Renormalization of four-fermion operators for higher twist calculations

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The evaluation of the higher twist contributions to Deep Inelastic Scattering amplitudes involves a non trivial choice of operator bases for the higher orders of the OPE expansion of the two hadronic currents. In this talk we discuss the perturbative renormalization of the four-fermion operators that appear in the above bases.

1. INTRODUCTION

We present here the first steps of a more general and large scale research program, which aims at the evaluation of the contributions of higher twist operators in Deep Inelastic Scattering (DIS) processes\textsuperscript{[1-4]}. We consider in particular the next-to-leading terms in the OPE, which are the next order corrections to the presently known DIS amplitudes and structure functions. The matrix elements of higher twist operators are outside the reach of perturbative QCD, and lattice QCD provides at present the only way to compute them in a reliable way and from first principles. Renormalization factors are then necessary to relate the numbers extracted directly from the lattice to the physical matrix elements.

2. HIGHER TWIST

In the light-cone expansion of the T-product of two hadronic currents in DIS, the leading behavior of the Wilson coefficients is given (up to logs) by \( C_{n,i}^{\text{leading}}(x^2) \sim (x^2)^{d_{g(n,i)} - n - 2d_f}/2 \), and is governed by the twist \( \tau = d_{g(n,i)} - n \) (dimension minus spin) of the corresponding operators. The various terms in the expansion can thus be classified according to their twist: operators of twist-2 are the leading ones, and the higher twist contributions are suppressed as \( (\Delta x^2)^{-(\tau_{\text{min}})/2} \).

Our main interest is \( \Delta \tau \) for the next-to-lowest twist operators. Twist-4 operators of spin \( n \) are related to the \( 1/Q^2 \) power corrections to the \( n \)-th moment of the structure functions:

\[
\int_0^1 dx x^{n-2} F_2(x, Q^2) = C_n^{(2)}(Q^2/\mu^2) A_n^{(2)}(\mu) + C_n^{(4)}(Q^2/\mu^2) A_n^{(4)}(\mu) + O(\frac{1}{Q^4}). \tag{1}
\]

While twist-4 effects are negligible for very large \( Q^2 \), at energies of a few GeV they could be relevant and refine the present QCD predictions. In fact, QCD cannot be tested unambiguously unless these contributions are known.

For twist 4, the Wilson coefficients are known at leading order from continuum QCD\textsuperscript{[1-4]}, but this does not exclude the possibility to compute them also non-perturbatively on the lattice\textsuperscript{[5]}. The real physical non-perturbative effects are contained in the matrix elements of the operators in the OPE, and the main problem is the lack of quantitative knowledge about these matrix elements.

\textsuperscript{2}We do not consider renormalon ambiguities, instanton and other possible non-perturbative effects here.
For the evaluation of these effects a non-trivial choice of operator bases for higher twist operators needs to be made\(^7\). The set of all possible twist-4 operators is in fact an incomplete set, and there are different ways to eliminate the redundant operators (using the equations of motion). A “canonical” basis proposed by Jaffe and Soldate\(^2\) involves totally symmetric and traceless operators with no contracted derivatives, but even this basis contains more operators than needed, as only part of them actually appear in the expansion of the T-product at tree level. The relevant basis of non-singlet operators for minimal spin is\(^3\)

\[
\begin{align*}
\overline{\psi} \gamma_\mu t^A \psi & \quad \overline{\psi} \gamma_\mu \gamma_5 t^A \psi \\
\overline{\psi} F_{\mu \nu} \gamma^\nu \psi & \quad \overline{\psi} F_{\mu \nu} \gamma^\nu \gamma_5 \psi \\
\overline{\psi} F_{\mu \nu} F_{\alpha \beta} \gamma_\rho \psi & \quad \overline{\psi} e^{\alpha \beta \lambda \rho} F_{\mu \alpha} F_{\nu \beta} \gamma_\lambda \gamma_5 \psi.
\end{align*}
\]

One can also choose a (non-minimal) basis in which operators do not contain gluon field strengths explicitly, but then contracted derivatives appear\(^2\). Most likely, the practical choice of a basis is to be done case by case, looking at the complexities of the mixing patterns, at the goodnes of the signals and at other practical issues. The situation can be further complicated by additional mixings due to the lattice.

Sometimes one can simplify the mixing structures. Purely gluonic operators can be at once eliminated by including only non-singlet operators from the start (as in the basis above), but there also exist particular non-trivial combinations of operators that have protection from singlet mixing. As an example, the difference between the second moments of longitudinal structure functions \(M_2^f(2, Q^2) - \frac{d}{dQ^2} M_2^f(2, Q^2)\) is a pure non-singlet quantity also at twist-4 level\(^3\).

Of special importance in the twist-4 case are the 4-fermion operators. At 1-loop level they do not mix with operators of the kind \(\overline{\psi} F \psi\), and the mixing matrix is in this respect triangular. Furthermore, only 4-fermion operators can transform as the flavor 27-plet, and this can be exploited to reduce strongly the room for renormalization mixing, since the flavor 27-plet is not present at twist-2 level (where only the singlet and the octet can contribute). Flavor symmetries can be in general very useful: for example the isospin-2 combination of the pion structure functions\(^5\),

\[
F_{I=2} = F_{3} + F_{3} - 2F_{2},
\]

gets contributions only from the 27-plet, and therefore cannot mix with twist-2 operators.

