Low-energy Spectra of Gamma-Ray Bursts from Cooling Electrons

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Abstract

The low-energy spectra of gamma-ray bursts’ (GRBs) prompt emission are closely related to the energy distribution of electrons, which is further regulated by their cooling processes. We develop a numerical code to calculate the evolution of the electron distribution with given initial parameters, in which three cooling processes (i.e., adiabatic, synchrotron, and inverse Compton cooling) and the effect of a decaying magnetic field are coherently considered. A sequence of results is presented by exploring the plausible parameter space for both the fireball and the Poynting flux–dominated regime. Different cooling patterns for the electrons can be identified, and they are featured by a specific dominant cooling mechanism. Our results show that the hardening of the low-energy spectra can be attributed to the dominance of synchrotron self-Compton cooling within the internal shock model or to decaying synchrotron cooling within the Poynting flux–dominated jet scenario. These two mechanisms can be distinguished by observing the hard low-energy spectra of isolated short pulses in some GRBs. The dominance of adiabatic cooling can also lead to hard low-energy spectra when the ejecta is moving at an extreme relativistic speed. The information from the time-resolved low-energy spectra can help to probe the physical characteristics of the GRB ejecta via our numerical results.

Key words: gamma-ray burst: general – methods: numerical – radiation mechanisms: non-thermal – relativistic processes

1. Introduction

The radiation mechanism responsible for the prompt emission of gamma-ray bursts (GRBs) remains unidentified since the discovery of GRBs. A typical spectrum of the GRB prompt emission can usually be well fit by the so-called Band function (Band et al. 1993), which smoothly joins low- and high-energy power laws. Except for the Band component, the superposition of multiple spectral components was also observed in some GRBs, such as the thermal component (e.g., Ghirlanda et al. 2003; Ryder 2005; Ghirlanda et al. 2007; Ryder et al. 2010; Zhang et al. 2011) or an additional power-law component (e.g., Abdo et al. 2009a; Ackermann et al. 2011). Since the Band function is still an empirical description for the GRB spectra, further studies are needed to manifest its physical origin.

Synchrotron radiation of electrons has been suggested to be a possible mechanism. However, one problem (called the fast-cooling problem; see Ghisellini et al. 2000; Zhang & Yan 2011) remains in a simple synchrotron model; i.e., the low-energy spectral index \( \alpha \) (\( F_\nu \propto \nu^\alpha \)) is predicted to be \(-1/2\) for fast-cooling electrons (Sari et al. 1998), which is incompatible with the fact that the observed value is \(-0.1\) in the majority of GRBs (Band et al. 1993; Preece et al. 2000; Nava et al. 2011; Zhang et al. 2011; Geng & Huang 2013). Modified synchrotron models have been proposed to ease this conflict. When electrons cooled mainly via inverse Compton (IC) scattering in the Klein–Nishina (KN) regime, it was suggested that the energy loss rate of electrons is roughly \( \propto \gamma_e^{-4} \) (\( \gamma_e \) is the electron Lorentz factor), and the corresponding flux density is \( F_\nu \sim \nu^0 \) (see Derishev et al. 2001; Bošnjak et al. 2009; Nakar et al. 2009; Wang et al. 2009; Fan 2010; Daigne et al. 2011). Subsequent detailed analytical study of this solution shows that it is impossible to obtain a spectrum with \( \alpha > -0.1 \) using IC cooling in the KN regime (Barniol Duran et al. 2012). Recently, by solving the continuity equation of electrons in energy space numerically, Uhm & Zhang (2014) found that the fast-cooling electrons could have a harder energy spectrum when the surrounding magnetic field is decreasing. Their numerical results show that \( \alpha \) can be even harder than zero in a certain parameter regime. However, IC cooling of electrons was not included in their calculations. It is then crucial to take IC cooling into account when solving the continuity equation of electrons.

Alternative models based on photospheric emission have also been proposed to explain the GRB prompt emission (e.g., Rees & Mészáros 2005; Pe’er et al. 2006; Giannios & Spruit 2007; Beloborodov 2010; Ryder et al. 2011). Indeed, the spectra of some GRBs are found to be consistent with a photospheric component (e.g., Ryder et al. 2010; Larsson et al. 2011; Pe’er et al. 2012). However, the photosphere typically predicts \( \alpha \sim 1.4 \) (Deng & Zhang 2014). Some additional effects should be considered to explain the observed index of \( \alpha \sim 0 \) with the photosphere model (e.g., Lundman et al. 2013). In general, given that the main spectral component of a typical burst is the Band component, it is still rational to suppose that the emission comes from a nonthermal mechanism in an optically thin region (Veres et al. 2012; Zhang et al. 2012; Kumar & Zhang 2015). So, in this study, we work within the framework of synchrotron radiation and focus on the low-energy spectra of GRBs.
In principle, the profile of the synchrotron spectra of the GRBs observed is directly determined by the distribution of electrons (called the electron spectrum/distribution for short) in the ejecta. The initial spectrum of the electrons that are accelerated somehow will soon be modified by cooling processes; thus, electron cooling is a key factor in the prompt emission, especially when we focus on the low-energy spectra of GRBs. Although only synchrotron emission is observed by us in the keV–MeV band, the electrons actually can be cooled in three ways, which are adiabatic, synchrotron, and IC cooling. A numerical solution of the electron distribution considering the three processes has been presented in previous researches within the internal shock scenario (Bošnjak et al. 2009a; Daigne et al. 2011). They found that the majority of observed GRB prompt spectra can be reconciled with a synchrotron origin. Some useful constraints on the microphysics of internal shocks were presented. This numerical approach can also be extended to the Poynting flux–dominated jet scenario. In this study, we investigate electron cooling in different physical situations and give a clue to distinguish them.

In addition to modeling the time-integrated spectra analytically, the analyses of the time-resolved spectra of the prompt emission (e.g., Lu et al. 2012; Jiang et al. 2016; Zhang et al. 2016a) can also provide important information on the radiation process. On the other hand, the numerical method has the advantage over the analytical method in that it can incorporate different radiation mechanisms and follow the evolution of physical properties in the emitting region. For example, Daigne et al. (2011) predicted that the high-energy (>100 MeV) light curve may display a prolonged pulse duration due to the IC emission. Uhm & Zhang (2014) revealed that as a jet expands rapidly from the central engine, the magnetic field in the emission region decreases, resulting in harder (than the case of constant magnetic field) spectra. Therefore, in our study, we develop a numerical code to calculate the evolution of the electron spectra and the corresponding flux spectra with different parameter sets. Adiabatic, synchrotron, and IC cooling of electrons (also see Bošnjak et al. 2009a) and the geometric effect of the emitting shells are considered properly in our code. Using this code, we can explore the resulting spectra in the plausible parameter space, which may provide clues to help relate the observed GRB spectra with the physical processes in the GRB ejecta. Different kinds of cooling patterns for electrons obtained in different scenarios can also serve as the baseline for further explorations.

The structure of this article is as follows. The three main cooling processes considered are briefly described in Section 2. The constraints from observations on the parameters involved in our calculations are presented in Section 3. In Section 4, we derive the conditions under which one particular cooling process will be dominant analytically. The analytical results are then compared with the numerical results in Section 5, where we generally study the roles played by different cooling processes in determining the evolution of the low-energy electron distribution. Finally, in Section 6, we summarize and discuss our results. The details of the numerical method and some relevant formulations used are given in Appendices A and B, respectively.

### 2. Cooling of Electrons

In the comoving frame of a relativistic jet, when an electron with a Lorentz factor \( \gamma^1_e \) is moving in the magnetic field of strength \( B^e \), it will lose energy by synchrotron radiation at a rate of (Rybicki & Lightman 1979)

\[
\dot{\gamma}_e^{\text{syn}} = -\frac{\sigma_T B^e \gamma^2_e}{6\pi m_e c},
\]

where \( \sigma_T \), \( m_e \), and \( c \) are the Thomson cross-section, electron mass, and speed of light, respectively. Hereafter, the superscript prime (‘) is used to denote the quantities in the comoving frame. The electron also undergoes adiabatic cooling (Uhm & Zhang 2012; Geng et al. 2014), i.e.,

\[
\dot{\gamma}_e^{\text{adi}} = \frac{1}{3} \gamma_e^{'} \frac{d \ln n_e^{'} dR}{dt} = -\frac{2}{3} \frac{\gamma_e^{'} dR}{R dt'},
\]

where we have taken the comoving electron number density \( n_e^{'} \propto R^{-2} \) for an expanding shell.

Additionally, the electrons will be cooled by the IC scattering of self-emitted synchrotron photons, which is referred to as the synchrotron self-Compton (SSC) process. The SSC cooling rate is given by (Blumenthal & Gould 1970; Fan et al. 2008)

\[
\dot{\gamma}_e^{\text{SSC}} = -\frac{1}{m_e c^2} \frac{3\sigma_T c}{4\gamma_e^{'} \nu_{\nu_{\text{min}}}} \int_{\nu_{\nu_{\text{min}}}}^{\nu_{\nu_{\text{max}}}} \frac{n_e \nu' d\nu'}{\nu'} \times \int_{\nu_{\nu_{\text{min}}}}^{\nu_{\nu_{\text{max}}}} h\nu' d\nu' F(q, g),
\]

where \( F(q, g) = 2g \ln q + (1 + 2q)(1 - q) + \frac{1}{2} \frac{(2gq)^2}{1 + 4g} (1 - q) \),

\[
g = \frac{\gamma' h\nu'}{m_e c^2}, \quad w = \frac{h\nu'}{\gamma' m_e c^2}, \quad \text{and} \quad q = \frac{w}{3g(1 - w)}. \]

The upper limit of the internal integral can be derived as \( \nu_{\nu_{\text{max}}} = \gamma' m_e c^2 / 4g + \gamma' \), and the lower limit is \( \nu_{\nu_{\text{min}}} = \nu' \). Overall, the total cooling rate of an electron can be obtained by summing up the processes.

