Transport coefficients in the early universe

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Abstract

We calculate numerically the electrical conductivity $\sigma$, heat conductivity $\kappa$ and shear viscosity $\eta$ of the hot plasma present in the early universe for the temperature interval $1 \text{ MeV} \lesssim T \lesssim 10 \text{ GeV}$. We use the Boltzmann collision equation to compute all the scattering matrix elements and regulate them by the thermal masses of the $t$- and $u$-channel particles. No leading order approximation is needed because of the numerical integration routines used.
1 Introduction

The transport properties of the hot plasma present in the early universe are of great interest. The transport coefficients play a significant role in the phase transitions of the early universe [1, 2, 3, 4], the creation and development of the primordial magnetic fields [3, 4] and finally in the creation of the primordial density perturbations and therefore in the galaxy formation [5, 6].

There has been many attempts to estimate the transport coefficients. (Textbook estimates for thermal conductivity, shear and bulk viscosity in the ultrarelativistic plasma were given in [9] and [10].) In [11] the viscosities of a pure gluon plasma and of a quark-gluon plasma were computed in the weak coupling limit from a variational solution to the Boltzmann equation. In [12] the transport coefficients were calculated for plasmas interacting through strong, electromagnetic and weak interactions to leading order in the interaction strength. The rates of momentum and thermal relaxation, electrical conductivity and viscosities of quark-gluon and electrodynamic plasmas were also included in [12]. In [13] the transport coefficients and relaxation times were calculated to leading orders in the coupling constant for degenerate quark matter within perturbative QCD for temperatures and inverse screening lengths much smaller than the quark chemical potential. The viscosities of the QCD-plasma were considered in [14]. Textbook estimates for thermal conductivity and viscosity in the ultrarelativistic plasma were given in [9] and [10]. The conductivity of a relativistic plasma has also been considered in [15] by reformulating the collision operator for a relativistic plasma in terms of an expansion in spherical harmonics.

However, all the previous estimates involve approximations. The calculations have been performed only to the leading (logarithmic) orders and certain scattering reactions present in the hot primordial plasma have been neglected. In the present paper, the only approximation we make is to assume that when particles appear in the heat bath of the primordial plasma, their thermal velocities are high enough to allow us to treat them ultrarelativistic and therefore massless. We consider radiation dominated plasma with temperatures from about 1 MeV to about 10 GeV.

The viscous damping and heat conducting effects affect the first order phase transitions in the early universe. During such phase transitions, instabilities may occur when the transport of latent heat is dominated by the fluid flow. In [1] this was studied in the EW transition in the small velocity limit, in [2] in the QCD transition, in [3] for cosmological detonation fronts and in [4] for general first order transitions with either large or small bubble wall velocities. The instabilities can be damped by finite viscosity and heat conductivity due to the diffusion of radiation on small length scales.

When considering the creation and development of primordial magnetic fields it is also of importance to know the transport coefficients of the surrounding primordial...
plasma \[3, 4\]. Finite electrical conductivity leads to the diffusion of magnetic fields and therefore is of importance when trying to explain the further evolution of an initial seed magnetic field (see e.g. \[16\]). The instabilities in the first order phase transitions can be shown to create seed magnetic fields \[3\]. Therefore the seed primordial magnetic fields also depend on the size of the transport coefficients in the primordial plasma in a crucial way. In particular, the seed fields created in the QCD phase transition are highly dependent on the neutrino viscosity \[3\].

Galaxy formation \[7\] is likewise affected by the thermal coefficients in the primordial plasma \[8\]. Viscosity tends to heat up the plasma, and on the other hand thermal conduction transfers heat from regions of high temperature to regions at lower temperature. The effects affect the structure formation and thus in order to make precise models of the galaxy formation it is important to know the values of the transport coefficients.

It should be noted that two major effects are involved when heat transport and viscosity are studied. These effects are the number of possible scattering reactions, i.e. the number of particles present in the plasma, and the Debye mass present in the propagator. As we will discuss in Section 4, the first effect will decrease the transport coefficients while the second effect will increase them. The net behaviour of the coefficients depend on the exact interplay between these two effects.

The different interactions give contributions of different strengths to the electrical conductivity as was shown in \[17\]. However, in \[17\] only strong and electromagnetic interactions were considered. Here we also deal with the scattering reactions involving neutrinos.

The paper is organized as follows. In section 2 we introduce the reader to the method of calculation and, using the full definition of the energy-stress tensor $T^{\mu\nu}$, we define the transport coefficients presented here. In section 3 we reconsider electrical conductivity. We first discuss what is the proper way to introduce the Debye screening to the propagators in the matrix elements. We then include additional scattering reactions not present in previous work and present a new value for the electrical conductivity. We also compare our results with other recent estimates for the electrical conductivity in the hot plasma. In Section 4 and 5 we apply our technique to calculating thermal conductivity and shear viscosity in the hot plasma, respectively, and compare our results with other recent estimates. Section 6 contains a summary of the results and a brief discussion on their relevance for the early universe.

## 2 The general formalism
2.1 The energy-stress tensor

We start by defining the transport coefficients. We do this by using the energy-stress tensor $T^{\mu\nu}$:

$$T^{\mu\nu} = (\rho + p)U^{\mu}U^{\nu} - pg^{\mu\nu} + T^{\mu\nu}_{TP} + T^{\mu\nu}_{EM},$$

where $\rho$ is the total energy density, $p$ the pressure, $U^{\mu}$ the velocity four-vector ($U^{\mu}U_{\mu} = -1$) and $g^{\mu\nu}$ the metric tensor. $T^{\mu\nu}_{EM}$ is the usual electromagnetic energy-stress tensor (see e.g. [18]) and $T^{\mu\nu}_{TP}$ represents the non-ideal contributions from a finite viscosity and heat conductivity caused by the diffusion of the various particles present in the plasma.

