Electrically tunable two-channel Kondo fixed points in helical liquids

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We study a quantum dot coupled to two edge states of a quantum spin Hall insulator through electron tunnelings in the presence of a Rashba spin-orbital interaction induced by an external electric field. We show that if the electron interactions on the edge states are repulsive, there are two possible phases, depending on the Luttinger liquid parameter $K$. For $1/2 < K < 1$, the low-temperature physics is controlled by a previously identified two-channel Kondo fixed point. For the edge states with even stronger repulsive interactions, i.e. $1/4 < K < 1/2$, the system reaches another phase at low temperatures, described by a new two-channel Kondo fixed point. This phase is separated from the original one by a continuous phase transition upon varying the value of $K$ through the external electric field. The corresponding critical point is described by a free Dirac fermion backscattered by a local potential. We investigate the low-temperature properties associated with this new fixed point and also discuss the scaling behaviors of the system at the critical point.

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I. INTRODUCTION

The experimental discovery of the quantum spin Hall (QSH) insulator in HgTe quantum wells following its theoretical predictions paved a new road in the study of topological phases. The QSH state is a member of the topologically nontrivial states of matter with the symmetry protected topological order. These states of matter have a finite bulk gap and, in the mean time, support gapless edge or surface excitations. For the QSH insulator, the edge states are helical in the sense that, on each edge, there is a counter-propagating Kramers’ pair of states with opposite spin polarizations. The stability of the helical edge states against potential scattering is protected by the time-reversal (TR) symmetry. Contrary to the case of simple potential scattering, the edge electrons can backscatter from the magnetic impurities through spin exchange, and thus the TR symmetry can no longer protect the helical states from mixing. The one-channel Kondo (1CK) effect of helical edge states was studied in Refs. 8-12.

In a recent work, a quantum dot (QD) coupled to two helical edge states was studied.13-15 It is well known that such a system realizes the usual 1CK effect for non-interacting electrons on the edges.14-15 In Ref. 13, it is shown that for weakly repulsive interacting electrons on the edges, with the Luttinger liquid (LL) parameter $K < 1$, the system is driven to a two-channel Kondo (2CK) fixed point. This result is non-trivial since in the context of a QD coupled to two LL leads, a much stronger Coulomb repulsion ($K < 1/2$) is warranted in order to realize the 2CK physics.

A more complete description of the 2CK physics in a QD coupled to two helical edge states must take into account the Rashba spin-orbital interaction because this interaction, which can be tuned by an external gate voltage, is a built-in feature of a quantum well.16 Moreover, the HgTe quantum wells exhibit some of the largest known Rashba couplings of any semiconductor heterostructures.17 In fact, it was found very recently that the presence of a Rashba coupling has profound effects on both the Kondo temperature and the transport properties of the helical liquids in the presence of a single magnetic impurity.

In this work, we consider a QD coupled to two helical edge states in asymmetric HgTe/CdTe quantum wells, as shown in Fig. 1(a). The Rashba coupling will be present in this system and we would like to investigate the effect of it on the 2CK physics studied in Ref. 13. We find that if the electron interactions on the edge states are repulsive, there are two possible phases, depending on the value of $K$. For $1/2 < K < 1$, the low-temperature physics is controlled by the 2CK fixed point identified in Ref. 13. In this region, we find that the impurity entropy at zero temperature $S_{imp}$ and the temperature dependence of the tunneling conductance between the two edges are not affected by the presence of the Rashba coupling. On the other hand, the Rashba coupling reveals its presence through the temperature dependence of the impurity specific heat $C_{imp}$ and the dynamical structure factor $S_i(\omega)$ of the spin in the dot at low temperatures, i.e., $C_{imp} \propto T^{K+1/K-2}$ for $1/\sqrt{3} < K < 1$, $C_{imp} \propto T^{1K-2}$ for $1/2 < K < 1/\sqrt{3}$, and $S_i(0) \propto T^{K-1}$. More interestingly, we show that the system reaches another phase at low temperatures, described by a new 2CK fixed point when $1/4 < K < 1/2$. In this region, $S_{imp} = \ln (2K)$, $C_{imp} \propto T^{1/K-2}$ or $T$, and $S_i(0) \propto T^{2K-1}$. The two phases are separated by a quantum critical point (QCP), which is described by a free Dirac fermion backscattered by a local potential.
In the 2CK phase, the system can be described by the Kondo Hamiltonian with a partially screened spin (denoted by the dashed arrow). The symmetric sector is a spinless LL, while the antisymmetric sector is a spinless LL.

