$1/f$ noise from vortex-antivortex annihilation

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The magnetic flux noise induced by vortices in thin superconducting films is studied. Rigid vortices in thin films as well as pancake vortices in the pancake gas regime are addressed. The vortex dynamics is described by a Feynman path integral which fully accounts for the balance between vortex entropy and vortex energetics. We find that vortex pair creation (annihilation) in the presence of pinning is a natural source for noise with a $1/f$ spectrum.

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Over the last decade it was noticed that the theory of strong interactions (QCD) and type II superconductors share a common feature: it was realized that the QCD vacuum sustains percolating vortices \cite{1}. The dynamics of the vortex texture is thereby related to important properties of QCD such as quark confinement. For instance, quark deconfinement at high temperatures appears as vortex depercolation transition \cite{2} and the universality class of the transition can be anticipated by investigating the vortex dynamics only \cite{3}. It turned out \cite{4} that a vortex model which essentially incorporates the vortex entropy and an energy penalty for vortex curvature already captures the dynamics of the confining vortices. Below, we will apply this idea to describe vortices of type II superconductors: In the case that the superconductor partition function is dominated by the vortex entropy, the vortex dynamics is described by a Feynman “path integral” which correctly accounts for the vortex entropy. The only energy penalty is given for the vortex mobility.

A central issue in the context of type II superconductors is the magnetic flux noise and voltage noise generated by the dynamics of vortices in superconductors. This topic has been investigated both theoretically and experimentally throughout the last four decades, starting with the pioneering work by Van Ooijen and Van Gurp \cite{5,6,7}. Early work focused on so-called flux-flow noise \cite{8} due to density and velocity fluctuations of vortices driven by a bias current, which causes voltage fluctuations in the flux flow regime with a noise power spectrum often scaling with frequency $f$ as $1/f^\alpha$ ($\alpha \approx 1 - 2$). More recently, various groups have performed experiments probing directly the magnetic flux noise in superconductors which is sensed by nearby magnetic field or flux sensors \cite{9,10,11,12,13,14}. Within such an approach, the magnetic flux noise at low magnetic fields has also been detected by placing a superconductor in close vicinity to a superconducting quantum interference device (SQUID) which senses directly the motion of vortices \cite{15}. Similarly, one can simply detect the motion of vortices located in the SQUID structure itself \cite{16}, e.g., combined with imaging of such vortices \cite{17}. Such measurements in low magnetic fields show also typically scaling of the spectral density of flux noise as $1/f$.

Apart from the attempt to understand the origin and mechanisms of such noise as a prerequisite for improving the properties of superconducting devices, the investigation of noise was also driven by the wish to exploit vortex dynamics as a probe of the superconducting state, in particular after the discovery of the high transition temperature $T_c$ cuprate superconductors, which show very rich phenomena associated both with the static and dynamic properties of ‘vortex matter’ \cite{18,19,20,21}. An interesting aspect is related to the layered structure of the cuprate superconductors and very thin superconducting films, which presumably leads to enhanced two-dimensional fluctuations near the Kosterlitz-Thouless transition as determined from simulations of XY-models with time-dependent Ginzburg-Landau dynamics \cite{22,23}.

In this letter, the magnetic flux noise induced by a vortex to a slot, similar to the situation of vortices trapped in a SQUID washer \cite{16}, is studied on the basis of a statistical vortex model. The model applies to the case of rigid vortices in thin films as well as to pancake vortices \cite{24} in the pancake gas regime close to $T_c$. Disregarding vortex pair creation (annihilation), we find that the noise spectrum shows a $1/\sqrt{T}$ behavior. Taking into account vortex pair creation (annihilation), we observe a $1/f$ law if the vortex pinning is active, while the spectrum behaves like $1/f^2$ if pinning is absent. This demonstrates that vortex pair creation (annihilation) of pinned vortices can be a natural source for $1/f$ noise.
Let us consider a single “pancake” vortex which moves in the 2-dimensional $xy$ plane. Vortex motion induces a time dependent magnetic flux to the slot parallel to the $y$-axis (see figure 1). The vortex “zitterbewegung” causes a frequency dependent noise to the SQUID, which measures the flux induced to the slot. The derivation of the noise spectrum $S(f)$ will be our goal below. The motion of the vortex is described in the present case by a 2-dimension vector $\vec{x}(t)$ which depends on the time $t$. In order to describe the pinning of the vortex to the origin $\vec{x} = 0$, we constrain the vortex motion by the weak pinning condition:

$$\frac{1}{t} \int_0^t dt' \vec{x}(t') = 0. \quad (1)$$

The latter condition implies that the “center of mass” of the vortex is tied to the origin of the coordinate system. The quantity of primary interest is the probability distribution

$$P(\vec{x}_0, \vec{x}; 0, t) = \langle \vec{x}(0) \vec{x}(t) \rangle \quad (2)$$

of finding a vortex at $\vec{x}$ at time $t$, when the vortex was located at $\vec{x}_0$ at time $t = 0$. The average in (2) is taken over all vortex world lines which satisfy the pinning condition (1) and the boundary conditions $(\vec{x}_0, 0)$ and $(\vec{x}, t)$. The noise spectrum is obtained from the probability distribution by $(\omega = 2\pi f)$

$$S(\omega) = \int dt \, d^2 x_0 \, d^2 x \, T_d(\vec{x}) \, P(\vec{x}_0, \vec{x}; 0, t) \, e^{i\omega t}. \quad (3)$$

