Definitions of Natural-Language Spatial Relations:
Combining Topology and Directions

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ABSTRACT Because SQL for querying data from spatial databases is ineffective, the query based on natural or visual language becomes an attractive research field gradually. However, how to define and represent natural languages related to spatial data are still gigantic problems. Because existing models of direction relations can't describe by use of some common concepts. First of all, detailed direction relations are proposed to describe the directions related to the interior of spatial objects, such as “east part of a region”, “east boundary of a region”, and so on. Secondly, by integrating the detailed directions with exterior direction relations and topological relations, several NLSRs are defined, such as “a road goes across the east part of a lake”, “a river goes along the east boundary of a province”, etc. Finally, based on the NLSRs abovementioned, a natural spatial query language (NSQL) is formed to retrieve data from spatial databases.

KEYWORDS geographic information system; topological relations; direction relations; detailed direction relations; natural-language spatial relations

CLC NUMBER TP391; TP181

Introduction

In many applications (CAD and geographical information system, GIS), data are managed by spatial databases, which store point, line, region objects and spatial relations between them. Spatial relations (topological relations, direction relations and approximate distance relations) not only are helpful to organize and manage spatial data, but also play an important role in spatial query language [1], image retrieval based on context [2], image understanding based on features and matching between spatial data sets, etc. Therefore, spatial relations become attractive research fields of image understanding, qualitative spatial reasoning and GIS, etc. People use natural language to communicate with each other, when these languages are associated with spatial relations, we call them natural-language spatial relations (NLSRs), which are on more high level and constructed from basic spatial relations, and easier to be exploited by people than spatial relations.

Chang et al. use 2D-string to express spatial relations between image objects and retrieve images [3]. In theory, however, 2D-string exploits two 1D strings to represent relations in high dimension space, which leads to that reliability and completeness of spatial relations can not be guaranteed and detailed information can not be measured any more [4]. As a result, 2D-string is unsuitable to represent NLSRs. Egenhofer [5] and Mark [6] et al. only discuss the association between qualitative spatial relations and spatial recognition, but not NLSRs. Frank proposes a projection-based method to formalize direction relations [7], which uses eight words—east, west, north, south, northwest, northeast, southwest and southeast to express direction relations around the reference object. But the projection-based method can not describe some direction...
concepts which inside the reference object, such as "east part of a region" and "west boundary of a region", etc. Mark and Egenhofer discuss how to integrate topological relations into definitions of NLSRs\cite{8}, and the influences of the metric information of topological relations on NLSRs are investigated in References \cite{9} and \cite{10}. However the roles of direction relations are not taken into account in their definitions.

In fact, many NLSRs are the combinations of multiple kinds of spatial relations. For example, in the sentence, "the Yellow River goes across east part of Gansu Province", there are two terms: one is direction relation—"east part", and the other is topological relation—"go across". Only the combination of topological relations or direction relations can not describe the meanings of this kind of NLSRs. On the basis of a new kind of direction relation—detailed direction relations, some NLSRs are defined by combining direction relations and topological relations, and then integrated into expanded SQL.

1 Spatial relations

1.1 Topological relations

Topological relations denote those unchanged characteristics under the topological transforms, such as translation, rotation and scaling, etc. According to 9-intersection model which is based on point-set topology\cite{5}, a spatial object A can be decomposed into three parts: the interior ($A^\circ$), the boundary ($\partial A$) and the exterior ($A^\complement$). The topological relations between objects A and B are determined by a 3 X 3 matrix formed by intersection of three parts of A and B.

Mark and Egenhofer have performed a paper test for the line/region topological relations discerned by 9-intersection model, and the results indicate that the types and the number of topological relations decided by 9-intersection model are logical and complete. The 9-intersection model can decide 512 topological relations in theory, but most of them are no sense except 19 relations. Fig. 1 only lists four relations, which represent the following meanings “a line goes across a region”, “a line is inside a region”, “a line goes along the boundary of a region" and “a line enters a region” respectively.

1.2 Direction relations

There are many models for describing direction relations, such as projection-based, cone-based, 2D-string and direction-relation matrix\cite{11}, etc. In general, all of these models use the approximate shapes of the reference and the target object to compute direction relations, so the results are not always accurate. Because the direction-relation matrix not only considers the sizes and the shapes of spatial objects, but also is more cognitively approximate with those concepts people used than other models, the direction-relation matrix is considered as a popular model.

