An estimation of local bulk flow with the maximum-likelihood method

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\begin{abstract}
A maximum-likelihood method, tested as an unbiased estimator from numerical simulations, is used to estimate cosmic bulk flow from peculiar velocity surveys. The likelihood function is applied to four observational catalogues (ENEAR, SFI++, A1SN and SC) constructed from galaxy peculiar velocity surveys and Type-Ia supernovae data at low redshift ($z \leq 0.03$). We find that the Spiral Field $I$-band catalogue constrains the bulk flow to be $V = 290 \pm 30 \, \text{km} \, \text{s}^{-1}$ towards $l = 281^\circ \pm 7^\circ$, $b = 8^\circ \pm 6^\circ$ on effective scales of $58 \, h^{-1}\text{Mpc}$, which is the tightest constraints achievable at the present time. By comparing the amplitudes of our estimated bulk flows with theoretical prediction, we find excellent agreement between the two. In addition, directions of estimated bulk flows are also consistent with measurements in other studies.

\textbf{Key words:} methods: data analysis – methods: statistical – galaxies: kinematic and dynamics – cosmology: observations – large-scale structure of Universe
\end{abstract}

\section{INTRODUCTION}
Cosmic bulk flow is the coherent motion of galaxies and galaxy clusters towards a particular direction. Since magnitude and direction of bulk flow are determined by the underlying density field at large scales, it serves as a direct probe of the large-scale structure of the Universe. There have been a lot of recent studies focusing on estimating bulk flow from a variety of observational probes, such as galaxy peculiar velocity surveys (Sarkar, Feldman, & Watkins 2007; Springob et al. 2007; Watkins et al. 2008; Feldman et al. 2011; Nuess & Davis 2011), Type Ia supernovae data (Sandage et al. 2007; Dai et al. 2011; Turnbull et al. 2012) and galaxy clusters with observations of the cosmic microwave background (CMB) radiation (Kashlinsky et al. 2011, 2011). However, amplitudes and directions of bulk flows at different depths in our local Universe obtained from different measurements do not reach good convergence. Some works have argued that the amplitude of the bulk flows they found is too high compared to the standard $\Lambda$ cold dark matter ($\Lambda$CDM) predictions (Watkins et al. 2009; Feldman et al. 2010; Kashlinsky et al. 2011, 2012), which has stimulated a lot of interest in looking for possible explanation in new physics (Afshordi et al. 2003; Mersini-Houghton & Holman 2005; Ma, Gordon, & Feldman 2011).

However, any analysis which claims to strongly rule out the simple inflationary $\Lambda$CDM model should be subject to careful examination, since a confirmed large-scale flow would have profound impact on our understanding of the large-scale structure of the Universe. Watkins et al. (2009) and Feldman et al. (2010) adopted the minimal variance weighting method to estimate bulk flows from their combined galaxy catalogues, declaring discovery of an excess power of flow $V = 407 \pm 81 \, \text{km} \, \text{s}^{-1}$ towards $l = 287^\circ \pm 9^\circ$, $b = 8^\circ \pm 6^\circ$ on a Gaussian window of $50 \, h^{-1}\text{Mpc}$ (corresponds to a top-hat window function of $\sim 100 \, h^{-1}\text{Mpc}$). But, by correcting Malmquist bias, selecting high-quality samples, and combining different data sets with the Bayesian hyper-parameter method, Ma & Scott (2013) found that there is no real excess power of flow on $50 \, h^{-1}\text{Mpc}$ ($V \sim 310 \, \text{km} \, \text{s}^{-1}$, $l = 280^\circ \pm 8^\circ$, $b = 5^\circ \pm 6^\circ$) and the estimated amplitude of density fluctuation $\sigma_8 = 0.65^{+0.17}_{-0.15} (1\sigma)$ is consistent with \textit{Wilkinson Microwave Anisotropy Probe} (\textit{WMAP}) 7-yr results (Komatsu \textit{et al.} 2011). In Ma & Scott (2013), the minimal variance method is extended to include bulk flows in shells at different distances (20–100 $h^{-1}\text{Mpc}$) and a likelihood function is formulated to combine all of these reconstructed shell velocities, the multishell likelihood method yields constraints on cosmological parameters of $\sigma_8 = 1.01^{+0.20}_{-0.20}$ and $\Omega_m = 0.31^{+0.25}_{-0.14}$ (based on the Spiral Field $I$-band catalogue, in abbreviation SFI++), which are consistent with WMAP 7-yr results very well.

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The recent estimation of bulk flow based on the ‘First Amendment’ compilation of 245 Type Ia supernovae found that bulk flow in the nearby Universe (a Gaussian window of 58 h$^{-1}$Mpc) is of 249 ± 76 km s$^{-1}$ in the direction $l = 319^\circ ± 18^\circ$, $b = 7^\circ ± 14^\circ$ (Turnbull et al. 2012), which is in good agreement with the expectation for the ΛCDM model ($V \sim 250$ km s$^{-1}$).

