Relations between Hamiltonian mechanics and quantum mechanics are studied. It is stressed that classical mechanics possesses all the specific features of quantum theory: operators, complex variables, probabilities (in case of ergodic systems). The Planck constant and the Fock space appear after putting a dynamical system in a thermal bath. For harmonic oscillator in a thermal bath, the probability amplitudes can be identified with the complex valued phase functions $f(q+ip)$ describing small deviations from the equilibrium state, when the time of relaxation is large. A chain of such oscillators models both the one-dimensional space (or string), and one-dimensional quantum field theory.

1 Introduction

At the present time it becomes clear that to uncover the nature of quantum description one has to turn to the Planck scales $l_P \sim 1.6 \cdot 10^{-33}$ cm [1,2]. For example, ’t Hooft came to conclusion that to construct self-consistent quantum gravity one should 1) admit discrete space-time and physical variables, 2) postulate at the Planck scales deterministic classical mechanics (CM), 3) admit (locally) dissipative processes [1]. Of course, at the atomic level one cannot obtain quantum mechanics (QM) from the classical one. It may be possible only in the models where matter, mechanics and space appear simultaneously [2]. For the recent activity in this direction see [3,4]. The problem of the Planck constant (what is the nature of $h$?) was not discussed in [1,3,4].

The aim of the article is to show that evolution of non-equilibrium states of a harmonic oscillator in a thermal bath is described by probability amplitudes (if the deviation from the equilibrium is small, and the relaxation time is large). It is the heat bath that is responsible for appearance of probabilities and the Planck constant $h$. A chain of such oscillators models one-dimensional quantum field theory or quantized bosonic string. One-particle excitations of the field play the role of particles and are described by wave functions. The distances between the oscillators are supposed to be of order $l_P$. Thus, 1D relativistic quantum world can be built in the framework of non-relativistic classical mechanics. The harmonic oscillator plays the prominent role in this model (as is well known fields are ordered sets of harmonic oscillators).

In section 2 the attention is drawn to the fact that Hamiltonian mechanics possesses all the specific properties of quantum theory: operators, complex valued functions, probabilities (for ergodic systems). Moreover, in case of complex systems there appears also a constant with dimension of action and the Hilbert space.

In section 3 equilibrium and non-equilibrium distributions for a system in a thermal bath are discussed. Variations of canonical variables are divided into two classes. Those, preserving the Gibbs distribution satisfy equations analogous to the Hamiltonian equations of motion. All the other variations give rise to non-equilibrium states.
Study of harmonic oscillator in a thermal bath (section 4) shows that phase functions $f(z)$, $z = (q + ip)/\sqrt{2}$, describing non-equilibrium states, in case of large relaxation time $t$, compose the Fock space and may be identified with the probability amplitudes.

In section 5 a chain of harmonic oscillators in a heat bath is considered. It is shown that in the continuum limit one obtains 1D quantum field theory. It elucidates the nature of quantum fields, matter (excitations of the fields) and physical space (the chain of oscillators).

In Conclusion (section 6) the main results of the paper are summarized, and the consequence of the proposed model for the Universe is pointed out (disappearance of coherent excitations of the vacuum, i.e. ”disappearance of matter”).

2 Hamiltonian mechanics and mathematical apparatus of quantum mechanics

To define Hamiltonian mechanics, one has to define 1) phase space (PS) — an even-dimensional manifold, 2) non-degenerate closed symplectic form on PS, 3) the Hamilton function $H(q,p)$. The symplectic form

$$\omega^2(q,p) = \sum_{k=1}^{n} \omega^{-1}_k(q,p) dq_k \wedge dp_k$$

defines the Poisson bracket for phase functions $f(q,p), g(q,p)$

$$\{f,g\} = \sum_{k=1}^{n} \omega_k(q,p) \left( \frac{\partial f}{\partial q_k} \frac{\partial g}{\partial p_k} - \frac{\partial f}{\partial p_k} \frac{\partial g}{\partial q_k} \right) \equiv \sum_{k=1}^{n} \omega_k(q,p) \frac{\partial(f,g)}{\partial(q_k,p_k)}.$$  

(2)

Because we are going to use non-canonical transformations, the following symbol is proved to be useful

$$\{f,g/q,p\} = \frac{\partial(f,g)}{\partial(q,p)}.$$  

(3)

The equation of motion reads

$$\dot{f} = \{f,H\}.$$  

(4)

Hamiltonian mechanics contains all the essential constituents of quantum mechanics. For the readers convenience we give some details used in what follows.

