A note on the polarization of the laser field in Mott scattering

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Abstract

In the first Born approximation and using an elliptically polarized laser field, the Mott scattering of an electron by a Coulomb potential is investigated using the Dirac-Volkov states to describe the incident and scattered electrons. The results obtained are compared with the results of S.M. Li et al [1] for the case of a linearly polarized laser field and with the results of Y. Attaourti et al [2] for the case of a circular polarization.

PACS number(s): 34.80.Qb, 12.20.Ds

1 Introduction

In this note, we give definite analytical results concerning the process of Mott scattering in the presence of a strong laser field and compare our results to previous theoretical works, namely the work of S.M. Li et al [1] and the work of Y. Attaourti et al [2]. We hope this contribution will be useful to all researchers working in this field and will bring an end to the controversy raised by the expression found by C. Szymanowski et al [3] for the case of a circular polarization of the laser field. An analytical expression for the spin-unpolarized differential cross section is derived using trace calculations. The electric field strength as well as the frequency of the laser field and the kinetic energy of the incoming electron being key parameters, the study of the process of Mott scattering in the presence of an elliptically polarized laser field introduces a new key parameter, namely the degree of ellipticity $\eta$. The cross section dependency on this new key parameter is reported. The general features of the
Mott scattering process are qualitatively modified when a laser field is present and this is particularly true when one study the spin-dependent relativistic Mott scattering. Not only it is important to take care of the fact that the electron is a fermion but also to describe this particle by the appropriate wave function in a non-perturbative way. This is done by using the Dirac-Volkov wave functions [4] which contain the interaction of the electron with the laser field to all orders. The organization of this paper is as follows. In section II, we present the theory in the first Born approximation. In section III, we discuss the analytical results for the spin-unpolarized differential cross section modified by the laser field and analyze their dependencies on the new relevant parameter, that is the degree of ellipticity $\eta$. We end by a brief summary and conclusion in section IV. Throughout this work, we use atomic units $\hbar = m = e = 1$ and work with the metric tensor $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

## 2 Theory

We treat the laser field classically since we are considering intensities that do not allow pair creation [3]. The four potential corresponding to the laser field satisfies the Lorentz condition $\partial^\mu A_\mu = 0$ and is given by

$$A = a_1 \cos(\phi) + a_2 \sin(\phi) \tan(\eta/2),$$

with $\phi = k.x = k^\mu x_\mu = wt - k.x$ and where $\eta$ is the degree of ellipticity of the laser field. The four vectors $a_1$ and $a_2$ satisfy the following relations $a_1^2 = a_2^2 = a^2$ and the Lorentz condition $k.A$ implies $a_1.k = a_2.k = 0$. The linear polarization is obtained for $\eta = 0$ and the circular polarization is obtained for $\eta = \pi/2$. The electric field associated with the potential of the laser field is

$$E = -\frac{1}{c} \frac{\partial}{\partial t} A.$$

### 2.1 Differential cross section

The interaction potential is the Coulomb potential of a target nucleus of charge $Z$

$$A^\mu_{Coul} = \left(-\frac{Z}{|x|}, 0, 0, 0\right),$$

with $x = (x, y, z)$. The dot product $\cdot$ denotes the usual 3-dimensional Euclidean inner product. The Lorentz transformation is given by $A^\mu' = L_{\mu\nu} A^\nu$, where $L_{\mu\nu}$ is the Lorentz transformation matrix.
and in the first Born approximation, the transition matrix element for the transition \((i \rightarrow f)\) is

\[
S_{fi} = \frac{i Z}{c} \int d^4 x \bar{\psi}_{q_f}(x) \frac{\gamma^0}{x} \psi_{q_i}(x).
\]  

(4)

The Dirac-Volkov wave functions \(\psi_{q_i}(x)\) and \(\psi_{q_f}(x)\) describe the incident and scattered electron respectively. Such wave functions normalized to the volume \(V\) are [4]

\[
\psi_q(x) = R(q) \frac{u(p, s)}{\sqrt{2QV}} e^{i S(q, x)},
\]  

(5)

where \(u(p, s)\) represents a Dirac bispinor normalized as \(u(p, s)u(p, s) = 2c^2\) and \(q^\mu = (Q/c, q)\) is the quasi-impulsion acquired by the electron in the presence of the laser field

\[
q^\mu = p^\mu - \frac{1}{2c^2(k.q)} \tilde{A}^2 k^\mu.
\]  

(6)