Although the detailed analysis with the minimal variance and multishell likelihood methods in Ma & Scott (2013) is in itself already a strong support to dispense the suspicion of a very large local bulk flow, it is still worthwhile to apply a different method to the same set of catalogues to check the robustness and reliability of the reconstructed bulk flow, and test the consistency between different methods. In this paper, we will use a different bulk flow reconstruction method, aka the maximum-likelihood method to calculate the bulk flows of several peculiar velocity catalogues. Furthermore, we will compare the reconstructed flows with the theoretical prediction for the ΛCDM cosmology model and investigate the tendency of the cosmic flow as a function of sample depths with currently available peculiar velocity catalogues.

This paper is organized as follows. Theoretical prediction of bulk flow on various depths $R$ and the maximum-likelihood method are presented in section 2. Introduction to the peculiar velocity samples and Malmquist bias correction is in section 3 together with specification on calculating effective depth of a sample from the geometry of the peculiar velocity survey. Section 4 shows the results of the constraints on the cosmic bulk flows by applying the likelihood function to the velocity catalogues, and the comparison against theoretical predictions. Our conclusion is in the last section.

Throughout the paper, we assume a spatially flat cosmology with WMAP 7-yr best-fitting parameter values (Komatsu et al. 2011), i.e. fractional matter density $\Omega_m = 0.2735$, fractional baryon density $\Omega_b = 0.0455$, Hubble constant $h = 0.704$, power-law index of scalar power spectrum $n_s = 0.967$ and amplitude of fluctuation $\sigma_8 = 0.811$.

## 2 Bulk Flow Model

### 2.1 Theoretical Prediction

In the linear theory of structure formation, the velocity field $\mathbf{v}(r, t)$ is related to the underlying density field by Peebles (1993)

$$
\mathbf{v}(r, t) = \frac{f(t)H(t)}{4\pi} \int \text{d}^3r' \delta_m(r', t) \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3},
$$

where $\delta_m(r) = (\rho(r) - \langle \rho \rangle)/\langle \rho \rangle$ is the density contrast at position $\mathbf{r}$, $f(t) = d\log D(t)/d\log a(t) \simeq \Omega_m^{1/2} + (1 + \Omega_m/2)/\Omega_m/70$ is the logarithmic derivative of the linear growth rate (Lahav et al. 1991; Dodelson 2003) and $H(t)$ is the Hubble parameter. Since the bulk flow we investigate is the streaming motion of very nearby objects, and the samples are within distance of 150 h$^{-1}$Mpc of our local volume, we take the cosmic time $t'$ in Eq. 1 to be our present time $t_0$, thus the Hubble parameter becomes Hubble constant $H_0$ at $a = 1$. The bulk flow $\mathbf{V}$ is the coherent motion of observed galaxies or galaxy clusters. Mathematically, $\mathbf{V}$ is the velocity field filtered by the window function defined by the geometry of the observational sample, and is actually determined by mass distribution outside the sample space (Juszkiewicz, Vittorio, & Wyse 1990; Nusser & Davis 1994; Li et al. 2012). The root-mean-square(hereafter rms) of the bulk motion ($V_{\text{rms}}$) on scale of $R$ is the velocity power spectrum filtered by the observational window function (Coles & Lucehind 2002)

$$
V_{\text{rms}}^2(R) = \langle |\mathbf{V}(r, t, R)|^2 \rangle = \frac{1}{(2\pi)^3} \int P_{\text{vv}}(k)W^2(R) d^3k ,
$$

where the $W(kR)$ is the Fourier transform of the real space selection function with size $R$. In linear regime, the velocity power spectrum at present epoch $a = 1$ is (Sarkar, Feldman, & Watkins 2007)

$$
P_{\text{vv}} = \frac{(H_0 f(t_0))^2}{k^2} P(k),
$$

where $P(k)$ is the linear power spectrum which in our calculation is generated by the software package CAMB (Lewis et al. 2000). Substituting Eq. (3) into Eq. (2) and adopting the simple top-hat window function $W(x) = 3(\sin x - x \cos x)/x^3 = 3J_1(x)/x$ (where $J_1(x)$ is the first spherical Bessel function), the rms of bulk velocity in a spherical region $R$ becomes (see also Ma, Ostriker, & Zhao 2012) for derivation

$$
V_{\text{rms}}^2(R) = \frac{(H_0 f(t_0))^2}{(2\pi)^3} \int W^2(kR) \frac{P(k)}{k^2} d^3k = \frac{(3H_0 f_0)^2}{2\pi^2} \int P(k) \left( \frac{J_1(kR)}{kR} \right)^2 dk .
$$

Equation (4) is the filtered velocity power spectrum in real space, which retains large-scale modes of perturbations. Bulk velocity rms of wider window is smaller than the one for narrower size window, because more modes are smeared out. Typical rms of bulk velocity $V_{\text{rms}}$ in ΛCDM model from top-hat window at 20 h$^{-1}$Mpc is $\sim 350$ km s$^{-1}$, while at 60 h$^{-1}$Mpc is $\sim 240$ km s$^{-1}$.

