Unified Wall Function and Application for Steady Turbulent Pipe Flow with Uniform Wall Roughness

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Abstract. A newly developed wall function for the velocity-profile of steady turbulent flow in various hydraulic regimes near rough walls was created. The wall function was then applied into two approaches to obtain the total flow energy loss (mechanical energy loss) of turbulent pipe flow. The energy loss values calculated through the above two approaches both predicted the experimental results well. The energy loss coefficient calculated through the second approach gave a more precise definition of the total flow characteristics than the Manning roughness coefficient as it was proved to be a function of the field statistics. It was furtherly found that this energy loss can be decomposed into the energy loss due to dissipation caused by molecular viscosity and the dissipation of turbulent kinetic energy, and that its density distribution falls mainly into the near-wall region.

Keywords. Wall function, pipe flow, wall roughness.

1. Introduction
The wall function method is a widely used approach in numerical simulations of wall-bounded turbulent flows as it is convenient and sufficiently precise. As the incoming flow conditions and wall conditions vary, the turbulent flow may fall into any one of the three hydraulic regimes: hydraulically smooth, transitionally rough, or fully rough. Even the inner region of turbulent boundary layers over smooth surfaces can be divided into the viscous sublayer, buffer region and logarithmic region [1].

Given this information, an accurate description of the near wall velocity profile under complex boundary conditions is essential for creating accurate numerical simulations of wall-bounded turbulent flow.

In existing wall functions of velocity profile, a common approach is to assume that the velocity profile is divided into a logarithmic profile and a linear profile at a certain distance (usually described by the near wall length scale, \( y^+ \)) above the wall in such a way that the buffer region is simplified [2]. In order to give a more accurate description of the various vertical regions, Van Driest [3] and Spalding [4] suggested velocity profiles in the forms of the integration and power laws, respectively, for smooth walls. For rough walls, the flow may instead fall into any one of the hydraulic regimes according to variations in the geometric roughness, distribution, scales, and incoming flow conditions. To give a unified description of the flow in various hydraulic regimes, Schlichting [1] and Scholz [5] carried out extensive measurements of flow over different roughness forms. Clauser [5] performed a review of their works and concluded that the geometry, shapes, and distributions of the roughness elements would all influence the near wall flow velocity. Based upon this data, White [4] suggested a
unified equation for various hydraulic regimes, which deviated from the results seen in the transitionally rough regime. In further studies, Grass [6], Pimenta et al. [7], Healzer et al. [8], and other researchers [9-10] performed numerous experiments on near wall flows using various roughness forms. Applying the results of their work, Ligrani et al. [11] developed a velocity profile wall function that agreed well with the experimental results from each hydraulic regime. Unfortunately, his wall function is restricted in the sense that it is piecewise function. In additional research, Dou [12] developed a continuous velocity profile wall function applicable to various hydraulic regimes; however, it must be noted that the criterion for each hydraulic regime in this function are extremely different from the above experimental results as well as those found in Nikuradse’s [13] experiments.

In engineering practice, the total flow energy loss or flow capacity (commonly quantified with the Manning roughness coefficient, $n$), can be obtained by integrating of the flow velocity over the cross sections of the pipe (or the channel depth) on the basis of the Darcy-Weisbach equation [14]. Therefore, determining the wall function of the velocity profile would have great effects on the total flow characteristics of both pipe and channel flows. However, there is no precise description of this wall function in conventional hydraulics; instead, the momentum roughness height, $y_0$, was introduced under the assumption that the full velocity profile is logarithmic and the velocity reaches zero at a certain height, $y_0$, above the wall. Based on this, simple functions for the values of $y_0$ and $n$ can be obtained by integrating this velocity profile over the pipe cross section. For example, Katul [15] gave a power law between $y_0$ and $n$ in hydraulics application based on the atmospheric boundary layer experimental results of Bradely [16] and Sutton [17].