II. MODEL

We consider two helical edge states of a QSH insulator brought close to each other at a tunneling junction. A QD is then placed at the middle of the junction and coupled to the two edge states through electron tunnelings. When the number of electrons in the dot is odd and the repulsive interaction between the electrons in the dot is much larger than the tunneling amplitude, the system can be described by the Kondo Hamiltonian $H = H_0 + H_t + H_K^{11,19}$

$$H_0 = \sum_{m=1,2} \int dx \left( v_F \Psi_m^\dagger \sigma_3i\partial_x \Psi_m + \alpha \Psi_m^\dagger \sigma_2i\partial_x \Psi_m \right),$$

and

$$H_K = \sum_{m=1,2} \Psi_m^\dagger(0) \left( \sum_{i=x,y} J_{i1} S_i + J_{i2} S_i \sigma_3 \right) \Psi_m(0) + \sum_{m \neq n=1,2} \Psi_m^\dagger(0) \left( \sum_{i=x,y} J_{i1} S_i + J_{i2} S_i \sigma_3 \right) \Psi_n(0).$$

In the above, $m = 1, 2$ label the two edges, $\Psi_m = [\psi_m^+, \psi_m^-]^T$, $\psi_m^+ = \psi_m L$, $\psi_m^- = \psi_m R$, $\sigma = \uparrow, \downarrow$, and $S$ is the spin operator for the spin-1/2 impurity. Here, $H_0$ and $H_t$ denote the kinetic energy (including the Rashba coupling with the strength $\alpha$) and the electron Coulomb interaction of the helical edge states, respectively. $H_K$ describes the Kondo interaction between electrons in the edges and a spin-1/2 magnetic impurity at $x = 0$. The magnetic anisotropy, i.e. $J_{i1} \neq J_{i2}$ with $l = 1, 2$, is induced by spin-orbital coupling.

The Rashba term in $H_0$ can be absorbed into the kinetic term by a spinor rotation $\tilde{\Psi}_m = e^{-i\frac{\theta}{2} \sigma_3} \Psi_m^{12,21}$ where $\theta = \tan^{-1}(a/\nu_F)$. By rotating also the impurity spin $S = e^{i\theta/2} S e^{-i\theta/2}$, $H_0$ becomes $\tilde{v}_F \sum_{m=1,2} \int dx \tilde{\Psi}_m^\dagger \sigma_3 i\partial_x \tilde{\Psi}_m$, where $\tilde{v}_F = \sqrt{v_F^2 + \alpha^2}$, and

$$H_K = \sum_{m=1,2} \Psi_m^\dagger(0) \left( \sum_{i=x,y} J_{i1} \tilde{S}_i + J_{i2} \tilde{S}_i \sigma_3 \right) \Psi_m(0) + \sum_{m \neq n=1,2} \Psi_m^\dagger(0) \left( \sum_{i=x,y} J_{i1} \tilde{S}_i + J_{i2} \tilde{S}_i \sigma_3 \right) \Psi_n(0),$$

where $J_{1x} = J_{i1}$, $J_{1y} = J_{i1} \cos^2 \theta + J_{i2} \sin^2 \theta$, $J_{2x} = J_{i1} \cos^2 \theta + J_{i2} \sin^2 \theta$, and $J_{2y} = (J_{i1} - J_{i2}) \sin \theta \cos \theta$ with $l = 1, 2$. Since the Rashba coupling respects the TR symmetry, we expect that the helical liquid on each edge is still described by the LL. Hence, the allowed electron-electron interaction is of the form

$$H_t = \sum_{m=1,2} \int dx \left( g_1 \sum_{\sigma} J_{m\sigma} J_{m\sigma} + g_2 J_{m+} J_{m-} \right),$$

