The great emptiness at the beginning of the Universe

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We propose the great emptiness as a possible beginning of the Universe in the infinite past of physical time. In the beginning particles are very rare and effectively massless. Only expectation values of fields and average fluctuations characterize the lightlike vacuum of this empty Universe. Our observed inhomogeneous Universe can be extrapolated backwards to the lightlike vacuum in the infinite past, and therefore can have lasted eternally. There is no physical big bang singularity. Standard inflation models admit a primordial flat frame for which spacetime is flat in the infinite past.

**Lightlike vacuum**

While the property of emptiness of the universe in the inflationary stage is well known, the lightlike behavior of excitations needs a more detailed discussion [8]. We aim here for physical quantities that are at least in principle observable by a gedankenexperiment. In a quantum field theory observable quantities should not depend on the choice of fields used to describe them. In more technical terms they should not depend on the “frame” used for the metric field. Observable quantities have to be dimensionless. In our case the relevant dimensionless quantity is the ratio of a particle mass over momentum.

$$m/p$$

In the familiar Einstein frame with fixed particle momentum, decreases exponentially,

$$\exp\left(-\frac{m}{p}\right) \approx \exp\left(-\frac{10^{-10}m}{10^{10}}\right) \approx 10^{-16},$$

At early times the particle is relativistic. For nucleons with the same momentum the ratio remains larger than one at any given finite time. We can repeat the argument by placing $$t_i$$ at some arbitrary moment during inflation. If inflation lasts long enough before $$t_i$$, the particle will again be ultrarelativistic at sufficiently early $$t$$.

A slow time dependence of $$\dot{H}$$ does not change the situation.

Consider now at the time $$t_0$$ at the end of inflation a superheavy particle with mass $$m$$ of the order of the Planck mass and a very small momentum, say $$p(t_0) = 10^{-10}m$$. At this time the particle is non-relativistic, $$m/p = 10^{10}$$, and has a momentum much smaller than the expansion rate of the Universe $$\dot{H}$$. Looking at a time $$t$$ sixty e-folds before the end of inflation, one finds already a rather small ratio $$(m/p) \approx e^{-60}10^{10} \approx 10^{-16}$$, and the particle is ultrarelativistic. Going back further the ratio further decreases rapidly. For nucleons with the same momentum the ratio is a factor $$10^{-18}$$ smaller at any time. We can repeat the argument by placing $$t_i$$ at some arbitrary moment during inflation. If inflation lasts long enough before $$t_i$$, the particle will again be ultrarelativistic at sufficiently early $$t$$.

In particular, if the inflationary epoch has no “beginning event” at some $$t_i$$, any nonzero momentum $$p(t_i)$$ will diverge as $$t$$ goes to minus infinity. A more detailed discussion would consider the evolution of momentum distributions, but the sense in which we speak about a “lightlike” vacuum should already be clear: towards the beginning particles propagate similar to photons. (There is always a tail of extremely small momenta $$p(t_i)$$ for which $$(m/p)(t)$$ remains larger than one at any given finite $$t$$. See ref. [8] for a discussion of different limits.) We will focus here on inflationary scenarios without a “beginning event” and discuss alternatives at the end of this note. Towards the “beginning” $$t \to -\infty$$ all particles then become massless in physical terms, justifying the notion of a lightlike vacuum. Massless particles are an indication of the possibility of unbroken scale symmetry.

**Physical time**

The statement that such a Universe can last since ever may encounter more doubts. It has been argued that the limit $$t \to -\infty$$ corresponds to a singularity which is necessarily reached at a finite proper time [9–12]. This “geodesic incompleteness” has given rise to the opinion that in standard inflationary cosmology the Universe starts with a singularity and that the inflationary epoch only lasts for an extremely short physical time, say $$10^{-40}$$ seconds. Many alternative beginnings have been proposed that aim to avoid this “initial singularity”.

