CP Violation from Surface Terms in the Electroweak Theory without Fermions

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Abstract

We consider the effect of adding a CP-odd, $\theta F \tilde{F}$-term to the electroweak Lagrangian without fermions. This term affects neither the classical nor perturbatively quantum physics, but can be observed through non-perturbative quantum processes. We give an example of such a process by modifying the theory so that it supports Higgs-winding solitons, and showing that the rates of decay of these solitons to specific final states are CP violating. We also discuss how the CP symmetry is restored when fermions are included.

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I. INTRODUCTION

In Quantum Chromodynamics (QCD) it is well known that violation of the charge-parity (CP) symmetry can appear through the inclusion of a CP-odd total divergence

\[ \Delta S_{QCD} = \frac{\theta_{QCD}}{16\pi^2} \int d^4x \text{Tr} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \]  

(1.1)

in the action \([1,2]\). Here \(F\) is the \(SU(3)\) field strength tensor and \(\theta_{QCD}\) is a dimensionless parameter. This modification does not affect the classical equations of motion and introduces no additional Feynman graphs in perturbation theory. It is only through nonperturbative quantum phenomena that this CP-odd surface term can be observed. In particular, the electric dipole moment of the neutron is affected by the presence of this term. Nevertheless, no dipole moment is observed and the tight experimental upper bound on this quantity translates into the constraint \(\theta_{QCD} \leq 10^{-9}\) \([3]\).

The manifestation of (1.1) in physical observables is complicated. Calculations deriving the relevant effects invoke dynamical chiral symmetry breaking and other phenomenological insights into strong physics. It seems useful to identify other manifestations of this type of total divergence to ascertain how they work.

If one introduces a term analogous to (1.1) in the standard electroweak theory, with \(F\) being the \(SU(2)\) gauge field strength, it has no observable effect. The chiral coupling of fermions to the \(SU(2)\)-gauge fields allows the term to be eliminated by performing a phase rotation on quark and lepton fields \([4]\). Thus a total divergence of the form (1.1) is unobservable in the standard electroweak theory.

Nevertheless, if one considers the electroweak theory without including fermions, then it should be possible to observe the total divergence. We will look for CP violating effects which arise from this term. Generally it is difficult to demonstrate such effects unless nonperturbative phenomena are identified. In this paper we alter the electroweak theory so that it supports classically stable solitons \([5-7]\), thereby introducing nonperturbative objects into the spectrum. We show that the rate of decay of these solitons to specific final states is CP violating and briefly discuss how the CP symmetry is restored when fermions are included.
II. PRELIMINARIES

Consider the bosonic sector of the electroweak theory as an effective field theory. In addition to the usual terms, assume that the Lagrangian density for this theory contains a gauge invariant, CP-even, Skyrme term [5,6] which stabilizes Higgs winding configurations as solitons. For simplicity consider only the $SU(2)$ gauge fields. The action is

$$ S_0[\Phi, A_\mu] = \int d^4x \left[ -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \text{Tr} D_\mu \Phi \Phi^\dagger D^\mu \Phi - \frac{\lambda}{4} \left( \text{Tr} \Phi \Phi^\dagger - v_0^2 \right)^2 + \frac{1}{32e^2 v^4} \text{Tr} \left( D_\mu \Phi \Phi^\dagger D_\nu \Phi - D_\nu \Phi \Phi^\dagger D_\mu \Phi \right)^2 \right], \quad (2.1) $$

where

$$ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], $$

$$ D_\mu \Phi = \partial_\mu \Phi - igA_\mu \Phi $$

and $\Phi(x)$ is related to the standard Higgs-doublet $(\varphi_1, \varphi_2)^T$ by

$$ \Phi = \begin{pmatrix} \varphi_2^* & \varphi_1 \\ -\varphi_1^* & \varphi_2 \end{pmatrix}. \quad (2.2) $$

Here $g = 0.65$ is the gauge coupling constant, the Higgs vacuum expectation value (VEV) is $v = 247\text{GeV}$, and the gauge and Higgs boson masses are $m = \frac{1}{2}gv$ and $m_H = \sqrt{2}\lambda v$ respectively. Note that the action $S_0$ is invariant under the usual CP-transformation.

