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Development of Identical Delivery Quantity Model in Inventory Management with Delayed Payments based on Bank Interest Rates and Murabaha Systems

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Abstract. To compete in international market, companies must produce better quality products with minimum cost. In order to achieve it, one of the supporting factors is a good and efficient inventory management among company and its distributions. For paying system in the inventory management, the companies are using syari’at and conventional scheme. Identical Delivery Quantity (IDQ) model was employed, to calculate minimum inventory cost for both company and its distributors that are mutually coordinated and collaborated. This model was developed by considering four different interest rates in the credit scheme, such as annuities, flat rate, sliding rate, and murabaha. The objective is to calculate total inventory cost for both company and its distributors with the credit schemes as the payment method. Numerical example of a real case was applied in the developed models. It shows that the murabahah is cheaper than another interest rates and it is more applicable in real system to calculate inventory cost with credit scheme as the payment method.

1. Introduction

In the current era of global economy with the rapid information flow, it is undeniable that the level of competition faced by the business and industry is so tight and sensitive. The company must produce in better quality with competitive cost. The common issue nowadays is high inventory cost in company and distributors, especially for the fashion products such as clothes. Hence, those parties require high capital to store the products. Recently, the company realize that the inventory among distributors must have better coordination to reduce cost and lead time without sacrificing customer service [1]. The business operations nowadays has strategy to share marketing and trade credit financing among manufacturers, suppliers, transporters, warehouses, retailers and final consumers in order to increase sales and profits [2]. Usually, the trade credit facilities are employed as capital loan. This credit scheme gives seller for a certain period to buyer without penalty, in this case company or its distributor, to pay the ordered products in order to reduce on-hand inventory and increase profits. This scheme is less desirable in trading because the usage of interest rates that results in imbalance rates between two parties.

A company that works with its distributors often faces problems of inventory on both sides. Therefore, a production and inventory policy, for the types of items to be produced and supplied by the
company, coordinated between the two parties, the objective is to minimize the total combined costs between the company and its distributors [3]. Mishra and Shaikh [4] proposed an integrated inventory model with capacity utilization dependent holding cost to optimize joint profit of supplier and retailer by using MATLAB software. Then, Sarkar et al. [5] developed an integrated inventory model between suppliers and customers with variable lead time, defective units, and delay in payments. Next, Lou and Wang [6] established an integrated inventory model with trade credit financing with permissible delay in payments in order to minimize the costs.

The basic model of Identical Delivery Quantity (IDQ) was introduced by Goyal [7]. This model determines the number of shipments to distributors same on every shipment. Then, this model is extended by Viswanathan [8]. He calculated combined cost for IDQ model. On the other hand, Delivery What is Produced (DWP) model has objective to calculate the total integrated inventory with assumption is the number of shipments are different as developed by Goyal [9]. Next, Shah and Shah [10] and Jamal et al. [11] proposed they continue the Aggarwal and Jaggi models applied to the state of shortage. The shortage has significance part especially in a model that considers delayed payments. If linked to in relation to facts in the field shortage may affect the quantity of the order, which is beneficial for the delay of payment. Pal and Ghosh [12] developed inventory models where pending payments are allowed depends on order quantity.

In this paper, the Identical Delivery Quantity (IDQ) model was employed to calculate minimum inventory cost for both company and its distributors that are mutually coordinated and collaborated. This model was developed by considering conventional and syari’at in the credit scheme, such as annuities, flat rate, sliding rate for conventional payment; and murabaha for syari’at payment. The objective is to calculate total inventory cost for both company and its distributors with the credit schemes as the payment method.

2. Base Model

Mathematical Model of IDQ (Identical Delivery Quantity)

The notations and definitions are as follows.

