Vector Gravity Theory

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Abstract

The vector theory of gravity is constructed in the framework of the special theory of relativity. Calculated anomalous procession of the Mercury’s perihelion and deflection of light near the Sun coincide with the experimental values.

1 Introduction

In this paper an attempt to describe the gravitational phenomena using the vector field approximation is made. Some attempts to describe the gravity using vector models were made earlier [1], however a number of difficulties arise in this approach. The main problems are: 1) the absence of the deflection of light in the gravitational field; 2) an incorrect value of the anomalous procession of the Mercury’s perihelion; 3) the problem of the sign of gravitational energy [2]. In this work the author tries to solve the mentioned problems. In particular, the correct value of the anomalous procession of the Mercury’s perihelion is received using the vector gravitational field Lagrangian up to the second order terms. To describe the deflection of light the generalization of this Lagrangian on the case of ultrarelativistic velocities has been used; in a result, the received value for the deflection of light near the Sun coincides with the experimental one. The components giving negative contribution into the stress-energy tensor of the gravitational field are excluded by imposing of a specific relativistically invariant condition. Also in this paper the effective geometrization of the given theory is made.

2 General model

We will match gravitation field 4-potential $A^i = (\varphi, c\vec{A})$, where $\varphi$ - usual scalar potential, $\vec{A}$- vector potential, $c$ - speed of light. Lagrangian of gravitation field with provision for matters is of the form of:

$$L = -A_i \dddot{A} + \frac{1}{8\pi\gamma} \frac{\partial A_i}{\partial x^k} \frac{\partial A^i}{\partial x_k}$$  (1)
where \( \gamma \)- gravitation constant, \( j^i = \mu \frac{1}{c} \frac{dx^i}{dt} \) - 4-vector to density of current of masses , \( \mu \)- density of mass of bodies. First summand describes interaction of field and matters, second describes characteristics of field without particles. Type second composed given that it recorded disregarding Lorence condition, also follows to note sign a plus before composed in change from electrodynamic.

Instead of scalar Lorence condition, which with transverse condition excludes negative contribution to tensor of energy-pulse zero scalar components, as well as contribution third components in the electrodinamic, in the event of gravitation necessary to use other condition, excluding negative contribution to tensor of energy-pulse vector component. The relativistically invariant condition for the vector field can be written in a 4-form:

\[
\left( \frac{\partial A^i}{\partial t} \frac{\partial A_i}{\partial t} - \frac{\partial A^i}{\partial x^i} \frac{\partial A_i}{\partial x_i} \right) \delta^i = 0
\]

where \( \delta^i \) is a unit vector. For \( i = 0 \) there is no condition on the scalar component. For \( i = 1, 2, 3 \) the condition has the following form:

\[
\left( \frac{d\vec{A}}{dt} \right)^2 - (\text{div}\vec{A})^2 = 0
\]

This condition can be introduced into the theory by using the method of the Lagrange’s multipliers.

From action (1) possible get the system of equations of gravitation field:

\[
\frac{\partial^2 A}{\partial x_k \partial x^k} = 4\pi \gamma j^i
\]

To get equations of gravitation field to similar Mackswell equations we will choose Lagrangian in the following type:

\[
L = -A_i j^i + \frac{1}{16\pi \gamma} F_{ik} F^{ik} + \frac{1}{8\pi \gamma} \left( \frac{\partial A^k}{\partial x^k} \right)^2
\]

which differs from (1) an unessential divergenton. Where \( F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \) anti-symmetric tensor of the gravitation field. As a result we will get the equations of gravitation field:

\[
\frac{\partial F^{ik}}{\partial x^k} + \frac{\partial^2 A^k}{\partial x^k \partial x^i} = 4\pi \gamma j^i
\]

In stationary event write the equation (6) in three-dimensional type for \( i = 0 \):

\[
\triangle \varphi = 4\pi \gamma \mu
\]

The decision (7) is of the form of:

\[
\varphi = -\gamma \int \frac{\mu}{r} dV
\]
Potential for one particles of mass $m$ $\varphi = -\frac{2m}{r}$. Consequently power acting in given field to other particle of mass $m'$

$$F = -\frac{\gamma mm'}{r^2}$$

(9) - there is known Newton’s law of gravity

Because of the stationarity from (3) we have $\text{div}\vec{A} = 0$. Then the equation for the vector potential can be written as

$$\nabla \vec{A} = 4\pi \gamma \vec{j}.$$  

(10)

From here we receive

$$\vec{A} = -\gamma \int \frac{\vec{j}}{r} dV.$$  

(11)

This field can be called cyclic. The induction of the field is

$$\vec{C} = \text{rot} \vec{A} = -\gamma \int \frac{[\vec{j} \vec{r}]}{r^3} dV = -\gamma \frac{[\vec{p} \vec{r}]}{r^3}.$$  

(12)

where $\vec{p}$ is a momentum of a particle.

