An answer checking method for quantum annealers

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Abstract. We present a generic approach for checking the validity of the solutions returned by quantum annealing devices to aid in the analysis of whether the solution is the true ground state of the desired problem. The underlying principle is to embed a mirrored graph $G'$ of the original graph $G$, and connect the two graphs via strong ferromagnetic/antiferromagnetic couplings that span across the mirror plane. This allows one to dismiss solutions that do not agree with the underlying mirror symmetry inherent to the true ground state of the composite graph. Using a 1000 qubit D-Wave 2X device, we demonstrate this method by applying it to spin glass problems defined on the device’s native Chimera architecture.

1. Introduction

Combinatorial optimization [1] represents a generic class of problems that has direct applications in a broad array of fields ranging from manufacturing and transportation to machine learning. A vast majority of combinatorial optimization problems, including the boolean satisfiability ($k$-SAT) problem, the traveling salesman problem, knapsack problem etc. can be formulated as quadratic unconstrained binary optimization (QUBO) problems [2]. Each QUBO problem, in turn, can be mapped onto an Ising spin glass [3, 4] of the form

$$H_{\text{prob}} = - \sum_{(i,j) \in E} J_{ij} s_i s_j - \sum_{i \in V} h_i s_i, \quad s_i \in \{\pm 1\},$$

where the graph $G = (V, E)$ with vertices $V$ and edges $E$ captures the structural information pertaining to the problem, with $J_{ij}$ and $h_i$ representing the exchange couplings and the local fields, respectively. Finding the ground state of the Hamiltonian (1) through exact enumeration is a non-deterministic polynomially (NP) hard problem [5], and hence in practice it is tackled via heuristic methods such as simulated annealing (SA) [6].

A relatively new optimization scheme called quantum annealing (QA) [7, 8, 9, 10] has recently gained widespread attention, particularly with the introduction of a line of commercial programmable devices by the D-Wave Systems Inc. [11] which utilize QA at the hardware level [12]. Unlike SA which circumvents local minima by thermal hopping over energy barriers, QA strives to avoid local minima with the aid of artificially introduced quantum fluctuations that allow the system to tunnel though energy barriers. These quantum fluctuations are introduced...
by mapping the classical Ising spins $s_i$ in the problem Hamiltonian to Pauli z-matrices $\sigma^z_i$ and incorporating a transverse magnetic field in the $x$ direction, which leads to the time-dependent Hamiltonian

$$H(t) = -A(t) \sum_i \sigma^z_i + B(t) H_{\text{prob}},$$

(2)

where the parameters $A(t)$ and $B(t)$ represent the annealing schedule. $A(0) \gg B(0)$, and hence at $t = 0$, the transverse field term dominates the Hamiltonian. At $t = 0$, the system is prepared in the ground state of the transverse field, which is an equal superposition state $(1/\sqrt{2}) \prod_i (|\uparrow\rangle + |\downarrow\rangle)$ of all $n$ qubits in the system. The goal of QA is to gradually decrease $A(t)$ while increasing $B(t)$ so that at the end of the annealing time $t = \tau$, $H_{\text{prob}}$ dominates the Hamiltonian, i.e. $H(\tau) \simeq H_{\text{prob}}$. If the annealing is performed sufficiently slowly, the time evolution of the system would follow the instantaneous ground state of $H(t)$ at all $t \in [0, \tau]$, which would ultimately result in the ground state of $H_{\text{prob}}$ at $t = \tau$.

In August 2015, D-Wave Systems Inc. announced the breaking of the 1000 qubit barrier with the release of their latest quantum annealing device, the D-Wave 2X [13]. The device operates at a temperature of about 15 mK, with the annealing time in the range of $20 - 2000 \mu s$. In D-Wave devices, the connectivity between the qubits is constrained to the so-called Chimera graph (See Fig. 1 for an illustration of a $4 \times 4$ Chimera graph.). A Chimera unit cell is a $K_{4,4}$ complete bipartite graph with each of the four qubits on the left/right connected to all the four qubits on the right/left. In addition, each qubit on the left is vertically connected to the corresponding qubit of the unit cells directly above and below, while each qubit on the right is coupled horizontally with the respective qubit of the unit cells directly to the left and to the right. The D-Wave 2X utilizes a $12 \times 12$ chimera graph, resulting in a total number of 1152 physical qubits, among which some of them are inaccessible to the users. This study used the device located at the NASA Ames Research Center which consists of 1097 accessible qubits.

