The SNO Solar Neutrino Data, Neutrinoless Double-Beta Decay and Neutrino Mass Spectrum

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Abstract

Assuming 3-ν mixing and massive Majorana neutrinos, we analyze the implications of the results of the solar neutrino experiments, including the latest SNO data, which favor the LMA MSW solution of the solar neutrino problem with tan²θ⊙ < 1, for the predictions of the effective Majorana mass |<m>| in neutrinoless double beta ((ββ)0ν) decay. Neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type are considered. For cos²θ⊙ ≥ 0.26, which follows (at 99.73% C.L.) from the SNO analysis of the solar neutrino data, we find significant lower limits on |<m>| in the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, |<m>| ≥ 0.03 eV and |<m>| ≥ 8.5 × 10⁻³ eV, respectively. If the spectrum is hierarchical the upper limit holds |<m>| ≤ 8.2 × 10⁻³ eV. Correspondingly, not only a measured value of |<m>| ≠ 0, but even an experimental upper limit on |<m>| of the order of few × 10⁻² eV can provide information on the type of the neutrino mass spectrum; it can provide also a significant upper limit on the mass of the lightest neutrino m₁. A measured value of |<m>| ≥ 0.2 eV, combined with data on neutrino masses from the ³H β-decay experiment KATRIN might allow to establish whether the CP-symmetry is violated in the lepton sector.

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1 Introduction

With the publication of the new results of the SNO solar neutrino experiment [1, 2] (see also [3]) on i) the measured rates of the charged current (CC) and neutral current (NC) reactions, $\nu_e + D \rightarrow e^- + p + p$ and $\nu_l (\bar{\nu}_l) + D \rightarrow \nu_l (\bar{\nu}_l) + n + p$, ii) on the day-night (D-N) asymmetries in the CC and NC reaction rates, and iii) on the spectrum of the final state $e^-$ in the CC reaction, further strong evidences for oscillations or transitions of the solar $\nu_e$ into active neutrinos $\nu_{\mu,\tau}$ (and/or antineutrinos $\bar{\nu}_{\mu,\tau}$), taking place when the solar $\nu_e$ travel from the central region of the Sun to the Earth, have been obtained. The evidences for oscillations (or transitions) of the solar $\nu_e$ become even stronger when the SNO data are combined with the data obtained in the other solar neutrino experiments, Homestake, Kamiokande, SAGE, GALLEX/GNO and Super-Kamiokande [4, 5].

Global analysis of the solar neutrino data [1, 2, 3, 4, 5], including the latest SNO results, in terms of dominant neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the latest results of the SNO experiment for the predictions of the effective Majorana mass $|\langle m \rangle| \neq 0$, characterizing the solar neutrino transitions, found in [1] at 99.73% C.L. reads:

$$\text{LMA MSW} : \quad 2.2 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 2.0 \times 10^{-4} \text{ eV}^2 \quad (99.73\% \text{ C.L.}).$$

The best fit value of $\Delta m^2_\odot$ obtained in [1] is $(\Delta m^2_\odot)_{BFV} = 5.0 \times 10^{-5} \text{ eV}^2$. The mixing angle $\theta_\odot$ was found in the case of the LMA solution to lie in an interval which at 99.73% C.L. is determined by [1]

$$\text{LMA MSW} : \quad 0.26 \lesssim \cos 2\theta_\odot \lesssim 0.64 \quad (99.73\% \text{ C.L.}).$$

The best fit value of $\cos 2\theta_\odot$ in the LMA solution region is given by $(\cos 2\theta_\odot)_{BFV} = 0.50$.

Strong evidences for oscillations of atmospheric neutrinos have been obtained in the Super-Kamiokande experiment [6]. As is well known, the atmospheric neutrino data is best described in terms of dominant $\nu_{\mu} \rightarrow \nu_\tau$ ($\bar{\nu}_{\mu} \rightarrow \bar{\nu}_\tau$) oscillations. The explanation of the solar and atmospheric neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., [7, 8]).

Assuming 3-$\nu$ mixing and massive Majorana neutrinos, we analyze the implications of the latest results of the SNO experiment for the predictions of the effective Majorana mass $|\langle m \rangle|$ in neutrinoless double beta $(\beta\beta_0\nu)$ decay (see, e.g., [9, 10, 11]):

$$|\langle m \rangle| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}.$$  

Here $m_{1,2,3}$ are the masses of 3 Majorana neutrinos with definite mass $\nu_{1,2,3}$, $U_{ej}$ are elements of the lepton mixing matrix $U$ - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [12, 13], and $\alpha_{21}$ and $\alpha_{31}$ are two Majorana CP-violating phases $^2$ [14, 15]. If CP-invariance holds, one has [16, 17] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, $k, k' = 0, 1, 2, ...$, and

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1,$$  

represent the relative CP-parities of the neutrinos $\nu_1$ and $\nu_2$, and $\nu_1$ and $\nu_3$, respectively.

The experiments searching for $(\beta\beta_0\nu)$-decay test the underlying symmetries of particle interactions (see, e.g., [9]). They can answer the fundamental question about the nature of massive

\footnote{We assume that the fields of the Majorana neutrinos $\nu_j$ satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$, where $C$ is the charge conjugation matrix.}
neutrinos, which can be Dirac or Majorana fermions. If the massive neutrinos are Majorana particles, the observation of (ββ)νν-decay \(^3\) can provide unique information on the type of the neutrino mass spectrum and on the lightest neutrino mass \([10, 11, 21, 22, 23, 24, 25, 26]\). Combined with data from the \(^3\)H β-decay neutrino mass experiment KATRIN \([27]\), it can give also unique information on the CP-violation in the lepton sector induced by the Majorana CP-violating phases, and if CP-invariance holds - on the relative CP-parities of the massive Majorana neutrinos \([10, 11, 22, 28]\).

