Triple Regge exchange and transverse single-spin asymmetries of the very forward neutral pion production in polarized $p + p$ collisions

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Recently, the RHICf Collaboration measured the transverse single-spin asymmetries of the very forward neutral pion in polarized $p + p$ collisions at $\sqrt{s} = 510$ GeV, produced at large pseudorapidity ($\eta \gtrsim 6$). The data show large asymmetries both in longitudinal momentum fraction $x_F$ and transverse momentum $p_T$ at $p_T < 1$ GeV/c. Employing baryonic triple Regge exchanges, we describe the complete RHICf data for the first time and show that the neutral pion production at low $p_T$ can be interpreted as a diffractive one.

Introduction – The spin of the nucleon has been one of the most crucial issues in hadronic physics since the “spin crisis” caused by the EMC experiment [1]. Since the nucleon consists of not only three valence quarks but also other partons such as antiquarks and gluons, the nucleon spin should originate from the partons inside it and their orbital angular momenta [2]. “How does the spin of the nucleon arise?” This profound question motivated the future plan for the Electron-Ion Collider (EIC) [3]. Meanwhile, the transverse spin of the nucleon provides yet another aspect to the internal structure of the nucleon. The transverse momentum-dependent functions (TMDs) together with the generalized parton distributions (GPDs) furnish the multi-faceted aspect of the structure of the polarized nucleon in the transverse plane (see recent reviews [4, 5]). Furthermore, sizable transverse single-spin asymmetries (TSSA) of the neutral pion in inclusive $pp$ collisions have been continuously reported well over decades [6-11] (see also recent reviews [5, 12]). Since the experimental data from the PHENIX and STAR Collaborations were obtained at higher values of the transverse momentum ($p_T \gtrsim 2$ GeV/c) in the mid-rapidity coverage, where the pseudorapidity is given as $2 < \eta < 4$ [8, 11], QCD-based approaches have been employed such as the TMDs [14-16] and collinear twist-3 factorization [17, 23] to describe the experimental data. The Jefferson Lab Angular Momentum (JAM) Collaboration [24] has carried out the simultaneous QCD global analysis, considering the data on the TSSA from various high-energy processes.

The TSSA at low transverse momentum in the large pseudorapidity display the nonperturbative diffractive nature. The RHICf Collaboration measured the TSSA of the neutral pion in transversely polarized $p^1 + p$ collision at $\sqrt{s} = 510$ GeV and reported that the TSSA increased rapidly as functions of both the longitudinal momentum fraction $x_F$ and low transverse momentum $p_T$ ($p_T < 1$ GeV/c) at the pseudorapidity larger than $6$ ($\eta > 6$) [25]. The RHICf experiment data posed a question of whether the large values of the TSSA of $\pi^0$ are due to diffractive scattering: The values of TSSA rise as $p_T$ increases and reach around 25 % at $p_T \simeq 0.8$ GeV/c. The dependence of the longitudinal momentum fraction or the Feynman-$x$ variable $(x_F)$ reveals even a drastic feature. In this Letter, we will answer for the first time the question addressed by the RHICf Collaboration: Considering the $p^1 + p \rightarrow \pi^0 + X$ process at low $p_T$ as diffractive scattering and introducing the baryonic triple-Regge exchanges, we explain the RHICf data very well.

Triple-Regge exchange – The applications of the Regge approach to inclusive hadronic reactions is dated back to the 1970s [26-32]. Mueller generalized the optical theorem for inclusive reactions: the differential cross section for the two-body inclusive reaction, $a + b \rightarrow c + X$, can be written in terms of discontinuity of the three-body process $abc \rightarrow ab\bar{c}$ along the missing mass $M_X^2 = (p_a + p_b - p_c)^2$. It was shown that the three-body amplitude has the Regge singularities similar to those for the two-body process [26, 34]. The triple-Regge exchange is obtained from an asymptotic behavior of the Mueller amplitude in the kinematic boundary. It was shown that the unpolarized cross section was successfully described by triple-Regge pole contributions [30]. The triple-Regge formalism is still employed as a robust tool for understanding diffractive processes [31, 36].

