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Citation for published version (APA):
Nesti, T., Sloothaak, F., & Zwart, A. P. (2020). Emergence of Scale-Free Blackout Sizes in Power Grids. Physical Review Letters, 125(5), [058301]. https://doi.org/10.1103/PhysRevLett.125.058301

DOI:
10.1103/PhysRevLett.125.058301

Document status and date:
Published: 31/07/2020

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
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Download date: 23. Jul. 2023
Emergence of Scale-Free Blackout Sizes in Power Grids

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(Received 12 September 2019; revised 7 June 2020; accepted 10 July 2020; published 31 July 2020)

We model power grids as graphs with heavy-tailed sinks, which represent demand from cities, and study cascading failures on such graphs. Our analysis links the scale-free nature of blackout sizes to the scale-free nature of city sizes, contrasting previous studies suggesting that this nature is governed by self-organized criticality. Our results are based on a new mathematical framework combining the physics of power flow with rare event analysis for heavy-tailed distributions, and are validated using various synthetic networks and the German transmission grid.

DOI: 10.1103/PhysRevLett.125.058301

Securing a reliable power grid is of tremendous societal importance due to the highly disruptive repercussions of blackouts. Yet the study of cascading failures in power grids is a notoriously challenging problem due to its sheer size, combinatorial nature, mixed continuous and discrete processes, and physics and engineering specifications [1–5]. Traditional epidemics models [6–9] are unsuitable for its study, as the physics of power flow are responsible for a nonlocal propagation of failures [10]. This challenge has created extensive interest from the engineering and physics communities [11–17]. Analytic models determining the blackout size ignore the microscopic dynamics of power flow, while the analysis of more realistic networks typically does not go beyond simulation studies. Therefore, a fundamental understanding of blackouts is lacking.

The total blackout size, measured in terms of number of customers affected, is known to be scale free [18–21], meaning there exist constants C, α > 0 such that

$$P(S > x) \approx Cx^{-\alpha}, \quad (1)$$

where $\approx$ means that the ratio of both quantities approaches 1 as $x \to \infty$. This law, also known as the Pareto law, occurs in many applications of science and engineering [22–26]. Its significance in our context lies in the fact that big blackouts are substantially more likely than one would infer from more conventional statistical laws. As a result, mitigation policies cannot write off extremely large blackouts as virtually impossible events, and should focus on those in equal proportion to the small, frequent ones. Given the tremendous societal impact of large blackouts, understanding why Eq. (1) occurs can lead to focused prevention and/or mitigation policies and is therefore of major significance.

Several attempts to explain Eq. (1) have appeared in the literature. Using simulations, previous studies suggest that Eq. (1) may occur as a consequence of self-organized criticality [1,18,19,27,28]. Specifically, Ref. [18] compares simulation traces of a model for blackouts with those of a model that is known to exhibit self-organized criticality, and shows that the autocorrelation functions are similar. Such indirect analogies of different observables do not provide direct explanations into the precise mechanism behind Eq. (1).

Other strands of literature model the cascading mechanism as a branching process with critical offspring distribution [29], without taking physical laws of electricity into consideration. Such models lead to blackout sizes with infinite mean, corresponding to a value of $\alpha = 0.5$. While a naive parametric estimation procedure using all data would lead to values of $\alpha$ in the range (0,1), modern statistical techniques focusing on the tail end of the distribution clearly indicate a finite mean blackout size [20,21].

In this letter, we propose a radically different and much simpler explanation than the aforementioned suggestions. Our central hypothesis is that Eq. (1) is inherited from a similar law for the distribution of city sizes [26,30–32]. We support this claim with a careful analysis of actual data, a new mathematical framework, and supporting simulations for additional insight and validation.

To develop intuition, we view the power grid as a connected graph where nodes represent cities, which are connected by edges modeling transmission lines. Initially, this is a single fully functioning network with balanced supply and demand. After several line failures, the network breaks into disconnected subnetworks, referred to as islands. The balance between supply and demand is not guaranteed to hold in each island, and at least one island is facing a power shortage. As the sum of total demand will be proportional to the total population in the island, the size of the power shortage is proportional to the total population, which is the sum of cities in that island. We now invoke a property of sums of Pareto distributed random variables, which informally says that the sum is dominated by the...
Our formulation may be extended to handle multiple
initial failures, correlated city sizes, generator failures,
simultaneous failures, generation limits, other strictly con-
vex supply cost functions, and other load-shedding mech-
anism. Such variations would affect the value of the
prefactor \( C \), but not the exponent \( \alpha \): the tail of the blackout
distribution is dominated by the scenario where there is a
single city that has a large power demand, while the
demand of the other cities is negligible. A formal version
of this statement is that, for sufficiently small \( \epsilon \),

\[
\min_{\mathbf{f}} \frac{1}{2} \sum_{i=1}^{n} g_i^2 \quad \text{such that } \sum_{i=1}^{n} g_i = \sum_{i=1}^{n} X_i,
\]

subject to the reliability constraint

\[
-\bar{f} \leq \mathbf{V}(\mathbf{g} - \mathbf{X}) \leq \bar{f}.
\]

