Renormalization group improvement of the effective potential in massive $\phi^4$ theory

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Abstract

Using the method of renormalization group, we improve the two-loop effective potential of the massive $\phi^4$ theory to obtain the next-next-to-leading logarithm correction in the $\overline{\text{MS}}$ scheme. Our result well reproduces the next-next-to-leading logarithm parts of the ordinary loop expansion result known up to the four-loop order.

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I. INTRODUCTION

The groundbreaking paper \[1\] of Gell-Mann and Low was in a large part directed to the problem of improving perturbation theory, i.e., to the problem of using the ideas of the renormalization group and the results of perturbation theory to a given order to say something about the next order of perturbation theory. The method of the renormalization group improvement of perturbation theory can be applied to the computation of the Green’s functions and other predictions of Feynman diagram perturbation theory including the effective potential.

The effective potential in quantum field theory is a convenient tool in probing the vacuum structure of the theory. The usual way of computing the effective potential is a loop expansion \[2\], for which an elegant method called the field shift method was developed by Jackiw \[3\]. It has been recognized for a long time that ordinary loop-wise perturbation expansions of important physical quantities are not only restricted to small values of the couplings but are often rendered useless by the occurrence of the large logarithms. Renormalization group resummation of these logarithms is then crucial to establish a region of validity for the perturbative results.

Renormalization-group-improved effective potentials were first considered in the context of massless models by Coleman and Weinberg \[2\]. In the massive case it has been demonstrated that this treatment also works provided one takes into account the running of the vacuum energy (or cosmological constant) \[4,5,6,7\]. In the papers of Ref. \[7\], the multiscale problems were studied.

The purpose of this paper is to improve the effective potential of the massive $\phi^4$ theory through the second-to-leading logarithm, i.e., next-next-to-leading logarithm, order in the modified minimal subtraction (\text{MS}) scheme. We compare the structures of the loop expansion and leading-logarithm series expansion of the effective potential in Sec. II. In Sec. III, the running parameters are determined through the three-loop order from their evolution equations with three-loop renormalization group functions and the result of the next-next-to-leading logarithm improvement is given. The final section is devoted to the concluding remarks.

II. LOOP EXPANSIONS VS. LEADING-LOGARITHM SERIES EXPANSION

The Lagrangian of the massive $\phi^4$ theory is given as

$$\mathcal{L} = \frac{1}{2}(\partial \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 - \Lambda + \frac{\delta Z}{2}(\partial \phi)^2 - \frac{\delta m^2}{2}\phi^2 - \frac{\delta \lambda}{4!}\phi^4 - \delta \Lambda.$$  \hspace{1cm} (1)

Here $\delta Z$, $\delta m^2$, and $\delta \lambda$ are the so-called counterterms of the wave function, mass, and coupling constant, respectively. Their values are known up to the five-loop order for the massive $O(N)$ $\phi^4$ theory in the \text{MS} scheme from the renormalization of the two- and four-point (one-particle-irreducible) Green’s functions \[3\]. The constant $\Lambda$, usually called the cosmological constant term, is included in the above Lagrangian, Eq. (1). The corresponding counterterm is calculated up to the five-loop order for the massive $O(N)$ $\phi^4$ theory in the \text{MS} scheme \[10\].
Such a term becomes relevant to the $\phi$-dependent terms of the effective potential \[5\]. We note that these renormalization counterterms can be determined either in the renormalization of the 1PI Green’s functions $\Gamma^{(0)}$, $\Gamma^{(2)}$, and $\Gamma^{(4)}$, or in the renormalization of the effective potential $V$. In particular, the vacuum energy renormalization constant up to three-loop order is given as

$$\delta \Lambda = \frac{m^4}{2(4\pi)^2 \epsilon} + \frac{\lambda m^4}{2(4\pi)^4 \epsilon^2} + \frac{\lambda^2 m^4}{(4\pi)^6} \left( \frac{5}{6\epsilon^3} - \frac{5}{18\epsilon^2} + \frac{1}{48\epsilon} \right).$$

The effective potential of the theory defined by Eq. (1) up to two-loop order in the $\overline{\text{MS}}$ scheme \[11,12\] is given as

$$V = V^{(0)} + V^{(1)} + V^{(2)},$$

$$V^{(0)} = \frac{m^2 \phi^2}{2} + \frac{\lambda \phi^4}{4!} + \Lambda,$$

$$V^{(1)} = \frac{\lambda}{(4\pi)^2} \left[ -\frac{3m^4}{8\lambda} + \frac{3m^2 \phi^2}{8} - \frac{3\lambda \phi^4}{32} + \left\{ \frac{m^4}{4\lambda} + \frac{m^2 \phi^2}{4} + \frac{\lambda \phi^4}{16} \right\} \ln \left( \frac{m^2_{\phi}}{\mu^2} \right) \right],$$

$$V^{(2)} = \frac{\lambda^2}{(4\pi)^4} \left[ \frac{m^4}{8\lambda} + \frac{m^2 \phi^2}{4} + \frac{3\lambda \phi^4}{32} \right] \ln \left( \frac{m^2_{\phi}}{\mu^2} \right) + \left\{ \frac{m^4}{8\lambda} + \frac{m^2 \phi^2}{4} + \frac{3\lambda \phi^4}{32} \right\} \ln^2 \left( \frac{m^2_{\phi}}{\mu^2} \right),$$

where $m^2_{\phi} \equiv m^2 + \lambda \phi^2/2$ and $A$ is a constant, $A = -1.171953619 \cdots$.

