The influence of the conical incidence on the waveguide-type colour-separating backlight for liquid crystal display

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Abstract. The lightguide assisted by diffraction gratings is used to implement colour-separating backlight for Liquid Crystal Display (LCD). In the configuration, the light incidence angle is required to be a precise designed one. However, the light incidence is conical always. In this paper, by rigorous coupled wave theory, the influence of conical input on the colour-separating backlight is investigated. It shows that, when the azimuthal angle of the light increases, the diffraction efficiency could be enhanced by a factor of 2, which means that the conical input will benefit the diffraction efficiency of the lightguide. However, it also leads to a larger output spot area at the same time, which could result in the mixing of RGB pix. When the azimuthal angle is smaller than 12°, the maximum diffraction angles of RGB light can be completely separated, which means that, with a designed distance or with an assisted micro-lens layer between the grating and the pix layer, the RGB light can reach their corresponding pix. The influence of the conical input on the polarized light is also investigated. The conical input will result in the output with composite polarization, which, in principle, disables the use of polarized light source in LCD.

1. Introduction
Liquid crystal display (LCD) technology has been wildly used in the TV, computer and various mobile terminals. However, the light management in present LCD is not efficient. Only around 5% of the light generated in the backlight of the LCD reaches the eyes of the user [1]. It has been suggested that, by a diffraction grating, the energy efficiency of the LCD can be greatly enhanced since the grating can separate the colours of the backlight to RGB light [2], and thus avoid using the colour filter, whose absorption is estimated to be over 66% [1]. This backlight requires that the light is guided in a lightguide and captured inside due to the total internal reflection. When the grating is applied on the surface of the lightguide, the light can be partially diffracted out the lightguide [3] [4] [5], at the same time, the light is separated according to the colour due to the dispersion of the grating [6]. To implement this idea, the light should be input in a design angle [7]. However, the light incidents are conical always, i.e. the plane of the incidence is not exactly perpendicular to the grating, therefore, it is important to investigate the influence of the conical incidence on the colour separated backlight. By rigorous coupled wave analysis theory [8], the influence of conical input on the colour-separating backlight is investigated. The results show when the azimuthal angle of the incidence light increases, the diffraction efficiency could be enhanced by a factor of 2, which means that the conical input can even benefit the diffraction efficiency of the lightguide. However, it also leads to a larger output spot area at the same time, which could result in the mixing of RGB pix. When the azimuthal angle is smaller than 12°, the maximum diffraction angles of RGB light can be completely separated, which
means, with a designed distance or with an assisted micro-lens layer between the grating and the pix layer, the RGB light can reach their corresponding pix. The influence of the conical input on the polarized light is also investigated. The azimuthal angle will result in output light with composite polarization, which, in principle, disables the use of polarized light source in LCD.

2. Theory

The study is based on the rigorous coupled wave analysis (RCWA) [8]. The incident electric field is given by

$$E_{inc} = U \exp[-i(k_{inc}x + k_y y + k_z z)],$$

where $U = (\cos \psi \cos \theta \cos \varphi - \sin \psi \sin \phi)x + (\cos \psi \cos \theta \sin \phi + \sin \psi \cos \phi)y + \cos \psi \sin \theta z$.

Here $\psi$ is the angle between the incident plane and the electric field vector, $\theta$ and $\phi$ as shown in Figure 1. The electrical fields in region 1 and region 2 are

$$E_1 = E_{inc} + \sum_j R_j \exp[-i(k_{yj}x + k_y y - k_{1j}z)] \text{, and } E_2 = \sum_j T_j \exp[-i(k_{yj}x + k_y y - k_{2j}(d - z))],$$

where $k_{yj} = k_0[n_1 \sin \theta \cos \phi - j(\lambda_0 / \Lambda)]$, $k_j = k_0 n_1 \sin \theta \sin \phi$, and

$$k_{1j} = \begin{cases} \left(\frac{(k_{yj}^2 + k_y^2)^{\frac{1}{2}}}{k_{yj}} - k_y^2 \right)^{\frac{1}{2}} & k_{yj}^2 + k_y^2 < k_{0n_1}^2, \\ \left(-\frac{(k_{yj}^2 + k_y^2)^{\frac{1}{2}}}{k_{yj}} - k_y^2 \right)^{\frac{1}{2}} & k_{yj}^2 + k_y^2 > k_{0n_1}^2, \end{cases}$$

$$k_{2j} = \begin{cases} \left(\frac{(k_{yj}^2 + k_y^2)^{\frac{1}{2}}}{k_{yj}} - k_y^2 \right)^{\frac{1}{2}} & k_{yj}^2 + k_y^2 < k_{0n_2}^2, \\ \left(-\frac{(k_{yj}^2 + k_y^2)^{\frac{1}{2}}}{k_{yj}} - k_y^2 \right)^{\frac{1}{2}} & k_{yj}^2 + k_y^2 > k_{0n_2}^2. \end{cases}$$