To find out the meaning of the transport coefficients we now follow [9] and derive the form of $T^{\mu\nu}_{TP}$. Because of dissipation, we must also add a correction term to the particle current $N^{\mu}$:

$$N^{\mu} = nU^{\mu} + N^{\mu}_{d},$$

where $n$ is the particle number density and $N^{\mu}_{d}$ the correction term. To avoid the ambiguities generated by the adding of extra terms induced by dissipation and diffusion, we should define what is meant by the number and energy densities:

$$n \equiv -U^{\mu}N_{\mu},$$

$$\rho \equiv U^{\alpha}U^{\beta}T_{\alpha\beta},$$

$$U^{\mu} \equiv (-N^{\nu}N_{\nu})^{-1/2}N^{\mu}.$$  

Inserting the definitions Eq. (3) into Eq. (1) and Eq. (2) we can easily see that the following conditions must be satisfied:

$$N^{\mu}_{d} = 0,$$

$$U_{\alpha}U_{\beta}T_{TP}^{\alpha\beta} = 0.$$  

Using the conservation laws for the fluid:

$$\partial_{\nu}T^{\mu\nu} = 0,$$

$$\partial_{\nu}N^{\nu} = 0.$$  

and making use of Eq. (2) and Eq. (4) we find that

$$\partial_{\nu}U^{\nu} = -\frac{U^{\nu}}{n}\partial_{\nu}n.$$  

Using Eq. (1), Eq. (5), Eq. (6) and the second law of thermodynamics, we can write a formula for the rate of entropy production using the total entropy current four-vector $S^{\mu} \equiv nsU^{\mu} - U_{\nu}T^{\mu\nu}_{TP}/T$, where $s$ is entropy per particle and $T$ the temperature:

$$\partial_{\mu}S^{\mu} = -\frac{1}{T}(\partial_{\nu}U_{\mu})T^{\mu\nu}_{TP} + \frac{1}{T^2}(\partial_{\nu}T)U_{\mu}T^{\mu\nu}_{TP}.$$  

Now the task is to construct $T_{\mu\nu}^{TP}$ in the way that the rate of entropy production per unit volume given by Eq. (7) is positive for all possible fluid configurations. As explained in [9], we may take the dissipative term to be linearly dependent on the space-time derivatives of the four-velocity, energy and number densities. Also, the effects considered here are of first order and therefore the adiabatic equations of motion can be used to find out the form of $T_{\mu\nu}^{TP}$.

It is easiest to continue in the locally moving Lorentz-frame, where $U^{\mu} = (1, 0, 0, 0)$. As explained, $T_{\mu\nu}^{TP}$ can now be constructed as a linear combination of the derivatives $\partial U^{\mu}/\partial x^{\nu}$, $\partial U^{\mu}/\partial t$, $\partial T/\partial x^{\nu}$ and $\partial T/\partial t$. Of these, $\partial U^{0}/\partial x^{i}$ vanishes in this frame and $\partial T/\partial t$ can be expressed in terms of $\nabla \cdot U$ because of adiabacity. Then the most general structure allowed by rotational and space-inversion is

$$
T_{ij}^{\mu\nu} = -\eta \left( \frac{\partial U^{i}}{\partial x^{j}} + \frac{\partial U^{j}}{\partial x^{i}} - \frac{2}{3} \nabla \cdot U \delta^{ij} \right) - \zeta \nabla \cdot U \delta^{ij},
$$

$$
T_{i0}^{\mu\nu} = -\kappa \frac{\partial T}{\partial x^{i}} - \xi U^{i} \frac{\partial T}{\partial t},
$$

$$
T_{00}^{\mu\nu} = 0.
$$

(8)

By comparing Eq. (8) with the usual non-relativistic hydrodynamics we can now recognize $\eta$, $\zeta$ and $\kappa$ to be the shear and bulk viscosity and the heat conduction coefficients, respectively. $\xi$ is a relativistic contribution with no non-relativistic counterpart. By using Eq. (8) in Eq. (7) and demanding that the rate of entropy production is positive for all fluid configurations, we get the result $\xi = T \kappa$ together with the fact that all the transport coefficients are non-negative.

Again, following [9], we consider a radiation dominated plasma having very short mean free times. The energy-stress tensor to first order in the free time $\tau$ for such a fluid reads [9]:

$$
T_{\mu\nu}^{\tau} = - \left( \frac{4 b \tau T^{3}}{(\partial \rho/\partial T)} \right) \left( \frac{T}{3} \frac{\partial U^{\gamma}}{\partial x^{\gamma}} + U^{\gamma} \frac{\partial T}{\partial x^{\gamma}} \right) \times \left( \frac{\partial U^{\mu}/\partial T}{\partial \rho} + (\partial U^{\mu}/\partial T) \frac{\partial U^{\nu}}{\partial \rho} \right) - 4b\tau T^{3} \left\{ \left( 2 U^{\mu} U^{\nu} U^{\gamma} + \frac{1}{3} g^{\mu\nu} U^{\gamma} + \frac{1}{3} \delta^{\mu\nu} U^{\gamma} + \frac{1}{3} \delta^{\mu\gamma} U^{\nu} + \frac{1}{3} \delta^{\nu\gamma} U^{\mu} \right) \frac{\partial T}{\partial x^{\gamma}} + \frac{T}{15} \left( 6 \frac{\partial U^{\mu} U^{\nu} U^{\gamma}}{\partial x^{\gamma}} + g^{\mu\nu} \frac{\partial U^{\gamma}}{\partial x^{\gamma}} + \frac{\partial U^{\mu}}{\partial x^{\nu}} + \frac{\partial U^{\nu}}{\partial x^{\mu}} \right) \right\} + O(\tau^{2}),
$$

(9)

where $b$ is a constant. Since $T_{\mu\nu}^{\tau}$ is a tensor, it is sufficient to study it in the locally comoving Lorentz frame. Using once again the properties of this frame, $U^{\mu} = (1, 0, 0, 0)$ and $\partial U^{0}/\partial x^{\mu} = 0$, we can somewhat simplify the energy-stress tensor, Eq. (9):

$$
T_{ij}^{\tau} = - 4 b T^{4} \left( \frac{1}{3} - \frac{\partial \rho}{\partial \rho} \right) \nabla \cdot U - \frac{4}{15} b T^{4} \tau \left( \frac{\partial U^{i}}{\partial x^{j}} + \frac{\partial U^{j}}{\partial x^{i}} - \frac{2}{3} \delta^{ij} \nabla \cdot U \right) + O(\tau^{2}),
$$

$$
T_{i0}^{\tau} = \frac{4}{3} b T^{3} \tau \left( \frac{\partial T}{\partial x^{i}} + T \frac{\partial U^{i}}{\partial t} \right) + O(\tau^{2}),
$$

(10)
\[ T_{\text{mp}}^{00} = O(\tau^2). \] (10)

From Eq. (10) it is easy to extract the values for the transport coefficients:

\[
\begin{align*}
\zeta &= 4bT^4 \tau \left( \frac{1}{3} - \left( \frac{\partial p_i}{\partial \rho} \right)_n \right)^2, \\
\eta &= (4/15)bT^4 \tau \quad \text{and} \\
\kappa &= (4/3)bT^3 \tau.
\end{align*}
\] (11)

As can be seen from Eq. (11), the transport coefficients are indeed proportional to the mean free time (or equivalently the mean free path) of particles in the plasma, in which interactions are sufficiently strong so that particles have a high scattering frequency. For various species of particles the coefficients are obviously different because their cross sections vary and thus they have different free paths in the plasma. In Sections 4 and 5 we calculate heat conductivity and shear viscosity considering all the particles and their scattering reactions in the hot primordial plasma. We note that the values obtained in Sections 4 and 5 are self-consistent with the assumption of the high scattering frequency, i.e. the mean free times estimated from the values of Table 5 are very short. Let us finally note that because the plasma considered here is highly relativistic, and therefore \( p \approx \frac{1}{3} \rho \), the bulk viscosity \( \zeta \) is approximately zero.

