 Scalar leptoquarks and the rare $B$ meson decays

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Abstract

We study some rare decays of $B$ meson involving the quark level transition $b \to ql^+l^-$ ($q = d, s$) in the scalar leptoquark model. We constrain the leptoquark parameter space using the recently measured branching ratios of $B_{s,d} \to \mu^+\mu^-$ processes. Using such parameters, we obtain the branching ratios, direct CP violation parameters and isospin asymmetries in $B \to K\mu^+\mu^-$ and $B \to \pi\mu^+\mu^-$ processes. We also obtain the branching ratios for some lepton flavour violating decays $B \to l_i^+l_j^-$. We find that the various anomalies associated with the isospin asymmetries of $B \to K\mu^+\mu^-$ process can be explained in the scalar leptoquark model.

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I. INTRODUCTION

The rare decays of $B$ mesons involving flavor changing neutral current (FCNC) transitions $b \to s/d$, provide an excellent testing ground to look for new physics. In the standard model (SM), these transitions occur at one-loop level and hence, they are very sensitive to any new physics contributions. Although, so far we have not seen any clear indication of new physics in the $b$ sector, but there appears to be some kind of tension with the SM predictions in some $b \to s$ penguin induced transitions. It should be noted that the recent measurement by LHCb collaboration [1] shows several significant deviations on angular observables in the rare decay $B \to K^{*0}\mu^+\mu^-$ from their corresponding SM expectations. In particular, the most significant discrepancy of $3.7\sigma$, arises in the variable $P_5^\prime$ [2] (the analogue of $S_5$ in [3]) provides high sensitivity to new physics (NP) effects in $b \to s\gamma$, $sl^+l^-$ transitions. Further results from LHCb experiment in combination with the critical assessment of the theoretical uncertainties will be necessary to clarify whether the observed deviations are a real sign of NP or simply the statistical fluctuations [4, 5].

Another indication of new physics is related to the recent measurement of isospin asymmetry in $B \to K\mu^+\mu^-$ process by LHCb experiment [6], which gives a negative deviation from zero at the level of $4\sigma$ taking into account the entire $q^2$-spectrum. The isospin-asymmetry in $B \to Kl$ is expected to be vanishingly small in the SM and hence, the measured asymmetry provides another smoking-gun signal for new physics.

More recently another discrepancy occurs in the measurement of the ratio of branching ratios of $B \to Kl^+l^-$ decays into dimuons over dielectrons by the LHCb collaboration [7],

$$R_K = \frac{\text{BR}(B \to K\mu^+\mu^-)}{\text{BR}(B \to Ke^+e^-)}, \quad (1)$$

and the obtained value in the dilepton invariant mass squared bin $(1 \lesssim q^2 < 6)$GeV$^2$ is

$$R_{K}^{LHCb} = 0.745^{+0.090}_{-0.074} \pm 0.036 \quad (2)$$

Combining the statistical and systematic uncertainties in quadrature, this observation corresponds to a $2.6\sigma$ deviation from its SM prediction $R_K = 1.0003 \pm 0.0001$ [8], where corrections of order $\alpha_s$ and $(1/m_b)$ are included. In contrast to the anomaly in the rare decay $B \to K^*\mu^+\mu^-$, which is affected by unknown power corrections, the ratio $R_K$ is theoretically clean and this might be a sign of lepton flavour non-universal physics.
Although it is conceivable that these anomalies mostly associated with $b \to s l^+l^-$ transitions are due to statistical fluctuations or under-estimated theory uncertainties, but the possible interplay of new physics could not be ruled out. These LHCb results have attracted many theoretical attentions in recent times [2, 4, 5, 9] both in the context of some new physics model or in model independent way. In this paper we would like to investigate some of the rare decay modes of $B$ meson involving the FCNC transitions $b \to (s, d) l^+l^-$, e.g., $B \to Kl^+l^-$, $B \to \pi l^+l^-$ and $B \to l_i^+l_j^-$ using the scalar leptoquark (LQ) model. In particular, we would like to see whether the leptoquark model can accommodate some of the anomalies discussed above, in particular the ones associated with $B \to Kl\ell$ processes. It is well known that leptoquarks are color-triplet bosonic particles that can couple to a quark and a lepton at the same time and can occur in various extensions of the standard model [10]. They can have spin-1, which exist in grand unified theories based on $SU(5)$, $SO(10)$ etc., but are most commonly assumed as scalars. Scalar leptoquarks can exist at TeV scale in extended technicolor models [11] as well as in quark and lepton composite models [12]. The phenomenology of scalar leptoquarks have been studied extensively in the literature [13–15].

In this paper we consider the model where leptoquarks can couple only to a pair of quarks and leptons and hence may be inert with respect to proton decay. Hence, the bounds from proton decay may not be applicable for such cases and leptoquarks may produce signatures in other low-energy phenomena [15].

The paper is organized as follows. In section II we briefly discuss the effective Hamiltonian describing the process $b \to s l^+l^-$ and the new contributions arising due to the exchange of scalar leptoquarks. The constraints on the leptoquark parameter space are obtained using the recently measured branching ratios of the decay modes $B_{s,d} \to \mu^+\mu^-$ and $B \to X_se^+e^-$ process in sections III. The branching ratios and various asymmetries of the rare decay modes $B \to Kl^+l^-$ and $B \to \pi l^+l^-$ are discussed in sections IV and V respectively. In Section VI we present the lepton flavour violating decays $B \to l_i^+l_j^-$ and Section VII contains the conclusion.
II. EFFECTIVE HAMILTONIAN FOR $b \rightarrow s l^+ l^-$ PROCESS

In the standard model effective Hamiltonian describing the quark level transition $b \rightarrow sll$ is given as \cite{16}

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{6} C_i(\mu) O_i + C_7 \frac{e}{16\pi^2} \left( \bar{s} \sigma_{\mu\nu} (m_s P_L + m_b P_R) b \right) F^{\mu\nu} + C_9^{\text{eff}} \frac{\alpha}{4\pi} \left( \bar{s} \gamma^\mu P_L b \right) \bar{\ell} \gamma_\mu \ell + C_{10}^{\text{eff}} \frac{\alpha}{4\pi} \left( \bar{s} \gamma^\mu P_L b \right) \bar{\ell} \gamma_\mu \gamma_5 \ell \right],$$

(3)

where $G_F$ is the Fermi constant and $V_{qq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\alpha$ is the fine-structure constant, $P_{L,R} = (1 \mp \gamma_5)/2$ and $C_i$'s are the Wilson coefficients. The values of the Wilson coefficients are calculated at the next-to-next-leading order (NLL) by matching the full theory to the effective theory at the electroweak scale and subsequently solving the renormalization group equation (RGE) to run them down to the $b$-quark mass scale \cite{3}, and the values used in this analysis are listed in Table-1.

| $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  | $C_6$  | $C_7^{\text{eff}}$ | $C_8^{\text{eff}}$ | $C_9$ | $C_{10}^{\text{eff}}$ |
|--------|--------|--------|--------|--------|--------|---------------------|---------------------|------|----------------------|
| -0.3001 | 1.008  | -0.0047 | -0.0827 | 0.0003 | 0.0009 | -0.2969            | -0.1642            | 4.2607 | -4.2453             |

TABLE I: The SM Wilson coefficients evaluated at the scale $\mu = 4.6$ GeV \cite{17}.

A. New Physics Contributions due to Scalar Leptoquark exchange

The effective Hamiltonian \cite{3} will be modified in the leptoquark model due to the additional contributions arising from the exchange of scalar leptoquarks. Here, we will consider the minimal renormalizable scalar leptoquark models \cite{15}, containing one single additional representation of $SU(3) \times SU(2) \times U(1)$ and which do not allow proton decay at the tree level. It has also been shown that this requirement can only be satisfied by two models and in these models, the leptoquarks can have the representation as $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$ under the gauge group $SU(3) \times SU(2) \times U(1)$. Our objective here is to consider these scalar leptoquarks which potentially contribute to the $b \rightarrow (s, d)\mu^+ \mu^-$ transitions and constrain the underlying couplings from experimental data on $B_{s,d} \rightarrow \mu^+ \mu^-$. The
details of these new contributions are explicitly discussed in Ref. [18], and here we simply outline the main points.

