Robust programming for basin-level water allocation with uncertain water availability and policy-driven scenario analysis

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Received: 9 April 2021 / Accepted: 25 May 2021 / Published online: 24 June 2021 © The Author(s) 2021

Abstract
Uncertainties from hydrological and meteorological environments constantly pose disturbances to water sustainability. Programming under such uncertainties aims at finding solutions to this risky condition. From the sight of uncertain water availability, this paper builds a water life cycle model to reduce the risks of inappropriate estimations of water availability within a river basin and incorporates the results in robust programming. Then, a policy-driven scenario analysis is conducted to provide managerial implications in terms of ongoing water-saving policies. With Min–Tuo river basin as the case, we finally reach the conclusions that: (1) Equity is a necessity when considering the water allocation in a river basin, which enables a more sustainable mode of local water use. (2) Local citizens’ willingness to follow the policies is a key to relieve the water pressure, while the progress of water-saving techniques could add to its effectiveness.

Keywords Water resources management · Robust programming · Water life cycle · Scenario analysis

Introduction
Uncertainties of water-related decisions exist in both engineering and ecological environment, from the sight of sustainable water management [1]. It has become a necessity that urban decision makers take water sustainability into account, as recommended in United Nations Sustainable Development Goal 6 (UN SDG6) [2]. Generally, we measure the risks of inappropriate decisions through the analysis of uncertainties. They usually appear with something we do not know, and something we thought is known [3]. For those unknowns we do not know, predictions and estimations can be applied to reduce the uncertainties. But with those unknowns that are pretended known, insufficient information often leads to over-fitting scenarios or unsustainable resource planning [4]. Due to the ambiguity of parameters that should be known before decisions, uncertain programming has become critical.

In general, parameters included in a programming model can be classified into two primary forms, i.e. deterministic and uncertain. Conventional studies applying optimization models in decision-making usually incorporate deterministic parameters. However, it is often the case that optimization parameters are not fully resolved at the very first [5], the same with water resources management. With unknown variations in future water demand and supplies, deterministic information is no longer applicable for future sustainable development, raising the need for uncertain decision-making [6, 7]. For example, the basin manager cannot anticipate the exact water availability, since meteorological factors like temperature and precipitation are out of control [8]. The alarm for ongoing climate change is ringing, and intensified temperature rising has altered both the meteorological and hydrological environment into more complicated. We must reconsider the water resources planning in a changing environment.

Since the optimization results can be significantly affected by uncertainties, even drawing infeasibility on programming, a long-term exploration into this field has lasted for decades. Stochastic optimization (SO) and robust optimization (RO) are two main branches of mathematical programming under uncertainties. As a typical solution to decision-making con-
sidering uncertainties, SO requires exact PDFs of uncertain parameters; however, lacked for most cases in practice due to the information barriers. In comparison, a RO problem starts with an uncertainty set, which can be shaped in multiple ways, e.g. box, ellipsoidal, polyhedral, and sets with uncertain moments [9]. Also, the feasibility within a RO problem must be guaranteed for any realization of the uncertain parameters [10], i.e. power of robustness. Though RO is a powerful tool for decisions under uncertainties, it has not grasped much attention in water resources management, compared with SO. This paper takes a trial to apply RO in a real-world water planning to provide more robust regional water allocation.

To date, uncertainties in water planning are commonly considered as interval, probability, and fuzzy sets. Stochastic programming searches for optimal solutions in an uncertainty set generated from PDFs that should be explicitly acknowledged. For fuzzy programming, uncertainties within fuzzy sets can be divided into different memberships or grades [11]. Interval programming defines uncertain variables and parameters as interval numbers, with known upper and lower bounds [12]. Intervals are also effective in building distribution functions and resolving the uncertainties in tests [13]. A summary of recent studies that focus on uncertain water planning is shown in Table 1, and some other studies also develop a more complex uncertainty set with hybrid types. In a combination of interval uncertainty and fuzzy programming, Li et al. [14] applied interval sets in defining uncertain parameters, and designed a fuzzy objective in programming. Fu et al. [15] considered the uncertainties in water availability as discrete intervals and probability distributions in exploring water planning alternatives. From the existing literatures, we can see that interval uncertainty is commonly considered in water planning, due to its simplicity and wide acceptability. Therefore, we follow the former literature and use the interval uncertainty set to represent the uncertain water availability.

Table 1 Classification of recent studies in uncertain water planning

| Branches                  | Sub-branches                  | Articles | Uncertainty presented as                                    |
|---------------------------|-------------------------------|----------|-------------------------------------------------------------|
| Uncertain programming     | Interval programming          | [16]     | Interval numbers                                            |
|                           |                               | [17]     | Intervals with random boundaries                             |
|                           | Stochastic programming        | [18]     | Probability distributions and fuzzy variables               |
|                           |                               | [19]     | Interval numbers with probability distributions             |
|                           |                               | [20]     | Interval numbers with probability distributions             |
|                           |                               | [21]     | Interval numbers with probability distributions             |
|                           | Fuzzy programming             | [22]     | Fuzzy sets                                                  |
|                           |                               | [23]     | Fuzzy sets                                                  |
|                           |                               | [24]     | Fuzzy interval sets                                         |
|                           | Deterministic programming     | –        | Deterministic parameter                                     |
|                           |                               | –        | Deterministic parameter                                     |
|                           |                               | –        | Deterministic parameter                                     |

Considering the uncertainties, SO requires exact PDFs of uncertain parameters; however, lacked for most cases in practice due to the information barriers. In comparison, a RO problem starts with an uncertainty set, which can be shaped in multiple ways, e.g. box, ellipsoidal, polyhedral, and sets with uncertain moments [9]. Also, the feasibility within a RO problem must be guaranteed for any realization of the uncertain parameters [10], i.e. power of robustness. Though RO is a powerful tool for decisions under uncertainties, it has not grasped much attention in water resources management, compared with SO. This paper takes a trial to apply RO in a real-world water planning to provide more robust regional water allocation.

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Practical programming of limited natural resources is considered critical under ambiguous information of availability, e.g. water availability, especially in developing countries with a large population but already insufficient resources. Total available water (AW) is a key for any programming model, since it is the major limiting factor in water allocation [28]. Traditional studies commonly treat AW as deterministic according to historical records of water allocation. In other words, the estimation for AW is experience-oriented. With the applications of forecasting techniques, e.g. time series analysis [29], linear regression [30], and artificial neural networks [31], related studies have enriched their explanation for AW. This paper mainly considers uncertainties existing in AW, under the development of RO model, and applies the water life cycle to provide more accurate estimations for AW’s nominal value, as a complement of existing literature.

This paper aims at solving the robust programming problem in a real-world case within a bilevel water management framework. As for the main contributions, (1) this paper builds a water life cycle model based on historical records before the decision-making, to provide a more accurate estimation for AW’s nominal value in the interval uncertainty set. (2) The robust solutions of a leader–follower problem are given, providing robust decisions that could be enforced to optimize overall water equity and local water use profits. (3) Different scenarios that could further influence the effectiveness of water-saving policies are built and then analyzed in the discussion, enabling insights into the future implications of final water allocation schemes.

Methodologies applied in this paper will be depicted in “Methodology”. Then, a case study is designed in “Case study”. Results of robust programming are shown in “Results and discussions”, followed by further discussions on the pol-
icy implications through policy-driven scenario analysis. In the last section, conclusions and existing limitations of this paper will be given.

