Analyzing lepton flavor universality in the decays $\Lambda_b \to \Lambda_c^{(*)}(\frac{3}{2}, \frac{1}{2}^-) + \ell \bar{\nu}_\ell$

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Lepton flavor universality can also be tested in the semileptonic decays $\Lambda_b \to \Lambda_c^{(*)}$ where $\Lambda_c^{(*)}$ denotes either the ground state $\Lambda_c(2286)$ (with $J^P = 1/2^+$) or its orbital excitations $\Lambda_c(2595)$ (with $J^P = 1/2^-$) and $\Lambda_c(2625)$ (with $J^P = 3/2^-$. We calculate the differential decay rates as well as the branching fractions of these decays for both tauonic and muonic modes with form factors obtained from a covariant confined quark model previously developed by us. We present results for the rate ratios of the tauonic and muonic modes which provide important tests of lepton flavor universality in forthcoming experiments.

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I Introduction

In the standard model (SM) the three charged lepton generations ($\ell = e, \mu, \tau$) together with their neutral and massless neutrinos interact with the weak gauge bosons universally. This SM feature is called lepton flavor universality. Recent experimental studies of the leptonic $B \to \tau \ell Q$ and semileptonic $B \to D^{(*)}\ell \bar{\nu}_\ell$ decays have shown deviations from the predictions of lepton flavor universality in the tauonic modes (for a review see Ref. [1]). If the observed deviations will be confirmed in future experiments, then it will open a new window in the search for new physics (NP) beyond the SM. There are many theoretical papers which consider different scenarios for the implementation of NP. Some studies extend the SM by introducing new particles and new interactions. Other studies adopt a model-independent approach by adding a set of NP operators to the effective Hamiltonian for the $b \to c \ell \bar{\nu}_\ell$ transition. The numerical values of the new Wilson coefficients are determined by fitting available experimental data. Nonperturbative hadronic effects in the $B \to D^{(*)}$ transitions are encoded in form factors which most often are evaluated using methods of heavy quark effective theory (HQET) [2,3].

Lepton flavor universality can also be tested in the semileptonic $\Lambda_b \to \Lambda_c^{(*)}$ decays where $\Lambda_c^{(*)}$ denotes either the ground state $\Lambda_c(2286)$ (with $J^P = 1/2^+$) or its orbital excitations $\Lambda_c(2595)$ (with $J^P = 1/2^-$) and $\Lambda_c(2625)$ (with $J^P = 3/2^-$. Following previous work [4] the authors of Ref. [5] discussed the transition form factors for decays into the above two excited $\Lambda_c^{(*)}$ states up to $O(1/m_b, 1/m_b)$ corrections in the heavy quark mass expansion using methods of HQET. In their $O(1/m_b, 1/m_b)$ analysis they showed that all relevant form factors are expressible through a single universal baryon Isgur-Wise function.

The semileptonic transition into the $J^P = 1/2^+$ ground state $\Lambda_c(2286)$ plus a heavy lepton pair $\tau \bar{\nu}_\tau$ has been studied in a number of theoretical papers. Among these is Ref. [6] the authors of which predicted the partial decay width starting from rather general assumptions. The effects of five possible new physics interactions were analyzed by adopting five different form factors. In Ref. [7] the effects of adding a single scalar or vector leptoquark to the SM have been investigated. It was shown that the best-fit solution for the Wilson coefficients obtained in the corresponding $B$ decays leads to similar enhancements in the branching fractions of the $\Lambda_b$ decays. The decay widths as well as the ratios of branching fractions for the $\tau$ and $e/\mu$ modes have been calculated in Ref. [8] by using QCD sum rule form...
factors. The decays $\Lambda_b \to p\ell^-\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$ were studied in Ref. [3] by using form factors from lattice QCD with relativistic heavy quarks. In Ref. [10] the authors presented predictions for the ground-state to ground-state $\Lambda_b$ decay in extensions of the SM by adding NP operators with different Lorentz structures. The scope of Ref. [10] was extended in Ref. [11] by adding a tensor operator. Both Refs. [10, 11] used form factors from lattice QCD in their analysis. We mention that semileptonic $\Lambda_b$ decays were also investigated in the framework of a relativistic quark model based on the quasipotential approach and a quark-diquark picture of baryons [12].