### 2.2 The Boltzmann equation

To actually calculate the transport coefficients, we need some tool to treat all the scattering reactions in the plasma. This tool is the Boltzmann equation, which in the expanding Robertson-Walker universe reads:

\[
\frac{\partial f(p^0)}{\partial t} + \dot{p}^i \left( \frac{\partial f}{\partial x^i} - 2 \frac{\dot{R}}{R} p^0 \frac{\partial f}{\partial p^i} - e \frac{\partial f}{\partial p^0} E^i \right) - \frac{\partial f}{\partial p^0} p^0 R \dot{R} - e \frac{\partial f}{\partial p^0} p^0 E^i = C(p, t)p^0, \quad (12)
\]

where \( C(p, t) \) is the collision integral. Here we have assumed a Robertson-Walker metric with a scale factor \( R \) for the background, and we have defined \( F^i_0 = -E_i \) and \( F^{ij} = \epsilon^{ijk} B_k \). The Robertson-Walker metric is valid here if the mean free path of the particles is sufficiently small i.e. no large flows are present in the universe. The resulting values in Sections 4 and 5 suggest that the usage of this metric is a self-consistent assumption. We also prefer to use co-moving coordinates and define

\[
\tilde{f}(p, t) = \int \delta(p_0 - (p^2 R^2 + m^2)^{1/2}) f(p, p_0, t) dp_0.
\] (13)

Inserting this into Eq. (12), making use of the local momentum defined as \( \tilde{p} = R p \) and assuming adiabatic expansion in which \( E^i \gg (\dot{R}/R) |p| \equiv H |p| \) we can use the
Lorentz-frame and write Eq. (14) in the form (from now on we drop the tildes and we also assume that the electric field is sufficiently small, $E^i \ll T^2$)

$$\frac{\partial f}{\partial t} + \frac{p^i}{p_0} \frac{\partial f}{\partial x^i} - e \frac{\partial f}{\partial p^i} E^i = C(p, t) .$$

(14)

To get a sufficiently good estimate, it is only necessary to consider $2 \rightarrow 2$ reactions, and thus we may use as the collision integral the following expression:

$$C(p, t) = \frac{1}{p_0} \int dP_b dP_c dP_d (2\pi)^4 \delta^4(p + p_b - p_c - p_d) |M(Ab \rightarrow cd)|^2 \mathcal{F},$$

(15)

where $p$ is the four-momentum of the charged test particle $A$, and we have defined $dP_i \equiv d^3p_i/(2\pi)^3 2E(p_i))$. The factor $\mathcal{F}$ contains the distribution functions of the initial and final states. The Pauli blocking factors must be included for the fermions and the enhancement factors for the bosons. Below are two examples of the $\mathcal{F}$-factor.

The first is for the case when all the particles in the reaction are fermions, and in the second case particles $b$ and $c$ are bosons, such as is the case for Compton scattering:

$$\mathcal{F} = \left\{ \begin{array}{ll}
[1 - f(p)][1 - f_b(p_b)] f_c(p_c) f_d(p_d) - [1 - f_c(p_c)][1 - f_d(p_d)] f(p) f_b(p_b), \\
[1 - f(p)][1 + f_b(p_b)] f_c(p_c) f_d(p_d) - [1 - f_d(p_d)][1 + f_c(p_c)] f(p) f_b(p_b).
\end{array} \right.$$

(16)

The time dependence is not shown explicitly above. Let us now assume that there exists a constant electric field in the early universe (Of course, we do not claim such a field actually exists but rather use it as a probe of the plasma properties. Furthermore, we take it to be constant in the sense that its coherence length is larger than the mean free path of charged particles) and treat its effect as a perturbation on the distribution of the test particle. We write $f = f_0 + \delta f$, where $f_0$ is the equilibrium distribution and $\delta f$ the small perturbation. Inserting this into the collision integral Eq. (15) results in

$$C(p, t) = -\frac{1}{p_0} \int dP_b dP_c dP_d (2\pi)^4 \delta^4(p + p_b - p_c - p_d)
\times |M|^2([f_{bc} f_{cd}[1 - f_{bd}] + f_{bd}[1 - f_{bc}][1 - f_{cd}])\delta f \equiv \hat{C}(p)\delta f(p, t).$$

(17)

Eq. (17) assumes that all the particles in the reaction are fermions; generalization to other cases is straightforward.

We have assumed that all the particles $b$, $c$ and $d$ have an equilibrium distribution. Therefore, if also the test particle has an equilibrium distribution, the collision integral vanishes and only the perturbation term in Eq. (17) survives. Note that the equilibrium distribution depends only on the momentum.

The Boltzmann equation can now be linearized in order to calculate the electrical conductivity. Assuming that $\delta f$ depends only on $p$, the linearized Boltzmann equation reads

$$\nu^i \frac{\partial f_0}{\partial x^i} - e \frac{\partial f_0}{\partial p^i} E^i = \hat{C}(p)\delta f(p),$$

(18)

where $\nu^i$ is the velocity of the probe particle $a$. Using Eq. (18) we can now begin to calculate the transport coefficients in the early universe.
3 Eletrical conductivity

There has been many attempts to estimate $\sigma$. In [19] this was done in the relaxation time approximation and in [20] and [21] a Coulomb correction was applied. Recently a numerical work was carried out in [17] which showed that the electrical conductivity in the early universe is mainly due to the leptonic contribution and its value for $T \leq 100$ MeV $\sigma \simeq 0.76T$ while at $T \simeq M_W$ it was found that $\sigma \simeq 6.7T$.

However, in [17] the contribution from lepton-quark scatterings was neglected, which was correctly pointed out in [22]. We have now included these scatterings and thus acquired a better estimate for the electrical conductivity. The additional reactions now included are of the form $l^\pm q \rightarrow l^\pm q$ and $l^\pm \bar{q} \rightarrow l^\pm \bar{q}$, and these affect only the leptonic contribution because the quarks interact via the strong interaction and thus the electromagnetic scattering processes mentioned above can still be neglected when considering the electrical conductivity created by the quarks.

