The Importance of Few-Nucleon Physics at Low Energy

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Abstract

This manuscript originates from the discussion at the workshop on the "Future of Few-body Low Energy Experimental Physics" (FFLEEP), which was held at the University of Trento on December 4-7, 2002 and has been written in its present form on March 19, 2003. It illustrates a selection of theoretical advancements in the nuclear few-body problem, including two- and many-nucleon interactions, the three-nucleon bound and scattering system, the four-body problem, the A-body (A>4) problem, and fields of related interest, such as reactions of astrophysical relevance and few-neutron systems. Particular attention is called to the contradictory situation one experiences in this field: while theory is currently advancing and has the potential to inspire new experiments, the experimental activity is nevertheless rapidly phasing out. If such a trend will continue, advancements in this area will become critically difficult.
1 Introduction

In nuclear physics one studies interacting hadrons in the nonperturbative regime of quantum chromodynamics (QCD). This field faces two natural frontiers. The first one is devoted to the origin and the fundamentals of the strong force between the nucleons, as well as its parametrization in terms of effective degrees of freedom (d.o.f.), suitable for the nonperturbative regime. The other is the completely microscopic, quantum mechanical treatment of many-body systems (nuclei) in terms of these effective d.o.f. with the strong force as dynamical input. Within these two frontiers, few-nucleon physics at low energy represents quite a rich and flourishing research field describing a great variety of strong interaction phenomena. The particular role of this field for the whole of nuclear physics is illustrated and underlined by the following three statements:

- “Why few nucleons?” The relatively small number of particles allows accurate solutions of the quantum mechanical many-body problem without the need of approximations, necessary and unavoidable for more complex systems. Therefore the comparison of such theoretical results with experimental data of comparable accuracy can lead to conclusive statements with respect to the assumptions for the underlying nuclear dynamics on which the theory is based.

- “Why at low energy?” The main interest is to understand strong hadron dynamics in the nonperturbative regime in a well controlled nonrelativistic framework. Therefore observables at low energy and momentum transfers represent the best testing grounds. Furthermore, possible relativistic effects can be handled quite reliably by incorporating leading
order relativistic contributions consistently without the need of a completely covariant approach.

• “Why could it be flourishing?” It is a field of “low cost investments” with “high revenues” in terms of fundamental insight. The theoretical methods are accurate and under control, and experiments are considerably much less costly than is the case for high-energy observables.

In this note we would like to point out the major achievements of few-nucleon physics at low energy with respect to the two above mentioned frontiers, and what are the open problems at present. But this note is not intended to give a comprehensive review of the impressive progress which has been obtained in recent years in this field, rather a few, particularly illustrative examples have been selected in order to underline the punch line of this note, namely to prevent a complete shut-down of low-energy machines for the study of few-nucleon systems and to fuel a more substantial support of experiments using such machines. Besides illustrating the progress, the selected examples will point out important questions and open problems of fundamental nature that have to be addressed in the future and for which a continuation of ongoing experiments is mandatory at appropriate existing accelerator facilities as well as at new dedicated ones.

The note is organized as follows. Sec. 2 is devoted to the nature of the strong interaction and its underlying degrees of freedom. Present models of the nucleon-nucleon (NN) potential will briefly be reviewed as obtained by studying bound and scattering states of the two-body system. Electromagnetic interactions with the two-nucleon system provide further insight into the properties of those forces and will be discussed in this section, too. In particular, we will suggest possible future experiments needed for a more detailed analysis of the underlying hadron dynamics. Sec. 3 is devoted to three- and four-body systems with special emphasis on the three-nucleon force (3N-force). The specific role of three-body nuclei for analyzing models of the 3N-interaction is discussed in detail. Also the four-body system is still "light" enough allowing accurate microscopic calculations in order to shed additional light on the 3N-force as well as on possible manifestations of four-nucleon forces (4N-forces). Furthermore, its characteristics of large binding energy, large central density and “closed-shell” structure closely resembles heavier systems. Therefore, it can be considered as an instructive testing ground for many-body approaches, models and approximations. The accurate treatment of many-body systems larger than four will be addressed in Sec. 4. It will be pointed out how few-nucleon physics can contribute to understand the dynamics of more complex nuclei on a microscopic basis. In particular, the phenomena of clusterization and collective motion will be discussed. A few examples will be shown, where ab initio microscopic calculations of bound states and reactions in the continuum in \(A > 4\)-systems start showing typical cluster or collective features. Again the need of appropriate experiments to clarify these issues will be emphasized. In Sec. 5 the important role of few-body physics for other fields, like elementary particle physics and astrophysics, is described very briefly by pointing out the use of very light nuclei as effective neutron targets and giving a few examples of few-body reactions of astrophysical relevance. Finally, concluding remarks are given in Sec. 6.
\section{Study of the Hadronic Force in the Two-Body System}

The first step for developing a model for the strong nucleon-nucleon (NN) interaction is to study the two-nucleon system, its bound and scattering states. The bound state properties are well known and nowadays a wealth of NN-scattering data exist, accumulated over many years. These data serve as input for fitting models of the NN-interaction. In the early days of nuclear physics very simple and purely phenomenological models have been introduced. After the basic idea of Yukawa of mesons as mediators of the strong interaction more sophisticated models have been developed. In fact, for many years two-nucleon physics has dealt with the design of more and more refined NN-potentials, parametrizing the NN-force in terms of meson and nucleon degrees of freedom, which, with the advent of QCD, are now considered as effective only.

In order to construct such NN-potentials one needs as a first step a partial-wave analysis (PWA) of the scattering data. In recent years, new methods of energy-dependent PWA have made possible the accurate determination of the NN phase-shift and mixing parameters. In order to achieve a high-quality description of the presently available database, a sophisticated treatment of the long-range electromagnetic interaction is required. Also the one-pion exchange interaction needs to be included, as well as a model for the medium-range interaction (heavy-meson exchange or two-pion exchange).

At present, a variety of so-called high-precision NN-potentials is available which fit the NN-scattering data (available at the time of construction) with a $\chi^2$ per datum of about one. Thus they describe those NN-data with magnificent accuracy. Beyond the longest range one-pion-exchange (OPE) part, which all these potentials contain, the medium and short range region is parametrized either purely phenomenologically or semiphenomenologically by the exchange of heavier mesons and introduction of hadronic vertex form factors. These modern conventional NN-forces contain typically 40-50 parameters but describe NN scattering data with high precision up to 350 MeV laboratory energy. With respect to the short range region, also more microscopic models with quark and gluon d.o.f. have been introduced.

Besides these phenomenological and meson-theory based models, recently another approach has been put forward in the framework of effective field theory (EFT) using only nucleon and pion d.o.f. It is based on chiral perturbation theory and starts from the most general Lagrangean for pion and nucleon fields obeying spontaneously broken chiral symmetry of QCD. The nuclear forces are constructed via a systematic expansion with respect to a small scale parameter, which is given by the ratio of generic external momenta $Q$ and a mass scale $\Lambda$ of the order of the $\rho$-meson mass. For momenta below $\Lambda$, nuclear forces can be evaluated systematically in increasing order of that ratio at the cost of an increasing number of contact forces. In leading order (LO) there is the well established one-pion exchange supplemented by contact forces which subsume unknown short range physics and which are fixed by fitting low-energy observables. The strength factors of those contact forces are called low-energy constants (LEC’s). In the next to leading order (NLO) there are various two-pion exchange diagrams and additional contact forces with new unknown LEC’s. Even higher order forces with more LEC’s have been already worked out.

