Analysis of microwave propagation in arrays of dielectric cylinders

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(Dated: March 22, 2022)

We rigorously compute the propagation and scattering of microwaves by regular and defected arrays of dielectric cylinders in a uniform medium. Comparison with the previous experimental results is made, yielding agreements in the transmission coefficients for a certain range of frequencies. The results also show that localized states are possible for defected arrays. However, the experimentally claimed localized states are not observed. Moreover, comparison is made with previous theoretical results. The agreements and differences are pointed out.

PACS numbers: 42.25.Hz, 41.90.1e, 71.55.Jv

Waves surround us. Direct human communication is mainly conveyed by acoustic waves, and is enriched by gestures which are passed into our eyes through optical waves. Nowadays electronic waves are also everywhere in our daily experiences such as audio & video systems, computers, and Nintendo games. Among many interesting properties pertinent to waves, the phenomenon of wave localization is probably most intriguing and remains as one of the unsolved mysteries of last century.

The concept of wave localization was first proposed by Anderson nearly half a century ago, initially for electronic systems\textsuperscript{1}, then extended to classical waves including microwaves to be addressed in this paper. It refers to situations that purely due to multiple scattering of waves by disorders, transmitted waves are confined or frozen in space and the wave envelopes decay nearly exponentially along any direction\textsuperscript{2}. Since its inception, wave localization has attracted substantial efforts from all areas of expertise, signified by the great body of literatures ranging from small scales such as electronic systems, nano-structured materials, to large scales such as seismic and ocean waves, as reviewed and summarized in, for instance, Refs.\textsuperscript{3}–\textsuperscript{12}. In spite of the efforts, however, some fundamental issues concerning wave localization are still open\textsuperscript{13}.

A few prominent problems in the investigation lie in the localization in two dimensional (2D) random media. First, although it has been conjectured, thought to be genuinely valid, by the scaling analysis\textsuperscript{14} that all waves are localized in 2D random media for any given amount of disorders or impurities, the experimental observation of 2D localization is scarce. Even for the limited experimental results that are available\textsuperscript{15}, the disparity\textsuperscript{16,17} still remains in the interpretation of the results. A main concern is whether the localization in 2D has indeed been observed or not\textsuperscript{18}. Some authors further pointed out that there still lacks a definite observation of 2D wave localization, particularly for classical waves\textsuperscript{18}. In addition, how waves are localized in 2D and whether the localization features can be explained or fitted by the existing theory\textsuperscript{19} are among a few important issues yet to be addressed.

It would be too ambitious to answer all the unsolved issues dwelling on 2D localization in a single endeavor. However, a further look at some of previous experimental observations of 2D localization, in the hope of clarifying some confusions, seems more realistic and imperative, and will therefore be the main theme of the present paper.

Here we would like to concentrate on the experimental observation of microwave propagation in 2D dielectric lattices, reported by McCall et al.\textsuperscript{15}. The reason is not only because the experiment is the first ever reported, but it has stimulated so many later investigations, and its importance could be inferred from the constantly increasing citation number. In\textsuperscript{15}, the authors measured transmission of microwaves through regular and defected arrays of dielectric cylinders. The transmission bands and photonic band gaps for the regular arrays are found to be in excellent agreement with theoretical predictions. For the defected arrays, the authors suggested the observation of a localized state. This observation, however, has been contrarily discussed and supported by two independent theoretical inspections\textsuperscript{16,17}. In this paper, we will use the standard multiple scattering approach first formulated by Twersky\textsuperscript{20} to further examine the experimental results. We will show that localized states are indeed possible for the defected arrays. However, the experimentally claimed localized states are not observed. Comparison will also be made with previous theoretical results. Agreements and differences are found and discussed. Some contraries in the previous analysis are clarified.