Furthermore, $y_0$ can be determined by the local wall materials or the sand grading of the channel bed. For example, Schlichting+, Hey [18], and Yen [19] researched the relationship between $y_0$, the Colebrook [20] equivalent roughness size, $\Delta$, and characteristic sand grain sizes $D_{50}$ and $D_{90}$. Given the behavioral tendencies defined by this research, the Manning roughness coefficient, $n$, may be characterized as a single-value function based on the wall materials and roughness sizes. However, due to the variation in cross sectional shapes and wall roughness, the total flow characteristics of pipes, channels, and natural rivers have important connections with the variation in flow boundary conditions. Therefore, it is worth research and investigation to discover a precise approach for obtaining the total flow characteristics from these field characteristics.

Considering that the Manning roughness coefficient, $n$, is not connected to the flow field characteristics, Liu [21] derived a total flow energy equation for steady pipe flow with smooth walls directly from the incompressible viscous fluid motion theory. Additionally, an explicit calculation for the total flow energy loss equation was given. Pan [22] gave a further total flow energy equation and energy loss calculation equation for turbulent pipe flow with consideration of wall roughness. Using this equation, the energy loss between two cross sections of a rough pipe can be calculated directly from flow field statistics such as the mean velocity, mean pressure, fluctuating velocity, and fluctuating pressure. Accurately obtaining behavioral calculations for a flow field with complex boundary conditions is of great importance for the application of the above approach. A revised wall function that includes the near wall velocity profile is needed for application in the numerical simulation of turbulent flow fields, and a precise numerical simulation should be used to describe field statistics required for defining the total flow characteristics. This approach would provide a unified work capable of describing of both the total flow and field characteristics.

In this paper, a wall function of the velocity profile of various hydraulic regimes was developed through theoretical analysis; an integration equation is presented in which the total flow energy loss was obtained by integrating the wall function over a selected cross section; a equation for calculating energy loss from the flow field is presented (in this model, the total flow energy loss was calculated directly from the flow fielded statistics predicted by numerical simulation) in which the wall function was applied for near wall treatment; variations in the energy loss obtained through the two models with the Reynolds number, $Re_d$, and relative wall roughness, $\Delta/R$, were researched; and further
research was done to investigate the constitution of this energy loss as well as the density distribution of energy loss over the radius of the pipe.

2. Materials and Methods: Developing the Unified Wall Function

Regarding steady turbulent flow in a uniform pipe with constant wall roughness, shown in Figure 1(a), a wall coordinate can be built to describe the velocity profile of the flow, as shown in Figure 1(b).

Considering that the roughness scale is much smaller than the pipe diameter, \( d \), a virtual origin (where the flow velocity is assumed to be zero) is commonly used to determine the highly variable position of the rough wall. In Figure 1(b), \( \delta \) represents the distance from the virtual origin to the roughness crests, and can be calculated as \( \delta = \beta \Delta \), where \( \beta \) is a coefficient [23]; \( y \) represents the distance between the virtual origin to the wall; and \( R \) is the effective pipe radius. It is apparent that \( y = R - r \).

Figure 1(a). Sketch of turbulent flow in pipes with wall roughness.

Figure 1(b). Sketch of turbulent flow over a rough wall.

The turbulent flow over a rough wall shown in Figure 1(b) may fall into any one of the following regimes according to the varied wall roughness and flow conditions: hydraulically smooth, transitionally rough, or fully rough. Even the inner region of the turbulent boundary layer over a hydraulically smooth wall can be divided into the viscous sublayer (\( y \leq 5 \nu / u_\tau \)), buffer region (\( 5 \nu / u_\tau < y < 30 \nu / u_\tau \)), and logarithmic region (\( y \geq 30 \nu / u_\tau \)). A wall function which is applicable to all the vertical regions as well as the various hydraulic regimes is required for describing the complete velocity profile.

