Interaction of compressible barrette with surrounding soil and raft

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Abstract. The statement and analytical solution are handled simultaneously with the comparison with the results gained by the numerical method for addressing the problem of quantifying the stress-strain state (SSS) of a multi-layer soil cell consisting of a barrette interacting with the raft-plate. The solution to the assigned problem is considered by the authors for two cases, without and taking into account the compressibility of the underlying soil layer. In the first case, when the underlying soil layer is non-compressive, the formulas are formed in order to determine the SSS in the body of the barrette with the surrounding soil and the equivalent deformation modulus of the barrette-raft-surrounding soil system, so that engineers can absolutely estimate the bearing capacity of the underlying soil under the heel of barrettes. The second case, when the compressibility of the underlying soil layer is taken into account, the solution to the assigned problem is found by means of establishing and solving differential equations of the second order. As a result of the approach above, the formulas for determining the SSS in the body of the barrette are confirmed and the formulas for estimating the settlement of the raft and the equivalent deformation modulus for the barrette-raft-surrounding soil system are obtained. It is noted that taking into account the downward movement of the compressible barrette into the underlying soil layer lead to the decrease in the equivalent deformation modulus of the system.

1. Introduction
Upon considering the interaction of compressible barrette with surrounding multi-layer soil system of limited dimensions and the raft, a complex SSS arises in the barrette-raft-surrounding soil system, which determines the settlement of the system and the bearing capacity of the barrette. The load applied to the raft is distributed between the barrette, the surrounding soil, and the raft. Quantifying the distribution of the applied load is one of the most challenging problems in soil mechanics. Though the solution to the assigned problem the geometric parameters of the barrette-raft-surrounding soil system such as the laying deep of barrettes, the spacing among barrettes and dimensions of barrettes as well as the critical settlement of the underlying soil layer, the barrette-raft-surrounding soil system and the bearing capacity of barrettes are determined as well as estimated. In addition, by means of solving the assigned problem, we also can find the equivalent value of the deformation modulus to the barrette-raft-surrounding soil system.
2. **Statement problem**

Considering a barrette embedded in a soil cell, the applied load to the raft is distributed between the surrounding soil and the underlying soil layer. Let the following parameters be known: the dimensions of the barrette and the soil cell, its embedded length, the deformation modulus of the barrette, and the surrounding soil, underlying soil.

The following problems are needed to be solved:
- The distribution of the applied load to the raft between the barrette and the surrounding soil;
- The distribution of the applied load to the raft between the barrette and the underlying soil layer;
- The settlement of the barrette-raft-surrounding soil system;
- The equivalent value of deformation modulus of the barrette-raft-surrounding soil system.

The solution to the assigned problem is conducted in two cases.

In the first case, when the underlying soil layer is possessed of vast rigid, i.e., $E_o >> E_i$, the downward movement of the barrette’s heel can be neglected ($S_b = 0; \sigma_b \neq 0$); in the second case, when the underlying soil layer is relatively compressive, i.e., the settlement of the barrette due to the punching of the heel into the underlying soil layer ($S_b \neq 0; \sigma_b \neq 0$). It is essential to be noted that, the stress at the surface of the barrette’s heel is not allowed to exceed the critical load $p^*$ proposed by L. Prandtl. i.e., $\sigma_b < p^*$.

3. **Solving the problem in the condition of the settlement under the barrette’s heel $S_b = 0$**

According to the proposed calculation scheme below, the vertical displacements of the raft, the surrounding soil, and the barrette are equal, thereby it can be roughly assumed that the compression of the surrounding soil, in this case, behaves as in oedometer test. On the other hand, the vertical stress in the body of the barrette, and the shear stress at the contact surface between the barrette and the surrounding soil remain constant, as a result of the listed evidence, the condition of the equilibrium settlement of the raft $S_r$, the barrette $S_b$, and the surrounding soil $S_s$ is fulfilled ($S_r = S_b = S_s$).

