Family Symmetries in the era of the LHC

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Abstract. The LHC is set to achieve a total 14 TeV collision energy for every pair of protons colliding. At this energy it will be possible to study interactions of the SM particles that may contain particles beyond it. We therefore will continue improving our understanding of particle physics, at the very least. Ideally many particles would be discovered and its interactions would be studied. A framework study for understanding the decay, mixing and CP breaking processes of these and the SM particles are the family symmetries. Proposed to explain the hierarchy of fermion masses and mixing in the SM but posed for explanations and predictions of what may happen beyond the SM. In this talk I will review the present bounds on flavour changing neutral currents and make a summary of how family symmetries can explain them.

1. Introduction

The flavour problem in the Standard Model (SM) is our ignorance in understanding the origin of the hierarchical values of quarks and charged leptons and the structure of the mixing in the quark sector. On the other hand, the CP violation problem is the fact that the Yukawa Lagrangian is not CP invariant, that is Charge and Parity transformations, respectively, are not conserved.

The CP violation problem can be put in very easy terms: if the Yukawa matrices were real, there would be no CP violation. However, since there is indeed CP violation this implies that there is a not trivial phase in one of the Yukawa matrices, either in the up or the down sectors or a combination of both. Specifically there is a not a trivial phase in the allowed redefinitions of the Yukawa matrices, Im \[ Y^d Y^d Y^d Y^d + Y^d Y^d Y^d Y^d \] \( \neq 0 \). Family symmetries (FS) address the two problems above. However, since the observables that we know are just quark masses and their mixing matrix, the Cabibbo-Kobayashi-Masakawa (CKM) matrix, Yukawa matrices cannot be uniquely determined and hence the way to address the flavour problem in the SM is not unique. Furthermore, we know that neutrino matrices are not zero and their mixing is not trivial. Therefore any unifying attempt to describe the way fermion masses are generated must include right-handed neutrinos. The biggest group which is candidate for a FS group that includes also three right-handed neutrinos is \( U(3)^6 \). This is simply just because for each kind of quarks, \( Q_L, u_R, d_R \) and leptons, \( L_L, e_R \) and \( \nu_R \), there is a factor \( U(3) \). The factor \( U(3) \) includes the three generations when no breaking of the FS has taken place. Hence the possible FS subgroups are many and quite varied: discrete or continuous. One should also decide the nature of the groups: global or gauged. We will refer in this contribution just to the supersymmetric FS models that have addressed both problems above mentioned, that is, aim to explain both

(i) the hierarchy of fermion masses and mixing and
(ii) the CP problem.
In the SM these problems are intimately related to the Glashow-Iliopoulos-Maiani (GIM) mechanism, which is effectively a framework to understand why even though we do not have a way to explain flavour and CP problems in the SM, we understand why flavour changing processes are under control. The reason is because there are three species of quarks with a unitary mixing matrix whose off-diagonal CKM elements are really small and because quark masses are so hierarchical.

Enter supersymmetry and the flavour and CP problems are magnified. Not only there are many more sources of flavour, that is matrices with a non trivial mixing and not understood eigenvalues, but it is also hard to understand how to control the many sources of CP violation.

In this contribution I point out why Supersymmetric Family Symmetries (SFS) are a relevant feature in the way we think supersymmetric spectra and their CP and flavour sources can be controlled and what we could expect from them in the era of the LHC. Furthermore, I point out the added benefits of constructing an effective framework, a la GIM mechanism, to be able to relate directly the structures of masses and mixing, both of fermions and sfermions with the way flavour changing processes are suppressed.

2. Family Symmetries

The ambiguity in determining the Yukawa matrices

\[ Y = U_L^\dagger \hat{Y} f U_R, \quad f = u, d, \]  

has to do with the fact that we just know the mass eigenvalues (in the SM we have \( \hat{Y}^u = \sqrt{2} \) Diagonal\([m_u, m_c, m_t]/v, \hat{Y}^d = \sqrt{2} \) Diagonal\([m_d, m_s, m_b]/v, v^2 = (\sqrt{2} G_F)^{-1} \) and the CKM matrix, \( V_{CKM} = U_{uL} U_{dL}^\dagger \). Hence the assumptions that should be made are reflected in the right-handed diagonalizing matrices \( U^u_R \) and \( U^d_R \). In the FS that are the subject of this note, the SM Yukawa Lagrangian, \( L = -Y^u_{ij} \bar{Q}_i H u_j - Y^d_{ij} \bar{Q}_i [i\sigma_2 H^*] d_j + h.c. \), is understood as an effective infrared completion where the Yukawa matrices are functions of fields, \( \theta \),

\[ Y^f = Y^f(\theta_i). \]  

Usually these \( \theta \) fields, charged under the FS group, are scalars which are fancily called flavons. Alternatively, the Higgs sector of the SM can be extended such that there is not only one Higgs scalar \( H \) but many that couple in a non trivial way to the matter fields of the SM and such that the sum of their contributions to the Yukawa couplings produce the observed hierarchy of masses and mixing. If, in addition one attempts to explain the hierarchy of neutrino masses and their non trivial mixing the problem gets more involved.

