Microscopic study of superdeformation in the A=150 mass region

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Abstract

The cranked Hartree-Fock-Bogoliubov method presented in previous studies of superdeformed bands in Hg and Pb isotopes is applied to the study of superdeformed bands in $^{150}$Gd, $^{152}$Dy, $^{151}$Dy and $^{151}$Tb. The same density-dependent zero-range pairing interaction used in the A=190 mass region leads to a similarly good agreement with the experimental data. In particular, our results confirm the importance of a correct treatment of pairing correlations in $^{150}$Gd to reproduce its experimental dynamical moment of inertia. The quasi-particle spectra obtained for all the nuclei studied here are compatible with the experimental results obtained for excited bands in this mass region. The quality of our results opens the possibility of studying microscopically very subtle phenomena like the properties of identical bands.

1 Introduction

The $A=150$ mass region is the first one where superdeformed (SD) bands have been observed. The nucleus $^{152}$Dy at the center of this region benefits optimally of the stability generated by the shell effects at $N=86$ and $Z=66$. For large angular momenta, these allow the formation of a SD well. However, at low spins, quantal stabilization is not strong enough to compensate the increase of surface energy associated with the large quadrupole deformation. The physical situation is therefore different from that encountered in the $A=190$ mass region where one believes that SD bands disappear for values of spins close to $10\hbar$ or sometimes even smaller. In $^{152}$Dy and neighbouring nuclei, the SD bands which have been detected exist probably only at spins larger than $20\hbar$. In this angular momentum range, pairing correlations are strongly weakened by the fast rotation and thus expected to play a minor role. They become necessary only for a detailed understanding of the data.

For this reason the SD bands of this region can be classified within models which neglect pairing correlations. This was first attempted by Bengtsson et al [1] using the Nilsson-Strutinsky approximation. In particular, they showed that the characteristic behavior of the second moment of inertia $J_2$ of SD bands could be understood in terms of the number of occupied intruder orbitals ($N=7$ for neutrons and $N=6$ for protons). There are indeed close analogies between the frequency dependence of the $J_2$’s of SD bands of different nuclei when they can be assigned configurations with the same number of occupied intruder orbitals. Nazarewicz et al [2] have used a phenomenological mean-field based on a Woods-Saxon potential with pairing correlations treated within the Bogoliubov method and shell effects calculated according to the Strutinsky procedure. They have investigated the importance of shape changes as well as that of pairing correlations. and confirmed the adequacy of a classification of SD bands according to the occupancy of intruder orbitals. They have also shown that in some nuclei, like $^{150}$Gd, pairing correlations are necessary to explain the frequency dependence of the dynamical moment of inertia. Their study has been further extended within the Woods-Saxon Strutinsky shell-correction method in the work of Satula et al [3] where the self-consistency between shape changes and dynamical pairing correlations is fully taken into account. This work also includes an approximate treatment of particle number projection by means of the Lipkin-Nogami prescription. To analyze the same self-consistency,
Shimizu et al. [4] have used a model in which pairing correlations are treated dynamically within the RPA approximation. All these studies indicate that among the nuclei of this mass region, $^{150}$Gd is one for which pairing should play an important role.

In a series of previous papers [5, 6, 7], we have presented a cranked Hartree-Fock-Bogoliubov method in which pairing correlations are treated dynamically thanks to the Lipkin-Nogami prescription. A Skyrme force was used in the particle-hole channel and a seniority interaction in the particle-particle channel [5, 6]. The introduction of a density-dependent zero-range interaction to describe pairing correlations greatly improved the agreement with the experimental data [7]. Transition energies were reproduced within 10 keV over more than 10 transitions in the four isotopes $^{190}$Hg, $^{192}$Hg, $^{194}$Hg, $^{194}$Pb. $^{150}$Gd provides an opportunity to test the properties of this pairing interaction at much higher spins, in a regime where pairing correlations are weakened. For this reason, we have decided to study pairing effects in this nucleus and in the magic SD nucleus $^{152}$Dy where their effect should be less important. We will also test the quality of the neutron and proton single-particle spectra that we obtain in two odd nuclei, $^{151}$Dy and $^{151}$Tb.

