The colored flux tube

Vladimir Dzhunushaliev

Institut für Mathematik, Universität Potsdam, D-14469, Potsdam, Germany
and
Dept. Phys. and Microel. Engineer., Kyrgyz-Russian Slavic University
Bishkek, Kievskaya Str. 44, 720021, Kyrgyz Republic

Abstract

It is shown that in the SU(2) Yang-Mills-Higgs theory with broken gauge symmetry a flux tube solution filled with a color longitudinal electric field exists. The origin of the gauge symmetry breakdown for this case is discussed.

1 Introduction

The quantum chromodynamics (QCD) describes the forces between quarks which are elementary blocks of hadrons. In contrast with quantum electrodynamics QCD has a very strong interaction between quanta of gauge fields. Probably the consequences of such nonlinearity is the appearance of a flux tube filled with a longitudinal color electric field between interacting quark and antiquark. The existence of such tube leads to a confinement of quarks: quark and antiquark are connected with the flux tube which gives a constant force between these particles. The quarks are the sources of a color electric field filling the flux tube. Therefore the derivation of the flux tube solutions in QCD is very important problem. Evidently this problem is quantum not classical one as many years of the attempts to obtain such solutions in the classical gauge theories have not any success. Although it is well known that in dual theories exist so-called Nielsen-Olesen flux tube solutions [1] which are filled with an Abelian magnetic field and BPS strings [2]. Mathematically these solutions are identical to the flux tubes in a superconductor where the magnetic field is pushed out from the superconductor. Such correspondence between color flux tube in QCD and magnetic flux tube in superconductor allows us to suppose that the QCD flux tube in some sense is a dual picture of a Meissner effect in the superconductivity [3], [4]: magnetic field is changed on electric one and a condensate of Cooper pairs on a condensate of (probably) magnetic monopoles.

In Ref. [5] it was shown that in the Euclidean spacetime there is a flux tube solution (colored flux tube) of the SU(2) Yang-Mills-Higgs gauge theory with broken gauge symmetry. The derived solution is filled with a color longitudinal SU(2) electric field. The analysis shows that the symmetry breakdown is necessary for the existence of colored flux tube as this solution is regular only for a discrete spectrum of mass $m \neq 0$ which describes the gauge symmetry breakdown. In this letter we would like to show that the same mechanism is in the action for the Lorentzian spacetime, too.

2 Flux tube equations

We will start from the SU(2) Yang-Mills-Higgs field equations with broken gauge symmetry

$$D_\nu F^{a\mu\nu} = g e^{abc} \phi^b D_\mu \phi^c - (m^2)^{ab} A^{b\mu},$$

$$D_\mu D^\mu \phi^a = -\lambda \phi^a (\phi^b \phi^b - \phi^\infty)$$

(1)

(2)

here $F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g e^{abc} A^b_\mu A^c_\nu$ is the field tensor for the SU(2) gauge potential $A^a_\mu; a,b,c = 1,2,3$ are the color indices; $D_\nu[\cdots]^a = \partial_\nu[\cdots]^a + g e^{abc} A^b_\mu[\cdots]^c$ is the gauge derivative; $\phi^a$ is the Higgs field; $\lambda, g$ and $\phi^\infty$ are some constants; $(m^2)^{ab}$ is a masses matrix which destroys the gauge invariance of the Yang-Mills-Higgs theory, here we choose $(m^2)^{ab} = diag \{m_1^2, m_2^2, 0\}$.

*E-mail: dzhun@hotmail.kg
The solution we search in the following form

\[ A_1^i(\rho) = \frac{f_1(\rho)}{g}; \quad A_2^i(\rho) = \frac{v_1(\rho)}{g}; \quad \phi^3(\rho) = \frac{\phi(\rho)}{g} \]  

(3)

here \( z, \rho, \varphi \) are cylindrical coordinate system. The substitution into the Yang - Mills - Higgs equations gives us

\[ f'' + \frac{f'}{x} = f \left( \phi^2 + v^2 - m_1^2 \right), \]  

(4)

\[ v'' + \frac{v'}{x} = v \left( \phi^2 - f^2 - m_2^2 \right), \]  

(5)

\[ \phi'' + \frac{\phi'}{x} = \phi \left[ -f^2 + v^2 + \lambda \left( \phi^2 - \phi_{\infty}^2 \right) \right] \]  

(6)