The supersymmetric version of the approach that Froggatt and Nielsen proposed [1] (FN) is is illustrated in the diagrams of Figure 1. In this example there are two scalars \( \theta \) and \( \bar{\theta} \), with charges 1 and −1, respectively, under the FS group and singlets under the \( G_{SM} \) group. They are coupled to the fermions \( f_R \) and \( f_L \) of the SM in different ways, through heavy fermion fields \( \chi \) with a mass term \( M \). For the fermion \( f \) coupled exactly in the way of Figure 1 (a) to \( \chi, \chi^*, H \) and \( \theta \); after \( H, \theta \) and \( \bar{\theta} \) take vacuum expectation values (VEVs) and assuming \( \langle \theta \rangle < M \), we obtain the following mass term

\[ m_f = \langle H \rangle \frac{\langle \theta \rangle}{M} \equiv \langle H \rangle \epsilon. \]  

Giving \( U(1)_{FS} \) charges to the MSSM plus right-handed neutrinos, as they appear in Table 1,

1 Roughly of order \( \lambda, \lambda^2 \) and \( \lambda^3 \), where \( \lambda \approx 0.226 \).
the Yukawa couplings for the different kinds of quarks are then given by

\[
Y_{ij}^u = a_{ij}^u e^{Q_{ij}^u}, \quad Y_{ij}^d = a_{ij}^d e^{Q_{ij}^d} \\
Q_{ij}^u = -(q_i - q_3) - (u_j - u_3), \quad Q_{ij}^d = -(q_i - q_3) - (d_j - d_3), \\
Y_{ij}^\nu = a_{ij}^\nu e^{Q_{ij}^\nu}, \quad Y_{ij}^e = a_{ij}^e e^{Q_{ij}^e} \\
Q_{ij}^\nu = -(l_i - l_3) - (n_j - n_3), \quad Q_{ij}^e = -(l_i - l_3) - (n_j - n_3),
\]

where \(a_{ij}^F\) are \(O(1)\) coefficients. This example illustrates pretty well the way hierarchical Yukawa matrices can be obtained: the smaller the Yukawa coupling, the bigger the power needed and so the bigger the combination of charges that appear in Eq. (4). This corresponds simply to more insertions of the heavy fermion pairs \(\chi\) and \(\chi'\) and the \(\theta\) or \(\bar{\theta}\) fields, that can be represented by diagrams like those in Figure 1. Using the Fayet-Iliopoulos term that any supersymmetric \(N = 1\) theory containing a \(U(1)\) gauge factor can have, we can fix the VEVs of the theta fields and we could even determine the charges of the MSSM fields through cancellation of anomalies (e.g. [2]).

Now, the principal problem with this approach alone is that the coefficients \(a_{ij}^F\) cannot be related to each other and if they have to be entirely fixed by phenomenological requirements, then the productivity of such scenarios is in trouble. In combination with a Supersymmetric Grand Unified Theory (SGUT) some of the problems are alleviated and both ingredients blend into a rich Family Symmetry SGUT that can give interesting predictions [3].

To relate the coefficients \(a_{ij}^F\) among the same kind of fermions, one can think on non-Abelian FS groups such as \(U(2)\) [4], \(SU(3)\) [5] and \(SO(3)\) [6]. Their structures are richer because matter fields can be put in their fundamental representations such that when the FS groups are broken the \(a_{ij}^F\) coefficients of the fermions in the same representation will be related. This reduces neatly the number of parameters and that is why they are so attractive. All the examples in this paragraph satisfy the conditions mentioned in the Introduction.

The prime example of discrete models are based on the group \(A_4\) but they concentrate in describing very well only the mixing in the neutrino sector, leaving aside the question of fermion hierarchy, except for example [7]. To describe the hierarchy of fermion masses they use a GUT [8].

When we would like to relate the hierarchy of mixing in the quark masses with that of the
eigenvalues, we can estimate of how the Yukawa matrices will look like [9]

\[
Y_d \propto \begin{bmatrix}
\leq \epsilon_d^4 & \epsilon_d^3 & \leq \epsilon_d^3 \\
\leq \epsilon_d^3 & \leq \epsilon_d^2 & \leq \epsilon_d^2 \\
\leq \epsilon_d^2 & \leq \epsilon_d & \leq \epsilon_d \\
\leq \epsilon_d & \leq 1 & \leq 1
\end{bmatrix}, \\
Y_u \propto \begin{bmatrix}
\leq \epsilon_u^4 & \epsilon_u^3 & \leq \epsilon_u^3 \\
\leq \epsilon_u^3 & \leq \epsilon_u^2 & \leq \epsilon_u^2 \\
\leq \epsilon_u^2 & \leq \epsilon_u & \leq \epsilon_u \\
\leq \epsilon_u & \leq 1 & \leq 1
\end{bmatrix},
\] (4)

for \( \epsilon_d = 0.13 \) and \( \epsilon_u = 0.04 \). These Ansatzs are the ones that \( SO(3) \) and \( SU(3) \) models reproduce.

