Chapter 1
Introduction

1.1 Overview of Supply Chain Disruption Management

A typical global supply chain in modern industry is a cyber-physical network of multiple part suppliers at different geographical locations and multiple production plants and distribution centers, where supplied parts are assembled into finished products and next distributed to customers. Figure 1.1 shows a schematic diagram of a multi-tier supply chain network, where each vertical level (suppliers, producers, distribution centers, customers) is called a tier or echelon, and the arcs represent material flows. For example, a large and complex multi-tier supply chain network of Ford Motor Company (e.g., Simchi-Levi et al. 2015) consists of over 50 manufacturing plants, ten tiers of suppliers, including 1400 tier 1 supplier companies with 4400 manufacturing sites in over 60 countries. Six million vehicles produced annually, require 55,000 different part types with a complex bill of materials.

In order to achieve a high customer service level at a low cost, a variety of complex, interconnected decision-making problems should be solved. The decision-making problems are strictly associated with the control and optimization of material flows (as well as financial and information flows) in the network, in particular, optimization of disrupted flows. Different types of material flows (e.g., flows of parts from suppliers to producers, flows of semi-finished products at producers, flows of finished products from producers to distribution centers and from the distribution centers to customers) should be coordinated in an efficient manner. In global supply chains, the control and optimization of material flows are accomplished by scheduling of manufacturing and supplies of parts, scheduling of production and customer orders for finished products, and scheduling of deliveries to customers. All those scheduling decisions should be efficiently coordinated to fulfill customer demand, especially in the presence of supply chain disruption risks. The schedule of customer orders immediately depends on the schedule of parts supplies, which in turn depends on supplier selection and order quantity allocation, that is, on supply
portfolio. On the other hand, the schedule of customer orders implicitly defines the schedule of deliveries of finished products to customers, which in turn depends on customer order allocation among assembly plants, that is, on demand portfolio. In view of the recent trend of outsourcing, offshoring and globalization, coordinated decision-making, e.g., selection of primary and recovery part suppliers and allocation of order quantities, selection of primary and recovery assembly plants and allocation of customer demand, and scheduling of customer orders in the assembly plants may significantly improve performance of a multi-tier supply chain network under disruption risks. However, most work on coordinated supply chain scheduling focuses on coordinating non-disrupted flows of supply and demand over a supply chain network to minimize the inventory, transportation, and shortage costs. The research on quantitative approaches to coordinated scheduling of disrupted flows in supply chains has not been sufficiently reported in the literature and is mostly limited to separate investigation of supply, production, or distribution stage.

![Fig. 1.1 A multi-tier supply chain network](image)

In contemporary global and lean supply chains, disruption risk management has become a vital part of supply chain management strategy. Material flows in supply chains can be disrupted by unexpected natural or man-made disasters such as earthquakes, fires, floods, hurricanes or equipment breakdowns, labor strikes, economic crisis, bankruptcy or by a deliberate sabotage or terrorist attack. The low-probability and high-impact flow disruptions and the resulting losses may threaten financial state of firms. For example, the Taiwan earthquake of September 1999 created huge losses for many electronics companies supplied with components by Taiwanese manufacturers, e.g., Apple lost many customer orders due to supply shortage of DRAM chips (Sheffi 2005). The Philips microchip plant fire of March 2000 in New Mexico resulted in 400 million Euros in lost sales by a major cell phone producer, Ericsson (Norrman and Jansson 2004). The disruptions in the automotive and electronics supply chains that occurred in 2011 after the Great East Japan Tohoku earthquake on March 11 and then the Thailand flooding in October, resulted in huge losses of major automakers and electronics manufacturers, e.g., Haraguchi and Lall (2015), Park et al. (2013). Similar effects were observed after the Kyushu earth-
quake in April 2016, e.g., Marszewska (2016). Toyota supply chain has been again severely disrupted when two plants of Aisin Seiki, a key supplier of car body and engine components were destroyed, and Renesas, a key supplier of semiconductors for Toyota engines, had to halt production in Kumamoto plant.

The external complexity of outsourcing and offshoring of multinational corporations may additionally contribute to disruption risks of contemporary global multi-tier supply chain networks. For example, Boeing’s global supply chain of 787 Dreamliner consists of over 50 tier 1 suppliers, including 28 suppliers outside of the USA. The external complexity disrupted the multinational supply chain and the final assembly of an aircraft, planned for 2–3 days was delayed by 4 years (e.g., Celo et al. 2018).

