Probing Heavy Higgs Boson Models with a
TeV Linear Collider

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Abstract

The last years have seen a great development in our understanding of particle physics at the weak scale. Precision electroweak observables have played a key role in this process and their values are consistent, within the Standard Model interpretation, with a light Higgs boson with mass lower than about 200 GeV. If new physics were responsible for the mechanism of electroweak symmetry breaking, there would, quite generally, be modifications to this prediction induced by the non-standard contributions to the precision electroweak observables. In this article, we analyze the experimental signatures of a heavy Higgs boson at linear colliders. We show that a linear collider, with center of mass energy $\sqrt{s} \lesssim 1$ TeV, would be very useful to probe the basic ingredients of well motivated heavy Higgs boson models: a relatively heavy SM-like Higgs, together with either extra scalar or fermionic degrees of freedom, or with the mixing of the third generation quarks with non-standard heavy quark modes.
1 Introduction

The origin of electroweak symmetry breaking (EWSB) constitutes the major open problem in high energy physics. It is commonly assumed that experiments at Run II of the Tevatron collider and at the Large Hadron Collider (LHC) will provide the hints necessary to distinguish between scenarios wherein the electroweak symmetry is broken via strong interactions (as in Technicolor theories) as opposed to the case where the mechanism responsible can be described in terms of a perturbative theory (as in the Standard Model or its supersymmetric extension). If the breakdown of the electroweak symmetry is induced by the formation of condensates of techniquarks via QCD-like interactions at energy scales of about a few TeV, one expects the Higgs particle to be replaced by several heavy states, including possibly a broad resonance, with the same quantum numbers as the Standard Model (SM) Higgs and a mass larger than the weak scale. On the contrary, supersymmetric extensions of the SM predict lighter Higgs boson states, with masses in the same range as those consistent with a SM interpretation of the precision electroweak observables.

It might be argued that the nature of the theory operating beyond the weak scale could be deduced (or constrained) from a study of the electroweak precision measurements. In supersymmetric models though, any additional contributions to such observables progressively disappear as the superparticle masses increase. In strongly interacting models, on the other hand, the new contributions could very well conspire to cancel the effects due to the large mass of the (composite) Higgs boson in the spectrum.

A. Topcolor models [1, 2] provide an interesting example of strongly interacting theories which, while allowing the presence of a heavy Higgs boson state, may still be consistent with precision electroweak measurements. In the simplest top condensate models [3], only the top quark is involved in the formation of the condensate leading to electroweak symmetry breaking. In this case, a top quark mass of the right order of magnitude can be achieved only by pushing the cutoff scale to very large values. This naturally leads to Higgs boson and quark masses of about 200 GeV, thereby creating a hierarchy problem similar to the one present in the SM. A supersymmetric version [4,5] solves the hierarchy problem and, for values of the compositeness scale of order of the grand unification scale, easily accommodates the measured top quark mass values. In such an extension, however, a four Fermi interaction scale of the order of the soft supersymmetry breaking masses is required, leaving us with the problem of understanding the origin of such a scale.

Lowering the effective scale of the standard top quark four Fermi interactions reduces the fine-tuning problem [6], but pushes the top quark mass to large values. A solution to this problem was provided by the authors of Ref. [7]. They postulated the existence of a heavy quark, $\chi_R$ and its mirror partner $\chi_L$, with $\chi_R$ carrying the same quantum numbers as the right-handed top quark. The new fermion $\chi$ has a large Dirac mass and the top quark becomes light via an effective see-saw mechanism involving the top and the $\chi$ degrees of freedom. For effective four-Fermi scales of about a few TeV, the model also predicts a heavy Higgs boson state. This does not lead to a conflict with electroweak precision observables since the mixing of the top quark with the heavy state induces modifications to the $\rho$ parameter, which counteract the effects of the heavy Higgs boson present in the spectrum [8].

Since the Dirac mass of the heavy state tends to be quite large (about of 3 to 10 TeV), and the effective theory at energies below this mass is a SM one with a heavy Higgs, it is
obvious that only a very large hadron collider would be able to directly probe the fundamental
degrees of freedom. In this work we show, that by detecting a heavy Higgs and measuring
the precise value of the top quark coupling to the $Z$ gauge boson, a TeV Linear Collider can
provide the means to analyze these models and even predict the presence of a heavy fermion
in the spectrum by establishing its mixing with the top.

**B.** Two Higgs doublet models provide the simplest extension of the SM, and arise naturally in extensions of the top-seesaw scenario described above. As in the SM, there is generically no prediction for the values of the charged, CP-even and CP-odd Higgs bosons (for simplicity we shall assume that there is no violation of CP in the Higgs effective potential). Constraints on the Higgs boson masses can still be achieved from the comparison of the two Higgs doublet predictions with the measured values of the precision electroweak observables. However, while in the SM the Higgs boson needs to be light, no such constraint exists in the two Higgs doublet models. The reason is that the extra degrees of freedom can provide the contributions necessary to compensate for the heavy Higgs boson. This is achieved by splitting the components of the effective non-standard Higgs doublet. Therefore, one can easily imagine a scenario in which all the Higgs bosons are heavy without implying any conflict with the precision electroweak observables. However, the heavier the SM-like Higgs boson, the larger is the effective quartic coupling leading to its mass. Moreover, for a fixed given value of the non-standard contribution to the precision electroweak observables, the heavier the non-standard Higgs boson, the larger the necessary splitting of masses, induced by additional quartic couplings, needs to be. Therefore, a heavy Higgs spectrum, with only a heavy SM-like Higgs at the reach of TeV Linear Collider would imply the presence of very large quartic couplings. In this article we show that requiring perturbative consistency of the theory up to a scale of order of a few TeV demands the presence of at least one SM-like Higgs boson with mass lower than 600 GeV. Moreover, for a SM-like Higgs boson mass larger than 450 GeV, extra non-standard scalar degrees of freedom should appear at a linear collider.

