Aspects of Non-Identical Multiple Pulse Compression

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Abstract—Some radar systems utilize pulse agility to achieve higher range resolution or mitigate range ambiguities. However, transmitting different waveforms on a pulse-to-pulse basis can have deleterious effects when traditional pulse-Doppler processing is employed. In this paper a non-adaptive technique, entitled Non-Identical Multiple Pulse Compression (NIMPC), is derived that facilitates pulse-agile clutter cancellation and is readily implementable via fast convolution techniques. The new method is extended to account for clutter-Doppler spread as well as multiple range-ambiguous clutter techniques. The new method is extended to account for clutter-Doppler spread as well as multiple range-ambiguous clutter intervals. NIMPC is assessed via simulation of a synthetic wideband pulse train.

I. INTRODUCTION

Pulse-Doppler radar systems typically employ pulse compression and Doppler processing to achieve sufficient SNR to detect, range, and determine the velocity of moving targets. Additionally, clutter cancellation is used to remove the returns from stationary and slow moving objects, for example, land and sea clutter. Transmitting an identical waveform on each pulse allows standard clutter cancellation to be implemented in the slow-time domain, where the available degrees of freedom are dictated by the number of pulses within a coherent processing interval (CPI). Great care is taken to ensure that there is very little timing error from pulse to pulse, as any jitter will limit the ability of the radar system to cancel clutter. Note that in this scenario the physically transmitted waveform may have some distortion relative to the presumed ideal waveform, though this distortion is nearly identical for each of the transmitted pulses and thus does not affect clutter cancellation. In addition, fill pulses are used to ensure that returns from range-ambiguous clutter or multiple-time-around clutter (MTAC) are present in each pulse receive interval and therefore possess the same slow-time structure as unambiguous clutter returns.

Some radars transmit different waveforms on each pulse for various reasons, for example, to synthesize a wider bandwidth or resolve range ambiguities, thus preventing the use of standard non-adaptive clutter cancellation techniques. In this paper, a new non-adaptive framework entitled Non-Identical Multiple Pulse Compression (NIMPC) is presented for which the available degrees of freedom for clutter cancellation are significantly higher than traditional techniques. A waveform-based clutter cancellation technique within the NIMPC framework is discussed, and simulation results are shown for the synthetic wideband scenario wherein each of the transmitted pulses has a progressively higher center frequency. Previously, pulse-to-pulse phase changes have been considered as a means to resolve range ambiguities, for which clutter cancellation can be performed using a more traditional approach [1] that is applied in the slow-time (pulse-to-pulse) domain. However, in this paper, both pulse-to-pulse waveform and center frequency changes are addressed.

It should be noted that the formulation presented in the following is related to a more general framework presented in [2]. The general framework can be parameterized in several ways, as seen in [2], to arrive at different solutions. The approach taken in this paper is to introduce a new algorithm from an implementation standpoint, whereas [2] provides a general model for which the appropriate parameterization and assumptions will result in the signal model used in the following section. The dimensionality of the NIMPC filter is consistent with the pulse compression matched filter and is shift invariant, that is, the same filter is applied to each range cell, resulting in an efficient implementation strategy.

II. NON-IDENTICAL MULTIPLE PULSE COMPRESSION

A. NIMPC Framework

The NIMPC signal model is similar to the model for the Time-Range Adaptive Processing (TRAP) algorithm in [3]. However, the non-adaptive benefits of this structure are considered in this paper. Specifically, the authors show how NIMPC can be used to implement deterministic clutter cancellation techniques.

The waveforms transmitted in a CPI can be represented by the $N \times M$ matrix $S$ where the $m$th column $s_m$ is the $m$th length-$N$ discretized waveform. The waveforms in $S$ are arbitrary and may vary with respect to modulation, center frequency, etc. from pulse to pulse. The received signal at the $\ell$th range cell from all $M$ pulses in the CPI is expressed as the row vector

$$y(\ell) = [y_0(\ell) \ y_1(\ell) \ \cdots \ y_m(\ell) \ \cdots \ y_{M-1}(\ell)], \quad (1)$$

the $m$th element of which is denoted as

$$y_m(\ell) = \sum_{n} [x_\ell(n) s_m e^{j2\pi n / N}],$$

$$n(\ell), \quad (2)$$

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where \( \mathbf{x}_\theta(\ell) = [x(\ell, \theta) \ x(\ell-1, \theta) \ \cdots \ x(\ell-N+1, \theta)]^T \) is a collection of the \( N \) complex scattering coefficients associated with the scatterers in the range profile corresponding to the Doppler phase shift \( \theta \) with which the \( m \)th waveform convolves at delay \( \ell \), and \( n(\ell) \) is additive noise. Collecting \( N \) contiguous fast-time (range) samples of the received signal model in (1) yields a received signal matrix representation