Now, given the filtered velocity rms on scale of $R$ (Eq. (4)), what is the probability distribution of the bulk flow magnitude on this scale? To address this question, we start from the 3D probability function of the bulk flow velocities in Cartesian coordinate. The Cartesian components of bulk flow should be Gaussian distributed, with zero means and variances of $V_{\text{rms},x}$, $V_{\text{rms},y}$, $V_{\text{rms},z}$ respectively, assuming null correlation between the three components, the probability distribution function of bulk flow $\mathbf{V}$ is

$$
p(\mathbf{V}) = p(V_x, V_y, V_z)
\propto \exp \left[-\frac{1}{2} \sum_{x,y,z} \left( \frac{V_x}{V_{\text{rms},x}} \right)^2 \right].
$$

In an isotropic and homogeneous universe, the velocity field possesses the property of $V_{\text{rms},x}^2 = V_{\text{rms},y}^2 = V_{\text{rms},z}^2 = V_{\text{rms}}^2/3$, therefore the probability distribution of bulk flow with magnitude $V$ becomes (Bahcall et al. 1994; Coles & Luehind 2002)

$$
p(V) dV = \int |p(\mathbf{V})| d\Omega_V |V|^2 dV = \sqrt{\frac{34}{\pi}} \left( \frac{V}{V_{\text{rms}}} \right)^2 \exp \left[-\frac{3}{2} \left( \frac{V}{V_{\text{rms}}} \right)^2 \right] \left( \frac{dV}{V_{\text{rms}}} \right) .
$$

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where the final line of equation is properly normalized. So the amplitude of the bulk flow actually follows the Maxwell-Boltzmann distribution, which is skewed and has long tail on the large velocity branch. The peak of the distribution is $V_0 = \sqrt{2/3} V_{\text{rms}}$, which is obtained by taking $dP(V)/dV = 0$. One can also calculate the asymmetric variance of velocities on different depths (Li et al. 2012).

In Fig. 3 we plot the peak and $\pm 1\sigma$ variance of the bulk velocity magnitude as a function of scale $R$ in solid line and dashed lines respectively. One can clearly see that the bulk motion amplitude decreases with increasing $R$. This is because for the top-hat window function $W(x) \simeq 1$ if $x < 1$, but $W(x) \simeq 0$ if $x > 1$, the upper limit in Eq. (2) is $R^{-1}$.

Therefore, a large volume (large $R$) would result in a relatively small value of bulk flow rms. In addition, the smaller the scale is, the larger the variance of the bulk flow is. This is the effect of the sample variance, because if the velocity field is filtered on a smaller scale, larger variance of this filtered velocity will be. If one averages the peculiar velocity over the whole Universe ($R \to \infty$), the average velocity should be fairly close to zero, if the primordial perturbations are adiabatic Gaussian as assumed in the concordance LCDM model.

We need to mention that, the model of Equations (3) and (4) for peculiar velocity and subsequent bulk flow is made under the ‘single-particle’ assumption, i.e. the galaxies do not strongly correlate with each other and therefore Maxwellian-Boltzmann distribution can be used to describe its behaviour. Since for our peculiar velocity catalogues, the data are quite sparse and not very correlated on small scales, our assumption is a good approximation. On the other hand, in the regime where gravitational clustering of the galaxies and their collisions cannot be negligible, one needs to look in to the scale dependence of the small-scale modes and then consider the correlation between small and large scales (i.e. the gravitational quasi-equilibrium distribution method (Ravindhury & Saslaw 1996; Ahmad et al. 2002; Leong & Saslaw 2004; Sivakoff & Saslaw 2005; Saslaw & Ahmad 2010; Leong & Saslaw 2004). Since we are most interested in large-scale bulk flows of which the small-scale velocity dispersion is smoothed out, we will not get involved into details of gravitational clustering properties in this paper.

### 2.2 The maximum-likelihood method

Now, let us move on to the issue of computing the likelihood of the magnitude and direction of the bulk flow. In general, for a peculiar velocity survey with $N$ number of objects (galaxies, galaxy clusters or Type Ia supernovae), of the $n$th object we can obtain its redshift $z_n$, distance $r_n$ (utilizing the empirical relation such as Tully-Fisher relation or the Fundamental Plane method, see Section 3.1), the line of sight velocity $V_n$ and its measurement error $\sigma_n$, and the Galactic

$$L(V, \Omega_V, \sigma_V) = L(V, \sigma_V)V^2$$

$$= \prod_{n=1}^{N} \frac{V^2}{\sigma_V^2 + \sigma_n^2} \exp \left[ -\frac{1}{2} \frac{1}{\sigma_V^2 + \sigma_n^2} \left( S_n - V \right) \right]$$