Up to now chiral NN-forces have been fitted to NN-scattering data within
NNLO, which amounts to fixing nine LEC’s. A good description of the standard and well established NN-phase shift parameters has been achieved up to about 100 MeV. In contrast to conventional high-precision NN-forces, which are not based on a systematic expansion, the fit is not perfect. In fact it makes no sense to expect high-quality fits at low chiral order. More precise fits are expected for higher order at the expense of more LEC’s. It remains to be seen up to which order one has to go for reaching the same precision and whether the number of LEC’s will then be comparable to the typically 40-50 parameters of the most modern conventional NN-forces. At present an interesting and lively debate between the traditional meson theory point of view and the EFT approach is going on and will presumably continue in the years to come where one important question is related to the model dependence or model error.

With respect to the experimental scattering data we would like to remark that the presently available database for $pp$ scattering is of good quality. However, for $np$ scattering the situation is not quite as good. New experiments, both $pp$ and $np$, should be aimed at improving the existing phase shifts. What new types of data are still useful is a question that can be answered in a quantitative manner by a collaborative effort of theorists and experimentalists, by using the existing partial-wave analysis as a benchmark.

Having determined the NN-force, one can proceed to study the theoretical predictions on the hadronic and electroweak structure of light nuclei. A central question is: how well do we understand those properties which were not used in fixing the parameters of the NN-interaction? In this sense, few-body nuclei serve as test laboratories for the study of a variety of hadronic and electroweak properties. A particularly well suited tool is provided by the electromagnetic (e.m.) probe, and in fact e.m. reactions on the two-nucleon system have allowed further insight into the structure of the hadronic force. The reason for this is that the e.m. interaction proceeds via the e.m. current of the hadronic system which requires the explicit knowledge of the d.o.f. underlying the interaction. In fact, the e.m. current has to be consistent with the Hamiltonian, because gauge invariance alone is not sufficient for its full determination.

For the two-body sector quite elaborate theoretical results are available describing a variety of e.m. processes on the deuteron like, e.g., static moments, elastic electron scattering, photo- and electrodisintegration and meson production. Present most advanced theories include consistent two-body meson exchange currents (MEC), isobar configurations and relativistic contributions of leading order in a $p/M_N$ expansion.

At low energy and momentum transfers, gross properties like, e.g., total photoabsorption cross section, elastic form factors and inclusive response functions are in satisfactory agreement with experimental data on the 5 percent level, and within this accuracy one finds almost no sensitivity to the theoretical input, i.e., to the realistic NN-interaction model and corresponding induced currents. For these gross properties, differences with respect to different interaction models appear on a much smaller scale, namely on the one percent level, requiring experimental data of at least the same accuracy which unfortunately do not exist. For deuteron photodisintegration a critical assessment of existing experimental data on various observables and a detailed comparison to the theory was given about ten years ago. Thus for a more detailed comparison between experiment and theory one desperately needs new
experimental data of considerably higher accuracy. This is particularly true for more exclusive observables like differential cross sections and polarization observables which show in certain kinematic regions a considerably higher sensitivity on the level of 10-20 % to the theoretical input. In particular, polarization observables bring often a much richer insight and stimulate in addition experimental interest due to technical challenges. Of special interest is also the transition between nonrelativistic and relativistic regime which appears in some observables already at quite low energies.

A few examples might be useful in order to illustrate the urgent need of precise experimental data:

(i) Total photodisintegration cross section and spin asymmetry. The latter determines the much disputed Gerasimov-Drell-Hearn sum rule and in fact, a very large negative contribution is expected very close to the break-up threshold because of the dominant magnetic transition to the $^1S_0$ scattering state which contributes only when photon and deuteron spins are opposite. In particular, a sizeable influence of relativistic contributions has been found in the spin asymmetry at low energies, say a few MeV above break-up threshold.

(ii) Nucleon polarization in $d(\gamma, N)N$ at low energy. There exist very few data which are of rather low accuracy. Also very few data on vector and tensor target asymmetries exist.

(iii) Exclusive electrodisintegration at low energy and momentum transfer $d(e, e'p)n$. Recent experimental results indicate a problem in the longitudinal-transverse interference structure function $f_{LT}$ which seem to be present also in the transverse one $f_T$. The latter, however, could not be separated from the longitudinal one in that experiment. It requires a Rosenbluth separation.

Finally, we would like to point out that NN bremsstrahlung is another very interesting electromagnetic reaction in the two-body sector, which only recently, with the advent of high-precision data for $pp$ bremsstrahlung, has become a good testing-ground for NN-interaction models. At present, theoretical models fail to describe the available $pp\gamma$ cross sections. The size of the discrepancy between theory and experiment is surprisingly large.

3 Many-Body-Forces in Three- and Four-Body Systems

An important new feature appears in few-nucleon systems with more than two particles: in the framework of presently known force models, these systems cannot be described using NN-forces only. It is necessary to include three-nucleon forces (3N-force). The origin and the explicit form of the 3N-force will be a central issue of future few-nucleon physics.

While the development of realistic NN-force models was possible thanks to a large amount of experimental NN-scattering data at a wide range of energies, the study of the 3N-force relies at present on a very small data basis only (binding energies, low-lying excited states of light nuclei, and a few reactions). It is clear that for a reliable and realistic 3N-force model, based on a deeper understanding of the underlying dynamics, the experimental data
set has to be enlarged considerably if one wants to reach the same degree of accuracy as in the NN case.

It is important to note that 3N-forces are not introduced in an ad hoc manner. Meson exchange theory as well as EFT and a more fundamental view in terms of quark-gluon dynamics naturally suggest that additional many-body forces between nucleons exist which cannot be reduced to pair-wise NN-interactions. Three-nucleon forces were proposed already in the thirties. Much later, first quantitative studies were carried out, where an important 3N-force contribution, arising from the intermediate excitation of a nucleon into a $\Delta$ in a two-$\pi$-exchange, was included in the three-nucleon ground state. It became clear that such effects cannot be neglected for the three-nucleon binding.

Without theoretical guidance, however, the phenomenological approach towards 3N-force is rather hopeless. In contrast to NN-forces, where only a few basic operators can contribute, the number of possible three-body operators is prohibitive, leading to a tremendously large number of terms, which moreover are accompanied by unknown scalar functions depending on four relative momenta.

Although the natural testing ground for the 3N-force is the three-nucleon system it turns out that some scattering observables over a broad range of energy show little sensitivity to the NN-force model choice or the inclusion of the 3N-force. This creates additional requirements for precision data needed to validate ongoing theoretical research. As will be shown below four-nucleon forces seem to be very small and thus also the four-nucleon system can play an important role in the investigation of the 3N-force. In addition, as will be pointed out in Sec. 4 important additional information on the 3N-force comes from the study of the low-lying spectra of light nuclei with $A > 4$.