Methodology

“Problem statement” illustrates the water allocation problem under a hierarchical framework with uncertain water availability. “Robust water allocation model” provides a global robust programming model that is also general for other basins under similar conditions. With special cases, parameters and constraints could be modified according to the features of water allocation.

**Problem statement**

As shown in Fig. 1, a leader–follower problem appears within a bilevel water management framework. Generally, a bilevel programming contains an upper level problem that has multiple lower level problems [32]. The leader, authority of river basin, owns the highest priority in water resources management and makes the final decision. The followers, i.e. subarea managers, hold the responsibility of allocating water to different sectors. According to former researches and the water use statistics in China, we consider four main water consumption sectors, i.e. ecological, industrial, domestic, and agricultural sectors. Under the proposed framework, both the overall equity of river basin and local profits of subareas can be optimized within a Stackelberg game, in which the authority of river basin leads the upper decision first, owning a complete knowledge of possible reflections of subareas.

In contrast, the subarea managers act accordingly right after the upper decisions are made [33]. Based on water allocation results, trade-offs of mutual gains from water allocation will be further discussed.

Before constructing the bilevel model for basin-level water allocation, the following assumptions were made for basin-level water allocation program, referring to the former works of [34, 35]:

1. Subareas own the shared water resources supplied by one single river basin and other supplies are not included, e.g. remained water resources of subareas.
2. Allocated water to subareas is all distributed to sectors, with no reservation.
3. There is no water exchanging or trading among the subareas or subordinate sectors.
4. River basin authority fully understands the objectives and constraints of bilevel decisions while subarea managers act accordingly.
5. Subareas are under an uncooperative situation where no information is shared and conflicts among competitive subareas are not included.

**Robust water allocation model**

In this paper, we put forward the robust programming in water resources management and apply solvable linear programming (LP) to obtain solutions, i.e. robust counterpart approach. The theory of robust counterpart (RC) begins with an equivalent LP form of the original robust programming [36, 37]. With the evolution of RC approach, adjustable robust counterpart (ARC) that aims at relieving the con-
servativeness of robust solutions was introduced. However, the ARC is computationally intractable (NP-hard) in most cases. Therefore, Ben-Tal et al. [38] proposed an improved affinely adjustable robust counterpart (AARC) to transfer ARC problem into solvable LP, while maintaining robustness. Basically, AARC introduces the affine function of uncertain data in the new problem.

With historical data of AW, we can define the support set \( \Omega \) as follows:

\[
\Omega = \left\{ \tilde{AW} | AW_{\min} \leq \tilde{AW} \leq AW_{\max} \right\}
\]  

(1)

where \( AW_{\min} \) and \( AW_{\max} \) are the upper and lower bounds of uncertain \( AW \).

Then, an interval uncertainty set can be defined with adjustable coefficient \( \theta \):

\[
\Omega = \left\{ \tilde{AW} | AW^*(1-\theta) \leq \tilde{AW} \leq AW^*(1+\theta) \right\}
\]  

(2)

where \( AW^* \) is the nominal value of \( \tilde{AW} \). Generally, the nominal value is present based on assumptions, e.g. average value of historical records. In this paper, a water life cycle model will be applied to determine the \( AW^* \). \( \theta \) represents the likelihood of managers willing to accept the uncertainties of climate change. When \( \theta = 0 \), the problem is equal to a deterministic problem where \( \tilde{AW} = AW^* \). Uncertainties that may exert impacts on \( \tilde{AW} \) cover both meteorological and hydrological factors [39]. In this paper, we set \( \theta \) as the measurement of meteorological factors, e.g. natural factors that may cause variations in water availability. In a wider range, \( \theta \) represents temperature, wind speed, humidity, and other related reflections of climate change. Following the instruction of AARC, we establish the bilevel robust programming model.

**Upper level decision**

In this paper, the leader (authority as a leader of river basin) first decides under what principles could the limited water resources be allocated rationally. Then, constraints that may influence the feasibility of allocation should be considered.

**Upper level objective function: maximizing the equity of water resource allocation** Equity refers to an unbiased situation where individuals under competition are treated equal, and in water allocation, it is defined as equitable access to water resources. Though equity is an uncountable term, we can follow what Gini has defined in the exploration of income inequality [40]. Hu et al. [35] measured water allocation equity by the equitable sharing of the used water quantity for each unit of economic benefits. In this paper, we focus on the equitable access to water of all water population in subareas, to relieve the gap between high water pressure and low water pressure, i.e. gap between a large population sharing limited water resources and a small population sharing abundant water resources.

Considering the water loss existing in transportation, distribution etc., the total amount of water from the river will be more than those reaching to the terminal users. Let \( Q_i^{ef} \) represents the efficiently allocated water which excludes the total of loss in subarea \( i \).

\[
Q_i^{ef} = Q_i(1 - \beta_i^{loss})
\]  

(3)

where \( \beta_i^{loss} \) is the ratio of total loss during the transportation from water plants to users.

Considering the total amount of water users \( S_i \) in subarea \( i \), we set the Gini coefficient of water allocation as follows:

\[
G = \frac{1}{2I} \sum_{i=1}^{I} \left( \sum_{u=1}^{I} \sum_{z=1}^{I} \left| \frac{Q_i^{ef}}{S_u} - \frac{Q_z^{ef}}{S_z} \right| \right)
\]  

(4)

where two different subareas are denoted by \( u \) and \( z \) among all the subarea \( i \). In specific, water allocation among subareas is regarded perfectly equal if \( G = 0 \), under which the water use per capital of each subarea shares no difference, i.e. entirely equitable access to water.

**Upper constraint 1: water availability** For the very first feasibility, total allocated water \( \sum_{i=1}^{I} Q_i \) of upper level to subarea \( i \) cannot exceed \( AW \), which is uncertain.

\[
\sum_{i=1}^{I} Q_i \leq \tilde{AW}
\]  

(5)

where critically, \( \tilde{AW} \) is presented as a random variable which shares no explicit information of probability distributions but historical records.

**Upper constraint 2: water demand** There’s always a tradeoff between water supply and demand before conducting the water allocation. Thus, we set a constraint on the range of \( Q_i^{ef} \) based on the minimal water demand of subareas. That is, the total amount of water allocated to subarea \( i \) should necessarily exceed or equal to \( D_i^{min} \).

\[
D_i^{min} \leq Q_i^{ef}
\]  

(6)

where minimal water demand \( D_i^{min} \) is based on historical water demand of study areas.

**Lower level decision**

From the sight of Stackelberg game, the followers (subarea managers) make the decisions right after the upper decision is
done, i.e. total available water allocated to subareas is settled. Then, lower decisions start with the priority of sectorial water use, i.e. objective of water allocation. Also, constraints that could exert disturbances on water allocation are included.

**Lower objective function: maximizing the overall profits**

Subarea managers consider more for the overall profits of water allocation to different sectors. As for the industrial sector, water resources are mainly distributed to production, manufacturing, and other industrial activities, denoted as $q_i^{\text{Ind}}$. Ecological water use $q_i^{\text{Eco}}$ ensures the protection of hydrological environment. $q_i^{\text{Dom}}$ and $q_i^{\text{Agr}}$, i.e. domestic sector and agricultural sector, are critical for local citizens. For subareas, unit profits gains are treated as deterministic, for which we can use statistical data to represent. Unit returns of water consumption in three economic sectors are represented by $p_i^{\text{Agr}}$, $p_i^{\text{Ind}}$ and $p_i^{\text{Dom}}$, which are estimated through historical records and shown in Appendix 1 (Table 7).