$$a(t) \approx \exp\left(\dot{H}(t-t_i)\right)a(t_i)\),$$

with $$a(t)$$ the scale factor in the Robertson-Walker metric and $$t$$ cosmic time. As a consequence the physical momentum of a particle, $$p = k/a$$, with $$k$$ the comoving momentum, decreases exponentially,
We argue here that in many models of inflation there is no physical initial singularity and physical time $t_{ph}$ extends to the infinite past, $t_{ph} \to -\infty$. While we will demonstrate later explicitly the regular behavior in a different metric frame with variable particle mass, we want to show here first how infinite physical time is found in the familiar Einstein frame with fixed particle masses. We note that proper time is useful for many purposes, but it is not a reasonable physical time when we consider the Universe towards its “beginning”. As is well known, proper time cannot be used for massless particles, and we have just seen that all particles become effectively massless for $t \to -\infty$. Furthermore, proper time is not a frame invariant quantity but rather depends on the specific choice of a metric field. A detailed discussion \cite{ref8} reveals that proper time is indeed inappropriate for the limit $t \to -\infty$.

Physical time should be based on oscillatory phenomena and a counting of oscillations. It is no accident that some type of “oscillation time” has been employed since the earliest descriptions of nature by humans. Today we use it by counting the oscillations of photons with an energy given by some particular atomic transition. The number of oscillations of the photon wave function with a given comoving momentum $k$ remains a valid physical time for all epochs of the Universe, including the beginning. Since the counting is discrete, it does not depend on the choice of coordinates. Neither does it depend on the choice of fields or the metric frame.

Expressed in terms of conformal time $\eta$, the wave equation for a massless particle in a homogenous isotropic Universe reads

\begin{equation}
(\partial^2_\eta + 2H a \partial_\eta + k^2)\varphi_k = 0,
\end{equation}

with complex $\varphi_k$ an appropriate component of the wave function in an eigenstate of comoving momentum $k$, $H = \partial_\eta a / a$, $\mathcal{H} = Ha = \partial_\eta \ln(a)$, $\partial_\eta \eta = dt$. The Hubble damping can be factored out,

\begin{equation}
\tilde{\varphi}_k = a \varphi_k, \quad \left(\partial^2_\eta + k^2 - \frac{a^2 R}{6}\right)\tilde{\varphi}_k = 0,
\end{equation}

with $R$ the curvature scalar. For $|a^2 R| \ll k^2$, which holds at the beginning of inflation, the number of oscillations $n_k$ is proportional to conformal time

\begin{equation}
n_k = \frac{k \eta}{2\pi}.
\end{equation}

We can therefore consider conformal time $\eta$ as a good proxy for oscillation time. For homogeneous isotropic cosmologies we can take it as physical time. Conformal time is indeed invariant under conformal transformations of the metric and therefore the same in all frames related by Weyl scaling.

For typical inflationary cosmologies without a “beginning event” both conformal time $\eta$ and oscillation time $n_k$ go to minus infinity as $t \to -\infty$. The Universe exists therefore since the infinite past if physical time is used – it is eternal. Only the mapping to proper time becomes singular for $t \to -\infty$, as may be expected for particles becoming massless. (See ref. \cite{ref8} for a discussion of physical time for massive particles.) Measured in proper time the duration of oscillations approaches zero very rapidly for $t \to -\infty$, whereas the number of oscillations goes to infinity. While the time between two ticks of the “photon clock” is frame dependent, the number of ticks is not.

Expressed in conformal time the history of the hot big bang Universe and inflation looks less dramatic. Measured in physical (conformal) time the “conformal age” of the Universe since the end of inflation amounts to around 46 billion years. For a cosmological epoch where

\begin{equation}
\frac{a(t)}{a(t_{in})} = \left(\frac{t}{t_{in}}\right)^{\frac{2}{n}},
\end{equation}

with $n = 3(4)$ for matter (radiation) domination, one finds

\begin{equation}
\eta(t_1) - \eta(t_2) = \left(1 - \frac{2}{n}\right)^{-1} t_{in}^{-\frac{2}{n} - 1} \left(t_1^{-\frac{2}{n}} - t_2^{-\frac{2}{n}}\right).
\end{equation}

For some time $t$ in the radiation dominated epoch one has

\begin{equation}
\eta_{eq} - \eta(t) = 2 zeq \left(t_{eq} - t^2\right).
\end{equation}

The difference in conformal time is much larger than the difference $t_{eq} - t$ in cosmic time, being enhanced by the redshift $2 zeq \approx 7000$ for matter radiation equality. In physical time the radiation dominated epoch between the end of inflation and matter-radiation equality lasts for $3.3 \cdot 10^9$yr or around one percent of the conformal age of the Universe.

