Accounting among the Natural Sciences

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Abstract

This paper develops research in order to clarify whether accounting theory belongs to the natural sciences. Three features characterizing these sciences are identified and evaluated. These are the relationships of accounting theory with respect to the fundamental laws of nature, the possible existence of necessary constants, and the abstractness of the categories introduced, which requires the use of mathematics. Studies have shown that these characteristics are fully visible. The second principle of thermodynamics has shown its significant influence leading to the indication of the existence of a constant quantity, necessary in capital accounts. It also turned out to be the basis of the original method of calculating the depreciation of assets. The principle of dualism, fundamental for accounting, revealed that it is, among other things, equivalent to the first principle of thermodynamics. And the principle of minimum action has always stimulated the development of cost accounting and management accounting. The practical significance of the research presented in the second part of the article consists in revealing of the economic constant. This constant determines the actions of nature in economic processes. As a result of the research, the knowledge of the possibilities of the accounting system is being expanded, in particular by incorporating the measurement of human capital of employees and opening the way to financial reporting in this area.

Keywords

Accounting, Natural Science, Fundamental Principles, Economic Constant

1. Introduction

The links between accounting theory and mathematics are clear and repeatedly described. In this study, it is examined and it is shown that at the basis of accounting theory, there are also physical laws, such as fundamental principles, in particular the principles of thermodynamics and the law of minimum action. It
is therefore a continuation of the research initiated in the article by M. Dobija and B. Kurek (2013a). The significant role of the first and second principle of thermodynamics is highlighted in order to explain the theoretical foundations contained in the principle of dualism, as well as to the progressive development of accounting as a scientific research program. The research reveals the economic constant on the pattern of physical constants, which determines the actions of nature in economic processes and is the basis for the calculation of human capital of employees and the theory of equitable remuneration.

2. Main Features of Natural Sciences

It is generally accepted that natural (natural) sciences try to explain how the world and the universe around us work. There are five main types: physics, chemistry, astronomy, Earth science and biology, but nothing stands in the way of engineering joining this group, as an extension of the knowledge emerging in natural sciences. However, it is not entirely obvious that this important group of sciences may include an economic discipline called accounting. It is therefore necessary to capture the main inalienable characteristics of natural sciences in order to identify and resolve this important issue. This is very important, since a positive outcome will cause economic thinking to be positively dominated by models of hard science of accounting rather than enigmatic political economics, whose name itself indicates unscientific conditions.

In the search for the characteristics of natural sciences, a statement by M. Gleiser (2008: p. 124) is helpful. This well-known physicist uses a customs metaphor to present scientific research and the expectations of scientists. He recognizes that scientists simply create descriptions of world phenomena, with three characteristics of their works. In these descriptions they appear: 1) physical constants, 2) fundamental laws, and 3) scientific method. The author states that physical constants are letters of the alphabet in these descriptions and that the fundamental laws of nature correspond to grammatical rules. When constructing these descriptions, scientists are guided by a well-known scientific method. M. Gleiser also points to the need for moderation as to the possibility of complete and final description of reality and its phenomena, which is natural and in accordance with the requirements of the scientific method defined by Karl Popper and Imre Lakatos epistemologies. Common sense also dictates moderation with regard to the possibilities of the human mind.

What are physical constants and how do scientists perceive them? The well-known scientist S. Hawking (1990: p. 120) characterizes the role of physical constants as follows:

…The laws of science, as we know them today, contain many basic physical constants, such as electron charge or proton to electron mass ratio. We cannot, at least today, calculate these constants on the basis of some theory; we have to determine them experimentally. It is possible that one day we
will discover a complete, uniform theory, capable of predicting the values of these numbers, but it is also possible that they change depending on the place in the Universe... It is worth noting that these values seem to be chosen very carefully to enable the development of life...

As writes H. Kragh (2016: p. 301) “...the fundamental constant (...) does not derive from any known physical theory and cannot be expressed by any other constants (...) It can be said that the fundamental constant is determined by our ignorance of it - not by its greatness, but by its very existence. ...” The author specifies basic constants of nature: speed of light c, Planck constant h (action quantum), gravity constant G, mass of electron m, mass of proton M, elementary charge e.

J. Barrow (2003: pp. 290-291) indicates an extremely important role of constants in theories. The author states that:

...our discovery of the rules of operation of Nature and the rules of change has led us to the discovery of mysterious numbers, which determine the structure of everything that exists. Permanent Natures provide the Universe with the possibility of its perception and its existence. (...) The Permanent Natures provide a bastion to protect science from unbridled relativism. They define the structure of the Universe in a way that can remove our prejudices resulting from our focus on purely human perception of things and matters. If we were to come into contact with the intelligentsia somewhere in the Universe, we would first learn about the constant Natures in order to obtain a common ground of understanding...

The constants discovered and calculated are like a link to higher knowledge, inaccessible to human minds. After all, we do not have a theory that reveals full knowledge of gravity, but calculating constants G has made it possible to determine the magnitude of earth’s acceleration and trigger rapid technical progress. On the other hand, since there are gravitational interactions, it can be assumed that there is knowledge about this phenomenon, but beyond the present intellectual and discursive range of man. Despite the incomplete knowledge, the constant allows positive actions. As scientists prove, the existing system of physical constants determines the existence of conscious life on Earth. These beliefs indicate the existence above the human intellect.

