1. Introduction

1.1. Charge order

The last decade has seen a strong revival of interest in cuprate superconductors, with the observation of charge orders in the underdoped regime of these materials. Maybe the starting point of this intense period of investigation was the observation by scanning tunneling microscopy (STM) of checkerboard-type patterns inside the vortices in Bi-2212 [1, 2]. Subsequent studies with Fermi surface reconstruction showed that this feature was generic [3, 4] (also verified in Bi-2201 [5, 6]) and that the charge patterns corresponded to two axial wave vectors \((Q_x, 0)\) and \((Q_y, 0)\), incommensurate with the lattice periodicity, and the magnitude of the wave vectors decreases with oxygen-doping. The charge excitation was also found to be non-dispersive in temperature, and correlated with the ‘hot-spots’—the points of the Fermi surface where the antiferromagnetic (AF) zone boundary is intersected. The picture refined itself a bit later, and we now believe the charge order emerges at the tip of the Fermi arcs [7, 8]. Another evidence for charge-density-wave (CDW) order comes from soft x-ray measurements [9].

The study of charge order in underdoped cuprates stayed in a status quo until the observation of quantum oscillations (QO) under a strong magnetic field in YBCO [10, 11]. This result pointed directly to a reconstruction of the Fermi surface induced by magnetic field and received several explanations in terms of stripe and charge patterns until the link was made with the bi-axial charge order observed by STM [12–16]. In particular, models for the reconstruction of the Fermi surface involved charge ordering with bi-axial wave vectors similar to those unveiled by STM. A subsequent nuclear magnetic resonance (NMR) study finally found some charge splitting under a magnetic field, which brought the final confirmation that charge order under a finite magnetic field is coherent, static and long-ranged [17–19]. The field versus temperature phase diagram was later completed by ultrasound experiments, which showed evidence for a flat
transition line at $B_c = 17$ T [20]. For $B \leq B_c$, YBCO is a $d$-wave superconductor. The increase of the magnetic field then creates vortices whose cores show the typical charge ordering [18]. For $B \geq B_c$, YBCO shows long range charge order with a typical ordering temperature remarkably similar in magnitude with the SC ordering temperature ($T_c$). In the pseudogap (PG) regime at $B = 0$, both hard x-ray [21, 22] and soft x-rays [23–26] studies showed the presence of a sizable short range excitation at the bi-axial wave vectors. A softening of the phonon spectrum has been observed in the cuprate superconductors, the pseudogap (PG) regime. The PG phase resides in an underlying emergent SU(2) symmetry, that is still centered around 41 meV, but shows a typical ‘hour-glass’ shape centered around 41 meV at $Q = (\pi, \pi)$ as a function of energy and wave-vector. It was later shown that the resonance exists as well in the PG phase above $T_c$, where it is still centered around 41 meV, but shows a typical ‘$Y$’-shape with a long energy-extension at $Q = (\pi, \pi)$ [73–76]. Many theoretical approaches have been invoked to describe the resonance below the superconducting (SC) transition [77–80]. This observation of the resonance around similar typical energies in the SC and PG phases, however, has never received a theoretical description, and constrains theories of the PG to keep some reminiscence of the SC phase. The neutron resonance was also observed in mono-layer tetragonal compounds (Hg-1201), where the long energy extension at $Q = (\pi, \pi)$ persists below $T_c$ [81]. Earlier, a neutron resonance study in Hg1201 cuprate close to optimal doping also observed a relation between the resonance and the SC gap, indicating a coupling between charge and magnetic fluctuations [82].

1.2. Pseudo gap regime

We turn now to one of the most enduring mysteries of cuprate superconductors, the pseudogap (PG) regime. The PG phase was observed in 1989 by NMR experiments [31, 32], where a gradual drop in the Knight-shift was observed at a crossover temperature $T^*$. This gap was attributed to a loss of density in the electronic carriers, and it was shown to decrease when the oxygen-doping increases, but no obvious symmetry breaking was associated with this phase transition. We focus here on a few properties of the PG phase which we will use later in the SU(2)-interpretation of the experiments. The first remark that one can make is that the PG is an extremely robust feature of the phase diagram. It seems insensitive to disorder [33, 34] and magnetic field [35] and is closely associated to a regime of linear-in-$T$ resistivity on its right hand side [36–38]. Another very striking observation in the PG regime detected by angle-resolved photoemission spectroscopy (ARPES) is the formation of Fermi arcs, instead of a closed-contour Fermi surface [39–44]. Recently, a momentum scale of similar magnitude as the one observed in the CDW was associated to the opening of the PG in the anti-nodal region of the Brillouin zone (BZ) [44–46], and led to an interpretation in terms of a pair-density-wave (PDW) [47, 48]—or a finite momentum superconducting state Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) [49, 50] at zero magnetic field.

Moreover, coherent neutron scattering showed a $Q = 0$ signal [51–56], which was interpreted in terms of intra-unit-cell loop currents [57–59], predicting a broken time-reversal symmetry with $Q = 0$ magnetic order. Although a $Q = 0$ phase is unable to open a gap in the electronic density of states, the doping dependence of the associated characteristic temperature of the loop-current state surprisingly follows the $T^*$-line. Note that NMR [60, 61] and $\mu$SR [62, 63] techniques were not able to detect such loop current. An explanation could be the longer time scale of local probes ($\approx 10^{-5}$–$10^{-6}$ s) compared with the inelastic neutron scattering (INS) time scale ($\approx 10^{-11}$ s). At lower temperature, a Kerr effect signal has been reported, hinting at a breaking of time-reversal (TR) symmetry inside the PG [64]. TR breaking in the PG phase was proposed earlier already from ARPES measurements [65]. This last observation is widely discussed by the community, but it is necessarily related to the $Q = 0$ loop currents [66, 67]. The INS is also interesting for revealing collective modes of the system. A resonance at 41 meV was found in YBCO in the early days of cuprate superconductivity [68] and at similar energies in other compounds [69–72]. It was first believed that this collective excitation existed only in the SC phase, where it has a typical ‘hour-glass’ shape centered around 41 meV at $Q = (\pi, \pi)$ as a function of energy and wave-vector. It was later shown that the resonance exists as well in the PG phase above $T_c$, where it is still centered around 41 meV, but shows a typical ‘$Y$’-shape with a long energy-extension at $Q = (\pi, \pi)$ [73–76]. Many theoretical approaches have been invoked to describe the resonance below the superconducting (SC) transition [77–80]. This observation of the resonance around similar typical energies in the SC and PG phases, however, has never received a theoretical description, and constrains theories of the PG to keep some reminiscence of the SC phase. The neutron resonance was also observed in mono-layer tetragonal compounds (Hg-1201), where the long energy extension at $Q = (\pi, \pi)$ persists below $T_c$ [81]. Earlier, a neutron resonance study in Hg1201 cuprate close to optimal doping also observed a relation between the resonance and the SC gap, indicating a coupling between charge and magnetic fluctuations [82].