Thereby, $T_d(\vec{x})$ is the transfer function which describes the coupling of the magnetic field of the vortex located at $\vec{x} = (x, y)$ at time $t$ to the slot, which is located at distance $d$ from the $y$-axis. A simple model for the transfer function is provided by the choice

$$T_d(\vec{x}) = \delta(x - d), \quad \vec{x} = (x, y). \quad (4)$$

Thereby, it is assumed that only “pancake” vortices which possess overlap with the slot induce a signal. The $\vec{x}$ integration in (3) sums up all contributions to the slot, whereas the $\vec{x}_0$ integration can be understood as an average over all initial conditions of the vortex.

In the following, we will assume that the partition function of the vortex matter is solely given by the interplay between vortex entropy and vortex energy. In this regime, the dynamic of the vortices is captured by a statistical model. Correlation functions are obtained by averaging over “pancake” vortex ensembles: The probability distribution is effectively described in terms of the “Feynman” path integral

$$P(\vec{x}_0, \vec{x}; 0, t) = N \int \mathcal{D}\vec{x}(t) \delta\left(\frac{1}{t} \int_0^t dt' \vec{x}(t')\right) \exp\left\{-\int_0^t dt' \left[\frac{m}{2} |\dot{\vec{x}}(t')|^2\right]\right\}, \quad (5)$$

where $m$ is the only parameter of the model. $m$ parametrizes the mobility of the “pancake” vortex moving in the 2d plane. All vortex worldlines, which contribute to the path integral (5) start at $\vec{x}_0$ at time $t = 0$ and end at $\vec{x}$ at time $t$. The normalization constant $N$ can be obtained from the condition $\int d^2 x \, P(\vec{x}_0, \vec{x}; 0, t) = 1$. Let us calculate the auxiliary quantity (the dependence on $\vec{x}_0$, $\vec{x}$ is suppressed)

$$Z(\vec{a}, t) = \int \mathcal{D}\vec{x}(t) \exp\left\{\frac{i}{t} \int_0^t dt' \vec{x}(t')\right\} \exp\left\{-\int_0^t dt' \left[\frac{m}{2} |\dot{\vec{x}}(t')|^2\right]\right\} \quad (6)$$

$Z(\vec{a}, t)$ corresponds to the probability distribution of freely moving vortices, while integration over $\vec{a}$ installs the weak pinning condition (1), i.e.,

$$P(\vec{x}_0, \vec{x}; 0, t) = \int \frac{d^2 \vec{a}}{(2\pi)^2} \, Z(\vec{a}, t). \quad (7)$$

It is tedious, but straightforward to calculate the function $Z(\vec{a}, t)$ in (6). For this purpose, the time interval $[0, t]$ is divided into $N$ pieces of equal distance:

$$t_n = \frac{t}{N}, \quad n = 0 \ldots N, \quad \vec{x}(t_n) = \vec{x}_n, \quad \vec{x}_n \equiv \vec{x}.$$

The discretized version of the path integral (6) is given by

$$Z(\vec{a}, t) \to e^{-N/2} \int d\vec{x}_1 \ldots d\vec{x}_{N-1} \exp\left\{\frac{i}{N} \sum_{n=0}^{N-1} (\vec{x}_{n+1} - \vec{x}_n)^2\right\} \exp\left\{-\frac{m}{2t} \sum_{n=0}^{N-1} (\vec{x}_{n+1} - \vec{x}_n)^2\right\}.$$

where the prefactor of the functional integral arises from the measure $\mathcal{D}\vec{x}(t)$. After performing the Gaussian integrals and taking the limit $N \to \infty$, we obtain up to an unimportant numerical factor

$$Z(\vec{a}, t) \propto \exp\left\{-\frac{m}{2t} (\vec{x} - \vec{x}_0)^2\right\} \exp\left\{-\frac{t}{24m} \vec{a}^2 + \frac{i}{2} \vec{a}(\vec{x} + \vec{x}_0)\right\}. \quad (8)$$

Setting $\vec{a} = 0$ in the last expression, we find that $Z$ only depends on $\vec{x} - \vec{x}_0$ reflecting translation invariance in the absence of the pinning center at the origin. Extending the correlation function to negative times, we replace $t$ in (8) by $|t|$. Inserting (8) in (5) and introducing the relative momentum $\vec{q}$ which is the conjugate variable to $\vec{x} - \vec{x}_0$, the integration over $t$, $\vec{x}_0$ and $\vec{a}$ can be performed yielding

$$S(\omega) \propto \int \frac{d^2 \vec{q}}{(2\pi)^2} \, d^2 x \, \Re \left\{\frac{1}{\vec{q}^2 + i\omega}\right\} \, T_d(\vec{x}) \, e^{i\vec{q}\vec{x}}. \quad (9)$$
The vortex statistical ensemble is now approached by means of the path integral

\[ S(\omega) \propto \sqrt{\frac{6m}{\omega}}. \]  

(10)

Hence, statistical ensembles, consisting out of single and weakly pinned vortices, cannot account for the $1/f$ type spectrum.