Now let us compare the structures of the loop expansion and the leading-logarithm series expansion of the effective potential. In the usual loop expansion, the $l$-loop quantum correction to the effective potential has the following general structure \[4\]:

$$V^{(l)}(\phi, \lambda, x, y) = \lambda^{l+1} \phi^4 \sum_{m=0}^{l-1} \sum_{n=0}^{l} a_{lmn} x^{m-2} y^n,$$

where

$$x \equiv \frac{1}{1 + 2m^2/(\lambda \phi^2)}, \quad y \equiv \ln \frac{m^2_{\phi}}{\mu^2}.$$  \hspace{1cm} (3)

Introducing a function $G_{n}^{(l)}$ as follows:

$$G_{n}^{(l)}(\phi, \lambda, x) = (4\pi)^2 \lambda \phi^4 \sum_{m=0}^{l-1} a_{tm(n)} x^{m-2},$$

\[1\] The two-loop effective potential for the massive $O(N) \phi^4$ theory in the $\overline{\text{MS}}$ scheme and the three-loop effective potential for the single-component massive $\phi^4$ theory were calculated in Ref. \[11\] and Ref. \[12\] respectively. In Ref. \[12\], there is no difficulty in obtaining the $\overline{\text{MS}}$ result because at the intermediate stage of calculation, finite constants of the renormalization counterterms in dimensional regularization scheme are kept as symbols, not as numerical values.
Eq. (3) can be rewritten as

$$V(l)(\phi, \lambda, x, y) = \lambda l \left( \frac{4\pi}{2} \right)^{l} \sum_{n=0}^{l} G^{(l)}_{n}(\phi, \lambda, x) y^{l-n}. \tag{5}$$

In terms of the original parameters, this equation can be expressed as

$$V(l)(\phi, \lambda, m^2, \mu^2) = \lambda l \left( \frac{4\pi}{2} \right)^{l} \sum_{n=0}^{l} G^{(l)}_{n}(\phi, \lambda, m^2) \ln^{l-n} \left( \frac{m^2_\phi}{\mu^2} \right). \tag{6}$$

For example, the four-loop contribution, $V(4)$, takes the following form

$$V^{(4)} = \frac{\lambda^4}{(4\pi)^4} \left[ A_1 \frac{m^4}{\lambda} + A_2 m^2 \phi^2 + A_3 \lambda \phi^4 + A_4 \frac{\lambda^4}{1 + 2m^2/(\lambda \phi^2)} \right]$$

$$+ \left\{ B_1 \frac{m^4}{\lambda} + B_2 m^2 \phi^2 + B_3 \lambda \phi^4 + B_4 \frac{\lambda^4}{1 + 2m^2/(\lambda \phi^2)} \right\} \ln \left( \frac{m^2_\phi}{\mu^2} \right)$$

$$+ \left\{ C_1 \frac{m^4}{\lambda} + C_2 m^2 \phi^2 + C_3 \lambda \phi^4 + C_4 \frac{\lambda^4}{1 + 2m^2/(\lambda \phi^2)} \right\} \ln^2 \left( \frac{m^2_\phi}{\mu^2} \right)$$

$$+ \left\{ D_1 \frac{m^4}{\lambda} + D_2 m^2 \phi^2 + D_3 \lambda \phi^4 + D_4 \frac{\lambda^4}{1 + 2m^2/(\lambda \phi^2)} \right\} \ln^3 \left( \frac{m^2_\phi}{\mu^2} \right)$$

$$+ \left\{ E_1 \frac{m^4}{\lambda} + E_2 m^2 \phi^2 + E_3 \lambda \phi^4 + E_4 \frac{\lambda^4}{1 + 2m^2/(\lambda \phi^2)} \right\} \ln^4 \left( \frac{m^2_\phi}{\mu^2} \right). \tag{7}$$

In addition to the $l$-loop correction, Eq. (5) or Eq. (6), there is a tree-level potential, called classical potential, which can be expressed as

$$V^{(0)} = \phi^4 \left( a + \frac{b}{x} \right) + \Lambda \equiv G^{(0)}_{0}(\phi, \lambda, x, \Lambda) = G^{(0)}_{0}(\phi, \lambda, m^2, \Lambda). \tag{8}$$

The (complete) effective potential is given as the sum of all loop corrections and the above tree-level potential $V^{(0)}$:

$$V = \sum_{l=0}^{\infty} \frac{\lambda^l}{(4\pi)^{2l}} \sum_{n=0}^{l} G^{(l)}_{n} \ln^{l-n} \left( \frac{m^2_\phi}{\mu^2} \right). \tag{9}$$

Notice that the zero-point energy level is set to be

$$V(\phi = 0) = \Lambda + \frac{\lambda^l}{(4\pi)^{2l}} \sum_{l=1}^{\infty} G^{(l)}_{l}(0, \lambda, m^2) = -\Gamma^{(0)}. \tag{10}$$

This choice was taken correctly by the authors of Ref. [5].

Rearranging the summation order in Eq. (10), we can reexpress the effective potential as follows:

$$V = \sum_{l=0}^{\infty} \sum_{n=0}^{l} G^{(n)}_{l} \left[ \lambda^n/(4\pi)^{2n} \right] \ln^{n-l} \left( \frac{m^2_\phi}{\mu^2} \right)$$
\[ \sum_{l=0}^{\infty} [\lambda^l/(4\pi)^{2l}] \sum_{n=l}^{\infty} G_l^{(n)} \tilde{y}^{n-l} \]
\[ \equiv \sum_{l=0}^{\infty} [\lambda^l/(4\pi)^{2l}] F_l(\phi, \lambda, x, \tilde{y}) \]
\[ \equiv \sum_{l=0}^{\infty} V^{(l)} , \quad (11) \]

where \( \tilde{y} \equiv \lambda y/(4\pi)^2 = (\lambda/(4\pi)^2) \ln(m^2/\mu^2) \). This form of expansion, Eq. (11), in powers of \( \lambda \), was first derived by Kastening [4]. The term proportional to \( \lambda^l \) in \( V \) is referred to as an \( l \)th-to-leading logarithm term [3]. The functions \( F_0, F_1, ... \) correspond to the leading, next-to-leading, ... logarithm terms, respectively. This concept of leading-logarithm series expansion is shown schematically in Fig. 1.