- \( Z \): total combined cost per year
- \( R \): estimated storage cost of capital invested in percentage (unit / year)
- \( C_v \): manufacturing costs per unit (Rp / unit)
- \( C_b \): purchase cost at distributor per product unit (Rp / unit)
- \( H_v \): inventory cost per unit of product at company per year after interest (Rp / unit)
- \( H_b \): inventory cost per unit of product at distributor per year (Rp / unit)
- \( S \): production cost at company per setup after interest rate (Rp / setup)
- \( A \): distributor's order cost for each shipment (Rp / message)
- \( P \): average production at company per year (unit)
- \( D \): number of orders from distributors per year (unit)
- \( \Gamma = D / P \): comparison between demand and average production
- \( N = 1 / \gamma = P / D \): comparison between production and demand averages
- \( \alpha = A / S \): comparison between order cost and setup cost
- \( \beta = H_b / H_v \): comparison of inventory cost between company and distributor
- \( Q_l \): number of shipments from company to distributor
- \( Q \): number of production in company per production run (unit)
- \( T = Q / D \): time interval between production run (year)
- \( K \): the number of shipments from the distributor in one production.
- \( P_l \): credit plafond (Rp)
- \( S \): production cost at company per set up before interest (Rp)
- \( H_v \): holding cost on the company before interest (Rp)
- \( T_b \): total interest for sliding rate model (Rp)
- \( I \): interest rate per year
- \( N \): term of credit (year)
M : number of payment period (month)
P1 : cost of product (rupiah per unit)
P (M) : variable unit cost when payment is delayed
P2 : expected cost of product desired by seller (rupiah)
P3 : agreed purchase cost (rupiah)
P4 : expected sales cost desired by seller (rupiah)
P5 - P1 : expected profit of buyer (rupiah)
P2 - P1 : expected profit of the seller (rupiah)
P3 - P1 : agreed profit (rupiah)

According to Goyal [7], the annual cost held by the company can be derived as follows:

\[ Z_1 = \frac{S.D}{Q} + \frac{1}{2} D.r.C_s \cdot \frac{Q}{D} \left[ 1 - \frac{D}{P} + \left( \frac{2D - 1}{P} \right) \right] \] (1)

While, the annual cost that held by the distributor can be formulated as follows:

\[ Z_2 = \frac{A.D.k}{Q} + \frac{1}{2} D.r.C_b \cdot \frac{Q}{Dk} \] (2)

Thus, the total combined costs held by firms and distributors, for a given value of T and k is a composite of \( Z_1 \) and \( Z_2 \):

\[ Z(T, k) = Z_1 + Z_2 \] (3)

For a given value k, the economic value of T is the optimal value Z to T. It can be derived as follows:

\[ \frac{dZ}{dT} = 0 \]

\[ T^2 = \frac{2(Ak + S)}{D \left( H_v \left[ 1 - \gamma + \left( \frac{2\gamma - 1}{k} \right) \right] + H_b \left[ \frac{1}{k} \right] \right)} \] (6)

For a certain value k, the optimum value of Z can be derived as follows:

\[ Z(T, k) = \frac{(Ak + S)}{T} + \frac{1}{2} D.T \left( H_v \left[ 1 - \gamma + \left( \frac{2\gamma - 1}{k} \right) \right] + H_b \left[ \frac{1}{k} \right] \right) \] (7)

where, T is same as equation (6). Thus the optimal Z value is:

\[ Z(k) = \frac{(Ak + S)}{\sqrt{2(Ak + S)}} \] (8)

It also can expressed as:

\[ Z(k) = \sqrt{2DSH_v} \left( (ek + 1) \left[ 1 - \gamma + \left( \frac{2\gamma - 1 + \beta}{k} \right) \right] \right) \] (9)

The optimum value k, say as \( k_1 \), can be found by minimizing \( Z_2 (k) \) from equation (9), as follows:
\[ Z^2(k) = 2D.S.H_v \left[ a_k(1 - \gamma) + (1 - \gamma) + \alpha \frac{k^{2\gamma - 1 + \beta}}{k} \right] \] (10)

After ignoring the independent variables and constants of \( k \), the minimization problem can be simplified into:

\[ Z^2(k) = a_k(1 - \gamma) + \frac{2\gamma - 1 + \beta}{k} \] (11)

The economic value of \( k = k_1 \) is obtained if:

\[ Z^2(k) = a_k(1 - \gamma) + \frac{2\gamma - 1 + \beta}{k} \] (12)

\[ Z^2(k_1) \leq Z^2(k_1 - 1) \] (13)

And

\[ Z^2(k_1) \leq Z^2(k_1 + 1) \] (14)

By substituting equations (13) and (14) into equation (12), it obtains:

\[ k_1(k_1 - 1) \leq \frac{2\gamma - 1 + \beta}{(1 - \gamma)\alpha} \] (15)

And

\[ k_1(k_1 + 1) \geq \frac{2\gamma - 1 + \beta}{(1 - \gamma)\alpha} \] (16)

Then, by combining equations (15) and (16), it will be obtained as follows:

\[ k_1(k_1 + 1) \leq \frac{2\gamma - 1 + \beta}{(1 - \gamma)\alpha} \leq k_1(k_1 + 1) \] (17)

Then the optimal value of total combined costs for IDQ strategies is:

\[ Z^*(IDQ) = \sqrt{2DSH_v} \left[ (ak + 1) \left\{ 1 - \gamma + \frac{2\gamma - 1 + \beta}{k} \right\} \right] \] (18)

Based on the equation above, there are some variables that associated with the cost, such as: \( S \) is the production cost in the company per setup (Rp/setup), and \( H_v \) is the inventory cost per unit of product at the company per year (Rp/unit).

