The Topological AC and HMW Effects, and the Dual Current in 2+1 Dimensions

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Abstract

We study the Aharonov-Casher (AC) effect and the related He-McKellar-Wilkens (HMW) effect in 2+1 dimensions. In this restricted space these effects are the result of the interaction of the electromagnetic field tensor with the dual of a current. Transferring the dual operation from the current to the field tensor shows that this interaction may be reinterpreted as due to the interaction of an effective vector potential and a current, and the AC and HMW effects follow immediately. A general proof of this for particles with an arbitrary spin is provided.

The restriction to 2+1 dimensions, with this interpretation, provides a unified way of treating the AC and HMW effects for an arbitrary spin. Perhaps more interestingly the treatment shows that a spin-0 particle can show AC and HMW effects, although it has no magnetic or electric dipole moment in
the usual sense.

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I. INTRODUCTION

The study of topological phases in quantum mechanics has provided deep understanding of quantum systems. A particularly interesting case of a topological phase is the Aharonov-Bohm (AB) effect, discovered in 1959 by Aharonov and Bohm [1]. The AB effect is due to a charged particle traveling in electric and magnetic field free region, developing a topological phase which influences interference patterns. Although the electromagnetic field in the region traversed by the particle vanishes there is a non-zero vector potential in that region — in quantum mechanics a vector potential can produce observable effects. The AB effect has been observed experimentally [2]. In 1984 Aharonov and Casher discovered [3] another configuration where a topological phase can develop, giving rise to what is now called the Aharonov-Casher (AC) effect. In this case, the effect occurs for a neutral spin-1/2 particle with anomalous magnetic dipole moment interacting with a two dimensional electric field. This is again a non-classical effect and a topological phase is developed. This effect has also been observed experimentally [4]. In 1993 another topological phase related to electric dipole interaction of a neutral spin-1/2 particle was discussed by He and MecKellar [5] and a year later, independently by Wilkens [6]. This effect, the He-McKellar-Wilkens (HMW) effect, is the dual of the AC effect with the replacements of electric field by magnetic field, and electric charge density by magnetic monopole density. Since this effect involves magnetic monopoles, it may be difficult to study it experimentally. However, in 1998, Dowling, Williams and Franson [7] pointed out that the HMW effect can be partially tested using metastable hydrogen atoms.

All the phases mentioned have some similarities and some of them are dual to each other [8–11]. The AC and HMW effects are however different in many ways to the AB effect. In particular, the AC and HMW effects have been shown to be phenomena involving two spatial dimensions in an essential way [9,14]. For this reason we study the AC and HMW effects in 2+1 dimensions directly, in the hope that this will reveal some of their hidden properties. In this paper we report the results of this study. We show that in 2+1 dimensions the
magnetic and electric dipole interactions for the AC and HMW effects can be interpreted as
due to a dual current interacting with the electric and magnetic field, respectively. This way
of viewing the AC and HMW effects provides an easy way to understand the topological
nature of these effects and the dual nature of these two effects, and to study these effects for
other spin systems in an unified way. We provide a general proof for arbitrary spin, and give
specific examples using spin-1/2, spin-1 and spin-0 particles. An interesting consequence
of this new understanding is that even a spin-0 particle, where no dipole moment can be
defined in the ordinary sense, can have AC and HMW effects.

II. THE AC, HMW EFFECTS AND THE DUAL CURRENT

We begin by studying the familiar case of the AC effect of spin-1/2 particles, but restrict-
ing considerations to $2 + 1$ dimensions from the beginning. The Lagrangian for a neutral
particle of spin-1/2 with an anomalous magnetic dipole moment $\mu_m$ interacting with the
electromagnetic field is given by

$$L = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - \frac{1}{2} \mu_m \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}. \quad (1)$$

The last term in the Lagrangian is responsible for the AC effect.

