Your Blockchain Needn’t Care
How the Message is Spread

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Abstract. In a blockchain system, nodes regularly distribute data to other nodes. The ideal perspective taken in the scientific literature is that data is broadcast to all nodes directly, while in practice data is distributed by repeated multicast. Since correctness and security typically have been established for the ideal setting only, it is vital to show that these properties carry over to real-world implementations. This can be done by proving that the ideal and the real behavior are equivalent. In the work described in this paper, we take an important step towards such a proof by proving a simpler variant of the above equivalence statement. The simplification is that we consider only a concrete pair of network topologies, which nevertheless illustrates important phenomena encountered with arbitrary topologies. For describing systems that distribute data, we use a domain-specific language of processes that is embedded in a general-purpose process calculus. This allows us to leverage the rich theory of process calculi in our proof, which is machine-checked using the Isabelle proof assistant.

Keywords: Blockchain · Networking · Process calculus · Bisimilarity · Isabelle · Formal methods

1 Introduction

Blockchains are becoming increasingly relevant in a variety of fields, such as finance, identification, logistics, and real estate. Incorrect behavior of a blockchain system may often result in serious damage. Therefore, machine-checked proofs of correctness and security are of high value in the blockchain field.

The fundamental task of a blockchain system is to maintain data consistency among distributed agents in an open network. To facilitate this, nodes have to regularly distribute data to other nodes. The perspective usually taken in the scientific literature is that such data is broadcast to all nodes directly. For example, the descriptions of the blockchain consensus protocols of the Ouroboros family \cite{ouroboros1,ouroboros2,ouroboros3} assume such direct communication. In practice, however, data is distributed via repeated multicast. Since correctness and security typically have...
been established for the ideal setting only, it is vital to show that they carry over to real-world implementations.

In this paper, we take an important step in this direction by showing that the ideal behavior of direct broadcast and the real behavior of broadcast via multicast are equivalent in an appropriate sense. Concretely, we make the following contributions:

– In Sect. 2, we define a restricted language of processes that are able to describe network communication. Processes in our language are closely connected to hierarchical Petri nets with exactly one input place per transition. We define our language via an embedding in a general-purpose process calculus. This approach enables us to leverage the rich theory of process calculi while allowing us to use an intuitive graphical notation similar to the one of Petri nets.

– In Sect. 3, we devise a notion of behavioral equivalence of networks that does not distinguish between different patterns of packet arrival. Our approach is to start with bisimilarity and weaken it by amending the involved processes to allow for additional behavior. Building on bisimilarity permits us in particular to reason in a modular fashion.

– In Sect. 4, we present a proof of behavioral equivalence of broadcast via repeated multicast and direct broadcast under the assumption that network communication may involve packet loss and duplication. Our proof is about a concrete pair of networks that nevertheless captures important general phenomena. The proof works by rewriting a process describing the former form of broadcast into a process describing the latter. For the individual rewriting steps, we rely on certain fundamental lemmas, not all of which have been proved so far. We present only the first part of our proof in detail, using the graphical notation for processes, but we have formalized the whole proof in Isabelle/HOL.

Afterwards, we discuss related work in Sect. 5 and give a conclusion and an outlook on ongoing and future work in Sects. 6 and 7.

2 A Language for Communication Networks

For describing communication networks, we use a custom language of processes that communicate via asynchronous channels. Let uppercase letters denote processes and lowercase letters denote channels. The syntax of our communication language is given by the following BNF rule:

\[
\text{Process ::= } 0 \mid P \parallel Q \mid \nu a. P \mid a \rightarrow b \mid a \Rightarrow [b_1, \ldots, b_n] \mid \square^2 a \mid \square^+ a \mid \square^* a
\]

A process is one of the following:

– The stop process 0, which does nothing
– A parallel composition $P \parallel Q$, which performs $P$ and $Q$ in parallel
– A restricted process \( \nu a. P \), which behaves like \( P \) except that the channel \( a \) is local
– A bridge \( a \rightarrow b \), which continuously forwards packets from channel \( a \) to channel \( b \)
– A distributor \( a \Rightarrow [b_1, \ldots, b_n] \), which continuously forwards packets from channel \( a \) to all channels \( b_i \)
– A loser \( \square \!, a \), which continuously drops packets from channel \( a \)
– A duplicator \( \square ^+ a \), which continuously duplicates packets in channel \( a \)
– A duploser \( \square ^* a \), which continuously drops packets from and continuously duplicates packets in channel \( a \)

Our language is embedded in the \( \Pi \)-calculus\(^4\), a general-purpose process calculus that is itself embedded in Isabelle/HOL. The first three constructs of our language stem directly from the \( \Pi \)-calculus, distributors are defined in terms of the more primitive send and receive constructs the \( \Pi \)-calculus provides, and the remaining constructs are defined in terms of the former ones.

The sublanguage formed by \( 0, \|, \nu, \text{ and } \Rightarrow \) corresponds closely to hierarchical Petri nets with exactly one input place per transition, and the other constructs can be analogously derived for such Petri nets. Based on this close connection to Petri nets, we introduce a graphical representation of communication processes as communication nets, which is shown in Fig. 1. Note that \( 0 \) and \( \| \) are represented by absence and composition, respectively.

