Two-dimensional topological solitons in rectangular magnetic dots.

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A general approach allowing to find the analytical expressions for equilibrium magnetic structures in small and flat magnetic nano-sized cylinders of arbitrary shape made of soft magnetic material is presented. The resulting magnetization distributions are two-dimensional topological solitons and have a non-zero topological charge. The approach is illustrated here on an example of a thin rectangular particle.

Introduction. The small magnetic cylindrical elements (dots) made of a soft magnetic material became the focus of attention due to their potential applications in the Magnetic Random Access Memory (MRAM) devices. These elements can be made of different shapes and sizes (usually of the order of 10 – 100 nm) and found to display a variety of magnetic structures (equilibrium distributions of magnetization vector field). The structures are solutions of non-linear and non-local (integral partial differential) equations of micromagnetics and were a recent subject of extensive study by finite element computer simulations (see e.g. [1] and references therein).

Recently, a new method allowing to treat the problem of finding the magnetic structures of thin dots analytically became available [2,3]. It is based on the existence of the well defined hierarchy of energies in small flat magnetic cylinders made of soft magnetic material [3]. The resulting analytical magnetization distributions minimize the exchange energy exactly, have no magnetic charges on the cylinder faces (except in a few points of topological singularities) and no magnetic charges on the cylinder sides.

Briefly, the general recipe of Ref. [3] can be formulated as follows. In a thin ferromagnetic cylinder (with thickness $L \leq L_E$, where $L_E = \sqrt{C/M_S}$ is the exchange length, $C$ is the exchange constant, $M_S$ is the saturation magnetization of the material) the dependence of the magnetization distribution $\vec{m}(\vec{r}) = \vec{M}(\vec{r})/M_S$ on the coordinate $Z$ perpendicular to the cylinder face can be neglected (so that $\vec{m}(X,Y)$, where $X$ and $Y$ are Cartesian coordinates on the planes parallel to the cylinder face). Also, because the magnetization vector has a fixed length $|\vec{m}| = 1$ it has only two independent components. Both these facts make it convenient to parametrize the magnetization vector field by a complex function $w(z)$ of the complex variable $z = X + iY$, $i = \sqrt{-1}$, so that $m_Z = (1 - w(z)\overline{w(z)})/(1 + w(z)\overline{w(z)})$, $m_X + im_Y = 2w(z)/(1 + w(z)\overline{w(z)})$, where a line over symbol means the complex conjugation. The requirement of minimum face magnetic charges makes the function $w(z)$ non-analytical (in the sense of the Cauchy-Riemann conditions), but it is possible to express it through another, analytical, function $f(z)$. For a particle with corners this can be done as

$$w(z) = \begin{cases} f(z) & |f(z)| \leq 1 \\ f(z)/\sqrt{f(z)f(\overline{z})} & 1 < |f(z)| \leq d \\ f(z)/d & |f(z)| > d, \end{cases} \tag{1}$$

where $d$ is an arbitrary constant defining the size of regions of the out-of-plane magnetization vector at topological singularities at cylinder corners.

Let us denote the conformal transformation of the interior of the unit circle $|t| \leq 1$ to $z \in D$ ($D$ is the set of points of the $X-Y$ plane belonging to the cylinder face) as $z = T(t)$, according to Riemann theorem such transformation exists for any simply connected region $D$. Then, the analytical function $f(z)$ corresponding to the magnetization vector field with no magnetic charges on the cylinder side is given by

$$f(z) = (tc + A - 4t^2)T'(t), \quad t \to T^{-1}(z), \tag{2}$$

where $c$ and $A$ are arbitrary real and complex constants respectively, $T^{-1}(z)$ is the inverse function of $T(t)$. The function $T^{-1}(z)$ exists and is unique because the transformation is conformal. The expression (2) contains essentially all the solutions minimizing the exchange energy and having no magnetic charge on the cylinder sides. The magnetostatic energy of the resulting solutions is also very small compared to the exchange energy (but this depends on the particle size) and it can be expected (for sufficiently small particles) not to introduce any new solution types but just to modify slightly.

a. The magnetic structures in a rectangular particle.

