Application of Saving Matrix Methods and Cross Entropy for Capacitated Vehicle Routing Problem (CVRP) Resolving

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Abstract. Distribution activity is one part of Supply Chain Management and very important component. It can improve profit company with a decrease in transportation cost. This activity has many problems like capacity of vehicle, difference of consumer demand, and difference of consumer location. These problems must be handled by determine the route to minimize distribution cost. My research is the company engaged in printing service where the product has been printed will be sent to several customers with different locations. In determining the route will be used Saving Matrix and Cross Entropy method. The purpose of them is to form an optimal route where saving matrix method is processed manually and Cross Entropy method by Matrix Laboratory software. The result obtained by calculation of Saving Matrix method are four sub routes and Cross Entropy method are three sub routes. The comparison of two methods used is Cross Entropy method 5.81% more efficient than Saving Matrix method based on total transportation cost.

1. Introduction

Distribution is the activity of moving and storing goods from the level of suppliers to the level of consumers in the supply chain. The distribution will be at each level of the supply chain where the material flow moves from supplier to factory, then the finished product from the factory moves to the consumer so this will directly affect the supply chain cost [1]. This research takes the object of a company that has goods distribution activity between producers and end users (users) located in Surabaya City, East Java. This company is engaged in printing services that produce based on orders from consumers.

The problem in this study is how to determine distribution routes that are able to make optimal distance and costs. One thing that often happens in distribution activities is Vehicle Routing Problem. namely the problem of determining the vehicle route to serve several customers with different locations. The VRP is to find the minimum cost routes to be traveled by a fleet of vehicles. All of the vehicles must start and terminate at a central depot. Each customer must be visited once, by one vehicle only [2]. The total demand for each route does not exceed the vehicle's carrying capacity which is commonly called a capacitated vehicle routing problem (CVRP).

In this study, the optimal distribution of products will be determined to solve distribution problems. This is so that the product can meet market demand well, simplify and accelerate distribution to reach consumers quickly so that it can minimize the total cost of transportation. The approach used through the saving matrix method and the Cross Entropy method where both have the ability to solve
transportation and distribution problems. Transportation in companies produces the highest costs in
the distribution system, which is one third to two thirds of the total distribution costs [3].

2. Theory

2.1 Distribution
Distribution is an activity to move products from a factory or distribution center to retailers or
consumers. The distribution process is generally carried out by transportation systems and modes or
can also be through other media [4].

2.2 Capacitated Vehicle Routing Problem (CVRP)
Capacitated Vehicle Routing Problem (CVRP) is the basic model of VRP problems. This is
consistent with the constraints experienced by having a vehicle with a limited capacity. In its
implementation, CVRP is more difficult to solve than TSP (Traveling Salesman Problem). All
requests from customers are known at the beginning, for each customer and carried out on the same
route, so all customer requests are entered on the same route [5].

The classic CVRP can be described as follows: N customers geographically dispersed in a planar
region must be served from a unique depot. Each customer asks for a quantity qi (i=1, 2, …, n) of
goods. The transport cost from node i to node j is Cij. m vehicles with a fixed capacity Q are available
to deliver the goods stored in the depot. Each customer must be visited just once by only one vehicle.
The objective of the problem is minimizing the total cost of all routes without violating the individual
capacity of each vehicle. The depot is denoted by i=0 [6].

The model of the vehicle routing problem is established based on the following assumptions [7]:
1. The vehicles in each warehouse are of the same type (the same load), and the number of
   vehicles varies with customer demand.
2. There is a limit to the load of the vehicle, which cannot be overloaded.
3. Each customer must be accessed and can only be serviced once; no repeat access is allowed.
4. After completion of the vehicle distribution task, the vehicle fleet must return to the original
   warehouse.

