Effects of Gravitational-wave Radiation of Eccentric Neutron Star–White Dwarf Binaries on the Periodic Activity of Fast Radio Burst Sources

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Abstract

We revisit the eccentric neutron star (NS)–white dwarf (WD) binary model for the periodic activity of fast radio burst (FRB) sources, by including the effects of gravitational-wave (GW) radiation. In this model, the WD fills its Roche lobe at the periastron and mass transfer occurs from the WD to the NS. The accreted materials can be fragmented and arrive at the NS episodically, resulting in multiple bursts through curvature radiation. Consequently, the WD may be kicked away owing to the conservation of angular momentum. To initiate the next mass transfer, the WD has to refill its Roche lobe through GW radiation. In this scenario, whether the periodic activity can show up relies on three timescales, i.e., the orbital period $P_{\text{orb}}$, the timescale $T_{\text{GW}}$ for the Roche lobe to be refilled, and the time span $T_{\text{frag}}$ for all the episodic events corresponding to each mass-transfer process. Only when the two conditions $T_{\text{GW}} \lesssim P_{\text{orb}}$ and $T_{\text{frag}} < P_{\text{orb}}$ are both satisfied, the periodic activity will manifest itself and the period should be equal to $P_{\text{orb}}$. In this spirit, the periodic activity is more likely to show up for relatively long periods ($P_{\text{orb}} \gtrsim$ several days). Thus, it is reasonable that FRBs 180916 and 121102, the only two sources having been claimed to manifest periodic activity, both correspond to relatively long periods.

Unified Astronomy Thesaurus concepts: Compact binary stars (283); White dwarf stars (1799); Neutron stars (1108); Radio transient sources (2008)

1. Introduction

Fast radio bursts (FRBs) are a kind of energetic radio transient with short durations (~ms) and extremely high brightness temperatures. The discovery of the first FRB (010724) by Lorimer et al. (2007) opens a new window in astronomy. Recently, the number of the detected FRBs has rapidly increased owing to improved observational techniques and new facilities, while the physical origin of FRBs remains a mystery (Katz 2018; Cordes & Chatterjee 2019; Petroff et al. 2019; Platts et al. 2019). It was found that some FRBs showed repeating behaviors (called repeating FRBs), while most of them are one-off bursts (see Cordes & Chatterjee 2019, for a review). It is also unclear whether repeating FRBs and one-off FRBs have the same physical origin (Palaniswamy et al. 2018; Caleb et al. 2019).

The Canadian Hydrogen Intensity Mapping Experiment Fast Radio Burst Project (CHIME/FRB; CHIME/FRB Collaboration et al. 2019) has detected a large number of FRBs and the first source with periodic activity (FRB 180916) has been reported (CHIME/FRB Collaboration et al. 2020). Since the report of the periodic activities, the physical origin has been widely investigated. Several kinds of models have been proposed to explain the periodic activities (for a review, see Xiao et al. 2021). First, FRBs occur in a binary system containing a stellar compact object (a neutron star (NS) or a black hole (BH)), where the observed period corresponds to the orbital period (Dai et al. 2016; Zhang 2017; Dai & Zhong 2020; Gu et al. 2020; Deng et al. 2021; Geng et al. 2021; Kuerban et al. 2021; Li et al. 2021; Voisin et al. 2021; Wada et al. 2021).

Second, an extremely slow rotation of an NS causes the ultralong rotational period of the bursting source (Beniamini et al. 2020; Xu et al. 2021a). Third, FRBs are emitted from the NS magnetosphere due to internal triggers or external triggers, and the long-lived precession of the emitting region causes the observed periodic activities (Levin et al. 2020; Tong et al. 2020; Yang & Zou 2020; Li & Zanazzi 2021; Sridhar et al. 2021).

