A hybrid control strategy based on CSA and ELM for uncertain nonlinear chaotic system

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Abstract
The study of nonlinear chaotic systems and their control is an important topic. In this paper, a hybrid control strategy based on cuckoo search algorithm and extreme learning machine is proposed. Cuckoo search algorithm is used in a hybrid control strategy in order to optimise the weights and biases in extreme learning machine leading to the improvement of its performance. Simulations indicate that the proposed method is able to fit nonlinear chaotic systems and control chaotic systems effectively. Data used in the nonlinear chaotic system are also tested for uncertainty and unknown systems. Simulation results confirm that the proposed method shows robustness for noisy data and perturbed parameters.

Keywords
Chaotic system, cuckoo search algorithm, extreme learning machine, control strategy, map

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Introduction
As a special nonlinear phenomenon, chaos has received much more attention since first proposed by Lorenz in 1963.¹ For unpredictability and sensitivity to initial condition, chaos has been excessively applied in various areas with satisfactory results, such as image encryption.²⁻⁴ However, in many electric systems and industries, chaos is severely unexpected for its complex characteristics. Hence, it is vital to design the control strategy in nonlinear system. Many methods have been researched and applied by domestic and abroad scholars in past decades, including adaptive control (AC), output feedback control (OFC), and sliding mode control (SMF) etc. Li⁵ designed an adaptive track controller to control chaos in fractional-order chaotic systems. Simulation revealed that the strategy has same performance in chaotic system with uncertain parameters and external perturbation. The idea of OFC is that by regarding output data as feedback variables, controller can be added into chaos system at any time.⁶ Unfortunately, the equilibrium points of chaotic system can be confirmed only if its mathematical model is known.⁶ Shen and Li⁷ introduced the discrete rippling sliding mode control (DSMC) to stabilize the continuous unified chaotic system by discretization of continuous chaotic systems. However, the premise matrices A and B have been known, that is, parameters of system should be known as prior information. Although the above methods have resolved the issue of controlling or restraining chaotic systems, the precondition is that the mathematical model of chaotic system has been known evidently. However, it is difficult or even impossible to achieve the accurate models for most of the chaos in actual systems or industries. For uncertain chaotic systems, neural network (NN) based on radial basis function (RBF) is an effective approach that can restrain chaos.⁸,⁹ Even so, the drawback of slow convergence and premature convergence in NN may limit the whole performance.

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In this paper, a hybrid method based on cuckoo search algorithm (CSA) and extreme learning machine (ELM) is explained in detail. The structure of the paper is as follows. The next section provides a brief overview of the existing methods. The subsequent section describes the proposed control strategy. Then contains the results of some numerical simulations to cover different chaotic systems, referencing signals, noise disturbances and perturbed parameters. Finally, the paper is concluded with conclusions.

**The proposed control strategy based on CSA and ELM**

**Problem formulation**

As a special nonlinear phenomenon, chaos with sensibility and unpredictability appears when simple conditions are satisfied, which would cause damage for systems. For an uncertain nonlinear single input and single output chaotic system, it can be described as follows

\[ i(k + 1) = \phi(i(k), o) \]  

where \( i \in \mathbb{R} \) is the system state variable, \( o \) is the system parameter, and \( \phi \) is the system function.

According to the above equation, outputs of nonlinear chaotic system are offset and target signals are traced indirectly if proper controller exists. As shown in Figure 1, a modified system with controller is described as follows

\[ i(k + 1) = \phi(i(k), o) + \Psi(k) \]  

where \( \Psi \) is the control section which is defined as

\[ \Psi(k) = -\hat{\phi}(i(k), o) + e^{\text{rand}}i_r(k + 1) + \text{rand} * (i(k) - i_r(k)) \]  

where \( \hat{\phi}(i(k), o) \) is the fitting function of nonlinear chaotic system by CSA-ELM, \( i_r \) is the reference signal and \( \text{rand} \) is the random parameter in (0,1).