### Table 1

| Model | \( \Gamma \) | \( \gamma_m^{'}(10^5) \) | \( B^e_0 (\text{G}) \) | \( N_{\text{m}}^{'}(10^{27} \text{s}^{-1}) \) | \( q \) | Adiabatic | SSC |
|-------|-------------|-----------------|-----------------|-----------------|-------|---------|------|
| M1    | 300         | 1               | 30              | 1               | 0     | No      | No   |
| M2    | 300         | 1               | 30              | 1               | 1     | Yes     | No   |
| M3    | 300         | 1               | 30              | 1               | 1     | Yes     | Yes  |
| M4    | 300         | 1               | 30              | 1               | 0     | Yes     | Yes  |

Note. In this group of calculations, one should note that \( R_0 = 10^{15} \text{ cm} \) is adopted, whereas the starting radius at which the jet begins to produce emission is \( R_1 = 10^{14} \text{ cm} \). This is only to achieve the same initial conditions as those in Uhm & Zhang (2014). However, \( R_1 = R_0 \) is commonly used all through this paper.
Figure 1. Evolution of the electron energy spectrum for four cases in the testing calculations (see Table 1). The orange dashed lines are the standard fast-cooling pattern, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-2}$, and the gray dashed lines present the expected cooling pattern according to the observations, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-1}$. The epochs shown for each case are different, since the electron-cooling timescales are different. The lower panel of each case is the negative spectral index of the electron spectrum, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-\gamma_e}$. In the standard calculation, M1, the electron spectrum shows a typical broken power-law profile, with the spectral indices being $-(p+1)$ above and $-2$ below $\gamma_e$. In other cases, the spectral indices below $\gamma_e$ are notably harder than $-2$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Evolution of the electron energy spectrum for four cases in the testing calculations (see Table 1). The orange dashed lines are the standard fast-cooling pattern, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-2}$, and the gray dashed lines present the expected cooling pattern according to the observations, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-1}$. The epochs shown for each case are different, since the electron-cooling timescales are different. The lower panel of each case is the negative spectral index of the electron spectrum, i.e., $dN_e/d\epsilon_e \propto \epsilon_e^{-\gamma_e}$. In the standard calculation, M1, the electron spectrum shows a typical broken power-law profile, with the spectral indices being $-(p+1)$ above and $-2$ below $\gamma_e$. In other cases, the spectral indices below $\gamma_e$ are notably harder than $-2$.}
\end{figure}
Figure 2. Corresponding synchrotron flux density spectra $F_\nu$ from the electrons with the energy distribution presented in Figure 1. The lower panel of each case shows the negative local spectral indices ($-\alpha$, $F_\nu \propto \nu^\alpha$) of $F_\nu$ in the upper panel. In the standard calculation, M1, it gives the typical fast-cooling spectrum $F_\nu \propto \nu^{-1/2}$ below $\nu_m$ ($\nu_m \propto \gamma_m^2$). For an even lower frequency, the spectrum is $F_\nu \propto \nu^{1/3}$, which is the profile of the low-frequency part of a single electron’s synchrotron spectrum. In other cases, the spectral indices below $\nu_m$ can be harder than $-1/2$ and can approach zero.
mentioned above, i.e.,
\[ \dot{\gamma}_e = \dot{\gamma}_{\text{syn}} + \dot{\gamma}_{\text{adi}} + \dot{\gamma}_{\text{SSC}}. \]

Heating of low-energy electrons due to synchrotron absorption (Ghisellini & Svensson 1991; Gao et al. 2013), which may pile up electrons in the low-energy range, is not considered here. In this work, we focus on the cooling processes in order to investigate their roles clearly. Heating or acceleration of electrons will be incorporated in our future studies.

The GRB prompt emission comes from a group of electrons, of which the instantaneous spectrum can be denoted as \( dN_e / d\gamma_e \). This electron distribution can be obtained by solving the continuity equation of electrons in energy space (Longair 2011),
\[ \frac{\partial}{\partial \gamma_e} \left( \frac{dN_e}{d\gamma_e} \right) + \frac{\partial}{\partial \gamma_e} \left[ e_{\text{tot}} \left( \frac{dN_e}{d\gamma_e} \right) \right] = Q(\gamma_e, \gamma'), \]
where \( Q(\gamma_e, \gamma') \) is the source function that describes the electrons injected into the emitting region. If the bulk Lorentz factor of the jet is \( \Gamma \), the comoving time \( \tau' \) can be related to the observer’s time by
\[ dt_{\text{obs}} = (1 + z) \Gamma (1 - \beta) \tau' \simeq \frac{1 + z}{2 \Gamma} d\tau', \]
is increasing. This group of calculations is within the Poynting flux-dominated jet scenario and corresponds to the case of \( L_c \approx L_B \). It differs from Table 2 mainly in the values of \( R_0 \) and \( \gamma_m^\prime \).

Assuming that the bulk Lorentz factor of the GRB ejecta is \( \Gamma \), the Lorentz factor of the electrons that radiate at the GRB spectral peak energy \( E_{\text{peak}} \) is \( \gamma_m^\prime \); then we have

\[
E_{\text{peak}} = \frac{1}{1 + z} \frac{3h\nu_cB^2}{4\pi m_e c} \Gamma \gamma_m^2,
\]

where \( h \) and \( q_e \) are the Planck constant and electron charge, respectively, and \( z \) is the redshift of the burst. The radiative cooling time for an electron of \( \gamma_e^\prime \) in the observer frame is

\[
t_c = \frac{3m_e c}{\Gamma \sigma_T B^2 \gamma_e^2} (1 + z),
\]

where \( Y \) is the Compton \( Y \) parameter. The dynamical time of the jet can be expressed as \( t_d \sim R(1 + z)/(2\Gamma^2c) \).

For typical parameters, the magnetic field strength in the fast-cooling regime (also see Equation (12)). The fast-cooling condition requires that \( t_c(\gamma_m^\prime) \approx t_d \), which implies

\[
\frac{B^2\gamma_m}{\Gamma} \geq \frac{6\pi m_e c^2}{\sigma_T R(1 + Y)},
\]

Taking typical values of \( E_{\text{peak}} \approx 500 \text{ keV} \) and \( R \approx 10^{15} \text{ cm} \), we can get

\[
B^2\Gamma \gamma_m^\prime = 2.9 \times 10^{13} (1 + z) \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right) G,
\]

\[
\Gamma \gamma_m^\prime \lesssim 3.3 \times 10^7 (1 + z)^{2/3} (1 + Y)^{1/3}
\times \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{2/3} \left( \frac{R}{10^{15} \text{ cm}} \right)^{1/3},
\]

by using Equations (8) and (10).

On the other hand, the specific flux at \( E_{\text{peak}} \) in the observer frame can be expressed as (e.g., Beniamini & Piran 2013, 2014; Kumar & Crumley 2015)

\[
E_{\text{obs}} = N_e \frac{\sqrt{3} q_e^2 B^2 \Gamma^2 (1 + z)}{4\pi D_L^2},
\]

where \( N_e \) is the total (already corrected for \( 4\pi \) solid angle) number of electrons with \( \gamma_e^\prime > \gamma_m^\prime \) and \( D_L \) is the luminosity distance of the burst. Then, we can estimate the number of electrons needed to produce a given observed flux by

\[
L_B = \frac{N_e}{10^{51} \text{ erg s}^{-1}}.
\]
combining Equations (11) and (13):

\[ \frac{N_e}{d_t(\gamma_m)} \approx \frac{N_e}{2\Gamma d_t/(1+z)} \times \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{-1} \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2, \]  

(14)

The corresponding average injection rate of electrons is

\[ N_{\text{inj}}' = \frac{N_e}{d_t(\gamma_m)} = \frac{N_e}{2\Gamma d_t/(1+z)} \times \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{-1} \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2 \times \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right) \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2 \text{s}^{-1}, \]

(15)

where we have used \( \tau' (\gamma_m') \) as the injection timescale in order to maintain the electron distribution and the intensity of the radiation flux within this period. One can see that in a synchrotron model, the typical value of \( N_{\text{inj}}' \) is almost model-independent and may be compared/verified with further detailed simulation results.

