Brane curvature and supernovae Ia observations

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It is well known that modifications to the Friedmann equation on a warped brane in an anti de Sitter bulk do not provide any low energy distinguishing feature from standard cosmology. However, addition of a brane curvature scalar in the action produces effects which can serve as a distinctive feature of brane world scenarios and can be tested with observations. The fitting of such a model with supernovae Ia data (including SN 1997ff at \(z \approx 1.7\)) comes out very well and predicts an accelerating universe.

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In recent times there has been considerable activity in large extra dimensions and the recovery of the four dimensional General Relativity (GR) as an effective theory from a more fundamental theory. The basic idea in these theories is the existence of a higher dimensional bulk in which our universe, called 3-brane is sitting as a hypersurface. The standard model matter fields are confined to the brane while gravity can, by its universal character, propagate in all (including extra) dimensions. The large extra dimensions also promise to solve the mass hierarchy problem of the standard model of particle physics. A particular example is the model of Randall and Sundrum [1], which has a warped extra dimension and has attracted a lot of interest.

In this model the authors considered a 5-D anti de Sitter (AdS) bulk with a \(Z_2\)-symmetric extra dimension and by a proper fine tuning of bulk parameters they obtained a flat static brane with a vanishing effective cosmological constant on the brane. The standard Newtonian potential is recovered in the model with \(r^{-3}\) corrections arising from the massive Kaluza-Klein (KK) modes on the brane. The \(Z_2\)-symmetry is motivated by the reduction of M theory to \(E_8 \times E_8\) heterotic string theory [2].

There have been many generalizations of this model and various cosmological issues have been addressed therein (see for example [3]). The recovery of Newtonian force law in this kind of models is a non-trivial task and requires solving the perturbations of the bulk metric. It is not clear a priori whether Newtonian gravity can always be recovered in such models. A counter example is provided by the conformally non-flat Nariai metric, for which there exists no massless graviton on the brane [4]. On the other hand, this recovery has been established for the Schwarzschild-AdS (S-AdS) bulk (which is also conformally non-flat) with an FRW brane metric and it gives rise to various possible physical universes including the one with effective positive cosmological constant [5]. Hence it could be envisioned that our FRW universe may be expanding out of a Schwarzschild black hole in an AdS bulk space.

Using the Israel junction conditions [6], one can write the analogue of the Friedmann equation on the brane [7]

\[
H^2 = \frac{8\pi G}{3} \rho \left( 1 + \frac{\rho}{2\sigma} \right) + \frac{\lambda}{3} + \frac{C}{S^4} - \frac{k}{S^2}, \tag{1}
\]

where \(G\) and \(\lambda\) are respectively 4-D gravitational and cosmological constants, \(\rho\) is the matter density on the brane, \(S\) is the scale factor, \(\sigma\) is the brane tension and \(C\) is the mass parameter of the bulk black hole. The four dimensional effective constants are related to the five dimensional bulk constants through the Israel junction conditions,

\[
\lambda = \frac{\lambda_5}{2} + \frac{16\pi^2}{3} G_5^3 \sigma^2, \quad G = \frac{4\pi}{3} G_5^2 \sigma \tag{2}
\]

where \(\lambda_5 = -6/l^2\) and \(G_5\) are the 5-D cosmological and gravitational constants. Here \(l\) is the radius of curvature of the bulk spacetime.

The model differs from the standard cosmology in the following two terms in eq. (1). First, the density-squared term representing the local effects, which arises due to corrections in stress tensor by imposing junction conditions. This term decays as \(1/S^4\) in the radiation dominated epoch and would be insignificant even at the time of big bang nucleosynthesis (BBN) [8]. Second, the term varying as \(1/S^4\), commonly known as the dark radiation, represents the non local bulk effect in terms of mass of the bulk horizon. This term enters into eq. (1) through the projection of bulk Weyl curvature on the brane and behaves like radiation. It can affect the BBN and hence can be constrained by observations. As we shall show in the following, the dark radiation term can be safely neglected at the epoch even as remote in the past as \(z \approx 1.7\), which is the highest redshift observed so far in the case of the supernovae Ia. Thus there survives no observational effect of these brane world modifications and the model reduces to the standard FRW model without having any distinguishing features of its own.

