TITLE:
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CITATION:
Nakanishi, Toshihiro ...[et al]. Efficient second harmonic generation in a metamaterial with two resonant modes coupled through two varactor diodes. APPLIED PHYSICS LETTERS 2012, 100(4): 044103.

ISSUE DATE:
2012-01

URL:
http://hdl.handle.net/2433/160630

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Citation: Appl. Phys. Lett. 100, 044103 (2012); doi: 10.1063/1.3679652
View online: http://dx.doi.org/10.1063/1.3679652
View Table of Contents: http://apl.aip.org/resource/1/APPLAB/v100/i4
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Efficient second harmonic generation in a metamaterial with two resonant modes coupled through two varactor diodes

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(Received 12 November 2011; accepted 6 January 2012; published online 26 January 2012)

We present an effective method to generate second harmonic (SH) waves using nonlinear metamaterial composed of coupled split ring resonators (CSRRs) with varactor (variable capacitance) diodes. The CSRR structure has two resonant modes: a symmetric mode that resonates at the fundamental frequency and an anti-symmetric mode that resonates at the SH frequency. Resonant fundamental waves in the symmetric mode generate resonant SH waves in the anti-symmetric mode. The double resonance contributes to effective SH radiation. In the experiment, we observe 19.6 dB enhancement in the SH radiation in comparison with the nonlinear metamaterial that resonates only for the fundamental waves. © 2012 American Institute of Physics. [doi:10.1063/1.3679652]

Metamaterials, which are composed of artificial sub-wavelength structures, exhibit extraordinary electromagnetic properties. Numerous studies have focused on the linear response characteristics of metamaterials. Recent studies have also reported the development of nonlinear media and the control of the nonlinear properties of the metamaterials through the introduction of nonlinear elements into the constituents of metamaterial. High nonlinearity can be achieved in resonant-type metamaterials such as split ring resonators (SRRs) by placing nonlinear elements at locations where the electric (or magnetic) field is concentrated due to the resonance effect. Nonlinear metamaterials have been studied for property tuning, 3-4 frequency mixing, 5 imaging beyond diffraction limit, 6 and developing bistable media. 7,8 Several kinds of nonlinear metamaterials for generating second harmonic (SH) waves have been reported. Most of the metamaterials are designed such that they resonate with the incident waves or the fundamental waves. 2,9-13 These are called singly resonant metamaterials. If the structure has resonant modes not only for the fundamental frequency but also for the SH frequency, SH radiation could be enhanced. 14,15 In this paper, we propose a method to implement metamaterials satisfying the doubly resonant condition, or a doubly resonant metamaterial, using coupled split ring resonators (CSRRs) with two varactor diodes, which generate SH waves due to nonlinearity. There are two resonant modes in the CSRR structure: one for the fundamental waves; the other for the SH waves. They are coupled owing to the nonlinearity of the diodes. The SH oscillation excited in the varactors directly excites the SH resonance mode through the nonlinearity-assisted coupling. Second harmonic generation through the CSRRs is more efficient than that through the previously proposed metamaterial with two resonant modes, where SH oscillation indirectly excites the resonant mode designed for the SH waves through magnetic coupling. 15 We demonstrate experiments in the microwave region to estimate the SHG efficiency of the CSRRs, comparing the singly resonant metamaterial.

The varactor loaded SRR shown in Fig. 1(a) is a typical example of singly resonant nonlinear metamaterials. It can be modeled as a series resonant circuit composed of an inductor, varactor, and resistor, as shown in Fig. 1(b). The electromotive force induced by the external magnetic field \( B \) is represented by the voltage source \( V(t) = V_0 \cos \omega t \) in the circuit model. If we consider the varactor as a linear capacitor, the circuit is a simple harmonic oscillator driven by the external force and the voltage across the capacitor, \( V_C \), reaches a maximum at the resonant angular frequency \( \omega_0 = 1/\sqrt{LC} \). However, the varactor has nonlinearity in the capacitance and its voltage \( V_C \) with respect to the charge \( q \) can be represented as \( V_C = q/C(q) = q/C + aq^2 \). An anharmonic term \( aq^2 \) contributes to the generation of second- or higher-order harmonic waves. Using the perturbation method under a weak nonlinearity condition, 16 the amplitude of the SH oscillation is obtained as

\[
\tilde{q}(2\omega) = \frac{aV_0^2}{\omega^3 Z(2\omega) Z(\omega)^2}.
\]

where \( Z(\omega) = R - j[\omega L - 1/(\omega C)] \) is the impedance of the circuit. When the circuit resonates at \( \omega = \omega_0 \), \( Z(\omega) \) takes a minimum value and \( |\tilde{q}(2\omega)| \) is maximized. As a result, the enhanced SH signal is radiated from the singly resonant

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where large due to resonance. On the other hand, the anti-
form two resonant modes with resonant angular frequencies
current (dashed line) and anti-symmetric current (solid line)
in the symmetric mode. This is because the induced SH
ð\(V\) induced by the magnetic flux through the rings and voltage
Figure 1(d) represents the circuit model of the CSRR. It
It is found that the resonant oscillation
mode induces the SH current in the anti-symmetric mode
angular frequency \(\omega \) and \(\omega_a = \omega_d/2\). Consequently,
the amplitude of the second harmonic oscillation can be sig-
nificantly enhanced in comparison to the singly resonant
metamaterial, where the SH current has to flow through the
high impedance \(|Z(2\omega)|\) in Eq. (1).