Operators. Equation (4) can be rewritten in the form

$$\dot{f} = \{H, /p,q\} f \equiv \hat{H}_{cl} f,$$  

(5)

where $\{H, /p,q\}$ is an operator: $\{H, /p,q\} f = \{f,H/q,p\}$. For simplicity, we took $n = 1$ and $\omega(q,p) = 1$. $\hat{H}_{cl}$ is an operator in CM [5].

Complex functions. Let us take as an example harmonic oscillator with the Hamiltonian

$$H = \frac{1}{2} \left( \frac{\tilde{p}^2}{m} + \gamma q^2 \right) = \frac{\omega}{2} (p^2 + q^2), \quad \omega = \sqrt{\frac{\gamma}{m}}, \quad p^2 = \frac{\tilde{p}^2}{\sqrt{\gamma m}}, \quad q^2 = q^2 \sqrt{\gamma m}.$$  

(6)
The equations of motion are
\[
\dot{x}(q,p) = \omega \hat{J} x(q,p), \quad \dot{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \hat{J}^2 = -1 \quad (x(q,p) : x_1 = q, x_2 = p).
\] (7)

In normal coordinates \( z = (q + ip)/\sqrt{2}, \bar{z} = (q - ip)/\sqrt{2} \) we have
\[
\dot{x}(\bar{z}, z) = i \hat{S} x(\bar{z}, z), \quad x(\bar{z}, z) = \hat{U} x(q, p), \quad \hat{S} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}, \quad \hat{U}^+ \hat{U} = 1,
\]
i.e. Eqs. (7) decouple, and one may use e.g. only the second equation
\[
\dot{z} = -i\omega z;
\] (8)

the first one can be obtained by complex conjugation. Complex canonical variables \( z, \bar{z} \) are useful in general case [6,7].

**The imaginary unit.** Notice, that 1) matrix \( \hat{J} \) in (7) is transformed into the imaginary unit \( i \) in (8): \( \hat{U} \hat{J} \hat{U}^+ = i \hat{S} \), 2) transformation \( q, p \rightarrow z, \bar{z} \) is not canonical one: \( \{ z, z/q, p \} = i \) (though \( \hat{U} \) is a unitary matrix; for a canonical transformation \( q, p \rightarrow Q, P \) by definition \( \{ Q, P/q, p \} = 1 \)).

**Probabilities.** Complex ergodic systems are stochastic ones; the time average of phase functions is equal to their average over an ensemble. Thus, in deterministic classical theories there appear probabilities and (for subsystems, see e.g. [8]) the Gibbs distributions. This is one of the most important points of our consideration.

**Hilbert space.** The Gibbs distribution allows to define the Hilbert space; for complex valued phase functions \( f(q,p), g(q,p) \) one can introduce the inner product [5,9]
\[
(f,g) = Z^{-1} \int \tilde{f} g e^{-\beta H(q,p)} dq \, dp
\] (9)

(here \( \beta = 1/kT, k \) — the Boltzmann constant, \( T \) — the temperature, \( Z \) — the normalizing constant). It was stressed however [9], that in QM wave functions are defined in configuration (or momentum) space, not in PS. The connection between the Hilbert space with the inner product (9) and the Fock space was not noticed. The point is: arbitrary phase function \( f(q,p) \) cannot be a wave function because \( \hat{q} \) and \( \hat{p} \) do not commute. But the phase functions \( \tilde{f}(q,p) \equiv f(z) \) can be considered (and are considered) as wave functions in the Fock space — now canonical variables are \( z, \bar{z} \) (see section 4).