The space regions occupied by cardinal directions, such as NW, N, NE, E, SE, S, SW, W and O, can be constructed through rectangle partition around the reference object (Fig. 2). Direction O means that the reference object is coincide with the minimum bounding rectangle (MBR) of the reference object. The direction relation matrix uses a 3 X 3 matrix to describe direction relations. If a target object intersects with a direction region, then the corresponding element in direction-relation matrix is non-empty, otherwise, empty. Symbol $E(A, B)$ means that object B is in the east to object A, other symbols, $W(A, B)$, $S(A, B)$ and $N(A, B)$, have similar meanings. The direction relations between any two objects can be represented as a set composed of cardinal directions that the target object falls inside. For example, the direction relation between A and B in Fig. 2 can be represented as $R(A, B) = \{N, NE, E, SE\}$. In addition, direction relations are granular in semantic resolution, so direction relations constructed from eight cardinal directions can be generalized into complex direction formed from four cardinal directions. For example, cardinal direction N in four directions system can be constructed by combining NW, N and NE cardinal directions in eight directions system.
2 Definitions of natural-language spatial relations

2.1 Detailed direction relations

Although direction relation matrix takes into account the sizes of reference and target objects and the shape of target objects, it still can not describe directions about the interior and boundary of a region, such as “east boundary of a region”, “east-central part of a region”, etc. Direction relation matrix only represents these concepts such as O direction, which is not logical clearly.

The O cardinal direction results in that some direction relations can not be discernable. The five combined spatial relations in Fig. 3 are identical according to direction relation matrix and topological relations. The reason lies in that all of them can be represented by a direction relation-(NE, E, SE, O), and a topological relation “go across”. However, these situations are inconsistent with people’s recognition. The membership degree of the first relation in Fig. 3 belonging to concept “a line goes across east part of a region” is larger than others’, the fourth and fifth relation even can not be expressed by the sentence to some extent. These differences between existing combined spatial relations and people’s recognition bring difficulties on retrieving spatial data in terms of natural language or expanded SQL based on spatial relations, even result in the return results which are not expected for users. Accordingly, it is necessary to take into account and resolve the problems led by O direction.

As showed in Fig. 4, detailed direction relations, which also consist of nine cardinal directions at most, are formed by partitioning the MBR of the reference region into nine parts further according to projection-based method. The central zone is a rectangle whose width and height are $\lambda_w$ and $\lambda_h$. The central point of the central zone coincides with the central point of the reference region.

Definition 1 The interior of a reference region is partitioned into nine cardinal directions at most, they are EP, WP, SP, NP, NEP, NWP, SEP, SWP and CP. The interior direction EP means east part of a region. Other interior directions have similar meanings. This kind of direction relations are called interior direction relations (Fig. 4(a)).

Definition 2 The boundary of a reference region is partitioned into nine cardinal directions at most, they are EL, WL, SL, NL, NEL, NWL, SEL, SWL and CL. The boundary direction EL means east boundary of a region. Other boundary directions have similar meanings. This kind of direction relations is called boundary direction relations (Fig. 4(b)).
Definition 3 The ring region denotes the difference between the MBR and the reference object. It is partitioned into nine directions at most, such as ER, WR, SR, NR, NER, NWR, SER, SWR and CR. The ring direction ER means east ring, other ring directions have similar meanings. This kind of direction relations are called ring direction relations (Fig. 4(c)).

Definition 4 The interior, boundary and ring direction relations can be called as detailed direction relations. Accordingly, the conventional direction relations and directions are called exterior direction relations and exterior directions respectively.

\[\begin{align*}
\text{EP}(A, p) & \equiv X(p) \geq X, \land Y(p) > Y_t, \land \text{Inside}(A, p) \\
\text{WP}(A, p) & \equiv X(p) \leq X, \land Y(p) > Y_b, \land \text{Inside}(A, p) \\
\text{SP}(A, p) & \equiv X(p) > X, \land X(p) < X, \land Y(p) \geq Y, \land \text{Inside}(A, p) \\
\text{SEP}(A, p) & \equiv X(p) \geq X, \land X(p) < X, \land Y(p) \geq Y, \land \text{Inside}(A, p) \\
\text{NEP}(A, p) & \equiv X(p) \geq X, \land Y(p) \geq Y, \land \text{Inside}(A, p) \\
\text{SWP}(A, p) & \equiv X(p) \leq X, \land Y(p) \leq Y_b, \land \text{Inside}(A, p) \\
\text{NWP}(A, p) & \equiv X(p) \leq X, \land X(p) < X, \land Y(p) < Y, \land \text{Inside}(A, p) \\
\text{CP}(A, p) & \equiv X(p) > X, \land X(p) < X, \land Y(p) > Y_b, \land \text{Inside}(A, p)
\end{align*}\]

