Research on the Calculation Method of Harmonic Contribution in the Case of Dominant System-side Harmonics

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Abstract. Existing methods for quantifying the responsibility of harmonic sources assume a dominant user side and use a harmonic source equivalence circuit to calculate the equivalent system impedance and background harmonic voltage, which in turn assesses the harmonic contribution of that source to the bus of concern. For users who actively participate in harmonic governance, it is very important to evaluate the responsibility of injecting harmonics into users. This paper assumes system-side is dominant, constructs a partial linear regression model and a constant impedance model, and tracks the regression error. The equivalent fundamental impedance is doubly screened to calculate the harmonic impedance for the corresponding number of times, which in turn quantifies the harmonic voltage duty. The results of simulation and the analysis of measured data show that this method has simple calculation model, small regression error (0.0037), high accuracy and practical engineering significance.

Keywords: Power quality; Harmonic impedance; Harmonic voltage responsibility; Partial linear regression model; Constant impedance model.

1. Introduction
With the rapid growth of smart grid, ultra-high voltage DC transmission and new energy sources with high penetration rates, the problem and form of harmonic pollution in power system has taken on new characteristics, modern power equipment puts forward higher requirements for power quality. Harmonics is an important measure of power quality and is commonly found in power systems. However, the continuous access of nonlinear loads and rectification equipment aggravates the harmonic pollution of the power grid, and it becomes more difficult to locate the harmonic sources and quantitative division of harmonic responsibility.

For the current harmonic management problems in power grids, international related organizations have developed a series of standards such as EMC 61000 one after another\cite{1-3}, and a considerable part of them have been adopted as national standards or guiding technical documents in China\cite{4,5}. Harmonic national standards have been implemented for more than two decades now, and have achieved great success in controlling harmonics in power grids, but many problems have also been exposed\cite{6,7}. The current standards and specifications cannot effectively manage loads that do not exceed the standards but continue to inject harmonics into the system; the limits for individual voltage levels are too strict and the control costs are large, and there is a lack of relevant indicators to assess the degree of harmonic pollution, moreover, The identification of harmonic sources, how to divide the responsibility for harmonics, and how to share the economic loss are not clearly mentioned.

At present, domestic and international studies on the problem of harmonic contribution classification are centered on the equivalent system harmonic impedance. The main methods for system harmonic impedance assessment at this stage are linear regression methods\cite{8-11}, fluctuation methods and...
improved algorithms. Their theoretical premise is to assume the dominance of the user side and negligible background harmonic fluctuations, construct a mathematical model to calculate the system harmonic impedance and the equivalent background harmonic voltage, and then quantify the harmonic voltage contribution injected from the user side to the system side. Several harmonic impedance calculation methods mentioned above, either "intervention" or "non-intervention" methods, estimate harmonic impedance for the system side, with the theoretical premise of assuming dominant harmonic fluctuations on the user side. For customers who actively use filters and other equipment to control harmonics, the calculation methods described above are not applicable if system-side harmonic fluctuations are dominant in the presence of a large number of harmonic sources. Therefore, this paper assumes that the system side is dominant and constructs a partial linear regression model based on the measured sample data. The regression error is tracked and the data that do not satisfy the given error accuracy are removed. The data where system-side fluctuations dominate is obtained by a second Nair screening, then the values of the user harmonic impedance series are updated by a constant impedance model, and finally the harmonic voltage liability injected by the system into the user during the monitoring period is quantified. This approach aims to avoid discussing the harmonic emission levels of a particular harmonic source load in isolation, but rather in relation to the direction of harmonic injection at the PCC (common connection point). The experimental results show that the method can effectively estimate the user-side harmonic impedance further accurately divide the harmonic voltage responsibility, provide data reliance for harmonic reward, punishment and governance, so it has practical engineering application value and research significance in the absence of a better method at present.

