Symplectic quantization for reducible systems

J. Barcelos-Neto\textsuperscript{*} and M.B.D. Silva
Instituto de Física
Universidade Federal do Rio de Janeiro
RJ 21945-970 - Caixa Postal 68528 - Brasil

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Abstract

We study an extension of the symplectic formalism in order to quantize reducible systems. We show that a procedure like ghost-of-ghost of the BFV method can be applied in terms of Lagrange multipliers. We use the developed formalism to quantize the antisymmetric Abelian gauge fields.

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\textsuperscript{*}Electronic mails: ift03001@ufrj and barcelos@vms1.nce.ufrj.br
1 Introduction

Systems where some of its constraints (first or second class) are not independent are said to be reducible. The use of the Dirac method [1] to quantize these systems requires the elimination of dependent constraints which leads to a final set where just independent ones are present. We mention that this is an opposit procedure related to the quantization method due to Batalin, Fradkin and Vilkovisky (BFV) [2, 3]. There, the elimination of dependent constraints (supposing that they are first-class) cannot be done. This would imply a lower number of ghost fields. In consequence, it would be impossible to achieve any final covariant result. The solution of this problem in the BFV method is the introduction of extra ghosts, called ghost-of-ghost, one for each relation among constraints. An interesting example involving this subject is the quantization of superparticle and superstrings, where the covariant quantization is just achieved after the introduction of an infinite tower of ghost-of-ghosts [4].

Recently, Faddeev and Jackiw [5] have remind us of the possibility of using the symplectic formalism [6] as an alternative quantization method to the Dirac one. They use it to quantize the chiral-boson [7], which is not a constrained system in the symplectic point of view, eventhough it is in the Dirac case [8]. Incidentally we mention that one of the interesting aspects of the symplectic method is that primary constraints of the Dirac formalism are not constraints in the symplectic case. That is why chiral bosons have no constraints in the symplectic formalism. Another example is the fermionic Dirac theory [9]. On the other hand, when the Dirac constraint theory has second-class constraints, these may also be constraints in the symplectic procedure (we call them true constraints). In this case, the symplectic formalism can also be applied since the phase space is properly modified in order to consistently define the symplectic tensor [10, 11, 12].

The purpose of the present paper is to consider the symplectic formalism for reducible systems. The elimination of dependent constraints, as is done in the Dirac case, is not possible to be applied here because this would imply in a reduction of the number of Lagrange multiplies, which are important in the absorbtion of superfluous degrees of freedom (The symplectic tensor can only be defined after the elimination of these superfluous fields). The solution we propose for this problem is to parallel the BFV formalism. We also introduce Lagrange multipliers-of-Lagrange multipliers for each reducible relation. As an example, we quantize the antisymmet-ritic Abelian tensor gauge fields. The consistency of our final results are verified in the comparation with the ones obtained by using the Hamiltonian Dirac procedure [1].

Our paper is organized as follows. In Sec. 2 we make a brief review of the symplectic formalism including the case with (true) constraints. At the end we address the way of dealing with the reducible case. In Sec. 3 we apply the developed formalism to the antisymmetric Abelian gauge fields.
2 Brief review of the simplectic formalism with constraints

Let us consider a dynamical system evolving in a phase space and described by the canonical set of variables \((q_i, p_i)\) \((i = 1, \ldots, N)\). These satisfy the fundamental Poisson brackets

\[
\{q_i, q_j\} = 0 = \{p_i, p_j\}, \\
\{q_i, p_j\} = \delta_{ij}.
\]

Considering that the bracket of some quantity \(A(q, p)\) with anything satisfies the relation

\[
\{A(q, p), \cdots\} = \frac{\partial A}{\partial q_i} \{q_i, \cdots\} + \frac{\partial A}{\partial p_i} \{p_i, \cdots\},
\]

and using the fundamental brackets (1), one can write the usual Poisson bracket relation involving two arbitrary quantities, say \(A(p, q)\) and \(B(p, q)\), as

\[
\{A(p, q), B(p, q)\} = \frac{\partial A}{\partial q_i} \{q_i, q_j\} \frac{\partial B}{\partial q_j} + \frac{\partial A}{\partial q_i} \{q_i, p_j\} \frac{\partial B}{\partial p_j} \\
+ \frac{\partial A}{\partial p_i} \{p_i, q_j\} \frac{\partial B}{\partial q_j} + \frac{\partial A}{\partial p_i} \{p_i, p_j\} \frac{\partial B}{\partial p_j},
\]

\[
= \frac{\partial A}{\partial q_i} \frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q_i}.
\]

