Filter Method and Its Consistency of Double-Star Position/SINS Integrated System

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Abstract  According to the characteristic of Beidou Double-star poiting system (for short: Double-star position), the optimal predication model of Double-star position/SINS integrated system is put forward, which can make use of the delayed position information from Double-star poiting system to predicate optimally for the integrated system, and then to correct SINS, and affords integrated results of some navigation parameters. In order to validate the consistency of the filter, the criteria for consistency of a filter is also studied, and the tested statistics are given, the experiment based on practical measured data shows that the filtering method is consistent with the integrated system.

Keywords  Double-star position; optimal predication; SINS; innovation; consistency

Introduction

The Beidou Double-star poiting system is one district position system built up by China. It adopts active poiting mechanism, and has the characteristic of delayed position and bad concealment [1], but it can poition accurately for the users in our territory and the around. The Double-star position/SINS integrated system is the developing trend in the field of navigation position in our country.

The integrated method based on position compensation in Reference [2] can improve the precision of navigation system effectively, but it depends on horizontal speed of the high precision integrated system. When the horizontal speed is low (for example, when the Beidou Double-star poiting system does not work), the precision of the integrated system is also low, at the same time, the filter method consider the position compensate error as colored noise, thus the system’s dimension will increase, and the real-time of the system will decrease.

So, this paper takes full advantage of the delayed position information provided by the Beidou position system to study the filter method of the integrated system and puts forward the optimal prediction model of integrated system. According to this model, the consistency of state estimation is studied, and the test statistics to prove consistency criteria using actually measured data are put forward.

1 Filter method of integrated system

1.1 Creation of optimal prediction model

Because for the users in the ground and in the sky, the way that the Double-star poiting system provides the height is different, the altitude channel is neglected here. While the “the north east ground” coor-
\[ \dot{X}(t) = A(t) \cdot X(t) + G(t) \cdot W(t) \]  
where \( X = [\phi, \psi, \theta, \delta \phi, \delta \psi, \delta \theta, \delta \phi, \delta \psi, \delta \theta, \epsilon_x, \epsilon_y, \epsilon_z, \epsilon_x, \epsilon_y, \epsilon_z] \) are the platform’s error angles; \( \delta \phi, \delta \psi, \delta \theta \) are the level speed errors; \( \delta L, \delta \lambda \) are the position errors; \( \epsilon_x, \epsilon_y, \epsilon_z \) are the gyro’s random constant; \( \epsilon_x, \epsilon_y, \epsilon_z \) are the first-order Markov process of gyro drift; \( \bar{V}_x, \bar{V}_y \) are the first-order Markov process of accelerometer drift. The definitions of the other parameters can refer to the Reference [3].

During the integrated interval, when the first integration takes place, because the receiver can be controlled, we can give the receiver an artificial order to prepare to send, the receiver receives this order, and transmits a position request order some times later, and then can receive the position information. After receiving the position information, the receiver sends the position request order immediately, then receives the position information again, the later process is the same as before. The detailed illustration is shown in Fig.1.

![Fig 1 Schedule of first combination in combination district](image)

where \( t_{k-1} \) prepares to send the order; \( t_k \) sends the position request order; \( t_{k+1} \) receives the position information which is the user’s position at \( t_k \) in fact.

Both the output position information of SINS at \( t_k \) and the information received by the Double-star positioning receivers at the \( t_{k+1} \) (namely the user’s position information at \( t_k \)) can be expressed as follows:

\[
\begin{align*}
\lambda_t &= \lambda_k + \delta \lambda \\
L_t &= L_k + \delta L \\
\lambda_{315} &= \lambda_k - \frac{N_E}{R_N \cos L} \\
L_{315} &= L_t - \frac{N_E}{R_N}
\end{align*}
\]  
(2)

where \( (N_E, N_N) \) are the measured noises of the Double-star positioning system, and the mean square deviation can refer to Section 1.2. The measurement state at \( t_k \) can be obtained based on Eq.(2):

\[
Z(k) = \begin{bmatrix} (L_t - L_{315})R_N \\ (\lambda_t - \lambda_{315})R_N \cos L \\ R_N \cdot \delta L + N_N \\ R_N \cos L \cdot \delta \lambda + N_E \\ H(k) \bar{X}(k) + V(k) \end{bmatrix}
\]  
(3)

Because the position information that received at \( t_{k+1} \) time is actually the user’s position information at \( t_k \), according to each information at \( t_{k+1} \) time, we make use of both Kalman filter’s time updating equations \((P(k|k-1), \bar{X}(k|k-1))\) and the state updating equations \((K(k), \bar{X}(k|k), P(k|k))\) to optimally filter at. Then using the optimal prediction value at \( t_{k+1} \) (namely using the time updating equations \( P(k|k-1), \bar{X}(k|k-1) \)), then use \( \bar{X}(k+1|k) \) to correct the result of SINS and navigation. The later filtering process is just the same as the process above.