The most important qualitative difference between physical time and cosmic time $t$ occurs for the (almost) exponential expansion (1) during inflation. For $t_i = t_0$ the end of inflation and $t$ some time during inflation one has

\begin{equation}
\eta(t_0) - \eta(t) = \frac{1}{H} \left(\frac{1}{a(t)} - \frac{1}{a(t_0)}\right) = \frac{1}{H a(t_0)} \left(\exp\{H(t_0 - t)\} - 1\right).
\end{equation}

Physical time diverges for $a(t) \to 0$, and the Universe lasts since ever when time is measured in physical units.

**Field relativity**

The lightlike behavior of all particles towards the beginning of the Universe suggests to use a frame where particle masses are not kept constant, but rather vanish towards the beginning. If the ratio between particles masses and the Planck mass remains constant, the Planck mass $M$ also has to vanish towards the beginning. This can be realized by replacing the Planck mass by a scalar field $\chi$, and all particle masses becoming proportional to $\chi$. In such a “scaling frame” the beginning with vanishing physical particle masses finds a simple description if $\chi \to 0$. We will see that in the scaling frame the big bang singularity is absent, demonstrating that the big bang singularity is actually a “field singularity” due to an inappropriate choice of fields, rather than a physical singularity. In some respect it is
analogous to the coordinate singularity at the south pole in Mercator projection coordinates, except that we speak now about “coordinates in field space”.

In the familiar Einstein frame the effective action describing the inflationary epoch involves the metric and a scalar “inflaton” field $\sigma$,

$$\Gamma = \int_x \sqrt{g_E} \left\{ -\frac{M^2}{2} R_E + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + V_E(\sigma) \right\},$$  \hspace{1em} \text{(10)}

with $V_E$ the effective scalar potential in the Einstein frame. Performing a Weyl transformation we choose a different metric field

$$g_{E,\mu\nu} = w^2 g_{\mu\nu}, \hspace{1em} w^2 = \frac{\chi^2}{M^2},$$  \hspace{1em} \text{(11)}

with $g_{E,\mu\nu}$ and $g_{\mu\nu}$ the metric in the Einstein and scaling frame and $\chi$ a scalar field that will be related to $\sigma$. Expressed in terms of $g_{\mu\nu}$, the action reads, with $\bar{\sigma} = \sigma/M$,

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{F(\chi)}{2} R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + U(\chi) \right\},$$  \hspace{1em} \text{(12)}

where

$$F(\chi) = \chi^2, \hspace{1em} U(\chi) = \lambda(\chi) \chi^4, \hspace{1em} \lambda(\chi) = \frac{V_E(\bar{\sigma})}{M^4}, \hspace{1em} K(\chi) = \chi \left( \frac{\partial \bar{\sigma}}{\partial \chi} \right)^2 - 6.$$  \hspace{1em} \text{(13)}

We assume a monotonic behavior

$$\left( \frac{\partial \bar{\sigma}}{\partial \ln \chi} \right)^2 = B = K + 6 > 0,$$  \hspace{1em} \text{(14)}

such that during inflation the $\chi$-dependence of $\lambda$ is directly related to the slow roll parameter $\epsilon$,

$$\left( \frac{\partial \ln \lambda}{\partial \ln \chi} \right)^2 = B \left( \frac{\partial \ln V_E}{\partial \bar{\sigma}} \right)^2 = 2B \epsilon.$$  \hspace{1em} \text{(15)}

The solutions of field equations of “variable gravity” based on the action (12) are discussed in ref. [13].

