Assessment and Mitigation of Dynamic Instabilities in Single-Stage Grid-Connected Photovoltaic Systems With Reduced DC-Link Capacitance

MOHAMMAD ADNAN K. MAGABLEH¹, (Graduate Student Member, IEEE), AMR RADWAN², (Senior Member, IEEE), AND YASSER ABDEL-RADY I. MOHAMED¹, (Fellow, IEEE)
¹Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 1H9, Canada
²Engineering and Design Department, College of Science and Engineering, Western Washington University, Bellingham, WA 98225, USA

Corresponding author: Mohammad Adnan K. Magableh (magableh@ualberta.ca)

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ABSTRACT Single-stage utility-scale photovoltaic (PV) systems are usually interfaced with the host grid via a centralized voltage-source converter (VSC). Recently, and due to their reliability, the dc-link film capacitors are favored over electrolytic types in grid-connected applications. However, the capacitance per unit volume of film capacitors is significantly smaller than electrolytic capacitors. The overall system stability might be compromised by the reduction of the dc-link capacitance, particularly in PV systems that have a dynamic resistance that varies with operating conditions. Using a detailed small-signal model of the grid-connected PV system, it is shown in this paper that the reduction of the dc-link capacitance interferes with the dynamic resistance of the PV array, which eventually leads to instabilities. The minimum dc-link capacitance that preserves the overall system stability is determined. A simple and effective active compensator is developed to mitigate the instabilities with the reduced dc-link capacitance. Detailed time-domain simulations are presented to validate the analytical results and show the proposed compensator’s effectiveness in preserving the system stability.

INDEX TERMS Active damping, DC-AC power converters, DC-link stabilization, grid-connected inverters, photovoltaic, single-stage, small-signal analysis.

I. INTRODUCTION

Due to the global climate change concerns and the continuous growth of energy demand, renewable energy is becoming increasingly popular all over the world. At the end of 2018, the world’s total installed photovoltaic (PV) capacity has reached 486 GW out of 2351 GW of the global total renewable energy capacity. By 2050, the PV global cumulative capacity is expected to contribute 11% of the worldwide electricity generation to avoid 2.3 gigatons of carbon dioxide emissions per year [1], [2].

Centralized voltage-source converters (VSCs) are usually used to interface the large PV power plants to the host grid [3], [4]. Generally, three types of dc-link capacitors are available for power converters; namely, they are: multi-layer ceramic capacitors (MLC-Cs), metalized polypropylene film capacitors (MPPF-Cs), and aluminum electrolytic capacitors (Al-Cs) [5]. Even though MPPF capacitors are the best choice for reliability, their utilization is still challenging due to the high cost and the limited capacitance per unit volume. It is well-known that the large dc-link capacitance enhances the dc-link voltage profile and increases the stability margin. Previous work has been conducted to preserve the dc-link stability under a reduced dc capacitance to pave the way to adopt the MPPF capacitors [6], [7]. In [8], a high-frequency capacitive ac-link is employed in a PV inverter to implement the film capacitors with their small capacitance per unit volume. Similar research has been conducted in [9] to use the small film capacitors in the dc-link to transfer the power from the input to the output of a new ac-ac converter topology.

As shown in Fig. 1, a single-stage VSC is used to interface a PV array to the utility-grid. The VSC adopts the...
standard vector control strategy with cascaded outer and inner control loops [10]. However, it is reported in the literature that, with the reduced dc-link capacitance, the VSC dynamics might involve right-half plane (RHP) poles in many applications, such as motor drives, wind turbines, and PV systems [11]–[14].

In [13], the dynamic resistance of the PV generator is analyzed. The instabilities due to the RHP poles in the dc-link voltage control loop are investigated. The unstable behavior is expected in both single- and two-stage systems, especially with the reduced dc-link capacitance. Similar conclusions have been drawn in [14] with further investigations on the minimum dc-link capacitance required to guarantee the system stability. This study reported that the dynamic stability of the dc-link voltage control might be affected when the PV generator operates in the constant current region (CCR). However, this effect is alleviated when the PV generator operates at the maximum power point (MPP) or the constant voltage region (CVR). In [15], a PV generator interfaced by a current mode-controlled boost dc–dc converter is investigated. A robust control method is proposed to regulate the terminal voltage due to the PV dynamic resistance and the dc-link capacitance interactions.

Analytical impedance-based models are developed in [16]–[18] to study the dynamic interactions between the dc-side and ac-side of the VSCs and to identify the influence of each side on the system stability under different operating conditions. The analysis reveals that the dc-link voltage controller induces low-frequency-oscillations (LFOs) in the rectification mode of operation and high-frequency-oscillations (HFOs) in the inversion mode. Moreover, a describing function method is introduced to overcome the discontinuity and the nonlinearity in the dc-side of the single-stage PV systems, and hence the system stability can be analyzed accurately.

To the best of the authors’ knowledge, the impact of the dynamic resistance of PV arrays on the single-stage grid-connected VSC with a reduced dc-link capacitance was briefly and solely addressed in [13], [14]. However, the developed single-stage model is not comprehensive and only comprises three states. Moreover, the dynamic model does not consider the influence of the step-up transformer, dc-cables, the grid-stiffness, the phase-locked loop (PLL), and ac-voltage control dynamics. More importantly, there is no proposed solution or mitigation technique to enhance the stability of the single-stage system and facilitate the integration of the PV generators with the reduced dc-link capacitance.

Motivated by the preceding challenges, this paper introduces a new yet simple and effective compensator to overcome the dynamic instabilities due to the variable operating conditions, especially under the minimized dc-link capacitance. The contributions of this paper to the research field are as follows.

1) The development of a comprehensive small-signal model for the single-stage PV system. The developed model is validated using large-signal nonlinear time-domain simulations in MATLAB/Simulink. The models are used to investigate the influence of PV dynamic resistance on the system stability at different operating regions.

2) The characterization of the dynamic interactions between the interfacing VSC with the reduced dc-link capacitance at different operating points using time-domain simulations and small-signal analysis.

3) The development a new compensator to enhance the integration of the single-stage PV systems under the minimized dc-link capacitance. The operation is investigated under different operating conditions, including grid faults.

The remainder of this paper is organized as follows. Section II presents the modeling, control details, and the small-signal stability analysis of the single-stage utility-scale grid-connected PV system. The PV dynamic resistance and the existence conditions of the RHP poles under different operating conditions are presented in Section III.
In Section IV, the proposed compensator is introduced to improve the system damping and eliminate instabilities under the reduced dc-link capacitance conditions. Section V provides time-domain simulations to evaluate the effectiveness of the proposed compensator. The conclusions are drawn in Section VI.

II. MODELING AND CONTROL OF THE THREE-PHASE GRID-CONNECTED VSC-PV SYSTEM

As shown in Fig. 1, the grid-connected PV array is interfaced by a three-phase two-level VSC consisting of six switching cells; each comprises an insulated-gate-bipolar transistor (IGBT) in parallel with a freewheeling diode. The PV system is connected to the dc-link capacitor via a dc transmission cable [19], whereas the VSC is connected to the utility-grid via an ac filter and a three-phase transformer.

