The Associated Production of Weak Bosons and Jets 
by Multiple Parton Interactions

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Abstract

The sources of $W + n$-jet events in hadron collisions are higher-order QCD 
processes, but also multiple-parton interactions. A subprocess producing a 
$W + k$-jet final state, followed by one producing $l$ jets in the same 
nucleon-nucleon interaction, will result in a $W + n$-jet event if $k + l = n$. In the simplest 
case a $W + 2$-jet event can be produced by a quark-antiquark annihilation into 
$W$ and a 2-jet event occurring in the same proton-antiproton interaction. We 
compute that this happens at the 10\% level of the higher-order QCD processes 
for the type of cuts made by the Tevatron experiments. For jet $p_T$ values of 
order 5\~10 GeV, multiple-parton interactions dominate higher-order QCD- 
processes. The emergence of this new source of $W + n$-jet events towards lower 
$p_T$ simulates the running of $\alpha_s$; it is imperative to remove these processes 
from the event sample in order to extract information on the strong coupling 
constant. Also, BFKL studies of low-$p_T$ jet cross sections are held hostage to 
a detailed understanding of the multiple-parton interactions. We perform the 
calculations required to achieve these goals. A detailed experimental analysis 
of the data may, for the first time, determine the effective areas occupied by 
quarks and gluons in the nucleon. These are not necessarily identical. We 
also compute the multiple-parton contribution to $Z + n$-jet events.
I. INTRODUCTION

The very clean samples of $W, Z + n$-jet events produced in proton-antiproton interactions at the Tevatron [1] have become a laboratory to study QCD [2]. In principle, these events can be used to determine the strong coupling $\alpha_s$. In this paper we point out that the same events can be used to study multiple-parton interactions. It would, in fact, be treacherous to determine $\alpha_s$ without removing their contribution to the data sample. We will show that their contribution strongly increases relative to the higher-order QCD processes with decreasing values of $p_T$. This behavior obviously mimics the running of the strong coupling. Furthermore, studies of BFKL physics with jets cover the low-$p_T$ region where multiple-parton processes eventually dominate and will have to await a detailed understanding of these processes.

We have computed the contributions of multiple interactions and higher-order QCD processes to the processes $W, Z + 2$-jets; see Fig. 1. In the simplest case of a $W + 2$-jet event the final state can be produced by a quark-antiquark annihilation into $W$ and a 2-jet event, both occurring in a single beam crossing [3]. The two-parton subprocess may involve the same, or different beam particles. In most cases the latter process can be identified by the detector and removed from the data set [4]. Our conclusions are that: i) higher-order QCD processes and multiple-parton interactions are similar in magnitude for $p_T$ values of the jets of order $5\sim 10$ GeV and ii) even for jets with a minimum $p_T$ of 20 GeV the multi-parton contribution is at the $10\sim 20\%$ level. Notice that a contribution of this magnitude is sufficiently important to interfere with a determination of $\alpha_s$. The obvious way to remove it is to study the event sample as a function of the minimum $p_T$ of the jets. We have also calculated the multiple interaction and higher-order QCD contributions to $W + 3$-jet event and comment on their relative importance when the number of jets produced in association with the $W$ increases. Finally, we repeated the calculations for $Z + 2, 3$-jet events.

Multi-parton scattering is of theoretical interest because it probes the partonic structure of hadrons in novel ways. The joint probability that a quark and antiquark annihilate into a $W$, and that another pair of partons produces a 2-jet final state in a single proton-antiproton interaction is

$$\sigma_{W \ 2\text{-jet}} = \sigma_W\left(\frac{\sigma_{2\text{-jet}}}{\pi R^2}\right).$$

(1)

The result is obvious except for the factor $\frac{1}{\pi R^2}$. It takes into account the fact that, once the first parton process has taken place, the area of the beam has been reduced to the area of the proton in which the first interaction occurred. In principle, there could exist correlations in the double-parton-scattering structure functions that would invalidate this expression. For instance, we are neglecting the correlation due to longitudinal momentum conservation, which should introduce factors of the form $(1 - x_1 - x_2)$, with $x_i$ being the fraction of momentum carried by the parton $i$. In the small $x$ regions, covered in our calculations, Eq. (1) is valid. The effective area $\pi R^2$ should be of order of the inelastic cross section, or about 50 mb. Earlier studies of final states with multiple jets and multiple Drell-Yan pairs [5], although admittedly plagued by difficult systematics or low statistics, have agreed on the much smaller value of roughly 15 mb. This may indicate that hard quarks and gluons only occupy a small area of the high-energy proton. Anticipating the higher quality of the Tevatron data, it may ultimately be possibly to separately measure the areas occupied by
quarks and gluons. Several authors have speculated that gluons may be clustered in the vicinity of valence quarks \[6\].