### 1.2 Model building of measurement noise

The factors that affect the Double-star positioning system accuracy are the ephemeris error, the distance-measurement error, the height error[^4][^5]. The ephemeris error affects the accuracy of the Double-star positioning system greatly, but the Chinese Double-star positioning system uses differential technique, after differential revision, the bigger ephemeris error is equivalent to a smaller distance-measurement error[^5]. At the same time, after differential revision, the distance-measurement error decreases its influence on the accuracy of the Double-star positioning system greatly, but the height error can not be reduced or eliminated its influence on the accuracy of the Double-star positioning system by differential technique, so after differential revision, the ephemeris error and the distance-measurement error can be equivalent to a integrated distance-measurement error.

The measurement error’s mean square deviation of Double-star positioning system (unit: meter) can express as follows:

\[
\sigma_{\text{ rms}}^2 = \sigma_{\text{ evo}}^2 + \sigma_{\text{ cvo}}^2, \quad \sigma_N^2 = \sigma_{\text{ nvo}}^2 + \sigma_{\text{ cvo}}^2
\]  
(4)

where \( \sigma_{\text{ evo}}, \sigma_{\text{ nvo}} \) represent the measurement error’s mean square root of Double-star positioning system in the direction of east-west and south-north caused by integrated distance measurement error; \( \sigma_{\text{ cvo}}, \sigma_{\text{ cvo}} \)
represent the measurement error’s mean square root of Double-star positing system in the direction of east-west and south-north caused by height error. According to the positing measurement equation and the position relationship between the user and the satellites in the space rectangle coordinate system \([4,5]\), we can deduce the equations as follows:

\[
\begin{align*}
\sigma_{r_{ew}} &= R_{N} \cdot \cos L \cdot \Delta \lambda = R_{N} \cdot \Delta H \cdot \left( \frac{\cos(\lambda_i - \lambda) - \cos(\lambda_i - \lambda)}{\sin(\lambda_i - \lambda)} \right) \\
\sigma_{r_{ns}} &= R_{N} \cdot \Delta L = \tan L \cdot \Delta H \cdot \left( \frac{\sin(\lambda_i - \lambda) - \sin(\lambda_i - \lambda)}{\sin L \cdot \sin(\lambda_i - \lambda)} \right) \\
\sigma_{m_0} &= \frac{m_0}{2 \sin L \sin(\lambda_i - \lambda)} \\
\sqrt{\sin^2(\Delta \lambda_i - \Delta \lambda_i) + 5 \sin^2(\lambda_i - \lambda_i) + 2 \sin(\lambda_i - \lambda_i) \sin(\lambda_i - \lambda_i)} \\
\sigma_{m_0} &= \frac{m_0}{2 \sin L \sin(\lambda_i - \lambda)} \\
\sqrt{\cos^2(\Delta \lambda_i - \Delta \lambda_i) + 5 \cos^2(\lambda_i - \lambda_i) + 2 \cos(\lambda_i - \lambda_i) \cos(\lambda_i - \lambda_i)}
\end{align*}
\]

where \((\lambda_i, L_i, h_i)(i = 1, 2)\) represent the longitude and latitude the user stays at; \((\lambda_i, L_i, h_i)\) represent the longitude, latitude and height the satellite \(i\) stay at; \(\Delta H\) represents the user’s height error; and \(m_0\) represents the equivalent distance-measurement error after differential revision.

2 Consistency test of filter method

2.1 Standard of consistency test

In the problem of estimating a time-invariant parameter, consistency of an estimator was defined as stochastic convergence of the estimate to true value. When one estimates the state of a dynamic system, since it is changing, stochastic convergence of the estimate to the true state will generally not occur. What is available for checking consistency, in addition to the “current” estimate of the state at time \(k\), \(\hat{X}(k|k)\), is the associated covariance matrix \(P(k|k)\).

Under the linear-Gaussian assumptions, the conditional PDF of the state \(X(k)\) at time \(k\) is:

\[
p[X(k)|Z^k] = N[\hat{X}(k|k), P(k|k)]
\]

The model of the system consists of the dynamic equation, the measurement equation, and the statistical properties of the random variables entering into these equations. If all these are completely accurate, Eq.(5) holds exactly.

In practice, of course, all models contain some approximations. Therefore, it is of interest to what extent Eq.(5) holds in practice. The Gaussian condition Eq.(5) is usually replaced by the condition that the expected value of the estimation errors achieved by the filter should match the filter-calculated covariance:

\[
E\{[X(k) - \hat{X}(k|k)][X(k) - \hat{X}(k|k)]^T \} = P(k|k) \quad (6)
\]

where \(Z^k = \{Z(j), j = 1, \ldots, k\}\). A state estimator is called consistent if it is unbiased and its state estimation errors satisfy Eq.(6). The most common criteria for consistency of a filter are as follows.