**III. RUNNING PARAMETERS AND THE NEXT-NEXT-TO-LEADING LOGARITHM IMPROVEMENT**

In his remarkable paper [4], Kastening has introduced two methods — the power series method and the differential equations method — to improve the effective potential in massive \( \phi^4 \) theory. Though we readily get a closed-form expression for \( F_0 \) from the obtained recursion relations for \( a_{lmn} \), the power series method is too much complicated to extend it beyond leading-logarithm correction. The differential equations method, which is a smarter one, does not rely on being able to sum up a power series involving coefficients gotten through complicated recursion relations. Instead, we obtain the recursion relations for the Kastening’s function \( f_l \) themselves from the renormalization group equation.

We use here the method of characteristics to improve the effective potential of the massive \( \phi^4 \) theory up to the next-next-to-leading logarithm correction for the first time: though there is an elegant method of Ref. [3], in which we use only \( G_l^{(l)} \) as boundary functions, we follow closely the method used in Sec. II of the second paper in Ref. [7].

Since we assume the effective potential \( V(= \sum_{l=0}^{\infty} V^{(l)} = \sum_{l=0}^{\infty} V^{(l)}) \) is independent of the renormalization scale \( \mu \) for the fixed values of the bare parameters, the effective potential \( V(\mu, \lambda, m^2, \phi, \Lambda) \) obeys the renormalization group equation

\[ \left[ \mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} - \gamma_\phi \phi \frac{\partial}{\partial \phi} + \beta_\Lambda \frac{\partial}{\partial \Lambda} \right] V(\mu, \lambda, m^2, \phi, \Lambda) = 0 \, . \quad (12) \]

The various \( \beta \) and \( \gamma \) functions (\( \beta_\lambda, \gamma_m, \gamma_\phi, \) and \( \beta_\Lambda \)) introduced in the above equation are known up to the five-loop order [8,9,10]. Here we quote these values up to the three-loop order:

\[ \beta_\lambda = -\frac{1}{16\pi^2} \, , \quad \gamma_m = -\frac{1}{12} \, , \quad \gamma_\phi = 0 \, , \quad \beta_\Lambda = 0 \, . \]

2The function \( F_l \) defined in Eq. (11) is related to the Kastening’s function \( f_l \) as follows:

\[ F_l = (4\pi)^{2l} (\lambda \phi^4)^{f_{l+1}} \, . \]
\[ \beta_\lambda = \frac{3\lambda^2 h}{(4\pi)^2} - \frac{17\lambda^3 h^2}{3(4\pi)^4} + \frac{\lambda^4 h^3}{(4\pi)^6} \left( \frac{145}{8} + 12\zeta(3) \right) \]
\[ \equiv \beta_1 \lambda^2 h + \beta_2 \lambda^3 h^2 + \beta_3 \lambda^4 h^3 , \]
\[ \gamma_m = \frac{\lambda h}{(4\pi)^2} - \frac{5\lambda^2 h^2}{6(4\pi)^4} + \frac{7\lambda^3 h^3}{2(4\pi)^6} \]
\[ \equiv \gamma_{m1} \lambda h + \gamma_{m2} \lambda^2 h^2 + \gamma_{m3} \lambda^3 h^3 , \]
\[ \gamma_\phi = \frac{\lambda h}{(4\pi)^2} \times 0 + \frac{\lambda^2 h^2}{12(4\pi)^4} - \frac{\lambda^3 h^3}{16(4\pi)^6} \]
\[ \equiv \gamma_1 \lambda h + \gamma_2 \lambda^2 h^2 + \gamma_3 \lambda^3 h^3 , \]
\[ \beta_\Lambda = \frac{m^4 h}{2(4\pi)^2} + \frac{m^4 \lambda h^2}{(4\pi)^4} \times 0 + \frac{m^4 \lambda^2 h^3}{16(4\pi)^6} \]
\[ \equiv m^4 (\beta_{\Lambda1} \lambda h + \beta_{\Lambda2} \lambda^2 h^2 + \beta_{\Lambda3} \lambda^3 h^3 ) . \] (13)

Note that in the above equation, \( h \) factors are inserted; we will use the \( h \) factor as a counting parameter in the leading-logarithm series expansion. \(^3\) Applying the method of characteristics to Eq. (12), we can write the solution of Eq. (12), \( V(\mu, \lambda, m^2, \phi, \Lambda) \) as follows:
\[ V(\mu, \lambda, m^2, \phi, \Lambda) = V(\bar{\mu}, \bar{\lambda}, \bar{m}^2, \bar{\phi}, \bar{\Lambda}) , \] (14)
where the running parameters satisfy
\[ \bar{h} \frac{d\bar{\mu}}{dt} = \bar{\mu} , \quad \bar{h} \frac{d\bar{\lambda}}{dt} = \beta_\lambda (\bar{\lambda}) , \quad \bar{h} \frac{d\bar{m}^2}{dt} = \gamma_m (\bar{\lambda}) \bar{m}^2 , \]
\[ \bar{h} \frac{d\bar{\phi}}{dt} = -\gamma_\phi (\bar{\lambda}) \bar{\phi} , \quad \bar{h} \frac{d\bar{\Lambda}}{dt} = \beta_\Lambda (\bar{\lambda}, \bar{m}^2) , \] (15)
and at the boundary point, \( t = 0 \), their values are given as \( \bar{\mu}(t = 0) = \mu, \bar{m}^2(t = 0) = m^2, \)
\( \bar{\phi}(t = 0) = \phi, \) and \( \bar{\Lambda}(t = 0) = \Lambda. \)

The solution to \( \bar{\mu} \) differential equation is given as
\[ \bar{\mu}^2(t) = \mu^2 \exp(2t/h) . \]

In order to solve \( \bar{\lambda} \) differential equation, we try a perturbative solution by writing
\[ \bar{\lambda} = \bar{\lambda}^{(0)} + \bar{h} \bar{\lambda}^{(1)} + \bar{h}^2 \bar{\lambda}^{(2)} + O(h^3) , \]
with the boundary conditions \( \bar{\lambda}^{(0)}(0) = \lambda, \bar{\lambda}^{(1)}(0) = \bar{\lambda}^{(2)}(0) = 0. \) Then, with \( \beta_\lambda \) in Eq. (13),
the equation we want to solve splits into three equations within the desired order:

\(^3\)In Eq. (12), \( h \) factors are inserted, too, as follows:
\[ V = V^{(0)} + h V^{(1)} + h^2 V^{(2)} + O(h^3) . \]
\[
\frac{d\bar{\lambda}^{(0)}}{dt} = \beta_1 \bar{\lambda}^{(0)2}, \\
\frac{d\bar{\lambda}^{(1)}}{dt} = 2\beta_1 \bar{\lambda}^{(0)}\bar{\lambda}^{(1)} + \beta_2 \bar{\lambda}^{(0)3}, \\
\frac{d\bar{\lambda}^{(2)}}{dt} = 2\beta_1 \bar{\lambda}^{(0)}\bar{\lambda}^{(2)} + \beta_1 \bar{\lambda}^{(1)2} + 3\beta_2 \bar{\lambda}^{(0)2}\bar{\lambda}^{(1)} + \beta_3 \bar{\lambda}^{(0)4}.
\]

Solutions to these equations are obtained as
\[
\bar{\lambda}^{(0)} = \frac{\lambda}{T}, \\
\bar{\lambda}^{(1)} = -\frac{\beta_2 \lambda^2}{\beta_1 T^2} \ln T, \\
\bar{\lambda}^{(2)} = \frac{\lambda^3}{T^2} \left(\left(-\frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1}\right)\left[T^{-1} - 1\right] - \frac{\beta_2^3}{\beta_1^3} \ln T + \frac{\beta_2^3}{\beta_1^3} \ln^2 T\right),
\]
where \(T \equiv 1 - \beta_1 \lambda t\). Similarly, we write \(\bar{m}^2\) as
\[
\bar{m}^2 = \bar{m}^{(0)} + \hbar \bar{m}^{(1)} + h^2 \bar{m}^{(2)} + O(h^3),
\]
and with \(\gamma_m\) in Eq. (13), obtain splitted equations:
\[
\frac{d\bar{m}^{(0)}}{dt} = \gamma_{m1} \bar{\lambda}^{(0)} \bar{m}^{(0)}, \\
\frac{d\bar{m}^{(1)}}{dt} = \gamma_{m1} \bar{\lambda}^{(0)} \bar{m}^{(1)} \bar{m}^{(0)} + \gamma_{m1} \bar{\lambda}^{(0)} \bar{m}^{(0)} + \gamma_{m2} \bar{\lambda}^{(0)} \bar{m}^{(2)} \bar{m}^{(0)} + \gamma_{m2} \bar{\lambda}^{(0)} \bar{m}^{(0)} + \gamma_{m1} \bar{\lambda}^{(0)} \bar{m}^{(2)} \bar{m}^{(0)} + 2\gamma_{m2} \bar{\lambda}^{(0)} \bar{\lambda}^{(1)} \bar{m}^{(0)} + \gamma_{m2} \bar{\lambda}^{(0)} \bar{m}^{(2)} + \gamma_{m3} \bar{\lambda}^{(0)} \bar{m}^{(2)}.
\]
With the \(\bar{\lambda}\) solutions, Eq. (14), together with the boundary conditions \(\bar{m}^{(0)}(0) = m^2\), \(\bar{m}^{(1)}(0) = \bar{m}^{(2)}(0) = 0\), we obtain the following \(\bar{m}^2\) solutions:
\[
\bar{m}^{(0)} = \frac{m^2}{T^{\gamma_{m1}/\beta_1}}, \\
\bar{m}^{(1)} = \frac{\lambda m^2}{T^{\gamma_{m1}/\beta_1}} \left[\left(-\frac{\beta_2 \gamma_{m1}}{\beta_1^2} + \frac{\gamma_{m2}}{\beta_1}\right)\left[T^{-1} - 1\right] - \frac{\beta_2 \gamma_{m1} \ln T}{\beta_1^3}\right], \\
\bar{m}^{(2)} = \frac{\lambda^2 m^2}{T^{\gamma_{m1}/\beta_1}} \left[\left(-\frac{\beta_2 \gamma_{m1}}{2\beta_1^2} + \frac{\beta_3 \gamma_{m1}}{2\beta_1^2} + \frac{\beta_2^2 \gamma_{m1}}{2\beta_1^2} - \frac{\beta_2 \gamma_{m2}}{2\beta_1^2}\right)\left[T^{-2} - 1\right] \\
+ \left(\frac{\beta_2 \gamma_{m1}}{\beta_1^3} - \frac{\gamma_{m2}}{2\beta_1^2} + \frac{\gamma_{m3}}{2\beta_1}\right)\ln T + \left(\frac{\beta_2^2 \gamma_{m1}}{\beta_1^4} - \frac{\beta_2 \gamma_{m2}}{\beta_1^3} - \frac{\beta_2 \gamma_{m1} \gamma_{m2}}{\beta_1^2}\right)\ln T \\
+ \left(\frac{\beta_2 \gamma_{m1}}{2\beta_1^4} + \frac{\beta_2 \gamma_{m2} \gamma_{m2}}{2\beta_1^4}\right)\ln^2 T\right].
\]
If we note that \( \bar{\phi} \) differential equation is of the same structure except the minus sign on the right-hand side, one may readily read off the perturbation solutions for \( \bar{\phi} = \bar{\phi}^{(0)} + h\bar{\phi}^{(1)} + h^2 \bar{\phi}^{(2)} + O(h^3) \) as follows:

\[
\bar{\phi}^{(0)} = \frac{\phi}{T - \gamma_1/\beta_1},
\]

\[
\bar{\phi}^{(1)} = \frac{\lambda \phi}{T - \gamma_1/\beta_1} \left[ \left( \frac{\beta_2 \gamma_1}{\beta_1^2} - \gamma_2 \right) \left( T^{-1} - 1 \right) + \frac{\beta_2 \gamma_1 \ln T}{\beta_1^2} \right],
\]