A special case of global supply chain disruption risk is an outbreak of a pandemic disease that may fast disperse over many geographic regions. The pandemic disease outbreak is characterized by long-term disruption existence and its unpredictable scaling. Simultaneously with epidemic outbreak propagation in the population, propagation of disruptions in global supply chains can be observed. Recent examples include SARS, MERS, Ebola, Swine flu, and most recently, pandemic of coronavirus disease COVID-19, e.g., Ivanov (2020).

The transboundary dynamics of the COVID-19 pandemic, which has caused severe global socioeconomic disruption, may have an impact on the scale of using outsourcing and offshoring in the future supply chains. An overreliance on lean manufacturing and just-in-time methodology may also be significantly limited in the future and pre-positioning of risk mitigation inventory more frequently used in practice.

In order to minimize losses caused by the shortage of material supplies, customer companies (firms) apply different disruption management strategies. For example, the firms may participate in supplier’s recovery process after disruption to reduce recovery time. When the Tohoku earthquake and tsunami disrupted Toyota supply chain and, in particular, supply chain of automotive semiconductors, Toyota supported recovery of its suppliers, Fujimoto and Park (2013). The automotive semiconductors were manufactured by Renesas Electronics, who shares over 44% of world-wide automobile microcontroller units, and its main plant in Naka was severely damaged by the earthquake. The shipping of automotive semiconductors was expected to be stopped for 8 months. In order to shorten the expected recovery time, Toyota and other Japanese automotive, electronics and semiconductor equipment manufacturers sent to Naka over 2500 engineers to support plant recovery. As a result, the recovery time to start shipping of automobile microcontroller units was shortened from 8 to 5 months, Matsuo (2015). Another example of helping by customer companies in supplier’s recovery process was the case of Riken Corporation, the largest supplier of piston rings to all Japanese automobile manufacturers. In July 2007, Riken plant in city of Kashiwazaki was hit by a strong earthquake and severely damaged. Immediately after the shutdown, the Japanese automakers coordinated by Toyota sent over 650 people including many equipment engineers to help its recovery and as a result the stoppage of piston rings production was shortened to 2 weeks only, Whitney et al. (2014). A similar action has been applied to help recovery of Aisin Seiki plants after the Kyushu earthquake in April 2016, (Marszewska 2016).
The above real-world examples illustrate a well known disruption management strategy of helping a primary supplier recover more quickly. However, when a primary supplier is hit by disruption, the customer company may choose either to support recovery of disrupted primary supplier, rely on a preselected backup supplier or select an alternate (recovery) supplier, non-disrupted or disrupted less severely than the primary supplier. In a similar way, a firm whose primary plant is hit by disruption may either stop production until recovery process is finished or move production to alternate (recovery) plants along with transshipment of parts to the recovery plants. The complex decision-making that involves various characteristics of a supply chain should be supported by optimization models to minimize cost (or maximize profit) and maximize service level as typical objective functions. The objective of this book is to present such optimization models and to stimulate further research on fundamental understanding of various mitigation and recovery policies in the presence of flow disruption risks in global supply chains.

### 1.2 Value-at-Risk Versus Conditional Value-at-Risk

A common tool for supply chain optimization under disruption risks is stochastic programming, in particular stochastic mixed integer programming (stochastic MIP or SMIP), e.g., Heckmann et al. (2015). Stochastic MIP is an exact mathematical modeling approach that allows for the inclusion of uncertainty by probabilistic scenarios of disruptive events and for finding the optimal solutions with respect to multiple objective functions. In this book the stochastic combinatorial decision-making problems will be formulated as multi-period, multi-objective stochastic mixed integer programs with expected or expected worst-case performance measures and trade-offs between various objective functions. In order to mitigate the impact of supply chain disruptions, the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), will be applied for the decision-making, e.g., Sawik (2008, 2009, 2010, 2011b, 2012a,b,c, 2013a,b, 2016). This section briefly defines and compares VaR and CVaR, e.g., Uryasev (2000), Sarykalin et al. (2008).

Let $F_W(u) = \text{Prob}\{W \leq u\}$ be the cumulative distribution function of a random variable $W$ representing cost. The VaR (Value-at-Risk) of $W$ with the confidence level $\alpha \in [0,1)$ is defined as a lower $\alpha$-percentile of the random variable $W$