To assess performance of this likelihood function, we simulate 300 mock catalogues and test the behaviour of the likelihood with these simulated data. In each mock catalogue, we simulate 100 Type-Ia supernovae data as one data set. The way we simulate each data set is as follows. We assume that in each data set, the supernovae share a bulk flow velocity ($V = 500 \text{ km s}^{-1}$) towards the direction of $(\cos(\theta) = 0.5, \phi = 4.0)$, while each line of sight veloc-

$$\theta = \text{the angle measured from } z = 0, \text{ and } \phi = \text{the azimuthal}$$

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ity has $\sigma_\star = 400 \text{ km s}^{-1}$ random motion. We also take the measurement error of 100 samples in the “First Amendment Type-Ia Supernovae” (in abbreviation ‘A1SN’) catalogue (see Section 3.1) as the measurement errors in our simulated data sets because these quoted errors are realistic representatives of the noise of Type-Ia supernovae. Therefore, in each mock catalogue, we simulate the line of sight velocity of 100 Type-Ia supernovae samples, which share a streaming motion while each has both random error and measurement error.

We then use each mock catalogue to constrain $(V, \sigma_\star, \cos(\theta), \phi)$ parameters, and plot the marginalized likelihood of each parameter. The constraints from four mock catalogues are demonstrated with green, black, blue and brown lines in Fig. 1. All of these distribution functions are perfectly peaked at the input values of preset parameters, which at least numerically prove that the maximum-likelihood method (Eq. 8) can produce unbiased estimates of the bulk flow.

There are other methods designed to measure bulk flow from peculiar velocity surveys than the maximum-likelihood method. The ‘All Space Constrained Estimate’ (ASCE, Nusser & Davis 2011) is one of such methods. The method is proposed in consideration of the observational limitation that distance indicators of peculiar velocity survey, such as Tully-Fisher relation and the Fundamental Plane method, can only probe a small fraction of galaxies around our local Universe. Nusser & Davis (2011) confirmed the validity of this method with their mock catalogues.

Another method, proposed by Branchini et al. (2012), is to use the galaxy luminosity function at different redshifts to fit the bulk flow velocity. Redshifts of the object may be biased by the Kaiser rocket effect, but Branchini et al. (2012) provides an analytical tool to correct this bias, and claims that it can lead to an unbiased reconstruction of bulk flows.

We will compare our reconstruction of the bulk flow.

Figure 1. A test of the likelihood function (Eqs. 7 and 8). $(\cos(\theta),\phi)$ is the direction of the bulk flow ($\theta = \pi/2 - b, \phi = l$). The description of this test is in Section 2.2.
with those found in previous studies [Watkins et al. 2000, Feldman et al. 2010, Nusser & Davis 2011; Branchini et al. 2012] (Turnbull et al. 2012, Ma & Scott 2013) in section 4.1.

3 PECULIAR VELOCITY SAMPLES AND RELEVANT TREATMENT

3.1 Peculiar velocity catalogues

Four different peculiar velocity catalogues from recent surveys are adopted to estimate bulk flows. The four samples are Early-type NEAR by galaxies (in abbreviation ENEAR, characteristic depth $d = 29 h^{-1}$Mpc, typical distance error $\sim 20$ per cent; da Costa et al. 2004; Bernardi et al. 2002; Wenger et al. 2003; Hudson 1994), SFI++ ($34 h^{-1}$Mpc, $\sim 23$ per cent; Springob et al. 2007), A1SN ($58 h^{-1}$Mpc, $\sim 8$ per cent; Jha et al. 2007; Hicken et al. 2009; Folatelli et al. 2011; Turnbull et al. 2012) and SC catalogue ($57 h^{-1}$Mpc, $\sim 20$ per cent; Giovanelli et al. 1998; Dale et al. 1999)). For details of these samples, including characteristic depths, typical distance errors and data compilation, please refer to section 3 of Ma, Branchini, & Scott (2012) and section 2 of Watkins et al. (2000).

In Feldman et al. (2010) and Watkins et al. (2009), there are five other catalogues employed, namely the SBF (Tonry et al. 2003), SN (Tonry et al. 2003), SMAC (Hudson 1994; Hudson et al. 2004), EFAR (Colless et al. 2001) and Willick sample (Willick 1999). Here we opt to abandon these five catalogues, and the reasons are as follows. For SMAC, EFAR and Willick, these samples are either very distant, in which case the distance errors are very large, or too sparse to support robust estimation, and their survey geometry is so complicated that make it hard to measure. In addition, as the survey goes deeper, the simple model of assuming Gaussian errors of distances is almost certainly inappropriate, and will become a dominant systematic effect in the distance estimation; velocity data beyond 100 $h^{-1}$Mpc are thus too noisy to reliably reconstruct bulk flow. For SBF data, it is too close to our own galaxy, some galaxies fall into our local non-linear structures, therefore it could strongly bias our estimation of bulk velocity on large scales. Since we will use the newly compiled A1SN catalogue (see Turnbull et al. 2012 and Ma, Branchini, & Scott 2012) which includes three Type-Ia supernovae data sets, we will not use its old sub data set, the SN set (Tonry et al. 2003), in our study.