In the present study we have used a Skyrme effective force with the SkM* parametrization to describe the interaction between nucleons in the particle-hole channel. In the particle-particle channel, several treatments of pairing correlations have been tested. First they have been neglected; in the following, the corresponding Hartree-Fock (HF) results will be referred to as (i). Then, three types of Hartree-Fock-Bogoliubov (HFB) calculations have been performed. In two of them, a seniority interaction is used in the particle-particle channel. In one case the strength is adjusted to reproduce the two neutron separation energies ($S_{2n}$) at zero spin (ii). The intensity of the other seniority force is smaller and has been chosen to reproduce the behaviour of the moment of inertia of $^{150}$Gd for $\hbar \omega \approx 500$keV (iii). The third kind of HFB calculation is performed with the density-dependent zero-range interaction obtained in a previous study of the lead-mercury SD bands [7] (iv).

## 2 The nucleus $^{150}$Gd

On Figure 1, we have plotted the $^{150}$Gd dynamical moment of inertia as a function of the rotational frequency $\hbar \omega$. The HF calculation leads to an underestimation of the moment of inertia at all spins. In this region of high angular momentum, the introduction of pairing correlations induces an increase of the $J_2$ moment of inertia which allows to locally reproduce the data. For instance, this is realized by the seniority force (iii) for $\hbar \omega \approx 500$keV. However, the agreement is only obtained in the near vicinity of this angular frequency. The first experimental point (see Fig. 1) is significantly above the smooth trend of $J_2$, which indicates a change in the structure of the band. This feature is not reproduced by this HFB calculation (curve iii). From the result (ii) one sees that the qualitative behavior of $J_2$ near $\hbar \omega = 400$keV can be obtained by a further increase of the seniority pairing strength. Unfortunately this is obtained at the cost of an increase of $J_2$ which leads to a systematic overestimation. It appears that the density dependent pairing interaction (iv) leads to a much better global agreement: the HFB calculation reproduces the
rapid decrease at low frequency while the magnitude at large $\hbar \omega$ remains close to data.

Figure 2 shows the pairing energies obtained in the three HFB calculations. For the density-dependent interaction, the absolute value of the pairing energies for neutrons are larger than those for protons. With seniority forces we obtain the opposite result. This can be understood as more neutrons orbitals are localized in the vicinity of the nuclear surface than protons ones. We note also that the two interactions (ii) and (iv), which lead to a rapid decrease of the dynamical moment of inertia at low frequencies, are also those which display the fastest decrease of the neutron pairing energy at least up to $\hbar \omega \approx 0.55$ MeV. Beyond this value of the angular frequency the behaviour of the pairing energy is much smoother.

Figures 3 and 4 give the particle routhians for neutrons and protons respectively. The left-hand side of the figures corresponds to the pure HF case. The right-hand side shows the routhians defined as the eigenstates of the cranked mean-field obtained with the density-dependent zero-range interaction (iv). The seniority interactions (ii) and (iii) lead to very similar spectra. The second N=7 neutron orbital is very close to the Fermi level. It is fully occupied in the HF calculation. On the other hand, the diagonal matrix element of the density matrix in the basis associated with the cranked mean-field is a rapidly varying function of $\hbar \omega$ when pairing correlations are included. Above the Fermi level these spectra show some significant differences with those obtained with either a modified oscillator [9, 10] or a Woods Saxon [2]. For instance, the [512]3/2 and [523]5/2 neutron orbitals are pushed up at higher excitation energies while the [514]9/2 and the [402] 5/2 orbitals are very close. On the other hand all calculations agree qualitatively for orbitals below the Fermi level which are the one related to SD bands in the lighter Gd isotopes. The proton routhians are nearly identical in all calculations. They differ only by the relative positions of the [301]1/2 and [660]1/2 orbitals. In our calculation, the first level is closer to the Fermi level, while their order is reversed with both the modified oscillator and the Woods Saxon potentials. The first orbitals above the Fermi level are in good agreement.