1.3. Symmetry guided approaches

Collective modes of a material give useful insights to probe symmetries of an effective model. One example is a resonance observed in the Raman $A_{1g}$ channel, that appears at energies very similar to the ones where a collective mode was observed by INS [83–85]. Raman scattering typically probes the symmetries of the Fermi surface and the presence of ‘two gaps’ in the underdoped regime of the cuprates was observed below $T_c$ [86–88]. This fact was corroborated in a series of ARPES experiments on BSCO from which the gap velocity $v_\Delta$ at the nodes was extracted and shown to differ from the Fermi velocity. Three regions in the phase diagram were identified [89]. Starting from the over-doped region and decreasing the doping, $v_\Delta$ is shown to first increase then to reach a plateau in the underdoped region down to dopings of the order of 5%, and after that it drops at lower dopings when the system gets close to the insulating Mott-transition. The key question associated with the PG phase is whether it is a ‘strong-coupling’ phenomenon, emerging as a direct consequence of the Mott transition [90–94], or whether it is a very unusual collective phenomenon which is sensitive to other peculiarities of the physics of the cuprates, like its low dimensionality, the antiferromagnetic fluctuations or its fermiology [95–98]. In this work, we argue that the key to explain the mystery of the PG phase resides in an underlying emergent SU(2) symmetry, which produces SU(2) pairing fluctuations at intermediate energy scales. These fluctuations are in turn unstable toward the formation of a new kind of excitonic state, the (RES) state,
which is responsible for gapping out the Fermi surface in the anti-nodal region of the BZ [98].

Let us compare the SU(2) proposal in the context of phase transitions and crossovers. The pseudogap state is peculiar, since the loss in the density of states in the antinodal region is neither accompanied by an obvious symmetry breaking nor significant change in the specific heat that would point to a usual phase transition. An alternative transition could involve a symmetry breaking at $Q = 0$, without explicitly breaking translational invariance, but no opening of a gap is found in this case [57]. In addition, non-continuous symmetries like the discrete $Z_2$ time reversal symmetry [48] or $(k \to -k)$ inversion symmetry [99] could be broken and involve a gap opening. The SU(2) proposal, on the other hand, describes the transition to the PG as a crossover, where a non abelian SC state with SU(2) preformed pairs is involved. The SU(2) fluctuations that affect the phase of the complex order parameter while the amplitude is fixed, eventually lead to the formation of local objects [98].

The paper is organized as follows: in section 2, we present the basics of the emergent symmetry model with SU(2) symmetry. Section 3 discusses the competition between the U(1) and SU(2) paring fluctuations in the framework of the non-linear $\sigma$ model. In particular, we propose to explain the PG state as a new type of charge order: the resonant excitonic state (RES) coming from the SU(2) fluctuations. We also demonstrate that the CDW state is a secondary instability produced by U(1) fluctuations mediated by a Leggett mode. In section 4, we discuss the possible experimental evidence of this phase before we conclude in section 5.

2. The emergent SU(2) symmetry

The concept of emergent symmetry in the context of the cuprate superconductors can be traced back to the work of Yang and later Zhang [100, 101] where a representation with pseudo-spin operators was introduced which rotated the d-wave superconductor to the d-density wave state [103]. The concept of SU(2)-symmetry was used later on in an effective theory of the PG leading to a rotation from the d-wave superconductor to the d-density wave state [103]. Here the generators of the symmetry are simply $\eta^z$, $\eta^y$ and $\eta^x$ and the effective theory is the O(4) non-linear $\sigma$-model. Let us mention a similar rotation between the nematic d-wave bond order $\Delta_{\text{bnd}} = 1/2 \sum_{\mathbf{k}, \sigma} d_\mathbf{k}^\dagger c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\overline{\sigma}}$ and d-wave states $\Delta_{\text{dc}} = -1/2 \sum_{\mathbf{k}} d_\mathbf{k}^\dagger c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k} \overline{\sigma}}$, where $d_\mathbf{k} = \cos k_x - \cos k_y$ [104]. The pseudo-spin generators in this case take the form $L^+ = L^0 = 1/2 \sum_{\mathbf{k}, \sigma} (c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\overline{\sigma}} - 1)$. Note that the chemical potential couples to the generator $\eta^z$ (or $L_0$) and thus a finite chemical potential breaks the SU(2) symmetry in favor of the SC state.

Another rotation, this time from the SC state towards the AF state, was introduced early on and became famous as the SO(5) theory [102, 105, 106]. The SO(5) theory is the one of a non-linear $\sigma$-model which operates on a five state ‘super spin’ $(n_1, n_2, n_3, n_4, n_5)$: two SC states $(n_1 = \Delta_1, n_5 = \Delta_0)$ and three AF vectors $(n_2 = s^+, n_3 = s^-, n_4 = s^0)$ [102]. The super-spin $n_\sigma$ is a vector representation of the SO(5) algebra. The SO(5) theory was based on the idea that both the SC and the AF states, that are direct neighbours in the phase diagram, are key players of the physics of these compounds. When both states are close enough in energy, one could perform a rotation between these two states, so that in between their respective phase transition an SO(5)-symmetric state is found where SC and AFM are undistinguishable. This phase was naturally associated with the PG of the cuprates. A typical SO(5) non-linear $\sigma$-model was introduced to describe the effective physics of the system, and four typical phase diagrams were derived which are depicted in figure 1. The mechanism favoring one of the states in the non-linear $\sigma$-model can be understood as a spin-flip transition—or called ‘super spin flop’ transition for the SO(5) symmetries. As mentioned above, one gets a very accurate picture by thinking of the
spin-flop transition of the antiferromagnetic state in a uniform magnetic field $\mathbf{B}$ along the easy $z$-axis [107, 108]. The magnetic field creates an easy plane $xy$, so that at a critical value of the field, the Néel wave vector changes its orientation abruptly from the $z$-axis to the $xy$-plane. Hence, although in each of the above cases the symmetries are different, the underlying physics is as simple as the one on a spin-flop transition. The four typical phase diagrams show the various phases as a function of temperature and an external parameter which breaks the symmetry and anisotropy of the compound. The $SO(5)$-symmetry is broken in scenario (1) – disconnected. In cases (1) and (3), although the symmetry is broken at zero temperature, thermal fluctuations lead to a restoration below the mean-field critical temperature $T_{MF}$. Adapted with permission from [102]. Copyright 1997 American Association for the Advancement of Science.

