Movement of electron when recombining in hydrogen plasma

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Abstract. An analytical model and the results of modeling are presented for movement of electrons in recombining hydrogen plasma. It is shown that in case of taking into account the magnetic moment and angular momentum as well as spin flip of electron in magnetic field the electron comes to the orbit with angular momentum $\hbar/2$. If azimuthal and radial components of kinetic energy of electron are equal then the full energy of such the orbits is 13.6 eV.

1. Analytical model
We consider electrons as charged particles with mechanical momentum $\hbar/2$ and magnetic moment. It is assumed that the direction of magnetic moment of electron coincides with the direction of its angular momentum. It is likewise assumed, that absorption and radiation of energy is due to spin flip of electron. The following equations can be used to describe the motion of electron with electrical charge and magnetic moment [1–3]:

$$m \frac{dv}{dt} = qE_0 + \frac{q}{c}[v \times H_0] + \nabla (\mu \cdot H) - \frac{1}{c} \frac{d}{dt}[\mu \times E_0],$$

$$\frac{dr}{dt} = v, \quad \frac{d\phi}{dt} = \frac{Q}{cr^2} \mu z,$$

where $v$, $r$, $q$, $\mu$ – velocity, radius vector, charge and magnetic moment respectively, $E_0$ and $H_0$ – strength of electric and magnetic fields (outer magnetic fields are considered) in laboratory coordinate system, $H$ – strength of magnetic field (outer) in coordinate system of moving electron:

$$H = H_0 - \frac{1}{c} [v \times E_0].$$

Let we consider electron motion in electric field of the proton. Such a motion happens in a plane $r$, $\phi$ which also contains the proton, spin of electron is perpendicular to $r$, $\phi$ plane, $z$-coordinate which is perpendicular to $r$, $\phi$ plane. We describe the motion of electron in this plane with the following equations, taken from (1, 2):

$$m_e \frac{d^2r}{dt^2} = m_e v^2 \left( \frac{d\phi}{dt} \right)^2 - \frac{Q e}{r^2} - \frac{Q}{cr^2} \mu z \frac{d\phi}{dt},$$

$$\frac{d}{dt} \left( m_e v^2 \frac{d\phi}{dt} + \frac{Q \mu z}{cr} \right) = 0.$$
Equations (4) and (5) describe electron motion in equatorial plane. Same equations are used in Bohr model as well as in the model, given by Gryzinski [4, 5].

As far as spin flip process we should admit, that to construct the detailed theory is not a trivial theoretical job. In the present considerations, we assume that spin flip happens almost immediately and we do not take into account the details of this process. The state of electron before spin flip and after it was considered instead. In this case instead of equation (5) the following equation is used:

$$\frac{d}{dt} \left( m_e r^2 \frac{d\varphi}{dt} + s \left( \frac{Q \mu_z}{cr} + \frac{\hbar}{2} \right) \right) = 0,$$

where $s$ – value equal to +1 or −1 depending on the direction of spin of electron with respect to the direction of magnetic field, surrounding the electron.

The important point, which should be especially mentioned, is that energy of electron might be changed during the process of spin flip. In case if the energy is decreased after the flip, then such a flip is accompanied by radiation of energy. In the opposite case (energy is increased after the flip), the flip process is accompanied by the absorption of energy. The latter case was not considered in our modelling.

Trajectories of electrons are shown in figure 1 for the electrons with same initial velocities but different angular momentum with respect to the proton. It is seen, that at the larger distances the electron is not trapped by the proton while at the smaller distances (where momentum is close to $\hbar$) the electron is trapped by the proton and the distance between them is compared to Bohr radius. This latter case is considered as recombination of electron.

It is shown that electron rapidly comes to the quite definite orbit near the proton, so called orbit of ground state. Such the orbit has a remarkable property – the orbital momentum of the electron is strictly equal to $\hbar/2$. This conclusion comes from the conservation law for total momentum (orbital momentum plus own momentum), since during the flip of angular momentum electron moves azimuthally and in the opposite direction but with the same velocity.
As a result the orbital momentum of stable state electron trajectory is equal to $-\hbar/2$ or $+\hbar/2$ at any time moment. In this steady state condition, the arbitrary disturbance of electron velocity is compensated or extinguished by radiation. It should be mentioned that we do not have an analytical model describing the process of energy exchange between radial and azimuthal degrees of freedom of electron. However we assumed this statement working a priori. The condition of equality of kinetic energies in radial and azimuthal degrees of freedom is only possible for the single electron orbit which energy is $-13.6$ eV.

2. Summary
Numerical modeling of motion of the electron near the proton is made on the base of well-known equations given for charged particles with its own magnetic moment. It follows from our study that angular momentum of electron in the ground state is $\hbar/2$ while the direction of electron spin is the opposite with respect to the direction of orbital angular momentum.

Therefore, to explain the recombination process and to understand the phenomena of ground state in hydrogen atom we take into account the following items:

- Columb attraction with the proton,
- own magnetic moment and angular momentum,
- the influence of spin flip of electron,
- the equality of kinetic energies of orbital and azimuthal degrees of freedom.

Recombining with the proton electron sits on the following steady state trajectory:

- full energy (kinetic plus potential) is equal to $-13.6$ eV,
- electron moves in radial direction from 0.14 to 1.86 Bohr radius,
- angular momentum due to spin flip varies from $-\hbar/2$ to $\hbar/2$ and contrary from $\hbar/2$ to $-\hbar/2$.

References
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