Stick-slip Avalanches in Steady Shearing:

Signature of Transition between Granular Fluid and Solid

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Abstract -- By observing the fluctuations of fluid-immersed granular particles upon steady shearing, we identify a transitional zone that sets the system apart from fluidic sliding and signals the onset of solid mechanics, as the shear rate decreases. Toward the slow extreme, statistical analyses of the avalanche events combined with internal imaging capture the continuous yet distinctive change of behaviors, and offer test grounds for theories on the development of plasticity. We link such transition with the velocity weakening of inter-particle frictions, and propose a three-state phase diagram that bridges our discoveries on tightly packed granular systems and previous understanding of suspension rheology.
Granular-fluid systems are excellent examples exhibiting solid-fluid duality. In the past decade, paradigms have been established for understanding the rheology of granular suspensions [1] where fluid-mediated particle interactions warrant a smooth flow at low shear rates. At densities well above the jamming point, the same system can instead withstand a shear stress indefinitely, behaving like a solid. The macroscopic deformation of granular solid often rely on different traditions such as elastoplastic modelling [2] to describe. Understanding the transition between these two extreme remains profound challenges, not only in the sophistication of theories treating partial fluidizations [3–5], but also due to the lack of experimental observation along such transition [6]. As a quasi-static problem, one recent consensus since the initial proposal of the jamming phase diagram [7] has been that the jamming point depends on inter-particle friction [8], and it is friction that makes a granular packing go beyond “ideal jamming” and exhibit fragile states where imposed shearing plays decisive roles [9–12]. Growing pieces of evidence have shown that tangential force between particles can also lead to dramatic changes in dynamics of rapid flows: Examples include experiments [13–15] or simulations [16,17], as a result of either actively turning on the friction, or of increasing the collisional stress above a critical load such that abrupt shear-thickening would occur.

For a tightly-packed system exhibiting long-lasting inter-particle contacts, most notably in geophysics, velocity-dependent frictions are often considered the root of instabilities leading to catastrophic consequences like earthquakes [18–20] or hysteresis in laboratory studies [21]. Most existing work, however, do not address how the velocity dependence of friction between single grains could affect the flow at the macroscopic level. Nevertheless, one recent work reports the correlation between the mean flow of sheared emulsion and the tribology of constituent particles [22].

In this Letter, we demonstrate that high packing fraction is just a necessary condition for creating solid-like response. The imposed shear rate plays a crucial role on the regime transition. We identify a transitional zone that is intimately connected to the velocity-dependent friction at the grain level. Along the transition, prominent stick-slip avalanches provide an unambiguous sign setting apart viscous sliding (like a fluid) and plastic yielding (like a solid). Based on our observations, we propose a three-state diagram that integrate the flows of interlocked solid, lubricated sliders, and suspended granular particles on one single landscape controlled by the shear rate and the confining volume (or pressure).
Setup and overview of transitions --- Shown as in Fig.1(a), our particles fill the space between two cones. The sidewall is formed by a stack of free-sliding acrylic rings with an inner diameter $2R=22$cm. Two cones are geometrically roughed at the scale of the particle with diameter $d\approx9$mm. The upper cone is set at a fixed height, with the mechanism specially reinforced to provide a smooth rotation at an angular resolution of $2\pi\cdot10^{-4}$ with a wide range of angular speeds $\Omega$. The base of the system is supported by the arrangement of six independent force sensors, in order to determine the total forces and moments that keep the base stationary. The gap between the rotating boundary and the sidewall is 2mm such that all particles cannot escape. The nominal volume fraction is defined as $\phi = Nv_1/V_{\text{access}}$ in which $N=O(1000)$ is the number of particles. $V_{\text{access}}$ represents the total volume accessible by the particles, and $v_1$ stands for the volume for the single particle (which has been determined by Archimede’s method and checked for the mono-dispersity). Simultaneously, we take internal images of particles through the bottom, when the interstitial space is filled with aqueous solution of 60% glycerol such that the refraction index is matched to the particles [23,24]. We illuminate the system at its mid-height, with a horizontal laser sheet at the wavelength of 532nm going through the 1mm gap between the tips of two cones. Image contrasts are generated by dying either particles or the fluid with Rodamine. Data reported in this paper are based on spherical particles of polydimethylsiloxane (PDMS) elastomer that we create by molding: We use the default 10:1 ratio with crosslinking agent [25] ; the particles exhibit a Hertzian response to compression with a Young’s modulus around 1.5MPa [33].