In order to allow for the merge of a vortex antivortex pair or for the vortex pair creation in the vortex statistical ensemble, the vortex world line is addressed by an implicit parameterization:

\[ \vec{x}(\tau) = (x(\tau), y(\tau)) , \quad t(\tau) , \quad \vec{x} := (x, y, t) , \]

where the parameter \( \tau \) will be called “proper time” below. If \( t(\tau) \) is a non-monotonic function, vortex pair creation and annihilation is take into account. The weak pinning condition (11) is generalized to an average over the vortex world line

\[ \frac{1}{\tau} \int_0^\tau d\tau' \ \vec{x}(\tau') = 0. \]  

(11)

The vortex statistical ensemble is now approached by means of the path integral

\[ Z_{CA}(\alpha, t) = \int D\vec{x}(\tau) \ \exp \left\{ \frac{i}{\tau} \int_0^\tau d\tau' \ \vec{x}(\tau') \right\} \exp \left\{ -\int_0^\tau d\tau' \ \left[ \frac{m}{2} \vec{x}'^2 + \frac{\mu}{2} t^2 \right] \right\} , \]  

(12)

where the dot denotes the derivative with \( \tau \). If \( \mu \) is small, \( t(\tau) \) possesses many non-monotonic parts. This implies that \( 1/\mu \) is related to the vortex pair creation rate. The ensemble average must be taken over all vortex world lines starting at \( \vec{x}_0 := (x_0, y_0, 0) \) and ending at \( \vec{x} := (x, y, t) \). There is no restriction for the proper time \( \tau \) implying that we will take the limit \( \tau \to \infty \) below. A calculation, analogous to that in the previous subsection, yields

\[ Z_{CA}(\alpha, t) \propto \int_0^\infty d\tau \ \exp \left\{ -\frac{m}{2\tau} (\vec{x} - \vec{x}_0)^2 - \frac{\mu}{2\tau} t^2 \right\} \exp \left\{ -\frac{\tau}{24m} \vec{\alpha}^2 + \frac{i}{2} \vec{\alpha}(\vec{x} + \vec{x}_0) \right\} . \]  

(13)

After switching to the momentum space, the \( \tau \) integration can be performed, i.e.,

\[ Z_{CA}(\alpha, t) \propto \int \frac{d^3q}{(2\pi)^3} \ \exp \left\{ i\vec{q}((\vec{x} - \vec{x}_0) + \vec{q}t) + i\vec{q}t\vec{\alpha} + \vec{q}^2 \right\} \]  

\[ \exp \left\{ -\frac{\tau}{24m} \vec{\alpha}^2 + \frac{i}{2} \vec{\alpha}(\vec{x} + \vec{x}_0) \right\} . \]  

(14)

The noise spectrum \( S(\omega) \) is now easily obtained by inserting (14) in (3). Setting \( \vec{x} = (x, y) \) and \( \vec{q} = (q_1, q_2) \), the \( \vec{x}_0, \vec{\alpha}, q_0 \) and \( t \) integration is straightforward:

\[ S(\omega) \propto \int \frac{d^2q}{(2\pi)^2} \ d^2x \ T_d(x) \ \exp \left( \frac{i2\vec{q}\vec{x}}{3m} + \frac{q^2}{2\mu} \right) . \]  

(15)

For the idealized transfer function in (4), our final result is

\[ S(\omega) \propto \exp \left\{ -\sqrt{\frac{3m}{\mu}} \omega d \right\} \sqrt{\frac{3m}{2\mu}} \omega . \]  

(16)

This is the central result of our letter: allowing for vortex pair creation and annihilation, the noise spectrum shows \( 1/\omega \) dependence at small frequencies if the vortex is pinned sufficiently close \( (\omega d \ll 1) \) to the slot.

How does pinning affect the noise spectrum? Let us consider the vortex statistical ensembles which include vortex pair creation and annihilation, but where the pinning...
constraint is absent. The noise spectrum is given in this case by

\[ S(\omega) \propto \int dt \, d^2x_0 \, d^2x \, T_d(\vec{x}) \, Z_{CA}(0, t) \, e^{i\omega t}. \]  

(17)

Note that we have set \( \vec{\alpha} = 0 \) in the auxiliary function \( Z_{CA}(\vec{\alpha}, t) \) rather than integrating over \( \vec{\alpha} \). Using (14), the frequency dependent part of \( S(\omega) \) is given by \( S(\omega) \propto 1/\omega^2 \). Obviously, the pinning of the “pancake” vortices is important to obtain a \( 1/f \) noise spectrum.

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