Now, we consider two leading models to obtain the plausible range of the key parameter \( \gamma_m \). First, if the jet is magnetically dominated, i.e., a relativistic Poynting flux-dominated jet\(^9\) (e.g., Zhang & Yan 2011; Beniamini & Piran 2014; Kumar & Crumley 2015), its isotropic equivalent magnetic luminosity is \( L_R \simeq \frac{B^2}{8\pi} \Gamma^2 c R^2 \). The kinetic energy power of the accelerated electrons is \( L_e \simeq N_{\text{inj}}' m_c c^2 \gamma_m' \Gamma^2 \). In principle, the ratio of

\[ \frac{L_e}{L_R} \simeq N_{\text{inj}}' m_c c^2 \gamma_m' \Gamma^2 \simeq \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{-1} \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2 \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right) \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2 \text{s}^{-1}. \]

(15)

\( \text{Figure 5. Evolution of the electron energy spectrum for the five cases in calculations of Group PJR15 (see Table 2).} \)

\( \text{9 In this article, by saying the scenario of the Poynting flux-dominated jet, we mean the regime invoking a large emission radius (the magnetization parameter is not necessarily very large; see Zhang & Yan 2011), which is consistent with a Poynting flux-dominated regime. Note that besides the synchrotron mechanism, emission from the Poynting flux-dominated jet has also been discussed in the photosphere context (e.g., Drenkhahn & Spruit 2002; Metzger et al. 2011).} \)
should be less than 1, since the jet is magnetically dominated. The upper limit of $\gamma_m'$ can thus be derived from Equation (16).

In the framework of the internal shock model (e.g., Rees & Mészáros 1994; Daigne et al. 2011), the dominant energy of the jet should be the kinetic energy carried by protons. If we consider that there are $\eta_p$ protons for every accelerated electron and assume that the accelerated protons remain nonrelativistic, then we get the kinetic energy power of protons as $L_p \approx \eta_p n_p m_p c^2 T^2$ (Bošnjak et al. 2009; Daigne et al. 2011; Beniamini & Piran 2013), and the ratio between $L_e$ and $L_p$ is

$$\xi_e = \frac{L_e}{L_p} \approx 4.1 \times 10^{12} \gamma_m'^{-3} \eta_p^{-1} (1 + z)^2 (1 + Y)^{-1} \left( \frac{F_{\nu_{\text{obs}}}}{1 \text{ mJy}} \right) \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right) \left( \frac{R}{10^{15} \text{ cm}} \right)^2 \left( \frac{D_L}{10^{28} \text{ cm}} \right)^{-2}.$$

Here $\xi_e$ can be equivalently treated as the familiar magnetization parameter $\sigma$ at the emission radius $R$. On the other hand, the ratio between $L_e$ and $L_p$ is

$$\xi_e = \frac{L_e}{L_p} = \frac{\gamma_m'}{\eta_p m_p}.$$

Figure 6. Corresponding synchrotron flux density spectra $F_\nu$ from the electrons with the energy distribution presented in Figure 5.
This gives an upper limit of $\gamma'_m$, i.e., $\gamma'_m = \xi_e \eta_p m_p c < \eta_p m_e c$
(Barniol Duran et al. 2012; Beniamini & Piran 2013; Kumar & Crumley 2015), since $\xi_e$ should be less than 1. Combining this limit with Equation (17), we obtain the plausible range of $\xi_B$,

$$1 > \xi_B > 6.6 \times 10^2 \eta_p^{-4}(1 + z)^2(1 + Y)^{-1} \left(\frac{E_{\text{peak}}}{1 \text{ mJy}}\right)^{-1} \times \left(\frac{500 \text{ keV}}{10^{15} \text{ cm}}\right)^2 \left(\frac{D_L}{10^{20} \text{ cm}}\right)^{-2}.$$  

Consequently, the range of $\gamma'_m$ could be derived from Equations (17) and (19). One may notice that the free parameter $\eta_p$ is crucial to determine the ranges of other quantities in the internal shock model. Previous studies indicate that $\eta_p \geq 10$ should be satisfied for the internal shock model to explain the GRB spectra (see Kumar & Zhang 2015 for a review). Whether this requirement can be fulfilled within the simulation of collisionless ion-electron shocks is still under debate (Sironi et al. 2015). This issue goes beyond the scope of our current study. In this work, we admit $\eta_p \geq 10$ first and see whether the low-energy spectra can be explained naturally.

### 4. Different Regimes

With the estimates on the ranges of the key parameters shown above, we now discuss the possible cooling behaviors of electrons in the emission region of a GRB analytically. Conventionally, we first take the synchrotron radiation as the main cooling process for electrons, since the spectra of the observed prompt emission resemble the synchrotron spectra. However, it is possible that the electrons could also lose energy largely through the other two processes. For instance, the adiabatic cooling rate for an electron of $\gamma'_e$ will dominate the synchrotron cooling rate if

$$\frac{\gamma'_e}{\gamma'_e,\text{ syn}} \geq 1,$$

which further gives

$$\Gamma^3 \gamma_{m,1}^{-1} \geq \frac{4 \pi m_e c \sigma_T (1 + z)^2 E_{\text{peak}}^2}{9 h^2 q_e^2} \simeq 5.3 \times 10^{22} (1 + z)^2 \left(\frac{E_{\text{peak}}}{500 \text{ keV}}\right)^2 \left(\frac{R}{10^{15} \text{ cm}}\right)$$

by using Equations (1), (2), and (11). For electrons of $\gamma'_e \lesssim 10^3$, one would find that this situation occurs when $\Gamma \geq 10^3$ and $\gamma'_m \geq 10^4$ or when $R$ is significantly smaller than $10^{15}$ cm. We will see that adiabatic cooling does dominate in some cases in the following calculations.

Moreover, SSC cooling is dominant if

$$\frac{\gamma'_e,\text{ SSC}}{\gamma'_e,\text{ syn}} \geq 1.$$  

The corresponding physical requirements are not straightforward, since $\gamma'_e,\text{ SSC}$ involves a double integral in Equation (3). Before giving accurate numerical results, some simple estimates can be done primarily. The magnetic energy density in the comoving frame of the ejecta is

$$U'_B = \frac{B^2}{8\pi},$$

while the radiation energy density in the comoving frame can be calculated as

$$U'_\gamma \simeq \frac{N \epsilon_{\gamma,\text{ syn}}}{4\pi R^2 c}.$$
Figure 8. Evolution of the electron energy spectrum for the four cases in calculations of Group PJR14 (see Table 3).
Figure 9. Corresponding synchrotron flux density spectra $F_\nu$ from the electrons with the energy distribution presented in Figure 8.
If scattering between an electron of energy \(e\) and photons is always in the Thomson regime, it is well known that the scattered energy \(\epsilon_{\mathrm{SSC}}\) and synchrotron energy \(\epsilon_{\mathrm{syn}}\) can be approximated as 

\[
U_{\mathrm{SSC}} = \frac{e}{2} E_{\text{peak}} \quad \text{and} \quad U_{\mathrm{syn}} = \frac{m_e c^2}{2} \frac{E_{\text{peak}}}{(1+z)}. 
\]

We define \(\gamma_T\) as the Lorentz factor of the electrons above which the scattering with the \(E_{\text{peak}}\) photons is in the KN regime (Nakar et al. 2009; Wang et al. 2009), i.e.,

\[
\gamma_T = \frac{\Gamma m_e c^2}{E_{\text{peak}}(1+z)} \simeq \Gamma (1+z)^{-\frac{1}{2}} \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{-1}. 
\]

Here \(\gamma_T\) will be smaller than \(\gamma_m\) only if \(\Gamma\) is not very large. So, for electrons within the range of \(\gamma_e \leq \gamma_m < \gamma_T\), IC cooling is in the KN regime. We also define \(h\nu_{\text{KN}} = m_e c^2 / \gamma_m^2\) as the critical photon energy of which the IC scattering between the electrons of \(\gamma_e\) is in the KN regime. The scattering between the electrons with photons of frequency \(\nu' < \nu_{\text{KN}}\) can effectively cool the electron. Assuming a low-energy photon spectrum of \(\nu' F_{\nu'} \propto \nu'^{\delta} (h\nu < E_{\text{peak}})\), then we have

\[
\frac{\gamma_{\mathrm{SSC}}}{\gamma_{\mathrm{syn}}} \simeq \frac{U'_{\nu}}{U_B} \approx \left( \frac{\gamma_T}{\gamma_m} \right)^{-\delta} \frac{U'_{\nu}}{U_B} = \left( \frac{\gamma_T}{\gamma_m} \right)^{-\delta} N_{\nu} \sigma_T \frac{\gamma_{\mathrm{m}}^2}{3\pi R^2}. 
\]

Therefore, the condition for the dominance of SSC cooling turns out to be

\[
1.3 \times 10^{-16} \left( \frac{\gamma_T}{\gamma_m} \right)^{-\delta} \gamma_m^4 (1+z)^{-2} \left( \frac{F_{\nu_{\text{obs}}}}{1 \text{ mJy}} \right) \times \left( \frac{E_{\text{peak}}}{500 \text{ keV}} \right)^{-1} \left( \frac{R}{10^{15} \text{ cm}} \right)^{-2} \left( \frac{D_L}{10^{28} \text{ cm}} \right)^2 > 1 
\]
by substituting Equation (14) into Equation (26). If the system is steady (d/dt = 0), the lower limit of δ is 0.5 when electrons are in fast cooling due to the synchrotron radiation (\(dN_e/d\gamma_e \propto \gamma_e^{-2}\) for \(\gamma_e < \gamma_{m}\)), while the upper limit is 1 when electrons are cooled by the SSC radiation (\(dN_e/d\gamma_e \propto \gamma_e^{-1}\) for \(\gamma_e < \gamma_{m}^{1/2}\); Wang et al. 2009). One will then find that for \(\gamma_{m} \geq 10^{4}\), the condition in Equation (27) can be met at least for electrons of \(\gamma_e \simeq \gamma_{m}\). Moreover, for an even smaller emission radius (e.g., \(R \approx 10^{14}\) cm), this condition will be relaxed significantly. So it is essential to consider SSC cooling during the evolution of the electron distribution. In summary, Equations (21) and (27) are useful explicit criteria on judging how an electron cools with given relevant parameters.