![Diagram](image)

**Figure 1.** Calculation scheme of barrette interaction with surrounding and incompressible underlying soil layer.

From the calculation scheme (figure 1) we have an equilibrium condition

$$p = \sigma_s \omega + \sigma_r (1 - \omega)$$

(1)

in which $\sigma_b$ is the stress in the body of the barrette, and $\sigma_r$ is the stress under the raft and

$$\omega = \frac{ab}{AB}$$

(2)
The value of settlement of the compressible barrette corresponding to the settlement due to the compression of the soil cell can be written as follows

$$S_v = \sigma_m L$$  \hspace{1cm} (3)

$m_v$ is the coefficient of volume compressibility of soil. The distribution of the stress under the raft $\sigma_s$ along the depth $z$ is shown in the calculation scheme (figure 1).

Considering the equilibrium condition $S_v = S_b$ from the calculation scheme (figure 1) we can get

$$\sigma_s = \sigma_v \frac{m_v}{m_b}$$  \hspace{1cm} (4)

$m_v$ and $m_b$ are coefficients of volume compressibility of soil and barrette, respectively.

Substituting the expression (4) into (1), we obtain

$$\sigma_b = p \frac{m_v}{m_{\omega} + m_b (1 - \omega)}$$  \hspace{1cm} (5)

By means of the equilibrium condition between the settlement of the soil under the raft and the settlement of the barrette $S_v = S_b$, we achieve $p \bar{m} L = S_m L$, in which $\bar{m}$ is the equivalent value of the coefficient of volume compressibility of the barrette-raft-surrounding soil system. Substituting $\sigma_b$ from expression (5) into the received expression above, we get

$$\bar{m} = p \frac{m_v}{m_{\omega} + m_b (1 - \omega)}$$  \hspace{1cm} (6)

$$S = p \bar{m} L$$  \hspace{1cm} (7)

It is vitally essential to confirm the condition $\sigma_s < p$ under the barrette’s heel, so that in this case the assigned problem is totally solved.

4. Solving the problem in the condition of the settlement under the barrette’s heel $S_R = 0$

![Figure 2. Calculation scheme of barrette interaction with surrounding and compressible underlying soil layer.](image)

The settlement of the barrette and surrounding soil under the effect of the applied load to the raft occurs due to the shear deformation of the soil within the active zone around the barrette with the range from $2a \times 2b$ to $2A \times 2B$ along the embedded length ($l$) of the barrette and the length ($L$) of the soil cell. In this case, the increment of the arising settlement at the depth $z$ can be determined by the following dependency:
\[ dS(r, z) = -\gamma(r, z) \, dr \] (8)
in which
\[ \gamma(r, z) = \tau(r, z) / G \] (9)
\[ \tau_a(r, z) = \tau_a(z) \frac{(B - r)^2}{(B - b)^2}, \quad \tau_b(r, z) = \tau_b(z) \frac{(A - r)^2}{(A - a)^2} \] (10)

Substituting the gained value of the shear stresses into expression (8) we can find the distribution of the maximum value of the settlement on the side surfaces corresponding to the edges 2a and 2b of the barrette \( S_{\text{side}_a} \) and \( S_{\text{side}_b} \), respectively
\[ S_{\text{side}_a}(z) = -\tau_a(z) \int \frac{(B - r)^2}{G} \frac{dz}{(B - b)^2} = \frac{B - b}{3G} \tau_a(z) \] (11)
\[ S_{\text{side}_b}(z) = -\tau_b(z) \int \frac{(A - r)^2}{G} \frac{dz}{(A - a)^2} = \frac{A - a}{3G} \tau_b(z) \] (12)
The settlement of the barrette's heel is obtained from the known dependency of the vertical displacement of a rectangular rigid stamp taking into account the depth coefficient \( K_1 < 1 \) on the applied load, so we have
\[ S_R = \frac{\sigma_R (1 - v_0) a K_i w}{G_0} = \sigma_K K \] (13)
\( G_0 \) and \( v_0 \) are parameters of deformability of the underlying soil layer, and \( w \) is the coefficient considering the shape of the stamp (square, rectangle).
\[ K = \frac{(1 - v_0) a K_i w}{G_0} \] (14)
\[ K_i = \frac{1}{(1 - v_0)^w} \left[ \frac{(1 + v_0)(3 - 4v_0)}{4\pi(1 - v_0)} \left[ \ln \left( \frac{n^2 + 1}{2} + n \right) + \ln \left( \frac{n^2 + 1}{2} + n \right) / n \right] \right. \\
+ \left. \frac{(1 + v_0)(8v_0^2 - 12v_0 + 5)}{4\pi(1 - v_0)} \left[ \ln \left( \frac{m^2n^2 + m^2 + 16}{2} + mn \right) + \ln \left( \frac{m^2n^2 + m^2 + 16}{2} + mn \right) / mn \right] + \ln \left( \frac{m^2n^2 + m^2 + 16}{2} + m \right) / mn \right] \\
+ \frac{2mn(1 + v_0)}{\pi(1 - v_0) \left( m^2n^2 + m^2 + 16 \right)^{1/2}} \left[ \left( m^2 + 16 \right) + \left( m^2n^2 + 16 \right)^{1/2} \right] \\
+ \frac{2(1 + v_0)(2v_0^2 - 1)^2}{\pi m(1 - v_0)} \left[ \arcsin \left( 4n \left( m^2 + 16 \right)^{1/2} \left( n^2 + 16 \right)^{1/2} \right) + \arcsin \left( 4n \left( m^2n^2 + 16 \right) \left( n^2 + 1 \right)^{1/2} \right) - \frac{\pi}{2} \right] \right] \] (15)