Can there appear constants like natural sciences in economic deliberations? To some extent, this is indicated by physiocrats referring to natural forces. It is difficult to calculate the impact of photosynthesis on the results of the economy, but they do exist. The human body acting on the model of a heat engine loses some of the energy gained from food and must have a heated shelter to live and act. This is not without an impact on the amount of wages necessary for the existence and preservation of human capital. The commodity and money economy appears to be a system of flows triggered by work and forces of nature, as shown in Figure 1.
M. Krauze (2020) describing the thermodynamics of the human body states:

…From the thermodynamic point of view, the human body is a kind of engine. It requires, as in technical engines, fuel supply. The fuel in this case is food, i.e. carbohydrates, proteins and fats. When introduced into the body, the nutrients are drawn into the metabolic modes (metabolism) and burned into simple compounds, mainly CO₂ and H₂O. In this process, energy is released. In the muscles, this energy can be partially converted into mechanical energy, manifesting itself in the form of isometric or isotonic contractions. The vast majority (over 80%) is, as in the engine, heat...

The author calculates that the release of heat by an adult, healthy person, staying at rest in the comfort zone, is 4184 kJ × kg body weight⁻¹ × h⁻¹. Thus, a person with a body weight of 70 kg, staying in a lying position, in total immobility, releases about 293 kJ of heat within an hour, i.e., approximately as much as 80 W bulb. During the effort, the release of heat increases rapidly. As explained by P. Atkins (2005: p. 157-158) “…The abstract heat engine in our body is dispersed among all the cells in our body and takes thousands of different forms…”.

The author explains the operation of the heat engine in the body through the ATP molecule (adenzintriposphate), which, thanks to enzymes, transforms into adezenodiphosphate and releases energy.

In anticipation of further considerations, it can be said that many studies, initiated by the estimation of the financial risk premium, have identified and determined an economic constant of \( p = 0.08 \ [1/\text{year}] \), which is related to the issues raised above, but it is not possible to create an appropriate theoretical model. It is, therefore, an example of a constant whose applications prove indispensable in human capital measurement theory. Referring to M. Kragh’s statement quoted above, it is worth noting that it is difficult to bring about a more pronounced tame of relativism in economic matters than that resulting from an understanding of the existence of the constant \( p = 0.08 \ [1/\text{year}] \). This figure controls the fairness of wages, prices, profit rates, and is a reference figure for discount and interest rates.
The second distinguishing feature of the natural sciences is the discovery of and respect for fundamental principles, the laws of Nature\(^1\). These are the principles to which the natural phenomena of our world are subject. These laws are fundamental, which means that they are discovered, but are not subject to command, and without them nothing will be explained properly. The fundamental principles in the economic context are set out in the article (Dobija & Kurek, 2013b), along with an explanation of the categories of capital, labour and the basis for their measurement. A deeper theoretical basis of knowledge about fundamental principles is contained in the works of P. Atkins (2005, 2007). Special attention should be paid to I and II principle of thermodynamics and their role in explaining various phenomena.

Research and investigations have shown that the original Sir Kelvin’s Principle II is already directly applicable in solving important economic problems. As we know, according to the Second Principle of Thermodynamic (SPoT), a thermal engine cannot run without a radiator. The point is that the human body also functions by converting food into energy according to the principles of the heat engine. This means that a human being, while living and working, dissipates part of the energy to the environment, which acts as a radiator. In this state of affairs, a hypothesis naturally arises that a fair remuneration should balance this dispersion, so that the body will keep the existing resources without experiencing their depreciation. The theory of autonomous systems by M. Mazur (1976) and its multiple applications is also worth noting. The autonomous system, like the human organism, maintains balance by balancing the loss of ability to perform the work by increasing the number of biological generators, actually different tissues. This extremely important theory is also based on the second principle of thermodynamics.

3. Fundamental Principles and Accounting Theory

For physics and economic sciences, the principle of minimum action also plays an important role. This fundamental principle states that changes in nature take place with a minimum of action, i.e. with a minimum of energy and time, so this kind of attitude is a fundamental imperative for management and control. The discovery of this principle, whose authorship is ultimately attributed to Pierre-Louis Moreau de Maupertuis (1698-1759), is associated with a group of eminent scholars, such as William G. Leibnitz (1646-1717), Leonard Euler (1707-1783) and Joseph Louis Lagrange (1736-1813). With respect to economic issues, action determines the product of capital and time. The mere determination of costs, which takes into account the intentional expenses in accordance with a specific normative and qualifies the above-normal consumption of economic resources as losses, indicates the imperative of minimum action. The algorithm of management accounting leads to maximization of profit with a minimum of assets

\(^1\text{Schwartz, N., “Laws of Nature”, Internet Encyclopedia of Philosophy,}
\text{http://www.iep.utm.edu/lawofnat/} \text{ (access date: 14.01.2013).}
and living labour. It can be said that the whole accounting, even of budgetary units, is directed to respect this principle, i.e. to achieve the greatest effects with minimum expenditure. Further considerations will indicate the significant role of equipping with appropriate assets and appropriate management to maximize economic effects with minimum effort.

