One-loop corrections to the curvature perturbation from inflation

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Abstract. An estimate of the one-loop correction to the power spectrum of the primordial curvature perturbation is given, assuming it is generated during a phase of single-field, slow-roll inflation. The loop correction splits into two parts, which can be calculated separately: a purely quantum-mechanical contribution which is generated from the interference among quantized field modes around the time when they cross the horizon, and a classical contribution which comes from integrating the effect of field modes which have already passed far beyond the horizon. The loop correction contains logarithms which may invalidate the use of naive perturbation theory for cosmic microwave background (CMB) predictions when the scale associated with the CMB is exponentially different from the scale at which the fundamental theory governing inflation is formulated. This may have important consequences for the comparison of chaotic inflationary models with forthcoming high-precision satellite data.

Keywords: cosmological perturbation theory, inflation, quantum field theory on curved space, physics of the early universe

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1. Introduction

For some years, the theory of inflation [1]–[7] has represented our most successful approach to the very early universe. Although we do not know with certainty what physical conditions applied in the very distant past, it is commonly believed that a number of fundamental properties of our universe were determined at that time. Among the most important of these is the ensemble of small density perturbations which later condensed into galaxies and gave rise to the network of cosmic structure we see today. Therefore, one question which must be answered by any theory which purports to describe the very early universe concerns the origin and nature of these perturbations. In inflationary models, the answer is provided by using an era of quasi-exponential expansion at very early times to generate small fluctuations from the vacuum. Remarkably, the properties of these perturbations can be calculated by applying a minimal extension of quantum field theory to the expanding universe [8]–[15].

The inflationary perturbations are generated by quantum-mechanical interference among field modes of wavelength $k$ around the time when such modes are of the order of the Hubble scale, that is, $k \sim aH$. If the inflationary expansion is still ongoing when these perturbations are generated then the local Hubble scale continues to contract, so that eventually they are far outside the horizon, giving $k \ll aH$. In this regime the field behaves in an approximately classical fashion [16]. Therefore the evolution of the curvature perturbation can be calculated reliably until the end of inflation, when it is
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gen generally assumed to settle down to a time-independent final value, unless isocurvature
modes exist which drive its evolution on large scales [17]–[19]. This apparently implies
that we have a robust prediction of the properties of the perturbations, regardless of the
details of the inflationary era.

However, this simple picture may not be entirely adequate. The above account
neglects at least one possible source of trouble, namely the question of quantum mechanical
corrections. In any quantum system, including inflation, we are obliged to study the
role of so-called loop corrections, which may modify or destroy the leading semiclassical
prediction. Loop corrections are systematic corrections in powers of a coupling constant,
which is small if perturbation theory is to be a good approximation. However, perturbation
series are generally asymptotic rather than convergent, and if the coefficients of higher-
order terms are large then such corrections need not be negligible even if the coupling
constant is small. In such circumstances, these corrections must be taken into account
when performing accurate comparisons of theory with observational data; for example,
their use is routine when interpreting the outcome of particle physics experiments.
Modern cosmological datasets have attained a remarkable degree of precision [20]–[22], and
dramatic observational improvements imply that future datasets will possess even greater
constraining power. It is of outstanding importance that our theoretical predictions
keep pace with observational developments. Therefore, as in particle physics, it is now
necessary to ask whether loop corrections can modify the predictions we use to compare
with experiment. Such refinements will certainly be required in order to extract the
maximum value from upcoming experiments, beginning with the data returned by the
forthcoming Planck satellite.

If such corrections are to play a role, what form should we expect them to take?
Models of inflation make predictions concerning the properties of the primordial curvature
perturbation, \( \zeta \). This perturbation is communicated to the cosmic microwave background
(CMB) at decoupling. The statistical properties of \( \zeta \) are predicted theoretically by
supposing that the universe is geometrically close to a patch of the classical, eternal
de Sitter universe, in which the expansion is exactly exponential. The de Sitter universe
has a special simplicity as a solution of the Einstein equations since it is maximally
symmetric, implying that its curvature is constant. However, as an arena for physics the
de Sitter universe is considerably less simple [23]. As is well known, there is a horizon
which surrounds any freely falling observer and screens a portion of the spacetime from
view. This horizon is associated with a finite entropy, \( S_{\text{dS}} \), which can be thought of
as representing a finite state space of size \( \sim \exp S_{\text{dS}} \) available to each inertial observer.
One therefore expects that perturbative predictions made over a volume \( V \gtrsim \exp S_{\text{dS}} \) (in
Planck units) are untrustworthy, since the volume \( V \) carries more perturbative degrees of
freedom than can actually be available in the full quantum gravity [24,25]. An inflationary
epoch lasting for \( N \) e-folds causes the universe to expand by a factor \( e^N \), so we expect
that the inflationary era cannot be described in perturbation theory for an arbitrarily
long time; in order not to violate the volume bound, it is necessary that \( N \lesssim S_{\text{dS}} \) for
reliable calculations. Related limits on \( N \) of various sorts have been reported in the
literature [26]–[28].

The above intuitive argument implies that we should expect the coefficients in the
loop expansion to become large when \( N \gg 1 \). For sufficiently large \( N \), we will have
a breakdown of perturbation theory and thus an apparent loss of predictivity from the
early universe. The purpose of the present paper is to explore the details of this loss of predictivity, quantified in terms of the loop corrections in the observable perturbation $\zeta$. Although a proper understanding of the breakdown of perturbation theory is important simply as a matter of principle, it is also crucially important in practice. The apparent success of the inflationary prediction means that it has become routine for experiments which measure temperature anisotropies in the cosmic microwave background to report their findings in terms of constraints on inflationary models. These constraints are usually derived using tree-level predictions for $\zeta$ which ignore the possibility of a breakdown in perturbation theory.

The important ramifications which attach to a failure of the perturbative description mean that loop corrections have received attention from many authors, beginning with 't Hooft and Veltman [29]. The possibility of an infrared perturbative breakdown was explored by Sasaki et al [30, 31], and later authors computed loop corrections in a variety of different settings; for example, [32]–[53]. We will derive an estimate for the loop correction which takes into account all relevant effects for an arbitrary inflationary potential, using the slow-roll approximation to control the calculation. A central feature of this analysis is the use of the nonlinear $\delta N$ expansion [54]–[57]. This expansion provides a systematic method for computing the evolution of $\zeta$ (and its correlation functions) in terms of the field perturbations at earlier times (and their correlation functions). Moreover, as will be described below, the $\delta N$ formulation has many useful properties for the purposes of loop calculations. In order to apply this method consistently, it is necessary to account for the presence of loop corrections in the field perturbations at early times. The requisite loop correction has recently been calculated [58], and in this paper all these corrections are assembled into a loop correction for $\zeta$, which allows an estimate of the onset of any breakdown in perturbation theory in a number of popular models of inflation.

This paper is organized as follows. In section 2 the nonlinear $\delta N$ formula is briefly reviewed, and in section 3 it is used to derive the complete one-loop correction to the power spectrum of $\zeta$. Our result agrees with that of Byrnes et al, who recently gave an expression for the loop correction valid to two loops. A feature of the $\delta N$ expansion is that one is free to construct the initial hypersurface at any time after the relevant k-modes have left the horizon. This freedom is discussed in section 3.3. In section 4 we use the formal $\delta N$ expression to compute an estimate for the loop correction. The loop correction is shown to be divergent on large scales in section 4.1, and must be regularized by restricting the computation to some finite box of size $\ell$. In sections 4.2 and 4.3 estimates for the regularized loop correction are given in two approximations. In section 4.2 the spectrum is assumed to be defined by a constant tilt, whereas in section 4.3 we specialize to the case of a monomial potential and compute an explicit, but model-dependent, estimate. Finally, we conclude with a discussion of these results in section 5.

We use natural units in which the speed of light $c$, Planck’s constant $\hbar$ and the Planck mass $M_P \equiv (8\pi G)^{-1/2}$ are set equal to unity. The metric signature is $(-, +, +, +)$ and the unperturbed background metric is taken to be of Friedmann–Robertson–Walker form

$$ds^2 = -dt^2 + a(t)^2 dx \cdot dx,$$

where $t$ is cosmic time and $a$ is the scale factor of the universe. However, it is often more convenient to use a conformally rescaled time variable $\eta$ (so-called conformal time),
defined by $\eta \equiv \int_{t}^{\infty} dt/a(t)$. The Hubble parameter satisfies $H \equiv \dot{a}/a$, where an overdot denotes a derivative with respect to $t$, and inflation occurs whenever $\epsilon \equiv -\dot{H}/H^2 < 1$.

