A Simple Non-Markovian Computational Model of the Statistics of Soccer Leagues: Emergence and Scaling effects

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Abstract

We propose a novel algorithm that outputs the final standings of a soccer league, based on a simple dynamics that mimics a soccer tournament. In our model, a team is created with a defined potential(ability) which is updated during the tournament according to the results of previous games. The updated potential modifies a teams’ future winning/losing probabilities. We show that this evolutionary game is able to reproduce the statistical properties of final standings of actual editions of the Brazilian tournament (Brasileirão). However, other leagues such as the Italian (Calcio) and the Spanish (La Liga) tournaments have notoriously non-Gaussian traces and cannot be straightforwardly reproduced by this evolutionary non-Markovian model. A complete understanding of these phenomena deserves much more attention, but we suggest a simple explanation based on data collected in Brazil: Here several teams were crowned champion in previous editions corroborating that the champion typically emerges from random fluctuations that partly preserves the gaussian traces during the tournament. On the other hand, in the Italian and Spanish leagues only a few teams in recent history have won their league tournaments. These leagues are based on more robust and hierarchical structures established even before the beginning of the tournament. For the sake of completeness, we also elaborate a totally Gaussian model (which equalizes the winning, drawing, and losing probabilities) and we show that the scores of the Brazilian tournament “Brasileirão” cannot be reproduced. This shows that the evolutionary aspects are not superfluous in our modeling and have an important role, which must be considered in other alternative models. Finally, we analyse the distortions of our model in situations where a large number of teams is considered, showing the existence of a transition from a single to a double peaked histogram of the final classification scores. An interesting scaling is presented for different sized tournaments.
1. Introduction

Soccer is an extremely popular and profitable, multi-billion dollar business around the world. Recently, several aspects regarding the sport and associated businesses have been the subject of investigation by the scientific community, including physicists who have devoted some work and time to describe statistics related to soccer. In the literature about soccer models, one can find applications of complex networks [1] and fits with generalized functions [2]; however, they oftentimes have only one focus: goal distribution (see e.g. [3, 4, 5]). Outside the soccer literature, it is important to mention other interesting studies which do not necessarily focus on the scores of the games, such as models that investigate properties of patterns emerging from failure/success processes in sports. In the case of basketball, it has been suggested [6] that the “hot hand” phenomenon (the belief that during a particular period a player’s performance is significantly better than expected on the basis of a player’s overall record), a definitively a non-random pattern, can be modeled by a sequence of random independent trials. Returning to soccer, some authors [7] have devoted attention to the influence of the perceptual-motor bias associated with reading direction in foul judgment by referees.

However, it is interesting to notice that there is a void in the literature: few studies have been carried out under the game theoretic approach of considering the outcome of a tournament from a simple dynamics among the competing teams. In other words, in looking at the statistics that emerge from this complex system called soccer, one can ask if the properties of the distribution of final tournament classification points can be seen as an emerging property of a soccer tournament dynamics established by simple rules among the different competing teams, or how these classification point distributions emerge from a soccer tournament by considering all “combats” among the teams. Here, we propose a model that combines previous studies concerning goal distribution [5] and a game theoretic approach to football tournaments that produces realistic final tournament scores and standings.

In this paper, we explore the statistics of standing points in the end of tournaments disputed according to the “Double Round Robin System” (DRRS) [1] in which the team with the most tournament points at the end of the season is crowned the champion, since many soccer tournament tables around the world are based on this well-known system. In general, 20 teams take part in the

[1] http://en.wikipedia.org/wiki/Round-robin_tournament
first tier tournament, such as “Serie A” in Italy, the English “Premier League”, the Spanish “La Liga” and the Brazilian “Brasileirão” (from 2003 onwards) soccer tournaments. During the course of a season, each team plays every other team twice: the “home” and “away” games. Moreover the points awarded in each match follows the 3-1-0 points system: teams receive three points for a win and one point for a draw; no points are awarded for a loss. The Serie A Italian soccer tournament, or simply the “Calcio”, has been played since 1898, but only from 1929 was it disputed in its current format and system. Their main champions have been Juventus, winner of the league 27 times, and Milan and Internazionale which won the league 18 times each. The Spanish “La Liga” also started in 1929, and over its history, the tournament has been widely dominated by only two teams: Real Madrid and Barcelona.