Two complete communication nets are shown in Fig. 2. They characterize the two networks whose behavioral equivalence we will show in Sect. 4. In both communication nets, channels \( s_i \) and \( r_i \) form the interface of a network node \( i \), with \( s_i \) accepting packets for sending and \( r_i \) providing received packets. The local channel \( m \) in the left communication net represents the broadcast medium, and each local channel \( l_{ij} \) in the right communication net represents a multicast

\(^4\) See https://github.com/input-output-hk/thorn-calculus.
link from node $i$ to node $j$. Note that we assume any communication to be unreliable, which is reflected by the channel $m$ and all channels $l_{ij}$ having duplosers attached.

Fig. 2. Example of direct broadcast (left) and broadcast via multicast (right)

The two communication nets correspond to processes $D$ (direct broadcast) and $M$ (broadcast via multicast) defined as follows:

$$D = \nu m. (\Box^* m \parallel s_0 \to m \parallel \ldots \parallel s_3 \to m \parallel m \to r_0 \parallel \ldots \parallel m \to r_3) \quad (1)$$

$$M = \nu l_{01}, \nu l_{02}, \nu l_{13}, \nu l_{23}, \nu l_{30}. (M_* \parallel M_i \parallel M_o) \quad (2)$$

$$M_* = \Box^* l_{01} \parallel \Box^* l_{02} \parallel \Box^* l_{13} \parallel \Box^* l_{23} \parallel \Box^* l_{30} \quad (3)$$

$$M_i = s_0 \Rightarrow [l_{01}, l_{02}] \parallel s_1 \Rightarrow [l_{13}] \parallel s_2 \Rightarrow [l_{23}] \parallel s_3 \Rightarrow [l_{30}] \quad (4)$$

$$M_o = l_{01} \Rightarrow [r_1, l_{13}] \parallel l_{02} \Rightarrow [r_2, l_{23}] \parallel l_{13} \Rightarrow [r_3, l_{30}] \parallel l_{23} \Rightarrow [r_3, l_{30}] \parallel$$

$$l_{30} \Rightarrow [r_0, l_{01}, l_{02}] \quad (5)$$

3 Loss-Agnostic Behavioral Equivalence

Weak bisimilarity would be a natural choice for the kind of equivalence that should hold between the two network processes shown in the previous section. Weak bisimilarity is a well-established notion of behavioral equivalence that provides a fine-grained distinction of observable behavior and allows for modular
For our communication language, its notion of behavior characterizes essentially how handing over packets to global channels may result in packets appearing on possibly other global channels.

Unfortunately, weak bisimilarity turns out to be too strict for our situation, as it is able to distinguish between broadcast via multicast and direct broadcast. To see why, assume in each of the networks shown in Fig. 2 a packet is sent by node 0 and this packet makes it to node 3. With direct broadcast, it is possible that neither node 1 nor node 2 receives the packet as well. With broadcast via multicast, however, node 1 or node 2 must receive it and must do so before node 3 receives it.

To remove this constraint on arrival patterns, we make the receive channels lossy. This way, intermediate nodes are no longer guaranteed to receive packets. It is not sufficient, however, to introduce this lossiness for broadcast via multicast only; we need to introduce it also for direct broadcast. This is because packet loss in the receive channels is observable and consequently unilateral introduction of such loss would create another behavioral mismatch between the two networks. The approach of making receive channels lossy leads to a notion of weak bisimilarity up to loss, which is derived from weak bisimilarity (written \( \approx \) in the following):

**Definition 1 (Weak bisimilarity up to loss).** Two processes \( P \) and \( Q \) are weakly bisimilar up to loss in channels \( r_1 \) to \( r_n \) exactly if

\[
\square r_1 \ || \ \ldots \ || \ \square r_n \ || \ P \ \approx \ \square r_1 \ || \ \ldots \ || \ \square r_n \ || \ Q.
\]

### 4 A Proof of Correctness of Broadcast via Multicast

Broadcast via multicast is expected to behave equivalently to direct broadcast as long as the multicast network is strongly connected. In this work, however, we prove this equivalence only for the particular networks depicted in Fig. 2, which nevertheless capture important phenomena that show up in other cases:

- The multicast network has a node with several outgoing and a node with several incoming links.
- In the multicast network, some nodes are reachable from certain other nodes only via more than one hop.

Concretely, our goal is to prove that \( M \), defined in Eq. 2, and \( D \), defined in Eq. 1, are weakly bisimilar up to loss in the receive channels, which is expressed by the statement

\[
\square r_0 \ || \ \ldots \ || \ \square r_3 \ || \ M \ \approx \ \square r_0 \ || \ \ldots \ || \ \square r_3 \ || \ D.
\]

We prove this statement by turning its left-hand side into its right-hand side through a series of transformation steps, each of which replaces subprocesses with bisimilar ones. The individual transformation steps rely on several fundamental lemmas about the communication language, not all of which we have proved yet.
The first transformation step deals with the distributors that forward packets from link channels. These distributors deliver packets from the link channels \( l_{ij} \) to the receive channels \( r_j \) and also relay them to the follow-up link channels \( l_{jk} \). The first transformation step splits each of these distributors into multiple bridges and thus in particular separates delivery and relaying of packets. The subsequent transformation steps collapse the relaying part into a single channel: the broadcast medium. We only discuss the first step in detail.