The $t$- and $u$-channels give singular contributions to the collision integral and that is why one needs regularization. The full finite temperature calculations with higher order Feynman graphs and resummation would provide the regularization correctly but the expressions for the matrix elements become quite long. Another technical complication is the correct handling of the real and imaginary parts. A suitably accurate calculation can be, however, performed by using thermal masses as regulators in the $t$- and $u$-channel propagators. This approach can be expected to give roughly the same results as the one using the full thermal propagator.

The thermal masses used in the $t$- and $u$-channels for the lepton, quark, photon and gluon propagators, respectively, are the following [23]:

\begin{align}

m_{l}^2 &= \frac{e^2}{8}T^2 \simeq 0.0115T^2, \\
m_{q}^2 &= \frac{g^2_s}{6}T^2 \simeq 0.251T^2, \\
m_{\gamma}^2 &= \frac{e^2}{3}(\Sigma_l Q_l^2 + 3\Sigma_q Q_q^2)T^2 \simeq 0.0306(\Sigma_l Q_l^2 + 3\Sigma_q Q_q^2)T^2, \\
m_{g}^2 &= g^2_s(3 + \frac{N_f}{3})T^2 \simeq 1.508(3 + \frac{N_f}{3})T^2.
\end{align}

(19)

where $N_f$ is the number of quark families present, the sums are over all particles with $m \leq T$, and we adopt the values $g^2_s = 4\pi\alpha_s \simeq 1.508$ and $e^2 = 4\pi\alpha_e \simeq 0.0917$, and $T_{QCD} = 200$ MeV.

The matrix elements for all the reactions used in the collision integral can be found in Tables 1, 2 and 3. All the elements are summed over final spins and averaged over initial spins; the QCD-processes are also summed over final color and averaged over initial color. Because we can treat every spin and color state as a different particle in the initial state we must, however, remove the averaging in the collision integral by multiplying the matrix elements with the number of initial spin and color states.
The proper introduction of the thermal regulators in the propagators of the $t$- and $u$-channels should also be given special attention. For example, the matrix element for the reaction $e^-e^+ \rightarrow e^-e^+$ is the sum $(u^2 + s^2)/t^2 + (u^2 + t^2)/s^2 + 2u^2/(st)$, which in [17] was first simplified into the form $(s^4 + t^4 + u^2(s + t)^2)/(s^2t^2)$ and only after that the regulators were inserted into the $t$- and $u$-channels. But one should be extremely careful when applying regularization because the adding of the regulators in different stages of the calculation changes the actual value of the matrix element, and therefore the value of the conductivity. This is so because the regulators are added only to the $t$- and $u$-propagators, not consistently everywhere in the matrix element. The proper way to introduce the screening terms is to apply the regulators to every term given by the Feynman graphs separately and before joining the terms together. The regulating should be done this way because the different terms in the matrix elements have singularities of different order and hence the terms should be regulated and integrated separately.

We consider the temperature interval from about 1 MeV to about 10 GeV and make the simple approximation that particles appear in the thermal bath when temperature is greater than their mass. The collision integral is performed numerically by evaluating the integral by a simple Monte Carlo simulation.

The results, however, are not very different from the previous ones [17]: the quark contribution is still negligible compared with the leptonic electrical conductivity and the actual value of the conductivity is still $\approx 10T$ in natural units. The effect of neutrino scatterings to the contribution of either QED or QCD electrical conductivity is negligible. The results are given in Fig. 1 and in Table 5.

The main difference between the value for the electrical conductivity here and in [22] is due to the difference in the definition of the electrical conductivity. In [22] an electric field extending itself over the whole universe is used to probe the electrical conductivity of the early universe. Because in [22] a homogenous field is assumed to cover the entire space, the leptons (quarks) and antileptons (antiquarks) are accelerated in the opposite directions. In this scenario, it is justified to neglect in the first order all other scattering reactions than the lepton-antilepton (quark-antiquark) annihilation. But the resulting electrical conductivity is a global one - one that is the same in the whole universe and does not take into consideration the small patch-like structure of the primordial magnetic field.

The primordial magnetic field $B$, if it exists, is most likely random: both its magnitude and direction vary. Therefore in the early universe it is more sensible to talk about the local electrical conductivity, $\sigma(x)$. The magneto hydrodynamic equation reads then

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times \left( \frac{\nabla \times B}{\sigma(x)} \right).$$

(20)
From Eq. (20) we see that to blindly treat the electrical conductivity strictly as a constant would disregard some aspects of the dissipation of the magnetic field. However, to a good approximation we can take the electrical conductivity to be a constant inside the patches of the magnetic field characterized by correlation length \( L \).

The magnetic field is coherent inside the correlation length \( L \). The different patches containing magnetic field are uncorrelated and thus a convenient way to define the local conductivity is the measure of the dissipation of the magnetic fields in the coherent patches. We measure the local electrical conductivity as experienced by a charged particle shot out of one of the magnetic patches to the next, uncorrelated one. The particles are ultrarelativistic because of the high temperature and therefore the different bulk velocities between the magnetic patches can be neglected.

The condition when this scenario is valid can be easily checked. The mean free path of a particle in the plasma is \( l_{\text{free}} = (\sigma_{tp}n_p)^{-1} \), where \( \sigma_{tp} \) is the transport cross section of the particle species \( p \), and \( n_p \) is its number density. The condition for validity is now that the mean free path is longer than the coherence length of the magnetic field, \( L < l_{\text{free}} \). Typical coherence length could be \( L \approx 1/T \) whereas \( l_{\text{free}} \approx 1/(\alpha^2T) \). In this case particles will typically be shot out of a given patch before they have time to interact. When they finally do, this will happen in a different patch where the other particles have a bulk motion of their own.

However, the coherence length of the primordial magnetic field is not known exactly. It might very well be of the order of the natural microphysical scale, the interparticle distance, but there could be processes and phenomena that would cause it to be much larger. For example, one such process studied recently is the inverse cascade [24], which transfers magnetic energy from small length scales to larger length scales.