**C.** The mixing of vector-like fermions with the SM quarks can produce important modifications to the precision electroweak parameters if they mix strongly with the third generation quarks. Due to the strong mixing, this could happen even if these extra fermions are relatively light. An example of this case are the Beautiful Mirrors, recently proposed by the authors to resolve the discrepancy between the measured forward-backward asymmetry of bottom quarks and its SM prediction. In one of the two cases studied in Ref. [14], the fit to the precision electroweak data is significantly improved when there is a heavy Higgs boson in the spectrum. This further implies that the new quarks must be lighter than roughly 300 GeV, rendering them easily observable at a Linear Collider, and allowing a measurement of the mixing angle between standard and mirror bottom quarks through the flavor-violating process in which one light and one heavy state are produced.

This article is organized as follows. In section 2 we review the experimental signatures of a heavy Higgs boson with SM-like properties. This implies, for example, that it decays predominantly into pairs of $W$, $Z$, and top. A heavy SM-like Higgs need not be a feature of any theory of physics beyond the Standard Model, but is approximately realized in the three models we proceed to analyze in detail. For definiteness, in the numerical simulations, we treat the Higgs as having precisely the properties of a heavy Higgs in the SM. We show

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1For the case of CP-violation see Ref. [12] and references therein.
that, for the mass range under consideration, the Higgs is a broad resonance and an accurate description of its signatures forbids the decoupling of the production and decay processes (the narrow width approximation). An estimate of the reach of a \( \sqrt{s} = 800 \) GeV collider is given. In section 3 we analyze the means to probe Top Seesaw Models. We show how the interplay of a Higgs search with an accurate measurement of the top quark coupling can shed light on the properties of this model and even predict the mass of a heavy quark with the same quantum numbers as the right-handed top quark. In section 4, we discuss the bounds on the two Higgs doublet model arising from the requirement of perturbative consistency of the theory up to a scale of the order of at least a few TeV combined with consistency with electroweak precision data. In section 5, we discuss the Beautiful Mirrors at a linear collider, and demonstrate that one can measure the mixing between the heavy vector-like quarks and the bottom quark. We reserve section 6 for our conclusions.

2 Heavy Higgs Boson

Since our concern is with theories containing heavy Higgs bosons, we begin by considering how well a heavy SM-like Higgs can be studied at a linear collider. This question is of interest in itself because, in the SM, a Higgs with mass greater than about 350 GeV has an appreciable width: for \( m_H = 350 \) GeV, the width \( \Gamma_H \) is roughly \( 10 \) GeV, and by \( m_H = 600 \) GeV, the width is more than 100 GeV. The reason for this large enhancement of the width lies in the origin of the longitudinal modes of the \( W \) and \( Z \) bosons. For the partial decay width of the a scalar Higgs boson into a pair of on-shell weak gauge bosons, we have

\[
\Gamma(H \to VV) \simeq \frac{G_F(|Q_V| + 1)}{\sqrt{2} 16\pi} m_H^3 \left( 1 - 4\frac{M_V^2}{m_H^2} + 3\frac{4M_V^4}{m_H^4} \right)^{1/2}
\]

where \( Q_V \) is the charge of the \( W \) (\( Z \)) while \( M_V \) is its mass. Therefore, for large Higgs boson masses, the width grows as \( m_H^3 \). This behavior can be easily understood from the equivalence theorem \([15–17]\), which dictates that, in high energy processes, weak bosons may be replaced by the corresponding pseudo-Goldstone bosons (pGB’s). The Higgs coupling to two pGB’s is given by the Higgs quartic interaction times the vacuum expectation value (VEV), \( \lambda v \).

Thus, the partial decay width into weak bosons is proportional (for large Higgs masses) to \( \lambda v^2 / m_H \), where \( m_H \) accounts for the phase space of the decay. Using the tree-level relation between the Higgs mass and \( \lambda \), namely \( m_H^2 = 2\lambda v^2 \), one then finds the partial width into weak bosons to be proportional to \( m_H^3 / v^2 \).

This should be compared to the partial decay widths into fermions, which are only linear in the Higgs mass.

\[
\Gamma(H \to f \bar{f}) \simeq \frac{N_c G_F m_H^2}{\sqrt{2} 4\pi} m_H \left( 1 - 4\frac{m_f^2}{m_H^2} \right)^{3/2}
\]

At these large masses, \( m_H \gtrsim 400 \) GeV, the decay is about 60% into \( W \) pairs, 20% into \( Z \) pairs, and the remainder into \( tt \) pairs\(^2\).

The simple picture of the Higgs as a particle that is produced with some cross section (for example, as \( ZH \)), and then decays into something or other breaks down at large Higgs masses. First, the large width means that, in order to study the production of a Higgs and its

\(^2\)These results conform with those obtained from HDECAY \([18]\).
subsequent decay products, one should take into account interference effects between contributions involving a Higgs boson and the continuum production of the same decay products through diagrams that do not directly involve a Higgs. Indeed, there is no meaningful way to separate the Higgs “signal” from the continuum “background” as is commonly done for narrow resonances and these interference effects may be quite significant. Moreover, approximating the Higgs signatures by a convolution of production and decay processes may lead to quantitatively bad estimates, particularly when the center of mass energy is close to the Higgs production threshold. The situation is further complicated in that the Higgs plays a key role in unitarizing the triple- and quadruple- gauge boson interactions at high energies, and thus the background “without the Higgs” is inconsistent. However, we are not sensitive to this problem, because our initial state consists of massless leptons which force some of the internal vector bosons to be transversely polarized, for which unitarity violations do not occur [16].