In Brazil, the national tournament, popularly known as “Brasileirão”, was first organized in a modern format in 1971. In 2010 the Brazilian Soccer Confederation (CBF) recognized as national champions the winners of smaller national tournaments such as the “Taça Brasil” (played from 1959 to 1968) and another tournament known as “Roberto Gomes Pedrosa” (played from 1967 to 1970). However, only in 2003 the Brazilian League started being disputed via the DRRS. In all past editions of the tournament the league table was based on the method of preliminaries, typically used in Tennis tournaments, which will not be considered in this paper. In the 10 editions played under the DRRS, the brazilian tournament has already been won by 6 different football clubs: Cruzeiro, Santos, São Paulo, Corinthians, Flamengo, and Fluminense.

The statistics as well as the fluctuations associated to the standings and scores of teams in tournaments with 20 teams playing under the DRRS can be very interesting. Moreover, if we are able to reproduce such statistics via a simple automaton considering the teams as “agents” which evolve according to definite “rules” based on their previous performances and conditions, one could use this information when preparing or building up a team before a competition. Thus, models (e.g. automata) of games in a tournament, whose results are defined by the evolving characteristics of the teams, could provide important knowledge. Therefore, by exploring the conditions under which the standing and scores of tournaments can be mimicked by a model, we propose a simple, but very illustrative, evolutionary non-Markovian process.

It is known that many events can alter the performance of teams during a season besides their initial strengths, such as the hiring of a new player, renewed motivation due to a change in coach, key player injuries, trading of players, among others. For the sake of simplicity, we consider that
the teams in the model initially have the same chance of winning the games and that the combination of events that can lead to an improvement of a team will be modeled solely by increasing the probability of a team winning future games after a victory. Similarly, a loss should negatively affect their future winning probabilities.

Our main goal is to verify if the Brazilian Soccer tournament has final standing scores with the same statistical properties that emerge from our simple model, and to check whether the properties of the Brazilian tournament differ from other leagues and, if so, the reasons for that behavior. In the first part of the paper we calibrate our model by using constant draw probabilities introduced ad hoc, based on data from real tournaments. In the second part, we have used draw probabilities that emerge from the model dynamics, being dependent on the teams “abilities”. Both situations are able to reproduce real tournament data. The advantage of the second approach is the independence of extra parameters, i.e., the first one uses pre-calculated rates from previous statistics. In addition, we analyze distortions of our model under hypotheses of inflated tournaments. Finally, we show a transition from single to double peaked histograms of final standing scores, which occurs when we analyze a small league and large tournaments. However, it is possible to obtain a scaling for different tournaments with different sizes.

2. A first Model: ad-hoc draw probabilities

In our model, each team starts with a potential \( \varphi_i(0) = \varphi_0 \), where \( i = 1, \ldots, n \) indexes the teams. Each team plays once with the other \( n - 1 \) teams in each half of the tournament; a team \( A \) plays with \( B \) in the first half of the tournament and \( B \) plays with \( A \) in the second, i.e. the same game occurs twice in the tournament and there is no distinction between home and away matches (the “home court advantage” could be inserted in the potential of the teams). In a game between team \( i \) and team \( j \), the probability that \( i \) beats \( j \) is given by

\[
\Pr(i > j) = \frac{\varphi_i}{(\varphi_i + \varphi_j)}.
\]

(1)

The number of games in the tournament is \( N = n(n - 1) \) and in each half of the tournament, \( n - 1 \) rounds of \( n/2 \) games are played. In each round, a matching is performed over the teams by a simple algorithm, that considers all circular permutations to generate the games. We give an illustration for \( n = 6 \) teams, starting with the configuration:
This configuration implies that in the first round, team 1 plays team 4, 2 plays 5 and team 3 plays team 6. To generate the second round, we keep team 1 fixed in its position and we rotate the other teams clockwise:

1 4 2
5 6 3

Now, team 1 plays team 5, team 4 plays 6 and team 2 plays 3. After \( n - 1 = 5 \) rounds, the system arrives at the last distinct configuration and all teams have confronted every other only once. We repeat the same process to simulate the second half of the tournament.