To date, FRBs 180916 and 121102 are the only two FRBs having been claimed to have periodic activity. FRB 180916 exhibits an activity period of 16.35±0.18 days with an activity window of about 5 days (CHIME/FRB Collaboration et al. 2020). Observationally, this source is not always active in all the predicted activity windows. FRB 121102 shows a possible 157 days of periodic behavior and an active window of ~100 days (Rajwade et al. 2020; Cruces et al. 2021). On the other hand, for some repeaters with tens of or even more than a thousand observed bursts, no periodic activity was found by time series analyses (Niu et al. 2021; Xu et al. 2021a).

In this paper, based on the NS–white dwarf (NS–WD) binary model with an eccentric orbit (Gu et al. 2020), we will investigate the effects of gravitational-wave (GW) radiation on the periodic activity of FRB sources. We will manage to understand (1) why the sources are not active in some predicted windows, (2) why no periodic activity was discovered for some repeaters with adequate burst observations, and (3) why the observed periods for the recurrence of activity (16.35 or 157 days) are significantly longer than the orbital periods of contact NS–WD binaries with moderate eccentricities (tens of minutes). The remainder of this paper is organized as follows. The NS–WD binary model with the GW radiation and the corresponding equations are described in Section 2. The results are shown in Section 3. Our conclusions and discussion are presented in Section 4.
2. NS–WD Binary Model with Gravitational Radiation

Gu et al. (2020) proposed a compact binary model with an eccentric orbit to explain the periodic activity of repeating FRB sources. The model includes a magnetic WD and an NS with strong dipolar magnetic fields. When the WD fills its Roche lobe at the periastron, mass transfer occurs from the WD to the NS, and the accreted and fragmented materials can produce multiple FRBs when traversing along the NS’s magnetic field. In this scenario, the period for the recurrence of activity should be equivalent to the orbital period $P_{\text{orb}}$. Owing to the conservation of angular momentum, the WD may be kicked away after a mass-transfer process for $q < 2/3$ (King 2007), where $q$ is the mass ratio defined as $q \equiv M_2/M_1$. Here, $M_1$ and $M_2$ are the NS and WD masses, respectively. As proposed by Gu et al. (2016), the WD can refill its Roche lobe through the GW radiation. Thus, it is essential to compare the two key timescales, i.e., $P_{\text{orb}}$ and $T_{\text{GW}}$, where $T_{\text{GW}}$ is the timescale for the Roche lobe to be refilled.

Figure 1 is a cartoon animation of our model that describes an NS–WD system with an eccentric orbit. Similar to Gu et al. (2020), a mass transfer occurs from the WD to the NS when the WD fills its Roche lobe near the periastron (panel (A)). Owing to the viscous processes, the accreted WD material can be fragmented and arrive onto the NS episodically. However, at other positions on the eccentric orbit, no mass transfer is supplied since the Roche lobe is not filled, as shown in panels (B), (C), and (D) of Figure 1. The fragmented materials travel along the magnetic field lines (onto the NS), resulting in multiple bursts by the curvature radiation.

We introduce two timescales during the accretion process. The first one is the accretion timescale $T_{\text{acc}}$, which is the timescale for the accreted materials to transfer from the inner Lagrangian point to the surface of the NS. Since the accreted materials have angular momentum and therefore viscous processes are necessary, $T_{\text{acc}}$ is normally longer than the orbital period $P_{\text{orb}}$. In our model, the mass-transfer process near the periastron in different cycles, the timescale $T_{\text{acc}}$ is regarded to be a certain fixed value. The second one is the timescale $T_{\text{frag}}$, which denotes the time span from the arrival of the first piece of material at the surface of the NS to the arrival time of the last piece. $T_{\text{frag}}$ corresponds to the activity window and can be either shorter or longer than $P_{\text{orb}}$. In this work, we will focus on $T_{\text{frag}}$ rather than $T_{\text{acc}}$, since the latter works as a unified time lag and may not have significant effects on the observations. On the contrary, $T_{\text{frag}}$ is an important parameter that can affect the periodicity of the observed FRBs, which will be discussed in Section 3.