We fabricated the CSRR illustrated in Fig. 1(c) with
35 \(\mu\)m-thick copper film on a polyphenylene ether (PPE)
substrate with a thickness of 0.8 mm. The dimensions
are \(a = 14 \text{ mm}, b = 24 \text{ mm}, c = 4 \text{ mm}, g = 0.5 \text{ mm},\)
and \(w = 1 \text{ mm}\). For comparison, we also prepared a reference
metamaterial, shown in Fig. 1(e), which has the same structure,
except for the absence of the central structure and the
direction of the diodes. The loop current in this structure also
resonates at the same fundamental frequency as that of the
CSRR, and the SH electromotive voltages generated in the
two varactors contribute to the loop current, which does not
resonate. Therefore, the ratio of SHG efficiency between
CSRRs and SRRs can be interpreted as the enhancement fac-
tor owing to the resonance effect for the SH waves.

Before SHG demonstration, we conducted transmission
measurements to identify the resonant modes and resonant
frequency of the metamaterials. We used a waveguide called
a stripline TEM-cell, as shown in Fig. 2.\(^{1,16}\) The waveguide
supports a TEM mode propagating in the \(z\) direction. The
tapered structures ensure that the wave impedance is main-
tained at 50 \(\Omega\) along the waveguide and the tips of the top
plate are connected to the input and output ports, which have
SubMiniature Type A (SMA) connectors. We placed three
SRRs or CSRRs at intervals of 5 cm inside the waveguide, as
shown in Fig. 2. In the transmission spectra, we observed
two resonant dips at 1.36 GHz and 2.72 GHz for the CSRRs
and a dip at 1.36 GHz for the SRRs. Thus, it is found that the
common dips at 1.36 GHz correspond to resonance in the
symmetric (or loop-current) mode and the higher resonant
mode at 2.72 GHz for the CSRRs is the anti-symmetric
mode. This CSRR structure satisfies the doubly resonant
condition, \(2\omega_a = \omega_d\).

Figure 3(a) shows the experimental setup to measure the
SH power generated in the metamaterials. A low pass filter
(LPF) was used to suppress residual harmonics from the

\[
\begin{align*}
L \frac{d^2 q_s}{dt^2} + R \frac{dq_s}{dt} + \frac{q_s}{C} + 2\omega_d q_s &= 2V_s \cos \omega t, \\
L \frac{d^2 q_a}{dt^2} + R \frac{dq_a}{dt} + \frac{q_a}{C_a} + \frac{\omega_c^2}{2} q_a^2 + \frac{\omega_a^2}{2} q_a^2 &= 2V_s \cos \omega t,
\end{align*}
\]

where \(1/C = 1/C + 2C\). The excitation voltage \(V_s\) is
induced by the magnetic flux through the rings and voltage \(V_a\)
is induced by the electric field at the central gap of the
structure. Equations (2) and (3) represent that the symmetric
current (dashed line) and anti-symmetric current (solid line)
form two resonant modes with resonant angular frequencies
of \(\omega_s = 1/\sqrt{LC}\) and \(\omega_a = 1/\sqrt{LC_a}\), respectively; these
modes are coupled through the nonlinearity \(x\). We set the
gap capacitance \(C = 2C\beta\), so that the doubly resonant
condition \(2\omega_a = \omega_d\) is satisfied.

We will solve \(q_s\) and \(q_a\) in the presence of a small nonli-
arity \(x\), using a perturbative approach. We assume that the
solution for \(x = 0\) is given by \(\tilde{q}_s(\omega) = \tilde{q}_a(\omega) = e^{-i\omega t} + c.c.
(i = s, a). From Eqs. (2) and (3), the amplitudes of \(q_s(\omega)\)
and \(q_a(\omega)\) oscillating at \(\omega\) are written as \(\tilde{q}_i(\omega) = V_i\omega_c^2(\omega)\) (\(i = s, a\)),
where \(Z_s(\omega) = R - 1/\omega C\) and \(Z_a(\omega) = R - 1/\omega C_a\) are the impedances for the symmetric
and anti-symmetric modes, respectively. When the excitation
angular frequency \(\omega\) is tuned close to \(\omega_a\), \(\tilde{q}_a(\omega)\) becomes
large due to resonance. On the other hand, the anti-
symmetric mode is hardly excited; \(\tilde{q}_s(\omega) \approx 0\).

