Mapping coherence in measurement via full quantum tomography of a hybrid optical detector

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Quantum states and measurements exhibit wave-like (continuous) or particle-like (discrete) character. Hybrid discrete–continuous photonic systems are key to investigating fundamental quantum phenomena1–3, generating superpositions of macroscopic states4, and form essential resources for quantum-enhanced applications5 such as entanglement distillation6–8 and quantum computation9, as well as highly efficient optical telecommunications9,10. Realizing the full potential of these hybrid systems requires quantum-optical measurements sensitive to non-commuting observables such as field quadrature amplitude and photon number11–15. However, a thorough understanding of the practical performance of an optical detector interpolating between these two regions is absent. Here, we report the implementation of full quantum detector tomography, enabling the characterization of the simultaneous wave and photon-number sensitivities of quantum-optical detectors. This yields the largest parameterization to date in quantum tomography experiments, requiring the development of novel theoretical tools. Our results reveal the role of coherence in quantum measurements and demonstrate the tunability of hybrid quantum-optical detectors.

Accurate knowledge about the operation of a quantum-optical detector is essential if it is to be used effectively, whether in foundational investigations or technological applications. Photodetectors are normally characterized by several parameters, including detectivity, spectral sensitivity and noise-equivalent power14. For quantum detectors, additional information is required for a complete specification of the detector, this being the set of operators that link the input quantum state to the classical detector output, known as the positive-operator-valued measure (POVM). This may be estimated by means of quantum detector tomography (QDT)15–18, and is necessary if the detector is to be used reliably. To date, QDT has been successfully applied to avalanche photodiodes (APDs)19, time-multiplexed detectors18,20,21, transition-edge sensors22 and superconducting nanowire detectors23. Matrix representations of the POVMs for these detectors are diagonal in the photon-number basis. Consequently, the reconstruction problem is linear and is therefore amenable to solution by means familiar to classical signal processing24. This is not true for a general quantum detector, where the POVM elements can have non-zero off-diagonals due to coherent superpositions. Even in conventional optical communications, coherent modulation and detection can increase the data transmission rate by an order of magnitude. Moreover, exploration and utilization of the full Hilbert space of a quantum system requires a detector capable of implementing a tomographically complete set of measurements25. Such a capability is also vital to fully harness the potential of hybrid quantum systems operating at the confluence of discrete and continuous variable regimes. To this end, phase-sensitive detectors that can measure coherent superpositions of photon-number states are essential to both quantum and classical optical applications. We focus on the particularly interesting example of the weak-homodyne photon detector.

Here, we introduce a QDT method for the reconstruction of the POVM of a coherent optical detector. This method is applied to two variants of a weak-homodyne detector: photon-counting and photon-number-resolving (PNRD). The POVM elements of such detectors have both phase and number sensitivity, denoted by their off-diagonal and diagonal matrix elements, respectively. Our experimental procedure, shown schematically in Fig. 1, can be applied universally to any optical detector. Although it uses only classical optical states as probes, the characterization is full in the sense that it spans the detector response in the photon-number Hilbert space. With the resulting POVM we can predict the detector response to any quantum state, including non-classical ones, in particular input modes that the detector is designed to register. Full quantum tomography is only realized by the development of a new recursive algorithm that radically reduces the computational complexity. The new recipe changes the complexity per recursion from quadratic to linear in dimension d of the POVM elements. This enables us to reconstruct a matrix of unprecedented size representing a quantum operation. In particular, we reconstruct a POVM with $1.8 \times 10^6$ parameters, almost two orders of magnitude larger than the largest quantum tomography ever performed26. By defining the transformed version of the Husimi distribution, we cast the reconstruction problem as a tractable semi-definite program, allowing us to determine both diagonal and off-diagonal elements. This makes our experiment the first full QDT.