The quantity \(R(q)\) is defined by \(R(q) = 1 + \frac{k.A}{2c(k.q)}\), and \(S(q, x) = -(q.x) - (a_1.q \sin(\phi) - (a_2.q \cos(\phi) \tan(\eta/2))/(c(k.q)), where the Feynmann slash notation is used [5]: for a given four vector \(A\), we have \(A = \gamma^\mu A_\mu\). Finally the averaged squared potential \(\overline{A^2}\) is given by

\[
\overline{A^2} = a^2(1 + \tan^2(\eta/2))/2,
\]  

(7)

from which one deduces \(\overline{A^2} = a^2\) for the case of a circular polarization of the laser field and \(\overline{A^2} = a^2/2\) for the case of a linear polarization of the laser field. The argument \(z\) of the ordinary Bessel functions is

\[
z = \sqrt{\alpha_1^2 + \alpha_2^2},
\]  

(8)

with

\[
\alpha_1 = \frac{(a_1.p_i)}{c(k.p_i)} - \frac{(a_1.p_f)}{c(k.p_f)}.
\]  

(9)

and

\[
\alpha_2 = \left[\frac{(a_2.p_i)}{c(k.p_i)} - \frac{(a_2.p_f)}{c(k.p_f)}\right] \tan(\eta/2).
\]  

(10)

A useful parameter that intervenes in the expression of the DCS is \(\phi_0 = \arctan(\alpha_2/\alpha_1)\). Using the standard procedures of QED [5], we obtain for the spin-unpolarized differential cross section evaluated for \(Q_f = Q_i + sw\)
Using REDUCE for the trace calculations \[6\], we obtain

\[
\frac{d\sigma}{d\Omega_f} = \sum_{s=-\infty}^{s=\infty} \frac{1}{d\Omega_f} \left( \frac{1}{2} \sum_{s_{1,s_f}} |M_f^{(s)}|^2 \right).
\]

(11)

with

\[
\frac{d\sigma^{(s)}}{d\Omega_f} = \frac{Z^2}{c^4} \left| q_f \right| \left| q_i - \mathbf{s}k \right|^4 \left( \frac{1}{2} \sum_{s_{1,s_f}} |M_f^{(s)}|^2 \right).
\]

(12)

Using REDUCE for the trace calculations \[5\], we obtain

\[
\frac{1}{2} \sum_{s_{1,s_f}} |M_f^{(s)}|^2 = 2 \left\{ J_2^2(z)A + (J_2^2(z) + J_2^2(z))B \right. \\
+ J_{s+1}(z)J_{s-1}(z)C + J_s(z)(J_{s+1}(z) + J_{s-1}(z))D \\
+ J_s(z)(J_{s+2}(z) + J_{s-2}(z))E + (J_{s+2}(z) + J_{s-2}(z))F \\
+ (J_{s-1}(z)J_{s+2}(z) + J_{s+1}(z)J_{s-2}(z))G \\
+ (J_{s+1}(z)J_{s+2}(z) + J_{s-1}(z)J_{s-2}(z))H \left. \right \}.
\]

(13)

The eight coefficients \(A, B, C, D, E, F, G\) and \(H\) are given respectively by

\[
A = c^4 - (q_i q_f) c^2 + 2Q_i Q_f - \frac{a^2}{2} \left( \frac{(k.q_f)}{(k.q_i)} + \frac{(k.q_i)}{(k.q_f)} \right) \\
+ \frac{a^2 w^2}{c^2 (k.q_i)(k.q_f)} ((q_i q_f) - c^2)(1 + \tan^2(\eta/2))/2 \\
+ \frac{(a^2)^2 w^2}{c^4 (k.q_i)(k.q_f)} \left( \frac{1}{8} \tan^4(\eta/2) + \frac{5}{8} + \frac{1}{4} \tan^2(\eta/2) \right) \\
+ \frac{a^2 w}{c^2} \left( \frac{Q_f}{(k.q_i)} + \frac{Q_i}{(k.q_f)} - \left( \frac{Q_i}{(k.q_i)} + \frac{Q_f}{(k.q_f)} \right) \times (1 + \tan^2(\eta/2))/2 \right),
\]

(14)

\[
B = \frac{w^2}{2c^2} \left( \frac{(a_1 q_i)}{(k.q_i)(k.q_f)} + \frac{(a_2 q_i)}{(k.q_i)(k.q_f)} \tan(\eta/2) \right) \\
- \left\{ \frac{a^2}{2} + \frac{(a^2)^2 w^2}{2c^4 (k.q_i)(k.q_f)} - \frac{a^2}{4} \left( \frac{(k.q_f)}{(k.q_i)} + \frac{(k.q_i)}{(k.q_f)} \right) \right. \\
+ \frac{a^2 w^2}{2c^2 (k.q_i)(k.q_f)} (q_i q_f - c^2) - \frac{a^2 w}{2c^2} (Q_f - Q_i) \times \left( \frac{1}{(k.q_f)} - \frac{1}{(k.q_i)} \right) \left(1 + \tan^2(\eta/2)/2 \right),
\]

