Modeling the longitudinal dynamics of electric multiple units with Xcos/Scilab software

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Abstract. During the traction and braking of trains, substantial longitudinal dynamic forces might occur in couplers. The method of modeling these forces for two different electric multiple units (EMUs) is presented in this study. For the EMUs consisted of independent vehicles, each of which rests on two bogies, computer simulations were carried out. Simulations were also executed for EMUs with Jacobs bogies, which support bodies of two adjacent carriages. The dynamic modeling of vibration protection train systems includes nonlinearities.

1. Introduction
The longitudinal train dynamics (LTD) studies the relationship between the motion of all types of rolling stock vehicles and forces acting on them in the longitudinal track direction [1]. Wu et al. [2] have documented in historical perspective an approach to LTD issues since 1831. That year, for the first time, Mohawk and Hudson had observed the dynamic behavior of 'slack' actions, referred to as a series of train vehicle impacts (during state transitions of connections—from tension to compression—or vice versa).

The EMUs can be divided into two categories [3]. The first EMU category includes trains consist of independent vehicles, each of which rests on two bogies, as shown in figure 1. Vehicles of such EMUs, which do not share bogies, can be longer. However, they need more bogies and must be equipped with additional anti-overlap systems between the two adjacent vehicles. The anti-overlap systems reduce the consequences of a railway accident.

Figure 1. Sketch of an exemplary train belonging to the first EMU category.

The second EMU category includes trains consist of such vehicles, each of which rests on one Jacob bogie shared between two adjacent carriages, at least, as shown in figure 2. The advantage of this category is that such EMU type has a reduced number of bogies in the train. Besides, shared bogies
allow to reduce lateral oscillations, less rolling, and pitching at high speeds. On the other hand, shorter vehicle bodies are recommended to meet the limit requirements concerning the smallest permissible radii of railway track curves.

Figure 2. Sketch of an exemplary train belonging to the second EMU category.

The longitudinal dynamic behavior of the EMU is described upon the assumption that there is no vertical or lateral movement of vehicles. Therefore the EMU model expressed by a system of differential equations has a much simpler structure.

2. Modeling of the LTD of the EMUs

Determination of the dynamic models for the EMUs plays an important role. Such models make it possible to carry out both simulations of their motion and the design of cruise control algorithms. The analysis of the dynamic models can be helpful during designing prototype EMU carriages as well as their couplers.

There are two basic methods for the derivation of EMU motion equations. The first method is based on the Lagrange formulation and is conceptually systematic and understandable. The second method based on the Newton–Euler formulation yields the EMU model in a recursive form. Hence, it is computationally more efficient since it exploits the typically open structure of the kinematic chain formulated for the multiple-unit train. Among the methods mentioned here, the first one is chosen.

With Lagrange formulation [4], variables: \( x_i \) \((i = 1, 2, 3)\) termed as generalized coordinates, describe the element positions of both EMU models. The first model shown in figure 1 consists of three separate carriages with the masses: \( m_i \). Note that \( k_i \) and \( c_i \) \((j = 1, 2)\) are the stiffness coefficients and the damping coefficients, respectively. At the same time, it is worth noting that the additional subscript ‘\( \alpha \)’ indicates that only linear characteristics of springs and dampers within ranges of stroke limitations and velocity-jump constraints are taken into account, respectively, as presented in [5]. Hence, the kinetic energy \( T_A \), the potential energy \( V_A \), and the dissipation energy \( D_A \) become

\[
T_A = \frac{1}{2} m_1 \ddot{x}_1^2 + \frac{1}{2} m_2 \ddot{x}_2^2 + \frac{1}{2} m_3 \ddot{x}_3^2 \quad (1a)
\]

\[
V_A = \frac{1}{2} k_1 (x_1 - x_2)^2 + \frac{1}{2} k_2 (x_2 - x_3)^2 \quad (1b)
\]

\[
D_A = \frac{1}{2} c_1 (\dot{x}_1 - \dot{x}_2)^2 + \frac{1}{2} c_2 (\dot{x}_2 - \dot{x}_3)^2 \quad (1c)
\]