Figure 1. Schematic phase diagram of the $SO(5)$ model [102]. Four types of scenarios are discussed in [102]: (1) a direct first order transition with a bi-critical point, (2) two second order transitions with an intermediate coexistence regime, (3) one single second order transition terminating at a QCP at zero temperature and (4) two second order transitions with a quantum disordered phase. Although the $SO(5)$ symmetry is broken in scenario (1), the $SU(2)$ symmetry which rotates from a $d$-wave SC state to an incommensurate $d$-wave charge order. The pseudo-spin generators have the form

$$\eta^+ = \sum_k c_k^\dagger c_k^\sigma,$$

and

$$\eta^- = \langle \eta^+ \rangle^\dagger,$$

(5)

$$\eta_1 = \frac{1}{2} \sum_k (c_k^\dagger c_k^\sigma + c_k^\dagger c_k^\tau - 1).$$

(6)

The $l=1$ irreducible representation is given by the $d$-wave version of equation (2) with $\Delta_0 = \chi_{CDW}$, $\Delta_1 = \Delta_{bC}$ and $\Delta_{-1} = -\Delta_1$, namely

$$\Delta_1 = -\frac{\sqrt{2}}{2} \sum_k \Delta_k c_k^\dagger c_{-k}^\sigma,$$

(7a)

$$\Delta_0 = \frac{1}{2} \sum_k \Delta_k c_k^\dagger c_k^\sigma,$$

(7b)

$$\Delta_{-1} = -\frac{\sqrt{2}}{2} \sum_k \Delta_k c_k^\dagger c_{-k}^\sigma,$$

(7c)

with $\Delta_k = (d_k^+ d_k) / 2$ and the momentum $\mathbf{k} = -\mathbf{k} + \mathbf{Q}$ and $(-\mathbf{k}) = -\mathbf{k}$.

The CDW ordering wave vector could be the axial CDW wave vector observed through many recent experiments (STM, quantum oscillations, x-rays, ARPES) or it could be another wave vector carefully chosen so that the $SU(2)$ symmetry is fully respected. As it turns out, the eight-hot-spot (EHS) model depicted in figure 2 provides an exact realization of such a symmetry, as was first mentioned by Metlitski and Sachdev in [113, 114]. This model is a simplified version of the spin-fermion model, which describes the vicinity of an AF QCP within a metallic substrate [115]. At this point, it is useful to recall that the spin-fermion model played an important role at the beginning of the theoretical investigation on the cuprates [116–118]. Two different views were (and still are) competing for the understanding of the phase diagram of these compounds. Observing that the SC phase is close to a Mott insulator, a first group of theoreticians consider that the system is fundamentally strongly correlated, namely that the Coulomb energy $U \approx 1$ eV is affecting the qualitative behavior down to very low temperatures [90, 94, 119]. This viewpoint has been extensively developed around the resonating valence bond (RVB) suggestion made by Anderson as early as 1987 [90], and now explored via extensive numerical calculations which can capture the strongly interacting behavior [94, 120]. Another part of the physics community defends the viewpoint that the Mott transition has a strong qualitative verified experimentally [111]. One reason invoked here was that the SC state is by nature itinerant while the AF state is an insulator in those compounds. The typical energy difference between those two states, the SC and the AFM one with characteristic energies of around 100 K and respectively 700 K is big, and hence of the order of the Coulomb $U$. The critique of the large energy difference $U$, among other points, was already elaborated in [112].

In the present work, we revive the concept of emergent symmetry, with an $SU(2)$ symmetry which rotates from a $d$-wave SC state to an incommensurate $d$-wave charge order.
influence up to 6–7% doping, beyond which the physics of the system is either mainly driven by the presence of AF fluctuations [121], or propose other weak coupling scenarios like e.g. [57, 96, 97, 122–124]. We will focus in the following on the spin-fermion (SF) model, a prominent weak coupling scenario, which couples conduction electrons to AF paramagnons through a simple spin-spin interaction term of equation (1) but with possesses an exact SU(2) symmetry defined by the operators

\[ \Phi = (\Phi^1, \Phi^2, \Phi^3) \] on the brink of criticality with the

\[ \Phi \]

within the EHS model, a further simplification is implemented with the reduction of the Fermi surface to eight 'hot spots' which are the points at \( T = 0 \) where the electrons scatter through the AF \( \Phi \)-modes. When the electron dispersion \( \varepsilon \) is linearized at the hot spots, the model possesses an exact SU(2) symmetry defined by the operators of equation (1) but with \( Q = Q_{\text{diag}} \) being the diagonal wave vector depicted in figure 2. This model was further studied in [126] and an SU(2) precursor of the AF state was found, where quadrupolar density wave (QDW) with diagonal wave vector, which is equivalent to a \( d \)-wave CDW with diagonal wave vector, is degenerate with the \( d \)-wave SC state. This new state can be understood as a kind of non-Abelian superconductor with order parameter \( b \) that, instead of having a U(1) phase, has an SU(2) unitary matrix fluctuating between the charge and SC sector [125, 126].

Subject to the constraint \( |\Delta_{\text{CDW}}|^2 + |\Delta_{\text{SC}}|^2 = 1 \). Within the framework of the EHS model, and the related O(4) non-linear \( \sigma \)-model, several experimental findings were successfully addressed [127–129]. The general picture follows closely the ideas expressed in the SO(5) theory, which are valid for all theories of emergent symmetries. A small curvature term in the electron dispersion breaks the symmetry in favor of the SC state. Hence at \( T = 0 \) the system is a superconductor. Once the temperature is raised, thermal fluctuations then excite the system between the two pseudo-spin states, restoring the SU(2) invariance below the PG temperature. Conversely, an applied magnetic field breaks the SU(2) symmetry in favor of the CDW state and beyond a certain critical field \( B_c \), a 'pseudo spin-flop' is observed where the ground state 'flips' from the SC state to CDW order. This 'pseudo spin-flop' was precisely observed in experiments performed under magnetic field, with a critical field \( B_c \sim 17 \) T [10, 14, 130]. In particular, the ultrasound experiment [20] shows that the typical B versus T phase diagram in figure 3 is very similar to the phase diagram (2) in figure 1. Within the EHS model, this experiment was addressed in [128]. Note that a coexisting phase is present in this phase diagram, which accentuates the similarity with the phase diagram (2) in figure 1 of the SO(5) theory. Notice as well that the CDW and SC temperatures are of the same order of magnitude, which was never the case for the AF and SC states. It is another indication that the SU(2) symmetry is more likely verified in the underdoped cuprates than the SO(5) symmetry.

Of course, a question can be raised at this point, which is that the exact realization of the SU(2) symmetry within the EHS model gives a charge wave vector on the diagonal, while only axial charge order was experimentally observed [1, 2, ...
6, 22, 23, 26, 44, 45, 131]. It is an important question in the SU(2) theory and we will address it in detail in the next section. For the moment, let us notice that similar rotations as in equation (1) can be generated for the axial wave vector $Q = \{Q_x, Q_y\}$ observed experimentally, which rotates similar multiplets as in equation (7) but for the axial wave vector. This idea of a rotation between the $d$-wave SC state and the axial charge order [129] was used to explain that the CDW signal is peaked at $T_c$ [22, 23]. It was also used in explaining the $A_{1g}$ mode observed in Raman scattering as a collective mode associated to this specific rotation [132–134].