Total profit gains of water allocated to economic sectors can be treated as the products of allocated water and unit profit gains:

$$\text{MaxP} = \sum_{i=1}^{I} q_i^{\text{Ind}} \times p_i^{\text{Ind}} + q_i^{\text{Dom}} \times p_i^{\text{Dom}} + q_i^{\text{Agr}} \times p_i^{\text{Agr}}.$$  \hspace{1cm} (7)

**Lower constraint 1: water availability**

The total amount of water allocated to sectors is equal to those initially allocated to subareas in the upper decisions:

$$q_i^{\text{Eco}} + q_i^{\text{Ind}} + q_i^{\text{Dom}} + q_i^{\text{Agr}} = Q_i^{\text{ef}}.$$  \hspace{1cm} (8)

**Lower constraint 2: minimal water demand**

To ensure the basic need of economic sectors, i.e. industrial, agricultural, and domestic sectors, allocated water to those sectors should be more than the minimal water demand.

$$q_i^{\text{Ind}} \geq (d_i^{\text{Ind}})_{\text{min}}$$  \hspace{1cm} (9)

$$q_i^{\text{Agr}} \geq (d_i^{\text{Agr}})_{\text{min}}$$  \hspace{1cm} (10)

$$q_i^{\text{Dom}} \geq (d_i^{\text{Dom}})_{\text{min}}.$$  \hspace{1cm} (11)

**Lower constraint 3: water-saving policies**

Water-saving has long been a critical issue in water resources management, with relevant regulations on total consumption from the local use. In this study, we consider limitations on total consumed water of industrial and agricultural sectors through maximum consumption before the planned year, i.e. quotas of water consumption, to support the water-saving policies:

$$q_i^{\text{Ind}} \leq c_i^{\text{Ind}}$$  \hspace{1cm} (12)

$$q_i^{\text{Agr}} \leq c_i^{\text{Agr}}$$  \hspace{1cm} (13)

where $c_i^{\text{Ind}}$ and $c_i^{\text{Agr}}$ represent the quotas of water consumption in the industrial and agricultural sector.

Specially when considering the fundamental requirement for living as primary principle, we treat the quota of domestic water use as a minimum guarantee:

$$q_i^{\text{Dom}} \geq c_i^{\text{Dom}}$$  \hspace{1cm} (14)

where $c_i^{\text{Dom}}$ represents the quotas of water consumption in domestic sector.

**Lower constraint 4: minimal ecological demand**

According to water report of Sichuan province, ecological water demand has long been compressed by economic sectors, which is harmful to the ecological environment of river basin. Thus, a constraint for minimal demand is set to ensure adequate supply of ecological water:

$$q_i^{\text{Eco}} \geq (d_i^{\text{Eco}})_{\text{min}}$$  \hspace{1cm} (15)

where $(d_i^{\text{Eco}})_{\text{min}}$ is the minimal water demand of ecological sector in subarea $i$.

In some cases, the scarcity of water could also be intensified due to the over-empahsized ecological water demand, and even the basic availability of living water could be influenced. Thus, we also attached another constraint on the maximum quantity of ecological water use:

$$q_i^{\text{Eco}} \leq (d_i^{\text{Eco}})_{\text{max}}$$  \hspace{1cm} (16)

where $(d_i^{\text{Eco}})_{\text{max}}$ is the maximum threshold of ecological water use in subarea $i$.

**Global model**

Finally, we reach to a global model composed of the upper and lower decisions. Given that $Q_i^{\text{ef}} = Q_i(1 - P_i^{\text{loss}})$, the total allocated water $\sum_{i=1}^{I} Q_i$ should be included in $\hat{\text{AW}}$ and the upper level objective is determined by $Q_i^{\text{ef}}$ that is effectively transported to subareas:

$$\min G = \frac{1}{2I \sum_{i=1}^{I} \frac{q_i^{\text{ef}}}{S_i}} \sum_{i=1}^{I} \sum_{z=1}^{N} \left| \frac{Q_i^{\text{ef}}}{S_i} - \frac{Q_i^{\text{ef}}}{S_z} \right|.$$  \hspace{1cm}
Conventional bilevel programming is regarded as NP-hard, which indicates observed possibility in compromising, in terms of the leader and followers’ shared profits. Basically, hierarchical problems can be transferred into a standard mathematical program by replacing each follower’s problem with its Karush–Kuhn–Tucker (KKT) condition [41]. We applied this strategy and transferred problem (17) to problem (18) to make it solvable:

\[
\begin{align*}
\sum_{i=1}^{I} Q_{i} & \leq \bar{A}W \\
Q_{i}^{ef} & \geq D_{i}^{\min} \\
\text{Max } P & = \sum_{i=1}^{I} \left( q_{i}^{\text{Ind}} \times p_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} \times p_{i}^{\text{Dom}} + q_{i}^{\text{Agr}} \times p_{i}^{\text{Agr}} \right) - Q_{i}^{ef} \\
\text{s.t.} \\
q_{i}^{\text{Eco}} + q_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} + q_{i}^{\text{Agr}} & = Q_{i}^{ef} \\
(d_{i}^{\text{Ind}})^{\text{min}} & \leq q_{i}^{\text{Ind}} \leq c_{i}^{\text{Ind}} \\
(d_{i}^{\text{Agr}})^{\text{min}} & \leq q_{i}^{\text{Agr}} \leq c_{i}^{\text{Agr}} \\
q_{i}^{\text{Dom}} & \geq c_{i}^{\text{Dom}} \\
(q_{i}^{\text{Ind}})^{\text{min}} & \leq q_{i}^{\text{Dom}} \leq c_{i}^{\text{Dom}} \\
Q_{i}^{ef}, q_{i}^{\text{Eco}}, q_{i}^{\text{Ind}}, q_{i}^{\text{Dom}}, q_{i}^{\text{Agr}} & \geq 0.
\end{align*}
\]

(17)