Assuming that the mathematical model of chaotic system is not known explicitly, so the target chaotic system is uncertain, its data are obtained easily and ELM can fit with the excellent fitting performance which can be depicted as

\[ i(k + 1) = \phi(i(k), o) \]  

The residual can be expressed as

\[ e(k) = i(k) - i_r(k) \]  

Since \( k \) is relatively large \( e^{\text{rand}}i_r(k + 1) \rightarrow i_r(k + 1) \), then

\[ e(k + 1) = i(k + 1) - i_r(k + 1) = \phi(i(k), o) - \phi(i(k), o) + e^{\text{rand}}i_r(k + 1) + \text{rand} * (i(k) - i_r(k)) - i_r(k + 1) \]  

If fitting function can match nonlinear system with high accuracy, equation (6) can be replaced by

\[ e(k + 1) = \text{rand} * (i(t) - i_r(k)) = \text{rand} * e(k) \]  

According to the system stability theory, with the constraint of \( |\text{rand}| < 1 \), the system is convergent and stable, and then the modified nonlinear system can gradually approach toward stability and the reference signal is traced.

**Basic theories in related algorithms**

As reviewed in section ‘Problem formulation’, the modified nonlinear system with controller can work for restrained chaos effectively only if the fitting function matches the nonlinear system with high accuracy.

ELM, as classifier and predictor, is a very useful and efficient tool. Proposed by Huang et al. in 2005,\(^1\) it has received growing attention with various application in many industrial processes, such as image processing,\(^11,12\) data prediction,\(^13\) fault classification,\(^14\) etc. ELM is single-hidden layer feedforward network, which can be expressed as follows

\[ \mathbf{NB} = \mathbf{\Xi} \]  

where \( \mathbf{N} \), \( \mathbf{B} \) and \( \mathbf{\Xi} \) are defined as follows

\[ \mathbf{N} = \begin{bmatrix} \rho(0, 1, i_1) & \rho(0, 2, \sigma_2, i_1) & \cdots & \rho(0, N, \sigma_N, i_1) \\ \vdots & \ddots & \ddots & \vdots \\ \rho(0, 1, i_1) & \rho(0, 2, \sigma_2, i_N) & \cdots & \rho(0, N, \sigma_N, i_N) \end{bmatrix}_{N \times N} \]
In the above formulas, \( i_t \), \( \tau_t \) and \( \rho(\cdot) \) are the input data, output data and activation function, respectively. \( \Xi \) are confirmed if the weight coefficient \( \omega \) and bias \( \sigma \) between the input layer and hidden layer are chosen.

In the hybrid control strategy, ELM will be adopted to fit uncertain nonlinear chaos systems. However, the method of choosing its weight coefficient and bias randomly in traditional mode would limit its performance. Motivating by optimization algorithm, the challenge can be overcome by using intelligent algorithm. In this paper, ELM is optimized by known swarm intelligent algorithm, specifically the algorithm called CSA.

Emulating the cuckoo breeding behavior, cuckoo search algorithm (CSA) is the novel optimization algorithm established by Yang.\(^{15}\) Owing to its excellent ability in optimization, it has been successfully applied for a wide variety of optimization problems,\(^ {16} \) such as facility layout design\(^ {17} \) and aircraft control.\(^ {18} \) The fundamental equations in CSA are

\[
\begin{align*}
\nu^\mu_{i+1} &= \nu^\mu_i + s \otimes \text{Levy}(\tilde{\alpha}) \\
s &= \lambda \otimes (\nu^\mu_i - \chi^\mu_i) \\
\text{Levy}(\tilde{\alpha}) &= \frac{\sigma}{|\omega|^2}
\end{align*}
\]

where \( \nu^\mu_i \) and \( \nu^\mu_{i+1} \) are the \( \mu \)-th iteration, the \( \nu \)-th and \( \tau \)-th nest, that is, two different potential solutions, \( s \) is the amplification parameter, \( \lambda \) is the scaling factor, \( \otimes \) shows entry-wise multiplications, \( \text{Levy}(\tilde{\alpha}) \) is the step length generated according to the Mantegna’s algorithm, and \( \tilde{\alpha} \) is the Lévy flights exponent. Also, \( \sigma \) and \( \omega \) are two random values from normal distribution with zero means.