The governing equation for mean velocity in a wall coordinate system, \( \overline{u_\tau} \), can be written as

\[
-ho u_\tau \overline{u_\tau} + \rho \nu \frac{\partial \overline{u_\tau}}{\partial y} = \rho u_\tau^2
\]

where \( u_\tau \) represents the frictional velocity and \( \nu \) represents the kinematic viscosity. The mixing length equation, \( -\rho u_\tau \overline{u_\tau} = \rho \nu \left( \frac{\partial \overline{u_\tau}}{\partial y} \right)^2 \), is used to solve for the Reynolds stress term, \( -\rho \overline{u_\tau} \overline{u_\tau} \). The velocity over a smooth wall can be derived from equation (1) as

\[
\frac{\overline{u_\tau}}{u_\tau} = \frac{2}{1 + \sqrt{1 + 4 f(y^+)^2}}
\]

The calculation of the mixing length, \( l \), has been suggested by Van-Deriest [3] to be

\[
l = \kappa y \left( 1 - \exp \left( -\frac{y^+}{\Lambda} \right) \right)
\]
where in \( f(y^+)=\left(\kappa y^+\right)^2\left(1-\exp\left(-\frac{y^+}{\Lambda}\right)\right)^2 \), \( \kappa \) is the Karman constant, and \( \Lambda=\frac{\Delta u}{v} \), and \( \Lambda \) is a characteristic length scale with the value 26 assigned to smooth walls.

To give a unified velocity profile for various hydraulic regimes, equation (2) can be revised by quantifying the velocity damping between smooth and rough walls in each hydraulic regime. The characteristic length scale, \( \Lambda^* \), can be determined based on experimental results quantifying the mixing length.

The generalization and dissipation of turbulent kinematic energy are equal in rough and smooth wall regions, meaning the physical mechanisms acting in logarithmic regions of the turbulent boundary layer are the same under both conditions. Thus, the velocity profile over both smooth and rough walls can be considered as approximately equal with the exception of damping over rough wall regions. This has also been verified by numerous experiments [24-27]. The longitudinal mean velocity for the various hydraulic regimes in which this applies can be written as

\[
\bar{u}^* = \int_{y^+}^{2} \frac{2}{1 + \sqrt{1 + 4f(y^+)}} dy^+ - \frac{\Delta u}{u_*}.
\]

Clauser’s [5] review of experiments using various roughness forms and sand grain gradings suggested that the ratio \( \Delta u/u_* \) exhibits a general relationship with the roughness approximated by the Reynolds number, \( \Delta^*=u_*\Delta/v \). When \( \Delta^* \) is large, \( \Delta u/u_* \) can be calculated using a log function of \( \Delta^* \); however, a single function describing this relationship is hard to determine for small \( \Delta^* \) (representing poorly sorted mixtures of sand and gravel or extreme uneven roughness). Fortunately, it is possible to define an averaged approximation with good relative fit to the extensive experimental measurements that have been taken during studies of this type of flow.

The wall roughness is typically more even and uniform for engineered pipes than for river beds, and as such the calculations for uniform and intensely distributed wall roughness can be considered representative for flows under such conditions. Figure 2 presents the experimental results from various roughness forms, including the sand grains used by Nikuradse [13] and the uniform sphere from the experiments by Ligrani et al. [11]. White [4] and Ligrani et al.’s [11] equation are also presented in figure 2.

White [4] gave a continuous function relating \( \Delta u/u_* \) and \( \Delta^* \), as follows

\[
\frac{\Delta u}{u_*} = \frac{1}{\kappa} \ln \left(1 + 0.3\Delta^* \right)
\]

However, Ligrani et al. [11] instead suggested a piecewise function in the form

\[
\frac{\Delta u}{u_*} = \frac{1}{\kappa} \left(3.5 - \frac{1}{\kappa} \ln \Delta^*\right) \sin \left(\frac{1}{2} \pi g\right)
\]

where \( g = \frac{\ln \left(\Delta^*/\Delta^*_s\right)}{\ln \left(\Delta^*/\Delta^*_r\right)} \), for \( \Delta^*_s < \Delta^* < \Delta^*_r \),

\[
g = 1, \text{ for } \Delta^* > \Delta^*_r.
\]

\[
g = 0, \text{ for } \Delta^* < \Delta^*_s.
\]

