Charged Black Holes in Phantom Cosmology

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Abstract

In the classical relativistic regime, the accretion of phantom-like dark energy onto a stationary black hole reduces the mass of black hole. Here we have investigated the accretion of phantom energy onto a stationary charged black hole and have determined the condition under which this accretion is possible. This condition restricts the mass to charge ratio in a narrow limit. This condition also challenges the validity of the cosmic censorship conjecture since a naked singularity is eventually produced as magnitude of charge increases compared to mass of black hole.

Keywords: Accretion; Black Hole; Phantom Energy.

1 Introduction

Accelerated expansion of the universe has been observed and confirmed by myriad of sources including analysis of cosmic microwave background radiation [1], large scale structure [2] and supernovae SNe Ia data [3, 4]. This expansion is supposedly driven by exotic vacuum energy having $\rho > 0$ and $p < 0$ or equivalently $p = -\rho$ (or $\omega = -1$), dominating the observable universe. Observations of WMAP data suggest that its magnitude is more than 70% of the total energy density of the universe [5]. Among other forms of exotic energies (e.g. quintessence, cosmological constant, k-essence, hessence etc), the phantom energy with $\omega < -1$ exhibits similar behavior.
on large cosmic scale. The genesis of phantom energy is not clear but it violates the null and weak energy conditions. As these conditions are the weaker one, they ensure that the stronger conditions (i.e. strong and dominant) will be violated automatically [6, 7, 8]. These energy conditions guarantee the positive definiteness of the energy densities and pressure densities of all the matter content in the universe. Recent observational data constrain the range of dark energy by $-1.38 < \omega < -0.82$ at 95% confidence level [9]. Rather the supernovae data favor an evolving $\omega(z)$ varying from quintessence ($\omega > -1$) to phantom regime ($\omega < -1$) [10]. Further, the extrapolation of WMAP data is best fitted with the notion of phantom energy [11].

The energy density and the pressure of the phantom energy can be represented by the minimally coupled spatially homogeneous and time dependent scalar field $\phi$ having negative kinetic energy term given by

$$
\rho = -\frac{\dot{\phi}^2}{2} + V(\phi), \quad p = -\frac{\dot{\phi}^2}{2} - V(\phi).
$$

(1)

Here $V(\phi)$ is the scalar potential and dot over $\phi$ represents the derivative with respect to time parameter $t$. Note that if the kinetic term in Eq. (1) is positive then it gives usual dark energy with satisfies all the energy conditions. The above parameters $\rho$ and $p$ are related to the Hubble parameter $H$ as

$$
H^2 = \frac{4\pi}{3} (-\dot{\phi}^2 + 2V),
$$

(2)

where $H(t) = \dot{a}/a$ and $a(t)$ is the scale factor which arises in the Friedmann-Robertson-Walker spacetime. From Eq. (2), we require the potential $V(\phi)$ to be positive. It is argued by using scalar field models of phantom energy, that it can behave as a long range repulsive force [12]. The phantom energy possesses some peculiar properties unlike normal matter e.g. (1) its energy density $\rho(t)$ increases with the expansion of the universe, (2) it ensures the existence and stability of traversable worm holes in the universe [13, 14, 15, 16, 17], (3) also self-gravitating, static and spherically symmetric phantom scalar fields with arbitrary potentials can generate a stable configuration of a regular black hole or apparently non-singular black hole which inherently possesses exactly Schwarzschild-like causal structure but the singularity is replaced by a de Sitter infinity, thereby generating an asymptotically de Sitter expansion beyond the black hole horizon [15, 19], (4) due to strong negative pressure the phantom energy can disrupt all gravitationally bound structures i.e from galactic clusters to the gravitationally collapsed objects including black holes [20, 21, 22, 23, 24, 25], (5) it can produce infinite expansion of the universe in a finite time thus causing the ‘big rip’ (i.e. a state when $a(t), \rho(t) \to \infty$ for $t < \infty$) [6, 11].