**The Planck constant \( h \).** For finite motion the integral
\[
Z = \int e^{-\beta H(q,p)} dq \, dp
\] (10)

exists. Then, the constant
\[
Z^{1/n} = h
\] (11)

has the dimension of action, and in certain cases it can be identified with the Planck constant \( h \) (see sections 4,5). Of course, in general case this \( h \) depends on \( H, \beta \) and is not a universal constant. But in a world made of identical elements, e.g. in 1D quantum field theory (linear chain of oscillators) this constant is universal one.


3 Equilibrium and non-equilibrium distributions

The Gibbs distribution and the Hamiltonian equations of motion. As was already mentioned, classical mechanics of complex ergodic systems is stochastic and bears the Gibbs distribution. Problem: find variations of canonical variables preserving the Gibbs distribution. From $\delta \exp(-\beta H) = 0$ one gets

$$\delta H(q,p) = \sum_i \left( \frac{\partial H}{\partial q_i} \delta q_i + \frac{\partial H}{\partial p_i} \delta p_i \right) \equiv \sum_i \nabla_i H \delta x_i \equiv \nabla H \delta x = 0 \quad (12)$$

($x_i = x(q_i,p_i), \ x = x(q_1,p_1,...,q_n,p_n)$). Solution of this equation is

$$\delta x = \hat{\Omega} \nabla H \delta t, \quad (13)$$

where $\hat{\Omega}$ is some $2n \times 2n$ antisymmetric matrix, such that

$$\nabla H \hat{\Omega} \nabla H = 0 \quad (14).$$

Among solutions (13) there are solutions of the Hamiltonian equations of motion. In the simplest case ($\hat{\Omega} = \otimes \hat{J}, \ \omega_k(q,p) = 1$) equations (13) are nothing but the familiar Hamiltonian equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \ \dot{p}_i = -\frac{\partial H}{\partial q_i}. \quad (15)$$

It follows from (12) that an arbitrary variation $\delta x$ is the sum

$$\delta x = \delta x_\perp + \delta x_\parallel \quad (16)$$

where $\delta x_\perp$ are given by (13). Variations $\delta x_\perp$ preserve the Gibbs distribution while variations $\delta x_\parallel$ give rise to non-equilibrium states.

Non-equilibrium distributions. The Gibbs distribution for harmonic oscillator introduces measure in PS

$$d\mu(z,z) = \frac{dz \wedge dz}{i\hbar e^{-\frac{\bar{z}z}{\hbar}}}, \ \hbar = \frac{\hbar}{2\pi} = \frac{1}{\beta \omega} \quad (17)$$

(see (6), (11)). Any other measure $\mu_p \neq \mu$ describes some non-equilibrium state

$$d\mu_p = (d\mu_p/d\mu)d\mu \equiv p(z,z)d\mu, \ \ p(z,z) \geq 0. \quad (18)$$

Now, consider variation of $\bar{z}, z$ with small non-zero $\delta x_\parallel$ in (16); we have

$$\omega \bar{z}z \rightarrow \omega \bar{z}z - \beta^{-1}(cz + \bar{c}z) + ..., \ \ e^{-\beta \omega \bar{z}z} \rightarrow |e^{cz} + ...|^2 e^{-\frac{\bar{z}z}{\hbar}},$$

and

$$d\mu(\bar{z},z) \rightarrow d\mu_f(\bar{z},z) = |f(z)|^2d\mu(\bar{z},z), \ \ f(z) = f_\omega(z) = e^{cz}; \quad (19)$$