Rather stringent upper bounds on \(\langle m \rangle\) have been obtained in the \(^76\)Ge experiments by the Heidelberg-Moscow collaboration \([29]\), \(\langle m \rangle < 0.35\) eV (90\%C.L.), and by the IGEX collaboration \([30]\), \(\langle m \rangle < 0.33 \pm 1.35\) eV (90\%C.L.). Taking into account a factor of 3 uncertainty in the calculated value of the corresponding nuclear matrix element, we get for the upper limit found in \([29]\): \(\langle m \rangle < 1.05\) eV. Considerably higher sensitivity to the value of \(\langle m \rangle\) is planned to be reached in several (ββ)νν-decay experiments of a new generation. The NEMO3 experiment \([31]\), which began taking data in 2001, and the cryogenic detector CUORE \([32]\), are expected to reach a sensitivity to values of \(\langle m \rangle\) \(\cong 0.1\) eV. An order of magnitude better sensitivity, i.e., to \(\langle m \rangle \cong 10^{-2}\) eV, is planned to be achieved in the GENIUS experiment \([33]\) utilizing one ton of enriched \(^76\)Ge, and in the EXO experiment \([34]\), which will search for (ββ)νν-decay of \(^{136}\)Xe. Two more detectors, Majorana \([35]\) and MOON \([36]\), are planned to have sensitivity to \(\langle m \rangle\) in the range of \(few \times 10^{-2}\) eV.

In what regards the \(^3\)H β-decay experiments, the currently existing most stringent upper bounds on the electron (anti-neutrino) mass \(m_{\nu_e}\) were obtained in the Troitzk \([37]\) and Mainz \([38]\) experiments and read, respectively, \(m_{\nu_e} < 2.5\) eV \([37]\) and \(m_{\bar{\nu}_e} < 2.9\) eV \([38]\) (95\% C.L.). The KATRIN \(^3\)H β-decay experiment \([27]\) is planned to reach a sensitivity to \(m_{\nu_e} \sim 0.35\) eV.

The fact that the solar neutrino data implies a relatively large lower limit on the value of \(\cos 2\theta_\odot\), eq. (2), has important implications for the predictions of the effective Majorana mass parameter in (ββ)νν-decay \([10, 11]\) and in the present article we investigate these implications.

\section{The SNO Data and the Predictions for the Effective Majorana Mass \(\langle m \rangle\)}

According to the analysis performed in \([1]\), the solar neutrino data, including the latest SNO results, strongly favor the LMA solution of the solar neutrino problem with \(\tan^2 \theta_\odot < 1\). We take into account these new development to update the predictions for the effective Majorana mass \(\langle m \rangle\), derived in \([10]\), and the analysis of the implications of the measurement of, or obtaining a more stringent upper limit on, \(\langle m \rangle\) performed in \([10, 11]\). The predicted value of \(\langle m \rangle\) depends in the case of 3-neutrino mixing of interest on (see e.g. \([10, 11, 25]\)): i) the value of the lightest neutrino mass \(m_1\), ii) \(\Delta m^2_{\odot}\) and \(\theta_\odot\), iii) the neutrino mass-squared difference which characterizes the atmospheric νμ (ν̅μ) oscillations, \(\Delta m^2_{\text{atm}}\), and iv) the lepton mixing angle θ which is limited by the CHOOZ and Palo Verde experiments \([39, 40]\). The ranges of allowed values of \(\Delta m^2_{\odot}\) and \(\theta_\odot\) are determined in \([1]\), while those of \(\Delta m^2_{\text{atm}}\) and θ are taken from \([41]\). Given the indicated parameters, the value of \(\langle m \rangle\) depends strongly \([10, 11]\) on the type of the neutrino mass spectrum, as well as on the values of the two Majorana CP-violating phases, present in the lepton mixing matrix.

We number the massive neutrinos (without loss of generality) in such a way that \(m_1 < m_2 < m_3\). In the analysis which follows we consider neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type \([10, 11, 21, 22, 23, 24, 26]\). In the case of neutrino mass spectrum with normal mass hierarchy \((m_1 \ll \langle \rangle m_2 \ll m_3)\) we have \(\Delta m^2_{\odot} \equiv \Delta m^2_{31}\) and \(\sin^2 \theta \equiv |U_{e3}|^2\), while in the case of spectrum with inverted hierarchy \((m_1 \ll m_2 \cong m_3)\) one finds

\(^3\)Strong evidences for (ββ)νν-decay taking place with a rate corresponding to 0.11 eV \(\lesssim \langle m \rangle \lesssim 0.56\) eV (95\% C.L.) are claimed to have been obtained in \([18]\). The results announced in \([18]\) have been criticized in \([19, 20]\).
\( \Delta m^2_\odot \equiv \Delta m^2_{32} \) and \( \sin^2 \theta \equiv |U_{ei}|^2 \). In both cases one can choose \( \Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} \). It should be noted that for \( m_1 > 0.2 \) eV \( \gg \sqrt{\Delta m^2_{\text{atm}}} \), the neutrino mass spectrum is of the quasi-degenerate type, \( m_1 \equiv m_2 \equiv m_3 \), and the two cases, \( \Delta m^2_\odot \equiv \Delta m^2_{21} \) and \( \Delta m^2_\odot \equiv \Delta m^2_{32} \), lead to the same predictions for \( |<m>| \).