Now consider adding the term

$$ \frac{\theta}{32\pi^2} \int d^4x \epsilon^{\mu\nu\alpha\beta} \text{Tr}(F_{\mu\nu} F_{\alpha\beta}) \quad (2.3) $$

to the action. In the absence of chirally coupled fermions, this term cannot be rotated away, and we assume throughout this paper that there are no fermions present. In particular, (2.3) is CP-odd, and thus we expect there to be CP violating processes associated with the new term. Moreover, as a consistency check, such processes should no longer be present if one introduces fermions, and we shall demonstrate that this is the case.
III. SOLITON DECAYS

Consider for the moment the action (2.1) in the limit where the Higgs self-coupling \( \lambda \to \infty \) and the gauge fields decouple, \( g \to 0 \). In this limit we recover the Skyrme action [5]. Such a theory is known to support Higgs-winding topological solitons

\[
\Phi(x) = \frac{v}{\sqrt{2}} e^{-i \sigma^a \hat{x} F(r)},
\]

(3.1)

where \( F(r = 0) = \pi \) and \( F(r \to \infty) \to 0 \). Defining

\[
U \equiv \frac{\Phi}{\sqrt{\text{Tr} \Phi^\dagger \Phi / 2}},
\]

(3.2)

the winding number is

\[
w[\Phi] = \frac{1}{24\pi^2} \int d^3 x \, \epsilon^{ijk} \left[ (U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U) \right],
\]

(3.3)

and the configuration (3.1) has \( w[\Phi] = 1 \).

Now, if we allow a finite Higgs self-coupling, one may identify a sequence of configurations which connects the soliton to a classical vacuum configuration. All such paths must go through a configuration where \( \Phi = 0 \) at some point in space. For this particular configuration, the winding number of the Higgs field is not defined. If \( \lambda \) is very large, the soliton remains classically stable and any configuration close to (3.1), but for which \( \Phi \) is not on the vacuum manifold (i.e., where \( \text{Tr} \Phi^\dagger \Phi \neq v^2 \)), has much larger energy than that of the soliton. However, if \( \lambda \to 0 \), the configuration (3.1) is no longer stable, and there exist sequences of configurations with monotonically decreasing energy that connect (3.1) to the vacuum. These opposite limits imply that there is a value \( \lambda^* (e) \) of the Higgs self-coupling for which the classical soliton is critically stable.

If we restore the gauge fields, there exists a second class of sequences of configurations that connect the soliton to a classical vacuum configuration [7,8]. For this class, the gauge field aligns with the Higgs field such that they both have the same winding. In the limit where the gauge coupling is small, the soliton (3.1) is classically stable. However, as the
parameter $\xi = \frac{4e^2}{g^2}$ decreases, the soliton becomes increasingly heavy, in units of $m_W$. Thus there exists a value for $\xi = \xi^*$ at which the soliton becomes critically stable to gauge alignment.

Consider the classical configuration space of the action, where gauge equivalent configurations are identified. We have identified two classes of paths that connect the soliton to the vacuum:

1. Paths that go through a zero of the Higgs field. Tunneling by such paths is controlled by the Higgs mass, $m_H = m_W \sqrt{\frac{3\lambda}{g^2}}$, since the barrier height is controlled by $(\lambda - \lambda^*)$.

2. Paths that do not go through a zero of the Higgs field (ie. paths for which $\int F \tilde{F} \neq 0$).

The barrier height in this direction is controlled by the parameter $(\xi - \xi^*) > 0$.

Now, let us identify the quantum transition amplitude $\langle \text{out} | T | s \rangle$ between asymptotic states, where $|s\rangle$ is the properly quantized soliton state and $|\text{out}\rangle$ is a state of a definite number of W particles built on top of a vacuum configuration in unitary gauge. This amplitude has two components

$$\langle \text{out} | T | s \rangle = \left( \int_{\text{class 1}} + \int_{\text{class 2}} \right) [d\Phi][dA] \exp \left( iS_0 + \frac{i\theta}{16\pi^2} \int F \tilde{F} \right).$$

(3.4)

The term $\frac{1}{16\pi^2} \int F \tilde{F} = 0$ for paths of class 1, and $\frac{1}{16\pi^2} \int F \tilde{F} = 1$ for paths of class 2. The previous expression then becomes

$$\langle \text{out} | T | s \rangle = \int_{\text{class 1}} [d\Phi][dA] e^{iS_0} + e^{i\theta} \int_{\text{class 2}} [d\Phi][dA] e^{iS_0}$$

$$\equiv A_1 + e^{i\theta} A_2,$$

(3.5)

where $A_1$ and $A_2$ are in general complex. Now compare this process to the CP conjugate process. Since $S_0$ is CP invariant, we obtain

$$\langle \text{out} | T | s \rangle = A_1 + e^{-i\theta} A_2,$$

(3.6)

1Note, although $\int F \tilde{F}$ is not well-defined, one can show that the approach taken is equivalent to evaluating the transition amplitude in the Hamiltonian formalism with a $\theta$-vacuum.
where we have written \( \text{CP}|s\rangle = |\bar{s}\rangle \), with \( |\bar{s}\rangle \) the antisoliton state, and \( \text{CP}|\text{out}\rangle = |\overline{\text{out}}\rangle \).