where \( m = 2a/l \) and \( n = b/a \).

Determining the calculated value \( \sigma_R \) in the linear statement, the degree of its approximation to the limit state should be checked \( (\sigma_R < \sigma_R^*) \). This test can be carried out by the formula of L. Prandtl:
\[ \sigma_R^* = (\gamma d + c_0 \cot \varphi_0) \left( 1 + \sin \varphi_0 \right) \exp \left( \frac{\pi \tan \varphi_0}{1 - \sin \varphi_0} \right) - c_0 \cot \varphi_0 \] (16)
\( c_0 \) and \( \varphi_0 \) are strength parameters of the underlying soil layer (kPa) and (radian); \( d \) is the depth at the level of barrette’s sole (m); \( \gamma \) is the unit weight of soil from the ground surface to depth \( d \) (kH/m³).
Considering the interaction of a compressible barrette with the surrounding and underlying soils, embedded into the soil cell with the origin of coordinates \( z = 0 \) at the barrette’s heel level, by dint of consideration the equilibrium conditions for the barrette’s element \( dz \), it is confirmed that the increment of the stress in the barrette’s body \( \text{d} \sigma_b(z) \) is taken equal to the increment of the shear stress on the contact surfaces between the barrette and the surrounding soil

\[
4ab \text{d} \sigma_b(z) = (4\tau_s a + 4\tau_s b) \text{d} z
\]

Hence, it is followed that

\[
\frac{\text{d} \sigma_b(z)}{\text{d} z} = \frac{\tau_s a + \tau_s b}{a \cdot b}
\]

(18)

Analyzing the equilibrium condition of the settlement on the barrette’s side surfaces \( S_{\text{side } a}(z) = S_{\text{side } b}(z) \), we get the dependency of the shear stress \( \tau_s(z) \) on \( \tau_s(z) \)

\[
\tau_s(z) = \tau_s(z) \frac{B-b}{A-a}
\]

(19)

The value \( \tau_s(z) \) in expression (8) is superseded by the received from (19), thereby we get

\[
\frac{\text{d} \sigma_b(z)}{\text{d} z} = \tau_s(z) \frac{(A-a)a + (B-b)b}{(A-a)ab} = \tau_s(z) I
\]

where

\[
I = \frac{(A-a)a + (B-b)b}{(A-a)ab}
\]

(21)

Through the condition of the linear deformability of the compressible barrette, which is characterized by the value of deformation modulus of the constituting material, \( E_b \) (kPa), we obtain

\[
\sigma_b(z) = \varepsilon_b(z) E_b = \frac{\text{d} S_b(z)}{\text{d} z} E_b
\]

(22)

Differentiating the equation (22) with respect to \( z \), we achieve:

\[
\frac{\text{d} \sigma_b(z)}{\text{d} z} = \frac{\text{d}^2 S_b(z)}{\text{d} z^2} E_b
\]

(23)