We will use the following conventions for the $2+1$ dimensional metric $g_{\mu\nu}$ and the anti-
symmetric tensor $\epsilon_{\mu\nu\alpha}$:

$$g_{\mu\nu} = \text{diag}(1, -1, -1) \quad \text{and} \quad \epsilon_{012} = +1. \quad (2)$$

While it is possible to find $2 \times 2$ matrices satisfying the $2+1$ dimensional Dirac algebra, there
are two inequivalent representations which generate different Clifford algebras. In particular
they may be distinguished by the sign in the relationship $\sigma_{12} = \pm \gamma^0$. If we use the two
component formalism we need these two inequivalent representations of the Dirac matrices
to be able to represent both spin up and spin down particles. To avoid this prescription we
will use a four component Dirac spinor which can describe spin up and down in the notional
z direction for a particle and for its anti-particle. In 2+1 dimensions these Dirac matrices satisfy the following relation:

\[ \gamma^\mu \gamma^\nu = g^{\mu\nu} + is\epsilon^{\mu\nu\lambda} \gamma_\lambda, \]  

(3)

where \( s = -i\gamma^0\gamma^1\gamma^2 = -\gamma^0\sigma^{12} \) which has eigenvalues \( \hat{s} = \pm \). A particular representation is

\[ \gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \]  

(4)

in which the operator \( s \) is diagonal and is given by

\[ s = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \]  

(5)

The interaction term in the Lagrangian is then

\[ \bar{\psi}\sigma^\mu\psi F_{\mu\nu} = -F^{\mu\nu} s\epsilon_{\mu\nu\lambda} \bar{\psi} \gamma^\lambda \psi, \quad \text{with} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 \\ E^1 & 0 & -B^3 \\ E^2 & B^3 & 0 \end{pmatrix}, \]  

(6)

where \( E^i \) and \( B^i \) are the electric and magnetic fields, respectively. The indices “1” and “2” indicate the coordinates on the \( x - y \) plane along the \( x \) and \( y \) directions. The index “3” indicates that the magnetic field in this configuration is normal to the \( x - y \) plane, in the notional \( z \) direction. The interaction is represented as the electromagnetic field tensor contracted with the tensor dual to the current \( \bar{\psi} \gamma^\mu \psi \).

We define an “effective vector potential” \( S_\mu \) as the dual of the field strength tensor

\[ S_\mu = (1/2)\epsilon_{\mu\alpha\beta} F^{\alpha\beta}, \]  

(7)

and write the Lagrangian as

\[ L = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi + s_\mu m S_\mu \bar{\psi} \gamma^\mu \psi. \]  

(8)
In the AC configuration, the magnetic field vanishes and \( E_1, E_2 \) are constant in time. Then \( S_\mu = (0, E_2, -E_1) \). The vector \( S_\mu \) satisfies
\[
\epsilon^{0\lambda \mu} \partial_\lambda S_\mu = -\partial_i E_i = -\lambda_e. \tag{9}
\]
Here \( \lambda_e \) is the surface density of electric charge in the \((x, y)\) plane. Making a transformation \( \psi' = \exp[-is\mu_m \int \vec{S} \cdot d\vec{r}]\psi \), one finds in \( \lambda_e = 0 \) region,
\[
L = \bar{\psi}' i \gamma^\mu \partial_\mu \psi' - m \bar{\psi}' \psi'. \tag{10}
\]
This is a Lagrangian for a free particle. The phase transformation of Eq. 10 is able to convert the interacting dipole system into a free system. We will refer to this phase transformation as the AC transformation. The effective vector potential \( S^\mu \) can be viewed as a pure “gauge” vector potential in charge free regions. The phase, relative to the configuration without any field, developed in the wave function when the particle travels along a closed path which encircles a total linear charge \( \Lambda_e \) is
\[
\theta_{AC} = \hat{s}_m \int \vec{S} \cdot d\vec{r} = -\hat{s}_m \int (\nabla \cdot \vec{E}) ds = -\hat{s}_m \Lambda_e. \tag{11}
\]
It is central to the existence of the AC effect that the magnetic dipole interaction can be written as an interaction between the electromagnetic field strength \( F_{\mu\nu} \) and the dual of the usual current \( \epsilon_{\mu\nu\lambda} j^\lambda \). From Eq. 3,
\[
L_{AC} = \frac{\hat{s}_m}{2} F_{\mu\nu} \epsilon_{\mu\nu\lambda} j^\lambda. \tag{12}
\]
For spin-1/2 Dirac particle \( j^\lambda = \bar{\psi} \gamma^\lambda \psi \) which is the non-vanishing current. This is the origin of the AC topological phase representation of the magnetic dipole interaction in this 2+1 dimensional configuration.

