An Overview on Game Theory and Its Application

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Abstract

This research paper looks into Game Theory while immensely addressing the historical background of this theory. The document also gives an overview and definition of relevant terminologies related to this theory like a game, Nash equilibrium, and dominance which form the basis of the theory concept. It also focuses on mixed strategies, extensive games with both perfect and imperfect information, auction bidding, and their relevant practical application of the concept as applied in the field of economics.

Keywords: Game theory, Zero-sum game, Nash equilibrium, Economics.

1 Literature Review

This theory has been of great importance in various fields, especially those of social sciences dating back to more than fifty years ago. It first looked into zero-sum games where one person’s gains are equal to the losses of the other player [1]. Game theory may be defined as a decision-making process in formal studies where various players make choices that directly or indirectly affect the other players’ interests.

The first application of the game theory was during the study duopoly by Cournt back in the year 1838. Other scholars like Camerer highlight the application of game theory in 1713 through a letter from James Waldegrave [2]. John Neumann, through his study of the theory of polar games, puts to the spotlight the application of this theory in the 20th century. Neumann, together with other neo-classical economists, brought with them a fresh way of looking into the theory as a competitive process with economic players applying strategic interactions.

The first published material on game theory was authored by John Forbes Nash in his thesis in 1949. The thesis was entitled ‘Non-Cooperative Games,’ where he introduced the widely used phenomenon of Nash Equilibrium or equilibrium point [1], [2]. Nash Equilibrium is noted to be a concept based on the principle where strategy combinations the players are most likely to choose represents one that none of the other players could do better if they chose other different strategy keeping in mind the strategy choice of the others.

Game theory is commonly used in economics, psychology, biology, and political science. Its application is majorly in studies of competitive scenes. As such, the stated problems are referred to as games while the participants are referred to as the players. A player may be defined as a person, or a group of persons that are involved in the decision-making process as Osborne
describes it [2]. Camerer adds on this definition and outlines specific assumptions stating that all players form beliefs with their basis on the actions of others, make the best response concerning the made beliefs, and finally adjust these responses until a point of equilibrium is achieved or until the beliefs are equal. He also notes that these assumptions are sometimes undermined and violated in the sense that some players tend to behave irrationally as the situation gets difficult [3].

The basic and most fundamental assumption that encourages game theory is that players are rational and have strategically reasoned. Players are also believed to be aware of the various alternatives available, and their choice of action is mostly after an optimization process.

Osborne views game in the light of the strategic interaction including all constraints on the player actions that may be taken by a player taking into consideration the interests of such a player but does not define the actions the decision-makers could take. Although most descriptions show a game as a situation involving at least two players, there exists one player game referred to as a decision problem [3]. It is important to note that the players could be individuals, firms, nations, or a combination of any of these. Game theory could be summarized as a language enabling structure formulation, analysis, and strategic scenario understanding. With numerous possible strategies to choose, games follow a sequence of moves where individual moves are considered crucial due to their contribution to the overall strategy [4]. A perfect example, as Pindyck and Rubinfeld states, is a situation where firms compete alongside each other through the setting of prices or a situation of auction bidding by a group of consumers.

2 Game Theory

Game theory is best applied in circumstances where various agents are interdependent [4]. This section mainly focuses on game theory with great emphasis on the dominance, Nash equilibrium, mixed strategies, max-min strategies, and extensive games, both with perfect and imperfect information, bidding in auctions, computation, and lastly, zero-sum games.

2.1 Games Definition

Since the major purpose of this study of game theory is the game, decision-makers involved are arranged according to their information, preferences, strategic actions availed to these players, and their respective influence on the outcome. By description, a game only specifies the payoffs of each player or group of players may achieve with its various member arrangement [5]. Game theory can be broken down into two broad categories, namely non-cooperative and cooperative game theory, with the distinction of these categories being whether the decision-makers are in a position to communicate with each other or not.

Non-cooperative game theory majorly looks into choices emanating from economic interactions among decision-makers. Every player independently and strategically makes choices intending to maximize their utility [6]. This simply means that players explicitly have their interests at heart without considering the interest of any other party. This model of non-cooperative game theory, the timing of players’ choices, and the details of orders are of utmost importance while determining the outcome of the game. Concepts such as Nash Equilibrium are applicable in solving non-cooperative games. The concept, as described by Lim, may be used both in the Normal and strategic form and only gives a solution in circumstances where each decision-maker tends to earn maximum payoff considering strategies applied by the other players. Unlike non-cooperative game theory, which majorly dwells on scenarios with competition, cooperative game theory applies analytical tools in the study of the behavior of rational players in the event where they cooperate [6]. Mainly, these games describe the
formation of cooperating groups of decision-makers that make better the position of the players in a game. Lim, in this case, views cooperative game theory scenarios as combination payoff sets satisfying group as well as individual rationality [7]. Cooperative game theory is most appropriately applied in events developing from international relations and political science, where the key focus is power.

