One Raised Product Prime Labeling of Some Star Related Graphs

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Available online at: www.isroset.org

Received: 9/May/2018, Revised: 23/May/2018, Accepted: 18/June/2018, Online: 30/June/2018

Abstract — One raised product prime labeling of a simple, finite and undirected graph G with p vertices and q edges is the labeling of the vertices of the graph with first p natural numbers, edges with product of the labels of the end vertices plus one. If the greatest common divisor of each vertex of degree greater than one is one then the graph is called one raised product prime graph. Here the greatest common divisor of the labels of the edges incident on a vertex is called greatest common incidence number of that vertex. Here we proved that star, bistar, splitting graph of a star, shadow graph of a star, jelly fish graph and some more star related graphs admit one raised product prime labeling.

Keywords — Graph labeling, product, prime labeling, prime graphs, star

I. INTRODUCTION

The concept of one raised product prime labeling was introduced in [5] by Sunoj B S and Mathew Varkey T K. Later in [6] and [7] they extend the study and proved the result for some path related graphs and some snake graphs. In this paper we proved that star, bistar, graph obtained by duplicating the apex vertex of star by a vertex, graph obtained by duplicating an edge of a star by an edge, graph obtained by duplicating the apex vertex of star by an edge, splitting graph of a star, shadow graph of a star, jelly fish graph and tensor product of star with path of length one admit one raised product prime labeling.

The paper contains three sections in which section I contains introduction to one raised product prime labeling, section II contains preliminaries and notations and section III contains main results and illustrations and section IV contains concluding remarks.

II. PRELIMINARIES AND NOTATIONS

In this paper we consider only those graphs which are simple finite and undirected. Graph is denoted by G, vertex set is denoted by V, edge set is denoted by E, p is the number of vertices of the graph G and q is the number of edges of the graph G. |V(G)| = number of vertices of graph G, |E(G)| = number of edges of graph G.

Definition 2.1[3] If the vertices and edges of the graph are assigned values subject to certain conditions is known as graph labeling.

Definition 2.2[8] The greatest common incidence number of a vertex is defined as the greatest common divisor [1] of the labels of the edges incident on that vertex.

Definition 2.3[5] Let G = (V(G), E(G)) be a graph with p vertices and q edges. Define a bijection f : V(G) → {1, 2, ..., p} by f(v) = i, for every i from 1 to p and define a 1-1 mapping f_{orpp} : E(G) → set of natural numbers N by f_{orpp}(uv) = f(u)f(v) + 1. The induced function f_{orpp} is said to be one raised product prime labeling, if for each vertex of degree at least 2, the gcin of the labels of the incident edges is 1.

Definition 2.4[5] A graph which admits one raised product prime labeling is called one raised product prime graph.

Definition 2.5[2] A tree with one internal node and n leaves is called star and is denoted by K_{1,n}.

Definition 2.6[4] For a graph G the splitting graph is obtained by adding to each vertex v a new vertex v’ such that v’ is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as S(G).

Definition 2.7[3] The shadow graph D_s(G) of a connected graph G is constructed by taking two copies of G say G’ and G” join each vertex v’ in G’ to the neighbors of the corresponding vertices in G”.

Definition 2.8[3] Let G be a cycle of length 4 with one chord. The graph obtained by joining the apex vertex of star K_{1,n} and star K_{1,m} to the vertices of degree 2 is called jelly fish graph and is denoted by JF(m,n).

Definition 2.9[3] Tensor product of path P_2 with star K_{1,n} is two copies of star K_{1,n+1}.
**Definition 2.10** [3]
Duplication of a vertex $v$ by a new vertex of graph $G$ produces a new graph $H$ by adding a vertex $u$ and joining $u$ to vertices of $G$ adjacent to $v$.

**Definition 2.11** [3]
Duplication of a vertex $v$ by a new edge of graph $G$ produces a new graph $H$ by adding an edge and joining the end vertices of the edge to $v$.

