Generation of travelling sine-Gordon breathers in noisy long Josephson junctions

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The generation of travelling sine-Gordon breathers is achieved through the nonlinear supratransmission effect in a magnetically driven long Josephson junction, in the presence of losses, a current bias, and a thermal noise source. We demonstrate how to exclusively induce breather modes by means of controlled magnetic pulses. A nonmonotonic behavior of the breather-only generation probability is observed as a function of the noise intensity. An experimental protocol providing evidence of the Josephson breather’s existence is proposed.

Introduction.—The sine-Gordon (SG) equation is one of the most celebrated nonlinear wave models, with a domain of application that includes pendula, Josephson junctions (JJs), gravity, high-energy physics, biophysics, and seismology [1–4]. Interestingly, solitonic excitations are sustained in a SG medium, kinks and breathers being the basic building blocks. Kinks, i.e., topological solitons, stem from an inherent degeneracy of the system’s ground state, while breathers are space-localized, time-periodic modes that can result from the attractive interaction between kinks and antikinks [5, 6]. Indeed, the SG equation provides an ideal environment for the exploration of soliton dynamics, with subsequent striking experimental realizations, thus it continues to attract great research interest in many scientific areas.

The electrodynamics of a long Josephson junction (LJJ) can be accurately described in the SG framework. Here, a kink (or fluxon) represents a quantum of magnetic flux \( \Phi_0 \). Since it can be stored, steered, manipulated, and made to interact with electronic instrumentation, such a nonlinear wave is featured in many applications [7–12]. Notably, the quantum dynamics of a single fluxon has been experimentally demonstrated [13].

Breathers are far more elusive than kinks, mainly because they radiatively decay due to dissipation, unless particular forms of driving are employed, and because their oscillatory nature gives rise to a practically null average voltage, i.e., beyond sensitivity of the existing high-frequency oscilloscopes [14]. As a consequence, despite the large variety of numerical and theoretical studies devoted to them [15–17], experimental evidence of breather modes has yet to be found in LJJs.

The interest towards breathers is also motivated by their significant applicative potential. In fact, such a mode could be effectively used to develop some novel applications in information transmission [18]. Furthermore, in contrast to kinks, they possess an internal degree of freedom, i.e., a proper frequency, which is particularly valuable for quantum computation purposes. More specifically, breathers behave as macroscopic artificial two-level atoms in an LJJ with a small capacitance per unit length, so that the realization of a Josephson breather qubit has been proposed [19, 20]. To these ends, detailed investigations concerning possible generation and control mechanisms are crucial.

The excitation and detection of breather-like objects has been deeply studied in the context of discrete systems as well. In particular, in JJ parallel arrays, which are modelled by the discrete SG equation (also known as the Frenkel–Kontorova model), the existence of oscillobreathers has been predicted, but due to their rapid pulsations, an experimental confirmation is still missing [1, 21]. Instead, the so-called rotobreather states can be tracked by measuring the local dc voltages throughout the JJ array, and they have been successfully observed in JJ ladders [22, 23].

This letter proposes the use of magnetic pulses for the generation of travelling breather modes into an LJJ by means of the nonlinear supratransmission (ST) effect. According to the latter phenomenon, a nonlinear system subjected to a sinusoidal driving with frequency laying in the forbidden band gap (FBG) can support energy transmission in the form of solitonic excitations, if the forcing amplitude is strong enough. Initially discussed in a discrete SG chain [24, 25], the ST mechanism appears to be the result of a generic nonlinear instability [26], and the corresponding activation threshold has been found in various situations [27–30].

Here, by looking at the average voltage drop across the junction, the external pulse’s frequency/amplitude space is thoroughly analyzed to exploit the ST process as a controllable source of breathers in LJJs. Interestingly, in the presence of dissipation, a current bias, and a thermal noise source, vast regions associated with the exclusive emergence of breather excitations are observed. In particular, the study reveals a sort of fluctuation-induced widening of these areas, which is seen to occur in correspondence with a nonmonotonic behavior of the breather-only generation probability, defined below, as a function of the noise amplitude. The latter result highlights the effectiveness of noise as a control parameter in the LJJ device.
Based on these findings, an experimental procedure providing evidence of the existence of breathers in LJJ is outlined. Moreover, given the degree of universality of the ST process, and its robustness against discreteness and finiteness, the application of the proposed approach in a discrete domain naturally comes to mind. Among the many contexts in which energy localization and transmission is being actively investigated, one could look, e.g., at oscillobreathers in a JJ parallel array.