**Primordial flat frame**

At this stage we still have a whole family of frames according to different possible choices for the relation between $\sigma$ and $\chi$, or the choice of the function $\bar{\sigma}(\chi)$. Many models admit a “primordial flat frame” by a choice of $\bar{\sigma}(\chi)$ for which

$$K < 0, \hspace{1em} K + 6 = \frac{\partial \ln K}{\partial \ln \chi} - \frac{\partial \ln \lambda}{\partial \ln \chi}.$$  \hspace{1em} \text{(16)}

With this choice there are cosmological solutions for which spacetime is flat.

Indeed, the metric field equations derived from the action (12) read for a Robertson-Walker metric ($R = 12H^2 + 6\dot{H}$)

$$3\chi^2 H^2 = \lambda \chi^4 + \frac{K}{2} \chi^2 - 6H \chi \dot{\chi},$$  \hspace{1em} \text{(17)}

$$\chi^2 R = 4\lambda \chi^4 - (K + 6) \chi^2 - 6\chi(\ddot{\chi} + 3H \dot{\chi}),$$  \hspace{1em} \text{(18)}

and the scalar field equation is given by

$$K(\ddot{\chi} + 3H \dot{\chi}) = -4\lambda \chi^3 - \chi^4 \frac{\partial \lambda}{\partial \chi} + R - \frac{1}{2} \frac{\partial K}{\partial \chi} \chi^2.$$  \hspace{1em} \text{(19)}

For frames obeying the condition (16) all three equations (17)–(19) can be solved for a flat Minkowski geometry, where

$$\dot{\chi} = \sqrt{-\frac{2\lambda}{K}} \chi^2, \hspace{1em} H = 0, \hspace{1em} R = 0.$$  \hspace{1em} \text{(20)}

For $\chi(t)$ one finds the formal solution

$$\chi(t) = \chi_0 \left( 1 + \chi_0 \int_t^{t_0} dt' \sqrt{-\frac{2\lambda}{K(t')}} \right)^{-1},$$  \hspace{1em} \text{(21)}

where $\chi_0 = \chi(t_0)$ and $(\lambda/K(t')) = (\lambda/K(\chi(t')))$. If $2\lambda/K$ reaches a constant $-c^2$ for $\chi \to 0$, one finds for the asymptotic behavior in the past infinity $t \to -\infty$ that $\chi$ vanishes according to

$$\chi(t) \to \frac{1}{c(t_0 - t) + \chi_0}, \hspace{1em} c = \sqrt{-\frac{2\lambda}{K}}.$$  \hspace{1em} \text{(22)}

The primordial flat frame has a regular geometry and all particles become massless in the infinite past as $\chi(t \to -\infty) \to 0$. The existence of such a frame clearly demonstrates the absence of a physical singularity. The singularity in the Einstein frame is a field singularity induced by the singularity in the field transformation (11) for $\chi \to 0$. While the metric $g_{\mu\nu}$ amounts to “regular field coordinates” for the infinite past, the Einstein metric $g_{E,\mu\nu}$ corresponds to “singular field coordinates”. The regular field coordinates provide for a more natural description of the physical properties of the lightlike vacuum. Conformal time is the same for both frames.

The “flat frame condition” (16) constitutes a differential equation for the function $B(\chi)$ or $B(\bar{\sigma})$ that defines the relation between $\bar{\sigma}$ and $\chi$ by eq. (14), namely

$$B = 2\epsilon \left( 1 \pm \frac{1}{\sqrt{2c(6 - B) \partial \bar{\sigma}}} \right)^2.$$  \hspace{1em} \text{(23)}

Here the minus sign applies if $V_E$ decreases with $\sigma$ and $\chi$ increases with $\sigma$, while the plus sign accounts for $V_E$ increasing with $\sigma$ and $\chi$ decreasing with $\sigma$. A primordial flat frame exists whenever for a given $V_E(\bar{\sigma})$ and associated $\epsilon(\bar{\sigma})$ a solution of eq. (24) with $0 < B(\bar{\sigma}) < 6$ exists. In particular, for constant $\epsilon$ one has constant $B = 2\epsilon \ll 1$, such that $K = B - 6$ is indeed negative. For small $\epsilon$ one finds the iterative solution

$$B = 2\epsilon \left( 1 \pm \frac{1}{3 - \sqrt{2c(6 - B) \partial \bar{\sigma}}} \right)^2.$$  \hspace{1em} \text{(24)}