$$\text{VaR}_\alpha(W) = \min\{u : F_W(u) \geq \alpha\}.$$

VaR represents the maximum cost associated with a specified confidence level of outcomes (i.e., the likelihood that a given portfolio’s costs will not exceed the amount defined as VaR). However, VaR does not account for properties of the cost distribution beyond the confidence level and hence does not explain the magnitude of the cost when the VaR limit is exceeded. Moreover, VaR is not a coherent measure of risk since it fails to hold the sub-additivity property ($f(x+y) \leq f(x) + f(y)$ where $f(.)$ is the risk measure). VaR of a portfolio can be higher than the sum of VaRs of the individual assets in the portfolio.
On the other hand, CVaR focuses on the tail of the cost distribution, that is, on outcomes with the highest cost. Assuming that \( F_W(u) \) is a continuous distribution function, the CVaR of \( W \) with the confidence level \( \alpha \in [0, 1) \), \( CVaR_\alpha(W) \), equals the expectation of \( W \) subject to \( W \geq VaR_\alpha(W) \). However, in the general case \( CVaR_\alpha(W) \) is not equal to an average of outcomes greater than \( VaR_\alpha(W) \) and is defined as the mean of the generalized \( \alpha \)-tail distribution

\[
CVaR_\alpha(W) = \int_{-\infty}^{\infty} u dF_W^\alpha(u),
\]

where

\[
F_W^\alpha(u) = \begin{cases} 
0 & \text{if } u < VaR_\alpha(W) \\
\frac{F_W(u) - \alpha}{1 - \alpha} & \text{if } u \geq VaR_\alpha(W).
\end{cases}
\]

\( CVaR_\alpha(W) \) can be considered as a generalization of the expected value, when \( \alpha = 0 \) they are equivalent. On the other hand, the higher is the confidence level \( \alpha \), the closer are values of \( VaR_\alpha(W) \) and \( CVaR_\alpha(W) \).

Alternatively, \( CVaR_\alpha(W) \) can be defined as the weighted average of \( VaR_\alpha(W) \) and the conditional expectation of \( W \) subject to \( W > VaR_\alpha(W) \).

When the distribution function has a vertical jump at \( VaR_\alpha(W) \) (the probability interval of confidence level \( \alpha \) with the same VaR), a probability “atom” is said to be present at \( VaR_\alpha(W) \). For example, when the distribution is modeled by scenarios, the probability measure is concentrated in finitely many points and the corresponding distribution function is a step function (constant between jumps) with jumps at those points. Since \( CVaR_\alpha(W) \) is obtained by averaging a fractional number of scenarios, the \( VaR_\alpha(W) \) atom can be split. When \( F_W(VaR_\alpha(W)) > \alpha \), then probability \( 1 - F_W(VaR_\alpha(W)) \) of the cost interval \( [VaR_\alpha(W), \infty) \) is smaller than \( 1 - \alpha \).

Note that if \( F_W(VaR_\alpha(W)) = 1 \), so that \( VaR_\alpha(W) \) is the highest cost that may occur, then \( CVaR_\alpha(W) = VaR_\alpha(W) \).

Summarizing the above definitions (from now on, VaR and CVaR will be denoted without subscript \( \alpha \), and with superscripts \( c \) and \( sl \) to denote cost and service level, respectively):

- Cost-at-risk (\( VaR^c \)) at a 100\( \alpha \)% confidence level is the targeted cost such that for 100\( \alpha \)% of the scenarios, the outcome will not exceed \( VaR^c \). In other words, \( VaR^c \) is a decision variable based on the \( \alpha \)-percentile of costs, i.e., in 100(1 - \( \alpha \))% of the scenarios, the outcome may exceed \( VaR^c \).
- Conditional cost-at-risk (\( CVaR^c \)) at a 100\( \alpha \)% confidence level is the expected cost in the worst 100(1 - \( \alpha \))% of the cases. In other words, we allow 100(1 - \( \alpha \))% of the outcomes to exceed \( VaR^c \), and the mean value of these outcomes is represented by \( CVaR^c \).

In other words, \( VaR^c \) is the acceptable cost level above which we want to minimize the number of outcomes and \( CVaR^c \) considers those portfolio outcomes, where costs exceed \( VaR^c \) (see, Fig. 1.2).
Generally, confidence level $\alpha$ indicates the level of conservatism that a decision maker is willing to adopt. As $\alpha$ approaches one, the range of acceptable worst-cases becomes narrower in the corresponding optimization problem. Figure 1.2 clarifies the concept of CVaR and demonstrates that CVaR is the conditional expected value exceeding the VaR.

Similar definitions of VaR and CVaR for service level are given below.

- **Service-at-risk (VaR$^{sl}$)** at a 100$\alpha$% confidence level is the targeted service level such that for 100$\alpha$% of the scenarios, the outcome will not be below VaR$^{sl}$. In other words, VaR$^{sl}$ is a decision variable based on the $\alpha$-percentile of service level, i.e., in 100$(1 - \alpha)$% of the scenarios, the outcome may be below VaR$^{sl}$.