### 3.1 Bound States

Nowadays three- and four-nucleon bound states and binding energies can be calculated with different methods based on any of the most modern high-precision NN-forces with an accuracy on the percentage level or less. For $A=3$ and $A=4$ a well founded formulation, the Faddeev-Yakubovsky scheme (FY), opened that avenue followed by alternative, equally accurate procedures: expansions in hyperspherical harmonics (HH) or gaussians (CRCGV), stochastic variational method (SVM) and path integral techniques in form of the “Green’s Function Monte Carlo” method (GFMC). Recently other very promising methods, based on the theory of effective interactions, have been developed: the “no-core shell model” (NCSM) and the “effective interaction HH” (EIHH) using expansions in harmonic oscillator and HH basis functions, respectively.

An example of the degree of accuracy reached by these methods in the four-body system is given in Table 1 where binding energy, expectation values of kinetic and potential energies and mean square radius of $^4$He are shown as they result from calculations of seven different techniques based on the same potential model. These most precise techniques lead to a clear cut answer: all modern realistic NN-forces underbind light nuclei significantly. Examples are displayed in Table 2. The results can be considered as evidence for the existence of at least a 3N-force.

As already stated above, the origin and the nature of 3N-forces as well as
Table 1: Kinetic \( \langle T \rangle \), potential \( \langle V \rangle \), binding energy \( E_b \) (all in MeV), and mean square radius of \(^4\)He as obtained by various methods.

| Method | \( \langle T \rangle \) | \( \langle V \rangle \) | \( E_b \) | \( \langle r^2 \rangle^{1/2} \) |
|--------|----------------|----------------|--------|----------------|
| FY     | 102.39         | -128.33        | -25.94(5) | 1.485          |
| CRCGV  | 102.25         | -128.13        | -25.89  |                |
| SVM    | 102.35         | -128.27        | -25.92  | 1.486          |
| HH     | 102.44         | -128.34        | -25.90(1) | 1.483          |
| GFMC   | 102.3(10)      | -128.25(10)    | -25.93(2) | 1.490(5)      |
| NCSM   | 103.35         | -129.45        | -25.80(20)|               |
| EIHH   | 100.8(9)       | -126.7(9)      | -25.944(10)| 1.486         |

Table 2: Binding energies of light nuclei in MeV based on the NN-potentials CD Bonn, Nijmegen I and AV18.

|       | CD-Bonn | Nijm I | AV18 | Exp  |
|-------|---------|--------|------|------|
| \(^3\)H | 8.01    | 7.74   | 7.6  | 8.48 |
| \(^4\)He | 26.26   | 24.98  | 24.1 | 28.30|
| \(^6\)He | 23.9    | 29.27  |      |      |
| \(^6\)Li | 26.9    | 31.99  |      |      |
| \(^7\)He | 21.2    | 28.82  |      |      |
| \(^7\)Li | 31.6    | 39.24  |      |      |
| \(^8\)He | 21.6    | 31.41  |      |      |
| \(^8\)Li | 31.8    | 41.28  |      |      |
| \(^8\)Be | 45.6    | 56.50  |      |      |

their description, consistent with the NN-force, is one of the central questions in the years to come. At present 3N-force models based on two-pion-exchange mechanism are available. Several of them, e.g., Tucson-Melbourne (TM) and Urbana IX have been employed in the above mentioned calculations and their parameters have been adjusted to the \(^3\)H binding energy. It is interesting to notice that the inclusion of 3N-force leads to a nearly correct \( \alpha \)-particle binding energy as we see in Table 3. There is a small overbinding which can presumably be removed by an additional fine tuning of the 3N-force parameters. In any case, in view of the high central density of the \( \alpha \)-particle, these results strongly indicate that four-nucleon forces presumably are small.

Besides the extensively investigated 3N-force based on two-pion-exchange mechanisms, interesting additional structures for the 3N-force appear for the exchange of heavier mesons. One finds a variety of short-range contributions including structures that are quite similar to those recently discussed also in the framework of EFT. Additional 3N-force terms appear if one combines pion-emission and re-absorption processes with the dynamics of the 3N system.

As an illustration of the status of the theory revealing properties of bound state wave functions, we show in Fig. 1 the two-nucleon relative momentum distribution for \(^3\)He, and in Fig. 2 the two-nucleon correlation functions \( C(r) \) for light nuclei. In Fig. 1 we see different populations of relative momentum...
Table 3: Theoretical binding energies for $^3\text{H}$ and $^4\text{He}$ in MeV based on the 3N-forces TM’ and Urbana IX.

|            | $^3\text{H}$ | $^4\text{He}$ |
|------------|--------------|---------------|
| CD-Bonn + TM’ | 8.48        | 28.4          |
| AV18 + TM’  | 8.45        | 28.36         |
| AV18 + Urbana IX | 8.48   | 28.50         |
| Exp.        | 8.48        | 28.30         |

Figure 1: The two-nucleon relative momentum distribution in $^3\text{He}$. The vectors $\vec{p}$ and $\vec{q}$ denote standard Jacobi momenta. Solid curve stands for AV18, the dashed curve for AV18 + Urbana IX and the dotted curve for CD-Bonn.

Figure 2: Two-nucleon correlation functions for light nuclei. The paired curves from below to above beyond the peak are for the $\alpha$-particle, for $^3\text{He}$, and for the deuteron, respectively. Solid curves represent results obtained with CD Bonn+TM (CD Bonn for the deuteron) and dashed curves with AV18+TM (AV18 for the deuteron).
distribution for CD Bonn and AV18. The 3N-force effects are rather small in all cases. Interestingly, the probabilities to find two nucleons at a distance $r$ in the deuteron, $^3\text{He}$, and $^4\text{He}$ are essentially identical for $r \leq 2$ fm and deviate at larger distances only due to different binding energies (Fig. 2).

Although these wave function properties help us having a ”pictorial” representation of the dynamical effects, they are not directly observable; nevertheless, they may leave their mark on certain experimental observables, allowing thus their study indirectly. To this end it is necessary to have a complete control on all other theoretical ingredients relevant to the considered observable so that the dependence on the remaining quantity may be studied in a reliable manner. The determination of the neutron electric form factor from electron scattering off $^2\text{H}$ and $^3\text{He}$ by studying polarization observables is a typical example for this (see Sec. 5.1).

### 3.1.1 Neutron-Neutron Interaction and Neutron Clusters

A recent experimental result suggests the possible detection of bound multineutron clusters ($A \geq 3$). Such a possibility, in particular concerning a tetra-neutron, has been recurrently considered in the literature both from the experimental and theoretical side but never confirmed.

The experimental method is based on beams of neutron-rich nuclei and differs from the previous ones in that the $4n$-formation is no longer based on double charge exchange processes (e.g. $\pi^- + ^4\text{He} \rightarrow \pi^+ + ^4n$) but rather in removing from already pre-existing $4n$-clusters the parasites which impede it to exist. Thus, studying the break-up of $^{14}\text{Be}$ by a carbon target, some events compatible with the reaction $^{14}\text{Be} \rightarrow ^{10}\text{Be} + ^4n$ were reported. The number of these events, though well separated from the background, is however very small. New runs with increasing statistics are necessary to confirm or disprove the findings.