Just as in any software-based heuristic method, a QA device may not necessarily find the global energy minimum for a problem during a given annealing run. In this paper, we present an “answer checking method” that allows one to perform one test to assess the validity of the solution as the true ground state of the problem. In section 2, we provide a detailed description of our technique, along with its limitations. In section 3, we demonstrate the application of this method to generally allowed spin glass problems on the D-Wave 2X device’s native Chimera topology.

2. Answer checking via mirrored graphs

Our answer checking method exploits the symmetry conditions associated with the ground state solutions of graphs with mirror symmetry. The steps involved in the method can be listed as follows:

- Split the native architecture of the annealing device into two identical halves by defining a mirror axis.
- Embed the desired graph $G$ onto one side of mirror axis.
- Embed the mirror image ($G'$) of the original graph $G$ onto the opposite side of the mirror axis.
- Connect a selected set of qubits of graph $G$ with the corresponding mirror qubits on $G'$ via strong ferromagnetic/antiferromagnetic couplings that span across the mirror axis (Henceforth denoted as “mirror couplings”).

Fig. 1 illustrates the application of this method to a hypothetical QA device with a $4 \times 4$ native Chimera architecture. Note that in order to make the two halves of the device identical,
one should mirror any inaccessible qubits and couplings present in the device onto the respective sides of the mirror axis prior to embedding the desired graphs.

Let $q_i \in G$ and $q'_i \in G'$ respectively denote a chosen qubit and the corresponding mirror qubit that are connected via a mirror coupling $M$. Since the energy associated with this interaction takes the form $E = -Mq_iq'_i$, during the annealing process, a sufficiently large value of $|M|$ will impose the constraint $q_i = q'_i$ for the case of ferromagnetic coupling ($M > 0$), and $q_i = -q'_i$ for the case of antiferromagnetic coupling ($M < 0$). To understand how these constraints imposed by the mirror couplings can assist one to validate solutions, let us first consider the simplest scenario in which the ground state of the original graph $G$ is non-degenerate. If the device succeeds at finding the true ground state of the composite graph $G_T = G + G'$, for $M > 0$, the corresponding classical spin configuration will possess reflection symmetry with respect to the mirror axis. For $M < 0$, the solution will satisfy spin-flip symmetry, i.e. if the spins on one side of the mirror are inverted, the resulting spin configuration will possess reflection symmetry about the mirror plane. If the ground state of the original graph $G$ is degenerate, in principle, it is possible to have ground state solutions of $G_T$ that are comprised of two different ground state configurations of $G$ respectively occupying the two sides of the mirror plane, which also happens to satisfy the constraints imposed by the mirror couplings. In such cases, the solution will not satisfy the aforementioned symmetry conditions. However, the probability of such asymmetric ground state solutions significantly reduces as the ground state degeneracy of $G$ decreases. Moreover, the spatial correlations induced by the mirror couplings increases the likelihood of occurrence of symmetric solutions during the annealing process.

The arguments presented above suggest that the symmetry of the spin configuration with respect to the mirror plane provides a means of validating the solutions returned by the device. However, it should be noted that the presence of reflection/spin-flip symmetry does not necessarily guarantee that the true ground state of $G_T$ has been obtained, since the occasional occurrence of an excited state of $G$ and its mirror counterpart respectively occupying the two sides of the mirror plane would also satisfy the same symmetry conditions. Therefore, the presence of symmetry should rather be regarded as a heuristic test that increases one’s expectation that the true ground state has been realized.