The notion of an emergent symmetry is more general than any of its specific representations. It is indeed very nice to have a model, although very simplified, where the SU(2) symmetry is exactly realized (at all energy scales), but the main concern is whether this symmetry is approximately realized at finite temperatures in the underdoped region of the phase diagram. That is the interest of the concept of emergent symmetry: although it can be exactly realized in only a few effective models, if the splitting between the two pseudo-spin states is smaller than the typical energy of each state, it can also be approximately realized at low energies in the more realistic 2D $t–t'$ Hubbard model (this was verified explicitly in [135, 136] using two-loop RG techniques).

Another remark that can be made at this stage, is that another type of SU(2) symmetry was identified early on, which consists of performing a particle-hole transformation on each site $c^\dagger_{\sigma} \rightarrow c_{\bar{\sigma}}$, which translates in the reciprocal space as $c^\dagger_{\sigma} \rightarrow c_{\bar{\sigma}}$ for all k vector. This symmetry is interesting for the phase diagram of the cuprates because it is exact at half-filling and will be gradually broken with doping [137, 138]. The operators for this symmetry group take the form

\[
\eta_{ph} = \sum_k \sum_{\sigma} c^\dagger_{\bar{\sigma}} c^\dagger_k, \tag{10a}
\]

\[
\eta_z = \sum_{k\sigma} (c^\dagger_{\bar{\sigma}} c_{\sigma} - 1), \tag{10b}
\]

while one irreducible $l = 1$ representation associated to it can be taken as $3^1$

\[
\Delta_{1} = - \frac{1}{\sqrt{2}} \sum_k d_k c^\dagger_{\bar{\sigma}} c^\dagger_{-\bar{\sigma}} - d_{-\bar{\sigma}} c_{\bar{\sigma}}. \tag{11a}
\]

\[
\Delta_0 = \frac{1}{2} \sum_{k,\sigma} d_k c^\dagger_{\bar{\sigma}} c_{\sigma}, \tag{11b}
\]

\[
\Delta_{-1} = \frac{1}{\sqrt{2}} \sum_k d_k c^\dagger_{\bar{\sigma}} c_{\bar{\sigma}}. \tag{11c}
\]

Interestingly, within the EHS model, the operators (10) and (11) are, respectively, the same as (1) and (7) with a diagonal wave vector, since there the summation over k is reduced to the eight hot spots. The EHS model is also an exact realization of the SU(2) symmetry associated with particle-hole transformation.

$3^1$ Note that a different representation was used in the slave-boson approaches [137, 138], where the rotation was performed from the $d$-wave SC state towards $\pi$-flux phases.

Using (10) one can also ask oneself what is the SU(2) partner of the observed axial CDW. To fix ideas, let us take a uni-axial CDW order with ordering wave vector $Q$, relating two hot spots. One can then construct the $l = 1$ irreducible representation using the particle- hole transformation. This gives

\[
\Delta_{1} = - \frac{1}{\sqrt{2}} \sum_k d_k c^\dagger_{\bar{\sigma}} c^\dagger_{-\bar{\sigma}} - d_{-\bar{\sigma}} c_{\bar{\sigma}}. \tag{12a}
\]

\[
\Delta_0 = \frac{1}{2} \sum_{k,\sigma} d_k c^\dagger_{\bar{\sigma}} c_{\sigma}, \tag{12b}
\]

\[
\Delta_{-1} = \frac{1}{\sqrt{2}} \sum_k d_k c^\dagger_{\bar{\sigma}} c_{\bar{\sigma}}. \tag{12c}
\]

which means that the SU(2) partner of the $Q_i$ CDW is the pair density wave (PDW), namely a non-zero center of mass SC state, with $Q_i$ wave vector. This notion of PDW state was introduced recently to explain the very unusual ARPES data tracing the formation of the PG in Bi-2201 [47, 139]. In this theory, the formation of the PDW is suggested as the primary mechanism for the formation of the PG state, which means that the observed CDW is a secondary order. As such, it should be observed at a wave vector twice as big as the PDW wave vector $Q_{PDW} = 2Q_{CDW}$. In contrast, if the mechanism governing the underdoped region is a hidden pseudospin SU(2) symmetry, then the partner of a bi-axial CDW is a bi-axial PDW with the same wave vectors $Q_{CDW} = Q_{PDW}$ (see [48, 136, 140, 141]). The latter scenario has recently been verified experimentally [142].

3. Non linear $\sigma$-model and SU(2) versus U(1) pairing fluctuations

The idea of emergent symmetries received a recent critique, that when the group of symmetry is large enough, the symmetric phase is unstable to smaller subgroups [143]. For example, the symmetric phase associated to the SO(5) symmetry which was intended to describe the PG shall decompose into the SU(2) $\times$ U(1) group describing fluctuations around the AF and SC phases respectively. Similarly, the SU(2) symmetry which rotates between the CDW and SC channel shall decompose into the U(1) $\times$ U(1) groups. In this section, we consider seriously the criticism that the symmetric phase of large non-abelian groups is unstable, but wonder more particularly about the fate of SC fluctuations.