### 2.1 Normal Mass Hierarchy: \( \Delta m^2_\odot \equiv \Delta m^2_{21} \)

If \( \Delta m^2_\odot = \Delta m^2_{21} \), the effective Majorana mass parameter \( |<m>| \) is given in terms of the oscillating parameters \( \Delta m^2_\odot \), \( \Delta m^2_{\text{atm}} \), \( \theta_\odot \) and \( |U_{e3}|^2 \), which is constrained by the CHOOZ data, as follows [10]:

\[
|<m>| \approx \left( m_1 \cos^2 \theta_\odot + \sqrt{m^2_1 + \Delta m^2_\odot} \sin^2 \theta_\odot e^{i\eta_{21}} \right) (1 - |U_{e3}|^2) + \sqrt{m^2_1 + \Delta m^2_{\text{atm}}} |U_{e3}|^2 e^{i\eta_{31}}.
\]

The effective Majorana mass \( |<m>| \) can lie anywhere between 0 and the present upper limits, as Fig. 1 (left panels) indicates. This conclusion does not change even under the most favorable conditions for the determination of \( |<m>| \), namely, even when \( \Delta m^2_{\text{atm}}, \Delta m^2_\odot, \theta_\odot \) and \( \theta \) are known with negligible uncertainty [11]. Our further conclusions for the case of the LMA solution of the solar neutrino problem [1] are illustrated in Fig. 1 (left panels) and are summarized below.

**Case A:** \( m_1 < 0.02 \) eV, \( m_1 \ll m_2 \ll m_3 \).

Taking into account the new constraints on the solar neutrino oscillating parameters following from the SNO data [1] does not change qualitatively the conclusions reached in ref. [10, 11]. The upper limit on \( |<m>| \) for given \( m_1 \) reads:

\[
|<m>|_{\text{MAX}} \approx \left( m_1 (\cos^2 \theta_\odot)_{\text{MIN}} + \sqrt{m^2_1 + (\Delta m^2_\odot)_{\text{MAX}} (\sin^2 \theta_\odot)_{\text{MAX}}} \right) (1 - |U_{e3}|^2_{\text{MAX}}) + \sqrt{m^2_1 + (\Delta m^2_{\text{atm}})_{\text{MAX}}} |U_{e3}|^2_{\text{MAX}},
\]

where \( (\cos^2 \theta_\odot)_{\text{MIN}} \) and \( (\sin^2 \theta_\odot)_{\text{MAX}} \) are the values corresponding to \( (\tan^2 \theta_\odot)_{\text{MAX}} \), and \( (\Delta m^2_{\text{atm}})_{\text{MAX}} \) is the maximal value of \( \Delta m^2_{\text{atm}} \) allowed for the \( |U_{e3}|^2_{\text{MAX}} \) [41].

For the allowed values of \( \Delta m^2_\odot \) and \( \tan^2 \theta_\odot \) from the LMA solution region [1], eqs. (1) and (2), we get for \( m_1 < 0.02 \) eV: \( |<m>| \leq |<m>|_{\text{MAX}} \approx 8.2 \times 10^{-3} \) eV. The maximal value of \( |<m>| \) corresponds to the case of CP-conservation and \( \nu_1, \nu_2 \) and \( \nu_3 \) having identical CP-parities, \( \eta_{21} = \eta_{31} = 1 \).

There is no significant lower bound on \( |<m>| \) because of the possibility of mutual compensations between the terms contributing to \( |<m>| \) and corresponding to the exchange of different virtual massive Majorana neutrinos. Furthermore, the uncertainties in the oscillation parameters do not allow to identify a “just-CP violation” region of values of \( |<m>| \) [10] (a value of \( |<m>| \) in this region would unambiguously signal the existence of CP-violation in the lepton sector, caused by Majorana CP-violating phases). However, if the neutrinoless double beta-decay will be observed, the measured value of \( |<m>| \), combined with information on \( m_1 \) and a better determination of the relevant neutrino oscillation parameters, would allow to determine whether the CP-symmetry is violated due to Majorana CP-violating phases, or to identify which are the allowed patterns of the massive neutrino CP-parities in the case of CP-conservation (for a detailed discussion see ref. [11]).

**Case B:** Quasi-Degenerate Neutrino Mass Spectrum \( (m_1 > 0.2 \) eV, \( m_1 \simeq m_2 \simeq m_3 \)), or Neutrino Mass Spectrum with Partial Hierarchy \( (0.02 \) eV \leq m_1 \leq 0.2 \) eV)

The new element in the predictions for the effective Majorana mass \( |<m>| \) is the existence of a lower bound on the possible values of \( |<m>| \) (Fig. 1, left panels). For \( m_1 \geq 0.07 \) eV this lower
bound is significant, \(|<m>| \gtrsim 10^{-2}\) eV. In the case of quasi-degenerate spectrum, \(m_1 > 0.2\) eV, the lower bound reads: \(|<m>| \gtrsim 0.035\) eV.

For a given \(m_1 \geq 0.02\) eV, the minimal value of \(|<m>|\), \(|<m>|_{\text{MIN}}\), is given by

\[
|<m>|_{\text{MIN}} \simeq m_1(\cos 2\theta_\odot)_{\text{MIN}}(1 - |U_{e3}|^2_{\text{MAX}}) - \sqrt{m_1^2 + (\Delta m^2_{\text{atm}})_{\text{MAX}} |U_{e3}|^2_{\text{MAX}}} + O\left(\frac{\Delta m^2_{\text{sol}}}{4m_1}\right),
\]

(7)

where again \((\Delta m^2_{\text{atm}})_{\text{MAX}}\) is the maximal allowed value of \(\Delta m^2_{\text{atm}}\) for the \(|U_{e3}|^2_{\text{MAX}}\) [41].