**Definition 2.12** [3]
Duplication of an edge $e = (ab)$ by a new edge $f = uv$ of graph $G$ produces a new graph $H$ by adding an edge and joining $u$ to the neighboring vertices of $a$ other than $b$ and joining $v$ to the neighboring vertices of $b$ other than $a$.

### III. MAIN RESULTS

**Theorem 3.1**
Star $K_{1,n}$ ($n > 2$) admits one raised product prime labeling.

**Proof:** Let $G = K_{1,n}$ and let $v_1, v_2, \ldots, v_{n+1}$ are the vertices of $G$. Here $|V(G)| = n+1$ and $|E(G)| = n$.

Define a function $f : V \to \{1, 2, \ldots, n+1\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, n+1$$

Clearly $f$ is a bijection. For the vertex labeling $f$, the induced edge labeling $f_{orrp}$ is defined as follows

$$f_{orrp}(v_i v_{i+1}) = i+2, \quad i = 1, 2, \ldots, n.$$ Clearly $f_{orrp}$ is an injection.

**gein** of $(v_i)$

$$= \gcd\{f_{orrp}(v_1 v_2), f_{orrp}(v_1 v_3)\}$$

$$= \gcd\{3, 4\} = 1.$$ So, **gein** of each vertex of degree greater than one is 1.

Hence $K_{1,n}$ admits one raised product prime labeling.

**Example 3.1** $G = K_{1,4}$

**Theorem 3.2**
Bistar $B(m,n)$ ($m$ and $n > 2$) admits one raised product prime labeling.

**Proof:** Let $G = B(m,n)$ and let $v_1, v_2, \ldots, v_{m+n+2}$ are the vertices of $G$. Here $|V(G)| = m+n+2$ and $|E(G)| = m+n+1$.

Define a function $f : V \to \{1, 2, \ldots, m+n+2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, m+n+2$$

Clearly $f$ is a bijection. For the vertex labeling $f$, the induced edge labeling $f_{orrp}$ is defined as follows

$$f_{orrp}(v_i v_{i+1}) = ni+i+1, \quad i = 1, 2, \ldots, m.$$ Clearly $f_{orrp}$ is an injection.

**gein** of $(v_{m+1})$

$$= \gcd\{f_{orrp}(v_m v_{m+1}), f_{orrp}(v_{m+1} v_{m+2})\}$$

$$= \gcd\{m^2+m+1, m^2+3m+3\}$$

$$= \gcd\{2m+2, m^2+3m+1\}$$

$$= \gcd\{m+1, m(m+1)+1\}$$

$$= 1.$$ So, **gein** of each vertex of degree greater than one is 1.

Hence $B(m,n)$ admits one raised product prime labeling.

**Example 3.2** $G = B(4,3)$

**Theorem 3.3**
Let $G$ be the graph obtained by duplicating the apex vertex of star $K_{1,n}$ ($n > 2$). $G$ admits one raised product prime labeling.

**Proof:** Let $G$ be the graph and let $v_1, v_2, \ldots, v_{m+n+2}$ are the vertices of $G$. Here $|V(G)| = m+n+2$ and $|E(G)| = 2m+n+2$.

Define a function $f : V \to \{1, 2, \ldots, n+2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, n+2$$

Clearly $f$ is a bijection. For the vertex labeling $f$, the induced edge labeling $f_{orrp}$ is defined as follows

$$f_{orrp}(v_i v_{i+1}) = ni+i+1, \quad i = 1, 2, \ldots, n.$$ Clearly $f_{orrp}$ is an injection.

**gein** of $(v_{m+1})$

$$= \gcd\{f_{orrp}(v_m v_{m+1}), f_{orrp}(v_{m+1} v_{m+2})\}$$

$$= \gcd\{ni+1, ni+2i+1\}$$

$$= \gcd\{ni+1, n+2i+1\}$$

$$= \gcd\{i, (n+1)i+1\}$$

$$= 1.$$ So, **gein** of each vertex of degree greater than one is 1. Hence $B(m,n)$ admits one raised product prime labeling.
\[= \gcd \{n+2, n+3\}\]
\[= 1.\]

So, \(\gcd\) of each vertex of degree greater than one is 1. Hence \(G\) admits one raised product prime labeling.