The SPoT is worth more attention, due to the numerous controversies connected with it. If someone learns that SPoT is talking about the inevitable growth of disorder, about the fact that everything is getting older, and if something is being built so somewhere else, ruin is growing at an even faster rate, it may become convinced of its destructive overtones. But this person should also know that this principle is the basis of biological life, its duration and development. S. Hawking (1990: p. 143) presents a shocking example and account of the growth of entropy in the Universe. Not very clear, it would be better to say that the scholar writes an important work that organizes knowledge and creates new opportunities for action, but in his body, inevitable destruction is progressing. This, among other things, results directly from the second principle.

The second law of thermodynamics can be formulated in three equivalent ways:

- (Entropic) Entropy of the isolated system does not decrease.
- (Kelvin) It is not possible to do a process whose only result would be to do a work equivalent to the heat received from the source. The thermal motor cannot operate without a radiator.
- (Clausius) The process is not possible whose only result would be to transfer heat from cold to hot body.

The application of the first phrase immediately leads to a theoretically justified calculation of fixed asset depreciation amounts.

$$W_p \times e^{-dt} = W_k$$

(1)

where, $W_p$—initial value of the object, $W_k$—value at the end of the period, $t$—number of periods, $d$—fixed percentage depreciation of the fixed asset in the period. When calculating the fixed amount $d$ you get the percentage that determines the depreciation amount from the current balance. The figure $d$ can be expressed in terms of months or years. Moreover, the formula allows for theoretically justified forecasting of the value of a fixed asset for any given date. Thus, the current balance method is created without the need to arbitrarily determine $d$. This method ensures that a fixed asset transfers more value to products when it has the most.

The application of the formulation of the second principle of thermodynamics defining the operating conditions of a thermal engine, and consequently the inevitable loss of energy of the source, applies directly to a human being. This source of energy is food, but in order to consume it one needs a properly equipped apartment. In order to get everything we need for ourselves and our

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2Adamczyk, A. (2008). Lectures and Animations in General Physics. Thermodynamics. http://www.if.pw.edu.pl/~anadam/WykLadyFO/FoWWW_27.html (access date: 03.10.2020).
offspring we have to work, so naturally the issue of decent remuneration for work arises. Research in this area has led to the identification of the aforementioned constant, so both the fundamental principle and the fixed amount are inherent in the human capital account, which is explained further in the article. The above contents also indicate that the discussed principle is rightly considered as an engine of change. The SPoT requires the human being to do useful work in order to protect him or her from unfavorable changes.

The principle of dualism, this discursive expression of the basic identity of accounting, is a theoretical basis and has clear links to the principles of thermodynamics. Its consistent application results in the need for a double rule which, in turn, forces neither capital nor assets to arise from nothing, which is a property of the first law of thermodynamics. If the principle of dualism is written in a form without taking into account property rights, i.e. without foreign capital ($A_0 = C_0$, $A$ and $C$ mean assets and capital), this form raises questions about the nature of capital. As already explained (Dobija, 2016), capital is an abstract, potential category of ability to do work. Therefore, according to the SPoT, capital is subject to spontaneous, random dispersion. Hence, this principle indicates how to reasonably estimate this dispersion (formula (1)). Moreover, the principle of dualism leads to the natural definition of work as a transfer of capital from the sources of living or clotted work (assets) to produced objects or services. In turn, the concentration of capital in objects determines their value. In accounting, different value measures are used, with the value measure being a real and positive number that meets the requirements of the measure (additionality and monotony). Types of measures: exchangeable value, cost value, present value of the revenue stream, and others.

By adding the demand and supply law, which shapes the exchangeable value, to the set of specified principles, we obtain a number of five fundamental principles that form the basis of accounting theory. Thus, there is a strong premise to claim that accounting theory belongs to the natural sciences. Let us also note that natural sciences are also characterized by a wide application of mathematics in its constructive expression. This is related to the abstract nature of the categories present in these sciences, therefore their discursive approach requires abstract concepts of mathematics. So, is accounting a field that creates conditions and the need to apply mathematical description?

4. Constructive Mathematics as a Tool for Developing Accounting Theory

Let us note at the outset that science uses abstract categories, elusive with the senses. We do not perceive with the senses: capital, value, profit, even work. This is the reason why mathematics is used in natural sciences and economic sciences as a tool for theoretical description of the studied phenomena of the real world. The thing is, however, what kind of mathematics, after all, mathematicians themselves perceive antinomies in theories, i.e., the existence of a conjunction of
a couple of sentences, each of which deserves to be accepted, but at the same time are contradictory to each other (Dadaczyński, 2000). Kurt Gödel demonstrated the existence of antinomies in axiomatic theories. As R. Murawski (2003) Gödel wrote, he showed that even the theory of natural numbers or any richer theory (including arithmetics) cannot be fully axiomatized. He also showed that there is no absolute proof of the contradiction of mathematical theories. Therefore, the basic feature of constructive mathematics is the principle that all mathematical statements have a numerical sense based on the intuition of natural number. In this understanding of mathematics, accounting, which is a theory of measurement of abstract capital and in practical applications creates data sets and time series, is a natural field of application of mathematical concepts and procedures.