Nazarewicz et al [2] relate the low frequency behavior of the moment of inertia to a crossing between the N=7 neutron quasiparticles around $\hbar \omega = 0.4$ MeV. According to our quasiparticle (qp) plots for the density-dependent zero-range interaction (see Figure 3), an interaction between the [770]1/2 and [761]3/2 can also take place for $\hbar \omega$ just under 0.4 MeV. Unfortunately, we could not extend the calculations below the lowest frequency shown on Figure 3 for numerical reasons. In any case the strength of this interaction would be large, of the order of 0.65 MeV. We find a similar value for the energy difference between the [770]1/2 and [761]3/2 qp’s in the HFB calculation with the seniority interaction of the largest strength (ii). On the contrary with the seniority interaction (iii) which predicts a smooth behavior of $J_2$ for low angular momentum frequencies, the energy difference between the two qp’s is always larger than 1.6 MeV. At high spins our calculation finds a very gradual alignment of the N=7 qp orbitals.

As we have discussed, our HF and HFB calculations predict rather different dependences on $\hbar \omega$ of the occupation of the [770]1/2 orbital in the canonical basis. As deformation is mostly driven by this orbital, an increase of pairing correlations which diminishes its occupation affects the deformation of the nucleus. This is
reflected in the evolution of the quadrupole moment against rotational frequency (see Figure 6). In the HF case, the quadrupole moment is steadily decreasing. For the three HFB calculations, the deformation increases very rapidly below a frequency which depends on the intensity of pairing correlations. Above this frequency, the quadrupole moment decreases with a slope steeper than that found in the HF case. The rise of $J_2$ at low frequencies appears therefore related to a complicated interplay between pairing correlations and changes of deformation with rotation. The value of the charge quadrupole moment that we obtain (16 eb) is close to that measured for band 2 of $^{149}$Gd (15.6±0.3 eb) which has the same number of intruder orbitals as the yrast band of $^{150}$Gd [3]. It is also significantly smaller than the value obtained by the calculation of ref [4] (16.9 eb).

Four excited SD bands have been observed in $^{150}$Gd. The nature of their qp excitation content and in particular the occupied intruder levels can be surmised from the analogies between the behavior of their moment of inertia with those of the yrast bands of neighbouring nuclei. For band 2, the standard assignment of a N=6 and a [301]1/2 proton qp excitations is compatible with our calculations. Note however that the qp’s corresponding to intruder states are strongly mixed so that their hole or particle nature is difficult to identify. Band 3 and 4 [11] are signature partners, with transition energies close to 1/4 and 3/4 of the yrast band of $^{149}$Gd. Therefore, the 2 qp excitations have been assigned to a N=7 neutron orbital and either a [514]9/2 or a [402]5/2 excitation. A possible interaction between band 4 and band 2 has made it more plausible that bands 3 and 4 have negative parity. Our qp diagrams are compatible with both assignements. However in our calculations the [402]5/2 qp’s interact with the [651]1/2 (s=+1) or the [642]5/2 (s=-1) qp’s around 0.55 MeV. No sign of such an interaction is visible in the data. Nevertheless, one must remember that the creation of qp’s may strongly modify the mean-field and shift the frequencies at which interactions occur. Band 5 [12] exhibits a pronounced discontinuity around $\hbar\omega=0.5$ MeV. Above this frequency, the moment of inertia presents similarities with that of $^{152}$Dy. It is therefore interpreted as a 4 proton excitation involving two N=6 and two [301]1/2 qp’s. Our results are not incompatible with these data. The HFB calculation using the surface peaked delta pairing obtains indeed an interaction between two positive signature N=6 qp’s at the correct energy. However, introducing such an excitation into the HFB calculations would strongly modify the qp spectrum. It is also probably as realistic to consider the $^{152}$Dy qp spectrum (see Figure [1]) in which the excitation of the [301]1/2 qp’s is larger than 1.5 MeV. This large value does not support the proposed scenario. However, because going from $^{152}$Dy to $^{150}$Gd implies a lowering of the Fermi level by approximately 2.0 MeV, the [301]1/2 excitation may become more competitive.