The interaction Lagrangian for the coupling of scalar leptoquark \( X = (3, 2, 7/6) \) to the fermion bilinears is given as

\[
L = -\lambda_{ij}^u \bar{u}_{aR}^i (V_{aL} \nu_L^j - Y_{aL} \nu_L^j) - \lambda_{ij}^e \bar{e}_{aR}^i \left( V_{aL}^\dagger u_{aL}^j + Y_{aL}^\dagger d_{aL}^j \right) + h.c. .
\] (4)

Using the Fierz transformation, one can obtain from Eq. (4), the contribution to the interaction Hamiltonian for the \( b \rightarrow s\mu^+\mu^- \) process as

\[
H_{LQ} = \frac{\lambda_{ij}^{32} \lambda_{ij}^{22\ast}}{8M_Y^2} [\bar{s}\gamma^\mu(1 - \gamma_5) b][\bar{\mu}\gamma_\mu(1 + \gamma_5) \mu] \equiv \frac{\lambda_{ij}^{32} \lambda_{ij}^{22\ast}}{4M_Y^2} \left( O_9 + O_{10} \right). \] (5)

One can thus write the leptoquark effective Hamiltonian (5) analogous to its SM counterpart (3) as

\[
H_{LQ} = -\frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* (C_9^{NP} O_9 + C_{10}^{NP} O_{10}) ,
\] (6)

with the new Wilson coefficients

\[
C_9^{NP} = C_{10}^{NP} = -\frac{\pi}{2\sqrt{2}G_F \alpha V_{tb} V_{ts}^*} \frac{\lambda_{ij}^{32} \lambda_{ij}^{22\ast}}{M_Y^2} .
\] (7)

Similarly, the interaction Lagrangian for the coupling of \( X = (3, 2, 1/6) \) leptoquark to the fermion bilinear can be expressed as

\[
L = -\lambda_{ij}^d \bar{d}_{aR}^i (V_{aL} \nu_L^j - Y_{aL} \nu_L^j) + h.c. ,
\] (8)

and after performing the Fierz transformation, the interaction Hamiltonian becomes

\[
H_{LQ} = \frac{\lambda_{ij}^{22} \lambda_{ij}^{32\ast}}{4M_Y^2} [\bar{s}\gamma^\mu P_{RB}][\bar{\mu}\gamma_\mu(1 - \gamma_5) \mu] = \frac{\lambda_{ij}^{22} \lambda_{ij}^{32\ast}}{4M_Y^2} \left( O'_9^{NP} - O'_{10}^{NP} \right) ,
\] (9)

where \( O'_9 \) and \( O'_{10} \) are the four-fermion current-current operators obtained from \( O_{9,10} \) by making the replacement \( P_L \leftrightarrow P_R \). Thus, due to the exchange of the leptoquark \( X = (3, 2, 1/6) \), one can obtain the new Wilson coefficients \( C_9^{NP} \) and \( C_{10}^{NP} \) associated with the operators \( O'_9 \) and \( O'_{10} \)

\[
C'_9^{NP} = C'^{NP}_{10} = \frac{\pi}{2\sqrt{2}G_F \alpha V_{tb} V_{ts}^*} \frac{\lambda_{ij}^{22} \lambda_{ij}^{32\ast}}{M_Y^2} .
\] (10)

The analogous new physics contributions for \( b \rightarrow d\mu^+\mu^- \) transitions can be obtained from \( b \rightarrow s\mu^+\mu^- \) process by replacing the leptoquark couplings \( \lambda^{32} \lambda^{22\ast} \) by \( \lambda^{32} \lambda^{12\ast} \) and the CKM elements \( V_{ib} V_{is}^* \) by \( V_{ib} V_{id}^* \) in Eqs. (6-10). After having the idea of new physics contributions to the process \( b \rightarrow (s,d)\mu^+\mu^- \), we now proceed to constrain the new physics parameter space using the recent measurement of \( B_{s,d} \rightarrow \mu^+\mu^- \).
III. $B_{s,d} \to \mu^+\mu^-$ DECAY PROCESS

The rare leptonic decay processes $B_{s,d} \to \mu^+\mu^-$, mediated by the FCNC transition $b \to s,d$ are strongly suppressed in the standard model as they occur at one-loop level as well as suffer from helicity suppression. These decay processes are very clean and the only nonperturbative quantity involved is the $B$ meson decay constant, which can be reliably calculated using the non-perturbative methods such as QCD sum rules, lattice gauge theory etc. Therefore, they are considered as one of the most powerful tools to provide important constraints on models of new physics. These processes have been very well studied in the literature and in recent times also they have attracted a lot of attention [19–25]. Therefore, here we will point out the main points. The constraint on the leptoquark couplings from $B_s \to \mu^+\mu^-$ are recently extracted by one of us in Ref. [18].

The most general effective Hamiltonian describing these processes is given as

$$H_{\text{eff}} = G_F \alpha \sqrt{2} \pi V_{tb} V_{tq}^* \left[ C_{10}^{\text{eff}} O_{10} + C_{10}' O_{10}' \right],$$

(11)

where $q = d$ or $s$, $C_{10}^{\text{eff}} = C_{10}^{SM} + C_{10}^{NP}$ and $C_{10}' = C_{10}'^{NP}$. The corresponding branching ratio is given as

$$\text{BR}(B_q \to \mu^+\mu^-) = \frac{G_F^2}{16\pi^3} \tau_{B_q} \alpha^2 f_{B_q}^2 M_{B_q} m^2_{\mu} |V_{tb} V_{tq}^*|^2 \left| C_{10}^{\text{eff}} - C_{10}' \right|^2 \sqrt{1 - \frac{4m^2_{\mu}}{M^2_{B_q}}}. \quad (12)$$

However, as discussed in Ref. [19], the average time-integrated branching ratios $\overline{\text{BR}}(B_q \to \mu^+\mu^-)$ depend on the details of $B_q - \bar B_q$ mixing, which in the SM, related to the decay widths $\Gamma(B_q \to \mu^+\mu^-)$ by a very simple relation as $\overline{\text{BR}}(B_q \to \mu^+\mu^-) = \Gamma(B_q \to \mu^+\mu^-)/\Gamma^q_H$, where $\Gamma^q_H$ is the total width of the heavier mass eigen state.

Including the corrections of $\mathcal{O}(\alpha_{\text{em}})$ and $\mathcal{O}(\alpha^2)$, the updated branching ratios in the standard model are calculated in [25] as

$$\overline{\text{BR}}(B_s \to \mu^+\mu^-)|_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9},$$

$$\overline{\text{BR}}(B_d \to \mu^+\mu^-)|_{\text{SM}} = (1.06 \pm 0.09) \times 10^{-10}. \quad (13)$$

These processes are recently measured by the CMS [26] and LHCb [27] experiments and the current experimental world average [28] is

$$\overline{\text{BR}}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}, \quad \overline{\text{BR}}(B_d \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}, \quad (14)$$
which are more or less consistent with the latest SM prediction (13), but certainly they do
not rule out the possibility of new physics as the experimental errors are still quite large.