The upper bound on \(|<m>|\), which corresponds to CP-conservation and \(\eta_{21} = \eta_{31} = +1\) \(\nu_1, \nu_2\) and \(\nu_3\) possessing identical CP-parities), can be found for given \(m_1\) by using eq. (6). For the allowed values of \(m_1 \geq 0.02\) eV (which is limited from above by the \(3^\text{H} \beta-\text{decay data}\) [37, 38]), \(|<m>|_{\text{MAX}}\) is limited by the upper bounds obtained in the \((\beta\beta)_{0\nu}\)-decay experiments [29, 30]: \(|<m>| < (0.33 - 105)\) eV.

For values of \(|<m>|\), which are in the range of sensitivity of the future \((\beta\beta)_{0\nu}\)-decay experiments, there exists a “just-CP-violation” region. This is illustrated in Fig. 2, where we show \(|<m>| / m_1\) for the case of quasi-degenerate neutrino mass spectrum \((m_1 > 0.2\) eV, \(m_1 \simeq m_2 \simeq m_3\), as a function of \(\cos 2\theta_\odot\). The “just-CP-violation” interval of values of \(|<m>| / m_1\) is determined by

\[
(cos 2\theta_\odot)_{\text{MAX}}(1 - |U_{e3}|^2_{\text{MAX}}) + |U_{e3}|^2_{\text{MAX}} < \frac{|<m>|}{m_1} < 1 - 2|U_{e3}|^2_{\text{MAX}}.
\]

(8)

Taking into account eq. (2) and the existing limits on \(|U_{e3}|^2\), this gives \(0.67 < |<m>| / m_1 < 0.85\). The mass \(m_1\) in the case of interest is \(m_1 \simeq m_{\nu_e}\) and can be measured in the KATRIN experiment, provided \(m_1 \gtrsim 0.35\) eV.

A rather precise determination of \(|<m>|\), \(m_1\), \(\theta_\odot\) and \(|U_{e3}|^2\) would establish whether CP-violation takes place [11] and would imply a non-trivial constraint on the two CP-violating phases \(\alpha_{21}\) and \(\alpha_{31}\) [10]. Given the allowed values of \(\cos 2\theta_\odot\), eq. (2), the observation of the \((\beta\beta)_{0\nu}\)-decay in the present and/or the future \((\beta\beta)_{0\nu}\)-decay experiments, combined with a sufficiently stringent upper bound on the electron (anti-)neutrino mass \(m_{\nu_e} \simeq m_1\) from the tritium beta decay experiments, \(m_{\nu_e} < |<m>|_{\text{exp}}/(\cos 2\theta_\odot)_{\text{MAX}}(1 - |U_{e3}|^2_{\text{MAX}}) + |U_{e3}|^2_{\text{MAX}}\), would allow us, in particular, to exclude the case of CP-conservation with \(\eta_{21} = \pm \eta_{31} = -1\) (Fig. 2).

### 2.2 Inverted Neutrino Mass Hierarchy: \(\Delta m^2_{\odot} \equiv \Delta m^2_{32}\)

If \(\Delta m^2_{\odot} = \Delta m^2_{32}\), the effective Majorana mass \(|<m>|\) is given in terms of the oscillating parameters \(\Delta m^2_{\odot}\), \(\Delta m^2_{\text{atm}}\), \(\theta_\odot\) and \(|U_{e1}|^2\) which is constrained by the CHOOZ data [10]:

\[
|<m>| \simeq m_1|U_{e1}|^2 + \sqrt{m_1^2 + \Delta m^2_{\text{atm}} - \Delta m^2_{\odot}} \cos^2 \theta_\odot (1 - |U_{e1}|^2)e^{i\alpha_{21}}
+ \sqrt{m_1^2 + \Delta m^2_{\text{atm}}} \sin^2 \theta_\odot (1 - |U_{e1}|^2)e^{i\alpha_{31}}.
\]

(9)

The new predictions for \(|<m>|\) again differ substantially from those obtained before the appearance of the latest SNO data due to the existence of a significant lower bound on \(|<m>|\) for every value of \(m_1\): even in the case of \(m_1 \ll m_2 \simeq m_3\) (i.e., even if \(m_1 \ll 0.02\) eV), we get \(|<m>| \gtrsim 8.5 \times 10^{-3}\) eV (see Fig. 1, right panels). Actually, the minimal value of \(|<m>|\), \(\min(|<m>|)\), depends on whether CP-invariance holds or not in the lepton sector, and if it holds - it depends on the relative CP-parities of the massive Majorana neutrinos.