Figure 1(b) presents an overview of transitions, by showing the statistical distributions of instantaneous torque (presented as an effective stress in kPa). Data include experiments using different rotation rates ranging from 0.05 to 0.0001 rev/s (rps), all at the same $\phi=0.60$. At either the fast or the slow end, the histogram exhibits a bell shape on the semi-log plot close to a parabola. Note that the slower runs is at the state of higher stress, in startling contrast with reported studies on most rheometric flows [26–28]. Interestingly, between the two extremes, the data show series of transitional states with histograms that are highly asymmetrical. We label the case at 0.005rps as state-T. The strong asymmetry can be understood by observing the time series of torque --- see Fig.1(c). It shows that state-T exhibits abrupt changes of torque, with several large drops per unit strain ($=\Delta \theta /2\tan \xi$, see Fig.1a), and the concavity of the curve connecting the drops provides a minimal explanation of the asymmetry. We also label the other representative high-stress (low-stress) state as state-$\beta(\alpha)$, for the convenience of subsequent discussions.
In analogy to conventional rheology, we define “flow curves” by time-averaging the effective stress $\sigma_S$ and plot them against the shear rate $\dot{\gamma} (\equiv \Omega/2 \tan \xi)$ ----see Fig.1(d). This includes data from experiments at a wider range of rotation rates than that in Fig.1(b), plus two additional experiments using fluids at higher viscosities (by adjusting the concentration of glycerol). The trend shows not only an interval of $\dot{\gamma}$ with negative slope, but also a minimum with an uprising of stress at higher shear rates. For all experiments up to 0.05rps, we also compute the normalized occurrence of the large drops (LD, defined as drops of torque that are larger than twice the root-mean-square of the total fluctuation --- see Online Supplements[33] for justifications) as a simple indicator to characterize the stick-slip fluctuations. We incorporate the measurement of normal stress $\sigma_N \equiv$ time-averaged $F_z/\pi R^2$ and plot the counts of LD per unit strain against the viscous number $J \equiv \eta \dot{\gamma}/\sigma_N$ [1], shown as Fig.1(e) for results with three different fluids. They show a consistent rise and fall over the change of $J$, and reassure that the state we label as $\alpha$- (at 0.05rps) belongs to the falling branch of this generic trend.

**Approaching the plastic regime** --- Stick-slip avalanches are prominent only in a limited range of driving rate. We find the gradual change from state-$\alpha$ to state-$\beta$ deserves special attention. The change demonstrates how the system becomes less fragile and settles to a state of higher stress, as the system-wise avalanches become infrequent. Fig.2(a) shows the cumulative distributions of drop events in torque: We count the number of drops with a magnitude larger than a variable threshold $\Delta_{\text{min}}$. Data includes multiple runs at different driving rates. The event counts are normalized by the total strain accumulated in each run, and are plotted against the variable threshold normalized by the mean torque at each driving rate. The change is striking: At state-$\alpha$, the statistics indicates that for an accumulation of strain close to unity, one can expect at least one drop with a magnitude over 40% of the mean. In contrast, at state-$\beta$, such probability is two-decade smaller: a comparable drop would be extremely rare such that it takes a total accumulation of strain $\sim O(100)$ to obtain one such event. The inset graph shows the log-log plot for three long runs using the same sample. At state-$\alpha$, the statistics exhibits a dramatic change in the logarithmic slope as $\Delta_{\text{min}}/<\text{torque}>$ approaches unity, showing the effect of a characteristic scale set by the mean stress. In contrast, state-$\beta$ presents a scale-free behavior with a constant slope (between -2.5 and -3) that spans over two decades in probabilities, because rearrangements are mostly localized such that the associated stress releases are never comparable to the mean.
The dramatic difference between state-T and state-\(\beta\) are further illustrated by the scattered plots of the multicomponent force data (Fig.2b-c) and the direct visual evidence from internal images (Fig.2d-e). At state-T, the torque-\(F_z\) plot in Fig.2b reveals multiple traces of “loading” processes between abrupt drops. By observing the time series of torque and \(F_z\) (not shown), we note that the growth of \(F_z\) with the angular displacement during the loading processes is closer to linear than that of torque; this explains the concavity of those traces on the torque-\(F_z\) plane. Meanwhile, the \(F_y-F_x\) plot can be regarded as a reduced phase space and reveals multiple clusters corresponding to processes of “loading”, as well as sudden jumps in-between. The stack of differential images in Fig.2(d) shows that system-wise landslides are in-sync with the drops of torque. We have also shown in Fig.1(f) that these landslides are quite abrupt: Particle movements develop and diminish within \(\Delta \theta < 2\pi\cdot10^{-3}\) or \(10^{-2}\) in strain, with flashes of differential images [33] as vivid demonstrations of dynamical heterogeneities [29] for systems near jamming. To a great extent, a system-wise landslide interrupts the stress build-up. For state-T, such “reset” occurs at the frequency of several times per unit strain. On the contrary, large landslides become extremely rare for state-\(\beta\): Fig.2(e) shows that all detected particle movements are highly isolated, often within the range of one single particle or just a few. Understandably, the lack of large-scale rearrangements successfully maintains state-\(\beta\) at a higher level of stress (than the mean value for state-T), with most fluctuations being infinitesimally small. The physical origin of such contrast will be explained with the connection with grain-level tribology as follows.