If the condition Inside\(A, p\) in Eq. (1) is replaced by Onside\(A, p\) and Inside\((\text{MBR}(A) - A, p)\) respectively, then the definitions of corresponding boundary and ring direction will be formed.

2.2 Definitions of natural-language spatial directions

Topological relations, direction relations (including detailed direction relations) provide NLSRs with the most basic words for constructing more advanced and complex spatial relations language. The complex terms of spatial relations can be composed of either single kind of spatial relation, such as “go across”, “east-central part”, “west-central part” and “east-central boundary”, etc., or multiple-kinds of spatial relations, such as “go across from east part”, “go across from central part vertically”, “go across from central part horizontally” and “enter from west part”, etc.

Regarding cardinal detailed directions as atomic directions, the complex directions can be constructed. For example, complex directions in four directions system can be formed by combining cardinal directions in eight directions system. In general, the complex direction is the union of full or part spatial region of several cardinal directions.
Let complex directions $NP_4$, $SP_4$, $EP_4$, $WP_4$, and $CP_4$ represent "north part", "south part", "east part", "west part" and "central part" respectively, where $CP_4$ can be divided into $CP_{NS}$ (north-to-south, Fig. 5 (e)), $CP_{EW}$ (east-to-west, Fig. 5 (f)), $CP_{SW-NE}$ (southwest-to-northeast, Fig. 5 (g)), $CP_{NW-SE}$ (northwest-to-southeast, Fig. 5 (h)) and $CP$. Some of them can be constructed by combining the cardinal interior directions (Fig. 5(a)-5(f)), while others are generated by partitioning spatial regions of directions (Fig. 5 (g)-5(j)). The definitions of complex boundary and ring directions are different from that of complex interior directions. The difference is that it is not necessary to repartition spatial regions of directions when generating complex boundary or ring directions, but only recombine neighboring cardinal boundary or ring directions respectively. According to the definitions of complex boundary directions, $NL_4$, $SL_4$, $EL_4$ and $WL_4$, and complex ring directions $NR_4$, $SR_4$, $ER_4$ and $WR_4$ are combinations of corresponding cardinal boundary or ring directions in eight directions system by the similar way of generating complex interior directions, they are omitted here.

2.3 Definitions of natural-language spatial relations

Definition 5 Let $U_I$ represent the universal set of cardinal interior directions {EP, WP, NP, SP, NWP, NEP, SWP, SEP, CP}. If $R_i$ is a cardinal interior direction in $U_I$, then $R_i$ is a set composed of all cardinal interior directions in $U_I$ except $R_i$, which represents the complement of $R_i$ about set $U_I$, i.e., $R_i = U_I - R_i$. In the similar way, we can define the complement set of exterior, boundary and ring directions about their universal set $U_D$, $U_B$ and $U_R$ as $R_D = U_D - R_D$, $R_B = U_B - R_B$ and $R_R = U_R - R_R$ respectively.

Definition 6 Let $R_i$ be a complex interior direction. If $R_i$ is the combination of cardinal interior directions, then $R_i$ is the intersection of complement sets of all cardinal interior directions in $R_i$, i.e., $R_i = \bigcap_{R_j} (S')$; otherwise, $R_i$ can not be represented as the combination of cardinal interior directions. If $O_{R_i}$ is the spatial region of cardinal interior direction $R_i$, and $A$ is the reference object, then the spatial region of $R_i$ is $A - R_i$. That is, if $p$ is any point in $R_i$, then $R_i (A, p) = \text{Inside}(A - O_{R_i}, p)$ holds. Similarly, $R_D = \bigcap_{R_i} (S'), R_B = \bigcap_{R_i} (S')$ and $R_R$.
In terms of Definition 5, WP \((A, B)\) means that object \(B\) falls into the complement of cardinal interior direction WP. If WP \(=\{EP, NP, SP, NWP, NEP, SWP, SEP, CP\}\), then WP \((A, B) = EP(A, B) \lor NP(A, B) \lor SP(A, B) \lor NWP(A, B) \lor NEP(A, B) \lor SWP(A, B) \lor SEP(A, B) \lor CP(A, B)\). We also can conclude that WP\(_i\) \((A, B) = EP(A, B) \lor NP(A, B) \lor SP(A, B) \lor NEP(A, B) \lor SWP(A, B) \lor SEP(A, B) \lor CP(A, B)\) holds true.