We will now consider the additional contributions arising due to the effect of scalar leptoquarks in this mode. Including the contributions arising from scalar leptoquark exchange, one can write the transition amplitude for this process from Eq. (11) as

$$M(B_0^0 \rightarrow \mu^+\mu^-) = \langle \mu^+\mu^- | H_{eff} | B_0^0 \rangle = -\frac{G_F}{\sqrt{2} \pi} V_{tb} V_{tq}^* \alpha_f B_q M_{B_0^0} m_\mu C_{10}^{SM} P,$$

where

$$P \equiv \frac{C_{10} - C'_{10}}{C_{10}^{SM}} = 1 + \frac{C_{10}^{NP} - C'_{10}^{NP}}{C_{10}^{SM}} = 1 + re^{i\phi_{NP}},$$

with

$$re^{i\phi_{NP}} = (C_{10}^{NP} - C'_{10}^{NP})/C_{10}^{SM},$$

$r$ denotes the magnitude of the ratio of NP to SM contributions and $\phi_{NP}$ is the relative phase between them. As discussed in section II, the exchange of the leptoquarks $X(3, 2, 7/6)$ and $X(3, 2, 1/6)$ give new additional contributions to the Wilson coefficients $C_{10}$ and $C'_{10}$ respectively. Thus, the branching ratio in both the cases will be

$$\text{BR}(B_q \rightarrow \mu^+\mu^-) = \text{BR}(B_q \rightarrow \mu^+\mu^-)|_{SM}(1 + r^2 - 2r \cos \phi_{NP}).$$

Using the theoretical and experimental branching ratios from (13) and (14), the constraints on the combination of LQ couplings can be obtained by requiring that each individual leptoquark contribution to the branching ratio does not exceed the experimental result. The allowed region in $r - \phi_{NP}$ plane which are compatible with the $1 - \sigma$ range of the experimental data are shown in Fig.-1 for $B_d \rightarrow \mu^+\mu^-$ (left panel) and for $B_s \rightarrow \mu^+\mu^-$ (right panel). From the figure one can see that for $B_d \rightarrow \mu^+\mu^-$ the allowed range of $r$ and $\phi_{NP}$ as

$$0.5 \leq r \leq 1.3, \quad \text{for} \quad (0 \leq \phi_{NP} \leq \pi/2) \quad \text{or} \quad (3\pi/2 \leq \phi_{NP} \leq 2\pi),$$

which can be translated to obtain the bounds for the leptoquark couplings using Eqs. (10), (14) and (17) as

$$1.5 \times 10^{-9} \leq \frac{|\lambda^{32} \lambda^{12*}|}{M_S^2} \leq 3.9 \times 10^{-9},$$
FIG. 1: The allowed region in the $r - \phi^{NP}$ parameters space obtained from the BR($B_d \to \mu^+\mu^-$) (left panel) and BR($B_s \to \mu^+\mu^-$) (right panel).

with $M_S$ as the leptoquark mass. For $B_s \to \mu^+\mu^-$ process for $0 \leq r \leq 0.1$ the entire range for $\phi^{NP}$ is allowed, i.e.,

$$0 \leq r \leq 0.1 , \quad \text{for} \quad 0 \leq \phi^{NP} \leq 2\pi . \quad (21)$$

However, in our analysis we will use relatively mild constraint as

$$0 \leq r \leq 0.35 , \quad \text{with} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2 . \quad (22)$$

This gives the constraint on leptoquark couplings as

$$0 \leq \frac{|\lambda^{32}\lambda^{22*}|}{M^2_S} \leq 5 \times 10^{-9} \quad \text{for} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2 . \quad (23)$$

One can also obtain the constrain on the leptoquark couplings $|\lambda^{32}\lambda^{22*}|/M^2_S$ from the inclusive measurements BR($B^0_d \to X_s\mu^+\mu^-$). However, as shown in Ref. [18], these constraints are more relaxed than those obtained from $B_s \to \mu^+\mu^-$. So in our analysis we will use the constraints obtained from $B_s \to \mu^+\mu^-$. For other leptonic decay channels i.e., $B_{s,d} \to e^+e^-$, $\tau^+\tau^-$ only the experimental upper limits exists [29]. Now using the theoretical predictions for these branching ratios from [25], we obtain the constrain on the upper limits of the various combinations of leptoquark couplings as presented in Table-II. However, the constraints obtained from such processes are rather weak.
Decay Process | Couplings involved | Upper bound of the couplings
--- | --- | ---
$B_d \rightarrow e^\pm e^\mp$ | $|\lambda^{31}\lambda^{11*}|/M_S^2$ | $< 1.73 \times 10^{-5}$
$B_d \rightarrow \tau^\pm \tau^\mp$ | $|\lambda^{33}\lambda^{13*}|/M_S^2$ | $< 1.28 \times 10^{-6}$
$B_s \rightarrow e^\pm e^\mp$ | $|\lambda^{31}\lambda^{21*}|/M_S^2$ | $< 2.54 \times 10^{-5}$
$B_s \rightarrow \tau^\pm \tau^\mp$ | $|\lambda^{33}\lambda^{23*}|/M_S^2$ | $< 1.2 \times 10^{-8}$

TABLE II: Constraints obtained from the leptoquark couplings from various leptonic $B_{d,s} \rightarrow l^+l^-$ decays.

For the analysis of $B \rightarrow Ke^+e^-$ process, we need to know the values of the leptoquark couplings $\lambda^{31}\lambda^{21*}/M_S^2$, which can be extracted from the inclusive decay rates $B \rightarrow X_se^+e^-$. To obtain such constraints, we closely follow the procedure adopted in Ref. [18]. Using the SM predictions and the corresponding experimental measurements from [30] for both low-$q^2$ ($1-6)GeV^2$ and high-$q^2$ ($>14.2GeV^2$) as

$$BR(B \rightarrow X_see)|_{q^2\in[1,6]GeV^2} = (1.73 \pm 0.12) \times 10^{-6} \text{ (SM prediction)}$$

$$= (1.93 \pm 0.55) \times 10^{-6} \text{ (Expt.)}$$

$$BR(B \rightarrow X_see)|_{q^2>14.2 GeV^2} = (0.2 \pm 0.06) \times 10^{-6} \text{ (SM prediction)}$$

$$= (0.56 \pm 0.19) \times 10^{-6} \text{ (Expt.)}$$

(24)

In Fig-2, we show the allowed region in $C'_{10}^{NP} - \phi^{NP}$ parameter space due to exchange of the leptoquark $X(3,2,7/6)$ in the left panel. The right panel depicts the allowed region in the $C_9^{NP} - C'_{10}^{NP}$ space due to exchange of the leptoquark $X(3,2,1/6)$, where green (red) region corresponds to high-$q^2$ (low-$q^2$) limits in both the panels. Thus, it can be noticed that the bounds coming from the high-$q^2$ measurements are rather weak. Considering the exchange of the $X(3,2,7/6)$ leptoquark as an example, we obtain the bound on $C_{10}^{NP}$ as $-2.0 \leq C_{10}^{NP} \leq 3.0$ for the entire range of $\phi^{NP}$, which gives the bound on $r$ as

$$0 \leq r \leq 0.7 .$$

(25)

After obtaining the bounds on various leptoquark couplings, we now proceed to study the rare decays $B \rightarrow K/\pi l^+l^-$ and $B \rightarrow l_i^+l_j^-$. 

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FIG. 2: The allowed region in $C_{10}^{NP} - \phi^{NP}$ parameter space (left panel) and $C_{9}^{NP} - C_{10}^{NP}$ (right panel) obtained from $BR(B \rightarrow X_{s}e^{+}e^{-})$, where the green (red) region corresponds to high-$q^{2}$ (low-$q^{2}$) limits.

IV. $B \rightarrow Kl^{+}l^{-}$ PROCESS

We now consider the semileptonic decay process $B \rightarrow Kl^{+}l^{-}$, which is mediated by the quark level transition $b \rightarrow sl^{+}l^{-}$ and hence, it constitutes a quite suitable tool of looking for new physics. The isospin asymmetries of $B \rightarrow K\mu^{+}\mu^{-}$ and the partial branching ratios of the decays $B^{0} \rightarrow K^{0}\mu^{+}\mu^{-}$ and $B^{+} \rightarrow K^{+}\mu^{+}\mu^{-}$ are recently measured as functions of the dimuon mass squared ($q^{2}$) by the LHCb collaboration [31]. In this paper we will study the process in the large recoil region i.e., $1 \leq q^{2} \leq 6 \text{ GeV}^{2}$, in order to be well below the radiative tail of the charmonium resonances, using the QCD factorization approach [32–34]. LHCb has measured the branching ratio in this region and the updated result is [31]

$$BR(B^{+} \rightarrow K^{+}\mu^{+}\mu^{-})|_{q^{2} \in [1,6] \text{ GeV}^{2}} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}. \quad (26)$$

This mode has also been analyzed by various authors [35–37] and the SM predictions is given as

$$BR(B^{+} \rightarrow K^{+}\mu^{+}\mu^{-})|^{SM}_{q^{2} \in [1,6] \text{ GeV}^{2}} = (1.75^{+0.60}_{-0.29}) \times 10^{-7}. \quad (27)$$

Although, there is no significant discrepancy between these two results, the SM predictions is slightly higher than the experimental measurement.