*Connection with tribology and a three-state diagram* – The intriguing rate-dependent transitions can be understood by considering two facts. (1) Illustrated as Fig.3(a), imposing a shear rate \(\dot{\gamma}\) to a densely packed system creates a range of relative speeds among particles. In a system that is strongly damped, the anticipated distribution should be around the most probable speed \(~d\dot{\gamma}\). (2) By independent measurements using the same fluid and PDMS elastomer with a radius of curvature \(R=d/2\) to mimic our main experiments, we characterize the speed-dependent friction between surfaces – see Fig.3(b). The results show a characteristic speed \(V_c\) beyond which the tangential force decreases dramatically. Such effects are consistent with the transition from the regime of solid-solid friction to that of mixed lubrication, that is commonly cited as part of the Stribeck curves in tribology involving fluids [14,30].

Figure 3(c) combines results from four datasets from our main experiments, and an interesting picture emerges as on a two-parameter landscape mimicking the “three states of matter”. For each of \(\Phi\), we characterize the occurrence of stick-slip fluctuations (#LD/Strain, shown as its vertical coordinate) and compute the normal stress (\(\sigma_N\), shown as one of the
horizontal coordinates), as functions of the shear rate. The shear rate is also converted to its dimensionless form \( \frac{d\dot{\gamma}}{V_c} \). The choice of the four volume fractions warrants that \( \sigma_N \) are high enough to override the effect of gravity (due to about 10% mismatch of density) and go beyond the reported limit for stress-controlled suspensions (~0.58, see Ref. [31]). The most notable feature is that the occurrence of stick-slips reveals a ridge on the landscape that separates the state of viscous sliding (state-\( \alpha \)) and that of plastic yielding (state-\( \beta \)).

The emergence of a ridge on the landscape can be understood by comparing the magnitudes of \( d\dot{\gamma} \) and \( V_c \). For \( d\dot{\gamma}/V_c \gtrsim 1 \), most contacts are in the status of low-friction and are expected to slide smoothly --- except that at higher driving rates the system would enter shear-thickening regime and jamming can be induced by collisional stress [13–17,27,28]. However, at the shear rates with \( d\dot{\gamma}/V_c \) smaller but close to unity, one would expect that a small but substantial fraction of contacts are sliding: As the shearing keeps changing the particle configuration, these sliding contacts gather by chance and create slip planes such that system-wise landslides occur. Lastly, in the slow extreme with \( d\dot{\gamma}/V_c \ll 1 \), all contacts are in a high-friction state such that particles are interlocked. Since cooperative sliding becomes probabilistically impossible, the only mechanism for the global rearrangement is the accumulation of localized yielding, i.e., the system responds to the macroscopic shearing with numerous isolated slips (Fig.2e).

Figure 3(c) also shows that the stick-slip fluctuations diminish as a result of reducing \( \Phi \) (and subsequently the mean stress) -- see the left-hand-side projection of the data and the gray dotted arrow. Understandably, the ridge on the landscape is expected to flatten as the system is reduced to a suspension. (Our ongoing experiments with precisely density-matched particles and fluid have shown that frictional stress is substantially lost at \( \Phi \) below 0.48.) To connect to prior studies on granular suspensions, we indicate five contours of constant \( J \) on the horizontal plane. While most studies reviewed earlier [1] are limited to \( J > 10^{-5} \), our findings reflect the consequence of further lowering the viscous numbers by several decades. One recent work with a rotating drum [21] also reports that smooth flows cease to exist and are replaced with hysteric avalanches, when \( J \) goes below a critical value \( J_c \sim 10^{-6} \). In our experiments, we consistently find that the peak occurrence of LD corresponds to viscous numbers between \( 10^{-6} \) and \( 10^{-7} \) (see Fig.1e as well). However, our fixed-volume shearing allows further extension beyond such occurrence and shows that avalanches can again vanish at the slow extreme (with \( J < 10^{-9} \)).
Summary and outlooks--- We design an experiment to clarify the relationship and transitions among three regimes of granular dynamics in one setup: suspension flows, lubricated sliding, and plastic yielding. Keeping a sufficiently high volume fraction, we identify an unambiguous solid-fluid transition which can be understood as the competition between the shear rate $\dot{\gamma}$ and the velocity-weakening threshold $V_c$ for particle pairs: (1) We show a “dangerous zone” of shear rates where the most probable speed between particles $d\dot{\gamma}$ lie slightly below $V_c$. In these cases, although the majority of contacts are in solid-solid friction that favors the build-up of shear stress, a small but non-negligible fraction of low-friction sliders can create system-wise avalanches that bring the system to a state of lower stress, in a frequency of several events per unit strain – presented as a ridge on our phase diagram. These events are visualized by the imaging of internal landslides, which are well in-sync with the stick-slip patterns of torque. (2) At sufficiently high shear rate when viscous lubrication overrides the solid-solid friction between particle pairs, smooth sliding generally occurs and can be seen as a continuation from the well-established dense suspension regime where the stress arise mainly from collisions [1]. (3) Upon reducing the shear rate, we demonstrate the evolution toward plastic yielding --- The system-wise “brittle” failures are gradually replaced with isolated slips, with corresponding stress drops being infinitesimally small. The change of event statistics should provide test grounds for theories in understanding the “flow” of granular solids --- For instance, it should be interesting to test whether incrementally reducing concentration of sliders in a population model [32] would reproduce what we observe.