From the theoretical side it seems clear that the $nn$-force alone cannot bind such a system. Despite a quasi-resonant $nn$-scattering length ($a_{nn} \approx -18$ fm), the Pauli principle generates a strong effective repulsion which prevents a few-neutron cluster to exist. The required additional attraction should eventually come from $3n$-forces only. In recent GFMC calculations a variety of $3n$-interactions have been explored for fitting the $A = 8$ spectrum, and the authors were very pessimistic about the existence of small neutron droplets. At the same time, they point out our poor knowledge of the $3n$-force. The Urbana TNI-model, for instance, which so successfully describes the $3N$- and $4N$-phenomenology, turned out to fail when extrapolated to neutron-rich nuclei. The different versions of the Illinois forces, built to overcome these difficulties, give binding energies which vary over a range of about 1.5 MeV for a $T = 2$ state like $^8\text{He}$. In the $4n$-system, the missing attraction is far beyond such uncertainties and its existence is very unlikely. The situation is less clear for $A > 4$.

It is certainly a very interesting and challenging task to draw a definite conclusion on the existence of small neutron clusters given the non-existence of a di-neutron and theoretical results advocating unbound neutron matter.
Figure 3: Angular distribution in elastic $nd$ scattering (light grey band: modern NN-force predictions only; dark grey band: including 3N-forces) compared to $pd$ data

3.2 Scattering States

Bound state properties provide only limited information on the nature of the hadronic force in nuclei, whereas scattering states, appearing in reactions with hadronic and electromagnetic probes, yield a much richer and much more detailed information. A particularly rich source is obtained by studying polarization observables.

3.2.1 Three-Body Continuum

In contrast to the two-body case, continuum states for three particles are a non trivial theoretical problem. However, several powerful tools have been developed in the past in order to calculate scattering states in the continuum using realistic potentials: the Faddeev approach or the reformulation by the AGS scheme as well as the Kohn-variational approach using hyperspherical expansions.

This progress has opened up the possibility to access the tremendous variety of spin observables and multi-nucleon fragmentations in hadronic as well as electromagnetic reactions. This will be illustrated by a few cases. Fig. 3 displays the angular distributions of elastic $nd$ scattering at 65 and 135 MeV using NN-forces alone, and showing the effect of adding 3N-forces like the ones in Table 2. One readily notes a clear cut discrepancy in the minimum of the data with NN-force predictions only, and a nice agreement when 3N-forces are added (parameters of the 3N-force have been fixed before on the $^3\text{H}$ binding energy). As to the discrepancy at forward angles one has to notice that the experimental data refer to $pd$ scattering and the Coulomb force should be included in the theory. Although this result is very promising, much work remains to be done. While some observables in elastic $nd$ scattering can be well described by NN-forces alone and addition of 3N-forces plays no significant effect, others are only well described if 3N-forces are included. Some of these effects can even be seen at low energy, as shown in Fig. 4 for $pd$ elastic scattering at 1 MeV. The Coulomb force between protons is included in a very precise calculation that uses a correlated hyperspherical expansion of the wave function in configuration space together with the complex Kohn variational principle to determine the $S$- or the $K$-matrix. Part of this effect may be
understood in terms of scaling with $^3\text{He}$ binding energy when the 3N-force is included but clearly demonstrates the need for very precise data. Nevertheless one finds that for specific spin observables there are striking discrepancies to the data with present force models.

There is a well-known discrepancy in the description of the vector analyzing powers $A_y$ ("$A_y$-puzzle") and $iT_{11}$ for $Nd$ scattering. The comparison of theoretical predictions with data at $E_{\text{lab}} \leq 2$ MeV shows that the calculations underpredict the data by $\approx 30\%$ in the maximum of the angular distributions. The disagreement becomes worse at lower energies where the difference increases to $\approx 40\%$ at $E_{\text{lab}} = 650$ keV. None of the modern NN-potentials is able to describe $A_y$. Even when 3N-forces such as TM or Urbana IX are added, the discrepancy is only slightly reduced (see Fig. 5). The discrepancy in $A_y$ and $iT_{11}$ decreases with increasing energy. All this clearly indicates that the spin-isospin structure of the 3N-force is not yet sufficiently well understood and that much more experimental and theoretical work is required. New theoretical ways are already being explored, where purely phenomenological, meson theoretical, or EFT approaches are used.

In a recent meson theoretical calculation, pion retardation effects are taken into account. That such retardation effects can contribute to the 3N-force has been known since at least for two decades. In this scheme, single-pion exchanges lead to a non-negligible 3N-force of tensor structure, similar to the OPE term of the NN-interaction but involving the coordinates of all three nucleons. It leads to a distinct energy dependence of the 3N-force. At the present stage, there are still theoretical uncertainties which require the introduction of one adjustable parameter, but for the future one hopes that this mechanism may be constrained theoretically in order to obtain a completely
Figure 5: Proton analyzing power $A_y$ for elastic $pd$ scattering showing the $A_y$-puzzle; curves: theoretical results using various interaction models (see inset).

Figure 6: Spin observables for $nd$ scattering at 8.5 MeV. Solid (dash-dot) line including (excluding) the 3N-force. For $T_{20}$ and $T_{21}$ comparison is made with $pd$ data measured at energies shifted by +0.7 MeV.
parameter-free description of this effect. With a suitable adjustment of this single parameter one can remove the discrepancies for the vector analyzing powers, with smaller (but significant) effects for the other spin observables, such as the tensor analyzing powers (see Fig. 6). This is another example on how additional precise measurements can help to clarify the role of new pion-exchange terms in the three-nucleon dynamics, and consequently, can lead to a deeper understanding of the role played by the pion itself in the three-nucleon system.

While suffering from a certain lack of systematics in the organization of the dominant contributions, the meson picture has the advantage of visualizing the process by means of a particle exchange diagram, and to interpret intuitively the origin of the various terms. It also provides a direct link between the behavior of the few-nucleon systems at low energies and nuclear phenomena at intermediate energies, above the meson-production threshold where retardation is indispensable. This is an important aspect: few-nucleon physics at low-energy does not represent an isolated research field, but is strongly connected to contiguous areas like hadronic physics at intermediate energies and many-body physics, with vast opportunities for cross-fertilization.

![Figure 7: Elastic \( nd \) scattering at 65 MeV in NLO (grey band) and NNLO (dark band) of EFT approach compared to \( pd \) scattering data](image)

The chiral approach on the other hand has the advantage of a consistent simultaneous determination of NN- and 3N-forces. In this respect, the guidance provided by chiral perturbation theory can be very useful. In the chiral approach the 3N-force appears at NNLO and depends on two new parameters. This 3N-force appears in three topologies: a two-pion exchange, a one pion exchange between a two-nucleon contact force and the third nucleon and a pure 3N-contact force. The two new parameters are related to the latter two pieces, whereas the two-pion exchange is parameter free. The two parameters are adjusted to the \( ^3\text{H} \) binding energy and the \( nd \) doublet scattering length. Although this approach cannot at this time provide a cure for the \( A_y \) discrep-
ancy at low energy, it gives rise to very promising results as documented in the example of elastic scattering observables at 65 MeV in Fig. 7. The NLO predictions are shown as a band which covers the weak dependence on the choice of the cut-off parameter $\Lambda$. One sees clear deviations for $d\sigma/d\Omega$ and the nucleon analyzing power $A_y$. Turning to the next order NNLO, which includes the 3N-forces adjusted as described above, the agreement for those two observables is very good. Also for the tensor analyzing powers $T_{20}$, $T_{21}$, and $T_{22}$, the agreement improves. But here unfortunately, the quality of the only existing data is by far too poor to allow a definite conclusion. It is a further important example where precise and new data are very much needed in order to put the theory on a critical test. It will be very interesting to proceed with this systematic approach for the study of 3N-force effects and to include furthermore systematically relativistic corrections showing up as additional NN- and 3N-force contributions.