The systems considered here are from Ref.\textsuperscript{15} and they are composed of almost ideal arrays of low-loss high dielectric constant cylinders. The cylinders are placed in parallel in an uniform medium to form a square lattice of lattice constant denoted as d. The ratio of dielectric constants between the medium and cylinders is 9. Two types of measurement are carried out in\textsuperscript{15}. One is to measure the E-polarized microwave transmission across the samples which are shaped as a rectangular slab with thickness of 9d and width of 18d and the lattice constant is 1.27 cm. The other is to measure the intensity distribution at 11.2 GHz for a sample size of 7.6 cm x 7.6 cm with a cylinder being removed from the center of the sample, i.e. defected array. However, the lattice arrangement for the second measurement differs from the
first in that the lattice constant is 1.59 cm; such an inconsistency in the lattice arrangements has been discussed in [10]. The authors in [13] suggested that the localized states are observed for the defected array.

In our computation, we will take the physical parameters from [13]. However, we will consider larger lattice sizes when appropriate to ensure the stability of the results. Both lattice arrangements, i.e., $d = 1.27$ and $1.59$ cm corresponding to filling factors 0.4487 and 0.2863 respectively, will be considered. The cylinder radius is 0.48 cm. In addition to computing the transmission across rectangular slabs, for comparison purpose we also compute the transmitted intensity when the source is placed inside a sample that takes a circular shape. The spatial distribution of energy will also be calculated to compare with the experiment. Moreover, the phase diagram method [21] will be used to investigate a coherence phenomenon associated with localization. To be consistent with the experiment, we only consider the E-polarized microwaves.

![Figure 1](image1.png)

**FIG. 1:** The band structures of two 2D square lattices: (a) for $d = 1.27$ cm and (b) for $d = 1.59$ cm respectively. The dielectric constant is 9. The complete band gaps are apparent for both lattices.

Figure 1 shows the band structures for the two 2D square lattices aforementioned. The results on the left panel reproduce that in [13], confirming our numerical codes. For later discussion, the band structures of lattice constant 1.59 cm are also plotted. Here we see that the two results are similar for frequencies below the second gap. There are three prominent complete bandgaps for $d = 1.27$, while there are only two complete bandgaps for $d = 1.59$.

The transmission intensity is plotted as a function of frequencies in Fig. 2. The situations in (a1), (a2) and (a3) are considered in [15]. From this figure, we observe the following. First for $d = 1.27$ cm: (1) The results for situations considered by the experiment agree both qualitatively and quantitatively well with the measurement for frequencies up to the beginning of the third bandgaps at about 14 GHz; (2) While we did not plot the case of propagation along the $[21]$ direction, we plot the case of the transmission from a source located inside the sample, with a cylinder being deleted at the center (a4). The result is in accordance with the case that the transmission is measured across the sample. In both cases, there is a transmission peak inside the valley at 11.27 GHz, slightly above the measured value, which is outside the complete gap, but within the gap along the $[10]$ direction. The reason for the difference in the location and magnitude of the peak from the experiment is due to the resolution of plotting. (3) There are some significant differences between the theory and experiment for frequencies starting from the third gap. For these frequencies, we observe clearly a well defined reduction in the transmission, well matching the band structures in Fig. 1 and the reduction is even deeper than the second gap. The experimental results, however, show some smearing. Further calculations point to a few factors contributing to the disagreement, such as disorders, inhomogeneities in the radii of the cylinders, and the size of samples.

![Figure 2](image2.png)

**FIG. 2:** The transmitted intensity is plotted as a function of frequency (in GHz) for (a) $d = 1.27$ cm shown in the left panel and (b) for $d = 1.59$ cm shown in the right panel respectively. (a1 & b1): Regular system with size of thickness 9d and width 18d; the wave propagates in the $[10]$ direction; both the source and the receiver are kept at one lattice constant away from the both ends of the sample. (a2 & b2): The same as (a1 & b1) except that the wave propagates in the $[11]$ direction. (a3 & b3): System with a single defect, i.e., the central cylinder is removed; the propagation is along the $[10]$ direction; both the source and the receiver are kept at one lattice constant away from the both ends of the sample respectively. (a4 & b4): The source is moved into the center of the sample with the central cylinder being removed; the sample takes a circular shape of radius 9d.