2. Two Scenarios of Harmonic Contribution Calculation

The purpose of the division of responsibility is to calculate the contribution of each load to the harmonic distortion at the common connection point. Discussing the harmonic emission level of a particular harmonic source load in isolation is not meaningful for responsibility apportionment. The harmonic emission level of the concerned load is high, but the shared responsibility is not necessarily large, it must also be discussed in relation to the overall degree of harmonic distortion at the PCC to be meaningful.

Let the harmonic voltage at the PCC be \( \bar{U}_{PCC} \), the \( h \) second harmonic current flowing into the user side is \( \bar{I}_{PCC} \), the background harmonic voltage generated at the point of common coupling on the system side is \( \bar{U}_{b,0} \), and the \( h \) second equivalent harmonic impedance is \( Z_h \).

According to the Superposition theorem and circuit knowledge, it is known that the harmonic voltage at the common connection point is equal to the sum of the harmonic voltage and the background harmonic voltage formed at the common connection point by the harmonic current injected on the user side. As shown in Equation 1:

\[
\bar{U}_{PCC} = Z_h \cdot \bar{I}_{PCC} + \bar{U}_{b,0}
\]

(1)

From (1), the phasor \( \bar{U}_{PCC} \) is the harmonic voltage at the PCC; the phasor \( \bar{U}_{ch} \) is the harmonic voltage formed at the point of common coupling by the harmonic current injected at the user side, and \( \bar{U}_{ch} = Z_{ch} \cdot \bar{I}_{PCC} \); the phasor \( \bar{U}_{b,0} \) is the background harmonic voltage, and the phasor relationship is shown in Figure 1.

![Figure 1. Phasor relationship.](image)
The fluctuation volume method assumes that the background harmonic source is small or the background harmonic fluctuates very little relative to the harmonic of interest. When the background harmonic source is small, \( \dot{U}_{h,0} \approx 0 \), (1) can be reduced to

\[
\dot{U}_{PCC} = Z_h \cdot I_{PCC}
\]

Considering the phase-angle \( \theta \), the harmonic impedance can be expressed as

\[
Z_h = \frac{U_{PCC}}{I_{PCC}} = \frac{U_{PCC}}{I_{PCC}} \cos \theta + j \cdot \frac{U_{PCC}}{I_{PCC}} \sin \theta
\]

It can also be expressed as

\[
Z_h = \frac{\Delta U_{PCC}}{\Delta I_{PCC}}
\]

When the background harmonic fluctuations are large, the fluctuation method will produce large calculation errors, so scholars proposed the dominant fluctuation method to solve the problem. Assuming that the above fluctuations obey a normal distribution, the Nair's test is used for screening. Firstly, the mean \( \mu_{SU} \) and variance \( \sigma_{SU}^2 \) of the harmonic voltage modes at the PCC are calculated, and the equation is as follows

\[
\mu_{SU} = \frac{1}{N} \sum_{i=1}^{N} \Delta U_i \quad , \quad \sigma_{SU}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\Delta U_i - \mu_{SU})^2
\]

where \( N \) is the number of samples, and \( \Delta U_i \) is the module of the harmonic voltage fluctuation of the \( i \)th sample.

The amount of volatility that dominates on the system side is filtered by Nair screening as

\[
\frac{(\Delta U - \mu_{SU})}{\sigma_{SU}} < \alpha
\]

where \( \alpha \) is the Nair coefficient, which is generally 1~1.5.

According to equation (6) filter out the \( h \) second dominant harmonic voltage \( \Delta U'_{PCC} \) and current fluctuations \( \Delta I'_{PCC} \), then the \( h \) second harmonic impedance is

\[
Z_h = \frac{\Delta U'_{PCC}}{\Delta I'_{PCC}}
\]

The dominant fluctuation method can remove the influence of background harmonic source fluctuations, but the error size cannot be controlled during the calculation process, and the sample data may be mistakenly deleted. In order to track the qualified error and screen out the sample data at the stable moment of the system, this paper proposes a harmonic impedance estimation method based on a partial linear regression model.