In our model, the outcome of each match is a draw with probability \( r_{\text{draw}} \) and one team will beat the other with probability \( (1 - r_{\text{draw}}) \); the winning team is decided by the probabilities defined by Eq. \( \text{(1)} \). After each match, we increase \( \varphi_i \) by one unit if team \( i \) wins, decrease \( \varphi_i \) by one unit if team \( i \) loses and \( \varphi_i \geq 1 \), and leave it unchanged in the case of a draw. Here, we used \( r_{\text{draw}} = 0.26 \) that is the average draw probability in actual tournaments around the world. Actually, we observe that \( r_{\text{draw}} \) ranges from 0.24 (Spanish La Liga) to 0.28 (Italian Calcio); see table \( \text{[I]} \).

Besides this, the team is awarded points according to the 3-1-0 scheme. In each new match, the updated potentials are considered and the second half of the tournament begins with the conditions acquired by the teams in the first half. The team evolution dynamics is briefly described by the following algorithm:

```plaintext
Main Algorithm
1 If \((\text{rand}[0,1] < r_{\text{draw}})\) then
2 \( p_i = p_i + 1 \) and \( p_j = p_j + 1 \);
3 else
4 if \((\text{rand}[0,1] < \frac{\varphi_i}{(\varphi_i + \varphi_j)})\) then
5 \( p_i = p_i + 3 \); \( \varphi_i = \varphi_i + 1 \); \( \varphi_j = \varphi_j - 1 \);
6 else
7 \( p_j = p_j + 3 \); \( \varphi_i = \varphi_i - 1 \); \( \varphi_j = \varphi_j + 1 \);
8 endif
9 Endif
```

Here, it is important to notice that the algorithm works under the constraint \( \varphi_j \geq 1 \), for every \( j \). It is important to mention that the arbitrary choice of increments equal to one unit is irrelevant, since
it is possible to alter the relative change in potential by assigning it different starting values. For example, if team \( A \) is matched against team \( B \) in a certain round, we can denote by \( N_A \) and \( N_B \) the difference in number of wins and losses up to that round (\( N_A \) and \( N_B \) can be negative or positive integers) for each team. We can then write \( \phi_A = \phi_0 + N_A \) and \( \phi_B = \phi_0 + N_B \), so our model works with unitary increments/decrements, i.e., \( \Delta \phi = 1 \). As can be observed, for arbitrary \( \Delta \phi \) we have invariance of probability:

\[
\Pr(A > B) = \frac{\phi_A}{\phi_A + \phi_B} = \frac{\phi_0 \Delta \phi + N_A \Delta \phi}{\phi_0 \Delta \phi + N_A \Delta \phi + \phi_0 \Delta \phi + N_B \Delta \phi} = \frac{\hat{\phi}_A + N_A \Delta \phi}{\hat{\phi}_0 + N_A \Delta \phi + \hat{\phi}_0 + N_B \Delta \phi} = \hat{\phi}_A \frac{\Delta \phi}{\phi_0 + \Delta \phi},
\]

where \( \hat{\phi}_A = \hat{\phi}_0 + N_A \Delta \phi \) and \( \hat{\phi}_B = \hat{\phi}_0 + N_B \Delta \phi \). This simple calculation shows that we can start from an arbitrary potential \( \hat{\phi}_0 \) for the players and have exactly the same results if we perform increments according to \( \Delta \phi \). In this case our main algorithm must be changed to increment/decrement by \( \Delta \phi \) instead of 1 and it is dependent on one parameter only, i.e., \( \phi_0 / \Delta \phi \).