On the other hand, it was anticipated that the nondiagonal triple-Regge pole diagram would be able to present the polarization effects [32, 37]. However, the statistics of the experiments was very poor, so that no significant data were reported at that time. Only very recently, the RHICf experiment accomplished a measurement of the TSSA in the very forward direction [25]. Since the final particles have very high pseudorapidities and low transverse momenta, one can use Regge-exchange of the initial proton as shown in Fig. 1(a). As mentioned previously, the generalized optical theorem leads to Fig. 1(b) with the discontinuity on the complex $M_X^2$ plane. When $M_X^2$ is sufficiently large, $ip \rightarrow jp$ scattering can be also expressed as a Regge pole. Thus one can consider triple-Regge exchange to derive the TSSA as drawn in Fig. 1(c). We extend the formalism in Ref. [32] with baryon...
Regge trajectories introduced. Note that the spin should be transferred by the baryon reggeons, since the produced pion does not carry any spin from the polarized proton.

The Lorentz-invariant differential cross section for the inclusive reaction \( p + p^h \rightarrow \pi^0 + X \) in the high energy limit is given by

\[
\frac{d\sigma^h}{d^3p} = \frac{E}{s} \sum_{i,j} |A_{i\rightarrow\pi^0}(s, p_T; h)|^2 ,
\]

where \( h \) is the helicity direction of the polarized proton beam. \( s \) denotes the square of the energy in the center of mass (CM) framework, which is one of the Mandelstam variables. \( p_T \) stands for the transverse momentum. When the energy is high enough to apply the Regge formalism, we can use the generalized optical theorem by Mueller [26] to express \( \frac{d\sigma^h}{d^3p} \) in terms of two reggeon exchange and the reggeon with the energy \( M_X^2 \) as depicted in Fig. 1.

In the limit \( M_X^2 \rightarrow \infty \), the discontinuity of the \( i + p \rightarrow j + p \) scattering will follow the Regge behavior as

\[
\text{Disc} \ A_{i\rightarrow j}(M_X^2) = \sum_k G^i_j(t) \gamma_{ij}^p(0) \left( \frac{M_X^2}{s_0} \right)^{\alpha_i(0)},
\]

where \( G^i_j(t) \) represents the triple-reggeon coupling given as a function of \( t \) that is a square of the momentum transfer (one of the Mandelstam variables), corresponding to the black blob in Fig. 1(c), which will be discussed later. \( \gamma_{ij}^p \) is the vertex function for the \( p\pi \) vertex in Fig. 1(c). \( s_0 \) denotes the scale parameter, which is traditionally given to be around 1 GeV\(^2\). Then \( d\sigma^h \) is written as

\[
d\sigma^h \sim \beta^i_{i\lambda} \beta^i_{i\mu} \mathcal{P}_i \mathcal{P}_j G^i_j(t) \gamma_{ij}^p(0) \left( \frac{M_X^2}{s_0} \right)^{\alpha_i(0)},
\]

where \( \beta^i(t) \) stands for the residue of \( i(t) \) exchange and \( \mathcal{P}_i(t) \) is the reggeon propagator defined as \[33, 40\]

\[
\mathcal{P}_i(t) = \alpha_B^{i} \left( J_i - \alpha_i(t) \right) (1 - x_F)^{-\alpha_i(t)}.\]

Here \( \alpha_i(t) \) and \( J_i \) denote respectively the Regge trajectory and the spin for exchange particle \( i \). The signature factor is given as

\[
\xi_i(t) = \frac{1 + \tau_i \exp \{-i\pi(\alpha_i(t) - 0.5)\}}{2},
\]

where \( \tau_i \) represents the signature of the corresponding reggeon, i.e., \( \tau_i = (-1)^{j_i-1/2} \).

We introduce the proton, the \( \Delta(1232) \) isobar, the excited baryon \( N^*(1520) \), and the \( \Delta(1600) \) isobar with negative parity to derive the TSSA of very forward pion production as shown in Fig. 1. Since the Regge approach does not provide the vertex structure, we need to employ the effective Lagrangians for \( NN\pi \), \( N\Delta\pi \), \( N^*\pi \) and \( N\Delta^*\pi \) given as

\[
\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_\mu \gamma_5 \frac{T}{2} \cdot \psi \partial^\mu \pi, \]

\[
\mathcal{L}_{\pi NN^*} = -\frac{f_{\pi NN^*}}{m_\pi} \bar{\psi} \gamma_\mu (g_{\mu\nu} + a \gamma_\nu \gamma_\lambda) \gamma_5 T \cdot \psi \partial^\nu \pi, \]

