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The \( g \) factor of highly charged ions

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Abstract. Recent years have witnessed a remarkable improvement in the theoretical description of bound-electron \( g \) factors, paralleled with a quantum jump in the experimental accuracy in the investigation of these quantities. In the present article we give a brief summary of the latest developments, with emphasis on the influence of quantum electrodynamic and nuclear effects on the \( g \) factor of few-electron highly charged ions, and on the possible determination of fundamental constants.

1. Introduction

Penning-trap \( g \)-factor experiments employing the continuous Stern-Gerlach effect enable a very precise measurement of the microwave Zeeman splitting in one- or few-electron ions. Therefore, the gyromagnetic – or \( g \) – factor of highly charged ions (HCI) provides an exciting possibility for testing fundamental theories. Effects of strong-field quantum electrodynamics (QED) are increasingly relevant at higher and higher ionic charges and at improved experimental precision. Due to the large overlap of the electron density with the nuclear matter, nuclear structural and motional effects are also of great importance in such systems. Furthermore, the high accuracy which can be achieved on the experimental as well as theoretical side enables the extraction of fundamental constants such as the electron mass and the fine-structure constant \( \alpha \).

2. Quantum electrodynamic corrections to the bound-electron \( g \) factor

At the beginning of the millennium, the magnetic sector of QED has been explored through \( g \) factor measurements with light one-electron ions \([1, 2]\) on the \( \text{one-loop} \) level. At low nuclear charges, an expansion in terms of the electron-nucleus coupling strength parameter \( Z\alpha \) (where \( Z \) denotes the nuclear charge number) typically yields satisfactory results for the self-energy and vacuum polarization radiative correction. However, in case of the \( g \) factor, the QED corrections, dominated by the Schwinger vertex correction term \( \frac{\alpha}{\pi} \) \([3]\) are very large on the scale of the experimental accuracy. Therefore, the first few terms of the \( Z\alpha \) expansion (see e.g. \([4, 5, 6]\) may not be sufficient, and an all-order treatment in this parameter \([7, 8]\) has to be applied even for the lightest elements. Recently, the evaluation of the one-loop radiative corrections has been completed \([9, 10]\) at order \( (Z\alpha)^5 \), and the numerical uncertainty of nonperturbative-in-\( Z\alpha \) calculations has been largely decreased in the low-\( Z \) range \([11]\).

A further improvement of the experimental accuracy and the increase of the accessible ionic nuclear charges yielded an even more stringent test of QED: the \( g \) factor of hydrogen-like silicon \((Z=14)\) with a \( 5 \cdot 10^{-10} \) fractional uncertainty has been determined \([12]\), allowing to scrutinize...
two-loop QED corrections up to order \((Z\alpha)^4\) [13, 14] and certain higher-order contributions of the one-loop terms, such as, e.g., the magnetic-loop vacuum polarization correction [15, 16]. The experimental accuracy was further enhanced soon by approximately an order of magnitude by using phase-sensitive measurement techniques [12, 17]. The theoretical description has been extended to include virtual light-by-light scattering effects [18] of order \(\left(\frac{\alpha}{\pi}\right)^2(Z\alpha)^4\), which turned out to be significant even for light elements. Very recently, the gargantuan task of evaluating all two-loop terms of order \((Z\alpha)^5\) has been finalized [10] (see also [9]), increasing the theoretical accuracy especially in the low-\(Z\) regime. Calculations of two-loop corrections for non-perturbative Coulomb fields have also been started [19, 20]. Here, the most challenging part is the evaluation of the two-loop self-energy diagrams, which contain intricate ultraviolet and infrared divergences [20].

3. Many-electron ions

While experiments with hydrogenic ions allow detailed tests of strong-field QED due to their comparably simple structure, in many-electron ions, electron correlations can also be studied, together with many-electron QED effects. The first measurement of the \(g\) factor of a many-electron HCl was done in an electron beam ion trap experiment [21]. The Zeeman splitting of the optical \(^2P_{3/2} \rightarrow ^2P_{1/2}\) ground-state fine-structure transition was resolved by emission spectroscopy, yielding \(g\) factor values largely limited by the inaccurate knowledge of the magnetic field strength. This was followed by a Penning trap measurement of the \(g\) factor of Li-like Si [22] with \(\delta g/g = 1.1 \times 10^{-9}\), confirming the need for a rigorous QED treatment [23] of electron-electron interaction at the level of two-photon exchange diagrams. In such ions, also QED screening effects have to be accounted for, which are described by Feynman diagrams in which both radiative loops as well as a virtual photon exchanged between two electrons are present [24, 25]. Later, the experiments have been extended up to Li-like Ca [26].