Besides elastic nd-scattering also the nd-break-up process will be a treasure to probe the nuclear forces because it has an overwhelmingly rich structure of break-up configurations leading to a five-fold differential cross section, and of course, a great variety of spin observables. While special break-up configurations have been measured in the past, sometimes with spectacular agreement between theory and experiments, sometimes with nagging discrepancies, predictions have been worked out for regions of phase space where 3N-force effects are especially large for cross sections as well as spin observables (up to factors of 2 to 3). It appears that data for the break-up provide the most complete information on 3N-dynamics and would be very much needed.

Likewise the electromagnetic probe represents a valid instrument to study the 3N-force. First evidence that an electromagnetic reaction is sensitive to the three-body interaction was shown in the calculation of the total photoabsorption cross section of $^3$H and $^3$He with realistic NN- and 3N-forces through the Lorentz integral transform (LIT) method. It was shown that 3N-force effects of the order of 10 %, resulting mainly from the change in the triton binding energy, were present at the dipole resonance peak and in the tail below the pion threshold. The accuracy of the results was later tested against a FY calculation. It turned out that the theoretical precision is much higher.

Figure 8: Total photoabsorption cross section of $^3$He in the LIT approach: AV18 (dash-dotted) and AV18 + Urbana IX (full curve).
than the precision of available experimental data. As shown in Fig. 8 even no
definite conclusions can be drawn with respect to the interplay between the
NN- and 3N-forces and on the corresponding two- and three-body currents as
long as new high-precision experimental data is not available.

Exclusive reactions (two- and three-body break-up) induced by real and
virtual photons are very likely more sensitive to 3-body dynamics. Therefore,
precise experimental data on two- and three-body photodisintegration cross
sections as well as separated (inclusive, semi-inclusive as well as completely
exclusive) electrodisintegration cross sections at lower momenta are needed.
The availability of calculations which take into account FSI in a complete and
accurate way removes the necessity to restrict the kinematics to the quasi-
elastic region. Thus it opens up the possibility to study the transition from
below to that regime. For instance, the spin-dependent momentum distribu-
tion of polarized proton-deuteron clusters in polarized $^3\text{He}$ have been studied
theoretically via the processes $^3\text{He}(e,e'p)d$ or $^3\text{He}(e,e'd)p$. It turned out that
only at small deuteron momenta ($p_d \leq 1 - 2 \text{ fm}^{-1}$) and for energies below
the pion threshold the process provides direct insight into this momentum
distribution. For larger momenta, the longitudinal and transversal response
functions are affected too strongly by FSI and MEC’s. But the latter effects
are equally interesting, and their theoretical description should be checked by
precise data. Exactly these type of data and their confrontation with theory
will reveal whether the complex hadron dynamics is theoretically understood.

### 3.2.2 The Four-Body Continuum

In the quest to go beyond three-nucleon scattering using ab initio calcula-
tions, four-body dynamics in the continuum is really a challenging issue. One
major goal involves an answer to the following question: can we understand
the simplest four-nucleon scattering problem using present NN- or NN+3N-
force models as we do for most $nd$ and $pd$ scattering observables? The $A = 4$
system is the lightest nuclear system displaying clear cut resonances in
physical observables. One of the best examples is the total neutron-triton
cross-section which shows a broad resonance peak centered at $E_n = 4 \text{ MeV}$.
Despite its nicely looking form, a careful R-matrix analysis reveals overlapping
broad resonances. None of the related scattering phase shifts passes ever
through 90°. This is a common feature of all ($A=4$)-nuclei $^4\text{He}$, $^4\text{He}$, and $^4\text{Li}$.

The study of $n+^3\text{H}$ elastic scattering (or $p+^3\text{He}$) below three-body break-
up threshold is the simplest 4N-scattering problem because its total isospin is
$I = 1$, and up to $E_n = 8.34 \text{ MeV}$ the problem reduces to the elastic channel
alone. In the past five years a number of calculations emerged using different
methods or force models. Although some small disagreement with data shows
up at threshold energies, no fully converged exact calculation using realistic
NN-interactions is able to describe the total $n+^3\text{H}$ cross section at the res-
onance peak ($E_n = 4 \text{ MeV}$). In the near future more calculations applying
different NN-forces are expected and studies are underway to understand the
role of the 3N-force and the sensitivity to changes in NN-force models. In
addition to accurate benchmark calculations to clarify possible discrepancies
between the results of different groups, one needs precision neutron data for
scalar and vector observables. The absence of Coulomb force in $n+^3\text{H}$ scatter-
ing opens a realm of theoretical possibilities to the solution of the 4N-problem
which leads to the possibility of using a wide variety of (2N + 3N)-force mod-

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16
Using the AGS equations and a rank one separable representation of realistic NN-interactions, a complete study of all four-nucleon reactions has been made at energies up to the four-body break-up threshold. These calculations uncover large disagreements in \(dd \rightarrow dd\) and \(dd \rightarrow p^3\text{H}\) tensor analyzing powers as shown in Fig. 9, as well as an \(A_y\) deficiency in \(n^3\text{He} \rightarrow n^3\text{He}\) and \(n^3\text{He} \rightarrow p^3\text{H}\). Nevertheless the total cross section data for \(n^3\text{He} \rightarrow n^3\text{He}\) elastic scattering in the resonance peak is well accounted for by these calculations. Confirmation of these findings requires the use of the full NN-potential.

Like in \(p+d\) elastic scattering, fully converged variational calculations also find an \(A_y\) discrepancy in \(p^3\text{He}\) scattering at low energy (Fig. 10), but in addition a fairly large disagreement with the existing data for the differential cross section (Fig. 11). One also finds that \(p^3\text{He}\) scattering lengths extracted from effective range phase shift analysis depend on the data sets one includes. In particular if cross section and analysing power measurements at proton energies below 1 MeV are added to the higher energy data \((E_p < 12\text{MeV})\), one gets two sets of solutions. This is a clear indication that one may need...
Figure 11: Differential cross section for the same reaction as in Fig. 10 with the same notation.

Figure 12: Comparison of various data for $p-^3\text{H}$ scattering at 120 degrees with the R-matrix analysis (solid) and the AV18 calculation (dotted).
high precision data at low energy to clarify the situation.

In the \( p+^3\text{H} \) sector one finds very old data such as shown in Fig. 12. This situation makes it impossible to determine the position of the related \( 0^+ \) resonance which enters into the determination of the nuclear compressibility coefficient. Another example is the charge exchange reaction \( ^3\text{H}(p,n)^3\text{He} \) or its inverse reaction. Here again the vector polarizations are very sensitive to the 3N-force as seen in Fig. 13. For the corresponding elastic scattering cross sections the sensitivity is quite reduced. The reason for this feature can be traced back to the isospin structure of various partial waves. The \( 0^+ \) and \( 0^- \) transition matrix elements reach almost the unitary limit of one, forcing the elastic one close to zero. Although this may lead to a reduced sensitivity in the elastic channels, one challenging issue involves the correct determination of the position and width of the \( 0^- \) resonance in \( I=0 \).