Let us identify an appropriate final state. First note that this is a quantum process, so it is necessary to properly quantize the soliton state \([\ref{0}]. \) This involves taking into account the zero modes of the classical soliton configuration in order to construct a state \( |s\rangle \) with definite quantum numbers. The elements of the translational zero mode may be superposed to create a definite momentum state with center of mass momentum \( \mathbf{P} = 0 \). Similarly, the elements of the rotational/isorotational zero mode may be superposed to create states with \( I = J = 0, \frac{1}{2}, 1, \ldots \). Which choice of weak-isospin/spin is appropriate depends on the specific physics underlying the Higgs sector. To be definite, here we choose \( I = J = 0 \). If the soliton mass is not much larger than its inverse size, we expect that the soliton will decay predominantly to states with a small number of W-particles. Consider the case where the final state contains two \( W_0 \) particles (those particles that become the \( Z^0 \)'s when hypercharge is included), and where both particles have a definite \( z \)-component of spin, \( m = 1 \). Then, we define our final state as

\[
|\text{out}\rangle = |\mathbf{k}, m_1 = 1; -\mathbf{k}, m_2 = 1\rangle. \tag{3.7}
\]

Note that, with this choice, the CP conjugate state \( |\overline{\text{out}}\rangle \) is identical to \( |\text{out}\rangle \).

We can now identify a parameter which characterizes the CP violation. Consider the difference between the differential probabilities to decay into a specific two-particle final state

\[
\Delta_{CP} \equiv \frac{d\Gamma}{d\Omega}(\cos 2\varphi)\Big|_{s\rightarrow|\text{out}\rangle} - \frac{d\Gamma}{d\Omega}(\cos 2\varphi)\Big|_{\bar{s}\rightarrow|\text{out}\rangle}.
\]

This may be expressed as

\[
\Delta_{CP} = \frac{k}{32\pi^2 M_s^2} \left[ |\langle \text{out}|\mathcal{T}|s\rangle|^2 - |\langle \overline{\text{out}}|\mathcal{T}|s\rangle|^2 \right]
\]

\[
= \frac{k \sin \theta}{16\pi^2 M_s^2} i(A_1^* A_2 - A_2^* A_1), \tag{3.8}
\]

where \( k = \sqrt{k \cdot k} = \sqrt{\frac{1}{4}M_s^2 - m_W^2} \) and \( M_s \) is the mass of the soliton. Note that, in the limit in which there is no CP violation \( (\theta = 0) \), this quantity vanishes.
IV. AMPLITUDES FROM SEMICLASSICAL METHODS

A. The Framework

We perform the estimate of $\Delta_{CP}$ in the following way. Describe the quantum soliton as a coherent state around the classical soliton configuration, in the spherical ansatz [11]. This coherent state tunnels along the minimal Euclidean path underneath the appropriate energy barrier and, once the state emerges, it evolves along a path in configuration space corresponding to a classical solution of the Euler-Lagrange equations. Such a solution then dissipates and asymptotically approaches a solution to the linearized spherical equations of motion. We write these linearized classical solutions as $f_{ia}^n(k)$, where $n = 1, 2$ labels the path. Thus, in unitary gauge, the field variables associated with these classical solutions are

$$A_{ia}^n(x) = \int d^3k \left[ f_{ia}^n(k)e^{-ik\cdot x} + f_{ia}^{n*}(k)e^{ik\cdot x} \right].$$  \hspace{1cm} (4.1)

Writing the coherent states built around $f_{ia}^1(k)$ and $f_{ia}^2(k)$ as $|f, 1\rangle$ and $|f, 2\rangle$ and taking the overlap between these states and the final state of interest, we obtain

$$A_1 = e^{iS_1}\langle \text{out}|f, 1\rangle$$
$$A_2 = e^{iS_2}\langle \text{out}|f, 2\rangle,$$  \hspace{1cm} (4.2)

where $e^{iS_1}$ and $e^{iS_2}$ are the respective saddle-point evaluations of the coherent state path integral around the extremal paths of type 1 and 2.

The coherent state expresses $|f, 1\rangle$ and $|f, 2\rangle$ as an expansion in Fock space with weights dependent on $f_{ia}^1(k)$ and $f_{ia}^2(k)$, respectively. After some algebraic manipulation, the matrix elements $\langle \text{out}|f, n\rangle$ can be written

$$\langle k, m_1 = 1; -k, m_2 = 1|f, n\rangle = B_n(k^2)\cos 2\varphi,$$  \hspace{1cm} (4.3)

where $\varphi$ is defined as the angle between the vector $k$ and the spin-quantization direction and

$$B_n(k^2) \equiv \frac{1}{2} \sqrt{\frac{5}{32\pi}} (3\hat{k}_i\hat{k}_j - \delta_{ij}) f_{ia}^n(k)f_{ja}^n(-k).$$  \hspace{1cm} (4.4)
Note that \( B_n \) is only a function of \( k^2 \) since the \( f_{ai}^n(k) \) are in the spherical ansatz.

