A fast Maximum Likelihood Estimation algorithm for demodulating Fiber White-Light Interferometry

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Abstract. Being introduced to upgrade the precision of demodulation for Fiber White-Light Interferometer, Maximum Likelihood Estimation faces the problem of low calculation speed. In this paper, a high-speed algorithm is proposed based on Newton Iteration for Maximum Likelihood Estimation to accelerate its fine-search process, and the process is precisely kept in the main peak of MLE spectrum with a numerical calculation. The simulation results show that the fast algorithm increase the speed for two orders of magnitude.

1. Introduction

As an absolute and accurate interferometer, optical Fiber White-Light Interferometry (FWLI), such as fiber F-P and fiber Michelson sensor, are widely used in many fields [1]. Signal demodulation is the key technology of FWLI, and many demodulation algorithms are presented. The period analysis in wavelength domain and FFT analysis in transform domain are the two typical representatives [2][3]. Although period analysis has been widely used in many applications, but its demodulation precision is not satisfied in some special fields. On the other hand, Cross-correlation Method, Total-phase Method and Maximum likelihood Method are reported in high precision demodulation of other interferometric signals[4][5][6]. Taking advantages of both frequency and phase of the signal, these algorithms have large computational volume which decreases its demodulation speed dramatically[7]. In some special applications of FWLI, both high precision and high speed demodulation is necessary. If the problem of computational volume can be addressed, fast maximum likelihood estimation is possible in high precision demodulation of FWLI.

2. Demodulation of FWLI with Maximum Likelihood Algorithm

Typical FWLI of Fiber F-P and fiber Michelson sensor are shown in Fig1. The output of FWLI is two-beam interference and can be written as Eq(1) after discrete sampling and DC filtering.

\[
S_n = A \cos \left( 2\pi \Lambda_0 \left( k_0 + n \cdot \delta k \right) \right)
\]

Where, \(S_n\) is the discrete signal of interference spectrum, \(A, k_0\) and \(\delta k\) are the amplitude and initial wavenumber and sampling interval of the signal respectively, \(\Lambda_0\) is wanted parameter of optical-path-difference (OPD), \(n=1, 2, 3, \ldots\).

The Maximum Likelihood Estimator (MLE) \(\hat{\Lambda}_0\) of the \(\Lambda_0\) can be expressed as the following:
\[ \hat{\Lambda} = \max \left\{ \ln p(\mathbf{S}_n, \Lambda) \right\} = \max \left\{ -\sum_{n=0}^{\infty} \left[ \mathbf{S}_n - A \cos(2\pi \Lambda k_0 + 2\pi \Lambda n \cdot \delta k) \right]^2 \right\} \]  

(2)

Because \((1/N) \sum \cos(2\pi \Lambda k_0 + \varphi_0) = 0\), Eq.(2) can be simplified into Eq.(3).

\[ \hat{\Lambda} \approx \max \left\{ \sum_n \mathbf{S}_n \cos \left( 2\pi \Lambda k_0 + 2\pi \Lambda n \cdot \delta k \right) \right\} = \max \left\{ \text{Re} \left[ e^{-2\pi \Lambda^2} \right] \cdot \text{Re} \left[ \mathcal{F}(\mathbf{S}_n) \right] \right\} \]

(3)

According to Eq(3), the MLE is the FT spectrum of \( \mathbf{S}_n \) modulated by a high-frequency factor \( \text{Re} \left[ e^{-2\pi \Lambda^2} \right] \), which is different from those of Discrete Fourier Transform (DFT) and Zero-Padded DFT (ZPDFT) as shown in Fig.2(b). According to Eq(3), \( \hat{\Lambda} \) is the maximum location of the spectrum and MLE can be realized in peak-seeking in Fig2. According to Fig2, MLE has a higher precision of \( \hat{\Lambda}_{MLE} \) due to its sharp waveform, while \( \hat{\Lambda}_{DFT} \) and \( \hat{\Lambda}_{ZPDFT} \) of DFT and ZPDTF have relative lower precision due to their gentle waveform as shown in Fig2(c). Otherwise, high precise demodulation of MLE is a time-consuming process in its in maximum peak-seeking due to multiple peaks screening.

In a deep investigation of Fig 2(c), it is clear that \( \hat{\Lambda}_{ZPDFT} \) is close to \( \hat{\Lambda}_{MLE} \). This implies that it is possible to cut the process of multiple peaks screening dramatically if the \( \hat{\Lambda}_{MLE} \) is limited in the section of its main peak by ZPDFT. In addition, if there is a fast algorithm in seeking \( \hat{\Lambda}_{MLE} \) from \( \hat{\Lambda}_{ZPDFT} \), it could further accelerate the demodulating process of MLE.

