DISORIENTED CHIRAL CONDENSATES

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ABSTRACT

The idea that a bubble of misaligned vacuum is sometimes produced in high energy collisions is reviewed.
This talk is devoted to the popular topic of disoriented chiral condensates. The idea has been put forward in refs. 1 - 3 and has received a further boost from ref. 4. At present, several dozens of papers on this subject are circulating and the literature is constantly growing. In the following, results from ref. 5 will often be used.

As you all know, QCD has an approximate SU(2) × SU(2) global symmetry. This symmetry is spontaneously broken and the relevant part of the order parameter $\psi^a \bar{\psi}^b$ ($a, b = 1, 2$) is a vector $\mathcal{M} = (\sigma, \vec{\pi})$ transforming under the O(4) subgroup of SU(2) × SU(2). The physical vacuum is a medium where $\mathcal{M}$ points in the $\sigma$ direction, because the pion mass $m_\pi$ is non-vanishing. However, the pion mass is small and the energy cost of tilting $\mathcal{M}$ is also small, of the order of $m_\pi^2 f_\pi^2$/(unit volume), where $f_\pi$ is the pion decay constant. The medium where $\mathcal{M}$ is misaligned has been baptized by Bjorken a ”disoriented chiral condensate” (DCC). It is clear that DCC must be insulated from the physical vacuum and that its existence can only be ephemeral.

![Diagram showing physical vacuum, large energy density, and pion field pointing in a random direction in isospace.](image-url)
energy hadronic and/or nuclear collisions. The energetic debris form a shell
shielding the inner region, where DCC resides, from the outer physical vac-
um. The phenomenon is truly interesting only when the pion field in the
inner region is coherent, i.e. essentially classical. It is then expected that
it points in a random direction in isospace. This implies, in turn, a striking
experimental signature. Let \( f \) denote the fraction of neutral pions among all
pions resulting from the decay of DCC. The random variable \( f \) is distributed
according to the law

\[
\frac{dP(f)}{df} = \frac{1}{2\sqrt{f}},
\]

written first explicitly in ref. 3 (but known earlier to Bjorken). The original
derivation of (1) is semi-classical, but there exists \( ^6 \) a quantum mechanical
version of the argument, which makes it clear that there is no conflict be-
tween (1) and isospin conservation. Thus, there is 10% probability that in a
cluster of 100 pions only one is neutral! This is very far from any naive statisti-
tical expectation and is of immediate phenomenological interest: DCC might
perhaps be an explanation for the mysterious Centauro events observed by
cosmic ray people \( ^7 \). Furthermore, if DCC is indeed produced in high energy
collisions it may convey relatively clean information about the initial dense
state, since it is so well signed.

We shall now discuss the dynamics of DCC in more detail, using the clas-
sical equations of motion of the linear \( \sigma \) model (cf. ref. 4). The Lagrangian
is

\[
L = \frac{1}{2}[(\partial \sigma)^2 + (\partial \vec{\pi})^2] - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - 1)^2 + H\sigma,
\]

For the sake of the argument we set \( \lambda \gg H \) (the fields are dimensionless and
an overall dimensionfull factor has been omitted, for simplicity of writing -
thus \( \lambda \) and \( H \) have the dimension of \([\text{mass}]^2 \)).

In order to proceed analytically, we adopt an idealization due originally
to Heisenberg \( ^8 \) : we assume that at time \( t = 0 \), the whole energy of the
collision is localized within an infinitesimally thin slab with infinite trans-
verse extent (instead of a pancake shaped region). The symmetry of the
problem then implies that the fields can only depend on the proper time
\( \tau = \sqrt{t^2 - x^2} \), where \( x \) is the longitudinal coordinate. Notice, that \( f = f(\tau) \)
implies \( \partial^2 f = \ddot{f} + f/\tau \). The action of the d’Alambertian produces both
an “acceleration” and a “friction” term, respectively. Of course, there is no true energy dissipation. The energy in a covolume decreases because of the expansion of the system.

We further assume random initial conditions for the fields and their gradients, such that chiral symmetry is initially unbroken: $\langle M \rangle = \langle \dot{M} \rangle = 0$ at $\tau = \tau_0$. This initial randomness can perhaps result from a rapid cooling (“quench”) of the quark-gluon plasma, as proposed in ref. 4, but we do not insist on this interpretation.

The equations of motion are

\begin{align}
\ddot{\pi} + \dot{\pi}/\tau &= -\lambda(\sigma^2 + \pi^2 - 1)\pi \\
\ddot{\sigma} + \dot{\sigma}/\tau &= -\lambda(\sigma^2 + \pi^2 - 1)\sigma + H
\end{align}

From these equations one easily finds that

\begin{equation}
\dot{\pi} \times \dot{\pi} = \vec{a}/\tau
\end{equation}

and

\begin{equation}
\pi \dot{\sigma} - \sigma \dot{\pi} = \vec{b}/\tau + \frac{H}{\tau} \int^\tau \vec{\pi} \tau \, d\tau
\end{equation}

The isovectors $\vec{a}$ and $\vec{b}$ are integration constants. Eq. (3) is a consequence of the conservation of the isovector current, while eq. (4) reflects the partial conservation of the iso-axial-vector current. The constants $a$ and $b$ measure the initial strength of these currents. The component of $\vec{\pi}$ along $\vec{a}$ necessarily vanishes, $\pi_a = 0$.