where $J_{m\sigma} = \psi_m^\dagger \psi_m \sigma \psi_m$. To proceed, we bosonize the Kondo Hamiltonian $H = H_0 + H_t + H_K$ according to the formula: $\psi_{m\sigma} = \sqrt{\pi a_0} \sum e^{i \pi \phi_{m\sigma}} \tilde{\phi}_{m\sigma}$, where $a_0$ is the short-distance cutoff. By defining the bosonic fields $\Phi_m = \phi_{m+} - \phi_{m-}$, $\Theta_m = \phi_{m+} + \phi_{m-}$, $\Phi_{s/\alpha} = (\Phi_1 \pm \Phi_2)/\sqrt{2}$, and $\Theta_{s/\alpha} = (\Theta_1 \pm \Theta_2)/\sqrt{2}$, the Hamiltonian for the edge states can be written as $H_0 + H_t = \sum_{s=+,-} \frac{1}{\nu_F} \int dx \left[ K(\partial_x \Theta_s)^2 + \frac{1}{2}(\partial_x \Phi_s)^2 \right]$, where the LL parameter $K$ and the speed of the collective excitation $v$ depend on both the Coulomb interaction and the Rashba coupling strength. On the other hand,
$H_K$ becomes

$$H_K = \frac{\tilde{J}_{1Z} \tilde{S}_z}{\pi \alpha_0} \cos \left[ \sqrt{2\pi} \Phi_s(0) \right] \cos \left[ \sqrt{2\pi} \Phi_a(0) \right] + \frac{\tilde{J}_{1y} \tilde{S}_y + \tilde{J}_{1E} \tilde{S}_z}{\pi \alpha_0} \sin \left[ \sqrt{2\pi} \Phi_s(0) \right] \cos \left[ \sqrt{2\pi} \Phi_a(0) \right] + \frac{\tilde{J}_{2Z} \tilde{S}_z}{\pi \alpha_0} \cos \left[ \sqrt{2\pi} \Phi_s(0) \right] \cos \left[ \sqrt{2\pi} \Phi_a(0) \right] \sin \left[ \sqrt{2\pi} \Phi_s(0) \right] + \frac{\tilde{J}_{2y} \tilde{S}_y + \tilde{J}_{1E} \tilde{S}_z}{\pi \alpha_0} \sin \left[ \sqrt{2\pi} \Phi_s(0) \right] \cos \left[ \sqrt{2\pi} \Phi_a(0) \right] - \frac{1}{\sqrt{2\pi}} \left( \tilde{J}_{1Z} \tilde{S}_z + \tilde{J}_{1E} \tilde{S}_y \right) \partial_z \Theta_a(0) \right)

(5)

In general, there are extra local backscattering terms caused by the QD in the Hamiltonian $H$. However, these terms are irrelevant as long as $K > 1/4$. In the following, we shall focus on the regime with $1/4 < K < 1$, so that these local backscattering terms can be neglected for low-energy physics.

### III. SCALING ANALYSIS

Near the Gaussian fixed point $(\tilde{J}_i = 0 = J_{1E}$ with $i = 1, 2$ and $j = r, y, z)$, the scaling dimensions of the various terms in Eq. (3) are $\Delta(J_{1r}) = K = \Delta(J_{1y}) = \Delta(J_{1E})$, $\Delta(J_{1z}) = 1$, and $\Delta(J_{2r}) = K/2 + 1/(2K) = \Delta(J_{2y}) = \Delta(J_{2z}) = \Delta(J_{2E})$. Thus, for $K < 1$, the $J_2$ terms decrease while the $J_1$ terms grow when the temperature is lowered.

In order to study the physics at the strong-coupling regime when $\tilde{J}_1$ is of order 1, we refer Ref. 13 by employing the Emery-Kivelson unitary transformation $U = \exp \left[ i \sqrt{2\pi K} \tilde{S}_a(0) \tilde{S}_z \right]^{1/2}$ where $\tilde{\Phi}_{s/a} = \Phi_{s/a}/\sqrt{K}$ and $\tilde{\Theta}_{s/a} = \sqrt{K} \Theta_{s/a}$. The transformed Hamiltonian $\tilde{H} = UHU^\dagger$ is of the form

$$\tilde{H} = H_s^{(0)} + H_a^{(0)} + \lambda \tilde{S}_z \partial_x \tilde{\Theta}_a(0)$$

$$+ \tilde{S}_x \left[ \tilde{J}_{11} \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] + \tilde{J}_{12} \cos \left[ \sqrt{2\pi K} \Theta_a(0) \right] \right]$$

$$+ \tilde{J}_{1E} \tilde{S}_z \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] + \delta H, \quad (6)$$