\[
\mathbf{Y}(\ell) = \sum_\theta [\mathbf{x}_\theta(\ell)(\mathbf{S} \odot \mathbf{V}_\theta)] + \mathbf{N}(\ell),
\]

where

\[
\mathbf{V}_\theta = \begin{bmatrix}
1 & \cdots & e^{j\theta} & \cdots & e^{j(M-1)\theta}
\end{bmatrix}
\]

is an \( N \times M \) matrix,

\[
\mathbf{X}_\theta(\ell) = \begin{bmatrix}
x(\ell, \theta) & x(\ell-1, \theta) & \cdots & x(\ell-N+1, \theta)
x(\ell+1, \theta) & x(\ell, \theta) & \cdots & x(\ell-N+2, \theta)
\vdots & \vdots & \ddots & \vdots
x(\ell+N-1, \theta) & x(\ell+N-2, \theta) & \cdots & x(\ell, \theta)
\end{bmatrix}
\]

is an \( N \times N \) matrix containing the complex scattering amplitudes corresponding to the \( 2N-1 \) range cells surrounding \( x(\ell, \theta) \), and \( \odot \) denotes the Hadamard product.

As in [3], reorganize the matrix of (3) into the single length-\( NM \) vector

\[
\mathbf{y}(\ell) = \text{vec}\left( \mathbf{Y}(\ell) \right) = \text{vec}\left[ \sum_\theta \mathbf{X}_\theta(\ell)(\mathbf{S} \odot \mathbf{V}_\theta) \right] + \mathbf{n}(\ell),
\]

in which \( \mathbf{n}(\ell) = \text{vec}(\mathbf{N}(\ell)) \). Based on (6), a (normalized) joint range-Doppler steering vector can be expressed as

\[
\mathbf{w}_\theta = \frac{1}{NM} \text{vec}\left[ (\mathbf{S} \odot \mathbf{V}_\theta) \right],
\]

which can subsequently be applied to obtain the normalized NIMPC estimate

\[
\hat{x}_{\text{NIMPC}}(\ell, \theta) = \mathbf{w}_\theta^H \mathbf{y}(\ell).
\]

It should be noted that applying the filter in (8) yields an identical result as standard range and Doppler processing filters when applied sequentially. However, in the next section, the added degrees of freedom in the NIMPC framework are exploited to achieve clutter cancellation for the non-identical pulse scenario, which is not easily achieved using separate range and Doppler processing.

**B. NIMPC Clutter Cancellation**

When identical pulses are transmitted, the relative slow-time (pulse-to-pulse) change between clutter returns is constant throughout the entire pulse duration; whereas in the non-identical case, the relative pulse-to-pulse phase difference between returns changes as a function of fast time. These waveform changes yield matched filter range sidelobes that are different for each waveform, which is referred to as range sidelobe modulation (RSM). The RSM may be highly correlated, as in the case of synthetic wideband waveforms, or uncorrelated, when for example random pulse-to-pulse coding is used [4]. Hence, standard clutter cancellation techniques, which are applied in the slow-time domain, do not typically possess the degrees of freedom necessary to cancel all of the different pulse-to-pulse phase progressions associated with RSM.

Clutter cancellation can be achieved using the NIMPC framework by deterministically modeling the clutter signal structure from an individual range cell as interference and by applying the maximum signal-to-interference plus noise ratio (SINR) solution. The interference covariance matrix can be constructed as

\[
\mathbf{R} = (\mathbf{P}_\theta \mathbf{P}_\theta^H + \varepsilon \mathbf{I}),
\]

where

\[
\mathbf{P}_\theta = \left[ \mathbf{c}_\theta^{(-N+1)} \mathbf{c}_\theta^{(-N+2)} \cdots \mathbf{c}_\theta^{(-1)} \mathbf{c}_\theta^0 \right],
\]

\[
\mathbf{c}_\theta^n = \text{vec}(\mathbf{S}_n \odot \mathbf{V}_\theta),
\]

\[
\mathbf{S}_n = \begin{bmatrix}
\mathbf{s}_n^0 \mathbf{s}_n^1 \cdots \mathbf{s}_n^{N-1} \\
\mathbf{s}_n^0 \mathbf{s}_n^1 \cdots \mathbf{s}_n^{N-1}
\end{bmatrix}
\]