To calculate the branching ratio, one use the effective Hamiltonian presented in Eq. (3) and obtain the transition amplitude for this process. The matrix elements of the various
hadronic currents between the initial $B$ meson and the final $K$ meson can be parameterized in terms of the form factors $f_+, f_0$ and $f_T$ as \cite{8, 38}

\begin{equation}
\langle K(p_K)|\bar{s}\gamma_\mu b|\bar{B}(p_B)\rangle = (2p_B - q)_\mu f_+(q^2) + \frac{M_B^2 - M_K^2}{q^2}q_\mu[f_0(q^2) - f_+(q^2)]
\end{equation}

\begin{equation}
\langle K(p_K)|\bar{s}i\sigma_{\mu\nu}q^\nu b|\bar{B}(p_B)\rangle = -[(2p_B - q)_\mu q^2 - (M_B^2 - M_K^2)q_\mu] \frac{f_T(q^2)}{M_B + M_K},
\end{equation}

where the 4-momenta of the initial $B$-meson and the final kaon are denoted by $p_B$ and $p_K$ respectively, $M_B, M_K$ are the corresponding masses and $q^2$ is the momentum transfer. In the large recoil region, the energy of the kaon $E_K$ is large compared to the typical size of hadronic binding energies ($\Lambda_{QCD}$) and the dilepton invariant mass squared $q^2$ is low. As a consequence, in this region the virtual photon exchange between the hadronic part and the dilepton pair and the hard gluon scattering can be treated in an expansion in $1/E_K$ \cite{8}, using either QCD factorization \cite{33, 34} or Soft Collinear Effective Theory (SCET) \cite{39}.

At the leading order in $1/E_K$ expansion, all the form factors $f_{+, 0, T}(q^2)$ can be related to a single form factor $\xi_P(q^2)$. Within QCDF approach, the form factor $f_+(q^2)$ is chosen to be $f_+(q^2) = \xi_P(q^2)$ and including subleading corrections, the other form factors can written as \cite{8}

\begin{equation}
\frac{f_0}{f_+} = \frac{2E_K}{M_B} \left[ 1 + O(\alpha_s) + O \left( \frac{q^2}{M_B^2 \sqrt{\Lambda_{QCD}/E_K}} \right) \right],
\end{equation}

\begin{equation}
\frac{f_T}{f_+} = \frac{M_B + M_K}{M_B} \left[ 1 + O(\alpha_s) + O \left( \sqrt{\Lambda_{QCD}/E_K} \right) \right].
\end{equation}

Thus, only one soft form factor $\xi_P(q^2)$ appears in the $B \to K$ transition amplitude due to the symmetry relations in the large energy limit of QCD \cite{32, 40} and the transition amplitude for the process $B \to Kl^+l^-$ can be written as \cite{8}

\begin{equation}
\mathcal{M}(\bar{B} \to Kl) = \frac{iG_F\alpha}{\sqrt{2}\pi} V_{td} V_{ts}^* \xi_P(q^2) \left[ F_V p_B^\mu(\bar{l}\gamma_\mu l) + F_A p_B^\mu(\bar{l}\gamma_\mu\gamma_5 l) + F_P(\bar{l}\gamma_5 l) \right],
\end{equation}

The functions $F_{V, A, P}(q^2)$ are given as

\begin{equation}
F_V = C_9 + \frac{2m_bT_P(q^2)}{M_B \xi_P(q^2)}
\end{equation}

\begin{equation}
F_A = C_{10}
\end{equation}

\begin{equation}
F_P = m_t C_{10} \left[ \frac{M_B^2 - M_K^2}{q^2} \left( \frac{f_0(q^2)}{f_+(q^2)} - 1 \right) - 1 \right].
\end{equation}

\[11\]
The parameter \( T_P(q^2) \) takes into account the virtual one-photon exchange between the hadron and the lepton pair and hard scattering contribution. At lowest order, it can be expressed as

\[
T_P^{(0)}(q^2) = \xi_P(q^2) \left( C_\gamma^{\text{eff}(0)} + \frac{M_B}{2m_b} Y^{(0)}(q^2) \right). 
\]  

(33)

The function \( Y(q^2) \) denotes the perturbative part coming from one loop matrix elements of the four quark operators and is given in Ref. [33]. The detailed expression for \( T_P(q^2) \), including the subleading corrections is presented in Appendix A.

With Eq. (31), the double differential decay rate with respect to \( q^2 \) and \( \cos \theta \) for the lepton flavor \( l \) is given as

\[
\frac{d^2 \Gamma_l}{dq^2 d \cos \theta} = a_l(q^2) + c_l(q^2) \cos^2 \theta, 
\]

(34)

where

\[
a_l(q^2) = \Gamma_0 \sqrt{\lambda \beta_l} \xi_P^2 \left[ q^2 |F_P|^2 + \frac{\lambda}{4} (|F_A|^2 + |F_V|^2) + 2m_l (M_B^2 - M_K^2 + q^2) \text{Re}(F_P F_A^* + 4m_l^2 M_B^2 |F_A|^2) \right],
\]

\[
c_l(q^2) = -\Gamma_0 \sqrt{\lambda \beta_l} \xi_P^2 \frac{\lambda}{4} \left[ |F_A|^2 + |F_V|^2 \right],
\]

(35)

\[
\lambda = M_B^2 + M_K^2 + q^4 - 2(M_B^2 M_K^2 + M_B^2 q^2 + M_K^2 q^2), \quad \beta_l = \sqrt{1 - 4m_l^2/q^2},
\]

(36)

and

\[
\Gamma_0 = \frac{G_F^2 \alpha^2 |V_{tb} V_{ts}^*|^2}{2^{9/2} \pi^5 M_B^3}. 
\]

(37)

The decay rate can be expressed as

\[
\Gamma_l = 2 \left( A_l + \frac{1}{3} C_l \right), 
\]

(38)

where the \( q^2 \)-integrated coefficients are given as

\[
A_l = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 a_l(q^2), \quad C_l = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 c_l(q^2). 
\]

(39)

The observable \( R_K \) which is the ratio of \( B \to K \mu^+ \mu^- \) to \( B \to Ke^+e^- \) decay rates with same \( q^2 \) cuts is

\[
R_K \equiv \frac{\Gamma_\mu}{\Gamma_e} = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 d\Gamma_\mu / \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 d\Gamma_e. 
\]

(40)
which probes the lepton flavour non-universality effects.

Another observable, which can be constructed from ratios or asymmetries where the leading form factor uncertainties cancel. The CP-averaged isospin asymmetry is such an observable which is defined as

\[ A_I(q^2) = \frac{d\Gamma(B^0 \rightarrow K^0 \mu^+ \mu^-)/dq^2 - d\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2}{d\Gamma(B^0 \rightarrow K^0 \mu^+ \mu^-)/dq^2 + d\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2} \]

\[ = \frac{d\text{BR}(B^0 \rightarrow K^0 \mu^+ \mu^-)/dq^2 - (\tau_0/\tau_+)}{d\text{BR}(B^0 \rightarrow K^0 \mu^+ \mu^-)/dq^2 + (\tau_0/\tau_+)} \frac{d\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2}{d\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2}. \] (41)