We stress that taking the velocity dependence of friction into account should be essential for making realistic predictions on the flow of granular solids, such as in geophysical contexts. For fundamental researches, this also leads to many open questions, such as whether the coupling between particle velocity distribution and the speed-dependent tribology might demand a modification on concept of a “critical” density of jamming, and whether the occurrence of avalanches can be understood by generalizing theories from the percolation of contacts [10] to that of low-friction sliders, or by imposing a distribution of frictional coefficients in quasi-static simulations [12].
FIG. 1 (a) Schematics for the setup and measurements, from left to right: the time-dependent torque and three net forces as determined from sensors around the base; cross-sectional view of the main setup; 3D illustration of the roughened cones; close-up of one original fluorescent image; and the differential image δf computed from two frames with δθ=8π·10⁻⁴. Pixels in positive (negative) values are displayed in blue (red). (b) Statistical distributions of measured torque, in steady states at different rotation rates Ω/2π (in revolution per second, rps). φ=0.60. (c) The time series of torque for three examples in (b), all plotted against the angular displacement. The high-frequency noise has been filtered with a cut-off at 40ms. (d) Time-averaged stress, plotted against the shear rate γ. (e) Counts of LD (defined in main text) per unit strain, plotted against the viscous number J. Both (d) and (e) include experiments with fluids at three different viscosities. φ=0.56. Drop counts are applied only to experiments up to 0.05 rps due to the high-frequency cut-off. (f) Sequence of δf with δθ=2π·10⁻⁴, featuring one typical avalanche corresponding to a drop indicated by the paired circles in (c). The angular movements of the driving cone are labeled in reference to a fixed θ₁.
FIG. 2 (a) Cumulative distribution of drop events with $|\Delta \text{torque}| > \Delta_{\text{min}}$, for experiments at different driving rates indicated in rps. Event counts are normalized by the strain accumulated. Data include experiments using two batches of particles (A2 and C) in the same fluid, $\phi=0.56$. (b-c) Scattered plots of $\{\text{torque vs. } F_z\}$ simultaneously with $\{F_y \text{ vs. } F_x\}$, for state-$T$ and state-$\beta$, respectively. Numbers represent time-ordered sequence; these graphs are also presented in animations online. Each axis spans $\pm 2 \text{RMS}$ of the variable it represents around its mean value. Data are accumulated with a total angular rotation of 0.3, in which an interval corresponding to unit strain is marked in green. Animated versions are provided online[33]. (d) Stacked sequence of $\delta f$ in-sync with $\text{torque}(t)$, for state-$T$. (e) Four typical frames of $\delta f$ with $\delta \theta = 2 \pi \cdot 10^{-4}$, for state-$\beta$. 
FIG. 3 (a) Illustration of the motion of densely packed particles under an imposed shear rate $\dot{\gamma}$; (b) Schematics and results of measurements on the average tangential force $f_x$, normalized by the normal force $f_z$, for a localized contact between PDMS surfaces with a radius of curvature $R$. Data are plotted as functions of the sliding speed, for experiments at multiple pressing depths (controlled by the distance $S$ between the base planes) up to about $0.1d$. Data reveals a critical velocity $V_c$ that appears insensitive to the pressing depth. (c) Proposed phase diagram presented with results from experiments with four different values of $\phi$. The 3D plot consists of the normalized occurrence of LD as the vertical coordinate, with a horizontal plane spanned by shear rate $\dot{\gamma}$ and normal stress $\sigma_N$. Contours of constant values of $d\dot{\gamma}/V_c$ and $J$ are also displayed. For the ease of referencing, data points are also projected onto the side plane and the bottom.
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