A DIFFERENTIAL METHOD OF MAXIMUM ENTROPY

A.T. Bajkova

We consider a differential method of maximum entropy that is based on the linearity of Fourier transform and involves reconstruction of images from the differences of the visibility function. The efficiency of the method is demonstrated with respect to the recovery of source images with bright components against the background of a sufficiently weak extended base. The simulation results are given along with the maps of an extragalactic radio source 0059+581, which were obtained using the standard and differential methods of maximum entropy for three observation dates and show that the principle of differential mapping allows us to increase considerably the dynamic interval of images.

1. THE PRINCIPLE OF DIFFERENTIAL MAPPING

The idea of differential mapping is not new. At present, it is realized most comprehensively in the program package "DifMap" [1] at the California Institute of Technology, where the "cleaning" procedure (CLEAN) is used as deconvolution operation.

The method of differential mapping is based on the fundamental property of the Fourier transformation. According to this method, the bright components of the source, which were reconstructed at the first stage, are subtracted from the initial visibility function, the subsequent reconstruction is performed using the residual visibility function, and the reconstruction results are added at the final stage.

In the case of CLEAN, such a mapping method mainly influences the convergence rate. However, if the maximum entropy method (MEM), which has pronounced nonlinear properties, is used as the deconvolution operation, the principle of differential mapping can improve the reconstruction quality, particularly when the source has bright compact components against the background of a sufficiently weak extended base. An example of such improvement due to subtraction of the bright component from the visibility function is shown in [2].

What is the cause of the reconstruction quality improvement? The fact is that after the subtraction of bright components, which were reconstructed at the first stage using the MEM, from the initial visibility function we obtain the residual visibility function, in which the proportion of the weak extended component becomes larger. (The subtraction result is called the first-order residual visibility function.) Therefore, we artificially decrease the dynamic interval of the map, which corresponds to the residual visibility function, and, thus simplify the image reconstruction at the second stage.
If the bright components of the image reconstructed at the second stage are subtracted from the first-order residual visibility function, we obtain a second-order residual visibility function, to which a map with a still lower dynamic interval corresponds. Using this procedure, we obtain residual visibility functions of an increasingly high order.

To obtain the desired source map, the map reconstructed at the last stage must be summed up with all the components subtracted from the visibility function at the previous stages.

Formally, the algorithm of two-stage differential mapping is formulated as

\[ F_{vis}^{(2)} = F_{vis}^{(0)} - F_{vis}^{(1)}, \]

where \( F_{vis}^{(0)} \) is the initial visibility function of the desired brightness distribution over the source \( I(x) \); \( F_{vis}^{(1)} \) is the visibility function corresponding to the bright component \( I^{(1)}(x) \) reconstructed at the first stage; \( F_{vis}^{(2)} \) is the residual visibility function corresponding to the source map \( I^{(2)}(x) \) reconstructed at the second stage.

The resulting map has the form

\[ I(x) = I^{(1)}(x) + I^{(2)}(x). \]

2. SIMULATION RESULTS

In Figs. 1 and 2 we show the results of simulation of the principle of differential mapping using the generalized maximum entropy method (GMEM) [3] for real images with both positive and negative values (the advantages of the GMEM compared with the MEM are discussed below in Section 3).

The experiment shown in Fig. 1 was carried out for three model sources in the form of one or several bright point components against the background of a weaker extended base that has a Gaussian form. In the figures we give the following source names: ”Model-1”, ”Model-2”, and ”Model-3”. For ”Model-1” the base amplitude is 20% of the amplitude of the bright point-like component, while for ”Model-2” it is only 2%. ”Model-3”, which has four bright components against the background of a 20% Gaussian, is more complicated. To form the visibility function, we used the symmetric coating of the UV surface with almost uniform filling over the entire aperture, which included 98 points. To study the reconstruction improvement due to use of the differential mapping principle in its pure form, no noise was added to the visibility function.

In Figs. 1 and 2, for each vertically located source, we show the maps that appear horizontally in the following order (from left to right): the map of the model source, a ”noisy” image resulting from the inverse Fourier transform of the initial visibility function, an image reconstructed at the first stage of differential mapping using the GMEM, and the resulting map of differential mapping. (It must be noted that the bottom level of the contour line corresponds to 1% of the peak value of the flux in all the above images.)

Obviously, the maps obtained at the first stage can be treated as the limiting performance of the GMEM applied to the initial visibility function. The subtraction of the bright components reconstructed at the first stage from the visibility function allows us to improve significantly the reconstruction quality at the second stage. In Table 1 we show the quantitative properties of
Fig. 1.

Fig. 2.
the image reconstruction quality from Fig. 1 after the first and second stages of the differential mapping.

As is obvious from the table, compared with the standard GMEM, the use of the differential GMEM allows us to increase the signal-to-noise ratio (SNR) of the output image by a factor of 3 to 5, such that the maximum effect is achieved for the source "Model-2", which has maximum dynamic interval.

In Fig. 2 we show the results of reconstruction of images of the sources "Model-1" and "Model-2" for the case of another filling of the UV plane, which includes 98 points distributed uniformly inside the central region. The above region is approximately a quarter of the aperture. This example of mapping is more complicated compared with the previous case because of the total absence of the high spatial harmonics of the source in the visibility function. As is obvious from the figure, in this case the differential GMEM also gives higher-quality results than the standard GMEM.