and

\[
\mathbf{s}_n^n = \begin{bmatrix}
\mathbf{0}_N^T \ s_n(0) \ \cdots \ s_n(N-1) \\
\mathbf{0}_N^T \ s_n(0) \ \cdots \ s_n(N-1)
\end{bmatrix}
\]

in which \( s_n(n) \) is the \( n \)th sample of the \( m \)th pulse and \( \mathbf{0}_N \) is an \( n \times 1 \) vector of zeros. In (9) \( \varepsilon \) is a diagonal loading factor to prevent ill-conditioning. It has been observed that the receiver noise power is a suitable value for \( \varepsilon \) in most scenarios. The middle column of \( \mathbf{P}_\theta \) corresponds to the contribution of clutter in the desired range cell; whereas the other \( 2(N-1) \) columns correspond to contributions from clutter in the surrounding range cells, thus accounting for the RSM. The resulting clutter-cancelled estimate is

\[
\hat{x}_{\text{NIMPC-C}}(\ell, \theta) = \mathbf{w}_\theta^H \mathbf{y}(\ell),
\]

where

\[
\mathbf{w}_\theta = \frac{\mu}{NM} \mathbf{R}^{-1} \text{vec}(\mathbf{S} \odot \mathbf{V}_\theta)
\]

is the clutter-whitened NIMPC filter and \( \mu \) is an arbitrary scale factor.

Extending the clutter notch to account for clutter-Doppler spread, achieved by placing multiple closely spaced notches, is implemented by replacing \( \mathbf{P}_\theta \) in (9) with
\[ \tilde{P}_\theta = \begin{bmatrix} P_{\theta_0} & P_{\theta_1} & \cdots & P_{\theta_{Q-1}} \end{bmatrix}, \]  
where \( Q \) is the total number of notches. However, each notch requires \( 2N-1 \) degrees of freedom (DOFs) and may necessitate the use of additional pulses to increase the available DOFs, given by the product \( NM \).

Standard clutter cancellation techniques rely on the ability of the transmitter to reproduce a nearly identical waveform on each pulse. When pulse agility is used, each waveform is different; thus any error between the transmitted waveforms and the digital representations used for processing will limit the amount of clutter that can be suppressed by NIMPC.

\section*{C. Extension to Range Ambiguous Clutter Intervals}

The clutter cancellation techniques in the previous section can be extended to account for multiple-time-around clutter (MTAC). MTAC is present in medium and high PRF surveillance radar systems where the clutter response is measured from distances greater than the unambiguous range

\[ r_u = \frac{c}{2 \text{PRF}}, \]

where \( c \) is the speed of light and PRF is pulse repetition frequency. The returns from beyond \( r_u \) will appear to originate from within the primary interval such that several different PRF values are normally used to decipher the true range of ambiguous targets. Typically in the range-ambiguous case, the radar will transmit several pulses before the receiver is turned on such that returns from each ambiguous clutter interval will be present when the receiver starts recording the first pulse used for processing. The pulses that are transmitted before the receiver turns on are referred to as fill pulses. When identical waveforms are used, a fill pulse is required for each expected range ambiguous interval; that is, the number of fill pulses is equal to the number of ambiguous intervals (not counting the primary unambiguous interval).

Although it is not necessary to transmit fill pulses when pulse agility is used since the clutter returns from each interval will inherently be different, they will be considered in the following analysis to ensure that the returns from each interval have uniform energy. For a given interval, targets and clutter from other intervals will not coherently integrate, thus producing interference in addition to the aforementioned effects of RSM. Clutter residue from ambiguous intervals can be addressed by modifying the covariance matrix from (9) to include the clutter response from multiple intervals. For \( K \) intervals and \( Q \) notches at each interval, the required DOF is given by

\[ \text{DOF} = KQ(2N-1), \]

which should not exceed the available DOF dictated by the product \( MN \).