In the beginning of modern accounting there were two scholars who were well acquainted with mathematics. It was Leonardo from Pisa, that is Fibonacci and Luca Pacioli, who, while writing a guide to the mathematical knowledge of the time, posted chapters containing complete knowledge about the organization and maintenance of a double, one-dimensional accounting system, that is, with only one layout of accounts. Although accounting was later formed as a discipline separate from mathematics, the participation of formal mathematicians and scholars with excellent knowledge of mathematics in the practice and development of accounting theory was always significant. Let us mention a British mathematician like A. Cayley (1894), who, while dealing with composite numbers, significantly developed the theory of analytical functions. He also wrote a book for accounting.

Accounting itself requires the understanding of a set of real numbers, which is the basis for the description of value measures, the concept of a measure being unambiguously mathematical. The mathematical form of the principle of dualism has the form of a basic equation, as is the case in other natural sciences. Balance equations of some accounts are used to determine the quantities needed. The very understanding of accounting as a theory for the measurement and analysis of economic quantities characterizing the activity of an economic unit indicates that mathematics is invariable for accounting systems. Let’s look at the question of profit measurement. Formally, there are three equivalent ways of measuring this quantity depending on the number of independent account layouts used: balance accounts, result accounts and economic power accounts (Table 1).

In one-dimensional accounting, as interpreted by Y. Ijiri (1986), only information about the growth of assets and changes in liabilities is generated. In the case of two dimensions, the reasons for the increase in profit are additionally explained by the revenue and cost streams, their average intensity. In three-dimensional accounting, the changes in the flow rate are also explained by the effect of elementary economic forces. It should be added that, as in natural sciences, meaningful interpretations are no further than the second derivative. For example,
Table 1. Three formulas for measuring profit in the differential calculus.

| Dimensions of Accounting | Formulas describing income measurement |
|--------------------------|----------------------------------------|
| One Dimension            | Income = $\Delta E = \Delta A - \Delta D$ |
| Two Dimensions           | $I = \Delta E/\Delta t = \Delta Q/\Delta t = \Delta S/\Delta t - \Delta K/\Delta t$ where: $I$—average value of economic momentum, $Q$—flow rate, $S$—total sales, $K$—total costs, $\Delta t$—number of sub periods in accounting period. Income = $\Delta E = I \times \Delta t = S - K$ |
| Three Dimensions         | $F = \Delta E/\Delta t = \Delta E/\Delta t = Q/\Delta t - Q_0/\Delta t$ where $F$—elementary economic force causing change of momentum. Income = $\Delta E = I \times \Delta t + \sum$ (impulses caused by force actions) |

the derivative of the distance travelled is speed, and the next derivative defines acceleration. In our system, the derivative after equity is economic momentum and the second derivative defines elementary force. Of course, there are limitations for the cost and force accounts (Dobija, 2001: p. 330-339).

In accounting, both in practice and in theory, mathematics is ubiquitous. What is important is that its use is also a factor stimulating cognition. When we consider the basic indicator $r = \text{ROA} = \Delta E/E_0$, we come to a compound percentage formula and at the same time a model showing non-linear capital growth in management. It is known that $E_1 = E_0(1 + r_1)$, and in the following year will be $E_2 = E_0(1 + r_1)(1 + r_2)$ unless the capital is withdrawn. Assuming that there are some averages, $r$ is given the general formulas:

$$E_n = E_0(1 + r)^n \quad E_r = E_0e^{st}$$ (2)

The second formula presents a situation in which the capital is constantly growing, not as a percentage added at the end of the year. This is the basic model of growth, which is consistent with the principle that capital does not come from nothing. This model provokes the question about the variable $r$, about the explanation of the nature and structure of this quantity. This is one of the important issues in accounting, economics and finance.

The structure of the rate $r$ is determined by the use of SPoT and a reminder that work is only a transfer of capital, so this capital is not created. Its cardinal meaning consists in diminishing the destructive influence, i.e. dispersion of capital. We paint the metal sheet with paint and rust will not destroy the windowsill as quickly as if it had happened without painting. Therefore, writing according to SPoT: $E_r = E_0e^{-st}$, where $s$ a random variable determining the rate of capital dissipation, we notice that in nature, there must be opposite effects if the economic activity usually brings profits and not systematic reduction. The first positive influence comes from nature $[e^{st}]$ and the second from work processes $[e^{mt}]$. Thus, the general model of capital change is:

$$E_r = E_0 \times e^{-st} \times e^{ms} \times e^{mt}, \text{ and } p = \hat{E}(s) = 0.08 \ [1/year]$$ (3)

where, $\hat{E}(s)$—average value of random variable $s$, $p$—economic constant of po-
Albert Einstein expressed his admiration for the folding percentage formula and, showing a deeper interest, he presented Rule 72. Probably, the appreciation would have been greater with the knowledge of the structure of the capital growth rate $r = p + m - s$, which identifies and quantifies the influences of nature, work and the second principle of thermodynamics. As the model (3) indicates; if the work completely eliminates the influence of SPoT, the annual growth rate of capital in management is 8.33%. The constant is an a priori figure of 0.08, which is verified by empirical studies. At the same time, the capital model determines the sources of profit that economists saw in risk, which F. Knight modified by differentiating uncertainty from risk, thus coming a little closer to including SPoT.

