Effect of the magnetic charge on weak deflection angle and greybody bound of the black hole in Einstein-Gauss-Bonnet gravity

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The objective of this paper is to analyze the weak deflection angle of Einstein-Gauss-Bonnet gravity in the presence of plasma medium. To attain our results, we implement the Gibbons and Werner approach and use the Gauss-Bonnet theorem to Einstein gravity to acquire the resulting deflection angle of photon’s ray in the weak field limit. Moreover, we illustrate the behavior of plasma medium and non-plasma mediums on the deflection of photon’s ray in the framework of Einstein-Gauss-Bonnet gravity. Similarly, we observe the graphical influences of deflection angle on Einstein-Gauss-Bonnet gravity with the consideration of both plasma and non-plasma mediums. Later, we observe the rigorous bounds phenomenon of the greybody factor in contact with Einstein-Gauss-Bonnet gravity and calculate the outcomes, analyze graphically for specific values of parameters.

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I. INTRODUCTION

In the theory of General Relativity (GR), black holes (BHs) are known as a great prediction of Einstein’s theory of gravity and their geometrical characteristics are illustrated by simple mathematical equations. The most recent experimental observation is about the verification of the existence of gravitational waves [1, 2] and also the very first image of the supermassive BH shadow [3, 4] has allowed to recognize the behavior of the geometry and to analyze the various models in the context of strong field limit. Inspite of the fact that the Einstein theory of GR has the restriction of feasibility where its predictive power has been loosened in such regions. Therefore, its applicability and validity of higher order theories have been proposed to make the possible expressions of GR [5]. Thus, the lovelock theory has considered as an induction of Einstein theory [6] as that is a higher order theory which consist of D-dimensions. Adding the Gauss-Bonnet (GB) term to the Einstein-Hilbert action seems to be another way to compensate for higher order curvature terms.

In Higher dimensions theory of gravity, lots of efforts have been made for the better understanding of the low-energy limit of String theory. An important higher dimensional generalization of Einstein gravity is the Einstein-Gauss-Bonnet gravity (EGBG) which is discovered by Lanczos in 1938 [7], and was rediscovered by LoveLock in 1971 [6]. The study of EGBG becomes important since it provides a vast set up to explore a lot of conceptual issues related to the gravity.

The four dimensional EGBG theory, gets much attention in recent times [8]-[23]. The Glavan and Lin [24] demonstrated the GB term in the EGBG theory as a topological invariant before regularization, later after regularization, the equation of motion has been discussed as a solution of EGBG [25]. The observational results in EGBG theory are different from GR due to the fact that it has surplus infinitely strongly coupled scalar [26, 27].

In 2015, LIGO detected the gravitational waves [28], and after this detection, there is renewed interest in the topic of gravitational lensing [29].

Firstly gravitational lensing proposed by Soldner in 1801 in the context of Newtonian theory [30]. As light emitted by distant galaxies, passes by massive objects in the universe, the gravitational pull from these objects can distort or bend the light’s path. This is called gravitational lensing. Three types of gravitational lensing has been classified in the literature: (i) strong gravitational lensing (ii) weak gravitational lensing (iii) micro-gravitational lensing [33]-[37]. From previous many years, we have seen many researches that linked gravitational lensing with the Gauss-Bonnet theorem.

Gauss-Bonnet theorem is the most prominent technique that is used for calculating the weak deflection angle by means of optical geometry, which is engaged by Gibbons and Werner (GW) [38, 39]. The deflection angle calculated in the framework of
GW is contemplated as a result of partially topological effect, the deflection angle is obtained by integrating the Gaussian optical curvature of the BH [38], which is given as follows

\[ \tilde{\alpha} = - \int \int_{D_\infty} K dS \]

where \( \tilde{\alpha} \) denotes the deflection angle, \( K \) denotes the Gaussian optical curvature, \( ds \) denotes the optical surface and \( D_\infty \) symbolizes the infinite domain surrounded by the photon ray, apart from the lens. Thus, the GW methodology has discussed in a unique prospect for BHs and wormholes [40]-[83].