Our theory will consist of Einstein gravity coupled to a scalar field $\phi$ with potential $V(\phi)$, which can be arbitrary except that it must support an epoch of inflation for some values of the field. The background is taken to be homogeneous with spatially dependent perturbations $\delta \phi$ which satisfy the smallness condition $|\delta \phi| \ll |\phi|$. When $\epsilon \ll 1$, the field $\phi$ rolls only a short distance in a Hubble time, and therefore this is known as the slow-roll regime. When slow-roll applies, it is often a good formal approximation to compute in powers of $\epsilon$ or related small quantities obtained by taking dimensionless time derivatives of $\epsilon$, although a key theme of this paper will be that such expansions are usually asymptotic rather than convergent. The most important such parameter is the so-called $\eta$ parameter, defined by $\eta \equiv \ddot{H}/H \dot{\phi}$. This measures the local curvature in the potential $V$, and in typical models it is small whenever $\epsilon$ is, although this need not be true in general.

2. The $\delta N$ formula

Our theories which govern cosmological evolution at high temperatures and energies are typically written in terms of a number of microscopic fields whose quanta are the particle species which populate the universe. The interesting cosmological observables are expectation values of products of these fields, often in the special case where the fields which participate in the expectation value are evaluated at equal times but at distinct spatial positions $\{x_1, \ldots, x_n\}$. It is usually more convenient to translate to Fourier space, and work instead with expectation values of fields evaluated at distinct wavenumbers $\{k_1, \ldots, k_n\}$.

2.1. The power spectrum and bispectrum

The lowest-order non-trivial expectation value is the one-point function:

$$\langle \delta \phi(k) \rangle = (2\pi)^3 \delta(k) O,$$

where $O$ is a dimensionless quantity which depends on the time of evaluation. At tree-level the one-point function is always zero if we have chosen to work in a perturbatively stable vacuum, but it is possible that a non-zero $O$ may be generated radiatively. It was shown in [58] that to one-loop order $O$ is given by a renormalization-scheme-dependent number, which can be absorbed into a redefinition of the background field.

The next-order expectation value is the two-point function:

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) P(k_1),$$

where $P(k)$ again depends on the time of evaluation and is known as the power spectrum. It is sometimes more convenient to work in terms of the so-called dimensionless power spectrum $P$, which is related to $P$ by the rule $P \equiv k^3 P/2\pi^2$.

The three-point expectation value can be written as

$$\langle \delta \phi(k_1) \delta \phi(k_2) \delta \phi(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) B(k_1, k_2, k_3),$$

where $B$ is referred to as the bispectrum, since in typical cases it is proportional to a sum of terms quadratic in $P$ [59, 60]. To express $B$ in terms of a dimensionless quantity, it is
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conventional to introduce the momentum-dependent parameter \( f_{\text{NL}} \) \[^{[60]}\] which is defined to satisfy \[^{[61,57]}\]

\[
B(k_1, k_2, k_3) \equiv -\frac{6}{5} f_{\text{NL}} \left\{ P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1) \right\}. \tag{5}
\]

Note that \( B \) (and \( f_{\text{NL}} \)) can always be written as a function of the three scalars \( \{k_1, k_2, k_3\} \), in terms of which it does not depend directly on the relative orientation of the \( \{k_i\} \); alternatively, one can use the momentum conservation condition \( \sum_i k_i = 0 \) to rewrite \( B \) as a function of only two of the momenta, but in this case it will explicitly depend on their relative orientation, making \( B \) (and \( f_{\text{NL}} \)) always a function of three independent degrees of freedom.

2.2. Tilt

The dimensionless quantities \( P \) and \( f_{\text{NL}} \) are generally scale-dependent. Their scale dependence can locally be characterized by two tilt parameters \( n \) and \( n_{\text{NL}} \) \[^{[62]}\], defined by

\[
n - 1 \equiv \frac{d \ln P}{d \ln k} \quad \text{and} \quad n_{\text{NL}} \equiv \frac{d \ln f_{\text{NL}}}{d \ln k}, \tag{6}
\]

where \( f_{\text{NL}} \) is taken to be evaluated at equilateral momenta \( k_i = k \) in the definition of \( n_{\text{NL}} \), and by convention \( n - 1 \) is chosen as the tilt of \( P \) rather than \( n \).\[^{[2]}\] These numbers are typically small \[^{[63]}\]. For a limited range of wavenumbers it may be an acceptable approximation to take \( P \) and \( f_{\text{NL}} \) as almost flat over the range, whereas if it is necessary to characterize \( P \) or \( f_{\text{NL}} \) over a larger range then one can write

\[
P \approx P|_{k_0} \left( \frac{k}{k_0} \right)^{(n-1)|_{k_0}} \quad \text{and} \quad f_{\text{NL}} \approx f_{\text{NL}}|_{k_0} \left( \frac{k}{k_0} \right)^{n_{\text{NL}}|_{k_0}}, \tag{7}
\]

where \( k_0 \) is some pivot wavenumber which is characteristic of the range of wavenumbers in question, and \( '|_{k_0}' \) denotes evaluation at this point. However, both these approximations break down over an exponentially large range of wavenumbers since in general none of \( P, f_{\text{NL}}, n \) or \( n_{\text{NL}} \) are constant. Over such an exponentially large range equation (7) substantially overpredicts the spectrum, which typically grows only like a power of \( \ln k \) rather than a power of \( k \) \[^{[44,64]}\]. Where possible we will use equations (6) and (7) to give model-independent estimates for tilted spectra, but it is apparently necessary to make more detailed model-dependent estimates where an exponentially large range of scales applies (see section 4.3).

\[^{[1]}\] Sometimes \( f_{\text{NL}} \) is defined by writing \( \zeta = \zeta_0 - \frac{4}{3} f_{\text{NL}} \ast \zeta_0^2 \) (in coordinate space), where \( \zeta \) is the full curvature perturbation, \( \zeta_0 \) is a Gaussian random field, ‘\( \ast \)’ denotes a convolution and \( f_{\text{NL}} \) is to be thought of as parametrizing a possible \( \chi^2 \)-type contribution to \( \zeta \). To linear order in \( f_{\text{NL}} \), this expansion for \( \zeta \) in terms of \( \zeta_0 \) produces the formula (5). However, since there is no motivation for such a specific type of nonlinearity in \( \zeta \), we follow Lyth and Rodríguez \[^{[57]}\] in adopting equation (5) for \( f_{\text{NL}} \), whatever its source.

\[^{[2]}\] Note that \( n_{\text{NL}} \) clearly does not capture the most general variation in \( f_{\text{NL}} \) under a change of the momenta, since \( f_{\text{NL}} \) is a function of three degrees of freedom. More generally, one could define the tilt of \( f_{\text{NL}} \) as a vector quantity obtained by taking the gradient with respect to the \( k_i \).
2.3. The $\delta N$ formula

The microscopic fields are not observable by themselves; only a specific combination of them can be observed as the gauge-invariant perturbation $\zeta$ [65] which is communicated to the CMB. An extremely powerful and computationally convenient way to obtain this combination is to employ the so-called $\delta N$ formula [54,55,66,56,57]. Consider the Arnowitt–Deser–Misner [67] line element

$$ds^2 = -N(t, x)^2 dt^2 + h_{ij}(t, x)\{dx^i + N^i(t, x) dt\}\{dx^j + N^j(t, x) dt\}, \quad (8)$$

which describes a perturbed universe, with the perturbations parametrized by the lapse function $N$, the shift vector $N^i$ and the spatial metric $h_{ij}$. For any choice of $t$, we can write the spatial metric on the three-dimensional slices of constant time in the form

$$h_{ij} = a(t)^2 \exp\{2\psi(t, x)\}\delta_{ij}. \quad (8)$$

The quantity $\psi$ is known as the curvature perturbation associated with this slicing.

Consider any two hypersurfaces at times $t_*$ (the initial hypersurface) and $t_c$ (the final hypersurface), where $t_c > t_*$. In the perturbed universe the number of e-foldings of inflation between these slices will vary from place to place, and satisfies

$$N(t_* \to t_c, x) = N_0(t_* \to t_c) + \psi_c(x) - \psi_*(x), \quad (9)$$

where $N_0$ represents the number of e-folds which would have taken place in the unperturbed background. By choosing the initial slice to be flat, so that $\psi_* = 0$, and the final slice to be uniform density, so that $\psi_c = \zeta_c$, one obtains the simple expression $\zeta = \delta N$, where $\delta N \equiv N(x) - N_0$ is the shift in e-folds between $t_*$ and $t_c$ generated by the perturbations. We expect widely separated Hubble volumes to evolve locally like independent universes, so on large scales this shift can be computed merely by accounting for the variation in initial conditions from place to place at time $t_*:

$$\zeta_c(x) = \sum_{n=1}^{\infty} (\delta \phi_*(x))^n \left( \frac{\partial}{\partial \phi_*} \right)^n N(t_c, t_*), \quad (10)$$

where the $\delta \phi_*$ are the field perturbations at $t_*$, evaluated on the flat slicing. (Note that $\zeta_c$ defined by this expression has no dependence on $t_*$. In principle $N$ depends on all the initial conditions on the initial slice. Therefore, since $\phi$ is governed by a second-order differential equation it will generally be necessary to specify the pair $\{\phi_*, \dot{\phi}_*\}$ at $t_*$. However, we will make the simplifying assumption that the slow-roll approximation applies at the initial time. This relates $\phi_*$ to $\dot{\phi}_*$, and hence it is unnecessary to include $\dot{\phi}_*$ separately.)