The natural-language spatial relations can be defined by combining existing cardinal or complex directions with topological relations. Assuming that Cross\((A, B)\), Enter\((A, B)\), Alongside\((A, B)\), EP\(_{o}\)\(\)Cross\((A, B)\), EP\(_{w}\)\(\)Cross\((A, B)\), EP\(_{w}\)\(\)Enter\((A, B)\), EP\(_{w}\)\(\)Alongside\((A, B)\) and Inside\((A, B)\) represent the topological sentences "\(B\) goes across \(A\)" "\(B\) enters \(A\)" "\(B\) goes along \(A\)" and "\(B\) is inside \(A\)" respectively; symbol "o" represent the combination operator between direction and topology, then the four topological sentences can be defined by Eq. (3).

\[
\text{Cross}(A,B) \equiv LR_1, \quad \text{Inside}(A,B) \equiv LR_2 \\
\text{Alongside}(A,B) \equiv LR_3, \quad \text{Enter}(A,B) \equiv LR_4
\]

In Eq. (3), \(LR_1, LR_2, LR_3\) and \(LR_4\) are topological relations whose meanings are in Fig. 1. Natural-language spatial relations combining topology and direction are formulized qualitatively by Eq. (4).

\[
\begin{align*}
\text{WP}_{o}\text{Cross}(A,B) & \equiv [\text{WP}_{o}(A,B) \land \neg \text{WP}_{o}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{EP}_{o}\text{Cross}(A,B) & \equiv [\text{EP}_{o}(A,B) \land \neg \text{EP}_{o}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{NP}_{o}\text{Cross}(A,B) & \equiv [\text{NP}_{o}(A,B) \land \neg \text{NP}_{o}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{SP}_{o}\text{Cross}(A,B) & \equiv [\text{SP}_{o}(A,B) \land \neg \text{SP}_{o}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{CP}_{NS}\text{Cross}(A,B) & \equiv [\text{CP}_{NS}(A,B) \land \neg \text{CP}_{NS}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{CP}_{EW}\text{Cross}(A,B) & \equiv [\text{CP}_{EW}(A,B) \land \neg \text{CP}_{EW}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{CP}_{SW-NE}\text{Cross}(A,B) & \equiv [\text{CP}_{SW-NE}(A,B) \land \neg \text{CP}_{SW-NE}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{CP}_{NW-SE}\text{Cross}(A,B) & \equiv [\text{CP}_{NW-SE}(A,B) \land \neg \text{CP}_{NW-SE}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\text{CP}_{o}\text{Cross}(A,B) & \equiv [\text{CP}_{o}(A,B) \land \neg \text{CP}_{o}(A,B) ] \lor [\text{Cross}(A,B) ] \\
\end{align*}
\]