\[
\begin{align*}
\text{Min} G & = \frac{1}{2I} \sum_{i=1}^{I} \left( \sum_{u=1}^{U} \sum_{z=1}^{Z} \frac{Q_{i}^{ef} - Q_{i}^{ef}}{S_{u} - S_{z}} \right) \\
\sum_{i=1}^{I} Q_{i} & \leq \bar{A}W \\
Q_{i}^{ef} & \geq D_{i}^{\min} \\
q_{i}^{\text{Eco}} + q_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} + q_{i}^{\text{Agr}} & = Q_{i}^{ef} \\
(d_{i}^{\text{Ind}})^{\text{min}} & \leq q_{i}^{\text{Ind}} \leq c_{i}^{\text{Ind}} \\
(d_{i}^{\text{Agr}})^{\text{min}} & \leq q_{i}^{\text{Agr}} \leq c_{i}^{\text{Agr}} \\
q_{i}^{\text{Dom}} & \geq c_{i}^{\text{Dom}} \\
(q_{i}^{\text{Ind}})^{\text{min}} & \leq q_{i}^{\text{Dom}} \leq c_{i}^{\text{Dom}} \\
(d_{i}^{\text{Eco}})^{\text{min}} & \leq q_{i}^{\text{Eco}} \leq c_{i}^{\text{Eco}} \\
q_{i}^{1+} & = s_{i}^{1-} + s_{i}^{2-} + s_{i}^{3-} + s_{i}^{4-} + s_{i}^{5-} = -p_{i}^{\text{Ind}} - p_{i}^{\text{Dom}} - p_{i}^{\text{Agr}} \\
q_{i}^{1+} & = ((q_{i}^{\text{Eco}} + q_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} + q_{i}^{\text{Agr}}) - Q_{i}^{ef}) - s_{i}^{1-}(Q_{i}^{ef} - (q_{i}^{\text{Eco}} + q_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} + q_{i}^{\text{Agr}})) \\
q_{i}^{1+} & = (q_{i}^{\text{Ind}} - (q_{i}^{\text{Ind}})^{\text{min}} - s_{i}^{1-}(c_{i}^{\text{Ind}} - q_{i}^{\text{Ind}}) + s_{i}^{3+}(q_{i}^{\text{Agr}} - (d_{i}^{\text{Agr}})^{\text{min}}) - s_{i}^{3-}(c_{i}^{\text{Agr}} - q_{i}^{\text{Agr}})) \\
q_{i}^{1+} & = (q_{i}^{\text{Dom}} - c_{i}^{\text{Dom}}) + s_{i}^{4+}(q_{i}^{\text{Dom}} - (d_{i}^{\text{Dom}})^{\text{min}}) + s_{i}^{5+}(q_{i}^{\text{Eco}} - (d_{i}^{\text{Eco}})^{\text{min}}) - s_{i}^{5-}(d_{i}^{\text{Eco}})^{\text{max}} - q_{i}^{\text{Eco}} = 0 \\
q_{i}^{1+} & = (q_{i}^{\text{Eco}} + q_{i}^{\text{Ind}} + q_{i}^{\text{Dom}} + q_{i}^{\text{Agr}}) = 0 \\
Q_{i}, q_{i}^{\text{Eco}}, q_{i}^{\text{Ind}}, q_{i}^{\text{Dom}}, q_{i}^{\text{Agr}} & \geq 0, s_{i}^{j \pm} \geq 0, j = 1 \ldots 6
\end{align*}
\]

(18)
where \( s_i^{\pm}, d_i^{\pm}, s_i^{4\pm}, d_i^{4\pm}, \) and \( s_i^{6}, d_i^{6} \) represent the dual variables of \( q_i^{\text{Eco}}, q_i^{\text{Ind}}, q_i^{\text{Agr}} \) and \( q_i^{\text{Dom}}, q_i^{\text{Eco}}, q_i^{\text{Ind}}, q_i^{\text{Agr}} \) under the typical situation of right-hand side uncertainty. Transferred AARC models are presented as problem (20). Note that, the original decision variables \( q_i^{\text{Ind}}, q_i^{\text{Agr}}, q_i^{\text{Dom}}, q_i^{\text{Eco}} \) are transferred to their affine functions considering AW, i.e., \( \pi_i, AW, \pi_i^{0,\text{Ind}} + \sum_{r \in R} \pi_i^{r,\text{Dom}} AW_r, \pi_i^{r,\text{Ind}} + \sum_{r \in R} \pi_i^{r,\text{Agr}} AW_r, \pi_i^{r,\text{Dom}} + \sum_{r \in R} \pi_i^{r,\text{Ind}} AW_r \) and \( \pi_i^{0,\text{Eco}} + \sum_{r \in R} \pi_i^{r,\text{Eco}} AW_r \). Final LP problem (21) is obtained based on the principles in Eq. (22) (shown in Appendix 2). Results could be processed through Lingo software.

### Case study

In “Study area” and “Data sources”, we introduce the study area according to its geographical features, water use conditions and the available data sources. Then in “Modeling the water life cycle”, we conduct a water life cycle analysis using statistical data of study area. Worth mentioning, the study area, i.e. Min–Tuo river basin, is facing the same water allocation issue that we mentioned in “Problem statement”. Also, other basins in China could also find a reference in this study.

### Study area

Numerous tributaries of Yangtze river form the Min–Tuo river basin (99°–106° E, 28°–34° N) and cover a total area of 16.3 \( \times 10^4 \) km\(^2\) at the upstream (shown in Fig. 2). Min river owns abundant water resources for 953.6 \( \times 10^9 \) m\(^3\) on average and supplies for more than 11 prefectures in Sichuan Province and Qinghai Province, China, with a coverage of 13.5 \( \times 10^4 \) km\(^2\) and full length of 735 km. It’s the main water supplier of Sichuan Province and serves water resources for about 1.9 \( \times 10^6 \) residents, more than half of which are in urban areas. Embedded with large population, water planning for Min river must find trade-offs between equity and local profits. Tuo river situates in central Sichuan Province and covers a total area of 2.56 \( \times 10^4 \) km\(^2\) at a length of 627.4 km. Since Tuo river flows through the industrial cities of Luzhou, Neijiang, Ziyang, Jianyang, Chengdu and Deyang in Sichuan Province, it has to balance the industrial water consumption among areas. Moreover, it has been faced with intensified water issues, e.g. inefficient water allocation and water pollution [42]. Since only 7% of the Min river locates in Qinghai Province while Tuo river fully involved in Sichuan Province, this paper takes Sichuan Province in the case study, and applies the bilevel RO model to deal with the real-world water allocation problem of this area. In all, 13 subareas and the water use sectors are included in further analysis (shown in Table 2).

For river basin, the uncertainties in total available water have been a long-term concern in related studies [43, 44]. In this paper, both meteorological and hydrological uncertainties of water availability are considered, through the incorporated water life cycle analysis and robust programming. For the principles of water allocation, managers of Min–Tuo river basin should find a sustainable path toward adequate water supplies with less water stress, with the implementation of water-saving policies. As shown in “Problem statement”, this paper considers basin-level water allocation under a hierarchical framework, in which a Stackelberg game should be satisfied with Nash equilibrium.

### Data sources

Statistics applied in this study are from Sichuan Statistical Yearbook (2010–2019), Sichuan Water Resources Bulletin (2009–2019), Report of Water resources planning in Sichuan province (2013), Key points of water conservation in Sichuan province in 2020 etc., as well as two official websites, i.e. the Sichuan Provincial Water Resources Department and Hydrology and Water Resources Survey Bureau of Sichuan Province.

Reference parameters in robust programming are set based on the historical water allocation results and statistics of Min–Tuo river basin. Statistics of water population in each subarea (2004–2019) are applied to predict the future water population. With serial records of water allocated to subareas from river, as well as water allocated to sectors from subareas, the minimal demand of subareas, minimal and maximum demand of different sectors (i.e. ecological, industrial, domestic, and agricultural sectors) could be obtained. Motivated by water-saving policies performed in Sichuan province, we also treat maximum quantity of water allocated.
to sectors before the planned year as the quotas of water use, as stated in “Robust water allocation model”.

Modeling the water life cycle

This paper refers to life cycle analysis (LCA) for modeling AW in a more accurate way. Cai et al. [45] consider water processing as the extraction, production, use, treatment and discharge/reuse among water reservoirs, users and treatment plants. Similarly, water life cycle (WLC) in this paper has five components in processing, i.e. upper stream, river basin, water plants, subareas and sewage treatment plants, with the natural environment as the boundary. Accordingly, five periods in water processing are covered, i.e. water flowing, extraction, production, use and recycling, from “cradle to grave.”

