Multi-order Accurate Stabilized Tracking System with Multi-DOF under Influence of Ocean Waves

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Abstract. In order to avoid influence of ocean wave motion to the tracking system, a tracking platform with multi-DOF and multi-level accurate stabilized system is designed. The position of platform and load on multiple degrees of freedom can be compensated, so as to meet the required stability conditions and realize the stable tracking of the platform to the target. The stability of the platform is analyzed and simulated. The simulation results show that the control effect of the stable platform is good, and the stability effect of roll and heave of the platform is obvious, which can slide in two degrees of freedom at XY directions, with engineering application prospect.

Keywords: Multi-DOF; Compensation method; Tracking platform.

1. Introduction
The stabilized platform is a kind of active isolation equipment, which can be installed on a variety of motion carriers to isolate the attitude disturbance meanwhile provide a stable datum for the controlled object on the platform [1]. With the development of Chinese military science and technology in recent years, the stable platform mechanism is also developing rapidly. It has a wide range of uses in aerospace, industrial control, military and commercial ships, such as aerial photography, shipborne missile launching platform, Shipborne Satellite receiving antenna, water surface tracking platform, etc. Due to the influence of sea waves, unstable movement of the tracking platform is inevitable, which will produce a large range of roll, pitch and heaving movements, will have adverse effects on the normal operation of the tracking system on the platform [2]. Therefore, a stabilized platform is needed to isolate the irregular movement of the platform caused by the sea waves, so as to ensure the stability of the tracking system.

Aiming at the design of stabilized system under the interference of sea waves, a stable tracking platform with multi degrees of freedom and multi-level accuracy based on the background of ocean tracking platform is proposed.

2. Wave Model
The characteristics of ocean wave is space-time and phase superposition of standing wave in both S-wave and P-wave directions [3-5]. The typical wave spectrum is P-M spectrum and the formula is Eq. 1.

$$S_\omega (\omega) = \frac{173.5^{2}}{T_1^{4}\omega^5} \exp \left( -\frac{691}{T_1^{4}\omega^4} \right)$$

Where: $a = 0.0081; \beta = 0.74; G$-acceleration of gravity; u-wind speed at 19.5m from the sea surface.
According to the p-m spectrum, the conclusions are as follows: (1) the frequency of general waves is less than 1Hz. (2) typical waves transmit to all sides in the way of superposition of P-wave and S-wave. As the main factor affecting the stabilized of a ship in waves is longitudinal wave, we can get the following equation by referring to relevant literature:

\[
\frac{d^2 h}{dt^2} + K \frac{dh}{dt} = C^2 \left( \frac{d^2 h}{dx^2} + \frac{d^2 h}{dy^2} \right)
\]  

(2)

According to formula Eq. 2, there is a coupling phenomenon between wave height h and distance in X axis and in Y axis in a particular time and place. In this range, the change of particle position can be obtained by the analytical solution of formula Eq. 1, so its pointing coordinates are \([H(T), X(T), y(T)]\).

3. Platform Structure and Kinematics Analysis

Adopt the secondary servo system structure as shown in Figure 1. The mechanical structure of the whole system consists of three parts: the photoelectric pod in the upper part, the buoy in the middle and in the lower part. Among them, The photoelectric pod on the upper part is the core part of the whole device, it includes fine adjustment platform, servo motors, extendable rods, counterweight and load, etc.; the buoy in the middle part provides the buoyancy of the whole system, so the accurate parameters of size and installation position needed to be obtained by mechanical calculation; the float on the lower part includes power supply, controller and driver. The connecting wires in the actuator and the pontoon are connected with various devices in the photoelectric pod through the collector ring.

![Figure 1](image-url)

**Figure 1.** Overall diagram of system structure.

There are platform coordinate system and inertial coordinate system in marine vehicle. A target on the ocean is in the inertial coordinate system, while the motion of the electric cylinder is in the platform coordinate system, as shown in Fig. 2.
Figure 2. Schematic diagram of adjustment platform at any position.

Solving the position of the input in the platform coordinate system by given the position and attitude of the output inertial coordinate system is called the inverse position solution of the mechanism. In the process of moving platform, the spatial attitude parameters of six platform coordinate systems are $x$, $y$, $z$, $\alpha$, $\beta$, $\gamma$ of the platform. To realize the control of the platform, the length $L_1$, $L_2$, $L_3$ and $L_4$ of the four electric cylinders are needed, which is the known output and input exactly. That is to say, in order to achieve the control of the platform spatial attitude, it is necessary to derive the inverse solution equation of the platform position.