Where \( \Delta^*_s \) and \( \Delta^*_r \) represent the lower and upper criteria of the transitionally rough regime, respectively. The suggested values are \( \Delta^*_s = 15 \), and \( \Delta^*_r = 55 \) [11]. It is shown in figure 2 that equation (5) deviates from the experimental data at small roughness Reynolds numbers. To improve precision under these conditions, this study suggests a revised function in the form

\[
\frac{\Delta u}{u_*} = \frac{1}{\kappa} \ln \left(1 + 0.0175\Delta^* + 0.0231(\Delta^*)^2\right)
\]
Variations of $\frac{1}{\kappa} \ln \Delta^+ - \frac{\Delta u}{u_\kappa}$, with $\Delta^+$ defined equation (8), are plotted in figure 2. It can be seen from this plot that equation (8) fits the experimental results better than equation (5). When comparing equations (6), (7), and (8), it can be seen that equation (8) presents a smoother function which is better fit to the experimental results for sand grains, uniform spheres etc. The exception to this evaluation is over wire mesh surfaces. As proved by Tachie et al. [26], the wire mesh surface generates greater influences to the velocity profile than the sand grain roughened surface.

Figure 2. Variations of $\Delta u/u_\kappa$ with $\Delta^+$. Figure 3. Variations of $\Lambda^+$ with $\Delta^+$. 

Figure 3 plots variations in the characteristic length scale, $\Lambda^+$, against $\Delta^+$, determined by experimental results. The least square fit of this data is given by

$$\Lambda^+ = 26 \exp \left( -0.0009 (\Delta^+) \right)$$

With the information discussed above, it can be determined that the wall function is a combination of equation (4), equation (8) and equation (9). The longitudinal velocity profile for various hydraulic regimes and vertical regions can be obtained by solving these equations together.

3. Results: Application of the Wall Function

The total flow characteristics are of great interest in conventional hydraulics, as the determination of the total energy loss of flow is of great significance in pipe design, flooding estimation, sediment routing, and other engineering practices. The wall function designed in this study can be applied in the following integration and unified description models to obtain the total flow energy loss.

3.1. Integration Model of Total Flow Energy Loss

The energy and momentum equations for steady turbulent flow in smooth pipes were built by Liu et al. [21]. Pan et al. [22] extended the application of this work into turbulent flow in rough-walled pipes and gave the mean shear stress over the wetted perimeter, $\chi$, by solving the total flow momentum and energy equations together

$$\tau_0 = \frac{\lambda}{8} \rho U^2 = \rho g \frac{R h_f}{2 L}$$

where $\tau_0 = \rho u_c^2$, $U$ is cross sectional averaged velocity, $h_f$ is the total flow energy loss of per unit mass of water per unit time between two cross sections, $L$ is the distance between the two sections, and $\lambda$ is the energy loss coefficient.

Substituting $\tau_0 = \rho u_c^2$ into equation (10) gives

$$\sqrt{\frac{8}{\lambda}} = \frac{U}{u_c}$$

(11)
Equation (11) has the same form as the Darcy-Weisbach equation in hydraulics. Using this equation, the total flow energy loss can be obtained from the cross sectional mean velocity, \( U \cdot U \) is obtained by integrating equation (4) over the cross section. After this step, the total flow energy loss is obtained as

\[
\lambda = \frac{8}{\pi} \frac{U}{u} = \frac{1}{\pi R^2} \int_0^R 2\pi r \frac{U}{u} \left( \int_0^r \frac{2}{1 + 4f(y)} dy - \frac{\Delta u}{u} \right) dr
\]

(12)

The total flow energy loss and its variation with the Reynolds number, \( Re \), and the relative wall roughness, \( \Delta / R \), in rough pipe flow obtained through Equation (12) are plotted in figure 4.