3 The nucleus $^{152}$Dy

The dynamical moment of inertia of $^{152}$Dy is plotted on Figure 7. Calculations have been performed for three of the four cases considered for $^{150}$Gd: (i), (ii) and (iv). In the last two cases, as in all the results reported in this work, the pairing strengths were kept equal to those used in the study of $^{150}$Gd. In all cases, the order of magnitude of the moment of inertia is slightly overestimated.
In the HF calculation, both the neutron and proton contributions to the moment of inertia are almost flat. When pairing correlations are included, the neutron contribution to $J_2$ is always decreasing with $\hbar \omega$, while the proton contribution is slightly increasing up to 0.5 MeV. The nearly flat moment of inertia results therefore from a quasi cancellation of both contributions. Such a cancellation between the neutron and proton contributions to $J_2$ has also been obtained in ref [3], although in this case the proton contribution to $J_2$ is increasing faster than in our calculations. With the Skm* parametrisation of the Skyrme force, it does not seem possible to decrease the mean value of the moment of inertia of $^{152}$Dy and thus obtain a better agreement with data by modifying only the pairing interaction. The HF moment of inertia shows the same quality of agreement with data as obtained by Satula [2] and as the relativistic Hartree calculation of Konig and Ring [13]. The calculation of Shimizu et al [4] leads also to an overestimation of the dynamical moment of inertia, probably related to a deficiency of the Nilsson potential parametrisation. In our HFB calculations we find that the pairing energies are always non zero, although their smooth variations with angular frequency lead only to small contributions to the moment of inertia.

The variation of the charge quadrupole deformations as a function of the rotational frequency for the three cases shown on Figure 7 is plotted on Figure 8. As for $^{150}$Gd, the HF quadrupole moment is always decreasing, while the pairing correlations lead to an increase of this moment. However, the effect is less pronounced for Dy than for Gd. The mean deformation that we obtain (17.5 eb) is very close to that measured for band 4 of $^{149}$Gd and for the lowest band of $^{152}$Dy [8]. It is lower than the value obtained in ref [2] (18.9 eb).

On Figures 9, 10 and 11 are plotted the energies of the neutron and proton routhians and of the quasiparticules respectively, as a function of the rotational frequency. The differences between the Gd and Dy neutron routhians are larger in the HF than in the HFB calculations. The main difference is the slightly deeper position of the [770]1/2 levels with respect to the Fermi level, so that the corresponding qp has always a positive energy. This prevents the crossing between the two N=7 qp’s. Our neutron diagrams differ from the cranked Woods-Saxon ones by the position of the [411]1/2 levels, which are lower and more separated from the N=6 levels in our calculation than in ref [2]. Similarly we find a [301]1/2 proton routhian more bound and not as close to the intruder N=6 levels.

Our particle and qp diagrams are compatible with the experimental data on excited bands in $^{152}$Dy [4] for bands based on 2qp neutron excitations (band 4, 5 and 6). Our results also support the contention that band 3 could be constructed on the excitation of the [651]3/2 and [530]1/2 proton qp’s. At the deformation of the yrast SD bands, the [301]1/2 proton qp is very excited. Because we note that at larger deformations this routhian becomes closer to the Fermi level we expect an energetically competitive excited band based on this qp to be more deformed. A simultaneous excitation of the rapidly downsloping and deformation driving N=7 proton orbital could realize this effect. Therefore the assignment of such a qp excitation content to experimental band 2 is plausible according to our spectrum. Note that our N=7 proton particle state is more excited than in calculations based on Saxon-Woods potentials and is always above the [530]1/2 qp routhian of negative signature. This feature is in agreement with the larger intensity observed for band
3 than for band 2.

4 The nucleus $^{151}$Dy

The neutron spectrum of $^{152}$Dy (Figure 3) indicates that the ground SD band of $^{151}$Dy should correspond to a hole in the $[770]1/2$ intruder state. Excited bands can be obtained by making another hole in one of the neutron states located 1.0 MeV below this intruder. The spectra displayed on Figure 9 are compatible with the assignments of two of the four excited bands observed experimentally [13] to the neutron $[642]5/2$ (band 2) and $[411]1/2$ (band 4) orbitals. The large splitting between the two $[411]1/2$ levels is also in agreement with the fact that the signature partner of band 4 is not observed. The other bands should correspond to the $[651]1/2$ orbitals. Based on these excitations on can construct up to three bands of the same parity and of both signatures with orbitals which are strongly mixed by rotation. The study of these excited bands is rather delicate and will not be performed here.

