The Low-Energy Dynamics of $\mathcal{N} = 1$ SUSY Gauge Theories with Small Matter Content

Witold Skiba

Department of Physics, University of California San Diego
La Jolla, CA 92093–0319, USA

Abstract

We describe the low-energy dynamics of $\mathcal{N} = 1$ supersymmetric gauge theories with the Dynkin index of matter fields less than or equal to the Dynkin index of the adjoint plus two. We explain what kinds of nonperturbative phenomena take place in this class of supersymmetric gauge theories.

I. INTRODUCTION

The knowledge of nonperturbative dynamics of SUSY gauge theories advanced significantly after the realization of how powerful the restrictions imposed by holomorphy are [1,2]. It turned out that holomorphy and symmetries are sufficient to determine the vacuum structure of a large number of supersymmetric theories. The full Lagrangian of $\mathcal{N} = 1$ theories contains both holomorphic and non-holomorphic quantities, but only the holomorphic ones can be determined beyond what is possible to compute in perturbation theory.

There are two kinds of holomorphic quantities in $\mathcal{N} = 1$ theories: the superpotential and the coefficient of the gauge-kinetic term. Symmetry restrictions and compatibility with various limits are often powerful enough to determine the exact form on nonperturbatively generated superpotentials. The gauge-kinetic term is important only when the gauge degrees of freedom are present in the infrared. When the gauge group is broken or exhibits confinement, there are no light vector bosons in the spectrum. The field dependent coefficient of the gauge-kinetic term can be determined using the electric-magnetic duality, for which explicit transformations are only known in the Abelian case. For non-abelian theories we have limited results about duality. The non-Abelian version of duality relates theories which flow to the same infrared fixed point and also gives a mapping between the gauge-invariant operators [3]. So far, the examples of non-abelian duality are known only in isolated cases, but no general arguments for

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finding dual theories have been found. We will not discuss theories related by such a duality here. On the other hand, $\mathcal{N} = 1$ theories can be studied systematically in a wide range of other nonperturbative phenomena including confinement, Abelian duality, quantum deformation of moduli spaces, lifting of the moduli space due to dynamically-generated superpotentials. We will describe these phenomena in theories based on simple gauge groups without tree-level superpotentials.

The classical moduli space of supersymmetric gauge theories can be described by the vacuum expectation values (VEVs) of gauge-invariant polynomials on the space of chiral superfields. For any given theory, there is always a minimal set of such polynomials which parameterizes the moduli space. Some of the operators in the minimal set can be subject to constraints, which are polynomials in the operators. The choice of the basic set of gauge invariants is not unique, but the number of such invariants in the minimal set and the number of constraints among them is unique in a given theory.

We now describe the restrictions imposed by symmetries on the form of the dynamically-generated superpotential. The $U(1)$ charges must be chosen so that the symmetry is free of anomalies. For example, the $R$ charge has to obey

$$\sum_i (r_i - 1)\mu_i + \mu_{adj} = 0,$$

where $(r_i - 1)$ is the R charge of the $i$-th fermion representation, $\mu_i$ the respective Dynkin index, and $\mu_{adj}$ is the Dynkin index of the adjoint representation. In our conventions gauginos carry $R$ charge one. Since we consider theories with no tree-level superpotential, there are as many non-anomalous global $U(1)$’s as chiral superfields. These symmetries restrict the dynamically generated superpotential to be of the form

$$W_{dyn} \propto \left( \frac{\Lambda^{(3\mu_{adj} - \mu)/2}}{\prod_i \phi_i^{\mu_i}} \right)^{\frac{2}{\mu_{adj} - \mu}},$$

where $\mu = \sum_i \mu_i$. Depending on the sign of $\mu - \mu_{adj}$ the characteristic scale of the theory, $\Lambda$, appears in the numerator or the denominator of the above equation. When $\mu = \mu_{adj}$ a superpotential cannot be generated since all fields carry zero R charge, while the superpotential must have R charge of two. We will now discuss nonperturbative phenomena dividing theories according to the value of $\mu - \mu_{adj}$.