In the following subsections, the modeling and control details of the overall system are provided.

A. POWER CIRCUIT MODEL

As shown in Fig. 1, the grid is modeled as a voltage source and a series-connected impedance. The grid impedance is defined by the short-circuit-ratio (SCR) and the reactance-to-resistance ratio (X/R). An inductive-capacitive (LC) low-pass filter is commonly used to attenuate the current and voltage harmonic components at the ac-side of the VSC [4]. The step-up transformer is generally utilized in the grid-connected PV systems to match the terminal ac voltage of the step-up transformer is generally utilized in the grid-connected transformer.

The dynamic model of the LC-filter, the grid impedance integrated with the transformer, and the dc transmission cable are implemented by the following equations in the direct-and-quadrature (dq)-grid reference frame.

\[
L_f \frac{dI_{dq}^g}{dt} + j \omega L_f I_{dq}^g + (R_f + r_{on}) I_{dq}^g = V_{dq}^g - v_{dq}^g
\]

\[
L_T \frac{dI_{gq}^g}{dt} + j \omega L_T I_{gq}^g + R_T I_{gq}^g = \frac{1}{N^2}(V_{dq}^g - V_{dq}^q)
\]

\[
C_f \frac{dV_{dq}^g}{dt} + j \omega C_f V_{dq}^g = i_{dq}^g - \frac{1}{N}i_{gq}^q
\]

\[
L_{dc} \frac{dI_{pv}^q}{dt} + R_{dc} I_{pv}^q = r_d I_{pv}^q - V_{dc}
\]

\[
R_f, L_f, C_f \text{ are the filter resistance, inductance, capacitance;}
\]
\[
r_{on} \text{ is the diode on-state resistance; } L_T = L_g + L_T; \text{ the total inductance of the grid (L_g) and the transformer inductance referred to the secondary side (L_T);}
\]
\[
R_T = R_g + R_T; \text{ the total resistance of grid side (R_g) and transformer resistance referred to the secondary side (R_T); }
\]
\[
N \text{ is primary to the secondary turns ratio of the step-up transformer;}
\]
\[
R_{dc}, L_{dc} \text{ is the dc-cable resistance, inductance; } V_{dc} \text{ is the dc-link voltage;}
\]
\[
r_d, I_{pv}, V_{pv} \text{ is the PV dynamic resistance, current, voltage;}
\]
\[
j \text{ is the imaginary unit number; } \omega \text{ is the angular speed of the utility;}
\]
\[
v_{dq}^g \text{ and } i_{dq}^g \text{ are the voltage and current in the dq-grid frame;}
\]
\[
I_{gq}^g \text{ and } I_{dq}^g \text{ are the current in the dq-grid frame, respectively. The terminal voltage and current of the VSC in the dq-grid frame are } v_{dq}^g \text{ and } i_{dq}^g, \text{ respectively.}
\]

B. CONTROL AND SYNCHRONIZATION OF THE GRID-CONNECTED VSC

As shown in Fig. 2, the detailed vector control loops of the grid-connected VSC are presented. The phase-locked-loop (PLL) is used to synchronize the VSC to the grid and generate the signals in the dq-converter frame. The outer dc-link voltage control loop, which is used with MPPT to transform the VSC voltage, is used to control the PCC voltage, to generate the reference values of the inner current control loop [4], [10], [20], [21].

The bandwidth of the inner control loop is selected to be around 10-20% of the VSC switching frequency (expressed in rad/s). Further, the bandwidth of the outer control loop is selected to be around 10-20% of the inner control loop [10]. The phase margin is selected to be no less than 45°, and as the phase margin increases, the system becomes unconditionally stable. The VSC controllers are discussed in detail in the following subsections.

1) GRID SYNCHRONIZATION

The vector control is characterized by the use of a PLL to synchronize the VSC with the utility-grid. A proportional and integral (PI) controller \( G_c = k_p + k_i \omega \) is implemented to set the q-component of the PCC voltage \( \left( v_q^g \right) \) to zero. As shown in (5)-(6), the synchronization angle \( \theta \) is generated by integrating the angular frequency \( \omega \) and is used to transform the three-phase grid-frame signals, denoted by the superscript “g”, to the converter-frame, denoted by the superscript “c”, and vice versa [10]. Under transient conditions, the synchronization angle oscillates to resynchronize the converter-frame with the grid-frame, and eventually, the angle difference \( \epsilon = \theta - \theta^c \) becomes zero in the steady-state conditions when the two frames are aligned.

\[
d\epsilon_c = \frac{\Delta \omega}{\omega_c^q} + \omega^c
\]

\[
d\epsilon_c = \omega^c
\]

Reiterating to Fig. 2, the measured quantities should be transformed and retransformed between the converter-frame and the grid-frame to accurately model the influence of the PLL on the system dynamics. The frame transformation is mathematically modeled as \( \left( f_{pq}^c = e^{-j \theta} f_{pq}^g \right) \) [25].

The open-loop transfer function of the PLL dynamics is \( \ell_{pll} = \frac{1}{V_{pv} \sqrt{1 + (\tan PM_{PLL})^2}} \), and the controller gains can be calculated as follows:

\[
k_i^{pll} = \frac{\omega_c^q}{\sqrt{1 + (\tan PM_{PLL})^2}}
\]

\[
k_p^{pll} = \frac{(k_i^{pll})^2 \tan PM_{PLL}}{\omega_c^q}
\]
In this paper, and as shown in Fig. 3, the selected phase margin \((PM_{pll})\) and the cut-off frequency \((\omega_{pll})\) of the PLL are \(84.4^\circ\) and \(181\) rad/s, respectively.

2) INNER CURRENT CONTROL
From the current control loop in Fig. 2, a PI controller \((G_i = k_{ip} + k_{ii}/s)\) is implemented to regulate the injected active and reactive power to the grid. The dynamic equation describing the \(dq\)-current controller is implemented as follows:

\[
v_{idq} = v_{dq} + j\omega_o L_f i_{dq} + G_i (i_{dq} - i_{dq}^*)
\]

where \(j\omega_o L_f i_{dq}\) and \(v_{dq}\) are the decoupling and feedforward terms, respectively.

The open-loop gain of the current control loop is \(\ell_{cc}(s) = G_i / (L_f s + (R_f + r_{on}))\), and the controller gains can be calculated as follows:

\[
k_{ip} = (R_f + r_{on}) / \tau_i
\]

\[
k_{ii} = L_f / \tau_i
\]

In this paper, and as shown in Fig. 3, the designed phase margin and the bandwidth \((1/\tau_i)\) of the closed-loop current control are \(90^\circ\) and \(1922.7\) rad/s, respectively.

