Informativeness Improvement of Hardness Test Methods for Metal Product Assessment

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Abstract. The paper presents a combination of theoretical suggestions, results, and observations allowing to improve the informativeness of hardness testing process in solving problems of metal product assessment while in operation. The hardness value of metal surface obtained by a single measurement is considered to be random. Various measures of location and scattering of the random variable were experimentally estimated for a number of test samples using the correlation analysis, and their close interaction was studied. It was stated that in metal assessment, the main informative characteristics of hardness testing process are its average value and mean-square deviation for measures of location and scattering, respectively.

1. Introduction

The assessment of metal products while in operation shows their degradation due to such negative factors [1–3] as wear abrasion, corrosions, and fatigue. One of the metal assessment methods used for products while in work and operation is their hardness testing [5–6]. For the last two decades, metal assessment methods have been intensively developed as well as instruments and scopes of their application [7–9]. The traditional use of durometers implies measurement of product at its several points, and the average value of obtained results should be used to assess the metal hardness. Ekici, Starikov, Muzyka, et al. [10–12] showed that the use of the additional informative parameter identified during the hardness testing process, namely, the hardness scattering, allows the improvement of the metal assessment method. Until recently, the implementation of this technique has been restricted by the instrument unavailability. However, the production of high-end portable durometers with a lower effect on test samples eliminated this restriction, and a parallel evaluation of the average value of hardness and its certain measure of scattering is now being used in transportation, construction and other industries [13–15]. The literature review concerning the use of other informative hardness parameters showed the insufficiency of the data.

2. Theory

Mechanical stresses, corrosions, and intensive tensile and compressive forces result in structural modifications of the surface layers of metals and alloys.
The following variants are the most visible manifestation of these changes: • surface roughness (the kind, direction, and level of mechanical stresses as well as their duration define the degree of the surface modification. It should be noted that surface roughness can be both increased and decreased); • small disoriented fatigue cracks; • explicit, oriented and rather deep cracks. Such cracks are usually induced by metal-to-metal contact of parts by repeated trajectories; • corrosion spots (oxides, salts of mother metal(s)). The specific density (relative fraction of corrosion spot area) and the size of spots depend on the duration the metal product exposure to such hazardous factors as humidity, acid and acid solution vapors, alkalis, and salt solutions; • oxide, salt and other films produced by corrosion.

The value of hardness $H$ obtained by a single measurement is naturally considered to be random. The random variable $x$ is completely defined in case its probability density function $F(x)$ is known [16]. The random variable collection of characters is divided into two large groups, namely: measure of location and measure of scattering. In the work of Korn et al. [16], measures of location of random variable $x$ include the average value $\bar{x}$, median $Me_x$ and mode $Mo_x$. Measures of scattering of random variable $x$ include dispersion $\sigma^2$, mean-square deviation $\overline{(x-\bar{x})^2}$, coefficient of variation $Vx$, coefficient of skewness $Ax$, and coefficient of excess $Ex$.

Note 1 In scientific literature, the distribution function of the random variable $H$ is described by different laws such as lognormal, and Weibull and Gumbel distributions. These laws are two-parameter distributions which comprise a measure of location and a measure of scattering.

All mentioned measures of location and scattering including those in Note 1 can be used for metal assessment when applied to the random variable $H$.

Note 2. The degree of metal surface uniformity of hardness, sometimes called the coefficient of uniformity $k$ [12–14], is connected with values of measure of location.

Note 3. The estimation of hardness value obtained by a single measurement depends on the contact area between the indenter and the material being tested.

From Note 3 it follows that the smaller the size of indenter the more evident is the scattering of hardness. Presently, the size of indenters and their penetration force can be varied. Depending on the size of indentation, hardness, micro-, and nano-hardness can be measured.

Let us put forward the basic statements which allow a substantiation of character of the random variable $H$ used in the capacity of metal product assessment provided by hardness test methods.

Statement 1. The change of metal surface properties under any impact conditions is caused by its structural modification.

Statement 2. For each of variants of visible manifestation of changes stated above, the surface layer modified during the product operation will possess at least one hardness parameter regarded as a random variable, dissimilar to the respective one of the original metal.

Statement 3. A sampling parameter of the random variable $H$ selected from the collection of those to be analyzed can be informatively excessive in case it is closely connected with one or more residual parameters.

Close connection between random variables $x$ and $y$ is evaluated using the correlation factor $r_{xy}$. Parameters $x$ and $y$ are considered to be closely connected in case the correlation factor $r_{xy}$ satisfies the condition of $r_{xy} \leq |r_{xy}| \leq 1$. The lower bound $r_l$ of this interval is determined experimentally. Since the authors present technical measurements, the value $r_l$ ranges between 0.85–0.95.