\section*{III. IMPLEMENTATION}

The NIMPC filter in (15) can be computationally expensive to apply due to the large dimensionality. However, fast convolution via FFT processing can be used to apply the whitened NIMPC filters (which can be computed offline). First, examine the relationship between \( \tilde{y}(\ell) \) and \( \tilde{y}(\ell+1) \):

\[ \tilde{y}(\ell) = \begin{bmatrix} y_0(\ell) \\ \vdots \\ y_0(\ell+N-1) \\ y_1(\ell) \\ \vdots \\ y_1(\ell+N-1) \\ \vdots \\ y_{M-1}(\ell) \\ \vdots \\ y_{M-1}(\ell+N-1) \end{bmatrix}, \quad \tilde{y}(\ell+1) = \begin{bmatrix} y_0(\ell+1) \\ \vdots \\ y_0(\ell+N-1+1) \\ y_1(\ell+1) \\ \vdots \\ y_1(\ell+N-1+1) \\ \vdots \\ y_{M-1}(\ell+1) \\ \vdots \\ y_{M-1}(\ell+N-1+1) \end{bmatrix}. \]

The colored blocks in (22) represent the contributions of the individual pulses to the received signal vector. Next, consider the whitened NIMPC filter which can be expressed as

\[ \tilde{w}_\theta = \left[ \tilde{w}_\theta^{(0)} \tilde{w}_\theta^{(1)} \cdots \tilde{w}_\theta^{(M-1)} \right]^T, \]

where the filter has been segmented into \( M \) separate \( N \)-length contiguous blocks. The application of the NIMPC filter for a single Doppler bin can be represented as a convolution of the received signal blocks in (22) with the corresponding segment of the NIMPC filter from (23). This convolutional implementation is expressed as

\[ \hat{x}_{\text{NIMPC-C}}(\ell, \theta) = \frac{1}{NM} \sum_{m=0}^{M-1} \sum_{n=-N}^{N-1} \tilde{w}_\theta^{m} y_m(\ell+N-1-n), \]

for \( \ell = 0, 1, \ldots, L-N \) where \( L-1 \) is the length of the range profile and \( (\cdot)^* \) denotes complex conjugation. Equation (24)
can be implemented efficiently using the fast Fourier transform (FFT) as

\[
\hat{x}_{\text{NIMPC-C}}(\ell, \theta) = \frac{1}{NM} \sum_{m=0}^{M-1} F^{-1} \{ F\{ \bar{\mathbf{w}}_{m,\theta} \} \times F\{ \mathbf{y}_m \} \},
\]  

in which \( \mathbf{y}_m \) is the received data from the \( m \)th pulse, \( \bar{\mathbf{w}}_{m,\theta} \) is the time-reversed complex conjugate of \( \mathbf{w}_{m,\theta} \) zero-padded to the length of \( \mathbf{y}_m \), and \( F \) and \( F^{-1} \) are the FFT and inverse FFT, respectively. Note that usually the received vector is padded with zeros before the FFT, but since the formulation in (24) does not include the convolutional tails which represent the eclipsed region, then the additional zero-padding is unnecessary. In fact, the first \( N-1 \) samples of the output from (25) should be discarded to produce a result equivalent to that produced by (24). While the NIMPC formulation naturally accounts for the range sidelobes associated with clutter in the eclipsed region that extend into the leading and trailing edges of the range profile, it does not provide a clutter-free estimate in the eclipsed region. Thus the convolutional tails associated with the eclipsed regions are discarded.

**IV. SIMULATION RESULTS**

Consider the synthetic wideband scenario for which each column of the matrix \( \mathbf{S} \) is a 40-chip phase-coded P3 waveform [5], over-sampled by a factor of two such that \( N = 80 \) and each waveform is at an offset center frequency. The CPI contains 50 (=\( M \)) pulses and three additional fill pulses, resulting in a total of 53 transmitted waveforms for which the normalized center frequency of the \( m \)th pulse is given by

\[
f_m = \frac{m}{2(M+K-2)} \]

for \( m = 0, 1, \ldots, M+K-2 \), where \( K = 4 \) is the number of range intervals (1 unambiguous and 3 ambiguous per (15)). The resulting bandwidth is twice as large as that of a single pulse, and consequently the range resolution twice as fine. The simulated scene consists of four small moving targets in the fourth interval with the ranges and Doppler phases listed in Table 1. For an X-band radar with a 10 kHz PRF, the Doppler phases in Table 1 are comparable to target velocities ranging from 37 to 65 mph. The moving targets are embedded in additive white Gaussian noise with a signal-to-noise ratio (SNR) of \(-13 \) dB, and the average clutter-to-noise ratio (CNR) is 57 dB (stated values are pre-processing) such that the clutter power is 70 dB higher than that of the targets. The clutter extends through the primary interval and three range ambiguous intervals and is present in all eclipsed regions. The coherent processing gain is 33 dB. For the synthetic wideband waveform described above, the added benefit is enhanced range resolution, and ambiguous ranges will still have to be unwrapped using a multiple staggered PRF scheme.