For light few-electron ions, significant advancement is anticipated by the use of the methods of Nonrelativistic Quantum Electrodynamics (NRQED). NRQED allows the construction of highly accurate variational many-electron wave functions (see e.g. [27, 28, 29]). Calculations have been started at the leading order in \(Z\alpha\) [30]. It has been shown that NRQED results, combined with terms from the relativistic \(\frac{1}{2}\) expansion, can indeed significantly improve the accuracy of the \(g\) factor of light Li-like ions.

4. Nuclear effects

As with the increasing strength of the binding Coulomb field the electronic wave function has a larger and larger overlap with the nucleus, accurate \(g\) factor measurements with such systems also deliver new insights into ground-state nuclear structural properties. These nuclear contributions are not only interesting on their own, but they are necessary to complete the overall theoretical understanding for a broad range of charge numbers.

The most dominant nuclear structural effect, namely, the finite size effect is rather well understood [31, 32]. It can be calculated by means of perturbation theory, yielding easily evaluable analytical formulas [31], or numerically in an all-order fashion by including the potential corresponding to the extended nucleus into the radial Dirac equation [32]. Formulas for higher-order corrections to the nuclear size contribution, such as the anomalous-magnetic-moment correction and the third Zeemach momentum of the proton charge distribution, have been derived very recently in Ref. [33]. The comparison of theoretical and experimental \(g\) factors may provide one with a new tool of determining nuclear radii: the accuracy of the experiment with Si\(^{13+}\) even allowed one to extract the value of the nuclear radius [12]. While the determined value is less accurate than the presently established one [34, 35], this proof-of-the-principle study encourages one to further improve the accuracy and to extend the investigations to higher charges and to different isotopes of the same element.
In recent studies, also the nuclear shape (or deformation) effect, i.e. the deviation of the nuclear shape from a perfectly spherical one, has been taken into account [36, 37]. While part of the nuclear deformation effect can be incorporated into the spherically averaged root-mean-square radius and thus it can be understood as a constituent of the dominant finite size effect, some terms explicitly depending on the nuclear deformation parameters prevail and show a relative contribution to the $g$ factor of the order of $10^{-6}$ for elements as heavy as uranium. Their contribution will be relevant in the interpretation of near-future experiments [38, 39, 40, 41].

Of the similar magnitude are typically nuclear polarization contributions, arising from virtual electromagnetic excitations of the nucleus due to the bound electron. These effects can be described to the lowest order by the exchange of two virtual photons between the electron and the fluctuating nuclear four current. While such virtual processes are largely off the energy shell due to the different energy scale of nuclear and electronic levels, they still may contribute noticeably to the $g$ factor in heavy elements [42]. Because the nuclear parameters entering the dynamic electromagnetic polarizability are not sufficiently well known, these terms, usually associated with large theoretical uncertainties, set a natural limit to the accuracy of testing bound-state QED effects. Recently, a calculation of the nuclear polarization effect extended for Li- and B-like heavy ions showed that these terms can be largely suppressed in a weighted difference of the $g$ factors for two different charge states of the same element [43].