With these formulae at hand, we now proceed for numerical estimation. To make predictions for SM observables, or to extract information about potentially new short distance physics, one should require the knowledge of associated hadronic form factors. For this purpose we use value of the form factors \( f_+(q^2) = \xi_P(q^2) \) calculated in the light-cone sum rule (LCSR) approach [38], where the \( q^2 \) dependence is given by simple fits as

\[ f_+(q^2) = \frac{r_1}{1 - s/m_{fit}^2} + \frac{r_2}{(1 - s/m_{fit}^2)^2}, \] (42)

and the values of the parameters used are taken from [38]. The particle masses and the lifetime of \( B_s \) meson, the decay constants are taken from [29] and the SM Wilson coefficients \( C_i \)’s are listed in Table-1. For the CKM matrix elements we use the Wolfenstein parametrization with the values \( A = 0.814^{+0.023}_{-0.024}, \lambda = 0.22537 \pm 0.00061, \bar{\rho} = 0.117 \pm 0.021 \) and \( \bar{\eta} = 0.353 \pm 0.013 \) [29]. The values of the quark masses used in our analysis are as follows. The \( b \)-quark mass used in \( T_P \) and \( F_V \) is the potential subtracted (PS) mass \( m_{PS}(\mu_f) \) at the factorization scale \( \mu_f \sim \sqrt{\Lambda_{QCD}m_b} \) and is denoted by \( m_b \). The function \( Y(q^2) \) is evaluated by using the pole mass \( m_{b_{pole}} \) and its relation to the PS mass is given as \( m_{b_{pole}} = m_{b_{PS}(\mu_f)} + 4\alpha_s\mu_f/3\pi \). The quark masses (in GeV) used are \( m_b = 4.6, m_c = 1.4 \), the fine structure coupling constant \( \alpha = 1/130 \). Using these values we show in Fig.-3 the variation of differential branching ratio for \( B^0 \rightarrow K^0 \mu^+ \mu^- \) (left panel) and the \( B^+ \rightarrow K^+ \mu^+ \mu^- \) (right panel) in the standard model with respect to the di-muon invariant mass. The variation of isospin asymmetry and \( R_K \) are shown in Fig- 4. The total branching ratios integrated over the range \( q^2 \in [1, 6] \text{GeV}^2 \) are summarized in Table-3. Our predictions are in agreement with the previous predictions [35], and the slight difference can be attributed to the difference in the values of input parameters used in the calculation. But these predictions are slightly larger than the experimental values.
FIG. 3: The variation of branching ratios with $q^2$ for the decay processes $B^0 \rightarrow K^0 \mu^+ \mu^-$ (left panel) and $B^+ \rightarrow K^+ \mu^+ \mu^-$ (right panel) in standard model and in leptoquark model.

FIG. 4: The variation of isospin asymmetry (left panel) for $B \rightarrow K \mu \mu$ and $R_K$ (right panel) with $q^2$.

In the leptoquark model, these processes will receive additional contributions arising from the leptoquark exchange and hence, the Wilson coefficients $C_{9,10}$ will receive additional contributions $C_{9,10}^{NP}$ as well as new Wilson $C'_{9,10}$ associated with the chirally flipped operators $O_{9,10}'$ will also be present as already discussed in Section II. The bounds on these new Wilson coefficients can be obtained from the constraint on $r$ [22] which has been extracted from the experimental results on BR($B_s \rightarrow \mu^+ \mu^-$). Thus, for the leptoquarks $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$, we obtain the value of $r \lesssim 0.35$ for $\pi/2 \leq \phi^{NP} \leq 3\pi/2$, which can be
translated with eqns (7), (10) and (17) to give the value of the new Wilson coefficients as

\[ |C_{9}^{LQ}| = |C_{10}^{LQ}| \leq |r C_{10}^{SM}| \quad \text{(for } X = (3, 2, 7/6)) \]

\[ |C_{9}^{\prime LQ}| = |C_{10}^{\prime LQ}| \leq |r C_{10}^{SM}| \quad \text{(for } X = (3, 2, 1/6)) \] \hspace{1cm} (43)

| Observables | SM Predictions | Values in LQ model |
|-------------|----------------|-------------------|
| \(B_d \to K^0 \mu^+ \mu^-\) | 1.82 \times 10^{-7} | (2.04 - 2.16) \times 10^{-7} |
| \(B^+ \to K^+ \mu^+ \mu^-\) | 1.99 \times 10^{-7} | (2.2 - 2.3) \times 10^{-7} |
| \(B^+ \to K^+ e^+ e^-\) | 1.82 \times 10^{-7} | (2.3 - 3.7) \times 10^{-7} |
| \langle A_I \rangle | -0.03 | -0.036 \to -0.024 |
| \langle R_K \rangle | 1.09 | 0.62 - 0.96 |

TABLE III: The predicted values for the integrated branching ratio and isospin asymmetry in the range \(q^2 \in [1, 6] \text{GeV}^2\) for the decay mode \(B \to K \mu \mu\) and the value of \(R_K\).

Using these values we show the variation of differential branching ratio and isospin asymmetry and \(R_K\) for \(X = (3, 2, 7/6)\) in Figs.-3 and 4. For the calculation of the \(B^+ \to K^+ e^+ e^-\) in the determination of \(R_K\), we have used the constraint on the leptoquark couplings obtained from \(B \to X_s ee\) inclusive decay rate. From these figures it can be seen that there is slight deviation in \(B \to K \mu \mu\) branching ratios from their SM values. The isospin asymmetry also has slight deviation from its SM prediction and this deviation is substantial in the low \(-q^2\) region. However, the \(R_K\) value deviates significantly and it is possible to accommodate the observed experimental value in the leptoquark model. The integrated branching ratios and the isospin asymmetries are presented in Table-III.

V. \(B \to \pi ll\) PROCESS

In this section we would like to study the decay mode \(B \to \pi \mu^+ \mu^-\) which is mediated by the quark level transition \(b \to d l^+ l^-\). This decay mode has been recently observed by the LHCb \[41\] collaboration and the measured branching ratio is

\[ \text{BR}(B^+ \to \pi^+ \mu^+ \mu^-) = (2.3 \pm 0.6 \text{ (stat)} \pm 0.1 \text{ (syst)}) \times 10^{-8}, \] \hspace{1cm} (44)

at 5.2\(\sigma\) significance.
For the calculation of branching ratio, we closely follow [17] and here we preview only the main results. In the standard model the effective Hamiltonian for $b \to d l^+ l^-$ transition is given by

$$H_{eff} = \frac{-G_F}{\sqrt{2}} \left[ \lambda_t^{(d)} H_{eff}^{(t)} + \lambda_u^{(d)} H_{eff}^{(u)} \right] + h.c.$$  \hfill (45)

where $\lambda_q^{(d)} = V_{qb} V_{qd}^*$ and

$$H_{eff}^{(u)} = C_1 (O_1^c - O_1^u) + C_u (O_2^c - O_2^u)$$

$$H_{eff}^{(t)} = C_1 O_1^c + C_2 Q_2^c + \sum_{i=3}^{10} C_i O_i.$$  \hfill (46)

It should be noted that for $b \to d l^+ l^-$ transitions $\lambda_u^{(d)}$ and $\lambda_t^{(d)}$ are comparable in magnitude with the phase difference $\phi_2 = \arg (-V_{td} V_{tb}^*/V_{ud} V_{ub}^*)$. Thus, one can write the transition amplitude for the process $B \to \pi l^+ l^-$ as

$$M(B(p) \to \pi(p') l^+ l^-) = \frac{G_F \alpha}{2 \sqrt{2} \pi} c_\pi^{-1} \xi_\pi \left[ \lambda_t C_{9,\pi}^{(t)} + \lambda_u C_{9,\pi}^{(u)} (p + p')^\mu (\bar{l} \gamma_\mu l) ight] + \lambda_t C_{10} (p + p')^\mu (\bar{l} \gamma_\mu \gamma_5 l),$$  \hfill (47)

where $c_\pi = 1/\sqrt{2}$ for $\pi^0$ and 1 for $\pi^\pm$ and

$$C_{9,\pi}^{(t)}(q^2) = C_9 + \frac{2m_b T_\pi^{(t)}(q^2)}{M_B \xi_\pi(q^2)}$$

$$C_{9,\pi}^{(u)}(q^2) = \frac{2m_b T_\pi^{(u)}(q^2)}{M_B \xi_\pi(q^2)}.$$  \hfill (48)