The first-order solution of \(q_a\) with respect to \(x\) satisfies
\[
L \frac{d^2 q_s^{(1)}}{dt^2} + R \frac{dq_s^{(1)}}{dt} + \frac{q_s^{(1)}}{C_a} + \frac{\omega_a^2}{2} (q_s^{(0)})^2 = 0.
\]
It is found that the resonant oscillation \(q_s^{(0)}\) in the symmetric
mode induces the SH current in the anti-symmetric mode
through the last term \(\frac{x}{2}\) \((q_s^{(0)})^2\). The nonlinearity of the varac-
tors results in a coupling between the two resonant modes. It
should be noted that if the diodes are arranged in the same
direction on the outer ring, the fundamental current and the
induced SH current flow in the symmetric mode, which reso-
nates only for the fundamental wave. From Eq. (4), the
amplitude of the SH oscillation is obtained as
\[
\tilde{q}_a(2\omega) = \frac{aV_s^2}{2\omega^2 Z_a(2\omega) Z_s(\omega)^2}.
\]
It is easily deduced from Eq. (2) that there are no SH waves
in the symmetric mode. This is because the induced SH

**FIG. 2.** Schematic of stripline TEM-cell waveguide.
FIG. 3. (a) Experimental setup for SHG measurements. (b) SHG power for −3 dBm input. (c) SHG peak power as a function of input power.

has been studied and 6.6 dB enhancement of SHG has been achieved.\(^\text{15}\) Both of the metamaterials are composite metamaterial composed of two resonant structures, a primary resonator and secondary resonator, which are magnetically coupled. The SH waves generated in the primary resonator indirectly excite the secondary resonator through magnetic coupling. Roughly speaking, the efficiency of the SHG for these metamaterials in the strong coupling limit could approach that of the CSRR metamaterial, where the SH oscillation directly excites the resonant mode owing to the arrangement of the nonlinear elements. However, it is actually difficult to attain strong magnetic coupling with finite element dimensions and also difficult to optimize the parameters of the metamaterials, considering the resonant frequency shifts induced by strong coupling. The CSRR metamaterial attains a much higher enhancement factor, just by designing the structures to satisfy \(\omega_s = 2\omega_a\).

In this paper, we proposed a method to enhance SHG with CSRR metamaterial. The arrangement of the nonlinear elements induces coupling between the two resonant modes, which satisfy the doubly resonant condition. Owing to the direct excitation in the resonant mode for SH waves, we attained significant enhancement of up to two orders of magnitude. The precise and quantitative evaluation of SHG efficiencies and optimization of the CSRR structure are left as future problems. We expect that metamaterials with nonlinearity-assisted coupling could be employed for investigating other types of nonlinear phenomena.

The present research was supported by Grants-in-Aid for Scientific Research Nos. 22109004 and 22560041, by the Global COE program of Kyoto University and by a research grant from The Murata Science Foundation. One of the authors (Y.T.) would like to acknowledge the support of a Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists.

1. J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. 47, 2075 (1999).
2. I. V. Shadrivov, A. B. Kozyshev, D. W. van der Weide, and Y. S. Kivshar, Appl. Phys. Lett. 93, 161903 (2008).
3. B. Wang, J. Zhou, T. Koschny, and C. M. Soukoulis, Opt. Express 16, 16058 (2008).
4. D. A. Powell, I. V. Shadrivov, and Y. S. Kivshar, Appl. Phys. Lett. 95, 084102 (2009).
5. D. Huang, A. Rose, E. Poutritina, S. Larouche, and D. R. Smith, Appl. Phys. Lett. 98, 204102 (2011).
6. Z. Wang, Y. Luo, T. Jiang, Z. Wang, J. Huangfu, and L. Ran, Phys. Rev. Lett. 106, 047402 (2011).
7. R. Yang and I. V. Shadrivov, Appl. Phys. Lett. 97, 231114 (2010).
8. M. W. Klein, C. Enkrich, M. Wegener, and S. Linden, Science 313, 502 (2006).
9. M. W. Klein, M. Wegener, N. Feth, and S. Linden, Opt. Express 15, 5238 (2007).
10. E. Kim, F. Wang, W. Wu, Z. Yu, and Y. Shen, Phys. Rev. B 78, 113102 (2008).
11. Z. Wang, Y. Luo, L. Peng, J. Huangfu, T. Jiang, D. Wang, H. Chen, and L. Ran, Appl. Phys. Lett. 94, 134102 (2009).
12. A. Rose, D. Huang, and D. R. Smith, Phys. Rev. Lett. 107, 063902 (2011).
13. M. V. Gorkunov, I. V. Shadrivov, and Y. S. Kivshar, Appl. Phys. Lett. 88, 071912 (2006).
14. T. Kanazawa, Y. Tamayama, T. Nakanishi, and M. Kitano, Appl. Phys. Lett. 99, 024101 (2011).
15. E. Poutritina, D. Huang, and D. R. Smith, New J. Phys. 12, 093010 (2010).
16. R. E. Collin, Field Theory of Guided Waves (IEEE, New York, 1991).