The shape of the particle enters the resulting distribution through the conformal transformation $T(t)$. The conformal transformation of the unit circle to the rectangle is

$$T(t) = C_1\left(\int_0^t \frac{1}{\sqrt{1 + t^4 - 2t^2\cos 2\delta}} \, dt + C_0\right), \tag{3}$$

which is a particular case of Schwartz-Christoffel integral. This integral can be expressed analytically through the elliptic integral of the first kind $F(\phi, m)$. The parameter $0 \leq \delta \leq \pi/4$ controls the aspect ratio of the rectangle, $\delta = \pi/4$ corresponds to square, $\delta \approx 0.172426$ to the 2:1 rectangle. The constants $C_1$ and $C_0$ were chosen in such
a way that the height of the rectangle is 1 and its upper left corner is situated at the point with \( X, Y = 0 \). For the \( 2 : 1 \) rectangle shown in the pictures \( C_1 \approx 0.63189 \) and \( C_0 \approx -1.849 + i0.787 \). This transformation is visualized in Fig. 1.

The magnetic structures in the particle are thus

\[
f(z) = (ctc + A - \overline{Ar}^2) \frac{1}{\sqrt{1 + t^4 - 2t^2 \cos 2\delta}}, \quad t \to T^{-1}(z),
\]

where \( C_1 \) is absorbed into \( c \) and \( A \).

To find the values of constants \( c \) and \( A \) it is required to minimize the total energy of the particle (including magnetostatics). This will be done elsewhere, let us now just treat these constants as independent parameters and analyze the possible types of solutions qualitatively.

As it was already shown for the cases of circular and triangular cylinders (and it is applicable to any soliton of type (2)) there are two basic classes of solutions depending on whether \(|c| < 2|A|\) or not. The first class, corresponds to the case \(|c| > 2|A|\) is a vortex, the sign of \( c \) controls the direction of the rotation of the magnetization vector in it. If the particle is square the centered vortex (Fig 2a) is stable. In elongated particles a state with displaced vortex (Fig 3, see also Figs 3,5 in Ref. [1]) can also be. The other class of solutions contained in (2) corresponds to the two skyrmions (hedgehogs) bound to the cylinder sides. If the particle is not of the circular shape positions of the hedgehogs on its boundary are not equivalent. Particularly, for particle with corners the total energy is lower if the hedgehogs are located in the corners (however states with hedgehogs at the middle of the edges can be metastable). For rectangle (because of the symmetry) there are two possibilities: either the hedgehogs are located at the neighbouring corners (Fig 2b), or at the opposite ones (Fig 3b). Both these structures, usually called “C” and “S”, already appeared in numerical micromagnetic simulations for rectangles.

Because of the completeness of the solution (2) no other shapes of the equilibrium magnetization vector field can be expected in the rectangular particle if it is sufficiently small (but larger then the single domain size, where absence of magnetic charges is not required) and if it is made of sufficiently isotropic magnetic material, which are the basic assumptions of the presented theory.

The expression (4) also describes the metastable states when the \( c \) and \( A \) deviate from their equilibrium values. These states can be useful for considering the magnetization reversal, excitations and the stability of the equilibrium solutions (see e.g. [4]).

Summary. A simple analytical framework for finding the profiles of two-dimensional topological solitons in small flat ferromagnetic cylinders of the arbitrary shape was presented. As an example, the case of the rectangular cylinder was considered.

The resulting magnetization vector fields may find their use in magnetic microscopy (for fitting the signal from the probe by changing the parameters \( A \) and \( c \)), and for building the phase diagrams of small magnetic particles outlining the regions of the particle geometrical parameters where each of the presented equilibrium solutions correspond to the ground state of the system.

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\[\text{FIG. 1. Conformal mapping of the unit circle to the rectangle, dots on both plots correspond to each other to show how the interior is transformed.}\]
FIG. 2. The equilibrium configurations of the magnetization vector (its projection to the cylinder face is shown) in the rectangular particle \([0, 1]\) with \(d = 6\): a) vortex \(c = 2, A = 0\); b) displaced vortex \(c = 2, A = 1.9\); c) "C" structure \(c = 2, A = 11.6\); d) "S" structure \(c = 0, A = 6e^{0.172426}\).