The general capital model leads directly to the determination of how the human capital of an employee ($H$) is measured. It is immediately known that $H = H(p)$, which allows to develop the theory of measuring employee’s personal capital. Having a fixed value of human capital, the natural and spontaneous dispersion of this capital ($R$) is caused and determined by the SPoT. In turn, in order to maintain the original value of this capital, a minimum wage $W(p)$ equal to the dispersion of the capital is determined. This reasoning leads to formulas:

$$R = s \times H(p) \quad W(p) = E[s \times H(p)] = p \times H(p)$$

The wage set by formula (4) is fair, because balancing the dispersion ensures that an employee’s human capital is not depreciated. It is empirically proven that formula (4) pay allows parents to bring their two descendants to the level of capital held by their parents at the start of their work.

Another field of accounting where mathematics acts as an important cognitive factor is cost accounting and management accounting. It allows to go deeper and create an object called economic activity function (FAE) useful in various applications. Functional description of manufacturing processes begins with the presentation of manufactured products and services ($G$) in the sales price as a function of manufacturing cost ($K$): $G = K(1+r)$, where $r$—cost profitability rate. Introducing $K = W + B$—division of manufacturing costs into wage costs $W$ and non-wage costs $B$, the formula $G = (W + B) \times (1+r)$ is obtained. Using the asset-to-cost ratio, the FAE formula is applied:

$$G = W[1 + (z \times A)/W] \times [1 + r] = W \times Q$$

where, $Q$ is an unchanged quantity that determines the productivity of work. On this basis we can enter models of variables $G$ and $Q$.

$$G = We^{AFW} \quad and \quad Q = e^{AFW}$$

In the model (6), variable $F$ synthesizes the effects of asset rotation and cost

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3Albert Einstein is credited with discovering the compound interest rule of 72. Referring to compound interest, Albert Einstein is quoted as saying: “It is the greatest mathematical discovery of all time”. [http://www.ruleof72.net/rule-of-72-einstein.asp](http://www.ruleof72.net/rule-of-72-einstein.asp)
profitability, so it represents the management level. We interpret A/W as technical equipment of work. The location of size A in the model in the power exponent indicates the immense strength of the impact of assets on the effects of minimum action management (Baier et al., 2002). A similar influence is exerted by the F variable representing level of management. The W variable (wages), on the other hand, is subject to human capital theory and fair remuneration and should not be arbitrarily determined. One of the many applications of the model (6) is to determine the bonus fund in accordance with the economic performance achieved (Dobija & Jędrzejczyk, 2016).

5. Selected Empirical Studies of the Economic Constant of Potential Growth

It follows from the considerations presented above that respecting the fundamental and constant principles in economic sciences leads to useful practical results, and brings these sciences closer to the natural sciences. When considering the economic constant, an important factor is the efficiency of one of the most important processes taking place in the biosphere of our planet - photosynthesis. T. Lincoln (2002) and K. Miyamoto (1997) in their research show that in the process of converting light energy into the energy of hemic carbohydrate bonds, not all 100% of solar energy reaching plants is fully used and efficient. The light must be high energy (wavelength 400 to 700 nanometers - photosynthetically active radiation) but there are also other limiting conditions. In total, the achievable efficiency of converting solar radiation into energy of stored biomass, as confirmed by studies (Zhu, Long, & Ort, 2008), amounts to a maximum of 8% of total solar radiation. This level of photosynthesis efficiency has become a natural limiter of size of the economic constant of potential growth and added value in the economy.

As demonstrated in previous empirical studies (Dobija, 2015; Kurek, 2012; Renkas, 2016) the value of economic constant is a key part of the human capital measurement model. Model presents themselves:

\[
H(p) = [K(p) + E(p)] \times [1 + Q(T)]
\]

where, \(H(p)\) — value of human capital, \(p\) — economic constant = 0.08 [1/year], \(K(p)\) — capitalized living costs using the capitalization rate \(p\), \(E(p)\) — capitalized costs of professional education using the capitalization rate \(p\), \(Q(T)\) — factor of experience increase over \(T\) years of professional work.

In the case of the human capital for person without professional education and professional experience (for example for a 17-year-old) the above model will be presented like: \(H(p) = K(p)\), which means that we take into account only the capitalized cost of living. Capitalized cost of living \(K(p)\) is determined using continuous capitalization and as demonstrated at (Renkas, 2017b) \(K(p) = k(e^{rt} - 1)/p\). This model determines the value of the absolute minimum wage in economy. Hence, on the basis of formula (4), we come to the formula for estimation of constant \(p\), which base on real minimum wage:
\[ W^e = p \times K(p) = p \times k \times e^p - 1 \]

where, \( W^e \)—real minimum wage, \( p \)—economic constant, \( t \)—number of years, \( k \)—monthly living costs.

Since the theoretical calculation of a fair minimum wage requires the use of the constant \( p \), we will estimate its value using the example of the actual minimum wage in the USA, denoting \( p \) in the above formula as the value sought. The economy of USA was chosen due to the widespread desire to migrate to this country, which indicates the fairness of wages there. The above formula (8) is transforming to the form:

\[ p = \frac{1}{t} \ln \left( \frac{W^a}{k} + 1 \right) \]

where, \( W^a \)—real minimum wage in USA.