More generally the interaction \( F_{\mu\nu} \epsilon_{\mu\nu\lambda} j^\lambda \) generates a topological phase regardless of the specific value of the spin of the particle if the AC conditions required for the electric field is satisfied. This can be seen by studying the change of the action \( \Delta S \) of the system due to \( L_{AC} \), for a closed trajectory from time 0 to time T for a point particle with velocity \( \vec{v} \propto \vec{j} \),
\[ \Delta S \sim \int_0^T F^{\mu\nu} \epsilon_{\mu\lambda j}^\lambda \sim \int_0^T (\vec{S} \cdot \vec{v}) dt = \oint \vec{S} \cdot d\vec{r} \sim \theta_{AC} \]  

(13)

when the AC conditions are satisfied.

The electric dipole interaction can also be interpreted as due to a dual current interaction, demonstrating in a simple and generic way the topological nature of the HMW effect.

To see this we first need to turn to the usual $3 + 1$ dimensional space, where the HMW effect is due to the interaction term

\[ L_{HMW} = -\frac{1}{2} \mu_e \bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu} \]  

(14)

in the Lagrangian. Using the identity $\sigma^{\mu\nu} \gamma_5 = (i/2) \epsilon^{\mu\alpha\beta} \sigma_{\alpha\beta}$, this becomes

\[ -i F_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi = \tilde{F}^{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi, \]  

(15)

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\alpha\beta} F_{\alpha\beta}$ is the $3 + 1$ dimensional dual of the electromagnetic field tensor.

Now go to $2 + 1$ dimensions, where $L_{HMW}$ can further be written as

\[ L_{HMW} = -\frac{1}{2} s_{\mu e} \tilde{F}^{\mu\nu} \epsilon_{\mu\lambda \gamma} \bar{\psi} \gamma^\lambda \psi, \quad \text{with} \quad \tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & -B^1 & -B^2 \\ B^1 & 0 & E^3 \\ B^2 & -E^3 & 0 \end{pmatrix} \]  

(16)

As was emphasized previously, the HMW effect is the dual of the AC effect, it is the interaction between the dual field strength $\tilde{F}^{\mu\nu}$ and the same dual current $\epsilon_{\mu\lambda \gamma} \bar{\psi} \gamma^\lambda$ which enters the AC effect.

Setting $T^\mu = (1/2) \epsilon^{\alpha\beta \gamma} \tilde{F}_\alpha^\gamma$, one can write the Lagrangian as

\[ L = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - s_{\mu e} T_\mu \bar{\psi} \gamma^\mu \psi. \]  

(17)

For the HMW configuration, with a vanishing electric field, $T_\mu = (0, B_2, -B_1)$. The vector $T_\mu$ satisfies, $\epsilon_{0ij} \partial^j T^i = -\partial_i B^i = -\lambda_m$. Here $\lambda_m$ is the line density of the magnetic monopole charge. The Lagrangian, expressed in terms of $\psi' = \exp[is_{\mu e} \int^T \vec{T} \cdot d\vec{r}] \psi$, is in the form of the Lagrangian of a free particle in a magnetic monopole free region, just as in the AC case discussed earlier. We will refer to this transformation as HMW transformation. $T^\mu$
is a pure “gauge” field in magnetic monopole free regions. Following previous discussions for the AC effect one finds that when the particle travels along a closed path on which $\lambda_m = 0$, the wave function develops a non-zero HMW phase given by, $\theta_{HMW} = \hat{s}\mu_e\Lambda_m$, where $\Lambda_m$ is the linear monopole density enclosed by the path.

The above analysis emphasizes that the AC and HMW effects have the same origin. In the AC case the topological term is due to the interaction of the dual current $\epsilon^{\mu\nu\lambda}j_\lambda$ with electric field, and in the HMW case the topological term is due to the interaction of the same dual current with magnetic field.