Using the mechanical momentum $\vec{M}$ we can write $\vec{A}$ and $\vec{C}$ as

$$\vec{A} = -\gamma \frac{\vec{M} \vec{r}}{r^3}, \quad \vec{C} = -\gamma \frac{3\vec{n}(\vec{M} \vec{n}) - \vec{M}}{r^3},$$  

(13)

where $\vec{n}$ is a unit vector in the $\vec{r}$-direction.

Thus two moving particles experience (besides the gravitational attraction one to another) the cyclic force. The latter can be attractive or repulsive, depending on the mutual direction of the velocities of the particles.

Notice the smallness of the effects conditioned by the vector potential; their values are almost inaccessible for the experimental registration.

### 3 Gravitation experiments

We will calculate importance of adjustment for planet perihelion and corner of deflection of light in field of power to gravity within the framework of offereded approach. We will find the type of Lagrange function for particles with provision for gravitation field in the second approach. The Lagrange function for body, residing in external gravitation field is of the form of:

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - m\varphi + m\vec{v} \vec{A}$$  

(14)
where $\varphi$ - scalar potential of gravitation field $\vec{A}$ vector potential the fields, which in analogy with magnetic possible to name cyclic to example. Source from expressions for lagging potentials, following from decision of equations (14)

$$\varphi = -\int \frac{\rho - R/c}{R} dV, \quad \vec{A} = -\int \frac{\vec{j} - R/c}{R} dV,$$

(15)
degrading scalar potential in row before members of second order, but for vector limiting first-order members [3] will find type of Lagrange function in second approach. We will consider given function for system two particles excluded from it moving the system as whole:

$$L = T + \frac{\gamma m_1 m_2}{r} + \frac{\gamma m_1 m_2 v^2}{c^2 r}$$

(16)
The Energy of system possible to write:

$$E = E_0 - \frac{\gamma M m}{r} - \frac{\gamma M J^2}{m c^2 r^3} = E_0 - V$$

(17)
where a speed $\vec{v} = r \frac{d\psi}{dt}$ denominated in moment of pulse $J = mr^2 \frac{d\psi}{dt}$, $\psi$- angle; $M = m_1, m = m_2$.

The calculations of adjustment a perihelion and angle of deflection of light in gravitation field comfortable to conduct the Runge - Lenz vector with use. For the first time Runge - Lenz vector for calculation the adjustments of the General theory of relativity there was aplying in work [4].

$$\vec{X} = \vec{v} \times \vec{J} - \gamma M m \vec{e}_r$$

(18)
where $\vec{e}_r$ - a unit vector in the $r$ - direction. The derivative on time from Runge - Lenz vector:

$$\frac{d\vec{X}}{dt} = (r^2 \frac{\partial V}{\partial r} - \gamma M m) \frac{d\vec{e}_r}{dt} = \left(\frac{3\gamma M J^2}{mr^2 c^2 X^2}\right) \frac{d\psi}{dt} \vec{e}_\psi$$

(19)
The direction of $\vec{X}$ changes with angular speed:

$$\vec{\omega} = \frac{\vec{X} \times \vec{X}}{X^2} = \left(\frac{3\gamma M J^2}{mr^2 c^2 X^2}\right) \frac{d\psi}{dt} \vec{X} \times \vec{e}_\psi$$

(20)
And its total change when the particle moves within from $\psi_1$ to $\psi_2$ (is expected that this change little and vector $\vec{X}$ is originally oriented toward $\psi = 0$):

$$\Delta \alpha = \int_{\psi_1}^{\psi_2} \omega dt = \frac{3\gamma M J^2}{mc^2} \int_{\psi_1}^{\psi_2} \frac{\cos \psi d\psi}{X r^2}$$

(21)
When $\vec{X}$ is constant and is oriented toward $\psi = 0$ we have

$$\vec{X} \vec{r} = X r \cos \psi = J^2 - \gamma M m r$$

(22)
From nonperturbation orbits (22) express $r$ and substitute in (21). For bound orbits ($m \neq 0$) with eccentricity $e = A/M$, semi-major axis $a = J^2/\gamma Mm^2(1 - e^2)$ find the perihelion precession:

$$\Delta \alpha = \frac{3\gamma Mm}{c^2J^2} \int_0^{2\pi} \frac{(X \cos \psi + \gamma Mm)^2}{X} \cos \psi d\psi =$$

$$= \frac{6\pi\gamma^2m^2M^2}{c^2J^2} = \frac{6\pi\gamma M}{c^2a(1 - e^2)}$$

(23)

The value of offset a perihelion for Merkury is $\Delta \alpha = 43''$.