- **Conditional service-at-risk (CVaR$^{sl}$)** at a 100$\alpha$% confidence level is the expected service level in the worst 100$(1 - \alpha)$% of the cases. In other words, we allow 100$(1 - \alpha)$% of the outcomes to be below VaR$^{sl}$, and the mean value of these outcomes is represented by CVaR$^{sl}$.

In other words, VaR$^{sl}$ is the acceptable service level below which we want to maximize the number of outcomes and CVaR$^{sl}$ considers those portfolio outcomes, where service levels do not exceed VaR$^{sl}$ (see, Fig. 1.3).

Since VaR and CVaR measure different parts of the cost (service level) distribution, VaR may be better for optimizing portfolios when good models for tails are not available, otherwise CVaR may be preferred. There are many practical reasons explaining the success of CVaR as a risk measure, e.g., Filippi et al. (2020). CVaR, which is a so-called downside risk measure, penalizes only the negative deviations with respect to a target, set by a decision-maker. CVaR is sensitive to the worst outcomes but it is not as conservative as a minimax approach. However, it simultaneously considers only a fraction of the worst outcomes, while neglecting the others. It is the decision-maker who decides on this fraction by choosing her confidence level. Computationally, CVaR can often be easily embedded in an optimization model by
1.3 Local, Regional, and Global Disruptions

Assume that the supply chain consists of \( I \) interconnected facilities (nodes in this network) that are located in \( \mathcal{R} \) disjoint geographic regions. A facility in a supply chain network may be a supplier of raw material or components, a manufacturer or an assembly plant, a distribution center, or a retailer. Denote by \( I^r \subseteq I = \{1, \ldots, I\} \) the subset of supply chain facilities in region \( r \in \mathcal{R} = \{1, \ldots, \mathcal{R}\} \), where \( \bigcup_{r \in \mathcal{R}} I^r = I \). The supply chain facilities are subject to random independent local disruptions that are uniquely associated with a particular facility, which may arise from equipment breakdowns, local labor strikes, fires, etc. Denote by \( p_i \) the local disruption probability for object \( i \in I \), i.e., the state of facility \( i \) is “on” (non-disrupted) with probability \((1 - p_i)\), or is “off” (disrupted) with probability \( p_i \).

In addition to independent local disruptions of each facility individually, there are also potential regional disruptive events that may result in correlated regional disruption of all facilities in the same geographic region, and global disruptive super events that may simultaneously impact all the facilities, i.e., the entire supply chain. For example, such regional disruptive events may include, flood, earthquake, regional strike in a transportation sector, whereas global disaster super events may include an economic crisis, widespread labor strike in a transportation sector, etc.

Let \( p^r \) and \( p^\star \) be the probability of correlated regional disruption, simultaneously of all facilities \( i \in I^r \) in region \( r \in \mathcal{R} \), and correlated global disruption, simultaneously of all facilities \( i \in I \), respectively. The global disruptive super event, the regional disruptive events in each region and the local disruptive events are assumed to be adding linear constraints and continuous variables, so that a linear structure of the model can be preserved.
independent events. Thus, the disruption probability, \( \pi_i \), of every facility \( i \in I^r, r \in R \) is
\[
\pi_i = p^* + (1 - p^*)p^r + (1 - p^*)(1 - p^r)p_i; \ i \in I^r, r \in R.
\] (1.1)

Denote by \( P_s \) the probability that disruption scenario \( s \) is realized, where each scenario \( s \in S \) is comprised of a unique subset \( I_s \subset I \) of facilities that are in state “on” (non-disrupted), and \( S = \{ 1, \ldots, \tilde{S} \} \) is the index set of all disruption scenarios. Each scenario \( s \in S \) can be represented by a 0-1 vector \( \lambda_s = \{ \lambda_{1s}, \ldots, \lambda_{\tilde{I} s} \} \), where \( \lambda_{is} = 0 \) denotes disruption of facility \( i \in I \) under scenario \( s \in S \), and \( \lambda_{is} = 1 \) denotes a normal, non-disruptive state of facility \( i \in I \) under scenario \( s \in S \). Given the number of facilities \( \tilde{I} \), the total number of scenarios in which at least one facility is non-disrupted is given by
\[
\sum_{i=1}^{\tilde{I}} \left( \begin{array}{c} \tilde{I} \\ i \end{array} \right) = 2^\tilde{I} - 1.
\] Including the scenario in which all facilities are disrupted, there are a total of \( \tilde{S} = 2^\tilde{I} \) potential disruption scenarios. For each scenario \( s \in S \), the facilities \( i \in I \setminus I_s \) can be disrupted either by a local, regional, or global disaster event.

