the quantized space.

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Abstract

It is shown, that the space quantum existence (SQE) non-ambiguously determines the metrical form for the space without a time, using weak condition of metrical additivity. The hypothesis is proposed, Riemann metric is only possible for quantized space classes. This statement probably can close the problem of metrical form.

During long time the space problem is discussed in many works from the point of view of connection between space and time coordinates[1]. It is reckoned, the metrical space form is proved by experimental data and this question was not discussed. But now many virtual spaces exist (see, for example [2]) and there is not guaranty, that space metrical form is unique.

The following approach probably closes the metrical form problem. Certainly, all spectrum of this problem is not discussed in this article, particularly, in cases, when global constants can be changed, etc, but the proposed method can be useful in future.

As known, Fermat’s theorem is formulated[1] as the follow statement: For any integer nonzero $N_x, N_y, N_L$ and natural $\alpha \geq 3$, it is not possible the follow equality:

$$N_x^\alpha + N_y^\alpha = N_L^\alpha$$

(1)

Using Plank’s hypothesis[2] about space quantum[3]: The minimal value of space length exists:

$$L = Nl_{Pl}$$

(2)

where $N$ is natural, $l_{Pl}$ is space quantum.

For the spaces having natural metrical degree the follow theorem can be proved:

**Theorem.** If (1) and (2) are correct, then the space metrical degree of quantized space have value $\alpha \leq 2$:

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1 See information about proof in [1].

2 In the most part, it’s written “for natural number”, but for “nonzero integer number” theorem is correct too.

3 The space quantum is $q = \sqrt{\frac{Gh}{c^3}} = 1.616 \times 10^{-33}$cm, where $G$ is gravity constant, $h$ is Planck’s constant, $c$ is light beam speed.
$$\|L\|^{\alpha} = \sum_{i} \|x_i\|^{\alpha}$$

(3)

The proof is made by the substitution $\|x_i\| = N_i l_{P1}$ to (3), then equation (3) reduces to (2).

For $L^p$ Banach’s space the theorem is proved.

Unfortunately, this approach has one shortcut: the metrics is limited by the form (3) only.

Let the vector $L$, which be decomposed as

$$L = x_i + y_j$$

(4)

and the metric be some function of $\|x\|, \|y\|$: $M = F(\|x\|, \|y\|)$.

In the begin, we make the assumption, which should be definitelty natural for the physical space: the “coordinate splitting”.

**Axiom.** A coordinate system exists, where metric is additive:

$$F(\|x + y\|) = F(\|x\|) + F(\|y\|)$$

(5)

or

$$F(\|L\|) = F(\|x\|) + F(\|y\|)$$

(6)

Assuming Tailor’s decomposition is possible for metric

$$F(\|L\|) = \sum_{i} a_i \|L\|^{\alpha} = \sum_{i} a_i N_i^{\alpha} l_{P1}$$

(7)

and using Borh’s statement about the equivalence of continuous and discrete approaches and accounting (7), we can rewrite (6) as:

$$a_1 N_L = a_1 N_x + a_1 N_y$$

$$a_2 N_L^2 = a_2 N_x^2 + a_2 N_y^2$$

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$$a_i N_L^{\alpha} = a_i N_x^{\alpha} + a_i N_y^{\alpha}$$

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( the grouping is made for degrees of $l_{P1}$, taking into account that $\|x\|, \|y\|$ are arbitrary ).

But (3) is possible only if $i \leq 2$ ( see previous theorem ) or for $a_i = 0$.

This means that:
\[ M(||L||) = a ||x||^2 + b ||x|| \] (9)

For our case \( b = 0 \).

So we nearly proved

**Hypothesis.** Any additive metric in quantized space is quadratic.

**Remark 1**

It is interesting, that \( a \) can be zero. One-dimensional space does not have decomposition (4). However, for any \( a, b \) in this space and \( c = a + b \) expression (2) is valid too: (2) does not rigoursly demand the decomposion (1), (2) is enough.

**Remark 2.**

In the framework of Borh’s conception we used (1), (2) and (8), that is not quite consistent, if one follows formal mathematical logic (\( N_x, N_y \) are not generally arbitrary for discrete approach). This contradiction is escaped, because the numbers \( N_x, N_y \) take “almost” all values: almost for any great value \( N_L \) we can find \( N_x, N_y \) which satisfy to (1). Their number is infinite, furthermore, they have the very dense spectrum of values, that is enough for the space construction. In this case, (2) can be used.

**Remark 3.**

The proposition, that linear operator exists, was not used: we did not introduce scalar multiplicity definition, only special system, where metric is additive. This indicates, our space needs not be Hilbert’s.

**Conclusion.**

For the quantized space it is enough to assume that the space system, having the additive metric, exists. Then we can conclude, that our space is Hilbert’s.

For the space coordinates the connection between SQE and the metric form is proved.

For the space, which includes the time, the problem is known can be solved on the basis, that space coordinates’ metric is quadratic. This definitely means that pseudoeuclidean metric for inertial coordinate systems and, consequently, Riemann’s metric for non-inertial systems is only possible for quantized spaces classes.

These theorems are also the additional indirect proof of SQE.

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