If the solution of eq. (23) does not remain within the allowed interval for $B(\bar{\sigma})$ as $\chi$ increases, it is actually sufficient to define $\chi(\bar{\sigma})$ such that the condition (16) holds in the limit $\chi \to 0$. In this case one finds solutions that approach flat space in the infinite past and are again free of singularities.
Chaotic inflation

As a simple example we may consider chaotic inflation [6] with

\[ V_E = \frac{1}{2} m^2 \sigma^2, \quad \chi = \frac{1}{2} \bar{\sigma}^2, \quad b = \frac{m^2}{M^2}, \quad \epsilon = \frac{2}{\bar{\sigma}^2}. \quad (25) \]

For the primordial flat frame \( \bar{B}(\bar{\sigma}) \) has to obey the differential equation \( (\bar{\sigma} > 0) \)

\[ \frac{\partial \bar{B}}{\partial \bar{\sigma}} = (6 - \bar{B}) \left( \sqrt{\bar{B}} - \frac{2}{\bar{\sigma}} \right). \quad (26) \]

For \( \bar{\sigma} \gg 1 \), as appropriate for the inflationary epoch, the solution reads

\[ \bar{B} = \frac{4}{\bar{\sigma}^2} - \frac{16}{3\bar{\sigma}^4} + \ldots \quad (27) \]

The field \( \chi \) vanishes for large \( \bar{\sigma} \) as

\[ \chi \approx \chi_0 \exp \left( -\frac{\bar{\sigma}^2}{4} \right), \quad (28) \]

such that the evolution equation (20) for \( \chi \) becomes

\[ \dot{\chi} = \sqrt{\frac{2b}{3} \bar{\chi}^2 \sqrt{\chi_0/\chi}}. \quad (29) \]

The combination \( c = \sqrt{-2\chi/\bar{\chi}} \) approaches for large \( \bar{\sigma} \) an increasing value

\[ c(t) \rightarrow \sqrt{\frac{\lambda}{3}} \equiv \sqrt{\frac{6}{\chi_0} \dot{\chi}(t)} \approx \sqrt{\frac{2b}{3 \ln \chi_0}} \chi(t). \quad (30) \]

Correspondingly, \( \chi(t \rightarrow -\infty) \) reaches zero somewhat faster than \( \sim 1/t \). Replacing the constant \( c \) in eq. (22) by the function \( c(t) \) actually becomes a good approximation for \( t \rightarrow -\infty \), since \( -\dot{c}(t_0 - t) / c = (1 - 1/\chi(t)) / (2 \ln \chi_0/\chi) \) vanishes for \( \chi \rightarrow 0 \). In flat space conformal time is proportional to cosmic time, \( \eta = t/a_0 \), and we find the implicit approximate solution for \( \eta \rightarrow -\infty \).

\[ \chi^{-1}(\eta) = \sqrt{\frac{2b}{3 \ln \chi_0}} \chi_0 \eta_0 - \eta + \chi_0^{-1}. \quad (31) \]

The same solution for \( \chi(\eta) \) is found in the Einstein frame if we translate the slow roll solution for \( \sigma(t) \) to conformal time and then to \( \chi(\eta) \) according to eq. (28). As it should be for physical time the behavior of \( \chi(\eta) \) does not depend on the choice of frame.

Inhomogeneous Universe

Our Universe is not homogeneous and isotropic. The question arises if our observed inhomogeneous Universe can have lasted since ever in physical time, or if the extrapolation backwards necessarily encounters a physical singularity. It is often believed that the latter is the case and therefore a physical big bang singularity is unavoidable in presence of the observed inhomogeneities. We will show here that inhomogeneities are compatible with a Universe existing since infinite physical time, with a big bang singularity being a field singularity similar to the homogeneous and isotropic solution.