3) DC-LINK VOLTAGE CONTROL
Referring to Fig. 1, the dynamic equation of the dc-link capacitor is given as follows:

\[
\frac{d}{dt} \left( \frac{1}{2} C_{dc} V_{dc}^2 \right) = -P_{dc} - P_{R_f} - P_{L_f} - P_{pcc}
\]

where \(P_{ext}\) is the incoming PV power; \(P_{dc}\) is the approximated injected power to the ac-side assuming a lossless power converter and is represented by the summation of the drop power of the filter resistance \(P_{R_f} = Real \{1.5 R_f i_{dq}^\text{conjugate}\}\), the drop power of the filter inductance \(P_{L_f} = Real \{1.5 L_f i_{dq}^\text{conjugate}\}\), and the PCC power

\[
P_{pcc} = Real \{1.5 v_{dq}^\text{conjugate}\}\]

\(C_{dc}\) is the dc-link capacitor and its nominal value is designed at \(5000\) µF based on the practical guidelines in [4].

As shown in Fig. 2, a PI controller \((G_{vdc} = k_{vdc} + k_{vi}/s)\) is implemented to regulate the squared value of the dc-link voltage \((V_{dc}^2)\) to the squared reference value \((V_{dc}^2)^*\) which is generated by the MPPT algorithm. The MPPT loop is a function of the weather conditions such as the temperature and the irradiance. In the literature, many MPPT methods have been introduced, where the most widely used techniques are the perturb-and-observe (P&O) and incremental conductance (IC) [4]. The dynamic equation describing the dc-link voltage controller is

\[
i_d^* = \eta P_{ext} - G_{vdc} (V_{dc}^2 - V_{dc}^{2*}) / 1.5 v_d^2
\]

where \(\eta\) is a constant used to enable the feedforward power compensation to reduce the nonlinearity effect of the PV generator.
The accurate open-loop gain of the dc-link voltage control loop is \( \ell_{\text{vdc}}(s) = \left(2G_{\text{vdc}}(\tau_s s + 1)\right) / (C_{\text{dc}} s (\tau_s s + 1)) \), where \( \tau_p = 2L_f P_{pv} / 3v_d^2 \) is a positive time-constant given in terms of the operating points [10]. The dc-link voltage controller gains can be calculated as follows:

\[
\begin{align*}
    k_i^{\text{vdc}} &= \frac{C_{\text{dc}} \omega_{\text{vdc}}^2 \sqrt{1 + (\omega_{\text{vdc}} \tau_i)^2}}{2 \sqrt{1 + (\tan (\theta_{\text{vdc}}))^2} \sqrt{1 + (\omega_{\text{vdc}} \tau_p)^2}} \\
    k_p^{\text{vdc}} &= \frac{k_i^{\text{vdc}} \tan (\theta_{\text{vdc}})}{\omega_{\text{vdc}}}
\end{align*}
\]

(14) (15)

where \( \theta_{\text{vdc}} \) is given by \( \theta_{\text{vdc}} = PM_{\text{vdc}} + \tan^{-1}(\omega_{\text{vdc}} \tau_i) - \tan^{-1}(\omega_{\text{vdc}} \tau_p) \).

Ignoring the instantaneous power of the VSC interface reactors, the simplified open-loop gain becomes \( \ell_{\text{vdc}}(s) = (2G_{\text{vdc}}(\tau_s s + 1)) / (C_{\text{dc}} s (\tau_s s + 1)) \). The phase margin of the simplified open-loop increases to the maximum value at a certain frequency \( \omega_{\text{max}}^{\text{vdc}} = \sqrt{k_i^{\text{vdc}} / (\tau_ip^{\text{vdc}})} \). As a result, the controller gains can be calculated at \( \omega_{\text{max}}^{\text{vdc}} \) as

\[
\begin{align*}
    k_i^{\text{vdc}} &= \frac{C_{\text{dc}}^2 \omega_{\text{vdc}}^4 \sqrt{\left(1 + (\omega_{\text{max}}^{\text{vdc}})^2\right)^2}}{4 \left(1 + (1 / (\omega_{\text{vdc}} \tau_i))\right)^2} \\
    k_p^{\text{vdc}} &= \frac{k_i^{\text{vdc}} \omega_{\text{vdc}}}{\omega_{\text{max}}^{\text{vdc}}}
\end{align*}
\]

(16) (17)

As shown in (12), small-signal models are developed with the consideration of the instantaneous power of the VSC interface reactors to accurately study the system stability.

As shown in Fig. 3, the achieved phase margin \((PM_{\text{vdc}})\) and the cut-off frequency \((\omega_{\text{vdc}})\) for both the accurate and the approximated models in (14)-(15) and (16)-(17), are 85.8°, 193.8 rad/s and 78.6°, 192.3 rad/s, respectively.

4) PCC VOLTAGE CONTROL

A PI controller \((G_{\text{vac}} = k_p^{\text{vac}} + k_i^{\text{vac}} s)\) is implemented to regulate the PCC voltage by generating a reference \(q\)-component of the injected current \(i_q^*\). The dynamic equation of the AC-voltage controller is given as follows:

\[
i_q^* = -G_{\text{vac}} (v_d^* - v_d) / 1.5v_d^*
\]

(18)

The open-loop transfer function of the AC-voltage control loop becomes \( \ell_{\text{vac}}(s) = (2\omega\nu^2L_T / 3v_d^2) (G_{\text{vac}} / (\tau_s s + 1)) \), and the ac-voltage controller gains can be calculated as follows:

\[
\begin{align*}
    k_i^{\text{vac}} &= \frac{3v_d^2 \omega_{\text{vac}} \sqrt{1 + (\omega_{\text{vac}} \tau_i)^2}}{2 \omega\nu^2L_T \sqrt{1 + (\tan (\theta_{\text{vac}}))^2}} \\
    k_p^{\text{vac}} &= \frac{k_i^{\text{vac}} \tan (\theta_{\text{vac}})}{\omega_{\text{vac}}}
\end{align*}
\]

(19) (20)

where \( \theta_{\text{vac}} \) is given by \( \theta_{\text{vac}} = PM_{\text{vac}} - 90 + \tan^{-1}(\omega_{\text{vac}} \tau_i) \).

As shown in Fig. 3, the designed phase margin \((PM_{\text{vac}})\) and the cut-off frequency \((\omega_{\text{vac}})\) of the ac voltage control open-loop are 86.4° and 130 rad/s, respectively.

C. SMALL-SIGNAL MODEL DEVELOPMENT AND VERIFICATION

To study the dynamic stability of the grid-connected PV system, a detailed small-signal model has been developed by linearizing the nonlinear equations \( \dot{x} = f(x(t), u(t)) \) in Section II. The linearized state-space model of the overall system is defined as follows:

\[
\begin{align*}
    \dot{x} &= Ax + Bu \\
    y &= Cx + Du
\end{align*}
\]

(21)

where \( A = (\partial f(x, u) / \partial x)|_{(x, u) = (x^*, u^*)} \) is the system matrix; \( B = (\partial f(x, u) / \partial u)|_{(x, u) = (x^*, u^*)} \) is the control matrix; \( C \) is the output matrix; \( D \) is the feedforward matrix; and the perturbation variables \( \Delta x, \Delta u, \) and \( \Delta y \) are the state, input, and output vectors, respectively. The resultant small-signal state-space model consists of 14 state variables, and it is shown in Appendix C.