5. Numerical Calculations

The main task is to solve the continuity equation of electrons in the energy space, i.e., Equation (5), which is also called the advection equation with a source term. This kind of partial differential equation can be efficiently solved by the constrained interpolation profile (CIP) method (Yabe & Aoki 1991; Yabe et al. 2001). A detailed discretization procedure can be found in Appendix A. In principle, the final results are determined by the initial and boundary conditions for Equation (5). On the other hand, we have already obtained the plausible range for relevant parameters according to the estimates in Section 3. As we mentioned before, we intend to give an overview of the evolution of the electron distribution under different physical conditions. So we explore the parameter space by performing several groups of calculations to investigate the roles played by different radiation mechanisms in different cases. In this paper, we adopt the assumption that the comoving magnetic field in the jet is decaying with radius as proposed in Uhm & Zhang (2014), i.e.,

\[
B' = B_0' \left( \frac{R}{R_0} \right)^{-q},
\]

where \(B_0'\) is the magnetic strength at \(R_0\), and \(R_0\) is the radius where the jet begins to produce the first photon that is observed by us.

![Figure 11. Parameter space for the internal shock model when \(R_0 = 10^{15}\) cm and \(\eta_p = 20\) are adopted. The shadowed region is the plausible region for the \(\gamma_{m} - \Gamma\) couple constrained from Equations (12) and (19). The magenta dashed line represents the condition for fast cooling, while the gray dashed and red dashed lines show the lower limit (\(\gamma_{m} \leq \eta_p m_p/c m_e\)) and upper limit (deduced by \(\xi_b \leq 1\) with Equation (17)) of \(\gamma_{m}^{1/2}\) respectively. Here \(\Gamma\) is assumed to range from 10 to 100, according to previous research on GRB jets (Liang et al. 2010, 2013). The solid lines are the relationship between \(\gamma_{m}^{1/2}\) and \(\Gamma\) using Equation (11), when different values of \(B'\) (denoted by different colors) are adopted and \(F_{peak}\) is set to be 500 keV typically. Particularly, the cyan dotted line represents the minimum value of \(B'\) for \(\gamma_{m}^{1/2} - \Gamma\) couples to overlap with the shadowed region. The positions of the parameters in the four calculations of Group IS20R15 (see Table 4) are marked by star symbols.

The injected electrons are assumed to be a power law

\[
Q(\gamma_e, t') = Q_0(t') (\gamma_e/\gamma_{m}^{1/2})^{-p}
\]

for \(\gamma_e > \gamma_{m}^{1/2}\), where \(Q_0\) is related to the injection rate by

\[
N_{inj} = \int_{\gamma_{m}}^{\gamma_{max}} Q(\gamma_e, t') d\gamma_e.\]

Here \(\gamma_{max}\) is the maximum Lorentz factor of electrons and is given by the approximation \(\gamma_{max} \simeq 10^{0.5} \left( \frac{B}{10^4} \right)^{0.5}\) (Dai & Lu 1999; Huang et al. 2000). So, authors may notice that \(\gamma_{max}\) is evolving with time in the results of some calculations.

| Model | \(\Gamma\) | \(\gamma_{m}^{1/2}\) (10^4) | \(B_0'\) (10^7 G) | \(N_{inj}\) (10^{27} s^{-1}) | \(L_e\) (10^{51} erg s^{-1}) | \(L_B\) (10^{51} erg s^{-1}) | \(L_P\) (10^{51} erg s^{-1}) |
|-------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| IS100R15A | 120 | 10 | 0.5 | 16 | 1.7 | 5.4 x 10^{-4} | 3.2 |
| IS100R15A | 60 | 10 | 1 | 58 | 1.7 | 5.4 x 10^{-4} | 3.2 |
| IS100R15A | 20 | 10 | 3 | 530 | 1.7 | 5.4 x 10^{-4} | 3.2 |
| IS100R15B | 460 | 5 | 0.5 | 2 | 1.7 | 8.0 x 10^{-3} | 6.4 |
| IS100R15B | 230 | 5 | 1 | 8 | 1.7 | 8.0 x 10^{-3} | 6.4 |
| IS100R15B | 77 | 5 | 3 | 71 | 1.7 | 8.0 x 10^{-3} | 6.4 |
| IS100R15B | 23 | 5 | 10 | 800 | 1.7 | 8.0 x 10^{-3} | 6.4 |
| IS100R15C | 1900 | 1 | 3 | 0.58 | 1.7 | 5 | 32 |
| IS100R15C | 580 | 1 | 10 | 6.2 | 1.7 | 5 | 32 |
| IS100R15C | 580 | 1 | 100 | 620 | 1.7 | 5 | 32 |
| IS100R15D | 3000 | 0.7 | 3 | 0.2 | 1.7 | 21 | 46 |
| IS100R15D | 1200 | 0.7 | 10 | 2.1 | 1.7 | 21 | 46 |
| IS100R15D | 1200 | 0.7 | 100 | 210 | 1.7 | 21 | 46 |

Note. This group of calculations is within the internal shock model and includes four subgroups, of which the values of \(\gamma_{m}^{1/2}\) are different and \(L_e/L_B\) ranges from 0.08 to 3000.

Table 5

Parameters Used in the Calculations of Group IS100R15 (\(\eta_p = 100\), \(R_0 = 10^{15}\) cm)
are eight free parameters in total in our calculations, i.e., \( \Gamma \), \( \gamma' \), \( B_0 \), \( p \), \( N'_{ij} \), \( q \), \( R_0 \), and \( \gamma_e \) (or \( \eta_e \)). Particularly, \( q = 1 \) is commonly adopted for all calculations unless explicitly stated, since the toroidal magnetic field in the ejecta decreases as \( R^{-1} \). As we will see, this treatment does not markedly impact our main conclusions. Also, \( p = 2.8 \) is commonly adopted, since the evolution of the low-energy electron distribution is nearly unaffected by \( p \) in fast-cooling cases. Therefore, there are still six free parameters left. Below, we choose reasonable values for these parameters and perform a sequence of calculations to represent various physical conditions. With the electron spectra being derived numerically, we can then calculate the corresponding synchrotron radiation spectra according to Appendix B.

### 5.1. Testing Calculations

In this section, we first check the significance of SSC cooling in the evolution of the electron distribution in the testing calculations. Four calculations are performed and named in form of “\( M_i \) (\( i = 1, ..., 4 \)).” In \( M_1 \), we set \( q = 0 \) and ignore SSC and adiabatic cooling so that this should give a “standard” evolution pattern for electrons under synchrotron cooling only. The values for the other parameters (shown in Table 1) are taken as the same as those in Uhm & Zhang (2014) in order to compare the results directly. In \( M_2 \), we set \( q = 1 \) to achieve similar results with the decaying \( B_0 \) case shown in Uhm & Zhang (2014). In \( M_3 \), we introduce the SSC cooling process and compare the results with \( M_2 \). This should be more close to the realistic situation. At last, we set \( q = 0 \) to ignore the effect of decaying \( B_0 \) in \( M_4 \), where the result can clearly show the role played by SSC cooling. The resultant electron distributions and radiation spectra are shown in Figures 1 and 2, respectively for these four calculations. Moreover, we present the cooling rates of different radiation mechanisms in Figure 3. It is not surprising that the indices of the low-energy electron spectra are always strictly \(-2\) for \( \gamma_e \) in \( M_1 \), and the indices in \( M_2 \) turn harder along with the decreasing value of \( B_0 \), as proposed by Uhm & Zhang (2014). In \( M_3 \) and \( M_4 \), it is interesting to find that the indices of the low-energy electron spectra are approaching \(-1 \) with the elapsing time, and the electrons with \( \gamma'_e < \gamma'_m \) in \( M_1 \), and the indices in \( M_2 \) turn harder along with the decreasing value of \( B_0 \), as proposed by Uhm & Zhang (2014). In \( M_3 \) and \( M_4 \), it is interesting to find that the indices of the low-energy electron spectra are approaching \(-1 \) with the elapsing time, and the electrons with \( \gamma'_e < \gamma'_m \) are being cooled mainly via the SSC process, as shown in Figure 3. The asymptotic value of \(-1 \) is consistent with what is predicted theoretically, as mentioned. Another natural result is that the electrons are cooled much faster after considering the SSC process. For example, in \( M_3 \), the minimum Lorentz factor of electrons has already reached \( \gamma_e \approx 20 \) at 0.03 s in the observer frame, while it takes \( 1.5 \) s for electrons to cool to \( \gamma'_e = 100 \) in \( M_2 \). With these testing calculations, we see that SSC cooling...
Figure 15. Evolution of the electron energy spectrum for the four cases of Group IS20R15 (see Table 4).
Figure 16. Corresponding synchrotron flux density spectra $F_{\nu}$ from the electrons with the energy distribution presented in Figure 15.
can play an important role in determining the electron distribution, at least in cases considered in previous research.

5.2. Cases in Different Scenarios

Next, we perform numerical calculations by taking the parameters in plausible ranges for GRBs corresponding to different physical scenarios.