It is interesting to note that if one considers the brane models with Minkowski bulk [9, 10, 11] the deviations from standard cosmology are expected at a cross over

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scale \( r_c \), defined as the ratio \( M_4^2/2M_5^3 \), where \( M_4 \) and \( M_5 \) are respectively the 4-D and 5-D Planck masses. The action in these models contains a term proportional to the curvature scalar on the brane which also helps in recovery of 4-D Newtonian gravity on the brane. The cosmological consequences of such an introduction in these models were first studied in Ref. \[10, 11\] and the Friedmann equation with the cosmological constant on the brane in these models has also been derived \[10, 12\]. The comparison of these models with cosmological observations has also been pursued \[13\].

The term proportional to curvature scalar can be viewed as a quantum correction and is also usually needed for a proper definition of stress-energy tensor on the boundary of S-AdS spacetime \[14\]. A variant of the AdS bulk brane model was considered by Kim, Lee and Myung \[15\] in which they incorporated a brane curvature. The brane in this model is a hypersurface given by the equation with the cosmological constant on the brane in these models has also been derived \[10, 11\]. The Friedmann equation in these models contains a term proportional to the scalar term in the bulk-brane action via a small coupling. In fact, the model fits the SN Ia data and study its cosmological consequences. This model provides a departure from the standard cosmology at late time evolution of the universe. Here, our main aim is to test the model against the supernovae Ia observations and study its cosmological consequences. In fact, the model fits the SN Ia data very well.

The brane in this model is a hypersurface given by the metric

\[
ds^2 = -dt^2 + S^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\phi^2 \right)
\]

embedded in a bulk composed of two patches of negative \( \lambda_5 \) spacetime whose action is

\[
S = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left( R + \frac{12}{l^2} \right) + \frac{1}{8\pi G_5} \int dx^4 \sqrt{-h} K + \sigma \int d^4 x \sqrt{-h} + \frac{b}{16\pi G_5} \int d^4 x \sqrt{-h} \frac{1}{2} R^{(4)},
\]

where \( K \) is the trace of the extrinsic curvature of the brane. The first term in the action is the standard Einstein-Hilbert action, the second term is the Gibbons-Hawking term which is necessary for the variational problem to be well defined and the third term is the contribution of brane tension to the action. We are interested in looking at the incorporation of the brane curvature scalar term in the action coupling through the parameter \( b \) which is small so that its higher order terms in effective Einstein’s equations on the brane can be neglected at the present epoch. For this model the modified Friedmann equation \[14\], in low energy limit (\( \rho \ll \sigma \)), becomes

\[
H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} + \frac{C}{S^4} - \frac{(k - \alpha)}{S^2}
\]

where

\[
\alpha = b \left[ \sqrt{\frac{\pi G \sigma}{3}} l - \frac{1}{16 \pi G \sigma l^2} \right].
\]

\[
\Lambda = \lambda - \frac{3b}{l} \sqrt{3\pi G \sigma} \left[ \frac{8\pi G \sigma l^2}{9} + \frac{1}{4\pi G \sigma l^2} - 1 \right],
\]

\[
C = C \left[ 1 + b \left( \frac{3 - \beta^2}{3\beta} \right) \right], \quad \beta = 4l \sqrt{\frac{\pi G \sigma}{3}}.
\]

There would also be higher order terms in \( S \) in eq. \[9\], which have not been included for their effect being insignificant at the epochs of redshifts up to 1.7. The term \( \alpha/S^2 \) would be dominant over the density and dark energy terms at late times of expansion of the universe. It may be noted that \( 1/S^2 \) fall off is typical of \( \rho + \Sigma p_i = 0 \) which is a characteristic of topological defects like cosmic strings and textures \[17\]. The inclusion of topological defects in standard cosmology has been widely considered (see for example \[17\]).