Hawking radiation is thermal radiation that is suspect to be released outside a BH’s event horizon due to relativistic quantum effects. It is named by the physicist Stephen Hawking who derived it’s actually in 1974 [84]-[89], moreover he predicted that BH eventually evaporate entirely. According to Hawking’s theory, BH is not perfectly “black” but instead actually emits particles. This radiation, could eventually lose the energy and mass of BHs to make them disappear. The Hawking radiations are changed due to the bending of continuum, while breeding to structurally infinity. The spill radiations are changed from at the geographically infinity radiations, the change can be analyzed by greybody factor [90]. There are many many ways to find greybody factor, such as WKB approximation method [91]-[100]. A blackbody that emits radiant energy and has same relative spectral energy distribution at the same temperature, the difference in the energy distributions in smaller amount, is called greybody factor.

This work is characterized as follows; In section 2, we inspect about EGBG. In section 3, with the concern of the Gauss-Bonnet theorem, we probe the deflection angle in the context of non-plasma medium. In section 4, we are concerning with the graphical analysis of deflection angle in non-plasma medium. In section 5, we calculate the deflection angle for EGBG in plasma medium, and in section 6 we analyze the graphical impact of deflection angle in the framework of plasma medium. In the section 7, we enlarge the investigation and compute rigorous bound of greybody factor for EGBG and observe its graphical impact in last section.

II. WEAK GRAVITATIONAL LENSING AND EINSTEIN-GAUSS-BONNET GRAVITY

The line-element of spherically symmetric \( D \)-Dimensional spacetime is defined as [101]:

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2. \] (1)

Here, \( d\Omega_{D-2}^2 \) denotes the line-element of the unit \((D - 2)\)-dimensional sphere

\[ f(r) = 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2} + \mathcal{O}(r^3) \quad r \gg 1, \quad d\Omega_{D-2}^2 = d\theta^2 + \sin^2 \theta d\phi^2; \] (2)

here, the magnetic mass of BH is denoted by \( M \), \( r \) expresses the radial coordinate, \( q_m^2 \) represents magnetic charge and \( G \) represents gravitational constant of BH. By placing the value of \( f(r) \) in Eq.(1), we attain the following equation

\[ ds^2 = - \left( 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2} \right) dt^2 + \left( 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2} \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \] (3)

By making the assumption that the source and the viewer both are in the same equatorial plane along the pathway of null photons which is also on the equivalent plane keeping \((\theta = \frac{\pi}{2})\), for the sake of null geodesic we place \( ds^2=0 \) and we acquire the optical metric given as follows

\[ ds^2 = - \left( 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2} \right) dt^2 + \left( 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2} \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \] (4)

Afterwards, we make change in optical metric into a new frame of coordinate system \( \tilde{r} \) composed as,

\[ dt^2 = g_{ab} dx^a dx^b = d\tilde{r}^2 + f^2(\tilde{r}) d\phi^2, \] (5)
It is notified that the previous system \((a, b)\) is transformed into new system \((r, \phi)\) and determinant value is given as 
\[
\det \tilde{g}_{ab} = \frac{r^2}{f(r)};
\]
make use of Eq.\((5)\), the value of remaining non-zero Christoffel symbols which are given as
\[
\Gamma_{\phi\phi} = \frac{r f'(r) - 2 f(r)}{2}, \quad \Gamma_{r\phi} = \frac{-rf'(r) + 2f(r)}{2rf(r)} = \Gamma_{\phi r}, \quad \Gamma_{rr} = -\frac{f'(r)}{f(r)}
\]
and for optical curvature value of non-zero components of Riemann tensor is characterized as
\[
R_{\phi\phi\phi} = -k f^2(r)
\]
where \(R_{\phi\phi\phi} = g_{\phi\phi} R_{\phi\phi\phi}\). Hence, the Gaussian optical curvature \(K\) can be calculated by
\[
K = \frac{R_{\text{total}}^{\text{Scalar}}}{2}.
\]
By taking into account the previous equation, the Gaussian optical curvature \(K\) in terms of radial coordinate \(r\) of Schwarzschild metric can written as [38]
\[
K = -\frac{1}{f(r)} \left[ \frac{dr}{dr} \frac{d}{dr} \frac{df}{dr} \right]^2 + \frac{1}{r^2} \frac{df}{dr} \left( \frac{df}{dr} \right)^2.
\]
Ultimately, the respective Gaussian optical curvature \(K\) of photon for Einstein-Gauss-Bonnet gravity can be calculated by using Eq.\((6)\) into Eq.\((8)\), we obtain such a conclusion
\[
K \approx -\frac{2GM}{r^3} + \frac{3Gq_0^2}{r^4} - \frac{6G^2Mq_0^2}{r^5} + O(M^2, q_0^3, G^3).
\]