5.2.1. The Poynting Flux–dominated Jet

Let us first consider the case that the jet is Poynting flux–dominated. According to Equation (16), we can have the upper limit of $\gamma_m$ in the calculations. One may note that $\eta_e = 1$ means that $L_e = L_B$, which seems to be contrary to the fact of a Poynting flux–dominated jet. However, we would see that the corresponding results are representative. In addition, we perform two groups of calculations, in which $R_0$ is taken to be $10^{15}$ cm (called Group PJR15) and $10^{14}$ cm (called Group PJR14), respectively. Then, in each group, we assume a series of values for $\Gamma$, and the corresponding $B_0$ can be obtained via Equation (11). For $R_0 = 10^{14}$ cm, we perform five calculations named in the form of “PJR15ΓN,” with N denoting the value of $\Gamma$, which ranges from 50 to $10^3$ (see details in Table 2). For $R_0 = 10^{15}$ cm, we perform four calculations named in the form of “PJR14ΓN,” in which $\Gamma$ ranges from 10$^2$ to $10^3$ (see details in Table 3).

From the results of PJR15Γ1000, we notice that the electron cooling is dominated by the synchrotron radiation for $\gamma_e > 10^5$, whereas the adiabatic expansion dominates the cooling of electrons with $\gamma_e < 10^5$ (see Figure 4). It indicates that SSC cooling is not significant in this case, which also holds for other cases with even greater $B_0$ in Group PJR15. On the other hand, calculating the SSC cooling rates is extremely time-consuming, since it is difficult to achieve the integral convergence in our code. So, we do not include the SSC cooling effect in this group of calculations. The corresponding results for electron distributions, flux spectra, and cooling rates are shown in Figures 5–7, respectively. In the results of PJR15Γ1000 and PJR15Γ600, it is seen that along with the decaying of $B_0$, adiabatic cooling becomes more and more dominant for low-energy electrons, since $\dot{\gamma}_e,\text{syn} \propto B^2 \propto R^{-2}$ and $\dot{\gamma}_e,\text{adi} \propto R^{-1}$. The decreasing of $\dot{\gamma}_e,\text{adi}$ along $R$ leads to an electron spectrum that is harder than $-1$, just like what is done by the decreasing $\dot{\gamma}_e,\text{syn}$ (see the results of M2). The low-energy indices of the flux density spectra in PJR15Γ1000 ($\alpha, F_0 \propto \nu^\alpha$) can be larger than zero, and $\alpha$ in PJR15Γ600 is within $[-0.5, 0]$. In the results of PJR15Γ300, PJR15Γ100, and PJR15Γ50, the cooling of all electrons is dominated by the synchrotron radiation only. In Figure 5, the decaying of $B_0$ leads the electron spectra to become harder than $-2$, as expected in PJR15Γ300. However, this does not occur in
Figure 18. Evolution of the electron energy spectrum for the 13 cases in Group IS100R15 (see Table 5).
Comparing with the results of Uhm & Zhang (2014), the electron spectrum seems to become harder when the electrons are cooled at a decreasing cooling rate. A history/experience of decreasing cooling rate for an electron is the key factor to result in a hard spectrum. For relatively large \( B' \) and \( R \), the cooling timescale of electrons is significantly extended.

Table 6
Parameters Used in the Calculations of Group IS10R14 (\( \eta_p = 10, R_0 = 10^{14} \) cm)

| Model           | \( \Gamma' \) | \( \gamma_m' \) \( \times 10^3 \) | \( B_0' \) \( \times 10^7 \) G | \( N_{\text{inj}}' \) \( \times 10^7 \) s\(^{-1} \) | \( L_{\text{e}} \) \( \times 10^{51} \) erg s\(^{-1} \) | \( L_B \) \( \times 10^{51} \) erg s\(^{-1} \) | \( L_p \) \( \times 10^{51} \) erg s\(^{-1} \) |
|-----------------|---------------|-------------------------------|----------------------------|------------------------|----------------|----------------|----------------|
| IS10R14AT'900   | 1900          | 1                             | 3                         | 0.58                   | 1.7            | 0.05           | 3.1            |
| IS10R14AT'580   | 580           | 1                             | 10                        | 6.2                    | 1.7            | 0.05           | 3.1            |
| IS10R14AT'58    | 58            | 1                             | 100                       | 620                    | 1.7            | 0.05           | 3.1            |
| IS10R14BT'2300  | 2300          | 0.5                           | 10                        | 0.79                   | 1.7            | 0.8            | 6.3            |
| IS10R14BT'770   | 770           | 0.5                           | 30                        | 7.1                    | 1.7            | 0.8            | 6.3            |
| IS10R14BT'230   | 230           | 0.5                           | 100                       | 79                     | 1.7            | 0.8            | 6.3            |

Note. This group of calculations is within the internal shock model and includes two subgroups, of which the values of \( \gamma_m' \) are different and \( L_{\text{e}}/L_B \) ranges from 2 to 30.
Figure 19. Corresponding synchrotron flux density spectra $F_\nu$ from the electrons with the energy distribution presented in Figure 18.
smaller than the dynamical timescale. As a consequence, the history of the decreasing cooling rate of an electron is too short to take effect.

In calculations of Group PJR14, the results for electron distributions, flux spectra, and cooling rates are shown in Figures 8–10, respectively. In the results of PJR14Γ1000, it can be seen that adiabatic cooling is dominant for low-energy electrons due to the relatively large $\Gamma$ and small $B', R_0$. The indices of the low-energy electron spectra in PJR14Γ1000 are harder than $-1$, and $\alpha$ also becomes harder than zero at a frequency of $\sim 1$ keV when $t_{\text{obs}} > 0.5$ s. In PJR14Γ600, synchrotron cooling is dominant at early times, while adiabatic cooling becomes dominant at late stages. The low-energy electron spectra and the flux spectra are harder than the standard ones, but not so hard as those in PJR14Γ1000. In the results of PJR14Γ300 and PJR14Γ100, we can find that synchrotron cooling is always dominant, and the low-energy electron spectra and the flux spectra are just similar to the standard ones. Here again, in PJR14Γ300 and PJR14Γ100, the cooling timescale of an electron under relatively large $B'$ is significantly smaller than the dynamical timescale, and the mechanism of “decreasing synchrotron cooling rate” cannot work to make the electron spectra significantly harder than $-2$.

For the Poynting flux–dominated jets considered here, we confirm that the decreasing synchrotron cooling rate (decreasing $B'$) will lead to hard electron and flux spectra, according to the results of our Groups PJR15 and PJR14. The values of $\gamma'_m$ used in PJR15 and PJR14 are one or two orders smaller than those used in Uhm & Zhang (2014) to meet the physical condition of $L_B \gtrsim L_\nu$. Although the $\gamma'_m$ used here is only the upper limit, the results from Groups PJR15 and PJR14 should be representative of three kinds of cooling patterns in this scenario, i.e., hard electron spectra caused by decreasing $\dot{\gamma}'_{\text{e, adi}}$ or decreasing $\dot{\gamma}'_{\text{e, syn}}$ and normal spectra under large $B'$. Moreover, we notice that $B'$ should not be too large in this scenario, otherwise the synchrotron cooling timescale would be much shorter than the dynamical timescale, and the effect of

Figure 19. (Continued.)
Figure 20. Comoving cooling rates of different cooling mechanisms for the electrons with the energy distribution presented in Figure 18.
decaying $B'$ is weakened. This is also the reason that we did not explore the regime of $L_B \gg L_0$ (when SSC cooling is not important). In other words, if we want to use the decreasing $\dot{\varepsilon}_{\text{syn}}$ to work for hard electron spectra, there should be a lower limit $\zeta$ for the extent of fast cooling, i.e., $\zeta \lesssim t_c/t_d < 1$. The "unsuccessful" results from PJR15–PJR15\textsuperscript{50} and PJR14\textsuperscript{300}–PJR14\textsuperscript{100} together indicate that $\zeta$ should be larger than $10^{-5}$. Since $B' \propto \varepsilon_{\text{syn}}^{-1/2}$, the proper range of $B'$ for decaying $\dot{\varepsilon}_{\text{syn}}$ to work is likely to be within roughly two orders of magnitude. If this mechanism is true for GRB spectral hardening, the narrow range of $B'$ indicates a potential way to probe the magnetic strength of the GRB jet using its spectral characteristics.

5.2.2. The Internal Shock Scenario

Now we turn to the internal shock model. In this scenario, since we still have little knowledge about the emission radii of GRBs and the crucial parameter $\eta_p$, we perform four groups of calculations to try to cover various possibilities. For $R_0 = 10^{15}$ cm, we consider two situations, i.e., $\eta_p = 20$ or\textsuperscript{11} $\eta_p = 100$. For $\eta_p = 20$, we show the allowed parameter region in the $\gamma'_{\text{ms}}$–$\Gamma$ diagram (see Figure 11) by combing the restrictions given in Equations (12) and (17). In this group of calculations, called Group IS20R15, four calculations are performed, and the positions of the corresponding parameters in the parameter space are marked as black stars in Figure 11. Detailed parameter values in each calculation are listed in Table 4, and each calculation is named in the form of "IS20R15N," with N denoting the value of $\Gamma$. Similarly, in the calculations of Group IS100R15 ($\eta_p = 100$), we perform 13 calculations, for which the corresponding information can be seen in Table 5 and Figure 12. These 13 calculations are classified into four subgroups according to different values

\textsuperscript{11} Here we use $\eta_p = 20$ rather than $\eta_p = 10$ due to the fact that $\eta_p = 10$ would slightly violate the underlying condition of $\xi_B + \xi_c \lesssim 1$. 