Our QDT is performed by probing a quantum detector with a tomographically complete set of coherent states $|\alpha_\text{in}\rangle$. The measured outcome statistics gives the sampled Husimi Q function of the detector operator $Q_\alpha(\alpha) = \langle \alpha | \hat{\Pi}_\alpha | \alpha \rangle / \pi$, where $|\alpha\rangle$ is the POVM of the detector. The POVM elements can be reconstructed from $Q$ functions using convex optimization. For optical detectors, POVM elements can be written in the photon-number basis as $\hat{\Pi}_n = \sum_k \Pi_k^{(n)} |j(k)\rangle$. Detector saturation allows truncation of the Hilbert space at a finite number of $d-1$ photons, leaving us with $d^2-d$ parameters to estimate for each POVM element. In quantum-state tomography, one can reconstruct the full density matrix because $d$ is typically

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Figure 1 | Experimental set-up. The output of a laser passes through a phase modulator (PM) and an amplitude modulator (AM) to prepare a set of probe states, which are injected into the detector. The magnitudes of the probe states $|\alpha|$ are controlled by a motorized half-wave plate (HWP) followed by a Glan-Thompson polarizer (GT). The phases $\theta$ of the probe states are set by a piezo translator (see Methods, ‘Experimental details’). From the measurement statistics, the detector POVM elements are reconstructed. ND, neutral density filter; LO, local oscillator; BS, beamsplitter.

small due to the lack of bright quantum-optical sources. In contrast, quantum-optical detectors can have a large dynamic range, with $d$ typically in the range of $10^2$ to $10^5$. This makes reconstructing $O(d^2)$ parameters extremely challenging. We overcome this problem by using the transformed version of the Husimi distribution

$$\int_0^{2\pi} Q_n(\alpha)e^{-i\theta}d\theta = 2e^{-|\alpha|^2} \sum_j \Pi_{j+l} |\alpha|^{2j+l} \sqrt{(j+l)!}$$

This reduced function enables the recursive reconstruction of the principal diagonal ($l = 0$) and then each leading off-diagonal ($l = 1 \ldots d$) (see Methods, ‘Reconstruction procedure’). The number of coefficients calculated per POVM element per recursion is now no more than $d$. For instance, $l = 0$ describes a phase-averaged coherent state as input, for which the detection probabilities involve only the principle diagonals of the POVM elements. In many situations, losses and phase fluctuations restrict the number of significant off-diagonals to $l < d$. Moreover, situations involving input states with a fixed photon number $N$, like N00N states$^{27}$ or Holland–Burnett states$^{28}$, require only $N$ leading diagonals of the POVM elements to predict all measurement outcomes.

We apply the above strategy to the tomography of a weak-homodyne detector (experimental data files are provided along with the Supplementary Information). This detector combines the input state with a local oscillator (LO), typically a small-amplitude coherently driven LO, and then each leading off-diagonal ($l = 1 \ldots d$) (see Methods, ‘Reconstruction procedure’). The number of coefficients calculated per POVM element per recursion is now no more than $d$. For instance, $l = 0$ describes a phase-averaged coherent state as input, for which the detection probabilities involve only the principle diagonals of the POVM elements. In many situations, losses and phase fluctuations restrict the number of significant off-diagonals to $l < d$. Moreover, situations involving input states with a fixed photon number $N$, like N00N states$^{27}$ or Holland–Burnett states$^{28}$, require only $N$ leading diagonals of the POVM elements to predict all measurement outcomes.