The big rip is characterized by a future singularity implying a finite age of the universe. It has been proposed that this future singularity can be avoided if the phantom energy is interacting with the dark matter [26, 27]. The interaction of phantom energy and dark matter leads to stable attractor solutions at late times and the big rip is avoided in the parameter space [28]. Also, it was argued that there are certain classes of unified dark energy models stable against perturbations, in which cosmic dooms day can be avoided [29]. Moreover in scalar tensor theories, quantum gravity effects may prevent (or, at least, delay or soften) the
cosmic doomsday catastrophe associated with the phantom \[30 \ 31\]. Also in Gauss Bonnet gravity theory and loop quantum cosmology, the big rip occurrence is also avoided \[32 \ 33\]. In order to avoid the big rip with phantom matter, it is sufficient to have a phantom scalar field with a potential bounded above by some positive constant \[34\]. It is also suggested that phantom dark energy with \(\omega < -1\) can effectively ameliorate the coincidence problem (i.e. why does the observable universe begin the accelerated expansion so recently and that why are we living in an epoch in which the dark energy and the matter energy density are comparable?) \[35 \ 36\]. In another model using vector like dark energy with a background of perfect fluid, it is demonstrated that the cosmic coincidence problem is fairly solved \[37\].

The fate of a stationary uncharged black hole in the phantom energy dominated universe was investigated by Babichev et al \[20\]. The phantom energy was assumed to be a perfect fluid. The phantom energy was allowed to fall onto the black hole horizon only in the radial direction. It was concluded that black hole will lose mass steadily due to phantom energy accretion and disappear near the big rip. We here adopt their procedure for a static, stationary and charged black hole. The gravitational units are chosen for this work.

The paper is organized as follows: In the second section, we have explained the relativistic model of accretion onto a charged black hole and obtained the black hole mass loss rate. In third section, we have determined the critical points of accretion model and have analyzed the dynamics about these points. Finally we conclude our paper.

2 Accretion onto Charged Black Hole

We consider a static and spherically symmetric black hole of mass \(M\) having electric charge \(e\), so-called Reissner-Nordström (RN), specified by the line element

\[
ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),
\]

where

\[
f(r) = 1 - \frac{2M}{r} + \frac{e^2}{r^2}.
\]

If \(e^2 > M^2\) then the metric is non-singular everywhere except at the curvature or the irremovable singularity at \(r = 0\). Also if \(e^2 \leq M^2\) then the function \(f(r)\) has two real roots given by

\[
r_{h\pm} = M \pm \sqrt{M^2 - e^2}.
\]

These roots physically represent the apparent horizons of the RN black hole. The two horizons are termed the inner \(r_{h-}\) and the outer \(r_{h+}\). The outer horizon is effectively called the event horizon while the inner one is called the cauchy horizon of the black hole. The metric (3) is then regular in the regions specified by the inequalities: \(\infty > r > r_{h+}, r_{h+} > r > r_{h-}\) and \(r_{h-} > r > 0\). Note that if \(e^2 = M^2\), then it represents an extreme RN black hole while if \(e^2 > M^2\), it yields a naked singularity at \(r = 0\) \[38 \ 39\].
The phantom energy is assumed to be a perfect fluid specified by the stress energy tensor

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. \]  

(6)

Here \( p \) is the pressure and \( \rho \) is the energy density of the phantom energy. Also \( u^\mu = (u^t(r), u^r(r), 0, 0) \) is the four velocity of the phantom fluid which satisfies the normalization condition \( u^\mu u_\mu = -1 \).

We assume that the in-falling phantom fluid does not disturb the global spherical symmetry of the black hole. Further the energy-momentum conservation \( T_{\mu\nu}^{\mu} = 0 = T_{r r}^{\mu} \) gives

\[ ur^2M^{-2}(\rho + p)\sqrt{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} = C_1, \]

(7)

where \( u^r = u = dr/ds \) is the radial component of the velocity four vector and \( C_1 \) is a constant of integration. For inward flow, we will take \( u < 0 \). Moreover, the second constant of motion is obtained by projecting the energy conservation equation onto the velocity four vector as \( u_\mu T^{\mu\nu} = 0 \), which yields

\[ ur^2M^{-2}\exp \left[ \int_{\rho_{\infty}}^{\rho_h} \frac{dp'}{\rho' + p(\rho')} \right] = -A. \]