3. Results Part I: Exploring the first model – Calibrating parameters

Before comparing our model with real data from tournaments played under the DRRS, it is interesting to study some of its statistical properties. Given \( n \) teams, one run of the algorithm will generate a final classification score for each team. For instance, starting with \( n = 20 \) teams with the same potential \( \phi_0 = 30 \), a possible final classification score generated by our algorithm in increasing order is \([23, 28, 39, 41, 44, 45, 47, 48, 49, 53, 54, 57, 60, 61, 62, 62, 64, 64, 65, 72] \). To obtain significant information from the model, it is necessary to average these data over different random number sequences. To that end, we compute histograms of final score distributions for \( n_{run} = 100 \) different final scores, for a varying number of teams.

In Fig. 1(a), we display the relative frequency of scores as a function of the rescaled score, considering all teams initially with \( \phi_0 = 2 \), for varying tournament sizes \( n \). Under this regime of low \( \phi_0 \), the changes in potential according to the algorithm generate large fluctuations in the winning/losing probabilities and a double peak pattern is observed in the histograms.

For a study of scaling size, we consider our histogram as a function of the variable \( \frac{\text{score}}{n} \).
Figure 1: Figure (a): The score histogram rescaled by the number of teams in the large fluctuation regime tournament for $\varphi_0 = 2$. The histogram was generated from an average over $n_{\text{run}} = 100$ different tournaments simulated by our automaton. We simulated tournaments with $n = 20, 40, 80, 160, 320,$ and $640$ different teams. Figure (b): The accumulated frequency of classification scores: number of teams with score smaller than a determined score divided by the number of teams. 

the larger the tournament the larger are the team scores (number of points). This double peak shows that our dynamics leads to two distinct groups: one that disputes the leadership and the other that fights against relegation to lower tiers. In Fig. 1(b), we plot the cumulative frequency as a function of $\frac{\text{score}}{n}$ (that essentially counts how many teams have scores smaller than, or equal to a given score). We can observe an interesting behavior due to the presence of extra inflection points that makes the concavity change sign and the non-gaussian behavior of the scores, independent of the size of the tournaments. Although clearly non-gaussian, because of the double peak and the “S” shaped cumulative frequency, the Kolmogorov-Smirnov (KS) and Shapiro-Wilk (SW) tests (references and routine codes of these tests are found in [8]) were performed to quantify the departure from gaussianity. An important point for methods applied is KS [9] can be applied to test other distributions differently from SW which is used for normality tests specifically.

By repeating the experiment for $\varphi_0 = 30$ a transition from a single peak to a double peak can be observed from $n \approx 40$ which is observed in Fig. 2(a). Under this condition, wins and losses cause small changes in the winning/losing probabilities simulating a tournament under the “adiabatic” regime.

We observe that this interesting behavior is reflected in the curves of cumulative frequencies
that change from single to double “S” shaped in Fig. 2 (b). It is interesting to verify whether this tournament model is able to mimic the score statistics of real tournaments and, if so, under what conditions? To answer this question, we need to explore real tournaments statistics. In Table 1, we show the compiled data of the last 6 editions of important soccer tournaments around the world: Italian, Spanish, and Brazilian. We collect data about scores of the champion teams (maximum) and last placed teams. We average these statistics for all studied editions and we analyze the Gaussian behavior of score data for each edition separately (20 scores) and grouped (120 scores) by using two methods: Shapiro-Wilk and Kolmogorov-Smirnov using a significance level of 5%. The draw average per team was also computed, which shows that $r_{\text{draw}} \approx 10^{10} \approx 0.26$ which corroborates the input used in our previous algorithm.