3.2 Malmquist bias correction

In the catalogues described above, there are three different classes of distance indicators, the Tully-Fisher relation (SFI++ as SC), the Fundamental Plane method (ENEAR) and the Type-Ia SN luminosity function (A1SN). These distance indicators all have their intrinsic errors. For Tully-Fisher selected samples, such as SFI++ and SC, their distance errors are around 23 per cent, which is slightly larger than the distance error of the Fundamental Plane-selected NEAR sample ($\sim 18$ per cent). For Type-Ia supernovae data, the luminosity function can be used to calibrate the distance in better precision, the distance errors of A1SN catalogue are only $\sim 7$ per cent. The uncertainty of distance indicators, especially for Tully-Fisher and Fundamental Plane-selected objects, suggests that an object with its measured distance $d$ may actually deviate from its true distance by a broad range of possible values. This is the effect of Malmquist bias [Malmquist 1920], which characterizes the fact that inhomogeneous distributions of matter and distance (or magnitude) errors can in general bias the distance (magnitude) measurement. As a result, the probability function of the true distance $r$ given the measured distance $d$ strongly depends on the intrinsic errors of distance indicators, and the underlying density distribution [Malmquist 1920; Lynden-Bell et al. 1985]. Taking the IRAS-PSCz (Point Source Catalogue with redshift) catalogue which probes the full-sky underlying density field out to 192 $h^{-1}$Mpc as the model of cosmic matter distribution, we follow the guideline in section 3.1 of Ma, Branchini, & Scott (2012) and section 2.3 of Ma & Scott (2013) to correct Malmquist bias for A1SN, SC and NEAR catalogues. Note that the SFI++ catalogue (Springob et al. 2007) is already corrected for Malmquist bias.

Once the Malmquist bias is corrected, our next step is to select samples. In the four catalogues, objects with distance beyond 100 $h^{-1}$Mpc are very sparse and suffer from large errors due to uncertainties in the distance indicators, which are consequently discarded from the sample. Additionally, several SFI++ galaxies with $d \lesssim 30 h^{-1}$Mpc are strongly affected by local non-linear structures, showing very large velocities [Ma, Branchini, & Scott 2012], we also excluded these high-velocity members ($|v| > 3000$ km s$^{-1}$) from the SFI++ catalogue since they are clearly close to some local non-linear structures. Our final samples for the maximum-likelihood analysis are listed in Table 1.

### Table 1. Final samples for analysis extracted from the four peculiar velocity catalogues

| Catalogue | $d \leq 100$ | $d > 100$ | $R_{\text{min}}$ | $R_{\text{max}}$ | $b_{\text{cut}}$ |
|-----------|-------------|-----------|-----------------|-----------------|-----------------|
| NEAR      | 690 7       | 6 100     | 3.6             |                 |                 |
| A1SN      | 175 100     | 5 100     | 0               |                 |                 |
| SFI++     | 2915 541    | 0 100     | 8               |                 |                 |
| SC        | 28 42       | 13 90     | 18              |                 |                 |

3.3 Geometry of the survey

In Fig. 2 we show the histogram of the distances for each sample along the radial direction (upper two panels), and spatial distribution of the four samples on the sky (lower two panels). From the upper two panels of Fig. 2 one can see that the four samples have different distance hist-
Figure 2. Upper two panels: the distance histogram of the four catalogues; lower two panels: the distribution of the ENEAR, A1SN, SFI++ and SC samples on the sky. The data are quite homogeneous across the full sky therefore robust tests of local anisotropy can be made.

togram, the ENEAR catalogue is effectively the shallowest sample with the median distance around $40\, h^{-1}\text{Mpc}$, while the A1SN and SFI++ catalogues all have median distance around $50\, h^{-1}\text{Mpc}$. The lower two panels show that the four catalogues have nearly coverage the full sky, except for the small blank region along the galactic plane. Geometry information of the four peculiar velocity catalogues are tabulated in Table D.