When \( pa < \psi \), the new solution is produced by local random walk which can be written as

\[
\nu^\mu_{i+1} = \nu^\mu_i + \psi \otimes H(pa - \delta) \otimes (\nu^\mu_i - \nu^\mu_{i+1})
\]

where \( H(\cdot) \) is the Heaviside function, \( \psi \) and \( \delta \) are two random numbers with a uniform distribution, and the term in the second bracket corresponds to the difference of two randomly solutions.

**Hybrid control strategy based on CSA and ELM**

According to the above interpretation, the hybrid control strategy based on CSA and ELM is shown in Figure 2. As the figure revealed, the related parameters are: \( i_t \) is the reference signal; \( m \) is the data of chaotic system used to train and test ELM; \( k \) is the current number of chaotic data; \( \zeta, N, h \), and \( \lambda \) are the number of input data, the number of hidden neurons, the weight and bias between the input layer and hidden layer in ELM, respectively; \( \xi, n, \lambda, \delta, \Upsilon \) and \( \psi \) are the parameters of nests, the maximum iterations, the step size, the random parameter, reference accuracy and discovered probability in CSA; similarly, \( r \) is the current accuracy.

Assuming the mathematical model of nonlinear chaotic system is unknown means its mathematical model is uncertain. In order to obtain fitting function of nonlinear uncertain chaotic system, vast amount of data is used in advance to train and test ELM. When the fitting function matches the actual system with high accuracy, outputs of system are offset, and the target signal is traced. As the schematic flowchart illustrated, the performance is determined by fitting accuracy. In our proposed strategy, CSA is employed into the hybrid strategy to improve the performance of ELM by optimizing the weights and biases in ELM, although ELM itself has satisfying fitting performance for nonlinear system.

The proposed hybrid control strategy can be summarized as follows

- **Step 1:** Initialize reference signal \( i_t \) and the number of training data \( m \) in nonlinear chaotic system.
- **Step 2:** Obtain training data \( i(k) \) for ELM.
- **Step 3:** Initialize the number of hidden neurons between the input layer and hidden layer, and establish the framework for ELM.
- **Step 4:** Initialize the number of nests \( \xi \), the maximum iterations \( n \) and the value of \( \lambda, \delta, \Upsilon \) and \( \psi \).
- **Step 5:** Determine the potential optimal parameter and generate the initial locations of nests.
- **Step 6:** Calculate the fitness value and update the global best location in nests.
- **Step 7:** Update locations of all nests, obtain fitness value and replace the worst nest.
- **Step 8:** Judge host bird has found cuckoo’s egg or not. If so, replace this nest with new one.
- **Step 9:** If \( r \) is equal or small to \( \Upsilon \), turn to Step 10. If not, \( k = k + 1 \), and back to Step 6.
- **Step 10:** Output \( h_{\text{opt}} \) and \( \lambda_{\text{opt}} \). And obtain fitting function \( \phi(i(k), \alpha) \) of chaotic system with optimal ELM.
- **Step 11:** Test chaotic system with controller.

**The simulation and discussion**

In order to test the performance of our proposed control strategy, cubic chaotic system with four various...
target signals is demonstrated as test nonlinear system. Then, the four various chaotic systems with same target signals are tested to verify the universality of proposed control strategy. Finally, the logistic chaotic system with noise disturbance and parameter disturbance is also tested to verify the robustness of control strategy.