In order to figure out the simplectic structure, we write coordinates and momenta in just one set of \(2N\) generalized coordinates which we denote by \(y^\alpha\) \((\alpha = 1, \cdots, 2N)\), in such a way that

\[
y^i = q_i, \\
y^{N+i} = p_i.
\]

Now, the fundamental Poisson brackets simply read

\[
\{y^\alpha, y^\beta\} = \epsilon^{\alpha\beta},
\]

where the antisymmetric tensor \(\epsilon^{\alpha\beta}\) is given by the matrix

\[\text{We develop this section just using discrete coordinates. The extension to the continuous case can be done in a straightforward way.}\]
\[
\begin{pmatrix}
\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}
\end{pmatrix}, \tag{6}
\]

and \( I \) is the \( N \times N \) identity matrix. The Poisson bracket involving two arbitrary quantities \( A(y), B(y) \) can also be directly obtained in terms of the tensor \( \epsilon^{\alpha\beta} \)

\[
\{A(y), B(y)\} = \partial_\alpha A \{y^\alpha, y^\beta\} \partial_\beta B ,
\]

\[
= \epsilon^{\alpha\beta} \partial_\alpha A \partial_\beta B , \tag{7}
\]

where \( \partial_\alpha = \partial/\partial y^\alpha \). The quantity \( \epsilon^{\alpha\beta} \) guarantees the usual antisymmetry of the Poisson bracket relations (for the bosonic case).

By the observation of (6) one might infer that the general form of the brackets in the case with constraints is

\[
\{y^\alpha, y^\beta\} = f^{\alpha\beta}(y) , \tag{8}
\]

where \( f^{\alpha\beta} \) is an antisymmetric tensor and that must be nonsingular (notice that this last property is also verified by \( \epsilon^{\alpha\beta} \)). Its inverse is the simplectic tensor related to the constrained system. We mention that the simplectic tensor can be used as a metric (simplectic metric) that raises and lowers indices in the simplectic manifold

\[2\]

Sympletic manifolds whose metric is \( \epsilon^{\alpha\beta} \) are related to systems without constraints. This is the case, for example, of the self-dual fields \[5, 7\].

Here, one can point out what are the general fundamentals of Dirac and simplectic methods. The first one is developed by looking at the left-hand side of expression (8), that is to say, it tries to generalize the Poisson brackets by including the constraints, until a final form is reached where constraints can be taken as strong relations \[1\]. In the case of the simplectic formalism, constraints (which are usually in a small number than in the Dirac method) are used to make deformation in the geometrical structures in order that the simplectic tensor can be consistently defined.

The FJ formalism deals with first-order Lagrangians. It is opportune to mention that this is not a serious restriction because all systems we know, described by quadratical Lagrangians, can always be set in the first-order formulation. This is achieved by extending the configuration space with the introduction of auxiliary fields. These are usually the momenta, but this is not necessarily so \[10, 11\]. We mention that systems with higher derivatives can be described in this same way \[12\].

Let us consider a system described by a first-order Lagrangian like

\[
L = a_\alpha(y) \dot{y}^\alpha - V(y) , \tag{9}
\]

where \( a_\alpha \) and \( V \) are functions of the self-dual fields \[5, 7\].
where $y^\alpha$ is a set of $2N$ coordinates. $y^{i+N}$ can be the momenta or other auxiliary quantities introduced in order to render the Lagrangian the first-order condition. From the expression above, the Euler-Lagrange equation of motion reads

$$f_{\alpha\beta} \dot{y}^\beta = \partial_\alpha V, \quad (10)$$

where

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha. \quad (11)$$

If $\det (f_{\alpha\beta}) \neq 0$, one can solve (10) for the velocities $\dot{y}^\alpha$, i.e.

$$\dot{y}^\alpha = f^{\alpha\beta} \partial_\beta V, \quad (12)$$

where $f^{\alpha\beta}$ is the inverse of $f_{\alpha\beta}$. This is the simplectic tensor reported earlier and, in fact, one can show that $f^{\alpha\beta}$ are the Dirac bracket between the coordinates $y^\alpha$, $y^\beta$.