To calculate the deflection of light in the gravitational field the Lagrangian (16) should be written without taking into account the assumption of small velocities, in order it can be used in the ultrarelativistic case. Similar Lagrangian up to the second order terms was found in [5] for ultrarelativistic particles in the electromagnetic field. For the gravitational field this Lagrangian can be written as

$$L = \frac{m_1m_2}{r_{21}} \left[ \frac{f(\eta_1^2) + f(\eta_2^2)}{2} \beta_1\beta_2 + \frac{h(\eta_1^2) + h(\eta_2^2)}{2} (\beta_1r_{21})(\beta_2r_{21}) + 1 \right]$$

(24)

where $\eta_a^2 = (r_{ab} \times \beta_a)^2$, $\beta = v_a/c$.

The functions $f$ and $g$ are defined as [5]

$$f(x) = \frac{1}{1 + \sqrt{1 - x}} \simeq \frac{1}{2} + \frac{1}{8} x + ...$$

(25)

$$h(x) = \frac{f(x)}{\sqrt{1 - x}} \simeq \frac{1}{2} + \frac{3}{8} x + ...$$

(26)

The expression for the energy of particles moving with the speed of light (photons) takes the form (16), but without taking into account the Newtonian interaction. The last term in (16) can be written as $\frac{2MJ^2}{\varepsilon r^3}$, where $\varepsilon$ is a photon frequency. Therefore for unbound orbit with mass of photon $m = 0$ we have:

$$\Delta \alpha = \frac{3\gamma M\varepsilon}{c^4J^2} \int_{-\pi/2}^{\pi/2} X \cos^3 \psi d\psi = \frac{4\gamma M}{c^2b}$$

(27)

where $b = \frac{\varepsilon J^2}{Ac^2}$ - aiming parameter. Therefore for the ray, going by edge of Sun $\Delta \alpha = 1,75''$. The given results for offset a perihelion and angle of deflection of light, got within the framework of vector theory gravitation field comply with similar results, got in the General theory of relativity [2, 3] and are confirmed experimental [1].

Also possible note that in givenened theory, either as in JTR exists the so-called effect of red offset, appearing when removing the photons from massive objects.
4 Effective geometrization

To describe all the basic gravitational effects it is enough to geometrize the Lagrangian (16). Rewrite this Lagrangian in the following form:

\[
L = -mc^2 (1 - v^2/c^2)^{1/2} - m\varphi - m\varphi v^2/c^2
\]  

(28)

In the gravitational theory the Lagrangian, from which the geodesic equations are received, is written as

\[
L = -mc^2 \left( -g_{ik} \frac{dx^i}{dt} \frac{dx^k}{dt} \right)^{1/2}
\]  

(29)

Write the metric tensor \(g_{ik}\) in a form \(g_{ik} = g_{ik}^0 + h_{ik}\), where \(g_{ik}^0\) is a metric of the Minkowsky spacetime, \(h_{ik}\) is corrections describing the gravitational field. Then the Lagrangian takes a form

\[
L = -mc^2 (1 - v^2/c^2 - h_{00} - 2h_{0j}v^j - h_{jk}v^jv^k)^{1/2}
\]  

(30)

where \(j, k = 1, 2, 3\). By expanding the expression under the square root and comparing (28) and (30) the metric \(g_{ik}\) can be found up to the second order terms, which corresponds to the postnewtonian approximation:

\[
g_{00} = -1 - 2\varphi \\
g_{0\alpha} = 1 - 2\varphi \\
g_{00} = 0.
\]

The anomalous procession of the Mercury’s perihelion and the deflection of light can be found by solving the Hamilton - Jacoby equation

\[
g^{ik} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^k} - m^2c^2 = 0.
\]

In a result the received values coincide with the experimental ones [1].

5 Conclusion

In this article the gravitational theory has been formulated in the framework of the special theory of relativity. In analogy with the electromagnetic interaction the gravitational interaction is described by the vector 4-potential. The new field, conjugated to the gravitational one and reminding of the magnetic field by description, appears in this theory. The field is generated by the momenta, the angular momenta and the spins of particles. Calculated in the article anomalous procession of the Mercury’s perihelion and deflection of light near the Sun coincide with the experimental values.

This article does not concern the cosmological and cosmogonical models that appear in the framework of the theory of the vector gravitational field. This question is a subject of the further studies.
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