3. Calculation of Harmonic Voltage Contribution in the Case of Dominant System-side Harmonic Fluctuations

The evaluation of harmonic contribution is not simply a matter of scalar magnitude, but must also be discussed in the context of directionality, and therefore requires consideration of the calculation of phase angle.

3.1. Calculation of Phase Angle

The absolute phase angle of harmonic voltage and harmonic current is difficult to obtain, but the angle of harmonic voltage ahead of harmonic current can be deduced from the power factor.
The phase angle difference is derived as follows.

(1) If the system side is predicted to be the dominant harmonic source, the angular phase difference of harmonic voltage ahead of harmonic current is projected as follows.

- When $P > 0$ and $Q > 0$, the phase angle difference is equal to $\arccos(\cos \phi_h)$;
- When $P > 0$ and $Q < 0$, the phase angle difference is equal to $-\arccos(\cos \phi_h)$;
- When $P < 0$ and $Q > 0$, the phase angle difference is equal to $\pi - \arccos(\cos \phi_h)$;
- When $P < 0$ and $Q < 0$, the phase angle difference is equal to $-\arccos(\cos \phi_h) - \pi$.

(2) If the user side is predicted to be the dominant harmonic source, the angular phase difference of harmonic voltage ahead of harmonic current is deduced as follows.

- When $P < 0$ and $Q < 0$, the phase angle difference is equal to $\arccos(\cos \phi_h)$;
- When $P < 0$ and $Q > 0$, the phase angle difference is equal to $-\arccos(\cos \phi_h)$;
- When $P > 0$ and $Q < 0$, the phase angle difference is equal to $\pi - \arccos(\cos \phi_h)$;
- When $P > 0$ and $Q > 0$, the phase angle difference is equal to $\arccos(\cos \phi_h) - \pi$.

Where $P$ is harmonic active power, $Q$ is harmonic reactive power and $\cos \phi_h$ is harmonic power factor.

3.2. Partial Linear Regression Model

Let the amount of sample data dominated by system-side harmonic fluctuations during the monitoring time period be $N$, divided into $l = N / n$ segments, and construct a partial linear regression model for the $k$th segment according to the rolling formula of equation (1).

$$
\begin{align*}
\hat{U}_{\text{PCC}}(k_i) &= Z(k)\hat{I}_{\text{PCC}}(k_i) + \hat{U}_{h,0}(k) \\
\hat{U}_{\text{PCC}}(k_2) &= Z(k)\hat{I}_{\text{PCC}}(k_2) + \hat{U}_{h,0}(k) \\
& \quad \ldots \\
\hat{U}_{\text{PCC}}(k_n) &= Z(k)\hat{I}_{\text{PCC}}(k_n) + \hat{U}_{h,0}(k)
\end{align*}
$$

(8)

where $\hat{U}_{\text{PCC}}(k_i), \hat{I}_{\text{PCC}}(k_i)$ is the $i$th sample data in segment $k$; $U_{\text{PCC}}(k_i), I_{\text{PCC}}(k_i)$ is the amplitude of the $i$th sample data in the $k$th segment; $Z(k), \hat{U}_{h,0}(k)$ is the user harmonic impedance and background harmonic voltage of the first paragraph, respectively.

The solution of the above regression equation is

$$
x = (A^T A)^{-1} A y
$$

(9)

Among them, $x = \begin{bmatrix} Z(q) \\ U_{h,0}(q) \end{bmatrix}$, $A = \begin{bmatrix} \hat{I}_{\text{PCC}}(q_1) \\ \hat{I}_{\text{PCC}}(q_2) \\ \vdots \\ \hat{I}_{\text{PCC}}(q_n) \end{bmatrix}$, $y = \begin{bmatrix} \hat{U}_{\text{PCC}}(q_1) \\ \hat{U}_{\text{PCC}}(q_2) \\ \vdots \\ \hat{U}_{\text{PCC}}(q_n) \end{bmatrix}^T$.