Now let us turn into the issue of calculating characteristic depth of each catalogue. The galaxy peculiar velocity survey can probe only limited depth with full or partial sky coverage. Therefore, the characteristic depth of the samples are strongly affected by this effective survey volume. Ma & Scott (2013) and Turnbull et al. (2012) calculated the effective depth as the average of distances of all member objects. They weighted the distance of every object with the square of the inverse of its distance error, i.e.

$$\mathcal{T} = \frac{\sum_n r_n / \sigma_n^2}{\sum_n 1 / \sigma_n^2} .$$

However, the weighted-average distance does not take into account the radial distribution of the survey, as well as the influence of partial sky coverage. In this work, we adopt an alternative approach for the characteristic depth calculation proposed by Li et al. (2013) to take care of these effects. Considering the real survey geometry (Table 1) and the radial distribution function, we identify that the ‘true’ survey window function gives

$$\hat{W}(k) = \int W_{\text{true}}(\mathbf{x}) e^{-ik\cdot\mathbf{x}} d^3\mathbf{x} ,$$

which can be plugged into Eq. (2) to yield an effective rms of bulk flow velocity (detailed calculation is in Appendix A)

$$\hat{V}_{\text{rms}}^2(R) = \frac{1}{(2\pi)^3} \int P_{\text{cc}}(k) \hat{W}^2(k) d^3k .$$

The value of this velocity rms is the expectation of linear theory for the true window function. The effective depth of the sample is defined as the radius $R$ of the top-hat window function which offers the same theoretical velocity rms (Eq. (9)) as the true window function does. The effective depth is so in the sense that it filters the same modes of perturbation as the true survey window function. We list our findings of effective depth in the first column of Table 2. This characteristic depth will be used to locate the position of the bulk flow magnitude on the velocity–distance diagram (Fig. 3).

4 RESULTS

4.1 Reconstructed bulk flow

The likelihood function (Eqs. 7 and 8) is applied for estimation of $(V,\cos(\theta),\phi,\sigma_{\phi})$ to the four peculiar velocity samples. In Fig. 3 we plot the constraints on the bulk flow $V$ (panels a, d and e), the small-scale dispersion $\sigma_{\phi}$ (panel b), and joint likelihood contours on planes of $(V,\sigma_{\phi})$ and $(\cos(\theta),\phi)$ (panels c and f).
Maximum likelihood of cosmic bulk flow

From Fig. 3, one can see that since different surveys probe different volumes of the Universe, peaks of the likelihood functions locate at different values, reasonable comparison ought to be made together by their characteristic depths. Comparison of the constraints with the theoretical model is in Section 4.2.

In Fig. 3b, we plot the likelihood of the small-scale intrinsic velocity dispersion. It is apparent that each catalogue prefers different $\sigma_*$. For the Type-Ia supernovae sample (A1SN) and the SC catalogue, $\sigma_*$ is around 250 km s$^{-1}$, but for the SFI++ and the ENEAR, $\sigma_*$ is around 400 km s$^{-1}$. The value of $\sigma_*$ reflects the disturbance on very small scales, whilst bulk motion reflects perturbation on large scales. Thus, the bulk motion $V$ and the $\sigma_*$ should not correlate with each other, which is verified by the (nearly) orthogonal contours shown in Fig. 3b.

We further plot the likelihood of the direction angles $\cos(\theta)$ (Fig. 3c) and $\phi$ (Fig. 3d), and their correlation contours (Fig. 3f). By comparing the $(\cos(\theta), \phi)$ contours in Fig. 3c, we can find that the direction angles constrained by our A1SN, ENEAR and SFI++ catalogues are pretty well consistent with the Type-Ia supernovae constraints by Dai et al. (2011), the SFI++ constraints by Nusser & Davis (2011), the combined catalogue constraints by Watkins et al. (2008) and Ma, Gordon, & Feldman (2011), and the reconstructed Two-Micron All-Sky Redshift Survey density field Kitaura et al. (2012).

We plot our results of the constraints (Table 2) in Fig. 4 together with the predictions of the $\Lambda$CDM model. The solid line is the peak ($V_0$) of the distribution (Eq. 3), and the dashed lines are the ±1σ confidence interval. One can see that the data are consistent with the expectation for the $\Lambda$CDM cosmology. Note that the SFI++ catalogue, with its nearly full-sky coverage and dense sampling, provides the tightest constraint of the bulk flow amplitude. In addition, by comparing our constraints with the other studies in Table 4 and the earlier works (Courteau et al. 1997; Willick et al. 1997; Courteau et al. 2001) by using Mark III and Shallflow catalogues, we can see that they all provide constraints on bulk flow amplitude ($\sim 300$ km s$^{-1}$) on scales of $50$ h$^{-1}$ Mpc that are consistent with the prediction for the $\Lambda$CDM model.

If the data are improved by prospective new surveys, such as 6dF survey Jones et al. (2003) or Square Kilometer...
5 CONCLUSION

As introduced in Section 4, bulk flow is the coherent motion of sampled galaxies, galaxy clusters or supernovae, which can be used as a test of the growth of structure. Yet there are some tentative observational evidences from the peculiar velocity surveys and the CMB observation suggesting possible excess power on scales around 50 h⁻¹Mpc (in radius). Here, we show that data of current peculiar velocity surveys actually do not provide strong evidence against the ΛCDM model.