We first explore experimentally the role of coherence in quantum measurements performed by a weak-homodyne APD (detection efficiency 39%) by modulating $|\alpha_{LO}|$ and $M$. We adjust $M$ by means of the relative delay between the input signal and LO. Similar results are expected for mode mismatches in other degrees of freedom. We present reconstructed one-click POVM elements for $|\alpha_{LO}|^2 = 5.5, 0.8, 5.5$ and $M = 0.99, 0.99, 0.16$ in Fig. 2a–c. The resulting variations in the relative magnitudes of the diagonals and off-diagonals of the POVM elements govern the number and phase sensitivities of the detector. Detection losses are present in the reconstructed POVMs, allowing us to estimate a quantum efficiency of 25.5%. When the detection efficiency is fixed, for perfect mode overlap as in Fig. 2a,b, reducing the LO strength leads to suppressed phase sensitivity. Mode mismatch leads to decoherence, resulting in an effective LO strength $M|\alpha_{LO}|^2$ (see Supplementary Information: ‘Effect of mode-mismatch’). This explains why the off-diagonals in Fig. 2a,b and c are similar. The difference in LO strengths is offset by their different mode overlaps. Mode mismatch also leads to modulation of the diagonals by adding a noise with Poissonian statistics (see Supplementary Information: ‘Effect of mode-mismatch’). This behaviour is present in the different diagonals of Fig. 2b and c.

Our results can be further elucidated through the Wigner functions of the POVM elements in Fig. 2d–f. The presence of off-diagonals in an operator leads to radial asymmetry in its Wigner function. Because the response of a detector to an input state is determined by the overlap of their respective Wigner functions, the phase sensitivity of a POVM element can be inferred immediately from the radial asymmetry of its Wigner function. At one extreme is the Wigner function corresponding to a detection outcome for a standard homodyne detector, which is a plane in the phase space. The opposite extreme is the Wigner function of a Fock state projection, which is a radial annulus. The experimentally reconstructed Wigner functions of our weak-homodyne detectors demonstrate the potential for tuning between the two extremes, that is, field quadrature and photon number measurements. The displacement of the dips of the Wigner functions from the origin
confirms the phase sensitivity of the POVM elements. This displacement is given by  

$$D = \sqrt{(1 - T)M |\alpha_{LO}|}$$  

(2)

matching our measurement results, which go up to 1.8, about 3.6 times the quadrature uncertainties of coherent states. Allied with the phase adjustment, this tunable region is sufficient to probe most of the non-classical state generated to date, and a mode overlap $M = 0.99, 0.99$ and 0.16, respectively. Only the relative phase between the input states and LO determines the measurement result. We assume the LO phase to be zero, making the matrix elements all real. a–c. Top: POVM elements in the Fock basis. The reconstructed coefficients are in dark blue, with the first leading off-diagonals in green for emphasis and presented again below each matrix. All other coefficients are negligible. As a reference, the POVM of an ideal PNRD has only one matrix element on the principle diagonal. Decreased effective LO strength $L$ due to the limited precision in the measurement of the experimental parameters $(T, M$ and $|\alpha_{LO}|)$, as well as the noise in the reconstructed Wigner functions that limits the precision with which $D$ can be estimated.

Figure 2 | Experimentally reconstructed one-click POVM elements of a weak-homodyne APD. Left to right: LO strengths $|\alpha_{LO}|^2 = 5.5,ur 0.8 and 5.5 photons, and a mode overlap $M = 0.99, 0.99$ and 0.16, respectively. Only the relative phase between the input states and LO determines the measurement result. We assume the LO phase to be zero, making the matrix elements all real. a–c. Top: POVM elements in the Fock basis. The reconstructed coefficients are in dark blue, with the first leading off-diagonals in green for emphasis and presented again below each matrix. All other coefficients are negligible. As a reference, the POVM of an ideal PNRD has only one matrix element on the principle diagonal. Decreased effective LO strength $L$ due to the limited precision in the measurement of the experimental parameters $(T, M$ and $|\alpha_{LO}|)$, as well as the noise in the reconstructed Wigner functions that limits the precision with which $D$ can be estimated.