**Different reference signals**
Cubic chaotic map can be defined as follows

\[ i(k + 1) = \ell t(k)(1 - t(k)^2), t(k) \in (0, 1) \] (13)

Correspondingly, the fitting function with CSA-ELM is

\[ \hat{i}(k + 1) = \hat{\phi}(i(k)) \] (14)

The controller can be designed as

\[ \Psi(k) = -\hat{\phi}(i(k)) + e^{m_1}t_i(k + 1) + \text{rand} \cdot (i(k) - t_i(k)) \] (15)

Finally, the modified nonlinear system can be established as

\[ i(k + 1) = \ell t_i(k)(1 - t_i(k)^2) + \Psi(k) \] (16)

In the modified nonlinear system, \( \ell = 2.59 \) and \( m = 500 \). And the controller will be added into the system when \( k = 500 \), and 300 of data is used to train ELM and the rest is used to test the fitting function. The size of nests \( \zeta \), the maximum iterations \( n \), the step size \( \lambda \), the random parameter \( \delta \), the reference accuracy \( \gamma \) and the discovered probability \( \psi \) are set as 10, 100, 0.01, 0.5, 1.0 \times 10^{-5} \text{ and } 0.4 \text{ in CSA, respectively; } \text{The number of hidden neurons is set as } 10; \text{ The four target signals are constant signals with an amplitude of } 0.5, \text{ and the sinusoidal wave, sawtooth wave and triangular wave have an amplitude range of } 0.4–0.6.

All the test data of ELM with various target signals and outputs of system are shown clearly in Figure 3. As the results showed, the test data match with the predicted data by the optimal ELM, and the outputs of modified nonlinear system are same to define various target signals. Apparently, the chaotic phenomenon is controlled and different target signals are also traced effectively which verify that the proposed control strategy is suitable for various target signals.

**Different chaotic systems**
In order to verify commonality of our proposed control strategy, another four various chaotic maps, Logistic map, Chebyshev map, Tent map and Sinusodial map, are tested with two typical target signals, sawtooth wave and triangular wave.

1. **Logistic map**

   Introduced by Robert May in 1976, the logistic map would lead to chaotic dynamics. Logistic map generates chaotic sequences in \((0, 1)\). It can be defined as follows

   \[ i(k + 1) = \ell t_i(k)(1 - i(k)) \] (17)

2. **Chebyshev map**

   Chebyshev chaotic is a classical chaotic map applied in machine learning and image recognition. It can generate chaotic phenomenon in \((-1, 1)\). The definition of Chebyshev map can be described as follows

   \[ i(k + 1) = \cos(\ell \cos^{-1} i(k)), \ell > 0 \] (18)

3. **Tent map**

   Similarly, tent map can generate chaotic phenomenon in \((0, 1)\). Its formula is

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**Figure 2.** The structure of hybrid algorithm based on CSA-ELM.
The typical behaviors of four chaotic maps are shown in Figure 4. In Logistic map, the initial value is 0.2 and $\ell=4$, and the behavior of the present chaos is shown in Figure 4(a). Figure 4(b) shows the result of Chebyshev map with $\ell=5$. For divided-interval, the tent chaotic map is a little different from the other; however, it is still a classical chaos as shown in Figure 4(c). Figure 4(d) shows the result of Sinusoidal chaotic map with an initial value of 0.3. For same sawtooth signal, all the results of four chaotic maps are shown in Figure 5. As is showed, the Logistic chaotic map, Chebyshev chaotic map and Sinusoidal chaotic map can be controlled by ELM with CSA. As expected, the target signal has been traced precisely by our proposed control scheme. It proved that the proposed approach to control nonlinear chaotic system has robustness for parameter perturbation.

**Noise disturbance**

Although unable to avoid, it is challenging to reduce the effect of noise and disturbance. Various control approaches have been proposed by researchers to resolve denoising problem, such as wavelet transform (WT). In this sub-section, noise disturbance is added into nonlinear system to verify the proposed modified control scheme.