The model.—An overlap-geometry Josephson tunnel junction is considered, assuming that the dynamics of the phase difference between the pair wave functions of the two superconductors $\varphi(x,t)$ follows the equation

$$\varphi_{xx} - \varphi_{tt} - \alpha \varphi_t = \sin \varphi - \gamma - \gamma_T(x,t).$$

(1)

The $x=0$ end of the junction is subjected to an oscillating magnetic field, perpendicular to the length of the junction and parallel to the plane of the barrier, thus the following boundary conditions apply

$$\varphi_x(0,t) = A(t) \sin(\Omega t), \quad \varphi_x(l,t) = 0.$$  

(2)

A subscript notation is used to denote partial differentiation, space is normalized to the Josephson penetration depth, $\lambda_J = \sqrt{\Phi_0/(2\pi J_c L_P)}$, and time is normalized to the inverse of the Josephson plasma frequency, $\omega_p = \sqrt{2\pi J_c/(\Phi_0 C)}$, where $J_c$ is the critical value of the Josephson current density, $L_P$ is the inductance per unit length, and $C$ is the capacitance per unit length. In Eq. (1), $\alpha = G/(\omega_p C)$ is a damping parameter, $G$ is an effective normal conductance, $\gamma = J_b/J_c$ is the normalized bias current, and $\gamma_T(x,t)$ is a Gaussian, zero-average noise source with the autocorrelation function given by

$$\langle \gamma_T(x_1,t_1)\gamma_T(x_2,t_2) \rangle = 2\alpha \Gamma \delta(x_1 - x_2) \delta(t_1 - t_2).$$

(3)

Here, $\Gamma = 2e k_B T/(h J_c \lambda_J)$ is the noise amplitude, which is proportional to absolute temperature $T$ ($e$ is the electron charge, $k_B$ is the Boltzmann constant, and $h$ is the reduced Planck constant). Finally, in Eq. (2), $A(t)$ and $\Omega$ are two dimensionless quantities related, respectively, to the external field’s amplitude and frequency, and $l = L/\lambda_J$ is the normalized length of the junction.

Equation (1) is numerically solved with the initial conditions

$$\varphi(x,0) = \arcsin \gamma, \quad \varphi_t(x,0) = 0;$$

(4)

also, in order to access the ST phenomenon, the external field has to oscillate at a frequency falling into the junction’s FBG, i.e., $\Omega < 1$, corresponding to $\omega < \omega_p \sim 100 \text{ GHz} - 1 \text{ THz}$ [14, 16]. Moreover, to reproduce a meaningful experimental pulse, Gaussian switching-on/off regimes are chosen as

$$A(t) = \begin{cases} 
A \exp \left[ -\frac{(t-t_{on})^2}{2\sigma_{on}^2} \right] & t < t_{on} \\
A & t_{on} \leq t < t_{off} \\
A \exp \left[ -\frac{(t-t_{off})^2}{2\sigma_{off}^2} \right] & t \geq t_{off}.
\end{cases}$$

(5)

In Eq. (5), the Gaussian distribution with standard deviation $\sigma_{on}$ provides a smoothly increasing signal envelope for $t < t_{on}$ (in practice, $t_{on} = 3\sigma_{on}$ is set). Then, the boundary of the junction is sinusoidally driven until an induced travelling excitation reaches a selected position [30], i.e., for $t < t_{off}$. If this event occurs, the driving amplitude is gradually decreased, with the typical scale $\sigma_{off}$. By doing so, one is mostly able to achieve the controlled generation of single breather modes in the system.

In what follows, the length of the junction is $l = 100$, which generally allows to ignore reflection effects at $x = l$, while keeping reasonable execution times. Also, the damping coefficient is $\alpha = 0.02$ [35], the bias current is $\gamma = 0$, and the width of the increasing (decreasing) Gaussian tail is $\sigma_{on} = 10$ ($\sigma_{off} = 2.5$). A typical simulation outcome is illustrated in Fig. 1.

**Breather detection.**—As stated above, one of the main aims of the present work is to identify the regions of the $(\Omega, A)$ parameter space in which the ST generation mechanism leads to breather excitations only. To this end, the meaning of the expression “breather-only” used throughout the letter shall now be operatively defined. Essentially, such a term includes all the cases in which breathers are the only solitons left in the junction from some point onwards, after a possible transitory phase that could involve kinks and antikinks. Conversely, the following situations are not considered to belong to the breather-only class: (i) at least one bound kink-antikink couple gets separated by the end of the simulation; (ii) at least one unpaired kink (or antikink) is produced; (iii) the breather state is not observed at all.

Different techniques can be constructed for the classification of the outcomes of the simulations. For instance, one can take advantage of the practically null average voltage drop across the junction produced by breathers. By virtue of the second Josephson relation [34], it is possible to define the (normalized) time-averaged voltage

$$\langle V \rangle = \frac{1}{t_{max}} \int_0^{t_{max}} \varphi_t dx.$$

(6)
Eventually, for $\Gamma \sim 1$, different kinds of excitations (in-
including breathers, kinks, and antikinks) begin to appear in the junction, in addition to the ST-induced ones, due to the magnitude of the stochastic perturbation, and the breather-only generation probability falls to zero in the \((\Omega, A)\) parameter space in a uniform way.