Our interest is in the case in which the heavy Higgs is SM-like, and we would like to see how heavy the Higgs might be and yet still be observed and identified as such. Hence we define the background to Higgs production to be that emanating from the full (interfering) set of resonant and non-resonant production graphs in the limit of large Higgs mass, \( m_H \to 1 \) TeV. We have verified that our results for the background do not differ significantly if we choose heavier Higgs masses. This is justified given the fact that our specific observables do not suffer from unitarity violations. In fact, because the unitarization of longitudinal gauge boson scattering occurs order-by-order in perturbation theory, one encounters problems with the Breit-Wigner prescription for the form of the propagator, which sums Higgs self-energy contributions at all orders. We have used the modified Higgs propagator proposed in [19] (see also [20]), which circumvents this problem, and observe that for our results, there is essentially no difference between the modified propagator and the naive Breit-Wigner.

The large branching ratio of \( H \to W^+W^- \) implies that polarization of the initial electron and/or positron beams can be very effective at reducing backgrounds to heavy Higgs searches, since it suppresses radiation of the \( W \) bosons from the \( e^- \) or \( e^+ \) lines, and can reduce these backgrounds by roughly an order of magnitude. Some of the Higgs production modes, such as \( W \)-fusion modes, will also be significantly affected by polarization, so the question as to whether or not polarization is beneficial depends on the production and decay mode of the Higgs under study.

In order to properly analyze the Higgs signatures, it is useful to analyze the naive Higgs production cross sections which, as explained above, can only serve as an indicator of the real production cross section to be obtained when full width effects are taken into account. In Fig. 1 we show the Higgs production cross sections in the modes:

- \( e^+e^- \to ZH \) (\( ZH \) production)
- \( e^+e^- \to \nu_e\bar{\nu}_eH \) (\( W \)-fusion)
- \( e^+e^- \to e^+e^-H \) (\( Z \)-fusion)

for a linear collider of \( \sqrt{S} = 500 \) GeV and 800 GeV, respectively. To demonstrate the effect of beam polarization, we have chosen the ideal case of 100% polarization (right- or left-handed) for both the positron and the electron. The tiny electron mass ensures that only two initial state polarization combinations may contribute in each of the first two cases above. Results
Figure 1: Cross sections for Higgs production as a function of Higgs mass through three different processes at an $e^+e^-$ collider with $\sqrt{s} = 500(800)$ GeV. The ordered subscripts (LR, RL and RR) refer to the (100%) polarizations of the $e^+$ and $e^-$ respectively.

for partially polarized beams may easily be obtained by forming an appropriate weighted sum of the two polarization combinations shown.

Note that the $W$-fusion process tends to be the dominant process with $Z$-fusion being relatively unimportant. While beam polarization is not very important in $ZH$ production, it certainly plays a crucial role in the other two, particularly for $W$-fusion. This is only to be expected as the $e^+_L e^-_R$ initial state cannot support $W$-fusion. Hence, lack of beam polarization would only serve to reduce the total $W$-fusion cross section by an approximate factor of 4.

We proceed now to examine the influence of the large Higgs width on these naive “on-shell” production curves in the context of specific decay modes of the Higgs. This forces us to include the non-resonant graphs that would be associated with the background in a narrow resonance search. We begin with $e^+e^- \rightarrow Z^* \rightarrow ZH \rightarrow ZW^+W^-$ which we expect from Fig. 1 to be an important search mode up to the kinematic limit of $\sqrt{s} - M_Z \geq M_H$. Clearly, this mode benefits from $RL$ polarization of the $e^-$ ($e^+$) which reduces the cross section only modestly, while significantly removing a large portion of the $ZW^+W^-$ non-resonant production. In Fig. 2 we show the production cross section for $RL$ polarization as a function of the collider energy, and for a variety of Higgs masses. For reference, the kinematic limits on production are also shown. The effect of the Higgs width, especially for masses above 500 GeV, is striking. The Higgs contributions begin much earlier than the kinematic limit imposed by the on-shell Higgs approximation. In Fig. 2 we also show the $ZH$ production cross section as a function of the Higgs mass for different center of mass energy. As is evident from the figure, above the kinematic limit, the on-shell approximation
Figure 2: Cross sections for $ZW^+W^-$ production as a function of (a) $\sqrt{S}$ for a variety of Higgs masses at a linear collider, assuming RL polarization of the $e^-$ ($e^+$). The dotted lines corresponding to each curve show the kinematic limit for on-shell Higgs production, and the dashed line indicates the cross section when the Higgs mass is very large, and is thus indicative of the background; (b) as a function of the Higgs mass for different $\sqrt{S}$. The solid and dashed line denote the full three body process simulation and the two body production plus Higgs decay approximation, respectively.

over-estimates the cross section. Higgs production, however, extends well beyond what is expected from the naive on-shell approximation, and therefore, for sufficient luminosity, the Higgs reach may be actually underestimated compared to the real reach once width effects are taken into account.

A very similar story holds for the other dominant Higgs production process, viz. $WW$ fusion. In Fig. 3 we present the cross sections for the three major final states emanating from a heavy Higgs thus produced. Although the cross sections may seem to be significant vis a vis the projected luminosities at a linear collider, the continuum background is very large too. So while discovering a relatively light Higgs may would be easy, the same is certainly not true for $m_H \gtrsim 550$ GeV. It is instructive to look at the relative deviations in the cross sections caused by the presence of the scalar. For the $\nu_e\bar{\nu_e}W^+W^-$ and the $\nu_e\bar{\nu_e}ZZ$ final states, a very large fraction of the cross section arises from non-resonant diagrams. Consequently, this deviation is more pronounced for an initial state starting with the ‘wrong’ polarization ($e^+_L, e^-_R$). Whether this advantage overcomes the drawback of much smaller statistics is of course a matter of a quantitative analysis.

As the preceding discussion demonstrates, the question as to how heavy a Higgs is still amenable to be discovered at a high energy linear collider cannot be answered trivially. Even a semi-realistic assessment of the experimental efficiency needs detailed simulations of the events. While such a study is beyond the scope of this work, it is interesting nevertheless to examine the event kinematics. We focus on the kinematics of the $W$-fusion process (with RL polarization), as it represents, for fixed collider energy, the largest production mode for
the highest Higgs masses, and is thus expected to provide the best hope of discovery for a very heavy Higgs. Apart from indicating possible ways to enhance the signal to background ratio, this will also allow us to gain some insight into the importance of the large width and its impact on the observability of a Higgs with a large mass and (approximately) SM-like interactions.