Here we shall assume that the mean free path of a particle is longer than the correlation length of the primordial magnetic field. To calculate the local conductivity, we assume one test particle from each species of particles being shot out of one patch of magnetic field to another, totally uncorrelated one. In the new patch, we assume a patch-wide electric field to give us a tool to measure the conductivity. Of course, this is just a test field, we do not claim such fields to exist in the early universe. Inside the patch, we also assume isotropy and therefore we can write Eq. (18) to the form

\[
- e \frac{\partial f_0}{\partial p^i} E^i = \hat{C}(p) \delta f(p). \tag{21}
\]

From Eq. (21) one easily obtains

\[
\delta f(p) = - \frac{e}{\hat{C}(p)} \frac{\partial f_0(p)}{\partial p^i} E^i. \tag{22}
\]

The induced current density is given by

\[
j = e \int \frac{\delta f(p) \mathbf{p}}{p} d^3 \mathbf{p}, \tag{23}
\]
and conductivity $\sigma_A$, associated with a given particle species $A$, is defined by

$$j_A = \sigma_A E.$$  \hspace{2cm} (24)

Thus the contribution of a single species $A$ to conductivity is $\sigma_A \sim 1/\sum |M(AX \rightarrow Y)|^2$, where the sum is over all the processes which scatter the test particle $A$. For the purpose of conductivity, we may view the mixture of different particle species of the early universe a multicomponent fluid. The flow of each component contributes to the total current and adds up to the total conductivity, which reads

$$\sigma_{\text{tot}} = \sum_A \sigma_A,$$  \hspace{2cm} (25)

where the sum is over all the relativistic charged species present in the thermal bath. Note that the total conductivity is dominated by the species that has the weakest interaction. This is because the weaker the interaction, the longer time the current flow is maintained.

We consider the temperature interval $1 \text{ MeV} \lesssim T \lesssim 10 \text{ GeV}$, and make the simple, crude assumption that particles appear in the thermal bath only when temperature is greater than their mass. Thus below 100 MeV, for example, the only charged particles present are the electrons and positrons. When $T \gtrsim T_{\text{QCD}}$, also the quarks should be counted in. Their main interactions are strong, so that their electromagnetic interactions may be neglected. The list of the relevant reactions involving QED and QCD charged particles can be found in Tables 1 and 2.
4 Thermal conductivity

To find out the thermal conductivity, we now abandon the isotropy requirement and the requirement of locality used in the previous Section in computing the electrical conductivity. Let us for a while let the equilibrium distribution \( f_0 \) depend also on the chemical potential \( \mu \):

\[
f_0 = \frac{1}{(1 \pm e^{(p - \mu)/T})^2}.
\]

The conditions for thermal equilibrium require the constancy of temperature and of the sum \( \mu + V \), where \( V \) is the energy of the particles in an external field, throughout the media considered. Here we take \( V = e\phi \), where \( \phi \) is the electric field potential. In a plasma with a non-uniform temperature distribution, the electric field \( E \) is not zero even if the current is zero. In general, when both the current density \( j \) and the temperature gradient \( \nabla T \) are not zero, the relation between these quantities and the electric field can be written as

\[
j = \sigma E - \alpha \sigma \nabla T + \frac{\sigma \nabla \mu}{e}, \tag{26}
\]

where \( \sigma \) is the electrical conductivity and \( \alpha \) the thermoelectric coefficient. We can now write the Boltzmann transport equation in the form

\[
-eE \cdot \frac{\partial f_0}{\partial \mathbf{p}} + \frac{p}{\mathbf{p}} \cdot \frac{\partial f_0}{\partial \mathbf{r}} = \hat{C}(p, T) \delta f. \tag{27}
\]

Assuming that the temperature \( T \) and the chemical potential \( \mu \) are functions of the spatial coordinates it is easy to solve \( \delta f \), the small perturbation to the equilibrium distribution \( f_0 \) from Eq. (27):

\[
\delta f = \frac{p}{pT\hat{C}(p, T)} \cdot \left( -eE \frac{\partial f_0}{\partial \mu} + \frac{\partial f_0}{\partial T} \nabla T \right) + \frac{p}{pT\hat{C}(p, T)} \cdot \left( eE + \nabla \mu + (p - \mu) \nabla T \right). \tag{28}
\]

In order to determine the thermoelectric coefficient \( \alpha \), we must now assume for a while that the condition \( eE + \nabla \mu = 0 \) holds. From the definition of the electric current Eq. (23) and from Eq. (26) one gets

\[
j = -\alpha \sigma \nabla T = -e \int \frac{p\delta f}{p} d^3\mathbf{p}. \tag{29}
\]

Remembering the earlier definition of \( \sigma \), Eq. (23) and Eq. (24), it is now easy to solve the thermoelectric coefficient:

\[
\alpha = \frac{1}{eT}(\mu - G), \tag{30}
\]

where

\[
G \equiv \int \frac{p^3 e^{(p - \mu)/T} dp}{(1 + e^{(p - \mu)/T})^2 \hat{C}(p)} \times \left( \int \frac{p^2 e^{(p - \mu)/T} dp}{(1 + e^{(p - \mu)/T})^2 \hat{C}(p)} \right)^{-1}. \tag{31}
\]
Now let us assume that $j = 0$, in which case we obtain from Eq. (26) that $\alpha \nabla T = \mathbf{E} + \nabla \mu / e$ and that

$$\delta f = \frac{\pm e^{(p-\mu)/T}}{1 \pm e^{(p-\mu)/T}} \frac{p - G}{T^2 C(p)} \frac{\mathbf{p} \cdot \nabla T}{p}. \quad (32)$$

From the energy flux relation

$$\mathbf{q} = \int \mathbf{p} \delta f d^3 \mathbf{p} \equiv -\kappa \nabla T, \quad (33)$$

and from noting that in the early universe we can put the chemical potential $\mu$ to zero, we get an expression for the thermal conductivity coefficient $\kappa$:

$$\kappa = \frac{\pm 4\pi}{3T^2} \int_0^\infty \frac{p^3}{C(p)} \frac{e^{p/T}}{(1 \pm e^{p/T})^2} (G - p) dp. \quad (34)$$

The plus-minus sign refers to fermions and bosons, respectively. The heat conductivity coefficient is, however, always positive because the $\mathcal{F}$ (Eq. (16)) factor also changes its sign in the collision integral when considering fermions or bosons, respectively.

As explained in Section 2, thermal conductivity is related to the mean free time, or equivalently, to the mean free path. The values for the heat conductivity obtained in this Section are self-consistent with the assumption of short mean free times in Section 2 as can be seen by comparing the values presented in Table 5 and Eq. (11).

From Eq. (34) it can be easily seen that $\kappa$ is proportional to the inverse of the square of the coupling constant of the scattering reaction in question. Thus the smaller the coupling constant is the bigger the value for $\kappa$ will be i.e. $\kappa$ indeed is related to the mean free path.

The subtle interplay between the increasing effect on thermal conductivity by the temperature dependent regulators and the decreasing effect due to the change in the amount of particles (potential scatterers) present should also be noted. As can be seen in Fig. 2 and Table 5, the case for the leptonic thermal conductivity, $\kappa_l$, is clear. The first effect dominates in the lepton case, increasing $\kappa_l / T^2$ all the way from the case where only electrons are present to the case where also the b-quark is present in the heat bath. The photon case is also straightforward. Because there are no temperature dependent regulators the only effect comes from the fact that more and more scatterers are present the higher the temperature is. Thus we see in Fig. 2 and Table 5 that the photonic thermal conductivity divided by the temperature squared, $\kappa_\gamma / T^2$, is decreasing steadily as the temperature rises.