3. Results
Here we demonstrate the application of the answer checking method to Ising spin glass problems defined on the device’s native Chimera topology. The $12 \times 12$ Chimera graph of the D-Wave 2X was divided into two $12 \times 6$ subgraphs via a vertical mirror axis located between the 6th and 7th columns of unit cells. The horizontal couplings that connect these two columns of unit cells were designated as ferromagnetic mirror couplings, with the magnitudes set to the maximum allowed value of 1. As the problem graph $G$, we considered Chimera graphs with different sizes, for which we varied the number of columns of unit cells ($N$) from 1 to 6 while keeping the number of rows fixed to 12. First, the local fields $h_i$ were set to zero, and the couplings were randomly chosen from three different instance classes: $J_{ij} \in \{\pm 1\}$ (U1 class), $J_{ij} \in \{\pm 5/7, \pm 6/7, \pm 1\}$ ($U_{5,6,7}$ class), and $J_{ij} \in \{\pm 8/28, \pm 13/28, \pm 19/28, \pm 1\}$ ($S_{28}$ class) [14, 15]. From each class, 1000 random instances were selected. For each of these instances, we performed 1000 annealing runs and examined the solution/solutions that correspond to the lowest energy. If at least one of the solutions that correspond to the lowest energy was found to be symmetric with respect to the mirror plane, it was considered as an indication that the true ground state might have been achieved. Based on the results of the 1000 random instances in each class, we calculated the probability $P_{\text{sym}}$ that at least one of the solutions corresponding to the lowest energy is found to be symmetric. Fig. 2 (a) shows $P_{\text{sym}}$ as a function of $N$ for the three instance classes. Since the number of qubits in the problem is directly proportional to $N$, for all three instance classes, we observe a rapid decrease in $P_{\text{sym}}$ with increasing $N$. Fig. 2 (b) demonstrates the influence
of the random local fields $h_i$ on $P_{\text{sym}}$. Here, we have compared the results obtained for the $U_1$ class with/without the presence of local fields randomly chosen to be either $+1$ or $-1$. Since non-zero local fields act as biases to the spins and render the problem easier to solve, we observe an increase in $P_{\text{sym}}$ with the inclusion of the fields.

![Figure 1](image1.png)

**Figure 1.** Application of the answer checking method to a hypothetical quantum annealing device with a $4 \times 4$ native Chimera architecture. The native architecture is divided into two identical halves via the vertical mirror axis. White-colored qubits and the couplings marked as dotted lines represent device’s inaccessible qubits and couplings and their mirror counterparts. The original graph (red) is embedded onto the left half of the device while its mirror image (blue) is embedded onto the right half. The horizontal couplings that connect the 2nd and the 3rd columns of unit cells in the device are designated as mirror couplings (green).

![Figure 2](image2.png)

**Figure 2.** Probability ($P_{\text{sym}}$) that out of 1000 annealing runs, at least one of the solutions corresponding to the lowest energy is found to be symmetric with respect to the mirror plane. Number of columns of unit cells ($N$) in the problem graph was varied from 1 to 6 while keeping the number of rows fixed to 12. (a) $P_{\text{sym}}$ vs. $N$ for three different instance classes with zero local fields: $J_{ij} \in \{\pm 1\}$ ($U_1$ class), $J_{ij} \in \{\pm 5/7, \pm 6/7, \pm 1\}$ ($U_{5,6,7}$ class), and $J_{ij} \in \{\pm 8/28, \pm 13/28, \pm 19/28, \pm 1\}$ ($S_{28}$ class). (b) $P_{\text{sym}}$ vs. $N$ for the $U_1$ class with/without random local fields $h_i \in \{\pm 1\}$.
4. Summary
We have introduced an answer checking method for quantum annealers that enhances one's expectation that the true ground state of the problem has been obtained. The method relies on symmetry requirements associated with the ground state solutions of graphs with mirror symmetry. Our approach has some of the same flavor as a recently introduced quantum annealing error correction scheme [16, 17] in that both require duplicating the original problem. However, in contrast to the aforementioned error correction scheme in which each qubit of the original and the duplicated graphs are ferromagnetically coupled to a penalty qubit, in our approach, a selected set of qubits of the original graph are directly coupled to the corresponding mirror qubits. Our method is independent of the details of the device’s native topology, and hence is readily applicable to any quantum annealing device with an arbitrary native graph structure.

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