Putting this all together yields

\[
\Delta_{CP} = \frac{k \sin \theta}{16\pi^2 M_s^2} \left( \cos 2\varphi \right)^2 i \left[ e^{i(S_2-S_1^*)} B_1^* B_2 - e^{i(S_1-S_2^*)} B_2^* B_1 \right]. \tag{4.5}
\]

It remains to estimate the quantities entering this expression in a controlled regime.

**B. The Limit**

In order to perform a controlled estimate, we work in the following limit: \( g \rightarrow 0; \ m_W \) fixed; \( e \) fixed, implying \( \xi = \frac{e^2}{g^2} \rightarrow \infty \); and \( \lambda \) fixed, implying \( m_H = \frac{\sqrt{\xi}}{g} \rightarrow \infty \). Focus first on tunneling via path 1. In the limit above, the gauge fields decouple from the soliton, and the properties of the soliton are completely determined by the Higgs dynamics. In particular, for the Skyrme model we are considering, this implies

\[
M_s \sim \frac{v}{e}, \quad L_s \sim \frac{1}{ev}. \tag{4.6}
\]

The amplitude for Higgs unwinding remains unsuppressed as \( g \rightarrow 0 \), even though it is a tunneling process, and this implies that \( \text{Re}(S_1) \sim \mathcal{O}(1) \) and \( \text{Im}(S_1) \sim \mathcal{O}(1) \) in the expression \( (4.2) \). Moreover, because the mass and length of the soliton scale as in \( (4.6) \), there is no further suppression from \( \langle \text{out}|f, 1 \rangle \) in \( (4.3) \).

Now focus on tunneling via path 2. Consider the dynamics in the unitary gauge. By rescaling the action, we see that \( g^2 \) plays a role analogous to \( \hbar \), and that the soliton approaches a pure winding configuration of size \( [m_W\sqrt{\xi}]^{-1} \). Similarly, the Euclidean tunneling path approaches the ’t Hooft instanton \([\Pi]\) of the same size, implying that the decay amplitude approaches the instanton amplitude \([\mathcal{I}\mathcal{O}]\). Because the configuration that emerges from under the barrier at the end of tunneling is governed by the same length scale, \( [m_W\sqrt{\xi}]^{-1} \), the number of quanta in the final state is approximately

\[
N_{\text{quanta}} \sim \frac{E}{L^{-1}} \sim \frac{m_W/(g^2\sqrt{\xi})}{m_W\sqrt{\xi}} \sim \mathcal{O}(1). \tag{4.7}
\]
Thus, the amplitude is not further suppressed by having a small number of particles in the final state, implying $|\langle \text{out}|\text{f},2 \rangle| \sim \mathcal{O}(1)$. Moreover, the configuration that emerges from under the barrier is already linearized, which implies that in \((1.2)\), $e^{iS_2} \sim \mathcal{O}(e^{-8\pi^2/g^2})$ is real.

Thus, our estimate yields

$$|\Delta(\cos 2\varphi)| \sim e^{-8\pi^2/g^2}(\cos 2\varphi)^2 \sin \theta.$$  \hspace{1cm} (4.8)

for the difference between the differential decay probability for soliton decay to a two $W_0$-particle $S = 2, M_S = 2$ state and that for antisoliton decay to the same two $W_0$-particle state.

\section*{V. CONCLUDING REMARKS}

In this paper, we have presented a relatively simple scenario by which a total divergence manifests itself in a physically observable processes. We considered the effect of adding a CP-odd total divergence to the electroweak theory without fermions. We showed that if Higgs winding solitons exist in such a theory, their decay into a specific final state violates CP through the interference of two topologically distinct decay channels. Finally, for consistency, we should demonstrate how the effect disappears when fermions are included.

If one introduces fermions, the axial anomaly implies that

$$\partial_{\mu}J^\mu_F = F\tilde{F},$$  \hspace{1cm} (5.1)

so that in any process for which $\int F\tilde{F} \neq 0$, fermions are created or destroyed. This implies that in the Hamiltonian picture the degeneracy of vacua with different winding is lifted and thus any interference between paths 1 and 2 goes away. Thus, any CP violation ceases to exist.

One may also describe the effect of the $\theta F\tilde{F}$-term as the inclusion of an extra phase on the fermion state, since the phase only appears when a fermion is produced. Thus, the role of the $\theta$-term may be considered as a redefinition of the phase of the fermion state. It should be no surprise that this is precisely the mechanics by which one formally removes the term from the functional integral \[3\].
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