The case is similar with gluons and quarks as can be seen in Fig. 3 and Table 5. The different behaviour of the quark thermal conductivity $\kappa_q$ and the gluonic thermal conductivity $\kappa_g$ can be understood when one looks at the different processes that are involved in the scatterings. All QCD matrix elements are listed in Table 2. The quarks get the biggest contribution to their collision integral from the potentially
Figure 2: $\kappa/T^2$ as a function of temperature for leptons and photons.

singular matrix element which has been regulated by thermal masses while the gluons are not that sensitive to thermal masses.

The thermoelectric coefficient $\alpha$ in Eq. (30) for leptons is found to be $\pm 39.1$ (depending on the sign of the lepton electric charge) and for quarks $47.4/e_q$, where $e_q$ is the electric charge of the quark.

The neutrinos do not have any singularities in their matrix elements and thus as the temperature grows and more and more scatterers appear in the heat bath the neutrino thermal conductivity divided by the temperature squared, $\kappa_\nu/T^2$, decreases steadily. $\kappa_\nu/T^2$ is shown in Fig. 4.

Earlier attempts to estimate the heat conductivity can be found, for example, in [10, 12, 13]. However, in [10] no actual matrix elements were calculated but heat conductivity was estimated to be proportional to the mean free time of a particle species studied. In [12] and [13] a more thorough calculation was performed to the leading logarithmic order. However, due to the approximations used in the formalisms of [12] and [13] we believe that values in the present paper are more accurate.

5 Viscosity

To find out the shear viscosity in the early universe we, once again, set the stage by starting from the Boltzmann equation Eq. (12) and the definition of the shear viscosity and assuming stationary viscous flow:

$$-\eta(\partial_i V_k + \partial_k V_i - \frac{2}{3}\delta_{ik} \nabla \cdot \mathbf{V}) = \int d^3 p p_i v_k \delta f,$$

(35)
Figure 3: $\kappa/T^2$ as a function of temperature for quarks and gluons.

Figure 4: $\kappa/T^2$ as a function of temperature for neutrinos.
Table 1: Matrix elements $|M|^2/(32\pi^2\alpha_{em})$ for the QED processes used in the collision integral. All matrix elements are summed over final spins and averaged over initial spins. $\alpha_{em}$ is the electromagnetic coupling constant, $i \neq j$, and $e_q$ is the charge of the quark in question.

| $l^- l^+ \rightarrow \gamma \gamma$ | $ut(\frac{1}{t^2} + \frac{1}{t^2})$ |
| $l^- l^+ \rightarrow l^- l^+$ | $-us(\frac{1}{t^2} + \frac{1}{t^2})$ |
| $l^- l^+ \rightarrow l^- l^+$ | $\frac{ut^2 + s^2}{t^2} + \frac{u^2 + t^2}{s^2} + \frac{2u^2}{s^2}$ |
| $l^- l^- \rightarrow l^- l^-$ | $\frac{ut^2 + s^2}{t^2} + \frac{u^2 + t^2}{s^2} + \frac{2u^2}{s^2}$ |
| $l^- l^- \rightarrow l^- l^-$ | $\frac{s^2 + u^2}{t^2}$ |
| $l^- l^+ \rightarrow l^- l^+$ | $\frac{u^2 + t^2}{s^2}$ |
| $l^- l^+ \rightarrow q\bar{q}$ | $3e_q^2 \frac{s^2 + t^2}{s^2}$ |
| $\gamma \gamma \rightarrow l^- l^+$ | $ut(\frac{1}{t^2} + \frac{1}{t^2})$ |
| $\gamma \gamma \rightarrow q\bar{q}$ | $3e_q^2 \frac{u(1 + \frac{1}{t})}{t}$ |
| $\gamma l^- \rightarrow \gamma l^+$ | $-us(\frac{1}{t^2} + \frac{1}{t^2})$ |
| $\gamma q \rightarrow \gamma q$ | $-3e_q^2 \frac{u(1 + \frac{1}{t})}{t}$ |

Table 2: Matrix elements $|M|^2/(16\pi^2\alpha_s)$ for the QCD processes used in the collision integral. All matrix elements are summed over final spins and colors and averaged over initial spins and colors. $\alpha_s$ is the strong coupling constant, and $i \neq j$.

| $q\bar{q} \rightarrow gg$ | $\frac{1}{2} \left( \frac{4}{9} \left( \frac{ut}{t^2} + \frac{ut}{u^2} \right) - 2 \left( \frac{2u^2 - ut}{s^2} \right) - \left( \frac{u^2}{u^2} + \frac{t^2}{s^2} \right) \right)$ |
| $qq \rightarrow gg$ | $2\left( \frac{t^2 - us}{t^2} \right) - \frac{4}{9} \left( \frac{ut}{u^2} + \frac{us}{s^2} \right) + (\frac{u^2}{u^2} + \frac{s^2}{s^2})$ |
| $q\bar{q} \rightarrow q\bar{q}$ | $\frac{4}{9} \left( \frac{ut + s^2}{t^2} + \frac{u^2 + t^2}{s^2} - 2\frac{u^2}{s^2} \right)$ |
| $qq \rightarrow gg$ | $\frac{4}{9} \left( \frac{u^2 + t^2}{t^2} + \frac{u^2 + t^2}{s^2} - 2\frac{s^2}{s^2} \right)$ |
| $q_i q_j \rightarrow q_i q_j$ | $\frac{4}{9} \left( \frac{s^2 + u^2}{t^2} \right)$ |
| $q_i q_i \rightarrow q_i q_i$ | $\frac{4}{9} \left( \frac{u^2 + t^2}{s^2} \right)$ |
| $gg \rightarrow gg$ | $\frac{3}{2} \left( \frac{4}{9} \left( \frac{ut}{t^2} + \frac{ut}{u^2} \right) - 2 \left( \frac{2u^2 - ut}{s^2} \right) - \left( \frac{u^2}{u^2} + \frac{t^2}{s^2} \right) \right)$ |
| $gg \rightarrow gg$ | $2\left( \frac{t^2 - us}{t^2} \right) - \frac{4}{9} \left( \frac{ut}{u^2} + \frac{us}{s^2} \right) + (\frac{u^2}{u^2} + \frac{s^2}{s^2})$ |
| $gg \rightarrow gg$ | $\frac{1}{2} \left( \frac{17t^2 - 8us}{2u^2} + \frac{17u^2 - 8st}{2u^2} + \frac{17s^2 - 8ut}{2s^2} + \frac{15ut - s^2}{2ut} + \frac{15st - u^2}{2st} + \frac{15us - t^2}{2su} - \frac{135}{4} \right)$ |
Table 3: Matrix elements $|M|^2/G^2$ for the neutrino processes used in the collision integral. All matrix elements are summed over final spins and averaged over initial spins. $G$ is the Fermi coupling constant, and $i \neq j$. For other abbreviations, see Table 4.