In Eq. (4), symbol EP\(_{o}\)\(\)Cross\((A, B)\), WP\(_{o}\)Cross\((A, B)\), NP\(_{o}\)Cross\((A, B)\), SP\(_{o}\)Cross\((A, B)\), CP\(_{NS}\)\(\)Cross\((A, B)\), CP\(_{EW}\)\(\)Cross\((A, B)\), CP\(_{SW-NE}\)\(\)Cross\((A, B)\), CP\(_{NW-SE}\)\(\)Cross\((A, B)\) represent "\(B\) goes across from east part of \(A\)", "\(B\) goes across from west part of \(A\)" etc. When the subscript \(i = 4\), WP\((A, B)\) represents complex interior direction in four directions system; while \(i = 8\), it represents the cardinal interior directions in eight directions system. The natural-language spatial relations combining topology and directions not only depend on topological relations and exterior direction relations, but also rely on interior directions where object \(B\) falls inside. Both the two points are indicated in Eq. (4). By similar way, we can also define some other NLSRs, such as NEP\(_{o}\)Cross\((A, B)\), SEP\(_{o}\)Cross\((A, B)\), SWP\(_{o}\)Cross\((A, B)\), NWP\(_{o}\)Cross\((A, B)\), EP\(_{o}\)Enter\((A, B)\), WP\(_{o}\)Enter\((A, B)\), NP\(_{o}\)Enter\((A, B)\), SP\(_{o}\)Enter\((A, B)\), CP\(_{NS}\)\(\)Enter\((A, B)\), CP\(_{EW}\)\(\)Enter\((A, B)\), CP\(_{SW-NE}\)\(\)Enter\((A, B)\) and CP\(_{NW-SE}\)\(\)Enter\((A, B)\).

In the view of the spatial extent perspective, topological relation Alongside\((A, B)\) and Inside\((A, B)\) coincide with boundary and interior direction relations. As a result, the combination of the two relations with directions can be conducted by modifying the definitions of cardinal boundary and interior directions respectively. Eq. (5) defines the NLSRs—"object \(B\) is inside east part of object \(A\)" and "object \(B\) is on the east boundary of object \(A\)"—as symbol EP\(_{o}\)Inside\((A, B)\) and ELOAlongside\((A, B)\) respectively. Definitions of other NLSRs can be obtained by similar way, so they are omitted.

\[
\begin{align*}
\text{EPoInside}(A,B) & \equiv \forall p \in B, X(p) \geq X, \land Y(p) \geq Y_b \land Y(p) \leq Y, \land \text{Inside}(A,p) \\
\text{ELOAlongside}(A,B) & \equiv \forall p \in B, X(p) \geq X, \land Y(p) \geq Y_b \land Y(p) \leq Y, \land \text{Onside}(A,p)
\end{align*}
\]
3 Refinement of natural-language spatial relations

Despite Eqs. (3), (4) and (5) formalize the NLSRs, it is inevitable to refine these definitions further in most cases. As shown in Fig. 6, both two figures in first column illustrate the definitions of the sentence — "go across from north part"— according to topological and direction relations. However, they are different clearly because, for the top one, the line object completely lies in the top side of the interior partition, while the line falls into south part of the top side of the interior partition in the bottom figure. This demonstrates that NLSRs associates with not only interior directions where the target object falls into, but also the exterior direction relations that the target object lies in. Therefore it is necessary to refine the definitions of NLSRs according to exterior direction relations. The figures of the second and third column have the same situations as that in first column. On the other hand, the two figures in the fourth column completely illustrate definitions of neither the sentence "go across from east part", nor the other "go across from central part", of NLSRs, because despite both two target objects fall into the interior direction CP and NP, the metric information that the two target object fall into NP and CP are different. Hence, it is necessary to refine the natural-language spatial relations according to the metric information to decide which sentence can be used to exactly describe the two figures in the fourth column.

3.1 Refinement of exterior direction relations

Because the definitions of natural-language spatial relations associate with exterior direction relations, while the existing exterior direction relations can not describe the path which the target object fall into effectively, it is necessary to refine the exterior direction relations. As illustrated in Fig. 7, the interior partition of interior direction relations and the extended line of MBR of the reference object refine the spatial extent outside the MBR. Exterior direction N can be partitioned into LN, CN and RN further, while E is divided into TE, CE and BE, and then direction W and S have similar partition. We use the symbol LN(A,B) to represent that the object B lies in the LN of object A. Other symbols, for example, CN(A,B) and RN(A,B), are similar to LN(A,B).