The probability \( P_s \) of each disruption scenario \( s \in S \) is derived as follows. First, the probability \( P_{sI} \) of realizing disruption scenario \( s \) for suppliers in \( I^r \) is determined. If there are non-disrupted suppliers in region \( r \), i.e., \( I^r \cap I_s \neq \emptyset \), then \( P_{sI} \) is the product of regional non-disruption probability, \( (1 - p^r) \), local probabilities of non-disrupted suppliers, \( (1 - p_i) \), \( i \in I^r \setminus I_s \), and local probabilities of disrupted suppliers, \( p_i \), \( i \in I^r \setminus I_s \). Otherwise, i.e., if all suppliers in region \( r \) are disrupted, \( I^r \cap I_s = \emptyset \), then either the entire region is disrupted with probability, \( p^r \), or the region is non-disrupted with probability, \( (1 - p^r) \), and every supplier \( i \in I^r \) is locally disrupted with probability, \( p_i \). Thus, the probability \( P_{sI} \) is
\[
P_{sI} = \begin{cases} (1 - p^r) \prod_{i \in I^r \setminus I_s} (1 - p_i) \prod_{i \in I^r \cap I_s} p_i & \text{if } I^r \cap I_s \neq \emptyset, \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } I^r \cap I_s = \emptyset. \end{cases}
\] (1.2)

The probability \( P_s \) for each disruption scenario \( s \in S \) with the subset \( I_s \) of non-disrupted facilities, and with all possible combinations of different disruptive events considered, is
\[
P_s = \begin{cases} (1 - p^*) \prod_{r \in R} P_{sI}^r & \text{if } I_s \neq \emptyset, \\ p^* + (1 - p^*) \prod_{r \in R} P_{sI}^r & \text{if } I_s = \emptyset. \end{cases}
\] (1.3)

If global and regional disruption probabilities are negligible, i.e., \( p^* = 0 \) and \( p^r = 0, r \in R \), the probability \( P_s \) of disruption scenario \( s \) in the presence of independent local disruptive events only, reduces to
\[
P_s = \prod_{i \in I_s} (1 - p_i) \cdot \prod_{i \notin I_s} p_i.
\] (1.4)
1.4 Two-Level Versus Multi-Level Disruptions

In this section the scenarios with the two-level, all-or-nothing disruptive events considered in Sect. 1.3 are enhanced for the multi-level (partial) disruptive events. In contrast to yield uncertainty (e.g., defective products) that occurs, for instance, when the quantity of supply delivered is a random variable, typically modeled as either a random additive or multiplicative quantity, the multi-level disruptions are modeled as events of different level (e.g., partial capacity available) which occur randomly and may have a random length, e.g., Schmitt and Snyder (2012).

Assume that each facility $i \in I$ is subject to random independent local disruptions of different levels, $l \in L_i = \{0, \ldots, \overline{L}_i\}$, where the disruption level refers to the available fraction of full capacity of a facility (e.g., a partial fulfillment of an order by a supplier, a partial fulfillment of a customer order by a producer, etc.).

Level $l = 0$ represents complete shutdown of a facility, i.e., no capacity available, (e.g., no order delivery) while level $l = \overline{L}_i$ represents normal conditions with full capacity available (e.g., full order delivery). The fraction of full capacity of facility $i$ available under disruption level $l$ is described by $\gamma_{il}$

$$\gamma_{il} = \begin{cases} 
0 & \text{if } l = 0 \\
\in (0, 1) & \text{if } l = 1, \ldots, \overline{L}_i - 1 \\
1 & \text{if } l = \overline{L}_i.
\end{cases} \quad (1.5)$$

Denote by $S = \{1, \ldots, S\}$ the index set of all disruption scenarios, and by $P_s$ the probability of disruption scenario $s \in S$. Each scenario $s \in S$ can be represented by an integer-valued vector $\lambda_s = \{\lambda_{i_1}, \ldots, \lambda_{i_S}\}$, where $\lambda_{i_s} \in L_i$ is the disruption level of facility $i \in I$ under scenario $s \in S$. When all potential disruption scenarios are considered, then $S = \prod_{i \in I} \overline{L}_i + 1$.

Assume that for each scenario $s \in S$, each facility can be disrupted either by a multi-level local disruptive event or by a two-level regional disruptive event. The probability $P_s$ for disruption scenario $s \in S$ with the subset $I_s$ of non-shutdown facilities is given by (1.3). Now, the probability, $P^r_s$, of realizing of disruption scenario $s$ for facilities in $I^r$ is (e.g., Sawik 2015b)

$$P^r_s = \begin{cases} 
(1 - p^r) \prod_{i \in I^r} (p_{i, \lambda_{i_s}}) & \text{if } I^r \cap I_s \neq \emptyset \\
p^r + (1 - p^r) \prod_{i \in I^r} p_{i,0} & \text{if } I^r \cap I_s = \emptyset,
\end{cases} \quad (1.6)$$

where $p_{i, \lambda_{i_s}}$ is the probability of occurrence of the disruption at level $l = \lambda_{i_s}$ at facility $i$. 
1.5 Risk-Neutral, Risk-Averse, and Mean-Risk Decision-Making

In this book we consider three types of decision-making policies.