The role of SC fluctuations in the physics of cuprates is indeed very mysterious. We know that they are a few orders of magnitude more intense that in standard metals like Al, or Cu [122], but experiments detecting the Josephson effect were observed only a few tens of degrees above $T_c$ [33, 144, 145]. In the deeply underdoped phase, U(1) SC fluctuations form a dome shape that we will discuss further in this section [145–149]. Direct observation of pre-formed pairs in the PG phase was always negative, but a giant proximity effect was observed in the Lanthanum-compounds induced in the PG phase when it is surrounded by optimal SC phases [150–152]. The very easy injection of pairs from optimally
and with \( kq \) it is thus a \(-\Lambda \)† is vanishes \( \chi \) + ″ ′′ \( F \) () (\( F \)) in the positive region of the first Brillouin zone. \( = \sum \right( -\Xi a ) /uni27E8 () () /uni27E9 \) ′′ \( \Delta = 2 \hat{C}_{\hat{k}} \), it grows at \( 2 \) \( 21 \) ′, with \( cckk q kk q \) \( \hat{\pi} G ik \) (b) Variation of crossing the hotspot, but grows in the nodal line to the upper edge. It is small in the blue region and vanishes for the two black lines are \( \SU(2) \) structure. The coefficients write which effectively kills the \( \SU(2) \) fluctuations can be found, in the context of the EHS model in \[126\], and in the context where regions of the Brillouin zone instead of points are ‘hot’ (or anti-nodal regions) in \[98\]. The massless O(4) effective free energy has result is depicted in figure 5. We obtained an excitonic state in \( J_{\hat{a},k} \) = (\( J_{\hat{a}}(\nu \cdot q)^2 + a_{\hat{a},k} \)) with \( n_{\hat{a},k',q} = \langle \hat{\Delta}_{\hat{a},q} \hat{\Delta}_{\hat{a}',q'} \rangle = \frac{\pi_0(\delta_{\hat{a},k-k'} + \delta_{\hat{a},k'})}{(i\omega_n + J(\nu \cdot q)^2 + a_{\hat{a},k})} \). (17) where the form of the SC fluctuations comes from equations (13) and (14). The self-consistent Dyson equation (or ‘gap equation’) writes \[ \chi_{\hat{a},k'} = - \sum_{q} n_{\hat{a},k',q}(\hat{\Delta}_{\hat{a},q} \hat{\Delta}_{\hat{a}',k'}) \] (18) with \( \chi_{\hat{a},k'} = \sum_{q} n_{\hat{a},k',q}(\hat{\Delta}_{\hat{a},q} \hat{\Delta}_{\hat{a}',k'}) \). \[ \langle \hat{G}_{\hat{a},k'} \rangle \] (19) \( = - \frac{\chi_{\hat{a},k'}}{(i\epsilon_n - \xi_{\hat{a}'})/(i\epsilon_n - \xi_{\hat{a}}) - \lambda_{\hat{a},k'}} \). To get some idea about the nonlocal nature the order parameter \( \chi_{\hat{a},k'} \), it is illustrative to consider one of the labels having a constant shift \( \hat{k}' = k + \hat{P} \). The order parameter can then be decomposed as \( \chi_{\hat{a},k'} = \hat{\chi}_{\hat{a}F \hat{P}} \) \[ [\hat{G}_{\hat{a},k'}] \] (20) \( = - \frac{\chi_{\hat{a},k'}}{(i\epsilon_n - \xi_{\hat{a}'})/(i\epsilon_n - \xi_{\hat{a}}) - \lambda_{\hat{a},k'}} \). Note that in equations (18) and (19) the external wave vectors \( k, k' \) are a priori not defined, but are left free to find self-consistently the most favorable solution. We studied numerically the possible excitonic solutions of the gap equations and the result is depicted in figure 5. We obtained an excitonic state in which a large number of wave vectors are degenerate with a typically \( k - k' = 2k_F \) which are spread out in the anti-nodal region of the Brillouin zone producing a depletion of the density of states in this region (see figure 6). Due to the angular

3.1. The SU(2) SC fluctuations

In this section, we assume that at an intermediate energy scale SC fluctuations are present, protected by an SU(2) symmetry between the CDW and SC channel. The microscopic derivation of the non linear σ-model describing the SU(2) fluctuations can be found, in the context of the EHS model in \[126\], and in the context where regions of the Brillouin zone instead of points are ‘hot’ (or anti-nodal regions) in \[98\]. The massless O(4) effective free energy has the following form

\[ F_{SU(2)} = \frac{T^2}{2} \sum_{\omega < 0} \int kq \, \tr \hat{\delta}^2 \hat{k},q [J_{\hat{a},k} \omega^2 + J_{\hat{a},k} q^2] \hat{\delta} \hat{k},q \] (13)

where \( \hat{\delta} \hat{k},q = (\Delta_{CDW} - \Delta_{SC} \Delta_{CDW}) \) is the SU(2) matrix associated with the condition \( |\Delta_{CDW}|^2 + |\Delta_{SC}|^2 = 1 \), and \( \tr \) runs on the SU(2) structure. The coefficients write \( J_{\hat{a},k} = |M_{\hat{a}}|/|G|^{-1} |G^\dagger| \), and \( J_{\hat{a},k} = J_{\hat{a},k} k^2 \), where \( M_{\hat{a}} \) is the magnitude of the mean-field SU(2) order parameter \( \hat{M}_{\hat{a},k} = (\hat{m}_{\hat{a},k})^\Lambda \), with \( \hat{m}_{\hat{a},k} = M_{\hat{a}} \hat{\delta}_{\hat{a},k} q \), which has a 4 × 4 structure in the SU(2) spaces \( \tau \otimes \Lambda \) \[126\]—where \( \tau \) is the particle-hole transformation and \( \Lambda \) is the \( \hat{Q} \)-translation. The Green’s function writes \( \hat{G}^{-1} = \hat{G}_{0,k} + \hat{M}_{0,k} \), with \( \hat{G}_{0,k} = i\omega - (\gamma \xi_k^a - \xi_k^c) \Lambda_k \), with \( \xi_k^a = (\xi_k + \epsilon_k - q) / 2 \), and \( \epsilon_k \) being the electron energy dispersion. Note that no information was given on the value of the \( \hat{Q} \)-wave vector for the CDW sector. It corresponds in all generality to the SU(2) operators (1) and (2). The exact SU(2) symmetry is verified when \( \xi_k^a = 0 \) which effectively kills the \( \tau \)-term in the equation for \( \hat{G}_{0,k}^{-1} \). Hence, \( \xi_k^a \) models the symmetry breaking term associated with this specific wave vector and contributing to the free energy as

\[ F_{SB} = \frac{T^2}{2} \sum_{\omega < 0} \int kq \, \tr [\hat{\delta} \hat{k},q, q \hat{\delta} \hat{k},q] \] (14)

with

\[ J_{\hat{a},k} = \frac{1}{4} \frac{\pi_0 \delta_{\hat{a},k-k'} + \delta_{\hat{a},k'}}{(i\omega_n + J(\nu \cdot q)^2 + a_{\hat{a},k})} \]. (15)

The shape of the symmetry-breaking term equation (15) is visualized in figure 4. One can observe the anisotropy of the mass in various directions in the Brillouin zone: the mass is much bigger in the nodal direction than in the anti-nodal one.

3.2. Resonant excitonic state (RES)

We now integrate the SU(2) SC fluctuations out of the partition function, and evaluate the consequences of them in the charge channel. We get the following effective action

\[ S_{eff}[\epsilon] = \sum_{kk'q} \pi_{kk'q} \epsilon_{kk'q} \hat{G}(q-k-k') \] (16)

with

\[ \pi_{kk'q} = \langle \hat{\Delta}_{kk'q} \hat{\Delta}_{kk'q} \rangle = \frac{\pi_0(\delta_{kk-k-k'} + \delta_{kk-k'})}{(i\omega_n + J(\nu \cdot q)^2 + a_{kk})} \]. (17)
dependence of the fluctuation mass ($a_{\text{node}} \gg a_{\text{anti-node}}$), we obtain a preferential gapping out of the anti-nodal region, which is characteristic of the SU(2) SC fluctuations compared to the original U(1).

3.3. Long range charge order

At this stage, we have proposed a theory for the PG phase of cuprates superconductors. In the following we will give some more arguments that this theory is a promising candidate to describe high-$T_c$ superconductors. We know from a body of experimental evidence, though, that the PG phase is distinct from the observed uniaxial CDW. At zero field, the CDW dome decreases when the oxygen doping decreases, which is at variance with the PG $T^*$ line that increases with the chemical potential (or oxygen doping) [23, 26, 131, 153–155]. Moreover, recent studies of the Fermi surface reconstruction under a magnetic field of 17 T infer that the PG is formed before the Fermi surface is reconstructed by CDW order -recall that the CDW becomes long range and three dimensional beyond $B = 17$ T. There are many proposals for the PG phase [47, 48, 94–96, 120, 129, 143, 156–159], but since ours consists of a special type of excitonic liquid, it is important to shed light on the relationship between the excitonic RES phase and the observed axial CDW which is stabilized under magnetic field.