**Example 3.3** Let \(G\) be the graph obtained by duplicating the apex vertex of star \(K_{1,4}\)

\[\text{Fig – 3.3}\]

**Theorem 3.4** Let \(G\) be the graph obtained by duplicating an edge of star \(K_{1,n}\) \((n \text{ is a natural number greater than } 2)\). \(G\) admits one raised product prime labeling.

**Proof:** Let \(G\) be the graph and let \(v_1, v_2, \ldots, v_{n+1}\) are the vertices of \(G\).
Here \(|V(G)| = n+3\) and \(|E(G)| = 2n+1\).
Define a function \(f : V \to \{1, 2, \ldots, n+3\}\) by
\[f(v_i) = i, \quad i = 1, 2, \ldots, n+3\]
Clearly \(f\) is a bijection.
For the vertex labeling \(f\), the induced edge labeling \(f_{\text{rpp}}\) is defined as follows
\[f_{\text{rpp}}(v_i, v_{i+1}) = ni+i+1, \quad i = 1, 2, \ldots, n.\]
\[f_{\text{rpp}}(v_i, v_{i+2}) = ni+2i+1, \quad i = 1, 2, \ldots, n.\]
\[f_{\text{rpp}}(v_{n+2}, v_{n+3}) = n^2+5n+7.\]
Clearly \(f_{\text{rpp}}\) is an injection.
\(\gcd\) of \((v_1)\) = \(\gcd\) \(\{f_{\text{rpp}}(v_1, v_{n+1}), f_{\text{rpp}}(v_1, v_{n+2})\}\)
\[= 1, \quad i = 1, 2, \ldots, n.\]
\(\gcd\) of \((v_{n+1})\) = \(\gcd\) \(\{f_{\text{rpp}}(v_1, v_{n+1}), f_{\text{rpp}}(v_2, v_{n+1})\}\)
\[= 1.\]
\(\gcd\) of \((v_{n+2})\) = \(\gcd\) \(\{f_{\text{rpp}}(v_1, v_{n+2}), f_{\text{rpp}}(v_2, v_{n+2})\}\)
\[= 1.\]
So, \(\gcd\) of each vertex of degree greater than one is 1. Hence \(G\) admits one raised product prime labeling.

**Example 3.4** Let \(G\) be the graph obtained by duplicating an edge of star \(K_{1,4}\)
**Theorem 3.6** Splitting graph of star $K_{1,n}$ ($n > 2$) admits one raised product prime labeling.

**Proof:** Let $G = S'(K_{1,n})$ be the graph and let $v_1, v_2, \ldots, v_{2n+2}$ are the vertices of $G$.

Here $|V(G)| = 2n+2$ and $|E(G)| = 3n$.

Define a function $f : V \rightarrow \{1, 2, \ldots, 2n+2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, 2n+2$$

Clearly $f$ is a bijection.

For the vertex labeling $f$, the induced edge labeling $f_{orpp}$ is defined as follows

$$f_{orpp}(v_i v_{n+1}) = i + n + 1, \quad i = 1, 2, \ldots, n.$$  

$$f_{orpp}(v_i v_{n+2}) = 2n + 1 + i, \quad i = 1, 2, \ldots, n.$$  

$$f_{orpp}(v_{n+2} v_{n+2+i}) = (n+2)^2 + i(n+2) + 1, \quad i = 1, 2, \ldots, n.$$  

Clearly $f_{orpp}$ is an injection.