Fig. 6 Refinement of natural-language spatial relations

Fig. 7 Refinement of exterior direction relations

According to the refinement of exterior direction relations, each interior direction corresponds to a limitation of exterior direction. The limitation confines the NLSRs to some exterior directions, which is helpful to distinguish the consistent degree between a figure and a NLSR. For example, the exterior limitation of the complex interior direction NP, is composed of N(A, B), TW(A, B), TE(A, B), NW(A, B), NE(A, B),
B), NWR(A,B), NER(A,B) and NR(A,B), which indicates that the limitation is still a complex direction. On the basis of the limitation of exterior direction, the two spatial relations in

\[ Ex - N(A,B) \equiv N(A,B) \lor TW(A,B) \lor TE(A,B) \lor NW(A,B) \]

\[ \lor NE(A,B) \lor NWR(A,B) \lor NER(A,B) \lor NR(A,B) \]

\[ Ex - S(A,B) \equiv S(A,B) \lor BW(A,B) \lor BE(A,B) \lor SW(A,B) \]

\[ \lor SE(A,B) \lor SWR(A,B) \lor SER(A,B) \lor SR(A,B) \]

\[ Ex - E(A,B) \equiv E(A,B) \lor RN(A,B) \lor RS(A,B) \lor NE(A,B) \]

\[ \lor SE(A,B) \lor NER(A,B) \lor ER(A,B) \lor SER(A,B) \]

\[ Ex - W(A,B) \equiv W(A,B) \lor LN(A,B) \lor LS(A,B) \lor NW(A,B) \]

\[ \lor SW(A,B) \lor NWR(A,B) \lor WR(A,B) \lor SWR(A,B) \]

\[ Ex - NE(A,B) \equiv NE(A,B) \lor RN(A,B) \lor TE(A,B) \lor NER(A,B) \]

\[ Ex - NW(A,B) \equiv NW(A,B) \lor LN(A,B) \lor TW(A,B) \lor NWR(A,B) \]

\[ Ex - SE(A,B) \equiv SE(A,B) \lor RS(A,B) \lor BE(A,B) \lor SER(A,B) \]

\[ Ex - SW(A,B) \equiv SW(A,B) \lor LN(A,B) \lor BS(A,B) \lor NW(A,B) \]

\[ Ex - CPNs(A,B) \equiv CPNs(A,B) \lor CN(A,B) \lor CS(A,B) \lor NR(A,B) \lor SR(A,B) \]

\[ Ex - CPew(A,B) \equiv CPew(A,B) \lor CE(A,B) \lor CW(A,B) \lor ER(A,B) \lor WR(A,B) \]

\[ Ex - CPswNE(A,B) \equiv CPswNE(A,B) \lor Ex - SW(A,B) \lor Ex - SW(A,B) \]

In addition, NLSRs with refinement of exterior direction relations can be defined by Eq. (7).

\[ WPoCross(A,B) \equiv [WP(A,B) \land \neg WP'(A,B)] \land [Ex - W(A,B)] \land [Cross(A,B)] \]

\[ EPoCross(A,B) \equiv [EP(A,B) \land \neg EP'(A,B)] \land [Ex - E(A,B)] \land [Cross(A,B)] \]

\[ NPoCross(A,B) \equiv [NP(A,B) \land \neg NP'(A,B)] \land [Ex - N(A,B)] \land [Cross(A,B)] \]

\[ SPoCross(A,B) \equiv [SP(A,B) \land \neg SP'(A,B)] \land [Ex - S(A,B)] \land [Cross(A,B)] \]

Eq. (7) only defines part of the NLSRs, others can be constructed by similar way. It is significant to point out that, from the concept extension, all spatial relations meeting the definitions of either Eq. (4) or Eq. (7) can be described by the same languages, but the relations meeting the refinement definitions (Eq. (7)) belong to the languages with a larger consistent degree than that meeting the definitions in Eq. (4). The purpose of refinement only is to distinguish the extent of the consistent degree between the NLSRs and the sentences they represent. In practice, whether the refinement is needed depends on the requirement of users and the purpose of applications.

3.2 Metric refinement of topological and detailed direction relations

Metric information contains two aspects: topology and direction. The metric information of topology refines the relation between the geometry and a topological term, while the metric information of direction reflects the relation between the geometry and a direction term.

In Eq. (8), \( \alpha, \beta, \lambda \) and \( \gamma > \alpha \) holds true. The definition of Cross’(A,B) means that only two following conditions are met simultaneously, the relation between object A and B can be expressed by the sentence “line B go across region A”. The first condition is that the topological relation between A and B must be LR1; the sec-
ond condition is that the ration between the part of B inside A and total part of B is larger than \( a \). It is clear that Eq. (8) is a modified version of Eq. (3).

