A modified homogeneous relaxation model for CO₂ two-phase flow in vapour ejector

M. Haida¹, M. Palacz¹, J. Smolka¹, A. J. Nowak¹, A. Hafner², K. Banasiak³

¹ Institute of Thermal Technology, Silesian University of Technology, Gliwice, Poland
² NTNU Department of Energy and Process Engineering, Trondheim, Norway
³ SINTEF Energy Research, Trondheim, Norway

Michal.Haida@polsl.pl

Abstract. In this study, the homogenous relaxation model (HRM) for CO₂ flow in a two-phase ejector was modified in order to increase the accuracy of the numerical simulations. The two-phase flow model was implemented on the effective computational tool called ejectorPL for fully automated and systematic computations of various ejector shapes and operating conditions. The modification of the HRM was performed by a change of the relaxation time and the constants included in the relaxation time equation based on the experimental result under the operating conditions typical for the supermarket refrigeration system. The modified HRM was compared to the HEM results, which were performed based on the comparison of motive nozzle and suction nozzle mass flow rates.

1. Introduction

Due to the restrictive legal regulations for environmental protection in refrigeration, common synthetic refrigerants are replaced by the environmentally friendly natural refrigerants, such as carbon dioxide (denoted as R744). Prof Lorentzen patented a transcritical carbon dioxide system for automotive air conditioning, which led to the design and manufacturing of rival refrigeration systems with CO₂ as the main working fluid [1]. R744 is classified as a non-toxic and non-flammable fluid with low global warming potential index GWP of 1, and ozone depletion potential index of 0. The R744 refrigeration systems are mostly introduced in cold climates as the result of the relatively low efficiency at the high surrounding temperature. One of the solution to improve the energy performance of the CO₂ transcritical refrigeration cycle is the implementation of the two-phase ejector as the main expansion device in the cycle.

Theoretical and experimental analyses have indicated that replacing the expansion valve with the ejector in the CO₂ transcritical vapour compression cycle improves the energy performance [2]. The CO₂ ejector expansion refrigeration cycle (EERC) was analysed in [3] based on the modified non-dimensional ejector solver proposed in [4]. The authors state that COP improvement of the EERC was more than 16% for typical air conditioning operating conditions. The energy performance improvement of the R744 ejector expansion transcritical cycle up to 18.6% is indicated in [5]. The experimental investigation of the transcritical R744 cycle with a prototype ejector was presented in [6]. Experimental results confirm the energy performance improvement of the EERC up to 7% and the influence of the
The one-dimensional model of the R744 two-phase ejector based on the delayed equilibrium model was presented in [7]. The numerical results are validated with the experimental results for a typical range of operating conditions. Therefore, the two-phase ejector with designed geometry, based on foregoing computational model, improves the energy performance of the R744 refrigeration system up to 8% [8].

The design concept of the R744 multi-ejector system with non-continuously controllable ejectors for supermarket application was shown in [9]. The steady-state simulation for constant ejector efficiency of 20% shows the COP improvement between 10% at 15 °C and 20% at 45 °C ambient temperature compared to the reference booster system. Transient simulations were performed based on the annual variable ambient temperature and annual variable load profiles for heating and cooling modes for three different climate regions. The COP for cooling mode increased between 20% and 30% during the winter and 17% in the Mediterranean region, 16% in Middle Europe, and 5% in Northern Europe during the summer.

The three-dimensional model of the single- and two-phase flow of a real fluid based on the enthalpy-based energy equation, in which the specific enthalpy is an independent variable was performed in [10]. The computational model is tested and validated for the single-phase R141b ejector, and for the two-phase R744 ejector. Foregoing homogenous equilibrium model [11] was used to design and manufacture the multi-ejector expansion pack with four different ejector with a binary profile for the R744 parallel-compression refrigeration system. The multi-ejector module was experimentally investigated in [12]. The experimental results indicate high value of the ejector efficiency up to 30% for all four vapour ejectors. The experimental investigation of the R744 multi-ejector refrigeration system based on energy and exergy analysis was investigated in [13]. The author state that COP and the exergy efficiency of the multi-ejector system increases up to 8% and 13.7% compared to the reference parallel-compression system under the typical supermarket operating conditions.