On the other hand, the WD may be kicked away after a mass-transfer process, and thus widen the orbital separation. The GW radiation is an essential mechanism to reduce the orbital separation and enable the WD to refill its Roche lobe, initiating the next mass-transfer event. Thus, the time interval between two adjacent mass-transfer processes $T_{\text{GW}}$ is a key timescale that may determine whether the periodic activity can show up. Obviously, $T_{\text{GW}} \lesssim P_{\text{orb}}$ is a necessary condition for the periodic activity to appear. On the contrary, for $T_{\text{GW}} \gg P_{\text{orb}}$, since $T_{\text{GW}}$ also varies with different amounts of mass transfer near the periastron (Equation (9)), the periodicity will be hard to discover.

The dynamic equation of a binary is given by

$$\frac{G(M_1 + M_2)}{a^3} = \frac{4\pi^2}{P_{\text{orb}}^2},$$

where $G$ is the gravitational constant, and $a$ is the semimajor axis of the eccentric orbit. The effective Roche lobe radius for the WD $R_{\text{L,2}}$ at the periastron can be simply described as (Paczyński 1971)

$$\frac{R_{\text{L,2}}}{a(1 - e)} = 0.462 \left(\frac{M_2}{M_1 + M_2}\right)^{1/3},$$

where $e$ is the eccentricity of the orbit. The WD radius $R_{\text{WD}}$ depends only on its mass and composition (Tout et al. 1997). For $M_2 \lesssim 0.2 M_\odot$, it is convenient to simplify Equation (17) of
Tout et al. (1997) as

$$R_{\text{WD}} = 0.0115 R_\odot (M_{\text{CH}}/M_2)^{1/3},$$  \hfill (3)

where $R_\odot$ is the solar radius, and $M_{\text{CH}} = 1.44 M_\odot$ is the Chandrasekhar mass limit. The orbital angular momentum $J$ of a binary system in an eccentric orbit can be expressed as (Peters 1964)

$$J = M_1 M_2 \left[ \frac{Ga(1 - e^2)}{M_1 + M_2} \right]^{1/2}. \hfill (4)$$

When the WD fills its Roche lobe at the periastron, the mass transfer occurs. With the assumption of the orbital angular momentum conservation ($\Delta J = 0$) and the assumption that the eccentricity $e$ does not change during the whole process, the variation $\Delta a$ due to the mass transfer can be derived from Equation (4):

$$\frac{\Delta a}{a} = 2(1 - q) \frac{\Delta M_2}{M_2}, \hfill (5)$$

where the positive $\Delta M_2$ is the transferred mass near the periastron. On the other hand, the WD radius will expand with decreasing mass. By Equations (2) and (3), we obtain the required variation of the semimajor axis $\Delta a_*$ for the WD just to refill its Roche lobe ($\Delta R_{\text{WD}} = \Delta R_{L2}$):

$$\frac{\Delta a_*}{a} = \frac{2}{3} \frac{\Delta M_2}{M_2}. \hfill (6)$$

The critical condition $\Delta a = \Delta a_*$ results in a critical mass ratio $q = 2/3$, as shown by King (2007). Thus, for $q > 2/3$, we have $\Delta a < \Delta a_*$ and $\Delta R_{\text{WD}} > \Delta R_{L2}$, which indicates that the expansion of the WD radius is more rapid than that of its Roche lobe (e.g., Dong et al. 2018). Thus, the Roche lobe will be overfilled at the periastron for each time. However, our study focuses on $M_2 < 0.2 M_\odot$ and therefore the values of $q$ are far below 2/3 in the NS–WD model. In other words, we only have $\Delta a > \Delta a_*$. In this case, the Roche lobe can be refilled only when the GW radiation is taken into account.