We expand the metric around a homogeneous isotropic averaged value

\[ g_{\mu\nu}(\eta, x) = a^2(\eta)(\eta_{\mu\nu} + \gamma_{\mu\nu}(\eta, x)), \quad (32) \]

with \( x \in \mathbb{R}^3 \) denoting spacelike coordinates and \( \eta \) conformal time, and similar for the scalar field

\[ \chi(\eta, x) = \chi_0(1 + \delta(\eta, x)). \quad (33) \]

The Weyl scaling (11) relates the scaling frame to the Einstein frame

\[ a^2(\eta_{\mu\nu} + \gamma_{\mu\nu}) = \frac{M^2 a_E^2}{\chi^2(1 + \delta)^2} (\eta_{\mu\nu} + \gamma^E_{\mu\nu}). \quad (34) \]

With \( a(\eta) = (M/\chi(\eta)) a_E(\eta) \) one has in linear order

\[ \gamma_{\mu\nu} = \gamma^E_{\mu\nu} - 2\delta\eta_{\mu\nu}. \quad (35) \]

We concentrate here on the graviton or traceless transverse tensor fluctuations \( \gamma_{mn}, m, n = 1 \ldots 3 \). They obey

\[ \gamma_{mn} \delta^{mn} = 0, \quad k^m \gamma_{mn} = 0, \quad (36) \]

where we have switched to a Fourier representation \( \gamma_{mn}(\eta, k) \), with \( k \) the spacelike comoving momentum, \( k^m = \delta^{mn} k_n, k^2 = k^m k_m \). The first relation (36) implies that \( \gamma_{mn} \) is invariant under conformal frame transformations (35), \( \gamma_{mn} = \gamma_{mn,E} \).

The linearized field equations for \( \gamma_{mn}(k) \) can be written in a frame invariant form [14]

\[ (\partial_\eta^2 + 2\bar{\Delta}_E \partial_\eta + k^2 + \bar{\Delta}_E) \gamma_{mn}(\eta, k) = 0, \]

\[ \bar{\Delta}_E = 2(\bar{\Delta}^2 + 2\partial_\eta \bar{\Delta}) - 2k^2 + \bar{K} (\partial_\eta \chi)^2. \quad (37) \]

They involve the frame invariant combinations

\[ \dot{V} = \frac{V}{F^2}, \quad \bar{K} = \frac{K}{F} + \frac{3}{2F^2} \left( \frac{\partial F}{\partial \bar{\chi}} \right)^2, \quad (38) \]

evaluated for the average metric and scalar field. In particular, in the Einstein frame one has \( F = M^2 \) and \( \dot{V} = V_E/M^4 \). Further frame invariant quantities are the scale factor in units of the variable Planck length, and suitable \( \eta \)-derivatives thereof,

\[ A = \sqrt{F} a, \quad \bar{\Delta}^E = \partial_\eta \ln A. \quad (39) \]

The frame invariant formulation (37) allows us to take over the solution for \( \gamma_{mn}(\eta, k) \) from the scaling frame to the Einstein frame and vice versa. In particular, if \( \gamma_{mn}(\eta \rightarrow -\infty) \) and its derivatives remain finite, there is no physical singularity.
The field equations for the homogeneous and isotropic average metric can also be written in a frame invariant form [14],

$$2\ddot{\mathcal{H}} + \partial_\eta \mathcal{H} = A^2 \dot{V}, \quad \ddot{\mathcal{H}} - \partial_\eta \mathcal{H} = \frac{K}{2} (\partial_\eta \chi)^2,$$

(40)

$$\dot{K} (\partial_\eta^2 + 2 \mathcal{H} \partial_\eta) \chi + \frac{\partial \dot{K}}{2} \partial_\chi (\partial_\chi \chi)^2 = -A^2 \frac{\partial \dot{V}}{\partial \chi},$$

(41)

resulting in $\dot{\Delta}_s = 0$. In the primordial flat frame with $F = \chi^2$, $\mathcal{H} = \partial_\eta \ln a$, one has

$$A = \chi a, \quad \mathcal{H} = \mathcal{H} + \partial_\eta \ln \chi.$$   

(42)