\[
\bar{\phi}^{(2)} = \frac{\lambda^2 \phi}{T - \gamma_1/\beta_1} \left[ \left( \frac{\beta_2 \gamma_1}{\beta_1^2} + \frac{\beta_3 \gamma_1}{\beta_1^2} - \frac{\beta_2 \gamma_2}{\beta_1^2} + \frac{\beta_2 \gamma_1 \gamma_2}{\beta_1^2} + \frac{\gamma_2}{\beta_1^2} - \frac{\gamma_3}{\beta_1^2} \right) \left( T^{-1} - 1 \right) 
+ \left( \frac{\beta_2 \gamma_1^2}{\beta_1^2} - \frac{\beta_2 \gamma_1 \gamma_2}{\beta_1^2} \right) \ln T + \left( \frac{\beta_2 \gamma_1^2}{\beta_1^2} + \frac{\beta_2 \gamma_2}{\beta_1^2} - \frac{2 \beta_2 \gamma_1 \gamma_2}{\beta_1^2} \right) \ln \frac{T}{T^2} 
+ \left( \frac{\beta_2 \gamma_1^2}{\beta_1^2} + \frac{\beta_2 \gamma_1 \gamma_2}{\beta_1^2} \right) \ln T + \frac{\beta_2 \gamma_1^2}{\beta_1^2} \right] \right) \left( T^{-1} - 1 \right).
\]

Finally, we try the solution to the \( \bar{\Lambda} \) differential equation as

\[ \bar{\Lambda} = \bar{\Lambda}^{(0)} + h\bar{\Lambda}^{(1)} + h^2 \bar{\Lambda}^{(2)} + O(h^3) \, . \]

Then from the splitted equations

\[
\frac{d\bar{\Lambda}^{(0)}}{dt} = \beta_{\Lambda 1} \bar{m}^{2(1)} \, ,
\]

\[
\frac{d\bar{\Lambda}^{(1)}}{dt} = 2\beta_{\Lambda 1} \bar{m}^{2(0)} \bar{m}^{2(1)} + \beta_{\Lambda 2} \bar{\lambda}^{(0)} \bar{m}^{2(0)} \, ,
\]

\[
\frac{d\bar{\Lambda}^{(2)}}{dt} = \beta_{\Lambda 1} \bar{m}^{2(1)} \bar{m}^{2(0)} + 2\beta_{\Lambda 1} \bar{m}^{2(0)} \bar{m}^{2(0)} + \beta_{\Lambda 2} \bar{\lambda}^{(1)} \bar{m}^{2(0)} + 2\beta_{\Lambda 2} \bar{\lambda}^{(0)} \bar{m}^{2(0)} \, ,
\]

and the boundary conditions \( \bar{\Lambda}^{(0)} = \Lambda, \bar{\Lambda}^{(1)} = \bar{\Lambda}^{(2)} = 0 \), we obtain

\[
\bar{\Lambda}^{(0)} = \Lambda - \frac{m^4 \beta_{\Lambda 1}}{\lambda (\beta_1 - 2\gamma_{m1})} \left[ \frac{T^{1-2\gamma_{m1}/\beta_1}}{T_{1-2\gamma_{m1}/\beta_1}} - 1 \right] ,
\]

\[
\bar{\Lambda}^{(1)} = m^4 \left[ \frac{2\beta_{\Lambda 1}}{(\beta_1 - 2\gamma_{m1})} \left( \frac{\beta_2 \gamma_{m1}}{\beta_1} + \gamma_{m2} \right) \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) 
- \left( \frac{\beta_2 \beta_{\Lambda 1}^2}{\beta_1^2} + \frac{\beta_3 \beta_{\Lambda 1}}{\beta_1^2} \right) \gamma_{m2} \left( \beta_1 \gamma_{m1} \right) - \frac{\beta_{\Lambda 2}}{2\gamma_{m1}} \right] \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) 
- \frac{\beta_{\Lambda 1} \beta_2}{\beta_1^2} \ln T \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) ,
\]

\[
\bar{\Lambda}^{(2)} = m^4 \left( \frac{2\beta_2 \beta_{\Lambda 2}}{\beta_1^2} + \frac{\beta_3 \beta_{\Lambda 1}}{\beta_1^2} - \frac{2\beta_2 \beta_{\Lambda 1} \gamma_{m1}}{\beta_1^2} + \frac{4\beta_2 \beta_{\Lambda 1} \gamma_{m2}}{\beta_1^2} \right) \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) 
- \frac{\beta_{\Lambda 2} \gamma_{m2}}{\beta_1^2} \left( \beta_1^2 + \frac{\beta_2 \beta_{\Lambda 1} \gamma_{m2}}{\beta_1^2} + \frac{\beta_2 \beta_{\Lambda 1} \gamma_{m2}}{\beta_1^2} \right) \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) 
- \frac{\beta_{\Lambda 2} \gamma_{m2}}{\beta_1^2} \left( \beta_1^2 + \frac{\beta_2 \beta_{\Lambda 1} \gamma_{m2}}{\beta_1^2} + \frac{\beta_2 \beta_{\Lambda 1} \gamma_{m2}}{\beta_1^2} \right) \left( T^{1-2\gamma_{m1}/\beta_1} - 1 \right) .
\]
\begin{equation}
\begin{aligned}
&\frac{\beta_{A1}}{\beta_1(\beta_1 - 2\gamma_{m1})} \left( \frac{\beta_2' \gamma_{m1}}{\beta_1^3} - \frac{2\beta_2' \gamma_{m1}^2}{\beta_1^3} - \frac{2\beta_2' \gamma_{m1}^2}{\beta_1^3} \right) \left[ T^{1-2\gamma_{m1}/\beta_1} - 1 \right] \\
&- \frac{\beta_2' \gamma_{m1}^2}{\beta_1^3} + \frac{4\beta_2' \gamma_{m1} \gamma_{m2}}{\beta_1^3} - \frac{2\beta_2' \gamma_{m2}}{\beta_1^3} + \gamma_{m3} \left[ T^{1-2\gamma_{m1}/\beta_1} - 1 \right] \\
&+ \frac{1}{\beta_1} + 2\gamma_{m1} \left( \beta_{A3} - \frac{\beta_2' \Lambda_{A2}}{\beta_1^2} - \frac{2\beta_2' \Lambda_{A2} \gamma_{m1}}{\beta_1^2} \right) \\
&+ \frac{\beta_2' \beta_{A1} \gamma_{m1}}{\beta_1^3} + \frac{3\beta_2' \beta_{A1} \gamma_{m2}}{\beta_1^3} + \frac{4\beta_2' \beta_{A1} \gamma_{m1} \gamma_{m2}}{\beta_1^3} + \frac{2\beta_2' \beta_{A1} \gamma_{m2}^2}{\beta_1^3} + \frac{2\beta_2' \beta_{A1} \gamma_{m2}^2}{\beta_1^3} + \frac{2\beta_2' \beta_{A1} \gamma_{m3}}{\beta_1^3} \left[ T^{1-2\gamma_{m1}/\beta_1} - 1 \right] \\
&+ \left( \frac{2\beta_2' \beta_{A1} \gamma_{m1}}{\beta_1^3} - \frac{2\beta_2' \beta_{A1} \gamma_{m1}}{\beta_1^3} \right) \frac{\ln T}{T^{2\gamma_{m1}/\beta_1}} \\
&+ \left( \frac{-\beta_2' \beta_{A2}}{\beta_1^2} + \frac{2\beta_2' \beta_{A1} \gamma_{m1}}{\beta_1^3} - \frac{2\beta_2' \beta_{A1} \gamma_{m2}}{\beta_1^3} \right) \frac{\ln T}{T^{1+2\gamma_{m1}/\beta_1}} \\
&+ \frac{\beta_2' \beta_{A1} \gamma_{m1}}{\beta_1^4} \ln^2 \frac{T}{T^{1+2\gamma_{m1}/\beta_1}} \right] .
\end{aligned}
\end{equation}