To further illustrate its behavior, the quantity \(P_{\text{breather-only}}(\Gamma)\) is calculated for \(N = 10^4\) realizations and the two sample \((\Omega, A)\) combinations circled in Fig. 3(a). In the first case \((\Omega = 0.55\) and \(A = 2.19\)), which falls outside of the breather-only region identified for \(\Gamma = 0\), a nonmonotonic profile is obtained; as shown in Fig. 3(a), the probability is practically zero when \(\Gamma \to 0\) and the dynamics is closer to the deterministic case, it reaches a peak for an optimal value of \(\Gamma\), and eventually goes back to zero when the stochastic influence becomes disruptive, i.e., when the fluctuations are sufficiently strong to both easily break the oscillatory bound state and produce additional excitations into the junction. The second combination \((\Omega = 0.52\) and \(A = 2.45)\), taken as a representative case for the high-probability core, leads to a decreasing step-like function, see Fig. 4(b). This determines the temperature values at which breather modes can be safely excited without noise disturbances.

In view of possible applications, the robustness of the induced breathers against the stochastic background is also quite relevant. Although not explicitly shown here, a detailed analysis indicates that there exists a wide range of noise intensities where their average persistence time above the level of fluctuations remains close to the quantity \(1/\alpha\). The latter is the perturbative prediction on the breather’s radiative decay lifetime for \(\Gamma = 0\) \cite{15,16}, which is seen to hold independently of the position in the \((\Omega, A)\) plane.

Now, based on these results, a test giving experimental evidence of breather modes in LJJJs is outlined. First, scanning a portion of the \((\Omega, A)\) space in the presence of a sufficiently high current bias allows for the characterization of the ST process, if suitable average voltage measurements can be performed. In fact, the power balance between dissipation and input from the current bias term leads to well-defined voltage patterns whenever fluxons emerge (even from a broken up breather state), while very low average voltages are expected for the \((\Omega, A)\) couples that cannot transmit energy in the medium. Once established that nonlinear (fluxon-based) excitations are indeed being induced above a certain \((\Omega, A)\) threshold, such a region can be inspected for breather-only \((\Omega, A)\) pairs, whose existence and robustness is demonstrated earlier in the letter. Specifically, in the unbiased junction a potential breather-only \((\Omega, A)\) combination produces a very low average voltage. If the latter case is encountered, a sufficiently high current bias \((\gamma \gtrsim 0.1, \text{see, e.g., Ref.} \ [14])\) should be switched on after a time interval much greater than \(1/\alpha\) from the magnetic pulse’s application. If the low average voltage persists, breathers only are quite likely being generated, and due to their radiative decay, no current-driven dissociation into the kink-antikink state can take place. Conversely, if fluxon-related voltage patterns arise after the current switch, indicating the presence of at least one kink (or antikink), another \((\Omega, A)\) couple can be tried. To summarize, in the case of a persisting low average voltage, one is observing high-frequency excitations which vanish within a time interval \(\sim 1/\alpha\). Linear (plasma) waves can be reasonably excluded since the junction’s FBG is being considered, and besides the preliminary \((\Omega, A)\) sweep confirms that fluxon-based modes are being excited. The existence of Josephson breathers would be supported by this observation.

Conclusions.—The present work shows that magnetic pulses can be a controllable source of travelling breather modes in an LJJ. More precisely, by looking at the average voltage drop across the junction, refined bifurcation diagrams are produced in the driving pulse’s \((\Omega, A)\) space, in the presence of dissipation, a current bias, and a thermal noise source. This allows for the characterization of significant regions where the exclusive formation of breathers occurs. Moreover, these areas are seen to maintain their identity in the noisy case, and a sort of noise-induced widening is found as well. The latter fact happens in correspondence with a nonmonotonic behavior of the breather-only generation probability, defined above, as a function of the noise amplitude.

Studies that focus on generation and control techniques are vital for all breather-related applications, e.g., in information transmission \cite{15} and quantum computation \cite{19,20}. Furthermore, given that the detection of Josephson breathers is a long-standing problem in the realm of mesoscopic soliton physics, an experimental strategy to confirm their existence is proposed.

Finally, the degree of universality of the ST process, together with its robustness against discreteness and finiteness, suggests applying this general approach to the discrete world. For example, interesting developments can be expected for oscillobreathers in JJ parallel arrays, but...
energy localization and transmission is being actively investigated in many other contexts as of today [31–33].