It is logically to select such a parameter which is easy to identify for further analysis or some arguments can be put forward in favor of its application in case it possesses an explicit physical interpretation.

Consequence of Statement 3 In metal assessment, the sampling $y$ parameter of the random variable $H$ is additional to the informative parameter $x$ in case it is independent of $x$ parameter.

Let us assume that connection between $x$ and $y$ parameters are negligible at the fulfillment of $0 \leq |r_{xy}| \leq r_1$ condition. The lower bound $r_1$ of this interval is also determined experimentally. The value $r_1$ ranges between 0.1–0.2.
3. Experimental
With a view to assess the informativeness of parameters describing the random variable $H$, a series of hardness tests was conducted for steel specimens exposed to different types of external actions. Specimens of different steel types were tested having average value of hardness ranging from 80 to 200 HB. Calibration measurements were conducted by Brinell hardness of 100 and 184 HB. Hardness measurements are characterized by the high-quality treatment of the surface. Therefore, the effect exerted by the surface roughness on the hardness properties assessment as a random variable, is minimized. Hardness tests were carried out with the dynamic indenter TEMP-4 in compliance with recommendations given by technical specifications and guidelines.

The original sampling was provided for each test specimen using Brinell hardness $(H_1, H_2, H_3, \ldots, H_n)$, where $n$ is the volume of sampling ($n \geq 200$).

In all, seven specimens were tested. Measures of location and scattering of the random variable $H$ were estimated for each specimen. All sampling parameters were subsequently considered as random variables. In accordance with the Consequence of Statement 3, the independence (level of dependence) of these values was verified by couples of parameters.

At the first stage of the experimental data processing the following parameters of the random variable were estimated: the average value $\overline{H}$; median $MeH$; and mode $MoH$. These parameters can be obtained from

$$\overline{H} = \frac{1}{n} \sum_{i=1}^{n} H_i, \quad \sum_1 = \sum_{H_i \geq MeH} 1, \quad f(MoH) = \max f(H). \quad (1)$$

Along with measures of location, measures of scattering were determined: mean-square deviation $\sigma_H$, coefficient of variation $V_H$, coefficient of skewness $A_H$, and coefficient of excess $E_H$. Scattering parameters [16] can be obtained form

$$\sigma_H = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (H_i - \overline{H})^2}, \quad V_H = \frac{\sigma_H}{\overline{H}}, \quad A_H = \frac{m_3}{m_2^{\frac{3}{2}}}, \quad E_H = \frac{m_4}{m_2^2} - 3. \quad (2)$$

where $m_k$ is the central $k$-order sampling moment of the value analyzed. It can be found from [16]:

$$m_k = \frac{1}{n} \sum_{i=1}^{n} (H_i - \overline{H})^k. \quad (3)$$

According to note 2, parameters of the assumed $H$ random distribution can act as measures of location and scattering. In the number of scientific papers devoted to the study of strength of materials, it is assumed that $H$ or $\ln H$ random variables are Weibull-distributed.

The random variable $x$ is Weibull-distributed [16] in case its distribution function is described by equation 4:

$$F(x) = 1 - e^{-\frac{(x)^\theta}{\lambda}}. \quad (4)$$

In addition to (1) and (2), the location of the random variable $H$ will be characterized by $\lambda_H$ and $\lambda_{\ln H}$ parameters, while its scattering by $k_H$ and $k_{\ln H}$ parameters.
Weibull distribution parameters $k$ and $\lambda$ can be obtained using the method of matching moments. Two initial moments $x$ and $x^2$ are to be obtained for the random variable $x$. Weibull distribution parameters $k$ and $\lambda$ are obtained from [17]:

$$
\frac{\bar{x^2}}{(\bar{x})^2} = \frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)}, \quad \lambda = \frac{\bar{x}}{\Gamma\left(1+\frac{1}{k}\right)}, \quad \Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt .
$$

(5)

In [11–13], the authors describe the method of estimation of the coefficient $k_{\text{lnH}}$:

$$
\ln\bar{H} = \frac{\sum_{i=1}^n \ln H_i}{n}, \quad \sigma_{\ln H} = \sqrt{\frac{\sum_{i=1}^n (\ln H_i - \ln\bar{H})^2}{n-1}} .
$$

(6)

Gumbel approximation for $k$ parameter is as follows:

$$
k = \frac{0.4343d}{\sigma_{\ln H}} .
$$

(7)

It is assumed that the larger values of $k_H$ and $k_{\text{lnH}}$ coefficients provide the low level of hardness scattering and the better structural organization and the lower degree of damage. Unlike the larger, the smaller values of $k_H$ and $k_{\text{lnH}}$ coefficients provide the highest degree of damage. It was shown in [11–13] that the level of scattering measures of metal property to be identified, including hardness, can correspond to another statistical criteria. In the present work, the authors verify the validity of this statement.