$\text{gcd }$ of $(v_i)$

$$= \text{gcd } \{f_{orpp}(v_i v_{n+1}),$$

$$f_{orpp}(v_i v_{n+2})\}$$

$$= \text{gcd } \{i + n + 1, 2n + 1 + i\}$$

$$= \text{gcd } \{i, (n+1)i + 1\}$$

$$= 1, \quad i = 1, 2, \ldots, n.$$  

$\text{gcd }$ of $(v_{n+1})$

$$= \text{gcd } \{f_{orpp}(v_1 v_{n+1}),$$

$$f_{orpp}(v_2 v_{n+1})\}$$

$$= \text{gcd } \{n+2, 2n+3\}$$

$$= \text{gcd } \{n+1, n+2\}$$

$$= 1.$$  

$\text{gcd }$ of $(v_{n+2})$

$$= \text{gcd } \{f_{orpp}(v_1 v_{n+2}),$$

$$f_{orpp}(v_2 v_{n+2})\}$$

$$= \text{gcd } \{n+3, 2n+5\}$$

$$= \text{gcd } \{n+2, n+3\}$$

$$= 1.$$  

So, $\text{gcd }$ of each vertex of degree greater than one is 1. Hence, $G$ admits one raised product prime labeling.

**Example 3.6** $G = S'(K_{1,4})$

**Theorem 3.7** Shadow graph of star $K_{1,n}$ ($n > 2$) admits one raised product prime labeling.

**Proof:** Let $G = D(K_{1,n})$ be the graph and let $v_1, v_2, \ldots, v_{2n+2}$ are the vertices of $G$.

Here $|V(G)| = 2n+2$ and $|E(G)| = 4n$.

Define a function $f : V \rightarrow \{1, 2, \ldots, 2n+2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, 2n+2$$

Clearly $f$ is a bijection.

For the vertex labeling $f$, the induced edge labeling $f_{orpp}$ is defined as follows

$$f_{orpp}(v_i v_{2n+1}) = 2ni + 1, \quad i = 1, 2, \ldots, n.$$  

$$f_{orpp}(v_i v_{2n+2}) = 2n+1 + i, \quad i = 1, 2, \ldots, n.$$  

$$f_{orpp}(v_{n+i} v_{2n+1}) = (n+i)(2n+1)+1, \quad i = 1, 2, \ldots, n.$$  

$$f_{orpp}(v_{n+i} v_{2n+2}) = (n+i)(2n+2)+1, \quad i = 1, 2, \ldots, n.$$  

Clearly $f_{orpp}$ is an injection.

$\text{gcd }$ of $(v_i)$

$$= \text{gcd } \{f_{orpp}(v_i v_{2n+1}),$$

$$f_{orpp}(v_i v_{2n+2})\}$$

$$= \text{gcd } \{2ni+1, 2n+1+i\}$$

$$= \text{gcd } \{i, (2n+1)i + 1\}$$

$$= 1, \quad i = 1, 2, \ldots, n.$$  

$\text{gcd }$ of $(v_{n+i})$

$$= \text{gcd } \{f_{orpp}(v_{n+i} v_{2n+1}),$$

$$f_{orpp}(v_{n+i} v_{2n+2})\}$$

$$= \text{gcd } \{(n+i)(2n+1)+1, (n+i)(2n+2)+1\}$$

$$= \text{gcd } \{n+i, (n+i)(2n+1)+1\}$$

$$= 1, \quad i = 1, 2, \ldots, n.$$  

$\text{gcd }$ of $(v_{2n+1})$

$$= \text{gcd } \{f_{orpp}(v_1 v_{2n+1}),$$

$$f_{orpp}(v_2 v_{2n+1})\}$$

$$= \text{gcd } \{2n+2, 4n+3\}$$

$$= \text{gcd } \{2n+1, 2n+2\}$$

$$= 1.$$  

$\text{gcd }$ of $(v_{2n+2})$

$$= \text{gcd } \{f_{orpp}(v_{2n+2})\}$$

$$= \text{gcd } \{2n+3, 4n+5\}$$

$$= \text{gcd } \{2n+2, 2n+3\}$$

So, $\text{gcd }$ of each vertex of degree greater than one is 1. Hence, $G$ admits one raised product prime labeling.
Example 3.7  $G = D(K_{1,4})$

Theorem 3.8 Tensor product of path $P_2$ and star $K_{1,n}$ admits one raised product prime labeling.