An interesting and instructive point occurs when the quantity $f_{\alpha\beta}$ is singular. In this case one cannot identify it as the simplectic tensor and, consequently, the brackets structure of the theory cannot be consistently defined either. This means that the system, even in the FJ approach, has constraints (true constraints). One way to solve this problem is to follow the standard Dirac formalism. However, this can also be achieved by working in a geometric manner. In this case, we use the constraints to conveniently deform the singular tensor quantity in order to obtain another tensor that may be nonsingular. If this occurs, this new quantity can be identified as the simplectic tensor of the theory. Let us briefly review the developments of the simplectic method when there are true constraints involved.

Let us denote the above mentioned singular quantity by $f_{\alpha\beta}^{(0)}$, and suppose that it has, say, $M$ ($M < 2N$) zero modes $v_m^{(0)}$, $m = 1, \cdots, M$, i.e.

$$f_{\alpha\beta}^{(0)} v_m^{(0)\beta} = 0. \quad (13)$$

The combination of (10) and (13) gives

$$\ddot{v}_m^{(0)\alpha} \partial_\alpha V^{(0)} = 0. \quad (14)$$

This may be a constraint. Let us suppose that this actually occurs (we shall discuss the opposite case soon). Usually, constraints are introduced in the potential part of the Lagrangian by means of Lagrange multipliers. Here, in order to get a deformation in the tensor $f_{\alpha\beta}^{(0)}$ we introduce them into the kinetic part instead. This is done
by taking the time-derivative of the constraint and making use of some Lagrange multiplier.

These Lagrange multipliers, which we denote by $\lambda_m^{(0)}$, enlarge the configuration space of the theory. This permit us to identify new vectors $a^{(1)}_\alpha$ and $a^{(1)}_m$ as

$$
\begin{align*}
  a^{(1)}_\alpha &= a^{(0)}_\alpha + \lambda_m^{(0)} \partial_\alpha \Omega^{(0)}_m, \\
  a^{(1)}_m &= 0,
\end{align*}
$$

where $\Omega^{(0)}_m$ are the constraints obtained from (14). In consequence, one can now introduce the tensor quantities

$$
\begin{align*}
  f^{(1)}_{\alpha\beta} &= \partial_\alpha a^{(1)}_{\beta} - \partial_\beta a^{(1)}_{\alpha}, \\
  f^{(1)}_{\alpha m} &= \partial_\alpha a^{(1)}_m - \partial_m a^{(1)}_\alpha = -\partial_m a^{(1)}_\alpha, \\
  f^{(1)}_{mn} &= \partial_m a^{(1)}_n - \partial_n a^{(1)}_m = 0.
\end{align*}
$$

Here $\partial_m = \partial/\partial \lambda^m$. If $\det f^{(1)} \neq 0$, where $f^{(1)}$ is a matrix which also involves the Lagrange multipliers, then we have succeeded in eliminating the constraints. If not, one should repeat the procedure above as many times as necessary.

It may also occur that we arrive at a point where we still obtain a singular matrix and the corresponding zero modes do not lead to any new constraint. This is the case, for example, of gauge theories. At this point, if we want to define the simplectic tensor, we have to introduce some gauge condition. For details, see reference [10].

In order to have a clearer idea of the problem to be circumvented in the case of reducible systems, we emphasize the role played by the Lagrange multipliers in the symplectic formalism. They absorb some superfluous degrees of freedom of the theory. This and the deformation of the symplectic structure make possible the definition of the symplectic tensor, which is the final goal of the formalism. Its inverse makes the bridge to commutators of the quantum sector. So, when constraints are not independent and we eliminate some of them as in the Dirac procedure, we have also a lower number of Lagrange multipliers. This implies that some superfluous degrees of freedom cannot be eliminated and, consequently, we are not able to identify the symplectic tensor.

We solve this problem by paralleling the BFV procedure. We introduce new Lagrange multipliers (Lagrange multiplier-of-Lagrange multiplier) for each relation among the constraints and manipulate these as in the previous case.