The new differential equation can be gained by putting \( \tau_s(z) = \frac{3GS_{\text{side } a}(z)}{B-b} \) from (11) into (19), and equating the received expression to (23)

\[
\frac{\text{d}^2 S_b(z)}{\text{d} z^2} - \lambda^2 S_b(z) = 0
\]

(24)

in which

\[
\lambda^2 = \frac{3GI}{E_b(B-b)} \left[ \frac{1}{cm^2} \right]
\]

(25)

The expression received by solving the differential equation (24) can be written as follows:

\[
S_b(z) = C_1 \text{sh} \lambda z + C_2 \text{ch} \lambda z
\]

(26)

\( C_1 \) and \( C_2 \) are the integration constants and determined from the boundary conditions when \( z = 0 \), \( S_b(0) = S_R \), \( \text{sh} 0 = 0 \), and \( \text{ch} 0 = 1 \)

\[
\frac{\text{d} S_b(z)}{\text{d} z} = C_1 \lambda \text{ch} \lambda z + C_2 \lambda \text{sh} \lambda z = \varepsilon_b(z)
\]

(27)
when \( z = l \), \( \varepsilon_b (l) = C \lambda l \lambda l + C_z \lambda l \lambda l = \sigma_N / E_b \).

With \( C_2 = S_R \) we have

\[
C_i = \frac{\sigma_N}{E_b \lambda l \lambda l} - S_R \frac{\lambda l}{\lambda l \lambda l}
\]

(28)

Substituting all the obtained expression above into expression (26), we have the expression, describing the settlement of the body barrette depending on the depth \( z \) (m)

\[
S_b (z) = C_1 \lambda l \lambda l + C_2 \lambda l \lambda l = \left[ \frac{\sigma_N}{E_b \lambda l \lambda l} - S_R \frac{\lambda l}{\lambda l \lambda l} \right] \lambda l \lambda l + S_R \lambda l \lambda l = \sigma_N / E_b \lambda l \lambda l
\]

(29)

\[
S_b (z) = \left[ \frac{\sigma_N}{E_b \lambda l \lambda l} - \sigma_R K \frac{\lambda l}{\lambda l \lambda l} \right] \lambda l \lambda l + \sigma_R K \lambda l \lambda l
\]

(30)

Conducting the differentiation of expression (30) with respect to \( z \), we get

\[
\frac{dS_b (z)}{dz} = \varepsilon_b (z) = \frac{\sigma_N}{E_b} - \frac{\sigma_R K \lambda l}{\lambda l \lambda l} \lambda l \lambda l + \frac{\sigma_R K \lambda l}{\lambda l \lambda l} \lambda l \lambda l
\]

(31)

\[
\sigma_b (z = 0) = \sigma_R = \frac{\sigma_N}{\lambda l \lambda l + K \lambda l \lambda l}
\]

(32)

Replacing the \( \sigma_R \) in expression (30) by the obtained \( \sigma_R \) from (32), we finally get the dependency of the compressible barrette’s settlement on the depth \( z \) due to the application uniformly distributed load to the raft:

\[
S_b (z) = \sigma_N \lambda l \lambda l \left[ \left( \frac{E_b \lambda l \lambda l}{\lambda l \lambda l} \right)^{-1} \lambda l \lambda l - \frac{\lambda l \lambda l}{\lambda l \lambda l + K \lambda l \lambda l} \lambda l \lambda l \right] + \frac{\sigma_N}{\lambda l \lambda l + K \lambda l \lambda l}
\]

(33)

in which

\[
V_1 = \left( \frac{E_b \lambda l \lambda l}{\lambda l \lambda l} \right)^{-1} - \frac{\lambda l \lambda l}{\lambda l \lambda l + K \lambda l \lambda l} \lambda l \lambda l
\]

(35)

Considering the equilibrium condition between the settlement of the surrounding soil at the head of the barrette (\( z = l \)), we achieve the expression

\[
S_R (l) + S_b = S
\]

(36)

\[
\sigma_R K + \int_0^l \left[ \frac{\sigma(z)}{E_b} \right] \lambda l \lambda l + \sigma_N \lambda l \lambda l \lambda l = \sigma_R \lambda l \lambda l + \sigma_N \lambda l \lambda l \lambda l
\]

(37)

where \( m_x \) and \( m_0 \) are the values of the coefficients of volume compressibility of the surrounding soil in the range of the barrette’s embedded length \( l \) and the underlying soil, respectively. \( S_R (l) = \sigma_R K \) is the settlement gained due to the punching of the barrette’s heel into the underlying soil layer; \( S_b \) is the settlement of the barrette’s body; \( S \) is the total settlement procured due to the compression of the surrounding soil around the barrette in the range of the barrette’s embedded length \( l \) and the underlying soil layer.