**Case A:** \(m_1 < 0.02\) eV, \(m_1 \ll m_2 \simeq m_3\).
The effective Majorana mass $\langle m \rangle$ can be considerably larger than in the case of a hierarchical neutrino mass spectrum [10, 23]. The maximal value of $|\langle m \rangle|$ corresponds to CP-conservation and $\eta_{21} = \eta_{31} = +1$ and for given $m_1$ reads:

$$|\langle m \rangle|_{\text{MAX}} \simeq m_1 |U_{e1}^2|_{\text{MAX}} + \left( \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}} - (\Delta m_{\text{sol}}^2)_{\text{MIN}}} (\cos^2 \theta_{\odot})_{\text{MIN}} + \frac{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}}}{m_1^2 + (\Delta m_{\text{sol}}^2)_{\text{MIN}} (\sin^2 \theta_{\odot})_{\text{MAX}}} \left(1 - |U_{e1}|^2_{\text{MIN}}\right) \right), \quad (10)$$

where $(\cos^2 \theta_{\odot})_{\text{MIN}}$ and $(\sin^2 \theta_{\odot})_{\text{MAX}}$ are the values corresponding to $(\tan^2 \theta_{\odot})_{\text{MAX}}$, and $|U_{e1}|^2_{\text{MIN}}$ is the minimal allowed value of $|U_{e1}|^2$ for the $(\Delta m_{\text{atm}}^2)_{\text{MIN}}$. For the allowed ranges of the neutrino oscillation parameters found in refs. [1, 41], the maximal allowed value of $|\langle m \rangle|$ is $|\langle m \rangle|_{\text{MAX}} \simeq 0.08 \text{ eV}$.

The existence of a relevant lower bound on $|\langle m \rangle|$ in the case of the LMA solution, $|\langle m \rangle| \gtrsim 8.5 \times 10^{-3} \text{ eV}$, is a consequence of the fact that $\cos 2\theta_{\odot}$ is found to be significantly different from zero. The minimal value of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MIN}}$, is reached in the case of CP-invariance and $\eta_{21} = -\eta_{31} = -1$, and is determined by:

$$|\langle m \rangle|_{\text{MIN}} \simeq m_1 |U_{e1}^2|_{\text{MAX}} - \left( \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}} - (\Delta m_{\text{sol}}^2)_{\text{MAX}}} (\cos^2 \theta_{\odot})_{\text{MIN}} - \frac{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}}}{m_1^2 + (\Delta m_{\text{sol}}^2)_{\text{MIN}} (\sin^2 \theta_{\odot})_{\text{MAX}}} \left(1 - |U_{e1}|^2_{\text{MAX}}\right) \right), \quad (11)$$

where $(\cos^2 \theta_{\odot})_{\text{MIN}}$ and $(\sin^2 \theta_{\odot})_{\text{MAX}}$ are the values corresponding to $(\tan^2 \theta_{\odot})_{\text{MAX}}$, and $|U_{e1}|^2_{\text{MAX}}$ is the maximal allowed value of $|U_{e1}|^2$ for the $(\Delta m_{\text{atm}}^2)_{\text{MIN}}$.

**Case B: Quasi-Degenerate Neutrino Mass Spectrum** ($m_1 > 0.2 \text{ eV}$, $m_1 \simeq m_2 \simeq m_3$), or Spectrum with Partial Inverted Hierarchy ($0.02 \text{ eV} \leq m_1 \leq 0.2 \text{ eV}$).

The discussion and conclusions in the case of the spectrum of quasi-degenerate type and with partial inverted hierarchy are identical to those in the same case for the neutrino mass spectrum with normal hierarchy given in sub-section 2.1, Case B, except for the maximal and minimal values of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MAX}}$ and $|\langle m \rangle|_{\text{MIN}}$, which for a fixed $m_1$ are determined by:

$$|\langle m \rangle|_{\text{MAX}} \simeq m_1 |U_{e1}^2|_{\text{MIN}} + \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}} \left(1 - |U_{e1}|^2_{\text{MIN}}\right), \quad (12)$$

$$|\langle m \rangle|_{\text{MIN}} \simeq m_1 |U_{e1}^2|_{\text{MAX}} - \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}}} \left(\cos 2\theta_{\odot}\right)_{\text{MIN}} \left(1 - |U_{e1}|^2_{\text{MAX}}\right), \quad (13)$$

where $|U_{e1}|^2_{\text{MIN}}$, $|U_{e1}|^2_{\text{MAX}}$, in eq. (12) (in eq. (13)) is the minimal (maximal) allowed value of $|U_{e1}|^2$ given the $(\Delta m_{\text{atm}}^2)_{\text{MAX}}$ $(\Delta m_{\text{atm}}^2)_{\text{MIN}}$.

For any value of $m_1 \geq 0.02 \text{ eV}$, the lower bound on $|\langle m \rangle|$ reads $|\langle m \rangle| \gtrsim 0.01 \text{ eV}$.

### 3 The Effective Majorana Mass and the Determination of the Neutrino Mass Spectrum

The existence of a lower bound on $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MIN}}$, in the quasi-degenerate and inverted mass hierarchy ($\Delta m_{\odot}^2 = \Delta m_{21}^2$) cases implies that the future $(\beta \beta)_{0\nu}$-decay experiments might allow to determine the type of the neutrino mass spectrum (under the general assumptions of 3-neutrino mixing and massive Majorana neutrinos, $(\beta \beta)_{0\nu}$-decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, neutrino oscillation explanation of the solar and atmospheric neutrino data). This conclusion is valid not only under the assumption that the $(\beta \beta)_{0\nu}$-decay will be observed in these experiments and $|\langle m \rangle|$ will be measured, but also in the case only a sufficiently stringent upper limit on $|\langle m \rangle|$ will be derived.