Similarly, assuming the nonlinear system is Sinusodial chaotic map, that is

$$i(k + 1) = \sin(\pi i(k))$$

(25)

Then the fitting function by CSA-ELM is given as

$$i(k + 1) = \hat{\Phi}(i(k))$$

(26)

So the controller is

$$\Psi(k) = -\hat{\Phi}(i(k)) + e^{\ell_1}i_r(k + 1) + \text{rand} \times (i(k) - i_r(k))$$

(27)

Finally, the modified nonlinear system is

$$i(k + 1) = \sin(\pi i(k)) + \Psi(k)$$

(28)

Assuming the data sampled from nonlinear system have been mixed by random noise, which amplitude is 0.01, then CSA-ELM is trained with the same other parameters mentioned in sub-section ‘Different chaotic
Figure 3. The test data of ELM and outputs of system with different target signals. (a) The test data of ELM with target signal 1. (b) The outputs of system with target signal 1. (c) The test data of ELM with target signal 2. (d) The outputs of system with target signal 2. (e) The test data of ELM with target signal 3. (f) The outputs of system with target signal 3. (g) The test data of ELM with target signal 4. (h) The outputs of system with target signal 4.
The target signal includes sawtooth wave and triangular wave. The reference accuracy is set to $1.0 \times 10^{-5}$ for the two tested signals. All the results are shown in Figure 7. As the results revealed, nonlinear system is still fitted by optimal ELM with high accuracy, and the two tested signals, sawtooth wave and triangular wave, are traced satisfactorily, although noise exists in nonlinear system which also proved that the proposed method for nonlinear chaotic system has robustness for noise perturbation.

**Comparison with the state-of-the-art algorithms**

In this paper, since CSA has the advantages of simple structure, high computational efficiency and strong reliability, it is natural to apply CSA to optimize ELM. However, after a lot of experiments, it is found that if the classical algorithm POS, FA, etc. are used to optimize the ELM, then almost the same control results will be obtained. Result shown in Figure 8 leads to the advantage that our proposed method is versatile.

**Conclusion**

Chaos control is a significant issue for integrated circuit system which ensures efficient work for important places, such as satellite communication, industrial power and aeronautics. Taking chaotic system into consideration, especially nonlinear uncertain chaotic system, in this paper, a hybrid control strategy based on CSA and ELM has been proposed. A modified controller is added into the nonlinear system and if fitting function matches the nonlinear system, outputs of system are offset, then the target signal is traced with high accuracy. In the test section, different chaotic systems, different target signals, parameter perturbation and noise disturbance in nonlinear chaotic system have been tested to verify the performance of our control strategy. Simulation results indicate that the nonlinear chaotic system is matched satisfactorily by the optimal CSA-ELM and the target signals are traced well. In conclusion, the proposed method has commonality which can be suitable for various and uncertain chaotic system, and also has robustness which can resist parameter perturbation and noise disturbance in a way.

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Figure 5. Fitness value curves and outputs of system for four chaotic maps. (a) The test data of ELM with chaotic map 1. (b) The outputs of system with chaotic map 1. (c) The test data of ELM with chaotic map 2. (d) The outputs of system with chaotic map 2. (e) The test data of ELM with chaotic map 3. (f) The outputs of system with chaotic map 3. (g) The test data of ELM with chaotic map 4. (h) The outputs of system with chaotic map 4.
Figure 6. The test data of ELM and outputs of system with parameter perturbation. (a) The test data of ELM with target signal 3. (b) The outputs of system with target signal 3. (c) The test data of ELM with target signal 4. (d) The outputs of system with target signal 4.

Figure 7. Fitness value curves and outputs of system with noise disturbance. (a) The test data of ELM with target signal 3. (b) The outputs of system with target signal 3. (c) The test data of ELM with target signal 4. (d) The outputs of system with target signal 4.
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![Figure 8. Compare results. (a) CSA and FA optimized ELM comparison. (b) CSA and POS optimized ELM comparison.](image-url)