The HF moment of inertia of the yrast SD band of $^{151}$Dy is displayed on Figure 12, it shows a smooth decrease as a function of $\hbar \omega$ which is slightly more pronounced than that found for $^{152}$Dy. This feature that our calculation appears to share with all other self-consistent calculations is in contradiction with the behavior which could be expected based on the qualitative discussion of ref.[1]. It disagrees also with data which shows a slow regular increase of $J_2$ over the entire range of observed angular momenta. It has been suggested that a gradual mixing with another configuration might resolve the discrepancy between calculation and data [2]. In principle pairing correlations can introduce such a mixing. A HFB calculation with the seniority force (ii) shows indeed a sensitivity of $J_2$ to pairing. On the other hand as in ref.[2] we find a curve with a maximum near $\hbar \omega =$500keV. An analysis of the contributions to $J_2$ shows that the neutron contribution is almost flat (instead of decreasing in $^{152}$Dy in agreement with the discussion of ref.[4]). The maximum is therefore due to the proton contribution. Finally we note that the charge quadrupole moment predicted by our HFB calculation is of 17 eb, a value smaller than the 18 eb obtained in ref.[2].

5 The nucleus $^{151}$Tb

Although many bands have been detected in this nucleus we have limited our calculations to the first three bands which will allow a test of our proton spectrum. Since pairing correlations are not expected to play a major role, we have only performed a HF study.

From the consideration of the proton routhian spectrum of $^{150}$Gd (Figure 4) it appears favorable to define bands in $^{151}$Tb by the creation of qp based on the $[651]3/2$ orbitals. On the other hand, the qp routhian spectrum of $^{150}$Gd suggests that at high spins, one considers also the qp associated with the orbital $[301]s=-$. Finally a fourth band could be build on the $[301]s=+$ qp. In fact four of the eight bands that have been observed [16, 17] fit rather well this assignment pattern. The excitations which have been used to construct the three bands studied in this work are presented on figure 13. This figure shows the effect that each breaking of a pair generates in the proton routhian spectrum. In the limit of small
angular frequency, the time-reversal-symmetry breaking always lead to a situation in which the empty orbit is more bound than its occupied signature partner. Apart from the two routhians of the broken pair, the rest of the spectrum is not very much modified.

The moments of inertia obtained for all these configurations are shown on the right-hand side of Figure 14 together with the experimental data (left). The overall agreement with data is quite good. Experimentally, band 2 which is contracted on the [301]1/2 s=+ orbital and has therefore the same proton intruder content as $^{152}$Dy is identical to the yrast band of this nucleus. Our calculation finds indeed that the $J_2$ curves of these two bands are almost the same. This confirms the special nature of the [301]1/2 orbital. It shows that mean-field calculations are relevant for the problem of identical bands whether high-K (see ref. [18]) or low-K (this work) are involved. However, for the [301]1/2 orbital, our calculation does not lead to identical bands because we do not predict the correct relative alignment. A discussion of this effect and the first indications on how it may be related to the time-odd components of the mean-field can be found in ref. [19]. The data on the relative magnitudes of the three moment of inertia are very well reproduced. Our calculations predict close values for the charge quadrupole moments of band 2 and of the identical yrast band of $^{152}$Dy. On the other hand the values of these moments for bands 1 and 3 are 1 eb lower. At $\hbar \omega$ around 0.7 MeV, the excitation energies of bands 2 and 3 with respect to band 1 are of the order of 1.1 MeV and 0.8 MeV respectively. This is in good agreement with the feeding pattern of band 3, but is probably too large for band 2.

6 Conclusion

In this work, we have studied the lowest superdeformed bands of four nuclei in the A=150 mass region. We have compared data with results obtained by the cranked Hartree-Fock method on the one hand and with the cranked Hartree-Fock-Bogoliubov method on the other hand. In the latter case, we have tested three different forces in the pairing channel.