where $H_s^{(0)} = \frac{\xi}{2} \int dx \left[ \left( \partial_x \tilde{\Phi}_{s/a} \right)^2 + \left( \partial_x \tilde{\Phi}_{s/a} \right)^2 \right], \lambda = \frac{\tilde{J}_{1s}}{\sqrt{2\pi K}} - \sqrt{2\pi K} \nu, \tilde{J}_{11} = (\tilde{J}_{1x} + \tilde{J}_{1y})/2, \delta \tilde{J}_{11} = \tilde{J}_{1x} - \tilde{J}_{1y} = (J_{1x} - J_{1y})/2, (J_{1x}, \sin^2 \theta (l = 1, 2),$ and

$$\delta H = g_1 \left\{ \tilde{S}_x \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] - \tilde{S}_y \sin \left[ \sqrt{2\pi K} \Phi_a(0) \right] \right\} \times \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] + g_2 \cos \left[ \sqrt{2\pi K} \Theta_a(0) \right]$$

$$\times \left\{ \tilde{S}_x \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] - \tilde{S}_y \sin \left[ \sqrt{2\pi K} \Phi_a(0) \right] \right\}$$

$$+ g_3 \tilde{S}_z \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] \cos \left[ \sqrt{2\pi K} \Theta_a(0) \right] + g_4 \tilde{S}_z \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] \sin \left[ \sqrt{2\pi K} \Theta_a(0) \right]$$

$$\times \left\{ \tilde{S}_y \cos \left[ \sqrt{2\pi K} \Phi_a(0) \right] + \tilde{S}_x \sin \left[ \sqrt{2\pi K} \Phi_a(0) \right] \right\}.$$
On the other hand, for $1/4 < K < 1/2$, the $g_1$ term becomes relevant, and it renders 2CK fixed point unstable. Since the $J_{1\perp}$ term also flows to strong coupling in this region, we expect that at low temperatures the system is described by the new fixed-point Hamiltonian

$$\tilde{H}_* = H_s^{(0)} + \frac{\delta J_{1\perp}}{2\pi a_0} \tilde{S}_x \cos \left( \sqrt{8\pi K} \tilde{\Phi}_s(0) \right) + H_a^{(0)} + \frac{\tilde{J}_{1\perp}}{\pi a_0} \tilde{S}_x \cos \left( \sqrt{2\pi K} \tilde{\Phi}_a(0) \right),$$

(8)

where $\xi_a = \left\langle \cos \left( \sqrt{2\pi K} \tilde{\Phi}_a(0) \right) \right\rangle$. We refer to this new fixed point as the 2CK' one in the following. Again, $\tilde{S}_x, H_a = 0$, and we may set $\tilde{S}_x = \pm 1/2$ so that $\tilde{H}_* = H_a(\tilde{S}_x = \pm 1/2)$. Both the symmetric and the antisymmetric sectors in $\tilde{H}_*$ amount to a spinless LL with an impurity backscattering term at $x = 0$ and the corresponding effective LL parameters $K_s = 2K$ and $K_a = K/2$, respectively. For $1/4 < K < 1/2$, the $\tilde{J}_{1\perp}$ and the $\delta \tilde{J}_{1\perp}$ terms both flow to the strong-coupling regime and eventually cut both sectors into two separated pieces at $x = 0$. At finite temperature, tunneling between the two half wires is allowed, leading to the perturbations $\lambda_1 \tilde{O}_1$ and $\lambda_2 \tilde{O}_2$, where $\tilde{O}_2 = \tilde{\Psi}_A(0) \tilde{\Psi}_B(0) + H.c.$ and $\tilde{\Psi}_A(B)$ referred to fermions in the two separated regions of the wire for the symmetric sector. The scaling dimensions of the various perturbations around the 2CK' fixed point is given in the third column in Table I. We see that all the perturbations are irrelevant for $1/4 < K < 1/2$, except the $g_3$ term, which is marginal. This confirms that $H_*$ is indeed the low-energy effective Hamiltonian in this region.

### IV. PHYSICAL PROPERTIES

One of the most remarkable features of the multichannel Kondo effect is the existence of the fractionally degenerate ground state, which reveals itself in the impurity entropy at zero temperature. To compute it, we notice that $\tilde{S}_x$ commutes with the fixed-point Hamiltonian, and thus we may write the partition function $Z$ as $Z = Z_+ + Z_-$ for $1/2 < K < 1$ and $Z = Z_+ + Z_-$. For $1/4 < K < 1/2$, where $Z_{\pm} = tr\{e^{-\beta H_{\pm}}\}$ and $\bar{Z}_{\pm} = tr\{e^{-\beta H_{\pm}}\}$, respectively. Following the same reasoning used in Ref. [13], it can be shown that $Z_+ = Z_-$ and $\bar{Z}_+ = \bar{Z}_-$. The impurity entropy of $H_+(\bar{H}_+)$ at $T = 0$ has been calculated in Ref. [25]. It is in $\sqrt{K/2}$ for the antisymmetric sector in both $H_+$ and $\bar{H}_+$ and in $\sqrt{2K}$ for the symmetric sector in $\bar{H}_+$.