\[
\mathcal{L}_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi} \bar{\psi} \gamma_\mu (g_{\mu\nu} + a \gamma_\nu \gamma_\lambda) T \cdot \psi \partial^\nu \pi, \]

\[
\mathcal{L}_{\pi N\Delta^*} = -\frac{f_{\pi N\Delta^*}}{m_\pi} \bar{\psi} \gamma_\mu (g_{\mu\nu} + a \gamma_\nu \gamma_\lambda) T \cdot \psi \partial^\nu \pi, \]

where \( \psi, \psi, \mu \) and \( \pi \) denote respectively the Dirac, Rarita-Schwinger, and pseudoscalar fields for the nucleon, \( \Delta \) isobar, and the pion. \( f_{NN}, f_{NN^*}, f_{N\Delta}, \) and \( f_{N\Delta^*} \) designate the strong coupling constants for the corresponding vertices and \( m_\pi \) is the pion mass. \( T \) represents the Pauli matrix for the spin 1/2 isospin operator and \( T \) stands for the isospin transition operator from isospin 1/2 to 3/2 states. \( g_{\mu\nu} \) is the metric tensor \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) and \( a \) is the off-shell parameter for the spin 3/2 baryon.

The Regge factorization implies that the Born amplitudes of the one-particle exchange (OPE) can be subdivided into residues for each vertex and reggeon propagator. The proton-baryon-pion vertex functions are computed from the given effective Lagrangian respectively as
follows:

\[ \beta_{\lambda\lambda'}(p_T) = \bar{u}_N(\lambda', q) k^0 \gamma_3 u_p(\lambda, p), \]
\[ \beta_{\lambda\lambda'}^+(p_T) = i \bar{u}_N(\lambda', q) (k_0 + a_{\lambda\mu} k^\mu \gamma_5 u_p(\lambda, p)), \]
\[ \beta_{\lambda\lambda'}(p_T) = \bar{u}_N^\dagger(\lambda', q) (k_\mu + a_{\lambda\mu} k^\mu) u_p(\lambda, p), \]
\[ \beta_{\lambda\lambda'}(p_T) = \bar{u}_N^\dagger(\lambda', q) (k_\mu + a_{\lambda\mu} k^\mu) u_p(\lambda, p), \quad (7) \]

where \( p, k, \) and \( q \) are the four-momenta of the proton, pion, and the exchanged reggeon. \( \lambda \) and \( \lambda' \) denote the helicities of the baryons. For simplicity, we will switch off the off-shell parameter \( a \) in Eq. (6). Note that \( \beta_{\lambda\lambda'} \) should be real-valued functions, so the signature factor determines the phase of \( d\sigma \). It plays a crucial role in deriving the TSSA. Note that by the Abarbanel-Gross theorem the transverse single spin asymmetry (TSSA) in the backward direction, which agrees with the RHICf experimental data [25]. The TSSA in the backward direction are almost consistent with zero [24].

**Transverse single spin asymmetry** — The transverse single spin asymmetry is defined by the ratio of spin-dependent and spin-average differential cross section:

\[ A_N = \frac{\Delta \sigma_\perp}{\sigma} = \frac{d\sigma^\perp - d\sigma^\parallel}{d\sigma^\perp + d\sigma^\parallel}, \quad (8) \]

where \( \perp (\parallel) \) indicates the polarization of the proton in the transverse direction. Inserting Eq. (4) into Eq. (8), we can straightforwardly compute \( A_N \). Before we derive the explicit expression for \( A_N \), we discuss the parity invariance of \( \beta_i \) that will provide two constraints. Firstly, \( d\sigma^\perp \) vanishes if state \( k \) has unnatural parity. A matrix element that consists of two fermions (1, 2) and a spinless particle (3) obeys the following parity relation

\[ \beta_{\lambda\lambda'}^{3\lambda_3\lambda_3} = \eta_1 \eta_2 \eta_3 (-)^{\lambda_1' - \lambda_2} \beta_{-\lambda_1\lambda_2}, \]