the higher degrees of $\bar{z}, z$ in the exponential are omitted. Then, instead of $p(\bar{z},z)$ in (18) we have $|f(z)|^2$, so to study evolution of a non-equilibrium state it is enough to consider evolution of $f(z)$. Evolution of a single analytical function $f(z)$ describes evolution of a non-equilibrium state; entire functions $f(z)$ are elements of the Fock space [10,11]. This is the key to the problem.
Remark. Introduction of \( h \) by Eqs. (11), (17) may look unconvincing at the first sight. Notice, however, that (17) can be obtained by projection of a sphere \((\varphi, \theta)\) on complex plane \( z \)

\[
|z|^2 = \frac{1}{\beta} \ln \left( \frac{1}{2\beta R^2 \sin^2 \theta / 2} \right), \quad \arg z = \varphi; \quad R^2 \sin \theta d\varphi \wedge d\theta = d\mu(\bar{z}, z),
\]

where \( R \) is radius of the sphere [2]. If PS is a sphere then its area can be identified with \( h = 4\pi R^2 \) — the universal constant with dimension of action for Hamiltonian dynamics on the sphere. A chain of such oscillators can model a bosonic string, while a network of strings models e.g. 3D space [2]. Hamiltonian mechanics in this space possesses the universal constant \( h \).

4 Harmonic oscillator in a thermal bath and quantum mechanics

It is easy to show now that for harmonic oscillator evolution of small (in the sense of section 3) deviations from the equilibrium state is described by quantum mechanics if time of relaxation is large \((t_r \gg \omega^{-1})\).

Evolution of non-equilibrium states. Hamiltonian (6) in complex variables takes the form

\[
H = \frac{\omega}{2} \left( \begin{array}{cc} q & 1 \\ -1 & 0 \end{array} \right) U^* U^+ \left( \begin{array}{cc} z & 0 \\ 0 & z \end{array} \right) = \frac{\omega}{2} \left( \begin{array}{cc} z & 1 \\ 0 & z \end{array} \right) \left( \begin{array}{cc} \bar{z} & 0 \\ 0 & \bar{z} \end{array} \right) = \frac{\omega}{2} (\bar{z}z + z\bar{z}),
\]

and the equation of motion for \( f(z) \) reads

\[
\dot{f} = \{f, H/q, p\} = i\{f, H/\bar{z}, z\} = -i\omega z \frac{d}{dz} f(z) = -i\omega \frac{d}{d\ln z} f(z) \equiv \hat{H}_{cl} f
\]

or, in terms of \( \varphi = \arg z \)

\[
(\partial_t + \omega \partial_{\varphi}) f(\varphi, t) = 0
\]

\((d|z|/dt = 0)\). Functions \( f(\varphi - \omega t) \) solve equation (22); they describe waves on the circle \(|z| = \text{const}\) and should be periodic functions. So, \( f(z) \) can be developed into the Laurent series. But in general case such functions are singular at \( z = 0 \) and do not suit for description of any sensible state of harmonic oscillator. The proper functions are those given by the Maclaurin series

\[
f(z) = \sum_{n=0}^{\infty} c_n Z_n(z), \quad Z_n(z) = z^n / \sqrt{n!}.
\]

Functions \( Z_n \) satisfy equations of motion

\[
\dot{Z}_n = -i\omega n Z_n,
\]

and evidently correspond to motion with frequencies \( \omega n \).

Probability amplitudes. Solutions of equations (21), (22) form a linear space. Now, define the scalar product

\[
(g, f) = \int d\mu(\bar{z}, z)g(\bar{z})f(z);
\]
then the entire functions \( f(z) \) of order \( \rho \leq 2 \) form the Fock space \( \mathcal{F} \) [10,11], i.e. the Hilbert space of harmonic oscillator. We conclude: the phase functions \( f(z) \), describing non-equilibrium distributions of harmonic oscillator, can be identified with the probability amplitudes. For example, it is well known [10,11] that functions \( Z_n(z/\sqrt{\hbar}) \) give the orthonormal basis in the Hilbert space

\[
(Z_n, Z_m) = \delta_{nm}, \tag{26}
\]

and we demand

\[
(f, f) = \sum_{n=0}^{\infty} |c_n|^2 = 1. \tag{27}
\]

The latter equality means that \( |f(z)|^2 \) may be identified with the probability density. Numbers \( |c_n|^2 \) also represent some (discrete) probability distribution. Evolution of a non-equilibrium distribution (i.e. of \( |f(z)|^2 \)) is given by evolution of a single function \( f(z) \).