### III. DEFLECTION ANGLE OF EINSTEIN-GAUSS-BONNET GRAVITY IN NON-PLASMA MEDIUM

Now we find, the deflection angle for Einstein-Gauss-Bonnet gravity with the help of Gauss-Bonnet theorem in the framework of non-plasma medium. Hence, by utilizing the Gauss-Bonnet theorem to the domain \(\mathcal{V}_{R}\), given as below [38]
\[
\int \int_{\mathcal{V}_R} KdS + \oint_{\partial \mathcal{V}_R} kdt + \sum \varepsilon_z = 2\pi \mathcal{F}(\mathcal{V}_R),
\]
where \(K\) symbolizes the Gaussian optical curvature and \(k\) symbolize geodesic curvature, characterized as \(k = \tilde{g}(\nabla_\gamma \tilde{\gamma}, \tilde{\gamma})\) such as \(\tilde{g}(\tilde{\gamma}, \tilde{\gamma}) = 1\), here unit acceleration vector designated by \(\tilde{\gamma}\), the exterior angle of \(z^{th}\) vertex symbolized as \(\varepsilon_z\). When \(R\) goes to infinity, as a result the jump angle calculate by estimate to \(\pi/2\) and we proceeds \(\theta_O + \theta_S \to \pi\). Here, the value of Euler characteristic number is \((\mathcal{F}(\mathcal{V}_R) = 1)\) and \(\mathcal{V}_R\) represents non-singular region. So, we conclude the following,
\[
\int \int_{\mathcal{V}_R} KdS + \oint_{\partial \mathcal{V}_R} kdt + \varepsilon_z = 2\pi \mathcal{F}(\mathcal{V}_R),
\]
Here, the total jump angle presented by \(\varepsilon_z = \pi\). When the value of \(R \to \infty\), then the remainders of the part yield \(k(E_R) = | \nabla_{E_{\text{R}}} \hat{E}_{\text{R}} |\). For the sake of geodesic curvature the value of radial component is given as,
\[
(\nabla_{E_{\text{R}}} \hat{E}_{\text{R}})^\nu = \hat{E}_{\nu} E_{\text{R}}^E + \Gamma_{\nu \phi}^E (Z_{\text{R}}^E)^2.
\]
By keeping \(R\) is very high, then \(E_R := r(\phi) = R = \text{const}\). Thus, the composition of the Eq.\((12)\) converts to \((\hat{E}_E^E)^2 = \frac{1}{f(r)}\). Memorizing \(\Gamma_{\phi \phi}^E = \frac{r f'(r) - 2 f(r)}{2}\), we yield
\[
(\nabla_{E_{\text{R}}} \hat{E}_{\text{R}})^\nu \to \frac{1}{R}.
\]
So, \( k(E_R) \rightarrow \frac{1}{\pi} \). Using optical metric Eq.(5), it can be written as \( dt = R d\varphi \). Hence;

\[
k(E_R) dt = d\varphi.
\] (14)

All of the above results taking into count, we acquire

\[
\int \int_{V_R} K ds + \oint_{\partial V_R} k dt = R \rightarrow \infty \int \int_{M_\infty} K dS + \int_{0}^{\pi+\Theta} d\varphi.
\] (15)

The photon ray at \( 0^{th} \) order in weak field deflection limit is determined as \( r(t) = b / \sin \varphi \). Thus, utilizing (2.13) and (16): then, the deflection angle determined as [39];

\[
\tilde{\delta} = \int_{0}^{\pi} \int_{b/\sin \varphi} K \sqrt{\det g} \, d\tilde{r} \, d\varphi,
\] (16)

where

\[
\sqrt{\det g} = r\left(1 + \frac{3GM}{r} - \frac{3Gq_m^2}{2r^2}\right).
\] (17)