Table 1. Target Parameters

| Target | Range Cell | Doppler Phase θ |
|--------|------------|-----------------|
| Target 1 | 770 | 70° |
| Target 2 | 820 | 40° |
| Target 3 | 870 | 65° |
| Target 4 | 920 | 50° |

Figure 1. NIMPC estimate (in dB) with no clutter cancellation (equivalent to standard range-Doppler processing)

Figure 2. NIMPC estimate (in dB) with clutter cancellation

The Doppler view of the range-Doppler maps in Figs. 1 and 2 are displayed in Fig. 3, which is formed by taking the maximum value over all of the range cells in each Doppler bin. The clutter has been suppressed by approximately 90 dB. Figure 4 shows the NIMPC filter response, defined as

U.S. Government work not protected by U.S. copyright.
\[
\chi(\theta) = \frac{\vec{w}_o^H \text{vec}(S \circ V_o) \vec{w}_o^T \text{vec}(S \circ V_o)}{\vec{w}_o^H \text{vec}(S \circ V_o)}.
\] (27)

which compares the processing gain of the NIMPC filter with that of the optimal (for a point target in white noise) range-Doppler matched filter. The filter response illustrates that for this scenario there is maximum SNR loss of \(-1\) dB, outside of the clutter notch, which is an acceptable trade-off for 90 dB of pulse-agile clutter cancellation.

To alleviate the effective Doppler spread associated with the range ambiguous clutter, the P3 code from the previous example is replaced with a single random phase code of the same length (40 chips, over-sampled by 2). Although the random phase code suffers from elevated range sidelobes, the Doppler intolerance of this waveform should alleviate the Doppler spreading of the clutter returns from the first three intervals when examining the fourth interval. Figures 5 and 6 are the simulated outputs from the fourth interval for NIMPC without and with clutter cancellation, respectively. One can see that the effect of the clutter-Doppler spread associated with the P3 code has been reduced to a width commensurate with a single interval of clutter illuminated with a stepped-frequency waveform.

Observe that the clutter response in Fig. 1 and corresponding clutter notch in Figs. 2-4 are offset from zero even though there is no clutter–Doppler spread. This artificial spread may result in an undesired suppression of slow moving targets. The asymmetry is caused by the Doppler tolerance of the P3 code in combination with the stepped-frequency waveform present in the ambiguous clutter from the three other intervals. The clutter returns from the first three intervals are composed of pulse trains that are higher in frequency than the pulse train used for processing the fourth interval. This frequency shift results in a negative range shift of the matched filter peak, which in conjunction with the range-Doppler coupling of the stepped-frequency waveform shifts the peak of the clutter response from the other intervals to a negative Doppler in the fourth interval.
The clutter notch in this case is much narrower as evidenced in the Doppler view of Fig. 7. Also, the clutter suppression performance is similar to when the P3 code is used. However, the minimum in the NIMPC filter response (Fig. 8) has decreased to $-2.2$ dB when the random phase code is used.

![Doppler view of NIMPC estimate with and without clutter cancellation when the P3 code is replaced with a random phase code](image1.png)

Figure 7. Doppler view of NIMPC estimate with and without clutter cancellation when the P3 code is replaced with a random phase code

![NIMPC filter response vs. Doppler phase when the P3 code is replaced with a random phase code](image2.png)

Figure 8. NIMPC filter response vs. Doppler phase when the P3 code is replaced with a random phase code

V. CONCLUSION

A new non-adaptive framework, entitled Non-Identical Multiple Pulse Compression (NIMPC), has been presented that allows clutter cancellation to be performed for scenarios when the radar waveforms within a CPI change from pulse to pulse. A novel implementation has been provided that enables a real-time realization of the NIMPC algorithm via fast convolution techniques. Simulation results for synthetic wideband waveforms were investigated, and it was determined that range-ambiguous clutter from Doppler-tolerant synthetic wideband waveforms induces an artificial clutter-Doppler spread, which can be cancelled using NIMPC but may result in an undesired suppression of slow moving targets. When a Doppler intolerant waveform is used in the synthetic wideband regime, the artificial clutter-Doppler spread is alleviated and the minimum discernable velocity can be improved. NIMPC was shown via simulation to suppress pulse-agile clutter by 90 dB; however, in reality the waveform error imposed by the transmitter will limit the effectiveness of the algorithm. The effect of waveform distortion is left as future work.

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