The planning stage concerns how the operational line limits
\( \bar{f} \) are set. For this, we solve Eq. (2) without Eq. (3), yielding
the uniform (across cities) solution \( g_j^{(\text{op})} = (1/n) \sum_{i=1}^{n} X_i \)
for all \( j \geq 1 \), and \( \bar{f}^{(\text{op})} = -\mathbf{V}X \) (see Ref. [35], Sec. IV).

Then, the operational line limits \( \bar{f} \) are set as

\[
\bar{f}_\ell = \lambda |f_\ell^{(\text{op})}| = \lambda |(\mathbf{V}X)_\ell|, \quad \ell = 1, \ldots, m,
\]

where \( \lambda \in (0, 1] \) is a safety tuning parameter, referred to as loading factor. In the operational stage, we solve Eq. (2)
subject to Eq. (3), yielding a different solution \( g^{(\text{op})} \) which is not uniform due to the constraint (3). Equation (4)
implies that line flows can have a heavy tail, which is consistent with impedance data [53]. This property is
essential, as it allows us to create a subnetwork in which the
mismatch between supply and demand is heavily tailed.

This mismatch is established in the emergency stage,
which is described next. We focus on cascades initiated by
a single line failure, sampled uniformly across all lines. A
line failure changes the topology of the grid and causes a
global redistribution of network flows according to power
flow physics. Consecutive failures occur whenever there are
one or more lines for which the redistributed power flow
exceeds its emergency line limit \( F_\ell = \bar{f}_\ell / \lambda \). Failures are
assumed to occur subsequently, and take place at the line
where the relative exceedance is largest. Whenever line
failures create additional islands, we proportionally lower
either generation or demand at all nodes to restore power
balance. The cascade continues within each island until
none of the remaining emergency line limits are exceeded
anymore.

Our framework consists of three stages called planning,
operational, and emergency. The first two stages determine
the actual line limits and line flows. We employ the widely
used direct current optimal power flow (dc OPF) formulation
with quadratic supply cost functions [1]:

\[
\min_{\mathbf{g}} \frac{1}{2} \sum_{i=1}^{n} g_i^2 \quad \text{such that } \sum_{i=1}^{n} g_i = \sum_{i=1}^{n} X_i,
\]

subject to the reliability constraint

\[
-\bar{f} \leq \mathbf{V}(\mathbf{g} - \mathbf{X}) \leq \bar{f}.
\]
We illustrate this in Fig. 2, which also shows how the property of Pareto tails is realized by means of a few load-shedding events, each of which is a fixed fraction of the largest node, and corresponds to a network disconnection.

Our analysis illustrates how heavy-tailed city sizes cause heavy-tailed blackout sizes. Our modeling choices allow for a precise exploration of the cascade sequence, and, inherently, an explicit formula for the blackout size tail. However, we emphasize that the essential elements that lead to heavy-tailed blackout sizes are that both the demands and the line limits are heavy tailed. The small nodes together generate a non-negligible fraction of the demand of the large node. When the power grid satisfies these properties, then Eq. (5) continues to hold, leading to a heavy-tailed mismatch whenever there is a disconnection. We illustrate this numerically by studying the effect of relaxing several assumptions in our framework.

The most delicate step, for which Ref. [35], Sec. IV.D provides a rigorous proof, is to show that the cascade sequence does not change when performing the normalization argument in the limit $x \rightarrow \infty$, which is nontrivial due to continuity issues.

In Ref. [35], Sec. IV, we show that the prefactor $C$ in Eq. (7) is discontinuous at a discrete set of values of $\lambda$. At such points, the number of possible scenarios leading to a large blackout is increasing and/or $|A_1|$ is decreasing in $\lambda$. We illustrate this in Fig. 2, which also shows how the principle of a single big jump (5), which links the total blackout size to the size of the largest city $X_1$, is realized by a few load-shedding events, each of which is a fixed fraction of $X_1$ and corresponds to a network disconnection.