In the paper [14], the authors used genetic algorithm and evolutionary algorithm to optimise the mixing chamber in the R744 two-phase ejector based on the HEM under the operating condition with acceptable accuracy of the model. The optimisation procedure was performed for two ejectors with different capacity. The author states that the optimal length of the mixing section increases approximately 50% for both ejectors. However, the baseline diameter of the mixer was already close to the optimal value and it was the crucial geometric parameter in ejector performance. According to [14] more complex mathematical models like homogenous relaxation model (HRM) should be used to improve the optimisation tool.

The homogenous relaxation model as a linearized expansion was proposed in [15]. The authors describe difference between HEM and HRM with special attention on a study of dispersion, characteristics, choking and shock waves. The relation between the empirically correlation of the relaxation time and the quality of the flashing fluid is described in [16]. The comparison between the results obtained from HRM, HEM and available experimental data was performed in [17]. The validation procedure was carried out for one-dimensional flashing water flow to predict the critical mass flow rates and the pressure distributions. The HRM predicts with good accuracy both the pressure distributions and the critical mass flow rates, when the HEM underestimates the critical mass flow rates over 20% for small value of inlet subcooling [17]. The investigation of the HRM of the CO2 supersonic two-phase flow through the ejector motive nozzle was presented in [18]. The numerical results are validated for three different converging-diverging nozzle diameters with the experimental results taken from [19]. In addition, the new correlation for the relaxation time of CO2 is proposed. In the paper [20], the authors applied HRM in the R744 condensing two-phase ejector based on the CFD open-source OpenFOAM model proposed in [21]. The CFD model was built using the Eulerian pseudo-fluid approach. The validation procedure of the pressure distribution among the ejector indicated high inaccuracy of the model in most cases, especially for the refrigeration system with the internal heat exchanger.
The accuracy of the HEM, which was applied to three-dimensional CFD-based simulations of the R744 expansion inside two-phase ejector, was presented in [22]. The computational results of the motive nozzle mass flow rate was compared to the measured mass flow rate under the different operating conditions. The accuracy analysis shows the acceptable application of the HEM for the operating regimes near or above the critical point. Unfortunately, the inaccuracy of the model increases with the decreasing motive nozzle temperature and decreasing distance to the saturation line.

The aim of presented paper is to improve the HRM accuracy applied into the two-dimensional axisymmetric CFD model of the R744 transcritical two-phase ejector. HRM was built based on own HEM, which the accuracy of the model was presented in [22]. The numerical simulations were performed by use of the ejectorPL platform. The investigation is performed by comparison of the relaxation time correlation of CO2 proposed in [18] to the constant values of the relaxation time in the whole domain together with HEM results. The validation of the numerical results is performed based on the motive mass flow rate and the suction mass flow rate accuracy compared to the experimental results. The experimental results were carried out in SINTEF Energy Research laboratory in Trondheim under the typical supermarket refrigeration operating conditions.

2. Methodology

2.1. HRM approach

The equations for the conservation of mass, momentum and enthalpy are given below.