By using the values of Gaussian curvature in Eq.(2.13) into Eq.(16). Then deflection angle is calculated as:

\[
\tilde{\delta} \approx \frac{4GM}{b} - \frac{8G^2Mq_m^2}{3b^3} - \frac{3G\pi q_m^2}{4b^2} + O(M^2, q_m^3, G^3)
\] (18)

IV. GRAPHICAL INSPECTION FOR NON-PLASMA MEDIUM

In this section, we observe the graphical influence of deflection angle. And also seek know about the vital importance of these graphs. Moreover, we investigate the impact of magnetic charge \( (q_m) \) and impact parameter \( (b) \) on the deflection angle.

A. Comparison between deflection angle \( \tilde{\delta} \) and impact parameter \( (b) \)

![Graph](image_url)

**Figure 1** Here, we can see the behavior of \( \tilde{\delta} \) with respect to \( b \) in order to varying the value of magnetic charge \( q_m \) and remain both \( G \) and \( M \) fixed.

1. In left panel (i), we examined that the deflection angle \( \tilde{\delta} \) gradually increase first for lower values of magnetic charge \( q_m \) and at the end slope decrease.
2. In right panel (i), we observed that angle shows the decline behavior at higher values of magnetic charge \( q_m \).
B. Comparison between deflection angle ($\tilde{\delta}$) and the magnetic charge ($q_m$)

![Graph showing correspondence between $\tilde{\delta}$ and $q_m$.]

Figure 2: Correspondence between $\tilde{\delta}$ and $q_m$.

- **Figure 2** Now, we check the behavior of $\tilde{\delta}$ with respect to magnetic charge $q_m$ by changing $b$ and $G,M$ being fixed.

1. Figure (i), shows that the value of $\tilde{\delta}$ continually decrease by using the large values of $b$.

V. GRAVITATIONAL LENSING FOR EINSTEIN-GAUSS-BONNET GRAVITY IN PLASMA MEDIUM

Now, this section is devoted to calculate the weak gravitational lensing for Einstein-Gauss-Bonnet gravity in the framework of plasma medium. For (EGB) gravity the value of refractive index $n(r)$ [102], given as:

$$n^2 (r, \omega_e(r)) = 1 - \frac{\omega_e^2}{\omega_e^2 (r)}$$

(19)

where the term $(\omega_e)$ characterize for electron plasma frequency and the term $(\omega_\infty)$ express the photon frequency which is observed by a viewer at infinity, then the relevant optical metric presented as [102];

$$dt^2 = g_{opt}^{ij} dx^p dx^f = n^2 \left[ \frac{dr^2}{f^2(r)} + \frac{r^2 d\Phi^2}{f(r)} \right],$$

(20)

with the value of determinant $g_{opt}^{ij}$.

$$\sqrt{g_{opt}} = r(1 - \frac{\omega_e^2}{\omega_\infty^2}) + GM(3 - \frac{\omega_e^2}{\omega_\infty^2}) - \frac{Gq_m^2}{2r}(3 - \frac{\omega_e^2}{\omega_\infty^2}).$$

(21)

By using Eq.(20),the value of only non-zero christoffel symbols are,

$$\Gamma^l_{ll} = (1 + \frac{\omega_e^2 f(r)}{\omega_\infty^2}) \left[ -f(r)'f(r)^{-1}(1 - \frac{\omega_e^2 f(r)}{\omega_\infty^2}) - \frac{f(r)'\omega_e^2}{2\omega_\infty^2} \right]$$

and

$$\Gamma^m_{ml} = (1 + \frac{\omega_e^2 f(r)}{\omega_\infty^2}) \left[ r^{-1}(1 - \frac{\omega_e^2 f(r)}{\omega_\infty^2}) - \frac{f(r)'f(r)^{-1}}{2}(1 - \frac{\omega_e^2 f(r)}{\omega_\infty^2}) - \frac{f(r)'\omega_e^2}{2\omega_\infty^2} \right]$$

and

$$\Gamma^l_{mm} = (1 + \frac{f(r)\omega_e^2}{\omega_\infty^2}) \left[ -f(r)(1 - \frac{f(r)\omega_e^2}{\omega_\infty^2}) + \frac{r^2 f(r)'}{2}(1 - \frac{f(r)\omega_e^2}{\omega_\infty^2}) + \frac{r^2 f(r) f(r)'}{2\omega_\infty^2} \right].$$