It is easy to see that smallness of \( c \) in (19) does not impose limitations on the set of functions \( e^{cz} \) as a basis in \( \mathcal{F} \). It is known [10] that for arbitrary small \( \epsilon > 0 \) vectors \( e^{cz} \) form a (over)complete basis in \( \mathcal{F} \) if an infinite sequence of points \( c_k \) in the complex plane converges to some limit \( c_0 \) (one can omit from the sequence any finite number of points). It means that if some function \( \psi(z) \in \mathcal{F} \) is orthogonal to all the functions \( e^{cz} \) then \( \psi(z) = 0 \). Indeed, functions \( e^{cz} = \langle z|\bar{c}\rangle = \bar{c}(z) \) play the role of \( \delta \)-functions in \( \mathcal{F} \), and \( (\bar{c}_k, \psi) = \langle c_k|\psi \rangle = \psi(c_k) = 0, \forall k \). But \( \psi(z) \) is entire function, so \( \psi(z) = 0 \).

**Commutation relations.** It is well known that for the Fock space with measure (17) operator \( \hat{\bar{z}} \) defined as \( \hat{\bar{z}}g(z) = \bar{z}g(z) \) has the Hermitian conjugate \( \hat{z} = \hbar d/d\bar{z} \) (it can be proved by integration by parts in (25) [10]). The commutation relation

\[
[\hat{z}, \hat{\bar{z}}] = \hbar \tag{28}
\]

allows to identify \( \hat{z}, \hat{\bar{z}} \) as the creation and annihilation operators for quantized harmonic oscillator. Then, the canonical variables \( q, p \) also become operators

\[
[\hat{q}, \hat{p}] = \frac{i}{2}[\hat{\bar{z}} + \hat{z}, \hat{\bar{z}} - \hat{z}] = i\hbar. \tag{29}
\]

Remember, that we are working in the framework of pure classical theory.

**The Schroedinger equation.** The classical equation of motion (21) can be identified with the Schroedinger equation if we multiply it by \( i\hbar \):

\[
i\hbar \dot{Z}_n(z) = \hat{H}_{cl} Z_n(z), \quad \hat{H}_{cl} = i\hbar \hat{H}_{cl} = \hbar \omega \hat{a}^+ \hat{a}; \tag{30}
\]

here \( \hat{a} = d/dz, \hat{a}^+ = z \) (because they act on functions \( f(z) \)).

The spectrum of operator \( \hat{H}_{cl} \) differs from that for quantized oscillator \( E_n = \hbar \omega(n + 1/2) \). We would obtained correct answer using the Hamiltonian (20). Surprisingly, Eq. (20) automatically gives correct ordering of operators. It is important to elucidate the nature of the "quantum energy" \( \hbar \omega/2 \). Functions \( Z_n \) correspond to classical periodic motion with frequencies \( \omega_n \) in the \( 1D \varphi \) space (non-equilibrium states). But there is another canonical variable \( |z| \), characterized by the Gibbs distribution, which also contributes to the total energy. So, the "zero energy" \( \hbar \omega/2 \) should be attributed to the universal influence of the thermal bath.
Restoration of the equilibrium state. To model restoration of the Gibbs distribution one may introduce, for example, friction. The equation of motion for harmonic oscillator then reads
\[ \ddot{q} + \alpha \dot{q} + \omega^2 q = 0, \] (31)
where \( \alpha > 0 \) specifies friction. For infinitesimal \( \alpha \) the general solution of (31) is
\[ q(t) = c_1 e^{-i(\omega - i\alpha/2)t} + c_2 e^{i(\omega + i\alpha/2)t}, \] (32)
Evidently, there is only one equilibrium distribution. We see that the classical deterministic motion disappears in the limit \( t \to \infty \) (\( t > t_r \sim \alpha^{-1} \)). It means that in this model of quantum oscillator all the probability amplitudes tend to zero when \( t \to \infty \).