3.2. Equation of Total Flow Energy Loss from Field Statistics

3.2.1. Explicit Equation of Total Flow Energy Loss from Field Statistics. Pan et al. [22] gave a calculation equation for the energy loss of turbulent flow in a pipe with measured wall roughness to be

\[
\rho g Q h = \int \int \left( 2 \mu s_i - \rho u_i u_j \right) s_j dV
\]

(13)

where \( Q \) is the flow discharge, \( \mu \) is the dynamic viscosity, \( s_i \) is the tensor of mean strain rate, and \(-\rho u_i u_j\) is the Reynolds stress. Giving the axial symmetry of pipe flow seen in figure 1(a), equation (13) can be transformed into cylindrical coordinates, which gives an explicit calculated equation for the total flow energy loss coefficient

\[
\lambda = \frac{16}{U^2 R} \int_0^R \left( \frac{dU}{dr} - u_j u_j \right) \frac{dU}{dr} dr
\]

(14)

Through Equation (14), the total energy loss of flow in a turbulent pipe can be calculated directly from turbulent field statistics including the mean velocity and Reynolds stress. These flow field statistics can be obtained by solving the system averaged equations of turbulent rough pipe flow, given above, together with turbulence models. In the near wall sections of these numerical simulation, the suggested wall function can be used to determine the near wall velocity between the fully rough regime and the wall.

3.2.2. Obtaining the Flow Field: Mathematical Modeling and Numerical Method. For turbulent uniform flow, the time mean quantities can be defined as system average quantities. Thus, the governing equations for turbulent pipe flow are determined to be Equation (15) and (16). The \( k-\varepsilon \) model can be used to solve for the Reynolds stress term in Equation (16). Quantities such as mean velocity are given at the pipe inlet, the pipe is assumed to be long enough to have fully developed turbulent flow at the outlet, and the near wall velocity can be determined using a wall function approach combining Equation (4), (8) and (9).

\[
\frac{\partial \overline{u}}{\partial x_j} = 0
\]

(15)

\[
\frac{\partial \rho \overline{u}_j u_i}{\partial x_j} = \rho f_i - \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2 \mu s_i - \rho u_i u_j \right)
\]

(16)

where \( f_i \) is the gravitational acceleration, and \( \overline{p} \) represents the mean pressure.

The finite volume method can be used to derive discrete forms of the governing equations. In this application, a first-order upwind difference scheme is used to determine the convection term, a central difference scheme is used for calculating the diffusion terms, and the SIMPLE scheme (based on a collocated grid) is used in conjunction to solve for the pressure and velocity. The resulting algebraic
equations are then solved using Gauss-Seidel iterations. The convergence conditions are set to require a residual of inlet flow discharge less than 0.01% and a residual of total flow discharge less than 0.5%.

3.3. Results of the Total Flow Energy Loss

3.3.1. Variation of the Energy Loss. The turbulent pipe flow with various relative wall roughness ($\Delta/R$ values of $1/126$, $1/60$, $1/30.6$, and $1/15$) and Reynolds numbers were numerically simulated. The total flow energy loss was then calculated from the simulated field statistics using equation (14). Variations in $\lambda$ with different $Re_d$ and $\Delta/R$ values are plotted in figure 4, together with the result of equation (12). Additionally, the Blasius [14] smooth equation (equation (17)) and Nikuradse smooth equation (equation (18)) are presented below.

$$\lambda = \frac{0.3164}{Re_d^{1/4}}$$

(17)

$$\frac{1}{\sqrt{\Delta}} = 3\log\left(Re_d \sqrt{\Delta}\right) - 0.8$$

(18)