If we insert the numerical values in Eq. (13) for the symbols \( \beta_1, \gamma_{m1}, \gamma_{m2}, \gamma_{m3}, \beta_1 \), and \( \beta_{A1} \) into Eqs. (16) — (19), we obtain the running parameters up to the lowest three orders:

\begin{align*}
\tilde{\lambda}(t) &= \lambda \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1} + \frac{17\hbar^2 \lambda^2}{9(4\pi)^2} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-2} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \\
&+ \frac{\hbar^2 \lambda^3}{(4\pi)^4} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-2} \left[ \frac{1603}{648} + 4\zeta(3) \right] + \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-3} \left[ \frac{1603}{648} + 4\zeta(3) \right] \\
&- \frac{289}{81} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) + \frac{289}{81} \ln^2 \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \bigg]\right) + O(\hbar^3) ,
\end{align*}

\begin{align*}
\bar{m}^2(t) &= m^2 \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3} \\
&+ \frac{\hbar^2 \lambda^2 m^2}{(4\pi)^2} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3} + \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-4/3} \left[ \frac{19}{54} + \frac{17}{27} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \\
&+ \frac{\hbar^2 \lambda^2 m^2}{(4\pi)^4} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3} \left[ \frac{1787}{11664} + \frac{2\zeta(3)}{3} \right] \\
&- \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-4/3} \left[ \frac{5531}{5832} + \frac{4\zeta(3)}{3} + \frac{323}{1458} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \\
&+ \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-7/3} \left[ \frac{9275}{11664} + \frac{2\zeta(3)}{3} - \frac{221}{729} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) + \frac{578}{729} \ln^2 \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \\
&+ O(\hbar^3) ,
\end{align*}

\begin{align*}
\tilde{\phi}(t) &= \phi + \frac{\hbar \lambda \phi}{36(4\pi)^2} \left( 1 - \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3} \right) + \frac{\hbar^2 \lambda^2 \phi}{(4\pi)^4} \left( \frac{7}{432} \right) \\
&- \frac{1}{1296} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1} - \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-2} \left[ \frac{5}{324} + \frac{17}{324} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] + O(\hbar^3) ,
\end{align*}
\[ \bar{\Lambda}(t) = \Lambda + \frac{m^4}{2\lambda} \left\{ 1 - \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} \right\} \]
\[ + \frac{\hbar m^4}{(4\pi)^2} \left\{ -1 + \frac{19}{54} \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} + \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-2/3} \left[ \frac{35}{54} + \frac{17}{54} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \right\} \]
\[ + \frac{\hbar^2\lambda m^4}{(4\pi)^4} \left\{ \frac{23}{30} + \frac{6\zeta(3)}{5} - \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} \left[ \frac{2509}{11664} + \frac{2\zeta(3)}{3} \right] \right\} \]
\[ - \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-2/3} \left[ \frac{10129}{11664} + \frac{2\zeta(3)}{3} + \frac{323}{1458} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \]
\[ + \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-5/3} \left[ \frac{9239}{29160} + \frac{2\zeta(3)}{15} + \frac{323}{1458} \ln \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) + \frac{289}{1458} \ln^2 \left( 1 - \frac{3\lambda t}{(4\pi)^2} \right) \right] \}
\[ + O(\hbar^3). \] (20)

If we choose \( t \) as
\[ t = \frac{\hbar}{2} \ln(\frac{m^2_0}{\mu^2}), \]
then \( \bar{\mu}^2(t) \) becomes
\[ \bar{\mu}^2(t) = m^2 + \frac{\lambda \phi^2}{2}, \] (21)

which is independent of \( \mu \). Substitute \( \bar{\lambda}, m^2, \bar{\phi}, \) and \( \bar{\Lambda} \) of Eq. (20), and \( \bar{\mu}^2 \) of Eq. (21) into the right-hand side of Eq. (14) with \( V \) as two-loop approximation in Eq. (4). Then rearrange resulting terms in \( \hbar \) order. In this rearranging process, the running scale \( t \) should not be replaced with \( \frac{\hbar}{2} \ln(\frac{m^2_0}{\mu^2}) \). This is very important for the correct collection of logarithms of various powers into a given leading-logarithm series order. By this rearrangement, we can write the effective potential \( V \) as
\[ V = V^{(0)}(\phi, \lambda, m^2, t, \Lambda) + \hbar V^{(1)}(\phi, \lambda, m^2, t) + \hbar^2 V^{(2)}(\phi, \lambda, m^2, t) + O(\hbar^3), \]