The dark radiation term in eq. \[4\] now contains additional contributions from the bulk parameters. In order to calculate the relative contribution of this term to the expansion dynamics, we write eq. \[5\] in a more convenient form as

\[
H^2(z) = H_0^2 \left[ \Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_{\text{dr}}(1+z)^4 - \Omega_{\text{keff}}(1+z)^2 \right],
\]

where \( k_{\text{eff}} \equiv k - \alpha \) and we have defined the various energy density components, in units of the critical density, in the following form.

\[
\Omega_m = \frac{8\pi G}{3H^2} \rho, \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}, \quad \Omega_{\text{dr}} = \frac{C}{S^4H^2}, \quad \Omega_{\text{keff}} = \frac{(k - \alpha)}{S^2H^2}.
\]

The subscript 0 denotes the value of the quantity at the present epoch. The BBN constrains the dark radiation density to be between \((-1.23 \text{ and } 0.11) \times \rho_0 \), where \( \rho_{\text{dr}} \) is the present energy density of the photon background with \( \rho_{\text{dr}} \approx 2.5 \times 10^{-5} \). Thus the upper BBN constraint imply that at the highest SN redshift \( z = 1.755 \), the contribution of the (positive) dark radiation term in eq. \[9\] is only 0.003 % of the contribution from the \( \Omega_m \) term for \( \Omega_m^0 \) as low as 0.33 (which comes from the various recent measurements of \( \Omega_m^0 = 0.330 \pm 0.035 \) at one sigma level \[18\]). Similarly for the lower constraint (i.e., negative dark radiation), the corresponding contribution is 0.03 %. The dark radiation term, hence, can be safely neglected while considering the SN Ia observations and eq. \[9\] reduces to

\[
H(z) = H_0 \left[ \Omega_m(1+z)^3 + \Omega_\Lambda - \Omega_{\text{keff}}(1+z)^2 \right]^{1/2}
\]

implying that \( \Omega_m + \Omega_\Lambda = 1 + \Omega_{\text{keff}} \). Note that \( \alpha \) does not enter into the metric on the brane but it enters in the brane Friedmann equation eq. \[5\] through the Israel junction conditions. This is equivalent to shifting the curvature index \( k \) of the standard cosmology by an amount \(-\alpha \). This term \( (k - \alpha)/S^2 \) can be neglected in the early universe. However, it dominates at the present
epoch and predicts significant departures from the standard cosmology, especially the expansion dynamics of the universe.

In order to test the model with the supernovae Ia observations, we derive, in the following, the magnitude-redshift relation. We note that the luminosity distance $d_L$, of a source of redshift $z$, located at a radial coordinate distance $r$, is given by $d_L = (1 + z) S_0 r$ where $r$ can be calculated from the metric as

$$\frac{1}{S_0} \int_0^1 \frac{dz'}{H(z')} = r, \quad \text{when} \quad k = 0$$

$$= \sin^{-1} r, \quad \text{when} \quad k = 1$$

$$= \sinh^{-1} r \quad \text{when} \quad k = 1. \quad (12)$$

The apparent magnitude $m$ of the source is given by

$$m(z) = \mathcal{M} + 5 \log[D_L(z)] \quad (13)$$

where $\mathcal{M} \equiv M - 5 \log H_0 + \text{constant}$, $M$ is the absolute luminosity of the source and $D_L \equiv H_0 d_L$, is the dimensionless luminosity distance. The present radius of the universe $S_0$, appearing in $d_L$, can be calculated from eq. (10) as

$$S_0^2 H_0^2 = \frac{k - \alpha}{\Omega_m + \Omega_\Lambda - 1} \quad (k \neq \alpha). \quad (14)$$

We can now calculate, by using eqs. (11, 12), the predicted value of the apparent magnitude at a given redshift if we know the values of the parameters $\Omega_m, \Omega_\Lambda, \alpha$ and $\mathcal{M}$. In order to test the model, we consider the data on the redshift and magnitude of a sample of 54 supernovae of type Ia considered by Perlmutter et al (excluding 6 outliers from the full sample of 60 supernovae) [13], together with the recently observed supernova 1997ff at $z = 1.755 \pm 0.05$ (with $m_i^{corr} = 25.05 \pm 0.34$) the highest redshift supernova observed so far [20].