Within the EHS model, or within all sorts of simple weak coupling RPA evaluation, we find that the axial charge order is a secondary instability, weaker than CDW with a wave vector on the diagonal. The question that is then raised, is why nothing at all is observed on the diagonal, whether it is by STM [2, 5, 6, 8] or by x-rays [22, 23, 26, 153]. The simplest explanation is that the pseudogap is forming, gapping out the anti-nodal region of the Brillouin Zone, and wiping out the CDW instability on the diagonal. When the mechanism of formation of the pseudogap instability has operated, then the secondary instability can be visible, at the tip of the Fermi arcs. Many suggestions have been made for the formation of axial CDW order. The fact that this wave vector is present as a secondary instability in any weak coupling theory, and stabilized for example, in the presence of additional effect like Coulomb interactions [140, 160, 161], within both one-loop

![Figure 5](image-url)  
Figure 5. (Left) Density of the charge order parameter $|\chi_{\text{node}}|$ in the first Brillouin zone from the RES state. The charge density follows the Fermi surface, but due to an SU(2) dependent mass contribution, does not stabilize in the nodal region. (Right) Charge order parameter around the hotspot position for a constant $2p_F$ ordering vector. Reproduced from [98].

![Figure 6](image-url)  
Figure 6. Set of degenerate $2p_F$ couplings between electrons on opposed Fermi surfaces in the antinodal region due to the RES state. Reproduced from [98].

![Figure 7](image-url)  
Figure 7. (Upper panel) Temperature-hole doping phase diagram deduced from Nernst effect experiments (reproduced with permission from [149]; copyright 2006 American Physical Society). The gray area corresponds to CDW phase where Nernst coefficient is not zero. (Lower panel) Temperature-hole doping phase diagram with the superconducting critical temperature (black dots) and the onset temperature of the CDW axial order (red dots) deduced from RXS experiments in YBa$_2$Cu$_3$O$_{6+x}$, (reproduced with permission from [26]; copyright 2014 American Physical Society).
and two-loop RG [135, 136, 162–164], starting from a three band model [157], or invoking the proximity to the van Hove singularity [165] has been outlined in many works, including ours. All these studies are based on the observation that axial CDW is distinct from the formation of the PG state and starts to get formed at the tip of the arcs. In all mentioned scenarios though, it is quite unclear why the CDW dome is increasing to get formed at the tip of the arcs. In all mentioned scenarios.

In the paper, we would like to offer an alternative scenario which to our knowledge has not been proposed yet. It is based on the observation that the CDW dome follows closely the U(1) phase fluctuations that can induce a charge order with the correct wave vectors, see figure 7. The gap equation for the axial CDW mediated by the Leggett mode reads

\[ \chi_{\kappa \sigma} = T \sum_{\omega, q} \pi_{\kappa \sigma, q} \{G_{\kappa \kappa + q - k - q} + \Delta_{\kappa \sigma}^0 \} \],

(20)

with \( \mathbf{Q}_0 = \mathbf{Q}_{xy} \) being the axial wave vector, \( G_{12} \) given by equation (19) with the replacement \( \xi_k \rightarrow \xi_k + \Delta_{\kappa \sigma}^0 \) to take account of the gapping of the FS into account, whereas \( \pi_{\kappa \sigma, q} \) is the U(1) correlation of the phase fluctuations at the tip of the arcs, as represented in figure 8, which is given by

\[ \pi_{\kappa \sigma, q} = \frac{T \sum_{\omega, q} \{G_{\kappa \kappa + q - k - q} + \Delta_{\kappa \sigma}^0 \} \} \],

(21)

with \( \mathbf{Q}_0 = \mathbf{Q}_{xy} \) being the wave vector at the tip of the one Fermi arc and \( \mathbf{Q}_0 = \mathbf{Q}_{xy} \) being the wave vector at the tip of the adjacent Fermi arc. We use a generic form of the propagator \( \pi_{\kappa \sigma, q} \), where \( \pi_0, J_0, J_1 \) and \( m_0 \) are non-universal parameters and the dependence on \( \mathbf{Q}_0 \) is neglected. We have performed a numerical study of equation (20) which confirms that U(1) SC fluctuations mediated by a Leggett mode produce axial CDW with the desired wave vector. This proposal has the merit to consistently link both the formation of the PG and the observed axial CDW to SC fluctuations, the former being described by the SU(2) non-linear \( \sigma \)-model while the latter are the standard U(1) phase fluctuations.

4. Discussion

In the phase diagram of high temperature cuprates a few key players can be identified [138, 168]. There is at half-filling the Mott insulating transition with typical energy of 1 eV associated to it. Antiferromagnetism is ubiquitous in the whole phase diagram, with an ordered phase of typically \( T_{\text{Neel}} \approx 700 \) K at half-filling, very close to the Mott transition, and strong but short range AF fluctuations in the underdoped regime. In the proposal of this paper, the mysterious PG phase of high temperature cuprates is attributed to a new kind of excitonic state, the RES, which can be understood as a new type of ‘liquid’ of excitons, with a superposition of degenerate wave vectors. This state is a consequence of integrating out the SC fluctuations, protected by an emergent SU(2) symmetry between the SC and charge channel. In the discussion of this proposal, the first thing to recall is that although antiferromagnetism is not directly responsible for the PG, it is nevertheless the underlying force driving the emergence of precursor orders. In the early version of this theory, the EHS model has been studied as a reference model where the SU(2) symmetry is verified [126, 169]. In this model the eight hot spots are singled out of the Fermi surface, and long range AFM fluctuations stabilize the composite SU(2) order parameter, composed by a diagonal quadrupolar density wave and SC. In more generic versions of this theory, the model is extended to ‘hot regions’ of the Brillouin zone—the anti-nodal regions, where AF acts predominantly and the SU(2) symmetry is most strongly verified [125]. Antiferromagnetism did not disappear from the phase diagram, but rather has a very special relation to the PG by defining the width of the ‘hot regions’, thus limiting the domain of action of the RES state, and also being the driving force both behind SC pairing and the SU(2) symmetry.
The concept of emergent symmetry, though, is more robust and general than even the idea of Quantum Criticality and it is under such a generic paradigm that we want to cast out the underdoped region of cuprate superconductivity. The main idea is that charge orders are the natural partner and competitors of SC pairing in the underdoped region of the cuprates, and typical pseudo ‘spin flops’ between the two orders are to be expected, and we believe already observed under magnetic field [20]. The consequences of a phase diagram controlled by an SU(2) emergent pseudospin symmetry are numerous, and can be tested by various experimental probes.