Integrated structure of WLC is shown in Fig. 3. In specific, this paper denotes the inflows and outflows in pairing during the water processing periods. Initially, streams from the upper river $Q_{\text{inflows}}$ flow into the target basin. Also, precipitation offers another main inflow $Q_{\text{in}-1}$ into the river basin, with unavoidable loss in evaporation $Q_{\text{out}-1}$. In the second stage, non-processed natural water $Q_{\text{in}-2}$ are extracted from the river into the water plants in subareas, where the natural water are transferred into usable water $\sum Q_i$. Also, unqualified water or water loss in this stage is denoted as $Q_{\text{out}-2}$.

In the third stage, i.e. water resources programming, usable water for subareas $\sum Q_i$ is further allocated to different water use sectors ($\sum Q_{ij}$). In the fourth stage, used water from different sectors $Q_{\text{in}-3}$ is transported into sewage treatment plants ($Q_{\text{out}-3}$), and can be divided into treated water $Q_{\text{in}-4}$ and untreated water $Q_{\text{out}-4}$. Then, treated water $Q_{\text{in}-4}$ will be further processed for recycled water $Q_{\text{in}-5}$, with the production of unqualified water $Q_{\text{out}-5}$. Generally, recycled water $Q_{\text{in}-5}$ can be directly allocated to subareas.

Assumptions for the WLC are shown as below:

1. Uncertainties mainly exist in the water flowing and precipitation periods, i.e. $Q_{\text{inflows}}$ and $Q_{\text{in}-1}$. Therefore, we further defined them as $\tilde{Q}_{\text{inflows}}$ and $\tilde{Q}_{\text{in}-1}$ in modeling AW.
2. $Q_{\text{in}-4}$ and $Q_{\text{in}-5}$ can only be known when decisions on water resources programming in last period are deter-
mined, since they depend on decision variables \( \sum Q_i \) and \( \sum Q_{ij} \).

3. Precipitations are considered as basin-level, and other sources of water resources, e.g. other inflows, are not included.

4. Ratio of water loss in precipitation, water production, sewage treatment and water recycling are deterministic in terms of annual statistical records, scilicet the technologies for water processing are mature. Also, ratio of sewage produced from used water is deterministic.

**Introduction to the variables and parameters in WLC is shown in the Appendix 1** (Table 6).

Then, we could easily reach to the estimation of \( AW^* \).

\[
AW^* = \sigma_6 (\tilde{Q}_{in-1} + Q_{inflows}) + Q_{in-5}
\]  

(19)

where \( \tilde{Q}_{in-1} \), \( Q_{inflows} \) and \( Q_{in-5} \) represent uncertain efficient precipitations, uncertain inflows from the upper stream, and qualified water after recycling, respectively. In general, the \( \tilde{Q}_{in-1} \) can be predicted through time series analysis based on historical records of precipitations. \( Q_{inflows} \) is viewed as annually stable, not considering extreme natural disasters, e.g. floods. \( Q_{in-5} \) is decided by the water use condition of last year, i.e. total amount of used water, sewage produced and treated, and effectively recycled water. Finally, the overall availability coefficient of river water is denoted by \( \sigma_6 \), i.e. utilization rate of total water resources.

In this paper, we incorporate Autoregressive Integrated Moving Average model (ARIMA) to obtain the estimation for future precipitations. In specific, it consists of three basic forecasting models, i.e. autoregressive model (AR), moving average model (MA) and difference model (I). It can also be viewed as an Autoregressive Moving Average model (ARMA) that incorporates difference effects. Commonly, this model is denoted as ARIMA\((p, d, q)\), where \( p \) is the order of autoregressive, \( d \) is the order of difference and \( q \) is the order of moving average. With SPSS 25 software, we input the annual precipitations in Min–Tuo river basin during 1998–2019, and use the ARIMA\((0, 2, 1)\) model for predicting regional precipitations in 2020 and the result is shown in Fig. 4. Estimation for precipitation in 2020 is \( 1574.56 \times 10^8 \) m\(^3\) (Stationary \( R^2 = 0.578 \)).

Based on historical records of total precipitations and estimations for effectively transferred water resources in Min–Tuo river basin from 1997 to 2019, we reach to the average of \( \sigma_1 \) as 0.57. According to the technological targets of Key points of water conservation in Sichuan province in 2020, the ratio of lost water in production and transportation is 0.1, i.e. \( \sigma_2 \). Referring to water consumption, sewage production and treatment records, we set \( \sigma_3 \) and \( \sigma_4 \) as 0.83 and 0.9, respectively. In the process of water recycling, the ratio of qualified recycled water \( \sigma_5 \) is 0.2, based on the technological targets of Key points of water conservation in Sichuan province in 2020. The average of \( \sigma_6 \), i.e. utilization rate of total water resources, in Min–Tuo river basin is 0.288, referring to [46].

With the prediction of annual precipitation in 2020, sum of allocated water in 2019 and estimations for parameters in WLC, we reach to the estimation of \( AW^* \) at \( 235.035 \times 10^8 \) m\(^3\).

**Results and discussions**

The results of bilevel robust programming are shown in “Allocation strategies based on predicted AW* in 2020” and in “Policy-driven scenario analysis”, a policy-driven scenario analysis is conducted and the managerial insights are provided in discussions.
Table 3 shows the optimization results of bilevel RO model as \( \theta \) changes. Considering \( \theta \) as the measurement of meteorological factors, we set the range of \( \theta \) as \([0, 0.2]\) and selected 0.05, 0.1, 0.15, 0.2 in the numerical examples, to describe to what extent the managers can accept the uncertainties of climate change. Since robust programming aims at optimizing the worst case within the interval uncertainty set (see Eq. 2), the total allocated water to subareas decreases as \( \theta \) increases. As shown in the results, the upper objective, i.e. minimal Gini coefficient of per capita water, reaches to its optimality at 0.03998 when \( \theta = 0 \), while the lower objective, i.e. local profits, reaches \( 44,047.50 \times 10^8 \) yuan at its optimality. Besides, the upper and lower objectives both get worse while \( \theta \) increases, arriving at 0.11596 and \( 31,465.57 \times 10^8 \) yuan respectively. In this case, \( \theta \) exerts great impacts on the bilevel decisions, because of the deduction in total available water.

To further explore the performance of lower decisions by water use sectors with economic returns, i.e. industrial, domestic and agricultural sectors, we can turn to Fig. 5. Due to the settings of prevailed domestic water use, economic returns from service industry, i.e. water use of local citizens, take the lead among the three sectors, varying from \( 28,587.77 \times 10^8 \) yuan to \( 21,377.64 \times 10^8 \) yuan. As for the industrial water use, it brings \( 8243.48 \times 10^8 \) yuan in return when \( \theta = 0.1 \) and meets a drop at \( 7980.54 \times 10^8 \) yuan when \( \theta = 0.15 \). Accordingly, the returns from agricultural water use vary from \( 2184.77 \times 10^8 \) yuan to \( 1954.16 \times 10^8 \) yuan as \( \theta \) changes.