All $g$ factor studies mentioned so far have been restricted to elements with a vanishing nuclear spin. In ions with a non-vanishing nuclear spin $I$, the electronic angular momentum $j$ couples to a total quantum number $F$. The associated $g_F$ factor may be easily constructed in a first approximation from the electronic $g_J$ factor and the nuclear $g_I$ factor. However, above certain accuracy, mixed Zeeman-hyperfine terms need to be taken into account as well. These nuclear magnetic shielding terms in hydrogenic ions have been calculated in a relativistic manner in Ref. [44]. A more accurate theory of the nuclear magnetic shielding was developed taking into account the self-energy and vacuum polarization corrections to the shielding parameter in Refs. [45, 46]. As it was theoretically suggested, $g$ factor experiments extended to ions with nonzero nuclear spin may even deliver more accurate or so far even unknown nuclear magnetic momenta without disturbing chemical shifts [47, 45]. These basic nuclear properties are anticipated to provide input for testing nuclear structure theory, and are relevant in nuclear magnetic resonance measurements. Furthermore, very recently, a redetermination of the magnetic moment of the $^{209}$Bi nucleus lead to the resolution [48] of the “hyperfine puzzle” [49, 50], a $7-\sigma$ deviation of experiment and theory for the hyperfine splitting of the H- and Li-like $^{209}$Bi ions. On the experimental side, $g_F$ factors may be accessed by employing a laser-microwave double-resonance technique (see e.g. [40, 47, 51]).

Nuclear recoil effects due to the relativistic interaction between the electron and the nucleus can be best tested in isotope shift experiments. Most of the QED and electron correlation contributions, which may not be understood to utmost precision, can be cancelled by comparing the $g$ factors of two isotopes, allowing a sensitive probe of the nucleus-related phenomena. The isotope dependence of the $g$ factor in Li-like Ca ions was measured and calculated in Ref. [26]. The good agreement between the predicted recoil contribution and the high-precision measurement paved the way to novel QED tests beyond the Furry picture. Recent experimental developments [38] are anticipated to enable such QED benchmarks in the near future. On the theoretical side, the understanding of the nuclear recoil effect in Li-like ions has been improved very recently in Ref. [52]. An inconsistency has been identified in the previous treatment of the many-electron nuclear recoil contribution. The new evaluation provides, in particular, a new value of the isotopic shift for Li-like calcium, which is in a significantly better agreement with the experimental result of Ref. [26]. The treatment of Ref. [52] has also been extended to B-like ions [53].
5. Determination of fundamental constants

$g$-factor measurements, in combination with theory [7, 8, 13, 14, 32, 54, 55], allowed an independent determination of the electron mass $m_e$ [56, 57]. The most accurate value [58, 59] of $m_e$ has been obtained from a recent measurement employing $^{12}$C$^{5+}$ ions, improving on previous measurements by more than an order of magnitude. The current CODATA value of $m_e$ is largely defined by this determination [60, 61]. The recently constructed ALPHATRAP Penning-trap setup [38], in combination with theory, is anticipated to push the boundaries even further. The use of the $^4$He$^+$ ion instead of or in addition to $^{12}$C$^{5+}$ may provide a more accurate $m_e$ value, as theoretical uncertainties are the lowest for the lightest possible H-like ion [62].

Very recently, in Ref. [63], a similar measurement of the $g$ factor of the muonic $^4$He$^+$ ion was put forward as an independent means to determine the muon’s magnetic moment anomaly and mass. The scheme presented may enable in future the increase of the accuracy of the muon mass by more than an order of magnitude, provided that such challenging experiments can be performed and certain two-loop light-by-light scattering diagrams can be calculated.

Experimental studies can be extended to the heaviest ions, including Pb$^{81+}$ and U$^{91+}$, which is expected in the forthcoming years by the use of the ALPHATRAP setup [38] and the HITRAP facility [40, 41]. One motivation of such studies is that $g$-factor measurements with heavy ions are anticipated to yield an alternative $\alpha$ value [64, 65, 66]. In Ref. [64], a weighted difference of the $g$ factors of heavy H- and B-like ions of the same element was put forward. It was demonstrated that the theoretical uncertainty of the nuclear effects in this difference can be decreased down to $4 \times 10^{-10}$ for heavy ions such as Pb, which was several times smaller than the uncertainty due to $\alpha$ at that time. Since then, however, the uncertainty of the fine-structure constant was reduced by an order of magnitude [60], making it more difficult to access it in bound-electron $g$-factor experiments. In Ref. [65, 66] a weighted difference of the $g$-factors of low-$Z$ H- and Li-like ions was put forward, for which a much stronger suppression of nuclear effects can be achieved, leading to an accuracy competitive with or better than the current value of $\alpha$.

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