Writing the AC and HMW interactions in the dual current interaction forms, $F^{\mu\nu}\epsilon_{\mu\nu\lambda}j_\lambda$ and $\tilde{F}^{\mu\nu}\epsilon_{\mu\nu\lambda}j_\lambda$, has several advantages. It enables one to easily understand the topological nature and the dual nature of these two effects, and also provides an unified way to study related topological interactions for other particles with different spins. One just needs to obtain the current $j_\mu$ for the particles concerned.

III. THE AC, HMW EFFECTS FOR ARBITRARY SPINS IN 2+1 DIMENSIONS

The AC effect with arbitrary spin has been studied in Ref. [10] for maximal projection of spin, the highest or lowerest spin projection on the notional z direction. Here we show that for arbitrary spins, one can also interpret the AC and HMW effects for any projection of the spin as interaction of a dual current with electric and magnetic field, respectively. We emphasis that AC and HMW effects exist as exact results in 2+1 dimensions in the appropriate field configuration, not only for the maximal spin projections, but for all projections. This is a more general result than that obtained in Ref. [10]. An easy way of proving our result is to use the Bargmann-Wigner formulation for a field with arbitrary non-zero spin,

$$L = \bar{\psi}^S_{\alpha(1)\ldots\alpha(2S)} \sum_{n=1}^{2S} \left( \gamma^{(n)} \mu \partial^\mu - m - \frac{1}{2} \mu_m F^{\mu\nu} \sigma^{(n)}_{\mu\nu} \right) \psi_{\alpha(1)\ldots\alpha(2S)},$$

for the AC effect. For the HMW effect one needs to replace $\mu_m F^{\mu\nu}$ by $-\mu_e \tilde{F}^{\mu\nu}$. In Eq. (18) $S$ labels the total spin of the particle, $S_m$ indicates the spin projection along the notional z
direction in the rest frame of the particle. \( \psi_{\alpha(1), \ldots, \alpha(2S)}^S \) is the wave function symmetric under exchange of \( \alpha^{(n)} \), and the \( \gamma^{(n)}_{\mu} \) and \( \sigma^{(n)}_{\mu\nu} \) act on the \( n \)th component of the Dirac index of \( \psi_{\alpha(1), \ldots, \alpha(2S)} \).

Using the relation \( F^{\mu\nu} \sigma^{(n)}_{\mu\nu} = -F^{\mu\nu} s^{(n)}_{\mu\nu} \epsilon_{\mu\nu\lambda\gamma} \), one can write the Lagrangian as

\[
L = \bar{\psi}_{\alpha(1), \ldots, \alpha(2S)}^S \left[ \sum_{n=1}^{2S} \left[ \gamma^{(n)}_{\mu} i \partial^\mu - mI^{(n)} + \mu_m S^\mu s^{(n)}_{\mu\nu} \gamma^{(n)}_{\nu} \right] \psi_{\alpha(1), \ldots, \alpha(2S)}^S \right],
\]

which is very suggestive that the dipole interaction has a form similar to the dual current form discussed in the previous section. However, it is not quite the same yet, one needs further to show that after making an AC transformation the Lagrangian expressed in the transformed field is the same as the Lagrangian for a free particle. We now show that this is indeed possible.

To this end we note that each of the \( s^{(n)} \) commute with the operator \( \sum_{n=1}^{2S} \gamma^{(n)}_{\mu} i \partial^\mu - mI^{(n)} + \mu_m S^\mu s^{(n)}_{\mu\nu} \gamma^{(n)}_{\nu} \). Therefore if \( \psi_{\alpha(1), \ldots, \alpha(n)}^S \) is a solution of equation of motion, \( \Sigma \psi_{\alpha(1), \ldots, \alpha(n)}^S \), where \( \Sigma = \sum_{n=1}^{2S} s^{(n)} \), is also a solution. However, the function obtained by the action of an individual \( s^{(n)} \) on a solution of the equation of motion is not generally a solution of the equation of motion. The reason for this is the symmetrisation of the wave function over the indices, and the exceptional cases are just those with a maximal spin projection. For each of the Dirac indices, \( s^{(n)} \) has eigenvalues \( \hat{s}^{(n)} = \pm \), when symmetrize the wave function, there are \( 2S + 1 \) ways to symmetrize the wave function depending on how many \( \hat{s}^{(n)} \) have “+” and “-”. These give the wave function for each value of \( S^m = \sum_n \hat{s}^{(n)} \). In particular the eigenfunctions of \( \Sigma \) can be constructed as