It can be seen from figure 4 that, when the flow is in the hydraulically smooth regime, the energy loss depends solely on the Reynolds number, $Re_d$; however, in the transitionally rough regime the energy loss varies both with the Reynolds number, $Re_d$, and the relative wall roughness, $\Delta/R$, and in the fully rough regime, it depends solely on $\Delta/R$ and no longer varies with changes in $Re_d$. In the hydraulically smooth regime, the results of equation (12) exhibit an overall agreement with the results of equation (17), equation (18), and Nikuradse experiment with the exception of a slight deviation at low Reynolds numbers. In the transitionally rough and fully rough regimes, both equation (12) and the equation of total flow energy loss from field statistics show a high level of agreement with the Nikuradse experiment. Generally, both these equations provide good predictions of the Nikuradse experimental results in terms of total flow energy loss.

![Figure 4](image.png)

**Figure 4.** Total flow energy loss in steady turbulent pipe flow.

3.3.2. Constitution of the Energy Loss. In conventional hydraulics, energy loss is defined as a total flow characteristic obtained from the force balance of a certain volume of water flow. It is not related to the flow field statistics, and its constitution and distribution have not yet been researched. It can be shown that the proposed equation of total flow energy loss from field statistics can also be realistically applied to calculating the constitution and distribution of the energy loss in such systems as equation (14) can be further divided into $\lambda = \lambda_1 + \lambda_2$, where $\lambda_1$ represents the energy loss due to dissipation
caused by molecular viscosity and $\lambda_2$ represents the energy loss resulting from turbulence-related dissipation.

$$\lambda_1 = \frac{8\nu}{U' R} \int_0^\infty \left( \frac{d\bar{u}}{dr} \right)^2 rdr$$  \hspace{1cm} (19a)$$

$$\lambda_2 = \lambda - \lambda_1$$  \hspace{1cm} (19b)$$

$\lambda_1$ and $\lambda_2$, calculated through equation (19), are plotted in figure 5. It can be seen that $\lambda_1$ represents a smaller percent of $\lambda$, increases proportional to the Reynolds number, and decreases slightly as the relative wall roughness increases. A larger percent of $\lambda$ is accounted for by $\lambda_2$, which decreases as the Reynolds number increases and increases proportional to the relative wall roughness. In the transitionally rough regime, both $\lambda_1$ and $\lambda_2$ vary significantly with the Reynolds number; however, when the flow develops into the fully rough regime, the constitution of the energy loss varies only slightly and tends to become constant.

3.3.3. Distribution of Energy Loss Density over the Pipe Radius. The variable $\lambda_r$ is defined as the energy loss density at any pipe radius, $r$. It can be written in the form of equation (20):

$$\lambda_r = \frac{\int_{r}^{r+\Delta r} \left( \nu \frac{d\bar{u}}{dr} - \bar{u}_{\tau} \frac{d\bar{u}}{dr} \right) rdr}{\int_{r}^{r+\Delta r} \left( \nu \frac{d\bar{u}}{dr} - \bar{u}_{\tau} \frac{d\bar{u}}{dr} \right) rdr}$$  \hspace{1cm} (20)$$

which can then be used to show that

$$\int_{0}^{R} \frac{\lambda_r}{\lambda} dr = 1$$  \hspace{1cm} (21)$$

The energy loss density is calculated through equation (20), and its distribution over the pipe radius is plotted in figure 6. It can be seen there that the energy loss mainly occurs in a very limited region near the pipe wall, and decreases to nearly zero at a distance from the wall. When $Re_d$ increases, the ratio $\lambda_r/\lambda$ at the wall increases and the distribution of the energy loss density is more intense closer to the wall.

![Figure 5. Variations in the constitution of energy loss with $Re_d$.](image)

![Figure 6. Distribution of energy loss density over the pipe radius.](image)

4. Discussion
Determining the flow field and the total flow characteristics of turbulent flow in rough pipes has received constant concern in both fluid mechanics and hydraulic research. As the flow over rough wall in engineering practice and natural rivers may fall into any flow regimes among the hydraulically smooth regime, the transitionally rough regime, and the fully rough regime, many equations [11, 27-28] have been developed in the past of the determination of the velocity profile and the energy loss coefficient. Most of these equations are piecewise or implicit for each flow regime except for Cheng’s [28] equation, which is an interpolation of the laminar, hydraulically smooth and fully rough regimes.