(8)

Here \( A \) is a constant of integration. Above \( \rho_h \) and \( \rho_{\infty} \) are the energy densities of the phantom energy at the horizon and at infinity respectively. From Eqs. (7) and (8) we have

\[ (\rho + p)\sqrt{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} \exp \left[ - \int_{\rho_{\infty}}^{\rho_h} \frac{dp'}{\rho' + p(\rho')} \right] = C_2, \]

(9)

where \( C_2 = -C_1/A = \rho_{\infty} + p(\rho_{\infty}) \). In order to calculate the rate of change of mass of black hole we integrate the flux of the fluid over the entire cross-section of the event horizon as

\[ \dot{M} = \oint T_r^t dS, \]

(10)

where \( T_r^t \) determines the momentum density in the radial direction and \( dS = \sqrt{-g}d\theta d\phi \) is the surface element of the horizon, where \( g \) is the determinant of the metric. From Eqs. (7 - 10), we get

\[ \frac{dM}{dt} = 4\pi AM^2(\rho_{\infty}(t) + p_{\infty}(t)), \]

(11)

which clearly demonstrates that mass of black hole decreases if \( \rho_{\infty} + p_{\infty} < 0 \). Note that Eq. (11) can be solved for any equation of state of the form \( p = p(\rho) \) or in particular \( p = \omega \rho \). In general, Eq. (11) holds for all \( \rho \) and \( p \) violating the dominant energy condition, thus we can write

\[ \frac{dM}{dt} = 4\pi AM^2(\rho(t) + p(t)). \]

(12)

In the astrophysical context, the mass of black hole is a dynamic quantity. The mass increases by the accretion of matter and can decrease by the accretion of the phantom energy. Since we are not incorporating matter in our model, the mass of black hole will decrease correspondingly.
3 Critical Accretion

We are interested only in those solutions that pass through the critical point as these correspond to the material falling into the black hole with monotonically increasing speed. The falling fluid can exhibit variety of behaviors near the critical point of accretion, close to the compact object. For instance, for a given critical point \( r = r_c \), we have the following possibilities [40]: (a) \( u^2 = c_s^2 \) at \( r = r_c \), \( u^2 \to 0 \) as \( r \to \infty \), \( u^2 < c_s^2 \) for \( r > r_c \) and \( u^2 > c_s^2 \) for \( r < r_c \). Thus for large distance, the speed of flow becomes negligible (subsonic), at the critical point it is sonic, while the flow becomes supersonic for very small \( r \). Other solutions for the flow near \( r_c \) are not of much interest due to their impracticality, like (b) \( u^2 < c_s^2 \) for all values of \( r \) and (c) \( u^2 > c_s^2 \) for all values of \( r \). Solutions (b) and (c) are not realistic since they describe both subsonic and super-sonic flows for all \( r \). Similarly, (d) \( u^2 = c_s^2 \) for all values of \( r > r_c \) and (e) \( u^2 = c_s^2 \) for all values of \( r < r_c \). Last two solutions are also useless since they give same value of speed at a given \( r \). Hence from this discussion, we see that solution (a) is the only physically motivated, near the critical point.

To determine the critical points of accretion we shall adopt the procedure as specified in Michel [41]. The equation of mass flux \( J_r = 0 \) gives

\[
\rho u r^2 = k_1,
\]

where \( k_1 \) is constant of integration. Dividing and then squaring Eqs. (7) and (13) give

\[
\left( \frac{\rho + p}{\rho} \right)^2 \left( 1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2 \right) = \left( \frac{C_1}{k_1} \right)^2 = C_3.
\]