To provide a more specific report on the numerical results of bilevel water allocation, we first focus on the results of deterministic model (\( \theta = 0 \)), shown in Fig. 6. It can be found that if the uncertain AW is considered as deterministic, the managers would make safer decisions while not considering the possible changes in water availability at all. Therefore, the results represent how original decisions are made when the impacts of climate change are not included. Figure 6 shows the allocated water to subareas, and then to sectors. Red curve Level 2019 reflects the water use per capital among different subareas before the planned year, where the gaps between Deyang and Ziyang are nearly shown by three times of total water per capital in Ziyang. Besides Ziyang, areas including Neijiang, Zigong, Yibin, Luzhou, Ngawa and Garzê are far behind, compared with other areas. While in the results of the planned year (blue curve), the gaps are prominently narrowed. Figure 6b represents the numerical results and corresponding proportions of allocated water.

For comparison, Figs. 7 and 8 show how results vary among managers’ different attitudes toward uncertainties of climate change. When \( \theta = 0.05 \), the managers are likely to support decisions under the variations of available water.
within $[-5\%, +5\%]$. In this occasion, worst case happens when total available water is reduced by 5\%. Similarly, the worst cases in robust programming refer to reduced available water by 10, 15 and 20% when $\theta = 0.1$, $\theta = 0.15$ and $\theta = 0.2$. As $\theta$ changes, the impacts of climate change exert limited variations within total allocated water to subareas and lead to a proportionally change in columns. Similarly, the blue curves in Fig. 7 are depicted according to water use per capita under different levels of meteorological disturbances. The gaps among different subareas are likely to expand when
θ gets bigger, while those left-behind areas, Ziyang, Neijiang etc., sharing a consistent level of water per capita. From Fig. 8, we can find that only the domestic water use is constantly shrunk as total available water being reduced, because of the priority of citizens’ livable environment designed in the constraints. In other words, with surplus water supplies considering the maximum demands from other three sectors, i.e. industrial, agricultural and ecological, local managers are willing to divert it to unconstrained domestic water sector to make more profits.

**Sensitivity analysis**

Since the future demands of four sectors are estimated according to historical data, the robustness of water allocation strategies could be further examined through a sensitivity analysis with increasing demands. In building the original strategies of water allocation, the maximum water demands are controlled under the maximum consumption before the planned year, and the minimal water demands are based on historical minimal demands. Therefore, we assume that both the maximum and minimal demands are increased by proportions and test the results with the varying θ. From the results (shown in Table 4), we can see that the solutions remain optimal when demands are increased by less than 50%, and we could still reach optimality by enlarging the acceptance for uncertainties as demands increase, until the solutions become infeasible when increased by 80%.

**Policy-driven scenario analysis**

**Scenario design**

As stipulated by Report of Water resources planning in Sichuan province (2013), the total water consumption of Min–Tu river basin in 2020 is scheduled to be $175 \times 10^8$ m³, with detailed targets for domestic water use per capital, industrial water use per industrial production and agricultural water use per farm land. To test the strength of ongoing water-saving policies, as well as the possible water stress generated...
from increasing demands, a policy-driven scenario analysis is conducted from the sight of total available water and water-saving targets. On one hand, real-word disturbances, e.g. population growth and industrial production growth, could bring in extra water demands in comparison with prediction. On the other hand, the effectiveness of water-saving policies could largely alter the water use conditions, e.g. more local citizens are willing to take actions and then less water will be wasted. Progress in water-saving techniques could also add to the effectiveness of policies. Data applied in further discussions include forecasted population, farm land and industrial growth in 2020. Corresponding simulated data of policy targets (quotas) are then calculated according to Report of Water resources planning in Sichuan province (2013).

Scenario S0 is used as the control group, in which no extra measures are taken to relieve the possible stress of water use, reflected by total water use, industrial water use, domestic water use and ecological water use. Introduction to designed scenarios is shown below.

The first column denotes the proportion by which the maximum and minimal demands are increased.
unnecessary waste in water allocation and improving the water recycling efficiency. In specific, we improve $\beta_{\text{loss}}$ in robust programming and $\sigma_2$, $\sigma_4$ and $\sigma_5$ in WLC by 20%, restricted within $[0, 1]$ if the improved value exceeds the boundaries.

The third scenario (S3) is prepared to test how citizens’ willingness for water-saving performs under relevant policies. We illustrate citizens’ willingness by the effectiveness of policy implementation. In this paper, we assume that only 60% of the water population initially hold firmly the new water-saving policies while other 40% do not. For those local citizens with strong willingness, quotas of water use are exactly given by the government. Inversely, the other will keep up with the average water use before the planned year, i.e. in 2019.

The fourth scenario (S4) integrates both the technical progress of water-saving and citizens’ willingness toward ongoing policies, in comparison with S1.

Details for parameter settings of the four scenarios are shown in Table 5.

### Policy implications under different scenarios

Results of scenario analysis are shown in Figs. 9 and 10. First we notice that the results of robust programming mostly exceed the quotas of water use in subareas, and only some of the areas satisfy the baseline of water-saving policies. That is, the schemes we obtained through robust programming are not feasible considering the water-saving policies designed in 2013. Therefore, we have to reconsider the ongoing policies under the latest mode of water use. For areas like Ziyang, Neijiang and Luzhou, the gaps between expectation and demands at present are prominent, in comparison with Chengdu, Ya’an, Meishan and Liangshan.

In contrast to Fig. 9 that focuses on the existing gaps between expectation and demands at present, Fig. 10 indicates how technical progress and the actual effectiveness of water-saving policies influences gaps. First with scenario 1, the Fig. 10a illustrates the possible population growth and economic development influencing future water demands. Gaps between water supplies (color lines) and limited water use (red areas) are further narrowed in this occasion, considering the existing gaps shown in Fig. 9. It indicates a greater stress in water supplies, since ascending demands are approaching the actual supplies in planned year. Then, we try to find a solution to this condition and relieve the possible stress in water supplies. Two basic measures are included, i.e. relevant technical progress in water-saving and citizens’ willingness to take the actions. The former adds to the feasibility by lifting up supplies with more available water, while the latter works to lower down the demands through citizens’ water-saving actions. In scenario 2, techniques including reducing the unnecessary waste in production and improving the efficiency of recycling are applied. Through the advancement of techniques, the total available water has increased by 2.6%. As shown in Fig. 10b, the water pressure has been relieved to some extent, especially for areas including Ziyang, Yibin and Luzhou, while Deyang, Ya’an and Leshan still in great supply pressure. Then, we turn to improve citizens’ willingness to keep up with the water-saving policies in scenario 3. Shown in Fig. 10c, the gaps existing at present have been further narrowed, compared with scenario 2. In this case, 80% of the local citizens would like to save more water as stipulated by policies while only 20% stay still. Apparently, it’s more effective than promoting technical progress solely. Then, Fig. 10d shows how the two measures cooperate to relieve the water stress, as designed in scenario 4. We can see that the overall effectiveness of actions has been doubled in narrowing the gaps. In this condition, the overall water stress originated from speed-up population growth and economic development has been largely relieved.

### Conclusions

This paper applies RO in a real-world water allocation case under a bilevel water management framework. With the application of WLC, the estimations for future water availability are considered more accurate, since the uncertain factors have been transferred to annual precipitations to a large extent. Within the changes of meteorological factor $\theta$, optimization results under different attitudes toward risks are shown and reflect managerial implications for basin manager. Besides, a policy-driven scenario analysis is conducted to provide suggestions for managers of case area, as well as similar regions with water allocation issues. Main findings
Table 5 Parameter settings of scenarios

| Parameter settings                              | S0 (%) | S1 (%) | S2 (%) | S3 (%) | S4 (%) |
|-----------------------------------------------|--------|--------|--------|--------|--------|
| Population growth rate                        | –      | + 20   | + 20   | + 20   | + 20   |
| Industrial production growth rate              | –      | + 20   | + 20   | + 20   | + 20   |
| Urbanization rate                              | –      | + 20   | + 20   | + 20   | + 20   |
| Water-saving technical progress                | –      | –      | + 20   | –      | + 20   |
| Citizens’ willingness toward policies          | –      | –      | –      | –      | + 20   |

“–” denotes no extra changes to this parameter.