3. Flavour & CP Problems in SUSY Extensions of the SM

Let us consider the Minimal Supersymmetric Standard Model (MSSM) plus three right-handed neutrinos, its soft supersymmetric Lagrangian \( \mathcal{L}_{soft} \) is

\[
-\mathcal{L}_{soft \, \nu_{MSSM}} = \bar{q}_L^i (m_Q^2)^{ij} \bar{q}_L j + \bar{u}_R^j (m_u^2)^{ij} \bar{u}_R i + \bar{d}_R^j (m_d^2)^{ij} \bar{d}_R i \\
+ \bar{l}_L^i (m_L^2)^{ij} \bar{l}_L j + \bar{e}_R^j (m_e^2)^{ij} \bar{e}_R i + \bar{\nu}_R^j (m_\nu^2)^{ij} \bar{\nu}_R i \\
+ (b h_d h_u + \frac{1}{2} B^{ij} M_1^{ij} \bar{h}_R^i \bar{h}_R^j + \text{h.c.}) \\
+ \left( -a_d^{ij} h_d h_u \bar{u}_R^i \bar{u}_R^j - a_u^{ij} h_u h_u \bar{u}_R^i \bar{u}_R^j - a_u^{ij} h_d h_u \bar{u}_R^i \bar{u}_R^j \right) \\
+ \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 \tilde{W} \tilde{W} + \frac{1}{2} M_3 \tilde{G} \tilde{G} + \text{h.c.},
\] (6)

where SU(2) indices are not written explicitly. Then the flavour and CP supersymmetric problems are easy to identify: arbitrary values of masses and trilinear couplings in the supersymmetric breaking soft Lagrangian, give arbitrary flavour changing neutral currents (FCNC) and can easily exceed CP bounds. From \( W_{\nu_{MSSM}} \) we will not have more flavour or CP sources than those in the SM plus three right-handed neutrinos, except of course those coming from the coupling \( \mu_{\alpha\beta} H_u^\alpha H_d^\beta \).

To be concrete, we can consider the case of the CP asymmetry in the \( K^0 \) system, \( \bar{K}^0 - K^0 \), which is a mixture of \( s \bar{d} \) and \( \bar{s} d \). In the SM, this mixing can only be mediated by \( W^- \) and the up-type quarks, Figure 2. The mixing is small due to the GIM mechanism. That is the VEV of the effective Hamiltonian describing the \( \bar{K}^0 - K^0 \) mixing, \( (K^0|e^{eff}_{\Delta S=2}|\bar{K}^0) \), at \( M_W \),

\[
(K^0|e^{eff}_{\Delta S=2}|\bar{K}^0) = \frac{G_F^2 M_W^2}{4\pi^2} X \langle K^0|[d^\mu P_L s][d^\nu P_L s]|\bar{K}^0 \rangle, \\
X_{M_W} = \sum_{i,j=1}^3 V_{js} V_{id}^* V_{jd} V_{is} S \left( \frac{m_i^2}{M_W^2} \frac{m_j^2}{M_W^2} \right),
\] (7)

is small. The GIM mechanism manifest here simply as follows. Firstly due to the unitarity of the \( 3 \times 3 \) \( V_{\text{CKM}} \) matrix such that combinations of the type \( V_{js} V_{id}^* V_{jd} V_{is} \) are impossible to appear. Second, because of the value of the function \( S(x_1, x_2) = S \left( \frac{m_i^2}{M_W^2}, \frac{m_j^2}{M_W^2} \right) \) for the
Experimental values are and $B$ computation of different relevant quarks (c and t) $^2$

\[ X = [V_{es}V_{cd}]^2 S \left( \frac{m_e^2}{M_W^2} \right) + [V_{ts}V_{td}]^2 S \left( \frac{m_t^2}{M_W^2} \right) + V_{es}V_{cd}^* V_{ts}V_{td}^* S \left( \frac{m_e^2}{M_W^2}, \frac{m_t^2}{M_W^2} \right) \]

\[ S \left( \frac{m_e^2}{M_W^2} \right) = 2.49 \times 10^{-4}, \quad S \left( \frac{m_t^2}{M_W^2} \right) = 2.37, \quad S \left( \frac{m_e^2}{M_W^2}, \frac{m_t^2}{M_W^2} \right) = 1.97 \times 10^{-3}, \]

\[ |V_{es}V_{cd}|^2 \propto \lambda^2, \quad |V_{ts}V_{td}|^2 \propto \lambda^{10}, \quad V_{es}V_{cd}^* V_{ts}V_{td}^* \propto \lambda^7. \]  

(8)

The CP violation parameter

\[ \epsilon_K = \frac{-\text{Im} \left[ \langle K^0|\mathcal{H}_{\text{s}S=2}^{\text{eff}}|K^0 \rangle \right]}{\text{Re} \left[ \langle K^0|\mathcal{H}_{\text{s}S=2}^{\text{eff}}|K^0 \rangle \right]} \]

\[ = \frac{-2 \text{Im} \left[ \langle K^0|\mathcal{H}_{\text{s}S=2}^{\text{eff}}|K^0 \rangle \right]}{\Delta m_{K^0}}, \]

\[ = \frac{C_{\text{F}}^2 f_K^2 m_K M_W^2 B_K \text{Im}[X]}{6\sqrt{2}\pi^2 \Delta m_K} \]

is a great way to check the sensitivity to CP violation. In Eq. (9) $m_K$ is the mass of $K^0$, $f_K$ and $B_K$ constants of the Kaon system. The QCD NLO (next to leading order) [11] and the experimental values are

\[ \epsilon_K^{\text{SM}} = (0.00178 \pm 0.00025), \quad \epsilon_K^{\text{exp}} = (0.00229 \pm 0.00010), \]  