Eq. (37) reads for the flat space solution ($\mathcal{H} = 0$)

$$(\partial_\eta^2 + 2 \partial_\eta \ln \chi) \partial_\eta + k^2 \gamma_{mn} = 0.$$   

(43)

In this case we can employ cosmic time instead of $\eta$ if we replace $k^2$ by $k^2 = k^2 / \bar{a}^2$ with $\bar{a}$ the constant scale factor for the flat space solution. With eq. (20) the linear evolution of graviton fluctuations obeys

$$(\partial_t^2 + \chi G \partial_t + k^2) \gamma_{mn} = 0,$$

(44)

with

$$G = \sqrt{-8 \Lambda / K}.$$   

(45)

For constant $G$, or $G$ increasing logarithmically as for chaotic inflation ($G = (8b / 3) \ln (\eta_0 / \chi)$), or $G$ increasing or decreasing with a power $\chi^\gamma$, $\gamma > -1$, the term $\sim \chi G$ vanishes for $\chi \rightarrow 0$ or $\eta \rightarrow -\infty$. What remains in the limit $t \rightarrow -\infty$ are oscillations with constant amplitude

$$\gamma_{mn}(t) = e^{ik(t - \bar{\eta})} \gamma_{mn}^{(+)} (t_1) + e^{-ik(t - \bar{\eta})} \gamma_{mn}^{(-)} (t_1).$$   

(46)

This is the behavior of a free massless particle that remains completely regular for $t \rightarrow -\infty$. No singularity is expected for any finite $t$, such that the graviton fluctuations can be followed to the infinite past without encountering any singular behavior. The inhomogeneous cosmology describing graviton fluctuations around a homogeneous isotropic background or average metric is regular for all times in the past.

The asymptotic behavior (46) is easily translated to conformal time by replacing $t \rightarrow \eta$, $k \rightarrow k$. In this form it holds for arbitrary metric frames related by conformal transformations. In particular, one finds the same behavior in the Einstein frame. Thus inhomogeneous metrics characterized by

$$ds^2 = a^2_E (\eta) \left\{ -dt^2 + \left( \delta_{mn} + \gamma_{mn}(n, x) \right) dx^m dx^n \right\}$$   

(47)

can be extrapolated to $\eta \rightarrow -\infty$ as regular solutions.

It is often argued that the Weyl tensor $C_{\mu\nu\rho\sigma}$ becomes singular for the extrapolation of inhomogeneous cosmologies towards the “big bang singularity”. The squared Weyl tensor, multiplied by $\sqrt{g}$,

$$W = \sqrt{g} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma},$$   

(48)

is invariant under conformal transformations and therefore the same in all frames related by Weyl scaling. Even for regular $W$ the squared Weyl tensor alone may become divergent if $\sqrt{g}$ approaches zero, as is the case in the Einstein frame where $\sqrt{g} = a_E^2$. This divergence is, however, only an artifact of a singular choice of field coordinates, and does not indicate a physical singularity. In the primordial flat frame no singularity of the squared Weyl tensor occurs. In all frames $W$ remains finite for $\eta \rightarrow -\infty$.

**Arrow of time**

Not every arbitrary inhomogeneous Universe can be extrapolated backwards to the infinite past without encountering a singularity. This is related to the presence of decreasing fluctuation modes. We denote the amplitude of such a decreasing mode by $\varphi(\eta, k)$, that we take real and positive for convenience. Any interval of values at time $\eta_1$, $\varphi(\eta_1, k) < \varphi(\eta_1, k)$, is mapped to a smaller interval at $\eta_2 > \eta_1$, $\varphi(\eta_2, k) < \varphi(\eta_2, k)$, $\varphi(\eta_2, k) < \varphi(\eta_1, k)$. Assume now that for $\eta_1 \rightarrow -\infty$ arbitrary values of $\varphi(\eta_1, k)$ (e.g. $\varphi(\eta_1, k) \rightarrow \infty$) are mapped to a finite interval $\varphi(\eta_2, k)$ at $\eta_2$. Starting at $\eta_2$ and following the evolution backwards to $\eta < \eta_2$, only the amplitudes in the interval $\varphi(\eta_2, k) < \varphi(\eta_2, k)$ can be followed consistently to $\eta \rightarrow -\infty$. In contrast, for all values outside the allowed interval, $\varphi(\eta_2, k) > \varphi(\eta_2, k)$, the backwards solution has to diverge for some finite $\eta_n$. The solution becomes singular, and this singularity cannot be removed by field redefinitions.