From equation (9) can be found from the user-side harmonic impedance, and by $y = y - Ax$ calculating the $i$th segment regression error, the $k$th segment regression error is

$$
\epsilon(k) = \sum_{i=1}^{n} \left| \frac{\gamma(k_i)}{\hat{U}_{\text{PCC}}(k_i)} \right|
$$

(10)
The first filtering is performed given the error accuracy $\delta_0$. If the segment error is less than the given accuracy $\delta_0$, the data in this segment is retained; if it is not satisfied, the data in this segment is discarded. If there is no data meeting the requirement after screening, the algorithm is aborted.

Given the Nair coefficient $\alpha$, the mean $\mu_{z_h}$ and variance $\sigma^2_{z_h}$ of the user harmonic impedance of the remaining $m$ segments are calculated for the second screening.

$$
\mu_{z_h} = \frac{1}{m-1} \sum_{i=1}^{m} Z_h(i), \quad \sigma^2_{z_h} = \frac{1}{m-1} \sum_{i=1}^{m} (Z_h(i) - \mu_{z_h})^2
$$

(11)

When the harmonic impedance satisfies

$$
\frac{Z_h - \mu_{z_h}}{\sigma_{z_h}} < \alpha
$$

(12)

Then the data in this paragraph is retained and recorded as $Z_h(1), Z_h(2), \cdots, Z_h(s)$.

The double screening of the above methods inevitably affects the completeness of data and the accuracy of harmonic assessment during the monitoring period. In order to address the shortcomings of the method, this paper proposes a constant impedance model to estimate the user harmonic impedance to fill the data gap of the partial linear regression model.

### 3.3. Constant Impedance Model

Due to the time-varying nature of the load, there is no better method to accurately estimate the harmonic impedance of the user, especially for its phase angle is difficult to estimate, but its mode value can be estimated by the method of calculating the harmonic impedance of its corresponding times through the fundamental impedance equivalence.

Assuming that the impedance value remains constant for a short time during the simulation calculation, the user-side fundamental impedance is

$$
Z_{L0} = \frac{U^2_{L0}}{P_{L0} - jQ_{L0}}
$$

(13)

where $U_{L0} = U_{L0} \cdot (\cos \phi + j \sin \phi)$, $Z_{L0}$ is the user's impedance of fundamental frequency, $U_{L0}$ is the fundamental voltage amplitude, $\phi$ is the fundamental voltage phase angle, $P_{L0}$ is the active power of fundamental frequency, and $Q_{L0}$ is the reactive power of fundamental frequency.

From the circuit knowledge, the user harmonic impedance is

$$
Z_h' = \text{real}(Z_{L0}) + j \cdot h \cdot \text{imag}(Z_{L0})
$$

(14)

Where, $Z'_h$ is the user harmonic impedance estimated by the constant impedance model, $\text{real}(Z_{L0})$ is the real part of the user's impedance of fundamental frequency, $\text{imag}(Z_{L0})$ is the imaginary part of the user's impedance of fundamental frequency, and $h$ is the number of harmonics to be evaluated.

The constant impedance model is used to estimate the user-side harmonic impedance and fill in the data eliminated by double screening in the partial linear regression model to obtain updated values of the user harmonic impedance series $Z_{nh}$, ensuring that the magnitude of harmonic pollution injected into the user by the system can be assessed at any moment during the monitoring period.

The harmonic voltage contribution injected into the user on the system side $\beta_{nh}$ is

$$
\beta_{nh} = \frac{U_{L0} \cos \theta}{U_{PCC}} \times 100% = \frac{U^2_{PCC} + Z_{nh}^2 r_{PCC} - U^2_{L0}}{2U^2_{PCC}} \times 100%
$$

(15)
4. Example Analysis
The actual measurement data came from a 10kV circuit breaker in a substation in Qujing, Yunnan Province, and the test point was at the 10kV side of the 110kV power supply substation at the 098 circuit breaker inlet of the Tongfa line. The voltage, current, power and power factor data at the measurement point were collected using FLUKE 1760. The data collected are all 1ms statistics, i.e. the root mean square value of 50 periodicity. MATLAB is used to simulate the 3rd harmonic, and the data which meets $P > 0$ of harmonic active power is selected to calculate the harmonic voltage contribution by the method of this paper.
The analysis is performed with the 3rd harmonic, and the effective values of the fundamental voltage, 3rd harmonic voltage and 3rd harmonic current are shown in Fig. 2 - Fig. 4, respectively.