\section{W + 2-JETS EVENT RATES}

The multiple-interaction cross section for the production of a final state \(A\) in association with the state \(B\), is given by

\[
\sigma_{A-B} = \sigma_A \sigma_B \left[ \frac{1}{\pi R^2} + 2 \sqrt{\frac{1}{\pi R^2} \left( \frac{N-1}{\sigma_{inel}} + \frac{(N-1)}{\sigma_{inel}} \right)} \right],
\]

for \(A \neq B\). Here \(N\) stands for the average number of interactions per beam crossing. Summation over \(A\) and \(B\) states leading to the same \(A-B\) final state is understood. The first term corresponds to double-parton interactions involving the same beam particles, like in Eq. (1), while the last term describes simultaneous interactions between different beam particles. As for the second term in Eq. (1), it corresponds to the interaction of two partons in a proton in one bunch with two different protons in the other bunch. It can be understood as the interference term for the first two reaction mechanisms. In our calculation we use \(R = 0.7\) fm, which corresponds to an effective cross section \(\pi R^2 = 15.4\) mb, \(\sigma_{inel} = 51.7\) mb, and \(N = 1.8\). This value of \(R\) is also consistent with the the one derived from the analysis of the \(\gamma + 3\)-jet production by CDF \[4\].

In order to obtain the multiple-parton and QCD cross sections for the production of a \(W\) and two jets, we used the package MADGRAPH \[7\] to evaluate the relevant tree-level matrix elements. These agree with previous evaluations. We imposed a minimum-\(p_T\) cut on the transverse momenta of the jets and required that the jets (partons) be in the rapidity region \(|\eta_j| < 3\), and separated by \(\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} \geq 0.9\). We used the CTEQ3M structure functions \[8\] and evaluated the QCD scales at \(\hat{s}/2\), where \(\hat{s}\) is the subprocess center-of-mass energy.

Our results for the \(W + 2\) jets production are exhibited in Fig. 2 as a function of the minimum-\(p_T\) cut. We present in Fig. 3 the \(p_T\) spectrum of the jets produced in association with the \(W\) for both multiple-interaction and higher-order QCD processes. As can be seen from this figure, the jets produced in multi-parton scattering are much softer than those produced by higher-order processes. This is why multiple-particle interactions become more important with decreasing \(p_T\).

In Fig. 4 we plot the statistical significance (signal over square root of background) for multi-parton interactions assuming an integrated luminosity of 100 pb\(^{-1}\), \(\text{Br}(W \rightarrow e\nu_e) = 10.8\%\), and an efficiency of 34\% to detect the \(W\) \[1\]. The double-parton interactions can be observed in the \(W + 2\)-jet final state with a significance in excess of 5\(\sigma\) provided the minimum \(p_T\) of the jets is smaller than 10–12 GeV. Clearly, the signal for double-parton scattering can be further enhanced, e.g. by a cut requiring that the jets are back-to-back in the transverse plane in a \(W + 2\) jet event. The poorly known effective area \(\frac{1}{\pi R^2}\) dominates the uncertainty of the calculations. The results shown assumed a common value of 15.4 mb for quarks and gluons, corresponding to \(R = 0.7\) Fermi. It is clear from these results that the Tevatron has the opportunity to determine this quantity with much improved precision.
III. $W + 3$-JETS EVENT RATES

Double-parton interactions can also be studied in the production of $W + 3$-jets. The cross section for this process is given by Eq. (2) with

$$\sigma_A \sigma_B = \sigma_{W,\text{jet}} \sigma_{2\text{-jet}} + \sigma_{W} \sigma_{3\text{-jet}}.$$

The cross sections for this final state via higher-order QCD, double-parton interactions and multi-beam interactions are shown in Fig. 5 as a function of the minimum $p_T$ of the jets. As we can see from Fig. 6, the transverse momenta of the $W$ produced via higher-order QCD are larger than those generated in the double-parton interaction. This suggests that a cut like $p_T^W \lesssim 20$ GeV should increase the signal-to-background ratio.

Figure 4 also shows the statistical significance for the double-parton production of $W + 3$-jets as a function of the minimum transverse momenta of the jets, indicating that the signal should be visible for $p_{\text{cut}}^T \lesssim 12$ GeV. This curve was obtained using the same luminosity and efficiencies assumed for the $W + 2$-jet signal. No further cuts were introduced to enhance the signal. Notice that double-parton scattering is observable in both $W + 2$-jet and $W + 3$-jet channels for approximately the same range of transverse momenta. Therefore, the observation of a signal in one channel can be confirmed in the other. Moreover, it is possible to observe the presence of double-parton scattering by analyzing the ratio $R_{3/2} = \sigma_{W,3\text{-jets}}/\sigma_{W,2\text{-jets}}$. As shown in Fig. 6, double-parton interactions enhance this ratio by a factor 1.5–2 for low-$p_T$ cuts.

IV. $Z + n$-JETS

Multiple-parton scattering can also be studied through the production of a $Z$ accompanied by 2 or 3 jets. In order to study these processes we must evaluate

$$\sigma_A \sigma_B |_{Z,2\text{-jet}} = \sigma_Z \sigma_{2\text{-jet}}; \quad (3)$$

$$\sigma_A \sigma_B |_{Z,3\text{-jet}} = \sigma_{Z,\text{jet}} \sigma_{2\text{-jet}} + \sigma_Z \sigma_{3\text{-jet}}. \quad (4)$$

We exhibit in Table I(II) the cross sections for the production of $Z + 2$-jets ($Z + 3$-jets) from higher-order QCD processes, double-parton scattering and multiple-beam interactions. These results were obtained imposing a minimum-$p_T$ cut on the transverse momenta of the jets. As before, we required that the jets (partons) populate the rapidity region $|\eta_j| < 3$, separated by $\Delta R \geq 0.9$.