Here we redefined \( \phi/\alpha \to \phi, f/\alpha \to f, v/\alpha \to v, \phi_{\infty}/\alpha \to \phi_{\infty}, m_{1,2}/\alpha \to m_{1,2}, \rho \alpha \to x; \) the constant \( \alpha \) will be defined later. The similar equations with the presence of \( A_x^a \) and without masses \( m_{1,2} \) was investigated in Ref’s \( 4, 7 \) where it was shown that the corresponding solutions becomes singular either on a finite distance from the axis \( \rho = 0 \) or on the infinity. The solutions with the masses \( m_{1,2} \) and a magnetic field \( H_z \) were obtained in Ref. \( 8 \) and the result is that the following versions of the flux tube exist: (1) the flux tube filled with electric/magnetic fields on the background of an external constant magnetic/electric field; (2) the Nielsen-Olesen flux tube dressed with transversal color electric and magnetic fields.

3 Numerical algorithm

We will solve the equations set \( 4-6 \) with an iterative procedure which is described in Ref. \( 5 \). Eq. \( 5 \) on the \( i \) step has the following form

\[ v''_i + \frac{v'_i}{x} = v_i \left( \phi_{i-1}^2 - f_{i-1}^2 - m_{2,i}^2 \right), \]  

(7)

here the functions \( f_{i-1} \) and \( \phi_{i-1} \) was defined on \( (i - 1) \) step and the null approximation for the functions \( f(x) \) and \( \phi(x) \) is \( \phi_0(x) = 1.3 - 0.3/\cosh^2(x/4) \) and \( f_0(x) = 0.2/\cosh^2(x) \). The numerical investigation for Eq. \( 7 \) shows that there is a number \( m_{2,i}^* \) for which: for \( m_{2,i} > m_{2,i}^* \) the solution is singular \( v_i(x) \to -\infty \) by \( x \to \infty \) and for \( m_{2,i} < m_{2,i}^* \) the solution is also singular \( v_i(x) \to +\infty \) by \( x \to \infty \). It means that there is such number \( m_{2,i}^* \) for which the solution \( v_i(x) \) is regular one. The number \( m_{2,i}^* \) is defined with the method of the iterative approximation. Here is necessary that the functions \( f_{i-1}(x) \to 0 \) and \( \phi_{i-1}(x) \to \phi_{\infty,i-1}^* \) by \( x \to \infty \).

Eq. \( 6 \) on this step has the following form

\[ f''_i + \frac{f'_i}{x} = f_i \left( \phi_{i-1}^2 + v_i^2 - m_{1,i}^2 \right), \]  

(8)

here the function \( \phi_{i-1} \) was defined on \( (i - 1) \) step and the function \( v_i \) is the solution of Eq. \( 7 \). The numerical investigation for Eq. \( 8 \) shows that there is a number \( m_{1,i}^* \) for which: for \( m_{1,i} > m_{1,i}^* \) the solution is singular \( f_i(x) \to -\infty \) by \( x \to \infty \) and for \( m_{1,i} < m_{1,i}^* \) the solution is also singular \( f_i(x) \to +\infty \) by \( x \to \infty \). It means that exists such number \( m_{1,i}^* \) for which the solution \( f_i(x) \) is regular one. The number \( m_{1,i}^* \) is defined with the method of iterative approximation. It is necessary that the functions \( v_i(x) \to 0 \) and \( \phi_{i-1}(x) \to \phi_{\infty,i-1}^* \) by \( x \to \infty \).

The next step on the \( i \) iteration is solving of Eq. \( 7 \)

\[ \phi''_i + \frac{\phi'_i}{x} = \phi_i \left[ -f_i^2 + v_i^2 + \lambda \left( \phi_i^2 - \phi_{\infty,i}^2 \right) \right] \]  

(9)

here the functions \( f_i \) and \( v_i \) are the solutions of Eq’s \( 7, 8 \). The numerical investigation for this equation shows that there is a number \( \phi_{\infty,i}^* \) for which: for \( \phi_{\infty,i} > \phi_{\infty,i}^* \) the function \( \phi_i(x) \) oscillates with decreasing amplitude, for \( \phi_{\infty,i} < \phi_{\infty,i}^* \) the function \( \phi_i(x) \to +\infty \) by \( x \to x_0 \). The number \( \phi_{\infty,i}^* \) defines a regular solution \( \phi_i(x) \). It is necessary that the functions \( v_i(x) \to 0 \) and \( f_i(x) \to 0 \) by \( x \to \infty \).