The first model of three degrees of freedom expressed by second-order differential equations takes the following form:

\[
m_1 \dddot{x}_1 + c_{1(1)} (x_1 - \dot{x}_2) + k_{1(1)} (x_1 - x_2) = F_1 - R_1^A \quad (2a)
\]

\[
m_2 \dddot{x}_2 - c_{1(1)} \dddot{x}_1 + (c_{1(1)} + c_{2(1)}) \dddot{x}_2 - c_{2(1)} (x_2 - \dot{x}_3) - k_{1(1)} \dddot{x}_1 + (k_{1(1)} + k_{2(1)}) \dddot{x}_2 - k_{2(1)} (x_2 - x_3) = F_2 - R_2^A \quad (2b)
\]

\[
m_3 \dddot{x}_3 + c_{2(1)} (x_3 - \dot{x}_2) + k_{2(1)} (x_3 - x_2) = F_3 - R_3^A \quad (2c)
\]
In the above system of equations, $F_i^A$ are the non-conservative tractive forces assigned to each carriage. $R_i^B$ are the total drag force, which resist the motion of each carriage, i.e., they act opposite to the direction of train velocity. In addition, the spring coupler forces are $F_i^{s(A)} = k_i^A(x_1 - x_2)$ and $F_2^{s(A)} = k_2^A(x_2 - x_3)$, and the damping coupler forces are $F_1^{d(A)} = c_i^A(x_1 - x_2)$ and $F_2^{d(A)} = c_2^A(x_3 - x_2)$.

Similarly, the second EMU model shown in figure 2 consists of three separate carriages with the masses: $m_i^B$ and four bogies of the masses: $m_i^B$ $(k = 1, 2, 3, 4)$. To describe the motion of the rigid body elements of this EMU, the kinetic energy $T_B$, the potential energy $V_B$, and the dissipation energy $D_B$ should be first defined

$$T_B = \frac{1}{2} m_1^B x_1^2 + \frac{1}{2} m_2^B x_2^2 + \frac{1}{2} m_3^B x_3^2 + \frac{1}{2} m_4^B x_4^2$$

$$V_B = \frac{1}{2} k_1^B (x_1 - q_1)^2 + \frac{1}{2} k_2^B (q_1 - x_2)^2 + \frac{1}{2} k_3^B (x_2 - q_2)^2 + \frac{1}{2} k_4^B (q_2 - x_3)^2 + \frac{1}{2} k_5^B (x_3 - q_3)^2 + \frac{1}{2} k_6^B (q_3 - x_4)^2 + \frac{1}{2} k_7^B (x_3 - q_4)^2 + \frac{1}{2} k_8^B (q_4 - x_3)^2$$

$$D_B = \frac{1}{2} c_1^B (\dot{x}_1 - \dot{q}_1)^2 + \frac{1}{2} c_2^B (\dot{q}_1 - \dot{x}_2)^2 + \frac{1}{2} c_3^B (\dot{x}_2 - \dot{q}_2)^2 + \frac{1}{2} c_4^B (\dot{q}_2 - \dot{x}_3)^2 + \frac{1}{2} c_5^B (\dot{x}_3 - \dot{q}_3)^2 + \frac{1}{2} c_6^B (\dot{q}_3 - \dot{x}_4)^2 + \frac{1}{2} c_7^B (\dot{x}_3 - \dot{q}_4)^2 + \frac{1}{2} c_8^B (\dot{q}_4 - \dot{x}_3)^2$$

The second model of seven degrees of freedom expressed by second-order differential equations takes the following form:

$$m_1^B \ddot{x}_1 + \left(c_1^B + c_2^B\right) (\dot{x}_1 - \dot{q}_1) + c_3^B (x_1 - q_2) + \left(k_1^B + k_2^B\right) (x_1 - q_1) + k_3^B (x_1 - q_2) = -R_1^B$$

$$m_2^B \ddot{x}_2 + c_4^B (\dot{x}_2 - \dot{q}_2) + c_5^B (x_2 - q_3) + k_4^B (x_2 - q_2) + k_5^B (x_2 - q_3) = -R_2^B$$