While angular distributions of the final state particles do bear the mark of a scalar, we find that the invariant mass distribution of the Higgs ‘decay products’ is perhaps the best discriminant. This is not unexpected. Rather, this is the very variable that one would have concentrated on for a resonant production process. In Figs. 4 we present the corresponding distributions for each of the processes considered in Fig. 3. We note that polarization can widely influence the signal to background ratio, and thus can play an essential role in heavy Higgs studies at the LC. We further see a general trend that while a clear resonant peak is obtained for a Higgs mass of 500 GeV, for 600 GeV it is much smaller and wider. And for 650 GeV, a peak is almost non-existent with the excess spread out almost evenly.

In Table 2 we show the signal and background cross sections associated with different final states consistent with the weak-fusion production of a heavy Higgs at an 800 GeV linear collider. The optimal mass window for the discovery of such a heavy Higgs is displayed and the luminosity required for 3$\sigma$ evidence or a 5$\sigma$ discovery of the heavy Higgs boson is displayed for different choices of the electron and positron polarization. We stress again that careful study, including efficiencies and modeling of detector effects, is needed to draw firm conclusions, but it seems reasonable to expect that a SM-like Higgs with mass below 650 GeV could be identified in its principle decay modes from 100 fb$^{-1}$ of data at a polarized 800 GeV linear collider. This is to be compared with the expected discovery reach of up to 1 TeV by the LHC [21]. Of course, once the Higgs has been discovered at the LHC, it will be up to the linear collider to carefully study its properties, and thus the question of its observability there remains of the utmost importance.

Figure 3: Cross sections for $e^+e^- \rightarrow \nu_e\bar{\nu}_eW^+W^-$ (solid), $e^+e^- \rightarrow \nu_\ell\bar{\nu}_\ell ZZ$ (dashed) and $e^+e^- \rightarrow \nu_\ell\bar{\nu}_\ell t\bar{t}$ (dotted) as a function of the Higgs mass. The upper and lower sets correspond to ($e_R^+,e_L^-$) and ($e_L^+,e_R^-$) initial states respectively.
Figure 4: Invariant mass distribution of the ‘visible’ part in the processes $e^+ e^- \rightarrow \nu \nu W^+ W^-$, $\nu \nu ZZ$ and $\nu \nu t\bar{t}$ in the presence of a Higgs. Three representative masses are chosen. Also shown are the predictions in the “no Higgs” limit ($m_H = 1$ TeV). For each case, two different choices of positron and electron polarizations have been used.
Table 1: The optimal mass windows that maximize $S/\sqrt{B}$ for particular final states emanating from a given polarization combination for the $e^+e^-$ pair. Also shown are the luminosities required for obtaining a “3σ” and a “5σ” excess within the mass window. Within the given window, a 100% efficiency is assumed.

3 Top Seesaw Model

We now study a promising model of dynamical EWSB with a heavy Higgs that can be consistent with precision electroweak data: the Top Seesaw. Its simplest implementation starts with the gauge group $SU(3)_1 \times SU(3)_2$, which is broken to $SU(3)_c$ (QCD) by the introduction of a bi-fundamental field $\Phi$, which parameterizes the unspecified Topcolor-breaking dynamics. The left-handed component of the top quark transforms in the fundamental representation of $SU(3)_1$, while the right-handed component transforms in the fundamental representation of $SU(3)_2$. An additional singlet fermion $\chi$ has left- and right-handed components which transform in the fundamental representation of $SU(3)_2$ and $SU(3)_1$, respectively. After the breakdown of the strong gauge groups to $SU(3)_c$, mass terms involving the left-handed component of $\chi$ with the right handed components of $\chi$ and $t$ are allowed. We shall called these mass terms $m_{\chi\chi}$ and $m_{\chi t}$ respectively, and they are naturally of order of the scale of breaking of the strong gauge groups. Only one Dirac fermion acquires mass, with left handed component given by $\chi_L$ and right handed component given by a combination of $\chi_R$ and $t_R$.

The massive strong vector bosons (colorons) induce four-Fermi interactions involving the $t_L$ and $\chi_R$ currents. There are expected to be other interactions, but for natural values of
$m_{\chi t}$, they do not lead to any relevant phenomena in the process of electroweak symmetry breaking. After a Fierz transformation, the relevant interaction reads

$$\mathcal{L}_{\text{int}} \simeq \frac{g^2}{M^2} \left( \overline{\psi}_L \chi_R \right) \left( \overline{\chi}_R \psi_L \right)$$  \hspace{1cm} (3)

where $g$ is the strong Topcolor interaction strength, $M$ is the mass of the heavy coloron gauge boson, and $\psi_L = (t, b)_L$. For appropriate values of $\kappa = g^2/4\pi$, the model undergoes a spontaneous breakdown of the electroweak symmetry. In the massless limit, $m_{\chi\chi}, m_{\chi t} \to 0$, the Nambu-Jona-Lasinio (NJL) critical coupling for spontaneous symmetry breaking \cite{22} is given by $\kappa_c = 2\pi/N_c$. The effect of nonzero $m_{\chi\chi}$ and $m_{\chi t}$ is to require a somewhat larger coupling for EWSB \cite{11}. However, taking the Topcolor coupling arbitrarily strong represents fine-tuning, in the sense that one is forcing two a priori unrelated quantities (the Topcolor breaking scale $M$ and the scale at which Topcolor would have confined had it remained unbroken) to be very close to one another. Thus it is more natural to assume that the Topcolor interaction is only slightly super-critical. For definiteness, we will confine our discussion to $\kappa/\kappa_c < 1.2$.