The value \( f(0) \) can be arbitrary but we choose them by such a way that \( f(0)/\alpha = 0.2 \), the other initial conditions are \( v(0) = 0.5 \) and \( \phi(0) = 1.0 \). Thus in the equations set \( 4-6 \) we have three independent parameters \( \lambda, v(0) \) and \( \phi(0) \). In the calculations presented here we take \( \lambda = 0.1 \).

The iterative process described above gives us the \( m_{(1,2),i} \) and \( \phi_{\infty,i}^* \) presented on Table 1. The functions \( f_i(x), v_i(x) \) and \( \phi_i(x) \) for \( i = 1, 2, 3 \) are presented on Fig’s 1, 2, and 3.
Table 1: The iterative parameters $m^*_i$ and $\phi^*_\infty$.

By the definition the color electric and magnetic fields are

\begin{align*}
E^3_\rho(x) &= F^3_{\rho z} = \frac{f(x)v(x)}{g}, \\
E^1_\rho(x) &= F^1_{\rho \phi} = -\frac{f'(x)}{g}, \\
H^2_\phi(x) &= x \epsilon_{\rho \phi z} F^{2 \rho z} = -\frac{v'(x)}{g}.
\end{align*}

These fields are presented on Fig. 3.

From Eq's (10)-(12) it is easy to see that the symptotical behaviour of the regular solutions $f^*(x)$, $v^*(x)$ and $\phi^*(x)$ is

\begin{align*}
f^*(x) &= f_0 e^{-x\sqrt{\phi^2_\infty - m^2_1}} + \cdots, \\
v^*(x) &= v_0 e^{-x\sqrt{\phi^2_\infty - m^2_2}} + \cdots, \\
\phi^*(x) &= \phi^*_\infty - \phi_0 e^{-x\sqrt{2\lambda \phi^2_\infty}} + \cdots
\end{align*}

where $f_0$, $v_0$ and $\phi_0$ are some constants.

The energy density is

\begin{align*}
2g^2\epsilon(x) &= f'^2(x) + v'^2(x) + \phi'^2(x) + v^2(x)f^2(x) + v^2(x)f^2(x) + \phi^2(x) + f^2(x)\phi^2(x) + \\
&\quad m^*_1 f^2(x) - m^*_2 v^2(x) + \frac{\lambda}{2} \left(\phi^2 - \phi^2_\infty\right)^2
\end{align*}

and presented in Fig. 5. The linear energy density will be finite as all terms in (16) have exponential decreasing at the infinity.
The flux of the longitudinal electric field is

$$\Phi = \int E_z^2 ds = 2\pi \int_0^\infty \rho \frac{f(\rho)v(\rho)}{g} d\rho = \frac{2\pi}{g} \int_0^\infty x f(x)v(x) dx < \infty. \quad (17)$$

We see that the flux $\Phi$ depends on the parameters $\lambda, v(0)$ and $\phi(0)$ as the solution $f(x)$ and $v(x)$ depend on these parameters which is in contrast with such topological solutions as ’t Hooft-Ployakov monopole and Nielsen-Olesen flux tube. It is the indication of the fact that the colored flux tube solution is not the consequence of any differential equations of the first order as it is for the topological solutions. In other words the colored flux tube is a dynamical object not topological one.

4 Colored flux tube as a pure quantum object

In Ref. [5] was presented some arguments that the obtained there colored flux tube has a pure quantum origin. The arguments are that the SU(3) quantum Yang - Mills theory in some approximation can be reduced to the classical SU(2) Yang - Mills - Higgs theory plus some extra term which is zero for the ansatz [4]. We will describe this $SU(3) \rightarrow SU(2)$ reduction following to Ref. [9]. At first the SU(3) gauge potential on ordered and disordered phases should be decomposed:
1. The gauge field components $A^a_\mu \in SU(2)$, $a = 1, 2, 3$ belonging to the small subgroup $SU(2) \subset SU(3)$ are in an ordered phase:

$$\langle A^a_\mu (x) \rangle \approx (A^a_\mu (x))_{cl}. \quad (18)$$

The subscript $(\cdot \cdot \cdot)_{cl}$ means that the SU(2) components of the SU(3) gauge field can be considered as almost a classical field. $(\cdot \cdot \cdot)$ is a quantum average.