Some observations about this table are useful. The traditional European tournaments, based on the DRRS have non-Gaussian traces as opposed to the Brazilian league, an embryony tournament played under this system. This fact deserves some analysis: in Brazil, over the last 6 editions, (compiled data are presented in Table 1) 4 different football clubs have won the league. If we consider all 10 disputed editions, we have 6 different champions which shows the great diversity of this competition. The Brazilian League seems to be at a greater random level when compared to the European tournaments. A similarity among teams suggests that favorites are not always
Table 1: Compiled data of important tournaments around the world: Italian, Spanish, and Brazilian Leagues

|                | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | all       |
|----------------|------|------|------|------|------|------|-----------|
| **Italian* (Calcio)** |      |      |      |      |      |      |           |
| minimum        | 35   | 26   | 30   | 30   | 29   | 24   | 29(2)     |
| maximum        | 86   | 97   | 85   | 84   | 82   | 82   | 86(2)     |
| Kolmogorov-Smirnov | no  | yes | yes | yes | yes | yes | no        |
| Shapiro-Wilk   | no   | no   | yes | yes | yes | yes | no        |
| draws (average per team) | 12.5 | 11.4 | 11.2 | 9.5 | 10.2 | 9.7 | 10.8(5)   |
| **Spanish (La Liga)** |      |      |      |      |      |      |           |
| minimum        | 24   | 28   | 26   | 33   | 34   | 30   | 29(2)     |
| maximum        | 82   | 76   | 85   | 87   | 99   | 96   | 87(4)     |
| Kolmogorov-Smirnov | yes | yes | yes | yes | yes | yes | no        |
| Shapiro-Wilk   | yes  | yes | yes | yes | no  | no  | no        |
| draws (average per team) | 10.5 | 9.8  | 8.7  | 8.3  | 9.5  | 7.9  | 9.1(4)    |
| **Brazilian (Brasileirão)** |      |      |      |      |      |      |           |
| minimum        | 28   | 17   | 35   | 31   | 28   | 31   | 28(2)     |
| maximum        | 78   | 77   | 75   | 67   | 71   | 71   | 73(2)     |
| Kolmogorov-Smirnov | yes | yes | yes | yes | yes | yes | yes       |
| Shapiro-Wilk   | yes  | no   | yes | yes | yes | yes | yes       |
| draws (average per team) | 9.7  | 9.6  | 9.6  | 10.2 | 11.8 | 10.5 | 10.1(4)   |

* The 2006 year (which corresponds to season 2005/2006) was replaced by 2004/2005 in Calcio since cases of corruption among referees have led to changes in teams scores with points being reduced from some teams and assigned to others. Here “yes” denotes positive to normality test and “no” denotes the opposite, at a level of significance of 5%.

crowned champions and many factors and small fluctuations can be decisive in the determination of the champion. This may also indicate that the Brazilian tournament has an abundance of homogeneous players differently from the Italian tournament, in which the traditional teams are able to hire the best players or have well-managed youth teams, or even sign the ones who play for the national Italian team. Consider for example Real Madrid and Barcelona in Spain: they govern the tournament by signing the best players, even from youth teams from abroad (as is the case with the World’s Player of the Year, Lionel Messi who joined Barcelona at age 13 from Argentina). It is not uncommon for a player who has stood out in the Brazilian or other latin american championships to be hired to play in Europe for the next season, further contributing to the lack of continuity from one season to the next and to the “randomization” of the teams.

In Brazil, there is not a very large financial or economic gap among teams and although fa-
orites are frequently pointed out by sports pundits before the beginning of the tournament, they are typically not able to pick the winners beforehand. In fact, many dark horses, not initially pointed out as favorites, end up winning the league title. This suggests that, in Brazil, the champions emerge from very noisy scenarios, as opposed to other tournaments that only confirm the power of a (favorite) team. One could add to that the existence of a half-season long local (or state-based only) tournament making the predictions widely reported in the press not very trustful or reliable in any sense. Therefore, it is interesting to check our model by studying its statistical properties, changing parameters and then comparing the model with real data.