The immediate effect of the dynamical symmetry breaking is to induce a mass $\mu_{t\chi}$ mixing the left-handed component of $t$ with the right handed component of $\chi$. The diagonalization of the fermion mass matrix leads to two mass eigenstates; a heavy one of mass of order $m_{\chi\chi}$ and a light one, which should be identified with the top quark. In order to distinguish the interaction eigenstates from the mass eigenstates, we henceforth denote the interaction eigenstates as primed fields, i.e. $t'$, and the physical (mass) eigenstate fields as unprimed.

The mass parameter $\mu_{t\chi}$ required to generate appropriate $W$ and $Z$ masses is approximately given by the value that the top quark mass would acquire in the absence of the fermion $\chi$, starting with a large Yukawa coupling value at the scale $\Lambda \simeq \mathcal{O}(M)$. This value is of order 700 GeV for $\Lambda$ of order of a few TeV. For $m_{\chi\chi} \gg m_{\chi t} \gg \mu_{t\chi}$, the top quark mass is given by

$$m_t \simeq \frac{\mu_{t\chi} m_{\chi t}}{m_{\chi\chi}}.$$  \hspace{1cm} (4)

It is clear that the value of the top quark mass defines the ratio of the parameters $\mu_{t\chi}$ and $m_{\chi\chi}$. A detailed analysis of the parameter space of $\Lambda$, $m_{\chi\chi}$, and $m_{\chi t}$ which provide acceptable $W$, $Z$, and $t$ masses may be found in \cite{11}. These parameters also determine the mixing of the right- and left-handed components of the top and the heavy quark. Because $t'_L$ is a component of a weak doublet while $\chi'_L$ is a weak singlet, the left-mixing has a direct influence on the interaction of the physical top with the weak bosons. On the other hand, the right-handed mixing is between two states with the same quantum numbers, and thus has no impact on the coupling to the gauge fields.\footnote{The right-handed mixing does have a slight impact on the coupling with the composite Higgs boson(s), though generally the deviation is at the level of per cent, too small to measure at a linear collider \cite{23}.}

### 3.1 Topcolor Confronts Precision Electroweak Observables

In addition to the massive fermions, there is generally a heavy composite (predominantly of $t_L$ and $\chi_R$) scalar particle which plays the role of the Higgs in the low energy effective theory. For natural values of the Topcolor interaction strength (that is, values close to the critical value required for EWSB), the Higgs boson mass may be computed to be roughly
between 400 and 600 GeV \[11\]. One can understand this result as the value the mass would have if the quartic coupling would become strong at the scale \( \Lambda \), about 600 GeV for \( \Lambda \) of a few TeV.

A heavy Higgs boson seems to be at variance with the measured values of the precision electroweak observables \[24\]. However, it is also important to consider the combined radiative effects of the Higgs and the see-saw on the precision data, which can be neatly summarized in terms of the oblique correction parameters, \( S \), \( T \) (or equivalently, \( \Delta \rho \)), and \( U \) \[25\]. The left-handed mixing between \( t' \) and \( \chi' \) leads to a modified top-quark contribution to the \( T \) parameter. The new contribution is approximately given by replacing \( m_t^2 \) in the expression for \( T \) with the effective value, viz.

\[
\left( m_t^{\text{eff}} \right)^2 = s_L^4 M^2_\chi + c_L^4 m_t^2 + 2 s_L^2 c_L^2 m_t^2 \ln \left( \frac{M^2_\chi}{m_t^2} \right)
\]

\[
\simeq m_t^2 + s_L^4 M^2_\chi + 2 s_L^2 m_t^2 \left[ \ln \left( \frac{M^2_\chi}{m_t^2} \right) - 1 \right]
\]

where \( M_\chi \simeq m_{\chi\chi} \) is the value of the heavy quark mass and the mixing angle \( \theta_L \) is given by

\[
s_L \equiv \sin \theta_L \simeq \frac{\mu_{t\chi}}{m_{\chi\chi}}.
\]

Thus, the model leads to a new, positive, fermionic contribution to the \( T \) parameter, which can overcome the negative contribution induced by the presence of the heavy Higgs boson. In contrast, the fermionic contribution to the \( S \) parameter is very small, while the heavy Higgs boson (in comparison with the light Higgs seemingly favored by the data in the absence of further ingredients) produces a positive contribution. The net effect is that the fermions produce a slightly greater shift in \( T \) than the shift associated with the heavy Higgs. Together with the requirements that both EWSB and the correct top-quark mass are obtained, consistency with precision observables further constrains the mass of \( \chi \) to lie between roughly 3.8 and 7 TeV, for natural Topcolor models \[11\]. Such large masses, even for strongly interacting particles, are difficult to detect at either the LHC and a TeV linear collider.

The top coupling with the \( Z \) boson, relative to its SM prediction is,

\[
\frac{\delta g_L}{g_L} = \frac{s_L^2}{1 - 4 \sin^2 \theta_W/3}.
\]

For the viable range of parameters discussed above, the resulting shift in the coupling ranges between about 5% and 1%. A Linear Collider will provide an accurate determination of the top couplings to the weak vector bosons, by measuring the top quark production at threshold. Recently, there has been great theoretical progress in understanding this process, and it is safe to say that the typical deviations expected in the simplest Topcolor models, as described above, are far larger than the present theoretical uncertainties within the SM, with the latter being presently estimated to be less than about 1.5 percent \[27\].

\[4\] A similar mechanism renders the Topflavor Top Seesaw model \[26\] consistent with precision data by mixing top and bottom with heavy vector-like weak doublets, and predicts a Higgs mass above 200 GeV.
The values given above should be therefore be compared with the experimental precision on these couplings expected at a $\sqrt{s} = 500$ GeV TESLA linear collider [28], which are approximately 1.8 and 1.4 percent for the axial and vector couplings, respectively. Fig. 5 shows the predictions for the variation of the left handed coupling of the top quark to the $Z$ boson as a function of the heavy quark mass. These values are compared with the values of the precision measurement parameter $\Delta T$ necessary to reproduce the precision electroweak data within the minimal Top Seesaw model. In the region of parameters consistent with experiment, $\delta g_L/g_L$ varies from somewhat more than one up to about five percent. As the figure shows, such variations are testable at a 500 GeV collider for most of the allowed parameter space.