1. Risk-neutral that is based on expected value optimization approach, e.g., expected cost minimization or expected service level maximization approach. The risk-neutral policy focuses on an average performance of a supply chain.

2. Risk-averse wherein, rather that optimizing the expected value of an objective function, the decision maker uses a Conditional Value-at-Risk (CVaR) approach to measure and quantify risk and to define what comprises a worst-case scenario. The CVaR methodology allows the decision maker to evaluate worst-case values of an objective function, to specify to what extent worst-case scenarios should be avoided and to shape distribution of the resulting objective function values, associated with such a policy. The risk-averse policy focuses on worst-case performance of a supply chain.

3. Mean-risk wherein the decision maker seeks for a best trade-off between expected value and CVaR of an objective function. The mean-risk policy focuses on both the average and the worst-case performance of a supply chain, simultaneously.

In this book we utilize stochastic mixed integer programming (SMIP) approach, that leads to a two-stage optimization problem. The decisions that are made ahead of time are considered the first stage decisions and are represented by the first stage decision variables.

Typical first stage decision variables are

- binary selection variables, such as supplier selection variables, supplier protection variables,
- fractional allocation variables, such as order quantity allocation variables, risk mitigation inventory pre-positioning variables.

In the risk-averse decision-making, VaR (cost-at-risk, service-at-risk, etc.) can also be interpreted as first stage variables.

The second stage decision variables represent decisions that are made after the realization of the random events (e.g., supply disruptions) is known. The second stage variables are dependent on the realized random event.

Typical second stage decision variables are

- binary selection variables, such as recovery supplier selection variables, recovery plant selection variables,
- time-indexed binary assignment variables, such as production scheduling variables, distribution scheduling variables,
- fractional allocation variables, such as recovery order quantity allocation variables, recovery demand allocation variables, risk mitigation inventory usage variables, transshipment variables,
- continuous variables, such as tail cost, tail service level.
1.5 Risk-Neutral, Risk-Averse, and Mean-Risk Decision-Making

1.5.1 Risk-Neutral Decision-Making

The stochastic formulation of the risk-neutral decision-making problem aimed at loss (cost) minimization can be written as

\[
\min_{x \in X} c^T x + E[Q(x, \xi^s)], \quad (1.7)
\]

where \( c^T x + E[Q(x, \xi^s)] \) is the total cost function of the first stage problem and

\[
Q(x, \xi^s) = \min_{y^s \in Y^s} \{ (q^s)^T y^s \} \quad (1.8)
\]

is the optimal solution of the second-stage problem corresponding to the first stage decision variables \( x \) and the realization of the random data \( \xi^s \) for scenario \( s \in S \), denoted by \( \xi^s = (q^s, Y^s) \).

\( E[Q(x, \xi^s)] \) is the expected “cost” taken with respect to random scenario \( s \in S \).

The objective function \( Q(x, \xi^s) \) of the second-stage problem (1.8), also known as the recourse (cost) function, is a random variable.

Here \( x \) and \( y^s \) are the vectors of first stage and second stage decision variables, where the first stage decisions are deterministic and the second-stage decisions are dependent on random scenario \( s \). \( X \) denotes the feasible set of first stage decisions and \( Y^s \) is the feasible set of second stage decisions for random scenario \( s \in S \). The second-stage problem (1.8) may be infeasible for some first stage decisions \( x \in X \).

To deal with the uncertainty in the second stage, a scenario-based modeling approach is proposed that has been widely used in stochastic programming. In the second stage, let us consider random scenario \( s \in S \) to have a discrete distribution, where \( P_s \) is the probability of occurrence for scenario \( s \in S \), and \( S \) is a finite set of scenarios. Given a finite set of scenarios, \( S \), with associated probabilities, \( P_s, s \in S \), the expected value \( E[Q(x, \xi^s)] \) can be evaluated as \( E[Q(x, \xi^s)] = \sum_{s \in S} P_s Q(x, \xi^s) \).

Hence, we can present the deterministic equivalent of the stochastic formulation (1.7).

\[
\min \; c^T x + \sum_{s \in S} P_s (q^s)^T y^s \quad (1.9)
\]

s.t. \( x \in X \), \( y^s \in Y^s \), \( s \in S \).