As shown in Figure 2, the platform coordinate system $o$-xyz is established on the upper platform, while the platform coordinate system $o$-xyz is established on the lower platform. Then, the motion of the platform can be decomposed into translation ($x$, $y$, $z$) along the three coordinate axes of $o$-xyz along the origin of the inertial coordinate system $o$-xyz, and rotation ($\alpha$, $\beta$, $\gamma$) around the coordinate axis. In the case of only rotation without translation, $(X_{A1}, y_{A1}, z_{A1})$ is equivalent to $(x, y, z)$, which is regarded as static coordinate, while $(X_{A1}, y_{A1}, z_{A1})$ is equivalent to $(x'', y'', z'')$, as a moving coordinate static coordinate. Therefore, the rotation transformation matrix $[t] = [C] \times [b] \times [a]$ is as follows Eq. 3:

$$
\begin{bmatrix}
\cos \beta \cos \gamma & -\cos \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma \\
\cos \beta \sin \gamma & \cos \alpha \sin \gamma + \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma \\
-\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta
\end{bmatrix}
$$

(3)

By substituting the matrix $[t]$ into the previous transformation formula, the static coordinates $(X_{A1}, y_{A1}, z_{A1})$ of $A_1$ can be calculated, so the length $L_1$ of the expansion bar can be calculated as shown in Eq. 4-7.

$$
L_1 = \sqrt{(X_{A1} - X_{A1})^2 + (Y_{A1} - Y_{A1})^2 + (Z_{A1} - Z_{A1})^2}
$$

(4)

In the same way:

$$
L_2 = \sqrt{(X_{A2} - X_{A2})^2 + (Y_{A2} - Y_{A2})^2 + (Z_{A2} - Z_{A2})^2}
$$

(5)

$$
L_3 = \sqrt{(X_{A3} - X_{A3})^2 + (Y_{A3} - Y_{A3})^2 + (Z_{A3} - Z_{A3})^2}
$$

(6)

$$
L_4 = \sqrt{(X_{A4} - X_{A4})^2 + (Y_{A4} - Y_{A4})^2 + (Z_{A4} - Z_{A4})^2}
$$

(7)
4. Control Design

According to the system, the state of the whole servo system spatial model is constructed, so the control is designed by using the state feedback. The target tracking diagram of the servo system is shown in Fig. 3.

\[ \text{arcsin} \left( \frac{h}{d} \right) = \arctan \left( \frac{L}{H} \right) = \alpha \]  

(8)

The equivalent circuit parameters of the motor are as follows: R is the armature resistance, L is the armature inductance, j is the moment of inertia, KF is the viscous friction coefficient, Ka is the torque constant, KB is the back EMF constant, UA is the input voltage, UE is the back EMF, TD is the load torque, I is the armature current, \( \omega \) is the motor speed. The voltage balance equation and torque balance equation are shown in Eq.9 and 10, respectively.

\[
\begin{align*}
L \frac{di}{dt} + Ri &= E \\
T_m &= K_a i \\
J \frac{d\omega}{dt} + K_f \omega &= T_E \\
\omega &= \omega
\end{align*}
\]

(9)

(10)

From Eq.9 and 10, the state space model of the motor is shown in Eq.11

\[
\begin{align*}
\dot{X} &= AX + BU \\
Y &= CX + DU
\end{align*}
\]

(11)

In Eq.11, the matrix of each function module and state quantity x, control quantity u and observation measurement y are shown in Eq.12 and Eq.13 respectively:
The motor parameters are as follows: \( r = 2.6 \, \Omega \), \( l = 2 \times 10^{-3} \, \text{h} \), \( j = 1.2 \, \text{kg} \cdot \text{m}^2 \), \( K_F = 0.01 \, \text{N} \cdot \text{m} \cdot \text{s} \), \( K_a = 0.7 \, \text{n} \cdot \text{M} / \, \text{A} \), \( K_B = 0.7769 \, \text{v} \cdot \text{S} \). Take the above parameters into \( a, B, C, D \) and take the approximate results to get the Eq. 14:

\[
A = \begin{bmatrix}
    -1300 & -388.45 \\
    0.58 & -0.0083 
\end{bmatrix} \quad B = \begin{bmatrix}
    500 & 0 \\
    0 & 0.833 
\end{bmatrix} \quad C = \begin{bmatrix}
    0 & 1 
\end{bmatrix} \quad D = \begin{bmatrix}
    0 & 0 
\end{bmatrix} 
\]