Here $R_j$ is the normalized vector electric-field amplitude of the j-th backward-diffracted wave in region 1. $T_j$ is the normalized vector electric-field amplitude of the j-th forward-diffracted wave in region 2. We will get the normalized vector electric and magnetic field amplitude of the j-th backward-diffracted and forward-diffracted wave in region 1 and region 2 by Maxwell equation. The normalized vector electric and magnetic field amplitude of the j-th backward-diffracted and forward-diffracted wave in the grating are given by:

$$E = \sum_j \left[ D_{yj}(z)\vec{x} + D_{yj}(z)\vec{y} + D_{zj}(z)\vec{z} \right] \exp[-i(k_{yj}x + k_y y)],$$

and

$$H = -i\sqrt{\varepsilon_0 / \mu_0} \sum_j \left[ C_{yj}(z)\vec{x} + C_{yj}(z)\vec{y} + C_{zj}(z)\vec{z} \right] \exp[-i(k_{yj}x + k_y y)].$$

Here $D_j$ and $C_j$ are the amplitude of electric and magnetic field vector. They satisfy the Maxwell’s equation in the grating region: $\nabla \times E = -i\omega \mu_0 H$, $\nabla \times H = -i\omega \varepsilon_0 E$. Thus a set of coupled-wave equations in a matrix form is obtained:

![Figure 1 Geometry for the grating and the conical incidence.](image)
We can obtain that
\[
\frac{\partial D_x}{\partial (k_0 z)} = (K_x E^{-1} K_x - I) C_x - K_x E^{-1} K_y C_y, \quad \frac{\partial D_y}{\partial (k_0 z)} = K_y E^{-1} K_x C_x + (I - K_x E^{-1} K_y) C_y, \\
\frac{\partial C_x}{\partial (k_0 z)} = (K_x - E) D_x - K_x K_y D_y, \quad \frac{\partial C_y}{\partial (k_0 z)} = K_y K_x D_x - (K_y - E) D_y.
\]

It can be written by
\[
\begin{align*}
\frac{\partial^2 D_x}{\partial (k_0 z)^2} &= \begin{bmatrix} K_x^2 + (K_x E^{-1} K_x - I) E & (E^{-1} K_x E - K_y) K_x \\ (E^{-1} K_x E - K_y) K_y & K_y^2 + (K_x E^{-1} K_x - I) E \end{bmatrix} \frac{D_x}{D_y}, \\
\frac{\partial^2 C_x}{\partial (k_0 z)^2} &= \begin{bmatrix} K_x^2 + E(K_y E^{-1} K_x - I) & (K_y - EK_x E^{-1}) K_x \\ (K_y - EK_x E^{-1}) K_y & K_y^2 + E(K_y - EK_x E^{-1}) \end{bmatrix} \frac{C_x}{C_y}, \\
\end{align*}
\]

where $K_x$ is a constant matrix, then it can be
\[
\begin{align*}
\frac{\partial^2 D_x}{\partial (k_0 z)^2} &= \left( k_x / k_0 \right)^2 I + K_x^2 - E \left[ D_x \right], \\
\frac{\partial^2 D_y}{\partial (k_0 z)^2} &= \left( k_x / k_0 \right)^2 I + (K_x E^{-1} K_x - I) E \left[ D_y \right].
\end{align*}
\]

Thus the electric and the magnetic fields are given by
\[
\begin{align*}
D_j (z) &= \sum_{m=1}^{N} a_{1,j,m} \left\{ -c_{1,m} \exp(-k_0 q_{1,m} z) + t_{1,m} \exp(-k_0 q_{1,m} (z - d)) \right\}, \\
C_j (z) &= \sum_{m=1}^{N} a_{2,j,m} \left\{ c_{2,m} \exp(-k_0 q_{2,m} z) + t_{2,m} \exp(-k_0 q_{2,m} (z - d)) \right\}, \\
C_j (z) &= \sum_{m=1}^{N} g_{21,j,m} \left\{ c_{1,m} \exp(-k_0 q_{1,m} z) + t_{1,m} \exp(k_0 q_{1,m} (z - d)) \right\} \\
&\quad + \sum_{m=1}^{N} g_{22,j,m} \left\{ c_{2,m} \exp(-k_0 q_{2,m} z) + t_{2,m} \exp(k_0 q_{2,m} (z - d)) \right\}, \\
D_j (z) &= \sum_{m=1}^{N} g_{11,j,m} \left\{ -c_{1,m} \exp(-k_0 q_{1,m} z) + t_{1,m} \exp(k_0 q_{1,m} (z - d)) \right\} \\
&\quad + \sum_{m=1}^{N} g_{12,j,m} \left\{ -c_{2,m} \exp(-k_0 q_{2,m} z) + t_{2,m} \exp(k_0 q_{2,m} (z - d)) \right\},
\end{align*}
\]