**II. $\mu = \mu_{adj} + 2$**

An example of theory with with $\mu = \mu_{adj} + 2$ is supersymmetric $SU(N_C)$ with $N_F = N_C + 1$ fields $Q$ in the fundamental representation and $\bar{Q}$ in the antifundamental representation. This theory was found to confine without breaking chiral symmetries. Massless fields of the confined theory are the mesons $M = Q\bar{Q}$ and the baryons $B = Q^{N_C}$, $\bar{B} = \bar{Q}^{N_C}$. When $\mu > \mu_{adj}$ the superpotential of Eq. (3) has fields in the numerator and the scale $\Lambda$ in the denominator. The limit $\Lambda \to 0$ seems to give a singularity which is not present in the classical theory. Therefore,
most theories with $\mu > \mu_{\text{adj}}$ cannot generate a superpotential. Nevertheless, supersymmetric QCD with $N_F = N_C + 1$ generates a superpotential of the form

$$W = \frac{BM\bar{B} - \det M}{\Lambda^{2N_C-1}}. \quad (3)$$

This superpotential has a consistent classical limit because of constraints among the mesons and baryons. The equations of motions reproduce precisely the classical constraints among the $M$’s, $B$’s and $\bar{B}$’s. Thus, the numerator of the superpotential in Eq. (3) vanishes identically in the classical limit, and the singular behavior is avoided. The moduli spaces of the classical and quantum theories are identical since the equations of motion of the quantum theory reproduce the classical constraints. For example, the point where none of the operators has VEVs belongs to the quantum moduli space. At this point none of the global symmetries is broken, and one can verify that the ’t Hooft anomaly matching conditions are saturated by the mesons and baryons.

It is possible to identify other theories which confine without chiral symmetry breaking and whose classical and quantum moduli spaces are identical. Constraints present in the classical theory have to be reproduced by the equations of motion of the superpotential. All such theories, termed s-confining, can be found in a systematic manner [5]. The confining phase of such theories is continuously connected to their respective Higgs phase. For VEVs much larger than the characteristic scale, at generic points on the moduli space of theories with $\mu > \mu_{\text{adj}}$ the gauge group is completely broken [6]. Thus the low-energy degrees of freedom are the fields that are not eaten by the Higgs mechanism. These are parameterized by the gauge invariants made out of chiral superfields. Since there is no phase transition between the Higgs and confining phases, the same fields must comprise the infrared spectrum of the theory near the origin.

We now summarize the arguments that allow to identify the s-confining theories [5]. First, the confining and Higgs phases are indistinguishable whenever sources in all possible representations can be screened by massless dynamical fields. This requires that an s-confining theory has at least one field in a faithful representation. Second, the superpotential must be smooth at the origin. Comparison with Eq. (2) implies that $\mu = \mu_{\text{adj}} + 2$. A singularity of the superpotential would indicate that additional massless states are present. These two conditions limit the number of candidate theories to a relatively short list. The last criterion also comes form the fact that the same massless fields are present on the entire moduli space. Giving VEVs to some of the fields, such that the gauge group is only partially broken, leads to new effective theories. But these new theories should also be described in terms of gauge invariant operators alone. In other words, an s-confining theory has to flow to an s-confining theory. Let us see how this works in practice. For example an $SU(4)$ theory with $5\Box$ obeys the index restriction $\mu = \mu_{\text{adj}} + 2$. However the two-index antisymmetric representation is not faithful, therefore this theory cannot be s-confining. As another example consider $SU(4)$ with $\Box + 2 (\Box + \Box)$. Along a certain flat direction this theory flows to $SU(2)$ with $8\Box$. An $SU(2)$ theory with $8\Box$ is not s-confining as it can be described in terms of a dual gauge theory [3]. Therefore $SU(4)$ with $\Box + 2 (\Box + \Box)$ is not s-confining.
Using the criteria we have just described it is possible to analyze all theories with $\mu = \mu_{adj} + 2$ and eliminate the ones which are not s-confining. For the remaining candidate theories the low energy degrees of freedom were found, and these fields indeed satisfy 't Hooft anomaly matching conditions at the origin of the moduli space [5].

