealing to the number of counts per energy bin in the experimental data [12, 13, 14], the numbers in Eq. (14) can be used to obtain an estimate for the total number of hot spots created in supercritical heavy-ion collisions at given integrated luminosity.
Figure 1: Dimensionless one-particle spectral shapes $S_{l=1}$ for the emission of charged leptons of dimensionless energy $x = E/m_{l=1}$ due to SU(2) hot-spot evaporation for $y = 1/2$ (solid) and $y = 1$ (dashed).

3 Dilepton spectrum from hot-spot evaporation

Let us now derive predictions for the shape of the invariant mass spectrum of dileptons emitted from the surface of an evaporating hot-spot. Under certain kinematic constraints this is the situation relevant to the analysis presented in [8]. Considering a charged lepton far away from the hot-spot surface and thus in thermal equilibrium, it is reasonable to assume that it is not temporally and spatially correlated to another charged lepton emerging from the same hot-spot. Moreover, it is experimentally impossible to resolve the hot-spot spatially or temporally at presently available energies. Thus, the entire hot-spot evaporation is seen as an instant of extremely high-multiplicity emission of charged leptons emerging from a point.

Having said the above, it is clear that the number of charged lepton pairs $dN_{2(l=1)}$ with their members in momentum bins $dp_1$ and $dp_2$ and angular bins $d\phi_1$, $d\phi_2$,

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6For electrons/positrons and $\sqrt{s} \sim 1 \text{TeV}$ the evaporation time is $\sim 10^{-19}$ s and the initial radius $10^{-10}$ m [4]. For muons/antimuons and comparable values of $\sqrt{s}$ these time and distance scales are even smaller. Since the CDF II single-hit resolution is $10^{-6}$ m an evaporating hot-spot acts like a pointlike vertex experimentally.
\[ d \cos \theta_1, \text{ and } d \cos \theta_2 \text{ reads:} \]

\[ dN_{2(t=1)} = \left( 4\pi \int_0^{t_{ev}} dt \ r(t)^2 \right)^2 \times \frac{dp_1 \ dp_2 \ d \cos \theta_1 \ d \cos \theta_2 \ d \varphi_2 \times}{4\pi^6 \sqrt{p_1^2 + m_{i=1}^2} e^{\sqrt{p_1^2 + m_{i=1}^2}/T_{ti}} + 1 \sqrt{p_2^2 + m_{i=1}^2} e^{\sqrt{p_2^2 + m_{i=1}^2}/T_{ti}} + 1}. \] (15)

For the invariant mass \( M \) of a charged lepton pair we have

\[ M^2 = 2(m_{i=1}^2 + p_1^2 \sqrt{m_{i=1}^2 + p_2^2} - p_1 p_2 \cos \theta), \] (16)

where \( \theta \equiv \angle(p_1, p_2) \). Solving Eq. (16) for \( p_1 \), the only acceptable solution is

\[ p_1(M, p_2, \cos \theta) = \frac{(M^2 - 2m_{i=1}^2)p_2 \cos \theta + \sqrt{(m_{i=1}^2 + p_2^2)(M^4 - 4M^2m_{i=1}^2 - 4(1 - \cos^2 \theta)m_{i=1}^2 p_2^2)}}{2(m_{i=1}^2 + (1 - \cos^2 \theta)p_2^2)}. \] (17)

Moreover, we have

\[ \cos \theta = \cos \theta(\theta_1, \theta_2, \varphi_1, \varphi_2) = \frac{P_1 \cdot P_2}{p_1 p_2}. \] (18)

Appealing to Eqs. (15), (17), and (18), we have for the number \( dN_{2(t=1)} \) of lepton pairs per invariant mass bin \( dM \)

\[ \frac{dN_{2(t=1)}}{dM} = \left( 4\pi \int_0^{t_{ev}} dt \ r(t)^2 \right)^2 \times \int dp_2 \frac{dp_1}{dM} \ d \cos \theta_1 \ d \cos \theta_2 \ d \varphi_2 \times \frac{1}{4\pi^6 \sqrt{p_1^2 + m_{i=1}^2} e^{\sqrt{p_1^2 + m_{i=1}^2}/T_{ti}} + 1 \sqrt{p_2^2 + m_{i=1}^2} e^{\sqrt{p_2^2 + m_{i=1}^2}/T_{ti}} + 1}, \] (19)

where the integration over \( p_2, \theta_i, \text{ and } \varphi_i \) is subject to kinematic constraints. For example, towards the infrared one may cut off the modulus of each lepton’s momentum component \( p_{1,1} \) (\( i = 1, 2 \)) in the plane transverse to the beam direction. Moreover, the opening angle \( \theta \) of the lepton pair may be constrained by \( 1 \geq \cos \theta \geq \cos \theta_0 \) where \( \cos \theta_0 \) is a fixed number between 1 and -1. According to Eq. (18) the kinematic constraint on \( \cos \theta \) implies constraints on \( \theta_i \) and on \( \varphi_i \).

As in Sec. 2 it is advantageous to rescale \( p_2, M, \text{ and } T_{ti} \) by \( m_{i=1}^{-1} \) in Eq. (19) and to factor the result in analogy to Eq. (13) into the following form

\[ \frac{dN_{2(t=1)}}{dM} = c_{2(t=1);y,\delta} \ m_{i=1}^{-1} \times S_{2(t=1)}(X, y), \] (20)
Figure 2: Plots of the function $S_{2(l=1)}(X, y)$ (dimensionless spectral distribution in invariant mass of charged dilepton events) at $\cos \theta_0 = -1; 0.8$, $\pi_{1,\perp} = \pi_{2,\perp} = 0; 9.47$, and $y = 1/2; 1$ as a function of $X$.

where $c_{2(l=1);y,\delta} \equiv c_{l=1; y, \delta}^2$ and $X \equiv \frac{M}{m_{l=1}}$. Constraints on the transverse momentum are then expressed in terms of the dimensionless version $\pi_{i,\perp} \equiv \frac{p_i}{m_{l=1}}$. The integral associated with $S_{2(l=1)}(X, y)$ is performed using Monte Carlo methods.