3 This is not strictly true, because this is not the most general perturbation of a spatial slice. The curvature perturbation $\psi$ amounts to a fluctuation in the trace of $h_{ij}$, which describes the volume expansion in a local region of the slice. In principle there are further vector and tensor modes which describe vorticity and gravitational waves. These modes are set to zero. In general this is not a consistent procedure, because scalar, vector and tensor perturbations mix beyond linear order. Therefore the presence of scalar perturbations will induce perturbations in the vector and tensor modes. However, we expect that these effects will not destroy the leading order prediction for $\zeta$, which we can obtain by focusing simply on $\psi$. 

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2.4. Expectation values

Equation (10) can be used to compute \( n \)-point correlation functions of the curvature perturbation \( \zeta \) by first constructing a product of \( n \) copies of (10) and taking expectation values \( \langle \cdots \rangle \) in the appropriate vacuum state. After application of this process, and working to lowest non-trivial order in \( \delta \phi \), it follows that

\[
\langle \zeta(x_1)\zeta(x_2) \rangle_c = \left( \frac{\partial N(t_c, t_s)}{\partial \phi_*} \right)^2 \langle \delta \phi(x_1)\delta \phi(x_2) \rangle_* ,
\]

where the subscript \( c \) denotes evaluation on the final slice at time \( t_c \). Equation (11) shows that we can compute the power spectrum at any late time \( t_c \) using our knowledge of correlators only at some fixed early time \( t_s \). By repeating the above process for higher \( n \)-point expectation values and to higher orders in \( \delta \phi \), it is obvious that we can use the \( \delta N \) formula to generate predictions for any \( n \)-point function of \( \zeta \) to any required accuracy compatible with the neglect of spatial gradients.

The manipulations described above are now standard in the literature. However, the foregoing discussion makes it completely clear that the \( \delta N \) formula contains no physics; it is merely an identity which expresses \( \zeta \) in terms of geometrical quantities. From the geometrical point of view, \( \zeta \) is nothing more than a book-keeping object which describes how much one region of the universe has expanded (if \( \zeta > 0 \)) or contracted (if \( \zeta < 0 \)) relative to the mean expansion. Therefore, as has been observed before, it is a pure gauge mode in exact de Sitter space [61].

How are we to assign an interpretation to the notion of mean expansion? This is necessary to determine what question we actually answer when we compute properties of \( \zeta \) using the \( \delta N \) formula. It corresponds to specifying which region of the universe is supposed to be described by the unperturbed background metric (1). In practice, we do not imagine that the inflationary patch which gave rise to the large flat region we observe today fills the entirety of our spatial slice. For example, this may happen because the universe emerged from some primeval phase with initial conditions which were chaotically varying, and the requirements for successful inflation were only realized at a limited fraction of locations. Arguments of this sort are invoked for chaotic inflationary models, and arise in a similar way for cosmologies based on the so-called ‘landscape’ of string theory vacua. In such scenarios we should expect spacetime to retain a disordered, inhomogeneous and anisotropic character beyond the boundary of the inflationary patch.

It is quite clear that we should only associate the FRW background metric (1) with the region of spacetime containing the primordial inflationary patch, perhaps after a few e-foldings of inflation so that the patch is sufficiently close to spatial flatness and isotropy [68]. One then expects that whatever primordial unpleasantness lurks beyond the boundary of this patch is irrelevant when we make a prediction from inflation, provided we live sufficiently far from the boundary. This is analogous to the decoupling theorem in field theory (see, e.g., [69]). It follows from the simple point of principle, related to cluster decomposition, that in order to make a prediction at one point we should not need to know the disposition of the universe at some far distant location.

In practice this means that \( a(t) \) should only be interpreted as providing an effective description within some box which coincides with the inflationary patch. But we are quite free, if we wish, to imagine that we are working within some smaller box, as long as it is large enough to contain scales of the order of the present horizon size. Whichever box size...
we pick, $\zeta$ represents the modulation in local expansion as one varies one’s location within the box. It is now necessary to ask whether the answers to calculations depend on the box size. Certainly, $a(t)$ will generally shift in value between differently sized boxes \([70]–[73]\), because the split between background and perturbations is different on disparate scales. One can obtain the correct variation in $a(t)$ by demanding that the expectation value of $\zeta$ to vanish within the box, that is, $E_{\ell}(\zeta) = 0$, where $E_{\ell}$ represents the spatial expectation value within a box of characteristic size $\ell$. This process, which determines the physical perturbation mode within the box, is analogous to imposing a renormalization condition which determines a physical property of a particle, such as its mass.

This $\ell$ dependence of $a(t)$ and $\zeta$ imply that one is left with predictions for the properties of $\zeta$ which depend on the box size, $\ell$. As emphasized by Boubekeur and Lyth, one can aggregate predictions in boxes of size $\ell$ into larger boxes of size $\mu \gg \ell$ in such a way that the dependence of $\zeta$ on $\ell$ is cancelled by the variation of background quantities \([71,73]\). This is analogous to renormalization group invariance in field theory. However, the statistical properties of $\zeta$ itself do depend on $\ell$, which seems like a failure of locality. The resolution is that if the properties of $\zeta$ calculated within a small box are accessible to experiment, the properties of $\zeta$ calculated in some much larger box are not, because they require measurements to be made on scales which are larger than our present horizon. In order to make predictions for the CMB, we wish $\zeta$ to represent the perturbation which is actually present in the radiation when the universe becomes transparent. This entails making an optimal choice for $\ell$ in the same way that making a prediction for a scattering experiment at some characteristic momentum transfer entails making an optimal choice for the running coupling constant.

In what follows we compute the loop correction in $\langle \zeta(k_1)\zeta(k_2) \rangle$. This will enable us to discuss the $\ell$ dependence of the prediction, and select an appropriate box size for the purposes of CMB predictions.

3. The one-loop correction to the power spectrum of $\zeta$

3.1. A renormalized $\delta N$ formula

In this section, we compute the formal $\delta N$ expression for the one-loop correction to $\langle \zeta(k_1)\zeta(k_2) \rangle$. This formal expression will turn out to be divergent and will require regularization and renormalization, as outlined in section 2 above. In order to organize the calculation, it is convenient to express the various contributions to $\zeta$ using a set of graphical rules, analogous to the Feynman diagrams of quantum field theory. Similar diagrams were introduced by Crocce and Scoccimarro \([74]\) and adapted for use in the present context by Zaballa et al \([75]\). The use of such diagrams has recently been formalized to all orders by Byrnes et al \([76]\) using a slightly different notation. Their principal use is to keep track of various loop corrections which arise from the $\delta N$ algorithm described above; when equation (10) is translated to Fourier space, the products $\delta \phi(x)^n$ at a point $x$ become convolutions with independent integrals over $n - 1$ momenta. Some of these momenta are removed after taking expectation values, but this cancellation is not always exact and any unconstrained integrals can be thought of as making loop corrections to the expectation value.
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Figure 1. Diagrammatic representation of the $\delta N$ expansion. To represent an $n$-point correlator one draws all $m$-valent vertices attached to $n$ external legs. Internal lines terminate at special vertices marked by a cross ‘$\times$’. One then draws all possible $j$-gons which connect the $\times$-vertices, of which two possible examples are displayed. 2-gons are treated specially for notational convenience (see figure 2). Each $m$-valent vertex contributes a ‘coupling constant’ $\partial^{m-1}N/\partial \phi_m^{m-1}$.

Figure 2. Special representation for 2-gon diagrams. In the left-hand diagram the two $\times$-vertices should be connected by a 2-gon, but this is difficult to draw. Instead, we adopt the notation shown in the right-hand diagram, where the 2-gon is represented by two parallel lines transverse to the propagators in which they are inserted. In principle there should also be 1-gon initial conditions, where a single $\times$-vertex is tied off at a one-point function $\langle \delta \phi \rangle$, but because we are neglecting the one-point function on large scales such 1-gons do not appear in these diagrams.

The rules for drawing diagrams can be summarized as follows. In order to evaluate the contribution to an $n$-point correlator, one draws all diagrams with $n$ external legs which inject momenta $\{k_i\}$ into the diagram, where $i = 1, \ldots, n$. To each external leg one attaches a single $m$-valent vertex (where $m \geq 2$), at which momentum conservation applies. Each $m$-valent vertex contributes a ‘coupling constant’ $\partial^{m-1}N/\partial \phi_m^{m-1}$. The internal lines attached to such vertices terminate at any number of special vertices, which we label with a cross ‘$\times$’. One then draws all combinations of $j$-gons which can be used to connect the $\times$-vertices with no $\times$-vertex left over, replacing each $j$-gon by a $j$-point expectation value of the $\delta \phi$ as shown in figure 1. These expectation values should be evaluated at the momenta which flow into the corresponding $\times$ vertices. For clarity, we allow a special notation for 2-gons. (See figure 2.) Finally one integrates over all internal momenta in order to obtain the expectation value. In such diagrams, it is sometimes useful to imagine the various $j$-gons as representing initial conditions which flow through the graph, encountering time evolution at each $m$-valent vertex.

