Higher spin $m \neq 0$ excitations on curved backgrounds and cosmological supergravity

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Abstract

Some time ago, we showed that a weak (linear) massless spin 2 wave could only propagate on a Ricci-flat or Ricci-constant background: it must necessarily be a perturbation of General Relativity. This was just the continuation of the higher spin chain: massless $s = 3/2$ also required Ricci-flatness (which is the basis of supergravity) while $s > 2$ needs Riemann flatness. This note re-analyzes the problem in a perhaps more physical way: by considering massless and – only apparently – massive small excitations and showing how their parameters relate to the cosmological constant. We will thus prove, in a simple physical way, the necessity, as well as the sufficiency, of the known supergravity results.
1 Introduction

In [1], we showed that massless linear $s = 2$ excitations, of a very general a priori form, could only propagate on a Ricci $-\,$, or equivalently Einstein $-\,$, flat or constant space: they had to be effectively perturbations of the general relativity (GR) background. This result fits neatly into a well-known sequence: massive or massless $s < 3/2$ moves in any space, $s = 3/2$ only in a Ricci-flat (if massless) or “tuned” constant (if massive) one (whence supergravity (SUGRA) in the coupling to gravity), while all higher than $s = 2$ fields require a Riemann-flat background, hence no “hypergravities” with $s = 5/2$ or higher [2]. [Even if $s = 5/2$ can live on a fixed, non-dynamical, background such as AdS, that is as unsatisfactory as living in flat space, for the same reason.]

A deeper physics explanation for the above result is that spacetime in GR is not a fixed arena, flat or otherwise, but a dynamical system with its own action. When added to that of spin 2 $\sim \int D\epsilon D\epsilon$, the resulting new equations read $G_{\mu\nu}(g) = T_{\mu\nu}(h)$. Given the linear $\Box h + \ldots = 0$ equation, the now obvious result is that the total system is equivalent to pure GR, but with action $\sim \int R(g + h)$, to quadratic $h$-order. Without this, the spin 2 field would just be a test field, with negligible $T_{\mu\nu}$, hence of negligible interest.

A more involved, but similar, story holds for $s = 3/2$ and SUGRA rather than just GR. The combined action of the massless spin $3/2$ field plus GR is consistent because $-$ and only because $-$ the “Bianchi identities” of the $3/2$ equations, $\sim (\text{Ricci}) D\psi_\mu = 0$ hold using $\text{Ricci} \sim T_{\mu\nu}(\psi)$ and the rather involved cubic Fierz identities. Underlying this is of course the local supersymmetry of the combined system. Otherwise, the spin $3/2$ field would have to be a dull test-field, with a weak $T_{\mu\nu}$ of no effect as well! [Of course the Einstein Bianchi identities always work, since they merely express local coordinate invariance.]

In [1], we also considered the cosmological case, where identical arguments apply to prove that the background must be Ricci-constant. The purpose of this note is to provide physical insight into the derivation of [1], by emphasizing how the “mass” of the linear $s = 2$ excitations is required to connect to the cosmological constant for consistency.

2 Cosmological excitations

The basic point is that Cosmological General Relativity (CGR) involves a new, massive, parameter $\Lambda$, so consistent excitations on it must do so as well, simply by now being (apparently) massive. This lesson was learned in SUGRA, where the $s = 3/2$ excitation had to become massive to allow the more general SUGRA with cosmological term [3], the upshot being that $s = 3/2$ was actually apparently massive as a result of being in a tuned AdS space as explained in [4].
We thus let the linear \( s = 2 \) wave have an apparent \( m^2 \) of dimension \( \Lambda \). In the notations and implicitly assumed \((++++)\) signature of [1], this means adding to the Pauli-Fierz action

\[
I_2 = \int d^4x h^{\mu\nu} \theta_{\mu\nu\alpha\beta} h^{\alpha\beta}
\]

(where \( \theta \) is the second-order Hermitian operator yielding the gauge-invariant spin 2 field equation \( \Box h_{\mu\nu} + \cdots = 0 \)) the term

\[
I_C = -\frac{m^2}{4} \int d^4x \left( h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h^2 \right) (-g)^{-\frac{1}{2}}
\]

where proper background covariance is manifest, but where we do not assume a priori any relationship between \( m^2 \) and \( \Lambda \) (contrary to what was done in [1]). Note that the field \( h_{\mu\nu} \) carries density weight one, hence the factors \((-g)^{-\frac{1}{2}}\) in the action [5].

However, enforcing gauge invariance of the spin 2 field under \((-g)^{-\frac{1}{2}}\delta h_{\mu\nu} = D_{\mu} \xi_{\nu} + D_{\nu} \xi_{\mu} - g_{\mu\nu} D_{\alpha} \xi_{\alpha} \) yields, by a trivial extension of the calculation leading to Eq. (3) of [1] that the background must be Ricci-constant with \( \Lambda = -m^2 \). Indeed, the variation of the cosmological piece \( I_C \) of the action adds to the variation of \( I_2[h] \)

\[
\delta I_2[h] = \int d^4x \xi^\mu \left[ R_{\mu\rho} D_{\nu} h^{\rho\nu} + \frac{1}{2} (D_{\alpha} R_{\mu\beta} + D_{\beta} R_{\mu\beta} - D_{\mu} R_{\alpha\beta}) h^{\alpha\beta} \right] (-g)^{-\frac{1}{2}}
\]

the term

\[
\delta I_C[h] = m^2 \int d^4x \xi_{\mu} D_{\nu} h^{\mu\nu} (-g)^{-\frac{1}{2}}
\]

imposing through \( \delta I_2[h] + \delta I_C[h] = 0 \) the background equation

\[
R_{\mu\nu} = -m^2 g_{\mu\nu}
\]

equivalent to

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} = 0
\]

with

\[
\Lambda = -m^2
\]

(from which \( D_{\alpha} R_{\mu\nu} = 0 \) follows automatically).

We thus see that the background must be indeed Ricci-constant with \( \Lambda = -m^2 \), so that the total system is equivalent to a pure CGR one with \( g_{\mu\nu} + h_{\mu\nu} \), to quadratic \( h \) order. Anti-de Sitter space corresponds to \( m^2 > 0 \), i.e., real mass, while de Sitter space corresponds to imaginary mass.

Of course, as shown in [4], the excitation’s “mass” effectively vanishes as a result of gauge invariance. For \( s = 2 \) alone, either sign of \( m^2 \) is allowed, because while dS GR is also effectively massless, it is finite, involving a geometrical horizon. Real \( m \) requires
AdS just as in the $s = 3/2$ case where $m \sim \sqrt{-\Lambda}$ corresponds to $\Lambda < 0$, i.e., anti-deSitter (AdS) gravity in our conventions. Hence the uniqueness of AdS, vs absence of dS-SUGRA, corresponding to SO(3,2) vs SO(4,1) representations in group language, an alternate explanation. We stress, however, that the derivation of (7) given here does not require supersymmetry.

3 Conclusions

In summary then, we first rederived our original claim in [1] that massless $s=2$ excitations on Einstein-flat/constant spaces in GR are consistent if and only if they are perturbations of the latter, $g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$ by a physical argument treating those excitations purely dynamically, without assuming any a priori relationship between the cosmological constant and the mass of the spin 2 excitation. More centrally, we explained in a dynamical sense, how CGR and SUGRA arise from the corresponding (apparently massive) linear $s = 2$ and $3/2$ excitations through the derived relation $\Lambda = -m^2$ in the same way as did their massless counterparts, that is, this condition is not merely sufficient but necessary.

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