$$m_3^B \ddot{x}_3 + c_6^B (x_3 - q_3) + \left(c_7^B + c_8^B\right) (\dot{x}_3 - \dot{q}_4) + k_6^B (x_3 - q_3) + k_7^B (x_3 - q_4) = -R_3^B$$

$$m_4^B \ddot{q}_1 + \left(c_1^B + c_2^B\right) (\dot{x}_1 - \dot{q}_1) + \left(k_1^B + k_2^B\right) (q_1 - x_1) = F_1^B$$

$$m_5^B \ddot{q}_2 + c_3^B (\dot{q}_2 - \dot{x}_1) + c_4^B (\dot{x}_2 - \dot{q}_2) + k_3^B (q_2 - x_1) + k_4^B (q_2 - x_2) = F_2^B$$

$$m_6^B \ddot{q}_3 + c_5^B (\dot{q}_3 - \dot{x}_2) + c_6^B (\dot{x}_3 - \dot{q}_3) + k_5^B (q_3 - x_2) + k_6^B (q_3 - x_3) = F_3^B$$

$$m_7^B \ddot{q}_4 + c_7^B (\dot{q}_4 - \dot{x}_3) + k_7^B (q_4 - x_3) = F_4^B$$

In equations (4), the forces of second suspension springs are $F_2^{s(B)} = k_2^B (x_2 - q_2)$, $F_3^{s(B)} = k_3^B (x_3 - q_3)$, and $F_4^{s(B)} = k_4^B (x_4 - q_4)$. It is assumed that $k_1^B = k_2^B$, $k_3^B = k_4^B$, $k_5^B = k_6^B$, and $k_7^B = k_8^B$. The forces of second suspension dampers are $F_2^{d(B)} = c_2^B (\dot{x}_2 - \dot{q}_2)$, $F_3^{d(B)} = c_3^B (\dot{x}_3 - \dot{q}_3)$, and $F_4^{d(B)} = c_4^B (\dot{x}_4 - \dot{q}_4)$. Note that $c_1^B = c_2^B = c_3^B = c_4^B$, $c_5^B = c_6^B$, and $c_7^B = c_8^B$.

Although various external loads act on each train carriage, only these loads, related solely to the longitudinal direction, are significant to model the LTD. When the EMU moves on along a straight line, the rolling resistance forces are speed independent. However, increased rolling resistance at high speed also includes some components of laminar airflow, which is proportional to the speed. The aerodynamic drag of the EMU is proportional to speed-squared. As indicated in [5, 6], the aerodynamic drag of EMU consists of separate drag forces with a different value for each carriage. Therefore, the total drag force, $R_i$, acting only on the $ith$ carriage can be expressed as follows:

$$R_i = A_i + B_i \nu + C_i \nu^2$$

(5)
where $A_i \equiv (A_i + \mathbb{B}_i \eta_i)W_i$ and $B_i \equiv C_i W_i$, in which $W_i$ is the weight of the $i$th carriage and $A_i$ is the rolling resistance coefficient assigned to the $i$th carriage, $\mathbb{B}_i$ is its bearing resistance coefficient, $C_i$ is its flange resistance (in the case of a curved track), and $\eta_i$ denotes the axles’ number counted for the $i$th carriage. In equation (5), $C_i$ is the aerodynamic resistance coefficient of the $i$th carriage. Because equation (5) contains a nonlinear expression of the train velocity, therefore systems of differential equations (2) and (4) are also nonlinear (as discussed in [7]).

Figure 3. Cruise control system for the EMU.

3. Xcos LTD models of EMUs

Computer simulation of dynamic systems requires solving systems of differential equations, such as given by equations (2) and (4). The numerical integration of differential equations is a universal way to solve them. A software package of Scilab, called Xcos, for modeling and simulation of explicit and implicit dynamical systems comes with a variety of numerical solvers used to integrate the differential equations describing dynamic systems. Xcos provides a lot of functionalities to establish proper cruise control strategies.