where $a_{1,j,m}$ and $q_{1,m}$ are the element of the eigenvector matrix and the positive square root of the eigenvalues of the matrix $\left[ k_x I + K_x^2 - E \right], a_{2,j,m}$ and $q_{2,m}$ the element of the eigenvector matrix and the positive square root of the eigenvalues of the matrix $\left[ k_x I + (K_x E^{-1} K_x - I) E \right], c_{1,m}, t_{1,m}, c_{2,m}$ and $t_{2,m}$ is the unknown constant, $g_{11}, g_{12}, g_{21}$ and $g_{22}$ given as
\[
\begin{align*}
g_{11} &= \left[ K_x^2 - E \right]^{-1} W_1 Q_1, \\
g_{12} &= \left( k_x / k_0 \right) \left[ K_x^2 - E \right]^{-1} K_x W_2.
\end{align*}
\]
\[ g_{21} = (k_x / k_0) \begin{bmatrix} K_x E^{-1} K_x - I \end{bmatrix}^{-1} K_x E^{-1} W_1, \quad g_{22} = \begin{bmatrix} K_x E^{-1} K_x - I \end{bmatrix}^{-1} W_2 Q_2. \]

Here \( Q_1 \) and \( Q_2 \) are the diagonal matrices with the diagonal elements \( q_{1,m} \) and \( q_{2,m} \).

At the boundary \( z = 0 \),

\[
\sin \psi j_0 + R_{\\psi,j} = \cos \phi j D_{\\psi,j}(0) - \sin \phi j D_{\\phi,j}(0), \\
i \left[ \sin \psi n, \cos \theta j_0 \right] = \sin \phi j C_{\\psi,j}(0) - \cos \phi j C_{\\phi,j}(0), \\
-\sin \psi n, \cos \theta j_0 = \sin \phi j C_{\\psi,j}(0) - \cos \phi j C_{\\phi,j}(0), \quad \text{and} \\
\cos \phi j \cos \theta - i \left[ k_{1,j} / (k_0 n_1^2) \right] R_{pi} = \cos \phi j D_{\\phi,j}(0) - \sin \phi j D_{\\psi,j}(0). 
\]

It can be written as

\[
\begin{bmatrix} \sin \psi j_0 \\ \sin \psi n, \cos \theta j_0 \\ -\sin \psi n, \cos \theta j_0 \\ \cos \phi j \cos \theta j_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_{g_{11}} & F_{g_{12}} X_1 & F_{g_{12}} - F_{W_2} & \left( F_{g_{12}} - F_{W_2} \right) X_1 \end{bmatrix} c_{1m} \\
F_{g_{21}} + F_{g_{21}} X_1 & -\left( F_{W_1} + F_{g_{21}} \right) X_1 & F_{g_{22}} & -F_{g_{22}} X_1 \\
F_{g_{21}} - F_{W_1} & -\left( F_{g_{21}} - F_{W_1} \right) X_2 & F_{g_{22}} & -F_{g_{22}} X_2 \\
F_{g_{11}} & F_{g_{11}} X_1 & F_{g_{11}} + F_{g_{12}} & \left( F_{W_2} + F_{g_{12}} \right) X_1 \end{bmatrix} c_{2m}.
\]

When \( z = d \)

\[
T_{\\psi,j} = \cos \phi j D_{\\psi,j}(d) - \sin \phi j D_{\\phi,j}(d), \\
R_{\phi,j} = -\sin \phi j C_{\\psi,j}(d) - \cos \phi j C_{\\phi,j}(d), \quad \text{and} \\
T_{\phi,j} = \sin \phi j C_{\\psi,j}(d) - \cos \phi j C_{\\phi,j}(d), \quad \text{and} \\
i \left[ k_{1,j} / (k_0 n_1^2) \right] T_{pi} = \cos \phi j D_{\\phi,j}(0) + \sin \phi j D_{\\psi,j}(0). 
\]

It can be written as

\[
\begin{bmatrix} I & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} F_{g_{11}} X_1 & F_{g_{11}} & \left( F_{g_{12}} - F_{W_2} \right) X_1 & \left( F_{g_{12}} - F_{W_2} \right) X_1 \end{bmatrix} c_{1m} \\
\left( F_{W_1} + F_{g_{21}} \right) X_1 & -\left( F_{W_1} + F_{g_{21}} \right) X_1 & F_{g_{22}} & -F_{g_{22}} X_1 \\
\left( F_{g_{21}} - F_{W_1} \right) X_2 & -F_{g_{22}} X_2 \\
F_{g_{11}} & F_{g_{11}} X_1 & \left( F_{W_2} + F_{g_{12}} \right) X_1 & \left( F_{W_2} + F_{g_{12}} \right) X_1 \end{bmatrix} c_{2m}.
\]