A TeV linear collider will also be able to discover the heavy Higgs boson. The determination of the Higgs boson mass together with the precise measurements of the top quark couplings to the weak gauge bosons would serve to identify the two key ingredients of the Top Seesaw mechanism of EWSB: namely, a heavy Higgs boson, and extended interactions of top. The knowledge of the Higgs mass and top coupling deviations would provide a prediction for a range of $\chi$ masses compatible with the Top Seesaw model. Therefore, a TeV Linear Collider will become a Top Seesaw analyzer.
4 Two Higgs Doublet Model

A second example of a model in which the precision electroweak observables may be satisfied, even in the presence of a heavy SM-like Higgs boson, is a two Higgs doublet model. As we explained in the introduction, in this model, the contribution to the parameter $\Delta \rho$ accruing from the second Higgs doublet may be compensated by the one from the heavy SM Higgs, bringing the predictions for the precision electroweak observables back within the experimental $1\sigma$ contour.

A generic two Higgs doublet model, of course, is characterized by quite a few parameters. Of particular interest to us are $\tan \beta$ (the ratio of the two vacuum expectation values), the Higgs mixing angle $\alpha$, and the four masses: $m_A$ (the pseudoscalar), $m_{H^\pm}$ (the charged Higgs) and those for the light and heavy CP-even neutral states namely $m_h$ and $m_H$. Along with the common mass parameter $(\mu_{12}\phi_1^\dagger\phi_2)$, these determine all the properties of the Higgs including their self interactions.

The relevant question for such models relate to the observability of at least one of the non-standard Higgs bosons (along with the SM-like Higgs) at a TeV linear collider. If the mass of the SM-like Higgs boson is close to the present experimental bound on this quantity, $m_{H^{SM}} \geq 114$ GeV, then the non-standard Higgs bosons can obviously be arbitrarily heavy, provided they are sufficiently degenerate. On the other hand, if the mass of the SM-like Higgs boson is large,

$$M \equiv m_h^2 \sin^2(\alpha - \beta) + m_H^2 \cos^2(\alpha - \beta) \geq m_{\text{bound}}^2,$$

with $m_{\text{bound}}$ larger than about 400 GeV, then the extra contributions to the precision electroweak observables demand either light non-standard Higgs bosons, observable at a TeV Linear Collider, or very large mass splittings between the second-Higgs doublet components. This can be understood by noting that, for mass differences $\Delta m^2$ smaller than the average mass $m^2$, the contributions to the parameter $\Delta \rho$ are proportional to

$$\Delta \rho \sim \frac{(\Delta m^2)^2}{m^2},$$

and therefore, the larger $m^2$ is, the larger $\Delta m^2$ should be in order to produce the same effect. The precise one-loop contributions to the parameters $S$ and $T$ are given by [13, 29, 30]

$$\Delta \rho = \frac{\alpha}{16\pi \sin^2 \theta_W m_W^2} \left\{ \cos^2(\beta - \alpha) \left[ f(m_A, m_{H^\pm}) + f(m_{H^\pm}, m_h) - f(m_A, m_h) \right] ight.$$  

$$+ \sin^2(\beta - \alpha) \left[ f(m_A, m_{H^\pm}) + f(m_{H^\pm}, m_H) - f(m_A, m_H) \right] \right\}$$  

$$+ \cos^2(\alpha - \beta) \Delta \rho_{SM}(m_H) + \sin^2(\alpha - \beta) \Delta \rho_{SM}(m_h) - \Delta \rho_{SM}(m_{H^{SM}})$$

with

$$f(x, y) = \frac{x^2 + y^2}{2} - \frac{x^2 y^2}{x^2 - y^2} \log \frac{x^2}{y^2}$$

and

$$\Delta \rho_{SM} = \frac{3\alpha}{16\pi \sin^2 \theta_W m_W^2} \left[ f(m, m_Z) - f(m, m_W) \right] - \frac{\alpha}{8\pi \cos^2 \theta_W}$$ (12)
and where we have subtracted the SM contribution. Similarly,

\[
S = \frac{1}{12\pi} \left\{ \cos^2(\beta - \alpha) \left[ \log \frac{m_H^2}{m_{H^\pm}^2} + \log \frac{m_h m_A}{m_{H^\pm}^2} \right] \\
+ 2 \frac{m_h^2 m_A^2}{(m_h^2 - m_A^2)^2} + \frac{(m_h^2 + m_A^2) (m_h^4 + m_A^4 - 4m_h^2 m_A^2)}{(m_h^2 - m_A^2)^3} \log \frac{m_h}{m_A} \right\} \\
+ \sin^2(\beta - \alpha) \left[ (m_h \leftrightarrow m_H) \right] - \frac{5}{6} \right\}
\] (13)

Now, large splittings between the masses of the different non-standard Higgs bosons can only be obtained via large values of some of the quartic couplings. Similarly, a large value for the effective SM-like Higgs mass (eq. $S$) is again obtained only through large values of the quartic couplings. Thus, the requirement of perturbative consistency of the model up to some high energy scale $\Lambda_{\text{new}}$ serves to put constraints on the Higgs masses. Since we are interested in the case of a heavy Higgs boson, with mass $O(500 \text{ GeV})$, we should demand that the theory be perturbative up to at least a few TeVs.

The requirement of perturbative consistency of the theory has two important effects. On the one hand, it sets a limit on the SM-like Higgs boson mass. On the other, it sets a limit on the possible splitting of masses of the non-standard Higgs masses and therefore on the average mass of the non-standard Higgs bosons for a given value of their contribution to the precision electroweak observables.