5 Linear chain of harmonic oscillators in a heat bath

It is not difficult to show that the linear chain of harmonic oscillators in a heat bath models standard 1D quantum field theory. Again, it is the Gibbs distribution that is responsible for the non-trivial measure (see (17), (25)) in the Fock space.

Hamiltonian of a chain of harmonic oscillators can be written in the form
\[ H = \frac{1}{2} \sum_n \left( \frac{p_n^2}{m} + \tilde{\gamma}(q_n - q_{n-1})^2 + \gamma q_n^2 \right), \] (33)
In normal coordinates \( a(k), a^*(k) \) it reads
\[ H = \frac{1}{2} \int \Delta dk \omega(k) \left( (a^*(k)a(k) + a(k)a^*(k)) \right), \] (34)
where
\[ a(-k) = \frac{1}{\sqrt{2}} \left( u(k) \sqrt{m\omega(k)} + i \frac{p(k)}{\sqrt{m\omega(k)}} \right), \]
\[ q_n = \int \Delta dk u(k) \varphi_n^*(k), \quad p_n = \int \Delta dk p(k) \varphi_n(k), \quad \varphi_n(k) = \frac{1}{\sqrt{2\Delta}} e^{i\omega k}, \Delta = \frac{\pi}{a}; \] (35)
here \( a \) is the distance between the neighbour oscillators, \( \omega^2(k) = \gamma/m + 4(\tilde{\gamma}/m) \sin^2(\pi k/2\Delta) \). In the limit \( a \to 0, n \to \infty, an \to x, a^2\tilde{\gamma}/m \to 1, \gamma/m = M^2, q_n \sqrt{m/a} \to \varphi(x,t) \), one obtains the 1D theory of free scalar field \( \varphi \) with mass \( M \). It is important that
1) the Hamiltonian (34) presents a set of noninteracting oscillators with frequencies \( \omega(k) \),
2) this theory is Lorentz invariant (the Lagrangian is \( (\varphi^2 - \varphi'^2 - M^2\varphi^2)/2 \)), so all the fields \( a(k), a^*(k) \) with different \( k \) transform one into another by the Lorentz transformation.

Evidently, the Gibbs measure \( \sim \exp(-\beta \int dk \omega(k)a^*(k)a(k)/\hbar) \) differs from the Fock space measure \( \exp(-\int dk a^*(k)a(k)/\hbar) \). But in fact the difference is formal. Introducing new variables \( \tilde{a}(k) = \sqrt{\lambda_k} a(k), \lambda_k = \omega(k)/\omega, \omega = \omega(0) \), one rewrites (34)
\[ H = \omega \int \Delta \tilde{a}^*(k) \tilde{a}(k). \] (36)
The Gibbs measure now takes the form

$$\prod_k \lambda_k^{-1} \beta \omega \frac{d\tilde{a}^*(k) \wedge d\tilde{a}(k)}{2\pi i} e^{-\beta \omega \int dk \tilde{a}^*(k)\tilde{a}(k)}$$

(37)

(the integration limits are omitted). The infinite factor $\prod_k \lambda_k^{-1}$ is not essential — it is eliminated by normalization. The corresponding change of functionals $\Phi[a] \to \Phi[\lambda^{-1/2} \tilde{a}]$ leads to redefinition $\Phi[\lambda^{-1/2} \tilde{a}] = \tilde{\Phi}[\tilde{a}]$. Thus, we come to the scalar product for complex valued functionals $\Phi[a] \to \Phi[\lambda^{-1/2} \tilde{a}]$ leads to redefinition $\Phi[\lambda^{-1/2} \tilde{a}] = \tilde{\Phi}[\tilde{a}]$. Thus, we come to the scalar product for complex valued functionals $\Phi[a]$ into the Fock space

$$\langle \Phi_1, \Phi_2 \rangle = \int \prod_k da^*(k) \wedge da(k) e^{-\int dk a^*(k) a(k)/\hbar}$$

(38)

(in the limits $a \to 0, \Delta \to \infty$). From (38) it follows that $\hat{a}^+(k)\Phi[a^*] = a^*(k)\Phi[a^*], \quad \hat{a}(k)\Phi[a^*] = \hbar \delta \Phi[a^*]/\delta a^*(k)$, and $[\hat{a}(k), \hat{a}^+(k')] = \hbar \delta(k - k')$.