Figure 21. Evolution of the electron energy spectrum for the six cases in Group IS10R14 (see Table 6).
used for $\gamma_m$. Each calculation is named in the form of “IS20R15WTN,” with “W” (e.g., A, B, etc.) distinguishing different subgroups and N denoting the value of $\Gamma$. When $R_0 = 10^{14}$ cm is adopted, we also consider two situations, i.e., $\eta_p = 10$ or $\eta_p = 100$. Six calculations are performed in Group IS10R14 (see Table 6 and Figure 13), while 10 calculations are performed in Group IS100R14 (see Table 7 and Figure 14).

In the calculations of Group IS20R15, we notice that $1.5 \times 10^4$ and $L_e \approx L_B$. The results for electron distributions, flux spectra, and cooling rates are shown in Figures 15–17, respectively. From these results, we find that the electron and flux spectra can be hard enough to match the observations only when adiabatic cooling is dominant (in IS20R15Γ1300). For the other three cases (IS20R15Γ430–IS20R15Γ86), $\gamma_e$ is only comparable to $\gamma_{\text{e,syn}}$ for electrons of $\gamma_e \lesssim 10^2$, and the indices of the low-energy electron spectra are slightly harder than $-2$.

In the calculations of Group IS100R15, we can see that $\gamma_m$ ranges from $7 \times 10^3$ to $10^5$ and $L_e/L_B$ ranges from 0.08 to 3000. According to the results (see Figures 18–20), we find the following. (1) For Subgroup IS100R15A ($L_e/L_B = 3000$), $\gamma_{\text{e,SSC}}$ is always dominant for electrons of $\gamma_e < \gamma_m$, and the resulting indices of the electron spectra are $\sim -1.3$. (2) For Subgroup IS100R15B ($L_e/L_B = 200$), $\gamma_e$ is dominant for electrons of $\gamma_e < 10^4$, and the resulting indices of the electron spectra are $\sim -1.4$, except for IS100R15BΓ60, in which $\gamma_{\text{e,adi}}$ becomes dominant at the late time and the low-energy electron indices can be even harder than $-1$. (3) For Subgroup IS100R15C ($L_e/L_B = 0.3$), $\gamma_e$ is always larger than $\gamma_{\text{e,SSC}}$. The electron spectra resemble the standard ones in IS100R15CΓ580 and IS100R15CΓ58, while the electron indices in IS100R15CΓ1900 are becoming harder than $-1$ due to the dominance of adiabatic cooling. (4) For Subgroup IS100R15D ($L_e/L_B = 0.08$), $\gamma_{\text{e,syn}}$ is always larger than $\gamma_{\text{e,SSC}}$. 

Figure 22. Corresponding synchrotron flux density spectra $F_e$ from the electrons with the energy distribution presented in Figure 21.
The electron spectra resemble the standard ones in IS100R15D and IS100R15D-3900, while the electron indices in IS100R15D-3900 are becoming even harder than zero due to the dominance of adiabatic cooling.

In the calculations of Group IS100R14, we find the following (see Figures 21–23). (1) For Subgroup IS10R14A (L_e/L_B = 30), \( \gamma'_e \) is larger than \( \gamma'_e,SSC \) for electrons of \( \gamma'_e < 3 \times 10^3 \), and the resulting indices of the electron spectra are \( \sim -1.5 \) for IS10R14A-580 and IS10R14A-58. For IS10R14A-1900, \( \gamma'_e,adi \) is always dominant for electrons of \( \gamma'_e < \gamma'_m \), and the electron indices are becoming harder than zero. (2) For Subgroup IS10R14B (L_e/L_B = 2), \( \gamma'_e,SSC \) is slightly larger than \( \gamma'_e,SSC \) for electrons of \( \gamma'_e < 3 \times 10^3 \), and the resulting indices of the electron spectra are \( \sim -1.8 \) for IS10R14B-5770 and IS10R14B-2000. For IS10R14B-7730, \( \gamma'_e,adi \) is becoming increasingly dominant for \( \gamma'_e < \gamma'_m \), and the electron indices are becoming harder than zero.

In the calculations of Group IS100R14, \( \gamma'_m \) ranges from \( 5 \times 10^5 \) to \( 10^5 \), and \( L_e/L_B \) ranges from 2 to \( 3 \times 10^5 \). According to the results (see Figures 24–26), we find the following. (1) For Subgroups IS100R14A and IS100R14B (L_e/L_B = 3 \times 10^5 or 8.5 \times 10^3), \( \gamma'_e,SSC \) is always much larger than both \( \gamma'_e,SSC \) and \( \gamma'_e,adi \) for electrons of \( \gamma'_e < \gamma'_m \), and the resulting indices of the electron spectra are \( \sim -1 \). (2) For Subgroup IS100R14C (L_e/L_B = 30), \( \gamma'_e,SSC \) is larger than both \( \gamma'_e,SSC \) and \( \gamma'_e,adi \) for electrons of \( \gamma'_e < 3 \times 10^3 \), and the resulting indices of the electron spectra are \( \sim -1.5 \). (3) For Subgroup IS100R14D (L_e/L_B = 2), since \( \gamma'_e,adi \) is becoming increasingly dominant \( 5 \times 10^5 \) to \( 10^5 \), and \( L_e/L_B \) ranges from 2 to \( 3 \times 10^5 \). According to the results (see Figures 24–26), we find the following. (1) For Subgroups IS100R14A and IS100R14B (L_e/L_B = 3 \times 10^5 or 8.5 \times 10^3), \( \gamma'_e,SSC \) is always much larger than both \( \gamma'_e,SSC \) and \( \gamma'_e,adi \) for electrons of \( \gamma'_e < \gamma'_m \), and the resulting indices of the electron spectra are \( \sim -1 \). (2) For Subgroup IS100R14C (L_e/L_B = 30), \( \gamma'_e,SSC \) is larger than both \( \gamma'_e,SSC \) and \( \gamma'_e,adi \) for electrons of \( \gamma'_e < 3 \times 10^3 \), and the resulting indices of the electron spectra are \( \sim -1.5 \). (3) For Subgroup IS100R14D (L_e/L_B = 2), since \( \gamma'_e,adi \) is becoming increasingly dominant

Figure 23. Comoving cooling rates of different cooling mechanisms for the electrons with the energy distribution presented in Figure 21.

### Table 7

Parameters Used in the Calculations of Group IS100R14 (\( \eta_p = 100, R_0 = 10^{14} \text{ cm} \))

| Model               | \( \Gamma \) | \( \gamma'_e \) (10^4) | \( B_0 \) (10^11 G) | \( N_{0e} \) (10^{-21} cm\(^{-3}\)) | \( L_e \) (10^{51} erg s\(^{-1}\)) | \( L_B \) (10^{-3} erg s\(^{-1}\)) | \( L_p \) (10^{51} erg s\(^{-1}\)) |
|---------------------|--------------|--------------------------|----------------------|----------------------------------|---------------------------------|--------------------------------|--------------------------------|
| IS10R14A-120        | 120          | 10                       | 0.5                  | 16                               | 1.7                             | 5.4 \times 10^{-6}             | 3.2                             |
| IS10R14A-160        | 60           | 10                       | 1                    | 58                               | 1.7                             | 5.4 \times 10^{-6}             | 3.2                             |
| IS10R14A-20        | 20           | 10                       | 3                    | 530                              | 1.7                             | 5.4 \times 10^{-6}             | 3.2                             |
| IS10R14B-360        | 360          | 4                        | 1                    | 4                                | 1.7                             | 2.0 \times 10^{-4}             | 7.9                             |
| IS10R14B-120        | 120          | 4                        | 3                    | 36                               | 1.7                             | 2.0 \times 10^{-4}             | 7.9                             |
| IS10R14B-36        | 36           | 4                        | 10                   | 400                              | 1.7                             | 2.0 \times 10^{-4}             | 7.9                             |
| IS10R14C-580        | 580          | 1                        | 10                   | 6.2                              | 1.7                             | 0.05                           | 31                              |
| IS10R14C-58         | 58           | 1                        | 100                  | 600                              | 1.7                             | 0.05                           | 31                              |
| IS10R14D-2300       | 2300         | 0.5                      | 10                   | 0.78                             | 1.7                             | 0.81                           | 63                              |
| IS10R14D-230        | 230          | 0.5                      | 100                  | 78                               | 1.7                             | 0.81                           | 63                              |

**Note.** This group of calculations is within the internal shock model and includes four subgroups, of which the values of \( \gamma'_e \) are different and \( L_e/L_B \) ranges from 2 to \( 3 \times 10^4 \).
Figure 24. Evolution of the electron energy spectrum for the 10 cases in Group IS100R14 (see Table 7).
Figure 25. Corresponding synchrotron flux density spectra $F_{\nu}$ from the electrons with the energy distribution presented in Figure 24.
for low-energy electrons, the indices of the electron spectra are getting harder than zero. The results of IS100R14D are similar to those of IS10R14B, since their parameters are actually the same. So, we have not shown them as subfigures in the relevant figures of this calculation group.