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{U}) = 0
\]

\[
\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla (\rho \mathbf{UU}) = -\nabla p + \nabla \cdot \mathbf{t}
\]

\[
\frac{\partial (\rho E)}{\partial t} + \nabla (\rho \mathbf{UE}) = \frac{\partial p}{\partial t} + \nabla \cdot (k \nabla T + \mathbf{t} \cdot \mathbf{U})
\]

Where \( \rho \) is a density, \( \mathbf{U} \) is a velocity vector, \( \mathbf{t} \) is a stress tension, \( E \) is a total energy, \( k \) is a thermal conductivity, and \( \nabla T \) is a laplacian of the temperature. The additional vapour mass balance equation is defined as the total derivative of the quality equal to the vapour generation rate divided by the total density.

\[
\frac{\partial x}{\partial t} = \frac{\Gamma}{\rho}
\]

where \( \Gamma \) is the vapour generation rate and \( x \) is the quality. The total enthalpy defined in Eq. (3) is a sum of the specific enthalpy and the kinetic enthalpy.

\[
E = h + \frac{U^2}{2}
\]

where \( h \) is the specific enthalpy. The specific enthalpy and the total density of the mixture can be defined as follows [17].

\[
h = x \cdot h_{sv} + (1 - x) \cdot h_{ml}
\]

\[
\rho = \alpha \cdot \rho_{sv} + (1 - \alpha) \cdot \rho_{ml}(p, h_{ml})
\]

where \( h_{ml} \) and \( \rho_{ml} \) are the specific enthalpy and the density in metastable conditions, \( h_{sv} \) and \( \rho_{sv} \) are the specific enthalpy and the density in saturated vapour, \( \alpha \) is the void fraction of the mixture. The void fraction is the local quality of the mixture multiplied by the ratio of the local density and divided by the density of the saturated vapour.

\[
\alpha = \frac{x \cdot \rho}{\rho_{sv}}
\]

The vapour mass balance equation presented in Eq. (4) can be defined based on a linearised expansion proposed in [15].

\[
\frac{\partial x}{\partial t} = -\frac{x - \bar{x}}{\theta}
\]
where $x$ is the instantaneous quality, $\bar{x}$ is the equilibrium quality and $\theta$ is the relaxation time. The relaxation time formulation for R744 was proposed in [18].

$$\theta = \theta_0 \cdot \alpha^a \cdot \varphi^b$$

(10)

where $\theta_0$ is equal to $2.14 \times 10^{-7}$ s, $\varphi$ is the non-dimensional pressure difference, $a$ and $b$ are the constant coefficients set to -0.54 and -1.76, respectively. The non-dimensional pressure difference for greater pressures is given below [17].

$$\varphi = \left| \frac{p_{\text{sat}}(\bar{x}=0) - p}{p_{\text{crit}} - p_{\text{sat}}(\bar{x}=0)} \right|$$

(11)

where $p_{\text{crit}}$ is the critical pressure of CO$_2$.

2.2. The two-phase ejector parameters

The ejector performance is described by four specific parameters, i.e. mass entrainment ratio, pressure lift, pressure ratio, and the ejector efficiency. The mass entrainment ratio is the ratio between the suction mass flow rate and the motive mass flow rate.

$$\phi = m_{\text{suction}} / m_{\text{motive}}$$

(12)

The pressure ratio is the outlet pressure divided by the suction nozzle pressure.

$$\Pi = \frac{P_{\text{outlet}}}{P_{\text{suction}}}$$

(13)

Related to the pressure ratio, the pressure lift shows the difference between the outlet pressure and the suction nozzle pressure.

$$dP = P_{\text{outlet}} - P_{\text{suction}}$$

(14)

The ejector efficiency is defined as proposed in [6]. The benefit of using this definition is that it can be applied for an experimental investigation because it avoids the measured static pressure in the mixing chamber.

$$\eta_{e,j} = \phi \cdot \frac{[h_{C}(s=s_{\text{suction}}=P_{\text{outlet}}) - h_{D}(s=s_{\text{suction}}=P_{\text{outlet}})]}{[h_{D}(h=h_{\text{motive}}=P_{\text{outlet}}) - h_{D}(s=s_{\text{motive}}=P_{\text{outlet}})]}$$

(15)

3. Test campaign

The experimental investigation was performed on the test facility equipped with the multi-ejector module in SINTEF laboratory in Trondheim, Norway. The test campaign was prepared for four different vapour R744 ejectors (VEJ) of the performance increasing in the binary order (1:2:4:8) and applied in the multi-ejector [12]. The multi-ejector module is designed to ensure the maximum system flexibility [13]. The shape of the R744 vapour ejector applied in the multi-ejector module is shown in Figure 1. The main dimensions of the R744 vapour ejector are listed in Table 1.