We perform an analysis of our model considering different initial parameters $\phi_0 = 2, 10$ and $30$ and the different tournaments (see plot (a) in Fig. 3). We have used $r_{\text{draw}} = 0.26$ in our simulations. It is possible to observe that the model fits the Brazilian soccer very well for $\phi_0 = 30$ and $\Delta \phi = 1$. It is important to mention that extreme values (minimal and maximal) are reproduced with very good agreement. For example, plot (b) in Fig. 3 shows that minimal and maximal values obtained by our model (full squares and circles respectively, in black) are very similar to the ones obtained from the six editions of the Brazilian tournament (open squares and circles, in blue). We also plot continuous lines that represent the average values obtained in each case. This shows that our model and its fluctuations capture the nuances and emerging statistical properties of the Brazilian tournament which, however, seems not to be the case of Calcio and La Liga. Plot (a) of Fig. 3 shows that the cumulative frequency of these two tournaments are very similar to one another and that no value of the parameter $\phi_0$ (many others were tested) is capable of reproducing their data.

Now a question that can quickly come to mind to readers of this paper is: are we modeling something that is entirely random and non-evolutionary, i.e., could we use a simpler model? The answer, fortunately, is “not really”. To understand this, let us suppose a completely random and non evolutionary model (the probabilities do not change with time), in which a team should win, lose, or draw with the same probability: $1/3$.

A comparison of the best fit of our model (evolutionary) with the totally random (static) model, under the exact same conditions of 20 teams under the DRRS, is shown in Fig. 4. We observe that the static model does not reproduce the lower and upper values as well as the shape of cumulative frequency of the Brasileirão which, on the other hand, is very well fitted by our model.
Figure 3: Figure (a): A comparison between results produced by our model, using different initial parameters $\phi_0 = 2, 10$ and $30$ and different tournaments. We have used $r_{\text{draw}} = 0.26$. Results were obtained considering 6 runs of our artificial tournament. We can observe that the model fits the Brazilian league (black continuous curve obtained from 6 editions of the Brazilian league) precisely for $\phi_0 = 30$. On the other hand, Calcio and La Liga are not reproduced by our model indicating clear differences between such tournaments and the Brasileirão. Figure (b): This figure shows that minimal and maximal values obtained by our model (full squares and circles respectively, depicted in black) are very similar to the ones obtained in the six editions of the Brazilian League (open squares and circles, in blue). The continuous line corresponds to the average values obtained in each case for a comparison.

4. Second model: Draw probabilities emerging from the model itself

Previous authors (see for example [3] and [5]) claim that in a match between two teams $A$ and $B$, the probability that the result is $(n_A, n_B)$, where $n_i$ is the number of goals scored by team $i$, can be approximated by a Poisson distribution:

$$
\Pr[(n_A, n_B) | (\phi_A, \phi_B)] = \Pr(n_A, \phi_A) \cdot \Pr(n_B, \phi_B)
$$

$$
= \frac{\phi_A^{n_A}}{n_A!} e^{-\phi_A} \cdot \frac{\phi_B^{n_B}}{n_B!} e^{-\phi_B}
$$

(3)

where $\phi_A$ and $\phi_B$, the average number of goals in a game, are taken as the abilities of the teams.

It is very interesting to work with models that have as few free parameters as possible; the imposition of an ad-hoc draw probability in our previous version of the model can therefore be seen as a shortcoming. It is possible to overcome this problem, and at the same time maintain the same model properties, by using Eq. (3) as a means to calculate $r_{\text{draw}}$ in the previously defined algorithm. Given two teams, with potentials $\phi_A$ and $\phi_B$, we can calculate $r_{\text{draw}}$ making the direct
Identification of the abilities with our concept of potential, i.e., $\phi_A = \varphi_A$ and $\phi_B = \varphi_B$, so that

$$r_{draw} = \Pr [(n_a = n_b) | (\phi_A, \phi_B)]$$

$$= \sum_{n=0}^{\infty} \frac{\phi_A^n e^{-\phi_A} \cdot \phi_B^n e^{-\phi_B}}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(\phi_A \phi_B)^n e^{-(\phi_A + \phi_B)}}{n!},$$

leaving our model with only one free parameter.

