Local corner smoothing of linear toolpath based on B-spline fitting and its piecewise interpolation

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Abstract. Line segment toolpaths are commonly utilized in machining of complex parts with curved surfaces. However, the first-order discontinuity of the linear toolpath results in frequent variation of the feedrate, which is bad to the machining quality. To smooth the sharp corners of linear toolpath thus avoiding above problem, this paper presents a local corner smoothing approach. Compared with existing methods, the smoothed toolpath using the proposed is one integral B-spline, instead of alternative straight-line segments and spline segments, which make the continuity more easily to ensured. Additionally, the integral long B-spline is interpolated through a piecewise feedrate scheduling method, thus the computational burden can be released. Effectiveness of the presented method in corner smoothing and kinematic constraining is demonstrated by illustration test.

1. Introduction
Toolpaths generated by most Computer-Aided Manufacturing (CAM) applications are linear segments, although the desired machining paths are curves. Due to the fact that the toolpath consisted by linear segments travels abrupt tangential direction variation, the feedrate must have large fluctuations when controlling the feed axes according to the linear toolpath strictly, which is bad to the machining quality[1-2]. Therefore, smoothing of the sharp corners becomes significant.

In order to smooth the corners of linear toolpaths, many studies have been conducted. For example, Zhang et al. [3] approximated the sharp corner by a cubic B-spline, and scheduled the feedrate with S-shape velocity profile. Zhao et al. [4] blended the straight-line segments by a cubic B-spline with five control points, and the interpolation point on the linear-spline mixed paths was computed by jerk-limited look-ahead feedrate scheduling. Fan et al. [5] approximated one corner with two quartic Bezier splines whose curvature energy was minimalized, and a 15 phases acceleration/deceleration profile was employed to realize the jerk smooth interpolation. Fan et al. [6] and Zhang et al. [7] approximated the sharp corners with a nine-degree Bezier curve and a quartic B-spline curve, respectively. In addition, some local smoothing strategies for five-axis machine tools [8] or six-axis robots toolpaths [11] were presented.

Even though, the existing methods mainly focus on replacing the sharp corners with spline curves, and remaining flat regions as the straight lines. That is to say, the smoothed toolpath is composed by linear segment and spline segment. This makes it hard to guarantee the integral continuity and
smoothness of the feed motion. Therefore, this paper presents a new local smoothing approach by integral B-spline fitting and its piecewise interpolation. First, the linear toolpath is replaced by a long integral B-spline through control-point assigning. Then, the interpolation trajectories for feed axes are generated by piecewise feedrate scheduling of the smoothed B-spline. Illustration tests demonstrate the feasibility of the presented approach.

2. B-spline fitting of the linear toolpath by control-point assigning

For most of the existing corner smoothing approaches, the smoothed toolpath is consisted by line segments and spline segments alternately, which introduce a new issue, i.e. the consideration of the continuity between linear and spline segments. To avoid this issue, this paper fits the linear toolpath as one B-spline curve. However, if the curve-fitting approaches are used, the iterative computation is required, which increases the computation burden. Therefore, we fit the B-spline by directly assigning of the control points. As shown in Figure 1, the detail control-point assigning principle is provided as follows. First, the original cutter locations, i.e. \( Q_1, Q_2, \) etc. in Figure 1, are directly token as partial of the B-spline control points. Then, two control points are added neighboring each inner cutter locations. The final assigned control points are expressed as \( P_1, P_2, \) etc. in Figure 1. The length \( l_i \) between two inner added control points and the cutter location is determined as

\[
l_i = \min \left( \frac{||Q_{i-1} - Q_i||}{2}, \frac{||Q_{i+1} - Q_i||}{2}, \frac{e}{\sin \frac{\beta_i}{2}} \right)
\]

where \( e \) is the fitting error tolerance, and \( \beta_i \) is the angle of \( i \)-th corner (see Figure 1).

Except for the control points, the curve order, knot vector should also be given for a B-spline. To realize the \( C^0 \) continuity, the curve order is selected as 4, and the knot vector can be computed by the centripetal parameterization approach. Using this method, the fitted B-spline has a similar smoothing performance with the existing methods, i.e. 1) keeps straight at flat regions and 2) transits smoothly at sharp corners without exceeding the error tolerance \( e \). The reason is analyzed as follows. First, it is acknowledged that once three control points are colinear, the B-spline curve must pass through the middle point. Therefore, once there are four colinear control points, the middle two points will be passed through, which means that the spline curve segment between the middle two control points keeps straight. It can be seen from Figure 1, there exist four control points on each inner linear segment, such as \( P_2-P_5 \). Therefore, the spline between \( P_2 \) and \( P_5 \) must be straight. Note that although the smoothed path between \( P_2 \) and \( P_5 \) is straight, it is one part of an integral spline, but not a real straight line. This makes the global continuity can be easily ensured. Second, the distance from the transition curve to the desired cutter location must be smaller than the error tolerance \( e \), and the reason is proofed below. As shown in Figure 2, the distance from the spline curve to the cutter location must be smaller than \( d \) according to the convex hull property of the B-spline. According to Eq. (1), there must exists

\[
d = l_i \cdot \sin \frac{\beta_i}{2} \leq e
\]