Model (1.9) is also known as the wait-and-see model (e.g., Birge and Louveaux 2011; Kall and Mayer 2011). In contrast to the two-stage approach with the recourse...
model (1.7), in the wait-and-see approach both, the decisions on the first stage variables \( x \) and the second stage variables \( y^s \), are taken simultaneously only when the outcome of \( \xi^s = (q^s, Y^s) \) is known.

### 1.5.2 Risk-Averse Decision-Making

In the model proposed below, CVaR is represented by an auxiliary function (1.10) introduced by Rockafellar and Uryasev (2000), for a set of pre-defined scenarios \( s \in S \) with corresponding probabilities, \( P_s \). Using the wait-and-see approach, the stochastic formulation of the risk-averse decision-making problem aimed at CVaR of loss (cost) minimization, given confidence level \( \alpha \), can be written as

Minimize

\[
CVaR = VaR + (1 - \alpha)^{-1} \sum_{s \in S} P_s \tau_s
\]  

subject to

\[
\tau_s \geq c^T x + (q^s)^T y^s - VaR; \ s \in S
\]  

\[
x \in X
\]  

\[
y^s \in Y^s; \ s \in S
\]  

\[
\tau_s \geq 0; \ s \in S.
\]

In the above formulation, constraints (1.11) compute the tail cost, \( \tau_s \), for scenario \( s \) and condition (1.14) indicates that the scenarios in which the loss exceeds VaR are considered only.

### 1.5.3 Mean-Risk Decision-Making

In the mean-risk formulation for the wait-and-see approach, \( \lambda \) is a nonnegative trade-off coefficient representing the decision maker risk preference. For a given confidence level \( \alpha \) one can construct the mean-risk efficient frontier by the parameterization on \( \lambda \) the weighted-sum program presented below.
1.6 A Multi-Portfolio Approach

Disruptions in supply chain rarely occur as isolated events. Supply chain disruptions must be contained before they propagate through the supply chain and create greater losses. Supply chain disruption propagation is defined as the spread of the disruption effects beyond the initial disruption location. In the literature, supply chain disruption propagation is also known as domino effect, snowball effect, and more recently as the ripple effect, e.g., Blackhurst et al. (2011), Basole and Bel-lamy (2014), Scheibe and Blackhurst (2018), Ivanov et al. (2015, 2019). In order to contain disruptions and prevent them from spreading through the supply chain, the decision-making on disruption management should be coordinated and both proactive and reactive decisions, spatially and temporally integrated.

In this book an innovative multi-portfolio approach and SMIP formulations with an embedded network flow problem are developed for spatially and temporally integrated decision-making in a supply chain under disruption risks. The portfolio approach to supply chain disruption management was introduced by Sawik (2011b,c,d, 2013c), first to mitigate the impact of disruption risks, and later for a combined risk mitigation and recovery in supply chains (e.g., Sawik 2017b, 2019a). In the context of supply chain disruptions, the portfolio is defined as the allocation of demand for different part types among suppliers and the allocation of demand for different product types among assembly plants of final manufacturer.

The concept of a multi-portfolio approach to supply chain disruption management is illustrated in Fig. 1.4. Depending on the impact of disruptive events, different types of primary and recovery portfolios should be determined. When a disruptive event hits part suppliers only, then only supply portfolios need to be selected. In a
general case, however, a disruptive event may impact both primary suppliers of parts and primary assembly plants of the manufacturer. Then the manufacturer selects recovery suppliers, recovery assembly plants along with the allocation of demand for parts and products among selected facilities and transshipment of the inventory of parts from disabled primary plants to recovery plants.

The multi-portfolio approach, integrated over time and space, allows for a coordinated decision-making in supply chain networks under disruption risks. When the multi-portfolio approach is applied, the proactive and reactive decisions are spatially and temporally integrated: the primary supply and demand portfolios to be implemented before a disruptive event are optimized simultaneously with recovery supply, transshipment, and demand portfolios for each potential disruption scenario for the aftermath period. The portfolios for both part suppliers and product manufacturers in different geographic regions are determined simultaneously. The multi-portfolio approach can be particularly useful for material flow coordination in contemporary multi-tier supply chain networks, which are cyber-physical systems, where integrated decision-making and control over the entire network is of utmost importance. In the SMIP decision-making models, each portfolio type is determined using binary selection variables and fractional demand allocation variables. Without loss of generality, in the definitions provided below we assume single part type and product type environment with a unit requirement of part per product and a single primary assembly plant, $j = 1$. In addition, denote by $D$, total demand for parts/products, and by $I, J$ and $m, n$, respectively, the index set of suppliers, assembly plants and the total number of suppliers, assembly plants.