2.2 Dominance

Assuming that all the players in a game are deemed rational, their made choices must result in their desired outcome given the actions of their opponents. In such a case, the said players are said to have dominant strategies. Dominant strategies refer to the most suitable choice for any player given every choice by the opponent player [7]. Payoffs in a dominant strategy are in such a way that despite the choices made by other players, there be no other strategy with a higher payoff.

Dominance in game theory is best brought out in the concept of Prisoner’s Dilemma first introduced in the mid-20th century by Tucker.

The game, according to Kerk, shows the major tensions with both individuals and group actions and their respective outcomes, which are more likely than not a result of their actions.

In the scenario of the prisoner’s dilemma, Davis explains it where two criminals thought of committing a crime together after their arrest are taken into different police cells. The police gather enough evidence for their duo conviction unless either party informs on the other party. This means that each party is faced with the option of either confessing or remaining silent, and every party knows the consequence of their action. The police give the two of them the option that if they both agree and confess, they both serve a ten-year jail term while if only one confesses, he gets one-year jail term and the other twenty-five years. In the case where none of them confesses, then they both face a jail term of 3 years. This scenario narrated by Avinash and Nalebu in 1991 may be best tabulated and summarized as follows [8]

| Prisoner 1/Prisoner 2 | Admit | Hold out |
|-----------------------|-------|---------|
| Admit                 | 10 years | 25 years |
| Hold out              | 1 year | 3 years |

Table 1: Prisoner’s Dilemma

The table shows prisoner one getting a 10-year jail term upon confession and 25 years if he does not. This means that if the second prisoner decides to admit, then the best option is for the first prisoner to confess as well. Conversely, if the second prisoner holds out, the first prisoner’s best choice is to confess still to get the minimum time of one year. These decision-makers have dominant strategies where, despite the choice made by other players in the game, none of the other strategies would give a higher payoff. In this case, the optimal solution does not lie in the scenario where both prisoners admit.

2.3 Nash Equilibrium

This is an economics theoretic game solution concept involving two or more players where each decision-maker has ultimate knowledge on the equilibrium strategies of the other players, and there’s nothing gain by alteration of individual strategy. Any game may have several Nash equilibriums as described by Myerson, although there may be considered unreliable in
comparison to the expected outcome of any game [9]. Several studies show that this equilibrium major concern is the expected choice of actions by players in any game. Decision-makers have to have perfect knowledge of opponents’ choices, and for this to be possible, the concept of the rationality of all the players becomes inapplicable. Alternatively, when statistical information of the previous playing circumstance is available and reliable, then the game falls in place perfectly. Another example of the prisoner’s dilemma and Nash Equilibrium is in cases of buyer and seller transactions. Buyers, in this case, are seen to only transact once or repeatedly and anonymously with any seller. Each buyer’s action is based on their belief in the other party’s actions.

2.4 Mixed Strategies

According to Turocy and Von (2001), strategic form games may not necessarily have a Nash equilibrium whereby one of the parties gets to choose one of the strategies. However, decision-makers have to utilize certain probabilities as a basis of their random strategic selection. Simply, this can be described the distribution of probabilities over the specified combination of actions. Alternatively, some scholars viewed this strategy as other players’ belief in a specific player’s action [10].

A perfect example of the application of mixed strategies is seen in drunk driving. Police officers decided to put up roadblocks with a probability of one third. When an individual drinks Coca Cola he gets zero, with wine consumption the same individual gets -2 with 1/3 probability and 1 with the probability of 2/3, so,

\[
\frac{1}{3} \cdot (-2) + \frac{2}{3} \cdot 1 = 0
\]

In this case, the decision-maker is torn between taking a Cola or Wine, putting all probabilities in mind. With wine consumption with the probability of 1/2, the expected payoff is 1 while he decides not to then;

\[
\frac{1}{2} \cdot (-2) + \frac{1}{2} \cdot 0 = -1
\]

Consequently, the officers are indifferent about whether to set up roadblocks with any mixed strategy. As such, the mixed strategy equilibrium is achieved. This equilibrium discourages players in a strategic game to introduce behavioral randomness since decision-makers are known to randomize events in a bid to influence the opponents’ behavior. This emphasizes the absence of Nash equilibrium in mixed strategies. In this view, the mixed strategy is seen to be opportunities for common knowledge since most positive probability actions are deemed most ideal.