This type of singularity does not indicate a singular cosmology. It rather indicates a prediction of a certain cosmology, namely $\varphi(\eta_2, k) < \varphi(\eta_2, k)$. Universes for which at $\eta_2$ the prediction is violated are not allowed. If one tries, nevertheless, to extrapolate the forbidden cosmologies backwards, the singularity at $\eta_n$ reminds us the inconsistency of the “forbidden Universes”. The situation is analogous to a damped pendulum. If arbitrary initial conditions lead to a maximal amplitude of 1cm after an hour, the attempt to follow an oscillation with amplitude 5cm backwards will lead to a solution that becomes singular in less than an hour backwards.

We emphasize that this type of “singularity” can only occur for decreasing modes. Modes with constant or almost constant amplitude can be extrapolated backwards to the infinite past. These correspond precisely to the “observable modes” in the primordial fluctuation spectrum. The gauge invariant scalar modes typically contain an almost constant mode as well as decreasing modes. Examples for decreasing scalar modes are discussed in ref. [13]. The almost constant mode is responsible for the scalar part in the primordial fluctuation spectrum.

Setting all decreasing modes to zero at some time $\eta_2$ during inflation, the Universe remains inhomogeneous. All types of inhomogeneities that can be accounted for by the almost constant modes are allowed. This type of inhomogeneous Universe can be extrapolated to the infinite past without encountering a singularity, similar to the special
case of the graviton fluctuations discussed above. The Universe with vanishing decreasing modes is precisely the observed inhomogeneous Universe. This inhomogeneous Universe can therefore last since the infinite past.

The presence of decreasing modes or damped fluctuations constitutes an arrow of time [13]. While field equations are time reversal invariant, a given homogeneous isotropic solution for the average metric is not. A given non-static cosmology can be viewed as spontaneous breaking of time reversal symmetry. Fluctuations around a given homogeneous isotropic “background solution” define an arrow of time. The positive time direction is the one for which the decreasing modes get smaller. The presence of an arrow of time is a general property of fluctuations around a time dependent cosmological solution. It does not need concepts as increasing entropy, which does not play an important role in the lightlike vacuum at the beginning of the Universe. Later on, after the end of inflation, entropy increases in the positive time direction that is defined by the behavior of fluctuations.

Discussion

We have shown that standard inflationary cosmologies can be extrapolated backwards to the infinite past in physical time, as measured by the number of oscillations of photons. This applies to our observed inhomogeneous Universe. No physical big bang singularity is present for these models. The often discussed singularity is only apparent, being related to a singular, and therefore not very appropriate, choice of coordinates in field space. Field relativity permits us to use better adapted choices for the metric field. In particular, in a primordial flat frame the averaged geometry becomes flat Minkowski space in the infinite past. The absence of singularities is very apparent.

The lightlike vacuum in the beginning of the Universe can be associated to quantum scale symmetry [15]. Unbroken scale symmetry implies massless particles, as encountered in the lightlike vacuum. Quantum scale symmetry arises from an ultraviolet fixed point in the flow of couplings, functions or functionals in quantum gravity coupled to particle physics. For interesting “crossover cosmologies” [16, 17] the Universe reaches an ultraviolet fixed point in the infinite past, and makes a transition or crossover to a different infrared fixed point that is approached in the infinite future.

We emphasize that a beginning as a lightlike vacuum is possible for many standard inflationary cosmologies, but not mandatory. Other possible histories of the Universe, as a crossing of the apparent big bang singularity in a bouncing Universe [18, 19], or quantum creation in finite regions of a multiverse [6, 7], can be imagined. In this case the lightlike vacuum would not last forever towards the infinite past. It would rather be reached at some particular time characterizing the bounce or creation of a bubble. Nevertheless, no necessity for such an extension is visible at present.

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