**Proof:** Let $G$ be the graph and let $v_1,v_2,\ldots,v_{2n+2}$ are the vertices of $G$.

Here $|V(G)| = 2n+2$ and $|E(G)| = 2n$.

Define a function $f : V \to \{1,2,\ldots,2n+2\}$ by

$$f(v_i) = i, \ i = 1,2,\ldots,2n+2$$

Clearly $f$ is a bijection.

For the vertex labeling $f$, the induced edge labeling $f_{orpp}$ is defined as follows

$$f_{orpp}(v_{2i+1}) = i(2n+1)+1, \ i = 1,2,\ldots,n.$$  

$$f_{orpp}(v_{2i+2} v_{n+i}) = (2n+2)(n+i)+1, \ i = 1,2,\ldots,n.$$  

Clearly $f_{orpp}$ is an injection.

**gein** of $(v_{2n+1})$

$$= \gcd \{ f_{orpp}(v_1 v_{2n+1}), f_{orpp}(v_{2} v_{2n+1}) \}$$

$$= \gcd \{ 2n+2, 4n+3 \}$$

$$= \gcd \{ 2n+1,2n+2 \} = 1.$$  

**gein** of $(v_{2n+2})$

$$= \gcd \{ f_{orpp}(v_{n+1} v_{2n+2}), f_{orpp}(v_{n+2} v_{2n+2}) \}$$

$$= \gcd \{ 2n^2+4n+3, 2n^2+6n+5 \}$$

$$= \gcd \{ 2n+2(2n+2)(n+1)+1 \}$$

$$= 1.$$  

So, **gein** of each vertex of degree greater than one is 1. Hence $G$ admits one raised product prime labeling.

Example 3.8  $G = $ Tensor product of $P_2$ and $K_{1,3}$.

Theorem 3.9 Jelly fish graph $JF(m,n)$ admits one raised product prime labeling.

**Proof:** Let $G$ be the graph and let $v_1,v_2,\ldots,v_{m+n+4}$ are the vertices of $G$.

Here $|V(G)| = m+n+4$ and $|E(G)| = m+n+5$.

Define a function $f : V \to \{1,2,\ldots,m+n+4\}$ by

$$f(v_i) = i, \ i = 1,2,\ldots,m+n+4$$

Clearly $f$ is a bijection.

For the vertex labeling $f$, the induced edge labeling $f_{orpp}$ is defined as follows

$$f_{orpp}(v_{m+1} v_i) = (m+i)(m+i+1)+1, \ i = 1,2,3.$$  

$$f_{orpp}(v_{m+i} v_{m+i+1}) = (m+i)(m+i+1)+1.$$  

$$f_{orpp}(v_{m+i} v_{m+3}) = (m+1)(m+3)+1.$$  

$$f_{orpp}(v_{m+i} v_{m+4}) = (m+2)(m+4)+1.$$  

Clearly $f_{orpp}$ is an injection.

**gein** of $(v_{m+i})$

$$= \gcd \{ f_{orpp}(v_{m+i-1} v_{m+i}), f_{orpp}(v_{m+i} v_{m+i+1}) \}$$

$$= \gcd \{ (m+i)^2-(m+i)+1, (m+i)^2+(m+i)+1 \}$$

$$= \gcd \{ (m+i), (m+i)(m+i)+1 \}$$

$$= 1, \ i = 1,2,3,4.$$  

So, **gein** of each vertex of degree greater than one is 1. Hence $G$ admits one raised product prime labeling.

Example 3.9  $G = JF(4,3)$

IV. Conclusion

Every graph is not one raised product prime graphs. So it is very interesting to find graphs or graph families which admit one raised product prime labeling. Here 9 graphs related to star graph family are investigated. This can be extended to some other graphs.

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