When mixing with operators of leading twist is forbidden, one can also avoid renormalon ambiguities completely. But even in cases where this mixing is not prohibited, a computation of the coefficient functions in a non-perturbative way\(^5\) would get rid of renormalon problems.

### 3. Perturbative Renormalization

Renormalization factors are needed to relate the numbers obtained from lattice simulations to the corresponding physical matrix elements: \(O^{\text{cont}}(\mu) = \mathcal{Z}(\mu, g(\mu)) \cdot O^{\text{lat}}(a)\). In this way one can give “continuum” numbers, in the sense that a primary result obtained from the lattice can be “converted” (through these renormalization factors) to its continuum equivalent.

Perturbative lattice renormalization is important by itself and as a hint and reference for non-perturbative renormalization studies, especially when one has to understand lattice mixings, generally much more intricate than in the continuum case. Mixing patterns on the lattice are in fact more transparent when looked at in perturbation theory.

Here we would like to study the perturbative renormalization for an important class of twist 4 operators: the 4-quark operators. We consider the symmetrized operators

\[
O_{\{\mu \nu\}} = \sum_A \overline{\psi} \gamma_\mu t^A \psi \cdot \overline{\psi} \gamma_\nu t^A \psi - \text{traces}\quad (3)
\]

\[
O_{\{\mu \nu\}}^{(5)} = \sum_A \overline{\psi} \gamma_\mu \gamma_5 t^A \psi \cdot \overline{\psi} \gamma_\nu \gamma_5 t^A \psi - \text{traces}.
\]

We impose as renormalization conditions that the 1-loop amputated matrix elements renormalized at a reference scale \(\mu\) are equal to the corresponding bare tree-level quantities:
\[ \langle q(p)q(p')|O(\mu)|q(p)q(p') \rangle = \]
\[ = Z_O \langle q(p)q(p')|O(\alpha)|q(p)q(p') \rangle^{1-\text{loop}}_{p^2=p'^2=\mu^2} \]
\[ = \langle q(p)q(p')|O(\alpha)|q(p)q(p') \rangle^{\text{tree}}_{p^2=p'^2=\mu^2}. \]

We find it convenient to use forward matrix elements \((p=p')\), the renormalization factors being not affected by this choice, and the calculations being much simplified.

The computations have been performed using Form codes for the analytic part, and Fortran codes for the numerical integration. They have also been checked by hand. In principle Fierz transformations could be used (both for color and Dirac indices), together with charge conjugation transformations. However, they are not needed when suitable sets of diagrams are evaluated together. We find that in this way the computations are simpler, and since we can integrate numerically each diagram with a good precision in a very short time we are not bound to use the 2-fermion results to which the 4-fermion diagrams can be reduced by Fierz rearrangements. Furthermore, we avoid any problem related to \(d\)-dimensional Fierz transformations.

From our lattice results we can derive the \(Z\) factors to any continuum scheme. As an example, we give here the matching factors between the lattice and the \(\overline{\text{MS}}\) scheme at a scale \(\mu = 1/a\). With obvious notations, the result for the operator \(O_{(\mu\nu)}\) (with \(\mu \neq \nu\)) is:

\[ Z_{11}^{latt-\overline{\text{MS}}} = (t_A \otimes t_A) \cdot (1 - g_0^2) \cdot \]
\[ \cdot \left\{ (t_A \otimes t_A) \left( -0.001442 + 0.040022 \cdot (1 - \alpha) \right) \right. \]
\[ \left. + (1 \otimes 1) \left( -0.023980 + 0.021345 \cdot (1 - \alpha) \right) \right\}, \]

where \(\alpha\) is the gauge parameter. To complete the mixing matrix we also need the contribution

\[ Z_{11}^{latt-\overline{\text{MS}}} = (1 \otimes 1) \cdot (1 - g_0^2) \cdot \]
\[ \cdot \left\{ (1 \otimes 1) \left( -0.348170 \right) \right. \]
\[ \left. + (t_A \otimes t_A) \left( -0.107911 + 0.096053 \cdot (1 - \alpha) \right) \right\}. \]

Similar results are found for the operator \(O^{(5)}_{(\mu\nu)}\):

\[ Z_{11}^{latt-\overline{\text{MS}}} = (t_A \otimes t_A) \cdot (1 - g_0^2) \cdot \]
\[ \cdot \left\{ (t_A \otimes t_A) \left( -0.11619 + 0.040022 \cdot (1 - \alpha) \right) \right. \]
\[ \left. + (1 \otimes 1) \left( -0.023980 + 0.021345 \cdot (1 - \alpha) \right) \right\}. \]

For these calculations we have used a regularization scheme which involves a totally anticommuting \(\gamma_5\) \((\{\gamma_5, \gamma_\mu\} = 0)\), that is simple to implement in computer codes. We are considering performing the calculations also in the \(\text{'t Hooft-Veltman scheme.}\)

The calculation of the renormalization factors can also be done non-perturbatively and it will be interesting to compare the perturbative with the non-perturbative results. Other twist-4 operators will also be considered, and a choice of basis will have to be made, maybe according to the physical situations, so that a reliable estimate of twist-4 effects for some particular processes can be obtained.

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