Configurations of the EMUs can include a diversity of combinations of power carriages, which are usually not used in the same way as passenger carriages in a locomotive-hauled train. The power carriages are self-propelled with motorized bogies. They can be comparable to electric locomotives in a certain sense. Usually used connections between carriage bodies of the EMUs are couplers (i.e., mechanisms used to connect rolling stock in a train) or Jacobs’ trailer bogies.

Figure 3 shows an Xcos diagram of the master configuration of the cruise control system built for the EMUs. The Xcos diagram has the super-block labeled as the LTD model with one input data port and four output data ports. The role of this super-block illustrated in figure 4 is to encapsulate nested Xcos diagrams. The nested Xcos diagrams, corresponding to equations (2), are shown in figure 5. This figure illustrates the set of block diagrams $A_1$, $A_2$, and $A_3$, which solve the following differential equations: (2a), (2b), and (2c), respectively. The GOTO/FROM blocks shown in figure 4 transport signals from one to another block without connecting them physically.

Figure 6 shows the super-block of the LTD model of the EMU with two Jacobs’ bogies. This super-block is nested in the system diagram of the cruise control system given in figure 3. The nested Xcos
diagrams, corresponding to equations (4), are shown in figure 7. This figure depicts the set of block diagrams denoted by B1, B2, B3, B4, B5, B6, and B7, which solve the following differential equations: (4a), (4b), (4c), (4d), (4e), (4f) and (4g), respectively.

Figure 4. Super-block from figure 3 representing the LTD model for the EMU, which consists of three separate vehicles.
Figure 5. Set of block diagrams to solve the differential equations (2a), (2b), and (2c).
Figure 6. Super-block from figure 3 representing the LTD model for the EMU, which rests on two Jacob bogies shared between two adjacent vehicles.
Figure 7. Set of block diagrams to solve the following differential equations: (4a), (4b), (4c), (4d), (4e), (4f) and (4g).
Figure 7. (continued) Set of block diagrams to solve the following differential equations: (4a), (4b), (4c), (4d), (4e), (4f) and (4g).
4. Conclusions
The paper presents the method of modeling the longitudinal dynamics of two different EMUs with Xcos/Scilab software. This method was used to determine longitudinal dynamic forces in railway train couplers. These forces are most of all generated during the traction and braking of trains. A detailed discussion of the results of computer simulations is included in [5]. The simulations were carried out for both types of EMUs. The first type of EMU was composed of independent vehicles, each of which rests on two bogies. The second one was with Jacobs bogies, which support bodies of two adjacent carriages.

As all commercial software for scientific computations Xcos/Scilab software offers Runge-Kutta methods, that is a family of implicit and explicit iterative methods for solving ordinary differential equations. The direct or iterative solving methods for ordinary differential equations are selected depending on their linear or nonlinear forms. Implicit methods require finding a solution by solving an equation involving both current and subsequent system states. On the other hand, explicit methods compute the subsequent state of the system from the current one. The implicit integration methods are often used to study highly nonlinear quasi-static problems. The drawback of explicit methods is their lack of control of the truncation-error magnitude. It has been demonstrated that differential equations of the LTD can be graphically and at the same time conveniently modeled with Xcos.

With the emphasis on issues, which have a relationship with LTD problems [8, 9], it was possible to include combinations of diverse traction control models for the MUTs using computationally efficient numerical methods. These models are essential for wheels’ prevention from excessive slip (for traction) or sliding (for braking). However, simulation of the complex railway system, which includes a train and subsystems of a track, has become a routine activity with ever more powerful computers. Such a type of railway system is often defined as a multibody system [10, 11, 12]. Computer models of the complex railway system treated as the multibody system can be applied to optimize suspensions and other train parts and check dynamic forces and accelerations against standards to ensure safe operation and a comfortable ride. In addition, it is worth paying attention to the possibility of computer simulations with different braking systems. Besides the pneumatic braking system, other braking systems, such as electric brakes and eddy current brakes, are used. There are also advanced braking technologies such as the hydraulic brake or the wing plate brake. It can also be considered to build a modified hybrid kinetic energy recovery system with the flywheel.

Even today, a significant portion of railway lines with low traffic levels is still not electrified [13]. Therefore, the need to use self-propelled rolling stock, not powered through a contact line, can be taken into consideration.

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