The differential branching ratio is given as

$$\frac{dBR}{d\xi^2}(B \to \pi l^+ l^-) = S_\pi \tau_B \frac{G_F^2 M_B^3}{96 \pi^3} \left( \frac{\alpha}{4 \pi} \right)^2 \lambda_\pi^3 \xi_\pi(q^2)^2 \left| \lambda_l \right|^2$$

$$\times \left( \left| C_{9,\pi}^{(t)}(q^2) - R_{\text{ut}} e^{i\phi_2} C_{9,\pi}^{(u)}(q^2) \right|^2 + \left| C_{10} \right|^2 \right),$$  \hfill (49)

where

$$\lambda_\pi(q^2, m_\pi^2) = \left[ \left( 1 - \frac{q^2}{M_B^2} \right)^2 - \frac{2m_b^2}{M_B^2} \left( 1 + \frac{q^2}{M_B^2} \right) + \frac{m_\pi^4}{M_B^2} \right]^{1/2}.$$  \hfill (50)

with $S_\pi = 1/c_\pi^2$ and $\lambda_u^{(d)}/\lambda_t^{(d)} = -R_{\text{ut}} e^{i\phi_2}$. The branching ratio for the CP conjugate mode can be obtained by changing the sign of the weak phase $\phi_2$. One can then define the $q^2$.
dependence of the direct CP asymmetries as

\[
A_{CP}^+(q^2) = \frac{d BR(B^+ \to \pi^- ll)/dq^2 - d BR(B^+ \to \pi^+ ll)/dq^2}{d BR(B^- \to \pi^- ll)/dq^2 + d BR(B^+ \to \pi^+ ll)/dq^2}
\]

\[
A_{CP}^0(q^2) = \frac{d BR(B^0 \to \pi^0 ll)/dq^2 - d BR(B^0 \to \pi^0 ll)/dq^2}{d BR(B^0 \to \pi^0 ll)/dq^2 + d BR(B^0 \to \pi^0 ll)/dq^2}.
\] (51)

The $q^2$ dependent isospin asymmetry is defined as

\[
A_I(q^2) = \frac{\tau_{B^0}}{2\tau_{B^\pm}} \frac{d BR(B^+ \to \pi^+ l^+ l^-)/dq^2}{d BR(B^0 \to \pi^0 l^+ l^-)/dq^2} - 1.
\] (52)

where $BR$ is the CP averaged branching ratio.

The $B \to \pi$ form factor can be obtained using the light-cone QCD sum rule approach

\[
\xi_\pi(q^2) = \frac{\xi_\pi(0)}{(1 - q^2/m_{B^\pm}^2)(1 - \alpha_{BK}q^2/m_{B^0}^2)},
\] (53)

where the numerical value for the normalization constant is $\xi_\pi(0) = 0.26^{+0.04}_{-0.03}$ and the slope parameter $\alpha_{BK} = 0.53 \pm 0.06$. The light cone distribution amplitude is given by

\[
\phi_\pi(u) = 6u(1-u) \left[ 1 + a_2^\pi C_2^{(3/2)}(2u - 1) + a_4^\pi C_4^{(3/2)}(2u - 1) + \cdots \right],
\] (54)

where $C_n^{3/2}(x)$ are Gegenbauer polynomials and the coefficients $a_n^\pi$ are related to the moments of distribution amplitudes (DAs). The numerical values of these coefficients are $a_2^\pi = 0.25 \pm 0.15$, $a_4^\pi = 0.1 \pm 0.1$ as given in Refs. [17, 38].

The $B$ meson light cone distribution amplitudes can be given as

\[
\Phi_{B,+}(\omega) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}, \quad \Phi_{B,-}(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0}
\] (55)

with $\omega_0 = 2\bar{\Lambda}_{HQET}/3$ and $\bar{\Lambda}_{HQET} = m_B - m_b$. These enter only through the moments

\[
\lambda_{B,+}^{-1} = \int_0^\infty d\omega \frac{\Phi_{B,+}(\omega)}{\omega} = \omega_0^{-1},
\] (56)

\[
\lambda_{B,-}^{-1}(q^2) = \int_0^\infty d\omega \frac{\Phi_{B,-}(\omega)}{\omega - q^2/M_B - i\epsilon} = \frac{e^{-q^2/M_B\omega_0}}{\omega_0} \left[ -Ei(q^2/M_B\omega_0) + i\pi \right],
\] (57)

where $Ei(z)$ is the exponential integral function. Using these formulae, we show in Fig. 5 the differential branching ratios for $B^0 \to \pi^0 \mu^+ \mu^-$ (left panel) and $B^- \to \pi^- \mu^+ \mu^-$ (right panel) both in the SM and in leptoquark model, for which we have used the constraints on leptoquark couplings as extracted from $B_d \to \mu^+ \mu^-$. In this case the branching ratios in
the leptoquark model have significant deviations from their corresponding SM values. The integrated branching ratios in the range $q^2 \in [1, 6] GeV^2$ are presented in Table-IV. Similarly the variation of CP asymmetries are shown in Fig-6 and the CP asymmetries averaged over $q^2$ range are given in Table-IV. The variation of isospin asymmetry is shown in the left panel of Fig. 7.

| Observables       | SM Predictions | Values in LQ model |
|--------------------|----------------|--------------------|
| $B_d \to \pi^0 \mu^+\mu^-$ | $2.6 \times 10^{-9}$ | $(3.2 - 3.4) \times 10^{-9}$ |
| $B^+ \to \pi^+ \mu^+\mu^-$ | $5.6 \times 10^{-9}$ | $(7.2 - 7.3) \times 10^{-9}$ |
| $\langle A^0_{CP} \rangle$ | $-0.103$ | $-0.04 \to -0.065$ |
| $\langle A^+_{CP} \rangle$ | $-0.268$ | $-0.11 \to -0.06$ |
| $\langle A_I \rangle$ | $0.078$ | $0.04 - 0.07$ |

TABLE IV: The predicted branching ratios, $q^2$ averaged CP asymmetries and the isospin asymmetry for $B \to \pi \mu^+\mu^-$ process.

The ratio of branching ratios of $B^+ \to \pi^+ \mu^+\mu^-$ and $B^+ \to K^+ \mu^+\mu^-$ has recently been measured by LHCb experiment [41] as

$$\frac{BR(B^+ \to \pi^+ \mu^+\mu^-)}{BR(B^+ \to K^+ \mu^+\mu^-)} = 0.053 \pm 0.014(\text{stat}) \pm 0.001(\text{syst})$$  \hspace{1cm} (58)

We define

$$R_+(q^2) = \frac{dBR(B^+ \to \pi^+ ll)/dq^2}{dBR(B^+ \to K^+ ll)/dq^2}$$  \hspace{1cm} (59)

and show the variation of $R_+(q^2)$ with dimuon invariant mass in the right panel of Fig. 7.

VI. LEPTON FLAVOUR VIOLATING DECAYS $B_{s,d} \to l_i^+ l_j^-$

It is very well known that in the standard model the family lepton numbers ($L_e$, $L_\mu$, $L_\tau$) are exactly conserved. However, the experimental observation of neutrino oscillation implies that the family lepton numbers are no longer conserved quantum numbers and must be violated. Due to the violation of these lepton numbers, flavour changing neutral current (FCNC) processes in the lepton sector could in principle occur, analogous to the quark sector.
Some examples of FCNC transitions in the lepton sector are: \( l_i \rightarrow l_j \gamma \), \( l_i \rightarrow l_j l_k \bar{l}_k \), \( B \rightarrow l_i \bar{l}_j \), etc., where \( l_i \) is any charged lepton. Although there is no direct conclusive experimental evidence for such processes that have been observed so far, but there exist severe constraints on some of these lepton flavour violation (LFV) decay modes \[29\]. The LFV decays are well studied in the literature in various beyond the standard model scenarios. Here we would like to investigate the effect of scalar leptoquarks in predicting the branching ratios for the LFV decays \( B_{s,d} \rightarrow l_i^+ l_j^- \). These decay modes are previously investigated in \[42\].

The effective Hamiltonian for \( B_{s,d} \rightarrow l_i^+ l_j^- \) process will have similar structure analogous
FIG. 7: The variation of isospin asymmetry for $B \to \pi \mu^+ \mu^-$ process and $R_+(q^2)$ both in SM and LQ model.

to $B_{s,d} \to l^+ l^-$, which is given in the leptoquark model as

$$H_{LQ} = \left[ G_V \left( \bar{s} \gamma^\mu P_L b \right) \bar{l}_i \gamma_\mu l_j + G_A \left( \bar{s} \gamma^\mu P_L b \right) \bar{l}_i \gamma_\mu \gamma_5 l_j \right],$$

(60)

where the constants $G_V$ and $G_A$ are given as

$$G_V = G_A = \frac{\lambda^{j3} \lambda^{i2} + \lambda^{j2} \lambda^{i3}}{8 M_Y^2}.$$  

(61)

Here we have considered the exchange of the leptoquark as $X(3,2,7/6)$ and for $X(3,2,1/6)$, one will have the chirality-flipped operators.