Finally, we apply QDT to a weak-homodyne PNRD. Such detectors have demonstrated great potential in non-classical state as it allows us to account for the external degrees of freedom, which are difficult to control and may change the detector response in an unexpected way.
preparation and measurement\textsuperscript{11,13}, as well as coherent optical communication\textsuperscript{10}. For the PNRD in our set-up, a time-multiplexed detector with $N = 9$ time bins\textsuperscript{15,16}, convolution effects, limited detection efficiency (24%) and interference with the LO ($\langle a_{\text{LO}} \rangle^2 = 5$) requires us to extend the Hilbert space to $d = 450$ (see Supplementary Information: ‘Estimation of $d$’). The total number of real parameters involved $(N - 1)d^2 \approx 1.8 \times 10^5$ considerably exceeds the largest quantum tomography ever performed, that of an eight-qubit state with 65,536 parameters\textsuperscript{26}. Our recursive reconstruction method provides a tractable solution to this problem. Figure 3a,b presents the experimentally reconstructed POVM elements of the one- and three-click events. Their distinctive ranges of sensitivity are evident. The Wigner function of the one-click POVM element, shown in Fig. 3c, has an overlap of 98% with a $D = 1.62$ displaced single-photon state (having experienced 84.3% loss, indicating a detection efficiency of 15.7%).

Weak-field homodyne detectors with photon-number resolution provide a unique phase-sensitive measurement in that they respond concurrently to both wave-like and particle-like characteristics of input quantum states. They hold great potential for applications in quantum-information science and fundamental investigations of quantum mechanics. We have used QDT to elucidate the simultaneous wave and particle sensitivity of weak-homodyne photon-number-resolving detection. Our QDT scheme does not rely on the technical details of the measurement process, providing a universal or device-independent understanding of the role of quantum coherence in a measurement process. It foreshadows a new means of assessment and verification of more complex optical detectors\textsuperscript{9}.

Methods
Experimental details. The local oscillator and probe states were generated by splitting an amplified Ti:sapphire laser (Coherent Mira Seed, followed by a Coherent RegA regenerative amplifier, with an operating wavelength of $\lambda_0 = 830$ nm at a repetition rate of $f_0 = 256.752$ kHz, filtered by a Semrock interference filter with a bandwidth of $\Delta \lambda = 3$ nm) at a broadband beamsplitter (BBS, reflectivity 34.5%). A half-wave plate (HWP) and a polarizing beamsplitter (PBS) were used to set the power of the LO, as well as defining its polarization. The probe state amplitude $|a|$ was adjusted with a dynamic range of $1 \times 10^3$ by a HWP and a Glan–Thompson polarizer (GT). A beam sampler with low reflectivity was used to send a fraction of the probe beam to a NIST-traceable Coherent FieldMaxxi-T0 power meter to monitor $|a|$. The phase of the probe state $\theta$ was controlled with a variable delay line driven by a piezo translator (Physik Instrumente P-841.30).

The LO and probe interfered at another BBS. One output of this BBS passed through a set of precalibrated neutral density (ND) filters (see Supplementary Information: ‘Experimental details’) and was coupled into a single-mode fibre (with 44% coupling efficiency for LO and 40% for probes) to be detected by an APD or time-multiplexed PNRD. To monitor $\theta$, the other output of the recombination BBS was sent to a fast photodiode through a single-mode fibre, which was aligned in such a way as to ensure the photodiode was monitoring the same mode as that going into the weak-homodyne detector. The mode overlap $M$ was also measured with this photodiode by balancing the probe and LO, and calculating the visibility of the interference fringes when $\theta$ was scanned. Here, we adjusted $M$ using the relative delay between LO and the probe, that is, $M = \left| \int f(t) g(t) \mathrm{d}t / \int |f(t)|^2 \mathrm{d}t \right|$, where $f(t)$ and $g(t)$ are the normalized temporal amplitude of the LO and probe respectively, and $g(t) = f(t - \Delta t)$.

For each probe state amplitude $|a|$ we used 40 phase settings $\theta$, uniformly distributed between 0 and $2\pi$, and measured the clock statistics for 0.5 s.