### 3. Proposed Model

#### 3.1 Flat rate Calculations

The interest calculation is based on credit ceiling and the amount of interest that is charged proportionally in accordance with the loan period. By using this calculation, the amount of principal payments and loan interest each month is the same [13]. For each variables that related to interest, the total cost is the amount of required cost (i.e. \( S \) and \( H_v \) before interest) plus the amount of interest that must be paid. For instance, the production cost in the company per setup before interest is:

\[ S = s + tb \] (19)

\[ tb = Pl \times i \times n \] (20)

where, \( tb \) is the total interest for flat rate calculations.

Similarly for \( H_v \), for example the inventory cost in the company before interest is \( hv \). It can be formulated as:

\[ H_v = h_v + tb \] (21)
By assuming all required cost is the cost from the bank, hence the S and Hv values are equal to Pl. Then, by substituting equations (19) and (21) in equation (18), the total combined cost for the IDQ model with the addition factors of the flat rate model calculation:

\[ Z^*(IDQ) = \sqrt{2D(s + (s \times i \times n))(h_i + (h_i \times i \times n)) \left( (ak + 1) \left\{ 1 - \gamma + \left( \frac{2\gamma - 1 + \beta}{k} \right) \right\} ^2 \right)} \]  

(22)

### 3.2 Annuity Calculations

The monthly instalments paid by the debtor have not changed during the loan period. However, the composition of the major instalments and interest instalments each month will change where the interest instalment will decrease while the principal instalment will be more enlarged. Monthly instalments in the annuity model calculation are formulated as follows [13].

Monthly instalments = \( Pl \times \frac{i}{12} \times \frac{1}{1 - \left( \frac{1 + \frac{i}{12}}{m} \right)} \)  

(23)

The total amount paid is the monthly instalment payable a number of payment periods agreed upon, then the total amount to be paid in each loan may be written below:

Total Instalments = \( m \times Pl \times \frac{i}{12} \times \frac{1}{1 - \left( \frac{1 + \frac{i}{12}}{m} \right)} \)  

(24)

With the same assumption and notation, it can be written as:

\[ S = m \times s \times \frac{i}{12} \times \frac{1}{1 - \left( \frac{1 + \frac{i}{12}}{m} \right)} \]  

(25)

and

\[ H_v = m \times h_v \times \frac{i}{12} \times \frac{1}{1 - \left( \frac{1 + \frac{i}{12}}{m} \right)} \]  

(26)

By substituting equations (25) and (26) in equation (18), the total combined cost for the IDQ model with the addition of the annuity model calculation factor is:

\[ Z^*(IDQ) = \sqrt{2D \times s \times h_i \times \left( m \times \frac{i}{12} \times \frac{1}{1 - \left( \frac{1 + \frac{i}{12}}{m} \right)} \right)^2 \left\{ (ak + 1) \left\{ 1 - \gamma + \left( \frac{2\gamma - 1 + \beta}{k} \right) \right\} ^2 \right\}} \]  

(27)

### 3.3 Sliding rate Calculations

The interest is calculated at the end of instalment payment period. The loan interest is calculated from the final balance of each month (debit balance) so that the interest paid by debtor every month is decrease. Hence, the amount of instalment paid by debtor becomes smaller [13]. The formula to calculate the interest per month is \( \text{Final Balance} \times \frac{i}{12} \)  

From the above description, total interest can logically formulated by the following formula:

\[ \sum_{n=1}^{m} \left( \text{Principal} \times n \right) \times \frac{i}{12} \]  

dengan n = 1,2,3,...

and \( \text{Principal} = \frac{Pl}{m} \)

With the same assumption and notation, it can be written as follows:

\[ S = s + \text{total interest } s \]  

(28)

And

\[ H_v = h_v + \text{total interest } h_v \]  

(29)

By substituting equations (28) and (29) in equation (18), the total combined cost for the IDQ model with the addition factor of the sliding rate calculation is:
\[ Z'(IDQ) = \sqrt{2D(s + \text{total interest } s)(h_r + \text{total interest } h_r)} \left( \alpha k + 1 \right) \left[ 1 - \gamma + \left( \frac{2\gamma - 1 + \beta}{k} \right) \right] \]  