Expression for Gaussian curvature in the form of curvature tensor written as;

$$K = \frac{R_{\Phi+\Phi}(g_{opt})}{det(g_{opt})},$$

(22)
To applying Eq. (22), the following Gaussian curvature is calculated as,

\[
K \approx \frac{GM}{r^3} \left(-2 - 3 \frac{\omega_e^2}{\omega_{\infty}^2} + 4 \frac{\omega_e^4}{\omega_{\infty}^4}\right) + \frac{G^2 M q_m^2}{r^5} \left(-6 - 26 \frac{\omega_e^2}{\omega_{\infty}^2} + 28 \frac{\omega_e^4}{\omega_{\infty}^4}\right)
\]

\[
+ \frac{G q_m^2}{r^7} \left(3 + 5 \frac{\omega_e^2}{\omega_{\infty}^2} - 3 \frac{\omega_e^4}{\omega_{\infty}^4}\right) + O(M^2, q_m^3, G^3)
\]

(23)

To do so, we make use of the (GBT) to find out deflection angle but also compare it with the non-plasma. Thus, for evaluating the deflection angle in the framework of weak field limit, as photon beams behaves like a straight line. Consequently, the limitation at 0th order is 

\[
r = b \sin \phi.
\]

So, the (GBT) is comprises into the following form to calculate the deflection angle \(\tilde{\delta}\);

\[
\tilde{\delta} = - \lim_{R \to 0} \int_0^\pi \int_0^R K dS
\]

(24)

Using Eq.(5.5), so the value of deflection angle for Einstein-Gauss-Bonnet gravity in the framework of plasma medium is computed as;

\[
\tilde{\delta} \approx \frac{4GM}{b} + \frac{GM}{b} \left(2 \frac{\omega_e^2}{\omega_{\infty}^2} - 14 \frac{\omega_e^4}{\omega_{\infty}^4}\right) - \frac{8G^2 M q_m^2}{3b^3} + \frac{G^2 M q_m^2}{b^4} \left(2 \frac{\omega_e^2}{\omega_{\infty}^2} - \frac{130 \omega_e^4}{9 \omega_{\infty}^4}\right)
\]

\[
- \frac{3G q_m^2 \pi}{4b^2} + \frac{G q_m^2 \pi}{b^3} \left(- \frac{\omega_e^2}{2 \omega_{\infty}^2} + \frac{2 \omega_e^4}{\omega_{\infty}^4}\right) + O(M^2, q_m^3, G^3)
\]

(25)

VI. GRAPHICAL INSPECTION FOR PLASMA MEDIUM

This section is presented the graphical impression of the deflection angle for Einstein-Gauss-Bonnet gravity. We prosecute the effect of numerous parameters on the deflection angle. Furthermore, we also elaborate the physical significance of these types of graph to inspect the actions of plasma medium. For our simplicity we assign \(G=1, M = 1, \frac{\omega_e}{\omega_{\infty}}=10^{-1}\) and also vary the values of magnetic charge \(q_m\) and impact parameters \(b\) in order to obtain these graphs.

A. Comparison between deflection angle \(\tilde{\delta}\) and Impact parameter \(b\)

![Figure 3: Correspondence between \(\tilde{\delta}\) and \(b\).](image)

- Figure 3 Represents the impact of \(\tilde{\delta}\) with relates to impact parameter \(b\) and vary the magnetic charge \(q_m\) and taking the value of \(M, G\) been constant.

1. In left fig(i), we can viewed that the value of deflection angle \(\tilde{\delta}\) gradually increased first by putting different lower values of magnetic charge \(q_m\) and then tends to move positive infinity.