(9)

so both quantities are compatible within 2σ C.L. While it is truly amazing that within the SM it is possible to achieve this compatibility, we wonder if we have achieved enough precision in the computation of $\epsilon_K$ or if there is a hint of something beyond the SM (BSM). Certainly the error in its experimental value is a thought test for theories BSM and in particular for supersymmetry. For the general $\nu MSSM$ in consideration, it is easy to see the many sources of contributions to this observable. The box diagrams associated to $\Delta F = 2$ processes in this theory are given in Figure 3, for the $K^0$ mixing under consideration, $q_j = s$ and $q_l = d$. We recognize that only the charged Higgs bosons are mediated by the couplings of the SM, the elements of $V_{\text{CKM}}$, Figure 3 (a). In all the other diagrams of Figure 2 ((b),(c),(d)), there appear new mixing elements. In diagram (b) we need to compute the mixing of the quarks and squarks mass eigenstates, identified by the elements of the matrix $K$. This mixing matrix is the unitary matrix diagonalizing the

\[ ^2 \text{In Eq. (7) } \mathcal{P}_L \text{ is the left-handed projection operator and the loop functions are } S(x_i, x_i) = S(x_i) = \frac{x_i^2 - \text{log}(x_i)}{4(1-x_i)^2} - \frac{3x_i^2 \text{log}(x_i)}{2(1-x_i)^2}, \quad S(x_i, x_j) = x_i \left( -\frac{3x_i x_j}{4(1-x_j)^2} + \log \left( \frac{x_j}{x_i} \right) - \frac{3x_i^2 \text{log}(x_j)}{4(1-x_j)^2} \right). \]


effective mass matrix for sfermions \( \tilde{f} \) in the basis where Yukawa matrices are diagonal, the super CKM (SCKM) basis, for example for squarks:

\[
(M_f^{SCKM})^2_{ij} = \left[ \begin{array}{cc} M_{SCKM}^2_{LL} & M_{SCKM}^2_{LR} \\ M_{SCKM}^2_{LR} & M_{SCKM}^2_{RR} \end{array} \right]_{ij} = (\tilde{M}_f^2)_{ij}
\]

\[
= \left[ \begin{array}{c} (U_L^T M_Q^2 U_L^T)_{ij} + \bar{m}_f^2 \delta_{ij} + D^L_f \\ -(U_R^T M^2_{LR} U_R^T)_{ij} v_f + \mu^* \tan \beta \tilde{M}_f \delta_{ij} \end{array} \right]
\]

\[
\mathcal{L}_{\text{eff}}^{m_f} = -(\tilde{q}_L^* \tilde{q}_R^* (M_f^2)^{ij} (\tilde{q}_L^* \tilde{q}_R^*)_{ij} = [\tilde{q}_L, \tilde{q}_R]_{i=1,2,3} K_f^{ij} \left[ \begin{array}{c} m_{q_1}^2 \\ \vdots \\ m_{q_6}^2 \end{array} \right] K_f^T \left[ \begin{array}{c} \tilde{q}_L^* \\ \tilde{q}_R^* \end{array} \right]_{j=1,2,3}.
\]

Here \( D^f_{L,R} \) remain diagonal, and \( \bar{m}_f \) is the diagonal matrix of the \( f \) type fermions, the indices \( i,j \) run over the different quark generations.

The diagram of Figure 3 (c), brings along the mixing of all the neutral \( U(1)_{em} \) sparticles, the neutralinos. Their mixing is described the Lagrangian \( \mathcal{L} = -\frac{1}{2} \tilde{\psi}^0 N N^\dagger \tilde{\psi}^0 N^* N^T (\tilde{\psi}^0)^T + \text{h.c.,} \)
where the gauge eigenstates are \( \tilde{\psi}^0 = (-i\tilde{b}, -i\tilde{t}, \tilde{h}_d^0, \tilde{h}_u^0) \) and the mass eigenstates need to be...
computed from

\[
\mathcal{M}_{\tilde{\psi}^0} = \begin{bmatrix}
M_1 & 0 & -M_{Zc\beta SW} & M_{Zs\beta SW} \\
0 & M_2 & M_{Zc\beta CW} & -M_{Zs\beta CW} \\
-M_{Zc\beta SW} & M_{Zc\beta CW} & 0 & -\mu \\
M_{Zs\beta SW} & -M_{Zs\beta CW} & -\mu & 0
\end{bmatrix} = NM_\chi^0 N^T,
\]

where \(N\) matrices are diagonalizing matrices and the mass eigenstates, \(\chi_i^0\), are given by \(\tilde{\chi}_i^0 = (\psi^0 N)_{ij}\). Finally, the diagram of Figure 3 (d) introduces the mixing of all the charged \(U(1)_{em}\) sparticles, the charginos. Their effective mass Lagrangian is given by \(\mathcal{L} = -\frac{1}{2} \left[ \tilde{\psi}^+ VV^+ \mathcal{M}_{\tilde{\psi}^+} U^+ U^{T} (\tilde{\psi}^-)^T + \tilde{\psi}^- \mathcal{M}_{\tilde{\psi}^-} T (\tilde{\psi}^+)^T \right]\), where \(\tilde{\psi}^+ = (-i\tilde{\psi}^+ h_u^+, \tilde{h}_d^-), \tilde{\psi}^- = (-i\tilde{\psi}^- h_d^+, \tilde{h}_u^-)\).