More specifically, as is illustrated in Fig. 3, the following statements can be made:
1. A measurement of $|<m>| = |<m>|_{\text{exp}} > 0.20 \, \text{eV}$, would imply that the neutrino mass spectrum is of the quasi-degenerate type ($m_1 > 0.20 \, \text{eV}$) and that there are both a lower and an upper limit on $m_1$, $(m_1)_{\text{min}} \leq m_1 \leq (m_1)_{\text{max}}$. The values of $(m_1)_{\text{max}}$ and $(m_1)_{\text{min}}$ are fixed respectively by the equalities $|<m>|_{\text{MIN}} = |<m>|_{\text{exp}}$ and $|<m>|_{\text{MAX}} = |<m>|_{\text{exp}}$, where $|<m>|_{\text{MIN}}$ and $|<m>|_{\text{MAX}}$ are given by eqs. (7) and (6);

2. If $|<m>|$ is measured and is found to lie in the interval $8.5 \times 10^{-2} \, \text{eV} \lesssim |<m>|_{\text{exp}} \lesssim 2.0 \times 10^{-1} \, \text{eV}$, one could conclude that either
   i) $\Delta m_2^2 \equiv \Delta m_{21}^2$ and the spectrum is of the quasi-degenerate type ($m_1 > 0.20 \, \text{eV}$) or with partial hierarchy ($0.02 \, \text{eV} \leq m_1 \leq 0.2 \, \text{eV}$), with $8.4 \times 10^{-2} \, \text{eV} \lesssim (m_1)_{\text{min}} \leq m_1 \leq (m_1)_{\text{max}} \lesssim 1.2 \, \text{eV}$, where the maximal and minimal values of $m_1$ are determined as in the Case 1;
   or that ii) $\Delta m_2^2 \equiv \Delta m_{32}^2$ and the spectrum is quasi-degenerate ($m_1 > 0.20 \, \text{eV}$) or with partial inverted hierarchy ($0.02 \, \text{eV} \leq m_1 \leq 0.2 \, \text{eV}$), with $2.0 \times 10^{-2} \, \text{eV} \lesssim (m_1)_{\text{min}} \leq m_1 \leq (m_1)_{\text{max}} \lesssim 1.2 \, \text{eV}$, where $(m_1)_{\text{max}}$ and $(m_1)_{\text{min}}$ are given by the equalities $|<m>|_{\text{MIN}} = |<m>|_{\text{exp}}$ and $|<m>|_{\text{MAX}} = |<m>|_{\text{exp}}$, and $|<m>|_{\text{MIN}}$ and $|<m>|_{\text{MAX}}$ are determined by eqs. (13) and (12);

3. A measured value of $|<m>|$ satisfying $8.5 \times 10^{-3} \, \text{eV} \lesssim |<m>|_{\text{exp}} \lesssim 8.0 \times 10^{-2} \, \text{eV}$, would imply that (see Fig. 3) either
   i) $\Delta m_2^2 \equiv \Delta m_{21}^2$ and the spectrum is quasi-degenerate ($m_1 > 0.20 \, \text{eV}$) or with partial hierarchy ($0.02 \, \text{eV} \leq m_1 \leq 0.2 \, \text{eV}$), or with normal mass hierarchy ($m_1 \ll 0.02 \, \text{eV}$), with $(m_1)_{\text{min}} \gtrsim 10^{-3} \, \text{eV}$, $(m_1)_{\text{max}} \lesssim 0.48 \, \text{eV}$, where $(m_1)_{\text{max}}$ and $(m_1)_{\text{min}}$ are determined as in the preceding Case 1,
   or that ii) $\Delta m_2^2 \equiv \Delta m_{32}^2$ and the spectrum is quasi-degenerate ($m_1 > 0.20 \, \text{eV}$) or with partial inverted hierarchy ($0.02 \, \text{eV} \leq m_1 \leq 0.2 \, \text{eV}$), or with inverted hierarchy ($m_1 \ll 0.02 \, \text{eV}$), with only a significant upper bound on $m_1$, $(m_1)_{\text{min}} = 0$, $(m_1)_{\text{max}} \leq 0.48 \, \text{eV}$, where $(m_1)_{\text{max}}$ is determined by the equation $|<m>|_{\text{MIN}} = |<m>|_{\text{exp}}$, with $|<m>|_{\text{MIN}}$ given by eq. (13);

4. A measurement or an upper limit on $|<m>|$, $|<m>| \lesssim 8.0 \times 10^{-3} \, \text{eV}$, would lead to the conclusion that the neutrino mass spectrum is of the normal mass hierarchy type, $\Delta m_2^2 \equiv \Delta m_{21}^2$, and that $m_1$ is limited from above by $m_1 \leq (m_1)_{\text{max}} \sim 5.8 \times 10^{-2} \, \text{eV}$, where $(m_1)_{\text{max}}$ is determined by the condition $|<m>|_{\text{MIN}} = |<m>|_{\text{exp}}$, with $|<m>|_{\text{MIN}}$ given by eq. (7).

A measured value of (or an upper limit on) the effective Majorana mass $|<m>| \lesssim 0.03 \, \text{eV}$ would disfavor (if not rule out) the quasi degenerate mass spectrum, while a value of $|<m>| \lesssim 8 \times 10^{-3} \, \text{eV}$ would rule out the quasi degenerate mass spectrum, disfavor the spectrum with inverted mass hierarchy and favor the hierarchical neutrino mass spectrum.