\[
\begin{align*}
\psi_{\ldots, \alpha(n)}^{S2S} &\sim \psi_{\ldots, ++, \ldots, +}^{S2S}, \\
\psi_{\ldots, \alpha(n)}^{S2S-1} &\sim \psi_{\ldots, +++, \ldots, +}^{S2S-1} + \psi_{\ldots, +++, \ldots, +}^{S2S-1} + \ldots + \psi_{\ldots, +++, \ldots, +}^{S2S-1}, \\
&\vdots \\
\psi_{\ldots, \alpha(n)}^{S-2S} &\sim \psi_{\ldots, --, \ldots, -},
\end{align*}
\]

Here the subindices \( \pm \) indicate the eigenvalues of \( s^{(n)} \) on each individual component. It is clear that \( \psi_{\ldots, \alpha(n)}^{S^m} \) are not eigenfunction of each individual \( s^{(n)} \), but are eigenfunctions of \( \Sigma \).
with eigenvalues $S_m$: $2S, 2S-1, \ldots -2S$. There are total $2S+1$ eigenstates which accounts for all the spin projections. Because these properties, one can further rewrite Eq.18 as

$$L = \bar{\psi}^S_{m} \gamma^{(n)}(i\partial^{\mu} - \frac{\mu_m}{2S} S^{\mu}(\Sigma)) \psi^S_{m} \exp[-i(\mu_m/2S) \sum_{n=1}^{2S} \int d^r \vec{S} \cdot \vec{B}] + m^2 B^{\mu} B_{\mu}. \tag{21}$$

This is the desired form. Making a AC transformation, $\psi^S_{m} = \exp[-i(\mu_m/2S) \sum_{n=1}^{2S} \int d^r \vec{S} \cdot \vec{B}] \psi^S_{m} \exp[i(\mu_m/2S) \sum_{n=1}^{2S} \int d^r \vec{S} \cdot \vec{B}]$, one obtains

$$L = \bar{\psi}^{S\prime}_{m} \gamma^{(n)}(i\partial^{\mu} - \frac{\mu_m}{2S} S^{\mu}(\Sigma)) \psi^{S\prime}_{m} \exp[-i(\mu_m/2S) \sum_{n=1}^{2S} \int d^r \vec{S} \cdot \vec{B}]. \tag{22}$$

The above is true not only just for the maximal projections with $S_m = \pm S$, but also for other projections. We have obtained a more general result than that derived in Ref. [10].

Using the fact that $\Sigma \psi^S_{m} = S_m \psi^S_{m}$, one easily finds the AC phase to be

$$\theta_{AC} = -\mu_m \frac{S_m \Lambda_e}{S}. \tag{23}$$

Replacing $\mu_m F^{\mu\nu}$ by $-\mu_e \tilde{F}^{\mu\nu}$ one obtains the case for HMW effect and the topological phase is given by $\theta_{HMW} = \mu_e (S_m/S) \Lambda_m$.

**IV. THE AC AND HMW EFFECTS FOR SPIN-1 PARTICLES**

The AC effect for particles with spin-1 has been studied in Ref. [11] using the Bargmann-Wigner formulation, and in more detail by Swansson and McKellar [13], using the Proca and Duffin Kemmer formalisms for the spin one field. Here we work directly in 2+1 dimensions, to show how the dual current interacting with electric and magnetic fields induces AC and HMW effects. Our results are based on yet another formulation of the Lagrangian.