The first important point is that the probability independent of previous ad-hoc information obtained from tournaments, arising as a property of the teams themselves, i.e., their ability to score goals. A plot of $r_{draw}$ as function of $\phi_A$ and $\phi_B$ is shown in fig. 5. However it is important to mention this definition must be adapted if $\varphi$ is not exactly the average number of goals of the team per match ($\phi$), since the number of goals in a match is finite, its extension to infinity can have drastic consequences in the draw probabilities. It should be noted that the potential of the teams (which may be rescaled) represents the abilities of teams but can be very different from the average of the number of goals scored by teams in a given match. In this case, a solution to this

Figure 4: **Comparison of our model (evolutionary) and a totally random (static) model**: We can observe that the static model does not reproduce the lower and upper values as well as the shape of the Brazilian League which, on the other hand, is very well fitted by our model.
problem is to consider a truncated Poisson function

\[ f^{\text{trunc}}(n, \phi) = Z(\phi, m)^{-1} \Phi^n e^{-\phi} \]

where

\[ Z(\phi, m) = \sum_{j=0}^{m} \frac{\phi^j e^{-\phi}}{j!} \]

with \( m \) being the appropriate cutoff for modeling and must be suitably adjusted. Therefore, \( r_{\text{draw}} \) is now re-written as

\[ r_{\text{draw}} = Z(\phi_B, m)^{-1} \sum_{n=0}^{m} \frac{(\phi_A \phi_B)^n}{n!^2} e^{-(\phi_A + \phi_B)} \]  

This is a solution but \( m \) must be adjusted according to the initial potential \( \phi_0 \) if we use \( \Delta \phi = 1 \). However, it is also possible to solve this problem by suitably scaling the potential to be the average number of goals in a match. Therefore, if we start the simulations with the potential representing the average number of goals of a real tournament \( \hat{\phi}_0 = \lambda = \phi_0 \Delta \phi \), then the increment must be given by \( \Delta \phi = \lambda / \phi_0 \), so that the the win/lose probabilities are kept fixed, according to equation 2. In this case, \( m \to \infty \) presents the best fits and gives the correct draw probabilities \( r_{\text{draw}} \), making the model again more suitable, since \( m \) is not an experimental parameter.

In the next section, we will present our results based on this new approach for the calculation.
of \( r_{\text{draw}} \) and we show that real data are also reproduced by both methods presented in this section.

5. Results Part II: variable draw probabilities

We perform new simulations considering our previously described algorithm, but allowing for variable draw probabilities. As before, we organize the teams via the DRRS, starting with fixed potentials and take averages over many runs of the model. Our first analysis was to reproduce the final score of the Brazilian tournament tuning different values of \( m \).

![Figure 6: Plots of \( r_{\text{draw}} \) given by equation 4 for different values of \( m \).](image)

Figure 6 shows the surfaces corresponding to \( r_{\text{draw}} \) calculated by equation 4. We can see that higher \( m \) values result in higher draw probabilities for low potentials. Figure 7 shows the cumulative distribution of simulated final scores from our artificial tournament generated by the model considering the variable \( r_{\text{draw}} \) given by equation 4. Three different values of \( m \) (10, 20, and 30) were tested. We can show that best value to fit the real data extracted from the same 6 Brazilian soccer tournaments (continuous curve) is \( m = 20 \). All teams started the simulations with \( \phi_0 = \varphi_0 = 30 \), calibrated in the previous results developed in Results Part I.