Considering expression (20), we can draw out the distribution at depth \( z \) of the shear stress on the side surface corresponding edge \( 2a \) of the barrette.
\[ \tau_s(z) = \frac{1}{I} \frac{d \sigma_s}{dz} = \frac{1}{I} \left[ \frac{\sigma_s}{ch l} - \sigma_r K \lambda E_b \frac{sh \lambda l}{ch l} \right] \lambda sh \lambda z + \sigma_r \frac{K \lambda^2 E_b}{I} ch \lambda z \] (40)

From the equality of the settlement of the barrette’s body and the underlying soil layer at the level of the barrette’s heel \( z = 0 \), we obtain the following expression

\[ \sigma_r K = \sigma_s m_0 (L-l) + \tau_s(0) \frac{B-b}{3G} \]

\[ \sigma_r K = \sigma_s m_0 (L-l) + \sigma_N \frac{B-b}{3GI} \frac{K \lambda^2 E_b}{ch \lambda l + K \lambda E_b sh \lambda l} \] (42)

After transforming three unknowns, a system of equations, which consists of expression (39), (42), and (1) is constituted as follows

\[ \begin{align*}
\sigma_r K + \sigma_N V_i sh \lambda l &+ \sigma_s V_i (ch \lambda l - 1) = \sigma_m l + \sigma_m (L-l) \\
\sigma_N \omega &+ \sigma_s (1-\omega) = p \\
\sigma_r K &+ \sigma_m (L-l) + \sigma_N \frac{B-b}{3GI} \frac{K \lambda^2 E_b}{ch \lambda l + K \lambda E_b sh \lambda l}
\end{align*} \]

Solving this system of equations we get the solution for the unknowns

\[ \sigma_i = p \frac{V_i + V_4}{\omega m_l + (V_3 + V_4)(1-\omega)}; \quad \sigma_N = p \frac{m_l l}{\omega m_l + (V_3 + V_4)(1-\omega)}; \quad \sigma_r = p \frac{m_l V_4 + (V_3 + V_4) m_0 (L-l)}{K \omega m_l + (V_3 + V_4)(1-\omega)}; \]

\[ \begin{align*}
V_3 &= V_i sh \lambda l + V_2 (ch \lambda l - 1); \\
V_4 &= \frac{B-b}{3GI} \frac{K \lambda^2 E_b}{ch \lambda l + K \lambda E_b sh \lambda l}
\end{align*} \] (44)

As a result, the expression for the compressive stress \( \sigma_s(z) \) in the barrette’s body, the settlement of the barrette, and the shear stresses on the barrette’s side surfaces \( \tau_s(z) \) and \( \tau_t(z) \) are displayed as follows

\[ S_b(z) = \sigma_s V_i sh \lambda z + \sigma_N V_z ch \lambda z \]

\[ \tau_s(z) = \frac{1}{A} \frac{d \sigma_s}{dz} = \frac{\lambda^2 V_i E_b}{I} sh \lambda z + \frac{\lambda^2 V_z E_b}{I} ch \lambda z \]

\[ \tau_t(z) = \frac{B-b}{A} = \frac{B-b}{A-\alpha} \left[ \frac{\lambda^2 V_i E_b}{I} sh \lambda z + \frac{\lambda^2 V_z E_b}{I} ch \lambda z \right] \] (47)

where \( \sigma_s, \sigma_r, V_i, V_z, V_4 \) are described in expressions (35) and (44).