The mass matrix

\[
\mathcal{M}_{\tilde{\psi}^+} = \left( \begin{array}{cc}
M_2 & \sqrt{2} M_{Wc\beta} \\
\sqrt{2} M_{Wc\beta} & \mu
\end{array} \right)
\]

is diagonalized by two different unitary matrices \(U\) and \(U'\), \(M_{\chi_{\pm}} = UM_{\chi_{\pm}} U'^*\). It is easy to guess that light gluinos can give contributions that easily exceed the observed value of \(\epsilon_K\). This is because there is just one type of mixing matrix involved, \(K\). For example, for masses of gluinos and s-downs both less than 400 GeV and \(K_{\chi_{\pm},id} = V_{\text{CKM},id}\) this would give \(\epsilon_K \geq O(10^{-3})\)!

These observations bring along the minimal flavour violating (MFV) scheme [12], where although we do not have the relations \(K_{\chi_{\pm},id} = V_{\text{CKM},id}\), Yukawa matrices are assumed to be the only sources of flavour and CP violation. This is done by imposing as boundary conditions at a determined scale, \(\mu_B\), the relations

\[
(n_Q^2)^{ij} = (m_d^2)^{ij} = (m_u^2)^{ij} = (m_L^2)^{ij} = (n_e^2)^{ij} = \delta_{ij}, \quad a_f^{ij} = Y_f^i A_0, \quad (13)
\]

which in the constrained MSSM (CMSSM) they are set at \(\mu_B = M_G\). This will be satisfied at the Electroweak scale only if \(A_0 = 0\) [13]. This is a very strong restriction, which the FS do not in general satisfy, we will come back to this issue in the next section.

4. Interplay

4.1. Effective supersymmetric theories

In an effective supersymmetric theory with a FS, both the effective superpotential \(W_{FS}\) and the soft Lagrangian \(\mathcal{L}_{\text{soft}} FS\) will be invariant under the action of the FS group \(G_{FS}\),

\[
W_{\nu MSSM} = W_{FS} \rightarrow_g^{G_{FS}} W_{FS}
\]

\[
\mathcal{L}_{\text{soft}} \nu MSSM = \mathcal{L}_{\text{soft}} FS \rightarrow_g^{G_{FS}} \mathcal{L}_{\text{soft}} FS,
\]

where we now assume that Eqs. (5) and (6) are FS invariant. From the rest of this contribution I will assume that flavons are always scalars and that Yukawa couplings, trilinear terms and supersymmetric mass terms are always combinations of flavons, for example for two different flavons \(\phi_1\) and \(\phi_2\) we have

\[
\frac{\langle \phi_1^2 \rangle}{M_1} + \frac{\langle \phi_2^2 \rangle}{M_2} e_{\alpha\beta} H_u^\alpha E_j^\beta L_j^\beta \rightarrow Y_{ij}^{\phi} = \frac{\langle \phi_1^2 \rangle}{M_1} + \frac{\langle \phi_2^2 \rangle}{M_2} \quad (15)
\]

For concrete examples I will refer to \(SU(3)\) FS models which have been somewhat extensively studied [14, 15, 16] and in particular to the set up in [17]. In these models usually MSSM matter fields, e.g. \(E_i^\alpha\) and \(L_j^\beta\), sit in antitriplet representations of \(SU(3)\) and so \(\tilde{\phi}^2\) and \(\tilde{\phi}^2\) sit in triplet representations, assuming the Higgs bosons are singlets under \(SU(3)\). Once the flavons
take VEVs, \( W_{FS} \) is an effective renormalizable superpotential. This superpotential should have come indeed from a renormalizable superpotential before the FS is broken, for example

\[
W_O = M_1 \chi_1^f \chi_1^f + M_2 \chi_2^f \chi_2^f + F_i H_f \chi_i^f + \tilde{\phi}_1^f \chi_1^f + \tilde{\phi}_2^f \phi_i^f + \mu H_u H_d,
\]

\[
F = L, Q, \quad f^c = U, D, E^c, E^c,
\]

then Eq. (5) should be obtained by integrating out the heavy fields \( \chi_i^f \) and \( \chi_i^f \partial W_{FS}/\partial \chi_i^f = \partial W_{FS}/\partial \phi^i_1 = 0 \). These fields \( \chi_i^f \) are often called messenger fields.

The observables to which \( W_{FS} \), Eq. (5), is confronted are again the hierarchical fermion masses \( \langle \tilde{Y}^u = \sqrt{2} \text{ Diagonal}[m_u, m_c, m_t]/(v \sin \beta) \rangle, \langle \tilde{Y}^d = \sqrt{2} \text{ Diagonal}[m_d, m_s, m_b]/(v \cos \beta) \rangle \) and the CKM matrix \( V_{CKM} \). Now of course we have the free parameter \( \tan \beta \) but mass ratios among the same kind of quarks are not affected by it.

It is evident that since the FS controls the full superpotential \( W_{FS} \) at least from it, we will not have more flavour or CP sources than those in the SM, except of course just coming from the right-handed neutrinos and the coupling \( \mu \epsilon_{\alpha \beta} H_u^\alpha H_d^\beta \).