Together with the contribution from $H_- (or \bar{H}_-)$, we find that $S_{\text{imp}} = (2K)/2$ for $1/2 < K < 1$ and $S_{\text{imp}} = (2K)/4$ for $1/4 < K < 1/2$.

The impurity correction to the free energy is given by

$$\delta F_{\text{imp}} \equiv F - F_0 = -\frac{\pi}{a_0} \int d\tau C_2(\tau) + O(\lambda^3),$$

where $F_0$ is the bulk free energy at the fixed point, $C_2(\tau) \equiv \left\langle T\{\delta H(\tau)\delta H(0)\} \right\rangle \propto \left[ \frac{\pi/2}{\sin(\pi/2\gamma)} \right]^{2\Delta}$ with $\langle \cdots \rangle$ being the expectation value at the fixed point, $\delta H$ denotes the leading irrelevant operator (LIO), $\lambda$ is the corresponding coupling constant, and $\Delta$ is the scaling dimension of $\delta H$. Near the 2CK fixed point, the LIO is the $\frac{4\pi \tilde{S}_x}{\pi a_0} \tilde{\Phi}_a(0)$ term with scaling dimension $K/2 + 1/2(2K)$ for $1/\sqrt{3} < K < 1$ and $\frac{2\pi \tilde{S}_x}{2\pi a_0} \tilde{\Phi}_a(0)$ term with scaling dimension $2K$ for $1/2 < K < 1/\sqrt{3}$. (We notice that in the absence of the Rashba coupling, the LIO is the $\frac{4\pi \tilde{S}_x}{\pi a_0} \tilde{\Phi}_a(0)$ term with scaling dimension $1/K$.) On the other hand, near the 2CK' fixed point, the LIO is the $\lambda_{\tilde{J}}$ term with scaling dimension $1/(2K)$. From the above results, we may obtain the temperature dependence of the impurity specific heat $C_{\text{imp}} = -T \frac{\partial^2}{\partial T^2} \delta F_{\text{imp}}$ at low temperatures, yielding $C_{\text{imp}} \propto T^{K+1/K-2}$ for $1/\sqrt{3} < K < 1$, $T^{4K-2}$ for $1/2 < K < 1/\sqrt{3}$, $T^{1/K-2}$ for $1/3 < K < 1/2$, and $T$ for $1/4 < K < 1/3$.

If we apply a small bias across the two edges, a current will flow from one edge to the other. Since only the $J_2$ terms will contribute to this current, the leading temperature dependence of the conductance $G$ at zero bias reflects the renormalization-group (RG) flow of the $J_{1\perp}$ term. From the above discussions, we see that neither the qualitative RG flow of the $J_{1\perp}$ term or its scaling dimensions near the various fixed points are affected by the presence of the Rashba coupling. As a result, the temperature dependence of $G$ is identical to that without the Rashba coupling. (See Fig. 1(b) in Ref. [13].)

One way to distinguish the 2CK and 2CK' fixed point is to investigate the dynamical structure factor of the impurity spin $\chi_1(\omega, T)$, which can be obtained
from \( S_i(\omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} S_i^{(2)}(\tau) \) by analytic continuation \( \omega_n \to \omega + i0^+ \), where \( i = y, z \) and \( S_i^{(2)}(\tau) = -\langle \mathcal{T}_\tau \{ S_i(\tau) S_i(0) \} \rangle \). After performing the unitary transformation \( U \), we have

\[
S_y = \cos \theta \left\{ \sin \left[ 2\pi K \Phi_x(0) \right] \tilde{S}_z + \sin \left[ 2\pi K \Phi_x(0) \right] \tilde{S}_y \right\} + \sin \theta \tilde{S}_z ,
\]

\[
S_z = -\sin \theta \left\{ \sin \left[ 2\pi K \Phi_x(0) \right] \tilde{S}_x + \sin \left[ 2\pi K \Phi_x(0) \right] \tilde{S}_y \right\} + \cos \theta \tilde{S}_z .
\]