One can parameterize the fields with one radial and two angular variables:

\begin{align}
\pi_b &= r \sin \theta \\
\pi_c &= r \cos \theta \sin \omega \\
\sigma &= r \cos \theta \cos \omega
\end{align}

where $\vec{c} = \vec{a} \times \vec{b}$. Assuming, for a moment, that $H = 0$ one finds that the angle $\omega$ is constant: $\omega = \arctan(a/b)$.

The discussion of the quality of the approximation $H = 0$ requires more space. One finds 5) that this approximation is realistic provided $\tau \ll b/\sqrt{H}$.
and \( a \ll b \). Actually, the last condition only insures the planarity of the motion and is not truly essential. We assume that this condition is satisfied because this simplifies the discussion considerably: the component \( \pi_c \) of the pion field is then always much smaller than \( \pi_b \) and can be neglected altogether.

Thus, we consider a dynamical system described by one radial coordinate \( r \) and by one angular coordinate \( \theta \). Consider the radial motion first.

**The radial motion.** In this case the presence of the term \( \propto H \) in the equations of motion is an inessential complication. Setting \( H = 0 \) one gets after some algebra the following differential equation for \( r \) (we neglect \( a \) compared to \( b \)):

\[
\ddot{r} + \frac{\dot{r}}{\tau} = \frac{b^2}{\tau^2 r^3} - \lambda r (r^2 - 1) \quad (10)
\]

This can be regarded as an equation of motion of a material point in a time dependent potential well with friction. The mechanical energy of the point satisfies the inequality

\[
\frac{d}{d\tau} \left[ \frac{1}{2} \dot{r}^2 + \frac{\lambda}{4} (r^2 - 1)^2 + \frac{b^2}{2\tau^2 r^2} \right] = -\frac{b^2}{\tau^3 r^2} - \frac{\dot{r}^2}{\tau} < 0 \quad (11)
\]

and, therefore, decreases monotonically while \( r \to 1 \). Linearizing the force with respect to \( r - 1 \) one obtains a Bessel equation whose large time solution reads

\[
r = 1 + \text{const} \cos \left( \frac{\tau}{\sqrt{2\lambda}} \eta \right) / (\tau \sqrt{2\lambda})^{\frac{1}{2}} \quad (12)
\]

Eq. (12) exhibits damped oscillations around the equilibrium position \( r = 1 \). These oscillations have frequency \( \propto \sqrt{2\lambda} \), which should be regarded, in this context, as large. Averaging over a few periods of the radial motion one finds the expectation value \( \langle r \rangle = 1 \). This value is reached in a relatively short time, when the time dependent part of the potential ceases to be important, i.e. for \( \tau \sim b/\sqrt{2\lambda} \).

**The angular motion.** In the approximation \( H = 0 \), the equation governing the angular motion is

\[
\dot{\theta} = \frac{b}{r^2 \tau} \quad (13)
\]
Since the angular motion is generically slower than the radial one, we replace in (13) $r$ by its expectation value $\langle r \rangle = 1$:

$$\dot{\theta} = \frac{b}{\tau}$$  \hspace{1cm} (14)

The solution is $\theta \approx b \ln (\tau/\tau_0)$, identical to that found in ref. 3 using the non-linear $\sigma$ model.

When $H > 0$, (14) is replaced by

$$\ddot{\theta} + \dot{\theta}/\tau + H \sin \theta = 0$$  \hspace{1cm} (15)

which is the equation of a pendulum with friction. Now, the time derivative of the corresponding mechanical energy satisfies the inequality

$$\frac{d}{d\tau} \left[ \frac{1}{2} \dot{\theta}^2 + H (1 - \cos \theta) \right] = -\frac{\dot{\theta}^2}{\tau} < 0$$  \hspace{1cm} (16)

This energy decreases monotonically, until it becomes of the order of $H$. At that time the circular motion of the pendulum turns into an oscillatory one. One can show 5) that the change of regime occurs when $\tau \approx b/\sqrt{H}$ and that at large time the solution to (15) is

$$\theta \sim \sqrt{b} \cos \left( \tau \sqrt{H} + \delta \right) / (\tau \sqrt{H})^{1/2}$$  \hspace{1cm} (17)

Notice, that at large enough time $\pi_b \approx \theta$. Actually, (17) describes free propagation of a pion with mass $\sqrt{H}$. It can easily be seen that (15) reduces to the Klein-Gordon equation for $\theta \ll 1$.