The discussion in section 2 implies that we should generalize the formal $\delta N$ expansion, equation (10), by inserting a zero-momentum expectation value $\delta N_0$:

$$\zeta = \delta N_0 + \sum_{n=1}^{\infty} \left\{ \delta \phi_0(x) \right\}^n \left( \frac{\partial}{\partial \phi_*} \right)^n N(t_c, t_\ast),$$

which is to be adjusted so that $E_\ell(\zeta) = 0$, meaning that $\zeta$ accurately represents the perturbation in expansion over a box of characteristic size $\ell$. Enforcing this condition
One-loop corrections to the curvature perturbation from inflation shows that $\delta N_0$ must obey
\begin{equation}
\delta N_0 = -\sum_{j=2}^{\infty} \frac{1}{j!} \frac{\partial^j N}{\partial \phi^j} \langle \delta \phi^j \rangle,
\end{equation}
where we have assumed that $\langle \delta \phi(x) \rangle = 0$ and that statistical homogeneity forces $\langle \delta \phi^j(x) \rangle$ to be independent of $x$ for $j \geq 2$. This guarantees that $\delta N_0$ carries zero momentum. Angle brackets $\langle \cdots \rangle$ denote an expectation value in the quantum vacuum at past infinity, as usual, and $E_\ell$ denotes a spatial average over a comoving spherical box of characteristic size $\ell$.

The scale factor is not the only background quantity which will receive a renormalization. In general we can expect all parameters which specify the homogeneous background model to each receive renormalizations. For example, the various derivatives of $a(t)$ will also be renormalized. These time derivatives can be expressed in terms of the slow-roll parameters\(^4\). However, the presence of $\delta N_0$ does not affect $n$-point expectation values of $\zeta$ for $n > 1$, because it leads to disconnected contributions. These are not the contributions we wish to measure from CMB observations and therefore can be discarded in the present context.

3.2. The power spectrum of the primordial curvature perturbation

The simplest possible graph corresponds to equation (11), which arises from the product of the two linear factors in (10):
\begin{equation}
\begin{array}{c}
\quad \rightarrow \quad \left( \frac{\partial N}{\partial \phi} \right)^2 \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_{\ast},
\end{array}
\end{equation}
where the subscript ‘$\ast$’ denotes evaluation on the initial slice. Although equation (14) is clearly a tree diagram in terms of the $\delta N$ diagrammatic rules described in section 3.1, it implicitly contains loop corrections which are present in the initial correlator $\langle \delta \phi(k_1) \delta \phi(k_2) \rangle$. This illustrates a general feature which arises in calculating $\zeta$ correlators using $\delta N$; there are two distinct types of loop. The first sort are the quantum-mechanical loops (called $q$-loops in [16]) present in the expectation values of $\delta \phi$. The second sort are loops which arise from expanding $\zeta$ correlators using the $\delta N$ algorithm (called $c$-loops in [16]). To arrive at a consistent result at any given order in the loop expansion, it is necessary to retain loop contributions from both these sources. Therefore, we require the $q$-loop correction in $\langle \delta \phi(k_1) \delta \phi(k_2) \rangle_{\ast}$. This $q$-loop has been computed by Sloth [51, 52] in the limit where $t_\ast$ is long after the mode with wavenumber $k \sim k_1 = k_2$ exited the horizon, and has recently been given in [58] in the opposite case where $t_\ast$ is the horizon-crossing time associated with $k$. We are free to choose to base the $\delta N$ expansion on either of these slices, and a discussion of their merits is deferred to section 3.3 below.

\(^4\) This is similar to the proposal of Kolb $et$ $al$ [77], who suggested that a related renormalization of the expansion rate at the present day could be responsible for the presently observed accelerated expansion. This proposal was criticized on various grounds in [78]–[88] and elsewhere. The present discussion differs from that given by Kolb $et$ $al$ because there is no suggestion here that renormalization of the acceleration rate would have experimental consequences for observers within any given horizon volume; indeed, precisely the opposite is the case. The renormalization is only relevant for so-called ‘meta-observers’ who can simultaneously compare rates between many different Hubble patches; it is relevant for what we can compute, but not necessarily for what we can observe.
At whatever time it is evaluated, the $q$-loop associated with equation (14) is accompanied by a number of $c$-loops arising from $\delta N$ integrations. The first of these can be written

$$
\begin{array}{c}
\begin{array}{c}
\bigcirc \\
\end{array}
\end{array}
\mapsto (2\pi)^3 \delta(k_1 + k_2) \frac{\partial N}{\partial \phi^2} \frac{\partial^2 N}{\partial \phi^2} \int \frac{d^3 q}{(2\pi)^3} B_3(k_1, k_2 - q, q),
\end{array}
$$

(15)

where $B$ is the bispectrum of the field fluctuations, defined in equation (4), and the subscript $^*$ again denotes evaluation on the initial slice. This term gives a contribution to the loop correction which depends on the presence of non-Gaussianity in the field perturbations $\delta \phi$. We shall see in section 4.2 below that this term can, in fact, give the largest contribution to the loop correction if $f_{\text{NL}}$ grows on very large scales, although this does not happen in the usual scenario. The presence of a single unconstrained momentum integral, here labelled $q$, indicates that this is a one-loop contribution. The finite time of evaluation, at conformal time $\eta_s$, provides a natural upper cutoff in the momentum integral. On scales characterized by wavenumbers larger than $q \sim -\eta_s^{-1}$ no perturbations have yet been generated; on these scales, the spectrum and all higher correlation functions such as $B$ and the trispectrum $T$ are zero\(^5\). The upper limit on the integral therefore depends on when we take the initial slice to be constructed, whereas the comoving box size $\ell$ provides a natural (and time-independent) lower cutoff at $q \sim \ell^{-1}$.

The next contribution arises from the product of two tree-level spectra. There are two ways such a product can arise. The first corresponds to the diagram

$$
\begin{array}{c}
\begin{array}{c}
\bigcirc \\
\end{array}
\end{array}
\mapsto (2\pi)^3 \delta(k_1 + k_2) \frac{1}{2} \left( \frac{\partial^2 N}{\partial \phi^2} \right) \int \frac{d^3 q}{(2\pi)^3} P_s(|k_1 - q|) P_s(r),
\end{array}
$$

(16)

whereas the second corresponds to a different configuration:

$$
\begin{array}{c}
\begin{array}{c}
\bigcirc \\
\end{array}
\end{array}
\mapsto (2\pi)^3 \delta(k_1 + k_2) \frac{\partial N}{\partial \phi} \frac{\partial^2 N}{\partial \phi^2} \int \frac{d^3 q}{(2\pi)^3} P_s(k) P_s(q),
\end{array}
$$

(17)

and the remarks below equation (15) concerning the presence of an upper and lower cutoff in the momentum integral apply equally to both these loops.

These diagrams exhaust the possible one-loop contributions if we assume that the one-point function of the field fluctuations is zero, $\langle \delta \phi(k) \rangle = 0$. As discussed in section 2.1, the one-point function $O$ is given to one-loop order by a renormalization-scheme-dependent number which can be absorbed into the background field $\phi(t)$. This number is accompanied by other unknown renormalization-scheme-dependent numbers which are present in the two- and higher $n$-point correlation functions. The dominant loop corrections scale like powers of $\ln k\ell$, which implies that on very large scales these scheme-dependent quantities should be negligible in comparison. We will assume that, to a good approximation, all such unknown quantities can be ignored; since we will not keep track of scheme-dependent numbers in the higher $n$-point functions there is no point keeping track of $O$ or a renormalization of the background field $\phi(t)$ either. Therefore, all contributions from one-point insertions will be discounted in the remainder of this paper.

These expressions agree with those recently given by Byrnes et al in [76].

\(^5\) If we were computing these objects using the underlying quantum field theory of $\zeta$ rather than by following the $\delta N$ method, then these wavenumbers would lie in a region of $\eta$-integration where the integrand is strongly suppressed by decaying exponentials, leaving almost no net contribution.
3.3. Where should the initial slice be set?

In the discussion of section 3.2 the initial time at which the $\delta N$ expansion is to be constructed, $\eta^*$, was left arbitrary. The initial time plays two roles in the analysis: it determines whether the field correlators $\langle \delta \phi(k_1) \cdots \delta \phi(k_n) \rangle$ are to be evaluated at the approximate time when the mode with wavenumber $k \sim k_1 = k_2$ crosses the horizon, or if they are to be evaluated at a much later time; and it provides an upper cutoff in integrals over $c$-loop momenta. These two effects must combine in such a way that the curvature perturbation is independent of $\eta^*$.