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3It is well-known that constraints satisfy the consistency condition of not evolving in time, that is to say, if $\Omega$ is a constraint we have that $\dot{\Omega}$ is also a constraint. Another point is that one could, instead, take the time derivative of the Lagrange multiplier.
3 An example involving reducible contraints

Let us consider the case of abelian tensor gauge fields described by the Lagrangian

$$L = -\frac{1}{6} F_{\mu\nu\rho} F^{\mu\nu\rho},$$  \hspace{1cm} (17)

where $F_{\mu\nu\rho}$ is a totally antisymmetric tensor which can be written in terms of potential fields $A_{\mu\nu}$ (also antisymmetric) by the relation

$$F_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu}. \hspace{1cm} (18)$$

This tensor is invariant (and consequently the Lagrangian above) under the gauge transformations

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu. \hspace{1cm} (19)$$

The reducible aspect of this theory can be envisaged from the fact that if we choose the gauge parameter $\Lambda_\mu$ as the derivative of some scalar quantity we will obtain that $A_{\mu\nu}$ do not change under the gauge transformation.

This theory was already discussed from the Dirac [13] and the BFV [3] points of view. In order to use the symplectic method, it is necessary to write the Lagrangian (17) in the first order notation. First we rewrite it as

$$L = -\frac{1}{2} \dot{A}_{ij} \dot{A}^{ij} - 2 \partial_i A_{j0} \dot{A}^{ij} - \partial_i A_{0j} \partial^j A^{0i} + \partial_i A_{0j} \partial^j A^{0i} - \frac{1}{2} \partial_i A_{jk} \partial^j A^{ik}. \hspace{1cm} (20)$$

It is opportune to mention that the symplectic method is essentially a noncovariant one. A first order version of this Lagrangian reads

$$L^{(0)} = \pi_{ij} \dot{A}^{ij} + \frac{1}{2} \pi_{ij} \pi^{ij} + 2 \partial_i A_{j0} \pi^{ij} - \frac{1}{2} \partial_i A_{jk} \partial^j A^{ik} + \partial_i A_{jk} \partial^j A^{ik}, \hspace{1cm} (21)$$

where $\pi^{ij}$ is the auxiliary field (here it is the momentum conjugate to $A_{ij}$). Its equation is a constraint relation whose equation of motion leads to the initial Lagrangian (17).

From expression (21) one identifies the quantities

$$a^{(0)A}_{ij} = \pi_{ij},$$
$$a^{(0)A}_{0j} = 0,$$
$$a^{(0)\pi}_{ij} = 0, \hspace{1cm} (22)$$
and obtains the tensors

\[
    f^{(0)A\pi}_{ijkl}(x, y) = \frac{\delta a^{(0)\pi}_{kl}(y)}{\delta A_{ij}(x)} - \frac{\delta a^{(0)}_{ij}(x)}{\delta \pi_{kl}(y)},
\]

\[
    = -\frac{1}{2} (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) \delta^{(3)}(\vec{x} - \vec{y}).
\]  

(23)

The remaining terms are zero. We then construct the matrix

\[
    f^{(0)} = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & 0 & -\frac{1}{2}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) \\
    0 & \frac{1}{2}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) & 0
    \end{pmatrix} \delta^{(3)}(\vec{x} - \vec{y}).
\]  

(24)

We easily see that it is singular. Let us consider that a zero mode has the general form \( \tilde{v}^{(0)} = (v_k, u_{kl}, \omega_{kl}) \). We thus get the equations

\[
    (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) \omega_{kl} = 0 \Rightarrow \omega_{kl} = 0,
\]  

(25)

\[
    (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) u_{kl} = 0 \Rightarrow u_{kl} = 0,
\]  

(26)

and the quantities \( v_k \) remain indeterminated. Consequently, from the equation

\[
    \int d^3\vec{x} v_k(\vec{x}) \frac{\partial}{\partial A_{0k}(\vec{x})} \int d^3\vec{y} V^{(0)} = 0,
\]  

(27)

that is the corresponding in the continuous of the expression (14) with \( V^{(0)} \) given by

\[
    V^{(0)} = -\frac{1}{2} \pi_{ij} \pi^{ij} - 2\partial_i A_{j0}\pi^{ij} + \frac{1}{2} \partial_i A_{jk}\partial^j A_{jk} - \partial_i A_{jk}\partial^j A^{ik},
\]  