The statistical significance of the multi-parton contribution to the $Z + 2$-jet production is shown in Fig. 8 for $Z$ decaying into $e^+e^-$ or $\mu^+\mu^-$ pairs, and an integrated luminosity of 100 pb$^{-1}$. The $Z$ acceptance was taken to be 41% [1]. We conclude that the multi-parton scattering can be observed at a $3\sigma$ level for $p_T^{\text{cut}} \leq 14$ GeV. The signal should be very clean for low values of the $p_T$ cut, even without further cuts requiring the $Z$ and jets to have pair-wise balancing transverse momenta. The signal for multi-parton scattering in the production of a $Z + 3$-jets is also visible for a $p_T$ cut smaller than 13 GeV, as shown in Fig. 8. Analogously to the $W + n$-jets case, the signal for multi-parton interactions in $Z + n$-jets also appears in the ratio $R_{3/2} = \sigma_{Z,3\text{-jets}}/\sigma_{Z,2\text{-jets}}$, as shown in Fig. 8.
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TABLES

TABLE I. $Z + 2$-jet cross sections at the Fermilab Tevatron, with the cuts and parameters discussed in the text.

| $p_T$ cut (GeV) | $\sigma_{Z\text{jet}}^{QCD}$ (pb) | $\sigma_{Z\text{2-jet}}^{\text{mult. int.}}$ (pb) | $\sigma_{Z\text{2-jet}}^{\text{mult. part.}}$ (pb) |
|----------------|-------------------------------|---------------------------------|-----------------|
| 5              | 800.                          | 1578.                           | 710.             |
| 10             | 269.                          | 159.                            | 71.              |
| 15             | 126.                          | 36.                             | 16.              |
| 20             | 67.                           | 12.                             | 5.3              |
| 25             | 40.                           | 4.7                             | 2.1              |

TABLE II. $Z + 3$-jet cross sections at the Fermilab Tevatron, with the cuts and parameters discussed in the text.

| $p_T$ cut (GeV) | $\sigma_{Z\text{jet}}^{QCD}$ (pb) | $\sigma_{Z\text{3-jet}}$ (pb) | $\sigma_{Z\text{3-jet}}^{\text{mult. int.}}$ (pb) | $\sigma_{Z\text{3-jet}}^{\text{mult. part.}}$ (pb) |
|----------------|-------------------------------|-------------------------------|---------------------------------|-----------------|
| 5              | 2582.                         | 195.                          | 906.                            | 408.             |
| 10             | 1386.                         | 42.                           | 49.                             | 22.              |
| 15             | 868.                          | 14.5                          | 7.2                             | 3.2              |
| 20             | 586.                          | 6.2                           | 1.6                             | 0.72             |
| 25             | 412.                          | 3.0                           | 0.46                            | 0.21             |
FIG. 1. $W, Z + 2$-jet multiple-parton scattering.

FIG. 2. Total cross section for the production of a $W$ accompanied by 2 jets, as a function of the minimum-$p_T$ cut, for: QCD processes (open circles), double-parton interactions (triangles), multiple interactions (squares).
FIG. 3. $p_T$ distribution of the jets in mb/GeV, for: (a) QCD 2-jets production, (b) $W + 2$-jets production.
FIG. 4. Statistical significance (Signal/$\sqrt{\text{background}}$) of the double-parton interactions for the production of a $W + 2$-jets (squares) and $W + 3$-jets (triangles) as a function of the minimum transverse momenta of the jets.
FIG. 5. Total cross section for the production of a $W$ accompanied by 3 jets, as a function of the minimum-$p_T$ cut, for: higher-order QCD processes (open circles), double-parton interactions (triangles), multiple interactions (squares).
FIG. 6. \( p_T \) distribution of the W in mb/GeV, for: (a) \( W + 1 \)-jet production; (b) QCD \( W + 3 \)-jets production with the solid line standing for the total while the dashed, dotted, and dot-dashed lines stand for the contribution from the quark-gluon, quark-quark, and gluon-gluon subprocesses respectively.
FIG. 7. $R_{3/2}$ in $W+n$-jet production as a function of the jet minimum transverse momentum. The squares stand for the sum of the QCD higher-order and double-parton contributions, while the triangles represent only the higher-order QCD contributions.
FIG. 8. Statistical significance (Signal/$\sqrt{\text{background}}$) of the double-parton interactions for the production of a $Z + 2$-jets (squares) and $Z + 3$-jets (triangles) as a function of the minimum transverse momenta of the jets.
FIG. 9. $R_{3/2}$ in $Z + n$-jet production as a function of the jet minimum transverse momentum. The squares stand for the sum of the QCD higher-order and double-parton contributions, while the triangles represent only the higher-order QCD contributions.