In Ref. [13] we have provided a thorough analysis of the decay $\Lambda_b^0 \to \Lambda_c^+(2286) + \tau^- + \bar{\nu}_\tau$ with particular emphasis on the lepton helicity flip and scalar contributions which vanish for zero lepton masses. We have calculated the total rate, the differential decay distributions, the longitudinal, and transverse polarizations of the daughter baryon $\Lambda_c^+$ as well as that of the $\tau$ lepton, and the lepton-side forward-backward asymmetries.

In a series of papers we have studied possible NP effects in the exclusive decays $\bar{B}^0 \to D(\pm_s)\tau^-\bar{\nu}_\tau$ and $B_c \to (J/\psi, \eta_c)\tau\nu_\tau$ including right-handed vector (axial), left- and right-handed (pseudo)scalar, and tensor current contributions [14][16]. The $B^0 \to D(\pm)$ and $B_c \to (J/\psi, \eta_c)$ transition form factors were calculated in the full kinematic $q^2$ range by employing the covariant confined quark model (CCQM) previously developed by us. For more detail regarding the $R_{D(\pm)}$ puzzle we refer to the recent study in Ref. [17] and references therein.

In Ref. [18] we have calculated the invariant form factors and the helicity amplitudes for the transitions $\Lambda_b \to \Lambda_c(\pm_1, 3/2^-)$, where the $\Lambda_c(\pm_1)$ are $\Lambda(sud)$-type ground and excited states with $J^P = 1/2^\pm, 3/2^\pm$.

The purpose of the present paper is to calculate the differential decay rates and branching fractions of the semileptonic $\Lambda_b \to \Lambda_c(\pm_1)$ decays in the SM for both the $\tau$ and $\mu$ modes using form factors evaluated in the covariant confined quark model.

II Decay properties of the transitions $\Lambda_b \to \Lambda_c(\pm_1, 3/2^-) + \ell^+ \bar{\nu}_\ell$

The $J^P$ quantum numbers and the interpolating three-quark (3q) currents of the baryons involved in our calculations are shown in Table I. For the $P$-wave excitations with quantum numbers $J^P = 1/2^-, 3/2^-$ we have taken the simplest modifications of the ground state $J^P = 1/2^+$ interpolating current.

| TABLE I: Quantum numbers and interpolating currents of charm and bottom baryons. |
|-----------------|-----------------|-----------------|-----------------|
| Baryon          | $J^P$           | Interpolating 3q current | Mass (MeV)     |
| $\Lambda_c(2286)$ | 1/2^+          | $\epsilon^{abc} u^a b^b C\gamma_5 d^c$ | 2286.46        |
| $\Lambda_c(2593)$ | 1/2^-          | $\epsilon^{abc} \gamma^5 u^a b^b C\gamma_5 d^c$ | 2592.25        |
| $\Lambda_c(2628)$ | 3/2^-          | $\epsilon^{abc} u^a b^b C\gamma_5 d^c$ | 2628.11        |
| $\Lambda_b(5620)$ | 1/2^+          | $\epsilon^{abc} u^a b^b C\gamma_5 d^c$ | 5619.58        |

The hadronic matrix element $\langle A_2|\bar{c}O^\mu b|A_1 \rangle$ ($O^\mu = \gamma^\mu(1 - \gamma^5)$) is expressed in terms of six and eight dimensionless invariant form factors $F^{V/A}_i(q^2)$ for the transitions into the $A_2(1/2^\pm)$ and $A_2(3/2^\pm)$ states, respectively. The details of their definition and their evaluation in the framework of the CCQM can be found in our paper [18]. In Figs. 1-3 we show the behavior of the calculated form factors where we use a short-hand notation for the form factors such that $V_i = F^V_i$ and $A_i = F^A_i$. We want to emphasize that our transition form factors are calculated using finite quark masses. Thus they include the $1/m_b$- and $1/m_c$-corrections considered in Ref. [3] as well as all higher powers of the heavy mass expansion.