The bound on the SM-like Higgs boson is enhanced for smaller values of the cutoff scale. The bound on the non-standard Higgs boson masses is correlated with the exact value of the Higgs boson mass within the allowed range: the larger the SM-like Higgs boson mass, the larger the necessary new contributions to the precision electroweak observables. For a given value of the average mass of the non-SM Higgs boson, these large contributions can only be obtained via larger quartic couplings and, since these are restricted by the perturbative bound, a stricter upper bound on the non-standard Higgs boson masses is obtained.

In Fig. 6 we show, for $\Lambda_{\text{new}} = 10 \text{ TeV}$ and $\Lambda_{\text{new}} = 2 \text{ TeV}$, the necessary center of mass energy of a linear collider to ensure the observability of at least one non-standard Higgs boson, as a function of the SM-like Higgs boson mass. To obtain this figure we demand that

- The values of the predicted $S$ and $T$ parameters lie within the 90\% C.L. ellipse about the experimentally measured values;

- None of the five quartic couplings should become nonperturbative (under one-loop renormalization group evolution) at any scale below $\Lambda_{\text{new}}$ for $\alpha_S(M_Z) = 0.118$ and $m_{\text{top}} = 175 \text{ GeV}$;

- Since the theory is only an effective one below $\Lambda_{\text{new}}$, the masses should all be much lighter than $\Lambda_{\text{new}}$; we use the rather loose criterion suggested by Hasenftatz et al. [31], namely $M^2 < \Lambda_{\text{new}}^2/2$ for all particles in the spectrum;

- To be considered observable, the Higgs bosons must be produced in pairs ($H^+H^-$, $HA$, $hA$) or in conjunction with a gauge boson ($hZ$, $HZ$) with a cross section larger than 0.1 fb (corresponding to 50 events with 500 fb$^{-1}$).

$^5$A similar statement also holds for the one doublet case (SM).
The last condition is somewhat heuristic, and would be improved by detailed simulation of each production mode, including Higgs decays and backgrounds. Such details for the exotic Higgses are highly model-dependent, and beyond the scope of this work.

For values of the Higgs boson mass somewhat larger than 200 GeV, a non-trivial bound on the non-standard Higgs bosons develops, provided the only source of new physics with a relevant impact on precision electroweak observables is an additional Higgs doublet. This bound is quite weak for small values of the SM-like Higgs boson mass, but becomes stronger as this mass increases. For the values of the cutoff scale considered, $\Lambda_{\text{new}} \geq 2$ and 10 TeV, the effective Higgs boson mass $M$ cannot be larger than about 550 and 500 GeV, respectively. Therefore, the first conclusion obtained from such considerations is that at least one of the Higgs bosons will be observable at a TeV Linear Collider. The second conclusion, which can be extracted from the results displayed in Fig. 6, is that for $\Lambda_{\text{new}} \gtrsim 10$ TeV and SM Higgs boson masses larger than 300 GeV, a linear collider with center of mass energy of about 1.2 TeV will be enough to ensure the observability of at least one non-standard Higgs boson. Observe that for $M \gtrsim 450$ GeV, both CP even Higgses may be produced in association with a $Z$ boson, implying values of $\sin^2(\alpha - \beta)$ significantly different from one.

It might be be argued that the fourth requirement imposed in obtaining Fig. 6 is unnecessarily restrictive. The Higgs particles could as well be produced in association with fermions, particularly the bottom and the top. As is well known, such couplings may be significantly enhanced over their SM counterparts. However, we have desisted from considering these processes, simple as they are, in view of the additional model dependence. The results of Fig. 6 are thus conservative and the actual value of $\sqrt{s}_{\text{min}}$ may be significantly smaller within a given scenario of Higgs-fermion couplings.
5 Beautiful Mirrors

Until now, we have discussed the constraints coming from the precision electroweak data in terms of the oblique parameters S, T and U. The LEP and SLD precision electroweak measurements are also sensitive to non-oblique corrections. The most important one of these is related to the couplings of the left- and right-handed bottom quark to the neutral gauge boson. In particular, measurements of the partial hadronic width of the Z boson decays into bottom quarks and of the forward-backward and left-right asymmetries of bottom quarks at the Z peak have been obtained, which have led to a precise measurement of the bottom quark couplings in terms of the electroweak mixing angle.

The forward-backward bottom quark asymmetry measured at LEP ($A_{FB}^b$) presents a $2.9 \sigma$ deviation from the best fit value within the SM. Such a large deviation becomes particularly significant, since it pushes the Higgs boson mass toward values consistent with the constraints coming from direct searches. Indeed, if the hadronic asymmetries were ignored, the best fit to the Higgs boson mass would lead to a value which is well below the present experimental bound. Probably equally significant is the fact that if this deviation is to be explained by a modification of the effective couplings of the bottom quarks to the Z boson, the absolute value of the coupling of the right-handed bottom quark should be 26 percent larger than the value predicted in the SM. Such a large deviation can be obtained in a natural way via tree-level mixing of the bottom quark with new, heavier quarks, with the same charge and color quantum numbers as the bottom quark.

In Ref. [14], we performed an analysis of the possible impact on the fit to the electroweak precision data proceeding from the presence of additional vector-like quarks in the spectrum. We showed that a significant improvement in the fit to the precision electroweak observables could be obtained by demanding a correlation between the Higgs boson and the new quark masses. Moreover, we showed that the introduction of the new degrees of freedom may lead to an improvement of the condition of unification of couplings, at a sufficiently large scale in order to avoid a conflict with proton decay constraints.

The details of these models are in Ref. [14]. Here we concentrate on one of the two possibilities studied in [14], which leads to the presence of a heavy Higgs boson in the spectrum. In this case, apart from the heavy Higgs boson, with masses in the range 250 to 500 GeV, one encounters a new doublet of quarks and its mirror partner, with masses lower than 300 GeV. The doublet contains a new top-like quark ($\chi$) and a bottom-like quark ($\omega$), with vector-like weak interactions. The right-handed $\omega$ mixes with the right-handed bottom quark, with a large mixing angle

$$\sin^2 \theta_R \simeq 0.3,$$

resulting in predictions for $A_{FB}^b$ consistent with measurements.