For the binary with an eccentric orbit, the variation of $a$ due to the GW radiation can be written as (Peters 1964)

$$\frac{da}{dt} = - \frac{64}{5} G^3 M_1 M_2 (M_1 + M_2) \left( 1 - \frac{73}{24} e^2 + \frac{37}{96} e^4 \right).$$  \hfill (7)

The time interval $T_{\text{GW}}$ between two adjacent mass-transfer processes can be derived as

$$T_{\text{GW}} = \frac{\Delta a - \Delta a_*}{da/dt}. \hfill (8)$$

Thus, an analytic relation between $T_{\text{GW}}$ and $\Delta M_2$ can be expressed as

$$T_{\text{GW}} = \frac{5 e^5 a^4 \left( \frac{2}{5} - q \right) \left( 1 + q \right) \left( 1 - e^2 \right)^{7/2}}{32 q G^3 (M_1 + M_2)^3 \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)} \frac{\Delta M_2}{M_2}. \hfill (9)$$

Equation (9) enables us to calculate $T_{\text{GW}}$ once $M_1$, $M_2$, $e$, and $\Delta M_2$ are given. In our model, we adopt a typical value $M_1 = 1.4 M_\odot$ for the NS.

### 3. Results

According to the assumption that the WD fills its Roche lobe at the periastron, i.e., $R_{L2} = R_{\text{WD}}$, we can obtain the values of $P_{\text{orb}}$ by Equations (1)–(3) and $T_{\text{GW}}$ by Equation (9) once $M_2$, $e$, and $\Delta M_2$ are given. $T_{\text{GW}}$ and $P_{\text{orb}}$ are two key timescales that may determine whether the periodic activity can show up. We use the ratio $T_{\text{GW}}/P_{\text{orb}}$ to show the effects of the GW radiation on the periodic activity.

Hydrodynamic simulation of NS–WD binaries showed that only the systems with $M_2 < 0.2 M_\odot$ can undergo stable mass transfer, while the systems with $M_2 > 0.2 M_\odot$ experience unstable mass transfer, which may lead to a tidal disruption of the WD (Bobrick et al. 2017). Thus, a stable NS–WD binary is likely to have a mass range of 0.01 $M_\odot < M_2 < 0.2 M_\odot$ (Gu et al. 2020). With $M_1 = 1.4 M_\odot$ and 0.01 $M_\odot < M_2 < 0.2 M_\odot$, i.e., $q < 1/7$, the WD may be kicked away after a mass-transfer process (King 2007).

Figure 2 shows the $T_{\text{GW}}/P_{\text{orb}}$ diagram with a pair of physically plausible ranges $\Delta M_2 = 10^{-12} - 10^{-9} M_\odot$ (e.g., Gu et al. 2016) and $e = 0.1 - 0.999$ for three masses $M_2 = 0.02 M_\odot$ (red solid lines), $M_2 = 0.05 M_\odot$ (black solid lines), and $M_2 = 0.1 M_\odot$ (blue solid lines). The pink horizontal dashed line corresponds to $T_{\text{GW}} = P_{\text{orb}}$, and the black vertical dashed line represents the reported 16.35 day period of FRB 180916. It is seen from Figure 2 that the values of $T_{\text{GW}}/P_{\text{orb}}$ fall into the quadrilateral regions. As mentioned in Section 2, $T_{\text{GW}} \lesssim P_{\text{orb}}$ is a necessary condition for the periodic activity to appear. In addition, the period should be equivalent to $P_{\text{orb}}$, since $T_{\text{GW}} \gg P_{\text{orb}}$, the WD may be kicked away after a mass-transfer process (King 2007).

Moreover, since $T_{\text{GW}}$ is proportional to $\Delta M_2$ (Equation (9)), the condition $T_{\text{GW}} < P_{\text{orb}}$ may be satisfied only for some cycles with relatively low values of $\Delta M_2$, and not for some other cycles with relatively high values of $\Delta M_2$. In this scenario, we can understand why FRB 180916 has periodic activity but is not always active in all the predicted windows.