![Figure 2. Fundamental voltage effective value curve.](image1)

![Figure 3. 3rd Harmonic voltage effective value curve.](image2)

![Figure 4. 3rd harmonic current effective value curve.](image3)

The presence of a large disturbance in the fundamental current can be seen in Figure 2, and there is a corresponding disturbance in Figures 3 and 4 at the moment of the large disturbance. Therefore, when the harmonic active power is positive, the background harmonic voltage has a small effect on the voltage at the common connection point and the disturbance is system dominantly generated.
The calculated 3rd user harmonic impedance amplitude from the partial linear regression model and the constant impedance model is shown in Figure 5 and 6. As can be seen from them, the method proposed in this paper successfully complements the vacant values of harmonic impedance, although there is a stratification between the modal values derived from the constant impedance method and the results derived from the partial linear regression model, but this improves as the number of segments is reduced.

![Figure 5. The amplitude of the third harmonic impedance without adding the phase angle.](image4)
(1) Without adding phase angle: given an error accuracy of 0.1, the mean value of error of regression for the number of segments \( q=3 \) is 0.0041, and the mean value of error of regression for the number of segments \( q=10 \) is 0.0371.

![Figure 6. The amplitude of the third harmonic impedance considering the phase angle.](image)

Figure 6. The amplitude of the third harmonic impedance considering the phase angle.

(2) Consider the phase angle: given an error accuracy of 0.1, the mean value of error of regression for the number of segments \( q=3 \) is 0.0037, and the mean value of error of regression for the number of segments \( q=10 \) is 0.0301.

![Figure 7. Percentage of harmonic voltage without the phase angle.](image)

Figure 7. Percentage of harmonic voltage without the phase angle.

![Figure 8. Percentage of harmonic voltage considering the phase angle.](image)

Figure 8. Percentage of harmonic voltage considering the phase angle.

| Eigenvalue | Regression error | Average value | 95% probability value |
|------------|------------------|---------------|-----------------------|
| Given phase angle | \( Q=3 \) | 0.0037 | 31.5202 | 52.8318 |
| | \( Q=10 \) | 0.0301 |

Table 1. Statistical characteristic values of harmonic voltage contribution.

As can be seen from Figure 7, Figure 8 and Table 1, the larger the number of segments, the larger the regression error. The magnitude of harmonic voltage contribution is not only related to the magnitude of each vector, but also related to the phase angle and the number of segments. Therefore, the calculation result of harmonic voltage responsibility may exceed 100% or may be negative. In actual engineering, if the system side is connected to heavy polluting load such as electric arc furnace, the harmonic voltage responsibility injected into the user side is very likely to exceed 100%.

5. Conclusion
In this paper, assuming that the system side is dominant, a combination of partial linear regression model and constant impedance model is constructed to consider the angular phase difference, while tracking the regression error and calculating its corresponding times of harmonic impedance through fundamental impedance equivalence, and then dividing the harmonic responsibility. Theoretical analysis and arithmetic results show that the present method has a simple calculation model, the regression error is much smaller than the given accuracy, and the accuracy of the results is high, which solves some shortcomings of other methods. This study takes into account the harmonic responsibility on the system side, contributing to the division of harmonic responsibility and the apportionment of economic losses, and promoting the realisation of a multifaceted dynamic evaluation index system in China and abroad. Since the load is time-varying, there is no accurate calculation method for
estimating the harmonic impedance of users, and there are few related studies, the model constructed in this paper needs to be further optimized to make its application wider and the calculation results more accurate.

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