| Process                              | Expression                                                                 |
|--------------------------------------|---------------------------------------------------------------------------|
| $\nu_i \nu_i \rightarrow \nu_i \nu_i$ | $24s^2$                                                                  |
| $\nu_i \nu_j \rightarrow \nu_i \nu_j$ | $8s^2$                                                                   |
| $\nu_i \bar{\nu}_i \rightarrow \nu_i \bar{\nu}_i$ | $24u^2$                                                                  |
| $\nu_i \bar{\nu}_j \rightarrow \nu_i \bar{\nu}_j$ | $8u^2$                                                                   |
| $\nu_i \bar{l} \rightarrow \nu_i \bar{l}$ | $8((g_V + g_A + 2)^2u^2 + (g_V - g_A)^2t^2)$                              |
| $\nu_i l \rightarrow \nu_i l$ | $8((g_V + g_A)^2u^2 + (g_V - g_A)^2t^2)$                                  |
| $\nu_i l \rightarrow \nu_i l$ | $8((g_V + g_A + 2)^2s^2 + (g_V - g_A)^2u^2)$                              |
| $\nu_i l \rightarrow \nu_i l$ | $8((g_V + g_A)^2s^2 + (g_V - g_A)^2u^2)$                                  |
| $\nu_i l \rightarrow \nu_i l$ | $8((g_V - g_A)^2s^2 + (g_V + g_A + 2)^2u^2)$                              |
| $\nu_1 l_1 \rightarrow \nu_2 l_2$ | $16u^2$                                                                   |
| $\nu_1 l_2 \rightarrow \nu_2 l_2$ | $16s^2$                                                                   |
| $\nu_i l \rightarrow \nu_i l$ | $24((g_V^q + g_A^q)^2u^2 + (g_V^q - g_A^q)^2t^2)$                      |
| $\nu i q \rightarrow \nu i q$ | $24((g_V^q + g_A^q)^2s^2 + (g_V^q - g_A^q)^2u^2)$                     |
| $\nu i \bar{q} \rightarrow \nu i \bar{q}$ | $24((g_V^q + g_A^q)^2u^2 + (g_V^q - g_A^q)^2s^2)$           |
| $\nu i q_1 \rightarrow \nu i q_2$ | $48s^2V_{12}$                                                            |
| $\nu i \bar{l} \rightarrow \nu i \bar{l}$ | $48u^2V_{12}$                                                            |

Table 4: Abbreviations for Table 3

| Abbreviation | Expression                        |
|--------------|-----------------------------------|
| $g_V$        | $-\frac{1}{2} + 2\sin^2\Theta_W$ |
| $g_A$        | $-\frac{1}{2}$                    |
| $g_V^q$ (charge $e_q = +2/3$) | $\frac{1}{2} - \frac{4}{3}\sin^2\Theta_W$ |
| $g_A^q$ (charge $e_q = +2/3$) | $\frac{1}{2}$                    |
| $g_V^q$ (charge $e_q = -1/3$) | $-\frac{1}{2} + \frac{2}{3}\sin^2\Theta_W$ |
| $g_A^q$ (charge $e_q = -1/3$) | $-\frac{1}{2}$                    |
| $V_{ud}$     | 0.975                             |
| $V_{us}$     | 0.221                             |
| $V_{ub}$     | 0.000                             |
| $V_{cd}$     | 0.200                             |
| $V_{cs}$     | 0.979                             |
| $V_{cb}$     | 0.050                             |
where \( \eta \) is the shear viscosity, \( \mathbf{v} \) the velocity of a particle and \( \mathbf{V} \) the velocity of the flow. As in Section 4, we do not need the requirement of locality used in Section 3 in the computation of the electrical conductivity. \( \delta f \) can now be deduced from Eq. (18) with \( E = 0 \):

\[
\delta f = \frac{\nabla f_0 \cdot \mathbf{v}}{C(p)}. \tag{36}
\]

Substituting Eq. (36) into Eq. (35) we see that

\[
\int d^3 p p_i v_k \frac{\nabla f_0 \cdot \mathbf{v}}{C(p)} \simeq - \int d^3 p p_i v_k \mathbf{v} \cdot \nabla (\mathbf{p} \cdot \mathbf{V}) \frac{\partial f_0}{\partial \mathbf{p}} \frac{1}{C(p)}. \tag{37}
\]

Let us now assume a flow \( \mathbf{V} = a \mathbf{y} \mathbf{e}_x \), where \( a \) is a constant. Using \( \mathbf{V} \), Eq. (35) and Eq. (37) we can write the following expression for the shear viscosity:

\[
\eta = \int d^3 p v_x v_y p_x p_y \frac{\partial f_0}{C(p) \partial \mathbf{p}}. \tag{38}
\]

Using the fact that \( \mathbf{v} \) and \( \mathbf{p} \) have the same directions and therefore \( p_i v_k = p_k v_i \), and performing the integration in spherical coordinates, we get

\[
\eta = -\frac{1}{T} \int_0^\pi d\Theta \int_0^{2\pi} d\Phi \int_0^\infty p^4 \sin^2 \Phi \cos^2 \Phi \sin^5 \Theta \frac{\pm e^p/T}{C(p)(1 \pm e^p/T)^2}. \tag{39}
\]

The \( \pm \) sign stands for fermions and bosons, respectively. The shear viscosity is, however, always found to be positive. Just as in the case of the thermal conductivity, we see here that the smaller the interaction of the particle species in question is, the bigger the shear viscosity of that species in the plasma is i.e. the viscosity is related to the mean free time of the particles, too. Here we see a similar behaviour as with the thermal conductivity in Section 4: the viscosity depends on the number of scattering reactions and on the thermal regulators. The scattering reactions are the same as in the previous sections, presented in Tables 1, 2 and 3. In Fig. 6 we show the results for the shear viscosity of the leptons and photons, and in Fig. 7 the shear viscosity of the quarks and gluons are presented. In Fig. 8 the neutrino shear viscosity is presented; all the results are collected in Table 5. It should be noted that the shear viscosities are the same for particles and antiparticles. Quarks and gluons interact strongly and thus their shear viscosity is the smallest while, respectively, the neutrinos interact weakly so that their shear viscosity is the largest. This can be understood by considering the shear viscosity with the following example. Let us shoot a particle into the hot (primordial) plasma. The weaker its interactions with the surrounding plasma are, the longer it will travel without scatterings, and vice versa. This is consistent with how we defined the shear viscosity in Section 2: it is indeed related to the mean free time (the mean free path) of a particle in the plasma. The values for the shear viscosity
obtained in this Section are self-consistent with the assumption of short mean free times in Section 2 as can be seen by comparing the values presented in Table 5 and Eq. (11).