Near the fixed point, we expect that \( S_i^{(2)}(\tau) \propto |\tau|^{-x_i} \) at \( T = 0 \) as \( |\tau| \to +\infty \), and \( x_i \) is determined by the term in the above equations with the smallest scaling dimension. Near both the 2CK and the 2CK’ fixed point, it is the \( \tilde{S}_x \) term which determines \( x_i \), yielding \( x_i = K \) for \( 1/2 < K < 1 \) and \( 2K \) for \( 1/4 < K < 1/2 \). Hence, we find that at low frequencies \( \text{Re} \chi_i(\omega, 0) \approx C_i |\omega|^{K-1} \) for \( 1/2 < K < 1 \) and \( C_i |\omega|^{2K-1} \) for \( 1/4 < K < 1/2 \), as well as \( \text{Re} \chi_i(0, T) \approx C_i T^{K-1} \) for \( 1/2 < K < 1 \) and \( C_i T^{2K-1} \) for \( 1/4 < K < 1/2 \) at low temperatures, where \( C_y, \tilde{C}_y \propto \cos^2 \theta \) and \( C_z, \tilde{C}_z \propto \sin^2 \theta \).

\[ \text{V. QUANTUM CRITICAL REGIME} \]

From the above analysis, the difference between the 2CK and the 2CK’ phases lies at the Hamiltonian for the symmetric sector. Hence, at the critical point separating these two phases, the system can be described by the Hamiltonian for the symmetric sector with the LL parameter \( K = 1/2 \). It turns out that this Hamiltonian can be reformulated as

\[
H_s = v \int dx \left( \psi^\dagger_i i \partial_x \psi_i - \psi^\dagger_i i \partial_x \psi_i^\dagger \right) + m \left[ \psi^\dagger_i \psi_i(0) + \text{H.c.} \right].
\]

where \( m = \pm \delta J_{\pm} \xi_0/2 \) and \( \psi_i \propto e^{\mp i\sqrt{\pi}(\Phi_x \pm \Phi_y)} \). In Eq. (4), we have set \( \tilde{S}_x = \pm 1/2 \) because \( [\tilde{S}_x, H_s] = 0 \). \( H_s \) is nothing but the Hamiltonian of a one-dimensional Fermi liquid backscattered by a \( \delta \)-function like potential at \( x = 0 \). Therefore, we expect that its thermodynamical properties should resemble those of the Fermi liquid. In fact, straightforward calculations show that \( S_{\text{imp}}^{(s)}(T = 0) = 0 \) and \( C_{\text{imp}}^{(s)} \propto T \). Since the combined contribution to \( S_{\text{imp}} \) from the antisymmetric sector and \( \tilde{S}_x \) vanishes, we conclude that \( S_{\text{imp}} = 0 \) at the QCP. Moreover, the contribution to \( C_{\text{imp}} \) arising from the antisymmetric sector is given by the LIO, which will give a higher power in \( T \). Consequently, the leading temperature dependence of \( C_{\text{imp}} \) is dominated by the symmetric sector, i.e. \( C_{\text{imp}} \propto T \). Logarithmic corrections to this result are possible and can be obtained by the one-loop RG equation near the critical point \( K = 1/2 \). However, this is beyond the scope of the present work.

\[ \text{VI. CONCLUDING REMARKS} \]

To summarize, we have shown that although the low-temperature physics of a QD coupled to two helical edge states is still described by the 2CK fixed point in the presence of a Rashba coupling for \( 1/2 < K < 1 \), the leading temperature dependence of thermodynamical quantities is drastically changed due to the new LIO’s produced by the Rashba interaction. With stronger Coulomb repulsions between electrons on the edges, the system will be driven to the 2CK’ phase. This phase is characterized by a new set of fixed point (line) Hamiltonians, and it can be distinguished from the 2CK phase by examining the impurity spin susceptibilities. At the boundary between the two phases, the system exhibits scaling behaviors which are distinct from those in the 2CK and 2CK’ phases, as we have shown. Since the LL parameter \( K \) depends sensitively on the Rashba interaction strength, the new phase described by the 2CK’ fixed point and the quantum phase transition into this phase can be electrically controlled. Therefore, our results not only reveal a new 2CK fixed point that has not been analyzed before, but also serve as a useful guide for future experimental investigations on this system.

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