**III. $\mu = \mu_{adj}$**

In the previous section we have shown how to identify s-confining theories. Adding a mass term for a flavor (or any other real representation with $\mu_R = 2$) yields theories with $\mu = \mu_{adj}$. Decoupling the massive fields gives theories with a smaller field content. For example, integrating out a flavor from s-confining supersymmetric QCD, one obtains a theory with $N_F = N_C$. As we already mentioned when $\mu = \mu_{adj}$ a dynamical superpotential can not be generated. The classical moduli space of supersymmetric QCD with $N_F = N_C$ has one constraint among the gauge invariants, which is $(\text{det} M - B\bar{B}) = 0$. The quantum moduli space turns out to be different from the classical one, and it is described by the constraint $(\text{det} M - B\bar{B}) = \Lambda^{2N_C}$. The modified constraint forces at least one combination of fields to have non-zero VEVs. If $\langle \text{det} M \rangle \neq 0$, then non-abelian flavor symmetries are broken; if $\langle B\bar{B} \rangle \neq 0$ then the baryon number is broken. For small VEVs SUSY QCD with $N_F = N_C$ is confining, while for large VEVs it is in the Higgs phase. Because the fundamental representation is faithful there is no phase transition between the two phases. One can check that the same happens when integrating out a flavor from any other s-confining theory.

Not all $\mu = \mu_{adj}$ theories can be obtain by integrating out fields from s-confining theories. Nevertheless, the infrared dynamics of all other theories with $\mu = \mu_{adj}$ has been determined [7,8]. $\mu = \mu_{adj}$ theories which have constraints among the basic invariants that parameterize the classical moduli space have low-energy dynamics similar to that of SUSY QCD with $N_F = N_C$. The set of operators and appropriate quantum modification of the moduli space space have been found in Ref. [7]. In some cases the modification is field dependent, that is, a classical constraint picks up a modification proportional to the scale of the theory times a product of fields. The low-energy dynamics is the same irrespective of the form of the quantum constraint. Near the origin of the moduli space the theory is confining, far away it is Higgsed, and there is no phase transition between the phases.

Indeed, all theories with $\mu = \mu_{adj}$ and constraints among the basic gauge invariants have fields in a faithful representation. Such theories have completely broken gauge groups at large generic VEVs. $\mu = \mu_{adj}$ theories which do not have constraints are all theories with a single adjoint superfield and three other examples [7]. Theories with an adjoint superfield automatically have $\mathcal{N} = 2$ supersymmetry; we will not discuss them here. The remaining three $\mathcal{N} = 1$ examples are $SO(N)$ with $(N - 2)\mathbb{C}$, $SU(6)$ with $2\mathbb{C}$, and $Sp(6)$ with $2\mathbb{C}$. None of these theories contains fields in a faithful representation. Generic VEVs of these theories break the gauge group to a single $U(1)$ or a product of $U(1)$’s. It is therefore possible to determine the coefficient of the gauge-kinetic term using electric-magnetic duality [10]. The auxiliary Seiberg-Witten
curves which encode information about the holomorphic coefficient of the gauge-kinetic term have been found for these theories \[10,8\]. It turns out that the \(U(1)\) gauge bosons are present at every point of the moduli spaces.

It is interesting that these three examples exhaust the list of theories in the Abelian Coulomb phase everywhere on the moduli space \[8\]. Since the Seiberg-Witten curve is written in terms of chiral gauge-invariant operators and the scale \(\Lambda\), one can argue based on R symmetry considerations that \(\mu\) must equal \(\mu_{\text{adj}}\). It is easy to understand that there cannot be \(U(1)\) symmetries for generic VEVs when \(\mu > \mu_{\text{adj}}\) since the gauge is broken completely in such cases. For \(\mu < \mu_{\text{adj}}\), as we will explain in the next section, all theories have dynamically-generated superpotentials which lift the moduli space. In a few cases theories have multiple branches: a branch with a superpotential and a branch with zero superpotential and confinement near the origin. However, none of the theories with \(\mu < \mu_{\text{adj}}\) has Abelian factors in the low-energy spectrum.