However, this paper presents two approaches in obtaining the total flow energy loss by applying a suggested new wall function. A single explicit wall function of velocity profile for various hydraulic regimes was built in this paper using the results of theoretical analysis. This wall function was then applied to obtain the total flow characteristics of turbulent flow in a pipe accounting for wall roughness. An integration equation was built by integrating the wall function over the pipe radius. Meanwhile, the proposed wall function was applied in a numerical simulation to obtain a refined prediction of the flow field. The energy loss from the flow field was subsequently developed, which gave an additional, explicit equation for calculating the energy loss directly from previously determined field statistics. The energy losses calculated through the two approaches described above both predicted the Nikuradse experimental results reasonably well. The two approaches can be used without considering the hydraulic regimes of the flow.

Additionally, a realistic significance of the second approach, in which the energy loss is calculated directly from the predicted field statistics, is that it provides a unified description of both the total flow and field characteristics. In the past, the energy loss is considered only as the resistance loss determined by force balance of total flow. The suggested approach, instead, can predict the total flow energy loss directly from the field statistics, and the constitution and density distribution of the energy loss was analyzed. It was found that the energy loss can be considered to be constituted from the energy loss dissipated by molecular viscosity and the turbulent kinetic energy dissipation. Results from these calculations showed that the molecular viscous dissipation constitutes a smaller percent of the total energy loss in flow and decreases as the Reynolds number increases, whereas the turbulent kinetic energy dissipation is the main contributor to energy loss when the Reynolds number is sufficiently high. The calculated energy loss density distribution over the radius of the pipe showed that the energy loss is mainly distributed in the near wall region, and decreases to zero at a certain distance from the wall. The distribution becomes even closer to the wall as the Reynolds number increases. The energy loss coefficient calculated through this approach can provide a more precise definition of the total flow characteristics comparing with the Manning roughness coefficient since it is a function of the field statistics and varies in each hydraulic regime.

5. Conclusions
Through theoretical analysis and numerical simulation, this paper developed a wall function of velocity profile for various hydraulic regimes over rough walls, and made investigations into the energy loss of turbulent pipe flow. By using the newly developed wall function, the numerical simulation gave sufficiently precise predictions of the flow field statistics for turbulent pipe flow. The total flow energy loss can therefore be calculated directly from the predicted flow statistics using our suggested equation.

This equation for energy loss, together with the revised wall function, was then able to give a unified correlation between the total flow characteristics, which is concerned in hydraulic applications, and the flow field statistics, which draws attractions from fluid mechanics research.

It needs to be noted that this wall function was developed on the basis of experiments conducted with wall roughness in very limited roughness forms such as sand grains, uniform spheres, and microscale gravels. These roughness types are typical for engineering pipe flows and artificial channel flows. Regarding to wire mesh type roughness, the results deviate as the wire mesh generates greater influences on the velocity profile. Future research is needed on other types of roughness for wider applications into complex channel and river flows.
Acknowledgments
This paper was supported by the National key Research and development program (Grant No.2018YFC0407806).