Note that in fact the gain is obvious, we reduce the arbitrariness in selecting the possible Yukawa couplings by constraining them through observables directly related to the supersymmetric vertices produced by \( W_{FS} \). What it does introduce much more flavour violating terms and sources of CP violation is the soft-supersymmetric Lagrangian \( \mathcal{L}_{\text{soft FS}} \).

Since \( \mathcal{L}_{\text{soft FS}} \) must be FS invariant, just by FS arguments we can interpret the trilinear terms as functions of the flavons

\[
\frac{\langle \tilde{\phi}_1^i \rangle \langle \tilde{\phi}_2^j \rangle}{M_1 M_2} A_0 h_d c_i^j I_j \rightarrow a_i^{ij} = \frac{\langle \tilde{\phi}_1^i \rangle \langle \tilde{\phi}_2^j \rangle}{M_1 M_2} A_0,
\]

where \( A_0 \) is just a mass term like in the CMSSM. Note that in effective supersymmetric theories Eqs. (15) and (17) have many ways to be generated because the only restrictions are the way the flavon fields, \( \tilde{\phi}_i \), of the theory can be contracted with matter fields. Note however there is not any way to relate Yukawa couplings, Eq. (15), and trilinear terms Eq. (17) even if by FS arguments they are given by the same combinations of flavons, simply because we cannot determine the exact coefficients in front of them and the relation among them, that is

\[
c_{ij}^Y = c_{ij}^a, \quad Y_i^{ij} = c_{ij}^a \langle \phi_i^f \rangle \langle \phi_j^f \rangle M_1 M_2, \quad a_i^{ij} = c_{ij}^a \langle \phi_i^f \rangle \langle \phi_j^f \rangle M_1 M_2 A_0.
\]

The ambiguities produced by this underdetermination can be lifted by introducing free parameters but this of course turns in to a question of how predictive the theory would be and in particular how many couplings to the heavy fields \( \chi_i^f \) will be allowed.

These theories have been successful to a point where the details of the messenger fields are ignored, all their masses are assumed to be the same and all what it matters to get the right hierarchy of fermion masses are the VEVs of the flavon fields. This is done by studying the scalar potential of the theory, in the example given just as a supersymmetric theory with \( W_O \) of Eq. (16) and assuming the VEV of Higgs is set to a much lower scale than the scale of breaking of the FS, we would have

\[
U_{FS} = \sum_i \left[ W_{Oi} W_{Oi} + \sum_a D_i^a D_i^a \right]_{\text{VEVs}} = g_{FS}^2 \sum_a \langle \tilde{\phi}_1^i \rangle_1 T_{xy}^a \langle \tilde{\phi}_1^i \rangle_1 + \langle \tilde{\phi}_2^i \rangle_2 T_{xy}^a \langle \tilde{\phi}_2^i \rangle_2.
\]

It is easy to see that since \( W_{Oi} = \delta W_O/\delta \phi_i \), \( i = 1, 2 \), are not proportional to any scalar then they cannot take any role in setting the VEVs of the flavons and all what it matters are the

\[\text{Here } \tan \beta = \langle H_u \rangle / \langle H_d \rangle = v_u/v_d. \]
4.2. Effective FS supergravity theories

In effective supergravity theories one really needs to specify the superpotential $W_O$, like the one in Eq. (16), and the Kähler potential of the complete MSSM, flavons and messenger fields, for example [17]

$$K = F_i^\dagger F_i \left[ 1 + \xi_F + \frac{\lambda^2_F}{M^2} \left( 1 + \xi_{\phi_i} \right) H_f^\dagger H_f \right] + f_{ij}^c f_{ij}^c \delta_{ij} + \frac{\lambda^2_F}{M^2} \left( 1 + \xi_{\phi_i} \right) \phi_2^\dagger \phi_2^\dagger,$$

$$+ H_f^\dagger H_f \left( 1 + \xi_{H_f} \right) + (Z_H H_u H_d + h.c.) + \ldots + K_H,$$

(20)

Where $K_H$ represents the hidden-sector including the fields breaking supersymmetry and the family symmetry, that is the flavons. The soft-squared masses and trilinears are then given as functions of the gravitino mass $m_{3/2}^2 = \langle e^{K_{H}/M^2} W_H^2 \rangle / M^4 = \text{const.}$ and all the fields of the hidden sector. Assuming that there is no contribution from $D$ terms [19], we have

$$m_{3/2}^2 = m_{3/2}^2 (K_{\alpha\beta}) - \langle F^m_{*} \left( \partial^*_{\alpha} \partial_{\beta} K_{\alpha\beta} - (\delta^*_{\alpha} K_{\gamma\beta}) \right) K_{\alpha\beta} \rangle,$$

$$\alpha'_{\alpha\beta\gamma} = \langle F^m_{*} \left[ \left( \partial_{\alpha} K_{H} \right) \frac{Y_{\alpha'\beta\gamma}}{M^2} \right] \left( \delta_{\beta} \partial_{\gamma} \right) \left( \delta_{\alpha} \partial_{\beta} \right) \rangle,$$

(21a)