Fourier transforming (17) and squaring one gets the pion spectrum (per unit transverse area). One finds a rapidity plateau with height $\sim b$. Of course, the existence of the rapidity plateau is an artifact of the boost-invariant initial conditions and should not be treated seriously. The important point is that the energy released in the decay of the condensate is proportional to the initial strength $b$ of the iso-axial-vector current.

Assuming, as in ref. 4, that the initial fluctuations of the fields and of their gradients are Gaussian with variances $\sigma_{f,g}^2$ respectively, one finds 5) the probability distribution of $b$ :

$$\frac{dP(b)}{db} = \frac{A}{2(\pi \sigma_f \sigma_g \tau_0)^2} K_1 \left( \frac{b}{\sigma_f \sigma_g \tau_0} \right), \hspace{1cm} 0 < a < A \ll b$$  \hspace{1cm} (18)
where $K_1(z) \sim \sqrt{\pi/2ze^{-z}}$ for $z \to \infty$, is the modified Bessel function.

Thus, with Heisenberg’s idealization the problem becomes 1 + 1 dimensional and can be solved analytically. At fixed time $t$ one finds quite naturally an inner region in which DCC resides, insulated from the outer physical vacuum by two Lorentz contracted regions where $\mathcal{M}$ and $\mathcal{M}$ fluctuate around zero. The *proper* time evolution of the inner region is that of a simple dynamical system. We have identified several stages in the evolution of this system. First there is a short phase in which angular motion and radial motion are strongly coupled. This phase lasts for a time $\tau \sim b/\sqrt{2\lambda}$. Next, the fictitious particle rotates slowly, while oscillating rapidly about the equilibrium radial position $r = 1$. At some point there is a transition from circular to oscillatory motion, which actually corresponds to free propagation of final state pions. The transition takes place when the pion mass can no longer be neglected and it occurs typically when $\tau \sim b/\sqrt{H}$. The pion field has a random orientation in isospace (when $a \ll b$ it oscillates along the direction of $\vec{b}$) and therefore the fraction of neutral pions is distributed according to (1).

The two time scales, $(2\lambda)^{-\frac{1}{2}}$ and $H^{-\frac{1}{2}}$, when estimated using the phenomenological values of the pion and $\sigma$ masses, differ only by a factor of 4. This is presumably not large enough to insure a clean separation of regimes, unless $b$ is large enough (say of order 3 to 5, or so). The analytic discussion sketched above indicates that the formation of an observable DCC is likely to be a rather natural but rare phenomenon, since the probability distribution of $b$ decreases exponentially. Hence, it is not interesting to average over $b$.

Once the simplifying assumptions made in the above discussion are relaxed, the problem becomes rapidly untractable analytically and one has to use a computer. Numerical calculations have been carried out by several authors. In a 1 + 1 dimensional scenario, but without boost invariance, it is found that the rapidity interval in which the pion field is correlated in isospin, is finite, as one might expect, and could be as large as 2 to 3. The time evolution of a non-expanding quenched system in 1 + 3 dimensions has been studied numerically in refs. 4 and 10. The authors observe indeed a rather dramatic amplification of long wavelength pion modes in the period immediately following the quench. Analogous numerical simulations, but taking into account expansion, are being done in Cracow 11).

The authors of refs. 10, 12 have pointed out that the correlation length
extracted from the correlator $\langle \pi(\vec{x},t)\pi(\vec{0},t) \rangle$ is not large, typically of the order of 1 to 2 fm. They conclude that large domains of DCC are unlikely to be created following a quench. Consequently the yield of pions from DCC decay is small and it is very difficult to distinguish DCC formation from a trivial statistical fluctuation. A way out has been suggested in ref. 13, where an "annealed" scenario has been suggested. Actually, all these authors are averaging over the initial conditions. But we have argued before that this may be misleading. It is plausible that generically the bubble is hardly observable. The truly relevant question is: what is the probability that a large enough domain is formed, when one starts with a random initial state? Provided this probability is not too small, the signal of DCC formation may be above the trivial backgroud. In principle, the probability in question, an analogue of (18), can be found from numerical simulations while the background can be estimated using standard models of multiparticle production. Such a useful phenomenological analysis has not been carried out yet. Notice, that (1) is expected to hold within the sample of (perhaps) rare events corresponding to the formation of large domains of DCC, since the isospin orientation of the condensate is not expected to be correlated with its spacial extent.

One should mention, that an experiment $^{14}$ is presently been run at the TEVATRON, with the aim of observing a disoriented chiral condensate. One expects that some results will become available during this year. Hopefully, focusing on very high multiplicity events in $p\bar{p}$ (instead of heavy-ion) collisions one meets the regime required for the formation of DCC, but this is not granted.

Although DCC formation has not yet been established experimentally, this hypothetical phenomenon has already triggered an intense theoretical activity, being a good pretext for studying non-equilibrium aspects of complicated high-energy nuclear collisions. For a long time, following the seminal work of Landau $^{15}$ on hadron hydrodynamics, the subject has been dominated by the concept of (local) thermal equilibrium. The increasing attention devoted to the non-equilibrium aspects of the problem is certainly a very interesting developpment.
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