(28)

one has

\[
    \int d^3\vec{x} v_k(\vec{x}) \partial_i \pi^{ik} = 0.
\]  

(29)

Since \( v_k \) is a generic function of \( \vec{x} \), we obtain the constraints

\[
    \partial_i \pi^{ij} = 0,
\]  

(30)

which plays the role of Gauss’ laws in this extension of the electromagnetic theory. In consequence of the property of reducibility we have that the constraints above are not independent. Notice that \( \partial_i \partial_j \pi^{ij} = 0 \).
Now, what we have to do is to introduce the constraints \[ (30) \] into the kinetic part of the Lagrangian by means of Lagrange multipliers. Since these constraints are not independent, we restrict the Lagrange multipliers by means of some convenient relation. In virtue of the similarity with the BFV case, we restrict the Lagrange multipliers as

\[ \partial^i \lambda_i = 0 \] (31)

and use another Lagrange multiplier (Lagrange multiplier-of-Lagrange multiplier) to also introduce it into the kinetic part of the Lagrangian. We then have

\[ L^{(1)} = \pi_{ij} \dot{A}^{ij} - \dot{\pi}_{ij} \partial^j \lambda^i - \dot{\lambda}^i \partial_i \eta - V^{(1)}, \] (32)

where

\[ V^{(1)} = -\frac{1}{2} \pi_{ij} \pi^{ij} + \frac{1}{2} \partial_i A_{jk} \partial^j A^{ik} - \partial_i A_{jk} \partial^j A^{ik}, \] (33)

where the fields \( A_{0j} \) were absorbed in \( \dot{\lambda}_j \). Now, the new coefficients are

\[
\begin{align*}
a^{(1)}_{A}{}_{ij} &= \pi_{ij}, \\
a^{(1)}_{\pi}{}_{ij} &= \frac{1}{2} (\partial_j \lambda_i - \partial_i \lambda_j), \\
a^{(1)}_{\lambda}{}_{i} &= -\partial_i \eta, \\
a^{(1)}_{\eta} &= 0
\end{align*}
\] (34)

and the corresponding matrix \( f^{(1)} \) reads

\[
f^{(1)} = \begin{pmatrix}
0 & -\frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) & 0 & 0 \\
\frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) & 0 & \frac{1}{2} (\delta_{jk} \partial_l - \delta_{ik} \partial_j) & 0 \\
0 & \frac{1}{2} (\delta_{il} \partial_k - \delta_{ik} \partial_l) & 0 & \partial_k \\
0 & 0 & \partial_k & 0
\end{pmatrix} \delta(3)(\vec{x} - \vec{y}),
\] (35)

where rows and columns follow the order \( A, \pi, \lambda, \eta \). This matrix is still singular. Let us consider a zero mode like \( \tilde{v}^{(1)} = (v_{kl}, u_{kl}, \omega_{k}, h) \). This will be actually a zero mode if

\[
\begin{align*}
u_{ij} &= 0, \\
v_{ij} &= \frac{1}{2} (\partial_j \omega_i - \partial_i \omega_j), \\
\partial_i h &= 0.
\end{align*}
\] (36)
Using the expression (14) and the above conditions in order to obtain possible new constraints, we get
\[
\int d^3\vec{x} \left[ v_{ij}(\vec{x}) \frac{\delta}{\delta A_{ij}(\vec{x})} + \omega_i(\vec{x}) \frac{\delta}{\delta \lambda_i(\vec{x})} + \eta(\vec{x}) \frac{\delta}{\delta \eta(\vec{x})} \right] \int d^3\vec{y} V^{(1)} = 0 ,
\]
\[
\Rightarrow 0 = 0 . \quad (37)
\]

As one sees, the zero modes do not lead to any new constraint. This is a characteristic of gauge theories. So, in order to try to obtain a nonsingular matrix, we fix the gauge. Let us then choose the corresponding of the Coulomb gauge of the electromagnetic theory, i.e.
\[
\partial_i A^{ij} = 0 . \quad (38)
\]