where \( \eta_i \) denotes the naturality of a particle \( i \). Since the proton has a natural parity, we have \( \eta_1 \eta_2 = +1 \). So, the residue of the \( ppk \) vertex satisfies the parity relation \( \beta^{k\lambda}_k(0) = (-)^{\lambda - \mu} \beta^{k\lambda}_{-\lambda,-\mu} \). It leads to \( \gamma^{pp}(0) = \sum_{\nu \lambda} \beta^{k\lambda}_k(0) = (1 + \eta_k) \beta^{k\lambda}_{0,0} \), which becomes zero when state \( k \) has unnatural parity. For example, the following particles such as \( \pi, \alpha_1 \), and \( \omega \), etc., have unnatural parity. Thus only the particles with natural parity, \( k = P, \rho, a_2 \), etc., can contribute to \( A_N \). Here \( P \) represents the pomeron. Following Eq. (8), we find that pomeron exchange contributes to \( d\sigma^\perp \) dominantly over other meson exchanges that have \( \alpha_k(0) \) less than 0.5.

Secondly, \( \Delta \sigma_\perp \) vanishes when \( i \) and \( j \) have opposite naturality each other. As mentioned previously, the unpolarized proton does not take across any information on its spin to the final state. This implies that \( k \) exchange will not affect the spin polarization of particle \( i \). Since the transversely polarized state is expressed in terms of positive and negative helicity states quantized along the \( z \)-axis: \( |\uparrow\rangle = (|+\rangle + i|\downarrow\rangle)/\sqrt{2} \) and \( |\downarrow\rangle = (|+\rangle - i|\downarrow\rangle)/\sqrt{2} \),

the residue functions in Eq. (3) are expressed as

\[ \beta_i^{\lambda}(x) = \frac{1}{\sqrt{2}} (\beta_i^{\lambda,+} + i \beta^{\lambda,-}), \quad \beta_i^{\lambda}(x) = \frac{1}{\sqrt{2}} (\beta^{\lambda,+}_x - i \beta^{\lambda,-}_x). \quad (9) \]

Using the fact that \( \eta_p = + \) and \( \eta_\pi = - \), we observe

\[ d\Delta \sigma_\perp \sim \sum_{\lambda = -1/2}^{1/2} \left( \beta^{\lambda,\lambda}_x \beta^{\lambda,-}_x - \beta^{\lambda,-,\lambda}_x \beta^{\lambda,\lambda}_x \right) \]
\[ = (1 + \eta_\pi \eta_\rho) \beta^{\lambda\lambda}_x \beta^{\lambda\lambda}_x. \quad (10) \]

Thus \( d\Delta \sigma_\perp \) vanishes when \( \eta_\pi \eta_\rho = -1 \). The most dominant trajectory with natural parity is the proton one. The next one is the excited nucleon \( N^*(1520) \) of which the spin-parity quantum numbers are given by \( J^P = 3/2^- \). As for the unnatural parity states, the interference between \( \Delta \) and \( \Delta(1600) \) exchanges furnishes the most dominant contribution.

Then the spin-dependent and spin-averaged differential cross-sections read

\[ d\Delta \sigma_\perp = \frac{1}{s} \sum_{i,j} \beta^{i\lambda}_{i\lambda} \beta^{j\lambda}_{-\lambda} 2 \text{Im} \mathcal{P}_i \mathcal{P}_j^* \]
\[ \times \mathcal{G}_{\mathcal{P}}^{ij}(t) \mathcal{G}_{\mathcal{P}}^{ij}(0) \left( \frac{M_X}{s_0} \right)^{\alpha_2(0)}, \]
\[ d\sigma = \frac{1}{s} \sum_{i,j} \beta^{i\lambda}_{i\lambda} \beta^{j\lambda}_{-\lambda} 2 \text{Re} \mathcal{P}_i \mathcal{P}_j^* \]
\[ \times \mathcal{G}_{\mathcal{P}}^{ij}(t) \mathcal{G}_{\mathcal{P}}^{ij}(0) \left( \frac{M_X}{s_0} \right)^{\alpha_2(0)}, \quad (11) \]

where the triple-Regge coupling \( \mathcal{G}_{\mathcal{P}}^{ij}(t) \) is often parametrized as \( G(t) = G(0) e^{bt} \), because it cannot be theoretically determined. In the present work, we parameterize the form of the triple-Regge couplings so that we can describe the RHICf data: \( G_{\mathcal{P}}^{ij}(t) = G_{\mathcal{P}}^{ij}(0) \sqrt{|t|} e^{-B_i^P |t|}/m_\pi \). We define the following parameters

\[ g_{\mathcal{P}}^{ij} = \frac{G_{\mathcal{P}}^{ij}(0)}{G_{\mathcal{P}}^{NN}(0)}, \quad B_i^P \equiv B_i^P - B_i^{NN} \quad (12) \]

and fit them to the RHICf data. In Table 4, we list the numerical values of \( g_{\mathcal{P}}^{ij} \) and \( B_i^P \). Note that \( B_i^P \) is the subtraction given in Eq. (12). Except for the \( \mathcal{P}NN^* \) vertex, all the values of \( B_i^P \) are set to be zero.