Fig. 10 Water allocation under different scenarios (10^8 m^3)

of this paper provide some insights for hierarchical water resources management considering future uncertainties:

(1) Equity is a necessity for sustainable development, especially when considering areas with uneven water distribution. As for Min–Tuo river basin, water allocation at present has intensified the difference between areas with large population but limited water resources, and those with small population but adequate water resources. By adding overall equity of water allocation, i.e. equitable access to water of all population, to robust programming as the basic principle, we can find uneven water use per capita has been improved a lot, especially for those left-behind areas. Besides, with the increasing uncertainties of climate change, the equity of basin-level water allocation could be influenced since total available water is not stable.
(2) The effectiveness of water policy implementation could be largely influenced by local citizens, as well as the feasibility of relevant techniques. In this study, we conduct a policy-driven scenario analysis to test the resilience of robust decisions, in terms of ongoing water-saving policies taken in Min–Tuo river basin. Results show that local citizens’ willingness to take actions could relieve the existing water stress caused by increasing demands to a large extent. Moreover, the considerations for technical progress, i.e. reducing the unnecessary waste in water production and improving the efficiency of recycling, could do more benefits than taking one action only.

(3) Hydrological and meteorological uncertainties in water resources management are inevitable, but appropriate methodologies could be applied to reduce the uncertainties in decisions. This paper considers uncertain water availability in both the hydrological and meteorological environment. On one hand, the application of WLC works to visualize the evolution of water resources to provide more accurate estimation for uncertain variable in robust programming. On the other hand, the robust programming introduces an adjustable factor, i.e. meteorological factor $\theta$ in this paper, to reflect the impacts of climate change as rounded as possible. In this condition, both hydrological and meteorological uncertainties are considered.

(4) The approaches and models applied in this study are universal and also helpful in providing sustainable and robust water allocation schemes. We would support its generality in other basins under similar water use conditions. When special cases are considered, e.g. basins with different hydrological features, modifications could be conducted in the parameter settings and constraints. Also, more insights could be found if we look into more water allocation cases in other basins of China.

Limitations of this paper will inspire us to explore more for uncertain decision-making in water resources management: (1) this paper applies incorporated WLC and adjustable robust programming to model the water availability and the relationship between water pollution and water shortage could be further explored from the sight of hydrology. (2) Multi-dimensional and multi-source meteorological and hydrological data will be considered in future studies to provide more accurate estimations for water availability, referring to more advanced prediction models, e.g. interpretable prediction model [47]. (3) Further discussion on this topic will be held with more uncertain factors, e.g. water demand, water supply capability and environmental changes.

Acknowledgements The work is supported by the National Natural Science Foundation of China (Grant no. 71771157), the Fundamental Research Funds for the Central Universities, Sichuan University (Grant nos. 2019hhs-19, 2020CXQ22), Funding of Sichuan University (Grant no. skqz201726), and Social Science Funding of Sichuan Province (Grant nos. SC19TJ005, SC20EZD026), Ministry of Ecology and Environment (Grant no. 2020QT017-K2020A003).

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Appendix 1

See Tables 6 and 7.
### Table 6 Variables and parameters in WLC

| Variables     | Definition                  | Classification     | Equation                                      | Estimation |
|---------------|-----------------------------|--------------------|-----------------------------------------------|------------|
| $Q_{\text{inflows}}$ | Upper inflows              | Deterministic      | –                                             | –          |
| $Q_{\text{in}-1}$   | Efficient precipitations   | Uncertain          | –                                             | –          |
| $Q_{\text{out}-1}$  | Evaporation loss           | Deterministic in ratio | $Q_{\text{out}-1} = \sigma_1(Q_{\text{in}-1} + Q_{\text{out}-1})$ | $\sigma_1$ |
| $Q_{\text{in}-2}$   | Extracted water            | Uncertain          | –                                             | –          |
| $Q_{\text{out}-2}$  | Water loss in production   | Deterministic in ratio | $Q_{\text{out}-2} = \sigma_2(Q_{\text{in}-2} + Q_{\text{out}-2})$ | $\sigma_2$ |
| $\sum Q_i$       | Allocated water to subareas | Decision variables | –                                             | –          |
| $\sum Q_{ij}$    | Allocated water to sectors | Decision variables | –                                             | –          |
| $Q_{\text{in}-3}$ | Sewage produced            | Deterministic in ratio | $Q_{\text{in}-3} = \sum Q_{ij} \times \sigma_3$ | $\sigma_3$ |
| $Q_{\text{out}-3}$ | Sewage transported         | Deterministic in ratio | $Q_{\text{out}-3} = \sigma_3$ |            |
| $Q_{\text{in}-4}$ | Treated sewage             | Deterministic in ratio | $Q_{\text{in}-4} = \sigma_4$ |            |
| $Q_{\text{out}-4}$ | Untreated sewage           | Deterministic in ratio | $Q_{\text{out}-4} = \sigma_4(1 - \sigma_4)$ | $\sigma_4$ |
| $Q_{\text{in}-5}$ | Qualified recycled water   | Deterministic in ratio | $Q_{\text{in}-5} = \sigma_5$ |            |
| $Q_{\text{out}-5}$ | Unqualified water          | Deterministic in ratio | $Q_{\text{out}-5} = \sigma_5(1 - \sigma_5)$ | $\sigma_5$ |

### Table 7 Unit returns of water consumption by sectors (yuan/m³)

|                  | CD  | DY  | ZY  | YA  | MS  | NJ  | LE  |
|------------------|-----|-----|-----|-----|-----|-----|-----|
| Agricultural sector | 19.10 | 18.80 | 23.40 | 29.04 | 18.41 | 39.53 | 22.07 |
| Industrial sector  | 564.72 | 318.52 | 386.80 | 183.99 | 222.70 | 407.13 | 225.62 |
| Domestic sector    | 558.59 | 423.97 | 368.59 | 224.57 | 331.18 | 381.59 | 297.74 |
|                  | ZG  | YB  | LZ  | NG  | GZ  | LS  |
| Agricultural sector | 28.81 | 35.36 | 23.39 | 20.10 | 24.49 | 20.24 |
| Industrial sector  | 291.05 | 134.52 | 407.98 | 459.86 | 332.18 | 487.73 |
| Domestic sector    | 387.98 | 330.59 | 281.95 | 159.88 | 96.16 | 323.43 |
Appendix 2

Problem (20):

\[
\begin{align*}
\text{Min} G &= \frac{1}{S} \sum_{t=1}^{T} \sum_{t=1}^{l} \left| \frac{1}{S} (\pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r}) - \frac{1}{S} (\pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r}) \right| \\
&= \frac{1}{S} \sum_{t=1}^{T} \sum_{t=1}^{l} \left( \pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r} \right) - \frac{1}{S} \sum_{t=1}^{T} \sum_{t=1}^{l} \left( \pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r} \right) \\
&\leq \frac{1}{S} \sum_{t=1}^{T} \sum_{t=1}^{l} \left( \pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r} \right) - \frac{1}{S} \sum_{t=1}^{T} \sum_{t=1}^{l} \left( \pi_{0,t} + \sum_{r \in I_t} \pi_{r,t}^{AW_r} \right) \\
&= 0
\end{align*}
\]

s.t.