Alternative approaches to the quite challenging problem of four-body dynamics in the continuum are provided by integral transform methods. The LIT method is particularly suited for inclusive reactions, which are very complicated to compute in the traditional way due to the increasing number of contributing channels with increasing particle number. First applications have been considered for e.m. inclusive reactions, although promising results for exclusive as well as purely hadronic reactions have already been obtained in a simple two-nucleon test case. Particularly interesting are the results of a LIT calculation for the longitudinal \( ^4\text{He} \) \((e,e')\)-response function at various momentum transfers between 300 and 600 MeV/c using a semi-realistic NN potential. The theoretical results could show that the effect of FSI is rather large, even in the quasi-elastic peak region. The FSI effect decreases with increasing momentum transfer, but nonetheless it became clear that quasi-elastic approaches are justified only at momenta well beyond 500 MeV/c and only at peak kinematics. It was also found that kinematics at low energies and relatively high momenta, which were suggested in order to study bound state correlation effects are not at all exempt from FSI. The obtained results are in good agreement with experimental data (size of experimental error about \( 10\div20 \% \)). It remains to be seen if the good agreement, in particular at high

![Figure 13: Predictions of proton analyzing power in \( ^3\text{H}(\vec{p},n)^3\text{He} \) at 3 MeV \( E_{c.m.} \) from R-matrix analysis and calculations using AV18 alone (av-conv) and AV18 together with Urbana IX (au-conv). The data shown at 2.9 MeV is very energy dependent.](image)
energy and momentum, survives a more realistic calculation which includes also lowest order relativistic contributions and more precise data. We would like to point out that with higher quality $^4$He($e, e'$)-data one could study more specific questions, e.g., the importance of 3N-forces. Preliminary studies on 3-body systems indicate that 3N-forces reduce the quasi-elastic peak height by about 10% and increase the high-energy tail by about the same amount. Such effects could be even larger in the four-body system, but one will certainly need more precise data than the present ones in order to obtain a clear picture.

Additional important insights into the hadronic structure of the 4N-system could be found studying the inclusive transverse ($e, e'$)-response, e.g., about the role of MEC. Clear evidence of their importance as well as surprising differences between the case of $^4$He and those of 3- and 6-body systems have been seen using another integral transform approach analysing Euclidean responses. The results show that MEC contributions seem to be particularly amplified in $^4$He and increase considerably with decreasing momentum transfer. Thus one can certainly state that a separation of longitudinal and transverse responses in inclusive electron scattering experiments on $^4$He at lower momentum transfers would be very valuable. Important aspects of the hadronic force and questions related to the consistency of the MEC with such a force could be investigated.

Besides the inclusive $^4$He($e, e'$)-reaction, the two-body break up channels should be studied, also including polarization effects (electron and/or polarization of outgoing particles). Such studies could reveal other important details of the hadronic structure than seen in inclusive ($e, e'$) or purely hadronic four-body reactions. First investigations have already been carried out. Complete agreement between experiment and theory has not yet been obtained.

Figure 14: Total photoabsorption cross section of $^4$He obtained with the LIT method with TN and MT potentials; experimental results: sum of ($\gamma, n$) and ($\gamma, p$)$^3$H cross sections (dotted curve with indication of error range) and indirect determination via Compton scattering (shaded area).
For future studies the region of low momentum transfer is especially interesting. In particular, one could investigate the transition from virtual to real photon physics. We would like to mention that with respect to the latter, striking differences between theory and available old experimental data have been found as seen in Fig. 14 and need further investigation both theoretically and experimentally. An experimental program has already started at Max-Lab in Lund. Other basic properties like electric polarizabilities and magnetic susceptibilities are largely unknown. They are accessible both via Compton scattering or via integrated photoabsorption cross sections to the extent that one is able to separate electric and magnetic processes. Their study would be interesting not only as a test of the underlying forces, but also to get an intuitive representation of their structure.

4 Microscopic Description of Many-Body Phenomena

As we have seen above, very light nuclei \((A = 2 \div 4)\) are privileged systems for the study of fundamental issues like properties and origin of the strong force. However, the study of heavier nuclei has its own merit with respect to new many-body phenomena like, e.g., clusterization and collective motion. Furthermore, one can expect that in heavier systems some of the interaction phenomena found in light nuclei may be modified or amplified in view of the higher average nucleon density (“medium effects”).

One of the principal advantages of studying very light systems resides in the fact that calculations are accurate enough in order to make the comparison with precise data conclusive with respect to the issues considered above. Therefore, in order to reach the same conclusive strength for heavier nuclei, it is important to produce also for systems with \(A > 4\) on the one hand accurate theoretical results and on the other hand accurate experimental data.

4.1 Properties of Ground and Low-Lying Excited States

Going beyond four-body nuclei, one finds quite significant effects from the 3N-force. Allowing for additional spin-isospin dependences in the 3N-force, leading to the model IL4, in addition to \(^3\)H and \(^4\)He the low-lying spectra for up to \(A=8\) nucleons could be rather well described as shown in Table 4. These results clearly show that, in relation to the most modern NN-forces, 3N-forces are unavoidable in order to describe binding energies and low-lying excited states of light nuclei. Since the number of nucleon triplets overtakes more and more the number of nucleon pairs with increasing \(A\), it is clear that 3N-forces have to be included in any realistic description of complex nuclei, too.

Already for \(A = 4, 6, 8, \ldots\) one may observe phenomena which are precursors of the above mentioned many-body phenomena. To illustrate this point of view, we show in the upper panel of Fig. 15 the spectrum of \(^8\)Be as obtained in a microscopic “Variational Monte Carlo” - “Greens Function Monte Carlo” calculation (VMC-GFMC) which is based on one-body orbitals with four nucleons in an \(\alpha\)-core coupled to \((A-4)\) one-body (\(\ell = 1\)) wave functions.
Table 4: Experimental and GFMC energies (in MeV) of particle-stable or narrow-width nuclear states. Monte Carlo statistical errors in the last digits are shown in parentheses.

|       | AV18 | IL4 | Exp |       | AV18 | IL4 | Exp |
|-------|------|-----|-----|-------|------|-----|-----|
| $^3\text{H}$ ($\frac{1}{2}^+$) | -7.61(1) | -8.44(1) | -8.48 | $^7\text{Li}$ ($\frac{1}{2}^-$) | -31.1(2) | -39.0(2) | -38.77 |
| $^3\text{He}$ ($\frac{5}{2}^+$) | -6.87(1) | -7.69(1) | -7.72 | $^7\text{Li}$ ($\frac{7}{2}^-$) | -26.4(1) | -34.5(2) | -34.61 |
| $^4\text{He}$ (0$^+$) | -24.07(4) | -28.35(2) | -28.30 | $^8\text{He}$ (0$^+$) | -21.6(2) | -31.9(4) | -31.41 |
| $^6\text{He}$ (0$^+$) | -23.9(1) | -29.3(1) | -29.27 | $^8\text{Li}$ (2$^+$) | -31.8(3) | -42.0(3) | -41.28 |
| $^6\text{He}$ (2$^+$) | -21.8(1) | -27.4(1) | -27.47 | $^8\text{Li}$ (1$^+$) | -31.6(2) | -40.9(3) | -40.30 |
| $^6\text{Li}$ (1$^+$) | -26.9(1) | -32.0(1) | -31.99 | $^8\text{Li}$ (3$^+$) | -28.9(2) | -39.3(3) | -39.02 |
| $^6\text{Li}$ (3$^+$) | -23.5(1) | -29.8(2) | -29.80 | $^8\text{Li}$ (4$^+$) | -25.5(2) | -35.2(3) | -34.75 |
| $^7\text{He}$ ($\frac{3}{2}^-$) | -21.2(2) | -29.3(3) | -28.82 | $^8\text{Be}$ (0$^+$) | -45.6(3) | -56.5(3) | -56.50 |
| $^7\text{Li}$ ($\frac{3}{2}^-$) | -31.6(1) | -39.5(2) | -39.24 | $^8\text{Be}$ (1$^+$) | -30.9(3) | -38.8(3) | -38.35 |