The loop correction evaluated long after horizon crossing has been computed by Sloth [51, 52] in terms of the variance $\langle \delta \phi^2 \rangle \equiv \int \ln q \ P(q)$, which is in principle a infrared divergent quantity and should be cut off at a momentum scale of the order of the box size, $q \sim \ell$, if $P$ does not vanish as $q \to 0$. The loop correction is found to diverge essentially logarithmically with the time of observation,

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \sim (2\pi)^3 \delta(k_1 + k_2) P_s(k) \{1 + \alpha \text{Ci}(-2k\eta) \langle \delta \phi^2 \rangle \},$$

(18)

where $|k\eta| \ll 1$ by assumption, $\alpha$ is a numerical factor of order $O(\epsilon)$, $P_s$ is the tree-level power spectrum, which is assumed to be almost constant between horizon crossing and the time of observation and $\text{Ci}(x)$ is the real part of the imaginary exponential integral:

$$\text{Ci}(x) \equiv \text{Re}\{\text{Ei}^{(1)}(ix)\}, \quad \text{where } \text{Ei}_n(z) \equiv \int_1^{\infty} \frac{dt}{t^n} e^{-t} \sim -\gamma - i\pi - \ln z$$

for $z \ll 1$,

(19)

where $\gamma$ is Euler’s constant. Equation (18) apparently contains two distinct divergences.

One divergence is implicit in $\langle \delta \phi^2 \rangle$. In exact de Sitter space $P(q)$ is a constant, so $\langle \delta \phi^2 \rangle$ appears to contain a logarithmic divergence at zero momentum. This can be interpreted as a pile-up of modes outside the horizon, which is caused by gravitational redshifting and is ultimately traceable to the quasi-de Sitter expansion. As modes are redshifted outside the horizon they carry perturbations, and the ever-increasing number of modes in this infrared phase space leads to large effects. However, the logarithmic divergence itself is fictional because the original inflationary patch was presumably not infinite in extent. Working in a finite box cuts off the divergence on a momentum scale $q \sim \ell^{-1}$.

The second divergence is present in the $\eta_* \to 0$ limit, and becomes manifest when one observes the correlator long after the $k$-mode in question has passed outside the horizon. This divergence comes from the logarithmic term in $\text{Ci}(-2k\eta_*)$. As with the pile-up of modes, this divergence is fictional in the sense that no observer within the universe can really set $\eta_* = 0$. Instead, observations must be made at a finite time and this finite time gives a cutoff on $\eta_*$.

Although we do not encounter a genuine infinity in either case, these ‘divergences’ can generate large coefficients in the perturbation series. Consider first the divergence as $\eta_* \to 0$, which apparently signals a breakdown of perturbation theory at some time of order $N \sim 1/\epsilon$ e-folds after horizon crossing. (Related breakdowns have been reported elsewhere in the literature; see, for example, [46, 26, 27].) The presence of this term endows the correlator with a time dependence that is not visible at tree-level. Therefore one should worry whether this time dependence could spoil conservation of $\zeta$ on superhorizon scales. If so this would be cause for serious concern, because $\zeta$-conservation is known to hold to
all orders of the $\delta N$ formula [56] and therefore cannot be spoiled by $c$-loops, and since no field correlator is ever evaluated at the time $\zeta$ is observed it cannot be spoiled by $q$-loops among the $\{ \delta \phi \}$ either.

In fact, the time dependence in equation (18) is required to maintain conservation of $\zeta$. Consider any two widely separated initial times. When evaluated at the same time of observation, the coefficient $\partial N/\partial \phi_*$ evolves between these two initial slices because the classical field $\phi$ is rolling down its potential. The correlator $\langle \delta \phi(k_1)\delta \phi(k_2) \rangle$ must likewise evolve between the two initial slices to compensate, in order that $\zeta$ remains constant. It is this evolution which is represented by the logarithmic ‘divergence’ in (18). Indeed, equation (18) should be thought of as the beginning of a Taylor series in $\ln(-k\eta_*)$, with the higher-order terms in the series generated by higher-order loop corrections. This series is of the form studied by Gong and Stewart [89, 90]. In particular, it implies that perturbation theory in powers of slow-roll parameters breaks down at sufficiently late times, of order $N \sim 1/\epsilon$ e-folds after horizon crossing.

Despite this breakdown in perturbation theory, our ability to make predictions from theories of the early universe is not really impaired. In such situations, one would ordinarily attempt to resum the large logarithms using a form of the renormalization group equation. This resummation procedure effectively moves the large coefficients from higher-order terms in the perturbation series into the leading-order term. In the context of equation (18), resummation of the large logarithms clearly yields the superhorizon evolution of the correlator. Since the perturbations behave classically in the superhorizon limit, we should expect that this evolution will correspond to the classical evolution of the field [54], [91]–[93].

Now consider the alternative choice, where the loop correction associated with a mode of wavenumber $k$ is evaluated at the time when $k$ crosses the horizon. At horizon crossing it follows that $-k\eta_* \simeq 1$. The large logarithmic term $\mathcal{C}(1-k\eta_*)$, and any higher-order logarithms, are therefore completely negligible. This version of the loop correction has recently been computed in [58], with the result

$$\langle \delta \phi(k_1)\delta \phi(k_2) \rangle_*= (2\pi)^3 \delta(k_1 + k_2) P_*(k) \{ 1 + \mathcal{P}_*(\frac{k\ell}{\eta_*} \ln k\ell + \beta) \},$$

(20)

where $\beta$ is a renormalization-scheme-dependent number, but according to the discussion in section 3.2 is essentially irrelevant in comparison with the $\ln k\ell$ term. Equation (20) explicitly exhibits the second type of divergence present in equation (18). It becomes divergent in the limit $k\ell \to \infty$, which corresponds to the box size, $\ell$, being very much larger than the scale of interest, $k^{-1}$. In equation (18), this divergence is associated with $\langle \delta \phi^2 \rangle$. The question of how large such $k\ell$ logarithms can become is the central focus of section 4 below. We will return later to the issue of whether these logarithms can also be resummed into evolution on large scales, in analogy with the $|k\eta_*|$ logarithms.

When working with equation (20), it is of crucial importance that the divergence at late times, associated with powers of $\ln(-k\eta_*)$, has been entirely resummed into the $\delta N$ coefficient associated with this correlator. This $\delta N$ coefficient is obtained from the classical evolution, demonstrating that the resummation of $|k\eta_*|$ logarithms reproduces the classical evolution, as described above. It is a remarkable virtue of the $\delta N$ formulation that this resummation is implicit in our freedom to fix the initial slice at any convenient time, without any necessity to deploy a complex formalism based on the renormalization group. Equations (18) and (20) are therefore merely different representations of the same
One-loop corrections to the curvature perturbation from inflation physics. It does not matter which time of evaluation we pick, as long as we have the ability to perform the relevant calculations consistently.

In this paper, we take the view that for analytic calculations it is better to evaluate the correlators of the fields soon after horizon crossing, where the logarithmic term \( \text{Ci}(-2k\eta_*) \), and any higher-order logarithms, are negligible. There are several reasons to adopt this approach:

(i) The \( q \)-loop correlators given in equations (18) and (20) were derived on the assumption that slow-roll is a good approximation between the time of horizon crossing and the time of observation of the \( \delta \phi \). At any given order in the loop expansion one encounters a potentially large number of diagrams, but many of these diagrams will be subdominant when slow-roll applies. For example, as discussed in [58], the \( q \)-loop correction to the two-point function has a diagrammatic expansion of the form

\[
\text{loop correction} \supseteq \underbrace{\frac{}{}} + \underbrace{\frac{}{}} + \cdots.
\]

Both of these diagrams are one-loop and therefore of second order in cosmological perturbation theory, which is an expansion in powers of \( P_* \approx 10^{-10} \). However, the second diagram is suppressed by an extra factor of order \( \sim \epsilon \), since each three-point vertex carries a factor of \( \dot{\phi}/H \) [94] whereas the four-point vertex carries no suppression [95]. However, large logarithms such as \( \text{Ci}(-2k\eta_*) \) can compensate for the smallness of \( \epsilon \). Where large logarithms are present the first diagram is dominated by the terms of \textit{subleading} order in slow-roll, since it is these terms which are accompanied by large logarithmic factors \([51,52]\). However, when terms such as \( \text{Ci}(-2k\eta_*) \) are large one must be wary that the second diagram (among other possible sources of subleading slow-roll terms) does not also begin to contribute. As the logarithms become increasingly large, further diagrams at two loop and beyond may also become relevant. Therefore, in order to compute reliably with a slow-roll truncated \( q \)-loop such as equation (20) one should pick the point of evaluation in such a way that large logarithmic terms do not appear. This means that the initial hypersurface on which we base the \( \delta N \) expansion should be close to the horizon crossing hypersurface. The slow-roll expansion is then safe between these two hypersurfaces, and we can compute the \( q \)-loop to very good accuracy by assuming it is dominated by the leading slow-roll part of the first diagram. For the two-point function this difficulty is not severe, since for moderate values of \( \ln(-k\eta_*) \) one can account for the contribution of the second diagram, together with the subleading part of the first. However, for higher-order correlation functions this may become an extremely practical consideration.