\[
\begin{align*}
\sum_{t=1}^{T} \sum_{t=1}^{l} \pi_{0,t} + \left( \sum_{t=1}^{T} \sum_{t=1}^{l} \pi_{r,t}^{AW_r} - 1 \right) AW_r &\leq 0 \\
(d_{\text{Ind}}^{\text{min}}) &\leq 0 \\
(d_{\text{Ind}}^{\text{max}}) &\leq 0 \\
(d_{\text{Agr}}^{\text{min}}) &\leq 0 \\
(d_{\text{Agr}}^{\text{max}}) &\leq 0 \\
(d_{\text{Eco}}^{\text{min}}) &\leq 0 \\
(d_{\text{Eco}}^{\text{max}}) &\leq 0
\end{align*}
\]

s.l.

\[
\begin{align*}
&\left( \pi_{0,t}^{\text{Ind}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Ind} AW_r} - (d_{\text{Ind}}^{\text{min}}) \right) - s_{i} - (c_{i}^{\text{Ind}} - (\pi_{0,t}^{\text{Ind}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Ind} AW_r})) \\
&= 0
\end{align*}
\]

\[
\begin{align*}
&\left( \pi_{0,t}^{\text{Agr}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Agr} AW_r} - (d_{\text{Agr}}^{\text{min}}) \right) - s_{i} - (c_{i}^{\text{Agr}} - (\pi_{0,t}^{\text{Agr}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Agr} AW_r})) \\
&= 0
\end{align*}
\]

\[
\begin{align*}
&\left( \pi_{0,t}^{\text{Eco}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Eco} AW_r} - (d_{\text{Eco}}^{\text{min}}) \right) - s_{i} - (c_{i}^{\text{Eco}} - (\pi_{0,t}^{\text{Eco}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Eco} AW_r})) \\
&= 0
\end{align*}
\]

\[
\begin{align*}
&\left( \pi_{0,t}^{\text{Ind}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Ind} AW_r} \right) + \left( \pi_{0,t}^{\text{Agr}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Agr} AW_r} \right) + \left( \pi_{0,t}^{\text{Eco}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Eco} AW_r} \right) \\
&= 0
\end{align*}
\]

\[
\begin{align*}
&\sum_{r \in I_t} \pi_{r,t}^{AW_r} \geq 0, \pi_{0,t}^{\text{Ind}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Ind} AW_r} \geq 0 \\
&\sum_{r \in I_t} \pi_{r,t}^{\text{Agr}} AW_r \geq 0, \pi_{0,t}^{\text{Agr}} + \sum_{r \in I_t} \pi_{r,t}^{\text{Agr} AW_r} \geq 0 \\
&\sum_{r \in I_t} \pi_{r,t}^{\text{Eco}} \geq 0, \sum_{r \in I_t} \pi_{r,t}^{\text{Eco} AW_r} \geq 0, s_{i}^{j} \geq 0, j = 1 \ldots 5, s_{i} \geq 0
\end{align*}
\]
Problem (21):

\[
\text{Min} \ G = \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} \left( \frac{\pi_{ij}}{T} + \frac{\pi_{ij}}{T} \right) + \left( \sum_{t=1}^{T} \gamma_{1t}^i \omega^* - \sum_{t=1}^{T} \gamma_{1t}^j \omega^* \right) + \theta \left( \sum_{t=1}^{T} \rho_{1t}^i \omega^* + \sum_{t=1}^{T} \rho_{1t}^j \omega^* \right)}{2I \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{\pi_{ij}}{T} + \sum_{t=1}^{T} \gamma_{1t}^i \omega^* + \theta \sum_{t=1}^{T} \rho_{1t}^i \omega^*}
\]

\[
\begin{aligned}
D_{\text{min}} &= \left( \pi_{1t}^0 + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* - \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) \leq 0 \\
\sum_{i=1}^{I} \pi_{1t}^0 + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* &\leq 0 \\
(A_{\text{ind}}^0 \text{ min}) &= \left( \pi_{1t}^0 \text{ind} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* - \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) \leq 0 \\
\left( \pi_{1t}^0 \text{ind} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) - c_{1t}^\text{ind} &\leq 0 \\
(A_{\text{AGR}}^0 \text{ min}) &= \left( \pi_{1t}^0 \text{AGR} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* - \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) \leq 0 \\
\left( \pi_{1t}^0 \text{AGR} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) - c_{1t}^\text{AGR} &\leq 0 \\
(A_{\text{Dom}}^0 \text{ min}) &= \left( \pi_{1t}^0 \text{Dom} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* - \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) \leq 0 \\
\left( \pi_{1t}^0 \text{Dom} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) - c_{1t}^\text{Dom} &\leq 0 \\
(A_{\text{Eco}}^0 \text{ min}) &= \left( \pi_{1t}^0 \text{Eco} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* - \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) \leq 0 \\
\left( \pi_{1t}^0 \text{Eco} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^* \right) - (d_{1t}^\text{Eco})_{\max} &\leq 0 \\
\end{aligned}
\]

s.t.

\[
\begin{array}{ll}
(21) & \pi_{1t}^0 + \pi_{1t}^0 \text{ind} + \sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \sum_{i=1}^{I} \rho_{1t}^i \omega^* = 0 \\
\end{array}
\]

\[
(21) & \frac{s_{ij}^+ + s_{ij}^- + s_{ij}^\text{ind} + s_{ij}^\text{AGR} + s_{ij}^\text{Dom} + s_{ij}^\text{Eco} + s_{ij}^+ - s_{ij}^-}{\sum_{i=1}^{I} \gamma_{1t}^i \omega^* + \theta \sum_{i=1}^{I} \rho_{1t}^i \omega^*} = 0
\]
\[ γ_1 = \sum_{t=1}^{T} \pi_{i,t} - ρ_1 ≤ γ_1 ≤ ρ_1 \]
\[ γ_2 = \sum_{t=1}^{T} \pi_{r,t} - 1 - ρ_2 ≤ γ_2 ≤ ρ_2 \]
\[ γ_3 = \sum_{t=1}^{T} \pi_{r,\text{Ind}} - ρ_3 ≤ γ_3 ≤ ρ_3 \]
\[ γ_4 = \sum_{t=1}^{T} \pi_{r,\text{Agr}} - ρ_4 ≤ γ_4 ≤ ρ_4 \]
\[ γ_5 = \sum_{t=1}^{T} \pi_{r,\text{Dom}} - ρ_5 ≤ γ_5 ≤ ρ_5 \]
\[ γ_6 = \sum_{t=1}^{T} \pi_{r,\text{Eco}} - ρ_6 ≤ γ_6 ≤ ρ_6 \]
\[ γ_7 = x^T \left( \sum_{t=1}^{T} \pi_{r,\text{Ind}} + \sum_{t=1}^{T} \pi_{r,\text{Agr}} + \sum_{t=1}^{T} \pi_{r,\text{Dom}} + \sum_{t=1}^{T} \pi_{r,\text{Eco}} \right), \]
\[ -ρ_7 ≤ γ_7 ≤ ρ_7 \] (22)

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