If the minimal value of $\cos 2\theta_{\odot}$ inferred from the solar neutrino data, is somewhat smaller than that in eq. (2), the upper bound on $|<m>|$ in the case of neutrino mass spectrum with normal hierarchy ($\Delta m_2^2 \equiv \Delta m_{21}^2$, $m_1 \ll 0.02 \, \text{eV}$) might turn out to be larger than the lower bound on $|<m>|$ in the case of spectrum with inverted mass hierarchy ($\Delta m_2^2 \equiv \Delta m_{32}^2$, $m_1 \ll 0.02 \, \text{eV}$). Thus, there will be an overlap between the regions of allowed values of $|<m>|$ in the two cases of neutrino mass spectrum at $m_1 \ll 0.02 \, \text{eV}$. The minimal value of $\cos 2\theta_{\odot}$ for which the two regions do not overlap is determined by the condition:

$$ (\cos 2\theta_{\odot})_{\text{MIN}} = \frac{\sqrt{(\Delta m_{21}^2)_{\text{MAX}}}}{2 \sqrt{(\Delta m_{\text{atm}}^2)_{\text{MIN}}}} + \sqrt{\frac{(\Delta m_{\text{atm}}^2)_{\text{MAX}}}{4(\Delta m_{\text{atm}}^2)_{\text{MIN}}}} (\sin^2 \theta)_{\text{MAX}} + \mathcal{O}(\Delta m_{21}^2)_{\text{MAX}} 4(\Delta m_{\text{atm}}^2)_{\text{MIN}}), \quad (14) $$
where we have neglected terms of order \((\sin^2 \theta)^2\). For the values of the neutrino oscillation parameters used in the present analysis this “border” value turns out to be \(\cos 2\theta \approx 0.25\).

Let us note that [11] if the \((\beta\beta)_{0w}\)-decay is not observed, a measured value of \(m_{\nu_e}\) in \(^3\)H \(\beta\)-decay experiments, \((m_{\nu_e})_{exp} \gtrsim 0.35\) eV, which is larger than \((m_1)_{max}\), \((m_{\nu_e})_{exp} > (m_1)_{max}\), where \((m_1)_{max}\) is determined as in the Case 1 (i.e., from the upper limit on \(\langle m \rangle\)), \(\langle m \rangle\) is given in eq. (7)), might imply that the massive neutrinos are Dirac particles. If the \((\beta\beta)_{0w}\)-decay has been observed and \(\langle m \rangle\) is determined, the inequality \((m_{\nu_e})_{exp} > (m_1)_{max}\), would lead to the conclusion that there exist contributions(s) to the \((\beta\beta)_{0w}\)-decay rate other than due to the light Majorana neutrino exchange which partially cancel the contribution due to the Majorana neutrino exchange.

A measured value of \(\langle m \rangle\), \((\langle m \rangle)_{exp} > 0.08\) eV, and a measured value of \(m_{\nu_e}\) or an upper bound on \(m_{\nu_e}\), such that \(m_{\nu_e} < (m_1)_{min}\), where \((m_1)_{min}\) is determined by the condition \(\langle m \rangle\) for the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, \(\langle m \rangle\) is given in eq. (7)), might imply that the MASS neutrinos are Dirac particles. If the \((\beta\beta)_{0w}\)-decay has been observed and \(\langle m \rangle\) is measured, the inequality \((m_{\nu_e})_{exp} > (m_1)_{max}\), would lead to the conclusion that there exist contributions(s) to the \((\beta\beta)_{0w}\)-decay rate other than due to the light Majorana neutrino exchange which partially cancel the contribution due to the Majorana neutrino exchange.

4 Conclusions

Assuming 3-\(\nu\) mixing and massive Majorana neutrinos, we have analyzed the implications of the results of the solar neutrino experiments, including the latest SNO data, which favor the LMA MSW solution of the solar neutrino problem with \(\tan^2 \theta \sim 3\), for the predictions of the effective Majorana mass \(\langle m \rangle\) in \((\beta\beta)_{0w}\)-decay. Neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type are considered. For \(\cos 2\theta \sim 0.25\), which follows (at 99.73% C.L.) from the analysis of the solar neutrino data performed in [1], we find significant lower limits on \(\langle m \rangle\) in the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, \(\langle m \rangle\) is given in eq. (7)), might imply that the MASS neutrinos are Dirac particles. If the \((\beta\beta)_{0w}\)-decay has been observed and \(\langle m \rangle\) is determined, the inequality \((m_{\nu_e})_{exp} > (m_1)_{max}\), would lead to the conclusion that there exist contributions(s) to the \((\beta\beta)_{0w}\)-decay rate other than due to the light Majorana neutrino exchange which partially cancel the contribution due to the Majorana neutrino exchange.

A measured value of \(\langle m \rangle\), \((\langle m \rangle)_{exp} > 0.08\) eV, and a measured value of \(m_{\nu_e}\) or an upper bound on \(m_{\nu_e}\), such that \(m_{\nu_e} < (m_1)_{min}\), where \((m_1)_{min}\) is determined by the condition \(\langle m \rangle\) for the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, \(\langle m \rangle\) is given in eq. (7)), might imply that the MASS neutrinos are Dirac particles. If the \((\beta\beta)_{0w}\)-decay has been observed and \(\langle m \rangle\) is determined, the inequality \((m_{\nu_e})_{exp} > (m_1)_{max}\), would lead to the conclusion that there exist contributions(s) to the \((\beta\beta)_{0w}\)-decay rate other than due to the light Majorana neutrino exchange which partially cancel the contribution due to the Majorana neutrino exchange.