The Lagrangian for a free spin-1 particle is usually written in the following form

$$L = \frac{1}{2} G^{\mu\nu} G_{\mu\nu} - \frac{1}{2} [G^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) + (\partial^\mu B^{\nu} - \partial^\nu B^{\mu})G_{\mu\nu}] + m^2 B^{\mu} B_{\mu}. \tag{24}$$

In 2+1 dimensions, one can write, without lost of generality, $B_\mu$ and $G_{\mu\nu}$ in terms of two other fields [14], $\phi_{+\mu}$ and $\phi_{-\mu}$ with
\[ B_\mu = \frac{1}{\sqrt{2m}} (\phi_+ \mu + \phi_- \mu), \]
\[ G_{\mu \nu} = \left( \frac{m}{2} \right)^{1/2} \epsilon_{\mu \nu \alpha} (\phi_+^\alpha - \phi_-^\alpha), \] (25)

and one obtains,
\[ L = \left[ m \phi_+ s'_{\mu} \phi_-^{* \mu} - \frac{s'}{2} \epsilon_{\mu \nu \alpha} (\phi_+^\alpha \partial_\mu \phi_-^{* \nu} + \phi_-^\alpha \partial_\mu \phi_+^{* \nu}) \right]. \] (26)

Here we adopt a summation convention on \( s' \), which, when a repeated subscript, is summed over + and −.

It is usual to write the non-minimal interactions of a spin-1 particle with the electromagnetic field, when expressed in terms of the fields \( B^\nu \) and \( G^{\mu \nu} \), in terms of the two forms \[ i \kappa \frac{m}{2} B^s_B^\mu F^\mu \] and \[ i (\tau m/m^2) G_\alpha^s G_\mu^\alpha F^{\mu \nu} \] which contribute to both the magnetic dipole and electric quadrupole moments. The magnetic dipole moment \( \mu_m \) is \( \mu_m = e (\kappa_m + \tau_m)/2 \), and the electric quadrupole moment \( Q_e \) is \( Q_e = -e (\kappa_m - \tau_m)/m^2 \). The AC effect is purely due to the magnetic dipole moment interaction, and in fact does not appear as an exact result when the electric quadrupole interaction is non-zero. Eliminating the electric quadrupole moment implies that \( \kappa_m = \tau_m \). One then can express the magnetic dipole interaction in terms of the \( \phi_\pm \) fields as
\[ L_{AC} = i \frac{\kappa_m}{m} F^\mu_\nu \phi_+^{* \mu} \phi_-^{* \nu}, \] (27)

For the electric dipole interaction, one just replaces \( \kappa_m F^{\mu \nu} \) by \( -\kappa_e \tilde{F}^{\mu \nu} \), and obtains
\[ L_{HMW} = -i \frac{\kappa_e}{m} \tilde{F}^\mu_\nu \phi_+^{* \mu} \phi_-^{* \nu}, \] (28)

In order to show that in the AC and HMW configurations that the magnetic and electric dipole interactions are topological, one only needs to show that the dipole interactions can be written as the interactions between the dual current, and the electric and magnetic fields, respectively. The current obtained from Eq. 26 is given by
\[ j_\mu = s' \epsilon_{\mu \nu \alpha} \phi_{s' \alpha} \phi_{s' \nu} - \phi_{s' \alpha} \phi_{s' \nu} \].

(29)

With this identification of the current, it can be easily seen that the dual current \( \epsilon_{\mu \nu \alpha} j^\alpha \) can be written as

\[ \epsilon_{\mu \nu \alpha} j^\alpha = -s' \left( \phi_{s' \mu} \phi_{s' \nu} - \phi_{s' \nu} \phi_{s' \mu} \right). \]

(30)

One then obtains

\[ L_{AC} = i g'_{AC} F^{\mu \nu} \epsilon_{\mu \nu \alpha} j^\alpha, \]

\[ L_{HMW} = i g'_{HMW} \tilde{F}^{\mu \nu} \epsilon_{\mu \nu \alpha} j^\alpha, \]

(31)

where \( g'_{AC,HMW} = \mp s' \kappa_{m,e}/2m \). The phases developed when the AC and HMW conditions are satisfied are, \( \theta_{AC}(s) = s'(\kappa_m/m)\Lambda_e \) and \( \theta_{HMW} = -s'(\kappa_e/m)\Lambda_m \), respectively. This completes the proof that the magnetic and electric dipole interactions in the AC and HMW configurations are topological. If \( \kappa_{m,e} \) is not equal to \( \tau_{m,e} \), it is not possible to make a AC or a HMW transformation such that the dipole and quadrupole interactions are transformed away in charge free regions.