The low-lying states of $^8\text{Be}$ resemble a rotational spectrum which in fact can be described quite nicely in a cluster-model where two $\alpha$-clusters rotate around their common center of mass. However, it is also possible to recover this picture from the VMC wave functions by a modified Monte Carlo density calculation. This can be seen in the lower panel of Fig. 15 which shows contours of constant density plotted in cylindrical coordinates. The left side of this panel shows the lab frame density. For the $J = 0$ ground state it has to be spherically symmetric. On the other hand, the intrinsic density shown on the right side exhibits clearly two peaks, with the neck between them having only one-third of the peak density. It is obvious that this feature can be interpreted as the clusterization into two $\alpha$’s of $^8\text{Be}$.  

This interpretation is corroborated by looking at the densities of the $J = 2^+$ and $4^+$ states. Although the laboratory densities for these states (in $M = J$ states) are quite different, the intrinsic densities are, within statistical errors, the same as the $J = 0$ intrinsic density, which is typical for a rotational band. In fact, if the $0^+$, $2^+$, and $4^+$ states are generated by rotations of a common intrinsically deformed structure, then their electromagnetic moments and transition strengths should all be related to the intrinsic moments which can be computed by integrating over the projected body-fixed densities. This has been explored, and a consistent picture is obtained.

The ab initio quantum Monte Carlo calculations are also used to study the structure of some of the lightest halo nuclei like $^6\text{He},^8\text{He}$. Fig. 16 shows predicted densities compared to those deduced from experiment. It can be seen that, as more neutrons are added, the tails of the matter distributions broaden considerably because of the relatively weak binding of the $p$-shell neutrons. In addition, the central neutron and proton densities decrease rather dramatically. This effect does not necessarily require any changes to the $\alpha$-core but can be understood at least partially from the fact that the $\alpha$ no longer sits at the center of mass of the entire system. The motion relative to the center of mass spreads out the mass distribution relative to that of $^4\text{He}$. Proton-proton distribution functions have also been calculated and one can compare the ones of $^4\text{He}$ to those of $^6\text{He}$ and $^8\text{He}$.

If the $\alpha$-core of $^6,^8\text{He}$ is not distorted by the surrounding neutrons, all
Figure 15: Upper panel: calculated spectrum of $^8$Be; lower panel: calculated density contours of the $^8$Be ground state in the lab frame (left) and the intrinsic frame (right), labeled with densities in fm$^{-3}$.

three $pp$ distributions should be the same. One sees, however, that the $pp$ distribution spreads out slightly with neutron number in the helium isotopes, with an increase of the pair r.m.s. radius of 4% in going from $^4$He to $^8$He. Although this could be interpreted as a swelling of the $\alpha$-core, it might also be caused by charge exchange correlations. This needs to be further investigated.

Due to the complexity of complete many-body calculations for larger and larger nuclei one has often resorted to the much simpler descriptions in terms of clusters that grasp the essential physics of a given problem. The methods normally applied to few-nucleon systems are then applied to few-cluster systems. It is highly interesting to test cluster-model results to A-body calculations in order to be aware of the range of applicability and the limits of these descriptions of many-body systems. These comparisons are now possible. The VMC wave functions for the AV18-Urbana IX model has been used to calculate a variety of cluster-cluster overlap wave functions, such as $\langle dp|t\rangle$,
Figure 16: Proton and matter densities in $^4$He, $^6$He and $^8$He. Analysis of $^6,^8$He proton scattering are shown by the dashed and dot-dashed curves.

$\langle dd|\alpha\rangle$, and $\langle \alpha d|6\text{Li}\rangle$.

Recently also the overlaps $\langle ^6\text{He}(J^\pi) + p(\ell_j)|^7\text{Li}\rangle$ for all possible $p$-shell states in $^6$He were studied. The spectroscopic factors, obtained from these overlaps, are 0.41 to the ground state of $^6$He and 0.19 to the $2^+$ first excited state. These factors are significantly smaller than the predictions of the Cohen-Kurath (CK) shell model, which gives values of 0.59 and 0.40, respectively. The CK shell model requires that the possible $^6\text{He}+p$ states sum to unity within the $p$-shell, whereas in the VMC calculation, the correlations in the wave function push significant strength to higher momenta that cannot be represented as a $^6\text{He}$ state plus $P$-wave proton.

Proceeding with this kind of investigations to larger $A$ will allow one to show where and whether the transition to mean field structures will show up. It will be very interesting to see whether spectroscopic factors close to unity will at some point appear as a result of microscopic A-body calculations and single-particle features which may appear in one-body knock-out experiments will then have an interpretation in terms of a more fundamental theory.

First calculations in the region of larger $A$ up to $A=16$ have already been carried out with VMC predictions of spectroscopic factors and NCSM calculations of the low-lying spectra and some electromagnetic properties. In particular the NCSM method, which has proved to be very accurate in calculating ground state properties of the traditional light systems (see e.g. Table 1), has also been pushed to study the low-lying spectrum of $^{12}\text{C}$ (see Fig. 17) and recently even the Gamow-Teller excitations of $A=14$ nuclei.

Though at present the calculations are not fully convergent they represent the first examples of $ab\ initio$ calculations with realistic interactions in a mass region which has been studied extensively for many years with simplified and approximate methods. Thus one may hope that in the near future the microscopic interpretation of many-body phenomena will be available.
4.2 Continuum States and Collective Motion

Unambiguous interpretations of experiments in terms of nuclear structure properties require in most cases a reliable treatment of the continuum. Inelastic processes in fact constitute a much richer source of information. Indeed, if one wants to get at single-particle properties like, e.g., shell momentum distributions via one-body knock-out experiments, or to study the microscopic origin of collective motion, one needs to have the continuum under control.

Figure 18: Total photoabsorption cross section of the $^6$Li and $^6$He in the LIT approach with MT and Minnesota potentials.