It should be noted that it is very difficult to implement the MPPT algorithm mathematically in the linearized model due to its nonlinearity, complexity, and discontinuity, and so the MPPT is represented by the corresponding change in the dc-link voltage reference. To evaluate the accuracy of the small-signal state-space model in (21), a perturbation of a 5% step increase in \( V_{dc} \) is applied under the steady-state condition at \( t = 1 \) s. To compare the linearized model with the actual model and clearly validate the accuracy, the proportional gain of the dc-link voltage controller is decreased to increase the bandwidth of the dc-link voltage control loop, which in turn produces an oscillatory and lightly damped response. Therefore, the model can be verified even in the presence of the lightly damped oscillations. As shown in Fig. 4, the model accuracy is validated at the intentionally lightly damped condition and the nominal highly damped design. The corresponding dominant damped and lightly-damped eigenvalues of the small-signal model are \( \lambda \approx -20 \) and \( \lambda \approx -14 \pm j57 \), respectively, which are influenced by the dc-link states according to the participation factor analysis.

III. PV DYNAMIC RESISTANCE INTERACTIONS AT THE REDUCED DC-LINK CAPACITANCE

A. PV DYNAMIC RESISTANCE

The single-diode PV model is selected in this study due to many reasonable aspects, such as simplicity and accuracy [4]. An array of the model “Mitsubishi Electric PV-UD190MF5” has been considered [22]. The overall model parameters and equations are depicted in Appendix A and B, respectively.

As shown in Fig. 5(a), the characteristics of the PV array mainly reflect the nonlinear behavior of the p-n junction diode. Fig. 5(b) shows the static and dynamic resistances of the PV array, where the PV static resistance \((r_s)\) is the DC resistance defined as the ratio \((V_{pv}/I_{pv})\) and the PV dynamic
resistance \( r_d \) is the AC small-signal-resistance defined as the negative reciprocal of \((dI_{pv}/dV_{pv})\) and is given in (25) at the bottom of the next page [23]. The PV dynamic resistance can also be derived using the dynamic resistance of the p-n junction diode \( (r^D_d) \) as follows:

\[
r^D_d = \frac{N_s n_s A k T^*}{N_p I_{rs} q} \exp \left\{ -\frac{q}{N_s n_s A k T^*} \left[ V_{pv} + \left( \frac{N_p}{N_s} R_s \right) I_{pv} \right] \right\}
\]

(22)

\[
r_d = (r^D_d \parallel N_s R_{sh}) + \frac{N_s}{N_p} R_s
\]

(23)

Due to the high-nonlinearity of the PV characteristics and the continuous perturbation in the operating points with time, it is shown in this paper that the PV generator should be implemented in terms of its dynamic resistance to accurately study the dynamic performance of the grid-connected system at certain operating conditions.

As shown in Fig. 5(b), the dynamic PV resistance is larger than the static PV resistance under the CCR operation, which can negatively impact the system stability compared to the CVR operation, where the dynamic PV resistance is smaller and has no influence. Both dynamic and static PV resistances have the same value at the MPP as in (24) with no potential impact on the system stability.

\[
- \left( \frac{dI_{pv}}{dV_{pv}} \right)|_{V_{pv}^{\text{MPP}}}^{-1} = \left( \frac{V_{pv}}{I_{pv}} \right)|_{V_{pv}^{\text{MPP}}} - \left( \frac{V_{pv}}{I_{pv}} \right)|_{V_{pv}^{\text{MPP}}}
\]

(24)

The effect of the large PV dynamic resistance on the interfacing inverter under the CCR must be considered for the following reasons. 1) The input-voltage-feedback control is the method used to control the PV inverter; therefore, the inverter should be analyzed in such a way that it is fed by a current source. 2) The MPPT algorithm should respond to the variations in the atmospheric conditions within a short amount of time to avoid energy losses. However, under rapidly changing atmospheric conditions and slow tracking speed, the MPPT algorithm might operate at the CCR and lose its capability to quickly adjust the system to operate back at the MPP.

In this paper, the nominal voltage and power of the PV generator are designed at 1186 V and 1.5 MW, respectively. This matches the rating conditions of the recent centralized power electronic converters in PV applications [24].

B. PV DYNAMIC RESISTANCE AND REDUCED DC-LINK CAPACITANCE INTERACTIONS

The dynamic interactions between the PV generator and the dc-side of the VSC at different operating conditions are demonstrated using eigenvalues analysis. The PV system stability is preserved at the MPP and CVR of the \( V_{pv} - P_{pv} \) curve. On the contrary, the stability is violated in the CCR only when \( C_{dc} \) is reduced to below the stability margin value of the dc-link capacitance.

Fig. 6(a) shows the eigenvalue stability analysis of the uncompensated system at the CCR with different dc-link capacitance values. As shown, the high-frequency dominant eigenmode migrates to the right-half plane (RHP) in the s-domain as the dc-link capacitance decreases, and the stability margin is violated at a dc-link capacitance of 0.77 p.u. despite the robustness of the dc-link voltage controller. Note that the dc-link voltage controller is tuned at each operating point to maintain the phase margin and the bandwidth.

Figs. 6(b)-(c) show the eigenvalue analysis of the uncompensated system at the MPP and under the CVR operation, respectively. It is clearly shown that the system stability is preserved with different values of the dc-link capacitance. However, at highly dynamic weather conditions, the PV array would typically oscillate between the CCR, MPP, and CVR regions. Therefore, the consideration of the dynamic instabilities in Fig. 6(a), i.e., under the CCR mode, is crucial.

The participation factor analysis has been conducted at the reduced dc-link capacitance conditions similar to Fig. 6,
and the corresponding results are depicted in Table 1. The unstable conditions are reflected on \( \lambda_{2,3} \) on the RHP, which is influenced by the voltage controller states. Note that the yielded instabilities are not due to the resonance of the LC combination on the dc-side; it is solely correlated to the reduced dc-link capacitor. Furthermore, the frequency of the oscillation of the unstable mode in Table 1 is 5498.6 rad/s, whereas the dc-line influences only the farthest eigenvalue in the left-half plane (LHP) and has no significant effect on the system stability under any condition.

IV. PROPOSED ACTIVE COMPENSATION FOR PV-REDUCED DC-LINK INTERACTIONS

Passive damping techniques can be used to enhance dynamic stability [26]. However, these methods are not preferred due to the associated power losses [27]. In this work, an active compensator, implemented in the VSC dc-side, is proposed to mitigate the instabilities and enhance the dynamic performance under the reduced dc-link capacitance and the CCR operation.