This gives the branching ratio as

$$\text{BR}(B_{s,d} \to l_i^+ l_j) = \frac{|p|}{4\pi m_B^2} |F_{V,B_{s,d}}|^2 \left[ (m_j - m_i)^2 \left( m_B^2 - (m_i + m_j)^2 \right) + (m_j + m_i)^2 \left( m_B^2 - (m_i - m_j)^2 \right) \right]$$

(62)

where

$$|p| = \frac{\sqrt{(m_B^2 - m_i^2 - m_j^2)^2 - 4m_i^2m_j^2}}{2m_B}.$$  

(63)

is the center-of-mass momentum of the outgoing leptons in the initial $B_{s,d}$ rest frame.

For numerical estimation we need to know the values of the different couplings involved in the expression for branching ratio. Assuming the leptoquarks to have full strength coupling to a lepton and a quark of the same generation and its coupling with the quarks and
leptons of different generations are assumed to be Cabibbo suppressed. We use the values of these couplings extracted from the leptonic decays $B_{s,d} \rightarrow \mu^+\mu^-$ as the benchmark values and determine the other required couplings assuming that the couplings between different generation of quarks and leptons follow the simple scaling law, i.e. $\lambda^{ij} = (m_i/m_j)^{1/4} \lambda^{ii}$ with $j > i$. This assumption follows from the fact that in the quark sector the expansion parameter of the CKM matrix in the Wolfenstein parameterization can be related to the down type quark masses as $\lambda \sim \sqrt{m_d/m_s}$ where as in the lepton sector one can have the same order for $\lambda$ with the relation $\lambda \sim (m_e/m_\mu)^{1/4}$. With this simple ansatz, the predicted values of the branching ratios for various LFV decays are listed in Table-V, which are consistent with present experimental upper limits [29].

| Decay Process | Couplings involved | Predicted BR | Expt. Upper limit [29] |
|---------------|--------------------|--------------|------------------------|
| $B_d \rightarrow \mu^\pm e^\mp$ | $\frac{\lambda^{31}\lambda^{12}}{M_S^{2}}$, $\frac{\lambda^{32}\lambda^{11}}{M_S^{2}}$ | $(9.5 \times 10^{-13} - 6.4 \times 10^{-12})$, $(2.0 \times 10^{-10} - 1.3 \times 10^{-9})$ | $< 2.8 \times 10^{-9}$ |
| $B_d \rightarrow \mu^\pm \tau^\mp$ | $\frac{\lambda^{32}\lambda^{13}}{M_S^{2}}$, $\frac{\lambda^{33}\lambda^{12}}{M_S^{2}}$ | $(7.5 \times 10^{-10} - 5.1 \times 10^{-9})$, $(1.3 \times 10^{-8} - 8.5 \times 10^{-8})$ | $< 2.2 \times 10^{-5}$ |
| $B_d \rightarrow e^\pm \tau^\mp$ | $\frac{\lambda^{31}\lambda^{13}}{M_S^{2}}$, $\frac{\lambda^{33}\lambda^{11}}{M_S^{2}}$ | $(5.2 \times 10^{-11} - 3.5 \times 10^{-10})$, $(1.8 \times 10^{-7} - 1.2 \times 10^{-6})$ | $< 2.8 \times 10^{-5}$ |
| $B_s \rightarrow \mu^\pm e^\mp$ | $\frac{\lambda^{32}\lambda^{21}}{M_S^{2}}$, $\frac{\lambda^{31}\lambda^{22}}{M_S^{2}}$ | $< 1.5 \times 10^{-11}$, $< 3.2 \times 10^{-9}$ | $< 1.1 \times 10^{-8}$ |
| $B_s \rightarrow \mu^\pm \tau^\mp$ | $\frac{\lambda^{32}\lambda^{23}}{M_S^{2}}$, $\frac{\lambda^{33}\lambda^{22}}{M_S^{2}}$ | $< 1.2 \times 10^{-8}$, $< 2.0 \times 10^{-7}$ | $-$ |
| $B_s \rightarrow e^\pm \tau^\mp$ | $\frac{\lambda^{31}\lambda^{23}}{M_S^{2}}$, $\frac{\lambda^{33}\lambda^{21}}{M_S^{2}}$ | $< 8.5 \times 10^{-10}$, $< 2.9 \times 10^{-6}$ | $-$ |

TABLE V: The predicted branching ratios for various lepton flavor violating $B_{s,d}$ decays.

VII. CONCLUSION

In this paper we have studied the effect of the scalar leptoquarks in the rare decays of $B \rightarrow Kl^+l^-$, $B \rightarrow \pi\mu^+\mu^-$ and the lepton flavour violating decays $B \rightarrow l_i^+l_j^-$. We have
considered the simple renormalizable leptoquark models in which proton decay is prohibited at the tree level. The leptoquark parameter space has been constrained using the recent measurements on BR($B_{s,d} \rightarrow \mu^+\mu^-$) and the value of BR($\bar{B}_d^0 \rightarrow X_s e^+e^-$). Using such parameters we obtained the bounds on the product of leptoquark couplings and then estimated the branching ratios, isospin asymmetries for $B \rightarrow K\mu^+\mu^-$ process. We found that the observed anomaly of $R_K$ can be explained in the leptoquark model. This is because the couplings of leptoquarks are family dependent and one can have lepton flavour interaction in this model. For $B \rightarrow \pi\mu^+\mu^-$, we have studied the effect of leptoquarks on branching ratios, CP asymmetry parameters, isospin asymmetry parameter $A_I$ and $R_+$ parameter which corresponds to the ratio of the branching ratios of $B^+ \rightarrow \pi^+\mu^+\mu^-$ to $B^+ \rightarrow K^+\mu^+\mu^-$. For $B \rightarrow \pi\mu^+\mu^-$ decays, these observables deviate significantly from their corresponding SM values. We have also obtained the branching ratios for various lepton flavour violating decays $B \rightarrow l_i^+l_j^-$. Some of these decay modes, e.g., $B_d \rightarrow \mu^+\tau^-$ are expected to have branching ratios which are within the reach of LHCb, the observation of which would provide the hints for possible existence of leptoquarks.

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Appendix A: Amplitude for $B \rightarrow P\gamma^*$ process

Here we will present the expressions for $B \rightarrow P\gamma^*$ amplitudes from Refs. [33, 34]. Including corrections $O(\alpha_s)$, the $B \rightarrow P\gamma^*$ amplitude in the heavy quark limit is given by

$$T_P^{(i)} = \xi_P C_P^{(i)} + \zeta_P \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \int_0^1 du \phi_P(u) T_P^{(i)}(u, \omega),$$

where

$$\zeta_P = \frac{\pi^2 f_B f_P}{N_C M_B}.$$  \hfill (A2)

The expressions for the coefficient functions $C_P^{(i)}$ and $T_P^{(i)}$ are given as

$$C_P^{(i)} = C_P^{(0,i)} + \frac{\alpha_s C_F}{4\pi} C_P^{(1,i)},$$

$$T_P^{(i)}(u, \omega) = T_P^{(0,i)}(u, \omega) + \frac{\alpha_s C_F}{4\pi} T_P^{(1,i)}(u, \omega),$$

22
and \( i = t, u \). The \( B \to P \gamma^* \) amplitude can be related to the \( B \to V \gamma^* \) amplitude as

\[
C_P^{(i)} = -C_{\parallel}^{(i)}, \quad T_{P,\pm}^{(i)}(u, \omega) = -T_{\parallel,\pm}^{(i)}(u, \omega),
\]

(A5)