**Results and Discussion.**— The RHICf Collaboration has first measured \( A_N \) for \( p + p^* \rightarrow \pi + X \) as a function of \( p_T \) with several different ranges of \( x_F \) given. Since the data on \( A_N \) in the negative \( x_F \) region are almost equal to zero, we concentrate on \( A_N \) with positive values of \( x_F \). In Fig. 2, we show the numerical results for \( A_N \) given as a function of the transverse momenta \( p_T \) with four different ranges of \( x_F \), compared with the RHICf data [25]. The present results are in quantitative agreement with the data. The value of \( A_N \) starts to increase as \( p_T \) increases till \( p_T \) reaches 0.2 ~ 0.3 GeV/c. Then, it seems
TABLE I. Numerical values of the parameters $g_P^{ij}$ and $b_P^{ij}$. The first column lists the values of $g_P^{ij}$ with $i$ and $j$ given whereas the second column shows the values of $b_P^{ij}$.

| $\Delta N$ | $g_P^{ij}$ | $b_P^{ij}$ [GeV$^{-2}$] |
|------------|------------|-------------------------|
| $N^*$      | 0.028      | 0.2                     |
| $\Delta \Delta$ | -0.018 | 0                      |
| $N^* N^*$ | 0.10       | 0                       |
| $\Delta \Delta$ | 0.022 | 0                      |
| $\Delta^* \Delta^*$ | 0.079 | 0                      |

![FIG. 2. Numerical results for the TSSA as a function of $p_T$ with several ranges of $x_F$ given. The present results are depicted by the triangles. The open circles with error bars illustrate the RHICf data [25].](image)

![FIG. 3. The TSSA as a function of $x_F$ with several ranges of $p_T$ given. Notations are the same as in Fig. 2.](image)

saturated for a while and then enlarges again as $p_T$ further increases. Note that in general the experimental uncertainties become larger as $p_T$ increases.

Figure 3 displays the numerical results for $A_N$ as a function of $x_F$ with five different ranges of $p_T$ given [25]. The current results have an outstanding fit with the RHICf data, in particular, as $p_T$ becomes smaller. Note that when $p_T$ approaches zero, $A_N$ is suppressed.

To scrutinize the current results, we plot $A_N$ as a function of $p_T$ and $x_F$ in Fig. 4. It is notable to see the peak in the mid-$p_T$ range ($\sim 0.5$ GeV/$c$), in particular, when $x_F$ is small. We can understand this feature of $A_N$ by examining the characteristics of the signature factor. At certain values of $p_T$ and $x_F$, $A_N$ becomes very sensitive to signature factor of the proton. The peak structure of $A_N$ occurs because of this sensitivity. On the other hand, when $x_F$ is large, the diagonal terms such as $NN$, $N^*N^*$, $\Delta \Delta$, and $\Delta^* \Delta^*$ diagrams come into play, the peak structure gets smeared. As $x_F$ becomes very small ($x_F < 0.3$), all the signature factors bring about a rapid oscillation of $A_N$. It indicates that the current scheme of the triple-Regge exchange breaks down when $x_F$ is very small.

![FIG. 4. The 3d plot of $A_N$ as a function of $p_T$ and $x_F$.](image)

**Conclusions.** – In this work, we aimed at investigating the transverse single spin asymmetries for the neutral pion production from inclusive polarized proton and proton collision, emphasizing the triple-Regge exchange that consists of two baryons and a pomeron. The numerical results of the current work are in quantitative agreement with the RHICf data. We discussed the feature of the transverse single spin asymmetries with $p_T$ and $x_F$ varied. When the pseudorapidity is large and $x_F$ is not very small, we can interpret the neutral pion production from inclusive polarized proton and proton collision as a diffractive one.

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