As shown in Table 1, the unstable eigenmode is mostly influenced by the dc-link state. Following rigorous investigations and mathematical manipulations, the most effective way to actively damp the system performance is proposed in this paper by injecting a modified version of the dc-link voltage into the \( d \)-axis current reference [Fig. 2]. Note that no extra sensors are required in this method, and so it is low-cost and effective. The compensated signals are derived and implemented as follows.

Assume that there is a series resistor \( (R_{C_{dc}}) \) with the dc-link capacitor, the dynamic power equation in (12) becomes

\[
I_{ext} - I_{dc} = C_{dc} \frac{d}{dt} \left[ V_{dc} - R_{C_{dc}} (I_{ext} - I_{dc}) \right]
\]

(26)

The role of the passive damping effect in (26) can be actively emulated by modifying the \( d \)-axis current reference from the dc-link voltage control loop in (13) as follows:

\[
I_{d}^{\text{total}} = \frac{\eta P_{ext} - G_{dc} (V_{dc}^{2} - V_{dc}^{2})}{1.5 V_{dc}^{2}} + I_{dmp}
\]

(27)

\[
I_{dmp} = \alpha \frac{C_{dc} R_{C_{dc}}}{C_{dc} R_{C_{dc}} s + 1} \frac{1}{\tau s + 1} \left( s C_{dc} V_{dc} \right)
\]

(28)

where \( I_{dmp} \) is the added damping current; \( \alpha \) is a gain used to enhance the high-frequency attenuation; \( H(s) \) is a high-pass filter with a time-constant equals \( C_{dc} R_{C_{dc}} \); \( L(s) \) is a low-pass filter with a time-constant equals \( \tau \); and \( I_{dmp} \) is the equivalent dc-link capacitor current. As a result, the damping current

\[
r_d = -\left( \frac{dI_{pv}}{dV_{pv}} \right)^{-1} = \frac{1}{(N_r/N_p) R_{int}} \exp \left( \frac{q (V_{pv} + (N_r/N_p) R_{int})}{N_r n_r A K T} \right) + \frac{1}{N_p n_p q} \exp \left( \frac{q (V_{pv} + (N_r/N_p) R_{int})}{N_r n_r A K T} \right)
\]

(25)
can be obtained by applying the dc-link voltage to a double high-pass filter with suitable cut-off frequencies.

Two more states are added to the model to stabilize the system, as shown in Appendix E. The influence of the proposed compensator on the dynamic response of the system under CCR operation is shown in Fig. 7(a). As compared to the uncompensated case, the stability margin is significantly improved by moving the eigenvalues to the LHP with the reduced dc-link capacitance, and the system is robustly damped with a dominant damping ratio \( \lambda \approx -20 \) as in the normal operation when the dc-link capacitance is 1 p.u.

Figs. 7(b)-(c) show the eigenvalue analysis at the MPP and under the CVR operation, respectively. It is shown that the system stability of the compensated system is still preserved with different values of the dc-link capacitance as compared to the uncompensated system.

In (28), the input to \( H(s) \) can be \( V_{dc} \) or \( I_{C_{dc}} = (1/(\tau s + 1))sC_{dc}V_{dc} \), which is the equivalent dc-link capacitor current that is obtained practically by filtering the derivative component of the dc-link voltage using a low-pass filter with a time-constant \( \tau \). As shown in Fig. 8, the best time-constant can be selected to obtain \( I_{C_{dc}} \) that is similar to the actual dc-link capacitance current is \( \tau = 0.1 \) ms.

The frequency response of the uncompensated and compensated dc-link voltage transfer functions is plotted in Fig. 9. The undamped transfer functions have a response peak around 5500 rad/s, whereas the cut-off frequency \( (1/C_{dc}R_{C_{dc}}) \) of the high-pass filter is designed at the same high-frequency following that \( R_{C_{dc}} = X_{C_{dc}} \), as shown in Fig. 10. Referring to Fig. 9, the frequency responses with the active damped method is shown when \( R_{C_{dc}} = 47 \) m\( \Omega \) in the CCR region and under different dc-link capacitance values. The actively compensated system is similar to the passively damped system with an actual series resistance of 47 m\( \Omega \). Clearly, the high-frequency peak is successfully mitigated at \( \alpha = 1 \); however, the attenuation gain \( \alpha \) should be strictly chosen to be between 0.05 and 1.6 to avoid influencing the cut-off frequencies and then, inducing instabilities.

As shown in Fig. 11, the active compensator does not influence the dc-link voltage controller and the steady-state performance of the grid-connected VSC. Moreover, the proposed compensator enhances the transient response by mitigating the resonance frequency at \( \omega \approx 4904.2 \) rad/s, which is induced by the ac-filter. Further, and as shown in (28),
V. EVALUATION RESULTS

A large-signal time-domain simulation model of the system under study in Fig. 1 is implemented in the Matlab/Simulink environment to evaluate the analytical results and validate the performance of the proposed active compensator.

The complete model entities are built using the SimPowerSystem® toolbox. The VSC is simulated using the average-model-based blocks. The simulation type is discrete with a sample time of 50 µs, and the complete system parameters are given in Appendix A.

A. UNCOMPENSATED SYSTEM RESPONSE

Fig. 12 shows the uncompensated response at different dc-link voltage levels and different dc-link capacitance values. The system operates in the CVR, MPP, and CCR at \( t = 0 - 3 \) s, \( t = 3 - 4 \) s, and \( t = 4 - 7 \) s, respectively.

The dc-link voltage has significant high-frequency oscillations that affect the system dynamic stability when the system operates at the CCR and reduced dc-link capacitance values lower than 0.77 p.u.

B. INFLUENCE OF THE PROPOSED ACTIVE COMPENSATION

The proposed active compensated response under the same operating conditions is shown in Fig. 13. The dc-link voltage is highly damped under the CCR operation and the lower dc-link capacitance values when the proposed compensator is activated at \( t = 1.3 \) s.

Similar to Fig. 12 of the uncompensated system, Fig. 14 shows the compensated response under the entire PV operation regions and different dc-link capacitance values. The
system operates in the CVR, MPP, and CCR at \( t = 0 - 3 \) s, \( t = 3 - 4 \) s, and \( t = 4 - 7 \) s, respectively. It is clearly shown that the compensated system is stable when the proposed compensator is activated at \( t = 4 \) s.

C. OPERATION UNDER FAULT CONDITIONS

Due to the increased penetration levels of renewable energy resources, the fault ride-through capability is incorporated to guarantee stable grid operation during grid fault conditions.

Fig. 15(a) investigates the low-voltage ride-through (LVRT) capability of the compensated system where the active compensator is applied under the CCR at \( t = 1 \) s, whereas Fig. 15(b) investigates the LVRT capability of the compensated system where the active compensator is applied at the MPP at \( t = 1 \) s. The PCC is subjected to a single line to ground fault for five cycles at \( t = 1.5 \) s. Clearly, the system stability is preserved with the proposed compensator.