The results of the transmission through a slab with $d =$
1.59 cm are depicted on the right panel of Fig. 2. These diagrams show that the transmission is significantly prohibited in the second gap, consistent with the band structure calculation. There is also a peak in the reduction regime. The frequency at which the energy distribution is measured is outside this forbidden regime.

An important feature in Figs. (a3) and (a4) is the peak inside the second gap along the [10] direction but outside the complete bandgap, i.e. within the pseudo gap. McCall et al. [4] attempted to explain this peak as the result of a localized state. To support this view, McCall et al. further measured the energy spatial-distribution at this peak and attempted, believed to be successful, to show that there is indeed a localized wave at this frequency. Unfortunately, this time they used the different lattice with $d = 1.59$ cm. The inconsistency has been pointed out in [7]. Meade et al. [16] argued that the peak is due to the resonance of extended states, and the apparent localized wave shown by Fig. 4 in [15] is superficial and is due to the resonance.

To resolve the disagreement between the measurement and the previous theoretical analysis, we show in Fig. 3 the 2D spatial distribution of energy at 11.27 GHz for the lattice of $d = 1.27$ cm and 1.59 cm with one cylinder being taken out at the center of the sample. We also plot the associated phase diagram. The source transmits at the center.

According to Refs. 21, 22, there will appear a coherence for the phase of the localized wave. We brief this feature here. The energy flow of EM waves is $\vec{J} \sim \vec{E} \times \vec{H}$. By invoking the Maxwell equations to relate the electrical and magnetic fields, we can derive that the time averaged energy flow is $\langle \vec{J} \rangle = \frac{1}{T} \int_0^T dt \vec{J} \sim |\vec{E}|^2 \nabla \theta$, where the electrical field is written as $\vec{E} = \vec{e}_E |\vec{E}| e^{i\theta}$, with $\vec{e}_E$ denoting the direction, $|\vec{E}|$ and $\theta$ being the amplitude and the phase respectively. It is clear that when $\theta$ is constant, at least by spatial domains, while $|\vec{E}| \neq 0$, the flow would come to a stop and the energy will be localized or stored in the space. The phase can be represented by a vector defined as $\vec{v} = \vec{e}_x \cos \theta + \vec{e}_y \sin \theta$, and the phase vectors at various space points can be drawn on the 2D coordinates. The coherence in the phases is a unique indication of localization.

Fig. 3(a) shows that the wave is indeed localized at 11.27 GHz, as suggested in [15] for the case of $d = 1.27$ cm. Here is shown that the energy is mostly confined within the lattice and the prescribed phase coherence prevails. The localization due to scattering by the cylinders is so strong that the magnitude of the intensity surpasses the energy directly from the source. However, this localized wave is not what was observed in the experiment. We note that for all cases, the sample size considered here is larger than that in the experiment.
localization shown by both Figs. 3 and 5 occurs outside the complete band gap. These results will eventually give insight to the relation between gap and localization. The absence of localization in defected arrays may be due to the finite size of samples, in the view of previous studies. But it might also hint that waves are not necessarily always localized in 2D, as suggested before [21].

Finally, to examine the difference between the theoretical and experimental transmission results when comparing the left panel of Fig. 2 and Fig. 3 in [15], we investigate the effect of disorders. For brevity, we only show the effect of the positional disorder which is introduced by moving the cylinders around their regular lattice position to a certain extent. Here the disorder is measured as 12% of random displacement of the cylinders within a circle around their original regular positions with regard to the lattice constant. The result is shown in Fig. 6. The result indeed shows that the disorder tends to degrade the transmission reduction in the third gap to a certain degree, while the disorder effect is not so significant for the second gap.

In summary, in this paper we have shown a numerical analysis of the previous experimental results on microwave propagation through arrays of dielectric cylinders embedded in a uniform medium [16] by the standard multiple scattering method. Comparison has been also made with other theoretical results. The results reveal that localized states are possible for defected arrays. However, the experimentally claimed localized states are not observed. The disorder effects are also presented. The present work is hopped to stimulate further experimental observations.

Thanks to NSC and NCU for supports.

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