(ii) Equations (18) and (20) are computed from integrals over \( \eta \) whose integrands are rapidly oscillating when \( |k\eta| \gg 1 \). This kills any contribution from very early times. Instead, the integral is dominated for times where \( |k\eta| \lesssim 1 \). The pre-factors in equations (18) and (20) are derived by approximating slowly varying quantities such as \( H \) and \( \epsilon \) at the time of horizon crossing. This will be a good approximation if the correlator is observed just a few e-folds after horizon crossing but may be on the verge of breaking down if \( \ln(|k\eta_*) \) is large, since in general one would then expect \( H \) and \( \eta \) to evolve between horizon crossing and the time of observation. One can account for this evolution using the \( \delta N \) expansion.
One-loop corrections to the curvature perturbation from inflation

In the remainder of this paper only the ‘early’ initial slice is considered, where the correlator is evaluated just a few e-folds after horizon crossing.

4. One-loop renormalization of the $\zeta$ power spectrum

4.1. The loop correction

We are now in a position to apply these general principles to the object of central interest, the power spectrum $\langle \zeta(k_1)\zeta(k_2) \rangle$. After collecting terms in equations (14)–(17), the connected part of the power spectrum can be written

$$\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) \left\{ \left( \frac{\partial N}{\partial \phi^*} \right)^2 P_s(k) + \int \frac{d^3q}{(2\pi)^3} \Sigma(k, q) \right\},$$

where $\Sigma$ is defined by

$$\Sigma = \frac{\partial N}{\partial \phi^*} \frac{\partial^2 N}{\partial \phi^* \partial^2 \phi^*} B_s(k, |k - q|, q) + \frac{\partial N}{\partial \phi^*} \frac{\partial^3 N}{\partial \phi^* \partial^3 \phi^*} P_s(k) P_s(q) + \frac{1}{2} \left( \frac{\partial^2 N}{\partial \phi^*} \right)^2 P_s(|k - q|) P_s(q).$$

In this formula $k$ can be taken to be either $k_1$ or $k_2$, since its orientation is immaterial, and the integration $d^3q$ is restricted to momenta between $q \sim k$ and $q \sim \ell^{-1}$. As discussed above, the time $\eta_*$ at which we construct the initial slice for the $\delta N$ expansion is taken to be the horizon-crossing time for $k$.

To estimate the magnitude of the loop correction present in (21) we must be able to evaluate the $q$ integral, but this is not trivial. In general, $B(k_1, k_2, k_3)$ is a complicated function of its arguments [94, 96] and the integral is difficult to evaluate exactly. We can obtain a reasonable estimate by applying the method of Boubekeur and Lyth [71]. In this approximation, one supposes that the integral is dominated by its infrared singularities. Summing over all singularities, one finds that

$$P_\zeta(k) = P_s(k) \left\{ \left( \frac{\partial N}{\partial \phi^*} \right)^2 + \Pi \right\},$$

where $P_s$ is the tree-level power spectrum evaluated at horizon crossing and $\Pi$ is defined by

$$\Pi \equiv \left( \frac{\partial N}{\partial \phi^*} \right)^2 P_s \left( \frac{35}{6} \ln k\ell + \beta \right)$$

$$+ \left\{ \frac{\partial N}{\partial \phi^*} \frac{\partial^3 N}{\partial \phi^* \partial^3 \phi^*} + \left( \frac{\partial^2 N}{\partial \phi^*} \right)^2 \right\} A(k\ell) - \frac{24}{5} f_{NL} \frac{\partial N}{\partial \phi^*} \frac{\partial^2 N}{\partial \phi^*} B(k\ell).$$

The functions $A$ and $B$ are defined by

$$A \equiv \langle \delta \phi^2 \rangle = \int \frac{d^3q}{(2\pi)^3} P_s(q) \quad \text{and} \quad B \equiv \langle f_{NL} \delta \phi^2 \rangle = \int \frac{d^3q}{(2\pi)^3} f_{NL} P_s(q).$$

The central issue in the remainder of this section, which will also be important for the background renormalization described in section 4.2 below, is how one should obtain estimates of $A$ and $B$. 

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4.2. The approximation of constant tilt

First assume that the power spectrum and $f_{NL}$ can be approximated by equation (7) with constant tilt parameters $n-1$ and $n_{NL}$. The $A$ and $B$ functions now satisfy

$$A(k\ell) = \mathcal{P}_s y_{n-1}(k\ell) \quad \text{and} \quad B(k\ell) = \mathcal{P}_s y_{n_{NL}+n-1}(k\ell),$$

where $y_r(x)$ is Lyth’s $y$-function [72], defined by

$$y_r(x) \equiv \frac{1}{r} \left(1 - |x|^{-r}\right).$$

The parameters $f_{NL}$, $n_{NL}$ and $n-1$ are evaluated at $k$. Equation (26) is approximately valid provided that the box size is not too much larger than the length scale associated with the mode of interest, $k^{-1}$, so that the approximation of fixed tilt is reasonable throughout the box.

The loop correction is of second order in cosmological perturbation theory, which is an expansion in powers of $\mathcal{P}$. However, the coefficients in equation (24) generated by the $\delta N$ expansion enter at different orders in the slow-roll approximation. In particular, $(N')^2$ is of order $\epsilon^{-1}$ whereas $(N'')^2 + N'N'''$ is of order 1 and $f_{NL} N'N''$ is of order $\epsilon^{1/2}$. Which of these terms is dominant depends critically on the tilts $n-1$ and $n_{NL}$.

(i) **Positive tilt.** This is the case where the spectrum decreases on large scales, also known as a blue spectrum. It is not generically produced by single-field models of inflation. For positive $r$, the $y$-function defined in equation (27) satisfies $y_r(x) \sim 1/r$. If the spectral and bispectral tilts $n-1$ and $n_{NL}$ are both positive, then the loop correction $\Pi$ will be dominated by the horizon-crossing loop correction (20). To estimate its magnitude, suppose that the loop correction constitutes no more than some fraction $f\%$ of the tree-level. It follows that

$$\Pi = \frac{35}{6} \mathcal{P}_s (\Delta N - N_s) \approx 10\mathcal{P}_s \Delta N \lesssim \frac{f}{100},$$

where $\Delta N$ is the total number of e-foldings between the scale $\ell$ and the end of inflation, and $N_s \approx 60$ measures when the presently observable universe left the horizon in terms of e-folds before inflation ended. The second approximate equality is valid if the loop correction is large, since this entails $\Delta N \gg N_s$. Equation (28) can be interpreted as a bound on $\Delta N$ in order that the loop correction does not significantly modify the tree-level prediction. This gives $\Delta N \lesssim (10^{3} f^{-1} \mathcal{P}_s)^{-1}$, or $\Delta N \lesssim 10^7$ for $f \sim 1\%$.

(ii) **No tilt.** For $r = 0$, the $y$ function behaves like $y_r(x) \sim \ln |x|$. If $n-1 = n_{NL} = 0$, it follows that all terms in $\Pi$ behave like logarithms of $k\ell$. However, since the horizon-crossing $q$-loop term dominates the $c$-loop terms by a factor $1/\epsilon$, this case gives the same conclusion as case (i) above.

(iii) **Negative tilt.** This is the case where the spectrum increases on large scales, also known as a red spectrum. It is the generic type of spectrum produced by single-field models of inflation. For $r < 0$, the $y$ function has asymptotic form $y_r(x) \sim x^{|r|/|r|}$ when $x \gg 1$. In this regime one of the $c$-loop terms will dominate, depending whether $n_{NL} \gtrless 0$; the term involving $f_{NL}$ will be dominant if $n_{NL}$ is negative, meaning that

\[\text{Note that there is some tension with the constraint, described above, which requires } \ln k\ell \text{ to be sufficiently large that we can ignore the unknown constant term in equation (20).}\]
One-loop corrections to the curvature perturbation from inflation

the non-Gaussianity grows on large scales\(^7\). For the purposes of making an estimate, however, it does not matter which \(c\)-loop is the dominant one since in either case \(\Pi \sim (k\ell)^4\), where \(t > 0\) is the dominant tilt, and the difference in pre-factor is not significant. It follows that

\[
\Pi \sim \frac{\alpha}{t} P_P e^{Nt},
\]

where \(\alpha\) is a numerical constant which absorbs the pre-factor. Equation (29) gives an extremely powerful constraint on how strongly \(P\) and \(f_{NL}\) can tilt, but unfortunately it is untrustworthy precisely in the interesting limit where \(k\ell \gg 1\) or if the tilt is significant. As discussed in section 2, in this limit the approximation of constant tilt overpredicts the spectrum. In reality the spectrum does not grow like a constant power, but only a power of a logarithm.