This construction elucidates concepts of space, quantum mechanics and quantum fields. The physical space is nothing but the chain of harmonic oscillators, fields are excitations of the chain, particles (quanta) are one-particle excitations of the fields, and quantum theory appears as a method appropriate for description of non-equilibrium distributions characterizing the system. One should distinguish “the physical space” from the outer one in which it is embedded. The physical space is the linear chain of oscillators. Excitations of the oscillators ("matter") propagate in the physical space. The outer space is analogous to the multidimensional space ($10D$ or $26D$) in superstring theory. The $3D$ physical space also can be modeled in this way as a $3D$ network made of superstrings [2].

It is important that here the neighbours of an oscillator are responsible for taking it out from the equilibrium. The Planck constant $\hbar = 1/\beta \omega$ is universal for all the oscillators, i.e. it is a fundamental constant of the theory. Distributions

$$\overline{\Phi[a]}\Phi[a] e^{-\int dk a^*(k) a(k)/\hbar}$$

(39)

describe non-equilibrium states of the system (because they differ from the equilibrium one). To describe the evolution of density $\rho[a^*, a] = \overline{\Phi[a]}\Phi[a]$ it is enough to describe the evolution of $\Phi[a]$, i.e. the evolution of non-equilibrium states is described by probability amplitudes (wave functionals) $\Phi[a]$. In this model the Planck constant $\hbar$ and the wave functions (functionals) appear simultaneously. The chain of harmonic oscillators models also the bosonic string.

6 Conclusion

It was shown that small deviations from equilibrium states of harmonic oscillator in a thermal bath are described by probability amplitudes (in the time interval $t_r > t \gg \omega^{-1}$). Thus, quantum mechanics can be modeled in framework of classical theory. Of course, it does not mean that one can obtain quantum theory of e.g. electron from classical theory of a pointlike particle. The result is meaningful only on the fundamental level of the Planck scales. For example, a chain of such oscillators in a thermal bath models one-dimensional quantum field theory (or quantized bosonic string). Excitations of the chain (quanta, particles) move along the chain — they model "matter". In this case, one simultaneously models both the $1D$ physical space and quantum mechanics.
Another important feature of the model — natural appearance of the Planck constant \( h \) (see (11), (17)). The probabilities, probability amplitudes and constant \( h \) appear as consequences of putting oscillator into a heat bath. The other attributes of QM (operators, complex functions) are indispensable features of CM. Actually, there is no need to introduce a thermal bath by hand. Complex ergodic systems are stochastic by themselves, i.e probabilities pervade Hamiltonian mechanics, and the Gibbs distribution also follows from CM. The ”analogy of the evolution of classical correlation functions with quantum mechanics” was discussed in [12] (without introducing \( h \); see also [13]).

It follows from (32) that in the (1+1) universe, modeled by a chain of harmonic oscillators, quantum description will be actual only in the time interval \( (t_r, \omega^{-1}) \); here \( t_r \) is the relaxation time of the chain excitations (if it doesn’t exceed that of the oscillators). When \( t \to \infty \) the system tends to the equilibrium state (“vacuum”). Any coherent motion dies away. This is another kind of ”death” of the ”universe” in contrast with the ”thermal death” of XIX-th century. In the latter case matter becomes thermalized, while here matter ”disappears” because this is nothing but quantum excitation of the chain (i.e. vacuum) specified by some probability amplitudes.

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