The primary supply portfolio is determined by the primary supplier selection variables and primary demand for parts allocation variables:

$$u_i \in \{0, 1\},$$

such that the value 1 means that supplier $i \in I$ is selected as a primary supplier; otherwise $u_i = 0$;

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**Fig. 1.4** A multi-portfolio approach to supply chain disruption management
1.6 A Multi-Portfolio Approach

$v_i \in [0, 1]$, the fraction of total demand for parts ordered from primary supplier $i \in I$ (such that $u_i = 1$) to primary plant $j = 1$.

The **recovery demand portfolio** is determined by the recovery plant selection variables and recovery demand for products allocation variables:

$q^s_j \in \{0, 1\}$, such that value 1 means that assembly plant $j \in J$ is selected as a recovery plant under disruption scenario $s$; otherwise $q^s_j = 0$;

$Q^s_j \in [0, 1]$, the fraction of total demand for products to be completed by recovery plant $j \in J$ (such that $q^s_j = 1$) under disruption scenario $s$.

The **recovery supply portfolio** is determined by the recovery supplier selection variables and recovery demand for parts allocation variables:

$U^s_i \in \{0, 1\}$, such that value 1 means that supplier $i \in I$ is selected as a recovery supplier under disruption scenario $s$; otherwise $U^s_i = 0$;

$V^s_{ij} \in [0, 1]$, the fraction of total demand for parts ordered from recovery supplier $i \in I$ (such that $U^s_i = 1$) to recovery plant $j \in J$ (such that $q^s_j = 1$), under disruption scenario $s$.

The **recovery transshipment portfolio** is determined by the inventory of parts transshipment variables:

$w^s_j \in [0, 1]$, the fraction of total demand for parts, transshipped from the disrupted primary plant $j = 1$ to recovery plant $j \in J$ (such that $q^s_j = 1$), under disruption scenario $s$, where $w^s_1$ represents the parts that remain in primary plant $j = 1$.

In a general case of the supply chain-wide impact of disruption risks, the different types of portfolios that should be determined, may include:

The **primary supply portfolio**

$$(v_1, \ldots, v_m),$$

which specifies supplies of parts from selected primary suppliers to primary assembly plant $j = 1$, where

$$v_i \in [0, 1]; i \in I, \sum_{i \in I} v_i = 1.$$

The **recovery supply portfolio for each scenario** $s$

$$(V^s_{1j}, \ldots, V^s_{mj}); \ j \in J,$$

which specifies supplies of parts from recovery suppliers to recovery assembly plants, where

$$V^s_{ij} \in [0, 1]; i \in I, j \in J, \sum_{i \in I} (\gamma^s_i v_i + \sum_{j \in J} V^s_{ij}) = 1,$$

$\sum_{i \in I} \gamma^s_i v_i$ denotes actual delivery of parts from primary suppliers to primary plant under disruption scenarios $s$ and $\gamma^s_i$ is the fraction of an order delivered by supplier $i$ under scenario $s$. 
The recovery transshipment portfolio for each scenario $s$

$$(w_1^s, \ldots, w_n^s),$$

which specifies transshipment of parts from primary assembly plant, $j = 1$ to recovery plants, where

$$w_j^s \in [0, 1]; \ j \in J, \ \sum_{j \in J} w_j^s = \sum_{i \in I} \gamma_i v_i - \sum_{t \in T : t < t_s} x_{1t}^s / D,$$

and $\sum_{t \in T : t < t_s} x_{1t}^s$ denotes production at primary assembly plant $j = 1$, before disruption at time $t_s$.

The recovery demand portfolio for each scenario $s$

$$(Q_1^s, \ldots, Q_n^s),$$

which specifies an allocation of unfulfilled demand for products among recovery plants, where

$$Q_j^s \in [0, 1]; \ j \in J, \ \sum_{t \in T : t < t_s} x_{1t}^s / D + \sum_{j \in J} Q_j^s = 1.$$

In addition, the two decision-making strategies will be considered: an integrated strategy with the perfect information about all potential future disruption scenarios, and a hierarchical approach with no such information available ahead of time. In the integrated approach, which accounts for all potential disruption scenarios, the primary supply portfolio that will hedge against all scenarios is determined along with the recovery supply and demand portfolios and production and inventory scheduling of finished products for each scenario. In the hierarchical approach, first the primary supply portfolio is selected to optimize supplies and production in a deterministic environment (without a disruption). Then, when a primary supplier and/or primary assembly plant is hit by a disruption, the recovery supply and demand portfolios are determined along with transshipment of parts and production and inventory scheduling at recovery plants to optimize the supply chain recovery, given the unfulfilled demand and the inventory of parts available for transshipment.