IV. \(\mu < \mu_{\text{adj}}\)

Since the description of all theories with \(\mu = \mu_{\text{adj}}\) is known, one could in principle integrate out fields in the real representations and obtain the description of theories with \(\mu < \mu_{\text{adj}}\). Integrating out can sometimes be very difficult due to cumbersome algebra. There are, however, general arguments which allow one to classify all \(\mu < \mu_{\text{adj}}\) theories and explain their dynamics \[6\]. First of all, each theory with \(\mu < \mu_{\text{adj}}\) has an unconstrained set of basic gauge invariants. At the quantum level, such theories turn out to always have a branch with a dynamical superpotential described in Eq. (2). Such a superpotential forces fields to infinite VEVs and lifts the moduli space. Some theories have an additional branch on which there is no superpotential \[5,10–12\]. One can understand the multiple branch structure and why all theories have branches with non-zero superpotentials by examining the smallest unbroken subgroup on the moduli space \[6\].

There are two known mechanisms for generating dynamical superpotentials: instantons and gaugino condensation. When the gauge group is completely broken, one-instanton contributions generate a superpotential. It is possible to connect all such theories to an \(SU(2)\) theory with one flavor, in which an explicit instanton computation was carried out \[4\]. All theories with a completely broken gauge group flow along a flat direction to an \(SU(2)\). Instantons in that subgroup can be related to a nonvanishing superpotential in the original theory.

Gaugino condensation takes place in SUSY Yang-Mills theories with arbitrary gauge groups. Whenever an unbroken subgroup remains at generic values of the moduli fields, the gaugino condensate contributes to the superpotential. If the unbroken subgroup is simple, then gaugino condensation can be related by scale matching to a nonvanishing superpotential of the original theory. When the unbroken subgroup is semi-simple, gaugino condensations from each factor contribute to the superpotential. Depending on the relationship between the scales of different factors, these contributions do or do not cancel. If the contributions do not cancel, the original theory has only branches with non-zero superpotential. Otherwise one obtains theories with
both zero and non-zero branches \[6\].

V. CONCLUSIONS

The vacuum structure of $s$-confining theories, which are a large subset of the $\mu = \mu_{\text{adj}} + 2$ list, and of all theories with $\mu \leq \mu_{\text{adj}}$ has been determined. The nonperturbative phenomena exhibited by those theories are confinement with or without chiral symmetry breaking, Abelian Coulomb phase, and dynamical generation of superpotentials. Such theories with a small matter content provide the basic blocks for supersymmetric model building. The majority of model building efforts so far, including compositeness and dynamical supersymmetry breaking, use theories of this kind. It would be very interesting to find out about the dynamics of the remaining asymptotically-free supersymmetric theories with $\mu > \mu_{\text{adj}}$.

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REFERENCES

[1] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19; Nucl. Phys. B431 (1994) 484.
[2] N. Seiberg, Phys. Rev. D49 (1994) 6857.
[3] N. Seiberg, Nucl. Phys. B435 (1995) 129.
[4] I. Affleck, M. Dine, and N. Seiberg, Phys. Rev. Lett. 51 (1983) 1026;
   Nucl. Phys. B241 (1984) 493.
[5] C. Csáki, M. Schmaltz, and W. Skiba, Phys. Rev. Lett. 78 (1997) 799;
   Phys. Rev. D55 (1997) 7840.
[6] G. Dotti, A. Manohar, and W. Skiba, \texttt{hep-th/9803087}.
[7] B. Grinstein and D. Nolte, Phys. Rev. D57 (1998) 6471; \texttt{hep-th/9803139};
   P. Cho, Phys. Rev. D57 (1998) 5214.
[8] C. Csáki and W. Skiba, \texttt{hep-th/9801173}.
[9] K. Intriligator and N. Seiberg, Nucl. Phys. B431 (1994) 551.
[10] K. Intriligator and N. Seiberg, Nucl. Phys. B444 (1995) 125.
[11] P. Cho and P. Kraus, Phys. Rev. D54 (1996) 7640;
   C. Csáki, W. Skiba, and M. Schmaltz, Nucl. Phys. B487 (1997) 128.
[12] G. Dotti and A. Manohar, Phys. Rev. Lett. 80 (1998) 2758.