Because the Newton iteration method has the ability to quickly find the peak of a function within its iterative convergence range, it is possible to be introduced in the process of locating MLE spectrum within its main peak and speeds up the peak-finding process. In order to carry out this fast iterative algorithm to the estimation of OPD, two problems must be addressed. Firstly, the convergence interval of OPD must be determined. Secondly, the iteration converges must be kept in the main peak in the iterative process, instead of other peaks.

3. The interval of convergence in MLE

The convergence range of Newton method is the range which keeps the iteration process in converging at the local peak. In MLE spectrum, as long as the Spectral resolution of ZPDFT \( \delta \hat{\Lambda}_{padded-DFH} \) is smaller than Newton Range (\( \Delta N_R \)) of the main peak, the \( \hat{\Lambda}_{ZPDFT} \) will be located...
within the $\Delta NR$ as shown in Fig 3 (b). Then the $\hat{A}_{MLE}$ can be accurately calculated by Newton method starting from $\hat{A}_{PPPT}$.

Because the convenient way to find the maximum value of a function is correspondent to locate its derivation in zero, both MLE (black dot line and spot) and its derivative function (blue line and spot) are plotted in Fig 3 (c) comparatively.

Been limited in the main peak of MLE, locating its derivation in zero can be quickly done from an initial location “P” within the Newton convergence interval range. The Newton Range (NR) can be determined by finding the range of limit cycle as the green loop of the infinite iterative circle in Fig 3(c) shows. However, unlike the calculation of NR in traditional frequency estimation problems\cite{8}, the MLE of interference spectrum is modulated by a high-frequency factor $\text{Re}\{\exp(-jk_0\Lambda)\}$, which makes the Analytical expression of the NR cannot be obtained in OPD ML estimation. Therefore, this paper presents a numerical solution of Newton convergence interval range.

The calculation process of finding the high-precision Newton convergence interval range contains two circulations. The first is finding the Newton range under accuracy $d\lambda$ and the second is decrease the accuracy by $d\lambda = d\lambda/10$. The numerical calculation stops until the accuracy is high enough. A complete process of calculation shown in fig 4 (b).

![Algorithm flowchart]

(a) Algorithm flowchart    (b) A simulation process of algorithm

Fig 4 calculation process of Newton convergence interval range

![Newton Low and high convergence range](a) Newton Low and high convergence range

![Newton convergence range](b) Newton convergence range

Fig 5 The relationship between optical spectrum parameters and the convergence range

In OPD estimation problem, parameters of the optical spectrum will influence the Newton convergence range. Center wavelength ($\lambda_c$) and wavelength range ($\Delta\lambda$) are two most important parameters. The convergence range in different $\lambda_c$ and $\Delta\lambda$ conditions are shown in fig 5 by simulation. The results show that the convergence interval is approximately linear with the center wavelength. The $\Delta\lambda$ has only a small influence on the result.
4. Fast iteration method of OPD Maximum Likelihood Algorithm

Similar to the traditional MLE frequency estimation of a sinusoid algorithm, the fast OPD algorithm contains coarse and fine-search processes\textsuperscript{7}. The coarse search uses zero-padded M-point FFT and fine-search using Newton iterative method. As mentioned before, the resolution of zero-padded \(M_{\text{padded}-DF\text{F}}\) must smaller than Newton Range to make ensure that coarse-search result falls into the NR. Assume that the original optical spectrum contains N points, the sampling interval in wavenumber domain is \(\delta k\) and the Newton range is \(\Delta NR\). The value of \(M\) must follow Eq.(5) which can be calculated by digital Fourier transform principle easily.

\[
M > \frac{2}{\delta k \cdot \Delta NR} + 1
\]  

(5)

In order to compare the algorithm speed before and after using Newton method, table.1 shows the total calculation time of 1000 times demodulation.

| parameters | fine-search methods | Interpolation method | NR iteration method |
|------------|---------------------|----------------------|---------------------|
| \(\delta k\) | 5cm\(^{-1}\) | 100nm | 1550nm |
| Estimated OPD | 0.1um | 0.01um | 0.001um |
| fine-search calculation load of MLE spectrum | 1000 | 2000 | 3000 |
| Total time (s) | 0.952 | 1.901 | 3.049 |

5. Conclusion

In summary, a fast Maximum Likelihood Algorithm for Optical-Path-Difference Estimation in Spectral Interferometry is proposed. By using Newton iteration method in fine-search of estimated OPD, the algorithm increases the speed of the algorithm by two orders of magnitude in the simulation. The Newton convergence interval range is calculated by limit cycle method to ensure that the fine-search process convergence to the main peak of MLE spectrum. The research in this paper paves the way for the engineering application of MLE of OPD algorithm.

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