(21b)

where $K_{\alpha\beta} \equiv \frac{\partial^2 K}{\partial C_{\alpha} \partial C_{\beta}}$ with $C_{\alpha} = (F_i, f_{ij}^c, H_f)$ and $K_{\alpha'\beta'}$ denotes the elements of the inverse matrix. Besides, $\partial_{\alpha} \equiv \partial / \partial h_{\alpha}$, $\partial^*_{\alpha} \equiv \partial / \partial h^*_{\alpha}$, and e.g. $\langle F^{\phi_1} \partial_{\phi_1} \rangle \partial_{\phi_{1,\alpha}} \equiv \langle F^{\phi_{1,\alpha}} \partial_{\phi_1} \rangle$. We have expressed the formula for the trilinear couplings in terms of $Y_{f_{\alpha}^c F_i H_f} \equiv \langle \tilde{N} Y_{f_{\alpha}^c F_i H_f} \rangle$, where $W_O' = W_O \left( \frac{W_u^*}{|W_u|} e^{M^2/\sum_{m}|h_m|^2} \right) \equiv \tilde{N} W_O$. We see then that supergravity give us an exact prescription or how we can relate Yukawa and trilinear couplings to flavons and how the spectra of soft-squared masses can indeed be controlled by the FS.

5. Expectations at the LHC

What can we expect at the LHC from such SFS theories? The first thing to note is that for having a definite answer we must answer something about the way supersymmetry is broken and to set the scale of its breaking. This is the first limitation of this scenario, it would be great if one can combine the breaking of supersymmetry with the breaking of the FS. Putting aside this question, one needs then to assume a definite value of $m_{3/2}$ and the relation between the fields breaking supersymmetry and the flavon fields. Let us assume that this is trivial, that is that their mixing is zero. As we have said from concreteness we take the example of [17],

$\ldots$
approximation where the flavour violating terms are given by dimensionless parameters

\[ \hat{m}^2_{ij} \sim m_{3/2}^2 \begin{pmatrix} 1 & \epsilon_f^2 & \epsilon_d^2 \\ 1 + \epsilon_f^2 & 1 & \epsilon_d^2 \\ 1 & \epsilon_f^2 & 1 \end{pmatrix}, \quad f = u, d, Q, e, L, \]  

(22)

where \( \epsilon \) parameters have the following constraints

\[ |\epsilon_Q \epsilon_u| \sim |\epsilon_u|^2 \sim \left( \frac{Y_f^2 F_2 H_u}{Y_f^2 F_2 H_u} \right)^2 = 0.04^2, \quad |\epsilon_Q \epsilon_d| \sim |\epsilon_d|^2 \sim \left( \frac{Y_f^2 F_2 H_d}{Y_f^2 F_2 H_d} \right)^2 = 0.13^2, \]

\[ |\epsilon_L \epsilon_e| \sim |\epsilon_d|^2. \]  

(23)

As we can see, they cannot be uniquely determined, but exactly these kind of relations can be tested through FCNC processes, before doing so, there are a couple of observations worth mentioning here. First, the matrices Eq. (22) are not like in the MFV case, that is diagonal at a high scale \( \langle \hat{\phi} \rangle \gtrsim M_G \). Second, this is not a problem because the off-diagonal elements are controlled by parameters related to the Yukawa couplings, Eq. (23), even thought not uniquely determined. This is the weakest point of the SFS and so a further understanding on how these parameters could be controlled should be addressed. To have an idea of the type of amount of flavour violation that these models have, let us present it in the mass insertion (MI) approximation where the flavour violating terms are given by dimensionless parameters \( \delta \) defined as

\[ (\delta^f_{RR})_{ij} \equiv \frac{(\hat{m}^2_{f,RR})_{ij}}{(\hat{m}^2_{f,RR})_{ii}}, \quad (\delta^f_{LR,RL})_{ij} \equiv \frac{(\hat{m}^2_{f,LR,RL})_{ij}}{\sqrt{(\hat{m}^2_{f,LL})_{ii}(\hat{m}^2_{f,RR})_{jj}}}, \]  

(24)

we do not have to compute the exact mixing matrices \( K, N \) and \( U' \) of Eqs. (11), (11) and (12) respectively, it is just enough to get Eq. (22) roughly at the EW to have an idea of the flavour violation induced. Analogously we can compute the \( (\delta^q_{XY}) \) parameters

\[ (\delta^q_{LR,RL})_{ij} = \frac{v}{\sqrt{1 + \tan^2 \beta}} \left[ -\tilde{a}_{ij} \frac{Q_i H_f}{30 \tan \beta \tilde{Y}_{di}^{\text{diag}}} + \frac{\mu^* \tan \beta \tilde{Y}_{ui}^{\text{diag}}}{30 \tan \beta \tilde{Y}_{ui}^{\text{diag}}} \right], \]

\[ (\delta^q_{LR,RL})_{ij} = \frac{v \tan \beta}{\sqrt{1 + \tan^2 \beta}} \left[ -\tilde{a}_{ij} \frac{Q_i H_f}{30 \tan \beta \tilde{Y}_{ui}^{\text{diag}}} + \frac{\mu^* \tan \beta \tilde{Y}_{ui}^{\text{diag}}}{30 \tan \beta \tilde{Y}_{ui}^{\text{diag}}} \right], \]  

(25)

where \( \sim \) denotes the quantities in the SCKM basis and the factor 30 is a renormalization running factor. Once this is done on the basis where all the fields have canonically normalised kinetic terms, we can compute the flavour violating parameters \( \delta \)'s and confront them with bounds obtained from several experimental constraints. To illustrate the point, we present these parameters for the example of this section in Table 2.