To sum up, in cases with a large \( \Gamma (10^3 \mathrm{G}) \), adiabatic cooling is the most dominant process for low-energy electrons' cooling, making the electron indices harder than \(-1\) or even zero. For cases of \( L_{\text{B}} > 1 \), and when \( \Gamma \) is not too large, SSC cooling dominates over synchrotron cooling, resulting in electron spectra with indices ranging from \(-1\) to \(-2\) with the decreasing of \( L_{\text{c}}/L_{\text{B}} \). When \( L_{\text{c}}/L_{\text{B}} \leq 1 \) is met while \( \Gamma \) is still not too large, synchrotron cooling will take over, and the resulting spectra slightly deviate from the standard ones. These three kinds of cooling patterns are generally consistent with previous numerical results in Bošnjak et al. (2009) and Daigne et al. (2011). The conditions for the dominance of adiabatic or SSC cooling in these results are consistent with the analyses in Section 4. In general, the combination of a large \( \gamma'_{\text{m}} \) and small \( B' \), \( R \) will favor the dominance of adiabatic or SSC cooling, rather than synchrotron cooling, according to Equations (21) and (27).

In reality, the observed minimum variability timescales of GRBs can be used to derive the internal shock radii, which typically gives \( R_0 = 10^{14} \mathrm{cm} \) or smaller. We have only explored the region of \( R_0 \geq 10^{13} \mathrm{cm} \) in our calculations. However, according to Equations (26) and (27), one will realize that a smaller \( R_0 \) will enhance the SSC cooling rate due to the increase of the radiation energy density, i.e., \( U_\gamma \propto R^{-2} \). Therefore, for a smaller \( R_0 \), SSC cooling will be more significant. Also, we can expect that the results of a smaller \( R_0 \) can be well differentiated by values of \( L_{\text{L}}/L_{\text{B}} \) and \( \Gamma \).

6. Discussion and Conclusions

We have developed a code to solve the continuity equation of electrons in GRB ejecta and analyzed the roles played by three cooling mechanisms (synchrotron, SSC, and adiabatic) and the effect of decaying magnetic field in determining the electron/flux spectra in both the fireball and the Poynting flux-dominated regimes. By exploring the parameter space and calculating the corresponding electron spectra, flux spectra, and electron cooling rates, we find that the hardening of the electron spectra can be attributed to synchrotron radiation with a decaying \( B' \), the dominance of adiabatic cooling, or the dominance of SSC cooling. Therefore, it is essential to coherently consider them together in future studies of the

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In our results, the spectral indices \( \alpha \) are strictly smaller than zero for cases when SSC cooling dominates. This is consistent with the limit of \( \alpha > -0.1 \) given in Barniol Duran et al. (2012) with detailed analytical study. While the results in Daigne et al. (2011) violate this limit slightly (see their Figure 2), this difference does not much affect the consensus on the effect of SSC cooling.
GRB prompt emission. The numerical method, as proposed in this paper, has the advantage over the analytical method in solving equations involving several factors simultaneously. According to our results, the low-energy spectra of GRBs could be explained by the synchrotron radiation from either a Poynting flux–dominated jet or an internal shock, although some shortcomings may exist for the two scenarios.

SSC cooling of electrons in the KN regime has been previously proposed to solve the fast-cooling problem. In this paper, our analyses confirm that SSC cooling is crucial in the internal shock scenario when we take parameters deduced from typical observational characteristics. A sequence of numerical calculations further reveals that SSC cooling will result in electron spectra with low-energy indices ranging from $-2$ to $\sim-1$. The physical condition for SSC cooling to be dominant is $L_e/L_B > 1$. In order to match the observations, i.e., $dN_e/dE_e \sim \gamma_e^{-1}$, $F_B \sim \nu^0$, the condition should be more strict, i.e., $L_e/L_B \gtrsim 10^4$, according to the results from the calculation Groups IS100R15 and IS100R14. This condition means $\xi_e/\xi_B \gtrsim 10^4$ within the internal shock model. It is still hard to understand why the energy fraction in the magnetic field could be so small. However, a possible explanation is that the $B'$ that cools the electrons here is much smaller than the magnetic field in the acceleration region. As proposed by Rossi & Rees (2003), Pe`r`e & Zhang (2006), Lemoine (2013), and Zhao et al. (2014), the magnetic field may accumulate in a small region (but carries the majority of the magnetic energy created by the shock) just near the shock front, and the magnetic field behind the shock decays rapidly with the distance from the front. This possibility is hinted at in particle-in-cell simulations of shocks (e.g., Medvedev et al. 2005; Chang et al. 2008). Since the electrons may be cooled in the downstream region behind the shock, the $B'$ they stream through should be smaller than that near the shock. Moreover, to achieve the electron spectra with low-energy indices of $\sim-1$, $\eta_B > 10^2$ is preferred to take $\gamma_m > 10^4$. It indicates that only $\sim1\%$ of electrons are accelerated and that they carry $\sim10\%$ of the energy when they cross the shock. This should correspond to the cases in which the relativistic electron-ion shock is of low magnetizations ($\sigma \leq 10^{-3}$), according to the simulations of Sironi & Spitkovsky (2011) and Sironi et al. (2015).

Our numerical results also support the idea that the hardening of the electron spectra purely by SSC cooling in the KN regime can be only up to but not equal to $-1$ (Barniol Duran et al. 2012). So, the fast-cooling problem could not be fully solved by SSC cooling, since we have some GRB spectra with $\alpha > 0$. However, for this small fraction of GRBs, they may be well explained when adiabatic cooling is the dominant cooling process for low-energy electrons. This requires that the ejecta is moving at an extremely relativistic speed, i.e., $\Gamma > 10^3$. In the framework of the internal shock model, this condition may be naturally fulfilled, since the initial energy of the ejecta is mainly turned into kinetic energy. On the other hand, the lower limit for $\Gamma$ in three GRBs set by the Fermi team is $\sim 1000$ (Abdo et al. 2009a, 2009b, 2009c). Thus, it is possible that adiabatic cooling may be dominant in a few GRBs.

For a Poynting flux–dominated jet ($L_B \gtrsim L_e$), we confirm that the low-energy indices of the electron spectra can be harder than $\sim-1$ when $B'$ is underdecaying with $R$ (Uhm & Zhang 2014), as shown in the results of PJR15\Gamma600 and PJR14\Gamma600. By using the constraints from observations and performing a sequence of calculations (see Section 5.2.1), we further reveal that $B'$ should not be too large in this scenario, otherwise the synchrotron-cooling timescale will be much shorter than the dynamical timescale, and the effect of the decaying $B'$ is weakened. This feature provides a way to identify the effect of SSC cooling from the effect of decaying $B'$.

Although spectral hardening could be achieved by two mechanisms, i.e., SSC cooling in the KN regime or the effect of decaying $B'$, they may be distinguished in observations. For SSC cooling in the KN regime, the hardening of the electron spectra is a result of the “intrinsuc” scale relation of $\dot{\gamma}_e^\text{tot} \sim \dot{\gamma}_e^\text{SSC} \propto \dot{\gamma}_e^\text{-1}$ (Derishev et al. 2001; Bo`snjak et al. 2009; Nakar et al. 2009; Wang et al. 2009; Fan 2010; Daigene et al. 2011). It can be seen that the hardening of the electron spectra could be established within a rather short timescale, e.g., $t_{\text{obs}} < 0.1$ s, via SSC cooling, as in the results of Subgroups IS100R14A and IS100R14B. In contrast, decaying $B'$ is an “external” way, and the hardening of the electron spectra due to this effect needs a longer time, e.g., $t_{\text{obs}} \gtrsim 0.5$ s, according to the results of PJR15\Gamma600 and PJR14\Gamma600. Therefore, if a flux spectrum of the form $F_\nu \sim \nu^0$ is observed in a single, distinguishable, short pulse ($t_{\text{obs}} < 0.1$ s) for a particular GRB, then it is more likely that the SSC mechanism should be the dominant cooling process. On the other hand, spectral lags and $E_{\text{peak}}$ evolution patterns are found to be related to broad pulses (with durations of seconds, called slow-component; Gao et al. 2012) rather than quick variabilities (Zhang & Yan 2011; Uhm & Zhang 2016a). According to $R \sim \Gamma^2 \tau_{\text{ch}},$ the emission radius is large for large $\tau_{\text{ch}},$ which would weaken the effect of SSC cooling. The large emission radius is also contrived within the internal shock model. Thus, for these broad pulses, the effect of decaying $B'$ should be more important if the evolution of spectral hardening (and softening) could be well matched with numerical results.

No general consensus on GRB jet properties (e.g., jet composition, emission radius) has been reached in the community. The information from the time-resolved low-energy spectra can help to probe the physical characteristics of the GRB ejecta via our numerical results. As mentioned above, SSC cooling in the KN regime works in the scenario of internal shocks (baryon-dominated jet), while the effect of decaying $B'$ mainly happens in the scenario of the Poynting flux–dominated jet. Once the time-resolved low-energy spectral hardening is affirmed to be due to a specific mechanism, the jet composition could also be inferred simultaneously. Furthermore, an overall comparison of the results among the four calculation groups within the internal shock model indicates that $\alpha \sim 0$ is more likely to be achieved at a small emission radius ($\lesssim 10^{15}$ cm). This value is well consistent with the results from other methods (Rees & Mészáros 1994; Gupta & Zhang 2008; Kumar & Zhang 2015).