Derivation of equation (1). Using $|\alpha|e^{i\eta}$, the coherent state projector can be expressed as

$$|\alpha\rangle\langle\alpha| = e^{-|\alpha|^2} \sum_{r,s} \frac{a_r^* a_s}{\sqrt{r! s!}} \langle r | \langle s |$$

$$= e^{-|\alpha|^2} \sum_{r,s} \frac{|\alpha|^{r+s} e^{i(r-s)\eta}}{\sqrt{r! s!}} | r \rangle \langle s |$$

Because $Q_r(\alpha) = \text{Tr} \left( \hat{\Pi}_r |\alpha\rangle\langle\alpha| / \pi \right)$, with $\hat{\Pi}_r = \sum_{s} \hat{\Pi}_s^r |r \rangle \langle s |$, equation (1) follows from

$$\int_{0}^{2\pi} e^{i(r-s)(\eta-\theta)} \mathrm{d}\theta = 2 \pi |\alpha|^{r-s} e^{i(r-s)\eta}.$$
The physical POVM set consistent with the data can be estimated using the constrained convex optimization

\[ \min ||P - F\tilde{\Pi}||_1 + g(\tilde{\Pi}) \]

subject to \( \tilde{\Pi} \geq 0 \) \( \sum_{n=0}^{N-1} \tilde{\Pi}_{nn} = I \) \( (5) \)

where \( ||M||_1 = \sqrt{\text{Tr}(M^2)} \) is the Frobenius norm and \( g(\tilde{\Pi}) \) is the regularization function, the form of which will be given later.

The reconstruction proceeds recursively, starting with the diagonals of \( \tilde{\Pi} \).

We construct the matrix \( P \) by averaging the measured statistics according to the left-hand side of equation (1). A similar averaging of the input states leads to an input matrix \( F \), and satisfies the identity

\[ p^{(l)} = p^{(l)}(\tilde{\Pi}) \]

where \( \tilde{\Pi}^{(l)} \) corresponds to only the \( l \)th diagonal of the POVM matrices. For each \( l \), one can then set up an independent semi-definite program as in equation (5).

The constraints need to be tailored at each recursion according to the conditions in equation (5) (see Supplementary Information: 'Reconstruction procedure').

The reconstruction problem effectively deconvolves a coherent state from the statistics to obtain the POVM set. This is an ill-conditioned problem, as seen by the large ratio between the largest and smallest singular values of the matrix \( F \). This makes the POVM extremely vulnerable to small fluctuations in the statistics.

This instability is taken care of by the regularization function \( g(\tilde{\Pi}) \), a convex quadratic function that still allows us to cast the regularized problem as a semi-definite program\(^{23}\). The same regularization function is enforced for each \( l \), one that penalizes large differences \( |\tilde{\Pi}_{ll}^{(l+1)} - \tilde{\Pi}_{ll}^{(l+2)}| + |\tilde{\Pi}_{ll}^{(l+2)} - \tilde{\Pi}_{ll}^{(l+1)}| \). Note that the regularization makes no assumption about the details of the quantum detector or the actual value of \( \gamma \). Variations of \( \gamma \) over two orders of magnitude produce at most a 10% difference in the values of the matrix elements of the reconstructed POVM. This is the worst-case scenario. Indeed, over the same range of variation in \( \gamma \), variation in the fidelity of the reconstructed POVMs is only \( \sim 1\% \).

The results presented in this Letter are reconstructed with \( \gamma = 0.5 \).

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Author contributions

L.Z., H.B.C.-R. and A.D. contributed equally to this work. L.Z., H.B.C.-R., X.-M.J. and L.A.W. conceived the project and contributed to the design of the experiment and to laboratory measurements and data analysis. L.Z., A.D. and M.B.P contributed modelling and data analysis. G.P., J.S.L. and B.J.S. contributed to the initial conception of the project. All authors contributed to writing the manuscript.

Additional information

The authors declare no competing financial interests. Supplementary information accompanies this paper at www.nature.com/naturephotonics. Reprints and permission information is available online at http://www.nature.com/reprints. Correspondence and requests for materials should be addressed to L.Z.