What do we learn from here? That FS can control flavour violating effects without the need of assuming MFV and through parameters relevant to the Yukawa couplings. Furthermore, FCNC processes really put to test these symmetries because there are parameters ((\( \delta^f_{LR,RL})_{12} \) and \( \delta^q_{LL,12} \)) that are already at the limit of their corresponding bounds.

The next question, it is what could be the real impact on the supersymmetric spectra? This was probed in [13]. The basic results are as follows. When the running of off-diagonal Yukawa
couplings, trilinear terms and soft-squared masses is taken into account according to parameters defined by the SFS, e.g. Eq. (23), we can use precise measurements such as $\varepsilon_k$, $BR(b \to s\gamma)$ and $BR(b \to \ell^+\ell^-\gamma)$ to assess their impact on soft masses. It is true that the splitting of the s-fermions will be mainly controlled about what we assume of the way supersymmetry is broken, because Yukawa couplings are small and so the running cannot be greatly affected. However trilinear terms and soft-squared masses are quite sensitive to FCNC. In Figure (4) from [13] we plot $BR(b \to s\gamma)$ as a function of our parameter $\varepsilon_d = \varepsilon_{S_d}$, there it was let to vary from zero to 0.5. What is interesting about this plot? The points on the left-hand side of all the curves correspond effectively to the CMSSM predictions of $BR(b \to \ell^+\ell^-\gamma)$ for different $m_{1/2}$ masses, from 300 to 600 GeV. Important departures from what it can be obtained in the CMSSM take place for $\varepsilon_{S_d} \geq 0.4$. For example in the CMSSM the particular point shown for $m_{1/2} = 400$ GeV would be experimentally excluded by $BR(b \to s\gamma)$ but for a SFS $\varepsilon_{S_d} = 0.44$ would be allowed. $BR(b \to \ell^+\ell^-\gamma)$ it is also quite sensitive to these changes and it has the right sensitivity for the LHC, unfortunately $BR(b \to s\gamma)$ requires photon energies of the order of few GeVs, which could be overseen at the LHC. Apart from the works in [14, 13], interesting

| $\delta_{RR,LL}^{12}$ | Example of [17] | Bound |
|----------------------|-----------------|-------|
| $\delta_{LL}^{12}$  | $\varepsilon_{LL}^{12} \sim 7 \cdot 10^{-5}$ | $9 \cdot 10^{-3}$ |
| $\delta_{LL}^{12}$  | $\varepsilon_{LL}^{12} \sim 7 \cdot 10^{-5}$ | $1 \cdot 10^{-2}$ |
| $\delta_{LR,RL}^{12}$ | $\frac{w}{\sqrt{1+\tan^2\beta}} \left( \frac{n_{A_0}}{3m_0^2} \right) \sim 4n \cdot 10^{-6}$ | $1 \cdot 10^{-5}$ |
| $\delta_{LL}^{23}$  | $\varepsilon_{LL}^{23} \sim 6 \cdot 10^{-4}$ | $2 \cdot 10^{-1}$ |
| $\delta_{LL}^{23}$  | $\varepsilon_{LL}^{23} \sim 6 \cdot 10^{-4}$ | $6 \cdot 10^{-4}$ |
| $\delta_{LR,RL}^{23}$ | $\frac{w}{\sqrt{1+\tan^2\beta}} \left( \frac{n_{A_0}}{3m_0^2} \right) \sim 4n \cdot 10^{-4}$ | $1 \cdot 10^{-3}$ |

Table 2: An example for the flavour violating parameters $\delta$ for the SPS 1a point ($m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV and $\tan\beta = 10$), together with the corresponding experimental limit. For a detailed description of the formulas see the text in this section.
analysis of FCNCS of effective supersymmetric $SU(3)$ family symmetries are presented in [11] and constraints from electric dipole moments (EDM) are studied in [23]. They are an important step towards the characterization of this kind of models, but the task is far from complete. It is thus an important and challenging task to determine extensively the changes in the parameter space which are typical of Supersymmetric Flavour Symmetries.

6. Final comments
Giving the exciting opportunity of discovery of the Higgs boson and supersymmetric particles at the LHC, we need to make an extensive catalogue of the possible features of the different supersymmetric models we propose. We have argued here the case of the supersymmetric family symmetries (SFS) that attempt to explain, both the hierarchy of mixing and fermion masses, the CP violation and the structure of sfermion masses and mixing.

I would like to emphasize that SFS explored in the literature have not concerned too much about the messenger sector of the theory, which is a decisive point towards a really predictive model. There have been few attempts, [24, 16, 17], but a serious understanding within a formal supergravity context should be addressed. Nevertheless, some characteristics of the SFS have already gave us information about the way FCNC and EDMs are under control and their impact on the supersymmetric spectra. We have ordered some pieces of the puzzle of the GIM supersymmetric family symmetry mechanism. Also as in the SM model, in the MSSM the mixing in FCNC processes needs to be small. The amount of this mixing can be controlled through parameters that are related to the mixing and eigenvalues of the Yukawa matrices. Therefore SFS have found a way to control mixing in both sectors, for fermions and sfermions, however a missing piece is to determine if one could really understand the structure of the supersymmetric eigenvalues from a SFS point of view and the conditions to get the CP violating phases from minimizing an scalar potential.

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