**Note Added.**

After the work on the present study was essentially completed, few new global analyses of the solar neutrino data have appeared [43, 44]. The results obtained in [43] do not differ substantially from those derived in [1]; in particular, the (99.73% C.L.) minimal allowed values of \(\cos 2\theta \circ\) in the LMA solution region found in [1] and in [43] practically coincide. Smaller minimal allowed values of \(\cos 2\theta \circ\) of the order of \(\sim 0.1\) (99.73% C.L.) and larger maximal allowed values of \(\Delta m^2_{31}\) of the order of \(4 \times 10^{-4}\) eV\(^2\) (99.73% C.L.) have been obtained in the analyses performed in [44]. At present it is not clear to us what is the source of these differences. Using the LMA values of \(\cos 2\theta \circ\) and \(\Delta m^2_{31}\) found in [44], the maximal value of \(\langle m \rangle\) in the case of a hierarchical neutrino mass spectrum \((m_1 \ll 0.02\) eV\) increases somewhat to \(10^{-2}\) eV\(^2\), while the minimal one in the case of the spectrum with inverted hierarchy is \(few \times 10^{-3}\) eV.
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Figure 1: The dependence of $|<m>|$ on $m_1$ in the case of the LMA solution of the solar neutrino problem \([1]\) (99.73% C.L.), i) for $\Delta m^2 = \Delta m^2_{21}$ (left panels) and $|U_{e3}|^2 = 0.05$ (upper left panel), $|U_{e3}|^2 = 0.01$ (middle left panel), $|U_{e3}|^2 = 0.005$ (lower left panel), ii) for $\Delta m^2 = \Delta m^2_{32}$ (right panels) and $|U_{e1}|^2 = 0.05$ (upper right panel), $|U_{e1}|^2 = 0.01$ (middle right panel), $|U_{e1}|^2 = 0.005$ (lower right panel). For $\Delta m^2 = \Delta m^2_{21}$, the allowed values of $|<m>|$ are constrained to lie in the case of CP-conservation in the medium-grey regions i) between the two thick solid lines if $\eta_{21} = \eta_{31} = 1$, ii) between the two long-dashed lines and the axes if $\eta_{21} = -\eta_{31} = 1$, iii) between the dash-dotted lines and the axes if $\eta_{21} = -\eta_{31} = -1$, iv) between the short-dashed lines if $\eta_{21} = \eta_{31} = -1$. For $\Delta m^2 = \Delta m^2_{32}$ the allowed regions for $|<m>|$ correspond to: for $|U_{e1}|^2 = 0.005$ and $|U_{e1}|^2 = 0.01$, the medium-grey regions i) between the solid lines if $\eta_{21} = \eta_{31} = \pm 1$, ii) between the dashed lines if $\eta_{21} = -\eta_{31} = \pm 1$, and, for $|U_{e1}|^2 = 0.05$, the medium-grey regions iii) between the solid lines if $\eta_{21} = \eta_{31} = 1$, iv) between the long-dashed lines if $\eta_{21} = \eta_{31} = -1$, v) between the dashed-dotted lines if $\eta_{21} = -\eta_{31} = 1$, vi) between the short-dashed lines if $\eta_{21} = -\eta_{31} = -1$, In the case of CP-violation, the allowed region for $|<m>|$ covers all the grey regions. Values of $|<m>|$ in the dark grey regions signal CP-violation.
Figure 2: The dependence of $|\langle m \rangle| / m_1$ on $\cos 2\theta_\odot$ for the quasi-degenerate neutrino mass spectrum ($m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3$). If CP-invariance holds, the values of $|\langle m \rangle| / m_1$ lie: i) for $\eta_{21} = \eta_{31} = 1$ - on the line $|\langle m \rangle| / m_1 = 1$, ii) for $\eta_{21} = -\eta_{31} = 1$ - in the region between the thick horizontal solid and dash-dotted lines (in light grey and medium grey colors), iii) for $\eta_{21} = -\eta_{31} = -1$ - in the light grey polygon with long-dashed and long-dashed-double-dotted line contours and iv) for $\eta_{21} = \eta_{31} = -1$ - in the medium grey polygon with the short-dashed and long-dashed-double-dotted line contours. The “just-CP-violation” region is denoted by dark-grey color. The values of $\cos 2\theta_\odot$ between the doubly thick solid lines correspond to the lower and upper limits of the LMA solution regions found in ref. [1] at 99.73% C.L.
Figure 3: The dependence of $|<m>|$ on $m_1$ in the case of the LMA solution obtained in ref.[1] (99.73% C.L.), for $\Delta m_{\odot}^2 = \Delta m_{12}^2$ and for $\Delta m_{\odot}^2 = \Delta m_{32}^2$. The allowed values of $|<m>|$ are constrained to lie in the case of CP-conservation: for $\Delta m_{\odot}^2 = \Delta m_{12}^2$ in the medium-grey and light-grey regions i) between the two lower thick solid lines if $\eta_{21} = \eta_{31} = 1$, ii) between the two long-dashed lines and the axes if $\eta_{21} = -\eta_{31} = 1$, iii) between the two thick dash-dotted lines and the axes if $\eta_{21} = -\eta_{31} = -1$, iv) between the three thick short-dashed lines and the axes if $\eta_{21} = -\eta_{31} = -1$; and for $\Delta m_{\odot}^2 = \Delta m_{32}^2$ - in the light-grey regions i) between the two upper thick solid lines if $\eta_{21} = \eta_{31} = 1$, ii) between the dotted and the doubly-thick short-dashed lines if $\eta_{21} = -\eta_{31} = -1$, iii) between the dotted and the doubly-thick dash-dotted lines if $\eta_{21} = -\eta_{31} = +1$. In the case of CP-violation, the allowed region for $|<m>|$ covers all the grey regions. Values of $|<m>|$ in the dark grey region signal CP-violation.