4.1. Spectroscopic signatures

One can first ask about the spectroscopic signature of such an excitable state. What can be seen in STM or x-rays? Our claim here is that we can reproduce the very recent findings on Bi-2212 [7, 170], that the pure $d$-wave component of the axial CDW extends up to the PG temperature, see figure 9. In the RES state, indeed, the excitons form not only around many degenerate wave vectors, but with a finite width around each wave vector.

The real space picture is that the particle-hole pairing is non local in space, and modulated by many wave vectors. When the induced charge on the oxygen is evaluated and Fourier transformed, one finds that it is 90% $d$-wave (100% for the diagonal wave vector and a bit less for the others), and at the same time, the axial wave vectors are more favored compared to the diagonal due to its nesting properties in the anti-nodal region [171]. The consequence is that the charge on the oxygen shows a preponderant spectrum with axial wave vectors $\mathbf{Q}_x$ and $\mathbf{Q}_y$. At this stage our conclusion is that the RES state is already observed by STM and x-rays, which have captured its preponderant contribution on the axial wave vectors.

4.2. Proximity effect

A second remark is that emergent symmetries rotating the SC phase to another type of order predict proximity effects when the PG phase is sandwiched between two optimally doped superconductors. The intensity of the induced current in the junction persists for thickness of the gap material much greater than the superconducting correlation length. This ‘giant’ proximity effect (GPE) is not explicable by the standard theory of the proximity effect between two SC junction, but can be understood in the situation where the SC state is ‘quasi-degenerate’ to another phase of matter and Cooper pairs can thus be easily injected from the SC state to the other state. The situation is thus very promising for emergent symmetries, and has been extensively studied in the case of the SO(5) symmetry [106, 110], where specific predictions for the current as a function of the phase difference across the junction can be made as well for the SU(2) symmetry, see figure 10.

Both the SU(2) symmetry and SU(2) fluctuation can account for the experimental data [150–152, 172–175].

4.3. Magnetic field phase diagram

The phase diagram found as a function of magnetic field and temperature, derived with a variety of experiments [10, 14, 18–20] is typical for a super ‘spin-flop’ between two states related by a symmetry (see figure 11). Note that three dimensional CDW has been recently observed by x-ray scattering above $B = 17$ T [29, 30]. The CDW and SC orders have the same order of magnitude in this diagram, and the transition between the two is very sudden, like in a generic spin-flop XY model [127, 128]. Moreover, an SU(2) partner of the axial CDW has recently been reported, i.e. the PDW with the same wave vector.
4.4. Collective modes

Emergent symmetries also have signatures in terms of collective modes. In a recent work we argued that the $A_{1g}$-mode observed in Raman scattering very close in energy to the neutron mode is such a signature of the SC-CDW SU(2) symmetry [132], see figure 12. The collective mode used in this work was associated with the $\eta$-operator of equation (1) with axial wave vector, thus associated to the triplet representation equation (12). The presence of the two orders in conjunctions was needed in order for the Raman scattering vertex not to vanish. The model could account for the absence of observation of this order in the $B_{1g}$ and $B_{2g}$ channels.

Inelastic neutron scattering has been reported since the very early days the presence of a collective mode in the underdoped regime, centered around the AFM wave vector $(\pi, \pi)$, at a finite energy around $E = 41$ meV for the compound YBCO [68, 74]. Many theories, based on an RPA treatment of a magnetic spinon mode below the SC gap have been produced in order to explain this very characteristic feature of the cuprates [80, 106, 176]. The SO(5) theory was originally devoted to the study of this mode [80]. The RPA theories, reproduce successfully the position of incommensurate signal around $Q = (\pi, \pi)$, having the typical ‘hour-glass’ shape in the energy-momentum space. The present theories have difficulties to account for the fact that this signal remains inside the PG phase, changing form from the ‘hour-glass’ to a ‘Y’ shape, namely acquiring some extra low energy spectral weight at $Q = (\pi, \pi)$. The proposed RES state is an excitonic state with excitations around a bunch of $2k_F$ wave vectors in the anti-nodal region. Thus it behaves a little bit as a ‘charge superconductor’, that in the simplest models, will gap out the electronic degrees of freedom precisely as a superconductor would do. We believe the RES state can also account for the extra spectral weight at $Q = (\pi, \pi)$, which will be presented in a future work [171].

Figure 11. $B - T$ phase diagram obtained from the spin-fermion model considering order parameter fluctuations around the mean-field value with a nonlinear $\sigma$ model. Reproduced with permission from [127]. Copyright 2013 American Physical Society.

[142]. Although it is not a direct proof of the underlying symmetry, it seems to rule out other scenarios for the PG state where the PDW is primary while the CDW orders are secondary, and hence occur at twice the same wave vector as the PDW.
momentum larger than the momentum relating the two Fermi points $2k_F$. Moreover when the dispersion cuts get closer to the nodes, the PG closes from below rather than from above (see figure 13). It has been argued that this set of peculiar features can only be explained by a PDW state (a finite momentum SC state), since only the particle-hole reversal specific to the pairing state can account for the closing of the gap from below [47, 177]. We argue that the RES state provides another explanation for this fascinating set of data. Besides the multiplicity of the wave vectors, the key ingredient is the non locality of the excitations in the reciprocal space, they form within a finite window in the anti-nodal region, which can account for the natural closing in energy of the gap, both from above and below (ARPES does not see the positive energies), so that we have only to account for the negative part of the spectrum [171].

4.6. Loop currents

The observation of a $Q = 0$ signal in neutron scattering at a temperature line following $T^*$ [51] is one of the mysteries of the PG phase, which has been interpreted in terms of the formation of intra-unit-cell $\Theta_\text{II}$-loop currents [94]. Although it is commonly understood that a $Q = 0$ phase transition does not open a gap in the electronic spectrum, and thus the $\Theta_\text{II}$-loop-current phase alone cannot be responsible for the origin of the PG, any proposal for the PG phase has to account for the signal observed in the neutron scattering experiment. We have produced two studies within the EHS model regarding the possibility of coexistence of charge orders and loop currents [141, 164]. In [164], we have shown, within a saddle-point approximation, that the $\Theta_\text{II}$-loop-current order cannot coexist with a quadrupolar-density-wave with diagonal wave vectors. As a result, we have offered this scenario as the possible reason explaining why such a quadrupolar-density-wave was never observed along the diagonal direction in the cuprates.

In a subsequent work [141], we have shown that a similar behavior is displayed by the d-wave CDW along axial momenta described by uni-directional wave vectors (i.e. of the stripe-type), since the $\Theta_\text{II}$-loop-current order is also always detrimental to such a uni-directional charge order (see figure 14(a)). By contrast, we have determined that bi-directional (i.e. checkerboard) d-wave CDW and PDW along axial momenta, which are in turn related by the emergent SU(2) pseudospin symmetry pointed out previously, are indeed compatible with the $\Theta_\text{II}$-loop-current order. As clearly seen in figure 14(b), these orders can coexist with one another in the phase diagram that we obtained numerically in [141] for an effective hot-spot model relevant to the phenomenology of the cuprate superconductors. These theoretical predictions agree, most spectacularly, with recent STM results [142] and also with x-ray experiments [26].