3. Saving Matrix
The saving matrix method is one method that is easy to apply for complex distribution problems
quickly. The advantage of this method lies in its ease of modification. If there are obstacles such as
vehicle capacity, number of vehicles, delivery time or other constraints, it can provide a quick and
practical solution even though the results do not guarantee optimal solutions [8]. In the application of
the saving matrix method there are steps that must be taken, namely as follows;

| Step | Description |
|------|-------------|
| 1    | Determine the distance matrix |
| 2    | Determine Saving Matrix |
| 3    | Allocate each point of location to the route |
| 4    | Sort point of location for each route |

Figure 1. The steps of Saving Matrix Method
4. Cross Entropy Method (CE)

Cross-Entropy (CE) is a general stochastic optimization technology based on a fundamental principle of information theory called cross-entropy [8]. The CE method can be used to solve estimation and optimization cases. The advantage of the CE method is that it is very reliable in optimization problems if it is formulated as an estimation problem. In its development, there has been a modification to the CE method so that this method can be used also to solve optimization problems that have been successfully applied in the completion of VRP, TSP, orienteering problems, job scheduling, and others.

In addition, the advantage of this method lies in its relatively small iteration, the CE algorithm is also a very concise algorithm, and the larger the resulting sample the better the resulting solution [10]. Once the associated stochastic optimization is defined, then CE method follow these two phases [11]:

1. Generate a solution sample randomly (vectors, trajectories, etc.) according to a specified random mechanism.
2. Update of the parameters of the random mechanism, on the basis of the data, in order to produce a “better” sample in the next iteration.

The following is the CE algorithm for the case of Capacitated Vehicle Routing Problem [12]:

1. Determine the supporting parameters, namely the percentage of elite sample parameters (ρ), smoothing parameters (α), stop tolerance (β), and the number of generation samples (N).
2. Determine the initial value of the P. transition matrix.
3. Determine generating N paths from the P transition matrix and calculating the probability of a node being served by the following mechanism.
4. Calculate and select the path with the optimum value of ρ x N sample.
5. Updating the transition matrix from the P matrix based on the generation of the track.
6. If the stop criteria are met, then the iteration stops, if not then the process returns to step 2.

In the steps that must be carried out in this study as well as identifying problems, conducting a literature review, collecting and processing data and analyzing the results of a study, and the last is to conclude the results of this research. The research method flowchart used in this study can be seen in Figure 2.

![Figure 2. Research Methodology](image-url)
5. Results

5.1 Sub Route Using Saving Matrix Method

From the results of route savings, several proposed routes were found in accordance with the vehicle's carrying capacity so that the value of distance savings from the route obtained by using the nearest neighbor method was obtained. After obtaining the sub route, the calculation of distribution costs is done by multiplying the distance of each route with the price of the fuel of the vehicle used to distribute the product which can be seen in Table 1.

| No | Sub Route | Vehicle Capacity (Unit) | Mileage (Kilometers) | Fuel Ratio (Liter) | Fuel Cost (Rupiah) | Vehicle Used | Cost (Rupiah) |
|----|-----------|-------------------------|----------------------|-------------------|-------------------|--------------|--------------|
| 1  | P-PSRN-PRBLG-LMJG-JMBR-KPNJN-BLT-TLGM-KDR-MDN-PNRM-BJNGR-TBN-LMGN-P | 37.380  | 858  | 39   | 5.150               | Truck        | 200.850      |
| 2  | P-PUSAT-SIDOARJO-MOJOKERTO-JOMBANG-MALANG-P | 32.981  | 271  | 6.82 | 5.150               | Truckl       | 35.114       |
| 3  | P-SURABAYA-P | 22.180  | 14   | 2 x 0.56   | 5.150               | 2 x L300   | 5.768        |
| 4  | P-GRESIK-P | 8.824  | 70   | 7     | 8.000               | Pick Up     | 56.000       |
|    | TOTAL     | 1213   |      |       |                    |              | 297.732      |

By using the saving matrix method is expected to be able to complete and make distance savings to get the route to be formed. In determining the route also must pay attention to the transport capacity of the vehicle used which will be filled by the product requested by the consumer. The routes formed from the results of the Saving Matrix include:

1. Factory – Pasuruan – Probolinggo – Lumajang – Jember – Kepanjen – Blitar – Tulungagung – Kediri – Madiun – Ponorogo – Bojonegoro – Tuban – Lamongan – Factory
2. Factory – Pusat – Sidoarjo – Mojokerto – Jombang – Malang – Factory
3. Factory – Surabaya – Factory
4. Factory – Gresik – Factory

From the results of the sub route that was formed obtained by using the Saving Matrix method, the total distance was 1213 kilometers with a total cost of Rp. 297.732 every month.