V. THE AC AND HMW EFFECTS FOR SPIN-0 PARTICLES

For a particle with non-zero spin it was possible to write the dipole interaction in term of the dual of the current. This suggests adopting an alternative procedure — start with the interaction of the electromagnetic field with the dual of the current and see where that leads. We demonstrate this procedure by studying particles with spin-0. One particularly interesting consequence of this new approach is that spin-0 scalar particles may also have AC and HMW effects. At first sight this looks strange because for a spin-0 particle dipole interaction, in the ordinary sense, can not be defined. However when the AC and HMW effects are interpreted as due to dual current interacts with special configurations of field strength and the dual field strength, it is very natural to have AC and HMW effects because
the current is well defined for a spin-0 particle which carries an additive quantum number (not necessarily an electric charge).

For a scalar the free Lagrangian is given by

\[ L = (\partial^\mu \phi^*)(\partial_\mu \phi) - m^2 \phi^* \phi. \] (32)

The current obtained from the above is \( i(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) \). One immediately finds the interaction terms for the AC and HMW effects to be

\[ L_{AC} = g_{AC} S^\mu i(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*), \]

\[ L_{HMW} = g_{HMW} T^\mu i(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*). \] (33)

The above Lagrange can NOT be transformed to a free scalar Lagrangian by the transformations \( \phi' = \exp[-i g_{AC} \int \vec{S} \cdot d\vec{r}] \phi \) and \( \phi' = \exp[-i g_{HMW} \int \vec{T} \cdot d\vec{r}] \phi \) for the AC and HMW cases, respectively. In order to achieve the transformation, one needs to add terms \( g_{AC}^2 S^\mu S_\mu \phi^* \phi \) and \( g_{HMW}^2 T^\mu T_\mu \phi^* \phi \) to the two terms in eq.(33), respectively. These terms look like the “seagull” terms introduced in usual the QED theory of spin-0 particles to ensure gauge invariance. Here the \( g_{AC,HMW}^2 \) terms ensure that the dipole interactions are pure AC and HMW “gauge” interactions, respectively.

VI. DISCUSSIONS AND CONCLUSIONS

In this paper we have shown that in 2+1 dimensions the electric and magnetic dipole interactions for the AC and HMW effects can be interpreted as a dual current interacts with electric and magnetic field, respectively. This way of viewing the AC and HMW effects allows an easy understanding of the topological nature of these effects, and it provides a straightforward proof of the effects as exact results in 2+1 dimensions for particles of arbitrary spin. We have discussed specific examples using spin-1/2, spin-1, and spin-0 particles.

In all the cases studied, the AC and HMW interactions can be transformed away when the conditions for AC and HMW effects are satisfied, namely, in regions where there are not
electric or magnetic monopole charges the interactions can be viewed as pure AC and HMW “gauge” interactions.

We have worked with neutral particles for the AC and HMW effects. In fact the AC and HMW effects are not limited to neutral particles. They also exist for charged particles. One can easily obtain the formalisms for the corresponding effects for charged particles by replacing the partial derivative operator $\partial_\mu$ by the covariant derivative $\partial_\mu - ieA_\mu$. In this case the charged particles also experience the usual electric interaction. The AC and HMW transformations will not transform the equations of motion into free particle equations of motion, but transform the dual current interaction parts away into the wave function and produce topological phases when the AC and HMW conditions are satisfied [9].

An interesting consequence of the new understanding discussed in this paper is that even spin-0 particles, where no dipole moment can be defined in the ordinary sense, can have AC and HMW effects. One may ask if the effects for spin-0 particles can be realized in physical situations. It is not difficult to construct such an interaction theoretically. One easy way of achieving this is to supersymmetrize the AC and HMW effects. The super partners of the spin-1/2 particles are spin-0 particles. The AC and HMW phases for the spin-0 particles are directly related to those for the spin-1/2 particles. More detailed studies of supersymmetric AC and HMW effects, and related problems will be presented elsewhere. It is not clear to us at this moment that if in certain composite systems in two spatial dimensions such interaction can actually exist. The experimental observation of these effects remains to be studied.

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