Therefore we conclude that the significance of the loop correction depends on the number of e-folds of inflation between the fundamental scale, \(\ell\), and the scale of interest, \(k^{-1}\). Except where there is a significant negative tilt the loop correction is reasonably small, perhaps of the order of 1\% in a model with \(10^7\) e-folds of inflation. For \(N \gtrsim 10^7\) e-folds the loop correction may need to be taken into account in accurate analyses. However to study the case of negative tilt a more sophisticated analysis is required.

4.3. Monomial potentials

To obtain a better prediction for \(k\ell \gg 1\), or where there is negative tilt, consider any local field theory model of inflation, with potential given by the monomial \([97]\)

\[
V(\phi) = g\phi^r,
\]

where \(g\) is a coupling constant and \(r\) is usually taken to be an integer. For \(r = 2\) and small values of the field this corresponds to an Nflation-type scenario \([98]\), in which the quadratic behaviour arises as an approximation to a periodic axion potential, and for larger \(r\) or large fields such monomials fit into the general framework of so-called chaotic inflation, where \(\phi\) is taken to be descending from above the Planck scale with a spectrum of field values which may cause some regions of the universe to inflate at the expense of other regions in which the field value is not suitable for inflation to occur.

Inflation occurs for field values greater than \(\phi_f \equiv r/\sqrt{2}\). However, one cannot contemplate arbitrarily large values for \(\phi\). Local field theory presumably breaks down as an approximate description of nature at least when \(V \approx 1\) (in Planck units), which occurs at a field value \(\phi_P \equiv g^{-1/r}\). This is certainly the largest value of the field for which the tools used in the present analysis make sense, but there may be other effects which imply that our approximations break down even before one reaches the Planck scale. For example, the ‘self-reproduction’ phase of eternal inflation ends when \(2\delta\phi^2 \gtrsim 2\epsilon/3 [97]\), which implies a maximum field value \(\phi_{sr} \approx (2\pi^2 r^2 g^{-1})^{1/(r+2)}\). One can think of either of these field values as natural energy scales at which to specify the masses and coupling constants of an effective field theory which approximates some ultraviolet completion of the standard model plus gravity.

\(^7\) This is not the usual situation, since in single-field slow-roll inflation \(f_{NL} \propto \epsilon [61]\), and therefore the non-Gaussianity decays on large scales. At the times when the fluctuation on large scales was imprinted, \(\epsilon\) was much closer to zero.
One-loop corrections to the curvature perturbation from inflation

Let us restrict attention to a regime where local field theory is a good approximation. Any two arbitrary field values $\phi_a$ and $\phi_b$ separated by $\Delta N$ e-folds of inflation are related via the rule

$$\phi_a^2 = \phi_b^2 + 2r \Delta N. \quad (31)$$

In order to match the fluctuation amplitude $P_\zeta = (24.1 \pm 1.3) \times 10^{-10}$ observed\(^8\) in CMB experiments, the coupling must be tuned to precisely satisfy

$$g = \frac{12\pi^2 r^2 P_\zeta}{\phi_*^{r+2}}, \quad (32)$$

where $\phi_*$ is the field value when presently observable scales were leaving the horizon. This typically occurs roughly $N_* \approx 60$ e-folds before the end of inflation in order that the flatness and horizon problems of classical cosmology are resolved. Equations (31) and (32) therefore imply a very small coupling constant, which is hard to understand at the Planck scale on the basis of our present knowledge of physics.

Suppose that inflation begins at some field value $\phi_{UV}$. After a few e-foldings of inflation, we can suppose that the flat FRW metric (1) applies within the inflating volume. One therefore sets the box size $\ell$ to correspond to the horizon size at this time. The field value when any smaller scale $q$ exits the horizon is related to the field value at the onset of inflation by (31), giving\(^9\)

$$\phi^2(q) = \phi_{UV}^2 - 2r \ln q \ell. \quad (33)$$

It follows that $\langle \delta \phi^2 \rangle$ can be written \cite{99,51}

$$A \equiv \langle \delta \phi^2 \rangle = \frac{r}{r+4} P_\zeta \left\{ \left( \frac{\phi_{UV}}{\phi_*} \right)^{r+4} - 1 \right\} \approx \frac{r}{r+4} P_\zeta \left( \frac{\phi_{UV}}{\phi_*} \right)^{r+4}, \quad (34)$$

where the second approximate equality applies in the limit $\phi_* \ll \phi_{UV}$. If inflation lasts for a large total number of e-folds $N \gg r$, then equations (31) and (34) imply

$$A = \frac{r}{r+4} P_\zeta \left( \frac{N}{N_*} \right)^{2+r/2}. \quad (35)$$

This scaling of $\langle \delta \phi^2 \rangle \equiv A$ with the total number of e-folds, namely $A \propto N^{2+r/2}$, was given earlier by Sloth \cite{52}. A similar procedure allows us to estimate $B$, giving

$$B = \frac{r^3}{r+2} \frac{P_\zeta}{\phi_*^2} \left\{ \left( \frac{\phi_{UV}}{\phi_*} \right)^{r+2} - 1 \right\} \approx \frac{r^2}{2N_*(r+2)} P_\zeta \left( \frac{N}{N_*} \right)^{1+r/2}. \quad (36)$$

In making this estimate, we have approximated $f_{\text{NL}} \sim \epsilon$ at each scale. Although this is not strictly numerically accurate \cite{61}, it should be sufficient to capture the tilt of the non-Gaussian fraction. Indeed, equations (35) and (36) show that $A$ grows faster than $B$ by a power of $N$, and therefore that the non-Gaussian component is negligible for a large total number of e-folds. This is in agreement with our expectation that the contribution to the

\(^{8}\) The quoted value is for WMAP3 + SDSS, ignoring any possible tensor contribution \cite{20}.

\(^{9}\) In fact, $\phi_{UV}$ should be reduced by a small amount since we are assuming that $\ell$ does not correspond precisely to the inflationary patch when $\phi = \phi_{UV}$ but to some slightly later time. However, if the field is in slow-roll at this time then $\phi$ will not roll far during the few e-folds which elapse.
Table 1. Limits on the total number of e-folds, $N$, in monomial models with $r=2$ and 4. $N_P$ measures the number of e-folds between the Planck scale and the time presently observable scales were leaving the horizon, and $N_{sr}$ is a similar measure, with the Planck scale replaced by the ‘self-reproduction’ scale of eternal inflation, $\phi_{sr}$ (defined in the text). The number in brackets (· · ·) following the maximum $N$ allowed by the $c$-loop gives the percentage error involved in estimating $N$ by the approximate formula (38).

| Maximum $N$ | Significance (%) | $q$-loop | $c$-loop | $N_P$ | $N_{sr}$ |
|-------------|------------------|----------|----------|-------|---------|
| $r = 2$ model |
| 0.1 | 4.30 × 10$^6$ | 4.06 × 10$^4$ | 1.28 × 10$^{10}$ | 5.03 × 10$^5$ |
| 1 | 4.30 × 10$^7$ | 8.74 × 10$^4$ | |
| 10 | 4.30 × 10$^8$ | 1.88 × 10$^5$ | |

| $r = 4$ model |
| 0.1 | 2.17 × 10$^6$ | 8.65 × 10$^3$ | 6.31 × 10$^5$ | 2.50 × 10$^4$ |
| 1 | 2.17 × 10$^7$ | 1.54 × 10$^4$ | |
| 10 | 2.17 × 10$^8$ | 2.74 × 10$^4$ | |

Let us estimate how many e-folds can elapse before the loop correction becomes significant enough to spoil the tree-level prediction. The coefficient of the $A$ term in equation (24) is $O(1)$ in slow-roll$^{10}$:

$$\frac{\partial N}{\partial \phi} \frac{\partial^3 N}{\partial \phi^3} + \left(\frac{\partial^2 N}{\partial \phi^2}\right)^2 = \frac{1}{r^2}.$$  

(37)

Thus, in order that the loop contributes at no more than some fraction $f$ of the tree-level, we must require $\Pi(\partial N/\partial \phi)^{-2} \lesssim f$, or

$$N \lesssim N^6_{\phi}(6+r)/(4+r)\mathcal{P}_\zeta^{-2/(4+r)} \{2(4 + r)f\}^{2/(4+r)}.$$  

(38)

This gives constraints on $N$ which are summarized in table 1. Note that the bounds arising from the $c$-loop contribution are roughly compatible with the bounds derived by Wu et al [26] on the basis that amplification of the quantum geometry should not spoil scale invariance of the spectrum.