(15)

\[
C = \frac{w^2}{c^2 (k.q_i)(k.q_f)} \cos(2\phi_0) \left\{ (a_1 q_i)(a_1 q_f) \right. \\
\left. \right \}
\]
\[
D = \left( a^2 q_i (a_2 q_f) \tan^2(\eta/2) \right) + \sin(2\phi_0) \left\{ (a_2 q_i)(a_1 q_f) - \frac{a^2}{4} \left( \frac{k q_f}{k q_i} + \frac{k q_i}{k q_f} \right) \right\}
\]

\[
E = \cos(2\phi_0)(\tan^2(\eta/2) - 1)a^2 w \times \left\{ -\frac{(q_i q_f) w}{4c^2(k q_i)(k q_f)} + \frac{1}{4c^2} \left( \frac{Q_i}{k q_i} + \frac{Q_f}{k q_f} \right) \right\}
\]

\[
F = (\tan^2(\eta/2) - 1)^2 \left( \frac{(a^2)^2 w^2}{32c^4(k q_i)(k q_f)} \right),
\]

\[
G = (\tan^2(\eta/2) - 1) \frac{a^2 w^2}{8c^3(k q_i)(k q_f)} \left\{ \cos(3\phi_0)((a_1 q_i) + (a_1 q_f)) \right. \\
+ \left. \sin(3\phi_0)((a_2 q_i) + (a_2 q_f)) \tan(\eta/2) \right\},
\]
\[ H = \left( \tan^2(\eta/2) - 1 \right) \frac{a^2 w^2}{8c^3(k.q_i)(k.q_f)} \left\{ \cos(\phi_0)(a_1.q_i + a_1.q_f) - \sin(\phi_0)(a_2.q_i + a_2.q_f) \tan(\eta/2) \right\}, \]  

where \( \dot{A} = a_1 \cos(\phi_0) + a_2 \sin(\phi_0) \tan(\eta/2) \). In the absence of the laser field, all the contributions coming from the sum over \( s \) of the various ordinary Bessel functions vanish except for \( s = 0 \) where \( J_s(0) = \delta_{s0} \) and we recover the well known formula for Mott scattering in the absence of the laser field \[5\]

\[ \frac{d\sigma}{d\Omega_i} = \frac{1}{4} \frac{Z^2 \alpha^2}{|\mathbf{p}|^2 \beta^2} \frac{(1 - \beta^2 \sin^2(\theta/2))}{\sin^3(\theta/2)}, \]  

where \( \theta = (\mathbf{p}_i, \mathbf{p}_f) \). It can easily be checked that for the case of a linear polarization of the laser field (\( \eta = 0 \)), the phase \( \phi_0 = 0 \) and the results of S.M. Li et al [1] are straightforward to obtain whereas for the circular polarization (\( \eta = \pi/2 \)), we find the results previously found by Y. Attaourti et al [2]. Eq. (11) is the relativistic generalization of the Bunkin and Fedorov [7] treatment and it contains the degree of ellipticity of the laser field as a new key parameter.