Having determined the estimation formula we can compute the constant \( p \) for several selected states in USA. Public knowledge is that in Washington D.C. beginning July 1, 2020, the minimum wage will increase to USD 15.00 per hour for all workers, regardless of size of employer. Adding 7.65% of the employer’s costs to this amount, we obtain the value of the employee’s employment costs. On a monthly basis it is: 176 hours \( \times \) 16.15 USD/hour = USD 2,842.4. Monthly living costs (\( k \)) in Washington DC are estimated at USD 848.4. Thus, assuming for the calculation of a person at age 17 is obtained:

\[ p = \frac{1}{17} \ln \left( \frac{2842.4}{848} + 1 \right) = 0.0865 \]  

In other selected states estimation leads to similar results (Table 2).

The results of the calculations contained in Table 2 confirm that the minimum wage in the USA is determined by the constant \( p \), and its value is at a level close to 0.08 [1/year]. The use of a constant in economics allows for the determination of fair values, therefore in terms of wages it is told about fair wages, which eliminates the natural spontaneous dispersion of human capital.

Thus, the statutory minimum wage in the USA is fair, which means, that earnings of two working parents allow the two descendants to reach the level of human capital they achieve, i.e. this wage guarantees the preservation of human capital. This fairness of the minimum wage in the USA is manifested in practice by the desire to migrate to the USA, which is a known fact. Fairness of the minimum wage is confirmed by the calculations made on the average values of living costs and wages. The average living costs is estimated at USD 577.00 per month. Parents’ earnings are taken at the average minimum wage per hour (from all states) multiplied by 176 hours, i.e.: 2 \( \times \) USD 9.00 per hour \( \times \) 176 hours = USD 3168.00. After adding 6.2% Social Security Tax and 1.45% Medicare Tax paid by the employer, the total family income is USD 3410.35. Assuming 20% on pension fund and 10% on health care, the income remaining in a family of four is USD 2387.24. Per person will be: USD 2387.24/4 = USD 596.81, which is more than the average living costs USD 577.00 (see computations in the Table 3).

Cost of Living in USA. [https://www.expatistan.com/cost-of-living](https://www.expatistan.com/cost-of-living) (access date: 10.10.2020).
Table 2. Calculations of the constant for selected states (2020).

| City and State           | Washington D.C. | Boston, Massachusetts | Denver, Colorado | Omaha, Nebraska | Portland, Oregon |
|--------------------------|-----------------|-----------------------|------------------|----------------|------------------|
| Family of four estimated monthly living costs, USD | 3392.00 | 3113.00 | 3115.00 | 2309.00 | 2989.00 |
| Monthly living costs per person (k), USD | 848.00 | 778.00 | 779.00 | 577.00 | 747.00 |
| Capitalization years of living costs (t) | 17 | 17 | 17 | 17 | 17 |
| Minimum Wage by State (per hour), USD | 16.15 | 13.73 | 13.83 | 9.69 | 14.26 |
| Value of the constant (p) | 0.0865 | 0.0831 | 0.0834 | 0.0809 | 0.0866 |
| The average value of the constant p from 5 states | 0.0841 |

*The statutory hourly wage was increased by the percentage of contributions (Social Security Tax and Medicare Tax) paid by the employer (7.65%). Data of living costs were taken from [https://www.expatistan.com/cost-of-living](https://www.expatistan.com/cost-of-living) (access date: 10.10.2020).

Table 3. Calculations of the income remaining in a family of four (2020).

| Item | Calculations |
|------|--------------|
| Earnings of two parents | $2 \times (9.00 \text{USD/hour} \times 176\text{ hours}) = 3168.00$ |
| Contributions paid by the employer (6.2% Social Security Tax and 1.45% Medicare Tax) | $3168.00 \times 0.0765 = 242.35$ |
| Total family income | 3410.35 |
| 20% on pension fund | 682.07 |
| 10% on health care | 341.04 |
| Income remaining in a family of four | 2387.24 |
| Income remaining in a family per person | $2387.24/4 = 596.81$ |
| Average cost of living | 577.00 |

This calculation proves that the level of the minimum wage in USA guarantees the preservation of human capital, so examined wages are the fair pay.

The economic constant can also be identified and measured on the basis of data on periodic earnings and returns on stocks. Such research has previously been done in assessing “risk premium” by researchers. This amount, defined as the difference between the real rate of return and return on Treasury Bills in the USA, is among others, a component of the model CAPM (Goetzmann & Ibbotson, 2006). It is matter of fact that CAPM deteriorates its value at the present time. What is new in the approach of the research on “risk premium” presented...
below is that we are aware of the economic constant and natural forces with which the “risk premium” is associated. Income on an efficient market is the result of, among others, the action of natural forces. After all, raw material was paid, employees received their wages (costs), depreciation of fixed assets also increased costs, so it is possible that the forces of nature are the source of periodic increase in invested capital. Therefore, the value of economic constant of potential growth is assessed as a real rate of return achieved in an effective market (Table 4).