where
\[ V^{(0)} = \Lambda + \frac{m^4}{2\lambda} (1 - T^{1/3}) + \frac{m^2 \phi^2}{2} T^{-1/3} + \frac{\lambda \phi^4}{24} T^{-1}, \]
\[ V^{(1)} = \frac{\lambda}{(4\pi)^2} \left\{ m^4 \left\{ -1 + \frac{19}{54} T^{1/3} + T^{-2/3} \left( \frac{59}{216} + \frac{17}{54} \ln T + \frac{1}{4} \ln S \right) \right\} \right\} \]
\[ + m^2 \phi^2 \left\{ \frac{4}{27} T^{-1/3} + T^{-4/3} \left( -\frac{49}{216} + \frac{17}{54} \ln T + \frac{1}{4} \ln S \right) \right\} \]
\[ + \lambda \phi^4 \left\{ \frac{1}{216} T^{-1} + T^{-2} \left( -\frac{85}{864} + \frac{17}{16} \ln T + \frac{1}{16} \ln S \right) \right\} \],
\[ V^{(2)} = \frac{\lambda^2}{(4\pi)^4} \left\{ m^4 \left\{ \frac{23}{30} + \frac{6\zeta(3)}{5} - T^{1/3} \left( \frac{2509}{11664} + \frac{2\zeta(3)}{3} \right) \right\} \right\} \]
\[ - T^{-2/3} \left( \frac{7051}{11664} + \frac{2\zeta(3)}{3} + \frac{323}{1458} \ln T + \frac{19}{108} \ln S \right) + T^{-5/3} \left( \frac{5189}{29160} + \frac{2\zeta(3)}{15} \right) \]
\[ - \frac{731}{2916} \ln T + \frac{289}{1458} \ln^2 T - \frac{2}{27} \ln T + \frac{17}{54} \ln S + \frac{1}{8} \ln^2 S \right\} \]
\[ + m^2 \phi^2 \left\{ T^{-1/3} \left( \frac{973}{11664} + \frac{\zeta(3)}{3} \right) - T^{-4/3} \left( \frac{4025}{11664} + \frac{2\zeta(3)}{3} + \frac{68}{729} \ln T + \frac{2}{27} \ln S \right) \right\} \\
+ T^{-7/3} \left( \frac{1475}{1458} + \frac{A}{4} + \frac{\zeta(3)}{3} - \frac{850}{729} \ln T + \frac{289}{729} \ln^2 T - \frac{73}{108} \ln S + \frac{17}{27} \ln T \ln S + \frac{1}{4} \ln^2 S \right) \right\} \\
+ \lambda \phi^4 \left\{ \frac{5}{1728} T^{-1} + T^{-2} \left( -\frac{197}{1728} - \frac{\zeta(3)}{6} + \frac{17}{1944} \ln T + \frac{1}{144} \ln S \right) \right\} \\
+ T^{-3} \left( \frac{131}{288} + \frac{A}{8} + \frac{\zeta(3)}{6} - \frac{2023}{3888} \ln T + \frac{289}{1944} \ln^2 T - \frac{23}{72} \ln S + \frac{17}{72} \ln T \ln S + \frac{3}{32} \ln^2 S \right) \right\} \\
+ \frac{m^6}{\lambda W} \left\{ -\frac{19}{216} T^{-1} + T^{-2} \left( \frac{19}{216} + \frac{17}{108} \ln T \right) \right\} \\
+ \frac{m^4 \phi^2}{W} \left\{ -\frac{35}{432} T^{-5/3} + T^{-8/3} \left( \frac{35}{432} + \frac{85}{216} \ln T \right) \right\} \\
+ \frac{\lambda m^2 \phi^4}{W} \left\{ -\frac{13}{864} T^{-7/3} + T^{-10/3} \left( \frac{13}{864} + \frac{119}{432} \ln T \right) \right\} \\
+ \frac{\lambda^2 \phi^6}{W} \left\{ \frac{1}{576} T^{-3} + T^{-4} \left( -\frac{1}{576} + \frac{17}{288} \ln T \right) \right\} \right\} , \\
(22) \]

with

\[ T \equiv 1 - \frac{3\lambda t}{(4\pi)^2} , \]
\[ W \equiv m^2 T^{-1/3} + (\lambda \phi^2/2) T^{-1} , \]
\[ S \equiv \frac{W}{m^2 + \lambda \phi^2/2} . \]

IV. CONCLUDING REMARKS

In this paper, using the the method of renormalization group we have improved the two-loop effective potential for the (single-component) massive \( \phi^4 \) theory for the first time. In obtaining our result the various three-loop renormalization group functions have been used.

We first compare the existing result of lower-order calculations. Our result \( V^{(0)} \) and \( V^{(1)} \) correspond to the Kastening’s functions \( f_1 \) and \( f_2 \), respectively. We compare them by subtracting one from the other:

\[ V^{(0)} - \lambda \phi^4 f_1 = \Lambda + \frac{m^4}{2\lambda} , \]
\[ V^{(1)} - \lambda^2 \phi^4 f_2 = -\frac{m^4}{(4\pi)^2} . \]

Only differences are \( \phi \)-independent constant terms. These discrepancies between our result and Kastening’s result are due to his not introducing a vacuum energy term in the Lagrangian. He has made a peculiar Ansatz for it as a working means [4]. Thus he used even the two-loop effective potential in obtaining next-to-leading logarithm correction for fixing the coefficient \( (b_2) \) in the Ansatz, contrary to the following general principle [5]: with the \( L \)-loop effective potential and \( (L+1) \)-loop renormalization group functions, we can obtain an
renormalization-group-improved effective potential which is exact up to the \( L \)th-to-leading logarithm order. It is remarkable that he has obtained the correct \( \phi \)-dependent terms from the ansatz. Other calculations of the leading-logarithm corrections, which exactly agree with our result \( V^{(0)} \), can be found in Ref. \[5\] and in the second paper of Ref. \[7\].