For the ground-state to ground-state transition $\Lambda_b \to \Lambda_c$ the finite mass form factors depicted in Fig. 4 show a close likeness to the limiting form factors of the HQL (see, e.g., the review [19]). In particular, the finite mass form factors $V_1(q^2)$ and $A_1(q^2)$ show an approximate agreement with the zero recoil normalization condition $V_1(q_{\text{max}}^2) = A_1(q_{\text{max}}^2) = 1$ at zero recoil $q_{\text{max}}^2 = (M_1 - M_2)^2$. The form factors $V_{2,3}(q^2)$ and $A_{2,3}(q^2)$ are predicted to be zero in the HQL. Our finite mass form factors are small yet non zero.
FIG. 1: Vector and axial form factors for the transition $\Lambda_b(1/2^+) \to \Lambda_c(1/2^+)$. 

FIG. 2: Vector and axial form factors for the transition $\Lambda_b(1/2^+) \to \Lambda_c(1/2^-)$. 

FIG. 3: Vector and axial form factors for the transition $\Lambda_b(1/2^+) \to \Lambda_c(3/2^-)$. 
The differential decay rate is given by (see Refs. [13, 18] for details)

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 (q^2 - m_\ell^2)^2 |p_2|}{192 \pi^3 m_\ell^2 q^2} \mathcal{H}_{\text{tot}}, \tag{1}
\]

\[
\mathcal{H}_{\frac{3}{2} \to \frac{1}{2}} = |H_{\frac{3}{2}}|^2 + |H_{\frac{1}{2} - 1}|^2 + |H_{\frac{1}{2}}|^2 + |H_{\frac{1}{2} - 0}|^2 \\
+ \frac{m_\ell^2}{2q^2} \left( 3 |H_{\frac{3}{2}}|^2 + 3 |H_{\frac{1}{2} - 1}|^2 + |H_{\frac{1}{2} - 0}|^2 + |H_{\frac{1}{2} - \frac{3}{2}}|^2 \right), \tag{2}
\]

\[
\mathcal{H}_{\frac{1}{2} \to \frac{1}{2}} = |H_{\frac{1}{2}}|^2 + |H_{\frac{1}{2} - 1}|^2 + |H_{\frac{1}{2} - \frac{3}{2}}|^2 + |H_{\frac{1}{2} - \frac{1}{2}}|^2 + |H_{\frac{1}{2} - 0}|^2 \\
+ \frac{m_\ell^2}{2q^2} \left( 3 |H_{\frac{1}{2}}|^2 + 3 |H_{\frac{1}{2} - 1}|^2 + |H_{\frac{1}{2} - \frac{3}{2}}|^2 + |H_{\frac{1}{2} - \frac{1}{2}}|^2 + |H_{\frac{1}{2} - 0}|^2 \right), \tag{3}
\]

where \( G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi constant; \( m_\ell \) is the charged lepton mass; \( |p_2| = \lambda^{1/2}(M_1^2, M_2^2, q^2)/(2M_1) \) is the magnitude of the three-momentum of the daughter baryon in the rest frame of the parent baryon; \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) is the kinematical triangle Källen function; \( M_1 \) and \( M_2 \) are the masses of the parent and daughter baryon, respectively. The \( H_{\lambda_1 \lambda_W} \) denotes the helicity amplitudes which are linearly related to the relativistic \( \Lambda_b \to \Lambda_c^{(*)} \) transition form factors (for details see our recent paper [18]).

In Fig. 4 we display the \( q^2 \) dependence of the normalized differential decay rates in the full kinematical region for the \( \mu \) and \( \tau \) modes. The \( P \)-wave factor \( |p_2|^3 \) in the differential rate is clearly visible for the \( 1/2^+ \to 1/2^-, 3/2^- \) transitions at the zero recoil end of the spectrum (see, e.g., the review [20]).