In this paper we adopted a maximum-likelihood method to peculiar velocity catalogues for the bulk flow estimation. Different from just using the ‘peak’ of the maximum-likelihood method as in Kaiser (1988), we employ the full likelihood function with simulated data sets and the state-of-the-art peculiar velocity survey. Numerical test with simulations indicates that the estimator is unbiased in the limit that no complicated survey geometry is involved, which is approximately true for the four catalogues.

We apply our likelihood function to the four catalogues, ENEAR, SFI++, A1SN and SC all of which are Malmquist bias corrected and properly trimmed, to obtain the magnitude and direction of the bulk flows. We find: (1) for the largest and densest Tully-Fisher selected catalogue, SFI++ survey constrain the magnitude of bulk flow as $V = 290 ± 50$ km s⁻¹ towards $l = 281° ± 7°$ at an effective depth of 58 h⁻¹Mpc, $b = 8° ± 6°$; (2) for the largest Fundamental Plane selected catalogue, ENEAR samples constrain the bulk flow as $V = 250 ± 50$ km s⁻¹ towards $l = 314° ± 14°$, $b = 6° ± 1°$ at an effective depth 49 h⁻¹Mpc. Directions of the bulk flow we find here are well consistent with the previous probes, while amplitudes of estimated bulk flows confirm an earlier investigation with the same data sets but different estimation method [Ma & Scott(2013)].

From the geometry of the selected peculiar velocity sam-

![Graph showing velocity magnitude versus redshift](image)

**Figure 4.** The comparison between the velocity magnitude from the likelihood function (Eq. (5)), and the theoretical evaluation (Eq. (6)) given different depths of survey.

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Table 2. The results of constraints from four catalogues. The $R$ is the effective top-hat window size of the sample after the Malmquist bias correct and sample selection. The error bars listed are quoted for 1σ confidence level (CL).

| Catalogues | $R$ (h⁻¹Mpc) | $V$ (100 km s⁻¹) | $\sigma$ (100 km s⁻¹) | $l$ (°) | $b$ (°) | Method | References |
|------------|--------------|-----------------|-----------------------|--------|--------|--------|------------|
| ENEAR      | 49           | $2.5 ± 0.5$     | $4.2 ± 0.3$           | $314 ± 14$ | $-6°^{+11}_{-9}$ | MLE     | This study |
| SFI++      | 58           | $2.9 ± 0.3$     | $3.7 ± 0.2$           | $281 ± 7$  | $8°^{+6}_{-5}$   | ASCE    | Nusser & Davis (2011) |
| A1SN       | 62           | $2.3 ± 0.5$     | $2.5 ± 0.3$           | $296 ± 16$ | $15°^{+13}_{-12}$| ASCE    | Nusser & Davis (2011) |
| SC         | 63           | $1.9^{+1.2}_{-0.9}$ | $2.9^{+0.8}_{-0.6}$ | $231°$  | $-2°^{+35}_{-31}$| MCVF    | Haugboelle et al. (2007) |

Table 3. Comparison of the reconstructed bulk flow of the top-hat window function with the studies in the literature. The error bars listed are for 1σ CL. ‘MLE’ stands for ‘Maximum-Likelihood Estimate’; ‘ASCE’ stands for ‘All Space Constrained Estimate’; ‘MCVF’ stands for the method of extracting Multipole Components of the Velocity Field. The sample used in Haugboelle et al. (2007) is the Type-Ia supernovae data from Hicken et al. (2009) and Hsu et al. (2007) respectively.

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Array (Square Kilometre Array), the data can be used to constrain any possible deviation from general relativity, i.e. the standard gravity theory. This is because any alternative theory of gravity would change the growth rate of structure, which will boost or diminish the power of the velocity field on intermediate scales ($0.01 \text{hMpc}^{-1} \lesssim k \lesssim 100 \text{hMpc}^{-1}$) (see fig. 2 in Ma, Ostriker, & Zhao 2012). While doing the constraints on the modified gravity model, one needs to keep in mind of the sample variance at different scales, which is plotted as the dashed line in Fig. 4. For each scale $R$, there is a certain level of uncertainty of fluctuations which reflects the variation of the number of velocity modes filtered by the window function. The sample variance limits the capacity of the reconstructed bulk flow to infer the underlying physics, one needs to consider this variance term in the full covariance matrix when the bulk flow is used to constrain cosmology.
Maximum likelihood of cosmic bulk flow