Here \( C_3 \) is a positive constant. Differentiation of Eqs. (13) and (14) and then elimination of \( d\rho \) gives

\[
\frac{du}{u} \left[ 2V^2 - \frac{\frac{M}{r} - \frac{e^2}{r^2} + u^2}{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} \right] + \frac{dr}{r} \left[ V^2 - \frac{\frac{M}{r} - \frac{e^2}{r^2} + u^2}{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} \right] = 0,
\]

or

\[
\frac{du}{dr} = -u \frac{\left[ V^2 - \frac{\frac{M}{r} - \frac{e^2}{r^2} + u^2}{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} \right]}{\left[ 2V^2 - \frac{\frac{M}{r} - \frac{e^2}{r^2} + u^2}{1 - \frac{2M}{r} + \frac{e^2}{r^2} + u^2} \right]} = \frac{N}{D},
\]

where

\[
V^2 \equiv \frac{d\ln(\rho + p)}{d\ln \rho} - 1.
\]

We have assumed that the flow is smooth at all points of spacetime, however if at any point the denominator \( D \) vanishes then the numerator \( N \) must also vanish at that point. Mathematically this point is called the critical point of the flow [44]. Equating the denominator \( D \) and numerator \( N \) to zero, we can get the so-called critical point conditions given by

\[
u_c^2 = \frac{M r_c - e^2}{2r_c^2},
\]
and

\[ V_c^2 = \frac{Mr_c - e^2}{2r_c^2 - 3Mr_c + e^2}. \]  

(19)

Note that by choosing \( e = 0 \) in the above equations, we can retrieve the results for the accretion of fluid onto a Schwarzschild black hole [41]. All the quantities with subscript \( c \) are defined at the critical point correspondingly. Physically, the critical points represent the sonic point of the flow i.e. the point where the speed of flow becomes equal to the speed of sound, \( u_c^2 = c_s^2 \) or the corresponding Mach number \( M_c = 1 \). This transition may occur from the initial subsonic to the supersonic or trans-sonic speeds. For any spherically symmetric spacetime, a surface where every point is a sonic point is called a sound horizon which itself will be spherical. Any perturbation or disturbance generated in the flow inside the sound horizon (\( r < r_c \)) is eventually pulled towards the black hole singularity and hence cannot escape to infinity.

It can be seen that the speed of sound (squared) \( c_s^2 = \partial p/\partial \rho \) has no physical meaning if the EoS parameter \( \omega < 0 \) (in \( p = \omega \rho \)). Thus it will apparently make the exotic cosmic fluids like the cosmological constant, quintessence and the phantom energy unstable that can not be accreted onto the black hole. In order to avoid this problem, Babichev et al [45] introduced a non-homogeneous linear equation of state (nEoS) given by \( p = \alpha(\rho - \rho_o) \), where the constants \( \alpha \) and \( \rho_o \) are free parameters. The nEoS can describe both hydrodynamically stable \((\alpha > 0)\) and unstable \((\alpha < 0)\) fluids. The parameter \( \omega \) is related to the nEoS as \( \omega = \alpha(\rho - \rho_o)/\rho \). Notice that \( \omega < 0 \) corresponds to \( \alpha > 0 \) and \( \rho > \rho_o \), thus making the phantom energy as hydrodynamically stable fluid. Therefore the speed of sound \( c_s \) is now well-defined with the nEoS for the phantom energy. Hence, the phantom energy can fall onto the RN black hole and can reduce the black hole mass. Since phantom energy reduces only mass and not charge, a stage is reached when the the cosmic censorship conjecture becomes violated i.e. \( e > m \), the so-called emergence of a naked singularity.

Now physically acceptable solution of Eq. (16) is obtained if \( u_c^2 > 0 \) and \( V_c^2 > 0 \), hence we get

\[ 2r_c^2 - 3Mr_c + e^2 \geq 0, \]  

(20)

and

\[ Mr_c - e^2 \geq 0. \]  

(21)

Eq. (20) can be factorized as

\[ 2r_c^2 - 3Mr_c + e^2 = (r_c - r_{c+})(r_c - r_{c-}) \geq 0, \]  

(22)

where

\[ r_{c\pm} = \frac{1}{4}(3M \pm \sqrt{9M^2 - 8e^2}), \]  