Of course, since the Gauss' law constraints are not independent, the gauge fixing conditions above cannot be independent either. We are going to proceed as in the previous case, that is to say, we introduce the constraints (38) into the kinetic part of the Lagrangian by means of Lagrange multipliers and restrict these as in (31). The result is
\[
\mathcal{L}^{(2)} = (\pi_{ij} - \partial_i \xi_j) A^{ij} - \dot{\pi}^{ij} \partial_i \lambda_j - \dot{\lambda}^i \partial_i \eta - \dot{\xi}^i \partial_i \rho - V^{(2)} , \quad (39)
\]
where
\[
V^{(2)} = -\frac{1}{2} \pi_{ij} \pi^{ij} + \frac{1}{2} \partial_i A_{jk} \partial^j A^{ik} . \quad (40)
\]
The term \( \partial_i A_{jk} \partial^j A^{ik} \) of the previous Lagrangian was absorbed into the kinetic part of \( \mathcal{L}^{(2)} \). Once more we identify the new coefficients to calculate \( f^{(2)} \). The final result reads
\[
f^{(2)} =
\begin{pmatrix}
0 & -\frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) & 0 & \frac{1}{2} (\delta_{jk} \partial_i - \delta_{ik} \partial_j) & 0 & 0 \\
\frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) & 0 & \frac{1}{2} (\delta_{jk} \partial_i - \delta_{ik} \partial_j) & 0 & 0 & 0 \\
0 & \frac{1}{2} (\delta_{il} \partial_k - \delta_{ik} \partial_l) & 0 & 0 & \partial_i & 0 \\
\frac{1}{2} (\delta_{il} \partial_k - \delta_{ik} \partial_l) & 0 & 0 & 0 & 0 & \partial_i \\
0 & 0 & \partial_k & 0 & 0 & 0 \\
0 & 0 & 0 & \partial_k & 0 & 0 \\
\end{pmatrix} \cdot \delta^{(3)}(\vec{x} - \vec{y}) \quad (41)
\]
where rows and columns follow the order \( A, \pi, \lambda, \xi, \eta, \rho \). This matrix is not singular. It is the symplectic tensor of the constrained theory. From its inverse we directly identify the brackets
\[
\{ A_{ij}(\vec{x}, t), \pi_{kl}(\vec{y}, t) \} = \frac{1}{2} \left[ \left( \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk} \right) + \frac{1}{\nabla^2} \left( \delta_{ik} \partial_j \partial_l - \delta_{jk} \partial_i \partial_l \right) - \delta_{il} \partial_j \partial_k - \delta_{jl} \partial_i \partial_k \right] \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{ A_{ij}(\vec{x}, t), \xi_k(\vec{y}, t) \} = -\frac{1}{\nabla^2} \left( \delta_{ik} \partial_j - \delta_{jk} \partial_i \right) \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{ \pi_{ij}(\vec{x}, t), \lambda_k(\vec{y}, t) \} = \frac{1}{\nabla^2} \left( \delta_{ik} \partial_j - \delta_{jk} \partial_i \right) \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{ \lambda_i(\vec{x}, t), \xi_j(\vec{y}, t) \} = \frac{2}{\nabla^2} \left( \delta_{ij} + \partial_i \partial_j \right) \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{ \lambda_i(\vec{x}, t), \eta(\vec{y}, t) \} = -\partial_i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{ \xi_i(\vec{x}, t), \rho(\vec{y}, t) \} = -\partial_i \delta^{(3)}(\vec{x} - \vec{y}).
\]

The only really important bracket is the first one, that involves the physical fields of the theory. It is a Dirac bracket in a sense that it is strongly satisfied by the constraint relations. Since there is no problem with ordering operators we can directly transform it to commutator. Other brackets are not necessarily Dirac brackets. The role of Lagrange multipliers are just to enlarge the configuration space in order to make possible the definition of the symplectic tensor, but they do not play any physical role in the theory.

We mention that the first bracket above is the same one obtained in [13] where the Dirac formalism was used.

4 Conclusion

We have considered the use of the symplectic formalism for reducible systems. We have followed a similar procedure of the BFV one where it is necessary the introduction of ghosts-of-ghosts. Here we have also introduced Lagrange multipliers-of-Lagrange multipliers in order that the symplectic tensor could be defined. We have applied the formalism to the antisymmetric tensor gauge field as an example.

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