There is another way to solve the fast synchrotron cooling problem. Except for the cooling processes of electrons, their acceleration processes should also be important in determining the final electron distribution. A hard electron energy distribution with index $\sim-1$ may be produced by a slow heating process (e.g., Ghisellini & Celotti 1999; Stern & Poutanen 2004; Asano & Terasawa 2009). This possibility was confirmed by Xu & Zhang (2017) recently, who considered the second-order Fermi acceleration in the turbulent
reconnection (Zhang & Yan 2011). In future studies, we will incorporate the acceleration term into Equation (5) and investigate the effect in more detail.

Some parameters are fixed in our calculations, such as $N_{\text{ini}}$, $\Gamma$, etc. Although these parameters may be variable in reality, our results can still be a useful baseline for further detailed explorations. Actually, the variation of these parameters can be easily taken into account in our numerical code by setting the boundary conditions as time-dependent functions. Furthermore, more physical processes could be considered instantly. For example, the scattering of electrons by various external radiation fields (Yan et al. 2016) may also be an important cooling mechanism. Additionally, the electrons that are not efficiently accelerated (the low-energy electrons of Maxwellian distribution) may provide considerable low-energy seed photons. For the complete test of a GRB model, it is necessary to consider the dynamics of the GRB jet at much earlier stages. For example, a Poynting flux–dominated jet may undergo acceleration when it is emitting (Uhm & Zhang 2016b), while the final Lorentz factor of merged ejecta is determined by the momentum of early shells. In the future, we will improve our code to simulate the dynamical processes of GRBs more physically, deriving the light curves and spectra at the same time. Such complete theoretical fitting to the observed spectra and light curves will give more clues on the characteristics of the emission region. We note that the Hard X-ray Modulation Telescope (HXMT; Li 2007; Xie et al. 2015) launched by China recently can cover an energy range of 1–250 keV. It will be efficiently in collecting the time-resolved low-energy spectra of GRBs in the near future. Our methodology proposed here will be helpful to make use of the HXMT data.

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Appendix A

Numerical Method

In this appendix, we present the discretization procedure to solve Equation (5), i.e., the CIP method (see Yabe et al. 2001 for a general review). For a one-dimensional nonlinear equation,

$$
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = g,
$$

(29)

it is expedient to separate the solution procedure into two fractional steps. One is the advection phase,

$$
\frac{\partial \hat{f}}{\partial t} + u \frac{\partial \hat{f}}{\partial x} = 0,
$$

(30)

and the other is the nonconvection phase,

$$
\frac{\partial \hat{f}}{\partial t} = g - f \frac{\partial u}{\partial x} = G,
$$

(32)

where $\hat{f} = \partial f/\partial x$, $\hat{G} = \partial G/\partial x$ stands for the spatial derivative of $f$ and $G$. Here $\hat{f}$ should be solved together with $f$ in the CIP method, which is crucial to obtain the propagation of the spatial derivative during the evolution. If we assume that the profile between two adjacent points can be interpolated by the cubic polynomial $F(x) = ax^3 + bx^2 + cx + d$, then the solution at grid $i$ can be evolved from step $n$ to step $n + 1$ by

$$
f_{i}^{n+1} = a_i \xi^3 + b_i \xi^2 + c_i \xi + d_i,
$$

(34)

$$
f_{i}^{n+1} = 3a_i \xi^2 + 2b_i \xi + c_i + D
$$

(35)

where $a_i = \frac{f_{i+1} - f_{i-1}}{\Delta x}$, $b_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2}$, $c_i = \frac{f_{i+1} - f_{i-1}}{\Delta x}$, $\xi = -u \Delta t$, and $\Delta t$ is the time step. Note that if $u < 0$ (just the case for electron cooling), one should replace $i - 1$ with $i + 1$ and $\Delta x$ with $-\Delta x$. In Equations (34) and (35), $f_i^{n+1}$ and $f_i^n$ are the intermediate solutions from the nonconvection phase and can be obtained through the centered finite difference of Equations (32) and (33), i.e.,

$$
f_i^* = f_i^n + G_i \Delta t,
$$

(36)

$$
f_i^\dagger = f_i^n + \frac{G_{i+1} - G_i \Delta t}{2\Delta x} - \frac{2u_i^n f_{i+1} - f_{i-1} - u_i^n \Delta t}{2\Delta x} - \frac{2u_i^n f_{i+1} - f_{i-1} - u_i^n \Delta t}{2\Delta x}
$$

(37)

For the specific case in this paper, since the thermal Lorentz factor $\gamma'_e$ ranges from 10 to 10^7, it is necessary to solve Equation (5) in the logarithm space of $\gamma'_e$. We determine $\log_{10} \gamma'_e = x$, so that $\frac{dN}{dx} = \ln 10 \gamma'_e \frac{dN}{dx}$, and Equation (5) is transformed to

$$
\frac{\partial \gamma'_e}{\partial \xi} \frac{dN}{dx} + \frac{\partial}{\partial \xi} \left[ \frac{dN}{dx} \frac{dN}{dt} \right] = Q(x, t) \gamma'_e \ln 10,
$$

(39)

where $\frac{dN}{dx}$ is actually what we want to solve in our code. In all the calculations, we set the range of $x$ to be [1, 8] and the total grids number $N_{\text{num}}$ to be 401. The time step for every evolution is determined by the Courant condition,

$$
\Delta t' \leq \frac{\Delta \xi}{\gamma'_{\text{max}}},
$$

(40)

where $\gamma'_{\text{max}}$ is the maximum thermal Lorentz factor and can be given by the approximation $\gamma'_{\text{max}} \approx 10^{0.5 (\frac{N}{10})^{0.5}}$ (Dai & Lu 1999; Huang et al. 2000). However, the uncertainty of the real value of $\gamma'_{\text{max}}$ has little influence on the evolution of the electron distribution and the spectra we focus on.
Appendix B
Formulations for Radiation

In the comoving frame, the synchrotron radiation power at frequency $\nu'$ is (Rybicki & Lightman 1979)

$$P'(\nu') = \frac{\sqrt{3} q_e^2 B^2}{m_e c^2} \int_{\gamma'_{\text{min}}}^{\gamma'_{\text{max}}} \frac{dN_e}{d \gamma'} \frac{P'(\nu')}{\nu'} d\gamma',$$  \hspace{1cm} (41)

where $\nu' = 3q_e B^2/4(\pi m_e c)$ and $\gamma'_{\text{min}}$ is the ejection efficiency on the low-energy indices of the EATS effect in this paper.

The synchrotron seed photon spectra can then be calculated as (Fan et al. 2008)

$$n'_{\nu'} \approx \frac{T'}{h^2 \nu' m_e c^2} \int_{\gamma'_{\text{min}}}^{\gamma'_{\text{max}}} n'_{\gamma'} F(\nu'/\nu') d\gamma',$$  \hspace{1cm} (42)

where $T' \approx \Delta/c$ is the time that the synchrotron radiation photons stay within the ejection. If we neglect the Doppler shift, we can be derived for a given $\theta$ and $\gamma'_{\text{max}}$, the synchrotron electron number density, and $\Delta \approx R'/\Gamma$ is the comoving width of the ejection. One should notice that $\Delta$ does not appear in our calculations, since the $\Delta$ in $T'$ and $n'_{\gamma'}$ are canceled out. Here, for simplicity, we have considered only the single-scattering case for SSC cooling and ignored the multiple-scattering process.

If we assume the spectrum of the effective arrival-time surface (EATS; Waxman 1997; Granot et al. 1999; Huang et al. 2007; Geng et al. 2017), then the observed spectral flux can be expressed as

$$F_{\text{obs}} = \frac{1}{4\pi D_L^2} \left(1+z\right) \left(1+z\right) \left(1+z\right) P(\nu'(t_{\text{obs}}))$$  \hspace{1cm} (43)

where $\nu' = (1+z)\nu_{\text{obs}}/D$, and $D = 1/[1/(1-\cos \theta) / r_{\text{obs}}]$ is the Doppler factor. In this work, the luminosity distance $D_L$ is obtained by adopting a flat $\Lambda$CDM universe, in which $H_0 = 71$ km s$^{-1}$, $\Omega_m = 0.27$, and $\Omega_\Lambda = 0.73$. For all the calculations in this work, the burst is assumed to be at a cosmological redshift $z = 1$. If we use the EATS effect into account, the observed spectral flux should be

$$F_{\text{obs}} = \frac{1}{4\pi D_L^2} \int_{0}^{\theta} P(\nu'(t_{\text{obs}})) D_s \sin \theta / 2 d\theta,$$  \hspace{1cm} (44)

where $\theta$ is the half-opening angle of the jet. The integration of $\theta$ is performed over an elliptical surface (or a sequence of $R_0$), which is determined by Gao et al. (2016).

$$t_{\text{obs}} = (1+z) \int_{0}^{R_0} \frac{1-\beta\cos \theta}{\beta c} dr = \text{const},$$  \hspace{1cm} (45)

from which $R_0$ can be derived for a given $\theta$. However, we found that there is little difference between the spectrum calculated from the case with the EATS effect and the case without in our calculations. Also, the EATS effect has no significant influence on the low-energy indices of the flux spectra. So, we just show the spectra calculated without the EATS effect in this paper.
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