1) The state error should be zero-mean (unbiased) and compatible with their covariance as yielded by the filter.

2) The innovations should have the same property.

3) The innovations should be white (uncorrelated in time).

The last two criterias, which are consequences of the first, are the only ones that can be tested in real data applications. The first criterion, which is really the most important, can be tested only in simulations, where the true state is available for comparison\([6]\). Because the test uses real data, the second and the third criteria are discussed mainly in the following section.

2.2 Choice of statistics in consistency examination

Under the hypothesis that the filter is consistent (the condition is \(H_0\)), the normalized innovations squared:

\[
\varepsilon_{\nu}(k) = V^T(k)S^{-1}(k)V(k) \quad (7)
\]

Eq.(7) has a chi-square distribution with \(n_z\) degrees of freedom, where \(n_z\) is the dimension of the measurement, \(V(k), S(k)\) innovation (measurement residual), measurement prediction covariance:

\[
V(k) = \tilde{Z}(k|k-1) = Z(k) - \tilde{X}(k|k-1)
\]

\[
S(k) = E[\tilde{Z}(k|k-1)\tilde{Z}(k|k-1)^T] = H(k)P(k|k-1)H^T(k) + R(k)
\]

For the single-run tests that can be performed in
real time, the criterion 2 can be tested with the following statistic:

$$\overline{\varepsilon}_v = \frac{1}{K} \sum_{k=1}^{K} V^T(k) S^{-1}(k) V(k)$$

which is the time-averaged normalized innovations squared. Under the hypothesis of $H_0$, $K\overline{\varepsilon}_v$ has a chi-square distribution with $Kn_z$ degrees of freedom.

Under the condition of fiducial probability $(1 - \alpha)$, the condition of the assumption that the filter is consistent is:

$$P[\varepsilon_v \in [r_1, r_2]|H_0] = 1 - \alpha$$

where $[r_1, r_2]$ is the confidence interval.

For the single-run tests that can be performed in real time, the criterion 2 can be tested on a single run in time by writing the whiteness test statistic for innovations $l$ steps apart as the time-averaged sample autocorrelation:

$$\overline{\rho}(l) = \sum_{k=1}^{K} V^T(k) V(k + l) \cdot \left( \sum_{k=1}^{K} V^T(k) V(k) \sum_{k=1}^{K} V^T(k + l) V(k + l) \right)^{-\frac{1}{2}}$$

This statistic is, for large enough $K$, approximately normally distributed with variance $1/K$. It is assumed that the confidence region of $(1 - \alpha)$ is $[-r, r]$ , if $\overline{\rho}(l)$ falls into this confidence region, this filter is consider to match with the system, else, it is one mismatched filter.

### 3 Experiment and analysis

In order to test the above algorithm, we carried on the static test in certain place. After differential correction, we considered that the measurement-range error caused by drifting position of the geostationary satellite was 8 m, measurement-range error was 13 m, height error was 10 m [7], then substitute these values into the Eq.(4), we considered that the positional error of the Double-star positing system was 30 m. The angular error of IMU was $30''$, the velocity error was 0.1 m/s, the initial latitude and longitude error was 100 m, the gyroscope drift was $1°/h$, the creational time of the gyro first-order Markov process was 3 600 s. Because of the restriction of the paper, we choose the position navigate parameter variety curve shown in Fig.2.

![Fig.2](image1.jpg) Error curve of integrated system’s position parameter

![Fig.3](image2.jpg) Curve of filter’s measurement residual
If the steps is selected as 55(1 step equals 1.5 s), then the measurement residual’s curves in the direction of longitude and latitude are shown in Fig.3 where \( L \) means latitude and \( \lambda \) means longitude. According to Eq.(11) the \( \bar{p}(l) \) every \( l \) steps is calculated as follows:

\[
\begin{align*}
\bar{p}(1) &= -0.4054, \\
\bar{p}(2) &= -0.0259, \\
\bar{p}(3) &= -0.0411, \\
\bar{p}(4) &= -0.0881, \\
\bar{p}(5) &= 0.1637, \\
\bar{p}(6) &= -0.1903, \\
\bar{p}(7) &= 0.0842, \\
\bar{p}(7) &= 0.1781.
\end{align*}
\]

If \( K = 50 \), with the assumption that the filter was valid, the error in above estimation was Gaussian distribution with zero mean value and variance of 1/\( K \), namely \( \bar{p} - \rho \sim N[0,1/50] \), then fiducially interval range of 95% was \([-0.2772, 0.2772]\). Above \( \bar{p}(l) \) were in the fiducially interval range, then the filter match with the system.

We could also calculate the normalized innovation square which is \( \zeta = 0.893 \), the confidence interval of 95% based on a chi-square distribution with 100 degrees of freedom is \([0.74, 1.3]\), so the filter match with the system.

From the above analysis, it is concluded that this filter method can raise the accuracy of the system effectively with the condition of assurance real-time, and the filter is tested to be match with the system.

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