2. In right fig(i), we can analyze that angle value decrease at the higher inputs of magnetic charge \(q_m\).
B. Comparison between the deflection angle $\tilde{\delta}$ and magnetic charge $q_m$

![Figure 4: Correspondence between $\tilde{\delta}$ and $q_m$.](image)

- **Figure 4** To show the correspondence of $\tilde{\delta}$ with magnetic charge $q_m$ and alternating the values of impact parameter and being fixed the values of $G$ and $M$ respectively.

1. In figure (i), we obtained that firstly deflection angle $\tilde{\delta}$ showed the straight line path and then steadily moves to negative infinity by changing values of $b$.

VII. GREYBODY FACTOR FOR EINSTEIN-GAUSS-BONNET-GRAVITY

This section is based on the calculation of the rigorous bound of the greybody factor. The bound of greybody factor can be stated as [92]

$$T \geq sech^2 \left( \frac{1}{2\omega} \int_{-\infty}^{\infty} V(r) dr_{*} \right).$$  \hspace{1cm} (26)

The D-dimension spherically symmetric line element is given as [101];

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$  \hspace{1cm} (27)

where,

$$f(r) = 1 - \frac{2GM}{r} + \frac{Gq_m^2}{r^2}$$

The interior and exterior two event horizon value $r_{\pm}$ is given by

$$r_+ = GM + \sqrt{G^2M^2 - Gq_m^2},$$  \hspace{1cm} (28)

$$r_- = GM - \sqrt{G^2M^2 - Gq_m^2},$$

The Schrodinger-like equation has been illustrated as;

$$\left( \frac{d^2}{dr_{*}^2} + \omega^2 - V(r) \right) \psi = 0$$  \hspace{1cm} (29)

here, $r_{*}^2$ denote the "tortoise coordinate".

$$dr_{*}^2 = \frac{1}{f(r)} dr$$  \hspace{1cm} (30)
\begin{align}
V(r) &= \frac{(d-2)(d-4)}{4} f^2(r) \frac{1}{r^2} + \frac{(d-2)}{2} \frac{f(r) f'(r)}{r} + l(l + d - 3) \frac{f(r)}{r^2} \\
\text{(31)}
\end{align}

In order to calculate the lower bound value on transmission probability taking the value \( h = \omega \) is;

\begin{align}
T &\geq \frac{1}{\cosh^2} \left( \frac{1}{2\omega} \int_{-\infty}^{\infty} V(r) dr^2 \right) \\
&= \frac{1}{\cosh^2} \left[ \frac{1}{2\omega} \int_{r_+}^{\infty} \left( \frac{(d-2)(d-4)}{4} f(r) \frac{1}{r^2} + \frac{(d-2) f'(r)}{r} + l(l + d - 3) \frac{1}{r^2} \right) \right] \\
&= \frac{1}{\cosh^2} \left[ \frac{1}{2\omega} \left( \frac{GM}{r_+^2} - \frac{2Gq_m^2}{3r_+^3} + \frac{l(l + 1)}{r_+} \right) \right]
\end{align}

If we take \( d = 4 \) and put \( r_+ \) value, then this bound value is reduced into the following

\begin{align}
T &\geq \frac{1}{\cosh^2} \left[ \frac{1}{2\omega} \left( \frac{GM}{3(GM + \sqrt{G^2M^2 - Gq_m^2})^2} - \frac{2Gq_m^2}{3(GM + \sqrt{G^2M^2 - Gq_m^2})^3} \right) \right. \\
&\quad \left. + \frac{l(l + 1)}{(GM + \sqrt{G^2M^2 - Gq_m^2})} \right]
\end{align}

So, we have calculated the lower bound for EGBG. If the BHs have no magnetic or electric charges, then the bound which is given above is reduced into

\begin{align}
T &\geq \frac{1}{\cosh^2} \left[ \frac{2l(l + 1)}{8\omega GM} \right] \\
\text{(32)}
\end{align}

which is the same as the bound for the 4D Schwarzschild (BHs) emitting spinless particles [92].