![Figure 1](image-url)  
**Figure 1** Shape and dimensions of the R744 vapour ejectors
Table 1 Geometry of the considered vapour ejectors

| Parameter name                  | Symbol | Unit | EJ1 | EJ2 |
|---------------------------------|--------|------|-----|-----|
| Motive nozzle inlet diameter    | D_{MN,1} | [mm] | 3.80 | 3.80 |
| Motive nozzle outlet diameter   | D_{MN,2} | [mm] | 1.12 | 1.58 |
| Motive nozzle converging angle  | \gamma_{MN,1} | [°] | 30.00 | 30.00 |
| Motive nozzle diverging angle   | \gamma_{MN,2} | [°] | 2.00 | 2.00 |
| Motive nozzle outer angle       | \gamma_{MN,3} | [°] | 38.00 | 38.00 |
| Pre-mixer length                | L_{MCH} | [mm] | 2.70 | 3.70 |
| Mixer length                    | L_{MIX} | [mm] | 11.50 | 16.25 |
| Diffuser angle                  | \gamma_{DIFF} | [°] | 5.00 | 5.00 |

The operating conditions presented in Table 2 were set for typical supermarket refrigeration applications. The motive nozzle pressure varied from 50 bar to 92 bar. Therefore, the numerical investigation is performed for either transcritical conditions or subcritical conditions linked to the ambient temperature. The suction nozzle pressure is related to the evaporation temperature in the medium-temperature evaporator between -10 °C and -6 °C. The outlet conditions was set to analyse the accuracy of the HRM for different values of the pressure lift.

Table 2 Set of the operating conditions for two different R744 two-phase ejectors.

| Case No | Ejector | Motive nozzle | Suction nozzle | Outlet |
|---------|---------|---------------|----------------|--------|
|         |         | Pressure [bar] | Temperature [°C] | Pressure [bar] | Temperature [°C] | Pressure [bar] |
| #1      | VEJ1    | 64.64         | 22.25          | 28.05     | 3.36             | 34.83          |
| #2      | VEJ1    | 64.79         | 22.09          | 28.01     | 2.48             | 33.77          |
| #3      | VEJ1    | 59.30         | 17.67          | 28.49     | 5.42             | 33.87          |
| #4      | VEJ1    | 58.43         | 17.69          | 28.45     | 1.98             | 31.01          |
| #5      | VEJ1    | 75.10         | 23.70          | 32.01     | 5.98             | 37.34          |
| #6      | VEJ2    | 53.93         | 6.33           | 27.30     | 5.70             | 34.23          |
| #7      | VEJ2    | 58.41         | 10.00          | 27.82     | 4.56             | 34.83          |
| #8      | VEJ2    | 61.79         | 20.27          | 29.93     | 3.58             | 33.87          |

The accuracy of the selected parameter of each numerical model (HEM or HRM) is calculated as the relative error between the experimental results and the numerical results.

\[ \delta_x = \frac{x_{\text{exp}} - x_{\text{num}}}{x_{\text{exp}}} \times 100\% \]  

(16)

The acceptable relative difference between the experimental results and the numerical results was assumed as less than or equal to 10%.

4. Computational tool

The numerical investigation was performed using the commercial ANSYS software on the ejectorPL platform [22]. The purpose of the ejectorPL software is to automate the simulation process by combining and controlling the geometry together with the mesh generator ANSYS ICEM CFD, executing the solver in ANSYS Fluent for the flow simulation, and finally processing the results data for the ejector operation. The 2-D axisymmetric ejector geometry was discretised with a fully structured grid of less than 10,000 elements with a minimum orthogonal quality of 0.9. The real fluid properties of R744 were approximated based on data obtained using the REFPROP libraries [23]. The realisable k-ε model was used to model the turbulent flow inside the ejector.