At the second stage, verification of the availability, direction, and closeness of correlation between parameters of the random variable $H$ was carried out using the correlation analysis. In case of sampling $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ for $x$ and $y$ random variables, the correlation factor $r_{xy}$ takes the form

$$
r_{xy} = \frac{\bar{xy} - \bar{x}\bar{y}}{\sigma_x \sigma_y} .
$$

(8)

The sign and the absolute value of a correlation factor describe the direction and the magnitude of the relationship between two variables.

The random variables $x$ and $y$ run over all possible location and scattering parameters, respectively. In equations 2–3, the number $n$ equals to the number of specimens. Seven specimens ($n = 7$) are sufficient merely for a preliminary study of the problem.

The estimation of the coefficient of uniformity $k$ based on equation 5, is rather complicated and requires a preliminary construction of the additional function of $k$. In equation 5 this function is expressed by the right hand side of the equation. Mathcad intended for the verification, validation, documentation of engineering calculations, can be an alternative for solving this problem.

4. Results and discussion

Table 1 gives the results of statistical processing of experimental data on Brinell hardness $H$ of specimens.

Table 1 contains the values of parameters of the random variable $H$ location, namely: $\text{Me}_H, \text{Mo}_H, \lambda_H$ and $\lambda_{\text{lnH}}$. Also there are given parameters of $\sigma_{\text{H}}, \mu_H, A, E, k_H, k_{\text{lnH}}$, which determine the hardness scattering. Parameters of location and scattering of $H$ were identified using equations 1–2 and 5–7.
inverse; the absolute values of the correlation factor range from 0.47 to 0.55.

Dependent on each other; correlation factors are close to zero; the dependencies are strong and direct.

As a rule, parameters of random variable $k_E$, $A_H$, $V_H$, $E_H$, $k_H$, and $k_H^{inl}$ are strictly absent; the absolute values of the correlation factor are less than 0.39.

In order to analyze informativeness of one or another parameter of the random variable $H$, and identify the excessive parameters, one can use the cor relation analysis. It should be noted that the sampling volume is insignificant. However, it may turn to be enough to determine informativeness of the suggested process of hardness testing. The correlation factor $r_{xy}$ is calculated from equation 8. Here random variables $x$ and $y$ run over all parameters of the random variable $H$, namely: $\bar{H}$, MeH, MoH, $\lambda_{H}$, $\lambda_{inl}$, $\sigma_{H}$, $V_H$, $A_H$, $E_H$, $k_H$, and $k_H^{inl}$. Correlation factor values for hardness parameters are given in table 2.

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| Measure | Parameter | Specimen number |
|---------|-----------|-----------------|
| Location | $\bar{H}$ | 100.8 | 184.2 | 119.2 | 126.0 | 126.4 | 177.8 | 192.3 |
|         | MeH | 101.2 | 184.3 | 121.7 | 124.3 | 128.9 | 177.6 | 191.4 |
|         | MoH | 97.1 | 184.3 | 121.7 | 122.7 | 134.0 | 183.2 | 187.3 |
|         | $\lambda_{H}$ | 111.7 | 204 | 132 | 139.6 | 139.9 | 197 | 213 |
|         | $\lambda_{inl}$ | 106.9 | 189.2 | 129.3 | 138.2 | 139.7 | 191.5 | 203.1 |
| Scattering | $\sigma_{H}$ | 7.13 | 5.19 | 12.00 | 15.93 | 19.07 | 15.90 | 12.06 |
|         | $V_H$ | 0.0708 | 0.0282 | 0.1006 | 0.1263 | 0.1509 | 0.0894 | 0.0627 |
|         | $A_H$ | 0.984 | 0.462 | $-0.603$ | 0.144 | 0.839 | $-0.288$ | 0.197 |
|         | $E_H$ | 3.90 | 1.78 | 0.47 | $-0.81$ | 2.09 | 1.37 | $-0.36$ |
|         | $k_H$ | 17.485 | 44.917 | 12.103 | 9.521 | 7.935 | 13.697 | 19.817 |
|         | $k_H^{inl}$ | 5.104 | 20.131 | 5.359 | 4.446 | 3.841 | 6.11 | 9.014 |

Table 1. A summary table of sampling parameters of $H$.

From table 2, the following intermediate conclusions can be drawn relative to mutual parameter dependencies which characterize the random variable $H$.