Table 1

| Source    | Maximum error at the 1st stage | SNR at the 1st stage | Maximum error at the 2nd stage | SNR at the 2nd stage |
|-----------|-------------------------------|----------------------|-------------------------------|----------------------|
| Model-1   | 0.4701                        | 3.84                 | 0.0572                        | 15.86                |
| Model-2   | 0.0467                        | 11.43                | 0.0088                        | 52.00                |
| Model-3   | 0.1890                        | 4.10                 | 0.0594                        | 13.20                |

3. MAPPING FOR THE RADIO SOURCE 0059+581

To demonstrate the potentialities of the differential generalized method of maximum entropy used for the real data processing, we show the results of mapping the extragalactic radio source 0059+581, which is often used as a reference source in the VLBI geodetic programs [4]. This source is of astrophysical interest because it shows fast variation of both the total flux and the structure. The maps that are constructed using the GMEM for a number of dates in the interval from June 1994 to December 1995 are shown in Fig. 3. Variation of the total flux in the same interval is presented in Fig. 4 in [5].

In the time interval from the middle to the end of 1994 we observed an almost linear decrease in the total flux from $\sim 4$ Ja to 1.5 to 2 Ja. As is obvious from Fig.3, we failed to obtain a map with dynamic range sufficient for detecting an extended element from the data of June 26, 1994 when the flux was maximal over the interval in question. Using the standard GMEM, we detected extended elements only starting from October 4, 1994 when their share with respect to the total flux became sufficient to be detected by the selected reconstruction method. In Fig. 4, for comparison, we show maps obtained using both the standard and differential GMEMs for the three dates on which the total flux was sufficiently large and the flux share corresponding to extended elements was insignificant. As is obvious from the figure,
the use of differential mapping allowed us to increase the dynamic range of the maps such that extended structures became very distinguished. It must be noted that the minimal level of the contour line corresponds to 1% of the peak value of the flux everywhere in the images. The parameters of the maps obtained using the differential GMEM are given below in Table 2.

Table 2

Map parameters for the source 0059+581

| Date   | Total Flux(Ja) | Peak Flux(Ja) |
|--------|----------------|---------------|
| 25.06.94 | 4.23           | 2.40          |
| 25.08.94 | 3.40           | 1.76          |
| 04.10.94 | 3.28           | 1.86          |

The main criterion for choosing the method of maximum entropy (either classical or generalized, the MEM or the GMEM) is the type of the desired distribution. If the desired image is
real bipolar (with positive and negative values) or complex with both real and imaginary parts, only the GMEM [6] can be used.

If it is known a priori that the image is real and nonnegative, the classical MEM can be used. However, one should keep in mind that in case of large errors in the data, the GMEM allows us to obtain [7] higher-quality maps with a much lower level of nonlinear distortions, as compared with the MEM. At present, the researchers of the IAA RAS carry out research in making maps of the extragalactic sources using the observation data from the astrometric and geodetic VLBI programs. These data were not obtained directly for astrophysical mapping, and, as a rule, have poor calibration. Therefore, to obtain satisfactory maps, the GMEM is recommended instead of the MEM.

Another advantage of the GMEM originates from the fact that an image with negative values can correspond to the residual visibility function obtained after the subtraction of the bright component that is reconstructed at the first stage of the differential algorithm from the initial visibility function. This is possible if the bright component was reconstructed at the first stage with an overestimated amplitude, which is quite typical of any nonlinear method. To rule out the unwanted image distortions when a nonnegative solution is sought using the data assuming the presence of negative components, we should use the generalized method of maximum entropy for real bipolar images rather than the classical method.

4. CONCLUSIONS

Compared with the traditional methods, the nonlinear methods of differential mapping using the maximum entropy method as the deconvolution operation allow us to improve substantially the reconstruction quality of source images containing bright point-like components against the background of weak base. The maps obtained using the differential method of maximum entropy are characterized by a higher dynamic range. To eliminate possible nonlinear distortions in the case of differential mapping, the generalized method of maximum entropy is preferred to the classical method.

This work was supported by the Russian Foundation for Basic Research, grant No 96-02-19177 and the Ministry of Science’s program "Astronomy. Fundamental Space Research", grant No 2.1.1.3.

REFERENCES

1. G.Taylor, The Difmap Cookbook, California Institute of Technology, Pasadena (1994).
2. T.Cornwell, "Very Long Baseline Interferometry and the VLBA", in: ASP Conference Series (J.A. Zensus, P.J. Diamond, and P.J. Napier, eds) vol.82, 227 (1995).
3. A.T.Bajkova, Commun. IAA RAS, No. 58 (1993).
4. A.T.Bajkova et al, Trans. IAA RAS, No. 1, 22 (1997).
5. T.B.Pyatunina, A.T.Bajkova et al, Trans. IAA RAS, No. 1, 64 (1997).
6. A.T.Bajkova, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 34, No. 8, 919 (1991).
7. A.T.Bajkova, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 38, No. 12, 1267 (1995).