The expressions for \( \bar{B} \to P \gamma^* \) amplitudes are

\[
T_{P}^{(t)} = \xi_P \left( C_{P}^{(0,t)} + \frac{\alpha_s C_F}{4\pi} \left[ C_{P}^{(f,t)} + C_{P}^{(n,f,t)} \right] \right) + \zeta_P \lambda_{B,-}^{-1} \int du\phi_P(u) \hat{T}_{P,-}^{(0,f)} + \frac{\alpha_s C_F}{4\pi} \xi_P \left( \lambda_{B,+}^{-1} \int du\phi_P(u) \left[ T_{P,+}^{(f,t)}(u) + T_{P,+}^{(n,f,t)}(u) \right] \right) + \lambda_{B,-}^{-1} \int du\phi_P(u) \hat{T}_{P,-}^{(n,f,t)}(u) \right). \tag{A6}
\]

\[
T_{P}^{(u)} = \xi_P \left( C_{P}^{(0,u)} + \frac{\alpha_s C_F}{4\pi} \left[ C_{P}^{(n,f,u)} \right] \right) + \zeta_P \lambda_{B,-}^{-1} \int du\phi_P(u) \hat{T}_{P,-}^{(0,u)} + \frac{\alpha_s C_F}{4\pi} \xi_P \left( \lambda_{B,+}^{-1} \int du\phi_P(u) T_{P,+}^{(n,f,u)}(u) + \lambda_{B,-}^{-1} \int du\phi_P(u) \hat{T}_{P,-}^{(n,f,u)}(u) \right). \tag{A7}
\]

where use has been made

\[
T_{P,-}^{(0,i)}(u, \omega) = \frac{M_B \omega}{M_B \omega - q^2 \mp i\epsilon} \hat{T}_{P,-}^{(0,i)},
\]
\[
T_{P,-}^{(n,f,i)}(u, \omega) = \frac{M_B \omega}{M_B \omega - q^2 \mp i\epsilon} \hat{T}_{P,-}^{(n,f,i)}(u). \tag{A8}
\]

The form factor terms including \( \mathcal{O}(\alpha_s^0) \) contributions are

\[
C_{P}^{(0,t)} = C_{\gamma}^{eff} + \frac{M_B}{2m_b} Y(q^2), \quad C_{P}^{(0,u)} = \frac{M_B}{2m_b} Y^{(u)}(q^2), \tag{A9}
\]

where

\[
Y^{(u)}(q^2) = \left( \frac{4}{3} C_1 + C_2 \right) [h(s, \mu_c) - h(s, 0)]. \tag{A10}
\]

The first order corrections \( C_{P}^{(1,i)} \) are divided into a factorizable and a non-factorizable term and can be written as

\[
C_{P}^{(1,i)} = C_{P}^{(f,i)} + C_{P}^{(n,f,i)}. \tag{A11}
\]

The factorizable and nonfactorizable terms including \( \mathcal{O}(\alpha_s) \) corrections are

\[
C_{P}^{(f,t)} = \left( \ln \frac{m_b^2}{\mu^2} + 2L + \Delta M \right) C_{\gamma}^{eff}, \tag{A12}
\]

where \( L \) and \( \Delta M \) are defined in [33, 34].

\[
C_{P}C_{P}^{(n,f,t)} = -\bar{C}_2 F_2^{(7)} - C_8^{eff} F_8^{(7)} - \frac{M_B}{2m_b} \left[ \bar{C}_2 F_2^{(9)} + 2\bar{C}_1 \left( F_1^{(9)} + \frac{1}{6} F_2^{(9)} \right) + C_8^{eff} F_8^{(9)} \right]. \tag{A13}
\]
\[ C_{F}C_{P}^{(nf,u)} = -\bar{C}_{2}\left( F_{2}^{(7)} + F_{2,u}^{(7)} \right) - \frac{M_{B}}{2m_{b}} \left[ \bar{C}_{2}\left( F_{2}^{(9)} + F_{2,u}^{(9)} \right) + 2\bar{C}_{1}\left( F_{1}^{(9)} + F_{1,u}^{(9)} + \frac{1}{6}(F_{2}^{(9)} + F_{2,u}^{(9)}) \right) \right] \]

(A14)

The longitudinal amplitude receives a contribution from weak annihilation topology, where the photon couples to the spectator quark in the B meson and the \( \mathcal{O}(\alpha_{s}^{0}) \) contributions to hard spectator scattering from the weak annihilation diagrams are

\[
\hat{T}_{P,-}^{(0,t)} = e_{q}\frac{4M_{B}}{m_{b}}C_{q}^{34}, \quad \hat{T}_{P,-}^{(0,u)} = -e_{q}\frac{4M_{B}}{m_{b}}C_{q}^{12},
\]

(A15)

where \( e_{q} \) is the charge of the spectator quark \( q = u, d \) in the \( B \) meson and

\[
C_{q}^{34} = C_{3} + \frac{4}{3}(C_{4} + 12C_{5} + 16C_{6}), \quad C_{q}^{12} = 3\delta_{qu}C_{2} - \delta_{qd}\left( \frac{4}{3}C_{1} + C_{2} \right) .
\]

(A16)

The first order corrections can also be divided into a factorizable and a nonfactorizable term as

\[
T_{P,\pm}^{(1,i)} = T_{P,\pm}^{(f,i)} \pm T_{P,\pm}^{(nf,i)} .
\]

(A17)

The factorizable and nonfactorizable terms including \( \mathcal{O}(\alpha_{s}) \) corrections to the hard spectator scattering term are given by

\[
T_{P,\pm}^{(f,i)}(u) = -C_{7}^{eff}\frac{4M_{B}}{\bar{u}E},
\]

(A18)

\[
T_{P,\pm}^{(nf,i)}(u) = -\frac{M_{B}}{m_{b}}\left[ e_{ut}(u,m_{c})(\bar{C}_{2} + C_{4} - C_{6}) + e_{dt}(u,m_{b})(\bar{C}_{3} + C_{4} - C_{6}) + e_{dt}(u,0)\bar{C}_{3} \right] .
\]

(A19)

\[
T_{P,\pm}^{(nf,u)}(u) = -e_{u}\frac{M_{B}}{m_{b}}\left( C_{2} - \frac{1}{6}C_{1} \right) [t_{\parallel}(u,m_{c}) - t_{\parallel}(u,0)] .
\]

(A20)

\[
\hat{T}_{P,-}^{(nf,t)}(u) = -e_{q}\left[ \frac{8C_{8}^{eff}}{\bar{u} + uq^{2}/M_{B}^{2}} + \frac{6M_{B}}{m_{b}}\left\{ h(\bar{u}M_{B}^{2} + uq^{2}, m_{c})(\bar{C}_{2} + \bar{C}_{4} + \bar{C}_{6}) + h(\bar{u}M_{B}^{2} + uq^{2}, m_{b})(\bar{C}_{3} + \bar{C}_{4} + \bar{C}_{6}) + h(\bar{u}M_{B}^{2} + uq^{2}, 0)(\bar{C}_{3} + 3\bar{C}_{4} + 3\bar{C}_{6}) - \frac{8}{27}(\bar{C}_{3} - \bar{C}_{5} - 15\bar{C}_{6}) \right\} \right]
\]

(A21)

\[
\hat{T}_{P,-}^{(nf,t)}(u) = -e_{q}\frac{6M_{B}}{m_{b}}\left( C_{2} - \frac{1}{6}C_{1} \right) \left[ h(\bar{u}M_{B}^{2} + uq^{2}, m_{c}) - h(\bar{u}M_{B}^{2} + uq^{2}, 0) \right] .
\]

(A22)
where $C_i$ (for $i = 1, \cdots, 6$) are defined by

\[
\begin{align*}
\bar{C}_1 &= \frac{1}{2} C_1, \\
\bar{C}_2 &= C_2 - \frac{1}{6} C_1, \\
\bar{C}_3 &= C_3 - \frac{1}{6} C_4 + 16 C_5 - \frac{8}{3} C_6, \\
\bar{C}_4 &= \frac{1}{2} C_4 + 8 C_6, \\
\bar{C}_5 &= C_3 - \frac{1}{6} C_4 + 4 C_5 - \frac{2}{3} C_6, \\
\bar{C}_6 &= \frac{1}{2} C_4 + 2 C_6.
\end{align*}
\] (A23)

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