5. Results

Figure 2 presents the motive nozzle mass flow rate accuracy of HEM and HRM for the relaxation time defined in Eq. (10) and for seven different constant values of the relaxation time compared to the
experimental results. The HRM indicated better accuracy than the HEM, except Cases #6 and #7, respectively. The increase of the relaxation time improved the accuracy of the HRM. The acceptable motive nozzle mass flow rate relative difference of the HRM was reached for case #3, #4, #5, #6, and #7 for the relaxation time higher than 0.0001 s. For the relaxation time smaller than 0.0001 s, the relaxation of the expansion process and the metastable region influenced slightly on the increase of the motive mass flow rate. For each investigated cases, the HRM accuracy stabilised for constant values of the relaxation time. Hence, the maximum accuracy of the HRM possible to reach was obtained for the relaxation time of approximately 0.001 s.

The HRM results for the relaxation time defined in Eq. (10) show the accuracy improvement of the numerical model by approximately 5% compared to the results given by HEM. However, the correlation function of the relaxation time increased the inaccuracy of the model for case #6 and #7, where the motive nozzle pressure was close to the critical pressure. Therefore, the function of the relaxation depended on the void fraction and the non-dimensional pressure difference should be calibrated for different motive nozzle parameters.

![Figure 2](image-url)  
**Figure 2** The motive nozzle mass flow rate accuracy of the HEM and HRM for different value of the relaxation time under the operating conditions presented in Table 2.

The comparison of the suction nozzle mass flow rate accuracy of the HEM and the HRM with different relaxation time compared to the experimental results was shown in Figure 3. The value of the suction mass flow rate for the different relaxation time was very sensitive. For very small value of the relaxation time, the HRM reached higher value of the suction mass flow rate than HEM, which increased the inaccuracy of the model. However, the change of the relaxation time between 1e-05 s and 1e-04 s occurred significant decrease of the suction nozzle mass flow rate produced by the HRM formulation. Therefore, the improvement of the HRM accuracy can be performed by a set of the relaxation time in the range from 1e-05 s to 1e-04 s. For these times, the motive mass flow rate reached the improvement of the accuracy and the suction mass flow rate can be obtained with the acceptable discrepancy.

In similar to the results presented in Figure 2, the HRM for the relaxation time defined in Eq. (10) improved the accuracy of the numerical results for Cases #1, #3, and #8. For Cases #2, #4, and #7 the HRM obtained the same value of the suction nozzle mass flow rate as the HEM. The HRM results for Case #2 and #6 indicated the higher discrepancy than the HEM approach.
Conclusion

The modified HRM for two-phase flow inside the R744 vapour ejector under the operating conditions typical to the supermarket refrigeration application was investigated. The different constant relaxation time for whole vapour ejector was defined in order to evaluate the discrepancy difference of the motive nozzle mass flow rate and suction nozzle mass flow rate between different relaxation time. The validation of the numerical results was performed based on the experimental results obtained by the authors at the SINTEF Energy Research Laboratory. The validation of the modified HRM was performed for two different vapour ejectors VEJ1 and VEJ2.

The modified HRM with relaxation time of 0.001 s obtained the highest accuracy of the motive nozzle mass flow rate. However, the relaxation time should be varied in the range from $1 \times 10^{-5}$ s to $1 \times 10^{-4}$ s based on the accuracy of the motive mass flow rate and the suction mass flow rate. Thereby, the HRM indicated relatively high accuracy of both streams.

The comparison of the relaxation time defined as the empirical function presented in [18] between the different constant values of the relaxation time shown that the constants included to the function of the relaxation time should be calibrated in order to improve the accuracy of the HRM of the motive nozzle mass flow rate and the suction nozzle mass flow rate.