Earlier estimates for the shear viscosity in hot (primordial) plasma can be found, for example, in [9, 11, 12, 13, 14]. However, in [9] no matrix elements were calculated, and it was only noted that the shear viscosity is proportional to the mean free path of a particle species studied. [11, 12, 13, 14] contained much more thorough calculations of the shear viscosity to the leading logarithmic order. The results, however, differ somewhat from the ones presented in Table 5 and Figs. 5, 6 and 7. We believe that the proper introduction and numerical integration of the matrix elements in the collision integral in the present calculation yields more accurate values for $\eta$ than in the earlier publications.

As noted before, in the early universe $p \simeq \frac{1}{3} \rho$, so that the bulk viscosity (Eq. (11)) is approximately zero.

### 6 Summary

We find that the electrical conductivity is dominated by the leptonic current and that the quark and neutrino contribution to the electrical conductivity can be neglected. The values for the electrical conductivity can be found in Table 5. The definition of the electrical conductivity should be carefully considered. We started from the magneto hydrodynamic equation Eq. (20) and noted that the electrical conductivity defines the dissipation rate of the magnetic field in the plasma. But because of the short corre-
Figure 6: $\eta/T^3$ as a function of temperature for quarks and gluons

Figure 7: $\eta/T^3$ as a function of temperature for neutrinos.
lation lengths of the magnetic fields in the early universe, the electrical conductivity must be a local quantity. The different patches of the magnetic field are uncorrelated and therefore our way of defining and calculating the electrical conductivity seems natural: when the mean free path of the particles in the hot primordial plasma is longer than the correlation length of the magnetic field, particles will be shot out of the small magnetic patches to the nearby uncorrelated ones and we can calculate the electrical conductivity by examining how fast a charged particle loses its initial bulk momentum. To define a global electric field which would extend itself over the whole universe would disregard the small random structure of the primordial magnetic field. We assume in the paper that the mean free path of the particles studied in the primordial plasma is always longer than the correlation length of the primordial magnetic field.

To compute the heat conductivity and shear viscosity we use the same tools as in the computation of the electrical conductivity, but without the requirement of locality. The creation of the primordial magnetic fields can also depend on the viscosity and heat transportation of the hot primordial plasma. The viscous damping and heat conducting effects do also affect the first order phase transitions in the early universe. During the phase transitions, instabilities may occur when the transport of latent heat is dominated by the fluid flow. The instabilities can be damped by finite viscosity and heat conductivity due to the diffusion of radiation on small length scales. The galaxy formation can also be considered to be dependent on the transport coefficients in the early universe. A summary of the values for the heat conductivity and shear viscosity is given in Table 5.

Reliable estimates for the transport coefficients will help us to understand better the creation of the primordial magnetic fields, phase transitions in the early universe and the creation of density perturbations in the primordial plasma, which then would seed galaxy formation.

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Table 5: Summary of the values of the total conductivity $\sigma$ for leptons and quarks; thermal conductivity $\kappa$ and shear viscosity $\eta$ for each particle species separately. The row 'Particles' indicates which particles (and their antiparticles) are present in the heat bath.

| Particles | $e$ | $e\mu ud$ | $e\mu uds$ | $e\mu uds c$ | $e\mu \tau uds c$ | $e\mu \tau uds cb$ |
|-----------|-----|-----------|-----------|-------------|-----------------|------------------|
| $\sigma_{tot(QED)}/T$ | 2.22 | 5.90 | 5.98 | 6.18 | 9.44 | 9.47 |
| $\sigma_{tot(QCD)}/T$ | - | 0.110 | 0.133 | 0.223 | 0.223 | 0.247 |
| $\kappa_i/T^2$ | 4.51×10² | 6.01×10² | 6.07×10² | 6.25×10² | 6.36×10² | 6.37×10² |
| $\kappa_q/T^2$ | - | 15.4 | 15.6 | 15.7 | 15.7 | 15.9 |
| $\kappa_\gamma/T^2$ | 3.58×10⁴ | 9.76×10³ | 8.95×10³ | 6.71×10³ | 5.65×10³ | 5.37×10³ |
| $\kappa_g/T^2$ | - | 8.89 | 6.63 | 5.34 | 5.34 | 4.48 |
| $\kappa_{\nu_e}/T^2$ | 2.45×10⁹ | 6.63×10⁸ | 5.59×10⁸ | 3.80×10⁸ | 3.48×10⁸ | 3.28×10⁸ |
| $\kappa_{\nu_\mu}/T^2$ | 3.44×10⁹ | 6.63×10⁸ | 5.59×10⁸ | 3.80×10⁸ | 3.48×10⁸ | 3.28×10⁸ |
| $\kappa_{\nu_\tau}/T^2$ | 3.44×10⁹ | 1.72×10⁹ | 1.38×10⁹ | 1.19×10⁹ | 3.48×10⁸ | 3.28×10⁸ |
| $\eta_l/T^3$ | 4.29×10² | 5.71×10² | 5.76×10² | 5.94×10² | 6.04×10² | 6.07×10² |
| $\eta_\gamma/T^3$ | - | 17.8 | 18.0 | 18.2 | 18.2 | 18.4 |
| $\eta_\nu_e/T^3$ | 2.11×10⁴ | 5.75×10³ | 5.27×10³ | 3.95×10³ | 3.33×10³ | 3.16×10³ |
| $\eta_{\nu_\mu}/T^3$ | - | 4.61 | 3.50 | 2.84 | 2.84 | 2.40 |
| $\eta_{\nu_\tau}/T^3$ | 1.59×10⁹ | 4.29×10⁸ | 3.62×10⁸ | 2.46×10⁸ | 2.25×10⁸ | 2.12×10⁸ |
| $\eta_{\nu_\tau}/T^3$ | 2.22×10⁹ | 4.29×10⁸ | 3.62×10⁸ | 2.46×10⁸ | 2.25×10⁸ | 2.12×10⁸ |
| $\eta_{\nu_\tau}/T^3$ | 2.22×10⁹ | 1.12×10⁹ | 8.93×10⁸ | 7.73×10⁸ | 2.25×10⁸ | 2.12×10⁸ |
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