The best-fitting parameters are obtained by minimizing $\chi^2$, which is defined by

$$\chi^2 = \sum_{i=1}^{55} \left[ \frac{m_i^{corr} - m(z_i)}{\delta m_i^{corr}} \right]^2, \quad (15)$$

where $m_i^{corr}$ refers to the effective magnitude of the ith supernovae which has been corrected by the lightcurve width-luminosity correction, galactic extinction and the K-correction from the differences of the R- and B-band filters and the dispersion $\delta m_i^{corr}$ is the uncertainty in $m_i^{corr}$.

The model fits the data for a wide range of parameters. In Fig. 1, the conical volume shows the 68 % confidence region which gets contributions only from the closed and flat models. The open model contributes only at higher confidence levels, more than 85 %. In the case of flat ($k = 0$) model (where $\Omega_k = 0$ is not necessarily zero), $m$, and hence $\chi^2$, are independent of $\alpha$ which is constrained only through eq. (14). The best-fitting parameters are obtained as $\Omega_m = 0.35$, $\Omega_\Lambda = 1.12$, $\alpha = 0.69$ and $\mathcal{M} = 23.91$ with $\chi^2 = 56.85$ at 51 degrees of freedom (dof), which represents a fit as good as for the best-fitting standard model ($\Omega_m = 0.87$, $\Omega_\Lambda = 1.51$ and $\mathcal{M} = 23.9$ with $\chi^2 = 56.89$ at 53 dof [21]). However, these models are not consistent with the recent anisotropy measurements of the cosmic microwave background radiation (CMBR), which imply that $k \approx 0$ [22]. However, the parameter space is large enough (as in the case of standard model), as is clear from Fig 1, to accommodate with the CMBR predictions. Additionally, we find that the model also fits the data very well for the case $k = 0$ for a wide range of parameters giving $\Omega_m = 1.35$, $\Omega_\Lambda = 1.54$, $\mathcal{M} = 23.93$ with $\chi^2 = 58.8$ (at 52 dof) as the best-fitting solution. Though this best fitting $\Omega_m$ is higher than the favoured value 0.33, however the parameter space is sufficiently large to accommodate low $\Omega_m$ values as has been shown in Fig 2, where we have shown the 68 % and 95 % confidence regions for this case. Note that, unlike the standard model, the points in the figure are not confined only on a straight line since for a given pair $\Omega_m$ and $\Omega_\Lambda$, model is degenerate in $\alpha$ and $S_0$. For the case $k = 0$, $d_L$ (hence $m$ and $\chi^2$) is independent of $\alpha$. If we fix $\Omega_m$ to 0.33, then the best-fitting solution (for $k = 0$) is obtained as $\Omega_\Lambda = 0.7$, $\mathcal{M} = 23.96$ with $\chi^2 = 62.0$ at 53 dof. The allowed ranges of $\Omega_\Lambda$ at 68 % and 95 % confidence levels are obtained, respectively, as 0.53–0.84 and 0.34–0.97. It should be noted that for this geometrically flat ($k = 0$) case, the model is not dynamically flat ($\Omega_{\text{total}} \equiv \Omega_m + \Omega_\Lambda \neq 1$) and reduces to the standard flat model, $\Omega_{\text{total}} = 1$ only when $\alpha = 0$.

We also notice that the exclusion of SN 1997ff from the sample has only a marginal effect on our results. For example, the global best-fitting model from the Perlmutter et al sample of 54 points yields $\Omega_m = 0.34$, $\Omega_\Lambda = 1.14$, $\mathcal{M} = 23.91$, $\alpha = 0.7$ with $\chi^2 = 56.83$ at 50 dof. The best-fitting model for the case $k = 0$, from this sample,
yields $\Omega_{m0} = 1.7$, $\Omega_{\Lambda 0} = 1.9$, $\mathcal{M} = 23.91$ with $\chi^2 = 56.9$ (at 51 dof).