![FIG. 4: The normalized differential decay rates for \( \mu \) (solid) and \( \tau \) (dashed) modes.](image)

In Table III we present our predictions for the semileptonic branching fractions. We have used the central values for the lifetime and the mass of the \( \Lambda_b \) from the Particle Data Group [21] \( \tau_{\Lambda_b} = (1.470 \pm 0.010) \text{ ps} \) and
$M_{\Lambda_b} = (5619.58 \pm 0.17)$ MeV. The value of the CKM matrix element is set to $|V_{cb}| = 0.0405$. We also display the numerical values for the ratio

$$R(\Lambda_c^{(*)}) \equiv \frac{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^{(*)}+\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^{(*)}+\mu^-\bar{\nu}_\mu)}.$$  

(4)

**TABLE II:** The branching fractions (in %) and the ratios $R(\Lambda_c^{(*)})$.

|         | $\Lambda_0^+(\frac{1}{2}^+)$ | $\Lambda_c^+ (\frac{3}{2}^-)$ | $\Lambda_c^+ (\frac{1}{2}^-)$ |
|---------|-------------------------------|-----------------------------|-------------------------------|
| $e$     | $6.80 \pm 1.36$               | $0.86 \pm 0.17$            | $0.17 \pm 0.03$              |
| $\mu$   | $6.78 \pm 1.36$               | $0.85 \pm 0.17$            | $0.17 \pm 0.03$              |
| $\tau$  | $2.00 \pm 0.40$               | $0.11 \pm 0.02$            | $0.018 \pm 0.004$           |
| $R(\Lambda_c^{(*)})$ | $0.30 \pm 0.06$ | $0.13 \pm 0.03$ | $0.11 \pm 0.02$ |

**TABLE III:** The ratio $R(\Lambda_c)$ calculated in various approaches.

| This work | Ref. [6] | Ref. [7] | Ref. [8] | Ref. [9] | Ref. [10] | Ref. [12] | Ref. [13] |
|-----------|----------|----------|----------|----------|-----------|-----------|----------|
| $0.30 \pm 0.06$ | $[0.15,0.18]$ | $[0.27,0.33]$ | $0.31 \pm 0.11$ | $0.34 \pm 0.01$ | $0.29 \pm 0.02$ | $0.31$ | $0.29$ |

Finally, in Table [III] we compare our result for the ratios $R(\Lambda_c)$ with those obtained in other approaches. Our finite mass result $R(\Lambda_c) = 0.30 \pm 0.06$ is quite close to the leading order HQL estimate $R(\Lambda_c) = r(\tau)/r(\mu) = 0.244$, which follows from the results presented in Ref. [19]. The HQL rate ratio estimate $R(\Lambda_c) = r(\tau)/r(\mu)$ is true to $O(\delta^2 = 0.178)$ where $\delta = (M_1 - M_2)/(M_1 + M_2)$. The function $r(x_\ell)$ is given by

$$r(x_\ell) = \sqrt{1-x_\ell^2 - \frac{9}{2} x_\ell^2 - 4 x_\ell^4} - \frac{15}{2} x_\ell^4 \ln \frac{1-\sqrt{1-x_\ell^2}}{x_\ell},$$  

(5)

where $x_\ell = m_\ell/(M_1 - M_2)$.

In summary, we have calculated the ratios of the tauonic to the muonic modes in the semileptonic decays of the bottom baryon $\Lambda_b$ to the ground state charm baryon $\Lambda_c$ and the two lowest $P$-wave excitations with $J^P = 1/2^-, 3/2^-$ quantum numbers. We are looking forward to the forthcoming experimental results on the ratios of the $\tau$- and $\mu$-rates for the three transitions that were analyzed in this paper. At a later stage one could extend the comparison also to the differential $q^2$ distributions.

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