| Samples | Numbers | *R* (h$^{-1}$Mpc) | *V* (100 km s$^{-1}$) | *l* (°) | *b* (°) | Method | References |
|---------|---------|------------------|----------------------|---------|---------|--------|------------|
| SFI++   | 2404    | 50.0             | 3.4 ± 0.4            | 280 ± 8 | 5.1 ± 6 | MV     | Ma & Scott (2013) |
| SFI++   | 3401    | 34.0             | 4.3 ± 1.0            | N/A     | N/A     | MV     | Watkins et al. (2009) |
| ENEAR   | 669     | 50.0             | 2.2 ± 0.6            | 310 ± 30| −9.8 ± 14| MV     | Ma & Scott (2013) |
| A1SN    | 153     | 50.0             | 2.2 ± 0.7            | 290 ± 60| 12.1 ± 60| MV     | Ma & Scott (2013) |
| A1SN    | 245     | 58.0             | 2.5 ± 0.7            | 319 ± 18| 7 ± 14  | MV     | Turnbull et al. (2012) |
| SN      | 557     | 150.0            | 1.9$^{+1.2}_{-1.0}$  | 290$^{+39}_{-31}$| 20 ± 32 | MCMC  | Dai et al. (2011) |
| SN      | 112     | 40.0             | 5.4 ± 0.9            | 258 ± 10| 36 ± 11 | WLS    | Weyant et al. (2011) |
| SN      | 112     | 40.0             | 4.5 ± 1.0            | 273 ± 11| 46 ± 8  | CU     | Weyant et al. (2011) |
| COMPO   | 4356    | 50.0             | 4.1 ± 0.8            | 287 ± 9 | 8 ± 6   | MV     | Watkins et al. (2009) |

Table 4. Comparison of the reconstructed bulk flow of the Gaussian window function with the studies in the literature. The error bars listed are for 1σ CL. ‘MLE’ stands for ‘Maximum-likelihood Estimate’; ‘MV’ stands for the ‘Minimal Variance’ method; ‘MCMC’ stands for the Bayesian Markov Chain Monte Carlo method; ‘WLS’ stands for ‘weighted least squares’. ‘CU’ stands for the ‘coefficient unbiased’ method. The sample used in Weyant et al. (2011) is the Union2 catalogue of Type-Ia supernovae (Amanullah et al. 2010), and the samples used in [Weyant et al. 2011] are the Type-Ia supernovae data from Hicken et al. (2009) and Jha et al. (2007) respectively. The final row shows the result of COMPOSITE data set, which is a combined catalogue from eight different peculiar velocity surveys (SBF, ENEAR, SFI++, SN, SC, SMAC, EFAR, Willick).

A1SN

In Section 3.3, we want to take the specific survey volume into account and calculate its real window function and rms of the velocity. We identify the true survey window function as

\[
W_{\text{true}}(x) = W(x)n(x),
\]

where \(n(x) = n(r, \theta, \phi)\) is the normalized density distribution

\[
\int n(x) \, d^3x = 1.
\]

If assuming the angular distribution of samples is isotropic, the radial distribution becomes \(\tilde{n}(r) = 4\pi r^2 n(r)\). But what we are interested is the “true” survey window function, by taking into account of the effective sample depth \((R_{\text{min}}, R_{\text{max}})\) and partial sky coverage \(b_{\text{cut}}\)\(^7\).

\[
\tilde{W}(k) = \frac{1}{Vol} \int W(x)n(x) \cos(kr(\theta \cdot \hat{\theta}_k))
\times r^2 \sin(\theta) \, dr \, d\theta \, d\phi,
\]

where we have used the real part of the plane wave \(\exp(ik \cdot x)\) as the Fourier transform kernel. Here we express the \(k = (k, \theta_k, \phi_k)\), and \(Vol\) is the volume of the survey. Substituting the survey geometry and the cosine angle of \(\theta\) and \(\theta_k\) (i.e. \(\theta \cdot \hat{\theta}_k = \cos \theta \cos \theta_k + \sin \theta \sin \theta_k \cos \phi\)), the radial distribution Eq. (A3) becomes

\[
\tilde{W}(k) = \frac{1}{4\pi Vol} \int_{R_{\text{min}}}^{R_{\text{max}}} \tilde{n}(r)
\times \cos(\rho \rho) \Theta(\cos \theta - \sin(b_{\text{cut}})) \sin(\theta) \, dr \, d\theta \, d\phi,
\]

where \(\Theta(x)\) is the Heaviside step function. The \(Vol\) in the denominator is the average factor, which ensure the \(\tilde{W}(k)\) is properly normalized

\[
Vol = \int W(x)n(x) \, d^3x
\]

\[
= \left[ \int_{R_{\text{min}}}^{R_{\text{max}}} \tilde{n}(r) \, dr \right]
\times \left[ \frac{1}{2} \int_{-1}^{1} \Theta(\cos \theta - \sin(b_{\text{cut}})) \, d\cos \theta \right]
\times \left[ \int_{R_{\text{min}}}^{R_{\text{max}}} \tilde{n}(r) \, dr \right] \times (1 - \sin(b_{\text{cut}})).
\]

Then we can substitute Eqs. (A4) and (A5) into Eq. (11) to calculate the rms of the bulk flow velocity \(V_{\text{rms}}\) corresponding for the true survey volume and samples.

\(^7\) Since we are unclear about the real angular selection function, we assume that it is uniformly distributed above the sky-cuts. If the angular distribution of the survey is completely known, one can substitute it into Eq. (A3) and calculate the corresponding filter.
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