The metric information of directions can be captured by an interior direction relation matrix in Eq. (9), where function \( \text{area}(OEP \cap B) \) means the intersection set between target object B and spatial region of direction EP. For line/region relation, the intersection set is a line object, and function area is to compute the length of the intersection; for region/region relation, the intersection is a region object, and area means to compute the area of the intersection. Function \( \text{area}(A \cap B) \) is to compute the length or area of the intersection between object A and B. As such function \( \text{area}(OEP \cap B)/\text{area}(A \cap B) \) represents the region ratio between the intersection of the target object B with the spatial region of direction EP and the one of object A and B (Fig. 8 (a)). It is clear that the total sum of all elements in matrix \( M_i \) is 1.

\[
M_i = \begin{bmatrix}
\frac{\text{area}(O_{newp} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{np} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{ne} \cap B)}{\text{area}(A \cap B)} \\
\frac{\text{area}(O_{wep} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{cp} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{ce} \cap B)}{\text{area}(A \cap B)} \\
\frac{\text{area}(O_{newp} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{cp} \cap B)}{\text{area}(A \cap B)} & \frac{\text{area}(O_{se} \cap B)}{\text{area}(A \cap B)}
\end{bmatrix}
\]

![Fig. 8 Metric refinement of detailed direction relations](image)

We can define metrics matrices, \( M_b \) and \( M_r \), of boundary and ring direction relations to capture metric information of boundary and ring direction. Let function \( \text{len}(O_{El} \cap B) \) be the length of line B is on the east boundary of region A; function \( \text{len}(\partial A \cap B) \) represent the length that line B coincide with A’s boundary, then function \( \text{len}(O_{El} \cap B)/\text{len}(\partial A \cap B) \) is to compute the ratio between part that line B coincide with EI. of A and A’s boundary. For the complex direction composed of cardinal directions (for example, WP), the ratio is the sum of elements in the matrix corresponding to cardinal directions. If the ratio is larger than or equal to 0.5, we can say that line B goes along the east boundary of region A. If the complex direction is not the combination of cardinal directions, then a new function can be defined to do same work, for example, \( \text{area}(O_{cpw NE} \cap B)/\text{area}(A \cap B) \).

According to the metric matrix, the direction whose ratio is the largest among all nine directions holds true, or directions whose ratios are larger than 0 (between 0 and 1) holds true. Accordingly, the natural-language spatial relations constrained by refinement and metric information of topology and directions can be defined by Eq. (10).

\[
\begin{align*}
WP0Cross(A,B) & \equiv [WP(A,B) \land M_i(1,0) > \rho] \land [\text{Ex} - \text{West}(A,B)] \land [\text{Cross}'(A,B)] \\
WP_{oCross}(A,B) & \equiv [WP(A,B) \land (M_i(0,0) + M_i(1,0) + M_i(2,0)) > \rho] \land [\text{Ex} - \text{West}(A,B)] \land [\text{Cross}'(A,B)]
\end{align*}
\]

Eq. (10) only gives definitions of two NLSRs, others can be constructed by similar ways. The limitation condition increases gradually from the qualitative refinement to metric definitions of NLSRs, so they can meet various requirements.
4 Examples

The formal definitions of NLSRs construct some kinds of mapping between natural language people used and geometry relations among spatial objects: $\text{geometry} \times \text{geometry} \rightarrow \text{string}$, which can be used to query spatial data from spatial databases and spatial data matching based on spatial relations. Traditional SQL for relation database can not deal with spatial data, so it is necessary to design spatial query language. NLSRs can be integrated into the whole sub-sentence of SQL to conduct spatial retrieval.

Assuming that there is a spatial database $\text{Spatial_DB}$ including three tables: $\text{Park}$, $\text{Road}$ and $\text{City}$, which store geometry and attribute data of parks, roads and cities respectively, and the geometry information is stored in a special field-geometry in each table, then by expanding the traditional SQL, we can conduct following two queries in spatial database $\text{Spatial_DB}$:

1) Retrieving out all roads going across the east part of districts
   
   Select Road. geometry
   From River and Park
   Where Road. geometry go cross Park. geometry on east part

2) Retrieving out all roads whose level is 1 going across the mid-east part of cities
   
   Select Road. geometry
   From Road and City
   Where Road. geometry go cross City. geometry on mid-east and Road. level=1

These natural-language terms and sentences can be used for spatial data retrieval, such as spatial SQL, visual spatial query and image retrieval content-based.

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