The present value of the Hubble parameter $H_0$ sets the age of the universe. A large number of independent methods appear to converging on a value of $H_0$ in the range $(60 - 80)$ km s$^{-1}$ Mpc$^{-1}$ \cite{25}, which sets the age of the best-fitting flat standard model in the range $(11.4 - 15.2)$ Gyr. If $H_0$ is towards the lower side of this range, there is no serious age problem with the standard cosmology in the view that the age of the globular clusters $t_{GC} = 12.5 \pm 1.2$ Gyr \cite{26} and the age of Milky Way $t_{MW} = 12.5 \pm 3$ Gyr \cite{26}. However, if $H_0$ shifts on the higher side (as is claimed by the recent measurements of $H_0 = 72 \pm 7$ km s$^{-1}$ Mpc$^{-1}$ by using Hubble Space Telescope \cite{27}), the standard model might get uncomfortable with its age. However, one can obtain larger age $t_0$ if larger values of $\Omega_0$ are allowed, as is clear from Fig 3, where we have plotted $(t_0)$ as a function of $\Omega_0$ freezing $\Omega_{m0}$ at its observed value 0.33. The expression for the age of the universe is the same in both models and is given by

$$t_0 = H_0^{-1} \int_0^\infty \frac{(1 + z)^{-1} dz}{\{z(\Omega_{m0} + 1)(1 + z)^2 - \Omega_0 z(z + 2)\}^{1/2}}.$$  \hspace{1cm} (16)

In the favoured standard model ($\Omega_{\text{total}} = 1$), $t_0$ can be increased only by increasing $\Omega_{\Lambda 0}$ (i.e. by reducing $\Omega_{m0}$), which does not seem likely as the recent measurements give very narrow range of $\Omega_{m0}$, as mentioned earlier. However, there is no such constraint on the present model and fairly large values of $\Omega_{\Lambda 0}$ are allowed by the data: $\Omega_{\Lambda 0}$ can rise as high as 0.97 at 2 sigma level (by fixing $\Omega_{m0} = 0.33$), which can increase the age as large as 14 Gyr even for $H_0$ as high as 72 km s$^{-1}$ Mpc$^{-1}$.

In Fig 4, we have shown the actual fitting of the best-fitting geometrically flat model (fixing $\Omega_{m0} = 0.33$) with the data and compared it with the favoured standard model.

We further note that the estimate of the parameter $\mathcal{M}$, which is usually referred to as the Hubble constant-free absolute magnitude, seems to be model independent, unlike the other parameters ($\Omega_i$) which very much depend on the models considered. For instance, the SN dataset, which we are considering here, has been fitted to a variety of models — the standard FRW models \cite{28}, kinematical $\Lambda$-models \cite{27}, quintessence models \cite{21}, quasi-steady state model \cite{28}, etc. All give the same $\mathcal{M} \approx 24$, for low as well as high redshift supernovae \cite{23}. This is remarkable and consistent with the primary assumption that the supernovae Ia are standard candles. To fix ideas, let us note that $\mathcal{M} = 24$ gives the corresponding absolute magnitude $M$ at peak luminosity of the supernovae as
\[ M = -19.3 \text{ for } H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ and } M = -19.1 \text{ for } H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}, \text{ which are in the right region for SN peak magnitudes} \ \square. \]

Summarizing, the modification caused by the inclusion of the brane scalar curvature in the action amounts to defining an effective dynamical curvature which is different from the geometric curvature. This essentially shifts the curvature parameter \( k \) to the effective curvature parameter \( (k - \alpha) \) in the Friedmann equation, hence making the difference even at the present epoch, which can be tested against the present observations. It seems to come out very well. This provides a welcome leverage which could be maneuvered for having a comfortable age for the universe. Our ongoing investigations indicate that the model seems to explain the CMBR anisotropy measurements and radio sources data successfully \( \square \).

Although the degeneracy in the parameter space is large, which is due to inadequacy of the present data, the situation will improve when more precise data with more points at \( z > 1 \) become available. It would then narrow down the degeneracy pinning the parameters more accurately. This might be accomplished by the proposed SNAP (SuperNova Acceleration Probe) project.

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