The output speed \( \omega \) of the motor has the following relationship with the effective expansion length \( h \) of the expansion rod:

\[
h = k_\omega T_f
\]

In Eq. 15, \( K \) is the speed ratio, which is equivalent to the lead of the ball screw. According to Eq. 8 - 15, the tracking of \( l \) can be realized by controlling the rotational speed \( \omega \) of the motor. The state feedback control law is shown in Eq. 16, and the simulation results are shown in Figure 4.

\[
U = \begin{bmatrix}
    0.5 & 0 \\
    0 & 0 
\end{bmatrix}
\]

### Figure 4. Target tracking diagram of fine tuning servo system.

In Figure 4, the gray solid line is the slope signal, and the slope is \( 20^\circ / \, \text{S} \) (0.349 rad / s). It simulates the input signal of the target to be tracked for fine tuning, the blue solid line is the response curve of the system tracking slope signal, and simulates the actual expansion height of the platform expansion bar. It can be seen that the steady-state error is significantly reduced when the state feedback control law is added compared with that when the state feedback control law is not added. However, in the initial stage of the system, the dynamic error is large, which shows that the system still has room for further adjustment. Once the poles and zeros of the system to be tracked are determined, the ideal state feedback control law can be obtained by pole and zero assignment. This method is easy to control the state of the system, that is, speed.
At this time, the length $L_1$, $L_2$, $L_3$ and $L_4$ of the four expansion rods are known, and the space attitude $x$, $y$, $Z$, $\alpha$, $\beta$, $\gamma$ of the platform need to be solved. Let $x = (x, y, Z, \alpha, \beta, \gamma)^T$, $f(x) = [F_1(...), F_2(...), F_3(...), F_4(...), F_5(...), F_6(...)]$, then the superposition formula of positive position solution is:

$$X_{k+1} = X_k - \hat{F}(X_k)^{-1} F(X_k)$$  \hspace{1cm} (17)

The linear equations of four equations and four unknowns are obtained:

$$\hat{F}(X_k)(X_{k+1} - X_k) + F(X_k) = 0$$  \hspace{1cm} (18)

By solving the equations, $X_k+1$ can be obtained. Select the appropriate initial point $x_0$ \{such as: $x_0 = (0,0,0,0)$\} and the termination condition \{such as: $|X_k+1-x_k| < \epsilon$, where $\epsilon$ is the precision\}. After many iterations, the values of $X$, $y$, $Z$, $\alpha$, $\beta$, $\gamma$ that meet the precision requirements can be obtained.

The comprehensive simulation results of the above forward and inverse solutions are shown in Figure 5 and figure 6.

![Figure 5. Pitch roll condition.](image-url)
Figure 6. Yaw condition.
Figure 5 and figure 6 show that when the tilting platform is in pitch and roll angle, the inverse motion settlement can calculate the specific feed value of each electric expansion rod, so as to drive the platform to reach the balance state rapidly. When the platform is at the yaw angle, the electric expansion bar does not produce longitudinal displacement, and can still move quickly to ensure the upper plane to track the target to be measured. Our future work is to optimize the compensation algorithm and improve the adaptability of the platform.

5. Conclusion
In this paper, based on the practical engineering application of the ocean surface stabilized tracking platform, a kind of multi degree of freedom multi-level accurate stabilized system under the interference of sea waves is proposed. The working principle, simulation and practical application of the platform are introduced in detail. The motion state of the electric cylinder is obtained by the inverse solution algorithm, and the platform and its inverse solution algorithm are verified. The simulation results show that the method has obvious stability effect on pitch and heave of the platform, and has a wide application prospect in engineering in the future.

References
[1] ZHANG H, KEXZ, JIAO R, Experimental Research on feedback Kalman Model of MEMS Gyroscope. C. The 8th International Conference Electron Measurement and Instruments, 2007:253-256.
[2] Keller J A, Smith E C. Experimental and theoretical correlation of helicopter rotor blade-droop stop impacts, J. Journal of Aircraft, 1996, 36(2):443-450.
[3] Khatri S K, In the search of a coastal ocean wave model. OCEANS 97, MTS/IEEE Conference Proceedings,1997,213-218.
[4] M. Onorato, A.R. Osborne, M. Serio. Freely Decaying Weak Turbulence for Sea Surface Gravity Waves, J. Physical Review Letters. 89 (2002) 144501-1-144501-4.
[5] Komen G J, Gavaleri L, Donelan M. Dynamics and Modeling of Ocean Waves. Cambridge University Press,1996.