Another unresolved problem is that, for some repeaters with tens of or even more than a thousand observed bursts, such as FRBs 190520B (Niu et al. 2021) and 201124A (Xu et al. 2021a), no periodic activity was found based on time series analyses. In our opinion, the reason is related to $T_{\text{frag}}$, which denotes the time span from the arrival of the first piece of material at the NS surface to the arrival time of the last piece, as described in Section 2. Since $T_{\text{frag}}$ is quite an uncertain timescale in our model, it is regarded as a free parameter. For $T_{\text{frag}} < P_{\text{orb}}$, it is clear that $T_{\text{frag}}$ will work as a relatively narrow activity window in each cycle. In order to interpret the
observations of FRBs 180916 and 121102 by our model, $T_{\text{frag}}$ is required to be around 5 and 100 days, respectively. For $T_{\text{frag}} \gtrsim P_{\text{orb}}$, however, the periodic activity will be concealed and difficult to reveal. Thus, the frequently repeating sources without periodic activity may correspond to $T_{\text{frag}} \gtrsim P_{\text{orb}}$.

4. Conclusions and Discussion

In this paper, we have revisited the eccentric NS–WD binary model by including the effects of GW radiation on the periodic activity of FRB sources. We have shown that, even though our model indicates that the cycle time of the burst activity should be equivalent to the orbital period, whether the periodic activity can be present is related to the two timescales, i.e., the orbital period $P_{\text{orb}}$, the timescale $T_{\text{GW}}$ for the WD to refill its Roche lobe through the GW radiation, and the time span $T_{\text{frag}}$ related to the different arrival times for the fragmented materials. Our analyses indicate that (1) only when the conditions $T_{\text{GW}} \lesssim P_{\text{orb}}$ and $T_{\text{frag}} < P_{\text{orb}}$ are both satisfied, the periodic activity will appear. Otherwise, for either $T_{\text{GW}} \gg P_{\text{orb}}$ or $T_{\text{frag}} \gtrsim P_{\text{orb}}$, the periodicity will be hard to discover. (2) Since $T_{\text{GW}}$ is proportional to $\Delta M_2$, it is understandable that FRB 180916 is not always active in all the predicted activity windows. (3) FRBs with relatively long periods ($\gtrsim$ several days) are more likely to show up. Thus, it is reasonable that the only two sources having been claimed to have periodic activity, i.e., FRBs 180916 and 121102, both correspond to relatively long periods.

According to our model, a large eccentricity is required for the periodic activity to show up. In our opinion, there are two possibilities for the formation of an NS–WD binary with a large eccentricity. One is the evolution result of a binary channel by the natal kick of the supernova explosion. Since both gravitational waves and accretion tend to circularize the orbit, a highly eccentric orbit in this scenario should correspond to a young NS. The other possibility is a WD captured by an NS, where an old NS is quite possible. Whether the second possibility plays an important role may be inferred from recent statistical analysis. The CHIME/FRB Collaboration released their first FRB catalog that includes 536 FRB events (Amiri et al. 2021). This uniform large sample allows a better statistical analysis of the FRB population. Zhang & Zhang (2022) tested the new CHIME sample against the star formation rate density, cosmic stellar-mass density, and delayed models. They concluded that the CHIME FRB population does not track the star formation history of the universe, and their results indicate the old population as the origin of FRBs. In addition, Hashimoto et al. (2022) showed that old populations such as old NSs and BHs are more likely progenitors of nonrepeating FRBs. Thus, with regards to these statistical results, for our model, it is required that the capture mechanism should have significant contributions to the formation of the eccentric NS–WD binaries.

Notably, the periodic activity behavior may be much more complex and diverse than our current understanding. Recently, Pleunis et al. (2021) reported that the burst activity of FRB 180916 is systematically delayed toward lower frequencies by about 3 days (0.2 cycles) from 600 to 150 MHz. They also discussed a possible link between the frequency dependence of the observed activity and a radius-to-frequency mapping effect for various models. Such an issue is beyond the scope of this paper.

It should be noted that outflows are not taken into account in this work. Outflows may carry angular momentum and escape from the binary system, which corresponds to $\Delta J < 0$ rather than $\Delta J = 0$. Dong et al. (2018) investigated such an issue and
showed that a violent mass transfer may occur even for $q < 2/3$, such as a BH–WD system. It is easy to understand that, if outflows carry away significant angular momentum, the timescale $T_{GW}$ can be greatly shortened, which is helpful to satisfy the condition $T_{GW} \lesssim P_{orb}$, and therefore the periodic activity is more likely to show up.

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