D. INFLUENCE OF THE DC-LINK CABLE

Fig. 16 shows the dc-link voltage response of the uncompensated and compensated systems with no cables on the dc-side. It is clearly shown that the source of instabilities is not due to the resonance at the dc-side, and they occur due to interactions between the dc-link capacitance and PV dynamic resistance.

E. SENSITIVITY RESULTS

The influence of the proposed compensator on the dynamic and steady-state performance of the grid-connected VSC is investigated. A unit step has been applied to the dc-link voltage at \( t = 1 \) s, and the corresponding compensated and uncompensated responses are shown in Fig. 17. It is clear that both responses have the same steady-state performance and share a similar rise time. Further, the ac-filter resonance is enhanced in the compensated system.
VI. CONCLUSION
This paper has introduced comprehensive modeling and control of the single-stage grid-connected PV system. The dynamic resistance of the PV arrays is analyzed and defined under different operating regions. It is found that reduced dc-link capacitance affects the dynamic stability of the overall system due to interactions with the dynamic resistance of the PV array. As a result, a new and simple compensator is proposed to stabilize the system with a reduced dc-link capacitance. The small-signal stability analysis of the overall system is performed under different operating conditions.

The proposed compensators have the following advantages: 1) it is simple yet effective and can be easily designed using linear analysis tools, 2) it does not affect the steady-state operation of the VSC grid-connected PV system, 3) it improves the damping performance of the dc-link voltage and provides a robust and stable performance at different operating conditions of the PV system, and 4) it facilitates successful low voltage ride-through at different operating conditions.

APPENDIX
A. SYSTEM PARAMETERS
The entire system parameters are:

Utility-Grid: \( v_g = 12.47 \text{kV}, \omega^o = 2\pi (60) \text{ rad/s}, \text{SCR} = 20, X/R = 10. \)

Step-Up Transformer: \( N = (0.48 \text{kV}/12.47 \text{kV}), L_p = 122.23 \ \mu\text{H}, R_p = 4.608 \ \Omega, L_s = 27.499 \ \text{mH}, R_s = 1.0367 \ \Omega. \)

VSC: \( P = 1.5 \ \text{MW}, f_{sw} = 51 * (60 \text{Hz}), L_f = 100 \ \mu\text{H}, R_f = 2 \ \text{m\Omega}, T_{on} = 1 \ \text{m\Omega}, C_f = 369 \ \mu\text{F}, k_{p}^{\text{ill}} = 1.8014e2 \ V^{-1}s^{-1}, k_{i}^{\text{ill}} = 3.1969e3 \ V^{-1}s^{-2}, k_{p}^{\text{d}} = 0.1923 \ \Omega, k_{i}^{\text{d}} = 3.8453 \ \Omega s^{-1}, k_{p}^{\text{dc}} = 0.4807 \ \Omega^{-1}, k_{i}^{\text{dc}} = 9.2415 \ \Omega^{-1}s^{-1}, k_{p}^{\text{vac}} = 71.8162 \ \text{VAR}^{-1}s^{-1}, k_{i}^{\text{vac}} = 1.9947e6 \ \text{VAR}^{-1}s^{-2}. \)

DC-Link: \( C_{dc} = 5000 \ \mu\text{F}. \)

DC-Cable: \( L_{dc} = 0.34 \ \mu\text{H}, R_{dc} = 0.125 \ \text{m\Omega}. \)

PV: \( N_{p} = 164, N_s = 48, n_s = 50, A = 0.9447, \)
\( I_s = 8.23 \ \text{A}, I_{rs} = 7.6985e11 \ \text{A}, q = 1.6022e-19 \ \text{C}, \)
\( R_t = 0.3238 \ \Omega, R_{sh} = 236.4479 \ \Omega, \alpha_t = 0.0382 \ \text{A/K}, \)
\( I^r_{irr} = 1000 \ \text{W/m}^2, T^* = 298.15 \ \text{K}, E_g = 1.1 \ \text{eV}, \)
\( k = 1.3806e-23 \ \text{J/K}. \)

B. PV MODEL
The model of PV generator is given as follows:

\[
I_{pv} = N_p I_{ph} - N_p I_{D} - \frac{V_{pv} + (N_s/N_p) R_d I_{pv}}{(N_s/N_p) R_{sh}}
\]

\[
I_{ph} = \left( I_{irr}/I_{irr}^* \right) \left[ I_{sc} + \alpha_i (T - T^*) \right]
\]

\[
I_D = I_{rs} \left( \exp \left\{ \frac{q (V_{pv} + (N_s/N_p) R_d I_{pv})}{N_s n_s A.K.T} \right\} - 1 \right)
\]

\[
I_{rs} = I_{rs}^* \left( \frac{T}{T^*} \right)^3 \exp \left\{ \frac{q E_g}{A.K} \left( \frac{1}{T^*} - \frac{1}{T} \right) \right\}
\]

where \( I_{ph} \) is the photo-generated current of the PV module, \( I_D \) is the diode current of the PV module, \( R_{sh} \) and \( R_t \) are the equivalent shunt and series resistance of the PV module, \( N_s \) is the number of series-connected modules in the PV panel, \( N_p \) is the number of parallel panels in the PV array, \( I_{irr} \) and \( I_{irr}^* \) are actual and reference solar illumination levels, \( I_{sc} \) is the short-circuit current of the PV module, \( \alpha_i \) is the temperature coefficient, \( T \) and \( T^* \) are actual and reference temperatures in Kelvin, \( I_{rs} \) and \( I_{rs}^* \) are the reverse saturation current at the operating and reference temperatures, \( q \) is the unit charge, \( K \) is the Boltzmann’s constant, \( A \) is the ideality factor, \( n_s \) is the number of PV cells, and \( E_g \) is the band-gap energy of the PV cell.

C. VALIDATION STATE-SPACE MODEL
Small-signal modeling of the grid-connected VSC–Utility-Grid Interface Model

\[
\begin{bmatrix}
\Delta I_{d}^s \\
\Delta I_{q}^s \\
\Delta I_{d}^g \\
\Delta I_{q}^g \\
\Delta V_{d}^s \\
\Delta V_{q}^s \\
\Delta V_{d}^g \\
\Delta V_{q}^g
\end{bmatrix} =
\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
A_2 \\
B_2 \\
C_2 \\
A_3 \\
B_3
\end{bmatrix}
\begin{bmatrix}
\Delta \omega
\end{bmatrix}
\]
\[
\begin{bmatrix}
\Delta v_d^\phi \\
\Delta v_q^\phi
\end{bmatrix} =
\begin{bmatrix}
-A_3 \\
-A_5
\end{bmatrix}
\begin{bmatrix}
\Delta \dot{v}_d^\phi \\
\Delta \dot{v}_q^\phi
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{N} \left[ \begin{array}{cc}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{array} \right] \Delta \dot{v}_d^{\rho} \\
C_3
\end{bmatrix} [\Delta \omega]
\]