(23)

which are positive satisfying \( r_{c+} > r_{c-} > 0 \). In general, for \( e \leq m \), the inner critical point will lie between \( r_{h-} \leq r_{c-} \leq r_{h+} \), while the outer one will satisfy \( r_{c+} \geq r_{h+} \). It is obvious that these roots will be real valued if \( 9M^2 - 8e^2 \geq 0 \) or

\[ \frac{M^2}{e^2} \geq \frac{8}{9}. \]  

(24)
These roots physically represent the locations of the critical or sonic points of the flow near the black hole. Notice that both mass and charge have same dimension of length, therefore all the inequalities here and below represent dimensionless ratios. From Eq. (22), we can see that these critical points specify two regions for the flow: (1) \( r_c > r_{c+} \) or (2) \( 0 < r_c < r_{c-} \). We shall now solve Eq. (19) using (21) and then deduce a condition for the black hole mass and charge.

To get solutions about the critical points, we substitute \( r_{c\pm} \) in Eq. (21). For \( r_{c+} \), Eq. (21) gives

\[
M \sqrt{9M^2 - 8e^2} \geq 4e^2 - 3M^2, \tag{25}
\]

which is satisfied if

\[
\frac{M^2}{e^2} \leq 1, \tag{26}
\]

and

\[
\frac{M^2}{e^2} < \frac{4}{3}. \tag{27}
\]

A comparison of inequalities (24), (26) and (27) imply

\[
\frac{8}{9} \leq \frac{M^2}{e^2} < \frac{4}{3}. \tag{28}
\]

Thus accretion through \( r_{c+} \) is possible if the above inequality (28) is satisfied. It encompasses the two types of black holes in itself: regular and the extreme RN black hole. Interestingly, the naked singularity also falls within the prescribed limits. Thus for all these spacetimes, the accretion is allowed through the critical point \( r_{c+} \). We stress here that using \( e = 0 \) in the inequality (28) to retrieve same condition for the Schwarzschild black hole can be misleading. The inequality is deduced using the outer apparent horizon and a critical point. Since Schwarzschild black hole \((e \to 0)\) possesses unique horizon and the critical point, the above inequality cannot be reduced for an uncharged black hole.

Now we consider case (2) when \( 0 < r_c < r_{c-} \). Substitution of \( r_{c-} \) in Eq. (21) gives

\[
M \sqrt{9M^2 - 8e^2} \leq 3M^2 - 4e^2. \tag{29}
\]

If \( 3M^2 - 4e^2 < 0 \) then Eq. (29) does not yield any solution. So we need \( 3M^2 - 4e^2 > 0 \) which yields

\[
\frac{M^2}{e^2} > \frac{4}{3}. \tag{30}
\]

Further inequality (29) is satisfied if

\[
\frac{M^2}{e^2} < 1. \tag{31}
\]

Since Eqs. (30) and (31) are mutually inconsistent, there is no solution for \( r_c \) in case (2). Thus accretion is not possible through \( r_{c-} \).

Since the mass of black hole is decreasing by the accretion of phantom energy (see Eq. 12), it implies that at least one critical point must exist for the fluid flow, which is specified by \( r_{c+} \). This critical point yields the mass to charge ratio of the black hole in the range specified by (28) which allows that accretion onto all charged spherically symmetric black holes.
4 Conclusion

We have analyzed the effects of accretion of phantom energy onto a charged black hole. The analysis is performed using two critical points $r_{c\pm}$. It turns out that accretion is possible only through $r_{c+}$ which yields a constraint on the mass to charge ratio given by Eq. (28). This expression incorporates both extremal and non-extremal black holes. Thus all charged black holes will diminish near the big rip. Apparently this condition predicts the existence of large charges onto black holes, although astrophysically no such evidence has been successfully deduced from the observations. In theory, the existence of large charges onto black holes is consistently deduced by the general theory of relativity. It needs to be stressed that there is no analogous condition for the Schwarzschild black hole ($e = 0$). This analysis can be extended for a rotating charged black hole (so-called Kerr-Neumann black hole) to get a deeper insight of the accretion process. This work also serves as the generalization of Michel [41] in terms of the accretion of phantom dark energy onto a charged black hole.

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