FIG. 2. Cascade in a six-node network with $X_1 = 1$, $X_j = 0$ for $j \geq 2$, $\lambda > 3/4$. The four lower and upper line flows are $\lambda/24$ and $5\lambda/24$, respectively, with corresponding emergency limits $1/24$ and $5/24$. The failure of an upper line causes the load on the adjacent lower line to surge to $\lambda/6 > 1/24$, causing this line to trip (stage 2). This cutoff leads to the load on the three remaining lower lines to surge to $\lambda/18$, causing them to trip as well (stage 3). After isolating nodes 2 and 6, the cascade ends with $|A_1| = 4$ and a total load shed of $2\lambda/6$ (stage 4).
randomness in our framework, namely the location of the first failure, can be interpreted as a mechanism to bootstrap linear combinations of city sizes. It is well known [33] that bootstrap methods cannot recover heavy-tailed behavior if the dataset is small. In order to recover a Pareto tail, the frozen network therefore needs to be sufficiently large, e.g., $10^4$ nodes. To illustrate this, Fig. 3(b) shows simulation results for the SynGrid model, a random graph model designed to generate realistic power grid topologies [53].

Finally, Fig. 3(c) reveals that Pareto-tailed city sizes is a crucial assumption in order to recover the same scale-free behavior for blackout sizes, as light-tailed city sizes do not lead to heavy-tailed blackout sizes. Additional supporting experiments are reported in Ref. [35], Sec. VI.

We next present experimental results using the SciGRID network [54,55], a model of the German transmission grid that includes generation limits and relaxes several assumptions. We simulate blackout realizations by considering one year’s worth of hourly snapshots. For each snapshot, we solve the operational dc OPF and remove one line uniformly at random, initiating a cascade. To assign city sizes to nodes, we have cities correspond to German districts, and we assign a fraction of the population of each district to specific nodes based on a Voronoi tessellation procedure. In this way, we account for the feature that a single city can encompass multiple nodes in a network. For more details, see Ref. [35], Sec. VII.

The German SciGRID network has a relatively small number of nodes (less than 600), and city sizes are frozen. Therefore, we do not recover Pareto-tailed blackout sizes. However, uniformly across different loading factors $\lambda$, we found that the preponderance of blackouts involves just a single load-shedding event due to a network disconnection. For a moderate loading factor $\lambda = 0.7$, nearly 98% of blackouts only involve a single disconnection. Even for a high loading factor $\lambda = 0.9$, 90% of the blackouts involve a single disconnection, and the fraction of blackouts with four or more disconnections is below 4%. Figure 4 depicts the largest observed blackout, for different values of $\lambda$. Even in these massive blackouts, the bulk of the total load shed is the result of a few load-shedding events. These observations are typical properties that follow from our framework (see Fig. 2), and sharply contrast the branching process approximations where many small jumps take place.

Using data analysis, probabilistic analysis, and simulations, we have illustrated how extreme variations in city sizes can cause the scale-free nature of blackouts. Our explanation and refinement (7) of the scaling law (1) show that specific details such as network characteristics only appear in the prefactor (7). The main parameter $\alpha$, which determines how fast the probability of a big blackout vanishes as its size grows, is completely determined by the city size distribution. Decreasing the constant (7) by performing network upgrades (which in our framework is equivalent to decreasing $\lambda$) would only lead to a modest decrease in the likelihood of big blackouts. Consequently, it is questionable whether network upgrades, as considered in Refs. [19,56], are the most effective way to mitigate the consequences of big blackouts.
Instead, it may be more effective to invest in responsive measures that enable consumers to react to big blackouts. It is shown in Ref. [20] that durations of blackouts have a tail which is decreasing much faster than Eq. (1). At the same time, production facilities often lack redundancy—even short blackouts can lead to huge costs, suggesting that the costs associated to a blackout are concave up to a certain duration. Therefore, if the goal is to minimize the negative effects of a big blackout, it may be far more effective to invest in solutions (such as local generation and storage) that aim at surviving a blackout of a specific duration. This is consistent with recent studies on the importance of resilient city design [57].

Finally, our framework and insights suggest new ways of approaching scale-free phenomena in other transportation networks, such as highway traffic jams [58]. While transport network topologies are not scale free, they may still exhibit scale-free behavior, caused by the scale-free nature of nodal sizes.

We thank Sem Borst for useful discussions, and the Isaac Newton Institute for support and hospitality during the program “Mathematics of Energy Systems.” Financial support was received from Grants No. NWO 639.033.413, No. NWO 024.002.003, and EPSRC No. EP/R014604/1.

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