To calculate the rate of return based on the data in Table 4, the percentage of inflation was subtracted from the return on stocks, which gave the value: 12.39 − 3.12 = 9.27% calculated according to the arithmetic average. Whereas, according to the geometric average it is 10.43 − 3.04 = 7.39%. This range (7.39 - 9.27) includes the average long-term rate of return achieved on the American capital market. To arrive at score was calculated the arithmetic mean of these two numbers and was obtained a value of 8.285%. For stock market and reporting corporate profits, the data indicates the value at the end of the accounting year. Therefore, if capital grows at the rate of 8% (ex ante), then at the end of the year (ex post) it reaches level \( e^{0.08} − 1 \), which is about 8.33%. Thus, the estimation specifies \( p = 0.08 \) [1/year].

Rates of return on invested capital in business entities were tested by B. Kurek (2012). His research was particularly concerned the rate of capital increase in entrepreneurship. The survey was conducted on a sample of financial statements of companies belonging to the Standard & Poor's 1500 further period of 20 years. The component parts of the index were taken into account, i.e. companies grouped in indices: Standard & Poor's 1000, Standard & Poor's 900, Standard & Poor's 600, Standard & Poor’s 500, Standard & Poor’s 400. The total number of observations obtained was 22,952. The results of statistical research confirmed the hypothesis about the average value of the risk premium ex post in the dimension of 8.33%, which corresponds to the 8% risk premium ex ante. The study was conducted at a confidence level of 0.999, obtaining a confidence interval of 8.25% - 8.89%, with the average being 8.57%. Statistical inference was considered completely safe due to low relative random error (3.75%).

Table 4. Cumulative statistics for rates of return on Stocks, Bonds and Treasury Bills in USA (1926-2004).

| Specification       | Stocks | Long-Term Government Bonds | Treasury Bills | Inflation | Real rate of return |
|---------------------|--------|-----------------------------|----------------|-----------|---------------------|
| Arithmetic average  | 12.39% | 5.82%                       | 3.76%          | 3.12%     | 9.27%               |
| Geometric average   | 10.43% | 5.44%                       | 3.72%          | 3.04%     | 7.39%               |
| Standard deviation  | 20.31% | 9.30%                       | 3.14%          | 4.32%     | 8.33%               |

Search: own study based on data taken from (Goetzmann & Ibbotson, 2006: p. 35).
Also, the existence of the economic constant was examined on the basis of wage expectations in Ukraine. Estimating the constant was done on data that comes from five different regions of Ukraine. Surveys were prepared obtaining data from Governmental Offices for job seekers. 3920 people were surveyed by asking about expected earnings and collecting data necessary for calculating the value of human capital. Choosing place for collecting the research data for survey was dictated by the fact that the job seeker does not exhibit excessive expectations, but they take into regard mainly their costs of living.

The survey contained 5 questions about: gender; age; education; work experience and expected pay in the event of employment. This information constituted the basis for estimating the value of human capital of the respondents. Converting the model (4) to the form \( p = W_U / H(p) \), where \( W_U \) presents the value of the expected remuneration, a basic equation was obtained, into which substituting the expected remuneration \( W_U \) and the estimated value of human capital of the respondents \( H(p) \) were determined the size of the average value of constant \( p \) from the survey data set. The obtained calculation results are presented in Table 5.

As can be seen from Table 5 in the group of 3920 respondents the average value of \( p \) in the light of wage expectations is at a level close to 0.08.

Calculations of human capital and constant estimation \( p \) can be made using fixed point theorems (e.g. S. Banach theorem). Using the Banach simple iteration method for the survey data set, the basic equation \( W_U = p \times H(p) \) was transformed to the form \( p = g(p) \), obtaining the function: \( p = \left( W_U / \left[ H(p) + p \right] \right) / 2 \). This function ensures the convergence of subsequent iterations. Taking the value of the first approximation \( p_0 \) at the level of 0.1 it was found that the current \( p \) meets the condition: \(|p_{\text{current}} - p_{\text{previous}}| \leq \varepsilon\). After six approximations, the zero value of the function was obtained (a fixed point was reached) for all survey items. The obtained fixed points of the examined function were in the range \( [0.07441 - 0.01198] \) giving the average value of the searched worth at the level of 0.0799. Around this worth was identified the largest concentration of calculation results. Thus, application Banach’s iteration method resulting from theorem of the fixed point confirmed size of the constant \( p \) at 0.08 [1/year].

It is also worth mentioning that a similar study of the size of the economic constant was carried out by W. Koziol (2011), who, on the basis of the human capital account and the analysis of wages of a large number of employees in a Polish enterprise, statistically confirmed its size at the level of 8%.

| Statistical quantities | Average value | Lower confidence interval value | Upper confidence interval value | Standard deviation | Median |
|------------------------|---------------|--------------------------------|--------------------------------|-------------------|--------|
| The constant \( p \)    | 0.079977      | 0.079702                       | 0.080525                       | 0.005237          | 0.079658 |
As we can see, there are many areas in which the constant p manifests itself and its value can be estimated. For example, established laws. According to A. Pikulska-Robaszkiewicz (1999: p. 41-42) in the Republic of Rome the interest rate on granting a loan was legally limited and defined as 1/12 of capital, i.e. 8.33% per annum. Maintaining this limitation, Emperor Justinian freed contracts from unjustified ruining percentage. This decision was a reasonable compromise between humanitarianism and the necessary needs of trading, which revealed the effect of the natural rate of capital multiplication. Later, the introduction of a similar limit for interest on maritime loans enabled the development of maritime trade.