Thus, the distance from the curve to the desired cutter location must be smaller than \( e \). As a result, the proposed B-spline fitting method can achieve favorable performance without iterative computation.

![Figure 1. Principle of the control-point assigning-based B-spline fitting.](image-url)
3. B-spline interpolation by piecewise feedrate scheduling

After replacing the linear toolpath by one integral B-spline curve, axial trajectories at each interpolation period should be generated. Feedrate scheduling performs as a premise in this progress. Since the fitted B-spline is one long curve instead of short curve segments, simultaneous scheduling of integral feedrate must be so time consuming that cannot be fulfilled in real time. Therefore, a piecewise feedrate scheduling method is proposed here.

Detail of the feedrate scheduling is introduced as follows. First, look ahead for a distance of \( d_l \) on the B-spline curve, and the \( d_l \) is computed as

\[
d_l = 2 \cdot s_{req}(0, v_p)
\]  

where \( v_p \) stands for the programmed feedrate, and \( s_{req}(0, v_p) \) represents the required distance when the feedrate accelerates from 0 to \( v_p \) under the S-shape acceleration/deceleration rule.

Second, scan the B-spline within the length of \( d_l \) and find the minimum curvature radius \( \rho_{\text{min}} \). After that, calculate the allowable feedrate in this B-spline segment \( v_{\text{allow}} \) under normal acceleration and jerk constraints by

\[
v_{\text{allow}} = \min\left(v_p, \sqrt{\rho_{\text{min}} a_{\text{max}}} \right)
\]

where \( a_{\text{max}} \) and \( j_{\text{max}} \) are maximum allowable acceleration and jerk, respectively.

Finally, as shown in Figure 3, schedule the acceleration/deceleration process. If the allowable feedrate of the next interval of \( d_l \) length is larger than that of the current interval, acceleration starts at the beginning of the next interval, otherwise, if the next interval allowable feedrate is smaller than current interval allowable feedrate, execute the deceleration process at the end of current interval.

After the feedrate scheduling, the interpolation point parameter can be computed by the second-order Runge-Kutta method according to the scheduled feedrate \( v_{\text{sc}} \) as

\[
u_{k+1} = u_k + \frac{T_s}{2} \left( \frac{v_{\text{sc}}}{\|C'(u_k)\|} + \frac{v_{\text{sc}}}{\|C'(u_k+\frac{T_s}{2}v_{\text{sc}}/\|C'(u_k)\|)} \right)
\]

where \( u_k \) is the current interpolation point parameter; \( T_s \) is the interpolation period; \( C(u) \) is the fitted B-spline.
4. Illustration test
In order to verify the feasibility of the proposed local smoothing method based on integral B-spline fitting and piecewise interpolation, illustration test is conducted in this section. Taking a linear toolpath of a propeller as the testing objective, and setting the fitting error tolerance as 0.04 mm, the B-spline fitting result is shown in Figure 4. As can be seen, the sharp corners are effectively smoothed without exceeding the error tolerance.

The axial acceleration and jerk constraints are set as $a_{\text{max}} = 2000 \text{ mm/s}^2$ and $j_{\text{max}} = 5 \times 10^5 \text{ mm/s}^3$, respectively. The programmed feedrate $v_p$ is set as 15 mm/s. The scheduled feedrate on the smoothed toolpath, coupled with the axial kinematic parameters, are shown in Figure 5. As can be seen, axial acceleration and jerk can be effectively bounded, which demonstrate the feasibility of the presented approach.

![Figure 4. B-spline fitting result. (a) Geometry. (b) Fitting error.](image)

![Figure 5. Experimental results. (a) Scheduled feedrate. (b) Axial acceleration. (c) Axial jerk.](image)
5. Conclusion
In this paper, a local corner smoothing method for linear segment toolpath is proposed. Compared with the existing corner smoothing methods, the proposed one fits all linear toolpath as one integral B-spline, thus the overall continuity can be more easily ensured. In addition, a piecewise interpolation method is presented for online generation of the axial trajectories for the long B-spline curve. Illustration tests demonstrate the feasibility of the presented method.

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