## 3 Results and discussion.

In this section, we discuss the numerical simulations for the differential cross sections of the Mott scattering by an elliptically polarized laser field. We assume without loss of generality that the target is a proton having a charge \( Z = 1 \). The \( z \) axis is set along the direction of the field wave vector \( \mathbf{k} \), \( a_1' = (0, a_1) \) and \( a_2' = (0, a_2) \) with the vectors \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) such that \( \mathbf{a}_1 = |\mathbf{a}|(1, 0, 0) \) and \( \mathbf{a}_2 = |\mathbf{a}|(0, 1, 0) \) from which one deduces that \( \overline{A^2} = a^2(1 + (\tan(\eta/2))^2)/2 = -|\mathbf{a}|^2(1 + (\tan(\eta/2))^2)/2 \). Thus, as found by S.M. Li [1] for the linear polarization, we have \( \overline{A^2} = -|\mathbf{a}|^2/2 \) and for the case of circular polarization [2], \( \overline{A^2} = -|\mathbf{a}|^2 \). The laser frequency used is \( w = 0.043 \) (a.u) which corresponds to a photon energy \( \hbar w = 1.17 \) eV. The incident electron kinetic energy is \( T_i = 2.7 \) keV which corresponds to a relativistic parameter \( \gamma = (1 - \beta^2)^{-\frac{1}{2}} = 1.0053 \) and the electric field strength has the value \( \mathcal{E} = 0.05 \) (a.u) = 2.57 \( 10^8 \) V/cm. The various differential cross sections are plotted as functions of the angle \( \theta_i \). For small scattering angles, typically \( 1^\circ \leq \theta_i \leq 15^\circ, 1^\circ \leq \phi_i \leq 15^\circ \), \((-180^\circ \leq \theta_f \leq 180^\circ, \phi_f = \phi_i + 90^\circ \), the summed spin-unpolarized differential cross sections are sharply peaked around \( \theta_f = 0^\circ \) and are all close to the corresponding unpolarized laser-free
differential cross section given in Eq. (22). The three DCSs corresponding to \( \eta = 0, \eta = 2\pi/3 \)
and \( \eta = \pi/2 \) are given in Fig. 1 together with the laser-free DCS for the geometry \( (\theta_i = 15^\circ, \phi_i = 15^\circ), (-180^\circ \leq \theta_f \leq 180^\circ, \phi_f = \phi_i + 90^\circ) \). We obtain four almost indistinguishable curves. Thus, at small angles, the summed differential cross sections are almost unmodified by the laser field and its polarization does not play a key role. The physical explanation of this observation is that classically, when the particles are close to the small angle scattering region, this corresponds to large impact parameters and the incident electron does not deviate notably from its trajectory. For other scattering angles, \( (45^\circ \leq \theta_i \leq 89^\circ, 45^\circ \leq \phi_i \leq 89^\circ), (-180^\circ \leq \theta_f \leq 180^\circ, \phi_f = \phi_i + 90^\circ) \), the situation changes drastically since for medium and large scattering angles, the momentum transfer during the Mott scattering is large and a significant number of photons can be exchanged with the laser field.

In Fig. 2, we compare the three DCSs corresponding to \( \eta = 0, \eta = 2\pi/3 \) and \( \eta = \pi/2 \) together with the laser-free DCS for the geometry \( (\theta_i = 60^\circ, \phi_i = 0^\circ), (-180^\circ \leq \theta_f \leq 180^\circ, \phi_f = 90^\circ) \) for an exchange of \( \pm 150 \) photons. In this geometry, the effect of the laser
field polarization is clearly shown since the three DCSs are now well distinguishable. One has to sum over a very large number of photons to recover the laser-free DCS. Furthermore, all numerical simulations have shown the following. The DCS for linear polarization is always higher than the two others. The DCS for elliptical polarization is lower or higher than the DCS for circular polarization depending on the value of the degree of ellipticity $\eta$.

![Graph showing DCS for different polarizations](image)

Figure 2: The summed spin-unpolarized cross sections for an exchange of $\pm 150$ photons scaled in $10^{-5}$. As in Fig. 1, $E = 0.05$ (a.u) and $w = 0.043$ (a.u). The corresponding Mott-scattering geometry is explained in the text.

To have an idea about the behaviour of the DCS as a function of the degree of ellipticity $\eta$, we have obtained a three dimensional curve corresponding to the same geometry as in Fig. 2 but for a degree of ellipticity $\eta$ varying from 0 to $\pi/2$. There are oscillations of the DCS for the elliptical polarization and the DCS for linear polarization is always the higher DCS. These oscillations decrease as the number of photons exchanged is increased. The convergence towards the laser-free DCS is faster for the linear polarization of the laser field. For the circular polarization, this convergence is easily obtained when one finds the value for the convergence corresponding to the linear polarization whereas it is much more difficult to infer from the previous results for which value of the number of photons exchanged, the DCS for the elliptical polarization will converge to the laser-free DCS. However, depending on the value of $\eta$ and in the non relativistic regime we have chosen, this number is $\pm 1250$ photons. When the incident
electron relativistic parameter is increased from $\gamma = 1.0053$ to $\gamma = (1 - \beta^2)^{-\frac{1}{2}} = 2$, the previous mentioned results remain valid but the corresponding DCSs are very small indicating a small probability that the very fast projectile electron will exchange photons with the radiation field. As for the behaviour of the DCSs with respect to the degree of ellipticity, the elliptically and circularly polarized laser modified cross sections become more sharply peaked around the angle $\theta_f = 0^\circ$. A similar result has been reported [3].

4 Conclusions

In this work, we have extended the study of the Mott scattering process of an electron by a charged nucleus to the case of a general polarization. We have shown that the Mott-scattering geometry as well as the key parameters such as the electric field strength and the incident electron kinetic energy influence the behaviour of the DCSs. Moreover, the degree of ellipticity $\eta$ is also a key parameter for the description of the Mott scattering process particularly in the region of large momentum transfer and for a number of photons exchanged lower than that for which the DCSs tend to the laser-free one.

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