2. The gauge field components $A^m_\mu$ (m=4,5, ..., 8) and $A^m_\mu \in SU(3)/SU(2))$ belonging to the coset $SU(3)/SU(2)$ are in a disordered phase. i.e. they are a condensate of the coset components of the SU(3) gauge field. It means that

$$\langle A^m_\mu (x) \rangle = 0, \quad \text{but} \quad \langle A^m_\mu (x) A^a_\nu (x) \rangle \neq 0. \quad (19)$$

These degrees of freedom are pure quantum degrees and are involved in the equations for the ordered phase as an averaged field distribution of coset components.

In order to simplify the 2 and 4-points Green’s functions the following assumptions and simplifications was made:
1. The correlation between coset components $A_{\mu}^a(y)$ and $A_{\mu}^a(x)$ in two points $x^\mu$ and $y^\mu$ is

$$
\langle A_{\mu}^a(y)A_{\nu}^a(x) \rangle = -\frac{1}{3} f^{abc} f^{mpc} \eta_{\mu\nu} \phi^b(y) \phi^c(x).
$$

where $f^{abc}$ is the structural constants of the SU(3) group. Here we have to indicate that it is correct only for static fields, i.e. $x^0 = y^0$.

2. There is no any correlation between ordered (classical) and disordered (quantum) phases

$$
\langle f(a_{\mu}^a(g(A_{\mu}^a)) \rangle = f(a_{\mu}^a) \langle g(A_{\mu}^a) \rangle
$$

where $f$ and $g$ are arbitrary functions.

3. The 4-point Green’s function can be approximated as

$$
\langle A_{\mu}^a(x)A_{\nu}^a(y)A_{\rho}^b(z)A_{\sigma}^c(u) \rangle = 
\left( E_{1,abcd}^{mnpq} \eta_{\alpha\beta} \eta_{\mu\nu} + E_{2,abcd}^{mnpq} \eta_{\alpha\mu} \eta_{\beta\nu} + E_{3,abcd}^{mnpq} \eta_{\alpha\nu} \eta_{\beta\mu} \right) \phi^a(x) \phi^b(y) \phi^c(z) \phi^d(u)
$$

where $E_{1,abcd}^{mnpq}$, $E_{2,abcd}^{mnpq}$, $E_{3,abcd}^{mnpq}$ are some constants and this approximation is also valid for the static fields.

Eq’s 20 and 22 tell us that this approach can be called “one function approximation”.

The final result of Ref. 9 is that the initial SU(3) Lagrangian

$$
L_{SU(3)} = -\frac{1}{4} F_{\mu\nu}^{A} F^{A\mu\nu}, \ A = 1, 2, \ldots 8
$$

with these assumptions and simplifications can be reduced to the SU(2) Yang - Mills - Higgs Lagrangian

$$
L_{SU(2)} = -\frac{1}{4} F_{\mu\nu}^{A} F^{A\mu\nu} + \frac{1}{2} \left( \partial_{\mu} \phi^a + \frac{g}{2} \epsilon^{abc} A_{\mu}^b \phi^c \right)^2 + \frac{m^2}{2} (\phi^a \phi^a) - \lambda (\phi^a \phi^a)^2 + \frac{g^2}{2} \phi^b a_{\mu}^{ab} a^{\mu} \phi^c.
$$

It is necessary to note that the term $\frac{m^2}{2} (\phi^a \phi^a)$ here also presents the assumed gauge symmetry breakdown. We see that the first term $F_{\mu\nu}^{A} F^{A\mu\nu}$ is the SU(2) Lagrangian for the ordered phase $A_{\mu}^a$ and the Higgs Lagrangian is presented with the next three terms. Let us note that there is an additional gauge noninvariant term $\frac{g^2}{2} \phi^b a_{\mu}^{ab} a^{\mu} \phi^c$ but for the ansatz 3 the corresponding terms in field equations are zero.

Finally, in the context of offered here $SU(3) \rightarrow SU(2)$ reduction the colored flux tube obtained above is a pure quantum object in the SU(3) Yang - Mills theory.