In Fig. 2 plots of $S_{2(l=1)}(X, y)$ are presented as a function of $X$ at $y = 1/2$ and $y = 1$, for $\cos \theta_0 = -1; 0.8$ (latter cut chosen by CDF [8]), and $\pi_{1,\perp} = \pi_{2,\perp} = 0; 9.47$ (latter cut corresponding to a representative 1 GeV infrared cut on both transverse momenta in the muon pair analyzed by CDF). Notice how severely the kinematic cuts on $\pi_{i,\perp}$ and $\cos \theta_0$ influence the spectral shape in view of the localization of maxima and their widths. Notice also that for SU(2)$_\mu$ the unconstrained case ($\pi_{i,\perp} = 0$ and $\cos \theta_0 = -1$) corresponds to a spectral maximum of about 0.6 GeV while for $\pi_{i,\perp} = 9.47$ and $\cos \theta_0 = -1$ the maximum shifts to about 2.3 GeV.

Setting $\sqrt{s} = 1$ TeV (a TeV photon generated at the Tevatron and depositing its energy into an SU(2)$_\mu$ hot-spot by virtue of the CDF detector material) and $m_{l=1} = m_\mu = 105.6$ MeV, one has

$$c_{2(l=1);y=1,\delta=1/2} = 7.6 \times 10^4; \quad c_{2(l=1);y=1,\delta=1} = 3.04 \times 10^5;$$
$$c_{2(l=1);y=1/2,\delta=1/2} = 2.29 \times 10^7; \quad c_{2(l=1);y=1/2,\delta=1} = 9.14 \times 10^7. \quad (21)$$

For example, considering SU(2)$_\mu$, $\sqrt{s} = 1$ TeV, and $\delta = 1$ Eq. (21) and Fig. 2 imply
that the number $N_{\text{max}}^{2(l=1)}$ of muon pairs (regardless of the charge of their participants) emitted at the spectral maximum per invariant-mass bin of $m_\mu$ is

$$
N_{2(l=1)}^{\text{max}}(\delta = 1, y = 1, \pi_{i,\perp} = 0, \cos \theta_0 = -1) = 3.04 \times 10^5 \times 0.44 = 1.34 \times 10^5, \\
N_{2(l=1)}^{\text{max}}(\delta = 1, y = 1, \pi_{i,\perp} = 0, \cos \theta_0 = 0.8) = 3.04 \times 10^5 \times 0.07 = 2.13 \times 10^4, \\
N_{2(l=1)}^{\text{max}}(\delta = 1, y = 1, \pi_{i,\perp} = 9.47, \cos \theta_0 = 0.8) = 3.04 \times 10^5 \times 4 \times 10^{-4} = 122.
$$

A comparison of Eq. (22) with the experimental counts of dimuon events per bin in invariant mass [8] can be used to obtain an estimate for the total number of hot-spots created within a given value of total integrated luminosity in TeV-range collider experiments.

4 Summary and Conclusions

Concerning the data analysis in TeV-range collider experiments, the lesson of the present article is that certain prejudices (kinematic cuts) applied to experimental data in extrapolating known physics (the SM) into the unknown necessarily hides potentially important signatures. The possibility that metastable hot-spots of a new phase of matter are not only created in ultrarelativistic heavy-ion collisions (RHIC), where due to Quantum Chromodynamics they are expected, but also in isolated, TeV-range proton-antiproton or proton-proton collisions suggests a scenario completely different from the perturbative SM approach for the deposition of center-of-mass energy $\sqrt{s}$ into collision products. Rather than creating secondaries of energy comparable to $\sqrt{s}$, energy would then be re-distributed on a large multiplicity of charged, low-energy leptons stemming from hot-spot evaporation. Because of the missing confinement mechanism in products of SU(2) Yang-Mills theories with equal electric-magnetic parity [1] it is clear that only the leptonic sector exhibits these large multiplicities explicitly. In view of the large data stream that will be generated by the LHC we hope that the present work will contribute to the process of overcoming these prejudices.

An exception to this rule is the deposition of $\sqrt{s}$ into the mass of intermediate vector bosons propagating in the preconfining phase of SU(2) Yang-Mills theory. A genuine prediction here is that two more but much heavier copies of vector-boson triplets must eventually be excited. For SU(2)$_\mu$ the mass of the neutral boson $Z'$ is hierarchically larger than $m_\mu$ (since this vector mode decouples at $T_H \sim m_\mu$). In this context, it is worth pointing out the potential resonance peak at $M \sim 240$ GeV (hierarchy $M/m_\mu \sim 2 \times 10^3$) seen in the $e^+e^-$ invariant mass spectrum generated at Tevatron Run II and reported by both collaborations CDF [17] and D0 [18]. Notice that resonances in this channel are practically free of hadronic contaminations. If true then this resonance is a plausible candidate of the decoupling dual gauge-field mode $Z'$ of SU(2)$_\mu$ [1].
Concerning the interpretation of narrow-width and correlated single electron or positron peaks in supercritical heavy-ion collisions, we believe that the present article has established a plausible connection to the creation of SU(2)$_e$ preconfining hot-spots and their subsequent evaporation.

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