References
[1] Schlichting H 1968 Boundary Layer Theory McGraw-Hill New York.
[2] Launder B E and Spalding D B 1974 The Numerical Computation of Turbulent Flows Computer Methods in Applied Mechanics and Engineering 3 269-289.
[3] Van Driest E R 1956 On Turbulent Flow Near a Wall Journal of the Aeronautical Sciences 23 1007-1011.
[4] White F M 1982 Viscous Fluid Flow Beijing China: China Machine Press 12 473-489.
[5] Clauser F H 1956 The turbulent boundary layer Advances in Applied Mechanics 4 1-51.
[6] Grass A J 1971 Structural features of turbulent flow over smooth and rough boundaries Journal of Fluid Mechanics 50(02) 233-255.
[7] Pimenta M M, Moffat R J and Kays W M 1975 The Turbulent Boundary Layer: An Experimental Study of the Transport Of Momentum And Heat with the Effect of Roughness Thermosciences Division Department of Mechanical Engineering Stanford University.
[8] Healzer J M, Moffat R J and Kays W M 1974 The Turbulent Boundary Layer on a Rough, Porous Plate: Experiment Heat Transfer with Uniform Blowing Thermosciences Division, Department of Mechanical engineering Stanford University.
[9] Mckeon B J, Li J, Jiang W, et al. 2004 Further observations on the mean velocity distribution in fully developed pipe flow Journal of Fluid Mechanics 501 135-147.
[10] Perry A E, Hafez S and Chong M S 2001 A possible reinterpretation of the Princeton superpipe data Journal of Fluid Mechanics 439 395-401.
[11] Ligrani P M and Moffat R J 1986 Structure of transitionally rough and fully rough turbulent boundary layers Journal of Fluid Mechanics 162 69-98.
[12] Dou G 1980 generalized laws of turbulent flow in open channels and pipes for various regions Journal of Nanjing Hydraulic Research Institute (1) 3-14.
[13] Nikuradze J 1950 Laws of flow in rough pipes Washington: National Advisory Committee for Aeronautics.
[14] Chanson H 1996 The Hydraulics of Open Channel Flow, Butterworth-Heinemann Computer Science Logic.
[15] Katul G, Wiberg P L, Albertson J, et al. 2002 A mixing layer theory for flow resistance in shallow streams Water Resources Research 38(11) 1250.
[16] Bradleey F 1968 A micrometeorological study of velocity profiles and surface drag in the region modified by a change in surface Q. J. R. Meteorol. Soc. 94 361–379.
[17] Sutton O G 1953 Micrometeorology 333 pp McGraw-Hill New York.
[18] Hey R D 1979 Flow resistance in gravel-bed rivers Journal of Hydraulic Engineering 105 365–379.
[19] Yen B C 1992 Hydraulic resistance in open channels, in Channel Flow Resistance: Centennial of Manning’s Equation Water Resources Publication. Highlands Ranch Colo 1-135.
[20] Colebrook C F and White C M 1937 Experiments with Fluid Friction in Roughened Pipes Proceedings of the Royal Society of London Series A Mathematical and Physical Sciences 161(906) 367-381.
[21] Liu S, Xue J and Fan M 2013 On the calculation of mechanical energy loss for steady pipe flow of homogenous incompressible fluid Journal of Hydrodynamics 25(6) 912-918.
[22] Pan W and Liu S 2015 A new mechanical energy Equation for total flow in uniformly and densely roughed pipes and numerical simulation of the flow Journal of China Institute of Water Resources and Hydropower Research (05) 375-379.
[23] Liu S, Liu J, Luo Q, et al 2011 Engineering Turbulence Beijing China: Science Press.
[24] Shockling M A, Allen J J and Smits A J 2006 Roughness effects in turbulent pipe flow *Journal of Fluid Mechanics* **564** 267-285.

[25] Allen J J, Shockling M A, Kunkel G J, et al. 2007 Turbulent flow in smooth and rough pipes *Philosophical Transactions of the Royal Society of London A: Mathematical Physical and Engineering Sciences* **365**(1852) 699-714.

[26] Tachie M F, Bergstrom D J and Balachandar R 2004 Roughness effects on the mixing properties in open channel turbulent boundary layers **126** 1025-1036.

[27] McKeon B J, Zagarolam V, Smits A J 2005 A new friction factor relationship for fully developed pipe flow *Journal of Fluid Mechanics* **538** 429-443.

[28] Cheng N 2008 Equations for friction factor in transitional regimes *Journal of Hydraulic Engineering* **134**(9) 1357-1362.