Acknowledgment

The authors gratefully acknowledge financial support partially from the Polish Norwegian Research Fund through project No. Pol-Nor/196445/29/2013 and partially from Research Council of Norway through project No. 244009/E20.

References

[1] Lorentzen G 1990.Trans-critical vapour compression cycle device. Google Patents.
[2] Sumeru K, Nasution H, Ani FN 2012 Renew Sustain Energy Rev 16 4927-37
[3] Li D, Groll EA 2005 Int J Refrigeration 28 766-73
[4] Kornhauser AA. The Use of an Ejector as a Refrigerant Expander International Refrigeration and Air Conditioning Conference; Purdue University: Purdue e-Pubs; 1990. p. 11.
[5] Deng Jq, Jiang Px, Lu T, Lu W 2007 Appl Therm Eng 27 381-8
[6] Elbel S, Hrnjak P 2008 Int J Refrig Int Du Froid 31 411-22
[7] Banasiak K, Hafner A 2011 Int J Therm Sci 50 2235-47
[8] Banasiak K, Hafner A, Andresen T 2012 Int J Refrigeration 35 1617-25
[9] Hafner A, Forsterling S, Banasiak K 2014 Int J Refrig Int Du Froid 43 1-13
[10] Smolka J, Bulinski Z, Fic A, Nowak AJ, Banasiak K, Hafner A 2013 *Applied Mathematical Modelling* **37** 1208-24

[11] Hafner A, Hemmingsen AK, Van De Ven A 2014 R744 refrigeration system configurations for supermarkets in warm climates *3rd IIR International Conference on Sustainability and the Cold Chain, ICCC 2014*: International Institute of Refrigeration.

[12] Banasiak K, Hafner A, Kriezi EE, Madsen KB, Birkelund M, Fredslund K, et al. 2015 *Int J Refrigeration* **57** 265-76

[13] Haida M, Banasiak K, Smolka J, Hafner A, Eikevik TM 2016 *Int J Refrigeration* **64** 93-107

[14] Palacz M, Smolka J, Kus W, Fic A, Bulinski Z, Nowak AJ, et al. 2016 *Appl Therm Eng* **95** 62-9

[15] Bilicki Z, Kestin J 1990 *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* **428** 379-97

[16] Austin BT, Sumathy K 2011 *Renew Sustain Energy Rev* **15** 4013-29

[17] Downar-Zapolski P, Bilicki Z, Bolle L, Franco J 1996 *International Journal of Multiphase Flow* **22** 473-83

[18] Angielczyk W, Bartosiewicz Y, Butrymowicz D, Seynhaeve J-M. 1-D Modeling Of Supersonic Carbon Dioxide Two-Phase Flow Through Ejector Motive Nozzle *International Refrigeration and Air Conditioning Conference; Purdue University*: Purdue e-Pubs; 2010.

[19] Nakagawa M, Berana MS, Kishine A 2009 *Int J Refrigeration* **32** 1195-202

[20] Colarossi M, Trask N, Schmidt DP, Bergander MJ 2012 *Int J Refrigeration* **35** 290-9

[21] Schmidt DP, Gopalakrishnan S, Jasak H 2010 *International Journal of Multiphase Flow* **36** 284-92

[22] Palacz M, Smolka J, Fic A, Bulinski Z, Nowak AJ, Banasiak K, et al. 2015 *Int J Refrigeration* Application range of the HEM approach for CO2 expansion inside two-phase ejectors for supermarket refrigeration systems

[23] Lemmon EW, Huber ML, McLinden MO 2013. NIST Standard Reference Database 23: Reference Fluid Thermodynamic and Transport Properties-REFPROP. Standard Reference Data Program. 9.1 ed. Gaithersburg: *National Institute of Standards and Technology*. 

8