**Inner Control Model**

\[
\Delta \dot{v}_d^{\phi} = \begin{bmatrix} 0 & -\alpha_p l_d \\
\alpha_p l_d & 0 \end{bmatrix} \Delta \dot{v}_d^{\phi} + \begin{bmatrix} \Delta \dot{v}_d^{\rho} \\
\Delta \dot{v}_d^{\phi}
\end{bmatrix} + \begin{bmatrix} \Delta v_d^{\phi} \\
\Delta v_d^{\rho}
\end{bmatrix}
\]

\[
\Delta \dot{v}_q^{\phi} = \begin{bmatrix} k_p & 0 \\
0 & k_p \end{bmatrix} \Delta \dot{v}_q^{\phi} + \begin{bmatrix} \Delta \dot{v}_q^{\rho} \\
\Delta \dot{v}_q^{\phi}
\end{bmatrix} + \begin{bmatrix} \Delta v_d^{\phi} \\
\Delta v_d^{\rho}
\end{bmatrix}
\]

**PLL Model**

\[
\begin{bmatrix}
\Delta \dot{\epsilon} \\
\Delta \dot{\psi}^{\text{PLL}}
\end{bmatrix} = \begin{bmatrix}
A_6 \\
B_6
\end{bmatrix}
\begin{bmatrix}
\Delta \epsilon \\
\Delta \psi^{\text{PLL}}
\end{bmatrix} + \begin{bmatrix}
k_{p}^{\text{PLL}} \\
0
\end{bmatrix} \frac{\delta^{\text{PLL}}}{v_d^{\rho}} \begin{bmatrix}
\Delta \dot{v}_d^{\rho} \\
\Delta \dot{v}_q^{\rho}
\end{bmatrix}
\]

\[
[\Delta \omega] = \begin{bmatrix} 0 & 1 \\
0 & 0 \end{bmatrix}
\begin{bmatrix}
\Delta \epsilon \\
\Delta \psi^{\text{PLL}}
\end{bmatrix} + \begin{bmatrix} k_{p}^{\text{PLL}} \\
0 \end{bmatrix} \frac{\delta^{\text{PLL}}}{v_d^{\rho}} \begin{bmatrix}
\Delta \dot{v}_d^{\rho} \\
\Delta \dot{v}_q^{\rho}
\end{bmatrix}
\]

**Frame Transformation**

\[
\begin{bmatrix}
\Delta v_d^{\phi} \\
\Delta v_q^{\phi}
\end{bmatrix} = \begin{bmatrix}
A_7 \\
B_7
\end{bmatrix}
\begin{bmatrix}
\Delta v_d^{\xi} \\
\Delta v_q^{\xi}
\end{bmatrix} + \begin{bmatrix}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{bmatrix} \Delta v_d^{\rho}
\]

\[
\Delta \dot{v}_d^{\xi} = \begin{bmatrix} \frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f} \end{bmatrix} \Delta \dot{v}_d^{\xi} + \begin{bmatrix}
\Delta \dot{v}_d^{\rho} \\
\Delta \dot{v}_d^{\phi}
\end{bmatrix}
\]

**Outer AC and DC-Link Voltage Dynamics and Control Models**

\[
\begin{bmatrix}
\Delta \dot{v}_d^{\phi} \\
\Delta \dot{v}_q^{\phi}
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{bmatrix} \Delta \dot{v}_d^{\rho} + \begin{bmatrix}
B_8 \\
C_8
\end{bmatrix}
\begin{bmatrix}
\Delta v_d^{\xi} \\
\Delta v_q^{\xi}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta \dot{v}_d^{\phi} \\
\Delta \dot{v}_q^{\phi}
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{bmatrix} \Delta \dot{v}_d^{\rho} + \begin{bmatrix}
B_9 \\
C_9
\end{bmatrix}
\begin{bmatrix}
\Delta v_d^{\xi} \\
\Delta v_q^{\xi}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta \dot{v}_d^{\phi} \\
\Delta \dot{v}_q^{\phi}
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{bmatrix} \Delta \dot{v}_d^{\rho} + \begin{bmatrix}
B_{10} \\
C_{10}
\end{bmatrix}
\begin{bmatrix}
\Delta v_d^{\xi} \\
\Delta v_q^{\xi}
\end{bmatrix}
\]

**Photovoltaic Generator Interface Model**

\[
\begin{bmatrix}
\Delta \dot{v}_d^{\phi} \\
\Delta \dot{v}_q^{\phi}
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{c_f} & 0 \\
0 & \frac{-1}{c_f}
\end{bmatrix} \Delta \dot{v}_d^{\rho} + \begin{bmatrix}
B_{11} \\
C_{11}
\end{bmatrix}
\begin{bmatrix}
\Delta v_d^{\xi} \\
\Delta v_q^{\xi}
\end{bmatrix}
\]

Define the following block matrices in matrix A, as shown at the bottom of the next page.

\[
a_{13} = -B_1 + B_1 C_6 + B_1 + B_1 C_4 D_6 + B_1 B_4 D_{10}
\]

\[
a_{15} = (C_1 + B_1 C_4) (C_6 + D_6 A_7) + B_1 (C_7 + A_4 B_7 + A_7 + B_4 (D_{10} A_7 - B_7))
\]

\[
a_{71} = C_9 + F_9 (A_1 + B_1 (A_4 - B_4))
\]

\[
a_{73} = D_9 + F_9 (C_1 D_6 + B_1 + B_1 C_4 D_6 + B_1 B_4 D_{10})
\]

\[
a_{75} = F_9 (C_1 + B_1 C_4) (C_6 + D_6 A_7) + F_9 B_1 (C_7 + A_4 B_7 + A_7 + B_4 (D_{10} A_7 - B_7))
\]
\[ a_{77} = A_9 + F_9 B_1 B_4 A_{10} \]
\[ a_{78} = E_9 + F_9 B_1 B_4 F_{10} \]

\[
B = \begin{bmatrix}
  B_1 B_4 B_{10} & B_1 B_4 E_{10} & Z \\
  Z & Z^T & -B_2 \\
  Z & Z^T & Z \\
  A_5 B_{10} & A_5 E_{10} & Z \\
  Z & Z^T & Z \\
  Z_v & B_8 & Z_v \\
  B_9 + F_9 B_1 B_4 B_{10} & F_9 B_1 B_4 E_{10} & Z \\
  C_{11} & Z_v & Z_v^T \\
\end{bmatrix}
\]

Define C and D to find the dc-link voltage response.
\[
C = \begin{bmatrix}
  \text{zeros} (1, 11) & 1 & \text{zeros} (1, 2)
\end{bmatrix}
D = \text{zeros} (1, 5)
\]

where the states \((\Delta i_d^g, \Delta i_q^g), (\Delta v_d^g, \Delta v_q^g), (\Delta \epsilon, \Delta V_{dc}^d), \Delta I_{pv}\) are the d-q components of the VSC current, dq components of the grid current, dq components of the PCC, the synchronization angle, the squared value of the terms of the VSC current, the PCC, control loop, the ac voltage controller, and the dc-link voltage, respectively.