Our calculations reproduce well the magnitude of moments of inertia. With the notable exception of $^{151}$Dy they also describe correctly the evolution of $J_2$ versus angular momentum. Our results confirm also the general belief that pairing correlations are much less important in this mass region than for Hg and Pb isotopes. Only for $^{150}$Gd do we find that pairing correlations are crucial to describe the behaviour of its moment of inertia for angular momenta between 30 and 40$\hbar$. We nevertheless find that even at the highest observed spins where pairing correlations are much weakened, they still affect the magnitude of the $J_2$. We also checked that the moment of inertia of $^{150}$Gd depends sensitively on the features of the pairing force. In particular the steeply down going slope of $J_2$ is correctly reproduced with a zero-range density-dependent while a very poor agreement with experiment is obtained with seniority (monopole) interactions.

Our discussion of excited bands in Gd and Dy isotopes shows that a Skyrme force like Skm* leads to a correct description of the properties of the high spin behavior of the mean-field. The calculated charge quadrupole moments of the SD bands are
very close to the latest Eurogam values [8]. We believe however that this agreement is especially significant for the variation of the quadrupole moments between SD bands of $^{150}$Gd and $^{152}$Dy since the experimental uncertainties on the absolute values remain large. The lowest qp excitations that we predict are compatible with the characteristics of the excited bands found in nuclei of this mass region. In $^{151}$Tb, a calculation of the three lowest bands leads to a correct prediction of the relative values of the moments of inertia within this nucleus and also with neighboring isotopes.

Following encouraging results obtained in the A=190 mass region on the identity of the rotational bands associated with specific quasi-particle content of the mean-field [18], the present work reproduces also the identity of the ground state band of $^{152}$Dy and of band 2 of $^{151}$Tb. This gives us some confidence on the ability of mean-field studies at dealing with the identical-band question. On the other hand, a recent study [19] has shown that a detailed understanding may require a thorough investigation of the time-odd terms in the Skyrme functional.

A remarkable outcome of this study is that the parametrisation of the density-dependent zero-range interaction, adjusted on the dynamical moment of inertia of $^{194}$Pb at low spins, without any modification, leads to very satisfactory results in the A=150 mass region and at higher spins.

This parametrisation shown to be successful on the neutron-poor side of the stability valley remains to be tested in cases that are more sensitive to the isospin degree of freedom. Although we have chosen a force with identical neutrons and protons pairing strengths, we believe that a relative variation of a few percents would not significantly modify our predictions concerning the moments of inertia of SD bands of the nuclei studied in this work. The study of other regions of the mass table is thus necessary to better determine the isospin behavior of the pairing force. The present work can be extented in several other directions. For instance, we still have to test our pairing interaction on the properties of rotational bands at normal deformations and at low spins. In particular, it is important to address the question of low-spin identical bands as it should be more sensitive to the interplay between the mean-field and the pairing channels. In addition the study of a carefully selected subset of the observed excited SD bands should also provide a demanding test of the ability of our zero-range density-dependent pairing interaction to describe bands with various quasi-particle excitations leading to very different dependence of $J_2$ on the angular frequency. Work along these directions is in progress.

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Figures

Figure 1: Comparison of the experimental dynamical moment of inertia of $^{150}\text{Gd}$ (dots) as a function of angular velocity with the results of four mean-field calculations: pure HF (full line), HFB with a seniority pairing adjusted to the $S_{2n}$ (dashed line), with a reduced seniority pairing (see text) (dash-dotted line), and with a surface-active delta pairing (dotted line).

Figure 2: Neutron and proton pairing energies of the $^{150}\text{Gd}$ ground SD band obtained with different pairing interactions: two seniority pairing interactions (see text), case (ii) - dashed line-, case (iii) - dash-dotted line-, zero-range interaction - dotted line-. 

Figure 3: Neutron particle routhians of $^{150}\text{Gd}$ calculated without pairing (left) and with the zero-range pairing interaction (right). The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).
Figure 4: Proton particle routhians of $^{150}$Gd calculated without pairing (left) and with the zero-range pairing interaction (right). The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 5: Neutron (left) and proton (right) quasi particle routhians of $^{150}$Gd calculated with the zero-range pairing interaction. The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 6: Charge quadrupole moments of the SD band of $^{150}$Gd calculated in the HF case (full line), with the seniority pairing interactions (dashed line and dashed-dotted lines), and with the surface-active delta pairing (dotted line).