5 Discussion

The main result of this letter is that the confinement problem in QCD and the gauge symmetry breakdown can be closely connected. In our calculations the symmetry breaking term is presented by the mass term $(m^2)^{ab} A_{\mu}^b$. Of course this term is inserted by hand. The problem for the derivation of this term from the first principles is the absence of non-perturbative technique for the calculations in quantum field theories with strong interactions. Although some calculations 10 show us that such terms can be obtained in quantum gauge theories on the perturbative level. One can indicate the probable connection of our problem with a Coleman - Weinberg mechanism 11 in $\lambda \phi^4$-theory. This mechanism gives us an additional term which changes an initial potential term and makes from its something like a Higgs potential (Mexico hat). The origin of this phenomenon is the presence of the nonlinear potential $\lambda \phi^4$ in the theory. One can suppose that the similar mechanism will work in the quantum gauge theories where there is the potential term like to $(A)^4$, but of course that here we have to use a non-perturbative technique which probably will be similar to the quantization method applied by Heisenberg to a non-linear spinor field 12.

Another interesting property of the colored flux tube solution is the discreteness of masses $m_{1,2}$ and $\phi_{\infty}$ which gives the finiteness of the corresponding solution. We can make an assumption that it is connected with the fact that these parameters are quantum corrections and as it takes place in any quantum theory something should be quantized.

In conclusion we would like to note that the derived here the longitudinal electric filed is essentially nonlinear: it is not the gradient of some function but it appears from the nonlinear term $g^{abc} A_{\mu}^a A_{\nu}^b$ of the non-Abelian tensor $F_{\mu\nu}^{A}$, i.e. the Maxwell eletrodynamics can not have such flux tube solutions even with broken gauge symmetry. One can say that the nonlinearity and symmetry breakdown of non-Abelian gauge theories probably are the key ingredients of confinement problem in QCD.
6 Acknowledgments

I am very grateful to the Alexander von Humboldt foundation for the financial support of this work and H.-J. Schmidt for the invitation to research in Potsdam University.

References

[1] H.B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
[2] M. A. Kneipp and P. Brockill, Phys. Rev. D64, 125012 (2001).
[3] G. 't Hooft, in High Energy Physics, edited by A. Zichichi, Editorice Compositori, Bologna, 1975.
[4] S. Mandelstam, Phys. Rep, C23, 245 (1976).
[5] V. Dzhunushaliev, “Colored flux tube in the Euclidean spacetime”, hep-ph/0307156.
[6] Y. N. Obukhov, Int. J. Theor. Phys. 37, 1455 (1998).
[7] V. Dzhunushaliev and D. Singleton, “Confining solutions of SU(3) Yang-Mills theory,” Contribution to 'Contemporary Fundamental Physics', ed. Valeri Dvoeglazov (Nova Science Publishers). In *Dvoeglazov, V.V. (ed.): Photon and Poincare group* 336-346. hep-th/9902076.
[8] V. Dzhunushaliev, “Flux tube dressed with color electric $E^a_{\rho,\phi}$ and magnetic $H^a_{\rho,\phi}$ fields”, hep-ph/0306203. V. Dzhunushaliev, “Electric/magnetic flux tube on the background of magnetic/electric field”, hep-th/0302215.
[9] V. Dzhunushaliev and D. Singleton, Mod. Phys. Lett., A18, 955(2003); V. Dzhunushaliev and D. Singleton, “Effective 't Hooft-Polyakov monopoles from pure SU(3) gauge theory”, hep-ph/0306202.
[10] V. E. Lemes, M. S. Sarandy and S. P. Sorella, J. Phys. A 36, 7211 (2003); D. Dudal and H. Verschelde, “On ghost condensation and Abelian dominance in the Maximal Abelian Gauge”, hep-th/0209025.
[11] S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).
[12] W. Heisenberg, *Introduction to the unified field theory of elementary particles.*, Max - Planck - Institut für Physik und Astrophysik, Interscience Publishers London, New York, Sydney, 1966; W. Heisenberg, Nachr. Akad. Wiss. Göttingen, N8, 111(1953); W. Heisenberg, Zs. Naturforsch., 9a, 292(1954); W. Heisenberg, F. Kortel und H. Mütter, Zs. Naturforsch., 10a, 425(1955); W. Heisenberg, Zs. für Phys., 144, 1(1956); P. Askali and W. Heisenberg, Zs. Naturforsch., 12a, 177(1957); W. Heisenberg, Nucl. Phys., 4, 532(1957); W. Heisenberg, Rev. Mod. Phys., 29, 269(1957).