Thus, the use of economic constant in determining of fair wages leads to the preservation of human capital. Let’s consider it on the example of a fair wage for person without professional education and experience in the Polish and Ukraine economy (Table 6).

The first part of Table 6 shows the calculations of the fair minimum wage. As we can see, the wage calculated on the basis of the human capital theory for the economy of Ukraine is higher than the legally fixed minimum wage in this country. This means that, with the current regulations on the minimum wage, human capital of an employee is depreciated. The first effect is the reduction in fertility and labor migration. In the Polish situation we can observe 100% compatibility of wages.

Table 6. Minimum wage in Poland and Ukraine, calculated taking into account the economic constant of potential growth (2020).

| Calculating the value of human capital and minimum wage | Poland    | Ukraine    |
|--------------------------------------------------------|-----------|------------|
| Monthly living costs, k                                | PLN 1004.0 | UAH 2197.0 |
| Years of capitalization                                | 18        | 18         |
| Human capital value, $H(p) = K(p)$                     | PLN 485,655.0 | UAH 1,062,647.0 |
| Annual remuneration, $W(p) = H(p) \times 0.08$         | PLN 38,852.4 | UAH 85,012.0 |
| Remuneration per hour, $W(p)/12/176$                   | PLN 18.40  | UAH 40.2   |
| Legal minimum wage per hour*                           | PLN 17.82  | UAH 29.2   |
| Percentage of compliance                               | 97%       | 73%        |
| Settlement of income in the family                     |           |            |
| Family (2 adults + 2 children)                          | 2 + 2     | 2 + 2      |
| Income (2 adults)                                       | PLN 6475.4 | UAH 14,150.4 |
| Pension contributions 20%                              | PLN 1295.1 | UAH 2830.1 |
| Health insurance 10%                                   | PLN 647.5  | UAH 1415.0 |
| Total amount remaining in the family                   | PLN 4532.8 | UAH 9905.3 |
| Amount per person                                      | PLN 1133.2 | UAH 2476.3 |

*The statutory hourly wage was increased by the percentage of contributions paid by the employer (Poland—20.6%, Ukraine—19.5%).
In the second part of the Table 5 shows, that if we consider a family of four, where two parents work and earn a fair minimum wage, this guarantees the preservation of human capital in the family. The amount in the family per person exceeded the level of social minimum. Thus, with the cost of living preserved the family has funds for health care and parents capitalize their pension funds.

For research on fair wages, the labor productivity index $Q$ is very useful, the interpretation and formula of which have been presented above. The work (Renkas, 2017a) presents an econometric model that shows the percentage of compliance of the fair minimum wage as a function of labor productivity $Q$. This model shows the empirical fact that the minimum wage established on the basis of the theory of human capital and the legal wage, determined by legislation, is consistent in countries with labor productivity greater than 2.8. The econometric formula is as follows:

$$Z = 73.98 \times Q - 10.89 \times Q^2 - 21.80$$

where, $Z$—the percentage of compliance of the minimum wage established on the basis of the theory of human capital and the legal wage set by the legislation in the surveyed country; $Q$—labor productivity index in the analyzed country.

The use of a quadratic function (a second degree polynomial with a maximum) showed that the countries with the $Q$ index exceeding 2.8 ($R^2 = 89\%$) have 100% agreement. The developed formula allowed to conclude that in order to achieve 100% compliance of the legal minimum wage with the minimum wage determined by the human capital theory in a particular country, the $Q$ index in that country must be at least 2.8. In an economy with a $Q$ index of 2.8 and above, the worker receives a fair wage that compensates for the natural dispersion of his individual human capital.

To illustrate the practical potential of this model, we will use it to evaluate the increase of the minimum wage in Poland planned for January 1, 2021. Having the estimated labor productivity index $Q$ for Poland at the level of 2.19, we set the maximum achievable wage compliance value at the level:

$$73.98 \times 2.19 - 10.89 \times 2.19^2 - 21.80 = 87.9\% .$$

According to the calculations of the fair minimum wage for Poland (Table 6), this corresponds to the amount of PLN $2.685 \times 0.879 = PLN 2.360. Thus, in Poland in the current situation, with the labor productivity index $Q$ at 2.19, the maximum possible increases in the minimum wage are limited by the value of PLN 2.360. It’s known that from January 1, 2021, the minimum wage in the country will be increased to PLN 2800. The calculations with the use of the model show that the increase is too high for this value of the $Q$ index. For this increase to be theoretically justified, the labor productivity index should reach the level of at least 2.6. This state of affairs does not foreshadow any catastrophe. However, its points to the fact that in order to avoid economic perturbations, the increase in these wages must be compensated by an increase in productivity. It is a feasible but very difficult task.
6. Conclusion

The conclusion from the presented deliberations and empirical studies is clear. Accounting belongs to the group of natural sciences. Its theory fully takes into account the fundamental laws of nature, including the principles of thermodynamics and the least action. Considerations on the measurement of human capital have led to the discovery of a constant quantity indispensable in modeling capital growth in real economy. The role of work as a process of preventing depreciation of objects and prolonging their duration was also explained. Accounting theory creates abstract categories, so applications of constructive mathematics are necessary and common, as in natural sciences. Accounting develops systematically as a progressive scientific research program in response to the needs of civilization development.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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