5.2 Sub Route Using Cross Entropy Method

By using the Cross Entropy method with a value of \( \rho = 0.6, \alpha = 0.9, N = 30 \) which was carried out with the help of Matlab software version 2018, a sub route was obtained based on the best results, as follows:
Table 2. Sub Route of Product with Cross Entropy Method

| No. | Sub Route                  | Distance (Kilometer) | Fuel Ratio (Liter) | Fuel Cost (Rupiah) | Cost (Rupiah) |
|-----|----------------------------|----------------------|-------------------|-------------------|---------------|
| 1   | 1 – 7 – 2 – 21 – 16 – 5 – 14 - 10 – 18 – 1 | 613                 | 27.86             | 5.150             | 143.498       |
| 2   | 1 – 19 – 17 – 13 – 8 – 1     | 514                 | 23.36             | 5.150             | 120.323       |
| 3   | 1 – 12 – 9 – 20 – 3 – 1     | 71                  | 3.23              | 5.150             | 16.620        |
|     | Total                      | 1198                |                   |                   | 280.441       |
|     | computing time (detik)     |                      |                   |                   | 0.9375        |

By using the Cross Entropy method which is done with the help of the 2018 version of Matlab software, sub routes are obtained based on the best results including:
1. Factory – Kediri – Blitar – Tulungagung – Probolinggo – Jember – Pasuruan – Lumajang – Sidoarjo – Factory
2. Factory – Surabaya – Pusat – Mojokerto – Kepanjen – Factory
3. Factory – Malang – Lamongan – Tuban – Bojonegoro – Factory
From the results of the sub route formed, the total distance is 1198 kilometers with a total cost of Rp. 280.441 every month.

5.3 Comparison of Saving Matrix Method and Cross Entropy Method

Based on the results of data piloting, a comparison was made between the initial sub routes that became company policies with the proposed sub routes obtained using the Saving Matrix method and the Cross Entropy method. The following is a comparison table which can be seen in Table 3.

Table 3. Comparison of Saving Matrix and Cross Entropy Method

| Indicator | Saving Matrix | Cross Entropy | Company Policy |
|-----------|---------------|---------------|----------------|
| Distance (Kilometers) | 1213           | 1198          | 1677           |
| Cost (Rupiah/Month)    | 297.732        | 280.441       | 875.932        |

From the table above it can be seen that the comparison between company policy with the use of the Saving Matrix method and the Cross Entropy method has a considerable difference that is by using the Saving Matrix method, obtained route savings of 464 kilometers or 27.67%. Whereas by using the Cross Entropy method, obtained route savings of 479 kilometers or 28.56%.

In terms of the total costs incurred for the initial distribution route of Rp. 875,932.00 per month, the total cost spent on the proposed route using the Saving Matrix method is Rp. 297,732 every month or obtained a savings of 66.01%, while the total costs incurred by using Cross Entropy amounted to Rp. 280,441.00 or obtained savings of 67.98%.

6. Conclusion

From the results of the analysis, using the Cross Entropy method obtained three proposed sub routes with the first sub route namely Factory - Kediri - Blitar - Tulungagung - Probolinggo - Jember - Pasuruan - Lumajang - Sidoarjo - Factory, the second sub route is Factory - Surabaya - Pusat - Mojokerto - Kepanjen - Factory, and the third sub route is Factory - Malang - Lamongan - Tuban - Bojonegoro - Factory with a total distance of 1198 kilometers with a total cost of Rp. 280,441.00. To save total costs, the Cross Entropy method is more efficient at 5.81% from the Saving Matrix method.
From the comparison of the two methods, it can be said that the use of the Cross Entropy method is better than the use of the Saving Matrix method in this case. In addition, in terms of computing the Cross Entropy method has a very fast time with the help of Matrix Laboratory software, but depending on the number of samples generated (N), the more N, the longer the computation time. Whereas the Saving Matrix method takes a very long time because it still uses the manual method. So that the more points visited, the longer the computation time.

7. References

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