Figure 7: Comparison of the experimental dynamical moment of inertia of $^{152}$Dy (dots) as a function of angular velocity with the results of calculations without pairing (full line) with a seniority pairing pairing (dashed line) and surface-active delta pairing (dotted line).

Figure 8: Charge quadrupole moments of the $^{152}$Dy SD band calculated in the HF case (dotted line), with a seniority pairing interaction (dashed line), and with the surface-active delta pairing (dotted line).

Figure 9: Neutron particle routhians of $^{152}$Dy calculated without pairing (left) and with the surface-active (right) pairing interaction. The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 10: Proton particle routhians of $^{152}$Dy calculated without pairing (left) and with the surface-active (right) pairing interaction. The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 11: Neutron (left) and proton (right) quasi particle routhians of $^{152}$Dy calculated with the surface-active pairing interaction. The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 12: Comparison of the $^{151}$Dy experimental dynamical moment of inertia (open squares) as a function of angular velocity with the results of a pure HF calculation (full line) and a HFB calculation (case ii).
Figure 13: Proton particle routhians of $^{151}$Tb calculated without pairing for three different HF configurations. The filled and empty orbitals near the Fermi level are marked with filled and open circles respectively. The (parity, signature) combinations are indicated by a full line (+,+), a dashed line (+,−), a dot-dashed line (−,+), and a dotted line (−,−).

Figure 14: Dynamical moment of inertia of three bands in $^{151}$Tb as a function of angular velocity. Left: experiment, right: HF calculation. For comparison, the $J_2$ of $^{152}$Dy ground SD band is also shown.
$J_z (\hbar^2/\text{MeV})$ versus $\hbar \omega$ (MeV) for $^{150}$Gd.

Experimental data (circles), Hartree-Fock (solid line), $G=12.5$ (dashed line), $G=14.4$ (dotted line), and delta-pairing (dashed-dotted line) are shown.
**a) HF**

$$150^{\text{Gd}} \quad \text{(protons)}$$

- [523]$7/2$
- [532]$5/2$
- [660]$1/2$
- [651]$3/2$
- [411]$3/2$
- [404]$9/2$
- [523]$7/2$

**b) δ–pairing**

- [411]$3/2$
- [651]$3/2$
- [301]$1/2$
- [660]$1/2$
- [301]$1/2$
- [301]$1/2$

$\epsilon_F$
$J_2$ ($\hbar^2/\text{MeV}$)

$\hbar\omega$ (MeV)

- $152$ Dy

- Experiment
- H.F.
- $G=14.4$
- $\delta$-pairing

$25$

$50$

$75$

$100$
\[ ^{152}\text{Dy} \quad \text{(neutrons)} \]

\( a) \text{ HF} \)

\( b) \text{ } \delta \text{-pairing} \)

\[ e_i \text{ (MeV)} \]

\[ \hbar \omega \text{ (MeV)} \]

\[ [521]3/2 \]
\[ [514]9/2 \]
\[ [402]5/2 \]
\[ [761]3/2 \]
\[ [770]1/2 \]
\[ [642]5/2 \]
\[ [651]1/2 \]
\[ [413]5/2 \]
\[ [411]1/2 \]

\[ [651]1/2 \]
\[ [642]5/2 \]
\[ [413]5/2 \]
\[ [411]1/2 \]
The diagrams represent the energy levels for protons and neutrons in the actinide region. The left diagram shows the neutrons, while the right diagram shows the protons. The energy levels are labeled with specific states such as $p[402]5/2$, $h[514]1/2$, and $h[770]1/2$. The vertical axis represents $e_{qp}$ (MeV), and the horizontal axis represents $\hbar \omega$ (MeV).
$J_{\frac{2}{3}} (\hbar^2/\text{MeV})$ vs $h\omega (\text{MeV})$

- **HF**
- **Exp.**
- **G=14.4**

$^{151}\text{Dy} 1$
