Lifetime of the $B^+_c$ meson in relation to flavour anomalies

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The Standard Model decay rate of the $B_c$ meson is discussed together with a novel approach that uses experimental data in combination with an operator product expansion. In the new method differences of $B$, $D$ and $B_c$ meson decay rates are considered for which the free-quark contributions drop out, leading to a reduction of the theory prediction.
1. Introduction

The $B_c$ decay rate places stringent bounds on New Physics models in the context of the $R(D)$ and $R(D^*)$ anomalies [1–3], from scalar Leptoquarks and Two-Higgs-Doublet models for example.

The $B_c = (\bar{b}c)$ meson is made up of two different heavy quarks. It’s decay is calculated using Non-Relativistic QCD (NRQCD), expanding in the small velocities of the two non-relativistic quarks. After matching the relevant QCD operators onto the NRQCD Lagrangian by integrating out the anti-particles of the respective quarks, this approach allows to perform a systematic expansion of the resulting NRQCD operators, which can be carried out up to any given order and the truncation uncertainty can be estimated. Furthermore, the symmetry properties of NRQCD allow to relate the relevant matrix elements of the four-quark operators to two parameters.

For the calculation we use the Optical Theorem to apply an Operator Product expansion (OPE) performed on the forward scattering matrix element of the $B_c$ meson. This OPE approach [4–6] leads to similar results as those obtained using QCD Sum Rules [7] or Potential models [8], which give lifetimes close to the experimental value. Experimentally, the $B_c$ decay rate has been determined with rather small uncertainties by the LHCb [9, 10] and CMS [11] collaborations, yielding an average of

$$\Gamma_{exp}^{B_c} = 1.961(35) \text{ ps}^{-1}. \quad (1)$$

In the following we will summarize the results from an updated OPE computation of the $B_c$ decay rate [12, 13]. We then outline a new method to obtain $\Gamma_{B_c}$, using experimental results as well as a non-perturbative expansions of the $B_c$, $B$ and $D$ mesons’ lifetime.

2. Results

In Ref. [12], the $B_c$ decay width was calculated in the $\overline{\text{MS}}$, meson and Upsilon schemes. Neglecting the strange-quark mass the result there is:

$$\Gamma_{B_c}^{\overline{\text{MS}}} = (1.58 \pm 0.40|\mu| \pm 0.08|n.p. \pm 0.02| \pm 0.01| V_{cb} \text{ ps}^{-1},$$

$$\Gamma_{B_c}^{\text{meson}} = (1.77 \pm 0.25|\mu| \pm 0.20|n.p. \pm 0.01| V_{cb}) \text{ ps}^{-1},$$

$$\Gamma_{B_c}^{\text{Upsilon}} = (2.51 \pm 0.19|\mu| \pm 0.21|n.p. \pm 0.01| V_{cb}) \text{ ps}^{-1}. \quad (2)$$

Here the main uncertainties result from the scale dependence, indicated by $\mu$ in the above equation. It can be reduced by including higher-order QCD corrections to the free-quark decay rates, as well as to the Wilson coefficients involved. The second largest uncertainty, indicated by $n.p.$ in eq. (2), stems from neglected higher-order corrections in the NRQCD expansion as well as from uncertainties of the non-perturbative parameters. In the $\overline{\text{MS}}$ scheme a non-negligible part of the uncertainty results from the $\overline{\text{MS}}$ masses of the $\bar{b}$- and $c$ quarks. Finally, there is an uncertainty due to the input parameters, which is dominated by $V_{cb}$.

When including the strange-quark mass in the calculation for the free $c$-quark decays, the central values of the decay rate is reduced by about 7%. We obtain in the three different schemes
\[ \Gamma_{\text{meson}} = (1.70 \pm 0.24) \mu \pm 0.20 |m_p| \pm 0.01 |m_s| \pm 0.01 |V_{cb}| \text{ ps}^{-1} \]  
\[ \Gamma_{\text{meson}} = (1.70 \pm 0.24) |\mu| \pm 0.20 |m_p| \pm 0.01 |m_s| \pm 0.01 |V_{cb}| \text{ ps}^{-1} \]  
\[ \Gamma_{\text{meson}} = (2.40 \pm 0.19) |\mu| \pm 0.21 |m_p| \pm 0.01 |m_s| \pm 0.01 |V_{cb}| \text{ ps}^{-1} \] 

Besides the uncertainties mentioned above we have indicated the uncertainty due to \( m_s \).

Within uncertainties the results in Eq. (3) are consistent with each other and with the experimental value in Eq. (1). There is however a rather wide spread among the three different mass schemes used. One strategy to improve on the precision of the theory result is to reduce the uncertainty due to scale-dependence. In the next section we discuss a novel approach that reduces the dominant uncertainty, which as explained, is from the renormalization scale dependence.

### 3. Novel determination of \( \Gamma_{B_c} \)

The main theory uncertainty is mainly due to the free-quark decay rate, which is the leading term in the non-perturbative expansion of the decay rate of a meson \( H_Q \) with heavy quark \( Q \):

\[ \Gamma(H_Q) = \Gamma^{(0)}_Q + \Gamma^{n.p.}(H_Q) + \Gamma^{WA+PI}(H_Q) + O(\frac{1}{m_Q^2}) \]  

where the second term includes non-perturbative corrections and the third term contains Weak Annihilation and Pauli Interference contributions. The expansion in Eq. (4) can be carried out not only for the \( B_c \) meson, but also for the \( B \) and \( D \) mesons. Taking now the difference of the three different decay rates leads to:

\[ \Gamma(B) + \Gamma(D) - \Gamma(B_c) = \Gamma^{n.p.}(B) + \Gamma^{n.p.}(D) - \Gamma^{n.p.}(B_c) + \Gamma^{WA+PI}(B) + \Gamma^{WA+PI}(D) - \Gamma^{WA+PI}(B_c) \] 

Since the free quark decay rate is independent of the meson state, it drops out on the right-hand side of Eq. (5), thereby reducing the uncertainty due to scale-dependence. For the computation of \( \Gamma(B_c) \), the decay rates of the \( B \) and \( D \) mesons can be taken from experiment, whereas the right-hand side can be computed using non-perturbative methods. The computation can be carried out for charged or neutral \( B \) and \( D \) mesons, leading in principle to four different ways to compute \( \Gamma(B_c) \). In Tab. 1 we show the results for the \( B_c \) decay rate in the meson and \( \overline{\text{MS}} \) scheme, obtained using the four different channels [14].

The results from this novel approach are in tension with the experimental result in eq. (1). Several reasons can be put forward to explain this disparity: 1. The uncertainties from NLO
corrections to Wilson coefficients and free quark decay rates might be underestimated; 2. Eye-graph contributions, neglected in lattice computations of matrix elements that we use [15], but estimated to be small using HQET sum rules [16]; 3. Unexpectedly large contributions from higher dimension operators in the 1/mQ expansion [17]; 4. Violation of quark-hadron duality.

A thorough analysis of the above mentioned points is in order to determine the reason for the discrepancy between the results and experiment.

4. Summary

We have outlined the OPE approach to determine the $B_c$ decay rate in the Standard Model. The results in the $\overline{MS}$, the meson and the Upsilon scheme are compatible with each other and with the experimental value. There is however a wide spread among the central values in the three different mass schemes, where the main uncertainties arise from neglected NLO QCD corrections.

We discussed a novel method to determine $\Gamma_{B_c}$ based on differences of $B$, $D$ and $B_c$ decay rates that allows to reduce the scale-dependence uncertainty. The results deviate significantly from the experimental value, and we presented various possible reasons for this discrepancy.

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