The first transformation step takes the broadcast-via-multicast process depicted in Fig. 2 with the receive channels made lossy and turns it into the process depicted in Fig. 3. The key justification for this transformation is the existence of the distributor splitting lemma, which is shown in Fig. 4. Note that, in order to apply this lemma, the respective source channel must have an accompanying duplicator and the respective target channels must have accompanying losers. The link channels, which also act as target channels and additionally as source channels, have duplosers attached to them. Since a duploser is defined as a parallel composition of a loser and a duplicator, the link channels fulfill the conditions we require for source and target channels.

![Fig. 3. The broadcast-via-multicast process after distributor splitting](image)

We have conducted the complete proof using communication nets and additionally developed a formal version of it\(^5\) using the Isabelle proof assistant.

\(^5\) See [https://github.com/input-output-hk/network-equivalences](https://github.com/input-output-hk/network-equivalences).
The formal version is phrased in a style similar to equational reasoning, with the difference that we use weak bisimilarity instead of equality. Since Isabelle does not come with support for automated rewriting based on equivalence relations other than equality, we have developed a corresponding extension\(^6\), which we use in our proof.

While the formal proof closely follows its communication net counterpart, it contains more technical details. In particular, it includes applications of basic bisimilarities, like associativity and commutativity of parallel composition, which the communication net proof does not need, because the communication net notation identifies processes that are bisimilar according to these basic bisimilarities. However, all these bisimilarities together form a confluent and terminating rewrite system, allowing our engine for equivalence-based rewriting to automatically find any proof that consists of a chain of rewriting steps involving only these bisimilarities. As a result, we can bundle consecutive applications of said basic bisimilarities in our formal proof, which therefore is still reasonably concise and readable, while being machine-checked at the same time.

\[\text{Fig. 4. The distributor splitting lemma}\]

5 Related Work

The correctness of broadcast via multicast, commonly referred to as \textit{network flooding}, and similar techniques has been studied to some extent in the literature. Following are two examples:

- Bani-Abdelrahman [2] formally specifies synchronous and bounded asynchronous flooding algorithms using LTL and verifies them using the model checker NuSMV. His results are limited to small network sizes and fixed delays, though.
- Bar-Yehuda et al. [3] emulate a single-hop (direct-broadcast) network with a multi-hop network using a synchronous gossiping algorithm. With a gossiping algorithm, a node does not relay an incoming packet to all neighboring nodes, but only to a randomly chosen one.

The existing literature, however, seems to lack the study of an \textit{equivalence} of the two broadcasting approaches, which we provide in this work. The reason for

\(^6\) See [https://github.com/input-output-hk/equivalence-reasoner](https://github.com/input-output-hk/equivalence-reasoner).
this lack may be related to the complications that arise due to the behavioral mismatches explained in Sect. 3.

There is a vast amount of literature on the relationship between process calculi and Petri nets. In particular, a long line of research has been developed with the theme of giving Petri net semantics to process calculi [5,7,10], providing process calculi with operational semantics expressing true concurrency as opposed to the traditional interleaving semantics. Another line of research approaches the reverse problem, that is, finding process calculi that are suitable for modeling Petri nets of certain classes [4,8]. In the light of all this research, our tandem of the communication language and its communication net notation, described in Sect. 2, can be regarded as a restricted process calculus that comes with a Petri net semantics.

6 Conclusion

We have defined a language for describing communication networks, which is embedded in a process calculus and closely connected to a class of Petri nets. Based on the connection to Petri nets, we have devised a graphical notation for our language. Building on this foundation, we have proved behavioral equivalence of broadcast via multicast and direct broadcast for a typical pair of networks. The graphical notation has allowed us to reason in an intuitive way, while the embedding in a process calculus has permitted us to develop a fully machine-checked proof. For specifying the equivalence between the two realizations of broadcast, we have devised the notion of weak bisimilarity up to loss.

7 Ongoing and Future Work

At the moment, we are completing the proofs of the fundamental lemmas that the correctness proof shown in Sect. 4 uses. Furthermore, we are working on a variant of our correctness proof that deals with broadcast integrated with packet filtering according to a fixed predicate.

In the future, we want to prove a modified correctness statement where the receive channels of direct broadcast are not lossy. This will clarify that broadcast via multicast has the more constrained behavior, but will require adjustments to the specification of network behavior. Furthermore, we want to generalize our correctness proofs to apply to arbitrary strongly connected multicast networks and their direct-broadcast counterparts. Finally, we want to generalize the proof about broadcast integrated with filtering to work with state-dependent filters.

Acknowledgements We want to thank James Chapman, Duncan Coutts, Kevin Hammond, and Philipp Kant, who supported us in our work on network equivalences and the development of all the theory and the vast amount of Isabelle formalizations that underlie it. We appreciate IO Global funding this very interesting work.
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