4.7. Pump probe experiment

Recent pump probe experiments of the SC density suggest coherent particle-particle and particle-hole mixtures [178], and find strong SC fluctuations at an intermediate energy scale [179] in underdoped cuprates. In the first series of pump probe experiments [180, 181], the cuprate was excited up to 1.5 eV and relaxation at the pico-second scale—observed in the optical THz regime—destroyed the Cooper pairs and showed two typical energy scales, one related to the PG regime and one associated with the formation of the coherent SC phase. Those two scales are typically the ones observed, for example, in the $dI/dV$ response of STM microscope. But in a recent experiment, the excitation was much weaker, in the mid infrared regime [179], which enabled to scan the properties of the PG phase without destroying the Cooper pairs. What was found resembles a pico-second photograph of the SC fluctuations with the superfluid density $\rho_s \sim \omega \Delta \sigma_2$ shooting up in the PG phase, up to temperatures of 300 K (see figure 15). This pico-second ‘photograph’ of the superfluid density was shown to follow the PG temperature as a function of doping.
4.8. General phase diagram

A general look at the phase diagram of the cuprates singles out many enduring mysteries, and one of the most enduring one is the linear-in-$T$ resistivity around optimal doping. This phase was identified in a seminal work as a marginal Fermi liquid (MFL) [182, 183], and it is still a challenge for theories to account for this phenomenon. Recent in-depth experiments show a more complex behavior of the resistivity with temperature, with a part linear in $T$ and a residual part going like $T^2$ when the strange metal regime is approached from the right hand side of the phase diagram [36, 37]. Recent resistivity measurements in Hg compounds probe this transition in detail and find universal resistivity versus temperature prefactors in both the quadratic and the linear regime [184]. Relaxation time and in-plane magnetoresistance measurements support the Fermi-liquid character of the $T^2$ regime [185, 186]. On the theoretical side, two schools of ideas have been advanced to explain this very unusual linear-$T$ regime. In the first school of ideas, it is believed that the proximity to the Mott transition creates a very strongly correlated electronic medium where the electron mean free path is so weak that we are above the Ioffe–Regel limit for the MFL regime [91]. The second school of ideas advances that the resistivity slope as a function of temperature is very steep, so that the second MFL regime extends far beyond the Ioffe–Regel limit at low enough temperatures. Within this second viewpoint, the challenge is to suggest a QCP beyond the dome which could produce a very resistive scattering behavior for the conduction electrons. It is precisely what the quantum critical version of the RES state does. Electronic scattering through quantum critical excitonic modes shows a quasi-one dimensional behavior, each electron scattering preferentially through its most favorable $k^2$ wave vector [98], and produces a resistivity that behaves as $\rho \sim T/\log T$ within a Boltzmann semiclassical calculation and the electronic self-energy that reads $\Sigma(\imath \epsilon_n) \sim \imath \epsilon_n/\log|\epsilon_n|$ in the ‘strange metal’ (SM) regime (see figure 16). On the same line of thought, maybe one of the most difficult feature of the PG to account for in any theory is that it is insensitive to a large amount of Zn-doping or irradiation by electrons [34, 145, 187], which locate on Cu sites and

Figure 15. Schematic phase diagram for YBCO proposed in [179]. Under out-of-equilibrium conditions realized by optical pump–probes, a high mobility phase in the blue shaded area can be realized that extends much above the critical temperature of equilibrium SC. Reproduced with permission from [179]. Copyright 2014 American Physical Society.

Figure 16. Schematic phase diagram of cuprate superconductors with the RES state, as proposed in [98]. The quasi one-dimensional scattering in the vicinity of QCP_{RES} produces the resistivity and the electronic self-energy anomaly observed in the strange metal phase. Reproduced from [98].

Figure 17. ARPES experiments on BSCO performed in [89]. The doping dependence of the gap velocity $v_\Delta$ reveals three distinct regimes: two regions at low and high doping where $v_\Delta$ drops and a third regime in-between, where $v_\Delta$ reaches a plateau. Reproduced with permission from [89]. Copyright 2012 PNAS.
produce strong disorder that exclude the doped Cu-site in the unitary limit [188]. The $T^*$-line is not affected and also the slope of the resistivity in the strange metal regime does not change [34]. It is difficult for any state of matter to have the sufficient robustness to show no sensitivity to such a strong perturbation. One way the RES state could resist is through the non-locality of the excitons (i.e. the particles-hole pair), which can typically being created at site $r$ and removed at site $r'$ with the typical correlation $\langle \delta n_r \delta n_{r'} \rangle$ [171].

5. Conclusion

To conclude, within the two large views of the cuprates where, on the one hand, the physics of the PG is solely determined by strong correlations and the proximity to the Mott transition, and the other view where the qualitative features of the physics of the PG are governed by some hidden symmetry, the present work makes a clear discrimination in favor of the latter. It is claimed here that the physics of the PG and the strange metal phase are controlled by an emergent SU(2) symmetry. Many properties of the underdoped cuprates can be captured within the pseudo-spin theory, the non-linear $\sigma$-model associated to this symmetry and the pseudo spin-flop physics between the SC and charge channel. We also claim that SU(2) superconducting fluctuations proliferate at intermediate energy scales in the physics of these compounds, and are the key ingredient to understand the PG phase. At lower energy, they lead to the formation of the RES state, which we believe has a lot of promising features to be the solution for the PG. At even lower temperatures, the U(1) phase fluctuations enter the game and produce coherent axial CDW mediated by a Leggett-mode. The SU(2) symmetry is present in the background of the whole underdoped region, and it is amazing that the pseudo-spin partners of the various orders (such as the PDW partner of the CDW order) have recently been observed in state-of-the-art STM experiments [142]. Lastly, we pose the question: What is the influence of the Mott transition on the phase diagram of the cuprates? We believe it will qualitatively affect the physics up to roughly 6% of hole doping. Below 6% of hole doping, techniques adequate to describe the very strongly coupled regime will capture the physics [86, 94, 120]. Beyond 6% of hole doping, the physics is qualitatively protected by the emergent SU(2) symmetry. A very revealing experiment is the variation of the nodal velocity $v_\gamma$ as a function of doping extracted from ARPES data [89]. A tri-sected dome is observed with three distinct regimes (see figure 17): (1) below 6% doping, (2) between 6% and 19% of doping and (3) above 19% of doping. Within the SU(2) theory, as with all theories controlled by an emergent symmetry, the critical value of 6% of doping is precisely the point where the Mott physics becomes dominant. Within the strongly correlated viewpoint, the typical doping of 6% is difficult to interpret.

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