The numerical limits on $N$ cannot be taken literally for loop:tree ratios as large as 10%, because the estimates for $g$ and $\langle \delta \phi^2 \rangle$ (given in equations (32) and (34)) were derived assuming that the tree-level prediction dominated the power spectrum, and this is no longer true when the loop constitutes an appreciable fraction of the tree-level. Likewise, as the significance of the loop increases, contributions from two-loop

\textsuperscript{10} Note that in a monomial model there is a special simplification, since $N''' = 0$.  

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corrections and beyond may become relevant. Nevertheless, these limits show clearly that, if inflation is taken to persist for an exponentially long time, corresponding to initial conditions at the Planck scale or the ‘self-reproduction’ scale of eternal inflation, then the loop correction will be large and cannot be ignored. On the other hand, if inflation is only taken to consist of sufficient e-folds to solve the horizon and flatness problems, $N \sim N_\star \approx 60$, then the loop correction will be completely negligible. These conclusions are in qualitative agreement with (but somewhat larger than) the results of Sloth [52], who found corrections of $O(1)$ for inflation beginning at the self-reproduction scale. The qualitative disagreement between references [51,52] and the present paper, on the one hand, and the conclusions of Martineau and Brandenberger [87] on the other arises because in [87] the random distribution of phases present in the inflationary perturbations caused an overall cancellation of the effect. In the present formalism the distribution of phases is invisible; only the spectrum is relevant, not the modes themselves, and the large effect described in table 1 is effectively the phase-space volume divergence given in equation (46) of Martineau and Brandenberger [87].

5. Discussion

In this paper, the $\delta N$ formalism has been used to estimate the magnitude of the one-loop correction to the inflationary power spectrum in the case of a single-field, slow-roll model of the inflationary epoch. The calculation can be split into two separate parts, which both make relevant contributions to the loop correction. The first depends on a detailed estimate of the one-loop correction generated by interference among quantized field modes around the time of horizon crossing. This calculation has recently been performed in [58]. The second measures the pile-up of field modes outside the horizon which are generated by the quasi-de Sitter expansion. This accumulation of modes as inflation proceeds gives an ever-larger phase space for infrared divergences [87], which is measured by the so-called $c$-loops of the $\delta N$ formalism. In this paper these two parts are assembled, allowing an estimate of the magnitude of the loop correction which includes all relevant effects.

A feature of this calculation is the use of the $\delta N$ formula to compute loop corrections from the superhorizon modes. We have shown that the loop correction from Schwinger loops among the $\delta \varphi$ contributes at only around 1% of the tree-level if inflation lasts for $10^7$ e-folds. In models of inflation driven by monomial potentials, or more generally with a significant negative tilt, the $\delta N$ loops scale with the total number of e-folds faster than the Schwinger loops. Therefore in any model where the loop correction is significant, it is probably an acceptable approximation to ignore the Schwinger loops and include only the effect of the $\delta N$ loops provided that the initial surface is chosen close to the horizon-crossing time for the $k$-mode under consideration. This is entirely analogous to the situation when computing higher-order $n$-point correlators of $\zeta$ [94,95,100], where the intrinsic non-Gaussianity among the $\delta \varphi$ is likewise negligible, and the significant effect can be computed merely by using the classical $\delta N$ formula. This is likely to be an extremely valuable simplification in practice, since it implies that one can dispense with the complicated Schwinger formalism described in [58].

The results described above are entirely consistent with our expectations from flat space quantum field theory: we are free to formulate the fundamental microscopic theory
which governs inflation at any energy scale we wish, but if we choose to formulate it at an energy scale which is wildly different from that of the phenomena we wish to observe, then we must expect significant corrections to be generated by loop effects. Such corrections take the form of large logarithms that violate the predictions of naive dimensional analysis. In the present case the loop correction is given by equations (23) and (24), the fundamental theory is formulated at a scale $\ell$ and we wish to predict phenomena associated with the CMB at a scale $k^{-1}$. The large logarithm which gives a significant loop correction is $N \sim \ln k\ell$. If $k\ell$ is exponentially large, which occurs in monomial models of inflation if initial conditions are set at around the Planck scale, then the loop correction has a sensitivity to the tilt of the field perturbation power spectrum. The $\delta N$ loop from the convolution of two power spectra is the most sensitive and scales like $N^{2+n/2}$; the $\delta N$ loop from any intrinsic non-Gaussianity among the $\delta \phi$ is suppressed, because $f_{NL}$ scales in the opposite direction, giving $N^{1+n/2}$; and the Schwinger loop is blind to anything but the epoch of horizon crossing and scales only like $N$. In the case of a negative tilt, which produces perturbations of increasing amplitude on large scales, the loop correction rapidly becomes very significant. The correction is irrelevant only if inflation lasts for a small number of e-folds.

The physical effect which demands that loop corrections be taken into account is simply back-reaction. Long ago, Salopek and Bond [101] used the stochastic formalism to show that any initially homogeneous background field would develop wild fluctuations on ultra-large scales as inflation progressed. The large loop correction encountered in the limit of a large box is another manifestation of this effect. It measures the dispersion generated by comparing a correlator calculated in one region of the universe with the same correlator calculated in a different region, perhaps where the background fields have very different values. The correlator calculated in the large box is therefore not measurable in any experiment: it involves comparing the CMB temperature which would be seen in an ensemble of different universes, and no physical observer can make such a comparison. For this reason, the correlation is a ‘computable’ in the sense of Witten [23], but is not an observable.

This result has no consequences for how we make or report cosmological observations, but has very significant consequences for how such observations are compared with theoretical models. In models where the fundamental theory is not taken to be valid within an exponentially large region, we are entitled to entirely disregard the question of loop corrections, since in this case they yield a negligible shift in tree-level values. This class of models includes the familiar single-field examples where inflation ends when the field reaches a certain point on its potential. In such cases we know the conditions which lead to the end of inflation and we always have the option to work within a small box. On the other hand, where isocurvature fields are present, these fields will generally acquire large fluctuations within the inflating region. If these fields play a role in ending inflation, then we may lose the option to compute within a small box, whereas the correlator within a large box may be afflicted with large loop corrections and therefore not be measurable. This problem is perhaps most severe precisely for those models to which we presently attach the greatest interest—those motivated from fundamental particle physics. For example, one popular class of such models arise from the so-called landscape of string vacua [102]. In such models, one imagines the unwanted six dimensions predicted by string theory to be wrapped up on a Calabi–Yau threefold which may be topologically
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quite non-trivial, as measured by the existence of a large number of homological cycles. These cycles can be wrapped by fluxes of $p$-form fields, giving rise to regions of the threefold which have different values for the cosmological constant $\Lambda$, together with the other constants of nature [103], and in general many scalar fields will be present which measure the size and shape of the Calabi–Yau threefold. These different regions may exhibit substantially different low-energy physics. In such models, the natural scale at which one computes the masses and couplings in the low-energy theory would be the Planck scale, or perhaps the self-reproduction scale of eternal inflation. Equations (23) and (24) show that, to compare any such theory to observation, one must account for large loop corrections which are inevitably encountered in passing from the Planck scale to the scale relevant for CMB predictions.

This has serious implications for how large-field models of inflation are compared with CMB observations. To understand how the Planck scale theory is related to the theory when scales associated with the CMB exited the horizon, one would need to integrate out the relevant degrees of freedom between $M_P$ and the CMB scale. By analogy with conventional field theory, as one changes scale the effective Lagrangian should be controlled by a flow equation analogous to Polchinksi’s exact renormalization group equation [104]. In flat space theory this flow has well-understood effects, attracting the Lagrangian to an invariant hypersurface on which non-renormalizable couplings are suppressed. (This would refer to the effective field theory for $\zeta$, not to the couplings in the matter theory.) Unfortunately a complete understanding of this process in the cosmological case is not yet in hand, and it does not appear to be understood precisely how the Planck scale theory will be related to the theory in a small box. One possible approach is to use the method of Salopek and Bond [101], who gave a Fokker–Planck equation which determined the probability distribution of the classical fields on large scales. Having done so, it is possible to use a small box to determine the value of the correlation functions at each value of the allowed background fields, but one is then restricted to making probabilistic statements concerning what should be observed in the CMB. This issue deserves further study.

In any case, considerable work remains to be done. The present analysis should be generalized to the case of multiple scalar fields in order that it can be applied to the best-motivated models from fundamental physics. This involves an extension of the $q$-loop calculation given in [58] to the case of many scalar fields. The calculation given by Weinberg [44] shows that, where $\zeta$ is accompanied by $N$ isocurvature fields, we can expect the $\zeta$ loop to receive an enhancement by a factor $\sim N$. However, in a multiple field scenario a more involved process is required to estimate $\langle \delta \phi^2 \rangle$ [105]–[108], since the growth of $P$ on superhorizon scales is no longer simple to predict. These issues should be addressed with some urgency.

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