In this respect considerable progress has been achieved in recent calculations of reactions on 6-body systems using integral transform methods. In fact, a combination of the LIT and EIHH methods allows one to calculate inclusive cross sections on a full A-body microscopic basis. An example is shown in Fig. 18 displaying the results of the photoabsorption cross section $\sigma(\omega)$ of $^6$Li and $^6$He. The very interesting result of this calculation is the appearance of a single giant dipole resonance peak for $^6$Li in contrast to two well separated peaks for $^6$He. One can associate the low-energy peak of $^6$He at $\omega \simeq 8$...
MeV to the break-up of the neutron halo, whereas the second one, at about $\omega=35$ MeV, should correspond to the break-up of the $\alpha$-core. Interpreting this feature as a collective motion, one can distinguish a soft collective mode due to oscillations of the core against the halo part of the nucleus from a classical Goldhaber-Teller mode of the proton sphere against the neutron sphere. On the other hand, the $^6$Li cross section does not show such a substructure. This is probably due to the fact that the break-up into two three-body nuclei, $^3$He + $^3$H, fills the gap between the two modes. Note that in case of $^6$He a corresponding break-up into two identical nuclei, $^3$H + $^3$H, is not induced by the dipole operator. The comparison with experimental data in Fig. 19 shows that the theoretical cross sections with semirealistic central $S$-wave interactions (MT, Minnesota) miss some strength at very low energies. Inclusion of $P$-wave interaction (AV4') leads to much improved results for $^6$Li. There is only a small effect in $^6$He, contrary to what is obtained in a cluster-model with an inert $\alpha$-core and two neutrons interacting via a $P$-wave potential. The situation in the region of the giant dipole peak is much less clear, either because of lack of experimental data ($^6$He) or because existing data do not lead to a unique picture ($^6$Li).

This example shows how important and urgent it is to intensify on the one hand the theoretical investigations of the microscopic dynamical structure of such many-body systems, and on the other hand to start or at least to renew an experimental program for obtaining accurate data for such targets both for the inclusive process and for the various exclusive channels as well. To give an example, it would be particularly interesting to see whether the $^3$He-$^3$H channel has a peak in between the soft and hard modes. Of course, a rich field would be opened by detailed studies of the deformed structure of such nuclei in terms of collective motion, like in the case of $^8$Be.

5 Few-Body Physics Related to Other Fields

5.1 Two- and Three-Body Systems as Effective Neutron Targets

Another very important aspect of few-body nuclear physics is the use of lightest nuclei like deuteron or $^3$He as effective neutron targets in view of the fact that free neutron targets do not exist. The method rests on the assumption
that (i) the binding is weak, and (ii) FSI effects and other hadronic influences due to the presence of spectator nucleons can either be neglected or reliably corrected for. This means that only a full control of the nuclear few body dynamics allows one to obtain clear cut information on the intrinsic properties of the neutron.

A typical example is the quest for the electromagnetic form factors of the neutron, in particular the small electric one \( G_{En} \) in quasifree electron scattering on deuterium or \(^3\)He, exploiting polarization observables. For deuteron electrodisintegration a systematic study of polarization observables has been performed with respect to kinematical regions where effects of two-body dynamics from FSI and MEC were minimal and on the other hand the dependence on \( G_{En} \) was maximal with the intention to guide experimental efforts. It turns out that especially for low momentum transfers the medium effects are sizeable, meaning that the determination of \( G_{En} \) relies heavily on the theory for the evaluation of such medium effects in a reliable manner. In fact the determination of \( G_{En} \) is done in such a way that one calculates the corresponding observables for varying values of \( G_{En} \) and comparing the result with the experimental data. An interesting check of this method is provided by performing the corresponding experiment on a bound proton. Similar theoretical studies for \(^3\)He have been done and promising results have also been achieved using a polarized \(^3\)He target for the determination of \( G_{En} \).

Another example, shown in Fig. 20, is the asymmetry \( A_T' \) for quasi-elastic scattering of polarized electrons on polarized \(^3\)He, which is known to be sensitive to the magnetic form factor of the neutron \( G_{Mn} \). This very nice agreement with the data allowed one to extract \( G_{Mn} \) in perfect agreement with the value extracted on the deuteron using the ratio of the asymmetries related to the processes \( d(e, e'n) \) and \( d(e, e'p) \). It should be emphasized again that this was only possible because FSI and MEC’s were properly taken into account.

5.2 Few-Body Reactions of Astrophysical Relevance

The control of the continuum allows one to study also electroweak processes of astrophysical relevance. This is of particular importance for those reactions whose cross sections are either impossible or very difficult to measure in the laboratory. As an example, we mention the \( pp \) weak capture \( p + p \rightarrow d + e^+ + \)
\(\nu_e\), the most fundamental process for energy production in main-sequence stars, and the hep process \(p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e\). The knowledge of the reaction rate, therefore, is based on the development of realistic models for the nuclear dynamics and the electroweak transition operators. In fact, a recent calculation in the framework of EFT has improved considerably the precision for evaluating the threshold \(S\)-factors of these reactions.

Another example is the \(^2\text{H}(p, \gamma)^3\text{He}\) capture reaction. This process is extremely sensitive to the two-body electromagnetic transition operators and to the small components of the nuclear wave functions. This is because the one-body transition operators cannot connect the main \(S\)-state components of the deuteron and \(^3\text{He}\) wave functions. Accurate data for this reaction exist and, therefore, interesting tests can be performed of both the nuclear Hamiltonian from which the nuclear wave functions are obtained, and the model used to describe the nuclear currents. Since the nuclear e.m. current is related to the Hamiltonian through current conservation, it is clear that the two topics are inter-related.

![Figure 21: Astrophysical \(S\)-factor of the \(^2\text{H}(p, \gamma)^3\text{He}\) capture reaction at low energies. The curve shows the theoretical calculation using the AV18+Urbana IX interaction model. Data from TUNL and LUNA.](image)

For illustration we show in Fig. 21 the calculated and measured astrophysical \(S(E)\)-factor at low energy. The calculation has been performed using the MEC model derived by the Urbana-Argonne collaboration. As one readily notes, a good agreement with the LUNA data is found. Also at higher energies, a good agreement between theory and experiment is achieved in general, but with some notable exception, represented by the tensor analyzing powers. These observables are in fact rather sensitive to the details of the two-body e.m. current and therefore represent an optimal testing ground for the e.m. current model.

### 6 Conclusions

As a summary we may conclude that the achievements in the field of few-nucleon physics reached up to now are very promising in the sense that one can hope to finally understand the low-energy observables in light nuclei quantitatively in terms of the basic hadronic two- and three-body forces combined
with accurate theoretical tools for solving the quantum mechanical few-body problem. Besides properties of ground and low-lying excited states, hadronic and electromagnetic reactions will provide a great variety of experimental observables against which the theory can be checked. Many of the problems debated in the seventies and the eighties and left unsolved for lack of theoretical guidance, could be addressed again nowadays with a more fundamental point of view, which focuses on subnuclear non-perturbative physics. In view of the in many cases very sparse experimental information it is of utmost importance that experimental programs in low-energy nuclear physics should be supported on a broader scale than has been done in the recent past.

In addition, pushing the above mentioned methods to solve nuclear systems with increasing number of particles will give us a much deeper insight into nuclear many-body phenomena, which traditionally are described in shell or cluster-models or in an even more phenomenological collective model. In this way such models would be put on a more solid theoretical ground by establishing a quantitative relation to the basic nuclear forces. Furthermore, accurate information gathered in few-nucleon physics, like spin structures of light nuclei, will be very useful for the analysis of high-energy experiments.

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