The diffraction efficiency of the grating can be obtained by

\[
DE_{ni} = |R_{\psi,i}|^2 \text{ Re} \left[ k_{1,i} / (k_0 n_1 \cos \theta) \right] + |R_{\phi,i}|^2 \text{ Re} \left[ k_{1,i} / (k_0 n_1 \cos \theta) \right], \quad \text{and} \\
DE_{ni} = |T_{\psi,i}|^2 \text{ Re} \left[ k_{2,i} / (k_0 n_1 \cos \theta) \right] + |T_{\phi,i}|^2 \text{ Re} \left[ k_{2,i} / (k_0 n_2 \cos \theta) \right].
\]

### Simulation and results

The grating adopted in the study has following parameters: grating period \( \Lambda = 400 \) nm, depth \( d = 180 \) nm, fraction of the period \( f = 0.5 \), refractive index \( n = 1.5 \) [9]. The wavelengths of the incident light are: 450nm (B), 550nm (G) and 650nm (R). When incidence angle \( \theta = 66^\circ \) and azimuthal angle \( \phi = 0 \), only -1T order diffraction light can output from the light guide. The diffraction efficiencies for RGB lights are 5.18\%, 4.70\%, and 5.70\%. The diffraction angles are: -14.2 \(^\circ\), 0 \(^\circ\) and 14.8 \(^\circ\), respectively. They get well separated symmetrically. By using the configuration in Ref[4], colour-separating backlight can be implemented for the LCD. However, the light source, like LED, normally has a field angle. This means the light incidences are conical always, which leads to azimuthal angle \( \phi \neq 0 \). In this case, for LED light source, the light incidence angles span a range, rather than a single designed one. Correspondingly, the diffraction light will not output as expected.
The diffraction efficiency as the function of the light incidence angle ($\theta$ and $\phi$) are presented in Figure 2. One could notice that the -1T order diffraction light can output when the incidence angle lies in the bright region. When incident angle $\theta \approx 45^\circ$, with the increasing of azimuthal angle $\phi$, the diffraction efficiency could be enhanced by a factor 2, and the maximum efficiency can be obtained.

In Figure 3, the diffraction efficiencies as the functions of the field angles of the light source are shown. When the light source has a small filed angle, the diffraction efficiency is even larger. This can also be found from Figure 2, since larger azimuthal angle have higher diffraction efficiency. However, when the field angle is larger than a critical angle, where the -1T order diffraction cannot output, the diffraction efficiency decrease quickly. Thus, if one considers diffraction efficiency solely, a light source with moderate field angle is desirable.

However, although the conical incidence can enhance the diffraction efficiency, at the same time, we should consider its influence on the angle of -1T order diffraction light, which determines the spot size.
of the output light. In Figure 4, the influence of the field angle of the light source on the spot size is shown. The incident angle is $\theta = 66^\circ$. Solid, dotted, and dashed lines are for R, G, and B lights. Obviously the normalized spot area enlarges as the field angles increasing, which could results in the mixing of RGB pix. The spot size of the B-light enlarges much fast than those of the R- and G-light as shown in Figure 4.

![Figure 5. The overlap angle between the RGB diffraction light as a function of azimuthal angle.](image)

When $\phi < 12^\circ$, RGB light are completely separated. This means, with a designed distance or with an assisted micro-lens layer between the grating and the pix layer, RGB lights can reach their corresponding pix without mixing. Thus a conical light source, used in the lightguide, must have a field angle smaller than $12^\circ$.

![Figure 6. The influence of the conical input on the polarized light. (a) TE input, (b) TM input: Dotted line(TE), solid line(TM). As the field angle increase, the output light has a composite polarization.](image)

The influence of the conical input on the polarized light is also investigated as in Figure 6. Figure 6a and 6b show the TE and TM linearly polarized inputs. One can find, in both cases, as the field angle of the light source increases, the output light becomes composite polarized, which, in principle, disables the use of polarized light source in LCD. Or the light field of the light source is extremely small.

### 4. Conclusion

By rigorous coupled wave theory, we have studied the influence of conical input on the colour-separating of LCD backlight assisted by diffraction gratings. The results show that, when the azimuthal angle of the light incidence increases, the diffraction efficiency could be enhanced by a factor of 2, which means that the conical input can benefit the diffraction efficiency of the lightguide. However, azimuthal angle also leads to a larger output spot area at the same time. When the azimuthal angle is smaller than $12^\circ$, the maximum diffraction angles of RGB light can be completely separated. This means the conical light source must have a field angle smaller than $12^\circ$. The influence of the
conical input on the polarized light is also investigated. The azimuthal angle will result in output light with composite polarization, which, in principle, disables the use of polarized light source in LCD.

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