Since \( m \) is not an accessible parameter of tournaments, we can start from \( \hat{\phi}_0 = 1.57 \) as the initial potential of the teams, which corresponds to the average number of goals scored by a team in a match of the Brazilian tournament studied, and adjust \( \Delta \phi = 1.57/30 \) in our algorithm. In this case, an excellent fit (see figure 8) is obtained, considering the regular Poisson distribution \((m \to \infty)\). Naturally the number 30 follows from our initial calibration of our model (when we fixed \( r_{\text{draw}} = 0.26 \), and \( \phi_0 = 30 \) led to an excellent fit as we previously observed).
As can be seen in the figures, we can obtain good fits with this new version of model, which is a little more complicated than the previous version with constant drawing probabilities, but it uses information inherent in the model itself.

Finally, to test some scaling properties of the model, we reproduce the same plot of figure 2 using \( r_{\text{draw}} \) according to 4 which is shown in plot (a) in figure 9. This figure shows histograms of final scores generated by \( n_{\text{run}} = 100 \) different simulated tournaments using variable probabilities \( r_{\text{draw}} \). We can observe that the transition from one to two peaks is fully due to the imposition that a team in our model has a minimal potential \( \phi \geq 1 \). This effect can be overcome if we scale \( \phi_0 \) with size system and, it is possible to collapse the curves by re-writing the scores as normal standard variables, i.e.,

\[
\text{score}(n) = s(n) \rightarrow s^*(n) = \frac{s(n) - \langle s(n) \rangle}{\langle (\Delta s(n))^2 \rangle}.
\]

Thus, if \( H(s^*(n), \phi_0, n) \) denotes the histogram of normalized scores, we have the scaling relation

\[
H(s^*(n_1), \phi_0, n_1) = H(s^*(bn_1), b\phi_0, bn_1).
\]

Plot (b) in figure 9 shows this scaling. We take the logarithms of the histogram to show the collapse more explicitly. The small inset plot is taken without the logarithm. We can see that different tournaments can present the same properties as long as the teams’ potentials are rescaled.
Summary and Conclusions

In this paper, we have explored a new model that reproduces in detail the final classification score (standings) of the teams in the Brazilian Soccer tournament. The Brazilian tournament, as opposed to other tournaments such as the Italian and Spanish Leagues, has some peculiarities and seems to display scores that emerge from a dynamics that preserves its Gaussian traces. This can be justified by several reasons: Brazilian tournaments have many distinct champions and the competition is not dominated by a few teams. Favorite teams frequently perform badly, and there is an inexhaustible source of new players, making the tournament more balanced and very susceptible to small fluctuations. Our model also seems to be a good laboratory to study fluctuations that may happen in large tournaments. More specifically, the model presents a transition from a one to a two peaked distribution of the final scores (standings) histograms that correspond to disputes near the champion's region and another closer to the region of the last placed team. Moreover, we also presented results relative to scaling of histograms of final scores and showed that tournaments based in our model for different sizes collapse on the same curve when we consider normal standard deviations for final scores and a linear scaling for potentials.

Here, it is important to mention that after the present contribution was completed, we were alerted of the existence of a similar model with more parameters proposed to study statistical properties of tournaments [10]. However, our contributions is very different, because in that study, the
matches are generated under the mean field approximation regime based on a Markovian random walk. In such an approximation, therefore, the teams do not evolve in time.

**About data extraction for the validation of the model**

Table 1 shows the data from real tournaments used to compare with the results produced by our model, as illustrated in Figs. 3 and 4. The data (available publicly at http://www.wikipedia.org/) is based on records from the Italian, Spanish, and Brazilian tournaments during the 2005/2006 - 2010/2011 seasons. For the Italian Calcio, the year 2006 (which corresponds to season 2005/2006) was replaced by 2004/2005, since cases of corruption and a referee scandal in 2005/2006 have supposedly changed the scores of teams, and points were reduced from some teams and awarded to others. To obtain the data from our model, we implemented a simple algorithm in the FORTRAN language which computes the possible games according to the DRRS system and evolves the potential and points of teams producing a final classification score, or even a large number of final classification scores. This is used to plot Figures 1 and 2 which explore the details of the model. All other figures of the paper represent a comparison of the data extracted from Wikipedia and those produced by our model.

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