VIII. GRAPHICAL BEHAVIOR OF GREYBODY FACTOR

This section shows the graphical influences of greybody factor lower bound for (EGBG) and its potential while taking \( (G = M = 1) \) and different values of magnetic charge \( q_m \) with angular momenta \( l = 0, 1, 2 \) respectively.
Figure 5: The left figure corresponds the potential and the right figure corresponds the relevant greybody factor lower bound for (EGBG).

Now, we can examine how $T$ behavior depends upon the potential shape. This analysis can be achieved by altering the magnetic charge parameter, $q_m$ as well as the angular momenta $\ell$. By fixing both $M = 1$ and $G = 1$, the potential takes the higher amplitude when $q_m$ changes as shown by the left plot in Figure.5. The greybody factor decrease for a certain value of $\omega$ whereas it is more difficult for the wave to be transmitted through higher potential value as shown by the right plot of Figure.5.

**IX. CONCLUSION**

This recent work is about to investigate deflection angle for Einstein-Gauss-Bonnet gravity in both cases for plasma and non-plasma mediums. To do so, we evaluate the weak lensing by applying (GBT) and derive deflection angle of photons ray for (EGB) gravity. The resulted deflection angle given in Eq.(3.9) follows as;

$$
\delta \approx \frac{4GM}{b} + \frac{GM}{b} \left( \frac{2 \omega_c^2}{\omega_{\infty}^2} - 14 \frac{\omega_c^4}{\omega_{\infty}^4} \right) - \frac{8G^2Mq_m^2}{3b^3} + \mathcal{O}(M^2, q_m^3, G^3)
$$

We investigate whether the resulted deflection angle can be reduced by reducing some parameters which turned into the deflection angle of Schwarzschild (BH) up to the first order value. Also, we review the graphical impression of alternate parameters on the deflection angle for Einstein-Gauss-Bonnet gravity. Further, the value of deflection angle is examined in the occupancy of plasma medium which is given by Eq.(5.7);

$$
\delta \approx \frac{4GM}{b} + \frac{GM}{b} \left( \frac{2 \omega_c^2}{\omega_{\infty}^2} - 14 \frac{\omega_c^4}{\omega_{\infty}^4} \right) - \frac{8G^2Mq_m^2}{3b^3} + \frac{G^2Mq_m^2}{b^3} \left( 2 \frac{\omega_c^2}{\omega_{\infty}^2} - \frac{130\omega_c^4}{9\omega_{\infty}^4} \right) - \frac{3Gq_m^2 \pi}{4b^2} + \frac{Gq_m^2 \pi}{b^2} \left( - \frac{\omega_c^2}{2\omega_{\infty}^2} + \frac{2\omega_c^4}{\omega_{\infty}^4} \right) + \mathcal{O}(M^2, q_m^3, G^3)
$$

(33)
When the value of plasma effect $\omega \rightarrow \infty$ nearest to zero, then the effect of plasma removed. Moreover, the graphical behavior of the deflection angle for Einstein-Gauss-Bonnet gravity in plasma comparison with some parameters. We examined the greybody factor for (EGBG) with the help of rigorous bound. Firstly, we compute the horizon value and then using Schrodinger-like equation which is obtained from the radial part, of the solution. Consequently, we observe the behavior of potential in order to investigate the greybody factor. It is conclude that slope height of the potential decrease when lower the magnetic charge values. Further, the rigorous bound upon greybody factors have been computed. It is discovered that we analyze qualitatively greybody factor for (EGBG) with the help of rigorous bound. Firstly, we compute the horizon value and then using Schrodinger-like equation which is obtained from the radial part, of the solution. Consequently, we observe the behavior of potential in order to investigate the greybody factor. It is conclude that slope height of the potential decrease when lower the magnetic charge values.

**Deflection angle concern with impact parameter $b$:**

1. In our conclusion, we seen that firstly the value of deflection angle increasing and then after turned to decreasing for the smaller values of magnetic charge $q_m$.

**Deflection angle concern with magnetic charge $q_m$:**

1. We reviewed that the deflection angle firstly follows the straight path and then after tends to negative infinity for the values of impact parameter in range between $1 < b < 20$.

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