In order to make the correctness check for \( V^{(2)} \) richer, we add the three-loop correction, \( V^{(3)} \), which can be readily obtained from the Ref. \[12\], to the two-loop effective potential of Eq. (2):

\[
V^{(3)} = \frac{\lambda^3}{(4\pi)^6} \left[ Q_1 \frac{m^4}{\lambda} + Q_2 m^2 \phi^2 + Q_3 \phi^4 \right.
\]

\[
+ \left\{ \frac{41 m^4}{96\lambda} + m^2 \phi^2 \left( \frac{371}{96} + \frac{7A}{8} \right) + \lambda \phi^4 \left( \frac{701}{384} + \frac{9A}{16} + \frac{\zeta(3)}{4} \right) \right\} \ln \left( \frac{m^2\phi}{\mu^2} \right)
\]

\[
- \left\{ \frac{17 m^4}{48\lambda} + \frac{37 m^2 \phi^2}{24} + \frac{143 \lambda \phi^4}{192} \right\} \ln^2 \left( \frac{m^2\phi}{\mu^2} \right)
\]

\[
+ \left\{ \frac{5 m^4}{48\lambda} + \frac{7 m^2 \phi^2}{24} + \frac{9 \lambda \phi^4}{64} \right\} \ln^3 \left( \frac{m^2\phi}{\mu^2} \right) \right],
\]

(23)

where \( Q_1, Q_2, \) and \( Q_3 \) are constants and their numerical values \[13\] are given as

\[
Q_1 = -0.5123 \cdots, \quad Q_2 = -1.8105 \cdots, \quad Q_3 = -0.9428 \cdots.
\]

Further all the coefficients in Eq. (7), except \( A_1, A_2, A_3, \) and \( A_4 \) are determined \[13\], from the fourth order part of Eq. (12) in perturbation theory

\[
\mu \frac{\partial V^{(4)}}{\partial \mu} + \lambda \left\{ \beta_1 \frac{\partial V^{(3)}}{\partial \lambda} + \gamma_1 m^2 \frac{\partial V^{(3)}}{\partial m^2} - \gamma_1 \phi \frac{\partial V^{(3)}}{\partial \phi} + \beta_{A1} \frac{\partial V^{(3)}}{\partial \Lambda} \right\}
\]

\[
+ \lambda^2 \left\{ \beta_2 \frac{\partial V^{(2)}}{\partial \lambda} + \gamma_2 m^2 \frac{\partial V^{(2)}}{\partial m^2} - \gamma_2 \phi \frac{\partial V^{(2)}}{\partial \phi} + \beta_{A2} \frac{\partial V^{(2)}}{\partial \Lambda} \right\}
\]

\[
+ \lambda^3 \left\{ \beta_3 \frac{\partial V^{(1)}}{\partial \lambda} + \gamma_3 m^2 \frac{\partial V^{(1)}}{\partial m^2} - \gamma_3 \phi \frac{\partial V^{(1)}}{\partial \phi} + \beta_{A3} \frac{\partial V^{(1)}}{\partial \Lambda} \right\}
\]

\[
+ \lambda^4 \left\{ \beta_4 \frac{\partial V^{(0)}}{\partial \lambda} + \gamma_4 m^2 \frac{\partial V^{(0)}}{\partial m^2} - \gamma_4 \phi \frac{\partial V^{(0)}}{\partial \phi} + \beta_{A4} \frac{\partial V^{(0)}}{\partial \Lambda} \right\} = 0,
\]

as

\[
B_1 = -\frac{295}{192} + 4Q_1 + \frac{\zeta(3)}{4},
\]

\[
B_2 = -\frac{271}{32} - \frac{53A}{48} + 5Q_2 - \frac{15\zeta(3)}{8} - \frac{3\zeta(4)}{4},
\]

\[
B_3 = -\frac{1549}{768} - \frac{35A}{96} + 6Q_3 - \frac{9\zeta(3)}{4} + \frac{3\zeta(4)}{8} - \frac{5\zeta(5)}{2},
\]

\[
B_4 = \frac{A}{8} + \frac{\zeta(3)}{4},
\]

\[
C_1 = \frac{73}{48}, \quad C_2 = \frac{97}{8} + \frac{35A}{16} + \frac{3\zeta(3)}{4},
\]

\[
C_3 = \frac{583}{96} + \frac{27A}{16} + \frac{9\zeta(3)}{8}, \quad C_4 = -\frac{1}{16},
\]

12
\[ D_1 = -\frac{55}{96}, \quad D_2 = -\frac{277}{96}, \quad D_3 = -\frac{201}{128}, \quad D_4 = \frac{1}{48}, \]
\[ E_1 = \frac{5}{48}, \quad E_2 = \frac{35}{96}, \quad E_3 = \frac{27}{128}, \quad E_4 = 0. \] \hspace{1cm} (24)

Our result of \( V^{(2)} \) well reproduce the next-next-to-leading logarithm parts of \( V^{(2)} \) in Eq. (2), \( V^{(3)} \) in Eq. (23), and \( V^{(4)} \) in Eq. (7) and Eq. (24) too, when it is expanded in power series of \( t \), as it should.

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FIG. 1. The loop expansion and the leading-logarithm series expansion of the effective potential. It is understood that each vertical sum marked by a box should be multiplied by the common factors \[\lambda/(4\pi)^2\] in front of the horizontal sums to give the leading-logarithm series expansion; for example, \(V^{(0)} = G_0^{(0)} + [\lambda/(4\pi)^2]G_0^{(1)} y + [\lambda^2/(4\pi)^4]G_0^{(2)} y^2 + \ldots\), \(V^{(1)} = [\lambda/(4\pi)^2]G_1^{(1)} + [\lambda^2/(4\pi)^4]G_1^{(2)} y + [\lambda^3/(4\pi)^6]G_1^{(3)} y^2 + \ldots\), etc.