**D. UNCOMPENSATED STATE-SPACE MODEL**

The small-signal-value of the input PV power in the dc-side dynamic equations \((\Delta P_{pv})\) is replaced with the small-signal-value of the input dynamic resistance \((\Delta r_d)\) to study the impact of the dynamic resistance, where \(r_d^o\) represents the PV operational region. The dc cable equation becomes

\[
[\Delta I_{pv}] = \begin{bmatrix}
  A_{11} \\
- \frac{1}{2 L_{dc}} V_{dc}^2 & 0
\end{bmatrix} \begin{bmatrix}
  \Delta V_{dc}^2 \\
  \Delta \phi_{dc}^c
\end{bmatrix} + \begin{bmatrix}
  B_{11}
\end{bmatrix} \begin{bmatrix}
  \Delta r_d - \frac{R_{dc}}{L_{dc}}
\end{bmatrix}
\times [\Delta I_{pv}] + \begin{bmatrix}
  0 & \frac{f_{pv}}{T_{dc}}
\end{bmatrix} \begin{bmatrix}
  \Delta V_{dc}^2 \\
  \Delta V_{dc}^2 + \Delta V_{dc}^* + \Delta V_{dc}^*
\end{bmatrix} \begin{bmatrix}
  \Delta \phi_{pv}
\end{bmatrix}
\]

The state-space model of the uncompensated system is similar to the previous model in Appendix C.

\[
A = A_{Uncomp} \quad B = B_{Uncomp}
\]

To find the transfer function \((\Delta V_{dc}/\Delta V_{dc}^*)\) of the uncompensated system, define the following C and D.

\[
\Delta V_{dc}^2 = 2V_{dc}^2 \Delta \phi_{dc}^c \\
\Delta V_{dc}^2 = 2V_{dc}^2 + \Delta V_{dc}^* + \Delta V_{dc}^* \\
\Delta V_{dc}^2 = \Delta V_{dc}^2 + \Delta V_{dc}^* + \Delta V_{dc}^*
\]

\[
C = \text{eye}(14) \quad D = \text{zeros}(14, 5)
\]

\[
\Delta x = \begin{bmatrix}
  \Delta i_d^g & \Delta i_q^g & \Delta v_d^g & \Delta v_q^g & \Delta \epsilon & \Delta V_{dc}^d & \Delta I_{pv}
\end{bmatrix}
\]

\[
\Delta u = \begin{bmatrix}
  A_1 + B_1(A_4 - B_4) & Z & a_{13} & B_1 & a_{15} & B_1 B_4 C_{10} & B_1 B_4 A_{10} & B_1 B_4 F_{10}
\end{bmatrix}
\]

\[
\Delta x_{Comp} = \begin{bmatrix}
  \Delta i_d^g & \Delta i_q^g & \Delta v_d^g & \Delta v_q^g & \Delta \epsilon & \Delta V_{dc}^d & \Delta I_{pv}
\end{bmatrix}
\]

\[
\Delta u_{Comp} = \begin{bmatrix}
  A_1 + B_1(A_4 - B_4) & Z & a_{13} & B_1 & a_{15} & B_1 B_4 C_{10} & a_{17} & B_1 B_4 F_{10} & B_1 B_4 G_{10} A_{12}
\end{bmatrix}
\]
E. COMPENSATED STATE-SPACE MODEL

Another two states, $\Delta i_x$ and $\Delta i_y$, are added to the model. The reference dq currents equation and dc-cable models are modified as follows:

- Outer Control Model

$$\begin{bmatrix} \Delta i_d^* \\ \Delta i_q^* \end{bmatrix} = \begin{bmatrix} A_{10} & B_{10} \\ C_{10} & D_{10} \end{bmatrix} \begin{bmatrix} \Delta V_d^2 \\ \Delta \varphi_d^c \\ \Delta V_q^c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \begin{bmatrix} \Delta i_x \\ \Delta i_y \end{bmatrix}$$

- Photovoltaic Generator Interface Model

$$\begin{bmatrix} \Delta I_{pv} \end{bmatrix} = \begin{bmatrix} A_{11} \end{bmatrix} \begin{bmatrix} \Delta V_d^2 \\ \Delta \varphi_d^c \end{bmatrix} + \begin{bmatrix} B_{11} \end{bmatrix} \begin{bmatrix} r_{dc} - R_{dc} \\ L_{dc} \end{bmatrix}$$

- Active Compensation Model

$$\begin{bmatrix} \Delta i_x \\ \Delta i_y \end{bmatrix} = \begin{bmatrix} A_{12} \end{bmatrix} \begin{bmatrix} \Delta i_x \\ \Delta i_y \end{bmatrix} + \begin{bmatrix} B_{12} \end{bmatrix} \begin{bmatrix} \Delta V_d^2 \\ \Delta \varphi_d^c \end{bmatrix}$$

- To find the transfer function $(\Delta V_{dc}/\Delta V_{dc}^*)$ of the compensated system, define $C = \text{eye}(16)$ and $D = \text{zeros}(16, 5)$.

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MOHAMMAD ADNAN K. MAGABLEH (Graduate Student Member, IEEE) was born in Irbid, Jordan, in September 1993. He received the B.Sc. degree (Hons.) in electrical engineering from the Jordan University of Science and Technology, Irbid, in 2016, and the M.Sc. degree in electrical engineering from the University of Alberta, Edmonton, AB, Canada, in 2020, where he is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering.

His research interests include dynamics and controls of power electronic converters for renewable energy applications and the reliability of power electronic converters.

AMR RADWAN (Senior Member, IEEE) received the B.Sc. degree (Hons.) in electrical engineering from Ain Shams University, Cairo, Egypt, in 2007, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Alberta, Edmonton, AB, Canada, in 2012 and 2016, respectively.

He is currently an Assistant Professor with the Engineering and Design Department, Electrical and Computer Engineering Program, Western Washington University, Bellingham, WA, USA.

He is also a registered Professional Engineer in Saskatchewan. His research interests include smart active distribution systems and control of power electronic converters.

YASSER ABDEL-RADY I. MOHAMED (Fellow, IEEE) was born in Cairo, Egypt, in November 1977. He received the B.Sc. (Hons.) and M.Sc. degrees in electrical engineering from Ain Shams University, Cairo, in 2000 and 2004, respectively, and the Ph.D. degree in electrical engineering from the University of Waterloo, Waterloo, ON, Canada, in 2008.

He is currently a Professor with the Department of Electrical and Computer Engineering, University of Alberta, AB, Canada. He is also a Registered Professional Engineer in Alberta. His research interests include dynamics and controls of power converters, grid integration of distributed generation and renewable resources, microgrids, modeling, analysis, and control of smart grids, and electric machines and motor drives.

Dr. Mohamed is also an Associate Editor of the IEEE TRANSACTIONS ON POWER ELECTRONICS and an Editor of the IEEE TRANSACTIONS ON POWER SYSTEMS, IEEE TRANSACTIONS ON SMART GRID, and IEEE POWER ENGINEERING LETTERS.