1. All parameters of the random variable location, namely $\bar{H}$, MeH, MoH, $\lambda_{H}$, $\lambda_{inl}$, are strictly dependent on each other; correlation factors are close to zero; the dependencies are strong and direct.
2. The mean-square deviation $\sigma_{H}$ does not depend on any location parameter of the random variable $H$; the absolute values of the correlation factor do not exceed 0.12.
3. The correlation factor $V_H$ weakly depends on all parameters of location. These dependencies are inverse; the absolute values of the correlation factor range from 0.47 to 0.55.
4. Dependence between skewness $A_H$ and excess $E_H$ coefficients and location parameters is practically absent; the absolute values of the correlation factor are less than 0.39.
5. Dependence between coefficients $k_H$ and $k_H^{inl}$ is direct and takes the interfacial position between medium and weak dependencies; the absolute values of the correlation factor range from 0.57 to 0.64.

| Parameters | $\bar{H}$ | MeH | MoH | $\lambda_{H}$ | $\lambda_{inl}$ | $\sigma_{H}$ | $V_H$ | $A_H$ | $E_H$ | $K_H$ | $K_{H^{inl}}$ |
|-----------|-----------|-----|-----|----------------|----------------|-------------|-------|-------|-------|-------|-------------|
| $\bar{H}$ | 1.00 | 0.99 | 1.00 | 1.00 | $-0.12$ | $-0.55$ | $-0.25$ | $-0.35$ | 0.55 | 0.64 |
| MeH | 1.00 | 0.99 | 1.00 | 1.00 | $-0.12$ | $-0.54$ | $-0.25$ | $-0.34$ | 0.55 | 0.64 |
| MoH | 0.99 | 0.99 | 0.99 | 0.99 | $-0.05$ | $-0.48$ | $-0.27$ | $-0.32$ | 0.51 | 0.61 |
| $\lambda_{H}$ | 1.00 | 1.00 | 0.99 | 1.00 | $-0.12$ | $-0.55$ | $-0.25$ | $-0.35$ | 0.55 | 0.64 |
| $\lambda_{inl}$ | 1.00 | 1.00 | 0.99 | 1.00 | $-0.03$ | $-0.47$ | $-0.28$ | $-0.39$ | 0.47 | 0.57 |
| $\sigma_{H}$ | $-0.12$ | $-0.12$ | $-0.05$ | $-0.12$ | $-0.03$ | 0.88 | $-0.20$ | $-0.38$ | $-0.80$ | $-0.69$ |
| $V_H$ | $-0.55$ | $-0.54$ | $-0.48$ | $-0.55$ | $-0.47$ | 0.88 | $-0.02$ | $-0.18$ | $-0.87$ | $-0.80$ |
| $A_H$ | $-0.25$ | $-0.23$ | $-0.27$ | $-0.25$ | $-0.28$ | $-0.20$ | 0.63 | 0.17 | 0.08 |
| $E_H$ | $-0.35$ | $-0.34$ | $-0.32$ | $-0.35$ | $-0.39$ | $-0.18$ | 0.63 | 0.19 | 0.05 |
| $k_H$ | 0.55 | 0.55 | 0.51 | 0.55 | 0.47 | $-0.80$ | $-0.87$ | 0.17 | 0.19 | 0.98 |
| $k_{H^{inl}}$ | 0.64 | 0.64 | 0.61 | 0.64 | 0.57 | $-0.69$ | $-0.80$ | 0.08 | 0.05 | 0.98 |

Table 2. Correlation factor values for hardness parameters.
6. The scattering parameters show the following dependencies: direct $\sigma_H - V_H$ dependence with 0.88 correlation factor; inverse $\sigma_H - k_H$ dependence with −0.8 correlation factor; $V_H$ dependence on both $k_H$ and $k_{lnH}$ with −0.8–0.87 correlation factor.

5. Implications
From the intermediate conclusions stated above it follows that:

1. All of $\overline{H}$, MeH, MoH, $\lambda_H$, $\lambda_{lnH}$ parameters can be used as a measure of location of the random variable $H$. However, it is more feasible to use $\overline{H}$ since it is easily identified. All other location parameters of the random variable can be used to exclude random errors in calculations.

2. Parameters $\sigma_H$, $A_H$, $E_H$ can be used as a measure of scattering. All other scattering parameters show a strong dependence on location parameters. It is more feasible to use a mean-square deviation as a parameter least dependent on any location parameter.

6. Conclusion
The experiments showed that in metal assessment, the main informative characteristics of hardness testing process are its average value and mean-square deviation for measure of location and measure of scattering, respectively. All other parameters can be used to improve the metal assessment methods. For this improvement it is expedient to develop multi-channel durometers, the appropriate algorithms of experimental data visualization and processing.

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