The new bottom and top quarks will be produced and observed at the Tevatron collider. However, a hadron collider will have difficulty measuring the bottom mixing angle, which is only relevant for the weak interactions, and is thus likely to be swamped by large hadronic backgrounds. A linear collider can directly measure the mixing by observing the flavor violating process:

$$e^+e^- \rightarrow Z^* \rightarrow b\omega$$

which is directly proportional to the mixing parameters $\sin^2 \theta_R \cos^2 \theta_R$. The dominant decay mode of the $\omega$ is also through this mixing, $\omega \rightarrow Zb$ with a branching ratio of nearly one.
For a sufficiently heavy Higgs boson, decay modes involving the beautiful mirrors such as $H \rightarrow \omega b$ (or $\bar{b}\omega$) may be allowed. For the relevant model parameters consistent with precision electroweak data, the partial widths into these modes are less than the width into top quarks. Therefore, the branching ratio of Higgs into gauge bosons and top quarks will generally differ from the SM predictions by about 10%.

In order to estimate how well a linear collider could measure $\sin \theta_R$ through this process, we simulate both signal and background (including a 300 GeV Higgs boson appropriate for this model) and apply simple kinematic cuts to simulate the detection efficiency of the bottom quarks, $p_T^b \geq 15$ GeV and $|y^b| \leq 3$. One can further reduce the background by applying a very loose cut on the $\omega$ mass, requiring that the invariant mass of the $Z$ with one or the other of the bottoms is within 50 GeV of $m_\omega$. We do not assume a specific decay mode of the $Z$ boson. In Fig. 7, we present the resulting signal cross sections for three values of $m_\omega$ and the background after applying the cuts described above. Comparison of the signal rates with the background indicates that signal rates equal to or greater than the background are obtained for collider energies greater than roughly $m_\omega + 100$ GeV, with cross sections greater than 50 fb. Thus the entire parameter space favored by precision data can be studied at a 500 GeV linear collider.

We have chosen a large invariant mass window of $m_\omega \pm 50$ GeV, which Fig. 7 indicates is sufficient to discover the flavor-violating signal. The physical width of the $\omega$ is of the order of a few GeV, and thus once the signal is found, the cut on the invariant mass may be tightened. Once it is less than 20 GeV, still much larger than the expected jet resolution, the background becomes essentially zero at all energies, and the signal remains unchanged. We estimate the precision of the measurement of the cross section from its statistical uncertainty and find that with 100 fb$^{-1}$ we can measure $\sin^2 \theta_R$ at an accuracy of 2.3% when $\sqrt{s} = 500$ GeV and to 2.6% when $\sqrt{s} = 800$ GeV for all $\omega$ masses below 300 GeV. Together with the observation of the heavy Higgs, this would establish quite precisely the beautiful mirror interpretation of present-day electroweak data.

### 6 Conclusions

Precision electroweak observables suggest the presence of a light SM-like Higgs boson, with mass close to the present experimental limits. The study of such a light Higgs is one of the primary motivations for a TeV scale linear collider. However, the nature of indirect constraints is such that the simplest explanation need not be the correct one, and thus it behooves us to consider the alternative that the Higgs is heavy. Models with heavy Higgs bosons are able to obtain consistency with the data by introducing new contributions to the precision electroweak observables, which compensate the additional contribution induced by the heavy SM-like Higgs, restoring the agreement between the theoretical predictions and the experimental results.

From the phenomenological point of view, the analysis of the signatures associated with a heavy Higgs presents interesting challenges, since it will be associated with a broad resonance that cannot be described by the simple production and decay signatures usually used in the narrow width approximation. We have presented possible ways of disentangling these

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6While we choose to display our results for this particular value of the Higgs mass, the conclusions are quite robust and do not change qualitatively for $250$ GeV $\leq m_H \leq 600$ GeV.
signatures at a linear collider and provided an estimate of the possible reach of a 800 GeV linear collider in the most important channels.

In this article, we have also studied some the most natural models leading to the presence of a heavy SM-like Higgs boson in the spectrum: the simplest Topcolor model, a two Higgs doublet extension of the SM and an extension with vector-like quarks which lead to an improvement in the fit of the measured couplings of the bottom quark to the $Z$ gauge boson. In the first case, we show that, although the new physical degrees of freedom may remain beyond the reach of even the LHC, the linear collider has the potential of testing these models via the measurement of the Higgs mass and the coupling of the top quark to the massive weak gauge bosons. In the second case, we have shown that, if we demand perturbative consistency of the theory up to a few TeV scale, at least one Higgs boson should appear in the spectrum and, if the SM-like Higgs boson is heavy, with mass larger than 500 GeV, at least one non-standard Higgs boson should appear at the reach of a TeV linear collider. Finally, in the model with extra light vector-like quarks, a precise measurement of the associated flavor violating processes will lead to an accurate measurement of the non-trivial mixing of these quarks with the third generation quarks and therefore provide a crucial test of this kind of model.

Figure 7: Cross sections for $e^+e^- \rightarrow \omega b$ ($\omega b$) production as a function of collider energy (solid curves), for three $\omega$ masses consistent with the beautiful mirrors explanation of the LEP data. The dashed line indicates the background from $e^+e^- \rightarrow Zb\bar{b}$ (including a Higgs of mass 300 GeV). All cross sections are for unpolarized beams and the cuts described in the text.
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\sigma (\text{fb})

\sqrt{s} = 500 \text{ GeV}
\( \sigma (fb) \)

\[ m_H (GeV) \]

\( (\nu_e \nu_e H)_{RL} \)

\( (ZH)_{RL} \)

\( (ZH)_{LR} \)

\( (v_e v_e H)_{LR} \)

\( (e^+ e^- H)_{RL} \)

\( (e^+ e^- H)_{RR} \)

\( (e^+ e^- H)_{LR} \)

sqrt(s) = 800 GeV