The settlement of the soil cell consisting of the barrette, the raft, the surrounding soil and underlying soil layer is constituted of two terms

\[ S_{soil\ cell} = S_{barrette-raft-sur-soil\ system} + S_{underlying\ soil\ layer} \] (48)

\[ S_{soil\ cell} = \sigma_r K + \sigma_N V_i sh \lambda l + \sigma_N V_z (ch \lambda l - 1) + \rho m_0 (L-l) \] (49)

Considering example:

\[ p = 500 \text{ kN}; \quad 2a = 1.5 \text{ m}; \quad 2b = 3 \text{ m}; \quad 2A = 7.5 \text{ m}; \quad 2B = 9 \text{ m}; \quad l = 30 \text{ m}; \quad L = 50 \text{ m}; \quad \alpha = \beta = 45^\circ; \]

\[ K_s = 0.722; \quad G = 5000 \text{ kPa}; \quad G_0 = 30000 \text{ kPa}; \quad \omega = 1.22; \quad v_1 = 0.35; \quad v_0 = 0.3; \quad E_b = 30000 \text{ MPa}; \]

\[ m_s = 4.615 \times 10^{-4} \text{ kPa}^{-1}; \quad m_0 = 9.905 \times 10^{-6} \text{ kPa}^{-1}. \]

Then we get:

\[ \sigma_r = 1341.75 \text{ kPa}; \quad \sigma_s = 7267.69 \text{ kPa}; \quad \sigma_5 = 16.59 \text{ kPa}; \quad S_0 = 0.022 \text{ m}; \quad S_{sh} = 0.026 \text{ m}; \]

\[ \tau_s(0) = \tau_t(0) = 94.36 \text{ kPa}; \quad S_{soil\ cell} = 12.5 \text{ cm}. \]

The result obtained by the numerical method using the PLAXIS 3D in linear formulation with the value of the uniformly distributed load \( p = 500 \text{ kPa} \)
**Figure 3.** Iso-fields of the settlement of the soil cell consisting of a barrette, a raft, surrounding soil and underlying soil layer.

The average values of the stress under the barrette’ heel, at the head of the barrette, and the settlement of the soil cell achieved by the numerical method are displayed as follows:

\[ \sigma_{R}^{\text{plaxis}} = 1458 \text{ kPa}; \quad \sigma_{N}^{\text{plaxis}} = 7342 \text{ kPa}; \quad S_{\text{soil cell}}^{\text{plaxis}} = 13.43 \text{ cm}. \]

The difference of the values shown above between analytical method and numerical method varies in the range from 1% to 8%.

The dependencies curve of \( \sigma_{b}(z) \), \( S_{b}(z) \), \( \tau_{b}(z) \), and \( \tau_{s}(z) \) on \( z \) (m) corresponding to 3 cases of the embedded length of the barrette and the depth of the soil cell 30-40; 40-50; 50-60m, respectively.

**Figure 4.** Iso-fields of the vertical stress of the soil cell consisting of a barrette, a raft, surrounding soil and underlying soil layer.

**Figure 5.** Dependency curve of the settlement of the compressible barrette \( S_{b} \) on depth \( z \) (m).

**Figure 6.** Dependency curve of \( \sigma_{b} \) in the compressible barrette on depth \( z \) (m).
5. Conclusions

- Upon the interaction of a compressible barrette with surrounding soils, underlying soil layer as well as the raft, a complex stress-strain-state (SSS) is formed, which completely depends on the geometric parameters of the barrette, as well as the physical and mechanical properties of the barrette, the surrounding soils, and the underlying soil layer.
- The solution of the assigned problems in a linear formulation is conducted by establishing and solving a second-order differential equation for the settlement of the barrette’s body.
- It is drawn out that the uniformly distributed load applied to the raft transferred from the raft to the side surfaces and the barrette’s heel is proportional to their geometric, mechanical parameters and this proportion changes with depth. The proportion stress at the barrette’s heel accounts for approximately 15% of the stress at the pile head.
- Upon comparison of the result obtained by means of calculating by analytical and numerical method is shown (no more than 8%), and the solution of the assigned problem can be recommended for performing calculation of the settlement and bearing capacity of the barrette-raft foundation.
- In order to increase the proportion of stress at the heel of the barrette and mobilize radically the bearing capacity of the underlying soil layer, the ratio of the barrette’s dimension parameters to the soil cell should be adjusted appropriately.

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