(30)

3.4 Murabaha Calculations

In a transaction murabaha payment is done with negotiation of profit from both sides, which is owed by the customer is the agreed cost, i.e. the acquisition cost plus the agreed profits. If customer gives an advance in the sale and purchase transaction murabahah, then the customer debt is the selling cost minus the down payment. Customer debt is not related to other transactions. This is stated in the fatwa of the National Sharia Council Number 4/DSN-MUI/IV/2000 on 1 April 2000 [14]. The murabaha trading flow is shown in Figure 1. The uncertainty of interest value during the planning period employs the interest system. To avoid any uncertainty in the value of interest, with reference to the same assumption, this study will change the system of interest usage into profit value agreed by both parties, the seller and the buyer.

\[ P_3 - P_1 = \frac{P_3 - P_1}{P_2 - P_1} = \frac{P_4 - P_1}{P_4 - P_1} \]  

(31)

\[ P_3 = \frac{P_4^2 - P_1^2 - P_1(P_4 - P_2)}{P_2^2 - 2P_1 P_3 + P_4} \]  

(32)
Similarly, in the IDQ model if $S$ and $Hv$ are performed with murabaha transactions (with the same assumption and the same notation) then it can be written:

$$S = s + ks = P_3$$ for $S$  \hspace{1cm} (33)

with $ks$ and $khv = $ benefits agreed from $S$ and $Hv$, and

$$Hv = hv + khv = P_1$$ for $Hv$  \hspace{1cm} (34)

By substituting equations (33) and equation (34) in equation (18) then the total combined costs for the IDQ model with the addition factor of murabaha calculation are:

$$Z^*(IDQ) = \sqrt{2D(s + ks)(h_v + khv)} \left(\alpha k + 1\right) \left(1 - \gamma + \frac{2\gamma - 1 + \beta}{k}\right)$$

(35)

4. Case Study

This research is based on a real case study in XYZ company that produce Muslim clothes in Yogyakarta and ABC company as a distributor of Muslim clothes in Yogyakarta. The data was collected from both companies, such as number of requests, production cost, inventory cost, total shipments, total interest $s$ for model sliding rate $tb$, total interest $hv$ for model sliding rate $tb$, interest rate per year, loan period, number of period payment, agreed profits, costs as shown in Table 1.

| Table 1. The Data                        |
|------------------------------------------|
| Number of requests (D)  | 10,722 units / year |
| Production cost (S)      | Rp 15,500,000 / setup |
| Inventory cost (Hv)      | Rp 7,000 / unit |
| Total shipments (k)      | 2 times |
| Total interest $s$ for model sliding rate $tb$ | Rp 1,259,375 |
| Total interest $hv$ for model sliding rate $tb$ | Rp 568.75 |
| Interest rate per year (i)  | 15% |
| Loan period              | 1 year |
| Number of period payment (m) | 12 months |
| Agreed profits (ks)      | Rp 500,000 |
| Agreed profits (khv)     | Rp 250,000 |
| $A$                      | 0.7 |
| $B$                      | 1.48 |
| $\Gamma$                 | 0.68 |
5. Result and Discussion
After practicing all the new models with numerical example can be obtained that by this new model is easier for user in calculating total cost or the value of \( Z^*(IDQ) \) of inventory which with delayed payment, and then get murabaha system resulted the lowest total cost compared to others after calculating with Microsoft Excel according to Table 2. The concept of murabaha produces lower costs because in the concept both parties are given the same opportunity to make an agreement on the value of interest or it can be interpreted that the concept of murabaha has the principle of openness:

|                | Flat rate         | Sliding rate     | Murabahah       |
|----------------|-------------------|------------------|-----------------|
| \( Z^*(IDQ) \) | Rp 95.693.419,00  | Rp 90.126.537,00 | Rp 86.039.594,00 |

6. Conclusion
The difference in the interest calculation system affects the results of the total price expected by both the company and its distributors. This is influenced by the prevailing banking rules and policies of each bank. Development of the \( Z^* \) IDQ model is influenced by the selection of the interest calculation system. Based on the calculations carried out the calculation model with the Murabaha principle is the best among the other models, this is because the Murabaha principle accommodates negotiations between the two parties. This means that the principle of Shari'a can be said to be more acceptable than conventional models. From the calculations performed from the cases raised, it shows that Murabaha has a difference of 3.3% - 8.5% cheaper than the others. The \( Z^* \) IDQ model will be easier to implement if the company and its distributor are a group of companies because it will facilitate coordination in pricing and related costs.

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