2.5 Perfect Information Extensive Games

These games refer to the representation of all moves in any game and are best described in the form of a decision tree. Players in this scenario get to choose their various preferred strategies without any prior knowledge of the other players’ choices [11]. However, with time, players may get information about the actions of fellow decision-makers, and hence a scenario of perfect information arises over time as all players become aware of opponents’ previous choices. In this form of interaction, only one player moves at a time in a bid to avoid simultaneous movement. Players in this scenario have the opportunity not only to learn the opponent’s moves at the beginning but also at any point in the game. There is a restriction in the observation of ongoing game since every player chooses their action plan once and for all, and hence one is unable to reconsider their initial decision on the plan of action.
2.6 Imperfect Information Extensive Games

Since one player’s payoff is affected by the action of the other, then the preferred strategy of each player may also be dependent on the other decision-makers. Therefore, these are games that are believed not to be fully observable [12]. A player, in this case, needs not to know the actions of that their fellow players have previously taken in the case of extensive games with imperfect information. According to Gilpin and Sandholm (2007), decisions on the actions to be taken at any given time may not be optimally decided while not considering other decisions at all points in time because such decisions impact immensely on the probabilities of different states at present.

2.7 Zero-sum Games and Computation

This mathematical representation refers to the scenario where a player’s utility gain or loss balances with the gains and losses of the competing participant. It is the case whereby upon the summation of all gains and negation of all losses of a certain participant total up to zero, and hence the zero-sum game title [13]. A case of two players with completely conflicting interests demonstrates and extreme case of this game. In this case, one team must win while the other completely lose without any in-betweens and hence a closed system, as stated by the theory by Neumann and Morgenstern. In a mathematical representation of a two-player game, every set of payoffs sums to zero.

2.8 Auction Bidding

One of the greatest accomplishments of game theory is seen in the analysis and design of the auction process. Theory on Auction, which was stipulated by the economist William Vickrey, assisted in auctions generating billions of dollars in mobile telecommunication. The Combination of strategies emanating from a set of decision-makers apply set strategies presented to every individual and come up with a payoff vector enhancing maximum utility and profits. Practically, in auctions, a valuable object is placed where bidders signal their willingness to pay with perfect knowledge of the set rule of assigning the object. The object is assigned to the optimal bidder following the set rules, about which all bidders had perfect information. Von Stengel describes an English auction or an open ascending auction as the situation where an object is put up for sale in the presence of all buyers and price keeps rising as long as there are at least two or more interested bidders [14]. The winning bidder gets the object at the last price at which the last bidder dropped.

3 Application of Game Theory

In economics, game theory is widely applied in areas such as oligopolies, market equilibrium, general equilibrium, auctions, amongst other applications [15]. The major application lies in oligopolistic competition and hence the major concern in this research paper. Oligopoly market may simply be defined as the scenario where a small group of huge companies in unison acquires control of a large market share well informed on the effects of their nature of correlation on the profits and market shares. Due to this oligopolistic structure, decision-making in this case heavily relies on game theory. A dynamic model in which companies and buyers regularly interact in identical circumstances can be named the best application of game theory. The Below illustration describes a situation where two companies with three strategies achieve their payoff’s demonstrating game theory application.

| Minimum | Median (me) | Maximum |
|---------|-------------|---------|

5
| Minimum (mi) | Median (me) | Maximum (ma) |
|-------------|-------------|--------------|
| 15, 15      | 21, 5       | 10, 3        |
| 5, 21       | 12, 12      | 5, 2         |
| 5, 10       | 2, 5        | 0, 0         |

Table 2: Prisoner’s Dilemma

From the above diagram, (mi, mi) (me, me) (ma, ma) combinations may be shown to indicate a monopoly event while (me, me) describes a single-period game Nash Equilibrium.

4 Conclusion

It is evident from this paper that game theory goes beyond mathematical representations to the description of the real world events where decisions made by other players affect other players’ interests. Game theory may not be very efficient in the prediction of behavior like in the case of sciences though in some unique cases, it might take up that role as described (Vorob’ev 1994, p. 14). Game theory in this essay is looked at as a conceptual analysis applicable in the decision-making process as well as conflict resolution. Immense emphasis has been put on auction bidding a set of decision-makers achieve a payoff through strict observance of a set of laid out strategies that all players have perfect information about (Vorob’ev 1994, p. 16-17). In an attempt to bridge the gap between theory and real-life situations, game theory is seen being applied in sales and marketing and through observation of people’s behavior by the choices they make.

Game theory is shown in companies’ management strategies where important decisions need to be made. With game theory knowledge, managers can make sound decisions while maximizing industry payoff. This paper also describes the role of Nash equilibrium in describing the concept of the prisoner’s dilemma that forms the basis in the illustration of game theory (Owen 2013, p. 31). The only challenge with game theory lies in the tradeoff between realism and solvability in the real world, a problem that the common assumptions of rationality and common knowledge try to smoothen out.

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